Coherence and the Day - Night Asymmetry in the Solar Neutrino Flux

Amol S. Dighe\textsuperscript{1}, Q. Y. Liu\textsuperscript{1}, Alexei Yu. Smirnov\textsuperscript{1,2}

\textsuperscript{1} The Abdus Salam International Center for Theoretical Physics, Strada Costiera 11, 34100, Trieste, Italy.
\textsuperscript{2} Institute for Nuclear Research, RAN, Moscow, Russia.

Abstract

We consider the day-night asymmetries predicted by MSW solutions of the solar neutrino problem. The integration over the neutrino energy, as well as over the production region or over the time intervals of more than a day leads to the averaging of oscillations on the way to the earth. This is equivalent to considering the neutrino state arriving at the surface of the earth as an \textit{incoherent} mixture of the neutrino mass eigenstates (even if there is no divergence of wavepackets). As a consequence, the $\nu_e$-regeneration effect inside the earth is incoherent, in contrast with the result in hep-ph/9902435.

1. The day-night effect – difference in the $\nu_e$ fluxes during the day and the night due to the earth matter – is one of the key signatures of the MSW solutions of the solar neutrino problem \cite{1}. (The effect is negligible for the vacuum oscillation solution.) Moreover, the zenith angle distribution of the events during the night is different for the large mixing angle (LMA) and the small mixing angle (SMA) solutions. (For recent discussions, see \cite{2}.) Therefore, the day-night asymmetry and the zenith angle dependence of the flux, if established, will not only prove the MSW solution, but also distinguish among the different MSW solutions.

Existing Super-Kamiokande (SK) results on the day-night asymmetry and the zenith angle dependence \cite{3} already exclude a significant range of the parameters $\Delta m^2$ and $\sin^2 2\theta$ of the LMA solution and some part of the SMA solution \cite{4,5}. SK results after 708 days show a 6\% excess of the number of events during the night \cite{6}. This indication of the day-night effect is not yet statistically significant (1.3$\sigma$).
The earth matter effect has been calculated, taking into account the divergence of the neutrino wave packet \cite{1} and the averaging effects \cite{2, 3, 4}. The neutrino state arriving at the earth was considered to be an incoherent mixture of the mass eigenstates $\nu_1$ and $\nu_2$. The calculations give specific zenith angle ($\Theta_z$) distributions of the number of events during the night, depending on the solution. In the case of the SMA solution, the excess during the night is expected to be in the largest zenith angle bin $\cos \Theta_z = -1.0 \div -0.8$, when the neutrinos cross both the mantle and the core of the earth \cite{2, 8}. In the case of the LMA solution, the excess of events will be distributed nearly uniformly in all the night bins. Existing SK data \cite{3} shows that the excess is not concentrated in the largest $\Theta_z$ bin, but is distributed in all the bins, thus indicating towards the LMA solution.

In a recent paper \cite{9}, the beam of neutrinos arriving at the earth was taken to be in a coherent state so that the phase between the flavor eigenstates is independent of the neutrino energy, the point of production, or the time of the year. This leads to a significant (a factor of 6) enhancement of the day-night asymmetry and to the modification of the zenith (nadir) angle distribution of the events. (For the SMA solution, the distribution is predicted to be more uniform, with a wide maximum in the bins $\cos \Theta_z = -0.8 \div -0.4$.) Clearly, this would significantly change the implications of the SK data.

In this paper, we show that due to averaging, the coherence is effectively lost and the results of \cite{9} are not valid.

The need to take the averaging effects into account has already been pointed out in \cite{6, 7, 10}. Here we shall explicitly show where and how the averaging occurs and discuss the conditions that need to be satisfied in order to observe the coherence effects, at least in principle.

2. For simplicity, let us consider the 2-neutrino mixing case:

$$
\nu_e = \cos \theta \; \nu_1 + \sin \theta \; \nu_2, \quad \nu_\mu = \sin \theta \; \nu_1 - \cos \theta \; \nu_2,
$$

where $\theta$ is the mixing angle in vacuum. (We will also use the notation $c \equiv \cos \theta$ and $s \equiv \sin \theta$.) In general, the neutrino state at the surface of the sun can be written in terms of neutrino mass eigenstates as

$$
\nu_\odot = a_1 \; \nu_1 + a_2 \; \nu_2 \; e^{i \phi_S},
$$

where $a_1$ and $a_2$ are the absolute values of the amplitudes of $\nu_e \rightarrow \nu_1$ and $\nu_e \rightarrow \nu_2$ transitions inside the sun respectively. They can be expressed in terms of $P_\odot \equiv \overline{P(\nu_e \rightarrow \nu_e)}$, the averaged survival probability of $\nu_e$ inside the
sun. Indeed, according to (1) and (2), the amplitude of probability to find \( \nu_e \) at the surface of the sun equals
\[
\langle \nu_e | \nu_\odot \rangle = a_1 \cos \theta + a_2 \sin \theta e^{i\phi_S}.
\] (3)

Then
\[
P_\odot \equiv |\langle \nu_e | \nu_\odot \rangle|^2 = a_1^2 \cos^2 \theta + a_2^2 \sin^2 \theta,
\] (4)
where averaging over the phase \( \phi_S \) has been done. From (4), we find
\[
a_2^2 \equiv 1 - a_1^2 = \frac{\cos^2 \theta - P_\odot \cos 2\theta}{\cos 2\theta}.
\] (5)

Let us assume that there is no loss of coherence due to the spread of the wave packet on the way to the earth. (This loss of coherence would further dilute the interference effects \([3, 9]\), but we shall not consider it here.) Then, on the way from the sun to the earth, the state (2) evolves to
\[
\nu_E = a_1 \nu_1 + a_2 \nu_2 e^{i(\phi_S + \varphi)},
\] (6)
where
\[
\varphi = 2\pi \frac{L}{\ell_\nu} = \frac{L \Delta m^2}{2E},
\] (7)
is the (relative) phase acquired by the mass eigenstates on the way from the sun to the earth. Here \( L \approx 1.5 \times 10^{13} \) cm is the distance between the sun and the earth.

The key point is that, for the MSW solutions of the solar neutrino problem, the distance \( L \) is much greater than the oscillation length \( \ell_\nu \). Indeed, for the SMA solution characterized by \( \Delta m^2 \approx 5 \times 10^{-6} \) eV\(^2\), and for the typical energy of Boron neutrinos (\( E \sim 10 \) MeV), we get \( \ell_\nu \approx 5 \times 10^8 \) cm, so that
\[
\frac{\ell_\nu}{L} \sim 3 \times 10^{-5}.
\] (8)
For the LMA solution (\( \Delta m^2 \approx 2 \times 10^{-5} \) eV\(^2\)) this ratio is even smaller : \( \ell_\nu/L \sim 10^{-5} \). For the low \( \Delta m^2 \) solution (\( \Delta m^2 \sim 10^{-7} \) eV\(^2\)) this ratio is \( \ell_\nu/L \sim 1.5 \times 10^{-3} \), still quite small.

From (7), we find that the phase \( \varphi \) varies with the energy as
\[
\Delta \varphi = \frac{L \Delta m^2}{2E} \frac{\Delta E}{E} = 2\pi \frac{L}{\ell_\nu} \cdot \frac{\Delta E}{E}.
\] (9)
Therefore, the change in energy as small as
\[
\frac{\Delta E}{E} \approx \frac{\ell_\nu}{L}
\]
leads to the change in the oscillation phase by $2\pi$. As a consequence, the integration over even small intervals of neutrino energy will lead to the averaging of oscillations and the observed effect will be equivalent to that due to an incoherent mixture of $\nu_1$ and $\nu_2$.

3. Let us consider this in more details. Denoting the amplitudes of $\nu_1 \to \nu_e$ and $\nu_2 \to \nu_e$ transitions inside the earth by $b_{e1}$ and $b_{e2}$ respectively, we can write the amplitude for finding $\nu_e$ in the detector as

$$\langle \nu_e | \nu_D \rangle = a_1 b_{e1} + a_2 b_{e2} e^{i(\phi_S + \phi)} ,$$

(10)

then the probability of observing $\nu_e$ at the detector is

$$P = a_1^2 |b_{e1}|^2 + a_2^2 |b_{e2}|^2 + 2 \text{Re}[a_1 a_2^* b_{e1} b_{e2} e^{i(\phi_S + \phi)}] .$$

(11)

Introducing $P_{2e} \equiv |b_{e2}|^2$ and using (5), we get

$$P = \frac{P_\odot - s^2 + P_{2e}(1 - 2P_\odot)}{\cos 2\theta} + P_{coh}(E) ,$$

(12)

where

$$P_{coh} = \frac{1}{\cos 2\theta} \sqrt{(P_\odot - s^2)(c^2 - P_\odot)P_{2e}(1 - P_{2e})} \cos(\phi_S + \phi_E + \phi) ,$$

(13)

with $\phi_E = \text{Arg}(b_{e1}^* b_{e2})$. The first term in (12) represents the incoherent part of the probability and it coincides with the one used in the literature [6,7,2]. The second term $P_{coh}$ corresponds to the interference contribution, which appears in the case of coherence.

Let us consider the observable effects of $P_{coh}$. In the case of the SK detector, the number of events due to this “coherent” term is given by

$$N_e(E_e) \propto \int dE_\nu' f(E_e, E'_e) \int_{E_e}^\infty dE_\nu \frac{d\sigma(E_\nu, E'_e)}{dE_\nu} F(E_\nu)P_{coh}(E_\nu) ,$$

(14)

where $F(E_\nu)$ is the original flux of neutrinos at the detector without oscillations and $f(E_e, E'_e)$ is the (electron) energy resolution function of the detector.

Let us consider the integration over the neutrino energy:

$$I \equiv \int dE_\nu \cdot K(E_\nu) \cdot \cos[\phi_S(E_\nu) + \phi_E(E_\nu) + \phi(E_\nu)] ,$$

(15)

where

$$K(E_\nu) \equiv \frac{d\sigma(E_\nu, E'_e)}{dE_\nu} F(E_\nu) \frac{1}{\cos 2\theta} \sqrt{(P_\odot - s^2)(c^2 - P_\odot)P_{2e}(1 - P_{2e})} .$$

(16)
The crucial point is that $K(E)\nu)$ is a slowly varying function of energy as compared to $\phi(E)\nu)$. Indeed, the typical scale of the change of cross-section, flux and probability in (15) is $O(\Delta E/E) \sim O(1)$. In particular, the variation in the probability $P_{2e}$ is of the order of 1 over an interval of $\Delta E/E \sim \ell_\nu/(2R_E)$, where $R_E$ is the radius of the earth, and the latter ratio is $\sim 1$.

The integration of the smooth function $K(E)\nu)$ with the rapidly oscillating function $\cos(\phi_S + \phi_E + \varphi)$ gives a result very close to zero. Indeed, the integration in (15) can be performed first over small intervals $\Delta E$ and then the results can be summed:

$$I \approx \sum_{k=0}^{k_{max}} \int_{E_0+k\Delta E}^{E_0+(k+1)\Delta E} dE_\nu K(E_\nu) \cdot \cos[\phi_S(E_\nu) + \phi_E(E_\nu) + \varphi(E_\nu)] ,$$

where $E_0 = E'_\nu$. Choosing the interval $\Delta E$ such that $L \ll \Delta E \ll 1$, so that the change of $K(E)$ over this interval can be neglected, we can write

$$I \approx \sum_k K(E + k\Delta E) \int_{\Delta E} dE \cos(\phi_S + \phi_E + \varphi)$$

$$\approx \int dE' K(E') \frac{1}{\Delta E} \int_{\Delta E} dE \cos(\phi_S + \phi_E + \varphi)$$

$$\approx \frac{1}{2\pi} \frac{\bar{E}}{\Delta E} \frac{\ell_\nu}{L} \cdot \int dE' K(E') ,$$

where $\bar{E}$ is the typical energy in the interval of integration. The contribution of the coherent part of the integral in comparison with the incoherent part can thus be estimated as

$$\frac{N^{coh}}{N^{incoh}} \sim \frac{1}{\Delta E} \frac{\bar{E}}{L} \frac{\ell_\nu}{L} .$$

Taking $\frac{\Delta E}{\bar{E}} \sim 0.1$ we conclude, that to avoid the averaging, one should measure the energy of the neutrino with an accuracy of better than $\sim 10^{-5}$, which looks practically impossible.

In the case of the LMA solution, the Boron neutrino appears at the surface of the sun as a mass eigenstate $\nu_2$, which corresponds to $a_1 = 0$, and the regeneration in the earth is described by $P_{2e}$ only, without any additional interference effects.

In the case of the low $\Delta m^2$ solution, the high energy part of the Boron neutrino spectrum can be at the non-adiabatic edge, which means that $a_1 \neq 0$, though it is rather small. Again, we need to measure the energy with an accuracy better than $\ell_\nu/L \sim 10^{-3}$ to disentangle the coherence effects.
4. Notice that even for a fixed energy, an “averaging” occurs due to our lack of knowledge about the neutrino production point in the production region. The size of the production region for Boron neutrinos inside the sun is nearly $0.1 R_\odot = 7 \cdot 10^9$ cm, an order of magnitude larger than the oscillation length in vacuum.

One can estimate the averaging effects due to the production of neutrinos from different points of the solar disc in the following way. For a small mixing angle, the transitions occur in a very thin resonance layer. Before the neutrinos enter this layer, their oscillations are strongly suppressed due to the large density, and after they exit the layer, one can consider just vacuum oscillations. For the SMA solution, the resonance layer is situated near $R \sim 0.2 R_\odot$. The difference in the path length of neutrinos produced in different points of the solar disc after they have crossed the resonance layer is thus $\Delta L \sim 0.1 R_\odot$ and the averaging occurs since

$$\Delta \varphi = \frac{\Delta m^2}{2 E} \Delta L = 2 \pi \frac{\Delta L}{\ell_\nu}$$

(20)

and $\Delta L/\ell_\nu \gg 1$ for the SMA and LMA solutions. For the small $\Delta m^2$ solution, the effect due to this averaging will be small.

Note that in this approach, no averaging occurs due to the production of neutrinos along the line of sight.

Another origin of averaging is related to the eccentricity of the orbit of the earth. Indeed, $\varphi$ is also a function of the time $t$ of the year, with

$$\Delta \varphi \sim \frac{\Delta L}{\ell_\nu} \left( \frac{\Delta t}{1 \text{ year}} \right),$$

(21)

where $\Delta L$ is the variation in the distance from the sun to the earth over one year. $\Delta L \approx 5 \times 10^{11}$ cm so that $\Delta L/\ell_\nu \sim 10^3$ for the SMA and LMA solutions. The integration over time $t$ of the year is then reduced to the integration over the rapidly oscillating function

$$\int dt \cos[\phi_S + \phi_E + \varphi(t)].$$

The integration of the number of events over a period of more than one day already leads to the averaging out of the interference term (for the SMA and LMA solutions). If the events are binned in time with $\Delta t \lesssim (\ell_\nu/\Delta L)$ year, this averaging can be avoided.
5. The state which arrives at the earth can be written in terms of flavor eigenstates as

\[
\nu_E = \frac{1}{\sqrt{\cos 2\theta}} \times \left[ \left( c\sqrt{P_\odot - s^2} + s\sqrt{c^2 - P_\odot} \ e^{i(\phi_S + \phi)} \right) \nu_e \\
+ \left( s\sqrt{P_\odot - s^2} - c\sqrt{c^2 - P_\odot} \ e^{i(\phi_S + \phi)} \right) \nu_\mu \right].
\]

(22)

In the paper [4], the state which arrives at the earth has been taken as

\[
\nu_E = \sqrt{P_\odot} \nu_e + \sqrt{1 - P_\odot} \nu_\mu,
\]

(23)

so that the oscillation effects on the way from the sun to the earth have been overlooked. Moreover, the phase between the flavor eigenstates \(\nu_e\) and \(\nu_\mu\) has been fixed independent of the production point and the energy of the neutrino. This coincides with the correct equation (22), only with either (i) \(s = 0\) and \(\phi_S + \phi = \pi\), or (ii) \(P_\odot = s^2\), which corresponds to the case when the transitions inside the sun are completely adiabatic and the state arriving at the earth is pure \(\nu_2\).

For a correct description, the coherence effects should be discussed in terms of the mass eigenstates.

6. To summarize, the relative phase between the neutrino mass eigenstates arriving at the earth depends strongly on (i) the energy of the neutrino, (ii) its production point and (iii) the earth-sun distance. With a finite uncertainty in the measurements of these quantities, the coherence effects due to this relative phase get averaged out. In order to be able to observe these coherence effects, one needs to (i) measure the energy of the neutrino to an accuracy of \(\Delta E/E \lesssim 10^{-5}\), (ii) determine the direction of the neutrino to an accuracy of better than \(1/10^{th}\) the size of the disc of the sun and in addition, have some detailed knowledge about the production region, and (iii) bin the events in time with \(\Delta t\) of the order of a few hours.

In all the practical cases, one can use the expression (12) without the \(P_{\text{coh}}\) term, which corresponds to an incoherent mixture of the mass eigenstates. The calculations of the regeneration effect which neglect the coherence are then valid.

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