CONSTRANING PERTURBATIVE EARLY DARK ENERGY WITH CURRENT OBSERVATIONS

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ABSTRACT
In this work, we study a class of early dark energy (EDE) models, in which, unlike in standard dark energy models, a substantial amount of dark energy exists in the matter-dominated era. We self-consistently include dark energy perturbations, and constrain these models using current observations. We consider EDE models in which the dark energy equation of state is at least \( w_m \gtrsim -0.1 \) at early times, which could lead to an EDE density of up to \( \Omega_{DE}(z_{CMB}) = 0.03 \Omega_m(z_{CMB}) \). Our analysis shows that marginalizing over the non-DE parameters such as \( \Omega_m, H_0 \), and \( n_s \), current CMB observations alone can constrain the scale factor of transition from EDE to late-time dark energy to \( a_t \gtrsim 0.44 \) and width of transition to \( \Delta \lesssim 0.37 \). The equation of state at present is somewhat weakly constrained to \( w_0 \lesssim -0.6 \), if we allow \( H_0 \sim 60 \text{ km s}^{-1} \text{ Mpc}^{-1} \). Taken together with other observations, such as SNe, Hubble Space Telescope, and Sloan Digital Sky Survey luminous red galaxies, \( w_0 \) is constrained much more tightly to \( w_0 \lesssim -0.9 \), while redshift of transition and width of transition are also tightly constrained to \( a_t \lesssim 0.19 \) and \( \Delta \lesssim 0.21 \). The evolution of the equation of state for EDE models is thus tightly constrained to \( \Lambda \)CDM-like behavior at low redshifts. Incorrectly assuming dark energy perturbations to be negligible leads to different constraints on the equation of state parameters—\( w_0 \lesssim -0.8, a_t \lesssim 0.33 \), and \( \Delta \lesssim 0.31 \), thus highlighting the necessity of self-consistently including dark energy perturbations in the analysis. If we allow the spatial curvature to be a free parameter, then the constraints are relaxed to \( w_0 \lesssim -0.77, a_t \lesssim 0.35 \), and \( \Delta \lesssim 0.35 \) with \( -0.014 < \Omega_c < 0.031 \) for CMB + other observations. For perturbed EDE models, the 2\( \sigma \) lower limit on \( \sigma_8 \) \((\sigma_8 \gtrsim 0.59)\) is much lower than that in \( \Lambda \)CDM \((\sigma_8 \gtrsim 0.72)\), thus raising the interesting possibility of discriminating EDE from \( \Lambda \)CDM using future observations such as halo mass functions or the Sunyaev–Zeldovich power spectrum.

Key words: cosmic background radiation – dark energy

Online-only material: color figures

1. INTRODUCTION
Over the last decade, the unexpected faintness of distant Type Ia supernovae (SNe) has shown that the expansion of the universe is accelerating at present (Riess et al. 1998; Perlmutter et al. 1999; Tonry et al. 2003; Riess et al. 2005; Astier et al. 2006; Hicken et al. 2009; Lampeitl et al. 2010). This remarkable discovery points to the existence of dark energy (DE), a negative pressure energy component which dominates the energy content of the universe at present. Other, complementary, probes such as the cosmic microwave background (CMB) and various large-scale structure surveys have also confirmed the existence of this mysterious component of energy (Eisenstein et al. 2005; Komatsu et al. 2009; Reid et al. 2009). Several theories have been proposed to explain this phenomenon, the simplest of which is the cosmological constant \( \Lambda \), with a constant energy density and a constant equation of state \( w = -1 \). The cosmological constant is fit well by the current data (Hicken et al. 2009); however, there are no strong constraints on the time evolution of dark energy at present. Thus, evolving models of dark energy remain viable as alternative candidates for dark energy. Many non-cosmological constant phenomenological explanations for cosmic acceleration have been suggested (see reviews by Sahni & Starobinsky 2000; Carroll 2001; Peebles & Ratra 2003; Copeland et al. 2006; Nojiri & Odintsov 2007; Sahni & Starobinsky 2006; Frieman et al. 2008, and references therein). These are based either on the introduction of new physical fields (quintessence models, Chaplygin gas, etc.), or on modifying the laws of gravity and therefore the geometry of the universe (scalar–tensor gravity, \( f(R) \) gravity, higher dimensional “Braneworld” models, etc.). As of now, there is no consensus on the true nature of dark energy.

An interesting class of models which have been suggested in the literature are early dark energy (EDE) models, a class of dark energy in which the early universe contained a substantial amount of dark energy. These models were studied theoretically in Dodelson et al. (2000); Skordis & Albrecht (2002); Doran & Robbers (2006) and references therein, and have been analyzed with respect to observations extensively in recent times in Linder & Robbers (2008); Francis et al. (2009); Grossi & Springel (2009); Fedeli et al. (2009); Xia & Viel (2009). For now, there are no strong observational constraints on the EDE models, and it is especially difficult to discriminate EDE models which have \( w = -1 \) at present from the \( \Lambda \)CDM model of dark energy.

In this work we use a parameterization of the equation of state of dark energy to study and constrain EDE models using the currently available data. We attempt to see if bounds can be put on the transition from EDE to the present day dark energy content of the universe. Section 2 explains the methods and data used for this analysis, Section 3 shows the results, and in Section 4 we conclude.

2. METHODOLOGY
Dark energy perturbations for dynamic dark energy models have been studied in a number of works, usually under the formalism of a minimally coupled scalar field (See Ma et al. 1999; Hwang & Noh 2001; Hu 2002; Malquarti & Lidfolk 2002; Weller & Lewis 2003; Bean & Doré 2004; Dutta & Maor 2007; Mota et al. 2007; Jassal 2009; Novosyadlyj & Sergijenko 2009; and references therein). For practical purposes, e.g., analyzing
dynamic dark energy models in light of data, it has sometimes been the practice to consider dark energy perturbations as negligible, and sometimes not. As shown in Park et al. (2009), not taking into account the dark energy perturbations correctly can lead to erroneous gauge-dependent results. In our analysis, we self-consistently include the dark energy perturbations for the EDE models, and also show how the results differ if these perturbations are not included.

2.1. Dark Energy Perturbations

A homogeneous and isotropic large-scale universe can be described by the Friedman–Lemaître–Robertson–Walker (FLRW) metric
\[ ds^2 = a^2(\eta) [d\eta^2 + \delta_{ij} dx^i dx^j], \]
where \( \eta \) is the conformal time, \( dx \) is the length element, and \( a(\eta) \) is the scale factor. The speed of light \( c \) is set to unity, so that the time variable has dimensions of length.

First-order perturbations take the form
\[ ds^2 = a^2(\eta)((1 + 2\Phi(x, \eta))d\eta^2 - (1 + 2\Phi(x, \eta))\delta_{ij} dx^i dx^j], \]
where \( \Phi, \Psi \) are the Bardeen potentials. If proper isotropy of the medium is zero, then \( \Phi = -\Psi \).

We adopt two equivalent approaches to account for the dark energy perturbations, the first consists of considering the dark energy component as an additional fluid, while in the second dark energy is defined as a minimally coupled scalar field. Both approaches lead to the same result within the framework considered, and each has its usefulness in analyzing the results.

2.1.1. Dark Energy as a Fluid

In this section, we follow the treatment of Weller & Lewis (2003). Along with the matter and radiation components, we consider dark energy to be an additional fluid component, so that the dark energy perturbations are characterized by an equation of state, an adiabatic sound speed, and an intrinsic entropy perturbation:
\[ w_{DE} = \frac{\rho_{DE}}{\rho_{DE}}, \]
\[ c_{s,DE}^2 = \frac{\rho_{DE}}{\rho_{DE}} \]
\[ \Gamma_{DE} = \frac{\delta_{DE}}{\rho_{DE}} - c_{s,DE}^2 \cdot \delta_{DE}. \]

Defining the frame invariant quantity \( c_{s,DE}^2 \) (the fluid sound speed in the frame comoving with the fluid), the continuity and Euler equations giving the evolution of the density contrast and velocity of a fluid with equation of state \( w_i = p_i/\rho_i \), and adiabatic speed of sound \( c_{s,i}^2 = p_i/\rho_i \), may be written as (prime denotes derivative with respect to \( \eta \))
\[ \delta' = -3H(c_{s,i}^2 - w_i)\delta_i - 9H^2(c_{s,i}^2 - c_{s,DE}^2)(1 + w_i)\frac{v_i}{k} \]
\[ - (1 + w_i)kv_i - 3(1 + w_i)\Psi' \]
\[ v_i' = -H(1 - 3c_{s,i}^2)v_i + \frac{k c_{s,i}^2 \delta_i}{(1 + w_i)} - kA, \]
where \( A \) is the acceleration (\( A = 0 \) in the synchronous gauge, \( A = -\Psi \) in the Newtonian gauge) and \( \mathcal{H} = a'/a = aH \) is the conformal Hubble parameter. For the matter component, \( w_m = c_{s,m}^2 = c_{s,DE}^2 = 0 \). For the dark energy component, a fluid with varying \( w_{DE} \geq -1 \) has \( c_{s,DE}^2 = w_{DE} - [d w_{DE}/d(\ln a)]/3(1 + w_{DE}) \). For scalar field-like dark energy models, \( c_{s,DE}^2 = 1 \). For a more general class of models, such as k-essence, \( c_{s,DE}^2 \) could be variable as well. To reduce the number of parameters, we consider \( c_{s,DE}^2 = 1 \) in our analysis, which would still allow us to study a wide range of dark energy models. Thus, for a universe containing matter (CDM + baryons) and dark energy, a set of four perturbation equations may be defined for the gauge-independent variables \( \delta_m, v_m, \delta_{DE}, \) and \( v_{DE} \) and are solved using adiabatic initial conditions.

2.1.2. Dark Energy Perturbations in Scalar Field Formalism

An equivalent and convenient approach for studying the dark energy perturbations is to regard the dark energy component as a minimally coupled scalar field \( Q \) with self-interaction potential \( V(Q) \). The field dynamics are given by
\[ Q'' + 2HQ' + a^2 \frac{dV}{dQ^2} = 0, \]
and the perturbations of the scalar field evolve through the perturbed Klein–Gordon equation
\[ \delta Q'' + 2H\delta Q' + \left( k^2 + a^2 \frac{d^2V}{dQ^2} \right) \delta Q = 4Q\Psi' - 2a^2 \frac{dV}{dQ} \Psi', \]
The metric perturbations evolve as
\[ \Psi'' + 3H\Psi' + 8\pi Ga^2V\Psi = 4\pi G \left( Q\delta Q' - a^2 \frac{dV}{dQ} \delta Q \right). \]
The matter density contrast may be obtained from the above equations to be
\[ \delta_m = \frac{1}{4\pi G \rho_m} \left[ 3H\Psi' + 8\pi Ga^2(\rho_m + V) + k^2 \right] \Psi' \]
\[ + 4\pi G \left( Q\delta Q' + a^2 \frac{dV}{dQ} \delta Q \right). \]

The fluid parameters for the dark energy component (as defined in Section 2.1.1) are related to the scalar field variables by
\[ w_{DE} = \frac{Q^2 - 2a^2V(Q)}{Q^2 + 2a^2V(Q)} \]
\[ c_{s,DE}^2 = 1 + \frac{2 \frac{dV}{dQ}}{3\frac{dQ}{\mathcal{H}}} \]
\[ \Gamma_{DE} = \frac{1 - c_{s,DE}^2}{w_{DE}} \left[ \delta_{DE} - 3H(1 + w_{DE})v_{DE} \right], \]
and the gauge-independent perturbation variables by
\[ \delta_{DE} = \frac{1}{a^2 \rho_{DE}} \left[ Q\delta Q' + a^2 \frac{dV}{dQ} \delta Q - Q^2 \Psi \right] \]
\[ v_{DE} = \frac{k \delta Q}{Q}. \]
2.1.3. Imprint of Dark Energy on Observables

The two basic dark energy-dependent observables are distance and growth rate. Distance measures are based on standard candles, rulers, or number densities as a function of redshift; growth rate measures are based on density perturbations in linear theory. All distance measures are ultimately based on the comoving distance to redshift $z$:

$$ r = \int_0^z \frac{dz}{H(z)} = \int_a^1 \frac{da}{aH(a)}. $$

(17)

e.g., the SNe Type Ia observations measure the magnitude of distant SNe, given by $m_B(z) = 5\log_{10}[(1+z)r(z)] + \mathcal{M}$. The effect of dark energy for distance measures is through the background expansion of the universe, i.e., from the Hubble parameter $H(z) = \frac{\mathcal{H}(a)}{a}$. For CMB data, this comes in through the angular diameter distance and the sound horizon

$$ D_A(a) = a \int_1^a \frac{da}{a^2H(a)} = a \int_1^a \frac{da}{aH(a)}. $$

(18)

$$ s(a) = \int_0^a \frac{c_s(a) da}{a^2H(a)} = \int_0^a \frac{c_s(a) da}{aH(a)}. $$

(19)

The density perturbations are affected by the presence of dark energy first through the Hubble parameter, and second through the linear perturbation of dark energy, as in Equations (6)–(7), or Equations (9)–(11). For the CMB power spectrum, the effect of dark energy first through the Hubble parameter, and second through the linear perturbation of dark energy, as in Equations (6)–(7), or Equations (9)–(11). For the CMB power spectrum, the effect of dark energy is through the last scattering surface or the last scattering effect at low $l$, as well as in a shift of the peak positions.

The low $l$ observations can be understood as follows. The behavior of the temperature anisotropy power spectrum in the CMB is given by the covariance of the temperature fluctuation expanded in spherical harmonics:

$$ C_l = 4\pi \int \frac{dk}{k} \mathcal{P}_\nu |\Delta_\nu(k, \eta_0)|^2. $$

(20)

where $\mathcal{P}_\nu$ is the initial power spectrum, $\eta_0$ is the conformal time today, and $\Delta_\nu(k, \eta_0)$ is the transfer function at each $l$.

On large scales the transfer functions are of the form

$$ \Delta_\nu(k, \eta_0) = \Delta_{\nu^{SS}}^I(k) + \Delta_{\nu^{ISW}}^I(k), $$

(21)

where $\Delta_{\nu^{SS}}^I(k)$ are the contributions from the last scattering surface from the ordinary Sachs–Wolfe effect and temperature anisotropy, and $\Delta_{\nu^{ISW}}^I(k)$ is the contribution due to the change in the potential $\phi$ along the line of sight (LOS) and is called the ISW effect. The ISW contribution can be written as

$$ \Delta_{\nu^{ISW}}^I(k) = 2 \int d\eta e^{-\tau(\eta)} \phi' j_1[k(\eta - \eta_0)], $$

(22)

where $\tau(\eta)$ is the optical depth due to scattering of the photons along the LOS, and $j_1(x)$ are the spherical Bessel functions.

The frame-invariant potential $\phi$, defined in terms of the Weyl tensor, is equivalent to the Bardeen potential in the absence of anisotropic stress and given by the Poisson equation

$$ k^2 \phi = -4\pi G \mathcal{A}^2 \delta \rho, $$

(23)

while its derivative in a matter plus dark energy universe, which is the source term for the ISW contribution, is given by

$$ k^2 \phi' = -4\pi G \frac{\partial}{\partial \eta} [a^2(\delta \rho_m + \delta \rho_{DE})]. $$

(24)

From the above equations, it is clear that the magnitude of the ISW contribution is dependent on the late-time evolution of the total density perturbations, therefore on the dark energy perturbations. It should be noted however that these are not independent of other cosmological parameters, and the effect of dark energy could be masked due to the degeneracy of the dark energy parameters with other parameters such as $H_0$ and the curvature of the universe.

2.2. Parameterization of Equation of State of Dark Energy

To study EDE models under this formalism, we consider a $w$-parameterization which may represent a large class of varying dark energy models (Corasaniti et al. 2003):

$$ w(a) = w_0 + (w_m - w_0) \frac{1 + \frac{\mathcal{H}(a)}{H_0}}{1 + \frac{\mathcal{H}(a)}{H_0} - w_0}, $$

(25)

where $w_0$ is the equation of state of dark energy today, $w_m$ is the equation of state in the matter-dominated era, $a_t$ is the scale factor at which the transition between $w_0$ and $w_m$ takes place, and $\Delta_r$ is the width of the transition. If $w_m$ is allowed to be a free parameter this parameterization can encompass a large class of models, including $\Lambda$CDM and $w = \text{constant}$ models. Models with constant or slowly varying $w \simeq -1$ would be consistent with current observations, however these are not EDE models, as they have negligible amounts of dark energy at early times. For such models, there would be very poor constraints on the transition parameters, since no significant transition takes place between early-time and late-time dark energy. Allowing these models in the analysis would therefore cause the constraints on $a_t, \Delta_r$ to weaken. Leaving the amount of EDE free would be interesting when comparing EDE models with $\Lambda$CDM and other dark energy models. Such comparisons have previously shown that while it is possible to put an upper limit on the amount of EDE, it is not possible to put strong constraints on the evolution of dark energy if all the different dark energy models are considered. Previous studies (Doran & Robbers 2006; Xia & Viel 2009) have constrained early-time dark energy density to $\simeq 3\%$ of the matter density; however, as seen in Xia & Viel (2009), the evolution of dark energy is weakly constrained. In this work, we study the EDE models exclusively to put constraints on the transition from early to late-time dark energy. If we are able to constrain the minimum redshift (or maximum scale factor) at which such a transition occurs, we would know that any signature for EDE would be found only in observations beyond that redshift. This would also put a constraint on the evolution of dark energy at low redshifts. For studying EDE models with this parameterization, we therefore choose $w_m > -0.1$, to ensure the presence of adequate amounts of dark energy at early times, so that we may put constraints on the transition from early- to late-time dark energy for these models.

2.3. Observations

We use the latest version of COSMOMC (Lewis & Bridle 2002) for our analysis, modifying the CAMB module, as well as the various modules pertaining to large-scale structure and SN observations in COSMOMC, using the equations defined in Section 2.1. For the analysis using only CMB data, we use the 5 year Wilkinson Microwave Anisotropy Probe (WMAP; Komatsu et al. 2009), Cosmic Background Imager (CBI; Readhead et al. 2004), Very Small Array (VSA; Dickinson et al. 2004), BOOMERANG (Piacentini et al. 2006), and ACBAR
Figure 1. Growth of relative dark energy density $\rho_{DE}/\rho_m$ (panels (a) and (d)), dark energy density contrast $\delta_{DE}$ and velocity perturbation $v_{DE}$ (panels (b) and (e)), and cold dark matter density contrast $\delta_m$ (panels (c) and (f)) with the scale factor $a$ for two dark energy models: DE with $w_{DE} = -0.9$ and EDE with $w_0 = -1.0, w_m = -0.1, a_i = 0.3$, and $\Lambda_0 = 0.2$, respectively. The red line in panels (a) and (d) represents the DE and EDE models, respectively, the black line in panels (a), (c), (d), and (f) shows the $\Lambda$CDM model for comparison, the dark energy model without perturbation is shown in green in panels (c) and (f), while the case with perturbation is shown in red in these panels. The solid line in panels (b) and (e) represents the dark energy density contrast, while the dashed line shows the velocity perturbation. The filled circle in panels (a) and (d) represent matter–dark energy equality for the DE and EDE models, respectively, while the cross represents matter–dark energy equality for $\Lambda$CDM.

(Reichardt et al. 2009) data sets. In addition to the CMB data, we use other observations as well. For SNe, we use the Constitution data set (SALT; Hicken et al. 2009). This data set comprises 397 Type Ia SNe, of which about 200 are at redshifts $z \lesssim 0.1$, and the remaining are distributed between $z = 0.1$ and $z = 1.7$. We also use the latest Sloan Digital Sky Survey (SDSS) data release (DR7) luminous red galaxy (LRG) data (Reid et al. 2009), and the recent value of the Hubble constant from the SHOES (SNe and H0 for the Equation of State) program, $H_0 = 74.2 \pm 3.6$ km s$^{-1}$ Mpc$^{-1}$ (1σ; Riess et al. 2009), which updates the value obtained from the Hubble Key Project (Freedman et al. 2001). We incorporate a top-hat prior on the age of the universe, $10$ Gyr $< t_0 < 20$ Gyr. The addition of these other observations allows us to constrain parameters such as $H_0$ which might otherwise be degenerate with the dark energy parameters of interest to us.

3. RESULTS

We first study the effect of the different dark energy components on the observations. To this purpose we choose two dark energy models: (1) a dark energy model with constant equation of state $w_{DE} = -0.9$; and (2) an EDE model with $w_0 = -1.0, w_m = -0.1, a_i = 0.3$, and $\Lambda_0 = 0.2$. We compare the behavior of these two models of dark energy with that of a $\Lambda$CDM ($w_{DE} = -1$) model. All three models have identical parameters on the observations. To this purpose we choose two components on the observations. To this purpose we choose two

$\Omega_m, H_0$. The first model is chosen for comparing the behavior of non-perturbative and perturbative dark energy for a dark energy model close to the cosmological constant in behavior, while the second is chosen for specifically studying how EDE affects the results.

3.1. Effect of Dark Energy Perturbations on Observable Quantities

Following Weller & Lewis (2003), we look at the effect on observations using the dark energy as a fluid framework. We first study the effect of non-perturbative dark energy on the observations. In a universe containing matter and a smooth dark energy component, the matter perturbations may be calculated from Equations (9)–(11), using $\delta Q = \delta Q' = 0$, to be

$$\delta_m'' - \mathcal{H}\delta_m' - 4\pi G\rho_m\delta_m = 0 .$$

From this equation, we see that the dark energy component appears only in the second term which is effectively a damping term, therefore a non-negligible amount of smooth dark energy would suppress the clustering of matter at large scales. Thus, the only effect of dark energy for a smooth dark energy model arises through the dark energy density, for both geometric (e.g., Type Ia SNe) and perturbative (e.g., CMB, matter power spectrum) data. For a matter-dominated regime, the above equation would result in $\delta_m \propto a$. For the DE model with a constant $w_{DE} > -1$, the transition between matter and dark energy happens earlier than for $w_{DE} = -1$, and more slowly, thus constant $w_{DE} > -1$ models are expected to have a smaller contribution to the ISW effect than $\Lambda$CDM. In Figure 1(a), we show the expansion history of the DE model considered as well as that of $\Lambda$CDM. We see that the dark energy density equals matter density earlier in the dark energy model, and we expect this to have a noticeable
The effect in the scalar $C_i$’s for CMB data. For the EDE model, things are slightly different, as seen in Figure 1(d). Since the value of the equation of state today is $w_{DE} = -1$, the transition from matter to dark energy occurs at nearly the same time as on ΛCDM. Also, because initially the dark energy density is higher in this model, this transition is flatter. Therefore, we may expect that these models would have a larger contribution to the ISW effect. The effect on the matter perturbations is a mild suppression for both cases as expected from Equation (26), seen in Figures 1(c) and (f).

Setting the dark energy perturbations to zero artificially is however not consistent with the general relativity framework except in the case of a cosmological constant, $w_{DE} = -1$. We therefore now add the dark energy perturbations to the calculation. We consider the gauge comoving with dark matter, in which the acceleration is zero. If $\delta_{DE}$ is initially zero, we see from Equation (6) that it is sourced by the other perturbations if $w_{DE} \neq -1$ via the source term $3(1 + w_{DE})\Psi'$. An overdensity causes a decrease in the local expansion rate so that $\Psi' < 0$. In this case, a fluid starts to fall into overdensities if $w_{DE} > -1$. In the subsequent evolution of DE perturbations if $c_{DE}^2 = 1$, then the source term for the velocity $v_{DE}$ is positive, thus causing the velocities to be anti-damped. For the density contrast $\delta_{DE}$, when $k \ll H$, the term $(1 + w_{DE})k v_{DE}$ can be neglected and the velocity and wavenumber enter only via the combination $(1 + w_{DE}) v_{DE}/k$, which is small. Thus, the evolution of $\delta_{DE}$ is almost $k$-independent at large scales, and the two remaining source terms $-3H(c_{DE}^2 - w_{DE})v_{DE}$ and $-3(1 + w_{DE})\Psi'$ are of opposite signs with $\delta_{DE} > 0$ and $\Psi' < 0$ initially. Therefore, dark energy perturbations change sign at very early times and start decreasing, having the opposite sign to that of the matter perturbations, which is cosmic variance limited, therefore difficult to rule out observationally. For the matter power spectrum, as expected, there is a small suppression of power at high $k$ (since the normalization is done at low $k$). The value of $\sigma_8$ in the non-perturbative case is $\sigma_8 = 0.79$, while that in the perturbed case is $\sigma_8 = 0.80$, and that for ΛCDM is $\sigma_8 = 0.82$. Neither the effect on CMB nor that on the matter power spectrum is in itself good enough to rule out the DE model, even for the case where DE perturbations have been accounted for. For the EDE model, as seen in Figures 2(c) and (d), the non-perturbative case shows effect mostly in the low $l$ regime through the ISW effect, which is cosmic variance limited. The results for the matter power spectrum today also show a very slight difference from the cosmological constant. These results appear to suggest that just the non-perturbative effects of dark energy are not sufficient to discriminate this EDE model from ΛCDM, especially if we factor in degeneracies with other cosmological parameters, such as $H_0$. When we consider the perturbative case, the ISW effect is actually muted, however, there is a slightly larger shift in the CMB peak position, (see the inset of Figure 1(c)) which is a tightly constrained observable. The matter power spectrum at present shows a stronger suppression at small scales which leads to a much smaller value of $\sigma_8 = 0.69$ (as compared to the non-perturbative case, where $\sigma_8 = 0.81$, which is close to the ΛCDM value). Thus, although the background expansion of this model is very similar to ΛCDM at late times, its early-time behavior leaves signatures for discriminating it from the ΛCDM model provided the dark energy perturbations are accounted for strong enhancement off the transfer function and therefore the matter power spectrum at large scales (low $k$). Thus the matter power spectrum at late times, when normalized at low $k$, would show a strong suppression on the small scales (i.e., at high $k$), and this suppression is effected due to the variation of the DE equation of state.

The effect of dark energy perturbations can be understood also from the scalar field formalism. From Equation (9), the scalar field $Q$ can be viewed as a fluid with comoving Jeans mode given by the curvature of the potential, i.e., the mass of the field, $k_j = a\sqrt{2V/\dot{Q}^2}$. Therefore, scales which correspond to modes $k < k_J$ will collapse under gravitational instability, while modes $k > k_J$ will undergo a series of damped oscillations due to pressure waves in the quintessence fluid. This has two major effects. First, the large-scale clustering of dark energy enhances the amplitude of the ISW effect in CMB at low $l$. Second, as a consequence of the homogeneity of the dark energy component on small scales and the fact that the growth of the linear matter perturbations is suppressed due to the lower values of $\Omega_m$, the linear matter power spectrum at small scales will have an amplitude which is smaller than in ΛCDM. We thus expect that on the very large scales ($k < k_J$) the dark energy clustering enhances the matter power spectrum compared to the unclustered case, while on small scales ($k > k_J$) the opposite occurs. If we CMB normalize the matter power spectrum (i.e., normalize it at large scales), the small scale matter power spectrum will show a stronger suppression of power than in the no perturbation case, thus giving a smaller value of $\sigma_8$ at present.
Figure 2. Scalar $C_l$’s and matter power spectrum for the DE model with $w_{DE} = -0.9$ (panels (a) and (b)), and the EDE model with $w_0 = -1.0$, $w_m = -0.1$, $a_f = 0.3$, and $\Delta t = 0.2$ (panels (c) and (d)). The black line in each panel represents the corresponding $\Lambda$CDM model, the green line represents the dark energy model with no dark energy perturbations, while the red line represents the case with dark energy perturbations taken into account. The insets in the panels (a) and (c) show the shift in the position of the first peak for the dark energy model considered. Panels (e) and (f) show the difference between the perturbed and unperturbed cases for both the DE (solid line) and the EDE (dashed line) models for the two observables.

(A color version of this figure is available in the online journal.)

properly. The effect of adding the dark energy perturbations is seen in Figures 2(e) and (f) for both DE and EDE models. In obtaining the scalar $C_l$’s, for the DE model, there is a fairly large difference at low $l$, while at high $l$ the perturbed and non-perturbed models behave similarly. For the EDE model, there is a large difference at low $l$, and also a significant difference at the
higher $l$s. For the matter power spectrum today, the EDE model shows a larger difference in the perturbed and non-perturbed cases. Thus, a model close to $ΩCDM$ today as also in the past (as in the DE model chosen) would be difficult to discriminate from $ΩCDM$ from current observations, but a model with a different expansion history in the past, even if it is very similar to $ΩCDM$ today (such as the EDE model), could be discriminated using the perturbative observations such as CMB and the matter power spectrum provided the dark energy perturbations are not neglected. These results are commensurate with those found in Ma et al. (1999) where constant equation of state models of dark energy were considered, and those in Alimi et al. (2010), where quintessence models of dark energy were studied.

We note here that, since in addition to the ISW effect, dark energy also makes itself felt in a shift of the CMB first peak position, we expect that the dark energy parameters may be degenerate with $ΩCDM h^2$ if the flatness condition is removed in the analysis. We study the effect of curvature on the scalar $C_l$'s in Figure 3. A non-flat $ΩCDM$ model will differ from a flat $ΩCDM$ model with all other parameters identical mainly in a shift of the peak positions. Figure 3 shows this shift for a $ΩCDM$ model with $ΩCDM = 0.06$. An EDE model with $w_0 = -0.65$, $w_m = -0.1$, $α_t = 0.2$, $Δ_1 = 0.1$, and a curvature $Ω_k = 0.06$ is also shown. For the EDE model, the dark energy component compensates for the curvature of the universe, thus the peak position is the same as for the flat $ΩCDM$ model. However, as seen in the previous paragraphs, EDE manifests itself not only in the shift of the peaks, but also in the shape of the peaks and in the low-$l$ ISW effect. In this example, the height of the first peak is different for the EDE model, as is the low $l$ behavior, rendering it distinct from the flat $ΩCDM$ model. Thus, although we expect some degeneracy between the dark energy parameters and the curvature, this degeneracy is not very strong, since both the position and the height of the first peak are strongly constrained by current CMB data.

### 3.2. Constraints from Observations

We first study the results using only the CMB data. The primary parameters to be varied are the standard CMB parameters—$ΩCDM h^2$, $ΩDE h^2$, $θ$, $η$, $η_s$, $A_s$, and the equation of state parameters $w_0$, $w_m$, $α_t$, and $Δ_1$. Since we wish to study EDE models, we restrict the equation of state at early times to $w_m \leq 0$. This can give rise to a dark energy density of up to $ΩDE(z) \lesssim 0.03Ω_{CDM}(z)$ at early times. We assume a flat universe, i.e., $ΩCDM = 0$, and consider the full dark energy perturbations. The secondary parameters that we deduce from the analysis are $Ω_m h$, $H_0$, $ΩDE / Ω_{CDM}(z_{CMB})$, and $σ_8$. The first column of Table 1 shows the mean and 2σ boundaries for the primary and secondary parameters. We see that the EDE parameters are constrained at $w_0 < -0.61$, $α_t < 0.44$ (which means $z_t > 1.2$), and $Δ_1 < 0.37$. The constraints on the scale and width of transition are reasonable, however, the constraint on the equation of state today, $w_0$, is too broad. We note however, that this result is obtained by using CMB data alone; using other data would reduce degeneracies with the other parameters. For instance, SNe Ia data would affect the equation of state today more strongly. The Hubble parameter for which $v_0 \approx -0.6$ is allowed is $H_0 \approx 60$ km s$^{-1}$ Mpc$^{-1}$, much lower than the currently accepted measurement for it (Riess et al. 2009). Therefore, we expect that the addition of other observations to the analysis should improve the constraints on the EDE parameters significantly. It is interesting to note also that the 2σ lower bound on $σ_8$ for this analysis is as low as $σ_8 \approx 0.49$, whereas the $ΩCDM$ fit to the WMAP5 data has $σ_8 > 0.72$.

We now redo the analysis adding other data sets to see how the constraints improve. Three distinct cases are considered: (a) full dark energy perturbations are taken into account, $ΩDE = 0$; (b) dark energy perturbations are considered negligible, $ΩDE = 0$; and (c) full dark energy perturbations are considered, and the constraint on the flatness of the universe is lifted (i.e., $ΩCDM$ is a free parameter). The results are shown in the second, third, and fourth columns of Table 1. For the fully perturbed, flat case, when all...
Figure 4. Constraints from CMB (WMAP5, CBI, VSA, BOOMERANG, ACBAR) and other data sets (SNe Type Ia Constitution, LRG DR7, SHOES) on EDE parameters \( w_0, a_t, \Delta_t \), as well as \( \Omega_0m, H_0 \), showing marginalized one-dimensional distributions and two-dimensional 68% and 95% limits. Full dark energy perturbations are taken into account and the curvature of the universe is fixed at \( \Omega_\kappa = 0 \).

The data are considered, the EDE parameters are constrained to \( w_0 < -0.89, a_t < 0.19 \) (i.e., \( z_t > 4.2 \)), and \( \Delta_t < 0.21 \). The addition of other data sets clearly enhances the constraints on the EDE model. This is because the other parameters which could be degenerate with the EDE parameters, such as the Hubble parameter, are well-constrained by other observations. We note that the constraint on the equation of state today, \( w_0 \), is stronger than that obtained using the background data alone (e.g., for Type Ia SNe, we find \( w_0 \lesssim -0.75 \) for constant equation of state, when systematics are included (Hicken et al. 2009)). In Figure 4, we show the two-dimensional 68% and 95% confidence levels, as well as the marginalized one-dimensional distributions for the EDE parameters of interest, \( w_0, a_t, \Delta_t \), and the matter density \( \Omega_0m \) and the Hubble parameter \( H_0 \), which are expected to be degenerate with the EDE parameters. We see that all three EDE parameters are now strongly constrained, and the non-EDE parameters are close to the values expected in the \( \Lambda \)CDM model. The evolution of the equation of state of dark energy with redshift is shown in Figure 5. We see that at low redshifts (\( z \gtrsim 2 \)), the 2\( \sigma \) confidence level for \( w(z) \) is quite close to \( \Lambda \)CDM. Thus current observations already constrain the evolution of the equation of state for EDE models to \( \Lambda \)CDM-like behavior at present and in the near past. Studying the background expansion data (which are usually below redshift of two) will therefore not be able to distinguish these EDE models from \( \Lambda \)CDM with any success even if there is adequate amounts of dark energy at early times. In order to distinguish these EDE models (currently accepted by the data) from \( \Lambda \)CDM, we need to look at the perturbative data. Thus, we may conclude that even if there is significant amount of dark energy in the universe at early times, this has to reduce to dark energy very close to \( \Lambda \)CDM at present times, that this transition cannot take place too late (around redshift of four) and that the transition needs to be sharp (\( \Delta_t \lesssim 0.2 \)). However, we should note that even with these constraints, \( \sigma_8 \) is still significantly different from the typical \( \Lambda \)CDM value, with the 2\( \sigma \) lower bound being at \( \sigma_8 \gtrsim 0.6 \). This is because, as discussed earlier in Section 3.1, EDE has a strong effect on the matter power spectrum, leading to a much lower \( \sigma_8 \) than that in the cosmological constant model. This means that studying data which utilizes the matter power spectrum (such as the halo mass functions) even at low redshifts may allow us to discriminate between EDE and \( \Lambda \)CDM models.

We next look at the case where dark energy perturbations are neglected, for the full data set. We find that, although the results are similar for many of the parameters, they can be rather different for the EDE parameters. As seen in the third column of Table 1, the EDE parameters are constrained to \( w_0 < -0.8, a_t < 0.33 \) (i.e., \( z_t > 2 \)), \( \Delta_t < 0.31 \), thus, neglecting
the perturbations for an EDE model would result in rather broader constraints on its parameters. The value of $\sigma_8$ allowed at $2\sigma$ is also much closer to the ΛCDM value, with $\sigma_8 > 0.72$. Neglecting the dark energy perturbations in a dynamic dark energy scenario may therefore produce results very different from the true results when full dark energy perturbations are considered.

If the flatness of the universe constraint is removed, taking dark energy perturbations into account, the EDE parameters are mildly degenerate with the curvature of the universe $\Omega_k$. As seen in the last column of Table 1, the EDE parameters in this case are constrained to $w_0 < -0.77$, $a_r < 0.35$ (i.e., $z_r \gtrsim 2$), $\Delta_0 < 0.35$, while the curvature of the universe is still rather tightly constrained to $-0.014 < \Omega_k < 0.031$. Thus, relaxing the flatness constraint leads to a weakening of the constraints on the parameters of the EDE models, but can still lead to reasonable constraints on the EDE parameters.

Previous works that have studied EDE with perturbations have constrained the amount of EDE using current observations, e.g., Xia & Viel (2009) obtained $\Delta_{\text{DE}} < 1.4 \times 10^{-3}$. However, as explained in Section 2.2, this study allowed for models of dark energy that have negligible amounts of dark energy at early times. This led to a weakening of the constraints on the EDE transition parameters, with the parameter $w_m$ attaining peaks both at $\approx -1$ and $\approx 0$, and the equation of state today being close to ΛCDM. Thus, no strong constraint could be put on the evolution of the equation of state. In this work we have attempted to address the question of how to put constraints on the transition of early-time to late-time dark energy if the universe contains a certain amount of early dark energy. If we constrain EDE to $w_m \gtrsim -0.1$, we exclude models which do not have EDE behavior, and thus are able to put reasonable constraints on the transition parameters, which give us an insight into the evolution of dark energy for these models. It is difficult to rule out the presence of EDE altogether, due to the dearth of data at very high redshifts, but with this study we are able to put constraints on when the universe could have transitioned from such early-time dark energy to late-time, ΛCDM-like behavior. We find that late-time behavior of the equation of state of these models must be close to ΛCDM below redshift of few. However, since the $\sigma_8$ of these models is rather different from ΛCDM, they may be distinguished from ΛCDM using data such as the halo mass function, or the Sunyaev–Zeldovich power spectrum, even at the lower redshifts. In addition, this work also studies the degeneracy between the curvature of the universe and the dark energy parameters.

4. CONCLUSIONS

In this work, we have studied EDE models using current observations. We find that, if a sizeable amount of dark energy exists in early times ($\Omega_{\text{DE}}(z_{\text{CMB}}) \simeq 0.03\Omega_m(z_{\text{CMB}})$), we may put tight constraints on the transition of this dark energy to its present day value, and that the present day value of the dark energy equation of state must be close to the ΛCDM value. If the dark energy perturbations are correctly accounted for then the current dark energy equation of state is constrained to $w_0 < -0.89$, while the transition from EDE must occur at redshifts of $z_r > 4.2$ with a narrow transition width of $\Delta_r < 0.21$. Incorrectly assuming that dark energy perturbations are negligible leads to a different result— $w_0 < -0.8$, $z_r > 2$, and $\Delta_r > 0.31$—thus showing that it is vital to include the dark energy perturbations self-consistently in any analysis that uses perturbative data such as CMB or the matter power spectrum. Leaving $\Omega_k$ to be a free parameters leads to a weakening of the constraints on the dark energy parameters, with $w_0 < -0.77$, $a_r < 0.35$ (i.e., $z_r \gtrsim 2$), $\Delta_r < 0.35$ for $-0.014 < \Omega_k < 0.031$. We note that, for the flat universe in which dark energy perturbations are considered, the value of $\sigma_8$ is much lower than that in corresponding ΛCDM models. As will be shown in a companion paper (Alam et al. 2010), this may lead to interesting constraints from future large-scale structure data such as halo mass functions, as also from the Sunyaev–Zeldovich power spectrum.

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Figure 5. 2σ confidence levels from CMB (WMAP5, CBI, VSA, BOOMERANG, and ACBAR) and other data sets (SNe Type Ia Constitution, LRG DR7, SHOES) on the equation of state of dark energy, for the EDE models considered. Full dark energy perturbations are taken into account and the curvature of the universe is fixed at $\Omega_k = 0$. 

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