Can heavy neutrinos dominate neutrinoless double beta decay?

J. Lopez-Pavon, S. Pascoli and Chan-fai Wong

Abstract

We study whether a dominant contribution to neutrinoless double beta decay coming from extra heavy degrees of freedom, introduced to generate the light neutrino masses, can dominate over the light neutrino contribution. It has been shown that this may occur at tree-level if the light neutrino contribution partially cancels out. Here we focus on this case, specifically in the context of type-I seesaw models paying special attention to the one-loop corrections to light neutrino masses, their contribution to the process and correlation with the heavy sector. We perform a general analysis without restricting the study to any particular region of the parameter space, although interesting limits associated with inverse and extended seesawlike models are discussed in more detail. It turns out that the heavy neutrinos can dominate the process only in those limits. For the inverse seesaw limit, we find a very constrained allowed region of the parameter space, with heavy neutrino masses around 5 GeV. The extended seesaw case allows for a larger region, but in general, a hierarchical spectrum of heavy neutrinos with masses above and below $\sim 100$ MeV is required.
I. INTRODUCTION

The existence of neutrino masses, strongly supported by neutrino oscillation experiments, is the first experimental evidence of physics beyond the Standard Model (SM). Furthermore, the fact that neutrino masses are smaller than the masses of the other SM fermions by several orders of magnitude calls for a “natural” New Physics (NP) explanation. Most of the models, including the very popular seesaw [1–4] ones, assume that the lepton number is not a conserved symmetry and that light neutrinos are Majorana particles. An interesting experimental window to search for NP gets opened: lepton-number-violating processes, highly suppressed in the SM, among which neutrinoless double beta decay ($0\nu\beta\beta$ decay) experiments are the most promising. In combination with the information coming from neutrino oscillation experiments, absolute neutrino mass experiments, precision measurements, and cosmology, $0\nu\beta\beta$ decay experiments can give us precious clues in order to identify the mechanism responsible for the neutrino mass generation and provide a complementary way to look for NP, possibly not otherwise accessible at the LHC.

Although NP is necessary in order to have $0\nu\beta\beta$ decay, its effects are usually indirect since the light neutrinos generically dominate the process in most of the models, as is the case in type-I, type-II [5–9] and type-III [10] seesaw realizations and some extradimensional models [11–13]. The key point is that the light neutrino contribution and the NP one are usually correlated through the generation of the light neutrino masses and the second of these, suppressed by being short range, is thus constrained and generally subdominant [14]. The question of whether a measurable direct contribution to the $0\nu\beta\beta$ decay rate coming from NP is theoretically and phenomenologically viable is thus very interesting. This question has been addressed recently in Refs. [15, 16] in the context of different type-I seesaw models. In these publications, a relevant exception to the argument above has been pointed out: the case in which the tree-level light neutrino contribution, induced by the presence of heavy fermion singlets, partially cancels out. In this case, it is found that the direct heavy neutrino contribution to the process is indeed relevant and can be as large as current bounds. However, no detailed discussion about the correlation among the light and heavy contributions, once the one-loop corrections to neutrino masses are included in the analysis, is given. The main goal of this work is to analyze to what extent having this dominant contribution from heavy neutrinos is possible in the general framework of
type-I seesaw models when the relevant one-loop corrections and experimental constraints are carefully considered, paying special attention to its correlation with the light neutrino contribution induced by these corrections.

We will first very briefly review the aspects of the $0\nu\beta\beta$ decay phenomenology relevant for our analysis. Considering a general parameterization of the neutrino mass matrix without restricting the analysis to any region of the parameter space, we will then study under which conditions the light neutrino contribution can be canceled at tree-level. We will include the one-loop corrections and study if the heavy neutrinos can give a dominant and measurable (i.e. within reach of the next-to-next $0\nu\beta\beta$ decay experiments) contribution to the process. Finally, we will show that, even when the tree-level cancellation takes place, the light and heavy contributions are not completely decoupled once the one-loop corrections are included in the study and a dominant heavy contribution may occur only in specific regions of the parameter space.

This paper is organized as follows: In Sec. II, we briefly review the $0\nu\beta\beta$ decay phenomenology in the general context of seesaw models, introducing the notation and the Nuclear Matrix Elements (NMEs) we will use. In Sec. III, the parameterization of the neutrino mass matrix is presented, distinguishing some relevant limits and their relation with well-known models, such as the inverse and extended seesaw ones. Section IV is devoted to the study of the cancellation condition of the light neutrino contribution and its tree-level consequences on the heavy neutrino sector. Section V is dedicated to the analysis of the relevant corrections: higher-order corrections to the seesaw expansion and one-loop corrections. The combined analysis of the $0\nu\beta\beta$ decay phenomenology, when these corrections and the relevant experimental constraints are taken into account, is presented in Sec. V. Finally, in Sec. VI we draw our conclusions.

II. DOMINANT HEAVY NEUTRINO CONTRIBUTION TO $0\nu\beta\beta$ DECAY?

As we have already mentioned, in the context of seesaw models, the contributions to neutrinoless double beta decay from NP at scales much heavier than the exchanged momentum ($\sim 100$ MeV), namely the ones mediated by heavy fermion singlets or scalar/fermion triplets introduced to generate the light neutrino masses, is usually subdominant and the light neutrinos typically dominate the process [14].
Let us very briefly review how the above result is obtained and the possible exceptions. Following the notation in Ref. [14], and restricting the study to type-I seesaw models, the $0\nu\beta\beta$ decay rate can be written as

$$\frac{\Gamma_{0\nu\beta\beta}}{\ln 2} = G_{01} \left| \sum_j U_{ej}^2 \frac{m_j}{m_e} \mathcal{M}^{0\nu\beta\beta}(m_j) \right|^2,$$

where $G_{01}$ is a well-known kinematic factor, $U$ is the unitary matrix which diagonalizes the complete neutrino mass matrix both for active and sterile neutrinos, $m_j$ are the corresponding eigenvalues, i.e., the neutrino masses, and $\mathcal{M}^{0\nu\beta\beta}$ are the Nuclear Matrix Elements (NMEs) associated with the process. The sum should be made over all the neutrino masses, including the heavy ones.

The NMEs can be computed using different methods, the main two being the quasiparticle random phase approximation (QRPA) [17,18] and the interacting shell model (ISM) [19,20]. In this work we will make use of the NME data presented in Ref. [14] and available in Ref. [21]. They were computed for different nuclei in the context of the ISM as a function of the neutrino mass, something very convenient for our analysis. We use a notation in which the dependence on the neutrino propagator is included on $\mathcal{M}^{0\nu\beta\beta}(m_j)$, in contrast with the notation usually adopted in the literature where the propagator is expanded to factorize the mass dependence. In Fig. 1 of Ref. [14] the NME dependence on the mass of the neutrino mediating the process is depicted, showing two different regions separated by the scale of the process $\sim 100$ MeV:

- Below the $0\nu\beta\beta$ scale, the NMEs reach their maximum value and are mainly independent of the neutrino mass. For $m_i \ll 100$ MeV, $\mathcal{M}^{0\nu\beta\beta}(m_i) = \mathcal{M}^{0\nu\beta\beta}(0)$.

- The NMEs corresponding to neutrinos much heavier than 100 MeV are suppressed with the heavy neutrino masses and scale as $\mathcal{M}^{0\nu\beta\beta}(m_i) \propto 1/m_i^2$.

This behavior of the NMEs, showing two clearly different regimes, can be easily understood by expanding the propagator of the neutrino mediating the process. The transition region around 100 MeV is well described in Fig. 1 of Ref. [14] since no assumptions have been made on the neutrino masses in the NME computation.

We can distinguish the following two contributions to the $0\nu\beta\beta$ decay amplitude:
\[ A \propto \sum_{i=1}^{3} m_i U_{ei}^2 M^{0\nu\beta\beta}(m_i) + \sum_{I} m_I U_{eI}^2 M^{0\nu\beta\beta}(m_I), \] (2)

the first term corresponding to the mostly active neutrino contribution, and the second to the extra states of the model. Here and throughout the text, we use capital letters to denote the mass indices of the mostly sterile states and lowercase letters for those of the mostly active states.

On the other hand, since a Majorana mass coupling for the active neutrinos is forbidden by the gauge symmetry, the diagonalization of the complete mass matrix leads to the following relation:

\[ \sum_{i=1}^{3} m_i U_{ei}^2 + \sum_{I} m_I U_{eI}^2 = 0. \] (3)

This equation, which relates the light and extra degrees of freedom of the model, should always be fulfilled at tree-level and plays a fundamental role in the phenomenology of $0\nu\beta\beta$ decay.

For extra states with all the masses well above 100 MeV, using the relation given in Eq. (3), the contribution to $0\nu\beta\beta$ decay in Eq. (2) can be recast as

\[ A \propto - \sum_{I} m_I U_{eI}^2 \left( M^{0\nu\beta\beta}(0) - M^{0\nu\beta\beta}(m_I) \right) \]
\[ \approx - \sum_{I} m_I U_{eI}^2 M^{0\nu\beta\beta}(0) = \sum_{i=1}^{3} m_i U_{ei}^2 M^{0\nu\beta\beta}(0), \] (4)

where we have used the fact that $M^{0\nu\beta\beta}(0) \gg M^{0\nu\beta\beta}(m_I)$. The contribution from the light active neutrinos thus dominates. A similar argument applies to models which implement the type-II and type-III seesaw [14], and more generically to models in which heavy sterile neutrino mixing with $\nu_e$ is introduced. As sterile neutrinos contribute to light neutrino masses, $\sum_{I} m_I U_{eI}^2$ is constrained by the value of the light neutrino masses while their contribution to $0\nu\beta\beta$ decay is suppressed by $M^{0\nu\beta\beta}(m_I)$ making it subdominant, at least if fine-tuning is not invoked as we will see in the following.

These considerations apply generically to models with extra sterile neutrinos but there are some notable exceptions:

- The case of extra states below and above 100 MeV. In this case Eq. (2) can be rewritten
as

\[ A \propto \left( \sum_{i=1}^{3} m_i U_{ei}^2 + \sum_{I}^{\text{light}} m_I U_{eI}^2 \right) \mathcal{M}^{0\nu\beta\beta}(0) + \sum_{I}^{\text{heavy}} m_I U_{eI}^2 \mathcal{M}^{0\nu\beta\beta}(m_I) \]

\[ \approx \left( \sum_{i=1}^{3} m_i U_{ei}^2 + \sum_{I}^{\text{light}} m_I U_{eI}^2 \right) \mathcal{M}^{0\nu\beta\beta}(0), \tag{5} \]

and the new states below 100 MeV may give the dominant contribution. Notice that if all the extra states are below the $0\nu\beta\beta$ scale the cancellation driven by Eq. (3) forbids the process. The same behavior as in this type-I seesaw realization with sterile neutrinos below and above the $0\nu\beta\beta$ scale applies to a type-II or type-III scenario in combination with type-I light sterile neutrinos \[^{14}\]. In all these scenarios NP above the $0\nu\beta\beta$ scale, either heavy sterile neutrinos (in the type-I seesaw) or heavy triplets (in the just-mentioned mixed type-I/type-II and type-I/type-III seesaw), are needed to avoid the cancellation, while the “light” sterile neutrinos can give the dominant contribution to $0\nu\beta\beta$ decay. \(^1\)

- **Additional contributions to neutrino masses.** In this case the mass relation becomes

\[ \sum_{i=1}^{3} m_i U_{ei}^2 + \sum_{I}^{\text{heavy}} m_I U_{eI}^2 = m_{LL}, \tag{6} \]

where $m_{LL}$ is an effective Majorana mass term generated for the active neutrinos by some other mechanism. $m_{LL}$ and $\sum_{I}^{\text{heavy}} m_I U_{eI}^2$ could be very large and cancel nearly exactly, keeping light neutrino masses under control. In this way, even with the contribution to $0\nu\beta\beta$ of the heavy states being weighted by the corresponding NME, a dominant effect could arise. However, it would have to overcome the suppression coming from the NME ($\mathcal{M}^{0\nu\beta\beta}(0) / \mathcal{M}^{0\nu\beta\beta}(m_I) \gg 1$) and a very high level of cancellation among $\sum_{I}^{\text{heavy}} m_I U_{eI}^2$ and $m_{LL}$ in Eq. (6) would be required. This possibly implies an uncomfortably high level of fine-tuning and will not be studied in this work.

- **A cancellation in the light neutrino contribution:** $\sum_{i=1}^{3} m_i U_{ei}^2 = 0$. If this cancellation took place, the heavy neutrinos would trivially dominate the process (at least, at tree-level).

\[^{1}\] The scalar/fermion triplet contribution to $0\nu\beta\beta$ decay is subdominant in comparison with the light active neutrino one \[^{14}\].
In this work we are going to focus on this last possibility. This relevant exception was studied in Refs. [15, 16, 22] and not contemplated in Ref. [14]. Of course, this cancellation in the light contribution could be obtained by invoking some symmetry, and the most natural one in this context is the lepton number. The well-known inverse [23] or linear [24] seesaw models, which involve small violations of the lepton number, could in principle implement this scenario. However, generating a measurable heavy neutrino contribution to the $0\nu\beta\beta$ decay is not trivial even in these models. First of all, $0\nu\beta\beta$ decay is a lepton-number-violating process and consequently is expected to be suppressed in this context. Moreover, the suppression of the heavy neutrino contribution with the NME ($\sim 1/m_i^2$) makes having very low-scale heavy masses unavoidable in order to obtain a relevant effect. This possibility has been recently explored claiming that the heavy neutrinos could be very relevant for some particular neutrino mass textures [15, 16]. Indeed, the main goal of this note is to check to what extent this is possible, paying special attention to the stability of the light neutrino masses under one-loop corrections and their contribution to the $0\nu\beta\beta$ decay.

III. THE MODELS

We will focus on the study of SM extensions which consist of the addition of $n+n'$ fermion gauge singlets, $N_i$, to the SM particle content without imposing lepton number conservation, whose Lagrangian is

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{kin}} - \frac{1}{2} \overline{N}_i M_{ij} N_j^c - y_{ia} \overline{N}_i \phi L_\alpha + \text{h.c.},$$

where $\mathcal{L}_{\text{SM}}$ is the SM Lagrangian and $\mathcal{L}_{\text{kin}}$ are the kinetic terms of the new fields $N_i$. Here, and in the rest of the paper, the subindex $\alpha$ denotes flavor ($\alpha = e, \mu, \tau$). Without loss of generality, the neutrino mass matrix can be expressed as

$$M_\nu = \begin{pmatrix}
0 & Y_1^T v/\sqrt{2} & \epsilon Y_2^T v/\sqrt{2} \\
Y_1 v/\sqrt{2} & \mu' & \Lambda \\
\epsilon Y_2 v/\sqrt{2} & \Lambda^T & \mu
\end{pmatrix} \equiv \begin{pmatrix}
0 & m_D^T \\
M_D & M
\end{pmatrix}.$$  

Here $Y_1$ and $Y_2$ are the $n' \times 3$ and $n \times 3$ matrices that form the Dirac block $m_D$. The Majorana submatrix $M$ is composed of $\mu'$, $\mu$ and $\Lambda$: the $n' \times n'$, $n \times n$ and $n' \times n$ matrices
respectively. Notice that $\epsilon$, $\mu_{ij}$ and $\mu'_{ij}$ are lepton-number-violating parameters.\(^2\) Another helpful, and widely used, basis is the one in which the Majorana submatrix for the sterile neutrinos is diagonal, which we will denote with a tilde in the following discussion. In order to illustrate the relation between these two bases, let us consider the $n = n' = 1$ case. Both bases are related through the following rotation which diagonalizes the Majorana submatrix $M$

$$
\tilde{M}_\nu = OM_\nu O^T = \begin{pmatrix}
0 & \tilde{Y}_1 v/\sqrt{2} & \tilde{Y}_2 v/\sqrt{2} \\
\tilde{Y}_1 v/\sqrt{2} & \tilde{M}_1 & 0 \\
\tilde{Y}_2 v/\sqrt{2} & 0 & \tilde{M}_2
\end{pmatrix} \equiv \begin{pmatrix}
0 & \tilde{m}_D^T \\
\tilde{m}_D & \tilde{M}
\end{pmatrix}, \quad O = \begin{pmatrix}
1 & 0 & \\
0 & 0 & A
\end{pmatrix},
$$

(9)

with $A$ being a $2 \times 2$ orthogonal matrix with the rotation angle \(^3\)

$$
\tan \theta = \frac{\mu' - \mu + \sqrt{4\Lambda^2 + (\mu' - \mu)^2}}{2\Lambda}.
$$

(10)

The Majorana masses $\tilde{M}_1$ and $\tilde{M}_2$ and the Yukawa couplings $\tilde{Y}_i$ are then given by

$$
\tilde{M}_{2,1} = \frac{1}{2} \left( \mu' + \mu \pm \sqrt{4\Lambda^2 + (\mu' - \mu)^2} \right),
\tilde{Y}_1 = Y_1 \cos \theta - \epsilon Y_2 \sin \theta,
\tilde{Y}_2 = Y_1 \sin \theta + \epsilon Y_2 \cos \theta.
$$

(11)

Of course, the analysis can be performed in any basis, but we will mainly work in the one in which the neutrino mass matrix is given by Eq. (8).

Notice that the mass matrix given in Eq. (8) is completely general. A particularly interesting set of models, included in Eq. (8), are those studied and summarized in Ref. [25], and which include the so-called inverse or multiple seesaw models [23, 26–28]. The lepton number is assumed to be a good global symmetry only broken in the neutrino sector through the small lepton-number-violating terms $\mu$ and/or $\epsilon$. In these models the light masses are “naturally” proportional to $\epsilon$ and/or $\mu$. Therefore, thanks to the suppression of the light neutrino masses coming from $\epsilon$ and $\mu$, the scale of NP given by $\Lambda$ can be lowered to the TeV

\(^2\) This corresponds to the case in which $L(\nu_{\alpha L}) = L(N_{i=1,...,n}) = -L(N_{j=1,...,n'}) = 1$. There are other possible lepton number assignments for $N_i$ and $N_j$, which are broken by different terms in the Lagrangian.

Our analysis is completely general and the neutrino mass matrix given in Eq. (8) depends on all possible lepton-number-violating parameters.

\(^3\) For simplicity, all the Majorana submatrix parameters in $M$ have been considered real.
level or even below. This allows sizable NP effects coming from the dimension-6 operator which, contrary to the dimension-5 one, does not present any extra suppression with $\epsilon$ and $\mu$, as it does not violate the lepton number $[25]$. Lepton-conserving processes very suppressed in the SM as the rare decays are very promising channels to probe this kind of NP. Interesting recent analysis of the $\mu \rightarrow e\gamma$, $\mu \rightarrow eee$ and $\mu \rightarrow e$ conversions in the context of low-scale small lepton-number-violating models can be found in the literature $[29-31]$. In Ref. $[15]$, these sorts of models are studied in the context of the $0\nu\beta\beta$ decay using a different parametrization based on the Casas-Ibarra one $[32]$, which parameterizes the neutrino mass matrix in terms of the light and heavy masses, the $U_{\alpha i}$ matrix and an orthogonal matrix $R$. Notice that the parametrization considered here is totally general and includes the Casas-Ibarra limit in which an approximate decoupling of light and heavy sectors is assumed.

Another interesting model included in Eq. (8) is the so-called extended seesaw model $[33]$. In these models $\mu'$ is the key parameter and is assumed to be larger than the rest of the parameters in Eq. (8), and more specifically, much larger than $\mu$ and $\epsilon Y_{2\alpha} v$, defining the highest scale of the model. The term $\mu'$ introduces large lepton number violation which can help to achieve successful low-scale leptogenesis $[34]$ without the need of a degenerate heavy neutrino spectrum $[33]$. This large violation of the lepton number is not present in the inverse seesaw scenario, in which the lepton number is assumed to be a good approximate global symmetry.

In any case, we will not restrict our study to any particular value of the parameters or, in other words, to any of the above mentioned specific limits. Nevertheless, for simplicity, we will consider the case in which only two fermion singlets ($n = n' = 1$) are added. In any case, we expect that the general conclusions obtained in this work can be applied to models with larger number of right-handed neutrinos.

Finally, from neutrino oscillations, we know that it is not easy to accommodate the experimental data in the region of the parameter space between the limits: $M_i \gg \tilde{m}_D$ (seesaw limit) and $M_i \ll \tilde{m}_D$ (pseudo-Dirac limit). In fact, in Ref. $[35]$ it is shown how the constraints from neutrino oscillation experiments leave those limits as the only allowed regions for $n = n' = 1$ and $\tilde{M}_1 = \tilde{M}_2$. The region of the parameter space in between is ruled out and only the pseudo-Dirac and seesaw limits survive. Reasonably extrapolating these results to the more general case with $\tilde{M}_1 \neq \tilde{M}_2$ studied here, leaves the seesaw limit
\( \tilde{M}_i \gg \tilde{m}_D \) as the only relevant part of the parameter space in the 0νββ decay context.\(^4\) From now on, we will focus on the *seesaw* limit. Notice, however, that this does not necessarily mean that \( \tilde{M}_i \) have to be at the GUT or the TeV scale and can be considerably lighter \([36–38]\).

### IV. LIGHT NEUTRINO MASSES AND 0νββ DECAY

For \( \tilde{M}_i \gg \tilde{m}_D \), the light neutrino mass matrix is given at tree-level by

\[
m_{\text{tree}} \approx -m_D^T M^{-1} m_D \approx \frac{v^2}{2(\Lambda^2 - \mu \mu')} (\mu Y^T Y_1 + \epsilon^2 \mu' Y^T Y_2 - \Lambda \epsilon (Y^T Y_1 + Y^T Y_2)) ,
\]

where \( m_D \) and \( M \) are the 2 × 3 Dirac and 2 × 2 Majorana submatrices, respectively, in Eq. (8) for \( n = n' = 1 \). Here, we have performed the standard “*seesaw*” \( m_D / M \) expansion, keeping the leading-order terms. We will discuss later if the higher-order corrections can be relevant. The contribution of the mostly active neutrinos to the 0νββ decay amplitude is proportional to the “ee” element of this effective mass matrix as

\[
A_{\text{light}} \propto \sum_{i=1}^{3} m_i U^2_{ei} \mathcal{M}^{0\nu\beta\beta}(0) \approx - (m_D^T M^{-1} m_D)_{ee} \mathcal{M}^{0\nu\beta\beta}(0) = \mu Y^2_{1e} + \epsilon Y_{2e} (\epsilon \mu' Y_{2e} - 2\Lambda Y_{1e}) \frac{v^2}{2(\Lambda^2 - \mu' \mu')} \mathcal{M}^{0\nu\beta\beta}(0).
\]

Therefore, the light neutrino contribution is strictly canceled as long as the parameters of the model satisfy the following relation:

\[
\mu Y^2_{1e} + \epsilon Y_{2e} (\epsilon \mu' Y_{2e} - 2\Lambda Y_{1e}) = 0.
\]

This condition is fulfilled for

\[
\epsilon = \mu = 0.
\]

Of course, it may also be satisfied for other choices of parameters, but \( \epsilon = \mu = 0 \) is the most stable one under radiative corrections and higher-order terms in the expansion, as we will show later. From now on, we will assume that this cancellation condition is fulfilled. Setting \( \epsilon \) and \( \mu \) to zero also leads to vanishing tree-level active neutrino masses. However,

\(^4\) Of course, the Dirac limit will not be considered in this analysis where the 0νββ decay phenomenology is studied.
light neutrino masses are expected to be generated at one loop if $\mu'$ is different from zero and breaks the lepton number, as we will see.

One could naively think that taking into account Eq. (3) would lead us to the same cancellation for the heavy neutrinos (see Eq. (2)); however, the dependence of the NMEs on $m_I$ avoids a complete cancellation, if the heavy neutrinos are not too degenerate.

When the heavy neutrinos are above the $0^{\nu}\beta\beta$ scale, $m_4, m_5 \gg 100 \text{ MeV}$, the heavy contribution to the $0\nu\beta\beta$ decay amplitude can be approximated as

$$A_{\text{extra}} \propto \sum_{I} m_I U_{eI}^2 M^{0\nu\beta\beta}(m_I) \propto -\left(m_D^T M^{-3} m_D\right)_{ee},$$

which reduces to

$$A_{\text{extra}} \propto \frac{v^2 \mu' Y_{1e}^2}{2 \Lambda^4},$$

if the light neutrino contribution is canceled ($\epsilon = \mu = 0$). Apparently, the above expression indicates that for large values of $\mu'$ and/or small enough $\Lambda$ the heavy neutrinos may give a relevant contribution to the $0\nu\beta\beta$ decay at tree level. At this point two interesting limits of Eq. (8) arise:

- **Extended seesaw limit** (**ESS limit**): $\mu' \gg \Lambda, m_D$. In view of Eq. (17), this possibility appears quite appealing. This limit is inspired by the so-called extended seesaw models and corresponds to a hierarchical spectrum for the heavy neutrinos:

$$m_4 \approx \bar{M}_1 \approx -\Lambda^2 / \mu', \quad U_{e4} \approx Y_{1e} v / \sqrt{2} \Lambda,$$

$$m_5 \approx \bar{M}_2 \approx \mu', \quad U_{e5} \approx Y_{1e} v / \sqrt{2} \mu',$$

where we also show the corresponding mixing with the active neutrinos. In this regime, the lightest of the two heavy neutrinos dominates the heavy contribution. Moreover, for large enough values of $\mu'$, $m_4$ becomes lighter than 100 MeV, the NME takes its maximum value and the heavy contribution to the $0\nu\beta\beta$ decay becomes independent of $\Lambda$:

$$A_{\text{extra}} \propto U_{e4}^2 m_4 M^{0\nu\beta\beta}(0) \approx -\frac{Y_{1e}^2 v^2}{2 \mu'} M^{0\nu\beta\beta}(0).$$

- **Inverse seesaw limit** (**ISS limit**): $\Lambda \gg \mu', m_D$. This limit corresponds to one of the Minimal Flavor Violation (MFV) models studied in Ref. [25]. It is also related to
the case analyzed in Ref. [15], where a different parameterization is used. In this case the heavy neutrino spectrum is quasi-degenerate, forming a quasi-Dirac pair:

\[
\begin{align*}
    m_4 &\approx -m_5 \approx \tilde{M}_1 \approx -\tilde{M}_2 \approx \Lambda, \\
    U_{e4} &\approx U_{e5} \approx Y_{1e}v/2\Lambda, \\
    \Delta \tilde{M} &\equiv |\tilde{M}_2| - |\tilde{M}_1| \approx \mu',
\end{align*}
\]

and we can expect lepton-number-violating processes such as neutrinoless double beta decay to be controlled by \(\mu'\).

If all the heavy neutrinos are located below the 0\(\nu\beta\beta\) scale, a cancellation driven by Eq. (3) is expected at tree-level, as we have already mentioned. This cancellation applies in general as long as all the heavy neutrinos are in the light regime, including the two limits distinguished above.

The approximation made in Eq. (16), \(M^{0\nu\beta\beta}(m_I) \propto 1/m_I^2\), does not apply if one of the heavy neutrinos (or both) is lighter than (or close to) \(\sim 100\) MeV. However, as we have already commented, we will not restrict the analysis to any particular value of the sterile neutrino masses. This is the reason why we have made use of a numerical computation for the NME in which no approximation for the neutrino mass dependence has been considered. Notice, for instance, that the phenomenology for heavy masses around 100 MeV can be very interesting and the approximation \(M^{0\nu\beta\beta}(m_I) \propto 1/m_I^2\) is not very accurate in that region.

In summary, at tree-level the light neutrino masses are independent of \(\mu'\) (and \(\Lambda\)) for \(\epsilon = \mu = 0\), being actually zero. However, lepton-number-violating processes such as 0\(\nu\beta\beta\) decay are sensitive to these parameters and \(\mu'\) in particular. The idea behind Refs. [15, 16] is to exploit this apparent decoupling between the heavy and light contributions in order to have a measurable effect in the 0\(\nu\beta\beta\) decay coming from the heavy side. In the following, we will check if a heavy dominant contribution is really possible once the relevant corrections and experimental constraints are taken into account.

V. HIGHER-ORDER CORRECTIONS IN THE SEESAW EXPANSION

Only the leading-order in \(m_D/M\) has been considered in the expansion performed in Eq. (12). We now check if the higher-order corrections may induce any relevant effects once the tree-level cancellation for the light masses takes place. The next-to-leading-order contributions to the light neutrino masses can be written as [39]:

\[12\]
\[ \delta m = \frac{1}{2} m_{\text{tree}} m_D^T M^{-2} m_D + \frac{1}{2} \left( m_{\text{tree}} m_D^T M^{-2} m_D \right)^T, \] 

(21)

where \( m_{\text{tree}} \) is the leading-order contribution given by \( m_{\text{tree}} = -m_D^T M^{-1} m_D \). As they are proportional to the leading-order active neutrino mass \( m_{\text{tree}} \), they are completely irrelevant for \( \mu = \epsilon = 0 \). In fact, the light neutrino masses vanish for \( \mu = \epsilon = 0 \) at all orders in the expansion \[39, 40\]. Contrary to the \( \mu = \epsilon = 0 \) case, other choices of the parameters which satisfy the cancellation condition given in Eq. (14) are flavor dependent, giving as a result nonvanishing higher-order corrections.

On the other hand, the factor \( m_D^T M^{-2} m_D / 2 \) is nothing but the coefficient of the effective \( d = 6 \) operator obtained when the heavy neutrinos are integrated out of the theory \[41\]. This coefficient, which induces deviations from the unitarity of the \( 3 \times 3 \) lepton mixing matrix, is independent of \( \mu' \) when the light neutrino cancellation \( (\mu = \epsilon = 0) \) takes place. Therefore, for \( \mu = \epsilon = 0 \), the \( d = 6 \) operator does not introduce any relevant \( \mu' \)-dependent deviation from unitarity, and \( \mu' \) can escape from the corresponding constraints \[42, 43\], even if \( \mu' \gg \Lambda \).

VI. ONE-LOOP CORRECTIONS

The one-loop corrections can be of two different types: renormalizable (i.e. the running of the parameters) or finite. In this section we will study both, starting with the renormalizable corrections. The analysis will be done after electroweak symmetry breaking (EWSB).

A. Renormalizable one-loop corrections

We are mainly interested in the running behavior of the parameters \( \mu \) and \( \epsilon \), since they drive the light neutrino mass cancellation, and \( \mu' \) and \( \Lambda \) which are the key parameters associated with the heavy contribution. We have performed the computation in the basis in which the neutrino mass matrix takes the form given by Eq. (8), in such a way that the one-loop running equations \[44–46\] for these parameters can be directly obtained:

\[
Q \frac{d}{dQ} (\epsilon Y_{2\alpha}) = \frac{\epsilon}{(4\pi)^2} \left[ \left( T - \frac{9}{4} g'^2 - \frac{3}{4} g^2 \right) Y_{2\alpha} - \frac{3}{2} Y_{2\beta} \left( (Y_l^\dagger Y_l)_{\beta\alpha} - Y_{1\beta}^* Y_{1\alpha} \right) + \frac{3}{2} \epsilon^2 Y_{2\beta} Y_{2\beta}^* Y_{2\alpha} \right], \\
Q \frac{d\mu}{dQ} = \frac{2\epsilon}{(4\pi)^2} \left[ \Lambda Y_{1\beta}^* Y_{2\beta} + \mu \epsilon Y_{2\beta}^* Y_{2\beta} \right],
\]
\[
\begin{align*}
Q^{d\mu'}_{dQ} &= \frac{2}{(4\pi)^2} \left[ \mu' Y^*_{1\beta} Y_{1\beta} + \epsilon Y^*_{2\beta} Y_{1\beta} \right], \\
Q^{d\Lambda}_{dQ} &= \frac{1}{(4\pi)^2} \left[ \Lambda Y^*_{1\beta} Y_{1\beta} + \epsilon \left( \mu' Y^*_{1\beta} Y_{2\beta} + \mu Y^*_{2\beta} Y_{1\beta} + \Lambda \epsilon Y^*_{2\beta} Y_{2\beta} \right) \right],
\end{align*}
\]

where \( T = \text{Tr} \left( 3Y_u^T Y_u + 3Y_d^T Y_d + Y_l^T Y_l + Y^T Y^* \right) \) and \( g \) and \( g' \) are the \( SU(2)_L \) and \( U(1)_Y \) gauge coupling constants of the SM. We do not need to solve the equations to realize that the effect of the one-loop renormalizable corrections to \( \mu \) and \( \epsilon \) is suppressed by the tree-level values of \( \epsilon \) or \( \mu \). This means that the cancellation of the light active neutrino masses is stable under one-loop renormalizable corrections, as expected, as a Majorana mass coupling for the active neutrinos is not allowed at tree-level. For vanishing \( \epsilon \) and \( \mu \) at tree-level, the light neutrino masses keep being zero independently of the running of the parameters (even for huge tree-level inputs of \( \mu' \)). This is no longer true once the finite corrections are taken into account, as we show in the next subsection.

**B. Finite one-loop corrections**

Indeed, after EWSB, a Majorana mass for the active neutrinos is generated through finite one-loop corrections. Of course, the other Yukawa and Majorana couplings among the active and sterile neutrinos also get finite corrections, but their contribution to the light neutrino masses vanishes for \( \mu = \epsilon = 0 \). This contribution is proportional to the finite one-loop corrections to \( \mu \) and \( \epsilon Y_{2\alpha} \) (see Eq. (12)). Since the sterile neutrinos only couple to the Higgs, via the Yukawas, the one-loop corrections to \( \mu \) (the Majorana coupling between \( N_2 N_2^c \)) and the Yukawa couplings between \( N_2 \) and \( \nu_{\alpha L} \) (\( \epsilon Y_{2\alpha} \)) are proportional to \( \epsilon Y_{2\beta} \) and vanish in the limit \( \mu = \epsilon = 0 \). Therefore, the dominant contribution to the light neutrino masses comes from the Majorana mass generated for the active neutrinos and is given by

\[
(\delta m_{LL})_{\alpha\beta} = \frac{1}{(4\pi v)^2} \left( \tilde{m}_D^T \right)_{\alpha i} \tilde{M}_i \left\{ \frac{3 \ln \left( \tilde{M}^2_i / M_Z^2 \right)}{M_i^2 / M_Z^2 - 1} + \frac{\ln \left( \tilde{M}^2_i / M_H^2 \right)}{M_i^2 / M_H^2 - 1} \right\} \left( \tilde{m}_D \right)_{i\beta},
\]

where \( \tilde{m}_D \) and \( \tilde{M} = \text{diag} \left( \tilde{M}_1, \tilde{M}_2 \right) \) are the Dirac and Majorana submatrices respectively, written in the basis in which the Majorana submatrix is diagonal, \( M_Z \) is the mass of the \( Z \) boson, and \( M_H \) is the Higgs boson mass. Notice that no expansion has been performed in
order to obtain this result. The structure of the correction is similar to the tree-level masses but in this case no cancellation takes place for $\mu = \epsilon = 0$.

![Graph](image)

**FIG. 1**: The colored region in the $\mu'-\Lambda$ plane corresponds to $|\delta m_{LL} (\mu = \epsilon = 0)| > 0.1$ eV. The yellow area (solid edges), orange (dashed), and red (dotted) stands for $Y_{1\alpha} = 10^{-1}$, $Y_{1\alpha} = 10^{-3}$, and $Y_{1\alpha} = 10^{-5}$, respectively.

In particular, Eq. (23) can be conveniently written in the $\mu = \epsilon = 0$ limit as

$$
\delta m_{LL} = \frac{1}{(4\pi)^2} \frac{Y_1^2 Y_1}{2} \left\{ \left( \frac{3 \tilde{M}_1 \ln (\tilde{M}_1^2 / M_Z^2)}{\tilde{M}_1^2 / M_Z^2 - 1} + \frac{\tilde{M}_1 \ln (\tilde{M}_1^2 / M_H^2)}{\tilde{M}_1^2 / M_H^2 - 1} \right) \cos^2 \theta 
\right.
\left. + \left( \frac{3 \tilde{M}_2 \ln (\tilde{M}_2^2 / M_Z^2)}{\tilde{M}_2^2 / M_Z^2 - 1} + \frac{\tilde{M}_2 \ln (\tilde{M}_2^2 / M_H^2)}{\tilde{M}_2^2 / M_H^2 - 1} \right) \sin^2 \theta \right\}.    \tag{24}
$$

where $\tilde{M}_{2,1}$ are the eigenvalues of the Majorana mass term given by Eq. (11), and $\theta$ is the rotation angle given by Eq. (10), both evaluated for $\epsilon = \mu = 0$. In Fig. 1, we show the region of the parameter space $\mu'-\Lambda$ given by $|\delta m_{LL} (\mu = \epsilon = 0)| > 0.1$ eV for different values of the Yukawa couplings. In order to understand better the implications of Eq. (24), we have obtained approximate expressions for two relevant limits:
• \( \Lambda \gg \mu', M_H, M_Z \). We have

\[
\delta m_{LL} \approx \frac{1}{(4\pi)^2} \left( \frac{Y_1^T Y_1}{2} \frac{M_H^2}{\Lambda^2} + 3M_Z^2 \mu' \right). \tag{25}
\]

As we have already discussed in Sec. IV, this case is included in the ISS limit and corresponds to a MFV model in which \( \mu, \epsilon \) and \( \mu' \) are lepton-number-violating parameters. What we observe here is that although the tree-level light neutrino masses cancel for \( \epsilon = \mu = 0 \), they are generated at one loop and are proportional to the only lepton-number-violating parameter different from zero, \( \mu' \), as expected, since the neutrino masses also violate this symmetry.

• \( \mu' \gg \Lambda \gg M_H, M_Z \). In this case, one finds

\[
\delta m_{LL} \approx \frac{1}{(4\pi)^2} \left( \frac{3M_Z^2}{\mu'} \ln \left( \frac{\Lambda^4}{M_H^4} \right) + \frac{M_Z^2}{\mu'} \ln \left( \frac{\Lambda^4}{M_H^4} \right) \right). \tag{26}
\]

This case is included in the ESS limit discussed in Sec. IV. Here, the one-loop light neutrino masses depend mildly on \( \Lambda \) and are suppressed by \( \mu' \). Again, this can be understood in terms of a lepton symmetry: \( \mu' \) is suppressing the violation of the lepton number at low energies in such a way that in the limit \( \mu' \to \infty \), the symmetry is completely restored in the effective theory.

In the next section, we will study the phenomenological consequences of Eq. (24) in the context of the 0\( \nu \)\( \beta \beta \) decay without considering any expansion on the parameters. It is important to remark here that once the tree-level cancellation takes place, only one mass is generated at one-loop, and at least two light masses are necessary to explain the light neutrino spectrum obtained in neutrino oscillation experiments. This is easy to solve: simply adding another fermion singlet to the model would allow us to generate the necessary extra light mass. However, for simplicity, we will keep studying the simpler case with only two extra sterile neutrinos.

VII. NEW PHYSICS DOMINANT CONTRIBUTION TO 0\( \nu \)\( \beta \beta \) DECAY AND ONE-LOOP NEUTRINO MASSES

Once the relevant one-loop corrections are taken into account, the Lagrangian is modified to

\[
\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{kin}} - \frac{1}{2} \bar{N}_i M_{ij} N_j^c - \frac{1}{2} (\delta m_{LL})_{\alpha\beta} \bar{\nu}_{\alpha L} \nu_{\beta L}^c - y_{i\alpha} \bar{N}_i \phi^3 L_\alpha + \text{h.c.}. \tag{27}
\]

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Consequently, Eq. (3), which comes from the diagonalization of the neutrino mass matrix, is also modified to the following one-loop version:

\[ \sum \text{light} m_i U_{ei}^2 + \sum \text{extra} m_I U_{eI}^2 = (\delta m_{LL})_{ee}. \] (28)

Notice that here \( U \) diagonalizes the neutrino mass matrix including the one-loop corrections. In the case of interest, when \( \epsilon = \mu = 0 \), the light neutrino masses associated with the mostly active neutrinos are determined by \( \delta m_{LL} \). The tree-level condition \( \sum \text{extra} m_I U_{eI}^2 = 0 \) remains true at the one-loop level but, as discussed in Sec. IV, heavy neutrinos could have a sizable effect on \( 0\nu\beta\beta \) decay thanks to the NME dependence on the heavy masses. However, at one loop, the NP contribution to the \( 0\nu\beta\beta \) decay and the light neutrino masses are related, as they depend on the same parameters, in particular \( \mu', \Lambda \) and \( Y_{1\alpha} \). Their decoupling achieved at tree-level does not remain true once radiative corrections are included. Consequently, the heavy parameters cannot be chosen arbitrarily as to dominate \( 0\nu\beta\beta \) decay but are constrained by light neutrino masses, which also contribute to the process.

In principle, the radiative corrections dependent on the transferred momentum \( p \) also have to be considered. This corrections are of two types: (i) proportional to \( p \) or \( p^3 \); (ii) dependent on \( p^2 \). The first come from the \( W \) and charged Goldstone boson corrections to the neutrino propagator and vanish by chirality. The second are associated with the \( Z \) and Higgs boson corrections to the propagator and are negligible in the region of heavy masses under consideration.

In the rest of the section, we will study under which conditions it may (or may not) be possible to have a dominant heavy neutrino contribution. We will pay special attention to the impact of the one-loop corrections and the experimental constraints on the parameters of the model. To illuminate the interplay among all these factors, we will first analyze the particular case of \( Y_{1\alpha} = 10^{-3} \) showing our results in Fig. 2.

First of all, we are assuming that the model under consideration provides the dominant source of light neutrino masses. In principle, they should be in agreement with neutrino oscillations data but, since we are generating at one loop only one light neutrino mass we only impose a conservative lower bound on the nontrivial eigenvalue of Eq. (24) given by the solar splitting, \( \sqrt{\Delta m^2_{\text{sol}}} \). Moreover, the absolute neutrino mass scale experiments impose an upper bound on the same combination of parameters, and as reference value we take the 95\% C.L. upper bound on the light neutrino masses from cosmology [50], \( m_\nu = 0.58 \) eV. Since
FIG. 2: Impact of one-loop corrections on $0\nu\beta\beta$ decay for $Y_{1\alpha} = 10^{-3}$. The red band is the 95\% C.L. allowed region for the one-loop generated light neutrino masses bounded by cosmology and neutrino oscillations. The orange band is the 95\% C.L. region of the parameter space in which the heavy neutrino contribution is between the present bound from EXO and the sensitivity of the next-to-next generation of $0\nu\beta\beta$ experiments. Blue (green) stands for the region in which the ratio $r$ between the heavy and light contributions is $r > 5$ ($1 < r < 5$). The grey region inside the dashed black line is the parameter space ruled out at the 95\% C.L. by the constraints on the mixing.

we are analyzing the case in which the tree-level active neutrino masses cancel ($\epsilon = \mu = 0$), these bounds can be directly translated into bounds on $\mu'$ and $\Lambda$ as a function of $Y_{1\alpha}$. They are shown in Fig. 2 as the red band. The Higgs mass, $m_H$, has been fixed to 125 GeV in all the calculations, as suggested by the recent LHC results [51, 52]. Notice that if no lower bound is imposed, as would be the case if the light neutrino masses came from some other mechanism, the outer region of the red band would not be excluded. Our conclusions remain valid also in this case, as we will discuss later. The constraint on the light neutrino masses shown in Fig. 2 can be understood by analytically taking into account the approximate
expressions derived in the previous section. In the ISS limit, \( \sqrt{\Delta m_{\text{sol}}^2} < \delta m_{LL} < 0.58 \text{ eV} \) scales as \( \mu'/\Lambda^2 \) in agreement with Eq. (25). For \( \mu' \gg \Lambda \), in the ESS limit, it becomes mainly independent of \( \Lambda \) according to Eq. (26).

The heavy neutrino contribution to \( 0\nu\beta\beta \) decay, given by \( A_{\text{heavy}} \propto \sum_{I=4,5} U_{eI}^2 m_I M^{0
u\beta\beta}(m_I) \), can be computed by diagonalizing the mass matrix in Eq. (8)\(^5\) and using the NME data calculated as a function of the neutrino masses [21]. The diagonalization can be easily performed in the \( \epsilon = \mu = 0 \) limit. This contribution has to respect the present experimental bound and, in order to be phenomenologically interesting, should be within the reach of future \( 0\nu\beta\beta \) decay experiments such as CUORE [53], EXO [54], GERDA [55], KamLAND-Zen [56], MAJORANA [57], NEXT [58] or Super-NEMO [59]. This constraint is shown as the orange band in Fig. 2: the 95\%C.L. region of the parameter space in which the heavy neutrino contribution is between the present bound from EXO [54] (\( 0\nu\beta\beta \) decay in \(^{136}\text{Xe} \)), which using the corresponding shell model NME is \( |m_{\beta\beta}| < 0.53 \) eV, and the future (optimistic) sensitivity of the next-to-next generation of \( 0\nu\beta\beta \) decay experiments, taken to be \( m_{\beta\beta} = 10^{-2} \) eV. The shape of the heavy contribution contour can also be easily understood from the discussion in Sec. [IV]. The heavy contribution scales as \( \mu'/\Lambda^4 \), following Eq. (17) closely until, for \( \mu' \gg \Lambda \), it becomes independent of \( \Lambda \) in agreement with Eq. (19). In the ISS region, both heavy neutrinos have masses larger than the \( 0\nu\beta\beta \) scale, and Eq. (17) holds. As expected from comparing Eq. (17) and Eq. (25), in this region the slope of the heavy contribution contour is twice the \( \sqrt{\Delta m_{\text{sol}}^2} < \delta m_{LL} < 0.58 \text{ eV} \) one.

For \( \mu' \gg \Lambda \), we enter the ESS limit and eventually the lightest of the two heavy masses becomes lighter than the \( 0\nu\beta\beta \) scale (\( \sim 100 \) MeV), while the heaviest one is too heavy to give a relevant contribution to the process. In this region, the “heavy” contribution is thus dominated by the sterile neutrino lighter than 100 MeV, for which the corresponding NME takes the maximum value, and is independent of \( \Lambda \) (see Eq. (19)). This dominant behavior of the sterile neutrino lighter than 100 MeV will be confirmed later in Fig. 4 as we will explain below.

Figure [2] also highlights the region of the parameter space for which the ratio \( r \) between the heavy and mostly active contribution to \( 0\nu\beta\beta \) decay, defined as \( r \equiv \)
\[ |A_{\text{heavy}}/A_{\text{light}}|,\] is between 1 and 5 (green region) or larger than 5 (blue region). The active contribution is determined by the one-loop correction to the light neutrino masses: \[ A_{\text{active}} \propto (\delta m_{LL})_{ee} M^{0\nu\beta\beta}(0). \] From Eqs. (17), (19) and (24), it is clear that \( r \equiv |A_{\text{heavy}}/A_{\text{light}}| \) should be basically independent of the Yukawa couplings.

\[ \begin{array}{c}
\text{FIG. 3: Same as in Fig. 2 for } Y_{1\alpha} = 10^{-4} \text{ (left), } Y_{1\alpha} = 10^{-5} \text{ (center), and } Y_{1\alpha} = 3 \cdot 10^{-6} \text{ (right).}
\end{array} \]

Finally, the information coming from the experiments that constrain the mixing between the active and heavy neutrinos is also included in Fig. 2. The grey region inside the dashed line is excluded at the 95\% C.L. by the constraints on the mixing extracted from weak decays (summarized in \[60\]) and non-unitarity bounds \[42, 43\].

As shown in Fig. 3, it is possible to have a dominant and measurable contribution from the heavy neutrinos to \(0\nu\beta\beta\) decay, keeping the light neutrino masses under control. Ignoring for the sake of discussion the constraints on the heavy mixing, this takes place in the two intersections among the red, the orange, and the blue regions, which lie in two interesting limits already discussed in Secs. III and IV:

- i) **ISS limit**: \( \Lambda \gg \mu', Y_{1\alpha}v \). The heavy neutrinos are quasidegenerate, and their contribution to the process is proportional to the splitting, given by \( \mu' \). Once the constraints on the mixing, \( U_{e4} \sim U_{e5} \sim Y_{1\alpha}v/2\Lambda \), are properly taken into account, the ISS limit is ruled out.

- ii) **ESS limit**: \( \mu' \gg \Lambda, Y_{1\alpha}v \). In this case, the lightest of the extra neutrinos has a mass lower than the neutrinoless double beta decay exchange momentum and dominates the process.
FIG. 4: Region of the parameter space, $\tilde{M}_2 \approx m_5$ vs $\tilde{M}_1 \approx m_4$, in which a dominant and measurable contribution of the heavy neutrinos is feasible, respecting bounds from neutrino oscillations, absolute neutrino mass scale experiments and weak decays. From top to bottom, the blue, cyan, green, yellow and red areas stand for $Y_{1\alpha} = 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5},$ and $3 \cdot 10^{-6}$, respectively. The black lines correspond to $\tilde{M}_1 = 100$ MeV and $\tilde{M}_2 = 100$ MeV.

We have chosen $10^{-3}$ as the input value of $Y_{1\alpha}$ in Fig. 2 as an example that allowed us to illuminate the discussion. The results for $Y_{1\alpha} = 10^{-2} - 10^{-3}$ are similar, but we have checked that for values of the Yukawa couplings larger than $10^{-2}$, a dominant contribution from the heavy neutrinos is not possible and can be at most of the same order as the contribution from light neutrinos. In Fig. 3, we show the plots analogous to Fig. 2 but for smaller values of the Yukawa couplings: $10^{-4}$ (left), $10^{-5}$ (center), and $3 \cdot 10^{-6}$ (right). For these values, the heavy neutrino mixing is small enough to satisfy the bounds coming from weak decays. We observe that the ratio between the light and heavy contributions is independent of the Yukawa couplings as expected. However, each of them separately depends strongly on that input. The region of the parameter space in which we have a measurable heavy contribution decreases with the Yukawa coupling, as is also the case for the red region, in which the
light neutrino masses keep being under control. We have checked that between $10^{-6}$ and $10^{-8}$, a dominant and measurable contribution of the heavy neutrinos may still be possible, but the light neutrino masses generated at one-loop are smaller than $\sqrt{\Delta m_{solar}^2}$. For values of the Yukawa couplings smaller than $10^{-8}$, the heavy contribution is too suppressed to be experimentally accessible.

The information given in Figs. 2 and 3 is summarized in Fig. 4 where we show the region of the parameter space in which a dominant and measurable contribution of the heavy neutrinos is possible, respecting at the same time the bounds on heavy mixing from weak decays and non-unitarity [42, 43, 60], and keeping light neutrino masses in the region between $\sqrt{\Delta m_{solar}^2}$ and their upper bound extracted from Ref. [50]. Although the tree-level cancellation for the light neutrino masses is taking place, once the one-loop corrections are taken into account, a dominant contribution from the heavy neutrinos cannot occur for larger or smaller values of the Yukawa couplings than the ones shown in Fig. 4. Notice that this dominant contribution is mainly possible only in the hierarchical seesaw scenario mentioned above ($|\tilde{M}_1| \lesssim 100 \text{ MeV} \ll |\tilde{M}_2|$), where the lightest sterile neutrino gets a mass smaller than (or around) 100 MeV and dominates the process. Indeed, this result is not surprising: in Ref. [14] it was shown that, in the case in which the cancellation of the light neutrino contribution does not occur, a hierarchical heavy spectrum like this is necessary in order to have a relevant contribution from the heavy neutrinos at tree-level. We have checked in this work that this conclusion, obtained at tree-level, can be extended to the case in which a cancellation of the light contribution takes place at tree-level if the one-loop level corrections are included in the analysis. Nevertheless, there is an exception to these conclusions. For $Y_{1\alpha} \approx 10^{-4} - 10^{-5}$, there is still a tiny region in which the heavy contribution could dominate when the heavy neutrinos are quasidegenerate and around 5 GeV (ISS region).

A comment is in order: the qualitative conclusions just depicted above are not affected significantly if the lower bound on the one-loop light neutrino masses imposed here ($\delta m_{LL} > \sqrt{\Delta m_{solar}^2}$) is not assumed in the analysis. In such a case, the allowed regions in Fig. 4 become a bit larger (vertically), and a dominant heavy neutrino contribution would be still possible for $Y_{1\alpha} = 10^{-6} - 10^{-8}$. Also in this case, a hierarchical spectrum with $|\tilde{M}_1| \lesssim 100 \text{ MeV} \ll |\tilde{M}_2|$ is required, with the possible exception of having a quasidegenerate spectrum with $|\tilde{M}_1| \sim |\tilde{M}_2| \sim 5 \text{ GeV}$ (in the same tiny region of the parameter space). This can be easily understood from Figs. 2, 3, eliminating the lower bound on $\delta m_{LL}$ would mean that the outer
region of the red bands would not be forbidden any more.

Finally, it should be remarked that the results presented in this section are not modified if a different upper bound on the light neutrino masses from the one used in our analysis \((m_\nu = 0.58 \text{ eV})\) is considered. Modifying this upper bound would be reflected in a slight modification of the inner boundary of the red bands in Figs. 2, 3, which have a marginal impact on the final results. (The intersection among the different contours is not affected.) In particular, we have checked that the conclusions drawn here remain valid if an upper bound from cosmology of \(m_\nu = 0.36 \text{ eV}\) is considered instead in the analysis.

**VIII. SUMMARY AND CONCLUSIONS**

The possibility of having a dominant contribution from heavy neutrinos to \(0\nu\beta\beta\) decay, when a cancellation of the tree-level light neutrino contribution takes place, has recently received much attention. In this work we have carefully analyzed this possibility in the general framework of type-I seesaw models. We have considered a general parameterization of the neutrino mass matrix which allowed us to explore the whole parameter space, identifying particularly interesting limits such as the inverse and extended seesaw models. We have shown which conditions have to be satisfied for a stable cancellation of the tree-level light neutrino contribution, allowing the heavy neutrinos to dominate the process at tree-level. We have studied the relevant corrections that may arise in this context. The finite one-loop corrections to the light neutrino masses turn out to be very relevant. Although logarithmic, their contribution to the \(0\nu\beta\beta\) decay rate tends to dominate very easily. We have found that the heavy neutrinos can give the main contribution to the process only for a very hierarchical heavy neutrino spectrum with masses below and above the \(0\nu\beta\beta\) scale \(\sim 100 \text{ MeV}\), which would match an extended seesaw like model. The “heavy” neutrino contribution is in fact completely dominated by the lightest sterile neutrinos with mass \(\lesssim 100 \text{ MeV}\), which is not suppressed by the NME. This result coincides with the general conclusions of the tree-level analysis performed when no cancellation takes place. Quantitatively, we have obtained that values of the Yukawa couplings between \(10^{-2}\) and \(10^{-6}\) \(\left(10^{-8}\right)\) are necessary, if a lower bound on the one-loop neutrino masses of \(\sqrt{\delta m^2_{\text{sol}}}\) (no lower bound) is imposed in the analysis. We qualitatively agree with part of the conclusions drawn in Ref. 16: the extended seesaw scenario might accommodate a relevant “heavy” neutrino
contribution. Nevertheless, our general conclusions clarify an important detail: in Ref. [16] it was hypothesized that this may happen with all the heavy neutrinos above the $0\nu\beta\beta$ decay scale while we conclude that the heavy spectrum needs to contain states in both regimes, below (or close to) and above 100 MeV.

An interesting exception arises for quasidegenerate heavy neutrinos with masses around 5 GeV which may give the dominant contribution in a tiny region of the parameter space for Yukawa couplings in the range $10^{-4} - 10^{-5}$. In agreement with Refs. [15, 16], we confirm that a relevant contribution to the $0\nu\beta\beta$ decay may come from a seesaw scenario with a quasidegenerate heavy neutrino spectrum, which corresponds to an inverse seesaw like model. However, we also show that this possibility is rather unlikely, since it can only take place in a very particular and small region of the parameter space.

Our results can be understood from the point of view of lepton number conservation. Even if the light neutrino contribution cancels out at tree-level, in order to have a measurable heavy contribution an important violation of the lepton number should be introduced through the heavy sector. This violation of the lepton number may not be reflected in the tree level light neutrino masses but appears naturally at the one-loop level, making more difficult a dominant heavy contribution.

Finally, we should remark that our analysis was performed considering two fermion singlets, and only one light neutrino mass was generated at one loop. In order to generate the two light neutrino masses required to explain neutrino oscillations, one has two options: (i) not considering a complete cancellation for the tree-level light neutrino masses, being only partial ($\mu$ and/or $\epsilon$ small parameters but different from zero), or (ii) adding more fermion singlets to the model. In case (i), tree-level light neutrino masses are generated. They might be very small, but the seesaw constraint given by Eq. (28) would leave the light active neutrinos as the dominant mechanism in $0\nu\beta\beta$ decay, at least if a fine-tuned cancellation between the tree-level and one-loop neutrino contribution is not invoked. Again, in this scenario a dominant contribution from the heavy neutrinos can be expected mainly for $|\tilde{M}_1| \lesssim 100$ MeV $\ll |\tilde{M}_2|$.

In case (ii), adding an even number of sterile neutrinos with the opposite lepton number ($n = n'$) would allow a complete cancellation for the light tree-level neutrino masses, generating at the same time two or more light neutrino masses at one loop. This kind of model was studied recently in Ref. [62], where six sterile neutrinos were considered ($n = n' = 3$). These models match the ISS limit studied here,
but obviously the number of free parameters is larger ($\mu'$ and $\Lambda$ are $n' \times n'$ matrices, and $Y_1$ is a $n' \times 3$ matrix). The lepton number violation, required to have a relevant heavy neutrino contribution, generates at the same time light neutrino masses at one loop and the latter will typically dominate in $0\nu\beta\beta$ decay, independently from the number of generations considered. Relevant exceptions are the cases highlighted in our study or possible further fine-tuned cancellations among the contributions due to different generations. Therefore, we expect similar results to the ones presented in this work, although a detailed analysis beyond the scope of this work would be necessary in order to take into account the just-mentioned fine-tuned cancellations and show the constraints in the corresponding larger parameter space.

**Note**

During the completion of this work an analysis which considered the generation of neutrino masses at the loop-level in inverse seesaw models was presented in Ref. [62].

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