Neutrino mass in a gauged $L_\mu - L_\tau$ model

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Abstract

We study the origin of neutrino mass through lepton-number violation and spontaneous $U(1)_{L_\mu - L_\tau}$ symmetry breaking. To accomplish the purpose, in addition to the $U(1)_{L_\mu - L_\tau}$ extension of the standard model, we include one Higgs triplet, two singlet scalars, and two vector-like doublet leptons. To reduce the number of free parameters, we employ the Frampton-Glashow-Marfatia (FGM) two-zero texture neutrino mass matrix as a theoretical input. It is found that when some particular Yukawa couplings vanish, a FGM pattern can be realized in the model. Besides the explanation of neutrino data, we can also obtain the absolute value of neutrino mass $m_j$; and their sum can satisfy the upper bound from the cosmological measurement with $\sum_j |m_j| < 0.17$ eV. Moreover, the effective Majorana neutrino mass for neutrinoless double-beta decay is below the current upper limit, and is obtained as $\langle m_{\beta\beta} \rangle = (0.34, 2.3) \times 10^{-2}$ eV. It is found that the doubly charged Higgs $H^{\pm\pm}$ decaying to the right-handed $\mu^{\pm\tau\pm}$ is induced from a dimension-6 operator and not highly suppressed, and its branching ratio is compatible with the $H^{\pm\pm} \rightarrow W^\pm W^\pm$ decay when the vacuum expectation value of Higgs triplet is $O(0.1)$ GeV.

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In spite of the mass hierarchy among the quarks and charged leptons, the particle masses, except the neutrinos, in the standard model (SM) can be attributed to the Brout-Englert-Higgs (BEH) mechanism \([1, 2]\), where the predicted Higgs boson was observed by ATLAS \([3]\) and CMS \([4]\) at a mass of 125 GeV. Based on the neutrino oscillation experiments, it was found that the neutrinos are also massive particles; however, the definite origin of their masses so far is unknown.

Moreover, although nonzero neutrino masses have been determined by the experiments, we still cannot tell their mass ordering, i.e., \(m_1 < m_2 < m_3\) or \(m_3 < m_1 < m_2\) is possible, where the former and latter are the mass spectrum with normal ordering (NO) and inverted ordering (IO), respectively. Hence, the current neutrino data can be shown in terms of the different mass ordering as \([5]\):

\[
\Delta m^2_{21} = (7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2, \quad \sin^2 \theta_{12} = 0.304 \pm 0.014,
\]
\[
\Delta m^2_{32} = (2.44 \pm 0.06, 2.51 \pm 0.06) \times 10^{-3} \text{ eV}^2 \text{ (NO, IO)},
\]
\[
\sin^2 \theta_{23} = (0.51 \pm 0.05, 0.50 \pm 0.05) \text{ (NO, IO)},
\]
\[
\sin^2 \theta_{13} = (2.19 \pm 0.12) \times 10^{-2},
\]

where \(m^2_{21} \equiv m^2_2 - m^2_1, \) \(m^2_{32} \) denotes \(m^2_3 - m^2_2 \) for NO or \(m^2_2 - m^2_3 \) for IO, and \(\theta_{ij}\) are the mixing angles of Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix \([6, 7]\). From the results, it is clearly seen that the PMNS matrix pattern is different from the Cabibbo-Kobayashi-Maskawa (CKM) for the quark-flavor mixing. In this work, we plan to study a model, where based on a flavor symmetry, the neutrino masses are dynamically generated without introducing singlet right-handed neutrinos, and all neutrino data can be explained. In addition, the model can also have interesting phenomenological implications on flavor and collider physics.

Inspired by the experimental indication of maximal \(\theta_{23}\), large \(\theta_{12}\), and small \(\theta_{13}\), various Abelian flavor-symmetry based models were proposed to understand the neutrino properties \([8–18]\). Among these flavor symmetries, we investigate the neutrino problems in an \(U(1)_{L_\mu - L_\tau}\) gauge symmetry. We focus on such gauge symmetry based on some phenomenological considerations, such as (i) gauge anomaly free is automatically satisfied \([19, 20]\); (ii) excess of muon anomalous magnetic dipole moment (muon \(g - 2\)) can be resolved \([21, 23]\); (ii) excesses in semileptonic \(B\)-meson decays can be explained \([24, 27]\); (iii) potential signals for the processes \(e^+e^- \rightarrow \gamma Z'\) \([28, 29]\) and \(\tau \rightarrow \mu Z'Z'\) \([30]\) can be observed at Belle II. Other
interesting studies can be found in \[31–38\].

In order to dynamically generate the neutrino masses, we require that each Majorana matrix entry has to be related to the effects of which arises from the lepton-number violation and spontaneous \(U(1)_{\mu - \tau}\) symmetry breaking. To achieve the lepton-number violation, we introduce a Higgs triplet, which carries hypercharge \(Y = 1\) and has no \(U(1)_{\mu - \tau}\) charge. Like the type-II seesaw \[39, 40\], the vacuum expectation value (VEV) of this triplet can dictate the lepton-number violating effects. It is found that due to the protection of \(U(1)_{\mu - \tau}\) gauge symmetry, we cannot obtain a realistic Majorana neutrino mass matrix without further discussing the \(U(1)_{\mu - \tau}\) breaking. Therefore, to break the gauge symmetry, we employ two singlet scalars, which carry different \(U(1)_{\mu - \tau}\) charges. Due to the chirality, the SM leptons cannot couple to the singlet scalars; therefore, we need to introduce proper exotic heavy leptons as the media. To avoid the anomaly cancellation, we employ two vector-like doublet leptons as the candidates. Based on the \(U(1)_{\mu - \tau}\) gauge symmetry, the number of singlet scalars and vector-like leptons (VLLs) in this approach is the minimal requirement to obtain a proper Majorana neutrino mass matrix.

It will be demonstrated later that not all Yukawa couplings appearing in the neutrino mass matrix are small. Therefore, in addition to the neutrino issue, the model can also provide interesting phenomena on flavor and collider physics. For instance, the lepton-flavor violating \(h \rightarrow \mu \tau\) decay can be as large as the current measurements; the excess of muon \(g - 2\) can be resolved by the mediation of \(Z'\) gauge boson and new light scalars; and the doubly charged Higgs decaying to \(\mu \tau\) and \(WW\) can be compatible each other without requiring the VEV of Higgs triplet to be the eV.

In the following, we start to introduce the model under the \(SU(2)_L \times U(1)_Y \times U(1)_{\mu - \tau}\) local gauge symmetry. In order to dynamically generate the neutrino mass in the \(U(1)_{\mu - \tau}\) extension of the SM, in addition to the SM particles, we include one Higgs triplet \((\Delta)\), two vector-like doublet leptons \((L_4, L_5)\), and two singlet scalars \((S, S')\). Their \(U(1)_{\mu - \tau}\) charges are given in Table I where the SM particles not shown in the table carry no such \(U(1)\) charges. Accordingly, the Yukawa couplings to the Higgs triplet are written as:

\[
-\mathcal{L}_Y^\Delta = \frac{1}{2} Y_{\mu e} L_e^T C i \tau_2 \Delta L_\mu + Y_{\mu \tau} L_\mu^T C i \tau_2 \Delta L_\tau + Y_{\tau 4} L_4^T C i \tau_2 \Delta L_4 + Y_{\tau 5} L_5^T C i \tau_2 \Delta L_5 + Y_{45} L_{4L}^T C i \tau_2 \Delta L_{5L} + Y'_{45} L_{4R}^T C i \tau_2 \Delta L_{5R} + H.c. \tag{2}
\]

From above equation, if the Higgs triplet \(\Delta\) carry two units of lepton number, the Yukawa
interactions are lepton-number conserved. However, when the Higgs triplet obtains a VEV, i.e. $\langle \Delta \rangle = v_\Delta / \sqrt{2}$, the lepton-number violating Majorana neutrino mass matrix for three light neutrinos is induced and expressed as:

$$
M^\nu = \begin{pmatrix}
\frac{Y_{ee}v_\Delta}{\sqrt{2}} & 0 & 0 \\
0 & 0 & \frac{Y_{\mu\tau}v_\Delta}{\sqrt{2}} \\
0 & \frac{Y_{\mu\tau}v_\Delta}{\sqrt{2}} & 0
\end{pmatrix},
$$

where the pattern of mass matrix leads to $m_2 = m_3$, $\theta_{13} = \theta_{12} = 0$, and $\theta_{23} = \pi/4$ [8, 16, 17]. Obviously, the results cannot explain the current neutrino data [5]. We clearly demonstrate that the neutrino mass matrix, which is arisen from the electroweak symmetry breaking and lepton-number violation, cannot explain the neutrino data due to the protection of $U(1)_{L_\mu - L_\tau}$ gauge invariance. In order to obtain a realistic neutrino mass matrix, we need to rely on other pieces of Yukawa interactions, which can break the $U(1)$ symmetry. Concerning the magnitude of $v_\Delta$, according to the electroweak symmetry breaking, the electroweak $\rho$-parameter at the tree-level can be written as [41]:

$$
\rho = \frac{m_W^2}{m_Z^2 c_W^2} = \frac{1 + 2v_\Delta^2/v_H^2}{1 + 4v_\Delta^2/v_H^2}.
$$

Taking the current precision measurement for $\rho$-parameter within 2$\sigma$ errors, the VEV of $\Delta$ has to be less than 3.4 GeV.

Besides Eq. (2), the gauge invariant Yukawa couplings to the Higgs and $S^{(0)}$ are given by:

$$
-L_Y = Y_{\ell} \bar{\ell} H \ell_R + y_\mu \bar{L}_\mu H \mu_R + y_\tau \bar{L}_\tau H \tau_R + y_{\mu'} \bar{L}_\mu L_{AR} S + y_{\tau'} \bar{L}_\tau L_{5R} S^\dagger \\
+ y_e \bar{L}_e L_{AR} S^\dagger + y_{e'} \bar{L}_e L_{5R} S^\dagger + y_S \bar{L}_5 L_{AR} S + y_{5'} \bar{L}_5 L_{5R} S^\dagger \\
+ m_{4L} \bar{L}_{4L} L_{4R} + m_{5L} \bar{L}_{5L} L_{5R} + H.c.,
$$

where $H$ is the SM Higgs doublet, only the first term is from the SM, and the other Yukawa interactions arise from the new particles. Although Eq. (5) can cause rich interesting phenomena on the lepton-flavor physics, since we focus on the neutrino physics in this work,
the detailed study on the flavor physics can be referred to [30]. Based on the Yukawa interactions in Eq. (5), it is found that the new entries of the Majorana mass matrix can be induced from higher dimensional operators, where the Feynman diagrams are sketched in Fig. 1 and the associated gauge invariant dimension-5 and -6 operators can be formulated as:

\[ -\mathcal{L}_\mu \supset \frac{Y_{\mu 4}y_*^s}{m_{4L}}L^T_\mu C\bar{\Delta}L_\mu S^\dagger + \frac{Y_{\tau 5}y_{\tau}^*}{m_{5L}}L^T_\tau C\bar{\Delta}L_\tau S + \frac{Y_{45}^s(y_e y_{\tau}^*)^s}{m_{4L}m_{5L}}L^T_\mu C\bar{\Delta}L_e S^\dagger S^\dagger \]

\[ + \frac{y_e y_{\tau}^* y_{5}^*}{m_{4L}m_{5L}}L^T_\mu C\bar{\Delta}L_\mu S^\dagger S^\dagger + \frac{y_e y_{\tau}^* y_{\tau}^*}{m_{4L}m_{5L}}L^T_\tau C\bar{\Delta}L_\tau SS^\dagger \]

\[ + \frac{(Y_{45}^s + Y_{\tau 5}^s y_{\tau}^*)}{m_{4L}m_{5L}}L^T_\mu C\bar{\Delta}L_\mu SS^\dagger + H.c. \]  

(6)

with $\bar{\Delta} = i\tau_2\Delta$. From the effective Lagrangian, when the $U(1)_{L_\mu - L_\tau}$ gauge symmetry is spontaneously broken by $\langle S \rangle = v_S/\sqrt{2}$ and $\langle S' \rangle = v_{S'}/\sqrt{2}$, the vanishing elements in Eq. (3) can be generated from Eq. (6) with $\langle \Delta \rangle = v_\Delta/\sqrt{2}$. We note that the dimension-6 operator $L^T_\mu C\bar{\Delta}\bar{\Delta}^\dagger\Delta L_\tau$ has been dropped due to $v_\Delta \ll v_S, v_{S'}$.

\[ \mathcal{V} = m^2_H H^\dagger H + m^2_{\Delta} \text{Tr}[\Delta^\dagger \Delta] + m^2_{S'} S'^\dagger S' + m^2_S S^\dagger S + \mu_\Delta[H^T(i\tau_2)\Delta^\dagger H + h.c.] \]

\[ + \mu_S[S'S'^\dagger S'^\dagger + h.c.] + \lambda_1[H^\dagger H]^2 + \lambda_2(\text{Tr}[\Delta^\dagger \Delta])^2 + \lambda_3 \text{Tr}[(\Delta^\dagger \Delta)^2] + \lambda_4[S'^\dagger S']^2 \]

\[ + \lambda_5[S'^\dagger S]^2 + \lambda_6(H^\dagger H)\text{Tr}[\Delta^\dagger \Delta] + H^\dagger (\lambda_7\Delta^\dagger \Delta + \lambda_8\Delta^\dagger \Delta) H + \lambda_9(S'^\dagger S')(H^\dagger H) \]

\[ + \lambda_{10}(S'^\dagger S)(H^\dagger H) + \lambda_{11}(S'^\dagger S')\text{Tr}[\Delta^\dagger \Delta] + \lambda_{12}(S^\dagger S)\text{Tr}[\Delta^\dagger \Delta] + \lambda_{13}(S'^\dagger S')(S^\dagger S). \]  

(7)
The VEVs of scalar fields are obtained by the minimal conditions \( \partial \langle V \rangle / \partial v_{H,S,S'} \Delta = 0 \), and each condition can be expressed as:

\[
\frac{\partial \langle V \rangle}{\partial v_H} \simeq m_H^2 v_H + \lambda_1 v_H^3 + \frac{1}{2} \lambda_9 v_S^2 v_H + \frac{1}{2} \lambda_{10} v_S^2 v_H \simeq 0, \tag{8}
\]

\[
\frac{\partial \langle V \rangle}{\partial v_S} \simeq m_S^2 v_S + \frac{1}{2} \lambda_S v_S^2 + \lambda_5 v_S^3 + \frac{1}{2} \lambda_{10} v_H^2 v_S + \frac{1}{2} \lambda_{13} v_S^2 v_S \simeq 0, \tag{9}
\]

\[
\frac{\partial \langle V \rangle}{\partial v_{S'}} \simeq m_{S'}^2 v_{S'} + \frac{1}{2} \lambda_{S'} v_{S'}^2 + \frac{1}{2} \lambda_{13} v_{S'}^2 v_{S'} \simeq 0, \tag{10}
\]

\[
\frac{\partial \langle V \rangle}{\partial v_{\Delta}} \simeq m_{\Delta}^2 v_{\Delta} + \frac{1}{2} \mu_{\Delta} v_H^2 + \frac{1}{2} (\lambda_6 + \lambda_7) v_H^2 v_{\Delta} + \frac{1}{2} \lambda_{11} v_{S'}^2 v_{\Delta} + \frac{1}{2} \lambda_{12} v_{S'}^2 v_{\Delta} \simeq 0, \tag{11}
\]

where we have ignored the \( v_{\Delta} \) terms in the first three equations and the \( v_{\Delta}^3 \) terms in the last equation due to \( v_{\Delta} \ll v_{H,S,S'} \). In order to avoid the precision Higgs measurements, we can adopt the mixing between \( H \) and \( S(S') \) to be small; then, the VEV of \( H \) can be simplified as \( v_H \simeq \sqrt{-m_H^2}/\lambda_1 \). If we further take \( \lambda_{13} \) and \( \mu_S \) to be small, the VEVs of \( S \) and \( S' \) can be found as \( v_S \simeq \sqrt{-m_S^2}/\lambda_5 \) and \( v_{S'} \simeq \sqrt{-m_{S'}^2}/\lambda_4 \) with \( m_{S,S'}^2 < 0 \). The \( v_S \) and \( v_{S'} \) are free parameters and their relation to the \( Z' \)-boson mass is given by \( m_{Z'}^2 = g_2^2 (4v_S^2 + v_{S'}^2) \); hence, their magnitudes can be taken as the value of the electroweak scale. From Eq. (11), the VEV of Higgs triplet can be determined as:

\[
v_{\Delta} \simeq - \frac{1}{\sqrt{2}} \frac{\mu_{\Delta} v_H^2}{\lambda_6 + \lambda_7} + \frac{1}{\sqrt{2}} \frac{m_{\Delta} v_{\Delta}^2}{\lambda_{11} v_S^2/2 + \lambda_{12} v_{S'}^2/2}. \tag{12}
\]

Because of \( v_{\Delta} < 3.4 \text{ GeV} \), in order to obtain the heavy Higgs triplet bosons, unlike Higgs doublet and \( S(S') \), \( m_{\Delta}^2 \) has to be positive and dictates the masses of Higgs triplet bosons. From Eq. (12), it can be seen that similar to the type-II seesaw model [39, 40], the Higgs triplet VEV is directly related to the lepton-number soft breaking term.

If we write the symmetric Majorana neutrino mass matrix as:

\[
M^\nu = \begin{pmatrix}
    m_{ee} & m_{e\mu} & m_{e\tau} \\
    m_{e\mu} & m_{\mu\mu} & m_{\mu\tau} \\
    m_{e\tau} & m_{\mu\tau} & m_{\tau\tau}
\end{pmatrix}, \tag{13}
\]

from the Yukawa couplings in Eqs. (2) and (6), each matrix element can then be expressed as:

\[
m_{ee} = \frac{Y_{ee} v_{\Delta} v_S v_{S'}}{\sqrt{2}} + \frac{Y'_{ee} y'_{e\mu} (y'_{e\mu})^* v_{S'} v_{S'} v_{\Delta}}{2\sqrt{2} m_{4L} m_{5L}}, \]

\[
m_{e\mu} = \frac{Y_{e\mu} y_{e\mu} v_{S} v_{\Delta}}{2 m_{4L}}, \quad m_{\mu\tau} = \frac{Y_{\mu\tau} y_{\mu\tau} v_{S} v_{\Delta}}{\sqrt{2}}, \quad m_{\tau\tau} = \frac{Y_{\tau\tau} y_{\tau\tau} v_{S} v_{\Delta}}{2 m_{5L}}, \tag{14}
\]
with \( \eta = Y_{\mu} y_{S}^{\prime} y_{\tau}^{\ast} + Y_{\tau} y_{S} y_{\mu}^{\ast} + (Y_{45} + Y_{45}^{\prime}) y_{\nu}^{\ast} y_{\mu}^{\ast} \). Although the neutrino mass matrix comes from the dimension-4, -5, and -6 operators, since the involving free parameters are different, the matrix entries in Eq. (14) can be taken as the same in the order of magnitude and have no particular hierarchy among them, unless there is a further indication. Due to the \( U(1)_{L_{\mu}-L_{\tau}} \) gauge symmetry, the light charged-lepton mass matrix in the first term of Eq. (5) has been diagonal. Although the other Yukawa interactions can induce the off-diagonal elements, these induced terms indeed are suppressed [30]. If we neglect these small off-diagonal effects as a leading approximation, the Majorana neutrino mass matrix can be diagonalized by the PMNS matrix as \( M'_{\text{dia}} = \text{diag}(\bar{m}_{1}, \bar{m}_{2}e^{i\alpha_{21}}, \bar{m}_{3}e^{i\alpha_{31}}) = U^{\dagger} M' U, \) where \( \alpha_{21,31} \) are the Majorana CP violating phases, the PMNS matrix is defined by \( U = U_{\nu} U_{\ell}^{\dagger} \approx U_{\nu} \) with \( U_{\nu,\ell} \) the unitary matrices for diagonalizing the charged-lepton and neutrino mass matrices, and the standard parametrization of PMNS matrix is given as [5]:

\[
U = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}c_{23}s_{13}e^{i\delta} & 0 \\
    s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix} \tag{15}
\]

with \( s_{ij} \equiv \sin \theta_{ij}, c_{ij} \equiv \cos \theta_{ij} \), and \( \delta \) being the Dirac CP violating phase.

From Eq. (13), there are six different complex matrix elements. After rotating three unphysical phases, we have nine independent parameters. Since neutrino oscillation experiments cannot observe the two Majorana CP phases, even \( \alpha_{21} = \alpha_{31} = 0 \), we still have seven free parameters. However, we only have six observables: \( \Delta m_{21,31}^{2}, \sin^{2} \theta_{12,13,23}, \) and Dirac CP phase \( \delta \); that is, we cannot determine all free parameters without further theoretical or experimental inputs. It has been investigated that a class of neutrino mass matrices may suffice to explain all neutrino experiments if the matrix textures have two independent zeroes [42]. The seven possible Frampton-Glashow-Marfatia (FGM) matrix patterns are classified as:

\[
\begin{align*}
A_{1} : & \begin{pmatrix} 0 & 0 & X \\ 0 & X & X \\ X & X & X \end{pmatrix}, \\
A_{2} : & \begin{pmatrix} 0 & X & 0 \\ X & X & X \\ 0 & X & X \end{pmatrix}, \\
B_{1} : & \begin{pmatrix} X & X & 0 \\ X & 0 & X \\ 0 & X & X \end{pmatrix}, \\
B_{2} : & \begin{pmatrix} X & 0 & X \\ 0 & X & X \\ X & X & 0 \end{pmatrix}, \\
B_{3} : & \begin{pmatrix} 0 & 0 & X \\ 0 & X & 0 \\ X & X & X \end{pmatrix}, \\
B_{4} : & \begin{pmatrix} X & X & 0 \\ X & 0 & X \\ 0 & X & X \end{pmatrix}, \\
C : & \begin{pmatrix} X & 0 & X \\ X & X & 0 \\ X & 0 & X \end{pmatrix} \tag{16}
\end{align*}
\]
TABLE II: Vanishing Yukawa (VY) couplings to realize the FGM two-zero textures in the model.

| Pattern | A₁ | A₂ | B₁ | B₂ |
|---------|----|----|----|----|
| VY      | \((Y_{ee}, y'e) \approx 0\) | \((Y_{ee}, y'e) \approx 0\) | \((y'e, Y_{\mu 4}) \approx 0\) | \((y'e, Y_{\tau 5}) \approx 0\) |
| Pattern | B₃ | B₄ | C  |    |
| VY      | \((y'e, Y_{\mu 4}) \approx 0\) | \((y'e, Y_{\tau 5}) \approx 0\) | \((Y_{\mu 4}, Y_{\tau 5}) \approx 0\) |    |

where the symbol \(X\) denotes a nonzero texture. The detailed study with two-zero textures can be found in [43–45]. In order to simplifying the analysis, it is a good approach to employ the FGM patterns as theoretical inputs.

As mentioned earlier that the neutrino mass ordering is still uncertain, i.e. \(m_1 < m_2 < m_3\) or \(m_3 < m_1 < m_2\) is allowed. With a FGM pattern, it helps understand what form of a neutrino mass matrix can lead to a certain mass ordering. According to the study in [46], it was concluded that by taking the neutrino data with 1σ errors, the NO spectrum can be achieved by the patterns \(A_{1,2}\) and \(B_{1,2,3,4}\), while the patterns \(B_{1,3}\) and \(C\) can conduct to IO spectrum. Accordingly, it is of interest to see how the matrix elements of Eq. (14) in our model realize each FGM matrix. It is found that when some Yukawa couplings are required to vanish, a definite FGM matrix pattern can then be accomplished. We show the vanishing Yukawa couplings for the corresponding FGM matrix in Table II. It is worth mentioning that the powerful FGM matrix pattern can also predict the absolute values of neutrino masses, where they so far have not yet observed in experiments.

Since our purpose is not to examine the all FGM patterns, for numerical analysis, we take the patterns \(A_1\) and \(C\) as the representatives of NO and IO mass spectra, respectively. To determine the non-vanishing entries of the neutrino mass matrix and \(|m_i|\), we use the neutrino data at the 1σ level as shown in Eq. (1). Due to the large experimental uncertainty, the values of Dirac CP phase are taken from a global data analysis with a \(\chi^2\) method [48], in which the results in the 1σ region are \(\delta/\pi = (1.18, 1.61)\) for NO and \(\delta/\pi = (1.12, 1.62)\) for IO. Combining the experimental inputs with two independent zero textures, we basically have eight known inputs; thus, we can completely constrain the four non-vanishing complex entries of the patterns \(A_1\) and \(C\).

According to the relation \(M^\nu = U^*M^\nu_{\text{dia}}U^\dagger\) and the zero textures in the \(M^\nu\), the mass
relations in the pattern $A_1$ can be expressed as:
\[ m_1^* \equiv \frac{U_{13}}{U_{11}} \left( \frac{U_{12}U_{23} - U_{13}U_{22}}{U_{11}U_{22} - U_{12}U_{21}} \right) m_3^*, \]
\[ m_2^* \equiv -\frac{U_{13}}{U_{12}} \left( \frac{U_{11}U_{23} - U_{13}U_{21}}{U_{11}U_{22} - U_{12}U_{21}} \right) m_3^*, \]
while in the pattern $C$, they are:
\[ m_1^* \equiv \frac{U_{22}U_{23}^2 - U_{23}^2 U_{32}^2}{U_{21}U_{32}^2 - U_{22}U_{31}^2} m_3^*, \]
\[ m_2^* \equiv -\frac{U_{22}U_{23}^2 - U_{23}^2 U_{32}^2}{U_{21}U_{32}^2 - U_{22}U_{31}^2} m_3^*, \]
where $m_k$s in general are complex; however, there are only two independent phases among $m_{1,2,3}$. If we take the central values of measured $\theta_{12,13}$ in Eq. (11), $\sin^2 \theta_{23} \approx 0.47$, and $\delta \approx 1.3\pi$, we can easily obtain:
\[
A_1 : \begin{cases} 
  m_1/m_3 \approx 0.022 + i 0.233, \\
  m_2/m_3 \approx 0.022 - i 0.102, \\
  |m_2|^2 - |m_1|^2 \approx 0.029|m_3|^2. 
\end{cases}
\]
However, it is found that the pattern $C$ is very sensitive to the values of mixing angles and CP phase $\delta$ when $\Delta m^2_{21}$ and $\Delta m^2_{32}$ are required to fit the data within 1$\sigma$ errors. If we take $\sin^2 \theta_{23} \approx 0.4515$ and $\delta \approx 1.59205\pi$, the results are:
\[
C : \begin{cases} 
  m_1/m_3 \approx -0.716 - i 0.955, \\
  m_2/m_3 \approx 1.124 - i 0.417, \\
  |m_2|^2 - |m_1|^2 \approx 0.0130|m_3|^2. 
\end{cases}
\]
When we further take $\Delta m^2_{21} \approx 7.53 \times 10^{-5}$ eV$^2$, the values of $|m_i|$ and $\Delta m^2_{23}$ can be determined as:
\[
A_1 : \begin{cases} 
  |m_1| \approx 5.5 \times 10^{-3} \text{ eV}, \\
  |m_2| \approx 1.03 \times 10^{-2} \text{ eV}, \\
  |m_3| \approx 5.06 \times 10^{-2} \text{ eV}, \\
  \Delta m^2_{32} \approx 2.45 \times 10^{-3} \text{ eV}^2; \end{cases} \quad C : \begin{cases} 
  |m_3| \approx 7.60 \times 10^{-2} \text{ eV}, \\
  |m_1| \approx 9.07 \times 10^{-2} \text{ eV}, \\
  |m_2| \approx 9.11 \times 10^{-2} \text{ eV}, \\
  \Delta m^2_{23} \approx 2.53 \times 10^{-3} \text{ eV}^2. \end{cases}
\]
From the analysis, the pattern $A_1$ and $C$ can fit the neutrino data for NO and IO mass spectra at the 1$\sigma$ level, respectively. However, if we compare the results with the cosmological limit on the sum of neutrino masses, which is given as [47]:
\[
\sum_\nu m_\nu < 0.17 \text{ eV}, \quad (22)
\]
it can be found that the resulting ∑|mj| in the pattern A_1 can satisfy the upper bound while that in the pattern C is higher than the limit. In order to understand whether the tension with the cosmological neutrino mass bound can be relaxed when the ranges of experimental measurements are extended, we adopt neutrino data at the 2σ level instead of 1σ level for the pattern C. For numerical analysis, we generate 5 × 10^8 sampling points by randomly selecting the experimental values of s_{12,23,13} and δ within 1(2)σ errors, and the values of m_1 in the range of [0.07, 0.17] eV; then, m_2 and m_3 are obtained via Eq. (18). In the end, the number of output points, which can fit the Δm_{21}^2 data in 1(2)σ range, is 552(3004). The resulting Dirac CP phase δ and ∑|mj| are shown in Fig. 2, where the dots in black and red denote the results with 1σ and 2σ errors, respectively. From the figure, it can be clearly seen that ∑|mj| in the pattern C can still satisfy the bound from the cosmological measurements when all neutrino data are taken at the 2σ level.

![Fig. 2: Scatter plot for the Dirac CP phase and ∑|mj|, where the dots in black and red denote the neutrino data with 1σ and 2σ errors, respectively.](image)

Since the uncertainties of sin^2θ_{23} and Δm_{32}^2 in Eq. (1) correspond to a 68% confidence level (CL), and the measurement of sin^2θ_{13} is associated with the value of m_{32}^2, in our following analysis we only use the pattern A_1, which can fit the neutrino data within 1σ errors, to show the constraints of the involving Yukawa couplings. From the mass diagonal relation \( M_{\ell\ell}' = (U_{tk}U_{ek})^* m_k \), when the PMNS matrix entries and m_k are known, \( M_{\ell\ell}' \) can then be determined. Following earlier discussions, where the FGM pattern A_1 can predict the value of each m_j, we therefore show the correlation between δ and |mj| in Fig. 3(a),
where the neutrino data within 1σ error have been satisfied. From the plot, it can be seen that each $|m_i|$ is located at around the value of Eq. (21) with a very narrow range. In the plot, we also show the effective Majorana neutrino mass $\langle m_{\beta\beta} \rangle$, which is related to the neutrinoless double-beta ($0\nu\beta\beta$) decay rate, and is defined by [18]:

$$\langle m_{\beta\beta} \rangle = \left| \sum_k U_{ek}^2 m_k \right|,$$  

(23)

where a 90% CL upper limit of $\langle m_{\beta\beta} \rangle < 0.061 - 0.165$ eV was obtained by the KamLAND-Zen collaboration [49]. Our result of $\langle m_{\beta\beta} \rangle \approx (0.34, 2.3) \times 10^{-2}$ eV clearly satisfies the bound. Following the results, we show the allowed ranges for $|m_{ij}|$ as a correlation of $|m_{\tau\tau}|$ in Fig. 3(b), where similar to Fig. 2 we use $10^7$ sampling points to generate the experimental inputs and set $m_1 \in [0.001, 0.1]$ eV. Accordingly, the magnitude ranges of $m_{ij}$ are given as:

$$m_{e\tau} = (0.99, 1.11) \times 10^{-2} \text{eV}, \quad m_{\mu\mu} = (2.5, 3.0) \times 10^{-2} \text{eV},$$

$$m_{\mu\tau} = (2.2, 2.4) \times 10^{-2} \text{eV}, \quad m_{\tau\tau} = (2.4, 2.8) \times 10^{-2} \text{eV}.$$

(24)

FIG. 3: (a) Predicted $|m_j|$ and effective Majorana neutrino mass for the $0\nu\beta\beta$ decay; and (b) allowed ranges for $m_{ij}$ as a correlation of $|m_{\tau\tau}|$, where FGM pattern $A_1$ is applied and neutrino data within 1σ errors are taken.

We now discuss the limits on the Yukawa couplings in Eqs. (2) and (5). To simplify the analysis, we take $m_{4L} \approx m_{5L} \equiv m_L$ and $v_S \approx v_{S'} \equiv v_X$, and define the parameters as:

$$a_L = \frac{y_r^* y_{1L}^* v_X}{2m_L}, \quad a_R = \frac{y_r y_{1R} v_X}{2m_L}, \quad \xi_{ab} = \frac{Y_{ab} v_\Delta}{\sqrt{2}},$$

(25)
where \( a_{R,L} \) can lead to the Higgs lepton-flavor violating \( h \to \mu \tau \) decay, and its branching ratio (BR) is given by (30):

\[
BR(h \to \mu \tau) = \frac{|a_L|^2 + |a_R|^2}{8\pi \Gamma_h} m_h. \tag{26}
\]

With \( m_h \approx 125 \text{ GeV} \) and \( \Gamma_h \approx 4.21 \text{ MeV} \), the limit on \( a_{L,R} \) can be obtained as

\[
\sqrt{|a_L|^2 + |a_R|^2} \approx 2.67 \times 10^{-3} \sqrt{\frac{BR(h \to \mu \tau)}{0.84 \times 10^{-2}}}. \tag{27}
\]

where \( BR(h \to \mu \tau) \) can be taken from the experimental data, and the current upper limits from ATLAS and CMS are 1.43% \([50]\) and 1.26% \([51, 52]\), respectively. Taking \( BR(h \to \mu \tau) \sim 10^{-3} \) and \( |a_L| \sim |a_R| \), we obtain \( |a_{L,R}| \sim 6.5 \times 10^{-4} \). Based on the new parameters, the neutrino mass matrix entries in Eq. (14) are expressed as:

\[
m_{\mu \tau} = \left( \frac{y_e}{y_{\mu}} \right)^* \frac{v_X}{m_L} a_L \xi_{45}, \quad m_{\mu \mu} = \frac{\sqrt{2}}{y_{\tau}} a_R \xi_{45}, \quad m_{\tau \tau} = \frac{\sqrt{2}}{y_{\tau}} a_L \xi_{55},
\]

\[
m_{\mu \tau} = \xi_{\mu \tau} + \frac{y_e}{y_{\mu} y_{\tau}} \frac{v_X}{m_L} a_L \xi_{45} + \frac{y_S}{y_{\tau}} \frac{v_X}{m_L} a_R \xi_{55} + \frac{2}{y_{\tau} y_{\mu}} a_L a_R^{*} (\xi_{45} + \xi_{45}') \tag{28}
\]

According to Eq. (24), if we take \( |m_{\mu \mu}| \approx |m_{\tau \tau}| \sim 2.7 \times 10^{-2} \text{ eV} \), \( |a_{L,R}| \sim 6.5 \times 10^{-4} \), the magnitudes of parameters are obtained as:

\[
\left| \frac{\xi_{\mu \mu}}{y_{\tau}} \right| \approx \left| \frac{\xi_{\tau \tau}}{y_{\mu} y_{\tau}} \right| \sim 2.9 \times 10^{-8} \text{ GeV}. \tag{29}
\]

Assuming \( \xi_{45} \approx -\xi_{45}' \) and taking \( |y_{\mu}^{(t)}| \approx |y_{\tau}^{(t)}| \approx 0.1 \), \( v_X \sim 100 \text{ GeV} \), and \( m_L \sim 1000 \text{ GeV} \), which can lead to a sizable \( BR(h \to \mu \tau) \), the magnitudes of \( \xi_{45}^{(t)} \) can be shown as:

\[
|y_{e}^{(t)}| \xi_{45}' \sim 1.5 \times 10^{-8} \text{ GeV} \text{, } |\xi_{\mu \mu}| \approx |\xi_{\tau \tau}| \sim 2.9 \times 10^{-9} \text{ GeV} \text{, } |\xi_{\mu \tau}| \sim 2.3 \times 10^{-11} \text{ GeV} \text{,} \tag{30}
\]

where \( |m_{\mu \tau}| = 10^{-2} \text{ eV} \) and \( |m_{\mu \tau}| = 2.3 \times 10^{-2} \text{ eV} \) are used, and the second and third terms in \( m_{\mu \tau} \) have been neglected due to \( y_S, y_{S}' \ll 1 \). To avoid the strict constraint from \( \mu \to e\gamma \) and \( \mu \to 3e \) rare decays, we can take \( y_e \ll 1 \); thus, the Yukawa couplings can have the hierarchy \( Y_{\mu \tau} \ll Y_{\mu \mu, \tau \tau} \ll Y_{45,45}' \).

After determining the magnitudes of the Higgs-triplet Yukawa couplings, which are used to explain the neutrino data, we make some remarks on the implications of this model in flavor and collider physics. If the new \( Z' \) gauge boson is in the MeV to GeV range, in addition to explaining the excess of muon \( g - 2 \) and the large BR for \( h \to \mu \tau \) decay, the sizable Yukawa couplings \( y_{e}^{(t)} \) and \( y_{\mu}^{(t)} \) can lead to the \( \tau \to \mu Z' Z' \) decay through the mediation.
of light scalar $S$. Unlike the type-II seesaw model, the doubly charged Higgs ($H^{\pm\pm}$) can decay to the right-handed $\mu^{\pm}\tau^{\pm}$ via the induced dimension-6 operator, expressed as

$$
\frac{Y_{45} y_{\tau} y_{\mu}}{m_{L}^{2}} \frac{\tau R H T \iota_{2} \Delta H }{2 m_{L}^{2}},
$$

where the corresponding $H^{\pm\pm}$ Yukawa coupling to $\mu^{\pm}\tau^{\pm}$ is $Y_{H^{\pm\pm}} = Y_{45} y_{\tau} y_{\mu} v_{H}^{2} / (2 m_{L}^{2})$. From Eq. (30), $Y_{45}$ can in principle be $O(1)$, depending on the $y_{e}$ value. Thus, with $m_{L} \sim 1000$ GeV, $v_{H} \sim 246$ GeV, $m_{H^{\pm\pm}} \sim 400$ GeV, and $y_{\tau} \sim y_{\mu} \sim 0.1$, the decay rate ratio of $H^{\pm\pm} \rightarrow \mu^{\pm}\tau^{\pm}$ to $H^{\pm\pm} \rightarrow W^{\pm}W^{\pm}$ can be estimated as $[53]$

$$
\frac{\Gamma(H^{\pm\pm} \rightarrow \mu^{\pm}\tau^{\pm})}{\Gamma(H^{\pm\pm} \rightarrow W^{\pm}W^{\pm})} \approx \left| \frac{Y_{H^{\pm\pm}}^{2} v_{H}^{2}}{2 v_{\Delta}^{2}} \frac{v_{H}^{2}}{m_{H^{\pm\pm}}^{2}} \right| \sim 10^{-3} \left| Y_{45} \right|^{2} / v_{\Delta}^{2}.
$$

It can be seen that with $|Y_{45}| \sim O(1)$ and $v_{\Delta} \sim 0.1$ GeV, the BR for $H^{\pm\pm} \rightarrow \mu^{\pm}\tau^{\pm}$ can be compatible with that for $H^{\pm\pm} \rightarrow W^{\pm}W^{\pm}$.

In summary, we studied the origin of neutrino mass in the gauged $L_{\mu} - L_{\tau}$ model. We learnt that although one Higgs triplet can violate the lepton number, it does not suffice to explain the neutrino data due to the $U(1)_{L_{\mu} - L_{\tau}}$ gauge invariance. It was found that a proper symmetric Majorana mass matrix can be obtained when a pair of vector-like leptons and two singlet scalars carrying the $L_{\mu} - L_{\tau}$ charges are introduced. In this model, a certain Frampton-Glashow-Marfatia matrix pattern can be realized when some Yukawa couplings are set to vanish. Using pattern $A_{1}$, we showed that when the neutrino data with $1\sigma$ errors and cosmological neutrino bound are satisfied, the involving Higgs-triplet Yukawa couplings have a hierarchy, i.e., $Y_{\mu\tau} \ll Y_{45}^{\mu\tau} \ll Y_{45}, Y_{45}'$, and $Y_{45}^{(\prime)}$ can be $O(1)$. As a result, the effective Majorana neutrino mass is below the current experimental upper limit. Moreover, the model can have interesting phenomena in flavor and collider physics, such as muon $g - 2$, $h \rightarrow \mu\tau$, $\tau \rightarrow \mu Z^{\prime}Z^{\prime}$, and $H^{\pm\pm} \rightarrow (W^{\pm}W^{\pm}, \mu^{\pm}\tau^{\pm})$ decays.

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