Single-spin asymmetries in SIDIS induced by anomalous quark-gluon and quark-photon couplings

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Abstract

Abstract: We calculate the contribution of the non-perturbative Pauli couplings in the quark-photon and quark-gluon vertices to the single-spin asymmetries (SSAs) in semi-inclusive deep inelastic scattering (SIDIS). We describe the nucleon using the spectator model with scalar and axial-vector diquarks. Both of the new couplings can cause the helicity flip of the struck quark. The helicity flip and the rescattering induced by the non-perturbative gluon exchange between the struck quark and diquark lead to the SSAs. Their azimuthal dependencies are the same as that usually ascribed to the Collins and Sivers effects. Our numerical results, based on the instanton model for the QCD vacuum, show that the non-perturbative quark-gluon and quark-photon interactions have a strong influence on these asymmetries.

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I. INTRODUCTION

One of the long-standing problems in the strong interaction theory is to understand the origin of spin effects in hadronic physics within Quantum Chromodynamics (QCD). To achieve this goal, a promising route is to investigate the mechanisms that are responsible for the large single-spin asymmetries (SSAs) in high energy hadronic reactions and in semi-inclusive deep-inelastic scattering (SIDIS) \[1-3\]. Two main mechanisms based on QCD factorization are usually considered. One of them is related to the spin-dependent Sivers distribution function \[4, 5\] and the other comes from the spin-dependent Collins fragmentation function \[6, 7\]. These functions should be either calculated within some nonperturbative approach based on QCD \[8-10\] or be extracted directly from experiments \[11-15\]. Another way to produce a non-zero SSA is through final or initial interactions between quarks by perturbative gluon exchange \[16\]. There is a relation between the two mechanisms \[17\]. Recently the twist-3 mechanism, based on quark-gluon correlations \[18-21\], was revisited \[22-26\] to solve the problem of the “sign-mismatch” between the Sivers function extracted from the inclusive production of pions in proton-proton collisions and SIDIS \[27, 28\]. Most of the approaches to calculate SSAs are based on the factorization assumption (see reviews \[29, 30\]). However, a proof of the validity of such an assumption is still lacking \[31, 32\]. Furthermore, the explicit mechanism for the breakdown of transverse-momentum-dependent (TMD) factorization was suggested in \[33\] (see also \[34\]). This mechanism is based on the existence of a small size strong gluonic fluctuation in the QCD vacuum called instantons (see reviews \[35, 36\]). These nontrivial topological solutions of QCD produce a very large anomalous quark chromomagnatic moment (AQCM) which flips the quark helicity \[37\]. This quark helicity flip is one of the important ingredients to generate SSAs. Indeed, it was demonstrated in Ref. \[33\] that the quark-gluon vertex induced by AQCM leads to a large SSA in quark-quark scattering (see also \[38, 39\] and references therein). Additionally, the instantons induce an anomalous quark magnetic moment \[40\] and, therefore, produce a nontrivial quark-photon vertex which flips the quark helicity and gives rise to SSA in SIDIS. It should be mentioned that the first qualitative discussion about possible effects of the Pauli-type soft quark-gluon interaction on SSAs in SIDIS was in Ref.\[41\]. The evidence for the existence of the smaller scale in QCD, compared to the confinement scale, was discussed in Refs. \[42, 45\].
In this paper we calculate SSAs in SIDIS using the instanton model for the helicity flip in the quark-gluon and quark-photon vertices \[37, 40\]. To describe the proton we use the spectator model \[46, 47\] with the non-perturbative final state interaction between the struck quark and both the scalar and axial-vector diquarks. We report the full set of helicity amplitudes, taking into account this novel helicity non-conserving mechanisms and study the angular dependence of SSA observables.

II. ANALYTICAL RESULTS IN SPECTATOR MODELS

In our model SSA comes from the interference of the tree and one-loop diagrams presented in FIG. 1. The general quark-gluon vertex for the interaction between the struck quark and the exchanged gluon has the following form

\[ V_{\mu}^{g,a} = t^a g_s \left( F_D^g \gamma^\mu - \frac{F_P^g}{2m_q} \sigma^{\mu\nu} (r_\nu - k_\nu) \right), \]

where \( t^a \) are the Gell-Mann colour matrices, \( g_s \) is the strong coupling constant, \( m_q \) is the constituent quark mass, \( \sigma^{\mu\nu} = [\gamma_\mu, \gamma_\nu]/2 \) and \( r - k \) is the gluon momentum. We also consider the general vertex for the interaction of the struck quark with the virtual photon

\[ V_{\mu}^{\gamma} = F_D^\gamma \gamma^\mu - \frac{F_P^\gamma}{2m_q} \sigma^{\mu\nu} q_\nu, \]

where \( q \) is the photon momentum, \( F_D^g(F_D^\gamma) \) and \( F_P^g(F_P^\gamma) \) are Dirac and Pauli form factors, which are functions of the corresponding gluon or photon momentum, respectively.
The interaction vertices of the gluon with the scalar and axial-vector diquarks are

$$\Pi^s_\mu = i e_s (2P - k - r)_\mu F^s_{\text{di}},$$

$$\Pi^a_{\mu,\alpha\beta} = -i e_a [(2P - k - r)_\mu g_{\alpha\beta} - (P - k)_{\alpha g_{\mu\beta}} - (P - r)_{\beta g_{\alpha\mu}}] F^a_{\text{di}},$$

where $e_s$ and $e_a$ are the color charges of the scalar and axial-vector diquarks and $F^s,a_{\text{di}}$ are the corresponding diquark form factors. The nucleon-quark-diquark vertices are chosen to be

$$V_s = ig^s_P \mathbf{1},$$

$$V^a_\mu = i g^a_P \gamma^5 \gamma^\mu,$$

where $\mathbf{1}$ is the unit matrix in the spinor space, $g^s_P$ and $g^a_P$ are the couplings in the proton-quark-scalar diquark vertex and the proton-quark-axial-vector diquark vertex. In general, these vertices depend on momenta of particles, i.e. they should include form factors (see [47] and references therein). However, we limit ourselves to the simplest point-like case.

To calculate helicity amplitudes we will use the approach presented by Hoyer and Jarvinen in [41]. Following their method, we choose a reference frame where the target proton is at rest and has the spin in the $y$ direction. The virtual photon momentum is along the $+z$ axis, i.e. the momenta of photon, proton and struck quarks are

$$q = (q^+, q^-, 0_\perp) \simeq (2\nu, -xM, 0_\perp),$$

$$P = (M, M, 0_\perp),$$

$$k = (xM, xM, k_\perp = k_\perp e^{i\phi}),$$

$$r = (xM, xM, r_\perp = r_\perp e^{i\psi}),$$

where $\nu = Q^2/2xM$ is the photon energy, $M$ is the proton mass and $x$ is the Bjorken variable. The polarization vectors of the photon and the axial-vector diquark are defined as

$$\epsilon^\lambda(q) = \frac{1}{\sqrt{2}} (0, 0, -\lambda, -i),$$

$$\epsilon_D(P - k, \lambda_a) = \frac{1}{\sqrt{2}} \left( \frac{2(\lambda_a k_x + i k_y)}{(1 - x)M}, 0, -\lambda_a, -i \right).$$

They satisfy the transversality condition, for instance $(P - k) \cdot \epsilon_D(P - k, \lambda_a) = 0$. The amplitude for the elastic quark-scalar diquark scattering process is $M_{s,s'}$. It corresponds
to the right-hand side of FIG. 1(b). \( s \) and \( s' \) are helicities of the initial and final quarks, respectively. In the limit of \( q^+ = Q^2/xM \to \infty \) at fixed \( k \) and \( r \), the helicity amplitudes are

\[
\mathcal{M}_{+,+} \simeq F_{di}^s F_D^y \frac{e_q e_s}{(k_\perp - r_\perp)^2} 2(1-x)Mq^+, \tag{7}
\]

\[
\mathcal{M}_{+,-} \simeq -F_{di}^s F_D^y \frac{e_q e_s}{2m_q (k_\perp - r_\perp)^2} 2(1-x)Mq^+ \left(k_\perp e^{+i\phi} - r_\perp e^{+i\psi}\right),
\]

where \( e_q \) is the color charge of the struck quark and \( F_{di}^s \) is the scalar diquark form factor. For the Born diagram presented in FIG. 1(a), the helicity amplitude is \( A_{s,s',+}^{\lambda,s,s'} \), where \( \lambda \) is the photon helicity. The amplitudes are

\[
A_{+,+}^{\lambda} \simeq Q_q g_P^s \sqrt{2Mq^+} \frac{1-x}{r_\perp^2 + B_R^2(m_q^2)} \times
\left[ \left( \frac{F_D^y}{2m_q} - \frac{F_P^y}{2m_q} D_Q \right) r_\perp e^{+i\psi} \delta_{\lambda,+1} + \frac{F_P^y}{2m_q} D_R r_\perp e^{-i\psi} \delta_{\lambda,-1} \right], \tag{8}
\]

\[
A_{+,-}^{\lambda} \simeq Q_q g_P^s \sqrt{2Mq^+} \frac{1-x}{r_\perp^2 + B_R^2(m_q^2)} \left[ -\frac{F_P^y}{2m_q} r_\perp^2 e^{2i\psi} \delta_{\lambda,+1} + \left( \frac{F_D^y}{2m_q} + \frac{F_P^y}{2m_q} D_R \right) D_R \delta_{\lambda,-1} \right],
\]

where \( D_Q = xM - m_q, D_R = xM + m_q, B_R^2(m_q^2) = (1-x)m_q^2 + xM^2 - x(1-x)M^2 \) and \( Q_q \) is the electric charge of the struck quark.

For the axial-vector diquark case the amplitude for the quark-diquark scattering is \( \mathcal{M}_{s,s'}^{\lambda_a,\lambda'_a} \). Here \( \lambda_a \) and \( \lambda'_a \) are helicities of the initial and final diquark, respectively. Amplitudes are

\[
\mathcal{M}_{+,+}^{\lambda_a,\lambda'_a} \simeq F_{di}^{a} F_D^y \frac{e_q e_a}{(k_\perp - r_\perp)^2} 2(1-x)Mq^+ \delta_{\lambda_a,\lambda'_a}, \tag{9}
\]

\[
\mathcal{M}_{+,-}^{\lambda_a,\lambda'_a} \simeq -F_{di}^{a} F_D^y \frac{e_q e_a}{2m_q (k_\perp - r_\perp)^2} 2(1-x)Mq^+ \left(k_\perp e^{+i\phi} - r_\perp e^{+i\psi}\right) \delta_{\lambda_a,\lambda'_a},
\]

where \( F_{di}^{a} \) is the axial-vector diquark form factor. For the diagram on FIG. 1(a) with the axial-vector diquark the amplitude is \( A_{s,s'}^{\lambda,\lambda_a} \), where \( \lambda \) and \( \lambda_a \) are photon and diquark helicities,
respectively. The full set of amplitudes is

\[
\begin{align*}
A_{++,}^+ &\simeq -Qg_P^a\sqrt{2Mq^+} \frac{1-x}{r_+^2 + B^2_R(m_q^2)} \left[ \left( F_D^0 + \frac{F_P^0}{2m_q} D_R \right) D_R + \frac{F_P^0}{2m_q} \frac{x}{1-x} r_+^2 \right], \\
A_{++,}^- &\simeq Qg_P^a\sqrt{2Mq^+} \frac{1-x}{r_+^2 + B^2_R(m_q^2)} \frac{F_P^0}{2m_q} \frac{x}{1-x} r_+^2 e^{+2i\psi}, \\
A_{++,}^- &\simeq Qg_P^a\sqrt{2Mq^+} \frac{1-x}{r_+^2 + B^2_R(m_q^2)} \frac{F_P^0}{2m_q} 1 \frac{x}{1-x} r_+^2 e^{-2i\psi}, \\
A_{++,}^- &\simeq -Qg_P^a\sqrt{2Mq^+} \frac{1-x}{r_+^2 + B^2_R(m_q^2)} \left( F_D^0 + \frac{F_P^0}{2m_q} (x D_R - D_Q) \right) \frac{r_+}{1-x}, \\
A_{++,}^- &\simeq 0.
\end{align*}
\]

The remaining helicity amplitudes are obtained through the relations

\[
\mathcal{M}_{s,s'}^{(\lambda_a,\lambda_{a'})} = (-1)^{s-s'} \left( \mathcal{M}_{-s,-s'}^{(-\lambda_a, -\lambda_{a'})} \right)^\ast \quad \text{and} \quad A_{s,s'}^{\lambda(\lambda_a)} = -(-1)^{s-s'} \left( A_{-s,-s'}^{-(\lambda_a)} \right)^\ast.
\]

Only the discontinuity (absorptive part) of the loop amplitude \(\mathcal{B}\) in FIG. \[\Pi(b)\] contributes to the asymmetry \(A_N\). According to the Cutkosky rules the discontinuity is given by a convolution of the amplitudes \(A_{s,s'}^{\lambda(\lambda_a)}\) and \(\mathcal{M}_{s',s}^{(\lambda_a,\lambda_{a'})}\) \[41\],

\[
\text{Disc } \mathcal{B}_{s,s'}^{\lambda(\lambda_a)} = i \int \frac{d^4k}{(2\pi)^4} 2\pi \delta ((k+q)^2 - m_q^2) 2\pi \delta ((P-k)^2 - M_B^2) \sum_{s',\lambda_a} A_{s,s'}^{\lambda(\lambda_a)} \mathcal{M}_{s',s}^{(\lambda_a,\lambda_{a'})}
\]

\[
= \frac{i}{2(1-x)Mq^+} \int \frac{d^2k}{(2\pi)^2} \sum_{\lambda_a} \left[ A_{s,-s'}^{(\lambda_a)} \mathcal{M}_{s',s}^{(\lambda_{a'},\lambda_a)} + A_{s,s'}^{(\lambda_a)} \mathcal{M}_{s',s}^{(\lambda_{a'},\lambda_a)} \right], \quad (11)
\]

which satisfies

\[
\text{Disc } \mathcal{B}_{s,s'}^{\lambda(\lambda_a)} = (-1)^{s-s'} \left( \text{Disc } \mathcal{B}_{-s,-s'}^{-(\lambda_a)} \right)^\ast.
\]

In the scalar diquark model considered in Ref. \[41\] only the first term in Eq. \[11\] was presented due to the fact that \(\mathcal{M}_{s,s} \simeq 0\) there. In contrast, \(\mathcal{M}_{s,s} \propto F_D^0\) in our calculation.
The spin asymmetry for a target polarized in the transverse $y$ direction is defined as

$$\mathcal{N} A_N \simeq \frac{2}{Q^4} \sum_{\lambda,(\lambda a),\lambda',(\lambda' a),s,s'} \text{Im} \left\{ L^{\lambda,\lambda'} \left[ A_{\lambda\lambda a,s,s'}^{(\lambda a)} + \frac{i}{2} \text{Disc} B_{\lambda\lambda a,s,s'}^{(\lambda a)} \right] \left[ A_{\lambda\lambda a,s,s'}^{(\lambda' a)} + \frac{i}{2} \text{Disc} B_{\lambda\lambda a,s,s'}^{(\lambda' a)} \right]^* \right\}$$

$$= \frac{4e^2}{y^2 Q^2} \text{Im} \sum_{\lambda,(\lambda a)} \left[ (1 + (1 - y)^2) \left\{ A_{\lambda,+,+}^{(\lambda a)} \text{Disc} B_{\lambda,+,+}^{(-\lambda a)} - A_{\lambda,+,+}^{(\lambda a)} \text{Disc} B_{\lambda,+,+}^{(-\lambda a)} \right\} - 2(1 - y)e^{-2i\lambda}\tau \left\{ A_{\lambda,+,+}^{(\lambda a)} \text{Disc} B_{\lambda,+,+}^{(\lambda a)} - A_{\lambda,+,+}^{(\lambda a)} \text{Disc} B_{\lambda,+,+}^{(\lambda a)} \right\} \right],$$

(12)

with the leptonic tensor in the helicity basis

$$L^{\lambda,\lambda'} = \frac{2e^2 Q^2}{y^2} \left\{ (1 + (1 - y)^2) \delta_{\lambda,\lambda'} - 2(1 - y)e^{-2i\lambda}\tau \delta_{\lambda,\lambda'} \right\}. \quad (13)$$

Here $\tau$ is the azimuthal angle of the lepton $l_{1\perp} = l_{2\perp} = l_1 e^{i\tau}$ and $y = P \cdot q/P \cdot l_1$ is the fraction of the beam energy carried by the virtual photon. Note that the $\delta_{\lambda,\lambda'}$ term in $L^{\lambda,\lambda'}$ generates the asymmetry in the third line of Eq. (12), which was in the calculation by Hoyer and Jarvinen [41]. The $\delta_{\lambda,\lambda'}$ term produces the second line of Eq. (12), and leads to an asymmetry similar to the Sivers effect as will be demonstrated below. This asymmetry arises in our model because the amplitudes $A_{\lambda s a,s s'}^{(\lambda a)}$ and the helicity non-flip amplitudes $M_{\lambda s a,s s'}^{(\lambda a)}$ are generally non-zero. The normalization $\mathcal{N}$ is given by the amplitude at tree order in FIG. (a):

$$\mathcal{N} = \frac{1}{Q^4} \sum_{\lambda,(\lambda a),\lambda',(\lambda' a),s,s'} L^{\lambda,\lambda'} A_{\lambda\lambda a,s,s'}^{(\lambda a)} \left( A_{\lambda\lambda a,s,s'}^{(\lambda' a)} \right)^*$$

$$= \frac{8Q_2^2(g_P^2)^2}{y^2 Q^2} M q^+ \left( \frac{1 - x}{r^2 + B_2^2(m_q^2)} \right)^2 \left[ 1 + (1 - y)^2 \right] \mathcal{N}_+^2,$$

(14)

where $\mathcal{N}_+^2$ depends on a diquark model. For the scalar diquark, we have

$$\mathcal{N}_+^2 = \sum_{\lambda} \{ |A_{\lambda,+,+}^\gamma|^2 + |A_{\lambda,+,+}^\gamma|^2 \}$$

$$\simeq \left( \mathcal{F}_D^\gamma - \frac{\mathcal{F}_P^\gamma}{2m_q} D_q \right)^2 r_\perp^2 + \left( \frac{\mathcal{F}_D^\gamma + \mathcal{F}_P^\gamma}{2m_q} D_R \right)^2 D_R^2 + \left( \frac{\mathcal{F}_P^\gamma}{2m_q} \right)^2 r_\perp^2 (r_\perp^2 + D_R^2),$$

(15)

which is in agreement with the results of Hoyer and Jarvinen [41] in the limit $\mathcal{F}_P^\gamma \to 0$. For
the axial-vector diquark model, we obtain

\[ N^v_+ = \sum_{\lambda,\lambda_a} \left\{ |A^\lambda_{\lambda_a} + |A^\lambda_{\lambda_a}|^2 \right\} \]

\[ \simeq \left[ \left( F_D^\gamma + \frac{F_P^\gamma}{2m_q} D_R \right) D_R + \frac{F_P^\gamma}{2m_q} x \frac{r_2^2}{1-x} \right]^2 + \frac{r^2}{(1-x)^2} \times \]

\[ \left[ (F_D^\gamma + F_P^\gamma)^2 x^2 + \left( F_D^\gamma + \frac{F_P^\gamma}{2m_q} (x D_R - D_Q) \right)^2 + \right. \]

\[ \left. \left( \frac{F_P^\gamma}{2m_q} \right)^2 \right) \left( (1 + 2 x^2) r_\perp^2 + (1-x)^2 D_R^2 \right] \right]. \] (16)

It can be shown that the unpolarized distributions \( N_{s,a}^v \) agree with well known results of conventional diquark models when \( F_P^\gamma \) vanishes. Using Eq. (12) we find that the asymmetry is

\[ A^s_{N,a} \simeq \frac{\epsilon_N \epsilon_{s,a} r_\perp^2 + B^2_{R}(m^2)}{[1 + (1-y)^2]} N_{s,a}^v \times \]

\[ \left\{ 2(1-y) \left[ F_{s,a}^+ J_{s,a}^+ \cos(3\psi - 2\tau) + F_{s,a}^- J_{s,a}^- \cos(\psi - 2\tau) \right] + \right. \]

\[ \left. [1 + (1-y)^2] F_{s,a}^0 J_{s,a}^0 \cos \psi \right\}, \] (17)

with the abbreviated notations \( F_{s,a}^n \) and \( J_{s,a}^n \) are explained in the Appendix. The \( F_{s,a}^n \) functions are combinations of Dirac and Pauli couplings, and the loop functions \( J_{s,a}^n \) are regularized by the form factors in the Dirac and Pauli couplings, which will be presented in Sec. III

In the Trento convention \([48]\), the angles \( \phi_s \) and \( \phi_h \) are used. The \( \phi_s \) is defined as the angle between the target spin direction and the lepton plane

\[ \phi_s = \pi/2 - \tau. \] (18)

The \( \phi_h \) is the angle between the hadron (or quark jet) and lepton planes

\[ \phi_h = \psi - \tau. \] (19)

Using these coordinates, the asymmetry in the both models is

\[ A_N = \epsilon A_N^{\sin(3\phi_h - \phi_s)} \sin(3\phi_h - \phi_s) + \epsilon A_N^{\sin(\phi_h + \phi_s)} \sin(\phi_h + \phi_s) + A_N^{\sin(\phi_h - \phi_s)} \sin(\phi_h - \phi_s), \] (20)
with the depolarization factor 
\[ \epsilon = 2(1 - y)/(1 + (1 - y)^2) \] 
and the definition:

\[ A_N^{\sin(3\phi_h - \phi_s)} = -e_q e_{s,a} \frac{r^2 + B_R^2(m_q^2)}{N_{s,a}^+} \mathcal{F}_{s,a}^+ J^+_s \tag{21} \]

\[ A_N^{\sin(\phi_h + \phi_s)} = e_q e_{s,a} \frac{r^2 + B_R^2(m_q^2)}{N_{s,a}^+} \mathcal{F}_{s,a}^- J^-_s, \tag{22} \]

\[ A_N^{\sin(\phi_h - \phi_s)} = -e_q e_{s,a} \frac{r^2 + B_R^2(m_q^2)}{N_{s,a}^+} \mathcal{F}_{s,a}^0 J^0_s. \tag{23} \]

The second line in Eq. (12) gives the asymmetry resulting from the relation 
\[ \cos \psi = -\sin(\phi_h - \phi_s) \] 
and the angular dependence of the asymmetry is identical to that which arises from the Sivers effect. The third line with \( \lambda = +1 \) in Eq. (12) gives the asymmetry resulting from 
\[ \cos(3\psi - 2\tau) = -\sin(3\phi_h - \phi_s) \] 
and when \( \lambda = -1 \) this gives the asymmetry resulting from 
\[ \cos(\psi - 2\tau) = \sin(\phi_h + \phi_s). \] 
Their angular dependence is the same as that from the Collins effect. The physical origin of the two asymmetries is, however, quite different from the original Collins and Sivers asymmetry. The helicity flip, which is the origin of all these asymmetries arises either from the tree level amplitudes \( A_{s,s'}^{\lambda,({\lambda}_a)} \) or from the loop amplitudes \( \text{Disc} B_{s,s'}^{\lambda,({\lambda}_a)} \). As a result, the Pauli couplings in both the quark-gluon and quark-photon vertices are able to generate the SSA. In the axial diquark model the Pauli couplings of the quark-photon vertex is required to induce the \( \sin(3\phi_h - \phi_s) \) asymmetry. This is because the factors \( \mathcal{F}_{a,1}^+ \), \( \mathcal{F}_{a,2}^+ \) and \( \mathcal{F}_{a,3}^+ \) are all proportional to \( \mathcal{F}_p^0 \) since they are related to the \( A_{s,s'}^{\lambda,({\lambda}_a)} \) amplitude in Eq. (12), which is also proportional to \( \mathcal{F}_p^0 \).

**III. FORM FACTORS**

For definiteness we will consider the case when the struck quark is a u-quark and the spectator is a (ud)-diquark. The Pauli form factor \( \mathcal{F}_p^q \) of the quark-gluon interaction was calculated using the instanton liquid model for the QCD vacuum in Refs. \[37, 49, 50\]. It is

\[ \mathcal{F}_p^q((r - k)^2) = \mu_a F_p^q((r - k)^2), \tag{24} \]

where

\[ \mu_a = \frac{-3\pi(\rho, m_q)^2}{4\alpha_s(\rho_c)} \tag{25} \]

and

\[ F_p^q((r - k)^2) \approx e^{-(r-k)^2/\Lambda_q^2} \tag{26} \]
with $\Lambda_q = 2/\rho_c = 1.2$ GeV and $\rho_c = 1/(600$ MeV) is the instanton size [40].

We take the the diquark radius from the instanton liquid model [51] which leads to the diquark form factor

$$F_{d_i}^{s,a}((r - k)^2) \approx e^{-(r-k)^2/\Lambda_{d_i}^2},$$

(27)

where $\Lambda_{d_i} \approx 0.7$ GeV for both the scalar and axial-vector diquark.

The Pauli form factor in the quark-photon vertex from the non-perturbative contribution has been calculated within the instanton model [40] and it can be approximated well by

$$F_{\gamma P}^{q}(Q^2) = \frac{\mu_q}{1 + \rho_c Q^2/(4.7 m_q)},$$

(28)

For the quark mass $m_q = 0.35$ GeV the anomalous magnetic moment is $\mu_q \approx 0.5$. We will also use the approximation that $F_{\gamma}^{q} \approx F_{\gamma}^{D} \approx 1$ for Dirac form factors in both the quark-photon and quark-gluon vertices. In recent papers by Roberts and collaborators [52, 53] the anomalous magnetic and anomalous chromomagnetic moments of the light quarks were calculated within the Dyson-Schwinger Equations (DSE) approach, with the non-perturbative quark and gluon propagators. The results are in qualitative agreement with the instanton model prediction. Unfortunately, the authors did not calculate both Pauli form factors at non-zero transfer momentum and, therefore, the calculation of SSA in SIDIS within their model is not possible at the present time.

**IV. NUMERICAL RESULTS**

We fix the value of $e_{s,a}e_{q} = 4\pi C_F\alpha_s$ with $C_F = 4/3$ and $\alpha_s = 0.5$ following the arguments in Refs. [16, 46, 47]. Additionally, we use $m_D = 0.6$ GeV for the mass of the scalar diquark and $m_D = 0.8$ GeV for the axial-vector diquark. The value of the quark mass $m_q = 0.35$ GeV is chosen as in the conventional diquark models [16, 46, 47]. This value is in agreement with the prediction for the dynamical quark mass within the Diakonov-Petrov instanton liquid model [36]. Finally, the physical nucleon mass $M = 0.94$ GeV is used. The numerical results for asymmetries $A_N^{\sin(3\phi_h-\phi_s)}$, $A_N^{\sin(\phi_h+\phi_s)}$ and $A_N^{\sin(\phi_h-\phi_s)}$ in both the spectator models for three different values of the $Q^2$ are presented in FIG. 2, FIG. 3 and FIG. 4. The SSAs are shown as a function of Bjorken’s variable $x$ with the value of transverse momentum of struck quark $r_\perp = 0.5$ GeV in panels (a) and (b) of those figures, and as a function of $r_\perp$
FIG. 2: The Collins-like asymmetry $A_N^{\sin(3\phi_h-\phi_s)}$ in the scalar (a,c) and axial-vector diquark (b,d) models. The black dashed, dotted, and dash-dotted lines are the full results with $Q^2 = 1.0$ GeV$^2$, 3.0 GeV$^2$ and 6.0 GeV$^2$ respectively. The solid curves are the results without the Pauli coupling in the photon-quark vertex ($F^\gamma_P = 0$).

at $x = 0.15$ in panels (c) and (d). The magnitudes of all three asymmetries in the axial-vector diquark model are smaller than those in the scalar diquark model. However, it can be seen that in most cases the induced Collins-like $A_N^{\sin(3\phi_h-\phi_s)}$ and $A_N^{\sin(\phi_h+\phi_s)}$ asymmetries are still of considerable size and the Pauli coupling in the quark-photon vertex contributes to both the magnitude and shape of the asymmetries. However, for the Sivers-like $A_N^{\sin(\phi_h-\phi_s)}$ asymmetry, the effect of $F^\gamma_P$ in the quark-photon interaction is small and as a result its $Q^2$ dependence is very weak.

V. CONCLUSION

We have investigated the contribution from the Pauli couplings in both the quark-gluon and quark-photon vertices to the SSA in SIDIS, adopting the scalar and axial-vector diquark models for the nucleon. The specific angular dependence of SSA induced by these couplings is the same as those usually called Collins and Sivers asymmetries. Our results show that these contributions are significant, especially for the Collins-like asymmetry. The important observation is that it is not only the helicity flip term from the quark-gluon in-
FIG. 3: The Collins-like asymmetry $A_N^{\sin(\phi_h+\phi_s)}$ in the scalar (a,c) and axial-vector diquark (b,d) models. The notation is the same as in FIG. 2.

FIG. 4: The Sivers-like asymmetry $A_N^{\sin(\phi_h-\phi_s)}$ in the scalar (a,c) and axial-vector diquark (b,d) models. The notation is the same as in FIG. 2.

teraction, considered by Hoyer and Jarvinen [41], contributes to the angular dependence of the asymmetry $A_N$. The helicity flip from the non-perturbative quark-photon vertex is also important. In this connection, we would like to stress that for the case of the axial-vector diquark, the Collins-like $A_N^{\sin(3\phi_h-\phi_s)}$ asymmetry is present only if the Pauli coupling of the photon with the struck quark is non-zero. Our results show that the Collins-like asymme-
tries have significant $Q^2$ dependence for both of the models. This effect is related to the strong $Q^2$ dependency of the quark electromagnetic Pauli form factor in Eq. (28).

Our estimations of SSA in SIDIS are based on the instanton model for the non-perturbative QCD vacuum. The instantons, nonperturbative fluctuations of the vacuum gluon fields, describe the non-trivial topological structure of the QCD vacuum and give a natural explanation of many of fundamental phenomena of the strong interaction, such as the spontaneous chiral symmetry breaking (SCSB). The average size of the instantons $\rho_c \approx 1/3$ fm is much smaller than the confinement size $R_c \approx 1$ fm and can be considered as the scale of SCSB. Furthermore, SCSB induced by the instantons is responsible for the formation of the constituent massive quark with size $R_q \approx \rho_c \approx 1/3$ fm. As a result, it leads to a helicity flip in both the quark-gluon and quark-photon vertices. Therefore, the study of the SSAs in SIDIS might give important information on the mechanism of SCSB in the strong interaction. Unfortunately, direct comparison of our results with experimental data on the SSA for the inclusive meson production in SIDIS is not possible at present. It requires to introduce a quark fragmentation into account. However, our results might be relevant to the SSA in jets production in SIDIS, which can be studied at future Electron-Ion Colliders.

**Appendix: Abbreviated functions in SSA**

The abbreviated notations $F_{s,a}^n$ and $J_{s,a}^n$ in Eq. (17) are explicitly expressed as:
Note that the loop functions $J^{\pm,0}_{s,a}$ are real. Substituting $k_\perp \to k_\perp + r_\perp$ after the transform-
mation $\phi \rightarrow \phi + \psi$, we find expressions

$$
\begin{align*}
J_{s,1}^+ &= \frac{J_{s,2}^0}{D_R} = \frac{(k_\perp e^{i\phi} + r_\perp)^2 k_\perp e^{+i\phi} r_\perp}{K} F_d^s(k_\perp^2), \\
J_{s,2}^+ &= \frac{(k_\perp e^{i\phi} + r_\perp)^2 k_\perp e^{-i\phi} r_\perp^2}{K} F_d^s(k_\perp^2), \\
J_{s,1}^- &= D_R J_{s,1}^0 = \frac{D_R^2 k_\perp e^{-i\phi}}{K} F_d^s(k_\perp^2), \\
J_{s,2}^- &= \frac{D_R^2 (k_\perp e^{-i\phi} + r_\perp)}{K} \frac{D_R^2 k_\perp e^{+i\phi} r_\perp}{K} F_d^s(k_\perp^2), \\
J_{s,3}^0 &= \frac{2 D_R k_\perp e^{i\phi} (k_\perp \cos \phi + r_\perp)}{K} \frac{D_R^2 k_\perp e^{-i\phi} r_\perp}{K} F_d^s(k_\perp^2), \\
J_{s,4}^0 &= \frac{D_R k_\perp e^{-i\phi} (k_\perp e^{i\phi} + r_\perp)^2 + r_\perp^2}{K} F_d^s(k_\perp^2),
\end{align*}
$$

(A.3)

with $K = (k_\perp^2 + r_\perp^2 + 2k_\perp r_\perp \cos \phi + B_R^2(m_q^2)) k_\perp^2$. For the axial-vector diquark model we
define the combinations of couplings

\[ F_{a,1}^+ = \mathcal{F}_D^g \left( \frac{\mathcal{F}_P^\gamma}{2m_q} \right)^2 \frac{x}{1-x} = \mathcal{F}_{s,2}^0 \frac{x}{1-x}, \]

\[ F_{a,2}^+ = \frac{\mathcal{F}_P^g \mathcal{F}_{D,1}^s}{2m_q} \left( \mathcal{F}_D^\gamma + \frac{\mathcal{F}_P^\gamma}{2m_q} D_R \right) \frac{x}{1-x} = \mathcal{F}_{s,4}^0 \frac{x}{1-x}, \]

\[ F_{a,3}^+ = \frac{\mathcal{F}_P^g (\mathcal{F}_D^\gamma)^2}{2m_q} \left( \frac{x}{1-x} \right)^2 = \mathcal{F}_{s,2}^+ \left( \frac{x}{1-x} \right)^2, \]

\[ F_{a,1}^- = \frac{\mathcal{F}_P^g}{2m_q} (\mathcal{F}_D^\gamma + \mathcal{F}_P^\gamma) \left[ \mathcal{F}_D^\gamma + \frac{\mathcal{F}_P^\gamma}{2m_q} (x D_R - D_Q) \right] \frac{x}{(1-x)^2}, \]

\[ F_{a,2}^- = \frac{\mathcal{F}_P^g (\mathcal{F}_D^\gamma)^2}{2m_q} \frac{x}{(1-x)^2} = \mathcal{F}_{s,2}^+ \frac{x}{(1-x)^2}, \]

\[ F_{a,3}^- = \frac{\mathcal{F}_D^g}{2m_q} (\mathcal{F}_D^\gamma + \mathcal{F}_P^\gamma) \frac{x}{(1-x)^2}, \]

\[ F_{a,4}^- = \frac{\mathcal{F}_D^g}{2m_q} \left[ \mathcal{F}_D^\gamma + \frac{\mathcal{F}_P^\gamma}{2m_q} (x D_R - D_Q) \right] \frac{x}{(1-x)^2}, \]

\[ F_{a,1}^0 = \frac{\mathcal{F}_D^g}{2m_q} (\mathcal{F}_D^\gamma + \mathcal{F}_P^\gamma) \left( \mathcal{F}_D^\gamma + \frac{\mathcal{F}_P^\gamma}{2m_q} D_R \right) \frac{x}{1-x}, \]

\[ F_{a,2}^0 = \frac{\mathcal{F}_D^g (\mathcal{F}_P^\gamma)^2}{2m_q} \frac{x}{1-x} = F_{a,1}^+, \]

\[ F_{a,3}^0 = \frac{\mathcal{F}_D^g}{2m_q} \left( \mathcal{F}_D^\gamma + \frac{\mathcal{F}_P^\gamma}{2m_q} (x D_R - D_Q) \right) \frac{x}{(1-x)^2} = F_{a,4}^-, \]

\[ F_{a,4}^0 = \frac{\mathcal{F}_D^g}{2m_q} (\mathcal{F}_D^\gamma + \mathcal{F}_P^\gamma) \left( \frac{x}{1-x} \right)^2 = x F_{a,3}^-, \]

\[ F_{a,5}^0 = \frac{\mathcal{F}_D^g}{2m_q} (\mathcal{F}_D^\gamma + \mathcal{F}_P^\gamma) \frac{x}{1-x}, \]

\[ F_{a,6}^0 = \frac{\mathcal{F}_D^g}{2m_q} \left( \mathcal{F}_D^\gamma + \frac{\mathcal{F}_P^\gamma}{2m_q} D_R \right) \frac{x}{1-x} = F_{a,2}^+, \]

\[ F_{a,7}^0 = F_{a,3}^+ = x F_{a,8}^0, \]

(A.4)
and the corresponding integrals are

\[
\begin{align*}
J_{a,1}^- &= D_R J_{s,1}^+ = J_{s,2}^0, \\
J_{a,2}^+ &= \frac{D_R k_1 e^{i\phi} [(k_1 e^{i\phi} + r_\perp)^2 + r_\perp^2]}{K} F_{d1}^a(k_\perp^2), \\
J_{a,3}^+ &= \frac{2k_1 e^{i\phi} (k_1 e^{i\phi} + r_\perp)(k_\perp \cos \phi + r_\perp) r_\perp^2}{K} F_{d1}^a(k_\perp^2), \\
J_{a,1}^- &= \frac{J_{s,3}^0}{D_R}, \\
J_{a,2}^- &= \frac{2k_1 e^{-i\phi} (k_1 e^{-i\phi} + r_\perp)(k_\perp \cos \phi + r_\perp) r_\perp^2}{K} F_{d1}^a(k_\perp^2), \\
J_{a,3}^- &= \frac{(k_1 e^{-i\phi} + r_\perp)^2 - r_\perp (k_1 e^{i\phi} + r_\perp) r_\perp}{K} F_{d1}^a(k_\perp^2), \\
J_{a,4}^- &= \frac{J_{a,2}^0}{D_R^2} = \frac{J_{a,2}^0}{D_R} = \frac{J_{a,4}^0}{2 D_R}, \\
J_{a,1}^0 &= \frac{J_{s,1}^0}{D_R}, \\
J_{a,6}^0 &= \frac{D_R k_1 e^{i\phi} (k_\perp^2 + 2r_\perp k_\perp \cos \phi + 2r_\perp^2)}{K} F_{d1}^a(k_\perp^2), \\
J_{a,7}^0 &= \frac{2k_1 e^{i\phi} (k_\perp^2 + 2r_\perp k_\perp \cos \phi + r_\perp^2) r_\perp^2}{K} F_{d1}^a(k_\perp^2), \\
J_{a,8}^0 &= \frac{2k_1 e^{i\phi} (k_\perp^2 \cos 2\phi + 2r_\perp k_\perp \cos \phi + r_\perp^2) r_\perp^2}{K} F_{d1}^a(k_\perp^2).
\end{align*}
\]  

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