Abstract—We study the problem of optimizing the cost of content delivery in a cooperative network of caches at end-nodes. The caches could be, for example, within the computers of users downloading videos from websites (such as Netflix, Blockbuster etc.), DVRs (such as TiVo, or cable boxes) used as part of video on demand services or public hot-spots (e.g. Wi-Fi access points with a cache) deployed over a city to serve content to mobile users. Each cache serves user requests locally over a medium that incurs no additional costs (i.e. WiFi, home LAN); if a request is not cached, it must be fetched from another cache or a central server. In our model, each cache has a tiered backhaul internet connection, with a usage cap (and fixed per-byte costs thereafter). Redirecting requests intended for the central server to other caches with unused backhaul capacity can bring down the network costs. Our goal is to develop a mechanism to optimally 1) place data into the caches and 2) route requests to caches to reduce the overall cost of content delivery.

We develop a multi-criteria approximation based on a LP rounding procedure that with a small (constant factor) blow-up in storage and upload limits of each cache, gives a data placement that is within constant factor of the optimum. Further, to speed up the solution, we propose a technique to cluster caches into groups, solve the data placement problem within a group, and combine the results in the rounding phase to get the global solution. Based on extensive simulations, we show that our schemes perform very well in practice, giving costs within 5 – 15% to the optimal, and reducing the network load at a central server by as much as 55% with only a marginal blow up in the limits. Also we demonstrate that our approach out-performs a non-cooperative caching mechanism by about 20%.

I. INTRODUCTION

The internet has seen a massive growth in traffic in recent years, and CDNs (Content Distribution Networks) play an important role in increasing the effective network bandwidth and reducing content access latency. A significant percentage of internet traffic is now driven by bulk traffic, i.e. large-sized files, such as video, software patches, software downloads etc. In this work we explore the design of a CDN - incorporating co-operating end-users - for bulk content delivery.

As a representative example of this class of traffic - we will motivate our work by focussing on video downloads - driven by the observation that popular video-sites are amongst the top (and fastest growing) contributors to overall Internet traffic.

Video applications are inherently bandwidth hungry, and with the increasing popularity of full-length, long duration video downloads - the management of video traffic has become an important problem in today’s internet. For example, consider a web-site (such as Netflix) from which users download and rent or purchase full-length movies. Each movie file will have a size of about tens of GBs, and assuming hundreds of thousands of active users, one can easily envision the web-site and the network having to serve peta-bytes of data (and increasing) monthly.

A natural direction towards alleviating this problem is to involve end-users in the content delivery architecture. Web-sites (and the network) can incentivize and manage users to act as content relays, i.e. requests can be redirected by the web-site to be served by another user who has a cached copy of the request. Such p2p based systems not only reduce the load on the content servers, but as has been pointed out by some recent work, they can also reduce load on the network (and enhance end-user performance) by exploiting the geographical locality of content and careful peer selection. There exist several practical examples of this approach - web-sites that push out large software updates (e.g. Windows patches) to millions of users, and BBC’s IPlayer video service have employed p2p based techniques to propagate content across their users.

Video files naturally lend themselves to a peer-peer based approach for content delivery. Video content is typically held in the end-user cache for an extended period of time (for later/multiple viewings or simply as part of the users library of content). Also - in case of systems such as TiVo or video-on-demand services, the ISP or the content provider directly manage the hardware cache. For these reasons, a carefully managed system (with appropriate incentives for end-users) through which locally cached content can be used as a content source for other users seems entirely feasible and appropriate - in terms of potential gains.

Managed peer-peer content delivery has been a studied problem in literature e.g. Telco-managed p2p TV, CDN with managed swarms, ISP managed p2p systems. With the rise of data hungry mobile devices (e.g. iPhone users), there is increased importance of Wi-Fi hot-spots used to “side-load” content on to the user device, faster and cheaper than over the cellular data network. Going forward - such Wi-Fi hot-spots may have the ability to locally cache (to reduce backhaul costs to the hot-spots) and serve rich content. Hence, we believe, another important use-case is that of content delivery over a network of “info-stations” at the edge of the network. Such a content-delivery model has also been explored earlier by several works, e.g. the Rutgers Infostations project, the Drive-thru-Internet and MIT CarTel projects (in contexts such as opportunistic content-delivery to moving vehicle), and for low-cost rich content delivery on mobiles (e.g. the mango project 1).

1\url{www.bbc.co.uk/iplayer/}
A managed p2p system of the kind we consider will comprise of a set of peer caches and one or more content servers. The end-user connects to one of the caches for content download. If the data is cached locally, it will be served immediately, free-of-cost. Otherwise, the request will be forwarded to the content server which will figure out the cache that retains a copy of the content and the download will resume in a p2p fashion.

Once the end-nodes are involved in the caching process, the content server needs to be even more mindful of the local constraints while making the global decisions e.g. constraints like node failures, storage capacity at the end-nodes become critical for survivability of the network. These issues have been studied throughly in the literature of delay tolerant network. In this work, we highlight another important aspect of end-node caching that has been previously ignored in the literature – given the deployment location of the end-nodes (homes, cafe, shops, malls etc.), it is likely that the back-haul connection will be a commodity broadband plan. Below we highlight some important facts regarding the commodity broadband plans that are of use today –

• A significant number of broadband plans are defined by a tiered cost model i.e. the end user pays a fixed amount every month with a usage limit, typically in the order of GBs. When this capacity is exceeded, the user will pay a penalty in terms of per-byte charge. Such a price model has been introduced/considered by US-based cable broadband providers like Comcast, Time Warner. Similarly in India, major ISPs like Airtel, BSNL have a range of tiered plans, with usage caps ranging from 1GB to 75GB. Finally, such limits and rate plans will be the norm for the increasing popular mobile broadband connections.
• In practical deployments, the content server has a high data rate connection that is bought in bulk from the service provider and hence has typically a lesser penalty than the content caches.

As per the facts mentioned above, the end-nodes are likely to have a monthly limits on the amount of uploads and downloads that can be allowed free-of-cost. In practice, not every node will reach this limit. This scenario gives rise to an interesting data management problem where a missed request for a cache can be opportunistically routed to another one that has a spare capacity to serve it, instead of always fetching it from the central server. In this paper, we formally model a system of cooperative caches with tiered cost function. The unique cost structure that we explore in this paper adds an intrinsic difficulty in the caching problem.

A. Contributions

In this paper we formally model a system for sideloading mobile content, and we make the following contributions:

• To the best of our knowledge, we are the first to model the back-haul costs of the caches as a tiered cost function (as is the case in practice), and show that it gives rise to a novel problem previously unstudied in literature.
• We prove the problem to be NP-hard, and propose an algorithm for (near) optimum placement of data on the caches. Our technique employs both a static as well as a dynamic component. Based on the request pattern, we statically place data on the caches to minimize the serving cost and supplement it with a cache replacement strategy to adapt to a dynamically changing stream of requests.
• We present a multi-criteria approximation algorithm for this problem, based on a novel LP rounding technique. Our algorithm produces a data placement that is within a constant factor of the optimum solution and results in a small (constant factor) blow-up in storage and upload limits of each cache. In experiments, our schemes give costs close to the optimum (within 5 − 15%), with a marginal blow up (< 1%) of the limits.
• Further, to speed up the solution we cluster caches into groups, solve the data placement problem within a group, and combine the results in the rounding phase to get the global solution. Experiments suggest that computationally tractable cluster sizes come well within 10% of a best possible solution.
• We demonstrate that co-operative caching with the tiered network cost model brings down the network delivery at a central server by as much as 55%, and is 20% better than a non-cooperative strategy.

II. RoadMap

The paper is organized as follows. In Section III we formally describe our assumptions and the system model and formulate the resulting optimization problem. We then survey the related work in this area in Section IV. Section V presents algorithms for a simplified data placement problem and analyze their properties and performance. In Section VI we present the complete solution for the data placement problem. We follow it up with a rigorous evaluation through simulations in Section VII. Finally we conclude with a summary of the work and future directions in Section VIII.

III. System Model and Problem Formulation

Our system of co-operating caches consists of three components - content caches at the end-nodes, a content server and a coordinator. Users are associated with one or more content caches, and can either access it directly (the cache is a disk on the user’s home computer), or via Ethernet LAN, WiFi or Bluetooth (the cache as a home networked storage device, or a public hot-spot).

The content server is a repository of all items requested, uploaded and shared by users. Any request at a cache is either

http://en.wikipedia.org/wiki/Delay-tolerant_networking

With a fixed monthly bill.
served locally, or via the content server, or by any other content cache. The coordinator also keeps track of the user behavior and the state of each cache (items stored, spare storage and capacity on the back-haul links).

A user’s content viewing habit will consist of sites visited regularly for popular content - e.g. online newspapers, video sites (such as TV episodes on Hulu), or recently popular user generated content on YouTube. In general, we believe there will be a strong predictive element to the popularly requested items in the system, and our system is designed to exploit this characteristic.

Based on usage statistics at each cache, sharing and mobility patterns of users, popularity of content items etc., periodically (over a time frame referred to as an epoch) the coordinator predicts the demands at each cache. Then, based on these predictions in each epoch, it i) places the items at each cache and ii) redirects subsequent user requests to suitable content caches, or to the content server.

The specific techniques required to predict user requests are an active and important area of research (and has been pursued in other works [12], [13]), but is orthogonal and beyond the scope and goal of this paper.

Also, typically, items popular in an epoch will retain their popularity for a period of time[14]. This can improve caching performance as the existing set of items in the cache need not be brought from the server in subsequent epochs. Hence, our system is designed to make data placement decisions keeping in mind the items already present in caches from previous epochs.

A. Problem Formulation

Consider a network of \(n+1\) nodes \(C = \{C_0 \ldots n\}\). Each node \(C_i\) (\(i > 0\)) represents a content cache. \(C_0\) represents the content server. For cache \(C_i\), let \(u_i\) and \(d_i\) specify the limit on data that can be uploaded/downloaded at the cache free-of-cost.

For any data transfer beyond the prescribed limits, \(\alpha_i\) and \(\beta_i\) denote the price per-byte paid by the cache for exceeding upload and download limits respectively. Let \(s_i\) denote the amount of storage at each cache. Table I summarizes the set of symbols for our problem formulation.

Let \(\{o_1, o_2 \ldots , o_m\}\) be a set of \(m\) data objects that are requested at the caches over an epoch[1]. In practice, accurate prediction of the time ordered sequence of requests at every cache may not be feasible. Instead, we assume the knowledge of an expected/average demand \(r_{ik}\) of object \(o_k\) at cache \(C_i\). As mentioned earlier, coarse level predictions of this kind is possible by mining user preferences and mobility patterns. However, the description of the specific prediction technique is beyond the scope of this paper.

At any moment of time, let \(d_i\) and \(u_i\) denote the amount of content that has been downloaded and uploaded respectively by cache \(C_i\), since the beginning of an epoch. A new request for content \(o_k\) at \(C_i\) will incur zero cost if served locally.

Otherwise, cache \(C_i\) will forward the query to \(C_j\) which has a copy of \(o_k\) and fetch it from there. This operation will increase both \(d_i\) and \(u_j\) by the size of \(o_k\). If no cache has \(o_k\) available then it will be downloaded from the server.

When all requests are served, any cache \(C_i\) incurs a cost of \(\max\{d_i - d_i, 0\} \cdot \beta_i + \max\{u_i - u_i, 0\} \cdot \alpha_i\) accounting for the extra downloads and uploads, respectively. The overall cost of content delivery in the system is thus \(\sum_{i=0}^n \{\max\{d_i - d_i, 0\} \cdot \beta_i + \max\{u_i - u_i, 0\} \cdot \alpha_i\}\). We now formally define the data placement problem studied in this paper.

**Data Placement Problem:** Given the set of caches \(\{C_{0 \ldots n}\}\) and demands \(R = \cup_{j,k}\{r_{jk}\}\), find the placement of items \(\{o_1 \ldots m\}\) into caches and a query forwarding policy that satisfies all the requests in the system with minimum total cost.

B. Solution Approach

We prove that the data placement problem is NP-hard by reducing it from the partition problem \(P(O, W, k)\), defined as follows: Given a set objects \(O = \{o_{1 \ldots n}\}\) of weights \(W = \{w_1 \ldots n\}\) (\(w_i \in Z^+\)) and a integer \(t < n\), is there a subset \(T \subseteq O\) of size \(|T| = t\), such that \(\sum_{i \in T} w_i = \sum_{i \in T}\)? We state the following result and refer the proof to Appendix A.

**Lemma 3.1:** The data placement problem defined above is NP-hard.
We now aim to find a good approximation algorithm to our problem. Due to the similarity of our problem with non-metric facility location problem [15], (which is known to be $O(\log(n))$ hard to approximate) we believe that our problem should be hard to approximate within a constant factor. Therefore, from a practical point of view, it would be useful to find a solution which is within constant factor of the optimum but stretches the storage and upload limits only by a constant factor. Such multi-criteria approximation has been adapted by other researchers [16] as well. With this motivation, we design a multi-criteria approximation algorithm for the data placement problem.

IV. RELATED WORK

Distributed, co-operative content-delivery [17] has been very successfully exploited by peer-peer applications (such as BitTorrent[8]). However, the model under which they operate is different - there is no centralized control, and any management of the end-node resources is purely local (e.g. cap outgoing BitTorrent sessions to 32Kbps). In the model we propose, a central entity collects the end-node information, and while mindful of the local constraints, makes optimum decision for the entire system.

The notion of co-operative caching has also been studied extensively in the literature [18], [19] before in the context of co-operating web proxies, or content distribution networks (such as Akamai). However, in our work, we assume the caches to be end-nodes, and the accompanying model for network bandwidth costs (tiered broadband model) is very different from the cost model assumed in previous studies (typically, per byte costs). This tiered cost model (particularly relevant for caching on end-nodes) creates a fundamentally different problem.

Baev et.al. [20], [21] study the data placement problem, where the aim is to map items and requests to caches to minimize the overall cost. However, unlike us, they consider only storage capacities, and not the upload/download capacity at caches. The paper closest related to our work is by Guha et.al. [16], where they consider the limit on number of users (or requests) that can be mapped to any cache. The crucial difference from our work is the assumption that each request served from a cache adds to the constraints or the cost (or both), while in our problem the requests served locally comes for free. This changes the problem structure significantly since in our case the optimal solution may serve arbitrary number of requests for free (locally), and makes it non-trivial to approximate using existing techniques.

Another related body of work is on data placement on parallel disks [19], [22], where the caches (media servers) have storage and load capacities and the aim is to map items and requests to caches; however, the aim here is to maximize the number of requests served, rather than the cost of serving all requests. Cooperative (proxy) caching has also been used for minimizing traffic in WWW [18], [23], however, again they do not consider upload/download capacities at caches.

Several researchers [24], [25] have analyzed the performance of peer-to-peer transfers for cooperative file transfers, downloading software patches, etc. Like us, they also use items (or parts of items) stored at one peer to serve request of another. However, since by design these systems are unmanaged, they mostly focus on either incentive mechanisms (to ensure that peers share) or in assisting users to pick “close by” peers to download from [26], [27]. In contrast, we have a managed system where a central controller predicts requests (based on historical data), takes into consideration the limits at caches and makes optimal data placement and routing decisions for the entire system.

Several projects [8], [28], [29], [30] have investigated issues emerging from serving content through a network of short-range hot-spots. Perhaps because the common use-case considered is to provide opportunistic Internet access from within moving vehicles, the primary focus of such work has been on the performance of the wireless link between the user and the hot-spots.

V. SOLUTION FOR A SIMPLIFIED DATA PLACEMENT PROBLEM

The aim of the data placement problem is to minimize the cost of satisfying all requests. This cost can be conceptually broken into two parts, the cost of putting the desired items in the caches at the beginning of an epoch and the cost of serving the requests (using the caches and the content server) throughout the epoch. We call them initial placement and serving costs, respectively. For ease of exposition, we start with solving a simplified problem that ignores the initial placement cost and optimizes only the serving cost[9]. The solution of the original problem builds upon the steps of this formulation, and we will discuss it in Section VI. Throughout this section, we will discuss the minimization of serving cost only and refer to it as data placement problem. We will also work with unit sized objects to keep things simple. However, our algorithms are easy to modify to account for non-uniform object sizes.

We will now describe our technique to solve the simplified data placement problem and get a multi-criteria approximation. The basic idea is to write the problem as an ILP (integer linear program), solve the relaxed LP to obtain a fractional solution to the data placement problem, followed by a novel rounding approach that ensures a constant factor increase in storage and upload limits that is within a constant factor of the optimum ILP cost.

A. Overall Approach

We will now describe the overall structure of our approach. Our first observation is that if we relax the storage limits slightly, the download costs are very easy to handle. For cache $C_i$, we will just use an extra storage of $s_i$ (i.e. double the storage of cache $C_i$) to store the most frequently requested items at $C_i$. Next, we consider a variant of our problem where any item (or copy of an item, to be precise) at cache $C_i$

[9]http://en.wikipedia.org/wiki/BitTorrent

[10]Strictly speaking, the problem is even more non-trivial, due to the items cached from previous epoch (hence requiring cache replacement); we will visit this issue in Section VI.
serves at most \( \tau_i \) requests, where \( \tau_i = \left( \frac{2k}{c} \right) \). We show (in Section \( V.C \)) that the solution to this has identical cost as the original problem. The advantage of this approach is that if any solution stays within storage limits, it is guaranteed to be within upload limits as well (or the upload increases by the same factor as storage). Rounding this constrained LP turns out to be considerably simpler, as it has no upload limit constraints, and so the main non-trivial step required is to handle the local requests while moving items across caches during rounding (see Section \( V.D \)).

We now mention a result due to Shmoys et.al. \cite{Shmoys} to solve the Generalized Assignment Problem (GAP), which can be used to find a feasible solution \( \{z_{ij}\} \) to the following equations.

\[
\sum_{i=1}^{m} z_{ij} = p_j \quad \forall j \in [1,n]; \quad \sum_{j=1}^{n} z_{ij} \leq q_i \quad \forall i \in [1,m]; \quad j \in [1,n]
\]

In \cite{Shmoys}, the authors show how to find an integer solution \( \{z_{ij}\} \) given any fractional feasible solution \( \{\hat{z}_{ij}\} \) for the above problem, where the \( q_i \) values increases to at most \( q_i + 1 \). We will refer to the above rounding algorithm as \( \text{GAP}(p,q,\hat{z}) \), which takes the set of parameters for GAP and a fraction solution \( \{\hat{z}_{ij}\} \) and returns the rounded integral solution \( \{z_{ij}\} \). Observe an interesting point that unlike the original GAP problem, we do not have a cost function to minimize, but to compute any feasible integer solution.

**B. ILP Formulation**

Before describing the ILP, we will make some observations that significantly simplify the structure of the ILP. First, notice that the server \( (C_0) \) does not download anything as it does not have any local requests to serve, and consequently the values of \( d_0 \) and \( \beta_0 \) do not play any role in our technique. Further, as mentioned earlier, the server has a high data-rate connection that is bought in bulk from the service provider, and hence has typically a much cheaper per byte transmission costs as compared to the caches. In our formulation, it essentially means that \( \alpha_0 < \alpha_i, \forall i > 0 \). As a consequence, once a node’s upload limit is reached, instead of further using its back-haul (for serving another cache), it will be cheaper to source the request directly from the server. This also intuitively follows from the motivation of this work as we want to use only the spare upload and storage capacities at nodes to reduce the load on the server; however, once no more spare capacity is left, the system will fall back to all caches getting content from the server.

Based on the above discussion, our caching problem can be formulated as the following integer linear program (\( I_1 \)):

\[
\min \left\{ \alpha_0 \cdot \max\{0, \sum_{j,k} x_{0jk} - u_0\} + \sum_{j \in C} \beta_j \cdot \max\{0, \sum_{i,k, i \neq j} x_{ijk} - d_j\} \right\}
\]

\( s.t. \quad \sum_{j \in C, k \in O, i \neq j} x_{ijk} \leq u_i \quad \forall i \in C \quad (1) \)

\[\sum_{k \in O} y_{ik} \leq s_i \quad \forall i \in C \quad (2)\]

\[\sum_{i \geq 0} x_{ijk} = r_{jk} \quad \forall j \in C; k \in O \quad (3)\]

\[0 \leq x_{ijk} \leq r_{jk}/y_{ik} \quad \forall i, j \in C; k \in O \quad (4)\]

\[y_{ik} \in \{0,1\}, \quad x_{ijk} \in \mathbb{Z}^+ \cup \{0\} \quad \forall i, j \in C; k \in O \quad (5)\]

Here \( y_{ik} \) denotes that the object \( k \) is stored at cache \( i \), and \( x_{ijk} \) is the number of requests for object \( k \) at cache \( j \) that are served from cache \( i \). The first two constraints express upload and storage limits. The third constraints specifies that all the requests must be satisfied, while the fourth makes sure that data is only served from a cache that stores it. Here \( C \) denotes the set of caches excluding the content server and \( O \) is the set of objects.

The cost function has two components, the first is the cost of serving all the requests from \( C_0 \) i.e. the content server. The second term captures the penalty incurred by the caches (excluding the content server) for exceeding the download limit, if any. As mentioned earlier, since \( \alpha_0 < \alpha_i \), once the upload limit is reached for a cache, the extra requests are served by the content server. Therefore, the upload limits are satisfied as hard constraints and there is no penalty term associated with the uploads. It is easy to see that the non-linear terms in the cost function can be converted to linear constraints, using standard techniques.

Note that the download cost must be accounted in the cost function, as it may not be possible to get a feasible solution where all requests are satisfied without crossing the download limits; however, our assumption that byte-cost of server is lower than those of caches implies that no requests are served by a cache after reaching its upload limit. We now consider both the terms in the cost function in turn. First we get rid of the second term through the following observation.

**Handling Download Costs:** We first show how to take care of the download limit constraints. For cache \( C_j \), let \( O_j \subseteq O \) be the set of \( s_j \) items that has the maximum total request \( (\sum_{k \in O_j} r_{jk}) \) at \( j \). Clearly, storing these items in \( C_j \) minimizes the amount of requests going out, and is at most the requests going out of \( j \) in the optimal solution of \( I_1 \). So we increase the storage at \( j \) by \( s_j \) (i.e. double the storage at \( C_j \)) and use the extra storage to store the item set \( O_j \). This ensures the second term in the cost is at most the second term in the optimal solution to \( I_1 \). For the rest of this paper we consider only the first term in the cost function of \( I_1 \). After this simplification, our cost function looks like

\[
\left\{ \alpha_0 \cdot \max\{0, \sum_{j,k} x_{0jk} - u_0\} \right\}
\]

We make a further observation that if one attempts to minimize the number of requests received at the content

\[11\text{Note that we do not really need to store } s_j \text{ items; we can compute the optimal solution to } F_1 \text{ and store only } \sum_k y_{jk} \text{ items at } j.\]
server, then it automatically minimizes the cost of the first term in $I_1$, and vice versa. The cost is zero if it is less than $u_0$, else positive. Ignoring the multiplicative factor $\alpha_0$, our cost function now has a simple form $\sum_{i,k} x_{ijk}$. In the next subsection we present an equivalent optimization problem which attempts to minimize this cost function.

C. $\tau$-Constrained Placement

Let us consider a variation of our problem where any item stored at a cache $C_j$ serves at most $\tau_j$ number of requests. Since the storage at cache $C_j$ is $1/\tau_j$ times the upload limit, this immediately implies that if any placement stays within the storage limit for a cache, it also stays within the upload limit. The resulting $\tau$-constrained ($\tau$-LP) $I_2$ is as follows.

$$\min \sum_{j,k} x_{ijk}$$

s.t.

1. $\sum_{j \neq i} x_{ijk} \leq \tau_i \cdot y_{ik}$ for all $i \in C, k \in O$ (6)
2. $\sum_{j} y_{ik} \leq s_i$ for all $i \in C$ (7)
3. $\sum_{j} x_{ijk} = r_{jk}$ for all $j \in C, k \in O$ (8)
4. $0 \leq x_{ijk} \leq \tau_{ijk} y_{ik}$ for all $i \in C, j \in C, k \in O$ (9)
5. $x_{ijk}, y_{ik} \in \mathbb{Z}^+ \cup \{0\}$ for all $i \in C, j \in C, k \in O$ (10)

Here the first constraint specifies that each copy of an item can serve only $\tau_i$ requests, while the last constraint gives freedom to open multiple copies of the same item at a cache. It is important to note that the restriction of $\tau_j$ is only on the requests served to other caches (constraint 6); a single copy of an item can serve any number of local requests, but not more than $\tau_i$ external requests.

The following lemma shows that a (approximate) solution to $I_2$ gives a (approximate) solution to $I_1$ with the same cost.

**Lemma 5.1:** For any $\tau$ s.t. $1/\tau \in Z^+$, any solution of $I_2$ with $\alpha$ times the optimum cost can be converted to a solution of $I_1$ also of $\alpha$ times the optimum cost, by increasing the storage at caches by at most a factor of 2.

**Proof:** Let $y_{ik}$ be any (even optimal) solution to $I_1$. For any item $o_k$ placed at cache $C_i$ (i.e., $y_{ik} = 1$), let $\ell_{ik}$ denote the number of requests of other caches satisfied by it. We create the solution to the $\tau$-constrained placement by opening $[\ell_{ik}/\tau_i]$ copies of $o_k$ at cache $C_i$, and dividing the $\ell_{ik}$ requests among these copies. Clearly, each copy serves at most $\tau_i$ requests. Moreover, at storage at cache $C_i$ is $\sum_k [\ell_{ik}/\tau_i] \leq 2 \sum_k \ell_{ik}/\tau_i \leq 2 u_i / \tau_i \leq 2 s_i$. To convert the solution of $\tau$-LP to $I_1$, we simply set all $y_{ik} > 1$ to $y_{ik} = 1$. Observe that this step only decreases the storage requirement. The total upload required for Cache $C_i$ is $\sum_k \sum_{j \neq i} x_{ijk} \leq \sum_k \tau_i \cdot y_{ik} \leq \tau_i \cdot s_i = u_i$. Therefore, it is a feasible solution for $I_1$ with the same cost as that of $\tau$-LP.

By combining these two facts, the result of the lemma follows.

D. Rounding the $\tau$-constrained LP

**Algorithm 1** ROUND($C = C_1 \ldots C_n, R = \{r_{ik}\}, y, x$)

1. for all $i,k \ s.t. \ y_{ik} \geq 1/2 \ do$
2. $y_{ik} = \lceil y_{ik} \rceil$;
3. end for
4. for all $k \ s.t. \ \exists i : y_{ik} < 1/2 \ do$
5. $n_k \leftarrow \sum_{i : y_{ik} < 1/2} y_{ik}$;
6. if $(n_k < 1/2)$ then reroute all demands coming to $n_k$ to any open copy of $o_k$;
7. else scale all $y_{ik} < 1/2$ to $y_{ik} \cdot \lceil n_k \rceil / n_k$;
8. end for
9. Let $s_i'$ be the amount of storage occupied by items $o_k$ such that $y_{ik} < 1/2$;
10. $y \leftarrow GAP(n_k, s_i', y)$;
11. reroute all demands coming to $n_k$ to the copies opened by GAP;
12. for all $i,j,k \ s.t. \ x_{ijk} < \hat{y}_{ik} \ do$
13. $x_{ijk} \leftarrow x_{ijk}/(r_{ik} - \hat{x}_{ijk}); /* handle extra demands */$
14. end for
15. for all $k \ do$
16. $x(\cdot)(\cdot)k \leftarrow GAP(\tau \cdot y(1 \ldots n), r(1 \ldots n)k, \hat{x}(1 \ldots n)(1 \ldots n)k)$;
17. end for

Let $F_2$ be the fractional version of the ILP $I_2$, that relaxes the last constraints to $y_{ik}, x_{ijk} \geq 0$. For subsequent discussions, let assume that $\{y_{ik}, \hat{x}_{ijk}\}$ denotes the optimum solution of the LP. The rounding procedure will convert this fractional solution to an integral solution $\{y_{ik}, x_{ijk}\}$. Rounding $\hat{y}_{ik}$ values in the fractional solution should be straightforward, as it is just an instance of the GAP problem. However, in our problem there is an extra complication—the requests satisfied locally ($\hat{x}_{ijk}$) do not add to the cost or the upload capacities, but due to reassignment of items to caches these will also start contributing to the cost and capacities. To avoid such extra requests, we first make sure that the items serving large values of $\hat{x}_{ijk}$ are not moved to other caches. Specifically, if we round big enough copies of any item ($\hat{y}_{ik} \geq 1/2$) to the closest higher integer, the extra requests are also bounded by $\hat{x}_{ijk} < r_{ik}/2$, due to constraint (9).

We now describe the steps in our procedure to round the fractional solution of $F_2$ (see Algorithm 1).

**(Step 1)** For any $\hat{y}_{ik} \geq 1/2$, round it to $\lceil \hat{y}_{ik} \rceil$.

**(Step 2)** Let $\hat{n}_k = \sum_{i : \hat{y}_{ik} < 1/2} \hat{y}_{ik}$ and $n_k = \lceil \hat{n}_k \rceil$. If $\hat{n}_k < 1/2$, since $\sum \hat{y}_{ik} \geq 1$ (due to constraints 3 and 4), there is at least one copy of $k$ opened in the last step. Route the demands coming to the copies of item contributing to $\hat{n}_k$ to the integral open copy.

**(Step 3)** For $\hat{n}_k \geq 1/2$, scale each $\hat{y}_{ik}$ by a factor of $n_k/\hat{n}_k$, and hence with the new values, $n_k = \sum_{i : \hat{y}_{ik} < 1/2} \hat{y}_{ik}$.

**(Step 4)** Let $s_i'$ be the amount of storage occupied by the items in step (3) at cache $C_i$. Find the integral placement of all objects in step (3), by solving the GAP problem: GAP$(n, s_i', \hat{y}_{ik})$. Note that the scaling in last step ensures that
Further, since the upload has increased to $u$. Note that since they are disjoint, the total upload is $u$ requests. Hence this step increases the total upload from cache use the entire GAP solution, see Section V-A). Since the solution of GAP can rounding procedure.

instance).

(Step 5) Now assign all the remaining demands ($\hat{x}_{ijk}$) proportionally to the caches that are already serving the rest of the demands for $r_{ik}$.

(Step 6) To round $\hat{x}_{ijk}$ values, note that only constraints (6) and (8) are important, since values of $\hat{x}_{ijk}$ do not affect (7), and (9) follows from (3). For each item $i$, this essentially means solving the GAP($r \cdot \sum_{1 \leq j \leq n} k \cdot \hat{x}_{i} \sum_{1 \leq k \leq n} k$) problem (note that $k$ is fixed in the subscripts of this GAP instance).

The following lemma states the approximation ratio of our rounding procedure.

**Lemma 5.2:** The ROUND algorithm finds a $\tau$-constrained placement with at most 8 times the optimal cost, and requires $(4s_{i} + 2)$ storage and $(8u_{i} + 4\tau_{max})$ upload at cache $C_{i}$, where $\tau_{max} = \max_{i} \tau_{i}$.

**Proof:** Let us first look at the upload limits. Let $u_{i}^{1}, u_{i}^{2}$ and $u_{i}^{3}$ be the upload used by items processed in steps (1), (2) and (3). Note that since they are disjoint, the total upload is $(u_{i}^{1} + u_{i}^{2} + u_{i}^{3}) \leq u_{i}$. The demands coming to copies of $y_{ik}$ closed in step (2) get rerouted to (one of) the copies opened in step (1). Further, since $n_{k} < 1/2$ and it is routed to a $y_{ik} < 1/2$, only the items contributing to $u_{i}^{1}$ get at most twice the number of requests. Hence this step increases the total upload from cache $i$ by an additional amount $u_{i}^{1}$. The scaling of $n_{k}$ values in step (3) increases the $s_{i}$ values passed to the GAP problem by a factor of 2, accompanied by another additive factor of 1 due to GAP solution, see Section VA. Since the solution of GAP can use the entire $s_{i}^{1} + 1$ storage, the upload by items contributing to $u_{i}^{3}$ can now grow to $2u_{i}^{3} + \tau_{max}$. Hence at the end of step (4), the upload has increased to $2(u_{i}^{1} + u_{i}^{2}) + \tau_{max} \leq 2u_{i} + \tau_{max}$.

Notice that in step (5) since we only assign requests $x_{ik} \leq r_{ik} y_{ik}$, and we have not moved any object with $y_{ik} > 1/2$, $x_{ik} < r_{ik}/2$. In other words, at least half of the demands of $r_{ik}$ is already satisfied. Hence this step can at most double the amount of upload from any cache. Further, rounding $x$ values in last step gives another at most 2 factor increase in the total upload, due to the GAP solution. Combining these, we get the final result.

Following a very similar argument for storage, we get the final storage at cache $i$ to be $4s_{i} + 2$. The increase in cost (basically the upload by the server), follows the same pattern as the uploads on other caches. Specifically, it is affected only in steps (2), (5) and (6), doubling each time. Hence the final cost is at most 8 times the optimum.

The above lemma along with the extra $s_{i}$ storage per cache to handle the download limit gives us the following theorem.

**Theorem 5.1:** The DATAPlacement algorithm gives a solution with at most 8 times the optimal cost, and requires $(5s_{i} + 2)$ storage and $(8u_{i} + 4\tau_{max})$ upload at cache $C_{i}$.

We consider $\tau_{max}$ as a system parameter that can be tuned as per the requirements. If $\tau_{max}$ is set to a constant, it will require storage at the caches $s_{i}$ to be proportional to the upload limits ($u_{i}$). This is not a concern since in general, per byte cost for storage is much lesser than the network cost.

## VI. Solution for the Data Placement Problem

The solution presented in the previous section needs two additions to solve the data placement problem defined in Section III. First, we need to include the initial placement cost in the objective functions of the LP. Further, as our aim is to determine the item placement at each epoch, the solution should take into account the items already in the caches due to placements in the previous epoch; this essentially implies designing a cache replacement strategy that determines which new item replaces which old item for the next epoch. In this section, we extend our rounding procedure to both these scenarios.

### A. Including the Initial Placement Cost

We first need to add the initial placement cost to the objective function in $\tau$-LP ($I_{2}$). Notice that in the solution of $\tau$-LP, there could be multiple copies of an object at a cache (as $y_{ik}$ could be greater than 1), so we could not directly use $y_{ik}$ to calculate the initial placement cost. Ideally, we need to change it to $\min \sum_{j,k} x_{ij} + \sum_{i,k} z_{ik}$, where $z_{ik}$ is 1 if a copy of $o_{k}$ is stored in cache $C_{i}$, 0 otherwise. The problem with this formulation is that it is not possible to express the $z_{ik}$ constraints in terms of linear inequalities, which makes linear programming approach not suitable for this.

We tackle this problem by using a modified function that approximates $z_{ik}$ within a constant factor. We use $\frac{y_{ik}}{y_{ik} + 1}$ as a 2-approximation to $z_{ik}$; as $\frac{y_{ik}}{y_{ik} + 1} \leq y_{ik} \leq z_{ik}$. We change the cost function in $I_{2}$ (while keeping all constraints same) to:

$$\sum_{i,j} x_{ij} + \sum_{i,k} \frac{y_{ik}}{y_{ik} + 1}$$

Let us call this new integer program as $I_{3}$. Clearly this function is 2-approximation to cost function in $I_{2}$ for all $x$ and hence any $\beta$-approximation to $I_{3}$ should give us $2\beta$-approximation to $I_{2}$. However, the relaxation of $I_{3}$ from integer values to real values for $x_{ijk}$’s and $y_{ik}$’s is unfortunately a concave optimization problem for which no general technique is known to solve at optimality. Therefore, we use standard branch and bound technique to get a probably good fractional solution and use a rounding technique to convert it to integral values.

Starting with a fractional (approximate) solution of $F_{3}$, we follow the very similar rounding steps outlined in Section V-D and convert it to an integral solution of $I_{3}$. We summarize the results in the following theorem and refer the details of the proof to Appendix B.
Theorem 6.1: The Branch and bound technique on $F_3$ followed by ROUND algorithm finds a solution to $I_3$ with cost at most 16 times the fractional cost and requires $(5s_i + 2)$ units of storage and $(8u_i + 4r_{\text{max}})$ units of upload limit at cache $C_i$.

B. Including Cache Replacements

In our model, at the beginning of every epoch, we have to recompute the data placement based on the predicted values of requests at each cache (Section III). Obviously, the cache will have many items already present due to placement of previous solution. We now modify our solution to account this previous state as well. Let $Y_i^t$ denote the set of items that are present in cache $C_i$ in epoch $t$. Clearly, the initial placement cost of putting any item in $Y_i^{t-1}$ in cache $C_i$ for epoch $t$ is zero. Therefore, in epoch $t$, we solve the data placement problem with the slightly modified objective function.

$$\left\{ \sum_{j,k} x_{0,jk} + \sum_{i} \sum_{k \in Y_i^{t-1}} \frac{y_{ik}}{y_{ik} + 1} \right\}$$

We again use branch and bound technique to get an approximate solution to the above program and use the ROUND algorithm to convert that solution to integer values.

Theorem 6.2: The branch-and-bound technique followed by ROUND algorithm finds a solution to the data placement problem with cost at most 16 times the fractional cost and requires $(5s_i + 2)$ units of storage and $(8u_i + 4r_{\text{max}})$ units of upload limit at cache $C_i$.

The above result states that we can get probably good solution for the data placement problem in every epoch. It does not, however, give guarantees on the solution over multiple (or all) epochs. For example, a scheme that has knowledge of requests across multiple future epochs may get items that would reduce the cost (specifically, the initial placement cost) not just for the current epoch, but for subsequent future epochs as well. We believe such a scheme would not benefit too much as compared to our scheme because of two reasons. First, it would require one to predict requests that are far into the future, which would be both impractical and inaccurate, limiting the usefulness of this scheme. Second, in our experiments (Section VIII), we observed that the initial cost is a very small fraction ($\leq 15\%$) of the total cost; hence the gain from optimizing it is not expected to result in significant cost reduction.

A Lower Bound: We now provide a simple lower bound on the total cost of the data placement problem. For cache $C_i$, define a function $f_{ik} = r_{ik} - 1$ if $k$ was present in cache in previous epoch and $f_{ik} = r_{ik}$ otherwise, that defines the amount of transmission saved by caching $o_k$ for the next epoch. Now, suppose we cache at $C_i$ the set $F_i$ (of size $|F_i| = s_i$) of items with the maximum $f_{ik}$ values. Clearly, this technique completely ignores cooperation among caches and minimizes (optimally) only the cost of serving local demands on each cache; we call it the local caching algorithm and compare our solution with it. The cost of this placement is $\sum_{i,k} r_{ik} - \sum_{i,k \in F_i} f_{ik}$, and it is easy to show that this is at most $\sum_i u_i$ away from the optimum. We compare our algorithms with this lower bound on the cost of the optimum solution (that we will call $C_{lb}$), and our results show that our approach very close to it (always with in 20%, in many cases with in 3% of $C_{lb}$).

Algorithm 3 CLUSTER($C,k$)

1: $F_{1,...,n} = C_{1,...,n}$
2: while $|F| > 1$ do
3: choose a pair of clusters $F_i, F_j \in F$ s.t. $|F_i \cup F_j| \leq k$ with maximum value of $|F_i \cap F_j|$;
4: if $|F_i \cap F_j| \leq 0$ break;
5: $F \leftarrow F - \{F_i, F_j\} \cup \{F_i \cup F_j\}$/merge clusters */
6: end while
7: return $F$;

VII. Optimization

Computation of a low-cost data placement strategy needs to solve the LP $I_2$ optimally. However, as the number of caches increases, the running time of the LP solver becomes prohibitively large (a sample run of 200 caches takes more than a day using the CPLEX solver). This increased running time is intuitive, owing to the large complexity of the LP solvers.

We propose a cache clustering technique to speed up the computation of the fractional LP solution. The idea is to group nodes into clusters, solve the fractional $\tau$-constrained LP for each cluster separately, and then apply the rounding procedure on the combined solution of the clusters. The key benefit is that running LP on small clusters is very fast; however, we risk losing benefits of cooperation among caches that were put in different clusters. Our clustering procedure attempts to maximize the intra-cluster cooperation by grouping caches based on the similarity of the items they request. The intuition is that if caches with similar requests are put into the same cluster, they will benefit more from cooperating with other caches in the cluster (in serving the common requests), than with caches in other clusters with dissimilar requests.

We use a simple hierarchical bottom-up clustering to compute the clusters. At each iteration, we greedily pick the pair of clusters which has the maximum similarity and merge them. We define similarity $\text{sim}(i,j)$ between clusters $F_i$ and $F_j$ as the Jaccard's coefficient of their request sets $R_i$ and $R_j$, respectively, $\text{sim}(i,j) = \frac{|R_i \cap R_j|}{|R_i \cup R_j|}$. Our technique takes only a single parameter $k$, which denotes the maximum size of any cluster. We merge clusters only if combining them produces a cluster of size at most $k$. Note that we do not merge clusters if the request sets do not overlap; this is because if the caches in the two clusters do not have common requests, they will not benefit from cooperative caching.

VIII. Simulations

We now describe an extensive set of simulations to evaluate our techniques. First, we describe our experimental set-up, and how the simulation inputs are generated. Then, we explore some crucial system design parameters that determine both the computational time and quality of our solutions. Once we
have determined suitable values of these parameters, we will evaluate our approach (and compare with a lower-bound on costs, and other heuristics) for a range of inputs values.

A. Input Generation

As described in Section III, our system inputs consist of \( n \) cooperating caches, each cache \( i \) has a storage capacity \( s_i \), upload and download usage caps \( u_i \) and \( d_i \) respectively. Also, there exist \( n_u \) users and \( m \) content items (requested and served) in the system. We consider two different system scenarios: 1) single association, no sharing (SANS): each user associates only with one cache and requests some items, but does not generate or share content with other users (this corresponds to the scenario of home users downloading movies, or software updates). To generate this input, we randomly associate each cache with a user, and the user randomly selects on average \( n_{\text{pop,items}} \) out of available \( m \) items; 2) multiple associations, with sharing (MAWS): a more general scenario, in which users on average associate with \( n_{\text{assoc}} \) caches, and can also generate and share \( n_{\text{shares}} \) pieces of content with \( n_{\text{friends}} \). The number of associations reflect the number of caches a user is likely to visit (and spend time) in the course of a day (e.g. bus-stops near home and work). While generating this input, our goal was also to preserve a notion of geographical locality of requests, i.e. some items are requested more frequently from a small set of caches, and also locality of social graph, i.e. friends have a greater chance of associating with the same caches. To ensure this, we created a mapping between items to caches, and with a higher likelihood, users requested items mapped to their associated caches. Similarly to create the social graph, with high likelihood, a user selected friends from caches with common associations.

Real World Traces: We also used real-world traces (from the CRAWDAD repository\(^{[2]}\)) collected at Wi-Fi access points distributed across the city of Montreal. These traces gave us realistic data on user association patterns with multiple public hot-spots. However, due to the limited scale of the data (only \( \approx 150 \) hot-spots), bulk of our experiments are performed with the synthetically generated input as described above.

Request patterns across epochs: Another key aspect of the simulated system is the dynamic nature of the request patterns over time. To capture this in our simulations, at each epoch we randomly select and expire 5% of items from the caches. We then randomly select and repopulate the expired items in the cache.

Based on these strategies, we associate users with caches, items and friends, and compute the number of requests for each item at each cache in the system. This constitutes the input to the CPLEX LP solver. The rounding method (and other post-processing steps) applied to the resulting solution provide the costs computed by our approach, and for ease of exposition, we will subsequently refer to it as \( C_{\text{round}} \).

B. Choosing Cluster Size

Our first experiment is designed to determine a reasonable cluster size for the clustering procedure. Intuitively, we expect the output costs from clustering to improve when the size of clusters is large (with a subsequent increase in execution time). In Fig. 2 we plot the impact of cluster size on the cost of the ROUND algorithm. To evaluate the marginal gains of increasing size, we normalized the costs by dividing it with the cost with cluster size 200 (as this is the largest cluster size that we could run). The input values for the SANS case are \( n = 1000, m = 100, u_i \) randomly selected between 1 – 5, \( \tau = 1/3, n_u = 10000, \) and \( n_{\text{pop,items}} = 10 \). Additionally, for MAWS, \( n_{\text{assoc}} = 2 \) and \( n_{\text{shares}} = 3 \). As can be seen, beyond cluster sizes \( > 50 \) the marginal gains of larger cluster size are insignificant. Moreover, the execution time on this particular input grows 5 times from a cluster size of 50 to 100. Thus, for subsequent experiments, we fix the cluster size to 50.

C. Comparison and evaluation over a range of inputs

Now that we have identified suitable values of design parameters, we now evaluate and compare the performance of our approach across different values of \( n \), both for the SANS scenario in Fig. 4 and MAWS in Fig. 3. The values for other input parameters are as before. The results are averaged over 5 different input graphs and 3 epoch periods. The goal of these experiments is two fold. We quantify the benefits of co-operative caching as proposed in our approach (ROUNDing) compared to when caching is only done locally, at each node (Local-caching), and find that our co-operative algorithm reduces content delivery costs by 20 – 30%\(^{[13]}\).

Moreover, we also found that our approach drastically reduces network load at the central server by 55% and 27% for both the MAWS and SANS cases respectively. The reduction in delivery costs is also significant in experiments with the real-life WiFi traces (Figure 5), where the ROUNDing approach

\(^{[13]}\)Note that in SANS, local caching is not expected to give any savings as there is at most one request of any item on a cache, hence this experiment is only for the MAWS scenario.
outperforms Local-caching by more than 50%. These gains are significant since they indicate a service provider has considerable buffer to incentivize users to set aside some of their resources for co-operative caching.

Then we compare our approach with $C_{lb}$ a lower-bound on the benefits of co-operative caching. The Rounding approach stays well within 20% of the lower bound in the MAWS case, and within 5% in the SANS case.

In our experiments, we have also computed the blow-ups in the upload and storage limits. Interestingly, we found that on average there is practically no blow-up in either limit. Specifically, we found storage is increased by less than 1%, while about 5% of upload limit remains unused. Further, even the worst-case blow-ups of storage and upload limits are 3 and 4.5%, much less than the worst case factors proved in Theorem 5.1.

D. Dynamic request patterns over time

We now study the cost varies over different epochs. As discussed earlier, a prediction of requests arriving at a cache is computed across an epoch. Across epochs, given the current state of the caches, and a new set of requests, our algorithm computes a data-placement solution that minimizes the cost of serving content fetched either from the central server or a co-operative cache (as the case may be). The data-placement costs of our Rounding algorithm (Figure 6 and Figure 7) are compared to that of a Local-caching scheme that computes the data-placement solution by considering what is already present in the caches. We make two observations, firstly, both in SANS and MAWS data-sets, the cost of data-placement significantly decreases after the first epoch (when the caches are empty), and stabilizes in subsequent epochs. Secondly, our scheme outperforms the Local-caching scheme by 20%, both in the first and the subsequent epochs.

To conclude, through a fairly extensive set of simulations we have explored the performance of our proposed co-operative caching scheme. We have explored the system design space, and understood various aspects of our approach such as the computational time and blowups. We believe these results convincingly indicate that through a careful management of co-operative end-nodes one can substantially lower the cost of content delivery, while respecting the limitations of the end-host network resources.

IX. CONCLUSIONS

In this paper we consider a co-operative caching system consisting of end-nodes. Each node brings to the table some spare storage capacity, and spare capacity on their network back-haul links. Keeping in mind the limits at each node, we devise a centralized strategy to efficiently manage the distributed system resources, while reducing the cost on the central node. We believe this work has important implications for any end-node based caching solution, with tiered network access costs. Moving forward, interesting directions for future work would include 1) predicting requests at a cache based on historical behavioral patterns of users 2) managing a dynamic system with the cooperating end-nodes coming in and going out of the system 3) a distributed approach towards co-operative caching with similar constraints as studied in this paper.

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In other words, node has to get them from one of the other nodes. Further, nodes must each serve exactly \( L \) requests per distinct item) then we start paying cost per download.

The if part is true as if there is such a partition and the caching problem is zero (when all transfers stay within the upload/download limits) if and only if there is a partition that on the nodes are as follows: for the server and costs on the nodes are as follows: for the server and storage costs for the server and help node 1 in serving its request (and hence reducing the load on the server). The limits and costs on the nodes are as follows: for the server and for nodes and for nodes and for nodes and for nodes and for nodes and for nodes . The extra costs are \( \alpha_i = \beta_i = 1 \) (\( \forall i \in [0, 3] \)). In other words, node 1 cannot cache any object locally, and has to get them from one of the other nodes. Further, nodes can download and store only \( t \) and \( n - t \) objects, respectively, before their limits are exhausted. The limit on the server ensures that if it serves more than \( m \) downloads (one per distinct item) then we start paying cost per download.

We want to prove a 1-1 correspondence between our caching problem and the partition problem. Specifically, the cost of caching problem is zero (when all transfers stay within the upload/download limits) if and only if there is a partition \( T \). The if part is true as if there is such a partition \( T \), we can download objects in \( T \) in cache \( C_2 \) and \( O - T \) in \( C_3 \) and then serve all requests from them, giving zero cost. For the only if part, let us assume that there is a zero cost solution. Clearly, in this solution all the requests are served from \( C_2 \) and \( C_3 \), since there are at least 2 requests (as \( w_i \geq 1 \)) for each object and if any of them is served directly from the server, its upload limit will be crossed. Further, since \( C_2 \) and \( C_3 \) together serve a total of \( 2W \) requests and each limit is \( W \), for a zero cost solution they must each serve exactly \( W \) requests. The set of objects in \( C_2 \) represents partition. This completes the proof.

### A. Proof of NP-Hardness

**Proof:** We reduce a given instance \( P(O, W, t) \) of the partition problem into our problem as follows. The cache placement instance contains four nodes with \( C_0 \) being the server. The node \( C_1 \) gets all the requests in \( R \), s.t. the number of requests for object \( o_k \) is \( r_{1k} = 2 \cdot w_k \). Nodes \( C_2 \) and \( C_3 \) get no request of their own and only help \( C_1 \) in serving its request (and hence reducing the load on the server). The limits and costs on the nodes are as follows: for the server and storage limits are \( u_0 = m \), and \( d_0 = 0 \); for \( C_1 \), \( s_1 = 0 \), \( d_1 = 2W \), \( u_1 = 0 \); for nodes \( C_2 \) and \( C_3 \), \( s_2 = s_3 = t \), \( d_2 = d_3 = s_3 = m - t \) and \( L_2 = L_3 = W \). The extra costs are \( \alpha_i = \beta_i = 1 \) (\( \forall i \in [0, 3] \)). In other words, node 1 cannot cache any object locally, and has to get them from one of the other nodes. Further, nodes \( C_2 \) and \( C_3 \) can download and store only \( t \) and \( n - t \) objects, respectively, before their limits are exhausted. The limit on the server ensures that if it serves more than \( m \) downloads (one per distinct item) then we start paying cost per download.

We want to prove a 1-1 correspondence between our caching problem and the partition problem. Specifically, the cost of caching problem is zero (when all transfers stay within the upload/download limits) if and only if there is a partition \( T \). The if part is true as if there is such a partition \( T \), we can download objects in \( T \) in cache \( C_2 \) and \( O - T \) in \( C_3 \) and then serve all requests from them, giving zero cost. For the only if part, let us assume that there is a zero cost solution. Clearly, in this solution all the requests are served from \( C_2 \) and \( C_3 \), since there are at least 2 requests (as \( w_i \geq 1 \)) for each object and if any of them is served directly from the server, its upload limit will be crossed. Further, since \( C_2 \) and \( C_3 \) together serve a total of \( 2W \) requests and each limit is \( W \), for a zero cost solution they must each serve exactly \( W \) requests. The set of objects in \( C_2 \) represents partition. This completes the proof.

**B. Proof of Theorem 6.1**

**Proof:** Observe that the effect of rounding on the first part of the cost function (i.e., \( \sum_{j,k} W_{jk} x_{ijk} \)) follows from the analysis presented in Lemma 5.2. Therefore, we only consider the second part in the cost function. Let \( \text{OPT}_f(Y) \) denote the second part in the cost function corresponding to the optimum fractional solution of the data placement problem. We follow the steps in the rounding procedure and bound the approximation factor of \( \text{OPT}_f(Y) \) at each such step.

1. **Step 1** If the minimum cost of the convex optimization problem is an increasing function of \( y_{ik} \), this step increases \( \text{OPT}_f(Y) \) only by a factor of \( \frac{f(1)}{f(0)} = 1.5 \).
2. **Step 2** This step only decreases \( \text{OPT}_f(Y) \), again because \( f(y_{ik}) \) is an increasing function of \( y_{ik} \).
3. **Step 3** This step has no effect on \( \text{OPT}_f(Y) \).
4. **Step 4** Let us now solve \( \text{GAP}(n, s', y, \text{OPT}_f(Y)) \). Since \( 0 \leq y_{ik} \leq 1 \) for all \( y_{ik} \) going in to \( \text{GAP} \) and \( \text{GAP} \) algorithm rounds each \( y_{ik} \) to at most 2, the rounded solution satisfies \( 1 \leq y_{ik} + 1 \leq 3 \) and therefore the rounded solution is a 3-approx to \( \text{OPT}_f(Y) \).
5 & 6 These steps have no effect on \( \text{OPT}_f(Y) \).

If the minimum cost of the convex optimization problem can be expressed as \( \text{OPT}_f(X) + \text{OPT}_f(Y) \), then the rounding step produces a solution of cost at most \( 8 \cdot \text{OPT}_f(X) + \frac{9}{2} \cdot \text{OPT}_f(Y) \). The factors for upload and storage follows from the analysis presented in Lemma 5.2.

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14Observe that this GAP instantiation refers to the original rounding technique described in Shmoys et al. [31]. This GAP problem has a cost function and the rounding technique ensures that the cost of the integral solution is at most the cost of the optimum fractional solution.