A test of $CPT$ symmetry in $K^0$ vs $\bar{K}^0 \to \pi^+\pi^-\pi^0$ decays

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Abstract I show that the $CP$-violating asymmetry in $K^0$ vs $\bar{K}^0 \to \pi^+\pi^-\pi^0$ decays differs from that in $K_L \to \pi^+\pi^-$, $K_L \to \pi^0\pi^0$ or the semileptonic $K_L$ transitions, if there exists $CPT$ violation in $K^0$-$\bar{K}^0$ mixing. A delicate measurement of this difference at a super flavor factory (e.g., the $\phi$ factory) will provide us with a robust test of $CPT$ symmetry in the neutral kaon system.

Key words $K^0$-$\bar{K}^0$ mixing, $CPT$ violation

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1 The motivation

The $CPT$ theorem claims that a Lorentz-invariant local quantum field theory with a Hermitian Hamiltonian must have $CPT$ symmetry [1]. It is so far so good, because there is no convincing experimental hint at $CPT$ violation [2]. The breaking of $CPT$ symmetry, as expected in some “exotic” scenarios of new physics beyond the standard model (e.g., string theory) [3], would be a big deal. In any case, much more experimental tests of this theorem are desirable.

The $K^0$-$\bar{K}^0$ mixing system has been playing an important role in particle physics for testing fundamental symmetries (such as $CP$, $T$ and $CPT$) and examining conservation laws (such as $\Delta S = \Delta Q$). The existing experimental evidence for $CPT$ invariance in the mixing and decays of neutral kaon mesons remains rather poor [2]: it is not excluded that the strength of $CP$-violating interactions could be as large as about ten percentage of that of $CP$-violating interactions. This unsatisfactory situation will be improved in the near future, in particular after a variety of more delicate measurements are carried out at a super flavor factory [4] (e.g., the $\phi$ factory [5]).

There are several possibilities of probing $CPT$ violation in $K^0$-$\bar{K}^0$ mixing with the decays of $K_S$ and $K_L$ mesons into the two-pion and (or) the semileptonic states [2]. A different approach towards testing $CPT$ symmetry, with the help of neutral kaon decays into the three-pion states, has also been pointed out in Ref. [6]. The idea is simply that the $CP$-violating effect induced by $K^0$-$\bar{K}^0$ mixing in $K^0$ vs $\bar{K}^0 \to \pi^+\pi^-\pi^0$ transitions should not be identical to that in $K_L \to \pi^+\pi^-$, $K_L \to \pi^0\pi^0$ or the semileptonic $K_L$ decays, if $CPT$ symmetry is broken. Thus a careful comparison between these two types of $CP$-violating effects may provide us with a robust test of $CPT$ invariance in $K^0$-$\bar{K}^0$ mixing.

An unfortunate fact is that no attention has so far been paid to the method advocated in Ref. [6]. In this talk, which is more or less an advertisement, I shall explain why a test of $CPT$ symmetry is possible by measuring the time-dependent $CP$-violating asymmetry between $K^0(t) \to \pi^+\pi^-\pi^0$ and $\bar{K}^0(t) \to \pi^+\pi^-\pi^0$ decays. My result is hopefully useful for the upcoming experiments of kaon physics.

2 The idea

Let me outline the main idea. The mass eigenstates of $K^0$ and $\bar{K}^0$ can in general be written as

$$|K_S\rangle = \frac{1}{\sqrt{|p_1|^2 + |q_1|^2}} \left( p_1 |K^0\rangle + q_1 |\bar{K}^0\rangle \right),$$

$$|K_L\rangle = \frac{1}{\sqrt{|p_2|^2 + |q_2|^2}} \left( p_2 |K^0\rangle - q_2 |\bar{K}^0\rangle \right),$$

(1)
in which \( p_i \) and \( q_i \) (for \( i = 1, 2 \)) are complex mixing parameters. Note that \( p_1 = p_2 \) and \( q_1 = q_2 \) follow from CPT invariance [7]. The traditional characteristic quantities of \( CP \) violation in the \( K^0-\bar{K}^0 \) mixing system [2], \( \eta_{+-}, \eta_{00} \) and \( \delta_L \), are all related to \( K_L \) decays and thus the \( (p_2, q_2) \) parameters. For example,

\[
\delta_L = \frac{|p_2|^2 - |q_2|^2}{|p_2|^2 + |q_2|^2}
\]

(2)

in the absence of \( \Delta S = -\Delta Q \) interactions. A measurement of \( CP \) violation associated with

\[
\delta_S = \frac{|p_1|^2 - |q_1|^2}{|p_1|^2 + |q_1|^2}
\]

(3)

has been assumed to be extremely difficult, if not impossible, due to the rapid decay of the \( K_S \) meson to the two-pion state or the semileptonic state. Nevertheless, I shall show that \( \delta_S \) can be measured from the rate asymmetry of \( K^0 \) and \( \bar{K}^0 \) mesons decaying into the three-pion state \( \pi^+\pi^-\pi^0 \). The difference between \( \delta_S \) and \( \delta_L \) signifies \( CP \) violation in \( K^0-\bar{K}^0 \) mixing. This point can be seen more clearly if one adopts the popular \((\epsilon, \delta)\) parameters to describe \( CP \)- and \( CPT \)-violating effects in the \( K^0-\bar{K}^0 \) mixing system [2]:

\[
P_1 = 1 + \epsilon + \delta,
\]

\[
P_2 = 1 + \epsilon - \delta,
\]

\[
q_1 = 1 - \epsilon - \delta,
\]

\[
q_2 = 1 - \epsilon + \delta.
\]

(4)

Then

\[
\delta_L = 2(\text{Re}\, \epsilon - \text{Re}\, \delta),
\]

\[
\delta_S = 2(\text{Re}\, \epsilon + \text{Re}\, \delta).
\]

(5)

It turns out that \( \delta_S - \delta_L = 4\text{Re}\, \delta \) is a clear signature of \( CP \) violation [6].

Let me quote two typical experimental constraints on the \( CPT \)-violating parameter \( \delta \) in \( K^0-\bar{K}^0 \) mixing: \( \text{Re}\, \delta = (2.9 \pm 2.6_{\text{stat}} \pm 0.6_{\text{syst}}) \times 10^{-4} \) obtained by the CPLEAR Collaboration [8] and \( \text{Im}\, \delta = (0.4 \pm 2.1) \times 10^{-5} \) obtained by the KLOE Collaboration [9]. A systematic analysis of the \( CP \)- and \( CPT \)-violating parameter space has already been done by the Particle Data Group in Ref. [2].

3 The approach

The \( CP \) eigenvalue for the \( \pi^+\pi^-\pi^0 \) final state is given by \((-1)^{l+1}\), where \( l \) is the relative angular momentum between \( \pi^+ \) and \( \pi^- \). Since the sum of the masses of three pions is close to the kaon mass, the pions have quite low kinetic energy \( E_{\text{CM}}(\pi) \) in the kaon rest-frame, and the states with \( l > 0 \) are suppressed by the centrifugal barrier [10]. Thus the \( K_L \) meson decays dominantly into the kinematics-favored \((l = 0)\) and \( CP \)-allowed \((CP = -1)\) component, and the kinematics-favored \((l = 0)\) but \( CP \)-forbidden \((CP = +1)\) component. This implies an interesting Dalitz-plot distribution for the \( K_L \) decays into \( \pi^+\pi^-\pi^0 \) transition: it is symmetric with respect to \( \pi^+ \) and \( \pi^- \) for the \( CP \)-violating amplitude, but anti-symmetric for the \( CP \)-conserving amplitude. Let the ratio of \( K_S \) and \( K_L \) decay amplitudes be

\[
\eta_{0-} = \frac{A(K_S \rightarrow \pi^+\pi^-\pi^0)}{A(K_L \rightarrow \pi^+\pi^-\pi^0)}.
\]

(6)

It is clear that \( \eta_{0-} \) depends only upon the \( CP \)-violating component of \( A(K_S \rightarrow \pi^+\pi^-\pi^0) \), if data are integrated over the whole Dalitz plot [10, 11]. The time-dependent rates for the initially pure \( K^0 \) and \( \bar{K}^0 \) states decaying into \( \pi^+\pi^-\pi^0 \), denoted by \( R(t) \) and \( \bar{R}(t) \) respectively, can be calculated with the help of Eqs. (1) and (6). I arrive at [6]

\[
\begin{align*}
R(t) & \propto \left| p_1^2 + |q_2|^2 \right| \eta_{0-} e^{-\Delta m t} + 2\text{Re}(q_1^* q_2 \eta_{0-} e^{i\Delta m t}) e^{-\Delta R t/2}, \\
\bar{R}(t) & \propto \left| p_1^2 + |q_2|^2 \right| \eta_{0-} e^{-\Delta m t} - 2\text{Re}(p_1^* p_2 \eta_{0-} e^{i\Delta m t}) e^{-\Delta R t/2},
\end{align*}
\]

(7)

where \( \Delta m > 0 \) and \( \Delta R > 0 \) denote the mass difference and the width difference of \( K_S \) and \( K_L \) mesons, respectively. To a good degree of accuracy, I obtain the following \( CP \)-violating asymmetry:

\[
\begin{align*}
A(t) & \equiv \frac{\bar{R}(t) - R(t)}{\bar{R}(t) + R(t)} = \delta_S - 2 e^{-\Delta R t/2} \left[ \text{Re} \eta_{0-} \cos(\Delta m t) - \text{Im} \eta_{0-} \sin(\Delta m t) \right] \xi - 2 e^{-\Delta R t/2} \left[ \text{Re} \eta_{0-} \sin(\Delta m t) + \text{Im} \eta_{0-} \cos(\Delta m t) \right] \zeta,
\end{align*}
\]

(8)

in which

\[
\begin{align*}
\xi & = \frac{\text{Re}(p_1 q_2^* - q_1 p_2^*)}{|p_1|^2 + |q_1|^2} = 1 + O(|\epsilon|^2) + O(|\delta|^2) + O(\text{Re}(\text{Re}\, \delta^*)) , \\
\zeta & = \frac{\text{Im}(p_1 q_2^* + q_1 p_2^*)}{|p_1|^2 + |q_1|^2} = O(\text{Im}(\text{Re}\, \delta^*)).
\end{align*}
\]

(9)

It is obvious that \( \delta_S \) can be determined through the measurement of \( A(t) \). In particular, the relationship \( \lim_{t \to \infty} A(t) = \delta_S \) holds.
As I have emphasized, the difference between $\delta_S$ and $\delta_L$ hints at CPT violation in $K^0$-$\bar{K}^0$ mixing. If $|\text{Re} \delta|/|\text{Re} \epsilon| \sim 0.1$, then the difference $\delta_S - \delta_L = 4\text{Re} \delta$ can be as large as 0.4 $\text{Re} \epsilon \sim 6.6 \times 10^{-4}$ in magnitude, where the experimental value $\text{Re} \epsilon \approx 1.65 \times 10^{-3}$ has been used [2]. Since both $\epsilon$ and $\delta$ are small quantities, it turns out that $\xi \approx 1$ and $\zeta \approx 0$ are good approximations. Eq. (8) is therefore simplified to

$$A(t) = \delta_S - 2e^{-\Delta t^2/2} \left[ \text{Re} \eta_{+0} \cos(\Delta mt) - \text{Im} \eta_{+0} \sin(\Delta mt) \right].$$

In the neglect of CPT violation, namely, $\delta_S = 2\text{Re} \epsilon$, Eq. (10) can simply reproduce the result obtained in Ref. [10]. For illustration, I plot the behavior of $A(t)$ in Fig. 1, in which $\delta_S = 3 \times 10^{-3}$ and $|\eta_{+0}| = 5 \times 10^{-3}$ have typically been input. One may observe that $A(t)$ approaches $\delta_S$ for $t \geq 5\tau_S$ and reaches $\delta_S$ if $t \geq 10\tau_S$, where $\tau_S$ is the mean lifetime of the $K_S$ meson. This implies a certain feasibility to determine $\delta_S$ from the time-dependent CP-violating asymmetry $A(t)$.

![Fig. 1. An illustrative plot for the CP-violating asymmetry $A(t)$ with the typical inputs $\delta_S = 3 \times 10^{-3}$ and $|\eta_{+0}| = 5 \times 10^{-3}$ [6].](image)

4 The discussion

In the above analysis I have taken an integration over the whole Dalitz plot, such that $\eta_{+0}$ solely contains the CP-violating part of $A(K_S \to \pi^+ \pi^- \pi^0)$. To look at the CP-conserving component of $A(K_S \to \pi^+ \pi^- \pi^0)$, one may study the phase-space regions $E_{CM}(\pi^+) > E_{CM}(\pi^-)$ and $E_{CM}(\pi^+) < E_{CM}(\pi^-)$ separately [10]. In this case the corresponding CP-violating asymmetries between $R(t)$ and $\bar{R}(t)$ take the same form as $A(t)$ in Eq. (8) or Eq. (10), but $\eta_{+0}$ should be replaced by $\eta_{+0} \pm \lambda$, where $\lambda$ denotes the CP-conserving contribution to the ratio of $K_S$ and $K_L$ decay amplitudes [10]. Certainly, the CP-violating parameter $\delta_S$ can still be determined from measuring the time dependence of the relevant decay rate asymmetries.

An accurate measurement of $\delta_S$ from $K^0 \to \bar{K}^0 \to \pi^+ \pi^- \pi^0$ should be feasible at the $\phi$ factory, where a huge amount of $K^0\bar{K}^0$ events can be coherently produced [5]. Choosing the semileptonic decay of one kaon to tag the flavor of the other kaon decaying into $\pi^+ \pi^- \pi^0$ on the $\phi$ resonance, one should be able to construct the time-dependent rate asymmetry between $K^0(t) \to \pi^+ \pi^- \pi^0$ and $\bar{K}^0(t) \to \pi^+ \pi^- \pi^0$ decays in a way similar to Eq. (8). It is also expected that other super flavor factories may measure $\delta_S$ and $\delta_L$ to a good degree of accuracy.

Note that Lorentz invariance has been taken for granted in what I have discussed. As pointed out by Greenberg [12], "If CPT invariance is violated in an interacting quantum field theory, then that theory also violates Lorentz invariance". In my discussions, the dependence of the CPT-violating parameter $\delta$ on the sidereal time should in general be considered, since CPT violation may simultaneously imply a violation of Lorentz symmetry in the neutral kaon system. For simplicity, here I take $\delta$ to be a constant by assuming that the boost parameters of both $K^0$ and $\bar{K}^0$ are small and the corresponding Lorentz-violating effect is rotationally invariant in the laboratory frame [13]. In this approximation, my results are essentially valid as the averages over the sidereal time, such that the effect of Lorentz violation due to the direction of motion is negligible.

Finally, I like to mention that different approaches have been discussed to test CPT symmetry in $D^0$-$\bar{D}^0$, $B^0_s$-$\bar{B}^0_s$, or $B^0$-$\bar{B}^0$ mixing [14]. The idea presented here cannot directly be applied to those heavy neutral-meson systems. In this sense, it represents a unique way applicable in the $K^0$-$\bar{K}^0$ mixing system to test the CPT theorem.

5 The conclusion

To conclude, the CP-violating effect induced by $K^0$-$\bar{K}^0$ mixing in $K^0 \to \pi^+ \pi^- \pi^0$ decays is possible to deviate to some extent from that in $K_L \to \pi\pi$ or the semileptonic $K_L$ transitions due to the violation of CPT symmetry. Measuring or constraining this tiny difference may serve as a robust test of CPT invariance in the neutral kaon system.

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