The Frontier of Estimator Comparison between MLE and MEboot Estimation: Application for Optimization Management of Macroeconomics

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Abstract. One of the most difficult problems that many quantitative researchers have been trying to computationally solve is the parametric prediction. Interestingly, is the maximum likelihood estimator really the best estimator for data predictive estimating in the recent moment? This question leads the authors to conduct the mathematically experimental study by using data generating processes (DGP), entropy calculating, and cross-entropy analyses for seeking the best estimator between the maximum likelihood method (MLE) and maximum entropy bootstrapping approach (MEboot). Furthermore, the experimental solution would be employed to the real application in a macro economical research. Consequently, the empirical results in this paper can be the sensible tool for mathematicians or even economists to improve the data prediction in time-series analyses.

1. Introduction

Historically, it is like saying that to let data stories presenting moments in the future is not proper anymore. When we are focusing on behavioural information in the future, data collections should be observed from data in the future. However, the problem is that it is extremely difficult to find the future observations which contain the conditions of robustness and consistency. The former condition regards to provide an alternative and potentially powerful explanation of the generalization ability of data distributions. The latter condition is about to re-prove the statistical consistency of the linear classifier [1] such as the autoregressive regression (AR model) or even the generalized autoregressive conditional heteroskedasticity (GARCH process). Consequently, mathematical simulated experiments such as Data Generating Processes (DGP) relied on [2] and [3], Entropy analyses [4], even Cross-Entropy analyses (CE) [5] are employed in this paper to seek the best experimental solution for this issue and apply it to an empirical application.

Statistically, multivariate linear models are estimated by mostly employing the maximum likelihood estimator (MLE) to seek a parametric solution for several decades. For example, [6], [7], [8], and [9]. However, is MLE the most precise estimator? Interestingly, the maximum entropy bootstrapping estimator (MEboot) introduced by [10] can be the potential tool that evaluates linear models more precisely, but it is rare that this estimator is empirically employed to both experimental analyses and real applications. For
example, [11] and [12]. Accordingly, the comparison of these two estimator in mathematical experiments conducted in this paper can indicate the sensible way to improve a forecasting research, especially in macroeconomics.

2. The Objective and Scope of Research
The scope of the research is divided into two sections. First, it is the experiment of mathematical calculations using two simulated observation sets (Data Generating Process: DGP), including the sets of 500 and 1,000 random samples. Afterward, the comparison between the maximum likelihood estimator (MLE) and maximum entropy bootstrapping estimator (MEboot) is originated by applying the entropy and cross-entropy methods. Second, the section of the empirical application for macroeconomic time-series variables is conducted to confirm that the results from the experimental comparison can improve the parametric estimation of the empirical research based on the Markovitz optimization management by [13]. Additionally, Bayesian statistics relied on [14], [15], and [16] was employed to estimate the econometric models. For instance, ADF unit-root testing [17], Markov switching modelling [18]. The yearly observations employed in this section are Thailand macroeconomic factors such as gross domestic products (GDP), inflation rates, interest rates, exchange rates, and money supplies. These are collected during 1965 to 2015.

3. The Overall of Methodologies

![Figure 1. The conceptual framework of the research](image.png)
3.1 The results of the mathematically experimental analysis.
The section of the computationally experimental analysis in this paper is originally proposed to investigate the best parametric estimator between the MLE and MEboot estimations. Simulated data sets were estimated by using the maximum likelihood estimator (MLE) and maximum entropy bootstrapping approach (MEboot). From the AR-GARCH model, residual terms were observed and transformed into the cumulative distribution function (CDF), which is a real-valued random variable. The CDF terms were then employed to calculate on the entropy analysis, including the general entropy and cross-entropy method. Empirically, the results of the cross-entropy comparison were shown in Table 1. In the case of 500 random observations, there is obvious that the MEboot estimation more provides the precise estimator rather than using the MLE approach. In other words, the CE results stated the value of cross-entropy calculating on the residual terms observed from the MEboot estimation (82.0112) is the nearest entropy value to the overall entropy (65.9109). Similarly, in the case of 1,000 random samples, the result still confirms that the MEboot estimator can give the precise parameter rather than the counterpart. Obviously, the value of cross-entropy calculating on the residual terms observed from the MEboot estimation (161.1296) is the shortest distance value from the overall value (65.4534). Accordingly, it is sensible to conclude that the MEboot estimator will be chosen to empirically apply to the realistic time-series indexes.

Table 1. The Data Generating Processes (DGP) in the AR-GARCH model based on the simulated random sampling and the Entropy analyses.

| Details | AR-GARCH (n = 500) | AR-GARCH (n = 1,000) |
|---------|-------------------|---------------------|
|         | Overall Entropy   | MLE (CE analysis)   | MEboot (CE analysis) | Overall Entropy | MLE (CE analysis) | MEboot (CE analysis) |
| Estimated entropy values (calculating from residual terms generated by AR-GARCH) | 65.9109 | 83.2470 | 82.0112* | 65.4534 | 161.2539 | 161.1296* |

Noted: * the nearest CE value to the overall entropy value

3.2 The Empirical Application in the Real Economic Research

The ADF test based on Bayesian inference analyses the null hypothesis that a time-series data is non-stationary I(1) against the alternative I(0). The result was represented in Table 3. The results of the selecting prior for the switching estimation based on Bayesian inference and all of divided regimes, defining as boom and recession periods, were shown in Table 2. Empirically, the optimization approach for portfolio managements is employed to clarify the proportion of Thailand economic factors. Descriptively, Table 3 shows average values, standard errors and variances of five variables, which are successfully estimated from the AR-GARCH model using the MEboot estimator. As Markowitz’s mean-variance optimization by minimizing standard errors, the empirical result shows that the optimal approach can efficiently maximize the expected growth of the whole economic system and minimize the risky value of the whole economic expansion simultaneously. The details in Table 4 are also consonant. In booming periods, the sharp ratio is a double increment from 0.090 to 0.169. Similarly, in recession periods, the ratio is a little bit improvement from 0.333 to 0.410. As a result, this can be implied the overall economic system in both booming and recessing periods is improved when we employ the optimal management approach.

Table 2. The MSBVAR estimation for Thailand macroeconomic factors during 1976 to 2015.

| Variables | Stage | Duration (Forecasting by MSBVAR) |
|-----------|-------|----------------------------------|
| Thailand economic system (including GDP, inflation, interest rate, exchange rate, money supply) | Economic Boom periods | 17 years |
| | Economic Recession periods | 22 years |
Table 3. Descriptive information estimated from the AR-GARCH model with the MEboot estimator.

| Stage     | Details                              | GDP I(0) (%) | Inflation I(0) (%) | Interest rate I(0) (%) | Exchange rate Bath per US dollar I(0) (%) | Money supply I(0) (%) |
|-----------|--------------------------------------|--------------|-------------------|------------------------|------------------------------------------|----------------------|
| Boom      | Mean from the AR model (\(a\))      | 0.485        | 0.322             | 0.246                  | -0.185                                   | 0.504                |
|           | Variances from the GARCH model (\(\sigma^2\)) | 0.432        | 0.387             | 0.298                  | 0.223                                    | 0.449                |
| Recession | Mean from the AR model (\(a\))      | 0.413        | 0.134             | 0.568                  | 0.339                                    | 0.449                |
|           | Variances from the GARCH model (\(\sigma^2\)) | 0.409        | 0.161             | 0.508                  | 0.018                                    | 0.518                |

Noted: I(0) represents the trends of collected variables are stationary

Table 4. the mean-variance portfolio optimization based on minimizing standard errors

| Stage     | Details                              | Neutral Economic Cycling movement | Optimal policy management for Economic Cycling movement |
|-----------|--------------------------------------|-----------------------------------|-------------------------------------------------------|
| Boom      | The expected value of whole system growth, \( \mu \), mean | 0.275                             | 0.504                                                 |
|           | The risky value of whole system growth, \( \sigma \), variance | 3.041                             | 2.991                                                 |
|           | The sharp ratio of whole system \( \frac{\mu}{\sigma} \) | 0.090                             | 0.169                                                 |
| Recession | The expected value of whole system growth, \( \mu \), mean | 0.381                             | 0.568                                                 |
|           | The risky value of whole system growth, \( \sigma \), variance | 1.142                             | 1.385                                                 |
|           | The sharp ratio of whole system \( \frac{\mu}{\sigma} \) | 0.333                             | 0.410                                                 |

For Table 5, in the booming periods, money supplies especially are the factor needed to be the intensively controllable indicator. The latent value, which equals 0.46, also implied that the economic system is not efficient since there is some information which is elusive. In the case of recession periods, on the other hand, the factor of interest rates is the predominant. The latent value, which is equivalent to 0.43, stated that the economic system is not going to be efficient in the downsizing situations.

Table 5. The proportion of systematically economic optimization.

| Stage     | Details                              | GDP | Inflation | Interest rate | Exchange rate | Money supply |
|-----------|--------------------------------------|-----|-----------|---------------|---------------|--------------|
| Boom      | Optimal portions of growth (forecasting interval value) | 0.36 | 0.31      | 0.28          | 0.14          | 0.37         |
|           | Summary                               | 1.46 |           |               |               |              |
|           | Latent value                          |      |           |               |               | 0.46         |
| Recession | Optimal portions of growth (forecasting interval value) | 0.29 | 0.23      | 0.33          | 0.28          | 0.30         |
|           | Summary                               | 1.43 |           |               |               | 0.43         |
|           | Latent value                          |      |           |               |               |              |

4. Conclusion
The succession of mathematical modelling comparisons to investigate the suitable estimator for data predicting is already clarified in this paper. Experimentally, the DGP approach in the AR-GARCH model indicates the empirical result that the parametric estimator calculated by the MEboot method can efficiently provide a precise parameter rather than using the MLE estimation. This is strongly confirmed.
by the outcomes of cross-entropy (CE) calculations. Applying in the Markovitz optimization management, this application shows the estimated results empirically represented that the major macroeconomic indexes should be emphasized in booming periods are money supplies, GDP, and inflation rates, respectively. On the other hand, the predominant factors should be intensive concerned are interest rates, money supplies, and GDP, respectively.

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