An Adaptive Low-illumination Image Enhancement Algorithm based on Weighted Least Squares Optimization

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Abstract. An adaptive low-illumination image enhancement algorithm based on the weighted least squares optimization is proposed to solve the difficulty of detailed feature recognition in low-illumination images that collected by visible light imaging equipment. First, the image is converted from RGB channel to LAB channel. Second, we use an edge-preserving smoothing operator based on the weighted least squares optimization to coarsen smooth base layer and extract multi-scale details in brightness channel. Then, an adaptive weight is proposed and applied to the weighted fusion of smooth base and detail features. Finally, the Retinex enhancement is performed to obtain a ultimate enhanced image. Experiments result show that the image enhanced by this method has suitable visual brightness and clear details. In terms of objective indicators, it has good and stable performance in NIQE, TMQI, and information entropy.

1. Introduction
Images play a vital role in people's daily information acquisition. However, the low light environment affects the quality of visible light imaging seriously. In low-illumination images, the contrast is low and a large number of details are difficult to recognize. Low-illumination image enhancement has become one of the hotspots in the field of image processing and many algorithms have been proposed. But some problems have been found in former low-illumination image enhancement algorithms.

Low-illumination image enhancement methods are mainly divided into histogram equalization (HE), methods based on the atmospheric scattering model, methods based on the Retinex theory, and methods based on depth learning.

HE is a method of increasing contrast by stretching the dynamic range of image’s gray level. Reference[1] proposed an adaptive histogram equalization method with limited contrast to solve the problem of nonuniform and unnatural on enhanced brightness. It’s enhancement results always have obvious blurred on boundary details.

The method based on the atmospheric scattering model is to flip the low illumination image and remove the fog by treating the dark area as a foggy area[2]. Reference[3] obtains an atmospheric light map replaced with a local atmospheric light value to address the oversaturation caused by global atmospheric light. When there are some scenes similar to atmospheric light in the image, the satisfactory visual effect can not be obtained.

The core idea of Retinex theory based on illumination reflection model is that the reflection component determines the essential information of the object, and the image enhancement is realized by accurately estimating the illumination component[4]. Reference[5] obtains a multi-scale Retinex(MSR) method for image color enhancement and dynamic range compression. In enhanced
images, edges with large brightness differences are prone to halo. LIME algorithm in reference[6] initialize the illumination image by selecting the maximum value in each pixel channel, and then refine the initial illumination by adding a structural priori. The algorithm solves the problem of oversaturation in bright areas and phenomenon of halos, but there is a certain degree of color distortion. SRIE algorithm in reference[7] propose a weighted variational model to estimate both the reflectance and the illumination from an observed image. The algorithm can keep bright areas free from distortion, but the fusion weight parameters are not derived from the learning method, do not have statistical regularity and have poor robustness.

Deep learning is a new way to recover image information by encoding and decoding. Combining convolution neural network and Retinex theory, a Retinexnet model is proposed in reference [8]. The network decouples the image, obtains the illumination map and reflection map, and enhances the illumination map. The enhanced illumination map is multiplied by the original reflection map to obtain the enhancement result. Methods based on deep learning often have some disadvantages, such as difficult to obtain datasets and poor generalization.

Reference[9] proposes an image decomposition method. Enhance and filter images in HSI color space and RGB color space respectively, then two images are weighted and merged to obtain the final enhanced image. This method keeps the details of the image well, but the suppression effect of strong light is slightly poor.

Based on the above factors, An adaptive low-illumination image enhancement algorithm based on the weighted least squares optimization is proposed in this paper. It aims to enhance the brightness of the image while maintaining the detail and color of the image. In this paper, the L-channel of lab color model is used for illumination estimation. Use multi-scale edge-preserving decompositions for detail enhancement and image smoothing, which the method mentioned in reference[10]. The smoothed image is used as the base layer, the detail extracted image is used as texture layer. Then base layer and texture layer are weighted fused to get an image with enhanced detail and smooth base layer. Finally, the Retinex model is applied to enhance the brightness, and the final result is obtained.

2. Basic model

2.1. CIELab color model

The CIELab color model is a color model based on human physiological characteristics. It has the widest color gamut, so as long as the number of bits of color is large enough, no color loss will occur when any color mode is converted to CIELab color mode. The Lab color model consists of three channels, “L” is the brightness channel and contains only light and dark information, “a” is the color channel from green to red and “b” is the color channel from blue to yellow. When only process brightness channels, the color of the image is not changed.

The RGB to Lab channel conversion formula[11] is as follows: First, convert the RGB model to XYZ model.

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} =
\begin{bmatrix}
0.412453 & 0.357580 & 0.190423 \\
0.212671 & 0.715160 & 0.072169 \\
0.019334 & 0.119193 & 0.950227
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

(1)

Then it is converted from XYZ color model to lab color model.

\[
\begin{align*}
L &= 116 \times f(Y/Y_c) - 16 \\
a &= 500 \times f(X/X_c) - f(Y/Y_c) \\
b &= 200 \times f(Y/Y_c) - f(Z/Z_c)
\end{align*}
\]

(2)

Where \(X_c, Y_c, Z_c\) are the parameters for range mapping. \(X_c=1, Y_c=0.95045, Z_c=1.088754\). Function \(f\) is defined as:
2.2. Multi-scale WLS filter

The algorithm was first seen in reference [10]. In order to solve the problem that bilateral filtering cannot extract good detail information on multi-scale and may produce halos, the reference proposes an edge-preserving smoothing operator, based on the weighted least squares (WLS) optimization framework. The idea of the algorithm is seeking a smoothed image $\mathbf{R}$ as close as possible to input-image $\mathbf{P}$, but keep as original as possible in the edge part. The mathematical expression [12] is:

$$f(\omega) = \begin{cases} \omega^3, & \omega > \left(\frac{6}{29}\right)^3 \\ \frac{1}{3} \times \left(\frac{29}{6}\right)^2 + \frac{4}{29}, & \text{otherwise} \end{cases}$$

(3)

Where the subscript $q$ denotes the spatial location $(i,j)$ of a pixel. The data term $(\mathbf{R}_q - \mathbf{P}_q)^2$ is to minimize the distance between $\mathbf{R}$ and $\mathbf{P}$ to make the filtered image conform to the input image. The second regularization term achieve smoothness by minimizing the partial derivatives of $\mathbf{P}$. The smoothness requirement is enforced in a spatially varying manner via the smoothness weights $a_x$ and $a_y$. $\lambda$ is the smoothing adjustment ratio, the larger the value of $\lambda$, the smoother the output image.

$$a_{x,q}(\mathbf{R}) = \left(\left|\frac{\partial l}{\partial x}(q)\right| + \varepsilon\right)^{-\alpha}$$

$$a_{y,q}(\mathbf{R}) = \left(\left|\frac{\partial l}{\partial y}(q)\right| + \varepsilon\right)^{-\alpha}$$

(5)

Where $l$ is the log-luminance channel of the input image $\mathbf{P}$, the exponent $\alpha$ determines the sensitivity to the gradients of $l$, while $\varepsilon$ is a constant that prevents division by zero in areas where $\mathbf{P}$ is constant. For the pixel with high gradient value(edge), a small weight is given; For pixels with low gradient value, a larger weight is given. At implementation time, $\alpha=1.2, \varepsilon=0.0001$.

Formula (4) is expressed by image matrix can be rewritten as:

$$(\mathbf{R} - \mathbf{P})^\top (\mathbf{R} - \mathbf{P}) + \lambda(\mathbf{R}^\top \mathbf{D}^s \mathbf{A} \mathbf{D}^s \mathbf{R} + \mathbf{R}^\top \mathbf{D}^m \mathbf{A} \mathbf{D}^m \mathbf{R} + \mathbf{R}^\top \mathbf{D}^f \mathbf{A} \mathbf{D}^f \mathbf{R})$$

(6)

Where $A_s$ and $A_y$ are diagonal matrices containing the smoothness weights $a_x(R)$ and $a_y(R)$, respectively, and the matrices $D_s$ and $D_y$ are discrete differentiation operators.

Formula (6) is minimized by a derivative equal to 0 to obtain the filtered image. Convert to solving linear equation:

$$\mathbf{R} = (\varTheta + \lambda \mathbf{L}_P)^{-1} \mathbf{P}$$

$$\mathbf{L}_P = \mathbf{D}_s^f \mathbf{A} \mathbf{D}_s^f + \mathbf{D}_m^f \mathbf{A} \mathbf{D}_m^f$$

(7)

Where $\mathbf{L}_P$ is a five-point spatially inhomogeneous Laplacian matrix. $\varTheta$ is the identity matrix.

Three-level edge-preserving smoothing decomposition of images based on the above formulas. New images with smoothing or detail enhancement can be obtained by multi-scale fusion of three layers of images with different smoothness. The mathematical expression [10] is:

$$\mathbf{R}_q = \mu + \sigma(\delta,\eta) - \mu + \sigma(\delta,\eta)$$

(8)

Where $b$ is coarse base level, $d^1$ is medium level and $d^2$ is fine level. $\eta$ is a coefficient controlling the exposure of the base layer. $\mu$ is the mean of the lightness range, and $\sigma$ is a sigmoid function.

$$\sigma(\delta, x) = \frac{1}{1 + e^{-\gamma}}$$

(9)

Where $\delta$ are boosting factors for three layers.
2.3. Retinex model
Retinex theory holds that the dynamic range of image pixels is affected by the incident light, while the color of the object is determined by the ability of the object to reflect light of different wavelengths, regardless of the intensity of light[13]. The Retinex model decomposes an image into illuminated image and reflected image, expression as follows:

\[ I(x,y) = R(x,y) \times L(x,y) \] (10)

Where \( I(x,y) \) is the original image, \( R(x,y) \) is the reflective image and \( L(x,y) \) is the illuminated image.

Because it is not easy to obtain the reflective component \( R(x,y) \) which reflects the essential information of the object directly, it is necessary to estimate the illuminated component \( L(x,y) \) to eliminate its impact on the original image, restore the original appearance of the object, and then achieve the purpose of image enhancement.

3. The algorithm of this paper
In this paper, we proposed an adaptive low-illumination image enhancement algorithm based on the weighted least squares optimization. First, the image is converted from RGB channel to LAB channel. Then, use the edge-preserving smoothing operator based on WLS to decompose illuminated image smoothly at multiple scales. multi-scale fusion of three illumination maps with different smoothness to obtain the smooth base component and detail texture component. Refine the smooth base layer using a bilateral filter. Weighted fusion of smooth base layer and fine texture layer using the adaptive weights presented in this paper. Transform the enhanced Lab image into RGB channel to get refined image. Finally, enhance the refined image with Retinex enhancement to get an enhanced image. The flowchart of the image enhancement method is shown in figure 1.

3.1. L-channel processing by Multi-scale WLS
Reference[10] has given a multi-scale fusion method based on WLS to do smooth filtering or detail extraction of an image. This multi-scale fusion operator is applied to L-channel in this paper to decompose smooth base layer and fine texture layer.

Firstly, the original L-channel image is transformed into three different smoothness images by WLS filter. The smoothest image is labeled \( L_1 \), and the medium smooth image is labeled \( L_0 \). Unsmoothed images use the original L-channel image \( L \). Calculate with formula (7):

\[
\begin{align*}
L_0 &= (I + 0.125 \cdot L) \times L \\
L_1 &= (I + 0.5 \cdot L) \times L \\
L &= L
\end{align*}
\] (11)
Fusion of three images using formula (8), \( L_{\text{base}} \) means coarse base level, \( D_1 \) means detail level and \( D_2 \) means secondary detail level.

\[
L_x = L_{\text{base}} + D_1 + D_2
\]

\[
\begin{align*}
L_{\text{base}} &= \mu + \text{sigmoid}(\delta_x, (\eta L_x - \mu) / 100) \times 100 \\
D_1 &= \text{sigmoid}(\delta_1, (L - L_x) / 100) \times 100 \\
D_2 &= \text{sigmoid}(\delta_2, (L_x - L) / 100) \times 100
\end{align*}
\]

The value range of L-channel in Lab model is 0~100, so \( \mu = 50 \). After repeated tests, when calculating smooth base layer \( \eta = 1.1, \delta_0 = 15, \delta_1 = 4, \delta_2 = 1 \). When calculating fine texture layer \( \eta = 1.0, \delta_0 = 1, \delta_1 = 10, \delta_2 = 1 \).

### 3.2. Bilateral filter refinement

Bilateral filter (BLF) is a non-linear filter, where each pixel in the filtered result is a weighted mean of its neighbors, with the weights decreasing both with spatial distance and with difference in value. The specific formula [14] is as follows:

\[
\text{BLF}[I_m] = \frac{1}{W_m} \sum_{n \in N(m)} G_{\sigma_s}([m-n]) G_{\sigma_r}([I_m - I_n]) I_n
\]

\[
W_m = \sum_{n \in N(m)} G_{\sigma_s}([m-n]) G_{\sigma_r}([I_m - I_n])
\]

Where \( I \) is an input image, the subscripts \( m \) and \( n \) indicate spatial locations of pixels. \( N(m) \) represents the adjacent position of location \( m \). The \( G_{\sigma_s} \) and \( G_{\sigma_r} \) are Gaussians kernel functions, where \( \sigma_s \) is spatial domain standard deviation and determines the spatial support, while \( \sigma_r \) is range standard deviation and controls the sensitivity to edges. \( ||m-n|| \) represents the spatial distance of pixels and \( ||I_m - I_n|| \) represents the gray-scale distance of pixels.

Although BLF is well suited for noise removal at a fine spatial scale, its edge-preserving property will be weakened with the enhancement of filtering ability [15]. By contrast, WLS filter can achieve progressive coarsening, so WLS filter has excellent edge-preserving characteristics, which also results in WLS filter preserves some high-frequency noise points on the image when smoothing the image. We find that we can make up for WLS filter by utilizing the fine smoothing property of the bilateral filter. The experimental comparison is as follows and the experimental image is from reference [10].

![Figure 2. Comparison of BLF and WLS filtering.](image)

When only BLF is used for filtering, the edges of the color map become blurred as the smoothing parameters increase. When using WLS only for filtering, increasing the smoothing factor does not eliminate severe noise, only the boundary of the noise becomes clearer. By applying a certain degree
of bilateral filtering to the image obtained after WLS filtering, the noises which similar to edges can be eliminated while the boundary is preserved. This avoid high-intensity noise in the smooth area of the enhanced picture. After experiment test, we choose $\sigma_s=3$, $\sigma_r=0.2$ to refine smooth base layer.

3.3. Adaptive weighted fusion

Considering the different brightness ranges of different low-illumination images, we propose an adaptive weighted fusion method to adjust the brightness of the output L-channel image to prevent the enhanced image from being too dark or too bright. Reference the gray-level histogram in image processing[16], we recording the brightness distribution of L-channel images. Experiments show that what really affects the brightness of the enhanced image is base smooth layer. First compress the brightness space of base smooth layer to 0–100 and calculate brightness histogram. Then find the brightness value with the most number to mark as $\beta$. At the time of the study, an optimal brightness value range of refined image is 14~16. Here we take the median value of 15.

$$L_x = \psi \cdot L_{\text{smooth}} + L_{\text{fine}}$$

$$\psi = \frac{15}{\beta}$$

Where $\psi$ is adaptive weight, $L_{\text{smooth}}$ is the L-channel image of base smooth layer, $L_{\text{fine}}$ is the L-channel image of fine texture layer and $L_R$ is the fused image.

3.4. Illumination map estimation of Retinex enhancement

Single Scale Retinex(SSR) is a commonly used traditional Retinex enhancement algorithm[17]. Due to the use of Gaussian low-pass filter for smoothing, the areas with low illuminance will be affected by adjacent high pixel values, resulting in high estimated illuminance, and halo artifacts will occur at the edges with large brightness difference. Transposition a Gaussian filter in SSR into a WLS filter. For acquiring better results, normalize by the maximum value in the RGB three-channel.

$$L(x,y) = \frac{\text{wlsFilter}(I_x / I_{\text{max}})}{\text{wlsFilter}(I_y / I_{\text{max}})}$$

$$L(x,y) = \frac{\text{wlsFilter}(I_z / I_{\text{max}})}{\text{wlsFilter}(I_y / I_{\text{max}})}$$

Where $L(x,y)$ is estimated illumination map, $I$ is refined image.

The corrected illuminated image obtained from Formula (16) is used as an estimate of the illumination component, and then (10) is used to calculate the reflection component, which is the result of final enhancement.

4. Experiments and Result Analysis

In order to prove the effectiveness of the algorithm, low contrast image enhancement contrast experiment is carried out. Comparison with MSR[5], LIME[6], SRIE[7] algorithm which using Retinex theory and Retinexnet[8] model which using deep learning. All algorithms are implemented under the platform of Matlab. The configure of computer is Inter Xeon E3-1505M v5 CPU, 32G RAM. Operating system is windows10. Low-illumination pictures used in the experiment are all from ExDark datasets.

4.1. Subjective evaluation indicators

There are many factors that affect image quality. Visual intuition can inspect and judge the quality of an image synthetically. Randomly select four pictures from the datasets for enhancement and comparison. The results are shown in the figure 3.
As can be seen from the above figure, Retinexnet-enhanced image has the highest overall brightness, but has poor contrast, and poor visual quality. MRS-enhanced image has severe color distortion and details in some shadows are not restored, such as the cat's back in Scene 1 and the people in Scene 2. The overall enhancement of LIME-enhanced image is brilliant, but there is noise and artifacts on the edge of the object, such as the building outside the window in Scene 1, the lines on the motorcycle in Scene 3 and the bedside in Scene 4. SRIE-enhanced image has the best retention of the original image features, but the overall brightness is low and many details are still not clearly visible. Such as the background behind the motorcycle in Scene 3. The algorithm in this paper has an excellent effect on brightness enhancement, edge detail division and color preservation. Our algorithm is superior to the others in terms of subjective indicators.

4.2. Objective evaluation indicators
Use the three indicators Entropy, TMQI and NIQE to objectively evaluate the above five scenes. In addition to these three indicators, PSNR[18] and SSIM[19] are also commonly used to evaluate the quality of enhanced images. But these two indicators are greatly affected by brightness. The enhanced image brightness is closer to original image the better calculation result will obtained. So it is not evaluated.

Entropy[20]:Information entropy is a common no-reference indicator used to measure the various of image information. The larger the value, the richer the information of the image.
TMQI\[^{21}\]: Tone Mapped Image Quality Index is an indicator to measure the color consistency between the enhanced image and the original image. The larger the TMQI value, the better the image quality.

NIQE\[^{22}\]: Natural image quality evaluator is a no-reference indicator. It is a set of quality-aware statistical features based on the spatial natural scene statistics (NSS) model. The lower the NIQE value, the better it accords with the subjective evaluation criteria of the human eyes.

The three objective evaluation indicators of our algorithm and the other four algorithms are shown in Table 1 to Table 3.

**Table 1. Result comparison of Entropy values of different algorithms.**

| Scene   | Our algorithm | MRS     | LIME    | SRIE    | Retinexnet |
|---------|---------------|---------|---------|---------|------------|
| Scene 1 | 7.5031        | 6.6915  | 7.4814  | 7.2690  | 7.7284     |
| Scene 2 | 7.9152        | 6.9858  | 7.9029  | 7.7541  | 7.7412     |
| Scene 3 | 6.3212        | 5.3851  | 6.9752  | 5.6543  | 7.2009     |
| Scene 4 | 7.7956        | 6.4837  | 7.7826  | 7.1013  | 7.7692     |

Table 1 shows that the information entropy of our algorithm is better than MSR and SRIE. In Scene 3, our algorithm is slightly worse than LIME. Retinexnet has the best information entropy, this is because the image color is oversaturated and the hue mutation.

**Table 2. Result comparison of TMQI values of different algorithms.**

| Scene   | Our algorithm | MRS     | LIME    | SRIE    | Retinexnet |
|---------|---------------|---------|---------|---------|------------|
| Scene 1 | 0.9716        | 0.8979  | 0.8640  | 0.8283  | 0.8369     |
| Scene 2 | 0.9889        | 0.8920  | 0.9757  | 0.9026  | 0.9143     |
| Scene 3 | 0.8167        | 0.7711  | 0.7984  | 0.7835  | 0.9069     |
| Scene 4 | 0.9845        | 0.8750  | 0.9803  | 0.9424  | 0.9003     |

Table 2 shows that the TMQI of our algorithm is better than MSR, LIME and SRIE. In Scene 3, our algorithm is worse than Retinexnet. On the whole, our algorithm has the best color retention.

**Table 3. Result comparison of NIQE values of different algorithms.**

| Scene   | Our algorithm | MRS     | LIME    | SRIE    | Retinexnet |
|---------|---------------|---------|---------|---------|------------|
| Scene 1 | 2.3209        | 2.4336  | 1.9760  | 2.6054  | 4.3826     |
| Scene 2 | 4.1044        | 4.2572  | 5.5930  | 4.0065  | 5.3936     |
| Scene 3 | 3.5268        | 6.2589  | 4.0934  | 3.7241  | 5.2742     |
| Scene 4 | 3.6128        | 4.6305  | 3.8096  | 3.3972  | 4.3259     |

Table 3 shows that the NIQE of our algorithm is better than MSR and Retinexnet. In Scene 1, our algorithm is worse than LIME. In Scene 2 and Scene 4, our algorithm is worse than SRIE. This is because the SRIE retains a lot of shadows and fits the natural lighting conditions better, but it also causes the problem that the details in the shadows are not completely recognizable.

Considering the above data indicators, our algorithm has the best comprehensive performance, especially in color retention.

5. Conclusion
In the case of insufficient light, the image contrast is low and details are blurred. A low-illumination image processing algorithm based on multi-scale WLS filter enhancement in brightness channel is presented in this paper. The L-channel of the Lab color model is enhanced in detail to suppress the noise and enhance the detail of the picture, ensuring that the image after brightness enhancement will not appear the noise caused by the oversaturation of brightness. Using the Retinex model to enhance brightness ensures that the color is not significantly distorted. A weighted fusion algorithm is used to
combine smooth base layer with fine texture layer to enhance the robustness of the algorithm under different lighting conditions.

Experiments show that the details and colors of the image enhanced by our algorithm are well maintained, the edges of the object are obvious, no halos appear, and the compression of high light is also excellent. On subjective evaluation, the image has a suitable brightness enhancement degree and a good visual perception. In the comparison of objective evaluation indicator, our algorithm has lower NIQE, higher information entropy and TMQI. It objectively proves that our algorithm is superior to the traditional algorithms based on Retinex theory.

In WLS filtering, the least squares estimation of each pixel of the image is required, which takes a long time. In the next work, we will optimize the algorithm for single low-illumination image enhancement, reduce the processing time of the algorithm, and apply it to video processing to achieve video image enhancement in hardware systems.

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