Study on bond selection under intuitionistic fuzzy conditions

Xiao-Guang Zhou¹, Yang-Fan Ding¹ and Mi Lu²

Abstract

Intuitionistic fuzzy preference relations can take membership degrees, non-membership degrees, and hesitancy degrees into account during decision making. It has good practicability and flexibility in dealing with fuzzy and uncertain information. As for analytic network process, it is performed by thinking over the interaction and feedback relationships between criteria and indices, so that an effective method is provided for multi-criteria decision making. An index system with network structure for evaluating the bonds is presented, and a comprehensive method by combining the advantages of intuitionistic fuzzy preference relations and analytic network process is proposed to select and rank the bonds. A case study is given by the proposed method as well.

Keywords

Bond selection, intuitionistic fuzzy preference relation, analytic network process, fuzzy decision making

Introduction

How to select bonds is a very important issue for bond investors. It is inevitable to take multiple factors into consideration during the process of bond selection, such as the rate of return of the bond, the risk of the bond, market environment, national policy, etc. For this reason, bond selection becomes a multi-criteria decision-making problem. Flanigan et al. investigated the market selection factors for issuing foreign bonds by logistic regression models.¹ Aquila and Ronchetti analyzed the discrimination power of a common model for stock and bond returns.² Ebner studied the spreads between 10-year Euro denominated Central and Eastern European government bonds and their German counterpart and pointed out that higher European Central Bank reference rate and market volatility increased bond spreads and turned out to be the main driving factors.³ The determinants of the domestic and the foreign bond biases and their evolution over time were studied in Ferreira and Miguel.⁴ Bhattacharyay identified the determinants of bond market development in Asian economies through examining the relationships between the key financial and economic factors for bond issuance.⁵ Huang et al. analyzed the influence of international political risk on government bond yields in 34 debtor countries using a comprehensive database of 109 international political crises.⁶ The determinants of sovereign spreads in 31 emerging economies from 1994 to 2014 were discussed in Tebaldi et al.⁷ Macroeconomic explains only one-tenth of the daily variation in bond yields; its explanatory power improves substantially at lower frequencies, accounting for one-third of quarterly variations.⁸ A combination of structural, financial, and institutional factors seems to exert a significant effect on bond markets according to Smaoui et al.⁹ In fact, economic size, trade openness, investment profile, GDP per Capita, bureaucratic quality, and size and concentration of banking system are positively related to bond market development, while interest rate volatility and fiscal balance are negatively associated with the development of bond markets.¹⁰ In the process of bond selection, not only do various decision-making factors...
need to be taken into account, but also the interaction and feedback relationships between criteria and indices should be considered.

Analytic hierarchy process (AHP) is a multi-criteria decision-making method that combines qualitative and quantitative analysis. However, the AHP only considers the impacts of superior criteria on subordinate factors, so that the mutual influences among indices are ignored. As a consequence, ultimate results may dramatically deviate from the reality. Analytic network process (ANP) was introduced by Saaty, which is the extension of AHP.\(^\text{11}\) According to ANP, its network structure that contains interaction and feedback relationships are far more complex and reasonable than that of AHP. Therefore, ANP is adopted here to solve the decision-making process of bond selection.

The decision makers tend to take different viewpoints toward a problem due to discrepancies in personal knowledge background, practical experience, subjective preference, and risk preference. Hence, diverse experts may appraise criteria and schemes of an identical problem differently, which makes problems greatly uncertain. Atanassov presented the concept of intuitionistic fuzzy set.\(^\text{12}\) To be specific, hesitancy degree is added so as to make information expressed in a more comprehensive manner.

In the process of decision making, it is rather difficult for experts or decision makers to make accurate judgments about the attributes of schemes. For this reason, judgment matrices are formed by intuitionistic fuzzy preference relations (IFPRs), which are used to describe the preference information of experts or decision makers. Xu first presented IFPRs and defined additive and multiplicative consistency of IFPRs.\(^\text{13}\) Gong et al. put forward additive consistency of different IFPRs and developed a synthetic approach of the least square method and objective programming to solve the weight vectors of the IFPRs.\(^\text{14}\) Wang constructed a consistent IFPR, so that linear goal programming models could be set up by minimizing the deviations between IFPRs and priority weights.\(^\text{15}\) A comprehensive survey on decision making with IFPRs was presented with the aim of providing a clear perspective on the originality, the consistency, the prioritization, and the consensus of IFPRs in Xu and Liao.\(^\text{16}\) Chen and Chang proposed a new method for fuzzy multi-attribute decision making based on the proposed transformation techniques between intuitionistic fuzzy values and right-angled triangular fuzzy numbers and the intuitionistic fuzzy geometric averaging operators.\(^\text{17}\) Xu and Zhao presented an overview of the existing intuitionistic fuzzy decision-making theories and methods from the perspective of information fusion.\(^\text{18}\) The intuitionistic fuzzy best–worst method for multi-criteria group decision making was proposed by Mou et al.\(^\text{19}\)

In this paper, bond selection and ranking method are proposed by IFPRs and ANP. The main contributions of this study are as below.

1. An index system for bond selection is presented from the four aspects: rate of return, risk, investment strategy, and regional difference. Different from the existing research results, an ANP network structure is presented to describe the interaction and feedback among indices.

2. Combining the advantages of intuitionistic fuzzy preference relations with ANP, an intuitionistic fuzzy ANP method for bond selection is proposed. In order to accurately express the preference information of experts or decision makers under fuzzy conditions, the IFPRs are applied to form judgment matrices. The unweighted supermatrix, the weighted supermatrix, and the limit supermatrix under intuitionistic fuzzy preference relations are constructed, and then the global weights of the indices are obtained.

3. Taking the selection and sequencing of four bonds as an example, the effectiveness of the proposed method is illustrated. Different investors are able to select an optimal bond investment scheme which is most appropriate for them.

**Intuitionistic fuzzy preference relations**

**Definition 1:** \(X=\{X_1, X_2, \ldots, X_n\}\) is a non-empty set, \(A=\{<x, \mu_A(x), v_A(x)> | 0 \leq \mu_A(x)+v_A(x) \leq 1, x \in X\}\) is called an intuitionistic fuzzy set of \(X,\)\(^\text{12}\) where, \(\mu_A: X \rightarrow [0, 1], \mu_A(x)\) indicates the membership degree of element \(x\) in \(X; v_A: X \rightarrow [0, 1], v_A(x)\) indicates the non-membership degree of element \(x\) in \(X\) to \(A.\)

Additionally, \(\pi_A(x) = 1 - \mu_A(x) - v_A(x)\) indicates the hesitancy degree of element \(x\) in \(X\) to \(A,\) and it describes to what extent the information is unknown to the decision maker during decision making.

**Definition 2:** An IFPR \(R\) in set \(X\) is denoted by an intuitionistic fuzzy judgment matrix \(R=(r_{ij})_{n \times n} = (\mu_{ij}, v_{ij}, \pi_{ij})_{n \times n},\) where, \(\mu_{ij}\) and \(v_{ij}\) respectively indicate that to what degree the element \(x_j\) is superior or inferior to \(x_i\) in opinions of the decision maker according to the relevant criteria; and \(\pi_{ij}\) signifies the hesitation degree for the comparison. Besides, \(\mu_{ij}+v_{ij}+\pi_{ij} = 1,\) and \(\mu_{ij} = (0.5, 0.5, 0)\) for any \(i, j \in X.\)\(^\text{13}\)

**Definition 3:** If an IFPR \(R=(r_{ij})_{n \times n}\) satisfies \(r_{ij} = r_{jk} - r_{ik} + 0.5,\) for \(\forall\ i, j, k = 1, 2, \ldots, n,\) then \(R\) is called an IFPR that meets additive consistency.
Definition 4: In the case that an IFPR $R = (r_{ij})_{n 	imes n} = (\mu_{ij}, v_{ij}, \pi_{ij})_{n 	imes n}$ satisfies the additive consistency, if and only if $\mu_{ij} + \pi_{ij} = \mu_{ik} + 0.5$, $v_{ij} + v_{jk} = v_{ik} + 0.5$ can be satisfied simultaneously.

Deduction 1: If score function of an intuitionistic fuzzy value $r_{ij} = (\mu_{ij}, v_{ij}, \pi_{ij})$ is defined as $S(r_{ij}) = \mu_{ij} - v_{ij}$, a necessary and sufficient condition for the IFPR $R = (r_{ij})_{n 	imes n}$ to satisfy additive consistency is

$$S(r_{ij}) = S(r_{ik}) - S(r_{jk})$$  \hspace{1cm} (1)

where $\forall i, j, k = 1, 2, \ldots, n$.

Definition 5: The weight vector of an intuitionistic fuzzy set is denoted as $W = (w_1, w_2, \ldots, w_n)^T$, where $w_i = (w^\mu_i, w^v_i, w^\pi_i), w^\mu_i, w^v_i, w^\pi_i \in [0,1]$, and $w^\mu_i + w^v_i + w^\pi_i = 1$, $\forall i, j = 1, 2, \ldots, n$. If the following condition is satisfied

$$\sum_{j=1, j \neq i}^n w^\mu_j \leq w^\mu_i, w^v_i + n - 2 \geq \sum_{j=1, j \neq i}^n w^\pi_j \forall i, j$$

$$= 1, 2, \ldots, n.$$ \hspace{1cm} (2)

then $W$ is called as a normalized vector.

Deduction 2: $R = (r_{ij})_{n 	imes n}$ is assumed to represent the IFPR, if $W = (w_1, w_2, \ldots, w_n)^T$, that stands for a normalized intuitionistic fuzzy weight vector exist, which makes

$$r_{ij} = (\mu_{ij}, v_{ij})$$

$$= \begin{cases} (0.5, 0.5), & i = j; \\ (0.5w^\mu_i + 0.5w^v_i, 0.5w^v_i + 0.5w^\pi_i) & i \neq j \end{cases}$$ \hspace{1cm} (3)

Then $R$ is called as an IFPR that satisfies the additive consistency.\textsuperscript{14}

Analytical network process

For many problems, interior elements of hierarchies usually have mutual influences. Under this circumstance, the index system is more similar to a network structure. ANP is presented based on feedback AHP,\textsuperscript{11} which is a decision-making method applicable to non-independent feedback systems, and it not only breaks through the restrictions of AHP, but also resolves the mutual dependences between criteria and indices.

According to ANP, elements are classified into two major categories. One is called as control layer, including the objectives of problem and criteria independent. Control layer are free of decision criteria, but must cover at least one objective. The control layer can be regarded as an AHP structure. The other is network layer, which is constituted by the elements that affecting and depending on each other. Figure 1 is a typical structure of ANP.

The application of ANP is roughly comprised of the following five steps:

1. Establish ANP structure based on the problem. That is, the index system is presented and the interaction and feedback relationships between criteria and indices are analyzed, and the network structure of the index system is formed.

2. Perform pairwise comparison for all elements existing interaction relationships. The results of judgment matrices are usually given by experts or decision makers. These matrices are qualified or not, and the evaluation standards are consistent or not need to be examined.

3. Figure out the local weights of the judgment matrices. The local weights generally solved by some linear programming or non-linear programming models, etc.

4. Construct an unweighted supermatrix, a weighted supermatrix, and a limit supermatrix respectively to derive the global weights.

5. Calculate the comprehensive scores and rank the alternatives.

ANP based on IFPRs

The steps of ANP based on IFPRs are similar to those of ANP, but the expressing manners of preference information and the calculation of local weights and global weights are different.

ANP structure establishment

According to the problem, criteria are divided into two layers. One is the control layer and the other is the network layer. Regarding the former, objectives of the problem are determined in the first place and the main criteria are then classified according to the suggestions of experts and decision makers. Subsequently, sub-criteria or indices corresponding to each main criterion are pinpointed, and the relationships among sub-criteria or indices are analyzed to construct a network structure. As shown in Figure 1, $n$ main criteria have been set up under the objective, and a control layer is formed by the objective and main criteria. As dominated by the main criteria, the sub-criteria or indices can be found out to constitute the corresponding element sets. Therefore, a network structure can be comprised by analyzing whether influences exist between criteria and indices.
Intuitionistic fuzzy judgment matrices construction

The preference information of experts or decision makers is given by language appraisals or numeric scales, further are turned into intuitionistic fuzzy values. In this paper, a remark set for the corresponding intuitionistic fuzzy set is presented, as shown in Table 1. According to Table 1, when language appraisals such as “Absolutely Not Important,” “Equally Important,” and “Absolutely Important” are concerned, their hesitancy degrees of the corresponding intuitionistic fuzzy values are zero ($\pi = 0$). In addition, two intuitionistic fuzzy values with the same distances from “Equally Important” have identical hesitancy degrees, while their membership and non-membership degrees are swapped.

According to a certain criterion, both parties involved with relationships are compared to form a comparison matrix. In this matrix, all elements are expressed by intuitionistic fuzzy numbers and thus an intuitionistic fuzzy judgment matrix is formed. If the intuitionistic fuzzy judgment matrix satisfies the definition of intuitionistic fuzzy preference relation, then the construction process of intuitionistic fuzzy judgment matrix is the establishment course of intuitionistic fuzzy preference relation. Such a process is the conversion from being qualitative to quantitative.

Local weights calculation

In most cases, subjective and objective preferences of decision makers or experts for a scheme have deviated, and such a deviation is incurred by their insufficient understanding and incomprehensive consideration of the problem as well as differences in their modes of thinking, etc. In this paper, local weights are derived by minimizing the total deviation between subjective

---

### Table 1. Remark set for intuitionistic fuzzy set.

| Language appraisal            | Numeric scale | Intuitionistic fuzzy value | Hesitancy degree |
|-------------------------------|---------------|---------------------------|-----------------|
| Absolutely not important      | 1             | (0, 1)                    | 0               |
| Not very important            | 2             | (0.1, 0.8)                | 0.1             |
| Not important                 | 3             | (0.2, 0.6)                | 0.2             |
| Relatively not important      | 4             | (0.35, 0.55)              | 0.1             |
| Equally important             | 5             | (0.5, 0.5)                | 0               |
| Very important                | 6             | (0.55, 0.35)              | 0.1             |
| Important                     | 7             | (0.6, 0.2)                | 0.2             |
| Very important                | 8             | (0.8, 0.1)                | 0.1             |
| Absolutely important          | 9             | (1, 0)                    | 0               |

---

Figure 1. Typical structure of ANP.
and objective preferences of the decision makers or experts. This approach can reflect a preference relation of subjective–objective consistency more favorably.

In order to acquire the local weights by total deviation minimization, two deviation variables \( \varepsilon \) and \( \eta \) are introduced. The IFPRs given by experts or decision makers are inclined to fail to satisfy the requirement of objective consistency. Therefore, variables \( \varepsilon \) and \( \eta \) should be adopted to erase such a deviation. Moreover, the smaller \( \varepsilon \) and \( \eta \) are, the higher the consistency between subjective and objective preferences will be. In this study, a goal programming method is employed to obtain the local weights and the consistency indices. According to Definition 5 and Deduction 2, a model is proposed as follows:

\[
\min J = \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \left( |w_{ij}| + |\eta_{ij}| \right)
\]

\[
\begin{align*}
(0.5w_i^0 + 0.5w_j^0) - \mu_{ij} - \varepsilon_{ij} &= 0, \\
0.5w_i^0 + 0.5w_j^0 - v_{ij} - \eta_{ij} &= 0, \\
0 \leq w_i^* \leq 1, 0 \leq w_j^* \leq 1, w_i^0 + w_j^0 &\leq 1, \\
\sum_{j=1, j\neq i}^{n} w_{ij}^* &= w_i^* + n - 2 \geq \sum_{j=1, j\neq i}^{n} w_{ji}^*, \\
i, j = 1, 2, \ldots, n; j \neq i.
\end{align*}
\] (4)

where the objective function indicates the minimum total deviation of subjective and objective preferences. The first two constraint equations indicate that the weight vectors should satisfy the additive consistency of IFPRs. The third constraint condition denotes that the weights need to satisfy the definition of intuitionistic fuzzy set. As for the fourth constraint condition, it makes the weights to be a normalized vector according to Definition 5. Solving the above goal programming model, a weight vector \( W = (w_1, w_2, \ldots, w_n)^T \) can be acquired, and each weight is an intuitionistic fuzzy number, that is, \( w_j = (w_{ij}^0, w_{ij}^*) \).

Under the impacts of main criteria in the control layer, judgment matrices are derived from different element sets in the network layer. At the same manners, the above goal programming is formed and solved, and the local weights can be obtained correspondingly.

**The global weights acquirement**

In order to acquire the global weights of indices, the unweighted supermatrix, the weighted supermatrix, and the limit supermatrix are constructed and calculated respectively.

In line with the permutations of influencing factors and the local weights, an unweighted supermatrix can be gained. In the case that two factors have no interactional relationship, their local weight is denoted as \((0, 0)\).

The unweighted supermatrix takes the interactional relationships between elements and element sets into account, while ignoring the domination of control layer. As far as each sub-block of the unweighted supermatrix is concerned, its column vectors are normalized. By contrast, as far as the overall unweighted supermatrix is concerned, the column vectors are not normalized. Therefore, elements from different factor sets cannot be compared directly. In order to compensate for such a deficiency of the unweighted supermatrix, the supermatrix should be normalized.

Under the main criteria, each element set is regarded as an element, and diverse element sets are chosen as sub-criteria independently to compare the relative importance for all element sets. The goal programming model in the previous section is utilized to obtain the weight vectors, and these vectors are called as weighted weights.

A weighted supermatrix can be acquired by conducting the multiplication for the weighted weights and the unweighted supermatrix. The following operation rules of intuitionistic fuzzy set may be conducted.

\[
\begin{align*}
\mu_{A \times B} &= \mu_A \times \mu_B, \quad v_{A \times B} = v_A + v_B - v_A \times v_B, \\
\mu_{A+B} &= \mu_A + \mu_B - \mu_A \times \mu_B, \quad v_{A+B} = v_A \times v_B
\end{align*}
\] (5)

(6)

After obtaining the weighted supermatrix, the ultimate global weights of all indices should be further gained. As a result, the weighted supermatrix is stabilized to achieve the limit supermatrix. That is, the limit of the weighted supermatrix is solved. The method of getting the limit is continuously multiplying the weighted supermatrix itself until the elements in each line converge and become a unique value. Then, each column in the limit supermatrix stands for the global weights of indices. For calculation simplicity, membership degrees and non-membership degrees should be computed separately to figure out the respective limit supermatrices.

**The comprehensive scores computing and the alternatives ranking**

To calculate the comprehensive scores of the alternatives, experts or decision makers still need to grade each index of the alternatives. The grades multiplied by the
global weights of indices, and thus comprehensive scores of the alternatives to the general objective can be achieved. Consequently, alternatives can be selected and ranked on the basis of the comprehensive scores.

**Case study**

**Problem description**

Investors who invest their funds in bonds are apparently aimed at acquiring high earnings and returns. However, different investors may have diverse viewpoints and considerations as far as early observations and final selections are concerned. For an outstanding bond investor, he/she carries out overall investigations on conditions of various bonds before investment, to select one or more which is/are most appropriate for him/her.

In order to make an optimal investment plan and to win maximum returns, all factors should be analyzed in a more thorough manner. Different factors may have an effect on bonds, such as the deadline, liquidity, market interest rate, macroeconomic policy, etc., which ought to be comprehensively considered and analyzed during decision making.

![ANP network structure](image-url)

**Figure 2.** ANP network structure.
An investment company is assumed to have a sum of money and hopes to invest it in bonds. Through preliminary analysis, the company keeps their eyes on four bonds (bonds #1, #2, #3, and #4). In order to select an optimal bond, the ANP method based on IFPRs is applied.

**ANP structure construction**

By asking the advice of investment counselors from security companies and experts in the ponds field, in combination with market analysis, an index system for bond investment is presented. Firstly, main criteria should be considered on the premise of general objective recognition. The general objective is to select an optimal bond, and the main criteria are: \( U_1 \) for the rate of return; \( U_2 \) for the risk; \( U_3 \) for the investment strategy; \( U_4 \) for the regional difference.

Subsequently, a hierarchical structure should be established by considering the influences of the main criteria to the sub-criteria subordinate. For example, when the rate of return is the main criterion, then the deadline \( u_{11} \), coupon rate \( u_{12} \), face value and issue price \( u_{13} \), mode of interest reckon \( u_{14} \), and tax cost \( u_{15} \) are the sub-criteria. For the rate of return, the impacts on deadline, coupon rate, face value, and issue price, and mode of interest are rather significant. While that of tax cost is mainly embodied in a fact that the interest tax of national debts is exempted in China, but the tax liability should be performed for corporate bonds or enterprise bonds. For this reason, the investor should compare and comprehensively think about such bonds before investment.

All sub-criteria or indices are listed as follows.

\( U_1 = \{ \text{deadline } u_{11}, \text{ coupon rate } u_{12}, \text{ face value and issue price } u_{13}, \text{ mode of interest reckon } u_{14}, \text{ tax cost } u_{15} \} \);

**Table 2. The judgment matrix based on \( u_{22} \) under the main criterion \( U_2 \).**

| \( u_{22} \) | \( u_{21} \) | \( u_{23} \) | \( u_{24} \) |
|---|---|---|---|
| \( u_{21} \) | 5 | 6 | 7 |
| \( u_{23} \) | 6 | 5 | 6 |
| \( u_{24} \) | 7 | 6 | 5 |

**Table 3. The intuitionistic fuzzy judgment matrix based on \( u_{22} \) under the main criterion \( U_2 \).**

| \( u_{22} \) | \( u_{21} \) | \( u_{23} \) | \( u_{24} \) | Local weights |
|---|---|---|---|---|
| \( u_{21} \) | \((0.5, 0.5)\) | \((0.55, 0.35)\) | \((0.6, 0.2)\) | \((0.43, 0.37)\) |
| \( u_{23} \) | \((0.35, 0.55)\) | \((0.5, 0.5)\) | \((0.55, 0.35)\) | \((0.33, 0.67)\) |
| \( u_{24} \) | \((0.2, 0.62)\) | \((0.35, 0.55)\) | \((0.5, 0.5)\) | \((0.03, 0.77)\) |

**Table 4. The unweighted supermatrix.**

|  | \( u_{11} \) | \( u_{12} \) | \( u_{13} \) | \( u_{14} \) |
|---|---|---|---|---|
| \( u_{11} \) | \((0.00, 0.00)\) | \((0.42, 0.37)\) | \((0.37, 0.37)\) | \((0.23, 0.55)\) | \((0.30, 0.60)\) | \((0.43, 0.37)\) | \((0.42, 0.46)\) | \((0.58, 0.27)\) | \((0.60, 0.32)\) | \((0.44, 0.47)\) | \((0.40, 0.49)\) | \((0.80, 0.20)\) | \((0.00, 0.93)\) | \((0.00, 0.90)\) | \((0.57, 0.27)\) |
| \( u_{12} \) | \((0.43, 0.37)\) | \((0.00, 0.00)\) | \((0.33, 0.67)\) | \((0.55, 0.45)\) | \((0.60, 0.40)\) | \((0.32, 0.67)\) | \((0.24, 0.50)\) | \((0.13, 0.40)\) | \((0.16, 0.60)\) | \((0.23, 0.47)\) | \((0.40, 0.40)\) | \((0.10, 0.70)\) | \((0.27, 0.53)\) | \((0.40, 0.50)\) | \((0.13, 0.40)\) |
| \( u_{13} \) | \((0.33, 0.67)\) | \((0.32, 0.67)\) | \((0.00, 0.00)\) | \((0.00, 0.78)\) | \((0.00, 0.90)\) | \((0.03, 0.77)\) | \((0.20, 0.68)\) | \((0.13, 0.71)\) | \((0.08, 0.84)\) | \((0.23, 0.67)\) | \((0.00, 0.80)\) | \((0.10, 0.90)\) | \((0.17, 0.70)\) | \((0.10, 0.72)\) | \((0.00, 1.00)\) |
| \( u_{14} \) | \((0.00, 1.00)\) | \((0.00, 1.00)\) | \((0.00, 1.00)\) | \((0.00, 1.00)\) | \((0.00, 1.00)\) | \((0.00, 1.00)\) | \((0.00, 1.00)\) | \((0.00, 1.00)\) | \((0.00, 1.00)\) | \((0.00, 1.00)\) | \((0.00, 1.00)\) | \((0.00, 1.00)\) | \((0.37, 0.43)\) | \((0.20, 0.70)\) | \((0.00, 1.00)\) |
| \( u_{15} \) | \((0.38, 0.62)\) | \((0.35, 0.55)\) | \((0.30, 0.57)\) | \((0.45, 0.57)\) | \((0.40, 0.49)\) | \((0.00, 0.00)\) | \((0.00, 0.00)\) | \((0.00, 0.00)\) | \((0.00, 0.00)\) | \((0.00, 0.00)\) | \((0.00, 0.00)\) | \((0.00, 0.00)\) | \((0.00, 0.00)\) | \((0.00, 0.00)\) | \((0.00, 0.00)\) | \((0.25, 0.62)\) |
there exists any redemption clause ($u_{32}$), whether there exists any mortgage guarantee ($u_{34}$); 

$U_3 = \{\text{finance condition } u_{31}, \text{investment risk preference } u_{32}, \text{liquidity requirement } u_{33}\};$

$U_4 = \{\text{national policy } u_{41}, \text{economic environment } u_{42}, \text{foreign exchange fluctuation } u_{43}\}.$

Among these indices, not only are subordinate factors influenced by superior factors, but interaction and feedback relationships also exist among factors of the same layer. Therefore, based on the literature reviews in conjunction with the practical bond market, a network structure is presented, as shown in Figure 2.

The decision-making process based on IFPRs and ANP

In accordance with the ANP network structure, the optimal bond will be selected from the perspective of ANP based on IFPRs.

Intuitionistic fuzzy judgment matrices construction and local weights calculation. Before solving the local weights, the preference information given by experts or decision makers is converted into intuitionistic fuzzy values according to the remark set in Table 1, so that the intuitionistic fuzzy judgment matrices are obtained. For instance, when risk ($U_2$) is selected as a main criterion, and market interest rate ($u_{22}$) is chosen as sub-criterion, then the status of the issuer ($u_{21}$), whether there exists any redemption clause ($u_{23}$) and whether there exists any mortgage guarantee ($u_{24}$) are compared respectively, as shown in Table 2.

According to the scales in Table 2 and the remark set in Table 1, the preference information can be converted into an intuitionistic fuzzy judgment matrix, as shown in Table 3.

To acquire the local weights of the intuitionistic fuzzy judgment matrix, the goal programming model can be formed as below according to formula (4).

$$\min J = |e_{12}| + |\eta_{12}| + |e_{13}| + |\eta_{13}| + |e_{23}| + |\eta_{23}|$$

| $U_1$ | $U_2$ | $U_3$ | $U_4$ |
|-------|-------|-------|-------|
| $U_1$ | (0.43, 0.37) | (0.33, 0.67) | (0.35, 0.55) | (0.23, 0.74) |
| $U_2$ | (0.33, 0.67) | (0.43, 0.37) | (0.0) | (0.23, 0.74) |
| $U_3$ | (0.03, 0.77) | (0.03, 0.77) | (0.55, 0.35) | (0.0, 0.87) |
| $U_4$ | (0.0) | (0.0) | (0.0) | (0.36, 0.47) |

Table 6. The limit supermatrix.
The above goal programming can be solved by Matlab software, and the local weights are achieved as shown in Table 3.

To obtain the global weights of indices, the unweighted supermatrix, the weighted supermatrix, and the limit supermatrix should be figured out respectively.

When all the local weights of judgment matrices are achieved, an unweighted supermatrix can be constructed based on the relationships among indices, as shown in Table 4. As mentioned before, the unweighted supermatrix needs to be normalized.

The weighted matrix is formed by the local weights of the four main criteria. For example, under the rate of return $U_1$, the importance degrees of the other criteria are compared. According to formula (4), the local weights $(0.43, 0.37), (0.33, 0.67), (0.03, 0.77), (0, 0)^T$ can be acquired. The importance degrees are successively compared for one main criterion to other main criteria. Then the weighted weights can be acquired, as shown in Table 5.

For other intuitionistic fuzzy judgment matrices of pairwise comparisons, they can be solved in a similar way.

### Global weights acquirement.

The unweighted supermatrix needs to be normalized.

The weighted matrix is formed by the local weights of the four main criteria. For example, under the rate of return $U_1$, the importance degrees of the other criteria are compared. According to formula (4), the local weights $(0.43, 0.37), (0.33, 0.67), (0.03, 0.77), (0, 0)^T$ can be acquired. The importance degrees are successively compared for one main criterion to other main criteria. Then the weighted weights can be acquired, as shown in Table 5.

| Table 7. The final scores. | Weights | #1 | #2 | #3 | #4 | S#1 | S#2 | S#3 | S#4 |
|---------------------------|---------|----|----|----|----|-----|-----|-----|-----|
| Membership                |         |    |    |    |    |     |     |     |     |
| $\mu_{11}$                | 0.1351  | 0.4200 | 0.3200 | 0.0200 | 0.0300 | 0.0568 | 0.0432 | 0.0027 | 0.0041 |
| $\mu_{12}$                | 0.1261  | 0.3700 | 0.3300 | 0.0300 | 0.0000 | 0.0467 | 0.0416 | 0.0038 | 0.0000 |
| $\mu_{13}$                | 0.1138  | 0.2300 | 0.5500 | 0.0000 | 0.0000 | 0.0262 | 0.0626 | 0.0000 | 0.0000 |
| $\mu_{14}$                | 0.0308  | 0.3000 | 0.6000 | 0.0000 | 0.0000 | 0.0092 | 0.0185 | 0.0000 | 0.0000 |
| $\mu_{15}$                | 0.0087  | 0.4300 | 0.3300 | 0.0300 | 0.0000 | 0.0037 | 0.0029 | 0.0003 | 0.0000 |
| $\mu_{16}$                | 0.1250  | 0.2600 | 0.4300 | 0.1300 | 0.0000 | 0.0325 | 0.0538 | 0.0163 | 0.0000 |
| $\mu_{17}$                | 0.1312  | 0.3000 | 0.4000 | 0.0000 | 0.0000 | 0.0394 | 0.0525 | 0.0000 | 0.0000 |
| $\mu_{18}$                | 0.1036  | 0.2000 | 0.5000 | 0.1000 | 0.0000 | 0.0207 | 0.0518 | 0.0104 | 0.0000 |
| $\mu_{19}$                | 0.0465  | 0.3000 | 0.4000 | 0.0500 | 0.0500 | 0.0140 | 0.0186 | 0.0023 | 0.0023 |
| $\mu_{20}$                | 0.0591  | 0.1900 | 0.3600 | 0.1600 | 0.1900 | 0.0112 | 0.0213 | 0.0095 | 0.0112 |
| $\mu_{21}$                | 0.0670  | 0.2000 | 0.5000 | 0.1000 | 0.0000 | 0.0134 | 0.0335 | 0.0067 | 0.0000 |
| $\mu_{22}$                | 0.0529  | 0.2000 | 0.5000 | 0.1000 | 0.0000 | 0.0106 | 0.0264 | 0.0053 | 0.0000 |
| $\mu_{23}$                | 0.0000  | 0.3900 | 0.2300 | 0.0000 | 0.2300 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| $\mu_{24}$                | 0.0000  | 0.2000 | 0.5000 | 0.1000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| $\mu_{25}$                | 0.0000  | 0.3000 | 0.4000 | 0.0500 | 0.0500 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

| Final score               | 0.2513  | 0.3537 | 0.0558 | 0.0175 | 1.0000 |

(continued)
Additionally, through multiple times of multiplying the weighted supermatrix itself, each row in the supermatrix converged to an identical value. Thus, the limit supermatrices of the membership and non-membership degrees are acquired separately, as shown in Table 6.

### Comprehensive scores computing and the bonds ranking.

In order to obtain the comprehensive scores of the four bonds, the global weight of each index is multiplied by the score of the alternative respectively. After calculation, the final scores of bonds #1, #2, #3, and #4 are \((0.2513, 0.0002), (0.3537, 0.0001), (0.0558, 0.0191), (0.0175, 0.4656)\)T, as shown in Table 7. According to formula (1), the score function of intuitionistic fuzzy set, these four bonds are ranked as: #2 > #1 > #3 > #4. Therefore, Bond #2 can be selected as the main object of investment.

### Comparing with AHP based on IFPRs.

To show the effectiveness of the proposed method, we compared it with the AHP based on IFPRs. The weights and scores are acquired in the same way for the bond selection according to AHP based on IFPRs. The comprehensive scores of the four bonds are \((0.2494, 0.0185), (0.3632, 0.0001), (0.0512, 0.0205), (0.0223, 0.4883)\). According to formula (1), the order of the four bonds is: #2 > #1 > #3 > #4. Therefore, Bond #2 is still the best one.

When compared with ANP based on IFPRs, although their order and the optimal bond are the same, the comprehensive scores are different. The main reason is that AHP based on IFPRs ignores the interaction and feedback between criteria and indices and loses some useful information during the process of decision making. ANP based on IFPRs considers the influential relationships among indices, but the calculation is more complex.

### Conclusions

An index system of bond selection is presented from the four aspects: rate of return, risk, investment strategy, and regional difference. Not only are impacts of uncertainties on bond selection considered, mutual effects and feedbacks between criteria and indices are also taken into account, which are more accordant with the practical situations.

In order to accurately express the preference information of experts or decision makers under uncertain conditions, intuitionistic fuzzy preference relations are adopted, which can describe the preference information from membership degrees, non-membership degrees, and hesitation degrees.

Combining the advantages of intuitionistic fuzzy preference relations with ANP, an intuitionistic fuzzy analytic network process method for bond selection is proposed. The local weights are solved by goal programming models, and the unweighted supermatrix is established according to the local weights and the network structure of ANP. The limit supermatrix is derived from the weighted supermatrix by multiplying itself. The comprehensive scores of bonds are calculated based on the global weights and the scores of each index.

### Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

### Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work was supported by “the National Natural Science Foundation of China (no. 71771023)”.

---

**Table 7.** Continued

| Weights | #1     | #2     | #3     | #4     | S#1 | S#2 | S#3 | S#4 |
|---------|--------|--------|--------|--------|-----|-----|-----|-----|
| \(\mu_{15}\) | 0.2567 | 0.3700 | 0.6700 | 0.7700 | 0.9300 | 0.5317 | 0.7547 | 0.8290 | 0.9480 |
| \(\mu_{21}\) | 0.0045 | 0.5700 | 0.5700 | 0.7000 | 1.0000 | 0.5719 | 0.5719 | 0.7014 | 1.0000 |
| \(\mu_{22}\) | 0.0047 | 0.4000 | 0.6000 | 0.7000 | 1.0000 | 0.4029 | 0.6019 | 0.7014 | 1.0000 |
| \(\mu_{23}\) | 0.0089 | 0.6000 | 0.5000 | 0.7000 | 1.0000 | 0.6036 | 0.5045 | 0.7027 | 1.0000 |
| \(\mu_{24}\) | 0.0138 | 0.6500 | 0.4000 | 0.8000 | 0.9500 | 0.6548 | 0.4083 | 0.8028 | 0.9507 |
| \(\mu_{31}\) | 0.1059 | 0.8100 | 0.5400 | 0.7400 | 0.8100 | 0.8301 | 0.5887 | 0.7675 | 0.8301 |
| \(\mu_{32}\) | 0.0762 | 0.6000 | 0.3000 | 0.7000 | 1.0000 | 0.6305 | 0.3533 | 0.7228 | 1.0000 |
| \(\mu_{33}\) | 0.1256 | 0.6000 | 0.3000 | 0.7000 | 1.0000 | 0.6502 | 0.3879 | 0.7377 | 1.0000 |
| \(\mu_{41}\) | 0.0000 | 0.4700 | 0.7000 | 0.8700 | 0.7100 | 0.4700 | 0.7000 | 0.8700 | 0.7100 |
| \(\mu_{42}\) | 0.0000 | 0.6000 | 0.5000 | 0.7000 | 1.0000 | 0.6000 | 0.5000 | 0.7000 | 1.0000 |
| \(\mu_{43}\) | 0.0000 | 0.6500 | 0.4000 | 0.8000 | 0.9500 | 0.6500 | 0.4000 | 0.8000 | 0.9500 |
| Final score | 0.0002 | 0.0001 | 0.0191 | 0.4656 |
References

1. Flanigan MA, Tondkar RH and Andrews RL. An empirical investigation of factors affecting the selection of markets for foreign bond issues. *Int J Account* 1999; 34: 71–92.
2. Dell Aquila R and Ronchetti E. Stock and bond return predictability: the discrimination power of model selection criteria. *Comput Stat Data Anal* 2006; 50: 1478–1495.
3. Ebner A. An empirical analysis on the determinants of CEE government bond spreads. *Emerg Market Rev* 2009; 10: 97–121.
4. Ferreira MA and Miguel AF. The determinants of domestic and foreign bond bias. *J Multi Financ Manag* 2011; 21: 279–300.
5. Bhattacharyay BN. Determinants of bond market development in Asia. *J Asian Econ* 2013; 24: 124–137.
6. Huang T, Wu F, Yu J, et al. International political risk and government bond pricing. *J Bank Financ* 2015; 55: 393–405.
7. Tebaldi E, Nguyen H and Zuluaga J. Determinants of emerging markets’ financial health: a panel data study of sovereign bond spreads. *Res Int Bus Finance* 2018; 45: 82–93.
8. Altavilla C, Giannone D and Modugno M. Low frequency effects of macroeconomic news on government bond yields. *J Monet Econ* 2017; 92: 31–46.
9. Smaoui H, Grandes M and Akindele A. The determinants of bond market development: further evidence from emerging and developed countries. *Emerg Mark Rev* 2017; 32: 148–167.
10. Eichler S and Plaga T. The political determinants of government bond holdings. *J Int Money Finance* 2017; 73: 1–21.
11. Saaty TL. Decision making with dependence and feedback: the analytic network process. Pittsburgh: RWS Publications, 1996.
12. Atanassov KT. Intuitionistic fuzzy sets. *Fuzzy Sets Syst* 1986; 20: 87–96.
13. Xu Z. Intuitionistic preference relations and their application in group decision making. *Inform Sci* 2007; 177: 2363–2379.
14. Gong Z, Li L, Forrest J, et al. The optimal priority models of the intuitionistic fuzzy preference relation and their application in selecting industries with higher meteorological sensitivity. *Expert Syst Appl* 2011; 38: 4394–4402.
15. Wang Z. Derivation of intuitionistic fuzzy weights based on intuitionistic fuzzy preference relations. *Appl Math Model* 2013; 37: 6377–6388.
16. Xu Z and Liao H. A survey of approaches to decision making with intuitionistic fuzzy preference relations. *Knowl Based Syst* 2015; 80: 131–142.
17. Chen S and Chang C. Fuzzy multi-attribute decision making based on transformation techniques of intuitionistic fuzzy values and intuitionistic fuzzy geometric averaging operators. *Inform Sci* 2016; 352–353: 133–149.
18. Xu Z and Zhao N. Information fusion for intuitionistic fuzzy decision making: an overview. *Inform Fusion* 2016; 28: 10–23.
19. Mou Q, Xu Z and Liao H. A graph based group decision making approach with intuitionistic fuzzy preference relations. *Comput Ind Eng* 2017; 110: 138–150.