Synthesis of modal control in servo system with a differentiating observer

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Abstract. A method of synthesis of modal control in servo systems with a differentiating observer of the reference action is proposed. It is shown that with the joint design of the modal controller and the observer, the uncertainty of the synthesis problem arises, which leads to the oscillation of the regulated value under step setting actions. To eliminate this uncertainty, the transition to the optimization problem of control synthesis was carried out. A numerical example of the formulation and solution of this problem is presented, which confirms the effectiveness of the proposed method.

1. Introduction
At present, the problem of increasing the accuracy of servo systems is being solved based on the use of models of setting actions [1-3]. There are known methods for constructing such models using an observer of the task signal and its derivatives [2-5]. At the same time, the problem of joint provision of not only the accuracy of the task signal processing, but also the quality indicators of the transient process, in particular, with step setting actions, remains relevant [6-10].

The proposed work is devoted to solving this problem by means of modal control, the use of which is effective in lightly damped and structurally unstable systems.

2. Formulation of the problem
Let us consider an object:

$$y^{(n)} + \ldots + a_1 \dot{y} + a_0 y = b_m \dot{u}^{(n)} + \ldots + b_1 \dot{u} + b_0 u; \quad m < n,$$

where $u$ is a control action, $y$ is a controlled value, $a_0, \ldots, a_{n-1}, b_0, \ldots, b_m$ are constant coefficients.

As a result of constructing a modal controller, the closed-loop system equation will take the following form:

$$y^{(n)} + \ldots + a_1 \dot{y} + a_0 y = b_m \dot{u}^{(n)} + \ldots + b_1 \dot{u} + b_0 u,$$

in which the coefficients $a_{0,m}, \ldots, a_{n-1,m}$ determine the desired location of $n$ roots of the characteristic equation of the system. In the general case, it can be assumed that a part $S_1$ of these roots forms the required character of the transient process and regulation time, the other part $S_2$ is used to compensate for the dominant zeros of the characteristic equation $b_m \dot{u}^{(m)} + \ldots + b_1 \dot{u} + b_0 u$, and the remaining $|S|$ free roots ($|S|=n-|S_1|-|S_2|$) can be located at a sufficient distance from $S_1$ and $S_2$ so that the roots of $S$ do not significantly affect the quality indicators of the transient process.

Now let us assume that the observer of the first derivative of the setting action is implemented in the system, i.e. the factor $c_1 s + c_0 = a_{1,m} s^n + a_{0,m}$ appears on the right-hand side of equation (2). If zero ($-c_0/c_1$) turns out to be dominant with respect to the poles of the servo system, then during the
development of the step setting action, the oscillation of the controlled variable \( y(t) \) will appear. To eliminate oscillation, it is necessary to compensate for zero \((-c_0/c_1)\), i.e. introduce an additional pole \( s_3=\left(-\frac{c_0}{c_1}\right) \) into the desired polynomial of the system.

In this case, the uncertainty of the task of synthesis of the servo system arises: to build a modal controller, you need to know the parameters of the reference channel \( c_1 \) and \( c_0 \), and to build the reference channel, you need to know the characteristic equation of the servo \( s \) system: in this example these are two lower coefficients \( a_{1,m} \) and \( a_{0,m} \), that is, you need to know the parameters of the controller.

It is required to find a method for resolving this uncertainty.

### 3. Proposed solution method

Using the fact that the quality indicators of the system are determined only by the dominant roots of the characteristic equation, and the part \( S \) of the roots is free, we can formulate an optimization problem for such a joint placement of zeros and poles of a closed-loop system, which would provide both accuracy indicators and the required nature of the transient process:

\[
F(S) = \left[ s_3 - \left( -\frac{a_{0,m}}{a_{1,m}} \right) \right] \rightarrow \min;
\]

\[ S_1 = S_{tr}; \]
\[ S_2 = \lambda_{ob}; \]
\[ s_j \in S; \]
\[ S_4 < 5S_1; \]
\[ |S_1| + |S_2| + |S| = n. \]

where \( S_{tr} \) — roots coming from regulation time, \( \lambda_{ob} \) — zero of the object.

We will graphically show expression (3) in Fig. 1.

![Fig. 1. Graphical interpretation of the optimality criterion](image)

The criterion (3) will ensure the proximity of the values of zero \((-c_0/c_1) = (-a_{0,m}/a_{1,m})\) and the pole \( s_3 \) in the transfer function of the closed-loop system after the observer of the setting action is introduced into it. As a result, the system overshoot will be eliminated and the transient process will take on the desired character while maintaining the accuracy indicators.

Thus, the construction of a modal controller was reduced to an optimization problem.

### 4. Checking the efficiency of the method.

Let us check the efficiency of the method using the example of a fifth-order object (an electric drive with elastic coupling):

\[
\dot{x} = Bx + Nu;
\]
\[
y = Ax;
\]
where $d$ is a matrix of sensors; $x = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]^T$ signifies state coordinates.

Let us introduce the modal controller $R = [r_1 \ r_2 \ r_3 \ r_4 \ r_5]$ into the system Fig 2.

The characteristic equation of the system $P(s,R)$ will take the following form:

$$ P(s,R) = \frac{s^5 + a_{4,m}(R)s^4 + \ldots + a_{1,m}(R)s + a_{0,m}(R)}{s^3 + 27337.3 + 26666.6 - 50000s + 1800s + 0} = 0, $$

where the inequalities $s_4 \leq 5s_1$; $s_5 \leq 5s_1$ reflect the requirement that the poles $s_4$ and $s_5$ are far from the dominant root $s_1$, and the condition $s_3 \geq -1$ localizes the search area for the extremum (3).

Let’s illustrate the obtained location of the roots in Fig. 3.
The following solution was obtained: $s_3 = -0.99; s_4 = -3.05 \cdot 10^6; s_5 = -3.05 \cdot 10^6$. The observer zero is $(-a_{0,\text{z}}/a_{1,\text{z}}) = -0.96$. The root $s_3 = -0.99$ is close to zero of the observer and performs the compensation function. Fig. 4 shows the transient processes in the system synthesized with the fulfillment of the compensation condition and without this condition.

5. Discussion of the results
The analysis of the transient processes shown in Fig. 3a shows that due to the application of the proposed synthesis method, the observer zero is partially compensated by the root of the closed part of the system and, as a result, the oscillation in the system during the development of a stepped reference effect decreases four times (from 10% to 2.5%). Along with this, indicators of the quality of the transient process are provided: namely the required regulation time (Fig. 3a) and zero steady-state errors with constant and linearly increasing input signals (Fig 3b), i.e. the system implements the second order of astaticism without the use of integrators. The obtained results confirm the effectiveness of the proposed method.

6. Conclusion
The synthesis of modal control in servo systems with a high astaticism order with respect to the setting action faces the problem of the oscillation of the transient process caused by the use of derivatives of the reference signal in such systems.

The method of solving this problem has been proposed which is based on the joint placement of zeros and poles of the system by setting and solving the optimization problem of synthesizing a modal controller according to the criterion of the proximity of the actual location of the system roots to their desired location.
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