Quantum key distribution with unconditional security for all optical fiber network

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ABSTRACT

Previously the present protocol was referred as Yuen-Kim second version in our papers. In this paper, it is called Yuen protocol (Y-00) and we present an efficient implementation method of physical layer of Y-00 which can support a secure communication and a quantum key distribution (more generally key expansion) by IMDD(intensity modulation/direct detection) or FSK(frequency shift keying) optical fiber communication network. Although the general proof of the security is not yet given, a brief sketch of security analysis is shown, which involve an entanglement attack.

Keywords: quantum cryptography, Yuen protocol, fiber network

1. INTRODUCTION

A quantum key distribution for legitimate two users (Alice and Bob) is one of the most interesting subjects in quantum information science, which was pioneered by C. Bennett and G. Brassard in 1984[1]. In the original paper of Bennett, single photon communication was employed for implementation of quantum key distribution for simple explanation. However, despite that it is not essential in the great idea of Bennett, many researchers employed single photon communication scheme to realize BB84, B92[2]. There exist limitations of communications by single photon state or other quantum state(squeezed state and et al) with strong quantumness in the practical sense[3]. So it was discussed whether one can realize a secure key distribution guaranteed by quantum nature based on conventional light wave communication or not. One of the answers is that one employs coherent state as transmitter state, which has a semiclassical nature and a robustness for energy loss. Recently, as an example, an implementation of BB-84 by using continuous variable coherent state has been proposed[4], which may communicate more long distance and higher efficiency than single photon scheme.

On the other hand, in the history of information theory, there were already several works on protocol with unconditionally secure communication. Let us introduce a short history on this subject. The most important problem is what is unconditional security. The condition of perfect secrecy \( I(X; C) = 0 \) was given by Shannon. It means that the plaintext \( X \) and the cipher text \( C \) as a function of \( X \) and a secret key \( K \) should be statistically independent. He assumed in his model that Eve has access to precisely the same information as the legitimate user. As a result, a condition: \( H(X) \leq H(K) \) is required. It looks like impractical, because it means that perfect secrecy is achieved only when the secret key is at least as long as the plaintext message. However, such a pessimism was solved by introducing the modified Shannon’s model.

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such that Eve cannot receive precisely the same information as Bob. There may be certain conditions in
physical world to realize the perfect secrecy. If we have no conditions for it, then it is called unconditionally
secure scheme. In general, we can define a channel model of cipher communication or key distribution by
the conventional information theory. That is, we have two channels. One is the channel from Alice to Bob,
and the other is the channel from Alice to Eve. Let $X, Y, Z$ be random variables of Alice, Bob, and Eve,
respectively. The channel behavior is completely specified by the conditional probability $P(Y, Z|X)$. For
broadcast channels described by $P(Y, Z|X)$, Wyner[5] clarified a fact that one can realize a scheme with
unconditional security in which Eve is assumed to receive signals from Alice over a channel that is noisier
than the legitimate users. Subsequently Csiszár-Körner[6] generalized Wyner’s result defining the secrecy
capacity $C_s(P(Y, Z|X))$ which means that the maximum bits that Alice can send to Bob in secrecy. These
schemes allow us realization of one way communication with confidential message $X$ without initial key,
and also distribution of key $K$. However, the assumption that Eve’s channel is worse than the legitimate
channel is also unrealistic. When the discussion is devoted to only the problem of key distribution, Maurer[7]
pointed out that this assumption is not needed if the legitimate users can communicate over insecure but
authenticated public channel like BB-84. Unfortunately the efficiency of communication is not so good,
because it requires small SNR. In addition, in such information theoretical results, the problem of finding
actual encodable and decodable codes that perform in a particular situation was remained. Yuen and Kim[8]
showed that, using a protocol like B-92, a practical system with unconditional security can be realized if
there exist some statistically independent noise or quantum shot noise in receivers of Eve and Bob, even the
system is conventional fiber communication. It is called YK protocol. A simple experimental demonstration
of YK protocol based on classical noise was reported[9]. Thus the unconditionally secure key distribution
does not require a realization of single photon communications and also even quantum phenomena. These results
verified that the key distribution with unconditional security is not proper matter of quantum theory. So one
can realize unconditionally secure scheme by classical communication systems if there exist unavoidable noise
such as "thermal noise in free space" and so on. In addition, $H(X) \leq H(K)$ is not essential. However, these
schemes are still inefficient, because it requires small SNR also. One should investigate protocol with more
high rate under high security. The best way may be that one seeks noisy channel governed by unavoidable
quantum noise based on quantum mechanics. As a result, an interesting question arises " Is it possible to
create a quantum system with current technology that could provide a communication in which always Bob’s
error probability is superior to that of Eve?" In 2000, Yuen gave a protocol(Y-00) to realize it as a positive
answer[10], in which he clarified a new scheme so called $M$-ary level cipher. This provides a theoretical basis
for communication with confidential message for more general channel models including Wyner’s broadcast
channel, but it requires initial shared key between Alice and Bob. Furthermore, his scheme supports also
secure key expansion, because the message can be replaced by random number. In proceedings paper of QCM
and C 2002, an implementation method was shown by using conventional optical communication system[11].
These results urged the problem of Wyner, Csiszár, Maurer to return from classical to quantum. It may be
expected that it will open a new trend of quantum cryptography as a replacement of single photon scheme.

Thus, the theory of quantum key expansion has been developed along two directions. One is to follow
BB-84 and to give rigorous proof of security of BB-84 and B-92. The other is to follow a basic idea of Wyner
and others for broadcast channels. Y-00 is close to the latter, but it is applicable to more general channels.

In this paper, we will present an implementation method of physical layer for so called Y-00 protocol which
supports a secure communication and a quantum key expansion applicable to IMDD(intensity modulation
and direct detection) or FSK(frequency shift keying) fiber communication network with fiber amplifiers as
optical repeater, and propose an error correction method to improve the performance of the total system.

2. YUEN PROTOCOL:Y-00

2.1. Basis of protocol
It is well known that the no cloning theorem is not necessary condition for unconditionally secure key
expansion in information theoretical cryptography, and it is only specific example of the realization. The
general information theoretical condition for unconditional security was discussed in the field of information
theory[5,6]. Following this line, a physical cryptography by conventional optical communication was proposed by Yuen and Kim[8] which is called YK protocol. Let us here briefly introduce a development from YK protocol to Y-00 protocol. A fundamental idea of YK protocol comes from the next remark.

**Remark:** If there are statistically independent noises between Eve and Bob, there exists a secure key expansion protocol based on communication.

This is an idea along the concept of advantage distillation of Maurer[7], and it is regarded as a method to realize a scheme such that a channel of legitimate users is reliable than that of Alice and Eve, even the signal to noise ratio of Bob is less than that of Eve. Yuen and Kim emphasized that the essential point of security of the key distribution comes from detectability of signals. This is quite different from the principle of BB-84, et al which are followed by the no cloning theorem. That is, BB-84 and related protocol employ a principle of unavoidable disturbance to quantum states to guarantee the security, but YK protocol employs a principle of communication theory to do so, and it can be realized as a modification of B-92 which has reconciliation protocol through a public channel. Although this can apply to long distance fiber system with optical repeaters, it cannot provide unconditional security when we extend communication distance and in some practical situations. Unfortunately, all of protocols proposed so far including the above scheme is inefficient for applying to commercial use. To cope with this defect, it was proposed [10] to use a fundamental theorem in quantum detection theory as follows[12]:

**Theorem 1:**

Signals with non commuting or the same density operators cannot be distinguished without error.

This means that if we assign non commuting density operators for bit signals 1 and 0, then one cannot distinguish without error. When the error is 1/2 based on quantum noise, there is no way to distinguish them. So the problem is to create such a situation on the channel between Alice and Bob, but the channel of Alice and Eve is kept as a normal communication channel. To realize it, Yuen employed a combination of a shared short key scheme for the legitimate users and a stream cipher instead of reconciliation protocol. This realizes an advantage distillation. A crucial new ingredient in his protocol is the explicit use of a shared short key for cryptographic objective of secure communication and key expansion. We call it Yuen protocol(Y-00).

Let us here give a short survey of original protocol and physical layer of Y-00 as direct encryption.

(a) The sender(Alice) uses an explicit key(seed key: $K$, expanded into a long random running key: $K^\ast$ by use of pseudo random generator as a stream cipher) to modulate the parameters of a multimode coherent state.

(b) State $|\Psi_0\rangle = |\alpha/\sqrt{2}\rangle_1 \otimes |\alpha/\sqrt{2}\rangle_2$ is prepared. Bit encoding can be represented as follows:

$$|\Psi_b\rangle = \exp\{-iJ_z\phi_b\}|\Psi_0\rangle = |e^{-i\phi_b/2}\alpha/\sqrt{2}\rangle_1 \otimes |e^{i\phi_b/2}\alpha/\sqrt{2}\rangle_2$$

where $J_z = (a^\dagger a_1 - a^\dagger a_2)/2$.

(c) Alice uses the running key $K^\ast$ to specify a basis from a set of $M$ uniformly distributed two-mode coherent state.

(d) The message $X$ is encoded as $Y_{K^\ast}(X)$. This mapping of the stream of bits is the key to be shared by Alice and Bob. Because of his knowledge $K^\ast$, Bob can demodulate from $Y_{K^\ast}(X)$ to $X$.

In order to realize secure communication or to expand the key stream under secure way, we have to guarantee that Eve cannot obtain any information from the legitimate channel. They applied that the ciphering angle $\phi_\nu$ could have $k$ in general as discrete or continuous variable determined by running key generated from pseudo random generator. A ciphered two mode state may be

$$|\Psi_{bk}\rangle = \exp\{-iJ_z(\phi_\nu + \phi_b)\}|\Psi_0\rangle$$

The corresponding density operator for all possible choices of $k$ is $\rho_b$, where $b = 1$ or 0, where $\rho_b$ are density operator for Eve, and mixed state consisting of two-mode coherent states. The problem for the security is to
find the minimum error probability that Eve can achieve in bit determination. To find the optimum detection process for discrimination between \( \rho_1 \) and \( \rho_0 \) is a problem of quantum detection theory. The solution is given by [12]

\[ P_e = \min_{\Pi} (p_1 \text{Tr}\rho_1 \Pi_0 + p_0 \text{Tr}\rho_0 \Pi_1) \] (3)

Here, \( \{\rho_i\} \) are pure states for Bob, and are mixed states for Eve who has no initial key. As an example of encoding to create \( P_e(E) \rightarrow 1/2 \) which is the error probability of Eve, Yuen et al., used a scheme such that closest values of a given \( k \) can be associated with distinct bits from the bit at position \( k \), and two closest neighboring states represent distinct bits which means a set of basis state. In this scheme, they assumed that one chooses a set of basis state(keying state for 1 and 0) for bits without overlap. As a result, the error probability for density operators \( \rho_1 \) and \( \rho_0 \) becomes \( 1/2 \), when the number \( M \) of a set of basis state increases. Asymptotic property of the error probability depends on the amplitude(or energy) of coherent state. So one requires large number of \( M \) or modes when one wishes to extend the communication distance. Thus in this scheme Bob has always a better channel than Eve. This means the advantage distillation.

2.2. Security analysis

We clarify an intuitive proof of the security for Y-00 given by Yuen[23]. The security analysis of Y-00 is considered separately for secure direct encryption and key expansion. For direct encryption, the seed key: \( K \) is fixed and the running key: \( K^* \) to modulate the optical modulator is constructed by blocks of bits of pseudo random number generated(PRG) by pseudo random number generator with the seed key. We assume that the PRG has sufficient properties for uniformness and so on. In this scheme, Eve cannot get information bits, because of Eq(3). But she can try to get directly key information through the partial information of running key based on her observation of quantum states as basis in modulation. This corresponds to known plain-text attack in the conventional theory, because she attacks based on running key information which get from her observation for quantum states. The problem is how much accuracy for the observed data is possible for getting ”running key”. Since Eve has no a priori information on properties of random sequence as running key, she cannot apply several detection scheme known as correlation detections. So she has to employ optimum single shot detection scheme to get the most accurate data of each bit sequences. Let us consider a detection scheme based on individual quantum measurement and classical collective(or correlation) attack. We can evaluate the security by the quantum detection theory for quantum state signals which was formulated by Holeve and Yuen[12]. In our case, the detection limit is formulated by quantum minimax formula of Hirota-Ikehara[24].

[Quantum minimax game between Alice and Eve]

\[ P^*_{e} = \min_{\Pi} \max_{p_i} (1 - \sum p_i \text{Tr}\rho_i \Pi_i) \] (4)

where \( \rho_i \) are quantum states which represent the basis state in quantum M-ary cipher, \( \{\Pi\} \) is positive operator valued measure(POVM) which correspond to receiver of Eve, \( p_i \) is randomization probability of Alice. Finally the accuracy of Eve’s data is given by the minimax value of this game. By this ultimate detection limit, Eve has to estimate the running key based on her data which involves error. If Eve can get the perfect information on running key, then the security level of this scheme is the same as the conventional stream cipher. Let \( n \) be length of the seed key. The security level of the ideal stream cipher is represented by \( 2^n - 1 \). In general, the security level of conventional stream cipher is weaker than it. However, by quantum M-ary cipher, the data of Eve involve inherent error based on quantum noise. Here, if the number \( M \) is enough large, the equation (4) becomes \( P^*_{e} = 1 \). That is, the sequences of running key are truly randomized. Hence nobody can get the information on the correlation and others among the running key sequences from the observed data. So the legitimate users have almost secret channel which is guaranteed by the ultimate quantum detection limit.

Thus, the security of classical stream cipher is enhanced by quantum M—ary cipher scheme. As a result, Eve cannot apply the conventional correlation attack and others for the structure of the generator. The possibility for Eve’s strategy is only pure guess for all kind of key \( 2^n - 1 \) which corresponds to Brute force attack, even the security of pseudo random number generator is weak. However, if Eve takes the strategy...
of Brute force attack, then she needs complete cipher text. A trial to get cipher text is equivalent to the cipher text attack in this case. The problem is how to do such a cipher-text attack. First Eve has to clone the following sequence of quantum states

$$|\Psi_1\rangle = (|\alpha_i\rangle_1|\alpha_j\rangle_2|\alpha_k\rangle_3\ldots$$ (5)

then this sequence is copied as follows:

$$|\Psi_1\rangle = (|\alpha_i\rangle_1|\alpha_j\rangle_2|\alpha_k\rangle_3\ldots$$
$$|\Psi_2\rangle = (|\alpha_i\rangle_1|\alpha_j\rangle_2|\alpha_k\rangle_3\ldots$$
$$|\Psi_3\rangle = (|\alpha_i\rangle_1|\alpha_j\rangle_2|\alpha_k\rangle_3\ldots$$ (6)

and all sequences must be stored in quantum memory during the trial to check all kind of text. However, each quantum bits are non orthogonal $$\langle \alpha_i | \alpha_j \rangle \not\equiv 1$$, and also $$\langle \Psi_i | \Psi_j \rangle \neq 0$$. So the clone has error due to quantum no cloning theorem when $$M$$ is enough large. As a result, Eve cannot get exact cipher-text. If the error is enough large, it is difficult to apply the cipher-text attack, and also Brute force attack, because they would need a super exponential search. Thus, this scheme has almost perfect secrecy in the individual quantum measurement. For the case of the quantum collective measurement which will provide the same result, we will show in elsewhere.

In the case of key expansion, the physical layer is the same as the case of direct encryption. The information data is replaced into the true random number, and they are divided to the blocks with length $$n$$. After communication of first blocks with length $$n$$, the legitimate users keep the block, and the seed key is refreshed by the bit block. Then the next random number with the length $$n$$ is sent by the same physical process with refreshed seed key. They keep them and repeat the process. The bit strings (key) accumulated by the above communication are used as one time pad cipher. It is difficult to get the information for the block bit, because of the same reason for the case of the direct encryption. Since the seed key for M-ary cipher is always refreshed, this is stronger than the case of direct encryption. Even if Eve can get some information in this case, the legitimate user can eliminate it by the privacy amplification. So finally the unconditional security for quantum individual, collective measurements, and classical collective attack will be shown.

3. ENTANGLEMENT ATTACK

Analysis of security is an essential requirement for the discussion on cryptography. Various eavesdropping strategies have appeared in security analysis for BB-84, B-92[13-15]. A recent review is given by Gisin et al[16]. However, these are not always effective for other scheme, and depend on protocols. Here we discuss on the security for the present model. There is no way for getting bit information in the scheme, because the error of Eve is always 1/2 for bit sequence. However, in quantum system, Eve can attack by methods as not only passive processing, but also active processing. For free space communication, one does not need to consider opaque attack and others, because the strategy of Eve is only to receive a piece of light wave from Alice. On the other hand, for fiber system, one has to consider various eavesdropping strategies in the same way as BB-84. One of them is entanglement attack. Although there are several variation of entanglement attack, here it is sufficient to consider the most simple one. That is, since the legitimate users do not use public channel in Y-00, we do not consider general attacks as conventional collective and joint attack. The entanglement attack in this case is that Eve entangles a separate probe with coherent states from Alice and detect the probe. After entanglement operation, Eve has to send the mode A which comes from Alice and keep the mode B as probe. When Bob measures them, some effects come from the action. Even if the effects act completely, Eve cannot get information of bit, because the each state has possibility of 0 and 1(see section 4). It helps Eve only for information of quantum states which was sent by Alice. If Eve can know all quantum state used by the legitimate users by such an entanglement attack, it reduces to classical stream cipher system.
Here we show that Eve also cannot get information of quantum state used by the legitimate users. To give a simple theory of entanglement for coherent state is worth while, because all information is transmitted by coherent state. Let \( \mathcal{H}_A \) and \( \mathcal{H}_T \) be the Hilbert spaces of Eve and of the total signal plus probe system, respectively. If \( |\alpha_i\rangle, |\phi\rangle, U \) denote the signal, probe’s initial states and the unitary interaction, respectively, then the state of the signal received by Bob is given by the density operator obtained by tracing out Eve’s probe:

\[
\rho_B = \text{Tr}_E(U|\alpha_i\rangle\langle\alpha_i|U^\dagger)
\]

(7)

The unitary interaction operated by Eve should make the largest entanglement between the signals and probes. Hence it is reasonable to discuss the general properties of entanglement for coherent states. One of authors and van Enk gave a general analysis for entangled state of non orthogonal state like coherent state\([17, 18]\). Here we apply them to an evaluation of the security. The following quasi Bell states or unitary probes: Let us return to the discussion of the attacks. If Eve makes a scheme of the entanglement attack in front of the Bob’s receiver, the states come from Alice are non orthogonal, because the signal power is very small. It becomes a problem of the generation of entangled state for non orthogonal states. Eve cannot get complete entanglement by her probe according to the above statement. But she can send a trick state which is generated by Eve herself. The amplitude of Alice’s coherent state is one of \( M \), so Eve does not know the amplitude of signals. As a result, she has to prepare the entangled coherent states with \( M \) amplitudes. The probability for getting the appropriate entangled coherent state is order of \( 1/M^2 \), in addition she cannot know which is the complete one. Thus, the information on the quantum states of Eve involves inherent error. In addition, if the channel involves optical amplifiers as repeaters, then the received states are mixed state. Eve cannot make a good entanglement operation in this case.

If Eve makes the scheme at the Alice’s side, then almost states are regarded as orthogonal, because the signal power is large. Let us show a property of decoherence due to energy loss on the state \( |\Psi_2\rangle_{AB} \) which has complete entanglement. Eve has the source of entangled states, keep one part and transmit the other part to Bob through a lossy channel. Bob will receive the attenuated optical state. This is a model for specific sharing method of entangled state. In such a situation, Eve can prepare the following entangled state

\[
|\Psi_2(0)\rangle_{AB} = h_2^* (|\alpha\rangle_A - \frac{\alpha}{\sqrt{\eta}})B - | - \alpha\rangle_A (\frac{\alpha}{\sqrt{\eta}})B
\]

(10)
where \( h_2^* = 1/\sqrt{2(1-\kappa_A\kappa_B)} \), \( \kappa_A = \langle \alpha | - \alpha \rangle \), and \( \kappa_B = \langle \frac{\alpha}{\sqrt{\eta}} | - \frac{\alpha}{\sqrt{\eta}} \rangle \), and where \( \eta \) is transparency of the channel. When we employ a half mirror model for energy loss channel, the effect of energy loss is described by a linear coupling with an external vacuum field as follows:

\[
U_{BL}|\alpha\rangle_B|0\rangle_L = \sqrt{\eta}|\alpha\rangle_B|\sqrt{1-\eta}|\alpha\rangle_L.
\]

where the mode \( L \) is an external mode corresponding to energy loss, and \( \alpha \) is taken as real. If we use \(|\Psi_2^0\rangle_{AB}\) as the initial state, Alice will finally couple with the environment in

\[
\hat{I}_A \otimes U_{BL}|\Psi_2^0\rangle_{AB} \otimes |0\rangle_L
= h_2(|\alpha\rangle_A - |\alpha\rangle_B) - \sqrt{\frac{1-\eta}{\eta}}|\alpha\rangle_L - |\alpha\rangle_A|\alpha\rangle_B|\sqrt{\frac{1-\eta}{\eta}}|\alpha\rangle_L).
\]

Then the state shared by Eve and Bob is given by a super operator calculation as follows:

\[
\rho_{AB} = \tilde{h}_2^2 \left\{ \frac{1}{\sqrt{2(1-\kappa_A^2)}} (|\alpha\rangle_A - |\alpha\rangle_B - |\alpha\rangle_A|\alpha\rangle_B) \right\} (13)
\]

where

\[
|\Psi_2\rangle_{AB} = \frac{1}{\sqrt{2(1-\kappa_A^2)}} (|\alpha\rangle_A - |\alpha\rangle_B - |\alpha\rangle_A|\alpha\rangle_B),
\]

and where \( L = \exp{-4(1-\eta)|\alpha|^2} \), \( |\alpha\rangle_A - |\alpha\rangle_B = |\alpha\rangle_A - |\alpha\rangle_B \), and \( \tilde{h}_2^2 = 1/\{2(1-L\kappa_A^2)\} \). As a result, we have the following form:

\[
\rho_{AB} = \frac{(1-\kappa_A^2)}{(1-L\kappa_A^2)} |\Psi_2\rangle_{AB} \langle \Psi_2|
+ \frac{(1-L)}{(1-L\kappa_A^2)} (|\alpha\rangle_A - |\alpha\rangle_B - |\alpha\rangle_A|\alpha\rangle_B)
\]

(15)

Here let us discuss properties of entanglement of the above density operator. We can use the entangled fraction to measure the entanglement of mixed state. The fully entangled fraction of this density operator is given by

\[
f(\rho_{AB}) =_{AB} \langle \Psi_2 | \rho_{AB} | \Psi_2 \rangle_{AB}
\]

because \( |\Psi_2\rangle_{AB} \) has complete entanglement which is independent of \( \alpha \). As a result, we have

\[
f(\rho_{AB}) = \frac{(1-\kappa_A^2)}{(1-L\kappa_A^2)} + \frac{(1-L)(1-\kappa_A^2)^2}{(1-L\kappa_A^2)}
\]

(17)

In the special cases, the fraction of entanglement is that \( f(\rho_{AB}) = 1 \) for \( \eta = 1 \), and \( f(\rho_{AB}) \geq \kappa_A^2(1-\kappa_A^2) \) for \( \eta \neq 0 \). Although this is robust for energy loss compared with bi-photon entangled state, the entanglement is destroyed when \( \eta \neq 0 \). So it is useless in the sense for getting any information.

4. QUANTUM M-ARY CIPHER FOR FIBER NETWORK

An implementation of Y-00 in the above was discussed based on phase (or polarization) modulation which provides the same energy for bit[11]. To apply them to fiber communication system, it is better to realize them by intensity modulation/direct detection or frequency shift keying. But to keep the security, it will require some additional idea for implementation of Y-00. The essential role of modulation scheme in Y-00 is to assign non-commuting or same density operators to 0 and 1 for Eve. In the original implementation[11], they assumed that the selection of a set of the basis states to transmit 0 and 1 is as follows

\[
\{1, 0\} \rightarrow \{ |\Psi_1(1)\rangle, |\Psi_0(1)\rangle \}, \{ |\Psi_1(2)\rangle, |\Psi_0(2)\rangle \}, \ldots
\]

(18)
Figure 1. Scheme of selection of quantum state parameters

where \( \{ |\Psi_1(j)\rangle, |\Psi_0(j)\rangle \} \), \( j \in \mathcal{M} \) is a set of basis states. The bit or key information (1 and 0) is transmitted by one of \( M \) sets of basis states, controlling by initial shared key and running key. This idea comes from a fundamental principle in quantum detection theory such that we can construct non-commuting density operators only by set of non-orthogonal states when one does not allow overlap of the selection of a set of basis state for 1 and 0. The non commutative nature of density operators depend on strongly on amplitude or energy of coherent state. In order to improve this effect, huge number of set of basis states must be prepared.

On the other hand, when one allows overlap for selection of a set of basis state, it is easy to construct the same density operators for 1 and 0. That is, \( \rho_1 = \rho_0 \). This scheme is called overlap selection keying (OSK). As an advantage of the overlap selection keying, we can say that it provides a very simple system and Eve completely cannot estimate the bits at that time even if the set of the basis states is only one. Each set of basis state is used for \( \{1,0\} \), and \( \{0,1\} \), depending on running key.

\[
\text{Set } A_1 : 0 \rightarrow |\alpha(1)\rangle, \quad 1 \rightarrow |\alpha(M+1)\rangle
\]

\[
\text{Set } A_2 : 0 \rightarrow |\alpha(M+1)\rangle, \quad 1 \rightarrow |\alpha(1)\rangle
\]

So the density operators of 1 and 0 for Eve are

\[
\rho_1 = \rho_0 = \frac{1}{2}( |\alpha(1)\rangle\langle \alpha(1) | + |\alpha(M+1)\rangle\langle \alpha(M+1) | )
\]

However, if the signal energy is large, the coherent states become of orthogonal. So bit strings that Eve observed are perfectly the same as those of Bob, though Eve cannot know information bit. This gives still insecure situation. In order to cope with the above problem, we employ a combination of \( M \)-ary scheme and overlap selection keying. As a result, the energy dependency of the security may be alleviated, and it is enough to take into account only the average error probability for discrimination of \( 2^M \) pure states for the security.

Here we show an example of design for IMDD system. Let us assume that the maximum amplitude is fixed as \( \alpha_{\text{max}} \). We divide it into \( 2M \). So we have \( M \) sets of basis state \( \{(A_1, A_2), (B_1, B_2), \ldots \} \). Total set of basis state is given as shown in Fig.1. Here, let us assign 0 and 1 by the same way as Eqs(19),(20) for the sets of \( \{B_1, B_2\}, \{C_1, C_2\}, \ldots \). In this case, Eve completely cannot get bit information, while the knowledge of Eve depends on \( \alpha_{\text{max}} \) and \( M \) in the original scheme[11]. This fact gives a great advantage when we apply the Y-00 protocol to the practical system. Then she can try to know the information of quantum states used for bit transmission. This problem is of discrimination for \( 2M \) pure states. The error probability is given by Eq(4). Although we have many results for calculation of quantum optimum detection problems, to give the analytical solution of this problem is still difficult at present time, because the set of states does not have complete symmetric structure. However, the tight upper bound is given by applying square root measurement (SRM) to \( 2M \) pure states. It is well known that SRM provides the optimum or suboptimum
detection for many kinds of pure state signals \cite{19, 20}. The SRM is defined as follows:

\[ |\mu_i\rangle = \hat{G}^{-1/2} |\psi_i\rangle, \quad \hat{G} = \sum_{i \in \mathcal{M}} |\psi_i\rangle\langle\psi_i| \]  

(22)

where \( |\psi_i\rangle \) is signal state, \( \hat{G} \) is the Gram operator, and the detection operator is formed as

\[ \Pi_i = |\mu_i\rangle\langle\mu_i|, \quad i \in \mathcal{M} \] 

(23)

It is very easy to calculate the average error probability when we apply this detection operator, and these are very tight which can be proven by Helstrom’s Bayes cost reduction algorithm \cite{21} to check the tightness(see Fig.2). Thus if \( M \) increases, then her error for information on quantum states increases. In this case, pure guessing corresponds to \( \frac{2M-1}{2M} \). The lower bound is given by the minimum error probability: \( P_e^*(2) \) for signal set \( \{|\alpha_1\rangle, |\alpha_2\rangle\} \) which are neighboring states. It reads

\[ P_e(2) = \frac{1}{2}(1 - \sqrt{1 - \exp(-|\alpha_2 - \alpha_1|^2)}) \] 

(24)

For the error probability of Bob, if we can allow Bob to use the quantum optimum receiver, then the error probability of Bob is independent of the number of set of basis state, and it is given as follows:

\[ P_e(B) = \frac{1}{2}(1 - \sqrt{1 - |\langle\alpha_1|\alpha_{M+1}\rangle|^2}) \] 

(25)

We emphasize that Eve cannot get bit information in this stage, because the information for 1 and 0 are modulated by the method of Eqs(19),(20). We will show the detectability for the case of conventional receiver in the next section. Thus, it is achieved that the error of Bob is less than that of Eve even the SNR of Eve is greater than that of Bob, which corresponds to the advantage distillation. Let us compare with non-overlap and overlap selection keying. In the former case, Eve will try to get bit information, so the density operators for Eve become mixed states \( \rho_1 \), and \( \rho_0 \) consisting of set of quantum states which send 1 and 0, respectively. The error probability of Eve for bit information (\( \rho_1 \) or \( \rho_0 \)) depends on strongly power of laser light(see Fig 3). In the latter case, the error of Eve is always 1/2, and it dose not depend on power of laser light. On the information for set of basis states, the error probability of Eve becomes the same one given by the formulae Eq(4) in both cases. Thus in the latter case, it is enough to take the information on the set of basis states into account. Since, in the intensity modulation, we cannot keep the enough signal distance under the power constraint in comparison with phase modulation, the overlap selection keying has great advantage. In addition, it can send \( 2M \) bits by \( M \) sets of basis state. The above example was described as ASK(amplitude shift keying), but it is the same as the intensity modulation scheme. As simple modification, we can use frequency shift keying(FSK) and also combination of ASK and FSK. If one wishes more efficient scheme, then one can apply many modulation scheme, for instance Manchester scheme and so on.

### 5. DESIGN FOR PRACTICAL FIBER NETWORK

A special feature of YK and Y-00 protocol is that one can design the secure system based on only the abilities of Eve and Bob for detectability. That is, it is not important how many repeaters are used in communication channel from Alice to Bob. In general, the present fiber network consists of optical repeaters like fiber amplifier, and direct detection scheme(intensity detection for on and off). Although one can make the ability of Eve the level of pure guessing by increasing \( M \)(the number of basis set), it is difficult to improve the detectability of Bob when the communication distance is very long under the high bit rate, for example 1000 km and 1 Gbps. Here we show how we can solve it.

A practical fiber system consist of transmitter(laser diode), fiber cable, repeater(fiber amplifier), pre-amplifier, and photo-detector. The noise in receiver of the above scheme of IMDD is given as follows:

\[ \langle I_{on}^2 \rangle = \langle I_{th}^2 \rangle + \langle I_{sig}^2 \rangle + 2\langle I_{sp}^2 \rangle + \langle I_{sig-sp}^2 \rangle + 2\langle I_{sp-sp}^2 \rangle \] 

(26)
Figure 2. Error probability of Eve for discrimination of $M$ pure states. Horizontal line: upper is of the number of state, under is the number of basis state

Figure 3. Error probability of bit information for non overlap selection keying
where \( \langle I_{th}^2 \rangle, \langle I_{sig}^2 \rangle, \langle I_{sp}^2 \rangle, \langle I_{sig-sp}^2 \rangle, \langle I_{sp-sp}^2 \rangle \) are thermal noise in detector, quantum shot noise, noise by spontaneous emission from amplifier, beat noise between signal and spontaneous emission, beat noise between spontaneous emission itself, respectively. They are given as follows:

\[
\begin{align*}
\langle I_{sig}^2 \rangle &= 2e^2G_p\kappa_r\langle n \rangle B \\
\langle I_{sp}^2 \rangle &= 2e^2\kappa_rG_pN(G-1) + (G_p-1)\langle n \rangle B\delta f \\
\langle I_{sig-sp}^2 \rangle &= 4e^2G_p\kappa_rG_pN(G-1) + (G_p-1)\kappa_r\langle n \rangle n_{sp}B \\
\langle I_{sp-sp}^2 \rangle &= 2e^2\kappa_rG_pN(G-1) + (G_p-1)^2n_{sp}^2B^2\delta f
\end{align*}
\]

where \( e \) is the electron charge, \( G_p \) is the gain of pre-amplifier, \( G = \frac{1}{\kappa_r} \) is the gain of the repeater, \( \kappa_r \) is the transparency of the fiber, \( N \) is the number of repeater, \( \langle n \rangle \) is the photon number per second at transmitter, \( n_{sp} \) is the spontaneous emission factor, \( B \) is the bandwidth, and \( \delta f \) is the bandwidth of optical filter, respectively.

As a result, the bit sequence of Bob involves many error bits. That is, the error probability of Bob \( P_e \) in a real fiber system is not so good in comparison with quantum optimum receiver.

\[
P_e(B) \gg \frac{1}{2}(1 - \sqrt{1 - |\langle \alpha_1 | \alpha_{M+1} \rangle|^2})
\]

(31)

Here we give a method for the improvement of this unavoidable error in the direct detection. Since we can use high bit rate(Gbps), it does not give a degradation of the performance even if we use redundant codewords for sending bit information(0 and 1). In the following we show how it works. Let us assign codeword:\( u_i \) from the set \( W : \{ u_i \in W \} \) of the sequence of quantum states. For example, as error correction code, we assign one codeword from the following set of codewords:

\[
\begin{align*}
\text{Code}_1 : & \{ 0 \to u_1 = |\alpha_{(1)}\rangle |\alpha_{(1)}\rangle |\alpha_{(M+1)}\rangle, \quad 1 \to u_4 = |\alpha_{(M+1)}\rangle |\alpha_{(M+1)}\rangle |\alpha_{(1)}\rangle \} \\
\text{Code}_2 : & \{ 0 \to u_2 = |\alpha_{(1)}\rangle |\alpha_{(M+1)}\rangle |\alpha_{(1)}\rangle, \quad 1 \to u_5 = |\alpha_{(M+1)}\rangle |\alpha_{(1)}\rangle |\alpha_{(M+1)}\rangle \} \\
\text{Code}_3 : & \{ 0 \to u_3 = |\alpha_{(M+1)}\rangle |\alpha_{(1)}\rangle |\alpha_{(1)}\rangle, \quad 1 \to u_6 = |\alpha_{(1)}\rangle |\alpha_{(M+1)}\rangle |\alpha_{(M+1)}\rangle \} \\
& \vdots
\end{align*}
\]

(32)

If the set \( A_2 \) is chosen, then only bit 0 and 1 are changed, and the codeword is the same one. The structure of this codeword may be open for public, but the selection depends on initial key and running key.

Let us recall that the decision of legitimate user is binary. For simplifying the evaluation, we assume that the probability distribution of noise in the detector for bit is symmetric. That is, the conditional probabilities for 0 and 1 as the decision process are \( P(1|0) = P(0|1) \). Since the Hamming distance of this code is 3, it has a function of one bit error correction. So the average error probability is given as follows:

\[
P_e = 3P(0|1)^2 - 2P(0|1)^3
\]

(33)

If the bit error: \( P(0|1) = 10^{-4} \) in the practical receiver of direct detection, then it turns the value of average error into about \( 10^{-8} \) which is enough for practical use. This is effective rather than only to increase the transmitter energy of each light pulse for improving the error probability of Bob.

The security for bit information is kept in the case of overlap selection keying, because it is independent of signal power. For information on the set of basis states which are used for sending bit sequence, Eve has to design the quantum optimum receiver applying to total number of pure quantum states as all kind of codewords \( u_i \in W \), because Eve cannot know them. In this case, the number of codeword is \( 3M \). This corresponds to increasing the set of basis state. According to quantum detection theory and the numerical example of the error probability for \( M \)-ary pure states, the average error probability increases rapidly when the number \( M \) increases.

In this section, we showed that if we insert key for the selection of codeword into Y-00 protocol, then we can improve the detectability of Bob without the degradation of the security. More general discussion for design of error correcting code will be given in the subsequent paper.
6. CONCLUSIONS

We repeat our motivation here. Basically, one wants to realize one way secure communication. However, the situation of the eavesdropper is in general better than the legitimate user. To cope with such a situation, many proposals were devoted to discuss only a key distribution which allow two way communication or feedback to establish the advantage distillation. BB-84 and other quantum key expansion protocols provided a solution for such problems, introducing to detect the existence of eavesdropper and privacy amplification.

However, these may be unenterprising strategy in the practical system. Following such well known results, Yuen verified that more simple information theoretical secure communication and unconditionally secure key expansion exist. In this paper, we have shown an efficient physical layer of Y-00 protocol applicable to secure communication and quantum key expansion for fiber system. It was clarified that the overlap selection keying of set of basis states gives the very efficient way to establish the secure condition in Y-00 protocol. This work is an extension of reference 22.

Acknowledgment

We are grateful to H.P.Yuen, S.J.van Enk for helpful suggestions.

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