A natural fueling mechanism that helps to maintain the main core deuterium and tritium (DT) density profiles in a tokamak fusion reactor is discussed. In $H$-mode plasmas dominated by ion-temperature gradient (ITG) driven turbulence, cold DT ions near the edge will naturally pinch radially inward towards the core. This mechanism is due to the quasi-neutral heat flux dominated nature of ITG turbulence and still applies when trapped and passing kinetic electron effects are included. Fueling using shallow pellet injection or supersonic gas jets is augmented by an inward pinch of cold DT fuel. The natural fueling mechanism is demonstrated using the three-dimensional toroidal electromagnetic gyrokinetic turbulence code GEM and is analyzed using quasilinear theory. Profiles similar to those used for conservative ITER transport modeling that have a completely flat density profile are examined and it is found that natural fueling actually reduces the linear growth rates and energy transport.

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FIG. 1: (Color online). Density and temperature profiles of the Cyclone base case. Left: densities of main DT, helium ash and the DT fuel; right: temperature of the main species.

FIG. 2: (Color online). The Cyclone base case: the volume averaged particle fluxes of (a) the deuterium fuel, (b) the main deuterium, and (c) the helium ash for $C = 10^{-6}$, 0.01, 0.02, 0.03, 0.05, 0.07 and 0.1. The flux is normalized to $n_0 v_{th}$, where $v_{th}$ is the proton thermal velocity at $T_0$.

0.85 is discretized using a $128 \times 64 \times 32$ grid. The time step is $\Delta t = 2\Omega_i^{-1}$, where $\Omega_i$ is the proton gyrofrequency at $T_0$. We use 4,194,304 particles per species with realistic mass ratios, e.g., the helium mass is $4 \times 1837$ times of the electron mass. The simulation uses GEM, which is a global gyrokinetic $\delta f$ particle-in-cell code [7, 12].

We study the effects of natural fueling by running simulations with different concentrations of the DT fuel density. Figure 2 shows the the particle fluxes of the deuterium fuel, main deuterium, and the helium ash for concentrations from $C = 10^{-6}$ (no fueling) to $C = 0.1$. The particle fluxes are calculated as $\Gamma = \frac{n_0}{N} \sum_j w_j v_{E \times B}$, where $w_j$ is the weight of particle $j$, $v_{E \times B}$ is the particle’s $E \times B$ drift velocity in the $r$ direction, and $N$ is total number of particles. The fluxes are saved at 8 points along the minor radius $r$, and the results shown in Fig. 2 are volume averaged over all the points. The main and fueling tritium fluxes are similar to those of the deuterium for this case and not shown. From Fig. 2(a), the cold fuel flow always goes inward, and the level increases for bigger $C$. Hence, “natural fueling,” that is, the inward pinch of the cold fuel, is demonstrated for this simplified case.

FIG. 3: (Color online). The time-averaged particle fluxes of the main deuterium (MD), main tritium (MT), fuel deuterium (FD), fuel tritium (FT) and helium ash (HE) along $r$ for $C = 0.01$ and $C = 0.05$.

FIG. 4: (Color online). The density-normalized particle fluxes $\Gamma_\sigma/n_0 v_{th}$ of the five species for different $C$’s.

Estrada-Mila et al. [6] predicts using both theory and
gyrokinetic simulation that for $L_{\alpha He}/L_{\alpha e} < 0.84$, the helium ash flow should go outward, and this condition is well satisfied for the density profiles presented here. For the results in this paper, the helium ash always goes out. Assuming nearly adiabatic electrons, consistent with the properties of ITG turbulence, quasi-neutrality requires that the out-going helium ash flow must generate some inward going DT flow. Without the DT fuel, i.e., for the $C = 10^{-6}$ case, the particle fluxes of main DT go inward as the helium ash goes out. This phenomenon is clearly shown in Fig. 2(b) and (c). Therefore, edge fueling is needed to maintain the main DT density profile.

By adding the cold fuel from the edge, the inward particle flux of the main DT is reduced. As shown in Fig. 2(b), for $C = 0.05$, the particle flux of the main deuterium is nearly zero. Now, the helium ash goes out at the expense of the cold DT fuel instead of the hot main DT. Natural fueling is demonstrated and the main DT profile is maintained. The cold DT fuel, of course, must be heated as it migrates towards the core.

This process is better demonstrated in Fig. 3, where the time-averaged particle fluxes of all five species are shown across the minor radius. Comparing the results of $C = 0.05$ to $C = 0.01$, the increased DT fuel fluxes apparently cancel that of the main DT. We note that although this cancellation is significant, it is not complete. This is because the positions of the maximum flux are different for the main and fuel DT, due to their individual density profiles. The cold fuel concentration density should be kept below a threshold value above which the cold DT fuel fluxes go outward in this case.

Figure 4 shows the effects of fueling on all species. After normalizing by their individual densities, it is clear to see that fueling is most effective at pinching DT fuel inward. The particle flow of the main DT flux is altered from inward to near zero. The out-going helium ash flow appears insensitive to fueling. This is important, in that the natural fueling does not increase helium ash build up.

A rather crude assumption made in these simulations is that the cold fuel density is given and that it’s temperature profile is constant. For this natural fueling mechanism to work, the fuel should remain cold as it migrates towards the core. The fuel temperature will come in equilibrium with the core temperature on the ion-ion equilibration time. For a Tokamak fusion reactor at 15 keV in the core such as ITER, assuming the cold fuel temperature is 2 keV, the ion-ion thermal equilibration time is about 0.03 second. From Fig. 4, the averaged flow velocity of the fuel is about $7 \times 10^{-4}v_{th}$, where $v_{th}$ is the proton thermal velocity calculated at $T_0$. At fusion temperature, $v_{th}$ is more than $10^6$ m/s, therefore during the equilibration time, the fuel can go as far as 20m, which is much larger than the minor radius. The strong inward flux velocity is not simply caused by the negative density gradient of the fuel but rather due to the cold temperature. In fact, a pinch exists for cold fuel even in simulations with no density gradient. Indicating a convective pinch rather than diffusive density transport.

Next, a multi-species quasi-linear theory is used to explain the natural inward pinch of the DT fuel. For simplicity, assume there are only two ion species: the hot main deuterium labeled as ‘i’, and the cold fuel deuterium, as ‘f’. The linear dispersion relation is

\[ R_i + R_f = 1 - i\delta, \]

in which the $i\delta$ denotes the non-adiabatic electron response,

\[ R_\sigma = f_\sigma \left[ \frac{\omega_{s2} - \omega_d}{\omega} - \frac{\omega_{s1} \rho_{sf}}{\omega} + O(k_\theta \rho_s)^2 \right] \]

where $k_\theta = nq/r$, $n$ is the toroidal mode number, $\sigma = i, f$, $f_1 = \epsilon$, $f_1 = 1 - \epsilon$, and $\epsilon \equiv n_i/n_e$. The parameter $\omega_d = 2k_\theta \rho_s/R$ is a constant, $\omega_{s2} = k_\theta \rho_s/L_{n2}$, $\omega_{s1} = k_\theta \rho_s/L_I$, and $\omega_{s1} = \omega_{s2} + \omega_{s1}$. The quasi-linear particle flux given by

\[ \Gamma_\sigma = \text{Re}[i k_\theta \rho_s |\phi|^2 R_\sigma(\omega)n_e] \]

is proportional to $|\phi|^2$ at the nonlinear saturation level and the direction of the fuel flux is determined by $-\text{Im}(R_I)$, with the inflow of the fuel corresponding to a negative $\Gamma_I$.

The following conditions are generally satisfied in a tokamak plasma, and are assumed in this study: (i) $\omega_{s2} > 0$, meaning the main ion temperature is higher towards the core. (ii) The theory allows that for some Tokamak profiles, like in ITER [10], $\omega_{s1}$ could be negative. However, we require $\omega_{s2} = \omega_{s1} + \omega_{s1} > 0$, so the temperature gradient should dominate. (iii) The fuel ion density should concentrate at the edge, therefore $\omega_{s1} < 0$. (iv) Since the fuel ions are cold, its temperature gradient should be small compared to its density gradient, and we require that $\omega_{s2} = \omega_{s1} + \omega_{s1} < 0$. Conditions (iii) and (iv) are essential for fueling. We can simply assume $\omega_{s1} = 0$, as in simulations the fuel ion temperature is set to be the main ion temperature at the outer boundary point.

Neglect the $O(k_\theta \rho_s)^2$ term and define $\omega_N \equiv (1 - \epsilon)\omega_{s1} + \epsilon\omega_{s1}$, $\omega_P \equiv (1 - \epsilon)\omega_{s2} + \epsilon\omega_{s2}$, from Eq. 1 we have $\text{Im}(1/\omega^2) = (\omega_N - \omega_d)\text{Im}(1/\omega) + \delta_1/\omega_d$. With $\text{Im}(1/\omega) = -\gamma/|\omega|^2$ where $\gamma$ is the linear growth rate, the $-\text{Im}(R_I)$ term in Eq. 1 becomes

\[ -\text{Im}(R_I) = \frac{\epsilon \omega_{s2}}{\omega_P} \delta_1 - \omega_d \left( 1 - \frac{\omega_{s2}}{\omega_P} \right) \epsilon \frac{\gamma}{|\omega|^2} + \left( \omega_{s1} - \frac{\omega_{s2}}{\omega_P} \omega_N \right) \epsilon \frac{\gamma}{|\omega|^2} \]

Since $\epsilon$ is small, it is reasonable that $\omega_P > 0$. With the negative $\omega_{s2}$, the first term on the right hand side of
Eq. (4), which is proportional to \( \delta \), is therefore negative, meaning a non-adiabatic electron out flux should enhance fueling. The second term is proportional to \( \omega_d \) which comes from the toroidal geometry of the device, and this term is also negative, so toroidal effects are also favorable for natural fueling. In the limit of \( \omega_{nt} = 0 \), the third term becomes \( \epsilon(1-\epsilon)\omega_{ni}\omega_{TI}\gamma/(|\omega|^2\omega_p) \). Because of the density profile and the temperature gradient of the main ions, this term is also negative. We conclude that the fuel flux must be naturally negative and going inward towards the core.

Due to quasi-neutrality, adding a negative fueling flux should increase the main ion particle transport. This effect can also be seen by switching \( I \to i \) and \( \epsilon \to 1-\epsilon \) in Eq. (4), in which all three terms on the right-hand-side are positive for the main ion, satisfying \( \text{Im}(R_i) + \text{Im}(R_f) = -\delta \). In the case where only two ion species are present, e.g., hot main deuterium and cold fuel deuterium, the fuel ion goes in at the expense of hot ions going out, and therefore the natural fueling is of no value. However, when helium ash is present, the main DT pinch as the helium ash goes outward. Fueling is then useful to maintain the main DT profile, and balance the outgoing helium ash flux.

\[
\epsilon(1-\epsilon)\omega_{ni}\omega_{TI}\gamma/(|\omega|^2\omega_p) 
\]

![FIG. 5](image-url) (Color online). Profiles of the ITER-like case.

The global version of the Cyclone base case presented here have monotonically decreasing density profiles that peak at the magnetic axis. So far there’s no general consensus on the ITER density profile. One rather conservative study [10] shows ITER H-mode profiles with a completely flat density. The inward peaking helium ash then causes the main DT density to be hollow, or peak near the edge. We now examine the natural fueling mechanism in this situation. The next set of simulations have the same physical parameters as in the Cyclone base case, but use the ITER-like hollow DT density and temperature profiles, as shown in Fig. 5.

The simulation results for this flat density ITER-like case are shown in Fig. 6. This completely flat density profile is overall much less unstable and as a result the cold fuel flux is also lower. This reduces the travel length of the fuel to about 0.8m before they are heated by ion-ion thermal equilibration. The outward going helium ash flow still causes the main DT to go inward without fueling, but there is a different phenomenon. The main deuterium flux now separates from the main tritium flux. Similar to the previous case, adding cold DT fuel reduces the inward particle pinch of the main DT. However, due to the separation of the main D and T, it is very easy to overshoot with too large a \( C \), thereby, causing the main tritium to go outward. This overshoot is shown in the right panel of Fig. 6. Given this effect of DT separation in the ITER-like case, one may consider using different density profiles for the deuterium fuel and tritium, possibly less tritium and more deuterium, but it is beyond the scope of this Letter and will require future investigations.

Another significant new result of the ITER-like case presented here is that the linear growth rate of the ITG-induced instability is reduced by natural fueling, as shown in Fig. 7. This bonus is due to the hollow density profile of the main DT. In general, at low real frequency the linear growth rate \( \gamma \) from Eq. 1 is roughly \( \sqrt{\omega_{ni}\omega_p} \) with fueling and \( \sqrt{\omega_{ni}\omega_p}/\omega_{spi} \) for with main ions only. Since \( 0 < \omega_p < \omega_{spi} \), fueling decreases \( \gamma \) and hence is a stabilizing effect. This effect is insignificant for the Cyclone base case: from \( C = 10^{-6} \) to \( C = 0.1 \), \( \gamma \) is only reduced by 2\%, at about \( 6.8 \times 10^{-4} \Omega_i \). However, in the ITER-like case, since \( \omega_{ni} \) is negative, for comparable temperature profiles, \( 1/\omega_{spi} \) is bigger than the Cyclone base case and hence \( \omega_p/\omega_{spi} = 1 - \epsilon + \epsilon \omega_{spi}/\omega_{spi} \) is smaller. Therefore \( \gamma \) is reduced more by fueling. As a result, the heat fluxes of both main DT and helium ash are also reduced. For \( C = 0.05 \) in the ITER-like case, the heat fluxes are reduced by nearly 50\% from \( C = 10^{-6} \).

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FIG. 7: Linear growth rates for the ITER-like case.

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