Phase of Aharonov-Bohm oscillations in conductance of mesoscopic systems

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Motivated by a recent experiment we analyze in detail the phase of Aharonov-Bohm oscillations across a 1D ring with a stub coupled to one of its arms, in the presence of a magnetic flux. We demonstrate that there are two kinds of conductance extremas. One class of them are fixed at particular flux values and can only change abruptly from a maxima to a minima as incident energy is varied. We show a different mechanism for such abrupt phase change in conductance oscillation. We demonstrate that these extremas can exhibit “phase locking”. However, the second kind of extremas can shift continuously as the incident energy is varied.
Transport in mesoscopic samples has been studied extensively over the last decade. Technological developments has helped us to fabricate samples of sizes smaller than the single particle coherence length of the electron. Quantum mechanical interference effects drastically affect the transport in such systems and many non trivial phenomenon has been observed. Of particular interest are the universal conductance fluctuations, quantization of point contact conductance \cite{2}, normal state Aharonov-Bohm effect, \cite{3} current magnification effect, \cite{4} etc. A recent experiment \cite{5} reports some striking features of the conductance across an Aharonov-Bohm (A-B) ring with a quantum dot situated in one of its arms. The observations of the experiment are as follows. Transport through the dot in the Coulomb blockade regime has a coherent component. The phase of conductance oscillations change by $\pi$ over a finite energy scale as the Fermi energy or incident energy crosses the resonances of the dot. This scale is much smaller (an order of magnitude smaller) than the scale in which the phase of the wavefunction changes as the Fermi energy crosses the resonance of the dot. The observations were theoretically analyzed in ref \cite{6,7,8}. It has been argued \cite{6} that if at all the phase of conductance oscillations change, the change should be absolutely sharp i.e., should take place over an energy scale of zero width, or else it would mean a break down of micro-reversibility. However a finite width was indeed observed in the experiment. In ref. \cite{6} this was accounted to effects like noise and fluctuations. Other works \cite{7} attribute the finite width to the non-linear response. Ref \cite{6} tried to demonstrate such an abrupt change in the phase of conductance oscillations or parity of conductance oscillations \cite{6} by a model calculation in 1D. The effect can be physically explained as
follows. A-B effect of normal electrons in a ring cannot be interpreted in terms of partial waves propagating along the two arms of the ring. The ring has some bound states that exhibit strong resonance phenomenon in transport when the ring is coupled to leads. The resonances shift with the magnetic field. If it shifts away from the Fermi energy then conductance will decrease, and vice versa. So there will be an abrupt change of $\pi$ in phase of conductance oscillations as the Fermi energy crosses the resonance of a ring.

We give a completely different mechanism of such an abrupt change in the parity of conductance oscillations. This mechanism exhibits the change in parity as the Fermi energy crosses the resonance of the dot (as observed in the experiment) and not the resonance of the ring. We believe that this point is important for the following three reasons. i) Thermal smearing length in the experiment was estimated to be comparable to the ring’s length and it is unlikely that the resonances of the ring can manifest themselves in the real situation. However, the study of these resonances help us in understanding of the phase changes in the conductance oscillations. ii) If the phase of conductance oscillations is determined by the bound states of the isolated system then we do not expect any regular behavior at consecutive resonances because the E versus $\alpha$ curve of two consecutive bound states need not have opposite slopes. iii) Our mechanism is related to the coherent scattering by the actual geometry of the dot used in the experiment. So, it is likely that such parity changes are present in the actual experiment. iv) Besides we show a possible cause of not observing abrupt change of parity of conductance oscillations within the framework of Landauer formula without the violation of microreversibility.
A schematic diagram of the system on which we do the model calculation is shown in fig. 1. A one dimensional ring is connected to two ideal leads on two sides. The other ends of the ideal leads are connected to a reservoir. If the chemical potential of the reservoir on two sides are unequal then there will be a transport current through the ring. A stub or a side arm is situated on one arm of the ring. A flux $\phi$ pierce through the center of the ring. As a result an electron picks up a phase $\alpha = 2\pi \phi/\phi_0$ in going round the ring once. Here $\phi_0 = hc/e$ is the elementary flux quantum. The total circumference of the ring is $l = p + q + r$, where the various lengths $p$, $q$ and $r$ are denoted in the figure. The length of the stub is $L$. Throughout in our calculations we have chosen the parameters $p/l = 0.25$, $q/l = 0.25$, $r/l = 0.5$ and $L/l=1$. The geometry is a one dimensional representation of the actual system with which the experiment was performed. Persistent currents in such coupled geometries has been already studied in some details [10]. We solve for the transmission across this system exactly using the free electron wave guide theory on networks [11]. We use Griffith’s boundary conditions at the junction of the stub and the ring and the hard wall boundary condition at the end of the stub. This allow us to calculate the scattering matrix at the junctions from the first principles. Total transmission can be calculated analytically, Expression being too long to reproduce here, we analyze our results graphically. Transmission coefficient is directly related to the transmission conductance $G$ by Landauer formula and dimensionless conductance is $g = G/(2e^2/h)T$.

In fig 2 we have plotted the conductance ($g$) versus $\alpha$ for three values of incident energy or dimensionless Fermi wave vector $kl$ i.e., $kl=(\pi - 0.01)$ (thin
line), \( \pi \) (thick line), and \( \pi + .01 \) (thickest line). At \( kl=\pi \) the isolated stub has a bound state. We find that at the exact value \( kl=\pi \) there is no conductance oscillation. On the two sides of \( kl=\pi \) i.e., at \( kl=\pi + .01 \) and \( kl=\pi - .01 \) the conductance oscillations are out of phase by \( \pi \) or belong to opposite parity class. Both have a periodicity of \( 2\pi \) and both are symmetric in \( \alpha \) as required by microreversibility. This change of parity occurs for infinitesimal change of Fermi energy \( kl \) on the two sides of the value \( kL=\pi \) i.e., the change of parity occurs over an energy scale of zero width. Note that at \( \alpha=0 \) ring has no bound state at \( kL=\pi \) but the nearest ones are at \( kl=0 \) and \( 2\pi \). This sudden phase change can be understood if we map the stub into a delta function potential. It is known that the delta function potential is \( V(x) = k \cot(kL) \delta(x - x_0) \) [12] where \( x_0 \) is the position of the stub. At \( kL=\pi \) the effective potential is infinite. This implies that there is no propagation along the dot arm of the ring and so no interference induced by the magnetic flux. This is why the conductance oscillations disappear for this particular value of the Fermi energy. On the lower side of this value i.e., at \( kl=\pi - .01 \), the effective potential is an attractive delta function potential and on the higher side of this value of the Fermi energy i.e., at \( kl=\pi + .01 \) it is a repulsive delta function potential. This discontinuous change in the strength of the effective potential changes the phase of the wavefunction on the upper arm discontinuously by \( \pi \) and hence changes the parity of the conductance oscillations. Note that this sudden phase change also breaks the parity effect of persistent currents in isolated system [13]. But this parity change in conductance oscillations is not related to the slope of the eigenenergy.

In a certain Fermi energy range before the resonance is crossed the energy
dependent effective delta function potential changes very rapidly because \( \cot(kL) \) change very rapidly around the value \( kl=n\pi \). This also happens in a certain energy range after the resonance is crossed. This rapid change in the effective delta function potential incorporates large phase changes in the wave function of the electron. The phase of transmission amplitude across the stub is \( \theta = \arctan(\cot(kL)/2) \) and it is plotted in fig. 3 in the range \( kl=\pi+.01 \) to \( kl=\pi+.13 \) i.e., on the higher side of the resonance at \( kl=\pi \). The figure shows that the phase changes by a large amount in this range but the change is less than \( \pi \). Surprisingly, this phase change which is less than \( \pi \), cannot change the phase of the conductance oscillations. This is shown in fig. 4. Both for \( kl=\pi+.01 \) (thin line) and \( kl=\pi+.13 \) (thick line) the maxima occur in the same positions and so do the minima. The phase of the conductance oscillations get locked and cannot be affected by the phase changes created by the stub. This “phase locking” was conjectured in ref \[6\] and we can demonstrate it with an explicit example. Due to the non-locality of the electron, change in effective potential in the dot arm not only changes the phase of the wavefunction in the dot arm but also the phase of the electron wavefunction in the other arm. These phase changes balance each other in such a way that the parity of the conductance oscillations remain unchanged. Thus at these extremas the parity of the conductance oscillations can only change abruptly independent of the amount of phase acquired in traversing the dot. If this were true at all maxima and all minima then it is not possible to see a finite width in which the parity of the conductance oscillations change. But in the following we show that there can be some extremas that do not exhibit “phase locking” and can shift their positions
gradually as the incident energy is varied, without violating the condition that the conductance is symmetric in flux.

However if we increase the Fermi energy further away from $\pi$ then additional conductance extremas can appear. This is shown in fig 5 where we have plotted the conductance ($g$) versus $\alpha$ for $k_l=\pi - .2$ (thin line) and $k_l=\pi + .2$ (thick line). We find that for $k_l=\pi + .2$ at the point marked A there is a maxima at which there is no extrema for $k_l=\pi - .2$. So there can be new maximas opposite to which there is no minima and vice-versa. These new extremas are not restricted by micro-reversibility and can change in any fashion. They can change continuously in contrast to the extremas considered in figure 2 that can only change abruptly. This is shown in fig. 6 where we plot conductance ($g$) versus $\alpha$ for $k_l=\pi + .2$ (thin line) and $k_l=\pi + .3$ (thick line). The maximas are slightly shifted with respect to each other. Sometimes these new extremas are very prominent compared to the extremas that remain fixed in position. When the thermal smearing width is as large as the separation between resonances the small resonances may merge with the more prominent ones. If the resonances that survive are the ones that can shift their positions as the incident energy is varied, then it is not unlikely that maximas change to a minima in a finite range of incident energy.

At this point we want to stress that we are not interested in the actual origin of these second kind of extremas. There can be various reasons for their origin. Depending on the Fermi energy sometimes the conductance oscillations can show a $\phi_0/2$ periodicity and sometimes a $\phi_0$ periodicity [13]. So if for some energies the conductance oscillates more with the flux than at some other Fermi energy then we can have maximas opposite to which
there is not always a minima and vice versa. These additional extrema can also appear because as the energy is varied the strength of the effective delta potential also varies which results in changing the effective arm lengths of the ring. So, the system itself changes effectively as the incident energy changes. However, the point that we want to stress is that microreversibility does not guarantee that the extrema are always fixed in positions. Had they been so it would not be possible to see the parity of conductance oscillations changing over a finite energy range within the framework of Landauer conductance formula.

Finally in figure 7 we plot the conductance (g) versus incident wave vector kl when the flux through the ring is zero. It shows a very complicated sequence of conductance maxima and also conductance zeros. But they do not match with the resonances of the ring or zeros of the stub. The leads and the coupling of the dot to the ring changes them drastically. For example the isolated dot exhibits zero conductance at kl=π but the combined system show a large transmission.

The main result of our work can be summarized as follows. The conductance oscillations can exhibit two kinds of extrema. One class of them can change their parity abruptly as the incident energy is varied or cannot change their parity at all. The other class of extrema can change their parity continuously. Our study makes it possible in principle to observe the effect as observed in the experiment of Yacoby et al. We also show a different mechanism for abrupt change in parity of conductance oscillations.

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References

[1] permanent address: Institute of Physics, Bhubaneswar 751005, India.

[2] Quantum Coherence in Mesoscopic Systems, Vol 254 of NATO advanced study Institute, Series B; Physics, edited by Kramer B (Plenum, New York) 1991.

[3] Washburn S and Webb. R. A, Adv. Phys. 35 375(1986).

[4] A. M. Jayannavar and P. Singha Deo, Phys. Rev. B 51, 10175(1995); T. P. Pareek, P. Singha Deo and A. M. Jayannavar, Phys. Rev. B, 52 14657(1995).

[5] A. Yacoby, M. Heiblum, D. Mahalu and H. Shtrikman Phys. Rev. Lett. 74 4047(1995).

[6] L. Yeyati and M. Büttiker Phys. Rev. B 52 R14360(1995)

[7] G. Hackenbroich and H. A. Weidenmüller (unpublished)

[8] C. Bruder, R. Fazio and H. Schoeller (unpublished).

[9] P. A. Sreeram and P. Singha Deo (unpublished).

[10] P. Singha Deo, Phys. Rev B 52 5441(1995); T. P. Pareek and A. M. Jayannavar, Phys. Rev. B (in press); F. Pasceud and Montambaux (unpublished).
[11] A. M. Jayannavar and P. Singha Deo, Mod. Phys. Lett B, 8 301(1994); 
A. M. Jayannavar and P. Singha Deo, Phys Rev. B 49 13685(1994). P. 
Singha Deo and A. M. Jayannavar, Phys. Rev. B 50, 11629(1994)

[12] P. Singha Deo Phys. Rev. B (in press)

[13] J. D’Amato, H. M. Pastawski and J. F. Weisz, Phys. Rev. B 39, 
3554(1989).
Figure Captions

Fig. 1. Open metallic ring coupled to two electron reservoirs in the presence of magnetic flux $\phi$. A side arm of length $L$ is connected to one of the arms.

Fig. 2. Plot of conductance versus $\alpha$ for various values of incident dimensionless wave vector $k_l$.

Fig. 3. Plot of the phase of the transmission amplitude $\theta$ across the isolated stub versus $k_l$.

Fig. 4. Plot of conductance versus $\alpha$

Fig. 5. Plot of conductance versus $\alpha$

Fig. 6. Plot of conductance versus $\alpha$

Fig. 7. Plot of conductance versus $k_l$ in the absence of magnetic flux.