Status Report and first Light from CANNELX:
Casimir Force Measurements between flat parallel Plates

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Abstract. The Casimir And Non-Newtonian force EXperiment (CANNELX) was designed to detect Casimir and hypothetical fifth forces between truly parallel plates of cm size, set 10–30 µm apart. With sub-pN sensitivity and large interacting areas, the experiment aims to settle a long-standing question of Casimir physics regarding the role of dissipation at zero frequency in the description of dielectric functions. Active measurement and control of parallelism allows to accurately probe non-standard geometries, such as crossed cylinders or a cylinder opposing a plate. If the designed precision could also be reached in gas at pressures up to 500 mbar, CANNELX has been predicted to rule out completely the so-called chameleon model as explanation for dark energy. After a 6-year construction phase, the experiment has reached a first operational prototype state. In the present article, we give an overview of the setup and applied methods, present proofs of principle for key-technologies, and discuss technical hurdles yet to be overcome. Finally, we present first force gradient measurements between parallel plates in the range 6–40 µm.

1. Introduction
Force metrology is a way to test (new) physics in the nm to mm range of separations at low energy. In this sense, and for a limited range, force metrology could be seen complementary to the high-energy searches at large colliders. Most recent precision force measurements of short-range Casimir and Yukawa forces [1–6] have been performed between curved surfaces in order to avoid the technical difficulty of maintaining parallelism. As all known surface interactions depend inversely on the effective separation, even with large curvature radii, the by far largest contribution to these interactions is generated in a very small patch around the point of closest approach [7, 8]. Accordingly, the results of measurements depend strongly on the local surface properties in this patch, such as deformations, [7, 9] or surface potentials [10]. In metrological experiments, a detailed characterization of the interacting surfaces and the resulting local ‘patch effects’ is therefore of high importance, which can be challenging [11–13].
For perfectly parallel, flat surfaces, on the other hand, the entire surface intrinsically contributes to force generation, thereby leading to significantly (several orders of magnitude) stronger interactions [7, 14], as compared to curved test objects of similar size. At the same time, local electrostatic (as well as other) patch effects are suppressed in the parallel plate geometry due to multipole averaging [13]. This holds especially if the plate separation is much larger than the typical patch size [13, 15]. Finally, parallel plates closely resemble the one-dimensional geometric idealization often used in theory, for which experimental results are readily comparable to theoretical predictions.
These advantages of flat interacting surfaces over curved ones have motivated several attempts to
overcome the technical difficulties (required flatness, measuring and keeping parallelism, parallel displacement, dust) associated with the parallel plate geometry \([12-21]\). However, only few \([15, 17]\) groups have reported actual measurements. The long development times of such experiments and limited options to provide sufficient funding without immediate results represent high hurdles. In recent years, the development of micromachining technology has opened up an alternative way for force measurements between flat etched silicon structures \([22-24]\). However, three-body interactions with the underlying substrate, and limited control over the residual tilt angle render such measurements still unsuitable for metrology.

1.1. Physical motivation

Some motivation for force metrology comes from the Casimir effect. Theorized 1948 \([25]\), this force – arising from limiting the mode spectrum of virtual and thermal photons – has since received strong interest. A wide variety of aspects have been investigated over the past two decades, among which are the vanishing influence of thin layers \([26]\), lateral \([27]\) and repulsive configurations \([28]\) (review: \([29, 30]\)). Experiments are notoriously difficult, as Casimir forces are very short ranged, for which measurements are performed typically at separations \(\lesssim 100 \text{ nm}\) using atomic force microscopes \([31, 32]\), or torsion balances \([33-35]\). At such separations, surface roughness \([34-39]\), local variations of the Fermi potential (patches) \([13, 40, 41]\), and impurities have to be considered. Despite all efforts, there are still open questions regarding the Casimir effect. The first one regards a rather new ‘surface current’ approach promising accurate computations of Casimir interactions between arbitrarily shaped three-dimensional objects using finite element methods \([42]\). While making available a tool applicable even by non-experts, the accuracy of the approach cannot be tested properly, as nearly all available reference data have been obtained in the plate-sphere geometry \([1]\). A thorough verification of the surface current approach would require percent-level accuracy measurements of the Casimir force at separations \(\gtrsim 1 \mu \text{m}\) \(^3\) in geometries such as crossed cylinders or a cylinder vs. a plate, that all require control over the relative alignment and tilt of the interacting objects. Another interesting question regards a controversy \([3, 4, 45-48]\) concerning the role of dissipation at zero frequency. Experimental results at small separation \([39]\), where virtual photons give the major contribution to the Casimir effect, and for the special case of graphene \([50]\), suggest that dissipation has to be disregarded at zero frequency – in contradiction to a non-vanishing DC electrical resistance. At larger separation \([4]\), where thermal photons contribute most to the force (thermal Casimir effect), the situation is unclear \([38]\). While seemingly of technical nature, the problem is deep. If we would find that dissipation at zero frequency has to be included to describe experimental data taken at large separation, this would indicate that there exists a qualitative difference between thermal and virtual photons that, in this form, is not included in theory – possibly opening a door to new physics.

Another field motivating metrological force measurements is cosmology with its mystery of dark energy and the cosmological constant problem \([51]\). The 120 orders of magnitude mismatch between the measured value and the theoretical prediction for vacuum energy is pointing to a fundamental error in our understanding of physics. Introduced a century ago by Einstein as a temporary pragmatic remedy to avoid gravitational collapse in his theory of general relativity, the cosmological constant \(\Lambda\) still represents the homogeneous dark energy background in the successful cosmological ‘standard model’ \(\Lambda\)-CDM (cold dark matter with cosmological constant). While \(\Lambda\)-CDM is in agreement with the extensive experimental evidence \([52-55]\), the fact that \(\Lambda\) is an \textit{ad-hoc} introduced parameter with a value fine-tuned to fit our observations, remains unsatisfactory. One idea to find a more fundamental explanation for the measured dark energy density is to replace \(\Lambda\) in Einstein’s field equations by the dynamics of a new (scalar) field \(\phi\) (review: \([56]\)). Such a field would interact with other sectors of the standard model, for which this ‘quintessence’ approach is strongly constrained (early review: \([57]\)) by

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4 With the notable exceptions of Refs. \([22, 26, 43, 43]\), which, however, did not reach high accuracy, and measurements using corrugated plates opposing a sphere.

5 For 1 cm\(^2\) parallel plates, this would translate to an accuracy of the order 1 pN at 6 \(\mu\text{m}\), while for a cylinder of \(\sim 400 \mu\text{m}\) diameter at 2.5 \(\mu\text{m}\) from the plate, it would be around 0.1 pN.
experimental data [58, 59] and cannot explain dark energy. A way for quintessence to still give gravity-strength interactions on a cosmic scale, while avoiding excluded modifications of general relativity within galaxies are screening mechanisms (review: [53]). One particular way of screening is implemented by letting the mass $m_\phi$ of $\phi$ depend on the local energy density (chameleon screening [56, 60]). Limits on the chameleon coupling parameter $\beta_\phi$ to light have been found in afterglow experiments [61–63]. General couplings $\beta$ have been probed (among others, review: [64]) with neutron interferometry [65], bouncing neutrons [66–68], microspheres [69], and interferometry with Cs atoms [70, 71], the latter giving the presently lowest upper bounds on the interaction strength. Limits can also be derived from Casimir force experiments [2, 4, 5, 72]. Lower bounds ($0.01 \lesssim \beta < 1$) are imposed by torsion balance measurements [73].

1.2. The ‘business case’
Chameleon interactions are hard to probe, as the predicted strength lies at or below Casimir interactions and has a similar dependence on object separation as electrostatic effects. In 2010, it was proposed [74] that chameleon forces could be detected in a parallel plate force experiment, in which the separation is held constant while the pressure of ambient Xe gas is varied between 1 and 500 mbar. Electrostatic, Casimir, and gravitational forces would increase with the pressure, while the chameleon force is predicted to decrease. This qualitative difference would make the experiment highly sensitive to chameleon forces. If reaching a precision of 0.1 pN, for a plate area of 1 cm$^2$ and a separation of $\sim 10 \mu$m, the chameleon model could be excluded entirely as an explanation for dark energy [21]. Such an experiment could naturally also be used to find constraints on Yukawa forces, and to perform accurate measurements of Casimir forces, potentially yielding decisive data with respect to the problems described in section 1.1. This is the motivation for the construction of CANNEX.

In the present article, we describe the setup, detection methods, and major systematic effects of CANNEX in section 2. The present status, preliminary force measurements, and errors are discussed in section 3. Finally, we give a brief outlook on future developments in section 4.

2. Parallel plate force measurements
The setup and detection techniques are presented in section 2.1. Our method to detect and maintain parallelism is described section 2.2, followed by a discussion of vibration isolation in section 2.3, and the influence of some systematic effects in section 2.4.

2.1. Force sensing and measurement scheme
The parallel plates configuration is implemented by means of a rigid (lower) SiO$_2$ plate opposing a movable (sensor) Si plate supported by three helical spirals. We recently reported the geometry and design of the latter [21]. The lower plate was optically polished to a total waviness of less than 18 nm

| Symbol | Quantity                          | Value        | Method of determination          |
|--------|-----------------------------------|--------------|----------------------------------|
| $A$    | interacting area                  | 1.0834 ± 0.0005 cm$^2$ | by design                       |
| $t$    | plate thickness                   | 96 ± 5 μm    | manufacturer specification       |
| $m$    | effective sensor mass             | 31.7 ± 0.9 mg | electrostatic calibration        |
| $k$    | elastic constant                  | 0.132 ± 0.004 N/m | electrostatic cal./spectral analysis |
| $f_0$  | free resonance frequency          | 10.2432 ± 0.0028 Hz | spectral analysis               |
| $Q$    | mechanical quality factor         | up to 15000  | spectral analysis                |

Table 1: Main characteristics of the force sensor.
(peak-peak) \( Q \), and sputter coated from the top with 100 nm Au above a thin Cr contact layer. To create the sensor, silicon-on-insulator wafers were optically polished, wet etched, and coated by 10 nm TiW and 65 nm Au on both sides by a commercial contractor (Norcada Inc.). While we did not measure the actual waviness of the resulting sensor plate, the final thickness of 96 ± 5 µm and the optical polishing of the initial wafer are seen sufficient to guarantee negligible large-scale deformations of the same order as measured on the rigid plates \( \text{II} \). The main characteristics of the resulting sensor are collected in table \( \text{III} \). A schematic view of the geometric arrangement and a photograph of the core setup are shown in figures \( \text{II} \) and \( \text{III} \), respectively. We mount the lower plate in the center of a massive carrier plate supported symmetrically by three calibrated feedback-controlled piezo-electric stages (PI-601.3s) allowing us to set the separation \( d \) and tilt \( \theta \). The frame of the sensor is clamped to a rigid holder that can be moved in vertical direction via a drift-free stick-slip transducer (Attocube ANPz51) permitting a coarse setting of \( d \) with 5 mm range.

Our setup offers two independent ways to detect movements \( \Delta d \) of the upper plate. The first one relies on the capacitance between the plates, measured by an Andeen-Hagerling AH-2700A capacitive bridge. While this allows us in principle to sense also the absolute separation \( d \), measurements can be performed at a maximum rate of approximately 300 mHz and are, therefore, only suitable to detect low-frequency changes. The second way of detection is optical, via a Fabry-Pérot cavity created by the reflective upper surface of the sensor plate and the cleaved end of an optical fiber oriented at the center of the sensor. As source, we use a PI Pro800 adjustable wavelength laser at 1553 nm and 10 mW nominal output power, < 20% of which reach the sensor. The interferometric signal is frequency-limited only by the detector bandwidth of 3 kHz, for which the full mechanical spectrum of the sensor is accessible. As example, we have recorded the signal for several hours, with the core fixed to the ground. From a Fourier transform of these data (figure \( \text{III} \)), we can extract the free resonance frequency \( f_0 = \omega_0 / 2\pi = 10.2432 \) Hz and Q-factor \( \sim 10^4 \) \( \text{VI} \) by means of least-squares fitting (inset in the figure) to the vibration transfer function \( T_{\text{ex}}(\omega) = \omega^2/|\omega_0^2 - \omega_2 F_{ES} (m + i \omega \omega_0)/Q| \), with \( \omega_2 F_{ES} = e_0 AV^2/d^3 \) and the vacuum permeability \( e_0 \). Here, \( d \) is determined from a capacitance measurement, and voltages \( V \) are measured via dedicated lock-in amplifiers and ADCs. For measurements of the (Casimir) force gradient \( \partial F(d)/\partial d \equiv \partial \delta F \) in vacuum, we use a frequency shift technique \( \text{[VII]} \). We excite the sensor by applying a voltage \( V_{\text{ex}}(t) = V_{\text{ex}} \cos(\omega_{\text{ex}} t/2) \)

\[ f_0 = \frac{\omega_0}{2\pi} = 10.2432 \text{ Hz} \]

\[ Q \sim 10^4 \text{ \(\text{VI}\) by means of least-squares fitting (inset in the figure) to the vibration transfer function \( T_{\text{ex}}(\omega) = \omega^2/|\omega_0^2 - \omega_2 F_{ES} (m + i \omega \omega_0)/Q| \), with \( \omega_2 F_{ES} = e_0 AV^2/d^3 \) and the vacuum permeability \( e_0 \). Here, \( d \) is determined from a capacitance measurement, and voltages \( V \) are measured via dedicated lock-in amplifiers and ADCs. For measurements of the (Casimir) force gradient \( \partial F(d)/\partial d \equiv \partial \delta F \) in vacuum, we use a frequency shift technique \( \text{[VII]} \). We excite the sensor by applying a voltage \( V_{\text{ex}}(t) = V_{\text{ex}} \cos(\omega_{\text{ex}} t/2) \) \]
to the lower plate and track $\omega_{es}$ via a phase-locked loop (PLL) such that it equals

$$\omega_{es} = \omega_0(\partial_d F, d) = \sqrt{\omega_0^2 - \frac{\partial_d F(d)}{m}},$$

where $\omega_0 \equiv \omega_0(0, \infty) = \sqrt{k/m}$. $k$, and $m$ are the elastic constant and effective mass of the sensor, respectively (see table I). The latter two quantities are determined via an electrostatic calibration described in Appendix A. By inverting equation II, we can extract the total force gradient $\partial_d F = \partial_d (F_{ES} + F_C + F_G)$ from data of $\omega_0(\partial_d F, d)$, which we obtain from the PLL. In order to measure the Casimir force gradient $\partial_d F_C$, we need to subtract from $\partial_d F$ the electrostatic contribution $\partial_d F_{ES}$. Note that the gravitational attraction $F_G \ll \{F_C, F_{ES}\}$ for all approached $d$, for which we neglect $F_G$ subsequently. As is well known, the potential difference $V = V_0 + v_{es}(t)$ between the plates inherently contains an unknown component $V_0$ that arises from contact potentials in electric connections, and differences in the Fermi potential of the opposing surfaces. In order to cancel the global $V_0$, we apply an established feedback technique [52, 74, 28], which is described in detail in Appendix B. The basic concept is to add an oscillating potential $v_{AC}(t) = V_{AC} \cos \omega t$, with $\omega_1/(2\pi) = 4.8$ Hz, and a constant offset $V_{DC}$ to the potential applied to the lower plate. As shown in the appendix, the resulting sensor response at frequency $f_1 = \omega_1/(2\pi)$ (measured by a lock-in amplifier) is to first order proportional to the sum $V_0 + V_{DC}$. We use this signal to create an active feedback circuit driving $V_{DC} \rightarrow -V_0$, thereby eliminating the unknown $V_0$ and replacing it by a small well-known $v_{AC}(t)$. From the second harmonic signal generated by $v_{AC}(t)$, we can also obtain a reliable measurement of the separation $d$ (Appendix B). Similar to previous experiments [28-30], we perform measurements in series of steps in $d$. Starting from a nominal $d = 40\mu$m, we record at each position data on $\omega_{es}$, $V_0$, back-measurements of all applied voltages, signal amplitudes at $f_0$, $f_1$, $2f_1$, and environmental factors. Acquisition of each data point takes approximately 40 minutes, as the high Q-factor and low value of $f_0$ demand very long time constants of all lock-in amplifiers (around 30 s), and a bandwidth of a few mHz for the feedback loops. Data is only recorded after all feedback circuits have completed their swing-in. A complete series takes between 4 and 9 hours, depending on the seismic noise background (slowing feedback convergence). Before each series, we perform a re-calibration of $f_0$, since the latter may change with temperature (see section 2.4). To do so, we set $d \rightarrow d_{cal} \approx 80\mu$m (exact separation varies slightly but is known exactly), and perform

Figure 3: Fourier-transformed vibration spectrum of the sensor. The residual seismic background and the seismic filter resonance at 0.4 Hz are clearly visible at low frequency, as is the first harmonic at $f_0$, and the first tilt mode at 36.595 Hz. Resonance peaks at higher frequency correspond to various modes of the spring structure, as has been confirmed by numerical simulation of the sensor’s dynamics. Inset: Fit to the first harmonic frequency (see main text).
a precise measurement of $\omega_0(d, \partial dF)$. Using equation 1 and the known $V_{ex}$, and $V_{AC}$ we can compute $\omega_0$ under the assumption that $\partial dF_C(d_{cal}) \approx 0$. This assumption causes an acceptable error in the Casimir force gradient measured at $d < 30 \mu m$ of less than 2%. We have found that the plate tilt (section 2.2) stays fairly constant over long times within our detection error. Therefore, a tilt cancellation is only performed before a set of series measurements. For tilt measurements, we set $d \approx 20 \mu m$, and completely separate the plates from the potentials of the lock-in amplifiers via a mechanical switch (see figure 1). This configuration is necessary for proper operation of the AH2700 bridge. We do, however, bias the lower plate with the last known $V_{DC}$ to cancel $V_0$. The tilt cancellation mechanism itself is described in section 2.2.

While all of the above methods work well in vacuum, for future planned measurements of chameleon interactions, which require a varying gas pressure $\geq 1$ mbar, the sensor will be over-critically damped and frequency shift measurements are not an option. Instead, we intend to modulate isothermally the pressure in a controlled sinusoidal pattern with amplitude $\Delta p$ in the range $[0.1, 50]$ mbar, and record the movement of the sensor plate over several tens of cycles. We can then use a software lock-in to determine the modulation amplitude of the force $\Delta F_{ch} = \partial p F_{ch} \Delta p$. In order to cancel the surface potential $V_0$, the same technique as above can be applied, but the frequency $\omega_0$ must be chosen considerably smaller than in vacuum in order for the sensor to be able to react. The plate separation will be determined before filling the gas, and kept constant during the pressure modulation measurement. Constant $d$ is guaranteed by the calibrated strain-gauge sensors and feedback-circuits of the actuators, under the presumption that the setup is kept isothermal.

2.2. Ensuring parallelism

An important issue in CANNELX is to assure parallelism of the sensor and lower plates. In order to achieve parallelism, we use a capacitive detection and feedback technique. Analytic solutions for the problem of the capacitance between plates are available for idealized circular geometries [51, 52] and tilted rectangular ones [53, 54]. In agreement with the literature, and with numerical calculations of the very geometry used in CANNELX, we find that for $R/d > 100$, where $R$ is the plate radius, the approximation $C \approx \varepsilon_0 R^2 \pi / d$ for the capacitance is correct to within 1%. For small deviations $\theta$ from perfect parallelism, we may further assume the charge density on the plates to be homogeneous, which allows us to write

$$C(\theta, d) \approx \varepsilon_0 \int_{-R}^{R} \frac{2\ell(x)}{d(x)} dx, \quad \text{with } \ell(x) = \sqrt{R^2 - x^2}, \quad \text{and } d(x) = d + x \sin(\theta)$$

$$= 2\pi \varepsilon_0 \cos^2 \theta \left( d - \sqrt{d^2 - R^2 \sin^2 \theta} \right) = \frac{\varepsilon_0 R^2 \pi}{d} \left( 1 + \frac{R^2 \theta^2}{4d^2} \right) + O(\theta^4). \tag{2}$$

Although the tilt only enters at second order (of the expanded form on the last line of equation 2), a modest value of 1 mrad leads to an error of more than 10% in both electrostatic and Casimir force gradients at $d = 10 \mu m$. For percent-level accuracy in the Casimir force gradient at all approached $d$, $\theta \leq 50 \mu r a d$ is required, while measurements of chameleon forces at the 0.1 pN level would require $\theta \leq 300 \mu r a d$.\footnote{These limits can be obtained by applying the same approximation leading to equation 2 for the known distance dependence of Casimir [65], and chameleon [123] forces, respectively.}

In order to measure and reduce $\theta$ in the experiment, we first apply an additional small tilt $\alpha$ to the lower plate, and then slowly change the horizontal direction of $\alpha$ such that the tilt axis rotates with angular frequency $\omega_0/2\pi$ (see figure 3). Note that neither of the plates are rotated but only the surface normal of the lower plate performs a precession-like movement. As detailed in Appendix C, we may write the resulting capacitance as

$$C_{mod}(t) = \varepsilon_0 \frac{R^2 \pi}{d} \left( 1 + \alpha \frac{R^2}{2d^2} \cos \omega t \right)^2 + O(\alpha^3) + O(\theta^2). \tag{3}$$
Figure 4: Tilt compensation scheme. Note that the vertical direction is inverted here. The sensor plate always stays at rest.

Figure 5: Evolution of the measured tilt amplitude (lock-in amplifier output) during a typical run. The initial steep increase represents the lock-in swing-in until the feedback works efficiently. Performance at > 8 h, is limited by strong vibrations resulting in erroneous capacity measurements and overshoot of the feedback.

The amplitude and phase of the oscillating term $\propto a \theta$ in equation (3) can be measured using a four-quadrant lock-in amplifier, and be used to implement a feedback circuit that offsets the three piezos holding the lower plate, thereby driving $\theta$ to zero. We have first tested this method using a demonstrator setup with solid metal plates, and achieved long-term stability of roughly 3$\mu$rad with time constants of less than 100 s. In the actual annex setup, however, the sensor oscillates due to vibrational noise, for which the capacitive measurements have to be performed with significantly larger integration times, which in turn drastically increases the influence of drift. For these reasons, and the problems outlined in section 3, we could only obtain stability within 200$\mu$rad. Nonetheless, the basic principle also works under these far-from-optimum conditions, as shown in figure 5 for a typical run. Repeated parallelization runs showed consistent results (within the given stability).

2.3. Vibration isolation
As already mentioned in section 2.2, vibrations are critical in this experiment. Quantitative limits for the acceptable vibration level can be derived from the aimed accuracy. For chameleon measurements, which are performed at quasi-DC, one needs to minimize variations $d$ in the separation, as these will cause errors in the measured forces via non-linear effects [86, 87]. This becomes clear if we expand the expression for the force acting between the two plates

$$F(d + \delta d) \approx F(d) + \delta dd_F(d) + \frac{1}{2} \delta d^2 \frac{\partial^2}{\partial d^2} F(d).$$

While the expectation value $\langle \delta d \rangle = 0$ for stochastic noise, the second order term results in an offset. The maximum permissible $\delta d$ can then be computed by requiring that $(1/2) \delta d^2 \frac{\partial^2}{\partial d^2} F(d) < 0.1$ pN. Naturally, the strongest limits come from the smallest separations $d = 10 \mu$m, where for the sum of all expected forces inserted for $F$, we obtain a limit $\delta d < 20$ nm. Considering the known transfer function relating movements of the core chamber to relative movements between the two plates, $T_{xx,rel}(\omega) = \omega^2 / [\omega^2 + i\omega \Delta \omega_0 / Q - \omega^2]$, we can derive a limit for permissible vibrations of the core chamber $\delta d_c < T_{xx,rel}^{-1} \delta d$, leading to the solid red curve in figure 6. Note that this limit strongly depends on the Q-factor, which in turn changes with pressure and separation. Vibrations are also picked up as noise within the detection bandwidth. For Casimir measurements, this refers to the region around the resonance with range $\delta f_{BW}$, at which the sensor mechanically amplifies any vibrations. We can write the corresponding RMS vibration amplitude as $\Delta d_{RMS} = \left[ \int_{f_0-\delta f_{BW}}^{f_0+\delta f_{BW}} df \Delta d^2(f) \right]^{1/2} \approx \sqrt{\delta f_{BW} \Delta d(f_0)}$, where
$\Delta d(f)$ is the spectral vibrational noise. $\Delta d_{RMS}$ could be effectuated either by an external vibration or a force. We can relate the latter two quantities by $\Delta d_{RMS} = F_{RMS} T_{xx,rel} T_{Fx}^{-1}$. By requiring that the equivalent $F_{RMS} < 0.1 \text{ pN}$ we assure that the sensitivity in chameleon measurements is sufficient for the intended measurements, which leads to the dashed red upper limit in figure 6.

Figure 6: Recorded seismic background, performance of the attenuation system, and computed limits in a) horizontal and b) vertical direction.

In order to reduce seismic vibrations, we have implemented a passive five-axis isolation scheme, augmented by active feedback in vertical direction. Figure 7 gives an overview of this solution. In vertical direction, we use a geometric anti-spring (GAS) filter, consisting of three Euler springs holding a central ‘keystone’. By changing the radial position of the outer anchor points of the Euler springs, we are able to tune the stiffness, and thereby the filter’s resonance frequency down to $\sim 150 \text{ mHz}$, while supporting the entire weight of the core, equaling 86 kg. Extending from the keystone downwards, the core chamber hangs on a 1.6 m long pendulum, which provides isolation in horizontal direction. In an earlier version of the setup, we have used a 3 mm maraging steel wire. However, as can be seen in figure 6.

Figure 7: Simplified cut view of the CANNEX setup.

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(prev. perf.), this solution caused strong violin modes, and tilt of the core chamber. These effects could be mitigated by using a 2 mm steel wire instead, which shifts the violin modes to frequencies $\gtrsim 35$ Hz above the first harmonic frequencies of the sensor. The tilt resonances of the core chamber could be shifted just below the GAS filter resonance by mounting a tower on top of the core chamber, which raises the center of mass almost up to the point of suspension. In addition, we implemented a preliminary SISO feedback circuit sourced by a linear position sensor (LVDT) between the outer chamber and the keystone. The circuit is closed by a vertical voice coil actuator parallel to the LVDT. The present circuit aims to suppress the GAS resonance via positive feedback \[89\]. A prior attempt to use additional geophone sensors to implement a MISO circuit using a Kalman filter and feed-forward failed due to the high self-noise level of the available geophones. As can be seen in the figures, the environmental seismic background is very high \[10\]. Nonetheless, the obtained attenuation suffices to perform measurements using the current setup. However, this is only true during dry periods at nights, as the background is increased by a factor $\gtrsim 5$ when sewage pumps go into operation at the university.

### 2.4. Other systematics and error budget

Another disturbance is given by changes $\delta T$ in temperature. In previous setups \[52, 78\], we have reached mK thermal stability via simple feedback mechanisms. On the basis of the known thermal expansion coefficients and the geometry within the core, we estimate an error of $\delta d_{lh} \approx 60 \text{ pm/mK}$. Hence, for quasi-static chameleon force measurements at the 0.1 pN level (corresponding to $\Delta d \approx 1 \text{ pm}$, this estimation shows that a max. temperature variation of $15 \mu \text{K}$ is permissible (synchronized with the gas pressure modulation). Detailed numerical studies and tests will be required in order to assure this target.

Variations in temperature also alter the Young’s modulus of Si \[91\], and thereby the sensor resonance frequency. Considering equation \[1\] such errors appear in the same way as a measured force. Assuming changes $k \rightarrow k + \delta k = k(1 + a_Y \delta T)$ with a temperature coefficient of $a_Y = 6.4 \times 10^{-5} \text{ K}^{-1}$ \[91\], we can evaluate the ratio $\delta k/\partial_d F_C$, which is plotted in figure \[8\]. For $\delta T = 1 \mu \text{K}$, the error in the detected Casimir force would reach 10\% at around $20 \mu \text{m}$, which is acceptable for the present goals. However, these considerations imply that thermal stability is of utmost importance in CHannel.

![Figure 8: Relative error in the measured Casimir force gradient for different temperature variations $\delta T$. The 1\% line has been added as a guide for the eye.](image)

A source of errors known especially from Casimir measurements \[3, 103, 15, 19, 76\] are so-called ‘patch effects’ resulting from local variations in the Fermi potential of surfaces. Even for perfectly clean interfaces, the variations can reach several mV \[19, 92\] at lateral scales from a few nm to mm. In order to estimate the force generated by such potentials, we have performed Kelvin probe force measurements (KPFM) using a commercial Bruker Nanoscope AFM. As both the sensor and the rigid plates are too large to be accommodated in this device, we had to rely on measurements on fragments of an earlier prototype sensor treated in the same way as the device used for force measurements. One of two taken data sets is shown in figure \[9\]. From these data, we obtain a standard deviation of $\sigma_v = 639 \mu \text{V}$. The patch pressure gradient can then be computed via the model of Ref. \[40\].

In fact, the measured background at the VU Amsterdam even partially exceeds Petersen’s high noise model \[90\], being a reference for the maximum global expectable seismic background.
Figure 9: $10 \times 10 \mu m^2$ Kelvin probe image showing the surface potential on a sensor with residuals of a cleaning liquid underneath the gold coating (bubble-shaped structures). While the actually used sensor is free of these impurities, the data serves as a worst-case estimate of short-range patch structures. Data in the image have been filtered by a running mean over 10 pixel for better visibility of the bubbles.

$$\frac{\partial_d F_p}{A} = -\frac{4\sigma_v^2}{k_{max}^2 - k_{min}^2} \int_{k_{min}}^{k_{max}} dk k^4 \coth(dk) \text{csch}^2(dk),$$

where $k_{min} = 2\pi/\lambda_{max}$ and $k_{max} = 2\pi/\lambda_{min}$. Using the lateral resolution ($\lambda_{min} \approx 20 \text{ nm}$) and the size of the scans ($\lambda_{max} = 10 \mu m$) as conservative estimates for the max./min. patch sizes, respectively, and over-estimating $\sigma_v = 1.7 \text{ mV}$, equation 5 yields $8 \times 10^{-4} \text{ N/m}^2$ at $d = 10 \mu m$, less than 1% of the Casimir force gradient at this separation. Note that $\partial_d F_p$ computed on the basis of the data in figure 9 is falling off much faster than any of the forces of interest due to the averaging effect of parallel plates (see section 3). This, however, does not exclude patch effects entirely, as the limited size of our KPFM scans does not permit conclusions about larger scale variations. We discuss this topic further in section 3.

Other error sources, such as non-linear effects and offsets due to surface corrugations, roughness, bending of the sensor plate due to the applied forces, and piezo drifts have been investigated and found to be negligible in comparison to thermal and vibration-induced effects.

3. Status and first results

After more than six years of construction we have completed a first prototype version of CANNEX and performed first (Casimir) force gradient measurements in the range $6 - 40 \mu m$. However, due to administrative problems, we were forced to automatize the setup and take data remotely, without physical access to the experiment. A pipe leakage during this time led to damage at the thermal controllers, for which the core temperature could neither be measured nor controlled. The outer chamber could still be monitored, showing temperature variations of up to 1 K with the heating cycles of the building. Measurements were performed primarily at night time, where the ambient vibration level was lower and the temperature more stable. Figure 10 shows data on the measured force gradient from four different runs performed in Dec. 2017 and Jan 2018, together with the theoretical prediction of the Casimir force based on Lifshitz theory, and optical data from the literature. Errors from the uncertainty in frequency determination, the sensor mass, voltage measurements, and calibrations, are shown in dark gray. Light gray error bars indicate a correction applied on the separation (horizontal, based on the drift in $d_0$ measured between successive series), and shifts in $f_0$ due to thermal variations (not corrected for). While these errors are large, they could likely be eliminated from future measurements by placing the setup in a less noisy environment and restoring thermal control. The experimental data, however, also show a strong systematic effect of unknown origin, increasing the measured force gradient by a factor $\gtrsim 10$ with respect to the expected $\partial_d F_C$. A similar effect was found in Ref. [4] and explained by the presence of large-scale electrostatic patches. Our data on such patches (see section 2.4) leads to the red dot-dashed curve in figure 10. Patches of size $\lambda_p$ larger than the KPFM

11 The first author had to move and leave the setup after his contract at the VU Amsterdam expired.

12 Casimir force (gradient) calculations are well documented in the literature. See for example Refs. [11, 85, 93].
scan size 10µm cannot be excluded a priori, and there is evidence that on metallic surfaces such large-scale variations do exist [19]. However, if such patches existed on our plates and \( \lambda_p < R \), the effective compensation potential \( V_{DC} \) should vary with separation [76, 95], especially if \( \lambda_p > d \). As shown in figure 11, any such dependence is \( \lesssim 1 \) mV, and hence insignificant. If, on the other hand, \( \lambda_p \gtrsim R \) (i.e. a single large patch on either or both of our plates) then the effect would be compensated at least partially by our \( V_0 \) feedback [13]. We have performed a least-squares fit of our data to the function \( \partial_d F = \partial_d F_C + \partial_d F_p \) (see equation 5) on a double-logarithmic scale, taking \( \sigma_v, \lambda_{\text{min}} \) and \( \lambda_{\text{max}} \) as free parameters. The best result was \( \sigma_v = 29 \) mV, and \( \lambda_{\text{min}} \approx \lambda_{\text{max}} = 1.5 \) m, which indicates that influential patches must be of the order \( \lambda_p \gtrsim R \), such that they act as a global potential offset, which in turn would be compensated in our experiment. Hence, despite the fact that these considerations need to be verified by numerical simulation and KPFM measurements of the full areas of the two plates, we ponder that patches may not be the cause of the systematic effect seen here. Instead, we suspect that parasitic AC potentials at frequencies \( > f_0 \) may be the cause, which would appear as a constant electrostatic force but could not be compensated by our feedback. We have searched the setup for disturbances at the AC line frequency and its first harmonic, and found a signal of 2.7 mV which can, however, not entirely explain the observed effect. Further investigations are not possible, as the experiment had to be removed from its location and taken apart.

We would like to emphasize that the presented force gradient data have to be considered preliminary due to the presence of thermal problems, strong vibrations, and systematic effects. We therefore refrain from any further analysis and interpretation of our data. Nonetheless, our results still demonstrate one important fact: CANNEX is capable of detecting force gradients between parallel plates at the level of Casimir interactions at plate separations larger than 10µm. Reliable estimates of the achievable sensitivity and accuracy will only be possible after the setup has been repaired and rebuilt in a quieter and more controlled environment.

13 We have used different algorithms leading to slightly different fit results and uncertainties but all with the qualitatively same result of \( \lambda_{\text{min}} > 100 \) µm and \( \lambda_{\text{max}} > R \). Fixing \( \lambda_{\text{min}} < 10 \) µm results in worse agreement between equation 5 and our data.
4. Conclusion and outlook

CANNEX has been designed to perform accurate measurements of force (gradients) between truly parallel plates and in other geometries involving parallelism at object separations of roughly 10–30 µm. The experiment aims to provide data that could help to settle a long-standing discussion in the Casimir community regarding the role of dissipation at zero frequency in the description of dielectric functions, and to validate new theoretical approaches by exploring geometries other than a sphere opposing a plate. If eventually a precision of 0.1 pN can be reached for measurements in Xe gas of variable pressure, CANNEX has also been predicted to be able to rule out the chameleon model as an explanation for dark energy.

After six years of development, the setup has reached a first prototype state in which force gradients between parallel plates could be measured in the separation range 6–40 µm. We have presented proofs of concept for the measurement and control of parallelism, compensation of effective surface potentials, and the calibration of the instrument. Due to technical problems, our first data as well as calibrations contained large errors from thermal drift and vibrations. We also found a strong systematic effect that may be of electrostatic origin. In contrast to other reports in the literature we suspect, however, that this effect may not be caused by local spatial variations in the surface potential of the plates, but account it to the presence of parasitic AC potentials. Further investigation is required before a final conclusion can be achieved. Despite far-from-ideal conditions and results, the present article proves that CANNEX is indeed able to measure force gradients at the level of Casimir interactions between parallel plates at 10–30 µm separations.

At the time of writing, the setup is being moved from its old location at the VU Amsterdam to Vienna, where it will be rebuilt and refurbished. With vibrations and temperature under control, the setup will then hopefully reach its design targets and enable us to measure the Casimir force gradient between parallel plates separated by roughly 10–30 µm with an error of a few percent. In order to implement measurements in gases, additional techniques have to be tested to allow for an isothermal and controlled variation of the pressure.

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Appendix A. Calibration

For the extraction of force gradients from experimental data according to equation [1], knowledge of the effective mass \( m \) is required. We determine this parameter from a DC voltage sweep at \( d_c \approx 75 \mu m \), in which we shut off the \( V_0 \) compensation by setting \( V_{AC} = 0 \), and record \( \partial F(V_{DC}) \) for \(-0.85 \text{ V} < V_{DC} < 0.85 \text{ V} \). \( m \) is then obtained from a least-squares fit to equation [11], where we insert the known \( \omega_0 \) and \( \partial F(V_{DC}) = \varepsilon_0 \omega_0^2 \pi V_{DC}^2 d_c \). A typical dataset is shown together with the best fit in figure [XI], resulting in \( m = 31.7 \pm 0.9 \) mg, which compares well with the value 43 mg, for the lumped mass \( m_{lg} \) and 23.7 mg for the sensor plate alone, as computed from the known geometry and materials. Assuming \( \omega_0 = \sqrt{K/m} \), we obtain \( k = 0.132 \pm 0.004 \) N/m. Note that the horizontal shift of the \( V_{DC} \) range is due to an amplifier offset. As such offsets can appear in all DC and AC voltages, we generally only use back-measured values instead of nominal outputs for analysis throughout this work.

During regular force measurements, the \( V_0 \) feedback automatically compensates the \( V_{DC} \) offset. The shift \( V_0 = 70 \pm 21 \) mV of the zero position of the frequency shift in figure [XII] agrees well with results of other measurements shown in figure [XI]. Note further that the uncertainty in \( m \) is mainly due to PLL errors resulting from increased vibrational noise during working hours and thermal drift in \( k \) during the measurement. The acquisition time for the data shown in the plot was 20 h.

The lumped mass \( m \) of a mechanical oscillator comprised of a mass \( m_{lg} \) supported by a massive spring \( m_c \) is \( m = m_{lg} + m_c/3 \).
Appendix B. Electrostatic compensation and distance determination

We add a voltage $v_{AC}(t) = V_{AC} \cos \omega t$ and a constant $V_{DC}$ to the signal applied to the lower plate (see figure 11). As $|2\omega_1 - \omega_0|$ is much larger than the sensor’s bandwidth, we may approximate $\Delta d = \frac{\xi_0 A_v(t)^2}{[2(d - \Delta d)]^2}$, which can be resolved for $\Delta d$. Expanding for $v(t) \ll 1$, the solution is (within 1% error)

$$\Delta d = \frac{\xi_0 A_v(t)^2}{2k} + \frac{\xi_0 A_v^2}{2k^2} v(t)^4 + O\left[V(t)^6\right].$$

(B.1)

Inserting now the different components of $v(t)$, we arrive at the lengthy expression

$$\Delta d = \frac{\xi_0 A_v(V_0 + V_{DC})^2}{2k d^2} + \frac{\xi_0 A_v^2}{2k^2 d^5} \left((V_0 + V_{DC})^4 + 3(V_0 + V_{DC})^2 V_{AC}^2 + V_{AC}^4\right)$$

$$+ \frac{\xi_0 A_v(V_0 + V_{DC}) V_{AC}}{k d^2} \cos \omega t t + \frac{\xi_0 A_v^2}{2k^2 d^5} \left(4V_{AC}(V_0 + V_{DC})^3 + 3V_{AC}^3(V_0 + V_{DC})\right) \cos \omega t t$$

$$+ \frac{\xi_0 A_v^2}{4 k d^2} \cos 2 \omega t t + \frac{\xi_0 A_v^2}{2k^2 d^5} \left(V_{AC}^4 + 3V_{AC}^2 V_{DC}^2\right) \cos 2 \omega t t$$

(B.2)

where consequently the colored terms up to second order are relevant in the experiment. The optical interferometer detects a signal $S \equiv \xi d$ [13], where $\xi = \partial_d S$ is the optical conversion factor of the interferometer. Using a lock-in amplifier referenced to $f_1$, we can thus measure the amplitude $S_{\omega_1}$ of the term $\propto V_{AC}(V_0 + V_{DC})$ on the second line of equation B.3. By taking $S_{\omega_1}$ as the error input of a PID controller setting $V_{DC}$, we create a feedback loop that drives $V_{DC} \to -V_0$. This procedure practically cancels all terms $\propto (V_0 + V_{DC})$, leaving only small and well-known contributions of $V_{AC}(t)$. Note that $v_{AC}(t)$ results in cross terms at frequencies $n \omega_1 \pm m \omega_2$ (for $m, n$ being integers) but has no influence on the concept. Higher order signals have a strong dependence on $d$. In order to prevent such signals from becoming dominant at small $d$, we always keep $V_{AC}/d \ll 100$ constant and small. Note also that we keep $\xi$ constant by maintaining the interferometer’s ‘quadrature’ position (at which $\partial_d S$ reaches its maximum) using a separate feedback circuit that adapt the laser wavelength.

While we can perform accurately calibrated steps in $d$ on the scale $d_{pc}$ of the piezoelectric actuators, the offset $d_0 = d_{pc} + d$ of the actuator scale with respect to $d = 0$ is a priori unknown, equation B.3, however, provides us with the means to determine this offset. Assuming that $V_0 + V_{DC} = 0$, the second harmonic signal $S_{2 \omega_1}$ detected by another lock-in amplifier, is mainly contributed by the first term on the third line, which we invert to obtain

$$d_0 - d_{pc} = \sqrt{\frac{\xi_0 A_v V_{AC}^2}{4k} S_{2 \omega_1}}$$

(B.3)

equation B.3 contains only two unknown quantities ($d_0$ and $\xi$), which can be determined from a least square fit to the data on $V_{AC}$ and $S_{2 \omega_1}$ recorded during a complete series of steps in $d_{pc}$.

15 Non-linear effects are neglected here, which is only valid for small vibrational background and small signal amplitudes. While in the present measurements, vibrations may invalidate this approximation, in future measurements $bd \ll 10$ nm shall be maintained to assure linearity of the signal.
Appendix C. Capacitance for two inclined plates

We consider two circular plates of radius $R$, opposing each other at a center separation $d$. The upper plate is tilted upwards by an *a priori* unknown angle $\theta$ in the vertical ($xz$) plane (figure 3). Without loss of generality we assume here the $xz$ plane to coincide with the unknown horizontal ($xy$) orientation of $\theta$. The lower plate is tilted by a constant vertical angle $\alpha$ around an axis in the $xy$ plane, defined by an angle $\omega t$ with respect to the positive $x$ axis. We can write the separation between one point on the lower plate and another on the sensor plate in cylindrical coordinates $r = \sqrt{x^2 + y^2}$, $\phi = \arctan y/x$, as $d(r, \phi) = d + r \sin \theta \cos \phi - r \sin \alpha \cos(\phi - \omega t)$. The capacitance between the plates is then $C(d, \theta, \alpha) = \varepsilon_0 \int_0^{2\pi} d\phi \int_0^\infty dr r d(r, \phi)$. Since, this integral is not analytical, we expand the integrand for both, small $\theta$ and $\alpha$, and integrate all terms separately, leading to equation 2 in the main text.

References

[1] Decca R S, López D, Fischbach E, Klimchitskaya G L, Krause D E and Mostepanenko V M 2007 Phys. Rev. D 75 077101
[2] Masuda M and Sasaki M 2009 Phys. Rev. Lett. 102(17) 171101
[3] Kim W J, Sushkov A O, Dalvit D A R and Lamoreaux S K 2009 Phys. Rev. Lett. 103(6) 060401
[4] Sushkov A O, Kim W J, Dalvit D A R and Lamoreaux S K 2011 Nat. Phys. 7 230
[5] Decca R S 2016 Int. J. Mod. Phys. A 31 1641024
[6] Chen Y J, Tham W K, Krause D E, López D, Fischbach E and Decca R S 2016 Phys. Rev. Lett. 116(22) 221102
[7] Sedmik R I P, Almasi A and Iannuzzi D 2013 Phys. Rev. B 88(16) 165129
[8] van Zwol P J, Svetovoy V B and Palasantzas G 2009 Phys. Rev. B 80(23) 235401
[9] Bezerra V B, Klimchitskaya G L, Mohideen U, Mostepanenko V M and Romero C 2011 Phys. Rev. B 83(7) 075417
[10] Garcia-Sanchez D, Fong K Y, Bhaskaran H, Lamoreaux S and Tang H X 2012 Phys. Rev. Lett. 109(2) 027202
[11] Decca R, López D, Fischbach E, Klimchitskaya G, Krause D and Mostepanenko V 2005 Ann. Phys. 318 37
[12] Decca R S, Fischbach E, Klimchitskaya G L, Krause D E, López D, Mohideen U and Mostepanenko V M 2009 Phys. Rev. A 79(2) 026101
[13] Behunin R O, Dalvit D A R, Decca R S, Genet C, Jung I W, Lambrecht A, Liscio A, López D, Reynaud S, Schnoering G, Voisin G and Zeng Y 2014 Phys. Rev. A 90(6) 062115
[14] Brax P, van de Bruck C, Davis A C, Mota D F and Shaw D 2007 Phys. Rev. D 76(12) 124034
[15] Behunin R O, Intravaia F, Dalvit D A R, Neto P A M and Reynaud S 2012 Phys. Rev. A 85(1) 012504
[16] Spaarnay M J 1958 Physica 24 751
[17] Bressi G, Carugno G, Onofrio R and Ruoso G 2002 Phys. Rev. Lett. 88 041804
[18] Lambrecht A, Nesvizhevsky V V, Onofrio R and Reynaud S 2005 Class. Quant. Grav. 22 5397–5406
[19] Antonini P, Bimonte G, Bressi G, Carugno G, Galeazzi G, Messineo G and Ruoso G 2009 J. Phys.: Conf. Ser. 161 012006
[20] Nawazuddin M B S, Lammerink T S J, Berenschot E, Boer M d, Ma K C, Elwenspoek M C and Wiegerink R J 2012 Challenges 3 261
[21] Almasi A, Brax P, Iannuzzi D and Sedmik R I P 2015 Phys. Rev. D 91(10) 102002
[22] Zou J, Marcat Z, Rodriguez A W, Reid M T H, McCauley A P, Kravchenko I I, Lu T, Bao Y, Johnson S G and Chan H B 2013 Nat. Commun. 4 1845 (Preprint 1207.6163)
[23] Ardito R, Frangi A, Corigliano A, Masi B D and Cazzaniga G 2012 Microel. Reliab. 52 271 2011 Reliability of Compound Semiconductors (ROCS) Workshop
[24] Tang L, Wang M, Ng C Y, Nikolic M, Chan C T, Rodriguez A W and Chan H B 2017 Nat. Photonics 97
[25] Casimir H B G 1948 Proc Ned Ak Wet 51 793
[26] Lisanti, M, Iannuzzi, D and Capasso, F 2005 PNAS 102 11989
[27] Chen F, Mohideen U, Klimchitskaya G L and Mostepanenko V M 2002 Phys. Rev. A 66 032113
[28] Munday J N, Capasso F and Parsegian V A 2009 Nat. 457 170
[29] Iannuzzi D and Sedmik R 2015 Casimir effect between solid surfaces, ch 13 in Physics of Solid Surfaces (Landolt Börnstein, New Series, subvol. A)(Springer Berlin) p 692
[30] Bordag M, Klimchitskaya G L, Mohideen U and Mostepanenko V M 2009 Advances in the Casimir effect (Oxford University Press)
[31] Mohideen U and Roy A 1998 Phys. Rev. Lett. 81 4549 (Preprint physics/9805038)
[32] de Man S, Heeck K and Iannuzzi D 2009 Phys. Rev. A 79 024102
[33] Decca R S, López D, Fischbach E and Krause D E 2003 Phys. Rev. Lett. 91 050402
[34] Iannuzzi D, Lisanti M and Capasso F 2004 PNAS 101 4019
[35] Chan H B, Bao Y, Zou J, Cirelli R A, Klemens F, Mansfield W M and Pai C S 2008 Phys. Rev. Lett. 101 030401
[36] de Man S, Heeck K and Iannuzzi D 2009 Phys. Rev. A 79 024102
[37] Klimchitskaya G L, Roy A, Mohideen U and Mostepanenko V M 1999 Phys. Rev. A 60 3487 (Preprint quant-ph/9906033)
[38] Neto P A M, Lambrecht A and Reynaud S 2005 Europhys. Lett. 69 924
[39] van Zwal P J, Palasantzas G, van de Schootbrugge M and Hosson J T M D 2008 Appl. Phys. Lett. 92 054101
[40] Broer W, Palasantzas G, Knoester J and Sveto V B 2012 Phys. Rev. B 85(15) 155410
[41] Speake C C and Trenkel C 2003 Phys. Rev. Lett. 90 164043
[42] Kim W J, Sushkov A O, Dalvit D A R and Lamoreaux S K 2010 Phys. Rev. A 81 022505 (Preprint 0905.3421v2)
[43] Reid M T H, White J and Johnson S G 2013 Phys. Rev. A 88(2) 022514
[44] Garrett J L, Somers D A T and Munday J N 2018 Phys. Rev. Lett. 120(4) 040401
[45] Mostepanenko V M, Bezerra V B, Decca R S, Geyer B, Fischbach E, Klimchitskaya G L, Krause D E, López D and Romero C 2006 J. Phys. A 39 6589
[46] Klimchitskaya G L, Mohideen U and Mostepanenko V M 2011 Int. J. Mod. Phys. B 25 171–230
[47] Palasantzas G, Dalvit D A R, Decca R, Sveto V B and Lambrecht A 2015 J. Phys. 27 210301
[48] Klimchitskaya G L and Mostepanenko V M 2015 Phys. Rev. A 92(4) 042109
[49] Bimonte G, López D and Decca R S 2016 Phys. Rev. B 93(18) 184434
[50] Mostepanenko V M 2015 J. Phys.: Condens. Matter 27 214013
[51] Weinberg S 1989 Rev. Mod. Phys. 61 1
[52] Copeland E J, Sami M and Tsujikawa S 2006 Int. J. Mod. Phys. D15 1753 (Preprint hep-th/0603057)
[53] Joyce A, Jain B, Khoury J and Trodden M 2015 Phys. Rept. 568 1 (Preprint 1407.9059)
[54] Planck Collaboration, Ade P A R et al 2015 ArXiv e-prints (Preprint 1502.01589)
[55] Kwan J et al 2016 Mon. Not. Roy. Astron. Soc. 464(4) 4045(Preprint 1604.07871)
[56] Brax P and Davis A C 2015 Phys. Rev. D 91(6) 063503
[57] Carroll S M 1998 Phys. Rev. Lett. 81 3067 (Preprint astro-ph/9806099)
[58] Bertotti B, Jess L and Tortora P 2003 Nat. 425 374
[59] Williams J G, Turyshhev S G and Boggs D 2012 Class. Quant. Grav. 29 184004 (Preprint 1203.2159)
[60] Khoury J and Weltman A 2004 Phys. Rev. Lett. 93 634
[61] Chou A S, Wester W, Baumbaugh A and Gustafson H R 2009 Phys. Rev. Lett. 102
[62] Steffen J H, Upadhye A, Baumbaugh A, Chou A S, Mazur P O, Tomlin R, Weltman A and Wester W 2010 Phys. Rev. Lett. 105(26) 261803
[63] CAST Collaboration, Anastassopoulos V et al (CAST) 2015 Phys. Lett. B749 172 (Preprint 1502.04561)
[64] Burrage C and Sakstein J 2016 J Cosmol. Astropart. Phys. 2016 045
[65] Lemmel H, Brax P, Ivanov A, Jenke T, Pignol G, Pitschmann M, Potocar T, Wellenzohn M, Zawisky M and Abele H 2015 Physics Letters B 743 310
[66] Brax P and Pignol G 2011 Phys. Rev. Lett. 107(11) 111301
[67] Jenke T, Cronenberg G, Burgdörfer J, Chizhova L A, Geltenbort P, Ivanov A, Lauer T, Lins T, Rotter S, Saul H, Schmidt U and Abele H 2014 Phys. Rev. Lett. 112(15) 151105
[68] Pignol G 2015 Int. J. Mod. Phys. A30 1530048 (Preprint 1503.03317)
[69] Rider A D, Moore D C, Blakemore C P, Louis M, Lu M and Gratta G 2016 Phys. Rev. Lett. 117 101101 (Preprint 1604.04988)
[70] Hamilton P, Jaffe M, Haslinger P, Simmons Q, Müller H and Khoury J 2015 Science 349(6250)
[71] Jaffe M, Haslinger P, Xu V, Hamilton P, Upadhye A, Elder B, Khoury J and Holger M 2017 Nat. Phys. (Preprint 1612.05171)
[72] Geraci A A, Smullin S J, Weld D M, Chiaverini J and Kapitulnik A 2008 Phys. Rev. D 78(2) 022002
[73] Adelberger E, Gundlach J, Heckel B, Hoedl S and Schlamminger S 2009 Prog. Part. Nucl. Phys. 62 102
[74] Brax P, van de Bruck C, Davis A C, Shaw D J and Iannuzzi D 2010 Phys. Rev. Lett. 104 241101
[75] Timoshenko S and Woinowsky-Krieger S 1987 Theory of plates and shells 2nd ed (New York: McGraw-Hill) ISBN 0-07-064779-8
[76] Chang C C, Banishev A A, Castillo-Garza R, Klimchitskaya G L, Mostepanenko V M and Mohideen U 2012 Phys. Rev. B 85(16) 165443
[77] de Man S, Heeck K, Wijngaarden R J and Iannuzzi D 2010 J. Vac. Sci. Technol. B 28(3) C4A25
[78] Sedmik R I P, Borghesani A F, Heeck K and Iannuzzi D 2013 Phys. Fluids 25 042103
[79] de Man S, Heeck K, Wijngaarden R J and Iannuzzi D 2009 Phys. Rev. Lett. 103 040402
[80] de Man S, Smith K, Heeck K, Wijngaarden R J and Iannuzzi D 2010 Int. J. Mod. Phys. A25 2231
[81] Carlson G T and Illman B L 1994 Am. J. Phys. 62 1099
[82] Norgren M K and Jonsson B L G 2009 PIER 97 357
[83] Xiang Y 2008 J. Electrostat. 66 366–368
[84] Patla B 2013 PIER 97 (Preprint 1308.2983)
[85] Bordag M, Fialkovsky I V, Gitman D M and Vassilevich D V 2009 Phys. Rev. B 80 245406
[86] Albrecht T R, Grütter P, Horne D and Rugar D 1991 J. Appl. Phys. 69 668–673
[87] Antezza M, Pitaevskii L P and Stringari S 2004 Phys. Rev. A 053619
[88] Cella G, Sannibale V, DeSalvo R, Märka S and Takamori A 2005 Nucl. Instr. and Meth. A A540 502 – 519
[89] Mantovani M and DeSalvo R 2005 Nucl. Instrum. Meth. A 554 546 – 554
[90] Peterson J 1993 U.S. Dept. Int. Geol. Surv., Open-File Rept. 93-322
[91] Hopcroft M, Nix W and Kenny T 2010 J. Microelectromech. S., 19 229–238
[92] Robertson N A, Blackwood J R, Buchman S, Byer R L, Camp J, Gill D, Hanson J, Williams S and Zhou P 2006 Class. Quantum Grav. 23 2665
[93] Klimchitskaya G L, Mohideen U and Mostepanenko V M 2009 Rev. Mod. Phys. 81 1827 (Preprint 0902.4922)
[94] Lynch D W and Hunter W R 1985 Handbook of optical constants of solids vol 1 ed Palik E D (Academic Press, Elsevier San Diego) p 286
[95] Lamoreaux S K and Sushkov A O 2011 Preprint 1106.3549