Self-Organized Criticality in Particle Production

A. Paramonov, A. Rostovtsev

Institute f. Theoretical and Experimental Physics, ITEP, Moscow, Russia

Abstract

Self-Organized Criticality paradigm is a plausible picture for hadron production. A power-law behavior of hadron transverse momentum spectra and an approximate scaling observed for different hadrons in high energy hadronic collisions are discussed.

Multi-particle production in high energy hadronic collisions is understood in Quantum Chromodynamics as a result of irradiation and recombination of many strongly interacting quarks and gluons. A system of hadronizing soft partons is an open dynamical system (the number of partons is not conserved) with many degrees of freedom which is far from equilibrium. Even though we can calculate exactly an elementary parton-parton interaction, a complexity of the system of hadronizing partons doesn’t allow the extension of these calculations to predict particle states, in which the vast majority of the particles are produced with low transverse momenta ($P_t$). In addition, cascading decays of produced hadronic resonances make the picture even more complex. In practice, particle production in high energy hadronic collisions is modeled using statistical techniques.

The seemingly structureless multi-particle final states, however, are found to display regularities, namely, long-range particle-particle correlation and self-similarity in the particle spectra. Interestingly, a similar behavior is found for a broad class of phenomena including turbulence, earthquakes,
forest fires, cloud formation, etc. The phenomena from this class are described by power-law distributions. To explain the occurrence of power-laws in nature P.Bak et al.\cite{1} have proposed a Self-Organized Criticality (SOC) paradigm. In the SOC the dissipative systems spontaneously evolve into barely stable critical states, with long range correlations. The critical state is distinguished from the fully deterministic states by the response of a system to an external perturbation. A reaction of deterministic system is described by a characteristic response at a given scale depending on the system’s parameters. The distribution of the resulting events is narrow and is well described by an average value. For a critical system, the same perturbation can lead to an event of any scale. Although large power events are comparatively rare, the mechanism is the same to explain the rare large events and the smaller, more common ones. This scaling behavior is described by a power law distribution.

In it’s relatively short history SOC patterns have been found in myriads of fields including biology, sociology, computer science, economics, etc. The SOC models are widely used in modern physics to describe processes, such as occurrence of solar flares, earthquakes, and windstorms. The SOC gives rise to fractal dimensions of natural objects. The ubiquitous “fingerprints” of the SOC states are a power-law distribution of the events and long-range correlations. In high energy physics the long-range correlations in particle production have been found and studied in details before\cite{2}. In the present paper we discuss a power-law behavior of the transverse momentum spectra of different hadrons measured in collider experiments.

At low transverse momenta an exponential behavior of the hadronic spectra has been observed. It was interpreted using a thermodynamic analogy, with single particles distributed within a hot hadronizing matter according to R. Hagedorn\cite{3}

$$E \frac{d^3\sigma}{d^3p}_{y=0} \sim \sqrt{m^2 + P_t^2} \cdot exp\left(-\frac{\sqrt{m^2 + P_t^2}}{T}\right),$$

where $m$ is a particle mass and $T$ is characteristic temperature of the interaction\cite{2}. The data on high-$P_t$ particle production show a significant deviation

\footnote{Since the most of the data on hadron production are available for central rapidities ($y_{lab} \approx 0$) we consider the invariant differential cross sections for central rapidity only.}
from the exponential form towards a power-law

\[ E \frac{d^3 \sigma}{d^3 p} \sim \left( \frac{1}{P_t} \right)^n, \tag{2} \]

anticipated by the perturbative QCD calculations for parton scattering. These calculations, however, fail in the low-\( P_t \) region. An alternative statistical hadronization model \cite{4} based on the formation and decay of pre-hadronic clusters was found to give a good data description within a broad range of values of hadron \( P_t \), though it fails to describe the interactions with collision energy larger than 30 \( GeV \) \cite{5}. To describe high energy data within the whole \( P_t \) interval Gaźdзicki \textit{et al} \cite{6} have suggested a new statistical parameterization of hadron spectra which approximates the exponential form at low-\( P_t \) and the power-law at high-\( P_t \)

\[ E \frac{d^3 \sigma}{d^3 p} \bigg|_{y=0} \sim \left( \frac{1}{\sqrt{m^2 + P_t^2}} \right)^D. \tag{3} \]

In practice, to describe the hadron spectra an empirical power-law function is widely used

\[ E \frac{d^3 \sigma}{d^3 p} \bigg|_{y=0} \sim \left( \frac{1}{P_0 + P_t} \right)^D. \tag{4} \]

The form (4) is a very close approximation of the following expression

\[ E \frac{d^3 \sigma}{d^3 p} \bigg|_{y=0} \sim \frac{\sqrt{m^2 + P_t^2}}{(T_0 + (q - 1)\sqrt{m^2 + P_t^2})^q}. \tag{5} \]

obtained in a framework of non-extensive statistical mechanics \cite{7, 8}. In (5) the \( T_0 \) is a temperature of hadronizing matter, \( m \) is a hadron mass and \( q \) is an entropic index. For \( q \to 1 \) the expression (5) reduces to the Hagedorn form (1). The non-extensive statistical mechanics generalizes the standard thermodynamic approach for a presence of a long-range strong interaction and is deeply related to the SOC \cite{9}. It was shown \cite{10, 8} that non-extensive statistical model describes the hadron spectra in \( e^+ e^- \) interaction. It is of interest to investigate which parameterization from (3) to (5) is preferred by the high energy hadron collider data.

To compare the quality of the data description by the forms (3)-(5) we fitted these forms to the charged particle invariant cross section measured
in the UA1 experiment [11]. These data are selected as they cover a broad interval of $P_t$ values starting from very low $P_t$. The UA1 data are shown in Fig. 1(a) as function of $P_t$. Fig 1b and Fig 1c show the relative differences between the data and the fit results using form (3) with free mass parameter $m$ and the form (4) correspondingly. It is seen from the Fig. 1 that the form (3) doesn’t describe well the shape of the charged particle $P_t$ distribution, while the power-law form (4) is in a good agreement with the data over almost whole $P_t$ interval except of the very high $P_t$. Similarly good agreement with the data was found for the expression (5), however, for simplicity, we use the form (4) only in the rest of the paper.

The power-law behavior of the data is a formal signature of the SOC pattern. The SOC model itself doesn’t predict the values of the parameters $P_0$ and $D$. These parameters are defined by the underlying QCD dynamics. The exponent $D$ has to match the pQCD calculations at large $P_t$. Since QCD is flavor blind the exponent $D$ is likely to be the same for different hadrons.
produced at the same collision conditions. If the SOC approach is applied to particle production, $P_0$ defines a minimal scale at which the $P_t$ scaling starts to be broken and is likely to be related to the mass (as the only one dimensional macroscopic parameter available) of the produced hadron. To test this hypothesis we compare the spectra for different long-lived hadrons assuming, for simplicity, $P_0$ in (4) to be equal to the hadron mass $m$. A compilation of the invariant cross sections for $\pi^+$ [12], $K^+$, $\Lambda$ [13], $D^+_s$ [14] and $D^{*+}$ [15] measured in photoproduction at HERA ($\sqrt{s_{\gamma p}} \approx 200$ GeV) is shown in Fig.2 as function of $(m + P_t)$. To make the comparison of different species of hadrons the cross sections are given for the particle’s isospin and spin projections. The pions were not identified in the HERA experiments and have been recalculated from the measured charged particle spectrum by reducing this spectrum by 40% to take into account an admixture of kaons, protons and long lived charged leptons. Surprisingly, the spectra shown in Fig.2 for different hadrons can be described with a good approximation by single power-law function. The production mechanism for different long lived particles and for different intervals of $P_t$ is self-similar, i.e. once established for charmed mesons, this mechanism is valid for light pions also at low-$P_t$. We test further this universality with the high energy $p\bar{p}$ interaction. In Fig.3 the invariant cross sections for $\pi^+ [17]$, $K^0 [18]$, $D^{*+} [19]$ and $B^+$ [20] measured in $p\bar{p}$ collisions at the Tevatron ($\sqrt{s} = 1800$ GeV) are presented as function of the $(m + P_t)$ scaling variable. The Tevatron data show an approximate scaling behavior for the $\pi$, $K$ and $D^*$ mesons similar to that found at HERA $\gamma p$ collisions.

As seen from the Fig. 3 the $B$-meson invariant cross section is a factor of 10 higher than that of the lower mass hadrons when plotted at the same values of $(m + P_t)$ variable. At HERA, though the $B$ mesons are not yet directly reconstructed, the $B$-meson inclusive cross section was estimated using the Lund model [21] with a normalization to the $b$-quark production cross section measured at HERA [22]. The result of these calculations is shown in Fig.2 with a shaded band. The band width is defined by the experimental uncertainty of the $b$-quark production cross section measurement. The calculated $B$-meson yield at HERA is again a factor of 10 higher than that expected for other hadrons shown in Fig. 2. Noteworthy, the QCD calculations for open beauty production strongly underestimate the measured cross section at the

---

2Recently, the H1 has reported [16] the preliminary proton photoproduction cross section which supports the observed scaling.
Tevatron and HERA \cite{23} as well. Recently, a new parton $k_t$-factorization approach was used to describe the open beauty production \cite{24}, but it is still to be demonstrated that this approach can be consistently applied to other hadron species. Within the statistical approach the observed excess of $B$-meson production over the universal behaviour of the scale dependence of other mesons signals a different dynamics or the existence of an additional mechanism of $B$-production.

In summary, we noted that particle production in high energy hadronic collisions has the formal properties of a Self-Organized Critical process, namely: a) the hadronizing partons are known to represent an open strongly interacting dynamical system far from equilibrium – a typical system to deal with the SOC; b) the resulting hadronic state shows long-range particle-particle correlations; c) the particle spectrum is described by a power-law distribution as function of transverse momentum and mass of the particle; d) this distribution is the same for different long-lived hadrons and is defined by macroscopic parameters like mass and spin of the particle as expected for statistical models. Whether the latter is a pure coincidence or a demonstration of a statistical (alike SOC) nature of the hadron production is an open question. To prove an eligibility of the SOC mechanism in particle production, a successful quantum cellular automaton describing a quark-gluon system has to be built. It is possibly could be done in the future using QCD lattice calculations.

For completeness we give also the earlier applications of the SOC in high energy physics known to us. The SOC was suggested \cite{25} to describe inelastic diffractive scattering. Fractal properties of the hadronic collisions have been discussed in \cite{26}. In \cite{27} a concept of fractal dimensions of the proton structure function was introduced. This gave an excellent description of the low-$x$ HERA data, both in the non-perturbative and the deep-inelastic domain. These observations together with the self-similarity in particle production discussed in this paper make the SOC approach to be a candidate for an effective picture to describe a variety of non-perturbative QCD phenomena.

\section*{Acknowledgments}

The work was partially supported by Russian Foundation for Basic Research, grant RFBR-01-02-16431 and grant RFBR-00-15-96584.
References

[1] P.Bak, C.Tang and K.Wiesenfeld, Phys.Rev.Lett. 59, 381, 1987.
[2] E.A. De Wolf, I.M. Dremin and W. Kittel, Phys.Rept. 270, 1, 1996.
[3] R.Hagedorn, Suppl.Nuovo Cimento 3, 147, 1965.
[4] F.Becattini and U.Heinz, Z.Phys., C76, 269, 1996.
[5] F.Becattini, L.Bellucci and G.Passaleva, Nucl.Phys.Proc.Suppl. 92, 137, 2001.
[6] M.Gaździcki and M.I.Gorenstein, Phys.Lett. B 517, 250, 2001.
[7] C.Tsallis, J.Stat.Phys., 52, 479, 1988.
   C.Tsallis, Brazilian J. of Rhys., 29, 1, 1999.
[8] C.Beck, Physica A286, 164, 2000.
[9] C.Tsallis, e-Print Archive: cond-mat/0205571
[10] I.Bediaga, E.M.F.Curado and J.M. de Miranda, Physica, A286, 156, 2000.
[11] UA1 Collab., G.Bocquet et al, Phys.Lett. B 366, 434, 1996.
[12] ZEUS Collab., M.Derrick et al, Z.Phys. C67, 227, 1995.
[13] H1 Collab., C. Adloff et al, Z.Phys. C76, 213, 1997.
[14] ZEUS Collab., J. Breitweg et al, Phys.Lett. B481, 213, 2000.
[15] ZEUS Collab., J. Breitweg et al, Phys.Lett. B 401, 192, 1997.
[16] D. Ozerov, In Proceedings of the DIS2002 conference, Cracow,Poland, May 2002.
[17] CDF Collab., F. Abe et al, Phys.Rev.Lett. 61, 1819, 1988.
[18] CDF Collab., F. Abe et al, Phys.Rev. D 40, 3791, 1989
[19] V.Papadimitriou, FERMILAB-CONF-00-234-E, Published in *La Thuile 2000, Results and perspectives in particle physics* 185.
[20] CDF Collab., F. Abe et al, Phys.Rev.Lett. 75, 1451, 1995.

[21] B. Andersson et al., Phys. Rep. 97 31, 1983.

[22] H1 Collab., C. Adloff et al, Phys.Lett. B 467, 156, 1999.

[23] O. Behnke, In Proceedings of the 31st International Symposium on Multiparticle Dynamics (ISMD 2001), Datong, China, 2001. e-Print Archive: hep-ph/0111405.

J. Tseng, In Proceedings of the 16th Les Rencontres De Physique De La Vallee D’Aoste, La Thuile, Aosta Valley, Italy, 2002.

[24] A.V. Lipatov, V.A. Saleev and N.P. Zotov, e-Print Archive: hep-ph/0112114.

H. Jung, Phys.Rev. D65, 034015, 2002.

[25] C. Boros, et al, Phys.Rev. D61, 094010, 2000.

[26] I. Zborovsky, M.V. Tokarev, Yu.A. Panebratsev and G.P. Skoro, Phys.Rev. C59, 2227, 1999.

[27] T. Laštovička, DESY-02-036, e-Print Archive: hep-ph/0203260.
Figure 2: The invariant cross-sections for different long-lived hadrons as function of $m + p_T$, for rapidity $y = 0$ as measured in the laboratory system at H1 and ZEUS detectors. The cross-sections are given for one spin and isospin projections. The shaded bar corresponds to the $B$-meson cross section estimated from the measurement of inclusive $b\bar{b}$-production.
Figure 3: The invariant cross-sections for different long-lived hadrons as a function of $m + p_T$, for rapidity $y = 0$ as measured in the laboratory system at CDF detector.

\[
\sqrt{s_{\bar{p}p}} = 1800 \text{ GeV}
\]