Enhanced current fluctuation in Coulomb blockade regime of multilevel quantum dot

To cite this article: Atsushi Iwasaki and Mikio Eto 2017 J. Phys.: Conf. Ser. 864 012034

View the article online for updates and enhancements.

Related content
- Mesoscopic fluctuation in a multilevel quantum dot in the Kondo regime
  Seungjoo Nah
- Phase Relaxation and Non-Equilibrium Transport Properties through Multilevel Quantum Dot
  Yasuhiro Funabashi, Kazuhiko Ohtsubo, Mikio Eto et al.
- Current Fluctuations in a Semiconductor Quantum Dot with Large Energy Spacing
  Jeong Heejun
Enhanced current fluctuation in Coulomb blockade regime of multilevel quantum dot

Atsushi Iwasaki and Mikio Eto

Faculty of Science and Technology, Keio University, 3-14-1 Hiyoshi, Kohoku-ku, Yokohama 223-8522, Japan

E-mail: eto@rk.phys.keio.ac.jp

Abstract. Nonequilibrium current fluctuation is theoretically studied through a multilevel quantum dot in the Coulomb blockade regime, where the higher-order tunneling processes, “cotunneling,” are dominant in the transport. The current fluctuation shows a Poisson (super-Poisson) noise in the elastic (inelastic) cotunneling regime when the bias voltage \( eV \) between the leads is smaller (larger) than the excitation energy \( \Delta \) from the ground level in the quantum dot. The super-Poisson noise is remarkably enhanced by the degeneracy of the excited states around the beginning of inelastic cotunneling at \( eV \approx \Delta \), qualitatively in good agreement with a recent experimental result. An analytical expression is given for the current fluctuation as a function of degeneracy.

1. Introduction

Nonequilibrium current fluctuation through a quantum dot yields information on the dynamic motion of single electron transport [1]. In the present paper, we theoretically examine the current fluctuation in the Coulomb blockade regime, where the transport takes place by the higher-order tunnel processes, cotunneling. Although the nonequilibrium fluctuation of the cotunneling current was reported by several groups [2, 3, 4], we pay special attention to the roles of excited levels in the quantum dot.

Let us consider a quantum dot with \( N \) energy levels \( (E_1, E_2, \cdots, E_N) \) attached to two leads \( L \) and \( R \), as shown in Fig. 1. The bias voltage \( V \) is applied, \( eV = \mu_L - \mu_R \) with \( \mu_L \) and \( \mu_R \) being the electrochemical potentials in the leads (\( \mu_R < \mu_L \)). At low temperatures, only the elastic cotunneling through the ground level \( E_1 \) takes place when \( eV \) does not exceed \( \Delta_{12} = E_2 - E_1 \), whereas inelastic processes are also possible when \( eV > \Delta_{12} \). In the latter regime, Okazaki et al. found a large current fluctuation when an excited level is within the bias window; \( E_1 < \mu_R < E_2 < \mu_L \) [5]. This is attributable to the sequential transport through level \( E_2 \), besides the cotunneling through level \( E_1 \). A large difference in the time scale between sequential and cotunneling transports results in a super-Poisson noise (Fano factor \( \approx 3 \)) for the current fluctuation. Recently, an enhancement of the current fluctuation (Fano factor \( \approx 1.5 \)) was observed when the excited levels are out of the bias window \( (E_1, \cdots, E_N < \mu_R) \) in the transport through a carbon-nanotube (CNT) quantum dot [6]. The degeneracy of the energy levels should play a role in this current fluctuation, which stems from the spin and valley degrees of freedom in the CNT.

To understand the mechanism of enhanced current fluctuation in CNT, we examine a simple model of a quantum dot with \( N \) energy levels for spinless electrons. We show that a super-Poisson noise in the inelastic regime can be significantly enhanced by the degeneracy of excited levels.
IOP Conf. Series: Journal of Physics: Conf. Series 864 (2017) 012034  doi  :10.1088/1742-6596/864/1/012034

In our model, spinless electrons are described by the Hamiltonian,
\begin{equation}
H = H_0 + H_R + H_T,
\end{equation}
where
\begin{align}
H_0 &= \sum_{i=1}^{N} E_i n_i + U \sum_{i>j} n_i n_j + \sum_{\alpha=L,R} \sum_{i=1}^{N} \sum_{k} \epsilon_k c_{\alpha k}^\dagger c_{\alpha ki}, \\
H_R &= \sum_{\alpha=L,R} \sum_{i=1}^{N} V_{\alpha i} d_i^\dagger c_{\alpha ki} + \text{h.c.},
\end{align}
with \( n_i = d_i^\dagger d_i \), where \( d_i \) and \( d_i^\dagger \) are annihilation and creation operators of an electron in the quantum dot with energy level \( E_i \) (\( i = 1, \cdots, N \)), respectively. \( c_{\alpha ki} \) and \( c_{\alpha ki}^\dagger \) are those in lead \( \alpha \) with wavenumber \( k \) and channel \( i \). (We assume that level \( i \) in the quantum dot is coupled to channel \( i \) in lead \( \alpha \) by \( V_{\alpha i} \) in the tunnel Hamiltonian \( H_T \), which is the situation in CNT with spin and valley degrees of freedom.) The Coulomb interaction is denoted by \( U \) in the quantum dot.

We examine the Coulomb blockade regime with one electron in the quantum dot, \( E_1 < \mu_L, \mu_R < E_1 + U \). The bias voltage \( eV = \mu_L - \mu_R \) is so small that all the excited levels are out of the bias window, \( E_i < \mu_R \) (\( i = 1, \cdots, N \)). First, we study a model for the quantum dot with two or three levels [Fig. 1(a)]. The level spacings are denoted by \( \Delta_{i,i+1} = E_{i+1} - E_i \). Second, the degeneracy of the excited levels is studied by another model in Fig. 1(b).

The current \( \langle I \rangle \) and current fluctuation \( S(\omega = 0) \), which is the zero-frequency component of Fourier transformation of the current-current correlation function, \( \langle I(t)I(0) \rangle - \langle I \rangle^2 \), are evaluated using the full counting statistics to solve the master equation for the stationary state [4]. The transition rate \( \Gamma_{i \rightarrow j}^{\alpha \rightarrow \beta} \) from level \( E_i \) to level \( E_j \) in the quantum dot, accompanied by the single-electron transport from lead \( \alpha \) to \( \beta \) is given by the second-order perturbation with respect to \( H_T \). For \( i = j \), it is
\begin{equation}
\Gamma_{i \rightarrow i}^{\alpha \rightarrow \beta} = \frac{2\pi}{\hbar^2} \int d\epsilon_k \left[ |V_{\alpha i}|^2 |V_{\beta i}|^2 \frac{(E_i - \epsilon_k)^2}{(E_i - \epsilon_k)^2 + \sum_{l \neq i} |V_{\alpha l}|^2 |V_{\beta l}|^2} \right] f_{\alpha}(\epsilon_k)[1 - f_{\beta}(\epsilon_k)],
\end{equation}
where \( v \) is the density of states in the leads and \( f_{\alpha}(\epsilon) \) is the Fermi distribution function in lead \( \alpha \). For \( i \neq j \)
\begin{equation}
\Gamma_{i \rightarrow j}^{\alpha \rightarrow \beta} = \frac{2\pi}{\hbar^2} |V_{\alpha i}|^2 |V_{\beta i}|^2 \int d\epsilon_k \left( \frac{1}{E_i - \epsilon_k} + \frac{1}{E_j - \epsilon_k - U} \right)^2 f_{\alpha}(\epsilon_k)[1 - f_{\beta}(\epsilon_k - E_j - E_i)].
\end{equation}
Figure 2. (a) $dI/dV$ and (b) Fano factor $F$ as a function of the bias voltage $eV$ for the model in Fig. 1(a). The case of two levels ($\Delta_{12} = \Delta$) is denoted by dotted lines, whereas that of three levels ($\Delta_{12} = \Delta_{23} = \Delta$) is by solid lines. $dI/dV$ is normalized by its value at $eV/\Delta = 0$. $E_1 = -10\Delta$, $U = 20\Delta$, and temperature $k_B T = 0.02\Delta$. All the tunnel couplings $V_{ij}$ are identical to each other.

The Fano factor, $F = S(\omega = 0)/2e\langle I \rangle$, is useful to characterize the nonequilibrium current fluctuation at $eV > k_B T$, so-called shot noise. $F = 1$ for the Poisson processes, whereas $F > 1$ ($F < 1$) for the super-(sub-) Poisson processes.

3. Calculated Results

Figure 2 presents (a) $dI/dV$ and (b) Fano factor $F$ as a function of bias voltage $eV$ for the model in Fig. 1(a). In the case of two levels ($\Delta_{12} = \Delta$; dotted line), $dI/dV$ is almost constant at $eV < \Delta$, reflecting the elastic cotunneling through the lower level $E_1$ only, whereas $dI/dV$ suddenly increases around $eV = \Delta$ owing to the possible inelastic processes at $eV > \Delta$. A small peak in the $dI/dV$ curve is seen at the beginning of inelastic cotunneling, which is caused by the increase in the probability $P_2$ that an electron occupies level 2 in the quantum dot with increasing $eV$ ($dP_2/dV > 0$) [7]. The Fano factor is almost unity at $k_B T < eV < \Delta$ (large $F$ at $eV < k_B T$ is due to the thermal noise). $F = 1$ in the elastic cotunneling regime means that the cotunneling takes place as a Poisson process from lead $L$ to $R$. $F$ is enhanced to 1.25 at $eV > \Delta$. This super-Poisson noise is caused by more than one time-scale for both elastic and inelastic processes.

In the case of three equi-distant levels ($\Delta_{12} = \Delta_{23} = \Delta$; solid line), $dI/dV$ shows two rapid increases around $eV = \Delta$ and $2\Delta$. Note that the Fano factor is larger than that in the two-level system at $\Delta < eV < 2\Delta$, which is ascribable to the additional transport accompanied by the transition from level $E_2$ to $E_3$. Since the probability $P_2$ to occupy level $E_2$ is finite, an electron can be excited to level $E_3$ from $E_2$, which enlarges the current fluctuation further.

Figure 3 is devoted to the results for the model in Fig. 1(b), where the higher level is $n$-fold degenerate in the quantum dot ($n = 1, 2$, and 4). The peak in the $dI/dV$ curve at the beginning of inelastic cotunneling becomes larger for larger $n$. This is because the probability to occupy the higher level, $P_2$, increases with $V$ more rapidly ($dP_2/dV$ is larger) for larger $n$.

We find a larger peak in the Fano factor at $\Delta < eV < 2\Delta$ for larger $n$. This is due to the larger difference in the time scales for the transport through level $E_1$ and $E_2$ for larger $n$. In contrast, the Fano factor becomes smaller for larger $n$ at $eV > 3\Delta$, where the role of the lower level in the transport is less for larger $n$. 
Figure 3. (a) $dI/dV$ and (b) Fano factor $F$ as a function of the bias voltage $eV$ for the model in Fig. 1(b). The degeneracy of the higher level is $n = 1$ (solid line), 2 (dotted line), and 4 (broken line). $dI/dV$ is normalized by its value at $eV/\Delta = 0$. $E_1 = -10\Delta$, $U = 20\Delta$, and temperature $k_B T = 0.02\Delta$. All the tunnel couplings $V_{ai}$ are identical to each other.

We obtain an analytical expression for the Fano factor at $eV > \Delta$:

$$F = 1 + \frac{8n(n+1)(x-1)[3(n+1)x + n + 9]}{[(n+1)x + 3 - n]^2[(5n+1)(n+1)x^2 - (5n+1)(n-3)x - 16n]};$$

(5)

where $x = eV/\Delta$. This expression is justified when $-E_1, E_1 + U \gg \Delta, eV$. The curves in Fig. 3(b) are well reproduced by this expression for $eV > \Delta$.

4. Conclusion

We have studied the nonequilibrium current and its fluctuation through a multilevel quantum dot in the Coulomb blockade regime, where the cotunneling processes are dominant in the transport. The current fluctuation shows a Poisson noise in the elastic regime and a super-Poisson noise in the inelastic regime. The super-Poisson noise is remarkably enhanced by the degeneracy of the excited states around the beginning of inelastic cotunneling, which is qualitatively in good agreement with a recent experimental result using a CNT quantum dot. The analytical expression for this enhanced noise has been given as a function of degeneracy.

Acknowledgments

This work is partly supported by a Grant-in-Aid for Scientific Research (S) (No. 26220711) from Japan Society for the Promotion of Science and MEXT Grant-in-Aid for Scientific Research on Innovative Areas “Science of hybrid quantum systems” (No. 15H05870).

Reference

[1] Blanter Y M and Büttiker M 2000 Phys. Rep. 336 1
[2] Sukhorukov E V, Burkard G and Loss D 2001 Phys. Rev. B 63 125315
[3] Thielman A, Hettler M H, König J and Schön G 2005 Phys. Rev. Lett 95 146806
[4] Kaasbjerg K and Belzig W 2015 Phys. Rev. B 91 235413
[5] Okazaki Y et al. 2013 Phys. Rev. B 87 041302(R)
[6] Fujiwara R, Arakawa T and Kobayashi K private communications.
[7] Eto M 2001 Jpn. J. Appl. Phys. 40 1929