Dynamics of self-similar pulse generated in normal dispersion-decreasing fiber

Ge Xia $^{1}$, Youceng Feng $^{2}$

$^{1}$ School of Electronic and Electrical Engineering, Wuhan Textile University, Wuhan, 430073, China
$^{2}$ College of Electronics Information Engineering, South-central University for Nationalities, Wuhan, 430074, China

E-mail: xiaboat@wtu.edu.cn, fengyc@mail.scuec.edu.cn

Abstract. Self-similar pulse can be generated in a normal dispersion-decreasing fiber. We show that the dynamics of self-similar pulse evolution is governed by the different spectrum broadening mechanisms in the different stages, which is confirmed by the typical pulse spectral structures with amplitude noise added.

1. Introduction

Self-similar solutions to the nonlinear equations have been a subject of close attention in various physical systems [1]. Self-similar evolution of short pulses propagating in a fiber is of particular interest in fiber optics [2]. Mathematically, the exact and asymptotically stable self-similarly evolving solutions are governed by the generalized nonlinear Schrödinger equation (NLSE) describing propagation dynamics of short pulses in the fiber [3]. Specifically, it was demonstrated that the pulse with an arbitrary shape after propagation in a passive fiber with decreasing normal dispersion transforms into an asymptotically parabolic self-similar pulse [4], and can be used for pulse compression and supercontinuum [5]. However, optical wave breaking (WB) may occur during the self-similar pulse evolution in a fiber with normal dispersion under highly nonlinear condition for a given picosecond seed pulse [6].

In this work, we show that self-similar pulse evolution in a normal dispersion-decreasing fiber can be characterized by a dimensionless pulse shape function in the frequency domain as well as in the time domain. In particular, the dynamics of pulse evolution is governed by the different spectrum broadening mechanisms in the different stages once WB takes place, which is confirmed by the typical pulse spectral structures with amplitude noise added.

2. Self-similar pulse evolution in normal dispersion-decreasing fiber

The propagation of short optical pulses in a normal dispersion-decreasing fiber (NDDF) with varying group-velocity dispersion (GVD) may be described in terms of the dimensionless NLSE with varying coefficients

$$i \frac{\partial q}{\partial z} - \frac{\beta_s}{2} D(z) \frac{\partial^2 q}{\partial t^2} + \gamma |q|^2 q = 0$$

(1)

Where the $q(z,t)$ is the complex amplitude of the slowly varying optical field, $\beta_s > 0$ is the GVD value at $z = 0$, and $\gamma$ is the nonlinearity coefficient, which is considered to be uniform along NDDF, and...
higher-order dispersion is not included in Equation (1) while it is possible to fabricate NDDF with third-order dispersion by introducing a W-shaped core according to [4]. The normalized function \( D(z) > 0 \) describes the variation of the fiber GVD along its length. Specifically, with the choice of the hyperbolic profile of dispersion

\[
D(z) = \frac{1}{(1 + \Gamma_0^2)}
\]  

Equation (1) can be solved by using numerical approach of the symmetrical split step Fourier method with all the pulse and fiber parameters are chosen from [4]: A Gaussian chirp-free input pulse with full width half maximum (FWHM) 1.0ps and pulse energy of 40pJ, the fiber parameters: \( \gamma = 3.33 \text{km}^{-1} \text{W}^{-1}, \beta_2 = 1.25 \text{ps}^2 \text{km}^{-1}, \Gamma_0 = 0.028 \text{m}^{-1} \). From these parameters, one obtains dispersion length \( L_D = 288 \text{m} \) and nonlinearity length \( L_{NL} = 8 \text{m} \), \( N^2 = L_D/L_{NL} = 36 >> 1 \), indicating a highly nonlinear condition for the given picosecond seed pulse propagating in the fiber. Figure 1(a) and Figure 1(b) show the pulse temporal and spectral evolution along the fiber at different lengths.

**Figure 1.** The self-similar pulse evolution along the fiber in the time and frequency domain.

To describe the pulse evolution quantitatively, a dimensionless function \( K_t \) is usually introduced to characterize the pulse temporal shape, defined as

\[
K_t[q(z,t)] = \frac{\int_{-\infty}^{\infty} t^2 |q(z,t)|^2 \exp \left( \int_{-\infty}^{t} |q(z,t)|^4 \, dt \right) dt}{\left( \int_{-\infty}^{\infty} |q(z,t)|^2 \, dt \right)^2}
\]  

\[
K_s[q(z,t)] = \frac{\int_{-\infty}^{\infty} |q(z,t)|^2 \exp \left( \int_{-\infty}^{t} |q(z,t)|^4 \, dt \right) dt}{\left( \int_{-\infty}^{\infty} |q(z,t)|^2 \, dt \right)^2}
\]  

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K_s[q(z,t)] = \frac{\int_{-\infty}^{\infty} |q(z,t)|^2 \exp \left( \int_{-\infty}^{t} |q(z,t)|^4 \, dt \right) dt}{\left( \int_{-\infty}^{\infty} |q(z,t)|^2 \, dt \right)^2}
\]  

**Figure 2.** The evolution curves of \( K_t \) and \( K_s \) along the fiber in the time and frequency domain.
$K_t$ has been associated with an invariant of the propagation equations of the second-order moments of the pulse amplitude and effective power within the parabolic approximation for the pulse phase. This invariant indirectly expresses the conservation of energy [7]. It has been pointed out that $K = 0.0796$ for a Gaussian pulse and $K = 0.0720$ for an ideal parabolic pulse. The closer $K$ approaches 0.0720, the better pulses evolve to self-similar parabolic pulses. Here we further introduce it into the frequency domain as follows

\[
K_s[s(z,\omega)] = \frac{\int_{-\infty}^{\infty} \omega^2 |s(z,\omega)|^2 d\omega \left(\int_{-\infty}^{\infty} |q(z,\omega)|^4 d\omega\right)^2}{\left(\int_{-\infty}^{\infty} |s(z,\omega)|^2 d\omega\right)^3}
\]  \tag{4}

Where $s(z,\omega)$ denotes the pulse spectrum envelope, and it is the Fourier transformation of $q(z,t)$. The validity of Equation (4) is guaranteed by the Energy conservation law.

The evolution curves of $K_t$ and $K_s$ versus the pulse transmission distance are illustrated in Figure 2(a) and Figure 2(b) respectively. From the Figures, we can see both curves approach asymptotically towards the 0.0720 level line during pulse evolution though $K_s$ curve fluctuates more severely than $K_t$ curve, which indicates that the pulse will gradually evolve into a self-similar parabolic shape pulse eventually if the pulse propagates a long enough fiber length (far above 650m).

3. Dynamics of self-similar pulse evolution
As it can be seen in Figure 2(a) and Figure 2(b), both curves have an obvious oscillation before it come into the later steady self-similar evolution, which reveals that WB starts to take place. It is known that WB results from the temporal overtaking of different parts of pulse due to their different velocity forward in the pulse caused by the nonmonotonic pulse chirp. When the shifted light overrun the pulse tails, the leading and trailing regions of the pulse then contain light at two different frequencies, which interfere and generate new frequencies. The underlying mechanism of this kind of pulse spectral broadening is considered as four-wave mixing (FWM) [3]. In addition, it is worth noting that once WB appears, it occurs synchronously in both time and frequency domain no matter the oscillation of which curve appears ahead. From Figure 2, we can determine the distance where WB happens is at a fiber length about 70m from the valley of curve oscillation.

![Figure 3. Time and spectral profiles of the pulse at three different distances during the evolution.](image)

In Figure 3, the pulse temporal and spectral profiles at three typical distances (65m, 150m, and 650m) are chosen from the evolution for comparison. We see pulse broadens with the distance in both time and frequency domain. However, the profiles of the pulse spectral intensity vary much more severely than that of the pulse temporal intensity, which reveals the dynamics of self-similar pulse evolution
more clearly. As illustrated, the process of the pulse spectral evolution can be classified into about three stages: In the initial stage (from 0m to 65m), it is dominated by the expansion of ripples in the central part of pulse shown at 65m, which is the unique manners of the pulse spectral broadening caused by self-phase modulation (SPM); In the last stage (after 650m), it is dominated by the widening of sidelobes in the pulse wings shown at 650m, which is the particular manners of the pulse spectral broadening caused by FWM; in the middle stage (from 65m to 650m), it is dominated by both the expansion of ripples in the central part of pulse and the widening of sidelobes in the pulse wings shown at 150m, which signifies that both SPM and FWM take effect.

![Vibrating spectral profiles of the pulse at 65m (a) and 650m (b) with amplitude noise added.](image)

**Figure 4.** Vibrating spectral profiles of the pulse at 65m (a) and 650m (b) with amplitude noise added

The results can be further verified by adding amplitude noise to the seed pulse as shown in figure 4. The amplitude jitter boundary is set within 5 percent of the pulse peak power as usual. The response of vibrating spectral profiles to the noise manifests the characteristic spectral structures that just determined by the distinctive spectral broadening mechanisms as discussed above.

4. Conclusion

In conclusion, we have introduced a dimensionless pulse shape function in the frequency domain as well as in the time domain to describe the self-similar pulse evolution. WB will take place for a given picosecond pulse under highly nonlinear condition during the evolution. The pulse spectral profile varies along the propagation reveals that the dynamics of self-similar pulse evolution is governed by the different spectrum broadening mechanisms in the different stages. This is confirmed by the typical pulse spectral structures with amplitude noise added.

References

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