Non-linear waves in Fermi liquids

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Abstract

I show that when non-linearities are taken into account the Landau theory of Fermi liquids predicts the existence of hyperbolic waves in fermionic systems. The zero sound is described by a infinite set of coupled non-linear partial differential equations, one for each harmonic of oscillation of the Fermi surface. When just a few harmonics are included it is possible to describe the dynamics of the first sound in a very simple way. These results lead to the interesting experimental possibility of having non-linear effects in quantum liquids. In particular I discuss the problem of generation of second harmonic in a Fermi system.

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It is already well established that the dynamics of sound waves in neutral systems such as $He^3$ or plasmons in charged systems can be well described by the Landau theory of Fermi liquids [1]. The usual approach is based on the linearized Boltzmann equation from which response functions and the dispersion of the collective modes are obtained. It is known that the same type of linearized equation leads to the correct description of the low energy excitations of the Tomonaga-Luttinger model in one dimension [2]. This result means that in one dimension, due to the lack of a particle-hole continuum at low energies and long wavelengths, the bosonic excitations are simply sound waves. It has been shown recently by the generalization of the bosonization procedure in higher dimensions that the actual bosonic excitations of fermionic systems at low energies are the displacements of the Fermi surface, that is, particle-hole continuum plus collective modes [3]. Thus although the quasiparticle residue is zero in an one dimensional model with interactions, the Landau expansion for the energy and the Boltzmann equation are still valid.

Amazingly enough, the Landau theory is capable of predicting the existence of a first order phase transition. This transition happens at the point where the compressibility vanishes (spinodal line) and the system becomes unstable to long wavelength fluctuations. The Landau theory predicts that there is an exponential growth of the collective mode inside the many-body system leading to phase separation. This is also known as Pomeranchuk’s instability [4] and has been used to study phase separation in mixtures of helium, processes involving heavy ion collisions at high energies [5] and more recently, the problem of phase separation in two dimensional systems such as some cuprates and nicklates [6,7].

The main result of the linearization of the Boltzmann equation is that it leads to plane waves which propagate in space with a dispersion defined by the poles of the density-density response function [1]. In the case of instabilities the frequency of oscillation of the wave is found to be imaginary leading to an exponential growth of the amplitude [5]. In principle, for short range interactions, the growth in unbounded. However, in real systems we always have long range forces (such as the Coulomb force) or dissipative mechanisms which tend to inhibit the growth of the wave leading to saturation of the wave amplitude [7].
I am going to show that, within the framework of Landau theory, if non-linear terms are taking into account, the physics of the waves which propagate in Fermi systems is not described by ordinary plane waves but by the so-called hyperbolic waves [8]. The non-linearities appear when we consider fluctuations of the Fermi surface which are comparable to the Fermi momentum. This is similar to the problem of the inclusion of large momentum scattering in one dimension (backscattering or umklapp) which leads to Sine-Gordon theories [9]. The terms that we are going to keep shall lead to non-linear bosonic theories in dimensions higher than one [3].

I am going to review the main points on the physics of waves in Landau theory. In this way it will become clear the origin of the non-linearities I am talking about. The starting point is the Landau theory which assumes that the local change in energy due to a local change in the quasiparticle distribution, \( \delta n(\vec{k}, \vec{r}, t) \), is given by,

\[
\Delta E(\vec{r}, t) = \sum_{\vec{k}} \left( \epsilon^0_k + U(\vec{r}, t) \right) \delta n(\vec{k}, \vec{r}, t) + \frac{1}{V} \sum_{\vec{k}, \vec{k}'} f_{\vec{k}, \vec{k}'} \delta n(\vec{k}', \vec{r}, t) \delta n(\vec{k}, \vec{r}, t) \tag{1}
\]

where \( \epsilon^0_k \) is the bare dispersion relation, \( U(\vec{r}, t) \) is an external potential applied to the system, \( V \) is the volume and \( f_{\vec{k}, \vec{k}'} \) is the quasiparticle two-body interaction (for simplicity I am going to assume that these interactions are homogeneous, that is, do not depend on the position of the quasiparticles).

Then the actual quasiparticle dispersion is obtained from (1),

\[
\epsilon_{\vec{k}}(\vec{r}, t) = \epsilon^0_k + U(\vec{r}, t) + \frac{1}{V} \sum_{\vec{k}'} f_{\vec{k}, \vec{k}'} \delta n(\vec{k}', \vec{r}, t). \tag{2}
\]

It will become clear that non-linearities appear because the quasiparticle dispersion depends on the quasiparticle distribution. This is equivalent to problem of flow of fluids where the velocity of propagation depends on the density. At this level the equations of motion for the quasiparticles are derived from (2) as,

\[
\frac{\partial \vec{r}}{\partial t} = \nabla_{\vec{k}} \epsilon_{\vec{k}}(\vec{r}, t) \\
\frac{\partial \vec{k}}{\partial t} = -\nabla_{\vec{r}} \epsilon_{\vec{k}}(\vec{r}, t). \tag{3}
\]
The main problem now is how to describe the evolution in time of the quasiparticle distribution given the equations (3). In the Landau theory the quasiparticle distribution, $n(\vec{k}, \vec{r}, t)$, changes because the scattering of quasiparticles and it is described by the Boltzmann equation,

$$\frac{dn(\vec{k}, \vec{r}, t)}{dt} = I(n)$$

(4)

where $I(n)$ is the collision integral. Eq.(4) is written in terms of the partial derivatives with help of (3) as,

$$\frac{\partial n(\vec{k}, \vec{r}, t)}{\partial t} + \vec{v}_k \cdot \nabla_{\vec{r}} n(\vec{k}, \vec{r}, t) - \nabla_{\vec{k}} \epsilon_k n(\vec{k}, \vec{r}, t) \cdot \nabla_{\vec{r}} \epsilon_k(\vec{r}, t) = I(n).$$

(5)

As usual we expand the distribution around its equilibrium value, that is,

$$n(\vec{k}, \vec{r}, t) = n_0(k) + \delta n(\vec{k}, \vec{r}, t)$$

(6)

where $n_0(k) = \Theta(\mu - \epsilon^0_k)$ with $\mu$ the chemical potential. Substituting (2) and (6) in (5) and using that $\nabla_{\vec{k}} \epsilon^0_k = \vec{v}_k$ and $\nabla_{\vec{k}} n_0(k) = -\vec{v}_k \delta(\mu - \epsilon^0_k)$ we easily find,

$$\frac{\partial \delta n(\vec{k}, \vec{r}, t)}{\partial t} + \vec{v}_k \cdot \nabla_{\vec{r}} \delta n(\vec{k}, \vec{r}, t) + \delta(\mu - \epsilon^0_k) \left( \vec{v}_k \cdot \nabla_{\vec{r}} U(\vec{r}, t) + \frac{1}{V} \sum_{\vec{k}'} f^\dagger_{\vec{k}', \vec{k}} \vec{v}_{\vec{k}'} \cdot \nabla_{\vec{r}} \delta n(\vec{\vec{k}'}, \vec{r}, t) \right)$$

$$+ \nabla_{\vec{k}} \delta n(\vec{k}, \vec{r}, t) \cdot \nabla_{\vec{r}} U(\vec{r}, t) + \frac{1}{V} \sum_{\vec{k}'} \left( \nabla_{\vec{k}} f^\dagger_{\vec{k}', \vec{k}} \cdot \nabla_{\vec{r}} \delta n(\vec{k}, \vec{r}, t) \right) \delta n(\vec{\vec{k}'}, \vec{r}, t)$$

$$- \frac{1}{V} \sum_{\vec{k}'} f^\dagger_{\vec{k}', \vec{k}} \nabla_{\vec{k}} \delta n(\vec{k}, \vec{r}, t) \cdot \nabla_{\vec{r}} \delta n(\vec{\vec{k}'}, \vec{r}, t) = I(n).$$

(7)

I have kept all terms in the equation and I am not assuming that the deviations from equilibrium are small. Observe that the last two terms on the l.h.s. of (7) are not present in the usual approach of the Fermi liquid theory because they are second order in the deviations. These terms give rise to non-linear effects as I will show.

Since it is expected that the main physics of the problem depends only on the dynamics at the Fermi surface (which is usually the case in fermionic systems in any number of dimensions) the solution of (7) can be written as,

$$\delta n(\vec{k}, \vec{r}, t) = \delta(\mu - \epsilon^0_k) \rho(\vec{k}, \vec{r}, t)$$

(8)
where $\rho(\vec{k}, \vec{r}, t)$ obeys the following equation,

$$
\frac{\partial \rho(\vec{k}, \vec{r}, t)}{\partial t} + \vec{v}_k \cdot \left( \nabla \! \cdot \! U(\vec{r}, t) + \nabla \! \cdot \! \rho \rho(\vec{k}, \vec{r}, t) \right) + \frac{1}{V} \sum_{\vec{k}'} f_{\vec{k}, \vec{k}'} \vec{v}_k \cdot \nabla \! \cdot \! \rho(\vec{k}', \vec{r}, t)
$$

$$
+ \frac{1}{V} \sum_{\vec{k}'} \left( \nabla \! \cdot \! f_{\vec{k}, \vec{k}'} \cdot \nabla \! \cdot \! \rho(\vec{k}', \vec{r}, t) \right) \rho(\vec{k}', \vec{r}, t)
$$

$$
+ \frac{1}{V} \sum_{\vec{k}'} \left( \nabla \! \cdot \! f_{\vec{k}, \vec{k}'} \cdot \nabla \! \cdot \! \rho(\vec{k}', \vec{r}, t) \right) \rho(\vec{k}, \vec{r}, t) = \tilde{I}(\rho),
$$

where I have used $\rho_{\vec{k}} f_{\vec{k}, \vec{k}'} \nabla \! \cdot \! \delta(\mu - \epsilon_0^k) = -\delta(\mu - \epsilon_0^k) \nabla \! \cdot \! \delta(\rho_{\vec{k}} f_{\vec{k}, \vec{k}'}$ and defined $\sum_{\vec{k}'} = \sum_{k'} \delta(\mu - \epsilon_0^{k'})$.

For simplicity I assume a spherical Fermi surface. In this way I can simplify further the problem by expanding $\rho_{\vec{k}}$ in Legendre polynomials in three dimensions (a completely similar procedure can be done in two dimensions in terms of Fourier series [7]),

$$
\rho(\theta, \vec{r}, t) = v_F \sum_{l=0}^{\infty} u_l(\vec{r}, t) \ P_l(\cos \theta)
$$

where $\theta$ is the angle on the Fermi surface that parameterizes $\vec{k}$ and $v_F$ is the Fermi velocity. Analogously the quasiparticle interaction can also be expanded as,

$$
f_{\vec{k}, \vec{k}'} = \sum_{l=0}^{\infty} f_l \ P_l(\cos \theta_{\vec{k}, \vec{k}'}).
$$

Since in this paper I will consider the problem of instabilities and non-linearities in Fermi liquid theory I will keep only the two first harmonics in the interaction. Firstly it can be seen from (9) that in order to have non-linearities in the problem it is necessary to consider an interaction which has dispersion at the Fermi surface ($\nabla \! \cdot \! f_{\vec{k}, \vec{k}'} \neq 0$) that is, at least $f_1$ must be different from zero. Also, due to Galilean invariance [9][10], the effective mass of the quasiparticles depends only on $f_1$, namely, $m^* = m + \frac{N(0) f_1}{3}$, where $N(0) = \frac{1}{V} \sum_{k} \delta(\mu - \epsilon_0^k) = \frac{m^* v_F}{\pi^2}$ is the density of states at the Fermi surface. Secondly, the compressibility of a Fermi liquid, $\kappa$, depend on the parameter $f_0$ [9] ($\kappa = \frac{N(0)}{n(1 + N(0) f_0)}$ where $n$ is the density) and the instabilities to phase separation, or Pomeranchuk’s instabilities, appear for $N(0) f_0 < -1$ when the systems crosses the spinodal line [8]. Thus, in this approach it will be sufficient to consider only the first two harmonics, that is,
\begin{equation}
F(\vec{k}, \vec{k}') = F_0 + F_1 \cos(\theta_{\vec{k}, \vec{k}'})
\end{equation}

where $F_{\vec{k}, \vec{k}'} = N(0)f_{\vec{k}, \vec{k}'}$.

Notice that in eq. (9) we just need the following summation,

\begin{equation}
\frac{1}{V} \sum_{\vec{k}'} f_{\vec{k}, \vec{k}'} \rho_{\vec{k}'} = v_F \left( F_0 u_0 + \frac{1}{3} F_1 u_1 \cos \theta \right),
\end{equation}

which will be used later in the paper.

In momentum space the Fermi velocity and momentum are radial and written as, $\vec{v}_k = v_F \hat{r}$ and $\vec{k} = k_F \hat{r}$, where $v_F$ and $k_F$ are the Fermi velocity and momentum, respectively. For simplicity I will assume that the external potential depends only on the $z$ direction in real space, that is, $U(\vec{r}, t) = U(z, t)$. Thus it is natural to choose a reference frame in real space where the axis coincide with the axis in the momentum space, that is, $\hat{r} = \cos \theta \hat{z} + \sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y}$, $\theta = -\sin \theta \hat{z} + \cos \theta \cos \phi \hat{x} + \cos \theta \sin \phi \hat{y}$ and $\phi = -\sin \phi \hat{x} + \cos \phi \hat{y}$ where $\phi$ is the azimuthal angle. Thus the gradient in momentum space is simply $\nabla_{\vec{k}} = \frac{1}{k_F} \frac{\partial}{\partial \theta} + \frac{\hat{\phi}}{k_F \sin \theta} \frac{\partial}{\partial \phi}$. With this choice the propagation is one dimensional. Of course, more general frames can be chosen but this simple choice will help us to understand the dynamics of this system. Using the above expressions, from (9) and (13), one finds,

\begin{equation}
\frac{\partial \rho(\theta, z, t)}{\partial t} + v_F \cos \theta \left( \frac{\partial U(z, t)}{\partial z} + \frac{\partial \rho(\theta, z, t)}{\partial z} \right) + \\
+ v_F^2 \left[ F_0 \cos \theta \frac{\partial u_0(z, t)}{\partial z} + \frac{1}{3} F_1 \cos^2 \theta \frac{\partial u_1(z, t)}{\partial z} \right] \\
+ \frac{F_1 v_F}{3k_F} \sin^2 \theta \frac{\partial (u_1(z, t) \rho(\theta, z, t))}{\partial z} = I(u)
\end{equation}

which is the final set of equations which must be solved in order to get the dynamics of the system. Observe that the non-linearity (last term in the l.h.s. of (14)) comes with higher powers in $k_F^{-1}$ which shows that this term is important for fluctuations out of equilibrium. It is also easy to see that by projecting the above equation in each of its Legendre components the final set of coupled partial differential equations has the form $(U = 0)$,

\begin{equation}
\frac{\partial u_n(z, t)}{\partial t} + \sum_{m=0}^{\infty} C_{nm}\{u\} \frac{\partial u_m(z, t)}{\partial z} = I(u)
\end{equation}
where $C_{nm}(\{u\})$ is a velocity matrix which depends on the components $u_n$. This set describes a system of coupled hyperbolic waves (and, in particular, shock waves) where the damping depends now on the collision integral $I(u)$. Notice that this is a general result of the Landau theory which is obtained just from the fact that the quasiparticle dispersion depends on the quasiparticle distribution, eq.(2).

In order to understand and simplify the problem I consider the hydrodynamic limit where the collisions dominate, that is, when it is possible to have local thermal equilibrium in the system. In this case the sound mode that propagates is the first sound [1]. The dispersion of the sound mode can be obtained in Landau theory straight from the compressibility, $\kappa$ [1-7], and it has the dispersion,

$$\omega_q = c_s q$$

$$c_s = \sqrt{\frac{(1 + F_0) (1 + \frac{F_1}{3})}{3}} v_F.$$  \hfill (16)

The first sound propagates like a plane wave and at the spinodal line, that is, when $F_0 < -1$, the amplitude of the wave grows exponentially given rise phase separation. In the hydrodynamic limit, since the collisions dominate, the quasiparticles and collective modes decay. In this case it is sufficient to consider only the two first Fourier components in (10), that is,

$$\rho(\theta, z, t) \approx v_F \left[ u_0(z, t) + u_1(z, t) \cos \theta \right].$$  \hfill (17)

It is worth mention that in the collisionless limit, when the sound mode is the zero sound, it is necessary to solve the complete set (14), or at least, to calculate the non-linear response function of the system [11].

By substitution of (17) in (14) and projecting into the Legendre components we find the following set of coupled equations,

$$\frac{\partial u_0}{\partial t} + \frac{2F_1 v_F}{9k_F} u_1 \frac{\partial u_0}{\partial z} + \left[ \frac{v_F}{3} \left( 1 + \frac{F_1}{3} \right) + \frac{2F_1 v_F}{9k_F} u_0 \right] \frac{\partial u_1}{\partial z} = 0$$

$$\frac{\partial u_1}{\partial t} + \frac{4F_1 v_F}{9k_F} u_1 \frac{\partial u_1}{\partial z} + v_F (1 + F_0) \frac{\partial u_0}{\partial z} = -\frac{\partial U}{\partial z} - \frac{u_1}{\tau},$$  \hfill (18)
where I have introduced in the place of the collision integral the relaxation time $\tau$ in the second equation. This term is absent in the first equation because of the conservation of number of particles. Here I am going to assume that this term is independent of the position. In some cases we can consider $\tau$ to be momentum dependent which will lead to some interesting physics. Notice that (18) resembles the Navier-Stokes equation for a fluid with a frictional term. If $\tau^{-1}(\vec{r}) = \nu \nabla^2 \vec{r}$ where $\nu$ is the viscosity of the quantum fluid then we end up with a viscous term exactly as in the Navier-Stokes case. Already in this form, eq.(18) has an incredibly large number of interesting physical applications and can be studied in different ways. It would be interesting to apply the methods of critical phenomena to understand phase transitions in fermionic systems via renormalization group methods and relate these equations to the problem of formation of patterns. It is also worth to point out that if we use a viscous term in the equations we can define a Reynolds number, $R = c_s L/\nu$ where $L$ is some characteristic length of the system and we can talk about turbulence in Fermi liquids.

Observe that for $k_F \to \infty$ (18) reduces to the usual linear problem,

$$\begin{align*}
\frac{\partial \tilde{u}_0}{\partial t} + \frac{v_F}{3} \left(1 + \frac{F_1}{3}\right) \frac{\partial \tilde{u}_1}{\partial z} &= 0 \\
\frac{\partial \tilde{u}_1}{\partial t} + v_F (1 + F_0) \frac{\partial \tilde{u}_0}{\partial z} &= -\frac{\partial U}{\partial z} - \frac{u_1}{\tau}
\end{align*}$$

which can be solved by Fourier transform,

$$\begin{align*}
\tilde{u}_0(q, \omega) &= \frac{v_F}{3} \left(1 + \frac{F_1}{3}\right) \frac{q^2 U(q, \omega)}{\omega^2 - c_s^2 q^2 - i\frac{\omega}{\tau}} \\
\tilde{u}_1(q, \omega) &= \frac{q \omega U(q, \omega)}{\omega^2 - c_s^2 q^2 - i\frac{\omega}{\tau}}
\end{align*}$$

where $c_s$ is the sound velocity, given in (16), and $U(q, \omega)$ is the Fourier transform of $U(z, t)$. Observe that the poles of the solutions above, that is, the eigenfrequencies of the problem are given by,

$$\omega_q^\pm = -\frac{i}{2\tau} \pm \sqrt{\frac{(1 + F_0)(1 + F_1)}{3} v_F^2 q^2 - \frac{1}{4\tau^2}}.$$
When $\tau \to \infty$ the spectrum is the one expected for a hydrodynamic sound, eq.(16), there is no dissipation and the sound propagates as a plane wave. If $\tau$ is finite and $F_0 > -1$ ($\kappa > 0$) the the roots are pure imaginary numbers for $q \geq (\tau v_F \sqrt{4/3(1 + F_0)(1 + F_1/3)})^{-1}$, that is, $\omega_q^\pm = -i\Gamma^\pm(q)$ where $\Gamma^\pm(q)$ is a positive real number. It means that the plane wave disappears exponentially, that is, impurities kill the sound mode which is overdamped. For $F_0 > -1$ and $q \leq (\tau v_F \sqrt{4/3(1 + F_0)(1 + F_1/3)})^{-1}$ the sound mode is underdamped. However, if $F_0 < -1$ ($\kappa < 0$) the roots are purely imaginary and amplitude of the wave grows exponentially in time (in a time scale is $(\Gamma^-(q))^{-1}$). That is, impurities alone cannot stop the phase separation unless $\tau$ depends on the momentum $q$. However with a long range Coulomb repulsion one changes $F_0 \to F_0 + \frac{2\pi N(0)F^2}{\epsilon q}$ and the solution of the wave equation grows exponentially only for momentum transfer larger than $\sqrt{\frac{4\pi^2 m k_F (1 + F_1/3)}{\epsilon [1 + F_0]}}$. That is, there is a maximum size for the formation of an unstable droplet [7]. Once the instability develops in the system, the set (19) is not valid anymore and it is necessary to use the complete set (18) and consider non-linear effects.

In order to understand these effects I will consider the response of this system to a small perturbation $U$. I am going to write an expansion for $u_0$ and $u_1$ as,

\begin{align*}
u_0(z, t) &= \tilde{u}_0(z, t) + \delta u_0(z, t) \\
u_1(z, t) &= \tilde{u}_1(z, t) + \delta u_1(z, t)
\end{align*}

(22)

where $\tilde{u}_0(z, t)$ and $\tilde{u}_1(z, t)$ are given in (20) and are the linear response to the applied external potential. $\delta u_0(z, t)$ and $\delta u_1(z, t)$ are the non-linear response of the system to $U$. In particular we are interested in the case where they are proportional to $U^2$. By substitution of (22) in (18) and using (19) we easily find that,

\begin{align*}
\frac{\partial \delta u_0(z, t)}{\partial t} + \frac{v_F}{3} \left(1 + \frac{F_1}{3}\right) \frac{\partial \delta u_1(z, t)}{\partial z} &= -\frac{2F_1 v_F}{9k_F} \frac{\partial \tilde{u}_0(z, t) \tilde{u}_1(z, t)}{\partial z} \\
\frac{\partial \delta u_1(z, t)}{\partial t} + v_F (1 + F_0) \frac{\partial \delta u_0(z, t)}{\partial z} &= -\frac{2F_1 v_F}{9k_F} \frac{\partial \tilde{u}_1^2(z, t)}{\partial z}
\end{align*}

(23)

these equations can be solved by Fourier transform.
Suppose, for simplicity, that the external potential is monochromatic with some characteristic frequency $\Omega$ and wave vector $p$, that is, $U(k, \omega) = U_0 \delta(k - p) \delta(\omega - \Omega))$. Then we have,

$$
\tilde{u}_0(k, \omega) = \frac{c_s^2 U_0}{v_F (1 + F_0)} \frac{p^2}{\Omega^2 - c_s^2 p^2 - i \frac{\Omega}{\tau}} \delta(k - p) \delta(\omega - \Omega) \\
\delta u_0(k, \omega) = \frac{F_1 c_s^2 U_0^2}{9 \pi^2 (1 + F_0) k_F} \frac{p^2 \Omega^2}{\left(\Omega^2 - c_s^2 p^2\right)^2} \delta(k - 2p) \delta(\omega - 2\Omega) \tag{24}
$$

which means that the second harmonic is generated in the system. Thus a second peak must appear in an experiment which probes density fluctuations. The strength of this peak is proportional to the square of the strength of the imposed external potential. Observe that the ratio between the height of the peak of the first harmonic to the height of the second harmonic is given by,

$$
\frac{\tilde{u}_0(p, \Omega)}{\delta u_0(2p, 2\Omega)} = \frac{m^* - m}{3 \pi^2 m m^*} \frac{p^2 \Omega^2}{\left(\Omega^2 - c_s^2 p^2\right) \left(\Omega^2 - c_s^2 p^2 - i \frac{\Omega}{\tau}\right)} U_0 \tag{25}
$$

Experiments in $He^3$ have dealt only with linear effects which are described in (20) [16]. As far as I know, there have been no measurements of these interesting non-linear effects in bulk $He^3$. In particular it would be interesting to verify the existence of the second harmonic generation in $He^3$. The same effect was observed more than thirty years ago in non-linear optical systems [17].

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