Long range triplet Josephson current and 0–π transitions in tunable domain walls

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Abstract
The order parameter of superconducting pairs penetrating an inhomogeneous magnetic material can acquire a long range triplet component (LRTC) with non-zero spin projection. This state has been predicted and generated in proximity systems and Josephson junctions. We show, using a realistic domain wall of an exchange spring bilayer, how the LRTC emerges and can be tuned with the twisting of the magnetization. We also introduce a new kind of Josephson current reversal, the singlet-LRTC 0–π transition, that can be observed in one and the same system either by tuning the domain wall or by varying temperature.

Keywords: magnetic Josephson junction, long range triplet component, magnetic domain wall, current reversal, singlet-triplet mixing

1. Introduction

A decade ago, Bergeret, Volkov and Efetov (BVE) predicted that under certain conditions a triplet component with non-zero spin projection ($S_z = \pm 1$) of the pair amplitude might arise in a magnetic material of a proximity system, even if the adjacent superconductor (S) has singlet pairing interactions [1–3]. The BVE prediction has since garnered great interest [4–23] and recent measurements of the critical current in magnetic Josephson junctions [24–30] and the critical temperature in proximity systems [31, 32] confirmed the existence of the triplet component. One interesting consequence of the BVE prediction is that the superconducting
condensate may penetrate into a magnetic material much deeper than expected for a pure $S_z = 0$ state. This is due to the fact that this long range triplet component (LRTC) is unaffected by the internal magnetic field. The LRTC extends over distances similar to those of singlet Cooper pairs in a normal metal.

In this paper, we consider a structure that allows tuning the LRTC and that leads to a new kind of singlet-LRTC $0-\pi$ transition of the Josephson current. We use an exchange spring (XS) with a realistic and mathematically known closed form [33, 34], tunable magnetic domain wall. This structure provides a unique way to harness the LRTC and offers new opportunities to study the coexistence of superconducting and magnetic phases in proximity systems. Superconducting-magnetic hybrid structures are versatile and a timely subject of study. Not only have they been shown to exhibit LRTC, they have also been studied more recently, mostly in the clean limit, to search for Majorana fermions or the anomalous Josephson effect. For example, [35–37] analyze the existence of a triplet state and Majorana fermions in magnetic systems with spin–orbit interaction on top of a superconductor. Reference [38] shows that Majorana fermions exist when the cryptoferromagnetic state (magnetic helical structure coexisting with the superconducting state) is established in the superconductor. The helical magnetic state of conduction electrons in a semiconducting nanowire with strong spin–orbit interaction also displays a variety of other effects among which the reduction of the Josephson critical current when the helical state is established [39]. Finally, several papers show the existence of a $0-\pi$ transition and an anomalous Josephson effect (the presence of a current at vanishing phase difference) through a nanowire that has strong spin–orbit interaction and is subject to a magnetic field along the wire [40–43]. These examples bear some relation to the present work in part because in an appropriately chosen basis the magnetic domain wall can be interpreted as a spin–orbit interaction. However, it is also worth noting that in the XS Josephson junction proposed here the superconducting gap and the rotating magnetic configuration are found in physically distinct layers. The structure is thus not suited for Majorana fermion physics. The anomalous Josephson effect may on the other hand be present and is a subject of future investigations.

Here we predict three effects occurring in a wide (with respect to $\xi_F$, see below) S/XS/S Josephson junction. First, we show that starting with a wide homogeneous XS, which has no LRTC and no Josephson current, the progressive winding up of the domain wall generates an increasing current directly attributable to the LRTC. Second, we propose a new kind of Josephson current reversal, the singlet-LRTC $0-\pi$ transition, that occurs because the current contribution emerging from the LRTC with the tuning of the domain wall opposes and overcomes the current produced by the singlet component. Third, we observe that for specific conditions the singlet-LRTC $0-\pi$ transition can be induced by varying temperature.

As explained below, these effects require a magnetization continuously rotating in space and have not been observed in multilayers of homogeneous ferromagnets or half metals, or in the presence of spin–orbit and spin–active interfaces of the type discussed in the works mentioned above. We also emphasize that the current reversal presented here is qualitatively different from the $0-\pi$ transition proposed by Buzdin and Bulaevskii, and discussed in the literature [4, 44]. Previously, both theory and experiment that studied the $0-\pi$ transition of the Josephson current at fixed temperature varied the thickness of a thin F over a few nanometers and used the oscillatory behavior of the short range component of the superconducting order parameter to reverse the direction of the current [45–50] ([3, 12] are an exception discussed later). This requires fabricating a new sample for each thickness under identical experimental
conditions. The tunable XS avoids the latter and leads to a singlet-LRTC 0–π transition in wide junctions.

2. The model

We consider the S/XS/S magnetic Josephson junction of figure 1(a). The XS is a bilayer of homogeneous Fs with uniaxial anisotropy (along $\hat{z}$) and different magnetic anisotropy energies that interact at their interface. In the classical continuum limit the energy of each layer ($j = 1, 2$) of the XS is given by

$$E_j = \int_C \left[ A_j \left( \frac{d\phi_j}{dx} \right)^2 - K_j \cos^2 \phi_j - H \cos \phi_j \right] dx,$$

where $\phi_j(x)$ is the angle of the magnetization with respect to the anisotropy axis (see figure 1(a)), $A_j$ is the exchange energy and $K_j$ is the anisotropy energy of layer $j$ and $H$ is an external field applied along the anisotropy axis. The integration $C$ is over the intervals $(-t_1, 0)$ for $j = 1$ and $(0, t_2)$ for $j = 2$ and $t_1$ ($t_2$) is the thickness of the hard (soft) ferromagnetic layer. The two layers interact at the interface with an energy given by $V_f = -A_f \cos (\phi_1 - \phi_2)$, where
$\phi_2 = \lim_{\eta \to 0} \phi_{2/1}(\pm \eta)$ and $x = 0$ is the location of the interface between hard and soft layers. A hard (soft) $F$ has a large (small) value $K_1(K_2)$ [51]. Minimizing the energy for different values of $H$ leads to the curves shown in figures 1(b) and (c) (these figures were obtained assuming a strong interaction $A_I$ between the soft and hard $F$ and thus no phase slip at the interface). The important parameters of the XS for the following considerations are the anisotropy constants ratio $K_1/K_2$, the intrinsic domain wall width $\delta_j = \sqrt{|A_j/K_j|}$ and the saturation magnetization $h_j$, where $j = 1, 2$ refer to the hard and soft $F$, respectively. Applying a small magnetic field $H$ in direction opposite to the equilibrium magnetization induces a partial to full domain wall. In an earlier work by one of the authors an analytic description of the domain wall in a XS was derived from the above energy [33]. That model provides an excellent description of observables, such as the hysteresis loop or the magnetoresistance of experimentally realized XSs [33, 51, 52]. The great advantage of the XS is that the magnetic configuration can be smoothly tuned from a homogeneous $F$ to a Bloch domain wall. We leverage this knowledge to offer a clear picture of the conditions under which a magnetic domain wall generates a LRTC of the superconducting order parameter. To keep the focus on the study of the LRTC in a domain wall, we perform calculations in the absence of the small magnetic field $H$ that is usually used to generate it in the XS. We address the practical implementation in the discussion section below.

We discuss two XSs with different magnetic properties: (1) Co/Py is a strong XS with moderate anisotropy ratio $K_1/K_2$ familiar in the field of magnetism. The magnetization can be tuned from homogeneous to a half Bloch domain wall (figure 1(b)) that leads to small but observable triplet currents, (2) Ni$_3$Mn/Ni is proposed here as a weak XS that under certain annealing conditions presents a strong anisotropy ratio [51]. The magnetic configuration in this XS ranges from a homogeneous to a full Bloch domain wall (figure 1(c)). We predict the latter system to generate a current an order of magnitude larger than presently measured on other systems or predicted with Co/Py.

All considerations are made in the diffusive limit where the elastic scattering length is much smaller than the superconducting coherence length $\xi_S = D_S/2\pi T_c$ ($D_S$ is the diffusion length of the $S$ and $T_c$ the critical temperature of the proximity system). In this quasi-classical regime the state of the system is determined by the Usadel equations [54] for the scalar Green function $g_0$, the Gor’kov function $f_0$ for singlet components and the vector functions $g, f$ to account for the possible presence of triplet components of the order parameter. Following Ivanov and Fominov, the functions can be parametrized as [11]

$$\begin{align*}
g_0 &= M_0 \cos \theta, \\
f_0 &= M_0 \sin \theta,
\end{align*}$$

and

$$\begin{align*}
g &= iM \sin \theta, \\
f &= -iM \cos \theta,
\end{align*}$$

(2)

where $M_0$ is the scalar component and $M = (M_x, M_y, M_z)$ denotes the triplet amplitude vector. All five unknowns depend on position in the $S$s and the $F$. For this parametrization the generalized Usadel equations take the form [11]

$$\begin{align*}
\frac{D}{2} \nabla^2 \theta - M_0(\omega_n \sin \theta - \Delta \cos \theta) - (h \cdot M) \cos \theta &= 0, \\
\frac{D}{2} (M \nabla^2 M_0 - M_0 \nabla^2 M) + M(\omega_n \cos \theta + \Delta \sin \theta) - h M_0 \sin \theta &= 0,
\end{align*}$$

(3)

together with the normalization condition $M_0^2 - |M|^2 = 1$ [55]. The equations are written in the Matsubara formalism with $\omega_n = (2n + 1)\pi T$ ($n \in \mathbb{Z}$) [11].
The five unknown functions have to be determined self-consistently with the pair potential $\Delta(x)$ in the S and the magnetization profile in the XS

$$\mathbf{h}_j(x) = -h_j \left[ \sin \phi_j(x) \hat{y} + \cos \phi_j(x) \hat{z} \right],$$

where $j = 1$ and 2 are the hard and soft F, respectively. The angle $\phi_j(x)$ represents the tunable domain wall of figure 1. For a BCS type S in the wide diffusive limit $\Delta(x) \equiv \Delta_{\text{BCS}}$ in the S and vanishes in the XS.

Equation (3) is solved numerically with the normalization condition and standard boundary conditions:

$$M_0 = 1, M_0 = 0, \theta = \arctan (\Delta_{\text{BCS}}/\omega_n)$$

at the outer edge of the S and transparent interfaces

$$\eta_\text{BCS} = \pm 0, \eta_\text{BCS} \to + 0 \text{ for } X = \theta, M_j (j = 0, x, y, z).$$

The two systems, S/XS and XS/S, are solved separately since in the wide limit the two contributions to Gor'kov functions are essentially additive (see e.g. [55]). Once the solutions are obtained one can determine the Josephson critical current with

$$I_c(x) = \frac{\pi T_{\text{eff}}}{e} \sum_{n \geq 0} \sum_{a = \{0, y, z\}} \text{Im} \left( f_{a, n}^* \frac{\partial f_{a, n}}{\partial x} \right),$$

where $\sigma_F$ is the conductivity of the ferromagnetic metal.

3. The LRTC

Some insight into the behavior of the LRTC can be gained from plotting the relative orientation of the Gor'kov vector function $\mathbf{f}(x)$ with respect to the magnetization $\mathbf{h}(x)$ in the XS. Figure 2 shows $\mathbf{f}(x)$ obtained from equation (3) starting from the S/XS interface (left side of the figure) and the magnetization vector $\mathbf{h}$ for a particular partial domain wall. A similar figure is obtained starting at the XS/S interface. Due to the FFLO effect [56, 57], the Gor'kov vector function (green arrows with variable length) is anti-aligned with $\mathbf{h}(x)$ at the interface (blue arrows with constant magnitude). This indicates that only $S_z = 0$ components are present. Deeper into the XS, $\mathbf{f}(x)$ rotates and decreases in magnitude due to the rotation of $\mathbf{h}(x)$. For this particular tuning of the domain wall, only the component of $\mathbf{f}$ perpendicular to $\mathbf{h}$ is left near the inflection point of $\phi(x)$. The latter indicates the presence of the LRTC ($S_z = \pm 1$) [11]. There remains a small component of $\mathbf{f}$ parallel to $\mathbf{h}$ that is associated with the short-range $S_z = 0$ contribution that will be discussed later (see figures 3 and 4, and section 4) [19].

The physical understanding of the generic behavior of the order parameter $\mathbf{f}$ in the XS domain wall shown in figure 2 is that, as singlet Cooper pairs penetrate into the F, the $S_z = 0$ triplet component is generated first and has a maximum close to the interface. Then, the rotation of the magnetization partially transforms the $S_z = 0$ triplet component into an LRTC with $S_z \neq 0$. The figure and the calculations emphasize that the required magnetization profile for the observation of an $S_z \neq 0$ LRTC is highly nonlinear with fairly constant values over the characteristic length $\xi_{\text{eff}} = \sqrt{D_F/h}$ ($D_F$ is the diffusion length) near the interfaces where a maximal $S_z = 0$ triplet component can be generated, followed by a rapid rotation that transforms the $S_z = 0$ component into an $S_z \neq 0$ LRTC. This confirms statements made in [11, 18].

The behavior described above is found in the XS and contrasts with the stiff functional form $h(x) = \cos (Qx)$ that is usually introduced to model a domain wall [2, 3, 9, 15, 16]. We also note that the simplified cosine-shaped domain wall only satisfies the magnetic boundary conditions at the edges of the ferromagnet associated to (1) for discrete values of $Q$ at given
thickness of the magnetic material. In contrast, the XS domain wall (figure 1) satisfies the magnetic boundary conditions at arbitrary twist [33]. This implies a change of the functional form of the domain wall as a function of the twist and a departure from the cosine form, as seen in figure 1. As mentioned above, this matters for the generation of the various Gor’kov components and their relative weight in particular near the superconductor-XS interface. We therefore expect our domain wall to offer a more accurate physical description of experiments performed with inhomogeneous magnetic Josephson junctions.

4. Tunable LRTC and singlet-LRTC 0–π transition

4.1. Tunable LRTC

The precise knowledge of the Green and Gor’kov functions determined from the known analytic form of the XS allows the calculation of the Josephson critical current through the wide S/XS/S junction using equation (5) and integrating over the thickness of the XS. This leads to the first prediction shown in figure 3 where the Josephson voltage \( V_c = I_c R_N \) (\( R_N \) is the normal state resistance of the junction) is plotted against the normalized twist angle of the domain wall across the XS \( \phi/\pi \equiv [\phi(-t_1) - \phi(t_2)])/\pi \).

Figure 3(a) is obtained for the strong XS Co/Py with a thickness much larger than \( \xi_F \) (\( d_F/\xi_F = 22.5, 25 \) and 27.5, \( d_F = t_1 + t_2 \)). Hence, in absence of a domain wall when \( \phi/\pi = 0 \), there is no measurable current flowing through the homogeneous magnetic Josephson junction. This reproduces the expected behavior of an S/F/S junction in the wide F limit. A remarkable effect occurs as one induces the domain wall into the XS (0 < \( \phi/\pi \) < 1): A current appears
and increases with the winding of the domain wall! From the discussion above, the growing current with increasing $\Delta \phi$ has its origin in the emergence of a LRTC. This is substantiated by the fact that the singlet component to the current also plotted on the figure is an order of magnitude smaller. It is noteworthy that an $S_z = 0$ component of the current is observed at all in such wide junction since this is the so-called short range component expected to decay over distances determined by $\xi_F$, which is of the order of the nanometer for figure 3(a). The existence of this contribution is related to the continuous rotation of the quantization axis and the mixing of the individual spin components [58].

The main implication of figure 3(a) is that an increasing current with the winding of the domain wall offers a new way to prove the existence of an LRTC generated by the inhomogeneous magnetization in the XS and to tune that component. Finally, the figure inset shows that the current decreases with increasing thickness of either parts of the XS. This is due to the associated decrease in curvature of $h(x)$ (or $\phi(x)$) with increasing thickness of the XS and the resulting damping of the $S_z = 0$ triplet component near the S/XS interfaces.

4.2. Singlet-LRTC $0-\pi$ transition with domain wall twist

The second prediction is shown in figure 3(b), where we plot $|V_c|$ as a function of domain wall twist for the weak XS Ni$_3$Mn/Ni. We note three salient features of this result. The total current (black solid line) undergoes a $0-\pi$ transition with increasing twist of the domain wall, as shown by the V-shape curve. This current reversal is qualitatively different from those discussed earlier in the literature in that the present transition is solely due to the emergence of the LRTC and its competition with the singlet component deep in the magnetic material. The reversal of the current with increasing twist of the domain wall results from the growing $S_z = \pm 1$ component current opposing and overcoming the $S_z = 0$ contribution (of which the singlet part turns out to
be dominant, see [58]). Note that the presence of an \( S_z = 0 \) current for \( \Delta \phi = 0 \) is due to the fact that the Ni\(_3\)Mn/Ni is a weak XS, revealing the importance of both the hardness and magnitude of the magnetization of the Fs composing the XS.

The second feature displayed in the inset of figure 3(b) is the shift of the 0–\( \pi \) transition to smaller torsions of the magnetization profile with increasing thickness of the XS. This shift emphasizes that the singlet-LRTC 0–\( \pi \) transition cannot be observed for arbitrary thicknesses of the XS. For a thick enough soft F layer, the 0–\( \pi \) transition disappears and we recover the behavior of figure 3(a).

The last feature of figure 3(b) is interesting for experimental endeavors: at fixed temperature the 0–\( \pi \) transition can be observed in one and the same sample by tuning the domain wall in the XS. A similar observation can be made from calculations on a trilayer [12]. Note, however, that the mechanisms by which the transition is generated in the latter case is discernibly different from that described here, as can be clearly seen by examining the role of the singlet and triplet contributions to the Gor’kov function. Our 0–\( \pi \) transition is due to competing singlet and triplet components deep in the wide bilayer while in the trilayer of [12] it results from a sign change of the triplet Gor’kov function on either side of the trilayer for large enough mismatch of the magnetization direction in the thin outer layers; the short-range \( S_z = 0 \) components are irrelevant in that system because they are completely screened out by the homogeneous middle layer for the thickness considered here. Reference [12] and our work contrast with all other models and experiments in which the 0–\( \pi \) transition was predicted and observed as a function of the variable thickness of the homogeneous F in the S/F/S junction, and therefore required a different sample for each thickness (this statement is naturally made for figure 3(b) discussed here and does not apply to the temperature dependence of figure 4 discussed in the next section).

**4.3. Singlet-LRTC 0–\( \pi \) transition with temperature**

In figure 4, we analyze the temperature dependence of the Josephson current for representative partial domain walls in XSs with weak (figure 4(a)) and strong (figure 4(b)) ferromagnetism and using the BCS temperature dependence of the pairing potential.
The third prediction is that the singlet-LRTC 0–π transition shown in figure 3(b) can also be induced with temperature, similarly to conventional 0–π transitions [48, 49]. Figure 4(a) shows that in absence of a domain wall (Δϕ = 0) the voltage decreases monotonically with temperature up to the critical temperature $T_c$ where it vanishes. As one induces the domain wall, a 0–π transition appears at low temperature. This transition moves to higher temperatures with increasing twist of the magnetic profile, reflecting a stronger triplet component of the order parameter.

Contrasting with this result, the strong XS Co/Py displays no 0–π transition (figure 4(b)) because the singlet component is very small (figure 3(a)). This is another consequence of different anisotropy ratios and magnitudes of the magnetization in the XS. Figure 4(b) reveals three further interesting properties. First, the magnitude of the critical current decays exponentially fast over a large temperature range (linear decay in the figure). Second, the higher the twist, the higher the voltage and the more linear the curve on the log-linear scale. Finally, in accordance with the wide limit considered here the critical temperature remains unchanged [59].

The calculations leading to figure 4 lead us to one last remark about the importance of parameter values and domain wall shape. A study of the 0–π transition with temperature in a single inhomogeneous layer was conducted in [3], using a cos ($Qx$) model for the domain wall. In contrast with our findings, they observe the disappearance of the temperature-induced 0–π transition with increased inhomogeneity. Next to the general comments made at the end of section 3 about the cosine-shaped domain wall model, we also note that the F in [3] has a thickness of the order of $\xi_F$ and thus has an important oscillating singlet contribution that leads to the conventional 0–π transition when $Q = 0$. Our continuous inhomogeneity generates an entangled state where a LRTC appears but also further increases the singlet contribution as shown in [58]. Also, our XS is an order of magnitude thicker than the magnetic layer in [3] and allows for a true competition between LRTC and $S_z = 0$ components. This leads to finding the new kind of 0–π transition presented here.

5. Discussion and conclusions

Figures 3 and 4 emphasize that the singlet-LRTC 0–π transition is not observable in just any S/XS/S junction. The intrinsic properties of the XS, and in particular the anisotropy ratio, the thicknesses of the hard and soft Fs, and the magnitude of the magnetization are determinant parameters. A large anisotropy ratio, soft Fs and thicknesses chosen to optimize the $S_z = 0$ triplet formation near the S/XS interfaces offer favorable conditions to observe the above phenomena.

Three corollaries of our analysis are worth mentioning. The LRTC can actually be observed in an FF’ bilayer system and leads to observe a 0–π transition in first harmonic provided that the two Fs interact magnetically, as is the case of the XS. Our proposed bilayer allows flexible modeling of different FF’ systems realized experimentally since a tunable magnetization slip can be generated at the interface of the XS by placing a non-magnetic layer between the Fs to adjust their mutual magnetic interaction [60]. The special case with no interaction is generally studied and in particular in a recent paper [23]; however, our results are obtained on a more involved magnetization profile that allows, for example, for the observation of the 0–π transition in first harmonic in our bilayer and only in second harmonic in [23]. The second corollary is that the triplet component can emerge in the XS with a domain wall stretching over thicknesses far exceeding the coherence length $\xi_F$. Finally, our calculations
demonstrate that it is not necessary to have a symmetric multilayer to have a Josephson current generated by the LRTC. The main requirement is that the Gor’kov functions of both Ss overlap in the magnetic barrier.

We studied theoretically a wide XS Josephson junction and demonstrated that as one increases the winding of the domain wall in the XS a LRTC appears that competes with the short-range components to bring about a new kind of 0–π transition. The question arises as to how one can implement the idea experimentally. The twist of the domain wall in the XS is usually obtained by applying a small magnetic field in direction opposite to the initial equilibrium magnetization and thus parallel to the surface of the multilayer structure (along \(-\mathbf{z}\) in the calculation). If the superconducting leads feel the applied field, a Fraunhofer pattern is generated and the measurement of the critical current is modified by the field\(^1\). Several approaches can be taken to address this issue. One is to assume that the magnetic field will only be present in the magnetic material, as was done for example in [41–43] where only the nanowire is subject to a Zeeman term. Another more realistic approach is to assume that the field will affect the superconducting films but to realize that the fields typically required to generate the domain wall are of the order of a few tens of Oersted, at least an order of magnitude smaller than the critical field of Nb thin film (\(\sim 10\) T) used in the calculations. This difference could be further enhanced by choosing another conventional superconductor with higher critical field, such as for example MgB\(_2\). The effects presented in the paper should thus be measurable provided one corrects for the Fraunhofer effect. A last and more interesting suggestion is that one may be able to avoid the magnetic field altogether by intercalating a third, very thin antiferromagnetic layer at the interface between the soft ferromagnet and the superconductor, and use the exchange bias effect to pin the direction of the soft ferromagnet’s magnetization away from its anisotropy axis. The standard procedure to achieve this goal can be followed. Starting at a temperature above the Néel temperature \(T_N\) of the antiferromagnet one applies a field at an arbitrary but fixed angle with respect to the uniaxial anisotropy axis of the XS. Lowering the temperature below \(T_N\) in the presence of the field determines the orientation of the antiferromagnet and the direction of magnetization at the outer edge of the soft F. The field can then be switched off. Due to the pinning of the soft ferromagnet by the antiferromagnet, one generates a stable domain wall in the XS in absence of an applied magnetic field. Following this procedure for different angles of the applied magnetic field, one can continuously vary the twist of the domain and reproduce the configuration discussed in the paper. We trust in the perspicacity of experimentalists to find further ways to implement the ideas enunciated in the paper. The multilayer structure involving an XS proposed here has already motivated the work done in [17] and also studied in [22], although these papers focused on the variation of \(T_c\). The study of the Josephson current through the domain wall of an XS is an experimental challenge that remains open.

As a final remark, we note that the results presented in this work have been obtained in the diffusive limit. We expect some of the effects to remain in the clean limit, especially regarding the new kind of 0–π transition. Studies are underway on XS Josephson junctions in that limit.

In conclusion, the Josephson junction comprised of two singlet-pairing Ss and a magnetic XS is unique in that it allows varying the long range triplet supercurrent through wide junctions.

\(^1\) To calculate the Fraunhofer pattern requires adding a dimension to the motion of quasiparticles. Since we solve the Usadel equations without simplifying assumptions (such as the usual linearization of the equations), the computational time incurred by this extension requires a different numerical approach.
by tuning the domain wall in the XS. Using an exact analytic expression for the domain wall in the XS [33], we offer insight in the relevant parameters and inhomogeneity required for the emergence of the LRTC. We propose an experiment where a triplet current emerges with the increasing twist of the domain wall. We also predict that in junctions involving weak XSs, the tuning of the domain wall or varying the temperature allows for the observation of a singlet-LRTC $0-\pi$ transition.

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