Research on Bi-objective Vehicle Routing Problem Considering Empty Loading Ratio

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Abstract. The vehicle routing plan in logistic distribution minimizing the transportation cost merely may lead to high empty loading ratio and a waste of logistic resources. This research studies the bi-objective vehicle routing problem considering empty loading ratio. The definitions about the empty loading ratio are proposed firstly. A bi-objective vehicle routing model is developed, minimizing the empty loading ratio and transportation cost simultaneously. Then, the solution method based on $\varepsilon$ constraint method is provided to compute the Pareto optimal solutions. Finally, numerical experiments are reported, and the results show that the $\varepsilon$ constraint method is better than the weighting method for the proposed problem.

Keywords. Vehicle routing problem; bi-objective; empty loading ratio; Pareto optimal solutions.

1. Introduction

Logistic distribution is the activity of distributing goods by reasonable transportation plan. Generally speaking, logistic companies mainly pursue the minimization of their own costs. However, the vehicles’ paths decided in this way are easy to lead to high empty loading degree. In the process of distribution, it is impossible to have no empty loading at all. Since the empty loading of vehicles is a process of pure consumption, the high empty loading degree means the waste of logistic resources. For example, the empty loading degree of logistic vehicles in China is as high as 37%. Therefore, the empty loading ratio of vehicles has attracted more and more attentions.

Whether the vehicle routing arrangement is reasonable or not has a crucial impact on the efficiency of distribution. Researchers have done a good many of work on the vehicle routing problem (VRP). As a classic problem, VRP was proposed by Dantzig in the middle of the last century [1]. The general objective of VRP is minimizing the total transportation cost of vehicles. With the promotion of green development in the whole world, if logistic companies only consider the minimization of transportation cost, it may lead to high empty loading degree. Therefore, scholars began to consider the utilization of vehicles in their researches. In these studies, the utilization of vehicles is mainly considered from two perspectives. The first one is the VRP considering the departure cost of vehicles.
Some scholars believe that there is a fixed departure cost when using each vehicle and consider the departure cost in the total transportation cost. For example, Favaretto studied a VRP with departure cost and multiple time windows [2]. Ceschia investigated a heterogeneous VRP with departure cost and time windows [3]. Androutsopoulos and Bula studied bi-objective VRP for HAZMAT considering departure cost [4-5]. This kind of research can only improve the utilization of vehicles indirectly. The second one is the VRP considering empty driving cost. These studies mainly focused on full load VRP, since the empty driving cost per unit distance differs. For example, Currie studied a full load VRP with backhauling [6]. Zhang probed into a full load VRP with multiple trips [7]. Currie studied a full load, multi-terminal VRP [8]. This kind of research is out of touch with the reality after the toll has changed to be charged by vehicle type.

Through the above analysis, we know that the related studies are lack of enough attention to the empty loading ratio, and the trade-off between transportation cost and empty loading ratio is ignored. In this paper, we are going to study a bi-objective VRP considering empty loading ratio, aiming to minimizing the transportation cost and empty loading ratio simultaneously. The solution method to obtain Pareto optimal solutions will be provided. The problem definition is given firstly in the next section.

2. Problem Definition

2.1. Problem Description

\( G(V,E) \) denotes a transportation network. \( V=\{v_0,v_1,\cdots,v_n\} \) denotes the vertexes, where \( v_i,v_2,\cdots,v_n \) denote the customers, respectively. \( E=\{e(v_i,v_j)\} \) denotes the edges. \( d_{ij} \) is the length of \( e(v_i,v_j) \), \( q_i \) is the quantity of goods required by customer \( v_i \). All vehicles have a same capacity \( Q \), \( c \) is the unit transportation cost. There are \( K \) vehicles in the depot. The goal of the problem is to decide the paths of vehicles, minimizing the empty loading ratio and transportation cost simultaneously.

2.2. Empty Loading Ratio

The empty loading ratio in logistic practice in the traditional sense means the proportion of empty part in all loadable part of a vehicle. This kind of empty loading ratio can be expressed as \( \lambda = \frac{\alpha}{\beta} \), where \( \alpha \) denotes the empty part of a vehicle, \( \beta \) denotes all loadable part of a vehicle, respectively. The empty loading ratio in the traditional sense has nothing to do with the travel distance of the vehicle, thus the impact of empty loading ratio on distribution cannot be well reflected. Besides, the empty loading ratio in the traditional sense also cannot effectively reflect the empty loading degree of the entire fleet. Thus, in view of the above defects, we propose the following definitions.

**Definition 1.** The empty loading degree of vehicle \( k \) at departure is \( \lambda_k = \frac{\alpha_k}{\beta_k} \), where \( \alpha_k \) denotes the empty part of vehicle \( k \) at departure, \( \beta_k \) denotes all loadable parts of vehicle \( k \), respectively.

**Definition 2.** The empty loading ratio of vehicle \( k \) is \( r_k = \sum_{i=0}^{n} \sum_{j=0}^{n} \frac{\alpha_i}{\beta_k} d_{ij} x_{ijk} \), where \( x_{ijk} \) is a 0-1 variable which is intended to represent the path of the vehicle \( k \).

As can be seen from Definition 2, the empty loading ratio \( r_k \) consider the empty loading degree of vehicle \( k \) at departure and the travel distance of empty part (a pure consumption procedure). On the basis of definition 2, we have the definition as follows.

**Definition 3.** The empty loading ratio of a fleet is \( R = \sum_{k=1}^{K} r_k = \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=1}^{K} \frac{\alpha_i}{\beta_k} d_{ij} x_{ijk} \).

Since all vehicles are with a capacity \( Q \), the empty loading ratio of a fleet should be \( R = \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=1}^{K} \frac{\alpha_i}{\beta_k} d_{ij} x_{ijk} = \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=1}^{K} \frac{Q-l_k}{Q} d_{ij} x_{ijk} \), where \( l_k \) denotes the loading of vehicle \( k \) at departure.
One of the objectives is to minimize the empty loading ratio, that should be minimizing
\[ \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=1}^{K} \frac{Q - l_i}{Q} d_{ijk} x_{ijk}. \]

### 3. Mathematical Model and Solution Method

#### 3.1. Decision Variables

Before developing the mathematical model, we first introduce the decision variables to be used, as shown below.

- \( x_{ijk} \): A 0-1 variable, equals to 1 if \( e(v_i, v_j) \) is on the path of vehicle \( k \)
- \( l_k \): Loading of vehicle \( k \) at departure
- \( y_k \): Loading of vehicle \( k \) when leaving \( v_i \)

#### 3.2. Mathematical Model

Integrating all the above analysis, the model of the proposed problem is developed, as shown below.

Model \( M \):

\[
\begin{align*}
    f_1 &= \min \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=1}^{K} \frac{Q - l_i}{Q} d_{ijk} x_{ijk} \quad (1) \\
    f_2 &= \min \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=1}^{K} c_{ij} x_{ijk} \quad (2) \\
    \text{s.t.} & \quad \sum_{i=0}^{n} x_{hid} = \sum_{j=0}^{n} x_{ijk} \quad (i = 0,1,\ldots,n; k = 1,2,\ldots,K) \quad (3) \\
    & \quad \sum_{i=0}^{n} x_{ijk} \leq 1 \quad (j = 1,2,\ldots,n; k = 1,2,\ldots,K) \quad (4) \\
    & \quad Q(1 - x_{ijk}) + y_{ik} \geq y_{jk} + q_j \quad (i = 0,1,\ldots,n; j = 1,2,\ldots,n; k = 1,2,\ldots,K) \quad (5) \\
    & \quad y_{ik} \leq Q \quad (i = 0,1,\ldots,n; k = 1,2,\ldots,K) \quad (6) \\
    & \quad l_k = y_{ik} \quad (k = 1,2,\ldots,K) \quad (7) \\
    & \quad \sum_{i=0}^{n} \sum_{j=0}^{n} x_{ijk} \leq |S| - 1 \quad (k = 1,2,\ldots,K) \quad (8) \\
    & \quad x_{ijk} \in \{0,1\}; \quad y_{ik} \geq 0 \quad (i = 0,1,\ldots,n; j = 0,1,\ldots,n; k = 1,2,\ldots,K) \quad (9)
\end{align*}
\]

Objective (1) is to minimize the empty loading ratio of a fleet. Objective (2) is to minimize the transportation cost. Constraint (3)-(4) means the feasible paths of vehicles. Constraint (5) shows the changing relationship of decision variable \( y_{ik} \). Constraint (6) is the limit of vehicles’ capacity. Constraint (7) shows the loading of a vehicle at departure. Constraint (8) removes the sub-tours.

#### 3.3. Solution Method

Model \( M \) is a bi-objective optimization programming model, and a frequently-used solution method is the weighting method. The weighting method will set weights to the two objectives, and sum them up to a single objective. By solving the proposed single objective model several times, we can obtain an approximate Pareto front. However, the weighting method is not an exact method in strict sense, as
the appropriate weights are not available most of the time and some points on the Pareto front will be ignored. According to [9-12], the $\epsilon$ constraint method is capable of computing the entire Pareto front, which means we can develop an exact method based on the $\epsilon$ constraint method.

(1) Obtain a single objective model with $\epsilon$ constraint

We will transform objective (1) into the $\epsilon$ constraint, that is

$$\sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=1}^{K} \frac{Q-l_k}{Q} d_{ijk} x_{ijk} \leq \epsilon$$

(10)

Then, the single objective model with $\epsilon$ constraint is shown below.

Model $M_0$:

$$f_2 = \min \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=1}^{K} c_{ijk} x_{ijk}$$

s.t. Constraints (3)-(10)

(11)

(2) Determine the range of $\epsilon$

We have to determine the range of $\epsilon$ before the solution process. Before we start, we obtain two single objective models, as shown below.

Model $M_1$:

$$f_1 = \min \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=1}^{K} \frac{Q-l_k}{Q} d_{ijk} x_{ijk}$$

s.t. Constraints (3)-(9)

(12)

Model $M_2$:

$$f_2 = \min \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=1}^{K} c_{ijk} x_{ijk}$$

s.t. Constraints (3)-(9)

As we can see, $M_1$ and $M_2$ are two models with only one objective function. The lower bound of $\epsilon$ can be easily obtained by solving Model $M_1$. But for the upper bound of $\epsilon$, we have to solve Model $M_2$ firstly and obtain a solution $x^*$. Then, calculate the value of $f_1(x^*)$, and record it as $R^*$. Actually $R^*$ is the upper bound of $\epsilon$.

(3) Entire algorithm

After obtaining the single objective model with $\epsilon$ constraint and determining the bounds of $\epsilon$, we can initiate the procedure of the $\epsilon$ constraint method. The entire algorithm is shown below, where $\delta$ is a very small positive number.

Algorithm A:

Step 1: Transform the proposed model and obtain Model $M_0$, $M_1$, $M_2$.

Step 2: Solve Model $M_1$ and $M_2$, and obtain the upper bound $\epsilon_U$ and lower bound $\epsilon_L$ of $\epsilon$.

Step 3: Set $\epsilon = \epsilon_U$, $m = 1$.

Step 4: Solve Model $M_0$ and obtain the solution $x_m$. Add $x_m$ to the set of solutions $F$.

Step 5: Compute $f_1(x_m)$, set $\epsilon = f_1(x_m) - \delta$. If $\epsilon \geq \epsilon_L$, $m = m + 1$, return to Step 4.

Step 6: Delete dominate points from $F$. Output.
4. Computational Experiments
These instances for the experiments are generated on the basis of Golden’s test instances. All experiments are carried on a PC with 2.6 GHz CPU and 8 GB RAM. We apply Algorithm A to all these instances; the computed Pareto fronts are shown in figure 1 and the computation times are shown in table 1 (5 runs for each instance). We can see from table 1 that the Pareto fronts of instances can be computed in reasonable times. Besides, we compare the proposed Algorithm A with the weighting method (record it as Algorithm B). When applying Algorithm B, we increase the numbers of weight changes according to the number of solutions computed by Algorithm A, the results are also shown in figure 1. We can see from figure 1 that Algorithm B may ignore some points on the Pareto front, which confirms the defect of weighting method.

| Instance | Source | $n$ | Computation time (Second) |
|----------|--------|-----|---------------------------|
| I1       | Part of Golden 12 | 10  | 22                       |
| I2       | Part of Golden 12 | 11  | 33                       |
| I3       | Golden 12      | 12  | 82                       |
| I4       | Part of Golden 20 | 13  | 192                      |
| I5       | Part of Golden 20 | 14  | 384                      |
| I6       | Part of Golden 20 | 15  | 672                      |

Table 1. Computation times of all instances.

![Figure 1. Pareto fronts of all instances.](image-url)
5. Conclusions
Generally speaking, logistic companies mainly pursue the minimization of their own costs. However, the vehicles’ paths decided in this way are easy to lead to high empty loading ratio and a waste of logistic resources. In this research, we have studied a bi-objective VRP considering empty loading ratio. The definitions about the empty loading ratio are proposed firstly. A bi-objective routing model is developed, simultaneously minimizing the empty loading ratio and transportation cost. Then, the solution method based on $\varepsilon$ constraint method is proposed to compute the Pareto optimal solutions. Finally, the proposed mathematical model and solution method are tested by a series of instances; the results are compared with the weighting method.

The future researches may include three directions. First, we have provided the definition of empty loading ratio in this paper, and a more appropriate definition of empty loading ratio needs further study. Second, the proposed problem is a static problem, and more stochastic factors can be considered in order to be close to the reality. Lastly, it still needs a lot of work to develop more effective algorithms to solve bi-objective optimization problems.

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