On the way to fault detection method in moving load dynamics problem by modified recurrent neural networks approach

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Received: 3 July 2020 / Accepted: 30 January 2021

Abstract. Parameters identification on structure subjected to moving load can be predicted by using the accurate and reliable data. The concepts of recurrent neural networks (RNNs) approach have been used in parameters (crack locations and severities) identifications in structure subjected to moving load in the present methodology. This methodology has incorporated the knowledge based Elman’s recurrent neural networks (ERNNs) and Jordan’s recurrent neural networks (JRNNs) jointly for the identification of parameters. This approach has been addressed as the inverse problem for predicting the locations and quantification of cracks in the structure in a supervised manner. The Levenberg-Marquardt’s back propagation algorithm is implemented to train the proposed networks. To check the robustness of the present method, Numerical studies followed by Finite Element Analysis (FEA) and experimental verifications (Forward problems) are presented as a case study by considering a multi-cracked simply supported structure under a moving mass. The estimated crack locations and severities obtained from the proposed RNNs model converge well with those of FEA and experiments. From the demonstration of the case study, it concludes that the proposed analogy can identify and quantify the crack locations and severities effectively.

Keywords: Crack locations / crack severities / L-M algorithm / ERNNs / JRNNs

1 Introduction

Vibration based fault detection approaches are global methods which have been considered to spot the positions and severities of cracks in structures. The fault detection methods have achieved a wide range of attentions from engineering and scientific communities due to the unexpected structural failure. A consistent and efficient fault detection approach is essential to preserve the protection and reliability of structures. The fault or crack detection approaches have been carried out by using various conventional and intelligent methods for the past few years. In contrast, attention in the literature survey was raised significantly over the recent years in neural networks (NNs) based fault detection approaches. The NNs are quietly used to improve the abilities of conventional methods. Numerous studies have been carried out by various researchers and engineers to spot the positions and severities of cracks in structure by using the vibration based dynamical characteristics of structural responses.

A brief literature study on vibration-based condition monitoring is carried out here with more inclusive reviews. Bagherahmadi and Seyedpoor [1] proposed a proficient damage index approach for multiple damages in structure using the strain energy and frequency response function matrix of the structure. They have considered the change in strain energy of the structure as damage index parameter. The concepts of natural frequencies and mode shapes analysis mechanism are applied by Chasalevris and Papadopoulos [2] for detection of multiple cracks on structure. Chang et al. [3] developed a structural health monitoring approach based on ANNs modeling to interpret the damage parameters using the modal properties of the structures. The proposed approach has been demonstrated successfully to a 7DOF building structure for damage identification. Duininlin [4] proposed a fault detection approach on railway damper vehicle suspension system by considering the vertical bogie acceleration.

Eroglu and Tufekce [5] have developed a FEM model for an edge cracked Timoshenko straight beam. They have solved this problem by using genetic algorithm in an inverse manner. Hou et al. [6] have developed Genetic Algorithm (GA) based Optimal Sensor Placement (OSP) approach for damage recognition in structure with...
regularization damage detection. They have conducted experiments for a cantilever beam and three-storey frame structures to check the concept of the developed method. Hakim et al. [7] have developed a damage identification procedure in the domain ANNs by taking into account an I-beam structure. Jena et al. [8–11] have carried out numerous studies like theoretical, FEA and experimental to determine the responses of different types of structures subjected to transit mass. They have determined the structure’s response at various damage and moving load parameters. In the later part, Jena and Parhi [12,13] have also developed damage detection procedures for structure under traverse load as inverse methods using the concepts of different types of rule-based recurrent neural networks. (ERNNs and JRNNs).

Mao and Lu [14] have explored an effective method to determine the critical speed and resonance condition of bridge structure subjected to moving trains with different speeds. Their method is very much effective to study the effects of resonance in accordance with resonance speed. Pala and Reis [15] have investigated the effects of inertial, centripetal and Coriolis’s forces on the response of a cracked beam subjected to a moving load. Pathirage et al. [16] have developed an auto encoder based approach for structural damage identification approach in a steel frame structure. The concepts of NNs and deep learning approaches are applied here for pattern recognition problem. Yang et al. [17] have explored a method based on the combination of ANNs and dynamics strain mechanism to identify the damage parameters in moving load problem. Yang et al. [18] have developed a static algorithm by implementing the moving load induced response and bending theory mechanism to locate damage in structure. Wu et al. [19] have developed a 3-dimensional finite element model to determine the dynamic response of a concrete structure due to the excitation of a moving mass. They have calculated the stress and deflection of the structure at the critical positions of the moving load. Zhang et al. [20] presented a noble non-model based approach for crack detection in bridge structure under vehicular load based on the concept of phase trajectory alteration of multi-type induced vibration measurement. The proposed mechanism is very useful to recognize the shear failure in composite bridge structure.

So far from the literature studies, the authors have got the attentions that numerous works have been carried out for structural health monitoring problem using different techniques as forward and inverse methodologies. Even if many works are also carried using NNs based approach, but using the concepts of RNNs methodologies are few in the applications of mechanical engineering era like structural dynamics. Again as per the concern of authors’ knowledge, the literatures are very scanty on damaged detection in structure under moving mass using modified RNNs approach. The novelty of the proposed work is to develop a structural damage detection procedure using the combined model of the modified ERNNs and JRNNs approaches. The results obtained from the modified RNNs (combined model of the modified ERNNs and JRNNs) model are corroborated with those of FEA and experimental works. The entire damage detection procedure has been carried out in a supervised manner. The numerical, FEA and experimental works which are already validated with each other earlier by the authors [8–10] are considered as forward problems, while the modified RNNs analogy is considered as inverse method in the present formulation. The present method has been also compared with author’s previous works [12,13].

2 The problem definition

In the present investigation, a simply supported beam (Fig. 1) with multiple cracks subjected to a traversing load has been considered for the analysis. The Euler Bernoulli’s beam analogy is applied in the present problem without inclusion of the damping effect of the beam. The problem formulation has already described by the authors [8–10] in their previous work.

The equation of motion of a structure subjected to traversing load is given by:

$$EI \frac{d^4y}{dx^4} + m \frac{d^2y}{dt^2} = F(x, t)$$  (1)

where $F(x, t) = P(t) \delta(x - \gamma)$ = Interactive force due to traversing mass. 
$\delta$ = Dirac delta function.

$P(t) = Mg - M \left( \frac{d^2}{dt^2} + \nu \frac{d}{dt} \right) y(\gamma, t) = Applied \ force.$

$\gamma = vt$ = Position of the traversing mass at any moment of time ‘t’.

Fig. 1. Schematic presentation of cracked simply supported beam under traversing load.
Table 1. Frequency ratio for the cracked beam at different modes.

| Mode no | $\alpha_{1,2,3} = 0.13, 0.28, 0.41$ | $\alpha_{1,2,3} = 0.23, 0.37, 0.48$ | $\alpha_{1,2,3} = 0.13, 0.28, 0.41$ | $\alpha_{1,2,3} = 0.23, 0.37, 0.48$ |
|---------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\eta_{1,2,3} = 0.358, 0.503, 0.634$ | 1.0000 | 0.9647 | 0.9812 | 0.9952 |
| 1       | 0.9861                          | 0.9921                          | 0.9795                          | 0.9875 |
| 2       | 0.9907                          | 0.9795                          | 0.9947                          | 0.9875 |
| 3       | 0.9921                          | 0.9795                          | 0.9947                          | 0.9875 |

$L_1, L_2, L_3 = L_{1,2,3} =$ Locations of the first, second and third cracks from the left supported end of the cracked simply supported structure respectively.

d_1, d_2, d_3 = d_{1,2,3} =$ Deepness of the first, second and third cracks respectively.

$M =$ Mass of the traversing mass, $v =$ Traversing speed.

$EI$ = Flexural rigidity of the structure.

$m =$ Beam mass per unit length. $g =$ gravitational constant, $L =$ Length of the beam.

The solution of the above equation (1) has been written in series from i.e.

$$y(x, t) = \sum_{n=1}^{\infty} Y_n(x) T_n(t)$$

where $Y_n(x) =$ Shape function of the structure without the transit mass.

$T_n(t) =$ Amplitude function, $n =$ Number modes of vibrating structure.

For the evaluation of $Y(x)$, the equation (2) has been expressed as-

$$Y_n''(x) - \lambda_n^2 Y_n(x) = 0$$

where $\lambda_n^2 = \rho A \frac{v^2}{EI} + m \frac{v^2}{L^2}$.

The governing equation (2) which has been solved earlier by Jena and Parhi [8–10] is represented below:

$$EI \lambda_n^4 T_n(t) + m T_n''(t)$$

$$- \left( \frac{M}{S_n} \right) \left[ g - \sum_{q=1}^{\infty} \left( \frac{\partial}{\partial t} + v \frac{\partial}{\partial \gamma} \right) \frac{2}{Y_q''(t)} \right] Y_n(\gamma) = 0$$

A numerical example has been formulated to determine the responses of the simply supported beam subjected to the traversing load. The forward problem (Numerical, FEA and experimental works) has been considered in the same way of Jena and Parhi [8–10] as of their earlier approach and responses of the structure are also verified using FEA and experimentation methods by the authors in their previous works. The transient dynamic analysis method (Newmark-β integration method) has already been carried out to determine the responses of the simply supported structure under transit mass in ANSYS WORKBENCH 15 domain. The laboratory test method was also conducted for the proposed work. The test procedures (Fig. 2) remain same as the procedures of the author’s [10] previous work.

The frictional force between the moving mass and the beam is neglected here. The responses (amplitudes) of the beam are recorded at various desired locations. The responses of the structure are also observed from the test method. The FEA model of the present problem is shown in Figure 3 (Tab. 1).

### 3 Numerical formulations

To corroborate all the adopted method, a numerical example has been formulated.

- Beam type-Rectangular (Mild steel specimen)
- Beam size-145 cm × 4.5 cm × 0.5 cm.
- Moving speed- 5.23 and 6.41 m/s.
- Moving mass- 1.13 and 2.6 kg.
- Relative crack locations-$\eta_{1,2,3} = \frac{L_{1,2,3}}{L} = 0.358, 0.503, 0.634$ and $0.331, 0.545, 0.703$.

Fig. 2. Laboratory test model for simply supported structure.

Fig. 3. FEA model for the moving load problem for $\eta_{1,2,3} = 0.331, 0.545, 0.703, \alpha_{1,2,3} = 0.13, 0.28, 0.41. M = 1.13$ kg

Table 1. Frequency ratio for the cracked beam at different modes.
Relative crack depth $\alpha_{1,2,3} = \frac{d_{1,2,3}}{h} = 0.13, 0.28, 0.41$ and 0.23, 0.37, 0.48.

The natures of the cracks on the structure are open and transverse and the detailed analysis of cracks are already analyzed by Jena and Parhi [11]. The cracks are open and transverse in nature, the load is assumed to be acting on the transverse direction only. From the analysis of Figure 4, the sudden rise in dynamic amplification on the beam response is occurring due to only the presence of cracks. From the observed dynamic amplification, the positions and severities of cracks are determined by FEA and experimental analyses. The severities of cracks are quantified using the concepts of natural frequencies and mode shapes analysis mechanism using FEA approach by Chasalevris and Papadopoulos [2] and Bagherahmadi and Seyedpoor [1]. Based upon the analysis of the response, the crack positions and depth are observed.

4 Implementation of modified recurrent neural networks (RNNs) approach for fault detection

RNNs are those kinds of networks where the output data from the preceding step can be fed as input data to the present step. In case of conventional NNs, the data from input and output are independent with each other. But in RNNs, the data are dependent with each other and can be used whenever it’s required. RNNs have additional memory (context layer) which remembers the information that has been required later. In the present analogy, a modified RNNs based approach has been developed to identify the positions of cracks and determine the severities of crack in a supervised manner. The modified RNNs structure is the combination of the modified ERNNs and JRNNs structures. The architecture of the proposed RNNs structure has been represented in Figure 5. Based upon the architectural structure, the equations are developed for the modified RNNs. The different terminologies used in the network structure are described here. Primarily, the proposed RNNs model is trained with the training data parameters like relative deflections of the beam, number of cracks, speed of the transit mass, length of the structure, mass of the transit mass as input data and crack locations & severities as output data.

- $RD$ = Relative deflection at the crack position
- $RD_{1}, RD_{2}$ and $RD_{3}$ are the relative deflections of the structure at the first, second and third crack positions respectively.
- $n$ and $n'$ are the number of cracks in the input and output layers of the network model respectively.
- $W_{1} = RD_{1}, W_{2} = RD_{2}, W_{3} = RD_{3}.$
- $W_{4} = $ Length of the beam $(L)$.
- $W_{5} = $ Speed of the moving mass $(v)$.
- $W_{6} = $ Magnitude of the moving mass $(M)$.
- $W_{7} = $ Number of cracks.
- $\phi_{1} = $ First relative crack location $(\eta_{1})$.
- $\phi_{2} = $ First relative crack depth $(\alpha_{1})$.
- $\phi_{3} = $ Second relative crack location $(\eta_{2})$.
- $\phi_{4} = $ Second relative crack depth $(\alpha_{2})$.
- $\phi_{5} = $ Third relative crack location $(\eta_{3})$.
- $\phi_{6} = $ Third relative crack depth $(\alpha_{3})$.
- $\phi_{7} = $ No of cracks present in the structure $(n')$.

where, $i = 1, 2, \ldots, N$, 'N' is the total input nodes.

$r = 1, 2 \ldots r_{1}, r_{2}$ is the total number of neurons in the context layers, the context layers 1 & 2 having same number of neurons.

$k = 1, 2 \ldots k$, 'k' is the number of nodes in the output layer.

$j = 1, 2 \ldots j'$, 'j' is the number of neurons in each of the hidden layers, that is, first, second and third hidden layers which is constant throughout the layer.

$'B'$ is the self-recurrent value for each node in the output and context layers that varies from 0 to 1.

$Z^{-1}$ is the unit delay.

$w$ is weight of the connection.

$U_{1-7}$ and $V_{1-13}$ are the context unit values in the context layers-3 and 4 respectively.

$V_{1-13}$ and $V_{1-13}$ are the context unit values in the context layers-1 and 2 respectively.

$\phi_{1}'$ and $\phi_{13}'$ are the output values of the nodes at the context layer-1 at time index $'t'$ and $'t-1'$ respectively.

$\phi_{2}'$ and $\phi_{12}'$ are the output values of the nodes at the context layer-2 at time index $'t'$ and $'t-1'$ respectively.

$\phi_{3}'$ and $\phi_{13}'$ are the output values of the nodes at the context layer-3 at time index $'t'$ and $'t-1'$ respectively.

$\phi_{4}'$ and $\phi_{14}'$ are the output values of the nodes at the context layer-4 at time index $'t'$ and $'t-1'$ respectively.

$\phi_{j}'$ are $\phi_{j}'$ are the output values of the nodes at the hidden layer at time index $'t'$ and $'t-1'$ respectively.

$\phi_{i}'$ and $\phi_{i}'$ are the output values of the nodes at the output layer at time index $'t'$ and $'t-1'$ respectively.

$'t-1'$ is the time index that is deferred by one-time step due to the feedback connections which is represented by $Z^{-1}$ in the network architecture.

$'w'$ is the weight of connection.
By analyzing the modified RNNs structure (Fig. 5), the mathematical equations for the structure are extended as follows:

\[
\phi_{c1}^{I} = \phi_{j}^{I-1} + \beta \phi_{c1}^{I-1} \tag{5}
\]

\[
\phi_{c2}^{I} = \phi_{c1}^{I-1} + \beta \phi_{c2}^{I-1} \tag{6}
\]

\[
\phi_{c3}^{I} = \phi_{k}^{I-1} + \beta \phi_{c3}^{I-1} \tag{7}
\]

\[
\phi_{c4}^{I} = \phi_{c3}^{I-1} + \beta \phi_{c4}^{I-1} \tag{8}
\]

The net input to the first hidden layer of the network model is given by-

\[
\phi_{j1}^{I} = \sum_{i=1}^{N} W_{i,j1}^{I} \phi_{i}^{I-1} + \phi_{j1}^{I-1} + \beta \phi_{j1}^{I-1} + \phi_{c1}^{I} + \phi_{c2}^{I} + \phi_{c3}^{I} + \phi_{c4}^{I} \tag{9}
\]

Fig. 5. Modified architecture of RNNs model.
The net input to the second hidden layer of the network model is given by-

$$\phi_{j2} = \sum_{j=1}^{S} \phi_{j1}^t w_{j1,j2} + \beta \phi_{j2}^{t-1} + \phi_{j3}^{t-1}$$  \hspace{1cm} (10)

The net input to the third hidden layer or network model is given by-

$$\phi_{j3} = \phi_j = net_j = \sum_{j=1}^{S} \phi_{j2}^t w_{j2,j3} + \beta \phi_{j3}^{t-1}$$  \hspace{1cm} (11)

$$\phi_j^t = f(\text{net}_j^t)$$

The net output of the network model is given by-

$$\text{net}_k^t = \sum_{j=1}^{S} \phi_{j3}^t w_{j3,k}$$  \hspace{1cm} (12)

$$\phi_k^t = g(\text{net}_k^t)$$

The applied activation functions in the hidden and output layers are denoted by $f(.)$ and $g(.)$ respectively. The ‘tan-sigmoid’ activation function has been applied for the hidden layers, while that of output layer is ‘purelin’. The error functions $e$ of the output neurons are estimated approximately by employing the efficient weight factors relation, that is, $w_{new} = w_{old}^\Delta + \phi \Delta w$, where ‘$\phi$’, the learning rate coefficient that is varies from 0 to 1. The training procedures of the proposed network model are carried out using sum square error function mechanism. The L-M (Levenberg-Marquardt) back propagation algorithm by Yu and Wilamski [21] is implemented to the proposed model in Table 2. The patterns are generated for both for the damaged and healthy structures. The present RNNs model is trained with 750 numbers of patterns with different parameters of the moving load dynamic problem. From the 750 numbers of patterns, 650 patterns are used for training procedures of the network while 100 patterns are for testing procedures of the networks.

While training the network model, the number of neurons and hidden layers are selected on the basis of cross-validation analogy. The model data (neurons, layers) has been determined from the available training data set or patterns (650 numbers). The remaining 100 patterns are reserved to test the model. The number of neurons in each hidden layer is found to be 13, while the number of hidden layer to be 3. The numbers of neurons in the context layers (3 & 4) are 7 while those in context layers (1 & 2) are 13 respectively. It’s due to the reason that the data or information can be copied and transformed exactly from the respective preceding layers and forward to the proceeding layer. The training method has been conducted using the L-M algorithm. The patterns generated for the training process is exemplified in Table 2. The patterns are generated for both for the damaged and healthy structures. In Table 2, in the input parameters, the values of RD characterize to 1 indicates that there is no crack at that particular location which leads the values of are $\eta$ and $\alpha$ zero in the output parameters. The present methodology can identify up to three numbers of cracks.

5 Results and discussion

In the present investigation, a parameter identification (crack locations and severities) problem has been developed using modified recurrent neural networks in a supervised manner. The modified RNNs blend the combination of modified ERNNs and JRNNs. The L-M back propagation algorithm has been implemented to train and test the networks. The present formulation is based to explore an indirect approach for fault detection in moving load dynamic problem in the domain RNNs. To check the convergence of the developed method (Modified RNNs based approach), a numerical study has been carried out.
followed by FEA and laboratory corroboration. To substantiate all the adopted method, the same numerical example has been considered which has been analyzed in the numerical formulation part. The detailed analysis of the problem in the forward manner has been earlier discussed by the authors earlier. The severities of cracks are found out by analyzing FEA model. 750 numbers of patterns are generated for the proposed analysis, where 650 patterns are used for training process while rests of these are used for testing procedures.

The L-M back propagation methodology has been applied to train the modified RNNs structure. The present methodology is carried out in a supervised way as an inverse problem. 900 numbers of iterations are carried out. The results (relative crack locations and severities) found out from each of the analogies (FEA, experiment and RNNs) are corroborated with the numerical values and represented in Tables 3 and 4. The percentages of error or deviation values are calculated with the exemplified numerical values. The percentage of error values of FEA and experiments with numerical values are about 1.9% & 2.8% for relative crack locations while those for relative crack depth are about 2.1% & 3.0% respectively.

A graphical representation between sum square error values and number of iteration for both relative crack locations and severities are shown in Figure 6. The results obtained from the modified RNNs analogy vary with approximately 3.5% with the numerical values for relative crack locations while the results for crack severities vary about 3.9% approximately with the numerical values which are considered as good convergent. In the earlier works, the authors have also applied the analogy of the modified ERNNs [12] and JRNNs [13] based approaches individually with L-M back propagation algorithm.

The authors have also compared the present results with their earlier works [12,13] and found that the present approach yields better results. It has been observed that the modified recurrent neural networks approach yield better accuracy as comparison to the individual ERNNs and JRNNs analogies. So the proposed method can be used efficiently to identify, locate and quantify cracks on structure subjected to travelling mass.

6 Conclusions

The present study has quietly elaborated a structural health monitoring problem in the domain of moving load dynamics problem using modified RNNs based approach. The developed RNNs based analogy has been carried out in

Table 2. Training Patterns for modified RNNs model.

| RD-1 | RD-2 | RD-3 | L (cm) | M (kg) | v (m/s) | n | η₁ | η₂ | η₃ |
|------|------|------|-------|--------|--------|---|----|----|----|
| 1.299 | 1.409 | 1.144 | 150 | 3 | 4.5 | 3 | 0.3214 | 0.5714 | 0.7628 |
| 1.147 | 1.273 | 1.386 | 140 | 1.5 | 5 | 3 | 0.1171 | 0.2285 | 0.3357 |
| 1.18 | 1.39 | 1 | 130 | 2 | 5.5 | 2 | 0.267 | 0.342 | 0.28 |
| 1.313 | 1 | 1 | 160 | 2.5 | 6 | 1 | 0 | 0.672 | 0.75 |
| 1 | 1 | 1 | 155 | 3.5 | 7 | 0 | 0 | 0 | 0 |

Table 3. Comparison of results for relative crack locations (η) with various methods.

| Numerical | FEA | Experiments | RNNs |
|-----------|-----|-------------|------|
| η₁ | η₂ | η₃ | η₁ | η₂ | η₃ | η₁ | η₂ | η₃ | η₁ | η₂ | η₃ |
| 0.358 | 0.503 | 0.634 | 0.3516 | 0.494 | 0.623 | 0.348 | 0.489 | 0.6175 | 0.345 | 0.485 | 0.612 |
| 0.331 | 0.545 | 0.703 | 0.3352 | 0.535 | 0.6904 | 0.3215 | 0.5305 | 0.6843 | 0.319 | 0.525 | 0.678 |
| Average percentage of deviation | 1.75 | 1.86 | 1.68 | 2.81 | 2.75 | 2.62 | 3.58 | 3.5 | 3.4 |
| Total percentage of deviation | 1.76 | 2.72 | 3.49 |

Table 4. Comparison of results for relative crack depth (α) with various methods.

| Numerical | FEA | Experiments | RNNs |
|-----------|-----|-------------|------|
| α₁ | α₂ | α₃ | α₁ | α₂ | α₃ | α₁ | α₂ | α₃ | α₁ | α₂ | α₃ |
| 0.13 | 0.28 | 0.41 | 0.127 | 0.274 | 0.401 | 0.1263 | 0.272 | 0.397 | 0.125 | 0.269 | 0.394 |
| 0.23 | 0.37 | 0.48 | 0.225 | 0.362 | 0.4708 | 0.223 | 0.358 | 0.465 | 0.220 | 0.3554 | 0.461 |
| Average percentage of deviation | 2.07 | 1.98 | 2.03 | 2.97 | 2.9 | 2.96 | 3.95 | 3.85 | 3.87 |
| Total percentage of deviation | 2.02 | 2.94 | 3.88 |
a supervised manner. The numerical, FEA and experimental studies are considered as the forward problem while the RNNs based approach as inverse or indirect approach in the present investigation. The proposed RNNs structure has been trained by implementing the mechanism of L-M back propagation algorithm. The existence, relative crack locations and depth of the simply supported structure under travelling mass are found out by both the forward and inverse approaches. The obtained results ($\alpha$ and $\eta$) from RNNs methodology are compared with the numerical problem values and found to be about 3.9% and 3.5% for relative crack depth and positions respectively which seems to be convergent. The proposed method can be used effectively for crack detection in structure under moving load.

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