Thermodynamics in Higher Dimensional Vaidya Space-Time

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In this work, we have considered the Vaidya spacetime in null radiating fluid with perfect fluid in higher dimension and have found the solution for barotropic fluid. We have shown that the Einstein’s field equations can be obtained from Unified first law i.e., field equations and unified first law are equivalent. The first law of thermodynamics has also been constructed by Unified first law. From this, the variation of entropy function has been derived on the horizon. The variation of entropy function inside the horizon has been derived using Gibb’s law of thermodynamics. So the total variation of entropy function has been constructed at apparent and event horizons both. If we do not assume the first law, then the entropy on the both horizons can be considered by area law and the variation of total entropy has been found at both the horizons. Also the validity of generalized second law (GSL) of thermodynamics has been examined at both apparent and event horizons by using the first law and the area law separately. When we use first law of thermodynamics and Bekenstein-Hawking area law of thermodynamics, the GSL for apparent horizon in any dimensions are satisfied, but the GSL for event horizon can not be satisfied in any dimensions.

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I. INTRODUCTION

The connection between gravity and thermodynamics was discovered by Jacobson[1] by deriving the Einstein equation together with the first law of thermodynamics where the entropy is proportional to the horizon area. The horizon area of black hole is associated with its entropy and the surface gravity which is related with its temperature in black hole thermodynamics [2,3]. Frolov et al[5] calculated the energy flux of a background slow-roll scalar field through the quasi-de Sitter apparent horizon and used the first law of thermodynamics \(dE = TdS\), where \(dE\) is the amount of the energy flow through the apparent horizon. Using the Hawking temperature \(T_A = \frac{1}{2\pi R_A}\) and Bekenstein entropy \(S_A = \frac{\pi R_A^2}{G}\) (\(R_A\) is the radius of apparent horizon) at the apparent horizon (\(G\) is the Newton’s constant), the first law of thermodynamics (on the apparent horizon) is shown to be equivalent to Friedmann equations [6] and from this, the generalized second law (GSL) of thermodynamics is obeyed at the horizon. Gibbons and Hawking [7] first investigated the thermodynamics in de Sitter spacetime. Verlinde [8] found that the Friedmann equation in a radiation dominated FRW universe can be written in an analogous form of the Cardy-Verlinde formula. Padmanabhan [9] formulated the first law of thermodynamics on the horizon, starting from Einstein equations for a general static spherically symmetric space time.

In a spatially flat de Sitter spacetime, the event horizon and the apparent horizon of the Universe coincide and there is only one cosmological horizon for this space time. When the apparent horizon and the event horizon of the Universe are different, it was found that the first law and generalized second law (GSL) of thermodynamics hold on the apparent horizon, while they break down if one considers the event horizon [10]. Considering FRW model of the universe, most studies deal with validity of the generalized second law of thermodynamics starting from the first law when universe is bounded by the apparent horizon [11,13]. In the reference [14], a Chaplygin gas dominated expansion was considered and the GSL was investigated taking into account the existence of the observers event horizon in accelerated expanding universe and from this, it was concluded that for the initial stage of Chaplygin gas dominated expansion, the GSL of gravitational thermodynamics is satisfied.

Most of the cosmological thermodynamics have been studied in open, closed and flat FRW models of the universe. Recently, the generalized second law of thermodynamics (GSL) have been studied for spherically
symmetric LTB model [15] in four and higher dimensional quasi-spherical Szekeres’ model [16]. Unified first law and thermodynamics of dynamical black hole in n-dimensional Vaidya spacetime have been discussed in [17]. Generalized Vaidya spacetime in Lovelock gravity and thermodynamics on the apparent horizon have also been studied [18]. In 1996, Husain [19] gave non-static spherically symmetric solutions of the Einstein equations for a null fluid source with pressure \( p \) and density \( \rho \) related by the equation of state \( p = k\rho \). Wang et al [20] has generalized the Vaidya solution which include most of the known solutions to the Einstein equation such as anti-de-Sitter charged Vaidya solution. Husain solution has been used to study the formation of a black hole with short hair [21] and can be considered as a generalization of Vaidya solution [20].

In this work, we briefly describe the generalization of Vaidya solution (i.e., Husain solution) in \((n+2)\)-dimensional spherically symmetric space-time for a null fluid source with barotropic fluid. Using unified first law, the first law of thermodynamics has been derived in higher dimensional Vaidya spacetime model. The expressions of apparent horizon and event horizon radius have been obtained. The surface gravity and temperature on the apparent and event horizons have been calculated. The validity of GSL of thermodynamics has been examined using first law and area law of thermodynamics separately in presence of barotropic fluid with Vaidya null radiation.

II. BRIEF REVIEW OF \((n+2)\)-DIMENSIONAL VAIHYA SPACE-TIME

The spherically symmetric inhomogeneous metric in \((n+2)\)-dimensional space-time can be taken as [22]

\[
ds^2 = -\left(1 - \frac{m(v, r)}{r^{n-1}}\right)dv^2 + 2dvdr + \frac{r^2}{d\Omega_n^2}
\]

where \( r \) is the radial co-ordinate \((0 < r < \infty)\), \( v \) is the null co-ordinate \((-\infty \leq v \leq \infty)\) which stands for advanced Eddington time co-ordinate, \( m(v, r) \) gives the gravitational mass inside the sphere of radius \( r \) and \( d\Omega_n^2 \) is the line element on a unit \( n \)-sphere. For the matter field, we now consider two non-interacting components of energy momentum tensors namely the Vaidya null radiation and a perfect fluid having form

\[
T_{\mu\nu} = T^{(n)}_{\mu\nu} + T^{(m)}_{\mu\nu}
\]

where

\[
T^{(n)}_{\mu\nu} = \sigma l_{\mu} l_{\nu}
\]

is the component of the matter field which moves along null hypersurface and

\[
T^{(m)}_{\mu\nu} = (\rho + p)(l_{\mu} \eta_{\nu} + l_{\nu} \eta_{\mu}) + pg_{\mu\nu}
\]

represents the energy-momentum tensor of matter. Here \( \rho \) and \( p \) are the energy density and thermodynamic pressure while \( \sigma \) is the energy density corresponding to Vaidya null radiation. In the comoving co-ordinates \((v, r, \theta_1, \theta_2, ..., \theta_n)\), the two eigen vectors of energy-momentum tensor namely \( l_{\mu} \) and \( \eta_{\nu} \) are linearly independent future pointing null vectors having components

\[
l_{\mu} = (1, 0, 0, ..., 0) \quad \text{and} \quad \eta_{\mu} = \left(\frac{1}{2} \left(1 - \frac{m}{r^{n-1}}\right), -1, 0, ..., 0\right)
\]

and they satisfy the following relations

\[
l_{\alpha} l^{\lambda} = \eta_{\lambda} \eta^{\lambda} = 0, \quad l_{\lambda} \eta^{\lambda} = -1
\]

The non-vanishing components of the Einstein’s field equations (choosing \( 8\pi G = c = 1 \))
\[ G_{\mu\nu} = T_{\mu\nu} \]  

(7)

for the metric (1) with matter field having stress-energy tensor given by (2) are obtained as

\[ \rho = \frac{nm'}{2r^n}, \quad p = -\frac{m''}{2r^{n-1}} \quad \text{and} \quad \sigma = \frac{nm}{2r^n} \]  

(8)

where an dot and dash stand for partial derivatives with respect to \( v \) and \( r \) respectively. Now we assume the matter fluid satisfies the barotropic equation of state

\[ p = k\rho, \quad (k < 1, \text{ a constant}) \]  

(9)

From (8) and (9), we obtain the explicit solution for the gravitational mass function \( m(v, r) \) as [22]

\[
m(v, r) = \begin{cases} 
  f(v) - \frac{g(v)}{(nk-1)r^{nk-1}}, & nk \neq 1 \\
  f(v) + g(v) \log r, & nk = 1 
\end{cases}
\]  

(10)

where the integration functions \( f(v) \) and \( g(v) \) are arbitrary functions of \( v \) alone. This generalized Vaidya solution is also known as Husain solution in \((n+2)\)-dimensions. Since \( \dot{m} = \dot{f}(v) + \dot{g}(v) \log r \) becomes negative (i.e., \( \sigma < 0 \)) for very small \( r \), so the energy conditions are not always satisfied for all \( r \) for the solution with \( k = 1/n \). Hence we shall not consider this solution in future, because this is physically unrealistic. Since \( f(v) \) and \( g(v) \) are arbitrary functions of \( v \), so without any loss of generality we may assume [22] \( f(v) = f_0 v^{n-1} \) and \( g(v) = g_0 v^{n(k+1)-2} \), where \( f_0 \) and \( g_0 \) are chosen to be positive.

III. STUDY OF THERMODYNAMICS IN VAIDYA SPACE-TIME

In this section, we shall discuss the unified first law for higher dimensional Vaidya space-time. In this model, the first law of thermodynamics can be generated from unified first law. The validity of generalized second law of thermodynamics will be examined for apparent and event horizons using first law and using area law in the subsequent sections. Now we consider the metric (1) in the double null form [18, 23]

\[ ds^2 = h_{ab}dx^a dx^b + r^2 d\Omega_n^2, \quad a, b = 0, 1 \]  

(11)

where \( h_{ab} = \left( -\left(1 - \frac{m(v, r)}{r^{n-1}} \right), 1, 1, 0 \right) \). The unified first law is defined by [18] the following form

\[ dE = A \Psi + W dV \]  

(12)

where the surface area \( A \) is given by

\[ A = (n + 1)\Omega_{n+1} r^n \]  

(13)

and the volume \( V \) is defined by [2]

\[ V = \Omega_{n+1} r^{n+1} \quad \text{where} \quad \Omega_{n+1} = \frac{\pi^{n+1}}{\Gamma\left(\frac{n+2}{2}\right)} \]  

(14)

From (2), we get the components of energy-momentum tensor as
\[ T_{00} = \sigma + \rho \left( 1 - \frac{m(v, r)}{r^{n-1}} \right), \quad T_{01} = T_{10} = -\rho, \quad T_{11} = 0 \] (15)

The work density function \( W \) is given by

\[ W = -\frac{1}{2} h^{ab} T_{ab} = \rho \] (16)

The energy-supply vector is given by

\[ \Psi_a = h^{bc} T_{ac} \partial_b (r) + W \partial_a (r) = (\sigma, 0) \] (17)

So we have the energy flux or momentum density in the Vaidya spacetime model as

\[ \Psi = \Psi_a dx^a = (\sigma, 0) \] (18)

The Misner-Sharp energy \( E \) inside the Vaidya space-time surface is given by [25]

\[ E = \frac{n(n+1)}{2} \Omega_{n+1} r^{n-1} \left[ 1 - h^{ab} \partial_a (r) \partial_b (r) \right] = \frac{n(n+1)}{2} \Omega_{n+1} m(v, r) \] (19)

Now using (13), (14), (16) and (18), we get

\[ \mathcal{A} \Psi + W dV = (n+1) \Omega_{n+1} r^n (\sigma dv + \rho dr) \] (20)

Taking the total differential of (19), we have

\[ dE = \frac{n(n+1)}{2} \Omega_{n+1} (ndv + m' dr) \] (21)

Using (20), (21) and the unified first law (12), comparing the coefficients of \( dv \) and \( dr \), we obtain

\[ \rho = \frac{nm'}{2r^n} \quad \text{and} \quad \sigma = \frac{n m}{2r^n} \] (22)

which are the two field equations given in (8) for Vaidya space-time. But it is not possible to find the field equation \( p = -\frac{m''}{2r^n} \) directly from the unified first law. If conservation of energy is considered then using (22) we shall get this field equation. So we may conclude the unified first law and the Einstein’s field equations of \((n + 2)\) dimensional Vaidya space-time are equivalent.

Now the Gibb’s law of thermodynamics states that [10]

\[ T_h dS_I = pdV + d(E_I) \] (23)

where, \( S_I, \ p, \ V \) and \( E_I \) are respectively entropy, pressure, volume and internal energy within the horizon of the Vaidya spacetime. Here the expression for internal energy can be written as \( E_I = \rho V \). Since the null radiation moves along null hypersurface, so \( E_I \) does not depend on null radiation density \( \sigma \). Here we consider the equilibrium thermodynamics, so the temperature inside the horizon = temperature on the horizon = \( T_h \).

Using (8) and (14), the above expression (23) can be simplified to the form

\[ \dot{S}_I = \frac{\Omega_{n+1}}{2T_h} \left[ \{nm' - (n + 1)m''r_h\} \dot{r}_h + n\dot{m}r_h \right] \] (24)

In the following two sections, we shall examine the validity of generalized second law (GSL) of thermodynamics of the universe bounded by apparent and event horizons using first law and area law of thermodynamics separately for higher dimensional Vaidya spacetime in presence of barotropic fluid with null radiation.
IV. GSL USING FIRST LAW

We know that heat is one of the form of energy. Therefore, the heat flow $\delta Q$ through the horizon is just the amount of energy crossing it during the time interval $dv$. That is, $\delta Q = -dE$ is the change of the energy inside the horizon. So from equation (21) we have the amount of the energy crossing on the horizon as

$$-dE_h = -(n+1)\Omega_{n+1} r_h^n (\sigma + \dot{r}_h \rho) dv$$

(25)

The first law of thermodynamics (Clausius relation) on the horizon is defined as follows:

$$T_h dS_h = dQ = -dE_h$$

(26)

From these equations, the time variation of entropy on the horizon is given by

$$T_h \dot{S}_h = -(n+1)\Omega_{n+1} r_h^n (\sigma + \dot{r}_h \rho) = -\frac{n(n+1)}{2} \Omega_{n+1} (\tilde{m} + m' \dot{r}_h)$$

(27)

From above result, we may conclude that there is no role of pressure of the perfect fluid on the variation of horizon entropy for Vaidya space-time. Using (8), (24) and (27) we obtain the rate of change of total entropy as

$$\dot{S}_I + \dot{S}_A = \frac{\Omega_{n+1}}{2T_h} \left[ n \tilde{m}' r_h - n(n+1) \tilde{m} - \{n^2 m' + (n+1)m'' r_h\} \dot{r}_h \right]$$

(28)

A. Apparent horizon

The dynamical apparent horizon $r_A$ can be found from $h_{00} = 0$ [13], i.e.,

$$r_A^{n-1} = m(v, r_A)$$

(29)

The rate of change of total entropy (using (28) and (29)) for apparent horizon is

$$\dot{S}_I + \dot{S}_A = \frac{\Omega_{n+1}}{2T_A} \left[ n \tilde{m}' (v, r_A) r_A - n(n+1) \tilde{m}(v, r_A) - \{n^2 m' (v, r_A) + (n+1)m'' (v, r_A) r_A\} \frac{\tilde{m}(v, r_A)}{(n-1) r_A^{n-2}} \right]$$

(30)

If the r.h.s. of the above expression is non-negative, then we can say that the second law for the apparent horizon will be fulfilled. For barotropic EOS, the mass function $m(v, r)$ is defined in equation (10). In particular, we consider dark energy dominated space-time (say, $k = -2/3$), the figure 1 shows the variation of $\dot{S}_I + \dot{S}_A$ against $v$ for some dimensions ($n = 2$ (i.e., 4D), $n = 3$ (i.e., 5D) and $n = 4$ (i.e., 6D)). The graphical representation of above expression shows that the variation of total entropy for apparent horizon is positive oriented. Thus the second law of thermodynamics for apparent horizon is valid for all dimensions when we use the first law of thermodynamics.

B. Event horizon

The radius of the event horizon can be found from the metric (1), i.e. [15], $(ds^2 = 0 = d\Omega_n^2)$

$$-\left(1 - \frac{m(v, r)}{r^{n-1}}\right) dv^2 + 2dvdr = 0$$

(31)
Fig. 1 shows the variation of $\dot{S}_I + \dot{S}_A$ against $v$ from eq.(30) for barotropic fluid with $k = -2/3$, $f_0 = 1$, $g_0 = 1$. The red, green and blue lines denote for $n = 2$ (i.e., 4D), $n = 3$ (i.e., 5D) and $n = 4$ (i.e., 6D) respectively.

or,

$$r_{E}^{n-1} = \frac{m(v, r_{E})}{1 - 2r_{E}}$$

(32)

The rate of change of total entropy (using (28) and (32)) for event horizon can be obtained as

$$\dot{S}_I + \dot{S}_E = \frac{\Omega_{n+1}}{2T_E} \left[ m\dot{m}(v, r_{E}) r_{E} - n(n+1)\dot{m}(v, r_{E}) - \frac{1}{2} \left( 1 - \frac{m(v, r_{E})}{r_{E}^{n-1}} \right) \left\{ n^2 m'(v, r_{E}) + (n+1)m''(v, r_{E})r_{E} \right\} \right]$$

(33)

If the r.h.s. of the above expression is non-negative, then we can say that the second law for the event horizon will be fulfilled. Figure 2 shows the variation of $\dot{S}_I + \dot{S}_E$ against $v$ for some dimensions ($n = 2$ (i.e., 4D), $n = 3$ (i.e., 5D) and $n = 4$ (i.e., 6D)) for dark energy dominated space time (say, $k = -2/3$). The graphical representation of above expression shows that the second law of thermodynamics for event horizon can not be satisfied for all dimensions when we use the first law of thermodynamics.

V. GSL USING AREA LAW

In this section we have not considered the first law of thermodynamics, so we discard equation (27) for horizon entropy. In this case, we shall consider the horizon entropy and horizon temperature which are in the following.

A. Apparent horizon

The surface gravity is defined as [23, 24] follows

$$\kappa = \frac{1}{2\sqrt{-h}} \partial_a (\sqrt{-h} h^{ab} \partial_b (r)) = \frac{(n-1)m - rm'}{2r^n}$$

(34)

Hawking temperature on the apparent horizon is (using (34)) given by
Fig. 2 shows the variation of $\dot{S}_I + \dot{S}_E$ against $v$ from eq.(33) for barotropic fluid with $k = -2/3$, $f_0 = 1$, $g_0 = 1$. The red, green and blue lines denote for $n = 2$ (i.e., 4D), $n = 3$ (i.e., 5D) and $n = 4$ (i.e., 6D) respectively.

\[
T_A = \frac{\kappa}{2\pi} = \frac{(n-1)m - rm'}{4\pi r_A^n} = \frac{n-1}{4\pi r_A} \frac{m'(v, r_A)}{4\pi m(v, r_A)}
\]

(35)

For simple Vaidya space-time model (only null radiation), $\rho = p = 0$ i.e., $m' = 0$, we get the surface gravity on the apparent horizon as

\[
\kappa = \frac{(n-1)m(v, r_A)}{2 r_A^n} = \frac{n-1}{2 r_A}
\]

(36)

and in this case the temperature on the apparent horizon will be

\[
T_A = \frac{n-1}{4\pi r_A}
\]

(37)

The entropy on the apparent horizon can be found from standard Bekenstein-Hawking area law and is given by

\[
S_A = \frac{A}{4} = \frac{1}{4} (n+1) \Omega_{n+1} r_A^n
\]

(38)

Using (24), (29), (35) and (38), the rate of change of total entropy for apparent horizon is obtained as

\[
\dot{S}_I + \dot{S}_A = \frac{\Omega_{n+1}}{2T_A} \left[ \frac{nm'(v, r_A) - (n+1)m''(v, r_A)r_A}{(n-1)r_A^{n-2}} \right] + n m'(v, r_A)r_A + \frac{n(n+1)}{8\pi} m(v, r_A) \left\{ 1 - \frac{rm'(v, r_A)}{(n-1)m(v, r_A)} \right\}
\]

(39)

If the r.h.s. of the above expression is non-negative, then we can say that the second law for the apparent horizon will be fulfilled. Figure 3 shows the variation of $\dot{S}_I + \dot{S}_A$ against $v$ for some dimensions ($n = 2$ (i.e., 4D), $n = 3$ (i.e., 5D) and $n = 4$ (i.e., 6D)) for dark energy dominated space time (say, $k = -2/3$). The graphical representation of above equation shows that the second law of thermodynamics for apparent horizon is satisfied for all dimensions.
Fig. 3 shows the variation of $\dot{S}_I + \dot{S}_A$ against $v$ from eq.(39) for barotropic fluid with $k = -2/3, f_0 = 1, g_0 = 1$. The red, green and blue lines denote for $n = 2$ (i.e., 4D), $n = 3$ (i.e., 5D) and $n = 4$ (i.e., 6D) respectively.

**B. Event horizon**

The temperature on the event horizon is calculated as [24, 27–29]

$$T_E = \frac{(n - 1)r_E^{n-2}(1 - 2\dot{r}_E) - m'}{4\pi m} = \frac{n - 1}{4\pi r_E} - \frac{m'(v, r_E)}{4\pi m(v, r_E)}$$

(40)

The entropy on the event horizon is

$$S_E = A = \frac{1}{4}(n + 1)\Omega_{n+1}r_E^n$$

(41)

Using (24), (32), (40) and (41), the rate of change of total entropy for event horizon is obtained as

$$\dot{S}_I + \dot{S}_E = \frac{\Omega_{n+1}}{2T_E} \left[ \frac{1}{2} \left( 1 - \frac{m(v, r_E)}{r_E^{n-1}} \right) \left\{ nm'(v, r_E) - (n + 1)m''(v, r_E)r_E \right\} + nm'(v, r_E)r_E ight]$$

$$+ \frac{n(n + 1)}{16\pi} \left( r_E^{n-1} - m(v, r_E) \right) \left( \frac{n - 1}{r_E} - \frac{m'(v, r_E)}{m(v, r_E)} \right)$$

(42)

If the r.h.s. of the above expression is non-negative, then we can say that the second law for the event horizon will be fulfilled. Figure 4 shows the variation of $\dot{S}_I + \dot{S}_E$ against $v$ for some dimensions ($n = 2$ (i.e., 4D), $n = 3$ (i.e., 5D) and $n = 4$ (i.e., 6D)) for dark energy dominated space time (say, $k = -2/3$). The graphical representation of above equation shows that the second law of thermodynamics for event horizon can not be satisfied for any dimensions when we use the area law of thermodynamics.

**VI. CONCLUSIONS**

In this work, we have considered the Vaidya spacetime in null radiating fluid with perfect fluid in $(n + 2)$-dimensions and have found the solution for barotropic fluid with equation of state $p = kp$. This generalized
Fig. 4 shows the variation of $S_I + \dot{S}_E$ against $v$ from eq.(42) for barotropic fluid with $k = -2/3$, $f_0 = 1$, $g_0 = 1$. The red, green and blue lines denote for $n = 2$ (i.e., 4D), $n = 3$ (i.e., 5D) and $n = 4$ (i.e., 6D) respectively.

Vaidya solution is also known as Husain solution, which is physically realistic for $k \neq 1/n$. The solution contains two arbitrary functions $f(v)$ and $g(v)$, so without any loss of generality, we have assumed (suitably) $f(v) = f_0 v^{n-1}$ and $g(v) = g_0 v^{n(k+1)-2}$, where $f_0$ and $g_0$ are chosen to be positive. We have shown that the Einstein’s field equations can be obtained from Unified first law i.e., field equations and unified first law are equivalent. The first law of thermodynamics has also been constructed by Unified first law. From this, the variation of entropy function has been derived on the horizon. The variation of entropy function inside the horizon has been derived using Gibb’s law of thermodynamics. So the total variation of entropy function has been constructed at apparent and event horizons both. If we do not assume the first law, then the entropy on the both horizons can be considered by area law and the variation of total entropy has been found at both the horizons. Also the validity of generalized second law (GSL) of thermodynamics has been examined at both apparent and event horizons by using the first law and using the area law separately. When we use first law of thermodynamics, the GSL for apparent horizon in any dimensions is satisfied, but the GSL for event horizon can not be satisfied in any dimensions. For Bekenstein-Hawking area law of thermodynamics, the GSL for apparent horizon is satisfied in all dimensions the GSL for event horizon can not be satisfied in any dimensions.

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