Fundamental Comparative Study on Band-Gap Properties of 1-D Conventional Photonic Crystal and 1-D Function Photonic Crystal

Yungao Cai\textsuperscript{a1, *}, Yinghong Dong\textsuperscript{1}
\textsuperscript{1}College of Engineering Technology, Baoshan University, Baoshan Yunnan, 678000, China

Abstract. Photonic crystal is a kind of artificially material composed of two or more dielectrics arranged periodically, which can control the flow of photons due to the photonic band-gap. In this article, the band gap properties of the one-dimensional (1-D) conventional photonic crystal and the 1-D function photonic crystals are comparatively analyzed according to dispersion relation, periodicity and incident angle systematically. The results have some guiding significance for the 1-D photonic crystal and photonic crystal device fabrication.

1. Introduction
Since the concept of photonic crystal was proposed in 1987 by Yablonovitch [1] and John [2] respectively, it has many applications and has become one of the most important theses of the world because it can control the flow of photons due to the photonic band-gap. The author project group are concentrating on the 1-D conventional photonic crystal, and made some achievements, such as references [3-5]. In recent years, a kind of photonic cristal has been proposed, named function photonic cristal, which the refractive index is a function of the space position, this concept stems from the X.Y. Wu project group of Jilin University, China in 2012 [6-10].

In this article, the band gap properties of the 1-D conventional photonic crystal and the 1-D function photonic crystals are comparatively analyzed based on the preliminary work of the author project group and part referencs mainly from the X.Y. Wu project group systematically in order to guide the photonic crystal research and photonic crystal device fabrication better.

2. Comparative analysis and discussion
2.1. Difference in transfer matrix
As shown in figure 1, it’s schematic diagram of the 1-D function photonic crystal with the propagation of light; the author project group are concentrating on the 1-D conventional photonic crystal, the transfer matrix method had been used for the research process. According to references [3-5], the transfer matrix for the 1-D conventional photonic crystal could written as follow,

\[ \tilde{M} = \begin{bmatrix}
\cos \beta_2 \cos \beta_3 - \frac{p_2}{p_3} \sin \beta_2 \sin \beta_3 & -i \frac{p_2}{p_3} \cos \beta_2 \sin \beta_3 - i \frac{p_2}{p_3} \sin \beta_2 \cos \beta_3 \\
-i p_2 \sin \beta_2 \cos \beta_3 - i p_2 \cos \beta_2 \sin \beta_3 & \cos \beta_2 \cos \beta_3 - \frac{p_2}{p_3} \sin \beta_2 \sin \beta_3
\end{bmatrix} \]  

(1)
Here, $\beta_1 = \frac{2\pi n_1 b}{\lambda \cos \theta_1}$, $\beta_2 = \frac{2\pi n_2 b}{\lambda \cos \theta_2}$, $p_1 = n_1 \cos \theta_1$, $p_2 = n_2 \cos \theta_2$, $\theta_1$ and $\theta_2$ are the angles between the incident normal and the stratified direction of the two kinds of media with different refractive index in the basic periodic structure respectively.

According to the research achievements of the X.Y. Wu project group of Jilin University, China, the transfer matrix for the 1-D function photonic crystal could be written as follow [6-10],

$$M = M_1 M_2$$

$$= \begin{bmatrix}
\cos \delta_1 & -i \sin \delta_1 & \cos \delta_2 & -i \sin \delta_2 \\
- n_b(0) \frac{\varepsilon_0}{\mu_0} \cos \theta'_1 \sin \delta_1 & \frac{\varepsilon_0}{\mu_0} n_1(b) \cos \theta'_1 & - n_a(0) \frac{\varepsilon_0}{\mu_0} \cos \theta'_a \sin \delta_2 & \frac{\varepsilon_0}{\mu_0} n_2(a) \cos \theta'_a \\
\end{bmatrix}$$

Where,

$$\delta_b = \frac{w}{c} n_1(b)(\cos \theta'_1 b + \sin \theta'_1 \int_0^b \frac{dz}{\sqrt{(1 + k_0^2 n(z)^2 - 1)}})$$

Figure 1. Schematic diagram of the 1-D function photonic crystal.

2.2. Dispersion relation
The expression of the dispersion relation for the 1-D conventional photonic crystal are written as follow [3],

$$\cos kd = \cos \beta_1 \cos \beta_2 - \frac{1}{2} \left( \frac{n_2}{n_1} + \frac{n_1}{n_2} \right) \sin \beta_1 \sin \beta_2$$

For 1-D function photonic crystal, it’s written as follow [6]

$$\cos kd = \cos \delta_1 \cos \delta_2 - \frac{1}{2} \left( \frac{\eta_b}{\eta_a} + \frac{\eta_a}{\eta_b} \right) \sin \delta_1 \sin \delta_2$$

Here, $d = b + a$, $\beta = \frac{2\pi n_a n_b}{\lambda \left( \frac{w d}{2\pi c} \right)}$, $\eta_c = \sqrt{\frac{\varepsilon_0 - n^2}{\mu_0 b^2}} \sqrt{1 - \frac{n^2}{n_e^2} \sin^2 \theta'_0}$ ($x = a$ or $b$). 1-D conventional photonic crystal is a special case of the 1-D function photonic crystal. Since 2012, various types 1-D function photonic crystal has been studied by the X.Y. Wu project group [6], the type of refractivity are mainly include sinusoidal function, polygonal function and so on, the functional expressions are written as follow [6].
For sinusoidal function,

\[
\begin{align*}
    n_1(z) &= n_1(0) + A_1 \sin(z/b), \quad 0 \leq z \leq b \\
    n_2(z) &= n_2(b) + A_2 \sin(z-b/a), b \leq z \leq b + a
\end{align*}
\]

For polygonal function,

\[
\begin{align*}
    n_1(z) &= \begin{cases} 
    n_1(0) + \frac{2(m-1)n_1(0)z}{b}, & 0 \leq z \leq \frac{b}{2} \\
    n_2(b) + \frac{2(m-1)n_2(0)(b-z)}{b}, & \frac{b}{2} \leq z \leq b
\end{cases} \\
    n_2(z) &= \begin{cases} 
    n_2(0) + \frac{2(m-1)n_2(0)(z-b)}{a}, & b \leq z \leq \frac{b+a}{2} \\
    n_2(0) + \frac{2(m-1)n_2(0)(a+b-z)}{a}, & \frac{b+a}{2} \leq z \leq b + a
\end{cases}
\] (7)

The results shown that there will be bandgap in the disconnected region on the dispersion curve; and with the increase of frequency, the bandgap width decreases when \( kd = 3 \), but the bandgap increases when \( kd = 0 \) for the 1-D function photonic crystal [6]. But the bandgap width is constant with the increase of frequency for 1-D conventional photonic crystal [3].

2.3. Effect of periodicity on the band-gap property

Take the sinusoidal function as an example, as described in reference [7], the number of the period cycle were 4, 6, and 8, the relationship between transmittance and frequency were analyzed, figure 2 [4] and figure 3 [7] are the transmission spectra of 1-D conventional photonic crystal and 1-D function photonic crystal with different periodicity, as described in figure 2 and figure 3 in turn, no matter 1-D conventional photonic crystal or 1-D function photonic crystal, it shown that with the increase of the periodicity, the transmittance gradually decreases, and the transmittance in the bandgap will be kept unchangeable when the periodicity reach critical point [4-5].

![Figure 2](image_url)

**Figure 2.** Transmission spectra of the 1-D conventional photonic crystal with different periodicity.
2.4. Effect of incident angle on the band-gap property

In reference [4], when other parameters kept unchanged, and the incident angle were 0, $\frac{\pi}{18}$, $\frac{\pi}{6}$, $\frac{\pi}{3}$, and $\frac{4\pi}{9}$ for the 1-D conventional photonic crystal, the result shown that with the increase of the incidence angle, the bandgap width of the photonic crystal also increase as shown in figure 4, but the center position and the transmittance of the bangap remains unchanged. According to references [6-10], for the 1-D function photonic crystal, with the increase of the incidence angle, the bandgap width of the photonic crystal increase, this conclusion is in agreement with the 1-D conventional photonic crystal, but the transmittance of the bangap is getting lower, which is obviously different from the 1-D conventional photonic crystal.

3. Summary

Generally, fundamental study on band-gap properties of 1-D conventional photonic crystal and 1-D fuction photonic crystal1-D are contrasted based on part achievements of author project group and part references mainly from the X.Y. Wu project group according to expression of transfer matrix, dispersion relation, periodicity and incident angle. The conventional photonic crystal is a special case of the function photonic crystal, and the 1-D function photonic crystal has more, wider or narrower bandgap structures than the 1-D conventional photonic crystal. The results could have some guiding significance for the 1-D photonic crystal and specific photonic crystal device fabrication.
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