Local expansion of photonic W state using a polarization-dependent beamsplitter

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New Journal of Physics 11 (2009) 023024 (10pp)
Received 17 October 2008
Published 11 February 2009
Online at http://www.njp.org/
doi:10.1088/1367-2630/11/2/023024

Abstract. We propose a simple probabilistic optical gate to expand polarization entangled W states. The gate uses one polarization-dependent beamsplitter and a horizontally polarized single photon as an ancilla. The gate post-selectively expands $N$-photon W states to $(N+1)$-photon W states. A feasibility analysis considering the realistic experimental conditions shows that the scheme is within the reach of the current quantum optical technologies.

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1. Introduction

Entanglement is the most important resource to realize quantum information processing tasks that surpass the efficiency of their classical correspondences as well as provide solutions to problems that are intractable with classical resources [1]–[6]. The simplest form of this resource is the one between two parties, the so-called bipartite entanglement. Thanks to the theoretical and experimental efforts within the past two decades, we now have a clear understanding of the structure and characteristics of bipartite entangled states. It is well known that any bipartite state can be prepared from a maximally entangled bipartite state by local operation and classical communication (LOCC). It is generally accepted that the structure and dynamics of entanglement become more complex as the number of parties sharing entanglement increases. This sets a challenge in the theoretical and experimental studies of multipartite entanglement.

Among many interesting features of multipartite entangled states, a widely known one is the presence of inequivalent classes such as Greenberger–Horne–Zeilinger (GHZ), W and cluster states that cannot be converted into each other by stochastic local operations and classical communication (SLOCC) [7]. Experimental preparation and characterization of multipartite entangled states thus are not only essential for a better understanding of the quantum mechanics but also for the realization of state-specific information processing tasks. For example, the W state has been shown to be the only pure state to exactly solve the problem of leader election in anonymous quantum networks, whereas the GHZ state has been shown to be the only pure state to achieve consensus in distributed networks where no classical post-processing is allowed [8].

The photonic N-particle W state is represented by $|W_N\rangle = |(N - 1)_{H}, 1_{V}\rangle/\sqrt{N}$ with $|(N - k)_{H}, k_{V}\rangle$ being the superposition of all possible permutations of $N - k$ photons with horizontal (H) polarization and $k$ photons with vertical (V) polarization, e.g. $|W_3\rangle = |2_{H}, 1_{V}\rangle/\sqrt{3} = |1_{H}1_{H}1_{V}\rangle + |1_{H}1_{V}1_{H}\rangle + |1_{V}1_{H}1_{H}\rangle)/\sqrt{3}$. The W state has the peculiar property that every photon pair has the optimal amount of pairwise entanglement [9]–[11]. Such an entanglement structure forms a web-like system where every qubit has bonds with every other qubit, and the bipartite entanglement survives even if all the other $(N - 2)$-qubits are discarded. In recent years, there has been a number of theoretical proposals for the use of W states in multiparty protocols such as QKD [12], leader election in anonymous quantum networks [8] and teleportation, and for...
the preparation of W states in different systems including optics [13]–[18], ion traps [19] and NMR [20]. Some of these proposals have been realized in experiments [6], [19], [21]–[24].

In a recent study, we introduced an elementary optical gate to expand a state |W_N⟩ to a state |W_{N+2}⟩ by local manipulation on a single site without accessing all the qubits of the initial W state [18]. While such a local expansion was known for GHZ and cluster states, it was a challenge for W states as (i) the marginal states of the remaining untouched N − 1 qubits are different for |W_N⟩ and |W_{N+1}⟩ implying that the expansion process cannot be achieved unitarily, and (ii) the added new qubit should form pairwise entanglement with each of the untouched N − 1 qubits of the original W state (see figure 1). The gate proposed in [18] consists of two 50 : 50 beamsplitters and a half-wave plate (HWP), and expands any W state by two qubits as the ancillary state used in the gate is an H-polarized two-photon Fock state. In this paper, on the other hand, we propose a simpler probabilistic optical gate, which is based on post-selection, to expand a state |W_N⟩ by only one qubit. This new gate, which is shown in figure 2, is composed of a polarization-dependent beamsplitter (PDBS), an H-polarized single photon as an ancilla and a HWP for phase compensation. The gate operates as a one-input two-output gate that can expand the state |W_N⟩ to the state |W_{N+1}⟩.

This paper is organized as follows: In section 2, we describe the principles of the gate operation. Section 3 includes a discussion of how this basic gate structure can be used to
expand any polarization entangled \(W\) state. In section 4, we give a scheme for the experimental realization of this gate and carry out a feasibility analysis under realistic conditions. Finally, in section 5, we give a brief summary and conclusions.

2. Gate operation for expanding \(W\) state

The details of the proposed gate are shown in figure 2. The key component in this gate is the PDBS whose reflection and transmission coefficients depend on the polarization of the input light. The action of a PDBS for H-polarized photons and V-polarized photons can be written as

\[
\hat{a}_{1\text{H}}^\dagger = \sqrt{1 - \mu} \hat{a}_{3\text{H}}^\dagger - \sqrt{\mu} \hat{a}_{4\text{H}}^\dagger, \quad \hat{a}_{2\text{H}}^\dagger = \sqrt{\mu} \hat{a}_{3\text{H}}^\dagger + \sqrt{1 - \mu} \hat{a}_{4\text{H}}^\dagger
\]

and

\[
\hat{a}_{1\text{V}}^\dagger = \sqrt{1 - \nu} \hat{a}_{3\text{V}}^\dagger - \sqrt{\nu} \hat{a}_{4\text{V}}^\dagger, \quad \hat{a}_{2\text{V}}^\dagger = \sqrt{\nu} \hat{a}_{3\text{V}}^\dagger + \sqrt{1 - \nu} \hat{a}_{4\text{V}}^\dagger,
\]

where \(\hat{a}_{j\text{H}}^\dagger (\hat{a}_{j\text{V}}^\dagger)\) denotes the creation operator of H (V)-polarized photon in the \(j\)th mode of PDBS, and \(\mu (\nu)\) is the transmission coefficient for H (V)-polarization. The gate uses an H-polarized photon as the ancilla in mode 2, and a photon in mode 1 with an arbitrary polarization as the input. The successful operation of the gate is signalled by a coincidence detection that occurs when there is one photon in each of the output modes 3 and 4. In order to understand the working principle of this gate for \(W\)-state preparation and expansion, it is enough to consider its action on two possible cases: \(|1\text{H}⟩_1|1\text{H}⟩_2 = \hat{a}_{1\text{H}}^\dagger \hat{a}_{2\text{H}}^\dagger |\text{vac}⟩_{12}\) and \(|1\text{V}⟩_1|1\text{H}⟩_2 = \hat{a}_{1\text{V}}^\dagger \hat{a}_{2\text{H}}^\dagger |\text{vac}⟩_{12}\), where \(|\text{vac}⟩\) stands for the vacuum state. Using the relations given in equations (1) and (2) for the PDBS, we find that these input states are transformed into

\[
|1\text{H}⟩_1|1\text{H}⟩_2 \rightarrow \sqrt{2\mu(1 - \mu)}|2\text{H}⟩_3|0⟩_4 + (1 - 2\mu)|1⟩_3|1\text{H}⟩_4 - \sqrt{2\mu(1 - \mu)}|0⟩_3|2\text{H}⟩_4
\]

and

\[
|1\text{V}⟩_1|1\text{H}⟩_2 \rightarrow \sqrt{\mu(1 - \nu)}|1\text{V}⟩_3|1\text{H}⟩_4 + \sqrt{(1 - \nu)(1 - \mu)}|1⟩_3|1\text{H}⟩_4
\]

\[-\sqrt{\mu(1 - \nu)}|1⟩_3|1\text{V}⟩_4 - \sqrt{(1 - \mu)}|0⟩_3|1\text{V}⟩_4 \].

In the above equations, only the underlined terms lead to successful gate operation and we will focus only on those terms. It is seen that when the input photon is in V-polarization, the coincidence detection will post-select the state \(\sqrt{(1 - \nu)(1 - \mu)}|1\text{V}⟩_3|1\text{H}⟩_4 - \sqrt{\mu(1 - \nu)}|1\text{H}⟩_3|1\text{V}⟩_4\) which is a Bell state if the PDBS parameters are chosen such that \(\mu + \nu = 1\). It means that this gate works as an ‘entangling gate’. The probability of this event is then \(2\mu\nu\).

Next, let us assume that we have a Bell state in modes 0 and 0’ given as \(|W_2⟩ = (|1\text{V}⟩_0|1\text{H}⟩_0 + |1\text{H}⟩_0|1\text{V}⟩_0)/\sqrt{2}\). If the photon in mode 0’ is input to mode 1 of the gate (see figure 3), a triple coincidence at modes 0, 3 and 4 will post-select the state

\[
\frac{1}{\sqrt{2}}[(1 - 2\mu)|1\text{V}⟩_0|1\text{H}⟩_3|1\text{H}⟩_4 + \sqrt{(1 - \nu)(1 - \mu)}|1\text{H}⟩_0|1\text{V}⟩_3|1\text{H}⟩_4 - \sqrt{\mu(1 - \nu)}|1\text{H}⟩_0|1\text{H}⟩_3|1\text{V}⟩_4].
\]

If the weights of the components of this superposition state in equation (5) are made equal, then equation (5) will be of the form \(|W_3⟩\) except for a \(\pi\)-phase shift that can be compensated using a HWP in mode 4. The equalization of the weights occurs when

\[
1 - 2\mu = \sqrt{(1 - \nu)(1 - \mu)} = \sqrt{\mu\nu}.
\]
The second equality in equation (6) imposes the condition \( \mu + \nu = 1 \), that is the same condition obtained above for Bell state preparation. Solving the remaining equalities under the condition \( \mu + \nu = 1 \), we find that one should choose \( \mu = (5 - \sqrt{5})/10 \) and \( \nu = (5 + \sqrt{5})/10 \). Inserting these values of \( \mu \) and \( \nu \) into equations (3)–(4), and imposing the coincidence detection, we find that successful gate operation is characterized by the following transformations:

\[
\begin{align*}
|1_H\rangle_1|1_H\rangle_2 & \rightarrow \frac{1}{\sqrt{5}}|1_H\rangle_3|1_H\rangle_4, \\
|1_V\rangle_1|1_H\rangle_2 & \rightarrow \frac{1}{\sqrt{5}}|1_V\rangle_3|1_H\rangle_4 + \frac{1}{\sqrt{5}}|1_H\rangle_3|1_V\rangle_4,
\end{align*}
\]

(7)

where we have included the effect of the HWP in mode 4. Putting all together, we conclude that this gate can prepare the Bell state \(|W_2\rangle\) with a probability of 2/5 starting with a V-polarized photon in mode 1, and the \(|W_3\rangle\) state with a probability of 3/10 starting with the Bell state \(|W_2\rangle\) in modes 0 and 1. This success probability for \(|W_3\rangle\) state preparation is a significant improvement over other linear optics schemes existing in the literature. Among the already proposed schemes, the one in [18] has the highest success probability given as 3/16 which is less than that of the present scheme.

### 3. Expansion of polarization entangled \( W \) states

Here, we show that the same gate can be used to prepare and expand arbitrary \( W \) states. In the following, we will represent an \( N \)-partite \( W \) state \(|W_N\rangle\) as

\[
|W_N\rangle = [(|N-2\rangle_H, 1_V\rangle\rangle \otimes |1_H\rangle_1 + |(N-1)\rangle_H, 0_V\rangle\rangle \otimes |1_V\rangle_1)]/\sqrt{N}
\]

where the subscript 1 denotes the spatial mode of the photon that is input to the gate and 1 denotes the remaining \( N-1 \) modes of \(|W_N\rangle\). Using this notation, the transformation in equation (7) can be represented as

\[
|1_H\rangle_1|1_H\rangle_2 \rightarrow \sqrt{1/5} |2_H, 0_V\rangle
\]

Figure 3. Experimental set-up for realizing the proposed gate. PL, pulsed laser; SHG, second harmonic generator; Type-I BBO, phase matched \( \beta \)-barium borate crystal for SPDC; PDBS, polarization-dependent beamsplitters; \( D_j \), photon detectors; QWP, quarter-wave plate; HWP, half-wave plate; F, narrow-band interference filter; M, mirror.
and $|1_H⟩_1|1_H⟩_2 → \sqrt{\frac{1}{5}}|1_H, 1_H⟩$. Thus, we find that upon selection of the successful events, the action of the gate is given by

$$(N-2)|1_H⟩_1|1_H⟩_1 \rightarrow \frac{1}{\sqrt{5}}(|N-2⟩_1|2_H⟩_1 ⊗ |1_H⟩_1|1_V⟩_1),$$

$$(N-1)|1_H⟩_1|1_H⟩_1 \rightarrow \frac{1}{\sqrt{5}}(|N-1⟩_1|1_H⟩_1 ⊗ |1_H⟩_1|1_V⟩_1).$$

Using these relations, it is straightforward to show that the successful gate operations perform the following transformation on an initial $|W_N⟩$:

$$|W_N⟩ \rightarrow \frac{1}{\sqrt{5N}} [(|N-2⟩_1|1_H⟩_1 ⊗ |2_H⟩_1 + |N-1⟩_1|1_H⟩_1 ⊗ |1_H⟩_1]$$

$$= \sqrt{\frac{N+1}{5N}} |W_{N+1}⟩.$$

Thus, we conclude that the gate expands a given W-state $|W_N⟩$ to $|W_{N+1}⟩$ by one photon with a success probability of $(N+1)/5N$. The success probability will approach the constant $1/5$ when $N$ becomes very large. This analysis shows clearly that the proposed gate can be used in two different ways: (i) a given arbitrary-size W state $|W_N⟩$ can be expanded by one at each successful operation of the gate which takes place with the probability $(N+1)/5N$, e.g. a probability of $4/15$ for the expansion of $|W_3⟩$ to $|W_4⟩$ and (ii) starting from a V-polarized input photon, an arbitrary-size W-state can be prepared by cascade application of the gate. For example, cascading $k$ of this gate will prepare the state $|W_{k+1}⟩$ with a probability of $(k+1)5^{-k}$.

Note that in this cascaded operation of the gate, we need not confirm the successful operation of each gate separately. Coincidence detection among all output ports after the action of the last gate signals that all the gates have worked as expected. In this scheme, one cannot obtain coincidence detection if one or more of the gates fail to work properly.

4. Practical considerations for an experimental implementation

In this section, we introduce an experimental scheme for the implementation of this gate to expand the Bell state $|W_2⟩$ to $|W_3⟩$, and discuss the effects of realistic conditions on the performance of the gate. We will focus on the effects of imperfections in (a) the preparation of the $|W_2⟩$ and the ancillary state $|1_H⟩$, (b) the detection of the successful events and (c) the deviations of the parameters of PDBS from its optimal values.

4.1. Basic scheme

We propose the scheme given in figure 3 for the practical implementation of the proposed gate. In this scheme, the output of a pulsed laser (PL) with angular frequency $ω_0$ in the visible range of the spectrum is frequency doubled in a nonlinear crystal to produce pulses of ultraviolet (UV) light of angular frequency $2ω_0$. These UV pulses are then used to pump twice in forward and backward directions a pair of nonlinear crystals, which are stacked together such that their optical axes are orthogonal to each other [25]. The crystals are for Type-I spontaneous parametric down conversion (SPDC) to produce photon pairs in two modes (idler and signal) with the same polarization and at half the frequency of the pump beam. In the forward pumping direction, the polarization of the UV beam is set to vertical so that an H-polarized photon.
pair in modes 2 and 2′ is generated from which the required ancillary state \( |1_H⟩ \) in mode 2 can be prepared. The remaining (non-down-converted) portion of the UV beam first passes through a quarter wave plate (QWP) that changes its polarization into an ellipsoidal polarization. A mirror placed after the QWP back-reflects this beam and sends it through the QWP again which further changes the polarization of the beam into diagonal polarization. This diagonally polarized beam pumps the crystals in the backward direction creating the entangled photon pair \( (|1_H⟩₀|1_H⟩₀ + |1_V⟩₀|1_V⟩₀) / √2 \). Changing the polarization of the photon in the mode 0 (idler) of the SPDC output will prepare the \( |W_2⟩ \) in the spatial modes 0 and 0′. Then the ancillary photon in mode 2 and the photon in mode 1 of \( |W_2⟩ \) are mixed at the PDBS. The successful events are selected by a four-fold coincidence detection by ON/OFF detectors placed at modes 0, 2′, 3 and 4 as seen in figure 3. In order to have a high fidelity, it is crucial that the information on the source of the photons in modes 3 and 4 is erased. This can be done by placing narrowband interference filters, whose coherence times are much longer than the duration of the pump pulse, in front of the detectors [26], [27]. Indistinguishability of these photons can be further enhanced by spatial filters, which can be implemented using single-mode fiber-coupled photodetectors. It is to be noted that there is a trade-off between the indistinguishability and the success probability. The tighter is the spectral and spatial filtering the higher is the indistinguishability and hence the fidelity, but the lower is the efficiency. In the following analysis of the effects of imperfections in photon generation, photon detection and the PDBS on the performance of the gate, we assume for the sake of simplicity that the photons are perfectly matched and they are indistinguishable.

4.2. Effects of SPDC and imperfect detection

Imperfections in the photon detectors affect the gate in two ways: (i) recording some of the successful events as failures due to non-unit quantum efficiency and (ii) reporting some of the failures as successful due to dark counts and/or due to the fact that detectors cannot resolve the photon number. In the following, without loss of generality, we neglect the errors due to dark counts. This is acceptable as the dark counting rates of current detectors are very low [28]. Moreover, the requirement of four-fold coincidence detection in our scheme significantly reduces the probability of false events due to dark counts. Neglecting the dark counts, the positive operator valued measure (POVM) elements for ON/OFF photodetectors become

\[
\Pi_0 = \sum_{m=0}^{\infty} (1 - \eta)^m |m⟩⟨m|,
\]

\[
\Pi_1 = 1 - \Pi_0 = \sum_{m=1}^{\infty} [1 - (1 - \eta)^m] |m⟩⟨m|,
\]

where \( \Pi_0 \) and \( \Pi_1 \) are, respectively, elements for no click (OFF) and for a click (ON) [27]. Returning to our gate, we see that if there is only one photon in each of the modes 1 and 2, then the success probability of having one photon in each of modes 3 and 4 becomes \( 3\eta^4 / 10 \). Note that the error due to (ii) occurs when there is more than one photon in either or both of the modes 3 and 4. This takes place when either or both of the backward and forward SPDC processes prepare two or more photon pairs. In practical settings, SPDC suffers from the non-deterministic nature of the process. The output of the SPDC contains vacuum with high probability and the probability of photon pair generation is low. Moreover, although

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the probability is much lower, there are cases when multiple pairs of photons are generated. The generated state in the forward direction becomes

$$\Psi_{12} = \sqrt{g} (|\text{vac}\rangle_{12} + \gamma e^{i\phi} |1\text{H}\rangle_{1} |1\text{H}\rangle_{2} + \gamma^2 e^{i2\phi} |2\text{H}\rangle_{1} |2\text{H}\rangle_{2} \ldots ),$$

where $g = 1 - \gamma^2$ and $\gamma e^{i\phi}$ is proportional to the complex amplitude of the pump field. Assuming that the losses in the forward and backward pumping are negligible, the state in the backward direction can be written as

$$\Psi_{01} = \sqrt{g_1} (|\text{vac}\rangle_{01} + \gamma e^{i\phi} |W_2\rangle_{01} + \frac{1}{2} \gamma^2 e^{i2\phi} |\Lambda\rangle_{01} \ldots ),$$

where $|\Lambda\rangle_{01} = |1\text{H}\text{V}\rangle + |2\text{V}\rangle + |2\text{V}\rangle + |2\text{V}\rangle$ is unnormalized and $g_1 = (1 - \gamma^2/2)^2$. Combining the above expressions, we find that four-fold coincidence detection post-selects the state,

$$\Psi_{0122'} = \sqrt{g_2} [\gamma^2 e^{i2\phi} |W_2\rangle_{01} |1\text{H}\rangle_{1} |1\text{H}\rangle_{2}$$

$$+ \gamma^3 e^{i3\phi} (|W_2\rangle_{01} |2\text{H}\rangle_{2} |2\text{H}\rangle_{2} + \frac{1}{2} |\Lambda\rangle_{01} |1\text{H}\rangle_{1} |1\text{H}\rangle_{2} )] + O(\gamma^4)$$

$$= \sqrt{g_2} \left[ \frac{1}{2} \gamma^2 e^{i2\phi} (\hat{a}_{0\text{H}}^\dagger \hat{a}_{1\text{H}}^\dagger + \hat{a}_{0\text{H}}^\dagger \hat{a}_{1\text{V}}^\dagger)\hat{a}_{2\text{H}}^\dagger \hat{a}_{2\text{H}}^\dagger$$

$$+ \frac{1}{2} \gamma^3 e^{i3\phi} (\hat{a}_{0\text{V}}^\dagger \hat{a}_{1\text{H}}^\dagger + \hat{a}_{0\text{V}}^\dagger \hat{a}_{1\text{V}}^\dagger)(\hat{a}_{2\text{H}}^\dagger)^2(\hat{a}_{2\text{H}}^\dagger)^2$$

$$+ \frac{1}{4} \gamma^3 e^{i3\phi} (2\hat{a}_{0\text{H}}^\dagger \hat{a}_{0\text{V}}^\dagger \hat{a}_{1\text{H}}^\dagger \hat{a}_{1\text{V}}^\dagger + (\hat{a}_{0\text{V}}^\dagger)^2(\hat{a}_{1\text{H}}^\dagger)^2$$

$$+ (\hat{a}_{0\text{H}}^\dagger)^2(\hat{a}_{1\text{V}}^\dagger)^2)(\hat{a}_{2\text{H}}^\dagger)^2(\hat{a}_{2\text{H}}^\dagger)^2) \right]|\text{vac}\rangle_{0122'} + O(\gamma^4),$$

where we have focused on the terms up to $\gamma^3$ by considering that in practice, $\gamma^2 \sim O(10^{-4})$ is very small. The PDBS transforms modes 1 and 2 of $|\Psi_{0122'}\rangle$ according to the relations given in equations (1) and (2). Let $|\Psi_{0342'}\rangle$ be the state after the transformation. Using POVM given in equation (11), the four-fold coincidence detection probability $p_c$ can be calculated as

$$p_c = \frac{0342'}{0122'} \frac{|\Psi_{0342'}\rangle |\Pi_3^i \Pi_4^t \Pi_1^f |\Psi_{0342'}\rangle}{\langle \Pi_3^i \Pi_4^t \Pi_1^f |\Psi_{0342'}\rangle}$$

$$= \frac{1}{2} gg_1 \gamma^4 \eta^4 [2\mu (\mu - 1) + 1]$$

$$+ \frac{1}{2} gg_1 \gamma^6 \eta^4 (2 - \eta)^2 + \frac{1}{2} gg_1 \gamma^6 \eta^4 (2 - \eta)^2 [\mu (\mu - 1) + 1] + O(\gamma^8),$$

where $\Pi_1^f$ is the POVM for ‘click’ events at the detection in mode $j$. $p_t$ and $p_f$, respectively, correspond to probability of true and false coincidences. We see in equation (15) two contributions, $p_t$ and $p_f$. The true coincidences ($p_t$) are due to the $|W_2\rangle_{01} |1\text{H}\rangle_{1} |1\text{H}\rangle_{2}$ term in $|\Psi_{0122'}\rangle$ and the false coincidences ($p_f$) originate from multiple pairs of photons. Plugging the value $\mu = (5 - \sqrt{5})/10$ in these terms, we find that the ratio of true coincidences to total coincidence events becomes

$$p = \frac{p_t}{p_t + p_f} = 1 - 3\gamma^2 (\eta - 2)^2 + O(\gamma^4).$$
It is clearly seen that almost all the four-fold coincidence detections are true coincidences within the range of realistic values of $\eta$ and $\gamma$.

4.3. Effect of deviation in the PDBS parameter

In this section, we consider the effect of deviations in the parameters of PDBS from its ideal values of $\mu = (5 - \sqrt{5})/10$ and $\nu = (5 + \sqrt{5})/10$ on the probability and the fidelity of expanding $|W_2\rangle$ into $|W_3\rangle$. Let us assume that the reflection coefficients of PDBS for H- and V-polarized photons deviate from the ideal values by $\delta$ and $\Delta$, respectively. Then the action of the imperfect PDBS on H-polarized light and V-polarized light becomes

$$\hat{a}_{1H}^\dagger = \sqrt{1 - \mu - \delta \hat{a}_{3H}^\dagger} - \sqrt{\mu + \delta \hat{a}_{4H}^\dagger}, \quad \hat{a}_{2H}^\dagger = \sqrt{\mu + \delta \hat{a}_{3H}^\dagger} + \sqrt{1 - \mu - \delta \hat{a}_{4H}^\dagger},$$

and

$$\hat{a}_{1V}^\dagger = \sqrt{1 - \nu - \Delta \hat{a}_{3V}^\dagger} - \sqrt{\nu + \Delta \hat{a}_{4V}^\dagger}, \quad \hat{a}_{2V}^\dagger = \sqrt{\nu + \Delta \hat{a}_{3V}^\dagger} + \sqrt{1 - \nu - \Delta \hat{a}_{4V}^\dagger},$$

where $-\mu \leq \delta \leq \nu$ and $-\nu \leq \Delta \leq \mu$. Using these expressions, we calculated the probability of coincidence detection and the fidelity of the output state to the desired one. We omit the analytic expressions since they are rather lengthy and complicated. Instead, we depict the constant fidelity and constant probability contours as a function of $\delta$ and $\Delta$ in figure 4. We see that the effect of $\delta$ on the fidelity is much larger than that of $\Delta$. We can thus tolerate larger deviations from the ideal value for $\Delta$.

5. Conclusion

In this paper, we have proposed a simple probabilistic optical gate for expanding polarization entangled $W$ states and analyzed its feasibility taking into account the imperfections.
encountered in practice. The proposed gate is based on post-selection process to expand $|W_N\rangle$ by one qubit into $|W_{N+1}\rangle$ by locally acting on one of its qubits. A remarkable feature of this gate is that starting with a Bell state, it can prepare a tripartite entangled W-state with a success probability of $\frac{3}{10}$ which is the highest among all the proposed schemes so far. Moreover, the gate does not need stabilization of optical paths and does not employ sub-wavelength adjustments. Our feasibility analysis shows that the proposed gate can be implemented by the current experimental technologies. Thus this gate will provide a simple and useful tool to probe interesting features of multipartite W states.

Acknowledgments

This work was partially supported by JSPS Grant-in-Aid for Scientific Research(C) 20540389 and by MEXT Grant-in-Aid for the Global COE Program and Young scientists(B) 20740232.

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New Journal of Physics 11 (2009) 023024 (http://www.njp.org/)