The 21cm radiation from minihalos as a probe of small primordial non-Gaussianity

Sirichai Chongchitnan$^{1,2}$, Joseph Silk$^{1,3,4}$

$^1$ Oxford Astrophysics, Denys Wilkinson Building, Keble Road, Oxford, OX1 3RH,
$^2$ School of Engineering, Computing and Applied Mathematics, University of Abertay Dundee, Bell St., Dundee, DD1 1HG,
$^3$ Institut d’Astrophysique, UPMC, 98bis Boulevard Arago, Paris 75014,
$^4$ Department of Physics and Astronomy, The Johns Hopkins University, 3701 San Martin Drive, Baltimore MD 21218.

May 2012

ABSTRACT

We present a new probe of primordial non-Gaussianity via the 21cm radiation from minihalos at high redshifts. We calculate the fluctuations in the brightness temperature (measured against the cosmic microwave background) of the 21cm background from minihalos containing HI at redshift $\sim 6-20$, and find a significant enhancement due to small non-Gaussianity with amplitude $f_{NL} \lesssim 1$. This enhancement can be attributed to the nonlinear bias which is strongly increased in the presence of non-Gaussianity. We show that our results are robust against changes in the assumed mass function and some physical aspects of minihalo formation, but are nevertheless sensitive to the presence of strong radiation sources within or around the minihalos. Our findings are relevant for constraining and searching for small primordial non-Gaussianity with upcoming radio telescopes such as LOFAR and SKA.

1 INTRODUCTION

In this era of “precision cosmology”, many open questions in cosmology will be addressed with the highly anticipated results from the Planck satellite and a host of other ambitious ground and space-based experiments. One of the key goals of these experiments is to establish the statistical nature of the primordial cosmological perturbations, which, according to the single-field, slow-roll model of inflation, should be very close to a Gaussian field (see e.g. Maldacena (2003), for reviews see Bartolo et al. (2004), Chen (2010)). A detection of any primordial non-Gaussianity (NG) would therefore signify a deviation from the simplest model of inflation and hint at new physics in the early Universe.

At present, the most stringent constraint on NG comes from measurements of the anisotropies in the cosmic microwave background (CMB). In the “local” model of NG parametrized by a constant amplitude, $f_{NL}$, the WMAP satellite reported a limit $f_{NL} = 32 \pm 42$ (2σ) (Komatsu et al. 2011) whilst Planck is expected to improve this limit to $|f_{NL}| \lesssim 5$.

On smaller physical scales, galaxy clusters have been shown to be effective in constraining NG via the number counts of rare objects and measurements of the bias (see e.g. Scoccimarro et al. (2003); Dalal et al. (2008); Slosar et al. (2008); Desjacques & Seljak (2010); Chongchitnan & Silk (2011)). A positive $f_{NL}$, for instance, will increase the number of galaxy clusters at high redshifts and induce scale-dependence in the bias. Realistically, however, uncertainties in the mass-observable relation, redshift determination and other systematics will most likely limit the constraining power of cluster surveys to $|f_{NL}| > 10$. Slosar et al. (2008), for example, obtained the 2σ limit of $-29 < f_{NL} < 70$ from the analysis of various large-scale-structure datasets.

More recently, it has been shown that NG leaves imprints in the 21cm radiation due to spin-flip transitions of neutral hydrogen during the epoch of reionization. This transition occurs at an excitation temperature $T_e = 68$ mK and rest-frame frequency $v_0 = 1.42$ GHz (now redshifted to the radio band). These 21cm signals are expected to be measured over a wide range of redshifts by radio facilities such as LOFAR, MWA and SKA. It was first shown in Cooray (2006) that an ideal measurement of the 3-point correlation in the 21cm fluctuations from $z \sim 50$ can, in principle, probe $f_{NL}$ as small as $10^{-2}$ (see also Pillepich et al. 2007). Joudaki et al. (2011) considered a more conservative approach of measuring the 21cm power spectrum and concluded that $|f_{NL}| \sim 10$ is within reach of the next generation of radio telescopes. Tashiro & Sugiyama (2012) analyzed the number counts of ionized bubbles in 21cm maps and found them to be sensitive to $|f_{NL}| \sim 100$. Most recently, Tashiro & Ho (2012) argued that the correlation between the CMB and 21cm fluctuations will be capable of detecting $|f_{NL}| \sim 100$ and will be useful for removing foregrounds and systematics.

In this work, we present a new connection between NG...
21cm Emission from Minihalos

We follow the basic treatment of halos in Shapiro et al. (1999); Iliev & Shapiro (2001) where a MH of a given mass is modelled as a “truncated isothermal sphere” with radius \(r_\alpha\), temperature \(T_K\), dark-matter velocity dispersion \(\sigma_v\), and density profile \(\rho(r)\).

We begin by considering a single MH. Its 21cm signal may appear in emission or absorption against the CMB depending on the spin temperature, \(T_S\), which is determined by 1) energy exchanges between HI-bound electrons and CMB photons, 2) collisions between atoms, 3) interactions between neutral hydrogen to photons at rest-frame frequency, \(\nu\), and density profile \(\rho(r)\).

The observed brightness temperature at comoving distance \(r\) from the centre of the halo is given by

\[
T_b(r) = T_{\text{CMB}}(0) e^{-\tau} + \int_0^r T_S e^{-\tau} d\tau',
\]

where \(T_{\text{CMB}}(z) = 2.73(1 + z)\) K. The optical depth, \(\tau\), of neutral hydrogen to photons at rest-frame frequency, \(\nu\), along a line of sight with impact parameter, \(\alpha\) (in unit of \(r_\alpha\)), from the centre of the MH can be expressed as

\[
\tau(\nu) = \frac{3\nu^2 A_{10} T_e}{2\pi v_0^2} \int_{-\infty}^{\infty} \frac{n_{\text{HII}}(\ell)}{T_S(\ell)} \phi(\nu, \ell) d\ell,
\]

where \(A_{10} = 2.85 \times 10^{-15}\) s\(^{-1}\) and \(R\) and \(\ell\) are radial comoving distances satisfying \(\ell' = R' + (\alpha r)^2\), with \(R = 0\) at the centre of the MH. The number density of neutral hydrogen in the MH is \(n_{\text{HII}} \approx (1 - Y) \Omega_b/\Omega_m (\rho/m_H)\), with \(Y\) is the helium fraction and \(m_H\) is the mass of a hydrogen atom. The intrinsic line profile \(\phi(\nu)\) is modelled as a Doppler-broadened form

\[
\phi(\nu) = \left(\Delta \nu / \nu_0\right) \exp \left(-[\nu - \nu_0]/(\Delta \nu)^2\right),
\]

with \(\Delta \nu = (\nu_0/c) / 2k_B T(K) / m_H\), and \(k_B\) the Boltzmann constant.

In the special case when the line is unBroadened, \(\phi(\nu) = \delta(\nu - \nu_0)\), the optical depth reduces to that of the unperturbed IGM patch at redshift \(z\) (Madau et al. 1997)

\[
\tau_{\text{IGM}}(z) = \frac{3\nu^2 A_{10} T_e n_{\text{HII}}(z)}{2\pi v_0^2 T_S(z) m_H(z)}.
\]

Using (2)-3, we can rewrite the brightness temperature as

\[
T_b(\nu) = T_{\text{CMB}} e^{-\tau(\nu)} + \int_{-\infty}^{\infty} T_S(\ell) e^{-\tau(\nu, \ell)} \frac{\partial \tau}{\partial R} d\ell.
\]

Finally, the observed 21cm brightness temperature of a single MH with respect to the CMB is given by

\[
\delta T_b = \frac{T_b}{1 + z} - T_{\text{CMB}}(0),
\]

where \(T_b\) is averaged over the halo cross-section \(A = \pi r_\alpha^2\).

If we now consider an ensemble of MHs in the mass range \([M_{\text{min}}, M_{\text{max}}]\), the mean 21cm emission from an ensemble is given by (Iliev et al. 2002)

\[
\frac{\delta T_b}{T_b} = \frac{c(1 + z)^4}{\nu_0^4 H(z)} \int_{M_{\text{min}}}^{M_{\text{max}}} \nu_{\text{eff}}(M) \frac{dn}{dM} dM.
\]

where \(\nu_{\text{eff}} = |\phi(\nu_0)(1 + z)|^{-1}\) is the effective redshifted linewidth. Various prescriptions for the mass function, \(dn/dM\), will be compared later on. We set \(M_{\text{min}}\) to correspond to the virial temperature of \(10^4\) K whilst \(M_{\text{max}}\) is set by the Jeans mass, \(M_J\).

The key observable relevant for the upcoming radio arrays is the \(rms\) fluctuations in the 21cm emission. For a pencil-beam survey with frequency bandwidth \(\Delta \nu\) and angular size \(\Delta \theta\), the amplitude of the \(3\sigma\) fluctuation is

\[
(\delta T_b^2)^{1/2} = 3\sigma_p(\Delta \nu, \Delta \theta) \beta(z) \sqrt{T_b},
\]

(see e.g. Dodelson 2003) for the calculation of the variance \(\sigma_p\) in a cylinder). Here, \(\beta(z)\) is the weighted average of the bias \(b(M, z)\), defined as the ratio of the 2-point correlation for density peaks corresponding to a MH of mass \(M\), and that of dark-matter density fluctuations (detail in the next section).

\[
\beta(z) = \int_{M_{\text{min}}}^{M_{\text{max}}} b(M, z) F(m) \frac{dn}{dM} dM.
\]

where \(F(m) \propto T_r^2 \sigma_V\) is the effective flux from the MHs.

3 EFFECTS OF \(f_{NL} < 1\)

If Planck rules out \(|f_{NL}| > 2\) a few, then it would appear extremely difficult for large-scale-structure probes to ever improve on, or even corroborate, \(f_{NL}\) constraints from the CMB (unless \(f_{NL} is k\)-dependent, see e.g. LoVerde et al. 2008). Unlike galaxy clusters, MHs are neither very rare nor very massive, so naively we expect the enhancement in their number counts from \(f_{NL} \leq 1\) to be undetectably small. However, at redshift \(z \geq 6\), these MHs are strongly biased nonlinear objects and the scale-dependent effects on the bias can be much more dramatic than those on clusters.

In the Gaussian case, Iliev et al. (2003) used the nonlinear bias approach of Scannapieco & Barkana (2002) (based
on excursion set theory) to study 21cm emission from MHs. They found very good agreement for $(\delta T_b^2)$ between analytic prediction and simulation and concluded that, for practical purposes, one could, for instance, use the bias obtained in [Mo & White (1996)] (using the peak-background split approach). In this work, we extend this line of investigation to non-Gaussian scenarios.

A number of previous authors have investigated the effect of non-Gaussianity on the Fourier-space bias, $b(k)$, defined as the ratio of the power spectrum for density peaks and that of dark matter (e.g. Dalal et al. (2008); Matarrese & Verde (2008); Wagner & Verde (2012)). However, an arguably more intuitive measure of the bias is in real space, where we can directly obtain information on the clustering amplitude of density peaks separated by comoving distance $r$. Following the pioneering work of Kaiser (1984), we define

$$b(r) = \frac{\xi(r)}{\xi(r)}$$

(10)

where $r$ is the comoving length in Eulerian space. The correlation function $\xi(x_1, x_2) = (\delta(x_1), \delta(x_2))$ where $\delta(x)$ is the overdensity field and $r = |x_1 - x_2|$. Similarly, $\xi_{pk}$ is the 2-point correlation of density peaks corresponding to MHs of mass $M$. Whilst the real and Fourier space biases deal with the same physics of clustering in the overdensity field (and they are indeed equivalent in the Gaussian case), the real-space bias is related directly to the joint probability distribution of finding two overdense regions exceeding a collapse threshold in a given volume [Kaiser 1984]. As NG, by definition, distorts this probability distribution from the Gaussian, the change in the real-space bias seems a natural, measurable quantity which can be calculated, for instance, using a saddle-point expansion [Valageas 2010] or Edgeworth expansion about the Gaussian [Chongchitnan & Silk 2011]. The Fourier bias, on the other hand, would be more useful when working with the power spectrum and bispectra from different NG shape templates [Matsubara (2012), or when redshift-space distortions are incorporated [Mao et al. (2012)]. In this work, however, the real-space bias suffices for the calculation of the 21cm fluctuations [S].

To date there have only been a handful of calculations of the real-space bias in the presence of NG (and unfortunately $b(k)$ and $b(r)$ are not related via a straightforward Fourier transform). One such calculation is the saddle-point approach in [Valageas 2010, 2009]. Whilst this formalism has been shown to agree with simulations of massive halos ($\sim 10^{11} M_\odot$), it has yet to be tested against simulations of MH-scale resolution. We recognise that there may be limitations to the saddle-point formalism in the strongly non-linear regime. At the same time, there is not yet any convincing analytic model for the non-Gaussian bias in this regime, and thus we appeal to the saddle-point approach in this work as a first analytical approach to the problem. Recent progress in non-Markovian excursion-set theory [Adshead et al. (2012); Musso et al. (2012); Paranjape & Sheth (2012)] should soon allow a more accurate calculation of the NG bias down to much smaller masses, and high-resolution simulations will be needed to elucidate the gas physics on such scales.

In Chongchitnan & Silk (2012), we studied the saddle-point approach in detail and found that when $b(r)$ is averaged over all separation lengths $r$ within the volume that the MHs occupy (which in this case is a sphere with radius $L$), the result is the volume-averaged bias, $b(M, z)$, which is well-approximated by

$$b(M, z) \approx \left[1 + (6/5) f_{NL}(z) L^2 b_G(M, z)\right]$$

(11)

where $b_G$ is the Gaussian bias and the constant $K$ can be determined from calculating $b(r)$ on some fixed scale. $K(z)$ roughly grows as a linear function of $z$ as shown in the top panel of Fig. [I].

Using these results and combining it with the flux-weighting $P$, we plot the average bias, $\beta(z)$, for $f_{NL} = 0, 0.1$ and 1 in the middle panel of Fig. [I]. Clearly $\beta$ is sensitive to $|f_{NL}| \leq 1$, which boosts the integrand in the nominator of $P$ particularly on mass scales around $M_{\text{max}}$. We have also checked that the “Gaussian” curves can be closely reproduced using the linear bias of [Mo & White (1996), in agreement with [Iliev et al. (2003)].

Our main results are shown in the bottom panel of Fig. [I] which shows the redshift variation of the $rms$ 21cm fluctuations $S$ for $f_{NL} \leq 1$. The peak structure of these curves is the result of the competition between terms in Eq. [S] with $\sigma_0$ and $\delta T_b$ decreasing with $z$ (see fig. 2 of [Iliev et al. (2002)]), whilst $\beta$ increases as shown in the top panel. In this figure, we assume an observation bandwidth $\Delta \nu = 1$ MHz and beam angular diameter $\Delta \theta = 9$ arcminutes. Also overlaid are two increasing curves corresponding to the noise [Furlanetto et al. (2006)]

$$\delta T_{\text{noise}} \approx 20 \text{ mK} \frac{10^4 M_\odot}{A_{\text{tot}}} \left[\frac{1 + z}{10}\right]^{1.5} \left[\frac{\text{MHz 100hr}}{\Delta \nu \ t_{\text{int}}}\right]^{1/2}$$

The noise thresholds assume total effective areas $A_{\text{tot}} \approx 10^4$ m$^2$ (“LOFAR”) and $10^5$ m$^2$ (“SKA”), with integration time $t_{\text{int}} = 1000$ hours in both cases (see de Vos et al. (2009); Dewdney et al. (2009) for detailed specifications). These curves show that even $f_{NL}$ as small as 0.1 will boost the fluctuations from a few mKs to tens of mK. Hence, there are good prospects of detecting a small NG via the 21cm emission from high-redshift minihalos with upcoming radio telescopes.

Moreover, we have checked that the non-Gaussian effects on $(\delta T_b^2)$ cannot be easily reproduced by changing each of the fiducial cosmological parameters (within the observational limits). For instance, increasing $\sigma_0$ from 0.8 to 0.9 results in $\lesssim 0.1$ mK increase across the redshift range shown. A Fisher matrix analysis could be performed to shed light on parameter correlations, but we shall leave this for a future investigation.

4 DISCUSSION

Here we consider a number of caveats for the results in Fig. [I] mainly coming from the fact that MHs are relatively small objects whose dynamics are governed by nonlinear physics on small scales.

i) Mass Function: MHs are notoriously difficult to resolve in N-body simulations, requiring at least a $\sim 20$ Mpc comoving box and $\gtrsim 10^{10}$ Jeans-mass particles [Meiksin (2011)] (see [Iliev et al. (2003); Shapiro et al. (2006); Ciardi et al. (2006) for previous attempts). The large simulation in Iliev et al. (2012), in particular, was able to resolve down
to $10^4 M_\odot$ MHs and found their abundance to lie between the Press-Schechter [Press & Schechter 1974] and Sheth-Tormen [Sheth & Tormen 1999] predictions. We now consider how the 21cm emission from MHs is affected by the choice of mass function.

In the top panel of Fig. 1, we replot the 21cm fluctuations in the lower panel of Fig. 1 with $f_{\text{NL}} = 0.1$ using the above mass functions along with those of [Tinker et al. 2008] and [Warren et al. 2008] (note the linear scale here). The Tinker mass function is known to predict $n(z)$ lying between the Press-Schechter and the Sheth-Tormen predictions (e.g. [Chongchitnan & Silk 2012]). The Warren mass function was found to be accurate for high-redshift objects down to $10^7 h^{-1} M_\odot$ when compared with simulations ([Lukić et al. 2007]). We see that the Press-Schechter and Tinker prescriptions gave similarly high amplitudes of $\langle \delta T_b^2 \rangle$ for $z \lesssim 10$, whilst the Warren and Sheth-Tormen prescriptions favour lower amplitudes. The trends are reversed for $z \sim 20$. In any case, the uncertainty in the mass function does not appear to affect the detection prospects for LOFAR and SKA.

ii) Uncertainty in $M_{\text{min}}$: On MH scales there are a number of so-called “gastrophysical” effects which may overwhelm the imprints of NG. For instance, [Tseliakhovich et al. 2011] showed that for $z \gtrsim 10$, the velocity of dark matter relative to baryons is generally supersonic and thus baryons can advect out of dark matter potential, leading to the possibility that $M_{\text{min}} > M_J$. This may be further exacerbated by feedback mechanisms such as photoevaporation of MHs ([Iliev et al. 2005]) and shock heating in the IGM ([Oh & Haiman 2003]; [Pfnueli & Loeb 2004]).

The centre panel of Fig. 1 shows the effects on $\langle \delta T_b^2 \rangle$ when $M_{\text{min}}$ increases by a factor of 10, 50 and 100 (with $f_{\text{NL}} = 0.1$). The result is a suppression in $\langle \delta T_b^2 \rangle$ across all redshifts (although gastrophysical suppressions are expected to be more significant at $z \gtrsim 10$). Nevertheless, we see that the fluctuation amplitudes are generally robust against changes in $M_{\text{min}}$.

iii) Lyα pumping: The 21cm signals from MHs are unlikely to be completely immune to the Wouthuysen-Field mechanism, which redistributes the spin states and couples the spin temperature to that of radiation sources. This means that as the radiation intensity increases, $T_S \rightarrow T_K$ and the MH emission will be more and more suppressed.

Following [Chuzhov & Shapiro 2004], we introduce the
Lyo coupling of the form

\[ y_0 = 1.3 \times 10^{-12} \frac{J_\alpha T_*}{J_{10} T_K} \exp \left( \frac{-0.3(1 + z)^{1/2} T_K^{-2/3} \alpha}{1 + 0.4 T_K} \right), \]

where \( J_\alpha \) parametrizes the intensity of the radiation sources at the Lyo frequency (in units of \( 10^{-21} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1} \text{ sr}^{-1} \)). In the bottom panel of Fig. 2, we plot the effects of \( J_\alpha = 0.1, 0.5, \) and 1 on the 21cm fluctuations. Indeed we see a strong suppression of \( \langle \delta T^2 \rangle \), with \( J_\alpha \geq 1 \) capable of effectively canceling the boost from NG.

However, a more serious issue concerning Lyo pumping is that the MH signals will be completely submerged under a huge absorption signal from the IGM, which contains much more mass than that in MHs [Oh & Mack 2003; Furlanetto & Oh 2006; Yue et al. 2009]. In our example with \( J_\alpha = 0.1 \), we find an absorption amplitude of \( |\langle \delta T^2 \rangle| \sim 20 \text{ mK} \), increasing to 100 mK when \( J_\alpha = 1 \). We therefore conclude that if MHs are indeed exposed to strong Lyo pumping, their 21cm emission will not be visible unless \( f_{NL} \gg 1 \).

5 CONCLUSIONS

We have shown that a small amplitude of primordial non-Gaussianity may be detectable via the fluctuations in the 21cm emission of high-redshift MHs. Even with \( f_{NL} \lesssim 1 \), we showed that the strong enhancement in the bias leads to a significant increase in the amplitude of the fluctuations, as seen in Fig. 4. There are good prospects for such a detection by the next generation of large radio telescopes such as the SKA.

Our conclusions rely on a number of assumptions on the physics of MHs at high redshift. The analytic formalism used to calculate the bias assumes some extrapolations from cluster scales on which the theory has been well-tested. An improved calculation awaits a fuller understanding of nonlinear collapse in the presence of NG, perhaps with help from the resurgence of interests in excursion set theory. The results presented here are robust against the assumed mass function and minimum MH mass threshold. However, the fluctuations are sensitive to the presence of radiation background, since the Wouthuysen-Field effect is capable of overwhelming the MH signals with the IGM line-absorption signals.

Nevertheless, our conclusions still apply to MHs that are sufficiently isolated from UV sources with no strong feedback. This class of MHs provides a new probe for primordial non-Gaussianity which has so far been unexplored. For MHs that are subjected to strong background radiation, it may be possible for the MH signals to dominate if non-Gaussianity is much larger than the CMB bounds (perhaps with \( f_{NL} \sim 100 \)). However, this awaits high-resolution simulations to elucidate high-redshift gasphysics, cosmic reionization and the nonlinear bias given highly non-Gaussian initial conditions. Needless to say, this will be an extremely challenging task.

We thank Patrick Valageas and, in particular, Ilian Iliev for illuminating discussions. We also thank the referee for comments that led to major improvements of the first version. SC is grateful for the support of Lincoln College, Oxford, where part of this work was completed.

REFERENCES

Adhav P., Baxter E. J., Dodelson S., Lidz A., 2012, ArXiv: 1206.3306
Allison A. C., Dalgarno A., 1969, ApJ, 158, 423
Bartolo N., Komatsu E., Matarrese S., Riotto A., 2004, Phys. Rept., 402, 103
Chen X., 2010, Adv. Astron., 2010
Chongchitnan S., Silk J., 2011, Phys. Rev. D, 83, 083504
Chongchitnan S., Silk J., 2012, Phys. Rev. D, 85, 063508
Chuzhoy L., Shapiro P. R., 2006, ApJ, 651, 1
Ciardi B., Scannapieco E., Stoeck F., Ferrara A., Iliev I. T., Shapiro P. R., 2006, MNRAS, 366, 689
Cooray A., 2006, Phys. Rev. Lett., 97, 261301
Dala N., Dore O., Huterer D., Shirok A., 2008, Phys. Rev., D77, 123514
de Vos M., Gunst A., Nijboer R., 2009, Proceedings of the IEEE, 97, 1431
Desjacques V., Seljak U., 2010, Class. Quant. Grav., 27, 124011
Dewdney P. E., Hall P. J., Schilizzi R. T., Lazio T. J. L. W., 2009, Proceedings of the IEEE, 97, 5
Dodelson S., 2003, Modern Cosmology. Academic Press, San Diego
Field G., 1958, Proc. IRE, 46, 240
Furlanetto S. R., Loeb A., 2002, ApJ, 579, 1
Furlanetto S. R., Loeb A., 2004, ApJ, 611, 642
Furlanetto S. R., Oh S. P., 2006, ApJ, 652, 849
Furlanetto S. R., Oh S. P., Briggs F. H., 2006, Phys. Rep., 433, 181
Iliev I. T., Mellema G., Shapiro P. R., Pen U.-L., Mao Y., Koda J., Ahn K., 2012, MNRAS, p. 3013
Iliev I. T., Scannapieco E., Martel H., Shapiro P. R., 2003, MNRAS, 341, 81
Iliev I. T., Shapiro P. R., 2001, MNRAS, 325, 468
Iliev I. T., Shapiro P. R., Ferrara A., Martel H., 2002, ApJ, 572, L123
Iliev I. T., Shapiro P. R., Raga A. C., 2005, MNRAS, 361, 405
Joudaki S., Doré O., Ferramacho L., Kaplinghat M., Santos M. G., 2011, Phys. Rev. Lett., 107, 131304
Kaiser N., 1984, Astrophys. J. Lett., 284, L9
Komatsu E., et al., 2011, ApJS, 192, 18
LoVerde M., Miller A., Shandera S., Verde L., 2008, JCAP, 0804, 014
Lukić Z., Heitmann K., Habib S., Bashinsky S., Ricker P. M., 2007, ApJ, 671, 1160
Madau P., Meiksin A., Rees M. J., 1997, ApJ, 475, 429
Maldacena J., 2003, JHEP, 5, 13
Mao Y., Shapiro P. R., Mellema G., Iliev I. T., Koda J., Ahn K., 2012, MNRAS, 422, 926
Matarrese S., Verde L., 2008, ApJ, 677, L77
Matsubara T., 2012, ArXiv: 1206.0562
Meiksin A., 2011, MNRAS, 417, 1480
Mo H. J., White S. D. M., 1996, MNRAS, 282, 347
Musso M., Paranjape A., Sheth R. K., 2012, ArXiv: 1205.3401
Oh S. P., Haiman Z., 2003, MNRAS, 346, 456
Oh S. P., Mack K. J., 2003, MNRAS, 346, 871
Paranjape A., Sheth R. K., 2012, ArXiv: 1206.3506
Pillepich A., Porciani C., Matarrese S., 2007, ApJ, 662, 1
Press W. H., Schechter P., 1974, ApJ, 187, 425
Scannapieco E., Barkana R., 2002, ApJ, 571, 585
Scoccimarro R., Sefusatti E., Zaldarriaga M., 2004, Phys. Rev. D, 69, 103513
Shapiro P. R., Ahn K., Alvarez M. A., Iliev I. T., Martel H., Ryu D., 2006, ApJ, 646, 681
Shapiro P. R., Iliev I. T., Raga A. C., 1999, MNRAS, 307, 203
Sheth R. K., Tormen G., 1999, MNRAS, 308, 119
Slosar A., Hirata C., Seljak U., Ho S., Padmanabhan N., 2008, JCAP, 8, 31
Tashiro H., Ho S., 2012, ArXiv: 1205.0563
Tashiro H., Sugiyama N., 2012, MNRAS, 420, 441
Tinker J., Kravtsov A. V., Klypin A., Abazajian K., Warren M., Yepes G., Gottlöber S., Holz D. E., 2008, ApJ, 688, 709
Tseliakhovich D., Barkana R., Hirata C. M., 2011, MNRAS, 418, 906
Valageas P., 2009, Astron. Astrophys., 508, 93
Valageas P., 2010, Astron. Astrophys., 514, A46
Wagner C., Verde L., 2012, JCAP, 3, 2
Warren M. S., Abazajian K., Holz D. E., Teodoro L., 2006, ApJ, 646, 881
Wouthuysen S. A., 1952, AJ, 57, 31
Yue B., Ciardi B., Scannapieco E., Chen X., 2009, MNRAS, 398, 2122