Theoretical Interpretation of the NE18 Experiment on Nuclear Transparency in \(A(e, e'p)\) Scattering

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A b s t r a c t

The spectral function, measured in \(A(e, e'p)\) reactions, is distorted by the final-state interaction of the struck proton with the residual nucleus. This causes a broadening of the observed transverse-momentum distribution which is large even in the \(d(e, e'p)\) reaction. We discuss the effects of this \(p_T\)-broadening on the nuclear transparency measured in the recent NE18 experiment. Within conventional Glauber theory we can describe the measurements. Transparency effects are thus small in agreement with our earlier predictions.
The recently completed SLAC NE18 experiment [1] on $A(e, e'p)$ scattering concludes that, up to $Q^2 \lesssim 7 (\text{GeV}/c)^2$, there is no conclusive evidence for color transparency (CT). A slow onset of CT was predicted by us [2,3] in an approach which is based on CT sum rules and the quark-hadron duality. It was shown that CT and/or weak final-state interaction (FSI) in $A(e, e'p)$ reactions arises from delicate cancellations of contributions from elastic ($|i\rangle = |p\rangle$) and inelastic ($|i\rangle \neq |p\rangle$) intermediate states $|i\rangle$ propagating inside the nucleus [2,3]. At $Q^2 \lesssim 7 (\text{GeV}/c)^2$ the contributions from inelastic rescatterings is still small, however, and hence the very small signal in the NE18 experiment (for a general review on CT see [5-7]). Another very interesting aspect of the experiment is the evidence for substantial FSI effects already in the deuteron.

A quantitative description of the NE18 data [1] is an important check for our understanding of the FSI at large $Q^2$ and the purpose of this communication is to provide such a description. To do so, one should realize that, because of the finite spectrometer acceptance in the missing energy $E_m$ and missing momentum $\vec{p}_m = (p_{m,z}, \vec{p}_\perp)$ (the $z$-axis is chosen along the $\vec{q}$ direction), the sum-rule technique of refs. [2,3] is not applicable. It has been argued in [1] that one should compare the (partially integrated) measured spectral function $S(E_m, \vec{p}_m)$ to the calculated $S_{PWIA}(E_m, \vec{p}_m)$ in the plane-wave impulse approximation (PWIA). We will show, however, that the spectral function does not factorize as $S_{PWIA}(E_m, \vec{p}_m)$ times a $(E_m, \vec{p}_m)$-independent attenuation factor. This is due to the fact the FSI of the struck proton significantly distorts the $p_\perp$-distribution [8]. Evidently, elastic rescatterings deflect the struck proton and only the inelastic $pN$ cross section $\sigma_{in}(pN)$ contributes to the attenuation of the $p_\perp$-integrated cross section [2,8]. In parallel kinematics ($\vec{p}_\perp = 0$), on the other hand, the attenuation is controlled by $\sigma_{tot}(pN)$ [8]. Thus, because of the finite $p_\perp$-acceptance in the NE18 experiment, an accurate evaluation of the FSI effects is necessary for the quantitative interpretation of the data.

We begin with the analysis of the $d(e, e'p)$ scattering. At $Q^2 \gtrsim 1 (\text{GeV}/c)^2$ the kinetic energy of the struck proton is large, $T_{kin} \approx Q^2/2m_p$, and we can use the Glauber approximation for the FSI of the struck proton with the spectator neutron [8]. Then the
momentum distribution, \( f_d(p_{m,z}, \vec{p}_\perp) \), of the observed protons can be written as

\[
f_d(p_{m,z}, \vec{p}_\perp) = \left| \phi_d(p_{m,z}, \vec{p}_\perp) - \frac{\sigma_{tot}(pn)}{16\pi^2} \int d^2k \phi_d(p_{m,z}, \vec{p}_\perp - \vec{k}) \exp\left(-\frac{1}{2} B \vec{k}^2\right) \right|^2, \tag{1}
\]

where \( \phi_d(\vec{k}) \) is the momentum-space wave function of the deuteron, and \( B \) denotes the diffraction slope for elastic \( pn \) scattering. The measured transparency factor

\[
T_d(p_{m,z} = 0) = \frac{\int_{p_{\perp}}^{p_{\perp}^{\text{max}}} d^2p_\perp f_d(\vec{p}_\perp) / \int_{p_{\perp}}^{p_{\perp}^{\text{max}}} d^2\vec{p}_\perp |\phi_d(\vec{p}_\perp)|^2}{1 - T_d} \sim \frac{\sigma_{tot}(pn)}{2\pi R_d^2} \sim 0.07, \tag{2}
\]

explicitly depends on the values \( p_\perp \leq p_\perp^{\text{max}} \) accepted in the spectrometer. In NE18, \( p_\perp^{\text{max}} = 170 \text{ MeV/c} \) while the \( p_{m,z}\)-acceptance is very small [1]. Since \( B \ll R_d^2 \), where \( R_d \) denotes the radius of the deuteron, the rescattering term in (1) is almost constant over this range of \( p_\perp \). On the other hand, \( (R_d p_\perp^{\text{max}})^2 \gg 1 \). This leads to a simple estimate of the attenuation effect

\[
1 - T_d \sim \frac{\sigma_{tot}(pn)}{2\pi R_d^2} \sim 0.07, \tag{3}
\]

which is about twice as large as the Glauber’s shadowing effect in \( \sigma_{tot}(pd) \) [9,10]. Using a realistic Bonn wave function for the deuteron [11], the detailed predictions at \( p_{m,z} = 0 \) are shown in Fig. 1.

An extension to heavier targets is straightforward. With the broad \( E_m\)-acceptance of the NE18 experiment closure becomes applicable and, in the absence of FSI, one would obtain the single-particle momentum distribution, \( d\sigma_{PWIA} \propto n_F(\vec{p}) \) with

\[
n_F(\vec{p}) = \frac{1}{Z} \int dE_m S_{PWIA}(E_m, \vec{p}) = \frac{1}{A(2\pi)^3} \int d\vec{r}_1 d\vec{r}_1' \rho_1(\vec{r}_1, \vec{r}_1') \exp[i\vec{k}(\vec{r}_1 - \vec{r}_1')] \tag{4}
\]

where \( \rho_1(\vec{r}, \vec{r}') \) denotes the one-body density matrix while \( Z \) and \( A \) are the charge and mass number of the nucleus, respectively. With allowance for FSI [6,8,12], on the other hand,

\[
d\sigma_A \propto \frac{1}{Z} \int dE_m S(E_m, \vec{p}_m) = \int d\vec{r}' d\vec{r} \rho_1(\vec{r}, \vec{r}') \exp[i\vec{p}_m(\vec{r}' - \vec{r})] \cdot \exp\left[-\frac{1}{2}(1 - i\alpha_{pN})\sigma_{tot}(pN)t(\vec{b}, z) - \frac{1}{2}(1 + i\alpha_{pN})\sigma_{tot}(pN)t(\vec{b}', z')\right] \cdot \exp\left[t(\vec{b}, \text{max}(z, z'))\xi(\Delta)\right]. \tag{5}
\]
where $\vec{r} = (\vec{b}, z)$, $\vec{r}' = (\vec{b}', z')$, $\vec{\Delta} = \vec{r} - \vec{r}'$, $t(b, z) = \int_{z'}^\infty dz' n_A(b, z')$ and $n_A(\vec{r})$ is the matter density. Furthermore, $\alpha_{pN}$ denotes the ratio of the real and imaginary parts of the forward $pN$ scattering amplitude and $\xi(\vec{\Delta}) = \int d^2\vec{q} [d\sigma_{el}(pN)/d^2\vec{q}] \exp(i\vec{q}\vec{\Delta})$. Since $k_F R_A \gg 1$, Eq. (5) can be further simplified by employing the local-density approximation [13], $\rho_1(\vec{r}, \vec{r}') = \rho(\vec{\Delta}) n_A(\vec{R})$, where $\vec{R} = (\vec{r} + \vec{r}')/2$. The FSI has two effects: (1) besides the attenuation, there appears a phase factor $\exp[i\sigma_{tot}(pN)\alpha_{pN} n_A(b, z)(z - z')]$ which makes the measured missing momentum $p_{m,z}$ different from the longitudinal momentum $k_z$ of the target proton by an amount

$$k_z - p_{m,z} = \Delta p_{m,z} \approx \sigma_{tot}(pN)\alpha_{pN} n_A \sim \alpha_{pN} \cdot 70 \text{ (MeV}/c).$$  (6)

This momentum shift is the main contributor to distortion of the $p_{m,z}$-distribution as compared to the PWIA where $p_{m,z} = k_z$. All the other corrections are small [12]. (2) the factor $\exp[t(\vec{b}, z)\xi(\vec{\Delta})]$ in the integrand of Eq. (5) causes a broadening of the $p_\perp$-distribution [6,8]:

$$f_A(\vec{p}_\perp) = \frac{1}{Z} \int dE_m dE_{p_{m,z}} S(E_m, p_{m,z}, \vec{p}_\perp) = \sum_{\nu=0}^{\infty} W^{(\nu)} f^{(\nu)}(\vec{p}_\perp).$$  (7)

The probability, $W^{(\nu)}$, for $\nu$-fold elastic rescattering equals

$$W^{(\nu)} = \frac{1}{A} \int dz d^2\vec{b} \ n_A(\vec{b}, z) \exp \left[ -\sigma_{tot}(pN)t(\vec{b}, z) \right] \frac{[t(\vec{b}, z)\sigma_{el}(pN)]^\nu}{\nu!}$$  (8)

and

$$f^{(\nu)}(\vec{p}_\perp) = \int d^2\vec{s} [B/\nu\pi] \exp \left( -Bs^2/\nu \right) f_{PWIA}(\vec{p}_\perp - \vec{s}),$$  (9)

where $f_{PWIA}(\vec{p}_\perp) = \int dk_z n_F(k_z, \vec{p}_\perp)$ is the $p_\perp$-distribution in PWIA. For the purpose of the present analysis it is sufficient to assume $f^{(0)}(\vec{p}_\perp) = f_{PWIA}(\vec{p}_\perp)$ [12]. The normalization of $f_A$ is such that $\int d^2\vec{p}_\perp f_A(\vec{p}_\perp) = \sum_{\nu=0}^{\infty} W^{(\nu)} = T_A$, where

$$T_A = \frac{1}{A} \int dz d^2\vec{b} \ n_A(\vec{b}, z) \exp \left[ -\sigma_{in}(pN)t(\vec{b}, z) \right]$$

$$= \frac{1}{A\sigma_{in}(pN)} \int d^2\vec{b} \left[1 - \exp[-\sigma_{in}(pN)T(b)]\right]$$  (10)

is total transmission factor or ‘nuclear transparency factor’ [2]. The quasielastic knockout in parallel kinematics, $p_\perp = 0$, is dominated by the $\nu = 0$ (PWIA) component of
The above discussion proves that the distortion effects do not allow a factorization of the measured spectral function $S(E_m, \vec{p}_m)$ into $S_{PWIA}(E_m, \vec{p}_m)$ and an overall attenuation factor. Experimentally, one compares the observed nuclear cross section to the PWIA cross section integrated over the acceptance domain $D$ in the $(E_m, p_{m,z}, p_{\perp})$ space [1]. In order to extract the attenuation effect, it is therefore necessary to include the shift $\Delta p_{m,z}$ in the PWIA cross section, such that the proper definition of nuclear transparency, relevant to the NE18 experiment, should be

$$T_A(\text{NE18}) = \frac{\int_D dE_m dp_{m,z} dp_{\perp} S(E_m, p_{m,z}, p_{\perp})}{\int_D dE_m dp_{m,z} dp_{\perp} S_{PWIA}(E_m, p_{m,z} + \Delta p_{m,z}, p_{\perp})}.$$  

(11)

At values of $T_{\text{kin}}$ of the NE18 experiment the real part of the $pN$ elastic amplitude is rather large, $\alpha_{pN} \sim -0.5$ [14], and the effective shift (6) of the missing longitudinal momentum $\Delta p_{m,z} \sim -35 \text{ MeV}/c$. The effect of the shift $\Delta p_{m,z}$ vanishes for a wide $p_{m,z}$-acceptance, but for a narrow acceptance centered at $p_{m,z} = p^*$ one has

$$\frac{T_A(\Delta p_{m,z})}{T_A(\Delta p_{m,z} = 0)} \approx 1 + \frac{5}{2} \cdot \frac{(\Delta p_{m,z} + p^*)^2 - (p^*)^2}{k_F^2},$$  

(12)

which enhances $T_A$ by $\sim 5\%$ at $p^* = 0$. This is still within the NE18 error bars. At $Q^2 \lesssim 1 \text{ (GeV}/c)^2$, i.e., at $T_{\text{kin}} \lesssim 0.5 \text{ GeV}$, both $\sigma_{pN}$ and $\alpha_{pN}$ change rapidly with $T_{\text{kin}}$ [14] and the momentum shift may lead to a spurious $Q^2$ dependence of the spectral function and of the $y$-scaling function. This point has been missed in the discussion of FSI effects in [15]. The effects of the momentum shift (6) and the $p_{\perp}$-broadening effect approximately factorize. The NE18 cut $p_{\perp} \lesssim p_{\text{max}} = 250 \text{ MeV}/c$ [1] partly includes struck protons which are elastically rescattered, and we calculate $T_A(\text{NE18})$ for this cutoff making use of Eq. (7) for the $p_{\perp}$-distribution.

As can be seen from Fig. 1 we find very good quantitative agreement between our predictions for $T_A(\text{NE18})$ and the data. This leads to the following conclusions: (1) the Glauber theory gives an adequate description of the FSI at moderately large $Q^2$, (2) the
signal of CT in $A(e,e'p)$ reactions is still negligibly small at $Q^2 \leq 7 \text{ (GeV/c)}^2$, in agreement with our prediction ([2,3,5-7]). For $^{12}C(e,e'p)$ scattering we estimate the threshold for CT at $Q^2 \sim 5 \text{ (GeV/c)}^2$ and CT effects shown as the short-dashed curve in Fig. 1 increase $T_C$ by $\sim 5\%$ at the largest $Q^2$ of the NE18 experiment (Fig. 1). For heavier nuclei the CT signal is even smaller. The main reason is the smallness of the contribution from inelastic intermediate states because the amplitudes for the diffraction excitations $p + p \rightarrow i + p$ are small as compared to the amplitude of elastic rescattering $p + p \rightarrow p + p$ [12,3,5,7].

In summary, we have calculated the effects of distortions on the spectral function for the nuclear transparency experiment NE18 at SLAC [1]. It has been shown that the FSI, when treated in the Glauber approximation, is able to explain the large shadowing effect observed in $d(e,e'p)$ reaction. Glauber’s multiple-scattering theory is also found to provide a good description of the experimental results on heavier targets. In spite of the small CT effect in the data, even at the highest $Q^2$, the possibility of observing a signal in dedicated $^4He(e,e'p)$ experiments at CEBAF should not be ruled out. A discussion of this reaction will be presented elsewhere [18].

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References

[1] NE18 Collaboration: B.Fillipone, preliminary results reported at PANIC, Perugia, July 1993.

[2] N.N.Nikolaev, A.Szczurek, J.Speth, J.Wambach, B.G.Zakharov and V.R.Zoller, Jülich preprint KFA-IKP(Th)-1992-16 (1992), to be published in Nucl.Phys. A, and paper in preparation.

[3] N.N.Nikolaev, A.Szczurek, J.Speth, J.Wambach, B.G.Zakharov and V.R.Zoller, Phys. Lett. B317, 287 (1993).

[4] N.N.Nikolaev, A.Szczurek, J.Speth, J.Wambach, B.G.Zakharov and V.R.Zoller, Phys. Lett. B317, 281 (1993).

[5] N.N.Nikolaev and J.Speth, in: The ELFE Project on an Electron Laboratory for Europe, edited by J.Arvieux and E.De Sanctis, Editrice Compositori, Bologna, 1993.

[6] N.N.Nikolaev, Color Transparency: Novel Test of QCD in Nuclear Interactions, Landau Inst. preprint Landau 16-93, February 1993, Surveys in High Energy Physics (1993), in press; N.N.Nikolaev, High Energy Nuclear Reactions in QCD: Color Transparency Aspects, in: Lecture notes of 1992 RCNP Kikuchi School, 16-19 November 1992, Osaka Univ., RCNP-P-128, editors M.Fujiwara and M.Kondo.

[7] N.N.Nikolaev and B.G.Zakharov, Color transparency after the NE18 and E665 experiments, Jülich preprint KFA-IKP(Th)-1993-30; to be published in Proceedings of the Workshop on Electron Nucleus Scattering, Elba International Physics Center, Marciana Marina, Elba, July 5-10, 1993.

[8] N.N.Nikolaev, A.Szczurek, J.Speth, J.Wambach, B.G.Zakharov and V.R.Zoller, Jülich preprint KFA-IKP(Th)-1993-31 (1993).

[9] R.J.Glauber and G.Matthiae, Nucl. Phys. B21, 135 (1970).

[10] L.G.Dakhno, Sov.J.Nucl.Phys. 37, 590 (1983).
[11] R. Machleidt, K. Holinde and Ch. Elster, *Phys. Rep.* **C149**, 1 (1987).

[12] N. N. Nikolaev and B. G. Zakharov, paper in preparation.

[13] J. Negele and D. Vautherin, *Phys. Rev.* **C5**, 1472 (1971); **C11**, 1081 (1974).

[14] C. Lechanoine-LeLuc and F. Lehar, *Rev. Mod. Phys.* **65**, 47 (1993).

[15] O. Benhar, A. Fabrocini, S. Fantoni, G. A. Miller, V. R. Pandharipande and I. Sick, *Phys. Rev.* **C44**, 2328 (1991).

[16] N. N. Nikolaev, JETP Letters **57**, 82 (1993).

[17] N. N. Zotov and V. A. Zarev, *Sov. Phys. Uspekhi* **51**, 119 (1988); G. Alberi and G. Goggi, *Phys. Rep.* **C74**, 1 (1981).

[18] N. N. Nikolaev, A. Szczureck, J. Speth, J. Wambach, B. G. Zakharov and V. R. Zoller, to be published.
Figure captions:

**Fig. 1** Predictions for the $Q^2$ dependence of the $p_{\perp}$-integrated nuclear transparency $T_A$ (dashed lines), the transparency $W^{(0)}$ for parallel kinematics (dotted lines) and the transparency $T_A$(NE18) including the acceptance cuts of the NE18 experiment [1]. The dot-dashed curve in panel for $^{12}C$ indicates the effect from the onset of color transparency.