Invariant length scale in relativistic kinematics – Lessons from Dirichlet branes

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We show that Dirac–Born–Infeld theory possesses a hidden invariance that enhances the local $O(1,p)$ Lorentz symmetry on a Dirichlet $p$-brane to an $O(1,p) \times O(1,p)$ gauge group, encoding both an invariant velocity and acceleration (or length) scale. This enlarged gauge group predicts consequences for the kinematics of Dirichlet branes, with admissible accelerations being bounded from above. An important lesson beyond string theory is that a fundamental length scale can be implemented into the kinematics of general relativity, preserving both space-time as a smooth manifold and local Lorentz symmetry, contrary to common belief. We point out consequences for string phenomenology, classical gravity and atomic physics.

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Any candidate theory of quantum gravity must address the breakdown of the classical smooth manifold picture of space-time at distances comparable to the Planck length $\sqrt{\hbar G/c^3}$. The fundamental structure of physical space-time is therefore expected to be crucially different from smooth Lorentzian manifolds \cite{1}. Some proposals for quantum gravity theories address this issue \textit{ab initio} by employing a discrete structure to describe space-time, e.g., as a spin foam \cite{2}. Superstring theory, in contrast, is formulated on conventional smooth ten-dimensional Lorentzian space-time, but encodes a fundamental length scale $\ell$ in the dynamics. Models \cite{2} of the observable universe as a four-dimensional Dirichlet brane propagating in this higher dimensional manifold attract much attention in string phenomenology. At the level of D-branes, Seiberg and Witten \cite{3} observed a significant departure from the smooth manifold picture of space-time, in accordance with our expectations for quantum theories of gravity. They cast the D-brane dynamics at low energy into a gauge theory on a non-commutative space-time. The latter encodes the fundamental string length scale into the geometry without breaking Lorentz invariance, but at the cost of giving up the smooth manifold even in this low energy limit. The preservation of Lorentz invariance is crucial, as this is one of the most accurately tested symmetries in physics, with no indication that it is broken or deformed at any observable energy scale \cite{3}.

In this Letter, we show that Dirichlet $p$-branes, considered as smooth sub-manifolds of the string target space, possess a hidden $O(1,p)$ invariance in the low energy limit. Remarkably, this dynamical invariance can be absorbed into the brane geometry in a way that preserves both the smooth manifold structure and the local Lorentz symmetry. The resulting kinematics encode a maximal acceleration $a \sim 1/\ell^2$ in addition to the invariant speed of light, by an extension, rather than deformation, of the local Lorentz group $O(1,p)$ to $O(1,p) \times O(1,p)$. The strong equivalence principle is not violated, since the inertial frames coincide with those of general relativity. The derived geometry provides the relevant kinematical framework for observers in brane-world scenarios, with direct implications for string phenomenology. While the technique devised in this Letter is motivated by the low energy dynamics on D-branes, it can be applied independently of its string theoretical origin in order to geometrically encode the minimal length scale, expected in any theory of quantum gravity, into the kinematics of generally relativistic theories, preserving both local Lorentz symmetry and the strong principle of equivalence. In the context of string phenomenology, our approach describes the low energy dynamics on D-branes without giving up the concept of a smooth space-time manifold. In the context of general relativity, as we show below, it is related to the so-called non-symmetric gravity theory which features a ghost-free regularization of space-time singularities. Our formalism is finally sufficiently manageable to connect to well-known experiments in atomic physics. A correction to the Thomas precession \cite{6} provides an experimental lower bound of $a \geq 10^{22} m/s^2$.

Let us now turn to the detailed demonstration of our claims. For technical simplicity, we consider a Dirichlet $p$-brane in bosonic string theory, i.e., a $(p+1)$-dimensional Lorentzian sub-manifold $(\Sigma, g)$ of a 26-dimensional bulk space-time \cite{7}, with a gauge potential $A$ whose dynamics are governed by the Dirac–Born–Infeld action

\begin{equation}
\int_{\Sigma} \ell^{-(p+1)} \sqrt{|\det g_{\mu\nu}|} \sqrt{\det(\delta_{\mu\nu} + F_{\mu\nu})}
\end{equation}

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in the low energy limit \[8\]. The antisymmetric tensor \( F_{\mu\nu} = B_{\mu\nu} + \ell^2 F_{\mu\nu} \) contains the pull-back \( B_{\mu\nu} \) of the Neveu-Schwarz two-form field to the brane, and the field strength \( F_{\mu\nu} \) of the Abelian gauge potential \( A_\mu \) on the brane \( \Sigma \). Indices of fields living on the brane are raised and lowered using the induced metric \( g_{\mu\nu} \) on the brane. The action \[11\] is manifestly invariant under local \( O(1,p) \) transformations. In order to exhibit an additional hidden \( O(1,p) \) invariance, first observe that only even powers of \( F \) contribute to the Lagrangian, as \( \det(\delta + F) = \exp \tr \ln(1 + F) \), and the trace annihilates the odd powers of \( F \) in the series expansion of the logarithm. We are therefore free to multiply the field \( F \) by a unit square root \( \sqrt{\det g_{\mu\nu}} \). The Lagrangian density \( \mathcal{L}_{\text{DBI}} \) can be written as

\[
\mathcal{L}_{\text{DBI}} = \ell^{-(p+1)} \sqrt{\det g_{\mu\nu}} \sqrt{\det(\delta_{\mu\nu} + IF_{\mu\nu})}.
\]

Rather than assuming that \( I \) is real, it will turn out to be enlightening to consider an algebraic extension \( \mathbb{P} \) of \( \mathbb{R} \),

\[
\mathbb{P} := \{ a + Ib \mid a, b \in \mathbb{R}, I^2 = +1 \}
\]

with \( I \in \mathbb{P} \), but \( I \not\in \mathbb{R} \). The set \( \mathbb{P} \), equipped with addition and multiplication inherited from \( \mathbb{R} \), is a commutative ring with unit, which we term the \textit{pseudo-complex} numbers \[9\]. \( \mathbb{P} \) fails to be a number field due to the existence of zero-divisors

\[
\mathbb{P}^0 := \mathbb{P}_+ \cup \mathbb{P}_0 = \langle \sigma_+ \rangle_\mathbb{R} \cup \langle \sigma_- \rangle_\mathbb{R},
\]

where \( \sigma_{\pm} := \frac{1}{2}(1 \pm I) \). Note that the zero-divisors \( \mathbb{P}^0 \) do not have multiplicative inverses, and that \( \sigma_{\pm} \) are orthogonal projectors onto the eigenspaces of \( I \), \( I \sigma_{\pm} = \pm \sigma_{\pm} \), \( \sigma_+^2 = \sigma_+ \) and \( \sigma_-^2 = 0 \). As we will show in the following, taking the pseudo-complex form \[2\] of the Dirac–Born–Infeld Lagrangian seriously directly leads to the claimed extension of relativistic kinematics that incorporates a finite upper bound on accelerations. Mathematically, the addition in \[2\] requires the pseudo-complexification \( T_q \Sigma_\mathbb{P} \) of the tangent spaces \( T_q \Sigma \) \[10\], so that

\[
H^{\mu\nu} := \delta^{\mu\nu} + IF^{\mu\nu}
\]

is a well-defined tensor. A local frame at a point \( q \in \Sigma \) of a pseudo-complexified tangent space is specified by \((p+1)!\) orthonormal pseudo-complex vectors \( E_a \in (T_q \Sigma_\mathbb{P})^0 \cong \mathbb{P}^{p+1} \), i.e., \( g(E_a, E_b) = \eta_{ab} \), where \( q \) extends \( \mathbb{P} \)-bilinearly and \( \eta \) is the \((p+1)\)-dimensional Minkowski metric. On the dual space of \( (T_q \Sigma_\mathbb{P})^0 \), we choose the dual basis \( E^\alpha \), such that \( E^\alpha E_b = \delta_b^\alpha \). There is a gauge freedom associated with the choice of frame whose gauge group is the pseudo-complexified orthogonal group of the Minkowski metric \( \eta_{ab} \), i.e.,

\[
O_p(1,p) = \{ \Lambda \in \text{End}(\mathbb{P}^{p+1}) \mid \Lambda^m a \Lambda^n b \eta_{mn} = \eta_{ab} \},
\]

which, considered as a (real) Lie group, is of twice the dimension of the real Lorentz group \( O(1,p) \), and contains the latter as a subgroup. Of direct physical interest are only classes of frames related by transformations in the connection component of the identity \( SO_p^\mathbb{P} \) of the special orthogonal group. Its vector representation is easily obtained from the exponentiation of the matrices \( (M_{mn})_b := \eta^{mn} \delta^b_0 - \eta^{m0} \delta^b_n \), giving \( \Lambda(\omega) = \exp(\omega_{mn} M_{mn}) \), with pseudo-complex parameters \( \omega_{mn} \in \mathbb{P} \).

In order to see how the Dirac–Born–Infeld tensor \[3\] transforms under pseudo-complexified Lorentz transformations, it is necessary to study the representation theory of \( SO_p^\mathbb{P}(1,p) \) in some detail. First, we note from the definition \[10\] that the pseudo-complex Lorentz group as a Lie group decomposes into a direct product of two real (proper orthochronous) Lorentz groups,

\[
SO_p^\mathbb{P}(1,p) \cong SO^\mathbb{R}(1,p) \times SO^\mathbb{R}(1,p),
\]

by employing the zero-divisor decomposition \( \Lambda = \Lambda_+ \sigma_+ + \Lambda_- \sigma_-, \) where \( \Lambda_+ , \Lambda_- \in SO^\mathbb{R}(1,p) \). We can hence first apply the standard theory for (real) Lie groups, then identify the pseudo-complex vector representation \( (T_q \Sigma_\mathbb{P})^0 \cong \mathbb{P}^{p+1} \), and finally study its tensor powers. The irreducible real representations \( \mathcal{R} \) of \( SO_p^\mathbb{R}(1,p) \) are tensor products \( V_1 \otimes V_2 \) of real representations \( V_1 , V_2 \) of \( SO(1,p) \), on which \( \mathcal{R}(\Lambda) \) acts by

\[
\mathcal{R}(\Lambda)[v_1 \otimes v_2] := \rho_1(\Lambda_+)[v_1] \otimes \rho_2(\Lambda_-)[v_2].
\]

It is convenient to denote the ‘small’ representations of \( SO_p^\mathbb{R}(1,p) \) by pairs \([d_1, d_2]\) where \( d_j = \dim \mathcal{R}_j \). We define \textit{pseudo-complex conjugation} as the \( \mathbb{P} \)-linear map \( * : \mathbb{P} \to \mathbb{P} \), sending \( \sigma_\pm \mapsto \sigma_\mp \). The \textit{pseudo-complex conjugate} of a matrix is \( M^\dagger = (M^\ast)^\dagger \), and \( M \) is called \textit{pseudo-hermitian} if \( M^\dagger = M \). For each representation \( \mathcal{R} \), there exists a not necessarily equivalent \textit{pseudo-complex conjugate representation} \( \mathcal{R}^\ast \) on the same space, defined by \( \mathcal{R}^\ast(\Lambda) := \mathcal{R}(\Lambda^\dagger) \).

It is well known that the ‘smallest’ irreducible real representations of \( SO_p^\mathbb{R}(1,p) \) are of dimension \( d_1 = 1 \) (trivial), \( d_2 = p+1 \) (vector), \( d_3 = (p+1)/2 \) (antisymmetric rank-2 tensor) and \( d_4 = (p+1)(p+2)/2-1 \) (traceless symmetric rank-2 tensor). Considering direct sums of two irreducible pseudo-complex conjugate representations, we obtain a large class of \( \mathbb{P} \)-modules on which \( SO_p^\mathbb{P}(1,p) \) is represented. Clearly, the pseudo-complexified tangent spaces \( (T_q \Sigma_\mathbb{P})^0 \cong \mathbb{P}^{p+1} \) correspond to the vector representation \( \mathcal{R}_v \cong [d_v, 1] \oplus [1, d_v] \), whose elements \( v \) transform under \( \Lambda \in SO_p^\mathbb{P}(1,p) \) by

\[
v^m \mapsto \Lambda^m_n v^n = \sigma_+ \Lambda^+ \Lambda^m_n v^n + \sigma_- \Lambda^m_n v^n,
\]

where the last expression is given in the zero-divisor decomposition with \( v = v_+ \sigma_+ + v_- \sigma_- \), \( v \in \mathbb{P}^{p+1} \). There is also a \textit{trivial} (scalar) representation \( \mathcal{R}_t \) on the \( \mathbb{P} \)-module
\[ H^{mn} \mapsto \Lambda^m_k (\Lambda^*|^n_l H^{kl}) \]
\[ = \sigma_+ \Lambda^m_k \Lambda^n_l H^{kl} + \sigma_- \Lambda^m_k \Lambda^n_l H^{kl}. \]  

As the transformation maps pseudo-(anti-)hermitean tensors to pseudo-(anti-)hermitean ones, this representation decomposes further into pseudo-hermitean \( \mathcal{R}_H \) and pseudo-anti-hermitean rank-2 tensors \( \mathcal{R}_{\overline{H}} \), both of real dimension \( d^2 \). Since the vector representation \( \mathcal{R}_v \) is isomorphic to its pseudo-complex conjugate \( \mathcal{R}_{\overline{v}} \), we can find all irreducible second rank tensors of \( SO^P(1,p) \) in the decomposition,

\[ \mathcal{R}_v \otimes \mathcal{R}_v \cong \mathcal{R}_t \oplus \mathcal{R}_a \oplus \mathcal{R}_s \oplus \mathcal{R}_H \oplus \mathcal{R}_{\overline{H}}. \]

where an \( \mathbb{R} \)-bilinear tensor product must be used.

The Dirac–Born–Infeld tensor is pseudo-hermitean, i.e., of type \( \mathcal{R}_H \) and therefore transforms as \( \mathbb{R} \). As \( \mathcal{R}_H \) is irreducible, the \( \delta^{\mu\nu} \) - and the \( F^{\mu\nu} \) -part of \( \mathcal{R}_H \) will mix under generic pseudo-complex transformations. The space-time indices of \( H_{\mu\nu} \) are hence converted to indices with respect to a local orthonormal pseudo-complex basis by means of

\[ H_{ab} := E^\mu_a E^\nu_b H_{\mu\nu}. \]

Note that the alternative choice, \( H_{ab} := E^*_\mu a E^*_\nu b H_{\mu\nu} \), presents a transition to the isomorphic conjugate representation \( \mathcal{R}_{\overline{H}} \cong \mathcal{R}_H \), which physically corresponds to a charge conjugation, as pseudo-complex conjugation * acting on \( H \) inverts the sign of the electromagnetic field strength, which can be absorbed into a redefinition of charges \( q \mapsto -q \). Observe that the Dirac–Born–Infeld Lagrangian is charge conjugation invariant. If the Dirac–Born–Infeld Lagrangian is expressed in terms of \( H_{ab} \),

\[ \mathcal{L}_{DBI} = \ell^{-(p+1)} \sqrt{|\det g_{\mu\nu}|} \sqrt{(-1)^n \det E^\mu_a E^\nu_b H_{ab}}, \]

it is manifestly invariant under the \( SO^P(1,p) \) transformations

\[ H_{ab} \mapsto \Lambda^m_a \Lambda^n_b H_{mn}, \quad E_a \mapsto E^m_a. \]

Before we set out to reveal the physical interpretation of the pseudo-complex frame changes, we derive the most general form of pseudo-complex frames that can be reached from real frames by means of \( SO^P(1,p) \)-transformations. One can show that a pseudo-complex frame transformation \( \Lambda \in SO^P(1,p) \) uniquely decomposes into a product of a real Lorentz transformation \( L \in SO^R(1,p) \) and a pseudo-complex transformation \( Q = \exp(\omega_{mn} M^{mn}) \) with purely pseudo-imaginary parameters \( \omega_{mn} = -\omega^*_{mn} \). Therefore, the most general pseudo-complex frame \( \epsilon \) that can be reached from the real frame \( \epsilon \) is given by \( E_a = Q_a^b b_{b}^c e_c \). But \( b_{b}^c e_c \) simply presents a change of the real frame, so that it suffices to study the case where \( L \) is the identity. Now the transformation \( Q \) can be uniquely decomposed into a real and imaginary part, such that

\[ E_a = \gamma_a^b (\delta^c_b + \frac{I}{a} \theta^c_b) e_c, \]

where \( a \) is a parameter of inverse length dimension, to be determined below. Using that \( Q^* = Q^{-1} \), one easily finds that

\[ \gamma_{ab} = \gamma_{ba} \quad \text{and} \quad \theta_{ab} = -\theta_{ba}. \]

A subsequent real Lorentz transformation clearly preserves these symmetries, so that the restriction to \( L = \text{id} \) was without loss of generality. Note that \( \theta \) determines \( \gamma \) up to a real Lorentz transformation, because \( Q \) is invertible and hence injective when acting as a linear map.

In order to unravel the physical meaning of the pseudo-complex frame transformations, consider a solution of \((1 + 3)\)-dimensional Born–Infeld electrodynamics, i.e., assume a flat background \( \eta \), vanishing NS two-form \( B_{\mu\nu} = 0 \), and an electromagnetic field strength tensor \( F \) that, up to a real Lorentz transform \( L \) is given by \( F_{01} = -F_{10} = E, \quad F_{23} = -F_{32} = B \), while all other components vanish. Acting on \( H = \eta + I e^2 F \) with the transformation

\[ Q = \begin{pmatrix} \cos(I \alpha) & \sin(I \alpha) & 0 & 0 \\ \sinh(I \alpha) & \cosh(I \alpha) & 0 & 0 \\ 0 & 0 & \cos(I \varphi) & \sin(I \varphi) \\ 0 & 0 & -\sin(I \varphi) & \cos(I \varphi) \end{pmatrix}, \]

according to (13), one achieves the reduction of the generically pseudo-complex Born–Infeld tensor \( H \) to its ‘metric’ real part

\[ H'_{00} = -H'_{11} = 1/ \cosh(2 \alpha), \quad H'_{22} = H'_{33} = -1/ \cosh(2 \varphi), \]

if one chooses the transformation parameters \( I \alpha \) and \( I \varphi \) such that

\[ B = \ell^{-2} \tan(2 \varphi), \quad E = \ell^{-2} \tanh(2 \alpha). \]

Note that whereas (20) always has a unique solution, the condition (21) is only solvable for \( E < \ell^{-2} \), as is the case for solutions of Born–Infeld electrodynamics \((11)\) with vanishing magnetic field. Action of (17) on \( E_a \) according to (13) yields

\[ \gamma_{00} = -\gamma_{11} = \cosh(\alpha), \quad \gamma_{22} = \gamma_{33} = -\cos(\varphi). \]
and

\[ \theta_{01} = -\theta_{10} = a \tanh(\alpha), \quad \theta_{23} = \theta_{32} = a \tan(\varphi). \]  

So far, we have discussed the \( SO_F^c(1,p) \)-invariance of Born–Infeld electrodynamics. We now show how particles minimally coupled to the gauge field \( A \) inherit \( SO_F^c(1,p) \) as their kinematical group. It is well known that the Lorentz force on a particle of rest mass \( m \) and electric charge \( q \) can be expressed in the coordinates of a local orthonormal real frame \( e_a \) by \( m \Omega_0^b = q \eta^{bm} F_{mb} \), where \( \Omega_{ab} = -\Omega_{ba} \) is the Frenet–Serret tensor associated with a test particle of four-velocity \( u = e_0 \) and attached spatial frame \( e_a \), so that \( \nabla_u e_a = \Omega_{ab}^c e_b \). The components \( \Omega_{0\alpha} \) encode the 3-acceleration of the particle world-line, and \( e_{\alpha\beta\gamma} \) is the angular velocity of the spatial frame \( e_a \) with respect to a Fermi–Walker transported frame.

We now give the pseudo-complex frame \[ \text{a physical meaning by identifying the antisymmetric real Lorentz tensor } \theta \text{ with the Frenet–Serret tensor } \Omega, \text{ so that the scalar acceleration of a test particle, according to } (23), \text{ is given by } a \tanh(\alpha), \text{ for some } \alpha \in \mathbb{R}. \text{ This identification implies the modified Lorentz force law }

\[ m_a n^b \Omega = q_b H^{nm} F_{ma}, \]  

where the rest mass \( m \) and charge \( q \) are corrected by acceleration-dependent matrix-valued factors \( \gamma = \gamma(\Omega) \)

\[ m_a \gamma(\Omega) = m a, \quad q_a (\Omega) = q \gamma_{\alpha}^{-1} a^\gamma, \]  

and the magnetic field in the co-moving frame is used to determine the motion of the attached spatial frame. Note that by applying the pseudo-complex transformation \( Q \) of \[ \text{(15)}, \text{ we have passed to a co-accelerated frame which is determined up to a local real Lorentz transformation, given by the remaining freedom in } \gamma \text{ once } \theta \text{ has been fixed. The modified Lorentz force law } (24) \text{ holds in this frame so that indeed all quantities in } (24) \text{ are real. For small accelerations } \tanh(\alpha) \ll 1, (24) \text{ smoothly reduces to the standard relativistic Lorentz force law, if we set the so far undetermined parameter } a := 2q/m^2. \text{ This correspondence for low accelerations justifies the identification } \theta = \Omega. \text{ Note that if the proper acceleration tends to } a, \text{ or the angular velocity of the spatial frame to infinity, the dynamical charge } (24) \text{ of the test particle tends to zero. This switching-off of the charge is the dynamical explanation of why the test particle is not further accelerated by the electromagnetic field if it achieves the maximum acceleration.}

The kinematical meaning of the pseudo-complex Lorentz transformations is now evident from \[ (23). \text{ A rotation in the spatial } \alpha\beta-\text{plane with purely pseudo-imaginary parameter } I \varphi \text{ effects a transformation to a uniformly rotating frame with angular velocity } a \tan(\varphi). \text{ By means of local pseudo-imaginary rotations, one can therefore always arrange for non-rotating observers. Starting from such a Fermi–Walker transported observer, a boost transformation with purely pseudo-imaginary parameter } I \alpha \text{ effects a transformation to a translationally accelerated frame of scalar acceleration } a \tanh(\alpha), \text{ obviously respecting the invariant acceleration scale } a \text{ as an upper limit. The action of the real Lorentz group on inertial, i.e., real, frames is clearly as in standard relativity. In particular, the inertial observers of Dirac–Born–Infeld kinematics agree with the inertial observers of standard general relativity. Thus, there is no violation of the equivalence principle implied by the presence of a length scale in relativity. Compared to standard relativity, we impose one additional condition on admissible observers, namely that their co-accelerated, i.e., pseudo-complex, frames are continuously connected to inertial ones by \( SO_F^c(1,p) \), thus encoding the maximal acceleration scale. The term sub-maximally accelerated is justified because whenever the local frame of such an admissible observer is Fermi–Walker transported, then its scalar acceleration is indeed bounded by \( a \). Such relativistic kinematics with an invariant length scale can be considered independently of their origin in Dirac–Born–Infeld theory. We have therefore kinematically implemented a maximal acceleration, an idea going back to Caianiello \[ (12), \text{ but here derived from the low energy dynamics of D-branes.}

There is an intriguing connection to Moffat’s non-symmetric gravitational theory \[ (13). \text{ Using a generalized metric } g_{\mu\nu} + I g_{[\mu\nu]}, \text{ he formulates a theory which is free of ghost poles, tachyons and problems with asymptotic boundary conditions. A static spherically symmetric solution does not contain a black hole event horizon, so that the information loss problem is resolved at the classical level. His generalized metric is pseudo-hermitean } (10), \text{ and hence we now recognize that it transforms under the same representation } R_H \text{ as the Dirac–Born–Infeld tensor. This strongly suggests that Moffat found a gravity theory of Born–Infeld type, whose solutions indeed feature a regulation of gravitational singularities.}

\[ \text{References:} \]

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