Study on the vibration isolation characteristics of an anti-resonant hydropneumatic suspension

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Abstract: A novel anti-resonant hydropneumatic suspension system was presented in this study. The mathematical model of the presented suspension system was derived and the theoretical analysis was carried out. For lower frequency vibrations, the presented suspension system can achieve better vibration isolation performance comparing with the traditional hydropneumatic suspension system. The frequency spectrum on which the presented suspension achieves better vibration isolation performances can be widened by adjusting the length of the oil tube. Therefore, it is possible for the presented suspension system to achieve better vibration isolation performance on different road conditions.

1 Introduction

Nowadays, the suspension system is widely used in vehicles in order to separate the passengers from road shocks and the vibration that is arising due to the rough conditions of the road. The suspension system in any vehicle is most important because it provides ride comfort and safety towards the passenger and driver. Such a system reduces fatigue during the driving conditions. To improve the performance of the suspension system, the active semi-active control suspension technique was used recently [1–3].

A hydropneumatic suspension system is a type of semi-active controlled suspension. This type of suspension has some important and good properties which are mostly used in tracked vehicles and also it improves ride comfort. Such properties are non-linear stiffness and damping, convenient turning, vertical position locking [4, 5]. For traditional hydropneumatic suspension system, smaller stiffness must be adapted to isolate a vibration at a lower frequency. The smaller stiffness of the suspension system leads to larger deflection, which cannot allow them too much varying load to the vehicle. So there is a need for such a suspension system which could significantly counter the varying load problem.

Anti-resonant vibration isolation system, which uses the inertial force to cancel the spring force, can isolate a vibration at a lower frequency with higher stiffness. The description of the mechanism of the anti-resonant vibration isolator can be found in the references [6–8]. In the designing of anti-resonance vibration isolators, leverage was often used to adjust the effective mass of the system. Comparing with mechanical leverage, hydraulic ones have the advantage of a compact arrangement. Referring to [9, 10] can get the details of the hydraulic leverage.

In this paper, the mechanism of the anti-resonance was introduced to a hydropneumatic suspension system and presented a new anti-resonant hydropneumatic suspension. The mathematical model of the anti-resonant hydropneumatic suspension was derived. The non-dimensional transformation function was used to assess the vibration isolation performance of the hydropneumatic suspension. The spectrum width of the frequency at which the presented anti-resonant hydropneumatic suspension can achieve a better performance than a traditional one was discussed. The effects of the design variables on the non-dimensional transformation function were also presented.

2 Mathematical model

The presented anti-resonant pneumatic suspension system contains a piston with diameter $D$, which is inserted in an oil cylinder. The upper part of the piston is an air chamber with height $H$. The oil in the oil cylinder can flow into the air chamber through an oil tube with diameter $d$ and length $l$ (Fig. 1).

The piston and the oil cylinder connect the vehicle body and the wheels, respectively. As the piston moves down, the oil in the cylinder will move upward through the oil tube into the air chamber so that the air in the air chamber is compressed. On the opposite side, the air in the air chamber will expand.

In the static state, the force acted on the piston by the vehicle body is denoted as $G_0$ and the air pressure in the air chamber $p_g$ can be expressed as

$$p_g = \frac{4G_o}{\pi D^2} + \frac{4m_p g}{\pi D^2} - \rho_o g \frac{D^2 - d^2}{D^2} + p_a$$

(1)

where $\rho_o$ is the density of the oil, $m_p$ is the mass of piston and $p_a$ is the barometric pressure.

Assuming that the vehicle moves with a constant horizontal velocity $U$, the motion equation of piston is derived in the Cartesian coordinate system with a velocity of $U$. The momentum equation of the piston along the vertical direction can be written as

$$m_p \frac{d^2 z_p}{dt^2} = p_g \frac{\pi D^2}{4} - p_o g \frac{\pi}{4} (D^2 - d^2) + p_o g \frac{\pi}{4} (D^2 - d^2) - (G_0 + \Delta G) - m_p g$$

(2)

where $z_p$ is the vertical displacement of the piston, $p_g$ is the relative pressure acted on the top of the air chamber by the air, $p_o$ is the relative pressure acted on the bottom of the air chamber by the oil, $p_{ob}$ is the relative pressure acted on the bottom of the piston by the oil, and $\Delta G$ is the dynamic force acted on the piston by the vehicle body.

Considering the effect of the viscosity, the unsteady Bernoulli equation can be written as

$$p_{ob} = p_{ob} + \frac{\zeta}{\pi \rho_o l} \frac{dv}{dt} + p_o g \frac{d^2 z_p}{dt^2} + \rho_o g l$$

(3)

where $v$ is the relative velocity of the oil in the oil tube and $\zeta$ is the viscous dissipation coefficient. As the piston vibrates, the adiabatic process can be used to describe the variation of $p_{ob}$.
\[ p_{e} = p_{0} \left( \frac{V_{g}}{V_{e}} \right)^{\gamma} - p_{0} \]  

where \( V_{g} \) is the volume of the gas in the air chamber, \( V_{g0} \) is the value of \( V_{g} \) at static, and \( \gamma \) is the adiabatic index.

Substituting (1), (3), and (4) into (2), gives

\[
m_{e} \frac{d^{2}z_{e}}{dt^{2}} = p_{0} \left( \frac{V_{g}}{V_{e}} \right)^{\gamma} \pi D^{2} - \frac{1}{2} \pi D^{2} \]  

The volume of the gas in the air chamber can be calculated as

\[ V_{g} = V_{g0} + (z_{o} - z_{e}) \pi D^{2} / 4 \]  

where \( z_{o} \) is the vertical displacement of the oil cylinder. The relative velocity of the oil in the oil tube can be calculated as

\[
v = \left( \frac{dz_{o}}{dt} - \frac{dz_{e}}{dt} \right) \frac{D^{2}}{d^{2}} \]  

Substituting (6) and (7) into (5) gives

\[
m_{e} \frac{d^{2}z_{e}}{dt^{2}} = p_{0} \left( \frac{V_{g0}}{V_{e}} \right)^{\gamma} \frac{\pi D^{2}}{4} \cdot \frac{d^{2}v}{dt^{2}} + \rho_{d} \left( \frac{d^{2}z_{e}}{dt^{2}} \right) \frac{\pi}{4} (D^{2} - d^{2}) - \Delta G. \]  

The order ratio of the second term (viscous dissipation term) to the third term in the right-hand side of (8) is \( O(z_{o}/l)(D^{2}/d^{2}) \). Under the condition of small amplitude vibration \( O(z_{o}/l)(D^{2}/d^{2}) < 1 \), the viscous dissipation term can be ignored, and (8) can be written as

\[
m_{e} \frac{d^{2}z_{e}}{dt^{2}} + p_{0} \frac{\gamma \pi D^{4}}{4V_{g0}} z_{e} + \Delta G = F_{ex}. \]  

\[
F_{ex} = p_{0} \frac{\gamma \pi D^{4}}{4V_{g0}} + \frac{\pi}{4} (D^{2} - d^{2}) \rho_{d} \frac{d^{2}z_{e}}{dt^{2}} \]  

where \( F_{ex} \) represents the excitation force acted on the suspension system. Imaging that a vehicle runs on the smooth surface road \( (z_{o} = 0) \), there is no excitation force acted on the suspension system. Thus the vertical displacement of the piston and the dynamic force acted on vehicle body approach zero. Therefore, the comfortability of the vehicle can be improved by decreasing the value of \( F_{ex} \).

### 3 Performance of the suspension system

As a vehicle moves on a rough surface road, the vertical displacement of the oil cylinder can be expressed as

\[
z_{o}(t) = \text{Re} \left( \int_{0}^{\infty} A(\omega) e^{i\omega t} d\omega \right) \]  

where \( \omega \) is the angular frequency, \( A(\omega) \) is the amplitude of the variation of \( z_{o} \). The distribution of \( A(\omega) \) is obtained by the vehicle speed and the road surface roughness power spectrum. Substituting

\[
\omega \sim \omega_{cr} \left( 1 - \left( V_{g0} / p_{o} D (\rho_{g} V_{g0})^{1/2} \right) \right) \]  

where \( \omega_{cr} \) is the cut-off frequency of the traditional suspension. The value of \( \omega_{cr} \) is the transformation function. For the traditional hydrodynamic suspension, only spring force contributes to the excitation force. To compare the behaviour of the presented suspension to the traditional hydrodynamic suspension, only spring force contributes to the excitation force. Therefore, the lower frequency vibration \( \omega \) of the anti-resonant hydrodynamic suspension is better than that of the traditional one. The value of \( \omega_{cr} \) can be zero and the excitation force equals zero, which can be calculated as

\[
\omega_{cr} = \sqrt{2 \omega_{op}} \]  

Fig. 2 gives the variation of \( \omega_{cr} \) as the function of non-dimensional frequency \( \omega^{*} = \omega / (p_{o} D (\rho_{g} V_{g0})^{1/2}) \). It is seen that \( \omega^{*} < 1 \) as the non-dimensional frequency is less than a critical \( \omega_{cr} \), which can be calculated by \( \omega_{cr} = \sqrt{2 \omega_{op}} \).

Therefore, the lower frequency vibration \( \omega^{*} < \omega_{op} \) can be reduced by the anti-resonant hydrodynamic suspension. The width of the frequency spectrum on which the anti-resonant hydrodynamic suspension achieves better performance can be expanded by increasing \( \omega_{op} \). Adopting a smaller \( l \) can achieve a larger value of \( \omega_{op} \). However, the values of \( \omega_{cr} \) at lower frequencies \( \omega^{*} < \omega_{op} \) increase with the decreasing of \( l \).
variation of $|R'|$ at four frequencies ($\omega^* = 0.1, 0.2, 0.3$) as the function of $D/l$.

In the case of $d/D = 0.5$, $\omega_{op} > 0.4$ as $D/l > 0.22$. It is seen that $|R'|$ increases with the decreasing of $l$ as $D/l > 0.22$. As $l$ approaches zero, $|R'|$ at all frequencies approaches one, and the presented anti-resonant hydropneumatic suspension can be considered as a traditional hydropneumatic suspension. Therefore, the vibration isolation performance of the presented anti-resonant hydropneumatic suspension at different frequencies can be adjusted by changing the value of $l$. On comparing with the traditional hydropneumatic suspension, it is possible for the presented anti-resonant hydropneumatic suspension to get a better performance by adjusting $l$ to fit different road conditions.

### 4 Conclusion

A novel anti-resonant hydropneumatic suspension system was presented in this paper. The non-dimensional transformation function was used to assess the performance of the presented suspension. We concluded the following points.

i. As the vibration frequency is lower than a critical value, the vibration isolation performance of the presented anti-resonant hydropneumatic suspension is better than that of the traditional one.

ii. The value of critical frequency can be increased by decreasing the length of the oil tube to widen the frequency spectrum on which the presented suspension achieves better vibration isolation performances.

iii. The value of non-dimensional transformation function at lower frequencies increases with the decrease of oil tube length. To achieve the best performance of the presented suspension system, the oil tube length must be adjusted according to different road conditions.

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### 6 References

[1] Fang, Z.F., Shu, W.H., Du, D.J., et al.: ‘Semi-active suspension of a full-vehicle model based on double-loop control’, Procedia Eng., 2011, 16, pp. 428–437

[2] Qazi, A.J., Khan, A., Khan, M.T., et al.: ‘Optimization of semi-active suspension system using particle swarm optimization algorithm’, AASRI Proc., 2013, 4, pp. 160–166

[3] Sung, K.G., Seong, M.S., Choi, S.H.: ‘Performance evaluation of electronic control suspension featuring vehicle ER dampers’, Meccanica, 2013, 48, pp. 121–134

[4] Westhuizen, S.F., Els, P.S.: ‘Comparison of different gas models to calculate the spring force of a hydropneumatic suspension’, J. Terramechanics, 2015, 57, pp. 41–59

[5] Ansari, F.A., Ranjan, R., Koradea, D.N., et al.: ‘Characterization of the hydraulic suspension system, on the basis of accumulator pressure values for a special purpose vehicle’, Mater. Today, Proc., 2017, 4, pp. 709–716

[6] Desjardins, R.A., Hooper, W.E.: ‘Antiresonant rotor isolation for vibration reduction’, J. Am. Helicopter Soc., 1980, 25, (3), pp. 46–55

[7] Braun, D.: ‘Development of antiresonance force isolators for helicopter vibration reduction’, J. Am. Helicopter Soc., 1982, 27, (4), pp. 37–44

[8] Yilmaz, C., Kikuchi, N.: ‘Analysis and design of passive band-stop filter-type vibration isolators for low-frequency applications’, J. Sound Vib., 2006, 291, pp. 1004–1028

[9] Plooy, N.F., Heyns, P.S., Brennan, M.J.: ‘The development of a tunable vibration absorbing isolator’, Int. J. Mech. Sci., 2005, 47, pp. 983–997

[10] Liu, C.R., Xu, D.L., Ji, J.F.: ‘Theoretical design and experimental verification of a tunable floating raft vibration isolation system’, J. Sound Vib., 2012, 331, pp. 4691–4703