$\Sigma_b \to \Sigma^*_c$ weak decays in the light-front quark model with two schemes to deal with the polarization of diquark

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Abstract

Thanks to the remarkable achievements of LHC, a large database on baryons has been accumulated, so it is believed that the time for precisely studying baryons especially heavy baryons, has come. By analyzing the data, the quark-diquark structure which has been under intensive discussions, can be tested. In this work the decay widths of weak transitions $\Sigma_b \to \Sigma^*_c + X$ are calculated in terms of the light front quark model (LFQM). To carry out the calculations, the quark-diquark picture is employed where an axial-vector diquark composed of two light quarks serves as a spectator in the concerned processes. The first step of this work is to construct the vertex functions for $\Sigma^*_c$ and $\Sigma_b$, then the relevant form factors are derived. It is shown that under the heavy quark limit the Isgur-Wise functions for the transition are re-deduced. Indeed, how to properly depict the polarization ($\epsilon_\mu$) of the diquark is slightly tricky. In this work, we apply two schemes to explicitly determine the momentum-dependence of the diquark. The corresponding numerical results are presented which will be testified by the future experiments.

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I. INTRODUCTION

It is well noted, the data of mesons much exceed those of baryons, so the studies on the mesonic properties such as the mass spectra, production and decay mechanisms are abundant as well as many so called anomalies in the wide area. The reason is obvious that meson productions are more easily realized for electron-positron colliders. By contrast, the statistics on baryonic properties is relatively poor. Baryons possess three valence constituents whereas there are only two (quark-antiquark) in regular mesons. From the theoretical aspect, dealing with a three-body system is much more complicated than a two-body one. This is an old problem even exists in classical mechanics. Indeed, when a sufficiently large database lacked, an accurate study on baryons seemed not so compelling yet.

However at present, the situation is changed by emergence of great number of baryons, especially those containing one or two heavy quarks (anti-quarks). Researchers have opportunities to carry out more accurate studies on the baryonic processes. The standard approach for handling three-body systems is the Faddeev equations, which mathematically is rather difficult to be solved unless some simplification is taken into account. Among all the simplification schemes, the quark-diquark picture is more realistic especially for the heavy baryons. In the weak decay of a heavy baryon into another heavy baryon, only the heavy quarks take part in the transition while the light quarks can be regarded as spectators. At least at the leading order this is a good approximation. Thus the spins, flavors and isospins of the two light quarks remain as conserved quantum numbers during the transition, so that their combination can be considered as a loosely bound subsystem, i.e. a diquark whose inner structure can be manifested by a form factor. Generally, the form factor possesses a few free parameters which would be determined by fitting data or via some model-dependent calculations. The picture of baryon is most commonly adopted for calculating the mass spectra and transition rates. The authors of Refs.[1–6] studied the transitions between two heavy baryons in terms of the quark-diquark picture and the theoretical results are qualitatively accordant with the data. The picture is still in acute dispute although it is very effective and useful for studying the processes where baryons are involved. To confirm the validity of the picture, one may study typical transitions between heavy baryons. That is the goal of our paper.

In this paper we will study the transition $\Sigma_b \rightarrow \Sigma_c^{*}$ where the light diquark in the decay is regarded as a spectator. The hadronic matrix element for $\Sigma_b \rightarrow \Sigma_c^{*}$ is generally parameterized by a few form factors. Under the heavy quark limit[7] those form factors can be reduced into two Isgur-Wise functions[1, 2]. Some authors [1–5] calculated the form factors for $\Sigma_b \rightarrow \Sigma_c^{*}$ in various approaches where heavy quark limit was employed. Here we may consider a more general case beyond the heavy quark effective theory (HQET) and obtain the form factors. Parallel to numerous phenomenological models, the light-front quark model (LFQM) has been successfully applied to study transitions between different mesons [8–16]. For example in Ref.[11] the authors predicted some decay rates of $B$ and the results are consistent with data (Table XIV in Ref.[11]). The decay constants of several charmed mesons are calculated in Ref.[13] and are accordant to the data (Table II In Ref.[13]). The approach has also
been applied to deal with transition amplitudes between two baryons in the quark-diquark picture\cite{17,18}.

However the formulas in \cite{18} cannot be directly applied to $\Sigma_b \rightarrow \Sigma^*_c$ because the quantum number of $\Sigma^*_c$ is $\frac{3}{2}^+ [1,2]$. Since the isospin of both $\Sigma_b$ and $\Sigma^*_c$ is 1, the light $ud$ diquark must be in an isospin-1 and color 3 state. To guarantee the wavefunction of the $ud$ diquark to be totally antisymmetric for spin×color×flavor the spin of the $ud$ system should be 1. Thus we need to re-construct the vertex function for a $\frac{3}{2}^+$ heavy baryon which is regarded as a bound state of a heavy quark and a light axial vector diquark. The key problem is to validate the momentum dependence of the polarization of the axial diaquark. Naively, one should expect that the polarization vector in the vertex function directly depends on the momentum of the diquark. While the vertex function containing the polarization vector is deduced from the Clebsch-Gordan (CG) coefficients, the polarization uniquely depends on the total momentum of the baryon. In order to study the vertex function with an axial diquark we employ two schemes to deal with the polarization of the axial vector. We name them as Scheme I where the polarization vector depends on the momentum of the baryon and Scheme II where the polarization vector depends on the momentum of the diquark. Then using the Feynman rules in LFQM we write down the transition matrix element which is parametrized by eight form factors. Under the heavy quark limit, the transition matrix element of $\Sigma_b \rightarrow \Sigma^*_c$ is reduced into a simple form which is described by merely two generalized Isgur-Wise functions.

In this paper we will explore the semileptonic and non-leptonic decays of $\Sigma_b \rightarrow \Sigma^*_c$. By studying these decays of baryons we can get a better understanding on baryon structures and interactions among the constituents. These decays also provide valuable information of the CKM parameters and serve as an ideal laboratory to study non-perturbative QCD effects in the heavy baryon system. The semileptonic decay is relatively simple and not contaminated by the final state re-scattering effects, therefore one only needs to consider the pure hadronic transition between two hadrons, thus studies on semileptonic decays might be more favorable for testing the employed model and/or constrain the model parameters. Indeed, comparing our theoretical results with data the model parameters which are hidden in the vertex functions can be fixed. Even the widths of the non-leptonic decay $\Sigma_b \rightarrow \Sigma^*_c + M$ can also be evaluated in a similar way while assuming the interaction between the produced meson and heavy baryon to be weak, so that final state interactions can be ignored. Within the framework of the light-front model some parallel approached developed in Ref.\cite{19,22} were employed to study the decay of hadron.

This paper is organized as follows: after the introduction, in section II we construct the vertex functions of heavy baryons, then write down the transition amplitude for $\Sigma_b \rightarrow \Sigma^*_c$ in the light-front quark model and deduce the form factors with (without) using the heavy quark approximation, then we present our numerical results for $\Sigma_b \rightarrow \Sigma^*_c$ along with all necessary input parameters in section III. Section IV is devoted to our conclusion and discussions.
II. \( \Sigma_b \to \Sigma_c^* \) IN THE LIGHT-FRONT QUARK MODEL

Assuming the quark-diquark structure\[1, 2\], the heavy baryons \( \Sigma_b, \Sigma_c^* \) and \( \Sigma_c \) consist of a light 1\(^+\) diquark \([ud]\) and one heavy quark \( b(c)\). To ensure the right quantum numbers of \((\frac{1}{2})^+\) and \((\frac{3}{2})^+\), the orbital angular momentum between the two components is zero, i.e. \( l = 0 \), while the spin of the diquark is 1. First we need to construct the vertex functions of \( \Sigma_Q \) and \( \Sigma_Q^* \) according to their quantum numbers. We employ two schemes to deal with the polarization of the axial vector for its momentum dependence.

A. the vertex functions of \( \Sigma_Q \) and \( \Sigma_Q^* \) in Scheme I

In Refs.\[23, 24\] with the same model the authors calculated decay rates of pentaquark which is supposed to be in the antiquark-diquark-diquark structure. In their work appropriate vertex functions are constructed. Returning to our case where baryons are of a quark-diquark structure, thus, a similarity between baryon and pentaquark structures hints us that we may imitate the way for constructing the vertex functions of pentaquark\[23, 24\] to obtain the corresponding those for \( \Sigma_Q \) and \( \Sigma_Q^* \). The wavefunction of \( \Sigma_Q \) with a total spin \( S = 1/2 \) and momentum \( P \) is

\[
|\Sigma_Q(P, S, S_z)\rangle = \int \{d^3 \bar{p}_1\} \{d^3 \bar{p}_2\} 2(2\pi)^3 \delta^3(\bar{P} - \bar{p}_1 - \bar{p}_2) \times \sum_{\lambda_i, \pi} \Psi^{SS_z}(\bar{p}_1, \bar{p}_2, \lambda_1, \sigma) C_{\beta\gamma}^{\alpha} F^{ijij} |Q_\alpha(p_1, \lambda_1)[q_1, q_2](\sigma)(p_2)\rangle,
\]

where \( Q \) represents \( b \) or \( c \), \( [q_1 q_2] \) here is \([ud]\), \( \lambda \) denotes its helicity, \( \sigma \) stands for the polarization projection, \( p_1, p_2 \) are the on-mass-shell light-front momenta defined by

\[
\bar{p} = (p^+, p_\perp), \quad p_\perp = (p^x, p^y), \quad p^- = \frac{m^2 + p_\perp^2}{p^+},
\]

and

\[
\{d^3 \bar{p}\} = \frac{dp^+ dp_\perp^3}{(2\pi)^3}, \quad \delta^3(\bar{p}) = \delta(p^+) \delta^2(p_\perp),
\]

\[
|Q(p_1, \lambda_1)[q_1 q_2](p_2)\rangle = b^\dagger_{\lambda_1}(p_1) a^\dagger_{\lambda}(p_2)|0\rangle,
\]

\[
[a_\sigma(p'), a_\sigma^+(p)] = 2(2\pi)^3 \delta^3(p') \delta_\sigma \delta_\sigma',
\]

\[
\{d_{\lambda'}(p'), d_{\lambda}^+(p)\} = 2(2\pi)^3 \delta^3(p' - \bar{p}) \delta_{\lambda' \lambda}.
\]

The coefficient \( C_{\beta\gamma}^{\alpha} \) is a normalized color factor and \( F^{ijij} \) is a normalized flavor coefficient,

\[
C_{\beta\gamma}^{\alpha} F^{ijij} C_{\beta'\gamma'}^{\alpha'} F'^{j'i'} \langle Q_\alpha(p_1, \lambda'_1)[q_{1i}, q_{2j}](\sigma')(p'_2)\rangle \langle Q_\alpha(p_1, \lambda_1)[q_{1i} q_{2j}](\sigma)(p_2)\rangle = 2^2(2\pi)^6 \delta^3(\bar{p}_1 - \bar{p}) \delta^3(\bar{p}_2 - \bar{p}) \delta_{\lambda' \lambda} \delta_{\sigma \sigma'}.
\]

Intrinsic variables \((x_i, k_{i\perp})\) with \( i = 1, 2 \) are introduced to describe the internal motion of the constituents through

\[
p_1^+ = x_1 \bar{P}^+, \quad p_2^+ = x_2 \bar{P}^+, \quad x_1 + x_2 = 1, \quad p_{1\perp} = x_1 P_\perp + k_{1\perp}, \quad p_{2\perp} = x_2 P_\perp + k_{2\perp}, \quad k_{1\perp} = -k_{1\perp} = k_{2\perp},
\]

\[
\]
where $x_i$ are the light-front momentum fractions satisfying $0 < x_i < 1$. The invariant mass square $M_0^2$ is defined as

$$M_0^2 = \frac{k_{i\perp}^2 + m_i^2}{x_i} + \frac{k_{i\parallel}^2 + m_i^2}{x_i}.$$  \hfill (6)

The invariant mass $M_0$ is different from the hadron mass $M$ which satisfies the physical mass-shell condition $M^2 = P^2$. This is due to the fact that the baryon, heavy quark and diquark cannot be on their mass shells simultaneously. The internal momenta are defined as

$$k_i = (k_i^-, k_i^+, k_{i\perp}) = (e_i - k_{iz}, e_i + k_{iz}, k_{i\perp}) = \left(\frac{m_i^2 + k_{i\perp}^2}{x_i M_0}, x_i M_0, k_{i\perp}\right).$$ \hfill (7)

It is easy to obtain

$$M_0 = e_1 + e_2,$$

$$e_i = \frac{x_i M_0}{2} + \frac{m_i^2 + k_{i\perp}^2}{2 x_i M_0} = \sqrt{m_i^2 + k_{i\perp}^2 + k_{iz}^2},$$

$$k_{iz} = \frac{x_i M_0}{2} - \frac{m_i^2 + k_{i\perp}^2}{2 x_i M_0},$$ \hfill (8)

where $e_i$ denotes the energy of the $i$-th constituent. The momenta $k_{i\perp}$ and $k_{iz}$ constitute a momentum vector $\vec{k}_i = (k_{i\perp}, k_{iz})$ and correspond to the components in the transverse and $z$ directions, respectively. $m_1(m_2)$ is the mass of the heavy quark (the light diquark).

The momentum-space function $\Psi_{SSz}^{\Sigma_c}(\vec{p}_1, \vec{p}_2, \lambda_1, \sigma)$ in Eq. (1) is expressed as

$$\langle \lambda_1 | \mathbf{R}_M^\dagger(x_1, k_{1\perp}, m_1) | s_1 \rangle \langle \frac{1}{2} s_1; 1 \sigma | \frac{1}{2} S_z \rangle \psi(x, k_{\perp}),$$

where $\langle \frac{1}{2} s_1; 1 \sigma | \frac{1}{2} S_z \rangle$ is the C-G coefficients corresponding to the concerned transition and $s_1(m)$ are the spin projections of the constituents the heavy quark (diquark).

$$\langle \frac{1}{2} s_1; 1 \sigma | \frac{1}{2} S_z \rangle = A_1 \bar{u}(p_1, s_1) \frac{-\gamma_5 \not{\sigma} \not{P}}{\sqrt{3}} u(\not{P}, S_z).$$ \hfill (9)

Calculating the modular squares of the two sides and summing over all the polarizations, one obtains

$$A_1 = \frac{1}{\sqrt{2(M_0 m_1 + p_1 \cdot \not{P})}}.$$ \hfill (10)

$A_1$ also can be obtained by substituting the explicit expressions of $\bar{u}(p_1, s_1) \not{\sigma} \not{P} u(\not{P}, S_z)$ into Eq.(2), where $S_z$ is also spin projection of baryon \cite{23, 24}.

A Melosh transformation brings the the matrix elements from the spin-projection-on-fixed-axis representation into the helicity representation and is explicitly written as

$$\langle \lambda_1 | \mathbf{R}_M^\dagger(x_1, k_{1\perp}, m_1) | s_1 \rangle = \frac{\bar{u}(k_1, \lambda_1) u(k_1, s_1)}{2 m_1}. $$
Following Refs. [23, 24], the Melosh-transformed matrix is expressed as

\[
\langle \lambda_1 | R_M^\dagger(x_1, k_{1\perp}, m_1) | s_1 \rangle \langle \frac{1}{2} s_1; 1\sigma | \frac{3}{2} S_z \rangle = \frac{1}{\sqrt{6(p_1 \cdot \bar{P} + m_1 M_0)}} \bar{u}(p_1, \lambda_1)[-\gamma_5 \ell(\bar{P}, \sigma)] u(\bar{P}, S_z),
\]

which is the same as that given in those papers [23, 24].

For the baryon $\Sigma^*_Q$ with the total spin $S = 3/2$, the wavefunction is a bit more complicated as

\[
\Psi_{\Sigma^*_c}^{SS}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \sigma) = \frac{1}{\sqrt{6(p_1 \cdot \bar{P} + m_1 M_0)}} \bar{u}(p_1, \lambda_1)[-\gamma_5 \ell(\bar{P}, \sigma)] u(\bar{P}, S_z) \varphi(x, k_{\perp}),
\]

with $\varphi(x, k_{\perp}) = 4(\pi m_1)^3 e_x e_y \exp(-\frac{k_{\perp}^2}{2m_0^2})$. In this way, the final expression

\[
\Psi_{\Sigma^*_c}^{SS}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \sigma) = \frac{1}{\sqrt{6(p_1 \cdot \bar{P} + m_1 M_0)}} \bar{u}(p_1, \lambda_1)[-\gamma_5 \ell(\bar{P}, \sigma)] u(\bar{P}, S_z) \varphi(x, k_{\perp}),
\]

which is the same as that given in those papers [23, 24].

All other relevant notations can be found in Ref. [24].
we rewrite the expressions of $\Psi$ | normalizing the state

the authors present a similar vertex function for $\Sigma$

B. the vertex functions of $\Sigma_Q$ and $\Sigma_Q^*$ in Scheme II

For the axial vector diquark inside the baryons $\Sigma_c$ or $\Sigma_c^*$, a natural conjecture is that the polarization vector $\varepsilon$ in the final expressions of $\Psi_{\Sigma_c}$ and $\Psi_{\Sigma_c^*}$ should depend on the momentum of the diquark: $p_2$, instead of the total momentum $\bar{P}$. With this consideration, we rewrite the expressions of $\Psi_{\Sigma_c}$ and $\Psi_{\Sigma_c^*}$ as

$$\Psi_{\Sigma_c}^{SS'}(\bar{p}_1, \bar{p}_2, \lambda_1, \sigma) = \frac{A_2}{\sqrt{6(p_1 \cdot \bar{P} + m_1 M_0)}} \bar{u}(p_1, \lambda_1)[-\gamma_5 \varepsilon(p_2, \sigma)]u(\bar{P}, S_z) \varphi(x, k_\perp), \quad (18)$$

$$\Psi_{\Sigma_c^*}^{SS'}(\bar{p}_1, \bar{p}_2, \lambda_1, \sigma) = \frac{A_2'}{\sqrt{2(p_1 \cdot \bar{P} + m_1 M_0)}} \bar{u}(p_1, s_1)\varepsilon^{\alpha\beta}(p_2, \sigma)u(\bar{P}, S_z) \varphi(x, k_\perp), \quad (19)$$

with $A_2 = \sqrt{\frac{12(M_0 m_1 p_1 \bar{P})}{12M_0 m_1 + 4p_1 p_0 + 8p_1 p_2 p_2 + P^2 m_2^2}}$ and $A_2' = \sqrt{\frac{3M_0^2 m_2^2}{2M_0^2 m_2^2 + (\bar{P} \cdot p_2)^2}}$ which can be obtained by normalizing the state $|\Sigma_Q(P, S, S_z)\rangle$ as,

$$\langle \Sigma_Q(P', S', S'_z)|\Sigma_Q(P, S, S_z)\rangle = 2(2\pi)^3 P^+ \delta^3(\bar{P}' - \bar{P}) \delta_{S'S} \delta_{S'_z S_z}. \quad (20)$$

In fact, the coefficients $A_2$ and $A_2'$ would change to 1 when $p_2$ and $m_2$ are replaced by $\bar{P}$ and $M_0$ respectively, thus the expressions in Eq. [13] and [17] are recovered. In Ref. [25] the authors present a similar vertex function for $\Sigma_Q$ which adds a term $\varepsilon^\mu(p_2, \sigma) \cdot \bar{P}$ into Eq. (18) of this work. Since a normalization condition is required for every vertex function the difference would not seriously affect the results.

C. Calculating the form factors of $\Sigma_b \to \Sigma_c^*$ in LFQM

The leading order Feynman diagram responsible for the $\Sigma_b \to \Sigma_c^*$ weak decay is shown in Fig. [1]. Following the approach given in Ref. [23, 24] the transition matrix element can be computed with the wavefunctions of $|\Sigma_b(P, S, S_z)\rangle$ and $|\Sigma_c^*(P, S, S_z)\rangle$ in the Scheme I,

$$\langle \Sigma_c^*(P', S'_z) | \bar{c}\gamma^\mu(1 - \gamma_5) b | \Sigma_b(P, S, S_z)\rangle = \int \{d^3\bar{p}_2\} \phi_{\Sigma_c^*}(x', k'_\perp) \phi_{\Sigma_b}(x, k_\perp)$$

$$\times \bar{u}_a(\bar{P}', S'_z) [\varepsilon^\alpha(P', \sigma') (\gamma_1 + m_1)^\gamma^\mu(1 - \gamma_5)(\gamma_1 + m_1)[-\gamma_5 \varepsilon(P, \sigma)]u(P, S_z), \quad (21)$$
where
\[ m_1 = m_b, \quad m'_1 = m_c, \quad m_2 = m_{[ud]}, \] (22)
and \( Q' \) represents the heavy quark \( b \) (c), \( p_1 \) (\( p'_1 \)) denotes the four-momentum of the heavy quark \( b \) (c), \( P \) (\( P' \)) stands as the four-momentum of \( \Sigma_b \) (\( \Sigma'_c \)). Setting \( \bar{p}_2 = \bar{p}'_2 \), we have
\[ x' = \frac{P^+}{P'^+} x, \quad k'_1 = k_1 + x_2 q_1. \] (23)

Instead, with the Scheme II the transition matrix element is obtained by replacing \( \varepsilon^*(\bar{P}', \sigma) \) and \( \varepsilon(\bar{P}, \sigma) \) by \( A_2 \varepsilon^*(p_2, \sigma) \) and \( A_2 \varepsilon(p_2, \sigma) \) in Eq. (21). It is noted that one can use the formula \( \varepsilon^*(p_2, \sigma)^\mu \varepsilon(p_2, \sigma)_{\nu} = -g_{\mu\nu} + \frac{p_2^\mu p_2^\nu}{m_2^2} \) to deal with \( \varepsilon^*(p_2, \sigma) \) but he needs the components of the polarization vectors of \( \varepsilon^*(\bar{P}', \sigma) \) and \( \varepsilon(\bar{P}, \sigma) \) while summing over the polarizations. The transition matrix element is calculated in the \( q^+ = 0 \) reference frame.

The form factors for the weak transition \( \Sigma_b \to \Sigma'_c \) are defined in the standard way as
\[ \langle \Sigma'_c(P', S'_z, S'_{z'}) \mid \bar{c}\gamma^\mu(1 - \gamma_5)b \mid \Sigma_b(P, S, S_z) \rangle \]
\[ = \bar{u}_\alpha(P', S'_z) \left[ \gamma^\mu \frac{P_\alpha f_1(q_2^2)}{M_{\Sigma_b}M_{\Sigma'_c}} + \frac{f_2(q_2^2)}{M_{\Sigma_b}M_{\Sigma'_c}} P^\alpha P_\mu + \frac{f_3(q_2^2)}{M_{\Sigma_b}M_{\Sigma'_c}} P^\alpha P^{\mu\nu} + f_4(q_2^2) g^{\mu\nu} \right] u(P, S_z) - \]
\[ \bar{u}_\alpha(P', S'_z) \left[ \gamma^\mu \frac{P_\alpha g_1(q_2^2)}{M_{\Sigma_b}M_{\Sigma'_c}} + \frac{g_2(q_2^2)}{M_{\Sigma_b}M_{\Sigma'_c}} P^\alpha P_\mu + \frac{g_3(q_2^2)}{M_{\Sigma_b}M_{\Sigma'_c}} P^\alpha P^{\mu\nu} + g_4(q_2^2) g^{\mu\nu} \right] \gamma_5 u(P, S_z), \] (24)

where \( q \equiv P - P', Q \) and \( Q' \) denote b and c, respectively.

The baryon spinors \( u(P, S_z) \) and \( \bar{u}(P', S_z) \) which appear in the above formula are different from \( u(P, S_z) \) and \( \bar{u}(P', S_z) \). The spinors in Eq. (21) which are functions of momenta \( P \) and \( P' \) do not correspond to on-mass-shell states, whereas \( u(P, S_z) \) and \( \bar{u}(P', S_z) \) stand for the the on-shell baryons. Concretely, \( \bar{P}'^{(c)} + M^2_{\Sigma_b(M_{\Sigma'_c})} \neq E^2_{\Sigma_b(M_{\Sigma'_c})} \), as \( \bar{P} \) and \( \bar{P}' \) are the sums of the momenta of the involved constituents (quark and diquark) which are on their mass shells. Thus in principle one cannot obtain the form factors \( f_1, f_2, f_3, g_1, g_2, g_3 \) and \( g_4 \) from Eq. (21) at all. In Eq. (21), the hadronic matrix elements are calculated in the light-front-quark model in the un-physical regions. Since \( P \) and \( P' \) obey the relations \( E'^{(c)} = P'^{(c)} + M'^{(c)} \), the form factors in Eq. (24) are not directly extracted from Eq. (21). To remedy the conflict, if it is assumed that the form factors are the same in both physical and unphysical regions, we can extrapolate Eq. (24) to the following equation (Eq. (25)) where the spinors on the right side are off-shell, with the same form factors.

The Eq. (24) is re-written as
\[ \langle \Sigma'_c(P', S'_z, S'_{z'}) \mid \bar{c}\gamma^\mu(1 - \gamma_5)b \mid \Sigma_b(P, S, S_z) \rangle \]
\[ = \bar{u}_\alpha(\bar{P}', S'_z) \left[ \gamma^\mu \bar{P}_\alpha f_1(q_2^2) \frac{M_{\Sigma_b}}{M_{\Sigma'_c}} + \frac{f_2(q_2^2)}{M_{\Sigma_b}M_{\Sigma'_c}} \bar{P}^\alpha \bar{P}_\mu + \frac{f_3(q_2^2)}{M_{\Sigma_b}M_{\Sigma'_c}} \bar{P}^\alpha \bar{P}^{\mu\nu} + f_4(q_2^2) \bar{g}^{\mu\nu} \right] u(\bar{P}, S_z) - \]
\[ \bar{u}_\alpha(\bar{P}', S'_z) \left[ \gamma^\mu \bar{P}_\alpha g_1(q_2^2) \frac{M_{\Sigma_b}}{M_{\Sigma'_c}} + \frac{g_2(q_2^2)}{M_{\Sigma_b}M_{\Sigma'_c}} \bar{P}^\alpha \bar{P}_\mu + \frac{g_3(q_2^2)}{M_{\Sigma_b}M_{\Sigma'_c}} \bar{P}^\alpha \bar{P}^{\mu\nu} + g_4(q_2^2) \bar{g}^{\mu\nu} \right] \gamma_5 u(\bar{P}, S_z), \] (25)
here $f_1, f_2, f_3, f_4, g_1, g_2, g_3$ and $g_4$ are supposed to be invariant for any definite $q^2$ no matter $q = P' - P$ or $q = P' - \bar{P}$.

Multiplying the following expressions \( \bar{u}(\bar{P}, S_z)\gamma^\mu\bar{P}^\beta u_\beta(\bar{P}', S'_z), \) \( \bar{u}(\bar{P}, S_z)\bar{P}^\mu\bar{P}^\beta u_\beta(\bar{P}', S'_z), \) \( \bar{u}(\bar{P}, S_z)\gamma^\mu\bar{P}^\beta\gamma_5 u_\beta(\bar{P}', S'_z), \) and \( \bar{u}(\bar{P}, S_z)g^\mu\gamma_5 u_\beta(\bar{P}', S'_z), \) to the right sides of Eq.(21) and Eq.(25) ($\mu = +$), several algebraic equations are obtained. Then by solving them we achieve $f_1, f_2, f_3, f_4, g_1, g_2, g_3$ and $g_4$ (See Appendix for detail).

D. The generalized Isgur-Wise functions for the transition

Under the heavy quark limit ($m_Q \rightarrow \infty$)[26], the eight form factors $f_i, g_i$ (i=1,2,3,4) are no longer independent and the matrix elements are totally determined by two Isgur-Wise functions $\xi_1(v \cdot v')$ and $\xi_2(v \cdot v')$ and they are defined through the following expression[1, 2]

\[
\begin{align*}
< \Sigma_e^*(v', S'_z) | \bar{Q}'_\gamma \gamma_\mu (1 - \gamma_5) Q_v | \Sigma_b(v, S_z) > &= \frac{1}{\sqrt{3}} [g^{\alpha \beta} \xi_1(\omega) - v^\alpha v'^\beta \xi_2(\omega)] \bar{u}_\alpha(v', S'_z) \gamma^\mu (1 - \gamma_5)(\gamma_\beta + v_\beta) \gamma_5 u(v, S_z), \\
\end{align*}
\]

(26)

where $\omega \equiv v \cdot v'$, and

\[
\begin{align*}
f_1 &= -\frac{\xi_1 - \xi_2 - \xi_2 w}{\sqrt{3}}, & f_2 &= 0, & f_3 &= -\frac{2\xi_2}{\sqrt{3}}, & f_4 &= \frac{2\xi_1}{\sqrt{3}}, \\
g_1 &= -\frac{\xi_1 + \xi_2 - \xi_2 w}{\sqrt{3}}, & g_2 &= 0, & g_3 &= \frac{2\xi_2}{\sqrt{3}}, & g_4 &= -\frac{2\xi_1}{\sqrt{3}}. \\
\end{align*}
\]

(27)

Thus using these relations one can immediately obtain the form factors with the heavy quark limit from the Isgur-Wise functions.

With the re-normalized wavefunctions[23, 24]

\[
\begin{align*}
| \Sigma_Q(P, S_z) > & \rightarrow \sqrt{M_{\Sigma_Q}} | \Sigma_Q(v, S_z) >, \\
u(\bar{P}, S_z) & \rightarrow \sqrt{m_Q u}(v, S_z) \\
\phi_{\Sigma_Q}(x, k_\perp) & \rightarrow \sqrt{\frac{m_Q}{X}} \Phi(X, k_\perp),
\end{align*}
\]

(28)

and

\[
\begin{align*}
M_{\Sigma_Q} & \rightarrow m_Q, & M_0 & \rightarrow m_Q, \\
e_1 & \rightarrow m_Q, \\
e_2 & \rightarrow v \cdot p_2 = \frac{m^2 + k_\perp^2 + X^2}{2X}, \\
\vec{k}^2 & \rightarrow (v \cdot p_2)^2 - m_2^2, \\
\phi_{1 + m_1} & \rightarrow m_Q (\gamma + 1) \\
\frac{e_1 e_2}{x_1 x_2 M_0} & \rightarrow \frac{m_Q}{X} (v \cdot p_2),
\end{align*}
\]

(29)

9
the transition matrix elements obtained in the previous section are re-formulated under the heavy quark limit.

In the Scheme I

\[
\langle \Sigma^c_\alpha(v', S'_z) | \bar{c} v \gamma^\mu(1 - \gamma_5)b_v | \Sigma_0(v, S_z) \rangle = - \int \frac{dX}{X} \frac{d^2k_\perp}{2(2\pi)^3} \Phi(X, k_\perp) \Phi(X', k'_\perp) \frac{1}{\sqrt{3}} \bar{u}_\alpha(v', S'_z) \gamma^\mu(1 - \gamma_5)(\gamma_\beta + v_\beta) \gamma_5 \\
u(v, S_z) \varepsilon^\alpha(P', \sigma) \varepsilon^\beta(P, \sigma),
\]

with

\[
\Phi(X, k_\perp) = 4 \sqrt{v \cdot p_2} \left( \frac{\pi}{\beta_{\infty}^2} \right)^{\frac{3}{4}} \exp \left( - \frac{(v \cdot p_2)^2 - m_2^2}{2 \beta_{\infty}^2} \right),
\]

\[
\Phi(X', k'_\perp) = 4 \sqrt{v' \cdot p_2} \left( \frac{\pi}{\beta_{\infty}^2} \right)^{\frac{3}{4}} \exp \left( - \frac{(v' \cdot p_2)^2 - m_2^2}{2 \beta_{\infty}^2} \right),
\]

where \( \beta_{\infty} \) denotes the value of \( \beta \) in the heavy quark limit.

The transition matrix element is

\[
\langle \Sigma^c_\alpha(v', S'_z) | \bar{c} v \gamma^\mu(1 - \gamma_5)b_v | \Sigma_0(v, S_z) \rangle = - \int \frac{dX}{X} \frac{d^2k_\perp}{2(2\pi)^3} \Phi(X, k_\perp) \Phi(X', k'_\perp) \frac{1}{\sqrt{3}} \bar{u}_\alpha(v', S'_z) \gamma^\mu(1 - \gamma_5)(\gamma_\beta + v_\beta) \gamma_5 \\
u(v, S_z)(a_1 g^{\alpha\beta} + a_2 v^\alpha v^\beta + a_3 v^\alpha v^\beta + a_4 v^\alpha v^\beta + a_5 v^\alpha v^\beta).
\]

Using the relation \( \bar{u}' \gamma_5(\lambda' + 1) = (\lambda + 1) \gamma_5 u = 0 \), the terms involving \( a_3, a_4 \) and \( a_5 \) do not contribute to the transition, thus

\[
\langle \Sigma^c_\alpha(v', S'_z) | \bar{c} v \gamma^\mu(1 - \gamma_5)b_v | \Sigma_0(v, S_z) \rangle = - \int \frac{dX}{X} \frac{d^2k_\perp}{2(2\pi)^3} \Phi(X, k_\perp) \Phi(X', k'_\perp) \frac{1}{\sqrt{3}} \bar{u}_\alpha(v', S'_z) \gamma^\mu(1 - \gamma_5)(\gamma_\beta + v_\beta) \gamma_5 \\
u(v, S_z)(a_1 g^{\alpha\beta} + a_2 v^\alpha v^\beta),
\]

and

\[
a_1 = -1 \\
a_2 = \frac{1}{\omega + 1}
\]

In the Scheme II

\[
\langle \Sigma^c_\alpha(v', S'_z) | \bar{c} v \gamma^\mu(1 - \gamma_5)b_v | \Sigma_0(v, S_z) \rangle = - \int \frac{dX}{X} \frac{d^2k_\perp}{2(2\pi)^3} \Phi(X, k_\perp) \Phi(X', k'_\perp) \frac{1}{\sqrt{3}} \bar{u}_\alpha(v', S'_z) \gamma^\mu(1 - \gamma_5)(\gamma_\beta + v_\beta) \gamma_5 \\
u(v, S_z) \left( \frac{p_{\perp}^2 p_{\parallel}^2}{m_2^2} - g^{\alpha\beta} \right),
\]
with
\[ \Phi(X, k_\perp) = \sqrt{\frac{24}{16 + 8v \cdot p_2/m_2^2}} 4\sqrt{v \cdot p_2} \left( \frac{\pi}{\beta^2_{\infty}} \right)^\frac{1}{4} \exp \left( -\frac{(v \cdot p_2)^2 - m_2^2}{2\beta^2_{\infty}} \right), \]
\[ \Phi(X', k'_\perp) = \sqrt{\frac{3}{2 + v' \cdot p_2/m_2^2}} 4\sqrt{v' \cdot p_2} \left( \frac{\pi}{\beta^2_{\infty}} \right)^\frac{1}{4} \exp \left( -\frac{(v' \cdot p_2)^2 - m_2^2}{2\beta^2_{\infty}} \right), \] (36)

and
\[ a_1 = -\frac{(w^2 - 1)p_2^2 + 2v \cdot p_2v' \cdot p_2\omega - (v' \cdot p_2)^2 - (v \cdot p_2)^2}{2m_2^2(\omega^2 - 1)}, \]
\[ a_2 = -\frac{\omega(\omega^2 - 1)p_2^2 - 2v \cdot p_2v' \cdot p_2(2\omega^2 + 1) + 3\omega[(v' \cdot p_2)^2 + (v \cdot p_2)^2]}{2m_2^2(\omega^2 - 1)^2}. \] (37)

Comparing Eq. (32) with Eq. (26), we get
\[ \xi_1 = -\int \frac{dX}{X} \frac{d^2k_\perp}{2(2\pi)^3} \Phi(X, k_\perp)\Phi(X', k'_\perp)a_1, \] (38)
\[ \xi_2 = \int \frac{dX}{X} \frac{d^2k_\perp}{2(2\pi)^3} \Phi(X, k_\perp)\Phi(X', k'_\perp)a_2. \] (39)

Here \( \xi_1 \) and \( \xi_2 \) are similar to the forms appearing in Eq. (4.18) and Eq. (4.19) of Ref. [24]. We are able to directly evaluate them in the time-like region by choosing a reference frame with \( q_\perp = 0 \). It is noted that the components of \( \varepsilon(\hat{P}', \sigma) \) and \( \hat{P}' \) are different from those in the reference frame with \( q^+ = 0 \).

III. NUMERICAL RESULTS

In order to evaluate the transition rate of \( \Sigma_b \rightarrow \Sigma_c^* \) one needs to pre-set all input parameters. The baryon masses \( M_{\Sigma_b} = 5.811 \) GeV, \( M_{\Sigma_c^*} = 2.517 \) GeV are taken from [27]. The heavy quark masses \( m_b \) and \( m_c \) are set following Ref. [11]. In our calculation, the mass of the light axial vector diquark \( m_{[ud]_V} \) is set to be 770 MeV [1]. It can be conjectured that the diquark mass \( m_{[ud]_V} \) is close to the mass of an \( s \) quark (here we consider the constituent quark masses instead of current quark masses), thus we set \( \beta_{[ud]} = \beta_{bs} \) and \( \beta_{[ud]} = \beta_{cs} \) while the values of the corresponding model parameters are adopted from the meson cases [1]. The relevant input parameters are collected in Table II.

First of all, we need to numerically calculate the form factors, then using them we are able to predict the rates of semi-leptonic processes \( \Sigma_b \rightarrow \Sigma_c^* l\bar{\nu}_l \) and non-leptonic decays \( \Sigma_b \rightarrow \Sigma_c^* M^- \) (\( M \) represents \( \pi, K, \rho, K^*, a_1 \) etc.).
TABLE I: Quark mass and the parameter $\beta$ (in units of GeV).

| $m_c$ | $m_b$ | $m_{ud}$ | $\beta_{c[ud]}$ | $\beta_{b[ud]}$ |
|-------|-------|----------|-----------------|-----------------|
| 1.3   | 4.4   | 0.77     | 0.45            | 0.50            |

TABLE II: The $\Sigma_b \to \Sigma_c^*$ form factors given in the three-parameter form in Scheme I.

| $F$ | $F(0)$ | $a$   | $b$   |
|-----|--------|-------|-------|
| $f_1$ | 0      | 0.225 | -0.388 |
| $f_2$ | 0.100  | 2.23  | 8.57  |
| $f_3$ | -0.292 | 1.66  | 2.42  |
| $f_4$ | 0.561  | 1.80  | 1.50  |
| $g_1$ | -0.356 | 2.47  | 4.80  |
| $g_2$ | 0      | 0.044 | -0.167 |
| $g_3$ | 0.365  | 2.38  | 0.69  |
| $g_4$ | -0.784 | 2.07  | 1.49  |

A. $\Sigma_b \to \Sigma_c^*$ form factors and the Isgur-Wise functions in Scheme I

Since these form factors $f_i$ ($i = 1, 2, 3, 4$) and $g_i$ ($i = 1, 2, 3, 4$) are evaluated in the frame $q^+ = 0$ i.e. $q^2 = -q_1^2 \leq 0$ (the space-like region) one needs to extend them into the time-like region. As commonly adopted in literature, we may employ a three-parameter form factor\[24\]

$$F(q^2) = \frac{F(0)}{\left[1 - a \left(\frac{q^2}{M_{\Sigma_b}^2}\right) + b \left(\frac{q^2}{M_{\Sigma_b}^2}\right)^2\right]}$$

where $F(q^2)$ denotes the form factors $f_i$ ($i = 1, 2, 3, 4$) and $g_i$ ($i = 1, 2, 3, 4$). However, this extrapolation shown in Eq. (40) suffers a fatal disadvantage that if $F(0) = 0$, the form factor would remain zero for $q^2 \neq 0$, which obviously is invalid. Therefore we employ an alternative three-parameter extrapolation as

$$F(q^2) = F(0) - a \frac{q^2}{M_{\Sigma_b}^2} + b \left(\frac{q^2}{M_{\Sigma_b}^2}\right)^2.$$  \hspace{1cm} (41)

Using the form factors in the space-like region we may calculate numerically the parameters $a$, $b$ and $F(0)$, namely fixing $F_i(q^2 \leq 0)$. As discussed in previous section, these form factors are extended into the physical region with $q^2 \geq 0$ through Eq. (40) or (41). The fitted values of $a$, $b$ and $F(0)$ in the form factors $f_i$ ($i = 1, 2, 3, 4$) and $g_i$ ($i = 1, 2, 3, 4$) are presented in Table II. The dependence of the form factors on $q^2$ is depicted in Fig. 2.

Now let us turn to calculate the form factors in the HQET. In the heavy quark limit,
FIG. 2: (a) the form factors $f_i$ ($i = 1, 2, 3, 4$) and (b) the form factors $g_i$ ($i = 1, 2, 3, 4$) in Scheme I

$\beta^\infty = 0.50$ GeV is used for $\Sigma_b$ and $\Sigma_c^*$. The Isgur-Wise function is parameterized as

$$\xi(\omega) = 1 - \rho^2(\omega - 1) + \frac{\sigma^2}{2}(\omega - 1)^2,$$

where $\rho^2 \equiv -\left.\frac{d\xi(\omega)}{d\omega}\right|_{\omega = 1}$ is the slope parameter and $\sigma^2 \equiv \left.\frac{d^2\xi(\omega)}{d\omega^2}\right|_{\omega = 1}$ is the curvature of the Isgur-Wise function. To fit some available data, we write up the expressions with definite values as

$$\xi_1 = 1 - 1.90(\omega - 1) + 1.58(\omega - 1)^2$$

$$\xi_2 = 0.50[1 - 2.36(\omega - 1) + 2.28(\omega - 1)^2].$$

The dependence of the Isgur-Wise function $\xi_1$ on $\omega$ is depicted in Fig. 3(a). Let us compare $\xi_1$ obtained in this work with that given in Ref. [4]. One can notice that $\xi_1|_{\omega = 1} = 1$ holds, which is the mandatory normalization of the Isgur-Wise function. The dashed line (line 1) and the solid line (line 2) are our results in Scheme I and Scheme II respectively with the diquark mass being 770 MeV. The dotted line (line 3) and the dash-dotted line (line 4) correspond to $\Lambda = 750$ MeV and $\Lambda = 800$ MeV$^1$ respectively.

The dependence of the Isgur-Wise function $\xi_2$ on $\omega$ is depicted in Fig. 3(b). The dotted line (line 1) and the solid line (line 2) are the results obtained in Scheme I and Scheme II

---

$^1$ $\Lambda$ is the difference between the heavy baryon mass and the heavy quark mass. This figure is in somewhat analogue to that we achieved for the transition of a spin-1/2 baryon to another spin-1/2 baryon.
FIG. 3: (a) The Isgur-Wise function $\xi_1(\omega)$ for $\Sigma_b \to \Sigma_c^*$ where the line 1 and 2 are our results of $\xi_1(\omega)$ in Scheme I and II respectively, whereas the line 3 and 4 are given in Ref. [4] (b) The Isgur-Wise function $\xi_2(\omega)$ for $\Sigma_b \to \Sigma_c^*$ where the line 1 and 2 also are the results in Scheme I and II respectively.

respectively. We observe that $\xi_2|_{\omega=1} = 1/2$ in Scheme I is consistent with that obtained in Ref. [28, 29].

Using the aforementioned relations between the form factors $f_i \ (i = 1, 2, 3, 4)$ and $g_i \ (i = 1, 2, 3, 4)$ and the Isgur-Wise functions ($\xi_1$ and $\xi_2$), we obtain the form factors $f_i \ (i = 1, 2, 3, 4)$ and $g_i \ (i = 1, 2, 3, 4)$ in the heavy quark limit. According to the relations in Eq. (27) $f_2$ and $g_2$ are equal to 0. $f_3$ nearly overlaps with $f_2$ and $g_3$ overlaps with $g_4$.

By Fig. 2 and Fig. 4 we see that without taking the heavy quark limit the absolute values of the the form factors $f_i (i = 1, 2, 3, 4)$ and $g_1$ are slightly larger than those got with the heavy quark limit. Especially the values of $f_2(q^2)$ with the heavy quark limit are exactly equal to 0 but the values without the heavy quark limit slightly deviate from zero. One also can find that the values of $g_2(q^2)$ without the heavy quark limit are close to those with the heavy quark limit and the absolute values of the the form factors $g_3$ and $g_4$ without the heavy quark limit are slightly larger than those with the heavy quark limit at the small value of $q^2$. For the larger $q^2$, $g_3(q^2)$ and $g_4(q^2)$ deviate from the values under the heavy quark limit to a certain extent.
FIG. 4: (a) the form factors \( f_i (i = 1, 2, 3, 4) \) and (b) the form factors \( g_i (i = 1, 2, 3, 4) \) in heavy quark limit in Scheme I

TABLE III: The \( \Sigma_b \rightarrow \Sigma_c^* \) form factors given in the three-parameter form in Scheme II.

| \( F \) | \( F(0) \) | \( a \) | \( b \) |
|--------|--------|------|------|
| \( f_1 \) | -0.0744 | 0.967 | 0.755 |
| \( f_2 \) | 0.117 | 2.39 | 3.02 |
| \( f_3 \) | -0.203 | 2.13 | 2.52 |
| \( f_4 \) | 0.546 | 1.37 | 1.02 |
| \( g_1 \) | -0.367 | 1.88 | 1.70 |
| \( g_2 \) | 0.0205 | 1.19 | 1.29 |
| \( g_3 \) | 0.272 | 2.31 | 2.91 |
| \( g_4 \) | -0.763 | 1.54 | 1.18 |

B. \( \Sigma_b \rightarrow \Sigma_c^* \) form factors and the Isgur-Wise functions in Scheme II

Now we repeat the calculation done in last subsection within the framework of Scheme II. The fitted values of \( a, b \) and \( F(0) \) in the form factors \( f_i (i = 1, 2, 3, 4) \) and \( g_i (i = 1, 2, 3, 4) \) are presented in Table III. The dependence of the form factors on \( q^2 \) is depicted in Fig. 5. Comparing Fig. 5 and Fig. 2 one can find that \( f_i (i = 1, 2, 3, 4) \) and \( g_i (i = 1, 2, 3, 4) \) change more smoothly in Fig. 5 the values of \( f_i (i = 1, 2, 3, 4) \) in the two schemes are close to each other and the values of \( g_2 \) in the two schemes are close to 0 but there are some apparent differences between the values of \( g_1, g_3 \) and \( g_4 \) in the two schemes.
FIG. 5: (a) the form factors $f_i$ ($i = 1, 2, 3, 4$) and (b) the form factors $g_i$ ($i = 1, 2, 3, 4$) in Scheme II

In the heavy quark limit, the Isgur-Wise function is fitted as

$$\xi_1 = 1 - 2.08(\omega - 1) + 1.84(\omega - 1)^2$$

$$\xi_2 = 0.409[1 - 2.75(\omega - 1) + 2.98(\omega - 1)^2].$$

The dependence of the Isgur-Wise functions on $\omega$ is shown in Fig.3, $\xi_1$ in the two schemes and those given in Ref [4] are close to each other.

However from Fig.3(b), we observe that in Scheme II $\xi_2|_{\omega=1} = 0.41$ which is slightly lower than 1/2 [28, 29]. In fact under the heavy quark limit the mass of the heavy quark does not exist in the wavefunction, but the mass of the light diquark remains (Eq. (37)). Therefore the theoretical evaluation on the transition rate may weakly depend on its mass. For calculating hadronic matrix elements in terms of a concrete model with one or several parameters which are not determined by any underlying theory yet, we let $m_{ud|\nu}$ and $\beta^\infty$ vary within reasonable ranges and see how much the numerical results depend on it (them). In fact, it is the strategy which are generally adopted for model-dependent phenomenological studies.

The resultant figures show that $\xi_1$ almost does not change with respect to those variations, but the intercept $\xi_2|_\omega$ at $\omega = 1$ is not the same for different values of $m_{ud|\nu}$ and $\beta^\infty$. As a matter of fact the situation is exactly the same as for transitions of a spin-1/2 baryon into another 1/2-baryon under the heavy quark limit. Thus for simplicity let us recall what we gave in our earlier work as: $\xi_1|_{\omega=1} = 1$ and $\xi_2|_{\omega=1} = 0.47$ as $m_{ud|\nu} = 0.5$ GeV and $\beta^\infty = 0.4$. It is easy to understand that even though non-zero $m_{ud|\nu}$ breaks the heavy quark symmetry $SU_f(2) \otimes SU_s(2)$, the violation is still rather small. Therefore, one still can use the simplified expressions with only two Isgur-Wise functions to approach the hadronic matrix elements.
for either a spin-1/2 to spin-1/2 transition or a spin-1/2 to spin-3/2 transition under the limit.

FIG. 6: form factors in heavy quark limit in Scheme II

Using the aforementioned relations between the form factors $f_i(i = 1, 2, 3, 4)$ and $g_i(i = 1, 2, 3, 4)$ and the Isgur-Wise functions ($\xi_1$ and $\xi_2$), we obtain the form factors $f_i(i = 1, 2, 3, 4)$ and $g_i(i = 1, 2, 3, 4)$ in the heavy quark limit. From the Fig. 5 and Fig. 6 we observe that the absolute values of the form factors $f_i(i = 1, 2, 3, 4)$ and $g_i(i = 1, 2, 3, 4)$ without taking the heavy quark limit are slightly larger than those with the heavy quark limit at the same $q^2$. Especially the value of $f_2(q^2)$ with the heavy quark limit is exactly equal to 0 but the value without the heavy quark limit slightly deviates from zero.

C. Semi-leptonic decay of $\Sigma_b \rightarrow \Sigma_c^* + l\bar{\nu}_l$

Using the form factors obtained in last subsection, we evaluate the rate of $\Sigma_b \rightarrow \Sigma_c^* + l\bar{\nu}_l$ in two cases: with and without taking the heavy quark limit. We list our predictions in Table IV. The numerical results depend on the light diquark mass, even though not very sensitively. In Table IV as discussed above, with the same strategy, we let the diquark mass and parameters $\beta_{b[ud]}$, $\beta_{c[ud]}$ fluctuate up to 10%, and the corresponding changes are listed.

It is interesting to study a ratio of the longitudinal differential rate to the transverse one (which are integrated over $\omega$ and the ration $R$ is defined in the appendix), since it may provide more information about the model. $R \sim 1$ would imply the spatial distribution to be approximately uniform. Because $R$ is more sensitive to details of the employed models, a
FIG. 7: Differential decay rates \( d\Gamma/d\omega \) for the decay \( \Sigma_b \to \Sigma^*_c l\bar{\nu}_l \) in Scheme I(a) with heavy quark limit; (b) without heavy quark limit.

TABLE IV: The width (in unit \( 10^{10}\text{s}^{-1} \)) of \( \Sigma_b \to \Sigma^*_c l\bar{\nu}_l \) with \( m_{[u,d]} = 770 \pm 77\text{MeV} \), \( \beta_{c[u,d]} = 0.45 \pm 0.05 \) and \( \beta_{b[u,d]} = 0.50 \pm 0.05 \).

|                  | \( \Gamma \)     | \( \Gamma_L \)     | \( \Gamma_T \)     | \( R \)     |
|------------------|------------------|------------------|------------------|-----------|
| this work in Scheme I\(^a\) | 3.27±0.34        | 1.62±0.20        | 1.65±0.15        | 0.984±0.033 |
| this work in Scheme I\(^b\) | 4.03±0.40        | 1.76±0.16        | 2.27±0.24        | 0.775±0.013 |
| this work in Scheme II\(^a\) | 3.17±0.30        | 1.58±0.16        | 1.59±0.13        | 0.994±0.024 |
| this work in Scheme II\(^b\) | 3.34±0.26        | 1.51±0.13        | 1.83±0.13        | 0.825±0.024 |
| relativistic quark model\(^a\)[2] | 3.23             | 1.61             | 1.62             | 0.99      |
| relativistic three-quark model\(^a\)[4] | 4.56             | 2.49             | 2.07             | 1.20      |
| the Bethe-Salpeter approach\(^a\)[5] | 3.75             | -               | -               | -         |

\( ^a \) with the heavy quark limit
\( ^b \) without the heavy quark limit

Comparison of the theoretical prediction with the data which will be available at LHCb, can help to further constrain the model parameters.

We list our numerical results of \( R \) in Table IV, meanwhile the predictions achieved with other approaches [2] are also presented. One notices from Table IV that our results in the two schemes are close to that predicted by Ref. [2]. In Scheme I the total width without heavy quark limit is larger than that with heavy quark limit apparently. In Scheme II the total width without heavy quark limit is close to that with heavy quark limit.
FIG. 8: Differential decay rates $d\Gamma/d\omega$ for the decay $\Sigma_b \rightarrow \Sigma^*_c l \bar{\nu}_l$ in Scheme II(a) with heavy quark limit; (b) without heavy quark limit

The dependence of the differential decay rate of $\Sigma_b \rightarrow \Sigma^*_c l \bar{\nu}_l$ on $\omega$ is depicted in Fig. 7 and Fig. 8 for Scheme I and II respectively. Fig. 7 (Fig. 8) (a) and (b) are corresponding to the results with or without taking the heavy quark limits, and the two figures in Fig. 8 (Scheme II) are very similar to Fig. 7 (Scheme I) except the position of the crossing points of the dashed and the dotted lines. In addition the differential decay rate of $\Sigma_b \rightarrow \Sigma^*_c l \bar{\nu}_l$ on $\omega$ in the two schemes with heavy quark limit are close to each other but there exists a difference between the results in the two schemes when heavy quark limit is not taken into account.

D. Non-leptonic decays of $\Sigma_b \rightarrow \Sigma^*_c + X$

From the theoretical aspects, calculating the concerned quantities of the non-leptonic decays seems to be more complicated than the semi-leptonic ones, but can still shed lights on the properties of the chosen model. Our theoretical framework is based on the factorization assumption, namely the hadronic transition matrix element is factorized into a product of two independent matrix elements of currents,

$$
\langle \Sigma^*_c(P', S'_z)X | \mathcal{H} | \Sigma_b(P, S_z) \rangle
= \frac{G_F V_{bc} V^*_{qq'}}{\sqrt{2}} \langle X | \bar{q}^\gamma \gamma^\mu (1 - \gamma_5) q | 0 \rangle \langle \Sigma^*_c(P', S'_z) | \bar{c} \gamma^\mu (1 - \gamma_5) b | \Sigma_b(P, S_z) \rangle,
$$

(47)
there possibly exists a simple relation between the decay rates of \( \Sigma_b \) and the transition matrix element \( T_{\Sigma,b} \) as mentioned in the previous sections. Since the decay \( \Sigma_b \) with the same quark flavors, we can check the validity degree of the obtained form factors for the heavy bottomed baryon system.

Our numerical results are shown in Table V, where the uncertainties originate from the variation of \( m_{ud_{iv}} \) and \( \beta \) which are allowed to fluctuate by 10\%. The effective Wilson coefficient \( a_1 \) is set as 1 and the meson decay constants take the same values given in Ref. [17].

Some comments are made:

1. The ratio \( \frac{BR(\Sigma_b \to \Sigma^*_c \pi^0)}{BR(\Sigma_b \to \Sigma^*_c \rho^-)} \) without the heavy quark limit is 34.44 ± 6.56 (Scheme I) or 23.69 ± 5.37 (Scheme II) which will be experimentally tested and the consistency would tell us which scheme is the more realistic one.

2. The numerical results given in Table IV indicate that the decay rate \( \Gamma(\Sigma_b \to \Sigma^*_c V) \) is 2 to 3 times larger than \( \Gamma(\Sigma_b \to \Sigma^*_c P) \) where \( P \) and \( V \) are pseudoscalar and vector mesons with the same quark flavors.

**E. Comparison of the results on \( \Sigma_b \to \Sigma_c + \text{other(s)} \) with those on \( \Sigma_b \to \Sigma^*_c + \text{other(s)} \)**

Since the only difference between \( \Sigma_c \) and \( \Sigma^*_c \) is their total spins, it is natural to expect that there possibly exists a simple relation between the decay rates of \( \Sigma_b \to \Sigma_c + X \) and \( \Sigma_b \to \Sigma^*_c + X \). Now let us study the relation in terms of our numerical results.

From Table V one can notice:

1. The ratio \( \frac{BR(\Sigma_b \to \Sigma^*_c \pi^0)}{BR(\Sigma_b \to \Sigma^*_c \rho^-)} \) is about 2.

### Table V: widths (in unit 10^{10}s^{-1}) of non-leptonic decays \( \Sigma_b \to \Sigma^*_c X \) with the light diquark mass \( m_{ud_{iv}} = 770 \pm 77 \) MeV and \( \beta = 0.50 \pm 0.05 \).

| \( \Sigma_b \to \Sigma^*_c \pi^- \) | Scheme I\(^a\) | Scheme II\(^a\) | Scheme I\(^b\) | Scheme II\(^b\) |
|-----------------|--------|--------|--------|--------|
| 0.117 ± 0.019   | 0.141 ± 0.030 | 0.138 ± 0.030 | 0.132 ± 0.025 |
| 0.391 ± 0.060   | 0.447 ± 0.086 | 0.430 ± 0.091 | 0.411 ± 0.082 |
| 0.0094 ± 0.0015 | 0.0113 ± 0.0023 | 0.0109 ± 0.0024 | 0.0104 ± 0.0019 |
| 0.0211 ± 0.0032 | 0.0237 ± 0.0045 | 0.0227 ± 0.0047 | 0.0217 ± 0.0039 |
| 0.457 ± 0.064 | 0.486 ± 0.083 | 0.459 ± 0.141 | 0.437 ± 0.076 |
| 0.0163 ± 0.0016 | 0.0169 ± 0.0025 | 0.0144 ± 0.0023 | 0.0139 ± 0.0020 |
| 0.0535 ± 0.0057 | 0.0495 ± 0.0054 | 0.0452 ± 0.0067 | 0.0431 ± 0.0058 |
| 0.423 ± 0.038 | 0.433 ± 0.060 | 0.364 ± 0.260 | 0.351 ± 0.0048 |
| 1.25 ± 0.13 | 1.13 ± 0.12 | 1.03 ± 0.14 | 0.99 ± 0.13 |

\(^a\) without the heavy quark limit

\(^b\) with the heavy quark limit

where the part \( \langle X | \bar{q}\gamma^\mu(1 - \gamma_5)q | 0 \rangle \) is determined by the decay constant of meson \( X \) and the transition matrix element \( \langle \Sigma_c^*(P', S'_z) | \bar{c}\gamma^\mu(1 - \gamma_5)b | \Sigma_b(P, S_z) \rangle \) was studied in the previous sections. Since the decay \( \Sigma_b \to \Sigma^*_c \pi^- \) is the so-called color-favored transition, the factorization should be a good approximation. The study on these non-leptonic decays can check the validity degree of the obtained form factors for the heavy bottomed baryon system.
TABLE VI: widths (in unit $10^{10}$s$^{-1}$) of some decays $\Sigma_b \rightarrow \Sigma_c$($\Sigma^*_c$)+other(s) without the heavy quark limit in Scheme II.

|                  | $B$ represents $\Sigma_c$ | $B$ represents $\Sigma^*_c$ |
|------------------|---------------------------|-----------------------------|
| $\Sigma_b \rightarrow B\bar{\nu}l$ | 1.60±0.28                 | 3.34±0.26                   |
| $\Sigma^0_b \rightarrow B\pi^-$      | 0.140±0.037               | 0.141±0.030                 |
| $\Sigma^0_b \rightarrow B\rho^-$     | 0.396±0.091               | 0.447±0.086                 |
| $\Sigma^0_b \rightarrow BK^-$        | 0.0115±0.0030             | 0.0113±0.0023               |
| $\Sigma^0_b \rightarrow BK^{*-}$     | 0.0204±0.0041             | 0.0237±0.0045               |
| $\Sigma^0_b \rightarrow Ba^-\gamma$          | 0.369±0.068               | 0.486±0.083                 |
| $\Sigma^0_b \rightarrow BD^*_s$        | 0.727±0.150               | 0.433±0.060                 |
| $\Sigma^0_b \rightarrow BD^{*-}_s$      | 0.558±0.067               | 1.13±0.12                   |
| $\Sigma^0_b \rightarrow BD^-$          | 0.0266±0.0056             | 0.0169±0.0025               |
| $\Sigma^0_b \rightarrow BD^{*-}$       | 0.0261±0.0035             | 0.0495±0.0054               |

(2) The theoretically predicted widths of $\Sigma_b \rightarrow \Sigma_c$+light scalar and $\Sigma_b \rightarrow \Sigma^*_c$+light scalar are nearly equal.

(3) The ratio of $B(\Sigma_b \rightarrow \Sigma^*_c$+light vector) over $B(\Sigma_b \rightarrow \Sigma_c$+light vector) sways from 1 to 2, which is roughly consistent with the value 1.5 required by the $SU_s(2)$ symmetry in HQET.

It is noted that a factor 2 was missing in the formula for the transition $\frac{1}{2} \rightarrow \frac{1}{2} + V$ given by\[29\] $^2$. We used it to calculate the rate of $\Sigma_b \rightarrow \Sigma_c + V$ in Ref.\[18\]. In this work, by noticing that mistake, we rewrite the formula by adding up the missed factor and carry out the corresponding calculations. In table VI, the theoretical predictions of $\Sigma_b \rightarrow \Sigma_c + V$ are made in terms of the corrected formula.

IV. CONCLUSIONS AND DISCUSSIONS

In this paper, we explore the $\Sigma_b \rightarrow \Sigma^*_c$ transitions in the light front quark model. We calculate the widths of the semi-leptonic decays and non-leptonic two-body decays of $\Sigma_b \rightarrow \Sigma^*_c$ where the quark-diquark picture is employed so that the three-body inner structure of the baryons is reduced into a two-body one.

Since there exists an axial diquark in $\Sigma_b$ and $\Sigma^*_c$, one should expect its polarization vector in the vertex function is somehow momentum-dependent. The polarization vector deduced from the CG coefficients uniquely depends on the momentum of the baryon. With this momentum dependence, we name it as the Scheme I for later calculation. Alternatively, based on a physical consideration, we would set the Scheme II where the polarization vector of the diquark depends on its own momentum which may respect the identity $p_2 \cdot \epsilon \equiv 0$, since

$^2$ We have discussed this issue with the authors of Ref.\[29\], and then by having carefully checked this formula they agreed with us.
in our model, the diquark is on its mass shell. Then, we calculate the eight form factors and two generalized Isgur-Wise functions $\xi_1$ and $\xi_2$ under the heavy quark limit in the two schemes. The form factors $f_i$ ($i = 1, 2, 3, 4$) and $g_i$ ($i = 1, 2, 3, 4$) in Scheme II change more smoothly than those in Scheme I. The values of $f_i$ ($i = 1, 2, 3, 4$), $g_1$ and $g_2$ in the two schemes are close to each other but there are some apparent differences for $g_3$ and $g_4$ in the two schemes. $\xi_1$’s in the two schemes are close to each other and consistent with the results in references. However there is a discrepancy between $\xi_2$’s in the two schemes. $\xi_2|_{\omega=1}$ in Scheme I is exactly $1/2$ but $\xi_2|_{\omega=1}$ in Scheme II is slightly lower than $1/2$ predicted by the large $N_c$ theory. The deviation may be due to the non-zero mass of the light constituents in hadrons (baryon). We also evaluate the form factors $f_i$ ($i = 1, 2, 3, 4$) and $g_i$ ($i = 1, 2, 3, 4$) in the heavy quark limit in terms of the Isgur-Wise functions $\xi_1$ and $\xi_2$ in the two schemes and there are not apparently differences between them. Applying the form factors derived in the framework of the LFQM we evaluate the semi-leptonic decay rates of $\Sigma_b \to \Sigma_c^*$ with and without taking the heavy quark limit. The results in both cases do not decline much from each other, moreover, our numerical results are qualitatively consistent with that estimated in terms of different approaches.

It is noted that the Scheme I retains apparent Lorentz invariance for the vertex function whereas, even though the Scheme II seems to be more physical, the cost we pay is that the explicit Lorentz invariance is lost. This is a more profound question which we are going to address in our following work.

Since the RUN-II of the LHC is operating well and a remarkable number of data on $\Sigma_b$ (production and decay) is being accumulated by the LHCb collaboration, we have all confidence that in near future the rates and even the details of various decay modes would be accurately measured, so theorists will have a great opportunity to testify all available models and re-fix relevant model parameters.

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**Appendix A: Semi-leptonic decays of $\Sigma_Q \to \Sigma_Q^*l\bar{\nu}_l$**

The helicity amplitudes are expressed in terms of the form factors for $\Sigma_Q \to \Sigma_Q^*$ [30, 31]

$$H_{1/2,0}^{V,A} = \mp \frac{1}{\sqrt{q^2}} \frac{2}{\sqrt{3}} \sqrt{M_{\Sigma_Q} M_{\Sigma_Q^*}} (w \mp 1)[(M_{\Sigma_Q} w - M_{\Sigma_Q^*}) \mathcal{N}_1^{V,A}(w)$$

$$\mp(M_{\Sigma_Q} \mp M_{\Sigma_Q^*})(w \pm 1) \mathcal{N}_1^{V,A}(w) + M_{\Sigma_Q^*} (w^2 - 1) \mathcal{N}_2^{V,A}(w)$$

$$+ M_{\Sigma_Q^*} (w^2 - 1) \mathcal{N}_3^{V,A}(w)],$$
\[ H_{V,A}^{\lambda_{i}, \lambda_{W}} = \mp H_{V,A}^{\lambda_{i}, \lambda_{W}}. \]

Partial differential decay rates can be represented in the following form

\[
\frac{d\Gamma_T}{dw} = \frac{G_F^2}{(2\pi)^3} |V_{QQ}|^2 \frac{q^2 M_{\bar{Q}'}^2 \sqrt{w^2 - 1}}{12 M_{\Sigma}} \left[ |H_{1/2,1}|^2 + |H_{-1/2,-1}|^2 + |H_{3/2,1}|^2 + |H_{-3/2,-1}|^2 \right],
\]

\[
\frac{d\Gamma_L}{dw} = \frac{G_F^2}{(2\pi)^3} |V_{QQ}|^2 \frac{q^2 M_{\bar{Q}'}^2 \sqrt{w^2 - 1}}{12 M_{\Sigma}} \left[ |H_{1/2,0}|^2 + |H_{-1/2,0}|^2 \right].
\]

The ratio of the longitudinal to transverse decay rates \( R \) is defined by

\[
R = \frac{\Gamma_L}{\Gamma_T} = \frac{\int_{1}^{\Sigma_{\max}} d\Sigma \ q^2 \ p_c \left[ |H_{1/2,0}|^2 + |H_{-1/2,0}|^2 \right]}{\int_{1}^{\Sigma_{\max}} d\Sigma \ q^2 \ p_c \left[ |H_{1/2,1}|^2 + |H_{-1/2,-1}|^2 \right]}.
\]

Appendix B: the form factor of \( \Sigma_Q \rightarrow \Sigma_{Q'} \)

\[
\bar{u}(\bar{P}, S_z) \gamma^\mu \bar{P}^\beta u_{\beta}(\bar{P}', S'_z), \quad \bar{u}(\bar{P}, S_z) \bar{P}^\mu \bar{P}^\beta u_{\beta}(\bar{P}', S'_z), \quad \bar{u}(\bar{P}, S_z) g^\mu_{\beta'} u_{\beta}(\bar{P}', S'_z)
\]

are multiplied to the right side of Eq. (21), then

\[
F_1 = \int \frac{d^3 p_2}{(2\pi)^3} \phi_{\Sigma_2}^* (x, k_{\perp}') \phi_{\Sigma_1} (x, k_{\perp}) \text{Tr} \{ u_{\beta}(\bar{P}', S'_z) \bar{u}_{\alpha}(\bar{P}', S_z) \}
\]

\[
\times \left[ \epsilon^\alpha (\bar{P}', \sigma)|(\gamma^\nu (\gamma^\mu - \gamma_5)(\gamma^\beta - \gamma_5)| u(\bar{P}, S_z) \bar{u}(\bar{P}, S_z) u_{\beta}(\bar{P}', S'_z) \right], \quad \text{(B1)}
\]

\[
F_2 = \int \frac{d^3 p_2}{(2\pi)^3} \phi_{\Sigma_2}^* (x, k_{\perp}') \phi_{\Sigma_1} (x, k_{\perp}) \text{Tr} \{ u_{\beta}(\bar{P}', S'_z) \bar{u}_{\alpha}(\bar{P}', S_z) \}
\]

\[
\times \left[ \epsilon^\alpha (\bar{P}', \sigma)|(\gamma^\nu (\gamma^\mu - \gamma_5)(\gamma^\beta - \gamma_5)| u(\bar{P}, S_z) \bar{u}(\bar{P}, S_z) u_{\beta}(\bar{P}', S'_z) \right], \quad \text{(B2)}
\]

\[
F_3 = \int \frac{d^3 p_2}{(2\pi)^3} \phi_{\Sigma_2}^* (x, k_{\perp}') \phi_{\Sigma_1} (x, k_{\perp}) \text{Tr} \{ u_{\beta}(\bar{P}', S'_z) \bar{u}_{\alpha}(\bar{P}', S_z) \}
\]

\[
\times \left[ \epsilon^\alpha (\bar{P}', \sigma)|(\gamma^\nu (\gamma^\mu - \gamma_5)(\gamma^\beta - \gamma_5)| u(\bar{P}, S_z) \bar{u}(\bar{P}, S_z) u_{\beta}(\bar{P}', S'_z) \right], \quad \text{(B3)}
\]

\[
F_4 = \int \frac{d^3 p_2}{(2\pi)^3} \phi_{\Sigma_2}^* (x, k_{\perp}') \phi_{\Sigma_1} (x, k_{\perp}) \text{Tr} \{ u_{\beta}(\bar{P}', S'_z) \bar{u}_{\alpha}(\bar{P}', S_z) \}
\]

\[
\times \left[ \epsilon^\alpha (\bar{P}', \sigma)|(\gamma^\nu (\gamma^\mu - \gamma_5)(\gamma^\beta - \gamma_5)| u(\bar{P}, S_z) \bar{u}(\bar{P}, S_z) g_{\mu}^{\beta} \right], \quad \text{(B4)}
\]
\( \bar{u}(P, S_z)\gamma^\mu P^\alpha u_\alpha(P', S'_z), \quad \bar{u}(P, S_z)P^\alpha P^\beta u_\beta(P', S'_z), \quad \bar{u}(P, S_z)P^\mu P^\beta \quad u_\beta(P', S'_z), \quad \bar{u}(P, S_z)g^\mu u_\beta(P', S'_z) \) are timed on the right side of Eq. (25),

\begin{align}
F_1 &= \text{Tr} \{ u_\beta(P', S'_z) \bar{u}_\alpha(P', S'_z) \left[ \gamma^\mu \bar{P}^\alpha \frac{f_1(q^2)}{M_{\Sigma_b}} + \frac{f_2(q^2)}{M_{\Sigma_b}} p^\mu + \frac{f_3(q^2)}{M_{\Sigma_b}} P^\mu + f_4 g^\mu \right] \} \\
&\quad \times (B5)
\end{align}

\begin{align}
F_2 &= \text{Tr} \{ u_\beta(P', S'_z) \bar{u}_\alpha(P', S'_z) \left[ \gamma^\mu \bar{P}^\alpha \frac{f_1(q^2)}{M_{\Sigma_b}} + \frac{f_2(q^2)}{M_{\Sigma_b}} p^\mu + \frac{f_3(q^2)}{M_{\Sigma_b}} P^\mu + f_4 g^\mu \right] \} \\
&\quad \times (B6)
\end{align}

\begin{align}
F_3 &= \text{Tr} \{ u_\beta(P', S'_z) \bar{u}_\alpha(P', S'_z) \left[ \gamma^\mu \bar{P}^\alpha \frac{f_1(q^2)}{M_{\Sigma_b}} + \frac{f_2(q^2)}{M_{\Sigma_b}} p^\mu + \frac{f_3(q^2)}{M_{\Sigma_b}} P^\mu + f_4 g^\mu \right] \} \\
&\quad \times (B7)
\end{align}

\begin{align}
F_4 &= \text{Tr} \{ u_\beta(P', S'_z) \bar{u}_\alpha(P', S'_z) \left[ \gamma^\mu \bar{P}^\alpha \frac{f_1(q^2)}{M_{\Sigma_b}} + \frac{f_2(q^2)}{M_{\Sigma_b}} p^\mu + \frac{f_3(q^2)}{M_{\Sigma_b}} P^\mu + f_4 g^\mu \right] \} \\
&\quad \times (B8)
\end{align}

Then by solving these equations one can obtain the expressions of \( f_1, f_2, f_3, f_4 \).

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