Zero-sound in nuclear matter with the asymmetry parameter $-1 \leq \beta \leq 1$

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Abstract

Results for the zero-sound excitations in the isospin asymmetric nuclear matter are presented for the different parameters of asymmetry $-1 \leq \beta \leq 1$. The polarization operator is constructed within the random phase approximation. We use the Landau-Migdal quasiparticle interaction with the special isospin dependence of the phase volume in the normalization factor $C_0$. In the paper we present zero-sound branches of the dispersion equation solutions in the symmetric, asymmetric and neutron matter. The branches correspond to the different channels of decay of the zero-sound excitation in the nuclear matter or nuclei. In the asymmetric nuclear matter we obtain three branches of solutions on the physical and unphysical sheets of the complex frequency plane, $\omega_{s\tau}(k, \beta)$, $\tau = p, n, np$. We demonstrate how three branches turn into two branches in symmetric nuclear matter ($\beta = 0$) and into one branch in the neutron matter ($\beta = 1$). The solutions are applied to calculation of a strength function of the giant dipole resonance in $^{48}$Ca.

1 Introduction

At the present time a lot of attention is payed to the study of the nuclear matter properties at the different densities, asymmetry parameters, temperature, types of interactions, the particle composition. One of the goal of these investigations is to study of the linear response of the nuclear matter at different conditions [1] and the description of the strength functions of the different excitations in nuclei [2]-[8] to obtain the knowledge about the energies, nature and structure of excitations.

Our main results in this paper concern the different branches of zero-sound excitations in asymmetric nuclear matter. There are a lot of publications describing the different types of the excited collective states and their decays. In paper [2] the three types of excited states are
obtained in the framework of local isospin density approximation approach based on the density energy functional, their contribution to the energy-weighted sum rules in nuclei are evaluated.

In the papers [3, 4] the energies and damping rate of the giant excitations and the corresponding strength functions are considered in the asymmetric nuclear matter and in nuclei at different temperature. The zero-sound dispersion equation was constructed on the basis of the non-Markovian kinetic equation. Two (isoscalar and isovector) complex modes are obtained and a new mode predicted which exist in ANM only. The method of [3, 4] permits the transparent addition of physical processes that describe the resonance damping.

In [5] propagation of the sound modes is considered on the basis of the kinetic theory including collisions, temperature and memory effects. In [6] results for zero sound in nuclear matter obtained in the framework of Landau-Migdal theory, are applied to the giant resonances in nuclei. To calculate the width of resonances and its temperature dependence the developed kinetic theory was used.

The role of the effective nucleon-nucleon interaction in description of the giant resonances in the hot nuclei and dependence of the energies and widths on the temperature are investigated in [7].

In [8] a linear response theory starting from a relativistic kinetic equations is developed within a Quantum-Hadro-Dynamics effective field picture of the hadronic phase of nuclear matter. The dispersion relations are derived, they give the sound phase velocity and the internal structure of the normal collective modes, stable and unstable.

In our paper we investigate zero-sound excitations in the normal cold Fermi-liquid, consisting from the neutrons and protons at the different values of the asymmetry parameter. The dispersion equation for the collective excitations in the Fermi liquid theory [9, 10] is considered. In this equation we pay a special attention to the analytical structure of the polarization operators which contain the logarithmic functions. The cuts related to the logarithms are formed by the free particle-hole \( ph \) pairs. The stable collective solutions are placed on the right of cuts on the complex frequency plane at small wave vectors \( k \). When \( k \) increasing there is an overlapping of collective and \( ph \) modes. We look for solutions under the logarithmic cuts on the nearest unphysical sheet at these \( k \) and obtain a complex solutions placed on the unphysical sheet. The imaginary part of solutions is interpreted as a width of excitation appeared due to admixture of the free \( ph \) pairs to the collective modes.

Our approach was tested in the symmetric nuclear matter (SNM) where the branches of the nucleon zero-sound, isobar zero-sound and pion excitations were calculated [11, 12]. In [12] we considered the relation of the branches of zero-sound excitations in SNM to the energies and widths of the dipole giant resonances (GDR) in nuclei.
An isospin asymmetric nuclear matter is characterized by the density of the neutrons $\rho_n$ and protons $\rho_p$. An asymmetry parameter is determined as

$$\beta = (\rho_n - \rho_p)/(\rho_n + \rho_p).$$  

(1)

The Fermi-momentum for the protons and neutrons are

$$p_{Fn} = \left(\frac{3\pi^2}{2} (1 + \beta) \rho\right)^{1/3}, \quad p_{Fp} = \left(\frac{3\pi^2}{2} (1 - \beta) \rho\right)^{1/3}, \quad p_0 = \left(\frac{3\pi^2}{2} \rho_0\right)^{1/3}.$$  

(2)

where $\rho_0 = 0.17 \text{ fm}^{-3}$. We consider the zero-sound excitations with quantum numbers $L^\pi = 1^-$ in the asymmetric nuclear matter. They were investigated in paper [9] using the Fermi-liquid theory many years ago.

The external field $V_0^\tau(\omega, k)$ generates in the system the effective field $V_{\text{eff}}^{\tau_1\tau_2}(\omega, k)$. The system of equations for $V_{\text{eff}}^{\tau_1\tau_2}$ were written and studied in [9]. We investigate the solutions of the dispersion equation of this system. The frequencies and widths of the solutions in ANM are studied as the functions of the momentum $k$ and symmetry parameter $\beta$ at density $\rho = \rho_0$, and the effective Landau-Migdal quasiparticle interaction.

The effective fields $V_{\text{eff}}^{\tau_1\tau_2}(\omega, k)$ describe the two types of excitations, generated in the system by the external field: noninteracting (free) particle-hole pairs ($ph$-mode) $\omega_{ph}^\tau(k)$ and the zero sound collective mode $\omega_s^\tau(k)$.

In SNM at small $k$ the collective solutions $\omega_s(k)$ are real, but when $k$ increasing there is an overlapping of the collective and $ph$-modes. We obtain the solution in the region of overlapping using the analytical structure of polarization operator. The polarization operator contains the logarithmic functions and the logarithmic singularities related to them. We find that the branches $\omega_s(k)$, in the area of overlapping, are placed on the unphysical sheet neighboring with the physical one on the Riemann surface of the logarithm in polarization operator. The imaginary part $\text{Im}\omega_s(k)$ we interpret as the damping of the collective mode due to admixture to the free $ph$ excitations. The analogous process in the nuclei following by nucleon escape is characterize by the escape width. In SNM the isospin of the free $ph$ pairs and of the emitted nucleon is not determined in this approach.

In SNM we obtain one more branch of the collective solutions (we denote it $\omega_{s1}(k)$). It is placed on the unphysical sheet and starts at some $k = k_c$. The feature of this branch is its decay on the $ph$ pairs of the one isospin. In SNM it may be the proton or the neutron $ph$ pairs, but not the mixture. This branch is an important element of the relation of the solutions in SNM to the ones in ANM.

In ANM we consider three possible mechanisms of damping of the collective excitations, three decay channels. We consider separately the overlapping of the collective mode with 1) the free
neutron \( ph \)-pairs, 2) the free proton \( ph \)-pairs and 3) with the both the proton and neutron pairs. We obtain three different branches of solutions: \( \omega_{sn}(k, \beta) \) in the case 1; \( \omega_{sp}(k, \beta) \) when the overlapping with the proton \( ph \)-pairs is considered (case 2) and \( \omega_{np}(k, \beta) \) in the case 3. These branches are calculated on the different unphysical sheets (Sect. 3).

When we investigate the zero sound dispersion equations we can obtain several branches of the collective solutions (especially in the problem with pion dispersion equation \([11]\)). The question appears: which of the branches can be compared with the real physical excitations, stable or unstable? We consider as the physical sheet of the complex \( \omega \)-plane that one where the real collective solutions and the energies of the free \( ph \)-pairs are placed on the real axis. These solutions describe the free \( ph \) mode and the stable collective modes in the matter and we name them 'the physical solutions'. The solutions at the other parameters (the other \( k \), the densities, the asymmetry parameters, and so on) we consider as physical ones if it is possible to do the analytical continuation over the necessary parameter from the stable solutions to the point of the interest \([13]\). The analytical continuation can be made to the physical or unphysical sheets. This permits us to know the nature of the every branch and control how they behave themselves with changing of \( k \) and \( \beta \). When we consider the physical values, for example, the strength functions of the excitations in nucleus A (in Sect. 6), we take into account all physical solutions obtained at given \( k_A \).

It is important to note (Sect.3) that at \( \beta > 0 \) the stable solutions belong to the branches \( \omega_{sn}(k, \beta) \), at \( \beta = 0 \) they relate to \( \omega_s(k) \) and at \( \beta < 0 \) to \( \omega_{sp}(k, \beta) \). It means that at \( \beta > 0 \) the stable solutions begin to damp with \( k \) increasing due to the mixture with the neutron free pairs \( \omega_{ph}^n(k) \), at \( \beta < 0 \) due to mixture with \( \omega_{ph}^p(k) \) and at \( \beta = 0 \) the mixture with the both the proton and neutron pairs. Technically it is expressed in that at \( \beta > 0 \) the real solutions go with \( k \) increasing under the logarithmic cut of the neutron polarization operator and we name them \( \omega_{sn}(k) \). At \( \beta < 0 \) the real solutions go under the logarithmic cut of proton polarization operator. This is the branch \( \omega_{sp}(k) \). At \( \beta = 0 \) the real solutions go under the logarithmic cut of the both proton and neutron polarization operators. This is the branch \( \omega_s(k) \).

In our investigation there are four special values of the wave vector: \( k^p, k_t, k^{np}, k_c \). At small \( \beta \) we obtain \( k^p < k_t < k^{np} < k_c \). The character of branches and transitions to SNM and the neutron nuclear matter (NNM) depends of the interval where \( k \) is considered. The wave vectors \( k_t, k^p, k^{np} \) are functions on \( \beta (k_t(\beta) \) and so on) but \( k_c \) is determined at \( \beta = 0 \).

\[1\] In ANM the branches \( \omega_{si}(k, \beta), i = n, p, np \) depend on \( k \) and \( \beta \) but when it is not important we omit \( \beta \): \( \omega_{si}(k) \).
1.1 The relation of $\omega_s n(k), \omega_s p(k), \omega_s np(k)$ to the zero sound branches in SNM

A special study is devoted to the behavior of the branches at $\beta \to 0$. Here we need to define $k_t(\beta)$: this is the value of $k$ when the branch goes under the cut and the real branch becomes complex. Studying the pass from ANM to SNM we have to consider separately the part of $\omega_s(k)$ at small $k$ ($k < k_t$) when $\omega_s(k)$ is real and the part at large $k$ ($k > k_t$) when $\omega_s(k)$ is complex.

At $k < k_t$ the real solutions smoothly changes at $-1 \leq \beta \leq 1$. At $\beta = 0$ we attribute them to $\omega_s(k)$, at $\beta > 0$ attribute to $\omega_s n(k)$ and to $\omega_s p(k)$ at $\beta < 0$.

At $k > k_t$ a branch $\omega_s np(k)$ appears. It is a continuation of the complex part of $\omega_s(k)$ to $\beta \neq 0$. The branch $\omega_s(k)$ is a limit of a set of $\omega_s np(k, \beta)$ at $\beta \to 0$.

In the limit $\beta \to 0$ the branches $\omega_s n(k)$ and $\omega_s p(k)$ seem to come together and turn into the same branch. But our investigations demonstrate that the limit depends on $k$ and is related to $k_c, k_t$ and $\omega_s 1(k)$. At $k < k_t$ this is really true. At $k > k_c$ the branches $\omega_s n(k)$ and $\omega_s p(k)$ transit to $\omega_s 1(k)$ from the different unphysical sheets. But the limit $\beta \to 0$ does not exist at $k_t < k < k_c$ (Sect.5).

1.2 The relation of $\omega_s n(k), \omega_s p(k), \omega_s np(k)$ to the zero sound branches in the NNM

We do not consider NNM as a $\beta$-stable nuclear matter, but the matter consisting on the neutrons only. Passing to the neutron matter the density of the proton states disappears at $\beta \to 1$. We must include this fact in calculation of solutions. We can do this redefining the density of states in the normalization factor of the effective interaction. The density of states $N = d\rho/d\varepsilon_F$ is used in Eq. (3), it defines the factor $C_0$: $C_0 = N^{-1} = \frac{\pi^2}{p_0 m_0}$. We can define $C_0$ through the density of states with the isospin of the $ph$ pairs before and after the interaction.

$$C_{01,02} = (N_1 N_2)^{1/2} = \frac{\pi^2}{m_0 (pF_1 pF_2)^{1/2}}.$$  \hspace{1cm} (3)

We repeat the reducing of the dispersion equation [9] with $C_{01,02}$. In SNM the results do not changed. But in ANM when $\beta \to 1$ we have $pF_p \to 0$ and we obtain that $\omega_s p(k) \to 0, \omega_s np(k) \to 0$. Note, that $\omega_s p(k)$ and $\omega_s np(k)$ are the complex functions and both the real and imaginary parts go to zero. So in the neutron matter only the branch $\omega_s n(k)$ survives and $\omega_s p(k), \omega_s np(k)$ disappear. We demonstrate this in Sect.5.

In Sect. 2 we recall the system of equations for the dipole collective and non-interacting particle-hole excitations in the nuclear matter [9]. In Sect. 3 we discuss the location of the solutions on
the complex $\omega$-plane in SNM and ANM. In Sect. 4 we present the branches of solutions at the different $\beta, \rho = \rho_0$. In Sect. 5, 6 we consider the behavior of branches at $\beta \to 1$ and $\beta \to 0$. In Sect. 7 we present as example how to calculate the strength function of the GDR in $^{48}Ca$ using the zero sound branches calculated in the nuclear matter with $\beta$ corresponding to the $^{48}Ca$ and $k$ defined for this nucleus by Steinwedel and Jensen model \cite{16}.

2 Dispersion equation

We consider the isovector zero-sound excitations in the asymmetric nuclear matter. The equations for the effective fields which are the response of ANM to the E1 external field are written in paper \cite{9}:

$$
V_{\text{eff}}^{pp} = V^p_0 + F^{pp} A^p V_{\text{eff}}^{pp} + F^{pn} A^n V_{\text{eff}}^{np},
$$

$$
V_{\text{eff}}^{pn} = F^{pp} A^p V_{\text{eff}}^{pn} + F^{pn} A^n V_{\text{eff}}^{nn},
$$

$$
V_{\text{eff}}^{nn} = V^n_0 + F^{nn} A^n V_{\text{eff}}^{nn} + F^{np} A^n V_{\text{eff}}^{np},
$$

$$
V_{\text{eff}}^{np} = F^{np} A^p V_{\text{eff}}^{pp} + F^{nn} A^n V_{\text{eff}}^{np}.
$$

(4)

$V_{\text{eff}}^{pp}$ ($V_{\text{eff}}^{pn}$) is a component of the effective field which the proton feels when the external $V_0$ interact with proton (neutron). We define $V_{\text{eff}}^{np}$ ($V_{\text{eff}}^{nn}$) by analogy. $A^p, A^n$ are the integrals over the loops of the proton and the neutron particle-hole excitations. They are:

$$
A^p = A^p(\omega, k) + A^p(-\omega, k), \quad A^n = A^n(\omega, k) + A^n(-\omega, k).
$$

(5)

We use the effective Landau-Migdal interaction between the quasiparticles \cite{10}

$$
\mathcal{F}(\sigma_1, \bar{\tau}_1; \sigma_2, \bar{\tau}_2) = C_0 (F + F'(\bar{\tau}_1 \bar{\tau}_2) + G(\sigma_1 \bar{\sigma}_2) + G'(\bar{\tau}_1 \bar{\tau}_2)(\bar{\tau}_1 \bar{\tau}_2)),
$$

(6)

where $\sigma, \bar{\tau}$ are the Pauli matrices in the spin and isospin spaces. $C_0 = N^{-1} = \frac{\pi^2}{p_0 m_0}$ where $N$ is the density of states of one sort of particles, $m_0 = 0.94$ GeV. In \cite{10} the effective interaction between the similar (different) particles are $F^{nn}$, and $F^{pp}$ ($F^{np}$, $F^{pn}$). They are related to the constants in \cite{6} by

$$
F^{pp} = F^{nn} = C_0 (F + F'), \quad F^{pn} = F^{np} = C_0 (F - F').
$$

(7)

The matrix \cite{6} we rewrite as

$$
V_{\text{eff}} = V_0 + \mathcal{M}V_{\text{eff}}.
$$

(8)
Here $V_{\text{eff}}$ is a column, consisting from $V_{\text{eff}}^l, l = pp, pn, np, nn$. The matrix $\mathcal{M}$ is constructed from the elements $F^{pp} A^p, F^{pn} A^n$ and so on in (1). Reversing the matrix $(1 - \mathcal{M})$ we obtain for $V_{\text{eff}}$:

$$
V_{\text{eff}} = (1 - \mathcal{M})^{-1} V_0 = \frac{\tilde{\mathcal{M}}}{\det(1 - \mathcal{M})} V_0,
$$

(9)

$\tilde{\mathcal{M}}$ is the adjoint matrix. We study the frequencies of the collective particle-hole excitations which correspond to the solutions to the dispersion equation $\det(1 - \mathcal{M}) = 0$. This equation can be expressed as the product of two factors. For the dispersion equation we choose that one which has the correct limit in the symmetric nuclear matter.

$$
E(\omega, k) = 1 - C_0(F + F')A^p - C_0(F + F')A^n + 4FF'C_0^2A^pA^n = 0.
$$

(10)

We see that the isoscalar and isovector interactions contribute (in this paper we consider the case $F = 0$).

After integration over $ph$-loops we obtain for $A^\tau(\omega, k)$ the expression in the form of the Migdal function [9] :

$$
A^\tau(\omega, k) = \frac{1}{4\pi^2} \frac{m^3}{k^3} \left[ \frac{a^2 - b^2}{2} \ln \left( \frac{a + b}{a - b} \right) - ab \right]
$$

(11)

where $a = \omega - (\frac{k^2}{2m}), b = \frac{kp\varepsilon}{m}$. If we do not take into account the quasiparticle interaction then $V_{\text{eff}} = V_0$ and only the noninteracted (free) $ph$-pairs are excited. Inclusion of the interaction adds the collective mode to the excitations in the matter.

### 2.1 Normalization of the effective interaction Eq.(6)

We investigate the solutions of (10) in the asymmetric nuclear matter. A special attention is paid to the study of solutions at $\beta \to 0$ and $\beta \to 1$. When we go to SNM we can show a correspondence between the branches in SNM and ANM. But in the neutron matter the density of the proton states goes to zero and we expect the disappearance of the branches $\omega_{sp}(k)$ and $\omega_{snp}(k)$.

To include the dependence on the density of states we change the normalization of the effective quasiparticle interactions in the particle-hole channel by the following way: $C_0 \rightarrow (C_0_1C_0_2)^{1/2} = (N_1 N_2)^{-1/2}$ Eq.(3). We have included the density of states with the isospin of the $ph$ pairs before and after the interaction. Then

$$
F^{pp} = C_{0p}(F + F'), \quad F^{nn} = C_{0n}(F + F'),
$$

(12)

$$
F^{pn} = F^{np} = (C_{0p}C_{0n})^{1/2}(F - F').
$$

Here $C_{0p} = N^{-1}_p = \frac{x^2}{m_0pp}, C_{0n} = N^{-1}_n = \frac{x^2}{m_0pp_n}$. We repeat the reducing of the dispersion equation (10) with the redefined interaction (7) and obtain the following equation

$$
E(\omega, k) = 1 - C_{0p}(F + F')A^p - C_{0n}(F + F')A^n + 4FF'C_{0p}C_{0n}A^pA^n = 0.
$$

(13)
3 Location of solutions to Eqs. (10), (13)

In SNM we recall the main results on the energies of the particle-hole excitations \[14, 15\] and show the location of these results on the complex \(\omega\)-plane. Then we extend our consideration to ANM.

3.1 SNM

The system of equations (4) has two sorts of solutions corresponding to two sorts of excitations in nuclear matter: a set of the non-interacting particle-hole pairs \(\omega_{ph}(k)\) and the collective excitation \(\omega_s(k)\). At the beginning we consider the solution of Eq. (10) in the symmetric nuclear matter. In this case \(A_p(\omega, k) = A_n(\omega, k) = A(\omega, k)\), \(A = A(\omega, k) + A(-\omega, k)\). The dispersion equation (10) has the form:

\[
1 - 2 C_0 F' A = 0. \tag{14}
\]

This equation has the solutions corresponding to the collective excitations \(\omega_s(k)\).

The excitation energies of the free noninteracting pairs are \(p < p_F\) and \(|\vec{p} + \vec{k}| \geq p_F\)

\[
\omega_{ph}(k) = \varepsilon_{\vec{p} + \vec{k}} - \varepsilon_p, \quad \varepsilon_q = q^2/(2m). \tag{15}
\]

We show the part of the energies of the free \(ph\)-pairs in Fig.1 (left) as the dashed area. In SNM we can not distinguish the excitation of the neutron particle-hole pairs and the proton ones.

The branch of the collective excitations \(\omega_s(k)\) is shown by the solid curve in a schematic Fig. 1. We see that the collective excitation lies above the dashed area for the small \(k\). At a definite \(k = k_t\) there is an overlapping of two modes. The solutions \(\omega_s(k)\) become complex because of the damping of the collective mode due to admixture of \(ph\)-mode. In Fig.1 (left) at \(k > k_t\) only the real part Re \(\omega_s(k)\) is shown at the axis \(\omega > 0\) (Im \(\omega_s(k)\) is not shown).

We can consider \(\omega_s(k)\) and \(\omega_{ph}(k)\) from another point of view on the complex \(\omega\)-plane, Fig.1 (right). The function \(A\) (5) has the logarithmic cuts. At a fixed \(k\) the cut of \(A(\omega, k)\) is shown in Fig.1 (right) by the lines (1,1'). The cut of \(A(-\omega, k)\) correspond to the line (2,2').

\[
(1,1') : -\frac{kp_F}{m} + \frac{k^2}{2m} \leq \omega \leq \frac{kp_F}{m} + \frac{k^2}{2m}, \quad (2,2') : -\frac{kp_F}{m} - \frac{k^2}{2m} \leq \omega \leq \frac{kp_F}{m} - \frac{k^2}{2m}. \tag{16}
\]

The point of the cut are formed by the energies of the free \(ph\)-pairs \(\omega_{ph}\).

The boundaries of \(\omega\) in (16) at \(\omega > 0\) are the boundary of the dashed area in Fig.1 (left). At a fixed \(k\) the vertical line in the dashed area in Fig.1 (left) corresponds to the cut (1-1') in Fig.1 (right). The every point of the solid curve in Fig.1 (left) at \(k < k_t\) (point A) is the point on the real axis on the right from the cut in Fig.1 (right). But the every point at \(k > k_t\) in
Fig. 1 (left) (point B) is the point under the cut on the nearest lower unphysical sheet of the Riemann surface of the logarithm in $A(\omega, k)$ (11). The value $k_t$ is determined by the condition that there is a solution to (14) such that $\omega = \omega_s(k_t) = \frac{k p_F}{m} + \frac{k^2}{2m}$, (see (16)). At large $k$ we obtain on the unphysical sheet the complex solutions with Re $\omega_s(k)$ larger then the end of the cut (16) (point C): Re $\omega_s(k) \geq \frac{k p_F}{m} + \frac{k^2}{2m}$.

To obtain the solutions in the overlapping region we must go under the cut to the unphysical sheet of $A(\omega, k)$. We are failed to find solution on the physical sheet of the complex $\omega$-plane at $k > k_t$ [11, 12]. The transition of solutions of (10) to the unphysical sheet through the cut and appearance of their imaginary part we interpret as the damping of the collective excitations due to admixture of the free proton and neutron particle-hole pairs. In SNM we do not fix the isospin of the $ph$ pairs. The branch $\omega_s(k)$ is shown in Fig. 2 by the solid curves.

We have obtained the second collective branch of solutions to Eq. (10) (we denote it as $\omega_s(k)$). It is places on the unphysical sheet of the functions $A^p(\omega, k)$ or $A^n(\omega, k)$ but not of the both: $\omega_s(k)$ is calculated on the unphysical sheet of the logarithm of $A^n(\omega, k)$ but $A^p(\omega, k)$ is taken on the lower half plane of the physical sheet of the complex $\omega$-plane. Changing $n \leftrightarrow p$ we obtain the same $\omega_s(k)$ in SNM. This branch does not appears at the real axis. It is complex, damped quickly and started at some $k = k_c$. The value of $k_c$ is obtained numerically. Branches of solutions $\omega_s(k)$ and $\omega_s(k)$ are shown in Fig. 2. At the equilibrium density $\rho = \rho_0$ we obtain $k_c = 0.52 p_0$, $k_t = 0.34 p_0$. In calculations we use $\rho_0 = 0.17 f m^{-3}$, $p_0 = 0.268 GeV$, isovector constant $F'$ of the effective interaction Eq. (6) is $F' = 1.0$, effective nucleon mass $m^* = 0.8 m_0$.

### 3.2 ANM

In our model in asymmetric nuclear matter the excited collective states are damped due to mixture with $ph$-mode, $\omega^\tau_{ph}$. We can distinguish the mixture with $\omega^p_{ph}(k)$, with $\omega^n_{ph}(k)$ and with the both proton and neutron pairs. These three variants have the separate branches of solutions with the different widths.

In ANM the dispersion equation (13) has the form

$$1 - C_{0p} F' A^p - C_{0n} F' A^n = 0.$$  (17)

In Eq. (17) excitations $\omega^n_{ph}(k)$ and $\omega^p_{ph}(k)$ form the cuts in functions $A^p$ and $A^n$ (Eq. (5)) on the real axis.

The physical sheet and the cuts of Eq. (17) are shown in Fig. 3. The cuts of $A^\tau(\omega, k)$ and $A^\tau(-\omega, k)$ ($\tau = n, p$) are:

$$(1, 1') : \frac{k p_F}{m} + \frac{k^2}{2m} \leq \omega \leq \frac{k p_F}{m} + \frac{k^2}{2m}; \quad (2, 2') : -\frac{k p_F}{m} - \frac{k^2}{2m} \leq \omega \leq -\frac{k p_F}{m} - \frac{k^2}{2m}.$$  (18)
The cut of $A^r(\omega, k)$ is labeled by $(1, 1')$ and the cut of $A^r(-\omega, k)$ is labeled by $(2, 2')$. In Fig. 3 two sets of cut are shown for the $A^n$ and $A^p$ when $\beta > 0$ and $p_{Fn} > p_{Fp}$. In the symmetric nuclear matter the cuts are identical.

Now we consider solutions of Eq. (13) in the nuclear matter with $\beta > 0$. First, we obtain the zero-sound collective branch $\omega_s n(k)$. It is real at small $k$, then with $k$ increasing it is overlapping with the set of the neutron $ph$-pairs ($\omega_{ph}^n(k)$) and is damping due to the mixture with them. In nuclei this damping would correspond to the semi-direct decay of the collective state due to the neutron emission. Second, we obtain the zero-sound branch $\omega_s p(k)$. It is complex even at the small $k$, it is damping due to the mixture with the set of the proton $ph$-pairs ($\omega_{ph}^p(k)$). In nuclei this damping would correspond to the decay of the collective state due to the proton emission. Third, there is a $\omega_s np(k)$, it is damping due to overlapping both the proton and the neutron sets of $ph$-pairs.

4 Solutions of the dispersion equation in ANM

At the beginning we compare the branches of solutions in SMN and in ANM with small $\beta = 0.01$. In Fig. 4a we show the same two curves as in Fig. 2 ($\omega_s(k), \omega_{s1}(k)$) on the complex $\omega$-plane. In Fig. 4b,c,d we show the branches in ANM with a small $\beta = 0.01$: $\omega_{sn}(k), \omega_{sp}(k), \omega_{snp}(k)$. At small $\beta$ the branches in ANM are close to that in SNM, but there is not the direct correspondence between the whole branches. It will be discuss below how the branches in ANM approximate the solutions in SNM and their behavior at $\beta \to 0$.

In Fig. 4 we see that the branches start at the different $k$. There are four wave vector values which are related to branches, Fig 5

1) $k_t(\beta)$ - this is the value of wave vector when the real branch of solutions becomes complex. At $\beta = 0$ the real solutions turn into $\omega_s(k)$, at $\beta > 0$ into $\omega_{sn}(k)$ and at $\beta < 0$ into $\omega_{sp}(k)$.

2) $k_c$ is the wave vector value that the branch $\omega_{s1}(k)$ starts (Fig. 2), $k_c$ and $\omega_{s1}(k)$ exist in SNM only.

3) $k_{1,2}^{np}(\beta)$ are related to $\omega_{snp}(k)$; $k_{1}^{np}(\beta = 0) = k_t$. $k^{np}$ consists of two parts $k_{1}^{np}$ and $k_{2}^{np}$. Solutions $\omega_{snp}(k)$ exist at that $\beta$ and $k$ which are placed on the right and above the curve $k^{np}(\beta)$. There is not $\omega_{snp}(k)$ on the left and below of the curve $k^{np}(\beta)$.

4) $k^p(\beta)$ is the beginning of the $\omega_{sp}(k)$ at $\beta > 0$. For the every $\beta$ $\omega_{sp}(k)$ exist at $k > k^p$.

We obtain that at $\beta < 0.26$ there are inequalities $k^p(\beta) < k_t(\beta) < k_{1}^{np}(\beta) < k_c$ which are very sensitive to the input values of $F', m^*, p_0$.

To consider the solutions at $\beta < 0$ we change the protons to neutrons and vice versa.

In the Figs 6a - 6c the branches of solutions to Eq. (13) $\omega_{sn}(k, \beta)$ (Fig. 6a), $\omega_{sp}(k, \beta)$ (Fig. 6b)
and $\omega_{snp}(k, \beta)$ (Fig.6c) are shown for the different asymmetry parameter $\beta$. Calculations are made for the equilibrium density and $\beta=0.01, 0.2, 0.5, 0.8$. In the Figs.7a–7c we compare the branches $\omega_{sp}(k)$, $\omega_{sn}(k)$ and $\omega_{snp}(k)$ for the different values of the parameter of asymmetry $\beta$. The useful observation concerns the real parts of the solutions: at $\beta > 0.01$ Re$\omega_{sp}(k, \beta)$, Re$\omega_{snp}(k, \beta)$ are smaller than Re$\omega_{sn}(k, \beta)$ (Fig.7).

4.1 The branches $\omega_{sn}(k)$

As it was mentioned above at the small $k$ and $\beta > 0$ the dispersion equation has the real solutions. This takes place at $k < k_t$ (Fig.3). We name them $\omega_{sn}(k)$ because with $k$ increasing there is overlapping of $\omega_{sn}(k)$ with the neutron cut, Fig.6a. And $\omega_{sn}(k)$ decay due to the mixture with the free neutron $ph$ pairs.

After overlapping we continue the branch of solutions under the cut to the unphysical sheet of $A_p(\omega, k)$. It means that the Im$\omega_{sn}(k)$ appears at the wave vectors larger the definite one ($k > k_t(\beta)$). The function $k_t(\beta)$ is computed and shown in Fig.5. In nuclei with $N > Z$ it means that the semi-direct photoneutron channel ($\gamma, n$) of GDR decay is open.

Dependence of $\omega_{sn}(k)$ on $\beta$ is shown in Fig.6a. When the asymmetry parameter grows the real part of $\omega_{sn}(k)$ increase. The dependence of $|\text{Im}\omega_{sn}(k, \beta)|$ changes with $k$, see Fig.6a. We see the special behavior of $\omega_{sn}(k)$ at small $\beta$. This is explained by the transition to the solutions at $\beta = 0$ which will be discussed below.

4.2 The branches $\omega_{sp}(k)$

At $\beta > 0$ the branch $\omega_{sp}(k)$ is placed on the unphysical sheet completely. Since the real solution meets the neutron cut first when $k$ increase (Fig.4) we can not say that the real solution goes under the proton cut. We construct the unphysical sheet of $A^p(\omega, k)$ analogously the sheet of $A^n(\omega, k)$ and calculate the solutions on this sheet. This sheet is the nearest unphysical logarithmic sheet of the logarithm in $A^p(\omega, k)$. The imaginary part of $\omega_{sp}(k)$ corresponds to the damping of excitation due to mixture with the proton free $ph$-pairs. So $\omega_{sp}(k)$ is complex at the every $k$ for $\beta > 0$.

In the Figs.6 and 7 the branches $\omega_{sp}(k)$ are shown. The dependence of $\omega_{sp}(k)$ on $\beta$ differs from that of $\omega_{sn}(k)$: the real part of $\omega_{sp}(k)$ decrease with $\beta$, but $|\text{Im}\omega_{sp}(k)|$ increase. At the neutron matter $\beta = 1.0$, Eq.(1) the branch $\omega_{sp}(k)$ is absent, of course. A special behavior of $\omega_{sp}(k)$ at $\beta \to 1$, when it turns to zero is discussed in Sect.5. Again we see the special behavior of $\omega_{sp}(k)$ at small $\beta$.

Our calculations show that $\omega_{sp}(k)$ appears at the point $k = k^p(\beta)$ (Fig.5) and this $\omega_{sp}(k^p)$ coincides with the proton cut point $2'$ in Fig.3 for $A^p(-\omega, k)$ (16). Since at $\beta > 0$ the branch
\( \omega_{sp}(k) \) is completely placed under the proton cut of \( A^p(\omega, k) \), we are forced to conclude that 1) \( \omega_{sp}(k) \) at \( k < k^p \) is placed under the both (1,1') and (2,2') cuts in Fig.3 for \( A^p \) and we must take into account 2p2h states, or 2) accept that at \( k < k^p \) the collective excitation \( \omega_{sp}(k) \) annihilates with \( ph \) pairs. Decision is in progress. In nuclei emergence of \( \omega_{sp}(k, \beta) \) corresponds to the open semi-direct photoproton channel \( (\gamma, p) \) of GDR decay.

### 4.3 The branches \( \omega_{snp}(k) \)

In Fig.6 and 7 we show \( \omega_{snp}(k) \) for different \( \beta > 0 \). As it is shown in the next Section, at \( \beta = 0 \) and \( k > k_t \) the branch \( \omega_{snp}(k) \) coincides with complex part of \( \omega_s(k) \). Now \( \omega_{snp}(k) \) starts at \( k = k^np \) and is placed on the unphysical sheets only. At \( \beta = 0 \) we have \( k_t = k^np \) (Fig.5). In nuclei at \( k > k^np(\beta) \) this branch describes the semi-direct photonucleon \( (\gamma, N) \) channel of decay, but \( \omega_{snp}(k) \) does not define the isospin of the emitted nucleon.

As it is shown in Fig.5 the function \( k^{np}(\beta) \) consist of two parts \( k^{np}_1(\beta) \) and \( k^{np}_2(\beta) \). The behavior of \( \omega_{snp}(k) \) (shown in Fig.8) explains the behavior of \( k^{np}(\beta) \). At \( \beta < 0.26 \) there is no solutions at small \( k \); they exist at \( k \geq k^{np}_1(\beta) \) only. But at \( 0.26 \leq \beta \leq 0.36 \) the branch \( \omega_{snp}(k) \) consists of two parts. For example, at \( \beta = 0.3 \) (Fig.8) the first part exists at \( 0 \leq k \leq k^{np}_2(\beta = 0.3) \) and the second one at \( k > k^{np}_1(\beta = 0.3) \). For \( \beta > 0.36 \) the branch \( \omega_{snp}(k, \beta) \) exists at all \( k > 0 \) (relevant to our approach).

### 5 Behavior of solutions at \( |\beta| \to 1 \)

As it was explained in Introduction, we wait a special behavior of solutions at \( p_Fp \to 0 \). As the density of the proton states go to zero we wait disappearance of \( \omega_{sp} \) and \( \omega_{snp} \). To take into account the decreasing of the density of the proton states we change the normalization of the effective interaction, Eq.(12). The dispersion equation for the zero sound in ANM is presented in Eq.(13). In this subsection we compare solutions at \( |\beta| \sim 1 \) of the Eqs.(10) and (13).

The presentation of results is given in Fig.9. We fix \( k = p_0 \) and change the asymmetry parameter \( \beta \). The numbers 1 and 2 denote the solutions of Eqs.(10) and (13), correspondingly. We see that \( \omega_{sp} \) and \( \omega_{snp} \) marked by 2 turn to zero at \( \beta \to 1 \) and only \( \omega_{sn}(k) \) survives in Eq.(13). But the solutions of Eq.(10) do not disappear at \( |\beta| \to 1 \). We obtain the nonzero solutions and this contradict to the physical picture at \( \beta \to 1 \).
6 Behavior of solutions at \( |\beta| \to 0 \)

We demonstrate the behavior of branches \( \omega_s(k), \omega_{s1}(k), \omega_{sn}(k), \omega_{sp}(k), \omega_{snp}(k) \) at small \( |\beta| \). We consider \( \beta > 0 \) mainly.

6.1 \( \omega_s(k) \) and \( \omega_{s1}(k) \) as a limit of \( \omega_{sn}(k) \) and \( \omega_{sp}(k) \) at \( \beta \to 0 \)

At \( \beta = 0 \) in SNM we have 2 branches \( \omega_s(k) \) and \( \omega_{s1}(k) \) (Fig. 2). Figs. 4 is made for small \( \beta > 0 \). In this figure we see the splitting of \( \omega_s(k) \) and \( \omega_{s1}(k) \) into \( \omega_{sn}(k) \), \( \omega_{sp}(k) \) and \( \omega_{snp}(k) \). The real part of \( \omega_s(k) \) passes into the real part of \( \omega_{sn}(k) \). The complex part of \( \omega_s(k) \) transits to \( \omega_{snp}(k) \). The branch \( \omega_{s1}(k) \) turns into the complex parts of \( \omega_{sn}(k) \) and \( \omega_{sp}(k) \) on the different unphysical sheets.

First, we consider the dependence of the real solutions of Eq. (13) on \( \beta \). They are shown in Figs. 2, 10, 11(a, b), 13 at \( k < k_t \). They are the lines started at \( k = 0 \) and \( \omega = 0 \) and finished at \( k = k_t(\beta) \) and \( \omega(k_t, \beta) \) (Fig. 5). We attribute them to \( \omega_s(k) \) at \( \beta = 0 \) or to \( \omega_{sn}(k) \) at \( \beta > 0 \). We see that there is continuous transition between the branches at \( k < k_t \).

In Fig. 10 the solutions are shown on the complex \( \omega \)-plane. This presentation is a very sensitive to the distinction of branches. The branch \( \omega_{s1}(k) \) shown by the solid curve in Fig. 10, corresponds to the dashed curve in Fig. 4. In Fig. 10 we see that the branches \( \omega_{sn}(k) \) and \( \omega_{sp}(k) \) calculated for \( \beta = 0.01 \) envelope \( \omega_{s1}(k) \) (number '1'). This is more expressive for smaller \( \beta \) (but difficult to draw since they are too close). For the larger \( \beta \): \( \beta = 0.1 \) (number '2'), the branches do not feel \( \omega_{s1}(k) \) at all. This explains the distinct behavior of these branches at small \( \beta \), Figs. 6. So we conclude that \( \omega_{s1}(k) \) is the limit of \( \omega_{sn}(k) \) and \( \omega_{sp}(k) \) at \( \beta \to 0 \) (from different unphysical sheets). This is true at \( k > k_c \) when \( \omega_{s1}(k) \) exists.

Now we give the total picture for behavior of branches at \( -1 \leq \beta \leq 1 \), Figs. 11. We explained that there are the specific values of the wave vector \( k_t(\beta), k^p(\beta), k_c, k^{sp}(\beta) \) which determine the changes of branches on \( \beta \), Fig. 5. In Fig. 11 we demonstrate the \( \beta \)-dependence of the real and imaginary parts of branches \( \omega_{sn}(k), \omega_{sp}(k) \) at some values \( k_1 = 0.05p_0, k_2 = 0.4p_0, k_3 = 0.6p_0 \) in the three different intervals: \( k_1 < k_t(\beta), k_t(\beta) < k_2 < k_c \) and \( k_3 > k_c \).

At \( k = k_1 \) and \( \beta > 0 \) the branch \( \omega_{sn}(k_1) \) is real and changing \( \beta \) from \( \beta = 1 \) to negative values we go to the real \( \omega_{sp}(k_1) \) at \( \beta < 0 \). At \( \beta = 0 \) the branch goes through the point \( \omega_s(k_1) \). In the Introduction we declared that describing the strength functions of excitations in nuclei we take into account all physical solutions of the dispersion equation. Using Steinwedel-Jensen model [16] we can relate a definite wave vector \( k_A \) and \( \beta_A \) to the nucleus \( A \). We are interesting to know all physical solutions in our model at \( k = k_A \) and \( \beta = \beta_A \). So it is important to know is \( \omega_{sp}(k_1) \) at \( \beta = \beta_A \) a physical solution? Can we do the analytical continuation from physical solution
\( \omega_{sn}(k_1, \beta_A) \) to \( \omega_{sp}(k_1, \beta_A) \)? The answer will be presented below.

At \( k = k_1 \) and \( \beta > 0 \) the branch \( \omega_{sp}(k_1) \) is complex, it does not exist (at least, it is not found) at \( k < k^p(\beta) \) and cannot be continued to negative \( \beta \). The same we can say at \( \beta < 0 \) about \( \omega_{sn}(k_1) \) which cannot be continued to positive \( \beta \).

At \( k = k_2 \) and \( \beta > 0 \) the branch \( \omega_{sn}(k_2) \) is complex. As explained above we cannot continue it to \( \omega_{sp}(k_2) \) changing \( \beta \) from the positive to negative values. There is the discontinuity in dependence of the solutions on \( \beta \) at \( \beta = 0 \) and \( k = k_2 \). Both at \( k = k_1 \) and \( k = k_2 \) we cannot relate \( \omega_{sp}(k_2, \beta > 0) \) and \( \omega_{sn}(k_2, \beta < 0) \) changing \( \beta \) only.

At \( k = k_3 \) and \( \beta > 0 \) the branch \( \omega_{sn}(k) \) is a complex but we have a complex solution at \( \beta = 0 \) (\( \omega_{s1}(k_3) \)). We can move \( \beta \) to negative values and continue \( \omega_{sn}(k_3) \) to \( \omega_{sp}(k_3) \) through the point \( \omega_{s1}(k_3) \). Moreover we can do the analytical continuation from physical solution \( \omega_{sn}(k_3, \beta_A) \) to \( \omega_{sp}(k_3, \beta_A) \) through this point. In figure we see that we can continue \( \omega_{sn}(k_3, \beta) \) from \( \beta_A \) to \( \beta = 0 \), then we transit to the branch \( \omega_{sp}(k_3, \beta) \) at \( \beta = 0 \) and continue \( \omega_{sp}(k_3, \beta) \) back to \( \omega_{sp}(k_3, \beta_A) \).

As a result we have a receipt to know whether \( \omega_{sp}(k_1) \) is the physical solution at \( \beta_A \). We do continuation of \( \omega_{sn}(k_1, \beta_A) \) from \( k_1 \) to \( k_3 \), then transit to \( \omega_{sp}(k_3) \), and changing \( k \) go back to \( \omega_{sp}(k_1, \beta_A) \). Since there are not continuation on \( \beta \) at \( k_t < k < k_c \) we must go around the point \( k = k_c \).

In Fig.11 the imaginary parts of branches \( \omega_{sn}(k), \omega_{sp}(k) \) for the same \( k_1, k_2, k_3 \) are shown. At \( k = k_1 \) the branch \( \omega_{sn}(k) \) is real by the definition of \( k_1 \) and passes into the real \( \omega_{sp}(k) \) when \( \beta \) goes to negative values. At \( \beta > 0 \) \( \text{Im}\omega_{sp}(k_1) \) is very small (see Sect.(4.2)) and tends to zero at \( k \to k^p \) (but does not reach it since it exists at the unphysical sheet). At \( k = k_2 \) at \( |\beta| \to 0 \) the imaginary parts of branches go to zero, but there are not solutions of the dispersion equation at \( \beta = 0 \) for these \( k \), Fig.2. At \( k = k_3 \) \( \text{Im}\omega_{sn}(k) \) transit to \( \text{Im}\omega_{sp}(k) \) when we move \( \beta \) to negative values. And we can go for a given \( \beta_A \) from \( \text{Im}\omega_{sn}(k, \beta_A) \) to \( \text{Im}\omega_{sp}(k, \beta_A) \) similar to the real parts.

In Figs.11 for every \( k \) we see how the real and imaginary parts of \( \omega_{sp}(k) \) (\( \omega_{sn}(k) \)) go to zero when \( \beta \to 1 \) (\( \beta \to -1 \)).

### 6.2 The complex part of \( \omega_s(k) \) as a limit of \( \omega_{snp}(k) \) at \( \beta \to 0 \)

In Fig.10 we show a part of Fig.8 adding the branch at \( \beta = 0.1 \). At small \( \beta \) the branches \( \omega_{snp}(k) \) start at \( k_1^{np}(\beta) \) (see Fig.4) and \( k_1^{np}(\beta) \to k_t(\beta = 0) \) when \( \beta \to 0 \). We accept that the \( \omega_{snp}(k) \) and the complex part of \( \omega_s(k) \) start at the same \( k_3 \). In Fig.10 we see that at \( \beta \to 0 \) branches \( \omega_{snp}(k) \) go to \( \omega_s(k) \) (solid curve). We conclude that the complex part of \( \omega_s(k) \) appeared at \( k > k_t \) is a limit of \( \omega_{snp}(k) \) at \( \beta \to 0 \). It means that at \( \beta = 0 \) and \( k > k_t \) we have \( \omega_s(k) = \omega_{snp}(k) = \omega_{spp}(k) \).

\[ \text{This definition differs from \cite{[12]}} \]
Another demonstration of this statement is shown on the Fig. [12] where we present the dependence of $\omega_{\text{snp}}(k, \beta)$ on $\beta$ at three different values $k_1 = 0.3 p_0$, $k_2 = 0.355 p_0$, $k_3 = 0.4 p_0$. Looking at the curve $k^{np}(\beta)$ in Fig. [5] we expect the different dependence of $\omega_{\text{snp}}(k, \beta)$ on $\beta$ at these $k_{1,2,3}$ since $k_1 < k_1(\beta)$, $k_2 < k_{np}(max)$ and at $k_3 > k_{np}(max)$. In Figs. [12] we see that at $k = k_1$ the branch $\omega_{\text{snp}}(k)$ starts at $\beta > \beta_1$ where $k^{np}(\beta_1) = k_1$. This correspond to Fig. [5] there is not solutions on the left and below the curve $k^{np}(\beta)$. At $k = k_2$ there are two parts of solutions: at a small $\beta \simeq 0.02$ and $\beta > \beta_2$, where $k^{np}(\beta_2) = k_2$. In Fig. [12] the small part of solutions is shown at small $\beta$ and the main part at $\beta > \beta_2$. At $k = k_3$ we have solutions at all $\beta$ (see Figs. [5,8]). So when we go from positive $\beta$ to negative ones, the branch $\omega_{\text{snp}}(k)$ transits to $\omega_{\text{snp}}(k)$ crossing the point $\omega_s(k = 0.4 p_0)$ at $\beta = 0$. See Figs. [12] for the real and imaginary parts (solid curves).

As a result, at $k = k_3$ we have solutions at all $\beta$, at $k = k_1$ there are not solutions for small $\beta$ and there is a broken dependence of $\omega_{\text{snp}}(k, \beta)$ at $k = k_2$. In all cases we see that $\omega_{\text{snp}}(k, \beta)$ go to zero at $|\beta| \to 1$.

7 Calculation of the strength function of excitation in nuclei using $\omega_i(k, \beta)$

It is demonstrated above how the external electrical dipole photon excites the collective state in asymmetric nuclear matter and this state decays by three channels. In the every of the channels we can calculate the strength function $S(\omega, k_A)_i$. We can match the collective state to the GDR in a definite nucleus [9]. Here we give the example how to calculate the strength function for GDR in the nucleus $^{48}\text{Ca}$. Following Steinwedel-Jensen model [16] we can put into correspondence to the giant dipole resonance in nucleus $A$ a definite value of wave vector $k_A = \frac{2.08}{R_A}$. For $^{48}\text{Ca}$ we have $\beta_A = 0.21$, $k_A = 0.215 p_0$. For these values of $\beta_A$ and $k_A$ we have solutions $\omega_{sn}(k_A, \beta_A)$, $\omega_{sp}(k_A, \beta_A)$ and $\omega_{\text{snp}}(k_A, \beta_A)$. Structure function $S(\omega, k)$ is determined as [1]

$$S(\omega, k) = -\frac{1}{\pi} \text{Im} \left( \frac{\Pi^R(\omega, k)}{E(\omega, k)} \right).$$

(19)

$E(\omega, k)$ is given in Eq. (13). We present $S(\omega, k)$ as a sum of the strength functions $S(\omega, k)_i$ ($i = n, p, np, \text{reg}$) related to the separate contributions of the poles and the regular part. The residues in the poles are calculated on that unphysical sheets where the poles are placed.

$$S(\omega, k_A) = \sum_i S(\omega, k_A)_i = -\frac{1}{\pi} \text{Im} \sum_j \Pi^R(\omega, k_A) \frac{R_j(k_A, \beta_A)}{\omega - \omega_j(k_A, \beta_A)} + \text{Reg},$$

(20)

where $j = n, p, np$.

$$R_i(k_A, \beta_A) = \frac{1}{E'(\omega_{s1}(k_A, \beta_A))} = \frac{\text{Re}(E') - I \text{Im}(E')}{|E'|^2}.$$
\[ E'(\omega_{si}(k)) = dE(\omega, k)/d\omega|_{\omega \rightarrow \omega_{si}(k)}. \] (21)

In our calculations we have \[ E(\omega, k) \] numerically, \[ \Pi^R(\omega, k_A) = A^p_R + A^n_R = A^p_R(\omega, k) + A^n_R(-\omega, k) + A^p_R(\omega, k) + A^n_R(-\omega, k), \] Eq. (5). The regular part is formed by the logarithmic cuts of \[ A^p, A^n \] on the physical sheet and corresponds to the direct decay of GDR with emission of the proton on the cut of \[ A^p \] and neutron on the cut of \[ A^n \].

The strength functions \[ S(\omega, k_A)_i \] and their sum are presented in Fig. 13. The every solution of the Eq. (13) \[ \omega_{si}(k_A, \beta_A), i = p, n, np \] gives the maximum and their sum has an extended structure. In our approach the width of \[ \omega_{si}(k_A, \beta_A) \] corresponds to the escape width of excitations [12], so \[ S(\omega, k) \] in Fig. 13 is formed by emission of the one nucleon. The left maximum (long dashed curve) is given by the pole \[ \omega_{sp}(k_A, \beta_A) \] in Eq. (20) and points out to the proton emission. The short dashed curve corresponds to the neutron emission and other structures are determined by both the proton and neutron emission.

It is shown in [18] how the strength function is related to the cross sections. We can compare \[ S(\omega, k_A) \] with the cross sections of the semi-direct decay of GDR. We compare the contribution of \[ \omega_{sp}(k_A, \beta_A) \] to \[ S(\omega, k_A)_p \] with the maximum in the cross section of \((\gamma, p)\) reaction and contribution of \[ \omega_{sn}(k_A, \beta_A) \] to the maximum in \((\gamma, n)\). Since in our model \[ \text{Re} \omega_{sp}(k, \beta) < \text{Re} \omega_{sn}(k, \beta) \] (Fig. 4) for all \( k \) and \( \beta > 0 \) considered, we obtain that maximum generated by \[ \omega_{sp}(k_A, \beta_A) \] is placed at smaller energies than maximum corresponding to \[ \omega_{sn}(k_A, \beta_A) \].

But we see that the relative position of the gross maxima in the cross sections of the reactions \((\gamma, n)\), \((\gamma, p)\) is the opposite to that in Fig. 13, the maximum in \((\gamma, n)\) is placed at the smaller energies than in \((\gamma, p)\) [17]. This effect was intensively studied and related to the isospin splitting of giant resonances [17]. The contradiction observed can be related to some reasons: roughness in comparison with the experiment, the simplicity of our model, the absence of the isoscalar interaction and so on. A lot of the different effects are taken into account in [17], where a correct description of \((\gamma, n)\) and \((\gamma, p)\) cross section in \(^{48}\text{Ca}\) was obtained. In any case the further investigations are necessary.

As a result we have two or three maxima in the strength function in nuclei with \( \beta \neq 0 \) and one maximum in \( N = Z \) nuclei. We choose this nuclei \(^{48}\text{Ca}\) since all three branches \[ \omega_{s\tau}(k_A, \beta_A), \tau = p, n, np \] and continuum \( ph \) pairs give the contributions into the strength function. It should be mentioned that results in Fig. 13 are very sensitive in respect to the values of \( F', m^*, k_A \) and so on. To demonstrate the contribution of three solutions in Fig. 13 we have changed the value of effective interaction \( F' = 1.0 \) to \( F' = 0.9 \) since there is no solutions \[ \omega_{snp}(k_A, \beta_A) \] of Eq. (13) at \( F' = 1.0 \) (see Fig. 5).
8 Summary

We consider nuclear matter with asymmetry parameter \(-1 \leq \beta \leq 1\). We obtain the complex branches of the solutions of the dispersion equation (13). The imaginary part of them describes the decay of zero-sound excitations due to an admixture in the collective excitations of the free particle-hole pairs of a corresponding type.

The symmetric, asymmetric and neutron matter are considered. In SNM we obtain two branches of solutions \(\omega_s(k)\) and \(\omega_{s1}(k)\) (Fig. 2). In ANM three branches are found: \(\omega_{sn}(k)\), \(\omega_{sp}(k)\) and \(\omega_{snp}(k)\) for every \(-1 < \beta < 1\) (Figs. 6, 7) and one branch in neutron matter \(\omega_{sn}(k)\) (Fig. 9, 11). During the calculations the branches naturally appear as result of attempt to relate solutions of the dispersion equation Eq. (13) for different \(\beta\). The changes of solutions with \(\beta\) depend on the value of wave vectors. There are four values of \(k\) which determine the different regions. For small \(\beta\) we have \(k^p(\beta) < k_t(\beta) \leq k_{1np}^p(\beta) < k_c\).

At \(k < k_t\) solutions are real and can be calculated for all \(\beta\) (Fig. 5) and it is easy to pass from one \(\beta\) to another. We attribute them to different branches: for \(\beta = 0\) to \(\omega_s(k)\), for \(\beta > 0\) to \(\omega_{sn}(k)\) and to \(\omega_{sp}(k)\) for \(\beta < 0\). At \(k > k_t\) solutions become complex. An additional branch \(\omega_{snp}(k)\) is obtained. It is the continuation of \(\omega_s(k)\) to the \(\beta \neq 0\).

We investigated the behavior of branches when \(|\beta| \to 0\) and \(|\beta| \to 1\). It is shown that at \(\beta \to 1\) the branches \(\omega_{sp}(k)\) and \(\omega_{snp}(k)\) go to zero. Only \(\omega_{sn}(k)\) survives. We obtain this result in our calculations if to take into account the decreasing to zero of the proton states density at \(\beta \to 1\) (Eq. (3)).

At \(\beta \to 0\) and \(k > k_t\) the complex part of \(\omega_s(k)\) is the limit of the set of branches \(\omega_{snp}(k, \beta)\). When \(\beta \to 0\) and \(k > k_c\) the branches \(\omega_{sp}(k)\) and \(\omega_{sn}(k)\) go to \(\omega_{s1}(k)\). At \(k_t < k < k_c\) there is not the limit of \(\omega_{sn}(k)\) and \(\omega_{sp}(k)\) for \(\beta \to 0\).

At the end we demonstrate the calculation of the strength function of the giant dipole resonance in \(^{48}Ca\) using the solutions obtained. Imaginary part of solutions is related to the one nucleon emission. So the strength function (Fig. 13) is determined by the escape width of the excitations. In other words this strength function contains the contributions of the direct and semi-direct channels of GDR decay. Our attempt to turn to the experiments points out that the additional investigations of the structure of \(S(\omega, k_A)\) (Fig. 13) in our simple model are needed.
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Figure 1: A schematic figure. \textit{left:} The solid curve presents $\omega_s(k)$. The dashed area is occupied by the free \(ph\) pairs $\omega_{ph}$ corresponding to the cut (1-1'). \textit{right:} The cuts of the function $A$ in (14). The cut (1-1') is the cut of $A(\omega,k)$. The cut (2-2') is the cut of $A(-\omega,k)$. The point $A,B,C$ mark the solutions having the different location in respect to the cuts.

Figure 2: The branches of solutions in SNM. The solid curves is for $\omega_s(k)$, the dashed curves - $\omega_{s1}(k)$. At $\omega > 0$ we place $\text{Re}\,\omega_s(k), \text{Re}\,\omega_{s1}(k)$. At $\omega < 0$ there are placed the imaginary parts of branches.
Figure 3: A schematic figure at $\beta > 0$. The cuts of the functions $A^\tau$ ([13]) are presented on the complex $\omega$-plane. The cut (1-1') is the cut of $A^\tau(\omega, k)$. The cut (2-2') is the cut of $A^\tau(-\omega, k)$. In the upper (lower) part the cuts of $A^p$ ($A^n$) are demonstrated.
Figure 4: Comparison of zero-sound solutions in SNM and in ANM for small $\beta$. The complex $\omega$-plane: at the horizontal axis we show $\text{Re} \, \omega_{si}(k)$, at the vertical axis - $\text{Im} \, \omega_{si}(k)$. (a): $\beta = 0$, solid curve is $\omega_s(k)$, dashed curve is $\omega_{s1}(k)$; point '1' marks $\omega_s(k_t)$, '2' is for $\omega_{s1}(k_c)$. (b): $\beta = 0.01$, the branch $\omega_{sn}(k)$, '1' stands for $\omega_{sn}(k_t)$. (c): $\beta = 0.01$, the branch $\omega_{sp}(k)$, point '3' shows $\omega_{sp}(k_p)$. (d): $\beta = 0.01$, the branch $\omega_{snp}(k)$, '4' - $\omega_{snp}(k^{np})$. 
Figure 5: Dependence of a special wave vector values $k_t(\beta)$, $k_p(\beta)$, $k_{np}(\beta)$ on $\beta$. The curve $k_{np}(\beta)$ consists from two parts: $k_{np}^2(\beta)$ (solid curve) and $k_{np}^1(\beta)$ (dash-dotted curve). The dashed curves is for $k_t(\beta)$; dotted curves - $k_p(\beta)$; $\omega(k_t)$ is the final point of the real solutions at different $\beta$. The curves can be symmetrically reflected to the negative $\beta$. 
Figure 6: Dependence of the branches $\omega_{sn}(k)$ (a), $\omega_{sp}(k)$ (b) and $\omega_{snp}(k)$ (c) on $\beta$. At $\beta=0.01$ – solid curves; 0.2 – dashed; 0.5 – dot-dashed; 0.8 – dotted. Other notations are the same as in Fig. 2.
Figure 7: Comparison $\omega_s n(p)$, $\omega_s p(k)$ and $\omega_{snp}(k)$ for different $\beta$: $\beta=0.01 (a)$, 0.2(b), 0.5(c), 0.8(d). $\omega_{sn}(k)$ are shown by the solid curves; dashed curves show $\omega_{sp}(k)$; dotted curves - $\omega_{snp}(k)$. The number '1' means wave vector $k_t/p_0$, the number '2' - $k^p/p_0$, '3' - $k^{np}/p_0$. Other notations are the same as in Fig. 2.
Figure 8: Branches $\omega_{snp}(k, \beta)$. Complex $\omega$-plane. The values of $\beta$ is shown on the every curve. Solid bold curve stands for $\omega_s(k), \beta = 0$. For $\beta = 0.3$ the points $a$ and $b$ stand for $a = \omega_{snp}(k_{np}^2)$, $b = \omega_{snp}(k_{np}^1)$ (see Fig. 5).

Figure 9: Behavior of $\omega_{sp}(k), \omega_{sn}(k)$ and $\omega_{snp}(k)$ at $\beta \to 1$. The wave vector $k$ is fixed: $k = p_0$. Solid curves denote $\omega_{sn}(k = p_0, \beta)$, dashed curves – $\omega_{sp}(k = p_0, \beta)$ and dotted curves are for $\omega_{snp}(k = p_0, \beta)$. The number '1' marks solutions of Eq. (10) and '2' - solutions of Eq. (13).
Figure 10: Behavior of $\omega_{sp}(k)$, $\omega_{sn}(k)$ and $\omega_{snp}(k)$ at $\beta \to 0$. The complex $\omega$-plane. (a): The branches $\omega_{sn}(k)$ (dashed-dotted) and $\omega_{sp}(k)$ (dashed) for $\beta=0.01$ (noted by '1') and for $\beta=0.1$ (noted by '2') in comparison with $\omega_{s1}(k)$ (solid) calculated for $\beta=0$. (b): The branches $\omega_{s}(k)$ (solid curve) and $\omega_{snp}(k)$ at $\beta=0.1$, $0.2$, $0.3$, $0.4$ (the curves dot-dashed, dashed, dotted).
Figure 11: Dependence of the real and imaginary parts of $\omega_{sn}(k_i, \beta)$ and $\omega_{sp}(k_i, \beta)$ on $\beta$ at definite $k_i$, $i=1,2,3$. Solid lines $k_1 = 0.05p_0$, dotted lines $k_2 = 0.4p_0$; dashed lines $k_3 = 0.6p_0$; ‘n’ marks $\omega_{sn}(k_i, \beta)$, ‘p’= $\omega_{sp}(k_i, \beta)$; ‘s’= $\omega_s(k_1)$, ‘s1’= $\omega_{s1}(k_3)$. 
Figure 12: Dependence of the real and imaginary parts of $\omega_{sn}(k_i, \beta)$ on $\beta$ at definite $k_i$, i=1,2,3. Solid lines $k_3 = 0.4p_0$, dotted lines $k_2 = 0.355p_0$; dashed lines $k_1 = 0.3p_0$; ‘s’ marks $\omega_s(k_1)$ and $\omega_s(k_2)$. 
Figure 13: Strength function of the giant dipole resonance in $^{48}\text{Ca}$ (multiplied by $10^3$ after [1]). The wave vector $k_A = 2.08/A^{1/3}$, $\beta_A(k_A) = 0.167$, dotted - contribution of $\omega_{\tau}^\tau(k)$, $\tau = p, n$. Solid curve show the sum of these contributions.