Probing the small-scale structure of spacetime

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Abstract. Many theoretical attempts at understanding the Planck-scale structure of spacetime can accommodate minute departures from Lorentz and CPT invariance. At presently attainable energies, such effects are expected to be accurately described within effective field theory. Such a field theory, in turn, can be employed to identify currently feasible experimental tests. This work presents an overview of the theoretical motivations, the low-energy framework, and the phenomenological implications within this context.

1. Introduction
In the last two decades, there has been a renewed interest in experimental and phenomenological studies of the structure of spacetime at microscopic distance scales. The two primary reasons for this development are the following. First, novel ultrahigh-sensitivity experimental techniques have been established, and new and better observational data have become available [1]. Second, theoretical approaches to more fundamental physics have suggested the possibility of small deviations from our conventional spacetime picture. In particular, the symmetries underlying the spacetime-manifold structure—translation, Lorentz, and CPT invariance—may be violated by minuscule amounts [2]. The present work is primarily concerned with Lorentz- and CPT-symmetry breakdown.

For the identification and interpretation of suitable Lorentz and CPT tests, a theoretical framework allowing for deviations from these symmetries is needed. In the present context, the establishment of such a framework is hampered by the absence of fully realistic and complete theories underlying the Standard Model of particle physics (SM) and general relativity (GR). With these consideration in mind, a framework that captures a wide variety of Lorentz- and CPT-violating effects at presently attainable energies would be desirable and would have to be constructed by hand.

Over the last two decades, this goal has indeed been achieved: a framework has been developed that is based on effective field theory [3] and has become known as the Standard-Model Extension (SME) [4, 5, 6]. The SME contains the SM and GR as limiting cases; it can therefore be applied in all presently feasible experimental situations permitting a theoretical description of practically all current and near-future Lorentz and CPT tests. To date, the SME has provided the basis for over 100 phenomenological studies [1] across many physical systems, such as cosmic radiation [7], meson factories [8] and other particle colliders [9], resonance cavities [10], neutrinos [11], precision spectroscopy [12], and gravity [13].

In addition to experimental and phenomenological applications, the SME has provided a framework for numerous theoretical analyses of violations of Lorentz and CPT invariance. These analyses were concerned with the mathematical structure of effective field theories.
with spacetime-symmetry breaking [14], spontaneous Lorentz violation and Nambu–Goldstone modes [15], classical limits of Lorentz- and CPT-breaking physics [16], loop corrections and renormalizability [17, 18], as well as certain aspects of non-renormalizable contributions [19]. The above studies have added to the strength of the theoretical basis of the SME.

The outline of this work is as follows. Section 2 reviews the basic ideas behind the philosophy and the construction of the SME. A few remarks about possible mechanisms for Lorentz and CPT violation in more fundamental physics are contained in Sec. 3. Section 4 concludes this work with the discussion of some classes of experimental searches for departures from Lorentz and CPT symmetry.

2. SME basics

As mentioned in the introduction, the SME has to be constructed by hand, so some prior considerations are necessary. These include the physical framework, the scope, and the actual implementation of Lorentz and CPT breaking.

The physical framework for the description of deviations from spacetime symmetries at small distance scales should, for example, be microscopic; it should also be appropriate for the description of potential quantum-gravity features, such as the low-energy limit of an underlying Planck-level discreteness of spacetime. Effective field theory appears to be suitable in this context: it has been successfully used to describe non-relativistic many-body effects in condensed-matter systems. Note that such systems also exhibit a discrete structure at the atomic level, but the long wave-length limit is nevertheless governed by field theory. From a more general perspective, effective field theory is also a versatile tool in various other subfields of physics, such as nuclear and elementary-particle physics as well as hydrodynamics, and this tool is well understood from a theoretical perspective. For these reasons, a low-energy description of Lorentz and CPT violation in terms of effective field theory seems to be a general and well-founded assumption, and the starting point for the construction of the SME will be a field-theory Lagrangian.

The scope of the model is another important consideration. In the absence of a unique and fully realistic quantum-gravity theory, it is practical to perform broad experimental searches for Lorentz and CPT breakdown in a wide variety of physical systems. For this reason, the SME needs to allow for a general description various types of Lorentz and CPT violation, and it should do so for all known particle species. This can be achieved if the SME Lagrangian $L_{\text{SME}}$ contains both the usual SM Lagrangian $L_{\text{SM}}$ as well as the Einstein–Hilbert Lagrangian $L_{\text{GR}}$ for GR. This yields

$$L_{\text{SME}} = L_{\text{SM}} + L_{\text{GR}} + \delta L_{\text{LV}},$$

as a sensible ansatz for the SME model. Here, $\delta L_{\text{LV}}$ represents Lorentz- and CPT-violating departures from established physics across all presently accessible physical systems.

The Lagrangians $L_{\text{SM}}$ and $L_{\text{GR}}$ are known explicitly, so it remains to determine the Lorentz- and CPT-breaking piece $\delta L_{\text{LV}}$. Various ways can be envisaged to construct this term. Whatever this construction is, it should maintain coordinate independence of physics: coordinates are just a convenient mathematical tool for the description of physical quantities, but their choice should not affect the actual physics. With coordinate independence in mind, one might consider implementing Lorentz and CPT breaking via six spacetime transformations that are more general than the usual Lorentz transformations $\Lambda^\mu_{\nu}$, but contain $\Lambda^\mu_{\nu}$ as limiting cases at, say, low energies. We won’t follow this approach here because from a mathematical viewpoint, the Lorentz group is known to be stable. This essentially means that small group deformations are isomorphic to the original Lorentz group, so that it would be difficult to imagine observable effects. Another possibility for introducing Lorentz and CPT violation while maintaining coordinate independence is through non-dynamical background vectors and tensors.
$b^\mu, c^{\mu\nu}, (k_{AF})^\mu, \ldots$ that select preferred directions. This type of Lorentz and CPT breaking provides, in fact, the basis for the construction of the SME.

With the above considerations, we are finally in the position to write down explicitly sample terms contained in $\delta \mathcal{L}_{LV}$ in the flat-spacetime limit:

$$\delta \mathcal{L}_{LV} \supset b^\mu \bar{\psi} \gamma_\mu \psi, \ c^{\mu\nu} \bar{\psi} \gamma_\mu \partial_\nu \psi, \ (k_{AF})^\mu A^\nu \tilde{F}_{\mu\nu}, \ldots,$$

where $\psi$ is a SM fermion, and $\tilde{F}_{\mu\nu}$ denotes the dual field strength corresponding to the SM photon. The non-dynamical background vectors and tensors $b^\mu, c^{\mu\nu}, (k_{AF})^\mu, \ldots$ were introduced earlier; they control the type and extent of deviations from Lorentz symmetry. Both $b^\mu$ and $(k_{AF})^\mu$ have dimensions of mass. In addition to Lorentz invariance, they also violate CPT symmetry. The $c^{\mu\nu}$ coefficient is dimensionless and preserves CPT. These coefficients lead to physical effects that can be measured or constrained in experiments.

It is evident that the above construction possesses a large degree of generality. In fact, infinitely many terms of the form (2) can be added to $\delta \mathcal{L}_{LV}$. From a phenomenological perspective, the dominant terms are most interesting. These terms would be expected to be given by the subset of relevant and marginal operators one can construct. This yields a set of infinitely many contributions to $\delta \mathcal{L}_{LV}$ with a manageable number of Lorentz-violating coefficients. Other physically desirable features, like translation invariance and the usual gauge structure of the SM, can be imposed. This determines the most popular limit of the SME called the minimal Standard-Model Extension (mSME).

We remark that in curved-spacetime contexts involving non-negligible gravitational effects, it is generally difficult to impose spacetime independence on the Lorentz-breaking background vectors and tensors. Moreover, dynamical features must be added to the background Lorentz-violating quantities (such as spontaneous Lorentz breakdown) to ensure compatibility with the underlying geometric manifold structure.

3. Possible underlying sources for Lorentz breaking

Thus far, we have merely parametrized possible low-energy remnants of Lorentz violations within effective field theory. While such an approach was forced on us from a phenomenological perspective, it leaves unanswered the question concerning the origin of Lorentz breaking at a more fundamental level. As mentioned in the introduction, this question is presently difficult to address because of the absence of a completely satisfactory theory underlying the SM and GR. However, one can make some general remarks regarding possible mechanisms that can produce Lorentz-violating background fields, such as $b^\mu, c^{\mu\nu}, (k_{AF})^\mu, \ldots$ introduced above. This section provides brief overviews of some of the mechanisms cited in Ref. [2].

**Spontaneous Lorentz and CPT violation.**—From a theoretical point of view, spontaneous symmetry breaking (SSB) is an attractive mechanism for creating the Lorentz-violating $b^\mu, c^{\mu\nu}, (k_{AF})^\mu, \ldots$ coefficients in the SME. SSB is well established in condensed-matter physics. Moreover, in elementary-particle physics SSB is presently the preferred way for a theoretically consistent description of heavy spin-1 bosons, and it is therefore a major ingredient in the SM.

The basic idea behind SSB is that the lowest-energy state of the system, which is commonly associated with the vacuum, requires the field value to be non-zero. Such non-vanishing vacuum expectation values (VEVs) typically do not reflect the dynamical symmetries of the system and are the key observable effect associated with SSB. In the case of Lorentz violation, these VEVs correspond to the $b^\mu, c^{\mu\nu}, (k_{AF})^\mu, \ldots$ coefficients contained in the SME. The interactions required for the spontaneous breakdown of Lorentz symmetry are difficult to fit into the framework of conventional renormalizable gauge theories. String field theory, on the other hand, provides a setting in which such interactions are natural.

**Varying scalars.**—Various theoretical approaches to more fundamental physics involve additional scalar fields beyond the SM Higgs. Moreover, some astrophysical observations, such
as the flatness of the universe and its accelerated expansion, are often taken as experimental hints for novel scalars: the idea is that in such cosmological contexts, a scalar field can acquire a non-zero global value with a time evolution closely tied to that of the scale factor.

As an explicit example, let us consider \( N = 4 \) supergravity in four spacetime dimensions. Although unrealistic in detail (its particle content cannot be matched exactly to observations), it is contained in M-theory and may therefore yield insight into some generic features of a candidate quantum-gravity model. One such feature is the aforementioned presence of additional scalar fields: an axion \( a \) and dilation \( b \), which are coupled via a known function \( f(a, b) \) to the electromagnetic field strength, such that the term \( f(a, b) F \bar{F} \) is contained in the model’s Lagrangian. Within a simple cosmological setting, it is then possible to determine the dependence of the axion and the dilaton on the comoving time \( t \). This produces the effective lagrangian term \( f(t) F \bar{F} \). In a localized experiment, such a term would indeed be perceived as a varying coupling (in the present case as a time-dependent \( \theta \) angle).

A scalar field with a global spacetime dependence, regardless of the mechanism that drives this effect, is typically associated with a violation of spacetime-translation symmetry. Note also that Lorentz invariance is normally broken in such cases: since the gradient of a varying scalar field must be non-zero (at least in some regions of spacetime), it selects a preferred direction. Mathematically, this result is intuitively reasonable because the definition of the Lorentz-transformation generators involves the energy–momentum tensor, which ceases to be conserved. It follows that the ordinary time-independent rotation and boost operators may not exist. Effective Lorentz violation can also be explicitly exposed in our toy supergravity model: an integration by parts at the level of the action yields

\[
\frac{1}{2} \partial^\alpha f \to -2 (\partial^\alpha f) A^\beta \bar{F}_{\alpha \beta},
\]

which leaves unaffected the equations of motion under mild assumptions. The gradient \( \partial^\alpha f \) of the cosmological background \( f(t) \) is practically outside of experimental control for all local measurements. The gradient can then be taken as non-dynamical and—for time dependencies on cosmological scales—as approximately constant. Identifying \( -2 (\partial^\alpha f) \) with \( (k_A F)^\alpha \) in the SME context of Eq. (2), we explicitly see how this Lorentz- and CPT-violating Chern–Simons-type correction can arise in underlying physics.

**Non-commutative field theory.**—This popular approach to capture quantum aspects of the dynamics of spacetime is based on the idea that coordinates are no longer real numbers; they are promoted to operators that obey nontrivial commutation relations. A standard example often found in the literature is given by \([x^\mu, x^\nu] = i \theta^{\mu \nu} \). In this expression, the quantity \( \theta^{\mu \nu} \neq 0 \) is usually defined to be spacetime independent, and it therefore clearly exhibits the attributes of a coefficient in the SME, such as selecting preferred directions. For the physical interpretation of non-commutative models, one typically employs the Seiberg–Witten map, which transforms the model to a conventional field theory on Minkowski space. This Seiberg–Witten image of the non-commutative model still involves the non-dynamical and constant \( \theta^{\mu \nu} \), which acts precisely like a coefficient in the SME.

**Loop quantum gravity.**—A further widely studied approach to a quantized version of GR is loop quantum gravity. In certain semiclassical investigations, a number of results have been obtained that are incompatible with the maintenance of Lorentz symmetry. For example, under certain reasonable assumptions both electrodynamics and fermions acquire loop-quantum-gravity corrections that correspond at low energies to various Lorentz-violating SME coefficients.

### 4. Experimental studies

Once a consistent test framework for Lorentz and CPT breaking has been established and convincingly motivated, it can be employed for its primary purpose: the identification, analysis, and comparison of Lorentz-symmetry tests. As mentioned in the introduction, the last decade
or so has witnessed a revival of interest in Lorentz tests, and this section surveys some of the main ideas in this field.

**Kinematical tests involving particle reactions.**—One of the key effects of Lorentz and CPT violation is that one-particle dispersion relations are generally modified: they now contain the $b^{\mu}, c^{\mu\nu}, (k_{A\Phi})^\mu, \ldots$ coefficients from the SME. Since these are taken as spacetime independent in non-gravitational mSME contexts, the corresponding modified 4-momenta remain conserved. This will normally affect the details of the energy–momentum kinematics in particle reactions. For instance, decay thresholds may be shifted, collision processes kinematically forbidden in the Lorentz-invariant case may now occur, and certain conventional decays or reactions may no longer be permitted.

As an example, let us look at the spontaneous emission of a photon from a free charge. In ordinary Lorentz-symmetric physics, 4-momentum conservation forbids this process. However, certain mSME coefficients can act like a crystal on electromagnetic radiation causing an “index of refraction” even in vacuum. This, in turn, can lead to a photon speed that is smaller than that of a charge. Paralleling the ordinary Cherenkov effect (when photons propagate slower inside a macroscopic medium with index of refraction $n > 1$), charges are then unstable against the emission of Cherenkov light in such a Lorentz- and CPT-breaking spacetime.

Depending on the type of Lorentz violation, this “vacuum Cherenkov effect” may or may not involve a threshold. Let us continue by focussing on those mSME coefficients that would lead to a threshold for vacuum Cherenkov radiation. In such a case, we may determine an experimental limit on the size of this type of Lorentz breaking in the following way. Charges propagating at speeds larger than the modified speed of photons would not be stable: they would decelerate to speeds below threshold via the emission of vacuum Cherenkov photons. It then becomes apparent that if high-energy stable charges are observed, they have to have energies below the threshold. Thus, the simple observation of highly energetic electrons, for example, determines a lower bound for the threshold energy, from which a limit on Lorentz violation can be extracted. Employing LEP electrons with energies up to $104.5$ GeV in this context yields a (one-sided) constraint at the level of $1.2 \times 10^{-11}$ on the appropriate mSME coefficients.

Another effect along these lines is photon splitting in vacuum. In ordinary Lorentz-invariant physics, such a decay process is forbidden kinematically by energy–momentum conservation. However, certain types of Lorentz breaking may actually lead to photon speeds greater than $c$ instead of slowing light down. It is then possible for photons to propagate faster than the maximal attainable speed of charges. In analogy to the vacuum-Cherenkov case discussed above, in which charges beyond the threshold become unstable, one expects that highly energetic photons can now split into a charge–anticharge pair. A dispersion-relation analysis within the mSME test framework indeed confirms that this is the case. Like the vacuum Cherenkov effect, photon splitting in a Lorentz-breaking spacetime often occurs beyond a threshold, and it is then an excellent tool for the determination of observational constraints on this particular type of Lorentz breakdown. The line of reasoning parallels the Cherenkov case: if photons are observed to be stable, their energy must lie below the splitting threshold. One can then conclude that this threshold must be higher than the observed energy of these stable photons. This bound on the threshold energy can be used to establish a constraint on the value of the associated mSME coefficient. At the Tevatron collider, stable 300 GeV photons have been measured, which yields a (one-sided) limit of $5.8 \times 10^{-12}$ on the appropriate Lorentz-violating coefficient.

We remark that the derivation of the above constraints requires not only the kinematical possibility of the decays, but also a sufficient efficiency of these processes. But for calculations of the decay rate, the dispersion relation alone is inadequate, and dynamical aspects need to be considered. Since the SME is an effective field theory, dynamical investigations, such as decay-rate and cross-section studies, can be performed. In the above context of vacuum Cherenkov radiation and photon splitting, the efficiency required for a conservative extraction of bounds
has been confirmed validating our purely kinematical reasoning.

Spectropolarimetric studies of astrophysical sources.—The mSME’s photon sector contains the aforementioned \((k_{\text{AF}})^\mu\) coefficient. It has mass dimension 1, and the corresponding operator breaks both Lorentz and CPT symmetry. The \((k_{\text{AF}})^\mu\) contribution to the Lagrangian is sometimes called a Chern–Simons term. This term would lead to various effects, such as birefringence of light, vacuum Cherenkov radiation, and cavity-frequency shifts. These Lorentz- and CPT-violating effects would lead to measurable signatures and can therefore be employed for experimental tests. Observational searches for birefringence in astrophysical photons are a particularly promising tool in this context because the cosmological scale of the propagation time translates directly into extremely high sensitivities to the \((k_{\text{AF}})^\mu\) coefficient. Spectropolarimetry of electromagnetic radiation from astrophysical objects has placed constraints better than \(10^{-42}\) GeV on various components of \((k_{\text{AF}})^\mu\).

Investigations of cold antihydrogen.—Lorentz violation does not only affect free particles at high energies, but also bound-state energy levels. To gain some intuition, recall that departures from Lorentz symmetry are represented by background vectors and tensors in the SME, which is analogous to conventional physics in an external electromagnetic field. One therefore expects Lorentz breaking to affect bound systems via level shifts similar to the Zeeman and Stark effects, and this expectation can indeed be confirmed with SME calculations.

Lorentz breaking that is accompanied by CPT violation can be particularly well studied with matter–antimatter comparisons. In the present bound-state context, hydrogen (\(H\)) and antihydrogen (\(\bar{H}\)) spectroscopy provide an excellent probe for such CPT-violating effects. One of the transitions that can be considered is the unmixed 1S–2S transition: the expected experimental sensitivity for this measurement is at the \(10^{-18}\) level, a promising reach in the context of a possible Planck suppression of effects related to the quantum structure of spacetime. Although a study within the mSME reveals that there are indeed Lorentz-violating level shifts in hydrogenlike systems, they are identical in the initial and final states at leading order, so that the transitions frequency remains unaffected. From this viewpoint, it appears that the unmixed 1S–2S transition is actually less suitable to search for unsuppressed Lorentz- and CPT-violating signatures governed by effective field theory. Within the mSME, one can show that the leading non-vanishing correction to this transition arises from relativistic effects, and it contains two additional factors of the fine-structure constant \(\alpha\). The transition-energy shift, which is expected to be tiny already at order zero in \(\alpha\), therefore comes with a further suppression factor of roughly ten thousand.

A further transition suitable for Lorentz- and CPT-violation searches is the spin-mixed 1S–2S transition. When \(H\) or \(\bar{H}\) is confined with electromagnetic fields—such as in a Ioffe–Pritchard trap—both the 1S and the 2S states are split due to the conventional Zeeman effect. For this set-up, a study within the mSME establishes that the 1S–2S transition between the spin-mixed states is indeed altered at first order by Lorentz- and CPT-breaking coefficients. From a practical perspective, the magnetic-field dependence of this transition needs to be considered: it can limit the experimental sensitivity due to the inhomogeneity of the \(\vec{B}\) field in the trap, which represents a drawback for studying this particular transition. However, the development of new experimental techniques may circumvent this drawback, and a frequency resolutions in the vicinity of the natural linewidth may then be achievable.

A third possibility for a transition suitable for Lorentz and CPT tests lies in the hyperfine Zeeman transitions within the 1S state itself. Even in the limit of a zero \(\vec{B}\) field, the mSME establishes leading-order level shifts for two of the transitions between the Zeeman-split states. We note that this idea may also provide some experimental benefits: a variety of similar transitions, such as the usual H-maser line, can be well resolved in the laboratory.

Lorentz-violation searches involving Penning traps.—An analysis within the mSME demonstrates that not only energy levels in atoms can be shifted by departures from Lorentz
and CPT symmetry, but also, for example, the levels of protons and antiprotons confined in a Penning trap. A perturbation-theory study establishes that only a single mSME coefficient (the previously mentioned CPT-violating $b^a$-type background vector, which is contracted with the chiral current of a fermion) would lead to shifts in the transition frequency of the proton relative to the antiproton at first order. More concretely, the anomaly frequency of the proton is moved in a direction opposite to that of its antiparticle. This prediction can be employed to extract a clean experimental limit on the $b^a$ coefficient of the proton.

Tests at Neutral-meson factories.—Neutral-meson interferometry is widely appreciated to be an ultra-sensitive probe for CPT-symmetry: even minuscule differences between the meson and the antimeson mass would yield observable signatures suitable for interferometric experiments. Despite the fact that there is only a single mass parameter in the mSME for a each quark species and the associated antiquark species, these (anti)particles can nevertheless be affected differently by the Lorentz- and CPT-violating mSME coefficients. This can lead to a disparity between the dispersion relations for a meson and its antimeson, so that they can exhibit distinct energies although they have the same 3-momenta. Interferometric experiments are sensitive to such a difference in energy, and this idea provides the basis for measurements in this context. We note that in addition to the K-meson, also other neutral mesons can be studied with this idea. We remark in particular that not only CPT violation but also Lorentz breaking is present, so that boost- and rotation-dependent signals can be constrained.

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