We consider a logistics planning problem of prepositioning relief items in preparation for an impending hurricane landfall. This problem is modeled as a multi-period network flow problem where the objective is to minimize the logistics cost of operating the network and the penalty for unsatisfied demand. We assume that the demand for relief items can be derived from the hurricane’s predicted intensity and landfall location, which evolves according to a Markov chain. We consider this problem in two different settings, depending on whether the time of landfall is assumed to be deterministic (and known a priori) or random. For the former case, we introduce a fully adaptive multi-stage stochastic programming (MSP) model that allows the decision-maker to adjust the prepositioning decisions, sequentially, over multiple stages, as the hurricane’s characteristics become clearer. For the latter case, we extend the MSP model with a random number of stages introduced in [19], to the case where the underlying stochastic process is assumed to be stage-wise dependent. We benchmark the performance of the MSP models with other approximation policies such as the static and rolling-horizon two-stage stochastic programming approaches. Our numerical results provide key insight into the value of MSP, in disaster relief logistics planning.

**Keywords** disaster relief · hurricanes · logistics planning · multi-stage stochastic programs

1 Introduction

Over the past few decades, many significant hurricanes have hit the United States (US). These hurricanes have resulted in heavy loss of lives, numerous personal injuries, and consequential material damages. According to the National Oceanic and Atmospheric Administration (NOAA), a total of 273 hurricanes struck the US between 1851 and 2004. Since 2005 – the year in which the costliest hurricane in history, Katrina, made landfall in Louisiana – there have been at least 43 major hurricanes. Altogether, these major hurricanes have caused more than 600 billion dollars worth of damage in the US and the Caribbeans, making hurricanes to be among the costliest and the most frequently observed natural disasters.

Despite their tragic impacts, unlike other unpredictable natural disasters such as earthquakes, hurricanes can be detected a few days before they make landfall. Indeed, approximately five days before a hurricane predicted time of landfall, the National Hurricane Center (NHC) provides information about the hurricane’s predicted trajectory, forward speed, intensity, and endangered areas. This information comes in the form of forecast advisories, and they are often leveraged by humanitarian and governmental agencies to prepare and allocate hurricane relief supplies, such as first-aid commodities, food, water, housing, backup power generators, etc. Nevertheless, having accurate predictions about the hurricane characteristics is only a necessary condition for a successful disaster relief effort. As it happened, notwithstanding the unprecedented number of relief items prepositioned in preparation for Hurricane Katrina, the Federal Emergency Management Agency (FEMA) was heavily criticized for failing to prepare enough relief items [24], after the NHC provided accurate predictions about 48 hours before the storm hit [21]. Indeed, transportation and logistics play central roles in disaster relief efforts since they deal directly with the procurement,
allocation, and distribution of critical supplies and services to the affected areas [16, 29], as well as potential large-scale evacuation operations [26]. All of the aforementioned assertions, coupled with the dynamic evolution of a hurricane’s characteristics, emphasize the need for a reliable proactive, yet flexible, logistics operations plan.

In this paper, we consider a typical hurricane disaster relief logistics planning (HDRLP) problem where a decision-maker (DM) procures and prepositions relief items in anticipation of a hurricane that is predicted to make landfall in a few days. It is assumed that the DM can identify an affected potential area (APA) that is at risk of being struck by the hurricane. Within the APA, there is a set of demand points (DPs) that represent the locations from which demand for relief items is expected. These relief items can be procured from a major distribution center (MDC), and the procured items can be prepositioned at a set of supply points (SPs). Once the hurricane makes landfall, the prepositioned relief items can be shipped from the SPs to the DPs. We assume that the demand for relief items is determined based on two of the hurricane’s characteristics: its intensity and its location at the time of landfall. If these characteristics are known in advance, this problem can be modeled as a deterministic multiperiod network flow problem, defined over a logistics network (see Figure 1). In this problem, the goal of the DM is to find an optimal solution for the logistics operations in the network, including (i) procuring the relief items from the MDC; (ii) prepositioning the procured items at the SPs; and (iii) delivering the prepositioned items to DPs according to the incurred demand, which minimizes the overall logistics cost of procuring and shipping the relief items plus the penalty cost for failing to meet the demand for relief supplies (if any).

Since the demand for relief items is not known in advance – but can be derived from the hurricane’s characteristics which evolve according to a stochastic process with a known probability distribution – we propose a multi-stage stochastic programming (MSP) model for the HDRLP problem. MSP is a class of mathematical programming models for sequential decision-making under uncertainty. In a conventional MSP problem, the goal of a DM is to construct an optimal policy that controls the behavior of a probabilistic system over multiple decision stages as it evolves through a period known as the planning horizon. To that end, in each decision stage, the DM observes the current state of the system and chooses an action that is prescribed by the optimal policy. MSP models have a colorful range of applications in several areas such as energy [10], finance [14], and transportation and logistics [17], among others.

We assume that the hurricane’s characteristics evolve according to an Markov chain (MC), where we consider not only the uncertainty about the hurricane’s intensity and landfall location but also its time of landfall. To that end, we consider the HDRLP problem in two different settings, depending on whether the timing of the hurricane’s landfall is assumed to be deterministic or to follow a probability distribution. For each setting, we propose an MSP model that allows the DM to make adaptive logistics operational decisions over time as the characteristics of the hurricane at landfall become clearer.

Due to the uncertainty in the problem data and the sequential nature of the decision-making structure in MSP models, it is well-known that MSP models can be computationally challenging to solve. To that end, we consider alternative approximate approaches which are less computationally expensive than the MSP model. In particular, we consider static two-stage stochastic programming (S-2SSP) models, where the entire planning horizon is aggregated into two stages. We also consider a rolling-horizon (RH) approach where an S-2SSP model is constructed and solved in every period, but only the decisions pertaining to the current period are implemented. We evaluate the performances of these approaches in an extensive numerical experiment using synthetic data and perform various sensitivity analysis that leads to key insights into the value of MSP in hurricane relief logistics planning.

In sum, the contributions of our paper are threefold:

1. We propose novel MSP models to the HDRLP problem under demand uncertainty characterized by the hurricane’s evolving attributes.
2. We introduce and solve the HDRLP problem where the hurricane’s landfall time is assumed to be random in addition to its trajectory and intensity. This extends the scope of MSP modeling to address situations where the number of stages is random.
3. We provide insights into the value of MSP models in disaster relief logistics planning compared to alternative decision policies given by the S-2SSP and the RH approach.

The remainder of this paper is organized as follows. In Section 2 we review the literature on applying optimization models for dealing with hurricane relief logistics problems. In Section 3 we describe the problem setting, the proposed MSP models, and the alternative approaches. In Section 4 we discuss our numerical experiment results. We conclude with some final remarks in Section 5.
2 Literature Review

In the literature, there is a good deal of work on applying optimization models to address different aspects of disaster relief logistics planning. A significant part of the literature is related to the last-mile delivery problems [3], which focus on the scheduling and delivery of relief items to the affected individuals. Other aspects of the disaster relief effort have also been studied, including the social cost of unmet demand [20, 29, 30, 41], evacuation and rescue missions [26], disruptions in the power and supply chain systems [22, 40], among others. Our work pays less attention to the details of the logistics models, but focuses on the interplay between the dynamic evolution of the hurricane characteristics and the logistics operations, using a standard multiperiod logistics network flow model. To that end, we give a brief overview of the literature most relevant to these aspects.

2.1 Two-stage Stochastic Optimization Models for HDRLP

Most of the previous studies in the literature have focused on the logistics of operating the relief supply logistics network under uncertainty at the strategic level, using S-2SSP and robust optimization models [32]. In most of the S-2SSP models, see, e.g., [2, 15, 25, 28, 30, 33, 39], the first-stage decisions are typically defined as preparatory actions before the hurricane makes landfall, such as deciding on the locations of facilities for stockpiling relief items, delivery fleet assignment, and inventory levels. On the other hand, the second-stage decisions are typically defined as recourse actions after observing the realized hurricane attributes. One main concern with these S-2SSP models is that it does not adequately address the estimation errors inherited in the forecast uncertainty, especially during the early stages of the planning horizon. Additionally, a latent policy, where the mobilization of relief items is postponed until the forecast uncertainty is significantly reduced, might be costly (or even infeasible) due to the limited time that remains to accomplish these tasks. This dilemma underlines the need for a policy that can be adapted to the dynamic evolution of the hurricane.

2.2 Adaptive Decision Policies for HDRLP

One way to achieve an adaptive decision policy is to apply the so-called RH approach, where, a look-ahead model is constructed and solved in every period in the planning horizon, but only the decisions pertaining to the current period are implemented. This approach naturally gives rise to an adaptive decision policy as the look-ahead model is constructed based on the most up-to-date status of the system in each period [1, 8, 34]. In the context of disaster relief logistics, this procedure has mostly been applied by using a deterministic optimization model as the look-ahead model, where the underlying stochastic process is approximated by a point estimator [38]. Recently, two-stage stochastic programming (2SSP) models have been a popular choice for the look-ahead model in the RH approach [9, 27, 31, 40]. In our numerical study, we consider an RH framework where a 2SSP model is solved in each period, as an alternative approach to obtain approximate decision policies.

In addition to using stochastic programs as the look-ahead models, there are alternative approaches to achieve adaptive decision policies that are tailored for the application of HDRLP. For example, [23, 27] are among those that are most closely related to our work. They consider the problem of prepositioning relief items in preparation for a hurricane landfall. As is assumed that earlier advisories have greater uncertainty and that the logistics cost increases over time (see the discussion following Assumption 5 in Section 3), they view the process of making the pre-disaster planning decisions as having two layers: (i) when to start the prepositioning process; and (ii) how many relief items should be prepositioned at different DPs once the prepositioning process has started. From this perspective, in the first layer, the goal of the DM is to identify the “best” period to start the prepositioning process. Specifically, a period that strikes a balance between maximizing the level of confidence in the predicted hurricane characteristics and minimizing the logistics cost. [23] approach this problem as an optimal stopping problem, and [27] use a combination of decision theory and stochastic programming techniques. We use a similar Markovian structure for modeling the underlying stochastic process associated with the evolution of the hurricane’s characteristics. However, we impose the adaptability of sequential decision-making, to the arrival of new information, more explicitly via MSP models.

3 Stochastic Programming Models and Methods for Hurricane Relief Logistics Planning

This section is organized as follows. First, we give a general description of the HDRLP problem, introduce some basic assumptions, and provide a mathematical formulation in a completely deterministic setting. Second, we introduce a generic formulation for MSP models. Third, we introduce the MSP model and the alternative approaches for the situation where the hurricane time of landfall is assumed to be deterministic. Finally, we extend the MSP model and the alternative approaches to the situation where the hurricane time of landfall is assumed to be random.
3.1 The HDRLP problem: description, assumptions, and deterministic formulation.

We consider a typical HDRLP problem, which is modeled as a multiperiod network flow problem defined on a directed network $G = (V, A)$. In this network, the set of nodes $V = \{0\} \cup I \cup J$ consists of the MDC (denoted by node 0), the SPs (denoted by the set $I$), and the DPs (denoted by the set $J$). Assuming that the hurricane is set to make landfall at time $T$, the hurricane relief logistics process unfolds as follows. At any point in time $t = 1, \ldots, T$, the DM has the opportunity to procure relief items from the MDC and preposition them at different SPs. Prepositioned items at any supply point (SP) $i \in I$ can be rerouted to other SPs $i' \in I$ at any time $t$. At time $T$, once the hurricane makes landfall, the prepositioned relief items can be shipped from the SPs to the DPs, according to the demand incurred at different DPs. Figure 1 depicts an example of a logistics network for an APA with $|I| = 2$ and $|J| = 3$.

In this HDRLP problem, the objective is to minimize the total cost, which consists of the logistics cost (including the procurement and transportation costs) and the penalty cost for unsatisfied demand for relief items. The description of the parameters used in our model formulation is provided in Table 1.

| Parameter | Description |
|-----------|-------------|
| $c_{i,i'}^{b, t}$ | unit cost of transporting relief items from node $i \in \{0\} \cup I$ to node $i' \in I$, in period $t = 1, \ldots, T$. |
| $c_{i,j}^{a, t}$ | unit cost of transporting relief items from node $i \in I$ to node $j \in J$, at the time of landfall $T$. |
| $h_i$ | unit cost of holding items at SP $i \in I$ in period $t$, in period $t = 1, \ldots, T$. |
| $h_t$ | unit cost of procuring relief items from the MDC in period $t$, in period $t = 1, \ldots, T$. |
| $p$ | unit penalty cost for unsatisfied demand. |
| $q$ | unit salvage value for unused relief items. |
| $d_j$ | demand for relief items at the time of landfall at DP $j \in J$. |
| $u_i$ | inventory capacity of SP $i \in I$. |

Table 1: Description of the parameters in the HDRLP multiperiod network flow problem.

In addition, we make the following assumptions.

**Assumption 1** The MDC has unlimited capacity, whereas each SP $i \in I$ has an inventory capacity of $u_i$.

**Assumption 2** All shipments made at the start of a period $t$ will arrive at their destinations by the start of period $t + 1$.

**Assumption 3** The demand is only incurred at the time of the hurricane’s landfall, i.e., at time $T$.

**Assumption 4** We do not make decisions regarding the selection of the SPs, nor the timing of their activation. That is, all the SPs are assumed to be available starting from period $t = 1$.

**Assumption 5** The logistics costs $(c_{i,i'}^{b, t}, c_{i,j}^{a, t}, h_i, h_t)$ are nondecreasing over time $t = 1, \ldots, T$.

Assumption 1 is customary. Assumptions 2, 3, and 4 are introduced to simplify the underlying logistics optimization model. We note that, although Assumption 3 is a simplification, the demand for relief items at different SPs relies
heavily on the hurricane’s intensity and its location at the time of landfall. Hence, while it could be interesting to consider the situation where demand occurs over multiple periods, as we shall see in Section 4, imposing Assumption 3 does not undermine the conclusions of this paper. Assumption 4 is employed to avoid introducing binary decision variables to model the SP selection and activation, which would inflate the computational resources needed to solve the resulting multistage stochastic mixed-integer programming problem. Assumption 5 can be interpreted as the impact caused by the imminence of the hurricane on the day-to-day operations of the logistics network. In particular, the days before the hurricane landfall are typically associated with damage-inducing weather conditions (e.g., high winds, heavy rain, etc.), which may disrupt the logistics network. These disruptions can ultimately lead to a surge in the prices of relief commodities that are essential to the relief logistics efforts. Moreover, the imminence of a hurricane usually necessitates the need for urgent evacuation efforts by vehicles that can be deployed at short notice (e.g., general aviation aircraft). While such means of transportation have significant benefits, they often come with hefty prices. Nonetheless, as we show in Section 4, our proposed models and methods will remain valid even when Assumption 5 is lifted. The introduction of this assumption is, rather, to bridge the gap between the observations in our numerical experiments and the practical intuitions of HDRLP.

Deterministic formulation. The preamble to the stochastic optimization models presented in this section is the deterministic version of the problem. We may view this deterministic version from the perspective of a clairvoyant, i.e., the demand for relief items and the hurricane’s landfall time are both known before the logistics planning. We refer to the optimal solution to this deterministic optimization problem as the clairvoyance (CV) solution. To formulate this deterministic multiperiod network flow problem for HDRLP, we consider the decision variables shown in Table 2.

| Decision variable | Description |
|-------------------|-------------|
| \( x_t = (x_{ij})_{i \in I, t} \) | amount of inventory at SP \( i \in I \) at the end of period \( t \), \( \forall t = 1, \ldots, T \). |
| \( f_t = (f_{ij})_{i \in \{0\} \cup I, j \in J} \) | amount of relief items shipped from the MDC or SP \( i \in \{0\} \cup I \) to \( j \in J \), \( \forall i \in I, t = 1, \ldots, T \). |
| \( y = (y_{ij})_{i \in I, j \in J} \) | amount of relief items shipped from SP \( i \in I \) to DP \( j \in J \) at the time of landfall \( T \). |

Table 2: Description of the decision variables in the HDRLP multiperiod network flow problem.

Given the time of landfall \( T \), demand levels \( (d_{j})_{j \in J} \) and initial inventory levels \( (x_{i,0})_{i \in I} \) at the SPs, the corresponding deterministic multiperiod disaster relief problem is given by:

\[
\begin{align*}
\min & \quad \sum_{i \in I} \sum_{t=1}^{T} \left( \sum_{j \in \{0\} \cup J} c_{ij} x_{ij,t} + \sum_{i \in I} c_{ij} x_{ij,t} + h_{ij} f_{ij,t} \right) \\
& \quad + \sum_{i \in I} \sum_{j \in J} c_{ij} y_{ij} + \sum_{j \in J} p(d_{j}) - \sum_{i \in I} y_{ij} + \sum_{j \in J} q(x_{i,T} - \sum_{j \in J} y_{ij}) \\
\text{s.t.} & \quad x_{i,t-1} + \sum_{j \in J_{\neq i}} f_{ij,t} - \sum_{j \in J_{\neq i}} f_{ij,t} = x_{ij,t} \quad \forall i \in I, t = 1, \ldots, T, \quad (1a) \\
& \quad \sum_{j \in J_{\neq i}} f_{ij,t} \leq x_{ij,t-1} \quad \forall i \in I, t = 1, \ldots, T, \quad (1b) \\
& \quad 0 \leq x_{ij} \leq u_{i} \quad \forall i \in I, t = 1, \ldots, T, \quad (1c) \\
& \quad \sum_{j \in J} y_{ij} \leq x_{i,T} \quad \forall i \in I, \quad (1d) \\
& \quad \sum_{i \in I} y_{ij} \leq d_{j} \quad \forall j \in J, \quad (1e) \\
& \quad y_{ij} \geq 0 \quad \forall i \in I, j \in J, \quad (1f) \\
& \quad f_{ij,t} \geq 0 \quad \forall i \in I, j \in J, t = 1, \ldots, T. \quad (1g)
\end{align*}
\]

The first four terms in the objective function of formulation 1 represent the logistics costs. In particular, the first term represents the cost of shipping relief items from the MDC to the SPs, and rerouting the relief items between SPs; the second term represents the cost of holding the relief items at the SPs; the third term represents the cost of procuring relief items from the MDC, and the fourth term represents the cost for shipping relief items from the SPs to the DPs at the time of landfall \( T \). Additionally, the fifth term represents the penalty cost for unsatisfied demand at the DPs, and the last term represents the return from salvaging unused relief items.

Constraint 1a represents the flow balance constraint, which indicates that the amount of relief items at SP \( i \in I \) at time \( t = 1, \ldots, T \) is given by the amount of items stored in SP \( i \) from the previous time period, plus the items shipped from
other SPs ($\forall i' \in I, i' \neq i$) to SP $i$ and the MDC at the current period, and minus the items shipped from SP $i$ to other SPs at the current period. Constraint (1b) indicates that the total amount of items shipped from SP $i \in I$, at time $t = 1, \ldots, T$, to other SPs ($\forall i' \in I, i' \neq i$) cannot exceed the initial inventory level at the start of the current period. Constraint (1c) indicates that the amount of relief items stored in each SP $i \in I$ is nonnegative, and cannot exceed its capacity at any time. Constraint (1d) indicates that the total amount of items shipped from SP $i \in I$ at time period $t = 1, \ldots, T$ to all DPs ($\forall j \in J$) cannot exceed the inventory level at SP $i$ at the end of the same period. Constraint (1e) indicates that the total amount of items shipped to DP $j \in J$ should not exceed its demand level. Finally, constraints (1f) and (1g) impose the sign restrictions on decision variables $y_i$ and $f_t, \forall t = 1, \ldots, T$.

### 3.2 Generic MSP models

The proposed stochastic optimization models and methods are built based on the deterministic multiperiod network flow model (1). Before we discuss them in the context of the HDRLP problem, we first introduce a generic formulation for the MSP model.

**Generic MSP formulation.** The starting point of the MSP models introduced for the HDRLP problem is the following generic nested formulation.

$$\min_{a_1 \in \Xi(a_0, \xi)} z_1(a_1, \xi_1) + \mathbb{E}[\xi_1 | a_1] \left[ \min_{a_2 \in \Xi(a_1, \xi_2)} z_2(a_2, \xi_2) + \mathbb{E}[\xi_2 | a_2] \left[ \cdots + \mathbb{E}[\xi_T | a_T] \left[ \min_{a_T \in \Xi(a_{T-1}, \xi_T)} z_T(a_T, \xi_T) \right] \right] \right].$$

(2)

As customary, we assume that vectors $a_0$ and $\xi_1$ are given as input data. Moreover, $\xi_{|t} := (\xi_1, \xi_2, \ldots, \xi_t)$ denotes the history of the stochastic process up to time $t$, and $\xi_t$ is a random vector with a known probability distribution, supported on a set $\Xi_t \subset \mathbb{R}^{n_t}$, for $t = 2, \ldots, T$. We also make the following assumption on the stochastic process $\{\xi_t\}_{t=1}^{T}$.

**Assumption 6** The stochastic process $\{\xi_t\}_{t=1}^{T}$ follows a discrete-time MC, that is, $\mathbb{P}(\xi_t | \xi_1, \xi_2, \ldots, \xi_{t-1}) = \mathbb{P}(\xi_t | \xi_{t-1})$. Additionally, the state space $\Xi_t$ is finite and the one-step transition probability is given by a transition probability matrix $P$, where the $(k, k')$-entry is defined by $P_{k, k'} := \mathbb{P}(\xi_{t+1} = k' | \xi_t = k), \forall t = 1, \ldots, T$.

Under Assumption 6, the conditional expectation $\mathbb{E}[\xi_t | \cdot]$ will be replaced by $\mathbb{E}[\xi_t | \cdot]$. With a slight abuse of notation, we denote the conditional probability of $\xi_{t+1}$ given $\xi_t$ as $P_{\xi_{t+1} | \xi_t}$.

In formulation (2), the goal of the DM is to optimize a policy, which is a mapping from the state of the system to an action. In particular, at every decision period $t = 1, \ldots, T$, the DM makes a decision $a_t$ chosen from the feasible set $\mathcal{X}_t(a_{t-1}, \xi_t)$. The decision vector $a_t$ can be split into two types of decisions: a vector of state variables $u_t$ and a vector of local (control) variables $v_t$, i.e., $a_t := (u_t, v_t)$. The vector of state variables $u_t$ is what links different stages together, and the vector of local variables $v_t$ participates exclusively in the optimization problem defined for each period $t$. Under Assumption 4, the feasible set $\mathcal{X}_t(a_{t-1}, \xi_t)$ in each period $t$ is given by the set of linear constraints:

$$\mathcal{X}_t(a_{t-1}, \xi_t) = \left\{ a_t = (u_t, v_t) \mid A_t^e u_t + B_t^e u_{t-1} + C_t^e v_t = b_t^e \right\}.$$

(3)

As such, the decision-making process in formulation (2) proceeds as follows. In each period $t = 1, \ldots, T$, the DM observes the state of the system $s_t := S_t(u_{t-1}, \xi_t)$, which depends on: (i) the action from the previous period $a_{t-1}$ (and more specifically $u_{t-1}$), and (ii) the realization of random vector $\xi_t$ at the current period. When $t < T$, the DM aims to find an optimal action $a_t$ which minimizes the immediate cost given by $z_t(a_t, \xi_t)$, plus the expected future cost which is given by the nested expectation $\mathbb{E}[\xi_t | \cdot]$. Finally, when $t = T$, no more future cost is to be paid, and the DM’s goal is to minimize the immediate cost $z_T(a_T, \xi_T)$ only. A symbolic representation of the decision-making procedure in MSP models is shown in Figure 1 in the Appendix.

We refer to formulation (2) as the fully adaptive MSP (FA-MSP) model, as the optimal action $a_t$ can be adapted to the state of the system $s_t$ in every period $t = 1, \ldots, T$. We refer to note that, when referring to an FA-MSP formulation, we use the terms “period” and “stage” interchangeably. However, in other settings, as we describe later, a single stage may include multiple periods: decisions made in a single stage, which has the same level of adaptability, can include decisions for multiple periods in general.

**Dynamic programming equations.** A typical approach to proceed with the computation of an optimal policy for the FA-MSP model (2) is to use the so-called Bellman dynamic programming equations (DPE) (5):

$$Q_t(a_{t-1}, \xi_t) := \min_{a_t} \left\{ z_t(a_t, \xi_t) + \mathbb{E}[\xi_{t+1} | a_t] \mid a_t \in \mathcal{X}_t(a_{t-1}, \xi_t) \right\} \quad \forall t = 1, \ldots, T,$$

(4)
where $\Omega_{t+1}^5(a_t)$ is referred to as the expected cost-to-go function and is given by $\Omega_{t+1}^5(a_t) := \mathbb{E}[Q_{t+1}(a_t, \xi_{t+1}) | \xi_t], \forall t = 1, \ldots, T - 1$ with $\Omega_{T+1}^5(a_T) := 0$. Since by Assumption 6 the state space $\mathcal{Z}_t$ is finite, we can write the expected cost-to-go function as $\Omega_{t+1}^5(a_t) = \sum_{a_{t+1} \in \mathcal{Z}_{t+1}} P_{\xi_{t+1}|\xi_t} Q_{t+1}(a_t, \xi_{t+1}), \forall t = 1, \ldots, T - 1$ with $\Omega_{T+1}^5(a_T) := 0$. We care to mention that MSP models where the associated stochastic process $(\xi_t)^T_{t=1}$ follows a discrete-time MC are also referred to as the Markov-chain MSP. See, e.g., [11][12], for further discussions on this topic. A brief overview, and an algorithmic description, for how to solve the DPE [4], using nested Benders decomposition, is given in Section A in the Appendix.

### 3.3 Stochastic Programming Models with Deterministic Landfall Time

In this subsection, we assume that the hurricane landfall time $T$ is known a priori. First, we describe how the demand quantities at different DPs are determined in our model, and describe the stochastic process $(\xi_t)^T_{t=1}$ in the context of the HDRLP problem. Second, we complete the description of the FA-MSP formulation by juxtaposing the components of the deterministic multiperiod HDRLP problem [11] with the generic formulation provided in [2]. Finally, we discuss the alternative approximation policies.

#### 3.3.1 Modeling hurricane evolution and demand estimates

We assume that the demand for relief items at different DPs depends on two attributes of a hurricane at the time of its landfall: the intensity $\alpha_T \in A$ and the location $\ell_T \in L$. We also assume that this dependency structure is given by a deterministic mapping $D : A \times L \rightarrow \mathbb{R}^{\mathbb{Z}}$, that is, $d^\mathcal{Z}_{t} := (d^{\mathcal{Z}_{t}}_{1}, \ldots, d^{\mathcal{Z}_{t}}_{|\mathbb{Z}|}) = D(\alpha_t, \ell_t)$. It is intuitive to assume that DPs within closer proximity to the hurricane’s landfall location $\ell_T$ will prompt higher demand for relief items and vice versa. By contrast, hurricanes with higher intensity levels $\alpha_T$ will prompt higher demand for relief items and vice versa. As such, we assume that this mapping $D(\alpha_t, \ell_t)$ is increasing in $\alpha_t$ and decreasing in the distance between $\ell_t$ and the locations of DPs. The specifics of this function are provided, along with the remaining problem data, in Section C in the Appendix.

To model the stochastic evolution of the hurricane characteristics as an MC, we assume that the hurricane’s location and intensity are characterized by two independent random variables $\ell_t \in L$, $\alpha_t \in A$ with finite state space, such that $\xi_t := (\alpha_t, \ell_t)$ and $\mathcal{Z}_t := A \times L$, where $|\mathcal{Z}_t| < \infty \forall t = 1, \ldots, T$. We further assume that $\ell_t$ and $\alpha_t$ follow an MC. We denote the conditional probability of $\alpha_t$ given $\alpha_{t-1}$ as $P_{\alpha_t} | \alpha_{t-1}$, and the conditional probability of $\ell_t$ given $\ell_{t-1}$ as $P_{\ell_t} | \ell_{t-1}$. Consequently, $P_{\ell_t} | \xi_{t-1} = P_{\ell_t} | \alpha_{t-1} \times P_{\ell_t} | \ell_{t-1}$. We use matrices $P^\alpha$ and $P^\ell$ to denote the one-step transition probability matrices associated with the intensity $\alpha$ and location $\ell$, respectively.

#### 3.3.2 FA-MSP model with a deterministic time of landfall

Following the definitions of the decision variables in [11], at any point in time $t = 1, \ldots, T$, the state of the system in the HDRLP problem is characterized by two components: (i) the informational state $\xi_t$ representing the hurricane attributes, and (ii) the physical state $x_{t-1}$ representing the relief items inventory levels at SPs. Hence, the state variables are given by $x_t := x_t, \forall t = 1, \ldots, T$ and the local (control) variables are given by $\nu_t := f_t, \forall t = 1, \ldots, T - 1, \nu_T := (f_T, y)$ at the landfall stage $T$. That is, $x_t := (x_t, f_t), \forall t = 1, \ldots, T - 1$ and $\alpha_T := (x_T, f_T, y)$. Consequently, the policy is a mapping from the state of the system $(\xi_t, x_{t-1})$ to action $(x_t, f_t)$ for $t = 1, \ldots, T - 1$, and from $(\xi_T, x_{T-1})$ to $(x_T, f_T, y)$ for the landfall stage $T$.

We can then juxtapose the components of the CV deterministic formulation [11] with the remaining components of formulation [2]; namely, $z_t(a_t, \xi_t)$ and $\chi_t(a_t, \xi_t)$. To that end, we remark that based on Assumption 5 since the demand only occurs at the landfall stage $t = T$, one can remove the dependence on $\xi_t$ from $z_t(a_t, \xi_t)$ and $\chi_t(a_{t-1}, \xi_t)$ for every non-terminal stage $t = 1, \ldots, T - 1$, such that:

$$z_t(a_t, \xi_t) := \sum_{i \in [0]:t \neq f} \sum_{i \in [0]:t \neq f} c^x_{i,t,t} f^x_{i,t} + \sum_{i \in [0]:t \neq f} c^x_{i,t,t} x_{i,t} + \sum_{i \in [0]:t \neq f} h_{i,t} f^h_{i,t},$$

and

$$\chi_t(a_{t-1}, \xi_t) := \begin{cases} x_{i,t-1} + \sum_{i \in [0]:t \neq f} f_{i,t} - \sum_{i \in [0]:t \neq f} f_{i,t} = x_{i,t} & \forall i \in I, \\ \sum_{i \in [0]:t \neq f} f_{i,t} \leq x_{i,t-1} & \forall i \in I, \\ 0 \leq x_{i,t} \leq u_i & \forall i \in I. \end{cases}$$
We refer to the FA-MSP model introduced in this section, hereafter, as the FA-MSP model with a deterministic time of landfall (FA-MSP-D), to distinguish it from the one that we introduce later in Subsection 3.4 where the hurricane’s time of landfall $T$ is random.

### Alternate decision policies

MSP models are well known to be computationally challenging to solve. These computational challenges might especially be of concern when the solution effort is restricted by a computational budget (e.g., when a strict time limit is imposed). Consider a situation where the forming of a tropical storm was not detected in its early stages and, consequently, the initial forecast advisories were overdue. In this case, it is vital to have a model that can provide a “good” disaster relief logistics policy within a reasonable amount of time. In the remainder of this subsection, we discuss two alternative decision policies for mitigating some of the computational burdens of solving fully-adaptive MSP models. The first is the S-2SSP model, where the logistics decisions are obtained by solving a single 2SSP model and implemented statically. The second is the Rolling-horizon two-stage stochastic programming (RH-2SSP) approach, where an online policy is obtained by solving a sequence of 2SSP models and implementing the corresponding solutions in an RH fashion.

**Two-stage approximation.** As mentioned earlier, two terms that are often used interchangeably in MSP models are stages and periods. Both terms refer to different time points in the planning horizon $\{1, \ldots, T\}$ of an MSP problem. However, the two terms have the following subtle conceptual distinction which is of significant relevance to the context of the HDRLP problem. In this case, it is vital to have a model that can provide a “good” disaster relief logistics policy within a reasonable amount of time. In the remainder of this subsection, we discuss two alternative decision policies for mitigating some of the computational burdens of solving fully-adaptive MSP models. The first is the S-2SSP model, where the logistics decisions are obtained by solving a single 2SSP model and implemented statically. The second is the Rolling-horizon two-stage stochastic programming (RH-2SSP) approach, where an online policy is obtained by solving a sequence of 2SSP models and implementing the corresponding solutions in an RH fashion.

At the terminal (landfall) stage $t = T$, we have:

\[
    z_T(a_T, \xi_T) := \sum_{i \in I} \sum_{t \in T} c_{it, T} f_{it, T} + \sum_{i \in I} c_{i, T} x_{i, T} + h_T \sum_{i \in I} f_{0, T} + \sum_{i \in I} \sum_{j \in J} c_{ij} y_{ij}
\]

\[+ \sum_{j \in J} p(d_j^T - v_j) + \sum_{i \in I} q(x_{i, T} - y_{ij}), \quad (7)\]

and

\[
    x_T(a_{T-1}, \xi_{T-1}) := \begin{cases} 
    x_{i, T-1} + \sum_{t \in (0, T, i, \neq i)} f_{it, T} - \sum_{t \in (0, T, \neq i)} f_{it, T} = x_{i, T} & \forall i \in I, \\
    0 \leq x_{i, T} \leq u_i & \forall i \in I, \\
    \sum_{j \in J} y_{ij} \leq x_{i, T} & \forall i \in I, \\
    \sum_{i \in I} y_{ij} \leq d_j^T & \forall j \in J, \\
    y_{ij} \geq 0 & \forall i \in I, j \in J. 
    \end{cases} \quad (8)
\]

We refer to the FA-MSP model introduced in this section, hereafter, as the FA-MSP model with a deterministic time of landfall (FA-MSP-D), to distinguish it from the one that we introduce later in Subsection 3.4 where the hurricane’s time of landfall $T$ is random.

### Alternative decision policies

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\]

\[+ \sum_{j \in J} p(d_j^T - v_j) + \sum_{i \in I} q(x_{i, T} - y_{ij}), \quad (7)\]

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\[
    x_T(a_{T-1}, \xi_{T-1}) := \begin{cases} 
    x_{i, T-1} + \sum_{t \in (0, T, i, \neq i)} f_{it, T} - \sum_{t \in (0, T, \neq i)} f_{it, T} = x_{i, T} & \forall i \in I, \\
    0 \leq x_{i, T} \leq u_i & \forall i \in I, \\
    \sum_{j \in J} y_{ij} \leq x_{i, T} & \forall i \in I, \\
    \sum_{i \in I} y_{ij} \leq d_j^T & \forall j \in J, \\
    y_{ij} \geq 0 & \forall i \in I, j \in J. 
    \end{cases} \quad (8)
\]
The first-stage problem:

\[
\begin{align*}
\min_{x,f} & \quad \sum_{t=1}^{T-1} \left( \sum_{\ell \in \{0\} \cup J} \sum_{\ell' \notin I} c^b_{\ell' \ell} f_{\ell' T} + \sum_{\ell \in J} c^b_{\ell T} x_{\ell T} \right) + \sum_{t=1}^{T-1} h_t \sum_{\ell \in J} f_{\ell T} + \Omega^{S_1}(x_{T-1}) \\
\text{s.t.} & \quad x_{\ell T-1} + \sum_{\ell' \notin I} f_{\ell' T} - \sum_{\ell' \notin I} f_{\ell T} = x_{\ell T}, \quad \forall \ell \in I, \forall t = 1, \ldots, T-1 \\
& \quad \sum_{\ell' \notin I} f_{\ell' T} \leq x_{\ell T-1}, \quad \forall \ell \in I, \forall t = 1, \ldots, T-1 \\
& \quad 0 \leq x_{\ell T} \leq u_i, \quad \forall \ell \in I, \forall t = 1, \ldots, T-1.
\end{align*}
\]

Here, \(\Omega^{S_1}(x_{T-1}) := \sum_{\xi_T \in \Xi} P^{S_1}_{\xi_T}(x_{T-1}, \xi_T)\) gives the second-stage value function, where \(P^{T-1}_{\xi_T}\) gives the \((T-1)\)-step transition probability matrix associated with \(\xi\), and thus \(P^{T-1}_{\xi_T}\) corresponds to the conditional probability of \(\xi_T\) given initial state \(\xi_1\).

The second-stage problem:

\[
\begin{align*}
Q(\xi_T) := \min_{x_T, f_{T-1}} & \quad h_T \sum_{\ell \in J} f_{\ell T} + \sum_{\ell \in J} q_T \left( x_{\ell T} - \sum_{j \in J} y_{ij} \right) + \sum_{\ell \in J} \sum_{j \in J} c^d_{ij} y_{ij} \\
& \quad + \sum_{\ell \in J} c^b_{\ell T} x_{\ell T} + \sum_{\ell \in J} c^b_{\ell T} f_{\ell T} + \sum_{j \in J} p \left( d_{ij} - \sum_{i \in I} y_{ij} \right) \\
\text{s.t.} & \quad 0 \leq x_{\ell T} \leq u_i, \quad \forall \ell \in I, \forall t = 1, \ldots, T-1 \\
& \quad \sum_{j \in J} y_{ij} \leq x_{\ell T}, \quad \forall j \in J, \forall t = 1, \ldots, T-1 \\
& \quad y_{ij} \geq 0, \quad \forall j \in J, \forall i \in I.
\end{align*}
\]

We care to note that, other stage-aggregation approaches with less restrictive levels have also been proposed in the literature. For instance, in [42], the authors introduce a partially adaptive model, in which the decisions are fully adaptive up to a certain period, and then follow a two-stage approach thereafter.

While the S-2SSP model is easier to solve than the FA-MSP-D model, it is not difficult to see how the static nature of the S-2SSP could compromise the quality of the resulting policy. The policy obtained by the S-2SSP prescribes prepositioning specific levels of relief items, for each period \(t = 1, \ldots, T\), before seeing how the stochastic process will unfold. As such, if the stochastic process \((\xi_1, \ldots, \xi_T)\) end up evolving in a manner that is drastically different than the one initially predicted by the conditional probability \(P^{T-1}_{\xi_T}\), a large penalty might be incurred (or large quantities of relief items might be salvaged). To allow for some adaptability in the decision-making, we consider an RH procedure, which we discuss next. Note that we shall refer to the S-2SSP model introduced in this section, hereafter, as the S-2SSP model with a deterministic time of landfall (S-2SSP-D).

Rolling-horizon (RH) procedure. The RH procedure injects adaptability directly into the policy by solving a new optimization problem (usually referred to as a look-ahead model) in every period \(t \in \{1, \ldots, T\}\). An optimal policy for the MSP formulation \([4]\) is induced by the expected cost-to-go functions \(\Omega^{S_1}(\cdot), \forall \xi_T \in \Xi, t = 1, \ldots, T\), which can be approximated by a collection of cutting planes, in an offline fashion (see Algorithm \([4]\) in the Appendix). Instead, in the RH procedure, the policy is obtained in an online fashion, by solving a look-ahead model in each period of the planning horizon. In its simplest form, the RH proceeds as follows. In every period \(t = 1, \ldots, T\), the DM: (i) defines an optimization problem with \(t \in \{1, 2, \ldots, T - t + 1\}\) stages to serve as the look-ahead model that approximates the fully adaptive \((T - t + 1)\)-stage problem at period \(t\); (ii) solves this \(t\)-stage problem; (iii) implements only the decisions corresponding to the same period; and (iv) rolls forward one period.

Several methods have been proposed in the literature on how to define the \(t\)-stage problem that serves as the look-ahead model in every roll (see e.g., [1][8][34][37][32]). In this paper, we consider that \(t = 2\), i.e., we solve an S-2SSP-D model.
of the form (9) and (10) in every roll. We refer to this approach that implements the S-2SSP-D in the RH procedure as the RH-2SSP model with a deterministic time of landfall (RH-2SSP-D).

The advantage of using this framework is twofold. First, from a solution quality perspective, in each period $t$, by implementing the decisions pertaining to the current period $t$ only, the policy is naturally adaptive to the newly observed realizations of $\xi$, as opposed to the S-2SSP-D policy which commits to all decisions obtained from the model solved at the first period. Second, from a computational perspective, in each roll $t=1,...,T-1$, the DM solves a 2SSP model which is easier to solve than the FA-MSP-D model. We investigate these claims in the context of the HDRLP problem in our numerical study in Section 4.

### 3.4 Multi-stage Stochastic Programming Model with Random Landfall Time

One crucial assumption of solving the MSP formulation (2) is that it has a fixed number of stages $T$, which is known a priori by the DM. In the context of the HDRLP problem, and in light of Assumption 3, the value of $T$ corresponds to the landfall time of the hurricane. In practice, the time of landfall depends on several factors such as the hurricane’s forward speed and trajectory, which are random over time. For example, a rapid surge in the hurricane’s forward speed could cause the hurricane to arrive much earlier than anticipated – rendering the DM unprepared for its early arrival. There are other situations where the hurricane encounters circumstances hostile to its flourishing (e.g., cooler sea surface temperatures, shearing upper-level winds) which make it weaken or even dissipate altogether (7). In this subsection, we lift the assumption that the landfall time $T$ is fixed and known in advance. To that end, we extend the FA-MSP-D, S-2SSP-D, and RH-2SSP-D to the situation where $T$ is random. We do this, first, by proposing an MSP model with a random number of stages in the spirit of the one presented in (19), which we briefly review next.

(19) presents an MSP model with a random number of stages $T$, where $\xi$’s are assumed to be stage-wise independent. Note that with the stage-wise independence assumption, when $T$ is fixed and known a priori, it is only necessary to define a single expected cost-to-go function $Q_t(\cdot)$ for every stage in the planning horizon. To address the randomness of $T$, a parameter $T_{\max} < \infty$, referred to as the maximum possible number of stages, is introduced to replace $T$ in the MSP formulation (2). To facilitate DPE analogous to (4), the following two features are introduced. First, a new state variable $\zeta_t = \mathbb{I}_{t<T}$ is introduced in each stage $t=1,\cdots,T_{\max}$, which indicates whether or not the terminal stage has occurred. Second, the expected cost-to-go function $Q_t(a_{t-1})$ is replaced by $Q_t(a_{t-1}, \zeta_t)$. In other words, instead of using one expected cost-to-go function for each stage, the expected cost-to-go function is parameterized by whether or not the terminal stage has occurred, such that:

- $Q_t(a_{t-1}, 1)$ is the expected cost-to-go function when the terminal stage is yet to occur.
- $Q_t(a_{t-1}, 0)$ is the expected cost-to-go function when the terminal stage has already occurred. This is also referred to as a null function.

Letting $p_t = \mathbb{P}(t \geq T)$, the idea then is to rewrite the DPE as:

$$Q_t(a_{t-1}, \zeta_t; \zeta_{t-1}, 1) := \min_{a_t} \{ \zeta_t \mathbb{I}_t + Q_{t+1}(a_t, \zeta_t) + \mathbb{E}[Q_{t+1}(a_t, \zeta_t + 1, 1)] \} \quad \forall t=1,\ldots,T_{\max},$$

(11)

where

$$Q_{t+1}(a_t, 1) = (1-p_{t+1})\mathbb{E}[Q_{t+1}(a_t, \zeta_{t+1}, 1, 1)] + p_{t+1}\mathbb{E}[Q_{t+1}(a_t, \zeta_{t+1}, 1, 0)],$$

(12)

$$Q_{t+1}(a_t, 0) := 0, \text{ for } t=1,\cdots,T_{\max}-1 \text{ and } Q_{T_{\max}+1}(a_{T_{\max}}, \zeta_{T_{\max}}) := 0.$$

Note again that, since in (19) the stochastic process is assumed to be stage-wise independent, the $Q_t(\cdot)$ functions do not have the superscripts $\zeta_{t-1}$. We extend formulation (11) to the case where the $\xi$’s are assumed to be stage-wise dependent, following an MC model.

**DPE for an MSP model with a random $T$ and stage-wise dependent $\xi_t$.** The key idea in our proposed model is to interpret the function $Q_t(a_{t-1}, 1)$ in (12) as the expected cost-to-go function corresponding to a transient state of the MC: upon reaching this transient state, an immediate cost $\zeta_t(a_t, \xi_t)$ is incurred, and the process evolves into a new state in the next period. By contrast, the null function $Q_t(a_{t-1}, 0)$ can be interpreted as the expected cost-to-go function corresponding to an absorbing state in the MC: once the stochastic process reaches this absorbing state, it will remain there without incurring any future cost. Given this interpretation, we partition the state space into two sets $\mathcal{F}$ and $\mathcal{A}$, where $\mathcal{F}$ represents the set of transient states and $\mathcal{A}$ represents the set of absorbing states, such that $\mathcal{Z} = \mathcal{F} \cup \mathcal{A}$. We then rewrite the DPE (4) as follows.

- For a transient state $\xi_t \in \mathcal{F}$:

$$Q_t(a_{t-1}, \xi_t) := \min_{a_t} \{ \zeta_t(a_t, \xi_t) + Q_{t+1}(a_t) \} \quad \forall t=1,\ldots,T_{\max},$$

(13)
We now define the state space of the modified MC model to be
\[ \mathcal{X} = \{ (x, y) \mid x \in \mathbb{R}^2, y \in \mathbb{R} \} \]

We care to note that, with slight modification, the nested Benders decomposition, shown in Algorithm 1 in the Appendix, can be applied to solve the DPE (13) and (14), and the convergence of the algorithm follows by the argument shown in [19]. Nevertheless, we need to adapt our MC model, which characterizes the hurricane evolution, to incorporate the newly introduced temporal dimension of the hurricane random time of landfall. Additionally, since we do not know in advance when the hurricane makes landfall, we need to adapt our definition of the decision variables, the objective functions, and the constraint sets in the MSP model (2) to reflect the same. We discuss these modifications next.

Incorporating the temporal component to the MC model for hurricane evolution. We first split the random variable \( \ell_t \) denoting the hurricane location at time \( t \) into two independent random variables \( \ell_{x,t} \in L_x \) and \( \ell_{y,t} \in L_y \), representing the \( x \)-coordinate and \( y \)-coordinate of the hurricane location at time \( t \), respectively. We further assume that the evolution of each \( \ell_{x,t} \) and \( \ell_{y,t} \) is given by an MC model, where the one-step transition probability matrix is given by \( \mathbf{P}_{\ell_{x,t}|\ell_{x,t-1}} \) and \( \mathbf{P}_{\ell_{y,t}|\ell_{y,t-1}} \), respectively. For illustration, we extend the \( y \)-coordinate in the example shown in Figure 1 to include \( \ell_{y,t} < 0 \) values, discretize it, and let each point in the discretized space represent the location of the hurricane at time \( t = 1, \ldots, T_{\max} \) (see Figure 2), such that:

- \( \ell_{y,t} < 0 \) means that the hurricane has not made landfall yet at time \( t \).
- \( \ell_{y,t} = 0 \) means that the hurricane makes landfall at time \( t \).
- \( \ell_{y,t} > 0 \) means that the hurricane has already made landfall before time \( t \).

We now define the state space of the modified MC model to be \( \mathcal{X}_t = \mathbb{R} \times L_x \times L_y \), and let \( \xi_t = (\alpha_t, \ell_{x,t}, \ell_{y,t}) \) denote the Markovian state at time \( t \), where \( \ell_{y,t} \leq 0 \Rightarrow \xi_t \in \mathcal{T} \), i.e., \( \xi_t \) is a transient state. By contrast, \( \ell_{y,t} > 0 \Rightarrow \xi_t \in \mathcal{A} \), i.e., \( \xi_t \) is an absorbing state. Similarly to the MC model with a deterministic time of landfall, we can now redefine the joint probability distribution as \( \mathbf{P}_{\xi_{t+1}|\xi_t} := \mathbf{P}_{\alpha_{t+1}|\alpha_t} \times \mathbf{P}_{\ell_{x,t+1}|\ell_{x,t}} \times \mathbf{P}_{\ell_{y,t+1}|\ell_{y,t}} \).

We care to note that, in addition to making the time of landfall random, this construction can also allow for modeling other situations. For instance, one can define an absorbing state corresponding to the situation where the hurricane dissipates. This, for instance, could be because the wind slows down and the storm subsides (i.e., the intensity \( \alpha_t = 0 \)) or because the hurricane trajectory drifts outside of the APA. Figure 2 illustrates the MC model with a random landfall time, by showing different hurricane trajectories towards the APA.

The last missing piece for formulating this model is to define an appropriate value for \( T_{\max} \). This is, of course, case-by-case, and we discuss how to choose this value in Section 4 in the Appendix.

### 3.4.1 FA-MSP model with a random time of landfall

In this subsection, we present the fully adaptive MSP model with a random time of landfall (FA-MSP-R). Recall that by Assumption 3, the stage-\( t \) problems associated with the absorbing states are degenerate since, according to (14), the immediate cost functions are defined as \( g_t(\alpha_t, \xi_t) = 0 \), and the expected cost-to-go functions \( \Omega^\xi_{t+1}(\alpha_t) = 0 \), \( \forall \xi_t \in \mathcal{A}, t = 1, \ldots, T_{\max} - 1 \). As such, we focus on the definition of the stage-\( t \) problems associated with transient states only.

Since we assume the time of landfall \( T \) to be random, the demand can happen at any point before \( T_{\max} \). Hence, we need to index the demand quantities \( d \) by the period \( t \) for \( t = 1, \ldots, T_{\max} \), such that \( d_t := D(\alpha_t, \ell_{x,t}) \), when \( t = T \) and \( d_t := 0, \forall t \neq T \). Additionally, since it is not possible to decouple the stage-\( t \) problems, as we did in the FA-MSP-D into a set of non-terminal stages problems and a terminal-stage problem, all of the state and local variables need to be included in every stage-\( t \) problem, \( \forall t = 1, \ldots, T_{\max} \). To simplify the presentation, we introduce two auxiliary decision variables, \( \xi_t \) and \( \tau_t \) for \( t = 1, \ldots, T_{\max} \), such that:

- \( \xi_t := (\ell_{x,t})_{i \in J} \) denotes the amount of unsatisfied demand (if any).
- \( \tau_t := (\ell_{y,t})_{i \in J} \) denotes the amount of salvaged items (if any).
Note that, when \( d_{jt} \) is known, we can adopt the deterministic model \( (1) \) as follows.

\[
\begin{align*}
\min_{x_{i,t}, y_{i,t}} & \quad \sum_{t=1}^{T_{\text{max}}} \left( \sum_{i \in \{0,1\}, j \in I} c_i f_{i,t}^d + \sum_{i \in I} c_i^h x_{i,t} + h_t \sum_{i \in I} f_{0,t} + \sum_{i \in I} \sum_{j \in J} c_{ij,t} y_{i,j,t} + p \sum_{i \in I} \epsilon_{i,t} + q \sum_{i \in I} \bar{c}_{i,t} \right) \\
\text{s.t.} & \quad x_{i,t-1} + \sum_{\ell' \in I, \ell' \neq i} f_{i,t-1}^d - \sum_{\ell' \in I, \ell' \neq i} f_{i,t}^d - \sum_{j \in J} y_{i,j,t} - \bar{c}_{i,t} = x_{i,t}, \quad \forall i \in I, \quad t = 1, \ldots, T_{\text{max}}, \\
& \quad \sum_{\ell' \in I, \ell' \neq i} f_{i,t}^d \leq x_{i,t} - 1, \quad \forall i \in I, \quad t = 1, \ldots, T_{\text{max}}, \\
& \quad x_{i,t} \leq u_t, \quad \forall i \in I, \quad t = 1, \ldots, T_{\text{max}}, \\
& \quad \sum_{i \in I} y_{i,j,t} + \epsilon_{j,t} \geq d_{j,t}, \quad \forall j \in J, \quad t = 1, \ldots, T_{\text{max}}, \\
& \quad x_{i,t}, f_{i,t}, y_{i,j,t}, \epsilon_{j,t}, \bar{c}_{i,t} \geq 0, \quad \forall t = 1, \ldots, T_{\text{max}}.
\end{align*}
\]

Letting \( a_t := (x_{i,t}, f_{i,t}, y_{i,j,t}, \epsilon_{j,t}, \bar{c}_{i,t}) \), we can juxtapose the remaining components of the modified deterministic formulation \( (15) \) with the components of formulation \( (13) \) \( \forall t = 1, \ldots, T_{\text{max}} \), as follows.

\[
z_t(a_t, \bar{a}_t) := \sum_{i \in \{0,1\}, j \in I} c_i f_{i,t}^d + \sum_{i \in I} c_i^h x_{i,t} + h_t \sum_{i \in I} f_{0,t} + \sum_{i \in I} \sum_{j \in J} c_{ij,t} y_{i,j,t} + p \sum_{i \in I} \epsilon_{i,t} + q \sum_{i \in I} \bar{c}_{i,t},
\]

and

\[
\chi_t(a_{t-1}, \bar{a}_t) := \begin{cases} 
(x_{i,t-1} + \sum_{\ell' \in I, \ell' \neq i} f_{i,t-1}^d - \sum_{\ell' \in I, \ell' \neq i} f_{i,t}^d - \sum_{j \in J} y_{i,j,t} - \bar{c}_{i,t} = x_{i,t}, & \forall i \in I, \\
\sum_{\ell' \in I, \ell' \neq i} f_{i,t}^d \leq x_{i,t} - 1, & \forall i \in I, \\
x_{i,t} \leq u_t, & \forall i \in I, \\
\sum_{i \in I} y_{i,j,t} + \epsilon_{j,t} \geq -d_{j,t}^h, & \forall j \in J, \\
x_{i,t}, f_{i,t}, y_{i,j,t}, \epsilon_{j,t}, \bar{c}_{i,t} \geq 0. & \forall i, j.
\end{cases}
\]

### 3.4.2 Alternative decision policies for cases with random landfall time

We also consider alternative decision policies analogous to the S-2SSP-D and RH-2SSP-D discussed in Subsection 3.3.5.
**Static2SSP model with random landfall time** $T$. The S-2SSP model with a *random* time of landfall (S-2SSP-R) presented in this section is similar to the S-2SSP-D presented in Subsection 3.3; they both introduce a formulation where different periods across the planning horizon are aggregated into two stages only and they both render a static policy which entails making all the prepositioning decisions, in advance, before the hurricane making landfall. Specifically, in the first-stage problem (9) that includes all periods before the landfall time, where decisions concerning the preparation of the hurricane arrival are made; and a second-stage problem (10) that corresponds to the landfall time, where the DM’s reaction decisions are made once the uncertain demand is revealed. However, when $T$ is random, the static nature of a two-stage approximation has some delicate implications on defining the first-stage and second-stage problems.

The main concern is that, when $T$ is random, if the prepositioning decisions that are previously made cannot be adapted, it is possible that the DM might arrive at a static decision which prescribes prepositioning relief items after the hurricane’s landfall. That is, it could happen that $x_t > 0$ and $f_t > 0$ for some $T < t < T_{\text{max}}$. To see how this could be case, let $t'$ be an arbitrary period in the planning horizon such that $1 < t' < T_{\text{max}}$, and suppose that we solve an S-2SSP model with $T_{\text{max}}$ stages, such that the resulting policy prescribes to have the following prepositioning decisions: $x_t > 0$ and $f_t > 0$ for some $t \leq t'$, and $x_t = 0$ and $f_t = 0$, $\forall t' < t$. In other words, the DM starts (and continues) the prepositioning at any point in time before $t \leq t'$, and stops after time $t > t'$. Since $T$ is assumed to be random, there are two possible outcomes for the hurricane’s time of landfall: (i) the hurricane makes landfall *after* when all the prepositioning is finished, i.e., $t' < T$. This situation is similar to the one discussed for the deterministic landfall time case; (ii) the hurricane makes landfall *before* the prepositioning is finished, i.e., $T < t'$. This second situation has two further implications. First, as is the case for the model with a deterministic $T$, upon arriving at the landfall the DM pays a penalty for demand shortage (if any) or salvages the remaining items (if any). The second implication, which is of more relevance, is to start or continue prepositioning after the hurricane has made landfall. In light of Assumption 3 the act of starting or continuing to preposition relief items despite knowing that the hurricane has already made landfall is somewhat pathological. To circumvent this potential shortcoming, we introduce a set of “reimbursement” parameters $r_t, \forall t = 1, \cdots, T_{\text{max}}$ to the second-stage problem to compensate the DM for all the logistics costs that were counted in the first stage for every period $t > T$, such that

$$r_t := \begin{cases} \left( -\sum_{i \in I(0)} \sum_{t' \in J(t)} c_{i,t'} f_{i,t'} + \sum_{i \in I(t)} c_{i,t} x_{i,t} + h_i \sum_{t \in I} f_{i,t} \right) & \forall t > T, \\ 0 & \text{otherwise}. \end{cases} \quad (18)$$

Putting everything together, given initial data $x_0$ and $\xi_1$, we define the first-stage and second-stage problems in the S-2SSP-R model as follows.

The *first-stage* problem:

$$\begin{align} & \min_{x,f} \sum_{t=1}^{T_{\text{max}}} \left( \sum_{i \in I(0)} \sum_{t' \in J(t)} c_{i,t'} f_{i,t'} + \sum_{i \in I(t)} c_{i,t} x_{i,t} + h_i \sum_{t \in I} f_{i,t} \right) + \Omega^{\xi_1}(x,f) \\ & \text{s.t.} \quad \sum_{j \in I, j \neq i} f_{i,j,t} \leq x_{i,t-1}, \quad \forall i \in I, t = 1, \cdots, T_{\text{max}} \quad (19a) \\ & \quad x_{i,t} \leq u_i, \quad \forall i \in I, t = 1, \cdots, T_{\text{max}} \quad (19b) \\ & \quad x_{i,t} \geq 0, \quad \forall t = 1, \cdots, T_{\text{max}} \quad (19c) \end{align}$$

where $x = (x_1, \ldots, x_{T-1})$, $f = (f_1, \ldots, f_{T-1})$, and $\Omega^{\xi_1}(x,f) := \sum_{\xi_{\text{max}} \in \Xi_{\text{max}}} P^{\xi_{\text{max}}-1} \cdot Q(x,f,\xi_{\text{max}})$. Note that, since we do not know in advance when the hurricane makes landfall, the second-stage value function $Q(\cdot)$ is now a function of $\xi_{\text{max}}$, which gives the entire history of the stochastic process from $t = 1$ to $t = T_{\text{max}}$.

To define the second-stage problem, we let $I_{[t,T_{\text{max}}]}$ be an indicator function on whether or not period $t$ is after the hurricane’s landfall time $T_{\text{max}}$ associated with trajectory $\xi_{\text{max}}$. **
We benchmark the performances of the different policies using the following metrics. We implemented eight different approaches. The first four correspond to the case when the hurricane’s landfall time 

\( T \)

(vii) the S-2SSP-R described in (19) and (20); and (ix) the RH-2SSP-R discussed in Subsection 3.4.2 (and Section B in the Appendix).

The second-stage problem:

\[
Q(x, f, \tilde{x}_{1:T_{\text{max}}}) := \min_{y \in \mathcal{Y}} \sum_{t=1}^{T_{\text{max}}} \left( \sum_{i \in I} \sum_{j \in J} c_{ij,t} y_{ij,t} + \sum_{j \in J} \xi_{j,t} + \sum_{i \in I} \eta_{i,t} \right) \\
- \sum_{i \in I} \left( \sum_{j \in J} \sum_{t \in T} \sum_{i \in I} c_{ij,t} f_{t,i,j} + \sum_{i \in I} \sum_{t \in T} f_{0,i,j} \right) 1_{\{T > T_{\text{max}}\}} \\
\text{s.t. } \sum_{j \in J} y_{ij,t} + \xi_{j,t} = x_{i,t-1} + \sum_{j \in J, j \neq i} f_{j,t,i} - \sum_{j \in J, j \neq i} f_{i,t,j} - x_{i,t} , \\
\forall i \in I, t = 1, 2, \ldots, T_{\text{max}} \\
\sum_{i \in I} y_{ij,t} + \xi_{j,t} \geq \delta_{ij,t} , \\
\forall j \in J, t = 1, 2, \ldots, T_{\text{max}} \\
y_{ij,t} , \xi_{j,t} , \eta_{i,t} \geq 0 , \\
\forall t = 1, 2, \ldots, T_{\text{max}}.
\]

(20a)

(20b)

(20c)

Figure 3 shows a symbolic representation of the proposed S-2SSP-R formulation.

**RH-2SSP approach with random landfall time** \( T \). In general, the RH-2SSP model with a random time of landfall (RH-2SSP-R) presented here is similar to the RH-2SSP-D discussed in Subsection 3.3 for the deterministic landfall time case: in each period \( t \), the DM solves an S-2SSP model that serves as the look-ahead model, implements the decisions corresponding to period \( t \) only, and then rolls forward. Nonetheless, there is one, albeit small, remark we care to highlight for the sake of completeness, which we present in Section B in the Appendix.

### 4 Numerical Results

In this section, we summarize the performances of different approaches presented in Section 3. In our experiments, we implemented eight different approaches. The first four correspond to the case when the hurricane’s landfall time \( T \) is deterministic: (i) CV model with a deterministic time of landfall (CV-D) described in (1); (ii) the FA-MSP-D described in (4), (5), and (6); (iii) the S-2SSP-D described in (9) and (10); and (iv) the RH-2SSP-D described in Section 3.3.3. The other four correspond to the case when the hurricane’s landfall time \( T \) is random: (v) the CV model with a random time of landfall (CV-R) described in (15); (vi) the FA-MSP-R described in (13), (16) and (17); (vii) the S-2SSP-R described in (19) and (20); and (ix) the RH-2SSP-R discussed in Subsection 3.4.2 (and Section B in the Appendix).

We benchmark the performances of the different policies using the following metrics.

- \( \hat{x} \): the sample mean for the performance of a policy calculated using the out-of-sample evaluation (see 33 in Appendix D).
- \( \hat{p} \pm 1.96 \hat{\sigma} / \sqrt{N} \): the width of the 95%-confidence interval (CI) on the mean performance. Here, \( N \) is the number of scenarios used in the out-of-sample and \( \hat{\sigma} \) is the sample standard deviation (see 33 in Appendix D).
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Table 3: Performances of decision policies given by FA-MSP-D, S-2SSP-D and RH-2SSP-D.

| ψ | J | J | FA-MSP-D | RH-2SSP-D | static2SSP-D |
|---|---|---|---|---|---|
| 0.001 | 10 | 10 | 2333.37 180.31 200.80% 7015.46 1143.26 0.53 804.38% 2333.62 180.38 200.83% 7013.29 532.65 192.81% 19744.29 2724.49 2.13 663.79% | 2333.37 180.31 200.80% 7015.46 1143.26 0.53 804.38% 2333.62 180.38 200.83% 7013.29 532.65 192.81% 19744.29 2724.49 2.13 663.79% | 2333.37 180.31 200.80% 7015.46 1143.26 0.53 804.38% 2333.62 180.38 200.83% 7013.29 532.65 192.81% 19744.29 2724.49 2.13 663.79% |
| 0.6 | 10 | 10 | 4696.86 423.90 975.49 308.58% 9553.51 940.09 341.59% 10032.96 1397.54 2.66 363.75% 2784.98 334.82 1.39 658.78% | 4696.86 423.90 975.49 308.58% 9553.51 940.09 341.59% 10032.96 1397.54 2.66 363.75% 2784.98 334.82 1.39 658.78% | 4696.86 423.90 975.49 308.58% 9553.51 940.09 341.59% 10032.96 1397.54 2.66 363.75% 2784.98 334.82 1.39 658.78% |
| 5 | 10 | 10 | 9130.21 1412.26 1476.25 322.02% 9553.51 940.09 341.59% 10032.96 1397.54 2.66 363.75% | 9130.21 1412.26 1476.25 322.02% 9553.51 940.09 341.59% 10032.96 1397.54 2.66 363.75% | 9130.21 1412.26 1476.25 322.02% 9553.51 940.09 341.59% 10032.96 1397.54 2.66 363.75% |

- **gap**: the relative gap in the mean performance of each policy (\( \hat{z} \)) compared to the one obtained by the CV solution.
- **time**: time (in seconds) required for training each policy. We use notation “−” to indicate that the time-limit of three hours is reached. Note that this statistics is only applicable to the FA-MSP and S-2SSP models.

We note that the reason for not reporting the time for the training policies or performance evaluation for RH approaches is due to the fact that the decision policy is obtained in an online fashion during the out-of-sample evaluation. It is not a fair comparison between the computational time associated with the RH approaches and the time spent on training offline policies by the FA-MSP and S-2SSP models.

In all of our experiments, we use programming language Julia version 1.6.1, utilizing JuMP version 0.22.1 package [13], with commercial solver Gurobi, version 9.5.0 in our implementation. All of the numerical experiments are conducted on Clemson University’s primary high-performance computing cluster, the Palmetto cluster, where we used an R830 Dell Intel Xeon “big memory” compute node with 2.60GHz, 1.0TB memory, and 24 cores. The problem data used in our implementation can be found in Appendix C and the remaining implementation details can be found in Appendix D. All of the codes can be found at Githb [36].

4.1 Main results for the case of deterministic landfall time

In Table 3, we report the numerical results for FA-MSP-D, S-2SSP-D and RH-2SSP-D. The first three columns describe the test instances based on different values of \( \nu \), \( |I| \) and \( |J| \), and columns 4-7, 8-10 and 11-14 show the performance metrics results for FA-MSP-D, RH-2SSP-D and S-2SSP-D, respectively. Additionally, Figure 4(a) depicts the \( \hat{z} \) values of different policies, averaged across all the instances for different \( \nu \) values (see also Figure 8 in the Appendix).

From these results, we can see that on average, the relative gap in \( \hat{z} \) values to the CV-D solution is 264.01%, 340.73% and 573.99% for FA-MSP-D, RH-2SSP-D and S-2SSP-D, respectively: specifically, 183.73%, 183.75% and 722.71% when \( \nu = 0.001 \); 288.99%, 501.32% and 637.60% when \( \nu = 0.6 \); and 319.29%, 337.12% and 361.66% when \( \nu = 5 \).

In summary, the FA-MSP-D has the best overall performance, followed by the RH-2SSP-D, and finally the S-2SSP-D. This is expected due to the level of adaptability associated with the corresponding decision policy for each approach. We also see the trade-off between the solution quality and the computational effort, which is reflected in the computational time spent on policy training. One important observation is that, compared to when \( \nu = 0.6 \), the difference in the
average performance by the three policies shrinks when $\nu = 5$, and the difference in the average performance between
FA-MSP-D and RH-2SSP-D almost vanishes when $\nu = 0.001$. We discuss this observation in more details during our
discussion in the sensitivity analysis in Subsection 4.3.

### 4.2 Main results for the case of random landfall time

In Table 4, we report the numerical results for FA-MSP-R, S-2SSP-R and RH-2SSP-R. The first three columns describe
the test instances based on different values of $\nu$, $|I|$ and $|J|$, and columns 4-7, 8-10 and 11-14 show the performance
metrics results of FA-MSP-R, RH-2SSP-R and S-2SSP-R, respectively. Additionally, Figure 4(b) depicts the $\zeta$ values of
different policies, averaged across all the instances for different $\nu$ values (see also Figure 9 in the Appendix).

From these results, we can see that on average, the relative gap in $\zeta$ values to the CV-R solution is $100.14\%$, $307.56\%$
and $2061.64\%$ for the FA-MSP-R, RH-2SSP-R and S-2SSP-R, respectively: specifically, $0.00\%$, $12.08\%$ and $3342.29\%$
when $\nu = 0.001$; $75.78\%$, $586.67\%$ and $1901.92\%$ when $\nu = 0.6$; and $224.63\%$, $323.93\%$ and $940.71\%$ when $\nu = 5$.

In summary, most of the observations made in Subsection 4.1, for the case of deterministic landfall time, carry over
to the case of random landfall time: FA-MSP-R outperforms RH-2SSP-R, which in turn outperforms S-2SSP-R.
Additionally, the difference in the $\zeta$ values between FA-MSP-R and RH-2SSP-R almost vanishes when $\nu = 0.001$ and
gets smaller when $\nu = 5$. Nevertheless, the additional layer of uncertainty (randomness in $T$) gives rise to additional
insights. In particular, the randomness in $T$ emphasizes the influence of adaptability on the quality of the decision policy.
As we can see from Table 4 and Figure 4(b), the difference in the performance between S-2SSP-R and FA-MSP-R
(or RH-2SSP-R) is amplified. More importantly, the performance of FA-MSP-R is as good as the clairvoyant (CV-R
solution) across all of the instances when $\nu = 0.001$; this is also the case for the RH-2SSP-R approach, except for one
instance. We suspect that this is because when $\nu = 0.001$, the increase in the logistics cost over time is so small that it
encourages a “wait-and-see” policy, where the DM delays all the prepositioning of the relief items until the hurricane’s
attributes at landfall become clear. Nevertheless, the adaptability in the decision is incumbent on such a “wait-and-see”
policy, that is, the DM must have the leverage of being able to wait and adapt their decision according to what they
observe over time. We investigate these claims further in the next subsection.

### 4.3 Sensitivity analysis

In this subsection, we present and discuss the results for the sensitivity analysis performed on different policies for
instances with different cost-scaling factor $\nu$ and different initial intensity level of the hurricane. We discuss this for
both the deterministic and random landfall time cases, according to the following.
Additionally, as with how well FA-MSP-R performs against RH-2SSP-R and S-2SSP-R. As we can see from Figure 5, when At the first glance, these assertions could seem rooted in the An alternative perspective on the above assertions is that, for very large or very small values of ν\textsuperscript{\(1\)} the decision policies given by FA-MSP-D and FA-MSP-R models are similar to a two-stage optimization problem, where the first-stage corresponds to the period where all the procurement decisions are made, and the second-stage corresponds to the recourse decisions. From this perspective, this second-stage period is always defined as the landfall period \(t = T\), while the definition of the first-stage period depends on the value of \(\nu\). Specifically: when \(\nu \rightarrow 0\), the first-stage period \(t \rightarrow T - 1\) (the “wait-and-see” policy); and when \(\nu \rightarrow \infty\), the first-stage period \(t \rightarrow 1\) (the “early-commitment” policy).

At the first glance, these assertions could seem rooted in the \(\nu\) values only. However, these observations are concurrently owing to Assumption \(5\), which supposes that the demand occurs only at the landfall time, i.e., \(d_{t,j} = 0, \forall j \in J, t = 1, \ldots, T - 1\). This can be substantiated by comparing how well FA-MSP-D performs against RH-2SSP-D and S-2SSP-D with how well FA-MSP-R performs against RH-2SSP-R and S-2SSP-R. As we can see from Figure 5, when \(\nu\) gets close to 0, FA-MSP-D and RH-2SSP-D have very similar performances, and so does FA-MSP-R and RH-2SSP-R. Additionally, as \(\nu\) gets larger, the difference between FA-MSP-D, RH-2SSP-D and S-2SSP-D gradually gets smaller, and RH-2SSP-R and S-2SSP-R maintain their superiority over their S-2SSP-D and S-2SSP-R counterparts over the entire range of \(\nu\) values. Similarly, RH-2SSP-D and RH-2SSP-R maintain their superiority over their S-2SSP-D and S-2SSP-R counterparts over the entire range of \(\nu\) values. It is important to note, however, that their differences are not always paramount, especially in the deterministic landfall case. On one hand, as discussed in Subsection 4.2, the insignificance of a small cost scaling factor (e.g., \(\nu = 0.001\)) encourages a “wait-and-see” policy where all of the prepositioning decisions are postponed until the hurricane’s attributes at landfall become more clear. From Figures 6 and 7 we can see that this corresponds to postponing the procurement decisions until time \(T - 1\). On the other hand, we can see from Figures 5(a) and 5(b) that when \(\nu \rightarrow 5\), not only the FA-MSP and RH-2SSP policies yield similar performances, but the S-2SSP solution yields competitive performance as well. The reason for this is precisely opposite to what happens at the other end of the spectrum (when the value of \(\nu\) is small): as we can see from Figures 6 and 7 when \(\nu\) gets large, most (if not all) of the relief items are procured at the first period. In other words, the DM commits to a policy where all the prepositioning decisions are made as early as possible to hedge against the steep increase in the operational cost over time due to a large \(\nu\) value. This leads to similar performances obtained by different policies. Although small differences in their performances can still be observed, these differences can be attributed to the level of adaptability inherited in each policy which can be leveraged to modify the initial procurement decision slightly, depending on how the stochastic process unfolds. An alternative perspective on the above assertions is that, for very large or very small values of \(\nu\), the problem becomes similar to a two-stage optimization problem, where the first-stage corresponds to the period where all the procurement decisions are made, and the second-stage corresponds to the recourse decisions. From this perspective, this second-stage period is always defined as the landfall period \(t = T\), while the definition of the first-stage period depends on the value of \(\nu\). Specifically: when \(\nu \rightarrow 0\), the first-stage period \(t \rightarrow T - 1\) (the “wait-and-see” policy); and when \(\nu \rightarrow \infty\), the first-stage period \(t \rightarrow 1\) (the “early-commitment” policy).

4.3.1 Sensitivity analysis with respect to \(\nu\).

As we can see from Figures 5(a) and 5(b), the decision policies given by FA-MSP-D and FA-MSP-R models are superior to their RH-2SSP-D and RH-2SSP-R counterparts over the entire range of \(\nu\) values. Similarly, RH-2SSP-D and RH-2SSP-R maintain their superiority over their S-2SSP-D and S-2SSP-R counterparts over the entire range of \(\nu\) values. It is important to note, however, that their differences are not always paramount, especially in the deterministic landfall case. On one hand, as discussed in Subsection 4.2, the insignificance of a small cost scaling factor (e.g., \(\nu = 0.001\)) encourages a “wait-and-see” policy where all of the prepositioning decisions are postponed until the hurricane’s attributes at landfall become more clear. From Figures 6 and 7 we can see that this corresponds to postponing the procurement decisions until time \(T - 1\). On the other hand, we can see from Figures 5(a) and 5(b) that when \(\nu \rightarrow 5\), not only the FA-MSP and RH-2SSP policies yield similar performances, but the S-2SSP solution yields competitive performance as well. The reason for this is precisely opposite to what happens at the other end of the spectrum (when the value of \(\nu\) is small): as we can see from Figures 6 and 7 when \(\nu\) gets large, most (if not all) of the relief items are procured at the first period. In other words, the DM commits to a policy where all the prepositioning decisions are made as early as possible to hedge against the steep increase in the operational cost over time due to a large \(\nu\) value. This leads to similar performances obtained by different policies. Although small differences in their performances can still be observed, these differences can be attributed to the level of adaptability inherited in each policy which can be leveraged to modify the initial procurement decision slightly, depending on how the stochastic process unfolds. An alternative perspective on the above assertions is that, for very large or very small values of \(\nu\), the problem becomes similar to a two-stage optimization problem, where the first-stage corresponds to the period where all the procurement decisions are made, and the second-stage corresponds to the recourse decisions. From this perspective, this second-stage period is always defined as the landfall period \(t = T\), while the definition of the first-stage period depends on the value of \(\nu\). Specifically: when \(\nu \rightarrow 0\), the first-stage period \(t \rightarrow T - 1\) (the “wait-and-see” policy); and when \(\nu \rightarrow \infty\), the first-stage period \(t \rightarrow 1\) (the “early-commitment” policy).
and so does the difference between FA-MSP-R, RH-2SSP-R and S-2SSP-R. However, we can see that as \( \nu \) gets larger, this difference is shrinking at a slower pace in the random landfall time case (Figure 5(b)) than the deterministic (Figure 5(a)) landfall time case. In fact, we can reinterpret Assumption 3 as \( d_{j,t} = 0, \forall j \in J, t = 1, \ldots, T \) with probability 1 in the deterministic landfall time case, while in the random landfall time case, there is a positive chance that \( d_{j,t} > 0 \) for multiple periods \( t < T_{\text{max}} \). In other words, the situation that \( T \) is random resembles the situation where demand for relief items may occur in multiple stages. Hence, even when the cost-scaling factor \( \nu \) is large, we cannot treat the problem as a two-stage optimization problem due to the uncertainty of the timing of demand realization.

![Figure 5: Relative gap in \( \hat{z} \) values compared to the CV solutions as a function of \( \nu \) when \( T \) is deterministic (left) and random (right).](image)

(a) Relative gap with CV-D when \( T \) is deterministic.  
(b) Relative gap with CV-R when \( T \) is random.

Figure 5: Relative gap in \( \hat{z} \) values compared to the CV solutions as a function of \( \nu \) when \( T \) is deterministic (left) and random (right).

![Figure 6: \( \bar{f}_t \) obtained using the FA-MSP-D model as a function of \( \nu \) for \( t = 1, \ldots, T \), and \( \alpha_1 \in \{1, 3, 5\} \).](image)

(a) \( \alpha_1 = 1 \).  
(b) \( \alpha_1 = 3 \).  
(c) \( \alpha_1 = 5 \).

Figure 6: \( \bar{f}_t \) obtained using the FA-MSP-D model as a function of \( \nu \) for \( t = 1, \ldots, T \), and \( \alpha_1 \in \{1, 3, 5\} \).

![Figure 7: \( \bar{f}_t \) obtained using the FA-MSP-R model as a function of \( \nu \) for \( t = 1, \ldots, T \), and \( \alpha_1 \in \{1, 3, 5\} \).](image)

(a) \( \alpha_1 = 1 \).  
(b) \( \alpha_1 = 3 \).  
(c) \( \alpha_1 = 5 \).

Figure 7: \( \bar{f}_t \) obtained using the FA-MSP-R model as a function of \( \nu \) for \( t = 1, \ldots, T \), and \( \alpha_1 \in \{1, 3, 5\} \).

4.3.2 Sensitivity analysis with respect to \( \alpha_1 \).

From Figures 6 and 7 we can see that this experiment on the value of initial intensity level \( \alpha_1 \) amplifies the impact that the cost-scaling factor \( \nu \) has on the performances of different policies. For instance, when \( \nu \) is small (e.g.,
0.1 < ν < 0.5, we can see that when α₁ = 1 (very low intensity), most of the prepositioning decisions are postponed to the end. However, as the value of α₁ increases, less relief items are procured towards the end of the planning horizon and more are procured early on. In other words, a higher initial intensity level of the hurricane expedites the process of transitioning from the “wait-and-see” policy to the “early-commitment” policy as the value of ν increases. Furthermore, we can also see that this transition happens more swiftly in the case of a deterministic landfall time (see Figure[6]) in comparison with the case of a random landfall time (see Figure[7]). At the highest level of intensity, α₁ = 5, when ν = 0.001, f₁ = 0 for all the non-terminal stages except t = T − 1. However, in the random landfall case, this transition happens more slowly. We suspect that this can be attributed to the additional layer of uncertainty about the timing of the demand realization, which undermines the impact that ν and α₁ has on the sequential nature of the decision making process.

5 Conclusion

In this paper, we considered the problem of prepositioning relief items prior to an impending hurricane landfall, which we referred to as the hurricane disaster relief logistics planning (HDRLP) problem. We have studied this problem in two different settings, depending on if the time of landfall is deterministic or random. In each setting, we assume that the demand for relief items can be derived from the hurricane’s characteristics at the time of its landfall, namely its intensity and landfall location. We also assume that the evolution of the hurricane characteristics, including the timing of its landfall, is modeled as a discrete time MC model.

To solve the HDRLP problem, we have proposed two FA-MSP models, which we refer to as FA-MSP-D and FA-MSP-R when the time of landfall is deterministic and random, respectively. Both FA-MSP models induce optimal offline decision policies that allow the DM to make adaptive logistics operational decisions in every period of the planning horizon, depending on the most recently observed hurricane characteristics. In particular, our proposed FA-MSP-R model provides a novel extension to multistage stochastic programming with a random number of stages where the underlying stochastic process is assumed to be stage-wise dependent. Moreover, due to the computational challenges in exactly solving the MSP models, we have introduced alternative approximate decision policies. Specifically, we have introduced a 2SSP model where the corresponding decisions are implemented in a static manner, and an RH approach that employs the 2SSP model as the look-ahead model in each period of the planning horizon to achieve an online decision policy. These approaches are referred to, respectively, as S-2SSP-D and RH-2SSP-D when the time of landfall is deterministic; and S-2SSP-R and RH-2SSP-R when the time of landfall is random.

We have conducted an extensive set of numerical experiments to validate the proposed approaches. One key insight of our numerical results and sensitivity analysis is how the performances by different policies depend on how fast the unit logistics cost increases over the planning horizon. On one hand, when the cost increase is insignificant over time, the performances of decision policies associated with FA-MSP and RH-2SSP are similar for both situations when the time of landfall is deterministic and random. This is due to the fact that, insignificant increase in the cost encourages a latent “wait-and-see” policy where almost all of the logistics decisions are postponed until the hurricanes gets close to its landfall, so that its characteristics becomes more clear. However, this “wait-and-see” policy must be met with the DM’s ability to adapt their decisions at the later stages when the hurricane characteristics becomes clearer, which justifies why the same cannot be said about the S-2SSP-D and S-2SSP-R models. On the other hand, a similar observation can also be found when the unit logistics cost scales up significantly over time, where the performances of all different policies become more similar. This is due to the fact that the significant cost increase renders a DM who commits to making almost all the logistics decisions early on in the planning horizon (a “here-and-now” policy), before the logistics costs become untenable. Overall, the performances of different policies clearly depend on the levels of adaptability inherited in different approaches: with FA-MSP-D and FA-MSP-R having the best performances, followed by RH-2SSP-D and RH-2SSP-R, and finally S-2SSP-D and S-2SSP-R.

We have identified several directions to pursue for future research. One interesting avenue is to investigate cases where the demand for relief items may occur over multiple periods, rather than just occurring at the landfall stage only. Having to satisfy the demand over multiple periods will likely amplify the merits of the FA-MSP models. In addition, it would be interesting to consider the more realistic situations where the DM has to make decisions regarding the selection of SPs and the timings of their activation. Finally, from a risk preference perspective, it would be of interest to study risk-averse MSP models for the HDRLP problem.

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A Nested Benders decomposition.

The DPE (4) decomposes the nested formulation (2) over different stages \( t \), which serves as a basis for the so-called nested Benders decomposition (6). In a nutshell, the basic idea of nested Benders decomposition is to iteratively approximate the expected cost-to-go functions \( \mathcal{Q}_t^{\xi_t}(\cdot) \) by a collection of cutting-plane approximations. The algorithmic machinery for this procedure consists of the following two main steps.

1. **Forward** step: which simulates a trajectory of Markovian states by going forward in time from \( t = 1 \) to \( t = T \), solves the respective stage-\( t \) problem based on the current approximation of the expected cost-to-go function \( \mathcal{Q}_t^{\xi_t}(\cdot) \), and carries the corresponding optimal solution \( \bar{a}_t \) (also known as trial points) over to the subsequent stage (if any).

2. **Backward** step: which goes backward in time starting from \( t = T \), and places cuts on \( \mathcal{Q}_t^{\xi_t}(\cdot) \) which are obtained by the subgradient of \( \partial \mathcal{Q}_t^{\xi_t}(\cdot) \) at the points collected during the forward step. In particular, given a sequence of trial points \( (\bar{a}_1, \ldots, \bar{a}_T) \), one places a cut on \( \mathcal{Q}_t^{\xi_t}(\cdot) \) given by \( q_t(a_{t-1}) := \beta_t \top a_{t-1} + \pi_t \), where

\[
\beta_t \in \mathbb{E}[\partial \mathcal{Q}_t(\bar{a}_{t-1}, \xi_t)], \forall t = T, \ldots, 2
\]

and

\[
\pi_t := \begin{cases} 
\mathbb{E}[b_t \xi_t^\top \lambda_t] & \text{for } t = T \\
\mathbb{E} \left[ b_t \xi_t^\top \lambda_t + \sum_{c \in C_{t+1}} L_{t+1,c} \pi_{t+1,c} + \tilde{B}_{t,c} \right] & \text{for } t = T - 1, T - 2, \ldots, 2
\end{cases}
\]

where

- \( C_{t+1} \) is a collection of all the cutting planes placed on \( \mathcal{Q}_{t+1}(\cdot), \forall t = 1, \ldots, T - 1, \)
- \( \rho_{t,c} \) are the optimal dual vectors associated with \( \beta_{t+1,c} a_t + \pi_{t+1,c} \leq \mathcal{Q}_{t+1}(\cdot) \forall c \in C_{t+1} \) and \( t = 2, \ldots, T - 1, \)

Due to the polyhedral structure of the epigraph of \( \mathcal{Q}_t^{\xi_t}(\cdot) \), this iterative procedure builds a piece-wise linear lower approximation to \( \mathcal{Q}_t^{\xi_t}(\cdot) \) that converges in a finite number of iterations with probability one (35).

We care to note that, the cuts that one seeks to collect by the machinery of the nested Benders decomposition are generally categorized into two types of cuts: optimality cuts and feasibility cuts. In particular, in the forward step, when solving \( \hat{Q}_t(\bar{a}_{t-1}, \xi_t) \) given \( \bar{a}_{t-1} \) and \( \xi_t \): if the feasible set \( \chi_t(\bar{a}_{t-1}, \xi_t) = \emptyset \), a feasibility cut must be added; otherwise, the cut added to \( \mathcal{Q}_t^{\xi_t}(\cdot) \) is referred to as an optimality cut. In the HDRLP problem of our interest, we have what is known as the relatively complete recourse property which qualifies the following assumption.

**Assumption 7** We assume that for any given value \( \bar{a}_{t-1} \) and a realization of the random process \( \xi_t \in \Xi_t \), \( \chi_t(\bar{a}_{t-1}, \xi_t) \neq \emptyset \) and there always exists a feasible action \( \hat{a}_t = (\bar{a}_t, \tilde{v}_t) \) such that \( A_t^2 \tilde{u}_t + B_t^2 \hat{a}_t + C_t^2 \tilde{v}_t = b_t^2 \).

Assumption 7 eliminates the need to consider adding feasibility cuts. In Algorithm 1, we provide a pseudocode description of the nested Benders decomposition algorithm.

The nested Benders decomposition algorithm not only provides an optimal first-stage solution to be implemented here-and-now, but also induces a decision policy for the FA-MSP model. Contrary to the online RH policy, discussed in Subsection 3.3.3, policies induced by the expected cost-to-go functions \( \mathcal{Q}_t^{\xi_t}(\cdot) \) are referred to as offline policies. As the name suggests, offline policies are constructed in advance, before the unraveling of the stochastic process, whereas the online policies are constructed during the sequential realization of the stochastic process.
Algorithm 1 Nested Benders Decomposition.

\textbf{Input:} \( a_0, \xi_1, M \in \mathbb{R} \) which is a lower bound (LB) on \( \Omega^{\text{LB}}_{t+1}, \forall s_t \in \mathcal{S}_t, t = 1, \ldots, T - 1 \), and termination criterion (e.g., time-limit).

\textbf{STEP 0: (Initialization).}
- Set \( C^0_{t+1} = \emptyset, \forall \xi_t \in \mathcal{S}_t, t = 1, \ldots, T - 1, n = 0 \) be the iteration count.
- Let \( Q^n_t(\cdot, \cdot), \forall t = 1, \ldots, T \), be the problem solved at the \( n \)-th iteration and is defined according \( \text{[4]} \).
- Let \( \theta^n_{t+1} \) represent the epigraph of \( \Omega^{\text{LB}}_{t+1}(\cdot) \), and add constraint \( \theta^n_{t+1} \geq M \) to \( Q^n_t(\cdot, \cdot), \forall \xi_t \in \mathcal{S}_t, t = 1, \ldots, T - 1 \).
- Initialize \( t = 1, \xi^n_t = \xi_1, a^0_t = a_0, (\tilde{d}^1_t, \ldots, \tilde{d}^T_t) \) for the trial points, and a sample path \( (\xi^n_1, \ldots, \xi^n_T) \).

\textbf{STEP 1: (Forward Pass).}
- Solve the current problem \( Q^n_t(\tilde{d}^t_{t-1}, \xi^n_t) \) to obtain \( a^*_t \).
- Set \( \tilde{a}^t_t = a^*_t \) and let \( t \leftarrow t + 1 \).
- If \( t < T \), sample a new state \( \xi^n_t \) given \( \xi^n_{t-1} \) and repeat STEP 1. Otherwise, go to STEP 2.

\textbf{STEP 2: (Termination check).}
- If termination criterion is met, STOP; otherwise, go to STEP 3.

\textbf{STEP 3: (Backward Pass).}
- For every \( \xi_t \in \mathcal{S}_t \), solve the current problem \( Q^n_t(\tilde{d}^{t-1}_t, \xi_t) \).
- Generate a cut (if any) to \( \theta^n_{t+1} \) following the description in \( \text{[22] and [23]} \).
- Add the cut (if any) to \( C^0_{t+1} \) and let \( t \leftarrow t - 1 \).
- If \( t = 1 \), let \( n \leftarrow n + 1 \), set \( \tilde{a}^n_{t-1} = a_0, \xi^n_t = \xi_1 \) and go to STEP 1. Otherwise, repeat STEP 3.

\textbf{Output:} The collection of cutting planes \( \left\{ (C^n_2, \{C^n_3\}_{\xi_t \in \mathcal{S}_t}, \ldots, \{C^n_T\}_{\xi_{t-1} \in \mathcal{S}_{t-1}}) \right\} \).

### B RH-2SSP approach with random landfall time \( T \)

In this section we describe the RH-2SSP approach with random landfall time \( T \). In the deterministic landfall time case, the S-2SSP-D problem we solve in every roll is almost identical between different periods. However, because the planning horizon recedes as we roll forward, the aggregated number of periods shrinks by one period from one roll to the next. Note that the first-stage problem \( \text{[9]} \) does not contain any of the decision variables pertaining to the aftermath of the hurricane landfall, i.e., the delivery, shortage and demand for relief items. However, recall that in the random landfall time case, the first-stage problem in a given roll could be the stage when the hurricane makes landfall and the demand for relief items occurs. Therefore, in addition to reducing the length of the planning horizon in each roll, we move decision variables \( y, \xi, \tau \) of the first period in the current roll from the second-stage problem to the first-stage problem. That is, we move \( y_{\text{roll}}, \xi_{\text{roll}}, \tau_{\text{roll}} \) to the first-stage problem, where \( t_{\text{roll}} \) is the time index of the first period in the current roll. We then reformulate \( \text{[19] and [20]} \) as \( \text{[24] and [25]} \), as follows.

**First-stage:**

\[
\begin{align*}
\min_{x, f, y_{\text{roll}}, \xi_{\text{roll}}, \tau_{\text{roll}}} & \quad T_{\text{roll}} \sum_{t = t_{\text{roll}}}^{T_{\text{max}}} \left( \sum_{i \in \{0\}, j \in I} c^{b}_{ij, t} f_{ij, t} + \sum_{i \in I} c^{h}_{ij, t} x_{i, t} + h_t \sum_{i \in I} f_{ih, t} \right) \\
& \quad - \sum_{i \in I} \sum_{j \in J} c^{a}_{ij, t_{\text{roll}}} \xi_{ij, t_{\text{roll}}} + p \sum_{j \in J} \xi_{j, t_{\text{roll}}} + q \sum_{i \in I} \tau_{i, t_{\text{roll}}} \right) + \Omega^{\text{LB}} (x, f) \\
\text{s.t.} & \quad \sum_{j' \in J, j' \neq j} f_{ij', t} \leq x_{i, t-1}, \quad \forall i \in I, t = t_{\text{roll}}, \ldots, T_{\text{max}} \quad (24a) \\
& \quad x_{i, t} \leq u_t, \quad \forall i \in I, t = t_{\text{roll}}, \ldots, T_{\text{max}} \quad (24b) \\
& \quad \sum_{j \in J} \xi_{ij, t_{\text{roll}}} + \xi_{i, t_{\text{roll}}} = x_{i, t_{\text{roll}}} + \sum_{j \in \{0\} \cup J, j \neq t_{\text{roll}}} f_{ij, t} - \sum_{j \in J, j \neq t_{\text{roll}}} f_{ij, t_{\text{roll}}}, \quad \forall i \in I \quad (24c) \\
& \quad \sum_{i \in I} \xi_{ij, t_{\text{roll}}} + \xi_{j, t_{\text{roll}}} \geq a_{ij, t_{\text{roll}}}, \quad \forall j \in J, \quad (24d) \\
& \quad x_t, f_t \geq 0, \quad \forall t = t_{\text{roll}}, \ldots, T_{\text{max}} \quad (24e) \\
& \quad y_{\text{roll}}, \xi_{\text{roll}}, \tau_{\text{roll}} \geq 0. \quad (24f)
\end{align*}
\]
where $Q^\xi(x, f) := \sum_{i=1}^n \xi_{i} \xi_{\max} \mathbb{P}_{\xi_{\max}}^{T_{\max} - 1} \cdot Q(x, f, \xi_{\max}^{T_{\max}})$, and $\xi_{\max}^{T_{\max}}$ gives the trajectory of the stochastic process between $t = t_{\text{roll}}$ to $t = T_{\max}$.

Second-stage:

$$Q(x, f, \xi_{\max}^{T_{\max}}) := \min_{\xi \in \Xi} \sum_{t = T_{\max} + 1}^{T_{\max}} \left( \sum_{i = 1}^n \sum_{j \in J} c_{ij} \xi_{ij} + p \sum_{j \in J} \xi_{j} + q \sum_{t \in T} \xi_{t} \right)$$

$$- \sum_{t = T_{\max} + 1}^{T_{\max}} \left( \sum_{i = 1}^n \sum_{j \in J} c_{ij} \xi_{ij} + \sum_{i \in I} \sum_{j \in J} c_{ij} \xi_{ij} + \sum_{i \in I} \xi_{i} \right) \mathbb{I}_{t > t_{\text{roll}} + T_{\max}}$$

s.t. $\sum_{j \in J} \xi_{ij} + \xi_{ij} = x_{ij} - 1 + \sum_{j \in I(J \cup J)} f_{ij} - \sum_{j \in J} f_{ij} - x_{ij}, \forall i \in I, t = t_{\text{roll}} + 1, \ldots, T_{\max}$

$$\sum_{i \in I} \xi_{ij}, \xi_{ij}, \xi_{t} \geq 0, \quad \forall j \in J, t = t_{\text{roll}} + 1, \ldots, T_{\max}$$

$$y_i, \xi_i, \xi_{t} \geq 0, \quad \forall t = t_{\text{roll}} + 1, \ldots, T_{\max}.$$  \hspace{1cm} (25a)

$$y_i, \xi_i, \xi_{t} \geq d_{ij}, \quad \forall j \in J, t = t_{\text{roll}} + 1, \ldots, T_{\max}$$  \hspace{1cm} (25b)

$$y_i, \xi_i, \xi_{t} \geq 0, \quad \forall t = t_{\text{roll}} + 1, \ldots, T_{\max}.$$  \hspace{1cm} (25c)

### C Problem Data

This section is organized as follows. First, we describe how we generate the configuration of the disaster relief logistics network. Second, we describe how the logistics cost parameters in the multi-period network flow model are specified. Third, we discuss how the stochastic process on the evolution of the hurricane’s attributes is modeled. Fourth, we demonstrate how we define the appropriate $T_{\max}$ parameter for the case when the hurricane’s landfall time is random. Finally, we discuss how the hurricane’s attributes are mapped to the demand for relief items at different DPs. We note that many of the parameter settings follow those given in [18, 27].

**The coordinates of the MDC, SPs and DPs.** We consider an APA similar to the one depicted in Figure 1 when $T$ is deterministic, and the one depicted in Figure 2 when $T$ is random. In both cases, we randomly generate the DP locations by letting the $x$-coordinates of the SPs follow a uniform distribution $U(0, 700)$, and independently, the $y$-coordinates follow a uniform distribution $U(0, 100)$. Similarly, the $x$-coordinates of the DPs locations are randomly generated according to a uniform distribution $U(0, 700)$, and independently, their $y$-coordinates are randomly generated according to a uniform distribution $U(100, 200)$. Using these distributions, we generate the coordinates for different number of DPs $|J| \in \{10, 20, 30\}$ and SPs $|I| \in \{3, 6, 9\}$ to create network instances with different sizes. In all instances, we set the coordinates of the MDC to be (350, 450).

**Defining the logistics operational costs.** We assume that any transportation cost is a multiple of a unit cost $w = 0.0038$, scaled by an Euclidean distance-based metric. The unit cost of transporting a relief item from node $i \in \{0\} \cup I$ to node $i' \in I$ in period $t$ is given by:

$$c_{ij}^h = w (1 + v(t - 1)) \| (x_i, y_i) - (x_j, y_j) \|_2, \quad \forall i \in \{0\} \cup I, i' \in I, t = 1, 2, \ldots, T_{\max}.$$  \hspace{1cm} (26)

where $v$ is the scaling factor that captures the increasing operational cost over time as we discussed in Assumption 5, and $(x_i, y_i)$ and $(x_j, y_j)$ are the coordinates of SP $i \in \{0\} \cup I$ and SP $i' \in I$, respectively. Similarly, the unit cost of transporting a relief item from SP $i \in \{0\} \cup I$ to DP $j \in J$ in period $t$ is given by:

$$c_{ij}^h = w (1 + v(t - 1)) \| (x_i, y_i) - (x_j, y_j) \|_2, \quad \forall i \in \{0\} \cup I, j \in J, t = 1, 2, \ldots, T_{\max}.$$  \hspace{1cm} (27)

where $(x_i, y_i)$ and $(x_j, y_j)$ are the coordinates of SP $i \in I$ and DP $j \in J$, respectively. Moreover, we define the remaining parameters $h, c_{ij}^p, p$ and $q$ as a multiple of a base cost $\beta = 5$: $c_{ij}^h = 0.2 \beta, \forall i \in \{0\} \cup I, t = 1, 2, \ldots, T_{\max}$, $c_{ij}^p = -0.5 \beta$. Finally, the capacities of the SPs are generated randomly according to a uniform distribution $U(0.05 \beta, 0.5 \beta)$, where $d$ is the total maximum possible demand for relief items at any DP $j \in J$, and in our test instances we set $d = 400$. The rational behind this setup is that the capacity of each SP $i \in I$ is (i) proportional to the maximum possible demand that can occur at a given DP; (ii) increasing in the total number of DPs in the APA (i.e., $|J|$); and (iii) decreasing in the total number of SPs in the APA that could supply different DPs with the relief items (i.e., $|I|$).
Modeling the evolution of the hurricane’s attributes. We first specify the state space \( \mathcal{Z}_t \) of the MC model for the case of deterministic landfall time. Recall that when \( T \) is deterministic and known a priori, we only consider the hurricane’s horizontal location and define \( \mathcal{Z}_t = A \times L_x \), where \( A \) and \( L_x \) are the state spaces associated with the hurricane’s intensity and location along the \( x \)-coordinate, respectively. In our test instances, we set \( A = \{0, 1, \ldots, 5\} \), where 0 is an absorbing state corresponding to the situation that the hurricane dissipates, and states 1 to 5 correspond to the five categories pertaining to the Saffir–Simpson hurricane wind scale (SSHWS). The one-step transition probability matrix for the MC associated with the intensity level \( A \) is given by:

\[
P^\alpha = \begin{bmatrix}
1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.11 & 0.83 & 0.06 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.15 & 0.60 & 0.25 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.04 & 0.68 & 0.28 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.18 & 0.79 & 0.03 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.50 & 0.50
\end{bmatrix}.
\] (28)

Additionally, we define the state space for the hurricane’s landfall location along the \( x \)-coordinate as:

\[ L_x = \{(0, 100), (100, 200), \ldots, (600, 700)\}, \]

and the associated one-step transition probability matrix is given by:

\[
P^\ell_x = \begin{bmatrix}
0.004 & 0.300 & 0.395 & 0.198 & 0.049 & 0.038 & 0.016 \\
0.150 & 0.202 & 0.249 & 0.222 & 0.117 & 0.033 & 0.027 \\
0.198 & 0.249 & 0.029 & 0.168 & 0.206 & 0.099 & 0.051 \\
0.099 & 0.222 & 0.169 & 0.012 & 0.150 & 0.198 & 0.150 \\
0.025 & 0.117 & 0.206 & 0.15 & 0.004 & 0.150 & 0.348 \\
0.019 & 0.033 & 0.098 & 0.198 & 0.150 & 0.004 & 0.498 \\
0.008 & 0.019 & 0.025 & 0.098 & 0.198 & 0.150 & 0.502
\end{bmatrix}.
\] (29)

Consequently, when \( T \) is deterministic, the stochastic process associated with the hurricane’s evolution is characterized by an MC, where the transition probability matrix \( P \) is given by \( P_{\alpha|t-1} = P_{\alpha|t-1} \times P_{\ell_x}|\ell_x-1 \) with \( P^\alpha \) and \( P^\ell_x \) specified above.

We extended this to the case where \( T \) is random, as we discussed in Section 3.4 by introducing a temporal state that corresponds to the \( y \)-coordinate \( \ell_{y,t} \in L_y \), where \( L_y \) is defined as:

\[ L_y = \{[-350, -300), [-300, -250), \ldots, [-50, 0), [0, \infty)\}, \]

and the associated one-step transition probability matrix is given by:

\[
P^\ell_y = \begin{bmatrix}
0.0 & 0.6 & 0.3 & 0.1 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.6 & 0.3 & 0.3 & 0.1 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.6 & 0.3 & 0.1 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.6 & 0.3 & 0.1 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.6 & 0.3 & 0.1 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0
\end{bmatrix}.
\] (30)

Consequently, when \( T \) is random, the stochastic process associated with the hurricane’s evolution is characterized by an MC whose the transition probability matrix \( P \) is given by \( P_{\ell_{y,t-1}} = P_{\ell_{y,t-1}} \times P_{\ell_{x,t-1}} \times P_{\ell_{y,t-1}} \times P_{\ell_{x,t-1}} \), with matrices \( P^\alpha \), \( P^\ell_x \) and \( P^\ell_y \) specified above. Note that state \( \ell_{y,t} = [-50, 0) \) corresponds to the landfall event and state \( \ell_{y,t} = [0, \infty) \) is an absorbing state indicating that the hurricane has already made landfall. The rational behind this setup is to have an MC where the probability that the hurricane goes backward or remains in the same place from one period to the next is zero, i.e., \( P(\ell_{y,t-1} \leq \ell_{y,t})|\ell_{y,t} = 0 \), where \( \ell_{y,t} \) indicates the lower bound (LB) of the interval associated with \( \ell_{y,t} \). Additionally, we care to note that a dissipating hurricane corresponds to the state \( \alpha_t = 0 \). Accordingly, we have that:

- the set of transient states \( \mathcal{F} := \{\xi_t = (\alpha_t, \ell_{x,t}, \ell_{y,t}) \mid \alpha_t \neq 0, \ell_{y,t} < 0\} \), and
- the set of absorbing states \( \mathcal{A} := \{\xi_t = (\alpha_t, \ell_{x,t}, \ell_{y,t}) \mid \alpha_t = 0 \lor \ell_{y,t} \geq 0\} \).

**Defining \( T_{\text{max}} \) for the case of random landfall time.** To define the value of \( T_{\text{max}} \), we need to identify the largest number of steps required to transition from any initial state \( \ell_{y,t} \) to state \([0, \infty)\). Given the one-step transition probability matrix \( P^\ell_y \) specified in (30), since \( P(\ell_{y,t} \leq \ell_{y,t}|\ell_{y,t} = 0) = 0 \), the largest number of steps required to transition from
state $\ell_{s,1}$ to state $[0, \infty)$ corresponds to the situation where $\ell_{s,t+1} = \ell_{s,t}, \forall t = 1, \ldots, T_{\text{max}}$, where $\ell_{s,t}$ indicate the upper bound (UB) of the interval associated with $\ell_{s,t}$. In other words, in each period, the MC visits every subsequent intervals without skipping any of them. Assuming that $\ell_{s,1} = [-350, -300)$, we can set $T_{\text{max}} = |L_0| = 8$. Moreover, we define the value of $T$ used in our test instances for the case where the time of landfall is deterministic, from the one-step transition probability matrix $P^0$, as well. We do this by assuming that $T = \lceil \hat{T} \rceil = 5$, where $\hat{T}$ is the expected number of steps before reaching state $\ell_{s,1} = [0, \infty)$, starting from initial state $\ell_{s,1} = [-350, -300)$.

**Modeling the demand at DPs.** To define the demand value $d_{jt}$ associated with the hurricane’s state $\xi_t \in \mathbb{Z}_t$ at time $t$ for DP $j \in J$ with location $(j_x, j_y)$, we do the following. First, we discretize each of the possible landfall locations $\ell_{s,t}$ uniformly into a set of points $\{\ell_{m}^{m}\}_{m=1}^{M}$, where each point $\ell_{m}^{m}$ represents a realization of the hurricane’s landfall location (again, we consider the x-coordinates only and the y-coordinates are 0), $\forall m = 1, 2, \ldots, M$. Hence, for a given state $\ell_{s,t}$, the $m$-th realization for $x$-coordinate of the landfall location is given by:

$$\ell_{m}^{m} \doteq \ell_{s,t} + \frac{M}{2} + M(m-1) \equiv \ell_{s,t} + \frac{M}{2} + M(m-1),$$

where $\ell_{s,t}$ and $\ell_{s,t}^{+}$ indicate the lower and upper ends of the interval associated with $\ell_{s,t}$, respectively. We further assume that each of the $M$ realizations is equally likely. In our experiments, we assume that $M = 10$, and as an example, state $\ell_{s,t} = [0, 100)$ is discretized into a set of realizations $\{5, 15, \ldots, 95\}$. We then define the deterministic mapping $D(\alpha_t, \ell_T)$ as

$$D(\alpha_T, \ell_T) = \delta \times \left(1 - \frac{\delta}{\bar{\delta}}\right) \times \frac{\alpha_t}{(|A| - 1)^2},$$

where $\delta$ gives the distance between a DP $j \in J$ and $\ell_T$, and $\bar{\delta}$ is the maximum possible distance to a DP where demand for relief items occurs. Hence, given the hurricane’s attributes $\xi_t = (\alpha_t, \ell_T)$, we assume that the demand associated with DP $j \in J$ with location $(j_x, j_y)$ is given by:

$$d_{jt}^{\delta} = \begin{cases} 
\delta \times \left(1 - \frac{\delta}{\bar{\delta}}\right) \times \frac{\alpha_t}{(|A| - 1)^2} & \text{if } \delta(\ell_{s,t}, j) \leq \bar{\delta} \text{ and } \ell_{s,t} = 0 \\
0 & \text{otherwise} \end{cases}$$

where $\delta(\ell_{s,t}, j) := ||(\ell_{s,t}^{m}, 0) - (j_x, j_y)||_2$ gives the Euclidean distance between the hurricane’s landfall location and the location of DP $j \in J$. In our test instances, we set $\bar{\delta} = 300$. As we can see from (32), in accordance with the discussion in Section 3, the demand function $d_{jt}^{\delta}$ is increasing in the hurricane’s intensity $\alpha_t$ and decreasing in the distance $\delta(\ell_{s,t}, j)$ between the hurricane’s landfall location and the location of a DP.

**Initial values.** We assume that every SP is empty at the start of the planning horizon, that is, $x_0 = 0$. Additionally, for our initial experiments, we assume that $\alpha_1 = 1$ and $\ell_{s,1} = [100, 200)$ for the case of deterministic landfall time, and assume in addition that $\ell_{s,1} = [-350, -300)$ for the case of random landfall time.

We end this Section by noting that the above problem data is only considered for the purpose of our numerical experiments, and is by no means derived from historical data. As a summary, all parameters used in our numerical experiments are provided in Table 3 in Appendix A.

**D Implementation Details**

We organize the presentation of our implementation details as follows. First, we discuss the performance evaluation of these approaches. Next, we discuss how the stochastic optimization models are solved by: (i) describing the choice of initial values for some of the parameters; (ii) describing how the training sample paths are used for different stochastic optimization models; and (iii) discussing the termination criteria used for solving these models.

**Initial values.** For the FA-MSP-D, FA-MSP-R, and S-2SSP-D (including the S-2SSP-D solved in every roll of the RH-2SSP-D), we assume that the initial LB for the approximate expected cost-to-go function to be 0. For the S-2SSP-R (including the one solved in every roll of the RH-2SSP-R), because of the reimbursement variable (18), it is possible that we arrive at a negative recourse value, and thus we set the initial LB to be $-10^{10}$.

**Sample paths for out-of-sample evaluation.** To evaluate the performances of policies obtained from different approaches, we create a set of out-of-sample scenarios (sample paths) with a sample size of $N = 1000$. We compute a sample mean

$$\bar{\xi} = \frac{1}{N} \sum_{n=1}^{N} z(\xi^n),$$

(33)
and a sample standard deviation
\[ \hat{\sigma} = \sqrt{\frac{1}{N-1} \sum_{n=1}^{N} (z(\hat{\xi}_n) - \bar{z})^2}, \]
where \( \hat{\xi}_n := (\hat{\xi}_n^1, \hat{\xi}_n^2, \cdots, \hat{\xi}_n^T) \) is the realized trajectory of the stochastic process corresponding to sample path \( n \), and \( z(\hat{\xi}_n) \) is the cumulative objective value accrued by the employed decision policy at sample path \( n \). We compute a 95\% CI for the performance of the corresponding policy on the set of out-of-sample scenarios, which is given by \( \hat{\xi}_{\pm} = [\bar{z} - 1.96\hat{\sigma}/\sqrt{N}, \bar{z} + 1.96\hat{\sigma}/\sqrt{N}] \).

**Training scenarios.** We use nested Benders algorithm (see Algorithm 1) to obtain offline decision policies associated with the FA-MSP-D and FA-MSP-R models. In principle, during the training process of the nested Benders algorithm, every iteration of Algorithm 1 can be carried over a set of multiple sample paths. That is, in the forward step, one can collect trial points over multiple sample paths and, analogously, generate cuts for each of those sample paths when going backward. However, in all of our implementation, we observed that using a single sample path per forward/backward step works the best.

To solve the S-2SSP-D and S-2SSP-R models, we employ a standard implementation of the L-shaped method [6]. The basic idea of the L-shaped method is to successively build an outer-approximation of the recourse function \( \Omega^f_i(x_{T-1}) \) (or \( \Omega^f_i(x,f) \) in (19)). To do this, we can generate cuts in the spirit of the one generated at the terminal stage, in the backward step of Algorithm 1. One crucial distinction here is that, the recourse function for at the terminal stage in Algorithm 1 reflects the cost of the terminal period only, and to generate the cut, one needs to solve a scenario subproblem for each state \( \xi_i \in \Xi_i \). However, the recourse function in (19) (or in (19)) reflects the aggregated cost of the periods between \( t = 2 \) and \( t = T \) (or \( t = T_{\max} \) when \( T \) is random). Hence, to generate a cut, one needs to solve the second-stage problem (10) (or in (20)) for every possible sample path \( (\xi_2, \cdots, \xi_T) \) (or \( (\xi_2, \cdots, \xi_{T_{\max}}) \) when \( T \) is random). According to our problem data, this would correspond to a maximum of \( \left| \mathcal{A} \right| \times \left| L_a \right| T^{-1} \) (or \( \left| \mathcal{A} \right| \times \left| L_a \right| L_m \) \( T_{\max}^{-1} \) when \( T \) is random) sample paths. This exponentially large number of scenarios is especially prohibitive in the RH-2SSP-D and RH-2SSP-R approaches, where one solves the S-2SSP-D (or S-2SSP-R) model at each period \( t = 1, \ldots, T - 1 \). We circumvent this by generating 100 equally likely scenarios to serve as the training scenarios for the two-stage stochastic programming models. The same number of scenarios is also used in every roll of the RH-2SSP-D and RH-2SSP-R approaches. We performed an in-sample stability test in our experiments that shows that the size of 100 training samples is appropriate.

**Termination criteria.** We use the following termination criteria for the nested Benders decomposition algorithm: (i) the maximum number of iterations is set to be \( 10^5 \); (ii) the time limit is set to be three hours; and (iii) a stability test which keeps track of the progress of the LB for the optimal objective value (denoted by \( \bar{z} \)); if \( \bar{z} \) does not progress by more than \( \varepsilon = 10^{-3} \) (in the relative term) for more than \( \tilde{m} = 500 \) consecutive iterations, i.e., \( (\bar{z} - \bar{z} - \tilde{m})/\bar{z} < \varepsilon \), we terminate the algorithm. We use the same termination criteria when solving the two-stage SP model, both the for RH-2SSP-D and RH-2SSP-R in every roll of the rolling-horizon procedure and for S-2SSP-D and S-2SSP-R. Although criteria (i) and (ii) are very unlikely to be reached for S-2SSP-D and S-2SSP-R, we maintain them for the sake of consistency.

### E Acronyms

**Acronyms**

| Acronym | Description |
|---------|-------------|
| 2SSP    | two-stage stochastic programming |
| APA     | affected potential area |
| CI      | confidence interval |
| CV      | clairvoyance |
| CV-D    | CV model with a deterministic time of landfall |
| CV-R    | CV model with a random time of landfall |
| DM      | decision-maker |
| DPE     | dynamic programming equations |
| DPs     | demand points |
| FA-MSP  | fully adaptive MSP |
| FA-MSP-D| FA-MSP model with a deterministic time of landfall |
| Abbreviation | Description |
|--------------|-------------|
| FA-MSP-R     | fully adaptive MSP model with a random time of landfall |
| FEMA         | Federal Emergency Management Agency |
| HDRLP        | hurricane disaster relief logistics planning |
| LB           | lower bound |
| MC           | Markov chain |
| MDC          | major distribution center |
| MSP          | multi-stage stochastic programming |
| NHC          | National Hurricane Center |
| NOAA         | National Oceanic and Atmospheric Administration |
| RH           | rolling-horizon |
| RH-2SSP      | Rolling-horizon two-stage stochastic programming |
| RH-2SSP-D    | RH-2SSP model with a deterministic time of landfall |
| RH-2SSP-R    | RH-2SSP model with a random time of landfall |
| S-2SSP       | static two-stage stochastic programming |
| S-2SSP-D     | S-2SSP model with a deterministic time of landfall |
| S-2SSP-R     | S-2SSP model with a random time of landfall |
| SP           | supply point |
| SPs          | supply points |
| SSHWS        | Saffir–Simpson hurricane wind scale |
| UB           | upper bound |
| US           | United States |
\section*{F Figures}

\begin{figure}[h]
\centering
\begin{subfigure}[b]{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{figure8a.png}
\caption{$\nu = 0.001$.}
\end{subfigure}
\begin{subfigure}[b]{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{figure8b.png}
\caption{$\nu = 0.60$.}
\end{subfigure}
\begin{subfigure}[b]{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{figure8c.png}
\caption{$\nu = 5.00$.}
\end{subfigure}
\caption{$\xi$ values averaged across the different SPs and DPs for $\nu \in \{0.001, 0.6, 5.00\}$ when $T$ is deterministic.}
\end{figure}
Figure 9: $\hat{z}$ values averaged across the different SPs and DPs for $\nu \in \{0.001, 0.60, 5.00\}$ when $T$ is random.
Multi-stage Stochastic Programming Methods for Adaptive Disaster Relief Logistics Planning

G Tables

Table 6: The average amount of procured relief items $\bar{f}_t$ (see (21)) obtained from policies associated with the FA-MSP-D model as a function of the cost-scaling factor $\nu$ for different time periods $t = 1, \ldots, T$, and for different initial intensity levels $\alpha_1 \in \{1, 3, 5\}$.

| $\nu$ | $t = 1$ | $t = 2$ | $t = 3$ | $t = 4$ | $t = 1$ | $t = 2$ | $t = 3$ | $t = 4$ | $t = 1$ | $t = 2$ | $t = 3$ | $t = 4$ |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 0.1   | -       | -       | 249.63  | -       | -       | -       | -       | -       | -       | -       | -       | -       |
| 0.2   | -       | 14.76   | 19.83   | 205.58  | -       | -       | -       | -       | -       | -       | -       | -       |
| 0.3   | -       | 20.77   | 26.14   | 178.30  | -       | -       | -       | -       | -       | -       | -       | -       |
| 0.4   | -       | 22.66   | 26.74   | 167.85  | -       | -       | -       | -       | -       | -       | -       | -       |
| 0.5   | 163.07  | 10.91   | 21.58   | 68.75   | -       | -       | -       | -       | -       | -       | -       | -       |
| 0.6   | 175.08  | 28.68   | 16.46   | 54.86   | -       | -       | -       | -       | -       | -       | -       | -       |
| 0.7   | 190.76  | 31.12   | 22.18   | 40.58   | -       | -       | -       | -       | -       | -       | -       | -       |
| 0.8   | 195.77  | 32.90   | 22.26   | 33.08   | -       | -       | -       | -       | -       | -       | -       | -       |
| 0.9   | 200.02  | 33.69   | 21.90   | 31.70   | -       | -       | -       | -       | -       | -       | -       | -       |
| 1.0   | 204.91  | 34.15   | 21.74   | 30.66   | -       | -       | -       | -       | -       | -       | -       | -       |

Table 7: The average amount of procured relief items $\bar{f}_t$ (see (21)) obtained from policies associated with the FA-MSP-R model as a function of the cost-scaling factor $\nu$ for different time periods $t = 1, \ldots, T$, and for different initial intensity levels $\alpha_1 \in \{1, 3, 5\}$.

| $\nu$ | $t = 1$ | $t = 2$ | $t = 3$ | $t = 4$ | $t = 1$ | $t = 2$ | $t = 3$ | $t = 4$ | $t = 1$ | $t = 2$ | $t = 3$ | $t = 4$ |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 0.1   | -       | -       | 0.74    | 9.02    | -       | -       | 10.38   | 106.26  | -       | -       | 120.33  | 1203.89 |
| 0.2   | -       | 0.96    | 9.75    | 23.60   | -       | -       | 10.38   | 106.26  | -       | -       | 120.33  | 1203.89 |
| 0.3   | -       | 2.08    | 3.46    | 10.07   | -       | -       | 10.38   | 106.26  | -       | -       | 120.33  | 1203.89 |
| 0.4   | -       | 3.24    | 5.14    | 10.63   | -       | -       | 10.38   | 106.26  | -       | -       | 120.33  | 1203.89 |
| 0.5   | 29.21   | 2.16    | 4.55    | 7.78    | 18.57   | 19.59   | 3.78    | 65.57   | 1.8    | 5.00    | 35.44   | 55.17   |
| 0.6   | 40.77   | 1.95    | 4.55    | 7.20    | 16.50   | 17.47   | 3.28    | 65.57   | 1.8    | 5.00    | 35.44   | 55.17   |
| 0.7   | 48.94   | 3.30    | 3.89    | 6.54    | 14.88   | 16.13   | 2.95    | 65.57   | 1.8    | 5.00    | 35.44   | 55.17   |
| 0.8   | 56.21   | 5.86    | 3.39    | 6.05    | 13.55   | 15.12   | 2.70    | 674.71  | 3.15   | 21.58   | 44.99   | 39.78   |
| 0.9   | 59.72   | 7.05    | 3.67    | 6.00    | 12.57   | 14.35   | 2.65    | 700.32  | 2.68   | 14.97   | 41.97   | 37.02   |
| 1.0   | 62.87   | 7.73    | 4.14    | 5.71    | 11.99   | 14.01   | 2.49    | 763.03  | 2.00   | 15.82   | 34.51   | 30.22   |

Figure 10: A symbolic representation of the sequential decision-making process in MSP models.
| Parameter | Description | Value/Formula |
|-----------|-------------|--------------|
| $| J |$ | number of DPs | $\in \{10, 20, 30\}$. |
| $| J^I |$ | number of SPs | $\in \{3, 6, 9\}$. |
| $| j_i |$ | x-coordinates of the DPs | $\in U(0, 700)$. |
| $| j_y |$ | y-coordinates of the DPs | $\in U(0, 100)$. |
| $| i_x |$ | x-coordinates of the SPs | $\in U(0, 700)$. |
| $| i_y |$ | y-coordinates of the SPs | $\in U(0, 100, 200)$. |
| $| (0, 0, \ell) |$ | the x and y coordinates of the MDC | $(350, 450)$. |
| $A |$ | state space for $\alpha_i$ (hurricane intensity) | $[0, 1, \ldots, 5]$. |
| $| L_x |$ | state space for $\ell_x, j$ (landfall spatial dimension) | $[0, 100], \ldots, [600, 700]$. |
| $| L_t |$ | state space for $\ell_t, j$ (landfall temporal dimension) | $[-250, -300], \ldots, [-50, 0], [0, \infty)$. |
| $P^{\alpha}_{i,j} |$ | one-step transition probability matrix for $\alpha_i$ | see (25). |
| $P^{\alpha}_{i,t} |$ | one-step transition probability matrix for $\ell_x$ | see (29). |
| $P^{\alpha}_{t,j} |$ | one-step transition probability matrix for $\ell_t, j$ | see (30). |
| $\beta |$ | a set of transient states when $T$ is random | $\{ \beta = (\alpha_i, \ell_x, \ell_t) | \alpha_i \neq 0, \ell_t < 0 \}$. |
| $| T_{\max} |$ | maximum possible number of stages | 8. |
| $| | T^J |$ | floor of the expected number of steps before reaching the absorbing state $t_s = \{0, \infty \}$ | 5. |
| $| M |$ | number of discrete points (exact coordinates) for a potential landfall location ($\ell_x, j$) | 10. |
| $| d^p_{i,j} |$ | demand at DP $j \in J$ for given intensity $\alpha_i$, a horizontal location $\ell_x, j$, and vertical location $\ell_t, j$ | see (31). |
| $| P(\ell_{x,j} = \ell) |$ | probability of hurricane landfall in coordinates $\ell_{x,j}$ given state $\ell_x, j = \ell$ | $1/M$. |
| $| d |$ | maximum possible demand for relief items at a given DP | 400. |
| $| \bar{d} |$ | maximum possible distance to a DP which generates a request for relief items | 300. |
| $| p_{\ell} |$ | fuel cost — used in calculating the transportation costs | 0.0038. |
| $| \nu |$ | logistic costs scaling factor used in the main results | $\in [0.001, 0.60, 5.00]$. |
| $| c_{i,j}^{p} |$ | unit cost for prepositioning/rerouting the relief items from an SP $i \in \{0\} \cup I$ to an an SP $j \in J$ | see (26). |
| $| c_{i,j}^{p} |$ | unit cost for delivering the relief items from an SP $i \in I$ to a DP $j \in J$ | see (27). |
| $| \beta |$ | base cost used in the calculation of $h, c_p^q, p$ and $q$ | 5. |
| $| \gamma_{i,j}^{p} |$ | unit cost for holding a relief item at an SP $i \in I$ in period $t$ | $0.2\beta$. |
| $| h |$ | unit cost for procuring a relief item from the MDC in period $t$ | $\beta(1 + \nu(t - 1))$. |
| $| p |$ | unit cost for demand shortage (penalty for unsatisfied demand per item) | 80$. |
| $| q |$ | salvage value per item | -0.05$; |
| $| u_i |$ | inventory capacity at SP $i \in I$ | $\in U(0.05 d_{i,j}^{p}, 0.5 d_{i,j}^{p})$. |
| $| \bar{\xi} |$ | sample mean for the policy performance | $\Sigma_{n=1}^{N} \bar{z}(\bar{\xi}) / N$. |
| $| \bar{\sigma} |$ | sample standard deviation for the policy performance | $\sqrt{\Sigma_{n=1}^{N} \bar{z}(\bar{\xi})^2} / (N - 1)$. |
| $| \bar{\xi}_k |$ | 95% confidence level for the mean policy performance | $\bar{\xi} \pm 1.96 \bar{\sigma} / \sqrt{N}$. |
| $| x_0 |$ | initial inventory levels at different SPs | $\in (1, [100, 200], [-350, -300])$. |
| $| \xi_0 |$ | initial state in the initial experiments | $\in (1, [100, 200], [-350, -300])$. |
| $| N |$ | number of out-sample paths used to evaluate different policies | 1000. |
| $| K |$ | number of scenario sample paths used in the training of 2SSP models | 100. |
| $| \bar{n} |$ | maximum number of iterations used during the training of different models | $10^5$. |
| $| \varepsilon |$ | tolerance parameter | $10^{-3}$. |
| $| \bar{m} |$ | a stalling parameter used in the termination criterion | 500. |
| $| \bar{m} |$ | time limit the time limit used during the training of different models | $3 \times 60^2$ seconds. |

Table 8: Summary of the parameters and values used in the problem data and implementation.