Disorder and electronic transport in graphene

E R Mucciolo¹ and C H Lewenkopf²

¹ Department of Physics, University of Central Florida, Orlando, FL 32816-2385, USA
² Instituto de Física, Universidade Federal Fluminense, 24210-346 Niterói, RJ, Brazil

E-mail: mucciolo@physics.ucf.edu and caio@if.uff.br

Received 10 May 2010, in final form 24 May 2010
Published 23 June 2010
Online at stacks.iop.org/JPhysCM/22/273201

Abstract
In this review, we provide an account of the recent progress in understanding electronic transport in disordered graphene systems. Starting from a theoretical description that emphasizes the role played by band structure properties and lattice symmetries, we describe the nature of disorder in these systems and its relation to transport properties. While the focus is primarily on theoretical and conceptual aspects, connections to experiments are also included. Issues such as short- versus long-range disorder, localization (strong and weak), the carrier density dependence of the conductivity, and conductance fluctuations are considered and some open problems are pointed out.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

In just the few years since its first synthesis [1–3], graphene has become the center of attention of a substantial body of scientists in the fields of physics, materials science, chemistry, and electrical engineering. It is a fascinating material, made of a single atomic layer of carbon, with unique electrical, thermal, and mechanical properties. As such, it is likely to have a large number of practical applications. Even though the main features of its electronic band structure were described more than sixty years ago [4], only in recent years have we started to appreciate (and understand) the wealth of phenomena in which charge carriers in graphene can participate. There are already several comprehensive reviews in the literature covering the fundamentals of graphene [5–8] and the reader is strongly encouraged to consult them to acquire a background on the subject. Here we focus instead on how disorder in its various forms affects the electronic transport properties of graphene at low temperatures. This topic has become a vast subfield in its own right. Thus, rather than provide an exhaustive review, in the spirit of a topical review we try to cover just enough material to bring the reader up-to-date with key concepts and crucial results. As a consequence, to narrow the scope, we leave out topics such as ac transport, the quantum Hall effect, spin transport, and the effect of electron–electron interactions. Due to the limited space, we also do not discuss electronic transport in graphene nanoribbons.

The paper is organized as follows. In section 2 we describe the role of symmetries and symmetry breaking by disorder in graphene. The nature and differences among the various sources of disorder in graphene are discussed in section 3. In section 4 we look at transport properties at the neutrality or Dirac point, while in section 5 we discuss weak localization and conductance fluctuations caused by quantum interference. The case of doped (non neutral) graphene is considered in
2. Pristine graphene and the role of symmetry

To understand how disorder affects electronic transport properties in graphene, it is important to consider the ideal case of a pristine crystal [9]. Single-layer graphene is formed by carbon atoms disposed in a two-dimensional non-Bravais honeycomb lattice (namely, two intersecting hexagonal sublattices—see figure 1). The distance between nearest-neighbor sites is approximately 1.42 Å, while the lattice constant is \( a = 2.46 \) Å. The geometry and flatness of the lattice prohibit any overlap between the \( p_z \) orbital of a given atom and the \( s \), \( p_x \), and \( p_y \) orbitals of its neighbors. The orbitals \( s \), \( p_x \), and \( p_y \) hybridize to create the sp\(^2\) bonds that hold the atoms together, as well as a high-energy \( \pi \) band. The so-called \( \pi \) band is created by the overlap of \( p_z \) orbitals and can be treated independently of other bands. In graphene, the \( \pi \) band is responsible for most of the electron conduction. Thus, at low energies or doping, a single-band tight-binding model including only nearest-neighbor hopping between adjacent sites on distinct hexagonal sublattices (which are named A and B hereafter) provides a good approximation for studying the electronic properties in graphene. The hopping amplitude in this case is \( t \approx 2.7 \) eV [5].

When switching to a momentum representation, in addition to the spin degree of freedom, one has to introduce an isospin structure in the electron wavefunction to account for the two sublattices. The resulting Hamiltonian can be diagonalized to yield the energy band [4]

\[
E_{\pm}(k) = \pm \sqrt{1 + 4 f(k)},
\]

where

\[
f(k) = \cos^2(k_y a/2) + \cos(\sqrt{3} k_x a/2) \cos(k_y a/2).
\]

Neutral graphene has its Fermi energy at \( E_F = 0 \), cutting the\( \pi \) band exactly at the six corners of the hexagonal Brillouin zone. As shown in figure 1, only two sets of nonequivalent corner points exist and are denoted by K and K', with K and K' points alternating along the hexagon. Near any of these points, say K, the two (positive and negative) branches of the \( \pi \) bands in (1) can be approximated as

\[
E_{\pm}(k) \approx \pm \hbar v_F |k - K|,
\]

where \( v_F = \sqrt{3at/2\hbar} \approx 10^6 \) m s\(^{-1}\) is the Fermi velocity. This is a remarkable result: near \( E = 0 \) (the so-called Dirac point), carriers in graphene have a linear dispersion relation, much like massless relativistic particles. In addition, we can introduce another isospin index to the wavefunction in order to differentiate contributions from the K and K' points (also known as the two valleys), yielding the four-dimensional spinor

\[
\Psi = \begin{pmatrix} \psi^K_A \\ \psi^K_B \\ \psi^K'_A \\ \psi^K'_B \end{pmatrix}
\]

(real spin indices are omitted). Close to the Dirac point, the effective Hamiltonian acting on this four-dimensional spinor reads

\[
H = v_F (p \cdot \sigma) \tau_z
\]

\[
= \hbar v_F \begin{pmatrix} 0 & k_x - i k_y & 0 & 0 \\ k_x + i k_y & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_x + i k_y \\ 0 & 0 & -k_x - i k_y & 0 \end{pmatrix},
\]

where the linear momentum \( p = \hbar (k_x, k_y) \) is defined with respect to the corner points of the Brillouin zone and the Pauli matrices \( \sigma = (\sigma_x, \sigma_y) \) and \( \tau_z \) act on the AB and KK' subspaces, respectively. The eigenstates of equation (5),

\[
\begin{pmatrix} \psi^K_A \\ \psi^K_B \end{pmatrix} = \frac{1}{\sqrt{2}} e^{i\theta_k} \begin{pmatrix} e^{-i\theta_k/2} \\ \pm e^{i\theta_k/2} \end{pmatrix}
\]

\[
\begin{pmatrix} \psi'^K_A \\ \psi'^K_B \end{pmatrix} = \frac{1}{\sqrt{2}} e^{i\theta_k} \begin{pmatrix} \pm e^{-i\theta_k/2} \\ e^{i\theta_k/2} \end{pmatrix},
\]

provide a basis for solving transport problems in the presence of disorder. Here, \( \theta_k = \arctan(k_x/k_y) \) and the ± signs correspond to the conduction/valence bands, respectively.

One can use the eigenstates in equations (6) to calculate the conductance in pristine graphene. The results are strongly dependent on geometry and edge orientation [10, 11]. For later comparison, let us briefly comment on these results. The standard ballistic model considers a graphene ribbon of width \( W \) and length \( L \) connected to heavily doped graphene leads. In this geometry, the transverse momentum is quantized and free propagating modes exist at the contacts. This allows one to calculate the S-matrix and obtain the conductance \( G \) using the Landauer formula. The conductivity, defined as \( \sigma = (L/W)G \), has a minimum at the Dirac point described by

\[
\sigma \approx \frac{4e^2}{\hbar} \left( \frac{L}{W} \right) \left[ g_0 + \sum_{n=1}^{N} \frac{1}{\cosh^2(\pi n L/W)} \right],
\]
semiconducting ones [11]. For \( W \approx L \), \( \sigma \) is dominated by \( g_0 \). In general, \( \sigma \), as given by equation (7), depends on the aspect ratio \( W/L \). Hence, calling it conductivity is somewhat misleading, since the conductivity is usually expected to be a sample specific quantity and independent of geometry. Yet, for \( W/L \gg 1 \), \( \sigma \) is dominated by evanescent modes and one finds

\[
\sigma \approx \frac{4e^2}{\pi \hbar} \tag{8}
\]

This value is often (mis)quoted as the universal ballistic conductivity minimum, even though its derivation is only applicable to samples with a narrow aspect-ratio. The Landauer formula can also be used to calculate \( \sigma \) as a function of the carrier concentration \( n \) since the latter is related to the Fermi energy: \( n = E_F |E_\delta| / \pi (h V_y)^2 \). For sufficiently large values of \( |n| \), \( \sigma \) reads [12]

\[
\sigma(n) = \frac{e^2}{\hbar} L \sqrt{\pi |n|} \tag{9}
\]

Going back to equation (5), we notice that the effective Hamiltonian of pristine graphene is invariant under a large number of symmetry operations in the isospin spaces besides the usual spatial translation and rotation invariances [13]. First, the Hamiltonian commutes with the operators

\[
\Lambda_\pm = \sigma_z \tau_x, \quad \Lambda_y = \sigma_z \tau_y, \quad \Lambda_z = \sigma_0 \tau_z, \tag{10}
\]

which, together with \( \Lambda_0 = \sigma_0 \tau_0 \), form a group (here, the subscript 0 denotes the identity operator). Second, the Hamiltonian is also time-reversal and chiral symmetric; the corresponding operators are denoted by \( T_0 = \sigma_0 \tau_1 \) and \( C_0 = \sigma_0 \tau_0 \), respectively. Notice that time-reversal is also accompanied by an inversion in the sign of the linear momentum as well as a transposition of the Hamiltonian matrix (it basically takes K to K’ and vice versa). A total of fifteen distinct symmetry operations can be constructed by combining \( \Lambda_0, \tau_x, \tau_z, T_0 \), and \( C_0 \). For instance, \((T_0 \Lambda_0)^{-1} H (T_0 \Lambda_1) = H \) and \((C_0 \Lambda_0)^{-1} H (C_0 \Lambda_1) = -H \) (the latter exchanges \( \pi \) band branches but preserves the energy spectrum).

Disorder will not only break translation and rotation symmetries but also affect these other invariances. Thus, to describe static disorder in general, we add to the Hamiltonian in (5) a term of the form

\[
H_{\text{dis}} = \sum_{ij} V_{ij} \sigma_i \tau_j. \tag{11}
\]

Depending on which matrix elements \( \{V_{ij}\} \) are nonzero, certain pseudospin symmetries are broken while others are preserved. For instance, a generic nonmagnetic disorder may break all symmetries but time-reversal. In fact, the study of the effect of disorder on the transport properties of two-dimensional massless fermions described by Hamiltonians of the form (5) actually precedes graphene, and some important facts have been known for quite some time within the contexts of the integer Quantum Hall effect and d-wave superconductivity. Let us discuss a few important cases.

Chiral symmetry is associated with the block off-diagonal form of \( H \) in equation (5). Therefore, any disorder that introduces diagonal matrix elements or even a finite chemical potential (moving the Fermi energy away from the Dirac point) will break this symmetry and, in most cases, strongly affect the electrical conductivity. However, whenever \( V_{ij} = V_{ji} = 0 \), quantum corrections to the conductivity cancel each other to all orders in the disorder strength and there is no localization at the Dirac point [14–16]. The conductivity in this case takes a finite value which may depend on the disorder strength and whether the remaining symmetries of the Hamiltonian are broken or not [17, 13]. Bond disorder due to lattice distortions such as ripples, random magnetic fields, and dislocations and other lattice defects represented by non-Abelian gauge fields [18] fall into such a class of disorder.

When chiral symmetry is not present, two important situations arise. First, if the disorder is sufficiently smooth, varying significantly only over length scales larger than the lattice constant, there is very little mixing between the two valleys; the K and K’ points remain essentially decoupled and \( V_{kx} = V_{ky} = 0 \). For some time in the literature, the fate of massless Dirac fermions in the presence of long-range scalar disorder was unclear [19, 20]. In fact, some issues appeared even in the clean limit. For instance, a calculation of the Kubo conductivity of non-dis ordered Dirac fermions in the so-called \( \pi \) -flux phase model disagreed with the Einstein conductivity calculated in the thermodynamic limit [20]. Only recently an explanation for this discrepancy was provided [21], one reason being that since the neutral point is actually a critical point, the value of the clean conductivity depends on the order of limits. This indicates that one should also be careful when considering claims of universality in the disordered case. However, as we will discuss below, it became widely accepted recently that localization due to quantum interference is also absent in this case. Second, when disorder has a strong short-range component and intervalley mixing is significant, the opposite occurs and quantum corrections tend to normalize the conductivity to zero, turning graphene into an Anderson insulator [22–25]. In this case, as far as transport is concerned, charge carriers in graphene behave much like those in an ordinary two-dimensional electron gas.

Finally, it is worth remarking that far away from the Dirac point, graphene is no longer described by the Hamiltonian (5) since the dispersion relation ceases to be conic. For instance, trigonal warping changes the role of quantum corrections to the conductivity and enhances localization [26]. We return to this issue in section 5, when we address mesoscopic effects in graphene.

3. Nature of disorder in graphene

It is commonly accepted that graphene has very few lattice defects. Therefore, intrinsic disorder tends to be weak even in exfoliated samples. However, extrinsic disorder is invariably present and is basically dictated by the synthesis method and by way the graphene sheet is supported. Chemical contamination in the form of adsorbates can occur during lithographic processing of field-effect devices. The focused
electron beam used in the lithographic process, and imaging with SEM also introduce disorder, although its exact effect on the graphene lattice is still under debate [28, 27]. In addition, when graphene is laid on a substrate, lattice distortions (the so-called ripples) can appear due to a tendency in graphene to conform to the roughness in the substrate surface [29]. It has been recently suggested that wrinkles and other singularities can also be formed, causing lattice distortions [30]. Finally, most insulating substrates used in graphene devices are oxides which are rather prone to charge traps. These traps can be located either in the bulk of the substrate (but not too far away) or at the interface between the substrate and the graphene sheet and are a common source of disorder.

For clean, suspended graphene systems, extrinsic disorder can be substantially reduced [31, 32]. Nevertheless, some amount of disorder is still present, as shear and strain created by contacts and scaffolds typically induce corrugation in the graphene sheet [33] which can create electron scattering [34–40]. Ripples induced by thermal motion are rather unlikely at low temperatures, but can be an important source of scattering at high temperatures in the form of flexural phonon modes.

It was recognized early that screening in graphene is rather poor due to the low density of states near the neutrality point. Thus, at low doping, charge traps in the substrate as well as Coulomb impurities (e.g. charged adsorbates) act as long-range electron scatterers [41–45]. It was shown by several authors that Coulomb scattering (even when screened) leads to a linear increase of the conductivity with electron density, matching quite well the majority of the experimental data for field-effect graphene-based devices. However, there are also a few other scattering mechanisms that can also produce a linear dependence of the conductivity on carrier density.

There is one additional type of disorder in graphene that, albeit static, does not quite follow equation (11). It is induced by neutral adsorbates, such as atomic hydrogen, which tends to bind covalently to carbon atoms in the graphene sheet and locally distort both the lattice and electronic structure [46, 47]. The result is a mid-gap or resonant state near the Dirac point that acts as a short-range scatterer, similarly to (but not exactly like) vacancies. This type of disorder also induces a dominant linear dependence of the conductivity on carrier density, modulated by a weak logarithmic term [48–51]. Scattering by ripples in the graphene sheet can also, in principle, lead to a linear carrier density dependence [52], although this situation is considered unlikely since it requires free-standing, equilibrium fluctuations in the graphene sheet which are unlikely to dominate when a substrate is present. Finally, a finite amount of wrinkles may also induce the same linear dependence [30].

There is some recent experimental evidence that resonant scatterers may play a significant role in limiting mobility in graphene sheets deposited on oxide substrates by mechanical exfoliation [53, 54], but their microscopic nature is still unclear. In fact, at present, the dominant mechanism of electron scattering in graphene is still under debate for both suspended and non-suspended graphene. In particular, for the latter, despite some compelling evidence [55], the widespread view that Coulomb impurities are the most important scattering mechanism limiting mobility has been recently challenged [56].

Finally, we mention that disorder also plays a role in other characteristics of graphene devices, such as $1/f$ noise [57–59].

4. Neutral graphene

Graphene at the neutrality point has some remarkable properties. As mentioned earlier, the conductivity is finite even though, in the clean limit, the density of states vanishes. Early transport experiments found a conductivity minimum of the order of $e^2/h$, which raised the possibility that this may actually be a universal feature of graphene. It took some time for this issue to be settled, since several theories also pointed to some universality in the conductivity of Dirac fermions at the neutrality point. For instance, in the presence of certain types of chiral disorder such as random vector potentials, it was well known that the conductivity is essentially unaffected by disorder (as long as the disorder is not too strong), taking the universal value of $4e^2/\pi h$ [20, 60]. For other types of chiral disorder, the conductivity at the neutrality point is also of the order of $e^2/h$ [17]. In fact, more than two decades ago, Fradkin used a coherent potential approximation to show that two-dimensional zero-gap semiconductors have a universal Drude conductivity of the order of $e^2/h$ at zero energy when intervalley scattering is neglected [19]. Some years later, Lee used a self-consistent Born approximation (SCBA) to show that a similar result holds for fermions at the nodal points of dirty d-wave superconductors [61]. In an early theoretical work, Shon and Ando used the SCBA to show that for both short- and long-range impurity scattering, the conductivity in graphene is equal to $2e^2/h$ per spin degree of freedom [62] (see also [17]). Coincidentally, this is the same value as ballistic graphene when $W/L \gg 1$, as seen in section 2.

As more experiments began to show that the conductivity minimum can in fact vary widely from sample to sample [63], thus contradicting the idea of universality, it became clear on the theory side that the SCBA is inadequate to describe Dirac fermions near the neutrality point. First, it does not correctly incorporate quantum effects such as coherent scattering and localization. Second, it can hardly be justified, since at the Dirac point there is no clear expansion parameter controlling the approximation (the carrier wavelength diverges at the neutrality point for a uniform system). Third, and perhaps more importantly, real systems are far from being macroscopically homogeneous and uniform [64–67]. Strong fluctuations in the background potential can occur at length scales comparable to the mean free path $l$. This is not easily taken into account in the SCBA or in any semiclassical formulation, although some recent attempts have tried to circumvent this problem [68, 69].

The sample-dependent value of the conductivity minimum shows that the nature and the strength of the disorder plays a crucial role in determining the transport properties of graphene, and a careful theoretical analysis must be employed.

Let us first consider non-chiral, short-ranged potential disorder. Scattering induced by localized defects, neutral
impurities or adsorbates can transfer enough momentum to carriers so that coupling between valleys across the Brillouin zone (i.e. the K and K’ points) occurs. Several authors have considered this regime theoretically within the renormalization group approach [22–24, 17]. They have shown that, as in the case of standard two-dimensional metals with no spin–orbit coupling, coherent backscattering in graphene also leads to carrier localization when the samples are sufficiently large, namely, when the localization length \( l_{\text{loc}} \) is smaller than the sample linear size \( L \). Graphene in this case belongs to an orthogonal symmetry class [13, 22–24, 26]; in the one-parameter scaling language, it has a beta function \( \beta(\sigma) = \frac{d \ln \sigma}{d \ln L} \) that is always negative, as shown in figure 2(a). This result is supported by several numerical simulations [70–72]. However, it appears that, experimentally, \( L \) is either too small or the dephasing length \( \xi_{\phi} \) is shorter than \( l_{\text{loc}} \) (we note that, in two dimensions, the localization length can be exponentially larger than the classical mean free path). Thus, the most common effect of short-range disorder at the neutrality point is a reduction (but not a complete suppression) of the conductivity. However, when short-range disorder is strong, variable-range hopping conductivity is observed, indicating localization. We will get back to short-range disorder when we consider mesoscopic effects and doped graphene.

When the potential disorder is smooth at the atomic scale, intervalley scattering (and therefore backscattering) is suppressed. As a result, carriers can be described by a single-valley Dirac Hamiltonian with a random scalar potential [20, 73].

\[
H = v_F (\mathbf{p} \cdot \sigma) + V(r)\sigma_0. \tag{12}
\]

This Hamiltonian is symmetric under a pseudo time-reversal symmetry: \( H = \sigma_z H^\dagger \sigma_0 \) (notice that the real time-reversal symmetry operation would connect K and K’ points instead). This Hamiltonian belongs to a symplectic class [13, 20, 22–24, 26]. It is well known that normal metals with spin–orbit coupling fall in this class and show a metal–insulator transition [74], therefore, it seems plausible that the same could occur for graphene. In equation (12), the pseudospin (due to the sublattice structure) plays a role similar to the real spin in a normal metal and is coupled to the orbital motion through the \( \mathbf{p} \cdot \sigma \) term. From a theoretical viewpoint, until recently it was not clear whether a transition also exists for graphene in this class. Since it is difficult to fabricate samples with exclusively long-range disorder, experiments did not provide any insight into this question.

A field-theoretical treatment of this problem was proposed by two groups [13, 75, 76]. In particular, Ostrovsky et al have argued that the appropriate nonlinear sigma model includes a topological term which causes the appearance of a novel stable fixed point at a finite conductivity \( \sigma^* \) in addition to the usual symplectic unstable fixed point \( \sigma_S \approx 1.4e^2/h \) [75, 77]. The suppression of localization was also obtained by Ryu et al [76] using a combination of analytical and numerical calculations. These authors also derived a topological term which is explicitly constrained to be either 0 or \( \pi \) by symmetry. In contrast to [75], their treatment of the global anomaly encoded in the topological term yields only one stable metallic fixed point and no transition. Both results are illustrated in figure 2(b) using a one-parameter scaling beta function.

The derivation of topological terms [75, 76] indicates the validity of the single-parameter scaling of the conductivity, the existence of a novel stable fixed point and the consequent metal–insulator transition requires a nonperturbative investigation. This motivated several numerical studies that employed a variety of techniques [78–82]. Only a strictly positive beta function was obtained and no evidence of a new fixed point was found, thus corroborating the prediction of [76]. This supports the idea that the main role of the topological term is just to suppress the conventional symplectic metal–insulator transition. In all studies, the average conductivity followed a simple scaling law, increasing with the logarithm of the system size such that \( \beta(\sigma) = \alpha/\sigma \). While one could in principle expect the constant \( \alpha \) to be universal, the situation is somewhat more complex, particularly for finite-size systems (with the exception of [79]), since simulations are carried out at zero temperature and therefore in the fully coherent limit. In addition, most
simulations obtain the conductivity by first computing the
two-terminal conductance \( G \) and can be influenced by contacts and
group.

In our numerical simulations, we have found that the constant \( \alpha \) deviates from the universal value of \( e^2/\pi \hbar \) expected for the symplectic class. We use a single-band tight-binding model on a honeycomb lattice with an added long-range Gaussian correlated potential disorder [41], namely,

\[
\langle V(r)V(r') \rangle = K_0 \frac{(h \psi)^2}{2\pi \xi^2} \exp\left(-\frac{|r-r'|^2}{2\xi^2}\right),
\]

where \( K_0 \) parametrizes the disorder strength and \( \xi \) is the disorder correlation length. Figure 3 shows the finite-size scaling of the average conductivity using the recursive Green function method [81]. In essence, this method mimics a two-terminal transport measurement. Three different aspect ratios were considered with several values of \( K_0 \) and \( \xi/\mu \). The average conductivity obtained from the linear conductance is plotted versus \( L/1 \), the ratio between the system length and the disorder mean free path \( l \), which depends on \( K_0 \) and \( \xi \). Two clear regimes can be identified. For \( L/1 \ll 1 \), the probability of an electron being scattered by disorder as it traverses the sample is very small. This corresponds to the ballistic regime, where scattering occurs mainly at the sample edges and transport properties are dominated by the sample geometry. In contrast, when \( L/1 \gg 1 \), the system becomes diffusive. Now, electrons are scattered multiple times by disorder in their path across the sample. Scattering at the sample edges is no longer dominant and transport depends weakly on the geometry. Figure 3 clearly shows this crossover. In the \( L/1 \to 0 \) limit, the simulations approach the analytical result (indicated by arrows) for clean graphene as given by equation (7). For the diffusive regime, \( L/1 \gg 1 \), the conductivity is proportional to \( \ln(L/1) \), in agreement with the nonlinear sigma model prediction [17]. The mismatch between the numerical prefactor for the logarithm and the value characteristic of the symplectic class may be related to the finite contact resistance.

These simulations suggest an explanation for the transport experiments at the charge neutrality point. In the coherent diffusive regime, the conductivity has significant sample-to-sample fluctuations and its average shows a weak (logarithmic) dependence on the mean free path. Typical diffusive experimental samples have \( L/1 \approx 1-10 \) and \( \sigma \approx 4e^2/\hbar \), similar to what is seen in figure 3.

An intuitive, non-rigorous understanding of the lack of localization in the presence of long-range disorder is possible by invoking so-called Klein tunneling [83–85]. In the absence of intervalley scattering, as carriers move across charge puddles induced by long-range potential fluctuations, chirality prevents backscattering. When the Fermi energy cuts across a potential barrier, carriers are converted from a particle to a hole (or vice versa). Chirality conservation then requires a forward moving particle hitting the potential barrier at a normal angle to be scattered as backward moving hole (interband tunneling). In this case there is perfect charge transmission, as the real electron involved in the process continues to move in the same direction. For other incidence angles, a finite probability of particle backscattering appears and it is actually a periodic function of the angle. This effect has been recently observed experimentally [86]. Klein tunneling through charge puddles (p- and n-doped regions) is the basis of an attempt to explain the conductivity in graphene near the neutrality point using a random resistor network model [87].

5. Mesoscopic effects

Mesoscopic corrections to the conductivity have been studied for more than 20 years in numerous systems, such as metals and two-dimensional semiconductor heterostructures at low temperatures [88–92]. Weak localization and universal conductance fluctuations are the most ubiquitous manifestations of mesoscopic effects in transport. The weak localization correction to the conductivity can be understood semiclassically in terms of quantum interference between self-crossing paths originating from disorder scattering. In the loops, an electron can propagate in the clockwise and counterclockwise directions, and these two trajectories interfere. The effect is maximal when the system is time-reversal symmetric; for instance, in normal metals with negligible spin–orbit coupling it leads to constructive interference and a decrease in the probability of carriers to move forward. This interference effect gradually disappears as time-reversal symmetry is broken, for instance, by an external magnetic field. In diffusive systems, mesoscopic effects are also responsible for sample-to-sample fluctuations in the conductance, \( \delta G \), of the order of \( e^2/\hbar \), irrespective of the magnitude of \( G \) itself [93]. This phenomenon is known as universal conductance fluctuations (UCF). The magnitude of quantum interference effects strongly depends on \( L/\xi \), the ratio between the electron dephasing length and a sample specific length scale, such as the electron elastic mean free path or the linear size of the sample \( L \). When \( L/\xi \ll L \), this magnitude saturates to a constant value; in convention metals, this value is solely determined by the symmetry class and the sample geometry. Since, for any given sample, \( L/\xi \) decreases very rapidly with increasing temperature, low temperatures are usually required to observe mesoscopic effects.

The effects of quantum interference in the electronic transport in a honeycomb lattice had been theoretically addressed [94] even before graphene was first synthesized. Suzuura and Ando focused their analysis on the weak localization peak, namely, the enhanced resistivity at zero magnetic field. Using the standard disorder diagrammatic approach for diffusive systems, which amounts to a perturbative expansion in powers of \( (k_Fl)^{-1} \), it was shown that the weak localization correction depends strongly on the spatial range of the disorder. (Notice that this expansion is only formally justifiable away from the charge neutrality point, when \( k_F \) is finite.) As mentioned in section 2, due to chirality, backward scattering in graphene is suppressed for long-range disorder. In this case, electrons in the vicinity of distinct valley points do not couple and the system belongs to the symplectic class, leading to an anti-weak localization correction (negative magnetoconductance), namely, a valley rather than a peak in the curve of resistivity versus magnetic field. Short-range
(nonmagnetic) disorder, in contrast, can break all symmetries except the time-reversal one. In this situation, the system belongs to the orthogonal class and there is weak localization (positive magnetoconductance).

The theory was later extended to include trigonal warping effects [26]. The diagrammatic expression for the weak localization correction, \( \Delta \sigma(B) = \sigma(B) - \sigma(0) \), as a function of magnetic field reads

\[
\Delta \sigma(B) = \frac{e^2}{\pi \hbar} \left[ F\left( \tau_1^{-1}, \tau_2^{-1}, \tau_2^{-1}, \tau_3^{-1} \right) - F\left( \tau_1^{-1}, \tau_2^{-1}, \tau_2^{-1}, \tau_3^{-1} \right) \right] - 2F\left( \tau_1^{-1}, \tau_2^{-1}, \tau_2^{-1}, \tau_3^{-1} \right),
\]

(14)

where \( F(z) = \ln z + \Psi(1/2 + 1/z) \) and \( \Psi \) is the digamma function. The magnetic field is cast in terms of \( \tau_0^{-1} = 4eDB/\hbar \), where \( D = v^2/\tau_2 \) denotes the diffusion constant. The three other times scales correspond to: the dephasing time \( \tau_0 \), which is related to the dephasing length \( \ell_0 = (D\tau_0)^{1/2} \); the intervalley scattering time \( \tau_1 \), which is due to sharp, atomic-like disorder features; and the intra-valley scattering time \( \tau_3 \), which is related to extended defects, dislocations, ripples, etc. The derivation of this expression assumes that the momentum relaxation time, \( \tau_p \), is the shortest of all timescales and is caused by charge impurities (and not by atomically sharp defects), so that it does not affect chirality.

The first experimental report on mesoscopic interference effects in exfoliated graphene observed a strong suppression of the weak localization peak, as compared to the typical value of \( e^2/\hbar \), independently of doping even at very low temperatures [95]. This behavior was attributed to the presence of ripples in the graphene sheet: the effect of corrugations in the electronic structure can be translated into an effective random gauge field which destroys quantum interference in the same way as a random magnetic field. Further experiments on different exfoliated graphene samples [96], where temperature and carrier concentration were systematically varied, observed much larger weak localization peaks. Experiments in epitaxial graphene, on the other hand, showed an extremely sharp anti-localization peak [97].

These observations are consistent with the theory of [26] when the timescales appearing in equation (14) are properly adjusted. In line with the qualitative discussion presented above, a negative magnetoconductance correction or weak anti-localization, \( \Delta \sigma < 0 \), corresponds to the case when \( \tau_1 \to \infty \). Weak localization and positive magnetoconductance, \( \Delta \sigma > 0 \), occurs when \( \tau_1 \to 0 \).

More interestingly, the theory predicts a region in the parameter space where a transition from positive to negative magnetoconductance corrections occurs. To see this, it is convenient to consider a weak magnetic field, such that one can approximate the \( F(z) \) function by its leading order expansion, \( F(z) \approx \frac{z^2}{24} \). Then,

\[
\Delta \sigma(B) \approx \frac{8\pi e^2}{3h} \left( \frac{B \ell_0^2}{\Phi_0} \right)^2 \times \left[ 1 - \frac{1}{(1 + 2\tau_0/\tau_1)^2} - \frac{2}{(1 + \tau_0/\tau_1 + \tau_0/\tau_3)^2} \right],
\]

(15)

where \( \Phi_0 = h/e \) is the flux quantum. Based on this expression, Tikhonenko et al [98] proposed the diagram reproduced in figure 4. By following the dashed blue line, one can cross from a localization to an anti-localization region. Since the scattering times \( \tau_1 \) and \( \tau_3 \) are essentially temperature independent but \( \tau_0 \) is not, one can use temperature to move along this line. Another way to move in this parameter space is to vary the carrier density: low density and high resistivity is associated with short scattering times, while high densities and low resistivity go in the opposite direction. The experimental data [98] nicely support this analysis.

The picture that emerges from this analysis is that transport in exfoliated graphene on SiO\(_2\) substrates is influenced by both long- and short-range scattering processes. For standard experimental samples, the strength of short-range scattering processes is such that \( l_{\text{loc}} \gg L \). Recent experiments [99, 100] show that the conductivity is strongly suppressed when the concentration of atomistically sharp defects is artificially increased, likely because \( l_{\text{loc}} < L \).

A quantitative study of mesoscopic quantum corrections to the conductivity at the charge neutrality (Dirac) point lies outside the validity range of the diagrammatic approach. The interplay between localization length and the finite system length indicates that numerical tools are necessary to gain further theoretical insight in this case. Qualitatively, however, the main features are similar to those obtained at finite doping since, particularly when charge puddles are present, since in this case \( k_F \) is nonzero locally. In figure 5 we show the result of numerical calculations of the magnetoconductance for a strip with armchair edges, at the neutrality point and away, for both short- and long-range disorder. The calculations were performed with a tight-binding model and a Gaussian disordered potential, similarly to those presented in figure 3. Notice the transition from weak localization to anti-localization as the range of the potential is made larger than the lattice spacing.

Quantum interference among the multiple paths that carriers can take as they move across the sample can also cause conductance fluctuations. Despite their random, aperiodic nature, these fluctuations are reproducible when an external parameter is swept back and forth. They are in essence quantum fingerprints of the microscopic configuration of the disorder in the sample and are usually quite evident when \( L \ll \ell_0 \). A proper way to quantify UCF is to evaluate the
The autocorrelation function $C(\Delta X) = \langle \delta G(X + \Delta X)\delta G(X) \rangle$, where $X$ represents an external parameter such as a magnetic field and $\delta G$ is the deviation of the conductance from its average value.

Early transport measurements in graphene were able to identify mesoscopic fluctuations [101, 95, 102, 103], but only recently have more careful quantitative analyzes been carried out by several groups [104–108]. On the theory side, numerical simulations showed that long-range disorder in graphene can produce anomalously large conductance fluctuations [109], while in [108] no reduction is observed. In [107] this only occurs near the neutrality point, while in [108] no reduction is observed.

The temperature dependence of the UCF amplitude was measured by several groups. In particular, [105] finds a clear inverse power-law behavior at high doping, typical of conventional diffusive metals, combined with a saturation at low temperatures. However, the authors in [106] observed an exponential suppression of UCF with increasing temperature.

Finally, we mention a recent experiment where both weak localization and UCF were observed when an in-plane magnetic field was applied to graphene on SiO$_2$ [113]. The appearance of quantum interference in this case can be directly connected to rippling in the graphene sheet.

6. Doped graphene

Doped graphene shows remarkable high field-effect mobilities, even at room temperatures [114, 115]. This is a key property that makes this material a potential candidate for future carbon-based electronic devices. The dominant source of scattering limiting the mobility $\mu$ in graphene is still under debate. As already mentioned, at low temperatures, it is believed that charge impurities [41, 116, 69, 117], substrate effects, resonant scattering [48, 49], corrugations [52], and strain are the main sources of disorder.

With increasing doping, the conductivity of graphene depends only weakly on temperature and grows almost linearly.

**Figure 5.** Average magnetoconductance of an armchair strip ($W/L = 3$) at ($V_{\text{gate}} = 0$) and away from ($V_{\text{gate}} > 0$) the neutrality point for short- ($\xi = 0.5a$) and long-range ($\xi = 4.0a$) disordered Gaussian potentials. $\Phi$ is the total magnetic flux perpendicular to the graphene sheet, $\Phi_0 = \hbar/c$ is the flux quantum, and $t$ is the tight-binding nearest-neighbor hopping amplitude. A total of 500 realizations for each case have been used in computing the averages.

![Graph Showing Conductance vs Magnetic Flux](image-url)
with carrier density, in contrast with a $|n|^{1/2}$ dependence predicted for clean graphene, equation (9). These features were first explained by considering charge impurity disorder, likely due to charge trapped in the substrate [41, 116, 69, 117]. These theories also predict that the mobility should be inversely proportional to the impurity concentration $n_{\text{imp}}$ (for uncorrelated scatterers) and to strongly depend on the dielectric constant. So far, there is no experimental consensus on these properties. While experiments using potassium ions provide evidence that $\mu \propto 1/n_{\text{imp}}$ [118], experiments with gaseous adsorbates have only shown a weak dependence of $\mu$ on $n_{\text{imp}}$ [119]. There is also an experimental controversy regarding the effect of the dielectric constant on the conductivity: the Manchester group measured only a modest change in $\mu$ on immersing graphene in high-$\kappa$ environments, such as ethanol and water [56], while the Maryland group observed a significant change in $\mu$ on adding just a few monolayers of ice to a graphene sheet [55]. Recently, experiments with hydrogenated graphene revealed a strong short-range scattering component to the mobility, apparently more dominant than charge impurities in some samples [53, 54].

Let us now discuss the main theoretical approaches to address the mobility in graphene. For finite doping, such that $kF l \gg 1$, it is possible to calculate the conductivity within much a simpler theoretical framework than that used at zero doping, where one deals with a non-perturbative problem (see section 4). In addition, as one moves away from the charge neutrality point, mesoscopic corrections to the conductivity lose their importance, since $\sigma/\sigma_0 \ll 1$.

Assuming the system to be homogeneous and diffusive, $L \gg l$, the simplest approach to describe the linear transport properties of graphene is given by the Boltzmann theory [5–7], which is widely used as a guide for interpreting experimental results in doped graphene. This semiclassical approach gives the conductivity in terms of the transport scattering time $\tau_t$, namely

$$\sigma = \frac{e^2}{2} \int dE \left( \frac{\partial f}{\partial E} \right) v(E) v_F^2 \tau_t(E),$$  

(16)

where $v(E)$ is the density of states, $f(E)$ is the Fermi–Dirac distribution function, and $\tau_t$ is usually calculated using Fermi’s golden rule, namely,

$$\frac{\hbar}{\tau_t(k)} = \frac{n_{\text{imp}}}{4\pi} \int dk \left| \tilde{V}(q) \right|^2 (1 - \hat{k} \cdot \hat{k'}) F_{kk'} \delta(E(k) - E(k')), $$  

(17)

where $q = |k - k'| = 2k \sin(\theta/2)$ is the momentum transfer, $\tilde{V}(q)$ is the Fourier transform of the screening potential, and $\hat{k} \cdot \hat{k'} = \cos \theta$. Chirality effects are captured by $F_{kk'}$, which is $F_{kk'} = (1 + \cos \theta)/2$ for graphene.

By modeling short-range disorder with zero-range delta-like scatterers $V(r) = \sum_i u_i \delta(r - r_i)$ [62], one readily obtains that $\tau^{-1} \propto v(\epsilon)$. As a result, the conductivity does not depend on the chemical potential $E_F$ or on the carrier density $n$, which is obviously incompatible with the experimental results. This observation is frequently used to dismiss the importance of short-range scattering in graphene. Thus, one has to bear in mind the limited applicability of this model. At high doping and considering any kind of realistic of disorder with a spatial range $\xi_{\text{range}}$, such that $k_F \xi_{\text{range}} \neq 0$, modeling disorder by zero-range scatterers is hardly justifiable. This model is also not very helpful for small doping either, since the Boltzmann semiclassical approach breaks down for small $kF l$. Even more importantly, it was realized very early [120] that short- (and nonzero-) range disorder leads to resonant scattering. In this situation, the Fermi golden rule, equation (17), breaks down and $\tau_t$ should be calculated instead as follows [121]: (i) use resonant scattering to calculate the transport cross section $\sigma_r$, as has been done, for instance, in [6] for a model where randomly placed hard discs of radius $R$ are the disorder source. (ii) Use $\tau_t = n_{\text{imp}} v_F \sigma_r (kR)$ [121] instead of equation (17). By proceeding this way, it was shown [6, 48, 49] that resonant scatterers give rise to mid-gap states and lead to a conductivity quasi-linear in $n$, depending on the radius $R$ of the scattering discs.

The Boltzmann transport theory is customarily used to address the effects of charge impurities in graphene [116, 41, 44]. In this case, the disorder scattering potential $V(q)$ becomes $V_C(q)/\varepsilon(q)$, where $V_C(q) = 2\pi r^2/(kF)$ is the Fourier transform of the Coulomb potential in an effective background dielectric constant $\kappa$, whereas $\varepsilon(q)$ is the graphene dielectric function. Within this disorder model, the influence of the substrate on the electronic transport can be directly accessed by properly modifying $\kappa$. This is the theoretical basis for the experimental work in [55, 56]. The dielectric function

$$\varepsilon(q, T) = 1 + \varepsilon_C(q) \Pi(q, T)$$  

(18)

with $\Pi(q, T)$ denoting the irreducible polarizability function given by the standard bare bubble diagram [41, 122, 44]. The non-analytical behavior of $\Pi(q)$ at $q = 2kF$ leads to Friedel-like oscillations in the screened potential in real space, namely, $\phi(r) \sim \cos(2kF r)/r^3$ [84, 122]. Both $\Pi(q, T)$ and the chemical potential $\mu(T)$ temperature dependencies have a strong influence on the conductivity. For low temperatures, such that $T/T_F \ll 1$, the conductivity depends very weakly on $T$, whereas for $T/T_F \gg 1$ one finds [123] that $\sigma \propto (T/T_F)^{-2}$.

The Boltzmann theory has been also used to calculate the conductivity of graphene in the presence of static ripples [49] and phonons [48, 124, 125], with limited success in explaining the experiments, in particular the temperature dependence of $\sigma$. Let us also mention that the standard Boltzmann approach was used to calculate the graphene conductivity for the correlated Gaussian disorder model discussed in section 4, yielding [126]

$$\sigma(n) = \frac{2\sqrt{\pi} e^2}{K_0 h} \left[ (2\pi |n| e^2)^{3/2} + O(|n| e^2)^{1/2} \right].$$  

(19)

Since $K_0 \propto n_{\text{imp}}$ [109], the conductivity increases linearly with the inverse disorder concentration $n_{\text{imp}}^{-1}$. For high carrier concentrations, such that $n e^2 \gg 1$, $\sigma$ is proportional to $n^{-1/2}$.

The next standard approximation level beyond the Boltzmann transport theory is the self-consistent Born approximation, as mentioned in section 4. The starting point for the latter is the Kubo formula, which gives the conductivity in terms of Green’s functions. While the standard Boltzmann
approximation treats the scattering within the Fermi Golden rule, the SCBA prescribes an efficient way to encode the main scattering processes of an electron in a disordered potential into the Green’s function self-energy (see for instance [127]).

Ostrovsky et al [17] presented a thorough study of the conductivity due to generic (long- and short-range) Gaussian disorder in monolayer graphene at finite bias using the self-consistent Born approximation and a renormalization group (RG) analysis [23]. While for non-resonant (or weak) impurities the SCBA and the RG analysis give similar results, for resonant scattering the RG conductivity renders

$$\sigma(n) = \frac{e^2}{4\pi^2 n_{\text{imp}}} \ln^2 \frac{\Delta^2}{v_F^2|n|}$$  \hspace{1cm} (20)

where $\Delta$ is a momentum cutoff. This result is very similar to the one obtained for the hard disk model [6]. Note that the RG analysis for Gaussian disorder, including resonance scattering, is significantly different from the one obtained using Fermi’s golden rule, equation (19) [126].

The RG analysis [23, 17] has been also recently employed to study the conductivity in the presence of randomly placed adsorbates [128]. For resonant states far from the Fermi energy, it predicts quite distinct and asymmetric curves for $\sigma(n)$. For instance, $\sigma(n)$ can show an almost insulating-like behavior for p-type graphene and be metallic-like for n-type graphene. This is consistent with some, but not all, experiments with p-type graphene and be metallic-like for n-type graphene.

Before concluding this section, it is worth mentioning that although typical, good quality, micrometer-size graphene samples show $L/l > 1$, many do not belong to the regime $L/l \gg 1$. Hence, deviations from the diffusive theory presented above, such as in [63], should be expected. Numerical simulations provide insight into the ballistic–diffusive crossover regime at finite doping [81]: with increasing disorder strength, the conductivity dependence on the carrier concentration moves from a sublinear dependence, resembling that of equation (9), to a superlinear dependence.

7. Conclusions and outlook

In this review we attempted to highlight the most important developments related to the role of disorder in the electronic transport properties of graphene. We emphasized the role played by symmetry breaking and discussed the various types of disorder commonly found in exfoliated graphene sheets. Although some aspects of the interplay between disorder and quantum coherence still require theoretical and experimental investigation, much about strong and weak localization and mesoscopic fluctuations of conductance in graphene is already well understood. However, there is still no consensus about which scattering mechanism plays the dominant role in limiting mobility in both suspended and non-suspended graphene field-effect devices. The widespread view that charge traps in the substrate are the most effective scatterers of carriers in high-mobility graphene on oxide substrates has been challenged by recent experiments.

Several important topics were not covered in this review. For instance, disorder (intrinsic or extrinsic) plays an important role in inducing spin–orbit coupling in graphene and, consequently, in reducing spin relaxation times [129–132]. Thus, investigating how scattering mechanisms of various forms affect spin relaxation is critical for advancing graphene as a spintronics material. Another situation where disorder is critical is in nanoribbons [133–135]. In general, it is fair to say that there are still many open problems to explore in electronic transport in disordered graphene systems.

Acknowledgments

We would like to thank A H Castro Neto, M Ishigami, C Mudry, and N M R Peres for useful discussions and V M Pereira for carefully reading the manuscript. This work was supported in part by CNPq.

References

[1] Novoselov K S et al 2005 Proc. Natl Acad. Sci. USA 102 10451–3
[2] Berger C et al 2004 J. Phys. Chem. B 108 19912
[3] Zhang Y, Tan Y-W, Stormer H L and Kim P 2005 Nature 438 201–4
[4] Wallace P R 1947 Phys. Rev. 71 622–34
[5] Castro Neto A H, Guinea F, Peres N M R, Novoselov K S and Geim A K 2009 Rev. Mod. Phys. 81 109–62
[6] Peres N M R 2010 Rev. Mod. Phys. submitted
[7] Das Sarma S, Adam S, Hwang W H and Rossi E 2010 arXiv:1003.4731
[8] Abergel D S L, Apalkov V, Berashevich J, Ziegler K and Chkhrabarty T 2010 Adv. Phys. at press
[9] Saito R, Dresselhaus G and Dresselhaus M S 1998 Physical Properties of Carbon Nanotubes (Singapore: World Scientific)
[10] Katsnelson M I 2006 Eur. Phys. J. B 51 157
[11] Tworzydlo J, Trauzettel B, Titov M, Rycerz A and Beenakker C W J 2006 Phys. Rev. Lett. 96 266802
[12] Müller M, Bräuninger M and Trauzettel B 2009 Phys. Rev. Lett. 103 196801
[13] Ostrovsky P M, Gornyi I V and Mirlin A D 2007 Phys. Rev. B 75 245403
[14] Gade R 1993 Nucl. Phys. B 398 499–515
[15] Guruswamy S, LeClair A and Ludwig A W W 2000 Nucl. Phys. B 583 475–512
[16] Mudry C, Ryu S and Furusaki A 2003 Phys. Rev. B 67 064202
[17] Ostrovsky P M, Gornyi I V and Mirlin A D 2007 Phys. Rev. B 74 235443
[18] Morpurgo A F and Guinea F 2006 Phys. Rev. Lett. 97 196804
[19] Fradkin E 1986 Phys. Rev. B 33 3263–8
[20] Ludwig W A A, Fisher M P A, Shankar R and Grinstein G 1994 Phys. Rev. B 50 7526–52
[21] Ryu C, Mudry C, Furusaki A and Ludwig A W W 2007 Phys. Rev. B 75 205344
[22] Khveshchenko D V 2006 Phys. Rev. Lett. 97 036802
[23] Aleiner I L and Efetov K B 2006 Phys. Rev. Lett. 97 236801
[24] Altland A 2006 Phys. Rev. Lett. 97 236802
[25] Crespi A et al 2008 Nano Res. 1 361–94
[26] McCann E et al 2006 Phys. Rev. Lett. 97 146805
[27] Krasheninnikov A V and Nordlund K 2010 J. Appl. Phys. 107 071301
[28] Teweldebrhan D and Balandin A A 2007 Nano Lett. 7 1643–8
[29] Pereira V M, Castro Neto A H, Liang H Y and Mahadevan L 2010 arXiv:1004.5384
[119] Schedin F et al 2007 Nat. Mater. 6 652
[120] Peres N M, Guinea F and Castro-Neto A H 2006 Phys. Rev. B 73 125411
[121] Ziman J M 1972 Principles of the Theory of Solids (Cambridge: Cambridge University Press)
[122] Wunsch B, Stauber T, Sols F and Guinea F 2006 New. J. Phys. 8 318
[123] Hwang E H and Das Sarma S 2009 Phys. Rev. B 79 165404
[124] Fratini S and Guinea F 2008 Phys. Rev. B 77 195415
[125] Mariani E and von Oppen F 2008 Phys. Rev. Lett. 100 076801
[126] Adam S, Brouwer P W and Das Sarma S 2009 Phys. Rev. B 79 201404(R)
[127] Bruus H and Klensberg K 2004 Many-Body Quantum Theory in Condensed Matter Physics (Oxford: Oxford University Press)

[128] Robinson J P, Schomerus H, Oroszlany L and Fal’ko V I 2008 Phys. Rev. Lett. 101 196803
[129] Huertas-Hernando D, Guinea F and Brataas A 2009 Phys. Rev. Lett. 103 146801
[130] Castro Neto A H and Guinea F 2009 Phys. Rev. Lett. 103 026804
[131] Popinciuc M et al 2009 Phys. Rev. B 80 214427
[132] Pi K et al 2010 Phys. Rev. Lett. 104 187201
[133] Mucciolo E R, Castro Neto A H and Lewenkopf C H 2009 Phys. Rev. B 79 075407
[134] Evaldsson M, Zozoulenko I V, Xu H and Heinzel T 2009 Phys. Rev. B 78 161407
[135] Han M Y, Brant J C and Kim P 2010 Phys. Rev. Lett. 104 056801