Jeans instability of inhomogeneous dusty plasma with polarization force, ionization and recombination

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Abstract. The self-gravitational Jeans instability has been studied in dusty plasma containing significant background of neutral pressure and recombination of ions and electrons on the dust surface. The full dynamics of charged dust grains, ions and neutral species are employed considering the electrons as Maxwellian. We have derived the general dispersion relation for collisional dusty plasma with ionization, recombination and polarization force. The general dispersion relation describes the effects of considered parameters which are solved in different dusty plasma situations. Further, the dispersion relation is solved numerically. The present work is applicable to understand the structure formation of interstellar molecular clouds in astrophysical plasma.

1. Introduction
Over the past decades, the field of dusty plasma has become a great deal of interest. The dusty plasmas are ubiquitous in astrophysical plasmas and in low temperature laboratory plasmas. In dusty plasmas, the low frequency dust acoustic wave mode has been studied by many researchers [1,2] in which the thermal pressure effects are provided by the Maxwellian distributed electrons and ions while inertia is provided by dust particles. However, in the molecular clouds, the presence of massive dust and neutral particles give rise to gravitational force which involves the collapse of molecular cloud. The instability mechanisms and dispersion characteristics of dust acoustic wave mode incorporating the effects of free energy sources such as equilibrium ion streaming, collisions with the ions and neutrals, recombination of electrons and ions on the dust grain surface have also been investigated extensively [3,4]. Ostrikov et al. [5] have studied the dust acoustic wave instabilities with ionization and recombination in collisional dusty plasma. Dust acoustic modes have been discussed by Kaw and Singh [6] in presence of recombination and ionization in collisional dusty plasma. The effect of recombination and ionization on dust acoustic instability has been investigated by Bal and Bose [7] in partially ionized dusty plasma with strong coupling limits.

In the study of dust acoustic waves and instabilities, the polarization of plasma ions produces the dust polarization force around the dust grain which significantly modifies the dust dynamics and therefore is an important phenomenon of research. The effect of dust polarization force has been widely discussed by Hamaguchi and Farouki [8,9] and Khrapak et al. [10]. Recently, Jain and Sharma [11] have examined the influence of polarization force on Jeans instability in degenerate and non-degenerate collisional dusty plasma. Therefore, looking to the importance of polarization force, ionization and recombination we have studied the Jeans instability of collisional inhomogeneous dusty plasma incorporating the ionization, recombination and polarization force in present work.
2. Linearized model equations

The model includes a four component unmagnetized self-gravitating partially ionized dusty plasma which consists Maxwellian distributed electrons and ions, negatively charged dust grains and neutral particles to study dust acoustic wave propagation. In the present work, the sufficient background of neutral particles is taken; therefore, for the description of neutral particles we have taken the full dynamics of neutrals. The propagation of dust acoustic waves in collisional self-gravitating dusty plasma is described by the following set of equations.

The linearized electron perturbed density is

\[ n_{ei} = \frac{e n_{e0}}{k_B T_{e0}} \phi \]  

(1)

where \( n_{ei} \) and \( n_{e0} \) is perturbed and equilibrium electron number density, \( T_{e0} \) is electron temperature and \( \phi \) stands for perturbed electrostatic potential.

The continuity and momentum equations for dust and neutrals are

\[ D_j n_{j1} + n_{j0} \nabla u_{j1} = 0 \quad (j = d, n) \]  

(2)

\[ D_{u_{j1}} = -\frac{\nabla p_{j1}}{m_{j1} n_{j0}} - \frac{q_j \phi}{m_j} (1 - \Gamma) - \nabla \psi_{G1} - \nu_{d1} (u_{d1} - u_{n1}) \]  

(3)

\[ D_{u_{n1}} = -\frac{\nabla p_{n1}}{m_{n1} n_{n0}} - \nabla \psi_{G1} - \nu_{d1} (u_{n1} - u_{d1}) \]  

(4)

The symbol \( u_{j1}, \ p_{j1}, m_{j1}, n_{j0}, \) and \( n_{j1} \) represent perturbed velocity, pressure, mass, and equilibrium and perturbed number density of \( j \) particle respectively. The collision frequency of dust with neutral and neutral with dust is given by \( \nu_{d1} \) and \( \nu_{ad} \) respectively. The polarization force and charge of dust is given by \( \Gamma \) and \( q_d \) respectively.

The linearized first order self-gravitational potential is

\[ \nabla^2 \psi_{G1} = 4\pi G (m_{n1} n_{d1} + m_{d1} n_{n1}) \]  

(5)

where \( \psi_{G1} \) expresses the self-gravitation of dust and neutral.

The perturbed ion momentum and continuity equations are

\[ D_j (u_{i0} + u_{i1}) = -\frac{\nabla p_{i1}}{m_{i0} n_{i0}} - \frac{e \nabla \phi}{m_i} - \nu_e u_{i1} \]  

(6)

\[ D_j n_{i1} + n_{i0} \nabla u_{i1} + u_{i0} \nabla n_{i1} = -\nu_{R} \left( \frac{n_{d1}}{n_{d0}} \right) n_{i0} \]  

(7)

The first order charge neutrality condition is

\[ e n_{i1} = e n_{e1} - q_n n_{d1} \]  

(8)

The equilibrium ion velocity is symbolized by \( \bar{u}_{i0} \), the collision frequencies of ion with neutral is by \( \nu_{ae} \) and the frequency of recombination of ions and electrons on the dust grains surface is by \( \nu_{R} \).

3. Dispersion relation and discussions

We now consider that all the perturbed parameters are proportional to \( \exp (i k \cdot r + \omega t) \) where \( k \) is the wavenumber and \( \omega \) is the perturbation frequency. Using this perturbation in equations (1) – (7) we obtained the perturbed densities \( n_{e1}, n_{i1}, n_{d1} \) for electron, ion and dust respectively which are further combined with first order quasineutrality condition (8) and using \( i \omega = \sigma \) we obtained the general dispersion relation for collisional dusty plasma as
\[ \sigma^4 + \frac{\nu_m}{\omega_m^2} (\omega_{id}^2 + \omega_{jn}^2) \sigma^3 + \left( k^2 v_m^2 - \omega_{id}^2 + k^2 v_{id}^2 - \omega_{jn}^2 + k^2 (1 - \Gamma) \right) c_D^2 \left( 1 + \beta \nu_k \nu_m / k^2 v_m^2 \right) (1 + \alpha)^{1/2} \sigma^2 + \frac{\nu_m}{\omega_m^2} \left( k^2 v_m^2 - \omega_{id}^2 \right) \left( 1 + \beta \nu_k \nu_m / k^2 v_m^2 \right) (1 + \alpha) \right) - \left( \omega_{id}^2 + \omega_{jn}^2 \right) \sigma \right) \\
+ \left( k^2 v_m^2 - \omega_{id}^2 \right) \left( k^2 v_{id}^2 + k^2 (1 - \Gamma) \right) c_D^2 \left( 1 + \beta \nu_k \nu_m / k^2 v_m^2 \right) (1 + \alpha)^{1/2} - k^2 v_m^2 \omega_{id}^2 = 0. \tag{9} \]

where \( \omega_{id} = (4 \pi G M_n n_{do})^{1/2} \), \( \omega_{jn} = (4 \pi G M_d n_{do})^{1/2} \), \( v_m = (T_n / m_n)^{1/2} \), \( v_{id} = (T_d / m_d)^{1/2} \), 
\( \beta = n_{do} / Z_d n_{do} \), \( v_o = (T_i / m_i)^{1/2} \), \( c_D^2 = z_o^2 (n_{do} T_{do} / m_n n_{do}) \) and \( \alpha = (n_{do} T_{do} / n_{io} T_{io}) \).

We have assumed that the ions are initially at rest (i.e. \( u_{io} = 0 \)) and the ion neutral collision frequency is much less than the ion thermal speed.

Equation (9) represents the modified radiative condensation mode in partially ionized dusty plasma. Equation (9) is modified due to self-gravitation, polarization force and collision frequencies of different species. In the case when \( \omega_{id} = \omega_{jn} = 0, \nu_{id} = 0, \Gamma = 0 \) equation (9) reduces to the dispersion relation obtained by Bal and Bose [7] in weakly coupled state.

The Jeans criterion of instability is obtained from the constant term of equation (9) which is given as

\[ \left( k^2 - \frac{\omega_{id}^2}{v_m^2} \right) \left( v_{id}^2 + (1 - \Gamma) c_D^2 \left( 1 + \beta \nu_k \nu_m / k^2 v_m^2 \right) (1 + \alpha)^{1/2} \right) < \omega_{id}^2 \tag{10} \]

The modified Jeans instability criterion for radiative dusty plasma is shown by above equation (10). The modification is due to the dust polarization force, ionization recombination effect; ion and neutral thermal velocity and dust thermal velocity. If we ignore the effects of neutral particles, polarization force and ionization recombination effect the condition (10) recovers with the traditional condition of gravitational instability as described by Chandrasekhar [12].

4. Numerical estimation

We now examine the effects of polarization force and dust neutral collision frequency on growth rate of Jeans instability numerically. For this purpose, we normalize the dispersion relation (9) by neutral Jeans frequency. Therefore, the normalized form of dispersion relation (9) can be written as

\[ \sigma^4 + v_{id}^* \left( 1 + \omega_{id}^* \right) \sigma^3 + \left( k^2 v_m^2 + k^2 v_{id}^2 - \omega_{id}^* - 1 + k^2 (1 - \Gamma) \right) \left( k^2 v_m^2 + \omega_{id}^* + \omega_{jn}^* \right) \left( 1 + \beta \nu_k \nu_m / k^2 v_m^2 \right) (1 + \alpha)^{1/2} - \omega_{id}^* \sigma^2 + \left( k^2 v_m^2 - 1 \right) \left( k^2 v_{id}^2 + k^2 (1 - \Gamma) \right) \left( k^2 v_m^2 \right) (1 + \alpha)^{1/2} - k^2 v_m^2 \omega_{id}^* = 0 \tag{34} \]

In above equation we have used the following non-dimensional quantities

\[ \sigma^* = \frac{\sigma}{\omega_{jn}}, \quad k^* = \frac{k c_D^*}{\omega_{jn}}, \quad v_{id}^* = \frac{v_{id}}{c_D}, \quad v_m^* = \frac{v_m}{c_D}, \quad v_R^* = \frac{v_R}{c_D}, \quad \omega_{id}^* = \frac{\omega_{id}}{c_D}, \quad v_{id}^* = \frac{v_{id}}{c_D}, \quad v_m^* = \frac{v_m}{c_D}, \quad v_R^* = \frac{v_R}{c_D}. \]

In order to, for numerical estimation of our results we have plotted the normalized growth rate of Jeans instability against the normalized wave number for different values of dust neutral collision frequency and dust polarization force (using equation (11)). The numerical results are shown in figure 1 and 2. In figure 1 the variation of growth rate has been presented for \( v_{id}^* = 0.0, 0.4 \) and 0.8. From figure we find that the peak value of growth rate of Jeans instability decreases by increasing the value of collision frequency and therefore the presence of dust neutral collisions moves the system towards stabilization. Further, in figure 2 the influence of polarization force has been shown on growth rate of instability. It is clear from figure that it is the value of the dust polarization force increases the growth rate also due to this thus it tries to destabilize the system.
Figure 1. The growth rate of gravitational instability with normalized wave number for different values of dust neutral collision frequency. The values of constant parameters are $v_{nd} = 0.2$, $v_{ns} = 0.3$, $\Gamma = 0.4$, $\alpha = 0.1$, $\nu_\alpha = 0.1$, $\beta = 0.1$, $v_\nu = 0.2$ and $\omega_{nd} = 0.2$.

Figure 2. The growth rate of gravitational instability with normalized wave number for different values of polarization force. The values of constant parameters are $v_{st} = 0.2$, $v_{st} = 0.3$, $\nu_{st} = 0.2$, $\alpha = 0.1$, $\nu_\alpha = 0.1$, $\beta = 0.1$, $v_{st} = 0.2$ and $\omega_{st} = 0.2$.

5. Conclusions
In this work, we have examined the Jeans instability of collisional dusty plasma with polarization force, recombination and ionization. The general dispersion relation is obtained using the normal mode analysis method which is further discussed analytically and numerically. It is found that dust neutral collision frequency stabilizes the system whereas dust polarization force has destabilizing effect on dusty plasma system.

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