Numerical reconstruction of wave field spatial distributions at the output and input planes of nonlinear medium with use of digital holography

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Abstract. A numerical reconstruction of spatial distributions of optical radiation propagating through a volume of nonlinear medium at input and output planes of the medium was demonstrated using a scheme of digital holography. A nonlinear Schrödinger equation with Fourier Split-Step method was used as a tool to propagate wavefront in the volume of the medium. Time dependence of the refractive index change was not taken into account.

1. Introduction
It is known that optical systems have a limited field of view due to a finite value of the refractive index of lenses and a finite size of apertures. That is why, in the linear case, high spatial frequencies of the input radiation miss the detector. In the nonlinear case, these spatial frequencies after their nonlinear wave mixing with other components of the input radiation can create daughter waves propagating at a different angle, which allows these spatial frequencies to get to the detector [1]. The nonlinear processes are also accompanied by spectral broadening or spectral supercontinuum generation [2–4]. It is reported about a possibility to develop new methods to achieve the optical super-resolution based on the effect of the nonlinear wave mixing [5], because in the nonlinear case the recorded wavefront keeps the information about non-detected high spatial frequencies as well as evanescent waves of the input radiation. From this perspective, a development of a numerical model for imaging through a volume of nonlinear media has some point of interest.

2. Numerical reconstruction scheme
Wavefront distortions caused by nonlinearity depend on the parameters of the input radiation and the nonlinear medium [6]. In this paper, using digital holography method a numerical model was developed that allows to restore wave field at the output plane of the nonlinear medium from the known digital hologram and to get the original wave field at the input plane by back-propagation of previously calculated wave field. It is possible if a nonlinear response of the medium is known [6–8]. It was assumed that monochromatic laser source with continuous wave mode was used, therefore, a time dependence of the intensity induced refractive index change was not taken into account.

The wavefront reconstruction at the output plane of the nonlinear medium from the known digital hologram was performed in accordance with the method described in [9]. In the scheme of our numerical experiment two 4f-systems were used to transfer a wavefront from the test object O to the input plane of the nonlinear medium and from the output plane to the CCD photodetector (Figure 1).
Figure 1. An experimental setup scheme, illustrating the numerical experiment for wave field reconstruction in the volume of nonlinear media using an off-axis digital holography method: LM – laser, BE – beam expander, M – mirror, BS – beam splitter, O – test object, L – lenses, C – cuvette with tea in water solution (nonlinear medium), F – optical filter, CCD – photodetector.

To numerically restore a wave field at the input plane of the nonlinear medium using nonlinear digital holography, one needs to know a complex field of the radiation at some point. This complex field can be numerically propagated forward or backward [6] in the volume of the medium if the mathematical model of the propagating process is fairly accurate. Within a paraxial approximation the dynamics of the wave field of slowly varying wave function $U$ is well described with nonlinear Schrodinger equation [10,11]:

$$\frac{\partial U}{\partial l} = \left[\frac{i}{2k_0} \nabla^2 - i k_0 \Delta n(I)\right] U = \left[ D + N(U)\right] U$$

(1)

$U$ – complex wave field; $l$ – propagation direction; $i$ – imaginary unit; $k_0 = n_0(2\pi/\lambda)$ – wave number; $n_0$ – linear refraction index; $\lambda$ – wavelength; $\nabla^2$ – nabla; $\Delta n(I) = n_2 I$ – nonlinear refractive index change; $n_2$ – nonlinear refraction index; $I = n_0 E_0 c |U|^2/2$ – intensity; $D$ and $N$ – linear and nonlinear operators (the angular spectrum method [12,13] was used to calculate diffraction part $D$).

3. Results

Numerical simulation was performed with following parameters: $\lambda = 633$ nm; object size $2 \times 2$ mm ($512 \times 512$ pixels); thickness of the nonlinear medium $7.5$ mm; nonlinear step $\Delta l = 50$ um along the $l$ direction; optical power $1$ W; linear $n_0 = 1.33$ and nonlinear $n_2 = -1.8 \times 10^{-11}$ m$^2$/W refraction indexes.

Figure 2. Numerical retrieval amplitude and phase distributions of the wave field at the output (g,h) and input (i,j) planes of the nonlinear medium from the known digital hologram (e,f) of intersecting object (c,d) and reference beams (selections on pictures are enlarged by 10 times) with initial wave field distribution at the input plane (a,b).

Specified $n_0$ and $n_2$ parameters of the nonlinear medium correspond to the tea in water solution at the wavelength $\lambda \approx 633$ nm [14], the nature of its nonlinearity has a thermal origin. Results of the
numerical experiment are presented above (see Figure 2). The wave field distribution at the output plane of the nonlinear medium is determined by the balance of effects of diffraction and nonlinear defocusing that manifest themselves on the heterogeneity of initial distributions of amplitude (a) and phase (b). The numerical recovery algorithm can work with arbitrarily specified initial distributions.

4. Conclusion
A reconstruction of the wave field at the output and input planes of the nonlinear medium from the recorded digital hologram was numerically demonstrated. The method [9] was used to reconstruct wave field from the digital hologram. A wavefront back-propagation in the volume of the nonlinear medium was performed by the Fourier Split-Step method as a numerical solution of the nonlinear Schrodinger equation. Time dependence of the refractive index change was not taken into account.

The use of nonlinear methods in optics is of interest due to the fact that restoring nonlinear dynamics of the field and retrieving the missing spatial frequencies provides an opportunity to develop new methods for the optical super-resolution, including methods for optical sub-wavelength microscopy and lithography, microscopy with resolution exceeding diffraction limit. In this case, the effect of super-resolution results from a basic knowledge about the function of nonlinear propagation.

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