Measuring the Duration of Last Scattering

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The cosmic microwave background (CMB) fluctuations effectively measure the basic properties of the universe during the recombination epoch. CMB measurements fix the distance to the surface of last scatter, the sound horizon of the baryon-photon fluid and the fraction of the energy density in relativistic species. We show that the microwave background observations can also very effectively constrain the thickness of the last scattering surface, which is directly related to the ratio of the small-scale E-mode polarization signal to the small-scale temperature signal. The current cosmological data enables a 0.1% measurement of the thickness of the surface of last scatter: $19 \pm 0.065$ Mpc. This constraint is relatively model-independent, so it can provide a new metric for systematic errors and an independent test of the ΛCDM model. On the other hand, it is sensitive to models which affect the reionization history of the universe such as models with annihilating dark matter and varying fundamental constants (e.g., the fine-structure constant, $\alpha_{\text{EM}}$, and electron rest mass, $m_e$) and as such can be used as a viable tool to constrain them.

I. INTRODUCTION

The cosmic microwave background (CMB) stores a tremendous amount of information about the early history of the Universe and its subsequent evolution. Its large-scale anisotropies indicate the existence of tiny fluctuations in the primordial gravitational potential that are the seeds for the formation of galaxies and other large-scale structure. Shortly after the discovery of the CMB in 1965, it was also shown [1] that anisotropic Thomson scattering of photons and electrons induces a degree of linear polarization in the data. The polarization properties of the CMB provide yet another set of observables, the measurement of which enriches greatly our understanding of the Universe.

The CMB anisotropies were formed primarily around the epoch of hydrogen recombination at redshift $z \sim 1100$ defined by the peak of the visibility function, $v = |\tilde{r}| e^{-\tau}$, which describes the probability that a CMB photon last scattered off free electrons at a particular point in the history of the Universe. The shape of the CMB power spectrum is thus most sensitive to changes around the peak of the visibility function. For example, the location of that peak determines the distance to the last scattering surface, which in turn determines the positions of the peaks of the CMB power spectra. If we were to increase the width of the visibility function, that would correspond to a prolonged period of recombination, leading to more Thomson scatterings of photons off free electrons. On scales smaller than the recombination width, these scatterings lead to the cancellation of the CMB anisotropies along the line of sight, while on larger scales they lead to enhancement of the polarization signal. Similarly, changes in the ionization history, the photon-baryon sound speed, the gravitational potential around matter-radiation equality and other primordial properties which we have no direct way of probing would likely lead to measurable changes in the CMB power spectra and the baryon acoustic oscillation (BAO) peak, which would then allow us to put constraints on the features of the early Universe [2]. For example, models which involve annihilation of dark matter to Standard Model particles between the period of recombination and reionization result in a modification of the ionization history, as they lead to a heating of the baryons and ionization of the neutral hydrogen, and thus alterations in the CMB visibility function. Another such example are models which predict variations of the fundamental constants – e.g., the fine-structure constant, $\alpha_{\text{EM}}$, and the electron rest mass, $m_e$ provide another such example and can thus directly impact CMB observables [3–10]. Due to their nature of affecting mostly the large-scale observables, constraints on such models are not expected to become much more stringent with the drastic improvement in sensitivity expected of future cosmological surveys.

Over the past decade, cosmologists have been exploring many alternative ways to test our understanding of the early Universe. A validated approach they have taken is to introduce new parameters into the analysis of cosmological data, which can serve as powerful probes for discrepancies with our predictions and can help in the detection of systematic errors in our measurements or problems with the ΛCDM model. In principle, to show that a given new parameter may be of such use, one needs to check that it is not correlated with the standard parameters, i.e. that it reflects a different physical effect. The degree of correlation between the new and the standard parameters can be tested by studying their contour plots in a Monte-Carlo engine analysis [11].

An example of a parameter which probes the properties of the early Universe is the width of the visibility function during the “last scattering” of photons. By changing its width the strip of time from which the photons could have come would be broadened or narrowed. A longer period of last scattering would then result in a more polarized signal on large scales, since the photons would have scattered off the electrons a larger number of times [2] [12]. Therefore, an intriguing question to explore is: how well can we constrain the width of the last scattering surface with the most recent polarization data from
the Planck team. So far, the width of the last scattering surface has not been measured directly by CMB experiments, but its theoretical value is readily computed to be $\Delta \eta_s \approx 19$ Mpc ($\Delta z_s \approx 90$) using the latest cosmological codes, e.g. CLASS [13], and standard values for the cosmological parameters from Planck [14].

This paper is organized as follows. We first provide motivation for our work by stating a relationship between the polarization-to-temperature ratio of the power spectra and the width of the visibility function pointed out by [2]. We then parametrize this width through $\alpha_{\text{vis}}$ and explore how varying this new parameter affects the observable power spectra. Finally, we put constraints on its value using the latest CMB data from Planck, study its degeneracy with other parameters, and discuss its potential as a model-independent test of the $\Lambda$CDM model and of systematic errors. It also provides a powerful tool to constrain models which alter the reionization history of the Universe such as dark matter annihilation and decay and variable $\alpha_{\text{EM}}$ and $m_e$ models.

II. PHYSICAL MOTIVATION

The shapes of the temperature and polarization power spectra are affected by the width of the last scattering surface due to diffusion (Silk) damping [15]. Diffusion damping results from the scattering of photons off electrons during the free-streaming epoch [12]. The collisions of the free-streaming dipole produce a quadrupole moment in the photon distribution function, which in turn leads to a polarization of the CMB, as the polarization is proportional to the quadrupole moment of the photon distribution function [2] [12]. The polarization of the CMB is produced during the process of decoupling of matter and radiation, and is also proportional to the width of the last scattering surface $\Delta \eta_s$ and the conformal time of recombination $\eta_s$. The anisotropy in the CMB temperature also depends on these quantities, but differently. For instance, on large scales it is very insensitive to the values of $\Delta \eta_s$ and $\eta_s$ [2].

The ratio of the polarization spectrum to the temperature spectrum is shown to be strongly dependent on the width of the last scattering surface [2]:

$$\frac{C_{EE}^s}{C_{TT}^s} = \left[ \frac{\Delta \eta_s \eta_s}{(1 + R)^2} \right]^2 A_t,$$

where $r$ is the distance to the last scattering surface, $A_t$ is a scale-dependent factor, and $R = 3\rho_b/4\rho_r$ is the ratio between baryonic and radiation densities.

III. METHODS AND TOOLS

A. Parametrization

We parametrize the width of the visibility function, $\Delta \eta_s$, with our new parameter $\alpha_{\text{vis}}$ in the following way.

The visibility function $v_{\text{orig}}$ is well approximated by a Gaussian of width $\sigma \equiv \Delta \eta_s$ with some maximum value at recombination $v_{\text{max}}$ [10]:

$$v_{\text{orig}} \approx v_{\text{max}} \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\frac{(\eta - \eta_s)^2}{2\sigma^2}}.$$  (2)

To change the width of a Gaussian, simply multiply its width by a constant, i.e. $\sigma \to \alpha_{\text{vis}} \times \sigma$. In this case, the new visibility function becomes:

$$v_{\text{new}} \approx v_{\text{max}} \frac{1}{\sqrt{2\pi \sigma^2_{\text{vis}}}} e^{-\frac{(\eta - \eta_s)^2}{2\sigma^2_{\text{vis}}}}.$$  (3)

However, notice that we can write this as:

$$v_{\text{new}} = v_{\text{max}} \frac{v_{\text{orig}}}{v_{\text{max}}} \left( \frac{\sigma_{\text{vis}}}{\sigma} \right)^{-\alpha_{\text{vis}}}.$$  (4)

which can be applied to all Gaussian-like functions, as it does not assume anything specific about the function apart from an approximately Gaussian shape [17]. The results of this parametrization are shown in Fig. 1

![Figure 1](attachment:image.png) FIG. 1. The effect of changing the parameter $\alpha_{\text{vis}}$ on the numerically derived visibility function from CLASS [13] as a function of redshift. This parametrization preserves very well the shape of the visibility function and only changes its width. CLASS automatically normalizes the area under the visibility function to 1.
B. Data Analysis

We constrained the width of the last scattering surface, \( \alpha_{\text{vis}} \), utilizing the Monte-Carlo sampling engine MontePython \[11\] with the \textit{Planck} 2015 measurements of the CMB power spectra \[18\] through two sets of likelihoods:

The \textbf{first set} constrains it through the Silk damping effect and the large-scale polarization data, using only the \textit{lite} high \( \ell \) likelihoods:

- \textbf{low } \( \ell \) – consists of the CMB TT, EE, BB and TE power spectra from \( \ell = 2 \) to 29 (inclusive), and an extra nuisance parameter for the overall Planck calibration.

- \textbf{high } \( \ell \) \textit{lite} – consists of the CMB TT power spectrum from \( \ell = 30 \) to 2508 and the Planck absolute calibration nuisance parameter.

- \textbf{lensing} – consists of the \( \phi\phi \) lensing spectrum for \( \ell = 1 \) to 2048 (inclusive)

The \textbf{second set} uses both the full temperature and E-mode polarization high \( \ell \) spectra in order to constrain the width of the visibility function from the ratio of polarization to temperature:

- \textbf{low } \( \ell \)

- \textbf{high } \( \ell \) \textit{TTTEEE} – consists of the CMB TT, EE and TE power spectra from \( \ell = 30 \) to 2508 (inclusive), and a vector of 94 nuisance parameters.

- \textbf{lensing}

A simple \( \chi^2 \) test shows that current data should constrain \( \alpha_{\text{vis}} \) to less than 1% of the width of the visibility function, so we select a uniform prior on \( \alpha_{\text{vis}} \in (0.97, 1.03) \). The duration of the last scattering surface is then quantified by the product \( \alpha_{\text{vis}} \Delta \eta_* \), where \( \alpha_{\text{vis}} = 1 \) corresponds to the Standard Model prediction for its duration.

IV. RESULTS

A. Width of the Last Scattering Surface

In Fig. 2, we show the visibility function \( v = |\dot{\tau}| e^{-\tau} \) as output by CLASS \[13\] and the best-fitting Gaussian function with mean \( \eta_* \) and width \( \Delta \eta_* \). We find the width of the Gaussian fit to be approximately \( \Delta \eta_* \approx 19 \) Mpc (or \( \Delta z_* \approx 90 \) in redshift space), where we used standard values for the cosmological parameters \[14\].

In Fig. 3, we can see the effects of varying the width of the last scattering surface on the EE polarization power spectrum. As discussed in \[2\], we observe that the amplitude of the polarization power spectrum increases roughly quadratically with \( \alpha_{\text{vis}} \), while the peak locations are almost unchanged. We further see that the effect is stronger on the large angular scales, while the smaller scales are affected by the so called damping tail, which leads to a suppression of the amplitudes.

We have also studied the effect of varying the width of the visibility function on the temperature power spectrum and have found that, as expected, the resulting deviation is much smaller. On scales \( \ell > 200 \), the fractional difference is less than 10% when we perturb the width by \( \Delta \alpha_{\text{vis}} = 0.05 \). On larger scales, \( \ell < 200 \), contrary to the case of polarization, the effect on the power spectrum is quite negligible.

B. First Set: Constraints from Silk Damping and Low \( \ell \) Polarization-to-Temperature Ratio

As a first step, we test how well the current measurements of the temperature and large-scale polarization power spectra are able to constrain the duration of last scattering through \( \alpha_{\text{vis}} \). Measurements of the temperature power spectrum constrain the width because as argued in \[2\], the longer the photon last scattering lasts for, the stronger the damping effects would be on the smaller scales. On the other hand, the low \( \ell \) likelihood gives us the polarization-to-temperature ratio on large scales, and thus also helps us measure the width.

In Table I, we present the best-fit values for the parameters in the \( \Lambda \)CDM model and our new parameter \( \alpha_{\text{vis}} \). Notice that despite the fact that the low \( \ell \) data are affected by cosmic variance and systematics, the large-scale polarization data along with the measurements of
FIG. 3. Effects on the polarization power spectrum for different widths of the last scattering surface.

The temperature power spectrum from the Planck team on small scales, sensitive to second-order effects such as the damping, can constrain the width of the last scattering surface to very good precision (about 0.6%).

We can translate the measurement of the error of \( \alpha_{vis} = 0.9937^{+0.0056}_{-0.0062} \) into a measurement of the precision to which we can constrain the width of the last scattering surface. To do so, we multiply it by the value we obtained for its width from our theoretical prediction \( (\Delta \eta_s = 19 \text{ Mpc}) \). We find that our current measurements can constrain the value of \( \Delta \eta_s \) to within \( \sigma(\Delta \eta_s) \approx 0.11 \text{ Mpc} \) (68% CL), which is indeed very precise. The slightly lower value of \( \alpha_{vis} \) is most likely due to a combination of noise and cosmic variance, but it might also be suggesting that in our patch of the Universe, the polarization-to-temperature ratio (effectively) happens to be smaller than the overall, assuming the Standard Model is correct.

| Param     | best fit | mean±σ       | 95% low | 95% up  |
|-----------|----------|---------------|---------|---------|
| \( \alpha_{vis} \) | 0.993    | 0.994 ± 0.006 | 0.981   | 1.005   |
| \( \Omega_b h^2 \) | 0.0222   | 0.0222 ± 0.00025 | 0.02171 | 0.02268 |
| \( \Omega_{cdm} h^2 \) | 0.118    | 0.118 ± 0.002 | 0.114   | 0.122   |
| \( 100 \times \theta_s \) | 1.042    | 1.042 ± 0.0004 | 1.041   | 1.043   |
| \( \ln 10^{10} A_s \) | 3.08     | 3.08 ± 0.03  | 3.03    | 3.13    |
| \( n_s \) | 0.972    | 0.969 ± 0.006 | 0.957   | 0.981   |
| \( \tau_{reio} \) | 0.0697   | 0.0682 ± 0.015 | 0.04    | 0.0965  |
| \( H_0 \) | 68.2     | 68.0 ± 0.9   | 66.2    | 69.9    |

\( -\ln \mathcal{L}_{\text{min}} = 5355.23, \text{ minimum } \chi^2 = 1.071e + 04 \)

TABLE I. Constraints from Silk damping and low \( \ell \) polarization-to-temperature ratio.

C. Second Set: Constraints from Polarization-to-Temperature Ratio on all Scales

Equation [1] shows that the polarization-to-temperature ratio depends strongly on the width of the last scattering
surface, i.e. $C^{EE}_f/C^{TT}_f \propto \Delta \eta^2 \propto \alpha^2_{\text{vis}}$. For this reason, after including the measurements on the polarization of the power spectrum, we arrive at even more stringent constraints on the value of $\alpha_{\text{vis}}$. These are shown in Table I. The standard deviation of $\alpha_{\text{vis}}$ is nearly two times smaller compared with the first set, which corresponds to a precision of $\sim 0.3\%$). Translating the value of $\alpha_{\text{vis}} = 0.9974_{-0.0034}^{+0.0034}$ into the more physical quantity $\Delta \eta$, we find that the width of the visibility function is constrained to within $\sigma[\Delta \eta] \approx 0.065$ Mpc at 68% CL.

\begin{table}[h]
\begin{tabular}{lllll}
Param & best fit & mean$\pm\sigma$ & 95% low & 95% up \\
\hline
$\alpha_{\text{vis}}$ & 0.996 & 0.997$\pm$0.0034 & 0.991 & 1.004 \\
$\Omega_b h^2$ & 0.0222 & 0.0223$\pm$0.0002 & 0.0219 & 0.0226 \\
$\Omega_{\text{cdm}} h^2$ & 0.120 & 0.119$\pm$0.0015 & 0.116 & 0.122 \\
100$\times \theta_s$ & 1.042 & 1.042$\pm$0.003 & 1.041 & 1.043 \\
$\ln 10^{10} A_s$ & 3.03 & 3.07$\pm$0.03 & 3.02 & 3.12 \\
$n_s$ & 0.960 & 0.965$\pm$0.005 & 0.956 & 0.975 \\
$\tau_{\text{reio}}$ & 0.0453 & 0.0666$\pm$0.013 & 0.0422 & 0.090 \\
$H_0$ & 67.2 & 67.7$\pm$0.65 & 66.4 & 69.0 \\
\end{tabular}
\end{table}

$-\ln L_{\text{min}} = 5355.23$, minimum $\chi^2 = 1.071e + 04$

TABLE II. Constraints from polarization-to-temperature ratio on all scales.

In Fig. 4 we show the correlations between $\alpha_{\text{vis}}$ and the 6 standard parameters. Notice that while there appears to be a weak correlation between $\alpha_{\text{vis}}$ and the dark matter density $\omega_{\text{cdm}}$ and $\alpha_{\text{vis}}$ and the Hubble parameter, the dependence overall is not very strong, i.e. the changing of $\alpha_{\text{vis}}$ produces a different effect on the power spectrum than any of the other 6 parameters.

![FIG. 4. 2D contours of $\alpha_{\text{vis}}$ and the 6 standard parameters. The new parameter seems weakly correlated with the other parameters.](image)

V. CORRELATION WITH EXTRA PARAMETERS

We further tested the conjecture that $\alpha_{\text{vis}}$ measures a new physical effect on the CMB power spectrum by including parameters beyond the six standard ones in our Monte-Carlo sampling engine and looking for degeneracies. The parameters we added were the effective number of neutrino species $N_{\text{eff}} (\equiv N_{\nu r})$ and the energy density of curvature $\Omega_k$.

In Fig. 5 we show the 2D contours of $\alpha_{\text{vis}}$ and the two extra parameters. We do not find degeneracies with any of the parameters, which supports the claim that $\alpha_{\text{vis}}$ provides an independent test of the Standard Model as well as a test of the systematics of a given data set. We find that the mean value of $\alpha_{\text{vis}}$ is within 1$\sigma$ of the standard prediction for its value $\alpha_{\text{vis}} = 1$. If future experiments should find that its value differs by more than 1$\sigma$ from $\alpha_{\text{vis}} = 1$, that could be indicative of a system-
the width of the visibility function, and from the large-scale polarization, which is strongly dependent on the width. If we include the polarization data from Planck 2015 on all scales, we get the much tighter constraint of $\sigma[\Delta \eta_*] \approx 0.065$ Mpc. This is a consequence of the fact that the polarization-to-temperature ratio is proportional to the width of the last scattering surface squared. We believe that constraining this parameter is a good way to test our current model and probe for physics beyond the Standard Model.

In the near future, the new polarization data from upcoming experiments such as Simons Observatory (SO) and CMB-S4 [19] should allow us to measure the parameter $\alpha_{vis}$ with even greater precision. High-$\ell$ data from SPT and ACT should in principle also help measure better the diffusion damping tail and thus put constraints on $\alpha_{vis}$. Including $\alpha_{vis}$ when analyzing the new datasets will enable us to detect deviations from the $\Lambda$CDM model and look for systematic errors in these new datasets. In addition, it will be useful when constraining models which alter the reionization history of the Universe such as self-interacting dark matter models and variable-$\alpha$ models.

ACKNOWLEDGMENTS

We are very grateful to Joanna Dunkley for her hard and devoted work without which the completion of this project would have been barely possible. We would like to thank Blake Sherwin for providing us with useful comments, which helped us refine this paper. This work was completed as part of B. H.’s undergraduate senior thesis. The Flatiron Institute is supported by the Simons Foundation.