Single-atom interferometer based on two-dimensional spatial adiabatic passage

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In this work we extend the spatial adiabatic passage technique to the two-dimensional case by considering a single cold atom in a system of three identical harmonic traps in a triangular geometry. We show that the transfer of a single atom from the ground state of a harmonic trap to the ground state of the most distant one is successfully achieved in a robust way for a broad range of parameter values. Nevertheless, we find that there is a specific geometrical configuration of the traps for which a crossing of two energy eigenvalues occurs and the transfer of the atom fails, splitting its wavefunction into a coherent superposition between two of the traps. We take advantage of this situation to propose a single-atom interferometer based on spatial adiabatic passage and discuss its performance in terms of the final population distribution among the asymptotic eigenstates of the individual traps. The results have been checked with numerical simulations of the full two-dimensional Schrödinger equation.

I. INTRODUCTION

Atom interferometers are focus of current research interest due to their suitability to perform high-precision measurements [1–8]. This is mainly motivated by the small wavelengths associated to matter waves and by the wide range of atomic properties like mass, magnetic moment, and polarizability that make them suitable to measure fundamental constants, internal forces, accelerations and rotations.

The implementation of a matter-wave interferometer requires an efficient and robust method to split and recombine the matter wavefunction, i.e., an accurate control of the external degrees of freedom of the atom. Tunneling between trapping potentials constitutes a fundamental tool for the preparation and manipulation of single quantum particles states. However, direct tunneling between two resonant traps leads to Rabi-type oscillations of the atomic population, which are not experimentally easy to control since they are very sensitive to small variations of the parameter values of the system [9]. The spatial adiabatic passage technique in a system formed by three traps lying on a straight line has been proposed [9–10] as the spatial analogue of the stimulated Raman adiabatic passage (STIRAP) technique [17] well known in Quantum Optics. At variance with direct tunneling, spatial adiabatic passage offers a much richer behavior and a more robust performance than direct tunneling approaches since it does not require an accurate control of the system parameters. Three-well interferometry with Bose–Einstein condensates (BECs) using an analogue of fractional STIRAP has been recently addressed [18]. The spatial adiabatic passage has also been discussed for the transport of single atoms along dipolar waveguides [11] [19], for the transport of BECs in triple-well potentials [20] [21] and experimentally reported for the light transfer in coupled optical waveguides [22–24].

In this work, we extend the spatial adiabatic passage technique to a scheme that breaks the effective one-dimensionality that results from the direct analogy with the STIRAP processes. Recently, a fully two-dimensional adiabatic passage process in triple-well potentials without analogue in quantum optical systems of internal states has been reported [25], which addresses the generation of states carrying angular momentum by using non-identical harmonic traps. Here, we consider a single atom in a system of three not aligned identical two-dimensional harmonic traps and we focus on the analytical and numerical study of the conditions to achieve a complete transfer of the atom between the ground states of the most distant traps, showing that, under certain conditions in which the adiabatic transfer fails, a 50% superposition of the atom in two of the traps is robustly obtained, making possible the use of the system as a novel scheme for matter-wave interferometry.

The paper is organized as follows. In Section II we introduce the physical system that will be investigated for the two-dimensional spatial adiabatic passage, and diagonalize the Hamiltonian that governs the dynamics of a single atom in such a trapping potential. The conditions required to perform two-dimensional spatial adiabatic passage are derived in Section III. In Section IV we discuss the implementation and performance of a matter-wave interferometer using a level crossing in the eigenvalue spectrum. Finally, Section V is devoted to the conclusions.

II. PHYSICAL SYSTEM

We consider a system formed by three two-dimensional harmonic potentials (labeled A, B and C) with equal trapping frequencies ($\omega_A = \omega_B = \omega_C = \omega$). As schematically shown in Fig. 1, the three traps are not lying on a straight line but form a triangle, with the trap center positions being $x_A = -d_{AB} \cos \beta$, $y_A = -d_{AB} \sin \beta$, $x_B = d_{BC} \cos \beta$, $y_B = d_{BC} \sin \beta$, and $x_C = 0$. The Hamiltonian of the system is given by

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \sum_{i=A,B,C} \frac{1}{2} m \omega_i^2 (x_i^2 + y_i^2) + V(x, y),$$

where $V(x, y)$ is the potential energy of the system, which is a sum of harmonic potentials for each trap:

$$V(x, y) = \sum_{i=A,B,C} \frac{1}{2} m \omega_i^2 (x_i^2 + y_i^2) + V_{ext}(x, y).$$

The external potential $V_{ext}(x, y)$ is assumed to be a two-dimensional harmonic potential for simplicity.

The equations of motion for the quantum state $\psi(x, y, t)$ are given by the time-dependent Schrödinger equation:

$$i \hbar \frac{\partial \psi}{\partial t} = H \psi.$$

In the adiabatic limit, the Hamiltonian $H$ is replaced by its instantaneous diagonal part $H_{diag}$, and the state $\psi$ evolves according to

$$i \hbar \frac{\partial \psi}{\partial t} = H_{diag} \psi.$$

The diagonal part of the Hamiltonian is given by

$$H_{diag} = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \sum_{i=A,B,C} \frac{1}{2} m \omega_i^2 (x_i^2 + y_i^2) + V_{ext}(x, y).$$

The adiabatic approximation assumes that the off-diagonal terms in the Hamiltonian are small compared to the diagonal terms, and the state $\psi$ remains in the adiabatic state throughout the evolution. In this case, the wavefunction evolves according to the instantaneous Schrödinger equation:

$$i \hbar \frac{\partial \psi}{\partial x_i} = \left( \frac{\partial^2}{\partial x_i^2} + \frac{\partial^2}{\partial y^2} \right) \psi.$$

This equation can be solved numerically to obtain the time evolution of the wavefunction.

In summary, the two-dimensional spatial adiabatic passage technique allows for the transport of a single atom between two of the traps in a system of three identical harmonic traps in a triangular geometry. The conditions for achieving a complete transfer of the atom are derived analytically, and the performance of the interferometer is studied numerically. The results show that the technique is robust for a broad range of parameter values, making it a promising scheme for matter-wave interferometry.
where the couplings depend on the separation between the parameters and see the text.

respectively, where \( \phi \) are the ground states of the traps can be written as:

\[
\psi_A = \phi_0(x + d_{AB}\cos\beta)\phi_0(y + d_{AB}\sin\beta),
\]

\[
\psi_B = \phi_0(x)\phi_0(y),
\]

and

\[
\psi_C = \phi_0(x - d_{BC})\phi_0(y),
\]

FIG. 1. Schematic representation of the system of three harmonic traps, A, B and C with equal trapping frequencies. For the parameters definition see the text.

In Cartesian coordinates, the A, B and C asymptotic ground states of the traps can be written as:

\[
\psi_A = \phi_0(x + d_{AB}\cos\beta)\phi_0(y + d_{AB}\sin\beta),
\]

\[
\psi_B = \phi_0(x)\phi_0(y),
\]

and

\[
\psi_C = \phi_0(x - d_{BC})\phi_0(y),
\]

respectively, where \( \phi_0 \) is the single-particle ground state eigenfunction of the one-dimensional quantum harmonic oscillator.

In one-dimensional spatial adiabatic passage, three inline traps are considered such that the coupling between the outermost traps is neglected, i.e., only neighbor coupling is considered. In contrast, in the two-dimensional case we assume that all three traps are directly tunnel-coupled to each other. The tunneling rates between A and B, B and C, and A and C are denoted as \( J_{AB} \), \( J_{BC} \) and \( J_{AC} \), respectively. If the dynamics of the system are restricted to the space spanned by \( \{ \psi_A(t), \psi_B(t), \psi_C(t) \} \), the Hamiltonian that governs its evolution reads:

\[
H = \frac{\hbar}{2} \begin{pmatrix} 0 & -J_{AB} & -J_{AC} \\ -J_{AB} & 0 & -J_{BC} \\ -J_{AC} & -J_{BC} & 0 \end{pmatrix},
\]

where the couplings depend on the separation between the centers of the harmonic potentials as [9]:

\[
J_{ij} = \frac{-1 + e^{(\alpha d_{ij})/2} [1 + \sqrt{\pi} \text{erf}(\alpha d_{ij}/2)]/2}{\sqrt{\pi}(e^{(\alpha d_{ij})^2/2} - 1)/(\alpha d_{ij})},
\]

being \( i, j = A, B, C \) with \( i \neq j \), \( \alpha = \sqrt{(m\omega)/\hbar} \), and \( m \) the mass of the single cold atom.

New and richer phenomenology compared to the one-dimensional spatial adiabatic passage case is found by diagonalizing the Hamiltonian in Eq. (4). The energy eigenvalues of Eq. (4) are obtained from its characteristic polynomial which is a depressed cubic equation:

\[
E^3 + pE + q = 0,
\]

where

\[
p = -\frac{\hbar^2}{4}(J_{AB}^2 + J_{BC}^2 + J_{AC}^2),
\]

and

\[
q = \frac{\hbar^3}{4}J_{AB}J_{BC}J_{AC}.
\]

Since the energy eigenvalues of the Hamiltonian of Eq. (4) must be real, the solutions of Eq. (6) have to fulfill:

\[
4p^3 + 27q^2 \leq 0.
\]

In this case, the analytic expression of the energy eigenvalues reads:

\[
E_k = 2\sqrt{-\frac{p}{3}} \cos \left[ \frac{1}{3} \arccos \left( \frac{3q}{2p} \sqrt{-\frac{3}{p}} + \frac{2\pi}{3} \right) \right],
\]

where \( k = 1, 2, 3 \). For \( 4p^3 + 27q^2 < 0 \) three different energy eigenvalues exist, while \( 4p^3 + 27q^2 = 0 \) implies that the \( E_2 \) and \( E_3 \) eigenvalues become degenerates. In particular, the equality \( 4p^3 + 27q^2 = 0 \) is fulfilled if and only if

\[
J_{AB} = J_{BC} = J_{AC},
\]

and leads to an energy level crossing, \( E_2 = E_3 \). In our configuration this level crossing occurs for the angle \( \beta = 2\pi/3 \) when all the traps are equally separated. For any other angle \( \beta \), the distances between the traps cannot be all equal simultaneously and, therefore, as long as the traps are coupled, the system will have three different energy eigenvalues.

The eigenstates \( \Psi_k \) of Eq. (4) read:

\[
\Psi_k = \frac{1}{N} (a_k \psi_A + b_k \psi_B - c_k \psi_C),
\]

with

\[
a_k = J_{BC} - \frac{2E_k}{\hbar J_{AB}},
\]

\[
b_k = J_{AC} - \frac{2E_k}{\hbar J_{AB}},
\]

\[
c_k = J_{AB} - \frac{4E_k^2}{\hbar^2 J_{AB}},
\]
and

\[ N = \sqrt{a_k^2 + b_k^2 + c_k^2}, \]

where \( k = 1, 2, 3 \). For \( J_{AC} = 0 \), which means \( q = E_2 = b_2 = 0 \), Eq. (12) yields the same expression for the energy eigenstates as in the one-dimensional spatial adiabatic passage case. In particular, one of the eigenstates becomes the so-called spatial dark state, i.e., \( \Psi_2 = \cos \theta \psi_A - \sin \theta \psi_C \) with \( \theta = \tan^{-1}(J_{AB}/J_{BC}) \). In this case, the spatial adiabatic passage consists in adiabatically following the spatial dark state from the initial state \( \psi_A \) to the final state \( \psi_C \) by smoothly varying \( \theta \) from 0 to \( \pi/2 \).

III. TWO-DIMENSIONAL SPATIAL ADIABATIC PASSAGE

In this section we will make use of the previously derived eigenvalues and eigenstates of Hamiltonian (4) to investigate up to which extent spatial adiabatic passage works for the two-dimensional case where the coupling between the initial and final traps, \( J_{AC} \), is also present.

![Fig. 2](https://example.com/fig2.png)

**Fig. 2.** Temporal evolution of (a) the distances between traps \( d_{AB}, d_{BC} \) and \( d_{AC} \), and (b) the couplings \( J_{AB}, J_{BC} \) and \( J_{AC} \) during the spatial adiabatic passage process. The parameter values are: \( \beta = 0.5\pi \) and \( \delta = 0.2T \), where \( T \) is the total time of the process. Coupling rates are given in units of \( \omega \) and distances in units of \( \alpha^{-1} \).

In the two-dimensional case, a counterintuitive temporal sequence of couplings is applied with the single cold atom initially located in the vibrational ground state of trap \( A \). We call the temporal coupling sequence “counterintuitive” in analogy to the terminology used to describe the STIRAP technique for internal atomic levels [17]. In our case, with the \( B \) trap fixed in the position \( (0,0) \), the sequence consists in first bringing closer and then separating the \( C \) trap to the \( B \) trap and, with a certain temporal delay, approaching and then separating the \( A \) trap to the \( B \) trap, keeping the \( \beta \) angle fixed. Note that the distance \( d_{AC} \) depends on the two control distances, \( d_{AB} \) and \( d_{BC} \) and on the angle \( \beta \), so \( d_{AC} \) is not a free parameter. The couplings as a function of time can be easily calculated through their dependence on the separation between traps, see Eq. (4).

![Fig. 3](https://example.com/fig3.png)

**Fig. 3.** (a) Energy eigenvalues as a function of time and temporal evolution of the population of the asymptotic states of the traps \( \psi_A, \psi_B, \) and \( \psi_C \) for the three eigenstates of the system (b) \( \Psi_1 \), (c) \( \Psi_2 \), and (d) \( \Psi_3 \). Parameter values are the same as in Fig. 2.

Fig. 2 shows an example of the temporal evolution of the distances between traps and the corresponding coupling rates for the counterintuitive sequence of the spatial adiabatic passage. For this temporal evolution of the couplings, Fig. 3 shows the corresponding energy eigenvalues as well as the population of each asymptotic level of the individual traps for the three eigenstates of the system. From Eq. (12) and the example in Fig. 3 it is possible to observe that, when the counterintuitive coupling sequence is applied, the eigenstate \( \Psi_2 \) involves initially only the \( A \) trap. Thus, since the atom is initially located in the \( A \) trap, the system is in the state \( \psi(t = 0) = \Psi_2(t = 0) = \psi_A \). If the sequence is performed adiabatically [17], the system will be able to follow the eigenstate \( \Psi_2 \) during the whole process. At the end of the sequence (at a total time \( T \)), \( \psi(t = T) = \Psi_2(t = T) = \psi_C \). Therefore, by applying a counterintuitive coupling sequence the single atom is completely transferred from \( A \) to \( C \) traps. This is true for a range of angles from \( \beta = 0 \) (the one-dimensional spatial adiabatic case) to \( \beta < \beta_{th} = 2\pi/3 \). However, for \( \beta = \beta_{th} \) there is a level crossing between \( \Psi_2 \) and \( \Psi_3 \) and it is no longer possible to adiabatically follow the energy eigenstate \( \Psi_2 \). For angles larger than \( \beta_{th} \) the level crossing is again avoided. Nevertheless, the distance \( d_{AC} \) becomes shorter than \( d_{BC} \) at the beginning of the process, which leads to \( J_{AC} > J_{BC} \). This means that initially the eigen-
state \( \Psi_2 \) is a combination of \( \psi_A \) and \( \psi_B \), see Eqs. (13) and (14), which could prevent the complete transfer from \( A \) to \( C \) traps when a counterintuitive coupling sequence is applied.

\[
\begin{align*}
\dot{\Psi}_2 &= |\langle \psi_A | \Psi_k \rangle|^2, \\
\dot{\Psi}_3 &= |\langle \psi_C | \Psi_k \rangle|^2, \\
\dot{\Psi}_2 &= |\langle \psi_B | \Psi_k \rangle|^2.
\end{align*}
\]

Here \((x_i, y_i)\) with \( i = A, B, C \) are the positions of the individual trap centers, and \( \omega_A = \omega_B = \omega_C = \omega \).

Let us now investigate in detail the particular case for which there is an energy level crossing. Fig. 4 shows the evolution of the energy eigenvalues for the same temporal variation of the distances \( d_{AB} \) and \( d_{BC} \) as in Fig. 2 but with \( \beta = 2\pi/3 \).

Up to now, we have discussed two-dimensional spatial adiabatic passage restricting the dynamics to the space spanned by the three asymptotic ground states of the individual traps. In the following we will check the validity of this simplified model by numerically integrating the two-dimensional Schrödinger equation, which reads:

\[
\begin{align*}
\frac{i\hbar}{\partial t} \psi(x, y) &= \left[ -\frac{\hbar^2}{2m} \nabla^2 + V(x, y) \right] \psi(x, y),
\end{align*}
\]

where \( \nabla^2 \) is the two-dimensional Laplace operator and \( V(x, y) \) is the trapping potential, which we assume to be constructed from truncated harmonic oscillator potentials

\[
V(x, y) = \min_{i=A, B, C} \left\{ \frac{1}{2} m \omega_i^2 \left[ (x - x_i)^2 + (y - y_i)^2 \right] \right\}.
\]

Fig. 5 shows the population distribution at different times for a process of total time \( T = 5000\omega^{-1} \) with (a) \( \beta = \pi/2 \) and (b) \( \beta = 2\pi/3 \). One can see in Fig. 5(a) that a single particle is completely transferred from the \( A \) trap to the \( C \) trap, which corresponds to the adiabatic following of the eigenstate \( \Psi_2 \). Contrarily, in Fig. 5(b) we can see that the atom ends up in a superposition of \( A \) and \( B \) traps. This is due to the energy level crossing that occurs at \( t = 0.5T \) when the three traps are equidistant and \( J_{AB} = J_{BC} = J_{AC} \), which implies that the system is transferred from \( \Psi_2 \) to \( \Psi_3 \), following \( \Psi_3 \) until the end of the process.

Fig. 6 shows the final population in the \( C \) trap as a function of the angle \( \beta \), where the red curve represents the results of the numerical integration of the Hamiltonian (4) and the blue dots the results of the numerical integration of the two-dimensional Schrödinger equation.

\[
\begin{align*}
|\langle \psi_C | T \rangle|^2
\end{align*}
\]

FIG. 6. Final population in the \( C \) trap as a function of \( \beta \) for \( T = 5000\omega^{-1} \) and the parameter values as in Fig. 2. The red curve represents the results of the numerical integration of the Hamiltonian (4) and the blue dots the results of the numerical integration of the two-dimensional Schrödinger equation.

Fig. 6 shows the final population in the \( C \) trap as a function of the angle \( \beta \), where the red curve is found by numerically integrating the Hamiltonian (4) and the blue dots are the results of the numerical integration of the two-dimensional Schrödinger equation. The high agree-
ment of both results, supports the validity of the $4 \times 4$ Hamiltonian formulation. The fact that for the angle $\beta = \beta_{th}$ the population in $C$ drops to 0 corresponds to the level crossing between states $\Psi_2$ and $\Psi_3$. Moreover, we can see that for $\beta$ angles just above $\beta_{th}$ the complete transfer it is still efficient, which is consistent with the fact that, although that initially $J_{AC} > J_{BC}$, the value of $J_{AC}$ is very weak and the single cold atom is still in the $A$ trap when the $J_{BC}$ coupling becomes stronger than $J_{AC}$. For very large $\beta$ angles, the value of $J_{AC}$ is significant during the first stage of the process and prevents the eigenstate $\Psi_2$ to be equal to $\psi_A$ and, therefore, an efficient transfer to the $C$ trap is no longer possible.

**IV. SINGLE ATOM INTERFEROMETRY**

In the previous section we have seen that for $\beta = \beta_{th}$ the transfer of population between $A$ and $C$ traps fails, and the atomic wavefunction ends up in $\Psi_3$, which is a coherent antisymmetric superposition of $A$ and $B$ traps with equal probabilities. This coherent splitting of the atomic wavefunction and its eventual recombination can be used to implement a robust atomic interferometer as it will be discussed in the following lines.

The first step of the interferometer corresponds to the splitting process due to the level crossing already depicted in Fig. 5(b). At the end of the splitting, at time $T$, we perform the second step by imprinting a relative phase, $\varphi$, between the $A$ and $B$ traps. The last step is the recombination process that consists of reversing in time the evolution of the couplings performed during the splitting process, i.e., keeping $\beta$ fixed, we approach and separate first $A$ and $B$ traps, and with a certain time delay we approach and separate $C$ and $B$ traps.

At the final time, $2T$, the population distribution of the output atomic state among the asymptotic states of the traps will allow for a direct measurement of the imprinted phase.

**FIG. 8.** (a) Population at the end of the process ($t = 2T$) of $A$ trap, (b) sum of the populations of $B$ and $C$ traps, and (c) measured phase difference between $A$ and $B$ traps as a function of the imprinted phase difference at $t = T$. The solid lines correspond to the values of Eqs. (19), (20) and (21), whereas the dots correspond to the results of the integration of the two-dimensional Schrödinger equation. These results have been calculated for a total time $2T = 10000 \omega^{-1}$.

To check the performance of the interferometer, Fig. 7 shows the population distribution at different times during the recombination process for (a) $\varphi = 0$, (b) $\varphi = \pi/2$, and (c) $\varphi = \pi$. It is clearly shown in Fig. 7(a) that for $\varphi = 0$, at the end of the process ($t = 2T$), the atom returns to trap $A$. This is due to the complete reversibility...
of the splitting process that leads to the transfer back of the system from state \( \psi_3 \) to \( \psi_2 \) at the particular time for which the crossing of energy levels occurs. After the crossing, the system follows \( \psi_2 \), which at the end of the process has only contribution of \( \psi_A \).

When a phase difference, \( \varphi \), between the \( A \) and \( B \) traps is imprinted after the splitting process, the state of the system becomes \( \psi_3(T) = \frac{1}{\sqrt{2}}(\psi_A - e^{i\varphi}\psi_B) \) which can be decomposed at this particular time in a superposition of \( \psi_3(T) = \frac{1}{\sqrt{2}}(\psi_A - \psi_B) \) and \( \psi_1(T) = \frac{1}{\sqrt{2}}(\psi_A + \psi_B) \). By reversing the sequence of couplings, the \( \psi_3 \) contribution will be transferred to \( \psi_2 \) at the level crossing and it will end up in trap \( A \) while the \( \psi_1 \) contribution will evolve backwards according to Fig. 4(b) and at the end of the process will be in a superposition of traps \( B \) and \( C \). Thus, by measuring the population of the three traps at the end of the process one can infer the phase difference between \( A \) and \( B \) traps just before the recombination process.

The population of \( A \) trap is given by:

\[
|\psi_A(2T)|^2 = |\langle \psi_3|\psi_\varphi \rangle|^2 = \frac{1}{2}(1 + \cos \varphi),
\]

while the populations of \( B \) and \( C \) traps at the output of the interferometer read:

\[
|\psi_B(2T)|^2 + |\psi_C(2T)|^2 = |\langle \psi_1|\psi_\varphi \rangle|^2 = \frac{1}{2}(1 - \cos \varphi).
\]

Thus, the phase difference between \( A \) and \( B \) traps can be written as:

\[
\varphi = \pm \arccos \left[ \frac{|\psi_A(2T)|^2 - |\psi_B(2T)|^2 + |\psi_C(2T)|^2}{|\psi_A(2T)|^2 + |\psi_B(2T)|^2 + |\psi_C(2T)|^2} \right].
\]

In Fig. 8 we plot the analytic prediction and the numerically obtained population at the end of the process \( (t = 2T) \) of trap \( A \) (a), the sum of the populations of traps \( B \) and \( C \) (b) and the measured phase difference between \( A \) and \( B \) traps as a function of the imprinted phase difference at \( t = T \). We clearly see a full agreement between the results from Eq. (20), Eq. (19) and Eq. (21) and the corresponding numerical integration of the two-dimensional Schrödinger equation. The nearly linear behavior of the measured phase difference with the imprinted one evidences the excellent performance of the described system as a matter-wave interferometer.

We have also checked the robustness of the interferometer. First, we have added a shaking oscillation \( A_{\text{shake}}\sin(\omega_{\text{shake}}t) \) in the evolution of the approaching and separating distances \( d_{AB} \) and \( d_{BC} \). In Fig. 9 the results from the numerical integration of the two-dimensional Schrödinger equation show the difference in the measured phase comparing the case in which the shaking is added with the case without shaking. We can see that for a broad range of parameter values the measured imprinted phase is closer than 0.1 to the measured value without the shaking, which demonstrates the robustness of the interferometer in front of fluctuations in the approaching and separation process. Furthermore, we have also checked that, with a total time \( t = 2T = 10000\omega^{-1} \), a variation of the \( \beta \) angle up to 1% gives good results for the measured phase and would allow for the usage of the system as a single-atom interferometer.

The interferometry proposal here presented has been discussed at the single-atom level, which would require several realizations to perform statistics in order to obtain an accurate measurement of the phase difference \( \varphi \) between \( A \) and \( B \) traps. Nevertheless, the proposed scheme could also be performed at once by using a weakly interacting Bose–Einstein condensate (BEC) within a certain range of values of the nonlinearity [20, 26]. By numerically integrating the corresponding Gross–Pitaevskii equation, we have checked that for a BEC consisting of 1000 \(^3\)Li atoms in the \( |F = 1, m_f = 1 \rangle \) state, and using...
harmonic traps of trapping frequencies $\omega_\perp = 2\pi \times 400$ Hz and $\omega_z = 10\omega_\perp$, it is possible to measure the imprinted phase within an error of 10% for values of the s-wave scattering length between $\pm 0.003$ nm. These scattering length values can be reached by using a Feshbach resonance tuning a magnetic field around $B = 543.6$ G [27].

V. CONCLUSIONS

We have discussed two-dimensional spatial adiabatic passage for a single cold atom in a trapping potential consisting of three two-dimensional harmonic wells forming a triangular configuration. It has been shown analytically and numerically the successful performance of spatial adiabatic passage for a broad range of parameter values. However, there is a critical configuration for which the three tunneling rates become equal at a particular time during the dynamics which, implies a level crossing in the system’s eigenvalue spectrum. This level crossing produces a coherent splitting of the matter wave that we have used as the first step to build up a matter-wave interferometer. Once the matter wave is split between two of the traps we have imprinted a relative phase between these two traps showing that the recombination process results in a distribution of the matter wave among the asymptotic states of the traps that depends on the imprinted phase. Finally, we have numerically checked the excellent performance and robustness of the interferometer in order to measure the imprinted phase.

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