Torsional Behaviour of Irregular Buildings with Single Eccentricity

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Abstract. Torsional response in building structural systems arise from two sources: a) eccentricity between the centre of mass and centre of stiffness of vertical lateral force resisting elements and (b) accidental torsion due to the rotational component of ground motion about a vertical axis, the eccentricity between assumed and actual stiffness and mass, uncertainty in live load distribution, and uncertainty in dead loads due to variations in labour and materials. This paper addresses the effect of the ratio $\Omega$ between structure uncoupled torsional frequency ($\omega_\theta$) to uncoupled transition frequency ($\omega_c$ or $\omega_b$), on the torsional behaviour of the structure. A parametric study is carried out using the equivalent static force (ELF) method applied to one story building with single eccentricity by examining different values of $\Omega$. The study shows that structures with ratio $\Omega > 1.0$ are torsionally stiff and displacements values are not sensitive to the increment in eccentricity ratio, in other words, the structure shows planer frame behaviour and the deformations are mainly transitional rather than torsional. Conversely structures with ratio $\Omega < 1.0$ are torsionally flexible and displacements values are sensitive to increases in eccentricity ratio, therefore, the deformations are mainly torsionally twisting rather than transitional motion. It is also observed that when $\Omega = 1.0$, no significant increase in torsional response occurs. Torsional irregularities coefficient as defined by ASCE 7-10 building code, i.e., the ratio between maximum story drift at one end of the structure to the average story drifts at the two ends of the structure, have been calculated for different values of the ratio $\Omega$. Results shows that irregularity coefficient increases as the ratio $\Omega$ decreases. In addition, structures with $\Omega < 1.0$ are more sensitive to the value of eccentricity between centre of mass and centre of rigidity. Subsequently irregularity coefficient increases dramatically, and excessive torsion occurs as the value of eccentricity increases for structures with ratio $\Omega < 1.0$, while minor increment is observed on the value of irregularity coefficient when eccentricity increases. This paper discusses the significance of utilizing the frequency ratio $\Omega$ to control the torsional behaviour of building structures.

1. Introduction

Structures subjected to earthquake excitations experience lateral deformations that may be accompanied with torsional rotations. The magnitudes of lateral deflections are related to the stiffness and type of the structural system, magnitude and distribution of mass the structure, and mechanical properties of the structural materials. Irregularities in the structural system may amplify structural response leading to significantly higher damage compared to regular structures. Torsional irregularity, in particular, may amplify responses causing severe component or system damage. Structurally irregular buildings for instance may experience different drifts at adjacent stories coupled with excessive torsion. In order to
better understand the response of irregular structures, we define the parameter \( \Omega \) as the ratio between the rotational deformation \( \omega_\theta \) and translational frequency \( \omega_x \), as shown in Equation (1)

\[
\Omega = \frac{\omega_\theta}{\omega_x} \quad \text{or} \quad \Omega = \frac{\omega_\theta}{\omega_y}
\]  

(1)

The purpose of this study is to investigate the effect of the ratio of \( \Omega \) on the response of torsionally irregular structure with single eccentricity. To accomplish this aim, a parametric study has been carried out on one story case study building, subjected to various values of the two uncoupled frequency ratio \( \Omega \) and applied through horizontal ground motion in one direction perpendicular to the direction of eccentricity. The case study structure is symmetrical with respect to one plan view axis (x-axis). In the perpendicular axis (y-axis), the stiffness distribution is varied to generate various scenarios of the structure and different values of \( \Omega \). Therefore, the response of the structure will be examined for the lateral loading in one direct (y-direction). The displacements of the rigid diaphragm edge points are determined for various ratios of \( \Omega \) and the torsional irregularity factor, as defined by ASCE 7-10 code, is calculated and compared to the corresponding \( \Omega \) value using equivalent lateral force (ELF) method.

2. Torsional Vibration

In building structures, earthquake forces are assumed to act at the centre of mass of the floor while lateral force resistance acts at the centre of rigidity of the floor as determined by the layout of the rigid elements designed to resist lateral forces. Several studies reported demand imposed by torsion on concrete structures and the effect of building irregularities [1, 2]. In seismic response studies it is commonly assumed that all points of at building foundation are excited simultaneously. Therefore, if the centres of mass coincide with centres of rigidity of the floor diaphragms, a horizontal component of ground motion will induce translational motion without rotation. However, if the centres of mass and centre of rigidity do not coincide, a horizontal component of earthquake ground motion will generally induce both translational and rotational response about a vertical axis. Figure 1a shows a diaphragm where the centre of mass and centre of rigidity do not coincide and Figure 1b shows the diaphragm where the centre of mass and centre of rigidity are at the same location. It should not be forgotten that careful selection of location of vertical lateral force resisting system to coincide with centre of mass, significantly decreases torsional effects [3]. Seismic torsional response may lead to amplification of displacement responses and possibly damage at the perimeter elements of lateral force resisting system of a torsionally non-symmetric building systems. To account for potential amplification in seismic response due to torsion eccentricities of seismic forces are usually prescribed by building codes using static eccentricity. Further discussions on development and effect of equivalent static eccentricities of seismic forces can be found in the literature [4, 5].

![Figure 1](image_url)

Figure 1. (a) Eccentric Structure System. (a) Regular Structure System [9]
Eccentricity between the centre of mass and centre of rigidity in any particular diaphragm may be caused by non-symmetric distribution of stiff lateral force resisting elements with respect to the centre of gravity of the floor, or existence of large masses unsymmetrically with respect to the centre of rigidity. Twisting due to torsion, as shown in figure 2, may lead to total collapse as shown in figure 3.

![Building Torsional Vibration](image1)

**Figure 2.** Building Torsional Vibration. [9]

![Damage to buildings subjected to Earthquakes](image2)

**Figure 3.** Damage to buildings subjected to Earthquakes. [6]

### 2.1. Torsional provisions in seismic codes and standards

Seismic provisions in building codes typically define the design torsional moment at each storey as a product of the storey shear times a prescribed eccentricity. The code-prescribed eccentricities are presumed to exist between the centre of rigidity and the centre of mass. A static analysis of the structure is conducted for the applied story shears provides the design forces in the various elements of the structure. The code-prescribed eccentricities often include a multiplier intended to account for possible dynamic amplification of torsion, in addition to allowance for accidental torsion. Such torsion is caused by the rotational component of the ground motion and by possible eccentricity between the centre of rigidity and centre mass from their calculated positions [6].

Torsional irregularity problems were addressed in various Federal Emergency Management Agency (FEMA) publications as described by Mohamed and Khamwan [7]. ASCE 7-10 [8] requires consideration of amplifications in response due to torsional irregularities through the factor given by Equation (2) that are applied to accidental lateral load eccentricities of ±5%.

\[
A_x = \left( \frac{\delta_{\text{max}}}{1.2 \delta_{\text{avg}}} \right)^2
\]

Where, \( \delta_{\text{max}} \) is the maximum displacement at story level x and \( \delta_{\text{avg}} \) is the average of the displacements at the extreme points of the structure at story level x, respectively, computed by assuming \( A_x = 1 \). Maximum, minimum and average lateral displacements story level x are shown in ‘figure 4’.

The torsional amplification factor \( A_x \) shall not be less than 1 and is not required to exceed 3.0.

Alternatively we write the torsional irregularity coefficient \( \eta_t = \frac{\delta_{\text{max}}}{\delta_{\text{avg}}} \), such that:

- If \( \eta_t < 1.2 \), torsional irregularity does not exist, i.e., \( A_x = 1.0 \).
- If \( 1.2 < \eta_t < 2.083 \), then torsional irregularity exists and eccentricity amplification factor is computed by, \( A_x = \left( \frac{\eta_t}{1.2} \right)^2 \).
- If \( \eta_t > 2.083 \), then \( A_x = 3.0 \).
3. Free Vibration Uncoupled Frequency

The equation of equation for undamped free vibration of a structure including the eccentricity between the centers of mass and rigidity is described by Equation 3a, bb, and 3c [6].

\[ m\ddot{x} + K_x x = 0 \]  \hspace{1cm} (3a)
\[ m\ddot{y} - e_x \dot{\theta} + K_y y = 0 \]  \hspace{1cm} (3b)
\[ I_m \ddot{\theta} + K_\theta \dot{\theta} - m e_x (\ddot{y} - e_x \dot{\theta}) + m e_y (\ddot{x} + e_y \dot{\theta}) = 0 \]  \hspace{1cm} (3c)

Where,
- \( K_x = \sum_{i=1}^{n} k_{ix} \): total transitional stiffness in x-direction (\( n \) = number of vertical lateral force resisting elements in x direction),
- \( K_y = \sum_{j=1}^{m} k_{jy} \): total transitional stiffness in y-direction (\( m \) = number of vertical lateral force resisting elements in y direction),
- \( K_\theta = \sum_{i=1}^{n} k_{ix} l_i^2 + \sum_{j=1}^{m} k_{jy} l_j^2 \) is total rotational stiffness; \( m \) is the total mass; \( l_m \) is the mass moment of inertia of the system around center of mass (CM); and \( l_{iy} \) and \( l_{ix} \), are the distances of \( i^{th} \) and \( j^{th} \) resisting element from the center of rigidity along x and y axes, as shown in ‘figure 5’.

![Figure 5. Model of system of double eccentricity. [6]](image-url)
For free vibration analysis, the solution of Equations (3) can be described by Equation (b)

\[ x = X \sin(\omega t) \]  
\[ y = Y \sin(\omega t) \]  
\[ \Theta = \Theta \sin(\omega t) \]

where \( X, Y \) and \( \Theta \) are the displacements amplitudes in \( x, y \) and \( \Theta \) directions, respectively. The value of \( \omega \) is the circular natural frequency. Substitution of Equation (4) into Equation (3) gives Equation (5).

\[ (-m\omega^2 + K_x)X - m\omega^2 e_y \Theta = 0 \]  
\[ (-m\omega^2 + K_y)Y + m\omega^2 e_x \Theta = 0 \]  
\[ (- (I_m + m e_x^2 + m e_y^2) \omega^2 + K_\Theta) \Theta - m\omega^2 e_x X + m\omega^2 e_y Y = 0 \]

Equations (5) have a nontrivial solution only if the determinant of the coefficients of \( X, Y \) and \( \Theta \) are equal to zero. This condition yields the characteristic equation of describing such a system may be expressed by Equation (6).

\[ \omega^6 - \left[ \frac{K_x + K_y}{m} + \frac{K_x e_x^2 + K_x e_y^2}{I_m} \right] \omega^4 + \left[ \frac{K_x K_y}{m^2} + \frac{K_x (K_x + K_y) + K_x e_x^2}{m I_m} + \frac{K_x e_x^2}{I_m^2} \right] \omega^2 - \frac{K_x K_y K_\Theta}{m^2 I_m} = 0 \]

where, \( e_x \) is the static eccentricity (eccentricity between mass and rigidity centers) in the \( x \)-direction and \( e_y \) is the static eccentricity in the \( y \)-direction. Now considering the following expressions,

\[ \omega_x^2 = \frac{K_x}{m} \quad \omega_y^2 = \frac{K_y}{m} \quad \omega_\Theta^2 = \frac{K_\Theta}{I_m} \]
\[ e_x = \frac{e_x}{r_m} \quad e_y = \frac{e_y}{r_m} \quad c = \sqrt{1 + e_x^2 + e_y^2} \]
\[ \varepsilon = \frac{e}{r_m} \quad \varepsilon = \sqrt{e_x^2 + e_y^2} \]
\[ I_m = m r_m^2 \]

Hence, Equations (6) can be written as Equation (8).

\[ \left( \frac{\omega}{\omega_x} \right)^6 - \left[ 1 + e_x^2 + \left( \frac{\omega_y}{\omega_x} \right) (1 + e_y^2) + \left( \frac{\omega_y}{\omega_x} \right)^2 \right] \left( \frac{\omega}{\omega_x} \right)^4 + \left[ c \left( \frac{\omega_y}{\omega_x} \right)^2 + \left( \frac{\omega_y}{\omega_x} \right)^2 \left( 1 + \left( \frac{\omega_y}{\omega_x} \right)^2 \right) \right] \left( \frac{\omega}{\omega_x} \right)^2 \]

\[ \left( \frac{\omega_y}{\omega_x} \right)^2 \left( \frac{\omega_y}{\omega_x} \right)^2 = 0 \]

Where the values of \( \omega_x \) and \( \omega_y \) are the uncoupled circular natural frequencies of the system in \( x \) and \( y \)-directions, respectively. The value of \( \omega_\Theta \) is the uncoupled circular natural frequency of torsional vibration. Clearly, the coupled dynamic properties depend only on the four dimension-less parameters \( e_x, e_y, \omega_\Theta / \omega_x \) and \( \omega_y / \omega_x \).

4. Models Basic Parameters
In order to investigate the effect of frequency ratio \( \Omega \) on the torsional behaviour of the structure, a set one-story case study structures are selected. All structures are composed of 8.0 m x 8.0 m modules, Schematic floor plans are of structures are shown in ‘figures 6’. As can be seen in the figure, all the structures are symmetrical with respect to the x-axis. The horizontal rigid floor diaphragm is constrained in two lateral directions by vertical resisting elements. Site and building data are summarized in table (1).
Table 1. Building and Site data.

| Site and building Data                        | Value  |
|-----------------------------------------------|--------|
| Site class                                    | C      |
| Seismic design category                       | C      |
| 0.2 sec spectral acceleration (Ss)            | 0.6    |
| 1 sec spectral acceleration (S1)              | 0.18   |
| Risk category                                 | II     |
| Response modification factor (R)              | 5      |
| Deflection amplification factor (Cd)          | 4.5    |

Figure 6. a. Building (A) plan, b. Building (B) plan, c. Building (C) plan, d. Building (D) plan, e. Building (E) plan, f. Building (F) plan, g. Building (G) plan,
**Table 2a.** Building (A) columns properties.

| Building (A) | Dimensions (L*B*h) (mm) | Column Major Stiffness (N/mm) | Column Minor Stiffness (N/mm) |
|--------------|--------------------------|-----------------------------|-----------------------------|
| **Columns resisting in Y-direction** | 1000*250*4000 | 97092.11 | 6068.25 |
| **Columns resisting in X-direction** | 2000*250*4000 | 776736.8 | 12136.5 |
| **Leaning Columns** | - | - | - |

**Table 2b.** Building (B) columns properties.

| Building (B) | Dimensions (L*B*h) (mm) | Column Major Stiffness (N/mm) | Column Minor Stiffness (N/mm) |
|--------------|--------------------------|-----------------------------|-----------------------------|
| **Columns resisting in Y-direction** | 1200*250*4000 | 167775.16 | 7281.91 |
| **Columns resisting in X-direction** | 1900*250*4000 | 665954.78 | 11529.69 |
| **Leaning Columns** | - | - | - |

**Table 2c.** Building (C) columns properties.

| Building (C) | Dimensions (L*B*h) (mm) | Column Major Stiffness (N/mm) | Column Minor Stiffness (N/mm) |
|--------------|--------------------------|-----------------------------|-----------------------------|
| **Columns resisting in Y-direction** | 1600*250*4000 | 397689.28 | 9709.21 |
| **Columns resisting in X-direction** | 1900*250*4000 | 665954.78 | 11529.69 |
| **Leaning Columns** | - | - | - |

**Table 2d.** Building (D) columns properties.

| Building (D) | Dimensions (L*B*h) (mm) | Column Major Stiffness (N/mm) | Column Minor Stiffness (N/mm) |
|--------------|--------------------------|-----------------------------|-----------------------------|
| **Columns resisting in Y-direction** | 2000*250*4000 | 776736.8 | 12136.5 |
| **Columns resisting in X-direction** | 1000*250*4000 | 97092.11 | 6068.25 |
| **Leaning Columns** | - | - | - |

**Table 2e.** Building (E) columns properties.

| Building (E) | Dimensions (L*B*h) (mm) | Column Major Stiffness (N/mm) | Column Minor Stiffness (N/mm) |
|--------------|--------------------------|-----------------------------|-----------------------------|
| **Columns resisting in Y-direction (Grid-A)** | 2000*250*4000 | 776736.8 | 12136.5 |
| **Columns resisting in X-direction (Grid-B)** | 1000*250*4000 | 97092.11 | 6068.25 |
| **Leaning Columns (Grid-C)** | - | - | - |

The two columns on grid (C) are leaning columns and not participating in building lateral resistance.

**Table 2f.** Building (F) columns properties.

| Building (F) | Dimensions (L*B*h) (mm) | Column Major Stiffness (N/mm) | Column Minor Stiffness (N/mm) |
|--------------|--------------------------|-----------------------------|-----------------------------|
| **Columns resisting in Y-direction (Grid-A)** | 1000*250*4000 | 97092.11 | 6068.25 |
| **Columns resisting in Y-direction (Grid-B)** | 2000*250*4000 | 776736.8 | 12136.5 |
| **Leaning Columns (Grid-C)** | - | - | - |

The two columns on grid (C) are leaning columns and not participating in building lateral resistance.

**Table 2g.** Building (G) columns properties.

| Building (G) | Dimensions (L*B*h) (mm) | Column Major Stiffness (N/mm) | Column Minor Stiffness (N/mm) |
|--------------|--------------------------|-----------------------------|-----------------------------|
| **Columns resisting in Y-direction (Grid-A)** | 500*250*4000 | 12136.51 | 3034.13 |
| **Columns resisting in Y-direction (Grid-B)** | 2000*250*4000 | 776736.8 | 12136.5 |
| **Leaning Columns (Grid-C)** | - | - | - |

The two columns on grid (C) are leaning columns and not participating in building lateral resistance.
Lateral stiffnesses, rotational stiffness, transition circular frequencies, rotational circular frequency, radius of gyration, mass moment of inertia and uncoupled frequency ratio is calculated for buildings from (A) to (G) using equations (5) and tabulated in table (3).

| Building | Transition stiffness Kx (N/mm) | Transition stiffness Ky (N/mm) | Rotational stiffness Kɵ (N.mm) | Transition frequency ωx (rad/sec) | Transition frequency ωy (rad/sec) | Rotational frequency ωɵ (rad/sec) | Radius of gyration Im (m) | Mass moment of inertia Im (t.m²) | Uncoupled frequency ratio (Ω = ωɵ / ωy) |
|----------|-----------------------------|-----------------------------|------------------------------|---------------------------------|---------------------------------|-------------------------------|------------------------|-------------------------------|----------------------------------|
| (A)      | 2354483.7                   | 424777.98                   | 1.26*10¹⁴                   | 86.82                           | 36.88                           | 95.24                         | 6.66                   | 13607                         | 2.58                             |
| (B)      | 2026992                     | 705689.72                   | 1.3*10¹⁴                    | 80.47                           | 47.48                           | 96.42                         | 6.68                   | 13721                         | 2.03                             |
| (C)      | 2046410.4                   | 2023035.46                  | 2.1*10¹⁴                    | 80.32                           | 79.86                           | 120.82                        | 6.73                   | 14112                         | 1.5                              |
| (D)      | 351958.9                    | 5455362.9                   | 3.0*10¹⁴                    | 33.09                           | 130.27                          | 141.8                         | 6.82                   | 14656                         | 1.09                             |
| (E)      | 327685.87                   | 3901889.15                  | 1.02*10¹⁴                   | 31.93                           | 110.17                          | 82.73                         | 6.82                   | 14656                         | 0.75                             |
| (F)      | 54614.3                     | 2815671.17                  | 2.85*10¹³                   | 13.14                           | 94.35                           | 44.83                         | 6.69                   | 13890                         | 0.48                             |
| (G)      | 45511.93                    | 2390893.19                  | 5.86*10¹²                   | 12.08                           | 87.58                           | 20.69                         | 6.62                   | 13423                         | 0.24                             |

5. Results and discussions

The details of the seismic analyses performed on the buildings of the parametric study through “ETAB” software and using ELF method considering the previous given parameters are not shown herein for the sake of brevity. Ratio between displacement of building stiff edge (δs), near to center of rigidity, to center of mass displacement (δo) and ratio between displacement of flexible edge (δf), far from center of rigidity, to center of mass displacement(δo); In addition to torsional irregularity factor in accordance to ASCE 7-10 are calculated for different values of static eccentricity, and plotted in ‘figures 7 to 10’.

![Figure 7](image1.png) **Figure 7.** Displacement of Flexible edges (Right Edges) for different static anti-clock wise eccentricities.  

![Figure 8](image2.png) **Figure 8.** Displacement of Stiff edges (Lift Edges) for different static clock wise eccentricities.
It is observed from ‘figures 7&8’ that the displacement ratios for both stiff edge ($\frac{\delta_s}{\delta_o}$) and flexible edge ($\frac{\delta_f}{\delta_o}$) for structures with uncoupled frequency ratio $\Omega$ exceeding 1.0 are not sensitive to the increment of eccentricity ratio. Hence the structures show planer frame behaviour and the displacement are develop mainly by transition motion rather than torsional motion. Conversely structures with ratio $\Omega$ less than 1.0 are sensitive to the increment of eccentricity ratio; hence the displacements develop mainly by torsional motion rather transition motion.

![Figure 9](image)

**Figure 9.** Displacement of Flexible edges and Stiff edges for different static anti-clock wise eccentricities.

![Figure 10](image)

**Figure 10.** Torsional Irregularity Factor for different static eccentricities.

It is observed from ‘figures 9’, that when the frequency ratio $\Omega$ of a structure exceeds 1.0, the difference between structure flexible and stiff edge displacements decreases till the $\Omega$ exceeds 2.0, then both flexible and stiff edge displacement become approximately equal and the structure becomes under transition motion only. In other words the structure is torsionally stiff. Subsequently, torsional irregularity coefficient as shown in ‘figure 10’ decreases as the uncoupled frequency ratio increases.

Considering the case of static accidental eccentricity of 5%, it is interesting to note that buildings with uncoupled frequency ratio ($\Omega$) less than 1.0 are classified as Extremely Torsional Irregular type 1b according to ASCE 7-10 [8]. While buildings with uncoupled ratio ($\Omega$) less than 2.0 and exceeding 1.0 are classified as Torsional Irregular type 1a. On the other hand, buildings with uncoupled frequency ratio ($\Omega$) exceeding 2.0 have regular torsional response. Hence, a value of ($\Omega$) = 2.0 will be suitable value for structural designer consider during the selection and arrangement of the lateral resisting system in order to control the torsional behaviour of a building. Note that $\Omega = 1.0$ does not cause any significant increase in response.

6. Conclusions
In this study a parametric investigation is performed on different building configurations under variable ratio of uncoupled torsional to transition frequency. The effect of the frequency ratio on the torsional behaviour of buildings with single eccentricity and subjected to horizontal ground force perpendicular to the direction of eccentricity can be summarized as following:
The coupled dynamic properties of a structure depend only on the four dimension-less parameters \( \varepsilon_x, \varepsilon_y, \frac{\omega_b}{\omega_x} \) and \( \frac{\omega_y}{\omega_x} \).

Structures with ratio between the torsional frequency to the transitional frequency \( \frac{\omega_b}{\omega_x} \) or \( \frac{\omega_b}{\omega_y} \) exceeding 1.0 are torsionally stiff and displacements values are not sensitive to increase in eccentricity ratio. In other words, the structure shows planer frame behaviour and the displacements develop mainly by transition motion rather than torsional motion.

Structures with ratio between torsional frequency to transitional frequency \( \frac{\omega_b}{\omega_x} \) or \( \frac{\omega_b}{\omega_y} \) less than 1.0 are torsionally flexible and displacements values are sensitive increase in eccentricity ratio. In other words, the displacements develop mainly by torsional motion rather than transition motion.

When ratio between torsional frequency and translational torsional frequencies is 1, i.e. when \( \Omega = 1.0 \), response doesn’t increase significantly.

For the case study buildings examined, when the eccentricity is 5% of the building dimension and frequency ratio less than 1.0, the building classifies as type 1b extremely torsionally irregular according to ASCE 7-10. When the frequency ratio is greater than 1.0 and less than 2.0, the building classifies as type 1a torsionally irregular. Buildings with frequency ratio exceeding 2.0 have regular torsional response.

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