Analysis of topological structure of the QCD vacuum with overlap-Dirac operator eigenmode

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Coll. S. Hashimoto and G. Cossu
1. Introduction

2. Topological structure of QCD vacuum from overlap-Dirac eigenmodes

3. Flux-tube Formation by Low-lying Dirac Eigenmodes

4. Chiral Condensate in Flux-tube

5. Summary
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Vacuum Structure of QCD

QCD vacuum is filled with interesting objects.

instanton/anti-instanton

⇒ chiral symmetry breaking

⇒ quark confinement

Figs (left) JLQCD Coll. ’12 (right) www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/Nobel/
Overlap-Dirac Operator

\[ D_{ov} = m_0 \left[ 1 + \gamma_5 \text{sgn} \, H_W(-m_0) \right] \]

\( H_W(-m_0) \): hermitian Wilson-Dirac operator with a mass \(-m_0\) (Neuberger ’98)

overlap-Dirac eigenmode is an ideal probe to study QCD vacuum

- **exact chiral symmetry on the lattice**
- **Banks-Casher relation**
  — chiral symmetry breaking
  \[ \langle \bar{q}q \rangle = -\pi \rho(0) \]
- **index theorem** — topology of QCD
  \[ \frac{1}{2} \text{tr} \left[ \gamma_5 D_{ov} \right] = n_L - n_R \]

Dirac spectral density \( \rho(\lambda) \)

JLQCD Coll. ’10
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Dirac Eigenmode Decomposition of Field Strength

Dirac eigenmodes $\psi_\lambda(x) \implies$ Field strength tensor $F_{\mu\nu}$

Gattringer ’02

$$[\slashed{D}(x)]^2 = \sum_{\mu} D_{\mu}^2(x) + \sum_{\mu < \nu} \gamma_{\mu} \gamma_{\nu} F_{\mu\nu}(x)$$

$$\therefore F_{\mu\nu}(x) = -\frac{1}{4} \text{tr} \left[ \gamma_{\mu} \gamma_{\nu} \slashed{D}^2(x) \right] \propto \sum_\lambda \lambda^2 f_{\mu\nu}(x) \lambda$$

with Dirac-mode components $f_{\mu\nu}(x) \lambda$

$$f_{\mu\nu}(x) \lambda \equiv \psi_\lambda^\dagger(x) \gamma_{\mu} \gamma_{\nu} \psi_\lambda(x)$$

references Gattringer ’02, Ilgenfritz et al. ’07, ’08.

In this talk, we use dynamical overlap-fermion configurations by JLQCD Coll.

$\beta = 2.3$ (Iwasaki action), $a \sim 0.11$ fm, Volume $16^3 \times 48$,

overlap-Dirac operator $N_f = 2 + 1$, $m_{ud} = 0.015$, $m_s = 0.080$
Duality of Field Strength Tensor by Overlap-eigenmode

lowest eigenmode components of $f_{\mu\nu}$

\[ f_{12}(x)_0 \]

$f_{12}(x)_0$ around (8, 15, 7, 27)

negative peak

at this point, an anti-self-dual lump exists

\[ f_{12}(x)_0 \simeq -f_{34}(x)_0 \]

\[ f_{34}(x)_0 \]

$f_{34}(x)_0$ around (8, 15, 7, 27)

positive peak
**Action and Topological Charge from Dirac Eigenmodes**

**action density**

\[ \rho^{(N)}(x) = \sum_{i,j} \frac{\lambda_i^2 \lambda_j^2}{2} f_{\mu\nu}^a(x)_i f_{\mu\nu}^a(x)_j \]

**topological charge density**

\[ q^{(N)}(x) = \sum_{i,j} \frac{\lambda_i^2 \lambda_j^2}{2} f_{\mu\nu}^a(x)_i \tilde{f}_{\mu\nu}^a(x)_j \]

**Figure:** action and topological charge densities of lowest eigenstate

- Action Density #0 around (8, 15, 7, 27)
- Topological Charge Density #0 around (8, 15, 7, 27)

there is an “anti-instanton” at this point
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Flux-tube Formation Between Quarks

action density is “expelled” between quarks
⇒ formation of flux-tube ⇒ confinement potential

we discuss flux-tube structure from low-lying eigenmodes

flux-tube between quark and antiquark

Y-type flux-tube for 3Q-system

Bali-Schilling-Schlichter ’95

Ichie et al. ’03
Measurement of Flux-tube Structure

difference of action density $\rho(x)$ with or without Wilson loop $W(R, T)$

$$\langle \rho(x) \rangle_W \equiv \frac{\langle \rho(x)W(R, T) \rangle}{\langle W(R, T) \rangle} - \langle \rho \rangle < 0$$

N.B. action density is lowered in the flux-tube
Flux-tube Formation by Low-lying Eigenmodes

\( \rho^{(N)}(x) \): low-mode truncated action density

\[
\langle \rho(x) \rangle^{(N)}_W \equiv \frac{\langle \rho^{(N)}(x)W(R, T) \rangle}{\langle W(R, T) \rangle} - \langle \rho \rangle^{(N)}
\]

inter-quark separation \( R = 10 \)

- low-modes really contribute to flux-tube
  \[ \frac{100}{2,359,296} \sim 0.00004 \]
- \( \lambda_{\text{max}} \sim 300 \text{ MeV} \)

\((-\rangle \langle \rho(x) \rangle^{(N)}_W \quad N = 100 \text{ eigenmodes}\)
Flux-tube Formation by Low-lying Eigenmodes

\( \rho^{(N)}(x) \): low-mode truncated action density

\[
\langle \rho(x) \rangle^{(N)}_W \equiv \frac{\langle \rho^{(N)}(x)W(R, T) \rangle}{\langle W(R, T) \rangle} - \langle \rho \rangle^{(N)}
\]

inter-quark separation \( R = 9 \)

- low-modes really contribute to flux-tube
  \[
  \frac{100}{2,359,296} \sim 0.00004
  \]
- \( \lambda_{\text{max}} \sim 300 \) MeV

\[ \langle - \rangle \langle \rho(x) \rangle^{(N)}_W \quad N = 100 \text{ eigenmodes} \]
Flux-tube Formation by Low-lying Eigenmodes

\( \rho^{(N)}(x) \): low-mode truncated action density

\[
\langle \rho(x) \rangle^{(N)}_W \equiv \frac{\langle \rho^{(N)}(x)W(R, T) \rangle}{\langle W(R, T) \rangle} - \langle \rho \rangle^{(N)}
\]

inter-quark separation \( R = 8 \)

- low-modes \textit{really} contribute to flux-tube
  \[
  \frac{100}{2,359,296} \sim 0.00004
  \]
- \( \lambda_{\text{max}} \sim 300 \) MeV

\((-) \langle \rho(x) \rangle^{(N)}_W \) \( N = 100 \) eigenmodes
Flux-tube Formation by Low-lying Eigenmodes

$\rho^{(N)}(x)$: low-mode truncated action density

$$\langle \rho(x) \rangle^{(N)}_W \equiv \frac{\langle \rho^{(N)}(x)W(R,T) \rangle}{\langle W(R,T) \rangle} - \langle \rho \rangle^{(N)}$$

inter-quark separation $R = 7$

- low-modes really contribute to flux-tube
  \[
  \frac{100}{2,359,296} \approx 0.00004
  \]
- $\lambda_{\text{max}} \sim 300$ MeV

$(-) \langle \rho(x) \rangle^{(N)}_W \quad N = 100$ eigenmodes
Flux-tube Formation by Low-lying Eigenmodes

$\rho^{(N)}(x)$: low-mode truncated action density

$$\langle \rho(x) \rangle^{(N)}_W \equiv \frac{\langle \rho^{(N)}(x) W(R, T) \rangle}{\langle W(R, T) \rangle} - \langle \rho \rangle^{(N)}$$

inter-quark separation $R = 6$

- low-modes really contribute to flux-tube
  $$\frac{100}{2,359,296} \sim 0.00004$$
- $\lambda_{\text{max}} \sim 300$ MeV

(-) $\langle \rho(x) \rangle^{(N)}_W \quad N = 100$ eigenmodes
Flux-tube Formation by Low-lying Eigenmodes

\( \rho^{(N)}(x) \): low-mode truncated action density

\[
\langle \rho(x) \rangle^{(N)}_W \equiv \frac{\langle \rho^{(N)}(x) W(R, T) \rangle}{\langle W(R, T) \rangle} - \langle \rho \rangle^{(N)}
\]

Inter-quark separation \( R = 5 \)

- Low-modes really contribute to flux-tube
  \[
  \frac{100}{2,359,296} \approx 0.00004
  \]
- \( \lambda_{\text{max}} \approx 300 \text{ MeV} \)

\[ (-) \langle \rho(x) \rangle^{(N)}_W \quad N = 100 \text{ eigenmodes} \]
Cross Section of Flux-tube

inter-quark separation $R = 8$

inter-quark separation $R = 8$

$\sigma = 2.42 \pm 0.08$

$R = 8$

- $\sigma = 2.42 \pm 0.08$

- thickness of flux $\sim 0.6 \text{ fm}$

- gaussian form $\rho(y) \propto e^{-y^2/\sigma^2}$
About Dirac Eigenmodes and Confinement

Confinement remains without low or intermediate or higher eigenmodes

Refs. Gongyo-TI-Suganuma ’12, TI-Suganuma ’13

⇒ “seeds” of confinement seem to be widely distributed in eigenmodes

removing Dirac eigenmodes

interquark potential

Figs. Gongyo-TI-Suganuma ’12

Flux-tube formation by low-lying modes

⇒ direct evidence of the seed of confinement in the low-lying modes

≠ low-lying modes are essential for confinement
About Dirac Eigenmodes and Confinement

Confinement remains without *low* or *intermediate* or *higher* eigenmodes

Ref. Gongyo-TI-Suganuma ’12, TI-Suganuma ’13

“seeds” of confinement seem to be widely distributed in eigenmodes

Removing Dirac eigenmodes

Interquark potential

Flux-tube formation by *low-lying* modes

Direct evidence of the seed of confinement in the low-lying modes

Low-lying modes are essential for confinement
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Confinement remains without \textit{low} or \textit{intermediate} or \textit{higher} eigenmodes

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\( \Rightarrow \) “seeds” of confinement seem to be widely distributed in eigenmodes

removing Dirac eigenmodes

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Flux-tube formation by low-lying modes}
\end{figure}

\( \Rightarrow \) \textit{direct evidence} of the seed of confinement in the low-lying modes

\( \not \Rightarrow \) low-lying modes are essential for confinement

Figs. Gongyo-TI-Suganuma ’12
About Dirac Eigenmodes and Confinement

Confinement remains without low or intermediate or higher eigenmodes

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“seeds” of confinement seem to be widely distributed in eigenmodes

Removing Dirac eigenmodes

Interquark potential

Flux-tube formation by low-lying modes

Direct evidence of the seed of confinement in the low-lying modes

Low-lying modes are essential for confinement
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⇒ “seeds” of confinement seem to be widely distributed in eigenmodes.

Removing Dirac eigenmodes

\[
\rho(\lambda) V[a] = \begin{cases} 
1400 & \text{for } \lambda [a^{-1}] \geq 2.5 \\
1200 & \text{for } \lambda [a^{-1}] \geq 2.0 \\
1000 & \text{for } \lambda [a^{-1}] \geq 1.5 \\
800 & \text{for } \lambda [a^{-1}] \geq 1.0 \\
600 & \text{for } \lambda [a^{-1}] \geq 0.5 \\
400 & \text{for } \lambda [a^{-1}] \geq 0.0 \\
200 & \text{for } \lambda [a^{-1}] < 0.0 \\
0 & \text{for } \lambda [a^{-1}] < 0.0 
\end{cases}
\]

Interquark potential

\[
V_{Q\bar{Q}}(R[a^{-1}]) = \begin{cases} 
1.5 & \text{for } R[a] \geq 3.0 \\
1.0 & \text{for } R[a] \geq 2.0 \\
0.5 & \text{for } R[a] \geq 1.0 \\
0 & \text{for } R[a] < 1.0 
\end{cases}
\]

Flux-tube formation by low-lying modes.

⇒ Direct evidence of the seed of confinement in the low-lying modes.

⇒ Low-lying modes are essential for confinement.
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Dirac Eigenmodes and Chiral Condensate

chiral condensate $\langle \bar{q} q \rangle$

$$\langle \bar{q} q \rangle = - \text{Tr} \frac{1}{\slashed{D} + m_q} = - \frac{1}{V} \sum_{\lambda} \frac{1}{\lambda + m_q}$$

cf. Banks-Casher relation $\langle \bar{q} q \rangle = -\pi \rho(0)$

"local chiral condensate" $\bar{q} q(x)$ is given by

$$\bar{q} q(x) = - \sum_{\lambda} \frac{\psi_\lambda^\dagger(x) \psi_\lambda(x)}{\lambda + m_q}$$

with Dirac eigenmodes $\psi_\lambda(x)$

sample of a local condensate
Chiral Condensate in Flux-tube

difference of chiral condensate $\bar{q}q(x)$ with or without Wilson loop $W(R,T)$

$$\langle \bar{q}q(x) \rangle_W \equiv \frac{\langle \bar{q}q(x)W(R,T) \rangle}{\langle W(R,T) \rangle} - \langle \bar{q}q \rangle$$
Reduction of Chiral Condensate in Flux

difference of “truncated local chiral condensate”

$$\langle \bar{q}q(x) \rangle^{(N)}_W \equiv \frac{\langle \bar{q}q^{(N)}(x)W(R,T) \rangle}{\langle W(R,T) \rangle} - \langle \bar{q}q \rangle^{(N)}$$

- $\langle \bar{q}q(x) \rangle^{(N)}_W > 0$
- partially restored
- $|\langle \bar{q}q \rangle|_{\text{in flux}} < |\langle \bar{q}q \rangle|$
- “Bag-model” like

$m_q = 0.015$ using $N = 100$ eigenmodes

cf. chiral condensate in color-electro/magnetic fields (Suganuma-Tatsumi ’93)
Reduction of Chiral Condensate in Flux

difference of “truncated local chiral condensate”

\[
\langle \bar{q}q(x) \rangle^{(N)}_W = \frac{\langle \bar{q}q^{(N)}(x)W(R,T) \rangle}{\langle W(R,T) \rangle} - \langle \bar{q}q \rangle^{(N)}
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\( m_q = 0.015 \) using \( N = 100 \) eigenmodes

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difference of “truncated local chiral condensate”

\[ \langle \bar{q}q(x) \rangle_{W}^{(N)} = \frac{\langle \bar{q}q^{(N)}(x)W(R,T) \rangle}{\langle W(R,T) \rangle} - \langle \bar{q}q \rangle^{(N)} \]

- \[ \langle \bar{q}q(x) \rangle_{W}^{(N)} > 0 \]
- partially restored
- \[ |\langle \bar{q}q \rangle|_{\text{in flux}} < |\langle \bar{q}q \rangle| \]
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\]

\[ R = 6 \]

- \[ \langle \bar{q}q(x) \rangle_W^{(N)} > 0 \]
- partially restored
- \[ |\langle \bar{q}q \rangle|_{\text{in flux}} < |\langle \bar{q}q \rangle| \]
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\langle \bar{q}q(x) \rangle_{W}^{(N)} \equiv \frac{\langle \bar{q}q^{(N)}(x)W(R, T) \rangle}{\langle W(R, T) \rangle} - \langle \bar{q}q \rangle^{(N)}
\]

- \(\langle \bar{q}q(x) \rangle_{W}^{(N)} > 0\)
- partially restored
  \(|\langle \bar{q}q \rangle|_{\text{in flux}} < |\langle \bar{q}q \rangle|\)
- “Bag-model” like

\(m_q = 0.015\) using \(N = 100\) eigenmodes

cf. chiral condensate in color-electro/magnetic fields (Suganuma-Tatsumi ’93)
Reduction Ratio of Chiral Condensate in Flux-tube

\[ r(x) \equiv \frac{\langle \bar{q}q^{(\text{subt})}(x)W(R,T)\rangle}{\langle \bar{q}q^{(\text{subt})}\rangle\langle W(R,T)\rangle} < 1 \]

at center of flux
- reduction of |\(\langle \bar{q}q \rangle\)|
  \(\Rightarrow\) about 20 %

cf. subtracted condensate \(\langle \bar{q}q \rangle^{(N)} = \langle \bar{q}q^{(\text{subt})}\rangle + c_1^{(N)} m_q/a^2 + c_2^{(N)} m_q^3\)
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Summary

we discuss vacuum structure of QCD using overlap-Dirac eigenmodes

- low-lying overlap-Dirac eigenmodes
  ➞ show “instanton”-like behavior
    ▶ (anti-)self-dual field strength \( f_{\mu\nu} \simeq (-)\tilde{f}_{\mu\nu} \)
    ▶ (anti-)self-dual lump of action density / topological charge density

- flux-tube formation by low-lying Dirac eigenmodes
  ➞ low-lying eigenmodes contribute to the formation of flux-tube
    i.e., confinement

- chiral condensation in the flux-tube
  ➞ chiral symmetry is partially restored in the flux-tube
    reduction of \( |\langle \bar{q}q \rangle| \) is about 20% at the center of flux

outlooks

- chiral condensate in 3Q-system, quark-number densities in the
  flux-tube, light-quark content in quarkonia, topological susceptibility
  from eigenmodes, and so on
Appendix
Appendix
Lattice Setup

- gauge configurations
  - JLQCD dynamical overlap simulation $N_f = 2 + 1$
  - $16^3 \times 48$, $\beta = 2.3$, $m_{ud} = 0.015$, $m_s = 0.080$, $Q = 0$
  - $a^{-1} = 1.759(10)$ GeV ($m_\pi \sim 300$ MeV)
  - 100 low-lying Dirac eigenmodes ($\lambda_{UV} \sim 400$ MeV)
  - 50 configurations

- Wilson loop $W(R, T)$ measurement
  - APE smearing for spatial link-variable $U_i$ — 16 sweeps
  - $T = 4$ — ground-state component is dominant
  - measurement of action density/local chiral condensate at $T = 2$
Subtracted Chiral Condensate

\[
\langle \bar{q}q \rangle^{(N)} = \langle \bar{q}q^{(\text{subt})} \rangle + c_1^{(N)} \frac{m_q}{a^2} + c_2^{(N)} m_q^3
\]

Figure: (a) \(\langle \bar{q}q \rangle^{(N)}\) (b) \(\langle \bar{q}q \rangle^{(\text{subt})}\)

ref. J. Noaki et al., Phys. Rev. D81, 034502 (2010).