Development of a Statistical Model for the Effects of Air Blast Pressure, Melting Time and Fuel Consumed on Iron Melting Rate of an Erythrophleum Suave Lens Charcoal-Fired Cupola Furnace

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Abstract

This work was designed to investigate the effects of air blast pressure, melting time and fuel consumed on iron melting rate of an \textit{erythrophleum suaveolens} charcoal-fired cupola furnace. A regression model was formulated and the model validation established the existence of statistically significant relationships between the control variables and iron melting rate. By using the experimental data, $R^2$ value of 99.8 \% was obtained, thus confirming that the model is fit since 99.8 \% of the variation in melting rate could be explained by the control variables. The coefficients $b_{0ec}$, $b_{1ec}$, $b_{2ec}$ and $b_{3ec}$ are -434.656, 423.864, 1.188 and 1.858 respectively; and the output of the $t$-test showed that regression coefficients $b_{1ec}$, $b_{2ec}$ and $b_{3ec}$ were not equal to zero (table $t$-value = 2.201), hence they are statistically significant. The regression model developed in this work as explained by the model analysis can estimate the melting rate as a function of the control variables and form a basis for developing a computer software which may help energy and foundry managers to significantly monitor and improve on the melting rate of cupola furnace.

Key words: Melting rate, air blast pressure, melting time, fuel consumed, \textit{erythrophleum suaveolens} charcoal, cupola furnace, regression model.

1. Introduction

The efficient use of energy of any foundry largely depends on the efficiency of the melting process – a multi-step operation where the metal is heated, treated, alloyed, and transported into die or mold cavities to form a casting. The control of quality, composition, and the physical and chemical properties of the final product are melting process dependent just as the energy consumption and cost-effectiveness of producing the castings are equally accounted for by the melting process [1].

Mathematical models on the problems of choosing the optimum input parameters have been developed by several investigators. [2], developed the first mathematical model between carbon content in the charge and that of the tapped metal in cupola operations. [3], developed the first thermo chemical model for predicting cupola performance under various operating conditions. In prediction, function approximation and classification, artificial neural networks (ANN) are useful tools. It is equally useful in obtaining information from inconsistent and non-linear data. [4], modeled cupola furnace parameters with about 5\% error using ANN. For forecasting in multiple disciplines, regression techniques have a long history of use as tools.
[5], also developed for urban areas a practical crash prediction regression models for assessing the long-range safety impact of alternative freeway networks.

The selected properties of coal mixture and coke reactivity were statistically analysed for tightness and characteristics of the relationship between them [6]. Software Statgraphic using both simple linear regression and multiple linear regressions was used for the calculations. [7], forecasted and tracked furnace flame temperature selecting the most appropriate inputs that affect this process parameter by using artificial neural network (ANN), multiple linear regression (MLR), and autoregressive integrated moving average (ARIMA) models.

The aim of accurate data-driven modeling is to use machine learning techniques to tackle real-world problems [8] and it is popular for solving a variety of industry applications such as blast furnace iron making process [9] and laser welding process [10], [11]. Nonlinear statistical models such as kernel principal component analysis (KPCA), Kernel Fisher discriminant analysis (KFA), artificial neural network (ANN) and support vector machines (SVMs) have more powerful modeling ability than simple linear models in practical applications such as monitoring of complicated chemical plants and fault diagnosis by virtue of their strong approximation ability [12, 13]. However, lack of transparency and comprehensibility is a major limitation of such kinds of nonlinear models [14]. In other words, the understanding and interpretation of these nonlinear models are humanly difficult.

Hence, for simplicity and ease of interpretation multiple linear regression modeling method was adopted in this work. Statistical Package of Social Sciences (SPSS version 16.0) was employed to determine the coefficient of determination ($R^2$), which is the proportion of variation in the dependent variable explained by the regression model. The values range from 0 to 1; the value below 0.5 indicates that the model may not fit the data well [15]. Analysis of Variance (ANOVA), the sum of squares, degrees of freedom, and mean square are used to determine whether the model accounts for most of variation in the dependent variable or not [15]. The $F$-statistic helps in determining whether the independent variables are significant in explaining the variation in the dependent variable [16]. $T$ statistic helps to determine the relative importance of each variable in the model. The impression is favourable when $T$ values are below −2 or above +2. Co-linearity is the undesirable situation where the correlations among the independent variables are strong. In co-linearity diagnostics, a condition index greater than 15 indicates a possible problem and an index greater than 30 suggests a serious problem with multi-co-linearity [16].

This paper presents a model based on statistical multiple linear regression method to evaluate the effects of air blast pressure, melting time and fuel consumed on the iron melting rate of an *erythrophleum suaveolens* charcoal-fired cupola furnace. This work will provide input data for improving iron melting rate of the furnace by identifying, validating, and controlling independent variables (air blast pressure, melting time and fuel consumed) that are statistically significant in influencing iron melting rate.

2. Methodology

It is assumed that connections do exist between iron melting rate and other control variables relating to melting process. The development of statistical models like linear regression can serve as a means to verify the presence of relationships between interacting variables [17]. Therefore, two major steps are involved in the technical approach adopted. Firstly, multiple linear regression models were generated to represent expected relationships between iron melting rate and the independent variables (air blast pressure, melting time and fuel consumed).
Secondly, these models validity and adequacy were tested using statistical tools such as Coefficient of determination, $R^2$, hypothesis testing and Analysis of Variance (ANOVA).

2.1 Linear Regression

As shown in the linear model given by equation (1), a response variable ($Y$) is related to a set of control variables. The estimation of the parameters such as the intercept and regression coefficients associated with control variables are required when constructing a multiple linear regression model.

$$Y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + \ldots + a_kx_k + \epsilon$$

(1)

Where:
- $Y$ is a linear function of $k$ control variable $x_1, \ldots, x_k$ and $\epsilon$ is an error term.
- $a_0$ is intercept of the model, and $a_1, a_2, a_3, \ldots, a_k$ are regression coefficients associated with control variables $x_1, x_2, x_3$ respectively.

With the coefficients of the regression equation, associating response variable $Y$ with its control variables $x_1, x_2, x_3, \ldots, x_k$, model parameters can be estimated by using sample data as shown in equation 2:

$$Y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + \ldots + b_kx_k$$

(2)

Where; $b_0$ is intercept of the model, $b_1$ is regression coefficient associated with the control variable $x_1$, $b_2$ is regression coefficient associated with the control variable $x_2$ and $b_3$ is regression coefficient associated with the control variable $x_3$.

2.2 Variables Selection

The relationship of a dependent variable, melting rate ($\dot{M}$) with air blast pressure (P), melting time (T) and fuel consumed (F), all independent variables was derived. The units of measurements and definitions of these variables are defined as follows; Blast air pressure is the pressure of the air supplied by the blower in bar; Melting time (T) is the melting time of metal in Minutes.; Fuel Consumed (F) is the fuel consumed in melting the metals in kilogram, and Melting rate ($\dot{M}$) measures in kilogram of metal melted per minute.

The above factors were selected as control variables influencing melting rate of iron based on:

i) The presence of logical influence of these factors on Melting rate. For instance, increase in air pressure increases the velocity of the air in the tuyere and hence the rate of melting of the iron in contact with the solid fuel increases.

ii) It is logical to conclude that melting rate in kg/min would change as the amount of iron melted per unit time changes [18].

Scatter diagrams were used to validate the initial selection of control variables in order to justify the presence of such informative relationships between these factors.

2.3 Models Assumptions

The following statistical assumptions were made:

i. That the relationship between the melting rate and its related control variables was predictable (application of scatter diagrams).

ii. That multi-co-linearity was not present among air pressure, melting and fuel consumed.

iii. That the random errors ($\epsilon$) are independent and normally distributed with constant variance and zero mean.
3. Formulation of Multiple Linear Regression Models (FMLRM)

3.1 Models formulation.
Premised on the model assumptions and selected variables, the following multiple linear regression model was formulated for a cupola furnace using *Erythrophleum Suaveolens* charcoal (Esw) as fuel;

**Model:** for a cupola furnace using Esw as fuel.

\[
\text{Exp}(M/P,T,F) = b_{0\text{esw}} + b_{1\text{esw}} P + b_{2\text{esw}} T + b_{3\text{esw}} F
\]  

Where:

\[
\text{Exp}(M/P,T,F) \text{ is the expected value of melting rate in kg/min. given the independent variables P, T and F, } b_{0\text{esw}} \text{ is intercept of the model, } b_{1\text{esw}} \text{ is regression coefficient associated with air pressure of the model, } b_{2\text{esw}} \text{ is regression coefficient associated with melting time of the model and } b_{3\text{esw}} \text{ is regression coefficient associated with fuel consumed of the model.}
\]

3.2 Hypothesis 1: Testing model validity

**Model hypothesis:** for a cupola furnace using Esw as fuel is presented as in equation 4:

\[
H_0 : \beta_{jesw} = 0, \ j = 1, 2, 3
\]

If \( H_0 \) is rejected, then \( H_1 : \text{at least one } \beta_{jesw} \neq 0 \)  

3.3 Hypothesis II: Individual testing of coefficients of the multiple linear regression models.

**Hypothesis II** for any independent variable is as presented in equation 5.

\[
H_0 : \beta_{1-3\text{esw}} = 0 \ vs \ H_1 : \beta_{1-3\text{esw}} \neq 0 \text{ for the model}
\]

The null hypothesis was based on the assumption that there was no statistically significant relationship between melting rate and any of blast pressure, melting time and fuel consumed.

3.4 Data Collection

The cast iron scraps used in this work as melting objects were sourced from car engine blocks. While conducting the experiment, a predetermined quantity of metal (17 kg) was melted with 3 kg of Es charcoal per charge. In order to separate the slag from the molten iron, each charge was accompanied with 1 kg of limestone. For the purpose of improving the machinability of the cast iron, 1 kg of ferrosilicon was introduced to the charge at an interval before the iron was tapped. While using Es charcoal to melt the charge, the experiment was conducted at different values of air blast pressure of 0.103 and 1.02 bars, The variations of the rate of melting, fuel consumption and melting time with air blast pressure were recorded during the experiment as shown in Table 1.
Table 1: Es charcoal fuel based experiment without oxygen enrichment

| Air blast Pressure (P) (bar) | Melting Time (T) (min.) | Fuel Consumed (F) (Kg) | Melting Rate (\(\dot{M}_{1\text{exp}}\)) (Kg/min.) x 10^1 |
|-----------------------------|------------------------|------------------------|---------------------------------------------|
| 1.03                        | 10                     | 2.46                   | 17.00                                       |
| 1.03                        | 20                     | 5.0                    | 33.78                                       |
| 1.03                        | 30                     | 7.35                   | 51.02                                       |
| 1.03                        | 40                     | 9.86                   | 67.88                                       |
| 1.03                        | 50                     | 11.98                  | 85.03                                       |
| 1.03                        | 60                     | 14.66                  | 101.73                                      |
| 1.02                        | 10                     | 2.4                    | 15.60                                       |
| 1.02                        | 20                     | 4.77                   | 30.98                                       |
| 1.02                        | 30                     | 7.18                   | 46.77                                       |
| 1.02                        | 40                     | 8.77                   | 62.32                                       |
| 1.02                        | 50                     | 11.89                  | 77.50                                       |
| 1.02                        | 60                     | 14.03                  | 93.62                                       |

\[
\dot{M}_{1m} = \frac{1}{n} \sum_{i=1}^{n} M_i = \frac{1}{12} (683.23) = 56.94 \times 10^{-1} \text{kg/min.}
\]

Results of present research

4. Model Analysis and Discussion

The Statistical Package for Social Sciences (SPSS) used for this analysis is version 16.0. It was used to analyse the data obtained in Table 1 and the results are shown in Table 2.

Table 2: Model summary for a cupola furnace using Esw as fuel

| Parameter Value | Regression Sum of squares | Constant (b_{0ec}) | Condition index | Coefficient (\(\hat{b}_{1ec}\)) | VIF | T-Statistic | Predicted value | Mean (\(\mu\)) | Std. Deviation (\(\sigma\)) |
|-----------------|--------------------------|---------------------|-----------------|---------------------------------|-----|-------------|-----------------|----------------|--------------------------|
| R^2             | 0.998                    | 9349.393            | 1.00            | -434.656                        | -   | -3.955      | 56.936          | 29.154         |                          |
| F-Statistic     | 1603.882                 | 15.545              | 4.348           | 423.864                         | 1.778 | 3.950        | Residual 0       | 1.189           |                          |
| Significance of F-statistic | 0.000              | -                   | -               | T (b_{2ec})                      | 1.188 | 2.674        | -               | -                | -                       |

The plotted scatter diagram shown in Fig. 1 clearly indicates the validity of initial selection of variables. A computed \(R^2\) value of 0.998 as shown in Table 2, indicates that the regression was
significant as about 99.8% of the variation in melting rate could be accounted for by the control variables. A computed $F$-statistic value of 1603.882 as indicated in the ANOVA analysis result shown in Table 2 and the corresponding table value of 3.98 at 0.05 level of significance (q) and (2,11) degrees of freedom showed that the multiple linear regression model was significant and valid [10]. The model accounts for most of variation in the dependent variable as indicated by the large regression sum of squares (9349.393) in comparison to the residual sum of squares (15.545). The coefficients $b_{0ec}$, $b_{1ec}$, $b_{2ec}$ and $b_{3ec}$ shown in Table 2 are -434.656, 423.864, 1.188 and 1.858 respectively; and the results of the $t$-test (as given by hypothesis ii) at 0.025 level of significance and 11 degrees of freedom (table t-value=$t_{0.025, 11} = 2.201$) showed that regression coefficients $b_{1ec}$, $b_{2ec}$ and $b_{3ec}$ were statistically significant and not equal to zero [19].

Also in testing hypothesis I, the null hypothesis was rejected and the alternative accepted for all the independent variables since no value of the regression coefficients for all the independent variables ($P$, $T$ and $F$) was equal to zero. Therefore, the regression equation of melting rate of iron in kg/min. can be given by equation 6. The assumptions made were valid for this model with respect to multi co-linearity and residuals' distribution. As seen from Table 2, the condition indexes values of 4.348, 88.772 and 757.291 are for $P$, $T$ and $F$ respectively.

From Table 2 the standard predicted mean was 56.936 (i.e. 5.6936 kg/min) with standard deviation of 29.154 (i.e. 2.915 kg/min) implying that control variables were actually independent. Multi co-linearity was not a problem in this application (i.e. $VIF < 4$) [19], since the computed average variance inflation factor $VIF$ is 1.778 and thus indicates clearly that air pressure; melting time and fuel consumed were not significantly interacting factors.

\[ \text{Exp}(\frac{M_1}{P,T,F}) = -434.656 + 423.864P + 1.188T + 1.858F \]  \hspace{1cm} (6)
Figure 1: Scatter diagram of the model

5. Conclusion
The regression model developed in this work can adequately estimate the melting rate based on Air blast pressure; Melting time and Fuel consumption. The model equation may be used to develop a computer program which will help energy and foundry managers to significantly monitor and improve on the melting rate of cupola furnace which may in turn reduce the energy consumed in iron melting.

6. Recommendation
The model developed in this work is recommended to energy and foundry managers for prediction, monitoring and improving on the melting rate of cupola furnace.

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