1/$N_c$ Corrections to the Baryon Axial Currents in QCD

Roger Dashen and Aneesh V. Manohar

Department of Physics, University of California at San Diego, La Jolla, CA 92093

Abstract

We prove that the $1/N_c$ corrections to the baryon axial current matrix elements are proportional to their lowest order values. This implies that the first correction to axial current coupling constant ratios vanishes, and that the $SU(2N_f)$ spin-flavor symmetry relations are only violated at second order in $1/N_c$. 
In the large-$N_c$ limit of QCD \[1\], the lowest lying baryon states form an infinite tower of states with $I = J = 1/2, 3/2, \ldots$. The matrix element of the axial current between baryon states is of order $N_c$, and can be written as

$$\langle N | \bar{\psi} \gamma^i \gamma_5 \tau^a \psi | N \rangle = g_0 N_c \langle N | X^{ia}_0 | N \rangle,$$

where $X^{ia}_0$ and $g_0$ are of order one. In a previous paper \[2\], we showed how unitarity in pion-baryon scattering and consistency of chiral perturbation theory implied that the infinite tower of baryon states must be degenerate, and that the axial couplings must satisfy

$$[X^{ia}_0, X^{jb}_0] = 0,$$

a result also obtained previously by Gervais and Sakita \[3\]. This relation implies that the baryon states form a representation of a contracted $SU(4)$ spin-flavor algebra. The matrix elements of $X^{ia}_0$ can be written in terms of reduced matrix elements $X^0_{(J,J')}$ as

$$\langle J', m', \alpha' | X^{ia} | J, m, \alpha \rangle = X_{0}(J, J') \sqrt{\frac{2J + 1}{2J' + 1}} \left( \begin{array}{ccc} J & 1 & J' \\ m & i & m' \end{array} \right) \left( \begin{array}{ccc} J & 1 & J' \\ \alpha & a & \alpha' \end{array} \right),$$

where the normalization constant has been chosen so that $X_0(J, J') = X_0(J', J)$. The constraint eq. (2) implies that all the reduced matrix elements are equal. The choice $X_0(J, J') = 1$ for the normalization fixes the normalization of $g_0$ in eq. (1). In this paper, we will compute the $1/N_c$ correction to the axial current matrix elements. We will show that the order one contributions to the matrix element of the axial current between baryon states is proportional to the leading order $N_c$ contribution. Thus the pion-baryon coupling constant ratios in the baryon tower only get a correction at order $1/N_c^2$. This result is important because it simplifies the computation of the $1/N_c$ corrections of the weak and electromagnetic properties of baryons \[4\].

In studying the $1/N_c$ corrections, it is important to always work in the limit that the isospin $I$ and spin $J$ of the baryon are held fixed as $N_c \to \infty$. For $N_c = 3$, the $I = J$ tower of baryons is not infinite, but stops at the $\Delta$ resonance with $I = J = 3/2$. The finite height of the tower is automatically included as $1/N_c$ operators in the effective hamiltonian. For small $I$ and $J$, the $1/N_c$ corrections will be under control. As $I$ and $J$ get larger, the corrections increase, and eventually the $1/N_c$ expansion breaks down. The top of the baryon tower for finite $N_c$ is the point at which the $1/N_c$ expansion breaks down. Clearly, the larger the value of $N_c$, the larger the values of $I$ and $J$ before the perturbation expansion breaks down.
Consider the scattering process $\pi^a + N \to \pi^b + \pi^c + N$ at low energies. The nucleon pole graphs that contribute in the large-$N_c$ limit are shown in fig. 1. Graphs with multiple pions at the same vertex, such as those in figs. 1(c) and (d) are suppressed by $1/N_c^2$ relative to the leading terms. Define the matrix element of the axial current to order $1/N_c$ by

$$\langle N | \overline{\psi} \gamma^i \gamma_5 \tau^a \psi | N \rangle = g_0 N_c \langle N | X^{ia} | N \rangle,$$  

(4)

where $g_0$ is a constant independent of $N_c$. $X^{ia}$ then can be expanded in a series in $1/N_c$,

$$X^{ia} = X^{ia}_0 + \frac{1}{N_c} X^{ia}_1 + \ldots$$

(5)

The amplitude for pion-nucleon scattering from the diagrams in fig. 1 is proportional to

$$N_c^{3/2} [X^{ia}, [X^{jb}, X^{kc}]]$$

and violates unitarity unless the double commutator vanishes at least as fast as $N_c^{-3/2}$, so that the amplitude is at most of order one. (In fact, one expects that the double commutator is of order $1/N_c^2$ since the corrections should only involve integer powers of $1/N_c$. This result also follows from the large-$N_c$ counting rules which imply that each additional pion has a factor of $1/\sqrt{N_c}$ in the amplitude.) Substituting eq. (5) into the constraint gives

$$[X^{ia}_0, [X^{jb}_0, X^{kc}_0]] + [X^{ia}_0, [X^{jb}_1, X^{kc}_0]] = 0,$$

(6)

using $[X^{ia}_0, X^{jb}_0] = 0$. The only solution to the consistency equation (6) is that $X^{ia}_1$ is proportional to $X^{ia}_0$. This can be verified by an explicit computation writing $X^{ia}_1$ in terms of reduced matrix elements, or by using group theoretic methods discussed in ref. [4]. Thus we find that

$$X^{ia} = \left(1 + \frac{c}{N_c} \right) X^{ia}_0 + O\left( \frac{1}{N_c^2} \right)$$

(7)

where $c$ is an unknown constant. The first correction to $X^{ia}$ is proportional to the lowest order value $X^{ia}_0$, so the $1/N_c$ correction to the axial coupling constant ratios vanishes. The overall normalization factor $(1 + c/N_c)$ can be reabsorbed into a redefinition of the unknown axial coupling $g_0$ by the rescaling $g_0 \to g = g_0 (1 + c/N_c)$, $X^{ia} \to X^{ia}_0$, so there are no new parameters at order $1/N_c$ in the axial current sector.

Eq. (7) implies that the double commutator $[X^{ia}, [X^{jb}, X^{kc}]]$ is of order $1/N_c^2$, since the $1/N_c$ terms in the expansion of the double commutator vanish. The one-loop renormalization of the pion-baryon couplings from the graphs in fig. 2 is proportional to...
$N_c \left[ X^{ia}, [X^{ia}, X^{jb}] \right]$. The cancellation in the double commutator implies that the one-loop radiative correction is of order $1/N_c$, rather than $N_c$. Thus pion-loop effects are suppressed by $1/N_c$ in both the meson and the baryon sector.

The vanishing of the $1/N_c$ corrections to the axial couplings is similar to the Ademollo-Gatto theorem, and to the vanishing of $1/m_Q$ corrections in the heavy quark theory at zero recoil. There are two sources of $1/N_c$ corrections, $1/N_c$ corrections to the states, and $1/N_c$ corrections to the axial current operator. The $\pi - N$ scattering argument shows that the sum of these $1/N_c$ corrections is proportional to the lowest order axial current matrix element. This differs from the heavy quark theory, in which the $1/m_Q$ corrections vanish at zero recoil. In the heavy quark theory, the vector current is a symmetry of the full theory for any $1/m_Q$, and normalizes the form factors at zero recoil. We only have a contracted $SU(4)$ symmetry of the effective theory in the limit $N_c \to \infty$, with $\left[ X_0^{ia}, X_0^{jb} \right] = 0$. Thus the normalization of $X_0^{ia}$ cannot be determined from the commutation relations. What we have shown by studying $\pi - N$ scattering is that the $1/N_c$ corrections to the commutation relations of the contracted $SU(4)$ algebra vanish. This implies that the only possible $1/N_c$ correction is a rescaling of $X_0^{ia}$.

The non-relativistic quark model provides a representation of the $SU(4)$ algebra for finite $N_c$, and it is easy to verify that $\left[ X^{ia}, X^{jb} \right]$ is of order $1/N_c^2$ using eq. (11) and (12) of ref. [2]. This implies that $1/N_c$ corrections to the axial currents in the non-relativistic quark model are proportional to their lowest order values, a result that can also be obtained by explicit computation. There is no reason, however, why the quark model value for the $1/N_c$ correction (the value $c$ in eq. (1)) should be the same as the QCD value. The quark model predictions for the ratios of pion-baryon couplings differ from those of QCD only at second order in the $1/N_c^2$. This explains why the quark model predictions for the coupling constant ratios agree so well with experiment. The Skyrme model also gives a $1/N_c$ expansion of the axial currents, in which the first correction to the coupling constant ratios is of order $1/N_c^2$. There is no reason why the Skyrme model value for the $1/N_c$ correction should agree with QCD. The large-$N_c$ limit of QCD predicts that pion-baryon couplings should respect $SU(4)$ spin-flavor symmetry. We have seen that the first correction to this result vanishes, which explains why the experimental pion-baryon coupling ratios are close to their $SU(4)$-symmetric values.

We would like to thank E. Jenkins for helpful discussions. One of us (A.M.) would like to thank the Aspen Center for Physics for hospitality while this work was completed. This work was supported in part by DOE grant DOE-FG03-90ER40546 and by PYI award PHY-8958081.
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Figure Captions

Fig. 1. Diagrams contributing to $\pi N \rightarrow \pi \pi N$. Graphs (c) and (d) are suppressed by $1/N_C^2$ relative to the leading term.

Fig. 2. Diagrams contributing to the one-loop corrections of the pion-baryon couplings.
Figure 1

(a) + permutations (b) (c) (d)

Figure 2

(a) (b) (c)