Physics of the Electroweak Phase Transition at $M_H \leq 70$ GeV in a 3–dimensional $SU(2)$–Higgs Model

M. Gürtler$^1$, E.-M. Ilgenfritz$^2$, J. Kripfganz$^3$, H. Perlt$^1$ and A. Schiller$^1$

$^1$ Institut für Theoretische Physik, Universität Leipzig, Germany
$^2$ Institut für Physik, Humboldt-Universität zu Berlin, Germany
$^3$ Institut für Theoretische Physik, Universität Heidelberg, Germany

Physical parameters of the electroweak phase transition in a 3$d$ effective lattice $SU(2)$–Higgs model are presented. The phase transition temperatures, latent heats and continuum condensate discontinuities are measured at Higgs masses of about 70 and 35 GeV. Masses and Higgs condensates are compared to perturbation theory in the broken phase. In the symmetric phase bound states and the static force are determined.

1. Introduction

In this contribution some of the results given in [1] are presented.

Recent lattice studies of the electroweak phase transition [2]–[5] had been triggered by the interest in understanding BAU. The present quantitative understanding of possible mechanisms as well as the lower bounds for $M_H$ make BAU unlikely within the minimal standard model. Extensions, in particular supersymmetric ones, may still be viable, however.

A second reason for lattice investigations was the wish to control the behavior of perturbative calculations of the effective action. This quantity is the appropriate tool of (non–lattice) thermal quantum field theory for dealing with symmetry breaking. Infrared problems prevent a perturbative evaluation of the free energy in the symmetric phase to higher loops.

The $SU(2)$ gauge–Higgs model has become a test-field to control the validity of perturbative predictions over a broad range of Higgs masses. Lattice simulations make it possible to put both phases into coexistence near the phase equilibrium. Thus one is able to measure directly physical quantities quantifying the strength of the transition.

Furthermore, the 3$d$ effective model can be related to different 4$d$ theories, e.g. with top.

2. The strength of the phase transition

The action of the studied model is given by

$$S = \beta_G \sum_p \left( 1 - \frac{1}{2} Tr U_p \right) - \beta_H \sum_l \frac{1}{2} Tr (\Phi^*_x U_{x,a} \Phi_{x+a}) + \sum_x (\rho_x^2 + \beta_R (\rho_x^2 - 1)^2)$$

with

$$\beta_G = \frac{4}{ag^3}, \quad \beta_R = \frac{\lambda_3}{g^3}, \quad \beta_H = \frac{\lambda_3}{g^3}, \quad \beta_H = \frac{2(1 - 2\beta_R)}{6 + a^2 m^2}, \quad \frac{\lambda_3}{g^3} = \frac{1}{8} \left( \frac{M_H}{80 \text{ GeV}} \right)^2.$$

In Table 1 the results for the phase transition parameters are collected and tested on lattice spacing dependence at $M_H^* = 70$ GeV.

| $(\beta_G, M_H^*)$ | $\beta_{Hc}$ | $T_c$ | $M_H$ |
|-------------------|------------|-----|------|
| (12,70)           | 0.3435443(6) | 150.94(1) | 64.77 |
| (12,70)           | 0.3407942(6) | 151.27(2) | 64.77 |
| (12,35)           | 0.34140     | 76.2(1)  | 29.50 |

Table 1. $T_c$ and 4$d$ Higgs masses (in GeV) without top

The jumps of the Higgs length expectation values $\langle \rho^2 \rangle$ and $\langle \rho^4 \rangle$ at $T_c$ are connected to the
RG invariant discontinuities of the quadratic and quartic continuum Higgs condensates (Table 2)

\[
\Delta \langle \phi^+ \phi \rangle / g_3^2 = \frac{1}{8} \beta_G \beta_{Hc} \Delta \langle \rho^2 \rangle,
\]
\[
\Delta \langle (\phi^+ \phi)^2 \rangle / g_4^3 = (\frac{1}{8} \beta_G \beta_{Hc})^2 \Delta \langle \rho^4 \rangle.
\]

Table 2. The Higgs condensate discontinuities

| (βG, MH)   | Δ⟨φ+φ⟩/g3² | Δ⟨(φ+φ)²⟩/g4³ |
|------------|-------------|---------------|
| (12, 70)   | 0.250(3)    | 1.28(2)       |
| (16, 70)   | 0.250(4)    | 1.65(3)       |
| (12, 35)   | 3.20(1)     | 25.2(1)       |

The latent heat \(L_{\text{heat}}\) is calculated according to [6]

\[
L_{\text{heat}} T_4^c = M_H^2 \beta_{Hc} \beta_G \frac{\partial}{\partial \beta_{Hc}} \log \frac{w_s}{w_b} \bigg|_{\beta_{Hc}}.
\]

The change of the weights with \(\beta_H\) very close to the critical one is shown in Fig. 1. We obtain (without top) \(L_{\text{heat}}/T_4^c = 0.0182(9)\).

The coexistence of both phases opens the possibility to determine the surface tension \(\alpha\). The use of the equal weight method allows to estimate the contribution of the mixed phase state at \(w_b = w_s\).

We parametrize the relation between the weights at pseudo-criticality and \(\alpha\) for lattices \(L_x^2 \times L_z\) as follows

\[
\frac{w_{\text{mix}}}{w_s} = \frac{w_{\text{mix}}}{w_b} = b L_z^2 \log L_x \exp(-2\alpha a^2 L_x^2/T_c).
\]

\(L_z^2\) is an entropy factor (surface positions), \(\log L_x\) is the result (for \(d = 3\)) of capillary wave approximation (fluctuations of the surfaces), \(b\) counts different possible orientations of surfaces (cubic \((b = 3)\) and prolonged lattices \((b = 1)\)).

We find the upper bound (Fig. 2) \(\alpha/T_c^3 \approx 0.00023\).

These lattice results are smaller by one order of magnitude than the one-loop estimate for \(\alpha\) [7]. The surface tension is sensitive to the shape of the effective potential in the whole \(\varphi\) range between the symmetric and the broken phase. The
disagreement seems to indicate that the loop expansion to the effective potential gets out of control at intermediate \( \varphi \) values already, reflecting the infrared problems of the symmetric phase.

3. Broken phase and perturbation theory

Can the broken phase be understood perturbatively? It is known \[7\] that the appropriate effective expansion parameter is 
\[
g_{\text{eff}}^2 = \frac{g_3^2}{2m_W(T)}
\]
with the 3d gauge boson mass \( m_W(T) \). This coupling can be determined from the two-loop effective potential. Using Feynman gauge and \( \mu_3 = \frac{g_3^2}{g_4^2} \) for \( M^*_H = 70 \) GeV this effective coupling is found to be 1.10 at \( T_c \).

We have compared our MC data with two-loop continuum predictions (Feynman gauge) for the vector boson \( m_W(T) \) and Higgs boson \( m_H(T) \) masses (shown in Fig. 3) and the renormalized Higgs condensate using the effective potential \[7\]. The 3d masses are calculated from correlators of extended operators with appropriate quantum numbers.

![Figure 3. 3d masses at \( M_H^* = 70 \) GeV and \( \beta_G = 12 \) compared to perturbation theory](image)

Very good agreement is observed deeper in the broken phase, as should be expected. Wave function renormalization (in one-loop) is obviously required. Close to the phase transition we observe a systematic difference between lattice data and the continuum calculation as function of \( m_3^2 \). This may be an indication that higher loop terms start to play a significant role.

4. Some properties of the symmetric phase

Dynamical Higgs fields are expected to screen the static potential which can be defined in the 3d theory, too, and will be described below. Sufficiently away from the transition massive Higgs bound states become heavy. As a consequence, the symmetric phase should more and more resemble pure \( SU(2) \) gauge theory. However, in the \( \beta_H \) (resp. \( m_3 \)) region that we have explored the lowest Higgs bound state is still significantly lighter than the lightest \( 0^+ \) \( W \)-ball.

Results for the lowest Higgs \( (0^+) \) and vector boson bound states \( (1^-) \) are shown in Fig. 4 as function of \( m_3(g_3^2/g_4^2) \). Data from different \( \beta_G \) values nicely coincide. Our results for \( m_W \) should be considered as upper bounds, because of a possible admixture of states with a somewhat higher mass. One important conclusion from Fig. 4 is the scaling behavior of masses (no \( \lambda_3 \) dependence).

The data for the static potential \( V(R) \) are obtained from exponential fits to the Wilson loops. The potential (in \( g_3^2 \) units) is described assuming massless or massive perturbative \( W \)-exchange.

In Fig. 5 we present the string tension obtained from the ansätze for the potential at different \( m_3^2 \)
values. There is no significant $\lambda_3 (M_H^*)$ dependence of $\sigma/g_4^2$ on $\lambda_3 (M_H^*)$. At larger $m_3^2$ (temperatures) the result of the string tension based on the fit to massive perturbation theory is close to the value of pure 3d SU(2). For $R \leq 18a$ the expected screening behavior of the potential has not yet been observed. Still larger distances are difficult to study.

5. Summary

Our results provide evidence for the first order nature of the thermal phase transition in the SU(2)–Higgs system with Higgs masses up to 70 GeV.

We checked the reliability of perturbative calculations of the effective potential (masses and renormalized Higgs condensate in the broken phase). The effect of wave function renormalization cannot be ignored. Physics depending only on the potential in the vicinity of the broken minimum, can be systematically improved by higher order perturbation theory.

The surface tension which is sensitive to the barrier shape of the effective potential is systematically overestimated in the perturbative calculation. It is also very hard to measure for weak transitions.

The reduced model does not only make very precise predictions, but dimensional reduction seems to be reliable in the interesting range of Higgs masses. In order to learn about the necessity to include higher dimensional operators into the effective action one should explore the case of smaller or very much larger Higgs masses. The 3d lattice approach promises to be applicable as an effective formulation of nonstandard extensions of the Standard Model.

We have put a lot of emphasis to the properties of the symmetric phase. Within the 3d approach, information on the spectrum can only be accessed through 3d correlation lengths. The interplay between the confining properties of the 3d effective theory at high $T$ and the space–time structure of physical excitations needs further investigations, as well as the nature of this confinement itself for the 3d pure gauge theory and in the presence of scalar matter fields.

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