Free massive particles with total energy $E < mc^2$ in curved spacetimes

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We analyze free elementary particles with rest mass $m$ and total energy $E < mc^2$ in the Rindler wedge, outside Reissner-Nordstrom black holes and in the spacetime of relativistic (and non-relativistic) stars, and use Unruh-DeWitt-like detectors to calculate the associated particle detection rate in each case. The (mean) particle position is identified with the spatial average of the excitation probability of the detectors, which are supposed to cover the whole space. Our results are shown to be in harmony with General Relativity classical predictions. Eventually we reconcile our conclusions with Earth-based experiments which are in good agreement with $E \geq mc^2$.

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I. INTRODUCTION

The standard theory of quantum fields uses the fact that the Minkowski spacetime is maximally symmetric. The linear three momenta $(k^x, k^y, k^z)$ associated with the translational isometries on the spacelike hypersurfaces $t = \text{const}$ constitutes a suitable set of quantum numbers to label free particles, where we are assuming here that $(t, x, y, z)$ are the usual Minkowski coordinates. In this simple case, the dispersion relation $E \equiv \hbar \omega = \sqrt{|k| c^2 + m^2 c^4}$ imposes a simple constraint between the particle mass $m$, momentum $k$ and energy $E$, and, thus, free particles with well defined linear momenta must have total energy $E \geq m c^2$. Because Earth-based particle experiments assume in general the detection of asymptotic free states in Minkowski spacetime, the possibility of measuring particles with total energy

$$E < mc^2$$

is usually neglected. Moreover, in the classical context of General Relativity, the detection in loco of point particles satisfying Eq. (1.1) by direct capture is ruled out by the fact that an observer with four-velocity $u^\mu$ intercepting a particle with four-momentum $k^\mu = mv^\mu$ ascribes to the particle an energy $E = mv^\mu u_\mu \geq mc^2$. On the other hand, it is well known that, unlike standard Quantum Field Theory in flat spacetime, the field quantization carried over curved (stationary) backgrounds does not lead in general to any dispersion relation for the energy and other quantum numbers, avoiding thus the flat spacetime constraint $E \geq mc^2$. This can be understood by recalling that the concept of point particle finds no room in the context of Quantum Field Theory. Two main questions are raised, then, by the discussion above: Given a stationary spacetime (i) what is the minimum energy $E_{\text{min}} \equiv \hbar \omega_{\text{min}}$ allowed for a particle? and (ii) what is the probability density associated with the detection of particles with $E \in [E_{\text{min}}, mc^2)$ at different space points?

We analyze the questions above in the context of Quantum Theory of Linear Fields in Curved Spacetimes where the normal modes associated with our particles are seen as free. In order to define clearly what we mean by “observing a particle” we use Unruh-DeWitt detectors (endowed with an internal structure defined by a density of states). Our results are shown to be in agreement with General Relativity predictions concerning the position of
particles with $E < mc^2$ as defined by the same fiducial observers who proceed to the field quantization and with Earth-based experiments which usually assume $E \geq mc^2$.

The paper is organized as follows. In Section II we consider a two-dimensional Schwarzschild black hole which maintains a close relationship with the Rindler wedge where a full analytical investigation can be carried out. This section is important as a “theoretical laboratory” for the next ones and to provide a better understanding of some phenomena as, e.g., the decay of uniformly accelerated protons. This is so because the proton interaction with massive particles possessing energy $E < mc^2$ plays an important rôle according to coaccelerated observers with the proton. In Section III we investigate the questions (i) and (ii) outside Reissner-Nordstrom black holes. We discuss, in particular, the agreement of our results with General Relativity. In Section IV we consider a two-dimensional Schwarzschild black hole which plays an important rôle according to coaccelerated observers far away from the horizon, there is a strong detection damping.

Let us now consider in this spacetime a free scalar field $\hat\Phi(x)$ with mass $m$. The positive-frequency solutions of the Klein-Gordon equation $(\Box + m^2)\psi_\omega(x^\mu) = 0$ can be written as $u_\omega(x^\mu) = \psi_\omega(\rho)e^{-i\omega t}$ where

$$-\frac{d^2\psi_\omega}{dr^2} + V_{eff}\psi_\omega(\rho) = \omega^2\psi_\omega(\rho) .$$

Here $x = 4M \ln(\rho/4M)$ and $V_{eff} = (4M)^{-2}\rho^2m^2$. Note that the effective potential grows unboundedly at the infinity. By solving Eq. (2.3), we obtain

$$u_\omega(x^\mu) = \sqrt{(4M/\pi^2)}\sinh(4\pi M\omega)K_{4iM\omega}(\rho)e^{-i\omega t} ,$$

where $K_\nu(x)$ is the modified Bessel function and we have orthonormalized $u_\omega(x)$ according to the Klein-Gordon inner product $\langle u_\omega, u_\omega \rangle$. Note that $\omega \in (0, +\infty)$, i.e., there are massive Rindler particles with arbitrarily small energies.

Next, the field is expanded in terms of positive- and negative-frequency modes as usually:

$$\hat\Phi(x) = \int_0^{+\infty}d\omega \left[ \hat a_\omega \psi_\omega(x) + H.c. \right],$$

where the annihilation and creation operators satisfy $[\hat a_\omega, \hat a_\omega^\dagger] = \delta(\omega - \omega')$. Since $\partial_t u_\omega = -i\omega u_\omega$, the fiducial observer with respect to whom the quantization is second it provides a better understanding of some phenomena occurring in uniformly accelerated frames and third it allows a completely analytic discussion.

This fact is a reflection of the nonexistence of event horizons in the star spacetime. We show eventually that for Earth-based experiments $E_{\text{min}} \approx mc^2$, as expected. Our final remarks are made in Section V. We will assume hereafter natural units, $\hbar = c = G = 1$, unless stated otherwise.

II. DETECTION OF MASSIVE PARTICLES IN A TWO-DIMENSIONAL BLACK HOLE-LIKE SPACETIME

Let us begin considering the line element of a two-dimensional Schwarzschild spacetime:

$$ds^2 = (1 - 2M/r) dt^2 - (1 - 2M/r)^{-1} dr^2 . \tag{2.1}$$

Eq. (2.1) can be seen as describing a two-dimensional black hole with mass $M$. Close to the horizon, $r \approx 2M$, Eq. (2.1) can be written as

$$ds^2 = (\rho/4M)^2 dt^2 - d\rho^2 , \tag{2.2}$$

where $\rho(r) \equiv \sqrt{8M(r-2M)}$. (Note that in these coordinates the horizon is at $\rho = 0$.) Line element (2.2) is associated with the Rindler wedge (which is a globally hyperbolic spacetime) provided that $0 < \rho < +\infty$ and $-\infty < t < +\infty$. The advantage of considering spacetime (2.2) rather than (2.1) is three-fold: first it has the main relevant features (for our purposes) of the two-dimensional Schwarzschild spacetime (the fact that they differ asymptotically will not be important at this point),
performed is the one at $\rho = \rho_0 = 4M$, whose proper time is \(t\) [see Eq. (2.2)]. The Rindler vacuum $|0\rangle$ is defined from $\hat{a}_\omega |0\rangle = 0$.

Now, we introduce an Unruh-DeWitt detector [3] described by a localized monopole $\hat{m}(s)$ with proper time $s$ and worldline $z^\mu = z^\mu(s)$. Let $\hat{H}$ be the detector free Hamiltonian acting as $\hat{H}(E) = E|E\rangle$ on the detector energy eigenstates $|E\rangle$ and $\hat{m}(s) = e^{i\hat{H}s}\hat{m}(0)e^{-i\hat{H}s}$. We will denote by $|E_G\rangle$ the detector ground state and assume $E_G = 0$. The excited states of the detector will be ruled by some (normalized) density of states $\beta_{E_0}(E)$ peaked at $E = E_0$ and satisfying

$$\int_0^{+\infty} dE \beta_{E_0}(E) = 1. \quad (2.6)$$

The simplest choice is $\beta_{E_0}(E) = \delta(E - E_0)$ which characterizes a single excited-state detector. This will be enough in the black hole cases but not in the relativistic star one where we have to deal with particles with discrete energy spectra. For the later case it is more convenient to consider

$$\beta_{E_0}(E) = (n/E_0) \left[ \Theta(E - E_0 + E_0/2n) - \Theta(E - E_0 - E_0/2n) \right], \quad (2.7)$$

where $n = \text{const.} \gg 1$ and $\Theta(x)$ is the step function. We note that Eq. (2.7) satisfies Eq. (2.6) and the useful property $\beta_{E_0}(\epsilon c) = e^{-1}\beta_{E_0}(E)$ for $c \in \mathbb{R}$. Moreover, we can recover the single excited-state detector in the $n \to +\infty$ limit, since $\lim_{n\to+\infty} \beta_{E_0}(E) = \delta(E - E_0)$. Once the detector is defined, we couple it to a massive scalar field $\Phi(x^\mu)$ through the interaction action

$$\tilde{S}_I = \int_{-\infty}^{+\infty} ds c_0 \, \hat{m}(s) \, \Phi(x^\mu(s)), \quad (2.8)$$

where $c_0$ is a small coupling constant.

Let us now ask to our fiducial observer what is the total probability per (detector) proper time $\Gamma_\omega(\rho_d) \equiv P_\omega(\rho_d)/s_{d}^{\text{det}}$ of detecting a particle at some point $\rho_d$ with energy $\omega$. The excitation amplitude (at the tree level) $A^\text{det}_\omega \equiv \langle 0| \otimes \langle E|S_I|E_G\rangle \otimes |\omega\rangle$ associated with the particle detection can be shown to be

$$A^\text{det}_\omega = 4 c_0 \sqrt{M \sinh(4\pi M \omega)} K_{4iM\omega}(m \rho_d) \times \delta(E - 4M \omega/\rho_d), \quad (2.9)$$

where the detector selectivity was chosen such that $\langle E|m(0)E_G\rangle \equiv 1$. The detection rate is thus given by

$$\Gamma_\omega(\rho_d) = \frac{1}{s_{d}^{\text{det}}} \int_{0}^{+\infty} dE \beta_{E_0}(E) \int_{0}^{+\infty} d\omega' |A^\text{det}_\omega|^2 F_\omega(\omega') = \frac{2e^2}{\pi} \sinh(\pi E_0 \rho_d) \rho_d K^2_{4iE_0\rho_d}(m \rho_d) F_\omega \left( \frac{E_0 \rho_d}{4M} \right), \quad (2.10)$$

where we have chosen the density of states $\beta_{E_0}(E) = \delta(E - E_0)$ and $F_\omega(\omega')$ characterizes a mixed state peaked at the particle energy $\omega$ with the property that $F_\omega(\omega) = n_0 = \text{const.}$ for every $\omega$. (One can avoid the introduction of mixed states by considering the detection of wave packets, as shown below.) Now, we carefully adjust the detector at each point $\rho_d$ to maximize the detection probability $\langle 2.13 \rangle$. This is achieved by properly tuning its energy excitation gap: $E_0 = 4M \omega/\rho_d$ (note that $E_0$ and $\omega$ are related by a red-shift factor, as it should be). Hence Eq. (2.10) becomes

$$\Gamma_\omega(\rho_d) = \frac{2e^2}{\pi} n_0 \sinh(4\pi M \omega) \rho_d K^2_{4iM\omega}(m \rho_d). \quad (2.11)$$

Now, we may wonder what is the probability rate $\Gamma_\omega$ that a massive state $|\omega_0\rangle$ be measured by our observer at $\rho_0$ in the particular case where the detector is carried by our experimentalist, i.e., $\rho_d = \rho_0$ (and we recall that in the Rindler wedge $M = \rho_0/4$). In Fig. (3) we plot

$$\Gamma_\omega = \frac{2e^2}{\pi} n_0 \sinh(\pi \rho_0 \omega_0) \rho_0 K^2_{4i\rho_0\omega_0}(m \rho_0). \quad (2.12)$$

as a function of $\omega_0/m$. We clearly note that the farther away the observer is from the horizon, the steeper the detection damping for $\omega/m < 1$.

Next, let us define from Eq. (2.13) the normalized probability density

$$dP_\omega/d\rho_d \equiv \Gamma_\omega(\rho_d)/\int_0^{+\infty} \Gamma_\omega(\rho_d)d\rho_d. \quad (2.13)$$

$(dP_\omega/d\rho_d)d\rho_d$ is the probability that a particle with energy $\omega$ be found between $\rho_d$ and $\rho_d + d\rho_d$. We see from Fig. (3) that observers far away from the horizon will only be able to interact with the “tail” of the “wave functions” associated with particles with small $\omega/m$.

The physical content carried out by Eq. (2.13) can be reproduced by considering wave packets (rather than mixed states) as follows. Let us represent a particle with typical energy $\omega$ through a wave packet defined as

$$|\phi_\omega\rangle = \int_0^{+\infty} d\omega' G_\omega(\omega') \hat{a}_\omega^\dagger |0\rangle, \quad (2.14)$$

where $G_\omega(\omega')$ is a peaked function at $\omega' = \omega$ and $\int_0^{+\infty} d\omega' (G_\omega(\omega'))^2 = 1$ in order that $\langle \phi_\omega|\phi_\omega\rangle = 1$. The total probability for detecting $|\phi_\omega\rangle$ (at the tree level) at some $\rho_d$ between the constant-coordinate-time hypersurfaces $t = \text{const} \to -\infty$ and $t = \text{const} \to +\infty$ is $P_\omega^{\text{wp}}(\rho_d) = |\langle 0| \otimes \langle E_0|S_I|E_G\rangle \otimes |\phi_\omega\rangle|^2$, where we should tune the detector as before, $E_0 = 4M \omega/\rho_d$, to maximize
FIG. 2. We plot the probability density \( dP_\omega/d\rho \) for different \( \omega/m \) ratios, where we have assumed \( Mm = 1/4 \). We note that the smaller the \( \omega/m \) ratio, the closer to the horizon (in average) the particle lies, where the “gravitational potential” is stronger.

its detection probability. In order to obtain the probability between the constant-proper-time hypersurfaces \( s = \text{const} \to -\infty \) and \( s = \text{const} \to +\infty \), we must multiply both sides by the red-shift factor \( 4M/\rho d \):

\[
P^{wp}_\omega(\rho d) = 4c_0^2 |G_\omega(\omega)|^2 \sinh(4\pi M\omega) K^2_{4\Omega\omega}(m\rho d),
\]

(2.15)

where \( G_\omega(\omega) = \text{const} \). Note that Eqs. (2.15) and (2.11) differ only by a (dimensional) constant factor. Thus by replacing \( \Gamma_\omega(\rho d) \) by \( P^{wp}_\omega(\rho d) \) in Eq. (2.13), we obtain the same probability density \( dP_\omega/d\rho d \).

Now, in order to interpret Eq. (2.13) on General Relativity bases, let us first consider a row of detectors each of them lying at different \( \rho d \) and define the average detection position

\[
\langle \rho d \rangle \equiv \int_{0}^{+\infty} \rho d \ dP_\omega/d\rho d.
\]

(2.16)

By using Eq. (2.13), we obtain

\[
\langle \rho d \rangle = \frac{\pi \tanh(4\pi M\omega)(64M^2\omega^2 + 1)}{64M\omega} \approx \frac{\pi \ M \omega / m}{(\omega \gg a)},
\]

(2.17)

where \( a \equiv 1/4M \) is the proper acceleration of the fiducial observer.

Now, from General Relativity, a classical particle with mass \( m \) lying at rest at some point \( \rho p \) has, according to our fiducial observer at \( \rho 0 = 4M \), total energy \( \omega = m\rho p/4M \). By considering that the particle may have in addition some kinetic energy, the total energy would be \( \omega \geq m\rho p/4M \). By inverting this equation, we obtain

\[
\rho p \leq \rho p^\text{max} \equiv 4M\omega/m,
\]

(2.18)

which is expected to agree with \( \langle \rho d \rangle \), i.e., \( \langle \rho d \rangle \leq \rho p^\text{max} \), at least in the “high-frequency” regime \( \omega \gg a \) (where the quantum and classical behaviors may be compared). This conclusion is indeed in agreement with Eqs. (2.17) and (2.18) (see also Fig. 3). The smaller the \( \omega/m \) ratio, the more likely to detect the particle closer to the horizon where the “effective gravitational potential” decreases its total energy.

III. DETECTION OF MASSIVE PARTICLES OUTSIDE REISSNER-NORDSTROM BLACK HOLES

Let us consider now the line element of a globally-hyperbolic spherically-symmetric static spacetime

\[
ds^2 = f(r) dt^2 - h(r) dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2),
\]

(3.1)

where \( f(r) > 0 \) where the Killing field \( (\partial_\mu)^a \) is timelike. We may cast the positive-frequency solutions of the Klein-Gordon equation \( (\Box + m^2)u^a_{\omega ln}(x) = 0 \) in the form \( l \in \mathbb{N}, -l \leq m \leq l \)
FIG. 4. The scattering potential is plotted for $l = 10$ and $l = 0$. Full and dashed lines are related with $Q = 0$ and $Q = 0.9M$, respectively. $V_{\text{eff}}^{\text{RN}}$ vanishes at the horizon and tends asymptotically to $m^2$. We note that the larger the angular momentum, the higher the potential and the more difficult to find a particle with $\omega/m < 1$ far from the horizon. (We have assumed here $mM \gg 1$.

\[
\omega_{\text{eff}}^{\alpha}(r) = \sqrt{\frac{\omega}{\pi}} \frac{\psi_{\alpha}^0(r)}{r} Y_{\ell}(\theta, \varphi) e^{-i\omega t},
\]  

(3.2)

where $\psi_{\alpha}^0(r)$ satisfies

\[
- \frac{f}{h} \frac{d}{dr} \left( \sqrt{\frac{f}{h}} \frac{d}{dr} \right) \psi_{\alpha}^0(r) + V_{\text{eff}} \psi_{\alpha}^0(r) = \omega^2 \psi_{\alpha}^0(r)
\]

(3.3)

with the following scattering potential:

\[
V_{\text{eff}} \equiv \left[ \sqrt{\frac{f}{h}} \frac{d}{dr} \left( \sqrt{\frac{f}{h}} \frac{d}{dr} \right) + \frac{l(l+1)f}{r^2} + m^2 f \right].
\]

(3.4)

The label $\alpha$ was introduced to distinguish between the two independent solutions of the radial equation (3.3).

Outside a static black hole ($r > r_+$) with mass $M$ and charge $Q$ ($Q \leq M$), we have $f^{\text{RN}}(r) = 1/h^{\text{RN}}(r)$, where

\[
f^{\text{RN}}(r) = (1-r_+/r)(1-r_-/r) > 0
\]

(3.5)

and $r_\pm = M \pm \sqrt{M^2 - Q^2}$. (The label “RN” is a shortcut for Reissner-Nordstrom.) In this case, the scattering potential

\[
V_{\text{eff}}^{\text{RN}} = \left[ 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right] \left[ \frac{2M}{r^3} - \frac{2Q^2}{r^4} + \frac{l(l+1)}{r^2} + m^2 \right]
\]

(3.6)

vanishes at the horizon, $r = r_+$, and tends to $m^2$ asymptotically [see Fig. (4)]. Thus, only “outgoing” particles, $\alpha \rightarrow$, from the white hole horizon $H^+$ will be able to comply Eq. (3.1). (Eventually they will be totally reflected to the black hole horizon $H^-$.)

In contrast with the case $\omega = m$ is somewhat subler and will not be considered here (see Refs. [6]-[8] for the case $\omega = m = 0$.) Let us note that the larger the $l$, the higher the centrifugal barrier of the potential [see Fig. (4)]. As a consequence, particles with $l \gg 1$ which are found to verify Eq. (3.1) will be necessarily restricted to a close neighborhood around the horizon, where the gravitational field is strong enough to compensate the angular momentum effect. Hence, in order to find particles satisfying Eq. (3.1) “relatively” far from the horizon, we will focus on particles with $l = 0$.

For chargeless black holes, the local extrema of the scattering potential can be analytically determined. For the case $l = 0$ and $Mm < 1/4$, we have

\[
r_1 = \frac{1}{m} \left( \sqrt{3} \sin \frac{\xi}{3} - \cos \frac{\xi}{3} \right) \quad \text{and} \quad r_2 = \frac{2}{m} \cos \frac{\xi}{3}
\]

(3.7)

where $r_1$ and $r_2$ are associated with a local maximum and minimum ($r_1 < r_2$), respectively, and $\xi \equiv \pi - \arctan[\sqrt{1-16(mM)^2/(4mM)}].$ For the case $l = 0$ and $Mm > 1/4$ there are no local extrema [see Fig. (5)]. For $Mm = 1/4$, $r_1 = r_2$ is an inflection point and $V_{\text{eff}}^{\text{RN}} |_{r=r_1} = 3m^2/4$. For a black hole with $M \geq 3M_\odot$
and a particle with mass $m \geq m_{\epsilon,-}$, we have $Mm \geq 10^{16}$. This is why we will focus our simulations on values $Mm > 1/4$. An analytic investigation of the local extrema when $Q \neq 0$ is not possible since $dV_{\text{RN}}^\prime/dr = 0$ results in a full fifth-order algebraic equation but it can be seen numerically [see, e.g., Figs. 4 and 5] that the presence of charge does not change significantly the form of the potential $V_{\text{eff}}^\prime(r/r_{+})$.

Thus, close to and far from the horizon, we can write the outgoing $\psi_{\omega l}^\alpha$ functions as $(\omega \geq 0$, i.e., $\omega_{\text{min}} = 0$)

$$\psi_{\omega l}^\alpha(x) \approx A_{\omega l}^\alpha \left\{ e^{2iM\omega x} + \mathcal{R}_{\omega l} e^{-2iM\omega x} \right\} (x < 0, |x| \gg 1)$$

$(x \gg 1)$

(3.10)

and the incoming ones as $(\omega \geq m$, i.e., $\omega_{\text{min}} = m$)

$$\psi_{\omega l}^\alpha(x) \approx A_{\omega l}^\alpha \left\{ \mathcal{T}_{\omega l} e^{-2iM\omega x} + \mathcal{R}_{\omega l}^\ast e^{2iM\omega x} \right\} (x < 0, |x| \gg 1)$$

$(x \gg 1)$

(3.11)

where $\tilde{\omega} = \sqrt{\omega^2 - m^2}$ For $\omega \geq m$, $|\mathcal{R}_{\omega l}|^2$, $|\mathcal{T}_{\omega l}|^2$ and $|\mathcal{R}_{\omega l}^\ast|^2$, $|\mathcal{T}_{\omega l}^\ast|^2$ can be seen as reflection and transmission coefficients, respectively, satisfying the probability conservation equations

$$|\mathcal{R}_{\omega l}|^2 + \frac{\tilde{\omega}}{\omega} |\mathcal{T}_{\omega l}|^2 = 1 \quad \text{and} \quad |\mathcal{R}_{\omega l}^\ast|^2 + \frac{\omega}{\tilde{\omega}} |\mathcal{T}_{\omega l}^\ast|^2 = 1.$$  

(3.12)

For $\omega < m$, the outgoing modes fade exponentially far away from the horizon and $|\mathcal{R}_{\omega l}| = 1$. One of the tests performed to guaranty the reliability of our codes was the verification that Eqs. (3.10)-(3.11) were satisfied (with the appropriate relations for $\mathcal{R}_{\omega l}$ and $\mathcal{T}_{\omega l}$) along the numerical calculation.

We normalize $\psi_{\omega l}^\alpha$ such that $u_{\omega l m}^\alpha$ are Klein-Gordon orthonormalized [2]:

$$i \int_{\Sigma_t} d\Sigma n^\mu \left( u_{\omega l m}^\alpha \nabla_\mu u_{\omega l m}^\alpha \right) = \delta_{\alpha \alpha'}$$

(3.13)

where $\delta_{\alpha \alpha'} \equiv \delta_{\alpha \alpha'} \delta_{\mu \mu'} \delta_{\alpha \alpha'}$. The damping in the detection rate is steeper for observers far away from the horizon. (We have assumed here $mM = 2$.)

$$i \int_{\Sigma_t} d\Sigma n^\mu \left( u_{\omega l m}^\alpha \nabla_\mu u_{\omega l m}^\alpha \right) = 0,$$

(3.14)

where $\delta_{\alpha \alpha'} = \delta_{\alpha \alpha'} \delta_{\mu \mu'} \delta_{\alpha \alpha'}$ and $n^\mu$ is the future-pointing unit vector normal to the volume element of the Cauchy surface $\Sigma_t$. By using Eq. (3.9) to transform the left-hand side of Eq. (3.14) in a surface term, we obtain the normalization constants $A_{\omega l} = (2\omega)^{-1}$ and $A_{\omega l}^\ast = (2\sqrt{\omega})^{-1}$ (up to an arbitrary phase). The incoming and outgoing modes are orthogonal to each other with respect to the Klein-Gordon inner product (3.13)-(3.14). This can be seen by choosing $\Sigma_t = \mathcal{H}^- \cup \mathcal{J}^-$ in Eq. (3.13) and recalling that $\psi_{\omega l}^\alpha(x)$ and $\psi_{\omega l}^\alpha(x)$ vanish on $\mathcal{J}^-$ and $\mathcal{H}^-$, respectively.

Next we expand a massive scalar field $\hat{\Phi}(x)$ in terms of positive and negative frequency modes:

$$\hat{\Phi}(x) = \sum_{\alpha = 0}^{\infty} \sum_{l=0}^{\pm \infty} \sum_{m=-l}^{l} \int_{\omega_{\text{min}}}^{\infty} d\omega \left[ \hat{\phi}_{\omega l m}^\alpha u_{\omega l m}^\alpha(x) + \text{H.c.} \right],$$

(3.15)

where $\omega_{\text{min}} = 0$ when $\alpha = \text{and} \omega_{\text{min}} = m$ when $\alpha = \text{-}. As a consequence of orthonormalizing the normal modes with the Klein-Gordon inner-product, the annihilation $\hat{a}_{\omega l m}^\alpha$ and creation $\hat{a}_{\omega l m}^\ast$ operators satisfy the simple commutation relations

\[\ldots\]
related with \( Q \) estimation, namely, we have assumed \( M \) for the massive state \( \delta \alpha \), the detector at each point \( r \) for \( \alpha \), \( \omega \), \( l \), \( m \). We note that the smaller the \( \omega/m \) ratio, the closer to the horizon the particle should be.

\[
\left[ a_{\omega l m}, a_{\omega' l' m'}^\dagger \right] = \delta_{\alpha \alpha'} \delta_{l l'} \delta_{m m'} \delta(\omega - \omega') .
\] (3.16)

The Boulware vacuum \( |0\rangle \) is defined by \( a_{\omega l m}^\dagger |0\rangle = 0 \) for every \( \alpha, \omega, l \) and \( m \).

Let us assume an Unruh-DeWitt detector coupled to the massive scalar field as described by the interaction action (2.3). The detection rate (detection probability per detector proper time) of particles with quantum numbers \( \alpha, \omega, l \) and \( m \) (as defined by asymptotic fiducial observers) can be calculated (at the tree level):

\[
\Gamma_{\omega l m} = 2\beta_0 n_0 \omega \sqrt{f^{RN}(r_d)} \frac{|\psi_{\omega l m}^0(r_d)|^2}{r_d^2} |Y_{lm}(\theta_d, \varphi_d)|^2 .
\] (3.17)

where \( n_0 = F_\omega(\omega) = \text{const.} \), we have chosen \( \beta_0 \omega(E) = \delta(E - E_0) \) and we have tuned again the energy gap of the detector at each point \( r_d \) to maximize the detection probability, namely, \( E_0 = \omega / \sqrt{f^{RN}(r_d)} \).

As in the case of the Rindler wedge, we may calculate the detection rate \( \Gamma_{\omega l m}^\alpha \) in the particular case where the massive state \( |\alpha \omega_0 l m \rangle \) is defined by an experimentalist lying on the precise location of the detector, i.e., \( r_0 = r_d \). In principle, this would require that the scalar field were quantized with respect to a fiducial observer at \( r_0 \) (in which case \( e^{-i \omega t} \) in Eq. (3.2) would be replaced by \( e^{-i \omega \tau} \) with \( \tau = \sqrt{f^{RN}(r_0)} t \)). It is not difficult to see, however, that \( \Gamma_{\omega l m}^\alpha \) can be obtained directly from Eq. (3.17) with the following replacements: \( r_d \to r_0 \) and \( \omega \to \sqrt{f^{RN}(r_0)} \omega_0 \). [One can check this strategy in the Rindler wedge by obtaining directly Eq. (2.12) from Eq. (2.11)]. In Fig. (6) we plot \( \Gamma_{\omega^0 l^0 m^0} \) as a function of \( \omega_0/m \) for observers at different locations. (We recall that in four-dimensional spacetimes \( c_0 \) is dimensionless, in contrast to the case where the spacetime is two-dimensional.)

Now, let us define from Eq. (3.17) the (scalar) normalized probability density

\[
dP_{\omega l m}^{\alpha} / d\lambda = \Gamma_{\omega l m}^{\alpha} / \int \Gamma_{\omega l m}^{\alpha} (x') dV' .
\] (3.18)

Here \( dP_{\omega l m}^{\alpha} / d\lambda \) is the probability that a particle with quantum numbers \( \alpha, \omega, l, m \) be found at some spatial point \( x_d = (r_d, \theta_d, \varphi_d) \) in a proper volume \( dV_d = d\lambda_d r_d^2 \sin \theta_d d\theta_d d\varphi_d \), where \( d\lambda_d = \sqrt{f^{RN}(r_d)} dr_d \). The proper radial distance can be integrated from the horizon \( r = r_+ \) as a function of the radial coordinate:

\[
\lambda(r) = \int_{r_+}^r dr' \sqrt{f^{RN}(r')}
 = r \sqrt{f^{RN}(r)} + M \ln \left[ \frac{r + r \sqrt{f^{RN}(r)} - M}{r_+ - M} \right] .
\] (3.19)

Notice that \( \lambda = \lambda(r) \) is a regular function except in the case of extreme Reissner-Nordstrom black holes \( (Q = M) \). From Eq. (3.18), we obtain the radial-distance probability density

\[
dP_{\omega l m}^{\alpha} / d\lambda = \frac{\sqrt{f^{RN}(r_d)} |\psi_{\omega l m}^{\alpha}(r_d)|^2}{\int_{0}^{+\infty} d\lambda_d' \sqrt{f^{RN}(r_d')} |\psi_{\omega l m}^{\alpha}(r_d')|^2} .
\] (3.20)

Note that here \( dP_{\omega l m}^{\alpha} / d\lambda \) is the probability of detecting a particle with quantum numbers \( \alpha, \omega, l, m \) inside a shell between \( \lambda_d \) and \( \lambda_d + d\lambda_d \). In Fig. (7), we use Eq. (3.20) to plot \( dP_{\omega l m}^{\alpha} / d\lambda_d \) for particles with different \( \omega/m \) ratios (and \( l = 0 \)). We see that the smaller the \( \omega/m \) the closer to the black hole horizon the particle will be on average.

Next, in order to determine the mean particle radial distance, we calculate numerically [see Fig. (8)]

\[
\langle \lambda_d \rangle \equiv \int_{0}^{+\infty} d\lambda_d \lambda_d dP_{\omega l m}^{\alpha} / d\lambda_d .
\] (3.21)

Note that modes with \( \omega/m \geq 1 \) spreads over the whole space and thus \( \langle \lambda_d \rangle \) diverges. This is qualitatively different from the Rindler wedge case where the scattering potential grows unboundedly at the infinity what causes the normal modes to vanish asymptotically. As a consequence, at the Rindler wedge, \( \langle \rho_d \rangle \) is finite even for \( \omega/m > 1 \) [see Eq. (2.17)].
FIG. 8. \( \lambda(t) \) is plotted for \( \alpha \rightarrow \) and \( l = 0 \) and compared with \( \lambda_p^\text{max} \). We see, indeed, that \( \lambda(t) < \lambda_p^\text{max} \) in the “high-frequency” regime \( \omega > r_+^{-1} \) (i.e., at the right of the vertical broken line). (We have assumed here \( mM = 2 \) and \( \omega = 0 \)).

Now, by repeating our General Relativistic analysis performed in the Rindler wedge case in a more compact way, we obtain that according to (our fiducial) static asymptotic observers a particle with mass \( m \) at some point \( (r_p, \theta_p, \phi_p) \) with some four-velocity \( u^\mu \) has total energy \( \omega = mu^\mu(\partial_t)_{\mu} = m\sqrt{\gamma^{RN}(r_p)/1 + [u^mu_t]} \geq m\sqrt{\gamma^{RN}(r_p)} \). (The equality holds when the particle is at rest.) By inverting this relation, we obtain that a classical particle with mass \( m \) and total energy \( \omega \) (with \( \omega < m \)) will be at \( r_p \leq r_p^\text{max} \), where

\[
r_p^\text{max} = \frac{M[1 + \sqrt{1 - (1 - \omega^2/m^2)(Q/M)^2}]}{1 - \omega^2/m^2} \tag{3.22}
\]

(and \( r_p = r_p^\text{max} \) only when the particle lies at rest). Because the proper radial distance \( \lambda = \lambda(r) \) is a growing monotonic function, \( \lambda_p \leq \lambda_p^\text{max} \equiv \lambda(r_p^\text{max}) \). In the “high-frequency” regime, \( \omega \gg r_+^{-1} \), we expect as before \( \langle \lambda \rangle \leq \lambda_p^\text{max} \) (see Sec. 1). This is indeed verified in Fig. 8.

IV. DETECTION OF MASSIVE PARTICLES AT RELATIVISTIC STARS

Let us consider a static relativistic star with uniform density \( \alpha_0 = \text{const} \). The associated line element can be written as in Eq. (3.3) (see Ref. 8) with

\[
f^s(r) = \frac{1}{4} \left( 3\sqrt{1 - \frac{2M}{R}} - \sqrt{1 - \frac{2Mr^2}{R^3}} \right)^2 \Theta(R - r) \tag{4.1}
\]

and

\[
h^s(r) = (1 - 2m(r)/r)^{-1}\left(1 - 2Mr^2/R^3\right)^{-1}\Theta(R - r) \tag{4.2}
\]

where \( m(r) = 4\pi \int_0^r \alpha_0 \sigma^2 dr' \). We are using here the label “s” to denote quantities associated with the star spacetime. In these coordinates, \( r = R \) defines the star radius and \( M = (4\pi/3)\alpha_0 R^3 \) is the star total mass (which, of course, differs from the star total proper mass because of the binding energy contribution). Note that for sake of stability, \( R > R_c = 9M/4 \).

The positive-frequency solutions can be cast in the same form given in Eq. (4.2) where \( \psi_{\text{eff}}^s \) is defined by Eq. (3.3) with \( f(r) \) and \( h(r) \) given in Eqs. (4.1) and (4.2), respectively. The scattering potential can be obtained by using Eqs. (4.1) and (4.2) in Eq. (4.4):

\[
V_{\text{eff}}^s(r) = \left[ \frac{M}{R^3} \left( -9\frac{F(R)}{2} + \frac{9}{2} \frac{\sqrt{F(R)}}{h^s(r)} - \frac{1}{h^s(r)} \right) + \frac{1}{4} \left( \frac{l(l + 1)}{r^2} + m^2 \right) \left( 9F(R) + \frac{1}{h^s(r)} - 6\frac{\sqrt{F(R)}}{h^s(r)} \right) \right] \Theta(R - r) + f^s(r) \left( \frac{2M}{r^3} + \frac{l(l + 1)}{r^2} + m^2 \right) \Theta(r - R) \tag{4.3} \]

where \( F(R) \equiv 1 - 2M/R \). At the star surface the potential has a discontinuity:

\[
\lim_{r \rightarrow R_+} V_{\text{eff}}^s(r) = \frac{F(R)}{2} \left( \frac{l(l + 1)}{R^2} - \frac{M}{R^3} + m^2 \right) ,
\]

\[
\lim_{r \rightarrow R_+} V_{\text{eff}}^s(r) = F(R) \left( \frac{l(l + 1)}{R^2} + \frac{2M}{R^3} + m^2 \right) .
\]

In Fig. 9 we plot the scattering potential for different star parameters. (Note that the discontinuity of \( V_{\text{eff}}^s \) at \( r/R = 1 \) is very small for the parameters chosen in Fig. 9 to be self-evident.) It is interesting to observe that the gravitational field of the Earth is too faint to significantly influence the scattering potential for an electron-mass particle (\( m = 0.5\text{MeV} \)). At the center where the influence would be maximum, we would have for \( M = 6 \times 10^{27}\text{g} \)
will exist only one set of normalizable solutions and so of the discrete frequency spectrum. Moreover, there \( \omega = \omega_d \) of stars (absence of gravity. Notwithstanding, for quasi-extreme of a star would be of some importance for particle-physics possible to devise situations where the gravitational field (see Fig. 9). Indeed it is possible to devise situations where the gravitational field of a star would be of some importance for particle-physics processes.

The normal modes with \( \omega < m \) will be bounded states and \( \omega_{\text{min}} = \omega_1 \) will be the lowest eigenvalue of the discrete frequency spectrum. Moreover, there will exist only one set of normalizable solutions and so we will suppress the "\( \alpha \)" label hereafter. By defining \( d\tilde{r} = (2M)^{-1} \sqrt{h^2(r)/f^2(r)} dr \), we can write Eq. (3.3) as

\[
-\frac{d^2\psi_{\omega l}}{d\tilde{r}^2} + 4M^2 V_{\text{eff}}[\tilde{r}(r)]\psi_{\omega l} = 4M^2 \omega^2 \psi_{\omega l}.
\] (4.4)

The function \( \tilde{r}(r) \) is a growing monotonic function. At the star core (and choosing properly the integration constant) we have \( \tilde{r}|_{r=0} \approx (r/M)/(3\sqrt{F(R)} - 1) \) and the scattering potential becomes

\[
V_{\text{eff}}^s(\tilde{r})|_{r=0} \approx \frac{l(l+1)}{4M^2 \tilde{r}^2} + C^2 + O(\tilde{r}),
\] (4.5)

where

\[
C^2 = \frac{G(R)^2}{2M^2} \left[ \frac{(mM)^2}{2} + \frac{M^3 l(l+1)}{R^3 G(R)} + \frac{M^3}{R^3 G(R)} - \frac{M^3}{R^3} \right]
\] (4.6)

is a positive-definite constant and \( G(R) = -1 + 3\sqrt{F(R)} \).

Thus, close to the star center, \( \psi_{\omega l} \) is proportional to \( 2Mn\tilde{r} j_l(2M\tilde{r}) \) (since the solutions proportional to \( 2Mn\tilde{r} j_l(2M\tilde{r}) \) are non-normalizable), where \( j_l(x) \) is the spherical Bessel function and \( \nu \equiv \sqrt{\omega^2 - C^2} \). Moreover, far away from the star, \( V_{\text{eff}}^s(r) \approx m^2 \). Hence

\[
\psi_{\omega l}(\tilde{r}) \approx A_{\omega l} \left\{ \frac{B_{\omega l}(2Mn\tilde{r} j_l(2M\tilde{r}))}{C_{\omega l} e^{-2M\tilde{r}} + e^{2M\tilde{r}}} \right\} (\tilde{r} \approx 0)
\] (4.7)

where we recall that \( \tilde{\omega} \equiv \sqrt{\omega^2 - m^2} \). For \( \omega \geq m \), the modes are asymptotically free states and they can be normalized through Eqs. (3.13)-(3.14). Indeed we find

\[
A_{\omega l} = (2\sqrt{\omega\tilde{\omega}})^{-1} \text{ and } |C_{\omega l}| = 1.
\]

The normal modes fade exponentially and \( |C_{\omega l}| = 0 \), where we are using here latin indeces \( i, j, \ldots \) to label the discrete eigenfrequencies. The bounded states should be Klein-Gordon orthonormalized by imposing Eqs. (3.13)-(3.14) except for the fact that in this case \( \delta_{AA'} \equiv \delta_{ll'} \delta_{mm'} \delta_{\omega\omega_j} \).

Thus, we find

\[
\int_0^{+\infty} d\tilde{r} |\psi_{\omega l}(\tilde{r})|^2 = \frac{\pi}{4M\omega_1^2}.
\] (4.8)

The valid set of discrete eigenfrequencies \( \omega_1 \) is determined numerically [10]. Shortly, the strategy consists in evolving solutions with \( \psi_{\omega l}(0) = 0 \) [see Eq. (4.7)] (and arbitrary \( \psi_{\omega l}(\tilde{r})|_{\tilde{r}=0} = \text{const.} \) and search for the \( \omega_1 \)’s such that far enough from the star \( \psi_{\omega l} \sim e^{-2M\sqrt{m^2-\omega_1^2} \tilde{r}} \) as
TABLE I. We list the lowest eigenfrequencies \{\omega_1, ..., \omega_4\} for different star parameters. The lowest eigenfrequency \omega_1 is to be compared with the minimum classical energy \omega_{\text{class}}^{\text{min}}. We see that \omega_{\text{class}}^{\text{min}} < \omega_1, as expected, in the “high-frequency” regime \omega_{\text{class}} \gg M/R^2. By assuming that \(mM = 2\), then \(M/R^2 = 0.07\) and \(M/R^2 = 0.02\) for \(R/R_c = 1.2\) and \(R/R_c = 2\), respectively.

| \(mM = 2\) | \(R/R_c = 1.2\) | \(\omega_{\text{class}}^{\text{min}}/m\) | \(\omega_1/m\) | \(\omega_2/m\) | \(\omega_3/m\) | \(\omega_4/m\) |
|---|---|---|---|---|---|---|
| \(mM = 2\) | \(R/R_c = 2\) | \(0.264\) | \(0.351\) | \(0.456\) | \(0.556\) | \(0.648\) |
| \(mM = 2\) | \(R/R_c = 2\) | \(0.618\) | \(0.679\) | \(0.757\) | \(0.826\) | \(0.874\) |

The behavior of classical and quantum particles can be compared over the free states \(\chi\) and \(\delta\) for different star parameters. Namely, \(\omega_{\text{class}}^{\text{min}} = m\sqrt{f(r)}|_{r=0} = (m/2)(3\sqrt{1 - 2M/R} - 1)\). In the “high-frequency” regime, \(\omega_{\text{class}} \gg M/R^2\), where the behavior of classical and quantum particles can be compared, we expect \(\omega_1 \geq \omega_{\text{class}}^{\text{min}}\) because of the extra intrinsic “kinetic energy” of quantum origin (see Tab. I).

The massive scalar field can be cast as in Eq. (3.13)

\[
\hat{\Phi}(x) = \sum_{l=0}^{+\infty} \sum_{m=-l}^{l} \sum_\omega \left[ \delta_{\omega,m} \hat{\alpha}_{\omega,m}(x) + \text{H.c.} \right],
\]

where the summation in \(\omega\) was suppressed and we integrate over the free states \((\omega \geq m)\) and sum over the eigenfrequencies of the bounded states \((\omega_i < m)\). Here

\[
\left[ \delta_{\omega,m}, \delta_{\omega',m'} \right] = \delta_{\omega}\delta_{\omega',m'} = \delta_{\omega}\delta_{\omega',m'} = \delta_{\omega}\delta_{\omega',m'} = \delta_{\omega}\delta_{\omega',m'} = \delta_{\omega}\delta_{\omega',m'} = \delta_{\omega}\delta_{\omega',m'} = \delta_{\omega}\delta_{\omega',m'}.
\]

where \(\delta_{AA'}\equiv\delta_{\omega}\delta_{\omega,m}\delta_{\omega}\delta_{\omega',m}\) for free and bounded states, respectively.

Next we couple [see Eq. (2.8)] the massive scalar field to an Unruh-DeWitt detector characterized by the density of states \(\rho_{\omega} = \rho_{\omega}(\omega)\). The amplitude (at the tree level) associated with the detection of a bounded state with quantum numbers \(\omega_i, l\) and \(m\) is

\[
\mathcal{A}_{\omega_i,m}^{\text{det}} = \langle 0 | \langle \hat{\Psi}_m | \hat{E}_l | \hat{\Psi}_{\omega_i} \rangle | \omega_i,m \rangle.
\]

The detection rate (detection probability per detector proper time) is thus

\[
\Gamma_{\omega_i,m} = \frac{1}{\tau_d} \int_0^{+\infty} dE \beta_{E_0}(E) |\mathcal{A}_{\omega_i,m}^{\text{det}}|^2 = 2\omega_{\omega_i} \sqrt{f^*(r_d)\rho_{\omega_i}(r_d)^2 \omega_{\omega_i}/r_d^2} |Y_m (\theta_d, \varphi_d)|^2,
\]

where again the detector selectivity was chosen such that \(\langle E | m(0) | E_G \rangle \equiv 1\) and we have tuned the energy gap of the detector as usually: \(E_0 = \omega_i/\sqrt{f^*(r_d)}\). Notice the resemblance of Eqs. (4.11) and (3.17), where we recall that \(\delta_{\omega} = \delta_{\omega,0}\). As in the Reissner-Nordstrom case, we can calculate the detection rate \(\Gamma_{\omega_i,m}\) in the special case where the massive state \(\omega_i,m\) is defined by an experimentalist lying on the detector, as shown in Fig. II.

By using Eq. (II), we can define a normalized probability density \(dP_{\omega_i,m}/d\lambda_d\) analogously to Eq. (3.18), where \(d\lambda_d = d\lambda_d r_d^2 \sin \theta_d d\theta_d\) and \(d\lambda_d = \sqrt{r_d^2(r_d)}d\lambda_d\). The proper radial distance can be integrated from the center of the star as a function of the radial coordinate leading to

\[
\lambda = \frac{\arcsin(2M/r^2)^{1/2} \Theta[R - r]}{\sqrt{2M/r^3}} + \left\{ \frac{\arcsin(2M/R)^{1/2} \Theta[R - r]}{\sqrt{2M/R^3}} \right\} + M \left[ \frac{r + r \sqrt{1 - 2M/r - M}}{R + R \sqrt{1 - 2M/r - M}} \right] + r \frac{1 - 2M/r}{\sqrt{R}} \Theta[R - r].
\]

The radial-distance probability density \(dP_{\omega_i,m}/d\lambda_d\) is then calculated analogously to Eq. (3.20). In Fig. (II), we plot \(dP_{\omega_i,m}/d\lambda_d\) for particles with different \(\omega_i/m\) ratios. The mean particle radial distance \(\langle \lambda_d \rangle\) is defined analogously to Eq. (3.21) and was calculated numerically as shown in Fig. (II).
Although it is possible, in principle, to measure particles satisfying Eq. (11) even at Earth, these events are so rare in such weak gravitational fields that do not have to be considered for practical purposes. This can be seen from the analysis of the scattering potential [see discussion below Eq. (13)].

Notwithstanding the consideration of massive particles with arbitrary small total energy is just fundamental for a comprehensive understanding of some phenomena occurring in highly curved spacetimes (e.g., black holes) and accelerated frames. For example, the weak decay of uniformly accelerated protons in the Minkowski vacuum which is described by the $p \rightarrow n \ e^+ \ \nu$ channel (at the tree level) in an inertial frame was shown to be representable by the combination of the following three channels: $p \ e^- \rightarrow n \ \nu$ , $p \ \bar{\nu} \rightarrow n \ e^+$ and $p \ e^- \ \bar{\nu} \rightarrow n$ , in the proton coaccelerated frame. According to this description, the proton would decay by the absorption of a Rindler $e^-$ and/or $\bar{\nu}$ from the Fulling-Davies-Unruh thermal bath “attached” to the proton proper frame. However, in order that inertial and noninertial frame descriptions lead to the same observables (e.g. proper lifetime), we emphasize that the energy spectrum of the (massive) Rindler $e^-$’s and $e^+$’s must be $\omega \in [0, +\infty)$ (despite Rindler $e^-$’s and $e^+$’s have mass $m \approx 0.5$ MeV, as usually) [1]. Perhaps an even more extreme case concerns the importance of zero-energy particles to understand the radiation emitted from uniformly accelerated charges according to comoving observers. Namely, the emission of a (usual) Minkowski photon with transverse momentum $k_\perp$ from a uniformly accelerated charge as described by inertial observers corresponds, according to coaccelerated observers, to either the absorption from or the emission to the Fulling-Davies-Unruh thermal bath of a zero-energy photon with the same transverse momentum $k_\perp$. For obvious reasons, in Ref. [1] the photons considered were supposed to be massless but the same conclusion in terms of zero-energy particles would hold even if we considered that the photons were massive.

V. CONCLUSIONS

The detection rate for massive particles with total energy $E < mc^2$ was calculated for the Rindler wedge, black holes and star spacetimes. The mean particle positions were calculated and shown to be in qualitative agreement with General Relativity predictions in the “high-frequency” regime. The observation process is naturally taken into account in the formalism by considering Unruh-DeWitt-like detectors. In this way, we clearly define what we mean by “observing a particle”.

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