Squeeze film characteristics in synovial hip joint

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Abstract. In this paper general classification of the lubrication regimes in synovial hip joint was given. The discussion was focused on two main points, the calculation of film thickness and minimum film thickness for each of the (hydrodynamic lubrication, squeeze film lubrication and elasto-hydrodynamic lubrication), and the relationship between film thickness and gait cycle (stance phase and swing phase) also and the coefficient of friction. Physical properties affecting the film thickness will be discussed in each lubrication system.

Key word: Synovial hip joint, hydrodynamic lubrication, gait cycle.

1 Introduction:

The hip joint is one of the most important joints in the human body, it allows us to perform many of the movements as jumping, running and walking so it bears our body weight and the hip joint is also one of our most flexible joints and allows a greater range of motion (Flexion Extension, Abduction, Adduction) than all other joints. Lubrication synovial hip joint is a necessary process in order to reduce friction and wear between the surfaces of cartilage during daily activities, lubrication process depended on synovial fluid [1]. It has three main functions: lubrication, absorption and nutrition of the cartilage of the joint. In a healthy hip joint synovial fluid appear non-Newtonian, result in the relationship between viscosity and shear rate [2]. Lubrication mechanisms provide a protective film which allows for two surfaces to be separated and smoothed. In the absence of lubrication, surface would connect each one with another, thus would get damaged in the joint. This lubrication mechanisms differ by gait cycle (stance phases - swing phase). Also it is lubrication mechanisms adopted in the lubrication of joints in general and hip in particular to fluid film include (hydrodynamic, squeeze-EHL-weeping)boundary lubrication and mixed lubrication. In hydrodynamic lubrication a fluid film in was developed by the motion of the lubricant between the surface articular cartilages. The frictional forces in hydrodynamic lubrication affected by speed motion and lubricant viscosity. Squeeze lubrication describes the pressure generated by the flow of liquid oiler between the head of the fume and the acetabulum that impact on the viability of the joint to bear weights double as a result of the movement also describes the time compression of the liquid and out of the gap it and is one of the most important types of lubrication for being directly linked.
to stance phase Jing. L. [5] proposed an explanation for the synovial joints based upon a type of lubrication known to engineers as "hydrodynamic" Maroudas A.[6] was the first to propose an entirely new concept of lubrication "weeping lubrication" applied to synovial joint action, he considered unique and special properties of cartilage and how this could affect flow and lubrication. As pointed with elastic properties of synovial fluid and articular cartilage, as long as the synovial fluid is normal and as long as the articular cartilage remains elastic, the articular surfaces are never extremely close to each other and, consequently, the boundary lubrication does not take place. However, if with pathological changes in synovial fluid, a boundary-type of lubrication may exist. He found the similarity, and suggested that EHL was responsible for the lubrication of human joints, which was supported by later studies and researchers. Radin E.L, el at [8] clear that a low coefficient of friction can also be achieved without a fluid film through a mechanism known as boundary lubrication. In this case, molecules adhered to the surfaces were shared rather than a fluid film. It now appears that a combination of boundary lubrication (at low loads) and fluid film lubrication (at high loads) is responsible for the low friction in synovial joints. Mow. V [7] found EHL action is unlikely to occur in the knee or hip joints during the weight-bearing phase in walking. On the other hand, during periods of sliding under low load, as in an unloaded swing of the leg, a substantial fluid film is likely to be generated between the surfaces. Dowson D. [3] found that under physiological walking condition, as experienced in hip joints, the film thickness was slightly reduced, but the elasticity was not significantly affected. In other words, articular cartilage can be treated as a single phase material for the purpose of elasticity analysis. This assumption adopted in the EHL lubrication analysis of synovial joints by Dowson and his colleagues Albert. E [1] studied the hydrodynamic squeeze film lubrication of the human ankle joint, by modeling the joint by a partial porous journal bearing lubricated with a non-Newtonian couple stress fluid. Under squeeze film lubrication, the governing equations were solved numerically and showed increase in pressure, load capacity, and friction factor with a decrease in the time of approach.

2. Main characteristics of lubrication models:

Theoretical modeling of hip joint lubrication has been extensively investigated in the literature in the last years. In order to better classify and compare the main studies, a preliminary description of the major features of the problem and of the models proposed according to [Tawer (1966) and Dowson (1967)] hip joint can be represented by a solid sphere approaching a flat plate in squeeze film action.

2.1 Geometry [3]:

Hip joint consists of the femoral bone and acetabulum expressed in many sources with a ball-in-socket. Both of them have a modulus of elasticity (E), Poisson ratio (v) and radius (R). It is convenient to reduce the contact between two spheres to an equivalent contact of a sphere on a plane by simply defining the effective radius of contact R, and the effective elastic modulus E as:

\[
\frac{1}{R} = \frac{1}{R_1} \pm \frac{1}{R_2} \quad \frac{1}{E} = \frac{1}{2} \left[ \frac{(1 - v_1^2)}{E_1} + \frac{(1 - v_2^2)}{E_2} \right]
\]  

(1)
Where $E_1$, $v_1$ and $E_2$, $v_2$ are the Young’s modulus and Poisson ratio of the head and cup material, respectively. The (-ve) sign refers to internal contact, and the (+ve) sign refers to external contacts.

2.2 Elastic deformation [3]:

Even in a standard activity as walking, the articular cartilage surfaces of the implant were subjected to rather high levels of pressure that produces local deformation, which cannot be disregarded, both for soft-on-hard and hard-on-hard material couples. The entity of such deformations can be approximated applied Hertzian theory for a static dry contact; in particular the contact radius $a$ is defined as

$$a^3 = \frac{3WR}{4E}$$

(2)

3. Squeeze film characteristics

We will introduce squeeze film characteristics that depended on studying lubrication synovial hip joint.

(3.1) A sphere Approaching a Plane [4]:

In order to analyze the squeeze lubrication, Reynolds equation in polar coordinates may be written as:-

$$\frac{\partial}{\partial r}(r^2 \frac{\partial p}{\partial r}) + \frac{1}{r} \frac{\partial}{\partial \theta}(h^3 \frac{\partial p}{\partial \theta}) = 12 \eta \frac{\partial h}{\partial t}$$

(3)

Poisuille flow expresses the relationship between the rate of flow of a lubricant in a film and the pressure gradient in the film, $r$ the radial, $h$ is a function describing the separation between two interacting bodies (film
thickness) ; \( \eta \) is viscosity ; \( t \) is time approach spherical to plate. Because the problem is of polar symmetry ,that is \( \frac{\partial p}{\partial \theta} = 0 \) thus the equation (3) reduced to:-

\[
\frac{d}{dr} \left( rh^3 \frac{dp}{dr} \right) = 12\eta r \frac{dh}{dt}
\]

(4)

\[
\frac{dp}{dr} = \frac{6\eta r}{h^3} \frac{dh}{dt} + \frac{A}{rh^3}
\]

(5)

Where \( A \) is the integration constant. The boundary conditions for the fluid film pressure and radial in the human gap joint are:-

\[
\frac{dp}{dr} = 0 \quad \text{at} \quad r = 0
\]

(6)

Compensation boundary condition in the equation (5), hence we obtain the integration constant \( A \) (7) nd to compensate the value \( A \) in equation (7) we get the following expression

\[
\frac{dp}{dr} = \frac{6\eta r}{h^3} \frac{dh}{dt}
\]

The film thickness is \( h = h_o + R - (R^2 - r^2)^{1/2} \)

is radius \( R \) is minimum film thickness and \( h_o \) Where

\[
h_o + R - h = (R^2 - r^2)^{1/2}
\]

(8)

Now by differentiating Eq.(8) with respect \( r \) to obtain :

\[
\frac{dh}{dr} = \frac{r}{(R^2 - r^2)^{1/2}}, R \neq r
\]

(9)

\[
\frac{dh}{dr} = \frac{r}{h_o + R - h}
\]

(10)

\[(h_o + R - h) \, dh = r \, dr\]

\[
p = 6\eta \frac{dh}{dt} \int \frac{r}{h^3} \, dr + B
\]

(11)

\[
p = 6\eta \frac{dh}{dt} \left( \frac{h_o + R}{h^3} - \frac{h}{h^3} \right) dh + B
\]

(12)

Now we integrate Eq.(12) with respect to \( h \), pressure of the synovial fluid in the general form is found:
\[ p = 6\eta \frac{dh}{dt} \left[ -\frac{(h_o + R)}{2h^2} + \frac{1}{h} \right] + B \]  

\[ p = 3\eta \frac{dh}{dt} \left[ \frac{2h - h_o - R}{h^2} \right] + B \]  

(13)

We impose the boundary condition \( p = 0 \) where \( h = h_o + R \). Value of the constant been

\[ 0 = 3\eta \frac{dh}{dt} \left[ \frac{2h_o + 2R - h_o - R}{(h_o + R)^2} \right] + B = 3\eta \frac{dh}{dt} \left[ \frac{h_o + R}{(h_o + R)^2} \right] + B = 3\eta \frac{dh}{dt} \left[ \frac{1}{h_o + R} \right] + B \]  

(14)

\[ B = -3\eta \frac{dh}{dt} \frac{1}{h_o + R} \]  

(15)

When substituting the value of constant \( B \) in the equation (14) the pressure equation becomes:

\[ 6\eta \frac{dh}{dt} \int \left( \frac{h_o + \frac{\eta}{h}}{h_o + R} \right) dh - 3\eta \frac{dh}{dt} \frac{1}{h_o + h} \]  

(16)

(3.2) Load Capacity:

When the gap between the surfaces is filled with viscous lubricant, then positive pressure would be generated due to squeezing out of the lubricant from the gap and thus load carrying capacity would be generated.

\[ W = \int_0^R 2\pi r p \ dr \]  

(17)

\[ W = \frac{-6\pi\eta}{(h_o + R)} \int_0^R \left(1 - \frac{(h_o + R)^2}{h^2} \right) r \ dr \]  

(18)

From Eq (12)

\[ W = \frac{-6\pi\eta}{(h_o + R)} \int_{h_o}^{R+h_o} (h_o + R - h) \left(1 - \frac{(h_o + R)^2}{h^2} \right) dh \]  

(19)

\[ W = \frac{-6\pi\eta}{(h_o + R)} \int_{h_o}^{R+h_o} (h_o + R - h)(h_o + R - h) \left(1 - \frac{(h_o + R)^2}{h^2} \right) dh \]  

(20)

By integrating Eq (20) w. r. t. \( h \) we obtain the load carrying capacity in general from:
\[ W = 3\eta \pi h \frac{dh}{dt} \left( \frac{R}{h} \right)^2 \]

(3.3) Time of Approach:

The time that elapsed for a lubricating film thickness to be reduced to some minimum value, we use the equation given below.

\[ \int dt = \frac{3\eta \pi R^2}{W} \int_{h_2}^{h_1} \frac{dh}{h} \]

After integrating equation (1.23) with respect to variable \( h \) thus the time of approach for spherical to plane in general form will be found

\[ t = \frac{3\eta \pi R^2}{W} \ln \left( \frac{h_{01}}{h_{02}} \right) \]  \hspace{1cm} (23)

4. Film thickness Calculation:

The film thickness that separates the surfaces from each other changes thickness during the various events practiced by humans in (stance phase and swing phase). In this section, theories of hydrodynamic and Elastohydrodynamic lubrication all include the hydrodynamic factors \( (\eta, U, W) \), while in squeezes lubrication the lubricating film carries most of the load. The squeeze film time plays a major role in changing the magnitude of film thickness. In an attempt to evaluate the film thickness of synovial fluid using the various theories in the fluid film lubrication. The lubrication film thickness will be estimated for each type of lubrication mechanism using the numerical factors, seen in the table (1.1). There is a relationship between film thickness and gait cycle during swing phase where foot doesn’t contact with ground no weight occurs consequently there will be no pressure occur in the synovial film, thus the ball and cup will be separated. In this situation the lubrication film thickness will be greater than the surface roughness of the cartilage. In stance phase, occur developing where foot contact with ground bears the weight on synovial hip joint, thus causing pressure between ball and cup. And surfaces of the articular cartilage start to approach each other. Therefore the film thickness and hence the minimum film thickness may be lower than roughness

| Parameters                        | Symbols | Numerical values | Units   |
|-----------------------------------|---------|-----------------|---------|
| Effective modulus of Elasticity   | \( E \) | \( 10^7 - 10^9 \) | N/m²    |
| Effective radius of curvature     | \( R \) | 0.1-1           | m²      |
| Load                              | \( W \) | 4500            | N       |
4.1 Film thickness in hydrodynamic lubrication:

In isoviscous- rigid fluid film lubrication regime the magnitude of the elastic deformation of the surfaces is an insignificant part of the thickness of the fluid film separating them, and the maximum pressure in the contact is too low to increase fluid viscosity significantly. There are a number of equations for predicting lubricant film thickness in the regime (H.L.) for both line and elliptical contacts. Perhaps the best known of these is the Kapitsa equation for film thickness in elliptical contacts\[6\] :

\[
W = \frac{6 \pi U \eta}{5} \sqrt{\frac{2R^3}{h_0}} = 5.332 U \eta R^{3/2} h_0^{-1/2} \tag{24}
\]

when the assumed for a sphere with \(R_x = R_y\) and \(R = R_x / R_y = 1\) values for the above parameter are used it given \(h=3.16\mu m\). This again nearly equals the value of surface roughness of the articular cartilage in table (1.1).

4.2 Hydrodynamic Film thickness- characteristics:

The magnitude of full film thickness in hydrodynamic lubrication depends on some parameters such as viscosity, sliding speed relative radius of curvature and load. By using equation (24) that describe relationship load (\(W\)), speed (\(U\)) and viscosity (\(\eta\)) the results were analyzed in following figures. Figure (2) represents the effect of loads applied on joint during the swing phase for variance speeds chosen from chapter two (gait cycle), while loads were chosen from table (1). As seen the magnitude of the applied on the joint has a profound effect on the value of film thickness i.e when the load is low the value of film thickness high and vice versa. Figure (3) represents the pressure distribution in synovial joint operating under hydrodynamic lubrication conditions, the range of speed during the (stance and swing) phase of the walking cycle was obtained from chapter two. Figure (4) represents the effective viscosity of synovial fluid on film thickness. As seen in Figure (3) and (4) value of film thickness affected extrusive with velocity and viscosity when low velocity and viscosity value of film thickness is low and the vice versa.
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(4.3) Film thickness in squeeze film lubrication:

The approaching bearing material tends to squeeze synovial fluid where high viscous then out the synovial fluid of the gap. In squeeze film analysis, two basic parameters are usually required: load carrying capacity and time of approach. If it reconsider the approach of the sphere near the plan in that the radius of the Hertzian contact is given film thickness and the dimensionless film thickness formula [9].

\[
T = \frac{3\pi \eta a^4}{4Wh^2} \quad (25)
\]

\[
H = \frac{2.011 W^{1/6}}{T^{1/2}} \quad (26)
\]

Where \( T \) is dimensionless time, \( W \) dimensionless load and \( H \) dimensional film thickness. Using the assumed values for the parameter the film thickness is found to be about 3.5 \( \mu m \) at the start of stance phase. This implies initial contact so that time (0.05) seconds and continuity loading a reduce at a film thickness to about 0.238 \( \mu m \).
Results with increasing time of approach (ball to socket) after 10 seconds. Again, using the assumed values for the parameters, the dimensional film thickness becomes about 3.1 µm. At the start of stance phase this implies initial contact and continuity, load a reduction in dimensional film thickness to about 0.297 µm with increasing time, approach after 10 seconds. In both cases, the value of film thickness is found to be much larger than the value of surface roughness of the articular cartilage.

(4.4) Squeeze Film thickness – characteristics:

The hydrodynamic squeeze plays an essential role in the load carry capacity and time of approach, each of them major role in changing the value of film thickness as see in figures (1.5) and (1.6) using equation (25) and (26). Through daily of activities human (walking-running,...,Etc.), we find that the weight doubles several times after heel strike and before foot flat. Which negative affects film thickness in side and increasing the time approach of the ball on a plane on the other side.

(4.5) Film Thickness in elato-hydrodynamic film lubrication:

In isoviscous-elastic or soft Elasto-hydrodynamic (I-EHL) the lubricant thickness is independent of the lubricant pressure-viscosity characteristics but much more strongly dependent on the elastic properties of the contacting surfaces than for conventional (EHL). There are a number of regression equations for predicting lubricant film thickness in the (I-EHL) regime for both line and elliptical contacts. Perhaps the best known of these are the Hamrock and Dowson equation for film thickness in elliptical contact.
\[ H = \frac{h_{\text{min}}}{R} = 7.43(1 - 0.85e^{-0.3\frac{R}{R_s}}) U^{0.65} W^{-0.21} \quad (27) \]

Where

\[ U = \frac{\eta u}{ER_s} = \frac{2 \times 10^{-3} \times 0.075}{10^7} = 1.5 \times 10^{-11} \quad (28) \]
\[ W = \frac{w}{E(R_s)^2} = \frac{4500}{10^7} = 4.5 \times 10^{-4} \]

Substituting the value dimensionless viscosity and dimensionless load in Eq. (27) it was obtained the following value:

\[ h_{\text{min}} = 1.3006 \mu m \quad (29) \]

Where \( h_{\text{min}} \) minimum film thickness typical elastohydrodynamic film thicknesses predicted for articular cartilage surface were often in the range [1.3-1.4] \( \mu m \). Thus, it is unlikely that EHL alone could provide full fluid film lubrication. It found that the value of film thickness, is very close to the value of surface roughness of the articular cartilage. Figure (8) represents the effective viscosity of synovial fluid on film thickness during stance phase of the walking cycle. The variation in loads with film thickness for different values of speed is depicted in Figure (9). Finally figure (10) represents the effect radius of curvature on film thickness. It seems clear from the foregoing figures to that the increase in each factor (velocity, viscosity and radius of curvature) leads to an increase in film thickness. The percentage rate of increase in film thickness in figure (1.8) is 80% while percentage rate of increase in film thickness in figure (9) is 90%. Finally, in figure (10) the percentage rate of increase in film thickness is 93%.
5. Relationship lubrication with gait cycle:

The present study showed that in the two phases of the gait cycle during normal walking the stance and swing phase during which it is added force variation on the synovial human hip joint that varies from zero to several times the body weight. General as hydrodynamic pressure, creating a lubrication system either fluid film or boundary, that reduces friction and to facilitate mobility of the hip joint the relation between gait cycle and lubricant see in figure (11). It is studying the types of lubrication regimes in the hip joint during walking a great deal of importance. It also covers all related parameters affecting the gait cycle.

Consider the following experiment hypothetical mechanical spring bearing which represents film thickness inserted between ball and cup, spring load magnitude is different in the stance phase and swing phase therefore different lubrication mechanism.

(5.1) Heel strike in stance phase:

1-The weight of the body is applied when the heel strikes the ground see in figure ((a) 12).

2-Some of the lubricant is squeezed from between the surfaces. At the same time, the femoral head bearings starts rotating relative to the acetabular cup.
3- The external load becomes greater than the spring load.

4-This relative movement establishes squeeze lubrication. The fluid synovial fully or partially separates the surface articular cartilage.

(5.2) Mid stance and Toe off in stance phase

1- In mid-stance and toe off note that the speed sliding decrease so the film thickness and the surface roughness are high, see figure ((b) 12).

2- The load (body weight) increasing the result a squeeze-film situation may develop, leading to elasto-hydrodynamic lubrication and possibly both squeeze-film and boundary lubrication(after two or three gait cycle).

(5.3) Swing phases

1- In swing phase (toe off, mid-swing and terminal), note that foot doesn't touch the ground, it was found (body of weight) equals to spring load.

2- Sliding velocity is high

3- Final ball and cup separated from each other and this is illustrated in Figure ((c) 12)
Conclusions

1. Speed during gait cycle (stance phase - swing phase) lead to increasing flow synovial fluid therefore increasing hydrodynamic pressure inside hip joint.
2. Load spatula on the articular cartilage is effected to low film thickness.
3. Viscosity of synovial fluid is effected to increasing film thickness in hydrodynamic lubrication.
4. Time is divided between (stance phase - swing phase) where to end each phase low film thickness.
5. The gap between articular cartilage is differed during cycle of gait cycle.

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