**TOPSIS Based Two-Sided Matching Under Interval-Valued Intuitionistic Fuzzy Environment in Virtual Reality Technology Transfer**

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**ABSTRACT** Bodies’ behaviors remain important to the decision-making process of two-sided matching. The two-sided matching problem under interval-valued intuitionistic fuzzy environment is investigated from the perspective of bodies’ behavior, i.e., the behavior of matching willingness. For solving the problem, a normalized interval-valued score function is firstly presented. Through using this function, interval-valued intuitionistic fuzzy number (IvIFN) preference matrices are converted to normalized interval-valued score matrices. The normalized interval-valued score matrices are then converted to score matrices. Based on score matrices, the matching willingness can be obtained by solving an optimal model. Based on score matrices and matching willingness matrix, the weighting score matrices are set up. Aiming at weighting score matrices, we use the TOPSIS method to calculate the closeness degrees. Based on closeness degree matrices and matching matrix, a two-sided matching (TsM) model is developed. Considering the same or different statuses of bodies, the TsM model is transformed to the one-goal TsM model. Through model solution, the optimal scheme of TsM can be obtained. Ultimately, the availability of TOPSIS based TsM method is illustrated by using a matching example of virtual reality technology.

**INDEX TERMS** Two-sided matching, interval-valued intuitionistic fuzzy number, bodies’ behavior, model, virtual reality technology.

**I. INTRODUCTION**

Two-sided matching (TsM) is a well-known research direction of decision-making. It had been extensively employed in numerous fields, such as mechanical systems [1], complex product manufacturing tasks [2], green building technologies [3], content sharing in internet of vehicles [4], matching with the stars [5], stable job matching [6], marriage problems [7], and loan market [8]. As early as 1962, Gale and Shapley [9] have investigated two classical TsM model using preferences of ordinal numbers, i.e., stable marriage and college admissions. From reference [9], it is known that TsM focuses on obtaining the most suitable matching scheme(s) from two sets of bodies. Thereafter, various concepts, theories, methods, technologies, and applications are proposed [10]–[17]. Therefore, TsM research is very meaningful in theory and practice.

On the other hand, in decision-making process, owing to the increasing complexity of the society and economy, inadequacy of human knowledge and imprecise of judgement, the decision-makers or bodies may give their preferences using intuitionistic fuzzy sets (IFSs) rather than some exact values, such ordinal numbers, order relations, or linguistic variables [18]. Interval-valued intuitionistic fuzzy sets (IVIFSs) [19] can reflect human thinking more reasonable than IFSs, which is regarded as the generalization of IFSs. The degrees of membership, non-membership and hesitancy are expressed in interval numbers in [0, 1].
Hence, TsM research using IVIFSs also has very important meanings. The theory of IVIFSs proposed by Atanassov and Gargov [19] has gained a great attention in some research fields, such as distance measure of IVIFSs [18], ranking of IVIFSs [20], entropy of IVIFSs [21], aggregation operators of IVIFSs [22], correlation coefficient of IVIFSs [23], score functions of IVIFSs [24], envelopment analysis and preference fusion of IVIFSs [25]. And then, its application field is extended into decision-making. For example, the variable-based combinatorial optimization technique is presented to solve the IvIFN information MAGDM problems with combinatorial optimization characteristics [26]. With regarded to the multiple attribute decision making (MADM) problems under IvIF situations, the novel MADM approach is proposed [27]. A new interval-valued knowledge measure for the IVIFSs is presented firstly, and then it was applied into decision making problems in order to demonstrate it is simpler and more attractive in comparison with other existing measures [28].

However, the related theory of IFSs are rarely used in TsM research [29]. For example, a new similarity measure between TIFNs is proposed in reference [30], which has been used to develop an TsM approach. The developed new similarity measure between TsIFNs was also used to present an TsM approach [31]. The application of IVIFSs can be found in the following literature [29], [32]. A TsM model for the personnel-position matching problem is designed, which was based on several new intuitionistic fuzzy Choquet integral operators [32]. The TsM matching problem with IvIFNs and matching aspirations was studied [29]. But the TsM model developed in reference [32] may be only applied into the personnel-position matching problem. The interval-valued scores proposed in reference [29] may be less than zero, and the considered factors for determine matching aspirations in reference [29] may be incomplete. In addition, it is known that bodies’ behaviors play an important role in TsM process. For this reason, this paper extended the application scope of IVIFSs to TsM field, and studied the TsM problem from bodies’ behaviors using a matching example of virtual reality technology.

II. PRELIMINARIES

A. IvIFN

Definition 1 [19]: Let $D$ be a domain of discourse, and $P$ be a power set of interval $[0, 1]$. Then, an IvIFS could be represented in $\tilde{V} = \{ v, t_v, f_v \mid v \in D \}$, where $t_v = [t_v^-, t_v^+]$ and $f_v = [f_v^-, f_v^+]$ represent interval-valued membership degree and non-membership degree respectively, satisfying $t_v \in P$, $t_v^+ + f_v^- \neq 1$.

Definition 2 [19]: If $h_v = [h_v^-, h_v^+] = 1 - t_v - f_v$, then $h_v$ is known as hesitancy degree.

From Definition 2 and the operation rule of interval numbers, we know that for any $v \in D$, its hesitancy degree relative to $\tilde{V}$ can be computed below:

$$h_v = [h_v^-, h_v^+] = [1 - t_v^-, f_v^+]$$

Evidently, $h_v \in P$, $\forall v \in D$. In special, if $h_v = 0$, then $\tilde{V}$ is degraded as a fuzzy set.

For later expedience, one element of IvIFS $\tilde{V}$ is taken out separately, which is noted as $\tilde{v} = (t_{\tilde{v}}, f_{\tilde{v}})$. Then, $\tilde{v} = (t_{\tilde{v}}, f_{\tilde{v}})$ is called an IvIFN.

Although some score functions of IvIFN are proposed, several minor errors may exist when comparing two IvIFNs. Furthermore, a normalized interval-valued score function of IvIFN, which belong to $[0, 1]$, will be defined.

Definition 3: Let $\tilde{v} = (t_{\tilde{v}}, f_{\tilde{v}})$ be an IvIFN, an interval-valued score function of $\tilde{v}$ is expressed as

$$\tilde{s}_{\tilde{v}} = (t_{\tilde{v}} + \alpha_{\tilde{v}}h_{\tilde{v}}) - (f_{\tilde{v}} + \beta_{\tilde{v}}h_{\tilde{v}}) - \gamma_{\tilde{v}}h_{\tilde{v}}$$

where $\alpha_{\tilde{v}}$, $\beta_{\tilde{v}}$, and $\gamma_{\tilde{v}}$ (0 $\leq$ $\alpha_{\tilde{v}}$, $\beta_{\tilde{v}}$, $\gamma_{\tilde{v}}$ $\leq$ 1) represent support ratio, opposition ratio, and abstention ratio of $h_{\tilde{v}}$ respectively, satisfying

$$\alpha_{\tilde{v}} + \beta_{\tilde{v}} + \gamma_{\tilde{v}} = 1$$

Remark 1: Ratios $\alpha_{\tilde{v}}$, $\beta_{\tilde{v}}$, and $\gamma_{\tilde{v}}$ could be determined on the basis of preferences of IvIFNs, which should reflect the result of TsM and will be displayed in Section 4.1.

From Eqs. (2) and (3), we have

$$\tilde{s}_{\tilde{v}} = (t_{\tilde{v}} - f_{\tilde{v}}) + (2\alpha_{\tilde{v}} - 1)h_{\tilde{v}}$$

It is easy to conclude that $\tilde{s}_{\tilde{v}} \subseteq [-1, 1]$. In order to eliminate the influence of negative interval-valued score functions when they are aggregated, the normalized interval-valued score function is introduced below:

Definition 4: Let $\tilde{v} = (t_{\tilde{v}}, f_{\tilde{v}})$ be an IvIFN, the normalized interval-valued score function can be expressed as

$$\tilde{\tilde{s}}_{\tilde{v}} = \left(\frac{\tilde{s}_{\tilde{v}} + [1, 1]}{2}\right)^x$$

where parameter $x \geq 0$. Obviously, $\tilde{\tilde{s}}_{\tilde{v}} \subseteq [0, 1]$.

B. TsM

The concept of TsM is provided in many literatures. This paper uses the following related notations. Let $X = \{X_1, \ldots, X_k, \ldots, X_m\}$ and $Y = \{Y_1, \ldots, Y_l, \ldots, Y_n\}$ be two independent body set, where $X_k (Y_l)$ signifies the kth body on side X (the lth body on side Y). Give this hypothesis $2 \leq m \leq n$, and set $M = \{1, \ldots, k, \ldots, m\}$, $N = \{1, \ldots, l, \ldots, n\}$.

Definition 5 [33]: Let $\Omega : X \cup Y \rightarrow X \cup Y$ be a one-one mapping. If mapping $\Omega$ satisfies the following properties:
i) $\Omega(X_i) \in Y$, ii) $\Omega(Y_i) \in X \cup \{Y\}$, iii) $\Omega(X_i) = Y_i$ iff $\Omega(Y_i) = X_i \forall X_k \in X, Y_l \in Y$, then $\Omega$ is called a TsM. Therefore, $\Omega(X_i) = Y_i$ means that $\Omega(X_i, Y_i)$ is a matching pair (MP), which indicates body $X_k$ is matched with body $Y_l$, $\Omega(Y_l) = Y_i$ means that $\Omega(Y_l, Y_i)$ is a single matching pair (SMP), which indicates body $Y_l$ is single.

Definition 6 [33]: For a TsM $\Omega : X \cup Y \rightarrow X \cup Y$, it also could be represented in $\Omega = \Omega_{MP} \cup \Omega_{SMP}$, where $\Omega_{MP}$ signifies MP set, $\Omega_{SMP}$ signifies SMP set.

III. TOPSIS BASED TsM DECISION FOR IvIFNs

A. TsM PROBLEM FOR IvIFNs CONSIDERING MATCHING WILLINGNESS

The following problem of TsM is involved. Let $\hat{V}^X = [\hat{v}_{kl}]_{m \times n}$ be IvIFN matrix of side $X$, where IvIFN $\hat{v}_{kl} = < [r^x_{kl}, t^x_{kl}], [r^x_{kl}, t^x_{kl}] >$. Thereinto, $[r^x_{kl}, t^x_{kl}]$ indicates the interval-valued satisfied degree of body $X_k$ towards body $Y_l$, and $[r^y_{kl}, t^y_{kl}]$ indicates the interval-valued unsatisfied degree of body $Y_l$ towards body $X_k$. Let $\hat{W} = [w_{kl}]_{m \times n}$ be matching willingness matrix between $X$ and $Y$. Therefore, $w_{kl}$ indicates the matching willingness between body $X_k$ and body $Y_l$. Let $\Omega^*$ be the “optimum” TsM.

Remark 2: In this paper, bodies’ behaviors are represented by the matching willingness. Matching willingness $w_{kl}$ could be computed on the basis of satisfied degrees of bodies $X_k$ and $Y_l$, which should satisfy the features of non-negativity and standardization. Its computation approach will be displayed in Section 4.2.

Motivated by the above statement, the problem displayed here is how to acquire “optimum” TsM $\Omega^*$ on the basis of IvIFN matrices $\hat{V}^X = [\hat{v}_{kl}]_{m \times n}$ and $\hat{V}^Y = [\hat{v}_{kl}]_{m \times n}$, matching willingness matrix $\hat{W} = [w_{kl}]_{m \times n}$. The research procedure for the above-mentioned problem is presented in Figure 1.

B. ESTABLISHMENT OF SCORE MATRICES

First, by using Eqs. (4) and (5), IvIFN matrices $\hat{V}^X = [\hat{v}_{kl}]_{m \times n}$ and $\hat{V}^Y = [\hat{v}_{kl}]_{m \times n}$ are converted to normalized interval-valued score matrices $\bar{s}_V^X = [\bar{s}^X_{kl}]_{m \times n} = [[s^X_{kl}, s^X_{kl}]]_{m \times n}$ and $\bar{s}_V^Y = [\bar{s}^Y_{kl}]_{m \times n} = [[s^Y_{kl}, s^Y_{kl}]]_{m \times n}$, where normalized interval-valued scores $\bar{s}^X_{kl} = [\bar{s}^X_{kl}, \bar{s}^X_{kl}]$ and $\bar{s}^Y_{kl} = [\bar{s}^Y_{kl}, \bar{s}^Y_{kl}]$ can be computed by (6) and (7), as shown at the bottom of the next page.

![Figure 1. Research procedure for the above-mentioned problem.](image-url)
Thereinto, $\alpha_{v}^* Y$ can be interpreted as the support ratio of $\Omega(X_k) = Y_1$, $\beta_{v}^* Y$ can be interpreted as the opposition ratio of $\Omega(X_k) = Y_1$, $\gamma_{v}^* Y$ can be interpreted as the neutral ratio of $\Omega(X_k) = Y_1$; $\theta_Y^*$ represents the optimism coefficient of body $Y_i$.

Furthermore, normalized interval-valued score matrices $\tilde{S}_v^{X} = [\tilde{s}_{v}^{X}]_{m \times n}$ and $\tilde{S}_v^{Y} = [\tilde{s}_{v}^{Y}]_{m \times n}$ is converted to score matrices $S_v^{X} = [s_{v}^{X}]_{m \times n}$ and $S_v^{Y} = [s_{v}^{Y}]_{m \times n}$, where scores $s_{v}^{X}$ and $s_{v}^{Y}$ is expressed by:

$$s_{v}^{X} = (1 - \theta_Y^*)s_{v}^{X} + \theta_Y^* s_{v}^{Y}$$

(14)

$$s_{v}^{Y} = (1 - \theta_Y^*)s_{v}^{Y} + \theta_Y^* s_{v}^{X}$$

(15)

Thereinto, $\theta_Y^*$ represents the composite optimism coefficient of bodies on side $X$, and $\theta_Y^*$ represents the composite optimism coefficient of bodies on side $Y$.

**C. COMPUTATION OF MATCHING WILLINGNESS**

In above analysis, matching willingness $X_S$ is unknown. For obtaining $w_{kl}$, an analysis is provided as follows.

On one hand, if absolute difference $|s_{v}^{X} - s_{v}^{Y}|$ gets larger and larger, then the difference of satisfied degree between body $X_k$ and body $Y_1$ also gets larger and larger. Hence, matching willingness $w_{kl}$ should get smaller and smaller, and vice versa. On the other hand, if $s_{v}^{X}$ (or $s_{v}^{Y}$) gets larger and larger, then the satisfied degree of body $X_k$ towards body $Y_1$ (or the satisfied degree of body $Y_1$ towards body $X_k$) also gets larger and larger. Hence, matching willingness $w_{kl}$ should get larger and larger, and vice versa. Therefore, matching willingness $w_{kl}$ is inverse proportion to $|s_{v}^{X} - s_{v}^{Y}|$, and is proportional to $s_{v}^{X}$ and $s_{v}^{Y}$. Furthermore, selection of $w_{kl}$ should enable the following total weighting score for all bodies of two sides (noted as $R_{X \rightarrow Y}$) greatest:

$$R_{X \rightarrow Y} = \sum_{k=1}^{m} \sum_{l=1}^{n} \frac{s_{v}^{X} s_{v}^{Y} w_{kl}}{s_{v}^{X} - s_{v}^{Y}}$$

(16)

It is noted that $|s_{v}^{X} - s_{v}^{Y}| = 0$ occurs in some cases. In this case, Eq. (16) has no meaning. For dealing with the case, we change $|s_{v}^{X} - s_{v}^{Y}|$ into $\eta \cdot |s_{v}^{X} - s_{v}^{Y}|$ where $\eta$ is given in advance, $\eta > 1$. Hence, Eq. (16) can be represented by:

$$R_{X \rightarrow Y} = \sum_{k=1}^{m} \sum_{l=1}^{n} \frac{s_{v}^{X} s_{v}^{Y} w_{kl}}{\eta |s_{v}^{X} - s_{v}^{Y}|}$$

(17)

Therefore, an optimum model (T-1) for obtaining $w_{kl}$ could be set up, i.e.,

$$\max R_{X \rightarrow Y} = \sum_{k=1}^{m} \sum_{l=1}^{n} \frac{s_{v}^{X} s_{v}^{Y} w_{kl}}{\eta |s_{v}^{X} - s_{v}^{Y}|},$$

subject to $\sum_{i=1}^{m} \sum_{l=1}^{n} w_{kl}^2 = 1, k \in M; w_{kl} \in [0, 1], k \in M, l \in N$

**Theorem 1:** The optimum solution of model (T-1) is expressed by the following:

$$w_{kl} = \frac{s_{v}^{X} s_{v}^{Y} w_{kl}}{\eta |s_{v}^{X} - s_{v}^{Y}|} \left( \sum_{k=1}^{m} \sum_{l=1}^{n} \frac{s_{v}^{X} s_{v}^{Y} w_{kl}}{s_{v}^{X} - s_{v}^{Y}} \right)$$

(18)

**Proof:** Let Lagrange function $L = \sum_{k=1}^{m} \sum_{l=1}^{n} \frac{s_{v}^{X} s_{v}^{Y} w_{kl}}{s_{v}^{X} - s_{v}^{Y}} + \lambda \left( \sum_{i=1}^{m} \sum_{l=1}^{n} w_{kl}^2 - 1 \right)$. Furthermore, let $\frac{\partial L}{\partial w_{kl}} = 0$ and $\frac{\partial L}{\partial \lambda} = 0$, then we have

$$\frac{s_{v}^{X} s_{v}^{Y} w_{kl}}{\eta |s_{v}^{X} - s_{v}^{Y}|} + 2\lambda w_{kl} = 0$$

(19)

$$\sum_{i=1}^{m} \sum_{l=1}^{n} w_{kl}^2 - 1 = 0$$

(20)
Take Eq. (19) into Eq. (20), then we find:

\[
\lambda = -\frac{1}{2} \sum_{k=1}^{m} \sum_{l=1}^{n} \frac{\left(\sum_{i=1}^{n} s_{x_{ij}} s_{y_{ij}} - \bar{s}_{x_{ij}} \bar{s}_{y_{ij}}\right)^2}{\eta \|s_{x_{ij}} - \bar{s}_{x_{ij}}\|^{2m} \|s_{y_{ij}} - \bar{s}_{y_{ij}}\|^{2n}}
\]  

(21)

Take Eq. (21) into Eq. (19), then we find:

\[
w_{kl}^s = \frac{w_{kl}^s}{\sum_{j=1}^{n} w_{kj}^s}
\]  

(22)

Using Eq. (23), matching willingness matrix \( W = [w_{kl}^s]_{m \times n} \) is built.

**D. CONSTRUCTION OF CLOSEDNESS DEGREE MATRICES**

In this subsection, the TOPSIS technology is adopted to construct the closedness degree matrices. On the basis of score matrices \( S_{x_Y} = [s_{x_{Y_{ij}}}]_{m \times n} \) and \( S_{y_Y} = [s_{y_{Y_{ij}}}]_{m \times n} \), and matching willingness matrix \( W = [w_{Y_{ij}}]_{m \times n} \), the weighting score matrices \( \bar{S}_{x_Y} = [\bar{s}_{x_{Y_{ij}}}]_{m \times n} \) and \( \bar{S}_{y_Y} = [\bar{s}_{y_{Y_{ij}}}]_{m \times n} \) are constructed, where \( \bar{s}_{x_{Y_{ij}}} \) and \( \bar{s}_{y_{Y_{ij}}} \) are expressed by

\[
\bar{s}_{x_{Y_{ij}}} = w_{Y_{ij}} s_{x_{Y_{ij}}} \\
\bar{s}_{y_{Y_{ij}}} = w_{Y_{ij}} s_{y_{Y_{ij}}}
\]  

(24)

(25)

Aiming at weighting score matrix \( \bar{S}_{x_Y} = [\bar{s}_{x_{Y_{ij}}}]_{m \times n} \), its specific form can be demonstrated by

\[
\begin{bmatrix}
Y_1 & Y_2 & \ldots & Y_n \\
X_1 & \bar{s}_{x_{Y_{11}}} & \bar{s}_{x_{Y_{12}}} & \ldots & \bar{s}_{x_{Y_{1n}}} \\
X_2 & \bar{s}_{x_{Y_{21}}} & \bar{s}_{x_{Y_{22}}} & \ldots & \bar{s}_{x_{Y_{2n}}} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
X_m & \bar{s}_{x_{Y_{m1}}} & \bar{s}_{x_{Y_{m2}}} & \ldots & \bar{s}_{x_{Y_{mn}}} 
\end{bmatrix}_{m \times n}
\]

Then, the positive ideal vector from \( X \) to \( Y \) can be obtained, where \( s^{xY} = (s_{1Y}^{xY}, s_{2Y}^{xY}, \ldots, s_{mY}^{xY})^T \). Here, the element \( s_{kY}^{xY} \) can be computed as

\[
s_{kY}^{xY} = \max_{i \in N} \left\{ \bar{s}_{x_{Y_{ij}}} \right\}
\]  

(26)

The negative ideal vector from \( X \) to \( Y \) can also be obtained, where \( s^{xY} = (s_{1Y}^{xY}, s_{2Y}^{xY}, \ldots, s_{mY}^{xY})^T \). Here, the element \( s_{kY}^{xY} \) can also be computed as

\[
s_{kY}^{xY} = \min_{i \in N} \left\{ \bar{s}_{x_{Y_{ij}}} \right\}
\]  

(27)

Moreover, the ideal distance matrix from \( X \) to \( Y \) is obtained, where \( D^{xY} = [d_{kl}^{xY}]_{m \times n} \). Here, \( d_{kl}^{xY} \) is computed as

\[
d_{kl}^{xY} = s_{kY}^{xY} - \bar{s}_{x_{Y_{ij}}}
\]  

(28)

The negative distance matrix from \( X \) to \( Y \) is also obtained, where \( D^{xY} = [d_{kl}^{xY}]_{m \times n} \). Here, \( d_{kl}^{xY} \) is computed as

\[
d_{kl}^{xY} = \bar{s}_{x_{Y_{ij}}} - s_{kY}^{xY}
\]  

(29)

Then, the closeness degree matrix from \( X \) to \( Y \) is constructed, where \( C^{xY} = [c_{kl}^{xY}]_{m \times n} \). Here, closeness degree \( c_{kl}^{xY} \) is computed as

\[
c_{kl}^{xY} = \frac{d_{kl}^{xY}}{d_{kl}^{xY} + d_{kl}^{xY}}
\]  

(30)

From Eqs. (26)-(30), it is known that the larger \( c_{kl}^{xY} \) is, the larger satisfied degree of \( X_k \) to \( Y_l \) will be.

In a similar way, aiming at weighting score matrix \( \bar{S}_{Y_{ij}} = [\bar{s}_{y_{Y_{ij}}}]_{m \times n} \), its specific form can be demonstrated by

\[
\begin{bmatrix}
Y_1 & Y_2 & \ldots & Y_n \\
X_1 & \bar{s}_{y_{Y_{11}}} & \bar{s}_{y_{Y_{12}}} & \ldots & \bar{s}_{y_{Y_{1n}}} \\
X_2 & \bar{s}_{y_{Y_{21}}} & \bar{s}_{y_{Y_{22}}} & \ldots & \bar{s}_{y_{Y_{2n}}} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
X_m & \bar{s}_{y_{Y_{m1}}} & \bar{s}_{y_{Y_{m2}}} & \ldots & \bar{s}_{y_{Y_{mn}}} 
\end{bmatrix}_{m \times n}
\]

Then, the positive ideal vector from \( Y \) to \( X \) is obtained, where \( s^{xY} = (s_{1Y}^{xY}, s_{2Y}^{xY}, \ldots, s_{mY}^{xY})^T \). Here, the element \( s_{kY}^{xY} \) can be computed as

\[
s_{kY}^{xY} = \max_{k \in M} \left\{ \bar{s}_{y_{Y_{ij}}} \right\}
\]  

(31)

The negative ideal vector from \( Y \) to \( X \) can also be obtained, where \( s^{xY} = (s_{1Y}^{xY}, s_{2Y}^{xY}, \ldots, s_{mY}^{xY})^T \). Here, the element \( s_{kY}^{xY} \) can also be computed as

\[
s_{kY}^{xY} = \min_{k \in M} \left\{ \bar{s}_{y_{Y_{ij}}} \right\}
\]  

(32)

Moreover, the ideal distance matrix from \( Y \) to \( X \) is obtained, where \( D^{xY} = [d_{kl}^{xY}]_{m \times n} \). Here, \( d_{kl}^{xY} \) is computed as

\[
d_{kl}^{xY} = s_{kY}^{xY} - \bar{s}_{y_{Y_{ij}}}
\]  

(33)

The negative distance matrix from \( Y \) to \( X \) is also obtained, where \( D^{xY} = [d_{kl}^{xY}]_{m \times n} \). Here, \( d_{kl}^{xY} \) is computed as

\[
d_{kl}^{xY} = \bar{s}_{y_{Y_{ij}}} - s_{kY}^{xY}
\]  

(34)

Then, the closeness degree matrix from \( Y \) to \( X \) is constructed, where \( C^{xY} = [c_{kl}^{xY}]_{m \times n} \). Here, closeness degree \( c_{kl}^{xY} \) is computed as

\[
c_{kl}^{xY} = \frac{d_{kl}^{xY}}{d_{kl}^{xY} + d_{kl}^{xY}}
\]  

(35)
From Eqs. (31)-(35), it is known that the larger $c_{kl}$ is, the larger satisfied degree of $Y_l$ to $X_k$ will be.

E. BUILDING OF TsM MODEL CONSIDERING Closeness DEGREES

A 0-1 variable $z_{kl}$ is given in the first place, where $z_{kl} = \begin{cases} 1, & \Omega(X_k) = Y_l \\ 0, & \Omega(X_k) = Y_j \end{cases}$. After that, a matching matrix $Z = [z_{kl}]_{m \times n}$ is established. On the basis of closeness degree matrices $C^X = [c_{kl}^X]_{m \times n}$ and $C^Y = [c_{kl}^Y]_{m \times n}$, and matching matrix $Z = [z_{kl}]_{m \times n}$, a TsM model will be developed. To maximize the closeness degrees, the following TsM model (T-2) under the TsM constraints is built, i.e.,

$$
\begin{align*}
\text{(T-2)} & \max T_{X_k} = \sum_{i=1}^{m} \sum_{l=1}^{n} c_{kl}^X z_{kl}, \quad k \in M \\
& \text{s.t.} \quad \sum_{l=1}^{n} z_{kl} = 1, \quad k \in M; \\
& \quad \sum_{k=1}^{m} z_{kl} \leq 1, \quad l \in N; \\
& \quad z_{kl} = 0 \text{ or } 1, \quad k \in M, \quad l \in N
\end{align*}
$$

F. SOLUTION OF TsM MODEL CONSIDERING Closeness DEGREES

If we consider that all the bodies are in the same position, then model (T-2) is converted to the one-goal TsM model (T-3):

$$
\begin{align*}
\text{(T-3)} & \max T = \sum_{k=1}^{m} \sum_{l=1}^{n} (c_{kl}^X + c_{kl}^Y) z_{kl} \\
& \text{s.t.} \quad \sum_{l=1}^{n} z_{kl} = 1, \quad k \in M; \\
& \quad \sum_{k=1}^{m} z_{kl} \leq 1, \quad l \in N; \\
& \quad z_{kl} = 0 \text{ or } 1, \quad k \in M, \quad l \in N
\end{align*}
$$

If we consider that all the bodies are not exactly equal, then the technique of linear weighting can be employed. Let $w_k^X$ and $w_l^Y$ be the weights for bodies $X_k$ and $Y_l$ respectively, such that $\sum_{k} w_k^X + \sum_{l} w_l^Y = 1$, then model (T-2) is turned into the one-goal TsM model (T-4):

$$
\begin{align*}
\text{(T-4)} & \max T = \sum_{k=1}^{m} \sum_{l=1}^{n} (w_k^X c_{kl}^X + w_l^Y c_{kl}^Y) z_{kl} \\
& \text{s.t.} \quad \sum_{l=1}^{n} z_{kl} = 1, \quad k \in M; \\
& \quad \sum_{k=1}^{m} z_{kl} \leq 1, \quad l \in N; \\
& \quad z_{kl} = 0 \text{ or } 1, \quad k \in M, \quad l \in N
\end{align*}
$$

Via solving Model (T-3) or (T-4), optimum TsM matrix $Z^* = [z_{kl}^*]_{m \times n}$ is acquired. On the basis of matrix $Z^* = [z_{kl}^*]_{m \times n}$, optimum TsM scheme $\Omega^*$ can be acquired.

G. PROCEDURE FOR TOPSIS BASED TsM

Above all, a procedure for TOPSIS based TsM for IvIFNs is put forward below:

Step 1: Convert IvIFN matrix $V^X = [V_{kl}^X]_{m \times n}$ into normalized interval-valued score matrix $\bar{S}_{V}^X = [\bar{s}_{kl}^X]_{m \times n}^X$ via using Eqs. (6) and (8); Convert IvIFN matrix $V^Y = [V_{kl}^Y]_{m \times n}$ into normalized interval-valued score matrix $\bar{S}_{V}^Y = [\bar{s}_{kl}^Y]_{m \times n}^Y$ via Eqs. (7) and (11).

Step 2: Convert normalized interval-valued score matrices $\bar{S}_{V}^X = [\bar{s}_{kl}^X]_{m \times n}^X$ and $\bar{S}_{V}^Y = [\bar{s}_{kl}^Y]_{m \times n}^Y$ into score matrices $S_{V}^X = [s_{kl}^X]_{m \times n}^X$ and $S_{V}^Y = [s_{kl}^Y]_{m \times n}^Y$ via Eqs. (14) and (15) respectively.

Step 3: Acquire matching willingness matrix $W = [w_{kl}^X]_{m \times n}^X$ via Eqs. (22) and (23).

Step 4: Construct the weighting score matrices $\bar{s}_{V}^X = [\bar{s}_{kl}^X]_{m \times n}^X$ and $\bar{s}_{V}^Y = [\bar{s}_{kl}^Y]_{m \times n}^Y$ on the basis of score matrices $S_{V}^X = [s_{kl}^X]_{m \times n}^X$ and $S_{V}^Y = [s_{kl}^Y]_{m \times n}^Y$, matching willingness matrix $W = [w_{kl}^X]_{m \times n}^X$ via Eqs. (24) and (25) respectively.

Step 5: Compute the positive and negative ideal vectors $s^X = (s_1^X, s_2^X, \ldots, s_m^X)$ and $s^Y = (s_1^Y, s_2^Y, \ldots, s_m^Y)$ on the basis of weighting score matrix $\bar{s}_{V}^X = [\bar{s}_{kl}^X]_{m \times n}^X$ via Eqs. (26) and (27) respectively.

Step 6: Compute the closeness degree matrix $C^X = [c_{kl}^X]_{m \times n}^X$ via Eqs. (28)-(30).

Step 7: Compute the positive and negative ideal vectors $s^Y = (s_1^Y, s_2^Y, \ldots, s_n^Y)$ and $s^Y = (s_1^Y, s_2^Y, \ldots, s_n^Y)$ on the basis of weighting score matrix $\bar{s}_{V}^Y = [\bar{s}_{kl}^Y]_{m \times n}^Y$ via Eqs. (31) and (32) respectively.

Step 8: Compute the closeness degree matrix $C^Y = [c_{kl}^Y]_{m \times n}^Y$ via Eqs. (33)-(35).

Step 9: Construct a TsM Model (T-2) on the basis of closeness degree matrices $C^X = [c_{kl}^X]_{m \times n}^X$ and $C^Y = [c_{kl}^Y]_{m \times n}^Y$, and matching matrix $Z = [z_{kl}]_{m \times n}$.

Step 10: Transform TsM Model (T-2) into TsM Model (T-3).

Step 11: Acquire optimum TsM scheme $\Omega^*$ via solving model (T-3).

IV. A MATCHING EXAMPLE OF VIRTUAL REALITY TECHNOLOGY

An instance of supply-demand matching for virtual reality technology is provided in the section. An intermediary company in nanochip mainly provides the intermediary services for virtual reality technology to small and medium-sized manufacturing enterprises. Initially, five demanders on side $X$ (i.e., investors $X_1, X_2, \ldots, X_5$) release demand and self-preference information through the intermediary company in order to obtain the new technology for virtual reality. After three weeks, the intermediary company have received the supply and self-preference information of six suppliers on side $Y$ (i.e., enterprises $Y_1, Y_2, \ldots, Y_6$). Moreover, investors $X_1, X_2, \ldots, X_5$ evaluate enterprises $Y_1, Y_2, \ldots, Y_6$ by mainly considering the technical level, service level and credibility, and provide an IvIFN matrix of side $X$ ($V^X = [V_{kl}^X]_{5 \times 6}$),
which is demonstrated by Table 1. Enterprises $Y_1$, $Y_2$, ..., $Y_6$ can evaluate investors $X_1$, $X_2$, ..., $X_5$ by mainly considering ease of implementation, credibility and the potential for cooperation, and provide an IvIFN matrix of side $Y$ ($\tilde{V}_Y = [\tilde{v}_{ykl}]_{5 \times 6}$), which is demonstrated by Table 2. Ultimately, the intermediary company needs to decide the optimum TsM $\Omega^*$ combined with the above-presented preference information of investors and enterprises.
TABLE 4. Normalized interval-valued score matrix \( \hat{S}_{XY} = \left[ \frac{s_{X1Y}}{v_{kl}}, \frac{s_{X2Y}}{v_{kl}}, ..., \frac{s_{XnY}}{v_{kl}} \right]_{5 \times 6} \) when \( \theta_Y = 0.6 \) and \( \chi = 1. \\

| \( X_1 \) | \( Y_1 \) | \( Y_2 \) | \( Y_3 \) | \( Y_4 \) | \( Y_5 \) | \( Y_6 \) |
|---|---|---|---|---|---|---|
| [0.4383,0.5192] | [0.4408,0.6360] | [0.4634,0.5442] | [0.4400,0.5600] | [0.4128,0.6105] | [0.3425,0.5870] |
| [0.4942,0.6110] | [0.3386,0.5854] | [0.4510,0.6102] | [0.4113,0.6102] | [0.3252,0.6094] | [0.4257,0.5453] |
| [0.3392,0.5842] | [0.3832,0.5847] | [0.4374,0.6354] | [0.4421,0.5633] | [0.3083,0.5167] | [0.3581,0.5979] |
| [0.4261,0.5852] | [0.4267,0.5853] | [0.4800,0.5600] | [0.3325,0.6183] | [0.5453,0.5840] | [0.3758,0.5784] |
| [0.4650,0.5450] | [0.4405,0.5601] | [0.4088,0.6098] | [0.4942,0.6110] | [0.4378,0.5959] | [0.4437,0.6365] |

TABLE 5. Score matrix \( S_{XY} = \left[ \frac{s_{X1Y}}{v_{kl}}, \frac{s_{X2Y}}{v_{kl}}, ..., \frac{s_{XnY}}{v_{kl}} \right]_{5 \times 6} \) when \( \theta_Y = 0.6. \\

| \( X_1 \) | \( Y_1 \) | \( Y_2 \) | \( Y_3 \) | \( Y_4 \) | \( Y_5 \) |
|---|---|---|---|---|---|
| 0.5856 | 0.4983 | 0.4873 | 0.5250 | 0.5256 | 0.4988 |
| 0.4981 | 0.5512 | 0.4599 | 0.5512 | 0.5253 | 0.5598 |
| 0.5856 | 0.5246 | 0.4896 | 0.4671 | 0.5850 | 0.5251 |
| 0.4773 | 0.5433 | 0.5156 | 0.5261 | 0.5156 | 0.5261 |
| 0.5160 | 0.4778 | 0.5522 | 0.5435 | 0.4912 | 0.4874 |

TABLE 6. Score matrix \( S_{XY} = \left[ \frac{s_{X1Y}}{v_{kl}}, \frac{s_{X2Y}}{v_{kl}}, ..., \frac{s_{XnY}}{v_{kl}} \right]_{5 \times 6} \) when \( \theta_Y = 0.6. \\

| \( X_1 \) | \( Y_1 \) | \( Y_2 \) | \( Y_3 \) | \( Y_4 \) | \( Y_5 \) |
|---|---|---|---|---|---|
| 0.4868 | 0.5579 | 0.5119 | 0.5120 | 0.5314 | 0.4892 |
| 0.5643 | 0.4867 | 0.5465 | 0.5306 | 0.4957 | 0.4975 |
| 0.4862 | 0.5041 | 0.5562 | 0.5148 | 0.4333 | 0.4791 |
| 0.5216 | 0.5219 | 0.5280 | 0.5040 | 0.5685 | 0.4974 |
| 0.5130 | 0.5123 | 0.5294 | 0.5643 | 0.5327 | 0.5594 |

A brief solution process for the TsM problem with IvIFNs is demonstrated below.

Step 1: Convert the IvIFN matrix \( \hat{V} = \left[ \frac{v_{kl}}{s_{kl}} \right]_{5 \times 6} \) into normalized interval-valued score matrix \( \hat{S}_{XY} = \left[ \frac{s_{X1Y}}{v_{kl}}, \frac{s_{X2Y}}{v_{kl}}, ..., \frac{s_{XnY}}{v_{kl}} \right]_{5 \times 6} \) via using Eqs. (6) and (8) when \( \theta_Y = 0.6 \) and \( \chi = 1. \)

Step 2: Convert normalized interval-valued score matrices \( \hat{S}_{XY} = \left[ \frac{s_{X1Y}}{v_{kl}}, \frac{s_{X2Y}}{v_{kl}}, ..., \frac{s_{XnY}}{v_{kl}} \right]_{5 \times 6} \) into score matrices \( S_{XY} = \left[ S_{X1Y}, S_{X2Y}, ..., S_{XnY} \right]_{5 \times 6} \) via using Eqs. (14) and (15) respectively when \( \theta_Y = 0.6 \) and \( \theta_Y = 0.6. \)

Step 3: Acquire matching willingness matrix \( W = \left[ w_{kl} \right]_{5 \times 6} \) via Eqs. (22) and (23), as demonstrated in Table 7.

Step 4. Construct the weighting score matrices \( \hat{S}_{X1Y} = \left[ \frac{s_{X1Y}}{v_{kl}} \right]_{5 \times 6} \) and \( \hat{S}_{X2Y} = \left[ \frac{s_{X2Y}}{v_{kl}} \right]_{5 \times 6} \) on the basis of score matrices \( S_{X1Y} = \left[ S_{X1Y} \right]_{5 \times 6} \) and \( S_{X2Y} = \left[ S_{X2Y} \right]_{5 \times 6} \), matching willingness matrix \( W = \left[ w_{kl} \right]_{5 \times 6} \) via Eqs. (24) and (25), respectively.

Step 5-6. Compute the positive and negative ideal vectors \( s^{XY} = \left( s_{1Y}, s_{2Y}, ..., s_{6Y} \right)^T \) and \( s^{XY} = \left( s_{1Y}, s_{2Y}, ..., s_{6Y} \right)^T \) on the basis of weighting score matrix \( \hat{S}_{XY} = \left[ \frac{s_{X1Y}}{v_{kl}}, \frac{s_{X2Y}}{v_{kl}}, ..., \frac{s_{XnY}}{v_{kl}} \right]_{5 \times 6} \) via Eqs. (26) and (27) respectively. Compute the closeness degree matrix \( C^{XY} = \left[ c^{XY}_{kl} \right]_{5 \times 6} \) via Eqs. (28)-(30).

Step 7-8. Compute the positive and negative ideal vectors \( s^{XY} = \left( s_{1Y}, s_{2Y}, ..., s_{6Y} \right)^T \) and \( s^{XY} = \left( s_{1Y}, s_{2Y}, ..., s_{6Y} \right)^T \) on the basis of weighting score matrix \( \hat{S}_{XY} = \left[ \frac{s_{X1Y}}{v_{kl}}, \frac{s_{X2Y}}{v_{kl}}, ..., \frac{s_{XnY}}{v_{kl}} \right]_{5 \times 6} \) via Eqs. (31) and (32) respectively. Compute the closeness degree matrix \( C^{XY} = \left[ c^{XY}_{kl} \right]_{5 \times 6} \) via Eqs. (33)-(35).

Step 9-10: Construct a TsM Model (T-2) on the basis of closeness degree matrices \( C^{XY} = \left[ c^{XY}_{kl} \right]_{5 \times 6} \) and \( C^{XY} = \left[ c^{XY}_{kl} \right]_{5 \times 6} \), and matching matrix \( Z = \left[ z_{kl} \right]_{5 \times 6} \). Transform TsM Model (T-2) into TsM Model (T-3):

\[
\begin{array}{l}
\text{max } T = \sum_{k=1}^{5} \sum_{l=1}^{6} (c^{XY}_{kl} + c^{XY}_{kl})z_{kl} \\
\text{s.t. } \sum_{k=1}^{5} z_{kl} = 1, k \in M; \\
\sum_{l=1}^{6} z_{kl} \leq 1, l \in N; \\
z_{kl} = 0 \text{ or } 1, k \in M, l \in N
\end{array}
\]

where matrix \( \Psi = \left[ c^{XY}_{kl} + c^{XY}_{kl} \right]_{5 \times 6} \) is depicted by Table 8.

Step 11: Solve Model (T-3), optimum TsM matrix \( Z = \left[ z_{kl} \right]_{5 \times 6} \) is acquired, as shown in Table 9.
TABLE 8. Coefficient matrix $\Psi = [c_{kl}^X + c_{kl}^Y]$ of $5 \times 6$.

|    | $Y_1$ | $Y_2$ | $Y_3$ | $Y_4$ | $Y_5$ | $Y_6$ |
|----|------|------|------|------|------|------|
| $X_1$ | 1.2093 | 1.3664 | 0 | 0.8239 | 1.5426 | 0.0388 |
| $X_2$ | 1.4969 | 0.8203 | 0.125 | 1.6589 | 0.9124 | 1.4013 |
| $X_3$ | 1.4302 | 1.2349 | 1.3968 | 0 | 0.527 | 0.5819 |
| $X_4$ | 0 | 1.8148 | 1.2145 | 0.6956 | 1.8341 | 0.8406 |
| $X_5$ | 0.7388 | 0 | 1.7633 | 2 | 0.7335 | 1.2468 |

TABLE 9. Optimum TsM matrix $Z = [z^*_kl]_{5 \times 6}$.

|    | $Y_1$ | $Y_2$ | $Y_3$ | $Y_4$ | $Y_5$ | $Y_6$ |
|----|------|------|------|------|------|------|
| $X_1$ | 0 | 0 | 0 | 0 | 1 | 0 |
| $X_2$ | 1 | 0 | 0 | 0 | 0 | 0 |
| $X_3$ | 0 | 0 | 1 | 0 | 0 | 0 |
| $X_4$ | 0 | 1 | 0 | 0 | 1 | 0 |
| $X_5$ | 0 | 0 | 0 | 1 | 0 | 0 |

TABLE 10. 11 cases for weights $w_X$ and $w_Y$.

| Case | weight $w_X$ | Case | weight $w_Y$ |
|------|--------------|------|--------------|
| Case I | $w_X = 1, w_Y = 0$ | Case II | $w_X = 0.9, w_Y = 0.1$ |
| Case III | $w_X = 0.8, w_Y = 0.2$ | Case IV | $w_X = 0.7, w_Y = 0.3$ |
| Case V | $w_X = 0.6, w_Y = 0.4$ | Case VI | $w_X = w_Y = 0.5$ |
| Case VII | $w_X = 0.4, w_Y = 0.6$ | Case VIII | $w_X = 0.3, w_Y = 0.7$ |
| Case IX | $w_X = 0.2, w_Y = 0.8$ | Case X | $w_X = 0.1, w_Y = 0.9$ |
| Case XI | $w_X = 0, w_Y = 1$ |

FIGURE 2. Relationship of $T$ from Case I to IV.

FIGURE 3. Relationship of $T$ from Case V to VII.

FIGURE 4. Relationship of $T$ from Case VIII to XI.

FIGURE 5. Relationship of $T$ from Case I to XI.

On the basis of matrix $Z = [z^*_kl]_{5 \times 6}$, optimum TsM scheme $\Omega^*$ can be acquired, i.e., $\Omega^* = \Omega^*_{MP} \cup \Omega^*_{SMP}$, where $\Omega^*_{MP} = \{(X_1, Y_5), (X_2, Y_1), (X_3, Y_3), (X_4, Y_5), (X_5, Y_4)\}$, $\Omega^*_{SMP} = \{(Y_6, 0)\}$. In other words, investor $X_1$ matches with enterprise $Y_5$, investor $X_2$ matches with enterprise $Y_1$, investor $X_3$ matches with enterprise $Y_3$, investor $X_4$ matches with enterprise $Y_5$, investor $X_5$ matches with enterprise $Y_4$; enterprise $Y_6$ is not matched.

Next, model 3 is discussed further. Let $c_{kl} = w_Xc_{kl}^X + w_Yc_{kl}^Y$, then 11 cases of $c_{kl}$ can be listed in Table 10. From Table 10, it is known that the above Table 9 corresponds to Case VI. Other cases will be discussed from the view of relationships among weights $w_X$ and $w_Y$, objective function $T$, synthetical closeness degrees $c_{kl}$ and matching value $z^*_{kl}$.

Figure 2 reveals relationship of the values of object function $T$ from Case I to IV. Figure 3 reveals relationship of the values of object function $T$ from Case V to VII. Figure 4 reveals relationship of the values of object function $T$ from Case VIII to XI.
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According to Figures 2-4, the relationship of the values of object function $T$ from Case I to XI can be determined, as displayed in Figure 5. From Figure 5, the values of object function $T$ decreases first and then increases to five.

Figure 6 reveals relationship of synthetical closeness degrees $c_{kl}$ from Case I to IV. Figure 7 reveals relationship of synthetical closeness degrees $c_{kl}$ from Case V to VII. Figure 8 reveals relationship of synthetical closeness degrees $c_{kl}$ from Case VIII to XI.

According to Figures 6-8, the relationship of synthetical closeness degrees $c_{kl}$ from Case I to XI can be determined, as depicted in Figure 9. From Figure 9, the synthetical closeness degrees $c_{kl}$ are different in many cases.

V. CONCLUSION

An interval-valued intuitionistic fuzzy two-sided matching decision-making approach is put forward, where bodies’ behaviors are involved. Bodies’ behaviors here are characterized by the matching willingness. The IvIFN matrix is converted into normalized interval-valued score matrices, and then into score matrices. Based on score matrices, the matching willingness can be obtained. Based on score matrices and matching willingness matrix, the weighting score matrices are constructed. Furthermore, according to the TOPSIS technology, the closeness degree matrices are computed. Then a TsM model based on closeness degree matrices and matching matrix can be developed. Optimum TsM scheme is acquired through model solution. An instance of supply-demand matching for virtual reality technology is used to verify the validity of the presented approach.

Compared with previous studies, this paper makes contribution to the following two areas: (1) the application of IvIFSs was extended into TsM field, which are often neglected in previous studies; (2) the normalized interval-valued score function was given, which can make full use of hesitancy and eliminate the influence of negative interval-valued scores; (3) the view is based on bodies’ behaviors, and thus the obtained TsM scheme can reflect the matching willingness of bodies; (4) this paper puts forward a decision theory and method for TsM with IvIFNs. Limitations of the present study can be summarized below: it only investigated the TsM problem with IvIFSs preferences; but the related theories of stable matching with IvIFSs and other intuitionistic fuzzy information are not studied.

Therefore, the two aspects can be further studied. For one thing, when bodies’ preferences are other intuitionistic fuzzy forms, this type of TsM problem is worthy of attention. For another thing, the related theory of stable matching under
interval-valued intuitionistic fuzzy environment should also be considered.

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