Factor analysis in determining the quality of coal

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Abstract. The separation of coal material of three types of coals originating from three various Polish hard coal mines (types 31, 34.2 and 35, according to Polish nomenclature, which were steam coal, semi-coking coal and coking coal) into particle size fractions and then into particle density fractions was done and then the following parameters were measured for each particle size-density fraction: combustion heat, ash contents, sulfur contents, volatile parts contents, analytic moisture. In this way a 7-dimensional vector of data was created. Using methods of factor analysis the important features of coal were selected, which decide about their membership to individual types. To evaluate the appropriateness of the applied method the Bartlett’s sphericity test as well coefficient of Kaiser-Mayer-Olkin (KMO) were used. To select important factors the Kaiser criterion and Cattell’s scree test were used. The obtained results were compared with the results obtained in previous works by means of observation tunnels method. The results showed which particular features are crucial to define the type of coal what is also important to select appropriate method of its enrichment.

1. Introduction

Mineral raw materials which are beneficiated in purpose of their using characterize with many factors describing their features. In case of coal, these features are among others ash contents, sulfur contents, combustion heat, volatile parts contents or analytic moisture. The features mentioned above decide about coal quality also in economical aspect. Because of that the preciseness of determining values of these features is very important.

The most often researched properties of the coal are combustion heat, ash contents, sulphur contents, volatile parts contents and moisture. These features are very often highly correlated but also can occur independently. The selection of the necessary factors which influence on individual properties is the goal of the paper. To this purpose three types of coal (according to Polish nomenclature – coal types 31 (steam coal), 34.2 (gas-coking coal) and 35 (oro-coking coal)) were selected to the investigation which were divided into particle size and density fractions. The classification of coals is presented in Table 1.
Table 1. Classification of coal types according to Polish nomenclature [21].

| Coal type            | Coal number | Volatile parts contents [%] |
|----------------------|-------------|----------------------------|
| Steam coal           | 31          | Above 28                   |
| Gas-steam coal       | 32          | Above 28                   |
| Gas coal             | 33          | Above 28                   |
| Gas-coking coal      | 34          | Above 28                   |
| Orto-coking coal     | 35          | 20-31                      |
| Meta-coking coal     | 36          | 14-28                      |
| Semi-coking coal     | 37          | 14-28                      |
| Thin coal            | 38          | 14-28                      |
| Anthracite coal      | 41          | 10-14                      |
| Anthracite           | 42          | 3-10                       |
| Meta-anthracite      | 43          | Till 3                     |

The whole group of considered factors were measured for each size-density fraction [14]. The following variables were considered (X_i = 1, 2, …, 5).
X_1 – combustion heat [cal];
X_2 – ash contents [%];
X_3 – sulfur contents [%];
X_4 – volatile parts contents [V_a];
X_5 – moisture [W_a].

2. Materials and methods

The considered types of coal originated from three various Polish coal mines and all of them were initially screened on a set of sieves of the following sizes: -1.00, -3.15, -6.30, -8.00, -10.00, -12.50, -14.00, -16.00 and -20.00 mm. Then, the size fractions were additionally separated into density fractions by separation in dense media using zinc chloride aqueous solution of various densities (1.3, 1.4, 1.5, 1.6, 1.7, 1.8 and 1.9 g/cm^3). The fractions were used as a basis for further consideration and additional coal features were determined by means of chemical analysis. In purpose of appropriate identification of coal type many parameters are being measured which describe coal quality. For each density-size fraction such parameters as combustion heat, ash contents, sulfur contents, volatile parts contents and analytical moisture were determined, making up, together with the mass of these fractions, seven various features for each coal.

The example of obtained data is presented in Table 2.

Table 2. Data for size fraction 14.00-12.50 mm – coal, type 31.

| Density [Mg/m^3] | Mass [g] | Combustion heat [cal] | Ash contents [%] | Sulfur contents [%] | Volatile parts contents V_a | Analytical moisture W_a |
|------------------|----------|-----------------------|------------------|---------------------|----------------------------|-------------------------|
| <1.3             | 308.6    | 7048                  | 6.41             | 0.72                | 34.32                      | 3.23                    |
| 1.3-1.4          | 292.5    | 5859                  | 19.61            | 0.7                 | 29.22                      | 3.36                    |
| 1.4-1.5          | 36.1     | 2948                  | 16.55            | 0.76                | 28.92                      | 3.87                    |
| 1.5-1.6          | 10.7     | 5117                  | 26.10            | 1.55                | 31.08                      | 3.40                    |
| 1.6-1.7          | 25.6     | 4467                  | 35.78            | 2.28                | 26.71                      | 2.40                    |
| 1.7-1.8          | 139      | 3920                  | 37.20            | 1.23                | 29.24                      | 2.19                    |
| 1.8-1.9          | 12.7     | 3078                  | 48.20            | 1.13                | 24.05                      | 2.23                    |
| >1.9             | 601.2    | 457                   | 86.53            | 0.40                | 9.30                       | 0.91                    |
The measurements of $X_i$ were performed for each size-density fraction. Because of the fact that the individual features were measured in various units their standardization was done.

In purpose of selecting significant factors influencing on individual variables, the factor analysis method was applied. To evaluate adequacy of applying factor analysis to this problem two criteria were used: Bartlett’s test and Kaiser-Mayer-Olkin coefficient (KMO) [1, 2, 10, 11, 21].

The reduction of variables is done through the Cattell’s scree criteria and criterion of sufficient proportion which suggest to apply such number of factors that they explain together at least 85% of variance of all observed variables [22].

3. Results

Applying Bartlett’s test it occurred that for all researched cases the value of the test was significantly higher than the critical values on significance level being equal to $\alpha=0.0005$. The lowest value of the test $U$ was obtained for coal, type 35 in particle density fraction (1.9-2.0) and was equal to 84.74, while the critical value on this level is equal to 31.42. It can be said then that zero hypothesis (that correlation matrix is a unit matrix) should be rejected for all particle size and density fractions.

Furthermore, it can be noticed that in almost all cases the value of KMO coefficient was higher than 0.5. Only for density fraction lower than 1.3 g/cm$^3$ for coal, type 34.2 and density fraction (1.6-1.7) for coal, type 35 it occurred to be slightly lower than 0.5. That means that the results of Bartlett’s test and the values of KMO coefficient gave strong basis to apply factor analysis.

In the work, the reduction of variables is done through the Cattell’s scree criteria and criterion of sufficient proportion which suggest to apply such number of factors that they explain together at least 85% of variance of all observed variables [22].

The correlation matrix of the factor $Z_j$ with variable $X_i$ is obtained by creation of matrix $Z$, which elements are numbers

$$z_{ij} = \sqrt{\lambda_i}a_{ij}, \quad i, j = 1, 2, ..., 5. \quad (1)$$

where: $\lambda_i$ – $i$th eigenvalue of correlation matrix; $a_{ij}$ – elements of matrix $A$ which fulfills the condition $A^T = R$, where $R$ is correlation matrix of variables $X_i$.

The square of number $z_{ij}$ is the percentage of variance changeability explained by the factor $Z_j$. For example, considering coal, type 31 from the particle size fraction (10-12.5) it is obtained that matrix $Z$ is in form

$$Z = \begin{bmatrix}
-0.9813 & 0.1331 & -0.0962 & 0.0676 & 0.0747 \\
0.9828 & -0.1017 & 0.1145 & -0.0767 & 0.0700 \\
-0.0667 & -0.9963 & -0.0484 & 0.0246 & 0.0033 \\
-0.9793 & -0.0651 & -0.0297 & 0.1893 & -0.0019 \\
-0.9620 & -0.1035 & 0.2487 & 0.0063 & -0.0029
\end{bmatrix} \quad (2)$$

The eigenvalues of the correlation matrix are in this case numbers $\lambda_1=3.8177; \lambda_2=1.0355; \lambda_3=0.0875; \lambda_4=0.0488; \lambda_5=0.0105$.

The plot of scree is presented on Figure 1.
On the basis of the presented Cattell’s scree plot only these factors remain which are located to the left from the point in which a mild decline of eigenvalues is observed. In this case these are factors $Z_1$ and $Z_2$.

The group of factors ($Z_1$, $Z_2$) explain 98.07% of changeability of combustion heat, 97.12% of changeability of ash contents, 99.71% of changeability of sulfur contents, 96.33% of changeability of volatile parts contents and 93.62% of changeability of moisture.

It is obtained then that factor $Z_1$ is responsible for variables $\{X_1, X_2, X_4, X_5\}$ and factor $Z_2$ for variable $X_3$.

Let consider the particle density fraction (1.6-1.7) of coal, type 34.2

The matrix $Z$ is in form

$$Z = \begin{bmatrix} 0.8566 & -0.4196 & -0.1506 & -0.1824 & 0.3643 \\ -0.5854 & -0.6574 & 0.3863 & 0.1622 & 0.1400 \\ 0.5544 & -0.7491 & -0.2733 & 0.1882 & -0.2073 \\ -0.7229 & -0.5915 & -0.1757 & -0.2615 & -0.0447 \\ -0.4385 & 0.1454 & -0.8755 & 0.0956 & 0.0961 \end{bmatrix}$$

The eigenvalues of correlation matrix in this case are numbers $\lambda_1=2.0993; \lambda_2=1.5404; \lambda_3=1.0443; \lambda_4=0.1727; \lambda_5=0.1433$. The plot of Cattell’s scree is presented on Figure 2.

The Cattell’s scree plot suggests to take factors $Z_1$, $Z_2$ and $Z_3$ into consideration. The same factors explain sufficient percentage of changeability of all observed variables. Group of factors ($Z_1$, $Z_2$, $Z_3$) explains 93.25% of changeability of combustion heat, 92.41% of ash contents, 94.32% of sulfur.
contents, 90.33% of volatile parts contents and 97.99% of moisture, while factor $Z_1$ is related to variables $X_1, X_2, X_3, X_4$; factor $Z_2$ to variables $X_3, X_4, X_5$ and factor $Z_3$ to variable $X_5$.

Another criterion of limiting number of factors is determination of amount of percent of total variance explained by chosen factors (most often it is required to not be lower than 85%). In this case, for coal type 31, factors $Z_1$ and $Z_2$ explain 93.14% of variation of variable $X_1$ (combustion heat), 96.65% of variation of variable $X_2$ (ash contents), 99.00% of variation of variable $X_3$ (sulfur contents), 91.14% of variation of variable $X_4$ (volatile parts contents) and 89.14% of variation of variable $X_5$ (analytic moisture). For coal type 34.2, factors $Z_1$, $Z_2$ and $Z_3$ explain 95.21% of variation of variable $X_1$, 97.48% of variation of variable $X_2$, 99.95% of variation of variable $X_3$, 86.72% of variation of variable $X_4$ and 99.68% of variation of variable $X_5$. Finally, for coal type 35, these factors explain 98.21% of variation of variable $X_1$, 98.39% of variation of variable $X_2$, 99.87% of variation of variable $X_3$, 95.57% of variation of variable $X_4$ and 99.00% of variation of variable $X_5$.

The influences of individual factors on considered variables in all fractions of individual types of coal are presented in Tables 3-8. It was assumed that changeability of each feature should be explained by factors in at least 85%.

| Table 3. Influence of factors on properties of coal, type 31 by particle size fractions. |
|---------------------------------------------------------------|
| **Feature** | 0.5-1 | 1-3.15 | 3.15-6.3 | 6.3-8 | 8-10 | 10-12.5 | 12.5-14 | 14-16 | 16-20 |
| Combustion heat | | | | | | | | | |
| $Z_1$ | 97.49 | 95.39 | 97.57 | 97.63 | 94.13 | 96.29 | 81.52 | 95.41 | 99.89 |
| $Z_2$ | | | | | | | 11.42 | | |
| Ash contents | | | | | | | | | |
| $Z_1$ | 86.47 | 95.55 | 90.68 | 99.34 | 95.86 | 96.58 | 97.33 | 97.65 | 99.28 |
| Sulfur contents | | | | | | | | | |
| $Z_1$ | 95.74 | 68.65 | 94.59 | 7.99 | 5.00 | 0.04 | 2.31 | 22.45 | 2.06 |
| $Z_2$ | 30.83 | 91.96 | 94.63 | 99.26 | 90.13 | 77.28 | 97.19 | | |
| Volatile parts contents | | | | | | | | | |
| $Z_1$ | 94.71 | 95.60 | 84.65 | 88.34 | 93.41 | 95.90 | 96.09 | 88.56 | 86.71 |
| $Z_2$ | 12.11 | | | | | | | | |
| Moisture | | | | | | | | | |
| $Z_1$ | 93.25 | 95.21 | 75.25 | 95.19 | 95.90 | 92.34 | 79.90 | 97.35 | 94.22 |
| $Z_2$ | 18.16 | | | | | | 7.00 | | |
| **Table 4.** Influence of factors on properties of coal, type 34.2 by particle size fractions. |
|---------------------------------------------------------------|
| Feature | 0.5-1 | 1-3.15 | 3.15-6.3 | 6.3-8 | 8-10 | 10-12.5 | 12.5-14 | 14-16 | 16-20 |
|---------|-------|--------|----------|-------|------|---------|---------|-------|-------|
| **Combustion heat** |       |        |          |       |      |         |         |       |       |
| Z₁      | 97.89 | 97.73  | 88.58    | 65.37 | 94.14| 86.65   | 82.95   | 95.90 | 91.83 |
| Z₂      | 6.96  |        |          |       |      |         |         |       |       |
| Z₃      | 27.60 |        |          |       |      |         |         |       |       |
| **Ash contents** |       |        |          |       |      |         |         |       |       |
| Z₁      | 97.37 | 96.35  | 99.52    | 93.33 | 94.53| 91.83   | 87.25   | 91.83 | 85.34 |
| **Sulfur contents** |       |        |          |       |      |         |         |       |       |
| Z₁      | 97.51 | 62.60  | 1.84     | 9.92  | 31.49| 54.43   | 20.79   | 6.25  | 52.26 |
| Z₂      | 16.74 | 95.29  | 88.37    | 65.15 | 11.59| 72.53   | 91.43   | 44.28 |       |
| Z₃      | 5.85  |        | 31.21    |       |      |         |         |       |       |
| **Volatile parts contents** |       |        |          |       |      |         |         |       |       |
| Z₁      | 61.29 | 86.22  | 92.58    | 93.91 | 90.68| 77.51   | 68.14   | 84.89 | 82.95 |
| Z₂      | 25.35 |        |          |       |      | 11.59   | 26.65   | 0.09  | 14.36 |
| Z₃      |       |        |          |       |      |         |         | 7.59  |       |
| **Moisture** |       |        |          |       |      |         |         |       |       |
| Z₁      | 22.21 | 49.29  | 30.33    | 89.20 | 70.39| 24.35   | 38.26   | 64.02 | 66.19 |
| Z₂      | 73.23 | 32.06  | 3.01     | 16.54 | 69.87| 49.75   | 8.34    | 29.99 |       |
| Z₃      | 4.23  | 66.48  |          |       |      |         |         |       | 21.64 |

| **Table 5.** Influence of factors on properties of coal, type 35 by particle size fractions. |
|---------------------------------------------------------------|
| Feature | 0.5-1 | 1-3.15 | 3.15-6.3 | 6.3-8 | 8-10 | 10-12.5 | 12.5-14 | 14-16 | 16-20 |
|---------|-------|--------|----------|-------|------|---------|---------|-------|-------|
| **Combustion heat** |       |        |          |       |      |         |         |       |       |
| Z₁      | 92.23 | 98.46  | 96.29    | 92.87 | 90.70| 97.12   | 97.49   | 92.14 | 84.89 |
| Z₂      |       |        |          |       |      |         |         |       | 13.80 |
| **Ash contents** |       |        |          |       |      |         |         |       |       |
| Z₁      | 92.23 | 98.01  | 76.47    | 95.29 | 91.75| 96.80   | 96.68   | 93.89 | 95.08 |
| Z₂      |       |        |          |       |      |         |         |       |       |
| **Sulfur contents** |       |        |          |       |      |         |         |       |       |
| Z₁      | 12.18 | 4.59   | 6.48     | 1.16  | 24.70| 2.23    | 0.02    | 4.32  | 26.79 |
| Z₂      | 70.66 | 77.03  | 34.07    | 97.02 | 52.67| 69.80   | 67.99   | 75.00 | 62.85 |
| Z₃      | 15.35 | 18.35  | 48.24    |       | 21.51| 27.23   | 31.84   | 16.42 |       |
| **Volatile parts contents** |       |        |          |       |      |         |         |       |       |
| Z₁      | 78.35 | 87.90  | 99.64    | 92.14 | 76.51| 92.23   | 84.54   | 87.60 | 87.19 |
| Z₂      | 6.88  |        |          |       |      | 13.64   | 8.93    |       |       |
| **Moisture** |       |        |          |       |      |         |         |       |       |
| Z₁      | 15.14 | 36.10  | 54.49    | 40.60 | 44.32| 0.04    | 2.74    | 1.15  | 15.94 |
| Z₂      | 68.49 | 29.89  | 31.16    | 13.81 | 46.37| 89.85   | 63.66   | 83.50 | 72.35 |
| Z₃      | 14.06 | 26.68  | 43.74    |       |      |         | 33.50   | 15.26 |       |
Table 6. Influence of factors on properties of coal, type 31 by particle density fractions.

| Feature         | <1.3 | 1.3-1.4 | 1.4-1.5 | 1.5-1.6 | 1.6-1.7 | 1.7-1.8 | 1.8-1.9 | 1.9-2.0 |
|-----------------|------|---------|---------|---------|---------|---------|---------|---------|
| Combustion heat |      |         |         |         |         |         |         |         |
| $Z_1$           | 86.99| 87.47   | 35.10   | 84.97   | 59.66   | 28.46   | 87.94   | 75.69   |
| $Z_2$           | 60.40| 8.15    | 20.53   | 63.98   | 27.82   | 9.15    |         |         |
| $Z_3$           |      |         |         |         |         |         |         |         |
| Ash contents    |      |         |         |         |         |         |         |         |
| $Z_1$           | 92.42| 94.03   | 82.88   | 70.94   | 83.86   | 82.04   | 75.15   | 51.62   |
| $Z_2$           | 1.33 | 25.38   | 11.19   | 0.02    |         | 9.01    | 36.33   |         |
| $Z_3$           | 3.12 | 17.05   | 0.01    |         |         |         |         |         |
| $Z_4$           |      |         |         |         |         |         |         | 15.30   |
| Sulfur contents |      |         |         |         |         |         |         |         |
| $Z_1$           | 7.82 | 17.61   | 35.58   | 64.03   | 36.52   | 18.13   | 1.60    | 40.24   |
| $Z_2$           | 56.73| 80.64   | 48.87   | 4.86    | 36.97   | 67.04   | 87.51   | 54.30   |
| $Z_3$           | 34.85| 14.49   | 17.92   | 23.87   |         |         |         |         |
| Volatile parts contents |      |         |         |         |         |         |         |         |
| $Z_1$           | 89.88| 87.01   | 73.80   | 68.22   | 21.16   | 71.84   | 18.36   | 74.33   |
| $Z_2$           | 0.03 | 1.26    | 43.08   | 13.03   | 42.04   | 16.73   |         |         |
| $Z_3$           | 0.06 | 6.83    | 35.45   | 1.06    | 38.69   |         |         |         |
| $Z_4$           | 24.75| 23.66   |         |         |         |         |         |         |
| Moisture        |      |         |         |         |         |         |         |         |
| $Z_1$           | 1.87 | 79.85   | 63.42   | 46.36   | 93.10   | 60.04   | 39.66   | 66.11   |
| $Z_2$           | 67.24| 0.09    | 5.97    | 37.93   | 5.84    | 34.95   | 0.07    |         |
| $Z_3$           | 30.74| 18.61   | 27.06   | 15.37   | 31.75   | 0.06    | 37.06   |         |
| $Z_4$           |      |         |         |         |         |         |         | 24.35   |
### Table 7. Influence of factors on properties of coal, type 34.2 by particle density fractions.

| Feature          | <1.3 | 1.3-1.4 | 1.4-1.5 | 1.5-1.6 | 1.6-1.7 | 1.7-1.8 | 1.8-1.9 | 1.9-2.0 |
|------------------|------|---------|---------|---------|---------|---------|---------|---------|
| **Combustion heat** |      |         |         |         |         |         |         |         |
| $Z_1$            | 68.90 | 21.37   | 75.81   | 52.56   | 73.37   | 99.60   | 91.83   | 83.37   |
| $Z_2$            | 13.54 | 70.82   | 1.03    | 28.72   | 17.60   |         |         | 7.68    |
| $Z_3$            | 14.35 | 20.48   | 13.13   |         |         |         |         |         |
| **Ash contents** |      |         |         |         |         |         |         |         |
| $Z_1$            | 80.94 | 83.39   | 89.18   | 73.41   | 34.26   | 13.14   | 87.25   | 6.51    |
| $Z_2$            | 6.02  | 10.66   | 3.69    | 43.21   | 23.72   |         |         | 86.19   |
| $Z_3$            | 18.01 | 4.79    | 9.90    | 14.92   | 46.22   |         |         |         |
| $Z_4$            |       |         |         |         |         |         |         | 5.34    |
| **Sulfur contents** |      |         |         |         |         |         |         |         |
| $Z_1$            | 18.36 | 83.26   | 63.64   | 53.96   | 30.73   | 51.60   | 37.05   | 95.39   |
| $Z_2$            | 64.78 | 11.15   | 8.70    | 0.06    | 56.11   | 48.00   | 52.91   |         |
| $Z_3$            | 16.30 | 23.41   | 31.76   |         |         |         |         |         |
| **Volatile parts contents** |      |         |         |         |         |         |         |         |
| $Z_1$            | 82.88 | 71.58   | 56.23   | 11.26   | 52.25   | 1.46    | 75.65   | 87.32   |
| $Z_2$            | 15.10 | 0.06    | 0.01    | 81.28   | 34.98   | 72.72   | 19.64   |         |
| $Z_3$            | 26.70 | 34.32   |         |         |         |         |         |         |
| **Moisture**     |      |         |         |         |         |         |         |         |
| $Z_1$            | 49.75 | 47.22   | 2.97    | 43.19   | 19.22   | 43.02   | 40.90   | 64.67   |
| $Z_2$            | 25.45 | 42.22   | 86.52   | 8.72    | 2.11    | 32.11   | 48.26   | 24.86   |
| $Z_3$            | 20.63 | 39.77   | 76.65   | 4.38    |         |         |         |         |
| $Z_4$            |       |         |         |         |         |         |         | 20.13   |
Table 8. Influence of factors on properties of coal, type 35 by particle density fractions.

| Feature               | <1.3 | 1.3-1.4 | 1.4-1.5 | 1.5-1.6 | 1.6-1.7 | 1.7-1.8 | 1.8-1.9 | 1.9-2.0 |
|-----------------------|------|---------|---------|---------|---------|---------|---------|---------|
| Combustion heat       |      |         |         |         |         |         |         |         |
| Z₁                    | 36.22| 94.80   | 58.46   | 93.14   | 55.65   | 88.39   | 59.42   | 99.70   |
| Z₂                    | 47.32| 0.17    | 43.09   | 18.46   |         |         |         |         |
| Z₃                    | 13.34| 37.14   |         |         |         |         |         |         |
| Ash contents          |      |         |         |         |         |         |         |         |
| Z₁                    | 36.62| 97.91   | 15.03   | 71.14   | 76.54   | 77.59   | 59.87   |         |
| Z₂                    | 47.32| 56.07   | 23.93   | 9.48    | 5.65    | 32.02   |         |         |
| Z₃                    | 13.34| 27.41   |         |         |         |         |         |         |
| Sulfur contents       |      |         |         |         |         |         |         |         |
| Z₁                    | 22.05| 40.51   | 16.48   | 9.04    | 21.90   | 75.06   | 27.98   | 57.54   |
| Z₂                    | 70.30| 54.61   | 37.22   | 78.17   | 16.54   | 10.15   | 53.01   | 26.44   |
| Z₃                    | 17.61| 7.79    | 54.39   |         |         |         |         |         |
| Z₄                    | 25.56|         |         |         |         |         |         |         |
| Volatile parts contents|    |         |         |         |         |         |         |         |
| Z₁                    | 94.78| 55.74   | 4.00    | 49.97   | 28.64   | 39.06   | 31.14   | 26.50   |
| Z₂                    | 31.34| 42.35   | 19.51   | 44.55   | 35.14   | 10.68   | 44.03   |         |
| Z₃                    | 20.81| 15.70   | 11.12   | 13.42   | 51.94   | 12.22   |         |         |
| Z₄                    | 31.70| 4.93    |         |         |         |         |         |         |
| Moisture              |      |         |         |         |         |         |         |         |
| Z₁                    | 70.12| 23.27   | 86.19   | 7.81    | 27.96   | 5.03    | 52.51   | 6.35    |
| Z₂                    | 14.68| 1.38    | 33.79   | 44.98   | 68.92   | 18.25   | 0.21    |         |
| Z₃                    | 11.48| 72.67   | 48.87   | 20.76   | 25.99   | 1.18    | 89.98   |         |
| Z₄                    |      |         |         |         |         |         |         | 17.51   |

4. Conclusions
Because of the fact that the most often three factors occur in individual fractions and considering power of relations between individual properties the investigated variables can be divided into three subsets. First one contains combustion heat, ash contents and volatile parts contents, second one contains sulfur contents and the third one contains moisture. In scientific works [3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 24], through application of various visualization methods it was claimed that features being sufficient to identify coal type are sulfur contents, moisture and volatile parts contents. The conducted analysis confirms these results. The selection of variable $X_4$ (volatile parts contents) occurs from the fact that this variable is explained by other factor than mutual factor with variables moisture and combustion heat.

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Acknowledgement

The paper is a result of statutory project no. 16.16.100 215.