Intensity measures, fragility analysis and dimensionality reduction of rocking under far-field ground motions

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Abstract

Problems for which it is impossible to make a precise causal prediction are commonly tackled with statistical analysis. Although fairly simple, the problem of a rocking block on a rigid base subjected to seismic excitation exhibits a fascinating, complex response, making it extremely difficult to validate numerical models against experimental results, thus calling for a statistical approach. In this context, this paper statistically studies the rocking behaviour of rigid blocks, excited by synthetic far-field ground motions. A total of 50 million analyses are performed, considering rocking blocks of height ranging from 1 to 20 m and slenderness angle ranging from 0.1 to 0.35 rad. The results are used to explore the performance of different ground motion intensity measures (IMs), in terms of their ability to predict the maximum rocking rotation. By comparing the efficiency, sufficiency and proficiency of the IMs, it is found that the peak ground velocity (PGV) performs optimally. Then, fragility curves are constructed using different IMs, concluding again that the PGV is the most efficient IM. Impressively, the fragility curves for different block sizes collapse to a single curve, if a non-dimensional IM that involves PGV and the block geometry is used. Finally, the results produced on the basis of far-field synthetic motions are compared to results based on recorded ground motions.

KEYWORDS
dimensionality reduction, fragility curves, intensity measures, rocking

1 | INTRODUCTION

The rocking oscillator can be described as a rigid block that uplifts from its base when subjected to sufficiently strong ground motion. In the last few decades, it has attracted a lot of research interest for its use as an earthquake hazard mitigation strategy. The uplifting of a block or a structural element, such as a column, defines a limit value of inertia force, which can be transmitted between the foundation and the structure. Such a rocking mechanism limits the design loads and moments on both the structure and the foundation, offering great optimization potential. The effectiveness of rocking as an earthquake hazard mitigation strategy has been explored for various classes of structures, including rigid blocks,\textsuperscript{[1–8]} columns of ancient temples\textsuperscript{[9–18]} rocking frames,\textsuperscript{[19–27]} rocking bridges\textsuperscript{[28–36]} and rocking buildings.\textsuperscript{[36–44]} Makris and...
Vassiliou\textsuperscript{[19,20]} showed that the response of a rocking frame is equivalent to the response of a rigid free-standing solitary column with the same slenderness as the column of the frame, but of a larger size. For this reason, the rocking oscillator can be used to describe the dynamic response of a wide range of rocking isolated structures. Furthermore, the out-of-plane response of masonry walls is a form of rocking motion,\textsuperscript{[45–50]} and critical equipment or containers of hazardous material respond in purely rocking or rocking and sliding mode.\textsuperscript{[51–60]}

The very early studies on the subject point out the extreme complexity of the rocking block dynamics\textsuperscript{[6]} and the high sensitivity of the rocking response to the block size and slenderness, and to the details of the ground motion. In their pioneering work, Yim et al.\textsuperscript{[2]} concluded that from a probabilistic point of view the problem is much more ordered and that one should focus on the statistics of the responses to sets of ground motions, rather than the response to an individual ground motion. Following the same line of thought, it has been suggested that validation of rocking numerical models should be conducted in the statistical sense. Bachmann et al.\textsuperscript{[61]} showed that the model of Housner\textsuperscript{[1]} does not confidently predict the rocking response to an individual ground motion, but can nicely predict the probability of failure to a set of excitations. Vassiliou et al.\textsuperscript{[62]} and Manzo et al.\textsuperscript{[63]} extended the above conclusion to 3D rocking. The smoothening of the behaviour when the problem is treated statistically, explains the conclusion of Lachanas and Vamvatsikos\textsuperscript{[64]} that incremental dynamic analysis is a valid procedure for rocking structures.

In parallel, as there is no correlation between the rocking oscillator and any equivalent linear elastic oscillator,\textsuperscript{[65]} the question of optimal ground motion intensity measures (IMs) for rocking structures cannot be answered by resorting to the published work for linear or elastoplastic oscillators. To this end, Kafle et al.\textsuperscript{[66]} studied rocking blocks with heights between 1 and 3 m to conclude that the peak of the elastic displacement spectrum scales well with rocking-induced displacement. Studying rocking columns from ancient temples, Psycharis et al.\textsuperscript{[67]} concluded that peak ground velocity (PGV) performs better than peak ground acceleration (PGA) as an IM. Gelagoti et al.\textsuperscript{[68]} studied a two-storey frame with rocking foundations under analytical pulses and 18 recorded ground motions to conclude that the peak of the elastic displacement spectrum scales relatively well with the rocking-induced displacement. The same conclusion was drawn by Drosos and Anastasopoulos\textsuperscript{[69]} after shaking table testing of ancient rocking multi-drum columns, using 25 recorded ground motions.

Studying rocking blocks with heights of up to 5 m, Bakhtiary and Gardoni\textsuperscript{[70]} proposed a model that predicts the rocking rotation in function of several IMs. Dimitrakopoulos and Paraskeva\textsuperscript{[71]} performed numerical studies on rigid rocking blocks to understand the rocking potential of pulse-like ground motions using synthetic excitations. They proposed dimensionless univariate and bivariate IMs, based on PGA and PGV. In a follow-up paper, Giouvanidis and Dimitrakopoulos\textsuperscript{[72]} used unscaled recorded ground motions and proposed different IMs to predict the rocking angle and overturning potential. Studying an ancient rocking column, Pappas et al.\textsuperscript{[73]} concluded that both PGA and PGV correlate weakly with the maximum displacement. Kavvadias et al.\textsuperscript{[74]} proposed IMs based on integrating the elastic spectrum. Studying free and restrained rocking systems of negative systems under sets of far-field, near-field pulse-like and near-field non-pulse-like ground motions, Manzo et al.\textsuperscript{[75–77]} concluded that PGV performs overall better than PGA. Vassiliou and Sieber\textsuperscript{[78]} studied 3D rocking and concluded that the displacement response scales with a bivariate intensity measure that includes both PGA and PGV.

To extend these studies, this paper presents a rigorous probabilistic investigation of the rocking motion. Probabilities of overturning are derived on the basis of time history analyses using Housner’s numerical model and synthetic ground motions. The numerical model uses the Housner coefficient of restitution, which is found to be capable of predicting the statistical distribution of the rocking response,\textsuperscript{[62]} at least when impact forces are concentrated on the impacting corner. To the best of our knowledge, the present work considers the largest range of block geometries and far-field ground motion characteristics. A large number of conducted analyses allow the estimation of small failure probabilities with reasonable accuracy, which is essential for the derivation of the fragility of tall non-slender, and therefore, blocks of excellent stability. Various IMs are assessed on their capability of describing the rocking response and the overturning potential. Subsequently, a universal fragility curve is derived, which describes the probability of failure using non-dimensional quantities that involve both the block geometric characteristics and ground motion IMs.

2 | DESCRIPTION OF THE MODEL

2.1 | Numerical model of the rocking block

Assuming a large enough coefficient of friction between the block and the base, so that there is no sliding, the equation of motion of a free-standing block (Figure 1) with size $R$ and slenderness $\alpha$ is\textsuperscript{[79]}:
\[ I_O \ddot{\theta}(t) + mgR \sin(\alpha \text{sgn}(\theta(t)) - \theta(t)) = -ma_g(t)R \cos(\alpha \text{sgn}(\theta(t)) - \theta(t)) \] (1)

where: \( \dot{\theta} \) is the rocking angle; \( a_g \) the ground acceleration; \( \text{sgn}() \) the sign function; \( I_O \) the moment of inertia with respect to pivot point O or O'; and \( m \) is the mass of the block. For a homogenous block, the moment of inertia is \( I_O = \frac{4}{3}mR^2 \). Rocking initiates when the ground acceleration exceeds \( g \tan \alpha \). The frequency parameter \( p \) is defined as:

\[ p = \sqrt{\frac{mgR}{I_O}} \] (2)

For a homogenous block \( p = [(3g)/(4R)]^{1/2} \). Energy is assumed to be dissipated only during impacts and is modelled with a coefficient of restitution \( r \), defined as the ratio of the pre- to post-impact kinetic energy:

\[ r = \frac{\dot{\theta}_2^2}{\dot{\theta}_1^2} \] (3)

where: \( \dot{\theta}_1 \) is the angular velocity before and \( \dot{\theta}_2 \) the angular velocity after impact. Under Housner’s assumptions\(^1\):

\[ r = \left(1 - \frac{2mR^2}{I_O \sin^2 \alpha}\right)^2 \] (4)

For a homogenous block with \( I_O = 4/3mR^2 \), the coefficient of restitution becomes \( r = (1 - 3/2\sin^2 \alpha)^2 \). In this work, the equation of motion is integrated using the scripts provided in Vassiliou.\(^{80}\)

### 2.2 Ground motion model

The artificial ground motions are synthesized using the stochastic ground motion model developed by Rezaeian and Der Kiureghian.\(^{81–83}\) The model accounts for temporal and spectral nonstationarities of the ground motion. The procedure of synthesizing a single ground motion is illustrated in Figure 1. Ground motions are synthesized through subsequent filtering of Gaussian white noise. The first filtering with a linear filter introduces the spectral nonstationarity. The second filtering (time modulating filter) introduces the temporal nonstationarities, defining the time evolution of the energy release of the ground motion. The last filtering, applies a critically damped high pass filter to ensure zero velocity and displacement residuals. Because the ground motion model does not account for forward-rupture directivity and fling step effects, which are important for near-source ground motions, its application is limited to simulating far-field ground motions. Spectral versions of the Rezaeian and Der Kiureghian ground motion models have been published in Broccardo and Der Kiureghian\(^{84}\) and Broccardo and Dabaghi.\(^{85}\)
The ground motion model uses six mostly physically based parameters \((AI, D_{5-95}, t_{\text{mid}}, \omega_{\text{mid}}, \omega\dot{t}, \zeta_f)\), where \(AI\) is the Arias intensity, \(D_{5-95}\) is the effective duration (time between 5% and 95% levels of \(AI\)), \(t_{\text{mid}}\) is the time to the 45% level of areas intensity, \(\omega_{\text{mid}}\) is the filter frequency of the linear filter at \(t_{\text{mid}}\), \(\omega\dot{t}\) is the rate of change of the filter frequency with time, and \(\zeta_f\) is the filter damping ratio. In this study, the ground motion model parameters are selected quasi-randomly (Latin Hypercubes sampling), so as to follow a uniform distribution (Figure 2A). To ensure a large range of ground motion characteristics, the bounds of the distributions are selected such that they cover the largest part of the parameter distributions given in Rezaeian and Der Kiureghian,\(^{[83]}\) which are determined for a selection of ground motions from the NGA strong motion database. However, motions with low values of \(AI\) were not generated, as they result to very weak ground motions that would not even induce uplift.

Figure 2B graphically illustrates the correlations between the IMs of the simulated ground motion set, explored in this study. The values of the correlation coefficients are given by the colour and size of the dots. These correlations will help the interpretation of the results presented in Section 4.

### 3 | DESCRIPTION OF THE ANALYSIS

#### 3.1 | Points of simulation

Besides the ground motion parameters, the problem is defined by the two geometric parameters of the rocking block: the size \(R\) and the slenderness \(\alpha\). As summarized in Figure 3, in this work the size parameter is varied in the range \(R = [0.5 \text{ m}, 10 \text{ m}]\) and the slenderness in the range \(\alpha = [0.15 \text{ rad}, 0.35 \text{ rad}]\). As the response of rigid columns capped with a beam can be analysed with an equivalent solitary column,\(^{[19,20]}\) the selected size and slenderness intervals cover various applications, ranging from electrical equipment to rocking bridges or ancient Greco-Roman temples. As defined by Equation (4), Housner’s coefficient of restitution \(r\) is used, which is solely a function of \(\alpha\). Hence, the analysis contains a total of eight parameters, six of which describe the ground motion \((AI, D_{5-95}, t_{\text{mid}}, \omega_{\text{mid}}, \omega\dot{t}, \zeta_f)\), and the remaining two the geometry of the rocking block \((R, \alpha)\). Latin hypercube sampling is used to evenly populate this 8D parameter space with one million points. For each point, 50 analyses (ground motion simulation and dynamic analysis of the rocking block) using different Gaussian white noise are performed, resulting to 50 million analyses in total.

Figure 4 illustrates the distribution of the maximum rocking angle \(\theta_{\text{max}}\) for the complete dataset. Of the 50-million ground motions, 7.67-million overturned the block and 5.38-million did not even uplift it. In 87,790 simulations the block sustained a maximum rocking angle \(\theta_{\text{max}}\) larger than its slenderness \(\alpha\), with the maximum sustained rocking angle being \(\theta_{\text{max}} = 0.6515\) rad—almost twice the slenderness \(\alpha\) (in this specific case). Overall, the probability of failure is \(P_F = 0.1534\). Making the simplifying assumption that failure is reached when \(\theta_{\text{max}}\) exceeds the slenderness \(\alpha\), the probability of failure.
is estimated to be $P_F = 0.1552$. Hence, this assumption would have had a negligible effect on the estimation of failure probability. The same conclusion was drawn in Bakhtiary and Gardoni\cite{70} with a similar analysis using recorded ground motions. However, in the present study the “true” probability of failure is always derived, meaning that cases where $\theta > \alpha$ but without overturning, are not considered as failures.
3.2 Explored intensity measures

The above 50 million analyses correspond to 50 analyses per each one of the one million fixed points in the 8D space. However, the aim of this work is not to describe the response of the rocking oscillator in function of the six parameters of the Rezaeian and Der Kiureghian model, but in function of well-known IMs. Numerous ground motion IMs (and combinations of IMs)\textsuperscript{66–78} have been suggested for the rocking block. The present study concentrates on few, well-established IMs, which are relatively easy to compute. It neglects any IMs, which are based on the elastic spectrum, because of its incompatibility to the rocking oscillator. The selection mostly coincides with the one in Garini and Gazetas\textsuperscript{86} and Anastasopoulos et al.\textsuperscript{87}:

- The peak values of ground acceleration PGA, velocity PGV and displacement PGD.
- The mean period $T_M$, which is obtained from the Fourier amplitude spectrum as follows:

$$T_M = \frac{\sum C_i^2 (1/f_i)}{\sum C_i^2}$$

where: $C_i$ is the Fourier amplitude for each frequency $f_i$ within the range of 0.25–10 Hz.

- The area intensity $AI$, which is proportional to the integral of the squared acceleration time history $a_g(t)$:

$$AI = \frac{\pi}{2g} \int a_g^2(t) \, dt$$

- The root mean square acceleration ARMS:

$$ARMS = \sqrt{\int \frac{a_g^2(t) \, dt}{T_D}}$$

where: $T_D$ is the duration of the ground motion.

- The root mean square velocity VRMS:

$$VRMS = \sqrt{\int \frac{v_g^2(t) \, dt}{T_D}}$$

where: $v_g(t)$ is the ground velocity time history.

- The root mean square displacement DRMS:

$$DRMS = \sqrt{\int \frac{u_g^2(t) \, dt}{T_D}}$$

where: $u_g(t)$ is the ground displacement time history.

- The characteristic intensity IC, which is defined as follows:

$$IC = (ARMS)^{3/2} \sqrt{T_D}$$

- The specific energy density SED, which is defined as follows:

$$SED = \int v_g^2(t) \, dt$$
Figure 5  Schematic illustration of the data grouping into bins for the intensity measure variables

Table 1  Edges of bins for intensity measure variables

| Bin edges | Bin 1 | Bin 2 | Bin 3 | Bin 4 | Bin 5 | Bin 6 | Bin 7 | Bin 8 | Bin 9 | Bin 10 |
|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| PGA (g)   | 0.123 | 0.224 | 0.326 | 0.427 | 0.529 | 0.63  | 0.732 | 0.833 | 0.935 | 1.036  |
| PGV (cm/s)| 8.471 | 23.3  | 38.12 | 52.95 | 67.77 | 82.6  | 97.4  | 112.2 | 127.1 | 141.9  |
| PGD (m)   | 0.07  | 0.324 | 0.578 | 0.833 | 1.087 | 1.341 | 1.595 | 1.849 | 2.103 | 2.357  |
| T_M (s)   | 0.106 | 0.196 | 0.286 | 0.376 | 0.466 | 0.556 | 0.646 | 0.736 | 0.826 | 0.916  |
| AI (g/s)  | 0.039 | 0.192 | 0.345 | 0.498 | 0.651 | 0.804 | 0.957 | 1.11  | 1.263 | 1.416  |
| ARMS (g)  | 0.019 | 0.035 | 0.051 | 0.067 | 0.083 | 0.1   | 0.116 | 0.132 | 0.148 | 0.164  |
| VRMS (m/s)| 0.016 | 0.051 | 0.085 | 0.12  | 0.155 | 0.189 | 0.224 | 0.259 | 0.293 | 0.328  |
| DRMS (m)  | 0.02  | 0.089 | 0.157 | 0.226 | 0.294 | 0.362 | 0.431 | 0.499 | 0.568 | 0.636  |
| IC        | 0.022 | 0.06  | 0.097 | 0.135 | 0.172 | 0.21  | 0.247 | 0.285 | 0.323 | 0.36   |
| SED (m²/s)| 0.017 | 0.917 | 1.817 | 3.618 | 4.518 | 5.419 | 6.319 | 7.219 | 8.12  | 9.02   |
| CAV (m/s) | 5.219 | 9.811 | 14.4  | 18.99 | 23.59 | 28.18 | 32.77 | 37.36 | 41.95 | 46.55  | 51.14  |

Abbreviations: PGA, peak ground acceleration; PGV, peak ground velocity.

The cumulative absolute velocity CAV, which is defined as follows:

$$CAV = \sum_{i=1}^{N} H(PGA_i - \alpha_{min}) \int_{t_i}^{t_{i+1}} |a_g(t)| \, dt$$  \hspace{1cm} (12)

where: $N$ is the number of the 1-s time windows in the time series; $PGA_i$ is the PGA (in units of g) during the time window $i$; $\alpha_{min}$ is an acceleration threshold (usually taken as 0.025 g); $t_i$ is the start time of the time window $i$; and $H(x)$ is the Heaviside step function (unity for $x > 0$, zero otherwise).

3.3  Grouping the data set in intensity measures bins

Since the ground motion model does not produce ground motions with a specific intensity level (i.e., PGV = [10, 20, 30, 40, 50, 60, 70, 80, 90 cm/s]), to derive probabilities of overturn in function of IMs, the data set has to be binned according to IM values. Therefore, for each variable (PGA, PGV, PGD, T_M, AI, ARMS, VRMS, DRMS, or CAV) the data are grouped in 10 equally spaced bins (Figure 5). Because the distributions of the IMs can have thin tails, to avoid bins with almost no data inside, and to have an objective binning criterion, the spacing of the bins is selected such that 95% of the data always falls inside. Table 1 lists the edges of the bins.

The grouping of the block geometric variables ($R$, $\alpha$) into bins can be seen in Figure 3. The resulting 100 block geometries are depicted true-to-scale (centre of the bin interval). When computing the probability of failure as a function of the block variables and a single IM, the problem has three dimensions. In this three-dimensional space, each bin contains on average $0.95 \times 10^7 / 10^3 = 4.75 \times 10^4$ simulations. This number of simulations allows the estimation of small failure probabilities in the order of $P_F \sim 10^{-3}$ with reasonable accuracy.
### 3.4 Characteristics of optimal intensity measures

Previous studies\[^{[72,88–90]}\] identified optimal or proper IMs by examining several characteristics, such as efficiency, practicality, proficiency, sufficiency and hazard computability.\[^{[89,90]}\] An intensity measure is practical when there is a direct relation to the structural demand. Little or no dependence implies that the IM is not practical. Assuming that the relation between the engineering demand parameter EDP and the IM follows a power law:\[^{[91]}\]:

$$\text{EDP} = a \cdot \text{IM}^b \quad (13)$$

when the exponent parameter $b$ has a low value, the intensity measure has negligible influence on structural demand. Therefore, a practical intensity measure has a high value of exponent $b$, which can be estimated with linear regression analysis in the logarithmic plane, since:

$$\ln(\text{EDP}) = \ln(a) + b \cdot \ln(\text{IM}) \quad (14)$$

An efficient IM results in a small variability of structural demand, given the IM. In refs.\[^{[71, 72, 88–90]}\] efficiency of intensity measures is evaluated using $\beta_{D|IM}$, that is, the logarithmic standard deviation of the demand conditioned on the IM. A low value of $\beta_{D|IM}$ means less dispersion of the demand and consequently an efficient intensity measure. The coefficients of the linear regression analysis ($a, b$) can be used to obtain an estimate of $\beta_{D|IM}$:\[^{[92]}\]:

$$\beta_{D|IM} = \sqrt{\frac{\sum_i^N \left( \ln(\text{EDP}_i) - \ln(a \cdot \text{IM}_i^b) \right)^2}{N - 2}} \quad (15)$$

Apart from a linear relation between $\ln(\text{EDP})$ and $\ln(\text{IM})$, Equation (15) requires a constant variance, which may be a reasonable assumption only for a very limited range of IM levels.\[^{[93]}\] A relatively new metric, termed proficiency is presented in Padgett et al.\[^{[90]}\] It is a composite measure of efficiency and practicality, defined as:

$$\zeta = \frac{\beta_{D|IM}}{b} \quad (16)$$

A low value of the proficiency measure $\zeta$ indicates an optimal IM.

An intensity measure is considered sufficient, when it is statistically independent of ground motion characteristics, such as magnitude $M$ and the distance to the fault $R_{rup}$. The hazard computability of an IM refers to the easiness of assessing the probabilistic seismic hazard or determining the hazard curve. For example, in the case of PGA or PGV there are readily available hazard maps/curves or empirical relations that provide their values as a function of site and ground motion characteristics.

### 4 RESULTS

#### 4.1 Evaluation of practicality, efficiency and proficiency

To follow the above-described methodology an engineering demand parameter EDP has to be defined. As in Dimitrakopoulos and Paraskeva\[^{[71]}\] and Giouvanidis and Dimitrakopoulos,\[^{[72]}\] the absolute maximum response normalized with the block slenderness is used:

$$\text{EDP} = \frac{\|\theta_{\max}\|}{\alpha} \quad (17)$$

A value of $\text{EDP} = 0$ implies that the block does not uplift, while $\text{EDP} > 1$ implies that the block overturns (with few exceptions, see Section 3.1). As the value of maximum response $\theta_{\max}$ is categorical in nature, the evaluation of the IM in this subsection is restricted to simulations that uplift but do not overturn, which are referred to as safe-rocking.
FIGURE 6  Linear regression analysis of the maximum normalized response with respect to PGA, PGV, and $T_M$: (A) of the safe rocking simulations; and (B) of the safe rocking simulation with maximum normalized response larger than $\theta_{max}/\alpha = 10^{-3}$ (for plot clarity and file size of the figure each plot contains only 1000 randomly selected data points out of the $\sim 400,000$).

The linear regression analysis is performed by employing the *fitlm()*-function in MATLAB. Figure 6 shows the results for $(R_5, \alpha_5)$ and three intensity measures (PGA, PGV and $T_M$). Figure 6A presents the linear regression analysis using all the safe-rocking simulations. For the sake of clarity, the plots in Figure 6 contain only 1000 randomly selected data points out of the $\sim 400,000$. However, the linear regression analysis is performed always on the entire data set. One can observe that there are some simulations with very small response $\theta_{max}/\alpha = 10^{-3}$–$10^{-7}$ ($\ln(10^{-3}) \approx -7$), as in Giouvanidis and Dimitrakopoulos.[72] Usually, these are simulations using a ground motion where PGA is slightly higher than $g \tan \alpha$.

Although only few simulations, they have a significant effect on the linear regression analysis in the logarithmic space, which is consistent with the observation of Dimitrakopoulos and Paraskeva[71] for pulse-like ground motions. Therefore, Figure 6B plots the same analysis but using only simulations with a normalized maximum response larger than $\theta_{max}/\alpha > 10^{-3}$. Now, in the case of PGA, the exponent $b$ decreases from $b = 3.05$ to $b_{SR} = 1.55$ (subscript SR for ‘significant’ rocking). The coefficient of determination $R^2$, which is used to quantify efficiency in Giouvanidis and Dimitrakopoulos,[72] is also affected. In the case of PGA, $R^2$ reduces from $R^2 = 0.32$ to $R^2_{SR} = 0.16$, while in the case of PGV it increases from $R^2 = 0.49$ to $R^2_{SR} = 0.68$. Table 2 summarizes the results of the linear regression analysis for both: (a) all safe-rocking cases; and (b) safe-rocking, where $\theta_{max}/\alpha > 10^{-3}$. The displayed values are averaged over all the investigated 100 geometries.

When considering all the safe-rocking simulations, PGA (as well as ARMS and IC, which are strongly correlated to PGA—see Figure 2B) yields the largest $b$ and can therefore be considered as the most practical IM. With respect to efficiency, PGV shows the lowest $\beta_{DIM}$. However, the difference to the efficiency of other IMs is rather small. Furthermore, from Figure 6A–C it is obvious that for this large range of values the relation between EDP and IM is not linear, nor of constant variance. Hence, the quality of the $\beta_{DIM}$ estimate is at least questionable.

When restricting the set to simulations which show ‘significant’ rocking with $\theta_{max}/\alpha > 10^{-3}$($b_{SR}, R^2_{SR}, \beta_{DIM,SR}, \zeta_{SR}$), PGV and VRMS (which are strongly correlated) become clearly the most practical, most efficient and most proficient IMs. It is worth noticing that PGV shows for both cases the highest coefficient of determination $R^2 = 0.43$ and $R^2_{SR} = 0.16$, whereas for PGA the coefficient of determination is $R^2 = 0.33$ and $R^2_{SR} = 0.16$, indicating a relatively low goodness of fit.
TABLE 2  Linear regression coefficients $b$ and $R^2$, estimated dispersion of the demand $\beta_{DIM}$ and measure of proficiency $\zeta$: mean values over all geometries for both: (a) all safe-rocking cases; and (b) safe-rocking, where $\theta_{\text{max}}/\alpha > 10^{-3}$

| Reg. coeff. | PGA | PGV | PGD | $T_M$ | AI | ARMS | VRMS | DRMS | IC | SED | CAV |
|-------------|-----|-----|-----|------|----|------|------|------|----|-----|-----|
| (a) $b$     | 3.36| 2.52| 1.50| 1.32 | 1.99| 3.32 | 2.22 | 1.61 | 2.87| 1.01| 1.46|
| $R^2$       | 0.33| 0.43| 0.26| 0.12 | 0.33| 0.35 | 0.41 | 0.29 | 0.38| 0.34| 0.12|
| $\beta_{DIM}$ | 1.75| 1.62| 1.85| 2.01 | 1.75| 1.73 | 1.66 | 1.81 | 1.68| 1.76| 2.03|
| $\zeta$     | 0.57| 0.64| 1.25| 2.57 | 0.95| 0.57 | 0.75 | 1.13 | 0.66| 1.77| 1.47|
| (b) $b_{SR}$| 1.50| 1.97| 1.22| 1.51 | 0.92| 1.63 | 1.76 | 1.29 | 1.41| 0.82| 0.46|
| $R^2_{SR}$  | 0.16| 0.61| 0.41| 0.37 | 0.17| 0.19 | 0.60 | 0.45 | 0.21| 0.52| 0.06|
| $\beta_{DIM,SR}$ | 1.23| 0.83| 1.03| 1.05 | 1.22| 1.20 | 0.84 | 0.99 | 1.19| 0.92| 1.30|
| $\zeta_{SR}$| 0.84| 0.43| 0.86| 0.80 | 1.35| 0.76 | 0.48 | 0.78 | 0.88| 1.15| 77.7|

Abbreviations: PGA, peak ground acceleration; PGV, peak ground velocity.

As a side note, the value of the proficiency measure for CAV of $\zeta_{SR} = 77.7$ is not a mistake. For some not slender geometries, the regression coefficient $b$ is very close to zero, resulting in a very large value of $\zeta_{SR}$.

At this point, it should be noted that a rigid body model cannot confidently predict very small amplitude responses, as these are controlled by the deformability of the structure and by the unavoidable imperfections of the contact surface. Therefore, it is reasonable to focus on the safe rocking set, where $\theta_{\text{max}}/\alpha > 10^{-3}$—without disregarding that any conclusions on IM performance will not be applicable to small amplitudes. Based on this limitation, the most efficient, sufficient and proficient measure is PGV.

4.2 Efficiency assessment of the intensity measures with respect to the ground motions overturning potential

A limitation of the intensity measure evaluation is that it is performed exclusively on the safe-rocking simulations. Hence, simulations that did not uplift the block or induced overturning were not considered. This is an inherent limitation, as rocking is categorical in nature and the rocking angle assigned to overturning is no more than a convention. An alternative procedure, is to evaluate IMs based on their ability to predict the failure probability, $P_F$.

Figure 7 depicts the variation of $P_F$ with PGA, PGV, PGD and $T_M$ for some characteristic blocks out of the ones examined (Figure 3). Such curves are also known as fragility curves. The horizontal axes are the bin numbers used for the data grouping into bins. For each IM, the bin intervals are listed in Table 1. An ideal IM would produce a step function, clearly distinguishing between a safe domain ($P_F \sim 0$) and an unsafe domain ($P_F \sim 1$). In this ideal case, the logarithmic standard deviation of the IM would be $\beta = 0$. Hence, the steeper the curve, the lower the $\beta$, and the more efficient the IM. For the three geometries of Figure 7, it is obvious that the efficiency of PGA is relatively low, since its curves show an early plateau.
for \( P_F \leq 0.3 \). PGV consistently shows the steepest curve, with PGD and \( T_M \) following closely. Therefore, PGV, PGD, \( T_M \) seem to be more efficient intensity measures than PGA.

To systematically evaluate the efficiency of IMs, the approach adopted in Dimitrakopoulos and Paraskeva\(^{[71]}\) and Giouvanidis and Dimitrakopoulos\(^{[72]}\) is followed. The logarithmic standard deviation \( \beta \) of the intensity measure is estimated by fitting a lognormal cumulative distribution function to the empirical fragility curve (Figure 7) for each IM and block geometry. Using the lognormal cumulative distribution, the fragility function becomes:

\[
P(F|\text{IM} = x) = \phi \left( \frac{\ln(x) - \ln(\varphi)}{\beta} \right)
\]

where: \( P(F|\text{IM} = x) \) is the probability that a ground motion with IM level \( x \) causes overturning of the block; \( \phi(\ldots) \) is the standard normal cumulative distribution function; \( \varphi \) is the median of the distribution (the IM level with \( P_F = 50\% \)); and \( \beta \) is the standard deviation of interest.

The fragility function fitting is performed by following the maximum likelihood estimation (MLE) approach\(^{[94]}\) which is only briefly presented here: Given a set of \( n_j \) ground motions with IM levels \( x_j \) and probability of overturn \( p_j \), the probability \( P \) of observing \( z_j \) overturns of the block subjected to the set of \( n_j \) ground motions is given by the binominal distribution:

\[
P = \binom{n_j}{z_j} p_j^{z_j} (1 - p_j)^{n_j - z_j} \tag{19}
\]

Hence, the likelihood of \( p_j \) given \( n_j \) and \( z_j \) is:

\[
L(p_j; n_j, z_j) = \binom{n_j}{z_j} p_j^{z_j} (1 - p_j)^{n_j - z_j} \tag{20}
\]

For various IM levels \( x_j \), the likelihood becomes:

\[
L(p_j; n_j, z_j) = \prod_{j=1}^{m} \binom{n_j}{z_j} p_j^{z_j} (1 - p_j)^{n_j - z_j} \tag{21}
\]

where: \( m \) is the number of IM levels. The aim is to identify the fragility curve, which predicts \( p_j \) such that the likelihood is maximized. Substituting Equation (18) for \( p_j \):

\[
L(\varphi, \beta; n_j, z_j) = \prod_{j=1}^{m} \binom{n_j}{z_j} \phi \left( \frac{\ln(x_j) - \ln(\varphi)}{\beta} \right)^{z_j} (1 - \phi \left( \frac{\ln(x_j) - \ln(\varphi)}{\beta} \right))^{n_j - z_j} \tag{22}
\]

Since it is equivalent and numerically easier, the parameters of the lognormal distribution \( \varphi \) and \( \beta \) are estimated by maximizing the logarithm of the likelihood function:

\[
\{\hat{\varphi}, \hat{\beta}\} = \arg\max_{\varphi, \beta} \sum_{j=1}^{m} \left[ \ln \left( \binom{n_j}{z_j} \right) + z_j \ln \phi \left( \frac{\ln(x_j) - \ln(\varphi)}{\beta} \right) + (n_j - z_j) \ln \left( 1 - \phi \left( \frac{\ln(x_j) - \ln(\varphi)}{\beta} \right) \right) \right] \tag{23}
\]

Equation (23) does not require multiple observations at each IM level \( x_j \). It is possible to use ground motions, each having a unique IM level. In this case, the number of ground motions is \( n_j = 1 \), the number of observed overturns is \( z_j = 1 \) if the block overturns, and \( z_j = 0 \) otherwise. This avoids inaccuracies introduced through the grouping of data with respect to the IM level. The numerical optimization is performed using the MATLAB code supplement to Baker\(^{[94]}\) which can be accessed through\(^{[95]}\).

Figure 8 depicts the fragility function fitting for PGV for (a) different values of size \( R \) and (b) different values of slenderness \( \alpha \) using: (left) linear scale; and (middle) semi-logarithmic scale. This probabilistic representation of the rocking problem is remarkably ordered. The failure probability increases monotonically with an increase in PGV, decreases with an increase in size \( R \) and decreases with an increase in slenderness value \( \alpha \). Furthermore, the fitted lognormal CDFs seem
to be in good agreement with the failure probabilities derived from the grouping of the data. The discrepancies for small probabilities and large block sizes $R$, or large slenderness angles $\alpha$, can be attributed to the slightly different data sets. The lognormal CDFs are fitted using the entire dataset, while the data grouping uses only data that falls in between the 2.5% and the 97.5% quantile of the distribution of PGV. Overall, the assumption of lognormal distributed fragility curves seems reasonable, at least for PGV and the present ground motion set.

Table 3 lists the maximum likelihood estimates for the logarithmic standard deviation $\beta$ for a selection of different geometries. PGV and VRMS consistently show the lowest standard deviation and are consequently the most efficient IMs with respect to the overturning potential of ground motions. In the case of PGA, the standard deviation $\beta$ varies quite

Table 3 Maximum likelihood estimates of the logarithmic standard deviation $\beta$

| MLE       | PGA  | PGV  | PGD  | $T_M$ | AI   | ARMS | VRMS | DRMS | IC   | SED | CAV |
|-----------|------|------|------|-------|------|------|------|------|------|-----|-----|
| $\beta (R_1, \alpha_1)$ | 0.72 | 0.40 | 0.60 | 0.96  | 1.17 | 0.73 | 0.44 | 0.59 | 0.90 | 0.80 | 0.70 |
| $\beta (R_1, \alpha_5)$ | 0.81 | 0.39 | 0.73 | 0.73  | 1.12 | 0.75 | 0.44 | 0.68 | 0.83 | 0.99 | 1.03 |
| $\beta (R_1, \alpha_{10})$ | 0.86 | 0.43 | 0.99 | 0.62  | 1.40 | 0.69 | 0.48 | 0.84 | 0.70 | 1.39 | 1.83 |
| $\beta (R_5, \alpha_1)$ | 1.23 | 0.43 | 0.62 | 0.68  | 1.45 | 1.18 | 0.48 | 0.63 | 1.19 | 0.79 | 1.43 |
| $\beta (R_5, \alpha_5)$ | 1.70 | 0.37 | 0.68 | 0.55  | 1.59 | 1.44 | 0.40 | 0.63 | 1.32 | 0.89 | 1.70 |
| $\beta (R_5, \alpha_{10})$ | 1.45 | 0.38 | 0.87 | 0.49  | 1.17 | 1.01 | 0.36 | 0.70 | 0.86 | 1.09 | 5.95 |
| $\beta (R_{10}, \alpha_1)$ | 1.84 | 0.43 | 0.58 | 0.64  | 1.83 | 1.77 | 0.46 | 0.60 | 1.60 | 0.77 | 1.87 |
| $\beta (R_{10}, \alpha_5)$ | 2.50 | 0.40 | 0.62 | 0.59  | 1.97 | 2.27 | 0.38 | 0.56 | 1.82 | 0.80 | 3.02 |
| $\beta (R_{10}, \alpha_{10})$ | 2.07 | 0.38 | 0.69 | 0.62  | 1.11 | 1.42 | 0.35 | 0.57 | 1.03 | 0.84 | 4.36 |
| Mean($\beta$) | 1.63 | 0.39 | 0.70 | 0.60  | 1.52 | 1.38 | 0.41 | 0.64 | 1.25 | 0.91 | 2.66 |

Abbreviations: PGA, peak ground acceleration; PGV, peak ground velocity.
substantially with geometry. It is relatively low for small blocks, while for large blocks it is high. Due to the predominant periods of seismic motions, relatively small blocks will overturn if the ground motion is enough to induce uplift. Hence, PGA, which deterministically controls uplift, is expected to correlate more to the behaviour of smaller rather than larger blocks. Overall, for the block sizes considered, the efficiency of PGA is low as in Dimitrakopoulos and Paraskeva.\[71]

Unlike PGA and most of the remaining IMs, the standard deviation of PGV varies only little with geometry. Even VRMS (which yields the second best $\beta$), shows a range of values almost twice as large as the ones for PGV. This turns out to be a key feature of an IM, because the shape of the lognormal fragility curve is solely controlled by $\beta$, while the median or mean $\phi$ controls the horizontal location of the curve.

4.3 Dimensionality reduction of the rocking problem through the derivation of a universal fragility curve

As the fragility curves for different geometries are of similar shape, it is possible to make the curves collapse to a master curve, and hence reduce the dimensionality of the problem. Focusing on analytical pulses, Zhang and Makris\[79] showed that the maximum response of the rocking block to a pulse excitation of a given waveform, acceleration amplitude $a_g$ and frequency $\omega_g$ can be described by a function of the general form:

$$\theta_{\max} = f(\alpha, p, \omega_g, a_g, r)$$  \hfill (24)

where: $\alpha$ is the slenderness angle; $p$ the rocking frequency parameter; $g$ the gravitational constant; and $r$ the coefficient of restitution. By employing dimensional analysis (Buckingham’s $\Pi$ theorem), Equation (24) can be reduced to\[79]:

$$\theta_{\max} = f\left(\frac{\omega_g}{p}, \frac{a_g}{g}, \alpha, r\right)$$  \hfill (25)

In the case of slender blocks (small $\alpha$), employing orientational analysis\[96] leads to further dimensionality reduction:

$$\frac{\theta_{\max}}{\alpha} = f\left(\frac{\omega_g}{p}, \frac{a_g}{\alpha g}, r\right)$$  \hfill (26)

The above analysis holds for analytical pulses of a given waveform and cannot be directly extended to real ground motions, as IMs carry only part of the information that characterises a ground motion, leading to motion-to-motion variability of the response. However, it will be shown that by choosing appropriate IMs the dimensionality of the problem can be reduced in the statistical sense, if one aims at computing probabilities of overturn to sets of ground motions, rather than rocking angles to individual ground motions.

To this end, Figure 8A (right) plots the fragility curves for a given slenderness bin $\alpha = 0.17$–0.2 rad and for different $R$ bins, where $p$PGV is used as an IM. Remarkably the curves collapse to almost a single master fragility curve. A similar observation is made by Petrone et al.\[97] in an assessment of freestanding building contents for five block geometries and using $p$PGV$/\omega_g$ as an IM. In Figure 8B (right) the $R$ bin is kept constant and equal to $R_c = 5.25$–6.20 m, and collapse of the fragility curves to a single master curve is achieved by using $r^2$PGV$/\omega_g$ as an IM. Note, that in the present study the coefficient of restitution $r$ is not an independent variable, but the Housner coefficient of restitution is used, which is solely a function of slenderness $\alpha$. Hence, it is not possible to determine which part of the variation of the fragility curve is attributed to $\alpha$, and which part of the variation is attributed to $r$. Therefore, as $r$ was not modulated independently of $\alpha$, the conclusions drawn concern only block that dissipate energy according to Housner.

Figure 9 depicts the same 20 fragility curves (19 different curves, since the curve $R_6$ and $\alpha_4$ is displayed twice) of Figure 8 using the nondimensional term $r^2$PGV$/\omega_g$ as IM. The gravity constant $g$ is added to the denominator to non-dimensionalize the IM. It can be well observed that the fragility curves collapse to almost a single master curve.

Figure 10 shows the same representation as Figure 9, but now contains all 100 geometries. The data points and fragility curves are coloured with respect to the block slenderness $\alpha$, from red representing slender blocks to blue representing non-slender blocks. Compared to Figure 9, the scatter of the fragility curves expectedly increases. The curves that deviate
FIGURE 9  Variation of the probability of failure $P_F$ with $r^2 \frac{p_{PGV}}{g \tan(\alpha)}$ and size $R$ for blocks with the same slenderness $\alpha_s$ (blue colour) and slenderness $\alpha$ for blocks with the same size $R_s$ (red colour) in: (A) linear; and (B) semi-logarithmic scale.

FIGURE 10  Variation of the probability of failure $P_F$ with $r^2 \frac{p_{PGV}}{g \tan(\alpha)}$ for the investigated 100 geometries in: (A) linear; and (B) semi-logarithmic scale.

The most correspond to slender geometries. Looking at the same curves but coloured with respect to the block size, it can be observed that it is mainly the small blocks that contribute to the scatter—this is not shown in this paper for reasons of conciseness. The scatter may be caused by the grouping of the data. Since equally spaced bins are used, the data grouping is much more relevant for the small and slender blocks. Particularly in the case of block size, the first bin contains a range of $R = 0.5$–1.45 m. Hence, in the same bin, the larger blocks are almost three times the size of the smaller blocks. Overall, having in mind the large range of geometries, the amount of scatter is tolerable and the resulting curve can be understood as a universal fragility curve, such that it describes the overturning potential of a block independently of its size $R$ and slenderness $\alpha$.

Notably, Dimitrakopoulos and Paraskeva[71] concluded that $p_{PGV}/(g \tan(\alpha))$ is the best IM for rocking structures after studying pulse-like ground motions. Therefore, this study leads to the same result for far-field ground motions.
**Table 4** Comparison between the ML estimates of the logarithmic standard deviation $\beta$ of Giouvanidis and Dimitrakopoulos\(^{[72]}\) and the present study

| Exc. | Structure | PGA/$(\alpha_5)$ | pPGV/$(\alpha_5)$ | p$^2$PGD/$(\alpha_5)$ | $pT_M$ | pAI/$(\alpha_5)$ | ARMS/$(\alpha_5)$ |
|------|-----------|------------------|------------------|----------------------|--------|----------------|----------------|
| hor1 | $A \sim (R_1, \alpha_5)$ | 0.630 | 0.343 | 2.821 | 0.651 | 0.879 | 0.599 |
|      | $B \sim (R_2, \alpha_5)$ | 0.903 | 0.291 | 2.909 | 0.567 | 1.120 | 0.704 |
|      | $C \sim (R_3, \alpha_5)$ | 1.073 | 0.359 | 3.228 | 0.615 | 1.337 | 0.924 |
| hor2 | $A \sim (R_1, \alpha_5)$ | 0.556 | 0.354 | 3.109 | 0.599 | 0.898 | 0.590 |
|      | $B \sim (R_2, \alpha_5)$ | 0.800 | 0.435 | 3.034 | 0.554 | 1.278 | 0.881 |
|      | $C \sim (R_3, \alpha_5)$ | 1.214 | 0.477 | 3.259 | 0.531 | 1.615 | 1.045 |

**Ground motion set of this study**

| Geometry | PGA | PGV | PGD | $T_M$ | AI | ARMS |
|----------|-----|-----|-----|-------|----|------|
| $(R_1, \alpha_5)$ | 0.895 | 0.384 | 0.722 | 0.618 | 1.17 | 0.808 |
| $(R_2, \alpha_5)$ | 1.05 | 0.386 | 0.661 | 0.631 | 1.29 | 0.986 |
| $(R_3, \alpha_5)$ | 1.48 | 0.408 | 0.632 | 0.604 | 1.56 | 1.355 |

Abbreviations: PGA, peak ground acceleration; PGV, peak ground velocity.

5 | COMPARISON WITH RECORDED GROUND MOTIONS

This work used the model of Rezaeian and Der Kiureghian to generate 0.5 million far-field synthetic excitations per investigated geometry. Giouvanidis and Dimitrakopoulos\(^{[72]}\) determined optimal IMs and fragility curves using 12,686 recorded and unscaled ground motions from the NGA, KiK-NET, ESMD and ITACA databases, without distinguishing between pulse-like and non-pulse-like motions. In an effort to contribute to the debate around the use of synthetic or artificial ground motions, this section compares the results obtained with artificial ground motions to the ones of Giouvanidis and Dimitrakopoulos\(^{[72]}\).

Giouvanidis and Dimitrakopoulos\(^{[72]}\) focused on four specific rocking blocks, A–D. Three of them (A–C) fall within the range of geometries examined herein: $(R_A = 1.17 \text{ m}, \alpha_A = 0.2 \text{ rad})$, $(R_B = 2.27 \text{ m}, \alpha_B = 0.17 \text{ rad})$, $(R_C = 5.20 \text{ m}, \alpha_C = 0.14 \text{ rad})$. The relevant bins of this paper are $(R_1, \alpha_5)$, $(R_2, \alpha_3)$ and $(R_3, \alpha_2)$. Contrary to this work,\(^{[72]}\) assumes a constant coefficient of restitution of $r = \eta^2 = 0.846$—and this will be a source of discrepancy between the results. Furthermore, Giouvanidis and Dimitrakopoulos\(^{[72]}\) investigate the conditional probability of overturning given that the block uplifts. Therefore, the estimates of $\beta$ in Table 3 are not directly comparable to the estimates of Giouvanidis and Dimitrakopoulos\(^{[72]}\).

Table 4 re-derives ML estimates of $\beta$ for the relevant to\(^{[72]}\) bins, now conditioned on uplift, and it compares them to the values obtained by Giouvanidis and Dimitrakopoulos\(^{[72]}\). Notably,\(^{[72]}\) offers two values (hor1 and hor2), one for each horizontal component of the ground motions used. Except for PGD, the values exhibit remarkable similarity, despite the different characteristics of the ground motion sets used. Furthermore, the ML estimates of $\beta$ in Giouvanidis and Dimitrakopoulos\(^{[72]}\) are based on a small number of simulations, because a limited amount of intense real ground motions is available. For example, the whole fragility curve of block A was computed based on 525 simulations with 18 overturns in total. Therefore, its confidence interval should be fairly large. The values obtained by Giouvanidis and Dimitrakopoulos\(^{[72]}\) and this paper for the PGD-based fragility curves do not match. One possible reason could be that it is difficult to obtain reliable estimates of PGD, since it strongly depends on the ground motion correction method.

A key finding of the present study is that the estimate of $\beta$ for PGV, which is the shape parameter of the lognormal CDF, is independent of the geometry and therefore it is possible to make the lognormal CDFs collapse to a universal fragility curve. However, the estimates of $\beta$ for pPGV/$(\tan(\alpha))$ of\(^{[72]}\) (provided here in the upper part of Table 4) show a variation with geometry, which cannot be explained solely by the variation of the estimate that is due to the limited number of simulations of Giouvanidis and Dimitrakopoulos.\(^{[72]}\) A potential reason for this discrepancy is that\(^{[72]}\) provided $\beta$ for the conditional probability of overturning given the block experiences rocking—while such a condition was not set in the previous section of this manuscript where $\beta$ was shown to be independent of the geometry of the block.
CONCLUSIONS

This study examined the seismic behaviour of the rocking oscillator under far-field synthetic ground motions. To this end, it examined block geometries with heights ranging from 1 to 20 m and slenderness angles ranging from 0.1 to 0.35 rad, under 50 million synthetic ground motions. A variety of intensity measures were assessed, based on their capabilities of describing the induced rocking rotation and overturning potential of ground motions. To this end: (a) safe rocking cases were studied independently; and (b) fragility curves were derived and approximated by fitting lognormal cumulative distributions. It was found that:

1. Between all IMs studied, the induced rocking rotation scales the best (in terms of efficiency, sufficiency and proficiency) with PGV. This does not include very small rocking rotations, which are anyways not modelled adequately by a rigid body model. Moreover, PGV was proven to be the most efficient IM to predict the overturning probability of a block.

2. For all geometries considered, the fragility curves for different block geometries collapse to a universal fragility curve if the non-dimensional parameter $r^2 p\text{PGV}/(g\tan(\alpha))$ is used as an IM, where: $r$ is the coefficient of restitution; $p$ the rocking frequency; $g$ the gravity constant; and $\alpha$ the block slenderness.

3. The general character of the above conclusions has been examined by comparing the results of this paper with the results of Giouvanidis and Dimitrakopoulos [72] who used recorded and unscaled ground motions. However, Giouvanidis and Dimitrakopoulos [72] included a relatively low number of ground motions, and therefore this work remains to be validated against other ground motion sets with sufficiently large number of excitations.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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