Aspects of $\chi$ and $\psi$ Production in Polarized Proton-Proton Collisions

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Abstract

Several topics of relevance to low transverse momentum $\psi$ and $\chi_{1,2}(c\bar{c})$ production in polarized proton-proton collisions are discussed. The leading $\mathcal{O}(\alpha_s^3)$ contributions to the low $p_T$ $\chi_1$ production cross-sections via $gg$, $qg$, and $qq$ initial states are calculated as well as the corresponding spin-spin asymmetries. We find that $\chi_1$ production increases relative to direct $\psi$ and $\chi_2$ production, providing up to 25% of the observable $e^+e^-$ pairs arising from $\psi$ decays in $pp$ collisions at $\sqrt{s} = 500$ GeV. The spin-dependence of $\chi_1$ production, however, is much smaller than for either direct $\psi$ or $\chi_2$ production and so will likely be far less useful than either process in probing the polarized gluon structure function of the proton. A subset of the $\mathcal{O}(\alpha_s^3)$ radiative corrections to $\chi_2$ production involving initial state quarks are also performed and compared to leading order $gg \to \chi_2$ predictions.
1 Introduction

Even before the recent approval [1] of a program of polarized proton-proton collisions at collider energies at RHIC [2, 3, 4], there had been a rapid increase in the literature discussing the prospects for probing the spin-dependence of many QCD processes and their sensitivity to various polarized parton distributions in such high energy polarized hadron collisions.

Familiar processes such as jet production [5, 6], direct photon production [5, 7], and weak boson production [8, 9] have all been examined in this context in some detail. For a few processes, such as polarized Drell-Yan production [10] and direct photon production [11], the full set of radiative corrections have even been computed. In these cases, the next-to-leading (NLO) order corrections retain almost all of the features of the leading-order (LO) calculations, including their sensitivity to the sea quark and gluon polarizations respectively. On the other hand, heavy quark production has recently been studied [12] and it seems likely that NLO corrections (especially at large $p_T$) will change the spin-structure of $b$-quark production rather dramatically, pointing out the importance of the comprehensive study of NLO corrections to spin-dependent processes which might be measured with the proposed RHIC detectors.

Quarkonium (especially $J/\psi$) production, both at low transverse momentum [13, 14, 15, 16] and at high $p_T$ [17, 18], has also been examined for its possible spin-dependence and sensitivity to the polarized gluon content of the proton. One intriguing suggestion, due to Cortes and Pire [13] (hereafter CP), is to consider low $p_T \chi_2(\sigma\bar{\sigma})$ production where the dominant lowest-order subprocess would be $gg \to \chi_2$. To this order, the observable polarization asymmetry,

$$A_{LL} = \frac{(\sigma(++) - \sigma(+-))}{(\sigma(++) + \sigma(+-))} ,$$

would
(where $\pm$ refers to the proton helicity) in such quantities as $d\sigma/dy(y=0)$ (or $d\sigma/dx_F$) is directly proportional to $(\Delta G(x)/G(x))^2$ times a partonic level spin-spin asymmetry $\hat{a}_{LL}$. Here $\Delta G(x) \equiv G_+(x) - G_-(x)$ is the longitudinally polarized gluon density, where $+/-$ refers to a gluon with its helicity in the same/opposite direction as that of the proton. The partonic level asymmetries for $\chi_2/\chi_0$ production have been calculated in the context of potential models [16] and are maximally large, $\hat{a}_{LL} = -1/ + 1$ for $\chi_2/\chi_0$ respectively, as expected from simple angular momentum conservation arguments. The study of the angular distribution of the $\chi_2 \to \psi \gamma$ decay (which also provides the clean signal) can simultaneously measure the value of $\hat{a}_{LL}$ [13] as a further check. Similar experiments using $\chi_0$ production are hampered by the very small radiative branching ratio to $\psi \gamma$ final states. The spin-dependence of the other dominant $\psi$ production process at low $p_T$, the so-called ‘color-bleaching’ mechanism, $gg \to g\psi$, has also been examined [16].

In the context of the RHIC spin physics program, the PHENIX detector [2], with its excellent electron and photon detection, will likely be able to resolve $\psi$ and $\chi_{1,2}$ states very effectively so that further study of $\psi$ and $\chi_{1,2}$ production mechanisms at low $p_T$ in proton-proton collisions (polarized or not) is quite timely. (We note that the CDF Collaboration has presented promising preliminary results on such an analysis [19], demonstrating the ability to separate (at some level) direct $\psi$ from radiative $\chi_{1,2}$ decay in a high energy collider environment at large transverse momentum.)

Questions can easily arise, however, concerning the reliability of the theoretical understanding of $\chi(Q\overline{Q})$ production mechanisms in hadronic collisions. Issues such as the applicability of potential model analyses of quarkonium production in hadronic collisions at low values of $Q^2 = M^2_\chi$ (especially for charmonium) and, as mentioned
above, on the role of higher order QCD corrections are obvious ones. Both low transverse momentum inclusive $\psi$ and $\Upsilon$ production \cite{20, 21} and high $p_T$ $\psi$ production \cite{22, 23, 24, 25} have been well described by such models which do, however, require relatively large $K$ factors, namely $K \approx 2$, not unlike standard lowest-order jet, direct photon and heavy quark analyses. Recent calculations of radiative corrections to low $p_T$ $^1S_0(Q\overline{Q})$ (i.e., $\eta_Q$) production in hadron collisions \cite{26}, albeit for toponium production, confirm the presence of relatively large $K$-factors. These corrections would presumably be even larger for $\eta_c$ states because of the larger value of $\alpha_S$ at the lower charmonium mass scale. The $O(\alpha_S^3)$ corrections to the inverse process, i.e. $\chi_2 \rightarrow gg$ have been known for some time \cite{27} and were found to be quite large also indicating the possible importance of higher-order radiative corrections in P-wave quarkonium processes. In addition, data on the relative yields of $\psi$ and $\chi_{1,2}$ states in $pp$ collisions also seem to be compatible with such potential models as well \cite{28}.

The presence of large multiplicative $K$-factors is, by itself, not a strong argument against the use of the CP method as their analysis relies simply on the $2 \rightarrow 1$ kinematic structure of the $gg \rightarrow \chi_2$ subprocess. Purely virtual corrections to this process (retaining the $2 \rightarrow 1$ kinematics) are unlikely to change the LO predictions for the spin-asymmetry which is dictated by helicity conservation. The corrections arising from the $2 \rightarrow 2$, $gg \rightarrow \chi_2 g$ process at the same order (which are required to cancel the infrared divergences found in the virtual diagrams) can, however, change the kinematic structure of $\chi_2$ production and hence the underlying spin-spin asymmetry. Perhaps just as importantly, the NLO $O(\alpha_S^3)$ processes $qg \rightarrow q\chi_2$ and $g\overline{q} \rightarrow g\chi_2$, present at the same order, introduce a new dependence on the quark and antiquark distributions so that even if the contribution of quark/antiquark initiated processes is relatively small in the
unpolarized cross-section, it could make a significant impact on the spin-dependence depending on the relative sizes of $\Delta G(x)$ and $\Delta q(x)$.

A related process which appears for the first time at $\mathcal{O}(\alpha^3_S)$ is $\chi_1$ production. Because the Yang-Landau-Pomeranchuk theorem \cite{29} forbids $gg \to \chi_1$ at lowest (or any) order, the production of $\chi_1$ states begins at this order via the processes $gg \to g\chi_1$, $gg \to q\chi_1$, and $q\bar{q} \to q\chi_1$. While suppressed relative to $\chi_2$ production by an additional factor of $\alpha_S$, the relatively large radiative branching ratio to observable $\psi$ states ($Br(\chi_1 \to \psi\gamma) = 0.27$ compared to $Br(\chi_2 \to \psi\gamma) = 0.14$) partly compensates so we expect $\chi_1$ production to be not unreasonably small.

Because direct $\psi$ production (via $gg \to g\psi$) and $\chi_1$ production occur at the same order in $\alpha_S$, the ratio of their production cross-sections may well be less sensitive to assumptions about the momentum scale used in the perturbative calculations and so could constitute a further test of a potential model description of $\psi$ and $\chi$ state production as well as allowing for an improved confrontation with data on $\psi/\chi_1/\chi_2$ production ratios in hadroproduction experiments. The variable center-of-mass energy available at RHIC ($\sqrt{s} = 50 - 500 GeV$) will be quite useful in probing the energy dependence of these ratios as well in probing the $x$-dependence of $\Delta G(x)$.

In this work, we report on a set of NLO (i.e. $\mathcal{O}(\alpha^3_S)$) calculations of relevance to these (and related) questions. We calculate a subset of the spin-independent radiative corrections to $\chi_2$ production and include related results for $\chi_0$ production although the small radiative decay branching ratio to observable $\psi$ states make their production much less useful. Specifically we calculate the renormalized cross-sections for the subprocesses $qg \to q\chi_{0,2}$ and $q\bar{q} \to g\chi_{0,2}$ to assess the relative importance of initial states containing quarks and find that their contribution is likely always smaller than 15%
of the total cross-section. This implies that the dominant processes for $\chi_2$ production will still be from $gg$ initial states although a full set of NLO corrections will likely be necessary for a confident extraction of the polarized gluon distributions. We stress that we have not performed a complete set of radiative corrections to $\chi_2$ production but we do feel that our partial results will be useful in assessing the observability of various $\chi$ states and the likely spin-dependence of their production for the RHIC spin program.

Perhaps more importantly, we also present all of the $O(\alpha_s^3)$ contributions to $\chi_1$ production, including their spin-dependence and find that $\chi_1$ production increases in importance relative to direct $\psi$ and $\chi_2$ production, providing up to 25% of the lepton pair yield from charmonium states at the highest RHIC energies. The spin-dependence of $\chi_1$ production, however, is much weaker than for either direct $\psi$ and $\chi_2$ production and may not, therefore, contribute much to measurements of $\Delta G(x)$.

In the next section (2), we review the simplest $\psi$ and $\chi$ production mechanisms and describe the calculations of the partonic level cross-sections for the $\chi_{1,2}$ processes we consider while in Sec. 3 we present numerical results for $pp$ collisions in the energy range accessible to RHIC (namely 50 – 500 GeV). Finally, in Sec. 4, we discuss some aspects of the likely spin-dependence of these new production mechanisms and make some final comments.

2 Partonic Level Cross-sections

For reference, we begin by recalling the $O(\alpha_s^2)$ $gg \rightarrow \chi_{0,2}$ cross-sections which form the basis for the leading order description of $\chi$ production. They are well-known [30] and
are given by
\[ \hat{\sigma}(gg \to \chi_0) = \frac{12\pi^2\alpha_s^2|R_p'(0)|^2}{M^7}\delta(1-z) \] (2)
\[ \hat{\sigma}(gg \to \chi_2) = \frac{16\pi^2\alpha_s^2|R_p'(0)|^2}{M^7}\delta(1-z) \] (3)

where \( z \equiv M^2/\hat{s} \) and \( R_p'(0) \) is the derivative of P-state radial wavefunction at the origin. For completeness, we also find it useful to quote the result for \( gg \to g\psi \), namely
\[ \hat{\sigma}(gg \to g\psi) = \frac{5\pi\alpha_s^3|R_s(0)|^2}{9M^5}I(z) \] (4)
where
\[ I(z) = 2z^2 \left[ \frac{1 + z}{1 - z} + \frac{2z \ln(z)}{(1 - z)^2} + \frac{1 - z}{(1 + z)^2} - \frac{2z^2 \ln(z)}{(1 + z)^3} \right] \] (5)

where \( R_s(0) \) is the S-state radial wavefunction at the origin. (We note that, despite appearances, Eqn. 5 is finite as \( z \to 1 \).)

The first of the next-to-leading order corrections we consider are those for \( q\bar{q} \to g\chi_J \) for \( J = 0, 1, 2 \). The matrix elements for these processes are readily available \([30]\) and can be integrated (over \( d\hat{s} \)) to yield the finite results
\[ \hat{\sigma}(q\bar{q} \to g\chi_0) = \frac{128\pi\alpha_s^3|R_p'(0)|^2}{81M^7} \frac{z^2(1 - 3z)^2}{(1 - z)} \] (6)
\[ \hat{\sigma}(q\bar{q} \to g\chi_1) = \frac{256\pi\alpha_s^3|R_p'(0)|^2}{27M^7} \frac{z^2(1 + z)}{(1 - z)} \] (7)
\[ \hat{\sigma}(q\bar{q} \to g\chi_2) = \frac{256\pi\alpha_s^3|R_p'(0)|^2}{81M^7} \frac{z^2(1 + 3z + 6z^2)}{(1 - z)}. \] (8)

The factors of \((1-z)^{-1}\) are not artifacts of any renormalization but are symptomatic of P-wave bound state divergences which have been noticed previously \([20]\) and which can be regulated by restricting the region of integration to \( \hat{s} \geq (M + \Delta)^2 \) where \( \Delta \) is a typical binding energy, \( \Delta \approx 0.3\, GeV \). (Similar logarithmic divergences appear in the
production of $D$-states \cite{31} and in the multi-parton decays of $P$- and $D$-states \cite{32,33} as well.) We also note that the same kinematic factors appear here as in the related processes $Z^0 \to \chi_J + \gamma$ \cite{34}. New techniques for handling the related logarithms of binding energy which appear in $P$-wave decays in a rigorous way have been introduced \cite{35} and could eventually prove useful in this context as well.

The factor of $(1 - 3z)^2$ in Eqn. 6 implies a vanishing cross-section at some physical center-of-mass energy which is not obviously related to any symmetry. (Unlike radiation zeros \cite{36}, this zero appears in the cross-section when integrated over angles. As we will see below, the zero may persist beyond tree level, also unlike the case of radiation zeros.) For future reference, we note that in each of these cases, helicity conservation is sufficient to determine that the partonic level asymmetries are $\hat{a}_{LL} = -1$.

The next subprocesses we consider are $q + g \to q + \chi_J$. The cross-sections (in four dimensions) for these processes have also been calculated \cite{30} but when one attempts to integrate them over angles one encounters infinities, at least in the case of $\chi_0$ and $\chi_2$, which must be regulated. The $\chi_1$ case is more straightforward and we treat it first. Because of Yang’s theorem \cite{29}, when the $t$-channel gluon ($g^*$) becomes soft the amplitude describing $\chi_1 \leftrightarrow gg^*$ must vanish so that there is no divergence in the phase space integral for this case. The (already) finite result is

$$\hat{\sigma}(qg \to q\chi_1) = \frac{16\pi\alpha_s^3|R_P'(0)|^2}{9M^7}[(1 - z)(5 - 4z - 4z^2) - 3z^2 \ln(z)]. \quad (9)$$

For the $\chi_0$ and $\chi_2$ cases we must regularize the infinities encountered in the angular integration and for this we use familiar dimensional regularization techniques as in Ref. \cite{37}. (A complete set of calculations for the radiative corrections to $gg \to {}^1S_0(Q\overline{Q})$ have also been carried out using similar techniques \cite{26}.) The matrix elements for $qg \to$
\( q\chi_{0,2} \), summed/averaged over all final/initial polarizations, must then be calculated in \( N = 4 - 2\epsilon \) dimensions. After a long calculation using FORM [38], we find for the \( \chi_2 \) case

\[
\sum |M|^2 = \frac{64\pi^2\alpha_s^3|R'_p(0)|^2}{9M^3} \cdot \frac{F_4^{(2)}(s, t, u) + \epsilon F_\epsilon^{(2)}(s, t, u)}{(-t)(t - M^2)^4} \tag{10}
\]

where

\[
F_4^{(2)}(s, t, u) = (t - M^2)^2(t^2 + 6M^4) - 2us(t^2 - 6M^2(t - M^2)) \tag{11}
\]

and

\[
F_\epsilon^{(2)}(s, t, u) = (t - M^2)[t^2(M^2 - t) + 12(M^2(M^2 - t) - us)] \tag{12}
\]

while for \( \chi_0 \) we have

\[
\sum |M|^2 = \frac{32\pi^2\alpha_s^3|R'_p(0)|^2}{9M^3} \cdot \frac{F_4^{(0)}(s, t, u) - \epsilon F_\epsilon^{(0)}(s, t, u)}{(-t)(t - M^2)^4} \tag{13}
\]

where

\[
F_4^{(0)}(s, t, u) = (t - 3M^2)^2(s^2 + u^2) \tag{14}
\]

and

\[
F_\epsilon^{(0)}(s, t, u) = (t - 3M^2)^2(t - M^2)^2. \tag{15}
\]

To be consistent with more recent treatments of radiative corrections we choose to average over the initial state spins by using 2\((1 - \epsilon)\) degrees of freedom for the gluon which implies an additional factor of \((1 - \epsilon)^{-1}\) in Eqns. 10 and 13. (This choice has been discussed by Ellis and Sexton [39] and is used in Ref. [26].) Furthermore, we choose to work in the \( \overline{MS} \) scheme in which case the appropriate phase space factor is [40]

\[
PS = \frac{1}{8\pi} \left( \frac{1}{M^2} \right)^{\epsilon} z^{\epsilon}(1 - z)^{1-2\epsilon} \int_0^1 dy [y(1 - y)]^{-\epsilon} \tag{16}
\]

8
where the subprocess invariants are

\[ \hat{s} = \frac{M^2}{z}, \quad \hat{t} = -\frac{M^2}{z}(1-z)(1-y), \quad \hat{u} = -\frac{M^2}{z}(1-z)y. \] (17)

When one performs the angular integrals, the divergent $1/\epsilon$ term is proportional to the appropriate Altarelli-Parisi splitting function ($P_{gq}(z)$ in this case) and is absorbed into the scale-dependent parton distributions leaving a finite result. Then using the natural choice of scale $\mu = M$ we find the cross-sections

\[ \hat{\sigma}(qg \to q\chi_2) = \frac{16\pi\alpha_s^3|R'_p(0)|^2}{27M^7}G_2(z) \] (18)

where

\[ G_2(z) = 36(2 - 2z + z^2)\ln(1-z) - 3z^2\ln(z) - 53 + 69z - 18z^2 + 20z^3 \] (19)

and

\[ \hat{\sigma}(qg \to q\chi_0) = \frac{8\pi\alpha_s^3|R'_p(0)|^2}{27M^7}G_0(z) \] (20)

where

\[ G_0(z) = 54(2 - 2z + z^2)\ln(1-z) - 3z^2\ln(z) + 4(-35 + 42z - 9z^2 + 2z^3). \] (21)

We first note that, after renormalization, these contributions to the $\chi_2$ and $\chi_0$ cross-sections are, in fact, negative which is quite similar to the case of Drell-Yan production \cite{37} where the $gg \to q\gamma^*$ correction to the tree-level process $q\bar{q} \to \gamma^*$ is also negative. The contribution of the renormalized $gg \to q^1S_0(Q\bar{Q})$ cross-section to $\eta_Q$ production calculated in Ref. \cite{26} is similarly negative. We recall that the already finite $\chi_1$ contribution is positive.
As an aside, we note that the \( N = 4 - 2\epsilon \) matrix-element squared for \( q\bar{q} \to g\chi_0 \) can be obtained from Eqns. 13–15 by crossing and one can see that the zero in the matrix element at \( z = 1/3 \) (i.e. at \( \hat{s} = 3M^2 \)) in \( \chi_0 \) production is seemingly still present beyond tree level. We have no explanation for this fact.

The final case we then consider are the diagrams leading to \( gg \to g\chi_J \). For the case of \( \chi_{0,2} \) these diagrams must be combined with the virtual corrections to \( gg \to \chi_{0,2} \) to obtain a finite result and we do not attempt a complete analysis of those diagrams here. For the \( \chi_1 \) case, as there is no \( \mathcal{O}(\alpha_s^2) \) contribution, the \( gg \to g\chi_1 \) diagram gives the first non-vanishing contribution and it is well-behaved when integrated over angles.

The matrix element squared for \( gg \to g\chi_1 \) has been calculated in Ref. [10] and the total cross-section can be obtained by directly integrating the four-dimensional results to find

\[
\hat{\sigma}(gg \to g\chi_1) = \frac{4\pi\alpha_s^3|R'_P(0)|^2}{M^7} \frac{[(1 - z^2)H_1(z) + 12z^2\ln(z)H'_1(z)]}{(1 + z)^5(1 - z)^4}
\]

where

\[
H_1(z) = z^9 + 39z^8 + 145z^7 + 251z^6 + 119z^5 - 153z^4 - 17z^3 - 147z^2 - 8z + 10
\]

\[
H'_1(z) = z^8 + 9z^7 + 26z^6 + 28z^5 + 17z^4 + 7z^3 - 40z^2 - 4z - 4.
\]

Because there is an s-channel contribution to this processes, one obtains the bound-state divergences (the \( 1/(1 - z)^4 \) factor) which are also present in the \( q\bar{q} \to g\chi_J \) cases considered earlier and which are handled in the same way.
3 Numerical Results

We can now use these cross-sections to evaluate the contributions of quark (and anti-quark) initiated processes compared to the tree-level gluon-gluon fusion mechanism for $\chi_{0,2}$ states and all the $O(\alpha_s^3)$ contributions for $\chi_1$ production. In all our calculations we use $\alpha_s(Q^2 = M_\chi^2) = 0.26$ (corresponding to a leading-order $\Lambda = 200\,MeV$) and the wavefunction values $|R_S(0)|^2 = 0.7\,GeV^3$ and $|R'_P(0)|^2/M_\chi^2 = 0.006\,GeV^3$ [23]. We use a recently updated LO parameterization of parton distributions [41] evaluated at $Q^2 = M_\chi^2$ and the P-wave cutoff parameter $\Delta = 0.3\,GeV$. The effects of changing $\Delta$ by $\pm0.1\,GeV$ is at most a few percent and so is not a major source of theoretical uncertainty. A dependence on $\Delta$ will also arise in the renormalized $gg \rightarrow g\chi_{0,2}$ cross-sections and we expect a similar lack of sensitivity there as well.

In Fig. 1a, we plot the contributions to $\chi_2$ production in $pp$ collisions corresponding to the LO $gg \rightarrow \chi_2$ and NLO $qg \rightarrow q\chi_2$ and $q\bar{q} \rightarrow g\chi_2$ subprocesses. Since the contribution from the renormalized $qg$ initial state is negative, its absolute value is plotted. One can see that the $q\bar{q}$ processes are nowhere important in the RHIC energy range while the $qg$ initiated events comprise approximately 15% (30%) of the total (Born) level $\chi_2$ cross-section (assuming a constant $K = 2$ factor) which is not an overwhelmingly large effect. The contribution from $qg$ and $q\bar{q}$ initial states is similar to that found for $\eta_Q$ production found in Ref. [26] although the contributions from $qg$ states is somewhat larger. This implies that the total cross-section is still dominated by gluon fusion and so can still provide information on the polarized gluon distribution. A reliable extraction of $\Delta G(x)$, however, may well require, however, a full set of spin-dependent NLO corrections.
The various (already finite and positive) contributions to $\chi_1$ production from Eqns. 7, 9, and 22, are evaluated and shown in Fig. 1b. These results can then be combined with the direct $\psi$ production mechanism to provide estimates of the yield of $e^+e^-$ pairs from $\psi$, $\chi_2$, and $\chi_1$ states respectively once radiative branching ratios for the $\chi_{1,2}$ states are included in Fig. 2. We see that the relative amounts of $\psi$ and $\chi_2$ contributions to the observable lepton pair cross-section stays roughly constant over the RHIC range while the $\chi_1$ contributions increase to a rather substantial fraction at the highest RHIC energies. This fact offers yet another motivation for the usefulness of the high resolution electron and photon detection possible with the PHENIX detector at RHIC. We recall that an empirical K-factor of roughly 2 is required for the combined $\chi_2$ and $\psi$ contributions to the observable $\psi$ production data so that the prediction for $\chi_1$ contribution (as well as the other two) might well be underestimated here by a factor of 2.

4 Spin-spin Asymmetries

One of the strongest motivations for considering quarkonium production at RHIC is the possibility of measuring the spin-dependence of the various production mechanisms, part of which includes their possible sensitivity to the longitudinally polarized gluon distribution. A full treatment of the spin-dependent radiative corrections to $\chi_2$ production would be necessary to fully assess the extent to which the suggestion of Cortes and Pire really provides a truly direct measurement of $\Delta G$ but, barring that, some useful results can be already be gleaned from our partial study.

Since for the $\chi_1$ case, all of the individual cross-sections were finite at lowest order, we can make direct use of the individual helicity amplitudes for the contributing processes to derive the partonic-level spin-spin asymmetries for all of the $O(\alpha_s^3)$ processes giving rise to $\chi_1$ production.
For the case of $qg \rightarrow q\chi_1$, we can integrate the
individual helicity amplitudes found in Refs. [17] and [42] to find

$$\hat{a}_{LL}(qg \rightarrow q\chi_1) = \frac{3z(1 - z + z\ln(z))}{(1 - z)(5 - 4z - 4z^2) - 3z^2 \ln(z)}. \tag{25}$$

The corresponding spin-spin asymmetry for the total $gg \rightarrow g\chi_1$ cross-section is similarly calculable and is given by

$$\hat{a}_{LL}(gg \rightarrow g\chi_1) = \frac{z[(1 - z^2)A_1(z) + 12z\ln(z)A'_1(z)]}{(z^2 - 1)H_1(z) - 12z^2\ln(z)H'_1(z)} \tag{26}$$

where

$$A_1(z) = z^8 + 33z^7 + 145z^6 + 271z^5 + 43z^4 + 55z^3 - 273z^2 - 23z - 12 \tag{27}$$

$$A'_1(z) = z^8 + 8z^7 + 25z^6 + 29z^5 + 34z^4 - 32z^3 - 3z^2 - 21z - 1. \tag{28}$$

and the $H_1(z)$ and $H'_1(z)$ are given in Eqns. 23 and 24. As mentioned above, all of the $qq \rightarrow g\chi_J$ cross-sections have $\hat{a}_{LL} = -1$ from helicity conservation. The first two of these asymmetries are plotted versus the natural variable $z \equiv M^2/\hat{s}$ in Fig. 3 where it can be noted that they are positive, and clearly smaller in magnitude than the maximally large value of $\hat{a}_{LL} = -1$ for lowest order $\chi_2$ production. For comparison, we also plot the corresponding asymmetry for direct $\psi$ production, namely

$$\hat{a}_{LL} = -z \left[ \frac{(1 - z^2)F(z) + 2G(z)\ln(z)}{(1 - z^2)G(z) + 2z^2F(z)\ln(z)} \right] \tag{29}$$

where

$$F(z) = z^2 + 2z + 5 \quad \text{and} \quad G(z) = z^3 + 4z^2 + z + 2 \tag{30}$$
which is seen to be somewhat but not dramatically larger.

A useful measure of the average ‘analyzing power’ or spin-dependence in such a reaction is the average spin-spin asymmetry

\[
< \hat{a}_{LL} > = \frac{\sum_{ij} \int dx_1 \int dx_2 f_i(x_1, Q^2) f_j(x_2, Q^2) \hat{a}_{LL} d\hat{\sigma}}{\sum_{ij} \int dx_1 \int dx_2 f_i(x_1, Q^2) f_j(x_2, Q^2) d\hat{\sigma}}
\]  

(31)

which measures both the importance of the process to the total cross-section and its spin-spin asymmetry. (If the unpolarized parton densities, \( f_{i,j}(x, Q^2) \), in the numerator were replaced by the corresponding polarized distributions, \( \Delta f_{i,j}(x, Q^2) \), the result would be the observable spin-spin asymmetry.) The average spin-spin asymmetry for both \( \chi_1 \) production (including all three contributing processes) as well as that for direct \( \psi \) production are shown in Fig. 4 where we see the spin-dependence of \( \chi_1 \) production is not at all large. We comment that the average value of \( z \equiv M^2/\hat{s} \) for the \( gg \to g\psi \) (\( gg \to g\chi_1 \)) process is roughly 0.5 (0.25 – 0.15) over the energy range plotted which accounts for the size and relatively constant values of \( < \hat{a}_{LL} > \). So, while the measurement of \( \chi_1 \) production will provide another cross check on a potential model description of charmonium production, it will not likely give much information on the polarized gluon content of the proton.

As mentioned above, we have not evaluated the spin-dependent renormalized cross-sections for \( \chi_2 \) production (which in this case require a careful treatment of the helicity amplitudes in arbitrary dimensions, i.e. the \( \gamma_5 \) problem) but we can perhaps get some feel for the partonic level asymmetries for \( qg \to q\chi_2 \) by examining the known \( 2 \to 2 \) asymmetries [17] in the \( \hat{t} \to 0 \) limit. While the partonic level cross-sections are divergent in this limit, necessitating the renormalization procedure above, the asymmetries are well-behaved. We find in this limit (when \( \hat{t} \to 0 \) and \( \hat{u} \to (M^2 - \hat{s}) \))

\[
\hat{a}_{LL}(qg \to q\chi_2) = \frac{-2z + z^2}{2 - 2z + z^2} = -\hat{a}_{LL}(qg \to q\chi_0).
\]

(32)
where \( z \equiv M^2/\hat{s} \). We plot this function in Fig. 5 and note that at threshold \( z = 1 \) the partonic level asymmetry is equal to \(-1(+1)\) just as for \( gg \to \chi_2(\chi_0) \) at tree level.

For comparison, we also plot on the same figure the partonic level asymmetry for \( gg \to g\chi_2 \), again in the same limit, and note a similar behavior. As before, the \( \chi_{0,2} \) asymmetries are trivially related and we find

\[
\hat{a}_{LL}(gg \to g\chi_2) = \frac{-z(2 - 3z + 2z^2)}{(1 - z + z^2)^2} = -\hat{a}_{LL}(gg \to g\chi_0).
\]

(33)

The purely gluonic asymmetry, however, stays near its threshold value of \(-1\) over a wider range of \( z \) and so will contribute more to the observed asymmetry. (These expressions are also found in the \( \hat{u} \to 0 \) limit where divergences also occur in the \( gg \to g\chi_{0,2} \) processes.) If the fully renormalized gluon induced cross-section were described by these limiting values for the asymmetries, then an average value of \(<z>_{}\) of 0.5 (0.2), as for \( gg \to g\psi (gg \to g\chi_1) \), would correspond to \(<\hat{a}_{LL}>\approx -0.9 (-0.5)\).

These results suggest that the spin-dependence of the NLO \( gg \) and \( qg \) processes will be somewhat ‘softer’ but will still retain much of the same general structure (most especially the sign) of the tree-level spin-spin asymmetry in contrast to, for example, open heavy flavor production as discussed in Ref. [12].

As a final comment, we note that \( \chi_2(c\bar{c}) \) production RHIC probes the polarized gluon distributions in a kinematic region given by

\[
\sqrt{\tau} \equiv \frac{M}{\sqrt{\hat{s}}} \approx 0.07 - 0.007.
\]

We note that the production of a \( ^1S_0(t\bar{t}) \) toponium state (presumably detected via its two-photon decay) also has a maximally large partonic level spin-spin asymmetry given by \( \hat{a}_{LL} = +1 \). A future supercollider (such as the LHC) with a polarization
option would be expected to probe $\eta$ production in a kinematic range given by $\sqrt{t} \approx 0.0017 - 0.0025$ which would then be in the region of polarized gluon densities already probed by RHIC charmonium studies. Since the value of $\alpha_s$ probed by such heavy mass states would be much smaller than for charmonium, the effects of radiative corrections would be expected to be much smaller and the interplay between polarized gluon densities and the leading-order production mechanisms would be expected to be much more direct. In the same context, standard model Higgs boson production via gluon fusion, i.e. $gg \rightarrow H_0$ via quark loops, has the same leading-order spin-spin asymmetry, namely +1. Higgs boson production at the LHC via this mechanism would then be in the same kinematic region as $\chi_2(\tau)$ production at RHIC for Higgs boson masses in the range $200 - 2000 GeV$ which covers most of the expected range. In this regard, the spin-independent radiative correction calculations of Refs. [43, 44] are already useful in assessing the contributions of various NLO processes.

5 Acknowledgments

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Figure Captions

Fig. 1 The total cross-section, $\sigma(nb)$, for $\chi_{2,1}(c\tau)$ production in pp collisions versus $\sqrt{s}$ $(GeV)$. For 1(a), we plot the results for $\chi_2$ production with the solid (dotted, dashed) curves corresponding to the $\mathcal{O}(\alpha_s^2) \; gg \to \chi_2 \rightarrow q\bar{q}, \quad qg \to q\chi_2, \quad gg \to g\chi_2$ contributions. The negative of the $qg$ contribution is plotted for simplicity. For 1(b), the results for $\chi_1$ production are shown where the solid (dotted, dashed) curves correspond to the $gg \to g\chi_1 \rightarrow q\chi_1, \quad qg \to q\chi_1, \quad q\bar{q} \to g\chi_1$ contributions.

Fig. 2. The total cross-section for $\psi$ production (times leptonic branching ratio, $Br(\psi \to e^+e^-)$) in pp collisions versus $\sqrt{s}$(GeV). The contributions from the lowest order $gg \to g\psi$ (solid), $\mathcal{O}(\alpha_s^2) \; gg \to \chi_2$ (dotted), and total $\mathcal{O}(\alpha_s^2) \; \chi_1$ contributions (dashed) from $g, \quad gg, \quad qg$ initial states are plotted.

Fig. 3. The partonic level spin-spin asymmetry, $\hat{a}_{LL}$, (in the total integrated cross-section) versus $z \equiv M^2/\sqrt{s}$ for $\chi_1$ production. The solid (dotted) curves correspond to the asymmetry for $gg \to g\chi_1 \rightarrow q\chi_1, \quad gg \to g\chi_2, \quad qg \to q\chi_2$ production. The asymmetry for direct $\psi$ production via $gg \to g\psi$ production (dashed curve) is also shown for comparison. The partonic level asymmetries for $q\bar{q} \to g\chi_0,1,2$ and for $gg \to \chi_2$ are both $\hat{a}_{LL} = -1$.

Fig. 4. The average spin-spin asymmetry, $<\hat{a}_{LL}>$ versus the center of mass energy, $\sqrt{s} \; GeV$ for pp collisions. The results for total $\chi_1$ production (dots) (from $gg, \quad qg, \quad q\bar{q}$ processes) and direct $\psi$ production (solid) (via $gg \to g\psi$) are shown.

Fig. 5. The partonic level spin-spin asymmetries, $\hat{a}_{LL}$, versus $z \equiv M^2/\hat{s}$ for $\chi_2$ production in the limit that $\hat{t} \to 0$. The solid (dotted) curves correspond to the processes $gg \to g\chi_2 \rightarrow q\chi_2, \quad qg \to q\chi_2$. 

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\[ \text{Br}(\psi \rightarrow \text{e}^+ \text{e}^-) \cdot \sigma(pp \rightarrow \psi, \chi \rightarrow \gamma \gamma) \ (\text{nb}) \]
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