Comparison of IDW and GP models with application to spatiotemporal interpolation of rainfall in Bali Province, Indonesia

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Abstract. Precipitation is a critical weather component in our daily life. Spatiotemporal variation in rainfall is very influential in agriculture, health, tourism, and many more. Low or large precipitation volume or intensity can be a problem. However, precipitation is not easy to control, so minimize the negative impact of rainfall is an essential thing that can be done. Precipitation volume or intensity in an area is generally reported based on meteorology stations at several location points, which are then used to describe the value of precipitation in the broader region. However, the number of weather observation stations is often minimal. So using this average of little station information can produce incorrect rainfall information. Various approaches have been developed to be able to provide accurate information based on several observation points. These approaches include Inverse Distance Weighted (IDW) and Gaussian Process (GP). This study aims to compare the accuracy of the IDW and GP methods in conducting rainfall interventions in Bali's Indonesian province. Bali is one of the world's tourist destinations where rainfall is very influential on tourist visits. We found IDW provides a good prediction for close location, and GP is much better for a distant place.

1. Introduction
Precipitation is of prime important weather component in our daily life [1]. Spatiotemporal variation in rainfall affects agriculture, health, tourism, and many more. The accurate precipitation prediction for broad regions with limited rain observation stations is crucial for many environmental studies, mainly related to water resources. For small areas, the use of measurements from individual rain observation stations might be appropriate [2].

Precipitation is not easy to control. Low or large precipitation volume or intensity can be a problem. Minimize the negative impact of rainfall is the essential thing that can be done. Precipitation volume or intensity in an area is generally reported based on meteorology stations at several location points, which are then used to describe the value of precipitation in the broader area. However, the number of weather observation stations is often very limited. The average information from several
rain observation stations is widely used to inform the local regions' rainfall conditions. However, given the limited number of observation stations, the information might be misleading. The interpolation approaches from pint measurement are commonly used [3-5].

Various interpolation approaches have been developed to provide accurate local precipitation prediction. These approaches include Inverse Distance Weighted (IDW) and Gaussian Process (GP). This paper aims to compare and analyze IDW and GP methods on spatiotemporal rainfall interpolation concerning their suitability to produce spatial rainfall predictions on a monthly time step with a limited number of observations weather stations. Cross-validation is a standard method used to compare and evaluate the interpolation results [6]. This validation method's accuracy depends on the number and the locations of gauges within the study area, which should be representative of the spatial distribution of precipitation [2].

2. Material and Method
2.1. Study area
Bali is an Indonesian island (5,780 km²) located adjacent Java island (8.3405° S, 115.0920° E). There are only four rain observation stations in Bali (Figure 1).

Figure 1. Locations of observation weather stations in Bali, Indonesia

Figure 1 shows the location of observation weather stations in Bali, Indonesia. The position of station one looks very far apart compared to other stations.

2.2. Interpolation methods
Let $z_{it}$ denotes the precipitation data was observed from station $i$ at period $t$. Two methods are compared to predict the precipitation.

Inverse distance weighted (IDW)
IDW has been developed to extend the basic idea of using the data point's similarity weight. Instead, only value at the nearest data location being used, values at other data locations are considered but are
weighted. Weight is assumed to be inversely related to the distance between observed and predicted locations. The estimator is the form \[ z_o = \frac{\sum_{j=1}^{m} k_j z_j}{\sum_{j=1}^{m} k_j} \quad \forall t, t = 1, \ldots, 12 \] (1)

where \( z_j \) are from the "m" nearest positions and the weights \( k_j \) are chosen larger for data value near to where the estimate \( z_o \) is to be made. The denominator is a normalizing factor.

**Gaussian process (GP)**

We use independent GP model as an alternative of IDW. The Gaussian process (GP) dealing with nonlinear nonparametric regression with good Bayesian prediction performance. GP model is also known as nonparametric Bayesian approach [9, 10]. Bayesian method provides a flexible way in prediction modeling.

The hierarchical model for independent GP can be written as follows:

\[
\begin{align*}
\mathbf{Z}_{it} &= \mathbf{O}_{it} + \epsilon_{it} \\
\mathbf{O}_{it} &= \mathbf{X}_{it} \beta + u_{it}
\end{align*}
\]

for \( i = 1, \ldots, n \) and \( t = 1, \ldots, T \) with \( \epsilon_{it} \) and \( u_{it} \) were assumed identically independent with mean 0 and variances \( \sigma_\epsilon^2 \) and \( \sigma_u^2 \), respectively. Let \( S_\eta \) denotes the spatial correlation matrix obtained from Matern correlation function [11] defined as:

\[
\kappa(s_i, s_j; \phi, v) = \frac{1}{\Gamma(v)\nu} (2\sqrt{\nu}\|s_i - s_j\|\phi)^v K_v(2\sqrt{\nu}\|s_i - s_j\|\phi), \phi > 0, v > 0,
\]

where \( \Gamma(v) \) is the standard gamma function, \( K_v \) is the modified Bessel function of second kind with order \( v \), and \( \|s_i - s_j\| \) denotes the distance between areas \( s_i \) and \( s_j \). The parameter \( \phi \) is used to control the decay of the correlation when the distance \( \|s_i - s_j\| \) increases. The parameter \( \nu \) controls smoothness of the random field [12, 13]. Let \( \mathbf{O}_{it} \) denotes a random component and vector parameter model \( \theta = \{\beta, \sigma_\epsilon^2, \sigma_u^2, \phi, v\} \) with prior \( p(\theta) \). The logarithm joint posterior distribution is defined as [14]:

\[
\log p(\theta, \mathbf{O}, \mathbf{Z}|\mathbf{z}) \propto -\frac{n}{2} \log \sigma_\epsilon^2 - \frac{1}{2\sigma_u^2} \sum_{i=1}^{n} \sum_{t=1}^{T} (\mathbf{Z}_{it} - \mathbf{O}_{it})'(\mathbf{Z}_{it} - \mathbf{O}_{it}) - \frac{nT}{2} \log|\sigma_u^2 S_u|
\]

\[
-\frac{1}{2\sigma_u^2} \sum_{i=1}^{n} \sum_{t=1}^{T} (\mathbf{O}_{it} - \mathbf{X}_{it}\beta)'S_u^{-1}(\mathbf{O}_{it} - \mathbf{X}_{it}\beta) + \log p(\theta)
\]

Parameters \( \beta, \phi, \) and \( v \) are given independent normal prior distributions and \( (\sigma_\epsilon^2, \sigma_u^2) \) are signed inverse gamma. The GP model is estimated using Markov Chain Monte Carlo approach. It can be done using spTimer packages in R [14]. It uses Gibbs sampling technique [15].

2.3. Model comparison

For model comparison we use leave-one-out cross validation approach. We have four observation weather stations, therefore there were four difference schemes. The best model is selected based on predictive and goodness of fit performance measures.

**Predictive performance**

Predictive performance measures including mean absolute error (MAE), root mean squared error (RMSE), and mean absolute percentage error (MAPE) [14,16]:

\[
\text{MAE}_i = \frac{1}{T} \sum_{t=1}^{T} |\hat{z}_{it} - z_{it}|
\]

(5)
\[
RMSE_i = \sqrt{\frac{1}{T} \sum_{t=1}^{T} (\hat{z}_{it} - z_{it})^2}
\]
\[
MAPE_i = \frac{1}{T} \sum_{t=1}^{T} \left| \frac{\hat{z}_{it} - z_{it}}{z_{it}} \right|
\]

**Goodness of fit**

Goodness of fit measure is based on determination of coefficients \( R^2 \):

\[
R_i^2 = 1 - \frac{\sum_{t=1}^{T} (\hat{z}_{it} - z_{it})^2}{\sum_{t=1}^{T} (z_{it} - \bar{z}_i)^2}
\]

where \( \hat{z}_{it} \) denotes the prediction of precipitation for station \( i \) and time \( t \) with \( z_{it} \) is the observation value.

### 3. Result and Discussion

#### 3.1. Data exploration

The monthly precipitation data from four observation stations in Bali used as an application to compare the IDW and GP interpolation methods. The data were obtained from [http://dataonline.bmkg.go.id/home](http://dataonline.bmkg.go.id/home). The detailed is presented in Table 1.

| Month   | Station 1 (115.188; -8.582) | Station 2 (114.682; -8.312) | Station 3 (115.530; -8.374) | Station 4 (115.221; -8.669) |
|---------|------------------------------|------------------------------|------------------------------|------------------------------|
| January | 431.45                       | 257.70                       | 392.92                       | 424.92                       |
| February| 279.43                       | 190.37                       | 268.82                       | 289.57                       |
| March   | 227.30                       | 216.42                       | 306.47                       | 272.28                       |
| April   | 75.90                        | 133.52                       | 167.95                       | 97.55                        |
| May     | 94.47                        | 148.08                       | 86.05                        | 90.78                        |
| June    | 80.95                        | 80.12                        | 120.68                       | 61.33                        |
| July    | 54.18                        | 99.13                        | 109.57                       | 48.50                        |
| August  | 14.07                        | 35.92                        | 25.83                        | 7.67                         |
| September| 65.70                       | 32.40                        | 6.68                         | 44.17                        |
| October | 20.78                        | 107.75                       | 49.17                        | 37.02                        |
| November| 146.50                       | 196.22                       | 97.98                        | 179.03                       |
| December| 348.82                       | 290.73                       | 236.62                       | 325.15                       |

Station 1: Badung district
Station 2: Jimbaran district
Station 3: Karangasem district
Station 4: Badung city

The high precipitation was found between November to March. The precipitation data from observation station 2 relatively different from the other stations. In order to interpolate the precipitation data we generate 10,000 gird points.
3.2. Leave one out cross-validation for model comparison
In order to evaluate the prediction performance of IDW and GP methods we used leave one out cross-validation procedure. The results are presented below.

![Interpolation precipitation based on (a) IDW and (b) GP methods for stations 2, 3, and 4](image1)

**Figure 2.** Interpolation precipitation based on (a) IDW and (b) GP methods for stations 2, 3, and 4

![Interpolation precipitation based on (a) IDW and (b) GP methods for stations 1, 3, and 4](image2)

**Figure 3.** Interpolation precipitation based on (a) IDW and (b) GP methods for stations 1, 3, and 4
Figures 2-5 show the spatiotemporal interpolation precipitation for scheme 1-4. In general, GP model gives over smoothing result and IDW seems overfitting. The models comparison based on goodness of fit performance and predictive performance are presented in Table 2.
Table 2. Interpolation error monthly precipitation for each scheme

| Exclude   | IDW          | GP           |
|-----------|--------------|--------------|
|           | MAE | RMSE | MAPE | $R^2$ | MAE | RMSE | MAPE | $R^2$ |
| Station 1 | 14.58 | 18.02 | 21.13 | 0.982 | 31.34 | 42.81 | 0.999 |
| Station 2 | 49.45 | 64.95 | 35.94 | 0.989 | 35.67 | 45.43 | 0.970 |
| Station 3 | 44.2 | 53.5 | 88.49 | 0.994 | 44.5 | 50.87 | 94.400 | 0.980 |
| Station 4 | 13.09 | 15.71 | 23.23 | 0.999 | 30.47 | 33.9 | 50.070 | 0.959 |

Different set samples across the Bali province were used to compare rainfall interpolation methods and assess spatial rainfall variability over a 12-month monitoring period in 2018. Traditional and geostatistical interpolation methods, including inverse distance weighting (IDW), and Gaussian process (GP), were used to estimate wet and dry season rainfall. GP method produced the highest error, whereas IDW produced the lowest error in all schemes (i.e., MAE, RMSE, and MAPE). The GP method had a more fitted model with very high $R^2$ for the station one was excluded (see Fig. 1 (b)). In general, the IDW method outperformed in prediction performance (See Table 2 and 3). However, there is an exciting result of excluding station 2. Station 2 was located far away from the other stations. In this situation, the GP model provides a better result. It shows that the GP model is better used for sparse data and IDW for dense data.

Table 3. Interpolation error monthly precipitation for all schemes

|         | MAE  | MSE  | MAPE  | $R^2$ |
|---------|------|------|-------|-------|
| IDW     | 30.33 | 43.74 | 42.20 | 0.880 |
| GP      | 35.49 | 42.41 | 53.51 | 0.869 |

Table 3 shows the models comparison based on prediction performance indicators including MAE, RMSE, and MAPE, and goodness of fit based on $R^2$. IDW gives minimum error than GP model. The spatiotemporal interpolation of precipitation using 4 observation stations is presented in Figure 6.
Figure 6. Spatiotemporal interpolation given 4 stations based on IDW

Figure 6 shows the spatiotemporal interpolation using all observation stations based on IDW method. High precipitation occurs during periods and the highest volume of rainfall is found in the southern part of the island of Bali.

4. Conclusions
This paper is concerned with comparing the applied interpolators’ methods and finding a technique characterized by high accuracy of point estimations. The analyses conducted in this study enabled to provide the most reliable picture of the variability of precipitation quality parameters in a precipitation evolution in Bali province, Indonesia. We compared two spatial interpolation methods: IDW and GP. Monthly rainfall data from January – December 2018 were collected from 4 stations used. Both methods, IDW and GP models, were compared. We found they have similar goodness of fit performance; however, IDW outperforms based on prediction performance under MAE, MSE, and MAPE criterion. This paper did not develop a simulation study because the interpolation method may have good performance only for specific conditions and depend on its objective. In other words, there was not a universal superior method in the interpolation study [17]. Based on our research, if the point locations are dense, we can use IDW as the interpolation tool. However, if the point locations are sparses, the GP model could be the best alternative.

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