Incoherent interaction of light with electron-acoustic waves

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(Dated: October 22, 2018)

Abstract

The equations governing the interaction between incoherent light and electron-acoustic waves are presented. The modulational instability properties of the system are studied, and the effect of partially coherent light is discussed. It is shown that partial coherence of the light suppresses the modulational instability. However, short wavelength perturbations are less affected, and will therefore dominate in, e.g. pulse filamentation. The results may be of importance to space plasmas and laser-plasma systems.

PACS numbers: 52.35.Mw, 52.35.Kt, 52.35.Ra
The investigation of the interaction between light and ion-acoustic waves was pioneered by Karpman in the early seventies [1, 2]. Since then there have been numerous studies devoted to the corresponding governing equations, which are known as the Karpman equations. For example, the latter admit shock-like solutions as well as envelope light solitons [1, 2, 3]. Recently, motivated by observational and experimental studies [4, 5, 6, 7, 8], the governing equations for light interacting with electron-acoustic waves (EAWs) were derived [9], using a two-species electron fluid model. Since ions are assumed immobile in Shukla et al. [9], the phenomena of light-EAW interactions takes place on a timescale shorter than the ion plasma period.

In this Brief Communication, we will develop a formalism suited for treating effects of partial coherence of the light source. The modulational instability properties of the resulting system of equations are analyzed, and the coherent and incoherent cases are contrasted. It is shown that partial coherence lead to damping of the modulational instability. However, in the limit of short wavelengths, the effect of a finite spectral width is suppressed, and such short wavelength modes are therefore more likely to yield, e.g. pulse filamentation. The results can be of importance in space plasmas and laser-plasma systems involving short laser pulses.

In Ref. [9], the equations governing the nonlinear interaction between coherent light waves and electron-acoustic waves were obtained. We consider the propagation of electromagnetic waves in a two population electron plasma, where one of the electron species is hot, while the other electron species is cold. The electromagnetic wave is given by the vector potential \( A(t,z) = \psi(t,z) \exp(ik_0z - i\omega_0t) \). The evolution of the slowly varying light wave envelope \( \psi(t,z) \) is given by the equation

\[
2i\omega \left( \frac{\partial}{\partial t} - v_g \frac{\partial}{\partial z} \right) \psi + c^2 \frac{\partial^2 \psi}{\partial z^2} - \omega_{ph}^2 (N_h + N_c \delta) \psi = 0,
\]

where \( v_g = k_0c^2/\omega_0 \) is the group velocity of the light wave, \( N_h = n_h/n_{h0}, \ N_c = n_c/n_{c0}, \ \delta = n_{c0}/n_{h0}, \ n_c (n_h) \) is the cold (hot) electron number density perturbation, \( n_{c0} (n_{h0}) \) is the cold (hot) electron background number density, and \( \omega_{ph} = (4\pi n_{h0}e^2/m_e)^{1/2} \) is the hot electron plasma frequency. Moreover, the hot electron density distribution is given by the Boltzmann distribution, i.e.

\[
N_h = \exp(\varphi - \Psi) - 1,
\]

where \( \varphi = e\phi/T_h \) is the normalized electron-acoustic wave potential and \( \Psi = e^2|\psi|^2/2m_e c^2 T_h \).
is the normalized ponderomotive force potential due to the light source. The cold electrons are determined by

\[
\frac{\partial N_c}{\partial t} + \frac{\partial}{\partial z} [(1 + N_c)v_c] = 0,
\]

(3a)

and

\[
\left( \frac{\partial}{\partial t} + v_c \frac{\partial}{\partial z} \right) v_c = V_{Th}^2 \frac{\partial (\varphi - \Psi)}{\partial z},
\]

(3b)

where \( V_{Th} = (T_h/m_e)^{1/2} \) is the thermal speed of the hot electrons, and \( v_c \) denotes the cold electron fluid velocity. Finally, the electron-acoustic potential is determined by Poisson’s equation

\[
\lambda_{Dh}^2 \frac{\partial^4 \varphi}{\partial z^4} = N_h + N_c \delta,
\]

(4)

where \( \lambda_{Dh} = (T_h/4\pi n_0 e^2)^{1/2} \) is the hot electron Debye length.

From Eqs. (2)–(4) we obtain, using \(|\varphi - \Psi| \ll 1\), the equation

\[
\left( \frac{\partial^2}{\partial t^2} - C_e^2 \frac{\partial^2}{\partial z^2} - \lambda_{Dh}^2 \frac{\partial^4}{\partial t^2 \partial z^2} \right) \varphi = \left( \frac{\partial^2}{\partial t^2} - C_e^2 \frac{\partial^2}{\partial z^2} \right) \Psi,
\]

(5)

where \( C_e = V_{Th} \delta^{1/2} \). Thus, Eqs. (1), (4), and (5) form the desired system of equations for coherent interaction between light and EAWs. In Ref. [9] it has been shown that (1), (4), and (5) admits localization and collapse of light pulses.

In order to account for the effects of partial coherence in the light source, we employ the Wigner formalism. Starting from the two-point correlation function of the field of interest (in our case the light field), one performs a Fourier transform of this correlation function and obtains a generalized distribution function \([10]\). Thus, we let the generalized distribution function for the light quanta be \( \rho(t, z, k) = 1/2\pi \int d\zeta e^{ik\zeta} \langle \psi^*(t, z + \zeta/2) \psi(t, z - \zeta/2) \rangle \),

(6)

where the angular brackets denotes the ensemble average. Then the light intensity is given by

\[
\Psi = \frac{e^2 |\psi|^2}{2m_e c^2 T_h} = \frac{e^2}{2m_e c^2 T_h} \int dk \rho(t, z, k).
\]

(7)

Applying the time derivative on the definition (6), and using Eq. (1), we obtain the kinetic equation

\[
\frac{\partial \rho}{\partial t} + \left( v_g + \frac{c^2 k}{\omega_0} \right) \frac{\partial \rho}{\partial z} - \frac{\omega_{pe} \lambda_{Dh}^2}{\omega_0} \left( \frac{\partial^2 \varphi}{\partial z^2} \right) \sin \left( \frac{1}{2} \frac{\partial}{\partial z} \frac{\partial}{\partial k} \right) \rho = 0
\]

(8)
for the light pseudo-particles. Here the arrows denotes the direction of operation, and the
sin-operator is defined in terms of its Taylor expansion. By assuming a light distribution
function with a finite spectral width, i.e. a finite spread in the light power spectrum, the
effect of partial coherence can be incorporated into the light propagation through the two-
electron plasma. Thus, Eqs. (5), (7), and (8) describes the interaction between partially
coherent light and EAWs.

Next we analyze the stability properties of the system (5), (7), and (8) by perturbing
these equations around a stationary plasma state (see, e.g. [11] for a similar treatment of
quantum plasmas). We let
\[
\rho(t, z, k) = \rho_0(k) + \rho_1(k) \exp(iKz - i\Omega t), \quad |\rho_1| \ll |\rho_0|,
\]
\[\varphi = \varphi_1 \exp(iKz - i\Omega t).\]
Moreover, we have \(\Psi = \Psi_0 + \Psi_1 \exp(iKz - i\Omega z),\) where \(|\Psi_1| \ll |\Psi_0|\).
Linearizing the system (5), (7), and (8) with respect to the first order quantities, we obtain
the nonlinear dispersion relation
\[
1 = -\frac{e^2}{2m_e c^2 T_h} \frac{\omega_{ph}^2 \lambda_{Dh}^2 K^2 \Lambda}{2c^2} \int dk \frac{\rho_0(k + K/2) - \rho_0(k - K/2)}{\Omega - (v_g + c^2 k/\omega_0) K} \rho_0(k) K^2/\Omega - \Psi_0 \frac{c^2 \omega_{ph}^2 \lambda_{Dh}^2 K^4}{2\omega_0^2} (\Omega^2 - C_e^2 K^2),
\]
(9)

In the case of coherent light we have \(\rho_0 = |\psi_0|^2 \delta(k - \kappa_0),\) and the nonlinear dispersion
relation (10) yields
\[
1 = -\frac{e^2}{2m_e c^2 T_h} \frac{\omega_{ph}^2 \lambda_{Dh}^2 K^2 \Lambda}{2c^2} \left[\frac{|\psi_0|^2}{|\kappa_0 - i\Delta - \omega_0(\Omega - v_g K)/c^2 K|^2 - K^2/4}\right]
\]
(10)
i.e.
\[
(\Omega^2 - C_e^2 K^2 + \lambda_{Dh}^2 K^2 \Omega^2) \left[\Omega - \left(v_g + \frac{c^2 \kappa_0}{\omega_0}\right) K\right]^2 - \frac{c^4 K^4}{4\omega_0^2} = -\Psi_0 \frac{c^2 \omega_{ph}^2 \lambda_{Dh}^2 K^4}{2\omega_0^2} (\Omega^2 - C_e^2 K^2),
\]
(11)

Suppose now that the pulse phase \(\Phi(z)\) experiences a random variation, which satisfies
(12) \(\langle \exp[-i\Phi(z + \zeta/2)] \exp[i\Phi(z - \zeta/2)]\rangle = \exp(-\Delta |\zeta|),\) where \(2\Delta\) is the full wavenumber
width at half maximum of the power spectrum. Then the corresponding distribution function
is given by the Lorentzian [12]
\[
\rho_0(k) = \frac{|\psi_0|^2}{\pi} \frac{\Delta}{\left(k - \kappa_0\right)^2 + \Delta^2},
\]
(12)
where \(\kappa_0\) is a wavenumber shifting the location of the maxima of the distribution function
\(\rho_0 k\) Inserting the expression (12) into the nonlinear dispersion relation (9) we obtain
\[
1 = -\frac{e^2}{2m_e c^2 T_h} \frac{\omega_{ph}^2 \lambda_{Dh}^2 K^2 \Lambda}{2c^2} \left[\frac{|\psi_0|^2}{|\kappa_0 - i\Delta - \omega_0(\Omega - v_g K)/c^2 K|^2 - K^2/4}\right],
\]
(13)
Comparing the dispersion relations (11) and (14), it can be seen that the spectral broadening will reduce the growth rate.

From now on we put the wavenumber shift $\kappa_0$ to zero, thus centering the Lorentzian distribution (12) around 0, as well as transforming to a comoving system, such that $v_g \to 0$.

We may use normalized and dimensionless variables defined by

$$K \to \frac{cK}{\omega_0}, \quad \Omega \to \frac{\Omega}{\omega_0}, \quad C_e \to \frac{C_e}{c}, \quad \lambda_{Dh} \to \frac{\omega_0 \lambda_{Dh}}{c}, \quad \Psi_0 \to \frac{\omega^2_{ph} \lambda^2_{Dh} \Psi_0}{2c^2}, \quad \text{and} \quad \Delta \to \frac{c\Delta}{\omega_0}. \tag{15}$$

Using these dimensionless variables, we have plotted the typical behavior of the growth rate $\Gamma = i(\text{Re} \Omega - \Omega)$, as given by the dispersion relations (11) and (14), in Fig. 1. The uppermost curve corresponds to $\Delta = 0$, i.e. the dispersion relation as given by (10). The growth rate asymptotically tends to 1 for the parameter values chosen in Fig. 1, and characteristically approaches a constant value for large wavenumbers if other parameter values are chosen. Moreover, the three lower curves has successively higher values of the spectral width $\Delta$. The damping character of a nonzero spectral width can clearly be seen. Furthermore, it is clear that for long wavelengths, i.e. small values of $K$, the damping may completely suppress the growth rate, although small values may even enhance the growth rate (cf. the solid and dashed curved for small $K$). However, for short wavelength modes, i.e. higher values of $K$, even the cases with nonzero $\Delta$ asymptotically tends to the same value of the growth rate as in the monochromatic case. Thus, short wavelength modes seem almost unaffected by the spectral broadening, and such short wavelength perturbations would therefore dominate in the contribution to the filamentation, as well as in soliton and shock wave formation of the incoherent light interacting with EAWs. In principle, these observations could be of importance in situations where the two-species electron model plays an important role, such as in space plasmas and laser-plasma systems [4, 5, 6, 7, 8].

To summarize, we have derived the governing equations for the interaction between light (coherent and incoherent) and EAWs. The limit of mono-chromatic light was analyzed, and compared to the case where spectral broadening was taken into account. It was shown that
the effect of a finite width in the power spectrum of the light in general was to suppress the modulational instability. However, for short wavelength modes the spectral broadening was shown only to have a small influence on the modulational instability growth rate, thus making these modes more likely to dominate in the filamentation of pulses and the formation of shocks and solitons in space and laser plasmas.
[1] V. I. Karpman, Plasma Phys. 13, 477 (1971).
[2] V. I. Karpman, Phys. Scr. 11, 263 (1975).
[3] V. I. Karpman, *Nonlinear Waves in Dispersive Media* (Pergamon Press, Oxford, 1975).
[4] R. E. Ergun, Y.-J. Su, L. Andersson, C. W. Carlson, J. P. McFadden, F. S. Mozer, D. L. Newman, M. V. Goldman, and R. J. Strangeway, Phys. Rev. Lett. 87, 045003 (2001).
[5] S.V. Singh and G.S. Lakhina, Planet. Space Sci. 49, 107 (2001).
[6] B.B. Kadomtsev and O.P. Pogutse, Nucl. Fusion 11, 67 (1971).
[7] W.D. Jones, A. Lee, S.M. Gleman, and H.J. Douce, Phys. Rev. Lett. 35, 1349 (1975).
[8] W. Choe, C.S. Chang, and M. Ono, Phys. Plasmas 2, 2044 (1995).
[9] P. K. Shukla, L. Stenflo, and M. Hellberg, Phys. Rev. E 66, 027403 (2002).
[10] E. P. Wigner, Phys. Rev. 40, 749 (1932).
[11] D. Anderson, B. Hall, M. Lisak, and M. Marklund, Phys. Rev. E 65, 046417 (2002).
[12] R. Loudon, *The Quantum Theory of Light* (Oxford University Press, Oxford, 2000).
FIG. 1: The growth rate $\Gamma$ plotted as a function of the wavenumber $K$ with the wavenumber shift $\kappa_0 = 0$. Here we have used the normalized variables (15). Moreover, we have used the values normalized values $C_e = 0.5$, $\lambda_{Dh} = 0.5$, $\Psi_0 = 0.5$. The solid black curve has $\Delta = 0$, the dashed curve $\Delta = 0.1$, the dotted curve $\Delta = 0.5$, and the solid gray curve uses $\Delta = 1$. The damping effect of a finite spectral width can clearly be seen, although the short wavelength modes are less affected. The asymptotic value of the growth rate in this case is 1.