Temperature Dependence of the Symmetry Energy Components for Finite Nuclei

A N Antonov, D N Kadrev, M K Gaidarov, P Sarriguren, E Moya de Guerra

1Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia 1784, Bulgaria
2Instituto de Estructura de la Materia, IEM-CSIC, Serrano 123, E-28006 Madrid, Spain
3Departamento de Fisica Atomica, Molecular y Nuclear, Facultad de Ciencias Fisicas, Universidad Complutense de Madrid, E-28040 Madrid, Spain
E-mail: gaidarov@inrne.bas.bg

Abstract. We investigate the temperature dependence of the volume and surface components of the nuclear symmetry energy (NSE) and their ratio in the framework of the local density approximation. The results of these quantities for finite nuclei are obtained within the coherent density fluctuation model (CDFM). The CDFM weight function is obtained using the temperature-dependent proton and neutron densities calculated through the HFBTHO code that solves the nuclear Skyrme-Hartree-Fock-Bogoliubov problem by using the cylindrical transformed deformed harmonic-oscillator basis. We present and discuss the values of the volume and surface contributions to the NSE and their ratio obtained for the Ni, Sn, and Pb isotopic chains around double-magic $^{78}$Ni, $^{132}$Sn, and $^{208}$Pb nuclei. The results for the $T$-dependence of the considered quantities are compared with estimations made previously for zero temperature showing the behavior of the NSE components and their ratio, as well as with the available experimental data. The sensitivity of the results on various forms of the density dependence of the symmetry energy is studied. We confirm the existence of “kinks” of these quantities as functions of the mass number at $T = 0$ MeV for the double closed-shell nuclei $^{78}$Ni and $^{132}$Sn and the lack of “kinks” for the Pb isotopes, as well as the disappearance of these kinks as the temperature increases.

1. Introduction

The nuclear symmetry energy is related to the energy connected with the conversion of the isospin asymmetric nuclear matter into symmetric nuclear matter. As noted in [1] information about NSE from laboratory experiments can be obtained from quantities that are sensitive to it, such as static properties, nuclear excitations, collective motions, heavy-ion reactions and others. One can add also interesting phenomena including the supernova explosions, properties of neutron stars and rare isotopes, frequencies and strain amplitudes of gravitational waves from both isolated pulsars and collisions involving neutron stars that depend strongly on the equation of state of neutron-rich nuclear matter.

The method of the CDFM [2, 3] allowed us to make the transition from nuclear matter to finite nuclei in the studies of the NSE for spherical [4] and deformed [5] nuclei, as well as for Mg isotopes [6] using the Brueckner energy-density functional (EDF) of asymmetric nuclear matter [7, 8]. In [9] we used a similar method to investigate the $T$-dependence of the NSE for...
isotopic chains of even-even Ni, Sn, and Pb nuclei following the local density approximation (see, for instance, Ref. [10]) and using instead of the Brueckner EDF, the Skyrme EDF with SkM* and SLy4 forces. The $T$-dependent local densities $\rho_\sigma(r, T)$ and kinetic energy densities $\tau(r, T)$ were calculated within a self-consistent Skyrme HFB method using the cylindrical transformed deformed harmonic-oscillator basis (HFBTHO) [11, 12] with the same forces.

Agrawal et al. [10] pointed out the substantial change in the NSE coefficients for finite nuclei with temperature in comparison with the case of nuclear matter. Lee and Mekjian [13] emphasized the bigger sensitivity of the surface component of NSE to the temperature in respect to the volume energy term. The values of the temperature-dependent symmetry energy coefficients for sixty nine spherical and non-spherical nuclei with mass $36 \leq A \leq 218$ and charge $14 \leq Z \leq 92$ numbers, respectively, have been evaluated in Ref. [14] in the subtracted finite-temperature Thomas-Fermi framework by using two different energy-density functionals. A substantial temperature dependence of the surface symmetry energy was found, while the volume symmetry energy turned out to be less sensitive to the temperature. As a result, the symmetry energy coefficient of finite nuclei falls down as the temperature rises [14], an observation that has been confirmed in Ref. [9]. A study of the decomposition of NSE into spin and isospin components is carried out in Ref. [15] within the Brueckner-Hartree-Fock approximation with two- and three-body forces.

In our work [16] the volume and surface contributions to the NSE and their ratio were calculated within the CDFM using two EDF’s, namely the Brueckner [7, 8] and Skyrme ones. The CDFM weight function was obtained by means of the proton and neutron densities obtained from the self-consistent deformed HF+BCS method with density-dependent Skyrme interactions. The obtained results in the cases of Ni, Sn, and Pb isotopic chains were compared with results of other theoretical methods and with those from other approaches which used experimental data on binding energies, excitation energies to isobaric analog states (IAS), neutron-skin thicknesses and with results of other theoretical methods. We note that in [16] the obtained values of the volume and surface components of NSE and their ratio concern the case at $T = 0$ MeV.

The aim of the present work (see also Ref. [17]) is to evaluate the above mentioned quantities for temperatures different from zero. The $T$-dependent local density distributions $\rho_\sigma(r, T)$ and $\rho_\alpha(r, T)$ computed by the HFBTHO code are used to calculate the $T$-dependent CDFM weight function. Such an investigation of the thermal evolution of the NSE components and their ratio for isotopes belonging to the Ni, Sn, and Pb chains around the double-magic nuclei, will extend our previous analysis of these nuclei considering them as cold systems [16]. At the same time, the obtained results within the CDFM provide additional information on the thermal mapping of the volume and surface symmetry energies that has been poorly investigated till now (e.g., Ref. [14]). In addition, we study the sensitivity of the calculated $T$-dependent quantities on different available forms of the density dependence of the symmetry energy.

2. Theoretical formalism

The symmetry energy term in the expression for the nuclear energy given in the droplet model can be written in the form

$$S(T) \frac{(N - Z)^2}{A},$$

where

$$S(T) = \frac{S^V(T)}{1 + \frac{S^S(T)}{S^V(T)} A^{-1/3}} = \frac{S^V(T)}{1 + A^{-1/3}/\kappa(T)},$$

with

$$\kappa(T) = \frac{S^V(T)}{S^S(T)}.$$

2
In the case of nuclear matter, where $A \rightarrow \infty$ and $S^S/S^V \rightarrow 0$, we have $S(T) = S^V(T)$. From Eq. (2) follow the relations of $S^V(T)$ and $S^S(T)$ with $S(T)$:

\begin{align}
S^V(T) &= S(T) \left[ 1 + \frac{1}{\kappa(T)A^{1/3}} \right], \\
S^S(T) &= \frac{S(T)}{\kappa(T)} \left[ 1 + \frac{1}{\kappa(T)A^{1/3}} \right].
\end{align}

In what follows we use essentially the CDFM scheme to calculate the NSE and its components (see Refs. [2, 3, 16]). We have shown in our previous works [4, 5, 16] that the NSE in the CDFM for temperature $T = 0$ MeV can be obtained in the form

\begin{equation}
S = \int_{0}^{\infty} dx |F(x)|^2 S[\rho(x)],
\end{equation}

where the symmetry energy for the asymmetric nuclear matter that depends on the density $S[\rho(x)]$ has to be determined using a chosen EDF (in [16] Brueckner and Skyrme EDF’s have been used).

In this work the $T$-dependent NSE $S(T)$ is calculated by the expressions similar to Eq. (6) but containing $T$-dependent quantities:

\begin{equation}
S(T) = \int_{0}^{\infty} dx |F(x, T)|^2 S[\rho(x, T)].
\end{equation}

In Eq. (7) the weight function $|F(x, T)|^2$ depends on the temperature through the temperature-dependent total density distribution $\rho_{\text{total}}(r, T)$:

\begin{equation}
|F(x, T)|^2 = -\frac{1}{\rho_0(x)} \frac{d\rho_{\text{total}}(r, T)}{dr} \bigg|_{r=x},
\end{equation}

where

\begin{equation}
\rho_{\text{total}}(r, T) = \rho_p(r, T) + \rho_n(r, T),
\end{equation}

$\rho_p(r, T)$ and $\rho_n(r, T)$ being the proton and neutron $T$-dependent densities that in our work [9] were calculated using the Hartree-Fock-Bogoliubov method with transformed harmonic-oscillator basis and the HFBTHO code [11].

As mentioned, in the present work we consider the $T$-dependence of the NSE $S(T)$, but also of its volume $S^V(T)$ and surface $S^S(T)$ components and their ratio $\kappa(T)$ [Eq. (3)]. Following Refs. [16, 18, 19, 20] an approximate expression for the ratio $\kappa(T)$ can be written within the CDFM:

\begin{equation}
\kappa(T) = \frac{3}{R_0 \rho_0} \int_{0}^{\infty} dx |F(x, T)|^2 x \rho_0(x) \left\{ \frac{S(\rho_0)}{S[\rho(x, T)]} - 1 \right\},
\end{equation}

where $|F(x, T)|^2$ is determined by Eq. (8), $R = r_0 A^{1/3}$ [20] and $S(\rho_0)$ is the NSE at equilibrium nuclear matter density $\rho_0$ and $T = 0$ MeV. For instance, the values of $S(\rho_0)$ for different Skyrme forces in the Skyrme EDF are given in Table II of Ref. [16]. In what follows we use the commonly employed power parametrization for the density dependence of the symmetry energy (e.g., [19, 20, 21])

\begin{equation}
S[\rho(x, T)] = S^V(T) \left( \frac{\rho(x, T)}{\rho_0} \right)^\gamma.
\end{equation}

There exist various estimations for the value of the parameter $\gamma$. For instance, in Ref. [20] $\gamma = 0.5 \pm 0.1$ and in Ref. [19] $0.54 \leq \gamma \leq 0.77$. The estimations in Ref. [22] (given in Table 2
follows: isotopic chain.

and 0 using the SLy4 Skyrme force. The results are presented for two values of the parameter mass number (APR [23]). Another estimation of we would like to note that the values of increasing temperatures (1 = 1–3 MeV), while \( \kappa(T) \) slowly increases when \( T \). Here \( \gamma \) is around 2.03 (N"LO), 0.55 \pm 0.03 (N"LO), and 0.55 (DBHF) and 0.79 (APR [23]). Another estimation of \( \gamma \) is also given in Ref. [24].

Using Eq. (11) (and having in mind that \( S(\rho_0) = S^V \)), Eqs. (7) and (10) can be rewritten as follows:

\[
S(T) = S(\rho_0) \int_0^\infty dx |\mathcal{F}(x, T)|^2 \left[ \frac{\rho(x, T)}{\rho_0} \right]^\gamma, \quad (12)
\]

\[
\kappa(T) = \frac{S^V(T)}{S^S(T)} = \frac{3}{R\rho_0} \int_0^\infty dx |\mathcal{F}(x, T)|^2 \rho_0(x) \left\{ \left[ \frac{\rho_0}{\rho(x, T)} \right]^\gamma - 1 \right\}. \quad (13)
\]

We also study the sensitivity of the obtained results towards different density dependences of the symmetry energy. First, we note the Eq. (6) from Ref. [25]:

\[
S_2(\rho) = C_k \left( \frac{\rho}{\rho_0} \right)^{2/3} + C_1 \left( \frac{\rho}{\rho_0} \right) + C_2 \left( \frac{\rho}{\rho_0} \right)^{1.52}. \quad (14)
\]

This relationship, which coincides with the shape from the density-dependent M3Y interaction [26], is similar to that of Eq. (9) of Ref. [27], where the last term of Eq. (14) is exchanged by \( C_2(\rho/\rho_0)^{5/3} \). Second, we use also the dependence of the symmetry energy [25, 28]

\[
S_4(\rho) = 12.5 \left( \frac{\rho}{\rho_0} \right)^{2/3} + 17.6 \left( \frac{\rho}{\rho_0} \right)^\gamma. \quad (15)
\]

It was noted in [28] that the analysis of both isospin diffusion and double ratio data involving neutron and proton spectra by an improved quantum molecular dynamics transport model suggests values of \( \gamma = 0.4 \pm 0.05 \).

3. Results and discussion

Studying the \( T \)-dependence of the mentioned quantities we observed a certain sensitivity of the results to the value of the parameter \( \gamma \) in Eq. (11). In order to make a choice of its value we imposed the following physical conditions: i) the obtained results for the considered quantities at \( T = 0 \) MeV to be equal or close to those obtained for the same quantities in our previous works for the NSE, its components and their ratio (Ref. [9, 16]), and ii) their values for \( T = 0 \) MeV to be compatible with the available experimental data (see, e.g., the corresponding references in [16]).

In Fig. 1 the results for \( S(T) \), \( S^V(T) \), \( S^S(T) \), and \( \kappa = S^V(T)/S^S(T) \) are given as functions of the mass number \( A \) for the isotopic chain of Ni nuclei for temperatures \( T = \) 0–3 MeV calculated using the SLy4 Skyrme force. The results are presented for two values of the parameter \( \gamma = 0.3 \) and 0.4. The reason for this choice is related to the physical criterion mentioned above. It can be seen that at \( T = 0 \) MeV and \( \gamma = 0.4 \) the value of \( \kappa \) is around 2.6. This result is in agreement with our previous result obtained in the case of the Brueckner EDF in Ref. [16], namely 2.10 \( \leq \kappa \leq 2.90 \). The latter is compatible with the published values of \( \kappa \) extracted from nuclear properties presented in Ref. [20] from the IAS and skins [21] (2.6 \( \leq \kappa \leq 3.0 \)) and from masses and skins [18] (2.0 \( \leq \kappa \leq 2.8 \)). In the case of \( \gamma = 0.3 \) our result for \( T = 0 \) MeV is \( \kappa = 1.65 \) that is in agreement with the analyses of data in Ref. [20] (1.6 \( \leq \kappa \leq 2.0 \)), as well as with the results of our work [16] in the case of Skyrme EDF, namely, 1.5 \( \leq \kappa \leq 1.7 \) for the Ni isotopic chain.

It can be seen from Fig. 1 that the quantities \( S(T) \), \( S^V(T) \), and \( S^S(T) \) decrease with increasing temperatures (\( T = \) 0–3 MeV), while \( \kappa(T) \) slowly increases when \( T \) increases. Here we would like to note that the values of \( \gamma \) between 0.3 and 0.4 that give an agreement of the studied quantities with data, as well as with our previous results for \( T = 0 \) MeV, are in the
lower part of the estimated limits of the values of \( \gamma \) (e.g., in the case of \( \gamma = 0.5 \pm 0.1 \) [20]). It can be seen also from Fig. 1 that there are “kinks” in the curves of \( S(T) \), \( S^V(T) \), \( S^S(T) \), and \( \kappa(T) \) for \( T = 0 \) MeV in the case of the double closed-shell nucleus \( ^{76}\text{Ni} \). This had been observed also in our previous work for \( S(T) \) [9], as well as for its volume and surface components and their ratio \( \kappa \) at \( T = 0 \) MeV in Ref. [16].

In Fig. 2 are given the results for the \( T \)-dependence of \( S(T) \), \( S^V(T) \), \( S^S(T) \), and \( \kappa(T) \) for the double-magic \( ^{132}\text{Sn} \) nucleus obtained using both SkM* and SLy4 Skyrme forces. The results are presented by grey areas between the curves for the values of the parameter \( \gamma = 0.3 \) and \( \gamma = 0.4 \). It can be seen that \( S(T) \), \( S^V(T) \), and \( S^S(T) \) decrease, while \( \kappa(T) \) slowly increases with the increase of the temperature for both Skyrme forces.

Figure 3 illustrates the behavior of the density dependence of the symmetry energy \( S(\rho) \) by giving some of the curves for different functions, namely, \( S_2(\rho) \), three curves using Eq. (11) that we label as \( S_0(\rho) \) with \( \gamma = 0.3 \), 0.4, 0.7, as well as three curves for \( S_4(\rho) \) that correspond to \( \gamma = 0.2 \), 0.3, 0.7. At this point we emphasize that the CDFM weight function \(|F(x,T)|^2 \) which is used in Eqs. (7) and (10) has a form of a bell with a maximum around \( x = R_{1/2} \) at which the value of the density \( \rho(x = R_{1/2}) \) is half of the value of the central density equal to \( \rho_0 [\rho(R_{1/2})/\rho_0 = 0.5] \). So, namely in this region (around \( \rho/\rho_0 = 0.5 \)), the values of the different \( S(\rho) \) play the main role in the calculations of \( S(T) \) [Eq. (7)] and \( \kappa(T) \) [Eq. (10)].

In Fig. 4 we consider, as an example, the mass dependence of \( S(T) \), \( S^V(T) \), \( S^S(T) \), and \( \kappa = S^V(T)/S^S(T) \) in the case of the Ni isotopic chain for temperatures \( T = 0 \text{–} 3 \) MeV using the SLy4 Skyrme force. The results are given when the symmetry energy has the form of \( S_4(\rho) \) at \( \gamma = 0.2 \) and 0.3. One can see that, e.g., the value of \( \kappa \) at \( T = 0 \) MeV and \( \gamma = 0.2 \) is 1.90 that is close to the result obtained using Eqs. (11) and (13) for \( \gamma = 0.3 \) (it is \( \kappa = 1.66 \)) shown in the left panel of Fig. 1. The value of \( \kappa \) when \( \gamma = 0.3 \) is 2.69 which is similar to that in the
case of Eq. (11) and (13) for γ = 0.4 (it is κ = 2.64) shown in the right panel of Fig. 1. There exist in these cases similarities also of the behavior of the quantities S(T), S\(^V\)(T), and S\(^S\)(T) as functions of T in the corresponding cases. The reason for the mentioned similarities is the closeness of the corresponding curves shown in Fig. 3 in the region around \( \rho/\rho_0 = 0.5 \).

![Figure 3](image-url) Behavior of the density-dependent symmetry energy: \( S_2(\rho) \) with \( S(\rho_0) = 31.6 \) MeV [Eq. (14)] with \( C_k = 17.47, C_1 = 27.94, \) and \( C_2 = -13.81 \) (blue dash-dotted line); \( S_4(\rho) \) [Eq. (15)] with \( \gamma = 0.2 \) (red short-dashed line), \( \gamma = 0.3 \) (red solid line), \( \gamma = 0.7 \) (red dashed line), and \( S_0(\rho) \) [Eq. (11)] with \( \gamma = 0.3 \) (black solid line), \( \gamma = 0.4 \) (black dotted line), \( \gamma = 0.7 \) (black dashed line).

![Figure 4](image-url) Same as in Fig. 1, but with use of \( S_4(\rho) \) [Eq. (15)] for values of the parameter \( \gamma = 0.2 \) (left panel) and \( \gamma = 0.3 \) (right panel).

4. Conclusions

The results of the present work can be summarized as follows:

(i) With increasing T, the quantities \( S, S^V, \) and \( S^S \) decrease, while \( \kappa \) slightly increases for all the isotopes in the three chains for both Skyrme forces and for all used density dependences of the symmetry energy. The same conclusion can be drawn for the thermal evolution of the mentioned quantities in the cases of the three considered double-magic \(^{78}\)Ni, \(^{132}\)Sn, and \(^{208}\)Pb nuclei.

(ii) The results for \( S(T), S^V(T), S^S(T), \) and \( \kappa(T) \) are sensitive to the choice of the density dependence of the symmetry energy \( S[\rho(x, T)] \) in Eqs. (7) and (10). The sensitivity of the studied quantities towards the values of the parameter \( \gamma \) in Eq. (11) is shown. The results with use of other different density dependences of the symmetry energy [Eqs. (14) and (15)] are considered in detail. The similarities and differences between the results from various functional forms are related to the behavior of the corresponding values of \( S[\rho(x, T)] \) around the value of the ratio \( \rho(x, T)/\rho_0 = 0.5 \) for which the CDFM weight function has a maximum.

(iii) In the cases of double-magic \(^{78}\)Ni and \(^{132}\)Sn nuclei we observe “kinks” for \( T = 0 \) MeV in the curves of \( S(T), S^V(T), S^S(T), \) and \( \kappa(T) \), but not in the case of Pb isotopes. This effect was also observed in our previous works. It is also worth mentioning how the kinks are blurred and eventually disappear as \( T \) increases, demonstrating its close relationship with the shell structure.
5. Acknowledgments
P.S. acknowledges support from Ministerio de Ciencia, Innovación y Universidades MCIU/AEI/FEDER,UE (Spain) under Contract No. PGC2018-093636-B-I00.

References
[1] Li Bao-An 2017 Nucl. Phys. News 27 7
[2] Antonov A N, Nikolaev V A and Petkov I Zh 1979 Bulg. J. Phys. 6 151; 1980 Z. Phys. A 297 257; ibid 1982 304 239; 1985 Nuovo Cimento A 86 23; Antonov A N et al., ibid 1989 102 1701; Antonov A N, Kadrev D N and Hodgson P E 1994 Phys. Rev. C 50 164
[3] Antonov A N, Hodgson P E and Petkov I Zh 1988 Nucleon Momentum and Density Distributions in Nuclei (Clarendon Press, Oxford); 1993 Nucleon Correlations in Nuclei (Springer-Verlag, Berlin-Heidelberg-New York)
[4] Gaidarov M K, Antonov A N, Sarriguren P and Moya de Guerra E 2011 Phys. Rev. C 84 034316
[5] Gaidarov M K, Antonov A N, Sarriguren P and Moya de Guerra E 2012 Phys. Rev. C 85 064319
[6] Gaidarov M K, Sarriguren P, Antonov A N and Moya de Guerra E 2014 Phys. Rev. C 89 064301
[7] Brueckner K A, Buchler J R, Jorna S and Lombard R J 1968 Phys. Rev. 171 1188
[8] Brueckner K A, Buchler J R, Clark R C and Lombard R J 1969 Phys. Rev. 181 1543
[9] Antonov A N, Kadrev D N, Gaidarov M K, Sarriguren P and Moya de Guerra E 2017 Phys. Rev. C 95 024314
[10] Agrawal B K, De J N, Samaddar S K, Centelles M and Viñas X 2014 Eur. Phys. J. A 50 19
[11] Stoitsov M, Schunck N, Kortelainen M, Michel N, Nam H, Olsen E, Sarich J and Wild S 2013 Computer Physics Communications 184 1592
[12] Stoitsov M V, Dobaczewski J, Nazarewicz W and Ring P 2005 Comput. Phys. Comm. 167 43
[13] Lee S J and Mekjian A Z 2010 Phys. Rev. C 82 064319
[14] De J N, Samaddar S K and Agrawal B K 2012 Phys. Lett. B 716 361
[15] Guo W, Colonna M, Greco V, Lombardo U and Schultz H J 2018 arXiv: 1804.04827 [nucl-th]
[16] Antonov A N, Gaidarov M K, Sarriguren P and Moya de Guerra E 2016 Phys. Rev. C 94 014319
[17] Antonov A N, Kadrev D N, Gaidarov M K, Sarriguren P and Moya de Guerra E 2018 Phys. Rev. C 98 054315
[18] Danielewicz P 2003 Nucl. Phys. A 727 233
[19] Danielewicz P 2006 arXiv: 0607030 [nucl-th]
[20] Dieperink A E L and Van Isacker P 2007 Eur. Phys. J. A 32 11
[21] Danielewicz P 2004 arXiv: 0411115 [nucl-th]
[22] Sammarruca F 2017 Mod. Phys. Lett. A 32 1730027
[23] Akmal A, Pandharipande V R and Ravenhall D G 1998 Phys. Rev. C 58 1804
[24] Russotto P et al. 2016 Phys. Rev. C 94 034608
[25] Dong J, Zhuo W, Gu J and Lombardo U 2012 Phys. Rev. C 85 033308
[26] Mukhopadhyay T and Basu D N 2007 Nucl. Phys. A 789 201
[27] Agrawal B K, De J N, Samaddar S K, Colò G and Gulaksono A 2013 Phys. Rev. C 87 051306(R)
[28] Tsang M B, Zhang Y, Danielewicz P, Famiano M, Li Z, Lynch W G and Steiner A W 2009 Phys. Rev. Lett. 102 122701; Tsang M B et al. (2010) Int. J. Mod. Phys. E 19 1631