Camera Calibration of Stereo Photogrammetric System with One-Dimensional Optical Reference Bar

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Abstract. To carry out the precise measurement of large-scale complex workpieces, accurately calibration of the stereo photogrammetric system has becoming more and more important. This paper proposed a flexible and reliable camera calibration of stereo photogrammetric system based on quaternion with one-dimensional optical reference bar, which has three small collinear infrared LED marks and the lengths between these marks have been precisely calibration. By moving the optical reference bar at a number of locations/orientations over the measurement volume, we calibrate the stereo photogrammetric systems with the geometric constraint of the optical reference bar. The extrinsic parameters calibration process consists of linear parameters estimation based on quaternion and nonlinear refinement based on the maximum likelihood criterion. Firstly, we linear estimate the extrinsic parameters of the stereo photogrammetric systems based on quaternion. Then with the quaternion results as the initial values, we refine the extrinsic parameters through maximum likelihood criterion with the Levenberg-Marquardt Algorithm. In the calibration process, we can automatically control the light intensity and optimize the exposure time to get uniform intensity profile of the image points at different distance and obtain higher S/N ratio. The experiment result proves that the calibration method proposed is flexible, valid and obtains good results in the application.

1. Introduction
The stereo photogrammetric systems have dramatically increased with considerable stride of the algorithm and system. In order to measure a large-scale workpiece in the work field with stereo photogrammetric system, the two cameras are mounted on suitable position. So the system must be accurately and flexibly calibrated on line.

Much research has been done about the calibration of stereo photogrammetric systems. While conventional camera calibration techniques are either based on a stable point-field with known reference coordinates or on a temporarily stationary point-field with only approximately known 3-D coordinates\(^{[1-2]}\). Hans-Gerd Mass\(^{[3]}\) and P. Cerveri\(^{[4]}\) have applied a rigid bar, which have two features and the distance between the two features has been precisely calibration, as an image target. Due to taking two retro-reflective marks as image features, the size and intensity profile of the marks imaging points have dramatic difference at different location of a large-scale space, and this difference can not be compensated by increasing exposure time due to increasing exposure time, so the feature centroid location precision is not considerable high, and this feature form limits the calibration...
accuracy. At meantime, two marks can’t provide sufficient information to eliminate the affects of the
bad location, false match and other outliers on the calibration accuracy. Zhijing Yu [5] also presented
dynamic calibrating the stereo photogrammetric system based on optical reference bar with epipolar
constraint.

We proposed a flexible and reliable camera calibration of stereo photogrammetric system based on
quaternion with one-dimensional optical reference bar. The calibration method is carried out by
moving the optical reference bar which has three small collinear infrared LED marks, and the lengths
between these marks have been certified. The optical reference bar is imaged by stereo
photogrammetric system at a number of locations/orientations over the observation volume. In this
way, we can automatically control the light intensity and optimize the exposure time to get uniform
intensity profile of the image points at different distance and obtain higher S/N ratio and the constant
lengths provide enough additional geometric constraint to determinate the extrinsic parameters of the
stereo photogrammetric system. The calibration process consists of linear parameters estimation based
on quaternion and nonlinear refinement based on the maximum likelihood criterion.

2. Solving the stereo photogrammetric system calibration

2.1. Measurement theory of solo photogrammetric system

A camera is modeled by the usual pinhole: the relationship between a 3D point \( M \) and its image
projection \( m \) (perspective projection) is given by

\[
\lambda m = K[R T]M
\]

(1)

Where \( \lambda \) is an arbitrary scale factor, \( m=[u,v]^T \) is the coordinate of 2D image point, \( M=[X,Y,Z]^T \) is the
coordinate of 3D space point, \( [R T] \) is the extrinsic parameters, \( K \) is the intrinsic parameters.

The length of the stick \( AB \) is known to be \( L \), i.e.

\[
\|B - A\| = L
\]

(2)

The position of point \( C \) is also known with respect to \( A \) and \( B \), and, therefore,

\[
C = \lambda_A A + \lambda_B B
\]

(3)

Where the scale factor \( \lambda_A \) and \( \lambda_B \) are known.

Points \( a \), \( b \) and \( c \) on the image plane are the homogenous coordinates of the projections of space
points \( A \), \( B \) and \( C \), respectively. Without loss of generality, we choose the camera coordinate system
to the world coordinate system, therefore, \( R = I \) and \( T = 0 \) in equation (1). Let the unknown depths for \( A \), \( B \)
and \( C \) be \( Z_A \), \( Z_B \) and \( Z_C \), respectively. According to (1), we have

\[
A = Z_A K^{-1} a
\]

(4)

\[
B = Z_B K^{-1} b
\]

(5)

\[
C = Z_C K^{-1} c
\]

(6)

Substituting them into (3) yields

\[
Z_C C = Z_A \lambda_A a + Z_B \lambda_B b
\]

(7)

After eliminating \( K^{-1} \) from both sides, by performing cross-product on both side of the above
equation with \( c \), we have

\[
Z_A \lambda_A (a \times c) + Z_B \lambda_B (b \times c) = 0
\]

(8)

In turn, we obtain

\[
Z_B = -Z_A \lambda_A (a \times c) \cdot (b \times c)
\]

\[
\lambda_B (b \times c) \cdot (b \times c)
\]

(9)

From (2), we have

\[
Z_A \|K^{-1} h\| = L
\]

(10)

\[
h = \lambda_A (a \times c) \cdot (b \times c)
\]

\[
\lambda_B (b \times c) \cdot (b \times c) \cdot b + a
\]
According to equation (10) and (11), we can get
\[ Z_A = \pm \left( L^2 \left/ (h^T K^{-T} K^{-1} h) \right. \right)^{1/2} \]  
(12)

According to (12), (4) and (5), we can get the space points coordinate of \( A \) and \( B \).

2.2. Linear parameter estimation with quaternion

Moving the reference bar of known length, which is imaged by the two cameras at a number of locations/orientations over the observation volume, we obtain a set of image coordinates and the corresponding space coordinates of the image points in each camera.

If the image coordinates of the same space point \( M \) in two different cameras plane are \( m \) and \( m' \), respectively; and the coordinates of the same space point in the two cameras coordinate systems are \( M \) and \( M' \), respectively. The relationship between them can be written as:
\[ (m')^T K'^{-T} [T] R K^{-1} m = 0 \]  
(13)
\[ M = RM' + T \]  
(14)

Where \( K \) and \( K' \) are the two cameras intrinsic parameters matrix, respectively; \( R \) is the orthogonal rotation matrix, \( T = [t_x, t_y, t_z]^T \) is the translation vector.

The rotation matrix has can be parameterized with many methods, such as rotation matrix, rotation angle, rotation axis and quaternion. Quaternions have fewer parameters than rotation matrices, and it can economize account, enhance the linearization and avoid singular points. Because of these advantages, we adopt quaternion to parameterize the rotation matrix.

The rotation matrix \( R \) and the quaternion \( q = (\lambda_0, \lambda_1, \lambda_2, \lambda_3)^T \) satisfies the following equation
\[ R = \begin{bmatrix}
\lambda_0^2 + \lambda_1^2 - \lambda_2^2 - \lambda_3^2 & 2(\lambda_1 \lambda_2 - \lambda_0 \lambda_3) & 2(\lambda_1 \lambda_3 + \lambda_0 \lambda_2) \\
2(\lambda_1 \lambda_2 + \lambda_0 \lambda_3) & \lambda_0^2 - \lambda_1^2 + \lambda_2^2 - \lambda_3^2 & 2(\lambda_2 \lambda_3 - \lambda_0 \lambda_1) \\
2(\lambda_1 \lambda_3 - \lambda_0 \lambda_2) & 2(\lambda_2 \lambda_3 + \lambda_0 \lambda_1) & \lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2
\end{bmatrix} \]  
(15)

According to (15), the relation equation (14) can be parameterized by four quaternion parameters and three translation parameters.

If we know \( n \) corresponding points coordinates \( M_i \) and \( M'_i \) \((i=1,2,3\ldots n)\) in two coordinate systems, respectively, we can obtain the relation of the two cameras with equation (14).

To be simplification, we form two radial clusters of \( n \) radials generated by linking the \( n \) coordinate points with their center. The radial formed by the point and the center of \( n \) points is
\[ r'_i = M'_i - \frac{1}{n} \sum_{i=1}^{n} M'_i \]  
(16)
\[ r_i = M_i - \frac{1}{n} \sum_{i=1}^{n} M_i \]  
(17)

The equation (14) can be denoted by the two radials clusters, and it is rewritten as
\[ r_i = R(q) r'_i \]  
(18)

If \( R(q) \) is calculated, then
\[ T = \frac{1}{n} \sum_{i=1}^{n} M_i - R(q) \frac{1}{n} \sum_{i=1}^{n} M'_i \]  
(19)

In theory, the angle between the \( r_i \) \((i=1,2,3\ldots n)\) radials rotated with \( R(q) \) and \( r'_i \) \((i=1,2,3\ldots n)\) radials will be zero. But because of the effect of the noise, the two radials clusters with rotation can’t strictly superpose, we obtain the optimal rotation matrix \( R(q) \) with the minimum deviation. We can obtain the solution of the rotation matrix by seeking the maximum of the every two corresponding radials scalar quantity product represented as the following equation
\[ \chi^2 = \sum_{i=1}^{n} r_i \cdot (R(q) \times r'_i) = q^T W q \]  
(20)
Where $W$ matrix is represented as
\[
W = \sum_{i=1}^{n} \begin{bmatrix}
X'_iX_i + Y'_iY_i + Z'_iZ_i \\
Y'_iZ_i - Z'_iY_i \\
X'_iX_i - Y'_iX_i - Z'_iZ_i \\
X'_iY_i - Y'_iX_i \\
Y'_iZ_i + Y'_iX_i + Z'_iZ_i \\
Z'_iX_i + X'_iZ_i \\
X'_iY_i + Y'_iX_i - X'_iX_i + Y'_iY_i - Z'_iZ_i \\
Y'_iZ_i + Z'_iY_i - X'_iX_i - Y'_iY_i + Z'_iZ_i
\end{bmatrix}
\] (21)

The solution of $q=(\lambda_0, \lambda_1, \lambda_2, \lambda_3)^T$, which makes $\chi^2=q^TWq$ get the maximum, is the largest eigenvalue vector of the $W$ matrix. According to the above-mentioned method, we can get the rotation matrix $R$ and the translation vector $T$.

2.3. Nonlinear refinement based on maximum likelihood criterion

Ideally, according to measurement theory of solo photogrammetric system and linear parameter estimation based on quaternion, we can get the camera extrinsic parameters matrix which satisfies (13) and (14). In practice, it doesn’t because of noise in the extracted image points. Hence, we present a nonlinear refinement technique based on maximum likelihood estimation. The maximum likelihood estimation of camera parameters can be obtained by minimizing the following functional
\[
\sum_{i=1}^{n} \left( \|m_i - m_i(K, I, 0, M_j)\|^2 + \|m'_i - m'_i(K', R, T, M_j)\|^2 \right)
\] (22)

Where $M_i$ is the point space coordinate in the first camera coordinate system, $m_i$ and $m'_i$ are the first camera projection and the second camera projection of point $M_i$, respectively. Minimizing (22) is a nonlinear minimization problem, which is solved with the Levenberg-Marquardt Algorithm [6]. The LM iteration requires an initial value of the known two cameras intrinsic parameters $K$ and $K'$, and $[R,T]$ which can be obtained using the technique described in the previous subsection.

3. Experiment results

The stereo photogrammetric system consists of two high-resolution Kodak MegaPlus CCD photogrammetric cameras, video capture card, computer workstation and software, optical measure probe with infrared light emitting diodes(LEDS) and optical reference bar as shown in figure 1.

![Figure 1. The structure of stereo photogrammetric system.](image)

The experiment is performed in a measurement volume of $3m \times 3m \times 3m$. The two cameras are mounted on suitable position. Firstly, by moving the optical reference bar over the measurement volume, we applied our calibration algorithm to the stereo photogrammetric system and obtained the system calibration results. For every bar location, the LEDs light intensity are automatically controlled and guaranteed that the image points have uniform intensity profiles. We measured the standard gauge with the stereo photogrammetric system calibrated by the proposed method, and the experimental measurement results are shown in table 1.
Table 1. The experimental measurement results.

| Standard length (mm) | Measured length error (mm) |
|----------------------|----------------------------|
| 400                  | 0.029 0.041 0.021 0.061 0.052 0.036 0.027 0.030 0.021 0.057 |
| 700                  | 0.053 0.045 0.040 0.062 0.033 0.022 0.035 0.067 0.030 0.043 |
| 1000                 | 0.051 0.032 0.064 0.056 0.071 0.043 0.050 0.084 0.063 0.066 |

The experimental results show the average measured length error is very small, and the calibration method proposed is flexible, valid and obtains good results in the application.

4. Conclusion
In this paper, we proposed a flexible and reliable camera calibration of stereo photogrammetric system based on quaternion with one-dimensional optical reference bar. By moving the optical reference bar at a number of locations/orientations over the measurement volume, we calibrate the stereo photogrammetric systems with the geometric constraint of the optical reference bar. The calibration process consists of linear parameters estimation based on quaternion and nonlinear refinement based on the maximum likelihood criterion. The quaternion has the advantages of little parameters, quickness in calculating, avoiding singular points and enhancing the linearization. In the calibration process, we can automatically control the light intensity and optimize the exposure time to get uniform intensity profile of the image points at different distance and obtain higher S/N ratio. The experiment result proves that the average measured length error is little than 0.08mm, so the proposed calibration method is flexible, valid and obtains good results in the application.

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