Masses and magnetic moments of ground-state baryons in covariant baryon chiral perturbation theory

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We report on some recent developments in our understanding of the light-quark mass dependence and the SU(3) flavor symmetry breaking corrections to the magnetic moments of the ground-state baryons in a covariant formulation of baryon chiral perturbation theory, the so-called EOMS formulation. We show that this covariant ChPT exhibits some promising features compared to its heavy-baryon and infrared counterparts.

Keywords: mass; magnetic moment; ground-state baryons; chiral perturbation theory

1. Introduction

Effective field theories provide a way to understand low-energy strong-interaction phenomena in a systematic and controlled manner. In particular, chiral perturbation theory (ChPT) has long been deemed as the low-energy effective field theory of QCD. It has been widely used in studying many properties of hadrons (see Ref. and references therein). Historically, applications of baryon ChPT have suffered from the so-called power-counting-breaking (PCB) problem, which is due to the large non-zero baryon masses in the chiral limit, and different formulations to deal with this problem have been proposed, including the heavy-baryon ChPT, infrared ChPT, and its modified version, and the extended-on-mass-shell (EOMS) ChPT.
Over the past few years, the EOMS formulation of baryon ChPT has been successfully applied to study a number of physical observables\(^7\)–\(^{15}\) (see also Ref.\(^{16}\) and references cited therein). Compared to the more conventional HB and infrared formulations, the EOMS formulation not only satisfies all the symmetry constraints but also converges relatively faster, particularly in the three flavor sector and at large unphysical light quark masses in the two flavor space. In this talk we briefly introduce two recent works studying the magnetic moments and the masses of the lowest-lying octet/decuplet baryons. \(^a\)

### 2. Magnetic moments

The magnetic moments (MMs) of the octet baryons have long been related to those of the proton and neutron, i.e., the celebrated Coleman-Glashow (CG) relations.\(^{19}\) These relations are a result of (approximate) global SU(3) flavor symmetry. It was soon realized that ChPT may be employed to study SU(3) breaking effects on the MMs of the baryon octet. The first effort was undertaken by Caldi and Pagels in 1974,\(^{20}\) even before modern ChPT was formulated. It was found that at next-to-leading-order (NLO), SU(3) breaking effects are so large that the description of the octet baryon MMs by the CG relations tends to deteriorate, which was later confirmed by the calculations performed in Heavy Baryon (HB) ChPT\(^{21}\)–\(^{24}\) and Infrared (IR) ChPT.\(^{25}\) This apparent failure has often been used to question the validity of SU(3) ChPT in the one-baryon sector. In order to solve this problem, different approaches have been suggested, including reordering the chiral series\(^{26}\) or using a cutoff to reduce the loop contributions, i.e., the so-called long-range regularization.\(^{27}\)

We have shown that the above-mentioned apparent failure of baryon SU(3) ChPT is caused by the power-counting-restoration (PCR) procedure used in removing the PCB terms. In the following, we discuss the results for the octet \(^b\) baryon magnetic moments at NLO without considering the contributions of dynamical decuplet baryons, whose effects can be found in Ref.\(^8\)

Up to NLO, one has the diagrams shown in Fig. 1. The tree-level cou-

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\(^a\)These quantities have also been studied in the finite-range regularized ChPT. For details, see Refs.\(^{17,18}\)

\(^b\)For the study of the decuplet baryons in the EOMS formulation of baryon ChPT see Ref.\(^{10}\)
pling (a) gives the leading-order (LO) result

$$\kappa_B^{(2)} = \alpha_B b_6^D + \beta_B b_6^F,$$

(1)

where the coefficients \(\alpha_B\) and \(\beta_B\) for each of the octet baryons are listed in Table I of Ref. 7 This lowest-order contribution is nothing but the SU(3)-symmetric prediction leading to the CG relations.\textsuperscript{19,21}

The \(O(p^2)\) diagrams (b) and (c) account for the leading SU(3)-breaking corrections that are induced by the corresponding degeneracy breaking in the masses of the pseudoscalar meson octet. Their contributions to the anomalous magnetic moment of a given member of the octet \(B\) can be written as

$$\kappa_B^{(3)} = \frac{1}{8\pi^2 F_\phi^2} \left( \sum_{M=\pi,K} \xi_{BM}^{(b)} H^{(b)}(m_M) + \sum_{M=\pi,K,\eta} \xi_{BM}^{(c)} H^{(c)}(m_M) \right)$$

(2)

with the coefficients \(\xi_{BM}^{(b,c)}\) listed in Table I of Ref. 7 The loop-functions, which are convergent, read as

$$H^{(b)}(m) = - M_B^2 + 2m^2 + \frac{m^2}{M_B^2}(2M_B^2 - m^2) \log \left( \frac{m^2}{M_B^2} \right)$$

$$+ \frac{2m}{M_B^2} \left( m^4 - 4m^2M_B^2 + 2M_B^4 \right) \arccos \left( \frac{m}{2M_B} \right),$$

$$H^{(c)}(m) = M_B^2 + 2m^2 + \frac{m^2}{M_B^2}(M_B^2 - m^2) \log \left( \frac{m^2}{M_B^2} \right)$$

$$+ \frac{2m^3}{M_B^2} \left( m^2 - 3M_B^2 \right) \arccos \left( \frac{m}{2M_B} \right).$$

(3)

One immediately notices that they contain pieces \(\sim M_B^2\) that contribute at \(O(p^2)\) to the MMs, which break the naive PC. These terms have to be removed by applying a PCR scheme, such as the HB, the infrared, or the EOMS schemes.
Table 1. The baryon-octet magnetic moments (in nuclear magnetons) up to $O(p^3)$ obtained in different $\chi$PT approaches in comparison with data.

|     | $p$ | $n$ | $\Lambda$ | $\Sigma^-$ | $\Lambda^{0}$ | $\Sigma^+$ | $\Sigma^0$ | $\Xi^-$ | $\Xi^0$ | $\Lambda\Sigma^0$ | $\tilde{\chi}^2$ |
|-----|-----|-----|-----------|------------|---------------|------------|-----------|--------|--------|-----------------|-------------|
| Tree level | 2.56 | -1.60 | -0.80 | -0.97 | 2.56 | 0.80 | -1.60 | -0.97 | 1.38 | 0.46 |
| $O(p^2)$ |   |      |       |       |      |    |       |       |     |     |
| HB     | 3.01 | -2.62 | -0.42 | -1.35 | 2.18 | 0.42 | -0.70 | -0.52 | 1.68 | 1.01 |
| IR     | 2.08 | -2.74 | -0.64 | -1.13 | 2.41 | 0.64 | -1.17 | -1.45 | 1.89 | 1.86 |
| EOMS   | 2.58 | -2.10 | -0.66 | -1.10 | 2.43 | 0.66 | -0.95 | -1.27 | 1.58 | 0.18 |
| Exp.   | 2.793(0) | -1.913(0) | -0.613(4) | -1.160(25) | 2.458(10) | -0.651(3) | -1.250(14) | ± 1.61(8) |   |     |

In Table 1, we show the LO and NLO results obtained in the EOMS scheme. For the sake of comparison, we also show the NLO results obtained by using the HB and IR schemes. To compare with the results of earlier studies, we define

$$\tilde{\chi}^2 = \sum (\mu_{\text{th}} - \mu_{\text{exp}})^2,$$

while $\mu_{\text{th}}$ and $\mu_{\text{exp}}$ are theoretical and experimental MMs of the octet baryons. The results shown in Table 1 are obtained by minimizing $\tilde{\chi}^2$ with respect to the two LECs $b_6^D$ and $b_6^F$, renormalized $\tilde{b}_6^D$ and $\tilde{b}_6^F$. It is clear that the HB and IR results spoil the CG relations, as found in previous studies, while the EOMS results improve them.

The difference between the EOMS, HB, and IR approaches can also be seen from Fig. 2, where we show the evolution of the minimal $\tilde{\chi}^2$ as a function of $x = M_M/M_{M,\text{phys}}$, where $M_M$, $M_{M,\text{phys}}$ are the masses of the pion, kaon, eta used in the calculation and their physical values. It is clear that at $x = 0$, the chiral limit, all the results are identical to the CG relations. As $x$ approaches 1, where the meson masses equal to the physical values, only the EOMS results show a proper behavior, while both the HB and IR results rise sharply. This shows clearly that relativity and analyticity of the loop results play an important role in the present case.

3. Octet Baryon Masses

In the last few years, a number of lattice QCD collaborations reported the results of 2+1 and 2 flavor simulations of the octet and decuplet baryon masses, including the BMW, the PACS-CS, the HSC, the LHP, and the ETM collaborations. Surprisingly, when trying to describe their simulations with the NLO HB ChPT, both the PACS and LHP collaborations realized that they cannot obtain reasonable fits unless they allow some of
Fig. 2. SU(3)-breaking evolution of the minimal $\chi^2$ in the $\mathcal{O}(p^3)$ $\chi$PT approaches under study. The shaded bands are produced by varying $M_B$ from 0.8 to 1.1 GeV.

the LECs, e.g., $C$, $D$, and $F$ (with their definitions given below), to almost vanish, inconsistent with their empirical values.\(^{31,33}\) The findings are rather unexpected given the fact that in the two flavor sector, ChPT seems to work rather well, at least the IR formulation (see, e.g., Refs.\(^ {34,35}\)). To understand this situation, we performed a first NLO study of the octet and decuplet baryon masses employing the EOMS scheme.\(^ {11}\)

At $\mathcal{O}(p^2)$ the following terms in the chiral Lagrangian contribute to the octet and decuplet masses

$$
L_B^{(2)} = b_0 \langle \chi_+ \rangle \langle \bar{B} B \rangle + b_{D/F} \langle \bar{B} [\chi_+, B] \pm \rangle, \\
L_T^{(2)} = t_0 T_{\mu} T_{\nu} g_{\mu\nu} (\chi_+) + t_{D/F} T_{\mu} T_{\nu} (\chi_+, T_{\nu})_{\mu\nu},
$$

where $\langle X \rangle$ is the trace in flavor space and $(X, T_{\mu})_{\mu\nu} = (X)_{\mu\nu} T_{\mu}^{abc}$ +
\((X)_b^{abc} T^{\mu}_{abc} + (X)_c^{abd} T^{\mu}_{abd}\). In the Eqs. (5), \(\chi_+\) introduces the explicit chiral symmetry breaking, and the coefficients \(b_0, b_D, b_F, t_0, t_D\) are unknown LECs.

For the calculation of the leading loop contributions of Fig. 3 to the masses with pseudoscalar mesons \((\phi)\), octet- \((B)\) and decuplet-baryons \((T)\) we use the lowest-order \(\phi B\) Lagrangian and the \(\phi BT\) and \(\phi T\) ones of Refs.\(^{8,10}\). We take the empirical values \(D = 0.80\) and \(F = 0.46\) for the \(\phi B\) couplings, and \(C = 1.0^{8}\) and \(H = 1.13^{10}\) for the \(\phi BT\) and \(\phi T\) ones, instead of treating them as free parameters as in the LQCD analyses.\(^{31,33}\)

We calculate the loops in the covariant approach and recover the power-counting using the EOMS renormalization prescription.\(^6\) From the covariant results we have obtained the HB ones by defining \(M_D = M_B + \delta\) and expanding the loop-functions about the limit \(M_B \to \infty\).

In the following, we extend our analysis to the 2+1-flavor LQCD results of the PACS-CS collaboration.\(^{29}\) We choose the points for which both the pion and kaon masses are below 600 MeV and perform fits of the 7 parameters, \(M_{B0}, b_0, b_D, b_F, M_{D0}, t_0\) and \(t_D\), to the chosen 24 lattice points that we assume to have independent statistical errors \((\sigma_i)\) but fully-correlated errors propagated from the determination of the lattice spacing. The finite volume corrections have been included.\(^{12}\)

The results of the fits are shown in Table 2. The error bar quoted in the GMO and HB columns and the first one assigned to the covariant results are the uncertainties propagated from the fitted parameters. The second error bar in the covariant results is a theoretical uncertainty coming from the truncation of the chiral expansion which is estimated by taking \(1/2\) of the difference between the results obtained at LO and NLO.

Clearly the NLO EOMS ChPT describes much better the light-quark mass dependence of the octet baryon masses than the NLO HB ChPT. On the other hand, one need to study whether at NNLO the EOMS ChPT works still better than its HB (or IR) counterpart. Such a work is in progress. It should be noted that recently Semke and Lutz have made such an attempt using their formulation. With the help of a large-\(N_c\) operator analysis to relate the large number of LECs, they were able to describe reasonably well the latest lattice QCD simulations.\(^{36}\)

4. Summary and outlook

In the past few years, the EOMS formulation of baryon ChPT have been successfully applied to study a number of important physical quantities. Recently, the EOMS scheme has also been utilized to formulate a covariant
Table 2. Extrapolation in MeV from the fits to the PAC-S-CS results\(^{29}\) on the baryon masses using B\(\chi\)PT up to NLO. The \(\chi^2\) is the estimator for the fits to the lQCD results whereas \(\bar{\chi}^2\) include also experimental data. See Ref.\(^{11}\) for details.

|       | GMO     | HB      | Covariant | Expt. |
|-------|---------|---------|-----------|-------|
| \(M_N\) | 971(22) | 764(21) | 893(19)(39) | 940(2) |
| \(M_A\) | 1115(21) | 1042(20) | 1088(20)(14) | 1116(1) |
| \(M_S\) | 1165(23) | 1210(22) | 1178(24)(7) | 1193(5) |
| \(M_E\) | 1283(22) | 1392(21) | 1322(24)(20) | 1318(4) |
| \(M_G\) | 1319(28) | 1264(22) | 1222(24)(49) | 1232(2) |
| \(M_{G^*}\) | 1433(27) | 1466(22) | 1376(24)(29) | 1385(4) |
| \(M_{G^*_0}\) | 1547(27) | 1622(23) | 1531(25)(8) | 1533(4) |
| \(M_{G^*_1}\) | 1661(27) | 1733(25) | 1686(28)(13) | 1672(1) |

\(\chi^2_{d.o.f.}\) 0.63 9.2 2.1
\(\bar{\chi}^2_{d.o.f.}\) 4.2 36.6 2.8

ChPT for heavy-light systems and the results are very encouraging.\(^{37–39}\) We expect to see more applications of covariant baryon ChPT or heavy-meson ChPT in the near future.

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References

1. S. Scherer and M. R. Schindler, *Lect. Notes Phys.* **830**, 1 (2012).
2. J. Gasser, M. Sainio and A. Svarc, *Nucl.Phys.* **B307**, p. 779 (1988).
3. E. E. Jenkins and A. V. Manohar, *Phys.Lett.* **B255**, 558 (1991).
4. T. Becher and H. Leutwyler, *Eur.Phys.J.* **C9**, 643 (1999).
5. T. A. Gail, Chiral analysis of baryon form factors, ph.d. thesis, Technische Universität München, 2007.
6. T. Fuchs, J. Gegelia, G. Japaridze and S. Scherer, *Phys.Rev.* **D68**, p. 056005 (2003).
7. L. S. Geng, J. Martin Camalich, L. Alvarez-Ruso and M. Vicente Vacas, *Phys.Rev.Lett.* **101**, p. 222002 (2008).
8. L. S. Geng, J. Martin Camalich and M. Vicente Vacas, *Phys.Lett.* **B676**, 63 (2009).
9. L. S. Geng, J. Martin Camalich and M. Vicente Vacas, *Phys.Rev.* **D79**, p. 094022 (2009).
10. L. S. Geng, J. Martin Camalich and M. Vicente Vacas, *Phys.Rev.* **D80**, p. 034027 (2009).
11. J. Martin Camalich, L. S. Geng and M. Vicente Vacas, *Phys.Rev.* **D82**, p. 074504 (2010).
12. L. S. Geng, X.-l. Ren, J. Martin-Camalich and W. Weise, *Phys.Rev.* **D84**, p. 074024 (2011).
13. J. Alarcon, J. Martin Camalich and J. Oller, *Phys.Rev.* **D85**, p. 051503 (2012).
14. J. Alarcon, J. Martin Camalich and J. Oller, *Prog.Part.Nucl.Phys.* **67**, 375 (2012).
15. J. Martin Camalich, J. Alarcon and J. Oller, *Prog.Part.Nucl.Phys.* **67**, 327 (2012).
16. V. Pascalutsa, *AIP Conf.Proc.* **1388**, 60 (2011).
17. R. Young and A. Thomas, *Phys.Rev.* **D81**, p. 014503 (2010).
18. J. Hall, D. Leinweber and R. Young, *Phys.Rev.* **D85**, p. 094502 (2012).
19. S. R. Coleman and S. L. Glashow, *Phys.Rev.Lett.* **6**, p. 423 (1961).
20. D. Caldi and H. Pagels, *Phys.Rev.* **D10**, p. 3739 (1974).
21. E. E. Jenkins, M. E. Luke, A. V. Manohar and M. J. Savage, *Phys.Lett.* **B302**, 482 (1993).
22. L. Durand and P. Ha, *Phys.Rev.* **D58**, p. 013010 (1998).
23. S. Puglia and M. Ramsey-Musolf, *Phys.Rev.* **D62**, p. 034010 (2000).
24. U.-G. Meissner and S. Steininger, *Nucl.Phys.* **B499**, 349 (1997).
25. B. Kubis and U. G. Meissner, *Eur.Phys.J.* **C18**, 747 (2001).
26. M. Mojzis and J. Kambor, *Phys.Lett.* **B476**, 344 (2000).
27. J. F. Donoghue, B. R. Holstein and B. Borasoy, *Phys.Rev.* **D59**, p. 036002 (1999).
28. S. Durr, Z. Fodor, J. Frison, C. Hoelbling, R. Hoffmann et al., *Science* **322**, 1224 (2008).
29. S. Aoki et al., *Phys.Rev.* **D79**, p. 034503 (2009).
30. H.-W. Lin et al., *Phys.Rev.* **D79**, p. 034502 (2009).
31. A. Walker-Loud, H.-W. Lin, D. Richards, R. Edwards, M. Engelhardt et al., *Phys.Rev.* **D79**, p. 054502 (2009).
32. C. Alexandrou et al., *Phys.Rev.* **D80**, p. 114503 (2009).
33. K.-I. Ishikawa et al., *Phys.Rev.* **D80**, p. 054502 (2009).
34. M. Procura, T. R. Hemmert and W. Weise, *Phys.Rev.* **D69**, p. 034505 (2004).
35. M. Procura, B. Musch, T. Wollenweber, T. Hemmert and W. Weise, *Phys.Rev.* **D73**, p. 114510 (2006).
36. A. Senke and M. Lutz, *Phys.Rev.* **D85**, p. 034001 (2012).
37. L. S. Geng, N. Kaiser, J. Martin-Camalich and W. Weise, *Phys.Rev.* **D82**, p. 054022 (2010).
38. L. S. Geng, M. Altenbuchinger and W. Weise, *Phys.Lett.* **B696**, 390 (2011).
39. M. Altenbuchinger, L. S. Geng and W. Weise, *Phys.Lett.* **B713**, 453 (2012).