Time-dependent Solutions with Null Killing Spinor in M-theory and Superstrings

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Abstract

Imposing the condition that there should be a null Killing spinor with all the metrics and background field strengths being functions of the light-cone coordinates, we find general 1/2 BPS solutions in $D = 11$ supergravity, and discuss several examples. In particular we show that the linear dilaton background is the most general supersymmetric solution without background under the additional requirement of flatness in the string frame. We also give the most general solutions for flat spacetime in the string frame with RR or NS-NS backgrounds, and they are characterized by a single function.

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Study of time-dependent solutions in string theories is an important subject for its application to cosmology and understanding our spacetime [1, 2]. Among these, the solutions with partial supersymmetry (BPS solutions) are important because these allow us to discuss nonperturbative regions of our spacetime. It is known that the requirement of unbroken partial supersymmetry restricts the solutions to those with null or time-like Killing spinors [3]. In this paper, we focus on those solutions with null Killing spinors.

Recently such time-dependent solutions have been studied in the linear dilaton background in the null direction with $\frac{1}{2}$ supersymmetry [4, 5]. (Related solutions of null branes are considered in Ref. [6].) This is interesting not only in its BPS property but also in the possibility of realization in Matrix theory, which allows nonperturbative study of the solution. It has been suggested that the problem of singularity in the spacetime is under control in this setting.

The solution studied in [4] does not involve any background in the forms which are present in the string theories. This is certainly a simplification but the question then arises if the results are restricted to only this very special solution or they are valid for other related solutions with possible backgrounds. To answer this question, we have to know how a large class of solutions of this type are allowed in string theories. The purpose of this paper is to give more general class of solutions with nontrivial backgrounds.

Let us start with the general action for gravity coupled to a dilaton $\phi$ and an $n$-form field strength:

$$I = \frac{1}{16\pi G_D} \int d^Dx\sqrt{-g} \left[ R - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2n!} e^{a\phi} F_n^2 \right].$$

This action describes the bosonic part of $D = 11$ or $D = 10$ supergravities; we simply drop $\phi$ and put $a = 0$ and $n = 4$ for $D = 11$, whereas we set $a = -1$ for the NS-NS 3-form and $a = \frac{1}{2}(5 - n)$ for the form coming from the R-R sector. To describe more general supergravities in lower dimensions, we should include several scalars and forms, but for simplicity we disregard this complication in this paper.

We are interested in the metric

$$ds^2 = -2e^{2u_0}dudv + \sum_{i=2}^{D-1} e^{2u_i}(dx^i)^2,$$

where all metrics are assumed to be functions of $u = (t - x^1)/\sqrt{2}$ and $v = (t + x^1)/\sqrt{2}$.  

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Though we could consider more general metric in the \((u, v)\) part, we can always put the
metric in the above form without loss of generality since the two-dimensional metric is
(locally) conformally flat. The components of the Ricci tensor are

\[
R_{uu} = -\sum_{i=2}^{D-1} \partial_i^2 u_i + 2\partial_u u_0 \sum_{i=2}^{D-1} \partial_i u_i - \sum_{i=2}^{D-1} (\partial_i u_i)^2,
\]

\[
R_{uv} = -\partial_u \partial_v \left(2u_0 + \sum_{i=2}^{D-1} u_i\right) - \sum_{i=2}^{D-1} \partial_u u_i \partial_v u_i,
\]

\[
R_{vv} = -\sum_{i=2}^{D-1} \partial_i^2 v_i + 2\partial_v u_0 \sum_{i=2}^{D-1} \partial_i u_i - \sum_{i=2}^{D-1} (\partial_i u_i)^2,
\]

\[
R_{ij} = \delta_{ij} e^{2(u_i-u_0)} \left[2\partial_u \partial_v u_i + \partial_v u_i \sum_{k=2}^{D-1} \partial_u u_k + \partial_u u_i \sum_{k=2}^{D-1} \partial_v u_k\right].
\]

We consider mainly the \(D = 11\) supergravity and so there is only an \(n = 4\) form. We take
the background

\[
F = (\partial_u E du + \partial_v E dv) \wedge dx^2 \wedge dx^3 \wedge dx^4,
\]

leading to

\[
F^2 = -4! \cdot 2(\partial_u E \partial_v E) e^{-2(u_0+U)},
\]

where \(U = u_2 + u_3 + u_4\). This is an electric background and we could also consider magnetic
background, but that is basically the same as the electric case with the replacement

\[
g_{\mu\nu} \rightarrow g_{\mu\nu}, \quad F_n \rightarrow e^{a\phi} * F_n, \quad \phi \rightarrow -\phi.
\]

This is due to the S-duality symmetry of the original system (1). So we do not have to
consider it separately.

The field equations following from the general action (1) are as follows:

\[
0 = \partial_{\mu_1} \left(\sqrt{-g} e^{a\phi} F^{\mu_1 234}\right)
\]

\[
= -\partial_u \left\{ e^{a\phi - U + \sum_{i=2}^{D-1} u_i \partial_i E} - \partial_v \left( e^{a\phi - U + \sum_{i=2}^{D-1} u_i \partial_i E} \right) \right\},
\]

\[
\frac{1}{\sqrt{-g}} \partial_u \left[ e^{\sum_{i=2}^{D-1} u_i \partial_i \phi} \right] + \frac{1}{\sqrt{-g}} \partial_v \left[ e^{\sum_{i=2}^{D-1} u_i \partial_i \phi} \right] = \frac{1}{2} e^{a\phi - 2u_0 - 2U} \partial_u E \partial_v E,
\]

\[
- \sum_{i=2}^{D-1} \partial_i^2 u_i + 2\partial_u u_0 \sum_{i=2}^{D-1} \partial_i u_i - \sum_{i=2}^{D-1} (\partial_i u_i)^2 = \frac{1}{2} (\partial_u \phi)^2 + \frac{1}{2} e^{a\phi - 2U} (\partial_u E)^2,
\]

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\[ -\partial_u \partial_v (2u_0 + \sum_{i=2}^{D-1} u_i) - \sum_{i=2}^{D-1} \partial_u u_i \partial_v u_i = \frac{D - 8}{2(D - 2)} e^{a\phi - 2U} \partial_u E \partial_v E, \quad (10) \]

\[ - \sum_{i=2}^{D-1} \partial^2_v u_i + 2 \partial_v u_0 \sum_{i=2}^{D-1} \partial_v u_i - \sum_{i=2}^{D-1} (\partial_v u_i)^2 = \frac{1}{2} (\partial_v \phi)^2 + \frac{1}{2} e^{a\phi - 2U} (\partial_v E)^2, \quad (11) \]

\[ e^{2(u_i - u_0)} \left[ 2 \partial_u \partial_v u_i + \partial_v u_i \sum_{k=2}^{D-1} \partial_u u_k + \partial_u u_i \sum_{k=2}^{D-1} \partial_v u_k \right] = - \frac{D - 5}{D - 2} e^{a\phi + 2(-u_0 + u_i - U)} \partial_u E \partial_v E, \quad \forall i = 2, 3, 4, \quad (12) \]

\[ e^{2(u_i - u_0)} \left[ 2 \partial_u \partial_v u_i + \partial_v u_i \sum_{k=2}^{D-1} \partial_u u_k + \partial_u u_i \sum_{k=2}^{D-1} \partial_v u_k \right] = \frac{3}{D - 2} e^{a\phi + 2(-u_0 + u_i - U)} \partial_u E \partial_v E, \quad \forall i = 5, \ldots, D - 1. \quad (13) \]

From now on, we restrict our discussions to \( D = 11 \) supergravity for simplicity, but it is straightforward to repeat our analysis for other theories given our field equations. The supersymmetry transformation for \( D = 11 \) supergravity without dilaton is

\[ \delta \psi_\mu = \left[ \partial_\mu + \frac{1}{4} \omega_{ab} \Gamma_{ab} + \frac{1}{288} (\Gamma_{\mu}^{\nu \rho \sigma} - 8 \delta_\mu^{\nu} \Gamma_{\rho \sigma}) F_{\nu \rho \sigma} \right] \zeta, \quad (14) \]

where \( \Gamma_{ab} \) are antisymmetrized gamma matrices and

\[ \omega_{\mu ab} \equiv \frac{1}{2} \epsilon^F_{\mu} \left( \partial_\mu e_{ab} - \partial_e e_{b\mu} \right) - \frac{1}{2} \epsilon^F_{\mu} \left( \partial_\mu e_{ab} - \partial_e e_{b\mu} \right) - \frac{1}{2} \epsilon^F_{\mu} e_{\rho}^c (\partial_\rho e_{cc} - \partial_e e_{c\rho}), \quad (15) \]

is the spin connection. The Killing spinor equations are obtained by setting these to zero:

\[ \delta \psi_u = \left[ \partial_u - \frac{1}{6} \gamma_{234} e^{-U} \partial_u E + \gamma_- \left( \frac{1}{2} \partial_u u_0 + \frac{1}{12} \gamma_{234} e^{-U} \partial_u E \right) \right] \zeta, \quad (16) \]

\[ \delta \psi_v = \left[ \partial_v - \frac{1}{6} \gamma_{234} e^{-U} \partial_v E - \gamma_- \left( \frac{1}{2} \partial_v u_0 + \frac{1}{12} \gamma_{234} e^{-U} \partial_v E \right) \right] \zeta, \quad (17) \]

\[ \delta \psi_i = \left[ \partial_i + \gamma_i e^{u_i - u_0} \left( \frac{1}{2} \partial_u u_i - \frac{1}{6} \gamma_{234} e^{-U} \partial_u E \right) \right. \]

\[ \left. + \gamma_i e^{u_i - u_0} \left( \frac{1}{2} \partial_v u_i - \frac{1}{6} \gamma_{234} e^{-U} \partial_v E \right) \right] \zeta, \quad i = 2, 3, 4, \quad (18) \]

\[ \delta \psi_i = \left[ \partial_i + \gamma_i e^{u_i - u_0} \left( \frac{1}{2} \partial_u u_i + \frac{1}{12} \gamma_{234} e^{-U} \partial_u E \right) \right. \]

\[ \left. + \gamma_i e^{u_i - u_0} \left( \frac{1}{2} \partial_v u_i + \frac{1}{12} \gamma_{234} e^{-U} \partial_v E \right) \right] \zeta, \quad i = 5, \ldots, 10, \quad (19) \]

where \( \gamma_- = \frac{1}{2} (\gamma_- - \gamma_+ - \gamma_-) \) and \( \gamma_\pm = \frac{1}{\sqrt{2}} (\gamma_0 \pm \gamma_1) \).

We look for supersymmetric solutions for which eqs. (16)–(19) all vanish. We assume that \( \zeta \) can depend only on \( u \) and \( v \). It then follows from eq. (19) that

\[ [\gamma_- (6 \partial_u u_i + \gamma_{234} e^{-U} \partial_u E) + \gamma_+ (6 \partial_v u_i + \gamma_{234} e^{-U} \partial_v E)] \zeta = 0. \quad (20) \]
Multiplying $\gamma_-$ or $\gamma_+$, we find that nontrivial solutions can be obtained only if either $\gamma_-\zeta = 0$ or $\gamma_+\zeta = 0$, unless all $u_i$ and $E$ are constant. We choose

$$\gamma_-\zeta = 0. \quad (21)$$

Then we have

$$\partial_v E = \partial_v u_i = 0, \quad (i = 2, \ldots, 10), \quad (22)$$

where the last condition for $i = 2, 3, 4$ follows from eq. (18). The $uv$ component of the Einstein eq. (10) then requires that

$$\partial_u \partial_v u_0 = 0. \quad (23)$$

This means that $u_0 = P(u) + Q(v)$. We could then set $u_0 = 0$ by a gauge choice, but we find it more instructive to keep the $u$-dependence; we take $u_0 = u_0(u)$ in what follows.

Note that eq. (21) means that $\gamma_-\zeta = -\zeta$. Eqs. (16) and (17) now reduce to

$$\delta\psi_u = \left[ \partial_u - \frac{1}{4} \gamma_{234} e^{-U} \partial_u E - \frac{1}{2} \partial_u u_0 \right] \zeta = 0,$$

$$\delta\psi_v = \partial_v \zeta = 0. \quad (24)$$

Let us define

$$\zeta = \exp \left[ \gamma_{234} h + \frac{1}{2} u_0(u) \right] \zeta_0, \quad (25)$$

where

$$\partial_u h = \frac{1}{4} e^{-U} \partial_u E, \quad \partial_v h = 0. \quad (26)$$

Note that the Majorana spinor condition $\zeta = C(\bar{\zeta})^T$ is satisfied if $\zeta_0$ is a Majorana spinor. Namely for $\zeta = e^{\gamma_{234} h} \zeta_1$ with a Majorana spinor $\zeta_1$, we have $C\bar{\zeta}^T = C(\zeta_1^T e^{\gamma_{234} h} i\gamma_0)^T = C\gamma_0^T e^{\gamma_{234} h} \gamma_0^T \zeta_1^T = -i\gamma_0 e^{-\gamma_{234} h} C\zeta_1^T = -e^{\gamma_{234} h} i\gamma_0 C\zeta_1^T = e^{\gamma_{234} h} \zeta_1 = \zeta$ where we have used the Majorana property of $\zeta_1$: $\zeta_1 = C\gamma_0^T \zeta_1^T = -i\gamma_0 C\zeta_1^T$. We also note that the integrability condition $(\partial_u \partial_v - \partial_v \partial_u) h = 0$ is satisfied due to (22). Then, eq. (24) is equivalent to

$$\partial_u \zeta_0 = \partial_v \zeta_0 = 0. \quad (27)$$
Hence, eq. (21) is the only relevant condition on the Killing spinor and the number of
remaining supersymmetry is \( \frac{1}{2} \).

The \( uu \) component of the Einstein eq. (9) reduces to

\[
\sum_{i=2}^{10} \partial_u^2 u_i + \sum_{i=2}^{10} (\partial_u u_i)^2 + \frac{1}{2} e^{-2U} (\partial_u E)^2 = 2 \partial_u u_0 \sum_{i=2}^{10} \partial_u u_i, \tag{28}
\]

and others are trivial after using the conditions (22). Because these are all functions of \( u \)
only, this equation is an ordinary differential equation for 11 functions \( u_i \) \( (i = 0, 2, \cdots, 10) \)
and \( E \). We can regard eq. (28) as determining \( E \) for given metrics. This in fact gives a
very large class of solutions in \( D = 11 \) supergravity, generalizing those given in Refs. [4, 5].

Given a solution to eq. (28), the metric for our spacetime is given in eq. (2). Upon
dimensional reduction to 10 dimensions, we get the string frame metric by

\[
ds_{11}^2 = e^{-2\phi/3} ds_{st}^2 + e^{4\phi/3} (dx^{10})^2, \tag{29}
\]

where

\[
ds_{st}^2 = e^{u_{10}} \left[ -2 e^{2u_0} du dv + \sum_{i=2}^{9} e^{2u_i} (dx^i)^2 \right], \quad \phi = \frac{3}{2} u_{10}. \tag{30}
\]

The Einstein frame metric is then obtained as

\[
ds_E^2 = e^{-\phi/2} ds_{st}^2
= e^{u_{10}/4} \left[ -2 e^{2u_0} du dv + \sum_{i=2}^{9} e^{2u_i} (dx^i)^2 \right]. \tag{31}
\]

This is our main result. The solutions we have obtained appear slightly different from
those in Ref. [3] though ours are more explicit. It can be checked that the apparent
difference is due to the different basis. We have confirmed that they are consistent if we
make a Lorentz transformation.

Now in order for the theory to be solvable, the metric in the string frame should be
of that type. The simplest case is when it is flat.

Let us now discuss some interesting cases.

\[\text{The dilaton is absent for } D = 11 \text{ supergravity.}\]
Case (i): Flat metric in the string frame.

To have a flat metric in the string frame, we find from eq. (30)

$$u_0 = u_i = -\frac{u_{10}}{2} \equiv f. \tag{32}$$

Substituting this into eq. (28), we get

$$f'' = -\frac{1}{12} e^{-6f} (E')^2. \tag{33}$$

If we do not introduce the background, this yields

$$f = cu, \tag{34}$$

where we have dropped one integration constant which can be absorbed into the rescaling of coordinates. This is the linear dilaton, and most general solution which gives the flat space in the string frame without any background. Note that this is by no means trivial theory in the Einstein frame as long as the dilaton is nontrivial.

If we introduce background, any function $f$ satisfying (33) is allowed. For instance, the next simplest case is a constant field strength:

$$E' = 6a. \tag{35}$$

We get from eq. (33)

$$e^{3f} = \frac{a}{2c} \left( \frac{1}{b} e^{3cu} - be^{-3cu} \right). \tag{36}$$

Setting $b = a/(2c)$, we reproduce the solution (34) in the $a \to 0$ limit. If we put $b = \pm 1$ and take the $c \to 0$ limit, we get another simple solution

$$f = \frac{1}{3} \ln(3a|u|). \tag{37}$$

Study of string theory in this background may not be difficult because it has flat metric in the string frame.

Case (ii): Linear dilaton in the light-like direction.

As a generalization of the simple linear dilaton background obtained above, we can consider

$$\phi = -Qu, \quad u_{10} = -\frac{2}{3}Qu. \tag{38}$$
If we also assume $u_0$ and $u_i$ are all linear in $u$,

$$u_i = c_i u, \quad (i = 0, 2, \ldots, 9), \quad (39)$$

(28) gives

$$\sum_{i=2}^{9} c_i^2 + \frac{4}{3} Q^2 - 2c_0 \left( \sum_{i=2}^{9} c_i - \frac{2}{3} Q \right) = -\frac{1}{2} (E')^2 e^{-2(c_2+c_3+c_4)u}. \quad (40)$$

If we do not introduce the background and take all $c_i$ equal, (40) leads to

$$c_i = \frac{1}{3} Q, \quad -\frac{1}{6} Q. \quad (41)$$

The first one coincides with the linear dilaton in the flat metric in the string frame obtained above for $c = \frac{1}{3} Q$, and is the linear dilaton background considered in Ref. [4]. However, the metric in the string frame is not trivial. The second one, also considered in Ref. [5], has nontrivial background in the string frame, but we might study the string theory in this background by Matrix theory counterpart. For general $c$, we still have a solution with background

$$E' = \pm \frac{2}{3} \sqrt{2(3c - Q)(6c + Q)} e^{3cu}. \quad (42)$$

Here we comment on the general but formal solutions of (33). Since both $E$ and $f$ are functions of $u$, we could consider $E$ as an implicit function of $f$. The general solution is then given as

$$u = \int df \exp \left[ \frac{1}{12} \int df e^{-6f} \left( \frac{dE}{df} \right)^2 \right]. \quad (43)$$

**Case (iii): Solutions with 2 functions.**

If we set

$$u_0 = u_2 = \cdots = u_{10-d} = f, \quad u_{11-d} = \cdots = u_{10} = g, \quad (44)$$

eq (28) gives

$$(9-d)(f'^2 - f'') + d(2f'g' - g'^2 - g'') = \frac{1}{2} e^{-6f} (E')^2, \quad (45)$$

where we have assumed $d \leq 6$. For $g = E' = 0$, we get $f'' - f'^2 = 0$ and $f = -\ln u$ (the integration constants can be absorbed to the shift of $u$ and rescaling of the coordinates) [5].
After a change of variable, this yields the nontrivial background similar to the orbifold discussed in Ref. [1].

If we do not consider orbifold, this solution \( f = -\ln u \) corresponds to flat space. We can have more general solutions of this type interpolating two asymptotically flat spaces. If we set

\[
u_0 = 0, \ u_2 = u_3 = u_4 = f, \ u_5 = \cdots = u_{10-d} = g, \ u_{11-d} = \cdots = u_{10} = 0,
\] (46)

eq. (28) gives

\[ 3e^{-f}(e^f)'' + (6 - d)e^{-g}(e^g)'' = -\frac{1}{2}e^{-6f}(E')^2. \] (47)

Note that for \( d \geq 1 \), the metrics in the string frame and in the Einstein frame are identical.

In particular, for \( f = g \), this reduces to

\[ e^{-f}(e^f)'' = -\frac{1}{2(9-d)}e^{-6f}(E')^2. \] (48)

Because a spacetime with \( e^f, e^g \propto u \) or constant is (locally) flat in the gauge \( u_0 = 0 \), any solution to this equation yields a spacetime that is asymptotically flat at \( u \to \pm \infty \), provided \( E' \) does not vanish except in a finite range with respect to \( u \) or falls off sufficiently rapidly as \( u \to \pm \infty \). Such solutions can be regarded as representing a decompactification transition, so they may deserve further investigations.

For example, the solution

\[ e^f = 1 + u - (1 + u^4)^{1/4}, \ (u > 0), \] (49)

is obtained for

\[ E' = \frac{\sqrt{6(9-d)u}}{(1 + u^4)^{7/8}} \{1 + u - (1 + u^4)^{1/4}\}^{5/2}, \] (50)

which behaves as

\[ E' \propto \begin{cases} u^{-5/2} & \text{for } u \to \infty, \\ u^{7/2} & \text{for } u \to 0. \end{cases} \] (51)

The Riemann curvature components with respect to the normalized null basis are proportional to

\[ f'' + (f')^2 = -\frac{3u^2}{(1 + u^4)^{7/4}[1 + u - (1 + u^4)^{1/4}]}, \] (52)
which behaves for $u \to \infty$ and $u \sim 0$ as

$$f'' + (f')^2 \simeq \begin{cases} 
-3u^{-5} & \text{for } u \to \infty, \\
-3u & \text{for } u \to 0.
\end{cases} \tag{53}$$

Hence, the solution is flat for $u \to \infty$ and $u = 0$, and as a consequence it can be smoothly extended to the completely flat region in $u \leq 0$.

**Case (iv): Solutions with NS-NS backgrounds.**

The solutions we discussed up to this point except for those without backgrounds involve RR backgrounds, which are not easy to deal with. If we compactify in the 4-th direction, however, we can get solutions with NS-NS 2-form $B$:

$$ds_{st}^2 = e^{u_4} \left[ -2e^{2u_0} du dv + \sum_{i=2(\neq 4)}^{10} e^{2u_i} (dx^i)^2 \right], \quad \phi = \frac{3}{2} u_4, \quad B = E dx^2 \wedge dx^3. \tag{54}$$

To have flat metric in the string frame, we find from eq. (54)

$$u_0 = u_i = -\frac{u_4}{2} \equiv f. \tag{55}$$

Substituting this into eq. (28), we get

$$f'' = -\frac{1}{12} (E')^2. \tag{56}$$

This yields further interesting class of solutions whose spectrum can be investigated with string actions with these backgrounds. In fact we could also get similar results for linear dilaton and solutions with 2 functions easily. In particular we can have constant NS-NS 2-form background together with linear dilatons. It is known that such a constant NS-NS background yields noncommutative theory [7]. It is extremely interesting to examine what implications these backgrounds may have on the singular behavior in the spacetime.

It would be also interesting to explore the possible application of our solutions to cosmological models. Whether our general solutions have interesting applications to these problems remains to be seen.

In summary, imposing the condition that there should be a null Killing spinor with all the metrics and background field strengths being functions of the light-cone coordinates, we have found general 1/2 BPS solutions in $D = 11$ supergravity, and discussed several
examples. In particular we have shown that the linear dilaton background is the most
general supersymmetric solution without background with the additional requirement
of flatness in the string frame. We have also given the most general solutions for flat
spacetime in the string frame with RR or NS-NS backgrounds, and they are characterized
by a single function $f$ or $E$ related as (33) or (56). The fact that the solutions are flat
in the string frame suggests that they could be put in the Matrix theory, which allows
nonperturbative study. Considering the difficulty in the study of string theories in time-
dependent backgrounds, it should be very interesting to study these problems. We hope
to report on these issues in the near future.

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