On (Not)-Constraining Heavy Asymmetric Bosonic Dark Matter

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Recently, constraints on bosonic asymmetric dark matter have been imposed based on the existence of old neutron stars excluding the dark matter masses in the range from ~ 2 keV up to several GeV. The constraints are based on the star destruction scenario where the dark matter particles captured by the star collapse forming a black hole that eventually consumes the host star. In addition, there were claims in the literature that similar constraints can be obtained for dark matter masses heavier than a few TeV. Here we argue that it is not possible to extend to these constraints. We show that in the case of heavy dark matter, instead of forming a single large black hole that consumes the star, the collapsing dark matter particles form a series of small black holes that evaporate fast without leading to the destruction of the star. Thus, no constraints arise for bosonic asymmetric dark matter particles with masses of a few TeV or higher.

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The last few years stellar observations have been used in order to constrain specific dark matter candidates or predict interesting phenomena related to dark matter [1–13]. Specifically, compact stars such as white dwarfs and neutron stars have been found to impose severe constraints on some dark matter models [14–27]. In principle, there are two types of effects that can take place in compact stars and can give rise to constraints on dark matter. The first type is related to the thermal evolution of compact stars [13, 16, 17, 20]. In this case, annihilation of trapped weakly interacting massive particles (WIMPs) inside a compact star can produce significant amount of heat that can change the thermal evolution of the star at later times. As a result, stars old enough to be quite cold might maintain higher temperature due to the released heat.

The second type of constraints is related to asymmetric dark matter [14, 21, 23, 25]. In this case WIMPs carry a conserved quantum number and there is an asymmetry between the populations of WIMPs and anti-WIMPs, so that the annihilation is impossible in the present-day universe where only the WIMPs remain. Such kind of WIMPs can accumulate in a compact star quite efficiently as long as the WIMP-nucleon cross section exceeds a certain critical value which in the case of a neutron star is quite small ($\sigma \sim 10^{-46}$ cm$^2$) [12], several orders of magnitude smaller than current limits from direct searches. Under certain circumstances, the amount of WIMPs accumulated during the lifetime of the star is sufficient to result in a gravitational collapse of the the WIMPs into a black hole. In the case of fermionic dark matter the amount of WIMPs needed for collapse is $\sim M_{Pl}^2/m^2$, $M_{Pl}$ being the Planck mass and $m$ the mass of the WIMP. Unless one assumes very heavy WIMPs and/or extremely high background dark matter density at the location of a neutron star, this amount of WIMPs is difficult to accumulate during the lifetime of the star (although the situation may change in the presence of attractive self-interactions [23]).

On the contrary, if WIMPs are of bosonic nature, there is no Fermi pressure and the amount of WIMPs needed for gravitational collapse is significantly lower [28],

$$M > M_{\text{min}} = \frac{2 M_{Pl}^2}{\pi m} = 9.5 \times 10^{33} \text{GeV} \left( \frac{m}{10^3 \text{TeV}} \right)^{-1}. \quad (1)$$

This amount of WIMPs can be easily accumulated even by nearby known old neutron stars with the standard assumption about dark matter density near the Earth. As it was pointed out in [23], once $\sim 10^{36}$ WIMPs have been accreted and thermalized within the neutron star, a Bose-Einstein condensate (BEC) forms and all newly accreted WIMPs fall into the ground state. This state is very compact, so the WIMPs in the condensate start to self-gravitate way before the condition of Eq. (1) is satisfied. Thus, once the amount of WIMPs in the condensate meets the criterion for the gravitational collapse (Eq. (1)), they form a black hole. Note that only a fraction of all WIMPs (namely, those in the BEC) goes into a black hole. If the resulting black hole starts to grow and can destroy the host star in a reasonable time, constraints on dark matter arise as follows from the well known existence of old (more than a few billion years old) neutron stars.

The fate of the black hole inside the neutron star is determined by the two competing processes: the accretion of surrounding nuclear matter that increases the mass of the black hole and scales as $M_{BH}^2$ ($M_{BH}$ being the black
hole mass), and Hawking radiation that scales as \( M_{\text{BH}}^{-2} \). Therefore, there exists a critical value \( M_{\text{crit}} \) such that heavier black holes grow until the whole star is consumed, while lighter black holes evaporate completely. It follows from Eq. (1) that the mass of the black hole formed by this mechanism is inversely proportional to the WIMP mass, so that any black hole formed from WIMPs heavier than a few GeV is lighter than \( M_{\text{crit}} \) and evaporates completely, in which case no constraints arise.

In Refs. [24] [26] [27], it was suggested that for WIMPs heavier than a few TeV the above mechanism is modified and the constraints are recovered. The argument makes use of the observation that for sufficiently heavy WIMPs, the self-gravitation of the WIMP sphere sets in before the formation of BEC. At this point the total amount of accumulated WIMPs exceeds by many orders the amount of Eq. (1) required for the black hole formation, as well as the critical value \( M_{\text{crit}} \). So if one assumes that the collapse happens to the whole WIMP sphere at once the resulting black hole is heavy enough to grow and destroy the star. Hence the constraints reappear.

In this paper we demonstrate that this is not what happens. Although it is true that for heavy WIMPs the self-gravitation sets in without BEC formation, the collapse of the self-gravitating WIMP sphere is hampered by the released gravitational energy and happens on a timescale set by the cooling of the WIMPs. As the WIMP sphere gradually shrinks, its density increases and BEC forms on a timescale much shorter than the cooling time. From this stage on, the original scenario is reproduced: the BEC region grows, becomes self-gravitating and collapses into a black hole as soon as Eq. (1) is satisfied. The resulting black hole is too small to grow. Instead, it evaporates on a time scale much shorter than it takes for the rest of the WIMP sphere to collapse. The net result is the formation of tiny black holes one after the other which evaporate shortly after their birth without producing a heavy stable black hole. Thus no constraints on heavy WIMPs arise.

Let us consider this argument more quantitatively. The WIMPs become self-gravitating when their density becomes larger than that of the nuclear matter in the core of the neutron star. This happens when the total mass of the WIMPs exceeds

\[
M_{\text{BH}} = \frac{4\pi}{3} r_{\text{th}}^3 \rho_c = 2.2 \times 10^{40} \text{GeV} \left( \frac{10 \text{TeV}}{m} \right)^{3/2} \left( \frac{T}{10^9 \text{K}} \right)^{3/2}, \tag{2}
\]

where

\[
r_{\text{th}} \simeq 2.2 \text{ cm} \left( \frac{T}{10^9 \text{K}} \right)^{1/2} \left( \frac{m}{10^{10} \text{GeV}} \right)^{-1/2}, \tag{3}
\]

is the thermal radius of the WIMP sphere inside the neutron star at temperature \( T \) and \( \rho_c = 5 \times 10^{38} \text{ GeV/cm}^3 \) is the star core density. The density of WIMPs required to form BEC is [28]

\[
\rho_{\text{crit}} \sim 4.7 \times 10^{38} \text{GeV cm}^{-3} \left( \frac{m}{10^{10} \text{GeV}} \right)^{5/2} \left( \frac{T}{10^9 \text{K}} \right)^{3/2}, \tag{4}
\]

which implies that the total WIMP mass necessary for the formation of BEC is

\[
M_{\text{BEC}} = 2.1 \times 10^{40} \text{GeV} \left( \frac{m}{10^{10} \text{GeV}} \right) \left( \frac{T}{10^9 \text{K}} \right)^3. \tag{5}
\]

From the dependence of Eqs. (2) and (4) on the WIMP mass \( m \) one can see that for \( m \gtrsim 10 \text{ TeV} \) the self-gravitation sets in prior to the formation of BEC. Note that the total mass of the WIMPs in this case is large enough to overcome the constraint imposed by the uncertainty principle of Eq. (1).

Once the self-gravitation sets in, the WIMP sphere starts to collapse. However, it cannot collapse directly into a black hole because the WIMPs have to lose their energy and momentum. Instead, the collapse of WIMPs is likely to resemble the formation of dark matter halos, with the essential difference that constant interactions with nucleons provide an extra energy loss mechanism for the WIMPs. One may expect, therefore, that the WIMPs develop a cuspy profile similar to the dark matter halos, which shrinks as the WIMPs lose their energy in interactions with nucleons.

Even without the cusp, the shrinking WIMP sphere would form BEC long before it reaches the size comparable to its Schwarzschild radius. To make the argument as robust as possible, consider the worst (unrealistic) case where the WIMP sphere contracts maintaining a uniform density. In order to form a black hole of mass \( M \) the WIMPs have to reach the density \( \rho_{\text{BH}} \sim 3(32\pi G^3 M^2)^{-1} \sim 10^{74} \text{GeV/cm}^3 (M/10^{40} \text{GeV})^{-2} \), while the density required for BEC formation is much lower, c.f. Eq. (4). Note that the smaller is the black hole, the higher density is required, so the uniform contraction is indeed the worst case. Thus, BEC will unavoidably be formed before the WIMPs collapse into a black hole.

Consider now the formation of BEC in some more detail. Once the density of Eq. (4) is reached, the formation of BEC happens on time scales of order [29]

\[
t_{\text{BEC}} \sim \frac{\hbar}{k_B T} \sim 10^{-16} \text{s}, \tag{6}
\]

i.e. practically instantaneously. Further shrinking of the WIMP sphere results in increasing the mass of the condensate rather than the density of non-condensed WIMPs.

The size of the condensate can be estimated by noting that once the WIMPs become self-gravitating they dominate the gravitational potential. Taking their density approximately constant and equal to that required
for the condensate formation gives
\[ r_{\text{BEC}} = \left( \frac{8\pi}{3} G \rho_{\text{crit}} m^2 \right)^{-1/4} \]
\[ = 1.7 \times 10^{-6} \text{cm} \left( \frac{m}{10 \text{TeV}} \right)^{-9/8} \left( \frac{T}{10^4 \text{K}} \right)^{-3/8}. \]

This is much smaller than the size of the WIMP sphere.

As the mass of the BEC grows, it eventually becomes self-gravitating itself. This happens when the density of the condensate becomes equal to the non-condensed WIMP density, i.e., when its mass exceeds
\[ M_{\text{BEC}, \text{sg}} = \frac{4\pi}{3} \rho_{\text{crit}} r_{\text{BEC}}^3 = 9.6 \times 10^{21} \text{GeV} \left( \frac{m}{10 \text{TeV}} \right)^{7/8}. \]

Although self-gravitating, the condensate cannot yet collapse because it does not satisfy the constraint of Eq. (1). The latter is satisfied when the total mass of the condensed state reaches \( M_{\text{min}} \). Beyond that point stable configurations of the self-gravitating condensate do not exist and it collapses into a black hole [28]. The resulting mass of the black hole is given by Eq. (1). Note that this mass is much smaller than the total mass of the WIMP-sphere which is of order \( M_{\text{sg}} \), Eq. (2). Note also that the black hole mass becomes smaller as the mass of the dark matter particle, \( m \), increases.

The black hole of mass \( M_{\text{min}} \) is too small to survive the Hawking radiation. It evaporated on the time scale
\[ \tau = 5 \times 10^3 \text{s} \left( \frac{10 \text{TeV}}{m} \right)^3. \]

To conclude the argument, let us show that such burning of dark matter by the formation and evaporation of small black holes is more efficient than both the accretion of new black holes by the above mechanism. To this end consider the relevant time scales. The accumulation of WIMPs by the neutron star proceeds at a constant rate \( F \) that depends on the local dark matter density \( \rho_{\text{dm}} \) and equals [19]
\[ F = 1.25 \times 10^{29} \text{GeV/s} \left( \frac{\rho_{\text{dm}}}{10^3 \text{GeV/cm}^3} \right). \]

The time needed to accumulate the amount of dark matter equal to \( M_{\text{min}} \) is
\[ t_{\text{acc}} = 7.6 \times 10^4 \text{s} \left( \frac{m}{10 \text{TeV}} \right)^{-1} \left( \frac{\rho_{\text{dm}}}{10^3 \text{GeV/cm}^3} \right)^{-1}. \]

This is larger by an order of magnitude than the evaporation time even for \( m = 10 \text{ TeV} \) and even considering a \( 10^9 \text{ GeV/cm}^3 \) local dark matter density. For nearby neutron stars where the dark matter constraints are more reliable, this will give 5 orders of magnitude higher for 10 TeV WIMP mass.

Consider now the time needed to form a new black hole in the course of collapse of the self-gravitating WIMP-sphere. According to the picture described above, the contracting WIMP-sphere reaches at some point the density \( \rho_{\text{crit}} \) at which the formation of the BEC becomes possible. Further contraction happens at a constant WIMP density \( \rho = \rho_{\text{crit}} \) as the excess of the WIMPs goes into the BEC state. To form a black hole, the amount of WIMPs equal \( M_{\text{min}} \) has to be moved from the WIMP-sphere to the BEC. The corresponding release of potential energy is equal to the black hole mass times the difference of the gravitational potential between the surface and the center of the sphere,
\[ \delta E = \frac{1}{2} GM M_{\text{min}} \left( \frac{r_0}{M_{\text{min}}} \right). \]

where \( M \) is the total WIMP mass and \( r_0 \) is the radius of the WIMP-sphere,
\[ r_0 = \left( \frac{3M_{\text{sg}}}{4\pi \rho_{\text{crit}}} \right)^{1/3} = 2.2 \text{ cm} \left( \frac{10 \text{TeV}}{m} \right)^{4/3}. \]

The released energy has to be dispersed via collisions of WIMPs with nucleons, which sets the time scale of the process. Taking into account the Pauli blocking factor, the WIMP-nucleon collision time is
\[ \tau_{\text{col}} = \frac{1}{n \sigma v m_{\text{N}} v} = \frac{2p_F m}{3p_F \sigma \epsilon}, \]

where \( p_F \) is the Fermi momentum of the nucleons, \( m_{\text{N}} \simeq 1 \text{ GeV} \) is the nucleon mass, \( v \) is the WIMP velocity and \( \epsilon \) is the WIMP kinetic energy estimated as
\[ \epsilon \approx \frac{G M m}{2 r_0}. \]

from the virial theorem. Taking into account that the typical energy loss per collision is \( \delta \epsilon = 2 m_{\text{N}} \epsilon / m \) and assembling all the factors, the time required to lose the excessive energy and form a black hole is
\[ t_{\text{cool}} = \tau_{\text{col}} \frac{M \delta \epsilon}{M \delta \epsilon} = \frac{4 p_F}{3 \pi n_N \rho_{\text{crit}} M^2} r_0 M_{\text{pl}}^4. \]

One can see that this time is shorter for larger mass \( M \). Substituting \( M = M_{\text{sg}} \) into Eq. (16) one gets
\[ t_{\text{cool}} \approx 1.5 \times 10^3 \text{s} \]

This time is shorter by a factor of a few than the black hole evaporation time of Eq. (9). Note, however, the strong dependence of both quantities on the WIMP mass \( m \). Already for masses \( m \gtrsim 13 \text{ TeV} \) the black hole evaporation time becomes shorter.
Three comments are in order. The first comment is related to the black hole evaporation. The fate of a black hole of mass $M_{\text{BH}}$ is dictated by the equation

$$\frac{dM_{\text{BH}}}{dt} = CM_{\text{BH}}^2 - \frac{f}{M_{\text{BH}}^2},$$

(18)

where the first term in the left hand side corresponds to the Bondi accretion, and the second term corresponds to the Hawking radiation. Here $C = 4\pi \lambda_p G^2/c_s^3$, $\lambda$ being a constant of order one and $c_s$ the speed of sound. As we mentioned earlier, using Eq. (1) one can find that for WIMP masses above roughly 10 GeV, Hawking radiation wins over accretion and the evaporation time is given by Eq. (4). However, the newly formed black hole can also accrete from the dark matter population that, in the case examined here, can have densities significantly higher than the nuclear matter density $\rho_{\text{c}}$, as it can be seen from Eq. (4). The accretion of non-interacting collisionless particles in the non-relativistic limit onto a black hole is given by

$$F = \frac{16\pi G^2 M_{\text{BH}}^2 \rho_{\text{dm}}}{v_{\infty}},$$

(19)

where $\rho_{\text{dm}}$ is $\rho_{\text{crit}}$ of Eq. (4), and $v_{\infty}$ is the average WIMP velocity far away from the black hole. Using the virial theorem, we take $v_{\infty} = \sqrt{GM/r}$, $M$ being the total mass of the WIMP sphere and $r$ the radius of the WIMP sphere. It is understood that for the first black hole $M = M_{\text{sg}}$, and $r = r_0$. We have checked that for WIMP sphere masses ranging from Eq. (1) to Eq. (2) the Hawking radiation dominates overwhelmingly over the WIMP accretion despite the large WIMP density, and therefore the black hole evaporation time is given accurately by Eq. (4).

The second comment is that the cooling time derived above refers to the formation of the first black hole of mass given by Eq. (1). Subsequent black holes require progressively longer times. This is easily seen from Eq. (10) because $t_{\text{cool}}$ scales as $1/M^{5/3}$ (recall that $r_0$ scales as $M^{1/3}$ from Eq. (13)). Thus, the more black holes have formed and evaporated, the smaller is the remaining WIMP mass $M$ and the longer the time needed to form the next one.

The third comment has to do with the neutron star temperature. As one can see in Eq. (17), there is a strong dependence in $T$. So it is essential that $10^5$ K is a good estimate for the temperature of the neutron star at the time the star has accumulated $M_{\text{sg}}$ mass of WIMPs. Note that $M_{\text{sg}}$ depends on the temperature, cf. Eq. (2). The neutron star does not always have the same temperature, but becomes colder with time due to different cooling mechanisms. The cooling curves (i.e. the temperature of the star as a function of the age) for typical neutron stars can be found, e.g. in [31] and [32]. Knowing the cooling curves, $M_{\text{sg}}(T)$ and the WIMP accretion rate from Eq. (10), one can easily estimate the time it takes for the star to accumulate a total WIMP mass of $M_{\text{sg}}$. We found that in all cases of interest (i.e. for $m$ ranging from 10 to 1000 TeV and $\rho_{\text{dm}}$ from 0.3 to $10^3$ GeV/cm$^3$), the accumulation time is at least $1.5 \times 10^7$ years. By that time the temperature of the star should be below $10^5$ K, which justifies the use of this temperature in our estimates.

Finally, let us examine one more factor that can potentially affect our estimates. As the first black hole evaporates, it emits particles via Hawking radiation that reheat the core of the star. In principle such a deposit of heat can increase the temperature of nucleons and WIMPs and thus could potentially change our estimates. We show here that the increase in the temperature is small and the whole effect is negligible.

The energy produced by the black hole evaporation, once transferred to nucleons, propagates according to the diffusion equation,

$$\frac{\partial u}{\partial t} - D\nabla^2 u = Q,$$

(20)

where $u$ is the energy density due to the black hole evaporation, $D$ is the diffusion coefficient and $Q$ is the energy injection rate.

Since the black hole is tiny and the mean free path of the Hawking-emitted particles is short ($\sim 10^{-13}$ cm assuming a typical nucleon cross section $\sigma_N \sim \pi 10^{-26}$ cm$^2$), one may take $Q$ to be a delta-function in space, $Q = Q(t)\delta(\vec{x})$. We will also assume that the energy is injected at a constant rate. As we will see below, the time scale associated with the heat conductivity of nuclear matter is extremely short, so a constant rate is a good approximation for most stages of the black hole evaporation.

The diffusion coefficient $D = \kappa/c_V$ can be expressed in terms of the specific heat capacity $c_V = 1.4 \times 10^{-9}$ GeV$^3/(T/10^5$ K) of the nuclear matter [32] and the thermal conductivity $\kappa$. The latter can also be found in Ref. [32], $\kappa \sim 10^3$ GeV$^2$.

Assuming that the energy release by the black hole starts at $t = 0$, the solution to the above diffusion equation is

$$u(r, t) = \frac{Q}{4\pi D t} \text{erfc}\left(\frac{r}{\sqrt{4Dt}}\right).$$

(21)

The time-dependent factor saturates to one as soon as $r/\sqrt{4Dt} \ll 1$ and the solution of Eq. (21) becomes stationary at corresponding distances. The characteristic time scale for reaching the stationary solution at distances of order of the size of the WIMP cloud is

$$t_{\text{st}} = \frac{\pi}{4D} \approx 2.9 \times 10^{-9} s \left(\frac{10^3 \text{GeV}}{m}\right)^{8/3} \left(\frac{T}{10^5 \text{K}}\right).$$

(22)

This time scale is much shorter than $t_{\text{cool}}$ for all cases of interest. The same time scale determines the onset of the equilibrium once the heat source is switched off (the black hole has evaporated).

When the stationary solution is reached, the temperature of the nuclear matter is given by

$$T = \frac{Q}{4\pi \kappa T},$$

(23)
where $T_\infty = 10^5$ K. The first term describes the effect of the evaporating black hole. At distances comparable to the size of the WIMP-sphere $r \sim r_0$, the change in the temperature during the black hole evaporation is $\sim 10$ K $(m/10$ TeV$)^2$, where we have estimated $Q$ by dividing the black hole mass by its evaporation time. However, in cases where $t_{\text{cool}}$ is longer than the evaporation time, it is more appropriate to estimate the value of $Q$ by dividing the mass of the black hole by the $t_{\text{cool}}$ of Eq. (17). In this case, we find an increase in the temperature at most

$$\delta T \lesssim 35K \left( \frac{m}{10 \text{ TeV}} \right)^{-8/3}. \quad (24)$$

Thus, the effect of the black hole evaporation on the temperature in the center of the neutron star is completely negligible.

\section{I. CONCLUSIONS}

We demonstrated that it is not possible to obtain constraints on asymmetric bosonic dark matter with masses in the TeV range or higher from the collapse of WIMPs into black holes inside neutron stars and destruction of the latter. Although the self-gravitation of the accumulated WIMPs starts before the BEC formation, we showed that the WIMP sphere unavoidably passes through the stage of the BEC formation as it contracts. As a result, black holes with masses much smaller than the total WIMP mass form and evaporate in times shorter than both the accretion time scale and the WIMP sphere cooling time scale. This means that the WIMP sphere does not collapse to a single large black hole which would grow and consume the host star, but instead collapses piece by piece in such a way that every black hole that forms evaporates before another gets formed. The overall effect is the (unobservable) heating of the neutron star, but not its destruction. Thus no constraints on the TeV WIMP mass range can be imposed.

We would like to stress that the scenario we have studied is the most conservative one. We assumed a uniform constant density for the WIMP sphere. In reality, one should expect a more cuspy profile towards the center of the WIMP sphere. As a result, $\rho_{\text{BEC}}$ will be achieved earlier, and the cooling time for formation of subsequent black holes can increase substantially because the earlier the BEC forms, the smaller will be the energy loss of WIMPs colliding to nucleons due to smaller differences in the temperatures of the two species.

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