Hybrid and Exotic Mesons from FLIC Fermions

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The spectral properties of hybrid meson interpolating fields are investigated. The quantum numbers of the meson are carried by smeared-source fermion operators and highly-improved chromo-electric and -magnetic field operators composed with APE-smeared links. The effective masses of standard and hybrid operators indicate that the ground state meson is effectively isolated using both standard and hybrid interpolating fields. Focus is placed on interpolating fields in which the large spinor components of the quark and antiquark fields are merged. In particular, the effective mass of the exotic \(1^{-+}\) meson is reported. Further, we report some values for excited mesonic states using a variational process.

1. INTRODUCTION

Major experimental efforts are currently aimed at determining the possible existence of exotic mesons; mesons having quantum numbers that cannot be carried by the minimal Fock space component of a quark-antiquark pair. Of particular mention is the proposed program of the GlueX collaboration associated with the forthcoming upgrade of the Jefferson Laboratory facility. The observation of exotic states and the determination of their properties would elucidate aspects of QCD which are relatively unexplored.

The quantum numbers \(J^{PC} = 0^{-+}, 0^{--}, 1^{-+},\) etc. cannot be carried by a quark-antiquark pair in a ground-state \(S\)-wave. Lattice QCD calculations exploring the non-trivial role of explicit gluon degrees of freedom in carrying the quantum numbers of the meson suggest that exotic meson states do indeed exist and have a mass the order of 2 GeV \cite{1}. These findings are further supported here.

2. SIMULATION METHODOLOGY

Exotic quantum numbers can be constructed by merging standard local interpolating fields \(\bar{q}^a(x)\Gamma q^b(x)\) with chromo-electric, \(E_i^{ab}(x)\), or chromo-magnetic fields, \(B_i^{ab}(x)\). The \(J^{PC}\) quantum numbers of the interpolator are derived from the direct product of those associated with the quark bilinear and \(E_i^{ab} (1^{--})\) or \(B_i^{ab} (1^{+-})\). For example, combining the vector current of the \(\rho\) meson with a chromo-magnetic field, \(1^{--} \otimes 1^{+-}\) provides \(0^{-+} \oplus 1^{-+} \oplus 2^{-+}\) with the \(0^{-+}: \bar{q}^a \gamma_i q^b B_i^{ab}\) (\(\pi\) meson) and the \(1^{-+}: \epsilon_{ijk} \bar{q}^i \gamma_j q^k B_j^{ab}\) (exotic). We restrict ourselves to the lowest energy-dimension operators, as these provide better signal with smaller statistical errors. Table \(1\) summarizes the standard and hybrid interpolating fields explored herein.

The formulation of effective interpolating fields for the creation and annihilation of exotic meson states continues to be an active area of research. Here we consider local interpolating fields. Gauge-invariant Gaussian smearing \cite{28} is applied at the fermion source \((t=3)\), and local sinks are used to maintain strong signal in the two-point correlation functions. Our experience is that smeared-smeared correlation functions suffer an increase in error bar size relative to smeared-local correlation functions. Chromo-electric and -magnetic fields are created from APE-smeared links \cite{24} at both the source and sink using the highly-improved \(O(a^4)\)-improved lattice field strength tensor \cite{3}. The smearing fraction
Table 1

\[ \begin{array}{cccc}
J^{PC} & q^{a}q^{a} & q^{a}\gamma_{4}q^{a} & -i\bar{q}^{a}\gamma_{5}\gamma_{j}E_{j}^{ab}q^{b} \\
0^{++} & 0^{+-} & 0^{-+} & 0^{--} \\
-\bar{q}^{a}\gamma_{j}E_{j}^{ab}q^{b} & -\bar{q}^{a}\gamma_{5}\gamma_{j}B_{j}^{ab}q^{b} & -\bar{q}^{a}\gamma_{5}\gamma_{4}q^{a} & -i\bar{q}^{a}\gamma_{5}\gamma_{j}E_{j}^{ab}q^{b} \\
-\bar{q}^{a}\gamma_{5}\gamma_{4}B_{j}^{ab}q^{b} & -\bar{q}^{a}\gamma_{5}\gamma_{4}E_{j}^{ab}q^{b} & -i\bar{q}^{a}\gamma_{5}\gamma_{j}q^{a} & -i\bar{q}^{a}\gamma_{5}\gamma_{j}q^{a} \\
-\bar{q}^{a}\gamma_{5}\gamma_{4}E_{j}^{ab}q^{b} & -\bar{q}^{a}\gamma_{5}\gamma_{4}E_{j}^{ab}q^{b} & \bar{q}^{a}\gamma_{4}\gamma_{j}B_{j}^{ab}q^{b} & \bar{q}^{a}\gamma_{4}\gamma_{j}E_{j}^{ab}q^{b} \\
-\bar{q}^{a}\gamma_{5}\gamma_{4}B_{j}^{ab}q^{b} & -i\bar{q}^{a}\gamma_{5}\gamma_{j}q^{a} & \bar{q}^{a}\gamma_{5}\gamma_{4}E_{j}^{ab}q^{b} & \bar{q}^{a}\gamma_{5}\gamma_{4}E_{j}^{ab}q^{b} \\
\end{array} \]

\[ \alpha = 0.7 \text{ (keeping 0.3 of the original link).} \]

Propagators are generated using the fat-link irrelevant clover (FLIC) fermion action \( \tilde{P}_C \) where the irrelevant Wilson and clover operators of the fermion action are constructed using fat links while the relevant operators use the untouched (thin) gauge links. FLIC fermions provide a new form of nonperturbative \( \mathcal{O}(\alpha) \) improvement \(^8\) where near-continuum results are obtained at finite lattice spacing. Access to the light quark mass regime is enabled by the improved chiral properties of the lattice fermion action \(^9\).

The following results are based on 222 mean-field \( \mathcal{O}(a^2) \)-improved Luscher-Weisz \(^10\) gauge fields on a \( 16^3 \times 32 \) lattice at \( \beta = 4.60 \) providing a lattice spacing of \( a \approx 0.122(2) \) fm set by the string tension \( \sqrt{\sigma} = 440 \text{ MeV} \).

### 3. RESULTS

Of the hybrid interpolators listed in Table \(^1\) only the interpolating fields merging the large spinor components of the quark and antiquark fields provide a clear mass plateau. Figure \(^1\) illustrates the effective masses \( M(t) = -\log(G(t + 1)/G(t)) \), obtained from the second and fourth pion \((0^{-+})\) interpolators of Table \(^1\) for our intermediate quark mass \( (m_{\pi}^2 \approx 0.6 \text{ GeV}^2) \). Here \( G(t) \) is the standard two-point function projected to zero three-momentum. Excellent agreement is seen between the standard and hybrid interpolator-based correlation functions. Similar results are seen in Fig. \(2\) comparing effective masses obtained from the first and third \( \rho \)-meson \((1^{--})\) interpolators of Table \(^1\). The effective mass plot for the exotic \( 1^{-+} \) meson is illustrated in Fig. \(4\) where a plateau at early times is observed confirming the existence of the exotic \( 1^{-+} \).

Table \(\) summarizes these preliminary results. Further work on this topic will focus on extrapolating results to physical quark masses and using variational techniques to extract the masses of excited mesonic states. This research is supported by the Australian National Computing Facility for Lattice Gauge Theory and the Australian Re-
Table 2
Meson masses as a function of the hopping parameter $\kappa$.

| $J^{PC}$ | Operator                     | $\kappa = 0.1260$ | $\kappa = 0.1266$ | $\kappa = 0.1273$ | $\kappa = 0.1279$ | $\kappa = 0.1286$ |
|----------|-------------------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| $\pi: 0^{-+}$ | $\bar{q}^a\gamma_5q^a$ | 0.937 ± 0.005     | 0.861 ± 0.006     | 0.764 ± 0.006     | 0.672 ± 0.007     | 0.545 ± 0.007     |
|          | $\bar{q}^a\gamma_5\gamma_4q^a$ | 0.932 ± 0.006     | 0.856 ± 0.007     | 0.761 ± 0.007     | 0.670 ± 0.007     | 0.548 ± 0.007     |
|          | $-\bar{q}^a\gamma_5B_{ij}^bq^b$ | 0.999 ± 0.062     | 0.918 ± 0.066     | 0.816 ± 0.072     | 0.722 ± 0.076     | 0.561 ± 0.081     |
| $b_1: 1^{+-}$ | $-i\bar{q}^a\gamma_5\gamma_4\gamma_5q^a$ | 1.675 ± 0.012     | 1.633 ± 0.013     | 1.585 ± 0.013     | 1.548 ± 0.015     | 1.513 ± 0.017     |
|          | $i\bar{q}^aB_{ij}^bq^b$ | 2.373 ± 0.256     | 2.319 ± 0.246     | 2.264 ± 0.233     | 2.225 ± 0.222     | 2.223 ± 0.224     |
| $\rho: 1^{-}$ | $-i\bar{q}^a\gamma_5q^a$ | 1.184 ± 0.007     | 1.131 ± 0.008     | 1.070 ± 0.009     | 1.016 ± 0.010     | 0.954 ± 0.013     |
|          | $-i\bar{q}^a\gamma_5B_{ij}^bq^b$ | 1.255 ± 0.138     | 1.178 ± 0.117     | 1.077 ± 0.086     | 0.977 ± 0.138     | 0.836 ± 0.174     |
|          | $i\bar{q}^a\gamma_5B_{ij}^bq^b$ | 1.244 ± 0.086     | 1.172 ± 0.087     | 1.079 ± 0.088     | 0.987 ± 0.092     | 0.861 ± 0.102     |
| $1^{+-}$ | $-\epsilon_{ijk}\bar{q}^a\gamma_5B_{ij}^bq^b$ | 2.892 ± 0.475     | 2.881 ± 0.475     | 2.884 ± 0.479     | 2.908 ± 0.500     | 2.969 ± 0.516     |

Figure 2. Effective mass plot for correlation functions of the standard $\rho$-meson interpolator $\bar{q}^a\gamma_5q^a$ and the hybrid $\rho$ interpolator $\bar{q}^a\gamma_5\gamma_4B_{ij}^bq^b$.

Figure 3. Effective mass plot of the $1^{-+}$ exotic meson obtained from the hybrid interpolating field $\epsilon_{ijk}\bar{q}^a\gamma_5B_{ij}^bq^b$.

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