GLUINO AND NEUTRALINO CONTRIBUTION TO THE DIRECT CP VIOLATION PARAMETER $\epsilon'$

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ABSTRACT
I present a detailed and complete calculation of the gluino and neutralino contribution to the direct CP violating parameter $\epsilon'$ within the MSSM. I include the complete mixing matrices of the neutralinos and of the scalar partners of the left and right handed down quarks. I find that the neutralino contribution is generally small but can be larger than the gluino contribution for small values $m_S \leq 400$ GeV of the supersymmetric breaking scale.

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I. INTRODUCTION

Although the standard model (SM) predicts observable levels of direct CP violation in the K system for a large top quark mass, there is still a disagreement of the experimental results. Whereas the one of the NA31 collaboration at CERN [1] is given by
\[ \text{Re}(\epsilon'/\epsilon) = 23 \pm 6.5 \times 10^{-4}, \]
the collaboration E731 at Fermilab [2] reports a value of
\[ \text{Re}(\epsilon'/\epsilon) = 7.4 \pm 6.0 \times 10^{-4}. \]
For a top quark mass of \( m_t = 180 \pm 12 \text{ GeV} \) [3] the SM predicts \( \epsilon'/\epsilon \) to be in the range of \( (0 - 3) \times 10^{-3} \) [4–7], where the uncertainty lies among cancellations of strong and electroweak penguin diagrams, and thus is still in agreement with both experiments.

It is therefore of great interest to look at the contributions to this direct CP violating parameter in any kind of models beyond the SM of which the most promising nowadays is its minimal supersymmetric extensions (MSSM) [8]. Studies of the contribution to \( \epsilon'/\epsilon \) within this model were done a while ago [9–12 and references therein]. More recent work can be found in [13, 14 and references therein].

Since in the \( K^0 \) system \( (K_{21}^* K_{22})^2 m_c^2 \geq (K_{31}^* K_{32})^2 m_t^2 \) the charm quark contribution is larger than the top quark contribution, whereas it is vice versa in the \( B^0_d \) system. In recent papers [15, 16] we have shown that the largest contribution to the mass difference in the \( B^0_d \) system within the MSSM were given by the charged Higgses for small \( \tan \beta = v_2/v_1 \) (with \( v_{1,2} \) the vacuum expectation values, vev’s, of the neutral Higgs bosons) and the charginos. The gluino contributes non negligibly when its mass is small, of order 100 GeV, and the neutralino contribution was shown to be non neglectable for large \( \tan \beta \approx 50 \) and small values of the SUSY breaking mass parameter \( m_S \leq 300 \text{ GeV} \).

In the literature the SUSY contribution to the mass difference of the \( K_0 \) system and therefore to the parameter \( \epsilon \) (the imaginary part of the matrix elements leading to the mass difference) was supposed to be neglectable. As a consequence emphasis was put in calculating the penguin diagrams leading to \( \epsilon' \). However in [13, 14] the authors found non negligible interference between the box and penguin diagrams for certain ranges of \( x = m_\tilde{g}^2/m_\tilde{S}^2 \) in the proximity of 1.

In this paper I do a reanalysis of the penguin diagrams as shown in Fig.1 with gluinos and scalar down quarks within the loop, which contribute to the direct CP violation parameter \( \epsilon' \), reproducing the results of [9]. In the calculations I also include the neutralino contribution. I do not neglect any mixing of the scalar partners of the left and right handed down quarks nor any mixing in the neutralino sector. The goal of this paper is to compare the gluino contribution with the neutralinos one and to show that the latter in general can not be neglected. Here I do not give a complete analysis of the contribution of all particles within the MSSM (as there are the charged Higgs boson and charginos) or all sort of diagrams (including the box diagrams), which will be presented elsewhere [18].

\footnote{I therefore do not agree with the statement of [17] (analysing the contribution to the decay \( b \to sg \) within the MSSM), that the authors of [9] have neglected a crucial term}
In the next section I present the calculation and discuss the results in the third section. I end with the conclusions. Since in the literature [17] the correctness of the results in [9] was doubted I present the detailed calculation in the appendices.

II. GLUINO AND NEUTRALINO CONTRIBUTION TO THE DIRECT CP VIOLATING PARAMETER $\epsilon'$

In the SM there are two CP violation parameter: the indirect CP violation parameter $\epsilon$ and the direct CP violation parameter $\epsilon'$. Whereas the indirect one follows from the mass eigenstates of $K^0$ and $\bar{K}^0$ and is given by the imaginary parts of the diagrams leading to $\Delta m_{K^0}$, $\epsilon'$ describes the direct decay of the Kaons into two pions. The values are given by:

$$\epsilon = \frac{e^{i\pi/4}}{\sqrt{2\Delta m_{K^0}}} Im M_{12} \quad (1)$$

$$\epsilon' \approx -\frac{\omega}{\sqrt{2}} \xi (1 - \Omega) e^{i\pi/4} \quad (2)$$

$\Delta m_{K^0} = 2Re M_{12}, \omega = Re A_2 / Re A_0 \approx 1/22, \xi = Im A_0 / Re A_0$ and $\Omega = Im A_2 / \omega Im A_0$, where $A_0$ and $A_2$ are the amplitudes of the decays into two pions with $\Delta I = 1/2$ and $\Delta I = 3/2$ respectively. The uncertainty in the SM lies among certain cancellations between the electroweak and strong diagrams leading to the $\Omega$ term, which approaches 1 for $m_t \approx 220$ GeV and thus $\epsilon'/\epsilon$ obtains even negative values for higher top quark masses [6,7]. In [5] however the authors using chiral pertubation theory obtain positive values for all top quark masses.

To obtain the gluino and neutralinos contribution to the $\epsilon'$ parameter we have to calculate the diagrams as shown in Fig.1. In the calculation I consider the full mass matrices of the scalar down quarks including 1 loop corrections and the mixing terms of scalar partners of the left and right down quarks; that is I present the results in the mass eigenstates and not in the current eigenstates of the scalar down quarks. I also consider the full $4 \times 4$ matrix of the neutralinos and calculate its mass eigenstates and mixing angles numerically. For a detailed discription of the mass eigenstates, mixing angles of the scalar down quarks and neutralinos I refer the interested reader to [15, 16] and will not represent them here. Their couplings to the quarks can be found in Fig.24 in [19].

For the couplings of the gluinos to the gluons and to the quarks and scalar quarks it was shown a while ago, that there occur flavour changing strong interactions between the gluino, the left handed quarks, and their supersymmetric scalar partners, whereas the couplings of the gluino to the right handed quarks and their partners remains flavour diagonal [20, 21]. However in general this might not be the case and therefore the authors in [9] took both couplings to be flavour non diagonal, which leads to a term, which contributes to $\epsilon'$ proportional to the gluino mass $m_\tilde{g}$. As was pointed out in appendix B of [19] also the couplings of the neutralinos to the left- and right handed quarks and their superpartners are in general flavour non diagonal. To compare the neutralino contribution to $\epsilon'$ with the gluino one I therefore
include both couplings in the calculation.

The Lagrangians, which describe the couplings of the gluino to the gluon, of the gluon to the scalar down quarks and the flavour non diagonal one of the gluino to the left and right handed down quarks and their superpartners, needed for the calculation of the penguin diagrams, are given in the mass eigenstates of the scalar down quarks by:

\[
\mathcal{L}_{\tilde{g}\tilde{g}} = \frac{i}{2}g_s f_{abc} \bar{g}_a \gamma_\mu \tilde{g}_b G^\mu_c \\
\mathcal{L}_{\tilde{d}\tilde{d}} = -ig_s T^a G^{a\mu} \sum_{m=1,2} \tilde{d}_m^* \tilde{d}_m \\
\mathcal{L}_{FC} = -\sqrt{2}g_s T^a \sum_{m=1,2} \left[ K_{\tilde{d}_L}^m \bar{g}_a P_L \tilde{d}_m^* - K_{\tilde{d}_R}^m \bar{g}_a P_R \tilde{d}_m^* \tilde{d}_m \right] + \text{h.c.}
\]

Eq.(3) has to be multiplied by 2 to obtain the Feynman rules. \( f_{abc} \) are the SU(3)C structure constants, \( T^a \) its generators and \( g_s \) the strong coupling constant. \( P_L = (1 - \gamma_5) / 2 \) and \( P_R = (1 + \gamma_5) / 2 \) are the left and right handed helicity operators respectively. \( K_{\tilde{d}_L,R}^m \) is the supersymmetric version of the Kobayashi–Maskawa (KM) matrix. \( \tilde{d}_1, \tilde{d}_2 \) are the mass eigenstates of \( \tilde{d}_1, \tilde{d}_2 \) to the current eigenstates \( \tilde{d}_{L,R} \) (\( d \) stands for down, strange and bottom quarks, generation indices have been omitted) and is given by:

\[
\begin{pmatrix}
\tilde{d}_1 \\
\tilde{d}_2
\end{pmatrix}
= \begin{pmatrix}
\cos \Theta_d & \sin \Theta_d \\
-\sin \Theta_d & \cos \Theta_d
\end{pmatrix}
\begin{pmatrix}
\tilde{d}_L \\
\tilde{d}_R
\end{pmatrix}
\]

Following the philosophy of [9] I obtain after a lengthy but straightforward calculation of the penguin diagrams of Fig.1 the following results for the \( \xi \) parameter:

\[
\xi = - f \text{Im} M / \text{Re} M \\
M = M_{SM} + M_{\tilde{g}} + M_{\tilde{N}} \\
M_{SM} = \sum_{a=1-3} \frac{\alpha}{2 \sin^2 \Theta_W m_W^2} \left( A_{SM}^a + \eta_{QCD} B_{SM}^a \right) \\
M_{\tilde{g}} = - \sum_{a=1-3} \frac{\alpha_s}{m_{\tilde{g}}} \left( A_{\tilde{g}}^a + \eta_{QCD} B_{\tilde{g}}^a \right) \\
M_{\tilde{N}} = - \sum_{a=1-3} \sum_{i=1-4} \frac{\alpha}{2 \sin^2 \Theta_W \cos^2 \Theta_W m_{\tilde{N}_i}^2} \left( A_{\tilde{N}_i}^a - \eta_{QCD} B_{\tilde{N}_i}^a \right) \\
A_{SM}^a = \frac{2}{3} (1 + y_a) (1 - y_a - \frac{11}{4} y_a^2 - \frac{3}{4} y_a^3) \log(\frac{m_{u_a}^2}{m_W^2}) - \frac{3}{2} y_a (1 + \frac{25}{18} y_a + \frac{1}{3} y_a^2) \\
B_{SM}^a = \left[ \frac{3}{2} y_a^2 (1 + y_a)^2 \log(\frac{m_{u_a}^2}{m_W^2}) + \frac{1}{2} y_a (1 + \frac{9}{2} y_a + 3 y_a^2) \right] (m_s - m_Q) \\
y_a = \frac{m_{u_a}^2}{m_W^2 - m_{u_a}^2}
\]
\[ A_{\tilde{a}} = \sum_{m=1,2} \left[ C_{2G} A_G(x_{\tilde{a}m}^\tilde{g}) + 2 C_{2F} A_F(x_{\tilde{a}m}^\tilde{g}) \right] \left[ K_{a1L}^{*\tilde{g}} K_{a2L}^{\tilde{g}} \tilde{K}_{m1}^{a2} + K_{a1R}^{*\tilde{g}} K_{a2R}^{\tilde{g}} \tilde{K}_{m2}^{a2} \right] \]
\[ B_{\tilde{a}} = \sum_{m=1,2} \left\{ \left[ C_{2G} B_G(x_{\tilde{a}m}^\tilde{g}) - 2 C_{2F} B_F(x_{\tilde{a}m}^\tilde{g}) \right] \left[ K_{a1L}^{*\tilde{g}} K_{a2L}^{\tilde{g}} \tilde{K}_{m1}^{a2} - K_{a1R}^{*\tilde{g}} K_{a2R}^{\tilde{g}} \tilde{K}_{m2}^{a2} \right] \right\} \times (m_a - m_Q) - \left[ C_{2G} \bar{B}_G(x_{\tilde{a}m}^\tilde{g}) - 2 C_{2F} \bar{B}_F(x_{\tilde{a}m}^\tilde{g}) \right] m_{\tilde{g}} \left[ K_{a1L}^{*\tilde{g}} K_{a2L}^{\tilde{g}} - K_{a1R}^{*\tilde{g}} K_{a2R}^{\tilde{g}} \right] \tilde{K}_{m1}^{a1} \tilde{K}_{m2}^{a2} \}
\[ A_{\tilde{N}_i} = \sum_{m=1,2} A_F(x_{\tilde{a}m}^{\tilde{N}_i}) \left\{ K_{a1L}^{*\tilde{N}_i} K_{a2L}^{\tilde{N}_i} \tilde{K}_{m1}^{a2}(T_i^{dL} T_i^{sL} + T_m a T_m s) \right\} \]
\[ B_{\tilde{N}_i} = \sum_{m=1,2} \left\{ B_F(x_{\tilde{a}m}^{\tilde{N}_i}) \left[ K_{a1L}^{*\tilde{N}_i} K_{a2L}^{\tilde{N}_i} \tilde{K}_{m1}^{a2}(T_i^{dL} T_i^{sL} - T_m a T_m s) \right] \right\} (m_s - m_Q) \]
\[ x_{\tilde{a}m}^{\tilde{g}} = \frac{m^2_{\tilde{a}m}}{m^2_{\tilde{g}}} \]
\[ x_{\tilde{a}m}^{\tilde{N}_i} = \frac{m^2_{\tilde{a}m}}{m_{\tilde{N}_i}} \]
\[ T_{m a} = \frac{m_a}{m_Z \cos \beta} N_{i3} \]
\[ T_{i}^{aL} = e_a \sin 2 \Theta_W N_{i1}^l - (1 + 2 e_a \sin^2 \Theta_W) N_{i2}^l \]
\[ T_{i}^{aR} = - \left( e_a \sin 2 \Theta_W N_{i1}^l - 2 e_a \sin^2 \Theta_W N_{i2}^l \right) \]

where \( a \) runs over all 3 generations. Note that in the SM the up quarks are running in the loop, whereas in the MSSM the scalar down quarks are. Also note that \( T_{i}^{aL,R} = T_{i}^{aL,R} \cos \beta \) can be extracted from \( \tan \beta, N_{ij} \) and \( N_{ij}^l \) are the diagonalizing angles of the neutralinos as defined in eq.(A.20), eq.(A.23) and shown in Fig.24 in [19]. I take them to be real and put all unknown SUSY phases into \( K_{abL,R} \).
functions $A_a^{SM}$ and $B_a^{SM}$ are taken from [23]. When the mixing of the scalar down quarks is neglected, the functions $A_a^g$ and $B_a^g$ agree with [9] (see Appendices A+B) up to a relative minus sign of the term proportional to the gluino mass.

$\eta_{QCD}$ is a QCD factor obtaining the structure functions of the Kaons, pions as well as their masses with dimension 1/GeV and given in eq. (14) in [9]. There it was taken $f_K = f_\pi$ leading to $\eta_{QCD} \simeq 10.8$ and to a bit smaller value of $\simeq 8.3$ for $f_K = 1.27 f_\pi$ [27] for the masses taken there. $f \simeq 1/6$ and $m_Q$ is the constituent $u, d$ quark mass taken to be 0.3 GeV, $m_s = 0.5$ GeV, $m_b = 4.5$ GeV, $m_c = 1.3$ GeV and $m_t = 180$ GeV.

The functions $A_F$, $A_G$, $B_F$, $B_G$, $B_{\tilde{F}}$ and $B_{\tilde{G}}$ are given in the Appendix B. $C_{2G} = N$ and $C_{2F} = (N^2 - 1)/2N$ are the Casimir operators of the adjoint and fundamental $SU(N)$ representation respectively, obtained by the relations $T^bT^aT^b = [-1/2C_{2G} + C_{2F}]T^a$ and $f^{abc}T^bT^c = 1/2C_{2G}T^a$.

Before I discuss the results I have to make some comments. The final results of the calculation of the SUSY penguin and self energy diagrams are finite after summation. The infinite terms proportional to $C_{2F}$ and $C_{2G}$ are cancelled seperately. Whereas the $C_{2G}$ infinite terms are cancelled by summation over both penguin diagrams are the $C_{2F}$ ones cancelled by summation over the penguin diagram with two scalars and the self energy diagrams. The calculation for the penguin diagrams with neutralinos in the loop is similar to the $C_{2F}$ terms of the diagrams with the gluino in the loop.

In the literature it often can be found that the infinities are cancelled by the GIM mechanism (that is making use of $\sum_{a=1}^{3} K_{a1}K_{a2} = 0$). I find it not legitimate to do so since the model of the KM matrix is a model within the SM, which is a renormalizable model and therefore its divergencies should be removed by counter terms or by summation of all possible diagrams.

It is also well known that SUSY has new CP violation phases in the supersymmetric breaking sector (gaugino masses, scalar masses, $A, \mu$ term, vev’s etc), which are strongly bounded by the electric dipole moment of the neutron (EDMN) to be of the order of $10^{-2} - 10^{-3}$ [24, 25] or the SUSY masses are heavier than several TeV’s [26]. In eq.(7) the SUSY phase $\Phi_S$ only comes in when $K_{\tilde{N}i}$ is multiplied by $K_{\tilde{R}}^{a, \tilde{N}i}$ (the difference between the left and right part lies in a relative minus sign of the SUSY phase, see eq.(4) in [9]), that is the terms, which are proportional to $K_{\tilde{R}}^{a, \tilde{N}i}$ (the difference between the left and right part lies in a relative minus sign of the SUSY phase, see eq.(4) in [9]), that is the terms, which are proportional to $K_{\tilde{R}}^{a, \tilde{N}i}$. For example for flavour non diagonal couplings of the gluino to the left and right handed down quarks and their superpartners and after summation of the scalar down quark mass eigenstates the gluino mass term of $B_a^g$ in eq.(7) is proportional to $\sin(2\Phi_S)\sin2\Theta_a(G(x_{a1}^g) - G(x_{a2}^g))$ and therefore only contributes for non neglectable mixing of the scalar down quarks and for a non neglectable SUSY phase $\Phi_S$. In this paper I assume that CP violation occurs in the supersymmetric version of the KM matrix as well as in a SUSY phase but take $\sin(2\Phi_S) \leq 10^{-3}$, and the SUSY masses to be lower than the TeV range.

In the SM model we have $\xi_{SM} \approx -f_{c_2^g}s_2^gs_3^s \sin(2\Phi_2^{SM} - A_3^{SM})/c_1c_3(A_1^{SM} - A_2^{SM}) \simeq -4.34 \times 10^{-4}$ for a top quark mass of 180 GeV and the other quark masses.
taken as given above. For the KM angles I take $s_1 = 0.22$, $s_2 = 0.095$, $s_3 = 0.05$ and $\sin \delta = 0.2$ [28], which reproduces approximately the results of [23] (when the quark masses are taken as there and without the factor $f$). As was pointed out there the contribution from $B_\mu^{SM}$ can roughly be neglected even for higher values of the top quark masses and is less than 7% ($\xi_{SM} \simeq -4.69 \times 10^{-4}$ when included, which I will in the final results).

As was pointed out above I assumed that both couplings of the gluino and the neutralinos to the left and right handed down quarks and their superpartners are flavour non diagonal and to be of the same order, that is I take $K_{\tilde{g}abR} \approx K_{\tilde{N}abR} = e^{-i\Phi_S}K_{ab}$ and $K_{\tilde{g}abL} \approx K_{\tilde{N}abL} = e^{+i\Phi_S}K_{ab}$, where $K_{ab}$ is the supersymmetric version of the KM matrix diagonalizing the scalar down quark matrix.

The $\xi$ parameter in SUSY alone has a factor of $\varepsilon^2 \sin \delta_{SKM}$ compared to the SM one of $c_2 s_2 s_3 \sin \delta / c_1 c_3$ leading to the same order as in the SM if $\varepsilon \simeq 0.1$ and $\delta_{SKM} \simeq \delta$, where $\delta_{SKM}$ is the supersymmetric version of the phase of the KM matrix and $\varepsilon = \tilde{s}_a$ with $\tilde{s}_a$ the supersymmetric version of the angles in the KM matrix (see e.g. [15,16,20]). An enhancement of $\varepsilon$ can be compensated by a diminishing of $\delta_{SKM}$. I therefore take the same values for the SUSY KM parameters as in the SM and compare the results obtained from eq.(7) with the one obtained via the SM without the gluino and neutralino contributions.

I include the mixing of all generations of the scalar down quarks since as I have shown in [29] the mixing angles might also become important in the second generation. It can not be neglected here since the finite result in eq.(7) was obtained by an expansion in the down quark masses as shown in Appendix A; furthermore for large values of $\tan \beta \gg 1$ I obtain $\cos^2 \Theta_b \approx 1/\sqrt{2}$.

III. DISCUSSIONS

I now present those contributions for different values of $\tan \beta$ and the symmetry-breaking scale $m_S$. As input parameter I take the quark masses as given above, for $\sin^2 \Theta_W = 0.2323$, $\alpha = 1/137$ and for the strong coupling constant $\alpha_s = 0.1134$. Furthermore for not having too many parameters I use the well known GUT relations $m_{g_1} = \frac{5}{3} m_{g_2} \tan^2 \Theta_W$ and $m_{g_2} = (g_2/g_s)^2 m_{\tilde{g}}$ between the $U(1)$, $SU(2)$ and $SU(3)$ gaugino and gluino masses.

In Fig. 2, I show the ratio of the neutralino contribution and the SM model contribution to the direct CP violation parameter $\xi_{N+SM}/\xi_{SM}$ and compare them with the gluino contribution for three different values of $\tan \beta = 1$, 10 and 50 \footnote{Such high values for $\tan \beta$ are preferred in models, which require the Yukawa couplings $h_t$, $h_b$ and $h_\tau$ to meet at one point at the unification scale [31]} and a gaugino mass of $m_{\tilde{g}} = 200$ GeV. Here I have taken $\sin(2\Phi_S) = 10^{-3}$. The ratio of $\xi_{SM+N}/\xi_{SM}$ corresponds to the ratio $\epsilon^{SM+N}/\epsilon^{SM} \approx (\epsilon'/\epsilon)^{SM+N}/(\epsilon'/\epsilon)^{SM}$ if we assume that the SUSY contribution to $\epsilon_K$ is neglectable.

As we can see the neutralino contribution is more important than the gluino one for small values of $m_S \leq 400$ GeV. For higher values the neutralino contribution
is neglectable compared to the gluino and SM one. In the gluino case the most important term in eq.(7) is the one with the gluino mass \( m_{\tilde{g}} = 722 \text{ GeV} \) with the relation given above for \( \sin(2\Phi_S) \neq 0 \). For \( \sin(2\Phi_S) = 0 \) the gluino contribution is totally neglectable compared to the SM one, whereas the neutralino contribution is almost the same as before. I therefore only present the results with the upper limit for the SUSY CP violating phase. The gluino contribution becomes more important for higher values of \( \tan \beta \), whereas it is the opposite case for the neutralino contribution. For small values of \( m_S \) the SUSY contribution to \( \xi \) even becomes negative.

Smaller values as well as much larger values of the gaugino mass \( m_{g_2} \) in general lead to a gluino contribution almost indistinguishable from the SM one, whereas the neutralino contribution becomes somewhat bigger or smaller dependant on \( \tan \beta \). The shapes of the figure however are not changed. Changing the sign of the \( \mu \) parameter (the bilinear Higgs mass term in the superpotential), which enters the mass matrices of the neutralino and scalar quark masses, affects the sign of the \( \xi \) parameter in the neutralino and gluino case, leading to results above the ratio 1. However here only for large \( \tan \beta \) and for the gluino case the contribution is non neglectable to the SM one. Smaller values of \( c = -1 \) [15, 16] the parameter, which enters in the one loop corrections to the scalar down quark masses affects the results only slightly in the cases presented here.

As a result I have that the neutralino contribution becomes more important than the gluino one for small values of \( m_S \). Unfortunately this is only for a small range of \( m_S \) since for \( m_S \) smaller than about 300 GeV one or both of the lightest mass eigenvalues of the scalar bottom and scalar top quark mass becomes negative and so has to be discarded to avoid colour breaking.

IV. CONCLUSIONS

In this paper I presented the contribution of the neutralinos and gluino to the direct CP violating parameter \( \xi \). I have shown that for small masses of the SUSY breaking scalar mass \( m_S \leq 400 \text{ GeV} \) the neutralino contribution becomes more important than the gluino one. For a soft CP violation phase of zero the gluino contribution is totally neglectable compared to the SM contribution for a high gluino mass, whereas the neutralinos one remains important for a scalar mass smaller than 400 GeV. In the calculation I included the mixing of the neutralinos and of the scalar partner of the left and right handed down quarks.

Although it was generally believed that the SUSY contribution to the indirect CP violating parameter \( \epsilon_K \) can be neglected the authors in [15] found non neglectable interference between penguin and box diagrams for values of \( x = m_{\tilde{g}}^2/m_S^2 \approx 1 \) of the gluino diagrams. Since for a certain range of the parameter \( m_S \) the neutralino penguin diagrams cannot be neglected as I have shown in this paper I conclude that this might be also true for the box diagrams [18]. This is further supported by the fact that in the range mentioned above the neutralino contribution to the mass difference in the \( B_d^0 \) system cannot be neglected compared to the gluino contribution as we have shown in [16].

Finally I want to mention that the chargino and Higgs contribution to \( \epsilon'/\epsilon \) can
enhance the SM by at most 40% as was shown in [14]. For scalar masses $m_S$ higher than 400 GeV their contribution therefore becomes certainly the most important one, whereas for smaller values all particles within the MSSM have to be taken into account.

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VI. APPENDIX A

In this Appendix I present the complete calculations of the Feynman diagrams as shown in Fig.1 with flavour non diagonal couplings of the gluino to the left and right handed quarks and their scalar partners.

The first Feynman diagram of Fig.1 leads to the following expression after dimensional integration:

$$iM_1 = + \frac{2g_s^3}{(4\pi)^2} \left[ -\frac{1}{2} C_{2G} + C_{2F} \right] \int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \left\{ \left[ \frac{1}{\epsilon} - \frac{1}{\epsilon} - \gamma + \log(4\pi\mu^2) - \log(F_{\tilde{g}a_m}) \right] \right. $$

$$\left. \bar{u}d T^a \gamma_\mu (K^* \tilde{g} K^* \tilde{g} \tilde{K}^a_{m1} P_L + K_{a1L} K_{a2L} \tilde{K}^a_{m2} P_R) u_s \right. $$

$$\left. + \bar{u}d T^a \tilde{p}(K^* \tilde{g} K^* \tilde{g} \tilde{K}^a_{m1} P_L + K_{a1L} K_{a2L} \tilde{K}^a_{m2} P_R) \right. $$

$$\left. - m_{\tilde{g}} (K^* \tilde{g} K^* \tilde{g} \tilde{K}^a_{m1} P_L + K_{a1L} K_{a2L} \tilde{K}^a_{m2} P_R) \tilde{K}^a_{m1} \tilde{K}^a_{m2} \right) u_s (p_1 + p_2 - \tilde{p})_{\mu}/F_{\tilde{g}a_m} \right\} \epsilon^\mu_{\tilde{g}} $$

$$\tilde{p} = p_1 \alpha_1 + p_2 \alpha_2 $$

$$F_{\tilde{g}a_m} = m_{\tilde{g}}^2 - (m_{\tilde{g}}^2 - m_{\tilde{a}_m}^2) (\alpha_1 + \alpha_2) - p_1^2 \alpha_1 (1 - \alpha_1 - \alpha_2) - p_2^2 \alpha_2 (1 - \alpha_1 - \alpha_2) $$

$$- q^2 \alpha_1 \alpha_2 $$

$$\epsilon = 2 - d/2 \text{ and } q = p_1 - p_2. $$

For the second diagram in Fig.1 the result is:

$$iM_2 = + \frac{2g_s^3}{(4\pi)^2} \frac{1}{2} C_{2G} \int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \left\{ \left[ \frac{1}{\epsilon} - \frac{1}{\epsilon} - \gamma + \log(4\pi\mu^2) - \log(F_{\tilde{a}m\tilde{g}}) + \frac{m_{\tilde{g}}^2}{F_{\tilde{a}m\tilde{g}}} \right] \right. $$

$$\bar{u}d T^a \gamma_\mu (K^* \tilde{g} K^* \tilde{g} \tilde{K}^a_{m1} P_L + K_{a1L} K_{a2L} \tilde{K}^a_{m2} P_R) u_s \right. $$

$$\left. + \bar{u}d T^a \gamma_\mu (p_1 - \tilde{p})_{\mu}(K^* \tilde{g} K^* \tilde{g} \tilde{K}^a_{m1} P_L + K_{a1L} K_{a2L} \tilde{K}^a_{m2} P_R) \right. $$

$$\left. - m_{\tilde{g}} (p_1 - \tilde{p})_{\mu}(K^* \tilde{g} K^* \tilde{g} \tilde{K}^a_{m1} P_L + K_{a1L} K_{a2L} \tilde{K}^a_{m2} P_R) \tilde{K}^a_{m1} \tilde{K}^a_{m2} \right) $$

$$u_s/F_{\tilde{a}m\tilde{g}} \right\} \epsilon^\mu_{\tilde{g}} $$

(A.1)
Finally for the self energy diagrams the results are:

\[ iM_3 = -\frac{2g^3}{(4\pi)^2}C_{2F} \int_0^1 \left\{ \frac{1}{\epsilon} - \gamma + \log(4\pi \mu^2) - \log(H_{\tilde{g}\tilde{a},m}^{P_2}) \right\} \]

\[ \overline{\sigma}_d T^a \left\{ \phi_2 \alpha_1 (K_{a,1} K_{a,2L} K_{a,2L} P_L + K_{a,1} K_{a,2R} P_R) \right\} \]

\[ - m_{\tilde{g}} (K_{a,1} K_{a,2L} P_L + K_{a,1} K_{a,2R} P_R) \tilde{K}_{a,2L} \tilde{K}_{a,2R} \frac{p_2 + m_s}{p_2^2 - m_s^2} \gamma_{\mu} u_s \epsilon^* \mu \]  

(A.3)

\[ iM_4 = -\frac{2g^3}{(4\pi)^2}C_{2F} \int_0^1 \left\{ \frac{1}{\epsilon} - \gamma + \log(4\pi \mu^2) - \log(H_{\tilde{g}\tilde{a},m}^{P_1}) \right\} \]

\[ \overline{\sigma}_d T^a \left\{ \frac{\phi_1 + m_d}{p_1 - m_d} \left\{ \phi_1 \alpha_1 (K_{a,1} K_{a,2L} K_{a,2L} P_L + K_{a,1} K_{a,2R} P_R) \right\} \right\} \]

\[ - m_{\tilde{g}} (K_{a,1} K_{a,2L} P_L + K_{a,1} K_{a,2R} P_R) \tilde{K}_{a,2L} \tilde{K}_{a,2R} \frac{m_s}{m_s^2 - m_{\tilde{a},m}^2} \left( 1 - \alpha_1 \right) \]  

(A.4)

Neglecting all quark masses (that is with \( p_{1,2}^2 = 0 \)) the \( \gamma_{\mu} \) term is identical to zero after summation over all four terms. For the \( C_{2G} \) the result is:

\[ \int_0^1 d\alpha_1 \int_0^{1 - \alpha_1} d\alpha_2 \left\{ \frac{1}{\epsilon} - 1 - \gamma + \log(4\pi \mu^2) - \log(F_{\tilde{a},\tilde{g}}) + \frac{m_{\tilde{g}}^2}{m_{\tilde{a},m}} - \frac{1}{\epsilon} + \gamma - \log(4\pi \mu^2) + \log(F_{\tilde{g},\tilde{a}}) \right\} = 0 \]  

(A.5)

And for the \( C_{2F} \) term I obtain:

\[ \int_0^1 d\alpha_1 \int_0^{1 - \alpha_1} d\alpha_2 \overline{\sigma}_d T^a \left\{ \phi_2 \alpha_1 (K_{a,1} K_{a,2L} K_{a,2L} P_L + K_{a,1} K_{a,2R} P_R) \right\} \]

\[ - \frac{1}{\epsilon} - \gamma + \log(4\pi \mu^2) - \log(F_{\tilde{g},\tilde{a},m}) \]  

\[ = \int_0^1 d\alpha_1 \overline{\sigma}_d T^a \left\{ \phi_2 \alpha_1 \left( K_{a,1} K_{a,2L} K_{a,2L} P_L + K_{a,1} K_{a,2R} P_R \right) \right\} \frac{p_2 + m_s}{p_2^2 - m_s^2} \gamma_{\mu} \]

\[ + \frac{\phi_1 + m_d}{p_1 - m_d} \left\{ \phi_1 \alpha_1 \left( K_{a,1} K_{a,2L} K_{a,2L} P_L + K_{a,1} K_{a,2R} P_R \right) \right\} \frac{m_s}{m_s^2 - m_{\tilde{a},m}^2} \left( 1 - \alpha_1 \right) \]

\[ \times \alpha_1 \left\{ \frac{1}{\epsilon} - 1 - \gamma + \log(4\pi \mu^2) - \log(H_{\tilde{g},\tilde{a},m}) \right\} = 0 \]  

(A.6)

The term proportional to the gluino mass in eq.(A.3) and eq.(A.4) cancels after summation. To obtain eq.(A.6) and the cancellation of the gluino mass term of the self energy diagrams for zero quark masses we have to use the relations \( \overline{\sigma}_d P_{L,R} \phi_2 = m_d \overline{\sigma}_d P_{R,L} \phi_2 = m_s \overline{\sigma}_d P_{L,R} u_s = m_s P_{R,L} u_s \) and \( \phi_1 \phi_1 + m_d = m_d^2 \). Since the summation of
eq. (A.1-4) gives a zero result we have to expand the functions $F_{\tilde g a m}$, $F_{\tilde a m \tilde g}$ and $H_{\tilde g a m}^{P_q}$ with respect to $p_1^2$, $p_2^2$ and $q^2$. For the further calculation I strongly make use of similar relations as presented in eq. (A.6) and eq. (A.7) in [30]. Furthermore we have:

\[
\overline{\nu}_d \sigma_{\mu \nu} q^\nu (m_d P_L + m_s P_R) u_s = \overline{\nu}_d \{ \hat{p}_1 p_{1\mu} + \hat{p}_2 p_{2\mu} + \hat{p}_1 p_{1\mu} + \hat{p}_2 p_{1\mu} - 2 \hat{p}_2 \gamma_\mu \hat{p}_1 \\
- (p_1^2 + p_2^2) \} P_L u_s \tag{A.7}
\]

\[
\hat{p}_1 \gamma_\mu \hat{p}_2 = 2(\hat{p}_1 p_{2\mu} + \hat{p}_2 p_{1\mu}) + (q^2 - p_1^2 - p_2^2) \gamma_\mu - \hat{p}_2 \gamma_\mu \hat{p}_1 \tag{A.8}
\]

As a final result I obtain:

\[
i M = + \frac{2g_s^3}{(4\pi)^2} \frac{1}{m_{\tilde g}^2} \overline{\nu}_d T^a \left[ \frac{1}{2} C_{2G} M_G + C_{2F} M_F \right] u_s \tag{A.9}
\]

\[
M_G = \int_0^1 d\alpha_1 \left\{ \frac{1}{6} (q^2 \gamma_\mu - \hat{q} q_\mu) [2 - 3\alpha_1^2 - 3\alpha_1 (1 - \alpha_1)] \\
(K^{*\tilde g}_{a1L} K^{\tilde g}_{a2L} \tilde{K}^{a2}_{m1} P_L + K^{*\tilde g}_{a1R} K^{\tilde g}_{a2R} \tilde{K}^{a2}_{m2} P_R) \\
- i\sigma_{\mu \nu} q^\nu \frac{1}{2} \alpha_1 (1 - \alpha_1) [m_d (K^{*\tilde g}_{a1L} K^{\tilde g}_{a2L} \tilde{K}^{a2}_{m1} P_L + K^{*\tilde g}_{a1R} K^{\tilde g}_{a2R} \tilde{K}^{a2}_{m2} P_R) \\
+ m_s (K^{*\tilde g}_{a1L} K^{\tilde g}_{a2L} \tilde{K}^{a2}_{m1} P_R + K^{*\tilde g}_{a1R} K^{\tilde g}_{a2R} \tilde{K}^{a2}_{m2} P_L)] \\
+ i\sigma_{\mu \nu} q^\nu (1 - \alpha_1) m_{\tilde g} (K^{*\tilde g}_{a1L} K^{\tilde g}_{a2L} P_L + K^{*\tilde g}_{a1L} K^{\tilde g}_{a2R} P_R) \tilde{K}^{a1}_{m1} \tilde{K}^{a1}_{m2} / D_{\tilde g a m}
\right\}
\]

\[
M_F = \int_0^1 d\alpha_1 \left\{ \frac{1}{6} (q^2 \gamma_\mu - \hat{q} q_\mu) (K^{*\tilde g}_{a1L} K^{\tilde g}_{a2L} \tilde{K}^{a2}_{m1} P_L + K^{*\tilde g}_{a1R} K^{\tilde g}_{a2R} \tilde{K}^{a2}_{m2} P_R) \alpha_1^3 \\
+ i\sigma_{\mu \nu} q^\nu \frac{1}{2} \alpha_1^2 (1 - \alpha_1) [m_d (K^{*\tilde g}_{a1L} K^{\tilde g}_{a2L} \tilde{K}^{a2}_{m1} P_L + K^{*\tilde g}_{a1R} K^{\tilde g}_{a2R} \tilde{K}^{a2}_{m2} P_R) \\
+ m_s (K^{*\tilde g}_{a1L} K^{\tilde g}_{a2L} \tilde{K}^{a2}_{m1} P_R + K^{*\tilde g}_{a1R} K^{\tilde g}_{a2R} \tilde{K}^{a2}_{m2} P_L)] \\
- i\sigma_{\mu \nu} q^\nu \alpha_1 (1 - \alpha_1) m_{\tilde g} (K^{*\tilde g}_{a1L} K^{\tilde g}_{a2L} P_L + K^{*\tilde g}_{a1R} K^{\tilde g}_{a2R} P_R) \tilde{K}^{a1}_{m1} \tilde{K}^{a1}_{m2} / D_{\tilde g a m}
\right\}
\]

\[
D_{\tilde g a m} = 1 - (1 - x_{\tilde a m}^\tilde g) \alpha_1
\]

\[
x_{\tilde a m}^\tilde g = \frac{m_{\tilde a m}^2}{m_{\tilde g}^2}
\]

After Feynman integration eq. (A.7) leads to the results presented in eq. (7). The final functions after Feynman integration are shown in Appendix B. Up to a relative minus sign \(^3\) of the gluino mass the results presented in [9] are reproduced by the following replacements: $A_G = A/3$, $A_F = B/6$, $B_G = C$,

\(^3\) The relative minus sign is the relative sign between the couplings of the gluino to the left handed down quarks and their superpartners as explained in eq. (C89) in [8], which was not taken into account in [9].
$B_F = D/2$, $\tilde{B}_G = E$ and $\tilde{B}_F = 2C$, where $A_G$, $A_F$, $B_G$, $B_F$, $\tilde{B}_G$, $\tilde{B}_F$ are given in eq.(B.1-6) and $A$, $B$, $C$, $D$ and $E$ are the functions given in eq.11 of [9]. I therefore cannot confirm the statement of [17], that the authors in [9] neglected a crucial term. The function $F(x_j)$ of eq.7 in [17] is reproduced after Feynman integration of the term $2 - 3\alpha_1^2$ in $A_G$ however they omitted the term $-3\alpha_1(1-\alpha_1)$ there, which cancels the $\alpha_1^2$ leaving the term $2 - 3\alpha_1$ and thus reproducing the function $A$ of [9].

VI. APPENDIX B

$$A_G(x_{\tilde{a}m}) = \frac{1}{6} \int_0^1 d\alpha_1 \left[ 2 - 3\alpha_1^2 - 3\alpha_1(1-\alpha_1) \right]/D_{\tilde{g}\tilde{a}_m}$$

$$= \frac{1}{(1 - x_{\tilde{a}m})^2} \left\{ 1 - x_{\tilde{a}m} + \frac{1}{3}(1 + 2x_{\tilde{a}m}) \log(x_{\tilde{a}m}) \right\} \quad (B.1)$$

$$A_F(x_{\tilde{a}m}) = \frac{1}{6} \int_0^1 d\alpha_1 \alpha_1^3/D_{\tilde{g}\tilde{a}_m}$$

$$= \frac{1}{(1 - x_{\tilde{a}m})^4} \left\{ -11 + 18x_{\tilde{a}m} - 9x_{\tilde{a}m}^2 + 2x_{\tilde{a}m}^3 - 6 \log(x_{\tilde{a}m}) \right\} \quad (B.2)$$

$$B_G(x_{\tilde{a}m}) = \frac{1}{2} \int_0^1 d\alpha_1 (1 - \alpha_1)/D_{\tilde{g}\tilde{a}_m}$$

$$= \frac{1}{(1 - x_{\tilde{a}m})^3} \left\{ 1 - x_{\tilde{a}m}^2 + 2x_{\tilde{a}m} \log(x_{\tilde{a}m}) \right\} \quad (B.3)$$

$$B_F(x_{\tilde{a}m}) = \frac{1}{2} \int_0^1 d\alpha_1 \alpha_1^2(1 - \alpha_1)/D_{\tilde{g}\tilde{a}_m}$$

$$= \frac{1}{(1 - x_{\tilde{a}m})^4} \left\{ 2 + 3x_{\tilde{a}m} - 6x_{\tilde{a}m}^2 + x_{\tilde{a}m}^3 + 6x_{\tilde{a}m} \log(x_{\tilde{a}m}) \right\} \quad (B.4)$$

$$\tilde{B}_G(x_{\tilde{a}m}) = \int_0^1 d\alpha_1 (1 - \alpha_1)/D_{\tilde{g}\tilde{a}_m} = \frac{1}{(1 - x_{\tilde{a}m})^2} \left\{ 1 - x_{\tilde{a}m}^2 + x_{\tilde{a}m} \log(x_{\tilde{a}m}) \right\} \quad (B.5)$$

$$\tilde{B}_F(x_{\tilde{a}m}) = 2B_G(x_{\tilde{a}m}) \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (B.6)$$

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FIGURE CAPTIONS
Fig.1 The penguin diagrams with scalar down quarks, gluino and neutralinos within the loop.

Fig.2 The ratios $\xi^{SM+\tilde{N}}/\xi^{SM}$ and $\xi^{SM+\tilde{g}}/\xi^{SM}$ for different values of $\tan\beta = 1$ (solid line), $\tan\beta = 10$ (dashed line) and $\tan\beta = 50$ (dot-dashed line) as a function of $m_S$. $m_{g_2} = 200$ GeV ($m_{\tilde{g}} = 722$ GeV) and $\mu = 300$ GeV. The soft SUSY CP violating phase I took at its upper limit of $\sin(2\Phi_S) = 10^{-3}$. The lower curves for $m_S = 300$ GeV are for the neutralino contribution (except for $\tan\beta = 50$, here the neutralino one is the one closer to 1).
This figure "fig1-1.png" is available in "png" format from:

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