On the nuclear stopping in asymmetric colliding nuclei

Varinderjit Kaur and Suneel Kumar

School of Physics and Materials Science,
Thapar University Patiala-147004,
Punjab (India)

Rajeev K. Puri

Department of Physics,
Panjab University, Chandigarh (India)

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Abstract

Using an isospin-dependent quantum molecular dynamics (IQMD) model, nuclear stopping is analyzed in asymmetric colliding channels by keeping the total mass fixed. The calculations have been carried by varying the asymmetry of the colliding pairs with different neutron-proton ratios in center of mass energy 250 MeV/nucleon and by switching off the effect of Coulomb interactions. We find sizable effect of asymmetry of colliding pairs on the stopping and therefore on the equilibrium reached in a reaction.

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I. INTRODUCTION

In recent years, the study of heavy-ion collisions from low to relativistic energies has been focused on various phenomena that include the multi-fragmentation [1], anisotropy in the momentum distribution [2, 3], as well as global stopping of the nuclear matter [4]. As we know, nuclear stopping is one of the essential observables that depends crucially on the reaction dynamics. The stopping has also been linked with the degree of thermalization and equilibrium reached in a reaction. This problem has been handled both theoretically and experimentally in recent years. Some have also tried to correlate it with the production of light charged particles. Even the role of symmetry energy has also been explored in this domain. It still remains to be seen how stopping (and/or thermalization/equilibrium) is affected when the asymmetry of the reacting partners is altered. As noted, the dynamics and energy deposition in asymmetric reaction can be quite different than in a symmetric reaction. The asymmetry of a reaction has also been reported to affect the collective flow, balance energy as well as elliptical flow of the colliding pairs. It is worth mentioning that the outcome and physical mechanism behind the symmetric and asymmetric reactions are entirely different.

Following the establishment of radioactive beam facilities in many laboratories, it became possible to study the neutron-rich (or proton-rich) nuclear collisions at intermediate energies. The idea of studying nuclear stopping phenomena was introduced by Bass et al., [5] via the 'isospin-mixing' method. In 1988, Bauer [4], pointed out that the nuclear stopping at intermediate energies is determined by the mean-field as well as by the in-medium NN cross sections. In 1998, Li et al., [6] found that the degree of isospin equilibrium depends sensitively on both the in-medium NN cross section and equation of state of asymmetric nuclear matter. Another study in 2001 [7], explored the possibility of using nuclear stopping to probe the isospin dependence of in-medium NN cross-section. In 2002 [8], authors studied the behavior of excitation function \( Q_{zz}/nucleon \) and concluded that \( Q_{zz}/nucleon \) can provide information about the isospin dependence in terms of binary cross-sections. The recent work of many authors suggested [6-8] that the degree of approaching isospin equilibration helps in probing the nuclear stopping in HIC. Several more studies also focused in recent years on the isospin degree of freedom [9]. In 2006, one of us and co-workers [10] correlated the multifragmentation with global nuclear stopping. Their study revealed that the light charged particles (LCP's) act in a similar manner as the anisotropy ratio. They, however, excluded the isospin content of the colliding nuclei. In a recent communication [11], one of us and co-workers tried to study the effect of symmetry energy and isospin-dependent cross-section on nuclear stopping. Our findings revealed that the degree of stopping depends weakly on the symmetry energy and strongly on the
isospin-dependent cross-section. Most of the above mentioned theoretical and experimental studies concentrated on the dynamics for symmetric reacting partners. It, however, remains to be seen how asymmetry of a colliding pair affects the stopping and thermalization. As noted by FOPI collaboration [12], asymmetry of colliding pairs [13] can play decisive role in reaction dynamics. We plan to address this question in this present paper. In order to focus exclusively on the asymmetric aspects, we shall vary the masses of the projectile and target in such a manner that total system mass remains constant. Another hidden parameter that can affect outcome in asymmetric colliding nuclei is incident energy, we shall also keep center of mass energy fixed in present study. This study is conducted within the framework of IQMD model which is explained in the Sec.-II. The results are presented in Sec.-III, leading to the conclusions in Sec.-IV.

II. THE MODEL

The isospin-dependent quantum molecular dynamics (IQMD) [14] model treats different charge states of nucleons, deltas and pions explicitly, as inherited from the Vlasov-Uehling-Uhlenbeck (VUU) model. The IQMD model was used successfully in analyzing the large number of observables from low to relativistic energies. The isospin degree of freedom enters into the calculations via both cross sections and mean field.

In this model, baryons are represented by Gaussian-shaped density distributions

\[ f_i(r, p, t) = \frac{1}{\pi^2 \hbar^2} e^{-\frac{(r-r_i(t))^2}{2L^2}} e^{-\frac{(p-p_i(t))^2}{2\hbar^2}}. \] (1)

Nucleons are initialized in a sphere with radius \( R = 1.12A^{1/3} \) fm, in accordance with the liquid drop model. The Gaussian width which we are taking is in close agreement as given in ref. [15]. Each nucleon occupies a volume of \( \hbar^3 \) so that phase space is uniformly filled. The initial momenta are randomly chosen between 0 and Fermi momentum \( p_F \).

The nucleons of the target and projectile interact via two and three-body Skyrme forces and Yukawa potential. The isospin degrees of freedom is treated explicitly by employing a symmetry potential and explicit Coulomb forces between protons of the colliding target and projectile. This helps in achieving the correct distribution of protons and neutrons within the nucleus.

The hadrons propagate using Hamilton equations of motion:

\[ \frac{d\mathbf{r}_i}{dt} = \frac{d < H >}{d\mathbf{p}_i}; \quad \frac{d\mathbf{p}_i}{dt} = -\frac{d < H >}{d\mathbf{r}_i}. \] (2)
with
\[ < H > = < T > + < V > \]

\[
= \sum_i \frac{p_i^2}{2m_i} + \sum_i \sum_{j>i} \int \dot{f}_i(\vec{r}, \vec{p}, t)V^{ij}(\vec{r}, \vec{r})f_j(\vec{r}, \vec{p}, t)d\vec{r}d\vec{r}d\vec{p}d\vec{p}.
\]

(3)

The baryon-baryon potential \( V^{ij} \), in the above relation, reads as
\[
V^{ij}(\vec{r} - \vec{r}) = V^{ij}_{Skyrme} + V^{ij}_{Yukawa} + V^{ij}_{Coul} + V^{ij}_{Sym}
\]
\[
= t_1 \delta(\vec{r} - \vec{r}) + t_2 \delta(\vec{r} - \vec{r}) \rho^{-1}(\vec{r} + \vec{r})
\]
\[
+ t_3 \exp(|\vec{r} - \vec{r}|/\mu) + \frac{Z_i Z_j e^2}{|\vec{r} - \vec{r}|} + \frac{1}{\rho_0 T_3 T_3} \delta(\vec{r} - \vec{r}).
\]

(4)

Where \( \mu = 1.5 \text{fm} \), \( t_3 = -6.66 \text{MeV} \), \( t_4 = 100 \text{MeV} \). The values of \( t_1 \) and \( t_2 \) depends on the values of \( \alpha, \beta, \) and \( \gamma \). Here \( Z_i \) and \( Z_j \) denote the charges of the \( i \)th and \( j \)th baryon, and\( T_3, T_3 \) are their respective \( T_3 \) components (i.e. 1/2 for protons and -1/2 for neutrons). The parameters \( \mu \) and \( t_1, \ldots, t_6 \) are adjusted to the real part of the nucleonic optical potential.

### III. RESULTS AND DISCUSSION

In the present study, projectile mass is varied between 16 and 56 units and targets are chosen as different isotopes of \( Xe, Sn, Ru \) in such a way that total mass of the reaction remains constant (= 152) for all channels. For example, we take the reactions of \( \text{O}^{16} + \text{Sn}^{124}, \text{Si}^{28} + \text{Sn}^{120}, \text{S}^{32} + \text{Sn}^{112}, \text{Ca}^{40} + \text{Sn}^{112}, \text{Cr}^{50} + \text{Ru}^{102}, \) and \( \text{Fe}^{56} + \text{Ru}^{96} \) etc. Although, the total mass remains constant, the asymmetry of the reaction \( \eta = |(A_T - A_P)/(A_T + A_P)| \) keeps varying between 0.2 and 0.7. Nuclear stopping in HIC has been studied with the help of different variables. A direct measure of nuclear stopping is the rapidity distribution defined as

\[
Y(i) = \frac{1}{2} \ln \frac{E(i) + p_{\parallel}(i)}{E(i) - p_{\parallel}(i)},
\]

(5)

where \( E(i) \) and \( p_{\parallel}(i) \) are respectively, the energy and longitudinal momentum of the \( i \)th particle. For a complete stopping, one expects a single peaked Gaussian. Obviously, narrow Gaussian indicates better thermalization (equilibrium) compared to broader ones.
Fig. 1: The rapidity distribution $dN/dY$ as a function of reduced rapidity for free nucleons, LMF’s and IMF’s respectively at the center of mass energy $E_{C.M.} = 250$ MeV/nucleon. Different curves correspond to different asymmetries varying from 0.2 to 0.7 for semi-central impact parameter.

Another quantity used in the literature [16] is anisotropy ratio ($R$) defined as

$$R = \frac{2}{\pi} \left[ \frac{\sum_i |p_\parallel(i)|}{\sum_i |p_\perp(i)|} \right]$$

where summation runs over all nucleons. The transverse $p_\perp(i)$ and longitudinal $p_\parallel(i)$ momenta reads respectively as $\sqrt{p_x^2(i) + p_y^2(i)}$ and $p_z(i)$. Naturally, for a complete stopping $R$ should be close to unity. Some studies use quadrupole moment $Q_{zz}$ to analyze the stopping and thermalization. Quadrupole moment $Q_{zz}$ is defined as

$$Q_{zz} = \sum_i [2p_\parallel^2(i) - p_\perp^2(i)]$$

Naturally, for a complete stopping, $Q_{zz}$ should be close to zero.

Fig.1 shows the rapidity distribution $dN/dY$ for the emission of free nucleons as well as light mass fragments (LMF’s) [(2 $\leq A \leq 4$)], and intermediate mass fragments (IMF’s) [(5 $\leq A \leq A_{tot}/6$)] at a fixed centre-of-mass energy. Note that the asymmetry varies between 0.2 and 0.7 corresponding to the reactions of $^8$O+$^{54}$Xe (η = 0.7), $^{14}$Si+$^{54}$Xe (η = 0.6), $^{16}$S+$^{50}$Sn (η = 0.5), $^{20}$Ca+$^{50}$Sn (η = 0.4),...
$^{24}_{50}$Cr$^{102} + ^{44}_{44}$Ru (η = 0.3), $^{26}_{56}$Fe$^{56} + ^{44}_{44}$Ru (η = 0.2). The $Y_{C.M.}/Y_{beam} = 0$ corresponds to mid-rapidity (participant) zone and hence is responsible for the hot and compressed zone. On the other hand, $Y_{C.M.}/Y_{beam} \neq 0$ corresponds to spectator zone ($Y_{C.M.}/Y_{beam} < -1$ corresponds to target like (TL) and $Y_{C.M.}/Y_{beam} > 1$ corresponds to projectile like (PL) distributions). Interestingly, reaction corresponding to $\eta = 0.7$ yields peak at mid-rapidity which shifts towards negative side when one considers nearly symmetric reactions. We see that majority of free particles and LMF’s are emitted from the mid-rapidity region, whereas IMF’s are emitted from the spectator matter. From the shape of the Gaussian, one sees that free particles and LMF’s are better indicator for the thermal source at $\eta = 0.7$. If these reaction channels are analyzed at a fixed lab energy, the situation would have been entirely different. In that case, nearly symmetric reactions $\eta = 0.2$ would yield better thermalized source. It is worth mentioning that in many studies, one has kept the lab energy fixed. The absolute conclusion in these studies are not due to asymmetry alone.

In Fig.2, we display the impact parameter dependence of global variables (R and $Q_{zz}$) along with the multiplicity of LMF’s. The results are displayed at $E_{C.M.} = 250$ MeV/nucleon. Different curves in each panel represent different asymmetries. We observe that R and $1/Q_{zz}$ behave in a similar fashion (note that R and $Q_{zz}$ will behave in just opposite fashion). The amount of stopping/equilibrium increases with the asymmetry of the reaction. As we know, major contribution for the stopping of nuclear matter comes from the hot and compressed matter that decreases almost linearly with impact parameter. To correlate the degree of stopping with the multiplicity of light charged particles, we show in the last panel the impact parameter dependence of the multiplicity of light charged particles. The behavior of light charged particle with impact parameter is similar to that of anisotropic ratio and inverse of quadrupole moment. The decrease in the multiplicity of LMF’s complements with corresponding increase in the heavy fragments. These fragments are the remnant of the spectator matter. Therefore, a decrease in the multiplicity of LMF’s with impact parameter measures directly the decrease in the degree of equilibrium and hence global stopping. This implies that the LMF’s production can act as an indicator for nuclear stopping. At the same time, we also note that behavior with respect to asymmetry does behave in same fashion. This happens due to the fact that participant zone increases when one moves towards nearly symmetric reactions. Therefore, more LMF’s are produced for $\eta = 0.2$ compared to $\eta = 0.7$. It is worth mentioning that in the case of multiplicity of LMF’s, we talk about the number of particles while in the case of stopping, it is the momentum phase space (x, y and z components of momentum) that is responsible for the growth.
FIG. 2: The anisotropy ratio $R$, quadrupole moment $1/Q_{zz}$ and multiplicity of LMF’s as a function of impact parameters. Different curves correspond to different asymmetries varying from 0.2 to 0.7 for semi-central impact parameters.

The stopping at any time during the collision can be divided into the contributions emerging from the protons and neutrons. We decompose the stopping into the contributions due to protons and neutrons. Here, at each time step during the collision, stopping due to neutrons and protons is analyzed separately.

Fig.3(a) shows the final state quadrupole moment $1/Q_{zz}$ decomposed into contributions due to neutrons and protons as a function of asymmetry $\eta$. Fig.3(b) shows the final state anisotropy ratio $<R>$ as a function of the asymmetry of the system. We see no difference between the contributions due to neutrons and protons. This is due to the fact that $<R>$ is the ratio of the mean transverse momentum $p_{\perp}(i)$ to the mean longitudinal momentum $p_{\parallel}(i) = p_z(i)$. To see the clear contribution of neutrons and protons, one has to look into the contributions of transverse and longitudinal momenta as shown in Fig.3(c) and 3(d), respectively. A linear enhancement in the transverse and longitudinal momentum can be seen with asymmetry $\eta$. Further the contribution of neutrons exceeds the corresponding contributions due protons.

It will be of further interest to see whether the above findings depend on the isospin asymmetry (N/Z dependence) or not. For this, we display in Fig.4, the anisotropy ratio
FIG. 3: The quadrupole moment $1/Q_{zz}$ and anisotropy ratio $R$ as a function of asymmetry $\eta$ at a center of mass energy $E_{C.M.} = 250$ MeV/nucleon. The third and fourth panels show the variation of transverse and longitudinal momentum with asymmetry, respectively. Different curves correspond to the contribution of neutrons and protons along with total contribution.

$\langle R \rangle$, inverse of quadrupole moment $Q_{zz}$ and multiplicity of LMF’s as a function of neutron to proton ratio ($N/Z$) at different impact parameters ranging from central to peripheral one. We are also showing the results at impact parameter $\hat{b} = 0.1$ by changing the Gaussian width according to the formulae $\sigma = 0.16N^{1/3} + 0.49$ as given in ref [15]. Interestingly, we see no change in the results by the variation of Gaussian width. An increase in the number of neutrons will increase the number of collisions and hence the absolute value of $\langle R \rangle$, $1/Q_{zz}$ will increase with N/Z ratio. Our exclusive findings are: (1) maximum stopping is obtained for the systems with larger neutron content. This is true at all colliding geometries. This dependence diminishes as one moves from central to peripheral geometry. It is due to the fact that nuclear stopping is governed by the participant zone only. Moreover, in systems with more neutron content, role of symmetry energy could be larger, whereas effects due to isospin-dependent cross-section could play a dominant role in systems with less
neutron content (proton content). Therefore there is a possibility that the combined effect of symmetry energy and cross-section could be approximately the same for all systems with different neutron and proton content \[17\]. Also neutron-neutron or proton-proton cross-section is a factor of 3 lower than the neutron-proton cross-section. (2) On the other hand, the multiplicity of LMF’s follows the reverse trend. This is due to the reason that in case of \(N/Z = 1.1\ (\eta = 0.2)\), participant zone is more compared to \(N/Z = 1.4\ (\eta = 0.7)\). Therefore, more LMF’s are produced for \(N/Z = 1.1\) as compared to \(N/Z = 1.4\).

IV. CONCLUSION

Using the isospin-dependent quantum molecular dynamics (IQMD), we have studied the nuclear stopping in asymmetric colliding channels by keeping total mass fixed. The calculations have been carried out by varying the asymmetry of the colliding pairs with different neutron-proton ratios in the center of mass energy 250 MeV/nucleon and by switching off the effect of Coulomb interactions. The contribution of the neutrons and protons is checked in terms of anisotropy ratio \(< R >\) and quadrupole moment \(Q_{zz}\). The maximum stopping is obtained for the systems having maximum neutron-proton ratio because the contribution

FIG. 4: (Color online) Same as Fig. 2, but as a function of N/Z ratio. Different curves correspond to different impact parameters ranging between central to peripheral one.
of neutrons remains enhanced throughout the asymmetry range. Moreover, this dependence becomes weaker as one moves from central to peripheral geometry. Interestingly, reverse trend is obtained when we vary the multiplicity of LMF’s with N/Z ratio.

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