Fermionic Quantum Criticality with enlarged Fluctuation in Dirac Semimetals

Jiang Zhou\(^{1}\) (a) and Su-peng Kou\(^{2}\)

\(^{1}\) Department of Physics, Guizhou University, Guiyang 550025, PR China
\(^{2}\) Department of Physics, Beijing Normal University, Beijing 100875, PR China

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Abstract – The fluctuations-driven continuous quantum criticality has sparked tremendous interest in condensed matter physics. It has been verified that the fluctuations of gapless fermions can change the nature of phase transition at criticality. In this paper, we study the fermionic quantum criticality with enlarged Ising×Ising fluctuations in honeycomb lattice materials. The perturbative renormalization group approach is employed to investigated the flow equations and stable fixed-point structure. By including cubic terms of order parameter, we found that the phase transition is always continuous for any finite value of the number of Dirac fermions \(N_f\). Further, we also compute the critical exponents and predict the critical scaling behavior for \(N_f = 2\) case which corresponds to spin-1/2 fermions on graphene-like materials.

The quantum phase transitions (continuous) at zero-temperature driven by non-temperature parameters are believed to be key to understand unconventional properties of correlated many-body systems. For example, the strange metal in high-temperature superconductors. In a general quantum phase transition, the order parameter as a function of some tuning parameters which control the strength of the quantum fluctuations. Two different phases are separated by a quantum critical point (QCP), and the system displays some universal critical behavior in the vicinity of QCP, termed quantum criticality. In many cases, the quantum critical behavior can be well described within the Landau-Ginzburg-Wilson (LGW) framework.

Recently, the new paradigm of quantum matter represented by the frustrated magnet and interacting Dirac fermion systems present an exotic quantum criticality that beyond the LGW theory [1,2]. A prime example is the continuous transition between Neel phase and valence bond solid phase, termed deconfined quantum critical point at which emerge fractionalized ”spinon” degrees of freedom and noncompact U(1) gauge field [1]. Since the new degrees of freedom such as spinons emerge right at the QCP, the LGW theory could be fail to describe the deconfined QCP purely in terms of the space-time fluctuations of order parameter. Similar extra degrees of freedom also exist in the fermionic systems as massless Dirac fermionic excitation, such as graphene and three dimensional topological insulator [3,4]. The presence of fermion fluctuations at the critical point can dramatically change the nature of critical behavior and render a putatively first-order transition continuous. Evidence for this conclusion has been shown by the transition to \(Z_3\) Kekule valence bond solid (VBS) and \(Z_3\) nodal-nematic order in Dirac semimetals [5,6]. The \(Z_3\) Kekule VBS order gaps out the Dirac fermions while \(Z_3\) nematic order only shifts the positions of the nodes but does not gap out fermions. For such \(Z_3\) orders, the LGW theory implies a first-order transition, however, the recent RG analysis shows that the gapless fermions can perturbatively drive the phase transition into a continuous one, named fermion-induced quantum critical point [7]. Moreover, the quantum Monte Carlo simulations have been performed in ref. [8], which yields a continuous transition.

As a consequence, the interacting semimetal system may undergoes fluctuation-driven continuous phase transition from the semimetallic phase to a gapped broken phase when the interaction is sufficiently strong. At the fermionic critical point, the system involves bosonic (order-parameter) and massless fermionic critical fluctuations, both are low-energy fluctuations and thus are...
equally important for the quantum critical behavior. Therefore, a reasonable critical theory for the fermionic critical point may be the Gross-Neveu-Yukawa (GNY) field theory [9]. In condensed matter physics, a large number of interacting Dirac fermionic models exhibit the fluctuation-driven scenario [5–7,10–14] and share the same universal properties of the GNY field theory [15–16]. As the conventional $O(N)$ universality class, the critical behavior of a universality class is determined by the symmetry of the relevant degrees of freedom and the dimensionality we define the Hamiltonian. The participation of relativistic gapless fermions at the critical point results in a qualitatively different universality class called GNY universality class [15–16]. According to the symmetry of the order parameter, the GNY universality class comprises chiral Ising ($\mathbb{Z}_2$) class [13–20], chiral XY ($O(2)$) class [11,21–26] and chiral Heisenberg [$SU(2)$] class [13–16,27–30].

Once the fermion-induced continuous QCP is established, the question wether the enlarged fluctuations can alter the nature of phase transition is interesting, as this relate to some novel behavior such as Mott multicriticality [31,32]. The symmetry-enslarged scenario also exists in the deconfined QCP and competing fermionic criticality as emergent symmetry [35–34], as a result, the enlarged fluctuations driven QCP implies fermionization of the deconfined QCP. In this letter, we therefore perform RG analysis for the fermionic quantum criticality with enlarged Ising×Ising fluctuations in honeycomb lattice. We calculate the critical exponents characterizing the underlying continuous transition. The current one-loop RG result show that the enlarged fluctuation define a new universality class termed "chiral Ising plus Ising" universality class and the second-order phase transition emerges when the number of fermion flavors is finite. We hope our study may shed light on some competing quantum criticality in correlated many-body systems.

**Semimetal to Ising-order transition.** – We consider the spinless Dirac fermions on a honeycomb lattice, whose low-energy effective theory can be expressed as the Lagrangian density

$$\mathcal{L}_\psi = i \bar{\psi} \gamma^\mu \partial_\mu \psi,$$

where the conjugate fermionic field $\bar{\psi} = \Psi^\dagger \gamma^0$, the derivative operator $\partial_\mu = (\partial_0, \partial_i)$. Here, for convenience we have set the Fermi velocity $v_F = 3t/2$ as unity, and the summation convention over repeated indices is assumed. The $\gamma^\mu$ matrices satisfy the Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$, $\mu, \nu = 0, 1, 2$, and $g^{\mu\nu}$ is a Minkowski space metric. Explicitly, the four dimensional representation is given by

$$\gamma^0 = \tau^0 \otimes \sigma^3, \gamma^1 = \tau^0 \otimes i \sigma^1, \gamma^2 = \tau^3 \otimes i \sigma^2,$$

where the two-component identity matrix $\tau^0$ and the standard Pauli matrices $\tau^i$ act on the valley indices $K, -K$, the two-component Pauli matrices $(\sigma^0, \sigma^i)$ act in sublattice space $(A, B)$. In the free Dirac Lagrangian, the four-component Dirac spinor is defined as $\Psi = (\psi_{AK}, \psi_{BK}, \psi_{A-K}, \psi_{B-K})^T$. In the vicinity of Dirac points, then the Bloch Hamiltonian reads $H = \gamma^0 \gamma^i K_i$ with the Planck constant $\hbar = 1$. There are two matrices anticommute with all $\gamma^\mu$, namely,

$$\gamma^3 = \tau^1 \otimes i \sigma^2, \gamma^5 = \tau^2 \otimes i \sigma^2.$$

Furthermore, we can define $\gamma^{35} = \gamma^3 \gamma^5 = \tau^3 \otimes i \sigma^0$ which commutes with all $\gamma^\mu$ but anticommutes with $\gamma^3$ and $\gamma^5$. One can check that $[\gamma^{35}, \mathcal{H}] = 0$. The Hamiltonian possesses a symmetry implemented by $C\mathcal{H}^{-1} = -\mathcal{H}$, where $C$ is expressed as either $C = \gamma^0$ or $C = \gamma^0 \gamma^{35}$. This symmetry is conventionally called chiral symmetry or sublattice symmetry on a bipartite lattice, i.e., graphene lattice or π-flux square lattice. For generality, we introduce an arbitrary number $N_f$ of four-component fermion flavors. Then the $N_f = 2$ corresponds to spin-1/2 fermions on the graphene honeycomb lattice.

Now we consider the general symmetry breaking. To study the transition from semimetallic phase to insulating phase arising from the instability of electron interaction, we introduce the mean field order parameters of fermionic bilinear: $\chi = \langle \Psi^\dagger \gamma^0 \Psi \rangle$ and $\phi = \langle \Psi^\dagger \gamma^0 \gamma^{35} \Psi \rangle$, which can be triggered by sufficiently strong nearest-neighbor electron interaction. Both $\chi$ and $\phi$ break the chiral symmetry, $\chi$ corresponds to charge-density wave state and $\phi$ plays the role order parameter of quantum anomalous Hall phase. The symmetry-breaking can be described by the bosonic action density

$$L_{\chi\phi} = \frac{1}{2} (\partial^\mu \chi \partial_\mu \chi - m_\chi^2 \chi^2) + \frac{1}{2} (\partial^\mu \phi \partial_\mu \phi - m_\phi^2 \phi^2)
+ \lambda_\chi \chi^4 + \lambda_\phi \phi^4 + \lambda_{\chi\phi} \chi^2 \phi^2,$$

in which $\chi, \phi$ are Ising fields and we have introduced their interaction with strength $\lambda_{\chi\phi}$. Here the parameters $m_\chi^2$ and $m_\phi^2$ tune the phase transition from semimetallic phase to a phase with spontaneous $\mathbb{Z}_2$ symmetry breaking where the fermion masses are dynamically generated. The Ising-type order parameters couple to massless Dirac fermions take the form of Yukawa term

$$L_{\psi\chi\phi} = g_\chi \bar{\psi} \chi \Psi + g_\phi \bar{\psi} \phi \Psi.$$

Finally, the total action is given by $S = \int d^Dx (L_{\psi} + L_{\chi\phi} + L_{\psi\chi\phi})$, where $D = 2 + 1$ is the space-time dimensions. The final theory contains doubled Yukawa couplings is called dual GNY theory. We have set the boson and fermion velocity equally to preserve the Lorentz symmetry, which is reasonable since the the Lorentz invariance has been argued to emergent naturally in the infrared limit and the velocity difference between boson and fermion is always irrelevant in a large class of Yukawa theories.

**Renormalization group analysis.** – In this section, we perform RG analysis for the transition with enlarged Ising×Ising fluctuation within the Wilson formalism. Following the sprit of momentum-shell RG, we separate all
the fields into slow and fast frequency component and then integrate out the fast frequency component within the momentum shell $A/b < k < \Lambda (b > 1)$, $\Lambda$ is the momentum cut-off. Rescaling the system then gives the RG equations for the coupling constants and the boson masses.

**Flow equations.** We first consider the anomalous dimensions of the fermion and boson field respectively. At $O(g, \lambda)$ order, we have $g_{x\phi}(b) = g_{x\phi}^0 b^{2-\eta_{x\phi}-\eta_{\phi}}$, $\lambda_{x\phi}(b) = \lambda_{x\phi}^0 b^{2-\eta_{x\phi}-\eta_{\phi}}$ and $\lambda_{\phi}(b) = \lambda_{\phi}^0 b^{-2\eta_{\phi}}$ for $D = 4 - \epsilon$ dimensions. The relevant corrections at $O(g^2, \lambda^2)$ order yield the anomalous dimension

$$
\Delta_x = (D - 1 + \eta_x)/2, \Delta_{x\phi} = (D - 2 + \eta_{x\phi})/2,
$$

where $\eta_i \in \{\psi, \chi, \phi\}$ are anomalous dimension of the fermion and boson field respectively. At $O(g, \lambda)$ order, we have $g_{x\phi}(b) = g_{x\phi}^0 b^{2-\eta_{x\phi}-\eta_{\phi}}$, $\lambda_{x\phi}(b) = \lambda_{x\phi}^0 b^{2-\eta_{x\phi}-\eta_{\phi}}$ and $\lambda_{\phi}(b) = \lambda_{\phi}^0 b^{-2\eta_{\phi}}$ for $D = 4 - \epsilon$ dimensions. The relevant corrections at $O(g^2, \lambda^2)$ order yield the anomalous dimension

$$
\eta_{x} = 2Nfg_{x}^{2}\Omega_{D}, \quad \eta_{\phi} = 2Nfg_{\phi}^{2}\Omega_{D},
$$

where $\Omega_{D} = A_{D-1}/(2\pi)^D$ with $A_{D-1}$ as the area of $S_{D-1}$ sphere.

In the dual Yukawa GNY theory, all the couplings are marginal when $D \to 4$, which suggests that the fixed point can be accessible within $4 - \epsilon$ expansion. Upon rescaling theory, we have the following RG equations for the rescaled dimensionless coupling constants $g_i^2 \to g_i^2\Omega_D/\Lambda^2$, $\lambda_i \to \lambda_i\Omega_D/\Lambda^2$:

$$
\frac{dg_{x}^{2}}{d\ln b} = \epsilon g_{x}^{2} - (2N_{f} + 3)g_{x}^{4} - 3g_{x\phi}^{2}g_{\phi}^{2},
$$

$$
\frac{dg_{\phi}^{2}}{d\ln b} = \epsilon g_{\phi}^{2} - (2N_{f} + 3)g_{\phi}^{4} - 3g_{x\phi}^{2}g_{x}^{2},
$$

$$
\frac{d\lambda_{x}}{d\ln b} = \epsilon \lambda_{x} - 4N_{f}\lambda_{x}g_{x}^{2} - 36\lambda_{x}^{2} + N_{f}g_{x}^{4} - \lambda_{x\phi}^{2},
$$

$$
\frac{d\lambda_{x\phi}}{d\ln b} = \epsilon \lambda_{x\phi} - 4N_{f}\lambda_{x\phi}g_{x}^{2} - 36\lambda_{x\phi}^{2} + N_{f}g_{x}^{4} - \lambda_{x\phi}^{2},
$$

$$
\frac{d\lambda_{\phi}}{d\ln b} = \epsilon \lambda_{\phi} - 12\lambda_{x\phi}^{2} - 12\lambda_{\phi}\lambda_{\phi} + N_{f}g_{\phi}^{2}g_{x}^{2},
$$

First we study the flow in the $g_x^2$-$g_\phi^2$ plane. The zero of the beta functions of Yukawa couplings admit four fixed points, $A1$: $g_x^2 = 0$, $g_\phi^2 = 0$, $A2$: $g_x^2 = \epsilon/(2N_{f} + 3)$, $g_\phi^2 = 0$, $A3$: $g_x^2 = 0$, $g_\phi^2 = \epsilon/(2N_{f} + 3)$, and

$$
A4: \frac{\epsilon}{2N_{f} + 6}, \quad \frac{\epsilon}{2N_{f} + 6}.
$$

Among these fixed point, $A4$ is a stable fixed point. At the stable fixed point of the system, the flow equations for $g_x^2$ and $g_\phi^2$ are symmetric, so two Yukawa couplings have equal value at fixed point. Further, $\lambda_x$ and $\lambda_\phi$ also enjoy equal value for the same reason. Hence, we set $g_x = g_\phi$, $\lambda_x = \lambda_\phi$ in the remainder of the paper. For the physical case with $N_{f} = 2$, from Eqs.(10)-(14) we found the stable fixed point, numerically, resides at

$$
[(g_x^*)^2, \lambda_x^*, \lambda_{x\phi}^*] = (0.1\epsilon, 0.02597\epsilon, 0.03011\epsilon),
$$

which controls the critical behavior in the vicinity of the quantum critical point. The fixed points structure and the RG flow spanned by $g_x^2$, $\lambda$ and $\lambda_{x\phi}$ at the value of $N_{f} = 2$ is illustrated in Fig. In As shown as Fig.(b), the nonzero $\lambda_{x\phi}$ lead to a shifts off Gaussian fixed point. From the RG flow, we observe that the fluctuation bring the system flows to an dual Yukawa fixed point.

**Critical exponents.** We also derive the RG flow using the field-theoretic method as it is more convenient to obtain the scaling of the correlation length which is of the form $\xi \sim |\Delta|^{-\nu}$, $\Delta$ signs the distance to the critical point. To this end, we introduce the wave-function renormalizations $\Psi_0 = \sqrt{Z_{\Psi}}\Psi_R$, $\chi = \sqrt{Z_{\chi}}\chi_R$, $\phi = \sqrt{Z_{\phi}}\phi_R$, and coupling constant renormalizations $g_0 = Z_{g_0}\mu^{2/2g_R}$, $\lambda_0 = Z_{\lambda}\lambda^{1/2\mu}$, $m_0^2 = Z_{m0}m^2_{R}$, where $\mu$ is the renormalization energy scale. Specifically, one-loop calculations of renormalization constants using minimal subtraction in $D = 4 - \epsilon$ yield

$$
Z_{\chi} = 1 - \frac{2N_{f}g_x^{2}\Omega_{D}}{\epsilon}, \quad Z_{\phi} = 1 - \frac{2N_{f}g_x^{2}\Omega_{D}}{\epsilon}
$$

$$
Z_{\chi} = 1 - \frac{g_x^{2} + g_\phi^{2}}{2\epsilon} \Omega_{D}
$$
\begin{align}
Z_{m^2} &= 1 + (2N_f g_\lambda^2 \Omega_D + 6 \lambda_\chi \Omega_D) \frac{1}{\epsilon}, \quad (18) \\
Z_{m^2} &= 1 + (2N_f g_\phi^2 \Omega_D + 6 \lambda_\phi \Omega_D) \frac{1}{\epsilon}. \quad (19)
\end{align}

\begin{align}
Z_{\lambda_\phi \phi} &= 1 - \frac{N_f g_\phi^2 g_\lambda^2 \Omega_D}{\lambda_\chi \epsilon} + \frac{8 \lambda_\chi \Omega_D}{\epsilon} + \frac{2N_f g_\phi^2 \Omega_D}{\epsilon} \\
&+ \frac{2N_f g_\phi^2 \Omega_D}{\epsilon} + \frac{12 \lambda_\chi \Omega_D}{\epsilon} + \frac{12 \lambda_\phi \Omega_D}{\epsilon}. \quad (20)
\end{align}

The beta function for \( \lambda_\chi \phi \) is defined as the logarithmic derivatives with respect to the energy scale: \( \beta_{\lambda_\chi \phi} = \frac{d\lambda_\chi \phi}{d \ln \mu} \), considering the bare value \( \lambda_0 = Z_{\lambda_\mu} \mu^\lambda R \) is independent of \( \mu \), we obtain the flow equation as shown in Eq.(18). The anomalous dimension is defined as

\[ \eta_X = \frac{1}{Z_X} \frac{dZ_X}{d \ln \mu}, \quad (21) \]

where \( Z_X \in \{ Z_\psi, Z_\chi, Z_\phi, Z_{m^2} \} \). For a putative continuous phase transition, these anomalous dimensions govern the scaling behavior and thus universal. In particular, the fields acquire the anomalous dimensions [see also Eqs.(7),(8)]

\begin{align}
\eta_\chi &= \frac{\partial^2 \beta}{\partial \eta_\lambda \partial \eta_\chi} = (g_\chi^2)^* \Omega_D, \quad \eta_\phi = 2N_f (g_\phi^2)^* \Omega_D, \quad (22) \\
\eta_\psi &\equiv \frac{(g_\chi^2)^* + (g_\phi^2)^*}{2} \Omega_D, \quad (23)
\end{align}

the anomalous dimension for the mass square reads

\[ \eta_{m^2} = -12 \lambda_\chi \Omega_D - 2N_f g_\phi^2 \Omega_D. \quad (24) \]

The beta function for the dimensionless mass square is conventionally given as \( \beta_{m^2} = -(2 + \eta_{m^2}) m^2 \), and the inverse correlation length exponent \( \nu^{-1} \) associates with the anomalous dimension of mass square,

\[ \nu^{-1} = -d\beta_{m^2} / dm^2 |_{\lambda = 0} = 2 + \eta_{m^2}. \quad (25) \]

For the \( N_f = 2 \) case, the critical exponents read

\begin{align}
\eta_\chi &= \eta_\phi = 0.4 \epsilon, \quad \eta_\psi = 0.1 \epsilon, \quad (26) \\
\nu &= \frac{1}{2} + \frac{9}{50} \epsilon. \quad (27)
\end{align}

These exponents define a new universality class termed "chiral Ising plus Ising" universality class. Finally, the GNY model under investigation exhibits an emergent supersymmetry (SUSY) scenario at the stable fixed point. The SUSY scenario occurs for \( N_f = 1/2 \), quantitative estimates give \( \eta_\psi = \eta_\chi = \eta_\phi = 0.1 \epsilon \). The inverse correlation length exponent can be calculated from Eq.(25), yielding \( \nu = \frac{1}{2} + \frac{9}{50} \epsilon \). Our predictions have not yet been verified numerically, we hope the multicriticality driven by enlarged Ising fluctuations as well as other multicriticality could be checked by quantum Monte Carlo or related numerical methods in interacting electron system.

**The nature of phase transition.** The fixed point \( \alpha^*_i \) meets \( \beta_i(\alpha^*_i) = 0 \). Now linearize the flow equations about the fixed point, with the result:

\[ \beta_i = \beta_{\alpha_i} = B_{i,j}(\alpha_j - \alpha^*_j), \quad (28) \]

where \( B_{i,j} = \partial^2 \beta_i / \partial \alpha_j \partial \alpha_i \) is the stability matrix. The eigenvalues \( y_i \) of \( -B_{i,j} \) define the critical exponents which are universal at the putative continuous phase transition point. Explicitly, the critical index \( y_i > 0 \) implies the coupling \( \alpha_i \) is a relevant variable, this corresponds to a repelling flow in that direction. In turn, \( y_i < 0 \) corresponds to an attractive flow in that direction. In the RG theory, a genuine continuous critical point is characterized by a set of critical index among which only one is positive and all other are negative, the positive index (say \( y_1 \)) determines the correlation length exponent in terms of \( \nu = 1/y_1 \). That means that, the critical point is a stable infrared attractive fixed point if the index \( y_i(i > 1) \) are negative, then the phase transition becomes second order and the universal critical behavior emerges.

To confirm the transition is whether weakly first-order or second-order transition, we include the cubic terms \( \lambda_\chi^3 \chi^3 + \lambda_\phi \phi^3 + \phi^3 \) in the Lagrangian density. By definition, the nontrivial critical point is irrelevant under perturbations. The functional RG approach and perturbative RG approach show the cubic terms are always irrelevant when the flavors of fermion \( N_f \) is greater than a critical value \( N_c \). With respect to the critical index, we introduce an order \( y_1 > y_2 > y_i(i > 2) \), where the index \( y_1 \) determines the correction length exponent and is positive. Based on the RG equations in the presence of cubic terms, we show the second largest eigenvalues of stability matrix in Fig.\footnote{Fig.2: (Color online) The second largest critical index as the function of \( N_f \). The eigenvalues (numerical) of the stability matrix show the index remain negative for finite value of \( N_f \) in three space-time dimensions.}

Our current one-loop RG show \( y_2 \) remains negative for any finite value of \( N_f \). Consequently, the two-dimensional honeycomb lattice displays a continuous phase transition with critical scaling. For the critical value of \( N_f \), however, the functional RG predicted a second-order phase exist for \( N_f > 1.9 \). The current one-loop RG may predict a larger second-order regime, a more accurate value \( N_f \) may be obtained from the higher loop RG analysis.\footnote{\cite{55}}.
Conclusions. — In conclusion, we have studied the fermionic quantum criticality with Ising × Ising fluctuation in the two dimensional honeycomb materials. By tuning the nearest-neighbor interactions, the system undergoes a transition to “Ising plus Ising” phase and the transition is captured by dual Yukawa GNY model. With the help of one-loop RG analysis in $d = 4 - \epsilon$ dimensions, we have derived the RG equations for the coupling constants and investigated the stable infrared fixed-point structure. We found the quantum criticality is denominated by a dual Yukawa fixed point. At one-loop order, the critical exponents are $\eta_{\chi} = 0.4\epsilon, \eta_{\phi} = 0.1\epsilon, \nu = \frac{1}{2} + \frac{2}{5}\epsilon$. Further, we found a new emergent critical SUSY for the case of $N_f = 1/2$, and the exact result of anomalous dimensions for boson and fermion gives $0.1\epsilon$. To confirmed whether the phase transition is weak first-order or second-order transition, we have calculated the critical index of the stability matrix in the presence of cubic terms of order parameters. We found that the coupling are always irrelevant for any finite positive value of $N_f$, this observation means that the fluctuations of fermions qualitatively change the the nature of phase transition and render it continuous in Dirac materials \[20\]. Nevertheless, the spin-1/2 electrons on graphene lattice (corresponds to $N_f = 2$ case) with enlarged Ising × Ising fluctuations exhibit the second-order phase transition.

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