Traffic Light Planning Based on Distributed Constraint Optimization

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Abstract. Most of traffic light planning algorithms only consider the congestion of single intersection without the interaction between intersections. Therefore, it is difficult to apply to large-scale transportation network. In order to alleviate traffic congestion, the traffic planning problem is modeled as a distributed constrained optimization problem. The model is based on the waiting cost caused by the difference order of green lights in each direction of the intersection. On the simulation platform, complete algorithm and incomplete algorithm are used to solve the model, and the corresponding green light change order of all intersections is obtained when the waiting cost of vehicles is the lowest in the traffic network. The experimental results show that complete algorithm can find the optimal solution to optimize the vehicle travel time in the scene of no count the traffic flow; incomplete algorithm can find the sub-optimal solution in the traffic network with statistical traffic flow to meet the real-time requirements.

1. Introduction

In recent years, with the development of science and technology, traffic intersections in various countries and regions were more and more congested. For the congestion problem, the control of traffic lights is one of the key points.

At present, the research of traffic signal control is mainly divided into two categories: traffic flow detection and signal control. Traffic flow detection is mainly divided into magnetic induction detection[1], wave frequency detection and image detection[2]. Signal control is mainly divided into ZigBee technology control[3], fuzzy control[4] and deep learning[5] and other methods. Above schemes only controls a single intersection without considering adjacent intersections. This will lead to traffic congestion at adjacent intersections. Only by making use of the interaction and cooperation between traffic lights can traffic congestion be effectively alleviated.

The decentralized and interconnected relationship of traffic intersections is similar to the distributed constrained optimization problem (DCOP). DCOP is widely used in the cooperation of multi-agent systems. In DCOP, each agent controls a group of variables and assigns values to the variables to satisfy the constraints and objective functions. It can model many problems in production practice[6-7] based on multi-agent system theory.

In order to use multi intersection coordination to alleviate the waste of money and time caused by traffic congestion. Based on the urban traffic network, the traffic light planning problem is modeled as DCOP. And different types of algorithms are used to simulate the solution for different application
scenarios. The traffic light turn order of each intersection is obtained, and the planning time and cost of different algorithms in different scenarios are given, which verifies the effectiveness of the model.

2. Distributed constrained optimization problem

DCOPs can be divided into symmetric problem and asymmetric problem. In symmetric problem, the cost of variable is the same, while it is different in asymmetric problem. For the traffic network, due to the different green light sequence of connected intersections, the vehicle action is different. So, it is more consistent with the asymmetric problem. The framework of asymmetric distributed constrained optimization problems (ADCOPs) can be represented by four tuples $<A, X, D, F>$:

- $A = \{a_1, ..., a_n\}$ is the collection of agents, an agent can control one or more variables;
- $X = \{x_1, ..., x_m\}$ is the set of variables;
- $D = \{d_1, ..., d_m\}$ is the set of ranges from which variables take values;
- $F = \{f_1, ..., f_q\}$ is the set of constraint cost functions, and constraint $f_i : D_{i_1} \times \cdots \times D_{i_k} \rightarrow R^+ \cup \{0\}$ is the mapping of non-negative cost generated by different combinations of variables. In ADCOPs, the cost of $f_i$ is different for variables.

Generally, the goal of ADCOPs is to assign values to all variables to get the lowest sum of constraint costs among all agents. Namely:

$$X^* = \arg \min_{X \in D} \sum_{f_i \in F} f_i$$ (1)

In ADCOPs, agent connect with others. The structure can be visually represented by constraint graph, as shown in figure 1. In the constraint graph, nodes represent agents, while edges represent constraint relationships among agents. Agents connected by edges are named neighbors.

![Figure 1. An instance of ADCOP](image)

3. Traffic model

3.1. Establishment of traffic light constraint graph

The traffic network structure is composed of intersections that are connected by roads. Traffic lights can control the turning of vehicles and are set in directions. There is a great similarity between the traffic network and the constraint graph of ADCOPs. The traffic intersection is usually divided into three forks and intersections, that is, each intersection has three or four neighbors. As shown in figure 2(a), to convert the traffic network into constraint graph, we can take the traffic intersections as nodes, the roads between intersections as constraint edges, and the intersections connected by roads as neighbors. For boundary nodes $A_1, A_2, A_3, A_5, A_6$, and $A_7$, for the sake of display, some neighbors are hidden. The transformed constraint graph is shown in figure 2(b).
3.2. Change sequence of traffic light
As intersection $A_4$ shown in figure 2, it is an intersection with four directions: east, south, west and north. According to actual, vehicles of one direction can move straight, turn left and turn right. In general, the right-turn traffic light is set as circular and can pass without wait. Therefore, ignoring right-turn traffic lights, vehicles only move straight or left turn.

For ADCOP, the purpose is to plan the order of green light. Straight and left turn can be divided into synchronization (single direction vehicle straight and left turn) and asynchronization (opposite vehicle straight and left turn). Because the left turn traffic flow is usually less than straight, we choose asynchronization. So the operations that can be carried out at the intersection are: north-south straight $o_1$, north-south left turn $o_2$, east-west straight $o_3$, east-west left turn $o_4$.

The above four operations are controlled by traffic lights, which are mutually exclusive. By arranging the above operations in different order, we can get different green light conversion sequence values: $o_1o_2o_3o_4$, $o_1o_3o_2o_4$, ..., $o_4o_3o_2o_1$. For the three fork road $A_3$, we can also get the corresponding operation: north-south straight $o_1$, north-south left turn $o_2$ and east-west left turn $o_4$.

3.3. Optimization objective function
For traffic network, the waiting time of vehicles in all directions can be regarded as the total cost of ADCOP. Solution is the order of green light which makes the total waiting time lowest.

In the actual driving process, vehicles passing through two intersections are: arriving at the intersection $a_i$, waiting for the green light at $a_i$, driving and arriving at the intersection $a_j$, waiting for the green light at $a_j$. In this process, vehicles will have the cost of waiting time at both intersections. The hypotheses are as follows:

Supposed that the straight is $s$ and the turn left is $l$. Set the time stamp of arriving at $a_i$ as $t_0$, the time stamp of $a_i$ to $a_j$ straight green light as $t_{is}$, and the turning left green light as $t_{il}$. So, the cost of $a_i$ is the sum of the waiting time of straight direction and the turn left direction. Namely:

$$cost_i = cost_{is} + cost_{il}$$

$$cost_{is} = t_{is} - t_0$$

$$cost_{il} = t_{il} - t_0$$

Figure 2. Traffic network and constraint graph
Set the normal driving time of the road between $a_i$ and $a_j$ as $t_{ij}$. After arriving $a_j$, set the green time stamp of the latest through driving of vehicles from $a_i$ through direction $a_j$ be $t_{is,ja}$, and the green time stamp of the latest left turn be $t_{is,ja}$; set the green time stamp of the latest through driving of vehicles from $a_i$ through direction $a_j$ be $t_{il,ja}$, and the green time stamp of the latest left turn be $t_{il,ja}$. So, for the $a_j$, the straight or turning costs need to be considered according to different situations to judge whether the green time stamp of the same turn is the same. The cost of $a_j$ is equal to the sum of the costs of the straight and the left. For the straight, there are:

$$cost_{ij} = cost_{is,ja} + cost_{il,ja}$$ (5)

Among them, $cost_{is,ja}$ is the cost of vehicles from straight $a_i$ move straight at $a_j$. $cost_{il,ja}$ is the cost of vehicles from turning left $a_i$ move straight at $a_j$. There are:

$$cost_{is,ja} = t_{is,ja} - \min(t_{is}, t_{il}) - t_{ij}, \text{ while } t_{is,ja} = t_{il,ja}$$ (6)

$$cost_{il,ja} = t_{il,ja} - t_{is} - t_{il} - t_{ij} + t_{is,ja} - t_{ij}, \text{ while } t_{is,ja} \neq t_{il,ja}$$ (7)

In the same way, for the turning left at $a_j$, there are:

$$cost_{jl} = cost_{ls,ja} + cost_{ll,ja}$$ (8)

where $cost_{ls,ja}$ is the cost of vehicles from straight $a_i$ turn left at $a_j$, $cost_{ll,ja}$ is the cost of vehicles from turning left $a_i$ turn left at $a_j$. There are:

$$cost_{ls,ja} = t_{ls,ja} - \min(t_{ls}, t_{ll}) - t_{ij}, \text{ while } t_{ls,ja} = t_{ll,ja}$$ (9)

$$cost_{ll,ja} = t_{ll,ja} - t_{ls} - t_{ll} - t_{ij} + t_{ls,ja} - t_{ij}, \text{ while } t_{ls,ja} \neq t_{ll,ja}$$ (10)

The total cost of $a_j$ is:

$$cost_j = cost_{is} + cost_{il}$$ (11)

To sum up, when $a_i = d_i$ and $a_j = d_j$, the problem’s cost is:

$$cost_y(d_i, d_j) = cost_i + cost_j$$ (12)

### 3.4. ADCOP model of traffic light planning

After the cost is calculated, traffic network can be summarized as a ADCOP which gives different transformation order to all the green lights. According to the definition of ADCOP, it is necessary to determine the four tuples for traffic problems. Combined with the above, the ADCOP model of traffic light planning can be described as follows:

- **Agent**: every intersection can be regarded as an agent, which needs to interact with vehicles;
- **Variables**: traffic lights are set for each direction. A single traffic light controls a single direction. Traffic lights in all directions are used as variables;
- **Domains**: traffic light variables control different turning operations. As shown in the above, the order of green light transformation is taken as: $o_1o_2o_3o_4, o_1o_2o_4o_3, \ldots, o_2o_3o_4o_1$;
- **Optimization objective function**: make the vehicles in the traffic network pass the intersection as soon as possible, and the total waiting time is the shortest. Ideally, vehicles from the starting point to the destination can pass unimpeded without waiting.

ADCOPs algorithms can be divided into complete algorithm and incomplete algorithm according to whether the optimal solution is found. The complete algorithm can find the optimal solution stably.
with a long calculate time; incomplete algorithm can find the sub-optimal solution with less calculate time than complete algorithm. Considering the traffic network, for simple roads, they can set a fixed green light duration and solve it through the complete algorithm; For some important traffic networks with traffic flow statistics, after the traffic flow is detected, they can reasonable allocate green light time, dynamically adjust the traffic light sequence of each intersection.

4. Simulation experiment

Our simulation experiments are based on the simulation dcop solver platform, which uses multi threads to simulate the distributed environment. The simulation experiment uses the traffic network shown in Figure 2, in which there are six intersections and one fork. Considering that different road conditions will lead to different driving time, this paper assumes that the expected driving time between intersections is as shown in Table 1. In practice, the driving time of the road can be counted and set according to the statistical results. For the durations of green light, the traffic flow in each direction can be counted to dynamically adjust the green light duration. In this paper, we assume that the green light duration of straight and left turn at each intersection is shown in Table 2 (S: straight; L: left).

| Intersection | Driving Time(s) |
|--------------|----------------|
| A1 - A2      | 100            |
| A1 - A3      | 80             |
| A2 - A4      | 80             |
| A3 - A6      | 40             |
| A3 - A6      | 120            |
| A4 - A5      | 40             |
| A4 - A7      | 100            |
| A5 - A7      | 80             |

| Operations | Pass Time (s) |
|------------|---------------|
| A1 - S     | 60            |
| A1 - L     | 20            |
| A2 - S     | 80            |
| A2 - L     | 20            |
| A3 - S     | 80            |
| A3 - L     | 20            |
| A4 - S     | 100           |
| A4 - L     | 30            |

Table 1. The driving time of operation

Table 2. The pass time of operation

In this paper, the complete algorithms AsymPTFB[8], PTISBB and the incomplete algorithms ACLS[9], GCA_MGM[9] are selected respectively. The results of simulation are shown in Table 3. It can be seen from the table that the complete algorithms find the optimal solution. Its solution quality is higher than that of the incomplete algorithm, but the simulation time is longer; The simulation time of incomplete algorithm is much lower than that of complete algorithm, and the quality is still good. Therefore, the complete algorithm can be applied to solve the situation when no need of counting the traffic flow or difficult to count the traffic flow; In the traffic network with statistical traffic flow, the incomplete algorithm can be used to solve the congestion time to meet the real-time requirements.

| Algorithms | Simulation Time(s) | Cost | Algorithms | Simulation Time(s) | Cost |
|------------|--------------------|------|------------|--------------------|------|
| AsymPTFB   | 130                | 5742 | ACLS       | 23                 | 5826 |
| PTISBB     | 70                 | 5742 | GCA_MGM    | 28                 | 5824 |

Table 3. The results of simulation
5. Conclusion
The traffic congestion problem is of great significance to the economic environment. Considering the nature of cooperation between traffic intersections, this paper analyzes the traffic congestion problem and extends the problem to ADCOP. The traffic connection network is modeled as a distributed constraint graph, and the green light sequence of each direction of the traffic intersection is combined in different ways, which is unified with the neighbor intersection. The time cost of different planning combinations is calculated, and the distributed constraint problem algorithm is used to solve the problem. The simulation results show that: for the traffic network with fixed green time, the complete algorithm can be used to solve the problem at one time; For the dynamically adjustable traffic network, the incomplete algorithm can be used to dynamically solve the problem to update the green light transformation order in real time.

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