Radiative neutrino decays in very strong magnetic fields

M. Kachelrieß and G. Wunner

Theoretische Physik I, Ruhr-Universität Bochum, D-44780 Bochum, Germany

Abstract

The radiative decay $\nu_H \rightarrow \nu_L + \gamma$ of massive neutrinos is analyzed in the framework of the standard model with lepton mixing for very strong magnetic fields $B \gg B_{cr} = m_e^2/e \sim 4.14 \times 10^{13}$ G. The analysis is based on the approximate decay amplitude obtained by Gvozdev et al. Numerical results as well as analytical approximations for the decay rate are obtained for energies of the initial neutrino below and above the electron-positron pair creation threshold $2m_e$.

Key words: Decay of heavy neutrinos, neutrino mass and mixing, elementary particle processes in astrophysics.
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1 Introduction

In the Standard Model of electroweak interactions (SM) the neutrino masses are set to zero “by hand”. While all terrestrial experiments are consistent with this assumption, it is now clear that the solution of the solar neutrino problem requires nonzero neutrino masses and neutrino mixing [1]. Another motivation for neutrino masses comes from cosmology with the need of a hot dark matter particle in the few eV mass range or of a heavy ($\sim$ MeV) unstable particle [2]. In both cases, the $\tau$-neutrino is an ideal candidate. However, astrophysics and cosmology provide also stringent limits for neutrino masses and lifetimes [3]. In particular, the requirement that the neutrino energy density does not overclose the Universe restricts the sum of the masses of all stable neutrino species to be less than $92 \Omega_\nu h^2$ eV, where $\Omega_\nu h^2 \lesssim 1$.

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In extended SMs with neutrino mixing, neutrinos can decay by the process \( \nu_H \to \nu_L + \gamma \) and, if the mass \( m_H \) of the heavy neutrino is higher than \( 2m_e \), additionally by \( \nu_H \to \nu_L + e^- + e^+ \). The resulting lifetimes \( \tau = \Gamma^{-1} \) are, although strongly model-dependent, extremely long. In the simplest case where right-handed singlet fields \( N_{lR} \) are added to the SM the decay rate of the resulting Dirac neutrinos is [3]

\[
\Gamma_{SM} = \frac{\alpha}{2} \left( \frac{3G_F}{32\pi^2} \right)^2 \left( \frac{m_H^2 - m_L^2}{m_H} \right)^3 \left( m_H^2 + m_L^2 \right) \sum_{l=e,\mu,\tau} |U_{lH}U_{lL}^\ast m_l^2| ^2,
\]

(1)

where \( m_H, m_L \) are the masses of the heavy and of the light neutrino, respectively. If we assume that the \( \tau \)-lepton dominates the sum over the internal leptons \( l \) and that the elements of the mixing matrix \( U \) are of order unity, eq. (1) yields a lifetime \( \mathcal{O}(\tau_{SM}) = 10^{29} \) years for \( m_H = 30 \) eV and \( m_L \ll m_H \).

Since \( \tau_{SM} \gg T_{\text{Universe}} \), neutrinos in the SM with lepton mixing can be treated as stable and the limit \( m_\nu \lesssim 92 \) eV applies. The principal reason for the extremely small decay widths of neutrinos is their small mass. Obviously, the decay rate \( \Gamma \) for the radiative decay has to be proportional to \( \alpha G_F^2 \), and since for \( m_H \gg m_L \) the only energy scale available is the mass of the heavy neutrino, it follows \( \Gamma \propto \alpha G_F^2 m_H^5 \) on purely dimensional grounds.

This changes drastically if an external field is present. Then, in the low-field limit \( B \ll B_{cr,i} = m_i^2/e \) the decay rates in magnetic fields \( B \) depend only on the dynamical field parameters

\[
\chi_i = m_i^{-3} \sqrt{(p_ieF^{\mu\nu})^2} = \frac{p_\perp}{m_i} \frac{B}{B_{cr,i}},
\]

(2)

where \( p_\perp \) is the momentum perpendicular to \( B \) of the initial particle and \( B_{cr,i} \) are the critical fields of the charged particles, while in the strong field limit \( B \gg B_{cr,i} = m_i^2/e \) the decay rates can depend separately on \( p_\perp \) and \( B \) [5]. In both cases, the essential point is that the energy scale is set by the charged particles and not by the mass of the decaying particle.

The best example for this mechanism is photon splitting \( \gamma \to 2 \gamma \). This process is forbidden in vacuum not only due to the Furry theorem but also due to the zero mass of the photon. In an external field photon splitting is allowed and the energy scale is set by the charged particles running in the loop. For, e.g., \( B \ll m_e^2/e \) and \( \omega \ll 2m_e \) the dominating contribution to the loop is that of the electron, and the decay rate is given by [4]

\[
\Gamma \propto \alpha^3 \frac{m_e^2}{\omega} \left( \frac{k_\perp}{m_e} \right)^6 \left( \frac{eB}{m_e} \right)^6 = \alpha^3 \frac{m_e^2}{\omega} \chi_6.
\]

(3)
Consequently, the decay of all light particles should be enhanced in strong external fields. For the example of the radiative decay of neutrinos this was shown for various external fields configurations by Gvozdev et al. [6,7]. In the case of a weak magnetic field $B \ll B_{cr}$, they derived approximate decay rates valid for arbitrary momentum $p_\perp$ of the initial neutrino. However, in the strong field limit $B \gg B_{cr}$, they gave an asymptotic expression for the decay width valid only for $p_\perp \ll 2m_e$ [6].

Recently several authors proposed scenarios in which a strong magnetic field with $B \gg m_e^2/e$ in the early Universe was created. Among the ideas considered are the production of a magnetic field of $O(B) = 10^{-8}M_{GUT}^2/e \approx 10^{43}$ G by the ferromagnetic Yang-Mills vacuum [8], of $O(B) = m_W^2/e \approx 10^{24}$ G during the electroweak phase transition [9], of $O(B) = (\lambda' M_{Pl} m\phi_0)^{2/3} \approx 10^{48}$ G during hybrid inflation, where $\phi$ is the inflaton field with mass $m$ and Higgs-coupling $\lambda'$ [10], inflation in string cosmology [11], etc. Subsequently, several authors used nucleosynthesis to derive bounds on primordial magnetic fields [12–14]. In these works, the effect of the magnetic field on the weak interaction rates $n \leftrightarrow p + e^- + \bar{\nu}_e$ and on the expansion rate of the Universe was taken into account but the neutrinos were treated as stable. The limits obtained vary between $B \approx 2 \times 10^{13}$ G in ref. [13] and $B \gtrsim 1 \times 10^{15}$ G in ref. [14] at the beginning of nucleosynthesis ($T \approx 1$ MeV).

Obviously, if the neutrino lifetime becomes of the order of the age of the Universe at the time of nucleosynthesis or before, they can no longer be treated as stable. Since the effect of a magnetic field is equivalent to an increase of the effective number of neutrino species $N_\nu$, while the decay of a heavy neutrino before nucleosynthesis decreases $N_\nu$ from three to two, the limits obtained in ref. [13,14] could be even weakened by taking into account neutrino decays.

### 2 Decay rate of $\nu_H \rightarrow \nu_L + \gamma$

We consider the radiative decay of a heavy neutrino $\nu_H$ into a lighter neutrino $\nu_L$ and a photon $\gamma$ in a strong magnetic field $B \gg B_{cr} = m_l^2/e$. Without loss of generality, we can choose the magnetic field as $\vec{B} = B\hat{e}_z$ and the three-momentum $\vec{p}$ of the heavy neutrino perpendicular to $\vec{B}$, e.g. $p^\mu = (E, p_x, 0, 0)$. The momenta of the light neutrino and of the photon are denoted by $p'^\mu = (E', \vec{p}')$ and $k^\mu = (\omega, \vec{k})$; the components perpendicular to $B$ have the index $\perp$. If the energy of the heavy neutrino is small compared to the mass of the $W$-boson, $E \ll m_W$, the four-fermion interaction can be used and the decay is described by the Feynman diagram shown in Fig. 1. In this limit, Gvozdev
et al. [6] found for the matrix element $\mathcal{M}$ of the process $\nu_H \to \nu_L + \gamma$

$$\mathcal{M}(B \gg B_{cr}) = -\frac{e}{24\pi^2} \frac{G_F}{\sqrt{2}} C \sum_{l=e,\mu,\tau} U_{Hl} U_{Ll}^* \frac{eB}{m_l^2} f(x), \quad (4)$$

where an effective matrix element

$$C = k_1^2 \left( \varepsilon^*_\mu \tilde{\Phi}^{\mu\nu} j_\nu \right) - \left( k_\mu \tilde{\Phi}^{\mu\nu} j_\nu \right) \left( k_\mu \tilde{\Phi}^{\mu\nu} \varepsilon^*_\nu \right) \quad (5)$$

and a function $f(x)$ depending only on the ratio $x = 4m_e^2/k_\perp^2$ were introduced. The other abbreviations have the following meaning: The neutrino current $j$ is given by

$$j_\mu = \bar{\nu}_L (p_2)^\gamma_\mu (1 + \gamma^5) \nu_H(p_1), \quad \tilde{\Phi}^{\mu\nu} = \tilde{F}^{\mu\rho} F^\rho_{\nu}/B^2 - \tilde{F}^{\mu\nu}/B$$

and $\tilde{F}$ is the dual field tensor. For $x > 1$ the function $f$ is given by

$$f(x) = \frac{3}{2} x (x\Omega \arctan \Omega - 1) \quad (6)$$

with

$$\Omega = \frac{k_\perp}{\sqrt{4m^2 - k_\perp^2}}, \quad (7)$$

where we corrected a typographical error (in the argument of arctan) of ref. [6]; a check of the relative signs in eq. (6) is given by the requirement $\mathcal{M} \to 0$ for $\omega \to 0$. The function $f$ can be analytically continued in the complex $k_\perp^2$ plane. Then, for $x < 1$, the real part of $f$ follows simply as

$$\Re(f) = \frac{3}{2} x (-x\Phi \arctanh \Phi - 1) \quad (8)$$

with

$$\Phi = \frac{k_\perp}{\sqrt{k_\perp^2 - 4m^2}}, \quad (9)$$

while the imaginary part of $f$ is given by the discontinuity across the branching cut starting at $k_\perp^2 = 4m_e^2$, viz.

$$2i\Im(f) = f(k_\perp^2 + i\varepsilon) - f(k_\perp^2 - i\varepsilon),$$

$$\Im(f) = \frac{3}{4} i\pi x^2 \Phi. \quad (10)$$

Using $\text{arccoth} \ z = \frac{1}{2} \ln \frac{1+z}{1-z} + \frac{1}{2} \pi i$ for $\Im(z) < 0$, one sees that $f(x)$ for $x < 1$ given in ref. [6] coincides—except for the sign of the term $3/2x$—with the real part given by us\[4\], but that the authors of ref. [6] missed to evaluate the imaginary part of $f$.

In the following, we restrict ourselves to the case that the internal loop is dominated by the electron, i.e. that $m_e^2/e \ll B \ll m_\mu^2/e$. In this case, the

\[4\] This sign error has also been corrected by Gvozdev et al. in ref. [15].
contribution of additional charged particles, e.g. from an enlarged Higgs sector, to the SM can be neglected. Therefore the decay rates will hold in the limit $m_H \gg m_L$ not only for the SM with lepton mixing but also for more general models of massive neutrinos\footnote{If neutrinos are Majorana particles, the decay rates should be divided by two.}. Moreover, we set $U_{Hl}^*U_{Ll} = 1$ for simplicity.

The neutrino decay rate $\Gamma$ is given by

$$\Gamma = \frac{1}{16\pi E^2} \int_0^E d\omega |\mathcal{M}|^2.$$  

Averaging $|C|^2$ over spins, we obtain

$$|C|^2 = 16 \left[ k_\perp^2 \left( 2E\omega^2E' - 2E\omega k_z E' + E k_z p_z' + Ep'_z \omega^2 \right) + 2 \left( E' \omega - E' k_z + p'_z \omega + p'_z k_z \right) \left( k_z - \omega \right) E\omega k_z + E p'_z k_\perp^2 + \left( EE' - p_x p'_x \right) \right. \right.$$ 

$$\left. \left( - k_z^2 \omega^2 - k_x^2 k_z^2 - k_y^2 \omega^2 - k_y^2 k_z^2 + E\omega^3 k_z E' - 2E\omega^2 k_z^2 E' + E\omega k_z^3 E' + E\omega^3 k_z p'_z - E\omega k_z^3 p'_z \right) \right].$$  

In the limit $E \gg m_H, m_L$, we can set $E \approx \not{p}$ and $E' \approx \not{p}'$. Therefore the three particles propagate collinearly and the above expression reduces to

$$|C|^2 = 32\omega^4 EE'.$$  

The differential decay rate $d\Gamma/d\omega$ can be computed only for $k_\perp^2 \neq 2m_e$. Moreover, as the singularities of $|f(x)|^2$ at $k_\perp = 2m_e$ are not integrable, the total decay rate $\Gamma$ above the pair creation threshold is ill-defined. Physically speaking, we have not taken into account the finite lifetime of neutrinos and photons in magnetic fields. Usually, the finite lifetime is incorporated by the replacement of the energies $E$ by complexified energies $E - \frac{i}{2} \Gamma(E)$ in the denominators, where $\Gamma(E)$ is the decay width of the state with energy $E$. We note that, although this replacement seems natural, it is not unambiguous. To proceed, we assume that the decay widths of the neutrinos are neglectable compared to those of photons and insert the total decay width

$$\Gamma^{\text{tot}} = \Gamma_{\text{pair}}(\omega) + \Gamma_{\text{split}}(\omega)$$

in the energy denominators of $\Omega$ and $\Phi$. Here, $\Gamma_{\text{pair}}$ denotes the decay width of the process $\gamma \rightarrow 2e$ \footnote{If neutrinos are Majorana particles, the decay rates should be divided by two.} and $\Gamma_{\text{split}}$ the decay width of the process $\gamma \rightarrow 2\gamma$ \footnote{If neutrinos are Majorana particles, the decay rates should be divided by two.}.

Let us now consider the two analytically tractable limits $p_\perp \ll 2m_e$ and $p_\perp \gg 3$.
2m_e. The function f(x) behaves as f(x) \sim \frac{3}{2} x for x \to 0 and goes to 1 for x \to \infty. Inserting this and eq. (13) into eq. (11), we obtain

\[ \Gamma = \frac{\alpha G_F^2}{15 \cdot 288 \pi^4 E} (E \sin \theta)^6 \left( \frac{B}{B_{cr}} \right)^2 \] for \( m_H \ll p_\perp \ll 2m_e \) \hspace{1cm} (15)

\[ \Gamma = \frac{\alpha G_F^2 m_e^5}{8 \pi^4 E} \chi_e^2 \] for \( 2m_e \ll p_\perp \ll m_W \) \hspace{1cm} (16)

where \( \theta \) denotes the angle between the magnetic field \( \vec{B} \) and \( \vec{p} \). The first approximation was already derived in Ref. [6]. Note that the energy dependence of the total decay rate is the same as that for photon splitting.

We now discuss our numerical results, which were all computed using the approximation eq. (13) and with \( B = 10 B_{cr}, \theta = 90^\circ \). In Fig. 2 the differential decay rate \( d\Gamma / d\omega \) (normalized to unity when integrated over the abscissa) is shown for energies of the initial neutrino below (left) and above (right) the pair-creation threshold as a function of the energy \( \omega / E_1 \) of the emitted photon. The energy distribution between the photon and the light neutrino becomes more and more asymmetric for \( E \to 2m_e \), and finally, for \( E > 2m_e \), the resonance for \( \omega = 2m_e \) appears. For \( \omega = 100m_e \), the differential decay rate has a plateau similar to the case of photon splitting.

A comparison between the total decay rate eq. (11), its low-energy approximation eq. (15) and its high-energy approximation eq. (16) is made in Fig. 3. It can be seen that the exact rate is well reproduced by the approximations except for small region around \( 2m_e \).

Finally, we have computed the lifetime \( \tau \) of the heavy neutrino as a function of the age \( t \) of the Universe. Hereby we have assumed that the magnetic field scales like \( B(t) = B(t_0)(t_0 / t)^2 \), and used as the starting value \( B_0 \) the limits obtained in ref. [13] and [14] for the onset of nucleosynthesis at \( t_0 \approx 1s \). If the lifetime is below \( t = \tau \) and the temperature \( T \) of the Universe lower than \( m_H \), the heavy neutrino decouples. Decoupling occurs for \( m_H > 11 \text{MeV} \) using \( B_0 = 2 \times 10^{13} \text{G} \) and for \( 4 \text{MeV} \) using \( B_0 = 1 \times 10^{15} \text{G} \). Therefore, the frequently discussed \( \tau \)-neutrino with \( m_H \sim \text{MeV} \) [18] is possible in this scenario without introduction of new interactions.

### 3 Summary

We have presented the first analysis of the rate of radiative neutrino decay not restricted to momentum of the initial neutrino \( p_\perp \ll 2m_e \) in the limit of strong magnetic fields \( B \gg B_{cr} \). We have derived two simple approximations
for the total decay rate valid for $p_{\perp} \ll 2m_e$ and $p_{\perp} \gg 2m_e$ in this limit. The
decay rates obtained are extremely enhanced by the strong magnetic field. In
particular, if a strong primordial magnetic field existed in the early Universe,
it is possible that a heavy neutrino with only standard model interactions may
have decayed before the onset of nucleosynthesis.

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Fig. 1. Feynman diagram of radiative neutrino decay $\nu_H \to \nu_L + \gamma$ in the limit of four-fermion interaction.

Fig. 2. Differential decay rate $d\Gamma/d\omega$ (normalized to unity when integrated over the abscissa) as a function of $\omega/E_1$ for $B = 10m_e^2/e$ for (top) $E_1/m_e = 0.1$ (full), 1.5 (dashed) and 1.99 (dotted line), and (bottom) $E_1/m_e = 2.1$ (full), 10 (dashed) and 100 (dotted line).
Fig. 1. Total decay rate $\Gamma / s^{-1}$ for $B = 10m_e^2 / e$ as a function of $E / m_e$ (diamonds: exact rates, dashed straight lines: low-energy and high-energy approximation, respectively).