Why do we have parity violation?

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Abstract

We discuss here two of the questions posed at the beginning of the Bled 1998 workshop: Why is the weak charge dependent on handedness? Why do we have parity violation in the Standard Model? It is argued that the quarks and leptons must be protected from gaining a fundamental mass, very large compared to the electroweak scale, by gauge invariance and hence that their gauge charges must depend on handedness. Furthermore we argue that it is the conservation of parity in the electromagnetic and strong interactions rather than parity violation in the weak interactions that needs an explanation. We derive this parity conservation and indeed the whole system of Weyl fermion representations in the Standard Model from a few simple assumptions: Mass protection, small representations, anomaly cancellation and the Standard Model gauge group $SU(2) \times U(1)$.

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1 Introduction

Why do we have parity violation, or why is the weak charge dependent on handedness?

The short answer to this question is that we need at least some of the charges to be different for the observed right-handed and left-handed fermion states—i.e. handedness dependent or chiral—for the purpose of mass protection. That is to say our philosophy is that the particles we “see”—those we can afford to produce and measure—are very light (essentially massless) from the supposed fundamental (GUT, Planck,...) scale point of view. Consequently they need a mechanism for being exceptionally light, so that we have a chance to “see’ them. This mass protection mechanism is suggestively provided [1] by requiring that any pair of right and left (Weyl) components, for the observed quarks and leptons, should have at least one gauge quantum number different between them, so that any mass term is forbidden by gauge invariance.

Really we should rather ask why is parity conserved in the electromagnetic and strong interactions. Our philosophy would be that a priori there is no reason why these symmetries should be there at all, and it is the presence of the symmetries (rather than their breaking) that needs an explanation [2]. This is the philosophy of what we call random dynamics, which really means: all that is not forbidden occurs. It is a very natural assumption, since really to know a symmetry exists is much more informative than to know it not to be there. So a priori one should rather say that, if there is no reason for them, we should not expect symmetries to be present.

In the case of the question of whether the electroweak charges on the Weyl components of the quark and charged lepton fields should be the same for the two handednesses—right and left—we can say that, since the Weyl fields transform under Lorentz transformations without mixing into each other (i.e. they transform into themselves only), we should consider each Weyl field as essentially corresponding to a completely separate particle. As separate particles we expect them to a priori have completely different charges. You might of course object that when the particle has a mass, so that we are talking about a Dirac particle, there is a connection between the left and the right Weyl component fields. However in the Standard Model it is well-known that the masses come about as an effect of the Higgs field vacuum expectation value. So, before the effect of the Higgs mechanism, the fermions are massless and there is no association of the various Weyl fields with each other a priori. It is therefore not surprising that the weak charge
is dependent on handedness and, as emphasized above, it can then provide a mass protection mechanism.

The question that deserves and needs an answer is rather why there is parity conservation for the strong and electromagnetic interactions, in the sense that the electromagnetic and colour charges are the same on the right and the left components of the same Dirac particle. We show, in section 3, that this result follows from the requirements of small representations for the strong and electromagnetic interaction gauge group $U(3)$ and of no gauge anomaly for the photon and gluons. In addition the fact that the right-handed components are singlets, while the left-handed components are doublets, under the weak SU(2) gauge group needs an explanation. In section 4, we show that this result can also be derived, by extending the above requirements to the full Standard Model gauge group $S(U(2) \otimes U(3))$ and also requiring the quarks and leptons to be mass protected. Indeed we derive the complete representation pattern of the Standard Model fermions. We present our conclusions in section 5.

So let us begin by stating our basic assumptions about the full Standard Model and then specialise these to the assumptions necessary to derive parity conservation in the strong and electromagnetic interactions.

2 Starting assumptions

2.1 The assumptions to derive Standard Model representations of fermions

(a) As the starting point for the derivation of the Standard Model representations, we shall assume the gauge group and not only the gauge Lie algebra of the Standard Model to be $S(U(2) \otimes U(3))$.

(b) Further we shall make the assumption that the representations—realised by the Weyl fermions—of this group are “small”. More specifically we assume that the weak hypercharge charge $y/2$ is at most unity numerically, and that only the trivial and the lowest fundamental (defining) representations of the nonabelian groups SU(2) and SU(3) are used.

(c) Further we assume mass protection, i.e. we say that all particles for which a mass could be made, without the Higgs field being used, would be so heavy that we should not count them as observable particles.

(d) In our argument we shall also use the requirement that there shall be no gauge nor mixed anomalies. This is needed since otherwise there would be a breaking of the gauge symmetry.
These assumptions are of course known to be true in the Standard Model. Indeed they are rather suggestive regularities of the Standard Model, if one is looking for inspiration to go beyond the Standard Model. You could say that it might not be so difficult to find some rather general argumentation for why representations should be “small” in some way—not exactly how small perhaps.

We made a similar set of assumptions in the appendix of ref. [5] to derive the representations in a Standard Model generation of fermions. The assumption that the weak hypercharge $y/2$ is at most unity numerically was replaced there by the assumption that the sum of weak hypercharge squared for all fermions in a generation is as small as possible. Related discussions can be found in refs. [6, 7].

2.2 Slightly reduced assumptions for parity in strong and electromagnetic interactions

From the assumptions stated in the foregoing subsection we can indeed derive the fermion representations of the whole Standard Model and, thus, also the fact that there is parity conservation in electromagnetic and strong interactions. However, if we replace the requirement $|y/2| \leq 1$ by the slightly modified assumption that the electric charge $Q = y/2 + I_W^3$ (where $I_W^3$ is the third component of the weak isospin) has numerical value less than or equal to unity for all the Weyl fermion representations, the mass protection assumption is not needed for this parity derivation. The point of course is that the mass protection is performed by the weak interaction, and the electromagnetic and colour quantum numbers do not provide any mass protection themselves—they cannot with parity symmetry.

In other words, for the derivation of parity conservation in strong and electromagnetic interactions alone, we assume:

(a) Either the gauge group $U(3)$ for strong and electromagnetic interactions, or the total gauge group $S(U(2) \otimes U(3))$ as above.

(b) The “small” representations in the form $|Q| \leq 1$ and $|a| \leq |\mathbb{3}|$.

(c) Then of course there should still be no anomalies.
3 Derivation of parity for QCD and electromagnetism

The program of our proof of parity conservation for strong and electromagnetic interactions consists in showing that the Weyl fields must have quantum number combinations that will be paired into Dirac fields, so that parity in the electromagnetic and strong interactions gets preserved.

We should of course have in mind that, in four dimensions, one can consider the right-handed Weyl components as represented by their CP-conjugates so to speak, meaning a corresponding set of left-handed fields with the opposite charges. So we actually need only discuss the left-handed components, just letting them represent the right-handed ones too as antiparticles. What we then have to show is that there are always equally many left-handed Weyl field species with a given electric and colour charge combination and the opposite. In this way we could then say that at least the possibility is there for combining these Weyl fields into Dirac fields, so that the electric and colour charges on the right and the left Weyl components become the same.

Now, for anomaly calculations, it is easily seen that left-handed Weyl fields in conjugate representations give just equal and opposite contributions to the various anomalies. Thus we can only hope to say from anomaly considerations something about the number of species in one representation minus the number in the conjugate one. We should therefore introduce names for these differences:

We let the symbol $N(\frac{y}{2},a,I_W)$ denote the number of left-handed Weyl species with the weak hypercharge $\frac{y}{2}$, the colour representation $a$ and the weak isospin $I_W$ minus the number of species with the opposite (conjugate) quantum numbers. But, in the present section, we ignore the weak isospin and use $N(Q,a)$ to mean the difference between the number of Weyl-species with electric charge $Q$ and colour representation $a$ and the number of Weyl-species with the conjugate quantum numbers.

The requirement of the smallness of the representations means that $N(Q,a)$ is zero unless

$$|Q| \leq 1$$  \hspace{1cm} (1)
$$|a| \leq |3|$$  \hspace{1cm} (2)

Obviously by our definition $N(Q=0,1) = n - n = 0$, since the representation $(Q = 0, 1)$ is self-conjugate. The requirement of the gauge group being $U(3)$
means that the representations must satisfy the electric charge quantisation rule [4]

\[ Q + t/3 = 0 \pmod{1} \]  

where \( t \) is the triality of a representation.

What we have to show to get parity conservation for these interactions is that

\[ N(Q, a) = 0 \]  

for all the quantum number combinations \((Q, a)\).

The requirements of small representations and of the gauge group being \( U(3) \) leaves only the three differences of species numbers \( N_{(Q=1/3, \frac{2}{3})} \), \( N_{(Q=2/3, 3)} \), \( N_{(Q=-1/3, \frac{2}{3})} \) non-zero.

Now the anomalies in four dimensions come from triangle diagrams with external gauge fields for the gauge anomalies and with two gravitons and one gauge particle assigned in the case of the mixed anomaly. In order to get rid of the anomalies, so as to avoid breaking the gauge symmetry say, we must require that the relevant triangle diagrams have cancellations between the contributions coming from the different Weyl field species that can circle around the triangle. The only mixed anomaly diagram, not already vanishing for other reasons, is a triangle with Weyl particles circling around it having two gravitons attached and the photon at the third vertex. The cancellation required to get rid of this the mixed anomaly becomes

\[ N_{(Q=1/3)} + \frac{2}{3} \times 3N_{(Q=2/3, \frac{2}{3})} + (-\frac{1}{3}) \times 3N_{(Q=-1/3, \frac{2}{3})} = 0 \]  

To ensure no gauge anomaly there are three triangle diagrams that must have a cancellation: one with three external gluons, which gives

\[ N_{(Q=2/3, 3)} + N_{(Q=-1/3, \frac{2}{3})} = 0, \]  

one with one photon and two gluons attached, which gives

\[ 2/3 \times N_{(Q=2/3, \frac{2}{3})} + (-1/3) \times N_{(Q=-1/3, \frac{2}{3})} = 0 \]  

and finally one with three photons attached, which gives

\[ N_{(Q=1/3)} + \left(\frac{2}{3}\right)^3 \times 3N_{(Q=2/3, \frac{2}{3})} + \left(-\frac{1}{3}\right)^3 \times 3N_{(Q=-1/3, \frac{2}{3})} = 0 \]  

We have here got four linear equations for three unknowns, so it is no wonder that they lead to all the differences \( N_{(Q, a)} \) being zero. That then
means to every Weyl representation there is the possibility of finding just one with the opposite (conjugate) representation. This vanishing of the differences $N_{[Q,a]}$ shows that the gauge theory of colour and electromagnetism is vectorlike and thus automatically parity conserving, provided possible mass generation mechanisms do not violate the gauge symmetries. It means that one may directly construct a parity operator, by diagonalizing a perhaps present $U(3)$ gauge invariant mass matrix and letting it map the right-handed to the corresponding left-handed mass eigenstate and vice versa.

4 Deriving all standard model fermion representations

Using a very similar technique, but now within all the four assumptions stated in the subsection 2.1, we can show the fermion representations to be those of the Standard Model with some as yet not determined number of generations.

For this purpose the assumption about small representations can be taken to mean that $N_{[y/2,a,I_W]}$ is zero unless

$$|y/2| \leq 1$$  \hspace{1cm} (9)

$$|a| \leq |3|$$  \hspace{1cm} (10)

$$|I_W| \leq 1/2.$$  \hspace{1cm} (11)

Really it means that we assume zero species for the representations not fulfilling this and thus, of course, the same for the differences $N_{[y/2,a,I_W]}$. The requirement of the gauge group being $S(U(2) \times U(3))$ means that the species numbers are zero unless the congruence

$$y/2 + t/3 + d/2 = 0 \pmod{1}$$  \hspace{1cm} (12)

is fullfilled [4], where $t$ is triality and $d$ is duality.

Obviously by our definition $N_{(y/2=0,a,I_W=0)} = n - n = 0$, since the representation $(y/2 = 0, a, I_W = 0)$ is self-conjugate.

The small representation and the gauge group requirements now allow six $N_{[y/2,a,I_W]}$’s to be nonzero a priori, namely one for each of the allowed numerical values of $y/2$ which run from $1/6$, in steps of $1/6$, to 1.

As above we use the cancellation criteria for the anomalies, meaning the cancellation of triangle diagrams: This time the mixed anomaly cancellation
The diagram has two gravitons and one weak hypercharge coupling and it gives

\[
\frac{1}{6} \times 6N_{(y/2=1/6, I_W=1/2)} + \frac{1}{3} \times 3N_{(y/2=1/3, I_W=0)} \\
+ \frac{1}{2} \times 2N_{(y/2=1/2, I_W=1/2)} + \frac{2}{3} \times 3N_{(y/2=2/3, I_W=0)} \\
\quad + \frac{5}{6} \times 6N_{(y/2=5/6, I_W=1/2)} + N_{(y/2=1, I_W=0)} = 0 \quad (13)
\]

The no gauge anomaly triangle diagrams consist of one with three external gluons, while the one with three external W's is trivially zero and does not count. Then there are two diagrams with two gluons and one weak hypercharge coupling and two W's and one weak hypercharge coupling respectively. Finally there is one diagram with all three attached gauge particles being the abelian one (coupling to weak hypercharge). The corresponding anomaly cancellation conditions become:

\[
2N_{(y/2=1/6, I_W=1/2)} - N_{(y/2=1/3, I_W=0)} \\
+ N_{(y/2=2/3, I_W=0)} - 2N_{(y/2=5/6, I_W=1/2)} = 0, \quad (14)
\]

\[
\frac{1}{6} \times 2N_{(y/2=1/6, I_W=1/2)} + \frac{1}{3} \times N_{(y/2=1/3, I_W=0)} \\
+ \frac{2}{3} \times N_{(y/2=2/3, I_W=0)} + \frac{5}{6} \times 2N_{(y/2=5/6, I_W=1/2)} = 0, \quad (15)
\]

\[
\frac{1}{6} \times 3N_{(y/2=1/6, I_W=1/2)} + \frac{1}{2} \times N_{(y/2=1/2, I_W=1/2)} \\
\quad + \frac{5}{6} \times 3N_{(y/2=5/6, I_W=1/2)} = 0 \quad (16)
\]

and

\[
\left(\frac{1}{6}\right)^3 \times 6N_{(y/2=1/6, I_W=1/2)} + \left(\frac{1}{3}\right)^3 \times 3N_{(y/2=1/3, I_W=0)} \\
+ \left(\frac{1}{2}\right)^3 \times 2N_{(y/2=1/2, I_W=1/2)} + \left(\frac{2}{3}\right)^3 \times 3N_{(y/2=2/3, I_W=0)} \\
\quad + \left(\frac{5}{6}\right)^3 \times 6N_{(y/2=5/6, I_W=1/2)} + N_{(y/2=1, I_W=0)} = 0 \quad (17)
\]

Here we have got 5 equations linear in the N's of which there are 6. Thus it is not surprising that there is, up to the unavoidable scaling by
a common factor of all the unknowns—the N’s, just one solution. This must, however, be that of the Standard Model, since the latter satisfies the anomaly cancellation conditions. The scaling factor corresponds to the generation number we could say. So far we have only shown that the N’s are as in the Standard Model. We now need to use the assumption about mass protection to deduce that we cannot have both representations—i.e. a representation and its conjugate—associated with a given N present. That implies first that the cases of N’s that are zero imply that there will be no Weyl fermions at all associated with those quantum numbers—there will be no vector fermions. Also for the cases of nonzero N’s only one of the two associated representations will exist, depending on the sign of the N in question. With this conclusion we almost truly derived the Standard Model fermion representations. There are however still two ambiguities: 1) the generation number can be any integer, 2) we could have the opposite signs for the N’s which would correspond to a model that is, so to speak, a parity reflected version of the Standard Model.

Since we have now derived the whole representation system for the fermions in the Standard Model, we did not really need the exercise of deriving parity for the electromagnetic and colour interactions separately; we got it all at once after all, assuming though—as is needed—the Higgs mechanism for mass generation.

5 Conclusion

We have shown that, from the four requirements listed in subsection 2.1, it is possible to argue for the whole system of Weyl fermion representations in the Standard Model. So if one can just argue for these assumptions in some model beyond the Standard Model, one will have the fermion system for free.

Concerning the question of whether the charges depend on handedness, we saw that for the colour and electric charges no such dependence is allowed, by just using the smallness of electric charge and colour representation plus the no anomaly conditions. Concerning the question of why there is a dependence—namely for the weak charges—we saw that it was the mass protection requirement that enforced it. In fact each of the differences N had to be a difference between zero and another number, because the mass protection would not allow two sets of Weyl fields counted as left-handed having opposite (conjugate) quantum numbers. They would namely com-
bine to get a huge mass and be unobservable, according to the philosophy of mass protection. Thus indeed the charges must, in one way or another, be different for the right and left handnesses.

This really means that we take the point of view that the fundamental scale, or the next level in fundamentality, has so huge a characteristic energy or mass scale that all the particles we know must, in first approximation, be arranged to be massless, i.e. they must be mass protected.

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