Electron Capture in Early Gravitational Collapse – Nuclear Equation of State

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Abstract. We present the spectra of pre trapping neutrinos emitted from a core collapse supernova (having main sequence masses 15 and 25 $M_{\odot}$) within 1 kpc which can be detected by terrestrial detectors. The neutrino spectrum depends on the abundance of nuclei and free protons which undergo electron capture which in turn is determined by nuclear properties of the stellar core. The ambient temperature in the early pre trapping phase is not so high as to wipe out shell and pairing effects. We present results from Relativistic Mean Field (RMF) calculations, which we use to predict properties of the neutron rich nuclei which dominate the stellar composition at this stage of stellar collapse and compare the RMF results with the Baron et al (BCK) equation of state.

1. Detectable number of neutrinos and their spectra

Neutrinos emitted from the early phase of core collapse that precedes a type II/Ib/Ic SN explosion, are emitted mainly due to $e^{-}$-captures on free protons and heavy nuclei (in the f-p shell with $A \geq 60$). Up to core densities of $\sim 3 \times 10^{11}$ gm/cc (neutrino trapping density) these $\nu_e$ escape freely from the overlying stellar matter without any interaction that change their energy. Their spectroscopy by terrestrial detectors would yield important information on the physical and the nuclear configuration of the collapsing stellar core.

The total number of neutrinos emitted from a 1.4 $M_{\odot}$ stellar core as it evolves from a initial density of $\sim 4 \times 10^{9}$g/cm$^3$ to a neutrino-trapping density of $\sim 2 \times 10^{11}$g/cm$^3$ is $\sim 5 \times 10^{55}$. The charge-current reaction $\nu_e(d, pp)e^-$ on deuterium nuclei in the Sudbury Neutrino Observatory (SNO) $D_2O$ detector and the $\nu_e - e$ scattering reaction for the more massive Super Kamioka ($H_2O$ based) detector can facilitate the detection of a significant number of these neutrinos at 1 kpc (Burrows 1990).

The neutrino production rate (i.e. the spectrum of the emitted $\nu_e$) as a function of neutrino energy $E_{\nu_e}$ depends on the electron capture rate for heavy

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nuclei \((H)\) and free protons \((fp)\) which are given by

\[
\lambda_{fp,H} = \frac{\log 2}{(ft)_{fp,H} (m_e c^2)^6} < G > \frac{E_{\nu}^2 E_{\nu} + Q_{fp,H}}{(E_{\nu} + Q_{fp,H})^2 - (m_e c^2)^2} \frac{dE_{\nu}}{1 + \exp(E_{\nu} + Q_{fp,H} - \mu_e)}
\]

where \(< G >\) is the coulomb correction factor (taken to be \(\simeq 2\) for heavy nuclei and \(= 1\) for free protons, Fuller, 1982) and the ft’s are related to the nuclear transition matrix element. Fig 1(a) presents the incident cumulative spectra of \(\nu_e\) emitted up to a density of \(2.4 \times 10^{11} g/cm^3\), from a supernova explosion 1 kpc away. These calculations are done using a single zone approximation for the collapsing stellar core (Ray et al., 1984), and approximating the ensemble of neutron-rich heavy nuclei present in the core by a mean nucleus \(A\) (which is the most abundant nucleus present under the given thermodynamic core configuration – Bethe et al. 1979). The spectra for free proton has been weighted by the relative fraction of free protons \(X_p\) present in the core. \(Q_{fp,H}\) is the \(e^-\)-capture Q-value for free-protons and heavy nuclei respectively and \(Q_H\) is given as:

\[
Q_H = \hat{\mu} + 1.297 + E_{GT}\]

where \(\hat{\mu} = (\mu_n - \mu_p)\) is the difference in the neutron and proton chemical potentials and \(E_{GT}\) is the energy of the Gamow-Teller Giant Resonance centroid (the centroids in fp-shell nuclei, found from experimental data from \((n,p)\) reactions have been used for characterizing GT centroids in fp-shell nuclei (Sutaria and Ray, 1995) and are close to 3 MeV as used here). For free protons this reduces to \(Q_{fp} = \hat{\mu} + 1.297\).

Table 1. Pre-trapping neutrino detections in SNO and Super Kamioka with hardness ratios up to \(\rho_{10} = 24.16\) for indicated heavy nuclear e-capture matrix elements for 15 \(M_\odot\) Fuller (1982) and 25 \(M_\odot\) Weaver et al (1985) pre supernova stars.

| Star Mass | \(|M_{GT}|^2\) | \(t_{\text{collapse}}\) (ms) | Pre-trapping Variables | No. Detected | Hardness Ratio\(^a\) |
|-----------|----------|-----------------|----------------------|---------------|-----------------|
| \(15 M_\odot\) | 1.2/0.1 | 120 | 0.3969 | 1.0021 | 82 | 394 | 0.2786 | 0.8540 |
| | 2.5/0.1 | 120 | 0.4206 | 1.0085 | 66 | 444 | 0.2786 | 0.9537 |
| \(25 M_\odot\) | 1.2/0.1 | 190 | 0.3828 | 1.1080 | 120 | 566 | 0.2878 | 0.8319 |
| | 2.5/0.1 | 190 | 0.3813 | 1.1204 | 99 | 499 | 0.2916 | 0.9190 |

\(^a\)The hardness ratio denotes the number of neutrino events in the 5 MeV \(\leq E_{\nu_e} \leq 12\) MeV and 12 MeV \(\leq E_{\nu_e} \leq 25\) MeV bands.

Fig.1(b) and Fig.1(c) show the expected \(\nu_e\) spectra in SNO and Super Kamioka detectors respectively and are obtained by folding the spectra in Fig 1(a) with the energy dependent detection cross-sections quoted in Burrows (1990) and Sehgal (1974). Table 1 displays the total number of neutrinos which can be expected to be detected in the SNO and Super-Kamioka detectors. The neutrino production rate depends on the \(e^-\)-capture rate, and hence on the weak interaction strength \(\propto |M_{GT}|^2\) of the nuclei undergoing \(e^-\)-capture. The numbers in Table 1 have been calculated for two different values of \(|M_{GT}|^2\). Nuclei beyond \(^{74}\)Ge become neutron shell-blocked against \(e^-\)-capture (Fuller 1982), and
Figure 1. Neutrino spectra: Incident, in SNO and in Super-Kamioka.

Figure 2. Comparison of $\mu_n$ and $\hat{\mu} = \mu_n - \mu_p$ using RMF and BCK equation of state at $\rho = 1.0 \times 10^{10} \text{g/cm}^3$.

since capture can proceed only via either thermally excited shell unblocking or via forbidden transitions, the $e^-$-capture strength drops to $|M_{GT}|^2 = 0.1$ for neutron shell blocked nuclei (Kar and Ray 1983).

2. RMF calculations of core nuclear properties

The neutrino spectrum depends on the abundances of nuclei undergoing electron capture, and this in turn is determined by the nuclear properties specifically, the electron capture Q-values $Q_{fp,H}$ and the nuclear chemical potentials $\mu_n$ and $\hat{\mu}$. At the stage of collapse considered, the ambient temperature $T$ is typically less than about 1 MeV, and in this range of temperatures, the nuclei undergo transition from a region where shell and pairing affects dominate to the region
where the nucleus can be well approximated by the liquid drop model. To take into account the shell and pairing effects in the early stage of collapse, we have used the Relativistic Mean Field Theory (see e.g., Sheikh at al. 1993) to calculate $\mu_n$ and $\tilde{\mu}$ for a number of nuclei ranging from $^{52}\text{Mn}$ to $^{74}\text{Ge}$ (Sutaria et al. 1997).

In Fig. 2(a) we present the calculation of $\mu_n$ for isotopes of Ni in the ground state which are expected to have significant abundance in this early stage of stellar collapse. Fig. 2(b) presents the values of $\tilde{\mu}$ for the same set of nuclei. Earlier workers (Baron et al. 1985, Cooperstein, 1985) had developed nuclear equations of state which were based on the liquid drop model of the nucleus, with corrections for nuclear compressibility etc. In these models, the nuclear energy per nucleon $W_N$ is a function of the nuclear density $\rho_N$, the nuclear volume $V_N$, the proton fraction $x = Z/A$ and the ratio $u = \rho/\rho_N$ corresponding to stellar packing fraction. The chemical potentials of the neutrons and protons in equilibrium with nuclei under given thermodynamic conditions have been calculated in BCK code by the usual methods (see e.g. Bethe et al., 1979 and references therein). A comparison of the $\mu_n$ and $\tilde{\mu}$ from the BCK equation of state with the RMF computations is displayed in figures 2(a) and 2(b). In these figures, we have used outputs of BCK code at low temperature limit ($T \sim 0.1$ MeV) corresponding to the nuclear parameters: $a_V = -16.0$, $S_V$ (volume symmetry energy) = 30.34, and the nuclear compressibility factor $K_0 = 180.0$ MeV. Properties of Ni and other neutron rich nuclei in the fp-shell, such as binding energies, nuclear deformations etc, computed from the RMF code were discussed in relation to phenomenological nuclear models by Sutaria et al (1997).

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