Standard-like Model from $D = 4$ Type IIB Orbifolds

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Abstract

Based on the twisted R-R tadpole cancellation conditions at the singularities of $D = 4$ Type IIB orbifold $T^6/Z_3$, we propose a new bottom-up approach to embed standard model with three generations into string theory.

1 Introduction

The discovery of D-branes in string theory has largely extended our view on consistent string theory vacua. D-branes are the carriers of Ramond-Ramond (R-R) charges on which the open strings are allowed to end. Having open string sectors means having gauge interactions. Therefore, string theory with D-branes in its spectrum provides new scenarios for finding string theory vacuum configuration which is expected to be much closer to the observed standard model (SM). An interesting class of string theory vacua is the Type

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IIB orbifolds in $D = 4$ dimensions. They provide explicit string theory models where gauge fields and charged matter are localized on D-branes, whereas gravity and other closed string modes propagate in full spacetime. Several attempts have been made in this direction\cite{1,2,3}, among which the standard orientifolds belong to top-down approach while some $\mathbb{Z}_N$ orbifold models provide possibilities for bottom-up approach. In all D-brane models strong consistency conditions come from cancellation of R-R tadpoles\cite{4,5}. It is also expected in view of low-energy phenomenology that the twisted tadpole cancellation can guarantee the cancellation of nonabelian gauge anomalies.

Among the known D-brane constructions of $D = 4$ Type IIB orbifolds, the “bottom-up approach” pioneered in Ref.\cite{3} appears very interesting. By locating six D3-branes at one singularity of $T^6/\mathbb{Z}_3$ orbifold of Type IIB string theory and including a stack of D7-branes in the configuration as well, the authors of \cite{3} found a nontrivial solution of the R-R tadpole cancellation conditions which corresponds to a gauge group $U(3) \times U(2) \times U(1)$, very close to that of standard model, living in the worldvolumes of D3-branes at an orbifold singularity. Another significant characteristic of standard model, quark-lepton family replication with just three generations, is also ensured by the fact that there are only three complex planes in that compactified space. It is undeniable that such a bottom-up approach is attractive, however, there are some serious drawbacks. The massless chiral fermion spectrum of \cite{3} contains only 3 fermion states with three generations, their hypercharges are also not the same as those of the quarks and leptons in standard model. In addition, only the gauge fields living in D3-branes were supposed to describe the realistic interactions while there was no reliable interpretation for the gauge fields on D7-branes’ worldvolumes.

In this paper, we report a revised version of the above bottom-up approach, which may also provide a stringy realization of the phenomenological approach proposed in \cite{3}. Enlightened by the recent progress in the intersecting D-brane world\cite{6,7}, we now consider such a $T^6/\mathbb{Z}_3$ orbifold of Type IIB string theory (with D3-branes and three kinds of oriented D7-branes and anti-D7-branes in its non-perturbative spectrum) that there is at most one factorial gauge group $U(n) (1 \leq n \leq 3)$ embedded on the worldvolumes of each kinds of the oriented D-branes. We also postulate the presence of an additional Wilson line along the 4-th (complex) compactified dimension so that there is no a factorial gauge group $U(3)$ omnipresent on the oriented D-branes ($D7_{5,(0)}$-branes in our notation, See Section 2). Three-family replication of chiral fermion spectrum is achieved by the fact that
there are only three fixed points of the $\mathbb{Z}_3$ orbifold wrapped by D7$_{(5,0)}$-branes which can not feel the presence of the Wilson line. Near these 3 fixed points the total gauge group on the D-brane worldvolumes is then the expected $U(3) \times U(2) \times U(1) \times U(1)$, The chiral fermion states appear as the open string Ramond states stretched between two different kinds of D-branes. The requirement of cancellation of the twisted R-R tadpoles leads exactly to the massless left-handed fermion spectrum of standard model, with correct hypercharge assignment and correct family replication (3 generations), and without cubic $SU(3)$ gauge anomaly. To our surprise, we can further ensure the twisted R-R tadpole cancellation conditions and the cubic nonabelian gauge anomaly cancellation conditions at the remaining 24 $\mathbb{Z}_3$ orbifold fixed points without violating the above-mentioned properties. The only expense is that we have to include 12 extra massless left-handed fermion states into our chiral fermion spectrum, which are located at 6 fixed points separately but form the singlets of both “colour” group $SU(3)$ and “weak isospin” group $SU(2)$. As most of the similar approaches initiated from the intersecting brane configurations, our model is non-supersymmetric.

The paper is organized as follows. In section 2, we define our D-brane configuration at the $\mathbb{Z}_3$ orbifold origin. As a kind of bottom-up approach within Type IIB string theory, the guidance for our defining D3D7 brane configuration comes from the corresponding twisted R-R tadpole cancellation conditions and the gauge group structure of standard model of the low-energy particle physics. With the help of $\mathbb{Z}_3$ orbifold projection on Chan-Paton wavefunctions and the constraints from the cancellation of cubic $SU(3)$ gauge anomaly, we obtain five massless left-handed Ramond open string fermion states at origin which turn out to form an entire family of the chiral fermions of the standard model. In section 3, we study the consistency conditions and their solutions at remaining orbifold fixed points. In order to keep the phenomenological attraction of our model, we suppose that there is a discrete Wilson line along the 4-th complex plane of the orbifold. The tadpole cancellation conditions and their solutions are given point by point, which lead to another two families of standard model chiral fermions and 12 extra massless fermion states. In section 4, we give a detailed discussion about $U(1)$ anomalies and the corresponding cancellation mechanism. Section 5 is left as our conclusions and some remarks.
2 Chiral Fermionic Spectrum at Origin

We begin with the standard $D = 4, \mathcal{N} = 1$ supersymmetric $T^6 / \mathbb{Z}_3$ orbifold construction of Type IIB string theory\cite{3}. A set of D3-branes is located at some singularities (fixed points) of the considered orbifold. The configuration includes some kinds of oriented D7-branes as well. The D7-branes with different orientations are postulated to be independent of one another. We adopt light-cone gauge, in which we use complex planes representing the target space dimensions: the second complex plane corresponds to non-compact dimensions; the 3-rd, 4-th and 5-th complex planes correspond to $\mathbb{Z}_3$ invariant $T^6$ lattice. The $\mathbb{Z}_3$ orbifold action on open string states is given by a matrix

$$\theta = \text{diag}[\exp(2\pi i a_2/3), \exp(2\pi i a_3/3), \exp(2\pi i a_4/3), \exp(2\pi i a_5/3)] \quad (2.1)$$

for worldsheet fermions and

$$\theta = \text{diag}[\exp(2\pi i b_3/3), \exp(2\pi i b_4/3), \exp(2\pi i b_5/3)] \quad (2.2)$$

for (complex) worldsheet bosons. In addition, the orbifold action of the $\mathbb{Z}_3$ generators $\theta$ must be embedded on the Chan-Paton indices, defined by diagonal unitary matrices $\gamma_{\theta,3}$ and $\gamma_{\theta,7}$. In Eqs.(2.1) and (2.2) the twist parameters obey the group theory constraints $\sum_{i=2}^{5} a_i = 0(\text{mod}3)$ and

$$b_3 = -a_4 - a_5, \quad b_4 = -a_5 - a_3, \quad b_5 = -a_3 - a_4$$

The $\mathcal{N} = 1$ supersymmetry requires condition $\sum_{r=3}^{5} b_r = 0$. Although the model we will consider is non-supersymmetric, we impose this condition for simplicity. In what follows, we take

$$a_2 = 0, \quad a_3 = a_4 = 1, \quad a_5 = -2 \quad \left\{ \begin{array}{l}
b_3 = b_4 = 1, \quad b_5 = -2\end{array} \right. \quad (2.3)$$

to ensure a crystallographical orbifold action\cite{1, 3}.

The $\mathbb{Z}_3$ invariant $T^6$ can be realized as a root lattice of the Lie algebra $[su(3)]^3$. Let the simple roots $\{\tilde{\alpha}_{2i-2}, \tilde{\alpha}_{2i-1}\}$ of $su(3)_i$ be the basis vectors of the $i$-th complex plane of the $T^6$ lattice ($i = 3, 4, 5$), the orbifold actions are then equivalently defined by $\theta \tilde{\alpha}_{2i-2} = \tilde{\alpha}_{2i-1}$ and $\theta \tilde{\alpha}_{2i-1} = -\tilde{\alpha}_{2i-2} - \tilde{\alpha}_{2i-1}$. We can further express the position coordinates of $\mathbb{Z}_3$ fixed points as

$$x_f = \frac{m}{3}(\tilde{\alpha}_4 + 2\tilde{\alpha}_5) + \frac{n}{3}(\tilde{\alpha}_6 + 2\tilde{\alpha}_7) + \frac{p}{3}(\tilde{\alpha}_8 + 2\tilde{\alpha}_9) \quad (2.4)$$

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where \( m, n, p = 0, \pm 1 \). In what follows these fixed points will alternatively be denoted by \((m, n, p)\).

The local consistency of the considered D-brane configuration is guaranteed by the so-called twisted R-R tadpole cancellation conditions at all orbifold fixed points. Suppose that the D7-branes carry negative R-R charge while the other D-branes (namely D3-, D7\(_4\)- and D7\(_5\)-branes) carry positive R-R charge. Then at the fixed point \((m, n, p)\) the twisted R-R tadpole cancellation conditions for \( D = 4, \mathcal{N} = 1 \) supersymmetric Type IIB orbifold \( T^6/\mathbb{Z}_3 \) are[3, 8],

\[
\begin{cases}
Tr\gamma_{1,3,(m,n,p)} + \frac{1}{3}Tr\gamma_{1,7,4,(m)} - \frac{1}{3}Tr\gamma_{1,7,4,(n)} + \frac{1}{3}Tr\gamma_{1,7,5,(p)} = 0, \\
Tr\gamma_{2,3,(m,n,p)} - \frac{1}{3}Tr\gamma_{2,7,4,(m)} + \frac{1}{3}Tr\gamma_{2,7,4,(n)} + \frac{1}{3}Tr\gamma_{2,7,5,(p)} = 0
\end{cases}
\tag{2.5}
\]

where we have used the similar notation of Ref.[1], e.g., \( \gamma_{1,3,(m,n,p)} \equiv \gamma_{0,3,(m,n,p)} \) and \( \gamma_{2,3,(m,n,p)} \equiv (\gamma_{1,3,(m,n,p)})^2 \). D3\(_{(m,n,p)}\) are referred to as the D3-branes located at fixed point \((m, n, p)\), and D7\(_3\,(m)\), for example, stands for D7-branes located at the fixed point \((m)\) on the 3-rd complex plane whose worldvolumes are transverse to the 3-rd complex plane but full of the 4-th and 5-th planes. In this paper, we consider a non-supersymmetric version of above orbifold, in which the D7\(_3\)-branes are replaced by the corresponding anti-branes D7\(_3\). Branes and anti-branes carry opposite charges with respect to the R-R fields. Therefore, instead of Eqs.(2.5), the tadpole cancellation conditions for the considered orbifold should be[4]:

\[
\begin{cases}
Tr\gamma_{1,3,(m,n,p)} - \frac{1}{3}Tr\gamma_{1,7,4,(m)} - \frac{1}{3}Tr\gamma_{1,7,4,(n)} + \frac{1}{3}Tr\gamma_{1,7,5,(p)} = 0, \\
Tr\gamma_{2,3,(m,n,p)} + \frac{1}{3}Tr\gamma_{2,7,4,(m)} + \frac{1}{3}Tr\gamma_{2,7,4,(n)} + \frac{1}{3}Tr\gamma_{2,7,5,(p)} = 0
\end{cases}
\tag{2.6}
\]

The origin \((0,0,0)\) of \( T^6 \) lattice is naturally a \( \mathbb{Z}_3 \) fixed point, at which the twisted R-R tadpole cancellation conditions are written as,

\[
\begin{cases}
Tr\gamma_{1,3,(0,0,0)} - \frac{1}{3}Tr\gamma_{1,7,4,(0)} - \frac{1}{3}Tr\gamma_{1,7,4,(0)} + \frac{1}{3}Tr\gamma_{1,7,5,(0)} = 0, \\
Tr\gamma_{2,3,(0,0,0)} + \frac{1}{3}Tr\gamma_{2,7,4,(0)} + \frac{1}{3}Tr\gamma_{2,7,4,(0)} + \frac{1}{3}Tr\gamma_{2,7,5,(0)} = 0
\end{cases}
\tag{2.7}
\]

As a starting point of our bottom-up approach, we consider the following solutions of Eqs.(2.7),

\[
\begin{align*}
\gamma_{1,3,(0,0,0)} &= 1_1 \\
\gamma_{1,7,4,(0)} &= -\alpha 1_1 \\
\gamma_{1,7,5,(0)} &= -\alpha 1_2 \\
\gamma_{1,7,5,(0)} &= \alpha^2 1_3
\end{align*}
\tag{2.8}
\]
where $\alpha = \exp[(2\pi i)/3]$. This assignment leads to the gauge group $U(3) \times U(2) \times U(1)_{1,(0)} \times U(1)_{0,(0,0,0)}$ living in the worldvolumes of D-branes near origin:

\[ \begin{align*}
D3_{(0,0,0)} & : U(1)_{0,(0,0,0)} \\
\overline{D7}_{3,(0)} & : U(1)_{1,(0)} \\
D7_{4,(0)} & : U(2) = U(1)_{2,(0)} \times SU(2) \\
D7_{5,(0)} & : U(3) = U(1)_{3} \times SU(3) 
\end{align*} \]

We now consider the massless chiral fermion states in all possible open string sectors at origin.

### 2.1 $33$, $\tilde{7}_3\tilde{7}_3$, $7_47_4$ and $7_57_5$ sectors

In $33$ sector, the massless Ramond fermions are of the form $|s_2s_3s_4s_5\rangle$ (in light-cone gauge), where every $s_i$ is either of $\pm \frac{1}{2}$. The value of $s_2$ determines the spacetime fermion chirality. The worldsheet fermion number can be defined as $(-1)^F = \exp[i\pi(\sum_{i=2}^{5}s_i)]$ so that we can implement GSO projection $(-1)^F = 1$ by letting $\sum_{i=2}^{5}s_i = \text{even}$. Before $\mathbb{Z}_3$ orbifold projection, the left-handed massless states ($s_2 = -\frac{1}{2}$) are explicitly stated below,

\[ \begin{align*}
|\Psi_1\rangle &= \begin{pmatrix} -\frac{1}{2}, & -\frac{1}{2}, & -\frac{1}{2}, & -\frac{1}{2} \end{pmatrix} \\
|\Psi_2\rangle &= \begin{pmatrix} -\frac{1}{2}, & -\frac{1}{2}, & \frac{1}{2}, & \frac{1}{2} \end{pmatrix} \\
|\Psi_3\rangle &= \begin{pmatrix} -\frac{1}{2}, & \frac{1}{2}, & -\frac{1}{2}, & \frac{1}{2} \end{pmatrix} \\
|\Psi_4\rangle &= \begin{pmatrix} -\frac{1}{2}, & \frac{1}{2}, & \frac{1}{2}, & -\frac{1}{2} \end{pmatrix} 
\end{align*} \]  

(2.10)

Under orbifold projection,

\[ \theta^k|s_2s_3s_4s_5\rangle = \exp\left[\frac{2\pi i k}{3}(s_2a_2 + s_3a_3 + s_4a_4 + s_5a_5)\right]|s_2s_3s_4s_5\rangle \quad (k = 1, 2) \]

Explicitly,

\[ \begin{align*}
\theta|\Psi_1\rangle &= |\Psi_1\rangle \\
\theta|\Psi_j\rangle &= \alpha |\Psi_j\rangle \quad (j = 2, 3, 4) 
\end{align*} \]  

(2.11)

The open string spectrum is constructed by requiring the states $|\Psi, ij\rangle\lambda_{ji}$ to be invariant under the action of the orbifold projection. As a result, the Chan-Paton wavefunctions of the above massless open string states are determined by conditions [4],

\[ \begin{align*}
\lambda_1 = \gamma_{1,3,(0,0,0)}\lambda_1^{-1}\gamma_{1,3,(0,0,0)}^{-1} \\
\lambda_j = \alpha\gamma_{1,3,(0,0,0)}\lambda_j^{-1}\gamma_{1,3,(0,0,0)}^{-1} \quad (j = 2, 3, 4) 
\end{align*} \]  

(2.12)
This leads to one massless left-handed fermion state which forms the adjoint representation of Abelian gauge group $U(1)_{0,(0,0,0)}$. Similar situations appear in other three open string sectors $\tilde{T}_3\tilde{T}_3$, $T_4\tilde{T}_4$ and $T_5\tilde{T}_5$. Each of these sectors has such a left-handed massless fermion state that forms the adjoint representation of the corresponding unitary group. Actually, this is also true for the right-handed massless fermion states in these sectors. These states do actually form the non-chiral massless fermions with all $U(1)$ charges vanishing. In supersymmetric models, these states could be interpreted as the super-partners of the $U(1)$ gauge bosons. Because of the non-chirality of these states, we will simply ignore them from the discussion about the chiral fermion spectrum.

2.2 $\tilde{T}_3\tilde{T}_3$ and $\tilde{T}_3T_3$ sectors

Before orbifold projection, the massless Ramond states in these two sectors are $|s_2s_3\rangle$. Relying on the fact that $\tilde{D}\tilde{T}_3$ stands for an anti-brane, we should take $(-1)^F = -1$ as the GSO projection. This implies that $s_2 = s_3$. There is only one left-handed candidate ($s_2 = -\frac{1}{2}$) in each sector,

$$|\Psi\rangle = \left| -\frac{1}{2}, -\frac{1}{2}\right\rangle$$

which undergoes the following $Z_3$ orbifold action,

$$\theta |\Psi\rangle = \exp\left(-\frac{\pi i}{3}\right)|\Psi\rangle = -\alpha |\Psi\rangle$$

Therefore,

$$\lambda = -\alpha \gamma_{1,3,(0,0,0)} \gamma^{-1}_{1,\tilde{T}_3,(0)} = -\alpha \gamma_{1,\tilde{T}_3,(0)} \gamma^{-1}_{1,3,(0,0,0)}$$

Eqs. (2.15) lead to a massless Ramond fermionic state in $3\tilde{T}_3$ sector, which forms the fundamental representation of $U(1)_{0,(0,0,0)}$ and the anti-fundamental representation of $U(1)_{1,(0)}$.

2.3 $\tilde{T}_3T_4$ and $T_4\tilde{T}_3$ sectors

The massless Ramond fermionic states before orbifold action are $|s_2s_5\rangle$. The GSO projection $(-1)^F = -1$ is implemented by setting $s_2 = s_5$ and we have only one left-handed candidate in each sector,

$$|\Psi\rangle = \left| -\frac{1}{2}, -\frac{1}{2}\right\rangle$$
Under the $\mathbb{Z}_3$ orbifold projection,
\[
\theta |\Psi\rangle = \exp\left(\frac{2\pi i}{3}\right) |\Psi\rangle = \alpha |\Psi\rangle
\] (2.17)

Then,
\[
\lambda = \alpha \gamma_{1,7_3,(0)} \lambda^{-1} \gamma_{1,7_4,(0)} = \alpha \gamma_{1,7_3,(0)} \lambda^{-1} \gamma_{1,7_3,(0)}
\] (2.18)

There is no qualified massless left-handed Ramond state in either of these two sectors obeying Eqs.(2.18).

**2.4 $\tilde{7}_3 7_5$ and $7_5 \tilde{7}_3$ sectors**

The massless Ramond fermionic states before $\mathbb{Z}_3$ orbifold action are $|s_2 s_4\rangle$. The GSO projection $(-1)^F = -1$ is implemented by setting $s_2 = s_4$ and we have only one left-handed candidate in each sector,
\[
|\Psi\rangle = | -\frac{1}{2}, -\frac{1}{2} \rangle
\] (2.19)

Under the $\mathbb{Z}_3$ orbifold projection,
\[
\theta |\Psi\rangle = \exp\left(-\frac{\pi i}{3}\right) |\Psi\rangle = -\alpha |\Psi\rangle
\] (2.20)

Then,
\[
\lambda = -\alpha \gamma_{1,7_3,(0)} \lambda^{-1} \gamma_{1,7_5,(0)} = -\alpha \gamma_{1,7_3,(0)} \lambda^{-1} \gamma_{1,7_3,(0)}
\] (2.21)

Eqs.(2.21) lead to a massless Ramond fermionic state in $\tilde{7}_3 7_5$ sector. It forms the fundamental representation of $U(1)_{1,(0)}$ and the anti-fundamental representation of nonabelian gauge group $U(3)$.

**2.5 $37_5$ and $7_5 3$ sectors**

Before orbifold projection, the massless Ramond fermions are $|s_2 s_5\rangle$. These fermions are the Ramond open strings stretched between two D-branes rather than brane-antibrane pairs. So we should take $(-1)^F = 1$ as GSO projection in these sectors. In fact, such a GSO projection is also the requirement of cancellation of cubic $SU(3)$ gauge anomaly at origin. Obviously, $(-1)^F = 1$ in the considered sectors can be achieved via setting $s_2 = -s_5$. As before, there is only one left-handed ($s_2 = -\frac{1}{2}$) candidate in each sector,
\[
|\Psi\rangle = | -\frac{1}{2}, \frac{1}{2} \rangle
\] (2.22)
on which the $\mathbb{Z}_3$ orbifold projection acts as,

$$\theta|\Psi\rangle = \exp\left(-\frac{2\pi i}{3}\right)|\Psi\rangle = \alpha^2|\Psi\rangle$$ \hspace{1cm} (2.23)

So,

$$\lambda = \alpha^2 \gamma_{1,3,(0,0,0)}\gamma_{1,75,(0)}^{-1} = \alpha^2 \gamma_{1,75,(0)}\gamma_{1,3,(0,0,0)}^{-1}$$ \hspace{1cm} (2.24)

Eqs. (2.24) lead to a massless Ramond fermion state in $37\overline{5}$ sector. The state forms the fundamental representation of $U(1)_{0,(0,0,0)}$ and the anti-fundamental representation of $U(3)$.

### 2.6 $7_4\overline{7}_5$ and $7_5\overline{7}_4$ sectors

Before orbifold projection, the massless ramond fermions in these sectors are $|s_2s_4\rangle$. as explained in last paragraph, we have to set $s_2 = -s_3$ realizing GSO projection $(-1)^F = 1$. There is only one left-handed ($s_2 = -\frac{1}{2}$) candidate in each sector,

$$|\Psi\rangle = | -\frac{1}{2}\rangle$$ \hspace{1cm} (2.25)

on which the $\mathbb{Z}_3$ orbifold projection acts as,

$$\theta|\Psi\rangle = \exp\left(\frac{\pi i}{3}\right)|\Psi\rangle = -\alpha^2|\Psi\rangle$$ \hspace{1cm} (2.26)

Therefore,

$$\lambda = -\alpha^2 \gamma_{1,74,(0)}\gamma_{1,75,(0)}^{-1} = -\alpha^2 \gamma_{1,75,(0)}\gamma_{1,74,(0)}^{-1}$$ \hspace{1cm} (2.27)

Eqs. (2.27) lead to a massless Ramond fermionic state in $7_5\overline{7}_4$ sector which forms the fundamental representation of gauge group $U(3)$ but the anti-fundamental representation of $U(2)$.

### 2.7 $37_4$ and $\overline{7}_43$ sectors

In these two sectors, the massless ramond fermions before $\mathbb{Z}_3$ orbifold projection are of the form $|s_2s_4\rangle$. The GSO projection $(-1)^F = 1$ is implemented by setting $s_2 = -s_4$. There is only one left-handed ($s_2 = -\frac{1}{2}$) candidate in each sector,

$$|\Psi\rangle = | -\frac{1}{2}\rangle$$ \hspace{1cm} (2.28)
on which the $Z_3$ orbifold projection acts as,

$$\theta|\Psi\rangle = \exp\left(\frac{\pi i}{3}\right)|\Psi\rangle = -\alpha^2|\Psi\rangle$$  \hspace{1cm} (2.29)

Therefore,

$$\lambda = -\alpha^2\gamma_{1,3,(0,0,0)}\lambda\gamma_{1,7,4,(0)}^{-1} = -\alpha^2\gamma_{1,7,4,(0)}\lambda\gamma_{1,3,(0,0,0)}^{-1}$$  \hspace{1cm} (2.30)

They give rise to a massless Ramond fermion state in $7_43$ sector, forming the fundamental representation of gauge group $U(2)$ while the anti-fundamental representation $U(1)_{0,(0,0,0)}$.

Consequently, we obtain exactly one-family Standard-Model chiral spectrum from open strings stretching between different D-branes at $Z_3$ orbifold origin:

| State | Sector | Rep. of SU(3) | Rep. of SU(2) | $Q_3$ | $Q_{2,(0)}$ | $Q_{1,(0)}$ | $Q_{0,(0,0,0)}$ |
|-------|--------|---------------|---------------|------|-------------|-------------|----------------|
| $(3, 2)$ | $7_57_4$ | 3 | 2 | 1 | -1 | 0 | 0 |
| $(3, 1)$ | $7_37_5$ | 3 | 1 | -1 | 0 | 1 | 0 |
| $(3, 1)$ | $37_5$ | 3 | 1 | -1 | 0 | 0 | 1 |
| $(1, 2)$ | $7_43$ | 1 | 2 | 0 | 1 | 0 | -1 |
| $(1, 1)$ | $37_3$ | 1 | 1 | 0 | 0 | -1 | 1 |

### 3 Chiral Fermionic Spectrum at Other $Z_3$ Singularities

#### 3.1 Tadpole cancellation conditions

In the last section, we only discussed the cancellation of the twisted R-R tadpoles at $Z_3$ orbifold origin. Because each oriented D7-brane wraps nine fixed points, we have to ensure the tadpole cancellation at all of them. The simplest way to such a cancellation is by a replication at every orbifold fixed point of the D-brane configuration at the origin. This, however, obviously violates the phenomenological attraction of the considered orbifold because the standard model only contains 3 quark-lepton generations. Fortunately, in the model under consideration, three families of massless chiral fermion states can be obtained by introducing a Wilson line $e^{i\int A \cdot dl}$ passing the orbifold
origin and along the 4-th complex plane. The presence of such a Wilson line introduces probably additional gauge field structure on the worldvolumes of the $\overline{D7}_{3,(0)}$ and $D7_{5,(0)}$ branes, which will affect the parallel transport along the 4-th complex plane. The 6 fixed points $(0, \pm 1, p)$ with $p = 0, \pm 1$ wrapped by $\overline{D7}_{3,(0)}$ and 6 fixed points $(m, \pm 1, 0)$ with $m = 0, \pm 1$ wrapped by $D7_{5,(0)}$ can feel the presence of this Wilson line. Nevertheless, the fixed points $(m, 0, p)$ wrapped by $D7_{4,(0)}$ can not feel its presence $(m$ and $p$ do not vanish at the same time), either can not the fixed points $(0, 0, \pm 1)$ wrapped by $\overline{D7}_{3,(0)}$-branes and $(\pm 1, 0, 0)$ wrapped by $D7_{5,(0)}$-branes. Therefore, the twisted R-R tadpole cancellation conditions at the remaining $\mathbb{Z}_3$ fixed points are categorized into several classes. We list them point by point:

**At Fixed Points $(\pm 1, 0, 0)$:**

$$Tr\gamma_{1,3,(\pm 1,0,0)} - \frac{1}{3}Tr\gamma_{1,7,3,(\pm 1)} - \frac{1}{3}Tr\gamma_{1,7,4,(0)} + \frac{1}{3}Tr\gamma_{1,7,5,(0)} = 0 \quad (3.1)$$

**At Fixed Points $(0, \pm 1, 0)$:**

$$Tr\gamma_{1,3,(0,\pm 1,0)} = \frac{1}{3}Tr[\gamma_{1,7,3,(0)}\gamma_{W4,7,3}^\pm] - \frac{1}{3}Tr\gamma_{1,7,4,(\pm 1)} + \frac{1}{3}Tr[\gamma_{1,7,5,(0)}\gamma_{W4,7,5}^\pm] = 0 \quad (3.2)$$

**At Fixed Points $(0, 1, \pm 1)$ and $(0, -1, \pm 1)$:**

$$\begin{cases} 
Tr\gamma_{1,3,(0,1,\pm 1)} - \frac{1}{3}Tr[\gamma_{1,7,3,(\pm 1)}\gamma_{W4,7,3}] - \frac{1}{3}Tr\gamma_{1,7,4,(1)} + \frac{1}{3}Tr\gamma_{1,7,5,(\pm 1)} = 0 \\
Tr\gamma_{1,3,(0,-1,\pm 1)} - \frac{1}{3}Tr[\gamma_{1,7,3,(\pm 1)}\gamma_{W4,7,3}^{-1}] - \frac{1}{3}Tr\gamma_{1,7,4,(-1)} + \frac{1}{3}Tr\gamma_{1,7,5,(\pm 1)} = 0 
\end{cases} \quad (3.3)$$

**At Fixed Points $(m, 0, \pm 1)$ with $m = 0, \pm 1$:**

$$Tr\gamma_{1,3,(m,0,\pm 1)} - \frac{1}{3}Tr\gamma_{1,7,3,(m)} - \frac{1}{3}Tr\gamma_{1,7,4,(0)} + \frac{1}{3}Tr\gamma_{1,7,5,(\pm 1)} = 0 \quad (3.4)$$

**At Fixed Points $(\pm 1, 1, 0)$ and $(\pm 1, -1, 0)$:**

$$\begin{cases} 
Tr\gamma_{1,3,(\pm 1,1,0)} - \frac{1}{3}Tr\gamma_{1,7,3,(\pm 1)} - \frac{1}{3}Tr\gamma_{1,7,4,(1)} + \frac{1}{3}Tr[\gamma_{1,7,5,(0)}\gamma_{W4,7,4}] = 0 \\
Tr\gamma_{1,3,(\pm 1,-1,0)} - \frac{1}{3}Tr\gamma_{1,7,3,(\pm 1)} - \frac{1}{3}Tr\gamma_{1,7,4,(-1)} + \frac{1}{3}Tr[\gamma_{1,7,5,(0)}\gamma_{W4,7,5}^{-1}] = 0 
\end{cases} \quad (3.5)$$

**At Fixed Points $(m, n, p)$ with $m, n, p = \pm 1$:**

$$Tr\gamma_{1,3,(m,n,p)} - \frac{1}{3}Tr\gamma_{1,7,3,(m)} - \frac{1}{3}Tr\gamma_{1,7,4,(n)} + \frac{1}{3}Tr\gamma_{1,7,5,(p)} = 0 \quad (3.6)$$
Let the Wilson line action on Chan-Paton wavefunctions be given by the following diagonal unitary matrices [11, 12]:

\[
\begin{align*}
\gamma_{W4,\tilde{7}_3} &= 1_1 \\
\gamma_{W4,7_5} &= \text{diag}(1_1, \alpha 1_1, \alpha^2 1_1)
\end{align*}
\] (3.7)

This assignment defines nontrivial gauge field structure on the worldvolumes of D7_{5,(0)}-branes: at the six fixed points \((m, \pm 1, 0)\) with \(m = 0, \pm 1\) which can feel the presence of the Wilson line the full gauge group is reduced to \(U(1)_{31} \times U(1)_{32} \times U(1)_{33}\). Only at the remaining three fixed points \((m, 0, 0)\) with \(m = 0, \pm 1\), which can not feel the presence of the Wilson line, does the gauge group on the D7_{5,(0)}-branes turn out to become the “colour” group \(U(3)\). Consequently, there is not a gauge group \(U(3)\) omnipresent on the worldvolumes of D7_{5,(0)}-branes.

With the above Wilson line, we can achieve the cancellation of the twisted R-R tadpoles at all of the \(\mathbb{Z}_3\) orbifold fixed points. The corresponding solutions for the non-vanishing gamma matrices are as follows:

\[
\begin{align*}
\gamma_{1,3,(\pm 1,0,0)} &= 1_1 \\
\gamma_{1,3,(m,0,\pm 1)} &= -\alpha 1_1 \quad (m = 0, \pm 1) \\
\gamma_{1,\tilde{7}_4,(\pm 1)} &= -\alpha 1_1 \\
\gamma_{1,7_5,(\pm 1)} &= \alpha 1_1
\end{align*}
\] (3.8)

The other gamma matrices are supposed to be vanishing.

### 3.2 Gauge group and chiral fermion spectrum at fixed points \((\pm 1, 0, 0)\)

The twisted R-R tadpole cancellation conditions have been given in Eqs. (3.1). According the assignments in Eqs. (2.8) and (3.8), we have the solutions:

\[
\begin{align*}
\gamma_{1,3,(\pm 1,0,0)} &= 1_1 \\
\gamma_{1,\tilde{7}_4,(\pm 1)} &= -\alpha 1_1 \\
\gamma_{1,7_5,(0)} &= -\alpha 1_2 \\
\gamma_{1,7_5,(0)} &= \alpha^2 1_3
\end{align*}
\] (3.9)

which imply that the gauge groups

\[U(3) \times U(2) \times U(1)_{1,(\pm 1)} \times U(1)_{0,(\pm 1,0,0)}\]
have been arranged in the worldvolumes of the D3-, D7- and \( \tilde{D}7 \)-branes near these two fixed points. \( U(1)_{0,(\pm 1,0,0)} \) correspond to the gauge fields on D3-branes at fixed points \((\pm 1, 0, 0)\) and \( U(1)_{1,(\pm 1)} \) to the gauge fields on \( \tilde{D}7_{3,(\pm 1)} \)-branes. They are independent of one another and have nothing to do with the gauge groups near origin.

The similarities between solutions (3.9) and (2.8) result in the conclusion that there are two families of Standard Model left-handed massless fermionic states located at these two fixed points, and one family at each point. We summarize the massless chiral fermionic spectra into the following two tables:

**At Fixed point \( (1, 0, 0) \):**

| State | Sector | \text{Rep. of SU}(3) | \text{Rep. of SU}(2) | \( Q_3 \) | \( Q_{2,(0)} \) | \( Q_{1,(1)} \) | \( Q_{0,(1,0,0)} \) |
|-------|--------|---------------------|---------------------|----------|----------------|----------------|----------------|
| \((3, 2)\) | \( 7_5 7_4 \) | 3 | 2 | 1 | -1 | 0 | 0 |
| \((3, 1)\) | \( 7_3 7_5 \) | 3 | 1 | -1 | 0 | 1 | 0 |
| \((3, 1)\) | \( 3_7 5 \) | 3 | 1 | -1 | 0 | 0 | 1 |
| \((1, 2)\) | \( 7_{4,3} \) | 1 | 2 | 0 | 1 | 0 | -1 |
| \((1, 1)\) | \( 3_{7,3} \) | 1 | 1 | 0 | 0 | -1 | 1 |

**At Fixed point \( (-1, 0, 0) \):**

| State | Sector | \text{Rep. of SU}(3) | \text{Rep. of SU}(2) | \( Q_3 \) | \( Q_{2,(0)} \) | \( Q_{1,(-1)} \) | \( Q_{0,(-1,0,0)} \) |
|-------|--------|---------------------|---------------------|----------|----------------|----------------|----------------|
| \((3, 2)\) | \( 7_5 7_4 \) | 3 | 2 | 1 | -1 | 0 | 0 |
| \((3, 1)\) | \( 7_3 7_5 \) | 3 | 1 | -1 | 0 | 1 | 0 |
| \((3, 1)\) | \( 3_7 5 \) | 3 | 1 | -1 | 0 | 0 | 1 |
| \((1, 2)\) | \( 7_{4,3} \) | 1 | 2 | 0 | 1 | 0 | -1 |
| \((1, 1)\) | \( 3_{7,3} \) | 1 | 1 | 0 | 0 | -1 | 1 |

So far, we have exactly obtained the correct left-handed fermion spectrum of Standard Model with three generations. The weak hypercharge is defined as

\[
Y = \frac{2}{3} Q_3 + \frac{1}{2} Q_{2,(0)} + \sum_{m=0, \pm 1} Q_{0,(m,0,0)} \quad (3.10)
\]

The following is the hypercharge distribution in one family of the massless chiral fermion spectrum:
3.3 Gauge group and chiral fermion spectrum at fixed points \((0, \pm 1, 0)\)

The twisted R-R tadpole cancellation conditions have been listed in Eqs. (3.2). Corresponding to the assignment in (3.8), we have the following gamma matrices at these fixed points:

\[
\begin{align*}
\gamma_{1,3,(0, \pm 1,0)} &= 0 \\
\gamma_{1,\tilde{7}_3,(0)} \gamma_{W4,\tilde{7}_3}^{\pm 1} &= -\alpha 1_1 \\
\gamma_{1,\tilde{7}_4,(\pm 1)} &= \alpha 1_1 \\
\gamma_{1,\tilde{7}_5,(0)} \gamma_{W4,\tilde{7}_5} &= \text{diag}(\alpha^2 1_1, 1_1, \alpha 1_1) \\
\gamma_{1,\tilde{7}_5,(0)}^{-1} \gamma_{W4,\tilde{7}_5} &= \text{diag}(\alpha^2 1_1, \alpha 1_1, 1_1)
\end{align*}
\] (3.11)

which correspond to the following assignments of gauge groups

\[
\begin{align*}
\tilde{\text{D}}7_3 \text{ Wilson line} &: U(1)_{1,(0)} \\
\text{D}7_4 \text{ Wilson line} &: U(1)_{2,(\pm 1)} \\
\tilde{\text{D}}7_5 \text{ Wilson line} &: U(1)_{31} \times U(1)_{32} \times U(1)_{33}
\end{align*}
\] (3.12)

on the D-branes’ worldvolumes near these fixed points. The conditions of determining massless left-handed fermion spectrum at these fixed points are stated as,

\[
\begin{align*}
\tilde{\text{D}}7_3 \text{D}4 &: \lambda = \alpha \left[ \gamma_{1,\tilde{7}_3,(0)} \gamma_{W4,\tilde{7}_3}^{\pm 1} \right] \lambda \gamma_{1,\tilde{7}_4,(\pm 1)}^{-1} \\
\text{D}4 \tilde{\text{D}}7_3 &: \lambda = \alpha \left[ \gamma_{1,\tilde{7}_4,(\pm 1)} \gamma_{W4,\tilde{7}_3}^{\pm 1} \right] \lambda \gamma_{1,\tilde{7}_5,(0)}^{-1} \gamma_{W4,\tilde{7}_5}^{-1} \\
\tilde{\text{D}}7_3 \text{D}5 &: \lambda = -\alpha \left[ \gamma_{1,\tilde{7}_3,(0)} \gamma_{W4,\tilde{7}_3}^{\pm 1} \right] \lambda \gamma_{1,\tilde{7}_5,(0)} \gamma_{W4,\tilde{7}_5}^{-1} \\
\text{D}5 \tilde{\text{D}}7_3 &: \lambda = -\alpha \left[ \gamma_{1,\tilde{7}_5,(0)} \gamma_{W4,\tilde{7}_5}^{\pm 1} \right] \lambda \gamma_{1,\tilde{7}_3,(0)}^{-1} \gamma_{W4,\tilde{7}_3}^{-1} \\
\text{D}7_3 \text{D}7_5 &: \lambda = -\alpha^2 \left[ \gamma_{1,\tilde{7}_4,(\pm 1)} \gamma_{W4,\tilde{7}_3}^{\pm 1} \right] \lambda \gamma_{1,\tilde{7}_5,(0)}^{-1} \\
\text{D}7_5 \text{D}7_4 &: \lambda = -\alpha^2 \left[ \gamma_{1,\tilde{7}_5,(0)} \gamma_{W4,\tilde{7}_5}^{\pm 1} \right] \lambda \gamma_{1,\tilde{7}_4,(\pm 1)}^{-1}
\end{align*}
\] (3.13)

They lead to two massless left-handed Ramond fermion states at each fixed point. The chiral fermion spectrum can be summarized as,
3.4 Gauge groups and chiral fermion spectra at fixed points \((0, 1, \pm 1)\) and \((0, -1, \pm 1)\)

The twisted R-R tadpole cancellation conditions at these four fixed points have been given in Eqs. (3.3). With the assignments of Eq. (3.8), we get the following solutions for these conditions:

\[
\begin{align*}
\gamma_{1,3,(0,1,\pm 1)} &= \gamma_{1,3,(0,-1,\pm 1) = 0} \\
\gamma_{1,5_{3}(0)} \gamma_{W4_{3},5_{3}}^{\pm 1} &= -\alpha \mathbb{I}_1 \\
\gamma_{1,7_{4},(\pm 1)} &= \alpha \mathbb{I}_1 \\
\gamma_{1,7_{5},(\pm 1)} &= 0
\end{align*}
\]  

(3.14)

These orbifold actions on Chan-Paton wavefunctions imply that there are no D3- and D75-branes located at these fixed points. In other words, these fixed points are only wrapped by \(\tilde{\text{D}}7_{3,0}\)-branes and \(\tilde{\text{D}}7_{4,\pm 1}\)-branes with which the gauge groups \(U(1)_{1,0}\) and \(U(1)_{2,\pm 1}\) are associated respectively. The Chan-Paton wavefunctions at these fixed points are determined by,

\[
\begin{align*}
\tilde{\text{D}}7_{3}\text{D}7_{4} : \quad \lambda &= \alpha \left[ \gamma_{1,5_{3}(0)} \gamma_{W4_{3},5_{3}}^{\pm 1} \right] \lambda \gamma_{1,7_{4},(\pm 1)}^{-1} \\
\tilde{\text{D}}7_{4}\tilde{\text{D}}7_{3} : \quad \lambda &= \alpha \gamma_{1,7_{4},(\pm 1)} \lambda \left[ \gamma_{1,5_{3}(0)} \gamma_{W4_{3},5_{3}}^{\pm 1} \right]^{-1}
\end{align*}
\]  

(3.15)

There is no qualified massless chiral fermionic state subject to these constraints.

3.5 Gauge groups and chiral fermion spectra at fixed points \((m, 0, \pm 1)\)

The twisted R-R tadpole cancellation conditions at these six fixed points have been given in Eqs. (3.4). With the assignments of Eq. (3.8), we get the
following solutions for these conditions:

\[
\begin{align*}
\gamma_{1,3,(m,0,\pm 1)} &= -\alpha 1_1 \\
\gamma_{1,\tilde{7}_3,(m)} &= -\alpha 1_1 \\
\gamma_{1,7_4,(0)} &= -\alpha 1_2 \\
\gamma_{1,7_5,(\pm 1)} &= 0
\end{align*}
\]  

(3.16)

Therefore, there is one D3-brane at each (considered) fixed point. In addition, each of these six points is wrapped by a \( \tilde{D}_7 \)-brane and two D7-4-branes. On the worldvolumes of these D-branes and anti-branes the gauge groups are distributed as,

\[
\begin{align*}
\text{D3}_{(m,0,\pm 1)} : & \quad U(1)_{0,(m,0,\pm 1)} \\
\tilde{D}_7_{3,(m)} : & \quad U(1)_{1,(m)} \\
D_7_{4,(0)} : & \quad U(2) = U(1)_{2,(0)} \times SU(2)
\end{align*}
\]  

(3.17)

There is no massless left-handed fermionic state created from the open string sectors near these fixed points.

3.6 Gauge groups and chiral fermion spectra at fixed points \((\pm 1, 1, 0)\) and \((\pm 1, -1, 0)\)

The twisted R-R tadpole cancellation conditions at these four fixed points have been given in Eqs.(3.5). With the assignments of Eq.(3.8), we get the following solutions for these conditions:

\[
\begin{align*}
\gamma_{1,3,(\pm 1,1,0)} &= \gamma_{1,3,(\pm 1,-1,0)} = 0 \\
\gamma_{1,\tilde{7}_3,(\pm 1)} &= -\alpha 1_1 \\
\gamma_{1,7_4,(\pm 1)} &= \alpha 1_1 \\
\gamma_{1,7_5,(0)}\gamma_{W4,7_5} &= \text{diag}(\alpha^2 1_1, 1_1, \alpha 1_1) \\
\gamma_{1,7_5,(0)}\gamma_{W4,7_5}^{-1} &= \text{diag}(\alpha^2 1_1, \alpha 1_1, 1_1)
\end{align*}
\]  

(3.18)

Thereby, every fixed point in this set is wrapped by one \( \tilde{D}_7 \)-brane, one D7-4-brane and three D7-5-branes. There is no D3-branes located at these points. Because of the presence of the Wilson line passing through the origin and along the 4-th complex plane, the gauge groups near these fixed points at respectively \( U(1)_{1,(\pm 1)} \) on \( \tilde{D}_7_{3,(\pm 1)} \)-branes, \( U(1)_{2,(\pm 1)} \) on D7-4(\pm 1)-branes and \( U(1)_{31} \times U(1)_{32} \times U(1)_{33} \) on D7-5(0)-branes. The qualified Chan-Paton
wavefunctions at these fixed points satisfy constraint conditions:

\[
\begin{align*}
\bar{D}7_3 D7_4 : & \quad \lambda = \alpha \gamma_{1,\bar{7},3,(\pm 1)} \lambda \gamma_{1,\bar{7},4,(n)}, \quad (n = \pm 1) \\
D7_3 \bar{D}7_3 : & \quad \lambda = \alpha \gamma_{1,\bar{7},4,(n)} \lambda \gamma_{1,\bar{7},3,(\pm 1)}, \quad (n = \pm 1) \\
\bar{D}7_3 D7_5 : & \quad \lambda = -\alpha \gamma_{1,\bar{7},5,(\pm 1)} \lambda \gamma_{1,\bar{7},5,(0)} ]^{-1}, \quad (n = \pm 1) \\
D7_5 \bar{D}7_3 : & \quad \lambda = -\alpha \gamma_{1,\bar{7},4,(n)} \lambda \gamma_{1,\bar{7},3,(\pm 1)} \lambda \gamma_{1,\bar{7},5,(0)} ]^{-1}, \quad (n = \pm 1) \\
D7_3 D7_5 : & \quad \lambda = -\alpha \gamma_{1,\bar{7},4,(n)} \lambda \gamma_{1,\bar{7},5,(0)} ]^{-1}, \quad (n = \pm 1) \\
D7_5 D7_4 : & \quad \lambda = -\alpha \gamma_{1,\bar{7},4,(n)} \lambda \gamma_{1,\bar{7},5,(0)} ]^{-1}, \quad (n = \pm 1)
\end{align*}
\]

There are totally eight massless left-handed fermion states created from the open string sectors near these four fixed points (two states for each point). These chiral states form the unit representations for both SU(3) and SU(2). We summarize their U(1)-charges in the following table:

| Fixed Point | Sector | State | $Q_{31}$ | $Q_{32}$ | $Q_{33}$ | $Q_{22,(\pm 1)}$ | $Q_{1,1,(\pm 1)}$ | $Q_{1,(-1)}$ |
|-------------|--------|-------|--------|--------|--------|----------------|----------------|----------|
| $(1, 1, 0)$ | $7_3 \bar{7}_5$ | $(1, 1)$ | -1 | 0 | 0 | 0 | 1 | 0 |
| $(1, 1, 0)$ | $7_5 \bar{7}_3$ | $(1, 1)$ | 0 | 1 | 0 | 0 | -1 | 0 |
| (-1, 1, 0) | $7_3 \bar{7}_5$ | $(1, 1)$ | -1 | 0 | 0 | 0 | 0 | 1 |
| (-1, 1, 0) | $7_5 \bar{7}_3$ | $(1, 1)$ | 0 | 1 | 0 | 0 | 0 | -1 |
| $(1, -1, 0)$ | $7_3 \bar{7}_5$ | $(1, 1)$ | -1 | 0 | 0 | 0 | 1 | 0 |
| $(1, -1, 0)$ | $7_5 \bar{7}_3$ | $(1, 1)$ | 0 | 0 | 1 | 0 | -1 | 0 |
| (-1, -1, 0) | $7_3 \bar{7}_5$ | $(1, 1)$ | -1 | 0 | 0 | 0 | 0 | 1 |
| (-1, -1, 0) | $7_5 \bar{7}_3$ | $(1, 1)$ | 0 | 0 | 1 | 0 | 0 | -1 |

3.7 Gauge groups and chiral fermion spectra at fixed points $(m, n, p)$ with $m, n, p = \pm 1$

The twisted R-R tadpole cancellation conditions at these eight fixed points have been listed in Eqs. (3.16). With the assignments of Eq. (3.18), we have the following solutions:

\[
\begin{align*}
\gamma_{1,\bar{7},3,(m,n,p)} &= 0, \quad (m, n, p = \pm 1) \\
\gamma_{1,\bar{7},5,(m)} &= -\alpha 1_1, \quad (m = \pm 1) \\
\gamma_{1,\bar{7},4,(n)} &= \alpha 1_1, \quad (n = \pm 1) \\
\gamma_{1,\bar{7},5,(p)} &= 0, \quad (p = \pm 1)
\end{align*}
\]

There are totally eight massless left-handed fermion states created from the open string sectors near these four fixed points (two states for each point). These chiral states form the unit representations for both SU(3) and SU(2). We summarize their U(1)-charges in the following table:
D7-brane only. There is an abelian gauge group $U(1)_{1,m} \times U(1)_{2,n}$ on the D-brane worldvolume near each fixed point $(m, n, p)$ in this set. However, there is no massless left-handed fermion state created from the corresponding open string sectors.

Let us make a brief summary for the obtained massless fermion spectra. At fixed points $(m, 0, 0)$ with $m = 0, \pm 1$, we get exactly the non-supersymmetric Standard model chiral fermion spectra with three generations. At the other six fixed points $(m, \pm 1, 0)$ with $m = 0, \pm 1$, there are twelve extra massless left-handed fermions which nevertheless form the singlets of nonabelian gauge group $SU(3) \times SU(2)$. There are no massless chiral fermions located at remaining eighteen fixed points.

4 Gauge Anomalies and Cancellation Mechanism

According to the obtained chiral fermion spectra, there are no any cubic non-abelian gauge anomalies at all $\mathbb{Z}_3$ orbifold fixed points. There are also not any $U(1)$ anomalies at these fixed points except at $(0, 0, 0)$ and $(\pm 1, 0, 0)$. There exist potential $U(1)$ anomalies at these three fixed points. Relying on the coincidence of the massless chiral fermionic spectra, it is only necessary to discuss the $U(1)$ anomalies at orbifold origin and their cancellation mechanism.

4.1 Mixed $U(1)$-nonabelian anomalies

Taking the following normalization for $SU(n)$’s generators (where $n = 2, 3$),

$$Tr(\lambda_a \lambda_b) = \frac{1}{2} \delta_{ab}, \quad Tr(\lambda_a) = 0$$

we can easily get from the massless left-handed fermion spectrum at origin that,

$$Tr \left[ Q_0 \lambda^2_{SU(2)} \right] = -1 \quad Tr \left[ Q_0 \lambda^2_{SU(3)} \right] = 1$$

$$Tr \left[ Q_1 \lambda^2_{SU(2)} \right] = 0 \quad Tr \left[ Q_1 \lambda^2_{SU(3)} \right] = 1$$

$$Tr \left[ Q_2 \lambda^2_{SU(2)} \right] = -2 \quad Tr \left[ Q_2 \lambda^2_{SU(3)} \right] = -2$$

$$Tr \left[ Q_3 \lambda^2_{SU(2)} \right] = 3 \quad Tr \left[ Q_3 \lambda^2_{SU(3)} \right] = 0$$

(4.2)

For sake of convenience, we have simply denoted the $U(1)$ charges $\{Q_{0,(0,0,0)}, Q_{1,(0)}\}$ as $\{Q_0, Q_1\}$ in Eq.(4.2) (and thereafter). These traces describe the possi-
ble mixed $U(1)$-nonabelian anomalies at the orbifold origin. If we introduce alternatively another set of the orthogonal $U(1)$ charges

$$
\begin{align*}
Y &= \frac{2}{3}Q_3 + \frac{1}{2}Q_2 + Q_0 \\
\tilde{Q}_1 &= Q_1 \\
\tilde{Q}_2 &= 3aQ_3 + 2bQ_2 - (2a + b)Q_0 \\
\tilde{Q}_3 &= 3cQ_3 + 2dQ_2 - (2c + d)Q_0
\end{align*}
$$

(4.3)

where,

$$
a = \sqrt{21533} + 119 \quad b = -194
$$

(4.4)

c = \sqrt{21533} - 119 \quad d = 194

and interpret $Y$ as the weak hypercharge at the origin, we can prove that, among these four independent $U(1)$ charges, only the hypercharge is free of the $U(1)$-nonabelian gauge anomalies:

$$
Tr [Y^2_{SU(2)}] = Tr [Y^2_{SU(3)}] = 0
$$

(4.5)

The other three ones remain anomalous.

### 4.2 Pure $U(1)$ anomalies

The pure $U(1)$ anomalies can be categorized into three types: the cubic $U(1)$ anomalies, the mixed $U(1)$ anomalies and the triple mixed $U(1)$ anomalies. At $\mathbb{Z}_3$ orbifold origin, the cubic $U(1)$ anomalies are measured by traces:

$$
Tr [Q^3_0] = 4 \quad Tr [Q^3_1] = 4 \quad Tr [Q^3_2] = -8 \quad Tr [Q^3_3] = 0
$$

(4.6)

The mixed $U(1)$ anomalies are measured by the following traces:

$$
\begin{align*}
Tr [Q^2_0Q_1] &= -Tr [Q^2_1Q_0] = -2 \\
Tr [Q^2_0Q_2] &= -Tr [Q^2_2Q_0] = 4 \\
Tr [Q^2_0Q_3] &= -Tr [Q^2_3Q_0] = -6 \\
Tr [Q^2_1Q_2] &= -Tr [Q^2_2Q_1] = 0 \\
Tr [Q^2_1Q_3] &= -Tr [Q^2_3Q_1] = -6 \\
Tr [Q^2_2Q_3] &= -Tr [Q^2_3Q_2] = 12
\end{align*}
$$

(4.7)

In the obtained chiral fermionic spectrum, each left-handed fermion state form the fundamental representation of one factorial gauge group and the anti-fundamental representation of another factorial gauge group. There is
no such a fermion state that has three non-vanishing distinct $U(1)$ charges. Therefore, the model under consideration is free of the triple mixed $U(1)$ anomalies:

\[
\begin{align*}
Tr[Q_0Q_1Q_2] &= 0 \\
Tr[Q_0Q_1Q_3] &= 0 \\
Tr[Q_0Q_2Q_3] &= 0 \\
Tr[Q_1Q_2Q_3] &= 0
\end{align*}
\] (4.8)

Particularly, the hypercharge $Y$ is free of the cubic $U(1)$ anomaly:

\[
Tr(Y^3) = 0
\] (4.9)

### 4.3 Factorized anomaly traces and mechanism for $U(1)$ anomaly cancellation

It is remarkable that all the $U(1)$ anomalies appearing in our model can be cancelled through Green-Schwarz mechanism\[^{13}\]. To make this conclusion transparent, we now search the equivalent but more insightful expressions of the above anomaly traces\[^{13}\]. First, let us define an extended Delta symbol in $\mathbb{Z}_3$ orbifold case,

\[
\delta_{jl} = \begin{cases} 
1 & \text{if } j - l = 3N \quad (N \text{ is an arbitrary integer}) \\
0 & \text{otherwise}
\end{cases}
\] (4.10)

Then,

\[
\begin{align*}
\sum_{b_r=0}^{3}\left(\delta_{2,0+b_r} - \delta_{2,-b_r}\right) &= -3 \\
\sum_{b_r=0}^{3}\left(\delta_{2,3+b_r} - \delta_{2,-b_r}\right) &= -3 \\
\sum_{b_r=0}^{3}\left(\delta_{3,1+b_r} - \delta_{3,-b_r}\right) &= -3 \\
\sum_{b_r=0}^{3}\left(\delta_{3,2+b_r} - \delta_{3,-b_r}\right) &= 3
\end{align*}
\] (4.11)

With the help of the discrete Fourier transformation

\[
3\delta_{l,j\pm b_r} = \sum_{k=1}^{3} \exp\left[2\pi ik(j - l \pm b_r)/3\right]
\] (4.12)

and the mathematical identity

\[
\sum_{r=3}^{5} \sin\left(\frac{2\pi kb_r}{3}\right) = -\frac{1}{2} \prod_{r=3}^{5} \left[2 \sin\left(\frac{\pi kb_r}{3}\right)\right]
\] (4.13)
we can obtain from Eqs.(4.11) the following two significant mathematic expressions:

\[ 1 = \pm \frac{i}{9} \sum_{k=1}^{3} \left( \prod_{r=3}^{5} 2 \sin(\frac{\pi kb_r}{3}) \right) \exp(\pm \frac{2\pi ik}{3}) \quad (4.14) \]

On the other hand, it follows directly from Eq.(2.8) that,

\[ 1 = Tr\gamma_{k,3,(0,0,0)} \]
\[ \exp[ (2\pi ik)/3 ] = (-1)^k Tr\gamma_{k,\tilde{r}_3,(0)} \]
\[ 2 \exp[ (2\pi ik)/3 ] = (-1)^k Tr\gamma_{k,\tilde{r}_4,(0)} \]
\[ 3 \exp[ -(2\pi ik)/3 ] = Tr\gamma_{k,\tilde{r}_5,(0)} \quad (4.15) \]

and

\[ 1 = Tr\gamma_{k,3,(0,0,0)}^{-1} \]
\[ \exp[-(2\pi ik)/3] = (-1)^k Tr\gamma_{k,\tilde{r}_3,(0)}^{-1} \]
\[ 2 \exp[-(2\pi ik)/3] = (-1)^k Tr\gamma_{k,\tilde{r}_4,(0)}^{-1} \]
\[ 3 \exp[ (2\pi ik)/3 ] = Tr\gamma_{k,\tilde{r}_5,(0)}^{-1} \quad (4.16) \]

where \( k = 1, 2, 3 \). Therefore, the cubic \( U(1) \) anomaly traces in Eq.(4.6) can be rewritten as,

\[ \frac{1}{3} Tr[Q_0^3] = \frac{4}{9} \sum_{k=1}^{3} Tr[\gamma_{k,3,(0,0,0)}] Tr[\gamma_{k,3,(0,0,0)}] \]
\[ = \frac{4}{9} \sum_{k=1}^{3} Tr[\gamma_{k,3,(0,0,0)}] Tr[\gamma_{k,3,(0,0,0)}] \sum_{k=1}^{3} Tr[\gamma_{k,3,(0,0,0)}] \sum_{k=1}^{3} Tr[\gamma_{k,3,(0,0,0)}] \quad (4.17) \]

\[ \frac{1}{3} Tr[Q_1^3] = \frac{4}{9} \sum_{k=1}^{3} Tr[\gamma_{k,\tilde{r}_3,(0)}] Tr[\gamma_{k,\tilde{r}_3,(0)}] \]
\[ = \frac{4}{9} \sum_{k=1}^{3} Tr[\gamma_{k,\tilde{r}_3,(0)}] Tr[\gamma_{k,\tilde{r}_3,(0)}] \sum_{k=1}^{3} Tr[\gamma_{k,\tilde{r}_3,(0)}] \sum_{k=1}^{3} Tr[\gamma_{k,\tilde{r}_3,(0)}] \quad (4.18) \]

\[ \frac{1}{3} Tr[Q_2^3] = \frac{4}{9} \sum_{k=1}^{3} Tr[\gamma_{k,\tilde{r}_4,(0)}] Tr[\gamma_{k,\tilde{r}_4,(0)}] \]
\[ = \frac{4}{9} \sum_{k=1}^{3} Tr[\gamma_{k,\tilde{r}_4,(0)}] Tr[\gamma_{k,\tilde{r}_4,(0)}] \sum_{k=1}^{3} Tr[\gamma_{k,\tilde{r}_4,(0)}] \sum_{k=1}^{3} Tr[\gamma_{k,\tilde{r}_4,(0)}] \quad (4.19) \]

and \( \frac{1}{3} Tr[Q_3^3] = 0 \). The mixed \( U(1) \) anomaly traces can also acquire similar expressions:

\[ \begin{align*}
Tr[Q_0^2Q_1] &= -\frac{2}{9} \sum_{k=1}^{3} (-1)^k \left[ \Pi_{r=3}^{5} 2 \sin(\pi kb_r/3) \right] \cdot Tr[\gamma_{k,\tilde{r}_3,(0)}] Tr[\gamma_{k,\tilde{r}_3,(0)}] Tr[\gamma_{k,\tilde{r}_3,(0)}] \sum_{k=1}^{3} Tr[\gamma_{k,\tilde{r}_3,(0)}] \sum_{k=1}^{3} Tr[\gamma_{k,\tilde{r}_3,(0)}] \quad (4.20)
\end{align*} \]

\[ \begin{align*}
Tr[Q_0^2Q_2] &= \frac{2}{9} \sum_{k=1}^{3} (-1)^k \left[ \Pi_{r=3}^{5} 2 \sin(\pi kb_r/3) \right] \cdot Tr[\gamma_{k,\tilde{r}_4,(0)}] Tr[\gamma_{k,\tilde{r}_4,(0)}] Tr[\gamma_{k,\tilde{r}_4,(0)}] \sum_{k=1}^{3} Tr[\gamma_{k,\tilde{r}_4,(0)}] \sum_{k=1}^{3} Tr[\gamma_{k,\tilde{r}_4,(0)}] \quad (4.20)
\end{align*} \]

\[ \begin{align*}
Tr[Q_0^2Q_3] &= \frac{2}{9} \sum_{k=1}^{3} \left[ \Pi_{r=3}^{5} 2 \sin(\pi kb_r/3) \right] \cdot Tr[\gamma_{k,\tilde{r}_5,(0)}] Tr[\gamma_{k,\tilde{r}_5,(0)}] Tr[\gamma_{k,\tilde{r}_5,(0)}] \sum_{k=1}^{3} Tr[\gamma_{k,\tilde{r}_5,(0)}] \sum_{k=1}^{3} Tr[\gamma_{k,\tilde{r}_5,(0)}] \quad (4.20)
\end{align*} \]
The traces (4.2) measuring mixed \( U(1) \) nonabelian anomalies can also be expressed in the similar (factorized) fashion. Noticing, \( Tr[\lambda_{SU(2)}^{2}] = Tr[\lambda_{SU(3)}^{2}] = \frac{1}{2} \), we have:

\[
Tr[\gamma^{-1}_{k,74} \lambda_{SU(2)}^{2}] = \frac{1}{2} (-1)^k \exp(-2\pi ik/3) \quad Tr[\gamma^{-1}_{k,75} \lambda_{SU(3)}^{2}] = \frac{1}{2} \exp(2\pi ik/3)
\]

Moreover,

\[
\begin{align*}
Tr[\gamma_{k,3,(0,0,0)} \lambda_{U(1)_{0,0,0}}] Tr[\gamma^{-1}_{k,74} \lambda_{SU(2)}] & = \frac{1}{2} (-1)^k \exp(-2\pi ik/3) \\
Tr[\gamma_{k,3,(0,0,0)} \lambda_{U(1)_{0,0,0}}] Tr[\gamma^{-1}_{k,75} \lambda_{SU(3)}] & = \frac{1}{2} \exp(2\pi ik/3) \\
Tr[\gamma_{k,74,(0)} \lambda_{U(1)_{1,0}}] Tr[\gamma^{-1}_{k,74} \lambda_{SU(2)}] & = \frac{1}{2} (-1)^k \exp(-2\pi ik/3) \\
Tr[\gamma_{k,74,(0)} \lambda_{U(1)_{2,0}}] Tr[\gamma^{-1}_{k,74} \lambda_{SU(2)}] & = 1 \\
Tr[\gamma_{k,75,(0)} \lambda_{U(1)_{1,0}}] Tr[\gamma^{-1}_{k,75} \lambda_{SU(3)}] & = (-1)^k \exp(-2\pi ik/3) \\
Tr[\gamma_{k,75,(0)} \lambda_{U(1)_{3,0}}] Tr[\gamma^{-1}_{k,75} \lambda_{SU(2)}] & = \frac{3}{2} (-1)^k \exp(2\pi ik/3)
\end{align*}
\]

Rewriting the non-vanishing traces in Eqs. (4.2) with these mathematical
identities, we get:

\[
\begin{align*}
Tr[Q_0 \lambda^2_{SU(2)}] &= \frac{2i}{9} \sum_{k=1}^{3} (-1)^k \left[ \prod_{r=3}^{5} 2 \sin(\pi kb_r/3) \right] Tr[\gamma_{k,4,0}(0)\lambda_{U(1)_a,0,0,0}^1] Tr[\gamma_{k,7,5,0}^1 \lambda^2_{SU(2)}] \\
Tr[Q_0 \lambda^2_{SU(3)}] &= \frac{2i}{9} \sum_{k=1}^{3} (-1)^k \left[ \prod_{r=3}^{5} 2 \sin(\pi kb_r/3) \right] \cdot Tr[\gamma_{k,3,0}(0)\lambda_{U(1)_b,0,0,0}^1] Tr[\gamma_{k,7,5,0}^1 \lambda^2_{SU(3)}] \\
Tr[Q_1 \lambda^2_{SU(3)}] &= -\frac{2i}{9} \sum_{k=1}^{3} (-1)^k \left[ \prod_{r=3}^{5} 2 \sin(\pi kb_r/3) \right] \cdot Tr[\gamma_{k,3,0}(0)\lambda_{U(1)_b,1,0,0}^1] Tr[\gamma_{k,7,5,0}^1 \lambda^2_{SU(3)}] \\
Tr[Q_2 \lambda^2_{SU(3)}] &= \frac{2i}{9} \sum_{k=1}^{3} (-1)^k \left[ \prod_{r=3}^{5} 2 \sin(\pi kb_r/3) \right] \cdot Tr[\gamma_{k,7,5,0}(0)\lambda_{U(1)_a}^1] Tr[\gamma_{k,7,5,0}^1 \lambda^2_{SU(3)}] \\
Tr[Q_3 \lambda^2_{SU(2)}] &= \frac{2i}{9} \sum_{k=1}^{3} (-1)^k \left[ \prod_{r=3}^{5} 2 \sin(\pi kb_r/3) \right] \cdot Tr[\gamma_{k,7,5,0}(0)\lambda_{U(1)_a}^1] Tr[\gamma_{k,7,5,0}^1 \lambda^2_{SU(2)}]
\end{align*}
\]

and

\[
\begin{align*}
Tr[Q_2 \lambda^2_{SU(2)}] &= -\frac{2}{3} \sum_{k=1}^{3} Tr[\gamma_{k,4,0}(0)\lambda_{U(1)_a}^1] Tr[\gamma_{k,7,5,0}^1 \lambda^2_{SU(2)}]
\end{align*}
\]

In view of the above factorized expressions of the \(U(1)\) anomaly traces, we easily see that they can be cancelled by the coupling in the closed string sectors of the \(U(1)\) to a R-R field \(B_{k}^{\mu
u}\),

\[Tr(\gamma_{k,\lambda}(\lambda_{k})B_{k} \wedge F_{U(1)})\]

and that of the other \(U(1)\)'s or nonabelian gauge fields to the same R-R field \(B_{k}^{\mu
u}\),

\[Tr(\gamma_{k}^{-1} \lambda_{G}^2 (\partial^{[\mu} B_{k}^{\nu\rho]}) W_{\mu\nu\rho}^{CS})\]

This is because that these closed string low-energy amplitude will be proportional to\[3\]

\[
\begin{align*}
\sum_{k=1}^{3} C_k^{pq} Tr(\gamma_{k,p}\lambda_{k}^{(p)})Tr[\gamma_{k,q}(\lambda_{k}^{(q)})^2]
\end{align*}
\]

After the cancellation of \(U(1)\) anomalies via above Green-Schwarz mechanism, three of the four independent \(U(1)\) charges become into the global symmetries of the system, while the remaining one which is equivalent to weak hypercharge

\[Y = \frac{2}{3} Q_3 + \frac{1}{2} Q_2 + Q_0\]

survives as an (Abelian) gauge interaction. Needless to say, this is just what we expect. Because \(Q_3\) can be interpreted as the baryon number, its conservation guarantees the stability of proton.
5 Conclusions

In this paper, we have succeeded getting the chiral fermion spectrum of standard model and 3-generation replication of quark-lepton families from the $D = 4$ Type IIB orbifold $T^6/\mathbb{Z}_3$. A stack of D-branes (D3-, D74- and D75-) and anti-D-branes (D7̄3) are necessary to locate at orbifold singularities. The obtained model is non-supersymmetric but it is free of cubic $SU(3)$ gauge anomaly. The possible $U(1)$ gauge anomalies can be cancelled via a generalized Green-Schwarz mechanism. As a result, the proton decay is avoided by the global “baryon number” conservation.

It is plausible that the present paper provides a stringy realization of the phenomenological D-brane model of Ref.[2]. There is no unification for gauge coupling constants in this system. We suppose that the $U(1)_j$ charge is measured with respect to the corresponding coupling $g_j/\sqrt{2j}$, with $g_j$ the $SU(j)$ coupling constant. Then the expression for hypercharge leads to:

$$\frac{1}{g_Y} = \frac{6}{g_3}(\frac{2}{3})^2 + \frac{4}{g_2}(\frac{1}{2})^2 + \frac{2}{g_0}$$

This gives rise to the following expression for weak angle,

$$\sin^2 \theta_W \equiv \frac{g_Y^2}{g_2^2 + g_Y^2} = \left[ 2 + \frac{8}{3}(\frac{g_2}{g_3})^2 + 2(\frac{g_2}{g_0}) \right]^{-1}$$

In order to compare the above $\sin^2 \theta_W$ with low-energy data, running from the string scale to the weak scale should be taken into account. Just as what the authors of Ref.[2] did, we can postulate that the coupling constant $g_0$ is either $g_3$ or $g_2$. In this way, it seems possible for our model to accommodate the low-energy phenomenology.

The non-supersymmetric D-brane orbifold under consideration is free of the instability due to the brane-anti-brane annihilation. In fact there is no tachyon state in its NS open string spectrum. In view of the analysis of Ref.[7], tachyons only appear when the model contains coincident or very close branes and anti-branes of the same type, which does not happen in our orbifold. Relying on the absence of tachyon states, at string scale the Higgs mass squared would be positive and the spontaneous gauge symmetry breaking at the electroweak scale would be triggered by the radiative
corrections. This mechanism is conjectured to be very similar to the radiative electroweak symmetry breaking of the minimal supersymmetric standard model (MSSM), which will depend strongly upon the Yukawa couplings and the structure of the full global model. Furthermore, the global cancellation of R-R charges (the untwisted R-R tadpoles) may be achieved by identifying the considered \( Z_3 \) Type IIB orbifold as a subspace of \( Z_6 \) Type IIB orientifold. It is expected that in the \( Z_6 \) orientifold, which contains the \( Z_3 \) orbifold fixed points, the total R-R charge of the branes and anti-branes in our present model could be cancelled by the opposite R-R charge carried by the orientifold planes. We will discuss these issues in the near future.

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