Calculating Gravitational Radiation from Collisions

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**Abstract.** The development of both ground- and space-based gravitational wave detectors provides new opportunities to observe the radiation from binaries containing neutron stars and black holes. Numerical simulations in 3-D are essential for calculating the coalescence waveforms, and comprise some of the most challenging problems in astrophysics today. This article briefly reviews the current status of efforts to calculate black hole and neutron star coalescences, and highlights challenges for the future.

1. Introduction

Binaries containing black holes (BHs) or neutron stars (NSs) are among the most important and interesting sources for the gravitational wave detectors expected to start operation in the early part of the 21st century. These binaries spiral together due to the emission of gravitational radiation, leading to the final collision and coalescence of their components. The NS/NS, NS/BH, and stellar BH/BH binaries are target sources for ground-based interferometers such as LIGO, VIRGO, GEO600, and TAMA, while the space-based LISA is expected to be most sensitive to massive BH binaries (see the articles by Weiss and Kalogera in these Proceedings). Astrophysically, the data from these detectors can provide important insights into NS properties; the equation of state of matter at nuclear densities; models of gamma-ray bursts; active galactic nuclei and quasars; and the strong-field regime of gravity, including unambiguous detection of BH formation (Schutz 1997; Thorne 1998).

Calculations of the gravitational waves from such binaries focus on three physical regimes. During the *inspiral phase*, the components are widely separated and can be treated as point particles, allowing analytic calculations using post-Newtonian (PN) expansions. Templates constructed from the resulting waveforms are expected to form a key component of data analysis and source identification (Flanagan & Hughes 1998). The *coalescence phase* begins when the compact objects are close enough to suffer tidal deformation, and continues as a combination of relativistic and hydrodynamic effects drives the stars together on more rapid timescales. Calculation of the waveforms produced during this stage requires 3-D numerical simulations that solve the full Einstein equations and, for a NS, general relativistic hydrodynamics; in some cases, the formation of a BH must also be modeled. Finally, the *ringdown phase* encompasses the late-time behavior of the merger product or remnant, radiating in its normal modes (Thorne 1998).
Numerical simulation of the coalescence phase using full general relativity is one of the most challenging problems in astrophysics today. This article surveys the current status of efforts to calculate coalescence waveforms for NS/NS, NS/BH, and BH/BH binaries.

2. Requirements and Challenges for Successful Simulations

The difficulties in the 3-D numerical simulation of binary coalescence begin in specifying the initial state of the system, which must correspond to a physically realistic binary evolving through the PN inspiral phase (Brady, Creighton, and Thorne 1998). Specifically, there should be no incoming gravitational radiation in the system, and both the outgoing gravitational waves and the orbital parameters should correspond to those produced during the PN inspiral stage. These conditions must all be met within the constraints of the initial value Einstein equations, which are typically a set of coupled nonlinear equations. Although relativistic simulations to date have used highly simplified initial conditions that do not fulfill these requirements, progress is being made towards developing more plausible initial conditions (Alvi 2000; Buonanno & Damour 2000; Damour, Jaranowski, & Schäfer 2000; Marronetti & Matzner 2000).

Once the initial data has been specified, the remaining Einstein equations are solved to evolve the system in time. This evolution must be stable over timescales corresponding to at least a few binary orbits. Various formulations of the Einstein equations, notably the hyperbolic (Reula 1998; Alcubierre, et al. 1999) and conformal ADM (Shibata & Nakamura 1995; Baumgarte & Shapiro 1998) approaches, have been introduced in recent years to improve the stability of the evolution. Additional equations enter to specify the time-slicing of the spacetime and the evolution of the spatial coordinates. This latter gauge condition is likely to be quite important in keeping the binary in a co-rotating frame of reference; see below. In the case of a NS or WD, the equations of general relativistic hydrodynamics must also be solved, with high-resolution shock-capturing techniques playing an increasingly important role (Font, et al. 2000). If the binary contains a BH, additional challenges arise in moving the BH across the grid. While current efforts focus on excising the interior of the BH, which avoids the central singularity, the long-term stability of the inner boundary at the apparent horizon remains a challenge (Alcubierre & Bruegmann 2000). And, the code must be capable of handling the formation of a BH from the merger, an event that is generally detected numerically by finding an apparent horizon around the remnant (Alcubierre, et al. 2000; Huq, Choptuik, & Matzner 2000). The code should continue to evolve stably as the system enters the ringdown phase.

The gravitational radiation emitted during the coalescence produces its own set of challenges. Typically, the radiation will have wavelengths $\lambda_{GW} \sim 10 - 100L$, where $L$ is the scale of the source. The computational domain must be large enough to allow the signals produced near the source to propagate outward.

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1These formalisms assume that the initial data is given on a 3-D spacelike Cauchy surface. Methods employing characteristic or null slicings of spacetime are also under development (Bishop, et al. 1999).
and develop into gravitational waves in the radiation zone, and at the same time provide adequate resolution for the source dynamics. Such widely varying length and time scales point to the use of adaptive mesh refinement (AMR) as a critical component of success (Papadopoulos, Seidel, & Wild 1998; New, et al. 2000).

Gravitational wave detectors are located asymptotically far from the astrophysical sources. For a meaningful comparison with the observations, the radiation generated in the simulations must also be measured “at infinity.” One promising means of achieving this is to extract the waves on a spacelike slice at some distance \( \gg L\), and then propagate them along the characteristics to infinity using a different code (Bishop, et al. 1998). Of course, there will be some scattering of these waves back towards the source and thereby back into the original code. Currently, efforts are aimed at achieving this matching in a stable manner, and thereby producing good, physical outer boundary conditions for the source evolution codes.

Finally, numerical relativists generally use high performance computers and sophisticated visualization techniques to perform their simulations and extract the physics from the results. Further progress depends on continued access to advanced computing resources (Suen 1999).

3. Simulations of NS/NS Coalescence

Binaries containing 2 NSs have long been considered among the most promising sources for the first generation of ground-based interferometers. Such instruments could observe the inspiral phase of these systems during the last few minutes of their evolution, as the gravitational waves sweep upward in frequency from \( \sim 10 \) Hz to \( \sim 1000 \) Hz, yielding several thousand cycles of the signal across the broadband range of the detectors (Cutler, et al. 1993). The coalescence phase is expected to occur at rather high frequencies, typically \( \gtrsim 1\)kHz. Observations of coalescence are of considerable interest, as the waveforms are expected to contain information about NS structure and the nuclear equation of state at high densities (Zhuge, Centrella, & McMillan 1994, 1996), as well as general relativistic effects. Coalescing NS binaries are also astrophysically important as models for \( \gamma \)–ray bursts (Ruffert, Janka, & Schäfer 1996) and in the production of \( r \)-process nuclei (Rosswog et al. 1998).

Today, there are a variety of NS/NS coalescence simulations, ranging from those using the Newtonian limit to models with full general relativity. Simulations have been carried out using both synchronized (co-rotational) and non-synchronized (irrotational) binaries. The latter are considered more realistic, since NS viscosities are expected to be too small to cause synchronization before the stars coalesce (Kochanek 1992; Bildsten & Cutler 1992). In this article we will give a brief survey of coalescing NS simulations, highlighting important lessons that have been learned. For a more complete list, see the recent review by Rasio & Shapiro (1999).

Newtonian Models

In models with purely Newtonian gravitational fields, the gravitational waves are calculated using the quadrupole approximation (Misner, Thorne, & Wheeler 1973) with no gravitational back-reaction.Researchers in this area have
used either grid-based Eulerian hydrodynamics or Lagrangian smoothed particle hydrodynamics (SPH); see Faber, Rasio, & Manor (2000) for references. Such models are strictly valid only in the limit $GM/Rc^2 \ll 1$, for an object of mass $M$ and radius $R$. Since $GM/Rc^2 \sim 0.2$ for a typical NS, Newtonian simulations can only provide a rough guide to the physics expected in fully relativistic models of NS/NS coalescence. Nevertheless, Newtonian simulations remain an important tool for exploring gravitational wave phenomena and provide a simplified arena in which numerical techniques can be tested.

Coalescence in the Newtonian limit is driven by global hydrodynamic instabilities (Lai, Rasio, & Shapiro 1994; Rasio & Shapiro 1992) that cause the stars to plunge towards each other once they are close enough for tidal effects between them to become sufficiently strong. SPH simulations by Zhuge, Centrella, & McMillan (1994, 1996) in which the NSs are modeled as polytropes show that the gravitational wave energy spectrum $dE/df$ has a steep drop at a cutoff frequency, typically $f_{\text{cut}} \sim 2\text{kHz}$. The exact location of $f_{\text{cut}}$ depends on the NS radius $R$, the stiffness of the equation of state, the NS spin, and the mass ratio of the components. Ringing oscillations of the merged remnant can produce additional structure at even higher frequencies. New & Tohline (1997) carried out Eulerian simulations of synchronized binaries in both rotating and inertial (non-rotating) reference frames. They showed that the numerical viscosity caused by fluid flowing through the grid can produce spurious instability leading to coalescence. Thus a binary that is stable when evolved in the rotating frame, becomes unstable to merger when computed in the inertial frame. A recent study by Swesty, Wang, & Calder (1999) further underscores the importance of using 1PN quantities for consistency, all 1PN quantities must be small, requiring $GM/Rc^2 \ll 1$. To this end, Shibata, Oohara, & Nakamura (1997) and Ayal, et al. (1999) carried out 1PN mergers using $GM/Rc^2 \sim 0.03$. However, a realistic NS typically has $GM/Rc^2 \sim 0.2$. Also, since the 2.5PN radiation reaction terms scale as $(GM/Rc^2)^{2.5}$, imposing $GM/Rc^2 \ll 1$ results in very weak radiation reaction effects. Faber, et al. (2000) employ a hybrid scheme in which realistic NS parameters are used for the 2.5 PN radiation reaction terms, and the 1PN terms are scaled down artificially to keep them small.

Post-Newtonian Models

The next level of sophistication incorporates the lowest order post-Newtonian corrections to Newtonian gravity (1PN) and gravitational radiation reaction (2.5PN). Early calculations were carried out by Oohara & Nakamura (1992) using Eulerian techniques. The most recent PN simulations have been done by Faber, et al. (2000; see also Faber & Rasio 2000) using SPH, who find that the addition of 1PN corrections can lower the maximum gravitational wave luminosity and produce differences in the shape of the signal compared to Newtonian models.

The use of PN corrections in NS/NS simulations poses certain problems. For consistency, all 1PN quantities must be small, requiring $GM/Rc^2 \ll 1$. To this end, Shibata, Oohara, & Nakamura (1997) and Ayal, et al. (1999) carried out 1PN mergers using $GM/Rc^2 \sim 0.03$. However, a realistic NS typically has $GM/Rc^2 \sim 0.2$. Also, since the 2.5PN radiation reaction terms scale as $(GM/Rc^2)^{2.5}$, imposing $GM/Rc^2 \ll 1$ results in very weak radiation reaction effects. Faber, et al. (2000) employ a hybrid scheme in which realistic NS parameters are used for the 2.5 PN radiation reaction terms, and the 1PN terms are scaled down artificially to keep them small.
Additionally, the effects of PN corrections on the separation at which the NSs leave their quasi-circular inspiral orbits and begin to plunge toward the center are not yet clear. Faber & Rasio (2000) find that the PN corrections cause the plunge to begin at larger separations than in the Newtonian limit. However, Shibata, Taniguchi, & Nakamura (1998) have analyzed equilibrium sequences of binary stars, and find that the orbits generally become unstable at smaller separations as PN corrections are increased. This is an important issue to resolve, since a plunge beginning at larger separation will shift the coalescence waveform to lower frequencies. Newtonian simulations show the coalescence occurring at relatively high frequencies that are outside the broadband sensitivity of first-generation interferometers such as LIGO-1 (Zhuge, et al. 1994, 1996). New experimental techniques may improve the sensitivity of advanced interferometers at high frequencies (Meers 1988, Strain & Meers 1991; see also [http://www.ligo.caltech.edu/~ligo2/]).

**General Relativistic Models**

The earliest simulations of binary NS coalescence in full general relativity were carried out by Nakamura, Oohara, & Kojima (1987) using Eulerian techniques. Although there are currently a number of efforts worldwide aimed at developing relativistic coalescence models (e.g. Font, et al. 2000; Landry & Teukolsky 1999; Baumgarte, et al. 1999) this subject is still in its infancy. All fully relativistic codes to date are Eulerian.

The most advanced relativistic coalescence simulations have been carried out by Shibata & Uryu (2000). They solve the full Einstein equations using a conformal ADM formalism; these are coupled to the hydrodynamic equations that govern the sources, and the matter is taken to be a perfect fluid (Shibata 1999). Their initial data consists of 2 identical NSs, modeled as polytropes with $\Gamma = 2$, in a quasi-equilibrium orbit. Both irrotational and corotational models are considered. The orbit is then destabilized by reducing the angular momentum from its quasi-equilibrium value, and merger ensues. They find that a BH is formed on a dynamical timescale if the initial NSs are sufficiently compact. For corotational binaries, the remaining disk of matter around the BH can have a mass $\sim 5\% - 10\%$ of the total system rest mass. The merger of irrotational NSs to form a BH produces a disk with a much smaller mass, $< 1\%$ of the total system mass. For less compact initial NSs, the result of the coalescence is a differentially rotating massive NS.

4. **Simulations of NS/BH Coalescence**

Coalescence of a binary containing a NS and a stellar BH is another important source for ground-based gravitational wave detectors. Recent work suggests that observations of tidal disruption of a NS by a BH may provide measurements of the NS radius and equation of state (Vallisneri 2000). NS/BH systems are also astrophysically interesting as engines for $\gamma$—ray bursts (Janka, et al. 1999) and in the production of r-process nuclei (Lee 2000).

Simulations of NS/BH coalescence have been carried out in the Newtonian limit by Lee & Kluzniak (1995; 1999a,b; Kluzniak 1998) using SPH and by Janka, et al. (1999) using Eulerian techniques. In these efforts, the BH is
taken to be a Newtonian point mass surrounded by an absorbing boundary at
the Schwarzschild radius. These Newtonian codes are also extended to include
gravitational wave emission and backreaction using various techniques. Lee &
Kluźniak modeled the NS as a polytrope, whereas Janka, et al. used a more
physical description of NS matter.

The most recent NS/BH coalescence models have been calculated by Lee
(2000) using SPH with irrotational initial conditions. He finds that the fate of
the system depends strongly on the stiffness of the equation of state. After an
initial period of mass transfer, the NS is completely disrupted by the BH during
its second periastron passage for polytropic index $\Gamma = 2.5$. However, for a stiffer
model with $\Gamma = 3$, the NS is not completely disrupted; instead, a remnant is left
in a higher orbit about the BH. The resulting gravitational wave signals clearly
reflect the different outcomes.

Of course, relativistic effects are expected to be very important in NS/BH
systems. For this reason, simulations incorporating full general relativity may
introduce qualitatively new features in the waveforms and are eagerly awaited.

5. Simulations of BH/BH Coalescence

The merger of BH/BH binaries is, in many ways, the quintessential general rel-
ativistic source of gravitational radiation. Binaries containing BHs with masses
$\sim 10M_\odot$ are an important target for ground-based interferometers, while the
coalescence of binaries containing massive BHs $\sim 10^5 - 10^6 M_\odot$ is a prime candi-
date for observation by LISA. Since the dynamics of BH/BH binaries scales with
the total system mass $M$, observations in each of these mass ranges are pertinent
to the same basic BH physics (Thorne 1998). In particular, the gravitational
waves produced during these mergers are expected to have large amplitudes and
highly nonlinear characters, and to provide important tools for understanding
dynamical spacetime curvature. Astrophysically, the detection of BH/BH co-
alescences can yield important information on BH binary formation in dense
stellar systems such as galactic nuclei and globular clusters (Portegies Zwart &
McMillan 2000), as well as on active galactic nuclei and binary quasars (Mort-
lock, Webster, & Francis 1999).

Numerical simulations of BH/BH coalescence are, by nature, fully general
relativistic. The early work of Hahn & Lindquist (1964) was followed by a
major effort by Smarr and Eppley (Smarr 1978; Eppley 1977) that produced the
first simulations of gravitational radiation from axisymmetric, head-on collisions
of equal mass BHs in 2-D. Smarr and Eppley pioneered the use of singularity-
avoiding spacetime slicing in numerical relativity, which was used throughout the
1980s and 1990s. The most recent work using this approach is the simulation of
non-head-on collisions of spinning BHs in 3-D by Bruegmann (1999).

However, singularity-avoiding slicings typically give rise to problems, such
as grid stretching, that cause the simulations to fail after a relatively short
time. This technique is thus unsuitable for modeling BH/BH binaries for long
durations, such as the time from the late inspiral phase (when the point-mass
approximation breaks down) through coalescence and ringdown. A more promis-
ing approach is to use techniques in which the interior of the BH is excised from
the computational domain (Seidel & Suen 1992). Current efforts are focused on
developing appropriate “inner boundary conditions” at the excision surface that will allow long-term, stable evolutions (Alcubierre & Bruegmann 2000; Lehner, et al. 2000).

The first simulations of non-head-on BH collisions using singularity excision were recently carried out by Brandt, et al. (2000), with both spinning and non-spinning BHs. They used apparent horizon conditions to mark the excised regions, and were able to follow the BHs through merger, signalled by the formation of a single apparent horizon around the system. Preliminary estimates of the area of this horizon indicate that $\sim 2.6\%$ of the total mass is radiated away as gravitational waves. After a moderate time, however, instabilities caused these models to fail. Efforts are underway to determine the source of these instabilities, and to develop techniques to eliminate them.

6. Outlook

Gravitational wave astronomy offers many exciting opportunities to explore the physical universe. Significant challenges remain, however, in the experimental, theoretical, and computational arenas. In particular, fully general relativistic simulations are currently in their infancy; while important pieces of the puzzle are starting to take shape, considerable effort will be required to produce robust, reliable models. Ultimately, success in gravitational wave astronomy can be expected to involve partnerships and collaborations among experimentalists, relativity theorists, computational modelers, astronomers, and astrophysicists. The resulting synergy and creativity promise a bright and exciting future.

Acknowledgments. It is a pleasure to thank Kimberly New and Richard Matzner for comments on the manuscript. This work was supported by NSF grant PHY-9722109.

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