Abstract

We present qualitative pictures of the structures of magnetic monopoles (MMs), nuclearites (nuggets of strange quark matter, strangelets, surrounded by electrons) and Q-balls (supersymmetric states of squarks, sleptons and Higgs fields). In particular we discuss the relation between their mass and size. MMs, nuclearites and Q-balls could be part of the cold Dark Matter (DM); we consider astrophysical limits on the flux of these particles in the cosmic radiation.

1 Introduction

Magnetic monopoles (MMs), nuclearites (nuggets of strange quark matter + electrons) and Q-balls (supersymmetric states of squarks, sleptons and Higgs fields) could be part of the cold dark matter (DM) located in the halo of galaxies. They would have typical velocities of $\beta c \sim 10^{-3}c$; galactic magnetic fields may accelerate magnetic monopoles to larger velocities.

In this note we show some qualitative pictures of the structures of (i) superheavy GUT magnetic monopoles, (ii) intermediate mass magnetic monopoles, (iii) nuclearites and (iv) Q-balls. We discuss in particular the relation between the mass and the size of these particles.

Based on astrophysical considerations, we discuss phenomenological limits on the fluxes in the cosmic radiation, of magnetic monopoles, nuclearites and Q-balls.

2 Magnetic monopoles

The concept of magnetic monopole may be traced back to the origin of magnetism. In 1931 Dirac introduced the MMs in order to explain the quantization of the electric charge, obtaining the formula $eg = nhc/2$, from which $g = ngD = nhc/2e = 68.5e = 3.29 \cdot 10^{-8}$ c.g.s. symmetric system [1]. A MM possessing also an electric charge is called a dyon. A MM and an atomic nucleus may form a bound system with both magnetic and electric charges: also this system is called a dyon.

The energy losses of MMs and of dyons in matter, in the Earth and in different detectors were calculated in ref. [2]. An extensive bibliography of MMs is given in ref. [3].
2.1 GUT magnetic monopoles

In the context of the Grand Unified Theory (GUT) of electroweak and strong interactions, MMs with magnetic charges $g = ng_D$ are predicted to appear at the cosmic time of $\sim 10^{-34}$ s, during the phase transition corresponding to the spontaneous breaking of the Grand Unified group \[4\]. They should have been produced as point defects or in extremely high energy collisions of the type $e^+e^- \rightarrow M\overline{M}$ \[4\]. One of the main problems with GUT MMs is the too large abundance predicted by the standard cosmology. Models with inflation at the end of the GUT epoch reduce dramatically their number and we would be left mainly with MMs produced in very high energy collisions.

![Diagram of a GUT monopole](image)

**Figure 1:** Structure of a GUT monopole. The various regions correspond to: (i) Grand Unification ($r \sim 10^{-29}$ cm; inside this core one finds virtual X and Y particles); (ii) electroweak unification ($r \sim 10^{-16}$ cm; inside this region one finds virtual $W^\pm$ and $Z^0$); (iii) confinement region ($r \sim 10^{-13}$ cm; inside one finds virtual $\gamma$, gluons and a condensate of fermion-antifermion pairs and 4-fermion virtual states); (iv) for $r >$ few fm one has the field of a point magnetic charge.

The GUT MM mass is related to the mass of the X vector bosons, carriers of the unified interaction, by the relation $M_M \geq M_X/\alpha$, where $\alpha \approx 0.025$ is the dimensionless unified coupling constant and $M_X \approx 10^{14} \div 10^{16}$ GeV/c$^2$. Thus GUT magnetic monopoles should have $M_M \geq 10^{16}$ GeV/c$^2$. Due to their large masses, these MMs cannot be produced with existing and foreseen accelerators. They must be searched for in the penetrating cosmic radiation, using large area detectors \[5\]. GUT MMs should be characterized by relatively low velocities and relatively large energy losses. Direct searches for GUT MMs gave flux upper limits of few $10^{-16}$ cm$^{-2}$s$^{-1}$sr$^{-1}$ \[5\]; several indirect limits were obtained by different experiments \[3, 6\].

The predicted spatial structure of a GUT MM is illustrated in Fig. 1 \[6\]. The various regions are described in the figure caption. A GUT magnetic monopole may catalyse proton decay \[4, 6\]; the cross section for this process could be relatively large because of the size of the fermion-antifermion condensate, which may contain terms violating baryon number conservation (see Fig. 1).
2.2 Intermediate mass monopoles

Magnetic monopoles with masses of $10^{10} \div 10^{12}$ GeV/c$^2$ are predicted by theories with an intermediate mass scale [7] and would appear in the early universe at a time considerably later than the GUT time. Also these MMs are topological point defects; an undesirable large number of relatively light monopoles may be gotten rid of by means of higher dimensional topological defects (strings, walls, textures) [7].

![Diagram of an intermediate mass magnetic monopole]

Figure 2: Possible structure of an “intermediate mass magnetic monopole”. The inner region ($r \simeq 10^{-25}$ cm) corresponds to intermediate mass scales; inside this region one finds the intermediate mass bosons responsible for the symmetry breaking. The outer regions are as in Fig. 1 but without terms violating baryon number conservation in the fermion-antifermion condensate.

A possible structure of an intermediate mass MM is illustrated in Fig. 2. Notice that it has a larger core compared to the structure of a GUT monopole. The various regions correspond to: (i) intermediate mass scale of $R \simeq 10^{-25}$ cm; (ii) the electroweak scale; (iii) the condensate of fermion-antifermion pairs which could be the same as for a GUT monopole, but without any term violating baryon number conservation (thus these MMs cannot catalyse proton decay); (iv) the confinement region; (v) the outside region.

The number of intermediate mass monopoles could be considerably higher than that of GUT monopoles and galactic magnetic fields could accelerate them to high velocities. It has even be assumed that few of them could reach extremely high energies, interact in the upper earth atmosphere and lead to the highest energy cosmic ray showers [8, 9].

3 Nuclearites

Strange Quark Matter (SQM) should consist of aggregates of $u$, $d$ and $s$ quarks in approximately equal proportions [10]. The SQM is a colour singlet, thus it may have only integer electric charges. The overall neutrality of SQM is ensured by an electron cloud which surrounds it, forming a sort of atom. SQM may be the ground state of QCD.

The aggregates of $u$, $d$, $s$ quarks will be denoted with the terms strangelet, quark bag, SQM and nuclearite core; we shall use the word nuclearite for the core+electrons system.
Figure 3: Dimensions of the quark bag (R_N) and of the core+electrons system (nuclearite). The radii presented here (in a logarithmic scale) refer to the nuclearite quark bag. For nuclearite masses smaller than 10^9 GeV/c^2, the whole electron cloud is outside the quark bag and the core+electrons system has a global size of approximately 10^5 fm = 1 Å; for 10^9 < M_N < 10^{15} GeV/c^2 the electrons are partially inside the core; for M_N > 10^{15} GeV/c^2 all electrons are inside the core. The black dots indicate the electrons, the quark bag border is indicated by thick solid lines; the border of the core+electronic cloud system for relatively small masses is indicated by the dashed lines.

Strangelets could have been produced shortly after the Big Bang and may have survived as remnants; they could also appear in violent astrophysical processes, such as neutron star collisions. Nuclearites should have a constant density \[ \rho_N = M_N/V_N \simeq 3.5 \cdot 10^{14} \text{ g cm}^{-3}, \] somewhat larger than that of atomic nuclei.

They should be stable for all baryon numbers in the range between ordinary heavy nuclei and neutron stars (A \sim 10^{57}) \[ [11]. \] Nuclearites could contribute to the cold dark matter.

The structure of SQM can be described in terms of a bag model \[ [12]. \] In order to equilibrate the chemical potential of the quark species, SQM should have a number of s quarks slightly lower than the number of u or d quarks \[ [12]. \] Thus the nuclearite core should have a positive electric charge which would be balanced by a number of electrons \[ N_e \simeq (N_d - N_s)/3, \] where \( N_d, N_s \) and \( N_e \) are the numbers of quarks d, s and electrons, respectively, assuming \( N_d = N_u \) \[ [12]. \]

In the following, the radii \( R_N \) refer only to the nuclearite core (strangelet); the core+electronic cloud (nuclearite) system should have the constant radius of \( \sim 1 \text{ Å} \) for \( R_N < 1 \text{ Å} = 10^5 \text{ fm}. \)

For \( R_N \geq 10^5 \text{ fm}, \) all electrons must be inside the quark bag.

For \( 10^4 < R_N < 10^5 \text{ fm}, \) a fraction of the electrons are inside the quark bag, another fraction is external and gives the global dimension of \( \sim 10^{-8} \text{ cm} = 10^5 \text{ fm} \) to the nuclearite core + electrons system. In this condition a nuclearite is similar to a Bohr atom.

Fig. \( 3 \) illustrates qualitatively the space distributions for the system of nuclearite core+electrons. Notice that for \( R_N > 10^5 \text{ fm}, \) we picture the nuclearite as a sort of Thomson atom.

For nuclearite masses larger than \( 1.5 \cdot 10^{-9} \text{ g} \simeq 10^{15} \text{ GeV/c}^2, \) the relation between mass and radius should be (mass \( \propto \) volume, \( M_N \propto V_N \propto R_N^3 \))

\[
R_N = \left( \frac{3M_N}{4\pi \rho_N} \right)^{1/3} \tag{1}
\]

For \( M_N = 1.5 \cdot 10^{-9} \text{ g}, \) the nuclearite should have a radius

\[
R_N = \left( \frac{3}{4\pi} \frac{M_N}{3.5 \cdot 10^{14}} \right)^{1/3} = 8.8 \cdot 10^{-6} \text{ } M_N^{1/3} = 8.8 \cdot 10^{-6} \cdot (1.5 \cdot 10^{-9})^{1/3} \simeq 10^{-8} \text{ cm} = 10^5 \text{ fm} = 1 \text{ Å} \tag{2}
\]
Figure 4: Solid line: dependence of the nuclearite quark bag radius $R_N$ from its mass $M_N$. The dashed line gives the radius of the core+electrons system (nuclearite). We also indicate the mass regions accessible by the MACRO [15] and SLIM [23] experiments.

Figure 5: Qualitative dependence of the nuclearite core charge $Z_N$ from the nuclearite mass $M_N$. The solid line is computed in ref. [13]; the dashed line is a rough interpolation which takes into account that for $M_N \geq 10^{15}$ GeV/$c^2$ all the electrons are inside the quark bag.
For $M_N = 10^{18}$ GeV/c$^2$ one has $R_N \simeq 10^6$ fm $(1000)^{1/3} = 10^6$ fm.

Assuming that formula (1) is valid also for $M_N < 10^{15}$ GeV/c$^2$, one has, for the nuclearite core, the following values

\[
\begin{align*}
M_N &= 10^{18} \text{ GeV/c}^2 & R_N &\simeq 10^6 \text{ fm} \\
M_N &= 10^{15} \text{ GeV/c}^2 & R_N &\simeq 10^5 \text{ fm} = 1 \text{ Å} \\
M_N &= 10^9 \text{ GeV/c}^2 & R_N &\simeq 10^3 \text{ fm} \\
M_N &= 10^3 \text{ GeV/c}^2 & R_N &\simeq 10 \text{ fm}
\end{align*}
\]

Fig. 4 gives the dependence of the core radius $R_N$ and of the radius of the core+electrons system on the nuclearite mass $M_N$.

The expected relation between mass $M_N$ and charge $Z_N$ for nuggets of strange quark matter is shown in Fig. 5. The solid line is the curve computed in ref. [12], the dashed line is a rough interpolation taking into account that for $M_N \geq 10^{15}$ GeV/c$^2$ all the electrons are inside the quark bag. Nuclearites with $\beta \sim 10^{-3}$ could be detected by scintillators and nuclear track detectors, independently of the charge of the nuclearite core.

A “curious” problem arose in the discussion of the nuclearite production in heavy ion high energy colliders: there was a fear that possibly produced SQM would grow in size and destroy the earth. This was proven to be inaccurate in ref. [13].

The present flux upper limits on nuclearites in the cosmic radiation are discussed in Section 5 and given in ref. [14].

4 Q-balls

Q-balls [15] are aggregates of squarks $\tilde{q}$, sleptons $\tilde{l}$ and Higgs fields [16]. The scalar condensate inside a Q-ball core has a global baryon number $Q$ (may be also lepton number) and a specific energy much smaller than 1 GeV per baryon. We assume that the $Q$ numbers of quarks and squarks are equal to $1/3$ ($Q_q = Q_{\tilde{q}} = 1/3$) or $2/3$ ($Q_q = Q_{\tilde{q}} = 2/3$). Protons, neutrons and may be electrons can be absorbed in the condensate.

The vacuum expectation value inside a Q-ball core develops along “flat directions” of the potential [17]. These flat directions are parametrized by combinations of squarks and sleptons that are electrically neutral (otherwise they would not be flat directions). Supposing that the different flavour squarks are not mass degenerate, their numbers inside the Q-ball core would not be equal for the same reason as in the nuclearite core case.

By assuming that the baryon number is packed inside a Q-ball core, one can get upper limits for the Q-ball quantum number $Q$ and for the Q-ball mass $M_Q$: $Q \leq 10^{30}$ and $M_Q \leq 10^{25}$ GeV/c$^2$, respectively; Q-balls with $M_Q < 10^8$ GeV/c$^2$ are unstable [18].

In the early universe only neutral Q-balls were produced: SENS (Supersymmetric Electrically Neutral Solitons), which do not have a net electric charge, are generally massive and may catalyse proton decay. SENS may obtain an integer positive electric charge absorbing a proton in their interactions with matter; thus we may have SECS (Supersymmetric Electrically Charged Solitons), which have a core electric charge, have generally lower masses and the Coulomb barrier prevents the capture of nuclei. SECS have only integer charges because they are colour singlets. Some Q-balls which have sleptons in the condensate can also absorb electrons. The squarks $\tilde{q}$ inside the scalar potential bag have essentially zero mass.
The possible structures of SECS and SENS are shown in Fig. 6a,b. SENS may also interact with a proton of the interstellar medium, catalyse the proton decay leading to the emission of $2-3$ pions (or kaons) and transform quarks into squarks via the reaction $qq \rightarrow \tilde{q}\tilde{q}$. Thus some SENS may become SECS with a charge $+1$ emitting $2\pi^0$ with total energy of about 1 GeV.

When a SENS enters the earth atmosphere, it could absorb, for example, a nucleus of nitrogen which gives it the positive charge of $+7$ (SECS with $Z = +7$). The next nucleus absorption is prevented by Coulomb repulsion. If the Q-ball can absorb electrons at the same rate as protons, the positive charge of the absorbed nucleus may be neutralized by the charge of absorbed electrons. The incoming SENS remain neutral most of the time. Electrons may be absorbed via the reaction $u + e \rightarrow d + \nu_e$. If, instead, the absorption of electrons is slow or impossible, the Q-ball carries a positive electric charge after the capture of the first nucleus in the atmosphere (SECS). Other SENS could “swallow” entire atoms (remaining SENS). If a SENS could absorb an electron, it would acquire a negative charge (SECS with $Z = -1$). In the following we shall neglect the possibility that a neutral Q-ball (SENS) becomes charged (SECS) and viceversa.

The Q-ball mass $M_Q$, core size $R_Q$ and global quantum number $Q$ are related by the following relations [18], in theories with a bag potential $V = \phi(M_S^2)$,

$$M_Q = \frac{4\pi\sqrt{2}}{3} M_S Q^{3/4} \simeq 5924 \ M_S \text{(TeV)} \ Q^{3/4} \ (\text{GeV})$$

(3)

$$R_Q = \frac{1}{\sqrt{2}} M_S^{-1} Q^{1/4} \simeq 1.4 \cdot 10^{-17} \ M_S^{-1} \text{(GeV}^{-1}) \ Q^{1/4} \ (\text{cm})$$

(4)

where the parameter $M_S$ is the energy scale of the SUSY breaking symmetry. In the following we shall use $M_S = 100 \ \text{GeV}/c^2$ and $M_S = 1000 \ \text{GeV}/c^2$.

From Eq. (3) we have

$$Q^{1/4} = \left( \frac{3}{4\pi\sqrt{2} M_S} \right)^{1/3}$$

(5)
Figure 7: Dependence of the Q-ball radius $R_Q$ from its mass $M_Q$ for two values of the $M_S$ parameter. Q-balls with $M_Q \leq 10^8 \text{GeV/c}^2$ are unstable; Q-balls with $M_Q \geq 10^{25} \text{GeV/c}^2$ should be very rare. We also indicate the mass regions accessible by the SLIM and MACRO experiments for Q-balls with $\beta \sim 10^{-3}$ in the cosmic radiation.

Placing Eq. (5) into Eq. (4), we have

$$R_Q = \frac{1}{\sqrt{2}} M_S^{-4/3} \left( \frac{3M_Q}{4\pi\sqrt{2}} \right)^{1/3}$$

(6)

Note that, as for nuclearites, we have $R_Q \sim M^{1/3}$, once $M_S$ is fixed.

For $M_S = 100 \text{ GeV/c}^2$, we have from Eq. (6):

$$R_Q(m) \simeq 0.39 M_Q(\text{GeV/c}^2)^{1/3} \approx 0.39 (M_Q \cdot 0.197)^{1/3} \text{ fm} = 0.227 M_Q^{1/3} \text{ fm}$$

(7a)

The dependence of $R_Q$ from $M_Q$ is shown in Fig. 7 for $M_S = 100 \text{ GeV/c}^2$ and $1000 \text{ GeV/c}^2$.

Flux limits on Q-balls come mainly from the astrophysical dark matter limit given in Fig. 8. SECS with $\beta \simeq 10^{-3}$ and $M_Q < 10^{13} \text{ GeV/c}^2$ could reach an underground detector from above, SENS also from below [18, 20]. SENS may be detected by their almost continuous emission of charged pions (energy loss of about 100 GeV g$^{-1}$cm$^2$), while SECS may be detected by their large energy losses yielding light in scintillators, and possibly ionization. The energy losses of Q-balls in matter were computed in ref. [21]. Flux upper limits on SECS could be deduced from the limits for dyons with the same electric charge; flux limits on SENS could be obtained from limits on MMs which catalyse proton decay.
5 Astrophysical limits

Magnetic monopoles, nuclearites and Q-balls could be components of the galactic cold dark matter, required by the rotation curves of the stars in the outskirts of our galaxy and of other galaxies. Assuming a local DM energy density of $\rho = 0.3$ GeV/cm$^3$ and that MMMs, nuclearites and/or Q-balls could be part of it and have typical velocities of $\beta \simeq 10^{-3}$, we can obtain upper limits on their flux in the cosmic radiation.

Figure 8: Flux upper limits for nuclearites and for Q-balls versus their masses, assuming that they have $\beta = 10^{-3}$ and that each of them saturates the local dark matter density. Clearly, if the abundance of each of them is $10^{-3}$ of the cold dark matter, the quoted limits are $10^{-3}$ times smaller.

An experiment like MACRO has placed upper limits on the fluxes of MMMs and nuclearites and could place similar upper limits for SECS at the level of few $10^{-16}$ cm$^{-2}$s$^{-1}$sr$^{-1}$ for masses larger than approximately $10^{14}$ GeV/c$^2$ [5, 14], see Fig. 7. The experiment should also be capable to place limits on the MM catalysis of proton decay.

The SLIM experiment could reach a level of sensitivity of few $10^{-15}$ cm$^{-2}$s$^{-1}$sr$^{-1}$ for masses larger than approximately $10^{8}$ GeV/c$^2$ [22].

For superheavy MMMs the DM limits are considerably larger than the present experimental limits. But monopoles can gain energy in the galactic field; from the survival of the galactic field one has more stringent limits at the level of $\sim 10^{-15}$ cm$^{-2}$s$^{-1}$sr$^{-1}$ (Parker limit [23]), or even orders of magnitude lower (Extended Parker limits [24]). The limits, both experimental
and expected, are less well known for intermediate mass monopoles.

6 Conclusions

We have discussed qualitative pictures of the structures of superheavy and intermediate mass magnetic monopoles, of nuclearites and of Q-balls. In particular we have given the possible structure of these objects as function of their mass. We concluded with astrophysical considerations on their flux limits in the cosmic radiation. For nuclearites and Q-balls we assumed that they could be part of the cold DM and have $\beta = 10^{-3}$. Magnetic monopoles may be accelerated by galactic and intergalactic magnetic fields to higher velocities; for MMs, DM limits are considerably higher than limits based on the influence of magnetic fields.

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