Polar oscillations in magnetars

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Abstract. We study polar Alfvén oscillations of relativistic stars endowed with a strong global poloidal dipole magnetic field. Here we focus only on the axisymmetric oscillations which are studied by evolving numerically the 2D perturbation equations. We show that the spectrum of the polar Alfvén oscillations is discrete in contrast to that of axial Alfvén ones. We also show that the typical fluid modes are not significantly affected by the presence of the magnetic field.

1. Introduction
Up to now at least three giant flares have been detected in three different SGRs. The timing analysis of the decaying tail of the latest two giant flares revealed the existence of QPOs [1]. The most promising mechanism to produce these QPOs are the magnetar models. The attempts to explain the above QPO frequencies by using the crust torsional oscillations have been based both in Newtonian and in general relativistic dynamics (e.g., [2, 3]). These attempts were partially successful, but it has been also realized the difficulty to explain all the observed frequencies by using only the crust torsional oscillations. While, in an attempt to explain the QPOs without the crust torsional oscillations, Levin [4] using a toy model suggested that the torsional Alfvén oscillations could not form a discrete spectrum but instead they should form a continuum, while Sotani et al. [5] showed that the spectrum of the torsional Alfvén oscillations of relativistic magnetars forms a continuum and that there exist two distinct families as the so called “lower” and “upper” QPOs. They also figure out that the observed QPO frequencies in SGRs can be explained by using the frequencies of torsional Alfvén and the crust torsional oscillations.

One could actually argue that the features of torsional Alfvén oscillations are quite well understood. However, there are only a few studies of polar oscillations of the magnetized stars and it is still unknown whether spectrum of the polar Alfvén oscillations is discrete or continuous. Moreover, some of the higher QPO frequencies observed may be related to polar and not to axial Alfvén oscillations. Finally, polar oscillations are related to the emission of gravitational waves. So, in order to shed some light to the various features of polar Alfvén oscillations, we performed 2D numerical evolutions of the perturbation equation of relativistic magnetar models endowed with a global poloidal dipole magnetic field. The more detailed study can be seen in [6].

2. Equilibrium configuration
We assume that the background models of the strongly magnetized stars are spherically symmetric and non-rotating. This choice is justified since all known magnetars rotate very slowly and even for the case of magnetars the magnetic energy is much smaller than the gravitational
binding energy. On this stellar model we superimpose a dipole magnetic field. We assume that the star consists of a perfect fluid and in addition we adopt “the ideal MHD approximation”.

3. Numerical Results
For the time evolution of the perturbation equations, we use the iterated Crank-Nicholson scheme. In order to eliminate the spurious higher frequencies, we also add a 4th-order Kreiss-Oliger dissipation into the evolution equations. As numerical code tests, we compare the stellar oscillation frequencies without magnetic field with the previous results in [7].

The various features of polar oscillations are examined in two ways. The first involves checking of the FFT amplitude at various points inside the star. If the spectrum is continuous the peaks in FFT will depend on the position, while if the peaks are independent of the observer’s position then they correspond to a discrete spectrum. The second check involves the study of the phase of oscillation for each peak frequency in FFT, i.e., for the discrete spectrum the phase should be constant throughout the star. Figure 1 shows the FFT of the perturbation function \( v \) at various points inside the star with \( B = 10^{16} \) G, where \( v \) is corresponding to the displacement vector in the \( \theta \) direction. In this figure the three different lines correspond to different angular positions while the different panels correspond to different radial positions. It should be noted that the FFT of the other perturbation function, \( w \) which denotes the displacement vector in the radial direction, shows that the peaks correspond exactly to the same frequencies as in figure 1 and obviously the peaks are independent of the observer’s position. The peak at 2754 Hz corresponds to the \( 2f \) mode, while the lower peak at 300 Hz corresponds to the polar Alfvén oscillation mode \( 2a_0 \). Next we study the phase of the specific Alfvén oscillation mode \( 2a_0 \). In figure 2 we show the phase of the oscillation mode \( 2a_0 \) for a magnetar with \( B = 10^{16} \) G. The left and right panels correspond to the phases for the perturbation functions \( w \) and \( v \), respectively. We observe that the phases for both functions \( w \) and \( v \) are almost constant except for some regions around \( \theta \sim \pi/3 \) for the function \( w \) and a part of \( r/R \sim 0.8 \) for the function \( v \). We identify that these two regions with somehow strange behavior of the phase are related to the due to numerical loss of accuracy in the calculation of the effective amplitudes, where the eigenfunctions are almost zero because these are nodal points (see the figures 3 and 4 in [6]). Thus we can argue that the phase of the oscillation modes is independent from the position. With the above two tests we conclude that the Alfvén oscillations of polar parity are described by a discrete spectrum, in contrast to the spectrum of the axial Alfvén oscillations. This feature seems to be very similar to the case of inertial modes of rotating stars. That is, the spectrum of the axial inertial modes seems to be continuous, at least in the slow rotation approximation, while the polar modes admit a discrete spectrum.

Additionally, we examine how the fluid modes depend on the magnetic field strength. In
Figure 2. The phase of the fundamental polar Alfvén mode with $\ell = 2$ for a magnetar with $B = 10^{16}$ G. The left and right panels correspond to the perturbation functions $w$ and $v$.

Figure 3. The frequencies of the $f$ and $p_1$ modes with harmonic indexes $\ell = 2$, 3 and 4.

From figure 3, we plot the frequencies of the $f$ and $p_1$ modes with harmonic indexes $\ell = 2$, 3 and 4 as functions of magnetic field strength. The marks correspond to frequencies of the modes of the magnetized stars while the horizontal dashed lines are the frequencies for the non-magnetized ones. From this figure, we can see that the effect of magnetic field on the frequencies of fluid modes can be traced only when the magnetic field becomes stronger than a few times $10^{16}$ G.

4. Conclusion
In this work, we study the polar type oscillations of strongly magnetized neutron stars. By using the 2D time evolutions of the perturbation equations, we estimate the Alfvén oscillation modes as well as the fluid modes. Actually, by examining the FFT amplitude at various points inside the star and phase of the corresponding frequencies, we conclude that the Alfvén oscillations with polar parity are described by a spectrum consisting only by discrete modes. This is the first study of its kind for axisymmetric polar Alfvén oscillations, it remains open to understand the form of the spectrum for non-axisymmetric perturbations while more complicated forms of the magnetic field geometry should be assumed. For example, the inclusion of a toroidal component as in [8] will affect the form of the oscillation spectrum.

References
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