The cosmic Zevatron based on cyclotron auto-resonance: many-particle simulations

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Cyclotron auto-resonance acceleration has been recently advanced as a potential mechanism for accelerating nuclei to Zev energies (1 Zev = 10^{31} eV). All results have been based on single-particle calculations employing analytic solutions to the relativistic equations of motion in the combined magnetic and radiation fields. Here, many-particle simulation results are presented which lend support to the single-particle calculations. Each single-particle result is found to lie well within one standard deviation of the ensemble average of the corresponding many-particle simulation.

I. INTRODUCTION

Astrophysical environments which host, simultaneously, mega- and giga-tesla magnetic fields, beamed ultraintense radiation and pre-accelerated charged entities, are known to exist in the universe. Examples include a magnetar-powered supernova [1–3] and a merger-nova [4]. A more specific example is the binary neutron-start merger GW170817, source of the gravitational waves detected recently [5], which was followed by the emission of ultraintense radiation, with frequencies covering a substantial part of the known electromagnetic spectrum [6–9]. Beamed gamma-rays, in the form of a gamma ray burst (GRB), x-rays, and near-visible light were emitted in the afterglow. Among other things, these emissions are a clear indicator of stellar nucleosynthesis and the presence of atoms [10–14].

If the creation of matter by these processes is to be linked to earlier particle-antiparticle pair-production, due to the breakdown of the vacuum in the presence of electric and magnetic fields, then those fields must have been stronger than their Schwinger limits [15, 16]. For $e^+e^-$ pair-production the critical electric and magnetic field strengths are $E_c \approx 1.32 \times 10^{18} \text{ V/m}$ and $B_c \approx 4.41 \times 10^9 \text{ T}$, respectively, which correspond to the radiation intensity of $I_c \approx 2.3 \times 10^{33} \text{ W/m}^2$. When the mass of the electron $m_e$ in the expressions giving these critical values are replaced by the proton mass $m_p \sim 1936\ m_e$, the corresponding critical values for $p^+p^-$ pair-production are found to be $E_c \approx 4.95 \times 10^{24} \text{ V/m}$ and $B_c \approx 1.65 \times 10^{16} \text{ T}$, respectively, which give rise to the radiation intensity of $I_c \approx 3.2 \times 10^{46} \text{ W/m}^2$.

Radiation fields of the needed intensity in this work [17–23] may be associated with, for example, a gamma-ray burst (GRB). Consider a compact object or a binary neutron-star merger [24, 25] of peak isotropic luminosity $10^{48} \text{ W}$. Let half of the output energy [26, 27] of this event be radiant and beamed [28–30], as opposed to being emitted isotropically, through a circle of radius 100 m, centered on either polar cap of the object. Consequently the emitted radiation in this case can have an intensity around $I \sim 10^{43} \text{ W/m}^2$. Thus employing radiation field intensities, in this publication, as high as $10^{42} \text{ W/m}^2$, should come as no surprise.

Investigation of the emitted radiation can, in principle, be a source of valuable information about the merger and subsequent evolution of the newly formed entity. On the other hand, interaction of charged particles with the beamed radiation, especially in the added presence of superstrong magnetic fields associated with the merging entities, can drastically influence the subsequent kinetic energy evolution of such particles. The question thus arises as to whether atomic nuclei can be accelerated to Zev energies [31–36] and ejected as ultra-high-energy cosmic-rays (UHECR) as a result.

This work is part of efforts dedicated to answering this question [37]. Detection of such particles is quite rare. Only 72 events, with energies exceeding 57 EeV (1 EeV = $10^{18}$ eV) were detected by the Telescope Array experiment [38] between 2008 and 2013.

The mechanism of cyclotron auto-resonance acceleration (CARA) has recently been advanced [39, 40] as a possible explanation for the Zev energies of UHECR particles. Calculations have demonstrated Zev energy gains by the nuclei of hydrogen, helium and iron, due to interaction with ultraintense radiation and a superstrong uniform magnetic fields. The radiation-reaction effects were shown to be important in CARA, but not to lower the energy gain substantially from the ZeV level.

The investigations in [39] were general in nature and aimed at theoretical proof-of-principle demonstration of CARA in an astrophysical context [41–49]. They did not make specific reference to any known astrophysical environment where the resonance conditions (and ultraintense radiation and superstrong magnetic fields) may be found. These conditions may exist during the brief merger time of two compact objects, over the small areas that form the polar caps of the newly-formed object, during a magnetar-powered supernova explosion, among other possibilities [50–52]. Away from the polar caps, topology of the steady-state magnetic field of a compact object can be much more complex than uniform and its lines can be severely curved. On the other hand, the requisite radiation-field intensity for CARA to work deserves some discussion, too. This is offered at the very end of Sec. II below.

As such, CARA can be put forth as a mechanism for cosmic-ray acceleration, alternative to or complement-
ing the widely discussed models based on, for example, shock waves, magnetic reconnection and unipolar induction \[33\]. The existing models describe acceleration to energies close to the EeV level inside a potential cosmic-ray source, where a plasma background plays a central role. To reach the ZeV energy levels, it seems plausible to assume that a particle is first pre-accelerated inside the source by the shock wave mechanism, for example, and subsequently receives a big energy boost from CARA outside the source. This assumption will be made throughout this work.

The recent investigations employing CARA have also been single-particle \[39\]. In this work, many-particle simulations will be carried out to lend support to those single-particle calculations. To that end, the main working equations of CARA need to be amended slightly. The entities to be accelerated will be assumed to be initially picked randomly from an ensemble of \( N \) particles. The shape and size of the ensemble will be decided plausibly and \( N \) will be chosen so that the number of particles per unit volume will stay way below solid density \((\sim 10^{28} \text{ m}^{-3})\). This calls for a slight revision of the initial conditions adopted in our earlier work \[39\].

The aim of this work is two-fold: (a) to support the findings of the single-particle calculations in \[39\] with many-particle simulations, and (b) to strengthen the case for CARA by performing simulations which employ a more realistic set of astrophysical parameters than has been used in \[39\]. Included in the latter aim is also presenting, for the first time, results for acceleration by CARA of an ensemble of nickel nuclei.

In Sec. II, the CARA working equations will be revisited in order to incorporate the set of initial conditions appropriate for an ensemble of particles. Dynamics of the ensemble of particles will be investigated, based on the revised equations, in Sec. III, employing a parameter set (and for nuclei) the same as in \[39\]. In Sec. IV, similar simulations will be performed for: (a) nickel nuclei, not covered by the single-particle calculations in \[39\], (b) a more realistic parameter set, and (c) a smaller ensemble, to ensure that the particle-particle interactions may be considered negligible. A brief discussion of our results will be conducted, and some concluding remarks will be given, in Sec. V.

II. THE EQUATIONS

Figure 1 is a schematic diagram showing the initial ensemble of \( N \) identical particles, each of mass \( M \) and charge \( Q \), moving along the \( z \)-axis of a Cartesian coordinate system. Their positions are uniformly distributed inside a cylinder (or disk) of radius \( R \) and height \( H \). Their initial speeds are derived from a normal distribution of their injection kinetic energies, of mean \( K_0 \) and standard deviation \( \Delta K_0 \). This makes the number density \( n_u = N/(\pi R^2 H) \). The schematic diagram also shows a uniform magnetic field of strength \( B_s \), oriented along \( +z \), and a radiation wave propagating along the same direction. Only the size and shape of the initial ensemble, and the mean and spread of the initial kinetic energies, will be fixed. In all our calculations in this work, the initial ensemble size will be determined by the choices \( R = 5 \text{ m} \) and \( H = \lambda \), the wavelength of the radiation field employed.

For definiteness, the electromagnetic fields will be modeled by \[39\]

\[ E = iE_0 \sin \eta, \]
\[ B = jE_0/c \sin \eta + \hat{k}B_s. \]

In these equations, \( E_0 \) is the constant amplitude of the plane-wave radiation, \( c \) is the speed of light in vacuum, \( \eta = \omega t - kz \), \( k = \omega/c \), and \( i, j \) and \( \hat{k} \) are unit vectors in the positive \( x \), \( y \) and \( z \)-directions, respectively. Recall, at this point, the resonance condition that characterizes CARA. The condition ties the particle and radiation and magnetic field parameters by \( r = 1 \), where

\[ r = \frac{\omega_e}{\omega} \sqrt{\frac{1 + \beta_0}{1 - \beta_0}}; \quad \omega_e = \frac{QB_s}{M}, \]

in which \( \omega \) is the radiation frequency and \( \omega_e \) is the cyclotron frequency of the particle around the lines of the magnetic field. Resonance occurs when the cyclotron frequency matches the Doppler-shifted frequency of the radiation field, which the particle senses in its own rest frame. With \( Q, M, \) and \( \omega \) fixed, the resonance condition is essentially a relationship between \( B_s \) and \( \beta_0 \), the randomly-selected initial speed. This means that the dynamics of every single particle will get simulated in its own magnetic field, dictated by its velocity through the resonance condition. In other words, the particles get selected for acceleration based on the values of \( B_s \) and \( \beta_0 \) which satisfy the resonance condition. However, particles with initial transverse velocity components, and spreads thereof, will not be selected for acceleration by CARA.

We proceed now to amend the solutions to the equations of motion of a single particle in the presence of
the electromagnetic fields, by properly incorporating the above-mentioned initial conditions. The obtained equations will be used in the next section to carry out the promised many-particle simulations. On-resonance solutions to the relativistic Newton-Lorentz equations of motion follow essentially the same steps as in [39]. With the initial conditions on position expressed as \( \eta_0 = -kz_0 \), one finally obtains

\[
x(\eta) = x_0 + \frac{ca_0}{2\omega} \gamma_0(1 + \beta_0) \left[ (\sin \eta - \sin \eta_0) - (\eta \cos \eta - \eta_0 \cos \eta_0) \right],
\]

\[
y(\eta) = y_0 + \frac{ca_0}{2\omega} \gamma_0(1 + \beta_0) \left[ (\eta \sin \eta - \eta_0 \sin \eta_0) + 2(\cos \eta - \cos \eta_0) \right],
\]

\[
z(\eta) = z_0 + \frac{c}{\omega} \left( \frac{1 + \beta_0}{1 - \beta_0} \right) \left\{ \left( \frac{\beta_0}{1 + \beta_0} \right) (\eta - \eta_0) + \frac{a_0^2}{24} (\eta - \eta_0)^2 (\eta + 2\eta_0) \right. \\
+ \left. \frac{a_0^2}{16} \left[ (\eta - 2\eta_0) \cos 2\eta_0 + \eta \cos 2\eta + (2\eta_0^2 - 2\eta_0 - 1) \sin 2\eta_0 - \sin 2\eta \right] \right\},
\]

\[
\gamma(\eta) = \gamma_0 + \frac{a_0^2}{8} \gamma_0(1 + \beta_0) \left[ (\eta^2 - \eta_0^2) + (\sin^2 \eta - \sin^2 \eta_0) - (\eta \sin 2\eta - \eta_0 \sin 2\eta_0) \right].
\]

In these equations, \( \gamma_0 = (1 - \beta^2)^{-1/2} \) and \( a_0 \equiv QE_0/(Mc^2) \). Note that \( a_0^2 \) may be thought of as a dimensionless radiation-field intensity parameter, whereas the radiation field intensity in W/m^2 is \( I = ca_0E_0^2/2 \), where \( \epsilon_0 \) is the permittivity of free space. Equations (4)-(6) give a parametric representation of the particle’s trajectory. Equation (7) is the particle’s Lorentz factor (its energy scaled by \( Mc^2 \)).

![FIG. 2. Log-Log plot of the exit kinetic energy of the nuclide Fe^{26} with the radiation-field intensity. For acceleration by CARA using near-visible light, the wavelength is \( \lambda = 1 \) \( \mu \)m and the initial injection kinetic energy is \( K_0 = 150 \) MeV. For the GBR of wavelength \( \lambda = 5 \times 10^{-11} \) m, \( K_0 = 20 \) TeV. In both plots interaction is with 5 radiation-field phase-cycles. The horizontal dotted lines represent exit kinetic energies of 1 EeV and 1 ZeV, respectively.](image)

For the special case of initial position at the origin of coordinates, evolution of the kinetic energy of a particle with \( \eta \) may be written as [39]

\[
K(\eta) = K_0 + \left[ \frac{Q^2}{16\pi^2\epsilon_0 Mc^4} \right] \gamma_0(1 + \beta_0)(I\lambda^2) \\
\times \left\{ \eta^2 + \sin^2 \eta - \eta \sin 2\eta \right\},
\]

where \( K_0 = (\gamma_0 - 1)Mc^2 \) is the initial injection kinetic energy.

Figure 2 shows log-log plots of the exit kinetic energies against \( I \), the radiation-field intensity, at the end of interaction with 5 phase cycles of near-visible light and a GRB. The main assumption here is that the particle is pre-accelerated to kinetic energies of 150 MeV (near-visible) and 20 TeV (GRB). For these initial conditions, the resonance magnetic field strengths are 38.9316 MT and 1.09132 GT, respectively. Note that the flat parts of the \( K_e \) vs. \( I \) curves reflect those initial injection energies. The particle’s kinetic energy begins to increase substantially (due to interaction with the radiation field) after some threshold intensity has roughly been passed (~ \( 10^{25} \) W/m^2, for near-visible light, and ~ \( 10^{35} \) W/m^2, for the GRB). The energy range of 1 EeV to 1 ZeV is bounded by the two horizontal dotted lines in Fig. 2. The figure clearly shows that, for the chosen parameters, the minimum radiation-field intensities required to reach the EeV to ZeV kinetic energy levels are ~ \( 10^{35} \) W/m^2 (near-visible) and ~ \( 10^{42} \) W/m^2 (GRB). For other radiation-field wavelengths, different intensity levels would be needed, as will be demonstrated in Sec. IV below.

### III. THE SIMULATIONS

Viewed as functions of the radiation field phase, the equations will next be used to investigate some aspects of the dynamics of ensembles of particles in the combined
FIG. 3. Proton acceleration by CARA employing near-visible light. (a) Initial ensemble of $N = 100$ protons inside a disk of radius 5 m and thickness $\lambda = 1 \mu$m (number density $n_d \simeq 1.27324 \times 10^6$ m$^{-3}$). Initial ensemble kinetic energy: normal distribution of mean $K_i = 150$ MeV and standard deviation $\Delta K_0 = 1.5$ MeV. (b) Actual trajectories of the ensemble members during interaction with the radiation and magnetic fields. (c) Distribution of the ensemble particles at the end of an interaction time equivalent to 5 phase cycles ($\Delta \eta = 10\pi$) of the radiation field. (d) Kinetic energy evolution with the excursion distance for all of the particles in the ensemble. The radiation field intensity is $I = 10^{38}$ W/m$^2$ and the resonance magnetic field strength sensed by the ensemble members and calculated on the basis of Eq. (3) is $B_s = (11.2529 \pm 0.0354)$ MT.

radiation plus uniform magnetic fields. Without loss of generality, the examples will focus on the nuclei H$^{+1}$, He$^{+2}$ and Fe$^{+26}$, as in [39]. Cosmic rays are close to 90% protons, H$^{+1}$, the simplest atomic nucleus. Alpha particles, He$^{+2}$, account for about 10%, and the rest are heavier nuclei. Fe$^{+26}$ is one of the most stable nuclei in nature. Recent measurements by the Pierre Auger Observatory in Argentina [36] suggest that most UHECR particles are nuclei of elements heavier than the proton.

As shown schematically in Fig. 1, members of the initial ensemble are assumed to have already been pre-accelerated to relativistic velocities along the common directions of $B_s$ and $k$, the latter being the wavevector of the radiation field, by shock waves or any other means
For simplicity, it will be assumed that the wavefront of a radiation wave typically catches up with a particle at $t = 0$ when the latter is at the spatial coordinates $(x_0, y_0, z_0)$ and has a speed $\beta_0$, derived from the corresponding initial normal distribution of kinetic energies of mean $K_0$ and standard deviation $\Delta K_0$. In all cases considered, the interaction time will be equivalent to five radiation-field phase cycles, $\Delta \eta = \eta_e - \eta_0 = 10\pi$, with $e$ standing for exit.

As examples, we first investigate the dynamics of $N = 100$ protons accelerated by CARA, without reference to any astrophysical environment, known to a good degree
Numerical values will be displayed in tabular format, for along the same lines, albeit involving a much bigger en-

of certainty, where the conditions for acceleration may be met [54–62]. Figure 3 displays the results for acceleration using near-visible light of intensity $I = 10^{48}$ W/m$^2$ and wavelength $\lambda = 1 \mu$m. Speeds of the particles of the initial ensemble are derived from a normal distribution of kinetic energies ($K_0 = 150$ MeV, and $\Delta K_0 = 1.5$ MeV). Figure 3(a) shows the initial ensemble, a uniform distribution of initial positions ($x_0, y_0, z_0$). Interactions are assumed to commence at $t = 0$ (or, equivalently, at $\gamma_0 = -Kz_0$) causing the particles to follow the trajectories shown in Fig. 3(b). Figure 3(c) shows the positions through which the particles pass at the end of the interaction time. In other words, the initial ensemble in (a) evolves to the final spatial distribution of particles shown in (c) as a result of the acceleration process. Assuming that each particle’s own initial conditions launch it into cyclotron auto-resonance, not necessarily exactly, this will result in tremendous energy gain. In Fig. 3(d) the exit kinetic energy of each particle of the ensemble is shown as a function of its axial excursion along the $z$-direction. Exit (end-of-interaction) results for this example are displayed in the first row of Table I. Note, in particular, that the magnetic field strength shown in the last column is given as a mean ± some spread. This is due to the fact that once a value for $\beta_0$ has been picked at random, a value for $B_0$ will be dictated by the resonance condition, Eq. (3). Nevertheless, the spread in those values does not seem to disturb resonance appreciably and the particles end up attaining ZeV kinetic energies.

The second illustrative example also involves acceleration of $N = 100$ protons, albeit employing the fields of a GRB of intensity $I = 10^{43}$ W/m$^2$ and wavelength $\lambda = 5 \times 10^{-11}$ m. In this case, the initial ensemble kinetic energy (normal) distribution has mean $K_0 = 20$ TeV and spread $\Delta K_0 = 0.2$ TeV [25, 63]. Figure 4 displays results of simulations for this example similar to those of Fig. 3. Numerical values of the exit dynamical quantities pertaining to this example are displayed in the second row of Table I. The exit kinetic energies of the protons from interaction with the GRB are substantially larger than from interaction with the lower-intensity near-visible radiation, as expected. The resonance magnetic field in this case is lower than in the case of interaction with the lower-frequency near-visible light, as Eq. (3) predicts.

Further results from simulations performed essentially along the same lines, albeit involving a much bigger ensemble, two more nuclei, and exhibiting more exit numerical values, will be presented next. Only the exit numerical values will be displayed in tabular format, for the nuclei $\text{H}^{+1}$, $\text{He}^{+2}$, and $\text{Fe}^{+26}$. Table II shows simulation results for the acceleration of $N = 10^4$ particles by CARA employing near-visible light of intensity $I = 10^{48}$ W/cm$^2$ and wavelength $\lambda = 1 \mu$m. Fairly good estimates of the exit mean and spread of the kinetic energy and spatial coordinates, as well as the resonance magnetic field strength, may be read from the tabulated results. For example, from the last row for iron, one concludes that an ensemble of $N = 10^4$ nuclei uniformly distributed initially inside a cylinder of radius $R = 5$ m and height $H = \lambda = 1 \mu$m, evolves into roughly a cylinder bounded by a box of dimensions $2\Delta x_e \sim 5$ m, $2\Delta y_e \sim 5$ m, and $2\Delta z_e \sim 1.4$ km.

Table III is similar to Table II, but using the fields of a GRB of intensity $I = 10^{43}$ W/m$^2$ and wavelength $\lambda = 5 \times 10^{-11}$ m. The results presented in Tables II and III follow different patterns, as they correspond to two widely differing sets of parameters. The employed injection energies, radiation field intensities, and radiation wavelengths, differ by about 5 orders of magnitude in each category. This leads to different resonance magnetic field strengths. The exit kinetic energy decreases with increasing mass in Table III, as would be intuitively expected. However, the results shown in Table II follow the opposite trend, like in [39]. These opposing trends are, most probably, merely coincidental and, on account of the fact that they stem from two widely differing sets of parameters and initial conditions, cannot be compared or contrasted.

### IV. A MORE REALISTIC PARAMETER SPACE

It may be argued that the parameters employed in our calculations thus far have been unrealistic. The assumption has been made that a big portion of the energy output from the source, like a binary-star merger, is radiant and beamed [28–30] through a small circle, which leads to the GRB intensities exceeding $10^{42}$ W/m$^2$, for example, that have been employed in [39]. In the scientific literature of relevance, the assumption is often made that energy is radiated isotropically, not in a beam. The intensity calculated based on this assumption must, therefore, be many orders of magnitude smaller than $10^{42}$ W/m$^2$.

Another assumption made in our many-particle calculations has been that the particle-particle interactions are negligible. This has been justified by the fact that the number densities employed are very small compared to those in a typical solid, where such interactions can not be ignored.

Furthermore, the focus so far has been on examples already considered in [39]. The aim here has been to lend support to the single-particle results by performing simulations for the acceleration of non-interacting many particles. Not only do the non-interacting many-particle simulations agree, in general, with the single-particle calculations, but they do not seem to depend on the size of the initial ensemble employed ($N = 100$ and $N = 10^4$, in

| Intensity ($I$) | $\lambda$ | $K_e \pm \Delta K_e$ | $B_e \pm \Delta B_e$ |
|----------------|----------|----------------------|--------------------|
| $10^{38}$ W/m$^2$ | $10^{-6}$ m | $4.037 \pm 0.902$ (Zev) | $11.25 \pm 0.03$ (MT) |
| $10^{43}$ W/m$^2$ | $5 \times 10^{-11}$ m | $2.663 \pm 0.028$ (Zev) | $9.22 \pm 0.09$ (MT) |
they are almost straight, in light of the fact that they are
The trajectories appear to be semi-helical, but in reality
10 particles are shown, extending over about 1500 km.

The key departure from the old parameter set is employ-
ing ual distribution for the initial trans-
section).

The above examples, and \( N = 10 \) to be considered in this
section).

The initial conditions on position in the ensemble, adopted in the simulations, deserve further discussion. Employing a uniform distribution for the initial trans-
verse coordinates \( x_0 \) and \( y_0 \) did not play a significant
role. The end result has been to simulate the dynamics of each particle in its own uniform magnetic field. As is
well known, the magnetic field of a compact object cannot be uniform over a wide transverse area, and its lines
curve severely away from the polar caps [1–3]. Neither
is it even certain that the required ultrastrong magnetic fields can be uniform over many kilometers. Thus initial
transverse coordinates may suit a more general treatment that would simulate the magnetic field topology more re-
alistically than just being uniform, as has been assumed thus far [64]. A more economical course of action would have been to take \( x_0 = y_0 = 0 \) for all particles, as will
be done shortly. Clear departure from the results displayed in Figs. 3 and 4 will be in the subsequent particle
trajectories.

The above four points can be addressed together by employing a more realistic parameter set, and slightly
modifying the initial ensemble conditions on position. The key departure from the old parameter set is employ-
ing infrared radiation of wavelength 0.12 mm and intensity
\( I = 6 \times 10^{32} \) W/m\(^2\). An initial ensemble of only
10 nuclei, distributed along the \( z \)-axis between \( z = 0 \) and \( z = \lambda \), is employed, making the particle density
\( n_d \approx 8.33 \times 10^{27} \) m\(^{-1}\).

Simulations have been performed for the nickel nuclide
Ni\(^{28}\), one of the most stable nuclei in nature. All results from the simulations are displayed in Fig. 5, which is
similar to Figs. 3 and 4.

Figure 5(a) shows the initial ensemble of 10 particles distributed randomly along the line \( x_0 = y_0 = 0 \), of
length 0.12 mm. In Fig. 5(b) the trajectories of all
10 particles are shown, extending over about 1500 km.
The trajectories appear to be semi-helical, but in reality
they are almost straight, in light of the fact that they are
merely \( \sim 5 \) m in maximum transverse extension. As is
well known, very little energy is lost by radiation from an
accelerated particle which follows a quasi-linear traject-
ory [65]. At the end of the interaction with the electro-
magnetic fields, the initial linear distribution is shown in Fig. 5(c) to become 3D, roughly inside a box of di-

mensions 50 km \( \times \) 5 m \( \times \) 0.3 m. So, the accelerated
particles appear to be still beamed (they suffer very lit-
tle diffraction) even after traveling 1500 km from their
source. Despite the fact that detection on earth of UHE-
CRs is very rare, observation, by a single detector of some
that carry roughly the same ZeV-level kinetic energies
within a small time-interval, may confirm this prediction.

Finally, Fig. 5(d) shows that the kinetic energy of the
ensemble, at the end of interaction time equivalent to 5
radiation-field oscillations, is \( K_e \simeq (0.332 \pm 0.001) \) ZeV. Recall that the most energetic cosmic-ray particle to be
detected on earth had \( K_e \simeq 0.32 \) ZeV [66]. With the
resonance magnetic field strength sensed by the ensemble
members and calculated on the basis of Eq. (3) being
\( B_0 = (298784 \pm 194) \) T, the set of realistic parameters
employed in this section seem to lead to results that agree
with observations already made on earth [35, 36, 38].

V. CONCLUDING REMARKS

This work is part of an effort to lend support to conclu-
sions arrived at recently [39] regarding the acceleration to
ZeV energies, of protons and other bare atomic nuclei, by
cyclotron auto-resonance, in astrophysical environments
such as the merger of a binary neutron-star system or
a magnetar-powered supernova explosion. Those con-
clusions were based on single-particle calculations. The
current study has advanced the calculations to the level of
many particles, while keeping the overall number per
unit volume way below the number density of a typical
solid (\( \sim 10^{28} \) m\(^{-3}\)). The assumption here is that the pro-
cess takes place outside a compact object, or equivalent,
so that the particle-particle interactions may be ignored.
FIG. 5. Same as Figs. 3 and 4, but for Ni$^{+28}$ and employing the fields of infrared light of wavelength $\lambda = 0.12$ mm and intensity $I = 6 \times 10^{12}$ W/m$^2$. The initial ensemble has $N = 10$ nuclei all injected along the same line $x_0 = y_0 = 0$ and randomly distributed over the interval $z = 0$ and $z = \lambda$ (number density $n_0 \approx 8.33 \times 10^5$ m$^{-1}$). Initial ensemble kinetic energy: normal distribution of mean $\bar{K}_0 = 1$ GeV and standard deviation $\Delta K_0 = 10$ MeV. The resonance magnetic field strength sensed by the ensemble members and calculated on the basis of Eq. (3) is $B_s = (298784 \pm 194) T$.

Results of the many-particle simulations strongly agree with the single-particle calculations (with the former being more statistically significant). Let $X$ stand for a physical quantity pertaining to the particles accelerated by CARA. In [39] and in the present study $X \in \{K, x, y, z, B_s\}$. Denote by $X_e$ and $X'_e$ the exit values of $X$ obtained from the single-particle calculations and the many-particle simulations, respectively. In all cases considered, and for all quantities investigated, $X_e$ is found to lie within the range $X'_e \pm \Delta X'_e$, where $X'_e$ and $\Delta X'_e$ represent the mean and standard deviation of $X'_e$, respectively. Finally, inspection of the numbers (for the protons, in particular) displayed in Table I, on the one hand, and II and III (as well as the results of Sec. IV),...
on the other, reveals that the end results do not depend strongly on the size of the ensemble.

Assumptions regarding the simultaneous existence of ultraintense radiation fields and superstrong uniform magnetic fields aligned over many kilometers deserve further theoretical and observational justification. Taking the radiation field as plane-wave in nature led to the analytic solutions and to the resonance condition. The plane-wave assumption may not be accurate, but replacing the plane-wave description with a more realistic one, perhaps pulsed or tightly-focused, will make resonance approximate at best. Under these conditions, the plane-wave-based solution can still play a role in benchmarking the unavoidable numerical solutions to the equations of motion. Other possibilities for further progress in this field, and improvements to this model, may be accomplished by modeling the magnetic field more realistically. Lines of the super-strong magnetic field of a compact object curve very tightly and its strength can be a function of the time. These issues ought to be carefully addressed within a more realistic scenario. Studies employing Particle-In-Cell (PIC) simulations may be suitable for taking into account the interactions between the particles, the radiation and magnetic fields, and the particle-particle interaction effects, in a self-consistent way [67]. Finally, issues not thoroughly addressed by the current model regarding photo-disintegration and photo-production, among other things, must be addressed.

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