Analysis of the $P_c(4312)$, $P_c(4440)$, $P_c(4457)$ and Related Hidden-Charm Pentaquark States with QCD Sum Rules

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Abstract

In this article, we re-study the ground state mass spectrum of the diquark-diquark-antiquark type $uudc$ pentaquark states with the QCD sum rules by carrying out the operator product expansion up to the vacuum condensates of 13 in a consistent way. The predicted masses support assigning the $P_c(4312)$ to be the hidden-charm pentaquark state with $J^P = \frac{1}{2}^-$, assigning the $P_c(4440)$ to be the hidden-charm pentaquark state with $J^P = \frac{1}{2}^-$, $\frac{3}{2}^-$ or $\frac{5}{2}^-$, assigning the $P_c(4457)$ to be the hidden-charm pentaquark state with $J^P = \frac{1}{2}^-$ or $\frac{3}{2}^-$.

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1 Introduction

In 2015, the LHCb collaboration studied the $\Lambda_b^0 \to J/\psi K^- p$ decays and observed two pentaquark candidates $P_c(4380)$ and $P_c(4450)$ in the $J/\psi p$ mass spectrum with the significances of more than 9σ [1]. Recently, the LHCb collaboration studied the $\Lambda_b^0 \to J/\psi K^- p$ decays with a data sample, which is an order of magnitude larger than that previously analyzed by the LHCb collaboration, and observed a narrow pentaquark candidate $P_c(4312)$ with a statistical significance of 7.3σ [2]. Furthermore, the LHCb collaboration confirmed the $P_c(4450)$ pentaquark structure, and observed that it consists of two narrow overlapping peaks $P_c(4440)$ and $P_c(4457)$ with the statistical significance of 5.4σ [2]. The measured masses and widths are

$$P_c(4312) : M = 4311.9 \pm 0.7^{+6.8}_{-0.6} \text{ MeV}, \quad \Gamma = 9.8 \pm 2.7^{+3.7}_{-4.5} \text{ MeV},$$
$$P_c(4440) : M = 4440.3 \pm 1.3^{+4.1}_{-4.7} \text{ MeV}, \quad \Gamma = 20.6 \pm 4.9^{+8.7}_{-10.1} \text{ MeV},$$
$$P_c(4457) : M = 4457.3 \pm 0.6^{+4.1}_{-1.7} \text{ MeV}, \quad \Gamma = 6.4 \pm 2.0^{+5.7}_{-1.9} \text{ MeV}. \quad (1)$$

There have been several possible assignments of the $P_c$ states since the observations of the $P_c(4380)$ and $P_c(4450)$, such as the diquark-diquark-antiquark type pentaquark states [3, 4, 5, 6], the diquark-triquark type pentaquark states [7], the molecule-like pentaquark states [8, 9], the hadro-charmonium states [10], the re-scattering effects [11], etc.

In Refs. [4, 5], we construct the diquark-diquark-antiquark type pentaquark currents, study the $J^P = \frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^-$ hidden-charm pentaquark states with the strangeness $S = 0, -1, -2, -3$ systematically using the QCD sum rules, and explore the possible assignment of the $P_c(4380)$ and $P_c(4450)$ in the scenario of the pentaquark states. As the vacuum condensates are vacuum expectations of the quark-gluon operators, we take into account the contributions of the quark-gluon operators of the order $O(\alpha_s^k)$ with $k \leq 1$ and dimension $D \leq 10$ in carrying out the operator product expansion. Now we write down the contributions of the relevant vacuum condensates $D_3$,

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$D_4, D_5, D_6, D_7, D_8, D_9$ and $D_{10}$ explicitly,

\[
\begin{align*}
D_3 &= \langle \bar{q}q \rangle, \langle \bar{s}s \rangle, \\
D_4 &= \langle \frac{\alpha_s GG}{\pi} \rangle, \\
D_5 &= \langle \bar{q}g_{\bar{s}}Gq \rangle, \langle \bar{s}g_{s}Gq \rangle, \\
D_6 &= \langle \bar{q}q \rangle^2, \langle \bar{q}q \rangle \langle \bar{s}s \rangle, \langle \bar{s}s \rangle^2, \\
D_7 &= \langle \bar{q}q \rangle \left( \frac{\alpha_s GG}{\pi} \right), \langle \bar{s}s \rangle \left( \frac{\alpha_s GG}{\pi} \right), \\
D_8 &= \langle \bar{q}q \rangle \langle \bar{q}g_{s}Gq \rangle, \langle \bar{s}s \rangle \langle \bar{s}g_{s}Gq \rangle, \langle \bar{q}q \rangle \langle \bar{s}g_{s}Gq \rangle, \langle \bar{s}s \rangle \langle \bar{q}g_{s}Gq \rangle, \\
D_9 &= \langle \bar{q}q \rangle^3, \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle, \langle \bar{q}q \rangle \langle \bar{s}s \rangle^2, \langle \bar{s}s \rangle^3, \\
D_{10} &= \langle \bar{q}g_{s}Gq \rangle^2, \langle \bar{q}g_{s}Gq \rangle \langle \bar{s}g_{s}Gq \rangle, \langle \bar{s}g_{s}Gq \rangle^2, \langle \bar{q}q \rangle^2 \left( \frac{\alpha_s GG}{\pi} \right), \langle \bar{q}q \rangle \langle \bar{s}s \rangle \left( \frac{\alpha_s GG}{\pi} \right), \\
&\langle \bar{s}s \rangle^2 \left( \frac{\alpha_s GG}{\pi} \right).
\end{align*}
\]  

In calculations, sometimes we neglect the vacuum condensates $\langle \frac{\alpha_s GG}{\pi} \rangle$, $\langle \bar{q}q \rangle \langle \frac{\alpha_s GG}{\pi} \rangle$, $\langle \bar{s}s \rangle \langle \frac{\alpha_s GG}{\pi} \rangle$, $\langle \bar{q}q \rangle^2 \langle \frac{\alpha_s GG}{\pi} \rangle$, $\langle \bar{q}q \rangle \langle \bar{s}s \rangle \langle \frac{\alpha_s GG}{\pi} \rangle$, $\langle \bar{s}s \rangle^2 \langle \frac{\alpha_s GG}{\pi} \rangle$ due to the small values of the gluon condensate $\langle \frac{\alpha_s GG}{\pi} \rangle$. Those terms are not associated with the $\frac{\pi}{T^2}$, $\frac{\pi}{T^2}$ and $\frac{\pi}{T^2}$, where the $T^2$ are the Borel parameters, neglecting them cannot impair the predictive ability remarkably.

At the QCD side of the correlation functions, there are two heavy quark (or $c$-quark) propagators and three light quark propagators, if each heavy quark line emits a gluon and each light quark line contributes a quark pair, we obtain an operator $GG\bar{q}q\bar{q}q\bar{q}q$, which is of dimension 13, we should take into account the vacuum condensates at least up to dimension 13. The vacuum condensates $\langle \bar{q}q \rangle^2 \langle \bar{q}g_{s}Gq \rangle$, $\langle \bar{q}q \rangle \langle \bar{q}g_{s}Gq \rangle^2$ and $\langle \bar{q}q \rangle^3 \langle \frac{\alpha_s GG}{\pi} \rangle$ are associated with the $\frac{\pi}{T^2}$, $\frac{\pi}{T^2}$ and $\frac{\pi}{T^2}$, which manifest themselves at the small values of the $T^2$ and play an important role in determining the Borel windows, although at the Borel windows they play a minor important role.

In Refs. [12, 13, 14, 15, 16], we study the acceptable energy scales of the QCD spectral densities for the hidden-charm (hidden-bottom) tetraquark states and molecular states in the QCD sum rules in details for the first time, and suggested an energy scale formula

\[
\mu = \sqrt{M^2_{\chi/Y/Z} - (2M_Q)^2},
\]  

(3)

to determine the ideal energy scales of the QCD spectral densities, where the $X$, $Y$, $Z$ denote the four-quark states, and the $M_Q$ denotes the effective $Q$-quark masses [13, 15]. The energy scale formula works well for all the tetraquark states and molecular states. In Refs. [11, 15], we observe that the energy scale formula can be successfully applied to study the hidden-charm pentaquark states with a slight modification $\mu = \sqrt{M^2_{\pi} - (2M_c)^2}$ with the old value $M_c = 1.8$ GeV.

In this article, we study the ground state mass spectrum of the diquark-diquark-antiquark type $uudc\bar{c}$ pentaquark states with the QCD sum rules by taking into account all the vacuum condensates up to dimension 13 of the order $O(\alpha_s^k)$ with $k \leq 1$ in carrying out the operator product expansion, and use the energy scale formula $\mu = \sqrt{M^2_{\pi} - (2M_c)^2}$ with the updated value $M_c = 1.82$ GeV to determine the ideal energy scales of the QCD spectral densities [16], and update the analysis and explore the possible assignments of the $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ in the scenario of the pentaquark states.

The article is arranged as follows: we derive the QCD sum rules for the masses and pole residues of the ground state hidden-charm pentaquark states in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusion.
2 QCD sum rules for the hidden-charm pentaquark states

Firstly, we write down the two-point correlation functions $\Pi(p)$, $\Pi_{\mu\nu}(p)$ and $\Pi_{\mu\nu\alpha\beta}(p)$ in the QCD sum rules,

$$\Pi(p) = i \int d^4xe^{ipx} \langle 0| J(x)\bar{J}(0) |0 \rangle,$$

$$\Pi_{\mu\nu}(p) = i \int d^4xe^{ipx} \langle 0| J_\mu(x)\bar{J}_\nu(0) |0 \rangle,$$

$$\Pi_{\mu\nu\alpha\beta}(p) = i \int d^4xe^{ipx} \langle 0| J_{\mu\nu}(x)\bar{J}_{\alpha\beta}(0) |0 \rangle,$$

where $J(x) = J^1(x)$, $J^2(x)$, $J^3(x)$, $J^4(x)$, $J_\mu(x) = J^1_\mu(x)$, $J^2_\mu(x)$, $J^3_\mu(x)$, $J^4_\mu(x)$, $J_{\mu\nu}(x) = J^1_{\mu\nu}(x)$, $J^2_{\mu\nu}(x)$,

$$J^1(x) = \varepsilon^{iak} \varepsilon^{jlmn} u_j^T(x) C_{\gamma 5} d_k(x) u_m^T(x) C_{\gamma 5} c_n(x) C_{\gamma 5} T_a(x),$$

$$J^2(x) = \varepsilon^{iak} \varepsilon^{jlmn} u_j^T(x) C_{\gamma 5} d_k(x) u_m^T(x) C_{\gamma 5} \gamma_{\mu} c_n(x) \gamma_{\mu} C_{\gamma 5} T_a(x),$$

$$J^3(x) = \frac{2}{\sqrt{3}} \varepsilon^{iak} \varepsilon^{jlmn} u_j^T(x) C_{\gamma 5} d_k(x) C_{\gamma 5} c_n(x) \gamma_{\mu} \gamma_{\mu} C_{\gamma 5} T_a(x),$$

$$J^4(x) = \frac{2}{\sqrt{3}} \varepsilon^{iak} \varepsilon^{jlmn} u_j^T(x) C_{\gamma 5} d_k(x) C_{\gamma 5} \gamma_{\mu} c_n(x) \gamma_{\mu} C_{\gamma 5} T_a(x),$$

$$J^1_\mu(x) = \varepsilon^{iak} \varepsilon^{jlmn} u_j^T(x) C_{\gamma 5} d_k(x) u_m^T(x) C_{\gamma 5} \gamma_{\alpha} c_n(x) \gamma_{\alpha} C_{\gamma 5} T_a(x),$$

$$J^2_\mu(x) = \varepsilon^{iak} \varepsilon^{jlmn} u_j^T(x) C_{\gamma 5} d_k(x) C_{\gamma 5} \gamma_{\alpha} c_n(x) \gamma_{\alpha} C_{\gamma 5} T_a(x),$$

$$J^3_\mu(x) = \frac{1}{\sqrt{3}} \varepsilon^{iak} \varepsilon^{jlmn} u_j^T(x) C_{\gamma 5} d_k(x) C_{\gamma 5} \gamma_{\alpha} c_n(x) \gamma_{\alpha} C_{\gamma 5} T_a(x),$$

$$J^4_\mu(x) = \frac{1}{\sqrt{3}} \varepsilon^{iak} \varepsilon^{jlmn} u_j^T(x) C_{\gamma 5} d_k(x) C_{\gamma 5} \gamma_{\alpha} c_n(x) \gamma_{\alpha} C_{\gamma 5} T_a(x),$$

$$(\mu \leftrightarrow \nu),$$

$$J^1_{\mu\nu}(x) = \varepsilon^{iak} \varepsilon^{jlmn} u_j^T(x) C_{\gamma 5} d_k(x) u_m^T(x) C_{\gamma 5} \gamma_{\mu} \gamma_{\nu} c_n(x) \gamma_{\mu} \gamma_{\nu} C_{\gamma 5} T_a(x),$$

$$J^2_{\mu\nu}(x) = \varepsilon^{iak} \varepsilon^{jlmn} u_j^T(x) C_{\gamma 5} d_k(x) C_{\gamma 5} \gamma_{\mu} \gamma_{\nu} c_n(x) \gamma_{\mu} \gamma_{\nu} C_{\gamma 5} T_a(x),$$

where the $i, j, k, l, m, n$ and $\alpha$ are color indices, the $C$ is the charge conjugation matrix [4,5]. The currents $J(x)$, $J_\mu(x)$ and $J_{\mu\nu}(x)$ have negative parity, and couple potentially to both the negative parity and positive parity pentaquark states, as multiplying $i\gamma_5$ to the currents $J(x)$, $J_\mu(x)$ and $J_{\mu\nu}(x)$ changes their parity [17 [18 [19 [20].

Now we write down the current-pentaquark couplings (or the definitions for the pole residues) explicitly,

$$\langle 0| J(0)\bar{P}^- (\frac{1}{2}) \rangle = \lambda^- \bar{U}^-(p, s),$$

$$\langle 0| J(0)\bar{P}^+ (\frac{1}{2}) \rangle = \lambda^+ \bar{U}^+ (p, s),$$

(6)
where the superscripts ± denote the positive parity and negative parity, respectively, the subscripts \( \frac{1}{2}, \frac{3}{2} \) and \( \frac{5}{2} \) denote the spins of the pentaquark states, the \( \lambda, f \) and \( g \) are the pole residues. The spinors \( U_\mu(p,s) \) satisfy the Dirac equations \((\not{p} - M_\pm)U_\mu(p,s) = 0\), while the spinors \( U_\mu(p,s) \) and \( U_\mu(p,s) \) satisfy the Rarita-Schwinger equations \((\not{p} - M_\pm)U_\mu(p,s) = 0\) and \((\not{p} - M_\pm)U_\mu(p,s) = 0\), and the relations \( g^\mu U_\mu(p,s) = 0, \ g^\mu U_\mu(p,s) = 0, \ g^\mu U_\mu(p,s) = 0, \ g^\mu U_\mu(p,s) = 0\), respectively. For more details about the spinors, one can consult Ref.\[4\].

At the phenomenological side, we insert a complete set of intermediate pentaquark states with the same quantum numbers as the current operators \( J(x), i\gamma_5 J(x), J_{\mu
u}(x), i\gamma_5 J_{\mu
u}(x) \) into the correlation functions \( \Pi(p) \), \( \Pi_{\mu\nu}(p) \) and \( \Pi_{\mu\nu\alpha\beta}(p) \) to obtain the hadronic representation \[21 \ 22\]. After isolating the pole terms of the lowest states of the negative parity and positive parity hidden-charm pentaquark states, we obtain the following results:

\[
\Pi(p) = \lambda^1_2 \frac{\not{p} + M_-}{M_-^2 - p^2} + \lambda^1_3 \frac{\not{p} - M_+}{M_+^2 - p^2} + \cdots, \\
\Pi_{\mu\nu}(p) = \left( \lambda^2_2 \frac{\not{p} + M_-}{M_-^2 - p^2} + \lambda^2_3 \frac{\not{p} - M_+}{M_+^2 - p^2} \right) (g_{\mu\nu}) + \cdots, \\
\Pi_{\mu\nu\alpha\beta}(p) = \left( \lambda^3_2 \frac{\not{p} + M_-}{M_-^2 - p^2} + \lambda^3_3 \frac{\not{p} - M_+}{M_+^2 - p^2} \right) (g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}) + \cdots.
\]

In this article, we study the components \( \Pi^1_1(p^2), \Pi^1_2(p^2), \Pi^1_3(p^2), \Pi^1_4(p^2), \Pi^1_5(p^2) \) to avoid possible contaminations from other pentaquark states with different spins. For detailed discussions about this subject, one can consult Refs.\[4 \ 23\]. In Table 1 we present the quark structures of the negative parity hidden-charm pentaquark states and corresponding interpolating currents explicitly.

Now we obtain the spectral densities at the phenomenological side through dispersion relation,

\[
\frac{\text{Im} \Pi^1_1(s)}{\pi} = \lambda_j^1 \delta \left( s - M_+^2 \right) + \lambda_j^3 \delta \left( s - M_-^2 \right) = \rho_{j,H}^1(s),
\]

\[
\frac{\text{Im} \Pi^0_1(s)}{\pi} = M_- \lambda_j^1 \delta \left( s - M_-^2 \right) - M_+ \lambda_j^3 \delta \left( s - M_+^2 \right) = \rho_{j,H}^0(s),
\]

\[
\frac{\text{Im} \Pi^1_2(s)}{\pi} = \lambda_j^2 \delta \left( s - M_-^2 \right) + \lambda_j^3 \delta \left( s - M_+^2 \right) = \rho_{j,H}^1(s),
\]

\[
\frac{\text{Im} \Pi^0_2(s)}{\pi} = M_- \lambda_j^2 \delta \left( s - M_-^2 \right) - M_+ \lambda_j^3 \delta \left( s - M_+^2 \right) = \rho_{j,H}^0(s),
\]

\[
\frac{\text{Im} \Pi^1_3(s)}{\pi} = \lambda_j^2 \delta \left( s - M_-^2 \right) + \lambda_j^3 \delta \left( s - M_+^2 \right) = \rho_{j,H}^1(s),
\]

\[
\frac{\text{Im} \Pi^0_3(s)}{\pi} = M_- \lambda_j^2 \delta \left( s - M_-^2 \right) - M_+ \lambda_j^3 \delta \left( s - M_+^2 \right) = \rho_{j,H}^0(s),
\]

\[
\frac{\text{Im} \Pi^1_4(s)}{\pi} = \lambda_j^2 \delta \left( s - M_-^2 \right) + \lambda_j^3 \delta \left( s - M_+^2 \right) = \rho_{j,H}^1(s),
\]

\[
\frac{\text{Im} \Pi^0_4(s)}{\pi} = M_- \lambda_j^2 \delta \left( s - M_-^2 \right) - M_+ \lambda_j^3 \delta \left( s - M_+^2 \right) = \rho_{j,H}^0(s),
\]

\[
\frac{\text{Im} \Pi^1_5(s)}{\pi} = \lambda_j^2 \delta \left( s - M_-^2 \right) + \lambda_j^3 \delta \left( s - M_+^2 \right) = \rho_{j,H}^1(s),
\]

\[
\frac{\text{Im} \Pi^0_5(s)}{\pi} = M_- \lambda_j^2 \delta \left( s - M_-^2 \right) - M_+ \lambda_j^3 \delta \left( s - M_+^2 \right) = \rho_{j,H}^0(s),
\]
where $j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$, the subscript $H$ denotes the hadron side, then we introduce the weight functions $\sqrt{s} \exp\left(-\frac{s}{M^2}\right)$ and $\exp\left(-\frac{s}{M^2}\right)$ to obtain the QCD sum rules at the hadron side,

$$
\int_{4m^2}^{s_0} ds \left[ \sqrt{s} \rho_{j,H}(s) + \rho^{0}_{j,H}(s) \right] \exp\left(-\frac{s}{M^2}\right) = 2M_\lambda_j^{-2} \exp\left(-\frac{M^2}{T^2}\right),
$$

(14)

where the $s_0$ are the continuum threshold parameters. We separate the contributions of the negative parity and positive parity pentaquark states unambiguously.

In this article, we carry out the operator product expansion to the vacuum condensates up to dimension-13 and assume vacuum saturation for the higher dimensional vacuum condensates. We take the truncations $n \leq 13$ and $k \leq 1$ in a consistent way, the operators of the orders $O(\alpha^k)$ with $k > 1$ are discarded. The vacuum condensates $(\bar{q}q)^2(\bar{q}q,\sigma Gq)$, $(\bar{q}q)(\bar{q}q,\sigma Gq)^2$, $(\bar{q}q)^3(\bar{q}q,GG)$ are of dimension 11 and 13 respectively, and come from the Feynman diagrams shown in Fig.1 and play an important role in determining the Borel windows. Then we obtain the QCD spectral densities through dispersion relation,

$$
\rho_{j,QCD}(s) = \frac{\text{Im} \Pi^j_{\chi}(s)}{\pi},
$$

$$
\rho^{0}_{j,QCD}(s) = \frac{\text{Im} \Pi^{0,j}_{\chi}(s)}{\pi},
$$

(15)

where $j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$. Now we take the quark-hadron duality below the continuum thresholds $s_0$ and obtain the QCD sum rules:

$$
2M_\lambda_j^{-2} \exp\left(-\frac{M^2}{T^2}\right) = \int_{4m^2}^{s_0} ds \rho_{QCD,j}(s) \exp\left(-\frac{s}{T^2}\right).
$$

(16)

where $\rho_{QCD,j}(s) = \sqrt{s} \rho^{1}_{QCD,j}(s) + \rho^{0}_{QCD,j}(s)$,

$$
\rho_{QCD,j}(s) = \rho^{1}_{j}(s) + \rho^{2}_{j}(s) + \rho^{3}_{j}(s) + \rho^{4}_{j}(s) + \rho^{5}_{j}(s) + \rho^{6}_{j}(s) + \rho^{7}_{j}(s) + \rho^{8}_{j}(s) + \rho^{9}_{j}(s) + \rho^{10}_{j}(s) + \rho^{11}_{j}(s)
$$

$$
+ \rho^{12}_{j}(s),
$$

(17)
Figure 1: The diagrams contribute to the condensates $\langle \bar{q}q \rangle^2 \langle \bar{q}g_s \sigma Gq \rangle$, $\langle \bar{q}q \rangle \langle \bar{q}g_s \sigma Gq \rangle^2$, $\langle \bar{q}q \rangle^3 \langle \frac{\alpha_sGG}{\pi} \rangle$. Other diagrams obtained by interchanging of the heavy quark lines (dashed lines) or light quark lines (solid lines) are implied.

\[
\begin{align*}
\rho_j^0(s) &\propto \text{perturbative terms}, \\
\rho_j^1(s) &\propto \langle \bar{q}q \rangle, \\
\rho_j^2(s) &\propto \langle \frac{\alpha_sGG}{\pi} \rangle, \\
\rho_j^3(s) &\propto \langle \bar{q}g_s \sigma Gq \rangle, \\
\rho_j^4(s) &\propto \langle \bar{q}q \rangle^2, \\
\rho_j^5(s) &\propto \langle \bar{q}q \rangle \langle \frac{\alpha_sGG}{\pi} \rangle, \\
\rho_j^6(s) &\propto \langle \bar{q}q \rangle \langle \bar{q}g_s \sigma Gq \rangle, \\
\rho_j^7(s) &\propto \langle \bar{q}q \rangle^3, \\
\rho_j^8(s) &\propto \langle \bar{q}g_s \sigma Gq \rangle^2, \\
\rho_j^9(s) &\propto \langle \bar{q}g_s \sigma Gq \rangle^3 \langle \frac{\alpha_sGG}{\pi} \rangle. \\
\end{align*}
\]

The explicit expressions of the QCD spectral densities are too lengthy to be presented here, the interested reader can obtain them by contacting me via E-mail.

We derive Eq. (18) with respect to $\frac{1}{T^2}$, then eliminate the pole residues $\lambda_j^-$ and obtain the QCD sum rules for the masses of the hidden-charm pentaquark states,

\[
M^2 = \frac{-\int_{4m_c^2}^{s_0} ds \frac{d}{dT^2} \rho_{QCD,j}(s) \exp \left(-\frac{s}{T^2}\right)}{\int_{4m_c^2}^{s_0} ds \rho_{QCD,j}(s) \exp \left(-\frac{s}{T^2}\right)}. 
\]

3 Numerical results and discussions

We take the vacuum condensates to be the standard values $\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3$, $\langle \bar{q}g_s \sigma Gq \rangle = m_0^2 \langle \bar{q}q \rangle$, $m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2$, $\langle \frac{\alpha_sGG}{\pi} \rangle = (0.33 \text{ GeV})^4$ at the energy scale $\mu = 1 \text{ GeV}$ [21][22][24], and take the $\overline{\text{MS}}$ mass $m_c(m_c) = (1.275 \pm 0.025) \text{ GeV}$ from the Particle Data Group [25]. Moreover,
we take into account the energy-scale dependence of the quark condensate, mixed quark condensate and $\bar{M}\bar{S}$ mass,

\[
\langle \bar{q}q \rangle (\mu) = \langle \bar{q}q \rangle (1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{24\pi^2}},
\]

\[
\langle \bar{q}g_sGq \rangle (\mu) = \langle \bar{q}g_sGq \rangle (1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{2}{24\pi^2}},
\]

\[
m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{1}{24\pi^2}},
\]

\[
\alpha_s(\mu) = \frac{1}{b_0 t} \left[ 1 - \frac{b_1}{b_0^2} \left( \frac{\log t}{t} + \frac{b_2}{b_0^4} (\log^2 t - \log t - 1) + b_0 b_2 \right) \right], \tag{20}
\]

where \( t = \log \frac{\mu^2}{\Lambda^2} \), \( b_0 = \frac{33 - 2n_f}{12\pi} \), \( b_1 = \frac{153 - 19n_f}{24\pi^2} \), \( b_2 = \frac{2857 - 8033}{128\pi^4} + \frac{553}{24^n_f} \), \( \Lambda = 210 \text{ MeV}, 292 \text{ MeV} \) and 332 MeV for the flavors \( n_f = 5, 4 \) and 3, respectively \([25, 26]\), and evolve all the input parameters at the QCD side to the optimal energy scales \( \mu \) with \( n_f = 4 \) to extract the pentaquark masses.

In Refs. [20, 23], we study the heavy, doubly-heavy and triply-heavy baryon states with the QCD sum rules in a systematic way. In calculations, we observe that the continuum threshold parameters \( \sqrt{s_0} = M_{gr} + (0.5 - 0.8) \text{ GeV} \) work well, where the subscript \( gr \) denotes the ground state baryon states. The pentaquark states are another type baryon states due to the fractional spins \( \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \) and \( \frac{7}{2} \), respectively.

In this article, we take the continuum threshold parameters as \( \sqrt{s_0} = M_P + (0.55 - 0.75) \text{ GeV} \).

In this article, we choose the Borel parameters \( T^2 \) and continuum threshold parameters \( s_0 \) to satisfy the four criteria:

1. Pole dominance at the phenomenological side;
2. Convergence of the operator product expansion;
3. Appearance of the Borel platforms;
4. Satisfying the energy scale formula,

via try and error.

The hidden-charm or hidden-bottom four-quark and five-quark systems can be described by a double-well potential in the heavy quark limit \([4, 5, 12, 13, 14]\). The heavy quark \( Q \) serves as a static well potential and attracts a light quark to form a heavy diquark in color antitriplet. The heavy antiquark \( \bar{Q} \) serves as another static well potential and attracts a light antiquark or a light diquark to form a heavy antidiquark or triquark in color triplet. Then the diquark and antidiquark (or triquark) attract each other to form a compact tetraquark state (or pentaquark state).

The hidden-charm or hidden-bottom tetraquark and pentaquark are characterized by the effective \( Q \)-quark mass \( M_Q \) and the virtuality \( \sqrt{\sigma_{Gq} - (2M_Q)^2} \) or the energy scale \( \mu = \sqrt{M_{X/Y/Z/P} - (2M_Q)^2} \) of the QCD spectral densities \([4, 5, 12, 13, 14]\).

In this article, we carry out the operator product expansion up to the vacuum condensates of dimension 13, which is consistent with the dimension 10 in the tetraquark case \([4, 5, 12, 13, 14]\), and choose the updated value of the effective \( c \)-quark mass \( M_c = 1.82 \text{ GeV} \) determined in the QCD sum rules for the hidden-charm tetraquark states \([18]\).

The resulting Borel parameters or Borel windows \( T^2 \), continuum threshold parameters \( s_0 \), ideal energy scales of the QCD spectral densities, pole contributions of the ground state pentaquark states, and contributions of the vacuum condensates of dimension 13 are shown explicitly in Table 2.

In Fig 2 we plot the contribution of the vacuum condensates of dimension 13 with variation of the Borel parameter \( T^2 \) for the hidden-charm pentaquark state \([ud][uc]\bar{c}\ (0, 0, 0, \frac{1}{2})\) with the central values of the parameters shown in Table 2 as an example. Form the figure, we can see that
the vacuum condensates of dimension 13 manifest themselves at the region $T^2 < 2 \text{ GeV}^2$, we should choose the value $T^2 > 2 \text{ GeV}^2$. The higher dimensional vacuum condensates play an important role in determining the Borel windows, we should take them into account in a consistent way, while in the Borel windows, they play an minor important role as the operator product expansion should be convergent, for example, in the present case, the contribution of the vacuum condensates of dimension 13 is less than 1%.

In Fig.2 we plot the mass of the hidden-charm pentaquark state $[ud][uc]\bar{c}$ $\left(0, 0, 0, \frac{1}{2}\right)$ with variation of the Borel parameter $T^2$ for truncations of the operator product expansion up to the vacuum condensates of dimension 10 and 13, respectively. From the figure, we can see the vacuum condensates of dimension 11 and 13 play an important role to obtain stable QCD sum rules, we should take them into account.

From the Table 2, we can see that the pole contributions are about $(40-60)\%$ and the operator product expansion is well convergent, the first two criteria or the two basic criteria of the QCD sum rules are satisfied, so we expect to make reasonable predictions.

We take into account all uncertainties of the input parameters, and obtain the masses and pole residues of the negative parity hidden-charm pentaquark states, which are shown explicitly in Table 3. From Table 2 and Table 3 we can see that the energy scale formula $\mu = \sqrt{M_P - (2M_c)^2}$ is satisfied, the criterion 4 is satisfied.

In Figs 4-5 we plot the masses of the hidden-charm pentaquark states with variations of the Borel parameters $T^2$ in the Borel windows. From the figures, we can see that there appear very flat platforms, the criterion 3 is satisfied. Now the four criteria of the QCD sum rules are all satisfied, we expect to make robust predictions.

The predicted masses $M_P = 4.31 \pm 0.11 \text{ GeV}$ for the ground state $[ud][uc]\bar{c} \left(0, 0, 0, \frac{1}{2}\right)$ pentaquark state and $M_P = 4.34 \pm 0.14 \text{ GeV}$ for the ground state $[uu][dc]\bar{c} + 2[ud][uc]\bar{c} \left(1, 1, 0, \frac{1}{2}\right)$ pentaquark state are both in excellent agreement with the experimental data $M_{P(4312)} = 4311.9 \pm 0.7\text{MeV}$ from the LHCb collaboration [2], and support assigning the $P_c(4312)$ to be the hidden-charm pentaquark state with $J^P = \frac{1}{2}^{-}$.

The predicted masses $M_P = 4.45 \pm 0.11 \text{ GeV}$ for the ground state $[ud][uc]\bar{c} \left(0, 1, 1, \frac{1}{2}\right)$ pentaquark state, $M_P = 4.46 \pm 0.11 \text{ GeV}$ for the ground state $[uu][dc]\bar{c} + 2[ud][uc]\bar{c} \left(1, 0, 1, \frac{1}{2}\right)$ pentaquark state and $M_P = 4.39 \pm 0.11$ for the ground state $[ud][uc]\bar{c} \left(0, 1, 1, \frac{3}{2}\right)$, $[uu][dc]\bar{c} + 2[ud][uc]\bar{c}$
Figure 3: The mass with variation of the Borel parameter $T^2$ for the hidden-charm pentaquark state $[ud][uc]\bar{c} (0, 0, 0, \frac{1}{2})$, the $D = 10, 13$ denote truncations of the operator product expansion.

(1, 1, 2, \frac{5}{2}), [ud][uc]\bar{c} (0, 1, 1, \frac{5}{2})$ pentaquark states are in excellent agreement (or compatible with) the experimental data $M_{Pc(4440)} = 4440.3 \pm 1.3^{+4.2}_{-4.5}$ MeV from the LHCb collaboration [2], and support assigning the $P_{c}(4440)$ to the hidden-charm pentaquark state with $J^P = \frac{1}{2}^-, \frac{3}{2}^-$ or $\frac{5}{2}^-$. The predicted masses $M_{P} = 4.45 \pm 0.11$ GeV for the ground state $[ud][uc]\bar{c} (0, 1, 1, \frac{1}{2})$ pentaquark state, $M_{P} = 4.46 \pm 0.11$ GeV for the ground state $[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 0, 1, \frac{1}{2})$ pentaquark state and $M_{P} = 4.47 \pm 0.11$ GeV for the ground state $[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 0, 1, \frac{5}{2})$ pentaquark states are in excellent agreement the experimental data $M_{Pc(4457)} = 4457.3 \pm 0.6^{+4.1}_{-1.7}$ MeV from the LHCb collaboration [2], and support assigning the $P_{c}(4457)$ to the hidden-charm pentaquark state with $J^P = \frac{1}{2}^-$ or $\frac{3}{2}^-$. We can search for those hidden-charm pentaquark states in the $J/\psi p$, $\bar{D} \Sigma_c$, $\bar{D} \Sigma^*_c$, $\bar{D}^* \Sigma_c$ and $\bar{D}^* \Sigma^*_c$ invariant mass distributions and confront the present predictions to the experimental data in the future.

4 Conclusion

In this article, we restudy the ground state mass spectrum of the diquark-diquark-antiquark type $uudc\bar{c}$ pentaquark states with the QCD sum rules by taking into account all the vacuum condensates up to dimension 13 in a consistent way in carrying out the operator product expansion, and use the energy scale formula $\mu = \sqrt{M_{P} - (2M_{c})^2}$ with the updated effective $c$-quark mass $M_{c} = 1.82$ GeV to determine the ideal energy scales of the QCD spectral densities, and explore the possible assignments of the $P_{c}(4312)$, $P_{c}(4440)$ and $P_{c}(4457)$ in the scenario of the pentaquark states. The predicted masses support assigning the $P_{c}(4312)$ to be the hidden-charm pentaquark state with $J^P = \frac{1}{2}^-$, assigning the $P_{c}(4440)$ to be the hidden-charm pentaquark state with $J^P = \frac{1}{2}^-, \frac{3}{2}^-$ or $\frac{5}{2}^-$, assigning the $P_{c}(4457)$ to be the hidden-charm pentaquark state with $J^P = \frac{1}{2}^-$ or $\frac{3}{2}^-$. More experimental data and theoretical works are still needed to identify the $P_{c}(4312)$, $P_{c}(4440)$ and $P_{c}(4457)$ unambiguously.
Table 2: The Borel windows, continuum threshold parameters, ideal energy scales, pole contributions, contributions of the vacuum condensates of dimension 13 for the hidden-charm pentaquark states.

| $J^I(x)$ | $T^2 (\text{GeV}^2)$ | $\sqrt{s}_0 (\text{GeV})$ | $\mu (\text{GeV})$ | pole | $D_{13}$ |
|----------|-----------------|-------------------|----------------|-------|---------|
| $J^1(x)$ | 3.1 - 3.5       | 4.96 ± 0.10       | 2.3            | $(41 - 62)\%$ | $< 1\%$ |
| $J^2(x)$ | 3.2 - 3.6       | 5.10 ± 0.10       | 2.6            | $(42 - 63)\%$ | $< 1\%$ |
| $J^3(x)$ | 3.2 - 3.6       | 5.11 ± 0.10       | 2.6            | $(42 - 63)\%$ | $< 1\%$ |
| $J^4(x)$ | 2.9 - 3.3       | 5.00 ± 0.10       | 2.4            | $(40 - 64)\%$ | $< 1\%$ |
| $J^1_\mu(x)$ | 3.1 - 3.5   | 5.03 ± 0.10       | 2.4            | $(42 - 63)\%$ | $< 1\%$ |
| $J^2_\mu(x)$ | 3.3 - 3.7   | 5.11 ± 0.10       | 2.6            | $(40 - 61)\%$ | $< 1\%$ |
| $J^3_\mu(x)$ | 3.4 - 3.8   | 5.26 ± 0.10       | 2.8            | $(42 - 62)\%$ | $< 1\%$ |
| $J^1_\mu(x)$ | 3.3 - 3.7   | 5.17 ± 0.10       | 2.7            | $(41 - 61)\%$ | $< 1\%$ |
| $J^2_\mu(x)$ | 3.2 - 3.6   | 5.03 ± 0.10       | 2.4            | $(40 - 61)\%$ | $< 1\%$ |
| $J^3_\mu(x)$ | 3.1 - 3.5   | 5.03 ± 0.10       | 2.4            | $(42 - 63)\%$ | $< 1\%$ |

Table 3: The masses and pole residues of the hidden-charm pentaquark states.

| $qq'|q''\bar{c}$ ($S_L, S_H, J_{LH}, J$) | $M (\text{GeV})$ | $\lambda (10^{-3}\text{GeV}^6)$ | Assignments | Currents |
|-------------------------------|-----------------|----------------|-------------|----------|
| $[ud][uc]\bar{c}$ (0, 0, 0, $\frac{1}{2}$) | 4.31 ± 0.11 | 1.40 ± 0.23 | $P_c(4312)$ | $J^1(x)$ |
| $[ud][uc]\bar{c}$ (0, 1, 1, $\frac{1}{2}$) | 4.45 ± 0.11 | 3.02 ± 0.48 | $P_c(4440/4457)$ | $J^2(x)$ |
| $[uu][dc]\bar{c} + 2[ud][uc]\bar{c}$ (0, 1, 0, $\frac{1}{2}$) | 4.46 ± 0.11 | 4.32 ± 0.71 | $P_c(4440/4457)$ | $J^3(x)$ |
| $[uu][dc]\bar{c} + 2[ud][uc]\bar{c}$ (1, 1, 0, $\frac{1}{2}$) | 4.34 ± 0.14 | 3.23 ± 0.61 | $P_c(4312)$ | $J^4(x)$ |
| $[ud][uc]\bar{c}$ (0, 1, 1, $\frac{3}{2}$) | 4.39 ± 0.11 | 1.44 ± 0.23 | $P_c(4440)$ | $J^1_\mu(x)$ |
| $[uu][dc]\bar{c} + 2[ud][uc]\bar{c}$ (1, 0, 1, $\frac{3}{2}$) | 4.47 ± 0.11 | 2.41 ± 0.38 | $P_c(4440/4457)$ | $J^2_\mu(x)$ |
| $[uu][dc]\bar{c} + 2[ud][uc]\bar{c}$ (1, 1, 2, $\frac{3}{2}$) | 4.66 ± 0.11 | 5.13 ± 0.79 | $P_c(4440/4457)$ | $J^3_\mu(x)$ |
| $[uu][dc]\bar{c} + 2[ud][uc]\bar{c}$ (1, 1, 2, $\frac{5}{2}$) | 4.52 ± 0.11 | 4.49 ± 0.72 | $P_c(4440/4457)$ | $J^4_\mu(x)$ |
| $[ud][uc]\bar{c}$ (0, 1, 1, $\frac{3}{2}$) | 4.39 ± 0.11 | 1.94 ± 0.31 | $P_c(4440)$ | $J^1_\mu(x)$ |
|                     | 4.39 ± 0.11 | 1.44 ± 0.23 | $P_c(4440)$ | $J^2_\mu(x)$ |
Figure 4: The masses with variations of the Borel parameters $T^2$ for the hidden-charm pentaquark states, the $A$, $B$, $C$, $D$, $E$ and $F$ denote the pentaquark states $[ud][uc]\bar{c} (0, 0, 0, 1/2)$, $[ud][uc]\bar{c} (0, 1, 1, 1/2)$, $[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 1, 1, 3/2)$, $[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (1, 0, 1, 3/2)$ and $[uu][dc]\bar{c} + 2[ud][uc]\bar{c} (0, 1, 1, 3/2)$, respectively.
Figure 5: The masses with variations of the Borel parameters $T^2$ for the hidden-charm pentaquark states, the $G$, $H$, $I$ and $J$ denote the pentaquark states $[uu][dc][c] + 2[ud][uc][c] (1, 1, 2, \frac{3}{2})$, $[uu][dc][c] + 2[ud][uc][c] (1, 1, 2, \frac{5}{2})$ and $[ud][uc][c] (0, 1, 1, \frac{5}{2})$, respectively.
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