Superfluid phase of $^3$He–B near the boundary

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Abstract. Following our analysis of some older and most recent transverse sound experiments in superfluid $^3$He–B we have been able to solve one of the long-existing problem of superfluid quantum liquids in confined geometry, namely, answer a question what is the boundary state of $^3$He–B. We have devoted specific attention to the differences between transverse sound experiments data from that obtained in longitudinal sound experiments. In our analysis, we have considered several potentially possible explanations of the above experimental data: existence of a new superfluid phase in the vicinity of the boundary; excitation of different branches of squashing mode by longitudinal and transverse sounds and, finally, deformation of the B–phase near the boundary. The last possibility seems to be the most likely explanation implying that the boundary state of $^3$He–B is, in fact, the deformed B–phase, as was first suggested by Brusov and Popov about two decades ago for a case of presence of external perturbations such as a magnetic and an electric fields. Our result implies that influence of a wall or, in other words, a confined geometry does not lead to the existence of a new phase near the boundary, as had been suggested many years ago, but, instead, similarly to the case of other external perturbations (such as magnetic, electric fields etc.), the wall deforms the order parameter of the B–phase and this deformation leads to several very important consequences. For example, frequencies of the collective modes in the vicinity of the boundary change by up to about 20 percent.

1. Introduction

The problem of boundary state of superfluid quantum liquid with complex order parameter (OP) like superfluid $^3$He is quite interesting and has a long history. One of the problems here is a particular superfluid phase realizing in the vicinity of a boundary. Ultrasound experiments used longitudinal sound have played an important role in investigation of superfluid $^3$He. By these experiments all collective modes in both A– and B–phases of $^3$He (eighteen modes in each phase) have been observed$^{1,2)}$. Longitudinal sound experiments have also helped to identify the superfluid phases in bulk $^3$He. While these experiments give information about whole volume of liquid, transverse sound experiments via strong damping of transverse sound can be a probe of the state in the vicinity of the boundary. Thus the problem of boundary state could be solved by transverse sound experiments. In past, a few possibilities for a “boundary” state have been considered: A–phase (with $\mathbf{l}$ – vector perpendicular to surface) and 2D–phase. It has been shown that former phase is unlikely. We consider here a few additional superfluid phases as well as alternative picture exploiting Brusov et al.3 idea concerning deformation of a gap in B–phase. Our analysis of existing experimental data leads to conclusion that deformed B–phase is realized near the boundary.

2. Transverse sound experiments

More than ten year ago J. Ketterson etc.$^{4,5}$ have developed the transverse acoustic impedance technique and used it to investigate the superfluid $^3$He–B. They have used a fixed sound frequency

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and swiped temperature and were able to provide simultaneously measurements of longitudinal and transverse responses, that have allowed them to compare data from two these measurements. It turned out that while there is a correlation between the longitudinal and transverse responses they did not coincide exactly: there are peaks in the imaginary transverse acoustic impedance at temperatures above the squashing (sq)–mode, as well as below. At temperatures above the squashing –mode these features might be interpreted as due to a standing wave pattern. Based on this assumption, authors turned out to be able to obtain the change in the phase velocity associated with these oscillations. The nature of the contribution at temperatures below the squashing –mode was not clear that time as well as now. In this paper we will concentrate on interpretation of last features, accounting that they have been observed in recent transverse sound experiments as well. These recent Japanese experiments are very important, because they provide us with some numerical data allowing us to distinguish different possibilities. The main results of these experiments are as follows. The data have been obtained for two transverse sound frequencies: 28 MHz and 47 MHz. While temperature dependant squashing – mode frequency is equal to \( \omega_{sq} = 1.35 \Delta_0(T) \) for transverse sound frequency 28 MHz and at \( \omega \approx 1.55 \Delta_0(T) \) for transverse sound frequency 47 MHz. We consider a few possible interpretation of above experimental data: existence of new superfluid phases in the vicinity of a boundary, excitation of different branches of squashing mode by longitudinal and transverse sounds and deformation of B–phase by the wall. We come to conclusion that last possibility seems the most likely.

3. Possible new phases near the boundary

2D–phase

2D–phase with order parameter \( c_{i} \propto (\delta_{i} \delta_{a} + \delta_{a} \delta_{i}) \) exists at magnetic fields \( H>H_c \). Without magnetic fields in the vicinity of the wall the gradient terms in free energy can stabilized this phase, thus it could be consider as a candidate for “boundary” state.

The collective mode spectrum in 2D–phase has been calculated by Brusov et al., used path integral technique. In magnetic fields it consists of 18 collective modes, among them six Goldstone modes, four clapping–modes \( E = (1.17 - i0.13)\Delta_0 \), two pairbreaking–modes \( E = (1.96 - i0.31)\Delta_0 \), six modes, depending on magnetic fields: two \( E = 2 \mu H \), two \( E^2 = (1.96 - i0.31)\Delta_0^2 + 4 \mu^2 H^2 \), \( E^2 = (0.518)\Delta_0^2 + 4 \mu^2 H^2 \) and \( E^2 = (0.495)\Delta_0^2 + 4 \mu^2 H^2 \).

This spectrum contains a clapping – mode with frequency \( E = (1.17 - i0.13)\Delta_0 \). If we have B–phase in the most part of sample and 2D–phase in the vicinity of a boundary, thus our signal of transverse (longitudinal) sound is averaged signal from both regions. In bulk we have squashing – mode with frequency \( \omega_{sq} = 1.55 \Delta_0 \) and we have clapping–mode with frequency \( E = 1.17\Delta_0 \) in the vicinity of the boundary. The detecting signal will have frequency depending on both squashing – and clapping – mode frequencies and the amplitudes of two these modes. It should be two resonances in sound absorption: one from squashing–mode in bulk B–phase and other one from clapping–mode in boundary 2D–phase. Via large uncertainty in determination of the temperature of the peaks these two peaks can be seems as one peak. For longitudinal sound experiments main signal absorption takes place in a bulk liquid (region of “boundary” phase is much smaller than bulk phase one), thus attenuation amplitude peaked at squashing–mode frequency \( \omega_{sq} = 1.55 \Delta_0 \). In case of transverse sound experiments region of “boundary” phase
becomes comparable to bulk phase region, thus one should expect attenuation amplitude peaked at frequency intermediate between squashing–mode frequency \( \omega_{sq} = \sqrt{12/5} \Delta_0 \approx 1.55 \Delta_0 \) and clapping–mode frequency \( \omega_{cl} = 1.17 \Delta_0 \). From this point of view observed peak frequencies \( \omega \approx 1.25 \Delta_0 (T) \) and \( \omega \approx 1.35 \Delta_0 (T) \) are understandable. But let us consider dependence of peak frequency on sound frequency. The particular value of peak frequency depends on attenuation amplitudes of sound into these two modes, which finally depend on the relative regions occupied by frequency on sound frequency. The fact contradicts to the hydrodynamic attenuation mechanism of transverse sound, which predicts increased attenuation with frequency as \( \omega^2 \). Thus we should state that supposed picture when we have B–phase in bulk and 2D–phase near the boundary is inconsistent with Japanese data and we should rule out such possibility.

**Phases**

\[ c_{ia} \propto (\delta_{i1} \delta_{a1} - \delta_{i2} \delta_{a2}); \]
\[ c_{ia} \propto (\delta_{i1} \delta_{a2} + \delta_{i2} \delta_{a1}); \]
\[ c_{ia} \propto (\delta_{i1} \delta_{a1} - \delta_{i2} \delta_{a2}) \]

Let us consider these three phases. Spectra for them are identical and one gets the following results for high frequency modes (at zero momentum of excitations)

\[ E = \Delta_0 (T)(1.83 - i 0.06); \]
\[ E = \Delta_0 (T)(1.58 - i 0.04); \]
\[ E = \Delta_0 (T)(1.33 - i 0.10); \]
\[ E = \Delta_0 (T)(1.33 - i 0.08); \]
\[ E = \Delta_0 (T)(1.28 - i 0.04); \]
\[ E = \Delta_0 (T)(1.09 - i 0.22); \]
\[ E = \Delta_0 (T)(0.71 - i 0.05); \]
\[ E = \Delta_0 (T)(0.33 - i 0.34); \]
\[ E = \Delta_0 (T)(0.23 - i 0.71). \]

Two last modes have imaginary parts of the same order as real ones. This means that they are damped very strongly and could not be considered as resonances. Among the obtained spectra there are modes, which frequencies are closed to observed ones \( \omega \approx 1.25 \Delta_0 (T) \) and \( \omega \approx 1.35 \Delta_0 (T) \). These are well determined modes (imaginary part of frequency is much less than real part)

\[ E = \Delta_0 (T)(1.33 - i 0.10); \]
\[ E = \Delta_0 (T)(1.33 - i 0.08); \]
\[ E = \Delta_0 (T)(1.28 - i 0.04). \]

But there is no reason, why at one sound frequency one branch is excited while at other frequency another branch is excited. Thus, at this step we should rule out these phases as a candidate for “boundary” state.

**Phase**

\[ c_{ia} \propto (\delta_{i1} + i \delta_{i2})(\delta_{a1} + \delta_{a2}) \]

For spectrum of collective modes in this phase the following high frequency modes have been found (at zero momentum of excitations)

\[ E = \Delta_0 (T)(1.55 - i 0.32); \]
\[ E = \Delta_0 (T)(1.2 - i 0.06); \]
\[ E = \Delta_0 (T)(0.62 - i 0.05); \]
\[ E = \Delta_0 (T)(0.4 - i 0.55); \]
\[ E = \Delta_0 (T)(0.3 - i 1.0). \]

Two last modes have imaginary parts of the same order as real ones. They are damped very strongly and could not be considered as resonances. Among the collective modes of this state there is only one \( E = \Delta_0 (T)(1.2 - i 0.06), \) which frequency is closed to one of observed peaks \( \omega \approx 1.25 \Delta_0 (T) \). We can not explain the existence of peak at \( \omega \approx 1.35 \Delta_0 (T) \). Thus, similar to previous case, we should rule out these phases as a candidate for “boundary” state.

### 4. Different branches of squashing mode

One more possible reason for difference in longitudinal and transverse sound experiments data has been suggested by J. Ketterson et al. They supposed, that in accordance to Ref. 6 longitudinal and transverse sounds can be coupled to different branches of squashing–mode: namely, in the absence of magnetic field transverse sound couples to \( J_{\perp} = \pm 1 \) modes while longitudinal sound couples...
to the $J_Z = \pm 0$ mode. While frequencies of all branches of squashing–mode are the same and equal to 
$$\omega_{\text{sq}} = \sqrt{\frac{12}{5}} \Delta_0 \approx 1.55 \Delta_0 ,$$
dispersion corrections for branches with different projections of total moment of Cooper pairs $J$ turns out to be different. These dispersion corrections have been calculated by numerous authors, but complete calculations for all 18 collective modes in B–phase have been done by Brusov and Popov, who have obtained the following results for considering branches of squashing–mode

$$\omega_{\text{sq}}^2 = 12 / 5 \Delta_0^2 (T) + 0.418 \varepsilon^2 k^2 (2, \pm 1, i) , \quad \omega_{\text{sq}}^2 = 12 / 5 \Delta_0^2 (T) + 0.502 \varepsilon^2 k^2 (2, 0, i) .$$

From these results it follows that longitudinal resonance should takes place at higher frequencies than transverse one and this explains attenuation peak at temperatures below the squashing–mode in experiments on transverse sound attenuation. However if we will compare the real splitting of the squashing–mode branches via dispersion corrections and difference between squashing–mode peak (from longitudinal experiments) and transverse sound data it will turn out that former one is much less than later one. To estimate the dispersion induced splitting of the squashing–mode branches we can refer to similar observed splitting of real squashing–mode.

This splitting turned out of order 0.01 $T_C$. While the splitting of squashing–mode will be a little bit larger it should be the same order of magnitude. From the other side the difference between sq–mode peak (from longitudinal experiments) and transverse sound data is 0.2 $\Delta_0 (T) - 0.3 \Delta_0 (T)$ depending on transverse sound frequency. Accounting that $T_C$ is the same order of magnitude as $\Delta_0 (0)$, we come to conclusion that coupling longitudinal and transverse sounds to different branches of squashing–mode can not explain the observed features.

5. Deformed B–phase

Presence of boundary leads to deformation of the order parameter. Component of vector $d$, which is perpendicular to the boundary, becomes zero at the boundary in case of mirror, as well as diffusive reflection of atoms. By other words, Cooper pairs tend to move in the plane parallel to the boundary. While A–phase in a slab geometry should have the same order parameter as in bulk case (only $l$ should be perpendicular to the boundary), in case of B–phase order parameter of bulk B–phase can not satisfy to boundary condition, if it remains nondeformed. This deformation is coupled to appearance of additional gradient energy and decreasing of condensation energy.

More than twenty years ago Brusov and Popov have investigated the influence of gap distortion, caused by dipole interaction and by different type of external perturbations such as magnetic or electric fields, on the order parameter collective modes in B–phase. They have shown, that consequences of such influence are quite significant: it changes the frequencies of all collective modes and (that is especially important for us) splits the pairbreaking–, squashing– and real squashing–modes at zero momentum $q$ of collective excitations. At nonzero $q$ they predicted a branch crossing of these modes with different $J_Z$. Let us summarize their results and see what we have for squashing–mode. In presence of external perturbations such as magnetic or electric fields or boundary order parameter has the following form

$$A_i = \left[ \Lambda^{1/2} R(\vec{n}, \theta) \right] e^{i \phi},$$

where $\Lambda$ is diagonal matrix with elements

$$\lambda_1, \lambda_2, \lambda_3$$

and $\lambda_1 = \lambda_1' = \lambda_2' = \lambda_2 + \Omega^2, \lambda_3' = \lambda_3 - \alpha \Omega^2.$

We can rewrite order parameter

$$A_i \approx \Delta_1 [(\delta_{1a} \delta_{1l} + \delta_{1a} \delta_{a2}) \cos \theta + (\delta_{2a} \delta_{2l} - \delta_{1a} \delta_{a2}) \sin \theta] + \Delta_2 \delta_{a1} \delta_{a3} .$$

Here $\theta = \arccos(-1/4)$.

In case of dipole interaction and electric fields $\alpha = -2$, in case of magnetic fields $\alpha = -4$.

In case of confined geometry boundary will suppress the gap in a single particle spectrum in perpendicular direction, thus we suppose $\alpha$ to be negative in the vicinity of a
boundary. Thus, gap $\Delta_1$ along the boundary will be bigger than gap $\Delta$ in a bulk liquid, while gap $\Delta_2$ perpendicular boundary will be less than $\Delta$. Brusov and Popov results\(^8\) for the squashing–mode are as follows:

$$
E^2 = (12/5)\Delta^5_0(T) + (6\alpha + 11)\Omega^2 / 10 \quad \text{for } (2, \pm 1, i) \text{ branches,}
$$

$$
E^2 = (12/5)\Delta^5_0(T) + (2\alpha + 3)\Omega^2 / 2 \quad \text{for } (2, 0, i) \text{ branch,} \quad (12)
$$

$$
E^2 = (12/5)\Delta^5_0(T) + 3(\alpha + 3)\Omega^2 / 5 \quad \text{for } (2, \pm 2, i) \text{ branches.}
$$

The results for $(2, 0, i)$ branch is insufficient because in longitudinal sound experiments signal comes from whole volume of liquid and influence of region near the boundary is small or even negligible in this case. The essential for us is result for $(2, \pm 1, i)$ branches, which are excited in transverse sound experiments. This result can explain observed features of transverse sound experiments: resonant absorption of transverse sound below squashing–mode frequency, dependence of absorption frequency on transverse sound one. Thus, from results for $(2, \pm 1, i)$ branches it follows, that $(6\alpha + 11)\Omega^2 / 10 < 0$, thus $\alpha < -11/6 \approx -2$. We can estimate the value of extra term in squashing–mode spectrum appearing via boundary influence (through the gap distortion). From experiments\(^7\) it follows that $(6\alpha + 11)\Omega^2 / 8\sqrt{15}\Delta$ should be of order $0.3\Delta(T)$ at sound frequency $28\text{MHz}$ and of order $0.2\Delta(T)$ at sound frequency $47\text{MHz}$.

### 6. Conclusion

We have analyzed the old and recent transverse sound experiments in superfluid $^3\text{He}$–B, where some peaks in transverse sound absorption have been observed. These peaks are in disagreement with sq–mode frequency, obtained from longitudinal sound experiments. We consider a few possible explanations of above experimental data and come to conclusion that deformed B–phase is realized near the boundary. Farther transverse sound experiments at different sound frequencies are desirable to make picture more clear. These experiments turned out to be very important for further investigation of superfluid $^3\text{He}$ in confined geometry. Discussed problem has very closed connection with study of superfluid $^3\text{He}$ in aerogel\(^10\). More detailed discussion of considering problems can be found in\(^11\) and in a new monograph\(^12\). We thank J. Ketterson and Y. Okuda for useful discussions.

### References

[1]. P.N. Brusov, V.N. Popov, *Collective properties of quantum fluids*, Moscow, Nauka Publishing, 215 p., 1988.
[2]. P.N. Brusov, Phys. Rev. B, 43, 12888 (1991).
[3]. P.N. Brusov, V.N. Popov, Phys. Rev. B, 30, 4060 (1984).
[4]. S. Kalbfeld, D.M. Kucera and J.B.Ketterson, Phys. Rev. B, 48, 4160 (1993).
[5]. S. Kalbfeld, D.M. Kucera and J.B.Ketterson, Phys. Rev. Letts. 71, 2264 (1993).
[6]. G. F. Moores and J.A. Sauls, J. Low Temp. Phys. 91, 13 (1993).
[7]. Y. Aoki, Y. Wada, A. Ogino, M. Saitoh, R Nomura, and Y. Okuda, JLTP, 138, 783 (2005).
[8]. P.N. Brusov, V.N. Popov, ZhETP, 78, 234 (1980).
[9]. B.S. Shivaram et al, Phys. Rev. Letts., 49, 1646 (1982).
[10]. Peter Brusov, Jeeva Parpia, Paul Brusov et al, Phys. Rev. B Rapid Commun., 63, 140507 (2001).
[11]. Peter Brusov and Pavel Brusov, Phys. Rev. B 72, 134503 (2005)
[12]. Peter Brusov and Pavel Brusov, “Collective excitations in unconventional superconductors and superfluids”, Singapore, World Scientific Publishing, 500 p., 2009.