Electroweak Baryogenesis: A Brief Review *

Mark Trodden†

Particle Astrophysics Theory Group
Department of Physics
Case Western Reserve University
10900 Euclid Avenue
Cleveland, OH 44106-7079, USA.

Abstract

A brief review of the fundamental ideas behind electroweak baryogenesis is presented. Since a successful implementation of these ideas requires an extension of the minimal standard model, I comment on the necessary physics and how experimental constraints make these scenarios testable at the LHC, and perhaps at existing colliders.

CWRU-P18-98

*Review Talk, to appear in the Proceedings of the XXXIIId Rencontres de Moriond, “Electroweak Interactions and Unified Theories”, March 14-21,1998, Éditions Frontières.

†trodden@theory1.phys.cwru.edu.
I. INTRODUCTION

We’ve heard a lot of interesting talks in this conference about electroweak physics at high energies in colliders. In this talk, I want to describe one way in which electroweak physics, at high temperatures instead of energies, can play an important role in another arena, that of early universe cosmology.

A clear observational fact about the universe is that it is baryon-antibaryon asymmetric. In fact, a combination of results, ranging from everyday experience to cosmic γ-ray abundances, have demonstrated that there is negligible primordial antimatter in our observable universe \[1\]. To obtain a quantitative measure of this asymmetry we look to the standard cosmological model. One of the major successes of cosmology is an accurate prediction of the abundances of all the light elements; a calculation which requires a single input parameter, the \textit{baryon to entropy ratio}

\[ η ≡ \frac{n_B}{s} = \frac{n_b - n_\bar{b}}{s}, \]  

(1.1)

where \( n_b \) is the number density of baryons, \( n_\bar{b} \) is that of antibaryons, and \( s \) denotes the entropy density. If one compares calculations of elemental abundances with observations, then there is agreement between these numbers if

\[ 1.5 \times 10^{-10} < \eta < 7 \times 10^{-10}. \]  

(1.2)

We could just accept this as an input parameter for the evolution of the universe. However, it is part of the philosophy of modern cosmology to seek an explanation for the required value of \( η \) using quantum field theories of elementary particles in the early universe. While a number of different scenarios for generating \( η \) have been suggested, I’m going to describe those which make use of anomalous physics at the electroweak scale. These scenarios are collectively referred to as \textit{electroweak baryogenesis}, and have the virtue of relying on physics that is testable at terrestrial colliders. Because of space constraints I’ll only be able to describe the basic picture, but I hope to be able to give a feel for the fundamental physics and
challenges involved. Also, it is impossible to correctly reference a short article on a huge subject such as this. I have therefore only used references where I feel they are crucial, and hope that my colleagues will understand. For more detailed accounts of the subject and more complete referencing of the material, I refer the reader to existing longer reviews [2].

II. THE SAKHAROV CRITERIA

If we’re going to use a particle physics model to generate the baryon asymmetry of the universe (BAU), what properties must the theory possess? This question was first addressed by Sakharov [3] in 1967, resulting in the following criteria

- Violation of the baryon number \(B\) symmetry.

- Violation of the discrete symmetries \(C\) (charge conjugation) and \(CP\) (the composition of parity and \(C\))

- A departure from thermal equilibrium.

Of course, the first of these is obvious - no \(B\) violation, no baryon production. To understand the second condition, note that, roughly speaking, if \(C\) and \(CP\) are conserved, the rate for any process which generates baryons is equal to that for the conjugate process, which produces antibaryons, so no net excess is generated on average. Finally, in thermal equilibrium the number density of a particle species is determined purely by its energy, and since the masses of particle and antiparticle are equal by the CPT theorem, the number density of baryons equals that of antibaryons.

While there exist scenarios of baryogenesis which employ putative physics at very high energies, the central point of electroweak baryogenesis is that all three Sakharov conditions are satisfied within the relatively well-understood Glashow-Salam-Weinberg theory, as I’ll explain.
A. Baryon Number Violation

In the standard electroweak theory baryon number is an exact global symmetry. However, as realized by ’t Hooft [4], baryon number is violated at the quantum level through nonperturbative processes. These effects are closely related to the nontrivial vacuum structure of the electroweak theory. To see this, note two facts about the electroweak theory. First, one may write a vectorlike current for baryons as

\[ j_B^\mu = \frac{1}{2} \bar{Q} \gamma^\mu Q, \]  

(2.1)

where \( Q \) represents quarks, and there is an implied sum over the color and flavor indices. Now, due to quantum effects, any axial current \( \bar{\psi} \gamma^\mu \gamma^5 \psi \) of a gauge coupled Dirac fermion \( \psi \), is anomalous [5]. This is relevant to baryon number since the electroweak fermions couple chirally to the gauge fields. If we write the baryon current as

\[ j_B^\mu = \frac{1}{4} \left[ \bar{Q} \gamma^\mu (1 - \gamma^5)Q + \bar{Q} \gamma^\mu (1 + \gamma^5)Q \right], \]  

(2.2)

only the axial part of this vector current is important when one calculates the divergence. This effect can be seen by calculating triangle graphs and leads to the following expressions for the divergences of the baryon number and lepton number currents;

\[ \partial_\mu j_B^\mu = \partial_\mu j_l^\mu = n_f \left( \frac{g^2}{32\pi^2} W_\mu^a W^a_{\mu\nu} - \frac{g'^2}{32\pi^2} F_\mu^\nu F^\mu\nu \right), \]  

(2.3)

where \( W_{\mu\nu} \) is the SU(2) field strength tensor and, for simplicity, I’ve ignored the U(1) interactions. Also, \( n_f \) is the number of families, and a tilde denotes the dual tensor.

Now, the vacua of the theory may be labelled by the Chern-Simons number, defined by

\[ N_{CS}(t) \equiv \frac{g^2}{32\pi^2} \int d^3x \epsilon^{ijk} \text{Tr} \left( A_i \partial_j A_k + \frac{2}{3} i g A_i A_j A_k \right). \]  

(2.4)

Although this number is not gauge-invariant, we shall only be interested in changes in it, which are gauge-invariant.

For simplicity, consider space to be a 3-sphere and consider the change in baryon number from time \( t = 0 \) to some arbitrary final time \( t = t_f \). For transitions between vacua, the change in baryon number may be written using (2.4) as
\[ \Delta B = \Delta N_{CS} \equiv n_f[N_{CS}(t_f) - N_{CS}(0)]. \tag{2.5} \]

So, the change in baryon number is associated with a change in the vacuum state of the system.

At zero temperature, baryon number violating events are exponentially suppressed. This is because there exists a potential barrier between vacua and anomalous processes are thus tunneling events. The relevant barrier height is set by the point of least energy on the barrier. This point is known as the sphaleron, and has energy \( E_{sph} \sim 10 \text{ TeV} \). However, at temperatures above or comparable to the critical temperature of the electroweak phase transition, vacuum transitions over the sphaleron may occur frequently due to thermal activation.

Detailed calculations of the baryon number violating rate in the broken [6] and unbroken [7] phases around the critical temperature, taking into account fluctuations around the sphaleron and other effects of nonzero temperature have been performed, yielding

\[
\Gamma(T) = \begin{cases} 
\mu \left( \frac{M_W}{\alpha_W T} \right)^3 M_W^4 \exp \left( -\frac{E_{sph}(T)}{T} \right) & 0 \ll T < T_c \\
\kappa \alpha_W (\alpha_W T)^4 & T > T_c
\end{cases},
\tag{2.6}
\]

where \( \mu \) is a dimensionless constant. Here, the temperature-dependent “sphaleron” energy \( E_{sph}(T) \) is defined through the finite temperature effective potential. The important point here is that \( \Gamma(T) \) is large for \( T > T_c \) and extremely small for \( T < T_c \).

### III. C AND CP VIOLATION

Fermions in the electroweak theory are chirally coupled to the gauge fields. In terms of the discrete symmetries of the theory, these chiral couplings result in the electroweak theory being maximally C-violating. However, the issue of CP-violation is more complex.

CP is known not to be an exact symmetry of the weak interactions, and is observed experimentally in the neutral Kaon system through \( K_0, \bar{K}_0 \) mixing. Although at present there is no completely satisfactory theoretical explanation of this, CP violation is a natural feature of the standard electroweak model. The Kobayashi-Maskawa (KM) quark mass mixing matrix contains a single independent phase, a nonzero value for which signals CP violation.
While this is encouraging for baryogenesis, it turns out that this particular source of CP violation is not strong enough. The relevant effects are parametrized by a dimensionless constant which is no larger than $10^{-20}$. This appears to be much too small to account for the observed BAU and so it is usual to turn to extensions of the minimal theory.

There are two principal ways of doing this

- The Two-Higgs Doublet Model. Here there are two scalars $\Phi_1$ and $\Phi_2$, and the scalar potential is replaced by the most general renormalizable two-Higgs potential. To make the CP-violation explicit, we write the Higgs fields in unitary gauge as

$$
\Phi_1 = (0, \varphi_1)^T, \quad \Phi_2 = (0, \varphi_2 e^{i\theta})^T
$$

where $\varphi_1, \varphi_2, \theta$ are real, and $\theta$ is the CP-odd phase.

Changes in $\theta$ are dependent on changes in the magnitude of the Higgs fields. In particular, if a point in space makes a transition from false electroweak vacuum to true then $\Delta \theta > 0$, and sphaleron processes result in the preferential production of baryons over antibaryons. For the opposite situation $\Delta \theta < 0$, and sphaleron processes generate an excess of antibaryons. The total change in the phase $\theta$ (from before the phase transition to $T = 0$) is the quantity that enters into estimates of the BAU.

- Higher Dimension Operators. If we view the model as an effective field theory, valid at energies below some mass scale $M$, one expects extra, nonrenormalizable operators, some of which will be CP odd. A particular dimension six example is

$$
O = \frac{b}{M^2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(F_{\mu\nu} \tilde{F}^{\mu\nu}),
$$

with $b$ a dimensionless constant.

Whatever its origin, the effect of CP violation on anomalous baryon number violating processes is to provide a fixed direction for the net change in baryon number.
IV. THE ELECTROWEAK PHASE TRANSITION

The question of the order of the electroweak phase transition is central to electroweak baryogenesis. Since the equilibrium description of particle phenomena is extremely accurate at electroweak temperatures, baryogenesis cannot occur at such low scales without the aid of phase transitions.

For a continuous transition, the associated departure from equilibrium is insufficient to lead to relevant baryon number production \[9\]. The order parameter for the electroweak phase transition is \(\phi\), the modulus of the Higgs field. For a first order transition the extremum at \(\phi = 0\) becomes separated from a second local minimum by an energy barrier. At the critical temperature \(T = T_c\) both phases are equally favored energetically and at later times the minimum at \(\phi \neq 0\) becomes the global minimum of the theory. Around \(T_c\) quantum tunneling occurs and nucleation of bubbles of the true vacuum in the sea of false begins. At a particular temperature below \(T_c\), bubbles just large enough to grow nucleate. These are termed critical bubbles, and they expand, eventually filling all of space and completing the transition. As the bubble walls pass each point in space, the order parameter changes rapidly, as do the other fields and this leads to a significant departure from thermal equilibrium. Thus, if the phase transition is strongly enough first order it is possible to satisfy the third Sakharov criterion in this way.

There is a further criterion to be satisfied. As the wall passes a point in space, the Higgs fields evolve rapidly and the Higgs VEV changes from \(\langle \phi \rangle = 0\) in the unbroken phase to

\[
\langle \phi \rangle = v(T_c)
\]  

(4.1)
in the broken phase. Here, \(v(T)\) is the value of the order parameter at the symmetry breaking global minimum of the finite temperature effective potential. Now, CP violation and the departure from equilibrium occur while the Higgs field is changing. Afterwards, the point is in the true vacuum, baryogenesis has ended, and baryon number violation is exponentially suppressed. Since baryogenesis is now over, it is imperative that baryon number violation be
negligible at this temperature in the broken phase, otherwise any baryonic excess generated will be equilibrated to zero. Such an effect is known as washout of the asymmetry and the criterion for this not to happen may be written as

\[
\frac{v(T_c)}{T_c} \geq 1.
\] (4.2)

Although there are a number of nontrivial steps that lead to this simple criterion, (4.2) is traditionally used to ensure that the baryon asymmetry survives after the wall has passed. It is necessary that this criterion be satisfied for any electroweak baryogenesis scenario to be successful.

In the minimal standard model, in which the Higgs mass is now constrained \[10\] to be \(m_H > 89.3\) GeV, it is clear from numerical simulations \[11\] that (4.2) is not satisfied. This is therefore a second reason to turn to extensions of the minimal model.

V. MECHANISMS

Historically, the ways in which baryons may be produced as a bubble wall, or phase boundary, sweeps through space, have been separated into two categories.

1. local baryogenesis: baryons are produced when the baryon number violating processes and CP violating processes occur together near the bubble walls.

2. nonlocal baryogenesis: particles undergo CP violating interactions with the bubble wall and carry an asymmetry in a quantum number other than baryon number into the unbroken phase region away from the wall. Baryons are then produced as baryon number violating processes convert the existing asymmetry into one in baryon number.

In general, both local and nonlocal baryogenesis will occur, and the BAU will be the sum of that generated by the two processes. I’ll only have time to discuss one mechanism here and I’ll choose nonlocal baryogenesis and refer anyone interested to a recent discussion of local baryogenesis \[12\].
Nonlocal baryogenesis typically involves the interaction of the bubble wall with the various fermionic species in the unbroken phase. The main picture is that as a result of CP violation in the bubble wall, particles with opposite chirality interact differently with the wall, resulting in a net injected chiral flux. This flux thermalizes and diffuses into the unbroken phase where it is converted to baryons. In what follows I’ll just outline a simple nonlocal calculation [13].

First assume that the Higgs fields change in a narrow region at the face of the bubble wall. We call this the thin wall regime, and it is valid if the mean free path \( l \) of the fermions being considered is much greater than the thickness \( \delta \) of the wall. I’ll also ignore interactions in the broken phase, assuming here that baryon number violation turns off instantly after the wall passes.

The equation for the generation of baryon number may be written as

\[
\frac{dn_B}{dt} = -\frac{n_f \Gamma(T)}{2T} \sum_i \mu_i ,
\]  

(5.1)

where the rate per unit volume for electroweak sphaleron transitions is given by (2.6) for \( T > T_c \). Here, \( n_f \) is again the number of families and \( \mu_i \) is the chemical potential for left handed particles of species \( i \). The crucial question in applying this equation is an accurate evaluation of the chemical potentials that bias baryon number production. To be concrete, I shall focus on leptons [13]. If there is local thermal equilibrium in front of the bubble walls - as I am assuming - then the chemical potentials \( \mu_i \) of particle species \( i \) are related to their number densities \( n_i \) by

\[
n_i = \frac{T^2}{12} k_i \mu_i ,
\]  

(5.2)

where \( k_i \) is a statistical factor which equals 1 for fermions and 2 for bosons.

The source term in the diffusion equation is the flux \( J_0 \) resulting from the asymmetric reflection and transmission of left and right handed leptons off the bubble wall. For left-handed leptons, the relevant flux is

\[
J_0 \simeq \frac{v m_l^2 m_H \Delta \theta_{CP}}{4 \pi^2} .
\]  

(5.3)
where $v$ is the speed of the wall, $m_l$ is the lepton mass and I’ve written $\Delta \theta_{CP}$ to parameterize the CP-violation. Now, we feed $J_0$ into the diffusion equation for a single particle species

$$D_L L_L'' + v L_L' = \xi_L J_0 \delta(z), \quad (5.4)$$

where $D_L$ is the diffusion constant for leptons, $\xi^L$ is the persistence length of the current, and a prime denotes the spatial derivative in the direction $z$ perpendicular to the wall. This equation may be solved to give

$$L_L(z) = \begin{cases} J_0 \frac{\xi^L}{D_L} e^{-\lambda_D z} & z > 0 \\ 0 & z < 0 \end{cases}, \quad (5.5)$$

with the diffusion root

$$\lambda_D = \frac{v}{D_L}. \quad (5.6)$$

In the massless approximation the chemical potential $\mu_L$ can be related to $L_L$ by

$$\mu_L = \frac{6}{T^2} L_L \quad (5.7)$$

Inserting the sphaleron rate and the above results for the chemical potential $\mu$ into (5.1), the final baryon to entropy ratio becomes

$$\frac{n_b}{s} = \frac{1}{4\pi^2} \kappa A \nabla W (g^*)^{-1} \Delta \theta_{CP} \left(\frac{m_H}{T}\right)^2 \frac{m_H}{\lambda_D} \frac{\xi^L}{D_L}, \quad (5.8)$$

In some models this can be large enough to explain the BAU. However, the calculation must be carried through in detail separately for each model.

VI. EXTENSIONS OF THE MSM

As I mentioned earlier, the relevant extensions can be described roughly as

• Extra light scalars coupled to the Higgs particle, that lead to a more strongly first order phase transition and hence a strong departure from equilibrium and negligible washout.
• Two Higgs doublet models to get more CP-violation and enhance the phase transition.

• Higher dimension operators to provide extra CP-violation.

It is important to note that in electroweak baryogenesis, we may use particle physics to constrain these extensions. For example, the operator $\mathcal{O}$ induces electric dipole moments for the electron and the neutron, and the strongest experimental constraint on the size of such an operator comes from the fact that such dipole moments have not been observed. Working to lowest order (one-loop) \[^{[12]}\]

$$\frac{d_e}{e} = \frac{m_e \sin^2(\theta_W)}{8\pi^2} \frac{b}{M^2} \ln \left( \frac{M^2 + m_H^2}{m_H^2} \right).$$ \hspace{1cm} (6.1)

Then, using the experimental limit \[^{[14]}\] yields the bound

$$\frac{b}{M^2} \ln \left( \frac{M^2 + m_H^2}{m_H^2} \right) < \frac{1}{(3 \text{ TeV})^2}.$$ \hspace{1cm} (6.2)

Therefore, any baryogenesis scenario which relies on CP violation introduced via the operator $\mathcal{O}$ must respect the bound (6.2).

VII. THE MSSM

Here I just want to comment on how the minimal supersymmetric standard model is at present a viable and testable candidate theory for electroweak baryogenesis.

In the MSSM there are two Higgs fields. At one loop, a CP-violating interaction between these fields is induced through supersymmetry breaking. Alternatively, there also exists extra CP-violation through CKM-like effects in the chargino mixing matrix. Thus, there seems to be sufficient CP violation for baryogenesis to succeed.

Now, the two Higgs fields combine to give one light scalar Higgs $h$ such that $m_h < 125\text{GeV}$. In addition, there are also light stops (the superpartners of the top quark) in the theory. These light scalar particles can lead to a strongly first order phase transition if the scalars have masses in the correct region of parameter space. A detailed two loop calculation \[^{[15]}\], and lattice results \[^{[16]}\] indicate that the allowed region is given by
\begin{align}
75\text{GeV} \leq m_h \leq 105\text{GeV} \\
100\text{GeV} \leq m_{\tilde{t}} \leq m_t,
\end{align}

for \( \tan \beta \equiv \langle \Phi_2 \rangle / \langle \Phi_1 \rangle \sim 2 \). In the next few years, experiments at LEP should probe this range of Higgs masses and we should know if the MSSM is a good candidate for electroweak baryogenesis.

**VIII. SUMMARY**

I hope I’ve described how electroweak baryogenesis combines beautiful nonperturbative field theory with cosmology to possibly explain the BAU. By necessity, the account I’ve given is simplified and in reality a great deal of work has gone into understanding the relevant physics.

There exist several viable scenarios at present and, interestingly, there exists a window of parameter space in the MSSM that remains open, but that will be probed soon.

Ultimately, much of the physics of electroweak baryogenesis is testable in colliders, and it is possible that, with the next generation of colliders, we will understand where the matter we are made of came from.

**Acknowledgements**

I would like to thank the organizing committee for inviting me to speak, and for working so hard to make the conference run smoothly. This work was supported by the U.S. department of Energy, the National Science Foundation (NSF) and by funds provided by Case Western Reserve University. I am also grateful for a grant from the European Union Training and Mobility of Researchers Programme, and for additional help from the NSF.
REFERENCES

[1] G. Steigman, Ann. Rev. Astron. Astrophys. 14, 336 (1976); A.G. Cohen, A. De Rujula and S.L. Glashow, astro-ph/9707087 (1997).

[2] M. Trodden, “Electroweak Baryogenesis”, hep-ph/9803479, Submitted to Reviews of Modern Physics, (1998); V.A. Rubakov and M. E. Shaposhnikov, Phys. Usp. 39, 461 (1996); A.G. Cohen, D.B. Kaplan and A.E. Nelson, Ann. Rev. Nucl. Part. Sci. 43, 27 (1993); N. Turok, “Electroweak Baryogenesis”, in Perspectives on Higgs Physics, edited by G.L. Kane, p. 300 (1992).

[3] A.D. Sakharov, Zh. Eksp. Teor. Fiz. Pis’ma Red. 5, 32 (1967) [JETP Lett. 5, 24 (1967)].

[4] G. ’t Hooft, Phys. Rev. Lett. 37, 8 (1976).

[5] S. Adler, Phys. Rev. 177, 2426 (1969); J.S. Bell and R. Jackiw, Nuovo Cimento 51, 47 (1969).

[6] L. Carson and L. McLerran, Phys. Rev. D41, 647 (1990); L. Carson, X. Li, L. McLerran and R. Wang, Phys. Rev. D42, 2127 (1990).

[7] P. Arnold, D. T. Son and L. G. Yaffe, Phys. Rev. D55, 6264 (1997); P. Huet and D. T. Son, Phys. Lett. B393, 94 (1997); D. Son, “Effective Nonperturbative Real Time Dynamics of Soft Modes in Hot Gauge Theories”, hep-ph/9707351 (1997); G.D. Moore, C. Hu and B. Mueller, “Chern-Simons Number Diffusion with Hard Thermal Loops”, hep-ph/9710436 (1997).

[8] M. Shaposhnikov, Nucl. Phys. B299, 797 (1988); M. Dine, P. Huet, R. Singleton Jr. and L. Susskind, Phys. Lett. B257, 351 (1991; M. Dine, P. Huet and R. Singleton Jr., Nucl. Phys. B375, 625 (1992).

[9] V.A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B155, 36 (1985).

[10] S. de Jong, “Higgs Searches at LEP”, these proceedings.

[11] K. Kajantie, M. Laine, K. Rummukainen and M. Shaposhninkov, Phys. Rev. Lett. 77, 2887 (1996).

[12] A. Lue, K. Rajagopal and M. Trodden, Phys. Rev. D55, 1250 (1997).

[13] M. Joyce, T. Prokopec and N. Turok, Phys. Lett. B339, 312 (1994).
[14] E.D. Commins, S. B. Ross, D. DeMille and B. C. Regan, Phys. Rev. A50, 2960 (1994).

[15] M. Carena, M. Quiros and C.E.M. Wagner, 1997, “Electroweak Baryogenesis and Higgs and Stop Serches at LEP and the Tevatron,” hep-ph/9710401.

[16] M. Laine and K. Rummukainen, “A Strong Electroweak Phase Transition up to $m_H \sim 105$ GeV”, hep-ph/9804253.