Study of shaking table substructure test loading by inertial mass

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Abstract. It is hardly to carry out large-scale shaking table test of entire structure for the high cost of model construction and the capability limited of shaking table in the field of civil seismic research. In this paper, a shaking table substructure test method loading by inertial mass was studied, and the inertial mass parameter was designed based on the principle of frequency equivalence. Numerical simulation was performed and the applicability of the test method was analysed. A shaking table test model of 5-stories steel frame model and the substructure test loading by inertial mass have been carried out. Result shows that the phase of the time history curves could in good agreement, but the peak responses of the substructure model were larger than the entire structure tests. So, the shaking table substructure test loading by inertial mass based on the principle of frequency equivalence is not suitable for the structure with uniform distribution of mass and stiffness. Finally, it has been verified that the substructure test based on the principle of frequency equivalence is suitable for the structures with large rigidity and concentrated mass by analysing of another example. The research of this paper could be referred for substructure test of large and complex structure.

1. Introduction

In the research of earthquake engineering, theoretical analysis and numerical simulation can solve the linear problem of structural seismic responses. For nonlinear problem, those methods could not truly reflect the structural failure mechanism under strong earthquake, which needs to be verified by seismic test. Structural seismic testing method is still a very important research method for structural seismic studies. In the traditional seismic test, the quasi-static test and pseudo-dynamic test method cannot fully reflect the influence of rate-dependent dynamic appearances, and the shaking table test was limited by the size and loading capacity of shaking tables. In 1992, Nakashima [1] et.al proposed a real-time substructure test method, which is a development of traditional structural seismic test methods. It combines the advantages of experimental research and numerical simulation, and by splits the testing model to numerical substructure and experimental substructure, that is, the part in the elastic stage in the process of earthquake motion was taken as numerical substructures, and vulnerable parts or nonlinear parts were taken as experimental substructure [2, 3, 4, 5, 6]. The physical variables were exchanged real-time among substructures by loading device. The dynamic interactions between substructures were usually simulated by actuators and shaking table [7, 8] to accurately reflect the
performance of velocity dependent specimens. The real-time substructure test method fills the gap of the traditional seismic test method.

Recent years, real-time substructure test technology has developed rapidly. Schellenberg and Mahin [7] of the University of California-Berkeley developed and improved the real-time substructure test platform of Open-source Framework for Experimental Setup and Control (OpenFresco) based on the original distributed test computing collaborative platform. Cai Xinjiang [8,9] et.al constructed the OpenSEES-OpenFresco-MTS real-time substructure test system, in which the OpenSEES calculation unit was used for modelling and control of the entire test, the MTS loading system was used to load the test unit, the OpenFresco was used to transfer the data of the experimental substructure and the numerical substructure. The seismic performance of steel reinforced concrete frame with special-shaped columns was studied. Pan Peng and Wang Tao developed three different hybrid test systems: master-slave substructure hybrid test system, dynamic-static separation substructure hybrid test system and equal substructure hybrid test system. The flexibility, scalability and accuracy of these platforms were verified by a series of tests [10, 11, 12, 13]. Wu Bin and others developed HyTest, a hybrid seismic test platform for building structures by MTS secondary development programming library to connect the control system of test equipment. This platform uses MTS secondary development programming library to connect the test control system, and it is compatible with OpenSEES and ABAQUS finite element software to analyze the numerical substructure [14]. Although the real-time substructure test technology has developed rapidly, it still faces some problems. The boundary conditions of the experimental substructure are complex and hard to calculate accurately. When the loading components of multiple actuators are used to impose the boundary conditions, there is a problem of dynamic coupling [15]. In addition, there is a time delay in actuator, which not only affects the accuracy, but also affects the stability of test [16].

When using real-time substructure test to study large-scale complex engineering structure, the problem is more prominent: such as the calculation time of numerical substructure is too long, the boundary condition of experimental substructure is too complex. Therefore, the shaking table substructure test loading by inertial mass method, a simplified loading method, is studied in this paper. The lower portion of structure is regarded as the experimental substructure. The schematic diagram of shaking table substructure test loading by inertial mass is shown in Fig.1. A n-stories shear frame structure was taken as the entire system, and the lower i-stories of the frame structure were taken as the experimental substructure. The upper (n-i) stories (referred to as numerical substructure in the following) were replaced by the inertial mass equivalently.

![Figure 1. Schematic diagram of shaking table substructure test](image)

The expression of dynamic differential equation of the entire structure can be expressed as follows:
\[ M \ddot{x} + C \dot{x} + Kx = -MHg \]  

(1)

Where \( M, C \) and \( K \) represent the mass matrix, damping matrix and stiffness matrix of the numerical model of the entire structure respectively, \( \ddot{x}, \dot{x}, x \) are acceleration vector, velocity vector and displacement vector relative to the shaking table respectively. \( H \) is the unit column vector, \( \ddot{x}_g \) is the input ground motion acceleration.

In the shaking table substructure test, the horizontal force on the top of the experimental substructure is equal to the inertial force of the numerical substructure, that is, the base force of the numerical substructure, so the differential equation of the experimental substructure can be expressed as:

\[
\begin{bmatrix}
\vdots \\
m_1 \\
\vdots \\
m_n
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_1 \\
\ddot{x}_2 \\
\vdots \\
\ddot{x}_n
\end{bmatrix} +
\begin{bmatrix}
c_1+c_2 & -c_2 & 0 & \cdots & 0 \\
-c_2 & c_2+c_3 & -c_3 & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & -c_{n-1} & c_n & -c_n
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\vdots \\
\dot{x}_n
\end{bmatrix} +
\begin{bmatrix}
k_1+k_2 & -k_2 & 0 & \cdots & 0 \\
-k_2 & k_2+k_3 & -k_3 & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & -k_{n-1} & k_n & -k_n
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n
\end{bmatrix} =
\begin{bmatrix}
m_1 \\
m_2 \\
\vdots \\
m_n
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_1 \\
\ddot{x}_2 \\
\vdots \\
\ddot{x}_n
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]  

(2)

\[ f_d = -\sum_{e=1}^{n} m_e (\ddot{x}_e + \ddot{x}_g) = c_{e1}(\ddot{x}_{i+1} - \ddot{x}_i) + k_e(x_{i+1} - x_i) \]  

(3)

Where \( m_e, k_e, c_e, e = 1, 2 \cdots n \) represents the concentrated mass, damping and stiffness between layer and layer, respectively, \( x_e, \dot{x}_e, \ddot{x}_e, e = 1, 2 \cdots n \) are the displacement, velocity and acceleration of the corresponding layer relative to the shaking table, respectively, \( f_d \) is the interface force between the numerical substructure and the experimental substructure. In the shaking table substructure test loading by inertial mass, the inertial force generated by inertial mass replaces the interface force between the numerical substructure and the experimental substructure. In the motion equation of the simplified experimental substructure, \( f_d \) is:

\[ f_d = -m_d(\ddot{x}_i + \ddot{x}_j) \]  

(4)

Where \( m_d \) is the inertia mass.

3. Determination of inertia mass parameters

The inertial mass was designed according to the principle of frequency equivalence, in order to make the substructure test model loading by inertial mass close to the seismic response of the entire structure, The principle of frequency equivalence is to ensure that the fundamental frequency of the substructure loading by inertial mass is the same as the entire structure model.

The steps to calculate the inertial mass are: under the condition that the initial stiffness matrix and mass matrix of the test substructure are known, the fundamental frequency of the entire structure is obtained, so that the fundamental frequency of the experimental substructure by inertial mass is the same as the entire structure model.

The inertial mass parameters can be obtained by solving the following frequency equation.

\[
\begin{bmatrix}
(k_1 + k_2) - m_1 \omega^2 & -k_2 \\
-k_2 & (k_2 + k_3) - m_2 \omega^2 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
-k_i & \cdots & -k_{i-1} & k_i & -k_i \\
-k_i & \cdots & -k_{i-1} & k_i & -(m_1 + m_d) \omega^2
\end{bmatrix}
\]  

(5)

4. Numerical simulation

4.1 Numerical model

According to the theory of the short of mass similarity\[18\], the scale of 5-stories two-way single span steel frame structure was reduced scale, the similarity relationship of parameters is shown in Table 1, the parameters of entire structure are shown in Table 2, the column in the model was made of No.10 I-beam, and the beam was made of No.5 channel steel; SAP2000 software was used for the numerical analysis of the entire structure model and the substructure model loading by the inertial mass. According to the principle of frequency equivalence, the calculated inertia mass was 3900kg.
### Table 1. Model similarity factors.

| Physical Quantity       | Similarity Relationship | Similarity Coefficient |
|-------------------------|-------------------------|------------------------|
| Length $S_l$            | $S_l$                    | 1/3                    |
| Strain $S_e$            | $S_e$                    | 1                      |
| Elastic Modulus $S_E$   | $S_E$                    | 1                      |
| Area $S_A$              | $S_A$                    | 1/9                    |
| Density $S_\rho$        | $S_\rho$                | 2                      |
| Mass $S_m$              | $S_m = S_\rho S_l^{\frac{2}{27}}$ | 2/27                   |
| Time $S_t$              | $S_t = (S_E / S_\rho)^{0.5}$ | (2/9)^{0.5}            |
| Horizontal Acceleration $S_a$ | $S_a = S_E / (S_l S_\rho)$ | 2/3                    |

**Table 2. The parameters of entire structure model.**

| Parameters                        | Value   | Parameters               | Value |
|-----------------------------------|---------|--------------------------|-------|
| Mass of structure 1 to 4 floors   | 635kg   | Width                    | 1m    |
| Mass of structure top floor       | 481kg   | Height of storey         | 1m    |
| Length                            | 2m      | Damping ratio            | 1.6%  |

4.2 Numerical analysis

The substructure simulation analysis condition is the same as the entire structure, focusing on the response difference of the 1st-floor displacement and base shear of the entire structure and the experimental substructure model. 7 seismic waves with peak value of 2m/s² as excitation signals were input and the seismic response comparison between the substructure and the entire structure is shown in Table 3. Taking El-Centro seismic wave as an example, the time history curve comparison is shown in Fig.2.

**Table 3. Peak seismic response of full model.**

| Seismic Wave         | EBSP (kN) | SBSP (kN) | Relative Peak Error | ESDP (mm) | SDP (mm) | Relative Peak Error |
|----------------------|-----------|-----------|---------------------|-----------|----------|---------------------|
| Chichi               | 3.502     | 5.857     | 67.2%               | 6.805     | 10.138   | 48.9%               |
| Kobe                 | 4.877     | 7.235     | 48.3%               | 9.209     | 14.102   | 53.1%               |
| Chalfant Valley      | 4.074     | 6.347     | 55.8%               | 8.037     | 11.641   | 44.8%               |
| El-Centro            | 4.650     | 6.985     | 50.2%               | 8.847     | 13.373   | 51.2%               |
| Northridge           | 4.796     | 8.324     | 73.7%               | 9.846     | 16.061   | 63.1%               |
| Taft                 | 6.477     | 11.685    | 80.4%               | 13.073    | 22.075   | 68.9%               |
| Tianjin              | 8.108     | 15.571    | 92.0%               | 17.121    | 30.425   | 77.7%               |

EBSP, Entire Structure Base Shear Peak; SBSP, Substructure Base Shear Peak; ESDP, Entire Structure 1st-floor Displacement Peak; Substructure 1st-story Displacement Peak

![Displacement](image)

(a) Comparison of 1st-floor displacement

![Force](image)

(b) Comparison of base shear

**Figure 2.** Earthquake response under El-Centro wave excitation

It can be seen from Table 3 and Fig.2 that when the inertia mass is designed by the principle of frequency equivalence, their phase of the time history curve of the 1st-floor displacement and the base shear are in good agreement, but the relative peak value error is big. Under the action of 7 seismic waves, the average value of relative peak value error of 1st-floor displacement and base shear is 58.2% and 66.8%, respectively.
5. Experimental study

5.1 Shaking table and test model

The test equipment is a three-way six freedom electro-hydraulic servo shaking table, and the main technical indexes are shown in Table 4. The test model is the same as the numerical simulation model, and the entire structure model is shown in Fig.3 (a). The natural frequencies of the first two steps of the entire structure are 1.211Hz and 4.247Hz. The shaking table substructure test loading by inertial mass is shown in Fig.3 (b). The lower two floors of the entire structure were taken as the experimental substructure. A counterweight box was installed on the top of the substructure to simulate the influence of the upper structure on the experimental substructure.

Table 4. Technical index of the shaking table.

| Properties               | Value                     | Properties                      | Value       |
|--------------------------|---------------------------|---------------------------------|-------------|
| Table Size               | 2500mm×2500mm             | Maximum Acceleration            | 2g          |
| Load Weight              | 10Ton                     | Maximum Horizontal Displacement | ±125mm      |
| Frequency Range          | 0.1Hz~50Hz                | Maximum vertical displacement   | ±100mm      |

![Figure 3. Structural model](image)

Firstly, the inertia mass parameter was 3900kg calculated by the principle of same frequency, and the model identification of the experimental substructure was carried out under the white noise condition. Peak of the white noise is 0.5m/s² and frequency width is 0.1Hz-50Hz. The fundamental frequency of the experimental substructure was larger than the entire structure. The inertial mass parameter of the actual test was adjusted to 3780kg, and then, the fundamental frequency of the experimental substructure and the entire structure was equal.

5.2 Seismic response analysis

7 seismic waves with a peak value of 2m/s² were input to the entire structure and the substructure model. Table 5 shows the peak value of seismic response of the structure and the peak error relative to the entire structure. Taking El-Centro wave as an example, the seismic response comparison curve of substructure and entire structure is shown in Fig.4.

According to the Fig.4, when using the principle of frequency equivalence to design the inertia mass, the substructure and the entire structure’s phase of the time history curve of the 1st-floor displacement and the base shear are relatively consistent, and the trend of the two waveforms is basically the same. According to Table 5, under the action of 7 seismic waves, the average value of relative peak error of the 1st-floor displacement between the substructure model and the entire structure model is 70.8%, and the average value of relative peak error of the base shear is 71.3%.
Table 5. Peak seismic response of substructure structure.

| Seismic Wave  | EBSP (kN) | SBSP (kN) | Relative Peak Error | ESDP (mm) | SDP (mm) | Relative Peak Error |
|---------------|-----------|-----------|---------------------|-----------|----------|---------------------|
| Chichi        | 3.275     | 5.758     | 75.8%               | 3.908     | 6.924    | 77.2%               |
| Kobe          | 3.449     | 5.621     | 63.0%               | 4.073     | 7.209    | 77.0%               |
| Chalfant Valley | 3.203   | 4.941     | 54.3%               | 4.026     | 6.137    | 58.6%               |
| El-Centro     | 3.018     | 5.271     | 74.7%               | 3.869     | 6.415    | 59.3%               |
| Northridge    | 2.942     | 4.469     | 51.9%               | 2.924     | 5.378    | 83.9%               |
| Taft          | 3.253     | 6.197     | 90.5%               | 4.691     | 7.968    | 69.8%               |
| Tianjin       | 5.385     | 10.182    | 89.1%               | 7.278     | 12.355   | 69.7%               |

EBSP, Entire Structure Base Shear Peak; SBSP, Substructure Base Shear Peak; ESDP, Entire Structure 1st-floor Displacement Peak; Substructure 1st-story Displacement Peak

Figure 4. Earthquake response under El-Centro wave excitation

The test results show that when the upper structure is a multi-layer frame structure and the inertial mass is designed by the principle of frequency equivalence, the shaking table substructure test loading by inertial mass can get a good seismic response curve which is consistent with the phase of the entire structure, but there is a big peak error. The seismic response of the substructure is larger than the entire structure. The shaking table substructure test loading by inertial mass based on the principle of frequency equivalence is not suitable for the structure with uniform upper mass and stiffness distribution.

6. Example analysis

For water tower, lighthouse, pier, high pile cap, wind power tower and other structures with large concentrated mass, the shaking table substructure test loading by inertial mass can be considered. Taking the shaking table test of bridge pier structure as an example, the value of additional inertia mass was determined by the principle of frequency equivalence. the applicability was verified by numerical simulation.

Figure 5. The shaking table substructure test of bridge pier

As shown in Fig.5, inertia mass was placed on the top of the specimen to simulate the complex structure of the upper part of the bridge in a shaking table test. Taking the bridge as entire structure and the pier as substructure. Under the same earthquake action, the response difference between the pier with additional inertia mass and the pier in the entire bridge structure was analysed. The finite element software CSI Bridge was used for modelling. The bridge parameters are shown in Table 6,
and the entire structure model of the bridge is shown in Fig.6. Taking the middle pier as the experimental substructure, the inertia mass was loaded by defining the mass point. The substructure model is shown in Fig.7.

**Table 6.** The parameters of the bridge model.

| Parameters                  | Value                   |
|-----------------------------|-------------------------|
| Length                      | 80m                     |
| Mass                        | 1550.1t                 |
| Material                    | C30 concrete            |
| Pier Section                | 1525mm×1220mm           |
| Pier Height                 | 7.2375m                 |
| Total Width of Box Girder   | 10.98m                  |
| Total Height of Box Girder  | 1.525m                  |

*Figure 6. The bridge model*

*Figure 7. The substructure model*

According to the calculation by the principle of frequency equivalence, the inertial mass at the top of substructure was 273.4t. The El-Centro wave with the peak of 0.2g was input, and the response time history of the entire structure and substructure model was analysed. The displacement at the top of the pier was taken as the research object. The displacement peak error is 2.6%. The time history comparison curve of the displacement is shown in Fig.8.

*Figure 8. Earthquake response under El-Centro wave excitation*

It can be seen from the example that when the inertia mass designed by the principle of frequency equivalence, the response of substructure can reproduce the response of pier in the entire structure ideally. It has been verified that the shaking table substructure test loading by inertial mass is suitable for the structure with large upper stiffness and concentrated mass.

7. Conclusion

For the structure with uniform stiffness and mass distribution, the shaking table substructure test loading by inertial mass will have a big error, and even lead to the test results not reflect the actual seismic response of the entire structure.

For the structure with large rigidity and concentrated mass, the principle of frequency equivalence can be used and the part of large rigidity and concentrated mass of structure can be replaced by inertial mass. The difference of seismic response can be completely controlled within a satisfactory range.

The research in this paper provides a feasible simplified method for the shaking table substructure test. It can be used for structure with large stiffness and relatively concentrated mass, and has certain theoretical research and test application value.
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