Neutron Interferometry Using a Single Modulated Phase Grating

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NEUTRON INTERFEROMETRY USING A SINGLE MODULATED PHASE GRATING

A Thesis

Submitted to the Graduate Faculty of the Louisiana State University and Agriculture and Mechanical College in partial fulfillment of the requirements for the degree of Master of Science

in

The Department of Physics & Astronomy

by

Ivan Joseph Hidrovo Giler
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Abstract

Neutron grating interferometry provides information on phase and small-angle scatter in addition to attenuation. Previously, phase grating moiré interferometers (PGMI) with two or three phase gratings have been developed. These phase-grating systems use the moiré far-field technique to avoid the need for high-aspect absorption gratings used in Talbot-Lau interferometers (TLI) which reduce the neutron flux reaching the detector. We demonstrate through simulations a novel phase grating interferometer system for cold neutrons that requires a single modulated phase grating (MPG) for phase-contrast imaging, as opposed to the two or three phase gratings in previously employed PGMI systems. We compare the MPG system to experiments in the literature that use a two-phase-grating-based PGMI with a best-case visibility of around 39% by Pushin et al. 2017. The simulations of the MPG system show improved visibility in comparison to that two-phase-grating-based PGMI. For example, an MPG with a modulation period 120 µm, pitch of 1 µm, and grating heights with a phase modulation of \( \pi, \pi/4 \), illuminated by a monochromatic beam, produces a visibility of 85% with a comparable source-to-detector distance (SDD) as the two-phase-grating-based PGMI. Phase sensitivity, another important performance metric of the grating interferometer was compared to values available in the literature, viz. the conventional TLI with phase sensitivity of \( 4.5 \times 10^3 \) for a SDD of 3.5 m and a beam wavelength of 0.44 nm (Kim et al. 2014). For a range of modulation periods, the MPG system provides comparable or greater theoretical maximum phase sensitivity of \( 4.1 \times 10^3 \) to \( 10.0 \times 10^3 \) for SDD of up to 3.5 m. This proposed MPG system appears capable of providing high-performance PGMI that obviates the need for the alignment of 2 phase gratings.
Chapter 1. Introduction

Neutrons are a useful probing tool in measuring material properties and imaging bulk materials due to their dual particle and wave nature, the latter described quantum-mechanically by de Broglie wave packets showing interference phenomena [1]. As neutron waves pass through matter, they undergo phase-shifts due to interactions with local, spatially dependent potentials; the most common being neutron-nucleus interactions [2-3]. Small-angle neutron scattering (SANS) of neutrons, due to nuclear or magnetic interaction potential variations in the sample, also locally degrade the coherence of a well-defined neutron wave front [4-5]. Neutron beams are also attenuated by nuclear reactions and incoherent scattering. Thus, neutron grating interferometry can image variations in phase change (differential phase contrast image), small-angle scattering (dark-field image), and attenuation (transmission image) [1,5].

Currently, there are two grating interferometry methods at the forefront of neutron phase imaging: the Talbot-Lau interferometer (TLI) [6-7] and the phase-grating moiré interferometer (PGMI) [8-11]. The TLI can operate in the full field of a cold neutron beam (contrary to typical Mach-Zehnder interferometers) and has flexible chromatic coherence requirements ($\Delta \lambda / \lambda$). However, its high-aspect ratio absorption gratings are challenging to manufacture and reduce the neutron flux reaching the detector by about a factor 4 [9-10]. The PGMI also operates in the full-field of a neutron beam, but since it produces directly resolvable interference fringes in the far-field, only a source grating is required, thus reducing the intensity by about a factor of 2. In contrast to the TLI, the PGMI has relaxed grating fabrication requirements, has a broader wavelength acceptance, and permits control of the fringe period by varying the separation of the phase modulating gratings [10].
The PGMI is a far-field regime interferometer that employs the phase moiré effect, which is understood as the underlying mechanism for a neutron interferometer that only uses phase gratings [8-10]. In general, the phase moiré effect is an intensity effect that occurs when a phase grating induces a periodic spatial modulation in phase to a neutron wavefront. Specifically, this periodically modulated wavefront is the self-image of the first grating (Talbot effect). If a second phase grating is placed downstream from it, that self-image of the first grating beats with the second grating to produce a beat pattern (i.e., a beat pattern representing alternating strongly and weakly diffracting areas). This interference pattern can still be observed even if the two-phase gratings are in contact, provided that their periods are appropriately different. This is because the moiré effect arises from the slightly different size or orientation of the projected self-image of the first grating at the second grating either due to their difference in periods (spatial frequencies) or their inter-grating spacing (or a combination thereof) [8].

Figure 1.1. Schematic of a two-phase grating interferometer setup operating in the far-field (not to scale). Typically for cold neutrons (1 meV-25 meV), $L_1$ and $L_2$ are in the scale of meters and D is in the scale of millimeters. For the monochromatic beam configuration by Pushin et al. [9] the source-detector distance ($L = L_1 + D + L_2$) is 2.99 m. After exiting the slit, neutrons acquire a (caption cont’d.)
transverse coherence length of \( l_c = \frac{\lambda L_1}{s_w} \), where \( s_w \) is the width of the slit and \( L_1 \) is the distance between the slit and the \( G_1 \) grating. Interference in the form of a beat pattern is observed at the detector after neutrons pass through two pure phase gratings, \( G_1 \) and \( G_2 \) (made of Si). The working principle of this far-field interferometer is based on the universal moiré effect.

A non-interferometric grating system which uses only a single attenuation grating was reported by Strobl et al. [18] which has the advantages of its fringe visibilities being independent of the wavelength (achromatic) and its ability to extend the range of autocorrelation lengths probed in materials (particularly in nanometer scale) for dark-field imaging (DFI). However, the system as demonstrated has a SDD of 7.26 m with the grating-to-detector distances of only 50 mm to 300 mm and fringe visibilities of 10\% to 70\%, limiting the phase sensitivity with its small grating-to-detector distances. Fringe visibility also falls off with higher grating-to-detector distance. This system’s visibilities are highly dependent on the geometric blurring due to its pinhole source with the collimation ratio \( L/D \) (source-to-grating distance / pinhole size). Thus, the fringe visibilities are susceptible to dropping sharply if the source-to-grating distance is too small or the pinhole size is too large.

The purpose of this study is to investigate by simulation a novel PGMI system for cold neutrons that requires only a single modulated phase grating (MPG). The phase grating has a rectangular modulation in spatial width with a period of \( W \) and a finer “carrier” pitch \( P \). An advantage of using a single grating is reducing grating misalignment issues commonly found in multiple-grating systems, which can lead to fringe visibility or contrast loss [9-11]. We investigated the advantages of imaging performance of this system in terms of visibility and sensitivity of the MPG system over the two-phase-grating-based PGMI, which we will hereinafter refer to as the “standard-dual-grating” system.
Chapter 2. Methods and Materials

2.1. Important Performance Metrics for Grating Interferometry

The fringe contrast or “visibility” and phase sensitivity are important performance metrics for a grating interferometer. The signal-to-noise ratios (SNR) of differential phase contrast images (DPCI) and dark-field images (DFI) are strongly dependent on the visibility [12]. Thus, PGMI designs that are conducive in producing high fringe visibility measurements allow for the implementation of experiments with weaker signals or shorter exposure times [12]. The fringe visibility $V$ of the intensity modulation recorded by at an imaging detector is defined as

$$V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$

where $I_{\text{max}}$ and $I_{\text{min}}$ are the maximum and minimum intensities of the fringe pattern, respectively. The visibility can also be expressed as percentage since it ranges from 0 to 1. The phase sensitivity for an interferometer relates the ability to measure the phase difference $\Delta \phi$ of the interference fringes for a small refraction angle $\alpha$ imposed by an object on the neutrons [7,13,17]. It is defined as follows

$$S = \frac{\Delta \phi}{2\pi \alpha}$$

where the measured phase difference $\Delta \phi$ of the interference fringes is normalized by $2\pi$. The image of this phase difference $\Delta \phi$ is the DPCI, and the sensitivity is directly proportional to the contrast of the image as seen in Eq. (2). Using a geometric argument Xu et al. [10] have already derived the phase sensitivity for a single modulated phase grating interferometer as

$$S = \frac{D_{od}}{W'}$$

where $D_{od}$ is the object-to-detector distance and $W'$ is period of the interference fringe pattern at the detector.
2.2. Analytical Simulations for Neutron Interferometry

We have constructed an efficient wave-propagation simulator for neutron PGMI applications that can accept various designs (e.g. modulated phase gratings [13] or standard gratings). The code, named “N-SRDI”, was written in the C++ programming language and can simulate systems such as those shown in Figures 2.1-2 using the Sommerfeld-Rayleigh diffraction integrals (SRDI) [12]. These predict the observed complex-valued field amplitude $A(y)$ from a wave that has diffracted from a grating or aperture and which originated from a source wave function $U(P_s)$. The neutron field amplitude at a detector for a single-phase-grating system such as the MPG (Figure 2.1) can be expressed as

$$
A(y) = \frac{1}{j\lambda} \int U(P_s) \frac{e^{jkr_1}}{r_1} T_1(y_1) \cos(\theta_1) dy_1
$$

(4)

and the intensity is

$$
I(y) = |A(y)|^2
$$

(5)

where $U(P_s)$ is the neutron source wave function, $T_1(y_1)$ is the neutron transmission function though the MPG in a plane perpendicular to the neutron beam propagation direction along the $z$ axis, $k = \frac{2\pi}{\lambda}$ is the wave number, $\lambda$ is the wavelength, $r_1$ is the distance between the grating point $y_1$ to the detector point $y$ given by $r_1 = \sqrt{(Dga)^2 + (y - y_1)^2}$, and $\theta$ is the angle between $\vec{r_1}$ and the normal of the MPG. The transmission function is given by

$$
T_1(y_1) = A_1(y_1) e^{j\phi(y_1)}
$$

(6)

where $A_1(y_1)$ is the amplitude transmission of the neutron beam through the grating due to attenuation and $\phi(y_1)$ is the phase shift determined by the spatial heights of the MPG.
Figure 2.1. Schematic diagram of the simulated MPG system (not to scale). In our simulations the neutron source is a point source emitting a spherical wavefront along the $z$ axis (fringe patterns from a line source are later acquired with a convolution method). The source-to-MPG distance $D_{sg}$ was chosen to be 0.5 m or 1.0 m (depending on the pitch), and the MPG-to-detector distance $D_{gd}$ can range from 1.0 m to 5.0 m. The grating has a rectangular functional form that modulates in spatial height with a slow-varying period of $W$ and a smaller “carrier” pitch $P$. The duty cycle of the grating is 50%. The different heights ($h_1, h_2$) of the grating along the $z$ axis correspond to phase shifts ($\pi, \pi/4$) for neutrons of wavelength 0.44 nm. The grating simulated is an ideal phase grating so no attenuation of the beam is considered, i.e. $A_1=1$. The fringe period at the detector $W'$ is determined by the geometric magnification $D_{sd}/D_{sg}$ from the point source [13].

The SRDI shown in Eq. (4) is a representation of the Huygens-Fresnel principle since the observed complex-valued field amplitude at the detector is a superposition of diverging spherical waves $e^{jkr_1/r_1}$ originating from secondary sources located at each point $y_1$ along the phase grating [12]. In all simulations the neutron source wave function $U(P_s)$ is assumed to be a point source at point $y_0$; however, the fringe patterns from a line source are later acquired with a convolution method. Therefore, Eq. (4) was simplified to

$$A(y) = \frac{D_{gd}}{j\lambda} \int \frac{e^{jk(r_0+r_1)}}{r_0 r_1^2} T_1(y_1) dy_1$$

(7)
where $r_0$ is the distance between the point source at point $y_0$ and the point $y_1$ on the MPG given by

$$r_0 = \sqrt{(D_{sg})^2 + (y_1 - y_0)^2}$$

and the $\cos \theta$ (between $r_1$ and the $z$ axis) has been replaced by $D_{gd}/r_1$, explaining the $r_1^2$ term inside the SRDI show in Eq. (7).

To verify the accuracy of the N-SRDI code we also simulated standard-dual-grating systems for neutron PGMI applications and compared our results to experimental data from Pushin et al. [9]. Since two separate gratings are involved for this interferometer system, we use two SRDIs to calculate the observed complex-valued field amplitude $A(y)$ at the detector. Using the Sommerfeld-Rayleigh formulation for diffraction, the neutron field amplitude at a detector for a standard-dual-grating system (Figure 2.2) was simplified, using the same logic as before, to

$$A(y) = -\frac{DL_2}{\lambda^2} \int \int \frac{e^{ik(r_0+r_1+r_2)}}{r_0 r_1 r_2} T_1(y_1)T_2(y_2)dy_1 dy_2$$

Figure 2.2. Schematic diagram of the simulated standard-dual-grating system (not to scale). Simulations were done to compare to fringe visibility measurements done by Pushin et al. [9]. In our simulations the neutron source is a point source emitting a spherical wavefront along the $z$ axis (fringe patterns from a line source are later acquired with a convolution method). $L_1 = 1.2$ m and (caption cont’d.)
$L_2 = 1.79$ m. The inter-grating spacing $D$ ranges from 7 mm to 16 mm. The gratings have standard binary form with no modulation in spatial height. The periods of the gratings are $P_{G_1} = P_{G_2} = 2.4 \mu m$. The duty cycles of the gratings are 50%. The heights of the $G_1$ and $G_2$ gratings are $h_1$ and $h_2$, respectively. They both correspond to a phase shift of $0.27\pi$ for neutrons of wavelength 0.44 nm. The gratings simulated are ideal phase grating so there is no attenuation of the beam for either grating, i.e. $A_1 = A_2 = 1$. The fringe period at the detector $P_d$ is given by the ratio $LP_{G_2}/D$ as given in Ref. [8].

In all simulations (MPG or standard-dual-grating systems), we obtained results with a point source (that is $U(P_s)$ was a point source at point $y_0$ emitting spherical wave $e^{ikr_0/r_0}$) and then used a convolution method on the fringe pattern of a point source to simulate the line-source, as will be described next.

2.3. Fringe Pattern Blurring Due to Slit Width and Pixel Size via Convolution Method

2.3.1. Consideration of the Slit Width

In the first step of the simulation, the neutron source was assumed to be a point source emitting a spherical wave in the direction of the $z$-axis towards the imaging detector. A line source, such as that in Ref. [9], can be thought of as an array of point sources that are mutually incoherent. As shown in Figure 2.3, each point along a line source creates a fringe pattern at the detector that is spatially displaced with respect to the fringe pattern from the central point source. Their superposition causes the observed composite fringe pattern to be washed out, reducing the overall visibility of the fringe pattern. Consequently, a line source will have lower fringe visibility compared to a single, monochromatic point source.

Instead of repeating costly SDRI computations by taking a series of point sources along with the extent of a line source (Figure 2.3), we modeled the fringe pattern of a line source by first simulating the fringe-pattern due to a center-point source and then convolving it with a rectangular window function of the same size as the line source. This has the equivalent “wash-out” effect on
the fringe intensity pattern at the detector from simulating an array of point sources that make up a line source.

![Figure 2.3](image)

Figure 2.3. Schematic illustration demonstrating visibility loss of an approximated line source (not to scale). The red fringe pattern on the detector is due to the central point source, and the green fringe patterns are due to the endpoints of the approximated line source. Superposition of these fringe patterns will result in visibility loss due to the green fringe patterns being shifted away from the red fringe pattern. With more intermediate point sources added to approximate the line source, the visibility is improved since there are more fringe patterns that are closer in phase to the red fringe pattern. However, the visibility of this approximated line source will always be lower than that of the fringe pattern of a single point source because of the wash-out effect many slightly shifted fringe patterns create.

2.3.2. Consideration of the Pixel Size

Furthermore, to account for the fringe blurring due to the pixel resolution and to have the correct number of sampled data points given a certain pixel size, two steps are taken: (1) another convolution of the fringe pattern at the detector by a rectangular window function that is the same size as the pixel (2) subsampling of the fringe intensity pattern at pixel-size increments since the intensity pattern at the detector is originally sampled at 0.1 μm. This is followed by an interpolation to reconstruct the signal at higher sampling rates.
2.3.3. Assessment of Convolution Method

We simulated a case from the experimental setups from Ref. [9] to assess our convolution method that used a standard-dual-grating system with a monochromatic beam ($\lambda = 0.44$ nm) exiting a 200 µm slit source with the system parameters of $L_1 = 1.2$ m, $D = 12$ mm, and $L_2 = 1.78$ m (Figure 2.2). The identical phase gratings had pitch $P_{G_1} = P_{G_2} = 2.4$ µm and a $0.27\pi$ phase shift for 0.44 nm wavelength neutrons. Both gratings had duty cycles of 50%. The detector pixel size resolution was reported to be ~100 µm. In simulations the grating function is sampled at every 2 nm and the intensity pattern at the detector is sampled at every 0.1 µm. We used the two independent convolutions previously mentioned on the fringe pattern to account for blurring due to the slit width and pixel size. The fringe pattern is subsampled at 100 µm increments and intensity values are interpolated between the sampled values by a factor of 1000 using MATLAB’s interp function to return to a 0.1 µm sampling rate.

We compared our convolution method against two other methods: (1) the expected closed-form visibility function of slit size given in Ref. [9]: $V = V_o |sinc(\pi s/P_s)|$, where $V_o$ is the optimal fringe visibility from one point source, $s$ is the width of the slit and $P_s$ is the source period. The fringe visibility for a 200 µm source using our convolution method was 32.9% which closely matched the expected close form 32.7% visibility (blue and orange bars in Figure 2.4). (2) A brute force method of simulating many individual point sources on the slit, as illustrated in Figure 2.3. The yellow bar in Figure 2.4 represents the visibility for 21 points evenly space about the 200 µm width of the slit, yielding a 32.7% visibility. As more intermediate point sources were uniformly added as represented by the red bar (41 total point sources), the visibility remained at the expected visibility value of 32.7%. Thus, we verified that a central point source simulation followed by a convolution with a rectangular window function can adequately model the fringe pattern of a slit.
source, reducing computation cost considerably by avoiding brute force computations of several point sources.

Figure 2.4. Verification of convolution method visibility for a 200 μm slit source in a monochromatic beam. The fringe visibility using the convolution method is 32.9% closely matched the expected 32.7% visibility, theoretically calculated with Eq. 10 given in Ref. [9]: $V = V_0 |\text{sinc}(\pi s/P_s)|$ where $V_0$ is the optimal fringe visibility given by one point source (39.6%), $s$ is the slit size (200 μm), and $P_s$ is the source period (598 μm). Another method tested was modeling the slit as a series of evenly spaced point sources. The visibility converged after 21 points, with 21 and 41 points each yielding the expected visibility of 32.7%.

2.4. Evaluation of N-SRDI Simulations for a Standard-Dual-Grating System, Comparisons with Theory and Experiments

We evaluated the N-SRDI obtained fringe visibilities $V$ against experimental results with standard-dual-grating systems for the neutron PGMI in Ref. [9] using the same methods detailed in the previous section. We also evaluated whether the simulated fringe periods agreed with the theoretically predicted values [8].

The simulation parameters to match experiments are detailed in the previous section. The grating-to-grating separation distance $D$ was varied from 7 mm to 16 mm, while keeping the $L$ and $L_1$ fixed (Figure 2.2). The $L_2$ was adjusted accordingly. Our convolution method previously
described is applied to the fringe patterns, accounting for the 200 \( \mu m \) slit and 100 \( \mu m \) pixel used in the experiments.

2.5. N-SRDI Simulations for a Modulated Phase Grating (MPG) Neutron Interferometer Illuminated by a Monochromatic Beam

2.5.1. Evaluations of Fringe Visibility, Period and Maximum Phase Sensitivity

We next tested the novel MPG system illuminated by a monochromatic beam (\( \lambda = 0.44 \) nm) as outlined below in Figure 2.5. Fringe visibility, fringe period, and maximum phase sensitivity were evaluated for different modulation periods \( W \), pitches \( P \), slit widths \( S_w \), source-to-MPG distances \( D_{sg} \), and MPG-to-detector distances \( D_{gd} \) as explained in the results section 3.

Figure 2.5. Schematic of the MPG system (not to scale) with a slit source with width \( S_w \), MPG with grating heights with phase modulation of \((\pi/4, \pi)\), and detector with a phase object at an object-to-detector distance \( D_{od} \).

2.6. N-SRDI Simulations for a Modulated Phase Grating (MPG) Neutron Interferometer Illuminated by a Polychromatic Beam

We investigated how a polychromatic beam may degrade the visibility for our MPG system. We considered the configuration that produced the best visibility from the previous section. The polychromatic beam was modeled after the one at the NG6 Cold Neutron Imaging Facility (NCI) at the NCNR, which is approximately given by a Maxwell-Boltzmann distribution.
with a peak wavelength of $\lambda_c = 0.5$ nm [9]. The same MPG is used in both the monochromatic and polychromatic simulations. Since the polychromatic beam can be thought of as the aggregate of incoherent sources, the fringe patterns of each wavelength in the spectrum can be added in intensity according to their weight in the spectrum.

2.7. Single-Shot Phase Contrast Recovery with MPG

Phase objects, that is objects that introduce a pure phase shift on the wavefront on the path of the beam were simulated mathematically, and the effect on the interference fringe at the detector was analyzed.

The shift in the interference pattern $\Delta y$ at the detector is related to the refractive angle $\alpha$ imposed by an object on the wave field and therefore the object’s differential phase-shift $d\Phi/dy$ as follows [13,17].

$$\Delta y = D_{od} \tan \alpha \approx \frac{\lambda D_{od} d\Phi}{2\pi dy}$$

(9)

Estimating $\Delta y$, (which itself is a function of $y$ in general), can therefore yield the object’s differential phase shift $d\Phi/dy$. Note that $\Delta y$ also depends on system parameters: object-to-detector distance $D_{od}$ and the wavelength $\lambda$, which have to be corrected for a true estimate of the object’s differential phase.

Since we have large interference pattern period $W'$ (usually greater than the pixel size), and the detector sampling rate above the Nyquist sampling rate, we use a single-shot recovery using Fourier transforms to demonstrate our system properties, following method in Ref. [14]. The steps followed are shown below.

Step 1. Take Fast Fourier transform (FFT) of each simulated fringe pattern with and without object. Isolate the first harmonic by windowing the Fourier transforms on one side. A window-width of total width $1/W'$ is chosen symmetrically around the first harmonic peak located at $1/W'$
(i.e., the window extends $\pm 1/(2W')$ around $1/W'$). Perform Inverse Fast Fourier Transform (IFFT) of the one-sided windowed FFTs and take the angle difference of the two complex spatial domain signals. This angular difference $\Delta \phi$ is related to the spatial shift between the fringe pattern signals $\Delta y$ as

$$\Delta \phi = \phi - \phi_b = \frac{2\pi}{W'} \Delta y$$  \hspace{1cm} (10)

Note: the IFFT of the windowed FFT yields complex exponentials in the spatial domain, ideally the first harmonic signals. These have the frequency $2\pi/W'$. The difference of phase angles $\Delta \phi$ between the blank fringe pattern phase angle $\phi_b$ and the with-object fringe pattern phase $\phi$ is related to the spatial shift $\Delta y$ between these two signals as given by Eq. (7).

Note: the N-SRDI code outputs detector data with a detector pixel size of 0.1 $\mu$m. We convolve the fringe patterns by independent rectangular window functions to compensate for the fringe blurring due to the slit width and pixel size. Then we subsample the data at a rate equal to the pixel size which is 100 $\mu$m.

We interpolate the subsampled pixel-size detector data up by a factor of 1000 by using the MATLAB’s interp function to resample the function at 0.1 $\mu$m before performing the FFT and IFFT (these are the same processing steps used for the standard-dual-grating simulations). The FFT, IFFT, and angle difference of the fringe patterns are all performed in MATLAB. Each phase angle $\phi_b$ and $\phi$ are obtained modulo $\pm \pi$. The resultant phase angle difference has to be unwrapped before integration. Note for some small objects, the $\Delta y$ shift can be sub-pixel. However, since we sample the fringe pattern above the Nyquist rate, (i.e., pixel size $< W'/2$), we can always reconstruct the signals with interpolation and obtain the phase difference.
Step 2 (a). To obtain the object’s differential phase $d\Phi/dy$ at the detector space ($y$) from the measured phase difference $\Delta \phi$ we multiply by $(S\lambda)^{-1}$. We show this by combining Eq. (9) and Eq. (10) to obtain

\[
\Delta \phi = \frac{\lambda D_{od}}{W'} \frac{d\Phi}{dy} = S\lambda \frac{d\Phi}{dy}
\]

Where is $S = D_{od}/W'$ is the phase sensitivity for the interferometer. Expressed in terms of the object’s differential phase profile $d\Phi/dy$:

\[
\frac{d\Phi}{dy} = (S\lambda)^{-1} \Delta \phi
\]

Thus, the measured phase difference $\Delta \phi$ must be “corrected” by $1/(\lambda S)$ to obtain $d\Phi/dy$.

Step 2 (b). To obtain the object’s differential phase $d\Phi/dy_{ob}$ at the object space ($y_{ob}$), we have to scale (de-magnify) the spatial variable of the detector ($y$) by $1/M_{obj}$ where $M_{obj}$ = object magnification.

Step 3. We integrate $d\Phi/dy_{ob}$ to obtain the object phase profile. We use the MATLAB’s cumtrapz function for the integration. The scaled grid (object grid) is provided into MATLAB’s cumtrapz function as the grid argument. Note that we could have also magnified the true object’s grid and compared both phase profiles in the detector space ($y$), but we wanted to avoid any processing with the true object phase profile.
Figure 2.6. (a) Interference pattern with and without a triangle phase object with a peak phase of $8\pi$ rad using the system parameters $W = 50 \, \mu m$, Pitch = $2.1 \, \mu m$, $D_{sd} = 5 \, m$, $D_{sg} = 1 \, m$, and $D_{od} = 2.5 \, m$. Intensity values are originally sampled along the detector plane at $0.1 \, \mu m$ increments and are filtered by two independent rectangular window functions for blurring effects due to slit width and pixel size. (b) Shows both fringe patterns subsampled at $100 \, \mu m$ increments and interpolated back to a sampling rate of $0.1 \, \mu m$. The dark lines on the left and right are locations where the $\Delta y$ shift of the fringe pattern were checked manually.

| Fringe Pattern $\Delta y$ | Left Window $\Delta y$ ($\mu m$) | Right Window $\Delta y$ ($\mu m$) |
|---------------------------|-----------------------------------|-----------------------------------|
| Expected                  | 5.5                               | -5.5                              |
| Measured from Plot        | $5.4 \pm 0.2$                     | $-5.7 \pm 0.2$                    |

Figure 2.6 shows an example simulated blank and with-object fringe pattern. The phase object is a triangular object with a maximum phase shift of $8\pi$ rad, ramping up/down over $\pm 800 \, \mu m$ spatial extent. It is placed an object-to-detector distance of $2.5 \, m$. In reality, this phase profile could be a silicon wedge sample with a maximum height of $275.6 \, \mu m$ at the center and falling off to zero over $\pm 800 \, \mu m$ on either side of its peak. Figure 2.6(a) shows both fringe patterns just after being convolved with two independent rectangular filters to account for the blurring of the slit width of $200 \, \mu m$ and the pixel size of $100 \, \mu m$. Figure 2.6(b) shows the fringe patterns after being subsampled at every $100 \, \mu m$ and then interpolating between sampled values to return to a $0.1 \, \mu m$ sampling rate. In the N-SRDI code, the objects are represented simply as mathematical
phasors $e^{i\Phi(y_{ob})}$ which are applied to the wave field at the object-to-detector distance, where $\Phi(y_{ob})$ is the object phase profile.

The expected spatial shifts in the fringe pattern due to this triangular phase object are given by Eq. (9). Since the object phase consists of an up and down ramp, the spatial shift $\Delta y$ is a positive constant on the left side of the fringe pattern center (detector position 0 $\mu$m) and a negative constant on the right side of the fringe pattern center. As a manual check, we calculated the average $\Delta y$ in the two regions over three cycles each at the 90% normalized intensity line in Figure 2.6(b). In Table 2.1 we compare the expected $\Delta y$ and our measurements showing that the values are very close to each other.

Figure 2.7 (a) Shows the measured phase difference $\Delta \phi$ at the detector from Figure 2.6(b) obtained via the single-shot method explained in Step 1(b). The measured phase difference $\Delta \phi$ is corrected by a $(S\lambda)^{-1}$ factor to obtain the object differential phase $d\Phi/dy$ at the detector space ($y$).

(caption cont’d.)
(c) The grid of $d\Phi/dy$ is scaled by the inverse object magnification factor $D_{so}/D_{sd}$ to obtain (caption cont’d.) the object differential phase $d\Phi/dy_{ob}$ in the object space ($y_{ob}$). (d) Shows the object phase $\Phi$ retrieved from $d\Phi/dy_{ob}$ by integration (Step 3).

Figure 2.7 shows the Steps 1-3 recovery with the blank and with-object fringe patterns shown in Figure 2.6(b). First, we show the measured $\Delta \phi$ after Step 1 in Figure 2.7(a). This approximately shows the rectangular pattern expected from differential phase of the triangular object consisting of two ramps. The differential phase is shown in Figure 2.7(b) after correction by $(S\lambda)^{-1}$ as in Step 2. In Figure 2.7(c), the detector grid is scaled by the object magnification to switch to the objects grid space ($y_{ob}$) instead of the detector grid space ($y$). Then Figure 2.7(d) shows the integrated phase of the object (Step 3).

Figure 2.8. (a) Measured phase difference $\Delta \Phi$ at the detector for object-to-detector distances of 0.5 m to 3.5 m for the $8\pi$ rad triangle phase object. (b) Shows the corresponding object differential phase $d\Phi/dy_{ob}$ at the object plane (c) Measured phase shifts $\Delta \Phi$ at the detector for (caption cont’d.)
object-to-detector distances of 0.5 m to 3.5 m for the $8\pi \text{ rad}$ parabolic phase object (d) Shows the corresponding object differential phase $d\Phi/dy_{ob}$ at the object plane. $\Delta \phi$ are corrected by a $(S\lambda)^{-1}$ factor to obtain $d\Phi/dy$ and the detector grids are also corrected by the inverse object magnification factor $D_{od}/D_{sd}$.

Lastly, Figure 2.8 shows the Step 1 and Step 2(b) of the phase recovery process for the triangular and parabolic phase objects for different object-to-detector distances, which was varied from 0.5 m to 3.5 m. Figure 2.8(a) and Figure 2.8(c) shows the measured phase $\Delta \phi$ (Step 1) at the detector grid. Figure 2.8(b) and Figure 2.8(d) shows the object differential phase $d\Phi/dy_{ob}$ (Step 2(b)) on the object grid space ($y_{ob}$). They both approximately show the rectangular and linear $d\Phi/dy_{ob}$ expected for triangular and parabolic phase profiles, respectively.

In the results section 3, we show quantitative phase recovery error analysis of triangular, parabolic, and trapezoidal objects with object-to-detector distances of 0.5 m to 3.5 m and maximum object phases of $0.8\pi \text{ rad}$ and $8\pi \text{ rad}$.
Chapter 3. Results

3.1. Evaluation of N-SRDI Simulations for a Standard-Dual-Grating System, Comparisons with Theory and Experiments

To test our simulator N-SRDI, we simulated the standard-dual-phase grating system and compared our results to theory and experiments. We varied the grating-to-grating distance from 7 mm to 16 mm in increments of 1 mm and obtained fringe periods and fringe visibilities for the monochromatic experiments performed in Ref. [9] with $\lambda = 0.44$ nm. The fringe periods obtained via the simulations were compared to the theoretical closed-form prediction given by in Eq. (11) in Ref. [8] as shown in Figure 3.1. We observed an excellent agreement of the simulated fringe periods to the corresponding theoretical values.

Figure 3.1. Fringe period from standard-dual-grating simulations compared to theory. N-SRDI simulations were done with the monochromatic configuration parameters used in Ref. [9]. The theoretical fringe period is given by Eq. (11) in Ref. [8].

Figure 3.2 plots the N-SRDI with convolution fringe visibility results and the experimental fringe visibility results for the monochromatic experiments in Ref. [9]. Not all but every other grating-to-grating distance was simulated and shown below since we varied the grating-to-grating distance in spacings of 1 mm. The general trend was captured by the N-SRDI with convolution
simulations with the visibility peaking around 11 mm to 12 mm. The curve also fell within the statistical agreement of experiments (indicated by the error bars) except for the grating-to-grating distances D of 7 mm, 8 mm, and 11 mm.

Figure 3.2. Fringe visibility comparison of N-SRDI with convolution simulations to experiments performed in Ref. [9]. The dashed lines show the closed-form expected visibility given in Eq. 12 in Ref. [8] which was plotted in Ref. [9].

3.2. N-SRDI Simulations for a Modulated Phase-Grating (MPG) Neutron Interferometer Illuminated by a Monochromatic Beam

We simulated the MPG system with grating heights with a modulation of \( \pi/4 \) and \( \pi \), the same slit-source in a monochromatic beam (\( \lambda = 0.44 \text{ nm} \)) used for the standard-dual-grating system in the previous section, different modulation periods W, and the two different pitches of 2.1 µm and 1 µm. The pitch of \( P = 2.1 \) µm is the period of some of the gratings currently available at the National Institute of Standards and Technology Center for Neutron Research (NCNR). We show an example in Figure 3.3(a) of interference fringe “carpet” for \( W = 120 \) µm and \( P = 1 \) µm, where the carpet is obtained by placing the detector at different distances from the MPG. The carpet shows a diverging self-image of the modulation pattern. At the source-to-MPG distance of 0.5 m and MPG-to-detector distance of 2.39 m (SDD of 2.89 m), we are at a nearly equivalent SDD of...
2.99 m used in the setup in Ref. [9] for their monochromatic configuration. Our fringe visibility using our MPG is 85% as opposed to 39% for the standard-dual-grating setup in Ref. [9]. A fringe visibility analysis of the whole carpet is shown in Figure 3.3(b). The normalized fringe pattern with the maximum visibility of 85% at the MPG-to-detector distance of 2.39 m is shown in Figure 3.3(c).

![Figure 3.3](image)

**Figure 3.3:** (a) Interference fringe “carpet” generated with N-SRDI and convolution for the source-to-MPG distance of 0.5 m, \(W=120\ \mu m\), and Pitch = 1.0 \(\mu m\). The pixel size is 100 \(\mu m\) and the slit width is 200 \(\mu m\) (b) Fringe visibility analysis of the carpet with the maximum visibility at the MPG-to-detector distance of 2.39 m. The maximum visibility is 85%. (c) The normalized fringe pattern with maximum visibility of 85% at the MPG-to-detector distance of 2.39 m (SDD of 2.89 m).

The example shows that the MPG system may yield 85% visibility compared to the standard-dual-grating system’s maximum visibility ~39% in a nearly equivalent set-up geometry to the one used in Ref. [9]. In Figures 3.4 (a) and (b) we show the visibility and fringe period for different modulation periods \(W=50-600\ \mu m\) (all with a pitch of 2.1 \(\mu m\)) at varying MPG-to-detector distances. We make sure to not include cases where the pixel size > \(W'/2\), which would lead to insufficient sampling of the fringe pattern (Nyquist theorem) and cause aliasing. We
observed several operating points with fringe visibility $V > 40\%$ in Figure 3.4(a-b) for different fringe periods. The ability to control the fringe period is important as the fringe period determines the autocorrelation probing lengths in dark-field imaging (DFI) [16], and it is also inversely proportional to the phase sensitivity [13]. A variation in the fringe period of a factor of 4-5 is observed and several MPG configurations show high visibility.

Figure 3.4. Measured (a) fringe visibility and (b) fringe periods from simulations with varying MPG-to-detector distances of 1.0 m to 5.0 m. The fringe period $W'$ is given by the geometric magnification $D_{sd}/D_{sg}$ of $W$. The source-to-MPG distance is 1.0 m and the pitch is 2.1 $\mu$m. The pixel size is 100 $\mu$m and the slit width is 200 $\mu$m. Fringe sampling frequency is ensured to be greater than the Nyquist rate (pixel size $< W'/2)$.

Figure 3.5. Fringe visibility versus period for a source-to-MPG distance of 1.0 m, varying (caption cont’d.)
MPG-to-detector distances from 1.0 m to 5.0 m, and pitch of 2.1 μm. The pixel size is 100 μm and the slit width is 200 μm. The cases with a SDD of 2.99 m (≈ 3.0 m) like the monochromatic experimental configuration in Ref. [9] are shown with the red arrows.

Hence, to visualize the information in a meaningful way, we plot the visibility versus fringe-period in Figure 3.5, with the MPG-to-detector distance implicitly varied. The 9 circles in each curve correspond to the visibilities at a different MPG-to-detector distance (100 - 500 cm), i.e. the SDDs range from 200 cm to 600 cm. We observed that the MPG provided high visibility, V > 39%, for the green zone, and acceptable 20-39% in the light-blue zone. Comparing to the standard-dual-grating system, the peak visibility from those experiments (Figure 3.2) is shown as the asterisk with an error bar (falling on the “good visibility” zone). The dashed vertical line shows that for the same fringe period from the monochromatic setup in Ref. [9] with the best-case visibility of 39%, three MPG designs exists that can yield a better visibility V > 50%. Specifically, W = 300 μm yields approximately 59% visibility for MPG-to-detector distance of 100 cm (SDD of 200 cm), W=120 μm yields about 62% visibility for the larger MPG-to-detector distance of 400 cm (SDD of 500 cm), and W= 200 μm falls in-between with about 53% visibility and a SDD of 300 cm (like the monochromatic setup in Ref. [9]).

If one is looking for high visibility V > 39% with a compact setup geometry, we point out the operating points marked with yellow stars in Figure 3.5 where the SDD ≤ 300 cm. We, show several operating points of the MPG system with potentially higher visibility than the highest reported visibility of 39% from the standard-dual-grating system in Ref. [9], with similar fringe periods and similar or more compact geometry.

We observed that for smaller W cases, (yielding smaller periods) the visibility for the MPG was relatively poor except at larger MPG-to-detector distances. This zone is highlighted with a dashed square. We sought to improve the performance in this zone of operation by lowering the
pitch. The lower fringe period zone is shown in Figure 3.6 for the 1 µm cases. We make sure not to include cases where the pixel size > $W'/2$, which would lead to insufficient sampling of the fringe pattern (Nyquist theorem). The yellow stars highlight the operating cases with a SDD of 3.0 m or less that yielded visibility $V > 39\%$. Likewise, the dashed vertical line and the yellow stars suggest a range of W parameters (W between 70 µm to 120 µm) exists for the MPG to yield higher visibility than that of Ref. [9] for similar fringe periods and compactness.

![Figure 3.6](image.png)

Figure 3.6. Fringe visibility versus period for $D_{sg} = 0.5$, $D_{gd} = [1.0 \text{ m - 5.0 m}]$, and pitch = 1.0 µm. The cases with a SDD of 2.99 m ($\approx 3.0$ m) like the monochromatic experimental configuration in Ref. [9] are shown with the red arrows. The pixel size is 100 µm and the slit width is 200 µm. Fringe sampling frequency is ensured to be greater than the Nyquist rate (pixel size < $W'/2$).

We also investigated if the visibility for the smaller W cases (50 µm, 70 µm, and 120 µm) could be improved by using narrower slit widths $S_w$ (albeit knowing that realistically the flux reaching the detector would be reduced proportionately). We compare the previous visibility results from these small W cases which used a 200 µm slit to the visibility results that used 50 µm and 125 µm slit widths (Figure 3.7). We noticed that, in general, using a narrower slit width $S_w$
improved the visibility the most for the W = 50 μm and W = 70 μm cases as seen in Figure 3.7 (a-b). The narrower slit widths tended to improve the visibility the most for fringe patterns with smaller periods. In Figure 3.7(c) for the W = 120 μm we see that for fringe periods > 600 μm this improvement in visibility becomes less apparent when using narrower slits. We also note in Figure 3.7(c) the first two data points for S_w of 50 μm and 120 μm (inside purple oval) show some aliasing when observed at 100 μm pixel size because of the presence of higher harmonics in the fringe pattern.

Figure 3.7 (a-c) Fringe visibility for slit sizes S_w of 50 μm, 125 μm, and 200 μm, respectively. The source-to-MPG distance is 1.0 m and the pitch is 2.1 μm. The MPG-to-detector distances range from 1.0 m to 5.0 m. The pixel size is 100 μm. The purple oval in (c) encloses data points where pixel sampling frequency may not have been above the Nyquist rate.

Lastly, we calculated the maximum phase sensitivities of our MPG system. The maximum phase sensitivity is given by S_{max} = D_{gd} / W', (where the object is closest to the MPG, i.e. D_{od}
= D_{ga}). This is analyzed for different MPG-to-detector distances as shown in Figure 3.8(a) for different grating modulation periods. There is also an increase in phase sensitivity with lower $W'$ (lower fringe period) as shown in Figure 3.8(b) for a grating-to-detector distance of 5.0 m. Phase sensitivity information is sparse for the standard-dual-grating system but available in the literature for the conventional neutron Talbot-Lau interferometer (TLI). For modulation periods $W$ between 70 $\mu$m to 120 $\mu$m, we note that the theoretical maximum phase sensitivities are between $4.1 \times 10^3$ to $10.0 \times 10^3$ when using SDDs up to 3.5 m. These sensitivities are comparable or greater than that of the conventional neutron TLI of $4.5 \times 10^3$ with a SDD of 3.5 m and a beam wavelength of 0.44 nm [7,15].

![Figure 3.8](image)

(a) Maximum phase sensitivity for different MPG-to-detector distances (b) Phase sensitivity versus fringe period for the MPG-to-detector distance of 5 m. The pixel size is 100 $\mu$m and the slit width is 200 $\mu$m. Fringe sampling frequency is ensured to be greater than the Nyquist rate (pixel size < $W'/2$).
3.3. N-SRDI Simulations for a Modulated Phase Grating (MPG) Neutron Interferometer Illuminated by a Polychromatic Beam

We used a polychromatic beam for the setup which produced the best visibility of 85% in the monochromatic simulations ($\lambda = 0.44$ nm) to see how the fringe visibility would degrade. The polychromatic beam is approximately described by Maxwell-Boltzmann spectrum with a peak wavelength $\lambda_c = 0.5$ nm. The MPG used in both monochromatic and polychromatic simulations were the same and had fixed phases of $(\pi/4, \pi)$ at $\lambda = 0.44$ nm. The system parameters include a grating with modulation period $W$ of 120 $\mu$m, pitch of 1 $\mu$m, source-to-MPG distance of 0.5 m, and MPG-to-detector distance of 2.39 m. The pixel size is 100 $\mu$m and the slit width is 200 $\mu$m. While we note that the visibility dropped from 85% to 43% (Figure 3.9), this visibility is still higher than the standard-dual-grating system with a best-case visibility of around 39% in Ref. [9].

![Figure 3.9](image.png)

Figure 3.9. Fringe visibility degradation due to polychromatic beam for our MPG system. We consider the configuration that produced the best visibility of 85% using a monochromatic beam of $\lambda = 0.44$ nm (blue trace). The polychromatic beam described by a Maxwell-Boltzmann spectrum with peak wavelength $\lambda_c = 0.5$ nm produces a fringe pattern with visibility of 43% (orange trace). The system parameters include a grating with modulation period $W$ of 120 $\mu$m, pitch of 1 $\mu$m, source-to-MPG distance of 0.5 m, and MPG-to-detector distance of 2.39 m.
3.4. Single-Shot Phase Contrast Recovery with MPG

Figure 3.10 shows examples of the phase objects (triangle, trapezoid, and parabola) recovered using the single-shot method described in section 2.6. The parameters of MPG system used are $W = 50 \, \mu m$, $Pitch = 2.1 \, \mu m$, $D_{sg} = 1 \, m$, and $D_{sd} = 5 \, m$. The pixel size is $100 \, \mu m$ and the slit width is $200 \, \mu m$. In the top row triangular phase objects are recovered with maximum phases of $0.8\pi$ rad and $8\pi$ rad. In the middle row trapezoidal objects with flat regions in the center are recovered with maximum phases of $0.8\pi$ rad and $8\pi$ rad. Lastly, in the bottom row parabolic objects are recovered with maximum phases of $0.8\pi$ rad and $8\pi$ rad. The phase recovery is repeated for different object-to-detector distances, and the root-mean-square error (RMSE) of the recovered phase compared to the true phase is shown in Figure 3.11. The RMSE is defined as follows

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(\frac{\hat{y}_i - y_i}{n}\right)^2}$$  \hspace{1cm} (13)

where $\hat{y}_i$ are the true values of the object’s phase profile, $y_i$ are the recovered values of the object’s phase profile, and $n$ is the number of data points/observations in the phase profile. Figure 3.11(a) shows the RMSE analysis for the three phase objects with a peak phase of $0.8\pi$ rad, and Figure 3.11(b) shows the RMSE analysis for the three phase objects with a peak phase of $8\pi$ rad. When normalized to the peak phase of the phase object, the maximum RMSE for the triangle, trapezoid, and parabolic phase objects with $0.8\pi$ rad peak phases are $4.2\%$, $7.3\%$, and $7.6\%$, respectively. Likewise, the maximum RMSE for the triangle, trapezoid, and parabolic phase objects with $8\pi$ rad peak phases are $4.1\%$, $6.4\%$, and $8.4\%$, respectively.
Figure 3.10. Single-Shot phase acquisition using the parameters $W = 50 \mu m$, Pitch $= 2.1 \mu m$, $D_{sa} = 5 \text{ m}$, and $D_{sg} = 1 \text{ m}$. The pixel size is $100 \mu m$ and slit size $200 \mu m$. 
Figure 3.11. RMSE of recovered phase and true phase for (a) trapezoidal, triangular, and parabolic objects with peak phase $0.8\pi$ rad (b) and a peak phase of $8\pi$ rad.
Chapter 4. Discussion

We have used analytical simulations using Sommerfield-Rayleigh diffraction integrals (SRDI) to investigate a proposed novel PGMI and assess its imaging performance relative to standard-dual-grating systems that require more phase gratings. The major finding of this study was that the simulations show the MPG system to be a promising type of PGMI by producing higher visibilities (50% to 85%) than the standard-dual-grating system for a range of possible modulation period W parameters (Figure 3.5). At the lower end of the modulation periods, a lower pitch MPG or a narrower slit can aid in yielding higher visibilities (Figures 3.6 – 3.7); the former can add complexity to the fabrication of the grating and the latter would reduce the flux reaching the detector. We note that operating in the smaller W range improves the phase sensitivity, yielding as much 10.0 x 10³ for SDDs up to 3.5 m. The performance improvement with MPG comes with the added benefit of the MPG system being a single-phase-grating system, not requiring precise alignment of two gratings. For the phase recovery, a closer analysis of Figure 3.11 did not show an RMSE trend with respect to the object-to-detector distance. This suggests that the errors are dominated by the blurring due to the pixel size resolution and geometric unsharpness that grows with slit size rather than other factors such as sensitivity loss when the object is closer to the detector.

High fringe visibility and phase sensitivity are critical to imaging performance in grating interferometry. The signal-to-noise ratios (SNR) of differential phase contrast images (DPCI) and dark-field images (DFI) are strongly dependent on the visibility [12]. Our findings suggest that our MPG system could be conducive in producing high fringe visibility measurements and, consequently, images with high SNR. This could allow for the implementation of experiments with weaker signals or shorter exposure times when using our MPG system. The phase sensitivity
for an interferometer relates the ability to measure the phase difference $\Delta \phi$ of the interference fringes for a small refraction angle $\alpha$ imposed by an object on the neutrons, and the differential phase contrast of the DPCI is directly proportional to the phase sensitivity. Hence, when operating in the smaller $W$ range, our findings suggest out MPG system could be capable of producing images with high differential phase contrast. Moreover, our MPG system is PGMI that only requires a single grating; the advantage of using a single grating is reducing grating misalignment issues commonly found in multiple-grating systems, which can lead to fringe visibility or contrast loss [9-11]

The margin of improvement the MPG system displays is high for most cases and appreciable from what is currently reported in literature for the standard-dual-grating system. For example, $W = 120 \mu m$ and pitch $= 1 \mu m$ produces a visibility of 85% with a comparable SDD as the standard-dual-grating system operating in monochromatic configuration. When that same MPG configuration was illuminated by a polychromatic beam, the visibility dropped to 43% which is still higher than the best visibility case in Pushin et al. [9], and it demonstrates that our MPG system is potentially robust to polychromatic neutron sources. Phase sensitivity information is sparse for the standard-dual-grating system but available in the literature for the conventional neutron Talbot-Lau interferometer (TLI). For modulation periods $W$ between 70 $\mu m$ to 120 $\mu m$, we note that the theoretical maximum phase sensitivities are between $4.1 \times 10^3$ to $10.0 \times 10^3$ when using SDDs up to 3.5 m. These sensitivities are comparable or greater than that of the conventional neutron TLI of $4.5 \times 10^3$ with a SDD of 3.5 m and a beam wavelength of 0.44 nm [7,15].

This work has several strengths. The analytical simulations are based fully on diffraction theory (SRDI) which allow for relatively short computation time and low complexity. Their predictive power was verified by simulating the standard-dual-grating system with a
monochromatic configuration from Ref. [9], which produced visibility results in good agreement with experimental data and fringe periods which also agreed well with the theory given in Ref. [8]. Moreover, our N-SRDI simulator is capable of handling a variety of PGMI designs (e.g. modulated phase gratings and standard-dual-gratings) that are illuminated by either monochromatic or polychromatic neutron sources in cone beam geometry. SDRI computations are further reduced by using our convolution method, which accounted for blurring at the detector due to the slit width, by avoiding the need to model an extended source as a series of point sources.

The main limitation of this work is the need of experiments to verify the simulation results of the MPG system. Another limitation of this work is that we have only simulated ideal phase gratings (phase gratings with perfect rectangular profiles) without any defects that may reduce the visibility of the fringe pattern at the detector. Additionally, we are not taking into neutron coherence loss from air scattering over long distances and possible mechanical vibrations of the grating which may also have an impact on the fringe visibility.

Future work includes doing experiments at NIST in Gaithersburg, Maryland at the cold neutron imaging beamline NG6 using MPGs that have been made from gold and originally designed to have phase heights of \( (\pi, \pi/4) \) for 25 keV x-rays. Since we are repurposing these MPGs for cold neutrons with a wavelength of \( \lambda = 0.44 \text{ nm} \), preliminary simulations showed rather poor fringe visibility of 7% if we consider using a slit width of 200 \( \mu \text{m} \), an imaging detector with a pixel size of 100 \( \mu \text{m} \), source-to-MPG distance of 1.2 m, and a SDD of 2.99 m. However, fringe patterns may still be observed; and we will simulate other setups which may give us a better fringe visibility.
Chapter 5. Conclusion

We have proposed a novel phase-grating moiré interferometer system for cold neutrons that only requires a single modulated phase grating (MPG) for phase-contrast imaging, as opposed to the two or three phase gratings in previously employed PGMI systems. The MPG system promises to deliver a significantly better fringe visibility relative to previous experiments in the literature that use a two-phase-grating-based PGMI. When operating in the smaller $W$ range and using a SDD of up to 3.5 m, the MPG system could provide comparable or greater theoretical maximum phase sensitivity when compared to the conventional Talbot-Lau interferometer. The single MPG reduces the precise alignment requirements needed for multi-grating systems. Like other PGMI systems, the MPG system does not require the high-aspect-ratio absorption gratings used in Talbot-Lau interferometers which are challenging to manufacture and reduce the neutron flux reaching the detector.
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Vita

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