Strange Form Factors and Sum Rules of Baryon Decuplet

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\textbf{ABSTRACT}

Treating perturbatively the flavor symmetry breaking effects resided in the wave function as well as in the SU(3) Lagrangian, we calculate the strange form factors of decuplet baryons. The higher dimensional representation mixing components in the baryon wave function of the decuplet baryons are found to yield the important contributions to the strange form factors, especially of the $\Delta$ and $\Sigma^*$ baryons. The model independent sum rules among the baryon decuplet magnetic moments are also derived.
1 Introduction

Since Coleman and Glashow\footnote{1} predicted the magnetic moments of the baryon octet about forty years ago, there has been a lot of progress in both the theoretical paradigm and experimental verification for the baryon magnetic moments. Recently the measurements of the baryon decuplet magnetic moments were reported for \( \mu_{\Delta^+} \)\footnote{2} and \( \mu_{\Omega^-} \)\footnote{3} to yield a new avenue for understanding the hadron structure. The magnetic moments of baryon decuplet have been theoretically investigated in several models, e.g., in the quenched lattice gauge theory\footnote{4}, the quark models\footnote{5, 6}, the chiral bag model\footnote{7}, the chiral perturbation theory\footnote{8}, the chiral quark soliton model\footnote{9}, the QCD sum rules\footnote{10, 11} and the chiral quark model\footnote{12}.

On the other hand the chiral and SU(3) flavor symmetry breaking (FSB) effects in the chiral bag model (CBM)\footnote{13} are induced by the different pseudoscalar meson masses and decay constants outside and the quark masses inside the bag. Especially the SU(3) FSB originates from the strangeness degrees of freedom in the baryon structure, which has been significantly discussed after the EMC experiment on deep inelastic muon scattering\footnote{14} suggested a lingering question.

Quite recently, the SAMPLE collaboration\footnote{15} reported the experimental data of the proton strange form factor through parity violating electron scattering\footnote{16}. Moreover, McKeown\footnote{17} has shown that the strange form factor of proton should be positive by using the conjecture that the up-quark effects are generally dominant in the flavor dependence of the nucleon properties. This result is contrary to the negative values of the proton strange form factor which result from most of the model calculations\footnote{18, 19, 21, 21} except that of Hong and Park\footnote{22} based on the SU(3) CBM\footnote{23} and that of Meissner and co-workers\footnote{24} in the heavy baryon chiral perturbation theory. The CBM prediction on the positive strange form factor of the proton was also justified\footnote{25} by adjusting the inertia parameters in a systematic way.

In this paper we will report the result on strange form factor of the baryon decuplet in the minimal multi-quark structure where the symmetry breaking mass effects are treated in the perturbative scheme of the CBM, as an extension of our former work on the baryon octet.\footnote{22, 25}

In Section 2, the symmetry breaking in the CBM will be discussed in terms of the physical operators in the adjoint representation to yield the model independent sum rules. In Section 3, we will introduce the baryon wave functions in the multiquark Fock space of the higher representation mixing scheme to obtain the strange form factors of baryon decuplet.
2 Magnetic moments of baryon decuplet

In the CBM with the broken U-spin symmetry the Lagrangian is of the form

\[ L = \mathcal{L}_{CS} + \mathcal{L}_{CSB} + \mathcal{L}_{FSB} \]

\[ \mathcal{L}_{CS} = \bar{\psi} i \gamma^\mu \partial_\mu \psi - \frac{1}{2} \bar{\psi} U_5 \psi \Delta_B \]

\[ + (-\frac{1}{4} f_\pi^2 \text{tr}(l_\mu l^\mu) + \frac{1}{32\epsilon} [l_\mu, l_\nu]^2 + \mathcal{L}_{WZW}) \bar{\Theta}_B \]

\[ \mathcal{L}_{CSB} = -\bar{\psi} M \psi \Theta_B + \frac{1}{4} f_\pi^2 m_\pi^2 \text{tr}(U + U^\dagger - 2) \bar{\Theta}_B \]

\[ \mathcal{L}_{FSB} = \frac{1}{6} f_\pi^2 (\chi^2 m_K^2 - m_\pi^2) \text{tr}((1 - \sqrt{3} \lambda_8) (U + U^\dagger - 2)) \bar{\Theta}_B \]

\[ - \frac{1}{12} f_\pi^2 (\chi^2 - 1) \text{tr}((1 - \sqrt{3} \lambda_8) (U l_\mu l^\mu + l_\mu l^\mu U^\dagger)) \bar{\Theta}_B \]  

(2.2)

where the quark field \( \psi \) has SU(3) flavor degrees of freedom and the chiral field \( U = e^{i \lambda_a \pi_a} / f_\pi \in \text{SU}(3) \) is described by the pseudoscalar meson fields \( \pi_a \) (a=1,...,8) and Gell-Mann matrices \( \lambda_a \) with \( \lambda_a \lambda_b = \frac{2}{3} \delta_{ab} + (if_{abc} + d_{abc}) \lambda_c \) and \( \Theta_B (= 1 - \bar{\Theta}_B) \) is the bag theta function (one inside the bag and zero outside the bag). Here \( l_\mu = U^\dagger \partial_\mu U \) and \( \mathcal{L}_{WZW} \) stands for the topological Wess-Zumino-Witten (WZW) term. In the numerical calculation we will use the parameters \( e = 4.75, f_\pi = 93 \text{ MeV} \) and \( f_K = 114 \text{ MeV} \).

Here the chiral symmetry (CS) is broken by the quark masses \( M = \text{diag}(m_u, m_d, m_s) \) and pion mass \( m_\pi \) in \( \mathcal{L}_{CSB} \). Furthermore the SU(3) FSB with \( m_K/m_\pi \neq 1 \) and \( \chi = f_K/f_\pi \neq 1 \) is included in \( \mathcal{L}_{FSB} \).

Even though the mass terms in \( \mathcal{L}_{CSB} \) and \( \mathcal{L}_{FSB} \) break both the SU\(_L\)(3)×SU\(_R\)(3) and diagonal SU(3) symmetry so that chiral symmetry cannot be conserved, these terms without derivatives yield no explicit contribution to the electromagnetic (EM) currents \( J^\mu \) and at least in the adjoint representation of the SU(3) group the EM currents are conserved and of the same form as the chiral limit result \( J^\mu_{CS} \) to preserve the U-spin symmetry. However the derivative-dependent term in \( \mathcal{L}_{FSB} \) gives rise to the U-spin symmetry breaking conserved EM currents \( J^\mu_{FSB} \) so that \( J^\mu = J^\mu_{CS} + J^\mu_{FSB} \).

Assuming that the hedgehog classical solution in the meson phase \( U_0 = e^{i \lambda_i \phi^i} \) (i=1,2,3) is embedded in the SU(2) isospin subgroup of SU(3) and the Fock space in the quark phase is described by the \( N_c \) valence quarks and the vacuum structure composed of quarks filling the negative energy sea, the CBM generates the zero mode with the collective variable \( A(t) \in \text{SU}(3) \) by performing the slow rotation \( U \rightarrow AU_0 A^\dagger \) and \( \psi \rightarrow A \psi \) on SU(3) group manifold.
Given the spinning CBM ansatz, the EM currents yield the magnetic moment operators \( \hat{\mu}^i = \hat{\mu}^{i(3)} + \frac{1}{\sqrt{3}} \hat{\mu}^{i(8)} \) where \( \hat{\mu}^{i(a)} = \hat{\mu}^{i(a)}_{CS} + \hat{\mu}^{i(a)}_{FSB} \) with

\[
\hat{\mu}_{CS}^{i(a)} = -\mathcal{M} D^8_a - \mathcal{N}' d_{ipq} D^8_{ap} \hat{T}^R_{q} + \frac{N_c}{2\sqrt{3}} \mathcal{M} D^8_{a8} \hat{J}_i
\]

\[
\hat{\mu}_{FSB}^{i(a)} = -\mathcal{P} D^8_a(1 - D^8_{88}) + \frac{\sqrt{3}}{2} d_{ipq} D^8_{ap} D^8_{8q}
\]

where \( \mathcal{M}, \mathcal{N}, \mathcal{N}', \mathcal{P} \) and \( \mathcal{Q} \) are the inertia parameters calculable in the CBM.\(^{[25]}\)

Using the theorem that the tensor product of the Wigner D functions can be decomposed into sum of the single D functions, the isovector and isoscalar parts of the operator \( \hat{\mu}_{FSB}^{i(a)} \) are then rewritten as

\[
\hat{\mu}_{FSB}^{i(3)} = \mathcal{P}(-\frac{4}{5} D^8_{3i} + \frac{3}{10} D^2_{3i}) + \mathcal{Q}(\frac{3}{10} D^8_{3i} - \frac{3}{10} D^2_{3i})
\]

\[
\hat{\mu}_{FSB}^{i(8)} = \mathcal{P}(-\frac{6}{5} D^8_{8i} + \frac{9}{20} D^2_{8i}) + \mathcal{Q}(\frac{3}{10} D^8_{8i} - \frac{9}{20} D^2_{8i})
\]

(2.3)

Here one notes that the \( 1, 10 \) and \( \bar{10} \) irreducible representations (IRs) do not occur in the decuplet baryons while \( 10 \) and \( \bar{10} \) IRs appear together in the isovector channel of the baryon octet to conserve the hermiticity of the operator.

With respect to the decuplet baryon wave function \( \Phi_\lambda^b = \sqrt{\text{dim}(\lambda)} D^\lambda_{ab} \) with the quantum numbers \( a = (Y; I, I_3) \) (\( Y \); hypercharge, \( I \); isospin) and \( b = (Y_R; J, -J_3) \) (\( Y_R \); right hypercharge, \( J \); spin) and \( \lambda \) the dimension of the representation, the spectrum of the magnetic moment operator \( \hat{\mu}^i \) has the following hyperfine structure in the adjoint representation

\[
\mu_{\Delta^{++}} = \frac{1}{8} \mathcal{M} + \frac{1}{2}(N - \frac{1}{2\sqrt{3}} N') + \frac{3}{7} \mathcal{P} - \frac{3}{56} \mathcal{Q}
\]

\[
\mu_{\Delta^+} = \frac{1}{16} \mathcal{M} + \frac{1}{4}(N - \frac{1}{2\sqrt{3}} N') + \frac{5}{21} \mathcal{P} + \frac{1}{84} \mathcal{Q}
\]

\[
\mu_{\Delta^0} = \frac{1}{21} \mathcal{P} + \frac{13}{168} \mathcal{Q}
\]

\[
\mu_{\Delta^-} = -\frac{1}{16} \mathcal{M} - \frac{1}{4}(N - \frac{1}{2\sqrt{3}} N') - \frac{1}{7} \mathcal{P} + \frac{1}{7} \mathcal{Q}
\]

\[
\mu_{\Sigma^{++}} = \frac{1}{16} \mathcal{M} + \frac{1}{4}(N - \frac{1}{2\sqrt{3}} N') + \frac{19}{84} \mathcal{P} - \frac{17}{168} \mathcal{Q}
\]

\[
\mu_{\Sigma^{+0}} = \frac{1}{84} \mathcal{P} - \frac{1}{84} \mathcal{Q}
\]
\[
\begin{align*}
\mu_{\Sigma^{*-}} &= -\frac{1}{16} \mathcal{M} - \frac{1}{4} (N - \frac{1}{2\sqrt{3}} N') - \frac{17}{84} \mathcal{P} + \frac{13}{168} \mathcal{Q} \\
\mu_{\Xi^{*0}} &= -\frac{1}{42} \mathcal{P} - \frac{17}{168} \mathcal{Q} \\
\mu_{\Xi^{-}} &= -\frac{1}{16} \mathcal{M} - \frac{1}{4} (N - \frac{1}{2\sqrt{3}} N') - \frac{11}{42} \mathcal{P} + \frac{1}{84} \mathcal{Q} \\
\mu_{\Omega^-} &= -\frac{1}{16} \mathcal{M} - \frac{1}{4} (N - \frac{1}{2\sqrt{3}} N') - \frac{9}{28} \mathcal{P} - \frac{3}{56} \mathcal{Q}.
\end{align*}
\]

(2.5)

In the SU(3) flavor symmetric limit with the chiral symmetry breaking masses \(m_u = m_d = m_s, m_K = m_\pi\) and decay constants \(f_K = f_\pi\), the magnetic moments of the decuplet baryons are simply given by

\[
\mu_B = Q_{EM} \left( \frac{1}{16} \mathcal{M} + \frac{1}{4} (N - \frac{1}{2\sqrt{3}} N') \right)
\]

(2.6)

where \(Q_{EM}\) is the EM charge. Here one notes that in the CBM in the adjoint representation the prediction of the baryon magnetic moments with the chiral symmetry is the same as that with the SU(3) flavor symmetry since the mass-dependent term in \(\mathcal{L}_{CSB}\) and \(\mathcal{L}_{FSB}\) do not yield any contribution to \(J_{FSB}^\mu\) so that there is no terms with \(\mathcal{P}\) and \(\mathcal{Q}\) in (2.3).

Due to the degenerate d- and s-flavor charges in the SU(3) EM charge operator \(\hat{Q}_{EM}\), the CBM possesses the U-spin symmetry relations in the baryon decuplet magnetic moments, similar to those in the octet baryons

\[
\begin{align*}
\mu_{\Delta^-} &= \mu_{\Sigma^{*-}} = \mu_{\Xi^{-}} = \mu_{\Omega^-} \\
\mu_{\Delta^0} &= \mu_{\Sigma^{*0}} = \mu_{\Xi^{*0}} \\
\mu_{\Delta^+} &= \mu_{\Sigma^{++}}
\end{align*}
\]

(2.7)

which are subset of the more strong symmetry relations (2.6).

Since the SU(3) FSB quark masses do not affect the magnetic moments of the baryon decuplet in the adjoint representation of the CBM, in the more general SU(3) flavor symmetry broken case with \(m_u = m_d \neq m_s, m_\pi \neq m_K\) and \(f_\pi \neq f_K\), the decuplet baryon magnetic moments with \(\mathcal{P}\) and \(\mathcal{Q}\) satisfy other sum rules

\[
\begin{align*}
\mu_{\Sigma^{*0}} &= \frac{1}{2} (\mu_{\Sigma^{++}} + \mu_{\Sigma^{*-}}) \\
\mu_{\Delta^-} + \mu_{\Delta^{++}} &= \mu_{\Delta^0} + \mu_{\Delta^+} \\
\sum_{B \in \text{decuplet}} \mu_B &= 0.
\end{align*}
\]

(2.8-2.10)
Also one can easily see that the CBM shares with the nonrelativistic quark and chiral quark soliton models\cite{7,9} the following sum rules:\cite{7} \cite{9}

\begin{align}
-4\mu_{\Delta^{++}} + 6\mu_{\Delta^+} + 3\mu_{\Sigma^{++}} - 6\mu_{\Sigma^{*0}} + \mu_{\Omega^-} &= 0 \\
-2\mu_{\Delta^{++}} + 3\mu_{\Delta^+} + 2\mu_{\Sigma^{++}} - 4\mu_{\Sigma^{*0}} + \mu_{\Xi^-} &= 0 \\
-\mu_{\Delta^{++}} + 2\mu_{\Delta^+} - 2\mu_{\Sigma^{*0}} + \mu_{\Xi^-} &= 0 \\
\mu_{\Delta^{++}} - 2\mu_{\Delta^+} + \mu_{\Delta^0} &= 0 \tag{2.11}
\end{align}

and

\begin{align}
\mu_{\Delta^0} - \mu_{\Sigma^{*-}} = \mu_{\Sigma^{*+}} - \mu_{\Xi^0} = \frac{1}{2}(\mu_{\Delta^+} - \mu_{\Xi^-}) = \frac{1}{3}(\mu_{\Delta^{++}} - \mu_{\Omega^-}). \tag{2.12}
\end{align}

Here one notes that the $\Sigma^*$ hyperons satisfy the identity $\mu_{\Sigma^*}(I_3) = \mu_{\Sigma^{*0}} + I_3 \Delta \mu_{\Sigma^*}$, where $\Delta \mu_{\Sigma^*} = \frac{1}{16} M + \frac{1}{4}(N - \frac{1}{2} \sqrt{3} N') + \frac{3}{16} \bar{P} - \frac{5}{16} \bar{Q}$, such that $\mu_{\Sigma^{*+}} + \mu_{\Sigma^{*-}}$ is independent of $I_3$ as in (2.10). For the $\Delta$ baryons one can formulate the relation $\mu_{\Delta}(I_3) = \mu_{\Delta}^0 + I_3 \Delta \mu_{\Delta}$ with $\mu_{\Delta}^0 = \frac{1}{16} M + \frac{1}{4}(N - \frac{1}{2} \sqrt{3} N') + \frac{5}{112} \bar{P} + \frac{5}{112} \bar{Q}$ and $\Delta \mu_{\Delta} = \frac{1}{16} M + \frac{1}{4}(N - \frac{1}{2} \sqrt{3} N') + \frac{5}{3} \bar{P} - \frac{11}{168} \bar{Q}$, so that $\Delta$ baryons can be easily seen to fulfill the second sum rule in (2.10) and the last one in (2.11). Also the summation of the magnetic moments over all the decuplet baryons vanish to yield the model independent relation, namely the third sum rule in (2.10), since there is no SU(3) singlet contribution to the magnetic moments as in the baryon octet magnetic moments.

### 3 Strange form factors in multi-quark structure

Until now we have considered the CBM in the adjoint representation where the U-spin symmetry is broken only through the magnetic moment operators $\hat{\mu}_{FSB}^{(a)}$ induced by the symmetry breaking derivative term. To take into account the missing chiral symmetry breaking mass effect from $\mathcal{L}_{CSB}$ and $\mathcal{L}_{FSB}$, in this section we will treat perturbatively the symmetry breaking mass terms via the higher dimensional IR channels where, differently from the nonperturbative Yabu-Ando scheme\cite{28}, the CBM can be handled in the quantum mechanical perturbation theory with the higher IR mixing in the baryon wave function to yield the minimal multi-quark structure\cite{29}.

\footnote{In fact, the baryon decuplet magnetic moments in the nonrelativistic quark and chiral quark soliton models satisfy the model independent relations (2.7) and (2.10). Also the $V$-spin symmetry relations (2.12) hold in the most general SU(3) flavor symmetry broken case of the CBM with the higher representation mixing corrections (3.3).}
On quantizing the collective variable $A(t)$, one can obtain the Hamiltonian $H = H_0 + H_{SB}$ where

$$
H_0 = M + \frac{1}{2} \left( \frac{1}{I_1} - \frac{1}{I_2} \right) \hat{J}^2 + \frac{1}{2I_2} (\hat{C}_2^2 - \frac{3}{4} \hat{Y}_R^2)
$$

$$
H_{SB} = m(1 - D_{SS}^8)
$$

(3.1)

where $I_1$ and $I_2$ are the moments of inertia of the CBM along the isospin and the strange directions respectively and $\hat{J}^2$ and $\hat{C}_2^2$ the Casimir operators in $SU_R(2)$ and $SU_L(3)$ groups and $m$ the inertia parameter denoting the symmetry breaking strength. Minimizing the static mass $M$ the soliton profile function satisfies the Skyrme equation of motion in the meson phase

$$
(x^2 + 2 \sin^2 \theta) \frac{d^2 \theta}{dx^2} + 2x \frac{d \theta}{dx} + ((\frac{d \theta}{dx})^2 - 1 - \frac{\sin^2 \theta}{x^2}) \sin 2\theta - \mu^2 \pi x^2 \sin \theta = 0
$$

(3.2)

with $\mu = m_{\pi}/e\pi$ and $x = e\pi r$, the dimensionless quantities. Here one notes that the pion mass yields deviation from the chiral limit profile so that the numerical results in the massive Skyrmion theory can be worsened when one uses the experimental decay constant. Since $(m_u + m_d)/m_s \approx m_{\pi}^2/m_R^2 \approx 0.1$ we will neglect the light quark and pion masses.

For the baryon decuplet with $Y_R = 1$ and $J = \frac{3}{2}$ the possible $SU(3)$ representations of the minimal multi-quark Fock space are restricted by the Clebsch-Gordan series $10 \oplus 27 \oplus 35$ so that the baryon decuplet wavefunctions are described by $|B\rangle = |B\rangle^{10} - C_B^B |B\rangle^{27} - C_B^{35} |B\rangle^{35}$ where the representation mixing coefficients are given by

$$
C_B^\lambda = \frac{\lambda \langle B|H_{SB}|B\rangle^{10}}{E_\lambda - E^{10}}
$$

(3.3)

with the eigenvalues $E_\lambda$ and eigenfunctions $|B\rangle^\lambda = \Phi^\lambda_B \otimes \text{intrinsic}$ of the eigen equation $H_0 |B\rangle^\lambda = E_\lambda |B\rangle^\lambda$. Here $\Phi^\lambda_B$ is the collective wavefunction discussed above and the intrinsic state degenerate to all the baryons is described by a Fock state of the quark operator and classical meson configuration. Using the decuplet wavefunctions with the higher representation

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The algorithm for the Clebsch-Gordan decomposition of the tensor product of the two IRs in the $qq\bar{q}q$ is given by $(3 \otimes 3 \otimes 3) \otimes (3 \otimes 3) = (1 \oplus 8^2 \oplus 10) \otimes (1 \oplus 8) = 1^3 \oplus 8^4 \oplus 10^4 \oplus 10^2 \oplus 27^4 \oplus 35$ where the superscript denotes the number of the different IR's with the same dimension. Due to the baryon constraint $Y_R = 1$ coming from the WZW term, the spin-$\frac{1}{2}$ octet baryons are restricted to the IR's $8 \oplus 10 \oplus 27$ and the spin-$\frac{3}{2}$ decuplet baryons to $10 \oplus 27 \oplus 35$. 

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mixing coefficients \[3.3\] the additional hyperfine structure of the magnetic moment spectrum in the perturbative scheme is given by

\[\delta \mu^i_B = -2 \sum_{\lambda=2\ell,35}^{10} \frac{\langle B | \hat{\mu}^i | B \rangle \lambda \lambda \langle B | H_{SB} | B \rangle_{10}}{E_\lambda - E_{10}} \]  (3.4)

up to the first order of \( m \).

Then one can obtain the V-spin symmetry relations in the perturbative corrections of the decuplet magnetic moments

\[
\begin{align*}
\delta \mu_{\Delta^+} &= \delta \mu_{\Omega^-} = m I_2 \left( \frac{5}{672} M + \frac{1}{168} N + \frac{5}{336\sqrt{3}} N' \right) \\
\delta \mu_{\Delta^+} &= \delta \mu_{\Xi^-} = m I_2 \left( \frac{5}{112} M - \frac{1}{21} N - \frac{5}{336\sqrt{3}} N' \right) \\
\delta \mu_{\Delta^0} &= \delta \mu_{\Sigma^*\Xi^-} = m I_2 \left( \frac{5}{672} M - \frac{17}{168} N - \frac{5}{112\sqrt{3}} N' \right) \\
\delta \mu_{\Delta^-} &= m I_2 \left( \frac{5}{42} M - \frac{13}{384} N - \frac{25}{336\sqrt{3}} N' \right) \\
\delta \mu_{\Sigma^*-} &= \delta \mu_{\Xi^0} = m I_2 \left( -\frac{1}{224} M + \frac{1}{168} N + \frac{11}{336\sqrt{3}} N' \right) \\
\delta \mu_{\Sigma^*-} &= m I_2 \left( \frac{13}{336} M - \frac{1}{28} N - \frac{1}{168\sqrt{3}} N' \right). \tag{3.5}
\end{align*}
\]

Here one notes that including the above implicit FSB effects one can have the sum rules \[2.8\] and \[2.9\], and

\[\mu_{\Delta^+} - \mu_{\Delta^-} = 3(\mu_{\Delta^+} - \mu_{\Delta^0}) \]  (3.6)

which also holds in the chiral perturbation theory\[8\].

Now in the multiquark structure of the CBM with the chiral and SU(3) FSB, the magnetic moments of baryon decuplet can be broken up into three parts

\[\mu_B = \mu_{0,B}(M, N, N') + \delta \mu_{1,B}(P, Q) + \delta \mu_{2,B}(m I_2) \]  (3.7)

where the first term \( \mu_{0,B} \) comes from the chiral symmetric contribution, \( \delta \mu_{1,B} \) is due to the explicit FSB and \( \delta \mu_{2,B} \) is obtained from the implicit FSB in the representation mixing as shown in \[3.4\]. In Table 1 the baryon decuplet magnetic moments \( \mu_B \) are predicted in the CBM with the bag radius \( R \sim 0.6 \) fm corresponding to the magic angle \( \theta(R) = \pi/2 \), where the baryon number is shared equally with both quark and meson phases\[30\] and
the numerical values of the inertia parameters are given by $M = 0.66$, $N = 6.00$, $N' = 0.52$, $P = 1.11$, $Q = 1.27$ and $m\mathcal{I}_2 = 3.96$ as in the baryon octet case. In particular we obtain $\mu_{\Delta^{++}} = 1.29\mu_p$ comparable to the experimental value $\mu_{\Delta^{++}}^{exp} = (1.62 \pm 0.18)\mu_p$. For $\mu_{\Omega^-$ the CBM seems to well predict the value $-1.75$ n.m., consistent with the experimental data $-1.94 \pm 0.17 \pm 0.14$ n.m. Since $\mu_{\Omega^-$ could be mainly achieved from the strange quark and kaon whose masses are kept in our massless profile approximation $(m_u + m_d)/m_s \approx m_s^2/m_K^2 \approx 0$. Also one notes that the implicit FSB effects with the V-spin symmetry improve the prediction of $\mu_{\Delta^{++}}$, but that of $\mu_{\Omega^-$ seems worsened. In Table 2, the baryon decuplet magnetic moments obtained from other calculations such as the nonrelativistic quark model, relativistic quark model$^5$, lattice gauge theory$^4$, chiral perturbation theory$^8$ and Skyrmion model$^3$, are listed together with the experimental data.

Next in the SU(3) flavor symmetry broken case we decompose the EM currents into three pieces $J^\mu = J^\mu(u) + J^\mu(d) + J^\mu(s)$ where the q-flavor currents $J^\mu(q) = J^\mu(q)_{\text{CS}} + J^\mu(q)_{\text{FSB}}$ are given by substituting the charge operator $\hat{Q}$ with the q-flavor charge operator $\hat{Q}_q$.

$$J^\mu_{\text{CS}} = \bar{\psi}\gamma^\mu\hat{Q}_q\psi\Theta_B + \left(-\frac{i}{2}f_\pi^2\text{tr}(\hat{Q}_q l^\mu) + \frac{i}{8e^2}\text{tr}[\hat{Q}_q l^\nu][l^\mu, l^\nu] + U \leftrightarrow U^\dagger\right)\Theta_B$$

$$J^\mu_{\text{FSB}} = -\frac{i}{12}f_\pi^2(\lambda^2 - 1)\text{tr}((1 - \sqrt{3}\lambda_8)(U^\dagger \hat{Q}_q l^\mu + l^\mu \hat{Q}_q U^\dagger) + U \leftrightarrow U^\dagger)\Theta_B$$

to obtain the baryon decuplet magnetic moments in the s-flavor channel

$$\mu_{\Delta^s} = -\frac{7}{48}M + \frac{1}{12}(N - \frac{1}{2\sqrt{3}}N') + \frac{2}{21}P + \frac{5}{168}Q + m\mathcal{I}_2(\frac{85}{2016}M - \frac{25}{504}N - \frac{5}{252\sqrt{3}}N')$$

$$\mu_{\Sigma^*} = -\frac{1}{6}M - \frac{1}{126}P - \frac{1}{126}Q + m\mathcal{I}_2(\frac{13}{504}M - \frac{1}{42}N - \frac{1}{252\sqrt{3}}N')$$

$$\mu_{\Xi^*} = -\frac{3}{16}M - \frac{1}{12}(N - \frac{1}{2\sqrt{3}}N') - \frac{2}{21}P - \frac{5}{168}Q$$

$^3$For the Skyrmion model corresponding to the CBM with vanishing bag radius, we have used the numerical values of the inertia parameters, $M = 0.67$, $N = 5.03$, $N' = 0.91$, $P = 0.76$, $Q = 0.99$ and $m\mathcal{I}_2 = 1.79$. 

\[ \mu_\Omega^{(s)} = \frac{3}{224} M - \frac{1}{168} N' + \frac{1}{168 \sqrt{3}} N'' \]

\[ = -\frac{5}{24} M - \frac{1}{6} (N - \frac{1}{2\sqrt{3}} N') - \frac{3}{14} P - \frac{1}{28} Q \]

\[ + m I_2 \left( \frac{5}{1008} M + \frac{1}{252} N + \frac{5}{504 \sqrt{3}} N' \right). \] (3.8)

Here one notes that in general all the baryon decuplet magnetic moments fulfill the model independent relations in the u- and d-flavor components and the I-spin symmetry of the isomultiplets with the same strangeness in the s-flavor channel

\[ \mu_B^{(d)} = \frac{Q_d}{Q_u} \mu_B^{(u)}, \quad \mu_B^{(s)} = \mu_B^{(s)} \] (3.9)

with \( \bar{B} \) being the isospin conjugate baryon in the isomultiplets of the baryon.

Now the form factors of the decuplet baryons, with internal structure, are defined by the matrix elements of the EM currents

\[ \langle p + q | J^\mu | p \rangle = \bar{u}(p + q) (\gamma^\mu F_{1B}(q^2) + \frac{i}{2m_B} \sigma^{\mu\nu} q^\nu F_{2B}(q^2)) u(p) \] (3.10)

where \( u(p) \) is the spinor of the baryons and \( q \) is the momentum transfer. Using the s-flavor charge operator in the EM currents as before, in the limit of zero momentum transfer, one can obtain the strange form factors of baryon decuplet

\[ F_{1B}^{(s)}(0) = S \]

\[ F_{2B}^{(s)}(0) = -3\mu_B^{(s)} - S \] (3.11)

in terms of the strangeness quantum number of the baryon \( S = 1 - Y \) \((Y: \) hypercharge) and the strange components of the baryon decuplet magnetic moments \( \mu_B^{(s)} \).

As shown in Table 3, we have obtained the theoretical predictions of the CBM and the Skyrmion model for the strange form factors of baryon octet and decuplet. Here one notes that the implicit FSB contributions to the strange form factors in the CBM are quite significant in the \( \Delta \) and \( \Sigma^* \) decuplet baryons. Also the CBM prediction \( F_{2p}^{(s)} = 0.30 \) n.m. for the proton strange form factor is comparable to the SAMPLE Collaboration experimental result \( F_{2p}^{(s), exp} = 0.23 \pm 0.37 \pm 0.15 \pm 0.19 \) n.m. \[15\].
4 Conclusions

In this paper, we have considered the strange form factors of the baryon decuplet by including the implicit FSB effects via the higher dimensional IR mixing scheme. In this approach the CBM can be treated to yield the minimal multiquark structure in the quantum mechanical perturbation theory.

Using the chiral bag with the bag radius $R \sim 0.6$ fm corresponding to the magic angle $\theta(R) = \pi/2$ we have obtained the magnetic moments $\mu_{\Delta^{++}} = 3.59$ n.m. and $\mu_{\Sigma^-} = -1.75$ n.m., comparable to the experimental values $\mu_{\Delta^{++}}^{\text{exp}} = 4.52 \pm 0.50$ n.m. and $\mu_{\Sigma^-}^{\text{exp}} = -1.94 \pm 0.17 \pm 0.14$ n.m., respectively. Here one notes that the proton strange form factor was predicted$^{[25]}$ in the CBM with the value $F_{2p}^{(s)} = 0.30$ n.m. which is also comparable to the SAMPLE Collaboration experimental data $F_{2p}^{(s),\text{exp}} = 0.23 \pm 0.37 \pm 0.15 \pm 0.19$ n.m.$^{[15]}$.

From the SU(3) group structure of the chiral models, such as Skyrmion, MIT and chiral bag models, we have derived the sum rules among the magnetic moments of the baryon decuplet. Here one notes that these model independent sum rules also hold in the nonrelativistic quark and chiral quark soliton models and the chiral perturbation theory.

Substituting the charge operator in the EM currents with the s-flavor charge operator, we have obtained, in the zero momentum transfer limit, the strange form factors of the baryon decuplet in terms of the strangeness quantum number of the baryon and the strange components of the baryon decuplet magnetic moments. Here one notes that the strange form factors are degenerate in the isomultiplets with the same strangeness to respect the I-spin symmetry.

On the other hand, in the multiquark structure of the CBM with the chiral and SU(3) FSB, the strange form factors of baryon decuplet can be broken up into three parts: the chiral symmetric contribution, the explicit FSB one and the implicit FSB one with the representation mixing. The theoretical predictions for the strange form factors show that the implicit FSB effects are dominant in the $\Delta$ and $\Sigma^*$ decuplet baryons and in any case the higher dimensional IR mixing in the baryon wave function in the multiquark Fock space cannot be omitted in the strange form factors of the decuplet baryons.

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Table 1: The baryon decuplet magnetic moments \( \mu_B = \mu_{0,B} + \delta \mu_{1,B} + \delta \mu_{2,B} \) calculated in the chiral bag with bag radius \( R \sim 0.6 \) fm corresponding to the magic angle \( \theta(R) = \pi/2 \).

| \( B \)  | \( \mu_{0,B} \) | \( \delta \mu_{1,B} \) | \( \delta \mu_{2,B} \) | \( \mu_B \)  |
|--------|----------------|----------------|----------------|--------|
| \( \Delta^{++} \) | 3.00 | 0.41 | 0.18 | 3.59 |
| \( \Delta^{+} \)  | 1.50 | 0.28 | -1.03 | 0.75 |
| \( \Delta^{0} \)  | 0.00 | 0.15 | -2.24 | -2.09 |
| \( \Delta^{-} \)  | -1.50 | 0.02 | -3.45 | -4.93 |
| \( \Sigma^{++} \) | 1.50 | 0.12 | 0.73 | 2.35 |
| \( \Sigma^{+0} \) | 0.00 | 0.00 | -0.76 | -0.76 |
| \( \Sigma^{*-} \) | -1.50 | -0.13 | -2.24 | -3.87 |
| \( \Xi^{*0} \)  | 0.00 | -0.15 | 0.73 | 0.58 |
| \( \Xi^{*-} \)  | -1.50 | -0.28 | -1.03 | -2.81 |
| \( \Omega^{-} \)  | -1.50 | -0.43 | 0.18 | -1.75 |

Table 2: The baryon decuplet magnetic moments \( \mu_B^{CBM} \) and \( \mu_B^{SM} \) calculated in the chiral bag model (CBM) and Skyrmion model (SM), compared with the nonrelativistic quark model (NRQM), relativistic quark model (RQM), lattice gauge theory (LGT), chiral perturbation theory (CPT) and the experimental data.† The quantity used as input is indicated by ∗.

| \( B \)  | \( \mu_B^{NRQM} \) | \( \mu_B^{RQM} \) | \( \mu_B^{LGT} \) | \( \mu_B^{CPT} \) | \( \mu_B^{SM} \) | \( \mu_B^{CBM} \) |
|--------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( \Delta^{++} \) | 3.58 | 4.76 | 4.91 | 4.00 | 2.82 | 3.59 |
| \( \Delta^{+} \)  | 2.79 | 2.38 | 2.46 | 2.10 | 1.04 | 0.75 |
| \( \Delta^{0} \)  | 0.00 | 0.00 | 0.00 | -0.17 | -0.74 | -2.09 |
| \( \Delta^{-} \)  | -2.79 | -2.38 | -2.46 | -2.25 | -2.52 | -4.93 |
| \( \Sigma^{++} \) | 3.11 | 1.82 | 2.55 | 2.00 | 1.60 | 2.35 |
| \( \Sigma^{+0} \) | 0.32 | -0.27 | 0.27 | -0.07 | -0.28 | -0.76 |
| \( \Sigma^{*-} \) | -2.47 | -2.36 | -2.02 | -2.20 | -2.17 | -3.87 |
| \( \Xi^{*0} \)  | 0.64 | -0.60 | 0.46 | 0.10 | 0.18 | 0.58 |
| \( \Xi^{*-} \)  | -2.15 | -2.41 | -1.68 | -2.00 | -1.81 | -2.81 |
| \( \Omega^{-} \)  | -1.83 | -2.35 | -1.40 | -1.94* | -1.46 | -1.75 |

† For the experimental data \( \mu_\Delta^{exp} = 4.52 \pm 0.50 \) and \( \mu_\Omega^{-}^{exp} = -1.94 \pm 0.17 \pm 0.14 \) we have referred to the ref. [2] and ref. [3], respectively.
Table 3: The strange form factors of the baryon octet and decuplet $F_{2B}^{(s)} = F_{2B}^{(s),0} + \delta F_{2B}^{(s),1} + \delta F_{2B}^{(s),2}$ calculated in the chiral bag, compared with the Skyrmion model predictions $F_{2B}^{(s)SM}$.

| B    | $F_{2B}^{(s),0}$ | $\delta F_{2B}^{(s),1}$ | $\delta F_{2B}^{(s),2}$ | $F_{2B}^{(s)}$ | $F_{2B}^{(s)SM}$ |
|------|----------------|------------------------|------------------------|----------------|-----------------|
| $N$  | $-0.19$        | $-0.12$                | $0.61$                 | $0.30$        | $-0.02$         |
| $\Lambda$ | $0.55$     | $0.35$                 | $-0.41$                | $0.49$        | $0.51$          |
| $\Sigma$ | $-1.89$    | $-0.34$                | $0.69$                 | $-1.54$       | $-1.74$         |
| $\Xi$ | $0.07$        | $0.45$                 | $-0.27$                | $0.25$        | $0.09$          |
| $\Delta$ | $-1.17$    | $-0.43$                | $3.27$                 | $1.67$        | $0.04$          |
| $\Sigma^*$ | $-0.67$   | $0.00$                 | $1.51$                 | $0.84$        | $-0.10$         |
| $\Xi^*$ | $-0.17$      | $0.43$                 | $0.30$                 | $0.56$        | $-0.03$         |
| $\Omega$ | $0.34$      | $0.85$                 | $-0.36$                | $0.83$        | $0.24$          |