Anderson localization in optical lattices with speckle disorder

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We study the localization properties of non-interacting waves propagating in a speckle-like potential superposed on a one-dimensional lattice. Using a decimation/renormalization procedure, we estimate the localization length for a tight-binding Hamiltonian where site-energies are square-sinc-correlated random variables. By decreasing the width of the correlation function, the disorder pattern approaches a δ-correlated disorder, and the localization length becomes almost energy-independent in the strong disorder limit. We show that this regime can be reached for a size of the speckle grains of the order of (lower than) four lattice steps.

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I. INTRODUCTION

The Anderson model was proposed first to explain the absence of electronic diffusion in certain random lattices [1] and later to explain the absence of diffusion of light in certain amorphous materials [2]. In the first case, the electron energy is lower than the maxima of the lattice potential and the particles diffuse by tunnelling. In the second case the particles (photons in [2]) energy is higher than the potential and the particles are “free”. In both cases, in the absence of disorder, the eigenstates are delocalized states (Bloch waves in the first regime, plane waves in the second). The presence of a δ-correlated random potential freezes the wave propagation at a length of Lloc, the localization length, always in one (1D) and two dimension (2D), and depending on the disorder strength and the energy wave also in three dimension (3D) [3]. In the last few years the ultracold atom community has devoted a large effort to the experimental realization of Anderson localization. Anderson localization in the tight-binding regime was observed in momentum space with kicked-rotor set-ups, in 1D [4], and 3D [5], and in real space by using a quasi-periodic potential (thus not strictly-speaking a random potential). The key-ingredient for the experimental study of Anderson localization of ultracold atoms in the “free”-particle regime has been the speckle potential [6]. By using this handable optical potential, Anderson localization was observed in 1D [7] and in 3D [8, 9], and anomalous diffusion was observed in 2D [10]. The auto-correlation function of the speckle potential is a square-sinc, thus it decays algebraically as a long-range correlated disorder, but is also characterized by a finite size correlation length w, corresponding to the width of central bump of the square-sinc function.

In 1D, the presence of disorder patterns with auto-correlation functions decaying algebraically can mimic the presence of a mobility edge [11][14] and can even enhance localization, as shown in a microwave experiment [11]. Both phenomena are due to the fact that the disorder spectrum is non-zero in a finite momentum interval. Very recently Semmler and coworkers [15] have studied the phase diagram of correlated fermions in 2D and 3D optical lattices and in the presence of a speckle potential. From the analysis of the local Density Of States (DOS) they identify an Anderson-Mott and a Mott localized phase as functions of the interaction strength and the strength of the speckle potential. In this article we analyze the possibility of observing Anderson localization of a non-interacting wave, for example a non-interacting Bose-Einstein condensate [16], in a speckle potential superposed to a 1D lattice potential. By using a decimation/renormalization scheme [17] we analyze how the DOS of a lattice is modified by the presence of the speckle, and we estimate the localization length Lloc as function of the disorder strength and of the width w of the auto-correlation function. The speckle potential is introduced as an on-site disorder which has statistical properties which are the same as a genuine speckle potential. This is illustrated in Sec. II. In Sec. III we recall the reader of the decimation/renormalization procedure exploited to compute the DOS and Lloc. Our results show how the efficacy of the speckle potential to localize increases by increasing the disorder strength and by decreasing the correlation length w. These results can be a guide to choosing the experimental parameters to observe Anderson localization in the tight binding regime with speckle disordered patterns.

II. THE MODEL

To study the effect of a speckle potential in the presence of a 1D lattice on matter-wave transport we use the 1D Tight-Binding (TB) Hamiltonian,

$$H = \sum_{i=1}^{n_s} E_i |i\rangle\langle i| + \sum_{i=1}^{n_s-1} t(|i\rangle\langle i+1| + |i+1\rangle\langle i|)$$

where ns is the number of sites, Ein energy at the site i. The hopping term t is chosen site-independent. The
effect of the speckle potential is introduced in the on-site energy distribution by setting

\[ C_\ell = \langle \delta E_\ell \delta E_{\ell+i} \rangle = s^2 \left( \frac{\sin(2\pi \ell/w)}{2\pi \ell/w} \right)^2, \tag{2} \]

where \( \delta E_\ell = E_\ell - \langle E_\ell \rangle \) is the fluctuation of \( E_\ell \) with respect to the mean value \( \langle E_\ell \rangle \), \( s = \sqrt{(\langle \delta E_\ell \rangle)^2} \) is the disorder strength and \( w \) the width, in the units of the lattice step \( d \), of the correlation function in Eq. (2). The disorder spectrum is not uniform as in the Anderson model [1], but is described by the triangular function [7]

\[ S_\ell \propto s^2 (\kappa - |\ell|)\theta(\kappa - |\ell|) \tag{3} \]

where \( \kappa = 4\pi/w \), and \( \theta(x) \) is the Heaviside function.

A. Generation of the disordered potential

We use the Fourier Filtering Method (FFM) [18, 20] to generate the disorder pattern described by the correlation function (2). First we generate a sequence of \( N \) \( \delta \)-correlated random numbers \( \{u_j\} \), with \( j = 1, \ldots, N \) from a uniform distribution centered in zero and of width 1. The second step is the generation of the desired \( \{E_j\} \) distribution by “filtering”, in Fourier space, the uniform distribution \( \{u_j\} \). The filter being the spectral function \( S_\ell \), the \( E_j \)’s are evaluated directly from the expression

\[ E_j = \frac{1}{N_k} \sum_{j=0}^{N_k} \sum_{m=1}^{N} \sqrt{S_k} e^{ik(m-j)} u_m, \tag{4} \]

where \( N_k = 8N/w \) et \( k = -\kappa + (\pi/N) j_k \). By construction \( \langle E_j \rangle = 0 \), namely \( \delta E_j = E_j \), and \( (E_j\ell E_j+\ell) \) verifies Eq. (2) in the limit \( N \to \infty \). A different choice of \( \langle E_j \rangle \) would just shift the zero of the energy.

III. NUMERICAL RESULTS: THE DOS AND THE LOCALIZATION LENGTH

In the continuous limit, the single-particle DOS for an optical speckle has been studied in [21]. In the presence of a lattice (and in the absence of the speckle potential), the low-energy single-particle DOS has a typical saddle shape with two horns that correspond respectively to the center and the edge of the first Brillouin zone. To evaluate how the speckle potential modifies the DOS of the lattice, we compute the DOS, \( \mathcal{N}(E) \), regarding the Hamiltonian \( H \) by using the Kirkman-Pendry relation [22]

\[ \mathcal{N}(E) = \lim_{\varepsilon \to 0^+} \frac{1}{\pi} \text{Im} \left\{ \frac{\partial \ln[G_{n,s}(E+i\varepsilon)]}{\partial E} \right\}. \tag{5} \]

Here \( G(E) = (E - H)^{-1} \) is the Green’s function related to the Hamiltonian \( H \) at energy \( E \), and \( G_{i,j}(E) = \langle i|G(E)|j \rangle \). With the aim of computing the matrix element \( G_{1,n_s}(E) \), we reduce the dimensionality of the system by evaluating the effective Hamiltonian \( \hat{H} = \hat{E}_1|1\rangle\langle 1| + \hat{E}_{n_s}|n_s\rangle\langle n_s| + i\hat{t}|1\rangle\langle n_s+1| + |n_s+1\rangle\langle 1| \),

where \( \hat{E}_1, \hat{E}_{n_s}, \) and \( \hat{t} \) are functions of the energy \( E \) and of the Hamiltonian elements of the decimated states \( (2, 3, \ldots, n_s-1) \). \( G(E) = (E - \hat{H})^{-1} \) coincides with \( G(E) \) in the subspace \( \{1, n_s\} \) by construction.

The numerical results for the DOS as a function of the energy in units of \( |t| \) are shown in the first column of Fig. 1. One can observe that for large values of \( w \), the speckle disorder mainly affects the edge states of the DOS of the underlying perfect chain, while for smaller values of \( w \) the disorder mainly influences the central part of the spectrum.

The presence of the disorder modifies not only the DOS, but also the nature of the states, from extended to localized. In the continuous limit, the presence of the correlations described by Eq. (2) does not destroy localization but deeply modifies the behaviour of the localization length as a function of the energy [12–14]. To study the behaviour of the localization length \( \mathcal{L}_{loc}(E) \) in the tight-binding regime, we compute the Lyapunov coefficient \( \gamma(E) \), through the asymptotic relation

\[ \gamma(E) = [\mathcal{L}_{loc}(E)]^{-1} = \lim_{n_s \to \infty} \frac{1}{n_s d} \ln \frac{G_{n,s,n_s}(E)}{G_{1,s,1}(E)}. \tag{7} \]

The results shown in the second column of Fig. 1 have been computed for the case \( n_s = 200 \), but we have checked that the values obtained do not change significantly by increasing the value of \( n_s \) up to 1000. Analogously to the continuous case, we observe that all states are localized. In the limit of weak disorder, the localization length at the center of the spectrum, \( \mathcal{L}_{loc}(E = 0) \), is quite large, of the order of 50 lattice sites. By increasing the strength of the disorder, \( \mathcal{L}_{loc}(E = 0) \) decreases significantly only for small values of the correlation length \( w = \pi, w = 2\pi/3 \), and \( \mathcal{L}_{loc}(E) \) becomes almost energy-independent in the whole band. Longer-range correlations \( w = 2\pi \) and \( w = 4\pi \) act instead more efficiently on the edge states. To better understand these results we can refer to the continuous case, where

\[ \mathcal{L}_{loc}(k)^{-1} \sim \mathcal{L}_{loc}^B(k)^{-1} = \frac{w^2}{8k^2} S_{2k}, \tag{8} \]

in the Born approximation (see for instance [11]). From Eq. (8) we can expect to observe (i) a decrease of the localization length for large values of \( w \) in the limit \( k \to 0 \), since \( \mathcal{L}_{loc}^B(k \to 0) \sim k^2/(s^2 w) \), and (ii) an increase in the localization length for \( k > \kappa/2 \), namely where the Born approximation is no longer valid [12, 13]. Since in the TB case there is a \( k \to \pi/d - k \) symmetry in the DOS due to the presence of the underlying lattice and correlations act symmetrically with respect to the
FIG. 1: (Color online) DOS and localization length $L_{\text{loc}}$ (in units of the lattice step $d$) as functions of the particle energy (in units of the energy hopping) for a lattice of 200 sites where site energies are $t_{\text{tr}}$-distributed random variables [see Eq. (2)]. Each curve corresponds to an average over 100 configurations. The different lines correspond to: $w = 4\pi$ (continuous red line), $2\pi$ (dashed green line), $\pi$ (point-dashed blue line) and $2\pi/3$ (pointed magenta line). From top to bottom: $s/|t| = 1, 2, 5$ and 10.

In this article we have studied the effects of a speckle potential on the spectrum of a quantum particle (or a non-interacting wave) in a lattice potential. At fixed, large disorder strength ($s = 10|t|$), the localization efficacy of the speckle potential depends strongly on the width of the auto-correlation function $w$. Large values of $w$ enhance localization at very low energies and at the edge of the Brillouin zone. Shorter-range correlations ($w < 4$ lattice sites) act more efficiently on the center of the spectrum. More generally, our results show that a speckle superposed to an optical lattice is a suitable potential to study Anderson localization in the tight-binding regime: analogously to the continuous case, speckle correlations deeply modify the behaviour of the localization length as a function of the energy, but do not induce an insulator-metal transition.

IV. CONCLUSIONS

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