On the evolution of compact binary black holes

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(Dated: November 25, 2020)

Abstract

Based on the consideration of potential energy of the di-black-hole as a function of mass asymmetry (transfer) collective coordinate, the possibility of matter transfer between the black holes in a binary system is investigated. The sensitivity of the calculated results is studied to the value of the total mass of binary system. The conditions for the merger of two black holes are analyzed in the context of gravitational wave emission.

PACS numbers: 26.90.+n, 95.30.-k

Keywords: close binary stars, mass transfer, mass asymmetry
I. INTRODUCTION

The general point of view is that the binaries of compact object, composed of white dwarfs, neutron stars and black holes, eventually merge through gravitational wave emission, as observed recently with the LIGO and Virgo interferometers [1]. There is also a general consensus that the physical processes during the merger belong to the biggest unsolved problems of stellar evolution.

Mass transfer is an important process for close binary systems, because of its decisive role for the fate of the system. Hence, it is meaningful and necessary to study the evolution of binary stellar systems in the mass asymmetry coordinate \( \eta = (M_1 - M_2)/M \) \([M_k (k=1,2)\text{ are the object masses and } M = M_1 + M_2 \text{ is the total mass of system}]. \) In our previous works [2–4], we have analyzed the total potential energy \( U(\eta) \) and the orbital period as functions of \( \eta \) at fixed total mass \( M = M_1 + M_2 \) and orbital angular momentum \( L \) of the contact binary stars and galaxies. We have shown that the mass asymmetry (transfer) collective degree of freedom plays a comparable important role in macroscopic objects, being of comparable importance as in microscopic dinuclear systems. In close binary galaxies and stars, the mass asymmetry coordinate governs the asymmetrization (the transfer of mass from the lighter component to the heavy one) and symmetrization (the transfer of mass from the heavier component to the light one) of the system. As found, the symmetrization of binary object is the most favorable evolution channel. The symmetrization of binary galaxies and stars leads to the release of a large amount of energy. For example, in the case of binary galaxies, the released energy is about \( 10^{48−52} \text{ J} \), thus reaching the energy release of novae or even close to supernovae events [3]. Thus, the symmetrization of close binary galaxy due to the mass transfer is one of the important sources of the transformation of the gravitational energy to other types of energy, like radiation energy, in the Universe.

By this article, we intend to add a few aspects highlighting the role of mass transfer for binary black holes (BBH). Under all stellar binaries, BBH are undoubtedly a special class of objects. The presence of the event horizon inhibits the flow of mass between the core systems. On a speculative level, quantum tunneling as the Hawking radiation [5] may play a faint, but most likely negligible, role in view of the yet pending firm confirmation. However, BBH are embedded typically in a cloud of remnant matter, left over from the progenitor stars. Hence, a BBH system must always be considered together with the surrounding accretion...
disk. The material of that disk may serve for mass transfer between the core systems by
the so-called sloshing effect as illustrated clearly by the hydrodynamical calculations in Ref. 6.

In order to understand the conditions for matter exchange in a BBH system, we in-
vestigate the potential energy of a di-black-hole system as a function of mass asymmetry
coordinate $\eta$. We consider the possibility of this transfer based on the calculation of the
potential energy of BBH system – always to be understood together with its accretion disk
– is calculated as a function of mass asymmetry. Note that in addition to its own extreme
strong gravitational fields of two black holes, the potential energy contains the interaction
energy between two black holes. We use Newtonian mechanics, being well aware of the
strong gravitational fields. As found in [6], at the distance post–Newtonian effects may alter
the binary potential not more than 25% which will not affect the overall BBH properties.
Of course, general relativity will become essential at separations close to touching which is
a scenario beyond the scope of this work.

In section II we discuss the theoretical approach and the derived predictions, based on
a fix point analysis of the dependence of the total BBH energy functional on the mass
asymmetry coordinate. The work is summarized in section III.

II. THEORETICAL METHOD AND ITS APPLICATION

The total potential energy of the BBH system

$$U = U_1 + U_2 + V_{1R} + V_{2R} + V$$

is given by the sum of the potential $U_k$ ($k = 1, 2$) and rotational energies $V_{kR}$ ($k = 1, 2$)
of the two nonzero spin black holes, and black-hole-black-hole interaction potential $V$. The
energy of the black hole "$k$" is

$$U_k = -\frac{GM_k^2}{R_k},$$

where $G$, $M_k$, and $R_k$ are the gravitational constant, mass, and radius of the black hole,
respectively. The rotation energy of black-hole is

$$V_{kR} = \frac{S_k \omega_k}{2} = \frac{M_k c^2}{2},$$

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where \( S_k = M_k R_k c \) and \( \omega_k = c/R_k \) are the spin and rotational frequency of the black hole, respectively. Here, \( c \) is the speed of light.

The radius of the event horizon (distance from the gravitating mass \( M_k \) at which the particle velocity becomes equal to the speed of light \( c \)) is derived from the energy conservation law for a particle with mass \( m \):

\[
-\frac{G M_k m}{R_k} + mc^2 = 0.
\]

Note that the derivation of the radius \( R_k \) is based on classical mechanics and Newtonian law of gravity. Similarly, in Ref. [10], an expression for the radius \( R_k = 2GM_k / c^2 \) was obtained in the case of a non-rotating black-hole. Employing Eqs. (2) and (4), we obtain

\[
U_k = -M_k c^2.
\]

Because the two black holes rotate with respect to each other around the common center of mass, the black-hole-black-hole interaction potential \( V(R) \) contains, together with the gravitational energy of interaction \( V_G \) of two black holes, the kinetic energy of orbital rotation \( V_R \):

\[
V(R) = V_G + V_R = V_G + \frac{L^2}{2\mu R^2} = V_G + \frac{\mu v^2}{2},
\]

where \( v = (GM[2/R - 1/R_m])^{1/2} \) and \( R_m \) are the speed and the semimajor axis of the elliptical relative orbit, respectively, \( L \) is the orbital angular momentum of the di-black-hole system, and \( \mu = M_1 M_2 / (M_1 + M_2) = M_1 M_2 / M \) is the reduced mass. At \( R \geq R_t = R_1 + R_2 \), the black-hole-black-hole interaction potential is

\[
V_G(R) = -\frac{GM_1 M_2}{R}.
\]

From the conditions \( \partial V/\partial R|_{R=R_m} = 0 \) and \( \partial^2 V/\partial R^2|_{R=R_m} > 0 \), we find the relative equilibrium distance between two black holes corresponding to the minimum of \( V \):

\[
R_m = \frac{L^2}{G\mu^2 M}.
\]

We assume that the total angular momentum \( J = GM^2 / c \) of di-black-hole system is conserved during the conservative mass transfer and the orbital angular momentum of the di-black-hole is

\[
L = J - S_1 - S_2 = \frac{G(M^2 - M_1^2 - M_2^2)}{c}.
\]
Here the orbital and spin angular momenta are the parallel and the orbital angular momentum is minimal. Employing Eqs. (8) and (9), we derive the expression for relative distance

$$R_m = \frac{4GM}{c^2}. \quad (10)$$

As seen, the larger $M$ the larger $R_m$ is. Because $R_m/(R_1 + R_2) = 4$, the separation of components increases at $M_1 \to M_2$. From Eqs. (6), (7), and (10) one can obtain the simple formula

$$V(R_m) = -\frac{GM_1M_2}{2R_m} = -\frac{M_1M_2c^2}{8M} = -\frac{\mu c^2}{8} \quad (11)$$

for the interaction potential. Here, $V(R_m)$ depends only on the reduced mass $\mu$ and velocity of light.

So, using Eqs. (5), (3), and (11), we obtain the final expression

$$U = -\frac{Mc^2}{2} \left(1 + \frac{M_1M_2}{4M^2}\right) = -\frac{Mc^2}{2} \left(1 + \frac{\mu}{4M}\right) \quad (12)$$

for the total potential energy of the binary black hole system. For the BBH considered, $u(R_m) = (GM/R_m)^{1/2} = c/2$ and with a satisfactory accuracy one can neglect the relativistic effects and use the Newtonian law of gravity.

Using the mass asymmetry coordinate $\eta$ instead of masses $M_1 = \frac{M}{2}(1 + \eta)$ and $M_2 = \frac{M}{2}(1 - \eta)$, we rewrite the expressions (9) and (12) for the orbital angular momentum

$$L = \frac{GM^2}{2c} \left(1 - \eta^2\right) \quad (13)$$

and the total potential energy

$$U = -\frac{Mc^2}{2} \left(1 + \frac{1 - \eta^2}{16}\right). \quad (14)$$

Since the solution of equation

$$\frac{\partial U}{\partial \eta} = \frac{Mc^2}{16} \eta = 0$$

gives $\eta = \eta_m = 0$ and

$$\frac{\partial^2 U}{\partial \eta^2} |_{\eta=\eta_m} = \frac{Mc^2}{16} > 0,$$

the potential landscape has an oscillator shape and a global minimum at $\eta = \eta_m = 0$ for the arbitrary total mass $M$ of the BBH system (Fig. 1). So, the transfer of mass between
the black holes in the binary system is energetically favorable and, correspondingly, can occur. The initial asymmetric system is driven without additional driving energy to the symmetric BBH configuration (towards a global minimum). As seen, this conclusion does not depend on the choice of parameters, and the losses of the total mass and orbital angular momentum do not influence the symmetrization (the transfer of mass from the heavier component to the light one) of system. Due to the interaction energy between two black holes with their own extreme gravity fields, it becomes possible to transfer matter between them. However, the evolution of binary system in mass asymmetry coordinate depends also on the mass parameter in this degree of freedom. Since the surfaces of two black holes are spaced $R_1 + R_2 < R_m$ from each other, the mass parameter in $\eta$ is very large and, accordingly, to some extent prohibits the symmetrization of the di-black-hole system.

FIG. 1: The potential energy of the BBH (14) divided by $Mc^2$ as a function of mass asymmetry $\eta$.

The asymmetrization (merger due to the transfer of mass from the lighter component to the heavy one) of the binary black holes considered is not energetically favorable process and, correspondingly, the merger channel in mass asymmetry is strongly suppressed for di-black-hole. Thus, the question remains open on the mechanism of the merger of two black holes and the origin of gravitational waves [1].

In the case of the antiparallel orbital and spin angular momenta, the orbital angular momentum

$$L = J + S_1 + S_2 = \frac{G(M^2 + M_1^2 + M_2^2)}{c}$$

(15)
FIG. 2: The potential energy of the BBH (16) divided by $Mc^2$ as a function of mass asymmetry $\eta$.

is the maximal. For the binary black hole system with the orbital angular momentum \([15]\), the total potential energy is

$$U = -\frac{Mc^2}{2} \left( 1 + \frac{(1 - \eta^2)^3}{16(3 + \eta^2)^2} \right)$$

and $R_m/(R_1 + R_2) \geq 36$, $v(R_m) = (GM/R_m)^{1/2} \leq c/6$, $\eta_m = 0$, $\partial U/\partial \eta^2|_{\eta=\eta_m} > 0$, and all conclusions given above are also valid in this case. As seen from the comparison of Fig. 2 with Fig. 1, the same conclusions are also obtained in the case when the black holes of binary system are spinning in opposite directions.

III. SUMMARY

Employing the Newtonian law of gravity and considering the potential energy of the di-black-hole as a function of mass asymmetry (transfer) collective coordinate $\eta$, we have shown the possibility of matter transfer between the black holes in a binary system. The evolution of asymmetric binary system to the symmetric configuration with $\eta = \eta_m = 0$ is energetically favorable process. Although black holes have their own strong gravitational fields, the transfer of matter between two black holes arises from the energy of interaction between them. A di-black-hole system does not send any signals during its evolution. One can indirectly observe only the result of this evolution. In the course of time evolution the maximum of the distribution of expected BBH mass ratios shifts towards binary systems with equal masses.
The symmetrization of initially asymmetric BBH leads to the decrease of potential energy $U$, thus transforming the potential energy into internal kinetic energy. For example, for the binary black-holes $4M_\odot + 2M_\odot$ ($\eta_i = 0.33$) and $36M_\odot + 29M_\odot$ ($\eta_i = 0.11$), the internal energies of black-holes will increase during symmetrization by the amount $\Delta U = U(\eta_i) - U(\eta = 0) = Mc^2\eta_i^2 \approx 10^{47}$ J. For the comparison, in the cases of compact binary stars and compact binary galaxies, the released energies are about $10^{41}$ J and $10^{48}$–$10^{52}$ J, respectively [2, 3]. So, the BBH is the source of thermal energy.

The transfer of matter from a lighter component to a heavy one, leading to the merger of black holes in a binary system, is not an energetically favorable process and, accordingly, the question of the origin of gravitational waves remains open.

In the frame of our model, one can also perform the dynamic calculations of the evolution of binary system in mass asymmetry coordinate. But this extension of our model is the subject of future studies.

Acknowledgements

This work was partially supported by Russian Foundation for Basic Research (Moscow) and DFG (Bonn). V.V.S. acknowledge the partial supports from the Alexander von Humboldt-Stiftung (Bonn).

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