Renormalized spin coefficients in the accumulated orbital phase for unequal mass black hole binaries

László Á Gergely¹,², Peter L Biermann³,⁴,⁵,⁶,⁷, Balázs Mikóczi⁸ and Zoltán Keresztes¹,²

¹ Department of Theoretical Physics, University of Szeged, Tisza Lajos krt 84-86, Szeged 6720, Hungary
² Department of Experimental Physics, University of Szeged, Dóm tér 9, Szeged 6720, Hungary
³ Max Planck Institute for Radioastronomy, Bonn, Germany
⁴ Department of Physics and Astronomy, University of Bonn, Germany
⁵ Department of Physics and Astronomy, University of Alabama, Tuscaloosa, AL, USA
⁶ Department of Physics, University of Alabama at Huntsville, AL, USA
⁷ FZ Karlsruhe and Physics Department, University of Karlsruhe, Germany
⁸ KFKI Research Institute for Particle and Nuclear Physics, Budapest 114, PO Box 49, H-1525, Hungary

E-mail: gergely@physx.u-szeged.hu, plbiermann@mpifr-bonn.mpg.de, mikoczi@rmki.kfki.hu and zkeresztes@titan.physx.u-szeged.hu

Received 6 March 2009, in final form 12 July 2009
Published 6 October 2009
Online at stacks.iop.org/CQG/26/204006

Abstract
We analyze galactic black hole mergers and their emitted gravitational waves. Such mergers have typically unequal masses with a mass ratio of order 1/10. The emitted gravitational waves carry the imprint of spins and mass quadrupoles of the binary components. Among these contributions, we consider here the quasi-precessional evolution of the spins. A method of taking into account these third post-Newtonian (3 PN) effects by renormalizing (redefining) the 1.5 PN and 2 PN accurate spin contributions to the accumulated orbital phase is developed.

PACS numbers: 04.30.Db, 04.25.Nx

1. Introduction

Mankind has admired the sky with its Milky Way, stars, planets, moons and the Sun, the meteors and the Northern Lights. In the age of satellites, we map the sky through all electromagnetic frequencies and also through high-energy cosmic particles. Gravitational waves are the last great frontier to be reached and passed for the search for our physical understanding of the universe via the messengers reaching us with the speed of light or very close to it.

Gravitational waves, these fleeting distortions of space, the signature of accelerating masses in the universe, remain to be discovered. Therefore, it is of paramount importance to understand which objects are about to generate strong gravitational waves.
When the black holes residing in the centers of almost all galaxies merge, following the merger of their host galaxies, relatively strong gravitation waves are emitted and various aspects of this process were already investigated [1–6]. A recent review of the generic aspects of these galaxies nuclei as sources for ultra high-energy cosmic rays is in [7].

Starting from the broken powerlaw mass distribution of galactic central black holes [8–11], with simple assumptions on the cross section and relative velocity of the two galaxies, we have shown [6] that the most likely mass ratio \( \nu = m_2/m_1 \leq 1 \) of the merging black holes is between 1/3 and 1/30, thus a typical value will be at about 1/10. Within this range of mass ratios, the key stages of the evolution leading to and following the merger can all be treated with post-Newtonian (PN) approximations. A numerical analysis of such mass ratios was recently presented in [12].

In the following section, we describe the sequence of events which will accompany the merger of two supermassive galactic black holes. Then from section 3, we concentrate on the gravitational radiation-dominated dissipative regime. An important quantity in gravitational wave detection is the number of cycles spent in the frequency range of the detector. This quantity is proportional to the accumulated orbital phase (the total angle covered by the quasi-circular orbital evolution). The gravitational waves emerging from this process carry the imprint of the physical characteristics (finite size) of the black holes, like their spins and quadrupole moments. Spin precessions allow for increased accuracy and degeneracy breaking among the parameters characterizing the source [13].

We remark that according to [14], the mass quadrupole–mass monopole coupling for maximally rotating black holes (with quadrupole moment originating entirely in their rotation) gives rise to contributions in the accumulated orbital phase larger by a half to one order of magnitude than the spin–spin contribution, at least for the typical unequal mass ratios. Therefore for a successful detection this qualifies as the second most important finite mass contribution to be considered, after the spin–orbit effect.

In section 3, we discuss the effect of the quasi-precessional evolution of the spins on the accumulated phase of the gravitational waves, and we present estimates of the PN order in which various finite-size contributions to the change in the relative configurations of the spins and orbital angular momentum contribute. The cumbersome equations leading to these estimates are presented in the appendix.

Sections 4 and 5 explore a convenient analytical way to encompass the effect of spin and quadrupolar quasi-precessions even at a lower level accuracy. The leading-order contribution to the spin–spin (section 4) and spin–orbit (section 5) contributions to the accumulated orbital phase is discussed for both the equal mass and unequal mass cases in distinct subsections.

We summarize our findings in section 6.

2. The merger of two supermassive galactic black holes: an odyssey

First the dance. Two galaxies with central black holes approach each other to within a distance where dynamical friction keeps them bound, spiraling into each other. In the case when both galaxies with their central black holes were radio galaxies, their jets get distorted and form the Z-shape [15]: the radio emitting blobs and tails produce the appearance of dancing veils.

Second the meeting of the eyes. The central regions in each galaxy begin to act as one unit, in a sea of stars and dark matter of the other galaxy. During this phase, the central region can be stirred up and produce a nuclear starburst.

Third the lock. The black holes begin to lose orbital angular momentum due to the interaction with the nearby stars [16], and next by gravitational radiation. The spin axes tumble and
precess. This phase can be identified with the apparent superdisk, as the rapidly precessing jet produces the hydrodynamic equivalent of a powerful wind, by entraining the ambient hot gas, pushing the two radio lobes apart and so giving rise to a broad separation [17]. Due to the combined effect of precessions and orbital angular momentum dissipation by gravitational waves, the spin (and hence jet) direction of the dominant black hole is reoriented approximately in the direction of the original orbital angular momentum, leading to a spin-flip [6].

Fourth the plunge. The two black holes actually merge, with not much angular orbital momentum left to be radiated away, thus the direction of the spin remained basically unchanged (in the case of the typical mass ratio range we discuss; for other set-ups the final spin may vary [18–23]). The final stage in this merger leads to a rapid increase in the frequency of the gravitational waves, called ‘chirping’, but this chirping will depend on the angles involved.

Fifth the rejuvenation. Now the newly oriented more massive merged black hole starts its accretion disk and jet anew, boring a new hole for the jets through its environment (the GigaHertz peaked sources). In this stage, the newly active jet is boring through the interstellar clouds with a gigantic system of shockwaves that accelerate protons and other charged particles in a relativistic tennis game. These very energetic particles then interact in those same clouds that slow down the progression of the relativistic jet, and can so give rise to an abundance of neutrinos, TeV photons from pion decay, and yet other particles and photons. These particles and their interactions reach energies far beyond any Earth-bound accelerator.

Sixth the phoenix. The newly oriented jets begin to show up over some kpc, and this corresponds to the X-shaped radio galaxies, while the old jets are fading but still visible [24]. This also explains many of the compact steep spectrum sources, with disjoint directions for the inner and outer jets.

Seventh the cocoon. The old jets have faded and are at most visible in the low-radio frequency bubbly structures, such as seen for the Virgo cluster region around M87. The feeding is slowing down, while a powerful jet is still there, fed from the spin of the black hole.

Eighth the wait. The feeding of the black hole is down to catching some gas out of a common red giant star wind as presumably is happening in our Galactic center. This stage seems to exist for all black holes, even at very low levels of activity. This black hole and its galaxy are ready for the next merger, with the next black hole suitor, which may or may not come.

We predict that the super-disk radio galaxies should have large outer distortions in their radio images and should be visible in low keV x-rays. A detection of the precessing jet, and its sudden weakening, would then immediately precede the actual merger of the two black holes, and so may be a predictor of the gravitational wave signal.

3. The contribution of the spin precessions to the accumulated orbital phase

In the rest of the paper, we concentrate on the gravitational radiation-dominated dissipative regime, when the supermassive black hole binary radiates away gravitational waves.

If the binary system evolves unperturbed, the orbit circularizes faster than it shrinks in the radius. Therefore, we consider circular orbits for which the wave frequency is twice the orbital frequency.

Besides the Keplerian contribution to the accumulated orbital phase, there are general relativistic contributions, which scale with the post-Newtonian (PN) parameter \( \varepsilon \equiv m/r \approx v^2 \) (with \( m \) being the total mass and \( r \) being the radius of the reduced mass orbit about the total
mass; we take \( G = 1 = c \). It is generally agreed from comparison with numerical evolution that all contributions up to 3.5 PN orders have to be taken into account. To this order, beside the general relativistic contributions starting at 1 PN, there are various contributions originating in the finite size of the binary components. These are related either to their rotation (spin) or to their irregularities in shape (like a mass quadrupole moment).

Due to the continued accretion of surrounding matter into the supermassive galactic black holes they spin up considerably. By taking into account only the angular momentum transfer from accreting matter, the dimensionless spin parameters \( \chi_i = S_i / m_i^2 \) (with \( i = 1, 2 \)) grow to the maximally allowed value 1 even for initially non-rotating black holes [25]. By taking into account the torque produced by the energy input of the in-falling (horizon-crossing) photons emitted from the steady-state thin accretion disk, which counteracts the torque due to mass accretion, the limiting value of the dimensionless spin parameter is slightly reduced to 0.9982 [26]. Various refinements of this process, with the inclusion of open or closed magnetic field lines [27–35], also jets in the magnetosphere of the hole [36, 37], have not essentially changed this prediction. Therefore, we assume for the present considerations maximal rotation \( \chi_i \approx 1 \). Due to this rotation, the black hole is centrifugally flattened (it becomes an oblate spheroid), a deformation which can be characterized by a mass quadrupole scalar \( Q \), or its dimensionless counterpart \( p_i = Q_i / m m^2 \approx - (m_i / m)^2 \).

The accumulated orbital phase can be formally given as the PN expansion [14, 39, 40]:

\[
\phi = \phi_c + \phi_N + \phi_{1PN} + \phi_{1.5PN} + \phi_{2PN} + \phi_{2.5PN} + \phi_{3PN} + \phi_{3.5PN}.
\]  

(1)

Spin–orbit (SO) contributions appear at 1.5 PN, spin–spin (SS; composed of proper \( S_1 S_2 \) and self-contributions \( S_1^2 \) and \( S_2^2 \)) and mass quadrupole–mass monopole coupling (QM) contributions at 2 PN. They come with various PN corrections (PNSO at 2.5 PN; PNSS, PNQM and SO² at 3 PN; finally 2PNSO at 3.5 PN; these were not computed yet with the exception of PNSO [41, 42]). At 3 PN there are additional spin and quadrupole contributions, originating in the quasi-precessional evolution of the spin and orbital angular momentum vectors. The number of gravitational wave cycles \( N \) can be computed from the accumulated orbital phase as \( N = (\phi_c - \phi) / \pi \) (the gravitational wave frequency being twice the orbital frequency for quasi-circular motion).

The spin precession equations with SO, SS and QM contributions were given by Barker and O’Connell [43]. The relative geometry of the two spins \( S_i \) and orbital angular momentum \( L \) can best be described by the angles \( \gamma = \arccos(\hat{S}_1 \cdot \hat{S}_2) \) and \( \kappa_i = \arccos(\hat{S}_i \cdot \hat{L}_X) \) (an overhat denotes the direction of the respective vector). These angles are related by the spherical cosine identity

\[
\cos \gamma = \cos \kappa_1 \cos \kappa_2 + \cos \Delta \psi \sin \kappa_1 \sin \kappa_2.
\]

(2)

with \( \Delta \psi \) being the relative azimuthal angle of the spins. Due to quasi-precessions, \( \kappa_i \) and \( \gamma \) evolve. SupPLEMENTING the SO and SS contributions already given [44] with the QM contributions (with the quadrupole moment arising from pure rotation), we give these expressions to second-order accuracy:

\[
(\cos \kappa_i) = (\cos \kappa_i)_{SO} + (\cos \kappa_i)_{SS} + (\cos \kappa_i)_{QM},
\]

(3)

\[
(\cos \gamma) = (\cos \gamma)_{SO} + (\cos \gamma)_{SS} + (\cos \gamma)_{QM}.
\]

(4)

with the detailed contributions enlisted in the appendix.

The order of magnitude estimates of the terms in equations (A.1)–(A.9) (cf a footnote in [45], according to which \( O(\delta x) = O(\delta x) r e^{-1/2} \) and assuming maximal rotation lead to
\[ O(\delta_{SO}\kappa_1) \approx \varepsilon^{3/2}(1 + v^{-1})^{-2}(2 + v^{-1}) \approx \varepsilon^{3/2}v, \]
\[ O(\delta_{SO}\kappa_2) \approx \varepsilon^{3/2}(1 + v^{-1})^{-2}(1 + \varepsilon^{1/2}v^{-1}) \approx \varepsilon^{3/2}v(\varepsilon + \varepsilon^{1/2}), \]
\[ O(\delta_{SMK}\kappa_1) \approx \varepsilon^{3/2}(v + \varepsilon^{1/2}v^2) \approx \varepsilon^{3/2}v, \]
\[ O(\delta_{SMK}\kappa_2) \approx \varepsilon^{3/2}(1 + v)^{-2}(2 + v) \approx \varepsilon^{3/2}, \]
\[ O(\delta_{SS}\kappa_2) \approx \varepsilon^{3/2}(1 + v)^{-2}(1 - \varepsilon^{1/2}v) \approx \varepsilon^{3/2}, \]
\[ O(\delta_{SS}\gamma) \approx \varepsilon^{3/2}(1 + v^{-1})^{-2}(1 + v^{-1}) \approx \varepsilon^{3/2}, \]
\[ O(\delta_{SS}\gamma) \approx \varepsilon^{3/2}[1 + (1 + v^{-1})^{-2} - (1 + v)^{-2}] \approx \varepsilon^{3/2}. \]

Thus the angle \( \gamma \) varies at 1 PN, while the angles \( \kappa_i \) only at 1.5 PN during one orbital period. However \( \gamma \) appears only in the 2 PN order \( S_1S_2 \) contribution to the accumulated orbital phase as opposed to the angles \( \kappa_i \) which are present in all finite-size contributions, in particular in the 1.5 PN order SO contribution.

Therefore, 3 PN order quasi-precessional contributions to the accumulated orbital phase in the comparable mass case \( v \lesssim 1 \) arise from \( \delta_{SO}\kappa_1, \delta_{SS}\kappa_1, \delta_{SMK}\kappa_1 \) and \( \delta_{SO}\gamma \); while in the unequal mass case, found typical for supermassive galactic black hole binaries only from \( \delta_{SO}\kappa_2, \delta_{SS}\kappa_2 \) and \( \delta_{SO}\gamma \).

### 4. Leading-order evolution of the spin–spin coefficient

From among the quasi-precessional evolutions, we have recently integrated [14] the SO evolution (averaged over one orbit) of the angle \( \Delta \psi \), finding

\[ \Delta \psi = (\Delta \psi)_0 + \frac{3\mu n(v^{-1} - v)}{2a} \tau. \]

Here \( \mu = m_1m_2/m \) is the reduced mass, \( a \) is the radius and \( n = 2\pi/T_{\text{orbit}} = \pi/T_{\text{wave}} \). The time-dependent expression for \( \gamma \) is then given by equation (2), with \( \Delta \psi \) from equation (6).

The variation of both \( \Delta \psi \) and \( \gamma \) thus has a periodicity with period

\[ T_{3\text{PNS}} = F(v)\varepsilon^{-1}T_{\text{wave}}, \]

where

\[ F(v) = \frac{4(2 + v + v^{-1})}{3(v^{-1} - v)}. \]

#### 4.1. Equal masses

For equal mass binaries, this time scale becomes infinite \( \lim_{v \to 1} F(v) = \infty \) (see figure 1(a)) expressing the fact that the time-dependent part of \( \gamma \) goes to zero. Thus \( \gamma \) is a constant and there is no 3 PN quasi-precessional contribution to the accumulated orbital phase for equal mass binaries.

#### 4.2. Unequal masses

For the typical mass ratio, the factor \( F(v) \) is of the order of unity (see figure 1(b)), thus the variation time scale of \( \gamma \) is \( \varepsilon^{-1} \in (10, 1000) \) times larger than the wave period. The contribution of the quasi-precessions can be taken into account then by keeping only the first term in equation (2) when computing the 2 PN order \( S_1S_2 \) contribution to the accumulated
orbital phase, e.g., by introducing a renormalized spin coefficient. This coefficient arises by modifying the 2 PN contribution with the average of some of the 3 PN contributions. By using this renormalized coefficient in the context of the 2 PN accurate dynamics, one can bring closer the 2 PN analytical prediction to the numerical results [14].

Thus, in the unequal mass case we propose to replace

$$\sigma_{S_1 S_2} = \frac{S_1 S_2}{48 \eta m^4} (-247 \cos \gamma + 721 \cos \kappa_1 \cos \kappa_2),$$

(9)

with

$$\sigma_{S_1 S_2} = \frac{79 S_1 S_2}{8 \eta m^4} \cos \kappa_1 \cos \kappa_2,$$

(10)

in the respective 2 PN contribution to the accumulated orbital phase

$$\phi_{2PN} = -\frac{1}{\eta} \left( \frac{9275}{14450688} + \frac{284875}{258048 \eta} + \frac{1855}{2048 \eta^2} - \frac{15}{64} \right) \tau^{1/8},$$

(11)

$$(\text{Here } \eta = \mu / m \text{ is the symmetric mass ratio and } \tau \text{ is a dimensionless time parameter.})$$

5. Leading-order evolution of the spin–orbit coefficient

The 1.5 PN contribution to the accumulated orbital phase is

$$\phi_{1.5PN} = -\frac{3}{4 \eta} \left( \frac{1}{4} \beta - \pi \right) \tau^{1/4},$$

(12)

with the SO contribution given by

$$\beta = \frac{1}{12} \sum_{i=1}^{2} \chi_i \cos \kappa_i \left( 113 \frac{m_i^2}{m^2} + 75 \eta \right).$$

(13)

9 However we stress that there are other finite-size 3 PN order contributions to the phase, such as PNSS, PNQM and SO$^2$, which are still not computed.
5.1. Equal masses

The quasi-precessional equations with the SO, SS and QM contributions included were recently integrated by Racine for the equal mass case [46]. According to his analysis, there is a previously unknown constant of the motion

$$\lambda = \frac{2}{L_N} (S_1 \cos \kappa_1 + S_2 \cos \kappa_2). \quad (14)$$

As for equal masses $\beta = (47 L_N / 24) \lambda = \text{const}$, the quasi-precessional evolution at 3 PN does not affect the coefficient $\beta$ up to the 3 PN accuracy.

5.2. Unequal masses

In the unequal mass case, the leading-order evolutions of the angles $\kappa_i$ are $\delta_{SO} \kappa_2$ and $\delta_{SS} \kappa_2$. Averaged over one orbit they give [44]

$$\dot{\kappa}_2 = \frac{3(1 + \nu)m_2^2}{2a^3} \sin \kappa_1 \sin \Delta \psi. \quad (15)$$

For unequal masses $\nu \approx 10^{-1}$, the estimates (5) allow us to consider $\kappa_1$ as a constant (in a first approximation). The leading-order time dependence of $\Delta \psi$ is given by equation (6). Integration gives

$$\kappa_2 = (\kappa_2)_0 - \frac{\varepsilon^{1/2}}{1 - \nu} \sin \kappa_1 \cos \left[ (\Delta \psi)_0 + \frac{3 \mu n (v^{-1} - \nu)}{2a} t \right]. \quad (16)$$

(We have employed $\varepsilon \approx m/a$.) The time-varying part has the same period $T_{3\text{PN}}$ given by equation (7), which is of the scale of $\varepsilon^{-1}$, the orbital period. In agreement with this and the estimate (5)

$$\frac{O(\kappa_2 - (\kappa_2)_0)}{O(\delta_{SO,SS} \kappa_2)} = \frac{\varepsilon^{1/2}}{\varepsilon^{3/2}} = \varepsilon^{-1}. \quad (17)$$

As the time-varying part averages out, to leading-order accuracy we can simply identify the angle $\kappa_2$ with its constant initial value $(\kappa_2)_0$.

In conclusion, in the unequal mass case and in the leading-order approximation $\kappa_1 = \text{const}$ the time variation due to quasi-precessions renders $\kappa_2$ to another constant at 3 PN accuracy. There is no need to renormalize $\beta$, when taking into account the leading-order quasi-precessional contributions.

6. Concluding remarks

The process of the merger of two supermassive galactic black holes gives rise to exciting signals in all the messengers accessible now and in the near future, and will allow us to probe some of the most profound secrets of Nature, occurring at energies and circumstances far beyond anything on Earth. We have described the sequence of events during the process of merging of such supermassive galactic black holes with reference to related processes involving accretion, jets, energetic particles and electromagnetic signatures. We argued that the class of radio galaxies with a super-disk are particularly promising candidates for future powerful low-frequency gravitational wave emission.

The accumulated orbital phase is a quantity to be known precisely for successful detection of gravitational waves. Besides Keplerian, first and second PN order general relativistic contributions, the finite size of the black holes also contributes, first at 1.5 PN orders via the spin–orbit interaction, then at 2 PN by the spin–spin and quadrupole–monopole coupling.
These contributions are further modulated by quasi-precessional evolutions of the spins. However this is but a subset of all possible finite-size contributions at 3 PN, as stressed earlier. The way we propose to take into account the precessional 3 PN contributions is to add their average to the lower order terms, a procedure we call *renormalization* of the lower order coefficients.

We have proved here that there are no such quasi-precessional contributions in the equal mass case. Furthermore, we have proved for the unequal mass case that renormalizing the 1.5 PN spin–orbit coefficient will not change its value. At 2 PN there are quadrupolar and spin–spin contributions to consider. As the former can be regarded as constant up to 3 PN, the only modification due to quasi-precessions comes from the spin–spin interaction. The renormalized spin–spin coefficient at 2 PN is thus adjusted by the average of the 3 PN quasi-precessional contribution, taken over the period of change induced by these quasi-precessions (a time-scale one PN order lower than the orbital period). We have checked that the renormalized spin–spin coefficient gives closer results to the numerical estimates for compact binaries with the mass ratio of order $1/10$ [14].

Our results related to the renormalization of the spin coefficients are significant for supermassive black hole binaries, which are typically LISA sources. However as we have employed nothing but the mass ratio of the binary in our considerations, they also apply for binaries composed of an intermediate mass and a stellar mass black hole, which qualify as LIGO sources.

**Acknowledgments**

Participation of LÁG at the GWDAW13 meeting was supported by the Polányi Program of the Hungarian National Office for Research and Technology (NKTH). Support for work with PLB has come from the AUGER membership and theory grant 05 CU 5PD 1/2 via DESY/BMBF and VIHKOS. LÁG was also successively supported by a ROF Award of LSBU and by Collegium Budapest. The work of BM was supported by the Hungarian Scientific Research Fund (OTKA) grant no 68228, and of ZK by the OTKA grant no 69036.

**Appendix. Quasi-precessional evolutions of the relative spin angles**

The SO and SS contributions of the instantaneous variations of the relative spin angles [44] supplemented with the respective QM contributions (assuming that the quadrupole moment arises from pure rotation) to second-order accuracy are as follows:

\[
\cos \kappa_1 \dot{\gamma}_{SO} = \frac{3S_2}{2r^3} (2 + \nu^{-1}) \mathbf{L}_N \cdot (\mathbf{S}_1 \times \mathbf{S}_2),
\]

\[
\cos \kappa_1 \dot{\gamma}_{SS} = \frac{3S_2}{r^3} \left[ (\hat{r} \cdot \hat{S}_2) \mathbf{L}_N \cdot (\hat{r} \times \hat{S}_1) + \frac{S_1}{L_N} (\hat{r} \cdot \hat{S}_1) \hat{r} \cdot (\mathbf{S}_1 \times \mathbf{S}_2) \right],
\]

\[
\cos \kappa_1 \dot{\gamma}_{QM} = -\frac{3\mu m^3}{r^3} \left[ \frac{p_1}{S_1} (\hat{r} \cdot \hat{S}_1) \mathbf{L}_N \cdot (\hat{r} \times \hat{S}_1) + \frac{p_2}{L_N} (\hat{r} \cdot \hat{S}_2) \hat{r} \cdot (\mathbf{S}_1 \times \mathbf{S}_2) \right],
\]

\[
\cos \kappa_2 \dot{\gamma}_{SO} = -\frac{3S_1}{2r^3} (2 + \nu) \mathbf{L}_N \cdot (\mathbf{S}_1 \times \mathbf{S}_2),
\]

\[
\cos \kappa_2 \dot{\gamma}_{SS} = \frac{3S_1}{r^3} \left[ (\hat{r} \cdot \hat{S}_1) \mathbf{L}_N \cdot (\hat{r} \times \hat{S}_2) - \frac{S_2}{L_N} (\hat{r} \cdot \hat{S}_2) \hat{r} \cdot (\mathbf{S}_1 \times \mathbf{S}_2) \right].
\]
\[(\cos \kappa)^2_{QM} = - \frac{3\mu m}{r_3} \left[ \frac{p_2}{S_2} (\vec{r} \cdot \hat{S}_2) L_N \cdot (\vec{r} \times \hat{S}_2) \right. \left. + \frac{p_1}{L_N} (\vec{r} \cdot \hat{S}_1) \vec{r} \cdot (\hat{S}_1 \times \hat{S}_2) \right]. \quad (A.6)\]

\[(\cos \gamma)^{y_{SO}} = \frac{3L N}{2r^3} (v - v^{-1}) L_N \cdot (\hat{S}_1 \times \hat{S}_2), \quad (A.7)\]

\[(\cos \gamma)^{y_{SS}} = \frac{3}{r^3} \left[ S_2 (\vec{r} \cdot \hat{S}_2) - S_1 (\vec{r} \cdot \hat{S}_1) \right] \vec{r} \cdot (\hat{S}_1 \times \hat{S}_2), \quad (A.8)\]

\[(\cos \gamma)^{y_{QM}} = \frac{3\mu m}{r^3} \left[ \frac{p_2}{S_2} (\vec{r} \cdot \hat{S}_2) - \frac{p_1}{S_1} (\vec{r} \cdot \hat{S}_1) \right] \vec{r} \cdot (\hat{S}_1 \times \hat{S}_2). \quad (A.9)\]

References

[1] Peters P C 1964 Phys. Rev. 136 B1224
[2] Peters P C and Mathews S 1963 Phys. Rev. 131 435
[3] Thorne K S 1979 Proc. R. Soc. London A 368 9
[4] Biermann P L, Chirvasa M, Falcke H, Markoff S and Zier Ch 2005 Proc. Conf. on Cosmology (Paris, June 2000) ed N Sanchez and H de Vega pp 148–64 (arXiv:astro-ph/0211503) (invited review)
[5] Merritt D and Ekers R 2002 Science 297 1310–3
[6] Gergely L Á and Biermann P L 2009 Astrophys. J. 697 1621
[7] Biermann P L et al 2009 Proc. CRIS 2008—Cosmic Ray International Seminar: Origin Mass, Composition and Acceleration Mechanisms of UHECRs, Malia, Italy 2008 ed A Insolia Nucl. Phys. B Proc. Suppl. 190 61–78 (arXiv:0811.1848v3 [astro-ph])
[8] Press W H and Schechter P 1974 Astrophys. J. 187 425
[9] Lauer T R et al 2007 Astrophys. J. 662 808
[10] Wilson A S and Colbert E J M 1995 Astrophys. J. 438 62–71
[11] Ferrarese L et al 2006 Astrophys. J. Suppl. 164 334
[12] Gonzalez J A, Sperhake U and Bruegmann B 2009 Phys. Rev. D 79 124006
[13] Lang R N and Hughes S A 2006 Phys. Rev. D 74 122001
Lang R N and Hughes S A 2007 Phys. Rev. D 75 089902 (erratum)
Lang R N and Hughes S A 2008 Astrophys. J. 677 1184
[14] Gergely L Á and Mikóczki B 2009 Phys. Rev. D 79 064023
[15] Gopal-Krishna, Biermann P L and Wiita P 2003 Astrophys. J. Lett. 594 L103–6
[16] Zier Ch 2007 Mon. Not. R. Astron. Soc. 378 1309
[17] Gopal-Krishna, Wiita P J and Joshi S 2007 Mon. Not. R. Astron. Soc. 380 703
[18] Rezzolla L, Barausse E, Dorband E N, Polnay D, Reisswig C, Seiler J and Husa S 2008 Phys. Rev. D 78 044002
[19] Wasnik M C, Healy J, Herrmann F, Hinderer T, Shoemaker D M, Laguna P and Matzner R A 2008 Phys. Rev. Lett. 101 061102
[20] Tichy W and Marronetti P 2008 Phys. Rev. D 78 081501
[21] Barausse E and Rezzolla L 2009 Predicting the direction of the final spin from the coalescence of two black holes arXiv:0904.2577
[22] Healy J, Laguna P, Richard A, Matzner R A and Shoemaker D M 2009 Final mass and spin of merged black holes and the golden black hole arXiv:0905.3914
[23] Sperhake U, Cardoso V, Pretorius F, Berti E, Hinderer T and Yunes N 2009 Cross section, final spin and zoom-whirl behavior in high-energy black hole collisions arXiv:0907.1252
[24] Rottmann H 2001 Jet-reorientation in X-shaped radio galaxies PhD Thesis Bonn University (http://hss.ulb.uni-bonn.de/diss_online/math_pat_fak/2001/rottmann_helge/index.htm)
[25] Bardeen J M 1970 Nature 226 64
[26] Pathowski S and Thorne K S 1974 Astrophys. J. 191 499
[27] Blandford R D and Znajek R L 1977 Mon. Not. R. Astron. Soc. 179 433
[28] Camenzind M 1986 Astron. Astrophys. 156 137
[29] Camenzind M 1986 Astron. Astrophys. 162 32
[30] Camenzind M 1987 Astron. Astrophys. 184 341
[31] Takahashi M, Nitta S, Tamematsu Y and Tomimatsu A 1990 Astrophys. J. 363 206
[32] Nitta S Y, Takahashi M and Tomimatsu A 1991 Phys. Rev. D 44 2295
[33] Hirotani K, Takahashi M, Nitta S and Tomimatsu A 1992 Astrophys. J. 386 455
[34] Li L X 2000 Astrophys. J. 533 L115
[35] Wang D X, Xiao K and Lei W H 2002 Mon. Not. R. Astron. Soc. 335 655
[36] Falcke H and Biermann P L 1995 Astron. Astrophys. 293 665
[37] Kovács Z, Biermann P L and Gergely L Á 2009 Maximal spin and energy conversion efficiency in a symbiotic system of black hole, disk and jet (in preparation)
[38] Poisson E 1998 Phys. Rev. D 57 5287
[39] Blanchet L, Faye G, Iyer B R and Joguet B 2002 Phys. Rev. D 65 061501
Blanchet L, Faye G, Iyer B R and Joguet B 2005 Phys. Rev. D 71 129902 (erratum)
[40] Mikóczai B, Vasúth M and Gergely L Á 2005 Phys. Rev. D 71 124043
[41] Buonanno A, Cook G B and Pretorius F 2007 Phys. Rev. D 75 124018
[42] Blanchet L, Buonanno A and Faye G 2006 Phys. Rev. D 74 104034
Blanchet L, Buonanno A and Faye G 2007 Phys. Rev. D 75 049903 (erratum)
[43] Barker B M and O’Connell R F 1975 Phys. Rev. D 12 329
[44] Gergely L Á, Perjés Z and Vasúth M 1998 Phys. Rev. D 58 124001
[45] Gergely L Á 2000 Phys. Rev. D 62 024007
[46] Racine E 2008 Phys. Rev. D 78 044021