Wavelet-based numerical and semianalytical methods of computational local structural analysis

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Abstract. Numerical or semianalytical solution of problems of structural mechanics of high dimensionality is computationally costly process in many cases. However, structural engineer does not normally face the task of obtaining a solution of the problem with high accuracy at all points of the considered domain occupied by the structure. As a rule, all subdomains that are potentially dangerous in terms of structural strength are well known in advance. Operational and variational formulations of boundary problems of structural mechanics with the use of method of extended domain are presented. After corresponding (finite element or finite difference) discretization and passage to governing equations considering problems are transformed to a multilevel space with the use of multilevel wavelet transform (discrete Haar basis is used). Special algorithms of averaging are presented.

Keywords: numerical methods, semianalytical methods, local structural analysis, wavelet-based methods, boundary problem, reduction.

1. Introduction
Since the 90s of the last century, wavelet analysis has found its application in such areas of structural mechanics and solid mechanics as structural dynamics, structural health monitoring, detection of defects (fracture zones, cracks, etc.), probabilistic mechanics, data visualization, identification systems in civil engineering and others. We should mention here research works of A. Barinka, T. Barsch [1], P. Charton, A. Cohen [2], S. Dahlke [3], W. Dahmen [4], S. Dumont [5], S. Jaffard [6], A. Knooth [1], A.J. Kurdila [7], F. Lebon [5], P. Monasse, P. Oswald [8], V. Perrier [9], S. Prossdorf [10], S. Qian [11], R. Schneider [10], K. Urban [1] and J. Weiss [11].

Among conventional numerical methods (finite element method (FEM), boundary element method (BEM), Galerkin method, collocation methods) wavelets are used, as a rule, for approximation of unknown functions. The development of mathematical models, algorithms and program systems for performing wavelet analysis of complex oscillations for some distributed systems are presented in research works of M.V. Zhigalov [12], A.V. Krysko [12-15], V.A. Krysko [12-15] and V.V. Soldatova. The averaging method for elliptic differential equations based on wavelet transform and FEM for predicting effective properties and analyzing averaged solutions of equations for composites with known structure and component properties was developed by S.P. Kopysov and Yu.A. Sagdeeva
Wavelet-based methods were used in research works of O.V. Mkrtchyan, G.A. Ginchvelashvili and A.A. Reshetova within the study of nonstationary dynamic processes, modeling of accelerogram of earthquakes in the form of a nonstationary random process, development of nonlinear dynamic methods of structural analysis [17].

Historically, the first and most ineffective from the computational point of view is the use of a sufficiently detailed approximating mesh (grid) over the entire domain occupied by the considering structure. On the one hand, such an approach provides a high-precision solution in the selected local subdomains, occupied by the structure, on the other hand, it is associated with an unreasonably large amount of essentially computational work.

The second technique, which is widely recommended in the literature dealing with numerical methods (FEM) is the use of non-uniform approximating meshes, condensing in selected local subdomains. At the same time, usually there is no rigorous mathematical justification for the construction of such meshes (there are no rules and principles of the corresponding thickening). Some alternative is the use of finite elements with a special form function (for instance, high order polynomial).

The third technique, which widely used in modern practice of structural analysis includes “cutting out” the selected subdomain, specifying corresponding boundary conditions, based on the results of more rough analysis or on any a priori estimates. Thus, we have so-called fragmentation method developed by A. B. Zolotov and O.V. Garden for solution of problems of structural mechanics. As an alternative, we can consider using the technique of the method of super-elements.

The fourth technique is the use of the Monte-Carlo method, which has shown its effectiveness in finding local solutions even in multidimensional problems of high dimensionality, but which has, nevertheless, a major drawback in the form of low convergence rate.

This paper continues the series of papers of P.A. Akimov [18-21], D.N. Alekseev, M. Aslami [18,19], T.B. Kaytukov [19], M.L. Mozgaleva and A.B. Zolotov, devoted to the development of wavelet-based numerical methods of local structural analysis. In this case, the theoretical foundation for construction of local approximation meshes is the representation of solutions of the corresponding boundary problems of structural mechanics through the fundamental functions of differential operators with the subsequent use of the Saint-Venant principle [22]. Behavior of the fundamental function (including the decay rate) predetermines the nature of the enlargement of the mesh. In case of the absence of a decrease in the fundamental function, the construction of local approximating meshes is inexpedient.

2. Materials and methods

2.1. Basic formulas of fast discrete Haar transforms and averaging for one-dimensional problems

2.1.1. Algorithms of fast direct and inverse discrete Haar transforms. Let \( \omega = \{ x : a \leq x \leq b \} \) be considering one-dimensional domain, where \( x \) is coordinate, \( a \) is lower limit of interval; \( b \) is upper limit of interval. Domain \( \omega \) can be divided into \((n-1)\) equal parts, where \( n = 2^M \) and \( M \) is the number of levels of the Haar basis [23-27]. Coordinates of mesh nodes are defined by formulas.

\[
x_i = a + (i-1)h, \quad i = 1, 2, \ldots, n; \quad h = (b - a)/(n - 1).
\]

Haar mesh functions are defined by formulas (\( N_p \) is the number of Haar functions at level \( p \)):

\[
\psi^M_i(i) = \alpha^M_{-1}, \quad i = 1, 2, \ldots, n;
\]

\( \psi^p_i(i) = \alpha^p_{-1} \)

\[
\begin{cases}
1, & 2^p(j-1) < i \leq 2^p(2j-1) \\
-1, & 2^p(2j-1) < i \leq 2^p+1 j \quad i = 1, 2, \ldots, n, \quad 0 \leq p < M; \\
0, & i \leq 2^p+1(j-1) \bigcup i > 2^p+1 j,
\end{cases}
\]
\[ N_p = \begin{cases} n/2^{p+1}, & 0 \leq p < M \\ 1, & p = M; \end{cases} \quad \alpha_p = \begin{cases} \sqrt{2^{p+1}}, & 0 \leq p < M \\ \sqrt{2^M} = \sqrt{n}, & p = M. \end{cases} \]  

Let \( f(i) \) be arbitrary mesh function. Then we have

\[ f(i) = \sum_{p=0}^{M} \sum_{j=1}^{N_p} v_j^p \psi_j^p (i), \quad v_j^p = \sum_{i=1}^{n} f(i) \psi_j^p (i), \quad j = 1, 2, ..., N_p, \quad p = 0, 1, ..., M. \]  

where \( v_j^p, \quad j = 1, 2, ..., N_p, \quad p = 0, 1, ..., M \) are wavelet (Haar) expansion coefficients.

Let us consider algorithm of fast direct discrete wavelet (Haar) transform. We have

\[ u_j^0 = f(j), \quad j = 1, 2, ..., n; \quad \alpha_0 = \sqrt{2} \]  

and for all \( p = 0, 1, ..., M - 1, \) \( j = 1, 2, ..., N_p \) we can use the following formulas:

\[ v_j^p = \alpha_j^{-1} (u_{2j-1}^p - u_{2j}^p); \quad u_j^{p+1} = u_{2j-1}^p + u_{2j}^p; \quad \alpha_{p+1} = \sqrt{2} \alpha_p; \]

\[ \alpha_M = \sqrt{n}; \quad v_j^M = \alpha_M^{-1} u_j^M. \]  

where \( u_j^p, \quad j = 1, 2, ..., N_p, \quad p = 0, 1, ..., M \) are so-called auxiliary quantities.

Let us also consider algorithm of fast inverse discrete wavelet (Haar) transform. We have

\[ \alpha_M = \sqrt{n}; \quad \alpha_{M-1} = \sqrt{n}; \quad u_j^M = \alpha_M^{-1} v_j^M \]  

and for all \( p = M - 1, M - 2, ..., 0, \) \( i = 1, 2, ..., N_p \) we can use the following formulas:

\[ j = [(i + 1)/2]; \quad z = (-1)^i; \quad u_j^0 = \alpha_j^{-1} z v_j^0 + u_j^{p+1}; \quad \alpha_{p+1} = \alpha_p / \sqrt{2}. \]

Thus, we have

\[ f(i) = u_j^0, \quad i = 1, 2, ..., n. \]

2.1.2. Algorithm of averaging. The number of unknowns can be decreased with the use of special procedure of reduction of problems without significant loss of accuracy. Corresponding errors in local solutions are relatively small. Let us assume that it is necessary to use averaging at some level and \( q \).

Thus, for all \( p = 0, 1, ..., q \) and \( j = 1, 2, ..., N_p \) we can suppose

\[ (Du^p)_{2j-1} \approx (Du^p)_{2j} \approx (D\tilde{u}^p)_{2j-1}, \quad v_{2j-1}^p = v_{2j}^p, \quad j = 1, 2, ..., N_{p+1}; \]

\[ \tilde{u}_{2j-1}^p = (u_{2j-1}^p + u_{2j}^p) / 2; \quad (D\tilde{u}^p)_{2j-1} = (\tilde{u}_{2j-1}^p - \tilde{u}_{2j}^p) / (2^{-q} h); \]

Formulas of averaging have the following form

\[ v_{2j-1}^p = v_{2j}^p = \beta v_j^{p+1}, \quad j = 1, 2, ..., N_{p+1}; \quad \beta = 1/(2\sqrt{2}). \]

2.2. Basic formulas of fast discrete Haar transforms and averaging for two-dimensional problems

2.2.1. Algorithms of fast direct and inverse discrete Haar transforms. Let \( \omega \) be considering two-dimensional rectangular domain, \( \omega = \{(x_1, x_2): 0 \leq x_1 \leq l_1, 0 \leq x_2 \leq l_2\}, \) where \( x_1 \) and \( x_2 \) are corresponding coordinates; \( l_1 \) and \( l_2 \) are dimensions along \( x_1 \) and \( x_2 \) (length of domain along \( x_1 \).
and length of domain along \( x_2 \). Domain \( \omega \) can be divided into \((n-1)\) equal parts along coordinate \( x_1 \) and into \((n-1)\) equal parts along \( x_2 \) \((h_1 = l_1/(n-1), h_2 = l_2/(n-1))\), where \( n = 2^M \), \( M \) is the number of levels of the Haar basis. Coordinates of mesh nodes are defined by formulas

\[
x_{ij} = (i_1 - 1)h_1 , \quad i_1 = 1, 2, \ldots, n; \quad x_{2,ij} = (i_2 - 1)h_2 , \quad i_2 = 1, 2, \ldots, n.
\]

(15)

Haar mesh functions \( \psi^p_{s_1,s_2,j_1,j_2} (i_1,i_2) \), \( p = 1,2,\ldots,M \), \( j_1,j_2 = 1,2,\ldots,N_p \), \( s_1,s_2 = 0,1 \) (except \( s_1 = s_2 = 0 \)) are defined by formulas:

\[
\psi^p_{s_1,s_2,j_1,j_2} (i_1,i_2) = \alpha_p^{-1} \left\{ (-1)^{k_{i_1}+k_{i_2}} \sum_{q=1}^{2} \left( \bigcup_{k_q=0}^{2} \left( 2^p+1 \left( j_q - \frac{1}{2} + \frac{k_q}{2} \right) \right) \right) \right. \\
\left. \left\{ \begin{array}{cl}
0, & \text{in other cases}
\end{array} \right. \right. \\
\psi^0_{0,0,0,0} (i_1,i_2) = \alpha_0^{-1} \left( \begin{array}{c}
1
\end{array} \right).
\]

(16)

Let \( f(i_1,i_2) \) be an arbitrary mesh function and \( v^p_{1,0,j_1,j_2} , v^p_{0,k,j_1,j_2} , v^p_{1,1,j_1,j_2} , j_1,j_2 = 1,2,\ldots,N_p \), \( p = 1,2,\ldots,M \) are wavelet (Haar) expansion coefficients. Then we have

\[
f(i_1,i_2) = \psi^M_{0,0,1,1} \psi^M_{0,0,1,1} + \sum_{p=0}^{M-1} \sum_{j_1=1}^{N_p} \sum_{j_2=1}^{N_p} \left( v^p_{1,0,j_1,j_2} \psi^p_{1,0,j_1,j_2} (i_1,i_2) + v^p_{0,1,j_1,j_2} \psi^p_{0,1,j_1,j_2} (i_1,i_2) + v^p_{1,1,j_1,j_2} \psi^p_{1,1,j_1,j_2} (i_1,i_2) \right) + \\
\sum_{j_1=0}^{N_p} \sum_{j_2=1}^{N_p} f(i_1,i_2) \psi^p_{s_1,s_2,j_1,j_2} (i_1,i_2).
\]

(18)

Let us consider algorithm of fast direct discrete wavelet (Haar) transform. We have

\[
u^0_{j_1,j_2} = f(j_1,j_2) , \quad j_1 = 1,2,\ldots,n; \quad j_2 = 1,2,\ldots,n; \quad \alpha_0 = 2.
\]

(20)

and for all \( p = 0,1,\ldots,M-1 \), \( j_1,j_2 = 0,1,\ldots,N_p \), \( s_1,s_2 = 0,1 \) (except \( s_1 = s_2 = 0 \)) we can use:

\[
z_1 = (-1)^{s_1} , \quad z_2 = (-1)^{s_2} \quad \alpha_{p+1} = 2 \cdot \alpha_p.
\]

(21)

\[
v^p_{s_1,s_2,j_1,j_2} = \alpha_p^{-1} (v^p_{1,0,j_2,j_2} + z_1 u^p_{2,h_2,j_2-1} + z_2 u^p_{2,h_2,j_2-1} + z_1 z_2 u^p_{2,h_2,j_2});
\]

(22)

\[
u^p_{j_1,j_2} = u^p_{2,j_1,j_2-1} + u^p_{2,h_2,j_2-1} + u^p_{2,h_2,j_2} + u^p_{2,h_2,j_2} ;
\]

(23)

\[
\alpha_{M} = n; \quad v^M_{0,0,1,1} = \alpha_{M}^{-1} u^M_{1,1};
\]

(24)

where \( u^p_{j_1,j_2} , j_1,j_2 = 1,2,\ldots,N_p \), \( p = 1,2,\ldots,M \) are so-called auxiliary quantities.

Let us also consider algorithm of fast inverse discrete wavelet (Haar) transform. We have

\[
\alpha_{M} = n; \quad \alpha_{M-1} = n; \quad u^M_{1,1} = \alpha_{M}^{-1} v^M_{0,0,1,1}.
\]

(25)

and for all \( p = M-1, M-2,\ldots,0,i_1,i_2 = 1,2,\ldots,N_p \) we can use the following formulas:
\[ j_1 = [(i_1 + 1)/2]; \quad j_2 = [(i_2 + 1)/2]; \quad z_1 = (-1)^{i_1}; \quad z_2 = (-1)^{i_2}; \quad \alpha_{p-1} = \alpha_p/2; \]  \hspace{1cm} (26)

\[ u_{h,j_2}^p = \alpha^{-1}_p (z_1 v_{0,0,h,j_2}^p + z_2 v_{0,1,h,j_2}^p + z_1 z_2 v_{1,1,h,j_2}^p) + u_{h,j_2}^{p+1}. \]  \hspace{1cm} (27)

Thus, we have

\[ f(i_1, i_2) = u_{h,j_2}^0, \quad i_1 = 0, 1, \ldots, n, \quad i_2 = 0, 1, \ldots, n. \]  \hspace{1cm} (28)

2.2.2. Algorithm of averaging. Let us assume that it is necessary to make averaging at some level \( q \).

For all \( p = 1, 2, \ldots, q, \quad j_1, j_2 = 1, 2, \ldots, N_p, \quad s_1, s_2 = 0 \) (except \( s_1 = s_2 = 0 \)) we can suppose

\[ (D_1 u_1^p)_{2h_1,2j_1-1} = (D_1 u_1^p)_{2h_1,2j_1-1} = (D_1 u_1^p)_{2h_1,2j_1-1} \approx (D_1 \tilde{u}_1^p)_{2h_1,2j_1-1}; \]  \hspace{1cm} (29)

\[ (D_2 u_2^p)_{2h_2,2j_2-1} = (D_2 u_2^p)_{2h_2,2j_2-1} = (D_2 u_2^p)_{2h_2,2j_2-1} \approx (D_2 \tilde{u}_2^p)_{2h_2,2j_2-1}; \]  \hspace{1cm} (30)

\[ (D_1 D_1^* u_1^p)_{2h_1,2j_1-1} = (D_1 D_1^* u_1^p)_{2h_1,2j_1-1} = (D_1 D_1^* u_1^p)_{2h_1,2j_1-1} \approx (D_1 D_1^* \tilde{u}_1^p)_{2h_1,2j_1-1}; \]  \hspace{1cm} (31)

Formulas of averaging have the following form

\[ v_{s_1,s_2,2h_1,2j_1-1}^p = v_{s_1,s_2,2h_1,2j_1-1}^p = v_{s_1,s_2,2h_1,2j_1-1}^p = v_{s_1,s_2,2h_1,2j_1-1}^p; \]  \hspace{1cm} (32)

\[ \tilde{u}_{h_1,j_2}^p = (u_{h_1,j_2}^p + u_{h_1+1,j_2}^p + u_{h_1,j_2+1}^p + u_{h_1+1,j_2+1}^p)/4; \]  \hspace{1cm} (33)

\[ (D_1^* u_1^p)_{h_1,j_2} = (u_{h_1,j_2}^p - u_{h_1,j_2}^p)/(2^p h); \quad (D_2^* u_2^p)_{h_1,j_2} = (u_{h_1,j_2}^p - u_{h_1,j_2}^p)/(2^p h); \]  \hspace{1cm} (34)

\[ (T_1^* u_1^p)_{h_1,j_2} = u_{h_1,j_2}^p + u_{h_1,j_2}^p; \quad (T_2^* u_2^p)_{h_1,j_2} = u_{h_1,j_2}^p + u_{h_1,j_2}^p; \]  \hspace{1cm} (35)

\[ D_1 = 0.5 \cdot T_1^* D_1^*; \quad D_2 = 0.5 \cdot T_2^* D_2^*. \]  \hspace{1cm} (36)

2.3. About fast discrete Haar transforms and averaging for three-dimensional problems

This most cumbersome case is described in [28]. Corresponding algorithms of fast direct and inverse discrete Haar transforms and algorithm of averaging are similar to those described earlier.

2.4. Intro to multilevel wavelet-based numerical method of local structural analysis

In accordance with the method of extended domain [29], the domain \( \Omega \), occupied by considering structure, is embodied by extended one \( \Omega \) of arbitrary shape, particularly elementary. Operational formulation of the problem in extended domain \( \Omega \) normally has the form

\[ Lu = F, \]  \hspace{1cm} (41)
where $L$ is the operator of considering boundary problem with allowance for boundary conditions; $u$ is the unknown function; $F$ is the given right-side function.

Variational formulation of the problem can be obtained directly from operational formulation:

$$
\Phi(u) = 0.5 \cdot (Lu, u) - (F, u).
$$

Solution of (42) is the critical point of (41). $(f, g)$ denotes dot product of functions $f$ and $g$.

Obviously, it can be shown that discrete (numerical) formulation of the problem has the form:

$$
A\tilde{u} = \tilde{f},
$$

where $A = \{a_{i,j}\}_{i,j=1,2,\ldots,n_{gl}}$ is the finite difference (finite element) approximation of $L$; $\tilde{u} = [u_1 \ u_2 \ \ldots \ u_{n_{gl}}]^T$ is the unknown mesh function; $\tilde{f} = [f_1 \ f_2 \ \ldots \ f_{n_{gl}}]^T$ is the given right-side mesh function; $n_{gl}$ is dimension of considering problem.

Major peculiarities of specified method of basis (local) variations include universality and full adaptability to computer implementation. Particularly for linear problems we can use formulas

$$
a_{i,j} = \Phi(\tilde{v}^{(i)} + \epsilon^{(j)}) - \Phi(\tilde{v}^{(i)}) - \Phi(\epsilon^{(j)}) + \Phi(\tilde{0})]; \quad f_i = 0.5 \cdot [\Phi(\tilde{v}^{(i)}) - \Phi(-\tilde{v}^{(i)})],
$$

$$
\tilde{v}^{(i)} = [e_1^{(i)} \ e_2^{(i)} \ \ldots \ e_{n_{gl}}^{(i)}]^T, \quad i = 1,2,\ldots,n_{gl}; \quad e_j^{(i)} = \delta_{i,j}, \quad j = 1,2,\ldots,n_{gl};
$$

$\tilde{v}^{(i)}$, $i = 1,2,\ldots,n_{gl}$ are basis mesh vectors; $\tilde{0}$ is the null function; $\delta_{i,j}$ is the Kronecker delta.

Obviously, we can write the following wavelet-based (Haar-based) formulation of the problem:

$$
\Phi(\tilde{u}) = 0.5 \cdot (A\tilde{u}, \tilde{u}) - (\tilde{f}, \tilde{u}) = 0.5 \cdot (LQ\tilde{v}, \tilde{v}) - (\tilde{f}, \tilde{v}) = 0.5 \cdot (Q^*LQ\tilde{v}, \tilde{v}) - (Q^*\tilde{f}, \tilde{v}),
$$

where $Q$ is so-called transition matrix consisting from Haar basis vectors, located in rows; $\tilde{v}$ is vector of Haar expansion coefficients of the vector $\tilde{u}$.

Thus, expression (46) can be rewritten in the form:

$$
\tilde{v} = Q^*\tilde{f}, \quad \tilde{v} = Q^*\tilde{f}.
$$

Corresponding operational formulation of the considering problem has the form

$$
\tilde{L}\tilde{v} = \tilde{f}, \quad \tilde{L} = Q^*LQ; \quad \tilde{f} = Q^*\tilde{f}.
$$

Further reduction of the problem is based on the averaging algorithm specified above.

2.5. Intro to multilevel wavelet-based semianalytical method of local structural analysis

The objects of the multilevel wavelet-based semianalytical (discrete-continual) method are structures with piecewise constancy of physical and geometrical parameters in one dimension. This dimension or direction is called “basic direction”. As a result so-called wavelet-based discrete-continual design model is introduced. Wavelet approximation of extended domain along non-basic directions is introduces. However, along the basic direction problem remains continual. Analytical solution is apparently preferable in all aspects for providing high-precision structural analysis. It allows structural engineer considering boundary effects. Thus, we can correctly solve problems of structural mechanics when some components of corresponding solutions are rapidly varying functions. Due to dramatic changes of these components inside of mesh elements their rate of change can’t be correctly and adequately considered within conventional numerical methods. Analytical methods of solution are extremely effective here. Another feature of the proposing multilevel wavelet-based semianalytical method of local structural analysis is the absence of limitations on lengths of structures. It can be shown, that resultant multipoint boundary problem after corresponding reduction has the form [30-32]
\[
\begin{aligned}
\begin{cases}
\ddot{y}' = A_k \ddot{y} + \dddot{f}_k, & x \in (x^k_1, x^k_{n_k}), \quad k = 1, ..., n_k - 1 \\
B^+_k \ddot{y}(x^k_1 - 0) + B^+_k \ddot{y}(x^k_1 + 0) = g^+_k + g^+_n, & k = 2, ..., n_k - 1 \\
B^+_k \ddot{y}(x^k_1 - 0) + B^-_k \ddot{y}(x^k_{n_k} - 0) = g^+_1 + g^-_n,
\end{cases}
\quad (49)
\end{aligned}
\]

where \(x^k_j = x^k_{j,k}, \quad k = 1, ..., n_k\) are coordinates of boundary points; \(A_k, \quad k = 1, 2, ..., n_k - 1\) are matrices of constant coefficients of order \(n\); \(B^+_k, B^+_k, \quad k = 2, ..., n_k - 1\) and \(B^-_k, B^-_k\) are matrices of boundary conditions of order \(n\) at point \(x^k_j; \quad g^+_k, g^+_n\) are right-side vectors of boundary conditions at point \(x^k_j\); \(\ddot{y} = \ddot{y}(x) = [y_1(x) \quad y_2(x) \quad ... \quad y_n(x)]^T\) is the unknown vector function; \(\ddot{y}^{(n)} = \ddot{y}^{(n)}(x) = d^2y / dx^2; \quad \dddot{f}_k = \dddot{f}_k(x) = [f_{k,1}(x) \quad f_{k,2}(x) \quad ... \quad f_{k,n}(x)]^T, k = 1, 2, ..., n_k - 1\) are right-side vector functions.

Correct solution of considering multipoint boundary problem of structural analysis is rather complicated due to several factors. In the first place we should mention boundary effects and immense number of differential equations. Thus, stiff systems are under consideration. In the second place matrices of coefficients of considering type of systems have eigenvalues of opposite signs and corresponding Jordan matrices are not diagonal. It should be emphasized, that correct method of solution of multipoint boundary problems for systems of ordinary differential equations with piecewise constant coefficients in structural analysis has been already developed. This method overcomes all specified difficulties. Besides, we should mention the following advantages of the method: universality, full adaptability to computer implementation, computational stability, optimal conditionality of resultant systems, special algorithm of partial Jordan decomposition of matrix of coefficient, eliminating necessity of computing of root vectors.

3. Results

As is known, development, research and verification of correct mathematical models and methods of structural mechanics are the most important aspects of ensuring safety of buildings and complexes. Generally, in practice global solution in the whole domain is not required in many cases. On the contrary it is normally necessary to obtain high-precision solution of the problem in a certain set of prescribed (selected) subdomains. Wavelet-based numerical and semianalytical methods of corresponding computational local structural analysis are presented. These methods allow reducing dimensionalities of problems. Moreover high-precision results in selected subdomains are provided. Wavelets provide effective and powerful tool for such objectives.

4. Discussion

Results of the distinctive paper is certainly significant because so-called multilevel (local and global (conventional)) structural analysis is required for solution of various technical problems. On the one hand, defects and collapses are most commonly local (subdomains with edge effects deserve special attention in this regard because corresponding significant stresses can lead to destruction), but on the other hand, corresponding load-bearing capacity is determined by the global behavior of structure. Besides, it is associated with the condition of limit equilibrium. After expansion of the solution with the use of local wavelet basis corresponding components are considered at each level of the basis. Reduction of the problem with the use of original algorithm of averaging allows obtaining high accuracy solution of the problem in selected domains with an acceptable reduced size. This localization can be imposed to any selected subdomain by choosing an optimum reduction matrix.

5. Conclusion

Thus, considering wavelet-based multilevel numerical and semianalytical methods of local structural analysis are promising, practically and theoretically significant. Besides, semianalytical formulations are contemporary mathematical models which currently becoming available for computer realization.
Comparison between conventional FEM and wavelet-based numerical and semianalytical methods of local structural analysis must allow to conclude that the localization of the problem provide high-precision results for selected domains even in high level of reduction in wavelet coefficients. However, results of such local analysis may be unacceptable in the other (unselected) subdomains. Analysis of the behavior of the fundamental functions of boundary problems must be also under consideration in order to ensure the correct choice of reducing parameters.

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