Method Article

Transient convection experiments in internally-heated systems

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\textbf{Abstract}

Radioactive decay of unstable isotopes is one of the main heat sources in the early stages of planetary formation as well as in the mantle of terrestrial planets. Laboratory studies characterized by Rayleigh and Prandtl numbers in the range relevant for planetary bodies had remained beyond the ability of the experimental approach until the development of a new technique based on microwave heating. Using this technique, we performed a series of experiments focused on the thermal evolution of an internally heated viscous fluid cooled from above. We established a steady-state scaling law relying the internal temperature variation to the Rayleigh number and we showed that this scaling law remains valid during the transitory regime provided both internal heating and secular evolution of the temperature are taken into account. The result is a parameterized model describing the average internal temperature of the fluid as a function of time in terms of experimental conditions and fluid properties.

- We generated a uniform and stable volume heat source in a large volume tank, based on absorption of microwaves guided through an innovative design of microwave circuits.
- Automatic laser scanning of the tank coupled with image acquisition and processing enables us the measurement of the 3D temperature field in the convective fluid from which we extracted the volume average temperature and surface heat flux evolution in time.
- We validated a transient scaling law for the time evolution of the volume average temperature in an internally-heated convective system.

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Methodological details

Thermal convection in isoviscous fluids generated by bottom heating and top cooling (Rayleigh-Bénard convection) is characterized by two dimensionless numbers: the Rayleigh number and the Prandtl number. The Rayleigh number, Ra, defines the strength of convection and represents the ratio between the driving thermal buoyancy force and the opposing thermal and viscous dissipation effects:

\[ Ra = \frac{\rho g \alpha \Delta T d^3}{\kappa \mu}, \]  

(1)

where \( \Delta T \) is the temperature difference between the bottom and the top of the layer, \( d \) the layer thickness, \( g \) the acceleration of gravity, \( \rho \) the fluid density, \( \alpha \) the thermal expansivity, \( \kappa \) the thermal diffusivity and \( \mu \) the dynamic viscosity. Convection starts when Ra exceeds a critical value that depends on the geometry and the boundary conditions [1] and the convective pattern exhibits a series of transitions toward chaos as Ra increases. The second parameter, the Prandtl number, is a material property and represents the ratio of the fluid momentum diffusivity relative to the fluid heat diffusivity:

\[ Pr = \frac{\nu}{\kappa}. \]  

(2)

where \( \nu = \mu / \rho \) is the kinematic viscosity. When \( Pr >> 1 \) inertial effects are negligible compared to viscous effects and the fluid motion stops as soon as the heat source disappears. This is the case in laboratory experiments with \( Pr > 300 \) [2], and obviously valid for silicate planetary bodies, where \( Pr > 10^{23} \).

In the case of purely internally heated convection, the temperature scale is imposed by the internal heating rate:

\[ T_H = \frac{H d^2}{k}. \]  

(3)

where \( H \) is the heat generated per unit volume and \( k \) is the thermal conductivity. The resulting Rayleigh-Roberts number [3] is:

\[ Ra_H = \frac{\rho g \alpha H d^5}{k \kappa \mu}. \]  

(4)

Predicting the behavior of complex natural systems such as planetary mantles requires the determination of scaling laws derived from first principles relating some parameters of interest to the other parameters on which they depend. These functional relations are generally tested using either experimental or numerical results. They are applicable to natural systems only if the dynamic
similarity is respected, i.e. same thermal and mechanical boundary conditions, similar rheology of the fluid and similar dimensionless numbers.

**Transient laboratory experiments in the microwave oven**

To study the transient thermal evolution of an internally heated convective fluid, we performed eleven analogue experiments with different fluids and with different heating rates and initial temperatures (Table 1).

The working viscous fluids are obtained by the addition of between 0.3 wt% to 0.7 wt% of Hydroxyethyl cellulose (a thickener known as Natrosol) to distilled water. The fluid density and thermal expansion were measured with a DMA 5000 Anton Paar densimeter, its rheology was characterized with a RS600 Thermo Scientific Haake rheometer [4], and its thermal diffusivity was measured by the photopyroelectric method [5]. The absolute value of the viscosity at a reference temperature depends strongly on Natrosol concentration. Its activation energy obtained by fitting the viscosity by an Arrhenius law depends only slightly on the thickener concentration and has a value of $37 \pm 1$ kJ mol$^{-1}$ for the fluids used in this set of experiments. Experimental values of Prandtl number ($400 \ll \text{Pr} \ll 10^5$) are large enough for viscous effects to prevail over the inertial ones [2].

The prototype using a unique set-up of microwave (MW) internal heating of aqueous fluids [6,7] is shown in Fig. 1. A commercial microwave oven (Whirlpool AMW 848IX, cavity volume 40 l) was drastically modified to perform lab-scale experiments suited for the study of mantle convection. All the original electronic parts were removed, except for the magnetron and its power supply. A specially designed MW waveguide - homogenizer system was installed inside the oven’s chamber producing a homogeneous internal heating of the sample (contained in an optically and MW transparent tank covered with a cooling plate). The magnetron output power was monitored and its variations were compensated using proprietary, feedback based command and control hardware and software [8]. All MW circuits were maintained at constant temperature through a specially designed heatsink ensuring a stable output power deposited into the sample as heat. The various openings made into the walls allowing the flow visualization and thermostated bath fluid circulation were equipped by MW filters that ensure the safety of the user.

The experiments are performed in a $30 \times 30 \times 5$ cm$^3$ and 1 cm thick poly(methyl-methacrylate) tank so the bottom and lateral boundaries are insulating. The top surface is an aluminum plate whose temperature is fixed at $T_0$ (Table 1) using a thermostated bath. Once the fluid had reached a constant temperature, the microwave source was turned on at a prescribed power.

In situ temperature determinations in the fluid were performed using two methods: for experiments 1 to 9 by using thermochromic liquid crystals (TLC), and for experiments 10 and 11 via laser induced fluorescence (LIF) using a combination of two fluorescent dyes. Both methods were carefully calibrated as described previously (see for instance [9] for TLCs and [10] for the two-dyes

### Table 1

| Exp# | $\mu$ (Pa s) | $H$ (10$^4$ W m$^{-1}$) | $T_0$ (°C) | $T_\infty$ (°C) | $R_{\text{th}}$ | $T_0$ (°C) |
|------|--------------|-------------------------|------------|----------------|----------------|-------------|
| 1    | 1.63         | 1.11±0.39               | 18.0       | 10.4±1.7      | 6.3 $10^4$ | 46.3        |
| 2    | 1.70         | 3.56±0.38               | 6.7        | 25.4±0.9      | 2.2 $10^5$ | 148.3       |
| 3    | 0.55         | 1.91±0.09               | 16.4       | 10.9±0.3      | 3.4 $10^5$ | 79.6        |
| 4    | 0.54         | 2.00±0.14               | 16.4       | 11.9±1.3      | 3.5 $10^5$ | 83.3        |
| 5    | 0.26         | 3.11±0.32               | 16.4       | 16.4±1.2      | 1.3 $10^6$ | 129.6       |
| 6    | 0.18         | 3.55±0.38               | 16.0       | 13.5±0.2      | 2.0 $10^6$ | 147.9       |
| 7    | 0.17         | 5.67±0.48               | 12.0       | 20.0±0.2      | 3.5 $10^6$ | 152.9       |
| 8    | 0.06         | 3.49±0.26               | 18.0       | 10.3±0.2      | 5.6 $10^6$ | 145.4       |
| 9    | 0.06         | 3.71±0.48               | 12.1       | 16.1±0.2      | 9.2 $10^6$ | 154.6       |
| 10   | 0.07         | 2.22±0.12               | 18.7       | 8.0±0.2       | 3.0 $10^6$ | 92.5        |
| 11   | 0.11         | 1.24±0.07               | 9.0        | 6.3±0.1       | 6.5 $10^6$ | 51.7        |
Fig. 1. Microwave heating prototype: 1: cameras, 2: motorized scanning device, 3: laser entry point, 4: tubes to thermostated bath, 5: cooling plate, 6: tank, 7: MW antenna.

Fig. 2. Typical thermal structure (in °C) imaged in the course of experiment 11.

LIF). These methods give similar results as proven in [11]. Spatial resolution is set by the 0.2 mm pixel size of the digital camera, enabling excellent resolution of the temperature profile in thermal boundary layers at the top of the tank, which were always thicker than 2 mm. A laser sheet scans half of the tank whilst one (TLC) or two CCD cameras (LIF) acquire images in different spectral ranges. These 2D temperature maps can be used to reconstruct the 3D temperature field. An example of the 3D temperature field is shown in Fig. 2 for experiment 11, showing several cold downwellings starting from the top-boundary layer and a diffuse return flow.
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Fig. 3. Vertical profiles of the temperature in the tank for experiment 11. Profiles are taken every 5 min from green (0 min) to blue, red and black (150 min). The profiles tend to a steady state value in which the fluid layer is in equilibrium with its heat sources.

We extract vertical profiles of the horizontally-averaged temperature. Their evolution in time for experiment 11 is shown in Fig. 3; colors indicate time evolution from green to blue, red and black. The vertical temperature profile in the convecting layer can be split into a unique, upper thermal boundary layer and a convective interior. In the purely internally heated convection there is no basal thermal boundary layer since no heat is supplied from below. An important feature is that the convective fluid interior has a slightly negative temperature gradient.

We measured the time evolution of the average internal temperature over the entire volume T (Fig. 4) of the fluid and the heat flux $Q_k$ at its top using a least-squares linear fit to the uppermost three temperature values (Fig. 5). Both temperature and heat flux records tended towards steady-state values, noted $Q_{\infty}$ and $T_{\infty}$, that have been determined by exponential fitting. At steady-state, the integrated heat flux at the top surface is equal to the total amount of heat released in the tank interior, as required in equilibrium conditions (i.e. $P=Q_{\infty}S=HxV$, $Q_{\infty}=Hxd$). The internal heating rate in Table 1 represents the value determined from the steady-state heat flux. Errors on the determination of the steady-state heat flux values translated to errors on the heating rate $H$ and corresponded to the uncertainty of the exponential fit. This uncertainty can be larger that the experimental error if the steady-state is not completely reached. The experimental error on each individual experimental point is of the order of few percent since the temperature is measured with a precision of the order a few tenths of degree, and the position error is of the order of a pixel, which represents 0.2 mm. The viscosity value in Table 1, which is also used to calculate the value of $Ra_H$ corresponded to the value at $T_{\infty}$. Error bars on the dimensionless temperature indicated in Fig. 4 cumulate the uncertainty on the temperature determination and the heating rate uncertainty through the calculation of $T_H$.

A parameterized model for the thermal evolution of an internally-heated convective system

In the case of Rayleigh-Bénard convection, where convection results from cooling at the top of the system and heating at its base, scaling laws are expressed using the Nusselt number, Nu, defined as
Fig. 4. Time evolution of the experimental internal temperature during experiments. Dots represent experimental values and lines are theoretical prediction using Eq. (9) with $\beta=1/4$. The time was made dimensionless using the conductive timescale $t_c = d^2/\kappa$.

Fig. 5. Time evolution of the experimental output power during experiments. Dots represent experimental values and lines are theoretical prediction using Eq. (11) with $\beta=1/4$. The time was made dimensionless using the conductive timescale $t_c = d^2/\kappa$. 
Fig. 6. Comparison between the evolution as a function of Ra\textsubscript{H} of the experimental average internal temperature at steady state (points), made dimensionless using the temperature scale T\textsubscript{H}, and the theoretical scaling law Eq. (5) with an exponent \( \beta = 1/4 \) and proportionality constant \( c = 3.58 \) (line).

the ratio between the surface heat flux measured in the convective regime and the surface heat flux in the conductive regime. At steady state, Nu scales as the 1/3rd power of the Rayleigh number of the well-mixed convective layer. In the case of purely internal heating, at steady state the Nusselt number is unity [12] and the average internal temperature at steady state in an internally heated convective fluid [13] is:

\[ \frac{T_\infty}{T_H} = c \text{Ra}^{-\beta} \]  \hspace{1cm} (5)

with \( c \) and \( \beta \) dimensionless constants. Experiments yield \( \beta = 1/4 \) at steady state [14], in agreement with theory [15]. In principle, this law holds for the temperature difference across the unstable boundary layer. It turns out, however, that the boundary layer temperature which represents the maximum of the horizontally-averaged temperature and the average internal temperature can both be scaled to \( \text{Ra}^{-1/4} \) and hence are proportional to one another [14,16]. The same holds true even for a heterogeneous distribution of heat sources [11]. We confirmed that this scaling is valid in the present experiments over several orders of magnitude in Ra\textsubscript{H} (Fig. 6) with \( c = 3.58 \pm 0.15 \).

A fundamental question about the thermal evolution of an internally-heated convecting fluid is whether or not a scaling law established in steady-state remains valid during the transitory phase, provided one substitutes \( H \) by the effective heating:

\[ H^* = H - \rho C_p \frac{dT}{dt} \] \hspace{1cm} (6)

This hypothesis is often used in parameterized models [17] but it was not tested in real systems until now. Starting from this hypothesis, we propose an evolutionary model obtained simply by substituting the expression for \( H^* \) in the Eq. (5):

\[ \frac{T_\infty}{T_H^*} = c \text{Ra}^{-\beta}_{H^*} \] \hspace{1cm} (7)
Fig. 7. Global comparison between all the measurements of the surface heat flux performed as a function of time in the eleven experiments and the theoretical predictions of the scaling law of Eq. (11).

where

\[ T_{H^*} = \frac{H^* d^2}{k}, \]  

(8)

is the internal heating scale calculated with the effective heating rate \( H^* \).

A couple of algebraic manipulations yield the following ordinary differential equation for the volume average temperature of the system \( T \):

\[ \frac{dT}{dt} = \frac{H}{\rho C_p} - \frac{1}{\rho C_p} \left( \frac{d^2 c}{k} \right)^{\frac{1}{1-\beta}} \left( \frac{\rho g \alpha d^5}{k \kappa \mu} \right)^{\frac{\beta}{1-\beta}} T^{\frac{1}{1-\beta}} - \frac{Q_s}{d}. \]  

(9)

The solution of Eq. (9) with \( \beta = 1/4 \) is obtained through a 4th order Runge-Kutta integration using the same parameters (fluid properties and heating rate) as in the experiments (Table 1), no additional free parameters being introduced.

From the differential form of the global energy conservation equation:

\[ \rho C_p \frac{dT}{dt} = H - Q_s/d, \]  

we obtain an expression for the surface heat flux in the convective regime,

\[ Q_s = \left( \frac{kT}{\epsilon d} \right)^{\frac{1}{1-\beta}} \left( \frac{\rho g \alpha d^5}{k \kappa \mu} \right)^{\frac{\beta}{1-\beta}}. \]  

(11)

The agreement between the data (dots) and the model (lines) in Figs. 4 and 5 confirms that the secular evolution of the average internal temperature is fully equivalent to an additional contribution to internal heating. These results confirm the conclusion reached by [18] in their numerical study of the secular cooling of convective fluids.

The theoretical heat flux given by Eq. (11) is found in very good agreement with the experimental results over several orders of magnitude (Fig. 7). We applied these experimentally validated theoretical results to systems where the secular evolution of the average internal temperature is also related to
the time evolution of internal heating rate as in the case of a planetesimal [19]. In this accompanying paper we referred to [20] to transpose our scaling laws to spherical geometry by taking into account the curvature, either considering a whole sphere (in the case of an undifferentiated planetesimals) or a spherical annulus (in the case of a differentiated planetesimal). Their proportionality constant $c$ is coherent with our previously determined value obtained for free-slip top boundary conditions [16]. In [19] we also took into account the occurrence of a stagnant lid due to the silicates strong temperature dependence of viscosity. We did not address this aspect here, since the small temperature dependence of experimental fluids viscosity prevented the observation of a stagnant lid.

**Declaration of Competing Interest**

The Authors confirm that there are no conflicts of interest.

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