Unraveling information about supranuclear-dense matter from the complete binary neutron star coalescence process using future gravitational-wave detector networks

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Gravitational waves provide us with an extraordinary tool to study the matter inside neutron stars. In particular, the postmerger signal probes an extreme temperature and density regime and will help reveal information about the equation of state of supranuclear-dense matter. Although current detectors are most sensitive to the signal emitted by binary neutron stars before the merger, the upgrades of existing detectors and the construction of the next generation of detectors will make postmerger detections feasible. For this purpose, we present a new analytical, frequency-domain model for the inspiral-merger-postmerger signal emitted by binary neutron stars systems. The inspiral and merger part of the signals are modeled with IMRPhenomD\textsubscript{NRTidalv2}, and we describe the main emission peak of postmerger with a three-parameter Lorentzian, using two different approaches: one in which the Lorentzian parameters are kept free, and one in which we model them via quasi-universal relations. We test the performance of our new complete waveform model in parameter estimation analyses, studying simulated signals obtained from both our developed model and by injecting numerical relativity waveforms. We investigate the performance of different detector networks to determine the improvement that future detectors will bring to our analysis. We consider Advanced LIGO+ and Advanced Virgo+, KAGRA, and LIGO-India. We also study the possible impact of a detector with high sensitivity in the kilohertz band like NEMO, and finally we compare these results to the ones we obtain with third-generation detectors, the Einstein Telescope and the Cosmic Explorer.

\section{I. INTRODUCTION}

Neutron stars (NSs) can reach extremely high densities, creating conditions that cannot be reproduced by laboratory experiments. Hence, they provide a perfect environment to study supranuclear-dense matter and its Equation of State (EoS). Until a few years ago, the study of NSs was limited to electromagnetic (EM) observations, but since the first detection of a gravitational wave (GW) signal from a binary neutron star (BNS), GW170817 \cite{2}, GWs provide new ways to study NSs and their mergers. Since the EoS determines the NS’s macroscopic properties, such as its mass, radius, and tidal deformability, it can be constrained by measuring the imprint it leaves in the GW signal emitted during the coalescence \cite{3}.

Up to now, Advanced LIGO \cite{4} and Advanced Virgo \cite{5} detected two BNS systems, GW170817 \cite{1} and GW190425 \cite{7}. These detections already allowed to put constraints on the supranuclear-dense matter EoS, which was possible since the GW signal emitted during the inspiral phase provides information about the EoS through tidal deformability measurements \cite{2,8,10}. While the uncertainty on current measurements is still large, the higher sensitivities of future generation detectors such as the Einstein Telescope (ET) \cite{17,24} or the Cosmic Explorer (CE) \cite{24,25} will significantly improve them.

In addition to a more detailed analysis of the inspiral, 3rd generation (3G) GW detectors such as ET and CE are also expected to detect GWs from the postmerger phase of the BNS coalescence \cite{24,25}. This is of special interest, since the postmerger probes an even higher different density and temperature regime than the inspiral. While during the inspiral only densities up to the central density of the individual stars are probed, which corresponds to about 3 to 4 times nuclear saturation density, the postmerger phase probes densities even beyond five times nuclear saturation density, cf. Fig. 1 of \cite{31}. In addition, also temperatures of about 50 MeV are reached during the postmerger phase, which is large enough so that the effect of different transport coefficients will start to impact the data \cite{32,34}.

Unfortunately, postmerger studies pose numerous challenges. Firstly, the amplitude of the GW strain of the postmerger part of the observed GW signal is expected to be weaker than the inspiral one \cite{35,39}. Secondly, at higher frequencies the detectors’ sensitivity drops due
to quantum shot noise. For these reasons, it is not surprising that the dedicated searches for GWs emitted by a possible remnant of GW170817 [40, 41] found no evidence of such a signal, and showed that with the sensitivity of Advanced LIGO and Advanced Virgo the source distance should have been at least one order of magnitude less for the postmerger signal to be detectable. Finally, postmerger physics includes thermal effects, magnetohydrodynamical instabilities, neutrino emission, dissipative processes, and possible phase transitions [42–48], which make the postmerger particularly difficult to model, but, on the other hand, allow us to investigate a variety of interesting physical processes. Because of the complexity of the evolution, the study of the postmerger relies heavily on numerical-relativity (NR) simulations, which, however, are also limited due to their high computational cost and the fact that it is currently not possible to take into account all the physical processes that influence the postmerger.

Nonetheless, previous studies based on NR simulations showed some common key features of the postmerger GW spectrum, finding in some cases universal relations with the NS properties [36–39, 49–57], and some efforts have been made also to construct full inspiral, merger and postmerger models for BNS coalescences. Also morphology-independent analyses of the postmerger GW signal have been proposed in [51, 56, 58], while in [59] a hierarchical model to generate postmerger spectra was developed. With a different approach, [60, 62] construct analytical models for the postmerger signal, based on features found in NR simulated waveforms. Breschi et al. in [63] proposed a frequency-domain model for the postmerger, built with a combination of complex Gaussian wavelets, and showed in [64] how this model performs using a 3G detector network. Wijngaarden et al. [65] build a hybrid model, using analytical templates for the premerger phase and a morphology-independent analysis, based on sine-Gaussian wavelets, for the postmerger one.

Following similar ideas, in this paper we construct a phenomenological frequency domain model for the entire BNS coalescence consisting of the inspiral, merger, and postmerger phase. Our final aim is it to employ the developed model for parameter estimation analyses. To model the coalescence during the inspiral up to the merger, we rely on IMRPhenomD_NRTidalv2 [66]. The postmerger phase is modelled with a three-parameter Lorentzian describing the main emission peak of its spectrum, following Tsang et al. [67]. For the Lorentzian, we use two different approaches: in one case, we compute the parameters from quasi-universal relations, describing them as a function of the BNS’s properties, in the other one, we treat them as free parameters. Both versions can be directly employed by existing parameter estimation pipelines; see e.g. [68, 69].

This paper is structured as follows. In Sec. III we describe how our model is built, the methods used for parameter estimation, and the detectors we consider. Results are shown in Sec. IV and conclusions are presented in Sec. V. Appendix A shows the results obtained specifically with our postmerger model with free Lorentzian parameters, in Appendix B we discuss general parameter estimation with future detectors, and in Appendix C we provide more details about the validity and settings of the methods employed for our study.

II. METHODS AND SETUP

We construct a frequency-domain waveform model to describe the full inspiral, merger, and postmerger of a BNS coalescence. In this section, we describe how we model the postmerger part of the signal, and how we connect it to the inspiral-merger model to obtain the full waveform. We then describe the framework used for data analysis, explaining how we speed up parameter estimation using relative binning, the analysis setup, the BNS sources that we study, and the employed detector networks to determine to what extent future detector networks will enable postmerger studies.

A. Inspiral-merger-postmerger model construction

Multiple studies have shown that the postmerger GW spectrum includes various strong peaks [36–39, 49, 52–55, 70]. For simplicity, we limit ourselves to the main emission peak at a frequency $f_2$, which corresponds to the dominant GW frequency; see e.g. [49]. Following this approach, the postmerger can be described in time domain by a simple damped sinusoidal waveform [67], whose Fourier transform is a Lorentzian. Therefore, in frequency domain, we model the postmerger with a three-parameter Lorentzian

$$h_{22}(f) = \frac{c_0c_2}{\sqrt{(f - c_1)^2 + c_2^2}} e^{-i \arctan \left( \frac{f - c_1}{c_2} \right)},$$

where $c_0$ corresponds to the maximum value, $c_1$ to the dominant emission frequency $f_2$, and $c_2$ to the inverse of the damping time, which sets the Lorentzian’s width.

We determine the coefficients $c_i$ with two different approaches: (I) we treat them as free parameters, and try to measure $c_0$, $c_1$, and $c_2$ together with the other BNS’s properties; and (II) we compute the $c_i$ coefficients from quasi-universal relations that describe them as functions of the system’s parameters. Depending on its properties and EoS, a given BNS could undergo a prompt collapse to a black hole (BH), hence without a postmerger emission. In this scenario, while in case (I) we expect that the values recovered for the free parameters reflect the absence of a postmerger signal, in (II) the quasi-universal relations employed might lead to a bias in the estimation of the binary’s intrinsic parameters. For this reason, we ideally want to use the Lorentzian model with quasi-universal relations only when we know that a postmerger emission is present. Since the threshold mass for
a prompt collapse is EoS dependent and still unknown, following [60] we assume that a BNS system undergoes prompt collapse if the tidal polarizability parameter $\kappa_2^T$ is lower than a threshold value $\kappa_{\text{thr}} = 40$. The quantity $\kappa_2^T$ is defined as

$$\kappa_2^T = 3 \left[ \Lambda_A^4 X_A X_B + \Lambda_B^4 X_B X_A \right],$$  

where $\Lambda_j = \frac{2}{3} k_2 (R_j/M_j)^{\frac{5}{3}}$ with $j \in \{A, B\}$ are the dimensionless tidal deformabilities, and $X_j$ are respectively the radius and gravitational mass of the individual stars, and $M = M_A + M_B$ is the BNS’s total mass.

1. Quasi-universal relations for the Lorentzian parameters

For the approach introduced as method (II), we use quasi-universal relations, i.e. phenomenological relations that are independent of the EoS, to constrain the coefficients $c_i$ in Eq. (1). This provides a direct connection between the Lorentzian coefficients and the BNS’s properties.

Since the postmerger Lorentzian model extends the waveform used for inspiral and merger beyond its merger frequency $f_{\text{merg}}$, a straightforward way to find the value of $c_0$ is by rescaling the amplitude of the NRPhenomD_NRTidalv2 waveform at merger $A_{\text{NRPhenomD,NRTidalv2}}(f_{\text{merg}})$. Specifically, we use

$$c_0 = \sigma \times A_0 \times A_{\text{NRPhenomD,NRTidalv2}}(f_{\text{merg}}),$$

where $A_0$ is the mass and distance scaling factor employed in IMRPhenomD [72]. The prefactor $\sigma$ is added to obtain a better calibration to the NR waveforms, and we set $\sigma = 10.0$, which gives the lowest mismatch values (the definition of mismatch and details about its computation are provided in Sec. II B). Since $c_1$ represents the dominant postmerger oscillation frequency $f_2$, we resort to the fit in Eq. (8) of [67]

$$M c_1(\zeta) = \frac{\beta}{1 + A \zeta},$$

with $\beta = 3.4285 \times 10^{-2}$, $A = 2.0796 \times 10^{-3}$, and $B = 3.9588 \times 10^{-3}$. The parameter $\zeta$ is

$$\zeta = \kappa_2^{\text{eff}} - 131.7010 \frac{M}{M_{\text{TOV}}}.$$  

In the last equation, $\kappa_2^{\text{eff}} = 3/18 \tilde{\Lambda}$, with $\tilde{\Lambda}$ being the binary’s mass-weighted tidal deformability

$$\tilde{\Lambda} = \frac{16 (M_A + 12M_B) \Lambda_A^4 A_A + (M_B + 12M_A) \Lambda_B^4 A_B}{(M_A + M_B)^5}.$$  

Although $\zeta$, and therefore $c_1$, in Eq. (5) is a function of the maximum mass allowed for a non-rotating stable NS $M_{\text{TOV}}$, which depends on the specific EoS, we fix $M_{\text{TOV}} = 2M_\odot$ for the model version with quasi-universal relations in this work. The median relative error introduced on $\zeta$ by this approximation is 0.31, for the hybrid waveforms in the SACRA and CoRe database. This error propagates to the $c_1$ parameter causing a median relative error of approximately 5%.

With this choice for $c_0$ and $c_1$, a model for $c_2$ is built from a set of 48 non-spinning NR waveforms, from the CoRe database [77, 78]. For this, we first find the values of $c_2$ that minimize the mismatch of the Lorentzian waveform and the NR waveform between 0.75 $c_1$ and 8192 Hz using a flat noise power spectral density (PSD); see Sec. II B for details. The flat PSD ensures that no high-frequency information is suppressed in the match computation. For each waveform, $c_2$ minimization is performed using the ‘L-BFGS-B’, ‘SLSQP’, ‘TNC’ and ‘Powell’ methods available in SciPy [79] and the value of $c_2$ with the least mismatch value is used. It was seen that $c_2$ showed a similar trend against $\kappa_2^{\text{eff}} q^2$, with $q = M_A/M_B$ the mass ratio, as $c_1$ does against $\zeta$. Hence, a analogous ansatz was used to perform a fit. However, using the parameters obtained from doing a simple curve fit showed unphysical amplitude behaviour for a few of the NR waveforms. For further tuning, the mismatch was minimized for all the NR waveforms by varying the fit parameters and the parameters that gave the least mismatch were then recorded and added to the model. The functional form of $c_2$ and the values obtained for the fit parameters in this manner are

$$c_2 = 2 + \gamma \frac{1 + C \kappa_2^{\text{eff}} q^2}{1 + D \kappa_2^{\text{eff}} q^2},$$

with $\gamma = 19.4579017$, $C = -9.63390738 \times 10^{-4}$, and $D = 6.54926854 \times 10^{-5}$. The median relative error for $c_2$ during minimization is 0.56.

2. The full waveform

To obtain a model describing the full coalescence, the previously derived postmerger model is connected to the waveform describing the inspiral and merger part of the

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1 See also [71] for more updated relations which were not yet available when we started our work.

2 In principle we could treat $M_{\text{TOV}}$ as a free parameter, but this would impair the main benefit of this version of the model, namely to avoid additional parameters to sample over. However, in the future, given the increasing number of multi-messenger detections of binary neutron stars mergers and the possibility to observe high mass pulsars [73, 76], one can expect to have a significantly smaller uncertainty in $M_{\text{TOV}}$ than today. The value of the maximum supported mass estimated from this new information will then provide the fixed value of $M_{\text{TOV}}$ to employ in our model.
signal, for which we use the phenomenological waveform IMRPhenomD_NRTidalv2.

**Amplitude:** To ensure a smooth transition between the two models, we apply a Planck-taper window $\alpha_p$:

$$
\alpha_p = \begin{cases} 
0 & \text{for } f < f_{tr}, \\
\exp\left[ \frac{f_{end} - f_{tr}}{f_{tr} - f_{end}} + 1 \right]^{-1} & \text{for } f_{tr} < f < f_{end}, \\
1 & \text{for } f > f_{end}.
\end{cases}
$$

The window is applied just before the frequency of the main postmerger peak $f_2$, which corresponds to our model’s parameter $c_1$. The value of the window’s starting frequency $f_{tr}$ is chosen to ensure a good match with NR waveforms. In particular, in Ref. [67] one of the time-domain features identified in the postmerger signal morphology is the *first postmerger minimum*, which corresponds to a clear amplitude minimum present shortly after the merger, before the amplitude starts increasing again. By comparison with NR waveforms in the CoRe database [77][78], we found that this feature is best reproduced by our model when the Planck window is applied between $f_{tr} = 0.75c_1$ and $f_{end} = 0.9c_1$. Following [72], we add an exponential correction factor $\exp\left[ -p(f-c_1) \right]$ to the Lorentzian amplitude, in order to smooth out possible kinks arising when going to the time domain. We set $p = 0.01$, which is enough to reduce the kink, but not so large that it significantly influences the merger amplitude.

**Phase:** To ensure that the waveform phase is smooth, we introduce two coefficients $a$ and $b$, writing the phase as

$$
\phi_{IM}(f) = \phi_{Lor}(f) + a + bf,
$$

with $\phi_{IM}$ the phase of IMRPhenomD_NRTidalv2 waveform and $\phi_{Lor} = \arg(h_{22}(f))$ the Lorentzian one.

The values of $a$ and $b$ are computed at the same transition frequency $f_{tr} = 0.75c_1$ at which we start the Planck-taper window for the amplitude, such that

$$
\frac{d\phi_{IM}}{df}\bigg|_{f_{tr}} = \frac{d\phi_{Lor}}{df}\bigg|_{f_{tr}} + b, \\
\phi_{IM}(f_{tr}) = \phi_{Lor}(f_{tr}) + bf_{tr} + a.
$$

Finally, to reduce the Lorentzian contribution to the pre-merger and merger amplitude, we multiply the waveform by a factor $\exp[-iT\Delta t f]$, which will induce a time shift of $\Delta t$ in the time-domain waveform; $\Delta t$ is computed as the time interval between the merger and the *first postmerger minimum* described by Eq. (2) in [67].

The frequency-domain gravitational waveform can be written as

$$
\tilde{h}(f) = A(f)e^{i\phi(f)},
$$

with $A(f)$ the amplitude and $\phi(f)$ the phase. Therefore, in our model the full waveform is given by:

$$
\tilde{h}(f) = \begin{cases} 
A_{IM}(f)e^{i\phi_{IM}} & \text{for } f < f_{tr}, \\
A_{IM}(f) + \alpha_p A_{Lor}(f)e^{-\frac{c_1(f-f_{tr})}{2}} & \text{for } f_{tr} < f < f_{end}, \\
e^{i(\phi_{Lor} + bf + a) - iT\Delta t}} & \text{for } f > f_{end}.
\end{cases}
$$

where $A_{IM}(f)$ and $\phi_{IM}(f)$ are respectively the amplitude and phase of the IMRPhenomD_NRTidalv2 waveform, and $A_{Lor} = |h_{22}(f)|$ the amplitude of the Lorentzian one.

In the following, we refer to the IMRPhenomD_NRTidalv2/Lorentzian postmerger model with quasi-universal relations as QU-PM, to the one with free Lorentzian parameters as FREE-PM, and to the model without postmerger, IMRPhenomD_NRTidalv2, as NO-PM.

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3 We note that the employed approach neglects any contribution of the postmerger signal towards frequencies below the merger frequency.

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**B. Mismatch**

The mismatch between two waveforms $h_1$ and $h_2$ is defined as

$$
MM = 1 - \max_{\phi_c,t_c} \frac{\langle h_1(\phi_c,t_c)h_2 \rangle}{\sqrt{\langle h_1,h_1 \rangle \langle h_2,h_2 \rangle}},
$$

where $t_c$ and $\phi_c$ are an arbitrary time and phase shift, and the noise-weighted inner product is defined as

$$
\langle a | b \rangle \equiv 4\text{Re} \int_{f_{low}}^{f_{high}} \frac{\tilde{a}(f) \tilde{b}(f)}{S_a(f)} df,
$$

where $S_a(f)$ is the noise spectral density, $\tilde{a}(f)$ the Fourier transform of $a(t)$, and * denotes the complex conjugate. To validate the IMRPhenomD_NRTidalv2/Lorentzian model, we compute mismatches with the hybrid waveforms in the CoRe [77][78] and SACRA [80] database. The mismatch is computed with PyCBC [81] functions and
The frequency region is due to the fact that different values of \( c \) do not follow the trend of the ones computed only in the high frequency band, i.e., between \([1.1 f_{\text{merg}}, 4096]\) Hz, the bottom panel for the whole waveform, between \([30, 4096]\) Hz. In the latter case, for comparison we show also mismatches computed between the hybrids and the NO-PM model.

![FIG. 1. Mismatches between hybrid waveforms from the CoRe (in the gray-background band) and SACRA database, and our postmerger model, for both the versions FREE-PM and QU-PM. The top panel shows mismatches in the postmerger frequency band, i.e., between \([1.1 f_{\text{merg}}, 4096]\) Hz, and our postmerger model, for both the versions CoRe (in the gray-background band) and SACRA database.](image)

The top panel shows mismatches in the postmerger frequency band, i.e., between \([1.1 f_{\text{merg}}, 4096]\) Hz, the bottom panel for the whole waveform, between \([30, 4096]\) Hz. In the latter case, for comparison we show also mismatches computed between the hybrids and the NO-PM model.

To get the Lorentzian parameters that better describe the GW signal includes no signal at zero noise, i.e., with a flat PSD. For the FREE-PM model, to get the Lorentzian parameters that better describe each hybrid’s postmerger, we optimize the mismatch over \( c_1, c_2 \); we do not include the Lorentzian maximum value \( c_0 \) in the minimization, because, giving just an amplitude scaling factor, the mismatch is insensitive to it. The initial values for the optimization are found with a least-squares fit on the hybrid’s postmerger signal. We use the optimal values for the \( c_1 \) coefficients to generate the FREE-PM waveform, for which we compute the mismatch with the hybrid in different frequency ranges. For the QU-PM model, instead, the Lorentzian parameters are computed from the quasi-universal relations described in Sec. \[A.1\] using the values of the hybrids’ binary parameters. The top panel of Fig. \[1\] shows the mismatches in the frequency band between \([1.1 f_{\text{merg}}, 4096]\) Hz: despite our simple description of the postmerger, when using the FREE-PM model for almost all hybrids mismatches lie below 0.3. Mismatches values increase systematically by roughly a factor 3 when computing them with respect to the QU-PM model, which is expected since in this case the Lorentzian parameters are not optimized to the hybrid waveform. When considering the whole waveform, in the frequency range \([30, 4096]\) Hz, the mismatch is always below 0.005, as shown in the bottom panel of Fig. \[1\].

In the following, we focus on how to recover the source’s parameters given the detector data \( d \) and under the hypothesis of a specific model \( \mathcal{H} \) used to describe the waveform. In a Bayesian framework, this corresponds to evaluating the posterior \( p(\tilde{\theta}|\mathcal{H}, d) \), which, according to Bayes’ theorem, is

\[
p(\tilde{\theta}|\mathcal{H}, d) = \frac{p(d|\mathcal{H}, \tilde{\theta})p(\tilde{\theta}|\mathcal{H})}{p(d|\mathcal{H})}.
\]  

(16)

In Eq. \[16\], the prior probability density \( p(\tilde{\theta}|\mathcal{H}) \) encodes our prior knowledge about the source or the model; the evidence \( p(d|\mathcal{H}) \) describes the probability of observing the data \( d \) given the model \( \mathcal{H} \), independently of the specific choice of parameters \( \tilde{\theta} \); and the likelihood \( p(d|\mathcal{H}, \tilde{\theta}) \) represents the probability of observing \( d \) with the specific set of parameters \( \tilde{\theta} \).

The priors chosen for this work are described later in this section, while the evidence \( p(d|\mathcal{H}) \) serves as normalization constant of the posterior distribution, and is given by

\[
p(d|\mathcal{H}) = \int d\tilde{\theta}p(d|\mathcal{H}, \tilde{\theta})p(\tilde{\theta}|\mathcal{H}).
\]  

(17)

Assuming the data \( d \) consist of Gaussian noise and a GW signal \( h(\tilde{\theta}) \), the likelihood can be expressed as

\[
p(d|\mathcal{H}, \tilde{\theta}) \propto \exp\left[-\frac{1}{2} \left(d - h(\tilde{\theta})|d - h(\tilde{\theta})\right)\right]
\]  

(18)

with the noise-weighted inner product defined as in Eq. \[15\].
To sample the likelihood function, we use the nested sampling \cite{S2,S3} package \texttt{dynesty} \cite{S4,S5}, which is included in the \texttt{bilby} library \cite{S6,S9}, with 2048 live points.

1. Relative binning

The likelihood evaluations required at each sampling step are very expensive, since, in order to compute the inner product, we need to evaluate the waveform on a dense and uniform frequency grid. The size of the grid increases both with the duration of the signal and the maximum frequency used in the analysis. In our case, we set $f_{\text{max}} = 4096$ Hz, since the postmerger GW signal is expected to lie within the few kilohertz regime. Moreover, we study BNS systems, whose low masses imply a long signal duration. Although we set the starting frequency to $f_{\text{low}} = 30$ Hz, the typical duration of the signal in band is still roughly 200 s. To overcome the issue of the computational cost of the analysis needed for this work, we employ the technique of relative binning \cite{S0,S7}, which reduces the number of waveform evaluations from all the points on the grid to a limited number of frequency bins.

The underlying assumption in relative binning is that the set of parameters yielding a non-negligible contribution to the posterior probability produce similar waveforms, such that their ratio varies smoothly in the frequency domain. In each frequency bin $b = [f_{\text{min}}(b), f_{\text{max}}(b)]$, if we choose a reference waveform $h_0(f)$ that describes sufficiently well the data, the ratio with the sampled waveforms can be approximated with a linear interpolation

$$r = \frac{h(f)}{h_0(f)} = r_0(h, b) + r_1(h, b)(f - f_m(b)) + \mathcal{O}((f - f_m(b))^2),$$

with $f_m(b)$ the central frequency of the bin $b$.

This allows to approximate the likelihood inner product as

$$\langle d(f)|h(f)\rangle \approx \sum_b (A_0(b)r_0^2(h, b) + A_1(b)r_1^2(h, b)),$$

where the summary data

$$A_0(b) = 4 \sum_{f \in b} \frac{d(f)h_0^2(f)}{S_n(f)/T},$$

$$A_1(b) = 4 \sum_{f \in b} \frac{d(f)h_0^2(f)}{S_n(f)/T}(f - f_m(b))$$

are computed on the whole frequency grid, but only for the reference waveform. Also $\langle h(f)|h(f)\rangle$ is calculated with a similar approach. In this method, the evaluation of sampled waveforms is required only to compute the bin coefficients $r_0(h, b)$ and $r_1(h, b)$ in Eq. (19). In this paper, we follow the description and implementation of \cite{S4,S8}. To use relative binning with \texttt{bilby} inference, we employ the code in \cite{S9}. More details about the relative binning method applied to our analysis are given in Appendix C.

| Name                  | $\mathcal{M}_c$ | q   | $\tilde{\Lambda}$ | Injection           |
|-----------------------|-----------------|-----|-------------------|---------------------|
| Source1\text{[NR-inj]} | 1.17524         | 0.8 | 604               | NR: H121,151,00155 \[94\] |
| Source1\text{[qu-pm]} | 1.17524         | 0.8 | 604               | Bilby: quasi-universal |
| Source1\text{[free-pm]} | 1.17524         | 0.8 | 604               | Bilby: free parameters |
| Source2\text{[NR-inj]} | 1.08819         | 1.0 | 966               | NR: H125,125,00155 \[95\] |
| Source2\text{[qu-pm]} | 1.08819         | 1.0 | 966               | Bilby: quasi-universal |
| Source2\text{[free-pm]} | 1.08819         | 1.0 | 966               | Bilby: free parameters |
| Source3\text{[NR-inj]} | 1.17524         | 1.0 | 607               | NR: H135,135,00155 \[94\] |
| Source3\text{[qu-pm]} | 1.17524         | 1.0 | 607               | Bilby: quasi-universal |
| Source3\text{[free-pm]} | 1.17524         | 1.0 | 607               | Bilby: free parameters |

TABLE I. Properties of the sources used for injections. The NR hybrids are taken from the SACRA database \cite{S0}, where the employed EOSs of the NR data are simple two-piece polytropes as outlined in \cite{S0}. For the hybridization, we follow the procedure outlined in Sec. III C of \cite{19}. The inspiral waveform model with which we hybridize is \texttt{SEOBNRv4T} \cite{96}. For \texttt{bilby} injections, we used our \texttt{IMRPhenomD\_NR\_Tidalv2\_Lorentzian} model, both with quasi-universal relations and with free Lorentzian parameters. In case of injections with the free parameters model, the injected $c_0, c_1, c_2$ values are obtained from the best fit of the correspondent NR hybrid.

| Source1\text{[NR-inj]} | Source2\text{[NR-inj]} | Source3\text{[NR-inj]} |
|-------------------------|------------------------|------------------------|
| Name                  | PM | Total | PM | Total | PM | Total |
| LHV                    | 100 | 2.0 | 94 | 2.5 | 100 | 2.7 |
| LHVKI                  | 107 | 2.1 | 101 | 2.6 | 108 | 2.9 |
| LHVKIN                 | 126 | 6.8 | 119 | 8.8 | 126 | 9.9 |
| ETCE                   | 1267 | 10.2 | 1190 | 12.3 | 1268 | 13.3 |

TABLE II. SNR of the NR waveforms employed in our analysis for the different detector networks (with acronyms as shown in Fig. 2), considering the source at a distance of 68 Mpc; we show both the SNR for the whole waveform (in the 'Total' column), computed starting at 30 Hz, and the SNR of the postmerger part of the signal (in the 'PM' column), computed starting from the merger frequency.

2. Simulations

We test the performance of our model in parameter estimation analysis with simulated signals. We consider three different sources, and analyze them through \texttt{bilby} injections, i.e., using our own GW models, and through injecting NR hybrids with the same parameters; cf. Tab.II The employed hybrids have a postmerger signal duration of roughly 10 ms, and the postmerger contribution to their SNR for each detector network is shown in Tab.II.

All simulated signals are injected with zero inclination $i$ and polarization angle $\psi$, and with sky loca-
tion \((\alpha, \delta) = (0.76, -1.23)\). The sky location has been chosen such that none of the employed detector networks is particularly preferred. Depending on the analysis, we performed injections at three different distances: 225 Mpc, 135 Mpc, and 68 Mpc, which, in a network with Advanced LIGO+ and Advanced Virgo+, correspond approximately to a signal-to-noise ratio (SNR) of 30, 50, and 100 respectively; Table II reports the SNR for Source2 injections in the different detector networks and at different distances. We take priors uniform in \([0.5, 1.0]\) for mass ratio \(q\), and uniform in \([M_{c,s} - 0.05, M_{c,s} + 0.05]M_{\odot}\) for chirp mass, where \(M_{c,s}\) is the chirp mass of the source, and the prior width is given by the precision on chirp-mass measurements that we anticipate for future detectors. Regarding tidal deformability parameters, we sample over \(\tilde{\Lambda}\) and \(\Delta \tilde{\Lambda}\), with a prior uniform in \([0, 5000]\) and \([\Delta \tilde{\Lambda}]\) respectively, where \(\Delta \tilde{\Lambda}\) is defined in [97] as

\[
\Delta \tilde{\Lambda} = \frac{1}{2} \left[ \sqrt{1 - 4\eta(1 - \frac{13272}{1319}\eta + \frac{8944}{1319}\eta^2)(\Lambda_1 + \Lambda_2)} + (1 - \frac{15910}{1319}\eta + \frac{32850}{1319}\eta^2 + \frac{3380}{1319}\eta^3)(\Lambda_1 - \Lambda_2) \right].
\]

(23)

Luminosity distance priors are uniform in comoving volume, with \(D_L \in [1, 450]\) Mpc. Although all the sources considered are non-spinning, our baseline model IMRPhenomD_NRTidalv2 allows for aligned spins; we choose a uniform prior on the spin magnitudes \(|a_1|, |a_2| \in [0.0, 0.20]\). Finally, when using the postmerger model with free parameters for recovery, we choose uniform priors \(c_1 \in [2000, 4096]\) Hz and \(c_2 \in [10, 200]\) Hz, while for \(c_0\) we employ a logarithmic uniform prior in \([5 \times 10^{-27}, 1 \times 10^{-22}]\) s.
TABLE III. SNR values for zero-noise Source2\textsubscript{[qu–pn]} injections in the different networks (with acronyms as shown in Fig. 2) and for different distances.

| Network | Distance [Mpc] | SNR  |
|---------|----------------|------|
| ETCE    | 68             | 1239 |
|         | 135            | 624  |
|         | 225            | 355  |
| LHV KIN| 68             | 121  |
|         | 135            | 61   |
|         | 225            | 36   |
| LHV KI | 68             | 105  |
|         | 135            | 53   |
|         | 225            | 31   |
| LHV    | 68             | 98   |
|         | 135            | 49   |
|         | 225            | 30   |

D. Detector Networks

Earth-based GW detectors have the best sensitivity around a few tens to hundreds of Hz, which makes the inspiral and merger signal of coalescing compact objects the perfect candidate for detections. In this work, however, we are interested in the postmerger part of the signal, which is usually weaker and involves higher frequencies. Current detectors are strongly limited at these high frequencies, but the improvements planned for the future detectors’ upgrades and the next generation detectors are expected to make postmerger measurements feasible. Therefore, one of the goals of this work is to assess how future detectors can improve the studies we present. We include in our analysis the upgraded versions of existing detectors, Advanced LIGO+, Advanced Virgo+, and KAGRA, as well as new detectors whose construction has been planned for the next few years, LIGO-India and NEMO, and the next detector generation, Einstein Telescope and Cosmic Explorer. Advanced LIGO+ design \cite{98} will improve the current 4 km arm-length detectors in Hanford (H) and Livingston (L) sites, including a frequency dependent light squeezing and new test masses with improved coating. Advanced Virgo+ (V), similarly, is the planned upgrade for the current Advanced Virgo detector in Cascina \cite{5}. This transition will happen in two separate phases and include upgrades like the introduction of signal recycling and a higher laser power. Advanced LIGO+ and Advanced Virgo+ are the planned designs for the O5 observing run, which is scheduled to start roughly in 2025, and during which their BNS detection range will reach approximately 330 Mpc and 150-260 Mpc, respectively \cite{99}. KAGRA (K) \cite{102,103} is a 3 km arm-length interferometer built underground in the Kamioka mine in Japan, which already employs innovative technologies like cryogenic mirrors. For O5, its sensitivity at the end of the observing run is predicted to allow a BNS range of at least 130 Mpc \cite{99}. The LIGO network involves a third detector in India (I) \cite{103}, which is currently under construction and is expected to become operative approximately in 2025. Finally, the Neutron Star Extreme Matter Observatory, or NEMO (N), is an Australian proposal for a gravitational-wave detector with 4 km arm-length, specifically designed to have a high sensitivity in the kilohertz band \cite{104}. The possible location of NEMO has not been decided yet, therefore for this work we arbitrary place it at the location shown in Fig. 2. Although not officially approved yet, we include it in our analysis, since its high-frequency sensitivity is particularly interesting for postmerger studies.

3G detectors are expected to increase the sensitivity by a factor between 10 and 30 \cite{99} with respect to current LIGO detectors, but they require the construction of new facilities and are expected to start observing in the mid 2030s. At the moment, the planned 3G detector network includes plans for Cosmic Explorer (CE) in the US and Einstein Telescope (ET) in Europe. CE \cite{24,25} is planned as an L-shaped interferometer with 40 km arm-length.\footnote{Recently, also a configuration consisting of a 40 km and an additional 20 km detector has received attention and was considered as the reference concept for the recent Horizon study of \cite{25}. In \cite{105}, also a tunable design for the CE detector was proposed, which would enhance sensitivity in the kilohertz band.} For the purpose of this paper, we assume it placed at the current Hanford site. ET design \cite{17,18} includes a so-called ‘xylophone’ configuration, which guarantees an improved sensitivity at high and low frequencies at the same time \cite{91}. The two candidates for the ET site are Sardinia, in Italy, and Limburg, at the border between the Netherlands, Germany, and Belgium\footnote{In addition, recent interest arose for a third possible site located in the eastern part of Germany.}. For this work, we assume ET is placed at the current Virgo site. Although the final design of ET is still under development, here we consider it as a triangular detector, i.e., composed of three V-shaped interferometers with a 60 degree opening angle and 10 km arms.

In this work, we study four different detector networks: HLV, LHV KI, LHV KIN, and ETCE. The detectors’ locations and sensitivities are shown in Fig. 2.

III. RESULTS

In the following, we present the results of our simulations, for what concerns both the performance of our model and the improvement we obtain with future detectors. When using the postmerger model with quasi-universal relations, we are mainly interested in studying how well we can recover the tidal deformability parameter...
FIG. 3. Posterior probability density for $\tilde{\Lambda}$ in the case of bilby injections with the QU-PM model, for sources at 68 Mpc and with the ETCE network, and recovery with the three different models NO-PM, QU-PM and FREE-PM, in blue, orange and green respectively. The black dashed lines correspond to the injected values.

FIG. 4. Injected signal for Source1\textsubscript{[qu−pm]} (gray solid line), compared to the NO-PM waveform generated with the injected parameters (dashed blue line) and with the maximum likelihood parameters recovered with the NO-PM model (dash-dotted cyan line).

$\tilde{\Lambda}$. Since the quasi-universal relations that we derived depend on $\tilde{\Lambda}$, we expect that the postmerger part of the signal, when detected, brings additional information about this parameter. This will likely lead to a narrower posterior with respect to what we can obtain using a model without postmerger. In the case of the postmerger model with free Lorentzian parameters, we study how well the Lorentzian parameters $c_0, c_1, c_2$ can be recovered, and especially $c_1$, since it represents the frequency of the main postmerger emission peak.

A. Best-case scenario

We start by testing both versions of our model, FREE-PM and QU-PM, in the best-case scenario, i.e., for bilby injections in zero noise, for sources as described in Table I at a distance of 68 Mpc and with ETCE network. Figure 3 shows the posterior probability density of $\tilde{\Lambda}$ for signals obtained with QU-PM injections, and recovered with both our postmerger models, QU-PM and FREE-PM, and with the model without postmerger NO-PM. As expected, the $\tilde{\Lambda}$ posterior becomes tighter when going from the NO-PM to QU-PM model, with the width of the 90% confidence interval reducing by about 30%, from 23.11 to 15.84 in the case of Source2\textsubscript{[qu−pm]}, and from 15.42 to 11.07 for Source3\textsubscript{[qu−pm]}. In the FREE-PM recovery case, the posteriors become wider, with the width of the 90% confidence interval reaching 27.66 for Source2\textsubscript{[qu−pm]}. We also note that when recovering with this model, the median of $\tilde{\Lambda}$ is slightly underestimated with the respect to the injected values. Both these features are predictable due to the higher number of parameters we have to sample over. For Source1\textsubscript{[qu−pm]}, the injected value lies outside the NO-PM $\tilde{\Lambda}$ posterior distribution, but is well recovered with both the QU-PM and FREE-PM models. Given that the sampler converged to the maximum likelihood values for the parameters, this shift is not caused by sampling issues, but is probably due to the fact that injections are performed with a signal with postmerger, and when we recover with a model without the postmerger description, the waveform tries to latch on to the signal after the merger, causing a bias in the parameter estimation. This is confirmed by the comparison, shown in Fig.4, between the injected Source2\textsubscript{[qu−pm]} waveform, the NO-PM waveform generated with the maximum likelihood parameters recovered with the NO-PM model, and the one generated with the injected parameters. The maximum
FIG. 5. Posteriors of $c_1$ parameters for the three different sources, obtained when using the free-pm model both for injection and recovery. The black dashed lines show the injected values.

likelihood NO-PM waveform tries to recover part of the injected postmerger signal, resulting in a deviation with respect to the NO-PM waveform obtained from the injection parameters, which explains the bias in the $\tilde{\Lambda}$ posterior.

Figure 5 shows the posteriors for the $c_1$ Lorentzian parameter in the case of injection and recovery with the free-pm model, for the three different sources. The injected values of $c_0$, $c_1$, $c_2$ are the ones that give the best fit on the NR hybrid with the same binary parameters of the source considered. The $c_1$ parameter, which corresponds to the frequency of the main postmerger emission peak, is well recovered in all cases. Although we are mainly interested in the recovery of $c_1$, the free-pm model provides posteriors also for the $c_0$ and $c_2$ parameters, which are related to the maximum amplitude and width of the Lorentzian respectively. Note that the $c_0$ and $c_2$ parameters, which are not shown in the figure, are not recovered as well as the $c_1$ parameter, but their injected values lie in the posteriors 90% confidence interval in all cases, as reported in Table IV. While our model works for our main purpose of measuring the frequency of the dominant postmerger peak, the shifts that we see in the other parameters suggest that we can further improve the free-pm model; see e.g. [63, 65] for recent developments including postmerger features beyond the main emission frequency.

### B. Detector network performances in zero-noise

We want to investigate how future detector networks will improve our postmerger analysis. For this purpose, we inject signals obtained from the QU-pm model in zero noise, and recover both with the QU-pm and the NO-pm model. We analyze signals injected at three different distances (68 Mpc, 135 Mpc, and 225 Mpc), and we compare results for the four detector networks LHV, LHVKI, LHVKIN, and ETCE (as described in Sec. II D). Due to limited computational resources, we look only at two different sources, Source2[qu–pm] and Source3[qu–pm].

Table IV shows the Source2[qu–pm] injected signal, and the correspondent NR waveform: the signal injected with our QU-pm model describes well the main postmerger emission peak, but the NR waveform morphology includes also different sub-dominant emission peaks which our single Lorentzian cannot describe, and more structure in the frequency region right after the merger. Both these features should be addressed in future improvements of the model. In the same figure we show the maximum likelihood waveforms recovered both with the QU-pm and the NO-pm model, for a zero noise injection with the ETCE network. The recovered maximum-likelihood QU-pm signal overlaps with the injected one, showing how well 3G detectors will be able to recover this kind of signals. In the inspiral region, this applies also to the NO-pm maximum likelihood waveform. The inspiral signal, which we see is well recovered also with the NO-pm model, already contains information about the $\tilde{\Lambda}$ parameter; therefore, for a ETCE network with such

| Source   | $\text{log } c_0$ | $\text{log } c_0\text{, inj}$ | $c_2$ | $c_2\text{, inj}$ |
|----------|------------------|-------------------------------|-------|------------------|
| Source1  | $-56.79_{-0.34}^{+0.29}$ | $-56.65$                       | 96.40  | $-36.90\text{, inj}$ |
| Source2  | $-56.15_{-0.24}^{+0.21}$ | $-56.15$                       | 52.01  | $-14.19\text{, inj}$ |
| Source3  | $-55.90_{-0.21}^{+0.19}$ | $-55.90$                       | 41.26  | $-9.52\text{, inj}$ |

TABLE IV. Median with 5% and 95% quantile values of the posterior probability density for the $c_0$ and $c_2$ parameters, together with their injected values, for each of the three sources analyzed, in the case of injection and recovery with the free-pm model.
high SNR, we expect that little contribution to $\tilde{\Lambda}$ measurement comes from the postmerger part of the signal, given that this parameter is already very well constrained from the inspiral.

Figure 7 shows the uncertainty $\tilde{\Lambda}_{90\text{conf}}$, computed as the width of the 90% confidence interval of the $\tilde{\Lambda}$ posterior probability density, as a function of the different detector networks employed for the analysis, comparing the different distances and recovery models. As expected, Fig. 7 shows that for all the detector networks considered, and for both models, the width of the 90% confidence interval decreases with decreasing distance. In particular, for an LHV network, we find an improvement of $\sim 50\%$ when going from 225 Mpc to 135 Mpc, and of $\sim 25\%$ (for Source2$[\text{qu-pm}]$ even 56%) when going from 135 Mpc to 68 Mpc, for both models; for the ETCE network we find an improvement $\sim 45\%$ when going from 225 Mpc to 135 Mpc, and $\sim 55\%$ when going from 135 Mpc to 68 Mpc. Using the QU-PM model yields systematically tighter constraints on $\tilde{\Lambda}$, thanks to the additional information arising from the quasi-universal relations that describe the postmerger part of the signal. For both the sources, in the case of injections at 225 Mpc and with the LHV or LHVKI network, we see no significant differences in $\tilde{\Lambda}_{90\text{conf}}$ in the case of recovery with the QU-PM or NO-PM model. Considering that such injections generate an SNR $\sim 30$ in the case of LHV network, this is consistent with the fact that in these situations we do not detect the postmerger signal.

Interestingly, the best improvement when using the QU-PM model comes in the case of LHVKIN network. Going from the LHVKIN to the ETCE network, the constrain on $\tilde{\Lambda}$ improves of about $\sim 70\%$ for both models, while adding NEMO to the LHVKI network leads to an improvement in $\tilde{\Lambda}_{90\text{conf}}$ of $\sim 60\%$ for the QU-PM model, against the just $\sim 40\%$ for the NO-PM one. For both sources, we also see that for the LHVKIN network the constraint on $\tilde{\Lambda}$ obtained with the QU-PM model for injections at 135 Mpc is better than the one we retrieve with the NO-PM model for injections at 68 Mpc. 3G detectors are expected to have the best sensitivity over the whole frequency band, and indeed we see that for the ETCE network we get the smallest $\tilde{\Lambda}_{90\text{conf}}$ for both models. However, the high sensitivity at lower frequencies allows to obtain precise measurements of $\tilde{\Lambda}$ from the inspiral part of the signal alone, therefore reducing the impact of the possible information gained from postmerger. In the case of LHVKIN network, instead, the constraint on $\tilde{\Lambda}$ from the inspiral is the one of second-generation detectors, but the high sensitivity of NEMO in the kilohertz band leads to a better detection of the postmerger, and therefore to significantly tighter constraints when using the QU-PM model. If its realization is approved, adding NEMO to the network of second-generation detectors will significantly help the detection of postmerger signals and related studies. We note that for this work we analyze signals with a lower frequency cutoff $f_{\text{low}} = 30$ Hz.
Hz, missing many inspiral cycles; in reality, an additional improvement on $\Lambda$ measurements will be provided by the use of a lower $f_{\text{low}}$.

| Source | Model | $\Lambda_{\text{inj}}$ | $\Lambda_{\text{inj}}$ | $\Lambda_{\text{inj}}$ | $\Lambda_{\text{inj}}$ |
|--------|-------|-----------------------|-----------------------|-----------------------|-----------------------|
| Source2 | QU-PM | 956.68$^{+7.08}_{-8.37}$ | 959.93$^{+6.87}_{-8.71}$ | 966 | 966 |
|        | NO-PM | 966.35$^{+9.35}_{-11.82}$ | 953.10$^{+13.11}_{-19.11}$ | 966 | 966 |
| Source3 | QU-PM | 608.04$^{+11.65}_{-6.27}$ | 602.36$^{+7.86}_{-12.49}$ | 607 | 607 |
|        | NO-PM | 611.76$^{+6.66}_{-7.51}$ | 604.39$^{+6.84}_{-7.68}$ | 607 | 607 |

TABLE V. Median values with 90% confidence interval for the posterior probability density of $\Lambda$ in case of two different noise realizations, labeled as noise$\Lambda$ and noise$\Lambda_B$, for injections at 68 Mpc in the ETCE network and for the two different models QU-PM and NO-PM; the last column reports the injected value of $\Lambda$.

C. Detector Network Performances in non-zero noise

In the previous sections we focused on model and network performances, using injections in zero noise. Now we want to look at the influence of noise on our study. For this reason, we repeat the analysis using Gaussian noise. Due to limited computational resources, we restrict to only two sources, Source2[qu-pm] and Source3[qu-pm], and to one distance, 68 Mpc. We inject signals using the QU-PM model, and recover both with the QU-PM and NO-PM models, comparing results for the different detector networks LHV, LHVKI, LHVKIN, and ETCE. Figure 8 shows $\Lambda_{90\text{conf}}$ for the different detector networks. In order to assess the impact of noise fluctuations, we show results for two different noise realizations, which we call noise$\Lambda$ and noise$\Lambda_B$. Due to the noise impact on the analysis, we do not see the clear trends that we found in the zero noise runs, as described in the previous section Sec. III B. In the case of Source3[qu-pm] (bottom panels in Fig. 8), with the noise$\Lambda$ realization the constraints obtained with the QU-PM model are even wider than the ones recovered with the NO-PM model. The most extreme fluctuation is found for Source3[qu-pm], in the case of LHVKI network and QU-PM model, for which $\Lambda_{90\text{conf}} = 88.26$ in case of noise$\Lambda$ and $\Lambda_{90\text{conf}} = 4.84$ for noise$\Lambda_B$. However, we see that in general $\Lambda_{90\text{conf}}$ decreases with more advanced detectors, with an improvement between 80% and 90% when going from the LHV to the ETCE network. In most cases the QU-PM model allows us to better determine $\Lambda$, although the quantitative improvement strongly depends on the source and especially on the noise realization. Moreover, noise fluctuations impact also the median of the $\Lambda$ posterior probability density, causing different shifts with respect to the injected values (see Table V). Although such shifts appear to be small, they can cause the posterior’s median to lie outside the 90% confidence interval, especially in the case of ETCE network, where the $\Lambda_{90\text{conf}}$ is indeed very small.

D. Numerical-relativity injections

Finally, we analyze simulated signals obtained by injecting NR waveforms on top of Gaussian noise. Figure 9 shows the posterior probability density of $\Lambda$, for injections at 68 M pc in the ETCE network. For Source2[NR-inj], the recovered posteriors of $\Lambda$ peak at the injected value, but for the other sources the posterior is shifted with respect to it. For Source1[NR-inj], the $\Lambda$ injected value lies in the tail of the posteriors recovered with the QU-PM and FREE-PM model, and completely outside the posterior obtained with the NO-PM model; for Source3[NR-inj], the posteriors recovered with all the models peak at values between 575 and 578, with the injected value $\Lambda = 607$ lying completely outside their distributions. These shifts are due to noise fluctuations, as we showed in Sec. III C and possible limitations of our waveform models. The case analyzed here, using the ETCE network, generates a signal with a high SNR, and therefore a narrow posterior density for $\Lambda$; hence the shifts induced by noise fluctuations can result in the injected value being situated outside the 90% confidence interval. Using one of the postmerger models to analyze signals obtained with NR waveforms does not lead to a meaningful improvement in the $\Lambda$ constraints as the ones shown in Sec IIIA. This is consistent with the fact that mismatches computed over the whole waveform (cf. lower panel of Fig. 1) do not show significant improvements when using one of the postmerger models, considering that the noise and the complicated morphology of the NR injection make it more difficult for our models to recover the postmerger part of the signal, and therefore almost all the $\Lambda$ information comes from the inspiral. Nonetheless, when using the postmerger models, we see a modest improvement in the recovery of $\Lambda$ for Source2[NR-inj], with respect to the NO-PM one, and a clear improvement for Source1[NR-inj]. The latter is consistent with the results found in Sec. IIIA for Source1[NR-inj], where we concluded that, when using the NO-PM model, the presence of a postmerger signal, to which the NO-PM waveform tries to latch on, causes a bias in the $\Lambda$ parameter recovery. In Sec. III C we saw that noise fluctuations alone cannot impact the performance of our model, but in this case an additional issue is that the NR simulations contain a more complex GW structure in the postmerger, which is not fully recovered with our simple Lorentzian model. This appears clearly in Figure 10, which shows the injected NR waveform together with the maximum likelihood ones recovered with the different models and their 90% confidence interval. The postmerger peak obtained with the QU-PM model is slightly shifted with respect to the main postmerger peak of the NR waveform; however, the same shift was present also in the Source2[qu-pm] injected waveform in Fig. 6, and hence we conclude that it is due to the imperfection of the model.
FIG. 8. Width of the 90% confidence interval of $\hat{\Lambda}$ posterior for Source2$_{\text{qu-pm}}$ (top row) and Source3$_{\text{qu-pm}}$ (bottom row), as a function of the different detector networks, obtained with two different noise realizations, noise$_A$ for the left panels, and noise$_B$ for the right ones.

not to issues in the parameter estimation process. When optimizing the mismatches to compute the best values of the fit parameters for our quasi-universal relations, it is likely that the model tries to adapt to the whole morphology of the postmerger NR signal, thus shifting with respect to what would be the description of the main emission peak only. For the FREE-PM model maximum likelihood waveform, the postmerger peak lies at a higher frequency than the true one, is much wider and with a non-physical amplitude, though this does not affect the $\hat{\Lambda}$ recovery (cf. Fig. 9). Given the large bias in $c_0$, and the fact that the injected values vary in a small range, an improvement would probably be obtained already by restricting the prior range for this parameter. For comparison, in Fig. 10 we show also the waveform obtained from the FREE-PM model with the optimized parameters computed as explained in Sec. II B: the postmerger peak of the optimized FREE-PM waveform overlaps to the one of the NR waveform. Hence, the FREE-PM model can in principle describe the data well, but the additional information contained in the complex and more structured morphology of the postmerger in the hybrid signal makes it challenging for our simple model to recover all the parameters correctly. The fact that the postmerger Lorentzian parameters cannot be recovered with a good precision causes the 90% confidence interval of the recovered waveform to be very broad. The spectra recovered with the QU-PM model, instead, lie in a narrower interval because their values are determined by the binary’s parameters, which with 3G detectors are recovered with a very high precision (see Appendix B). We also note that the optimized FREE-PM model peak does not present the same shift as the QU-PM one, which is consistent with the fact that the mismatches in the the high-frequency region shown in Fig. 1 are systematically lower for the FREE-PM model. For this purpose, both our QU-PM and FREE-PM models need to be improved towards more structured signals. Moreover, hybridization of NR waveforms starts from the few last cycles of the inspiral, so that also the late-inspiral and merger waveform is based on NR simulations, and thus different from the model we employ. The difference between the hybrids and the waveform models in the late-inspiral region is visible also in Fig. 10 and can lead to biases, affecting the results obtained not only with our FREE-PM or QU-PM models, but also with the model without postmerger.
IV. CONCLUSIONS

We have developed an analytical, frequency-domain model to describe the GW emission during the inspiral, merger, and postmerger phases of a BNS coalescence. For the inspiral and merger, we employed the IMRPhenomD_NRTidalv2 waveform. We incorporate the postmerger part through modeling the main emission peak with a Lorentzian, whose parameters, in the two versions of our model, are either free or determined by quasi-universal relations. Due to the computational cost of the analysis, our study was limited to a restricted number of BNS systems. We have shown that in the best-case scenario of simulations with zero noise and high SNR, i.e. at a distance of 68 Mpc and with the ETCE network, the QU-pm model leads to better constraints on the \( \tilde{\Lambda} \) posteriors compared to the ones obtained with the NO-pm model, and the FREE-pm model grants an accurate measurement of the frequency of the main postmerger emission peak. Within our study, we find that noise fluctuations can significantly impact the results; as shown in Sec. III C, they produce both large differences on the accuracy of \( \tilde{\Lambda} \) measurements (quantified by the width of the 90% confidence interval of the recovered \( \tilde{\Lambda} \) posterior, e.g. Fig. 8), and shifts in the median value of such posterior, cf. Table V. In some cases, this overcomes the improvement on \( \tilde{\Lambda} \) measurements yielded by the use of the QU-pm model, and calls for caution in the interpretation of the results, to distinguish the effects of a different model from the ones of noise. It is important to note that the shifts in \( \tilde{\Lambda} \) recovery caused by noise fluctuations, which are evident especially in high-SNR injections, given the narrowing of the posterior, also affect the results obtained with the model without postmerger. In general, including the postmerger during the analysis provides tighter constraints on the \( \tilde{\Lambda} \) posterior than the original inspiral-only IMRPhenomD_NRTidalv2 model. Finally, we used our model to recover signals obtained by injecting NR waveforms. Although we still see improvements in some cases when using the postmerger models, they are not as significant as we found for the simulated signals. This is due to noise effects and the fact that NR waveforms include postmerger signals with a complex structure, which a simple Lorentzian model struggles to recover. Despite the promising results, we conclude that our model, in both its versions, still needs improvements in order to be employed in the analysis of real signals.

Another central point of our study was to assess the
performance of different detector networks, and to under-
stand how future detectors will improve the postmerger analysis. In particular, we considered four different net-
worfs: (i) Advanced LIGO+ in Hanford and Livingston together with Advanced Virgo+; (ii) the same network as (i) extended by KAGRA and LIGO-India; (iii) the same network as (ii) extended with NEMO; (iv) a net-
worfs consisting of a 40 km Cosmic Explorer and a 10km, triangular Einstein Telescope. Although 3G detectors,
as expected, will give the best constrains on $\tilde{\Lambda}$, we found that NEMO, thanks to its very high sensitivity in the kilohertz band, yields the biggest improvement when using the QU-PM model.

Our study showed how, with future detector networks, GW observations from the postmerger phase of a BNS coalescence will allow us to unravel information about the fundamental physics describing supranuclear-dense matter.

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Appendix A: Results for the free-parameter model

In the following, we show some results obtained with the postmerger model using free Lorentzian parameters. Performing parameter estimation analysis with the FREE-PM waveform requires sampling over three additional parameters, which implies even higher computational costs. For this reason, we could not run the same analyses with the FREE-PM model as we did for the QU-PM one. As shown in Sec. III A with high-SNR and zero-noise injections, we can recover $c_1$ accurately. In Fig. 11, we show how different detector networks can recover the $c_1$ param-
eter in the case of Gaussian noise injections, for simulated signals corresponding to Source3[free-PM] at 68 Mpc. In the case of second generation detectors, we basically recover the prior, although with a peak between [2500,3000] Hz, where also the injected value lies. Adding NEMO to the network leads to a strong improvement, resulting in a very sharp peak for the $c_1$ posterior. The recovered value of $c_1$ with the LHVKIN network is slightly overestimated with respect to the injected value. However, this happens also for the ETCE network, where again the posterior is a sharp peak, and the injected value lies in its lower tail, outside of the 90% confidence interval. In Sec. III A, we saw that, for the ETCE network, for the same simulated signal injected in zero noise, the value of $c_1$ is recovered very well. Therefore, we conclude that the shifts in the posterior peaks for ETCE and LHVKIN networks for the injections in Gaussian noise are most likely due to noise fluctuations, which, as reported in Sec. III C, for this source affect also the $\tilde{\Lambda}$ measurements. Finally, analyses of signals obtained by NR waveforms injections do not recover either of the Lorentzian parameters, mainly because of the complex structure of the postmerger signal in the NR waveforms, as already shown in Sec. III D. Although the FREE-PM model still needs improvement for the analysis of real signals, the results in Fig. 11 are promising, and especially show that adding NEMO to a network of second generation detectors will certainly make a difference for the study of BNS postmerger signals.

Appendix B: Parameter estimation with future detectors

Our discussion focused on the recovery of the $\tilde{\Lambda}$ pa-
parameter, or of the $c_1$ parameter in the case of FREE-PM
model, because these are the quantities that encode most of the information about the EoS. However, it is also interesting to look at the recovery of all the other binary parameters, to see how future detectors will help improving our knowledge of these systems. Fig. 12 shows the comparison between the normalized posterior probability density for $M_c$, $q$, $\Lambda$, $\chi_1$, $\chi_2$, $\alpha$, $\delta$ and luminosity distance $d_L$, obtained using different detector networks, for Source2[qu−pm] injections at 68 Mpc and in zero noise.

We find that 3G detectors will yield a strong improvement not only for what concerns $\Lambda$ recovery, but also in the estimation of $M_c$, $q$ and $d_L$; in particular, with the ETCE network we can estimate $M_c$ with a precision roughly 10 times better than the LHV one. We find only a slight improvement in the recovery of the spin magnitude values $\chi_1$ and $\chi_2$. The best estimation of the sky location parameters ($\alpha$, $\delta$) comes from the LHVKIN network, which is expected considering the larger number of detectors and their geographical distribution, as shown in Fig. 2. We also note that the improvement obtained by adding NEMO to the network is roughly of a factor 1.9 and 1.6 for $M_c$ and $q$ respectively, when computed in comparison with the LHVKI network, but it reaches a factor 4.4 for $\Lambda$ estimation. As discussed in Sec. IIIB, this is achieved thanks to the postmerger contribution to the signal, which for NEMO is significant as a result of its very high sensitivity in the kilohertz band. Overall, future detectors will grant very precise constraints on the BNS parameters, allowing us to better understand the properties and populations of these objects. We also point out that, for computational reasons, our analyses were performed starting from a frequency $f_{\text{low}} = 30$ Hz, and hence, in reality, additional information will be available by analyzing signals starting from lower frequencies. This will lead to a large improvement especially for the 3G detectors, because, for example, the xylophone configuration of ET, with the low frequency interferometer possibly operating at cryogenic temperatures, will ensure a good sensitivity down to $f_{\text{low}} = 5$ Hz. The additional information carried in the many inspiral cycles at low frequencies will further improve the constraints on the BNS parameters, being particularly beneficial for the spin parameters, considering that at low frequencies also spin-induced quadrupole moment effects become significant.

Appendix C: Relative binning settings and validity

The relative binning method allows us to greatly reduce the computational cost of our analysis. As explained in Sec. ICT, a fundamental requirement to employ this technique is having a reference waveform that describes the data sufficiently well. Although with real data we do not know the exact parameters of the source a priori, we can use information from low-latency analyses and quasi-universal relations to find the values to use as the fiducial parameters. Since there might still be biases in the parameters determined in such way, we checked the
influence of the choice of fiducial parameters, performing some tests with different fiducial values for $\Lambda_1, \Lambda_2$, and we found consistency between results.

In [86], the authors show results obtained with this method for GW170817, which, despite being a loud event, has an SNR much lower to the ones we study in this work (cf. Table III). The approximations used in relative binning are not expected to retain validity only in a given SNR range, but we tested the efficacy of this method applied to very loud signals by checking the consistency against results obtained with the nested sampling package LALInference [106] of the LIGO Algorithms Library (LAL) software suite [107].

Finally, when using the relative binning method, the choice of frequency bins in which the waveform is evaluated plays a crucial role. Following [86], this choice is dictated by the requirement that the differential phase change in each bin is smaller than some threshold $\delta_\phi$. In [86], the phase change is computed assuming a post-Newtonian (PN) description of the signal, in which the effect of the different binary parameters enter the phase with different powers of frequency. In the merger and postmerger part of the signal, the PN approximation is not valid anymore. It is not easy to find a similar way to properly describe the phase in the postmerger, without having to evaluate the waveform and incurring in computationally expensive processes that would undermine the speed-up advantage of this method. On the other hand, the phase computed with the PN approximation is then interpolated with frequency, and the frequency bins are determined by evaluating this interpolant over a grid of phases determined by the required precision $\delta_\phi$. Therefore, if such threshold is chosen small enough (for our analysis we set $\delta_\phi = 0.01$), we expect that the way in which the phase change is computed plays little role, and the dense frequency binning produced ensures that the bins' width is small enough to allow anyway a linear interpolation of the ratio between the generated waveform and the fiducial one, as in Eq. [19]. If this was not true, we

| $\delta_\phi$ | Total bins | PM bins |
|--------------|------------|---------|
| 0.005        | 6285       | 2767    |
| 0.007        | 4489       | 1976    |
| 0.01         | 3143       | 1384    |
| 0.03         | 1049       | 462     |
| 0.05         | 630        | 277     |

TABLE VI. Number of frequency bins employed in the relative binning method for different values of $\delta_\phi$, both for the frequency range [30, 4096] Hz and in the postmerger region, starting at the merger frequency.
would expect that changing the threshold $\delta$, and consequently the frequency bins, over which the waveform is evaluated, would give different results also if $\delta$ was kept small. Table 4 reports the number of frequency bins employed by the relative binning technique for different values of $\delta$, both in the whole frequency range considered for the analysis, and for the postmerger region only. Choosing small values of $\delta$ means increasing the number of bins over which we evaluate the waveform, and therefore the computational cost of the analysis; nevertheless, performing the analysis using relative binning with these settings is still much faster than running ‘standard’ parameter estimation analyses, which, for these kind of signals, are not computationally feasible. For standard parameter estimation, the waveform needs to be evaluated on a uniform grid that, with signals of the duration of roughly 200 s as the ones analysed here, includes approximately $8 \times 10^4$ points. Hence, considering that relative binning needs the evaluation of each sampled waveform only at the edges of the bins, this technique greatly reduces the number of required waveform evaluations. Figure 13 shows the posteriors recovered with the QU-PM model for the binary parameters of a Source2 injection at 135Mpc, with the LHVKIN network. We repeated the analysis multiple times, keeping the same settings but changing the frequency binning by using different values of $\delta$. We keep $\delta$ small, but look at both larger and smaller values with respect to the $\delta = 0.01$ used throughout this work. As the plot shows, we find great consistency between the results obtained with all the different values of $\delta$. Consequently, despite that the PN approximation does not hold in the postmerger phase, using it to determine the frequency bins for the relative binning method does not spoil the results, provided that the chosen $\delta$ results in small bin widths.

[1] B. P. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. Lett. 119, 161101 (2017), 1710.05832.
[2] T. Dietrich, T. Hinderer, and A. Samajdar, Gen. Rel. Grav. 53, 27 (2020), 2004.02527.
[3] K. Chatziioannou, Gen. Rel. Grav. 52, 109 (2020), 2006.03168.
[4] J. Aasi et al. (LIGO Scientific), Class. Quant. Grav. 32, 074001 (2015), 1411.4547.
[5] F. Acernese et al. (VIRGO), Class. Quant. Grav. 32, 024001 (2015), 1408.3978.
[6] B. P. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. X 9, 011001 (2019), 1805.11579.
[7] B. P. Abbott et al. (LIGO Scientific, Virgo), Astrophys. J. Lett. 892, L3 (2020), 2001.01761.
[8] T. Hinderer, B. D. Lackey, R. N. Lang, and J. S. Read, Phys. Rev. D81, 123016 (2010), 0911.3535.
[9] T. Damour, A. Nagar, and L. Villain, Phys. Rev. D85, 123007 (2012), 1203.4352.
[10] W. Del Pozzo, T. G. F. Li, M. Agathos, C. Van Den Broeck, and S. Vitale, Phys. Rev. Lett. 111, 071101 (2013), 1307.8338.
[11] B. D. Lackey and L. Wade, Phys. Rev. D91, 043002 (2015), 1410.8866.
[12] M. Agathos, J. Meidam, W. Del Pozzo, T. G. F. Li, M. Tompitsk, J. Veitch, S. Vitale, and C. Van Den Broeck, Phys. Rev. D92, 023012 (2015), 1503.05405.
[13] T. Dietrich et al., Phys. Rev. D99, 024029 (2019), 1804.02235.
[14] K. Kawaguchi, K. Kiuchi, K. Kyutoku, Y. Sekiguchi, M. Shibata, and K. Taniguchi, Phys. Rev. D97, 044040 (2018), 1802.06518.
[15] T. Hinderer, Astrophys. J. 677, 1216 (2008), 0711.2420.
[16] T. Damour and A. Nagar, Phys. Rev. D81, 084016 (2010), 0911.5041.
[17] M. Punturo et al., Class. Quant. Grav. 27, 194002 (2010).
[18] M. Maggiore et al., JCAP 03, 050 (2020), 1912.02622.
[19] A. Freise, S. Chelkowski, S. Hild, W. Del Pozzo, A. Perreca, and A. Vecchio, Class. Quant. Grav. 26, 085012 (2009), 0804.1036.
[20] S. Hild, S. Chelkowski, A. Freise, J. Franc, N. Morgado, R. Flaminio, and R. DeSalvo, Class. Quant. Grav. 27, 015003 (2010), 0906.2655.
[21] B. Sathyaprakash et al., in 46th Rencontres de Moriond on Gravitational Waves and Experimental Gravity (2011), pp. 127–136, 1108.1423.
[22] C. Pacilio, A. Maselli, M. Pasano, and P. Pani, Phys. Rev. Lett. 128, 101101 (2022), 2104.10035.
[23] P. K. Gupta, A. Puecher, P. T. H. Pang, J. Quarta, G. Koekoek, and C. Broeck Van Den (2022), 2005.01182.
[24] D. Reitze et al., Bull. Am. Astron. Soc. 51, 035 (2019), 1907.04833.
[25] M. Evans et al. (2021), 2109.09882.
[26] S. Küppel, L. Bovard, and L. Rezzolla, Astrophys. J. 872, L16 (2019), 1901.09977.
[27] A. Bauswein, T. W. Baumgarte, and H. T. Janka, Phys. Rev. Lett. 111, 131101 (2013), 1307.5191.
[28] L. Baiotti, B. Giacomazzo, and L. Rezzolla, Phys. Rev. D 78, 084033 (2008), 0804.0594.
[29] S. Bernuzzi, Gen. Rel. Grav. 52, 108 (2020), 2004.06419.
[30] L. Baiotti and L. Rezzolla, Rept. Prog. Phys. 80, 096901 (2017), 1607.03540.
[31] P. T. H. Pang et al. (2022), 2205.08513.
[32] P. Hammond, I. Hawke, and N. Andersson, Phys. Rev. D 104, 103006 (2021), 2108.08649.
[33] C. Raithel, V. Paschalidis, and F. Özel, Phys. Rev. D 104, 063016 (2021), 2104.07226.
[34] E. R. Most, A. Haber, S. P. Harris, Z. Zhang, M. G. Alford, and J. Noronha (2022), 2207.00442.
[35] W. Kastaun and F. Galeazzi, Phys. Rev. D 91, 064027 (2015), 1411.7975.
[36] A. Bauswein and N. Stergioulas, Phys. Rev. D91, 064001 (2012), 1204.1888.
[37] K. Takami, L. Rezzolla, and L. Baiotti, Phys. Rev. D 97, 064001 (2018), 1802.06518.
[38] A. Bauswein and N. Stergioulas, Phys. Rev. D 91, 064001 (2015), 1412.3240.
[39] S. Bernuzzi, T. Dietrich, and A. Nagar, Phys. Rev. Lett. 115, 091101 (2015), 1504.01764.
[40] A. Bauswein and N. Stergioulas, Phys. Rev. D91, 054040 (2020), 1905.01182.
[91] S. Hild et al., Class. Quant. Grav. 28, 094013 (2011), 1012.0908.
[92] M. Evans, J. Harms, and S. Vitale, Exploring the Sensitivity of Next Generation Gravitational Wave Detectors, https://dcc.ligo.org/LIGO-P1600143/public (2016).
[93] Y. Michimura, K. Komori, Y. Enomoto, K. Nagano, and K. Somiya, Example sensitivity curves for the KAGRA upgrade (2020).
[94] K. Kiuchi, K. Kawaguchi, K. Kyutoku, Y. Sekiguchi, M. Shibata, and K. Taniguchi, Phys. Rev. D 96, 084060 (2017), 1708.08926.
[95] K. Kawaguchi, K. Kiuchi, K. Kyutoku, Y. Sekiguchi, M. Shibata, and K. Taniguchi, Phys. Rev. D 97, 044044 (2018), 1802.06518.
[96] T. Hinderer et al., Phys. Rev. Lett. 116, 181101 (2016), 1602.00599.
[97] L. Wade, J. D. E. Creighton, E. Ochsner, B. D. Lackey, B. F. Farr, T. B. Littenberg, and V. Raymond, Phys. Rev. D 89, 103012 (2014), 1402.5156.
[98] J. Miller, L. Barsotti, S. Vitale, P. Fritschel, M. Evans, and D. Sigg, Phys. Rev. D 91, 062005 (2015), URL https://link.aps.org/doi/10.1103/PhysRevD; 91.062005
[99] B. P. Abbott et al. (KAGRA, LIGO Scientific, Virgo, VIRGO), Living Rev. Rel. 21, 3 (2018), 1304.0670.
[100] T. Akutsu et al. (KAGRA), PTEP 2021, 05A101 (2021), 2005.05574.
[101] K. Somiya (KAGRA), Class. Quant. Grav. 29, 124007 (2012), 1111.7185.
[102] Y. Aso, Y. Michimura, K. Somiya, M. Ando, O. Miyakawa, T. Sekiguchi, D. Tatsumi, and H. Yamamoto (KAGRA), Phys. Rev. D 88, 043007 (2013), 1306.6747.
[103] M. Saleem et al., Class. Quant. Grav. 39, 025004 (2022), 2105.01716.
[104] K. Ackley et al., Publ. Astron. Soc. Austral. 37, e047 (2020), 2007.03128.
[105] V. Srivastava, D. Davis, K. Kuns, P. Landry, S. Ballmer, M. Evans, E. D. Hall, J. Read, and B. S. Sathyaprakash, Astrophys. J. 931, 22 (2022), 2201.10668.
[106] J. Veitch et al., Phys. Rev. D91, 042003 (2015), 1409.7215.
[107] https://wiki.ligo.org/DASWG/LALSuite URL https://wiki.ligo.org/DASWG/LALSuite