Study on band gap properties of asymmetric locally resonant complex phononic crystals

Yingchao Zhao1*, Xiaodong Yang1

1Faculty of Materials and Manufacturing, Beijing University of Technology, Beijing, China

*Corresponding author’s e-mail: zhaoyingchao@emails.bjut.edu.cn

Abstract In this paper, the band structure and transmission characteristics of asymmetric locally resonant complex phononic crystals are studied. Based on the Bragg band gap mechanism, an asymmetric local resonance element was designed and the structural parameters were optimized to obtain a wide band gap. The results show that the range of the first complete band gap is obviously affected by the size of the scatterer, the radius and height of the locally resonant cylinder and the position of the cylinder. At the same time, the transmission characteristics of the 10*10 periodic arrangement structure are calculated to verify the accuracy of the energy band structure.

1. Introduction
Vibration suppression and noise reduction are urgent problems in daily life. Phononic crystal is a periodic structure composed of two or more materials[1]. When elastic wave propagates, it can propagate normally in some frequency ranges, but its propagation is inhibited in some frequency ranges[2]. Since its birth, phononic crystals have attracted extensive attention in the academic and engineering circles because of their powerful ability to manipulate elastic waves.

Nidhish Jain proposed a two-dimensional phonon crystal structure of soft particles, and solved the energy band structure by finite element method and discrete element method[3]. A fatty acid methyl ester solid-liquid phononic crystal sensing device and its numerical calculation were proposed by Hamed Gharibi[4]. By combining chiral structure with zigzag honeycomb structure, Wang obtained several new complete band gaps by opening degeneracy points between different bands, and optimized the parameters to obtain low frequency band gaps[5]. A two-dimensional tetragonal lattice phononic crystal structure composed of grid and air was proposed by Li, and the band gap range was adjusted by changing the length-width ratio of the grid[6]. Liu first proposed the concept of a locally resonant phononic crystal whose band gap corresponds to a wavelength far larger than the lattice size[7]. Wang proposed a two-component local resonance structure and calculated that the density ratio of the scatterer to the matrix was the main reason for the band gap variation[8]. The tubular local resonance structures were proposed by Shu, and compared with the ordinary columnar local structures, it was found that the tubular local resonance structures can generate wider range of band gaps.[9] The control of local resonance metamaterials on acoustic waves was studied by Wang, who studied the wave propagation effects in two waveguides with different shapes[10]. In this paper, an asymmetric complex phonon crystal including Bragg band gap mechanism and local resonance mechanism is proposed. The band structure and transmission characteristics are solved by finite element method, and the influence of structure parameters on band gap is studied.
2. Materials and Methods

2.1. Structural model of asymmetric locally resonant phononic crystals

The asymmetric complex phononic crystal model is shown in Figure 1. The structure consists of matrix, scatterer and resonance element. Lattice constant, dimension parameter of scatterer and radius of cylinder are shown in Figure 1. The lattice constant is $a$, the long side of the scatterer is $b$, the short side is $d$, the width of the hole is $c$, and the radius of the cylinder is $r$. The thickness of the matrix and the height of the cylinder are $f$. The top and bottom cylinders are in opposite positions on the diagonal.

The matrix and the scatterer form the band gap of Bragg scattering mechanism, and the cylinder structure generates the band gap through the local resonance mechanism. There are two band gap mechanisms, which is conducive to expand the parameter design, and more conducive to forming a low frequency wide range of complete band gap. The Brillouin region of phononic crystals arranged in a square lattice is shown in Figure 2. The black part is the first irreducible Brillouin region.

2.2. The principle of finite element method

The band structure was solved by Finite Element Method, and the cell of phononic crystal was meshing, and the characteristic equation of the cell was obtained as follows:

$$(K - \omega^2 M)U = 0$$

where $K$ is the overall stiffness matrix of a single cell node, $M$ is the overall mass matrix of a single cell node, and $U$ is the displacement of a single cell node. Combined with Floquet period conditions:

$$U(x + a, y) = U(x, y)e^{ik_x a}$$  \hspace{1cm} (2)

$$U(x, y + a) = U(x, y)e^{ik_y a}$$  \hspace{1cm} (3)

In Equations (2) and (3), the first $a$ is scalar, and the second $a$ is basis vector. $k = (k_x, k_y)$ is the wave vector of the first irreducible Brillouin region. Combining with the boundary conditions (2) and (3), the characteristic equation (1) is solved. The band structure can be obtained by taking the wave
vector \( k \) as the abaxial coordinate to take the boundary of the first irreducible Brillouin region along the 0-1-2-2-3 (0) path with the frequency as the vertical coordinate.

Infinite phonon crystal structure can prevent elastic waves from propagating in the band gap frequency range. In practice, the size of the structure is usually limited, so in the band gap range, the wave will be significantly suppressed, but still can propagate. By applying a certain background pressure field to the left side of the array periodic structure and measuring its input pressure, and measuring its output pressure to the right side of the array periodic structure, the transmission characteristic curve of the finite period structure can be obtained. The propagation of elastic waves in structures is generally expressed by transmittance, which is defined as follows:

\[
dB = 20 \log \left( \frac{P_{\text{in}}}{P_{\text{out}}} \right)
\]

where, \( P_{\text{in}} \) is the input pressure and \( P_{\text{out}} \) is the output pressure.

3. Results and Discussion

The matrix material is steel, and the scatterer and the locally resonant double-cylinder structure material is silicon rubber. Parameter \( a, b, c, d, r \) and \( f \) are 24mm, 4mm, 1.5mm, 2mm, 5mm and 2mm. The band structure is calculated as shown in Figure 3. In order to verify the accuracy of band structure, the transmission characteristic curve of 10*10 period array structure was calculated, as shown in Figure 4.

![Fig. 3 Band structure curve](image1)

![Fig. 4 Transmission characteristic curve](image2)

It can be found that there is a large complete band gap between 119.3Hz and 514Hz, and the band gap range is 394.7Hz. In the transmission characteristic curve, there are obvious wave troughs, indicating that the wave propagation is inhibited in the same frequency range, which further explains the accuracy of the band structure. In this section, we will study the effects of different scatterer size parameters, cylinder radius, cylinder height and cylinder position on the band gap.

3.1. The influence of different scatterer sizes on the first complete band gap

With other conditions unchanged, the whole size of the scatterer is multiplied by the coefficient \( p \), which is selected as 0.8, 0.9, 1.0, 1.1 and 1.2 respectively. The calculated variation of the first complete band gap is shown in Figure 5.
As can be seen from Figure 5, with the increase of the scatterer size, the initial frequency of the first complete band gap decreases from 125.5Hz to 109.2Hz, the cutoff frequency increases from 415.3Hz to 538Hz, and the band gap range gradually increases to 428.8Hz. This is because increasing the scatterer size is equal to increasing the scatterer filling rate, which is an important factor affecting the Bragg band gap.

3.2. The influence of cylindrical structural parameters on the first complete band gap

The radius and height parameters of the cylinder are important factors affecting the local resonant band gap, while other conditions remain unchanged (one parameter is changed at a time). The radius of the cylinder was selected as 1mm, 1.5mm, 2mm, 2.5mm, and 3mm, and the height of the cylinder was selected as 2mm, 4mm, 6mm, 8mm, and 10mm. The calculated variation of the first complete band gap was shown in Figure 6.

It is found that increasing the cylinder radius and the cylinder height will increase the range of the first complete band gap. The cylinder radius increases from 4mm to 6mm, the first complete band gap changes from 174Hz-331.7Hz to 74Hz-713.7Hz, and the starting frequency decreases while the cutoff frequency increases. Increasing the height of the cylinder mainly affects the initial frequency of the first complete band gap, which changes from 119.3Hz to 71Hz, and the cutoff frequency has almost no effect. This is because increasing the height and radius of the cylinder will reduce the frequency of local resonance and lead to a decrease in the band gap. In order to obtain the low frequency band gap, the
radius of the resonance element can be increased as much as possible during structural design.

3.3. The influence of the position of the cylinder on the first complete band gap

Four cylinders are on the diagonal. The center of the single cell was taken as the origin of coordinates, and the abscissa x of the center of the cylinder was the distance parameter. Moving along the diagonal, the position of the cylinder and its center changes. The x-coordinates are 4mm, 5mm, 6mm and 7mm. The change of the calculated first complete band gap is shown in Figure 7.

![Fig.7 The relationship between the complete band gap and the abscissa x of the cylinder](image)

When the cylinder moves along the diagonal, the initial frequency of the first complete band gap gradually decreases from 119.3Hz to 71Hz with little change, and the cutoff frequency increases significantly from 220Hz to 656Hz. The position of the resonance element mainly affects the cutoff frequency of the first complete band gap. In structural design, the resonance element is distributed along the diagonal as far as possible from the center, which is more conducive to obtaining low frequency broadband.

4. Conclusions

In this paper, an asymmetric local resonant complex phononic crystal model is proposed and the effects of different parameters on the first complete band gap are studied by using the finite element method. The results show that when the scatterer size changes, the initial frequency of the first complete band gap is small, the cutoff frequency increases, and the band gap frequency range increases. When the cylinder structure with local resonance is increased, the initial frequency range of the first complete band gap decreases obviously, and the cutoff frequency of the first complete band gap changes little. Moreover, it is found that the first complete band gap range increases with the increase of the cylinder radius and height. In the structural design, the size of the scatterer and the structural parameters of the resonant cylinder should be increased as much as possible, and the position of the cylinder should be distributed along the diagonal far from the center to form a low frequency band gap.

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