Abstract We use existing measurements of $D^- \rightarrow K^{*0}\rho^-$ and $B \rightarrow \psi + K^*$, coupled with flavor independence of QCD, and with vector meson dominance to show that long distance contributions to $B \rightarrow \rho + \gamma$ are potentially very serious. We note that long distance (LD) contributions can be appreciably different in $B^- \rightarrow \rho^- + \gamma$ and $B^0 \rightarrow \rho^0(\omega) + \gamma$. All radiative decays of $B$, $B_S$ are shown to be governed essentially by two LD and two short-distance (SD) hadronic entities. Separate measurements of $B^- \rightarrow \rho^- + \gamma$, $B^0 \rightarrow \rho^0(\omega) + \gamma$, along with $B \rightarrow K^* + \gamma$ appear necessary for a meaningful extraction of $V_{td}$. Measurements of $B_S \rightarrow \phi + \gamma$ and $K^{*0} + \gamma$ could also provide very useful consistency checks.
1 Introduction

The Cabibbo-Kobayashi-Maskawa (CKM) mixing angle $V_{td}$ is a parameter of crucial importance to the Standard Model (SM) and it is still very poorly known \cite{1}. Considerable experimental effort is directed towards its determination via the rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ \cite{2}. This process is considered to be theoretically clean for extraction of $V_{td}$ \cite{3}. However its branching ratio is extremely rare being about a few times $10^{-10}$ rendering a precise determination of $V_{td}$ very challenging. In $B$-physics one well known method for determining $V_{td}$ is via the experimentally measured $B$-$\bar{B}$ mixing. This requires a knowledge of the pseudoscalar decay constant $f_B$ and the “bag parameter” $B_B$. Neither of these quantities is directly accessible to experiment, at least not in the near future. $f_B$ could eventually be measured directly in $B$ decays, say via $B \rightarrow \tau + \nu_\tau$; but this will surely take a long time. The reliability of the theoretical calculations for $f_B$ and $B_B$ may therefore be a cause for concern. In any case the importance of $V_{td}$ demands that we determine it in many ways and with as much precision as possible.

One $B$-decay in which $V_{td}$ enters is $B \rightarrow \rho + \gamma$ \cite{4,5,6}. Since the related decay $B \rightarrow K^* + \gamma$ has already been detected \cite{7} it is useful to understand what we may learn about $V_{td}$ through a measurement of $B \rightarrow \rho + \gamma$. Rough estimates indicate that LD contribution to $B \rightarrow \rho + \gamma$ are potentially very serious. Since it is very difficult to accurately estimate these LD contributions a precise extraction of $V_{td}$ from $B \rightarrow \rho + \gamma$ \cite{8} therefore also appears rather difficult.

In this paper we try to quantify various LD and SD sources for radiative decays of all of the $B(B_S)$ meson, i.e. for:

\begin{alignat}{2}
B^- & \rightarrow \rho^- + \gamma & \quad (1) \\
B^0 & \rightarrow \rho^0 + \gamma & \quad (2) \\
B^0 & \rightarrow \omega + \gamma & \quad (3)
\end{alignat}
\[ B^- \rightarrow K^{*-} + \gamma \quad (4) \]
\[ B^0 \rightarrow K^{*0} + \gamma \quad (5) \]
\[ B_S \rightarrow \phi + \gamma \quad (6) \]
\[ B_S \rightarrow K^{*0} + \gamma \quad (7) \]

We show that two types of LD and essentially two types of SD contributions determine all of these decays. Thus separate experimental measurements of as many of these reactions as possible could allow a model independent determination of the hadronic entities and provide useful self consistency checks. Consequently, extraction of \( V_{td} \) to a meaningful level of accuracy in the long run may become possible. Clearly the necessary effort is then many times more than what is needed for a single measurement of \( B \rightarrow \rho + \gamma \).

On the other hand, we anticipate intense experimental activity in the area. Improvements at existing \( e^+e^- \) facilities such as CESR and LEP as well as construction of new \( e^+e^- \) based \( B \)-factories at SLAC and KEK will lead to an increased sample of \( B \)'s. Furthermore many dedicated \( B \) experiments are being proposed or planned at hadron machines. Bearing all that in mind we give a general strategy for attempting to extract \( V_{td} \) precisely from radiative \( B \)-decays.

2 A Close Look at \( B \rightarrow \rho + \gamma \).

2.1 The Long Distance Contribution from \( u\bar{u} \) States.

It has been known for a long time \([9]\) that for \( b \rightarrow d \) flavor-changing loop transitions (unlike for \( b \rightarrow s \)) the tree graphs (i.e. long-distance) become appreciably large and can easily dominate over the loop (i.e. the SD) process. A simple example is the process

\[ B^- \rightarrow d\bar{u}\gamma \rightarrow \rho^-\gamma \quad (2) \]
via the non-spectator (or the annihilation) mechanism shown in Fig. 1a. Notice that this graph goes via $V_{ub}$, i.e. another poorly known CKM parameter. So the reaction $B \to \rho + \gamma$ can occur, in principle, even if $V_{td}$ is vanishingly small. Although it is very difficult to accurately calculate such a contribution there are several ways of estimating its size, i.e. within a factor of two or three. We outline below two ways of calculating such contributions.

In the first method we invoke the correspondence of such annihilation graphs with spectator plus final state interactions (FSI) to note that Fig. 1(a) is exactly the same as Fig. 1(b). Fig. 1(b) shows the color allowed simple spectator contribution to $B^- \to \rho^- + \rho_0^0$, followed by $\rho_0^0 \to \gamma$ (where the subscript $V$ stands for virtual). The first step of $B^- \to \rho^- + \rho^0$ can be estimated by normalizing with the observed analogous decay: $D^- \to \rho^- + K^{*0}$ via Fig. 1(c). This correspondence between the two decays should hold because of the flavor ($b \leftrightarrow c$) symmetry of QCD. To the extent that $m_c(m_b) \gg \Lambda_{QCD}$ the effects of QCD do not care about the flavor-label charm or bottom [10].

Also SU(3) flavor symmetry ensures that the change from $K^{*0}$ to $\rho^0$ in $D$ versus $B$ decay is mild apart from phase-space correction. The conversion from $\rho \to \gamma$ can be dealt with by using vector-meson dominance. Thus

$$\frac{BR(B^- \to \rho^- \rho^0)}{BR(D^- \to \rho^- K^{*0})} = \left(\frac{m_b}{m_c}\right)^5 \frac{|V_{ub}|}{|V_{cs}|} \frac{\tau_{B^+}}{\tau_{D^+}} \chi_{ps}$$

where $\chi_{ps}$ is the phase space ratio.

The conversion from $\rho^0$ to photon can be crudely estimated by using VMD to amount a multiplicative factor of about $3 \times 10^{-3}$ [11, 12]. Using $|\frac{V_{ub}}{V_{cb}}| = .08, |V_{cb}| = .04$ we thus find:

$$BR(B^- \to \rho^- \gamma)_{L_{u_1}} = 6 \times 10^{-8}$$

where the subscript $L_{u}$ is to denote the LD contributions coming from $u\bar{u}$ state(s) such as $\rho^0$. 

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A second method for estimating the same contribution is to use bound state method of Ref. 13 for writing down the amplitude for \( B^- \to \bar{u}d\gamma \):

\[
A(B^- \to \bar{u}d\gamma)_{1a} \simeq \frac{f_B}{16} e_u e_{\bar{u}} \frac{g_s^2}{m_{\bar{u}}^2} \frac{1}{m_u} V_{ub} T r [(\hat{\rho}_B - m_B) \not{\rho}_B \gamma^\mu P_L] \]

(5)

\[
\bar{u}_d\gamma^\mu P_L v_u
\]

where \( m_u \) is the constituent mass of the \( u \) quark and \( e_u \) is its charge (i.e. 2/3). Now the light-quark current can be (vacuum) saturated by the \( \rho^- \) via use of:

\[
\langle 0 | \bar{u}_d\gamma^\mu \gamma_5 v_u | \rho^- (p_\rho, \epsilon_\rho) \rangle = f_\rho m_\rho \epsilon_\rho
\]

(6)

where \( f_\rho \) is the decay constant of \( \rho \) i.e. about 220 MeV. Thus:

\[
\frac{\Gamma(B^- \to \rho^- \gamma)_{1a}}{\Gamma(B^- \to \bar{u}d\gamma)_{1a}} \simeq 18\pi^2 f_\rho^2 m_\rho^2 m_B^4 \left( 1 - \frac{m_\rho^2}{m_B^2} \right) \simeq 7 \times 10^{-3}
\]

(7)

Using (5) and (6) we arrive at a second estimate for the LD correction due to \( \bar{u}u \) states

\[
BR(B^- \to \rho^- \gamma)_{L\bar{u}2} \simeq 8 \times 10^{-8}
\]

(8)

where we have used \( f_B = 180 \text{ MeV} \) \(^{[15]}\) and \( m_u = 330 \text{ MeV} \). In passing we note from eqns. (5) - (7) that the inclusive branching ratio for the reaction \( B^- \to \bar{u}d\gamma \) via the annihilation graph is given by:

\[
BR(B^- \to u\bar{d}\gamma) \approx 1.1 \times 10^{-5}
\]

(9)

Given the intrinsic uncertainties in each of the two methods outlined above the resulting numbers in eqns. (5) and (8) should be regarded as in qualitative agreement. Thus for one class of long distance contributions, namely those due to \( u\bar{u} \) states we will take the mean of the two numbers
from eqn. (4) and eqn. (8) and rather arbitrarily assign a factor of four uncertainty. Thus for the corresponding amplitude we get

$$A(B^- \to \rho^- \gamma)_{L_u} = (2 - 4) \times 10^{-4}$$ \hspace{1cm} (10)

Now let us address the case of the neutral $B_i.e.$ the corresponding LD contributions from $\bar{u}u$ states to $(B^0 \to \rho^0 \gamma)$. Then Fig. 1(a) gets redrawn as Fig. 1(d) and Fig. 1(b) gets redrawn as Fig. 1(e). In each case we see that the graphs for $B^0$ are color suppressed. Thus

$$A(B^0 \to \rho^0 \gamma)_{L_u} = -\left(\frac{1}{3} \times \frac{3}{2} \times \frac{1}{\sqrt{2}}\right) \times (2 - 4) \times 10^{-4} \hspace{1cm} (11)$$

$$A(B^0 \to \omega \gamma)_{L_u} = \frac{1}{6\sqrt{2}}(2 - 4) \times 10^{-4} \hspace{1cm} (12)$$

### 2.2 The Long Distance Contributions from $c\bar{c}$ States.

We next turn our attention to the LD contributions to $B^- \to \rho^- + \gamma$ from $c\bar{c}$ states. The most notable origin is the chain $B^- \to \rho^- + \psi_V$ followed by $\psi_V \to \gamma$. Using the measured rate

$$BR(B^0 \to K^{*0} \psi) = (1.6 \pm .3) \times 10^{-3} \hspace{1cm} (13)$$

we immediately get

$$BR(B^- \to \rho^- \psi) = 2BR(B^0 \to \rho^0 \psi) = \lambda^2 Br(B^0 \to K^{*0} \psi)p_{sK^*\rho} \hspace{1cm} (14)$$

where $\lambda \equiv \sin \theta_c = .22$ and $p_{sK^*\rho}$ is a phase space correction factor estimated to be about 1.4 due to the mass difference between $\rho$ and $K^*$ [14]. Following Ref. 11 conversion factor from $\psi \to \gamma$ is estimated at $5 \times 10^{-3}$. However, in this $\psi \to \gamma$ conversion we want to include only the transversely polarized fraction of $\psi$’s. These are estimated to be about 30% [17]. Thus, for the amplitude of LD contributions from $c\bar{c}$ states we get
\[ A(B^- \rightarrow \rho^- \gamma)_{\text{c}} = (2 - 6) \times 10^{-4} \]  
(15)

where in specifying the range we are again estimating about a factor of two uncertainty (in the amplitude).

\section*{2.3 The Short-Distance Contributions to $B \rightarrow \rho + \gamma$}

The SD (or penguin) contributions arise from loop graphs, such as Fig. 1(f) and 1(g). It is known for a long time that QCD corrections play an important role here. We recall that this is due to the fact that in the pure electroweak penguin (Fig. 1(f)) there is an accidental cancellation of the coefficients of terms that maintain GIM unitarity with a logarithmic dependence on the internal quark mass (i.e. $m_u, m_c, m_t$). As a result the leading terms exhibit a power law dependence on that mass. On switching on QCD the coefficient of the log term becomes nonvanishing and results in enhanced QCD radiative effects.

By now there is an extensive literature describing the effects of QCD on radiative decays of $B$’s. For our purpose it is useful to first discuss the $b \rightarrow s$ process namely the one relevant to $B^- (B^0) \rightarrow K^{*+} (K^{*0}) + \gamma$ (or to $B_S \rightarrow \phi + \gamma$). Recall the CKM unitarity for this channel:

\[ v_u^q + v_c^q + v_t^q = 0 \]  
(16)

where $v_j^q = V_{jb} V_{j}^{*q}$, $j = u, c, t$ and $q = s$ or $d$. Recall also that

\[ V_{us} = \lambda \simeq .22 \]  
(17)

\[ \frac{V_{ub}}{V_{cb}} = .08 \pm .03 \]  
(18)

\[ V_{tb} = .99 \pm .01 \]  
(19)

Thus for $b \rightarrow s$ case the up quark term ($V_{ub} V_{us}^{*}$) is negligible in comparison
to the other two terms in eqn. \([16]\). This has two important consequences. First is that one gets the usual relation:

\[
V_{ts} \simeq -V_{cb}
\]  

(20)

to a very good approximation. The second important consequence of the smallness of \(V_{ub}V_{us}^*\) is that in the \(b \to s\) penguin loop the \(u\) quark contribution is forced to become so small that the precise dependence on \(m_u\) is not at all important. Such is not the case for \(b \to d\) penguins as we will soon elaborate.

The penguin (SD) contributions can be written as

\[
A_{p}^q = \sum_j f_j v_j^q
\]  

(21)

For \(q = s\), we can use eqn. \([16]\) and rewrite

\[
A_{p}^s = (f_t - f_c) v_t^s + (f_u - f_c) v_u^s
\]  

(22)

Since \(v_u^s\) is extremely small the second term is bound to make a negligibly small contribution and consequently the assumption that \(f_c = f_u\) that one usually makes \([18]\) becomes a very safe assumption. Then for \(b \to s\) with a very good approximation one gets

\[
A_{p}^s = (f_t - f_c) v_t^s \equiv (f_t - f_c)V_{ts}
\]  

(23)

For the case of \(b \to d\) transitions the \(u\) quark in the loop no longer appears with the small parameter \(V_{us}(\equiv \lambda)\) multiplying its effects and the charm and the top quark both now have smaller CKM factors monitoring their contributions to the penguin amplitude. The \(u\) quark contribution is no longer necessarily negligible in comparison to the others and the assumption \(f_c = f_u\) is no longer a good approximation since it forces a potentially important (\(u\) quark) contribution to unnaturally vanish. Any reasonable deviation of \(f_c/f_u\) away from unity would have important corrections. To make the best use of
the experimental information that one gets from measurement of $B \to K^{*}\gamma$, it is prudent now to use unitarity and rewrite the $b \to d$ penguin as:

$$A_d^d = (f_t - f_c)v_t^d + (f_u - f_c)v_u^d$$  \hspace{1cm} (24)

Taking ratios of equations (23) and (24):

$$\frac{A_d^d}{A_d^s} = \frac{V_{td}^*}{V_{ts}}[1 + \Delta]$$

$$\Delta \equiv (f_u - f_c)(\frac{V_{ub}}{V_{cd}})$$  \hspace{1cm} (25)

Thus there are two hadronic entities:

$$f_u - f_c \equiv S_{uc}$$  \hspace{1cm} (26)

$$f_t - f_c \equiv S_{tc}$$  \hspace{1cm} (27)

monitoring all the SD contributions in $b \to s$ and $b \to d$ penguins. $f_t$ and $f_c$ have recently been calculated in Ref. [18]:

$$f_t \simeq -0.11$$  \hspace{1cm} (28)

$$f_c \simeq 0.16$$  \hspace{1cm} (29)

giving

$$S_{tc} = -0.27$$  \hspace{1cm} (30)

For extraction of $V_{td}$ from experiment the deviation from unity of the quantity in square parenthesis in equation (25) is important. First let us estimate the CKM ratio that enters there. We note that the use of [15]

$$f_B = 180 \pm 40 \text{ MeV}$$

$$B_B = 1 \pm 0.2$$  \hspace{1cm} (31)
emerging from lattice calculations along with the measured $B\bar{B}$ mixing gives

$$\frac{V_{td}}{V_{ts}} \simeq 0.22 \pm 0.08$$

(33)

Thus using (as 90% CL bounds)

$$|V_{ub}/V_{cb}| < 0.13$$

(34)

$$|V_{td}/V_{ts}| \geq 0.09$$

(35)

we get

$$|V_{ub}/V_{td}| \lesssim 1.5$$

(36)

A precise value for $f_u - f_c$ is extremely difficult to calculate. We will assume that $f_u$ and $f_c$ depend logarithmically on $m_u$ and $m_c$. Using constituent masses $m_u \simeq 0.3$ GeV, $m_c \simeq 1.8$ GeV and the numerical result (29) of Ref. [18] we then crudely estimate:

$$S_{uc}/S_{tc} = -0.30$$

(37)

Thus the ratio of the SD amplitudes for $b \to d$ and $b \to s$ may deviate appreciably from the CKM ratio $V_{td}/V_{ts}$. We note this deviation from the simple CKM scaling is controlled crucially by the ratio $V_{ub}/V_{td}$ just as the relative importance of the LD contributions due to $u\bar{u}$ states (i.e $L_u$) to $B \to \rho + \gamma$ is controlled by $V_{ub}/V_{td}$. If the mild indications from the current central values of $V_{ub}/V_{cb}$ and $V_{td}/V_{ts}$ (.08 versus .22) is confirmed then the extraction of $V_{td}$ from $B \to \rho + \gamma$ will indeed be easier than otherwise.

To gauge the relative importance of the LD and the SD contributions to ($B \to \rho + \gamma$) we need to estimate $A_p^s$ (i.e. SD amplitude for $b \to s$) so as to be able to use eqn. (25) to get $A_p^d$ (i.e. the SD amplitude for $b \to d$). We can try to use the experimental result on $B \to K^* + \gamma$ for that purpose; so we turn our attention to that reaction now.
2.4 The Long- and Short Distance Contributions to $B \to K^* + \gamma$

The LD contribution from $\bar{u}u$ states is easily estimated from eqn. (10)

$$A_s(B^- \to K^{*-}\gamma)_{Lu} \simeq (4 - 8) \times 10^{-5}$$

(38)

$$A_s(B^0 \to K^{*0}\gamma)_{Lu} \simeq (2 - 4) \times 10^{-5}$$

(39)

Similarly, from eqn. (13), with use of the $\psi \to \gamma$ conversion factor of $5 \times 10^{-3}$ and incorporating a factor of 0.3 for the fraction of transversely polarized $\psi'$s we get

$$A_s(B \to K^* + \gamma)_{Lc} = (1 - 3) \times 10^{-3}$$

(40)

So for $B \to K^*\gamma$ the LD contributions due to $c\bar{c}$ completely dominate over the $u\bar{u}$ ones [19].

Recall now the recent experimental result [7]

$$BR(B \to K^*\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$$

(41)

For the amplitude we translate this as

$$A_s(B \to K^*\gamma)_{expt} \simeq (6.7 \pm 1.7) \times 10^{-3}$$

(42)

From equations (40) and (42) we see that there can be about 15–50% LD contributions in the observed experimental result. Combining those two equations we arrive at the SD component

$$A^s_p \equiv A_s(B \to K^* + \gamma)_{SD} = (4.7 \pm 2.7) \times 10^{-3}$$

(43)

In arriving at eqn. (43) we have made a strong assumption that the SD and LD ($c\bar{c}$) amplitudes for $B \to K^* + \gamma$ have the same relative sign. This
assumption is based on the belief that an opposite choice of signs would make
the exclusive to inclusive ratio for the short distance component alone, i.e.

\[ H_{K^*} = \frac{\Gamma(B \to K^* + \gamma)}{\Gamma(b \to s + \gamma)} \]  

become uncomfortably large. The point is that lattice methods have been
used to calculate this hadronization ratio for the single operator

\[ \bar{s}_L \sigma_{\mu\nu} b_R F^{\mu\nu} \]  

that emerges from the short distance expansion. The results of the lattice
calculation are \[20\]:

\[ H_{K^*} = 6.0 \pm 1.2 \pm 3.4\% \]  

For our purpose we will adopt a very conservative interpretation of the lattice
results, namely

\[ H_{K^*} < 12\% \]  

Recall now the recent CLEO result \[21\]:

\[ BR(b \to \gamma + s) = (2.32 \pm 0.51 \pm 0.29 \pm 0.32) \times 10^{-4} \]  

Combining equations (41) and (48) indicates that the experimental value of
the exclusive to inclusive ratio is around 20% which tends to be on the high
side compared to the lattice expectation. By attributing a fraction of the
observed exclusive signal to come from LD sources as in equation (43) brings
the hadronization ratio for the SD piece i.e.

\[ \frac{(4.7 \times 10^{-3})^2}{2.3 \times 10^{-4}} \sim 9.6\% \]  

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to be more in the ball park of the lattice results. If, on the other hand, we take the LD and SD contributions to \( B \rightarrow K^*\gamma \) to have a relative negative sign then the SD fraction would have to be

\[
\frac{(8.7 \times 10^{-3})^2}{2.3 \times 10^{-4}} \sim 33\%
\]

which is too large from the lattice perspective.

### 2.5 Estimates for the Relative Importance of LD Contribution to \( B \rightarrow \rho + \gamma \)

Using eqn. (25) and (43) and invoking SU(3) gives us the SD contribution to (\( B \rightarrow \rho + \gamma \))

\[
A(B \rightarrow \rho + \gamma)_{SD} = \frac{V_{td}}{V_{ts}}[1 + \Delta] \times (4.7 \pm 2.7) \times 10^{-3}
\]

\[
\sim (5 - 15) \times 10^{-4}
\]

From eqn. (15) and (51) we see that for \( B^- \rightarrow \rho^- + \gamma \) the LD \( c\bar{c} \) states are at least 15% of (and could even dominate over) the SD ones. Indeed even that minimum value of 15% implies a contamination of these LD effects on the rate for \( B^- \rightarrow \rho^- \gamma \) to approach 30%. From eqn. (10) we see that the \( u\bar{u} \) states seem to be somewhat less important than the \( c\bar{c} \) but are roughly comparable. We emphasize again that the numbers given for \( L_u \) in eqn. (10) assume \( \left| \frac{V_{ub}}{V_{cb}} \right| = .08 \). Given the intrinsic difficulties of the LD estimates it appears doubtful that \( B^- \rightarrow \rho^- + \gamma \) alone in conjunction with \( B \rightarrow K^* + \gamma \) can be used to deduce reliable information on \( V_{td} \) before a lot more experimental data on radiative decays becomes available. In this regard a precise value of \( V_{ub} \) as well as the relative sign of \( V_{ub}V^*_{td} \) with \( V_{cb}V^*_{cd} \) is very important since a relative negative sign between these two CKM elements would result in (at least) a partial cancellation of the long distance \( L_u \) and \( L_c \) terms.

The LD contribution to \( B^0 \rightarrow \rho^0 + \gamma \) from \( u\bar{u} \) states are substantially less (see eqn. (11)) than for \( B^- \rightarrow \rho^- + \gamma \). The SD contributions are the
same for $B^0$ and $B^-$ (i.e. eqns. (50) and (51)). Thus $B^0 \to \rho^0 + \gamma$ may have appreciable advantages over $B^- \to \rho^- + \gamma$ for learning about $V_{td}$. In any event, it seems clear from the preceding estimates that the rates for $B^- \to \rho + \gamma$ may be quite different from that of $B^0 \to \rho + \gamma$. Since the SD contributions (which scale with $V_{td}$) are the same for $B^-$ and $B^0$ and the LD ones are not, separate measurements of $B^-$ and $B^0$ radiative decays are important to understanding the dynamics of these decays and they are essential for facilitating any reliable determination of $V_{td}$.

3 Four Hadronic Entities Essentially Determine all the Radiative $B$-Decays.

In the preceding section we have discussed the long and short distance contributions to charged and neutral $B$ decays to $\rho + \gamma$ and $K^* + \gamma$. During the course of that discussion we had to introduce two LD (namely $L_u$ and $L_c$) and two short distance (namely $S_{tc}$ and $S_{uc}$) entities. Indeed all the radiative $B$, $B_S$ decays to the seven final states given in eqn. (1) are governed by the same four hadronic entities [22]. Of course the dependence on CKM angles are not the same (also there are obvious differences in $N_c$ dependence and on flavor $SU(3)$) that have to be taken into account. Thus we can write

\[ A(B^- \to \rho^- + \gamma) = e_u \left[ (N_C - 1) v_u^d L_u + v_c^d L_c + k_b c_{B\rho} T_{1_{B\rho}} (v_t^d S_{te} + v_u^d S_{ue}) \right] \]

\[ A(B^0 \to \rho^0 + \gamma) = -\frac{1}{\sqrt{2}} \left[ (e_u - e_d) v_u^d L_u + e_u v_c^d L_c + k_b c_{B\rho} T_{1_{B\rho}} (v_t^d S_{tc} + v_u^d S_{ue}) \right] \]

(52)

(53)

Also

\[ A(B^0 \to \omega + \gamma) = \frac{1}{\sqrt{2}} \left[ (e_u + e_d) v_u^d L_u + e_u v_c^d L_c + k_b c_{B\rho} T_{1_{B\rho}} (v_t^d S_{tc} + v_u^d S_{uc}) \right] \]

(54)
\[ A(B^− \to K^{*−} + \gamma) = e_u [v_u^* N_c L_u + v_c^* L_c + k_b c_{B_K} T_{1_{B_K}} (v_t^* S_{tc} + v_u^* S_{uc})] + \]
\[ \simeq e_u [v_c^* L_c + k_b c_{B_K} T_{1_{B_K}} v_t^* S_{tc}] \] (55)
\[ A(B^0 \to K^{*0} + \gamma) = e_u [v_u^* L_u + v_c^* L_c + k_b c_{B_K} T_{1_{B_K}} (v_t^* S_{tc} + v_u^* S_{uc})] \]
\[ \simeq e_u [v_c^* L_c + k_b c_{B_K} T_{1_{B_K}} v_t^* S_{tc}] \] (56)

Similarly for related decays of \( B_S \):
\[ A(B_S \to \phi + \gamma) = e_u [v_u^* L_u + v_c^* L_c + k_b c_{B_\phi} T_{1_{B_\phi}} (v_t^* S_{tc} + v_u^* S_{uc})] \]
\[ \simeq e_u [v_c^* L_c + k_b c_{B_\phi} T_{1_{B_\phi}} v_t^* S_{tc}] \] (57)
\[ A(B_S \to K^* + \gamma) = e_u [v_u^* L_u + v_c^* L_c + k_b c_{B_{K^*}} T_{1_{B_{K^*}}} (v_t^* S_{tc} + v_u^* S_{uc})] \] (58)

Here \( k_b \) is a normalization constant designed so that the width for the flavor-changing transition coming from the short distance piece alone is related properly to the factors \( S_{tc} \) and \( S_{uc} \). Thus
\[ \Gamma(b \to d\gamma)_{SD} \equiv \Gamma(b \to d\gamma)_{\text{penguin}} = [e_u k_b (v_t^* S_{tc} + v_u^* S_{uc})]^2 \] (59)

\( T_1 \) is the only form factor (at \( q^2 = 0 \)) that determines the exclusive to inclusive ratio from the short-distance penguin part [19]. Thus
\[ \frac{\Gamma(B \to \gamma\rho)_{SD}}{\Gamma(b \to \gamma d)_{SD}} = C_{B_{\rho}}^2 \Gamma_{1_{B_{\rho}}}^2 \] (60)

where
\[ C_{B_{\rho}}^2 = 4 \left( \frac{m_B}{m_b} \right)^3 \left[ 1 - \frac{m_{\rho}^2}{m_B^2} \right]^3 \] (61)

4 Discussion

In Table 1 we give rough estimates for the radiative modes. For simplicity we have assumed that \( T_1(q^2 = 0) \) is the same for \( B \to K^*, \rho \) and \( B_s \to K^*, \phi \).
Table 1: Numerical Estimates

| Reaction          | |Amplitudes|/|10^{-4}| |Branching Ratio/10^{-4}|
|-------------------|-------------------|-------------------|-------------------|-------------------|
|                   | (u\bar{u})_{LD} | (c\bar{c})_{LD} | SD               | SD only           | Total             |
| B^+ \rightarrow \rho^+\gamma | 3 ± 1            | 4 ± 2            | 10 ± 6           | 2–25              | .4–68             |
| B^0 \rightarrow \rho^0\gamma | 1.1 ± .4         | 2.8 ± 1.4        | 7 ± 4            | 1–12              | 1–32              |
| B^0 \rightarrow \omega^0\gamma | .2 ± .1          | 2.8 ± 1.4        | 7 ± 4            | 1–12              | 2–23              |
| B \rightarrow K^*\gamma | .6 ± .3          | 20 ± 10          | 50 ± 30          | 40–640            | 90–1200           |
| B_S \rightarrow K^*\gamma | 1.5 ± .5         | 4 ± 2            | 10 ± 6           | 2–26              | 2–58              |
| B_S \rightarrow \phi\gamma | .6 ± .2          | 20 ± 10          | 50 ± 30          | 40–640            | 90–1200           |

There could easily be differences between these form factors amounting to 10 or even 20%. Future lattice and QCD sum rule calculations should be able to determine these quite reliably.

Notice that the spread in the range due to the SD piece alone is less than the spread after the LD contributions are included. This is in part because the relative signs are not known at this time. Thus typically the SD piece alone has a range of about one order of magnitude and that increases appreciably to the extent that in one case it becomes as much as two orders of magnitude when the LD pieces are also included.

We must also emphasize that the entries in the table are highly correlated so that as better experimental information on any mode(s) becomes available then it will effect the estimates for all of the modes. This is of course another way of saying that all of the decays involve only a few (i.e. four) hadronic quantities. In the case of $B \rightarrow K^*\gamma$ the recently obtained experimental measurement has been used to fix the relative sign between the SD piece and the LD $(c\bar{c})$ piece. There still remains an uncertainty in the theoretical expectation for the total $BR$ of about one order of magnitude. Measurements of $B \rightarrow \rho(\omega) + \gamma$, especially separate ones for charged and neutral, will significantly aid such an analysis in the future. Differences in the $BR$s for
\[ \rho^- + \gamma, \rho^0 + \gamma \text{ and } \omega + \gamma \text{ would be an excellent indicator of the extent of the LD contamination. If the LD contributions are small then the BRs for these modes should follow the expected factor of two difference due to the difference in their naive quark content.} \]

Indeed from eqns. (52–54) one finds:

\[ |A(B^- \to \rho^- + \gamma)| - \sqrt{2}|A(B^0 \to \rho^0 + \gamma)| = V_{ud}^d L_u [3\bar{e}_u - e_d] \quad (62) \]

\[ |A(B^- \to \rho^- + \gamma)| - \sqrt{2}|A(B^0 \to \omega + \gamma)| = V_{ud}^d L_u [3\bar{e}_u + e_d]. \quad (63) \]

Thus experimental determination of the differences in the BR’s can be used to quantitatively deduce the long distance piece due to \( u\bar{u} \).

Lattice calculations of \( B \to K^* + \gamma \) could also play a very useful role. If improved lattice calculations for \( B \to K^* \gamma \) also do not agree in their determination of the ratio \( H_{K^*} \equiv [BR(B \to K^* + \gamma)/BR(b \to s + \gamma)] \) with improved experimental measurements then the difference between the two must be attributed to long distance pieces (presumably due to \( c\bar{c} \) states) that the lattice calculations do not include.

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**Figure Captions**

**Fig. 1 a–e** A partial set of long distance contributions due to \( u\bar{u} \) states. Those due to \( c\bar{c} \) states typically result by replacing \( u \to c \) in Fig. 1e.
Fig. 1 f–g Show typical penguin (short-distance) contributions.

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