High-brightness single-crystal approach in quantum imaging with undetected photons

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Abstract. Quantum imaging with undetected photons is recently appeared promising branch of quantum optics. Due to properties of photon pairs (signal and idler), generated via spontaneous down-conversion (SPDC) process and phenomenon of induced coherence, one can obtain phase or intensity image of object without detecting of the photon, interacting with it. The theoretical treatment and limitations of such a system with an only crystal and increased brightness are given.

1. Introduction
Quantum imaging with undetected photons [1] arise from quantum interference experiment by Zou, Wang and Mandel [2,3] and their treatment of induced coherence without induced emission as an appearance of sine fringes in coincidence and single counting rates. Lemos et.al. proposed that image of the object could be subtracted from these fringes due to entanglement of SPDC-photon pairs (phase image) and the very idea of quantum interference – indistinguishability as its precondition [4]. This type of systems well could be implemented in spectroscopy under Abbe limit, biophysics and sensing. To improve this system further, higher signal-to-noise ratio (SNR) shorter image-acquisition time are demanded that explicitly means demand of sources with higher brightness [5].

2. Principle of proposed experiment
To provide higher brightness and, consequently, higher SNR and also make whole future system more cost-efficient, nonlinear crystal is placed in cavity, with mirrors M2-M3, aligned to reflect only idler mode (Fig.1). Idler photon propagates through the object and gains some phase information about it, then it reflects back to crystal and be means of the mirror M1, reflecting pumping photons, and the phenomenon of induced coherence and phase memory [2, 3], the new idler photon’s phase is “locked” by inducing field. Due to this fact and alignment of M4, reflecting signal mode, one can obtain sine fringes in the count rate of detector D [3]. This experimental arrangement is topologically similar to an experiment [6], and here its quantum imaging application feasibility was investigated, hence, mathematical treatment has common points.

Substantial increase of brightness is inextricably linked with presence of the mirrors M2-M3, that not only multiply the number of idler photons, inducing coherence, but also provide selection of vacuum states, which play essential role in probability of down-conversion and act like a “bridge” between photons in pair through of entanglement.
3. SPDC state and imaging

The process of SPDC and imaging was treated akin developed by Hong and Mandel one [7] and Lahiri et.al [4]: nonlinearity of non-centrosymmetric crystal leads to conversion of pump photon into correlated pair of photons with sum energy equal to the energy of pump photon (phasematching condition)

$$k_p = k_s + k_i$$
$$\omega_p = \omega_s + \omega_i$$

where $k_{p,i}$ and $\omega_{p,i}$ are pump, signal and idler wave vectors and frequencies respectively. Using approximation of strong enough classical pump field, SPDC-interaction Hamiltonian is

$$H_{in}(t) = \int_L d^3r \chi_{lmq} E_p(t) E_{sm} E_{iq}^{(-)} + H.c.$$ (2),

where $\chi_{lmq}$ is nonlinear susceptibility tensor, $L$ - crystal volume, $E_p$ - pump field, $E_{si}$ - negative-frequency parts of quantized signal and idler electric fields, respectively, $\chi_{lmq}$ - mutually orthogonal directions in space. Positive-frequency part of the field, obtained via second quantization formalism, is in general has the form

$$E^{(+)}_{ls}(r,t) = (E^{(-)}_{ls})^\dagger = \sum_{k,\sigma} A(k, \sigma) \exp \left( i(k_{ls}r - \omega_{ls}t) \right) a_{ls}(k_{ls}, \sigma)$$ (3),

where $i,s$ subscripts stand for idler signal photons and $A$ - for all constants “merged” together, $\sigma = 1,2$ is possible polarization direction, $\omega$ - frequency, $k$ - wave vector, $a(k, \sigma)$ - annihilation operator. As far as pump field is classical, then

$$E_p = \sum_{k,\sigma} V_p(k, \sigma) \exp \left( i(k_p r - \omega_pt) \right)$$ (4),

where $V_p$ stands for complex amplitude of pumping wave. The quantum state of the SPDC-field is described by well-known equation

$$|\psi(t')\rangle = \exp(-\frac{i}{\hbar} \int_0^t dt'H_{in} (t'))|0\rangle_i|0\rangle_s$$ (5).
Using (2)-(5), expanding exponential and neglecting second- and higher-order terms due to small amplitude of SPDC and considering spatially well-defined single mode case and propagation directions, we arrive to

\[ |\psi\rangle = |0\rangle_{s1} |0\rangle_{i1} + \alpha |1\rangle_{s1} |1\rangle_{i1} \]

(6), where \( \alpha \) stands for SPDC amplitude (\(|\alpha| \ll 1\)). As far as down-converted modes have strict angle intensity dependence, only mentioned modes will be considered further.

The key point in proposed experiment is to align idler modes \( i_{1,2} \) where subscripts stand for idlers generated due to down-conversion of pump field, propagating from left to right and retroflected back (Fig. 1), respectively. Meanwhile, the object (Fig. 1) can well be treated using quantum optical representation of beam splitter with transmission and reflection coefficients \( T \) and \( R \) (obviously, \(|T|^2 + |R|^2 = 1\)), respectively. So, annihilation operator for idler photon, generated by down-conversion of retroflected photon after alignment is

\[ a_{i2} = (Ta_{i1} + Ra_0) \exp(i\theta_i) \]

(7), where \( a_{i1} \) is annihilation operator for idler photon, generated by down-conversion of pump photon, propagating from left to right, \( \theta_i \) is the phase that \( i \) photon obtained while propagating through the object to mirror and back to crystal and \( a_0 \) stands for vacuum field on unused port of object-beam splitter.

Using (5)-(7), again, neglecting terms of order higher than one can find that final SPDC state is

\[ |\psi(t)\rangle = |0\rangle_{s1,i_1,i_2} + \alpha_1 V_p |1\rangle_{s1} |1\rangle_{i1} |0\rangle_{s2} + \alpha_2 V_p e^{-i\beta(T^*|0\rangle_{s1}|1\rangle_{i1}|1\rangle_{s2}|0\rangle_0 + R^*|0\rangle_{s1}|0\rangle_{i1}|1\rangle_{s2}|1\rangle_0} \]

(8).

As far as crystal is the same, \( \alpha_1 = \alpha_2 \). Count rate of detector \( D \) (i.e. intensity of one mode using perfect-detector approximation) is

\[ R_s = \langle \psi_s \mid E_s^{(-)} E_s^{(+)} \mid \psi_s \rangle \propto 2|\alpha_1 V_p|^2 |T|(1 + \cos(\theta_i + \theta_{s1} + \theta_{s2} - \theta_p)) \]

(9), where \( \theta_{s,i} \) is phase shift of both signal photons obtained during propagation to mirror M4 and to detector D, \( \theta_p \) stands for phase accumulated by pump photon while propagating to mirror M1 and back. Analysing (9) one can find that counting rate exhibits interference with visibility proportional to \( T \). All together it makes possible to obtain phase and intensity picture of the object from interference fringes without detecting of the photons, interacted with the object. Obviously, interference disappear if the object is opaque.

4. Brightness enhancement

The main advantage of quantum imaging with undetected photons counterpart - classical imaging with undetected photons, proposed by Shapiro et. al. [5] (nevertheless disadvantages, that are intrinsic for this kind of systems, such as absence of phase image acquisition possibility in principle), is higher SNR ratio and lower acquisition time. Both these features are the consequence of high brightness light sources, that are used in classical imaging systems and [5] gives clear analysis of these dependencies.

To explain increase of brightness of down-converted light source, let us analyse (9). Actually, it reflects the bosonic nature of photons and dependence of down-conversion probability on the mode population. Considering perfect alignment of mirrors M2 and M3, idler modes \( i_1 \) and \( i_2 \), could be treated as one mode with annihilation operator, described by (7), let us write the population of mode after down-conversion as

\[ N_s = F(N_i + 1) \]

(10), where \( N_{s,i} \) - population of signal and idler modes, \( F = \frac{4\pi^2}{c^2} \omega_i \omega_s |V_p|^2 = \beta^2 l^2 \) - efficiency of down-conversion [8], \( l \) - crystal length. It is clear from (10), that population of signal mode rises with population of idler mode. However, if one consider the case of signal and idler mode interaction while propagating in nonlinear crystal (detailed calculation could be found in [8]), then

\[ F = \beta^2 \frac{\text{sh}^2 l \left[ \frac{\beta^4 - \frac{\Delta^2}{4}}{\beta^4 - \frac{\Delta^2}{4}} \right]}{\beta^2 - \frac{\Delta^2}{4}} \]

(11),
where $\Delta = k_s + k_i - k_p$. Analysing (11) one can find the pump intensity threshold of SPDC to induced down-conversion, or superluminescence, transition that is of pivotal importance because it let us to estimate the threshold population of $N'_i$. Moreover, after this transition, photon pairs are no more entangled by vacuum, that makes impossible phase image acquisition. It is clear from (11) that if $|\beta| \gg 1$, then

$$N_s = \text{sh}^2 |\beta| \approx \frac{e^{2|\beta|}}{4}$$

(12).

To find the threshold population of $N'_i$, one need to equate (10) to (12) simply

$$N'_i = \frac{e^{2|\beta|}}{4F} - 1$$

(13).

Taking in a count, that $N_i$ rises constantly due to presence of mirrors M2 and M3, the care should be taken about their proper reflection coefficient, avoiding mentioned transition to parametric oscillator regime.

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