In medium properties of $K^0$ and $\phi$ mesons under an external magnetic field

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Abstract

The one loop polarization insertions for the kaon and the $\phi$ mesons are evaluated in a hadronic medium as functions of the field intensity and the baryonic density. For this purpose an effective chiral model of the hadronic interaction is used, supplemented with a phenomenological $K - \phi$ interaction. The propagators of the charged particles include the full effect of the coupling to the magnetic field which induces the quantum Landau levels. Consequently, the vacuum diagrams are defined within a zeta function regularization scheme. The effective masses as well as the decay widths are introduced and examined as functions of matter density and the magnetic intensity.
1 Introduction

The interaction between matter and strong magnetic fields is a subject of permanent research [1,2]. Among the different empirical manifestations studied, it deserves a special mention the detection of very strong radiation coming from the activity of certain astrophysical objects. In fact, the presence of intense magnetic fields has been deduced from the observational data of a class of neutron stars which have been generally included within the magnetar model [3-4]. The sustained X-ray luminosity in the soft (0.5-10 keV) or hard (50-200 keV) spectrum, as well as the bursting activity of these objects have been attributed to the dissipation and decay of very strong magnetic fields. Their intensity has been estimated around $10^{15}$ G at the star surface, but could reach much higher values in the dense interior. The origin of the magnetism, however, is still under debate [5,6].

As neutron stars are mainly composed of hadronic matter, the knowledge of the hadronic properties under a strong magnetic field is desirable to understand the composition and evolution of magnetars. A considerable number of publications have been focused on this subject, emphasizing aspects such as bulk properties of stellar matter [7,8], the quark structure of mesons by using lattice QCD [9], QCD sum rules for heavy mesons [10], non-relativistic potential for heavy mesons [11], an effective hamiltonian with QCD basis [12], the Nambu and Jona-Lasinio model [13,14], effective hadronic models for pions [15-19], and vector mesons [20,21,22,23].

The possible existence of kaon condensates in neutron stars has largely been debated [24-27]. Kaons have a significative role in the strong interaction because they are the lightest particles with open strangeness and they have a similar role as the pions have in the manifestation of the chiral symmetry. The $\phi$ meson, instead, has closed strangeness and its coupling to protons and neutrons would be excluded by the OZI rule. Therefore, the main channel of interaction with the hadronic medium in the low energy regime goes through the kaons. This is the reason why the $\phi$ meson would be one of the preferred probes of the formation of a quark gluon plasma phase in heavy ion collisions. Since it interacts scarcely with other hadrons, it is expected that the $\phi$ meson does not suffer considerable rescattering in the late hadronization stage, decoupling earlier from the surrounding hadronic medium [28]. In recent years the dynamics of the $\phi$ meson has been theoretically investigated in relation to the strange content of the nucleons [29], alternative couplings [30], effective mass [31], and exotic nuclei [32].

Unfortunately, the effects of magnetic fields on the properties of $K$ and $\phi$ have not received the attention they deserve. To my knowledge, only a few works deal with kaon condensates under magnetic fields, as the $K$-meson coupling model of [33] and the quark-meson coupling model of [34].

It must be pointed out that a Bose-Einstein condensate and a magnetic field could not locally coexist, however the lowest Landau level behaves as a ground state accommodating large particle densities [35]. In this sense, the search for boson condensates in magnetars is relevant [8].
The aim of this work is to investigate the effect of a uniform external field on the properties of the $K$ and $\phi$ mesons, immersed in a hadronic medium under conditions relevant for the study of neutron stars. In our approach the magnetic field is treated as a classical external field, therefore we neglect electromagnetic quantum corrections. We examine a range of matter densities where only protons and neutrons are the relevant baryons.

For this purpose we choose a model of the hadron interaction based on a non-linear realization of the SU(3) chiral symmetry [36, 37, 38, 39], where pions and kaons are regarded as Goldstone bosons. It also includes the lightest scalar and vector mesons, whose masses are generated dynamically, and a dilaton field to take account of the QCD trace anomaly. The model has been extended to treat open charm and bottom mesons [38, 39]. In addition, we include here a derivative coupling between kaons and $\phi$ meson [40].

The polarization insertion for the mesons is evaluated at the one loop level. The propagators for the charged particles includes the Landau quantum levels and, in the case of the baryons, also the anomalous magnetic moments. The kaon field couples directly to the baryons, hence the kaon selfenergy depends on the baryonic density. In contrast for a low density equilibrium state and assuming the OZI rules, the $\phi$ meson interacts only with the kaons and the corrections to its propagator would not depend on the matter density. This situation would cease as hyperons or a kaon condensate become stable.

The effective mass of the mesons in the hadronic medium is defined and analyzed at zero temperature for a wide range of densities $0 < n_B < 3n_0$, with $n_0$ the saturation density of nuclear matter, and magnetic intensities $10^{15} \text{ G} \leq B \leq 10^{19} \text{ G}$. The equilibrium conditions correspond to neutron star matter, i.e. electric neutrality and equilibrium against beta decay. Hence additional lepton fields are also included.

This work is organized as follows. In the next section the hadronic model is presented and the corresponding one-loop polarization insertions are evaluated. The results and discussion are given in Sec. III, and the conclusions are shown in Sec. IV.

## 2 In-medium meson polarization insertion

The effective model for the hadronic interaction described in [36] contains massless fields, which obtain their masses through the interaction with the scalar boson fields $\sigma$, $\zeta$ and the iso-triplet $\delta$. The second one has closed strange content. There is also a polynomial symmetry breaking term, and a scalar dilaton field.

In the mean field approximation (MFA) each of these scalar fields is decomposed as the sum of a vacuum expectation value plus a fluctuation. The only exception is the dilaton, for which zero fluctuation is assumed. These mean values are partially absorbed in the definition of the masses and in part they become parameters of the residual interaction.

The kaon field, instead, appears as a Goldstone boson. Within the same order of the chiral expansion it obtains non zero mass and a derivative meson-
baryon coupling given by

\[
\mathcal{L} = \frac{1}{8f_K} \left( -3iJ^\mu K \bar{\partial}_\mu K - J_\tau^\mu \bar{\partial}_\mu K + 4d_1 J \bar{\partial}^\mu K \partial_\mu K + 4d_2 J_\rho \bar{\partial}^\mu K^+ \partial_\mu K^- \right.
\]

\[
+ \frac{m_K^2}{2f_K} \left[ (\sigma + \sqrt{2}\zeta) \bar{K}K + \bar{K} \delta \cdot \tau K \right]
\]

\[+ \frac{1}{f_K} \left[ (\sigma + \sqrt{2}\zeta) \bar{\partial}^\mu K \partial_\mu K + \bar{\partial}^\mu \bar{K} \delta \cdot \tau \partial_\mu K \right] \tag{1}
\]

where the symbol \( K \) without an index stands for the duplet \( (K^+, K^0) \), and \( J^\mu = \bar{\Psi} \gamma^\mu \Psi \), \( J_\tau^\mu = \bar{\Psi} \gamma^\mu \tau_\alpha \Psi \), the bi-spinor \( \Psi = (\psi_p, \psi_n) \) contains proton and neutron fields. Furthermore \( J_p = \bar{\psi}_p \psi_p, \, J_n = \bar{\psi}_n \psi_n, \, J = J_p + J_n. \) It must be stressed that \( \sigma, \delta \) and \( \zeta \) are used here for the fluctuations of the scalar fields.

We complete Eq. (1) with an interaction between the vector meson \( \phi^\mu \) and the kaons \([40]\)

\[
\mathcal{L} = \frac{g}{\sqrt{2}} \phi^\mu \bar{K} \partial_\mu K \tag{2}
\]

At the one loop order the kaon propagator is modified by the diagrams shown in Fig. 1. The diagram (a) is first order and corresponds to the baryonic tadpoles. After regularization it gives zero vacuum contribution, and a finite density dependent part. The diagram (b) corresponds to kaon tadpoles mediated by the scalar mesons. Again its vacuum part is zero when it is properly regularized, and it will contribute only in the case of kaon condensation. Finally, diagram (c) corresponds to the one-meson exchange. In the present approach only contributions coming from mesons with masses below 1 GeV are considered and we focus on situations were kaon condensation is not probable. Under these conditions the one loop kaon polarization can be decomposed as a
sum of contributions coming from the different diagrams,

\[
\Pi_{K\alpha}(p) = \Pi_{\alpha}^{(a)}(p) + \Pi_{\alpha}^{(b)}(p) + \Pi_{\alpha}^{(c)}(p)
\]

\[
\Pi_{\alpha}^{(a)}(p) = \frac{i}{2f_K} \int \frac{d^4q}{(2\pi)^4} \left\{ d_2 p^2 \left[ \delta_{\alpha\alpha} \text{Tr} G^{(a)}(q) + \delta_{\alpha\beta} \text{Tr} G^{(n)}(q) \right] + \sum_{j=n,p} \left[ \frac{1}{2} (3 + I_\alpha I_j) p_\mu \text{Tr} \gamma^\mu G^{(j)}(q) + d_1 p^2 \text{Tr} G^{(j)}(q) \right] \right\}
\]

\[
\Pi_{\alpha}^{(b)}(p) = \frac{i}{4f_K} (2p^2 - m_K^2) \left[ \Delta_\alpha(0) + I_\alpha I_\alpha \Delta^{(3)}(0) \right] \times \sum_{\alpha'} \int \frac{d^4q}{(2\pi)^4} (2q^2 - m_K^2) \Delta^{(\alpha')}(q)
\]

\[
\Pi_{\alpha}^{(c)}(p) = \frac{i}{f_K} \int \frac{d^4q}{(2\pi)^4} \left( p^\mu q_\mu - \frac{m_K^2}{2} \right)^2 \left\{ \left[ \Delta^{(1)}_\alpha(p-q) + \Delta^{(2)}_\alpha(p-q) \right] \Delta^{(\alpha')}_K(q) + \left[ \Delta_\alpha(p-q) + \Delta^{(3)}_\alpha(p-q) \right] \Delta^{(\alpha')}_K(q) \right\}
\]

where \( \alpha = 1(2) \) corresponds to the charged (neutral) kaon, \( \tilde{\alpha} = 3 - \alpha, I_k = (-1)^{1+k}, p^2 = p_\mu p^\mu, G \) stands for the nucleon propagators and \( \Delta \) for the scalar mesons propagators. If needed, a superscript indicates the isospin component.

At the same order of approximation, the interaction of Eq. (2) induces the diagram of Fig. 1d for the \( \phi \) meson propagator. The corresponding polarization insertion is given by

\[
\Pi_{\phi^{\nu\nu}}(p) = -\frac{i}{2g^2} \sum_{\alpha} \int \frac{d^4q}{(2\pi)^4} \Delta^{(a)}_K(q) \Delta^{(a)}_\phi(q-p) (2q-p)^\nu (2q-p)^\nu
\]

Since particles are immersed in a uniform magnetic field, we include magnetic effects at the level of the propagators. Therefore, for the charged mesons we use

\[
\Delta(p) = 2\epsilon^{\phi} \sum_n (-1)^n e^{-\epsilon^2/qB} L_n(2p^2/qB) \left[ \frac{1}{p^2_n - \omega_n^2 + i\epsilon} + 2\pi i \delta(p^2_0 - \omega_n^2) n_B(p_0) \right]
\]

where the phase factor \( \Phi = qB(x + x')(y' - y)/2 \) embodies the gauge fixing, \( \omega_n = \sqrt{m^2 + p_0^2 + (2n + 1)qB} \), and \( n_B \) is the statistical occupation function for bosons.

For the nucleon propagators we have to deal with spin, hence an index \( s = \pm 1 \) is introduced to indicate spin projections along the direction of the uniform magnetic field, and we consider the effects of the anomalous magnetic moments \( \kappa \). For the neutron we have

\[
G^{(n)}(p) = \sum_s \Lambda_s \left[ \frac{1}{p_0^2 - E_s^2 + i\epsilon} + 2\pi i n_F(p_0) \delta(p_0^2 - E_s^2) \right]
\]
where \( E_s = \sqrt{p_x^2 + (\Delta - s \kappa_n B)^2}, \Delta = \sqrt{m_n^2 + p_z^2 + p_y^2}, \) and

\[
\Lambda_s = \frac{s}{2i} \gamma^1 \gamma^2 [\gamma^0 + i \gamma^1 \gamma^2 (s \Delta - \kappa_n B)] (\gamma^0 + m_n + is \Delta \gamma^1 \gamma^2)
\]

with \( u_\mu = (p_0, 0, p_x, p_y) \), and \( v_\mu = (0, p_x, p_y, 0) \).

The proton propagator is given by

\[
G^{(p)}(p) = e^{i\Phi} e^{-p_{F}^2 / m_B} \sum_{l, s} \Lambda_{ls} \left[ \frac{1}{p_{l}^2 - E_{ls}^2 + i\epsilon} + 2\pi i n_F (p_0) \delta(p_{l}^2 - E_{ls}^2) \right]
\]

where \( E_{ls} = \sqrt{p_x^2 + (\Delta_l - s \kappa_p B)^2}, \Delta_l = \sqrt{m_p^2 + 2lqB}, \Lambda_{ls} = (1+s) (\gamma^0 + m_p - \kappa_p B) \Pi^{(l)}, \)

and

\[
\Lambda_{ls} = \frac{1}{\Delta_l + sm_p} \left\{ (\gamma^0 - \kappa_p B + s\Delta_l) \Pi^{(l)} (2p_{l}^2 / qB) \right. \\
- (\gamma^0 + \kappa_p B - s\Delta_l) \Pi^{(l)} (-s\Delta_l - m_p) L_{l-1} (2p_{l}^2 / qB) \\
+ \left[ \gamma^0 + i\gamma_1 \gamma_2 (s \Delta_l - \kappa_p B) \right] i \gamma^0 \gamma^2 l \right\} \frac{s\Delta_l - m_p}{2p_{l}^2} \frac{1}{L_l (2p_{l}^2 / qB) - L_{l-1} (2p_{l}^2 / qB)} \}
\]

with \( \Pi^{(l)} = (1 \pm i \gamma^1 \gamma^2) / 2 \) and \( L_l \) stands for the Laguerre polynomial of order \( l \).

The propagators of the nucleons are constructed in a quasi particle scheme, where the mass \( m_k = m_0 - g_\sigma S - g_\delta I_k D \) and the energy spectrum \( E_k + g_\omega W + g_p R I_k \) are modified by the in-medium meson mean field values \( S = g_\sigma (N_{sp} + N_{sn}) / m_\pi^2, \ D = g_\delta (N_{sp} - N_{sn}) / m_\pi^2, \ W = g_\omega (N_p + N_n) / m_\omega^2, \ R = g_p (N_p - N_n) / m_p^2, \) and

\[
N_p = \frac{gB}{2\pi^2} \sum_{l, s} \int dp_z \left[ n_F (E_{ls}, \bar{\mu}_p) - n_F (-E_{ls}, \bar{\mu}_p) \right]
\]

\[
N_n = \sum_{s} \int \frac{dp}{(2\pi)^3} \left[ n_F (E_s, \bar{\mu}_n) - n_F (-E_s, \bar{\mu}_n) \right]
\]

\[
N_{sp} = \frac{gB}{2\pi^2} m \sum_{l, s} \int dp_z \frac{\Delta_l + s \kappa_p B}{E_{ls} \Delta_l} \left[ n_F (E_{ls}, \bar{\mu}_p) + n_F (-E_{ls}, \bar{\mu}_p) \right]
\]

\[
N_{sn} = \sum_{s} \int \frac{dp}{(2\pi)^3} \frac{\Delta + s \kappa_n B}{E_s \Delta} \left[ n_F (E_s, \bar{\mu}_n) + n_F (-E_s, \bar{\mu}_n) \right]
\]

The two first equations relate the conserved baryon number with the chemical potentials \( \mu_k \) by means of the effective chemical potentials \( \bar{\mu}_k = \mu_k - g_\omega W - g_p I_k R \).

The propagators just shown were derived within a thermal field theory [19, 43], more specifically within the real time formalism of Thermo-Field Dynamics. Here only the (1,1) component is exhibited because it suffices for the
present calculations at zero temperature. These results combine the gauge invariance of the proper time method \[41\] with the momentum representation of \[42\], furthermore they include the contributions of the anomalous magnetic moments.

The vacuum contribution for each of the polarizations in Eqs. (3)-(4) can be written as \( F_s(p^2) \), while for Eq. (6) it can be decomposed as \( g^{\mu\nu} F_0(p^2) + p^\mu p^\nu F(p^2) \). In general these functions are divergent, in order to extract meaningful contributions we impose the regularization conditions

\[
F_{s} (s = m^2) = 0, \quad \text{and higher order } \partial^n F / \partial s^n (s = m^2) = 0 \text{ if needed.}
\]

Here \( s = p^2 \) and \( m \) is the kaon or \( \phi \) meson mass as appropriate.

To handle the divergences we use dimensional regularization for uncharged particles, and zeta function regularization \[44\] for charged ones.

After regularization, we obtain for the kaon polarizations

\[
\Pi_1^{(s)} (p) = - \frac{1}{2f_K^2} \left\{ p_0 (2N_p + N_n) + p^2 [(d_1 + d_2) N_{sp} + d_1 N_{sn}] \right\}
\]

\[
\Pi_2^{(s)} (p) = - \frac{1}{2f_K^2} \left\{ p_0 (N_p + 2N_n) + p^2 [d_1 N_{sp} + (d_1 + d_2) N_{sn}] \right\}
\]

As mentioned before, we neglect the presence of a kaon condensate, hence we obtain \( \Pi_3^{(s)} = 0 \). The long expressions for the regularized \( \Pi_2^{(c)} \) are shown in the Appendix A.

The polarization insertion for the \( \phi \) meson can be written as a sum of contributions coming from the neutral and charged kaons

\[
\Pi_{\phi}^{\mu\nu} (p) = \sum_{\alpha} \left( A^{(\alpha)} \frac{p^\mu p^\nu}{p^2} + B^{(\alpha)} g^{\mu\nu} \right)
\]

(8)

Details of the functions \( A, B \) and their decomposition into longitudinal and transversal blocks are shown in the Appendix B.

A Dyson-Schwinger approach is used to define the effective masses of kaons and \( \phi \) mesons as poles of the corresponding corrected propagators. Hence the effective kaon mass \( m_K^* \) corresponds to the solutions of the equation

\[
p_0^2 - m_K^2 - \text{Re} \, \Pi_K (p_0, p = 0) = 0 \quad (9)
\]

in the unknown \( p_0 \). For the decay width we use \( \Gamma_K = \text{Im} \, \Pi_K (p^2 = m_K^* N_K) \).

The definition of the effective mass for the \( \phi \) meson requires some care. Using the inverse propagator of a free vector meson \( D_0^{-1} = - (p^2 - m_\phi^2) g^{\mu\nu} + p^\mu p^\nu \), the Dyson Schwinger equation becomes

\[
D_\phi^{-1} = - \left( p^2 - m_\phi^2 \right) g^{\mu\nu} + p^\mu p^\nu - \left( A \frac{p^\mu p^\nu}{p^2} + B g^{\mu\nu} \right)
\]

(10)

where we have assumed a structure similar to that described in Eq. (6) for the polarization insertion. In consequence we can write

\[
D_\phi^{\mu\nu} (p) = \frac{1}{p^2 - m_\phi^2 + B} \left( - g^{\mu\nu} + \frac{p^\mu - A}{m_\phi^2 - A - B} \frac{p^\mu p^\nu}{p^2} \right)
\]

(11)

7
Thus we adopt the lowest solution of the equation

\[ p_0^2 - m_{\phi}^2 + \text{Re} B(p_0, p = 0) = 0 \]

(12)
as the definition of the effective mass \( m_{\phi}^* \). Furthermore, the decay width is taken as \( \Gamma_{\phi} = \text{Im} B(p^2 = m_{\phi}^2) / m_{\phi} \).

It can be argued in favor of this definition that in the proximity of the solution \( p_0^2 \approx m_{\phi}^2 \) the propagator of Eq. (11) behaves as

\[ D_{\phi}^{\mu\nu}(p) \approx \frac{1}{p^2 - m_{\phi}^2} \left( -g^{\mu\nu} + \frac{p^{\mu} p^{\nu}}{m_{\phi}^2} \right) \]
namely a quasiparticle picture emerges, where the mass is dressed by the kaon interaction.

As explained in the Appendix B, the functions \( A, B \) are different for the longitudinal and transversal sectors. In the following we focus on the longitudinal meson branch.

### 3 Results and discussion

In this section we analyze the effective meson masses for a hadronic environment which can be found in situations of astrophysical interest. We consider electrically neutral matter in equilibrium against weak decay under an external magnetic field. We examine a range of matter densities below \( 7.5 \times 10^{14} \, \text{g/cm}^3 \) at zero temperature, hence protons and neutrons are the main components of the conserved baryonic number. Furthermore we consider magnetic intensities \( 10^{15} \leq B \leq 10^{19} \, \text{G} \), which cover the empirical data attributed to magnetars and approach to the QCD scale.

For the model parameters we use \( f_K = 122 \, \text{MeV}, \, m_{\phi} = 1020 \, \text{MeV}, \) for the \( \phi - K \) coupling we use \( g = 6.55 \) an average of the values adjusted in [40] to reproduce the decay widths \( \phi \rightarrow K^0\bar{K}^0 \) and \( \phi \rightarrow K^+K^- \). We neglect isotopic differences in the kaon mass adopting \( m_K = 495.5 \, \text{MeV} \), in addition to \( d_1 = 2.56/m_K, \, d_2 = 0.73/m_K \) [27] and a set of parameters [38] which predicts vacuum values for the nucleon and meson masses \( m_\sigma = 466.5 \, \text{MeV}, \, m_\delta = 899.4 \, \text{MeV}, \, m_\zeta = 1024.5 \, \text{MeV}, \, m_\omega = 782.5 \, \text{MeV}, \, m_\rho = 763 \, \text{MeV}, \) and the hadronic couplings \( g_\sigma = 10.567, \, g_\delta = 2.487, \, g_\zeta = -0.461, \, g_\omega = 13.326, \, g_\rho = 5.488 \).

This parametrization guarantees the KN scattering lengths and the binding properties of nuclear matter in the MFA, the saturation density \( n_0 = 0.15 \, \text{fm}^{-3} \), binding energy \( E_B = -15.3 \, \text{MeV} \), symmetry energy \( E_s = 31.6 \, \text{MeV} \), and slope parameter \( L = 65.9 \, \text{MeV/fm}^3 \).

As a first step we evaluate the MFA at zero temperature, in which case the Fermi occupation number becomes a step function. As a consequence, the Landau levels of the proton are occupied until a well defined maximum value. At the end of this calculation we obtain the chemical potentials, the effective nucleon mass, and the spin polarization of matter as functions of the magnetic intensity
and the baryonic density. The results of the MFA are inserted in the neutron and proton propagators, to evaluate the $\phi$ and $K^0$ polarization insertions.

Next we examine the effective mass of the neutral kaon as a function of the baryonic density. In Fig. 2 the results corresponding to $B = 10^{18}$ G are presented, there we compare the complete and the MFA treatments. The last one is obtained by including only the term $\Pi^{(\phi)}$, which is equivalent to the $B = 0$ calculations carried out for instance in Ref. [37]. Both results are very similar, exhibiting a monotonous increasing behavior with density. Only for densities $n/n_0 > 2$ the differences become appreciable. As explained in the Appendix A, in addition to the mean field contribution our results include the vacuum contributions coming from the coupling to two neutral mesons ($\Pi^{(\phi)}_{nn}$) and to two charged mesons ($\Pi^{(\phi)}_{cc}$). In the first case, there is no direct dependence on the magnetic intensity $B$, so we can distinguish the relative importance of this two terms by examining how the effective mass depends on $B$. That is the subject of Fig. 3 where the dependence of the kaon mass on the magnetic intensity at fixed baryonic densities $n = 0.075, 0.15$ and $0.3$ fm$^{-3}$ is shown in a logarithmic scale. A decreasing behavior is found, which is moderate at low and medium intensities but becomes more pronounced around $B \sim 10^{18}$ G. For all the cases is $m_K^*/m_K > 1$ even for the extreme value $B = 10^{19}$ G. For a given density the variation do not exceed a 3% between the extremes of the scale. Hence we conclude that the variation of the mass of the neutral kaon in a neutron star environment is dominated by density effects, while magnetic effects are moderate and become more appreciable at higher densities.

The behavior found for the kaon mass contrast with the results of Ref. [34], where an schematic model of the quark confinement is used. In that case a monotonous decrease with the baryonic density is obtained, reaching a 20% reduction at $n = 0.45$ fm$^{-3}$ for neutron star matter. This remarkable difference can be attributed, according to the previous discussion, to the direct coupling between kaons and baryons.

The decay width of the $K^0$ within this model is strictly zero because we have not included $K - \pi$ couplings. We have verified that at this order of approximation, mesons heavier than pions do not open alternative channels of decay.

Finally we consider the decay width $\Gamma_\phi$ of the $\phi$ meson. In the absence of a kaon condensate, $\Gamma_\phi$ does not depend on the matter density, hence the neutral mesons vertex contributes with a constant value of 1.92 MeV. The charged mesons vertex, instead, has a significative dependence of the applied magnetic field as can be seen in Fig. 4. The increase of the intensity causes a blocking of this channel with a threshold value $B_t$, given by $eB_t = m_\phi^2/4 - m_K^2$, that is $B_t \simeq 2.45 \times 10^{18}$ G. For $B < 4 \times 10^{17}$ G, the decay width exhibits rapid oscillations around 1.92 MeV, i.e. the same value as the neutral contribution. For stronger fields the periodicity as well as the amplitude of the oscillations increase quickly with $B$ until the threshold is reached. For intensities slightly below $B_t$ the decay width increases as far as 15% with respect to the $B = 0$
case. The oscillatory behavior is a typical effect of the occupancy of the Landau levels by the virtual mesons.

4 Conclusions

In this work an analysis of the effective mass and decay width of the lightest neutral mesons with strange content in a hadronic environment and in the presence of an external magnetic field has been carried out. As the results could depend on the composition of the hadronic medium, we have taken a situation of astrophysical interest, hadronic matter electrically neutral, in equilibrium against weak decay at zero temperature. We have focused on a range of magnetic fields $10^{15} \, \text{G} \leq B \leq 10^{19} \, \text{G}$, and baryonic densities which can be found in certain magnetars. However, the matter density is not so high to have a significative population of hyperons.

The calculations have been made within a covariant hadronic model based on chiral considerations, where kaons are regarded as Goldstone bosons coupled to other bosons, including a dilaton field. Their vacuum expectation values provide mass to the interacting fields and the fluctuations are included to account for quantum corrections. For this purpose we have used propagators which include the full effect of the external magnetic field, and the anomalous magnetic moments in the case of the baryons. Furthermore a phenomenological interaction between kaons and the $\phi$ meson is considered.

We have included one-loop corrections to the meson propagators. To extract meaningful results we have performed a regularization procedure of the polarization of the mesons subject to an external magnetic field. The finite one-loop approximation to the propagators is used to discriminate different contributions to the effective masses and decay widths in a Dyson-Schwinger approach.

The in-medium variation of the effective mass of the $K^0$ meson is dominated by density effects, while the dependence on the magnetic intensity is moderate. We have found a monotonous increase with the baryonic density, reaching a 13% enhancement at three times the normal nuclear density. The decay width of the neutral kaon involving scalar $\sigma$ and $\delta$ mesons is zero.

Within the present approach the in-medium properties of the $\phi$ meson do not depend on the matter density in the absence of a kaon condensate. Its decay width receives a constant contributions from the $\bar{K}^0 K^0$ channel of approximately 1.9 MeV, while the pair of charged kaons provides a characteristic behavior. It varies rapidly around the same value 1.9 MeV for relatively small intensities, and drops to zero at the threshold strength $B_t \simeq 2 \times 10^{18} \, \text{G}$. The decay channel for the $K^- K^+$ pair is blocked for $B > B_t$ due to the impossibility to accommodate a pair of virtual kaons within the discrete Landau levels. This fact is relevant for the analysis of non central heavy ion collisions, as stronger magnetic fields than $B_t$ have been predicted [46].

Further developments for this line of investigation will include the study of the in-medium properties of the charged kaons.
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6 Appendix A: $K^0$ polarization for the one-meson exchange

The $K^0$ polarization contains vertices for either two neutral mesons or two charged mesons $\Pi_{\alpha}^{(c)} = \Pi_{nc}^{(c)} + \Pi_{cc}^{(c)}$. In the first case the extraction of physical outcome can be carried out by dimensional regularization. Using standard procedures we obtain

$$-32\pi^2 f_K^2 \Pi_{nn}^{(c)} = \left( \frac{1}{\epsilon} - \frac{\gamma}{2} \right) \left( p^4 - m_K^2 p^2 + m_\chi^2 p^2 + m_\chi^2 \right) + \frac{p^2}{6} (3m_K^2 + 3m_\chi^2 - p^2)$$

$$- \int_0^1 dz \left[ z(3z-1)p^4 + zm_\chi^2 p^2 + (1-3z)m_K^2 p^2 + \frac{1}{2} m_K^2 \right]$$

$$\times \ln \left[ -z(1-z)p^2 + zm_\chi^2 + (1-z)m_K^2 \right] + \mathcal{O}(\epsilon)$$

The structure of the divergence as $\epsilon \to 0$ suggests a second order subtraction procedure, from which we obtain the regularized result

$$\text{Im} \Pi_{nn}^{(c)} = \sqrt{(p^2 + m_K^2 - m_\chi^2)^2 - 4m_K^2 p^2} \frac{(p^2 - m_\chi^2)^2}{64\pi^2 f_K^2} \Theta (p^2 - (m_K + m_\chi)^2)$$

$$64\pi^2 f_K^2 \text{Re} \Pi_{nn}^{(c)} = m_K^2 (p^2 - m_\chi^2)^2 \int_0^1 dz \left[ \frac{z(z-1)}{m_\chi^2 z + m_K^2 (1-z)^2} \right]$$

$$+ 2 \int_0^1 dz \left[ (z(3z-1)p^4 + zm_\chi^2 p^2 + (1-3z)m_K^2 p^2 + \frac{1}{2} m_K^2) \ln \left\{ \frac{zm_K^2 + (3z - 1)m_\chi^2}{m_\chi^2 z + m_K^2 (1-z)^2} \right\} \right]$$

$$-(p^2 - m_\chi^2) \int_0^1 dz \left[ \frac{m_K^2 (1 + 2z - 6z^2) + 2p^2 [(1 - 5z + 6z^2)m_K^2 + zm_\chi^2]}{m_\chi^2 z + m_K^2 (1 - z^2)} \right]$$

a sum over $\chi = \sigma, \delta$ is assumed, as stated in the second term between curly brackets in Eq. (13).

For $\Pi_{cc}^{(c)}$ we start with the first term between curly brackets in Eq. (5), it can be rewritten as

$$\frac{4i}{f_K^2} \sum_{n,l} (\frac{1}{2})^{n+l} \int d^4 q \left( p\mu q^n - \frac{m_K^2}{2} \right)^2 e^{-q_\perp^2 / \beta} L(q, (2q_\perp^2 / \beta)) e^{-(k_\perp^2 / \beta) L(2k_\perp^2 / \beta)} \int_0^\infty dt \int_0^1 dt' e^{\lambda t}$$

where $k = q - p$, $\Lambda = u_q^2 - 2z u_q + z u_p^2 - zm_\chi^2 - \beta - 2 \beta z n - (1 - z)(2 \beta l + m_K^2)$.

By changing the longitudinal variables of integration to $\bar{u} = u_q - z u_p$ followed by
a Wick rotation, we obtain from the exponential in Eq. (14) a gaussian factor. Furthermore, the discrete index \( n \) can be summed up by using the identity (see Eq.(8.975.1) of Ref. [45])

\[
\sum_{n=0}^{\infty} L_n(x) r^n = \frac{\exp[x r/(r-1)]}{1-r}
\]

Thus Eq.(14) can be rewritten as

\[
-\frac{1}{f_K(2\pi)^2} \int d^2 \vec{u}_E \int d^2 q_\perp \int_0^1 dz \int_0^\infty dt \ t \left[ (\vec{u}_\mu u_\mu^\nu)^2 + (zu_p^2 + v_q u_v^\nu - \frac{1}{2} m_K^2)^2 \right] e^{-t \vec{u}_E^2} \times \exp \left\{ t \left[ z(1-z)u_p^2 - zm_\perp^2 - \beta - (1-z)m_K^2 \right] - \frac{q_\perp^2}{\beta} \tanh(\beta z t) - \frac{k_\perp^2}{\beta} \tanh[\beta(1-z) t] \right\} \times [1 + \tanh(\beta z t)] [1 + \tanh(\beta(1-z) t)]
\]

Since linear terms in \( \vec{u} \) within the square bracket of the equation above will give zero contribution by symmetric integration, they have been omitted. A change on the perpendicular variables \( \vec{v} = v_q - \alpha v_p \) with \( \alpha^{-1} = 1 + \tanh(\beta z t) \coth[\beta(1-z) t] \) leads to a product of a gaussian times a polynomial in the new variables. Again linear terms will give zero contribution after integration. Hence we obtain

\[
-\frac{1}{f_K(2\pi)^2} \int d^2 \vec{u}_E \int d^2 q_\perp \int_0^1 dz \int_0^\infty dt \ t \left[ (\vec{u}_\mu u_\mu^\nu)^2 + (zu_p^2 + \alpha v_p^2 - \frac{1}{2} m_K^2)^2 + (\vec{u}_v u_v^\nu)^2 \right] \times \exp \left\{ t \left[ z(1-z)u_p^2 - zm_\perp^2 - \beta - (1-z)m_K^2 \right] + \alpha v_p^2 \tanh(\beta z t)/\beta \right\} e^{\delta^2 \tanh[\beta(1-z) t]/\alpha \beta \ t \vec{u}_E^2} \times [1 + \tanh(\beta z t)] [1 + \tanh(\beta(1-z) t)]
\]

By performing the integrations in \( \vec{u}_E, \vec{v} \) the following expression is obtained

\[
\frac{\beta}{32\pi^2 f_K^2} \int_0^1 dz \int_0^\infty dt \ \frac{\alpha v_p^2}{\tanh[\beta(1-z) t]} \left[ \frac{u_p^2}{t} + \frac{\alpha \beta v_p^2}{\tanh[\beta(1-z) t]} \right]^{2} - 2 \left( zu_p^2 + \alpha v_p^2 - \frac{1}{2} m_K^2 \right)^2 \times \exp \left\{ t \left[ z(1-z)u_p^2 - zm_\perp^2 - \beta - (1-z)m_K^2 \right] + \alpha v_p^2 \tanh(\beta z t)/\beta \right\} \times [1 + \tanh(\beta z t)] [1 + \tanh(\beta(1-z) t)]
\]

The integrand in this equation has a singularity at \( t = 0 \), to isolate the singularity we use the same procedure as in the zeta function regularization scheme [44]. We introduce a regularization factor \( t^s \), with \( 0 < s < 1 \) and anticipating \( v_p^2 = 0 \), a closed form in terms of the Hurwitz zeta function \( \zeta(z, q) \) can be obtained for the first integration

\[
(2\pi f_K)^2 \Pi^{(e)}_{cc} = -\frac{1}{4} \int_0^1 dz \left[ (zu_p^2 - \frac{1}{2} m_K^2) \right]^2 \Gamma(1 + s) \zeta(1 + s, \xi) - \beta u_p^2 \Gamma(s) \zeta(s, \xi)
\]

where

\[
\xi = \frac{zm_\perp^2 + (1-z)m_K^2 + \beta - z(1-z)u_p^2}{2\beta}
\]
From this expression a single pole can be isolated as \( s \to 0 \)

\[
-(4\pi f_K)^2 \Pi_{cc}^{(c)} = \int_0^1 dz \left\{ \begin{array}{l}
\left[ \beta u_p^2 B_1(\xi) + \left( z u_p^2 - \frac{1}{2} m_K^2 \right)^2 \left( \frac{1}{s} - \gamma \right) - \left( z u_p^2 - \frac{1}{2} m_K^2 \right)^2 \right] \psi(\xi) \\
- \beta u_p^2 \ln \left[ \frac{\Gamma(\xi)}{\sqrt{2\pi}} \right] + O(s) \end{array} \right\}
\]

where \( \psi, B_1 \) and \( \gamma \) are the di-gamma function, the Bernoulli function and the Euler-Mascheroni constant respectively. It must be pointed out that the coefficient of the pole is independent of \( B \).

The function \( \psi(\xi) \) is well defined for \( \xi > 0 \), in fact this is the case for the present calculations of the kaon effective mass due to the expected range of \( u_p^2 \).

Finally the following regularized version is obtained by making a subtraction at \( u_p^2 = m_K^2 \) and taking \( s \to 0 \)

\[
\Pi_{cc}^{(c)} = \frac{1}{(4\pi f_K)^2} \int_0^1 dz \left\{ \begin{array}{l}
\beta u_p^2 \ln \left[ \frac{\Gamma(\xi)}{\Gamma(\xi_0)} \right] + \left( z u_p^2 - \frac{1}{2} m_K^2 \right)^2 \left[ \psi(\xi) - \psi(\xi_0) \right] \\
- z(1-z)(1-2z) \left[ 4z u_p^2 - (2z+1)m_K^2 \right] (u_p^2 - m_K^2) m_K^2 \left[ 4\beta \psi'(\xi_0) + m_K^2 (2z-1)^2 \psi''(\xi_0) \right] \\
- \frac{z^2}{32\beta^2} \left( u_p^2 - m_K^2 \right)^2 m_K^2 \left[ 4\beta \psi'(\xi_0) + m_K^2 (2z-1)^2 \psi''(\xi_0) \right] \\
+ \frac{1}{2} z(1-z) u_p^2 (u_p^2 - m_K^2) \psi(\xi_0) \end{array} \right\}
\]

where \( \xi_0 = \left[ zm_3^2 + (1-z)^2 m_K^2 + \beta \right] / 2\beta \).

### 7 Appendix B: One loop phi meson polarization

At the one loop level, the phi meson polarization includes a two kaon vertex which can be either both neutral or both charged, so we can write \( \Pi = \Pi_{nn} + \Pi_{cc} \).

The first term can be decomposed as

\[
\Pi_{nn}^{\mu\nu} = \frac{g^2}{32\pi^2} \left[ p^\mu p^\nu A(p^2) + g^{\mu\nu} B(p^2) \right] \tag{16}
\]

the functions \( A, B \) have single poles which can be isolated by standard procedures of dimensional regularization

\[
A = \frac{1}{3} \left( \frac{2}{\epsilon} - \gamma \right) - \int_0^1 dz \left( 4z^2 - 4z + 1 \right) \ln \left[ -z(1-z) p^2 + m_K^2 \right] + O(\epsilon)
\]

\[
B = \frac{1}{3} \left( \frac{2}{\epsilon} + 1 - \gamma \right) (p^2 - 6m_K^2) + 2 \int_0^1 dz \left[ z(1-z) p^2 - m_K^2 \right] \ln \left[ -z(1-z) p^2 + m_K^2 \right] + O(\epsilon)
\]
and \( \nu \)

In contrast, I give. For instance, in the longitudinal sector we have

\[A_{\text{reg}} = -\frac{i\pi}{3} \Theta \left(p^2 - 4m_K^2\right) \left(1 - \frac{4m_K^2}{p^2}\right)^{3/2} - \int_0^1 dz \left(2z - 1\right)^2 \ln \frac{z(1 - z)p^2 - m_K^2}{z(1 - z)m_\phi^2 - m_K^2}

\]

\[B_{\text{reg}} = \frac{i\pi}{3} \Theta \left(p^2 - 4m_K^2\right) p^2 \left(1 - \frac{4m_K^2}{p^2}\right)^{3/2} - \frac{1}{3} \left(p^2 - m_\phi^2\right)

+ 2 \int_0^1 dz \left[\left(1 - z\right)p^2 - m_K^2\right] \ln \frac{z(1 - z)p^2 - m_K^2}{z(1 - z)m_\phi^2 - m_K^2}

\]

For \( \Pi_{cc} \) we follow the same steps described in the Appendix A and arrive to the expression

\[
\Pi_{cc}^{\mu\nu} = \frac{2g^2}{(2\pi)^4} \int_0^1 dz \int_0^\infty dt \left(1 - r\right) \frac{\frac{2\beta\mu}{3} \tan \left(\beta t\right) \nu_p^2}{\left(1 - r\right) \left(1 - e^{-2\beta t/r}\right)} \int d^2u_E \ e^{-u_p^2}

\times \int d^2v \ \exp \left\{\left[\tan \left(\beta t\right) + \tan \left(\beta t\left(1 - z\right)\right)\right] v^2 / \beta\right\} I^{\mu\nu}
\]

where \( \xi = \frac{m_K^2 + \beta - z(1 - z)u_p^2}{2}\beta \), \( \alpha \) was defined in the Appendix A and

\[
I^{\mu\nu} = g^{\mu\nu} \left(-u_2 g_6^\mu + v_1 g_4^\mu + v_2 g_2^\mu + u_3 g_3^\mu\right) + (1 - 4z + 4z^2)u_p^\mu u_p^\nu - \alpha(2z - 1) \left(u_p^\mu v_p^\nu + u_p^\nu v_p^\mu\right)

\times \left(1 - \frac{\tan \left(\beta t\right)}{\tan \left(\beta t(1 - z)\right)}\right) + \alpha^2 u_p^\mu u_p^\nu \left(1 - \frac{\tan \left(\beta t\right)}{\tan \left(\beta t(1 - z)\right)}\right)^2
\]

those terms giving zero contribution to either the integration over \( u_E \) or over \( v \) have been omitted in this equation. An analysis of Eq. (17) shows that, excepting the factor \( I^{\mu\nu} \), the integrand is invariant under the change of variable \( z \to 1 - z \). In contrast, \( I^{\mu\nu} \) is \( (2z - 1)\lambda \rho^\mu \rho^\nu \left(1 - \tan \left(\beta t(1 - z)\right)\right) / \tan \left(\beta t\right) \) for \( \mu = 0, 3 \) and \( \nu = 1, 2 \), is odd under the same change. Hence, integration over \( z \) gives \( \Pi_{cc}^{\mu\nu} = 0 \) for the mixing sector \( \mu = 0, 3 \) and \( \nu = 1, 2 \).

From the structure of \( I^{\mu\nu} \), a decomposition similar to that shown in Eq. (17) can be made for each of the longitudinal or transversal components of \( \Pi_{cc}^{\mu\nu} \). By taking \( v_p^2 = 0 \), closed forms for \( A \) and \( B \) in terms of the digamma function can be given. For instance, in the longitudinal sector we have

\[
A(v_p^2 = 0) = 2\beta \int_0^1 dz \left(1 - 2z\right)^2 \int dt \frac{e^{-2\beta t\xi}}{1 - e^{-2\beta t}}
\]

As in the cases previously discussed, we introduce a regularizing factor \( t^s \), \( 0 < s < 1 \), which allows the integration over \( t \) in terms of the Hurwitz zeta function \( \zeta(1 + s, \xi) \), whenever the condition \( 4(m_K^2 + \beta) - u_p^2 > 0 \) is satisfied. Otherwise, the parameter \( \xi \) could become negative as \( z \) varies. In such a case the result can be extended by using the functional relation

\[
\zeta(s, \xi) = \zeta(s, N + \xi) + \sum_{j=0}^{N-1} (j + \xi)^{-s}
\]
where $N$ is the minimum integer number which satisfies $m_K^2 + (2N+1)\beta - u_p^2/4 > 0$.

After this decomposition, we expand the result around $s = 0$ obtaining

$$ A\left(v_p^2 = 0\right) = \frac{1}{3} \left( \frac{1}{s} - \gamma \right) - \int_0^1 dz \left( 1 - 2z \right)^2 \left[ \psi(\xi + N) - \sum_{j=0}^{N-1} \frac{\Gamma(1+s)}{(j+\xi)^{1+s}} \right] + O(s) $$

In this formula an Sokhotski-Plemelj decomposition is used in each term of the finite sum and finally the meaningful result is obtained by regularizing at $p^2 = m_\phi^2$.

$$ \text{Re} A_{reg}\left(v_p^2 = 0\right) = - \int_0^1 dz \left( 1 - 2z \right)^2 [\psi(\xi + N) - \psi(\xi_0 + N)] + 2 \sum_{j=0}^{N-1} \frac{D_j}{u_p^2} \ln \left( \frac{1 - D_j}{1 + D_j} \right) $$

$$ \text{Im} A\left(v_p^2 = 0\right) = - 4\pi \beta \sum_{j=0}^{N-1} \frac{D_j}{u_p^2} \Theta \left( 1 - 4 \frac{m_K^2 + \beta(2j + 1)}{u_p^2} \right) $$

where $\xi_0 = \left[ m_K^2 + \beta - z(1 - z)m_\phi^2 \right] / 2\beta$ and $D_j = \sqrt{1 - 4[m_K^2 + \beta(2j + 1)]/u_p^2}$.

Proceeding in a similar way we obtain for $B$

$$ B\left(v_p^2 = 0\right) = - \left( \frac{1}{s} - \gamma \right) \left[ \frac{u_p^2 - 6m_K^2}{3} + 4\beta N \right] - 4\beta \int_0^1 dz \ln \left[ \frac{\Gamma(\xi + N)}{\sqrt{2\pi}} \right] $$

$$ + \ 4\beta \sum_{j=0}^{N-1} \ln(\xi + j) + O(s) \quad (21) $$

In this equation it can be seen that, in opposition to all the other cases studied in both Appendices, there is an explicit dependence of the main coefficient on the magnetic field.

From the above equation the regularized contributions are extracted as

$$ \text{Re} B_{reg}\left(v_p^2 = 0\right) = - 4\beta \int_0^1 dz \left[ \ln \frac{\Gamma(\xi + N)}{\Gamma(\xi_0 + N)} + (u_p^2 - m_\phi^2) \psi(\xi_0) \frac{z(1 - z)}{2\beta} - \sum_{j=0}^{N-1} \ln \left( \frac{\xi + j}{\xi_0 + j} \right) \right] $$

$$ \text{Im} B\left(v_p^2 = 0\right) = - u_p^2 \text{Im} A\left(v_p^2 = 0\right) $$

The imaginary parts of the polarizations are finite, so they do not need to be regularized.

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Figure 1: Feynman graphs included in the present calculations. Solid, dashed and dotted lines stand for kaon, non strange scalar mesons and baryon propagators respectively. In the case (d) the dashed line corresponds to the $\phi$ meson propagator.
Figure 2: The effective kaon mass as a function of the baryonic density at fixed magnetic intensity $B = 10^{18}$ G. Results corresponding to the mean field approximation (MFA) and the full treatment are compared.
Figure 3: The effective kaon mass as a function of the magnetic intensity for the fixed baryonic densities $n/n_0 = 0.5, 1,$ and $2$. 
Figure 4: The $\phi$ meson decay width as a function of the magnetic intensity.