TFAD: A Decomposition Time Series Anomaly Detection Architecture with Time-Frequency Analysis

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ABSTRACT
Time series anomaly detection is a challenging problem due to the complex temporal dependencies and the limited label data. Although some algorithms including both traditional and deep models have been proposed, most of them mainly focus on time-domain modeling, and do not fully utilize the information in the frequency domain of the time series data. In this paper, we propose a Time-Frequency analysis based time series Anomaly Detection model, or TFAD for short, to exploit both time and frequency domains for performance improvement. Besides, we incorporate time series decomposition and data augmentation mechanisms in the designed time-frequency architecture to further boost the abilities of performance and interpretability. Empirical studies on widely used benchmark datasets show that our approach obtains state-of-the-art performance in univariate and multivariate time series anomaly detection tasks. Code is provided at https://github.com/DAMO-DI-ML/CIKM22-TFAD.

CCS CONCEPTS
• Mathematics of computing → Time series analysis; • Computing methodologies → Anomaly detection.

KEYWORDS
time series anomaly detection, frequency domain analysis, data augmentation, time series decomposition

1 INTRODUCTION

With the rapid development of the Internet of Things (IoT) and other monitoring systems, there has been an enormous increase in time series data [11, 54]. Thus, effectively monitoring and detecting anomalies or outliers on the time series data is crucial to discovering faults and avoiding potential risks in many real-world applications. Generally, an anomaly is an observation that deviates from normality. Anomaly detection has been studied widely in different disciplines, including statistics, data mining, and machine learning [39], but how to perform it effectively on time series data is an active research topic and has received a lot of attentions recently [14, 27, 29, 33, 35, 49, 59] due to the special properties of time series.

Unlike ordinary tabular data, one distinguishing property of time series is the temporal dependencies. Usually, a point or a subsequence of time series is called an anomaly when compared to its corresponding “context”. Based on the relationship between the anomaly in time series and its context, we can define different types of anomalies, such as global point anomaly, seasonality anomaly, shapelet anomaly, etc. Thus, the first challenge in time series anomaly is how to model the relationship between a point/subsequence and its temporal context for different types of anomalies. Secondly, like other anomaly detection tasks, anomaly happens rarely, and there is usually limited labeled data for data-driven models. A possible solution is data augmentation, which is widely used in deep learning training [45]. Although some data augmentation methods have been proposed for time series data [53], how to design and apply data augmentation in time series anomaly detection remains an open problem.

As a typical signal, the time series data can be analyzed not only from the time domain but also from the frequency domain [18]. Most of the existing methods, including conventional and deep methods, focus mainly on time-domain modeling and do not fully utilize the information in the frequency domain. The frequency domain can provide vital information for time series, such as seasonality [52]. In addition, it is much easier to detect in the frequency domain than in the time domain for some complex group anomalies and seasonality anomalies. Recently, there have been some attempts to model time series from the frequency domain [31], such as the
data augmentation in the frequency domain [14]. Unfortunately, how to systematically and directly utilize the frequency domain and time domain information simultaneously in modeling time series anomaly detection is still not fully explored in the literature.

In this paper, to better detect various kinds of time series anomalies, we proposed a Time-Frequency domain analysis time series Anomaly Detection model, TFAD. It mainly contains two branches: the time-domain analysis branch and the frequency domain analysis branch. Specifically, with a well-designed window-based model structure, we implement a time series decomposition module to detect anomalies in different components with interference among different components reduced and a representation learning module with a neural network to gain richer sequence information. To deal with the challenges of insufficient anomaly data, we conduct data augmentation of TFAD in different views: normal data augmentation, abnormal data augmentation (not fully considered in existing works), time-domain data augmentation, and frequency domain data augmentation.

In summary, our main contributions are listed as follows:

- We integrate the frequency domain analysis branch with the time domain analysis branch to identify the temporal information and improve detection performance.
- We combine the time series decomposition module with a concise neural representation network. With the help of time series decomposition, a simple temporal convolution neural network performs well. Moreover, it makes the model easy to be implemented, and the anomaly results of different components give insights into why it is abnormal.
- Various data augmentation methods, besides normal data augmentation and time domain data augmentation, abnormal data augmentation and frequency domain data augmentation are also implemented to overcome the lack of anomaly data.

The rest of the paper is organized as follows. In Section 2, we review the related work. In Section 3, we briefly introduce the definitions of time series anomalies. In Section 4, we introduce our proposed TFAD algorithm, including motivations, architecture, and network design. In Section 5, we evaluate our algorithm empirically on both univariate and multivariate time series datasets in comparison with other state-of-the-art algorithms. An ablation study is also performed to analyze different modules in the network. And we conclude our discussion in Section 6.

2 RELATED WORK

Both traditional and deep methods have been applied in time series anomaly detection tasks. A survey of traditional techniques for time series anomaly detection is in [17]. Traditional techniques can be roughly classified as similarity-based [16, 28], window-based [13], decomposition-based [14, 51], deviations detection based methods [30], et al. With the rapid development of deep learning, a vast of deep anomaly detection methods emerge [19, 26, 39], including one-class type SVDD-based model [40], reconstruction type GAN/VAE-based model [37], et al. Deep methods are also applied in time series anomaly detection [44, 57, 63].

According to the input data type, there are methods designed for univariate time series, multivariate time series, or both. According to the access of labels, there are unsupervised, semi-supervised, and supervised time series anomaly detection methods. Online anomaly detection [4, 7] is also quite different from offline settings. For univariate time series anomaly detection, many traditional methods have been designed [35, 47, 55]. POT [47] proposes an approach based on Extreme Value Theory to detect outliers without the assumption of distribution. M-ELBO [55] designs a VAE-based algorithm on the Yahoo dataset. For multivariate time series anomaly detection, the development of representation learning stimulates blowout growth in the multivariate time series anomaly detection field. THOC [44] proposes a temporal classification model for time series anomaly detection by capturing temporal dynamics in multi scales. It follows the one-class classification framework generally used [39, 40]. Anomaly Transformer [57] designs unsupervised anomaly detection methods combining the transformer framework with a point-wise prior association by a mini-max strategy. Generative models, such as DAGMM [63], AnoGAN[42], and LSTM-VAE [32], also form a mainstream research direction. Some surveys compare different kinds of anomaly detection methods in time series [3, 12].

The most related work of this paper is [5], which introduces a window-based framework for anomaly detection in time series applied in unsupervised/supervised and univariate/multivariate settings. However, like most existing anomaly detection methods, it only works in the time domain. Although it works well for point-wise anomalies, it is usually hard to detect complex pattern anomalies, e.g., a sub-sequence time series. Instead, AutoITS [43] also takes the advantage of frequency domain analysis for period/seasonal forecasting. In summary, our designed TFAD model contains two branches: the time-domain analysis branch and the frequency domain analysis branch, which is different from existing works. Besides, the time series decomposition module used in traditional methods is also implemented in our TFAD architecture. Furthermore, several well-designed data augmentation methods are also considered in TFAD to gain rich, reasonable, and reliable dataset.

3 PRELIMINARIES

In this Section, we will first give a brief view of the definitions of general anomalies. Anomalies appear in many situations. Anomalies in time series are a special type as the point without information of neighbor points means nothing. Such characteristics make time series analysis different from others. Thus, we will also show the definitions of time series or more specially, saying, sequential anomaly definitions.

3.1 General Anomaly Definitions

In applications with non-sequential data, set $D \in \mathbb{R}$ as the data space and the normality follows distribution $N^*$. Assuming $N^*$ has a corresponding probability density function $p^*$, then the anomalies can be set as

$$A = \{d \in D \mid p^*(d) \leq \tau\}, \tau > 0,$$ (1)

where $\tau$ is the threshold of anomaly.

Anomalies in non-sequential data can be mainly classified into three types: point anomaly, contextual anomaly, and group anomaly.
A point anomaly is an individual data point that deviates from normality and is the most common case in anomaly detection. It can also be called a global anomaly. A context anomaly is also called a conditional anomaly. It is anomalous in a specific context. For instance, the temperature of 20 degrees is normal in most areas but abnormal in Antarctica. A group or collective anomaly is a group of points that abnormal.

### 3.2 Sequential Anomaly Definitions

Time series data is a sequence of data points \( X = (x_0, x_1, x_2, \cdots, x_n) \) where \( x_i \) is the point at timestamp \( i \). More specially, time series can be formally defined by structural modeling [27, 46] to include trend, seasonality and shapelets, as

\[
X = \sum_n \{A \sin(2\pi \omega_n T) + B \cos(2\pi \omega_n T)\} + \tau(T), \tag{2}
\]

where \( T = (1, 2, 3, \cdots, n) \) is a series of timestamps, \( \omega_n \) is the frequency of wave \( n \), \( A, B \) are coefficients, the combination of sinusoidal wave represents the shapelets and seasonality, and \( \tau(T) \) is trend component.

The definitions in a non-sequential context can not sufficiently define the group and context anomalies in sequential data. Instead, sequential anomalies can be classified as point anomaly (global point anomaly and context point anomaly) and pattern anomaly (shapelet anomaly, seasonal anomaly, and trend anomaly) [27], as shown in Figure 1.

Formally, point-wise anomalies can be defined as \( |x_t - \hat{x}_t| > \sigma \) where \( \hat{x}_t \) is the expected value and \( \sigma \) is threshold. Shapelet anomaly can be defined as \( s(\rho(\cdot), \hat{\rho}(\cdot)) > \sigma \), where function \( s \) measures the difference between two subsequences, \( \hat{\rho} \) is the expected shapelet and \( \sigma \) is a threshold. Seasonal anomaly refers to subsequence with abnormal seasonality and it is defined as \( s(\omega, \hat{\omega}) > \sigma \) where \( \hat{\omega} \) is expected seasonality. Trend anomaly is a subsequence whose trend alters the trend of \( X \). It can be defined as \( s(\tau(\cdot), \hat{\tau}(\cdot)) > \sigma \) where \( \hat{\tau} \) is the expected trend of subsequence.

A point anomaly is an individual data point that deviates from normality and is the most common case in anomaly detection. It can also be called a global anomaly. A context anomaly is also called a conditional anomaly. It is anomalous in a specific context. For instance, the temperature of 20 degrees is normal in most areas but abnormal in Antarctica. A group or collective anomaly is a group of points that abnormal.

### Figure 1: Anomalies in time series.

(a) shapelet anomaly  (b) seasonal anomaly  (c) trend anomaly  (d) global point anomaly  (e) context point anomaly

### Figure 2: Examples of point-wise anomaly in time and frequency domain.

(a) Point-wise anomaly  (b) Diff in time domain w/o point-wise anomaly  (c) Diff in frequency domain w/o point-wise anomaly

### 4 PROPOSED TFAD ALGORITHM

#### 4.1 Motivation: Time-Frequency Analysis

In this part, we will provide the design motivation of TFAD algorithm through the uncertainty principle of time-frequency analysis and demonstrate its effectiveness in time series anomaly detection.

##### 4.1.1 Uncertainty Principle for Time Series Representation in Time and Frequency Domains

The uncertainty principle expresses a fundamental relationship between the standard deviation of a continuous function and the standard deviation of its Fourier transformation [8]. Let the input signal \( s(t) \) have a spectrum \( S(\omega) \) as

\[
S(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} s(t) e^{-j\omega t} dt. \tag{3}
\]

The standard deviations of the time and frequency density functions, \( \sigma_t \) and \( \sigma_\omega \), are defined as the parameters that describe the broadness of the signal in time and frequency domains, respectively

\[
\sigma_t^2 = \int (t - \hat{t})^2 |s(t)|^2 dt, \quad \sigma_\omega^2 = \int (\omega - \hat{\omega})^2 |S(\omega)|^2 d\omega. \tag{4}
\]

Then we have the following result based on Schwarz inequality

\[
\sigma_t^2 \sigma_\omega^2 \geq \left| \int ts(t)s'(t) dt \right|^2 = \left| t^2 + j \text{Cov}_{t\omega} \right|^2 = \frac{1}{4} + \text{Cov}_{t\omega}^2. \tag{5}
\]

When we view the uncertainty principle from time series anomaly detection, it means that when one structure has a good catch of point-wise anomaly in the time domain, it would have a less sensitive detection power in the frequency domain, and vice versa. Designing a model with a single structure that can detect the time and frequency simultaneously is difficult. A better approach could be using different structures to detect them and merge them as a whole model.

##### 4.1.2 Understanding the Limitations of Single Domain Analysis

This subsection will show the significant difference in point-wise anomaly detection and pattern-wise anomaly detection intuitively by two simple examples and demonstrate the limitations of detecting anomalies only in the time or frequency domain. The illustration examples are plotted in Figure 2 and Figure 3, where Figure 2 shows the differences between the detection of point-wise anomaly in time domain (fig 2(b)) and frequency domain (fig 2(c)) by a global point anomaly example (fig 2(a)), and Figure 3 shows the differences between the detection of pattern wise anomaly in time domain...
works where augmented data should follow the original data distribution, noted as diff, between the original point (noted as Ori) and the point before it (noted as Past-1). Figure 2(b) and figure 3(b) show the different results in time domain of the point-wise anomaly example and pattern-wise anomaly example, respectively. It can be seen that point anomaly is easier to be detected than seasonality anomaly in time domain analysis. Detecting seasonality anomalies only through time-domain analysis is not easy.

For anomaly detection in the frequency domain, we compare the time series results with/without anomalies after time to frequency transform (by Fourier transform). Figure 2(c) shows results of the time series in frequency domain with and without point-wise anomalies. The difference is subtle and disperses in many channels, indicating it is hard to detect point-wise anomaly only with time domain analysis. Figure 3(c) shows the results of time series in frequency domain with and without seasonality-wise anomaly. Unlike the situation in point-wise anomalies, the difference is quite evident as different numbers of peaks are shown. Thus, seasonality anomaly is easier to detect than point anomaly in frequency domain analysis. It is impractical to detect point-wise anomalies only by frequency domain analysis.

**4.2 Motivation: Data Augmentation and Decomposition**

Besides the time-frequency analysis, we also consider data augmentation and decomposition to further improve the performance of time series anomaly detection.

**4.2.1 Time Series Data Augmentation.** The performance of machine learning usually relies on many training data. However, in reality, labeled data is usually limited. Data augmentation [53] contributes a lot to help mitigate these challenges. Unlike most existing works where augmented data should follow the original data distribution, we consider two kinds of data augmentation methods for the anomaly detection task: Data augmentation for normal data and anomaly data. In the anomaly detection task, anomalies are samples different from normal data. There are usually various kinds of anomalies. Thus, when anomaly data is augmented, there is no need to create anomalies identical to real anomalies in the dataset, which is also impractical. Diverse augmented anomalies generally contribute to the robustness of models.

**4.2.2 Time Series Decomposition.** Generally speaking, time-series data often exhibit various patterns, and it is usually helpful to split a time series into main components. It is a powerful technology for analyzing complex time series widely adopted in time series anomaly detection [14, 20, 59] and forecasting [6, 56, 62]. Furthermore, with the help of time series decomposition, simple temporal convolution neural networks can bring desirable performance (this will be discussed later). It makes the model easy to be implemented and provides insights based on anomaly results of different components.

**4.3 High Level Architecture of TFAD**

Following the aforementioned motivations, the high level architecture of our designed TFAD algorithm is summarized in Fig 4. Our method consists of two main branches, the time-domain analysis branch and the frequency-domain analysis branch. Besides that, both normal data augmentation module and anomaly data augmentation module are designed to increase the robustness of our method. Furthermore, the time series decomposition module is adopted to better detect anomalies in different components and provide insights into the explanation of anomalies.

**4.4 Network Design of TFAD**

In this section, we provide the detailed design of the TFAD algorithm. The whole network structure of TFAD is plotted in Fig. 5, where each module will be elaborated in the following parts.

**4.4.1 Data Augmentation Module.** As discussed before, we consider both normal and anomaly data augmentation.

For normal data augmentation, firstly, we generate data with low noise, which is more normal. Robust STL [51] is a desirable option to get trend and seasonal information of time series data, and the residual, which is in some way noise, can be ignored. Such more normal data keep the innate character of normal patterns and have larger differences with anomalies. It is easier to distinguish more normal data from anomalies and helps our model learn the intrinsic quality of normal data. To create diverse normal data, we transfer time series to the frequency domain by Fourier transform and make small changes in both the imaginary and the real parts to gain new data. In this way, diverse data is augmented. An example is shown in Figure 6.

Besides the typical anomalies in the data set for anomaly data augmentation, other possible anomalies should also be considered for anomaly data augmentation. It helps to detect new anomalies and contributes to the robustness of our method. Similar results also appear in the computer vision field [41], which demonstrates
that relatively few random outlier exposure images help to yield state-of-the-art detection performance. Specifically, we consider several data augmentation methods. Point scale modification in the time domain is adopted for point change anomaly, which is the most common type. For context anomalies, point/short sequence exchange and a mix-up between two different time series are considered. For anomalies in seasonal and some other complicated anomalies, several different data augmentation methods in the frequency domain are used to generate various sequence anomalies.

**4.4.2 Decomposition Module.** There are different kinds of methods to make time-series decomposition, for example, moving averages, classical decomposition methods such as additive decomposition and multiplicative decomposition, X11 decomposition method [10], seat decomposition method [10], and STL decomposition method [38]. In this paper, Hodrick–Prescott (HP) filter [21] is adopted for time series decomposition since it is easy to implement and works well in the real world.

Denote time series \( y_t, t = \{1, 2, \ldots, T\} \) contains a trend component \( \tau_t \), a residual part \( \epsilon_t \). That is, \( y_t = \tau_t + \epsilon_t \). Then, in HP filter, the trend component can be obtained by solving the following minimization problem

\[
\min_{\tau} \left[ \sum_{t=1}^{T} (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} \left( \tau_{t+1} - \tau_t - (\tau_t - \tau_{t-1}) \right)^2 \right],
\]

(6)
where the multiplier $\lambda$ is the parameter that adjusts the sensitivity of the trend to fluctuation and can be adjusted according to the frequency of observations [34]. After decomposition, both trend and residual components are utilized to improve performance since different anomalies may appear in different components.

We mainly consider the decomposition method for univariate time series in TFAD. For multivariate time series, we decompose each time series sequence separately. It may not be the best method for multivariate time series decomposition. However, significant improvement is gained, and we will leave a special design for multivariate time series as future work.

4.4.3 Window Splitting Module. In the time series anomaly detection task, temporal correlations among observations are significant, and sequence anomalies are usually harder to be detected than point anomalies. Furthermore, even point anomaly is hard to be detected without temporal correlations. Therefore, we adopt time series window to better gain sequence-wise/temporal correlation information [5, 49]. Specifically, a full time series window and a context time series window are set to detect anomalies in a suspect sequence, where the full window consists of a context window and a suspect window, as illustrated in Figure 7. The assumption is that suppose the context window is normal. If the pattern of the full window is consistent with the pattern of the context window, then there is no anomaly in the suspect window. If there is an anomaly in the suspect window, the pattern of the full window will not be consistent with the context window. With sliding windows, the label of each time point can be known and more details are in Section 4.4.5.

4.4.4 Time and Frequency Branches. In this section, the time and frequency branches will be discussed. As shown in Fig 5, the original time series will be first decomposed into trend and residual components. For each component, we set the full window sequence and context window sequence with the aforementioned window splitting. After that, time-domain representation learning and frequency-domain representation learning for each window sequence will be done to gain rich information on sequences. After that, the distance between the context window and the full window would be measured to calculate the anomaly score.

For the representation, most of the classical distances, such as Cosine distance, dynamic time warping (DTW) distance, are too susceptible to the length of time series to be used here. Instead, neural networks are widely used to gain the representation of complex samples in many tasks due to the power of representation ability. Thus, we utilize a neural representation network to overcome the above shortcoming of classical methods. Specifically, the temporal convolutional network (TCN) [2] is a simple and powerful architecture. Therefore, we adopt the TCN design as our representation network.

TFAD first transforms the time series from time-domain to frequency-domain for the frequency branch by discrete Fourier transform (DFT). For time series $\{x_n\} := x_0, x_1, \ldots, x_{N-1}$, its discrete Fourier transform $\{X_k\} := X_0, X_1, \ldots, X_{N-1}$ is defined as

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-\frac{2\pi i}{N} kn} = \sum_{n=0}^{N-1} x_n \cdot \left[\cos\left(\frac{2\pi}{N} kn\right) - i \cdot \sin\left(\frac{2\pi}{N} kn\right)\right].$$

(7)

The results of DFT contain real-part (denoted as $Re$) and imaginary-part (denoted as $Im$). To better get the information of time series in different locations, TFAD intersects $Re$ and $Im$ and gets $\{X_k\} = F(x_n) = \{Re_1, Im_1, Re_2, Im_2, \ldots, Re_n, Im_n\}$. After obtaining DFT results, representation learning is done by TCN similarly to the time branch.

There are different transform methods for time-frequency translation besides DFT, such as continuous Fourier Transform (CFT), short-time Fourier Transform (STFT), and wavelet transform. DFT is more suitable for our situation: the time-series data is usually discrete. As the length of the anomaly sequence is unknown, the window length of STFT is hard to set. Wavelet transform is considered to contain time and frequency information simultaneously, but evaluation shows that it does not gain a good performance as DFT. The main reason may be that, with the time-domain branch added already, a DFT with frequency information can gain more marginal utility than a wavelet.

4.4.5 Anomaly Score Module. Comparison between full window sequence and context window sequence is used to set anomaly score, which is a widely used metric for the degree of anomalousness [39]. Here, the cosine similarity between full window sequence and context window sequence is set as anomaly score. Specifically, denote anomaly score as $AS$, then

$$AS = \mathcal{F}\left\{\text{dis}(RV_{treT}, RV_{resT}, RV_{treF}, RV_{resF})\right\},$$

(8)

where $RV_{treT}, RV_{resT}$ are representation results of trend component and residual component in time domain respectively, $RV_{treF}, RV_{resF}$ are representation results of trend component and residual component in frequency domain respectively, and the distance function $\text{dis}()$ is the cosine similarity. The higher the anomaly score is, the higher the dissimilarity is, which means the suspect window is more likely to be abnormal. We can decide whether the suspect window is abnormal with a threshold for anomaly score. Thus, suspect windows can be labeled with an anomaly or not. However, we also want to know what label should be set for every time point. Therefore, a voting strategy is adopted. With sliding full and context windows, the suspect window is also sliding. Every point belongs to several suspect windows, and if more than half of them are labeled as an anomaly, the point will be set as an anomaly.

5 EXPERIMENTS

This section studies the proposed TFAD model empirically compared to other state-of-the-art time series anomaly detection algorithms on both univariate and multivariate time series benchmark.
datasets. We also investigate how each component in TFAD contributes to the final accurate detection by ablated studies and discuss the insights.

5.1 Baselines, Datasets, Metrics, and Evaluation

5.1.1 Baselines. For univariate time series anomaly detection, we compare our method with the state-of-the-art algorithms, including SPOT, DSPOT [47], DONUT [55], SR, SR-DNN, SR-CNN [35], and NCAD [5]. For multivariate time series anomaly detection, we compare recent deep neural network models like AnoGAN [42], DeepSVDD [40], DAGMM [63], LSTM-VAE [32], MSQLRED [58], OmniAnomaly [48], MTAD-GAT [60], THOC [44]. Note that some baselines above are not designed for temporal data, but they are extended for time series data with fixed lengths by sliding windows.

5.1.2 Datasets. We adopt the widely-adopted univariate and multivariate datasets for time series anomaly detection as follows:

- **KPI** [9] is a univariate time series dataset released in the AIOps anomaly detection competition. It contains dozens of KPI curves with labeled anomaly points. The points are collected every 1 minute or 5 minutes from Internet Companies, for instance, Sogou, Tencent, eBay, etc.
- **Yahoo** [36] is a univariate time series dataset for time series anomaly detection released by Yahoo research. Part of the dataset is synthetic, where the anomalies are algorithmically generated. Part of it is real traffic data to Yahoo services, where the anomalies are labeled manually by editors.
- **SMAP and MSL** [22] are two multivariate time series datasets published by NASA. SMAP and MSL have 55 and 27 unique telemetry channels, respectively, that is, 55 and 27 dimensions time series. More specifically, anomaly sequences in SMAP are composed of 62% point anomalies and 38% contextual anomalies, while MSL consists of 53% point anomalies and 47% contextual anomalies.

The summary of these datasets is shown in Table 1.

| model  | Yahoo (un.) | KPI (un.) | KPI (sup.) |
|--------|-------------|-----------|------------|
| TFAD   | 81.13 ± 0.52| 79.80 ± 0.74| 82.10 ± 0.42|

5.1.3 Metrics. Point adjusted F1 score [1, 44, 48] is the widely used metric in the time series anomaly detection task. In this metric, if one point is detected as an anomaly in a segment, the whole anomaly segment will be considered as detected. This metric fits well with real-world situations as, in most cases, the anomaly event affects more than one time point. For such an abnormal event, a single anomaly alarm is enough. Note that several other F1-type metrics [15, 23–25] have been proposed to provide a more precise evaluation of abnormal event detection. Either the first alarm of group anomalies is set with higher importance, or the proportion of alarms is evaluated. However, in this paper, our primary aim is not to discuss which metric is the best as different metrics are applied in different situations. Thus, we adopt the widely used point-adjusted F1 score as our metric and leave evaluations on more different metrics for future work.

5.1.4 Evaluation Details. We follow the common setting as in [5, 35] for better comparisons. Specifically, we split each dataset into the train part, validation part, and test part to choose models. For the Yahoo dataset, we split 50% as test data, 30% as train data, and 20% as validation data. The original KPI dataset contains train and test data, and we set 30% of the training data as validation data. For the evaluation of the KPI dataset, we apply both supervised setting (sup) where all labeled data are utilized and unsupervised setting (un) where the label information is not utilized. Every setting has been run ten times, and the mean and variance are reported.

5.2 Performance Comparisons

The performance comparisons of different baseline algorithms and our TFAD are summarized in Table 2 and Table 3 for univariate and multivariate time series anomaly detection tasks, respectively.

For the univariate time series anomaly detection in Table 2, it can be seen that deep learning methods usually bring better performance than the conventional methods like SPOT and DSPOT, due to their strong representation abilities. Note that both NCAD [5] and our TFAD introduce data augmentation, and the randomness of augmented data brings in fluctuation in the F1 score. The variance of TFAD is significantly lower than NCAD in most cases, which indicates our TFAD method would be more robust and stable in practical systems. In summary, our TFAD algorithm produces comparable performance to the best NCAD algorithm on the Yahoo dataset, and significantly outperforms all other competing algorithms on the KPI dataset.

For the multivariate time series anomaly detection in Table 3, we only compare TFAD with recent deep neural network models, since the conventional non-deep methods exhibit worse performance due to the limited ability for modeling the complex interaction and nonliterary of multivariate time series data. Note that all algorithms adopt unsupervised setting since none of multivariate datasets provides labels in the training data. It is interesting to find that our method even outperforms the state-of-the-art algorithms THOC [44] and NCAD [5] by a reasonable margin. This is mainly due to our novel architecture with both time and frequency branches, while existing works detect anomalies only in the time domain. In summary, our TFAD algorithm achieves the best F1 score.
Table 3: F1 score of anomaly detection on multivariate time series datasets. The best results are highlighted.

| Model           | SMAP (un.) | MSL (un.) |
|-----------------|------------|-----------|
| AnoGAN          | 74.59      | 86.39     |
| DeepSVDD        | 71.71      | 88.12     |
| DAGMM           | 82.04      | 86.08     |
| LSTM-VAE        | 75.73      | 73.79     |
| MSCRED          | 77.45      | 85.97     |
| OmniAnomaly     | 84.34      | 89.89     |
| MTAD-GAT        | 90.13      | 90.84     |
| THOC            | 95.18      | 93.67     |
| NCAD            | 94.45 ± 0.68 | 95.60 ± 0.59 |
| TFAD            | 96.32 ± 1.57 | 96.41 ± 0.34 |

among all competing algorithms in both SMAP and MSL datasets for multivariate time series anomaly detection.

5.3 Ablation Studies

To better understand how each component in TFAD contributes to the final accurate anomaly detection, we conduct ablation studies in the KPI dataset under supervised setting, and the results are summarized in Table 4.

Firstly, when only time branch (base TCN model) or frequency branch (DFT followed by TCN) is adopted, it performs not well. Secondly, with the decomposition module added in the time branch, the F1 score gains nearly 30% improvement compared with the same base TCN model, which demonstrates the benefits of decomposition in TFAD model. Thirdly, with the extra normal data augmentation module added where the ratio of augmentation data is set as 0.5, additional marginal improvement can be achieved. Similar improvements can be obtained with the time-domain abnormal data augmentation module added where the ratio of augmentation data is set as 0.4. An interesting phenomenon is that, when both NormAug and TimeAnAug modules are added in the previous version, the improvement of the F1 score is more than the sum of improvement when they are added respectively. It can be explained that such two directions of data augmentation make the distance between normality and anomaly larger with a nearly multiplicative effect. Note that the ratios of augmentation data may not be the best hyperparameters, but their performance improvements are still obvious. Lastly, after the frequency branch are added (corresponding to the full TFAD model), not only the F1 score improves, but the variance decreases. The reason is that, only with the time-domain branch without frequency branch, some sequence-wise anomalies are hard to be detected, and overfitting usually appears in the train set, which affects the performance in the test set. These ablation studies demonstrate the effectiveness of our TFAD design with time-frequency branches, data augmentation, and decomposition.

5.4 Model Analysis and Discussion

In this section, we provide visualizations and case studies to explain how the model works intuitively and obtain insights.

5.4.1 Contribution of time series decomposition module. Figure 8 shows an example in the Yahoo data set to help understand how time series decomposition contributes to anomaly detection. Figure 8(a) is the original time series without time series decomposition. Figure 8(b) shows the results of Figure 8(a) after time series decomposition. The blue line is the residual component, the yellow line is the trend component, and the anomalies detected are labeled with red circles. Obviously, with the decomposition module, the anomalies are easier to detect.

What is more, in our TFAD architecture as shown in Figure 5, representation results can be gained for trend component and residual component independently, which makes it possible to obtain an anomaly score for each component and explain in which component anomaly happens.

5.4.2 Effect of special anomaly data augmentation. Results in Table 4 show that, with data augmentation added, performance can be improved. The anomaly data augmentation methods above are general and not designed for specific datasets. However, if prior information of the dataset is given, special anomaly data augmentation methods can be designed to take advantage of the pattern of anomalies for further performance improvements.

To demonstrate it, one case study is summarized in Table 5 on SMAP dataset. The observation is that the first dimension of SMAP contains slow slopes when anomalies appear. By utilizing this prior information, we conduct slow-slop injection on the first dimension of SMAP datasets as special anomaly data augmentation. With this specially designed data augmentation method, it can be seen in Table 5 that significant extra performance gain is achieved in the TFAD model.
Table 4: Ablation studies of the proposed TFAD on KPI dataset. Denote the time series decomposition module as Dec, the norm data augmentation module as NorAug, the time domain anomaly data augmentation module as TimeAnAug, the frequency domain anomaly data augmentation module as FreqAnAug, and the frequency domain analysis module as FreqBran. The proposed TFAD algorithm combines all these modules. The best results are highlighted.

| case       | TCN | Dec | NorAug | TimeAnAug | FreqAnAug | FreqBran | Precision | Recall | F1 score |
|------------|-----|-----|--------|-----------|-----------|----------|-----------|--------|---------|
| Freq Branch|      | ✓   |        |           |           | ✓        | 13.8 ± 3.08 | 35.97 ± 4.40 | 19.59 ± 2.81 |
| Time Branch| ✓   |     |        |           |           |          | 44.888 ± 0.095 | 57.227 ± 0.0381 | 50.312 ± 0.0569 |
| (a)        | ✓   | ✓   |        |           |           |          | 57.557 ± 5.374 | 81.111 ± 5.339 | 66.968 ± 2.58 |
| (b)        | ✓   | ✓   |        |           |           |          | 57.869 ± 4.65 | 80.49 ± 4.85 | 67.099 ± 3.075 |
| (c)        | ✓   | ✓   |        | ✓         |           |          | 62.569 ± 7.059 | 89.45 ± 7.84 | 72.942 ± 2.644 |
| (d)        | ✓   | ✓   |        | ✓         | ✓         |          | 68.528 ± 9.412 | 85.949 ± 11.3698 | 74.934 ± 2.908 |
| (e)        | ✓   | ✓   |        | ✓         | ✓         |          | 69.444 ± 7.133 | 86.638 ± 8.044 | 76.385 ± 2.387 |
| TFAD       | ✓   | ✓   |        | ✓         | ✓         | ✓        | 79.176 ± 1.875 | 85.231 ± 1.464 | 82.058 ± 0.4199 |

Table 5: Results with slow-slop injection on first dimension of SMAP datasets as special anomaly data augmentation.

| Model      | Precision | Recall | F1 score |
|------------|-----------|--------|----------|
| TFAD       | 91.90     | 89.32  | 90.32    |
| TFAD+injection | 94.04    | 98.36  | 96.09    |

6 CONCLUSION

This paper proposes a time-frequency analysis-based model (TFAD) for time series anomaly detection. Although most of the traditional and deep methods in time series anomaly detection have achieved great success, how to take advantage of the time-frequency properties of time series is not well investigated. Our design with time and frequency branches fills in this gap. Besides, time series decomposition is implemented to bring insights into the explainability of the proposed model as well as simplify the neural network design. Furthermore, we also adopt data augmentation to overcome the lack of labeled anomaly data.

Based on these considerations, the proposed TFAD with time-frequency architecture can handle the challenges of various anomalies in time series. Although no complex neural network architecture is implemented, we gain better performance than most existing deep models in time series anomaly detection. Extensive empirical studies with four benchmark datasets show that our TFAD scheme obtains a state-of-the-art performance. Furthermore, ablation studies show that TFAD gains not only higher accuracy but also lower variance for time series anomaly detection in both univariate and multivariate scenarios.

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