Logical XOR gate response in a quantum interferometer: A spin dependent transport

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We examine spin dependent transport in a quantum interferometer composed of magnetic atomic sites based on transfer matrix formalism. The interferometer, threaded by a magnetic flux \( \phi \), is symmetrically attached to two semi-infinite one-dimensional (1D) non-magnetic electrodes, namely, source and drain. A simple tight-binding model is used to describe the bridge system, and, here we address numerically the conductance-energy and current-voltage characteristics as functions of the interferometer-to-electrode coupling strength, magnetic flux and the orientation of local the magnetic moments associated with each atomic site. Quite interestingly it is observed that, for \( \phi = \phi_0/2 \) (\( \phi_0 = ch/e \), the elementary flux-quantum) a logical XOR gate like response is observed, depending on the orientation of the local magnetic moments associated with the magnetic atoms in the upper and lower arms of the interferometer, and it can be changed by an externally applied gate magnetic field. This aspect may be utilized in designing a spin based electronic logic gate.

I. INTRODUCTION

With the rapid advancement in nanoscience and nanotechnology, specially in nanofabrication techniques [1], study of spin dependent transport in mesoscopic [2] systems has emerged as one of the most challenging topics in the last few decades. Analysis of spin transport and spin dynamics is essential to understand and develop the field - ‘spintronics’ [3, 4]. With the discovery of Giant Magneto-resistance (GMR) based magnetic field sensors [5] in 1994, remarkable development has taken place in the field of magnetic data storage applications and quantum computation techniques. A drastic enhancement in computation time has been made possible using the idea of quantum coherence and spin entanglement. Manifestation of coherence is one of the most important aspect of mesoscopic systems. It is evident from theoretical [6–12] and experimental [13–15] studies of spin transport through quantum confined nanostructures that the conductance of such systems depends on the spin state of electrons passing through the system and it can be controlled by an externally applied magnetic field. But measurement of current through these 1D nanostructures does not reveal the feature of quantum coherence, as it is detectable through interference experiments, most notably Aharonov-Bohm (AB) interferometry [16]. In order to study the effect of coherence, spin dependent transport has been studied in various types of ring type conductors or two path devices [17] such as an AB ring or AB type interferometer with embedded quantum dots, with a magnetic flux \( \phi \) penetrating the area enclosed yielding a flux dependent spin transmission probability. The study of spin dependent transport through interferometric geometries are important for further development in quantum information processing as well as for designing spin based nano-devices. The key idea of designing spin dependent nano-electronic devices is based on the concept of quantum interference effect [18–23], and it is generally preserved throughout the sample having dimension smaller or comparable to the phase coherence length. In realistic situation, experimentally sizable rings are typically of the order of 0.4-0.6 \( \mu m \). Therefore, ring type conductors or two path devices are ideal candidates where the effect of quantum interference can be exploited [24].

Recently, spin transport through AB type interferometers with embedded quantum dots has drawn much attention because of its demonstration of several physical phenomena e.g., quantum phase transitions, resonant tunneling and many body correlation effects. It opens a new area of study of spin transport, which includes spin dependent conductance modulation, spin filtering, spin switching, spin detecting mechanisms, etc. Conductance of such mesoscopic systems is associated with the transmission probability (\( T \)) of electrons, which can be calculated numerically by several methods like, mode matching techniques [25], Green’s function approach [26–28] or transfer matrix method [29, 30].

Aim of the present paper is to study the spin dependent transport through an AB type interferometric geometry made up of magnetic atomic sites. The interferometer, threaded by a magnetic flux \( \phi \), is attached symmetrically to two 1D semi-infinite non-magnetic electrodes. A simple tight-binding Hamiltonian is used to describe the system where all the calculations are done using transfer matrix formalism. Spin dependent conductance is calculated using the Landauer formula and also current-voltage characteristics are computed through the Landauer-Büttiker formalism [31, 32]. We explore several important features of spin transport with this simple, yet interesting geometry. Quite nicely we see that, at the half flux-quantum value of \( \phi \) (\( \phi = \phi_0/2 \)), the system exhibits XOR gate like response depending on the orientations of the local magnetic moments in the upper and lower arms of the interferometer, that can be changed by an external magnetic field. To the best of our knowledge, the spin based XOR gate response in such a simple geometry has

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II. MODEL AND SYNOPSIS OF THE THEORETICAL BACKGROUND

Let us begin by referring to Fig. 1 where a quantum interferometer, penetrated by an AB flux $\phi$ (measured in unit of the elementary flux-quantum $\phi_0 = ch/e$), is attached symmetrically to two non-magnetic electrodes, viz, source and drain. The magnetic conductor i.e., the interferometer is composed of six magnetic atoms, four of which are placed at four different corners of the AB type interferometer and the rest two are connected to the electrodes directly. The side-attached electrodes consist of infinite number of non-magnetic sites labeled as 0, $-1$, $-2$, ..., $\infty$ for the left electrode (source) and 7, 8, 9, ..., $\infty$ for the right electrode (drain). Each magnetic atomic site has a local magnetic moment associated with it. The direction of magnetization in each magnetic site is chosen to be arbitrary and specified by angles $\theta_n$ and $\varphi_n$ in spherical polar co-ordinate system for the $n$th atomic site. Here, $\theta_n$ represents the angle between the direction of magnetization and the chosen Z axis, and $\varphi_n$ represents the azimuthal angle made by the projection of the local moment on X-Y plane with the X axis.

The Hamiltonian for the full system i.e., the electrode-interferometer-electrode can be described as,

$$H = H_D + H_L + H_R + H_{LD} + H_{DR}$$  \hspace{1cm} (1)

where, $H_D$ corresponds to the Hamiltonian of the AB type interferometer device made up of magnetic atomic sites. $H_{L(R)}$ represents the Hamiltonian for the left electrode i.e., source (right electrode i.e., drain), and $H_{LD(DR)}$ is the Hamiltonian representing the device-electrode coupling.

The spin polarized tight-binding Hamiltonian for the interferometer can be written within the non-interacting electron picture in the form,

$$H_D = \sum_{n=1}^{6} c_n^\dagger \left( \epsilon_0 - \mathbf{h}_n \cdot \vec{\sigma} \right) c_n + \sum_{i=2}^{5} \left( c_{i-1}^\dagger t e^{i\Theta} c_i + + c_{i+1}^\dagger t e^{-i\Theta} c_i \right) + \left( c_4^\dagger t c_6 + c_6^\dagger t c_4 \right)$$  \hspace{1cm} (2)

where,

$$c_n = \begin{pmatrix} c_{n\uparrow}^\dagger & c_{n\downarrow}^\dagger \end{pmatrix}; \quad \epsilon_0 = \begin{pmatrix} \epsilon_0 & 0 \\ 0 & \epsilon_0 \end{pmatrix}; \quad t e^{i\Theta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix};$$

$$\mathbf{h}_n \cdot \vec{\sigma} = h_n \begin{pmatrix} \cos \theta_n & \sin \theta_n e^{-i\varphi_n} \\ \sin \theta_n e^{i\varphi_n} & -\cos \theta_n \end{pmatrix}$$

In Eq. 2, 1st term corresponds to the effective on-site energies of the interferometer. $\epsilon_0$’s are the site energies, while the $\mathbf{h}_n \cdot \vec{\sigma}$ term represents the interaction of the spin ($\sigma$) of the injected electron with the local magnetic moment placed at the site $n$ with strength $h_n$. $\theta_n$ and $\varphi_n$ represent the orientation of the local magnetic moment situated at the site $n$ as mentioned earlier. This term is responsible for spin flip scattering at the sites. Flipping of spin violates spin conservation in the transport process through the magnetic conductor which may provide much impact in spintronic applications and we will discuss about it in the forthcoming sub-sections. Second term describes the nearest-neighbor hopping integral between the sites, at the corners of the interferometer, modified due to the presence of AB flux $\phi$ which is incorporated by the term $\Theta = 2\pi\phi/4\phi_0$. The 3rd and 4th terms represent the nearest-neighbor hopping between the atomic sites 1, 2 and 4, 6, respectively.

Similarly, the Hamiltonian $H_{L(R)}$ can be expressed as,

$$H_{L(R)} = \sum_i c_i^\dagger \epsilon_{L(R)} c_i + \sum_i \left( c_i^\dagger t_{L(R)} c_{i+1} + \text{h.c.} \right)$$  \hspace{1cm} (3)

where $\epsilon_{L(R)}$’s are the site energies of the electrodes and $t_{L(R)}$ is the hopping strength between the nearest-neighbor sites of the left (right) electrode. In this Hamiltonian, $\epsilon_{L(R)}$ and $t_{L(R)}$ are in the form,

$$\epsilon_{L(R)} = \begin{pmatrix} \epsilon_{L(R)} & 0 \\ 0 & \epsilon_{L(R)} \end{pmatrix}$$

where, $\epsilon_{L(R)}$ is the energy of the left (right) electrode site.

FIG. 1: (Color online). Schematic view of a quantum interferometer, threaded by an AB flux $\phi$, attached to two semi-infinite 1D non-magnetic electrodes, viz, source and drain. The filled purple and blue circles correspond to the magnetic and non-magnetic atoms, respectively.
\[ t_{L(R)} = \begin{pmatrix} t_{L(R)} & 0 \\ 0 & t_{L(R)} \end{pmatrix} \]

In the same fashion, the conductor-electrode coupling Hamiltonian is described by,

\[ H_{L(D)DR} = \left( c_{0(6)}^\dagger t_{L(D)DR} c_{1(7)} + c_{1(7)}^\dagger t_{L(D)DR} c_{0(6)} \right) \]

where, \( t_{L(D)DR} \) being the device-to-electrode coupling strength.

Now, we start with the Schrödinger equation,

\[ H|\Phi\rangle = E|\Phi\rangle \]

where,

\[ |\Phi\rangle = \sum_i |\psi_i\rangle \]

Here, \(|\Phi\rangle\) is expressed as a linear combination of spin up and spin down Wannier states.

In order to calculate the spin dependent transmission probabilities through the interferometer, first we map the two-dimensional (2D) geometry into 1D structure by

![Diagram of renormalized 1D geometry](image)

FIG. 2: (Color online). Schematic diagram of the renormalized 1D geometry. In this renormalized geometry the total number of atomic sites \( N = 4 \), and, here we label the atomic sites in such a way (viz, 1, 2, 4 and 6) to understand the renormalized version of Fig. 1 much clearly. It shows that the atomic sites 3 and 5 of Fig. 1 get renormalized.

renormalization procedure. The schematic view of the 1D geometry is shown in Fig. 2.

We start renormalizing the interferometric geometry by writing down the difference equations at the six sites of the interferometer. They are given as follows,

\[ (E - \epsilon_1)\psi_1 = t\psi_2 + t_{LD}\psi_0 \]  
\[ (E - \epsilon_2)\psi_2 = t\psi_1 + t_{23}\psi_3 + t_{25}\psi_5 \]  
\[ (E - \epsilon_3)\psi_3 = t_{32}\psi_2 + t_{34}\psi_4 \]  
\[ (E - \epsilon_4)\psi_4 = t_{43}\psi_3 + t_{45}\psi_5 + t\psi_6 \]  
\[ (E - \epsilon_5)\psi_5 = t_{52}\psi_2 + t_{54}\psi_4 \]  
\[ (E - \epsilon_6)\psi_6 = t_{64}\psi_4 + t_{DR}\psi_7 \]

Here, \( \epsilon_n = (\epsilon_0 - \epsilon_n\sigma) \), \( t_{23} = t_{34} = t_{45} = t_{52} = t e^{i\theta} \), and \( t_{25} = t_{54} = t_{43} = t_{32} = t e^{-i\theta} \),

\[ E = \begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix} \]

and \( \psi_n = \begin{pmatrix} \psi_{n\uparrow} \\ \psi_{n\downarrow} \end{pmatrix} \).

Substituting \( \psi_3 \) and \( \psi_6 \) using Eqs. (9) and (11) we obtain the renormalized difference equations for sites 2 and 4 as,

\[ (E - \epsilon'_2)\psi_2 = t\psi_1 + t_{24}\psi_4 \]

\[ (E - \epsilon'_4)\psi_4 = t\psi_6 + t_{42}\psi_2 \]

Due to renormalization, the site energies of the 2nd and 4th sites get modified, and they are,

\[ \epsilon'_2 = \epsilon_2 + t_{23} \cdot (E - \epsilon_3)^{-1} \cdot t_{32} \]
\[ + t_{25} \cdot (E - \epsilon_5)^{-1} \cdot t_{52} \]

\[ \epsilon'_4 = \epsilon_4 + t_{43} \cdot (E - \epsilon_3)^{-1} \cdot t_{34} \]
\[ + t_{45} \cdot (E - \epsilon_5)^{-1} \cdot t_{54} \]

The hopping term between these sites (2 and 4) is also modified as,

\[ t'_{24} = t_{23} \cdot (E - \epsilon_3)^{-1} \cdot t_{34} \]
\[ + t_{25} \cdot (E - \epsilon_5)^{-1} \cdot t_{54} \]

and \( t'_{42} \) is the hermitian conjugate of \( t_{24}' \).

With this renormalized 1D geometry, we use transfer matrix method to calculate spin dependent transmission probabilities \((T)\) and current-voltage \((I-V)\) characteristics through the bridge system.

For an arbitrary site \( n \), the transfer matrix \((P)\) can be defined in terms of the wave amplitudes of its neighboring \((n+1)\) and \((n-1)\) sites as,

\[ \begin{pmatrix} \psi_{n+1\uparrow} \\ \psi_{n+1\downarrow} \\ \psi_{n\uparrow} \\ \psi_{n\downarrow} \end{pmatrix} = P \begin{pmatrix} \psi_{n\uparrow} \\ \psi_{n\downarrow} \\ \psi_{n-1\uparrow} \\ \psi_{n-1\downarrow} \end{pmatrix} \]

In our case, the transfer matrix equation for the renormalized geometry relating the wave amplitudes at sites 0, -1 and \( N + 1, N + 2 \) becomes,

\[ \begin{pmatrix} \psi_{N+2\uparrow} \\ \psi_{N+2\downarrow} \\ \psi_{N+1\uparrow} \\ \psi_{N+1\downarrow} \end{pmatrix} = M \begin{pmatrix} \psi_{0\uparrow} \\ \psi_{0\downarrow} \\ \psi_{-1\uparrow} \\ \psi_{-1\downarrow} \end{pmatrix} \]

where, \( N \) corresponds to the total number of sites in the 1D magnetic device after renormalization process. For our renormalized 1D system, \( N = 4 \). \( M \) being the transfer matrix for the full system and it can be expressed as,

\[ M = M_R \cdot P_6 \cdot P_4 \cdot P_2 \cdot P_1 \cdot M_L \]

where, \( P_1, P_2, P_4 \) and \( P_6 \) represent the transfer matrices for the sites labeled as 1, 2, 4 and 6, respectively. \( M_L \) and \( M_R \) correspond to the transfer matrices for the boundary sites at the left and right electrodes, respectively.
To evaluate the transmission probabilities of up and down spin electrons, we calculate the explicit form of \( M \), determining all the transfer matrices \( (M_R, P_6, P_4, P_2, P_1) \) and \( M_L \). The matrices are given below.

\[
P_1 = \begin{pmatrix}
E - \epsilon_0 + h_0 \cos \theta & h_1 \sin \theta e^{-i \varphi_1} & -i t_{LD} & 0 \\
h_1 \sin \theta, e^{i \varphi_1} & E - \epsilon_0 - h_1 \cos \theta & 0 & -i t_{LD} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0
\end{pmatrix}
\]

\[
P_6 = \begin{pmatrix}
E - \epsilon_0 + h_0 \cos \theta & h_0 \sin \theta e^{-i \varphi_0} & -i t_{DR} & 0 \\
- h_0 \sin \theta, e^{i \varphi_0} & E - \epsilon_0 - h_0 \cos \theta & 0 & -i t_{DR} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0
\end{pmatrix}
\]

\[
M_L = \begin{pmatrix}
t_{LD} e^{i \beta L} & 0 & 0 & 0 \\
0 & t_{LD} e^{i \beta L} & 0 & 0 \\
0 & 0 & e^{i \beta L} & 0 \\
0 & 0 & 0 & e^{i \beta L}
\end{pmatrix}
\]

\[
M_R = \begin{pmatrix}
e^{i \beta R} & 0 & 0 & 0 \\
0 & e^{i \beta R} & 0 & 0 \\
0 & 0 & e^{i \beta R} & 0 \\
0 & 0 & 0 & e^{i \beta R}
\end{pmatrix}
\]

\[
P_4 = \begin{pmatrix}
\frac{1}{t_{LD}} - \frac{\epsilon_0}{t_{LD}} & -\frac{\epsilon_0}{t_{LD}} & -\frac{t_{LD}}{t_{RD}} & -\frac{t_{LD}}{t_{RD}} \\
\frac{1}{t_{LD}} & \frac{1}{t_{LD}} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

\[
P_2 = \begin{pmatrix}
\alpha_1 - \frac{\alpha_2}{\beta_3} & -\frac{t_{LD} \alpha_2}{\beta_3} & -\frac{t_{LD} \alpha_2}{\beta_3} & 0 \\
\frac{1}{\beta_3} & \frac{1}{\beta_3} & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0
\end{pmatrix}
\]

where,

\[
\alpha_1 = t_{24}[2][2](E - \epsilon_0[1, 1]) + t_{24}[1, 2] \epsilon_2[2, 1] \\
\alpha_2 = \epsilon_2[1, 2] t_{24}[2, 2] + t_{24}[2, 1] (E - \epsilon_2[2, 2]) \\
\alpha = t_{24}[1, 1] t_{24}[2, 2] - t_{24}[1, 2] t_{24}[2, 1] \\
\beta_1 = \epsilon_2[2, 1] t_{24}[2, 1] + t_{24}[2, 1] (E - \epsilon_2[1, 1]) \\
\beta_2 = t_{24}[1, 1] (E - \epsilon_2[2, 2]) + t_{24}[2, 1] \epsilon_2[2, 1] \\
\beta = t_{24}[2, 2] t_{24}[2, 1] - t_{24}[2, 1] t_{24}[2, 1] = \alpha
\]

In the above expressions \( \epsilon'[i, j] \) corresponds to \( ij \)-th elements of the matrix \( \epsilon' \). Similarly we call the matrix elements of \( \epsilon' \).

The diagonal forms of \( M_L \) and \( M_R \) can be explained as follows. Due to translational invariance of the semi-infinite electrodes the wave amplitudes at the sites of the electrodes \( (L \text{and} \ R) \) can be written in Bloch wave form,

\[
\psi_{n} = A e^{i n k a} \\
= A e^{i n \beta (L,R)}
\]

Here, \( \beta(L,R) = k a, \) \( k \) is the wave vector and \( a \) being the lattice spacing of the discrete model. For the 0th site of the left electrode \( (L) \) we can write,

\[
\psi_{0L} = e^{i \beta L} \psi_{-1L} \\
\psi_{0L} = e^{i \beta L} \psi_{-1L}
\]

(22)

where \( \beta_L \) is defined by the following energy dispersion relation as,

\[
E = \epsilon_L + 2 t_L \cos \beta_L.
\]

(23)

Now, the difference equation for the 0th site (the boundary site of the left electrode) is in the form,

\[
(E - \epsilon_L) \psi_{0, \uparrow(\downarrow)} = t_L \psi_{-1, \uparrow(\downarrow)} + t_{LD} \psi_{1, \uparrow(\downarrow)}
\]

(24)

Thus,

\[
\psi_{1, \uparrow(\downarrow)} = \frac{(E - \epsilon_L)}{t_{LD}} \psi_{0, \uparrow(\downarrow)} - \frac{t_{LD} \psi_{-1, \uparrow(\downarrow)}}{t_{LD}} \\
\psi_{1, \uparrow(\downarrow)} = \frac{t_{LD} \psi_{0, \uparrow(\downarrow)} - t_{LD} \psi_{-1, \uparrow(\downarrow)}}{t_{LD}}
\]

(25)

Using Eqs. (22) and (25) we can construct \( M_L \) which shows the above diagonal form. In an exactly similar way we get the diagonal form of \( M_R \).

Let us first discuss the case of up spin incidence. Considering the whole system (left electrode-conductor-right electrode), the wave amplitudes at sites 0, \(-1\) and \(N + 1, \) \(N + 2\) can be written in terms of reflection and transmission amplitudes as,

\[
\psi_{-1\uparrow} = e^{-i \beta L} + \rho^{\uparrow \uparrow} e^{i \beta L} \\
\psi_{-1\downarrow} = \rho^{\uparrow \downarrow} e^{i \beta L} \\
\psi_{0\uparrow} = 1 + \rho^{\uparrow \uparrow} \\
\psi_{0\downarrow} = \rho^{\uparrow \downarrow}
\]

and,

\[
\psi_{N+2\uparrow} = \tau^{\uparrow \uparrow} e^{i (N+2) \beta_R} \\
\psi_{N+2\downarrow} = \tau^{\uparrow \downarrow} e^{i (N+2) \beta_R} \\
\psi_{N+1\uparrow} = \tau^{\uparrow \uparrow} e^{i (N+1) \beta_R} \\
\psi_{N+1\downarrow} = \tau^{\uparrow \downarrow} e^{i (N+1) \beta_R}
\]

(27)

where, \( \rho^{\uparrow \uparrow} \) represents the reflection amplitude of an up spin as an up spin, and \( \rho^{\uparrow \downarrow} \) denotes the the reflection amplitude of an up spin as a down spin. \( \tau^{\uparrow \uparrow} \) corresponds to the transmission amplitude of an up spin without any flipping, whereas \( \tau^{\uparrow \downarrow} \) denotes the spin flip transmission amplitude. We solve the transfer matrix equation (Eq. (19)) substituting the explicit expressions of the wave amplitudes from Eqs. (20) and (27).
The transmission probabilities $T_{\uparrow\uparrow}$ and $T_{\uparrow\downarrow}$ are defined by the ratio of the transmitted flux to the incident flux as,

$$T_{\uparrow\uparrow} = \frac{t_R \sin \beta_R}{t_L \sin \beta_L} |\tau_{\uparrow\uparrow}|^2$$

$$T_{\uparrow\downarrow} = \frac{t_R \sin \beta_R}{t_L \sin \beta_L} |\tau_{\uparrow\downarrow}|^2$$

(28)

Therefore, the total transmission probability of an up spin becomes,

$$T_{\uparrow} = T_{\uparrow\uparrow} + T_{\uparrow\downarrow}$$

(29)

In a similar way, we can calculate the total transmission probability for the case of a down spin incidence as,

$$T_{\downarrow} = T_{\downarrow\uparrow} + T_{\downarrow\downarrow}$$

(30)

Based on the Landauer conductance formula [2], the conductance $g_{\sigma\sigma'}$ through the interferometer can be calculated. At much low temperatures and bias voltage, it can be expressed in the from,

$$g_{\sigma\sigma'} = \frac{2}{\hbar} T_{\sigma\sigma'}$$

(31)

The spin dependent current flowing through the interferometric geometry can be determined from the expression [2],

$$I_{\sigma\sigma'}(V) = \frac{e}{\hbar} \int_{-\infty}^{+\infty} (f_S - f_D) T_{\sigma\sigma'}(E) \, dE$$

(32)

where, $f_{S(D)} = f \left( E - \mu_{S(D)} \right)$ gives the Fermi distribution function of the two electrodes with the electrochemical potential $\mu_{S(D)} = E_F \pm eV/2$.

Due to spin flip scattering individual spin currents ($I_{\uparrow\uparrow}$ and $I_{\downarrow\downarrow}$) are no longer constant quantities, whereas the total spin is conserved. However, we have defined spin currents by introducing the effect of spin-flipping. They are expressed as: $I_{\uparrow} = I_{\uparrow\uparrow} + I_{\uparrow\downarrow}$ and $I_{\downarrow} = I_{\downarrow\uparrow} + I_{\downarrow\downarrow}$. Non conservation of spin inside the magnetic quantum interferometer gives rise to a torque known as spin flip induced spin torque [35]. We can define this spin torque through the relation,

$$\tau_{\text{flip}} = \partial \langle \hat{S} \rangle / \partial t = \frac{i}{\hbar} \langle [\hat{H}, \hat{S}] \rangle$$

(33)

where, $\hat{S} = \frac{1}{2} \hat{\sigma}$. The spin torque may produce an angular displacement in the orientations of the local magnetic moments associated with each magnetic site.

III. NUMERICAL RESULTS AND DISCUSSION

Spin dependent transport properties through the magnetic conductor having interferometric geometry are studied in various aspects of interferometer-to-electrode coupling strength, AB flux $\phi$, external magnetic field and spin flipping. Here, we assume that the two non-magnetic (NM) electrodes are identical in nature. For our illustrative purposes, let us first mention the values of the different parameters those are considered for the numerical calculations. The on-site energies ($\epsilon_0$) in the interferometer are chosen to be 0. Magnitudes of all the local magnetic moments ($\hbar \phi_n$), associated with the atomic sites of the interferometer, are fixed at 0.5 and henceforth we call it simply as $\hbar$. The hopping integral between the nearest-neighbor sites of the interferometer is set at $t = 3$, while, for the NM electrodes it is chosen as $t_L = t_R = 4$. The site energies ($\epsilon_{L(R)}$) of all the sites in the electrodes are put to 0. The azimuthal angles $\varphi_n$ for all $n$ are fixed at zero and also the equilibrium Fermi energy $E_F$ of the conductor is set at 0. Here, we choose the units $\hbar = e = h = 1$ for the sake of simplicity.

Throughout the analysis, all the essential features of spin transport are studied for the two distinct regimes, depending on the strength of coupling of the interferometer to the NM electrodes.

Case 1: Weak-coupling limit.

This regime is typically defined by the condition $t_{LD(LR)} << t$. Here, we choose the values of the hopping parameters as, $t_{LD} = t_{DR} = 0.5$.

Case 2: Strong-coupling limit.

This limit is described by the condition $t_{LD(LR)} \sim t$. In this regime, we set the values of the hopping strengths as, $t_{LD} = t_{DR} = 2.5$. 

FIG. 3: (Color online). Typical conductance $g$ as a function of the magnetic flux $\phi$, where the green and magenta curves correspond to the weak- and strong-coupling limits, respectively. Other parameters are as follows: $E = 0$ and $\theta_4 = \theta_5 = \pi/3$. 
A. Variation of conductance with magnetic flux $\phi$

As representative examples, first in Fig. 4 we plot the variation of $g_{\uparrow\downarrow}$ and $g_{\uparrow\downarrow}$ for the interferometer as a function of magnetic flux $\phi$. The results are computed for the injecting electron energy $E = 0$, where the green and magenta curves correspond to the weak- and strong-coupling limits, respectively. The direction of local magnetization of the magnetic atoms (labeled as 3 and 5 in Fig. 1) in the interferometric arms are chosen to be oriented at an angle $\pi/3$ with respect to the preferred $+Z$ direction. All the other moments are aligned along $+Z$ direction. Figure 3 shows that both $g_{\uparrow\downarrow}$ and $g_{\uparrow\downarrow}$ vary periodically with $\phi$ showing $\phi_0$ flux-quantum periodicity.

For a symmetrically connected interferometer i.e., having identical configuration in the upper and lower arms ($\theta_1 = \theta_5$), the transmission probability drops exactly to zero at $\phi = \phi_0/2$ ($= 0.5$ in our chosen unit) and it can be shown very easily by simple mathematical calculation as follows.

For a symmetrically connected interferometer, the wave functions passing through the upper and lower arms of the interferometer are given by,

$$
\psi_1 = \psi_0 e^{\gamma_1} \int \vec{A} \, d\vec{r}
$$

$$
\psi_2 = \psi_0 e^{\gamma_2} \int \vec{A} \, d\vec{r}
$$

(34)

where, $\gamma_1$ and $\gamma_2$ are used to indicate the two different paths of electron propagation along the two arms of the interferometer. $\psi_0$ denotes the wave function in absence of magnetic flux $\phi$ and it is same for both upper and lower arms as the interferometer is symmetrically coupled to the electrodes. $\vec{A}$ is the vector potential associated with the magnetic field $\vec{B}$ by the relation $\vec{B} = \vec{\nabla} \times \vec{A}$. Hence the probability amplitude of finding the electron passing through the interferometer can be calculated as,

$$
|\psi_1 + \psi_2|^2 = 2|\psi_0|^2 + 2|\psi_0|^2 \cos \left( \frac{2\pi \phi}{\phi_0} \right)
$$

(35)

where, $\phi = \oint \vec{A} \, d\vec{r} = \oint \vec{B} \, d\vec{s}$ is the flux enclosed by the interferometer.

Here, it is clearly observed from Eq. 35 that at $\phi = \phi_0/2$, the transmission probability of an electron exactly drops to zero. This aspect can be utilized to design an XOR gate which we will describe in the forthcoming sub-sections. The $g$-$E$ spectra shows that the typical conductance gets enhanced significantly with the increase of coupling strength. Beside this, it is also observed that the transmission probability due to spin flipping is considerably smaller than the pure spin transmission.

B. Variation of conductance with polar angle $\theta$ of the magnetic moments

In Fig. 5 we present the variation of $g_{\uparrow\downarrow}$ and $g_{\uparrow\downarrow}$ with respect to $\theta$, where $\theta$ corresponds to the angle made by the local magnetic moments in the interferometric arms (sites labeled as 3 and 5) with the preferred $+Z$ direction. The orientations of local magnetic moments can be changed by applying an external magnetic field. All the other...
moments are aligned along +Z direction. The results are calculated for the typical magnetic flux \( \phi = \phi_0/4 \) and the injecting electron energy \( E = 0 \). In this case \( g_{\uparrow \uparrow} \) shows 2\( \pi \) periodicity as a function of \( \theta \) both for the weak- and strong-coupling limits. But, for up and down orientations of the local magnetic moments in the interferometric arms i.e., for \( \theta = 0 \) or any integer multiple of \( \pi \), no spin flip takes place, and accordingly, \( g_{\uparrow \uparrow} \) drops to zero for these configurations.

Explanation of zero transmission probability for spin flipping is given as follow. Spin flip occurs due to the presence of the term \( \hat{h}_s \sigma \) in the Hamiltonian (see Eq. (22)), \( \sigma \) being the Pauli spin matrix with components \( \sigma_x, \sigma_y \) and \( \sigma_z \) for the injecting electron. The spin flipping is caused because of the operators \( \sigma_+ (= \sigma_x + i\sigma_y) \) and \( \sigma_- (= \sigma_x - i\sigma_y) \), respectively. For the local magnetic moments oriented along \( \pm Z \) axes, \( \hat{h}_s (= h_x \sigma_x + h_y \sigma_y + h_z \sigma_z) \) becomes equal to \( h_z \sigma_z \). Accordingly, the Hamiltonian does not contain \( \sigma_x \) and \( \sigma_y \) and so as \( \sigma_+ \) and \( \sigma_- \), which provides zero flipping for up or down orientation of magnetic moments. For this typical configuration of the localized magnetic moments mentioned above (\( \pm Z \) axis) the spin flip torque vanishes which is clearly seen from Eq. (23).

On the other hand, for any other orientation (apart from up or down configuration) of local magnetic moments a non-vanishing spin transfer torque appears which can provide angular displacements of these moments. Due to these displacements, an additional contribution can occur to the spin transmission which is neglected in our present study.

C. XOR gate response

1. Conductance-energy characteristics

Let us now describe how such a simple geometric model can be implemented as an XOR gate. For the forthcoming discussion, we set the AB flux \( \phi = \phi_0/2 \).

In Figs. 5 and 6 we show conductance-energy \((g\cdot E)\) characteristics for the interferometric geometry both in the weak- and strong-coupling limits. With the help of external magnetic field, the orientation of the magnetic moments at sites 3 and 5 (measured by the parameters \( \theta_3 \) and \( \theta_5 \), respectively) can be changed. Here we will show that, depending on the values of \( \theta_3 \) and \( \theta_5 \), the interferometric geometry exhibits XOR gate response, keeping all the other moments oriented along \( \pm Z \) axis. These two \((\theta_3 \text{ and } \theta_5)\) are treated as the two inputs of the XOR gate. Let us first discuss the case of weak-coupling (Fig. 5). When both the two inputs to the gate are zero, i.e., \( \theta_3 = \theta_5 = 0 \), conductance exactly drops zero (see Fig. 5(a)). On the other hand, if any one of the two inputs is high i.e., \( \theta_3 \) or \( \theta_5 \) has a non-zero value, the conductance shows fine resonant peaks (see Figs. 5(b) and (c)) for some particular energy values. Finally, when both the inputs to the gate are high, the conductance again vanishes (Fig. 5(d)) for the entire energy range. The conductance peaks are associated with the energy eigenvalues of the interferometer. With the increasing number of magnetic atoms, comprising the interferometer, number of energy levels associated with the interferometer increases, therefore more resonant peaks appear in the conductance spectrum.

These features can be explained as follows. When both the two inputs are either low \((\theta_3 = \theta_5 = 0)\) or high \((\theta_3 = \theta_5 = \pi)\), the transmission probability exactly vanishes at the half flux-quantum value of \( \phi \) which provides zero conductance for the entire energy range. The vanishing behavior of conductance at \( \phi = \phi_0/2 \) for these symmetric configurations of the interferometer is clearly understood from our earlier discussion (sub-section IIIA)

If the symmetry in the orientation of the local magnetic moments in the two arms of the interferometer is broken by applying an external magnetic field i.e., \( \theta_3 \neq \theta_5 \) then the transmission probability becomes non-zero even at \( \phi = \phi_0/2 \). Since in this case \( \theta_3 \) and \( \theta_5 \) are chosen either as 0 or \( \pi \), no spin flipping takes place, and therefore, the contribution to \( g_{\uparrow \uparrow} \) comes only from the factor \( g_{\uparrow \uparrow} \). The contribution from the spin flipping to the conductance spectrum will be observed for any other values of \( \theta \) \((\theta = \theta_3 \neq \theta_5)\) apart from 0 and \( \pi \). Even for these orientations of the magnetic moments, the total conductance vanishes for the symmetric configuration, while it shows a finite non-zero value for the asymmetric one. Thus, we can conclude that the XOR gate like response will remain unchanged at the typical AB flux \( \phi = \phi_0/2 \) for any value of \( \theta \). In this interferometric geometry the asymmetry can be quantified by the term \( \Delta \theta \) (\( \Delta \theta = |\theta_3 - \theta_5| \)). The logical XOR gate response is obtained for any non-zero value of...
of $\Delta \theta$, while it is best observed for the maximum value of $\Delta \theta$ ($\Delta \theta_{\text{max}} = \pi$), which is presented in our case.

Another important thing to be mentioned here is that for $\theta = 0$ or $\pi$, spin flip transmission does not take place, and accordingly, the spin transfer torque $\tau_{\text{flip}}$ is zero which does not affect the magnetization direction, while for any other values of $\theta$ apart from 0 and $\pi$, $\tau_{\text{flip}}$ is nonzero which may cause a change in the direction of magnetization. But the change will be the same for all the magnetic sites having identical magnetic moments and hence XOR gate feature will not get affected. Thus, we can conclude that the violation of spin conservation due to spin flip scattering should not have any significant impact on XOR gate response. In short, we can say that the spin transmission probability becomes non-zero if and only if the moments embedded in the interferometric arms are oriented asymmetrically. Our numerical results clearly justify the XOR gate response.

In the same footing, here we also present the conductance-energy characteristics for the strong-coupling limit. The results are shown in Fig. 6. All the basic features in the four different choices of the two inputs ($\theta_3$ and $\theta_5$) are exactly similar to those as presented in Fig. 6, apart from the broadening of conductance peaks. The contribution to the broadening comes from the broadening of the energy levels of the interferometer in this strong-coupling limit. It provides a significant effect in the study of current-voltage ($I$-$V$) characteristics which we will describe in the following subsection. It is to be noted that the conductance-energy spectrum for down spin is exactly mirror symmetric to the spectrum observed for an up spin, and accordingly, we do not plot the results further for down spin.

2. Current-voltage characteristics

All the basic features of spin dependent transport obtained from conductance versus energy spectra can be explained in a better way through the current-voltage ($I$-$V$) characteristics. The current across the quantum interferometer is computed by integrating over the transmission probability varies exactly similar to that of the conductance spectrum, since we get the relation $g = T$ from the Landauer conductance formula (Eq. (31)) with $e = h = 1$ in our present formulation.

In Figs. 7 and 8 we plot the current ($I$) as a function of applied bias voltage ($V$) both for the symmetric and asymmetric orientations of the local magnetic moments in the upper and lower arms of the interferometer. The flux $\phi$ is set at $\phi_0/2$ i.e., $0.5$ in our chosen unit. Let us first start with the case of weak-coupling (Fig. 7). For symmetric configuration of the interferometric arms ($\theta_3 = \theta_5 = 0$ and $\theta_3 = \pi$, $\theta_5 = \pi$, $\theta_3 = \pi$ and $\theta_5 = 0$ and $\theta_3 = \theta_5 = \pi$), the current vanishes for the entire range of the bias voltage $V$ (Figs. 7(a) and (d)). This is due to the fact that, for these cases the transmission probability becomes zero for the entire energy range as we have studied earlier. On the other hand, for asymmetric configuration ($\theta_3 = 0$, $\theta_5 = \pi$ and $\theta_3 = \pi$, $\theta_5 = 0$) of the interferometric arms, the current is non-zero (Figs. 7(b) and (c)), because of non-vanishing spin transmission probability. This behavior becomes much different for strong-coupling limit. The results are shown in Fig. 6. All the basic features in the four different choices of the two inputs ($\theta_3$ and $\theta_5$) are exactly similar to those as presented in Fig. 6, apart from the broadening of conductance peaks. The contribution to the broadening comes from the broadening of the energy levels of the interferometer in this strong-coupling limit. It provides a significant effect in the study of current-voltage ($I$-$V$) characteristics which we will describe in the following subsection. It is to be noted that the conductance-energy spectrum for down spin is exactly mirror symmetric to the spectrum observed for an up spin, and accordingly, we do not plot the results further for down spin.

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more clearer from Table II where we make a quantitative estimate of typical current amplitude, computed at the bias voltage \( V = 10.52 \). It is observed that, the current \( I \) reaches the value to 0.294 only when any one of the two inputs are high and the other is low i.e., \((\theta_3 = 0 \text{ and } \theta_5 = \pi \text{ or } \theta_3 = \pi \text{ and } \theta_5 = 0)\), while for the other two cases \((\theta_3 = 0 \text{ and } \theta_5 = 0 \text{ or } \theta_3 = \pi \text{ and } \theta_5 = \pi)\), it \((I)\) gets zero. From these \(I-V\) characteristics the XOR gate like response is clearly visualized. In this weak-coupling limit, the current shows step-like behavior as a function of the applied bias voltage \( V \). This is due to the presence of sharp resonant peaks in the conductance spectra, as the current is obtained from integration method over transmission function \( T \). With the increase in applied bias voltage \( V \), the difference in chemical potentials of the two electrodes \((\mu_1 - \mu_2)\) increases, allowing more number of energy levels to fall in that range, and accordingly, more energy channels are accessible to the injected electrons to pass through the quantum interferometer from the source to drain. Incorporation of a single discrete energy level i.e., a discrete quantized conduction channel, between the range \((\mu_1 - \mu_2)\) provides a jump in the \(I-V\) characteristics.

TABLE I: XOR gate behavior in the limit of weak-coupling. The typical current amplitude is determined at the bias voltage \( V = 10.52 \).

| Input-I \( (\theta_3) \) | Input-II \( (\theta_5) \) | Current \( (I) \) |
|------------------------|------------------------|------------------|
| 0                      | 0                      | 0                |
| \( \pi \)              | 0                      | 0.294            |
| 0                      | \( \pi \)              | 0.294            |
| \( \pi \)              | \( \pi \)              | 0                |

TABLE II: XOR gate behavior in the limit of strong-coupling. The typical current amplitude is determined at the bias voltage \( V = 10.52 \).

| Input-I \( (\theta_3) \) | Input-II \( (\theta_5) \) | Current \( (I) \) |
|------------------------|------------------------|------------------|
| 0                      | 0                      | 0                |
| \( \pi \)              | 0                      | 0.785            |
| 0                      | \( \pi \)              | 0.785            |
| \( \pi \)              | \( \pi \)              | 0                |

The case of strong-coupling is depicted in Fig. A quantitative estimate of the current is given in Table II where the typical current is measured for the bias voltage \( V = 10.52 \). In this strong-coupling limit, all the basic features are exactly similar to that given in Table II only the magnitude of the output current gets enhanced to a value of 0.785. The non-zero currents show continuum behavior with respect to the change in bias voltage. As the sharp and discrete feature of the conductance peaks is lost in this strong-coupling limit, acquiring some broadening, the current changes continuously providing a much larger amplitude. Therefore, tuning the strength of the interferometer-to-electrode coupling, current can be enhanced significantly keeping the bias voltage constant. Exactly similar kind of behavior can also be observed for down spin current, and accordingly, we do not show the results here.

IV. CONCLUDING REMARKS

To summarize, in the present work we have explored spin dependent transport through interferometric geometry, penetrated by a magnetic flux \( \phi \) using transfer matrix formalism. A simple tight-binding framework has been adopted to illustrate the system, where the interferometer comprised of magnetic atomic sites is sandwiched between two non-magnetic electrodes, namely, source and drain. We have calculated numerically the spin dependent transmission probability including the effect of spin flip. Our numerical calculations describe conductance-energy and current-voltage characteristics as functions of the interferometer-to-electrode coupling strength, magnetic flux and the orientation of the local magnetic moments associated with each atom placed in interferometric arms.

First, we have observed the variation of conductance incorporating the effect of spin flip, as a function of magnetic flux \( \phi \) showing \( \theta_0 \) periodicity. It is noticed that, at half flux-quantum value of \( \phi \), transmission probability drops to zero for a symmetrically connected interferometer. Next, we have studied the variation of conductance with \( \theta \) (considering \( \theta_3 = \theta_5 = \theta \)) for symmetric configuration which shows that spin flipping is blocked for \( \theta = 0 \) or any integer multiple of \( \pi \). Finally, we have obtained XOR gate like response in \( g-E \) and \( I-V \) characteristics depending on the orientations of the local magnetic moments in the upper and lower arms of the interferometer at \( \phi = \theta_0/2 \).

Throughout our work, we have addressed all the essential features of XOR gate operation considering an interferometer with total 6 atomic sites. Among them 4 are placed at the corners to form a ring like structure and the rest two are coupled to the electrodes directly. In our model calculations, this typical number (6) is chosen only for the sake of simplicity. Though the results presented here change numerically with the ring size, but all the basic features remain exactly invariant. The main point of concern is that, whether the moments in the upper and lower interferometric arms are oriented symmetrically or not. The local magnetization direction can be changed by a rotation of the exchange field on the magnetic sites \([11, 36]\). Change of the local moment orientations at sites 1, 2, 4 and 6 in our present geometric model does not make any difference to the physical features of the results shown above. To be more specific, it is important to note that, in real situation the experi-
Eventually achievable rings have typical diameters within the range 0.4-0.6 \mu m. In such a small ring, unrealistically very high magnetic fields are required to produce a quantum flux. To overcome this situation, Hod et al. have studied extensively and proposed how to construct nanometer scale devices, based on Aharonov-Bohm interferometry, those can be operated in moderate magnetic fields [37].

In this work, we have calculated all the results by ignoring the effects of temperature, electron-electron correlation, electron-phonon interaction, disorder, etc. Here we fix the temperature at 0 K, but the basic features will not change significantly even in non-zero finite (low) temperature region as long as thermal energy ($k_BT$) is less than the average energy spacing of the energy levels of the quantum interferometer. Over the last few years a lot of efforts are made to incorporate the effect of electron-electron correlation in the study of spin dependent transport. Electronic correlation may cause decoherence among the waves passing through the interferometric arms. But, at low temperatures the decoherence produced by electron-electron correlation can be limited and in a very recent work Montambaux et al. [38] have justified it by studying electron transport for some arrays of connected mesoscopic metallic rings in presence of electronic correlation. The presence of electron-phonon interaction in Aharonov-Bohm interferometers provides phase shifts of the conducting electrons and due to this dephasing process electron transport through an AB interferometer becomes highly sensitive to the AB flux $\phi$ with the increase of electron-phonon coupling strength $\beta$ [39]. In the present work, we have addressed our results considering the site energies of all the atomic sites of the interferometer are identical i.e., we have treated the ordered system. But in real case, the presence of impurities will destroy phase coherence significantly which affects the transport properties. In this model it is also assumed that the two side-attached non-magnetic electrodes have negligible resistance. At the end, we would like to mention that we need further study in such systems by incorporating all these effects.

At the end, here we have designed a spin XOR gate using a quantum interferometer, based on the effect of quantum interference, which is a classical logic gate. On the other hand, quantum logic gates using ring geometries have already been proposed earlier which can be available in the reference [40].

All these predicted results may be utilized in designing tailor made spintronic circuits.

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