Does the Neptunian system of satellites challenge a gravitational origin for the Pioneer anomaly?

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ABSTRACT

If the Pioneer anomaly (PA) was a genuine dynamical effect of gravitational origin, it should also affect the orbital motions of the Solar system’s bodies moving in the space regions in which the PA manifested itself in its presently known form, i.e. as a constant and uniform acceleration approximately directed towards the Sun with a non-zero magnitude $A_{\text{Pio}} = (8.74 \pm 1.33) \times 10^{-10} \text{ m s}^{-2}$ after 20 au from the Sun. In this paper we preliminarily investigate its effects on the orbital motions of the Neptunian satellites Triton, Nereid and Proteus, located at about 30 au from the Sun, both analytically and numerically. Extensive observational records covering several orbit revolutions have recently been analysed for them, notably improving the knowledge of their orbits. Both analytical and numerical calculations, limited to the direct, Neptune–satellite interaction, show that the peak-to-peak amplitudes of the PA-induced radial, transverse and out-of-plane perturbations over one century are up to 300, 600 km, 8 m for Triton, 17 500, 35 000, 800 km for Nereid and 60, 120 km, 30 m for Proteus. The corresponding orbital uncertainties obtained from a recent analysis of all the data available for the satellites considered are, in general, smaller by one–two orders of magnitude, although obtained without modelling a Pioneer-like extra-force. Further investigations based on a reprocessing of the satellites’ real or simulated data with modified equations of motions including an additional Pioneer-type force as well are worth being implemented and may shed further light on this important issue.

Key words: gravitation – celestial mechanics – astrometry – ephemerides – planets and satellites: individual (Neptune, Nereid, Proteus, Triton).

1 INTRODUCTION

The Pioneer anomaly (PA; Nieto 2006) consists of an unmodelled, almost constant and uniform acceleration approximately directed towards the Sun and of magnitude (Anderson et al. 1998, 2002a)

$$A_{\text{Pio}} = (8.74 \pm 1.33) \times 10^{-10} \text{ m s}^{-2}. \quad (1)$$

It was detected in the radiometric data from the Pioneer 10/11 spacecraft after they passed the 20 au threshold moving along roughly antiparallel escape hyperbolic paths taken after their previous encounters with Jupiter (\sim 5 au) and Saturn (\sim 10 au), respectively. The PA’s existence has been subsequently confirmed by independent investigations by Markwardt (2002), Olsen (2007) and Levy et al. (2009) as well. Interestingly, latest data analyses are focussing on periodic variations of the anomaly, characterized as functions of the azimuthal angle $\varphi$ defined by the directions Sun–Earth antenna and Sun–Pioneer (Levy et al. 2009). Concerning the possibility that it started to manifest itself at shorter heliocentric distances (Nieto & Anderson 2005; Nieto 2008), efforts to retrieve and analyse early data from Pioneer 10/11 are currently being made (List & Mullin 2008; Toth & Turyshev 2008). The PA is one of some astrometric anomalies in the Solar system reported in recent years (Lämmertz, Preuss & Dittus 2008; Iorio 2009a; Anderson & Nieto 2010).

Attempts to explain some features of the PA in terms of mundane, non-gravitational effects, pertaining the Pioneer probes themselves like thermal forces among different parts of the spacecraft (Katz 1999; Murphy 1999; Anderson et al. 2002a; Mbelek & Michalski 2002; Scheffer 2003; Bertolami et al. 2008; Toth & Turyshev 2009) or external influences like anisotropic solar emission (Bini, Cherubini & Mashhoon 2004), have been undertaken, but some of them have not obtained full consensus so far (Anderson et al. 1999a,b; Bini et al. 2004). On the other hand, latest work by Bertolami et al. (2008) strongly points out that PA is a thermal effect due to the energy sources in the spacecraft; further studies on possible thermal effects like a potential asymmetric heat dissipation of the spacecraft surface are ongoing (Rievers et al. 2009). Conventional explanations of gravitational origin in terms of drag due to interplanetary dust, dark matter, Kuiper belt objects (Anderson et al. 2002a; Nieto 2005; Nieto, Turyshev & Anderson 2005; Bertolami &
Vieira 2006; de Diego, Núñez & Zavala 2006) have been found not satisfactorily as well. As a consequence, many suggestions invoking non-standard gravitational and non-gravitational physics like non-linear electrodynamics (Mbelek et al. 2007) have been proposed. For a review see e.g. Anderson et al. (2002a), Dittus et al. (2005), Bertolami & Páramos (2006), Rathke & Izzo (2006), de Diego (2008) and references therein. Among the various proposed exotic gravitational mechanisms we recall those by Brownstein & Moffat (2006), based on a long-range Yukawa-like extra-force, and by Jaekel & Reynaud (2008) who proposed metric extensions of the Einstein’s general theory of relativity (GTR). Attempts to find exotic gravitational explanations for PA did not even cease after the publication of the latest works on the non-gravitational effects like Bertolami et al. (2008); just to limiting to published works, see e.g. Avramidi & Fucci (2009), Wilson & Blome (2009), Greaves (2009) and Exirifard (2009). A dedicated spacecraft-based mission to test the PA in the outer regions of the Solar system has also been proposed and investigated (Dittus et al. 2005; Rathke & Izzo 2006; Bertolami & Páramos 2007).

The hypothesis that non-standard forces of gravitational origin are able to explain the anomalous behaviour of the Pioneer spacecraft must cope with the following crucial remark. If the PA was due to some modifications of the known laws of gravity, this should be due to a radial extra-force affecting the orbits of the astronomical bodies [planets and their satellites, comets, trans-Neptunian objects (TNOs), etc.] as well, especially those moving in the space regions in which the PA manifested itself in its presently known form. Otherwise, a violation of the equivalence principle largely incompatible with the present-day bounds of $\sim 10^{-13}$ from Earth-based laboratory experiments (Schlamminger et al. 2008) would occur. The impact of a Pioneer-like additional acceleration on the motion of planets and minor bodies in the outer regions of the Solar system interested by the PA was recently studied by numerous authors with different approaches (Anderson, Turyshhev & Nieto 2002b; Iorio & Giudice 2006; Page, Dixon & Wallin 2006; Pitjeva 2006; Rathke & Izzo 2006; Iorio 2007a,b; Tangen 2007; Wallin, Dixon & Page 2007; Standish 2008; Fienga et al. 2009; Iorio 2009b; Page, Wallin & Dixon 2009). In particular, Anderson et al. (2002b) discussed the impact of a Pioneer-like acceleration on the long-period comets and the form of the Oort cloud; however, such bodies are not particularly well suited to perform accurate tests of gravitational theories because of the impact of several aliasing non-gravitational perturbations like out-gassing as they approach the Sun. Page et al. (2006) investigated the potential offered by an analysis of the minor planets in the outer Solar system to confirm or refute the existence of a gravitational Pioneer effect. Wallin et al. (2007) used a well-observed sample of TNOs between 20 and 100 au from the Sun to constrain Pioneer-like deviations from Newtonian gravity in that region of the Solar system. By fitting the TNOs’ observations with modified equations of motion according to equation (1), Wallin et al. (2007) found $(0.87 \pm 1.6) \times 10^{-10} \text{ m s}^{-2}$, which is consistent with zero and whose upper bound is inconsistent with equation (1) at 4σ level. Rathke & Izzo (2006), Iorio & Giudice (2006), Iorio (2007a) and Tangen (2007) looked at the outer planets. Rathke & Izzo (2006) parametrized the PA in terms of a change of the effective reduced solar mass felt by Neptune finding it nearly two orders of magnitude beyond the current observational constraint. Moreover, they noted that the Pioneer 11 data contradict the Uranus ephemerides – obtained without explicitly modelling the PA – by more than one order of magnitude. Iorio & Giudice (2006), Iorio (2007a) and Tangen (2007) computed the secular effects induced by a uniform and radial extra-acceleration like that of equation (1) on the orbits of Uranus, Neptune and Pluto, located at 20–40 au from the Sun, and compared them to the present-day, unmodified ephemerides. Iorio & Giudice (2006) and Iorio (2007a) concluded that the resulting anomalous effects on all of them would be too large to have escaped from detection so far. Doubts concerning Neptune were raised by Tangen (2007) in the sense that the accuracy of the currently available observations for it would not, in fact, exclude the possibility that Neptune is acted upon by $A_{\text{Pio}}$. Other authors made a step further by including a Pioneer-like extra-acceleration in the force models and fitting again the planetary observations with such modified equations of motion. More specifically, Page et al. (2009) fitted modified dynamical models including equation (1) to observational records for Uranus, Neptune and Pluto showing that the current ephemeris of Pluto does not preclude the existence of the Pioneer effect because its orbit would not be well enough characterized at present to make such an assertion. Standish (2008) fitted planetary data records with a modified version of the JPL DE ephemerides with a uniform extra-acceleration directed towards the Sun acting on Uranus, Neptune and Pluto; a magnitude as small as just 10 per cent of equation (1) yielded completely unacceptable residuals for all the three outer planets. Fienga et al. (2009) added an extra-acceleration like that of equation (1) to the equations of motion of the outer planets and fitted the resulting modified ephemerides to their observations by finding that Uranus excludes the existence of Pioneer-like acceleration as large as equation (1) at a 4σ level. On the contrary, for Neptune and Pluto the effect of equation (1) is absorbed by the fit, so that the resulting residuals do not allow to exclude the existence of a Pioneer-like anomalous acceleration affecting such bodies. The existence of a standard PA in the regions crossed by Jupiter and Saturn has been ruled out by Iorio (2007b) and Standish (2008) with different approaches. For non-standard, velocity-dependent forms of the PA and their compatibility with different ephemerides of the outer planets, see Standish (2008), Standish (2010) and Iorio (2009b); such different approaches show that almost all of them are not compatible with the planetary observations.

In this paper we investigate a different astronomical laboratory with respect to those examined so far to put on the test the hypothesis of the gravitational origin of the PA independently of what detected in the Pioneer 10/11 telemetry. Indeed, we will look at the orbital effects induced by a Pioneer-like acceleration directed towards the Sun on the Neptunian satellites Triton, Nereid and Proteus in view of the recent improvements in their orbit determination (Jacobson 2009) based on the analysis of extensive data records covering several orbital revolutions. Their Keplerian orbital elements are listed in Table 1.

In this paper we will perform a preliminary sensitivity analysis by means of analytical and numerical calculations. Their goal

| Keplerian orbital element | Triton | Nereid | Proteus |
|---------------------------|--------|--------|---------|
| $a$ (km)                  | 354767 | 551714 | 117714  |
| $e$                       | 0.00003| 0.75428| 0.00090 |
| $I$ (°)                   | 130.9  | 5.0    | 28.9    |
| $\Omega$ (°)              | 213.2  | 320.3  | 48.1    |
| $\omega$ (°)              | 60.2   | 296.1  | 54.4    |
| $P_b$ (d)                 | 5.8    | 360.4  | 1.1     |
is to check if the scenario considered is worth further, more detailed investigations. They could involve, e.g. a re-processing of the Neptunian satellites’ real or simulated data sets with modified equations of motion including a standard gravitational Pioneer-like extra-acceleration radially directed towards the Sun as well.

In Section 2 we first analytically work out the anomalous PA-type orbital effects on Triton, Nereid and Proteus (Section 2.1). Then, we perform numerical integrations of their equations of motion with and without the PA. Finally, we compare our results to the latest determinations of the orbital accuracies for such satellites (Section 2.2). Section 3 is devoted to the conclusions.

2 EFFECTS OF A STANDARD PIONEER ANOMALY ON THE NEPTUNE’S SATELLITES

We use ICRF/J2000.0 with Neptune as centre body as reference frame. To be consistent with Jacobson (2009), we adopt 1989 October 31 as reference epoch. The reference system used has the ecliptic and mean equinox of reference epoch. In such a frame a standard Pioneer-like acceleration has the form

\[ A^{\text{Pio}} = A^{\text{Pio}} n_{\odot}, \]

(2)

where \( n_{\odot} \) is the unit vector pointing towards the Sun displayed in Table 2. As a consequence, \( A^{\text{Pio}} \) has the components shown in Table 3 at the reference epoch. It can be noted that it is mainly directed along the y-axis of the chosen frame. Since we are interested in its secular, i.e. averaged over one orbital period, effects on the motion of the Neptunian satellites, we can safely consider \( A^{\text{Pio}} \) as constant because of the short satellites’ periods (see Table 1) with respect to the Neptunian one amounting to 164.9 yr. In other words, each satellite faces the action of a constant and uniform disturbing acceleration directed along a generic direction in space which, in general, does not coincide with the Neptune-satellite radial one.

### 2.1 Analytical and numerical calculation

The orbital effects of such an anomalous acceleration can be worked out with standard perturbative techniques by using e.g. the Gauss equations for the variations of the elements (Bertotti, Farinella & Vokrouhlický 2003):

\[ \frac{d\theta}{dt} = \frac{2}{n\eta} \left[ eA_R \sin f + A_T \left( \frac{p}{r} \right) \right], \]

(3)

\[ \frac{d\eta}{dt} = \frac{\eta}{na} \left( A_R \sin f + A_T \left[ \cos f + \frac{1}{c} \left( 1 - \frac{r}{a} \right) \right] \right), \]

(4)

\[ \frac{dI}{dt} = \frac{1}{na\eta} A_N \left( \frac{r}{a} \right) \cos u, \]

(5)

\[ \frac{d\Omega}{dt} = \frac{1}{na \sin I \eta} A_N \left( \frac{r}{a} \right) \sin u, \]

(6)

\[ \frac{d\omega}{dt} = \frac{\eta}{nae} \left[ -A_R \cos f + A_T \left( 1 + \frac{r}{a} \right) \sin f \right] - \cos I \frac{d\Omega}{dt}, \]

(7)

\[ \frac{dM}{dt} = n - \frac{2}{na} A_R \left( \frac{r}{a} \right) - \eta \left( \frac{d\omega}{dt} + \cos I \frac{d\Omega}{dt} \right), \]

(8)

where \( \mathcal{M} \) is the mean anomaly of the orbit of the test particle, \( f \) is its true anomaly reckoned from the pericentre position, \( u \equiv \omega + f \) is the argument of latitude, \( n \equiv \sqrt{GM/a^3} = 2\pi/P_b \) is the unperturbed Keplerian mean motion (\( G \) is the Newtonian constant of gravitation and \( M \) is the mass of the central body), \( \eta \equiv \sqrt{1 - e^2} \) and \( p \equiv a(1 - e^2) \) is the semilatus rectum. \( A_R, A_T, A_N \) are the projections of the perturbing acceleration \( \mathbf{A} \) on to the radial \( R \), transverse \( T \) and out-of-plane \( N \) directions of the particle’s comoving frame whose time-varying unit vectors are (Montenbruck & Gill 2000)

\[ \mathbf{r} = \begin{pmatrix} \cos \Omega \cos u - \cos I \sin \Omega \sin u \\ \sin \Omega \cos u + \cos I \cos \Omega \sin u \\ \sin I \sin u \end{pmatrix}, \]

(9)

\[ \mathbf{i} = \begin{pmatrix} - \sin u \cos \Omega - \cos I \sin \Omega \cos u \\ - \sin \Omega \sin u + \cos I \cos \Omega \cos u \\ \sin I \cos u \end{pmatrix}, \]

(10)

\[ \mathbf{n} = \begin{pmatrix} \sin I \sin \Omega \\ - \sin I \cos \Omega \\ \cos I \end{pmatrix}. \]

(11)

A straightforward calculation shows that the \( R-T-N \) components of a constant and uniform perturbing acceleration, like our \( A^{\text{Pio}} \) over the time-scales involved here, are linear combinations of \( A_R, A_T, A_N \) with coefficients proportional to harmonic functions whose arguments are, in turn, linear combinations of \( u, \Omega \) and \( I \). Thus, in the satellite’s comoving frame \( A^{\text{Pio}} \) is time dependent through \( f \) in \( u \). In order to have the secular perturbations of the Keplerian orbital elements, \( A_R, A_T, A_N \) have to be inserted into the right-hand sides of the Gauss equations which must be evaluated on to the unperturbed Keplerian ellipse:

\[ r = a(1 - e \cos E), \]

(12)

where \( E \) is the eccentric anomaly, and integrated over a full orbital revolution by means of

\[ \frac{dE}{P_b} = \frac{1}{2\pi} \frac{1 - e \cos E}{\sqrt{1 - e^2 \sin^2 E}} \]

(13)

Other useful relations are

\[ \cos f = \frac{\cos E - e}{1 - e \cos E}, \quad \sin f = \frac{\sqrt{1 - e^2 \sin^2 E}}{1 - e \cos E}. \]

(14)
After cumbersome calculations it turns out that, apart from the semimajor axis $a$ whose secular rate vanishes, all the other Keplerian orbital elements $\psi$ experience non-vanishing secular precessions of the form

$$\langle \psi \rangle = \frac{C_{ij}^{(s)} A_j + C_{ij}^{(e)} A_j + C_{ij}^{(w)} A_j}{n a}, \quad \psi = I, \Omega, \omega, M.$$  

(15)

In it,

$$C_{ij}^{(s)} = \sum_k F_{jk}^{(s)}(e) \cos \xi_{jk}^{(i)}, \quad j, k = x, y, z,$$

(16)

where $F_{jk}^{(s)}(e)$ are complicated functions of the eccentricity and $\xi_{jk}^{(i)}$ are linear combinations of the longitude of the pericentre $\sigma \equiv \omega + \Omega + I$. In principle, they are time varying according to

$$\sigma = \sigma_0 + \sigma f,$$

(17)

$$\Omega = \Omega_0 + \Omega f,$$

(18)

$$I = I_0 + I f;$$

(19)

from a practical point of view, since their secular rates are quite smaller, especially for Triton and Nereid, we can assume $\sigma \approx \sigma_0$, $\Omega \approx \Omega_0$, $I \approx I_0$ in computing $\cos \xi_{jk}$, where $\sigma$, $\Omega$, $I$, $I_0$ are their values at epoch (see Table 1).

The $R-T-N$ shifts over a generic time interval $\Delta t$ can be exactly computed according to Casotto (1993). The radial perturbation is

$$\Delta R = K_a \Delta a + K_e \Delta e + K_M \Delta M,$$

(20)

where

$$K_a = \frac{r}{a},$$

(21)

$$K_e = -a \cos f,$$

(22)

$$K_M = \frac{ae}{\sqrt{1 - e^2}} \sin f.$$  

(23)

Equation (20) shows that it would be incorrect to identify the shift in the radial component of the orbit with the perturbation of the semimajor axis only. Otherwise, misleading conclusions concerning the mean motion $n$ and, thus, the transverse component as well could be traced. Indeed, if, say, a secular signature in $\Delta R$ was found, from the identification $\Delta R = \Delta a$ it could be argued that an analogous perturbation in the mean motion

$$\Delta n = -\frac{3}{2} \frac{\Delta a}{a}$$

(24)

would occur as well. As a consequence, a quadratic effect in the transverse component should occur through the perturbed mean longitude. Actually, this does not happen in our case: indeed, we will show that, although, as already noted, no secular effects on $a$ are present, both the radial and the transverse components exhibit cumulative perturbations with secular trends, without any quadratic signature in the transverse one. The transverse perturbation is

$$\Delta T = H_e \Delta e + H_M \Delta M + r (\Delta \omega + \cos I \Delta \Omega),$$

(25)

with

$$H_e = a \left(1 + \frac{1}{1 - e^2} \frac{r}{a}\right) \sin f,$$

(26)

1The semimajor axis $a$ does not appear in the denominator of the equation for the eccentricity rate.

$$H_M = \frac{a^2 \sqrt{1 - e^2}}{r}.$$  

(27)

The out-of-plane perturbation is

$$\Delta N = r (\Delta I \sin u - \Delta \Omega \sin I \cos u).$$

(28)

In equations (20), (25) and (28) the perturbations of the Keplerian orbital elements have to be intended as

$$\Delta \psi = \int_0^E d\psi, \quad \psi = a, e, I, \Omega, \omega, M,$$

(29)

where $d\psi$ is taken from equations (3)–(8). The explicit expressions of $\Delta R_{Pio}$, $\Delta T_{Pio}$, $\Delta N_{Pio}$ are rather cumbersome, so that we will not explicitly show them. It turns out that linearly growing signatures are present in all of them along with sinusoidal terms; quadratic terms are, instead, absent.

In Figs 1–3 we plot the Pioneer-induced $R-T-N$ perturbations for Triton, Nereid and Proteus over a century. In order to express the eccentric anomaly as a function of time we used a partial sum

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Analytically computed transverse, radial and out-of-plane Pioneer-induced $R-T-N$ perturbations for Triton over 100 yr according to equations (20), (25) and (28), and Table 1. In the expansion of equation (30) we retained just the first term because, in view of the extremely small eccentricity of the orbit of Triton, the second term is of the order of $10^{-10}$.}
\end{figure}
The Neptunian system and the Pioneer anomaly

40
20
0
20
40
60

t (y)

0
20
40
60

Nereid

△T (km)

-15000
-10000
-5000
0
5000
10000
15000

T km

40
20
0
20
40
60

t (y)

0
20
40
60

Nereid

△R (km)

-10000
-5000
0
5000
10000

R km

40
20
0
20
40
60

t (y)

0
20
40
60

Nereid

△N (km)

-400
-200
0
200
400

N km

Figure 2. Analytically computed transverse, radial and out-of-plane Pioneer-induced shifts for Nereid over 100 yr according to equations (20), (25) and (28), and Table 1. In the expansion of equation (30) we retained the first 40 terms because, in view of the large eccentricity of the orbit of Triton, the following ones are of the order of, or smaller than $10^{-6}$.

of the series

$$E = \mathcal{M} + 2 \sum_{s=1}^{\infty} \frac{J_s(\varepsilon_0)}{s} \sin(s\mathcal{M}), \quad \mathcal{M} = n(t - t_0),$$

(30)

where $J_s(\varepsilon_0)$ are the Bessel functions of the first kind.

The peak-to-peak amplitudes for the Pioneer-type $R-T-N$ perturbations are 300, 600 km, 8 m for Triton, 17 500, 35 000, 800 km for Nereid and 60, 120 km, 30 m for Proteus.

It should, now, be pointed out that we have only considered the direct perturbations induced by $\mathbf{A}_{\text{Pio}}$ on each satellite considered separately; in fact, the total effect may be even larger because of the mutual gravitational interactions among the satellites themselves and Uranus which are all allegedly influenced by the PA as well. Moreover, it can be argued that, over time intervals larger than one orbital period as those used here, the Pioneer-induced signatures are modulated by the slowly changing $\Omega, \omega, I$ because of Neptune’s oblateness and $N$-body interactions with the other giant planets and satellites themselves, and by the variation of the Sun’s position which reflects into slow changes in the components of $\mathbf{A}_{\text{Pio}}$.

To support our analytical calculation and to further clarify the issue of the semimajor axis, we also performed numerical integrations with MATHEMATICA of the equations of motion of Triton, Nereid and Proteus with and without the PA. Concerning the classical forces common to all the three satellites, we included the first two even zonal harmonics $J_2, J_4$ of Neptune and the attraction of Uranus, Saturn and Jupiter. The intersatellite interactions have been taken into account as well by considering Nereid and Proteus as massless point particles acted upon by a massive Triton, as done by Jacobson (2009). First, in Fig. 4 we plot the Pioneer-type perturbations on the semimajor axes $a$ of Triton, Nereid and Proteus over one Keplerian orbital period. As expected from our analytical calculation, no cumulative, net effects occur; this is neither in

Figure 3. Analytically computed transverse, radial and out-of-plane Pioneer-induced shifts for Proteus over 100 yr according to equations (20), (25) and (28), and Table 1. In the expansion of equation (30) we retained just the first term because, in view of the extremely small eccentricity of the orbit of Triton, the second term is of the order of $10^{-7}$.

2It converges for all $\varepsilon < 1$. See on the web: http://mathworld.wolfram.com/KeplersEquation.html

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2.2 Confrontation with the accuracy of the orbits

Although obtained differently, Figs 1–3 can be compared with figs 4–6 by Jacobson (2009), which yield a measure of the accuracy of the orbits of the Neptunian satellites considered. They have been obtained by fitting the observational data sets without including the PA in the dynamical force models of Triton, Nereid and Proteus, but are useful to give us an idea about a possible detection of Pioneer-type effects in their orbital dynamics. More specifically, figs 4–6 by Jacobson (2009) display the orbit uncertainties from the consider covariance mapped into the $R$–$T$–$N$ directions. In general, the solution covariance yields an optimistic measure of the orbit uncertainties because it does not account for possible systematic or unmodelled errors. They can occur in the dynamical force models, in the observation modelling or in the observation themselves; Jacobson (2009) believes that the dominant systematic errors mainly reside in the observations because the models adopted fit to them at their presumed accuracies. To include the effect of neglected errors, Jacobson (2009) added some ‘consider’ parameters to the estimation process. They are quantities which are not estimated, but whose uncertainty contributes to the uncertainty in the estimated parameters. The reliability of such a procedure was subsequently tested by Jacobson (2009) refitting the orbits with different data sets and comparing the consequent changes in the orbits to the uncertainties derived from the consider covariance for the modified data sets. It turned out that the changes in the orbits were at or below the level of uncertainties. Thus, Jacobson (2009) concluded that all important errors were properly accounted for, and that the consider covariance can reliably be adopted as a realistic measure of the orbit accuracy.

Fig. 4 by Jacobson (2009) deals with Triton. The middle panel shows the radial distance uncertainty which oscillates between 0.8 and 1.2 km over a time-span of one century. The middle panel of Fig. 1 tells us that the Pioneer-induced radial perturbation would be as large as 300 km for Triton, so that it seems reasonable to argue that the resulting overall anomalou shift should not have escaped from detection over a time-span of more than one century; note that the astrometry of Triton covers 161 yr from 1847, i.e. 1 yr after its discovery, through 2008. Even by rescaling the radial uncertainty by a factor of 10, the situation would not change. The upper panel of fig. 4 by Jacobson (2009) depicts the transverse uncertainty. It linearly grows reaching a level of about 150 km after 60 yr; the upper panel of Fig. 1 shows that $\Delta T^{\text{Pio}}$ is larger, although not by two orders of magnitude as in the radial case. Thus, also a Pioneer-type transverse effect may have remained undetectable with difficulty. The lower panel of fig. 4 by Jacobson (2009) displays the out-of-plane uncertainty which amounts to about 50 km after 60 yr, while the anomalous PA perturbation would be orders of magnitude smaller, as shown by the lower panel of Fig. 1.

The orbit accuracy of Nereid is shown in fig. 5 by Jacobson (2009). The radial distance uncertainty linearly grows up to about 1600 km after 60 yr (middle panel of fig. 5 by Jacobson 2009), while the peak-to-peak radial PA effect is as large as $\Delta R^{\text{Ner}} = 17.500$ km for Nereid whose data set covers 59 yr from its discovery in 1949 through 2008, i.e. one order of magnitude larger. The upper panel of fig. 5 by Jacobson (2009) displays the transverse accuracy which linearly grows up to about 3000 km. According to Fig. 2, $\Delta T^{\text{Ner}} = 35 000$ km, i.e. about one order of magnitude larger. The accuracy in the out-of-plane direction is displayed in the lower panel of fig. 5 by Jacobson (2009); it oscillates between a few km to 60 km. According to the lower panel of Fig. 2, the corresponding Pioneer-type out-of-plane peak-to-peak amplitude is as large as 800 km, more than 10 times larger. Thus, in the case of Nereid the PA perturbations over 59 yr would be more than one order of magnitude larger than the corresponding orbit accuracy in all the three directions.

Fig. 6 by Jacobson (2009) depicts the $T$–$R$–$N$ accuracy for Proteus whose observational record is about 20 yr long. The radial
distance uncertainty, shown in the middle panel of fig. 6 by Jacobson (2009), oscillates between 2.5 and 4.5 km, while the peak-to-peak amplitude of the anomalous PA radial signal over a similar time-span is about 20–30 km. The transverse accuracy (upper panel of fig. 6 by Jacobson 2009) linearly grows up to 200 km, making, thus, problematic a detection of the corresponding PA transverse shift ($\Delta T^{\text{freq}} = 120$ km). The anomalous out-of-plane effect is several orders of magnitude smaller than the corresponding orbit accuracy shown in the lower panel of fig. 6 by Jacobson (2009) which is as large as 50 km. Thus, in the case of Proteus a PA-type radial perturbation would be about one order of magnitude larger than the corresponding orbit uncertainty, while the transverse and out-of-plane PA signatures would have been overwhelmed by the corresponding orbit uncertainties.

Finally, let us conclude by noting that, concerning the direct effect of the PA on the orbital motions of the outer planets, Iorio & Giudice (2006) showed in their table 1 that the induced anomalous perihelion precessions $\dot{\psi}_{\text{pl}}$ are in the range 83.5–116.2 arcsec cty$^{-1}$ for Uranus–Pluto. Actually, latest determinations of the correction $\Delta \dot{\psi}$ to the standard perihelion precessions by Pitjeva (2010) with the EPM2008 ephemerides summarized in her table 8 are, instead, $-3.89 \pm 3.90, -4.44 \pm 5.40$ and $2.84 \pm 4.51$ arcsec cty$^{-1}$, respectively.

3 DISCUSSION AND CONCLUSIONS

We have investigated the impact that an anomalous, constant and uniform acceleration directed towards the Sun having the same magnitude of the PA would have on the orbital dynamics of the Neptunian satellites Triton, Nereid and Proteus which move in the deep PA region of the Solar system. Long data sets covering a large number of orbital revolutions are currently available for them.

We, first, used an analytical approach which only considered the direct PA-type perturbations on the three satellites taken separately to work out the corresponding shifts in the radial, transverse and out-of-plane orbit components. In fact, also the indirect effects caused by the PA-affected mutual gravitational interactions among them should be, in principle, considered. Then, we numerically integrated the equations of motion with and without an extra-PA acceleration confirming the analytical findings. It turned out that only secular and sinusoidal signatures are present in the three orbit components; we confirming the analytical findings. It turned out that only secular and sinusoidal signatures are present in the three orbit components; we confirming the analytical findings. It turned out that only secular and sinusoidal signatures are present in the three orbit components; we confirming the analytical findings.

Our analysis showed that the resulting anomalous orbital effects are much larger than the realistic orbit accuracies evaluated from a recent analysis of all the available astrometric observations by one–two orders of magnitude. However, it must be stressed that our investigation should be considered preliminary. Indeed, it would be necessary to refit the entire set of observations to the corresponding predictions computed by taking the anomalous PA effect into account. As an alternative approach, it would also be possible to fit the predicted observations without the PA to a set of simulated observations produced by including the PA. Our study demonstrates that such further investigations, which are beyond the scopes of this paper, should be considered worth the needed effort.

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REFERENCES

Anderson J. D., Nieto M. M., 2010, in Klioner S. A., Seidelman P. K., Soffel M. H., eds, Proc. IAU Symp. 261, Relativity in Fundamental Astronomy: Dynamics, Reference Frames, and Data Analysis. Cambridge Univ. Press, Cambridge, p. 189

Anderson J. D., Laing P. A., Lau E. L., Liu A. S., Nieto M. M., Turyshhev S. G., 1998, Phys. Rev. Lett., 81, 2858

Anderson J. D., Laing P. A., Lau E. L., Liu A. S., Nieto M. M., Turyshhev S. G., 1999a, Phys. Rev. Lett., 83, 1891

Anderson J. D., Laing P. A., Lau E. L., Liu A. S., Nieto M. M., Turyshhev S. G., 1999b, Phys. Rev. Lett., 83, 1893

Anderson J. D., Laing P. A., Lau E. L., Liu A. S., Nieto M. M., Turyshhev S. G., 2002a, Phys. Rev. D, 65, 082004

Anderson J. D., Turyshhev S. G., Nieto M. M., 2002b, BAAS, 34, 1172

Avramidi I. G., Fucchi G., 2009, Canadian J. Phys., 87, 1089

Bertolami O., Páramos J., 2004, Classical Quantum Gravity, 21, 3309

Bertolami O., Páramos J., 2007, Int. J. Modern Phys. D, 16, 1611

Bertolami O., Vieira P., 2006, Classical Quantum Gravity, 23, 4625

Bertolami O., Francisco F., Gil P. J. S., Páramos J., 2008, Phys. Rev. D, 78, 103001

Bertotti B., Farinella P., Vokrouhlicky D., 2003, Physics of the Solar System. Kluwer, Dordrecht, p. 313

Bini D., Cherubini C., Mashhoon B., 2004, Phys. Rev. D, 70, 044020

Brownstein J. R., Moffat J. W., 2006, Classical Quantum Gravity, 23, 3427

Casotto S., 1993, Celest. Mech. Dynamical Astron., 55, 209

de Diego J. A., 2008, Revista Mexicana Astron. Astrofisica Ser. Conf., 34, 35

de Diego J. A., Núñez D., Zavala J., 2006, Int. J. Modern Phys. D, 15, 533

Dittus H. et al., 2005, in Favata F., Sanz-Forcada J., Giménez A., Battrick B., eds, ESA SP-588. A mission to explore the Pioneer anomaly. ESA, Noordwijk, p. 3

Exirifard Q., 2009, Classical Quantum Gravity, 26, 025001

Fienga A., Laskar J., Kuchynka P., Manche H., Gastineau M., Lepicin-Lafitte C., 2009, in Heydari-Malayeri M., Reylé C., Samadi R., eds, SF2A-2009: Proceedings of the Annual Meeting of the French Society of Astronomy and Astrophysics. Besançan, France, p. 105

Greaves E. D., 2009, Revista Mexicana Astron. Astrofisica Ser. Conf., 35, 23

Iorio L., 2007a, Foundations Phys., 37, 897

Iorio L., 2007b, J. Gravitational Phys., 1, 5

Iorio L., 2009a, AJ, 137, 3615

Iorio L., 2009b, Int. J. Modern Phys. D, 18, 947

Iorio L., Giudice G., 2006, New Astron., 11, 600

Jacobson R. A., 2007, AJ, 137, 4322

Jaekel M.-T., Reynaud S., 2008, in Dittus H., Lämmerzahl C., Turyshhev S. G., eds, Lasers, Clocks and Drag-Free Controls. Exploration of Relativistic Gravity in Space. Springer-Verlag, Berlin, p. 193

Katz J. I., 1999, Phys. Rev. Lett., 83, 1892

Lämmerzahl C., Preuss O., Dittus H., 2008, in Dittus H., Lämmerzahl C., Turyshhev S. G., eds, Lasers, Clocks and Drag-Free Controls. Exploration of Relativistic Gravity in Space. Springer-Verlag, Berlin, p. 75

Levy A., Christophe B., Bério P., Métris G., Courty J.-M., Reynaud S., 2009, Advances Space Res., 43, 1538

List M., Mullin M., 2008, in Macias A., Lämmerzahl C., Camacho A., eds, AIP Conf. Proc. Vol. 977, Recent Developments in Gravitation and Cosmology. Am. Inst. Phys., New York, p. 284

Markwardt C. B., 2002, preprint (gr-qc/0208046)

Mbekle J. P., Michalski M., 2004, Int. J. Modern Phys. D, 13, 865

Mbekle J. P., Mosquera Cuesta H. J., Novello M., Salim J. M., 2007, Europhys. Lett., 77, 19001

Montenbruck O., Gill E., 2000, Satellite Orbits. Springer-Verlag, Berlin, p. 27

Murphy E. M., 1999, Phys. Rev. Lett., 83, 1890

Nieto M. M., 2005, Phys. Rev. D, 72, 083004

Nieto M. M., 2006, Europhys. News, 37, 30
Nieto M. M., 2008, Phys. Lett. B, 659, 483
Nieto M. M., Anderson J. D., 2005, Classical Quantum Gravity, 22, 5343
Nieto M. M., Turyshev S. G., Anderson J. D., 2005, Phys. Lett. B, 613, 11
Olsen Ø, 2007, A&A, 463, 393
Page G. L., Dixon D. S., Wallin J. F., 2006, ApJ, 642, 606
Page G. L., Wallin J. F., Dixon D. S., 2009, ApJ, 697, 1226
Pitjeva E. V., 2006, in Paper Presented at 26th Meeting of the IAU, The Dynamical Model of the Planet Motions and EPM Ephemerides. Prague, Czech Republic, Joint Discussion 16, no. 55
Pitjeva E. V., 2010, in Kliioner S. A., Seidelman P. K., Soffel M. H., eds, Proc. IAU Symp. 261, Relativity in Fundamental Astronomy: Dynamics, Reference Frames, and Data Analysis. Cambridge Univ. Press, Cambridge, p. 179
Tangen K., 2007, Phys. Rev. D, 76, 042005
Toth V. T., Turyshev S. G., 2008, in Macias A., Lämmerzahl C., Camacho A., eds, AIP Conf. Proc. Vol. 977, Recent Developments in Gravitation and Cosmology. Am. Inst. Phys., New York, p. 254
Toth V. T., Turyshev S. G., 2010, in Kliioner S. A., Seidelman P. K., Soffel M. H., eds, Proc. IAU Symp. 261, Relativity in Fundamental Astronomy: Dynamics, Reference Frames, and Data Analysis. Cambridge Univ. Press, Cambridge, p. 179
Standish E. M., 2008, in Macias A., Lämmerzahl C., Camacho A., eds, AIP Conf. Proc. Vol. 977, Recent Developments in Gravitation and Cosmology. Am. Inst. Phys., New York, p. 254
Wallin J. F., Dixon D. S., Page G. L., 2007, ApJ, 666, 1296
Wilson T. L., Blome H.-J., 2009, Advances Space Res., 44, 1345

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