FAQ about the “contextual objectivity” point of view.

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We discuss some Frequently Asked Questions about the “contextual objectivity” point of view on quantum mechanics introduced in two previous preprints [1,2].

I. INTRODUCTION

In a previous preprint [1], we introduced and discussed a “physical” (as opposed to mathematical) definition of a quantum state that reads in the following way:

The quantum state of a physical system is defined by the values of a complete set of physical quantities, which can be predicted with certainty and measured repeatedly without perturbing in any way the system.

As discussed in detail in ref. [1], this definition is in full agreement with the usual formalism of QM. It is also implies that some “objectivity” can be attached to the quantum state, because the quantum state is defined from a fully predictable course of events, that is independent of the observer. In [2] we tried to exploit this definition, together with some ideas about the system dimensionality, to propose a new axiomatic approach to QM, that is an attempt to spell out how the “quantum reality” is related with the “macroscopic reality”. While [1] is a straightforward rewriting of usual QM, [2] is much more tentative and its goal is mostly to stimulate some thinking. We recommend to the reader to have a look at least at [1] before reading the FAQ below.

II. FREQUENTLY ASKED QUESTIONS (AND ANSWERS)

Q: What is contextual objectivity?
A: It is an attempt to reformulate quantum mechanics (QM) in a more physical (as opposed to mathematical) way. The ultimate goal of such a reformulation, that is evoked in [2], would be to explain why QM is the way it is. In particular, the Hilbert space structure should be deduced, not postulated.

Q: The quantum state cannot be an objective property of the system, because if you are given the system it is not possible to recover the state.
A: You should be given not only the system, but also the set of relevant observables (i.e. the “context”, or in usual terms the measurement basis). Then the state can obviously be recovered with certainty.

Q: Does that mean that the set of relevant observables is “an intrinsic part of the reality” (cf Bohr’s answer to the EPR argument)?
A: In some sense yes. As explained in [3], a very specific quantum feature is the existence of “non-exclusive modalities” (in usual terms, non-orthogonal pure states): one cannot recover a state among a set of non-exclusive modalities, but it is quite possible to recover it among a set of exclusive ones (in usual terms, orthogonal ones). This is why the set of relevant observables is needed together with the system, and why we speak about “contextual” objectivity. We note that the appropriate set of relevant observables is also observer-independant.

Q: How to explain the EPR “paradox”?
A: In an EPR state, the initial quantum state is a state of the particles pair, and the fully predictable quantities are the results of Bell measurements, that are joint measurements on both particles. On the other hand, the states for each particle are undefined (they have no “reality”). When Alice performs a measurement, the state is redefined on her side. Given her measurement result, and assuming that she knew the initial entangled state, Alice can infer Bob’s state. On Bob’s side, nothing changed: there was no pure state before, and there is no pure state after, until Alice informs Bob about what she measured. For doing that she needs classical transmissions. There-

1 Throughout this paper “state” means “pure state”, unless it is specified as “mixed state”. To avoid confusion, pure states will also be called “modalities” [4].

2 The set of quantities is complete in the sense that the value of any other quantity which satisfies the same criteria is a function of the set values.
fore, Alice’s measurement does not “act upon” Bob’s particle in any sense. We note that Bob is also free to make a measurement on his side. If he does, it can be checked afterwards that his result is compatible with (but not determined by) Alice’s prediction.

With respect to Bell’s inequalities, one should notice that for an EPR state the strong correlations between measurements on the subsystems are due to global properties, while the properties of each subsystem are completely random. Such a situation is totally non-classical, because classically correlations must be “mediated” by properties of the subsystems. So Bell’s hypothesis contradict QM because of the failure of the “local reality” (or “separability”) assumption, that states that in order to explain the correlations, there should exist a property (a “local hidden variable”) that describes the polarization on each side. As said above, this is not the case in QM, but again there is no “action at a distance”, not even any “influence at a distance”.

The ultimate lesson from the EPR argument and Bell’s inequalities is that classical physics is unable to manage global properties of a system, that are not mediated by individual properties of the subsystems, while QM can perfectly do so. On the other hand, both classical and quantum correlations are due to “common causes in the past evolution”, and thus obey relativistic causality.

Q: Why not to say that a pure quantum state is a “state of knowledge” ?
A: An implicit consequence of the wording “state of knowledge” is that such a state should be contingent (i.e. observer-dependant), and that it should be associated with an “ignorance” of something. This is indeed true for a mixed state, but this is not true for a pure state, that is observer-independent, and that is not associated with the “ignorance” of anything (i.e., there is no hidden variables). Therefore, one might use a wording like “objective state of knowledge”, but this is not very clear, and this is why we prefer to say that a state is “real” in the contextual objectivity point of view. Actually, as said above, a pure state appears to be real and objective in the usual sense as long as one carries out measurements within the specified complete set of commuting observables, following the time evolution of the system.

3 Mathematically this is expressed by the dependance of the measurement result $A(\vec{a}, \lambda) = \pm 1$ on both the adjustable parameter $\vec{a}$ and the hidden variable $\lambda$. Classically, if there is nothing like $\lambda$, then $A(\vec{a}) = \pm 1$ is purely random, and the correlations should vanish, while this is not the case quantum mechanically. This emphasizes again that an essential hypothesis for Bell’s inequalities is “local reality”.

4 In case this sentence sounds too “holistic” to the reader, let us remind that in our approach, the quantum state is always “embedded” in a classical environment.

Q: How to deal with the problem of the boundary between quantum and classical “realities” ?
A: The discussion in [3] does not directly answer the question of the quantum-classical boundary, but rather makes it irrelevant, by claiming that the true basic postulate of QM is the existence of both a continuous classical world and a quantized quantum world. The structure of QM is then a consequence from the need to connect them together. As said above, this line of reasoning is an attempt to stimulate thinking rather than a proof, and we summarize below its main points.

A physical quantity is defined as an ensemble of possible measurements, that are connected between themselves by “geometrical” transformations that we call “knob transformations” (they may be standard geometrical transformations, such as rotations of a Stern-Gerlach magnet...). This definition of physical quantities, that cannot be avoided in our opinion, is essentially classical: it cannot be quantum, since it actually defines the parameters that will be used to measure the state of the quantum system. More precisely, we will assume that the “knob transformations” have the structure of a non-commutative continuous group. On the other hand, the quantum system is intrinsically quantized: though it may be in an infinite number of (non-exclusive) modalities, we postulate that for a given “context” (i.e. complete set of commuting observables), there is only a discrete number of exclusive modalities, that is a property of the system (its dimension). Not surprisingly, quantization is the main feature of the quantum system.

Given the two concepts of physical quantities and of a system, the most naive classical approach is to identify each physical quantity with the numbers given by the measurements, and to attribute “reality” to these numbers. EPR themselves realized that this definition of “reality” was too restrictive, and proposed instead their definition based upon predictability and reproducibility; this is just the idea that we use as our definition of a quantum state. But as soon as this is done, it appears that “reality” based upon predictability and reproducibility cannot be attributed simultaneously to all physical quantities: this is simply incompatible with the structure of the physical quantities described above.

One wants indeed to hold simultaneously that the physical quantities are defined by measurements, depending on continuous parameters, while the possible measurement results (the “exclusive modalities”) are quantized. How QM is able to make these two requirements fit together is discussed in [3] (see also [5]). We reach then

5 In an extreme view, what is required is the (continuous) “spacetime” in which the experiment is carried out, that is not a quantum space-time (at least, not in usual QM).

6 In a more precise definition, physical quantities will be associated with the infinitesimal generators of that group.
the conclusion that what is “real” at the macroscopic level is the definition of the physical quantities (i.e. of the possible measurements), and what is “real” at the quantum level (of the measured system) is the quantum state. These two “realities” are fully compatible - they are actually the only ones that can connect the experimental definition of a physical quantity and the measurement results in a consistent way.

Q: What about the “many-worlds” interpretation?
A: The so-called “many-worlds” interpretation of QM is a possible (the only possible?) alternative to our point of view. The difference between the two approaches is clear: while our approach is built to take into account that the measurement of \( S_z \) on a superposition \((|+\rangle + |-\rangle)/\sqrt{2}\) gives only one result, the “many-worlds” approach claims that it gives two (totally equivalent) results. For consistency with classical reality, it adds that our mind is made is such a way that it “follows” only one of these possibilities. Rather that introducing our mind at that point (whose’s mind actually?), we consider more fruitful to assume that physical reality is uniquely defined, within the framework of contextual objectivity.

[1] Philippe Grangier, “Contextual objectivity : a realistic interpretation of quantum mechanics”, arXiv: quant-ph/0012122
[2] Philippe Grangier, “Reconstructing the formalism of quantum mechanics in the contextual objectivity point of view.”, arXiv: quant-ph/0111154
[3] For the sake of completeness here is a short summary:
- For a given “knob settings” of the measurement apparatus, there exist \( N \) distinguishable quantum states \( \{ b_i \} \), that are called “exclusive modalities”. The value of \( N \), called the dimension, is a characteristic property of a given quantum system. It is not assumed that the \( \{ b_i \} \) are rays in an Hilbert space.
- Different knob settings are related between themselves by transformations \( g \) that have the structure of a continuous group \( G_K \). We introduce the representation of the group \( G_K \) by \( N \times N \) unitary matrices, and we denote as \( \bar{\Sigma}_g \) the unitary matrix corresponding to the group element \( g \).
- If the system is known to be in the state \( b_i \) from the set \( \{ b_i \} \), the probability that it is found in state \( b'_j \) from the set \( \{ b'_j \} \) corresponding to another knob settings obtained by the knob transformation \( g \) is:

\[
p_{i,j} = \text{Trace}(P_i \Sigma_g P_j \Sigma_g^*)
\]  

(1)

where the \( P_i \) are \( N \times N \) orthogonal projectors, and \( \Sigma_g \) is the unitary matrix corresponding to \( g \).

The Hilbert space structure can be deduced from the above statements, where the basic mathematical entities are the \( N \times N \) unitary matrices \( \Sigma_g \) that represent \( G_K \).