Targeting in Quantum Persuasion Problem

V. I. Danilov* and A. Lambert-Mogiliansky†‡

April 20, 2018

Abstract

In this paper we investigate the potential for persuasion arising from the quantum indeterminacy of a decision-maker’s beliefs, a feature that has been proposed as a formal expression of well-known cognitive limitations. We focus on a situation where an agent called Sender only has few opportunities to influence the decision-maker called Receiver. We do not address the full persuasion problem but restrict attention to a simpler one that we call targeting, i.e. inducing a specific belief state. The analysis is developed within the frame of a $n$-dimensional Hilbert space model. We find that when the prior is known, Sender can induce a targeted belief with a probability of at least $1/n$ when using two sequential measurements. This figure climbs to $1/2$ when both the target and the belief are known pure states. A main insight from the analysis is that a well-designed strategy of distraction can be used as a first step to confuse Receiver. We thus find that distraction rather than the provision of relevant arguments is an effective means to achieve persuasion. We provide an example from political decision-making.

1 Introduction

In this paper we build further on a theoretical result [7] that establishes the power of unconstrained belief manipulation or persuasion in the context of non-classical (quan-

---

*CEMI Russian Academy of sciences, Moscow, vdanilov43@mail.ru
†Paris School of Economics, alambert@pse.ens.fr
‡We would like to thank James Yearsley as well as an anonymous referee for very valuable discussions and comments.
tum) uncertainty. That result was a first step in the development of Quantum persuasion theory in the spirit of ‘Bayesian Persuasion’ by Kamenica and Gentskow [11], hereafter KG. Bayesian persuasion should not be confused with the theory of persuasion in communication games where an informed party (Sender) with certifiable information chooses what pieces of information to reveal to an uninformed decision-maker (Receiver). The subject matter of Bayesian persuasion is the choice by Sender of an ‘information structure’ (or measurement). In particular in KG, Sender is not better informed so they are not concerned with strategic revelation (or concealment) of information. The only choice variable of Sender is the measurement itself which is performed so the resulting signal is revealed publicly. And the question is how much can be achieved in terms of modifying a rational Receiver’s belief by an appropriate choice of measurement. The ultimate goal is to influence Receiver’s decision to act which depends on her beliefs about the world.

KG investigate the issue in the classical uncertainty framework. Receiver’s beliefs are expressed as a probability distribution over the set of states of the world and updating follows Bayes rule. However as amply documented the functioning of the mind is more complex and often people do not follow Bayes rule. Cognitive sciences propose various alternatives to Bayesianism. One avenue of research within cognitive sciences appeals to the formalism of quantum mechanics. A reason is that QM has properties that reminds of the paradoxical phenomena exhibited in human cognition. But the motivation for turning to QM is deeper. Indeed, the two fields share fundamental common features namely that the object of investigation and the process of investigation cannot always be separated. This similarity was already put forward by the fathers of quantum mechanics. Its mathematical formalism was developed to respond to that epistemological challenge (see [4]). In addition, quantum cognition has been successful in explaining a wide variety of behavioral phenomena (for a survey see Bruza and Busemeyer [7]). Finally, there exists by now a fully developed decision theory relying on the principle of quantum cognition (see [6, 8]). Therefore in the following we shall use the Hilbert space model of QM to represent the beliefs of an individual and capture the impact of new information on those beliefs. Clearly, the mind is likely to be even more complex than a quantum system but our view is that the quantum cognitive approach already delivers interesting new insights in particular with respect to persuasion.
In quantum cognition, we distinguish between the "world" and the decision-maker’s mental representation of it which is the basis for her decision. This representation of the world is modeled as a quantum-like system and characterized by a cognitive state. The decision relevant uncertainty is of non-classical (quantum) nature. As argued in Dubois et al. ([9]) this modeling approach allows capturing widespread cognitive limitations that people exhibit when constructing a mental representation of a ‘complex’ alternative.

The key quantum property that we use is that some characteristics (or properties) of a complex mental object may be ‘Bohr complementary’ that is incompatible in the decision-maker’s mind: they cannot have definite value simultaneously. A central implication is that measurements (new information) modifies the cognitive state in a non-Bayesian well-defined manner.

The kind of situations that we have in mind can be illustrated by the following story. A member of parliament (MP) is considering voting for a law to introduce a state of emergency in order to fight against terrorism. The terror threat can be either severe or moderate. If the threat is perceived as severe enough by our MP, she will support the law but not otherwise. Initially, she believes that the threat is severe with a high probability, so she would support the law. But a civil liberty activist wants her to vote down the law. Instead of trying to bring forward arguments about the actual threat, he brings up another topic: the EU process of decision-making and its intrusive regulatory policies. In particular, he proposes to find out whether the EU is regulating the weight of cucumbers - our MP represents farm owners. EU spent many months reflecting on cucumbers and ended up with a strict regulation which is broadly perceived as nonsensical. After this intermezzo, they return to the emergency issue and our MP does not feel as convinced as before and chooses to vote down the law. In section 3 we show how this story can be formalized and explained in terms of quantum persuasion.

Our results in [7] shows that with an unbounded sequence of measurements, the belief state of a quantum-like Receiver can be brought into any desirable state starting from any initial unknown state. Interestingly, the idea of turning to a sequence of measurements never arises in the classical context because any sequence of measurements can be merged into a single one (possibly very hard to implement in practice though). In the non-classical context it is generally not possible to merge several measurements into
one direct (projective) measurement unless they are compatible. But that framework allows accounting for incompatible measurements performed in a sequence. This incompatibility is an expression of the fact that measurements modify the system. How new information modifies the cognitive state is established in \[1\]. In order to extract results of more practical significance, we here investigate the polar case when only a limited sequence of measurement is feasible; more precisely we shall focus on the case of one or two measurements.

Because of the highly non-linear structure of the problem, we investigate a simpler task that we call “targeting”. The object of targeting is the transition of a belief state into another specified target state. We further confine attention to projective measurements and combinations of those. This is because our focus is on belief manipulation or state transition\[2\].

Our main result (Theorem 2) shows that if the initial belief state (prior) is known, any target state can be reached with a probability of at least $1/n$ (where $n$ is the dimension of the corresponding Hilbert space). This conclusion is also true for unknown prior when the target is a pure state. When both the prior and the target are pure states, the probability for successful manipulation increases to at least $1/2$.

The path of measurements is of interest on its own. The second measurement fully determines Receiver’s decision: we ask Receiver to determine herself with respect to the decision relevant uncertainty. For instance: do you think this used smartphone is worth more than 400 euro or less when the selling price is 400 euro. Clearly, if she thinks it is worth less than 400 euros, it is equivalent to a decision not to buy. However in a non-classical context, the elicitation (measurement) of the beliefs changes the state so it is not equivalent to making the decision on the basis on the non elicited (unmeasured) belief\[3\].

What we find most interesting is the characterization of the first measurement which aims at creating a state of uniform uncertainty. We show in Theorem 1 that Sender can always transform known Receiver’s prior into uniform uncertainty which explains appearance

\[1\] Updating follows Lüders’ rule which generalizes Bayes’ rule to the quantum context.

\[2\] The most general type of measurements called POVM (positive operator valued measurements) do not allow addressing state transition without additional restrictions.

\[3\] The impact of eliciting beliefs on decision-making has been exhibited in experimental set-ups see, e.g. [10].
of the probability $1/n$. This result is in clear contrast with the classical context where posteriors are constrained by Bayesian plausibility.\footnote{Bayesian plausibility entails that the expected posteriors must equal the priors.}

One interpretation is that the power of quantum persuasion is related to the possibility of creating confusion in Receiver’s mind. The strategy involves exploiting the incompatibility of perspectives (non-commutativity of measurements). Basically, this is a strategy of ‘calculated diversion’ to instigate confusion. Once Receiver is confused, it is much easier to persuade her to do what Sender wants.

The seminal paper by Kamenica and Gentskow \cite{KamenicaGentskow} analyzes the general problem of persuading a rational agent by controlling her information environment. It was followed by a series of other papers addressing competition in persuasion, costly persuasion, and most recently Bayesian persuasion with multiple senders. It also gave rise to a number of applications, see for instance “Persuading Voters” by Alonso Camara \cite{AlonsoCamara}. Our contribution extends that literature with a theoretical development to non-classical uncertainty. It also relates to more applied issues. Akerlof and Shiller \cite{AkerlofShiller} argue that manipulation is determinant to the functioning of markets. They suggest that it goes well beyond Bayesian updating: “Just change people’s focus and one can change the decisions they make” (p.173). In this paper we show that quantum persuasion is powerful even for short sequences of measurements and that it delivers a mechanism where merely changing the focus of the mind can significantly affect decision-making.

The paper proceeds as follows. In Section 2 we sketch the persuasion problem in the classical setting and thereafter formulate the corresponding problem in the quantum context. In Section 3 we analyze the targeting task and derive our main results. We provide an illustration in terms of the example presented in the Introduction. We conclude with some remarks.

\section{The model}

The persuasion problem has been formulated by Kamenica and Genskow (KG) \cite{KamenicaGentskow} as follows. There are two players called respectively Receiver and Sender. Receiver chooses
on action with uncertain consequences. To assess the value of the different actions Receiver uses her beliefs about the state of the world. Receiver’s action also has consequences for Sender. Therefore Sender may try to persuade Receiver so she chooses an action favorable to him. For that purpose Sender’s selects some information structure (or measurement) that generates new information (or signal) about the relevant uncertainty. Below we shortly remind of the formulation of the problem in the classical setting (in its simplest form). Thereafter, we describe in details the corresponding formulation in the context of non-classical (quantum) uncertainty.

2.1 The classical setting

Classical, or Bayesian, persuasion has been well described by KG. The uncertainty is formulated using a set Ω of states of Nature. For the sake of simplicity we assume that Ω is a finite set. An action is a function \( a : \Omega \rightarrow \mathbb{R} \); the number \( a(\omega) \) is a utility of Receiver from action \( a \) at the state \( \omega \). Receiver has to choose an action from some finite set of available actions. Since she does not know the true state of the world, Receiver relies on her belief to make her decision.

Receiver and Sender share a prior belief that is a probability distribution \( \beta \) on \( \Omega \) reflecting a common (objective) representation of the world i.e., a collection of probabilities \( \beta(\omega) \geq 0 \), \( \sum_{\omega} \beta(\omega) = 1 \). Let \( \Delta(\Omega) \) denote the simplex of probability distributions on \( \Omega \). The expected utility of an action \( a \) under a belief \( \beta \) is \( a(\beta) = \sum_{\omega} a(\omega)\beta(\omega) \). Thus an action can be represented as an affine function on \( \Delta(\Omega) \). Given a belief \( \beta \) Receiver chooses an action \( a^* \) that maximizes her expected utility \( a^*(\beta) \).

Action \( a \) brings Sender utility \( u(a) \). (Here we assume that Sender’s utility only depends on the action chosen by Receiver and not on the state of the Nature). Sender tries to influence receiver’s choice by providing some additional information. In order to obtain that information, Sender performs some measurement (of Nature) and informs Receiver about the result of the measurement.

An information structure is a map \( f : \Omega \rightarrow \Delta(S) \), where \( S \) denotes the set of signals. In state \( \omega \in \Omega \) the device generates a (randomized) signal \( f(\omega) \in \Delta(S) \). If we write this more carefully an information structure is given by a family \( (f_s, s \in S) \) of non-negative
function $f_s : \Omega \rightarrow \mathbb{R}_+; f_s(\omega)$ gives the probability of obtaining signal $s$ in a state $\omega \in \Omega$. Of course, $\sum_s f_s$ must yield the unit function $1_\Omega$ on $\Omega$.

When Receiver obtains a signal $s$, she, as a rational decision maker, updates her initial belief (or prior) $\beta$ using Bayes’ rule, i.e. she forms the posterior $\beta_s \in \Delta(\Omega)$, given as $\beta_s(\omega) = f_s(\omega) \beta(\omega)/p_s$, where $p_s = \sum_\omega \beta(\omega) f_s(\omega)$ is the probability of signal $s$. As a consequence, she chooses a new action $a^*(\beta_s)$ and Sender receives utility $u(a^*(\beta_s))$. When Sender uses such an information structure, his expected utility is equal to $\sum p_s u(a^*(\beta_s))$.

KG provide a characterization of the optimal measurement i.e., of the information structure that maximizes Sender’s expected utility. Bayesian plausibility, a property that says that the expected posterior is equal to the prior, $\beta = \sum_s p_s \beta_s$ plays a key role in their characterization.

### 2.2 The quantum setting

The description of a quantum system starts with the fixation of a Hilbert space (in our case finite dimensional) over the field of complex or real numbers. The choice of field does not affect our results so for the sake of simplicity we choose to limit ourselves to real numbers (the complex case obtains with minor changes). In this case $H$ is simply an Euclidian space equipped with a symmetric scalar product $(.,.)$.

We shall be interested not so much in the Hilbert space $H$ as in operators (that is linear maps from $H$ to $H$). For such an operator $A$ we denote by $A^*$ its conjugate which is defined by the following condition: $(v, Aw) = (A^*v, w)$ for all $u, w$ in $H$. Self-conjugate operators (for which $A = A^*$) are called **Hermitian**.

We model actions as Hermetian operators. The utility of an action for Receiver depends on her belief about the state of the relevant quantum system, see [8]. We elaborate on this below, after introducing two additional notions.

An Hermitian operator $A$ is **non-negative** if $(x, Ax) \geq 0$ for any $x \in H$. For example, any projector (that is a Hermitian operator $P$ with the property $PP = P$) is non-negative. Indeed, $(x, Px) = (x, PPPx) = (Px, Px) \geq 0$. More generally, for any operator $C$ the

---

More correctly we should speak here about symmetric operators; we use the term Hermitian holding in mind the extension to the complex case.
operator \( C^*C \) is non-negative.

The notion of trace of operators is a central instrument in what follows. It associates to an operator \( A \) the number \( \text{Tr}(A) \) called its trace. We collect in Proposition 1 below the properties of the trace that we use throughout the paper.

**Proposition 1.** i) \( \text{Tr}(A) \) is linear over \( A \);

ii) \( \text{Tr}(AB) = \text{Tr}(BA) \);

iii) If an operator is represented by a square matrix \( A = (a_{ij}) \), then \( \text{Tr}(A) = a_{11} + \ldots + a_{nn} \), that is \( \text{Tr}(A) \) equals to the sum of the diagonal terms of the matrix.

iv) If \( A \) is a non-negative operator then \( \text{Tr}(A) \geq 0 \), and equal to 0 only for \( A = 0 \).

**Definition.** A (cognitive) state is a non-negative (Hermitian) operator \( S \) with \( \text{Tr}(S) = 1 \).

The set of states is denoted as \( \text{St} \). It is the quantum analog to the classical set \( \Delta(\Omega) \). Indeed the notion of cognitive state reminds of the notion of belief in the classical context. The non-negativity of operator \( S \) is analogous to the non-negativity of a probability measure, and the trace 1 to the sum of probabilities which equals 1. In a way similar to the classical case, if the cognitive state of our DM is (operator) \( B \) she evaluates the (expected) utility of action \( A \) as \( \text{Tr}(BA) \).

**Examples of states.**

1. \( S_0 = E/\dim(H) \), where \( E \) denotes the identity operator on \( H \). This is the state of ‘uniform uncertainty’ or the ‘completely mixed’ state. In a sense, this state is a central point of the set \( \text{St} \). The utility of an action \( A \) in this state is equal to \( \text{Tr}(A)/\dim(H) \).

2. Pure states. Let \( e \) be an element of \( H \) with length 1 (that is \( \langle e, e \rangle = 1 \)). And let \( Pr_e \) be the orthogonal projector on \( e \), that is \( Pr_e(x) = \langle x, e \rangle e \) for any \( x \in H \). \( \text{Tr}(Pr_e) = \langle e, e \rangle = 1 \), therefore \( Pr_e \) is a state. Such states are called pure. The utility of an action \( A \) in such a state is equal to \( \langle e, Ae \rangle \).

3. Let \( S \) be a state, and \( U \) a unitary operator (that is \( U^{-1} = U^* \)), then the operator \( USU^{-1} \) is a state as well. Indeed, it is non-negative, and \( \text{Tr}(USU^{-1}) = \text{Tr}(U^{-1}US) = \text{Tr}(S) = 1 \). Note that the state \( S_0 \) is invariant under any unitary conjugation.

4. The set of states \( \text{St} \) is a convex space. If \( S_1, \ldots, S_r \) are states, and \( p_1, \ldots, p_r \) are non-negative real numbers such that \( p_1 + \ldots + p_r = 1 \), then the convex combination
$p_1S_1 + \ldots + p_rS_r$ is a state as well. The extreme points of this convex space are exactly the pure states. Any states can be represented as a convex combination of pure states; therefore non-pure states are also called mixed states.

As in the classical setting, the utility of an action $A$ is an affine function ($B \mapsto \text{Tr}(BA)$) on the convex state space $\text{St}$.

We understand cognitive states as Receiver’s beliefs about the state of some relevant quantum system. Receiver chooses an action from some (finite) set of available actions to maximize her utility. Her optimal action in state $B$ is denoted as $A^{opt}(B)$. As in the classical case, Sender may want to change receiver’s beliefs with the help of some information structure in order to induce a choice of action that is better for him (than the one Receiver would choose based on her prior). Below we define more precisely what we mean by information and its impact on Receiver’s belief. Essentially, this is the main and only difference between the quantum and the classical problem of persuasion.

*Information and its impact.* Information is generated by an information structure (IS) which consists of a measurement device (MD) and an informational channel (IC). The MD measures the system of interest and produces an outcome whereas the IC transforms the outcomes of the measurement device into signals. Receiver is informed about the performance of the MD, observes the signal and updates her prior belief. Formally, an IS is defined by three things: the set $S$ of signals, the collection of probabilities $p_s$ for the signals $s \in S$ (which depends on the state $B$) and the collection of associated posteriors $B_s$ that is the updated priors. In the present paper which focuses on updating, we have to distinguish between the two parts of the process (MD and IC). This is because the MD acts at the ‘quantum’ level (connected with state transition) whereas the IC is a purely ‘classical’ operation. Most clearly this is illustrated by the fact that a measurement whose different outcomes are collected into one and the same signal (which we call ‘blind’ measurement see below) nevertheless induces a change in Receiver’s belief. The measurement device (MD) is the core of any IS, and we have to describe it in more detail.

We next define a MD for the case the outcomes are directly communicated to Receiver that is when the signals are equal to the outcomes. Thereafter, we introduce information

---

6In our context, the relevant system is the represented world, see [9].
channels. The simplest MD consists of a single projective (or direct) measurement.

**Definition.** A *direct (or projective) measurement* is given by an orthogonal decomposition of the unit (ODU) \( H = \bigoplus_{i \in I} H_i \) of Hilbert space \( H \) (or equivalently, by a family \( P = (P_i, \ i \in I) \) of projectors, such that \( \sum_i P_i = E \)). The set of outcome of this measurement is the set \( I \); the probability \( p_i \) of outcome \( i \) in state \( B \) is given by Born rule as \( p_i = \text{Tr}(BP_i) = \text{Tr}(P_iBP_i) \). Upon obtaining a signal (outcome) \( i \) the prior \( B \) transits into the updated state \( B_i \) given by Lüders rule \( B_i = P_iBP_i/p_i \).\(^7\)

Note that \( p_i \geq 0 \) due to Proposition 1. Moreover, \( \sum_i p_i = 1 \); this is a simple consequence of the equality \( \sum_i P_i = E \). Lüders rule expresses the quantum nature of our setting. In the classical world, a measurement that generates new information changes beliefs but not the state itself. In the quantum world, measurements can essentially change the state of the measured system and, of course, the belief about the state. As a consequence the expected state \( B^{ex} = \sum_i p_iB_i = \sum_i P_iBP_i \) is generally not equal to the prior \( B \). This is in contradistinction with the classical case where \( B^{ex} = B \), a property called *Bayesian plausibility*. Note also that projective measurements are endowed with the property of *repeatability*: if we obtain an outcome \( i \) then a repeated measurement (with the same MD) gives the same outcome \( i \). When all the projectors \( P_i \) are one-dimensional (or pure states), we have a *complete* measurement; in this case \( B_i = P_i \) independently of \( B \).

Schematically, a direct measurement \( \mathcal{M} \) can be represented as a tree where the branches correspond to the possible outcomes. Which outcome obtains as the result of performing the measurement depends of course on the state of the system. An important feature of the quantum situation is that this relation is probabilistic: the state of the system only defines the probabilities for the different outcomes. We represent MD \( \mathcal{M} \) as follows

\(^7\)A behavioral justification for this rule is provided in [8]. And in [3] Lüders rule is shown to be a direct generalization of Bayes rule.
where $B$ is the initial state of quantum system, $p_i(B)$ the probability of outcome $i$, and the $B_i$ are the resulting (updated) states.

Above we addressed single measurements. However, Sender can also use a compound measurement consisting of a sequence of measurement devices (of course, the order in which the measurements are performed is important). Moreover, devices applied consecutively can be conditional, that is they can depend on the outcome of the previous one. The diagram below illustrates that point:

Here measurement $\mathcal{M}_1$ is applied when the outcome 1 of measurement $\mathcal{M}$ occurs, after outcome 2 the measurement stops, and after outcome 3, measurement $\mathcal{M}_3$ is applied. The set of outcomes of this compound MD is $\{a, b, c, 2, d, e\}$. The probability $p_a$ is equal $p_1(B)p_a(B_1)$ whereas $p_d = p_3(B)p_d(B_3)$. Upon receiving signal $a$ Receiver updates her beliefs to $B_a = \frac{p_a(P_1BP_1)P_2}{p_a}$.

Let us give a formal definition of a two-stage compound measurement. Suppose that the first measurement is given by an ODU $(P_i, i \in I)$, and that for every $i$ there is a ‘second’ measurement $M_i$ given by an ODU $(Q_j, j \in J_i)$. This compound MD has the set of outcomes $\bigcup_{i \in I}(\{i\} \times J_i)$. The probability of outcome $(i, j)$ (where $j \in J_i$) is $p(i, j) = \text{Tr}(Q_j(P_iBP_i)Q_j) = p_j(B_i)$. After obtaining outcome $(i, j)$ the system transits
into state $B_{i,j} = Q_j P_i B P_i Q_j / p(i, j)$.

In a similar way one can build more complicated trees of MDs. In our quantum context, we need to consider compound MDs because the composition of two (or more) non-commuting measurements is not a projective measurement.\footnote{Compound measurements belong to a general category of quantum measurements called POVM. However, nothing can be said in general about where such measurements takes the state. In order to address the issue of state transition and updating one needs to consider the details of the procedure.}

We now introduce the possibility that the outcomes of the measurement are not directly communicated. After the performance of a measurement (possibly compound), information about the outcome is transmitted to Receiver through an information channel which transforms the outcomes of the MD into signals.

**Definition.** A (random) informational channel (IC) is a mapping $f : I \to \Delta(S)$, where $S$ is the set of signals. In the other words, IC is a family $(f_s, s \in S)$ of non-negative functions on the set $I$ such that $\sum_s f_s = 1_I$. The number $f_s(i)$ is the probability for signal $s$ when outcome $i$ was obtained.

**Definition.** An information structure (IS) is a MD (with a set $I$ of outcomes) together with an IC $f : I \to \Delta(S)$.

Of course, using an informational channel only "garbles" the information extracted by the measurement device but this can be in Sender’s interest. Indeed, whenever the prior $B$ is a pure state, the posteriors $B_i$ following a projective measurement (or a sequence of direct measurements) are pure states too. But Sender may prefer Receiver to hold a mixed posterior. A random information channel can be used for the purpose of creating mixed posteriors.

Let us return to information structures. The probability $p_s$ for signal $s$ is equal to $p_s = \sum_i f_s(i) p_i$. In this case the initial state (prior) $B$ transits into the posterior $B_s$ given by the formula

$$B_s = \frac{\left( \sum_i f_s(i) p_i B_i \right) / p_s}{1}.$$  

Upon receiving signal $s$, Receiver understands (using Bayes rule) that the conditional probability $p(i|s)$ of outcome $i$ is $f_s(i) p_i / p_s$. She also understands that if she knew for sure that outcome $i$ occurred her belief would be represented by $B_i$. We can call $B_i$
‘intermediary beliefs’. Therefore the new cognitive state $B_s$ is a mixture of the states $B_i$ with weights $p(i|s)$, that is $B_s = \sum_i p(i|s)B_i = (\sum_i f_s(i)p_iB_i)/p_s$.

If the informational channel $f$ is the identity mapping from $I$ to $I$, we speak of a straightforward information structure. If the informational channel is degenerated (that is $S$ is a singleton); in other words, when Receiver gets no information except that the measurement was performed, we speak of a blind measurement. After a blind measurement, the posterior $B'$ is equal to $\sum_i p_iB_i$. As earlier emphasized the posterior $B'$ generally differs from the prior $B$.

The persuasion problem. Let us return to the persuasion problem, that is to the task of changing Receiver’s belief. Sender chooses an IS and announces it to Receiver. Thereafter the measurement is performed, the information channel is applied and the obtained signal is truthfully reported to Receiver. Receiver observes the signal, updates her beliefs, and then takes an (optimal given the new belief) action. Suppose that Sender chooses an IS with a measurement device $\mathcal{M}$ with the set of outcomes $I$ and the IC $f: I \rightarrow \Delta(S)$. Suppose further that the prior beliefs of Receiver are represented by a state $B$, and she observes a signal $s$. In this case she updates her prior $B$ to the posterior $B_s$. She chooses a new optimal action $A^{opt}(B_s)$. As a consequence of using this IS, Sender expects to get utility $\sum_s p_su(A^{opt}(B_s))$.

Since Sender’s objective is to maximize his expected utility, the question of selecting the optimal IS arises. In the quantum context, this problem is rather difficult due to the high degree of non-linearity. The problem can be all the more complicated as Sender can choose to use not a single MD, but a sequence of MDs. In the classical context we know that this possibility has no value because any sequence of measurements can be merged into a single one. This is generally not the case in the quantum context where opting for a sequence of measurements can significantly increase Sender’s persuasion power. We already mentioned an important result obtained in [7]. Namely, if Sender is not constrained with respect to the number of measurements, then starting from any prior he can with near certainty have Receiver’s belief transit into any targeted belief state. Of course this result has mostly a theoretical value. It shows that, in contrast with the

\footnote{Here we assume that Sender’s belief is equal to Receiver’s one.}
classical case, there exists in principle no obstacle to persuasion. In practice however the value of this result is strongly limited because Receiver is not likely to accept a large number of attempts to be persuaded. Moreover performing measurements is likely to be time and resource consuming. Therefore, is it interesting to investigate what Sender can achieve with very few measurements.

Below we consider a problem closely related to persuasion that we call the targeting.

3 Targeting

As mentioned above we do not address the question of optimal persuasion. We confine ourselves to a simpler task. Given a prior state $B$ and a ‘target’ state $T$, we evaluate the probability for the transition from $B$ into $T$ with one measurement or with a small series of sequential measurements. As usual we consider the case when Sender knows Receiver’s prior except in Proposition 3 where the result is established for unknown prior. The solution to this task can give a better understanding on how to solve the full persuasion problem. The connection between the two problems is most direct when Receiver chooses between two actions only i.e., ‘Good’ and ‘Bad’ (from the point of view of Sender). We can denote by $D_g$ the subset (in $\text{St}$) of ‘good’ states (leading Receiver to choose Sender’s preferred action) and by $D_b$ the subset of ‘bad’ ones. We can also assume that the prior $B$ lies in the domain $D_b$, because in the opposite case no measurement is needed. Then Sender chooses some point $T$ in the ‘good’ domain $D_g$ and looks for measurements which gives him this target $T$ with the largest possible probability $p$. This number and associated expected utility is a lower bound for what can be achieved in the full persuasion problem.\(^\text{11}\) We leave open the important question as to how to select the point $T$ in the ‘good’ domain $D_g$. Intuitively, $T$ should be as close as possible to the prior $B$.

The transition from prior $B$ to a target $T$ is realized with the help of an IS. More precisely suppose that we have an information structure consisting of an MD $\mathcal{M}$ with outcomes in $I$ and an information channel $f : I \to \Delta(S)$. We say that a signal $t \in S$\(^\text{10}\)In classical case Bayesian plausibility constitutes a major obstacle.

\(^{11}\)One can say that with ‘targeting’ the DM only cares about the target while ignoring all other consequences.
is “targeting” if \( t \) induces the transition of Receiver’s initial cognitive state \( B \) into the target \( T \) that is if the posterior \( B_t \) is equal to \( T \). The probability \( p_t \) for the signal \( t \) is called the probability of transition.

In other words, a signal \( t \in S \) is targeting if the operator \( \sum_i f_t(i)p_iB_i \) (see (1)) is proportional to the target state \( T \), \( pT = \sum_i f_t(i)p_iB_i \); in this case \( p = p_t = \sum_i f_t(i)p_i \) is the probability of transition. When Receiver learns signal \( t \) she updates her belief \( B \) into \( T \); the signal \( t \) comes with probability \( p_t \). Below we use a simplified notation \( w(i) = f_t(i) \) and call \( w(i) \) the targeting weights; of course, \( 0 \leq w(i) \leq 1 \), \( pT = \sum_i w(i)p_iB_i \), and \( p = \sum_i w(i)p_i \).

Thus, we shall be dealing with the following problem. Given an initial state (a prior) \( B \) and a final state (target) \( T \), we search for an IS (including one MD or a sequence of MDs) which transforms \( B \) into \( T \) with the largest possible probability. We know from (7) that with a large enough sequence of measurements, starting from any prior, the probability for that transition can be made arbitrary close to 1. In the next section we show that this probability can be made larger than \( 1/n \) with two measurements only. To this end we below consider in detail two cases: when only one measurement is feasible and when a series of two measurements is possible.

### 3.1 Transition with one measurement

It is intuitively clear that if the states \( B \) and \( T \) are close to each other then the probability \( p \) for the transition \( B \mapsto T \) can be made close to 1. Conversely, in [8] we proved that if \( p \) is close to 1 then the states \( B \) and \( T \) are close as well. On the other hand, the probability \( p \) can vanish. For example, if \( B \) and \( T \) are pure and mutually orthogonal states, one expects the probability for transition to be equal to 0. Indeed, the straightforward measurement \((T, ...)\) gives the posterior \( TBT \) which is proportional to \( T \), but the probability for the transition \( B \sim T \) is \( \text{Tr}(TB) = 0 \). Sender could use more subtle MD to achieve his target but the following general statement is true.

**Proposition 2.** Suppose that prior \( B \) and target \( T \) are orthogonal states (that is \( \text{Tr}(BT) = 0 \)). Then no information structure with a single measurement can transform \( B \) in \( T \).
Proof. Let \((P_i, i \in I)\) be a direct measurement, and \((w(i), i \in I)\) the corresponding targeting weights. We have \(pT = \sum_i w(i)P_iBP_i\). Multiplying this equation by \(B\), we obtain \(pBT = \sum_i w(i)BP_iBP_i\). Applying the trace, we get equality \(0 = \sum_i w(i)\text{Tr}(BP_iBP_i) = \sum_i w(i)\text{Tr}(P_iBP_iBP_i)\). Denoting \(A_i = P_iBP_i\) we have \(0 = \sum_i w(i)\text{Tr}(A_i^2)\). Each term of this sum is non-negative (see Proposition 1), from where it follows that \(\text{Tr}(A_i^2) = 0\) if \(w(i) > 0\). Proposition 1 implies now that \(A_i = 0\) for such \(i\). Hence \(p = \sum_i w(i)\text{Tr}(P_iBP_i) = 0\). \(\square\)

On the contrary, if \(B\) and \(T\) are reciprocally non-orthogonal states, then one can transform \(B\) into \(T\) with an IS that relies on a single MD. We shall not prove this here. Instead of, we consider three particular cases.

1A. Suppose that the target \(T\) is a pure state and is the projector on unit vector \(t\). Let \(e_1, \ldots, e_n\) be an orthonormal basis of \(H\) such that \(t = e_1\). Let \(M\) be the complete direct measurement on this base. If we make this measurement and obtain the outcome \(1\) then the prior \(B\) transits into pure state \(T\) with probability \(p = \text{Tr}(BT) > 0\).

Thus we have shown that if states \(B\) and \(T\) are non-orthogonal and \(T\) is pure then we can transform \(B\) into \(T\) with positive probability \(p\). Of course, \(p\) can be very small if \(B\) and \(T\) are almost orthogonal. On the contrary, if \(B\) and \(T\) are close then \(p\) is close to 1.

1B. Let us consider the case when the prior \(B\) is the ‘uniformly uncertain state’ \(S_0 = E/n\), where \(n = \dim H\). We assert that one can transform \(S_0\) into any target state \(T\) with probability at least \(1/n\), and explicitly indicate the corresponding MD.

Suppose that \((\text{in some orthonormal basis } e_1, \ldots, e_n)\) operator \(T\) has a diagonal form \(\text{diag}(a_1, \ldots, a_n)\). In the other words, \(T\) is the mixture \(a_1P_1 + \ldots + a_nP_n\), where \(P_i\) are projectors on \(e_i\), \(a_i \geq 0\), and \(a_1 + \ldots + a_n = 1\). As the MD we take the complete direct measurement associated with the basis \(e_1, \ldots, e_n\); as targeting weights, we take \(w(i) = a_i\). In this case the posterior \(B_i\) is equal to \((\sum_{i=1}^n a_i\text{Tr}(S_0P_i)/p_i\), where \(p_i = \sum_{i=1}^n a_i\text{Tr}(S_0P_i)\). But \(P_iS_0 = P_i(E/n) = P_i/n\), so that \(\text{Tr}(S_0P_i) = 1/n\) and \(p_i = \sum_{i=1}^n a_i/n = 1/n\). Therefore, the posterior is equal to \(\sum_i a_iP_i = T\), and the probability of transition is \(1/n\).

Note that we may be able to do better. If \(\max(a_i) < 1\) then we can take \(w(i) = a_i/\max(a_i)\) and obtain the desired transition with probability \(1/n \cdot \max(a_i)\).

1C. Note that in point 1B above the operators \(B\) and \(T\) are compatible (commuting).
Therefore this situation is essentially classical. The situation is completely different when the target \(T\) is equal to \(S_0\). We show (in contrast with the classical case) that there exists a blind measurement which transforms any arbitrary prior \(B\) (known to Sender) into \(T = S_0\) with probability 1. Of course, that measurement is incompatible with \(B\). The following Lemma provides the foundation for this assertion.

**Lemma.** Let \(p_1, \ldots, p_n\) be non-negative numbers, the sum of which is equal to 1. Then there exists a symmetric \(n \times n\) matrix \(A = (a_{ij})\) with spectrum \(p_1, \ldots, p_n\) and diagonal terms \(1/n\) (that is \(a_{ii} = 1/n\) for all \(i\)).

The Lemma above is a particular case of Horn’s wonderful theorem (see for example, [12, Theorem 9.B.2]).

**Corollary.** Let \(B\) be a state. Then there exists an orthonormal basis \(e_1, \ldots, e_n\) of the space \(H\) such that \((e_i, Be_i) = 1/n\) for any \(i = 1, \ldots, n\).

**Proof.** Let \(p_1, \ldots, p_n\) be the spectrum of \(B\); these numbers are non-negative and their sum is equal to 1. Let \(A\) be a symmetric matrix as in Lemma. Since \(A\) has the same spectrum as \(B\) there exists a unitary (indeed, orthogonal) matrix \(U\) such that \(B = UAU^{-1}\). Define vectors \(e_i\) \((i = 1, \ldots, n)\) as \(U(1_i)\), where \(1_i\) is the \(i\)-th canonical basis vector in \(R^n\). These vectors \(e_i\) form an orthonormal basis, and \((e_i, Be_i) = (U1_i, UAU^{-1}U1_i) = (U1_i, UA1_i) = (1_i, A1_i) = (a_{ij}) = 1/n\). □

Consider now the blind complete direct measurement \(\mathcal{M}\), given by the basis \(e_i\) from Corollary above. The posterior is the mixture of pure states \(e_i\) with weights \((e_i, Be_i) = 1/n\), that is the completely mixed state \(S_0 = E/n\). The probability of the transition (as for any blind MD) is equal to 1.

Thus, we proved the following

**Theorem 1.** Any state \(B\) can be transformed with one (blind) measurement into the completely mixed state \(S_0\) with probability 1.

This result is quite surprising because it shows that even in the case of a single measurement, the quantum setting yields results clearly distinct from those obtained in the classical case. Indeed in the classical context the support of the posterior must be in the support of the prior distribution. What is crucial here is that we use a measurement
which is incompatible with prior \( B \). We shall see next that this result which in itself is of little practical value since \( T \) seldom is equal to \( S_0 \) is very useful for targeting with two measurements. We also note that in contrast with the classical case the naive measurement \( T \) (which can be understood as a direct question on beliefs) because it changes the state has a power of persuasion as we illustrate in the example below.

### 3.2 The case of two measurements: instigating confusion

2A. Let us consider the following sequence of two measurements. First, we transform the prior \( B \) into completely mixed state \( S_0 \) applying the blind measurement \( \mathcal{M} \) defined above in point 1C. Thereafter, we apply the measurement in 1B above which transforms \( S_0 \) into the target \( T \) with a probability of at least \( 1/n \).

This proves the following

**Theorem 2.** Let prior \( B \) and target \( T \) be arbitrary states. Then there exists a sequence of two measurements transforming \( B \) into \( T \) with a probability of at least \( 1/n \).

We do not assert that the proposed sequence is optimal rather that it defines a lower bound. Often the transition is achievable with (much) higher probability; see Proposition 4 below.

2B. Note that in order to construct the measurement \( \mathcal{M} \) in 1B above, we need to know the prior \( B \). An interesting question arises: what is achievable when the prior \( B \) is unknown to Sender. Below we provide two particular results in this direction.

Suppose first that the target \( T \) is the completely mixed state \( S_0 \). Then, we can with probability 1, from any arbitrary unknown prior state \( B \), transit into \( T \) with the help of two measurements. We make an arbitrary direct measurement (with a base \((e_1,...,e_n)\)) as the first measurement \( \mathcal{M}_0 \). Depending on its result \( i = 1,...,n \) we make the second (blind) measurement \( \mathcal{M}_i \), given in Theorem 1, which transforms the state \( P_i = P_{e_i} \) into \( T = S_0 \).

The second case is more subtle. Namely, suppose that our target \( T \) is a pure state, that is the projector on a normalized vector \( t \). Due to the Corollary of the Lemma above (applied now to operator \( T \)), there exists an orthonormal basis \( e_1,...e_n \) such that
Let $e_i, Te_i = 1/n$ for any $i$. Since $Te_i = (e_i, t)t$, we have $1/n = (e_i, (e_i, t)t) = (e_i, t)^2$. In the other words, in basis $e_1, ..., e_n$ vector $t$ has the form $(1/\sqrt{n}, ..., 1/\sqrt{n})$.

Let $P_i$ be the projector on vector $e_i$. As the first measurement $\mathcal{M}$ we take the straightforward measurement in the basis $(e_1, ..., e_n)$. Applying this measurement, the prior $B$ transits into posteriors $B_i = P_i$ with probabilities $p_i = Tr(P_i B) = (e_i, B e_i)$.

What concerns the second measurement, we take it as the (naive) measurement $(T, ...)$. This measurement transits any posterior state $B_i = P_i$ into the state $T B_i T/Tr(TB_i) = T$ with probability $Tr(TB_i) = Tr(TP_i) = (t, e_i)^2 = 1/n$. In the average, we transit into the state $T$ with probability $\sum_i p_i \cdot 1/n = 1/n$.

Thus we proved the following

**Proposition 3.** If the target $T$ is a pure state then there exists a sequence of two measurements (depending only on $T$) which transits any (known or unknown to Sender) prior $B$ into $T$ with probability at least $1/n$.

We conjecture that the assertion in Proposition 3 is true for any arbitrary target $T$, i.e., pure or mixed.

**2C.** Let us consider more in detail the two-dimensional case i.e., when the dimension of Hilbert space $H$ is equal to 2. Moreover, we suppose that the prior $B$ and the target $T$ are pure states, given by normalized vectors $b$ and $t$. Without loss of generality, we can assume that these vectors have coordinates $t = (1, 0)$ and $b = (\cos(\varphi), \sin(\varphi))$.

As the first measurement $\mathcal{M}$ we take the direct measurement with orthonormal basis $(e_1, e_2)$, where $e_1 = (\cos(\varphi/2), \sin(\varphi/2))$ and $e_2 = (\sin(\varphi/2), -\cos(\varphi/2))$. After this measurement, the prior $b$ transits into the state $e_1$ with the probability $p_1 = (e_1, b)^2 = \cos^2(\varphi/2)$ and into the state $e_2$ with the (complementary) probability $\sin^2(\varphi/2)$.

Under an impact of the second (naive) measurement $(T, E-T)$ the state $e_1$ transits into $t$ with probability $(e_1, t)^2 = \cos^2(\varphi/2)$ whereas the state $e_2$ transits into $t$ with probability $(e_2, t)^2 = \sin^2(\varphi/2)$. As the result, with this sequence of two measurements, the prior $b$ transits into the target $t$ with probability $\cos^4(\varphi/2) + \sin^4(\varphi/2)$.

Since $\cos^2(\varphi/2) = 1/2 + \cos(\varphi)/2$ and $\sin^2(\varphi/2) = 1/2 - \cos(\varphi)/2$, we can rewrite $\cos^4(\varphi/2) + \sin^4(\varphi/2)$ as $1/2 + \cos^2(\varphi)/2$. Obviously, this number is no less than $1/2$. Summing up we have the following proposition

19
Proposition 4. When both the prior and the target are pure states known to Sender we can transform the prior $b$ into the target $t$ with the probability $(1 + \cos^2(\varphi))/2 = (1 + (b, t)^2)/2$, where $\varphi$ is the angle between the vectors $b$ and $t$.

One can easily see that a $n$-dimensional targeting problem essentially boils down to the two dimensional case when both the prior and the target are pure states known to Sender. Indeed, the vectors $b$ and $t$ lie in a two-dimensional subspace of $H$, and we can consider that they have the form $t = (1, 0, 0, ..., 0)$ and $b = (\cos(\varphi), \sin(\varphi), 0, ..., 0)$. As the first measurement $\mathcal{M}$ we take the direct measurement with orthonormal basis $(e_1, e_2, ..., e_n)$, where $e_1 = (\cos(\varphi/2), \sin(\varphi/2), 0, ..., 0)$, $e_2 = (\sin(\varphi/2), -\cos(\varphi/2), 0, ..., 0)$, $e_3 = (0, 0, 1, 0, ..., 0)$, $e_4 = (0, 0, 0, 1, 0, ..., 0)$ and so on. Further, the reasoning is the same as above.

3.3 Illustration: Persuading a MP to vote No

We now return to the story about the MP’s decision to support or not a law that introduces a state of emergency to combat terrorism (see Introduction). We below show how the theory developed above can be used to analyze the activist’s successful persuasion of the MP.

Let $H$ be a two-dimensional Hilbert space with an orthonormal basis $e_1, e_2$; the corresponding projectors are $P_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $P_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$. The MP (Receiver) has to choose between two actions: Yes and No. We represent the action No with Hermitian operator $N = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$. The utility of this action in the state $P_1$ is equal to 1, and is equal to -2 in the state $P_2$. The action Yes is given by Hermitian operator $Y = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.

If our MP has belief given by $B = \begin{pmatrix} a & b \\ b & 1 - a \end{pmatrix}$, then her expected utility of action No is $\text{Tr}(NB) = 3a - 2$ whereas the expected utility of action Yes is $\text{Tr}(YB) = 1 - 2a$. If $a < 3/5$ our MP votes Yes; if $a \geq 3/5$ the MP votes for No.

12The angle $\varphi$ is a measure of the distance between the prior and the target state.
We assume further that our MP holds initial belief (prior) \( B = \begin{pmatrix} 1/5 & 2/5 \\ 2/5 & 4/5 \end{pmatrix} \). With this belief MP votes Yes.

The activist (Sender) receives utility 1 when succeeding in rallying an MP to the NO vote whatever the true level of threat and utility 0 otherwise. With MP’s prior belief, the activist gets the utility 0. Can he persuade Receiver to vote NO by selecting an appropriate information structure? We would like to illustrate our results in Theorem 2 and Proposition 4 with this example as well as some issues pertaining to the choice of the target state.

1) To begin with, suppose that the activist naively asks MP whether she believes the threat is moderate (that is \( e_1 \)) or severe (that is \( e_2 \)). In the other words, he makes the direct measurement \((e_1, e_2)\). Already this simple and naive question changes the belief of our MP: with probability \( p_1 = \text{Tr}(BP_1) = 4/5 \) her posterior becomes \( P_1 \), and with probability 1/5 her posterior becomes \( P_2 \). So even this simple question increases the (expected) utility of the activist up to 1/5.

Next we show that with two measurements Sender can do much better.

2) Let us consider another perspective on the law which we call ‘the quality of public decision-making’ (cf. EU decision in the Introduction). A state of emergency gives extended new powers to public officials. This quality property is ”tested” by the following direct measurement \((Q_1, Q_2)\) with two possible outcomes \( Q_1 = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \) and \( Q_2 = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix} \). Such a measurement corresponds to a basis of the state space (of the mental representation of the issue) different from the basis \((e_1, e_2)\), a 45° rotation of \((e_1, e_2)\). This means that \((P_1, P_2)\) and \((Q_1, Q_2)\) are two non-commuting measurements. Or equivalently \((P_1, P_2)\) and \((Q_1, Q_2)\) correspond to properties of the system that are incompatible in the mind of Receiver. She can think in terms of either one of the two perspectives but she cannot synthesize (combine in a stable way) pieces of information from the two perspectives.

Assume now that Sender brings up a discussion about the quality perspective and performs measurement \((Q_1, Q_2)\) that is Receiver learns whether or not the EU regulates
cucumbers. With some probability \( p \) the belief \( B \) transits into \( Q_1 \) (and with the complementary probability into \( Q_2 \)). Thereafter, the activist again asks her whether she believes the threat is moderate or severe, i.e. performs measurement \((P_1, P_2)\). If the posterior \( B' \) is equal to \( Q_1 \) then with probability \( \text{Tr}(Q_1P_1) = 1/2 \) the new cognitive state becomes the target \( P_1 \). Similarly if the posterior is \( B'' = Q_2 \) with probability \( \text{Tr}(Q_2P_1) = 1/2 \) it transits into the target \( P_1 \). In the state of belief \( P_1 \) our MP votes NO. So with a probability 1/2 the target is reached and Sender gets an expected utility of 1/2 (instead of 0 without a persuasion or 1/5 in the case of a direct question).

3) Above we considered the case when the target was taken to be \( P_1 \). But we can take as a target another state \( T = \begin{pmatrix} 3/5 & \sqrt{6}/5 \\ \sqrt{6}/5 & 2/5 \end{pmatrix} \). This target (as a belief state of our MP) also induces action NO. As asserted in Proposition 4, Sender can transit into this target state with probability \( 1/2 + (b,t)^2/2 \). In our case \( b = (1/\sqrt{5}, 2/\sqrt{5}) \) and \( t = (\sqrt{3}/\sqrt{5}, \sqrt{2}/\sqrt{5}) \), so that \( (b,t) = \sqrt{3}/5 + \sqrt{8}/5 \), this probability is equal to 0.916!

4) But a little (quantum) marvel awaits us ahead. We shall show that with the help of one (blind) measurement Sender persuade MP to vote No with certainty.

For this purpose, Sender selects the blind measurement \((Q_1, Q_2)\), where \( Q_1 = \begin{pmatrix} 1+c \\ c \\ c \end{pmatrix} \) and \( c = 1/\sqrt{5} \). Correspondingly, \( Q_2 = \begin{pmatrix} 1-c \\ -c \\ 1+c \end{pmatrix} \). The probabilities are \( p_1 = (1+c)/2 \) and \( p_2 = (1-c)/2 \). The posterior is mixed state \( B^{ex} = p_1Q_1 + p_2Q_2 = \begin{pmatrix} 3/5 & 1/10 \\ 1/10 & 2/5 \end{pmatrix} \) and MP chooses to vote No with probability 1!

In our example, we assume that the danger perspective (threat level) which appeals to geopolitical arguments and emotional ones (fear) and the perspective on the quality of decision-making in public administrations which appeals to arguments about bureaucratic senselessness and the intrusiveness of the state, are two incompatible perspectives in the MP’s mind. She can think in either perspective but find it difficult to deal with them simultaneously although they are both relevant to the issue at stake.

The example illustrates how Sender can exploit the quantum indeterminacy of the cognitive state (expressed in the incompatibility of the two perspectives) to persuade
our (quantum-like) decision-maker. By performing a measurement on an incompatible perspective, the cognitive state is modified such that beliefs with respect to the severeness of the threat are updated so that Receiver prefers to vote NO with probability 1.

4 Concluding remarks

The theory of Bayesian persuasion establishes that it is often possible to influence a rational decision-maker by selecting a suitable information structure and making the corresponding measurement. However, in the classical uncertainty environment the persuasion power is strongly limited by Bayesian plausibility. The reality of persuasion or manipulation seems however far more extended. Therefore, we have here extended it to the non-classical (quantum) uncertainty environment. This corresponds to investigating persuasion with a quantum cognitive approach - an avenue of research that has experienced rapid growth under the latest 20 years. In a recent paper [7], we establish that provided Sender can make as many measurement as he wishes, full persuasion is always feasible. Sender can bring Receiver to believe whatever he wants. This theoretical result suggests that quantum persuasion is powerful indeed compared with Bayesian persuasion. But no one expects Receiver to accept attempts to persuasion under an indefinite time. This paper investigates what can be achieved with short sequences of measurements.

While Bayesian plausibility imposes a constraint, it also allows to simplify the persuasion problem so as to allow characterizing an optimal persuasion policy. In the non-classical context, no such simplification is available and the issue of optimality cannot be directly addressed. A simpler task that we call targeting is investigated and its results can be viewed as a lower bound for what can be achieved in a full persuasion problem. Our results show that even with a short sequence of two measurements, targeting is quite powerful. In particular when the prior is known (and is a pure state), whatever the initial belief, any target belief can be reached at least half of the time. Most interesting is to understand how this is made possible. The first step amounts to creating what we would like to call confusion (formally uniform uncertainty). This is possible precisely because any quantum belief state (even pure) can always be “broken” by using a non-commuting measurement. In quantum cognition this expresses the fact that people have difficulties to
process different kinds of information into a single and stable representation of the world. Our analysis suggests that this very feature makes a person easily influenceable. And that a person can be persuaded more effectively using well-designed distraction rather than by arguments of informational value to the decision. Distraction occurs when the mind is turned toward a perspective that Receiver finds hard to combine with her current perspective. It is a common experience that sellers use this tactic intuitively sensing that it is quite efficient. We have here provided an argument based on purely informational considerations that justifies a distraction policy in influence seeking activities.

References

[1] Akerlof G., and R. Shiller (2015) *Phishing for Phools - the economics of manipulation and deception*, Princeton University Press.

[2] Alonso, Ricardo, and Odilon Câmara. Persuading voters. *The American Economic Review* 106.11 (2016): 3590-3605.

[3] Beltrametti E, and Cassinelli G. (1981) *The logic of quantum mechanics*. Addison-Wesley Publising Company.

[4] Bitbol M. ed. (2009), *Physique Quantique et Sciences Humaines*, Edition CNRS Paris.

[5] Bruza P., and J. Busemeyer (2012) *Quantum Cognition and Decision-making*, Cambridge University Press.

[6] Danilov V. I., and A. Lambert-Mogiliansky (2010). Expected Utility under Non-classical Uncertainty. *Theory and Decision*, 68, 25-47.

[7] Danilov V. I., and A. Lambert-Mogiliansky (2018) ”Preparing a (quantum) belief system”, *Theoretical Computer Sciences* (in press), ArXiv [http://arxiv.org/abs/1708.08250](http://arxiv.org/abs/1708.08250)
[8] Danilov V. I., A. Lambert-Mogiliansky, and V. Vergopoulos (2018) Dynamic consistency of expected utility under non-classical (quantum) uncertainty. *Theory and Decision* (accessible on line) [http://rdcu.be/JrES](http://rdcu.be/JrES)

[9] Dubois F., and A. Lambert-Mogiliansky (2016). "Our (represented) world: a quantum-like object", in *Contextuality in Quantum Physics and Psychology*, ed. Dzhafarov et al, World Scientific, Advanced Series in Mathematical Psychology Vol. 6, 367-387.

[10] Erev, Ido, Gary Bornstein and Thomas S. Wallsten (1993). The negative effect of probability assessments on decision quality. *Organizational Behavior and Human Decision Processes* 55:78-94.

[11] Kamenica E., and M. Gentzkow (2011). Bayesian Persuasion. *American Economic Review*, 101(6): 2590-2615.

[12] Marshall A.W., and I. Olkin (1979) *Inequalities: Theory of Majorization and Its Applications*. Academic Press, New York.

[13] Nielsen M.A., and I.L. Chuang (2010) *Quantum Computation and Quantum Information*. Cambridge Univ. Press, Cambridge.

[14] Yearsley James (2016) "Advanced tools and concepts for quantum cognition: A tutorial" *Journal of Mathematical Psychology* 78, 24-39.