Robustness of competing climatic states

CHARLINE RAGON*, VALERIO LEMBO†, VALERIO LUCARINI†‡, CHRISTIAN VÉRARD§, JÉRÔME KASPARIAN*, MAURA BRUNETTI**†

ABSTRACT

The climate is a non-equilibrium system undergoing the continuous action of forcing and dissipation. Under the effect of a spatially inhomogeneous absorption of solar energy, all the climate components dynamically respond until an approximate steady state (or attractor) is reached. However, multiple steady states can co-exist for a given forcing and with the same boundary conditions. Here, we apply the Thermodynamic Diagnostic Tool (TheDiaTo) to investigate the statistical properties of five co-existing climates, ranging from a snowball to an ice-free aquaplanet, obtained in MITgcm coupled simulations. The aim is to explore the multistability of the climate model setup by highlighting differences in competing steady states and their characteristic signatures regarding the meridional transport of heat and water mass, the Lorenz energy cycle and the material entropy production. We also investigate how such attractors change when the model configuration is varied. We consider, in particular, the effect of changing the representation of the cloud albedo and of implementing an improved closure of the energy budget. We find that, even if the dynamics remains on the same attractor, state variables are modified. The set of metrics in TheDiaTo quantifies such modifications and represents a valuable tool for model evaluation.

1. Introduction

The climate is a highly complex and heterogeneous non-equilibrium system undergoing the continuous action of forcing and dissipation. The main source of external forcing is the inhomogenous absorption of incoming solar radiation. The atmosphere dynamically responds to such energy input by re-distributing the heat from lower to higher values of both latitudes and altitudes (Peixoto and Oort [1992], Lucarini et al. [2014a], Ghil and Lucarini [2020]). On the other hand, the ocean is essentially fueled by winds, tides and buoyancy forcing (Munk and Wunsch [1998], Wunsch and Ferrari [2004]). As a result, the inhomogeneous absorption of solar heat triggers a complex set of instabilities and feedbacks occurring at a wide range of temporal and spatial scales, redistributing heat and water mass, until approximate steady state conditions are achieved (Peixoto and Oort [1992]) where the dynamics of the system lies upon a high dimensional attractor (Saltzman [2001], Lucarini et al. [2014a], Ghil and Lucarini [2020]).

A generally accepted theoretical framework is that the climate is a multi-stable dynamical system, where multiple competing attractors can co-exist under the same forcing and boundary conditions (Budyko [1969], Sellers [1969], Ghil [1976], Saltzman [2001], Ghil and Lucarini [2020]). This implies that the phase space is partitioned among the basins of attraction corresponding to the various attractors and the basin boundaries (Lucarini and Bódai [2017]). Transition between the competing steady states is possible if the system undergoes forcing of deterministic or stochastic nature (Saltzman [2001], Ghil and Lucarini [2020], Margazoglou et al. [2021]). As an example, observational evidence suggests that during the Neoproterozoic era our planet flipped in and out of a so-called ‘snowball’ state (Hoffman et al. [1998], Pierrehumbert et al. [2011]). Multi-stability reflects the fact that there are different manners to redistribute the energy among the climate components in such a way that a stable climate can be established. It has been detected in models of different complexity, from energy balance models (Budyko [1969], Sellers [1969], Ghil [1976], Abbot et al. [2011]) and general circulation models (gcm) coupled to slab oceans (Lucarini et al. [2010], Boschi et al. [2013], Popp et al. [2016], Lucarini and Bódai [2017], Lucarini and Bódai [2019], Lucarini and Bódai [2020]) to fully coupled gcm (Ferreira et al. [2011], Rose [2015]). All these models have been able to reproduce the dichotomy between the competing warm and snowball climates. But, indeed, modelling exercises indicate the possible existence of additional climatic configurations, such as the slushball Earth (Lewis et al. [2007]) and the Jormungand state (Abbot et al. [2011]).
Recently, up to five attractors were found to co-exist under the same forcing and boundary conditions using the MIT general circulation model (MITgcm) ([Marshall et al. 1997a,b; Adcroft et al. 2004; Marshall et al. 2004]) in a coupled aquaplanet configuration ([Brunetti et al. 2019]). In terms of average surface temperature, these attractors range from the usual snowball state, which is completely covered by ice, to a hot, ice-free state, which is warmer than the usual warm state found in other studies. How can this large variety of co-existing states be physically described in a manner that sets the different states apart? To address this issue, we extend the analysis of such attractors performed in [Brunetti et al. 2019] with the construction of the entire bifurcation diagram for a solar constant ranging between 1336 and 1400 W m\(^{-2}\) and with the addition of, in particular, two new metrics, the intensity of the Lorenz energy cycle (LEC) and the material entropy production (MEP) of the atmosphere.

The extended set of metrics used to characterize the multiple attractors can furthermore be used to quantify biases in climate models, an important instrument for the understanding of climate dynamics ([Eyring et al. 2016; Balaji et al. 2017]). Despite the impressive progress in last decades, biases still remain in the new generation of climate models that are extremely difficult to reduce ([Wang et al. 2014; Rauser et al. 2015; Zhang et al. 2015; Palmer 2016; Stouffer et al. 2017; McKenna et al. 2020; Liao et al. 2021]). Such biases affect the description of key climatic features like the interplay of global modes of variability ([Yang et al. 2018]), their magnitude and frequency, or the occurrence of extremes ([Perkins 2011]), and reveal the limits of our ability to correctly reproduce the response to external (solar) or internal (volcanic and anthropogenic) forcings ([Rose et al. 2013; Gupta et al. 2019]).

A main source of biases in climate models comes from the use of parameterizations of unresolved processes. In the addition to the use of insufficiently observationally constrained parameters, parameterizations are often not constrained to conservation properties ([Anthes 1985; Haurin et al. 2016]). This can have an impact on the mean properties of the modeled system, inducing numerical drifts, and deserves particular attention. Here, we will focus specifically on two examples, by looking at the effect of different prescriptions for the cloud albedo and at the effect of improving the global energy budget of the model, to show the utility of considering a set of metrics including LEC and MEP.

To characterize the different attractors and address model biases in a comprehensive manner we use the Thermodynamic Diagnostic Tool (TheDiaTo, [Lembo et al. 2019]). TheDiaTo is a software tool that calculates a large set of metrics, including the aforementioned ones, and is thus ideally suited for the task at hand. TheDiaTo is also part of the CMIP6 ESMValTool 2.0 Earth system models diagnostic community effort ([Eyring et al. 2020]).

The paper is organized as follows. After the description of our setups and simulations used to construct the bifurcation diagram in Section 2 we apply TheDiaTo in Section 3 to study the five competing attractors. In Section 4 we discuss how re-injecting dissipated kinetic energy affects the physical budgets and the overall properties of the attractors, focusing on the hot state. In Section 5 we investigate the impact of changing the cloud albedo scheme on what we refer later to as the cold state, which is intermediate with respect to surface temperature. Finally, we discuss the importance of performing such diagnostics in climate models before drawing our conclusions in Section 6.

2. Methods and simulation setups

The simulations are performed using the MIT global circulation model (MITgcm, version e65q, [Marshall et al. 1997a,b; Adcroft et al. 2004]), a coupled atmosphere-ocean-ice model with a 15-levels dynamical ocean, a thermodynamic module for the sea ice component ([Winton 2000]), and a 5-layers atmospheric radiative module based on the SPEEDY model ([Molteni 2003]). The top layer represents the stratosphere, the bottom one the planetary boundary layer, and the remaining three the free troposphere. SPEEDY is an atmospheric module of intermediate complexity that provides a rather realistic representation of the flow despite the coarse vertical resolution and the simplified physical parametrizations, as results from the comparisons with ERA and NCEP/NCAR reanalysis fields ([Molteni 2003]), with the advantage of requiring less computer resources than state-of-the-art atmospheric models.

MITgcm is run in an aquaplanet configuration with no continents. The same dynamical kernel is used for both the ocean and the atmosphere, which are represented over the same cubed-sphere grid ([Marshall et al. 2004]). Each face of the cube includes 32 \(\times\) 32 cells, corresponding to an average horizontal resolution of 2.8°. The ocean depth is set to a fixed value of 3000 m. The \(CO_2\) concentration in the atmosphere is set to 326 ppm and various values for the solar irradiance \(S_0\) are considered, as detailed below. The orbital forcing is prescribed at present-day values, thus seasonality is taken into account. Albedos for snow, ice and ocean surface are within the observed range and correspond to those used in [Brunetti et al. 2019]. The relative humidity threshold for the formation of low clouds, a parameter denoted as RHCL2 in MITgcm, is set to the same value for all the simulations considered in the present study, \(RHCL2 = 0.7239\).

We extend the analysis performed in [Brunetti et al. 2019] by running 28 additional simulations with different incoming solar radiation \(S_0\). This allows us to construct the
The bifurcation diagram is obtained using the reference model setup where the dissipated kinetic energy is not re-injected in the system and low-cloud albedo is fixed to the constant value \( \alpha_{c0} = 0.38 \). This is denoted as SETUP-REF (and corresponds to SETUP2 in Brunetti et al. (2019)). We then alter one process at a time to better understand the impact of such changes on the model’s performance. In SETUP-FH, the dissipated kinetic energy is re-injected into the system as thermal energy (see Section 4) and this procedure is repeated until a tipping point is reached, where a shift to a different attractor is observed. We take the pragmatic approach of assuming that statistically steady-state conditions are realised within each attractor when its mean annual surface energy balance \( F_s \) becomes lower than 0.2 W m\(^{-2}\) in absolute value, corresponding to an ocean temperature drift \( dT/dt = F_s/(c_p \rho h) \) (Marshall and Plumb 2008, p. 229) lower than 0.05 °C/Century, with \( c_p = 4000 \) J K\(^{-1}\)kg\(^{-1}\) the specific heat capacity, \( \rho = 1023 \) kg m\(^{-3}\) the sea water density and \( h = 3000 \) m the ocean depth. Indeed, under such conditions, we see virtually no drift in the annual averages of the climatic observables of interest. Five stable branches exist, corresponding to the following climates: hot state (HS), warm state (WS), cold state (CS), waterbelt (WB) and snowball (SB). In particular, WS is stable within a narrow range of values of the solar constant\(^1\) \( 1368-1370 \) W/m\(^2\), while SB is stable for all the tested values ranging from 1336 to 1400 W/m\(^2\).

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\(^1\)Other bifurcation diagrams obtained with MITgcm with a horizontal resolution of 3.75° and simplified continental configurations can be found in Roser (2013), Gupta et al. (2019).

\(^2\)Green triangles are not regularly spaced in Fig. 1, especially near the tipping points WS \( \to \) HS and WS \( \to \) CS where we have explored in detail the narrow region of stability of WS.

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3. Characterization of the five aquaplanet climatic attractors

In order to quantify the relative importance of physical processes in multi-stable states, this section presents a detailed analysis of the five competing attractors obtained with \( S_0 = 342 \) W m\(^{-2}\) in SETUP-REF (as in Brunetti et al. (2019)); see Fig. 1

a. Energy and water-mass budgets and transports

Table 1. shows a selection of the average values of key global climatic observables for the five competing steady states. Average ocean temperatures range between \( T = -1.9 \) °C for the SB climate and \( T = 17.5 \) °C for the HS.
Correspondingly, the mean surface air temperature (SAT) ranges between -38.75 °C for the SB climate and 23.2 °C for the HS (see Fig. S1 in Supplemental Material). Let’s consider next the temperature difference between polar and equatorial regions (usually referred to as meridional temperature gradient, $\Delta T_{PE}$). This is computed by taking the difference between the average surface air temperature in the latitudinal belt $[30^\circ S, 30^\circ N]$ and in the region within $30^\circ$ and $90^\circ$ degrees latitude in the northern and southern hemispheres (Lucarini and Bodai 2017 [Margazoglou et al. 2021]). One finds the smallest value $\Delta T_{PE}$ in the HS (16.2 °C), which corresponds to conditions typical of so-called equitable climates (Huber and Caballero 2011). Instead, $\Delta T_{PE}$ is largest (34.3 °C) for the CS.

The Top-Of-Atmosphere (TOA) energy balance ranges between 0.3 and 2.9 W m$^{-2}$, which are typical values in coarse resolution simulations such as those performed in the climate models intercomparison project PCMDI CMIP3 (https://esgf-node.llnl.gov/projects/esgf-llnl/) in the preindustrial scenario (see for example Fig. 2a in Lucarini and Ragone (2011)). The presence of an imbalance is in apparent contradiction with steady state conditions; indeed, as discussed in Lucarini and Ragone (2011), Liepert and Previdi (2012), the spurious bias is due to physical processes that have been neglected, inconsistently treated or approximated in climate models, as well as to unphysical effects of numerical dissipation (Pascale et al. 2011, Mauritsen et al. 2012, Lauritzen and Williamson 2019, Trenberth 2020). In our setting, the main contribution comes from the frictional heating (see Section 4) and the fact that sea ice dynamics is neglected (Brunetti and Vérard 2018). Note that, as argued in Lucarini and Ragone (2011), the bias is positive in all cases.

The surface energy budget $F_s$ ranges between $-0.1$ and $0.2$ W m$^{-2}$ for all climates. These values are in general smaller than those reported for CMIP3 control runs (see Fig. 4 in Lucarini and Ragone (2011)) or for CMIP5 control runs (see Table 2 in Lembo et al. (2019)). The presence of such a small energy imbalance at the ocean’s surface strongly indicates that all our simulations have reached a steady state (Brunetti and Vérard 2018), with a drift in the average oceanic temperature of only few parts in $10^{-2}$ °C per century (see Table 1). We remark that such accurate steady-state conditions were not imposed, but have been reached after a few thousand years of simulation, which were required to remove transient effects.

Table 1 also shows the global water budget, given by the difference between global averages of evaporation $E$ and precipitation $P_{tot}$. We find that in all climates the bias in the water budget is negligible, so that approximately no water is lost during the numerical experiments, in agreement with Campin et al. (2008). This mirrors the presence of an accurately balanced latent heat budget $R_L$, also shown in Table 1 which is related to the phase transformation of water over different surfaces; see Appendix A. For com-
pelleness, we also list the global annual mean precipitation $P_{\text{tot}}$, whose maximum value is found in HS.

In all climates, the atmospheric and the oceanic enthalpy transports (Fig. 2) are poleward and anti-symmetric with respect to the equator as a result of the aquaplanet configuration, which is symmetric between the two hemispheres. The total transport balances the net TOA radiation influx in the equatorial region and the net TOA outflux in the polar regions (Trenberth et al. 2009, Lucarini and Ragone 2011). Going from HS to CS, peaks of meridional enthalpy transport increase in magnitude (Fig. 2), mean surface air temperature decreases and meridional temperature gradient increases. The enthalpy transport is strongest in CS attractor with a peak of 6.5 PW in the atmosphere and 3.3 PW in the ocean. This is not surprising because the largest part of the atmospheric enthalpy transport in the extratropics is carried out by mid-latitude eddies (Marshall and Plumb 2008, p. 155), in turn influenced by the meridional temperature gradient, while the oceanic heat transport is proportional to the strength of the circulation multiplied by the temperature gradient at the concerned latitudes (Boccaletti et al. 2005). Since HS has the steepest temperature gradient between pole and equator of the order of $\Delta T_{\text{PE}} = 34.3$ °C, it turns out that it also has the largest meridional enthalpy transport. Note that, despite the differences between HS, WS and CS, their meridional atmospheric enthalpy transport peaks at the same latitude ($\approx 40^\circ$ N/S) in agreement with the classical prediction by Stone (1978). The oceanic enthalpy transport is in all cases less intense than the atmospheric one and peaks at a lower latitude, as in the present-day Earth climate.

The much colder WB and SB climates are fundamentally different from the previous three in terms of transport profiles. In the WB climate, the atmospheric enthalpy transport peaks at the boundary of the water belt, where a large meridional temperature gradient is locally realised. Instead, the peak of the oceanic transport is obtained even closer to the equator and results from the intense overturning circulation inside the water belt. The meridional transports are negligible in the ice covered portion of the planet. In the case of the SB climate, the oceanic transport vanishes due to the absence of an ice-free ocean surface, and the atmospheric transport is extremely weak at all latitudes, which, in turn, agree with the fact that a) the meridional temperature differences are very small and that b) the atmosphere is dry, see Fig. 2c.

In the case of the three warmer climates, the annual mean meridional moisture transport - see Fig. 2a - is qualitatively similar to the present one, with the peak of the poleward transport occurring where the intensity of the meridional eddies is strongest - namely, in the storm track corresponding to the peak of the meridional enthalpy transport - whereas an equatorward transport is realised in the tropical region, coincident with the Hadley cells. Note that the largest poleward transport is obtained in the HS climate because higher surface temperature favours evaporation. Considering that the HS has the weakest total enthalpy transport (Fig. 2a), one deduces that large scale latent heat transport is relatively more important in the HS. Clearly, meridional moisture transport is almost vanishing in the SB climate, because evaporative processes are virtually absent, and is very weak and concentrated over and near the water belt in the WB climate.

b. Lorenz Energy Cycle (LEC)

The Lorenz energy cycle (LEC) describes the time-averaged transformation of energy between the available
potential form and the kinetic form. The reservoir of available potential energy $P$ is continuously replenished thanks to the inhomogeneous absorption of radiation, while the kinetic energy $K$ is continuously depleted as a result of dissipative processes (Lorenz 1955). This is the starting point for treating the atmosphere as a non-ideal engine, for defining its efficiency, and evaluating its entropy production (Peixoto and Oort 1992, Goody 2000, Pauluis and Held 2002, Lucarini 2009).

Specifically, a simplified version of the LEC can be represented in terms of two energy reservoirs and three conversion terms as in Pascale et al. (2011), where the available potential energy reservoir $P$ evolves due to generation by diabatic heating $\dot{Q}$ and conversion of potential to kinetic energy $C$. On the other hand, the kinetic energy reservoir $K$ is affected by the conversion term $C$ and the dissipation term $D$. For sake of clarity, the LEC can be formally summarised by the following budget equations:

$$ P = -C(P,K) + \dot{Q} \quad (1) $$
$$ K = -D + C(P,K) \quad (2) $$

The definitions of reservoir and conversion terms are given in Supplemental Material and in Lembo et al. (2019).

The reservoir of available potential energy $P$ is smaller in HS and WS than in CS, in agreement with the fact that, as far as simulations of the actual Earth are concerned, it decreases from pre-industrial to present-day conditions (Lembo et al. 2019). $P$ is proportional to the variance of temperature on isobaric surfaces in the free troposphere, and to the inverse of the difference in the lapse rate of dry vs. moist air, $\Gamma_d - \Gamma$ (Lorenz 1955, see eq. (10) in that paper). The former is dominated by the variance with respect to the zonal mean (see Supplemental Material), in turn related to meridional temperature gradients in the free atmosphere (Fig. 1), while $\Gamma_d - \Gamma$ is larger in warmer conditions (Marshall and Plumb 2008, p. 49). The combined effect of such contributions can explain the values of $P$ in the attractors. The fact, for example, that $P$ in HS is larger than in WS depends on weaker meridional gradients in WS at 250, 500 and 775 hPa, as shown in Fig. 4. On the other hand, the reservoirs of kinetic energy $K$, depending on the variance of wind speed, are very similar for HS, WS and CS (Fig. 5). Consequently, the sum of kinetic and available potential energy is different between the attractors.

The WB climate features the largest reservoirs of both available potential and kinetic energy. Indeed, the presence of a very intense temperature gradient localized at the ice edge (Fig. 4) and governing the dynamics (Brunetti et al. 2019) leads to large values of the available potential energy and allows for the presence of very intense zonal winds at low latitudes and at intermediate pressure levels, as shown in Fig. 5. In this attractor, jet streams are so intense that the mean kinetic energy reservoir amounts to almost 0.65 times the mean potential energy reservoir. The SB climate, instead, features the smallest reservoirs for both forms of energy as a result of the weak temperature gradients and weak atmospheric circulations throughout the domain.

Statistically steady state conditions imply that the long-term time-average values of the diabatic heating $\dot{Q}$ of the conversion term $C$ (which can also be seen as the mechanical work $W$ performed by the climatic engine or as the intensity of the LEC), and of the dissipation $D$ are identical. This is indeed the case for the values obtained for the five competing climatic attractors (Fig. 3 and Table 1). The intensity of the LEC is found to be considerably higher for the three warmer climates, where it ranges between 2.06 and 2.43 W m$^{-2}$. These figures are in broad agreement with the values obtained with seven climate models participating in CMIP5 using pre-industrial conditions (Lembo et al. 2019, Table 2), as well as with those found in additional CMIP5 model runs and in reanalysis datasets (Veiga and Ambrizzi 2013, Li et al. 2007). In contrast to the warmer climates, the intensity of the LEC is much weaker in the WB and in the SB climates, where weather variability is greatly reduced and localised in a narrow band (WB) or virtually absent (SB); see Lucarini et al. (2010) for a detailed analysis of the thermodynamics of the SB state. Note that, as well known (Lorenz 1955), there is no obvious relationship between the size of the energy reservoirs and the value of the conversion terms, so the fact that...
the WB state has the largest reservoirs of energy is not in contradiction with the very low intensity of the LEC.

c. Material-entropy production (MEP)

In the Earth system energy undergoes transport and transformation processes, and entropy is continuously produced by irreversible mechanisms. At steady state, while a zero net energy flux is reached at the boundary of the
Table 2. Contributions to material-entropy production in the five attractors at $S_0 = 342 \text{ W m}^{-2}$ in SETUP-REF.

| MEP [mW m^{-2} K^{-1}] associated to... | Hot state | Warm state | Cold state | Waterbelt | Snowball |
|-----------------------------------------|-----------|------------|------------|-----------|----------|
| Viscous processes                        | $8.3 \pm 0.1$ | $8.4 \pm 0.1$ | $7.32 \pm 0.08$ | $2.70 \pm 0.06$ | $1.39 \pm 0.04$ |
| Hydrological cycle                       | $47 \pm 4$ | $42 \pm 2$ | $30 \pm 1$ | $2.7 \pm 0.2$ | $0.89 \pm 0.09$ |
| Evaporation                              | $-371 \pm 2$ | $-342 \pm 1$ | $-272.0 \pm 0.6$ | $-40.3 \pm 0.1$ | $-9.62 \pm 0.04$ |
| Potential energy of droplets             | $6.30 \pm 0.05$ | $5.50 \pm 0.03$ | $3.77 \pm 0.01$ | $0.386 \pm 0.001$ | $0.1194 \pm 0.0004$ |
| Precipitation                            | $412 \pm 2$ | $379 \pm 1$ | $298.7 \pm 0.7$ | $42.6 \pm 0.1$ | $10.39 \pm 0.05$ |
| Sensible heat diffusion                  | $1.07 \pm 0.03$ | $1.55 \pm 0.02$ | $2.409 \pm 0.007$ | $2.558 \pm 0.007$ | $2.547 \pm 0.006$ |
| **Total MEP**                            | **56 \pm 4** | **52 \pm 2** | **40 \pm 1** | **8.0 \pm 0.3** | **4.8 \pm 0.1** |

For the sake of simplicity, ThedaTo neglects the phase changes occurring within the clouds during the formation and depletion of rain/snow droplets. Moreover, it deliberately focuses on MEP related to irreversible processes in the atmosphere, as it was previously shown (Pascale et al. [2011], Lucarini et al. [2014]). Moist processes are embedded in the hydrological cycle, namely the phase changes - e.g. evaporation, condensation, and sublimation taking place in non-saturated environment - and the dissipation of kinetic energy from precipitating hydrometeors (see eq. (A5)).

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Table 2 lists individual contributions to MEP for the five attractors. The total MEP increases with the mean surface air temperature (i.e., from SB to HS) (Lucarini et al. [2010]) mainly because the hydrological cycle is stronger in a warmer environment (Held and Soden [2006]) - see also Fig. 2. In contrast, the contribution coming from the diffusion of sensible heat is larger in colder climates, albeit it only represents a small fraction of the total MEP. This quantity depends on the product of the sensible heat flux, $H_s$, and the difference between top-of-boundary layer and skin temperatures, $T_{BL} - T_s$ (see second term in eq. (A5)). As can be seen in Fig. 4, it turns out that, while $T_{BL} - T_s$ is of the same order in all attractors, the sensible heat flux $H_s$ is larger in colder climates, since it depends on the difference between skin and near-surface temperatures (Marshall and Plumb [2008], p. 226).

d. Summary

The analysis performed so far is summarised in Fig. 6, where for each considered climatic variable the relative value is expressed with respect to the largest one realized among the five states. This visually highlights how, under...
the same forcing, the processes balance in a different way in co-existing attractors. It also helps in understanding how different variables are connected between each other, and which are the key quantities that identify a climatic state. In particular, we see that MEP due to the hydrological cycle (that represents the largest contribution to total MEP) is linked to total precipitation and the peak intensity in the water-mass transport. These quantities, together with the global temperature, are maximised in the HS, an ice-free steady state. On the other hand, the LEC intensity is closely linked to MEP due to viscous processes, which is largest in the WS. The meridional temperature gradient influences the enthalpy transport in both the ocean and the atmosphere, showing the most prominent peaks in the CS. Localised gradients of temperature and wind speed in the free troposphere increase the variance of such quantities, thus maximising values of $P$ and $K$, respectively, as in WB. Finally, SB minimises all the above quantities. The inclusion of LEC and MEP is crucial to distinguish the behaviour of WB and WS from the other steady states, thus showing the relevance of using TheDiaTo for attractors characterisation.

4. Effect of removing the energy bias by the re-injection of the dissipated kinetic energy

After having characterized the attractors, we can now evaluate their robustness against different model configurations. Kinetic energy within eddies is dissipated at small scales through an energy cascade (i.e., the dissipation term denoted as $DE$ in the complete LEC; see Supplemental Material). Energy conservation imposes that such dissipated mechanical energy should enter again into the energy budget as it is eventually converted into internal energy by friction. This term is usually ignored in general circulation models, as it is much smaller than other contributions, such as the latent heat exchanges. However, one should keep in mind that the frictional dissipation is positive definite, and, hence, does play a role in the overall energy budget. It has been shown that neglecting this term gives rise to a spurious thermal forcing of up to 2 W m$^{-2}$ ([Becker 2003]) and could explain part of the bias observed in TOA energy imbalance for the climate models ([Lucarini and Ragone 2011], [Wild 2020]). Thus, we have performed a sensitivity experiment, in which we evaluate the impact of the re-injection of dissipated kinetic energy on the modeled energy budget and the other thermodynamic diagnostics computed in TheDiaTo. In order to assess the relevance of this effect, we focus on the HS, comparing the standard setup SETUP-REF and the energy-consistent setup SETUP-FH for $S_0 = 342$ W m$^{-2}$; see Figure 1

While the imbalances of water mass and surface energy are similar in both cases, as shown in Table 3, the TOA imbalance is almost exactly reduced to zero (within the confidence interval) when frictional heating is re-injected

![Figure 7](image-url) **Fig. 7.** Comparison of hot states at $S_0 = 342$ W m$^{-2}$ in SETUP-REF and SETUP-FH in terms of northward enthalpy transport in the atmosphere (a) and in the ocean (b), and water-mass transport (c), where the inset is a zoom of the difference.
Table 3. Comparison of hot states at 342 W m$^{-2}$ in SETUP-REF and SETUP-FH. Statistically different values are in bold.

| Description                        | Name      | Units | SETUP-REF     | SETUP-FH     |
|------------------------------------|-----------|-------|---------------|--------------|
| Surface air temperature            | SAT       | °C    | 23.2 ± 0.2    | 24.92 ± 0.08 |
| Meridional temperature gradient    | ΔT$_{PE}$ | °C    | 16.2 ± 0.2    | 15.44 ± 0.09 |
| TOA energy budget                  | $R_s$     | W m$^{-2}$ | 2.5 ± 0.2    | 0.0 ± 0.2   |
| Surface energy budget              | $F_s$     | W m$^{-2}$ | 0.2 ± 0.3    | 0.2 ± 0.2   |
| Water budget                       | $E - P_{tot}$ | 10$^{-8}$ kg m$^{-2}$ s$^{-1}$ | 0 ± 2      | 0 ± 2      |
| Total precipitation                | $P_{tot}$ | 10$^{-5}$ kg m$^{-2}$ s$^{-1}$ | 4.47 ± 0.02 | 4.56 ± 0.02 |
| Mechanical work                    | $W$       | W m$^{-2}$ | 2.43 ± 0.03  | 2.33 ± 0.03 |

Table 4. Contributions to material entropy production in hot states with SETUP-REF and SETUP-FH at 342 W m$^{-2}$. Statistically different values are in bold.

| MEP [mW m$^{-2}$K$^{-1}$] associated to... | SETUP-REF | SETUP-FH |
|------------------------------------------|-----------|----------|
| Viscous processes                        | 8.3 ± 0.1 | 7.9 ± 0.1 |
| Hydrological cycle                       | 47 ± 4    | 48 ± 3   |
| Evaporation                              | −371 ± 2  | −377 ± 1 |
| Potential energy of droplets             | 6.30 ± 0.05 | 6.53 ± 0.03 |
| Precipitation                            | 412 ± 2   | 419 ± 2  |
| Sensible heat diffusion                  | 1.07 ± 0.03 | 0.91 ± 0.02 |
| Total MEP                                | 56 ± 4    | 57 ± 3   |

Fig. 8. Simplified Lorenz energy cycle for hot states at $S_0 = 342$ W m$^{-2}$ in SETUP-REF and SETUP-FH. Storage terms (boxes) in 10$^5$ J m$^{-2}$ and conversion terms (arrows) in W m$^{-2}$. Standard deviations are: 0.03 W m$^{-2}$ for $Q$, $C$ and $D$; 0.8 · 10$^5$ J m$^{-2}$ for $P$; 0.2 · 10$^5$ J m$^{-2}$ for $K$, in both setups.

as in SETUP-FH, confirming the importance of including such term in climate simulations. This improved conservation of energy gives rise to an increase of mean surface air temperature of 1.72 °C (see Table 3) and an average increase of 1.4 °C in the five atmospheric layers, as conjectured in Lucarini and Ragone (2011).

The transport of entropy is slightly less intense in SETUP-FH, as shown in Fig. 4a, b. The energy re-injection is local in space and time and takes place mostly in the mid-latitudes, which is where the strongest dissipation occurs. Hence, heat is added in a region where the annual mean TOA budget is negative (see Fig. S3 in Supplemental Material). As a result, the peak value of the meridional enthalpy transport as well as the meridional temperature gradient ΔT$_{PE}$ decrease in SETUP-FH, as shown in Fig. 7a, b and Table 3 respectively. Instead, the local increase in the surface temperature leads, as a result of enhancement of evaporation, to a slight strengthening of the meridional moisture transport in SETUP-FH with respect to the reference setup SETUP-REF (Fig. 7c).

The LEC for the two setups are compared in Fig. 9. Dissipation $D$ is 4% smaller in SETUP-FH than in the reference setup. In a steady state, the generation of available potential energy $Q$ and the dissipation $D$ have to balance and hence $Q$ has to decrease by an equal amount. This is consistent with a weaker meridional enthalpy transport in the atmosphere, as observed in Fig. 7a, and with a smaller meridional temperature gradient (from equator to poles), see Table 3.

While the intensity of the atmospheric circulation is lower in SETUP-FH, as measured by the mechanical work $W \sim D$, its hydrological cycle becomes slightly more effective, as already observed for the water-mass transport in Figure 7c, with statistically significant larger values of material-entropy production associated with precipitation and condensation, as shown in Table 4. Heating the lower levels of the atmosphere strengthens the hydrolog-
ical cycle and favours vertical transport of water vapour. Overall, the total MEP remains approximately constant because the slightly strengthening of the hydrological cycle in SETUP–FH is compensated by the reduced contribution of viscous processes and sensible heat diffusion.

In summary, re-injecting frictional heating improves the TOA budget, as expected, but has also several additional consequences, that have been revealed by TheDiaTo: on one hand, the increased mean temperature of the atmospheric column, particularly over the mid-latitudes, has strengthened the total precipitation and MEP associated to it; on the other hand, the reduced dissipation of energy has weakened the mechanical work of the LEC and the diabatic heating related to the meridional temperature gradient, and consequently the meridional enthalpy transport.

5. Testing different cloud albedo representations

A new representation for the cloud albedo has been introduced in the latest version of the atmospheric module SPEEDY (called ICTPAGCM, [Kucharski et al., 2006, 2013]) in order to reduce the net solar radiation at high latitudes and, hence, to have better agreement with observational data. The cloud albedo is assumed to depend on the latitude \( \phi \):

\[
\alpha_C(\phi) = \begin{cases} 
\alpha_{C0} + 0.2 \times |\sin(\phi)|^4 & \text{if } \cos(\phi) > \frac{1}{2} \\
\alpha_{C0} + 0.2 \times |\sin(\arccos(1/2))|^4 & \text{elsewhere},
\end{cases}
\]

so that \( \alpha_C(\phi) \) increases poleward from a value of \( \alpha_{C0} = 0.38 \) at the equator up to a maximum value of \( \approx 0.4925 \) at 60°S/60°N, and then holds constant up to the poles.

The physical rationale behind such a representation is that cloud cover appears effectively thicker when radiation is coming from lower geometrical angles. We have thus included such a modification in the atmospheric module of MITgcm and we call this configuration SETUP–FH–CL.

If we consider an incoming solar radiation of 342 W/m², the WB and SB states are observed in both SETUP–FH and SETUP–FH–CL (see Fig. 1). We will not investigate the effect of changing the cloud albedo representation for these two climates because they feature very weak dynamical processes and because they are in deep frozen state with high-albedo surface at high-latitudes, so that the effect of changing the albedo of high-latitude clouds can be understood as minimal.

In order to compare the effects of the cloud albedo prescription on attractors with stronger mechanical work, we consider an incoming solar radiation of 341 W/m² and look at the cold climates realized with SETUP–FH and SETUP–FH–CL; see red diamond and blue circle in Fig. 1.

\[\text{Fig. 9. Comparison of cold states at } S_0 = 341 \text{ W m}^{-2} \text{ in SETUP–FH and SETUP–FH–CL in terms of enthalpy transport in the atmosphere (a) and in the ocean (b), and of water-mass transport (c).}\]

\footnote{Note that for this value of the incoming solar radiation the HS becomes unstable in SETUP–FH and morphs into a climate that is close to the CS in SETUP–FH–CL.}
**Table 5.** Comparison of cold states at 341 W m$^{-2}$ in SETUP–FH and SETUP–FH–CL. Statistically different values are in bold.

| Description                        | Name | Units | SETUP–FH   | SETUP–FH–CL  |
|------------------------------------|------|-------|------------|--------------|
| Surface air temperature            | SAT  | °C    | 8.93 ± 0.08| 2.67 ± 0.09  |
| Meridional temperature gradient    | ΔTPE | °C    | 27.3 ± 0.2 | 33.5 ± 0.2   |
| Sea ice extent                     |      | 10$^6$ km$^2$ | 118.7 ± 0.8 | 159 ± 1      |
| TOA energy budget                  | $R_t$| W m$^{-2}$| −0.3 ± 0.2 | −0.3 ± 0.2   |
| Surface energy budget              | $F_s$| W m$^{-2}$| 0.0 ± 0.2  | 0.0 ± 0.2    |
| Water budget                       | $E - P_{tot}$| 10$^{-5}$ kg m$^{-2}$s$^{-1}$| −5 ± 8 | −5 ± 8  |
| Total precipitation                | $P_{tot}$| 10$^{-5}$ kg m$^{-2}$s$^{-1}$| 3.537 ± 0.009 | 3.168 ± 0.009 |
| Mechanical work                    | $W$  | Wm$^{-3}$| 1.98 ± 0.03| 2.02 ± 0.02  |

**Table 6.** Contributions to material entropy production in cold states with SETUP–FH and SETUP–FH–CL at 341 W/m$^2$. Statistically different values are in bold.

| MEP [mW m$^{-2}$K$^{-1}$] associated to... | SETUP–FH   | SETUP–FH–CL  |
|--------------------------------------------|------------|--------------|
| Viscous processes                          | 7.0 ± 0.1  | 7.18 ± 0.09  |
| Hydrological cycle                         | 35 ± 2     | 30 ± 2       |
| Evaporation                                | −299.4 ± 0.7| −270.0 ± 0.7 |
| Potential energy of droplets               | 4.39 ± 0.02| 3.75 ± 0.01  |
| Precipitation                              | 330.1 ± 0.8| 296.6 ± 0.8  |
| Sensible heat diffusion                     | 2.04 ± 0.02| 2.275 ± 0.008|
| Total MEP                                  | 44 ± 2     | 39 ± 2       |

![Flowchart](image)

**Fig. 10.** Simplified Lorenz energy cycle for cold states at 341 W m$^{-2}$ in SETUP–FH and SETUP–FH–CL. Storage terms (boxes) in 10$^5$ J m$^{-2}$ and conversion terms (arrows) in W m$^{-2}$. Standard deviations are: 0.03/0.02 W m$^{-2}$ for $Q$, $C$ and $D$; 0.8/0.8 · 10$^5$ J m$^{-2}$ for $P$; 0.2/0.1 · 10$^5$ J m$^{-2}$ for $K$, in SETUP–FH/SETUP–FH–CL, respectively.

Table 5 gives the key output characterizing the attractor in both configurations. First of all, it is important to verify that the new representation affects neither the global water-mass budget nor the energy budget. The increase of cloud albedo at high latitudes reduces the amount of incoming radiation, and the mean surface air temperature by nearly 6 °C. This is correlated to a larger sea ice extent in SETUP–FH–CL. However, both values of the sea ice cover are of the same order (120-160 millions of km$^2$) as those found in the cold climates at $S_0 = 342$ W/m$^2$ (see Table 2 in [Brunetti et al. (2019)]), suggesting that both attractors are of the same type. As a result of the more uneven absorption of solar radiation between high and low latitudes, the transports of enthalpy in both the atmosphere and the ocean (Figs. 9a, b) are stronger in SETUP–FH–CL. Moreover, the larger sea-ice extent determines an equatorward displacement of the peaks in the atmospheric transport. Consistently with what was found in Section 3, the SETUP–FH–CL climate has a higher reservoir of available potential energy $P$ (see Fig. 10) and a weaker moisture transport with its main peaks located closer to Equator, as shown in Fig. 9c. Indeed, the average temperature of the planet is greatly reduced with the new high-latitude cloud albedo prescription, hence the atmosphere as a whole becomes much drier, and a larger sea-ice extent reduces the ocean surface available for water evaporation.

The total MEP turns out to be significantly lower in SETUP–FH–CL, as a result of a smaller contribution in all components of the MEP budget, except for MEP associated to sensible heat diffusion and viscous processes, as shown in Table 6. This is due to the weaker hydrological cycle in SETUP–FH–CL, confirming the trend observed in Section 3 for attractors with colder temperatures. The change in MEP associated with viscous processes is not statistically significant, and the same holds for the intensity of LEC.
that is comparable in the two setups, of the same order as in CS at $S_0 = 342$ W/m$^2$ in SETUP-REF or SETUP-FH-CL, and lower than in WS (see Table 3 and Fig. 10), showing that the attractor is robust with respect to the new cloud prescription.

All in all, when looking at the CS, including the new representation leads to a decrease in the total MEP by $\approx 11.4\%$ due to a weaker hydrological cycle, and strengthens the enthalpy transport in both the atmosphere and the ocean, without introducing any spurious bias in water and energy budgets. Since Kucharski et al. (2006) showed that such cloud albedo prescription improves comparisons with observational data in present-day simulations, we can conclude that it is worth including it in the MITgcm atmospheric module.

6. Summary and conclusions

The climate is a nonequilibrium, multiscale system that features multistability. The occurrence of such phenomenon arises from its complex dynamics where forcing, dissipative processes, and nonlinear feedbacks can balance each other in different ways. The presence of multistability is intimately connected to the existence of tipping points coming with qualitative changes in the system dynamics for suitably defined forcings. In a deterministic setting, multistability is described by the presence of more than one competing steady state associated with different attractors, each included in a separate basin of attraction. While the dynamics of an autonomous system is confined to a single attractor, as its evolution is uniquely defined by its initial condition, the presence of stochastic forcings makes it possible for the system to explore the full phase space by performing transitions between the various basins of attraction (Benzi et al. 1983; Saltzman 2001; Lucarini and Bodai 2019; Lucarini and Bodai 2020; Ghil and Lucarini 2020).

We performed simulations using the MIT general circulation model with coupled aquaplanet configuration and different values of the incoming solar radiation $S_0$ in order to construct the bifurcation diagram (Fig. 1). Such methodology and the resulting graph are crucial to identify multistable regions, tipping points and the range of stability of each attractor. The model displays five attractors that co-exist for solar constant values in the range 1368-1370 W m$^{-2}$ (see Fig. 1), the classical "snowball" (SB) and "warm state" (WS) solutions, the very cold waterbelt (WB) state, where a water belt is present near the equator, the "cold state" (CS), where sea-ice extends to the mid-latitudes, and the "hot state" (HS), an equitable climate where no sea ice is present at all.

In order to describe the properties of such competing climatic states, we made use of a newly developed diagnostics tool, TheDiaTo (Lembo et al. 2019), that is part of the most recent version of the ESMValTool suite for evaluating Earth system models (Eyring et al. 2020). TheDiaTo provides flexible tools for the characterisation of fundamental thermodynamic and conservation laws, starting from first principles. More specifically, we have focused on global averages and patterns of near-surface temperatures, watermass, TOA and surface energy budgets, hence describing the strength of the meridional enthalpy and moisture transports, as well as the atmospheric Lorenz energy cycle (LEC) through its reservoirs, sources, sinks and conversion terms. The second law of thermodynamics has been assessed through the retrieval of material entropy production (MEP), i.e. the entropy change related to irreversible processes, such as the energy exchanges through sensible heat fluxes and the hydrological cycle. We have found the key processes that characterise each attractor: total precipitation, peak intensity in water-mass transport and global temperature are maximised in HS; LEC intensity and MEP in viscous processes are large in WS; meridional temperature gradient and enthalpy transport are dominant in CS; $P$ and $K$ are maximised when localised gradients of temperature and wind speed are present, as in WB; SB minimises all the previous quantities. These findings provide hints for using different climatic variables on reduced phase space for each climatic state, a subject that we will explore in further studies.

Exploring the multi-stability of the climate system requires to consider a wide range of initial conditions and detailed characterisation of the resulting climates. Such framework is of great relevance for modelling paleoclimate conditions (Pohl et al. 2014; Brunetti et al. 2015; Ferreira et al. 2018; Messori and Faranda 2020), where large uncertainty exists in initial conditions, as well as for the study of exoplanets, where the probability of finding an habitable planet is increased when multistability is allowed (Seager 2013; Boschi et al. 2013). Characterizing the basic properties of the modelled dynamics and thermodynamics is thus crucial, in order to suitably represent the tipping points of the climate system (Lenton et al. 2008) and allow for a more general definition of climate sensitivity (von der Heydt and Ashwin 2017; Ashwin and von der Heydt 2020).

The complete set of metrics in TheDiaTo can be used not only to quantitatively differentiate the attractors but also to highlight biases due to a simplified representation of physical processes in climate models. For testing this last point, we performed two numerical experiments. First, focusing on the HS, we forced re-injection of fractionally dissipated kinetic energy into the overall energy budget. On one hand, such correction effectively leads to a reduction in the TOA energy bias, as expected. On the other hand, it influences the dynamics of the atmosphere in a non-trivial way, as the re-injection is co-located with the frictional dissipation. The climate becomes warmer, the meridional temperature gradient is reduced, as well as the LEC intensity, while the total precipitation and MEP associated to it increase.
Second, focusing on the CS, we investigated the effect of introducing a dependency on latitude in the cloud albedo (cf. [Kucharski et al., 2006, 2013]). The implementation of such a scheme does not affect global conservation properties but causes changes in energy and moisture fluxes that lead to a cooler climate, with a weaker hydrological cycle and MEP. Despite these changes, TheDiaTo allowed us to verify that the attractors are of the same type (CS), showing that useful information can be inferred on the attractors nature even without the construction of the complete bifurcation diagram.

In summary, in both the numerical experiments the set of metrics in TheDiaTo has clearly shown that the climate state slightly changes, remaining on the same attractor, with non-negligible impacts on the dynamics and thermodynamics and, at the same time, preserving or improving the global conservation properties. Thus, TheDiaTo helps in controlling simulation quality and in evaluating different configurations. In particular, LEC and MEP, two metrics that are not routinely used for model evaluation, can provide additional benchmarks when constrained by observational data in present-day climate simulations.

While TOA energy imbalance can be improved though the inclusion of missing physical processes (such as frictional heating) or enhanced algorithms (with less numerical dissipation, for example), the energy imbalance at the surface $F_s$ is less affected by such procedures. As it is shown in several previous diagnostic studies, the *ghost energy* bias is concentrated in the atmosphere ([Lucarini and Ragone, 2011], [Lucarini et al., 2014a], [Lembo et al., 2017]). Instead, $F_s$ stabilizes as one considers longer simulation time, which allows the ocean to reach an approximate steady state. A vanishing surface energy imbalance (as assured in our simulations) guarantees that the drift of the mean ocean temperature is negligible. Thus, TOA and surface energy imbalance should be always monitored in climate models ([Brunetti and Vérard, 2018]) and we encourage to explicitly list them when presenting simulation results.

At present, TheDiaTo allows to perform a thermodynamic analysis of the atmosphere only. However, in principle, it is possible to establish a Lorenz energy cycle also for energy exchanges and transformations within the ocean ([von Storch et al., 2013]). As high resolution coupled models now allow for resolutions that are consistent with explicitly resolved mesoscale ocean eddies, a successive development would be creating a set of diagnostics that include the dynamics and thermodynamics of the oceans specifically, and it is left for future work.

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**Data availability statement.** The datasets generated during the current study are available from the corresponding author upon request.

## APPENDIX A

### Main equations used in TheDiaTo

The goal of this Appendix is to remind the reader how to calculate the main quantities discussed in this paper and to describe approximations/adaptations used to apply TheDiaTo on MITgcm outputs. We use (as much as possible) the same notation as [Lembo et al., 2019], where the reader can find all the details for the derivation of the equations reported here.

**a. Energy budget and transport**

Radiative fluxes at the surface ($F_s$) and at TOA ($R_t$) depend on latitude $\phi$, longitude $\lambda$ and time $t$ as follows:

\[
F_s(\phi, \lambda, t) = S_{\phi} - S_{\phi} + L_{\lambda} + L_{\lambda} - H_{\lambda} - H_{\phi} \quad \text{(A1)}
\]

\[
R_t(\phi, \lambda, t) = S_{\phi} - S_{\phi} + L_{\lambda} \quad \text{(A2)}
\]

where $S$ is shortwave radiation, $L$ longwave radiation, $H_{\phi}$ is latent heat flux and $H_{\lambda}$ is the sensible heat flux. Subscripts $t$ and $s$ denote top-of-atmosphere and surface, respectively. Upward (↑) or downward (↓) direction is also shown.

Global energy imbalances are computed by averaging $F_s$ and $R_t$ over the total surface and over a period of time of 20 years.

The meridional transport $\mathcal{T}$ is computed by taking the long-term temporal and zonal averages of eqs. (A1)-(A2), as follows:

\[
\mathcal{T}(\phi) = 2\pi \int_0^{\pi/2} a^2 \cos \phi' \langle F(\phi', \lambda, t) \rangle \, d\phi' \quad \text{(A3)}
\]

where $F$ is the radiative flux (at surface, $F_s$, or at TOA, $R_t$), $\phi'$ is the Earth’s radius. $\langle \rangle$ represents the long-term time mean and overline the zonal mean. The atmospheric transport is computed as the difference between the transport at TOA and that at surface, $\mathcal{T}_{atm} = \mathcal{T}_{TOA} - \mathcal{T}_t$. 

b. Budget and transport of water-mass and latent heat

The water-mass budget in the atmosphere corresponds to the difference between global averages of surface evaporation and precipitation, \( E - P_{\text{tot}} \), where \( P_{\text{tot}} \) includes both convective and large-scale precipitation. Since rainfall and snowfall precipitations are not differentiated in MITgcm, these are both accounted within \( P_{\text{tot}} = P_t + P_s \).

The evaporation \( E \) is an output of MITgcm and its calculation is based on different evaporation coefficients for ocean, land (when it is present) and ice surfaces. Thus, evaporation is not calculated from latent heat as proposed in TheDiaTo (see eq. (6) in that paper).

The meridional water-mass transport is obtained by calculating a cumulative integral over latitude of the zonal mean and long-term time mean of \( E - P_{\text{tot}} \), analogously to eq. (A.3).

The latent heat budget \( R_L \) depends on the latent heat flux \( H_L \), and precipitation \( P_{\text{tot}} \):

\[
R_L = H_L - L_v P_{\text{tot}}
\]

where \( L_v = 2.5008 \times 10^6 \) J/kg is the latent heat of evaporation. In MITgcm, the latent heat flux is deduced as a residual between the total energy surface budget (an output of MITgcm called TFLUX = \( F_L \)) and all the other components (as in eq. (A.1)). Note that in this way the heat associated with snow melting is taken into account in \( H_L \), assuring in general a well closed budget also for the latent heat, as shown in Table [II].

c. Material entropy production

In order to compute the material entropy production, we use the ‘direct method’ of TheDiaTo, where all contributions from irreversible processes are explicitly estimated as follows:

\[
\text{MEP} = \int \left[ \frac{K_s}{T_A} \right] dA - \int \left[ \frac{H_S}{T_S} \left( \frac{1}{T_S} - \frac{1}{T_{BL}} \right) \right] dA
\]

\[
- \int \left[ \frac{L_v}{T_s} \right] dA + \int \left[ \frac{L_v P_{\text{tot}}}{T_c} + g \frac{P_{\text{tot}} h_{\text{cl}}}{T_p} \right] dA\tag{A5}
\]

where \( T_A \) is the operating temperature (i.e., mean of near-surface \( (T_{2m}) \) and skin temperature \( (T_s) \)), \( T_{BL} \) is the temperature at the boundary layer, \( T_c \) is the working temperature at condensation, \( T_p \) is the mean of \( T_c \) and \( T_s \), \( K_s \) is the specific kinetic energy dissipation rate, \( g \) is the gravitational acceleration constant and \( h_{\text{cl}} \) is the distance covered by droplets. Contributions are integrated over the Earth’s surface area \( A \) or precipitation area \( A_v \). MEP associated to evaporation is directly derived from \( E \) given by the MITgcm diagnostics and not from the latent heat.

Note that material entropy production can also be estimated through an ‘indirect method’ separating vertical and horizontal energy transport terms [Lucarini et al., 2011; Lembo et al., 2019]. However, we have checked that in MITgcm such indirect method always overestimates vertical contributions. The low number of atmospheric level in MITgcm can be the source of this discrepancy with respect to the results obtained with the direct method.

d. Lorenz Energy Cycle (LEC)

Storage terms and conversion terms of LEC are computed in TheDiaTo using formulae given in Appendix A of Lembo et al. [2019] or in Ulbrich and Speth [1991]. Generation and dissipation terms are computed as residuals of the conversion terms at each reservoir.

e. Horizontal and vertical interpolation

TheDiaTo requires fields on a longitude-latitude grid. Thus we have interpolated the MITgcm output fields from the cubed-sphere grid with \( 32 \times 32 \) points per face (corresponding to an average horizontal resolution of \( 2.8^\circ \)) to a longitude-latitude grid at \( 2^\circ \).

MITgcm uses an Arakawa C-grid where scalar fields, like temperature and humidity, are saved at the center of grid cells, while vectorial fields are stored at the boundaries. This implies that, while all the scalar quantities and the horizontal components of wind speed are calculated at the centers \( k \), the vertical component is computed at levels \( k - 1/2 \), with the lowest on the sea surface. Since TheDiaTo requires that all the fields are calculated at the same point, we have linearly interpolated the vertical component of the wind speed on the same levels \( k \) as the other quantities. For the upper level (75 hPa), we have used a null vertical component at TOA.

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