Asymmetric two-output quantum processor in any dimension

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We propose two different implementations of an asymmetric two-output probabilistic quantum processor, which can implement a restricted set of one-qubit operations. One of them is constructed by combining asymmetric telecloning with a quantum gate array. We analyze the efficiency of this processor by evaluating the fidelities between the desired operation and the one generated by the processor and show that the two output states are the same as the ones produced by the optimal universal asymmetric Pauli cloning machine. The schemes require only local operations and classical communication, they have the advantage of transmitting the two output states directly to two spatially separated receivers but they have a success probability of 1/2. We show further that we can perform the same one-qubit operation with unity probability at the cost of using nonlocal operations. We finally generalize the two schemes for \(D\)-level systems and find that the local ones are successful with a probability of \(1/D\) and the nonlocal generalized scheme is always successful.

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I. INTRODUCTION

Quantum computers are machines that employ quantum phenomena, such as quantum interference and entanglement, to solve a particular problem. The computers have to execute a program, which is built with the help of a precise set of instructions, in order to give the desired solution. A specification of this set of instructions is called an algorithm. There are algorithms which can be performed faster on a quantum computer than on a classical one \cite{1}: the Deutsch-Jozsa algorithm for solving the oracle problem \cite{2,3,4}, the Shor algorithm for factoring large integers \cite{5}, and the Grover algorithm for searching unsorted databases \cite{6}.

The most important component of the computer architecture is the processor. One crucial property of the classical processor is that we keep the same circuit regardless of the instructions that we want to perform. One may then ask how to construct a universal quantum processor, a fixed device that implements any desired program on the information stored in quantum systems? This problem was originally investigated by Nielsen and Chuang \cite{7}, where they proposed a model of the quantum processor, which consists of a quantum gate array \(G\) acting on the data state |\(\psi\rangle_d\rangle\) and on the program state |\(P\rangle\). The dynamics of the quantum gate array is

\[ G(|d\rangle|P_U\rangle) = (U|d\rangle)\langle P'_U|, \tag{1.1} \]

where \(U\) is a particular unitary operator implemented by the processor. Nielsen and Chuang found two important results \cite{7}: (i) the state |\(P'_U\rangle\) is independent of the data register |\(d\rangle\) and (ii) no deterministic universal quantum gate array exists. Therefore they showed how to construct an one-output probabilistic quantum processor, whose operating principle is that of quantum teleportation \cite{8}. The outcome of a Bell measurement tells us when the desired operation succeeded.

In the last few years, much progress has been made on generalizations and applications of the probabilistic quantum processor. Huelga et al. found a generalization of the method of teleporting a quantum gate from one location to another \cite{9,10,11}. Two more proposals of probabilistic quantum processors were considered by Preskill \cite{12} and Vidal et al. \cite{13}. Vidal et al. analyzed a probabilistic gate, which performs an arbitrary rotation around the \(z\) axis of a spin-1/2 particle with the help of an \(N\)-qubit program state \cite{13}. More complex probabilistic programmable quantum processors have been proposed by Hillery et al. \cite{14,15} by investigating the case when an arbitrary linear operation \(A\) is performed. They have built the network for this processor by using a quantum information distributor \cite{16,17} for qubits and then for \(D\)-level systems (quDit). Hillery et al. have analyzed several classes of quantum processors,
which execute more general operations, namely completely positive maps, on quantum systems \[13\]. In addition, they have found two important results: one can build a quantum processor to perform the phase-damping channel and that this is not possible in the case of the amplitude-damping channel \[18\].

Further extensions have been developed. In Refs. \[13, 20\] a quantum processor, which executes SU(N) rotations was considered, and was found that the probability of success for implementing the operation is increased if conditioned loops are used. Recently Brazier et al. have investigated the case when we have access to many copies of the program state \[21\]. They have shown that the probability of success cannot be increased and that it is the same as the one obtained using two different schemes: VMC \[13\] and HZB \[19\]. Positive-operator-valued measures (POVM) are the most general measurements allowed by quantum mechanics \[11, 22\]. Therefore it would be interesting to study the possibility of realizations of POVMs on quantum processors. This problem was investigated by Ziman and Bužek \[23\], where they showed how to encode a POVM into a program state. Another important class of operations is the one of generators of Markovian dynamics, which are relevant in the context of quantum decoherence. Koniorczyk et al. have recently proposed a scenario for the simulation of the infinitesimal generators of the Markovian semigroup on quantum processors \[24\]. Approximate processors, i.e., processors which implement a set of unitary operations with high precision, have been introduced by Hillery et al. in Ref. \[25\]. The accuracy of the processor is given by the process fidelity, which was shown to be maximum if one chooses the program state to be the eigenvector corresponding to the largest eigenvalue of a certain operator. This operator depends on some operators \(A_{jk}\), which characterize the quantum processor, and the desired unitary operator \(U\) to be implemented. We emphasize that all the processors described above generate only one output state. A two-output processor was proposed by Yu et al. \[26\] by combining symmetric telecloning of qubits \[27\] with a programmable quantum gate array.

In this paper we present two different schemes for obtaining a two-output quantum gate for \(D\)-level systems. Suppose the following scenario: an observer Peter has to teleport the result of a certain operation to two distant parties, Alice and Bob. We show how this task can be accomplished using a shared entangled state, local operations, and classical communication (LOCC) by proposing two schemes. We restrict our study to a certain class of unitary operations, which depends on a parameter \(\theta \in [0, 2\pi]\). In the first scheme, described in Sec. II. A, we extend Yu et al.’s scheme by considering asymmetric telecloning of qubits. The program state consists of a four-particle entangled state shared between Peter, Alice, Bob, and another observer Charlie, who holds an ancillary system. (As Charlie’s particle is only used as a resource, and is discarded at the end, Peter and Charlie may be the same party in those schemes that do not need nonlocal operations between Alice’s, Bob’s and Charlie’s particles.) Peter measures the data qubit and his particle \(P\), which is included in the program register, in the standard Bell basis, and then communicates the outcome. With a probability of 1/2, Alice and Bob are able to recover the desired mixed output states, whose fidelities are identical with the ones obtained by the optimal universal asymmetric Pauli cloning machine. We also demonstrate that if Alice, Bob, and Charlie may use nonlocal operations, the protocol will always be successful with preserved fidelities. In Sec. II. B, we propose the second local protocol, which requires a four-particle entangled state as a quantum channel being distributed between Peter, Alice, Bob, and Charlie. The desired unitary operation is encoded in a generalized Bell basis. The protocol is as follows: Peter performs a measurement in the generalized Bell basis and then announces the outcome to Alice, Bob, and Charlie. The probability of success of this second scheme is 1/2 and the asymmetric outputs received by Alice and Bob are identical with the ones obtained in the previous scheme. Given the resources of the four-particle states, these local schemes present the advantage that they can be implemented in the lab.

In Sec. III we present the generalizations of these protocols for quDits. The two local generalized schemes are successful with a probability equal to \(1/D\), while unit probability of success again requires nonlocal operations. Finally, in Sec. IV we summarize our conclusions.

II. ASYMMETRIC QUANTUM GATE ARRAY FOR QUBITS

In this section we present two schemes for performing the following scenario: we start with an arbitrary qubit state

\[|\psi\rangle_d = \alpha_0|0\rangle + \alpha_1|1\rangle, \quad (II.1)\]

where \(|\alpha_0|^2 + |\alpha_1|^2 = 1\). We want to obtain two optimal universal asymmetric clones of a certain unitary computational operator described by:

\[U_\theta = \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix}, \quad (II.2)\]

with \(\theta \in [0, 2\pi]\), applied on the arbitrary input data state \(|\psi\rangle_d\).
A. Two-output quantum processor for qubits

We define a state required in the preparation of the program as

$$|\xi\rangle_{PABC} = \frac{1}{\sqrt{2}}(|0\rangle_{P}|\phi_0\rangle_{ABC} + |1\rangle_{P}|\phi_1\rangle_{ABC}), \tag{II.3}$$

where

$$|\phi_0\rangle_{ABC} = \frac{1}{\sqrt{2}}(000 + p|011\rangle + (1-p)|101\rangle), \text{ and}$$

$$|\phi_1\rangle_{ABC} = \frac{1}{\sqrt{2}}(111 + p|100\rangle + (1-p)|010\rangle), \tag{II.4}$$

with $0 < p < 1$. The two states $|\phi_0\rangle$ and $|\phi_1\rangle$ are obtained by applying an optimal universal asymmetric Pauli cloning machine on the states $|0\rangle|00\rangle$ and $|1\rangle|00\rangle$. The data system $d$ and the first qubit $P$ of the state $|\xi\rangle$ belong to an observer Peter, while the qubits $A, B, C$ are held by other distant parties, Alice, Bob, and Charlie, respectively. This “ownership” of the qubits (or, later, quDits) will hold for all our schemes. Note that, in the case of the “local” schemes, Peter and Charlie may be identically the same, as Charlie’s qubit is discarded at the end.

Peter wants to teleport the result of the desired unitary operator $U_\theta$ to Alice and Bob. Peter encodes the information carried by the unitary operator, in the program state $|P_U\rangle_{PABC}$ by locally applying $U_\theta$ on the qubit $P$ (see Fig. 1):

$$|P_U\rangle_{PABC} = U_\theta \otimes I_{ABC}|\xi\rangle_{PABC} = \frac{1}{\sqrt{2}}(e^{i\theta}|0\rangle_{P}|\phi_0\rangle_{ABC} + e^{-i\theta}|1\rangle_{P}|\phi_1\rangle_{ABC}). \tag{II.5}$$

Let us write now the input state with the help of the standard Bell basis $|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ and $|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$:

$$|\psi\rangle_d|P_U\rangle_{PABC} = \frac{1}{2}(|\Phi^+\rangle_{dP}(\alpha e^{i\theta}|\phi_0\rangle + \alpha_1 e^{-i\theta}|\phi_1\rangle)$$

$$+ |\Phi^-\rangle_{dP}(\alpha e^{i\theta}|\phi_0\rangle - \alpha_1 e^{-i\theta}|\phi_1\rangle)$$

$$+ |\Psi^+\rangle_{dP}(\alpha_1 e^{i\theta}|\phi_0\rangle + \alpha e^{-i\theta}|\phi_1\rangle)$$

$$+ |\Psi^-\rangle_{dP}(\alpha_1 e^{i\theta}|\phi_0\rangle - \alpha e^{-i\theta}|\phi_1\rangle)]. \tag{II.6}$$

Peter performs a measurement in the Bell basis on the data qubit and the first qubit $P$ in the program state as it is shown in Fig. 1. Then he communicates the outcome of the measurement to Alice, Bob, and Charlie. With a probability equal to 1/4 the output is $|\Phi^+\rangle_{dP}$ and therefore the output is projected to

$$|\eta\rangle_{ABC} = \alpha e^{i\theta}|\phi_0\rangle + \alpha_1 e^{-i\theta}|\phi_1\rangle. \tag{II.7}$$

Accordingly, after tracing over Charlie’s qubit, the two final states of Alice and Bob, respectively, are:

$$\rho_A = \text{Tr}_{B,C}|\eta\rangle\langle\eta| = \frac{1}{2(1-p+p^2)}\{(2p(|\alpha|^2 + |1-p|^2)|0\rangle\langle0| + [2p|\alpha|^2 + (1-p)^2]|1\rangle\langle1|$$

$$+2p\alpha_0\alpha_1^* e^{2i\theta}|0\rangle\langle1| + 2p\alpha_0^*\alpha_1 e^{-2i\theta}|1\rangle\langle0|\}; \tag{II.8}$$

$$\rho_B = \text{Tr}_{A,C}|\eta\rangle\langle\eta| = \frac{1}{2(1-p+p^2)}\{(2(1-p)|\alpha|^2 + p^2)|0\rangle\langle0| + [2(1-p)|\alpha|^2 + p^2]|1\rangle\langle1|$$

$$+2(1-p)|\alpha_0|^2|\alpha|^2|0\rangle\langle1| + 2(1-p)|\alpha_0|\alpha_1^* e^{2i\theta}|0\rangle\langle1|\}. \tag{II.8}$$

The efficiency of the quantum processor is evaluated with the help of the fidelities of the output states with respect to the exact data register outputs $U_\theta|\psi\rangle_d$:

$$F_A = d(\psi|U_\theta^\dagger \rho_A U_\theta|\psi\rangle_d = \frac{1+p^2}{2(1-p+p^2)}, \text{ and}$$

$$F_B = d(\psi|U_\theta^\dagger \rho_B U_\theta|\psi\rangle_d = \frac{2-2p+p^2}{2(1-p+p^2)}. \tag{II.9}$$
FIG. 1: The scheme for the quantum processor for qubits. The input states consist of: an arbitrary data register $|\psi\rangle_d$ and the program register $|P\rangle_P = U_0 \otimes I_{ABC} |\xi\rangle_P$. Alice and Bob will obtain two mixed states $\rho_A, \rho_B$ that are implemented by the quantum processor.

If the output of the measurement is $|\Phi^+\rangle_dP$, then the final state is

$$|\eta\rangle_{ABC} = \alpha_0 e^{i\theta} |\phi_0\rangle + \alpha_1 e^{-i\theta} |\phi_1\rangle. \quad (\text{II.10})$$

Alice, Bob, and Charlie can transform the state $|\eta\rangle$ to the state $|\eta\rangle$ of Eq. (II.10) by applying the local unitary operator $V = \sigma_A^z \otimes \sigma_B^z \otimes \sigma_C^z$.

For the other two outcomes, when Peter obtains $|\Phi^+\rangle_dP$ and $|\Phi^-\rangle_dP$, the result cannot be transformed to the state of Eq. (II.10) by local operations. Hence there is no chance to obtain the desired mixed outputs $\rho_A$ and $\rho_B$ if we don’t allow for nonlocal transformations. The processor hence succeeds with the probability $1/2$ and generates two asymmetric output mixed states. The fidelities of the final states given by Eq. (II.9) are identical with the ones of the clones emerging from the optimal universal asymmetric cloning machine given in Refs. [28, 29, 30]. In the particular case when $p = 1/2$, we recover the result of Yu et al. [26].

However, if we allow nonlocal operations between Alice, Bob, and Charlie, which either entails their respective particles to interact directly, or via a shared auxiliary entangled state, it is also possible to convert the states $|\Phi^+\rangle_dP$ and $|\Phi^-\rangle_dP$ to $|\eta\rangle$ and thereby always succeed with the protocol. This is generally true for all our proposed schemes.

B. Two-output local quantum gate for qubits

We use the same channel $|\xi\rangle_{PABC}$ of Eq. (II.3) initially shared by Peter, Alice, Bob, and Charlie as above, but this time Peter performs a measurement in a different Bell basis, which depends on $\theta$ (see Fig. 2):

$$|\tilde{\Phi}\pm\rangle = \frac{1}{\sqrt{2}} (e^{-i\theta} |00\rangle \pm e^{i\theta} |11\rangle),$$

$$|\tilde{\Psi}\pm\rangle = \frac{1}{\sqrt{2}} (e^{-i\theta} |01\rangle \pm e^{i\theta} |10\rangle). \quad (\text{II.11})$$

The input state can be written as follows

$$|\psi\rangle_d|\xi\rangle_{PABC} = \frac{1}{2} \left[ (|\tilde{\Phi}^+\rangle_dP (\alpha_0 e^{i\theta} |\phi_0\rangle + \alpha_1 e^{-i\theta} |\phi_1\rangle) + |\tilde{\Phi}^-\rangle_dP (\alpha_0 e^{i\theta} |\phi_0\rangle - \alpha_1 e^{-i\theta} |\phi_1\rangle) + |\tilde{\Psi}^+\rangle_dP (\alpha_1 e^{i\theta} |\phi_0\rangle + \alpha_0 e^{-i\theta} |\phi_1\rangle) + |\tilde{\Psi}^-\rangle_dP (\alpha_1 e^{i\theta} |\phi_0\rangle - \alpha_0 e^{-i\theta} |\phi_1\rangle) \right]. \quad (\text{II.12})$$

Peter communicates the outcome to Alice, Bob, and Charlie. If the result of the measurement of particles $d$ and $P$ is $|\tilde{\Phi}^+\rangle_dP$, then we get the same state $|\eta\rangle$ of Eq. (II.7):

$$|\eta\rangle_{ABC} = \alpha_0 e^{i\theta} |\phi_0\rangle + \alpha_1 e^{-i\theta} |\phi_1\rangle. \quad (\text{II.13})$$
FIG. 2: The scheme for the local quantum gate-array for qubits. The information of the operation to be performed is encoded in a generalized Bell basis. Peter performs a measurement in this generalized Bell basis and then communicates the outcome to Alice, Bob, and Charlie. By LOCC, Alice and Bob are able to get the mixed states $\rho_A, \rho_B$.

Alice and Bob easily obtain the two asymmetric outputs $\rho_A$ and $\rho_B$ given by Eq. (III.8). If the outcome is $|\tilde{\Phi}^\pm\rangle_dP$, then Alice, Bob, and Charlie apply the local operator $V = \sigma_A^z \otimes \sigma_B^z \otimes \sigma_C^z$ in order to recover the state $|\eta\rangle$. The other two outcomes $|\tilde{\Psi}^\pm\rangle_dP$ and $|\tilde{\Psi}^\mp\rangle_dP$ will lead to a different result and the procedure fails unless nonlocal transformations are used. Hence, the total success probability is $p = 1/2$. This protocol is constructed using only LOCC and therefore it is suitable for distant computation at two locations.

III. ASYMMETRIC QUANTUM GATE ARRAY FOR QUDITS

A. Two-output quantum processor for quDits

One-output probabilistic programable quantum processors have recently been analyzed by Hillery et al. [14, 19] in the case when the data are encoded on a $D$-level systems (quDits). More precisely, they have investigated the possibility to construct a processor which performs an arbitrary linear operation $A$:

$$ A = \sum_{m,n=0}^{D-1} q_{mn} U^{(mn)}, $$

(III.1)

where the operators $U^{(mn)}$ form a basis in the Hilbert-Schmidt space

$$ U^{(mn)} := \sum_{s=0}^{D-1} \exp \left( -\frac{2\pi i sm}{D} \right) |s-n\rangle \langle s|. $$

(III.2)

We now propose a generalization of the quantum processor presented in the Sec. II. A for quDits. An arbitrary data quDit-state is described by

$$ |\psi\rangle_d = \sum_{k=0}^{D-1} \alpha_k |k\rangle, $$

(III.3)

where $\sum_{k=0}^{D-1} |\alpha_k|^2 = 1$. We want to analyze the action of the restricted class of unitary operators given by

$$ U_\theta = \cos \theta I + i \sin \theta U(\frac{\pi}{D},0) = \sum_{s=0}^{D-1} \exp((-1)^s i \theta) |s\rangle \langle s | = \begin{pmatrix}
  e^{i\theta} & 0 & \ldots & 0 & 0 \\
  0 & e^{-i\theta} & \ldots & 0 & 0 \\
  \vdots & \vdots & \ddots & \vdots & \vdots \\
  0 & 0 & \ldots & e^{i\theta} & 0 \\
  0 & 0 & \ldots & e^{-i\theta} & 0 
\end{pmatrix}, $$

(III.4)
where $D$ is assumed to be even. We define a family of states which depends on a parameter $p$, $0 < p < 1$:

$$
|\phi_j\rangle = \frac{1}{\sqrt{1 + (D - 1)(2p^2 - 2p + 1)}}(|j\rangle|j\rangle|j\rangle + p \sum_{r=1}^{D-1} |j\rangle|j + r\rangle|j + r\rangle)
$$

$$
+(1-p)\sum_{r=1}^{D-1} |j + r\rangle|j\rangle|j + r\rangle),
$$

(III.5)

where $j = 0, \ldots, D - 1$, and $j + r = j + r$ modulo $D$. These states were found by one of us in Ref. 28 by considering the action of an optimal universal asymmetric Heisenberg cloning machine on the state $|j\rangle|00\rangle$. In addition, we introduce the state $|\xi\rangle_{PABC}$ as follows:

$$
|\xi\rangle_{PABC} = \frac{1}{\sqrt{D}} \sum_{j=0}^{D-1} |j\rangle_P|\phi_j\rangle_{ABC}.
$$

(III.6)

Peter encodes the operation $U_\theta$ in the state of a program register $|P_U\rangle_{PABC}$ as follows

$$
|P_U\rangle_{PABC} = U_\theta \otimes |ABC\rangle = \frac{1}{\sqrt{D}} \sum_{j=0}^{D-1} \exp((-1)^j i \theta) |j\rangle_P|\phi_j\rangle_{ABC}.
$$

(III.7)

We denote by $|\Phi_{m,n}\rangle$ the standard Bell basis for quDits:

$$
|\Phi_{m,n}\rangle = \frac{1}{\sqrt{D}} \sum_{k=0}^{D-1} \exp \left(\frac{2\pi i kn}{D}\right) |k\rangle|k + m\rangle.
$$

(III.8)

The input state can be written by using the standard Bell basis as

$$
|\psi\rangle_d |P_U\rangle_{PABC} = \frac{1}{D} \sum_{k,m,n=0}^{D-1} \alpha_k \exp((-1)^{k+m} i \theta) \exp \left(\frac{-2\pi i kn}{D}\right) |\Phi_{m,n}\rangle_{dP} |\phi_{k+m}\rangle_{ABC}.
$$

(III.9)

The desired output state should be

$$
|\Lambda\rangle = U_\theta |\psi\rangle_d = \sum_{j=0}^{D-1} \alpha_j \exp \left((-1)^j i \theta\right) |j\rangle.
$$

(III.10)

A measurement in the standard Bell basis onto the data state and the first qubit $P$ of the program state is performed by Peter. Then he announces the outcome of the measurement to Alice, Bob, and Charlie. With a probability $1/D^2$ he gets the outcome $|\Phi_{0,n}\rangle$ (where $n$ is a fixed number) and at the same time the state of the quDits $A, B$ and $C$ becomes

$$
|\eta\rangle_{ABC} = \sum_{k=0}^{D-1} \alpha_k \exp \left((-1)^k i \theta\right) \exp \left(\frac{-2\pi i kn}{D}\right) |\phi_k\rangle_{ABC}.
$$

(III.11)

Let us now define a local operator $V_n := V_n^A \otimes V_n^B \otimes V_n^C$, where

$$
V_n^X := \sum_{j=0}^{D-1} \exp \left(\frac{2\pi ijn}{D}\right) |j\rangle\langle j|,
$$

$X = A, B$;

$$
V_n^C := \sum_{j=0}^{D-1} \exp \left(-\frac{2\pi ijn}{D}\right) |j\rangle\langle j|.
$$

(III.12)

Depending on Peter’s outcome, Alice, Bob, and Charlie apply subsequently the local operator $V_n$ on the output state (III.11) and obtain

$$
V_n |\eta\rangle_{ABC} = |\eta\rangle_{ABC} = \sum_{k=0}^{D-1} \alpha_k \exp \left((-1)^k i \theta\right) |\phi_k\rangle_{ABC}.
$$

(III.13)
Since there are $D$ equiprobable measurement outcomes $|\Phi_{0,n}\rangle$ for $n = 0, \ldots, D - 1$, the total success probability is $1/D$. The two mixed output states of the quantum processor are, in all the $D$ cases,

$$
\rho_A = \text{Tr}_{B,C}|\eta\rangle\langle \eta| = \frac{1}{1 + (D-1)(2p^2 -2p+1)} \left( \sum_{j=0}^{D-1} \left\{ [2p + (D-2)p^2]|\alpha_j|^2 + (1-p)^2 \right\} |j\rangle\langle j| 
+ [2p + (D-2)p^2] \sum_{j,k=0\atop j \neq k}^{D-1} \alpha_j \alpha_k^* \exp \{ [(-1)^j +(-1)^{k+1}]i\theta \} |j\rangle\langle k| \right); 
$$

$$
\rho_B = \text{Tr}_{A,C}|\eta\rangle\langle \eta| = \frac{1}{1 + (D-1)(2p^2 -2p+1)} \left( \sum_{j=0}^{D-1} \left\{ [D-2(D-1)p + (D-2)p^2]|\alpha_j|^2 + p^2 \right\} |j\rangle\langle j| 
+ [D-2(D-1)p + (D-2)p^2] \sum_{j,k=0\atop j \neq k}^{D-1} \alpha_j \alpha_k^* \exp \{ [(-1)^j +(-1)^{k+1}]i\theta \} |j\rangle\langle k| \right). 
$$

(III.14)

The fidelities of the output states are given by:

$$
F_A = \langle \Lambda | \rho_A | \Lambda \rangle = \frac{1 + (D-1)p^2}{1 + (D-1)(2p^2 -2p+1)}, \quad \text{and} 
$$

$$
F_B = \langle \Lambda | \rho_B | \Lambda \rangle = \frac{1 + (D-1)(1-p)^2}{1 + (D-1)(2p^2 -2p+1)}. 
$$

(III.15)

They are identical with the ones generated by the optimal universal asymmetric Heisenberg cloning machine given in Ref. 28. In the case when the result of Peter’s measurement is not $|\Phi_{0,n}\rangle$, Alice, Bob, and Charlie are not able to recover the state $|\eta\rangle_{ABC}$ given by Eq. (III.13) only by LOCC, and then the computation fails. However, both in this and the following protocol, the success probability can be boosted to unity if the three parties may use nonlocal operations on their particles.

B. Two-output local quantum gate for quDits

Our input data state is an arbitrary quDit given by Eq. (II.5) and belongs to Peter. We define a different Bell basis for quDits depending on $\theta$:

$$
|\tilde{\Phi}_{m,n}\rangle = \frac{1}{\sqrt{D}} \sum_{k=0}^{D-1} \exp \{ (-1)^{k+1}i\theta \} \exp \left( \frac{2\pi ikn}{D} \right) |k\rangle |k+m\rangle. 
$$

(III.16)

The input state can be written with the help of this new basis as:

$$
|\psi\rangle_d|\xi\rangle_{PABC} = \frac{1}{D} \sum_{k,m,n=0}^{D-1} \alpha_k \exp \{ (-1)^{k+m}i\theta \} \exp \left( -\frac{2\pi ikn}{D} \right) |\tilde{\Phi}_{m,n}\rangle_d |\phi_{k+m}\rangle_{ABC}, 
$$

(III.17)

where the state $|\xi\rangle_{PABC}$ was introduced in Sec. III. A by Eq. (III.6) while $|\phi_j\rangle$ are given by Eq. (III.5). Peter performs a measurement in the new Bell basis on particles $d$ and $P$. The interesting measurement outcome is $|\tilde{\Phi}_{0,n}\rangle$ obtained with the probability equal to $1/D^2$. Accordingly, the projected state of particles $A$, $B$, and $C$ is $|\eta\rangle$ given by Eq. (III.11). Alice, Bob, and Charlie have to apply the same local unitary operator $V_n$ described in the previous subsection in order that Alice and Bob to obtain the two asymmetric outputs $\rho_A$ and $\rho_B$ of Eq. (III.14). The total success probability of this scheme is $1/D$, because there are $D$ outcome $|\tilde{\Phi}_{0,n}\rangle$.

IV. CONCLUSIONS

In this paper we have demonstrated the possibility of building a two-output programmable processor of qubits, which generates two asymmetric states with a probability of success of $1/2$. It is characterized by the same fidelities as the
ones generated by the optimal universal asymmetric Pauli cloning machine. We have analyzed a second protocol which performs the same task, being performed with the help of a generalized Bell basis. We implement these schemes by using only local operations and classical communication, and therefore they are suitable for distributed computation to two spatially separated receivers.

In addition, we have presented the generalizations of these two protocols to $D$-level systems. The generalizations present the advantage of sending the outcomes to different locations and they are achieved with a probability of $1/2$.

Finally, we have shown how to increase the success probability to unity by considering nonlocal operations. In this case the two output states are obtained only after the particles A, B, and C have interacted, either directly or through an auxiliary entangled state.

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