A simple Stochastic SIR model for COVID-19 Infection Dynamics for Karnataka after interventions – Learning from European Trends

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Abstract—In this short note we model the region-wise trends of the evolution to COVID-19 infections using a stochastic SIR model. The SIR dynamics are expressed using Itô-stochastic differential equations. We first derive the parameters of the model from the available daily data from European regions based on a ’x’-day history of infections, recoveries and deaths. The derived parameters have been aggregated to project future trends for the Indian subcontinent, which is currently at an early stage in the infection cycle. The projections are meant to serve as a guideline for strategizing the socio-political counter measures to mitigate COVID-19. This article considers the latest data for Europe and India.

I. INTRODUCTION

COVID-19 has brought misery to many regions in the world, especially in Europe. However, it has just taken off in other parts of the world. The remaining regions, including India hosts huge population approximately more than 50% of the world population. There is an eminent danger and most of these population are under the poverty-line or the countries do not have huge resources for medical interventions for these population especially in Europe. However, it has just taken off in other parts of the world. The remaining regions, including India hosts huge population approximately more than 50% of the world population. There is an eminent danger and most of these population are under the poverty-line or the countries do not have huge resources for medical interventions for these population especially in Europe.

The dynamics governing the evolution of the COVID-19 infections have been modeled using a stochastic differential equation SIR model [1]. The parameters of this model have been initially optimized for a set of European regions, individually. An aggregate of these parameters have then been used for projecting the future trends for the Indian region, specifically for Karnataka state. Multiple projections have been generated by varying the exposure factor $E_f$ that influences the growth rate of infections. This is done in order to provide insights for selective quarantining and lockdowns.

II. STOCHASTIC SIR MODEL

The evolution of COVID-19 infections in each region, has been modeled via the stochastic susceptible-infected-recovered (SIR) model [1] which is given as,

$$
\begin{align*}
    ds(t) &= -E_f\beta S(t)C(t)dt \\
    dC(t) &= (E_f\beta S(t)C(t) - \gamma C(t))dt + \sigma C(t)dW_t \\
    dR(t) &= \gamma C(t)dt - \sigma C(t)dW_t,
\end{align*}
$$

(1)

$$
\begin{align*}
    C(0) &= C_0, \quad S(0) = S_0 \\
    S(0) &= P_{total} - R(0) - C(0).
\end{align*}
$$

(2)

Model states

1) $t$ is a daily-time parameter.
2) $C(t)$ denotes the number of active infections at time $t$.
3) $S(t)$ denotes the total susceptible population at time $t$.
4) $R(t)$ denotes the total number of recoveries and deaths at time $t$.
5) $dC(t), dS(t), dR(t)$ denotes the change in the states at time $t$.
6) $dW_t$ is an incremental Weiner process (Brownian motion), which models the randomness in the evolution.
7) $E_f \in [0, 1]$ which multiplies $\beta$ is the exposure factor which models the interventions to subdue infection spread, such as lockdowns, quarantining, and preventive measures.

Model parameters

- Growth rate: the constant $\beta$ denotes the growth rate, which factors the rise in the number of infections, due to interactions between susceptible and infected population. This parameter is a lumped constants which is meant to account for: (a) the population size, (b) reproduction number $R_0$ of COVID-19, and (c) exposure-factor (which depends on mobility, precautionary measures, etc.).
- $\gamma$ is the rate of outcomes, i.e., the rate at which the infections are neutralized, which may be due to recovery.
TABLE I: Model parameters assumed. These numbers are slightly pessimistic.

| Region    | $P_{\text{total}}$ | $\beta$         | $\gamma$         | $\sigma$         |
|-----------|---------------------|-----------------|------------------|------------------|
| Germany   | $8.28 \times 10^7$  | $4.6 \times 10^{-10}$ | $0.005(1-28)$, $0.045(29-39)$ | 0.01             |
| Spain     | $4.67 \times 10^7$  | $7.5 \times 10^{-9}$  | 0.06  | 0.02             |
| France    | $6.7 \times 10^7$   | $4.4 \times 10^{-9}$  | 0.04  | 0.02             |
| Italy     | $6.055 \times 10^7$ | $4.5 \times 10^{-9}$  | 0.04  | 0.025            |
| India     | $133.92 \times 10^7$| $1.75 \times 10^{-10}$ | 0.02  | 0.01             |

or death. It is assumed that recovered persons would not spread the infections again (at least for a window of a month).

- $\sigma$ is a parameter used to model the stochasticity or randomness in the evolution, which may cause local deviations from the typical (exponential) trends.
- $P_{\text{total}}$ is the population of the region. $C_0$ and $S_0$ are initial number of infections and susceptible individuals.

III. Parameters based on European trends

The parameters of the SIR model were optimized based on the data obtained for different European regions and India. The criterion for optimization was to simultaneously minimize the square integral error, terminal error and terminal rate error, between the actual data and daily samples of the simulated data. Further, because we have data for more number of days for European countries, we try to use the parameters from those countries and appropriate it on the Indian data which is for lesser number of days. The assumption is that India may be in the catch game (which, of course, we do not want) if the behaviour of people is taken to be similar. We have to base our predictions based on some gross assumptions under the given circumstances.

Note: We have taken the Italian and German data 4 days earlier than France and Spain in order to reflect the earlier trend before their respective lockdown conditions. This will enable us to simulate for various levels of lockdown percentages (exposure factor).

IV. Simulation results

The SIR model has been simulated and the parameters have been optimized based on the infection trends obtained for European countries and India (after 15th March, when the infections started to show an exponential trend). The stochastic differential equations have been simulated using the Euler-Maruyama numerical integration method [2]. We present below how our model follows the statistics from various European countries.

![Fig. 1: SIR model for Germany. Day 1 = 24th Feb 2020. Interventions after day 26 resulted in $E_f = 0.28$](image1)

![Fig. 2: SIR model for Spain. Day 1 = 28th Feb 2020. Interventions after day 23 resulted in $E_f = 0.45$](image2)

V. Projections for India

A. Nationwide projection

The SIR model has been simulated beyond the ‘$x$’-days of considered data for India, with parameters averaged over the European regions considered above, as well as with the parameters obtained for the Indian region. This is done in order to demonstrate the possible growth of infections in India, based on the current trend so far (which is an optimistic case), or based on the European trend (which is the case to be prepared for). The exposure factor $E_f$ has been varied to scale down the rate of infections $\beta$. This factor illustrates the additional reduction in mobility due to state-wide lock-down and quarantining measures, which are essential for maintaining new infections within reasonable limits. For India we have two sets, (i) using the available data only from India and (ii) taking
Fig. 3: SIR model for France. Day 1 = 28th Feb 2020. Interventions after day 21 resulted in $E_f = 0.6$.

Fig. 4: SIR model for Italy. Day 1 = 24th Feb 2020. Interventions after day 19 resulted in $E_f = 0.45$.

Fig. 5: SIR model for India. Day 1 = 15th March 2020. Intervention: day 8 to day 15; $E_f = 0.6$, day 15 to day 22; $E_f = 0.6$ (lockdown violations), day 22 onwards $E_f = 0.6$. Projections are done from day 23 onwards (when the parameters were fixed) in order to assess the validity of the model. This figure will be continuously updated.

Fig. 6: Projections for India with varying rates of exposure using Indian parameters. Since we already have the data for previous days, our prediction was from the Day-1 = 3rd April.

Let us summarize very briefly what we observe.

- Below 25% exposure required for avoiding exponential growth for Karnataka, as well as for India using both Indian parameters and averaged European parameters.
- Fig. 9 suggests the inevitable need for an **Extended lockdown period for several months** for Karnataka, or until a vaccine is available.
- By ‘lockdown’ we mean restricted mixing i.e. fewer susceptible individuals $s(t)$ interact with fewer infected individuals $c(t)$, thereby introducing a reduction in the

Note: We have four graphs in each set. For example, when we use $E_f = 1$, that means there is no lockdown. When we use $E_f = 0.5$ exposure half of the country/state is in lockdown. This is to show the gravity of the situation if the lockdown is not respected.

**B. Karnataka Projections**

The average European parameters have been used to project the future trends for the state of Karnataka. The initial number of infections is assumed taken as 113 (3rd April) to account for additional unreported or undetected infections, over reported ones. We have made the assumption here that lockdown is in place and we have taken varied percentage of people obeying the rules.
FIG. 7: Projections for India with varying rates of exposure using Average European parameters. Since we already have the data for previous days, our prediction was from the Day-1 = 3rd April.

FIG. 8: Projections for Karnataka with varying rates of exposure using averaged European parameters. Initial active infections taken as 113 as on 3rd April. The random fluctuations are dominant in the fourth quadrant, due to lower $E_f$.

FIG. 9: Projections for Karnataka for 365 days with varying interventions using European average parameters.

VI. VARYING LOCKDOWN SCHEDULES

The usual tendency is to relax a bit when the situation is under control to address the economic issues. However in the case of COVID-19, we can not do that because we would be just postponing the issues. This we can see in the Figs 10 to 16. To understand what can happen if the lockdown is relaxed by 50% for sometime and again the lockdown is imposed. For example, we close all the schools, restaurants and theaters and allow only workers to move out. This idea looks like a good compromise since essential work and people having difficulties because of lockdown can carry out their work. Further, only 50% of the population are allowed. To understand such a situation, we simulated the ON-OFF model with 50% duty cycle.

Simulations are done by varying $E_f$ over time, in order to understand the infection response to time varying lockdown schedules over 6 months for Karnataka. Note that a lower $E_f$ could also indicate a mobile society, but with appropriate preventive measures, not just purely lockdown.

The various response curves have been plotted to serve as a guideline for selecting a particular lockdown schedule, or a mix of schedules, such that the infections are minimized simultaneously with the socio-economic losses incurred due to the lockdown. However, consistently the data shows that the effect of such a relaxation will only to shift the peaking and it does not help in any ways except getting a few months for preparation initially. We know that considering the scales for India we will not be able to handle even after six months the numbers that we see from the simulations. The trend is that after low infections over initial cycles, the infections will fluctuate with large magnitude. This method could be adopted if good medical support can be guaranteed. A similar simulation can be found in [4]. Our results Figs 10 to 16 are for Karnataka with initial condition of 113 infections on 3rd of April 2020.

Thus the only concrete step is to lockdown till we localize the cases with 99.999% accuracy and then allow people to cautiously start resuming their work.

- Note: The previous version of this draft uploaded on arxiv on 29th march (v2) predicted a total number of 10080 active infections in India, 30 days from 15th of March (See Fig.8 of v2 of [3]). As on 14th April, the number of recorded active infections is 9872. The number of active infections on 29th March was 902.
Fig. 10: For Karnataka with initial condition of 113 infections on 3rd of April 2020. 95% Lockdown for two weeks and 50% relaxation (for example schools are closed throughout, essential businesses are open).

Fig. 11: 3 weeks cycle similar to Fig. 10.

Fig. 12: 4 weeks cycle similar to Fig. 10.

Fig. 13: 5 weeks cycle similar to Fig. 10.

VII. Conclusions
This is a continuous work in which we are trying to find the model parameters everyday and project the possible scenarios, by varying the exposure factor for the rate of infection, as a result of evolving levels of quarantining. While this is not a completely verifiable projection, the model parameters look quite consistent, however they may reflect an over estimation of the projected number of infections in order to compensate for the unreported or undetected infections. Thus, we may take the numbers in this note to be a guide for further action by the law enforcing authorities.

References
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Fig. 14: 6 weeks cycle similar to Fig. 10.

Fig. 15: 7 weeks cycle similar to Fig. 10.

Fig. 16: 8 weeks cycle similar to Fig. 10.

Fig. 17: 6 weeks cycle for one year.

Fig. 18: 4 weeks cycle for one year.