From Digital to Analogue Magnetoelectronics: Theory of Transport in Non-Collinear Magnetic Nanostructures

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The date of receipt and acceptance will be inserted by the editor

Abstract Magnetoelectronics is mainly digital, i.e. governed by up and down magnetizations. In contrast, analogue magnetoelectronics makes use of phenomena occurring for non-collinear magnetization configurations. Here we review theories which have recently been applied to the transport in non-collinear magnetic nanostructures in two and multiterminal structures, viz. random matrix and circuit theory. Both are not valid for highly transparent systems in a resistive environment like perpendicular metallic spin valves. The solution to this problem is a renormalization of the conventional and spin-mixing conductance parameters.

1 Introduction

The giant magnetoresistance, as well as most of the current magnetoelectronics, can be understood in terms of the transport of electrons in either spin-up or spin-down state, since the magnetizations are collinear (parallel or antiparallel) with the spin-quantization axis. Both charge and spin transport can be described in terms of two “channels” with spin-dependent conductivities, scattering rates etc. [1]. This “digital” magnetoelectronics does not profit from the “analogue” freedom of a magnetization to point in any direction. Early seminal contributions by Slonczewski [2] and Berger [3].
revealed fundamentally new physics and technological possibilities of non-collinearity, which triggered a large number of experimental and theoretical studies. An important example is the non-equilibrium spin-current induced torque (briefly, spin torque) which one ferromagnet can exert on the magnetization vector of a second magnet through a normal metal in a biased spin valve structure. This torque can be large enough to dynamically turn magnetizations \[4\], which is potentially interesting as a low-power switching mechanism for magnetic random access memories \[5\]. Non-collinear magnetizations are also essential for novel magnetic devices like the spin-flip \[6,7\] and spin-torque transistors \[8\], detection of spin-precession \[9\], the Gilbert damping of the magnetization dynamics in thin magnetic films \[10\], and spin-injection induced by ferromagnetic resonance \[11\].

We are interested in heterostructures containing band ferromagnets that are accurately described by a Stoner spin-density functional model. Elemental metals and its alloys have high electron densities and their thin-film heterostructures are usually considerably disordered. Size quantization effects on transport can therefore mostly be disregarded \[12\]. Semiclassical methods are appropriate in slowly varying bulk regions of the structures, but heterointerfaces can be atomically sharp and must be treated fully quantum mechanically. There are basically two methods which are suitable to understand and compute transport properties of these systems from first principles, \textit{viz.} Green function theory with configurational averaging \[13\] and the scattering formalism for transport, combined with random matrix theory \[14\]. These two approaches have recently been extended to non-collinear magnetic structures, \textit{viz.} magnetoelectronic circuit \[6\] and random matrix theory \[15\]. Here, as in \[16\], we show that both approaches are closely related, but do not hold for transparent interfaces. Following Schep’s \[17\] strategy for collinear systems, both theories can be generalized, leading to analytical results for perpendicular spin valves with parameters that can be obtained by \textit{ab initio} band structure calculations, as well as determined quantitatively by experiments.

2 Boltzmann and Diffusion Equation

When a local non-equilibrium magnetization does not point in the direction of the spin-quantization axis, the distribution function for band states at the Fermi energy with index \(n\) is a matrix in Pauli spin space

\[
\tilde{f}_n (\mathbf{r}) = \begin{pmatrix}
    f_{\uparrow\uparrow} (\mathbf{r}) & f_{\uparrow\downarrow} (\mathbf{r}) \\
    f_{\uparrow\downarrow} (\mathbf{r}) & f_{\downarrow\downarrow} (\mathbf{r})
\end{pmatrix}
\Rightarrow f_n^c (\mathbf{r}) \mathbf{I} + \mathbf{\sigma} \cdot \mathbf{f}_n^s (\mathbf{r}).
\]  

On the right hand side the distribution is expanded into unit matrix and the vector of the Pauli spin matrices. \(f_n^c\) is charge accumulation and the spin accumulation \(f_n^s\) is a vector whose direction is always parallel to the magnetization vector \(\mathbf{m}\) in the bulk of a ferromagnet, but arbitrary in a normal metal depending on device configuration and applied biases. \(\tilde{f}_n\) can
be diagonalized by unitary rotation matrices in spin space, characterized by the polar angles $\theta$ and $\phi$. Let us assume for simplicity that these angles are piecewise constant in position space, thus disregarding magnetic domain walls \[18\] and magnetic field-induced spin precession in normal metals \[9\]. In the local spin quantization frame the distribution function is diagonal with two spin components $s = \pm 1$. Introducing spin-conserving and spin-flip scattering life times $\tau_s$ and $\tau_{sf}$, for the sake of argument taken to be state-independent, and separating the distribution into an isotropic electrochemical potential $\mu_s$ and an anisotropic term $\gamma_{ns}$ that vanishes when averaged over the Fermi surface, the Boltzmann equation for the stationary state reads \[19\]

$$
\mathbf{v}_{ns} \cdot \nabla \left( \gamma_{ns} + \mu_s \right) + \left( \frac{1}{\tau_s} + \frac{1}{\tau_{sf}} \right) \gamma_{ns} = \frac{\mu_{-s} - \mu_s}{\tau_{sf}}. 
$$

(2)

where $\mathbf{v}_{ns}$ is the group velocity of state $ns$. Charge and spin currents read

$$
\mathbf{j}_c = \frac{e}{h} A \sum_{ns} \mathbf{v}_{ns} \gamma_{ns},
$$

(3)

$$
\mathbf{j}_s = \frac{1}{4\pi A} \sum_{ns} \mathbf{v}_{ns} s \gamma_{ns}.
$$

(4)

The Boltzmann equation is still unnecessarily complicated for most realistic systems. In the presence of sufficient disorder, only the lowest harmonics of $\gamma_{ns}$ in reciprocal space survive. In that limit, the Boltzmann equation reduces to the diffusion equation

$$
\nabla^2 \left[ \mu_s (\mathbf{r}) - \mu_{-s} (\mathbf{r}) \right] = \frac{\mu_{-s} (\mathbf{r}) - \mu_s (\mathbf{r})}{\ell_{sd}^2}.
$$

(5)

$\ell_{sd} = \sqrt{D/\tau_{sf}}$ is the spin-flip diffusion length, which does not depend on spin index \[20\]. The spin-averaged diffusion coefficient $D$ can be written in terms of the density of states at the Fermi energy $N_s (E_F)$

$$
\frac{1}{(N_\uparrow (E_F) + N_\downarrow (E_F)) D} = \frac{1}{N_\uparrow (E_F) D_\uparrow} + \frac{1}{N_\downarrow (E_F) D_\downarrow}.
$$

(6)

in terms of the spin-dependent diffusion coefficients. In a simple two-band model $D_s = v_s \tau_s / 3$, where $v_s$ are the spin-dependent Fermi velocities. The average spin-flip relaxation time is defined as

$$
\frac{1}{\tau_{sf}} = \frac{1}{\tau_{sf,\uparrow}} + \frac{1}{\tau_{sf,\downarrow}}.
$$

(7)

The currents

$$
\mathbf{j}_s (\mathbf{r}) = -\frac{\sigma_s}{e} \nabla \mu_s (\mathbf{r})
$$

(8)

are governed by the spin-dependent conductivities

$$
\sigma_s = N_s (E_F) e^2 D_s.
$$

(9)
3 Boundary Conditions

The semiclassical approach is valid when the potential landscape varies slowly on the scale of the Fermi wave length. In heterostructures we often encounter regions in which materials change on an atomic scale, such as at intermetallic interfaces or tunnel junctions, which have to be treated quantum mechanically. The “nodes” are the bulk regions, in which the semiclassical distributions are well defined. The intermediate scattering regions, or “contacts”, can then be treated formally exactly by boundary conditions, which link the distributions of two neighboring nodes. Consider the spin valve structures in Fig. 1, which may be part of a larger circuit. We denote the distribution functions in the ferromagnetic terminals by subscripts $L$ and $R$. We explicitly allow for a drifting distribution by the superscript $\alpha = \pm 1$, which indicates whether drift is in ($\alpha = 1$) or against ($\alpha = -1$) the transport direction (from left to right). Taking into account the difference between the left- and right-moving distribution functions is the key generalization of the previous theories in our treatment here. In Sec. 5 it is discussed how the circuit theory can be recovered by renormalizing the conductance parameters. Here we concentrate on scattering theory.

In order to work with simple matrices instead of diadics, we follow Waintal et al. [15] and introduce the $4 \times 1$ vector representation $[f^\alpha_n(r)]^T = (f_{\uparrow\uparrow}^\alpha(r), f_{\uparrow\downarrow}^\alpha(r), f_{\downarrow\uparrow}^\alpha(r), f_{\downarrow\downarrow}^\alpha(r))^T$. The boundary conditions for the non-equilibrium distributions to the left and right of the scattering region then read:

$$f^+_{R,n} = \sum_{m \in L} (\hat{T}_{L \rightarrow R})_{nm} f^+_{L,m} + \sum_{m \in R} (\hat{R}_{R \rightarrow L})_{nm} f^-_{R,m},$$

$$f^-_{L,n} = \sum_{m \in L} (\hat{R}_{L \rightarrow R})_{nm} f^+_{L,m} + \sum_{j \in R} (\hat{T}_{R \rightarrow L})_{nm} f^-_{R,m}. \tag{11}$$

$\hat{T}$, $\hat{R}$ are $4 \times 4$ transmission and reflection probability matrices and the subscripts indicate the direction of the currents ($L \rightarrow R$ denotes transmission
from left to right, \( R \rightarrow R \) reflection from the right, etc. All matrix elements follow from the scattering matrix and are conveniently normalized, for example

\[
\left[ (\hat{T}_{L\rightarrow R})_{nm} \right]_{SS'} = \frac{1}{N^F_S} (t^{L\rightarrow R}_{nm})_S (t^{L\rightarrow R}_{nm})^{\dagger}_{S'}.
\] (12)

The transmission amplitudes, such as \( t^{L\rightarrow R}_{n,m,s',s} \) of a wave coming in from the left as mode \( m \) and spin \( s' \) and going out in mode \( n \) and spin \( s, \) are here collected in vectors \( t^{L\rightarrow R}_{nm} = (t^{L\rightarrow R}_{\uparrow \uparrow}, t^{L\rightarrow R}_{\uparrow \downarrow}, t^{L\rightarrow R}_{\downarrow \uparrow}, t^{L\rightarrow R}_{\downarrow \downarrow})^T_{nm} \). We also have \( S \in [1, 4], N^F_S = N^F_{\uparrow} (\delta_{S,1} + \delta_{S,2}) + N^F_{\downarrow} (\delta_{S,3} + \delta_{S,4}), \) where \( N^F_S \) is the number of modes for spin \( s, \) in the ferromagnet.

In a nutshell, this is a very general formulation of charge and spin transport, but it is not yet amenable for analytic treatment or analysis of experiments. The isotropy assumption that reduced the Boltzmann to the diffusion equation in the previous chapter, enormously simplifies the results, as demonstrated in the following.

We focus here on the electrical charge current as a function of the magnetization configuration in symmetric spin valves, as in Fig. 1(b),(c), in order to keep the analytical manipulations manageable. We will see later that we can derive rules from these results that are valid for general structures. \( \hat{T} \) and \( \hat{R} \) are functions of the magnetic configuration, which, disregarding magnetic anisotropies, can be parameterized by a single polar angle \( \theta. \) In first instance, we disregard spin-flip scattering and discuss later how it can be included. Integrating over the lateral coordinates leaves a position dependence only in the transport direction (\( x \)). The next step is the assumption that the distribution functions for incident electrons from the left and right are isotropic in space. The distribution functions for the outgoing electrons do not have to be isotropic, as long as they are subsequently scrambled in the nodes. The isotropy assumption may be invoked when the nodes are diffuse or chaotic, such that electrons are distributed equally over all states at the (spin-dependent) Fermi surfaces (which is equivalent to replacing state dependent scattering matrix elements by its average \[17\]). The Fermi surface integration is then carried out easily, and the distribution functions within left and right ferromagnet nodes (at locations \( x_L \) and \( x_R \), respectively) are matched via simplified boundary conditions

\[
\begin{align*}
    f^+ (x_R) &= \hat{T}_{L\rightarrow R} (\theta) f^+ (x_L) + \hat{R}_{R\rightarrow R} (\theta) f^- (x_R), \\
    f^- (x_L) &= \hat{R}_{L\rightarrow L} (\theta) f^+ (x_L) + \hat{T}_{R\rightarrow L} (\theta) f^- (x_R),
\end{align*}
\] (13a)

(13b)

where the \( 4 \times 4 \) transmission and reflection probability matrices have elements like \[17\]:

\[
\left[ \hat{T}_{L\rightarrow R} \right]_{SS'} = \frac{1}{N^F_S} \sum_{mn} (t^{R\rightarrow L}_{nm})_S (t^{R\rightarrow L}_{nm})^{\dagger}_{S'}. \]
(14)
In the coordinate systems defined by the magnetization directions, the transverse components of the spin accumulation in the ferromagnets vanish identically \[6,21\] and the distributions in the magnets depend on the local spin current densities \(\gamma_s\) and chemical potentials \(\mu_s\) only

\[
f^\pm(x_{L/R}) = \left((\pm\gamma^\uparrow + \mu^\uparrow)(x_{L/R}), 0, (\pm\gamma^\downarrow + \mu^\downarrow)(x_{L/R})\right).
\]

By this choice the explicit angle-dependence of transport is contained only in the matrices.

We can now link an arbitrary distribution on the left to compute the distributions on the right, subject to the constraint of charge current conservation. Here we focus on the simple case in which we apply a bias

\[
\Delta\mu = \sum_s (\mu_s(x_L) - \mu_s(x_R)),
\]

but no spin accumulation gradient \(\mu_s(x_L) - \mu_{-s}(x_L) = \mu_s(x_R) - \mu_{-s}(x_R)\) over the system. We then find that \(\gamma_s(x_L) = \gamma_s(x_R)\), i.e. the spin current component parallel to the magnetization on left and right ferromagnets are the same. The charge current

\[
I_c = \frac{e^2}{h} \sum_s N^F_s \gamma_s
\]

divided by the chemical potential drop is the electrical conductance \(G = I_c/\Delta\mu\). Eqs. \[13,15\] then lead to

\[
G = \frac{2e^2}{h} \sum_{S=1,4} \sum_{S'=1,4} \left\{N^F_S \left[\hat{T}_{L\rightarrow R} + \hat{R}_{R\rightarrow L}\right]^{-1} \hat{T}_{L\rightarrow R}\right\}_{SS'}.
\]

When the transparency is small, all transmission probabilities are close to zero, reflection probabilities are close to unity, and the Landauer-Büttiker conductance, starting point of \[15\], is recovered:

\[
G \rightarrow \frac{e^2}{h} \sum_{S=1,4} \sum_{S'=1,4} \left\{N^F_S \hat{T}(\theta)\right\}_{SS'}.
\]

Indeed, in this limit the distributions to the left and right are not perturbed by the current, the nodes are genuine reservoirs, and standard scattering theory applies. Also, when \(\theta = 0, \pi\), Eq. \[18\] is equivalent to results by Schep et al. \[17\] for the two-channel model.

The scattering region is still not specified and may be interacting and/or quantum coherent. We now discuss how analytical results can be obtained in the non-interacting, diffuse limit.
4 Semiclassical Concatenation

The scattering matrix of a composite system can be formulated as concatenations of the scattering matrix from separate elements, e.g., the scattering matrices of bulk layers and interfaces \[22\]. By assuming isotropy, i.e., sufficient disorder or chaotic scattering, Waintal et al. \[15\] proved by averaging over random scattering matrices that size quantization effects like the equilibrium exchange coupling or other phase coherent phenomena are destroyed by disorder and vanish like the inverse of the number of modes. Under these conditions we are free to define nodes in the interior of the device and link them via the boundary conditions \[13\]. This is equivalent to composing the total transport probability matrices in Eq. (18) in terms of those of individual elements by semiclassical concatenation rules \[23\]. The \(4 \times 4\) transmission probability matrix through a \(F(0)/N/F(\theta)\) double heterojunction as in Fig. 1 in which bulk scattering is absent, takes the form

\[
\hat{T}(\theta) = \hat{T}_{N\rightarrow F}(\theta) \left[ 1 - \hat{R}_{N\rightarrow N}(0) \hat{R}_{N\rightarrow N}(\theta) \right]^{-1} \hat{T}_{F\rightarrow N}(0),
\]

(20)

where the interface transmission and reflection matrices as a function of magnetization angle appear. The transformations needed to obtain \(\hat{T}_{N\rightarrow F}(\theta)\) and \(\hat{R}_{N\rightarrow N}(\theta)\) require some attention. In terms of the spin-rotation

\[
\hat{U} = \begin{pmatrix} \cos \theta/2 & -\sin \theta/2 \\ \sin \theta/2 & \cos \theta/2 \end{pmatrix}
\]

(21)

and projection matrices \((s = \pm 1)\)

\[
\hat{u}_s(\theta) = \frac{1}{2} \begin{pmatrix} 1 + s \cos \theta & s \sin \theta \\ s \sin \theta & 1 - s \cos \theta \end{pmatrix},
\]

(22)

the interface scattering coefficients (omitting the mode indices for simplicity) are transformed as follows \[6\]

\[
\hat{r}_{N\rightarrow N} = \hat{U} \hat{r}_{cN} \hat{U}^\dagger = \sum_s \hat{u}_s t_s^{cN},
\]

(23)

\[
t^{F\rightarrow N}_{ss'} = U_{ss'} t^{F}_{s'} ,
\]

(24)

\[
t^{N\rightarrow F}_{ss'} = t^{cN} U_{ss'} ,
\]

(25)

\[
t^{F\rightarrow F}_{ss'} = t^{cF} \delta_{ss'} .
\]

(26)

The superscript \(c\) indicates that the matrices should be evaluated in the reference frame of the local magnetization, and are thus diagonal in the absence of spin-flip relaxation scattering at the interfaces. Different transformation properties for the different elements of the scattering matrix derive from our choice to use local spin-coordinate systems that may differ for each magnet. Let us, for example, inspect a transmission matrix element from the normal metal into the ferromagnet with magnetization rotated by \(\theta\)

\[
[T_{N\rightarrow F}(\theta)]_{11} = \frac{1}{N_F} \sum_{mn} t^{R\rightarrow L}_{n\uparrow m\uparrow} (t^{R\rightarrow L}_{n\uparrow m\uparrow})^\dagger = \frac{1}{2} (1 + \cos \theta) \frac{1}{N_F} \sum_{mn} |t^{cN}_{n\uparrow m\uparrow}|^2
\]

(27)
and analogously for the other matrix elements as well as other matrices.

Transport through a more complex system can be treated by repeated concatenation of two scattering elements in terms of reflection and transmission matrices analogous to Eq. (20). In the presence of significant bulk scattering, we can represent a disordered metal \( B \) with thickness \( d_B \) by diagonal matrices like [17,15]

\[
\left( \tilde{T}_B \right)_{SS'} = \left( 1 + \frac{1}{N_s} + \frac{e^2 \rho_s^B d_B}{\hbar A_B} \right)^{-1} \delta_{SS'}, \tag{28}
\]

where \( \rho_s^B, A_B \) are the single-spin bulk resistivities and cross section of the bulk metal (normal or magnetic).

The interface parameters of the present theory are the spin-dependent Landauer-Büttiker conductances

\[
g_s = \sum_{lm} |t_{lm,s}^c|^2 = N_N - \sum_{lm} |r_{lm,s}^c|^2 = \sum_{lm} |r_{lm,s}^c|^2 = N_S - \sum_{lm} |r_{lm,s}^c|^2 \tag{29}
\]

and the real and imaginary part of the spin-mixing conductance

\[
g_{s-s} = N_N - \sum_{lm} r_{lm,s}^c \left( r_{lm,-s}^c \right)^* ,
\]

which can also be represented in terms of the total conductance \( g = g_\uparrow + g_\downarrow \), polarization \( p = (g_\uparrow - g_\downarrow) / g \), and relative mixing conductance \( \eta = 2g_\downarrow g_\uparrow / g \).

The actual concatenation of the \( 4 \times 4 \) matrices defined here is rather complicated even when using symbolic programming routines. This explains why in [15] analytic results were obtained only in special limiting cases. We found that final results are simple even in the most general cases, not only for Eq. (19) considered by [15], but also for Eq. (18). For the spin valves in Fig. 1, we find for the conductance as a function of angle

\[
G(\theta) = \frac{\tilde{g}}{2} \left( 1 - \frac{\tilde{\eta}^2}{1 + \tilde{\eta}^2 \frac{1 + \cos \theta}{1 - \cos \theta}} \right), \tag{30}
\]

where \( \tilde{\eta} = 2\tilde{g}_\uparrow / \tilde{g} \) and

\[
\begin{align*}
\frac{1}{g_s} &= \frac{1}{g_s} + \frac{e^2 \rho_F \rho_N}{\hbar 2A_F} + \frac{e^2 \rho_N d_N}{\hbar 2A_N} = \frac{1}{N_S^F} + \frac{1}{N_N^F} - \frac{1}{2N_{FS}^F} - \frac{1}{2N_{NN}^F} \tag{31} \\
\frac{1}{\tilde{g}_\uparrow} &= \frac{1}{g_\uparrow \downarrow} + \frac{e^2 \rho_N d_N}{\hbar 2A_N} = \frac{1}{2N_{NN}} \tag{32}
\end{align*}
\]

Equation (30) is identical to the angular magnetoresistance derived by circuit theory [4] after replacement of \( \tilde{g}_s \) and \( \tilde{g}_\uparrow \downarrow \) by \( g_s \) and \( g_\uparrow \downarrow \). Physically, in Eqs. (31,32) spurious Sharvin resistances are subtracted from the interface resistances obtained by scattering theory, whereas bulk resistances are added. These corrections are large for transparent interfaces and essential to obtain agreement between experimental results of transport experiments
in CPP (current perpendicular to plane) multilayers \cite{25,26,27} and first-principles calculations, for conventional \cite{17,28,29} as well as mixing conductances \cite{7}. The mixing conductance parameterizes the magnetization torque due to a spin accumulation in the normal metal, governed by the reflection of electrons from the normal metal. It is therefore natural that the mixing conductance is reduced by the bulk resistance of the normal metal and we can also understand that only the normal metal Sharvin resistance has to be subtracted. The real part of the mixing conductances is often close to the number of modes in the normal metal $g_{\uparrow \downarrow} \approx \frac{N}{N}$, in which case $\tilde{g}_{\uparrow \downarrow} \approx \frac{N}{N}/2$ \cite{30}. By letting $N_{s}^{F} \rightarrow \infty$ we are in the regime of \cite{15}. The circuit theory is recovered when, additionally, $N_{N} \rightarrow \infty$. The bare mixing conductance is bounded not only from below $\text{Re} g_{\uparrow \downarrow} \geq \frac{g}{2}$ \cite{6}, but also from above $|g_{\uparrow \downarrow}|^{2}/\text{Re} g_{\uparrow \downarrow} \leq 2N_{N}$.

5 Extended Circuit Theory

It is not obvious how these results should be generalized to more complicated circuits and devices as well as to the presence of spin-flip scattering in the normal metal. The magnetoelectronic circuit theory \cite{6} does not suffer from these drawbacks. Originally, it was assumed in Ref. \cite{6} that local spin and charge currents through the contacts only depend on the generalized potential differences, and the local node chemical potentials are obtained by a spin-generalization of the Kirchhoff laws of electrical circuits. This is valid only for highly resistive contacts, such that the in and outgoing currents do not significantly disturb the quasi-equilibrium distribution of the nodes. Fortunately we are able to relax this limitation and take into account a drift term in the nodes as well. In order to demonstrate this, we construct the fictitious circuit depicted in Fig. 2. Consider a junction that in conventional circuit theory is characterized by a matrix conductance $\hat{g}$, leading to a matrix current $\hat{i}$ when the normal and ferromagnetic distributions $\hat{f}_{L}$ and $\hat{f}_{R}$ are not equal. When the distributions of the nodes are isotropic, we know from circuit theory that

$$\hat{i} = \sum_{ss'} (\hat{g})_{ss'} \hat{u}_{s} \left( \hat{f}_{L} - \hat{f}_{R} \right) \hat{u}_{s'},$$

(33)

where the projection matrices $\hat{u}_{s}$ are defined in Eq. \cite{22} and $(\hat{g})_{ss} = g_{s}$, $(\hat{g})_{s,-s} = g_{s,-s}$. Introducing lead conductances, which modify the distributions $\hat{f}_{L} \rightarrow \hat{f}_{1}$ and $\hat{f}_{2} \leftrightarrow \hat{f}_{R}$, respectively, we may define a (renormalized) conductance matrix $\hat{\tilde{g}}$, which causes an identical current $\hat{i}$ for the reduced (matrix) potential drop:

$$\hat{i} = \sum_{ss'} (\hat{\tilde{g}})_{ss'} \hat{u}_{s} \left( \hat{f}_{1} - \hat{f}_{2} \right) \hat{u}_{s'},$$

(34)
When the lead conductances are now chosen to be twice the Sharvin conductances, and using (matrix) current conservation

\[
\hat{i} = 2N_N \left( \hat{f}_L - \hat{f}_1 \right)
\]

(35)
\[
= \sum_s 2N_s^F \hat{u}_s \left( \hat{f}_2 - \hat{f}_R \right) \hat{u}_s,
\]

(36)

straightforward matrix algebra leads to the result that the elements of \( \hat{g} \) are identical to the renormalized interface conductances found above [Eqs. (31,32) without the bulk resistivities]. By replacing \( \hat{g} \) by \( \hat{g} \) we not only recover results for the spin valve obtained above, but we can now use the renormalized parameters also for circuits with arbitrary complexity and transparency of the contacts. Also spin-flip scattering in \( N \) can now also be included \[6\]; it does not affect the form of Eq. (37) either, but only reduces the parameter \( \tilde{\chi} \). Other effects of the spin-flip scattering are discussed in detail by Kovalev et al. \[24\].

6 Applications

Intermetallic interfaces in a diffuse environment (see Fig. 1c) have been studied thoroughly by the Michigan State University collaboration \[25\] and others \[26,27\] in perpendicular (CPP) spin valves. These experiments provided a large body of evidence for the two-channel (i.e. spin-up and spin-down) series resistor model and a wealth of accurate transport parameters such as the spin-dependent interface resistances for various material combinations \[17,27,28,29\]. In exchange-biased spin valves, it is possible to measure the electric resistance as a function of the angle between magnetizations, which
has been analyzed experimentally and theoretically \cite{31,32}. Pratt c.s. observed that experimental magnetoresistance curves \cite{33} could accurately be fitted by the form \cite{6}

\[ \frac{R(\theta) - R(0)}{R(\pi) - R(0)} = \frac{1 - \cos \theta}{\chi (1 + \cos \theta) + 2} \]  

(37)

According to the new insights described above, the free parameter $\chi$ is a function of renormalized microscopic parameters

\[ \chi = \frac{1}{1 - \tilde{p}^2 \text{Re} \tilde{\eta}} - 1 \]  

(38)

in terms of the relative mixing conductance $\tilde{\eta} = 2\tilde{g}_f / \tilde{g}$, the polarization $\tilde{p} = (\tilde{g}_f - \tilde{g}_d) / \tilde{g}$, and the average conductance $\tilde{g} = \tilde{g}_f + \tilde{g}_d$.

Experimental values for the parameters for Cu/Permalloy (Py) spin valves are $\tilde{\chi} = 1.2$ and $\tilde{p} = 0.6$ \cite{33}. Disregarding a very small imaginary component of the mixing conductance \cite{7}, using the known values for the bulk resistivities, the theoretical Sharvin conductance for Cu (0.55 · 10$^{15}$ Ω$^{-1}$m$^{-2}$/spin \cite{17}), and the spin-flip length of Py as the effective thickness of the ferromagnet ($\ell_{sd}^F = 5$ nm \cite{25}), we arrive at the bare Cu/Py interface mixing conductance $G_{F}^{0} = 0.39 (3) \cdot 10^{15}$ Ω$^{-1}$m$^{-2}$. This value may be compared with the calculated mixing conductance for a disordered Co/Cu interface (0.55 · 10$^{15}$ Ω$^{-1}$m \cite{7}). The agreement is reasonable, but leaves some room for material and device dependence that deserves to be investigated in the future. The mixing conductance can also be determined from the excess broadening of ferromagnetic resonance spectra. A larger mixing conductance in Pt/Py can be explained by the larger density of conduction electrons in Pt compared to Cu \cite{30}. Reasonable agreement between experiment and theory has been also found by Zwierzycki et al. \cite{34} for Fe/Au multilayers.

The spin torque on a ferromagnet \cite{2,15} equals the spin current through the interface with vector component normal to the magnetization direction and its evaluation is closely related to the charge conductance \cite{6,15}. An analytical expression for the spin valve reads:

\[ L(\theta) = -\frac{\tilde{g}_f \lvert \tilde{\eta} \rvert^2 \sin \theta}{1 - \cos \theta + \lvert \tilde{\eta} \rvert^2 (1 + \cos \theta)} \frac{\Delta \mu}{8\pi} \]  

(39)

Note that here the imaginary part of the mixing conductance is taken into account explicitly, but the torque remains coplanar to the magnetization of the contacts, i.e. an out-of-plane “effective” field vanishes identically. Previous results \cite{2,15} are recovered in the limit that $\tilde{\eta} \to 2$ and $\tilde{p} \to 1$. By the generalized circuit theory it is now straightforward to compute the torque on the base contact of the spin-flip transistor with antiparallel source-drain magnetizations \cite{7}. Let us assume that the three contacts are identical, and the base contact magnetization lies in the plane of the source and drain.
Fig. 3 The spin-accumulation induced magnetization torque for a two-terminal spin valve and a three-terminal spin-flip transistor. $\Delta \mu$ is the source-drain bias and all contact parameters are taken to be the same, with $\text{Re} \eta = 2$ and $\text{Im} \eta = 0$

magnetizations. Assuming that we may disregard spin-flip scattering in the base contact, the in-plane torque $L_b$ turns out to be always larger than the spin valve torque $L$ in the two-terminal spin valve, Eq. (39), with a symmetric and flatter dependence on the angle of the base magnetization direction $\theta$ (see Fig. 3)

$$L_b(\theta) = -\frac{\tilde{g} \tilde{p} \text{Re} \tilde{\eta} \sin \theta}{(1 - \text{Re} \tilde{\eta}) \cos^2 \theta + \text{Re} \tilde{\eta} + \frac{6}{2 + |\eta|^2/(\text{Re} \eta)^2}} \frac{\Delta \mu}{4\pi}.$$  

(40)

In the presence of a significant imaginary part of the mixing conductance, we also find an out-of-plane (effective field) torque $L_\perp(\theta)$ with the same angular dependence and

$$\frac{L_\perp}{L_b} = -2 \frac{\text{Im} \tilde{\eta} \text{Re} \tilde{\eta}}{|\eta|^2 + 2 \text{Re} \tilde{\eta}}.$$  

(41)

Stiles and Zangwill [35] directly solved the Boltzmann equation for spin valves to obtain angular magnetoresistance and spin torque, approximating the mixing conductance by the number of modes (note that in a direct solution of the Boltzmann equation this parameters should not be renormalized). The numerical results agree well with the functional form (37) (M.D. Stiles, private communication). This function has been also derived by Slonczewski [36] and later by Shpiro et al. [37]. Slonczewski rederived this result with a simple circuit theory similar to that of [6] and also pointed out the relation between the angular magnetoresistance and the spin torque. Shpiro et al. [37] found the form (37) to be valid in the limit of vanishing exchange
splitting, thus in a regime different from the transition metal ferromagnets considered here [21].

7 Conclusions

We reported analytical results for the angular magnetoresistance of arbitrary spin valves, which, by comparison with experiments [33], leads to a value for the mixing conductance and spin torque for the Cu/Py interface of $G^{\uparrow\downarrow} = 0.39 (3) \cdot 10^{15} \Omega^{-1} \text{m}^{-2}$. The associated generalization of magnetoelectronic circuit theory opens the way to engineer materials and device configurations to optimize switching properties of magnetic random access memories. Mixing conductances determined by experiments or first principles theory are transferable to arbitrary devices and may be used for static as well as dynamic transport properties. The spin-dependent interface resistances determined by CPP-GMR transport experiments have played an important role in understanding “digital magnetoelectronics”. We hope that the spin-mixing conductances will play a comparable role in “analogue magnetoelectronics”.

Acknowledgements We profited from discussions with Bart van Wees, Paul Kelly, Alex Kovalev, and Yuli Nazarov, and have been supported by FOM, NSF Grant DMR 02-33773 and the NEDO joint research program “Nano-Scale Magnetoelectronics”.

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