Study on the influence of different discrete forms of DDA block on mechanical properties of coarse granular material

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Abstract. In the triaxial compression test of soil engineering, particle breakage may occur under the double action of axial and lateral loads. So, for the DDA simulation, the internal discretization of DDA block should be carried out at first. However, different discretization may affect the final simulation results. To solve this problem, the DDA numerical models based on triangular discretization, Voronoi discretization and no discretization are established to carry out the triaxial test simulation of coarse granular material, and the influence of different discretization of DDA block on mechanical properties of coarse granular materials was compared. An improved DDA was used to simulate the triaxial tests under different confining pressures, The numerical results show that the deviatoric stress-strain curves of the three discrete forms become steeper and the peak strength also increases with the increase of confining pressure. Under the same confining pressure, the deviatoric stress-strain curve with no discretization is the steepest and the peak strength is also the largest. Although deviatoric stress-strain curve of the triangular discretization is gentler, its peak strength is closer to that with no discretization compared with the Voronoi discretization. The triangular discretization model gradually changes from contraction to dilation in the loading process, and the higher the confining pressure is, the more obvious the contraction is. At the same time, the direction of contact force gradually changes from the initial minor principal stress direction to the major principal stress direction, forming an obvious force chain, and the final particle breakage mostly occurs in an ‘X’ shaped region. The developed DDA method can accurately reflect the evolution of mechanical properties of coarse granular materials under various working conditions, and the research results can provide reference for the discrete form of blocks in similar simulation.
1. Introduction

As an important engineering material, coarse granular material is widely used in engineering construction because of its wide distribution, convenience of materials, good permeability, good compactness and high shear strength. Coarse granular material is composed of sand, gravel and other unbounded materials. Due to its poor roundness and sharp edges, coarse granular particles are prone to be broken under the dual action of axial and lateral loads. However, the particle breakage of coarse granular materials as dam building materials will incur large deformation of the dam, which will further lead to the cracking of the face slab and the increase of seepage at the contact position endangering the safety of the dam. As known, China has nearly 100000 dams, of which more than 6000 are more than 30 meters high. Therefore, it is very significant to study the impact of particle breakage on mechanical properties of coarse granular materials.

Compared with the in-situ and laboratory tests of coarse granular materials, the numerical simulation test method has the advantages of no need to consider the scale effect, saving manpower and material resources, and low dependence on time and space. The most important thing is that it is not limited by the experimental conditions and scale, and can repeatedly and freely control the critical state of feature deformation. Therefore, it is better to study the mechanical properties of coarse granular materials by numerical simulation.

The essence of coarse granular material is a kind of discontinuous medium. Its shape and appearance are mostly convex polyhedron with edges and corners. There is no bond between the particles, and the particles can contact and bite each other. This is different from the numerical analysis of the random distribution of concrete aggregate [1-3]. In concrete, the aggregate mainly plays the role of skeleton and support, and there is no contact requirement. At present, the simulation of random placing of coarse granular material is mainly carried out based on the particle flow code (PFC) [4-6]. But there is a big defect for PFC, that is its basic element is a rigid and unbreakable disk. However, the shape of particle has a significant impact on mechanical properties of coarse granular materials. In reality, coarse granular materials will deform macroscopically under the action of external force, accompanied by the change of their own stress state. While the internal meso physical process will be characterized by particle breakage, sliding and rolling. Although the current PFC polymerizes the basic elements into fragile particle clusters using the cluster technology [7-9], the particle clusters cannot continue to be deformed and broken after they are broken into basic element. In addition, the friction problem caused by rotation in PFC is still controversial. Some scholars have made different attempts to solve these problems. Zhou et al. and Ma et al. [10-11] established the stochastic distribution model of rockfill by Monte Carlo method, and established stochastic granule discontinuous deformation (SGDD) model of rockfill from the micro perspective. Li et al. [12] used CT scanning technology to obtain the section image information of soil rock mixture, extracted the point cloud data of gravel surface through binarization and gravel boundary recognition technology, and reconstructed the three-dimensional model of gravel by reverse engineering software. Yan et al. [13] proposed a new generation method of coarse granular material model, that is, first scale the particles, then use the random algorithm to drop the scaled particles into a given area, further divide the particles into grids, at last enlarge the particles to the original size. Afterwards, a sample meeting the given void ratio can be obtained after using finite combined finite-discrete element method.
(FEM/DEM) to calculate its equilibrium state.

Discontinuous deformation analysis (DDA) [14-18] can use a variety of irregular polygons to characterize the shape of particles. Its analysis theory is relatively perfect, the relevant assumptions are more rigorous, and the calculation efficiency is higher. At the same time, the rotation and sliding of the block can be considered, and the open-close iterative algorithm can accurately simulate the contact between particles. In order to form a set of unified analysis software for random generation and mechanical analysis of coarse granular materials, a particle generation method was proposed by Guo et al. [19-20], which can randomly generate polygonal particles that meet the gradation requirements of coarse granular materials, and DDA method is used to simulate the accumulation and distribution of particles in the support tube under the action of gravity. After the initial stacking model is formed, DDA can be used to analyze the mechanical properties of coarse granular materials under test load and confining pressure. However, he did not consider the particle breakage. Later, Su [21] carried out a preliminary numerical test on the mechanical properties of coarse granular materials with single particle size, and the simulation results were consistent with the indoor test law. However, the specimen is single graded, and the grid is single and not uniform. In this paper, the uniformly graded coarse granular numerical specimens are generated, and the blocks of the specimens are discretized into sub-blocks in different forms. The improved DDA is used to simulate the triaxial test of coarse granular materials, and then the influence of different discrete forms of DDA block on mechanical properties of coarse granular materials is studied.

2. Foundations of DDA

In DDA [22], the displacement and deformation of the block are determined by six variables: 

\[(u_0, v_0 , r_0, e_x, e_y, \gamma_{xy})\].

Where \((u_0, v_0)\) is the rigid body displacement of the block, \(r_0\) represents the rotation angle of the rigid body around the centroid \((x_0, y_0)\), \(e_x, e_y\) and \(\gamma_{xy}\) represent the normal and shear strains.

The displacement \((u, v)\) is actually the accumulation of small displacement increment at each time step, and it can be expressed as

\[
\begin{pmatrix}
    u \\
    v
\end{pmatrix} = \begin{pmatrix}
    1 & 0 & -(y-y_0) & (x-x_0) & 0 & (y-y_0)/2 \\
    0 & 1 & (x-x_0) & 0 & (y-y_0) & (x-x_0)/2
\end{pmatrix} \begin{pmatrix}
    u_0 \\
    v_0 \\
    r_0 \\
    e_x \\
    e_y \\
    \gamma_{xy}
\end{pmatrix}
\]

(1)

The mutual constraints between the block interfaces constitute the block system. Assuming there are \(n\) blocks in the block system, the global equilibrium equation has the following form

\[
\begin{bmatrix}
    K_{11} & K_{12} & \cdots & K_{1n} \\
    K_{21} & K_{22} & \cdots & K_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    K_{n1} & K_{n2} & \cdots & K_{nn}
\end{bmatrix}
\begin{bmatrix}
    D_1 \\
    D_2 \\
    \vdots \\
    D_n
\end{bmatrix} = \begin{bmatrix}
    F_1 \\
    F_2 \\
    \vdots \\
    F_n
\end{bmatrix}
\]

(2)

Where element stiffness matrices \(K_{ii}\) and \(K_{ij}\) are 6 × 6 submatrix and force
vector $\{F_i\}$ is $6 \times 1$ submatrix.

According to the variational principle, the general equation (Equation 2) can be obtained by deriving the total potential energy $\Pi$ under the action of stress and external force, that is, the total potential energy is the minimum. The elements in row $r$ and column $s$ of $\left[K_{ij}\right]$ are:

$$\frac{\partial^2 \Pi}{\partial d_r \partial d_s} \quad (r, s = 1, 2, \ldots, 6) \quad (3)$$

The elements in row $r$ of $F_i$ are:

$$-\frac{\partial \Pi(0)}{\partial d_i} \quad (r = 1, 2, \ldots, 6) \quad (4)$$

Where: $\Pi$ is the total potential energy of the system, it is the algebraic sum of the potential energy of each block. After solving the Equation (2), the displacement of the deformed block can be calculated by Equation (1). The global equilibrium equation can be modified by adding or removing the rigid spring at the corresponding contact position under the conditions of no tension and no embedding conditions. The modified equation will be solved again until all contact conditions are satisfied.

3. Numerical simulation of triaxial test

3.1. Specimen preparation

First of all, an initial model with specified gradation, close contact and stable overall structure was generated according to the self-developed generation method of coarse granular material block system, as shown in Figure 1-(a). Obviously, for the initial model, DDA cannot simulate the breakage of coarse particles. Therefore, it is necessary to divide the DDA block in the initial model into sub-blocks, and embed joint elements between them to simulate the particle breakages. Here, we only discuss two common discretization forms, including triangular discretization and Voronoi polygonal discretization, as shown in Figure 1-(b) and (c).
3.2. Constraints and loading methods

According to the indoor triaxial compression test of coarse granular materials, the same loading mode and boundary conditions are applied to the numerical samples. A flexible emulsion film which is evenly divided into triangular blocks is applied on both sides of the sample allowing a certain elastic deformation. A rigid plate evenly divided into rectangular blocks is applied at the top and bottom respectively. The displacement mode of the top plate only considers the translation of the rigid body, which can avoid the rotation caused by the passive change of the load direction in the loading process. At the same time, a fixed constraint is imposed on the bottom rigid plate.

Firstly, the axial load is applied to the centre of the top loading plate, and the confining pressure is applied uniformly on both sides, then the numerical test of the mechanical properties of coarse granular materials will be carried out based on DDA. It is worth noting that this is a quasi-static problem. In order to simulate the relative motion and large displacement of the block, DDA adopts the dynamic method to solve the static problem, and obtains the static solution through step-by-step iteration, which is equivalent to the dynamic relaxation method. Therefore, it is necessary to apply the next level load after the deformation of the first level load is fully stable. The loading curve is shown in Figure 2.

3.3. Meso-parameters value

The values of material parameters of coarse granular materials adopted in DDA are shown in Table 1.

| Parameter                          | Value   | Parameter               | Value   |
|------------------------------------|---------|-------------------------|---------|
| Mass density $\rho$ ($kg/m^3$)     | $2.5 \times 10^3$ | Cohesion $c$ (MPa)     | 5       |
| Elastic modulus $E$ (GPa)          | 25      | Tensile strength $\sigma_b$ (MPa) | 2       |
| Poisson's ratio $\nu$              | 0.25    |                         |         |
| Friction angle $\varphi$ (°)       | 50      |                         |         |

In addition, the friction angle of fracture surface of coarse granular particles is $30^\circ$, and the cohesion and tensile strength are 0; The friction angle between particles is $30^\circ$, and the cohesion and
tensile strength are 0.

4. Simulation test results

Three different discrete forms are used in the simulation of geotechnical triaxial test based on DDA, including no discretization, triangular discretization and Voronoi polygonal discretization. The same loading mode and loading rate were adopted. Numerical tests were carried out under confining pressures of 0.5, 0.8, 1.0, 2.0 and 3.0 MPa.

4.1. Stress-strain curve

The relationship between axial stress and axial strain under various confining pressures is shown in Figure 3. The results show that the general trends of stress-strain curves under different confining pressures are basically the same. When the coarse granular material is compacted to some extent, the deviatoric stress increases rapidly with the increase of axial strain. With the continuous increase of vertical load, both the deviatoric stress and axial strain increase showing an obvious nonlinear relationship until reaching the peak strength. With the increase of confining pressure, the deviatoric stress-strain curves of the three discrete forms all become steeper and the peak strengths all increase. Under the same confining pressure, the deviatoric stress-strain curve with no discretization is the steepest and its peak strength is also the largest. Although deviatoric stress-strain curve of the triangular discretization is gentler, its peak strength is closer to that with no discretization compared with the Voronoi discretization.

The reason for the above phenomenon may be that the stress skeleton of the specimen with no discretization will not be broken, and the particles are large, so it is not easy to rotate and displace. Because of its stable structure, with the increase of axial load, the deviatoric stress-strain curve rises the fastest and the peak strength is the largest. However, for the specimen with particle breakage, after discretization, the skeleton particles will crack and break after a certain force. After the particles are broken, they will undergo continuous rotation and displacement, and the load at the contact point between particles redistribute and gradually reach stability. Therefore, the rising speed of deviatoric stress-strain curve will be relatively slow. The triangular discrete element is smaller in area and sharper in shape, easy to be embedded in the gap, strong in bite, forming a more stable stress structure. While the Voronoi discrete element has better roundness, easy to rotate, and the stress structure is not stable enough, so the peak strength of triangular discretization will be greater.
4.2. Volumetric strain curve

For the change of specimen volume, the compression is positive and the expansion is negative. It can be seen from the curve of volumetric strain and axial strain as shown in Figure 4 that under different confining pressures, the curve law is basically consistent, and the dilation rate with no discretization is the fastest, and the final volume expansion is the largest. By contrast, the dilation rate of the other two discrete forms is slower, and the final dilation of Voronoi discretization is larger than that of triangular discretization. During the loading process, the volume of triangular discretization model changes from shear contraction to dilation gradually, and the higher the confining pressure is, the more obvious the shear contraction is. However, the Voronoi discretization model and the model with no discretization do not have an obvious contraction process, and dilation is the dominant one.
The above phenomenon may be due to the formation of contact and stable structure of the specimen, during the preparation of the specimen, a certain degree of preloading process is carried out, which makes the particles closely arranged, and the porosity of the specimen is small, resulting in the shear contraction phenomenon is not obvious. However, each discrete element of triangular discretization model has a small area and a large number of acute angles, so it is easy to fill small gaps, and there is obvious shear contraction phenomenon. When the particle breakage is not considered in the shear process, the particles of the specimen will not break from large particles to small particles and fill the pores, so it shows stronger dilatancy. When the confining pressure increases, the shear contraction is more obvious. The reason may be that the contact between particles is relatively loose when the confining pressure is small, and the external force (mainly the indirect contact force of particles caused by confining pressure) to be overcome by the movement of rockfill particles under shear is small. With the increase of confining pressure, the particles contact more closely. Under the action of shear, the force between particles to be overcome in particle movement increases, which leads to the crushing of some particles and the increase of particle breakage rate, which leads to the increase of fine particle content. The broken fine particles are filled into the pores of the original larger particles, which makes the specimen denser. The volume of the specimen decreases gradually due to the filling of the pores between particles, which shows a more obvious phenomenon of shear contraction with the increase of confining pressure.
Figure 4. The Volumetric strain curves under different confining pressures (a) 0.5MPa, (b) 0.8MPa, (c) 1.0MPa, (d) 2.0MPa, (e) 3.0MPa

4.3. Development of particle breakage and formation of shear band

In this numerical model, each particle is simulated as rotatable and breakable, so the damage and breakup state of the specimen can be obtained at any loading stage. In the simulation process, the particles in contact with the loading plate at the corners of the specimen first break up and continue to develop towards the middle. The final particle breakage mostly occurs in an "X" shaped region, as shown in Figure 5. The figure shows the shape of the specimen when it reaches the peak strength under 1.0MPa confining pressure. By contrast, in the two models considering particle breakage, the "X" shaped region composed of white gaps can be seen clearly.

Figure 5. Comparison of particle crushing effect (a) Initial model (b) Triangular discretization model (c) Voronoi discretization model

Figure 6 and Figure 7 show the horizontal and vertical displacement distribution of particles when the specimen reaches the peak strength under 1.0MPa confining pressure. It can be seen that the horizontal displacement of the particles in the triangle area at the top and bottom of the specimen is smaller, and the horizontal displacement of the particles on both sides of the middle of the specimen is larger due to the restraint effect. The vertical displacement of the specimen shows obvious delamination, and the displacement in the top triangle area is the largest and decreases step by step. The horizontal and vertical displacement distribution is almost symmetrical. The maximum horizontal displacement of Voronoi discretization is slightly larger than that of triangular discretization, which may be due to the better roundness and easy rotation and displacement of polygonal granular element after particle crushing. However, the triangular discrete element is angular and prone to get stuck,
which leads to limited movement. At the same time, an 'X' shaped area is called the shear band, with an inclination of about 45°. This can be explained by the conclusion of Marsal [23]: The formation of this shear band is mainly due to the deformation concentration caused by strain localization, which leads to severe damage and breakage of coarse granular specimens.

4.4. Contact force distribution

A major advantage of numerical test over laboratory test is that the change process of some variables can be observed intuitively. For example, the change process of contact force distribution can be monitored, so as to understand the mechanical mechanism of specimen deformation and particle breakage. As shown in Figure 8, the figure shows the evolution process of contact force distribution under 1.0MPa confining pressure. In the initial stage, the size and direction of the contact force are more uniform, which indicates that the particles reach the denser state under the confining pressure; With the increase of deviatoric stress, the direction of contact force deflects gradually, mainly concentrated on the diagonal strip, from top left to bottom right and from top right to bottom left, which are two obvious concentrated traces. With the application of load, the direction of contact force gradually changes from the direction of initial minor principal stress to the direction of major principal stress. This shows that the distribution of particles is gradually directional, and the transmission of
force between particles has obvious direction, which is different from the uniform transmission of continuous force. At the same time, it can be seen that in the two discrete models, the contact force is closely connected with each block like a blood vessel, forming an obvious force chain, which represents the transfer network of the squeezing force between particles under the action of ballast. Because the large particles act as the skeleton of the sample, the red lines representing the contact force in the large particles are densely distributed. However, with the occurrence of large particle cracking and crushing, the contact force is dispersed and the red line distribution becomes sparse gradually. It shows that the influence of skeleton particle breakage on the mechanical properties of coarse granular materials is very important.

Figure 8. Contact force distribution evolution (a) Triangular discretization model (b) Voronoi discretization model

5. Conclusion and prospect
In order to study the influence of different discrete forms of DDA blocks on the mechanical properties of coarse granular materials, the DDA numerical models based on triangular discretization, Voronoi
discretization and no discretization were established to carry out the triaxial test simulation, and the corresponding stress-strain curve, volumetric strain curve, the development process of particle breakage, the formation of shear bands and the change of contact force distribution were discussed. The conclusions are as follows:

1) With the increase of confining pressure, the deviatoric stress curves of the three discrete forms all become steeper and the peak strengths all increase; Under the same confining pressure, the deviatoric stress-strain curve with no discretization is the steepest and the peak strength is also the largest. Although deviatoric stress-strain curve of the triangular discretization is gentler, its peak strength is closer to that with no discretization compared with the Voronoi discretization.

2) Under different confining pressures, the curve law is basically consistent, and the dilation rate with no discretization is the fastest, and the final volume expansion is also the largest. By contrast, the dilation rate of the other two discrete forms is slower, and the final dilation of Voronoi discretization is larger than that of triangular discretization. During the loading process, the volume of triangular discretization model changes from shear contraction to dilation gradually, and the higher the confining pressure is, the more obvious the shear contraction is. However, the Voronoi discretization model and the model with no discretization do not have an obvious contraction process, and dilation is the dominant one.

3) As the load applying step by step, the deformation concentration caused by strain localization leads to the severe damage of coarse particles, resulting in the formation of "X" shaped shear band. Particle breakages of the two discrete forms mainly occur in this "X" shaped region. This phenomenon is similar to the conjugate failure in traditional triaxial test, which shows that DDA simulation is reliable.

4) During the loading, distribution of particles gradually has the directionality. Different from the uniform transmission of continuous force, the transmission of force between coarse particles has obvious directionality. At the same time, the contact force of the particles considering particle breakage dispersed with the breaking of large particles. With the loading, the direction of contact force gradually changes from the direction of initial minor principal stress to the direction of major principal stress, forming an obvious force chain.

DDA has outstanding advantages in the numerical simulation test coarse granular material. In order to simulate the triaxial test of coarse granular materials more realistically, a 3D-DDA block model will be established in the next stage.

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