Is the Unitarity of the quark-mixing-CKM-matrix violated in neutron $\beta$-decay?

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Measurements by various international groups of researchers determine the strength of the weak interaction of the neutron, which gives us unique information on the question of the quark mixing. Neutron $\beta$-decay experiments now challenge the Standard Model of elementary particle physics with a deviation, 2.7 times the stated error.

1 The Standard Model, quark mixing and the CKM matrix

This article is about the interplay between the Standard Model of elementary particle physics and neutron $\beta$-decay. Since the Fermi decay constant is known from muon decay, the Standard Model describes neutron $\beta$-decay with only two additional parameters. One parameter is the first entry $|V_{ud}|$ of the CKM matrix. The other one is $\lambda$, the ratio of the vector coupling constant and the axial vector constant. In principle, the ratio $\lambda$ can be determined from QCD lattice gauge theory calculation, but the results of the best calculations vary by up to 30%. In neutron decay, several observables are accessible to experiment, which depend on these parameters, so the problem is overdetermined and, together with other data from particle and nuclear physics, many tests of the Standard Model become possible. $|V_{ud}|$ results significantly from the neutron lifetime $\tau$ and the $\beta$-asymmetry parameter $A_0$.

As is well known, the quark eigenstates of the weak interaction do not correspond to the quark mass eigenstates. The weak eigenstates are related to the mass eigenstates in terms of a $3 \times 3$ unitary matrix $V$, the so-called Cabibbo-Kobayashi-Maskawa (CKM) matrix. By convention, the $u$, $c$ and $t$ quarks are unmixed and all mixing is expressed via the CKM matrix $V$ operating on $d$, $s$ and $b$ quarks. The values of individual matrix elements are determined from weak decays of the relevant quarks. Unitarity requires that the sum of the squares of the matrix elements for each row and column be unity. So far precision tests of unitarity might be altered when the CKM matrix is expanded to accommodate more generations [11, 12]. A deviation $\Delta$ has been related to concepts beyond the Standard Model, such as couplings to exotic fermions [13, 14], to the existence of an additional $Z$ boson [15, 16] or to the existence of right-handed currents in the weak interaction [17]. A non-unitarity of the CKM matrix in models with an extended quark sector gives rise to an induced neutron electric dipole moment that can be within reach of next generation of experiments [18].

Due to its large size, a determination of $|V_{ud}|$ is most important. It has been derived from a series of experiments on superallowed nuclear $\beta$-decay through determination of phase space and measurements of partial lifetimes. With the inclusion of nuclear structure effect corrections a value of $|V_{ud}| = 0.9740(5)$ [19] emerges in good agreement of different, independent measurements in nine nuclei. Combined with $|V_{us}| = 0.2196(23)$ from kaon-decays and $|V_{ub}| = 0.0036(9)$ from B-decays, this leads to $\Delta = 0.0032(14)$, signaling a deviation from the Unitarity condition by 2.3 $\sigma$ standard deviation. The quoted uncertainty in $|V_{ud}|$, however, is dominated by theory due to amount, size and complexity of theoretical uncertainties. Although the radiative corrections include effects of order $Z\alpha^2$, part of the nuclear corrections are difficult to calculate. Further, the change in charge-symmetry-violation for quarks inside nuclei results in an additional change in the predicted decay rate which might lead to a systematic underestimate of $|V_{ud}|$. A limit has been reached where new concepts are needed to progress. Such are offered by studies with neutron and with limitations with pion $\beta$-decay. The pion $\beta$-decay has been measured recently at the PSI. The pion has a different hadron structure compared with neutron or nucleons and it offers an other possibility in determining $|V_{ud}|$. The preliminary result is $|V_{ud}| = 0.9971(51)$ [20]. The somewhat large error is due to the small branching ratio of $10^{-8}$. 
Further information on the CKM matrix and the unitarity triangle are based on a workshop held at CERN \[12\] and a workshop held at Heidelberg \[11\].

2 Neutron-β-decay

In this article, we derive $|V_{ud}|$, not from nuclear β-decay, but from neutron β-decay data. In this way, the unitarity check of (1) is based solely on particle data, i.e. neutron β-decay, K-decays, and B-decays, where theoretical uncertainties are significantly smaller. So much progress has been made using highly polarized cold neutron beams with an improved detector setup that we are now capable of competing with nuclear β-decays in extracting a value for $V_{ud}$, whilst avoiding the problems linked to nuclear structure. A neutron decays into a proton, an electron and an electron antineutrino. Observables are the neutron lifetime $\tau$ and spins $\sigma_e$, $\sigma_p$, $\sigma_{\bar{\nu}}$, and momenta $p_e$, $p_{\nu}$, $p_{\bar{\nu}}$ of the electron, antineutrino and proton respectively. The electron spin, the proton spin and the antineutrino are not usually observed. The lifetime is given by

$$\tau^{-1} = C |V_{ud}|^2 (1 + 3\lambda^2) f^R (1 + \Delta_R),$$

(2)

where $C = G_F^2 m_e^2/(2\pi^3) = 1.1613 \cdot 10^{-4} \text{s}^{-1}$ in h = c = 1 units, $f^R = 1.71335(15)$ is the phase space factor (including the model independent radiative correction) adjusted for the current value of the neutron-proton transition energy and corrected by Marciano \[13\]. $\Delta_R = 0.0240(8)$ is the model dependent radiative correction to the neutron decay rate \[17\]. The β-asymmetry $A_0$ is linked to the probability that an electron is emitted with angle $\vartheta$ with respect to the neutron spin polarization $P = <\sigma_e>$:

$$W(\vartheta) = 1 + \frac{v}{c} PA \cos(\vartheta),$$

(3)

where $v/c$ is the electron velocity expressed in fractions of the speed of light. $A$ is the β-asymmetry coefficient which depends on $\lambda$. On account of order 1% corrections for weak magnetism, $g_V - g_A$ interference, and nucleon recoil, $A$ has the form $A = A_0(1 + A_{\mu m}(A_1 W_0 + A_2 W + A_3/W))$ with electron total energy $W = E_e/m_ec^2 + 1$ (endpoint $W_0$). $A_0$ is a function of $\lambda$

$$A_0 = -2\frac{\lambda(\lambda + 1)}{1 + 3\lambda^2},$$

(4)

where we have assumed that $\lambda$ is real. The coefficients $A_{\mu m}$, $A_1$, $A_2$, $A_3$ are from \[14\] taking a different $\lambda$ convention into consideration. In addition, a further small radiative correction \[15\] of order 0.1% must be applied. For comparison, information about $|V_{ud}|$ and $\lambda$ are shown in Fig. (1). The bands represent the one sigma error of the measurements. The β-asymmetry $A_0$ in neutron decay depends only on $\lambda$, while the neutron lifetime $\tau$ depends both on $\lambda$ and $|V_{ud}|$. The intersection between the curve, derived from $\tau$ and $A_0$, defines $|V_{ud}|$ within one standard deviation, which is indicated by the error ellipse. Other information on $|V_{ud}|$, derived from nuclear β-decay and higher quark generation decays, assuming the unitarity of the CKM matrix, are shown, too. As can be seen from Fig. (1), both the nuclear β-decay result from \[9\] and the neutron β-decay from \[16\] do not agree with this unitarity value.

3 The experiment PERKEO and the result for $|V_{ud}|$

The following section is about our measurement of the neutron β-asymmetry coefficient $A$ with the instrument PERKEOII, and on the consequences for $|V_{ud}|$. The strategy of PERKEOII followed the instrument PERKEO \[21\] in minimizing background and maximizing signal with a 4π solid angle acceptance over a large region of the beam. Major achievements of the instrument PERKEO are:

- The signal to background ratio in the range of interest is 200.
- The overall correction of the raw data is 2.04%.
- The detector design allows an energy calibration with linearity better than 1%.
New polarizers and developments in polarization analysis led to smaller uncertainties related to neutron beam polarization. For a measurement of $\beta$-asymmetry $A_0$, the instrument PERKEO was installed at the PF1 cold neutron beam position at the High Flux Reactor at the Institut Laue-Langevin, Grenoble. Cold neutrons are obtained from a 25 K deuterium cold moderator near the core of the 57 MW uranium reactor. The neutrons are guided via a 60 m long neutron guide of cross section $6 \times 12$ cm$^2$ to the experiment and are polarized by a $3 \times 4.5$ cm$^2$ supermirror polarizer. The de Broglie wavelength spectrum of the cold neutron beam ranges from about 0.2 nm to 1.3 nm. The degree of neutron polarization was measured to be $P = 98.9(3)\%$ over the full cross section of the beam. The polarization efficiency remained constant during the whole experiment. The neutron polarization is reversed periodically with a current sheet spin flipper. The main component of the PERKEO II spectrometer is a superconducting 1.1 T magnet in a split pair configuration, with a coil diameter of about one meter. Neutrons pass through the spectrometer, whereas decay electrons are guided by the magnetic field to either one of two scintillation detectors with photomultiplier readout. The detector solid angle of acceptance is truly $2\pi$ above a threshold of 60 keV. Electron backscattering effects, serious sources of systematic error in $\beta$-spectroscopy, are effectively suppressed. Technical details about the instrument can be found in [18, 19]. The measured electron spectra $N_i^{\uparrow}(E_e)$ and $N_i^{\downarrow}(E_e)$ in the two detectors $(i=1,2)$ for neutron spin up and down, respectively, define the experimental asymmetry as a function of electron kinetic energy $E_e$ and are shown in Fig. 3.

$$A_{i\exp}(E_e) = \frac{N_i^{\uparrow}(E_e) - N_i^{\downarrow}(E_e)}{N_i^{\uparrow}(E_e) + N_i^{\downarrow}(E_e)} \quad (5)$$

By using (3) and with $< \cos(\theta) > = 1/2$, $A_{i\exp}(E)$ is directly related to the asymmetry parameter

$$A_{\exp}(E_e) = A_{1\exp}(E_e) - A_{2\exp}(E_e) = \frac{v}{c} APf. \quad (6)$$
The experimental function $A_{\nu, \exp}(E_c)$ and a fit with one free parameter $A_{\nu, \exp}$ (the absolute scale of $A_0$) is shown in Fig. 5. The total correction for the small experimental systematic effects is 2.04%.

With recent experiments from the University of Heidelberg [16,17], we obtain $A_0 = -0.1189(7)$ and $\lambda = -1.2739(19)$. With this value, and the world average for $\tau = 885.7(7)$ s, we find that $|V_{ud}| = 0.9724(13)$. With $|V_{us}| = 0.2196(23)$ and the negligibly small $|V_{ub}| = 0.0036(9)$, one gets

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \Delta = 0.9924(28). \quad (7)$$

This value differs from the Standard Model prediction by $\Delta = 0.0076(28)$, or 2.7 times the stated error. Earlier experiments [21,22,23] gave significant lower values for $\lambda$. Averaging over our new result and previous results, the Particle Data Group [1] arrives at a new world average for $|V_{ud}|$ from neutron $\beta$-decay which leads to a 2.2 s deviation from unitarity.

An independent test of CKM unitarity comes from W physics at LEP [20] where W decay hadronic branching ratios can be expressed in terms of

$$\frac{Br(W \rightarrow q\bar{q})}{1 - Br(W \rightarrow q)} = (1 + \frac{\alpha}{\pi} \sum |V_{ij}|^2). \quad (8)$$

Since decay into the top quark channel is forbidden by energy conservation one would expect $\sum |V_{ij}|^2$ to be 2 with a three generation unitary CKM matrix. The experimental result is 2.039(25), consistent with (7) but with considerably lower accuracy.

4 The future

The main corrections in the experiment PERKEO are due to neutron beam polarization (1.1%), background (0.5%) and flipper efficiency (0.3%). The total correction is 2.04%. With such small corrections to the data, we start to see a deviation from the Standard Model already in the uncorrected raw data. For the future, the plan is further to reduce all corrections. In the meantime, major improvements both in neutron flux and degree of neutron polarization have been made: First, the new ballistic supermirror guide at the ILL from the University of Heidelberg gives an increase of a factor of 6 in the cold neutron flux [24]. Second, a new arrangement of two supermirror polarizers allows to achieve an unprecedented degree of neutron polarization $P$ of between 99.5% and 100% over the full cross section of the beam [25]. Third, systematic limitations of polarization measurements have been investigated: The beam polarization can now be measured with a completely new method using an opaque $^3$He spin filter with an uncertainty of 0.1% [26,27]. As a consequence, we are now in the lucky situation to improve on the main uncertainties in reducing the main correction of 1.1% to less than 0.5% with an error of 0.1%. Thus, a possible deviation from the Standard Model, if confirmed, will be seen very pronounced in the uncorrected data. Future trends have been presented on the workshop "Quark-mixing, CKM Unitarity" in Heidelberg from 19 to 20 September 2002. Regarding the Unitarity problem, about half a dozen new instruments are planned or are under construction to allow for beta-neutrino correlation $\alpha$ and beta-correlation $\lambda$ measurements at the sub-10^{-3} level.

5 Summary

$|V_{ud}|$, the first element of the CKM matrix, has been derived from neutron decay experiments in such a way that a unitarity test of the CKM matrix can be performed based solely on physical data. With this value, we find a 2.7 $\sigma$ standard deviation from unitarity, which conflicts the prediction of the Standard Model of particle physics.

Future trends have been presented on the workshop "Quark-mixing, CKM Unitarity" in Heidelberg, September 19-20, 2002. Regarding the Unitarity problem, about half a dozen new instruments are planned or under construction to allow for beta-neutrino correlation $\alpha$ and beta-asymmetry $\lambda$ measurements at the sub-10^{-3} level. With next generation experiments measurements with a decay rate of 1 MHz become feasible [28].

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