3-D Modeling of Time Domain Electromagnetic (TDEM) Method to Analyze the Layered Earth Structure in the Geothermal Systems

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Abstract. 3-D Time Domain Electromagnetic (TDEM) using Finite-Difference Time-Domain (FDTD) method has been successfully conducted to illustrate the electromagnetic wave behavior in the Earth’s surface. In this modeling, we consider the attenuation of electromagnetic wave energy which is one of the physical properties. This is very important to determine the reservoir potential area of geothermal systems as one of renewable energy. FDTD modeling has also some advantages in solving EM field interaction problem on complex geometry and analyzing transient problems. TDEM method is used to model the value of electric and magnetic fields as a function of the time, distance, and depth. Medium conductivity in the subsurface which is used in this modeling are adjusted to the real condition on the geothermal system. The results of this modeling can be applied to describe the behavior of electromagnetic wave in the which is possible to determine the reservoir potential of geothermal systems.

1. Introduction

Electromagnetic (EM) is one of geophysical method induce electrical current in the earth using principal of electromagnetic induction. The electric field propagates in the sub-surface interacts with a conductive medium will generate an eddy current. This eddy current will generate a secondary magnetic field to be recorded by the receiver. Time Domain Electromagnetic (TDEM) methods have been used for geophysical exploration in the last several decades [1]. TDEM is one of electromagnetic method that utilize electromagnetic induction which varies with time. The time decay of electromagnetic field describes the existence of conductive medium beneath the earth’s surface.

TDEM methods have some advantages over the traditional DC resistivity techniques. When the electric is looping for once time from the transmitter, it takes only about a minute of measurement. TDEM methods will reach larger exploration than transmitter loop size and will result better lateral resolution [2]. TEM or TDEM method has been used in many areas such as determining the potential location of groundwater sources [3], delineating hydrogeological structure [4], determining potential aquifer [5], and determining the resistivity structure of volcano [16]. Therefore, a 3-D TDEM solution is important to solve and comprehend physical sense from the observed earth responses [7].

Finite-Difference Time-Domain (FDTD) methods have become one important tools for simulating TDEM responses and has been successfully used for solving EM problem [8]. The main advantages of FDTD are its good ability to solve EM interaction with complex geometry, analyzing transient problem and modeling non-homogeneous geometry [9]. The other one is penetration ability of EM in dielectric and or magnetic material [10]. FDTD method is relatively straightforward to implement, highly efficient, and able to provide accurate solutions over a wide range of TDEM simulations. FDTD has also been successfully applied for preliminary study of resistivity structure analysis of geothermal system [11].

We implement a 3-D FDTD method to simulate the diffusive trait of EM field in the earth. First, we discretize the spatial and temporal in 3-D domains. In this step, we implement Dirichlet boundary conditions with the value on each boundary edge is equal to zero. The primary computational problem is time stepping, which requires the solution of a matrix equation at every time step.
2. Methodology
Maxwell’s equation was implemented in this simulation and Yee algorithm was used to calculate the value on each node [8].

2.1. Basic Concept of Electromagnetic Field
Maxwell’s equations describe the interaction of EM fields at any conductive materials in the subsurface,

\[
\nabla \times E(r, t) = -\frac{\partial B(r, t)}{\partial t}
\]

\[
\nabla \times B(r, t) = \mu_0 J(r, t) + \mu_0 \varepsilon_0 \frac{\partial E(r, t)}{\partial t}
\]

With \(E\) and \(B\) are electric and magnetic induction field respectively, \(\mu_0\) is magnetic permeability, \(\varepsilon_0\) is electric permittivity, and \(J\) is current density.

By consider the constitutive relations for electric and magnetic fields,

\[
D(r, t) = \varepsilon E(r, t) = \varepsilon_0 \varepsilon_r
\]

\[
B(r, t) = \mu H(r, t) = \mu_0 \mu_r H(r, t)
\]

with \(D\) and \(H\) are displacement and magnetic field respectively. For any material that absorbs electric or magnetic field energy, equation (1) and (2) can be rewritten as follows:

\[
\frac{\partial B(r, t)}{\partial t} = -\nabla \times E(r, t) - M(r, t)
\]

\[
\frac{\partial D(r, t)}{\partial t} = \nabla \times H(r, t) - J(r, t)
\]

with \(M\) is the magnetization generated by the existence of magnetic field in the medium.

In linear, isotropic, non-dispersive, and lossy materials, we get the following equations:

\[
\frac{\partial H}{\partial t} = -\frac{1}{\mu} \nabla \times E - \frac{1}{\mu} \left( \sigma H + \sigma^* H \right)
\]

\[
\frac{\partial E}{\partial t} = -\frac{1}{\varepsilon} \nabla \times H - \frac{1}{\varepsilon} \left( \sigma \ J + \sigma^* E \right)
\]

Where \(\sigma\) is electric conductivity and \(\sigma^*\) is equivalent to magnetic loss.

2.2. Finite Difference and Yee Algorithm
For rectangular lattice, Yee denoted a space point in a uniform medium as [8]:

\[
(i, j, k) = (i \Delta x, j \Delta y, k \Delta z)
\]

Where \(\Delta x, \Delta y, \Delta z\) are the lattice space increments in the \(x\), \(y\), and \(z\) -axes, respectively, and \(i, j, k\) are integers. We also denote any function \(u\) of space and time evaluated at a discrete point in time as:

\[
u(i \Delta x, j \Delta y, k \Delta z, n \Delta t) = u_{i,j,k}^n
\]

Where \(\Delta t\) is the time increment and assumed to be uniform over the observation interval, and \(n\) is an integer.
Yee used centered finite-difference (central difference) expressions for the space and time derivatives that are both simply programmed and second-order accurate in the space and time increments.

\[
\frac{\partial u}{\partial t}(i\Delta x, j\Delta y, k\Delta z, n\Delta t) = \frac{u_{i+1/2,j,k}^{n} - u_{i-1/2,j,k}^{n}}{\Delta x} + O\left(\Delta x^2\right) \quad (11)
\]

By using Yee’s expression for the first partial derivative of \(u\), evaluated at the fixed space point \((i, j, k)\) follows the analogy:

\[
\frac{\partial u}{\partial t}(i\Delta x, j\Delta y, k\Delta z, n\Delta t) = \frac{u_{i,j,k}^{n+1/2} - u_{i,j,k}^{n-1/2}}{\Delta t} + O\left(\Delta t^2\right) \quad (12)
\]

the \(+1/2\) increment in the superscript denotes a time finite-difference over \(\pm 1/2\Delta t\).

3. Application and Result

Now we implement the idea and notation for achieving a numerical approximation of the Maxwell’s curl operations in 3-D.

3.1 Geometrical Space and Time Domain

![Figure 1](image.png)

We have the following attributes of the Yee space lattice, the location of the \(E\) and \(H\) components which are in the space lattice and the central-difference, that were determined by Gauss’s Law. The Finite-Difference expressions for the time derivatives are central-difference in nature and second-order accurate. The time-stepping algorithm is non-dissipative. The numerical wave propagating in mesh does not decay due to non-physical parameters.

3.2 Finite-Difference in Three Dimension Maxwell’s Equation

From the Maxwell’s equation and by considering only the \(E_x\) field component, yields:

\[
\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon} \left[ \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - (J_{source} + \sigma E_x) \right] \quad (13)
\]

By applying central differences for time and space derivatives in Equation (13), yield complete 6 components of electric and magnetic fields as follows:
3.3 3-D Finite-Differences Time-Domain Modeling

In this study, we generate an electromagnetic wave from a source which is located at the center of lattice. The electromagnetic wave source is generated by using sinusoidal source. The source is then turned off at time step $t = 100$ of the total 1000 time steps. The attenuation of electromagnetic wave is then observed by the time after the source is turned off. During the electromagnetic wave is generated by the source until turned off, we divide into 4 parts of total time step, i.e. time step = 1, 100, 500, and 1000. The picture of each time step shown by Figure 1.a; 1.b; 1.c; and 1.d respectively.

For full 3-D modeling on the XY and YZ cross section, the source is generated only at the initial time. In this modeling, we only observed at the last time step (time step = 1000). The result of this modeling are shown by Figure 3.a and 3.b.
Figure 2a. Amplitude of electric field at time-steps = 1

Figure 2b. Amplitude of electric field at time-steps = 100

Figure 2c. Amplitude of electric field (Ex) at time-steps = 500
Figure 2d. Amplitude of electric field (Ex) at time-steps = 1000

Figure 3a. Amplitude of electric field (Ex) in the-XY cross section at time-steps = 1000

Figure 3b. Amplitude of electric field (Ex) in the-YZ cross section at time-steps = 1000
4. Discussion

By observing the behavior of electromagnetic wave due to the interaction with any medium in the subsurface, the properties of medium can be determined. The attenuation of electromagnetic wave represents that medium has relatively high conductivity. From the figure above, we see that as the faster electromagnetic wave decays, show that it propagates in the more conductive medium. This condition is also influenced by medium conductivity factor which is presented in any electric and magnetic field components.

3-D visual modeling provides a more detail explanation about electromagnetic wave than by using 2-D scheme for the same case. Although need a longer computation time than 1-D and 2-D modeling. This is caused by the 3-D modeling need more space and iteration to construct the modeling domain.

5. Conclusion

We have successfully simulated Finite-Different Time-Domain (FDTD) modeling using Yee’s algorithm scheme. The electromagnetic wave was attenuated by the existence of conductive body in the subsurface. the greater the conductivity value of a medium, the electromagnetic waves will rapidly decay.

Acknowledgement

The authors gratefully thank to I Gede Putu Fadjar Soeya Djaja and all member of Modelling and Inversion Laboratory, Physics Department, Bandung Institute of Technology, for wonderful discussion about programming and suggestion for improving the manuscript. Hopefully that this research will be developed by field data from the geothermal and volcanic system area.

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