A simple combined projection method for conservative decision-making

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Abstract
Machine learning and artificial intelligence based techniques have brought great convenience to human life but along with a series of algorithmic “black box”, discrimination and ethical issues. One of the solutions is to integrate human and machine like the expert evaluation based research of multi-attribute decision-making where “human brain intelligence” is used for the support of “artificial intelligence”. In this article, we proposed a new and effective method to evaluate and rank alternatives in multi-attribute decision-making. Different from many existing approaches, this proposed method employs both the projection lengths and the projection angles of alternatives to make decisions. It supports psychological desirableness of decision makers and uses a Relu function to further enhance the output qualities. This proposed method is very simple to construct and applicable for much wider situations than the existing similar methods.

Keywords Conservative decision preference · Multi-attribute decision-making · Neutrosophic set · Combined projection method

1 Introduction
Artificial intelligence, machine learning and other big data technologies have gradually infiltrated and affected our daily life in a subtle way. However, its negative effects have also begun to receive increasing attentions from the public and media. For example, clinicians are often skeptical of the automatic image processing methods used in clinical, mainly because of the “black box” problem of these algorithms; marginalized groups or groups that are under-represented in training data may receive less attentions in some algorithms; some researchers argue that the opacity of the deep learning approach limits their scientific findings [1–3]. All these suggest that the decision algorithms should not only be limited to the demand of “accuracy”, but also need to consider robustness and interpretability etc.

A multi-attribute decision algorithm, which requires the input of individual or group evaluation data, can reflects the wisdom of the “human brain” with interpretable process and results. However, in many application scenarios such as the preference selection of different schemes in urban planning, risk portfolio schemes in investment strategies, and the evaluation of commodities in personalized recommendation, we also find out that a few individual preferences are often as important as the majority group preferences because of the existences of certain political, social, and economic realities. Therefore, it is necessary to construct simple and fast multi-attribute decision-making algorithms that achieve fully ranked alternatives with stable and flexible mental expressions.

Related works: Multi-attribute decision-making refers to the process of choosing the most satisfactory plan among the alternatives containing multi-type attributes. Because of the inherent and cognitive limitations of the decision subject, the diversity and complexity of decision attributes, it is usually difficult to assign the accurate values for the right judgments. Therefore, how to flexibly express decision information becomes one of the core problems of multi-attribute decision-making. In 1965, Zadeh [4] proposed fuzzy sets,
and then intuitionistic fuzzy sets, interval fuzzy sets, hesitant fuzzy sets, and other concepts to be expanded to describe information uncertainty [5 –11]. Many of these theoretical concepts have been successfully used for solving practical decision-making problems. To comprehensively describe the non-deterministic information, Smarandache [12] used neutrosophic set to depict the truth, the indeterminacy, and the falsehood of a statement simultaneously. Researchers then subsequently developed various concepts and aggregation operators of single-valued neutrosophic set (SVNS) [13, 14], interval neutrosophic sets (INS) [15], simplified neutrosophic sets (SNS) [16], multi-valued neutrosophic sets (MVNS) [17]. The study of multi-attribute decision-making methods based on neutrosophic set has been gradually developed, such as ELECTRE [18], VIKOR [19], TOPSIS [20, 21], MULTIMOORA [22], TODIM [23]. Multi-attribute decision-making methods on SVNS have been used in medical diagnosis, e-learning, image processing, data mining, supplier selection and other fields [24–26].

Among the multi-attribute decision-making methods, Projection Method (PM) [27] and Vertical Projection Method (VPM) [28] have gained special attentions from researchers because of the simplicity and intuitiveness. These two methods make decisions by comparing the projection lengths, and have been widely used for various neutrosophic sets [29–32] to solve many practical problems [33–37]. Although the VPM has been widely used in decision making in the uncertain information environments, it can lead incomparable alternatives. Extended from VPM, the Bi-directional Projection Model (BPM) [38–40] solves the incomparable problem with more complicated calculations by also considering projection distances of alternatives to the negative ideal solution to form bilateral distances. But these two methods fail to flexibly describe the psychological tendency as illustrated in the following.

**Incompatibility Problem:** Similar to TOPSIS and VIKOR methods, VPM compares alternatives by simply considering their projection length and distances to the positive ideal solution, and thus may lead to incomparable to different alternatives. For example, we have observe that (a) and (b) in Fig. 1 represent the comparable and incomparable cases of two solutions under the VPM. Since the VPM is only compared by the distance of the orthogonal projection points of the alternative solutions from the positive ideal solution, it is impossible to judge the superiority of the two solutions when the orthogonal projection points of the two solutions in Fig. 1 (b) are the same.

**Tendency Support:** The two above mentioned projection methods can be difficult to flexibly describe the psychological tendency in decision making. Generally, these two models use supplementary elements to express psychological tendencies. For example, they represent an aggressive (or a conservative) decision preference by adding a maximum (or minimum) value respectively. However, since the supplementary attribute values solely rely on the original elements in the set and are rather rigid, this supplementary approach is hard to support a differentiated description of the subjective psychology of the decision maker in case all the values of the original data are of equal length and no additional elements are needed. More recently, Ding, et al. [40] extend BPM in the context of hesitant triangular fuzzy sets (HTFS) for the group emergency decision making. Ni, et al. [41] use dual hesitant fuzzy sets (DHFS) in BPM to weigh the attributes for rational decision results. Although these two most recent research results support decision makers’ psychological mindset, they work on the different informative data sets with relatively heavier computation costs. These recent research endeavour also raised an open question: If VPM can be simply extended to eliminate the incompatibility and support psychological tendency? This motivates us for the research of this article and construct a new VPM-based method without the above mentioned two deficiencies.

**Contributions:** In this article, we shall consider how to improve the adaptability of VPM in the conservative decision making. We proposed a new simple Combined Projection Method (CPM) that eliminates the incompatibility among alternatives and can flexibly support the decision makers’ psychology tendencies. CPM extends VPM through intensively using projection angles with simple structure and computation costs. Its ranking results match the actual decision requirements well in practical situations.

**Fig. 1** a Comparison of alternative A₁ and A₂ under the VPM. b Alternatives A₁ and A₂ have the same projection point under VPM.
The rest of the article is organized as follows: Section 2 briefly reviews two major projection methods and identifies their deficiency aspects; Section 3 proposes the Combined Projection Method (CPM) and describes its major procedures; Section 4 is a running example that demonstrates the rationality and effectiveness of CPM; Section 5 concludes this article.

2 Vertical projection method (VPM)

In this section, we briefly review Vertical Projection Method (VPM) and the relevant concepts. To evaluate the impacts of different alternatives, VPM needs to compare the distances of their vertical projection points to the positive ideal point. As shown in Fig. 1, let $A_1$ and $A_2$ be two alternatives. For the positive ideal solution $A^+$ and the negative ideal solution $A^{-}$, vectors $A^+ A_1$, $A^+ A_2$ and $A^{-} A^{-}$ are used for ordering of $A_1$ and $A_2$ under VPM by the following 2 steps:

Step 1. Calculate and compare the lengths of the projections of vectors $A^+ A_1$ and $A^+ A_2$ on vector $A^+ A^{-}$:

$$|A^+ M_1| = Prj_{A^+ A^{-}}(A^+ A_1) = |A^+ A_1| \cos(A^+ A_1, A^+ A^{-}),$$

$$|A^+ M_2| = Prj_{A^+ A^{-}}(A^+ A_2) = |A^+ A_2| \cos(A^+ A_2, A^+ A^{-}),$$

where $|A^+ M_1|$ and $|A^+ A_1|$ are the modules of the vectors $A^+ M_1$ and $A^+ A_1$, $\cos(A^+ A_1, A^+ A^{-})$ is the cosine of the angle between vector $A^+ A_1$ and vector $A^+ A^{-}$.

Step 2. Decide the order of alternatives: The essence of the VPM is to compare the orthogonal projection distances of two alternatives $A_1$ and $A_2$ on the vector $A^+ A^{-}$, and the closer the projection point ($M_1$ or $M_2$) is to the positive ideal solution, the better the solution is. Since $|A^+ M_2| \preceq |A^+ M_1|$, point $M_2$ is closer to the positive ideal solution $A^+$ than that of $M_1$. We therefore define $A_2 \succ A_1$ indicating that $A_2$ is preferred to $A_1$.

However, as shown in Fig. 1(b) when the vertical projection points of $A_1$ and $A_2$ on the $A^+ A^{-}$ vector are identical ($|A^+ M_2| = |A^+ M_1|$), VPM is hard to define the preferential order for the two different alternatives.

VPM only considers the one-side distance between the alternative $A_1$ and the positive ideal solution $A^+$, leading to the incomparability problem. Although the Bi-directional Projection Model (BPM) extends VPM by solving the incomparability problem, it can be hard to extend to support risk preferences on neutrosophic set as explained in the following.

VPM and BPM models describe the risk preferences of decision makers generally by adding additional elements to shorter evaluation schemes. For example, suppose two schemes $A_1$ and $A_2$ with attribute values $0.5, 0.7$ and $0.3, 0.4, 0.5$ respectively, i.e., two hesitant fuzzy sets of unequal lengths. To make the attribute values $(0.5, 0.7)$ of $A_1$ have the same length of $A_2$, a complementary element is added to indicate conservative, aggressive, or neutral risk appetite of decision makers by picking up the minimum, the maximum or the mean of $0.5$ and $0.7$. That is, $(0.5, 0.7)$ is transformed into $(0.5, 0.5, 0.7), (0.5, 0.7, 0.7)$ or $(0.5, 0.6, 0.7)$ respectively. However, in case of SVNS (Definition 1 in the following) where each attribute is expressed as $(T, I, F)$ and does not need the complementary step, the decision makers’ tendency cannot be flexibly expressed.

Definition 1 [14] Let $X$ be the domain of points. A single valued neutrosophic set $K$ on $X$ is

$$K = \{(T_K(x), I_K(x), F_K(x)) \mid x \in X\}$$

where $0 \leq T_K(x) \leq 1$ is a truth membership function, $0 \leq I_K(x) \leq 1$ is an indeterminacy membership function and $0 \leq F_K(x) \leq 1$ is a falsity membership function. If $X$ has only one element, $K = (T_K(x), I_K(x), F_K(x))$ is called a single-valued neutrosophic number (SVNN) and its complement $K^C$ is defined as

$$K^C = \{(F_K(x), 1 - I_K(x), T_K(x)) \mid x \in X\}.$$  

3 Combined projection method (CPM)

In this section, we propose the Combined Projection Method (CPM) to solve the above mentioned shortcomings. We will first briefly describe general idea of CPM. We then illustrate the major functional steps of CPM in subsections, including the comparison with VPM in rankings.

As mentioned above, this proposed model CPM uses both the projection length and the projection angle for decision making where the projection angle describes the decision maker’s risk preference. With the belief that when the projection lengths are equal, a smaller projection angle implies a more stable evaluation result of the attribute values with less faced uncertain risk, our proposed CPM considers both the distances of projection points $M_1$ and $M_2$ of the two alternatives to the negative ideal solution $A^{-}$ (rather than to the positive ideal solution) and their angle $\beta_1$ and $\beta_2$. To rank the alternatives of the same projection length, CPM selects the alternative of smaller projection angle under the conservative preference. As shown in Fig. 2, alternatives $A_1$ and $A_2$ have the same orthogonal projection points. To avert risk, the proposed model defines $A_2 \succ A_1$ since angles $\beta_2 < \beta_1$ interpreting that $A_2$ is conservatively favorable.

3.1 Transform neutrosophic sets

In general, risk preferences can be classified as risk-loving (aggressive), risk-averse (conservative), and risk-neutral
For example, let alternative $A_1$ and $A_2$ be single-valued neutrosophic sets (SVNS), each SVNS contains two SVNN: $K_1$ and $K_2$.

$$A_1 = \{K_1, K_2\} = \{(0.50, 0.20, 0.20), (0.50, 0.30, 0.20)\},$$
$$A_2 = \{K_1, K_2\} = \{(0.49, 0.26, 0.21), (0.61, 0.43, 0.25)\},$$

we have

$$Z(A_1) = \{[0.56, 0.78], [0.50, 0.80]\},$$
$$Z(A_2) = \{[0.51, 0.78], [0.47, 0.81]\}.$$

When the mean values of the intervals are the same or very close, the intervals with shorter lengths indicate more stable in attribute values and less uncertainty in decision making. To derive a reasonable ranking order, $Z(A_1)$ and $Z(A_2)$ need to be normalized with Relu function to reject unqualified values.

### 3.2 Filtering the unqualified values

In the actual decision-making process, decision makers may have the knowledge of allowable possible values for certain attributes, which are called decision eligible values. For example, if the evaluation of $A_1$ scheme is $(-20, 120)$ and $A_2$ is $(40, 60)$, although the mean values of the evaluations are equal, the conservative decision maker will choose $A_2$. This is because that the project cannot have the risk of loss and 0 is the lower-bound of decision eligible value. The mental eligibility value expresses the decision maker’s risk-averse attitude toward the non-eligible scheme. The qualified reference value of attributes can be selected either subjectively or objectively. Subjective selection is generally based on experts’ experience or according to the industry standards, while objective selection can be based on the actual situation using mean, median, expected value, or other methods. In this article, the Relu function (Formula (4)) is used to filter the unqualified values of alternatives that beyond the acceptable (or tolerable) degree of the decision subject.

Let $Z(A) = [z_1, z_2] = [s, b]$, where $0 \leq s \leq b \leq 1$. Let the qualified reference value of the lower limit attribute be a. Then the lower limit Relu function of the interval is defined as

$$\text{Relu}(s) = \begin{cases} 0, & s < a, \\ s, & s \geq a. \end{cases}$$

Formula (4) in practical applications.
3.3 Computing projection lengths and angles

For any two alternatives $A_1$ and $A_2$, 
\[
Z(A_1) = \{ [s_{11}, b_{11}], [s_{12}, b_{12}], \ldots [s_{1n}, b_{1n}] \},
\]
\[
Z(A_2) = \{ [s_{21}, b_{21}], [s_{22}, b_{22}], \ldots [s_{2n}, b_{2n}] \},
\]
we define 
\[
Z(A_1) \cdot Z(A_2) = \{ \min\{ |s_{11} - s_{21}|, |b_{11} - b_{21}| \}, \max\{ |s_{11} - s_{21}|, |b_{11} - b_{21}| \}, \ldots, \]
\[
\min\{ |s_{1n} - s_{2n}|, |b_{1n} - b_{2n}| \}, \max\{ |s_{1n} - s_{2n}|, |b_{1n} - b_{2n}| \} \}\}.
\]

Assume $Z(A_i) = \{ [s_{i1}, b_{i1}], [s_{i2}, b_{i2}], \ldots [s_{in}, b_{in}] \},
(i = 1, 2, \ldots, m)$. The interval fuzzy positive ideal of $Z(A_i)$ is 
\[
A^+ = \{ [s_1^+, b_1^+], [s_2^+, b_2^+], \ldots, [s_m^+, b_m^+] \}
\]
\[
= \{ \max\{ s_1, b_1 \}, \max\{ s_2, b_2 \}, \ldots, \max\{ s_m, b_m \} \}.
\]
Similarly, the interval fuzzy negative ideal of $Z(A_i)$ is 
\[
A^- = \{ [s_1^-, b_1^-], [s_2^-, b_2^-], \ldots, [s_m^-, b_m^-] \}
\]
\[
= \{ \min\{ s_1, b_1 \}, \min\{ s_2, b_2 \}, \ldots, \min\{ s_m, b_m \} \}.
\]
The vector formed by the alternative solution $A_i$ and the negative ideal solution $A^-$ is $A^-A_i$, the vector formed by the negative ideal solution $A^-$ and the positive ideal solution $A^+$ is $A^+A^-$, and the cosine of the angle between the two vectors is 
\[
\cos (A^-A_i, A^+A_i) = \frac{A^-A_i \cdot A^+A_i}{|A^-A_i| \cdot |A^+A_i|}.
\]

Let $A^-A_i = \{ [s_{i1}^-, b_{i1}^-], [s_{i2}^-, b_{i2}^-], \ldots, [s_{in}^-, b_{in}^-] \}$, and $A^+A_i = \{ [s_{i1}^+, b_{i1}^+], [s_{i2}^+, b_{i2}^+], \ldots, [s_{in}^+, b_{in}^+] \}$. Then, for $1 \leq i \leq m,
\[
A^-A_i \cdot A^+A_i = \frac{1}{2} \sum_{q=1}^{n} (s_{iq}^-s_{iq}^+ + b_{iq}^-b_{iq}^+),
\]
\[
|A^-A_i| = \sqrt{\frac{1}{2} \sum_{q=1}^{n} (s_{iq}^-)^2 + (b_{iq}^-)^2},
\]
\[
|A^+A_i| = \sqrt{\frac{1}{2} \sum_{q=1}^{n} (s_{iq}^+)^2 + (b_{iq}^+)^2},
\]
\[
\Pr_{A^-A_i}(A^-A_i) = \sqrt{\frac{1}{2} \sum_{q=1}^{n} (s_{iq}^+)^2 + (b_{iq}^+)^2} \cos(A^-A_i, A^+A_i).
\]

\[\text{(8)}\]

The risk attitudinal parameter $0 \leq \alpha \leq 1$ describes the decision's risk preferences and determines the degree of conservativeness. A larger value of $\alpha$ indicates a greater emphasis on the decision maker's conservative preference. When $\alpha = 0$, it indicates that there is no need to emphasize the decision maker's conservative psychology in the model, or the decision maker is not conservative at all. When $\alpha = 1$, it means maximally emphasizing the decision maker's conservative psychology.

3.4 Comparing rankings under VPM and CPM

Suppose $A_1$ and $A_2$ are two alternatives and each of them is evaluated by two attributes $K_1$ and $K_2$. The original data and the transformed intervals $Z(A_1)$ and $Z(A_2)$ are shown in Table 1.

Table 2 depicts the ranking results of $A_1$ and $A_2$ under VPM and CPM where the ranks under CPM are derived

| Alternative | VPM (\(\alpha = 0\)) | VPM (\(\alpha = 0.5\)) | VPM (\(\alpha = 1\)) | CPM (\(\alpha = 0\)) | CPM (\(\alpha = 0.5\)) | CPM (\(\alpha = 1\)) |
|-------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| A_1         | 0.0851 ①               | 0.0851 ①               | 0.0733 ②               | 0.0632 ①               | ①                     | ①                     |
| A_2         | 0.0851 ①               | 0.0851 ①               | 0.0731 ②               | 0.0628 ①               | ①                     | ①                     |

*Ranking order: ①, ②
through Formula (5–9). Since the projection points of \( A_1 \) and \( A_2 \) are the same, these two schemes \( A_1 \) and \( A_2 \) have the same ranking under VPM.

Under CPM, the ranking is the same as that of VPM if \( \alpha = 0 \). However, when conservative risk preference is emphasized (i.e., \( \alpha = 0.5 \) or \( \alpha = 1 \)), the ranking changes indicating that \( A_1 \) scheme is superior to that of \( A_2 \). Clearly, this conclusion is not achievable just by simply observing the raw data. Observing that the mean values of the converted intervals of \( A_1 \) and \( A_2 \) are not significantly different and \( Z(A_1) \) is obviously less volatile than that of \( Z(A_2) \). Thus, the decision maker will prefer \( A_1 \) if they prefer less uncertainty. For conservative decision makers, the ranking of \( A_1 \) scheme is higher than \( A_2 \), which is obviously more reasonable.

In CPM, we employ the Relu function to filter out attribute values that do not satisfy the requirements. In the above example, if more than 10% certainty is required for the decision makers, the filtered attribute intervals by the Relu function are shown in Table 3 (depicted in bold), where \( A_1 \) is preferred in all situations regardless of the value of \( \alpha \) (Table 4). When the difference between \( A_1 \) and \( A_2 \) is significant under VPM, the ranking of CPM always remains consistent with VPM despite of \( \alpha \).

### 3.5 Computing procedures of CPM in general

Let \( A = \{A_1, A_2, \ldots, A_m\} \) be a set of alternatives, \( K = \{K_1, K_2, \ldots, K_n\} \) be a set of attribute, which may include both cost attribute and benefit attribute. \( W = \{w_1, w_2, \ldots, w_n\} \) be a weight set of attribute satisfying \( \sum_{i=1}^{n} w_i = 1 \) and \( w_i \geq 0 (i = 1, 2, \ldots, n) \). The computation steps of our proposed method as follows:

**Step 1.** Construct the normalized matrix on benefit attribute set.

1. Uniform each attribute evaluation rule by transforming each cost attribute into benefit attribute using its complement (Formula (2)). Normalized matrix \( R = (a_{ij})_{n \times m} \) is obtained by replacing each cost attribute \( K_i \) with \( K_i^C \);

2. Transform the normalized attribute SVNS matrix to interval decision matrix. Each element in the attribute set \( K \) of alternative set \( A \) is transformed into a normalized interval of decision matrix \( \hat{R} = ([s_{ij}, b_{ij}])_{n \times m} \) (Formula (3)).

### Table 3 Filtered interval values from the Relu function

| \( Z(A_1) \) | \( K_1 \) | \( K_2 \) | \( K_1 \) (Filtered) | \( K_2 \) (Filtered) |
|-------------|----------|----------|----------------------|----------------------|
| \( Z(A_1) \) | [0.2075, 0.7736] | [0.4189, 0.7027] | [0.2075, 0.7736] | [0.4189, 0.7027] |
| \( Z(A_2) \) | [0.0648, 0.9259] | [0.3419, 0.7607] | [0.0000, 0.9259] | [0.3419, 0.7607] |

### Table 4 Sorting under CPM using the Relu function

| Alternative | VPM | CPM |
|-------------|-----|-----|
| \( A_1 \)   | 0.0928 | 0.1261 |
| \( A_2 \)   | 0.1261 | 0.0928 |

*Ranking order: \( 0 \), \( 1 \)

**Step 2.** Set the qualified reference value of attribute to form a new decision matrix by considering psychological acceptance.

According to Formula (4), a qualified reference value is set for a specific attribute (or all attribute). A new interval decision matrix \( R' = ([s_{ij}', b_{ij}'])_{m \times n} \) is constructed to consider the psychological acceptance of decision-makers.

**Step 3.** Determine positive ideal and negative ideal.

According to Formulae (6) and (7), compute positive ideal \( A^+ \) and negative ideal \( A^- \).

**Step 4.** Computing weight.

According to the article [43], the information entropy of the original attribute value is calculated to form the objective weight.

1. For each evaluation of the original data of SVNN calculation information measure is

\[
E(a_{ij}) = \frac{1}{3(\sqrt{2} - 1)} \sum_{i=1}^{m} \left( \sin \frac{a_{ij}^c + 1}{4} - \cos \frac{a_{ij}^c + 1}{4} \right)
\]

where \( a_{ij} = (T_{a_{ij}}, I_{a_{ij}}, F_{a_{ij}}) = (a_{ij}^e, a_{ij}^b, a_{ij}^r), i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \) and \( t = 1, 2, 3 \). From Formula (2), we have \( (a_{ij}^e)^c = a_{ij}^e, (a_{ij}^b)^c = 1 - a_{ij}^b, (a_{ij}^r)^c = a_{ij}^r \).

2. Establish optimization model

\[
\min E(w) = \sum_{i=1}^{m} E(K_i)w_j^2
\]

where \( \sum_{j=1}^{n} w_j = 1, w_j \geq 0, \) and \( E(K_i) = \frac{1}{m} \sum_{j=1}^{m} E(a_{ij}), j = 1, 2, \ldots, n. \)

3. The weight of each attribute is obtained by Lagrange multiplication.
Step 5. Compute the comprehensive value $C_i$ of alternatives from Formula (8)∼(9) and sort them according to their values.

Step 6. Adjust the attitudinal parameter to determine the evaluation results. Assign attitudinal parameter $\alpha$ to a value of $[0, 1]$ to strength or weaken the conservative decision preferences.

Let $m$ be the number of alternatives, and $n$ be the number of attributes of an alternative. Since the time complexity of computing an $m \times n$ matrix and ordering $m$ elements are $O(mn)$ and $O(m \log m)$ respectively, the time complexity of our proposed algorithm is $O(\max(m \log m, mn))$. In practical applications, decision makers can determine the time costs from the sizes of alternatives and attributes.

4 Case study

In this section, we use two examples to illustrate the advantages of Combined Projection Method (CPM).

4.1 Rating city credit

To improve the social trust in China, the Chinese government has been vigorously promoting the social credit system recently. As to how to evaluate the degree of credit of each city, the Chinese government proposes to focus on four attributes: governmental credit ($K_1$), commercial credit ($K_2$), social credit ($K_3$), and judicial credit ($K_4$):

1. $K_1$: Governmental credit measures if the local government keeps the promise or not;
2. $K_2$: Commercial credit measures if a local enterprise has many bad reviews or perceptions;
3. $K_3$: Social credit measures if the local community members are honest or not;
4. $K_4$: Judicial credit measures if local justice is fair or not.

In the above, $K_2$ is a cost attribute and the rest are the benefit attributes.

We conducted a questionnaire survey for four cities (A,B,C,D) in Zhejiang province. The answers to each question have three choices: yes, no, and unsure. Therefore, each answer on each attribute is a neutrosophic set. For example, the evaluation result on the government credit of City A is expressed as eval $\left( A, K_1 \right) = (T_{A1}, I_{A1}, F_{A1}) = (0.82, 0.63, 0.20)$, indicating that there is 82% positive/true responding, 63% unsure/hesitate answer, and 20% negative responding. The evaluation results of the four cities are shown in Table 5. Based on this table, we will rank these four cities in six steps as mentioned in previous section in the following.

| Table 5 | Original evaluations data of four attributes and four cities |
|---------|-------------------------------------------------------------|
| City  | $K_1$ | $K_2$ | $K_3$ | $K_4$ |
| A     | (0.82, 0.63, 0.20) | (0.65, 0.51, 0.62) | (0.67, 0.43, 0.24) | (0.89, 0.26, 0.29) |
| B     | (0.76, 0.62, 0.29) | (0.55, 0.48, 0.21) | (0.58, 0.44, 0.24) | (0.79, 0.45, 0.26) |
| C     | (0.81, 0.45, 0.36) | (0.77, 0.64, 0.46) | (0.48, 0.53, 0.58) | (0.83, 0.37, 0.14) |
| D     | (0.87, 0.58, 0.28) | (0.61, 0.54, 0.65) | (0.68, 0.44, 0.21) | (0.92, 0.14, 0.29) |

| Table 6 | The normalized neutrosophic decision matrix of four cities |
|---------|-------------------------------------------------------------|
| City  | $K_1$ | $K_2$ | $K_3$ | $K_4$ |
| A     | (0.82, 0.63, 0.20) | (0.62, 0.49, 0.65) | (0.67, 0.43, 0.24) | (0.89, 0.26, 0.29) |
| B     | (0.76, 0.62, 0.29) | (0.21, 0.52, 0.55) | (0.58, 0.44, 0.24) | (0.79, 0.45, 0.26) |
| C     | (0.81, 0.45, 0.36) | (0.46, 0.36, 0.77) | (0.48, 0.53, 0.58) | (0.83, 0.37, 0.14) |
| D     | (0.87, 0.58, 0.28) | (0.65, 0.46, 0.61) | (0.68, 0.44, 0.21) | (0.92, 0.14, 0.29) |

| Table 7 | Interval decision matrix of four cities |
|---------|-------------------------------------------------------------|
| City  | $K_1$ | $K_2$ | $K_3$ | $K_4$ |
| A     | [0.4970, 0.8788] | [0.3523, 0.6307] | [0.5000, 0.8209] | [0.6181, 0.7986] |
| B     | [0.4551, 0.8263] | [0.1641, 0.5703] | [0.4603, 0.8095] | [0.5267, 0.8267] |
| C     | [0.5000, 0.7778] | [0.2893, 0.5157] | [0.3019, 0.6352] | [0.6194, 0.8955] |
| D     | [0.5029, 0.8382] | [0.3779, 0.6453] | [0.5113, 0.8421] | [0.6815, 0.7852] |

\[
w_j = \frac{(E(K_j))^{-1}}{\sum_{i=1}^{m} (E(K_i))^{-1}} \quad (j = 1, 2, \cdots n).\]
Step 1. Since $K_2$ is a cost attribute, it needs to be converted into benefit attribute (as depicted in bold in Table 6).

Step 2. With Formula (3), each neutrosophic set is then transformed into interval set as shown in Table 7.

Step 3. According to the experts’ opinions, among the four attributes from $K_1$ to $K_4$, the local government credit $K_1$ is considered as the most important attribute in ranking and should have a minimum requirement on the alternatives (cities). In this example, we use the mean value of the low-bound alternatives at $K_1(0.48875 = 0.4970+0.4551+0.5000+0.5029)\text{ as the qualified reference value.}$ We would adjust each low-bound alternatives of $K_1$ to be 0.0000 (as depicted in bold in Table 8) if it is less than the qualified reference value (i.e., the mean value).

Step 4. After the positive and negative ideal solutions are determined by using Formula (6) and (7), the weights of the four attributes are calculated and we can get the weight matrix: $(0.2584, 0.2127, 0.2250, 0.3039)^T$.

Step 5. Based on Formula (8) and Formula (9), we calculate $C_i$ of four cities (i=A, B, C, D) under $\alpha = 0, \alpha = 0.5, \alpha = 1$ respectively.

Step 6. Comparing $C_i$, we rank four cities as $D > A > C > B$ under CPM as depicted in Table 9.

The ranking results of CPM and VPM are listed in Table 9. We can see that the rank order of CPM is $A > D > C > B$, where the VPM value of CityD is ahead of that of CityA with only a very small margin. The rank order of CPM is always $D > A > C > B$ on three different $\alpha$ values. These two orders are only different in the order of CityA and CityD.

Actually, the rationality of CPM for ordering CityA and CityD can also be explained as follows:

For $K_1$ in Table 6, we have

\[
\begin{align*}
eval(A, K_1) &= (T_{A1}, I_{A1}, F_{A1}) = (0.82, 0.63, 0.20), \\
eval(D, K_1) &= (T_{D1}, I_{D1}, F_{D1}) = (0.87, 0.58, 0.28).
\end{align*}
\]

Since both $T_{D1} > T_{A1}$ and $(I_{D1} + F_{D1}) > (I_{A1} + F_{A1})$ hold, it can perceive neither CityD>CityA nor CityA>CityD. Similarly, for $K_2$ in Table 6

\[
\begin{align*}
eval(A, K_2) &= (T_{A2}, I_{A2}, F_{A2}) = (0.62, 0.49, 0.65), \\
eval(D, K_2) &= (T_{D2}, I_{D2}, F_{D2}) = (0.65, 0.46, 0.61).
\end{align*}
\]

We have $T_{D2} > T_{A2}$, $I_{D2} < I_{A2}$, and $F_{D2} < F_{A2}$ implying that CityD has a higher good review, a lower unsure review, and a lower bad review than that of CityA. Therefore, CityD>CityA holds in this situation.

For $K_3$, we can have $T_{D3} > T_{A3}$ and $(I_{D3} + F_{D3}) < (I_{A3} + F_{A3})$. This implies that CityD has a higher good review than that of CityA and the uncertainty review and bad review for CityA exceed those for CityD. These facts implicate that CityD>CityA for conservative decision makers.

For $K_4$, we can have $T_{D4} > T_{A4}$, $I_{D4} < I_{A4}$, and $F_{D4} = F_{A4}$ implying that CityD has a higher good review and a lower unsure review than that of CityA. Thus, CityD>CityA for $K_4$.

Observed from Table 7, the evaluated fluctuational intervals of $K_1$ and $K_2$ for CityD are both included by those for CityA. Again, the interval values of $K_2$ and $K_3$ for CityD are significantly higher than those for CityA. Thus, it is reasonable to conclude that CityD>CityA when the projection lengths are very close. For the rank of CityB and CityC, we can see that the ranking order of these two cities remains the same in both CPM and VPM. Because Relu function limits $K_1$, this leads to a large difference in the combined value ($C_i$) of CityB from the other cities. Thus, CityB always ranks as the worst for $\alpha = 0$, $\alpha = 0.5$ and $\alpha = 1$. This fact implies that CPM and VPM would output the same ranking results if the projection lengths are sufficiently different. When the projection lengths are very close, CPM can select favorable ranking through using risk attitudinal parameter $0 \leq \alpha \leq 1$ and subsequently find the most reasonable one for conservative decision makers. This indicates that CPM is stable, consistent with and extended of VPM.

### Table 8 Interval decision matrix with qualified reference value

| City | $K_1$       | $K_2$       | $K_3$       | $K_4$       |
|------|-------------|-------------|-------------|-------------|
| A    | [0.4970, 0.8788] | [0.3523, 0.6307] | [0.5000, 0.8209] | [0.6181, 0.7986] |
| B    | [0.0000, 0.8263] | [0.1641, 0.5703] | [0.4603, 0.8095] | [0.5267, 0.8267] |
| C    | [0.5000, 0.7778] | [0.2893, 0.5157] | [0.3019, 0.6352] | [0.6194, 0.8955] |
| D    | [0.5029, 0.8382] | [0.3779, 0.6453] | [0.5113, 0.8421] | [0.6815, 0.7852] |

*Ranking order: ①, ②, ③, ④*
4.2 Selecting enterprise alliance

To be successful in an innovative project, an enterprise needs to seek a strategic alliance to alleviate innovation risks and synthesize its technical capacity. Suppose that experts need to evaluate the three potential enterprises \( \{ \text{E}_1, \text{E}_2, \text{E}_3 \} \) in terms of innovation resources \( (K_1) \), market capacity \( (K_2) \), enterprise strength \( (K_3) \), financial expectation \( (K_4) \) and innovation risk \( (K_5) \). Here, innovation risk \( (K_5) \) is a cost attribute and needs to be minimized and required normalized.

Table 10 is the original evaluation results from experts. Following the previously mentioned Step 1 and Step 2, we have the normalized and transformed results as depicted in Tables 11 and 12 respectively.

In Step 3, since the innovation resources \( (K_1) \) dominate parties in the alliance, the certainty of the evaluation is expected to exceed 30%. Therefore, \( K_1 \) attribute of \( \text{E}_2 \) is filtered with Relu function as shown in Table 13.

Through Step 4∼Step 6, weights and comprehensive attribute values under different parameters are calculated and the final results are shown in Table 14.

In Table 14, \( \text{E}_2 \prec \text{E}_3 \prec \text{E}_1 \) holds for both VPM and CPM when \( \alpha = 0 \) when the conservative psychology of decision-makers is emphasized \( (\alpha \neq 0) \), the ordering of
and $E_3$ varies. However, in all situations, $E_2$ always remains as the most unfavorable one in orderings. Again, through observing the interval in Table 13, it can be seen that among the five attribute values of $K_1 \sim K_5$, the interval values of $K_1$, $K_3$ and $K_5$ of $E_3$ are significantly better than those of $E_1$. Therefore, it is more reasonable to rank $E_3$ better than $E_1$ in terms of risk avoidance psychology.

In Summary, the rationality and robustness of CPM have been verified in two examples with different data sets. Table 15 and Fig. 3 show the various effects of different projection angles and $\alpha$ values. Through analyzing the results on diverse data sets, the rationality, stability and interpretability of our proposed method has been confirmed. When the projection angles are between 0 and 90 degree, the comprehensive values decrease as $\alpha$ value goes up. This is used to measure the degrees of conservativeness.

### 5 Conclusions

In this paper, we proposed the Combined Projection Method (CPM) for decision making.

This new method can further distinguish risk preferences which are hard discriminative by the currently existing projection methods. It determines the order of alternatives with measured psychological intension by integrally considering projection length, angle and attitudinal parameter. Computationally, it is much simpler than that of the Bidirectional Projection Method (BPM). The proposed method convert a neutrosophic set ($T, I, F$) into an interval that well represents and interpreter the internal fluctuations of the attributes. The use of Relu function to constrain the lower bound of the interval can greatly support the requirements of conservative decision makers for certainty.

Although CPM has the above advantages over the VPM and BPM in conservative decision making, there are still some improvements that can be made. The proposed model uses the projection angle as the decision makers’ risk preferences. It is not very clear how to use the angle combined with other fuzzy expressions. This needs to be explored in our further research work.

### Appendix

In this Appendix, we compared PM, BPM, VPM and CPM methods by using the example in Section 3.4 with additional alternative $A_3$ as shown in Table 16.

Table 17 depicts the ranking results of $A_1$, $A_2$ and $A_3$ under PM, BPM, VPM and CPM. It can see that CPM and VPM have the most similar ranking results where $A_3$ is always ranked 1st. As described in Section 3.4, $A_1 > A_2$ in CPM but they are incomparable in VPM.

$A_1$ and $A_3$ are ranked differently in CPM and BPM. Observing these two alternatives on attribute $K_1$, we can see that their positive evaluation values are very close but the non-negative (positive + hesitant) evaluation value of $A_3$ is much higher than that of $A_1$ and the negative value of $A_3$ is lower than that of $A_1$. Similarly, for attribute $K_2$ of alternatives $A_1$ and $A_3$, their negative evaluation values are very close. The positive value of $A_3$ is higher than that of $A_1$ and the non-negative (positive + hesitant) evaluation value of $A_3$ is much higher than that of $A_1$. Therefore, the ranking $A_3 > A_1$ of CPM is reasonable.

In this example, we still select 10% as the qualifying reference value and filter the attribute values by with the Relu function. The filtered attribute intervals by the Relu function (depicted in bold) are shown in Table 18. Table 19 lists
the ranking results. Obviously, $A_2$ is the worst under BPM, VPM and CPM.

In conclusion, since the alternative projection used in PM differs from that in BPM, VPM and CPM, the obtained ranking results can be very different to those of BPM, VPM and CPM. Nevertheless, the ranks of BPM are determined through measuring the closeness of two projections, which can be difficult to interpret cognitively. Our proposed CPM algorithm is structurally simple and interpretable.

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**Table 16** The data of $A_1$, $A_2$, $A_3$, $Z(A_1)$, $Z(A_2)$ and $Z(A_3)$

| $A_i$ | $K_1$ | $K_2$ | $Z(A_i)$ | $K_1$ | $K_2$ |
|-------|-------|-------|----------|-------|-------|
| $A_1$ | (0.22, 0.60, 0.24) | (0.31, 0.21, 0.22) | $Z(A_1)$ | [0.2075, 0.7736] | [0.4189, 0.7027] |
| $A_2$ | (0.07, 0.93, 0.08) | (0.40, 0.49, 0.28) | $Z(A_2)$ | [0.0648, 0.9259] | [0.3419, 0.7607] |
| $A_3$ | (0.21, 0.83, 0.17) | (0.35, 0.35, 0.23) | $Z(A_3)$ | [0.1736, 0.8595] | [0.3763, 0.7527] |

**Table 17** The rankings under PM, BPM, VPM and CPM

| Alternative | PM | BPM | VPM | CPM |
|-------------|-----|-----|-----|-----|
| $A_1$ | 0.8078 ③ | 0.4149 ③ | 0.0851 ② | 0.0851 ② |
| $A_2$ | 0.8755 ② | 0.4136 ② | 0.0851 ② | 0.0851 ② |
| $A_3$ | 0.8587 ③ | 0.4101 ③ | 0.0575 ① | 0.1066 ① |

* Ranking order: ①, ②, ③

**Table 18** Filtered interval values from the Relu function

| $Z(A_i)$ | $K_1$ | $K_2$ | $K_1(Filtered)$ | $K_2(Filtered)$ |
|----------|-------|-------|-----------------|-----------------|
| $Z(A_1)$ | [0.2075, 0.7736] | [0.4189, 0.7027] | [0.2075, 0.7736] | [0.4189, 0.7027] |
| $Z(A_2)$ | [0.0648, 0.9259] | [0.3419, 0.7607] | [0.0648, 0.9259] | [0.3419, 0.7607] |
| $Z(A_3)$ | [0.1736, 0.8595] | [0.3763, 0.7527] | [0.1736, 0.8595] | [0.3763, 0.7527] |

**Table 19** Sorting under CPM using the Relu function

| Alternative | PM | BPM | VPM | CPM |
|-------------|-----|-----|-----|-----|
| $A_1$ | 0.8078 ③ | 0.4461 ② | 0.0928 ② | 0.1261 ② |
| $A_2$ | 0.8681 ③ | 0.3722 ③ | 0.1261 ③ | 0.1261 ③ |
| $A_3$ | 0.8587 ② | 0.4317 ② | 0.0584 ① | 0.1414 ① |

* Ranking order: ①, ②, ③
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