Optimal SQUID Loop Size in Arrays of HTS SQUIDs

Denis Crété¹, Yves Lemaître¹, Bruno Marcilhac¹, Eliana Recoba-Pawlowski¹,², Juan Trastoy¹ and Christian Ulysse³

¹Unité Mixte de Physique CNRS/THALES, Univ. Paris-Sud, Univ. Paris-Saclay, Palaiseau, France
²LPEM/CNRS, ESPCI Paris, PSL Research University, UPMC, Paris, France
³C2N/CNRS, Orsay, France
denis.crete@thalesgroup.com

Abstract. Arrays of Superconducting interference devices (SQUIDs) deserve much attention for high frequency magnetic field detection because of the combined advantages of wideband radiofrequency operation and improved dynamic range compared to single SQUID magnetometers. Indeed, in principle the dynamic range should scale as the square root of the number of SQUIDs. It is well-known that the size of a SQUID designed for magnetometry has an optimum resulting from a trade-off between large magnetic flux in its loop and small loop inductance. Among the factors affecting this optimum when using arrays of SQUIDs, we discuss the impact of Josephson junction characteristic dispersion, experimentally observed with high temperature superconductors (HTS) and wideband requirement. Both limit the SQUID size to lower values, in particular for arrays of SQUIDs connected in series.

1. Introduction

In the frequency range below 100 MHz, single Superconducting Quantum Interference Device (SQUID) circuits have demonstrated very good performance. They are generally optimized for maximum field to voltage transfer factor (thereafter called transfer-optimized) and use flux locked loops to achieve a high dynamic range. At high frequency (above 150 MHz) and wideband applications, a large dynamic range can be obtained with arrays of SQUIDs. Absolute magnetometers have been proposed with distributed (incommensurate) SQUID areas [1-3]. Usually, the SQUID loop areas are smaller or equal to the transfer-optimized size [3], corresponding approximately to

\[ \beta_L = \frac{L \cdot 2 I_C}{\Phi_0} \approx 1 \text{ or } 1/\pi, \]

where \( L \) is the SQUID loop inductance, \( I_C \) is the Josephson junction (JJ) critical current and \( \Phi_0 \) is the flux quantum (\( \Phi_0 = 2.07 \times 10^{-15} \text{Wb} \)). In addition, the layout complies with specific symmetries and the
Josephson junctions are identical ($\Delta I_c = 0$). Ideally, the optimal operation point ($T, I_b$) in the temperature-bias current space is the same for all SQUIDs. Smallest loop size depends on coupling/technical details for each microfabrication technology.

In this paper, we present several constraints which apply to (essentially series) arrays of SQUIDs, generally leading to a reduction of the SQUID loop size. Section 2 focuses on the impact of JJ parameter dispersion on series arrays of SQUIDs. In Section 3, we discuss the case of parallel and 2D JJ arrays. In Section 4, RF operation is discussed in terms of array impedance. Section 5 concludes this paper.

2. Series arrays of SQUIDs and JJ parameter dispersion

The following impacts of JJ parameter dispersion are expected for series arrays of SQUIDs:

i) because of the temperature dependence of the JJ parameters (mainly $I_C$ and depending on the technology, the normal resistance $R_N$ as well), the optimal conditions of operation of each SQUID are scattered in ($T, I_b$) space. As a result, at a given temperature and bias current, only a few SQUIDs are operating at their optimum; the other SQUIDs operate under degraded conditions. Therefore, the array does not perform as well as the scaling of a single SQUID. Let $V_{SQUID}$ (resp. $V_{SQA}$) be the output voltage amplitude of a single SQUID (resp. of the array) and $N_{SQUID}$, the number of SQUIDs in the array; then, $V_{SQA} < N_{SQUID}V_{SQUID}$.

ii) a self-induced flux, generally associated to an asymmetric distribution of the SQUID bias current, appears. As illustrated Fig. 1, two kinds of asymmetry may occur: Lay-out asymmetry (where the geometry of the loop branches and bias lines generate the asymmetry, resulting in inductance asymmetry), and Josephson asymmetry (where the critical currents of the Josephson junctions differ). The self flux induced by asymmetry is [4]:

$$\Delta \Phi = L.\Delta I_c + \Delta L. I_b$$  \hspace{1cm} (2)

Fig. 1. Self-induced flux in a SQUID, originating from different types of asymmetry; a) Layout (asymmetry of the SQUID), $\Phi_{top} + \Phi_{bottom} + \Phi_{left} + \Phi_{right} \neq 0$; b) Layout (asymmetry of the bias lines), $\Phi_{left} + \Phi_{right} + \Phi_{bias} \neq 0$; c) Josephson asymmetry, $\Phi_{left} + \Phi_{right} = L.(I_{C1} - I_{C2}) \neq 0$.

The first term in Eq. 2 originates from the JJ parameter scattering ($\Delta I_c$). Its impact is larger for larger SQUID inductance. The second term is associated to the layout asymmetry ($\Delta L$). It induces an evolution of the flux-shift with bias current and can be made small by proper design [5]. The self-induced flux causes a distribution of the phase $\phi$ of the periodic responses $V_{SQUID}(B)$,

$$\Delta \phi = 2\pi \frac{\Delta \Phi}{\Phi_0} \approx 2\pi \frac{L.\Delta I_c}{\Phi_0}$$  \hspace{1cm} (3)

and degradation of the resulting SQA output signal. Note that Josephson asymmetry also degrades the amplitude of the SQUID modulation [5]:

\[\text{...}\]
ΔV_{SQUID} ≃ \frac{R_N \sqrt{I_{C1} \cdot I_{C2}}}{L(I_{C1} + I_{C2})} \frac{1}{\Phi_0} \tag{4}

All samples are prepared with a single 150 nm-thick YBCO layer technology. The JJ are made by oxygen ion irradiation of the barrier [6]. We characterized the SQUIDs using a test bench described in [5], where the magnetic field is applied by feeding a current $I_{cc}$ in a pair of coils.

In fig. 2, we show experimental results obtained on a SQUID with a loop with inner dimensions 5µm×6µm, and a loop inductance of 9.1 pH, evaluated using InductEx and thus include kinetic inductance. For each temperature, the bias current $I_b$ has been adjusted for maximum voltage modulation. Its periodic response is well established, while the minima of the output voltage are not aligned and give an evaluation of $\Delta \phi$:

$$\Delta \phi = 2\pi \frac{I_{cc} |V_{\text{min}}|}{\Delta I_{cc}} \tag{5}$$

where $\Delta I_{cc}$ is the period of the oscillations of $\Delta V_{SQUID}$. Their positions depend on the experimental conditions: temperature $T$, critical current difference $\Delta I_c$ and/or bias current $I_b$. More detailed analysis is available in [5], and shows that the Josephson asymmetry (first term of Eq. 2) is dominating. As a consequence, the impact of $\Delta I_c$ is larger for SQUIDs with larger inductance, i.e. for larger SQUIDs.

Fig. 2: a) Layout of the 5 SQUIDs; JJ are represented by the yellow lines; b) voltage modulation $\Delta V_{SQUID}(B)$ of SQUID S1 for 50 < $T$ < 62 K (by steps of 1 K using a rainbow color scale with $T=50$ K for red and $T=62$ K for violet). The difference in self-induced flux at $T=62$ K and $T=50$ K almost reaches one flux quantum $\Phi_0$. For easier comparison, each curve has been shifted by subtraction of its average voltage.
In series arrays of SQUIDs, the antipeak is expected at \( B=0 \). Let \( \Delta \phi \) be the phase shift due to Josephson asymmetry of SQUID \( k \),

$$\Delta \phi = 2\pi \Delta \Phi / \Phi_0 = 2\pi L \Delta I_c / \Phi_0$$  \hspace{1cm} (6)

Disregarding the effect of \( \beta \) and \( \Delta I_c \) on SQUID modulation amplitude,

$$V_{SQA} \mid B=0 \mid = \sum_{k=1}^{M} I_c R_N \sqrt{\left( \frac{I_b}{2I_c} \right)^2 - \cos^2(\Delta \phi_k)} > \sum_{k=1}^{M} I_c R_N \sqrt{\left( \frac{I_b}{2I_c} \right)^2 - 1}$$  \hspace{1cm} (7)

Thus, \( \Delta V_{SQA} = V_{SQA}(B) - V_{SQA}(B=0) \) is smaller not only because of \( \beta \), but even smaller because of \( \Delta \phi \).

We have made a Monte-Carlo analysis of the impact of a normal distribution of \( \Delta \phi \) in a series array of identical SQUIDs, assuming that the individual SQUID response \( \Delta V_{SQUID} \) is sinusoidal. The results appear in Fig. 3, where \( \sigma_\phi \) is the standard deviation of the phase shift \( \Delta \phi \) due to self induced flux.

![Simulation with \( N_{SQUID} = 2000 \)](image)

Fig. 3. Modulation amplitude variation of 2000 SQUID series array output voltage vs. standard variance of the phase shift \( \Delta \phi \), and linear fit with a slope of -4.3 dB/rd².

Let \( \sigma_I \) be the absolute standard deviation of the critical current, Eq. (6) can be rewritten

$$\sigma_\phi = 2\pi L \sigma_I / \Phi_0$$  \hspace{1cm} (8)

and from Fig. 3, instead of \( \Delta V_{SQA} \sim N_{SQUID} \cdot \Delta V_{SQUID} \), we expect

$$\Delta V_{SQA} \approx N_{SQUID} \cdot \Delta V_{SQUID} \cdot e^{-\sigma_\phi^2 \left( \frac{\pi L}{\sigma_I} \right)^2}$$  \hspace{1cm} (9)

The effect of \( \Delta \Phi \) can be disregarded for single SQUID sensors, but is detrimental on a series array of SQUIDs and destroys the antipeak expected for such devices. The 3dB-degradation is reached for
\[ \sigma_I = \frac{\Phi_0}{2\pi L} \sqrt{\frac{3\log 10}{10}} \]  \hspace{1cm} (10)

i.e. \( \sigma_I L = 273 \mu A \cdot pH \) or \((\sigma_I/I_C)\beta_L = 0.13\). For example, a relative root mean square value for the critical current distribution of 13\% would cause a 3dB degradation for \( \beta_L \approx 1 \).

In order to reduce \( \Delta\phi \), it is essential to have a Josephson junction with small (absolute) critical current dispersion. Once the JJ technology is optimized and stabilized, the remaining degrees of freedom are the temperature of operation \( T \) and the SQUID loop inductance. As the critical current of SNS JJ decreases with temperature, \( \sigma_I \) typically tends to decrease while \( T \) raises in the domain showing Josephson behaviour. It is well known that large critical currents and/or loop inductance, associated to \( \beta_L > 1 \), degrade the SQUID response. Even more strikingly, the product \( L_\sigma \) appears to reduce the operation temperature range of a series array of SQUIDs, by raising its lower bound. As, very generally for SNS JJ, the \( I_C R_N \) product decreases with \( T \), we see from Eq. 7 that the amplitude of the SQUID voltage modulation will decrease: it is not a good solution to deal with \( \sigma_I \). Thus, it is generally easier to reduce/eliminate this contribution as

a) the layout asymmetry (\( \Delta L \)) is generally low (provided the situations depicted fig 1 a,b are avoided); and the difference between design values and real physical values has an important part of systematic error, which does not contribute to the dispersion of \( L \).

b) \( I_b \), being comparable to \( I_c \), is kept low in order to have \( \beta_L \approx 1 \) with maximum loop area. The limit of \( I_c R_N \) product decreases with \( T \), we see from Eq. 7 that the amplitude of the SQUID voltage modulation will decrease: it is not a good solution to deal with \( \sigma_I \). Thus, it is important to reduce \( L \). One may argue that it is important also to reduce \( \Delta L I_b \): while this is true, it is generally easier to reduce/eliminate this contribution as

We have investigated the effect of the SQUID loop geometry. The layout of the 20-SQUID circuit appears in fig. 4. Electrical connections appear as orange lines, while the superconducting material appears in red. The short yellow lines represent the JJ positions. Only 14 out of the 20 SQUIDs were tested. SQUIDs S3 to S7 are designed with a rectangular loop where the bias current flows essentially along the long side of the rectangular loop; SQUIDs S8 to S13 are designed with the same set of rectangular loops but the bias current flows essentially along the narrow side of a rectangular loop; S14 to S16 are SQUIDs with square washers. Characterization results are summarized in fig. 5 a (maximum SQUID voltage amplitude) and fig. 5 b (steepest slope of the transfer function). In the latter case, the slope of the modulation amplitude and the period of the oscillations \( \Delta I_{cc} \) (where \( S_{eq} = 1/\Delta L \) is proportional to the effective area of the SQUID). Here, we assume that the curves \( V_{SQUID}(B) \) can be fitted with the same model function \( V_{model} \) by \( V_{SQUID}(B) = A_{SQUID} V_{model}(S_{eq} I + \theta_{SQUID}) \), where \( A_{SQUID}, S_{eq} \) and \( \theta_{SQUID} \) are fitting parameters. Thus, the steepest slope, which we call “maximum transfer factor”, is proportional to the product of the amplitude \( A_{SQUID} \) by \( S_{eq} \). SQUID inductances are evaluated with InductEx. These preliminary results indicate that square SQUIDs are better than flat SQUIDs, which are better than long SQUIDs.
However, long SQUID S3 seems to perform as well as the square SQUIDs; one possible reason may be due to a good pairing of its JJ at the operating temperature (around 54 K). The factor 2 that we observe in fig. 5 b between the long and the square SQUIDs needs confirmation. In this case, it is important to choose the right shape for the SQUID loop.

Fig. 4. Layout of a series of SQUIDs with different loop geometries. For «LONG» and/or «FLAT» SQUID Sₖ, loop hole is 6 μm wide and 40-6×(k-1 modulo 7) μm long.

Fig. 5. Response of 14 different SQUIDs to a magnetic field; a) Amplitude of voltage modulation versus SQUID inductance; b) Maximum transfer factor versus $S_{eq}=1/\Delta I_{cc}$. 
3. 1D-parallel and 2D arrays

In the case of an ideal 1D-parallel array (i.e. without any dispersion), derivation of analytical expression is possible if self-field effects are negligible [3]. An array with $M$ identical JJ behaves as a single JJ with critical current $M I_C$, modulated by a form factor $S_M$:

$$S_M(B) = \frac{1}{M} \sum_{n=1}^{M} \frac{I_{C,n}}{I_C} e^{2\pi i \sum_{k=1}^{M} \sum_{i=1}^{N} \bar{a}_i}$$

(12)

where $a_i$ is the set of loop areas between neighbouring JJ. With over-damped JJs, the voltage at the array terminals is

$$\langle V_{SQA}(B) \rangle = I_C R_N \left( \left| \frac{I_b}{M \cdot I_C} \right|^2 - |S_M(B)|^2 \right)$$

(13)

However, in a more realistic case, self-field effects generally tend to reduce the maximum value of $|S_M(B)|$ [8]. Furthermore, when the critical current dispersion is taken into account, the self-induced fluxes in the different loops $\Delta \Phi_k$ become distributed at random:

$$S_M(B) = \frac{1}{M} \sum_{n=1}^{M} \frac{I_{C,n}}{I_C} e^{2\pi i \sum_{k=1}^{M} \sum_{i=1}^{N} \bar{a}_i + \Delta \Phi_k}$$

(14)

As a consequence, not only the distribution of $I_C$ increases the minimum value of $|S_M(B)|$ but also $\Delta \Phi$ reduces its maximum value: both contribute to reduce the voltage modulation amplitude. In addition, we expect that the critical current distribution will cause a shift in the position of the antipeak, i.e. the maximum value of $|S_M(B)|$ will be reached for $B \neq 0$. For most applications of 1D-parallel arrays, the latter effect can be disregarded. This is not the case for a 2D array with $N M$ JJ: because of antipeak misalignment, scaling with $N$ degrades.

4. RF operation

For SQUID operation above 150 MHz with a wide band and a large dynamic range, the best candidate is the SQUID array (SQA), with a number of SQUIDs connected in series, $N_{SQUID}$ as large as possible; but the output impedance of the array, $Z_{SQUID}$ varies as $N_{SQUID}$ and is limited by matching rules. To circumvent this limitation, 2D arrays have been proposed [7]. Their impedance can be tailored, as for 2D arrays with $M$ JJ in parallel,

$$Z_{SQUID} \approx \frac{N_{SQUID}}{M} Z_{JJ}$$

(15)

However, for wideband operation, the total length of the array must be much less than $\lambda_g$, the guided wavelength. As a consequence, flat geometry (cf. SQUIDs S8...S13) might be more favourable when inserted in both 1D-series and 2D arrays. A « flat » SQUID is at least 5µm long, and at $f=300$MHz, $\lambda_g \sim 30$ cm. This limits $N_{SQUID}$ to much less than 60000 (and scales as $1/f$). Another option,
which would not present this limitation but which is out of the scope of this paper, could be a travelling wave structure.

5. Conclusion

We have measured the impact of critical current dispersion on the response of individual SQUIDs to applied magnetic fields. The main effect is a self field induced by the asymmetric bias currents, which jeopardizes the principle of amplitude summation for SQUID arrays connected in series. For such arrays, if the SQUIDs are identical, we evaluate an amplitude degradation of $\sim -4.3$ dB/rd² with a phase standard deviation proportional to the product of the critical current standard deviation and inductance of the SQUID loop. This directly translates into smaller loop sizes for optimal design of series SQUID arrays.

For 1D-parallel arrays, self-field effects will not only degrade the antipeak amplitude, but will also shift its position in the curve of voltage modulation by a magnetic field. This shift also impacts the scaling of 2D arrays with the number of elements in series.

Preliminary results on SQUID loop geometry with comparable effective areas indicate that square loops (and flat loops) may operate better at DC than long loops. We expect this to remain true at RF, as long loops will have more reactance (contributing to a larger output impedance of the array).

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