Switchable next-nearest-neighbor coupling for controlled two-qubit operations

Peng Zhao, Peng Xu, Dong Lan, Xinsheng Tan, Haifeng Yu and Yang Yu

Institute of Quantum Information and Technology, Nanjing University of Posts and Telecommunications, Nanjing, Jiangsu 210003, China
State Key Laboratory of Quantum Optics and Devices, Shanxi University, Taiyuan, 030006, China

(Dated: April 21, 2020)

In a superconducting quantum processor with nearest neighbor coupling, the dispersive interaction between adjacent qubits can result in an effective next-nearest-neighbor coupling for which the strength depends on the state of the intermediary qubit. Here, we theoretically explore the possibility of engineering this next-nearest-neighbor coupling for implementing controlled two-qubit operations where the intermediary qubit controls the operation on the next-nearest neighbor pair of qubits. Specially, in a system comprising two types of superconducting qubits with opposite-sign anharmonicity arranged in an -A-B-A- pattern, where the unwanted static ZZ coupling between adjacent qubits could be heavily suppressed, a switchable coupling between the next-nearest-neighbor qubits can be achieved via the intermediary qubit, for which the qubit state functions as an on/off switch for this coupling. Therefore, depending on the adopted activating scheme, various controlled two-qubit operations such as controlled-iSWAP gate can be realized, potentially enabling circuit depth reductions as to a standard decomposition approach for implementing generic quantum algorithms.

I. INTRODUCTION

Implementing a gate-based quantum processor relies on arrays of qubits coupled together, and in a quantum processor with superconducting circuits, nearest neighbor (NN) coupling via a linear circuits (e.g., capacitor) provides a native architecture for satisfying this requirement [1]. But in practice, aside form the dedicated designed NN coupling, next-nearest-neighbor (NNN) coupling can also present in the superconducting quantum processor via several different mechanisms [2, 3], such as unintended static direct or indirect capacitive/inductive coupling described as a two-body interaction between NNN qubits via an effective reactance, and effective quantum coupling between NNN qubits resulting from the dispersive interaction between NN qubits. In general, these NNN couplings are considered as unwanted spurious interaction between qubits, leading to the gate infidelities, thus various approaches have been proposed for suppressing these spurious interactions [4, 5].

However, at the same time, the NNN coupling could also be utilized as a dedicated channel for implementing non-trivial tasks [5]. In particular, the effective NNN coupling mediated by the intermediary qubit, could be explored for realizing native three-qubit gate without resorting to the decomposition approach that involves a series of single- and two-qubit gates, thus reducing circuit depth for quantum algorithms and making them potentially attractive for NISQ application [6]. This is enabled by the fact that this NNN coupling is essentially a native three-body interaction [7], acting as a natural resource for implementing three-qubit gate operations. In fact, various theoretical and experimental studies have previously explored this three-body interaction, but in the situation where the three-body interaction is commonly used as a two-body interaction for two-qubit operations by setting the intermediary qubit (treated as a bus coupler) in its ground state [7–10]. And since this effective NNN coupling resulting from dispersive NN coupling is enabled by a second order process, its coupling strength has a similar magnitude as for the residual two-body interaction between adjacent qubit, such as ZZ coupling [11, 12], which imposes a limiting factor on the performance of the native three-qubit gate.

In this work, we theoretically explore the possibility of engineering the (intermediary) qubit-mediated NNN coupling in a superconducting quantum processor for implementing controlled two-qubit operations where the intermediary qubit control the operation on the NNN qubits. We demonstrate that, by

FIG. 1: (a) Sketch of a three-qubit system with NN coupling. Dispersive interaction between adjacent qubits (denoted as the round arrows) can results in an effective NNN coupling (denoted as the double-headed arrows) (b) Circuit diagram of a chain of three superconducting qubits capacitively coupled to each other, where the $Q_1$ and $Q_2$ are transmon qubit [19] and C-shunted flux qubit [20–22], that can be modeled as a weakly anharmonic oscillator with negative and positive anharmonicity, respectively. (c) For the intermediary qubit $Q_2$ in $|0\rangle$ state, the NNN exchange interaction with strength $J_{1(3)}$ is enabled by a single path (denoted as the dashed lines) involved with the intermediate state $|10\rangle$. (d) For $Q_2$ in $|1\rangle$ state, the NNN interaction with strength $J_{1(1)}$ is enabled by two paths (denoted as the dashed lines and dash dotted lines, respectively), and each involves with an intermediate state, i.e., $|101\rangle$ or $|020\rangle$.

*Electronic address: shangniguo@sina.com
†Electronic address: meisen0103@163.com
coupling two-type superconducting qubits with opposite-sign anharmonicity in an -A-B-A- pattern, on the one hand, the unwanted ZZ coupling between adjacent qubits can be heavily suppressed [13, 14], thus breaking the limitation on the performance of potentially implemented native three-qubit gates, on the other hand, a switchable coupling between NNN qubits can be realized [15], where the intermediary qubit state functions as an on/off switch for this NNN coupling. Thus, depending on the activating scheme, this switchable NNN coupling could be used to realize various controlled two-qubit operations [16], and a case study shows that controlled-iSWAP (C-iSWAP) gate [17, 18] with intrinsic gate fidelity (excluding the decoherence error) in excess of 99.9% can be achieved in 50 ns.

II. SWITCHABLE NNN EXCHANGE COUPLING

To start let us consider a system of three superconducting qubits (labeled as \(Q_{1,2,3}\)) capacitively coupled to each other, as depicted in Fig. 1, that can be modeled by a chain of three weakly anharmonic oscillators with NN coupling, described by (hereafter \(\hbar = 1\))

\[
H = \sum_i \left[ \omega_i q_i^\dagger q_i + \frac{\alpha_i}{2} q_i^\dagger q_i q_i^\dagger q_i + \sum_j g_j (q_j^\dagger q_i + q_i q_j^\dagger) \right],
\]

where the subscript \(l = 1, 2, 3\) labels qubit \(Q_l\) with anharmonicity \(\alpha_l\) and bare qubit frequency \(\omega_i\), \(q_i(q_i^\dagger)\) is the associated annihilation (creation) operator, and \(g_j\) \((j = 1, 3)\) denotes the strength of the NN coupling between \(Q_j\) and \(Q_2\).

We now consider that the system operates in the dispersive regime where the qubit frequency detuning \((\Delta_j = \omega_j - \omega_2)\) is far larger than the NN coupling strength, i.e., \(|\Delta_j| \gg g_l\). Thus, by using the Schrieffer-Wolff transformation [23, 24] which removes the NN coupling in the Hamiltonian given in Eq. (1), one can obtain an effective block-diagonal Hamiltonian for the full system [25]. Truncated to the qubit levels, the effective Hamiltonian has the following form [25]

\[
H_{\text{eff}} = \frac{\omega_1}{2} + \frac{\omega_2}{2} + \frac{\omega_3}{2} + \frac{\Delta_1}{2} + \frac{\Delta_2}{2} + \frac{\Delta_3}{2} + J_Z XZX + YZY + J_I XIX + YIY + J_Z ZZZ + \frac{\pi}{2} \right.
\]

where \((X, Y, Z, 1)\) represent the Pauli operator and identity operator, and the order indexes the qubit number, \(\omega_l\) and \(\Delta_l\) denote dressed qubit frequency of \(Q_l\) and the strength of the ZZ coupling between \(Q_j\) and \(Q_2\), respectively. The last four terms represent the effective interaction between NNN qubits \((Q_1\) and \(Q_3\)).

In Eq. (2), the terms \(XZX+YZY\) and \(XIX+YIY\) result in a net virtual exchange interaction between \(Q_1\) and \(Q_3\), for which the value of the net strength depends on the state of \(Q_2\). From the view of the second order-perturbation theory, the physics behind this feature is that setting by the state of \(Q_2\), the virtual exchange interaction results from different contributions. As shown in Fig. 1(c), where the frequency of \(Q_2\) satisfies \(\tilde{\omega}_2 > \tilde{\omega}_1\), the effective interaction between \(|100\rangle\) and \(|010\rangle\) with strength \(J_{1(0)} = J_I - J_Z = g_l g_3 (\Delta_1 + \Delta_3)/(2\Delta_1 \Delta_3)\) is enabled by the path given as \(|100\rangle \rightarrow |010\rangle \rightarrow |001\rangle\). However, as shown in Fig. 1(d), for the effective interaction \(|110\rangle \leftrightarrow |011\rangle\) with strength \(J_{1(1)} = J_I + J_Z\) given as

\[
J_{1(1)} = \frac{g_1 g_3}{2} \left[ \frac{\Delta_1 + \Delta_2}{\Delta_1 (\Delta_1 - \alpha_2)} + \frac{\Delta_3 + \alpha_2}{\Delta_3 (\Delta_3 - \alpha_2)} \right],
\]

there are two paths given as \(|110\rangle \rightarrow |020\rangle (|101\rangle) \rightarrow |001\rangle\), and since \(\tilde{\omega}_2 > \tilde{\omega}_1\), the two paths contribute with opposite-sign strength, enabling a competition between the positive and the negative contribution. Thus, one may reasonably expect that by engineering the anharmonicity of \(Q_2\), the strength of the dispersive interactions \(J_{1(1)}\) can take value of 0 when the two competitive contributions destructively interfere, while \(J_{1(0)}\) is intact.

According to Eq. (3), Figure 2(a) shows the calculated \(J_{1(1)}\)
and $J_{1(0)}$ versus the $Q_2$ anharmonicity $\alpha_2$ for system parameters given as: $g_j / 2\pi = 45$ MHz, $\Delta_j / 2\pi = -500$ MHz, and $\alpha_j / 2\pi = -350$ MHz. One can find that when $\alpha_2 / 2\pi$ takes value of 500 MHz, i.e., $\alpha_2 = -\Delta_{1(3)}$, the NNN coupling strength $J_{1(1)}$ is zero for $Q_2$ in $|1\rangle$, while for $Q_2$ in $|0\rangle$, since the coupling strength $J_{1(0)}$ is independent on $\alpha_2$, the interaction is intact, thus allowing us to control the NNN coupling with a high on/off ratio, where the state of $Q_2$ functions as an on/off switch. Therefore, at the working point $\alpha_2 = -\Delta_{1(3)}$, the effective NNN coupling can be described by

$$H_{CXY} = J_{1(0)}[|0\rangle\langle 0|]_2 \otimes (|01\rangle\langle 1|)_{1,3} + |10\rangle\langle 0|)_{1,3}. \quad (4)$$

Similar result can also be obtained for the terms $ZZZ$ and $IZZ$ in Eq. (2) [25], which describe the dispersive ZZ coupling between $Q_1$ and $Q_3$ resulted from the virtual exchange interaction between qubit states and non-qubit states, for which the interaction also depends on the state of $Q_2$, thus enabling the ZZ interaction controlled by the state of $Q_2$.

### III. IMPLEMENTING THE CONTROLLED-iSWAP GATE

| Table I: System parameters used for implementing the C-iSWAP gate. |
|---------------------------------------------------------------|
| **Qubits** | $Q_1$ | $Q_2$ | $Q_3$ |
|----------------------------------|--------|--------|--------|
| Anharmonicity $\alpha_j / 2\pi$ (MHz) | $-350$ | $350$ | $-350$ |
| Idle frequency $\omega_i / 2\pi$ (GHz) | $5.15$ | $6.35$ | $5.30$ |
| Interaction frequency $\omega_j / 2\pi$ (GHz) | $\sim 6.00$ | $6.35$ | $\sim 6.00$ |
| NN coupling strength $g_j / 2\pi$ (MHz) | 45 | 45 |

Having shown the switchable two-qubit exchange coupling, we now turn to use it to demonstrate controlled two-qubit operations. From Eq. (4), it becomes clear that the switchable NNN coupling at the working point could be used to realize the C-iSWAP gate with an arbitrary swap angle $\theta$, i.e.,

$$U_{CXY}(\theta) = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \cos \theta & 0 & 0 & -i \sin \theta & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & -i \sin \theta & 0 & \cos \theta & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix} \quad (5)$$

which becomes a C-iSWAP gate for $\theta = \pi/2$. However, we note that since this effective coupling results from the dispersive NN coupling (second-order process), its coupling strength has a similar magnitude as for the residual dispersive coupling between adjacent qubits that are presented in the full system effective Hamiltonian $H_{eff}$, i.e., ZZ coupling terms $ZZI$ and $IZZ$ [11, 25] (while terms $IZZ$ and $ZZI$ are originated from fourth-order process [12, 25]). Hence, these residual interactions impose a limiting factor on the fidelity of the native C-iSWAP gate.

As demonstrated in the previous study, in order to suppress the unwanted ZZ coupling between adjacent qubits in the present system where qubits are coupled together directly via a capacitor, the anharmonicity of $Q_2$ should have a similar magnitude as for the two adjacent qubits $Q_1$ and $Q_3$. This means that the optimal working point for realizing C-iSWAP gate using the NNN coupling is $\alpha_2 = -\Delta_{1(3)} = -\alpha_{13}$, as shown in Fig. 2(b), where the calculated $J_{1(1)}$ versus the anharmonicity $\alpha_2$ and the qubit frequency detuning $\Delta_j$ with system parameters $g_j / 2\pi = 45$ MHz and $\alpha_j / 2\pi = -350$ MHz is presented according to the Eq. (3).

Based on the above analysis, in the following discussion, we show a case study of exploring the switchable NNN coupling in a three-qubit system with always-on interaction for implementing the C-iSWAP gate. During the implementation of the three-qubit gate, the frequency of the intermediary qubit $Q_2$ is fixed, and the frequencies of the two NN qubits $Q_1$ and $Q_3$ vary from the idle frequency point $\omega_i$ to the interaction frequency point $\omega_j$ according to a time-dependent function as shown in Fig. 2(c), where the hold time is defined as the time-interval between the midpoints of the ramps [25, 26]. The list of system parameters are tabulated in Table I. Before going further, it is important to note that the above analysis, as well as the optimal working point, is derived based on the analytical expression for $J_{1(1)}$ given in Eq. (3), that is valid under the
Therefore, as shown in Fig. 3(a), by initializing the system in eigenstate state \( |\Psi\rangle \), the population versus hold time for system initialized in eigenstate state \( |100\rangle \) and \( |110\rangle \) at the idle point, respectively. One can find that for Q2 prepared in \( |0\rangle \), the NNN exchange interaction is turned on, enabling an almost complete population swap between Q1 and Q3 ((001) ↔ (100)), while for Q2 prepared in \( |1\rangle \), the exchange interaction is turned off, thus there is no population swap between Q1 and Q3. Moreover, although operating in the quasi-dispersive regime, during the time evolution, the population in Q1 or Q3 leakage to Q2 can still be strongly suppressed, as shown in Figs. 3(c) and 3(d).

As already mentioned before, a direct application of the switchable NNN coupling demonstrated in Fig. 3 is the implementation of the \( U_{CXY}(\theta) \) gate given in Eq. (5). Here, for illustration purpose, we consider the implementation of C-iSWAP gate, i.e., \( U_{CXX}(\pi/2) \), that is realized with a hold time of 43.2 ns, as shown in Fig. 3. By preparing system in eight logical basis states (logical eigenstates at the idle point), Figure 4(a) shows the output basis state, exhibiting a good agreement with the expected result from the ideal C-iSWAP. To quantify the performance of the implemented C-iSWAP gate, we consider the average gate fidelity defined as

\[
F = \frac{|\text{Tr}(U U^\dagger)|^2}{|\text{Tr}(U)|^2},
\]

where \( U \) is the actual evolution operator truncated to the qubit levels. Up to single-qubit phase gates and a global phase \( e^{i\theta} \), we find that our gate has an intrinsic fidelity of \( F = 99.97\% \) for gate time in 50 ns. Aside from the control error, this high intrinsic gate fidelity is enabled by (i) the low leakage error, as shown in Fig. 4(b), where one can find that the leakage to non-qubit state is suppressed below \( 10^{-5} \). (ii) lower coherence phase error, that is caused by parasitic ZZ coupling between qubits, as tabulated in Table II, the accumulated phase resulting from the interaction between qubit state and non-qubit is suppressed below 0.02 rad. Furthermore, from Table II, one can find that the coherence phase accumulated in state \( |011\rangle \) and \( |110\rangle \) are smaller than 0.005 rad. Considering the rather strong NN coupling, these rather lower accumulated phases demonstrate that by coupling two-type qubits with opposite-sign anharmonicity together, the residual ZZ interaction is heavily suppressed, as shown in previous studies \([13, 14]\). And since the coupling between NNN qubits is enabled by second-order process, and the qubits have a considerably larger anharmonicity, the ZZ coupling between NNN qubits is also suppressed (note that as mentioned above, the virtual exchange interactions between qubit state and non-qubit state are dependent on the state of Q2 \([25]\), thus the accumulated phases in \( |101\rangle \) and \( |111\rangle \) are different). However, we note that aside from the leakage error, in the case of the two-qubit iSWAP gate \([27, 36, 37]\), this residual ZZ coupling between NNN qubits imposes a fundamental tradeoff between the fidelity of C-iSWAP gate and the gate time.

In order to find the optimal working point numerically, firstly, according to the expression given in Eq. (3), we estimate the time \( T \) for realizing a full swap when the system is initialized in state \( |001\rangle \), giving \( T \approx \pi/2, J_{1}(\theta) = 45 \text{ ns} \). Therefore, as shown in Fig. 5(a), by initializing the system in eigenstate state \( |011\rangle \) at the idle point, and varying the qubit frequencies according to the pulse shown in Fig. 2(c) with hold time of 45 ns, we numerically study the swap error defined as \( 1 - P_{011} (P_{011} \text{ denotes the population in } |011\rangle \) after the time evolution) versus \( \Delta_{11} \) and \( \Delta_{10} \) that are defined as the frequency offset with respect to the ideal interaction point, as shown in the inset of Fig. 2(c). In Fig. 3(a), the square indicates the working point where the two offset are equal, i.e., \( \Delta_{11} = \Delta_{10} \), thus preserving on-resonance condition for iSWAP gate, and meanwhile, the swap error is much smaller than other point in the diagonal of the parameter space, means that at this point, the NNN coupling is turned off for Q2 in \( |1\rangle \) state. For enabling a complete swap in our fixed coupled system that is initialized in state \( |001\rangle \), we further consider a small overshoot \( \Delta_{s} \) applied on \( |1\rangle \) state. As shown in Fig. 3(b), the horizontal dashed line depicts the optimal value of the overshoot for this purpose.

![Fig. 4: C-iSWAP gate implementation.](image-url)
parameters, we note that these successes are based on a rather strong NN coupling (although feasible with present technology, but is larger than the typical NN coupling commonly used in practice [4,31]) and the qubit frequency with larger tunable range. In practice, the strong always-on coupling in present work may make single qubit addressing infeasible. However, we find that with a smaller NN coupling with strength of 30 MHz [32], the intrinsic gate fidelity above 99% (99.9%) can still be achieved in 50 (100) ns [25]. Moreover, the present protocol could also be applied to the system with tunable NN coupling [5,33–37], thus removing the above mentioned constraints.

| TABLE II: Accumulated coherence phase caused by the parasitic ZZ coupling between qubits during the gate implementation. |
|---|---|---|---|---|
| Phase (° × 10° rad) | 011 | 01 | 110 | 11 |
| 0.40 | -17.28 | -3.67 | 8.67 |

IV. CONCLUSION

In summary, we employ a scalable chain of nearest-neighbor-coupled superconducting qubit system comprising two-type superconducting qubits with opposite-sign anharmonicity to realize switchable coupling between next-nearest-neighboring (NNN) qubits. This switchable coupling is controlled by the state of the intermediary qubit, thus potentially enabling the implementation of various native controlled two-qubit operations. With realistic parameters, we show that it is possible to realize a C-iSWAP gate with an intrinsic average fidelity of 99.9% in 50 ns. These native implementations of three-qubit gates may find useful application for reducing circuit depth of NISQ algorithms [6] and for performing quantum simulations [38, 39].

Acknowledgments

This work was supported by the NKRDP of China (Grant No.2016YFA0301802), the NSFC (Grants No.11890704, No.11474152, No.61521001, and No.11847050), and the Key R&D Program of Guangdong Province (Grant No.2018B030326001). P. Xu was also supported by the Young fund of Jiangsu Natural Science Foundation of China (Grant No.BK20180750).

Appendix A: effective Hamiltonian for qubit system

As described in the main text, we consider a linear chain of three superconducting qubits with nearest-neighbor coupling, and here each qubit is treated as an ideal two-level system, thus described by the Hamiltonian $H = H_0 + V$ with $H_0 = \sum_{j=1}^{3} \tilde{\omega}_j \sigma_j^z / 2$ and

$$V = g_1 (\sigma_1^+ \sigma_2^- + \sigma_1^- \sigma_2^+) + g_3 (\sigma_2^+ \sigma_3^- + \sigma_2^- \sigma_3^+), \quad (A1)$$

where $\sigma_{\pm}^j$ denote the Pauli operators associated with the $j$th qubit labeled as $Q_j$ with bare qubit frequency $\tilde{\omega}_j$, and $g_1 (g_3)$ represents the coupling strength between nearest neighbor qubits, i.e., $Q_1 (Q_3)$ and $Q_2$.

We now turn to derive an effective Hamiltonian for this three-qubit system, and start from the original Hamiltonian $H$, which is composed of an unperturbed part $H_0$ with known eigenvalues and eigenstates and a small perturbation part $V$. We consider that the three-qubit system operates in the dispersive regime, where the detuning between nearest-neighbor qubit pair is larger than the coupling strengths between them, thus having $|\Delta_1| = |\tilde{\omega}_1 - \tilde{\omega}_2| \gg g_1$, and $|\Delta_3| = |\tilde{\omega}_3 - \tilde{\omega}_2| \gg g_3$.

For system operated in the dispersive coupling regime, we can eliminate the direct qubit-qubit coupling $V$ via a unitary transformation [23]

$$H_{\text{eff}} = \exp(-X)H\exp(X), \quad (A2)$$

where $X$ is chosen such that the direct coupling between the nearest-neighboring qubit pairs in the transformed Hamiltonian disappear. By choosing

$$X = \frac{g_1}{\Delta_1} (\sigma_1^+ \sigma_2^- - \sigma_1^- \sigma_2^+) + \frac{g_3}{\Delta_3} (\sigma_2^+ \sigma_3^- - \sigma_2^- \sigma_3^+), \quad (A3)$$

one can approve that it satisfies $[H_0, X] = -V$. Expanding to the second order of the small parameters ($\frac{g_1}{\Delta_1}, \frac{g_3}{\Delta_3}$), gives

$$H_{\text{eff}} = H_0 + \frac{1}{2} [V, X] + O(\lambda^3) \approx \sum_{j=1}^{3} \frac{\omega_j}{2} \sigma_j^z + J (\sigma_1^+ \sigma_3^- + \sigma_1^- \sigma_3^+) \sigma_2^z, \quad (A4)$$

with

$$\omega_1 = \tilde{\omega}_1 + \frac{g_1^2}{\Delta_1},$$

$$\omega_2 = \tilde{\omega}_2 - \left( \frac{g_1^2}{\Delta_1} + \frac{g_3^2}{\Delta_3} \right),$$

$$\omega_3 = \tilde{\omega}_3 + \frac{g_3^2}{\Delta_3},$$

$$J = -\frac{g_1 g_3}{2} \left( \frac{1}{\Delta_1} + \frac{1}{\Delta_3} \right),$$

where $\omega_j$ are the dressed qubit frequencies of $Q_j$, and $J$ is the effective three-qubit interaction strength (next-nearest-neighbor coupling) given as $J = -g_1 g_3 (\Delta_1 + \Delta_3)/(2\Delta_1 \Delta_3)$. From Eq. (A4), one can find that the magnitude of the dispersive XY interaction between $Q_1$ and $Q_3$ is independent on the state of $Q_2$, while the sign of the interaction is set by the $Q_2$ state, i.e., for $Q_2$ in $|0\rangle$ or $|1\rangle$, the value of the interaction strength has an opposite sign. Thus, the effective interaction
between $Q_1$ and $Q_2$ is in fact a three-body interaction.

We note that the original Hamiltonian $H$ is written in the usual bare basis of uncoupled system eigenstates, i.e., eigenstates of the unperturbed Hamiltonian $H_0$, but the derived effective Hamiltonian $H_{\text{eff}}$ given in Eq. (A4) is written in the transformed bare basis defined by the unitary transformation. In fact, for fixed coupled system, quantum information processing is commonly performed in the transformed basis [3, 26], i.e., eigenstates of the idle system Hamiltonian $H$, where qubits are all dispersively coupled to each other, i.e., the strength of the direct or indirect coupling between arbitrary pair of qubits are far smaller than the frequency detuning of the coupled qubits. In this way, at the idle point and in the interaction picture, the system states suffer no dynamics evolution. Therefore, throughout this work, we take all these factors into consideration, implicitly.

Appendix B: effective Hamiltonian for qubit system with higher energy levels

Since the superconducting qubit is naturally a multi-level system, in particular for qubits with weakly anharmonicity such as the transmon qubit and the C-shunted flux qubit, the higher energy levels of qubits have non-negligible effect on the effective coupling derived in the above section where superconducting qubits are treated as a ideal two-level system [19, 22]. In the following discussion, we will study the effect of the higher energy levels of qubits on the dispersive next-nearest neighbor coupling.

For a system consisting of three superconducting qubits (labeled as $Q_{1,2,3}$) as described in the main text, they can be modeled by a chain of three weakly anharmonic oscillators with nearest neighbor coupling [19, 22]. Thus, the Hamiltonian of this system can be described by $H = H_0 + V$, with

$$H_0 = \sum_{l=1}^{3} \left[ \tilde{\omega}_l q_l^2 + \frac{\alpha_l}{2} q_l^4 \right]$$

$$V = \left[ g_1 (q_1^2 q_2 + q_1 q_2^2) + g_3 (q_1^2 q_2 + q_3 q_2^2) \right],$$

where the subscript $l = 1, 2, 3$ labels superconducting qubit $Q_l$ with anharmonicity $\alpha_l$ and bare qubit frequency $\tilde{\omega}_l$, $q_l^4$ is the associated annihilation (creation) operator truncated to the lowest four-level (labeled as $\{0\}, \{1\}, \{2\}, \{3\}$)), and $g_j$ ($j = 1, 3$) denotes the strength of the coupling between adjacent qubits, i.e., $Q_j$ and $Q_2$. Again, in the following discussion, we consider that the system operates in the dispersive regime where the qubit frequency detuning ($\Delta_j = \tilde{\omega}_j - \tilde{\omega}_2$) is far larger than the NN coupling strength, i.e., $|\Delta_j| \gg g_j$.

1. Schrieffer-Wolff transformation

To derive an effective Hamiltonian for the three-qubit system, we turn to block-diagonalize the original system Hamiltonian $H = H_0 + \lambda V$, where $\lambda$ is introduced to show the orders in the perturbation expansion, and would be set to 1 after the calculations, thus the nearest neighbor coupling between qubits is eliminated and the next-nearest neighbor qubits are directly coupled to each other. By projecting the system onto the zero-excitation and one-excitation subspace of $Q_2$, we can further derive an effective Hamiltonian for the next-nearest neighbor qubits with the intermediary subspace in its ground state $|0\rangle$ or excited state $|1\rangle$. This is achieved by using the Schrieffer-Wolff transformation $[23, 24]$

$$H_{\text{eff}} = A^\dagger H A$$

$$A = e^{-iS}, \quad S = \sum_{n=1}^{\infty} S^{(n)} \lambda^n$$

Following the methods introduced in Ref.[24], the effective block-diagonal Hamiltonian for the three-qubit system has the following form

$$H_{\text{eff}} = \begin{pmatrix}
0 & 0 & 0 & \cdots \\
0 & H_1 & 0 & \cdots \\
0 & 0 & H_2 & \cdots \\
0 & 0 & 0 & H_3 \\
& & & \\
& & & \\
& & & \\
& & & \\
& & & \\
& & & \\
\end{pmatrix};$$

where $H_n$ ($n = 0, 1, 2, 3, ...$) denotes the effective Hamiltonian for the three-qubit system projected onto the n-excitation subspace of $Q_2$. Consequently, $H_0$ corresponds to the effective Hamiltonian projected onto the zero-excitation subspace of $Q_2$, i.e., describing the effective Hamiltonian for the next-nearest neighbor qubits with $Q_2$ in state $|0\rangle$. Truncated to the first three energy levels of qubits, i.e. operating with the basis $\{\{000\}, \{001\}, \{100\}, \{101\}, \{002\}, \{200\}\}$, $H_0$ reads

$$H_0 = \begin{pmatrix}
0 & 0 & 0 & \cdots \\
0 & \omega'_2 & J_{1(0)} & 0 & 0 & 0 \\
0 & 0 & \omega'_3 + \omega'_1 & J_{2(0), t} & J_{2(0), t} & 0 & 0 \\
0 & 0 & 0 & 2\omega'_3 + \alpha_3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2\omega'_1 + \alpha_1 \\
& & & & & & \\
& & & & & & \\
& & & & & & \\
& & & & & & \\
& & & & & & \\
\end{pmatrix}$$

where $\omega'_2 = \tilde{\omega}_2 + g_2^2 / \Delta_j$ ($j = 1, 3$) denotes the dressed transition frequency of $Q_j$, and $\omega'_1 = \tilde{\omega}_1 + g_2^2 / (\Delta_j + \alpha_j)$ is defined as an effective dressed qubit frequency, to which the coupling involved with higher energy levels of qubits contributes with an additional term $g_j^2 / (\Delta_j + \alpha_j)$.

$$J_{1(0)} = \frac{g_1 g_3 (\Delta_1 + \Delta_3)}{2 \Delta_1 \Delta_3}$$

represents the strength of coupling within one-excitation manifold $\{001\}, \{100\}$ of the NNN qubits, and

$$J_{2(0), t} = \frac{\sqrt{2} g_1 g_3 (\Delta_1 + \Delta_3 + \alpha_3)}{2 \Delta_1 (\Delta_3 + \alpha_3)}$$

$J_{2(0), t} \equiv J_{2(0), t} (1 \leftrightarrow 3)$

corresponds to the strength of coupling within the two-excitation manifold of NNN qubits $\{101\}, \{200\}, \{002\}$, i.e,
the interaction $|101\rangle \leftrightarrow |200\rangle$ and the interaction $|101\rangle \leftrightarrow |002\rangle$.

While $H_1$ represents the effective Hamiltonian projected onto the one-excitation subspace of $Q_2$, i.e., describing the effective Hamiltonian for the nearest-neighbor qubits with $Q_2$ in state $|1\rangle$. Then, truncated to the first three energy levels of qubits, i.e., operating with basis $\{ |010\rangle, |011\rangle, |110\rangle, |111\rangle, |012\rangle, |210\rangle \}$, $H_1$ is given as

$$H_1 = \begin{pmatrix}
E_{010} & 0 & 0 & 0 & 0 & 0 \\
0 & E_{011} & J_{1(I)} & 0 & 0 & 0 \\
0 & J_{1(I)} & E_{110} & 0 & 0 & 0 \\
0 & 0 & 0 & E_{111} & J_{2(1),I} & J_{2(1),II} \\
0 & 0 & 0 & J_{2(1),I} & E_{012} & 0 \\
0 & 0 & 0 & 0 & E_{210} & 0
\end{pmatrix}$$

(B7)

with

$$E_{010} = \omega_2',$$
$$E_{011} = \omega_2' + \omega_3' + \zeta_3, $$
$$E_{110} = \omega_2' + \omega_3' + \zeta_3' + \xi_1', $$
$$E_{111} = \omega_2' + \omega_3' + \zeta_3' + \zeta_3' + \xi_1', $$
$$E_{112} = \omega_2' + 2\omega_4' + \alpha_3 + \delta_1, $$
$$E_{210} = \omega_2' + 2\omega_4' + \alpha_3 + \delta_1,$$

$$J_{2(1),I} = \sqrt{g_1g_3} \frac{(2\Delta_1(\Delta_2 - \alpha_2) + (3\Delta_1^2 - \alpha_2^2)(\Delta_3 - \alpha_3 + \Delta_3) + (\alpha_2 + \Delta_1))(\alpha_3 - \Delta_3)}{2\Delta_1(\Delta_2 - \alpha_2)(\Delta_3 + \alpha_3)(\Delta_3 - \alpha_3 + \Delta_3)}.$$

From the above result, we can find that: (i) when the anharmonicity of $Q_2$ takes value of $\alpha_2 = 0$, i.e., $Q_2$ is linear resonator, the expression of $J_{1(I)}$ and $J_{1(I),II}$ given above will be reduced to $J_{1(0)}$ and $J_{2(0),II}$, respectively, agreeing to the well known fact that the linear-bus mediated coupling between two qubits is a two-body interaction between qubits. (ii) While $\alpha_2 \to \infty$, $Q_2$ can be safely treated as an ideal two-level system, thus $J_{1(1)} = -J_{1(0)}$ is obtained, as the result shown in Appendix. A.

2. Effective three-qubit Hamiltonian

For $Q_2$ in its ground state $|0\rangle$, the effective interaction among two-excitation manifold of the pair of nearest-neighborng qubits $Q_1$ and $Q_3$, i.e., interaction between $|101\rangle$ and $|002\rangle$ ($|200\rangle$) as shown in Fig. 5(a), for which the strength is given in Eq. (B6), causing the ZZ interaction between $Q_1$ and $Q_3$ with strength given as

$$\zeta_{101} = \frac{J_{2(0),I}^2}{\Delta - \alpha_3} - \frac{J_{2(0),II}^2}{\Delta + \alpha_1},$$

(B12)

where $\Delta = \omega_1 - \omega_3$. While for $Q_2$ in its excited state $|1\rangle$, the strength of the ZZ coupling (resulting from the interaction

where $\omega_2' = \omega_2 - g_2^2/\Delta_1 - g_3^2/\Delta_3$ denotes the dressed transition frequency of $Q_2$, $\zeta_i = 2g_i(\alpha_i + \alpha_2)/([\Delta_j - \alpha_2])$ represents the parasitic ZZ interaction between adjacent qubits $Q_j$ ($j = 1, 3$) and $Q_2$, which results from the interaction among higher energy levels of qubits, and

$$\delta_j = \frac{g_2^2(5\alpha_j + \Delta_j + 3\alpha_2)}{(2\alpha_j + \Delta_j)(\Delta_j + \alpha_j - \omega_2)} - \frac{g_2^2}{\Delta_j},$$

(B9)

represents frequency shift resulting from the interaction among higher energy levels of qubits. $J_{1(I)}$ describes the strength of coupling within the one-excitation manifold of nearest-neighbor qubits $\{ |110\rangle, |001\rangle \}$, and is given as

$$J_{1(I)} = \frac{g_1g_3}{2} \frac{\Delta_1 + \alpha_2}{\Delta_1(\Delta_1 - \alpha_2) + \Delta_3 + \alpha_2} \frac{\Delta_3 - \alpha_2}{\Delta_3(\Delta_3 - \alpha_2)}.$$

(B10)

while $J_{2(1),II}$ corresponds to the strength of coupling within the two-excitation manifold of nearest-neighbor qubits $\{ |111\rangle, |012\rangle, |210\rangle \}$, i.e., the interaction $|111\rangle \leftrightarrow |012\rangle$ and the interaction $|111\rangle \leftrightarrow |210\rangle$, is given as

$$J_{2(1),II} = J_{2(1),I}(1 \leftrightarrow 3).$$

(B11)

between $|012\rangle$ ($|210\rangle$) and $|111\rangle$, as shown in Fig. 5(b)) is

$$\zeta_{111} = \frac{J_{2(1),I}}{\Delta - \alpha_3 + \zeta_1' + \zeta_3' - \delta_3 - \Delta + \alpha_1 - \Delta_1 - \zeta_3' + \delta_1},$$

(B13)

Truncated to the qubit levels, the effective Hamiltonian of the full system has the following approximate form

$$H_{\text{eff}} = \omega_1 Z_{II} + \omega_2 Z_{II} + \omega_3 Z_{11} + \omega_4 Z_{11} + \zeta_1 Z_{II} + \zeta_2 Z_{11} + \zeta_3 Z_{11} + \zeta_4 Z_{11} + \zeta_5 Z_{21} + \zeta_6 Z_{21} + \zeta_7 Z_{21} + \zeta_8 Z_{21},$$

(B14)

where $(X, Y, Z, I)$ represent the Pauli operator and identity operator, and the order indexes the qubit number, $\omega_l$ denotes dressed qubit frequency given as

$$\omega_1 = \omega_1' + \frac{\zeta_1'}{2} + \frac{\zeta_{111} + \zeta_{101}}{4},$$
$$\omega_2 = \omega_2' + \frac{\zeta_2'}{2} + \frac{\zeta_{111} - \zeta_{101}}{4},$$
$$\omega_3 = \omega_3' + \frac{\zeta_3'}{2} + \frac{\zeta_{111} + \zeta_{101}}{4},$$

(B15)
\( \tilde{J}_j \) represents the strength of ZZ coupling between adjacent qubits, i.e., \( Q_{1(3)} \) and \( Q_2 \), given as

\[
\tilde{J}_1 = \frac{\tilde{J}_1}{2} + \tilde{J}_{111} - \tilde{J}_{101}, \quad \tilde{J}_4 = \frac{\tilde{J}_1}{2} + \tilde{J}_{111} - \tilde{J}_{101}.
\]

Taking

\[
\tilde{J}_Z = \frac{J_{1(4)} - J_{1(0)}}{2}, \quad J_I = \frac{J_{1(4)} + J_{1(0)}}{2},
\]

we recover the effective Hamiltonian of Eq. (2) of the main text.

In Eq. (B14), the term associated with \( XIX + YIY \) causes the excitation to be swapped between the next-nearest-neighboring qubits (iSWAP), and to which the term associated with \( XZX + YZY \) contributes with a swap rate for which the sign depends on the state of \( Q_2 \). The terms \( ZIZ \) and \( ZZZ \) correspond to the ZZ interaction between the next-nearest-neighbor qubits, i.e., \( Q_1 \) and \( Q_3 \), that is resulting from the virtual exchange interaction between qubit state and non-qubit states, as shown in Fig. 5.

**Appendix C: C-iSWAP gate**

As mentioned in the main text, in order to implement the C-iSWAP gate in our proposed system, the rounded trapezoid-shaped pulses are applied to adjust the frequency of \( Q_1 \) and \( Q_3 \), thus during the gate implementation, the qubit frequencies vary from the idle point to the interaction point, while the frequency of \( Q_2 \) keeps fixed. The rounded trapezoid-shaped pulse used in present work is described by a time-dependent function [26]

\[
\omega(t) = \omega_i + \frac{\omega_f - \omega_i}{2} \left[ \text{Erf} \left( \frac{t - t_{\text{ramp}}}{\sqrt{2} \sigma} \right) - \text{Erf} \left( \frac{t - T + t_{\text{ramp}}}{\sqrt{2} \sigma} \right) \right]
\]

where \( \omega_i \) and \( \omega_f \) denote the idle frequency point (where the logical states are defined as as the eigenstates of the system biased at this point, as discussed in Appendix A) and the interaction frequency point, respectively, the ramp time is defined as \( t_{\text{ramp}} = 4 \sqrt{2} \sigma \) with \( \sigma = 1 \text{ ns}, T \) represents the total time for implementing the gate operation, and the hold time \( t_{\text{hold}} = T - t_{\text{ramp}} \) is defined as the time-interval between the midpoints of the ramps.

### 1. Intrinsic gate fidelity

To quantify the intrinsic performance of the proposed C-iSWAP gate implementation, the metric of state-average gate fidelity is used in present work. The fidelity is defined as [28]

\[
F \equiv \frac{\text{Tr}(U_{\text{sys}}^\dagger U_{\text{target}}) + \text{Tr}(U_{\text{sys}}^\dagger U_{\text{target}})^2}{2}
\]

where \( U \) is the actual evolution operator in the logical eigenbasis at the idle point after applying an auxiliary single-qubit Z rotation on each of the three-qubits before and after the gate implementation, truncated to the qubit levels [26, 29, 30], and \( U_{\text{target}} = U_{CXY} (\pi/2) \) given as Eq. (5) of the main text.

According to the system Hamiltonian in Eq. (B1), and the control pulse of Eq. (C1), the actual evolution operator in the rotating frame with respect to \( H(0) \) is

\[
U_{\text{sys}} = \tilde{T} \exp \left( -i \int_0^T H_R(t) \, dt \right),
\]

where \( H_R(t) = e^{iH(0)t} H(t) e^{-iH(0)t} - H(0), \) and \( \tilde{T} \) denotes...
the time-ordering operator. Thus, \( U \) in Eq. (C2) is given as

\[
U = U_{\text{post}} P U_{\text{sys}} P \dagger U_{\text{pre}},
\]

where \( P \) is the projected operator defined in the computational subspace of the full system, and \( U_{\text{post}} \) and \( U_{\text{pre}} \) are

\[
U_{\text{post}} = e^{-i\phi_3 Z I I/2} e^{-i\phi_2 Z I I/2} e^{-i\phi_1 Z I I/2},
\]

\[
U_{\text{pre}} = e^{-i\phi_1 Z I I/2} e^{-i\phi_2 Z I I/2} e^{-i\phi_3 Z I I/2}.
\]

Hence, the gate fidelity is obtained as

\[
F = \max_{\phi_1, \phi_2} F(\phi_1, \phi_2),
\]

take value of 99.97\% for \( t_{\text{hold}} = 43 \text{ ns} \). Aside form the control error (as shown in Fig. 4(a)) and leakage to non-qubit states (as shown in Fig. 4(b)), the residual infidelity is caused by the coherence phase resulting from the ZZ interaction between qubits, as shown in the effective Hamiltonian of Eq. (B14). Hence, by assuming no control error and leakage, the actual implemented unitary operator can be described by

\[
U = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & e^{-i\phi_{11}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & e^{-i\phi_{11}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & e^{-i\phi_{11}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{-i\phi_{11}} \\
\end{pmatrix},
\]

where \( \phi_s \) (\( s = 011, 101, 110, 111 \)) represents the accumulated phase caused by the ZZ interaction during the gate operation.

In the Table II of the main text, we have also shown the accumulated phase caused by the ZZ interaction between qubits during the gate implementation, defined as

\[
\phi_s = \arg \langle s | U | s \rangle,
\]

i.e., the argument of the matrix element \( U_{ss} = \langle s | U | s \rangle \). Strikingly, in the Table II, one can find that the accumulated phase in state \( |101\rangle \) takes the largest value \( 0.01728 \), while in state \( |111\rangle \), it is below \( 0.01 \). This is caused by the fact that, as shown in Fig. 5, similar as the exchange interaction in qubit space \( \{ |100\rangle, |010\rangle \} \) and \( \{ |110\rangle, |011\rangle \} \), the strength of the virtual exchange interactions between qubit states and non-qubit states also depend on the state of \( Q_2 \), thus enabling the ZZ interaction (accumulated phase) controlled by the state of \( Q_2 \). As shown in Fig. 6, one can find that at the interaction point (indicated as the red dashed line), the strength of the interactions between qubit states and non-qubit states \( |101\rangle \leftrightarrow |002\rangle \) \( J_{2(1),1} \) is larger than that of the interaction \( |111\rangle \leftrightarrow |012\rangle \). Similar result can also be obtained for the interaction \( |101\rangle \leftrightarrow |200\rangle \) and the interaction \( |111\rangle \leftrightarrow |210\rangle \). This could explain the striking feature in Table II.

2. Gate operation with the typical coupling strength

As mentioned in the main text, although the implementation of the native three-qubit gates may benefit from the strong fixed coupling between adjacent qubits, this strong coupling may make single qubit addressing [32] and the implementation of two-qubit gates [4, 31], a challenge for system with limited frequency tunability. However, as shown in Fig. 7, with NN coupling strength of 30 MHz [4] and fixed ramp time \( t_{\text{ramp}} = 4 \sqrt{2} \text{ ns} \), the intrinsic gate fidelity above 99\% (99.9\%) can still be achieved in 50 (100) ns. We note that choosing larger ramp time, thus lengthening the gate time, should further reduce the leakage error [27, 37]. Therefore, the intrinsic gate fidelity could further be improved at the expense of increased gate time.

Appendix D: Switchable NNN exchange interaction for higher energy levels

From the expression in Eqs. (B6) and (B11), one can find that a switchable exchange interaction between qubit state and non-qubit states for \( Q_1 \) and \( Q_3 \), such as the interaction \( |11\rangle_1 \leftrightarrow |02\rangle_1 \) with strength \( J_{2(1),1} \), can be achieved by engineering the anharmonicity of \( Q_2 \), as shown in Fig. 5. Hence, as same as the case for the virtual exchange interaction in qubit space, this exchange interaction is controlled by the state of \( Q_2 \), i.e., when \( Q_2 \) in \( |0\rangle \) state, the interaction is turned on, causing the ZZ interaction between \( Q_1 \) and \( Q_3 \), while for \( Q_2 \) in \( |1\rangle \) state, the interaction is turned off.

According to Eq. (B11), Figure 8 shows the calculated \( J_{2(1),1} \) versus the anharmonicity \( \alpha_2 \) and qubit detuning \( \Delta_3 \) in the unit of the NN coupling strength \( g_j \) with \( \omega_1 = \omega_3 + \alpha_3 \). The system parameters used are \( g_j / 2 \pi = 45 \text{ MHz} \), and \( \alpha_3 / 2 \pi = -350 \text{ MHz} \). The regime indicated by the darker strip shows the working point, where the interaction
at the idle point, and the hold time takes value of 60 ns. The (orange) square indicates the working point where the NNN exchange interaction is turned off for \( Q_2 \) in state \(|1\rangle\). From the result shown in Fig. 8, one can find that in order to operate in the dispersive regime or quasi-dispersive regime, the value of \( \alpha_2 \) should be larger than 500 MHz. For \( \alpha_2 \) taking value below 500 MHz, the dispersive model, as well as the approximation adopted in present work, may break down. Thus, in the following discussion, \( \alpha_2 \) takes values larger than 500 MHz.

FIG. 8: Calculated values of \( J_{i(1),j} \) versus \( \alpha_2 \) and \( \Delta_3 \) in the unit of \( g_3 \) with \( \omega_1 = \omega_3 + \alpha_3 \). The anharmonicity \( \alpha_3 \) and NN coupling strength \( g_j \) take the same values as in Fig. 2(b) of the main text. The intersection (orange square) of the vertical cuts and the dark strip gives the ideal optimal working point for realizing switchable coupling between \(|11\rangle_{1,3} \) and \(|02\rangle_{1,3} \) with \( \alpha_2 / 2\pi \) take values of 665 MHz.

FIG. 9: Optimal working point for realizing switchable coupling between \(|11\rangle_{1,3} \) and \(|02\rangle_{1,3} \) with \( \alpha_2 / 2\pi \) take the value of 665 MHz. (a) Swap error \( 1 - P_{111} \) versus the frequency offset \( \Delta_{11} \) and \( \Delta_{13} \) that are defined with respect to the ideal interaction point, as shown in the inset of Fig. 8. The system is initialized in the eigenstate \(|111\rangle\) at the idle point, and the hold time takes value of 60 ns. The (orange) square indicates the working point where the NNN exchange interaction is turned off for \( Q_2 \) in \(|1\rangle\). (b) Population \( P_{101} \) versus the overshoot and the hold time for system initialized in the eigenstate \(|101\rangle\) at the idle point. The horizontal cuts (dashed lines) depict the optimal value of overshoot for enabling a full complete swap between \(|101\rangle\) and \(|002\rangle\). (c) Population swap \(|101\rangle \leftrightarrow |002\rangle \) and (d) \(|111\rangle \leftrightarrow |012\rangle \) versus hold time for system initialized in \(|101\rangle\) and \(|111\rangle\), respectively. With optimal frequency offset and overshoot obtained from (a) and (b), the NNN exchange interaction \(|11\rangle_{1,3} \leftrightarrow |02\rangle_{1,3} \) is turned on or off depending on the state of \( Q_2 \).

Following the same procedure as that for finding the optimal working for implementing C-iSWAP gate, we can also find the optimal working point for realizing switchable coupling between \(|11\rangle_{1,3} \) and \(|02\rangle_{1,3} \), as shown in Fig. 9. Hence, it is possible to implement the controlled-CZ gate with the switchable coupling between \(|11\rangle_{1,3} \) and \(|02\rangle_{1,3} \). As shown in Fig. 9(c) and 9(d), a controlled-CZ gate could be implemented in 120 ns. Similar result can also be obtained for coupling between \(|11\rangle_{1,3} \) and \(|20\rangle_{1,3} \). However, for the directed coupled system considered in present work, only if the magnitude of anharmonicity of the two adjacent qubits are compatible to each other, and having an opposite sign, the residual parasitic ZZ coupling between adjacent qubits could be heavily suppressed [14]. Since the anharmonicity of transmon qubit is around 200—400 MHz, for the directed coupled system with \( \alpha_2 \) of 665 MHz as shown in Fig. 9, the residual parasitic ZZ coupling between adjacent qubits cannot be heavily suppressed, thus limiting the performance of the controlled-CZ gate. However, for indirect coupled system, such coupled via a bus or a tunable coupler, the residual parasitic ZZ coupling can be heavily suppressed without the requirement that the magnitude of anharmonicity of the two adjacent qubits are compatible to each other [13, 14].
[1] A. G. Fowler, M. Mariantoni, J. M. Martinis, and A. N. Cleland, Surface codes: Towards practical large-scale quantum computation, Phys. Rev. A 86, 032324 (2012).
[2] B. R. Johnson. Controlling Photons in Superconducting Electrical Circuits. PhD thesis, Yale University, May 2011.
[3] A. Galiautdinov, A. N. Korotkov, and J. M. Martinis, Resonator-zero-qubit architecture for superconducting qubits, Phys. Rev. A 85, 042321 (2012).
[4] R. Barends, J. Kelly, A. Megrant, A. Veitia, D. Sank, E. Jeffrey, T. C. White, J. Mutus, A. G. Fowler, B. Campbell, Y. Chen, Z. Chen, B. Chiaro, A. Dunsworth, C. Neill, P. O’Malley, P. Roushan, A. Vainsencher, J. Wenner, A. N. Korotkov, A. N. Cleland, and J. M. Martinis, Superconducting quantum circuits at the surface code threshold for fault tolerance, Nature 508, 500 (2014).
[5] F. Yan, P. Krantz, Y. Sung, M. Kjaergaard, D. L. Campbell, T. P. Orlando, S. Gustavsson, and W. D. Oliver, Tunable Coupling Scheme for Implementing High-Fidelity Two-Qubit Gates, Phys. Rev. Applied 10, 054062 (2018).
[6] J. Preskill, Quantum Computing in the NISQ era and beyond, Quantum 2, 79 (2018).
[7] M. Roth, P. Ganzhorn, N. Moll, S. Filipp, G. Salis, and S. Schmidt, Analysis of a parametrically driven exchange-type gate and a two-photon excitation gate between superconducting qubit, Phys. Rev. A 96, 062323 (2017).
[8] D. C. McKay, S. Filipp, A. Mezzacapo, E. Magesan, J. M. Chow, and J. M. Gambetta, Universal gate for fixed-frequency qubits via a tunable bus, Phys. Rev. Applied 6, 064007 (2016).
[9] M. Roth, N. Moll, G. Salis, M. Ganzhorn, D. J. Egger, S. Filipp, and S. Schmidt, Adiabatic quantum simulations with driven superconducting qubits, Phys. Rev. A 99, 022323 (2019).
[10] M. Ganzhorn, D.J. Egger, P. Barkoutsos, P. Ollitrault, G. Salis, N. Moll, M. Roth, A. Fuhrer, P. Mueller, S. Werner, I. Tavernelli, and S. Filipp, Gate-Efficient Simulation of Molecular Eigenstates on a Quantum Computer, Phys. Rev. Applied 11, 044092 (2019).
[11] F. W. Strauch, P. R. Johnson, A. J. Dragt, C. J. Lobb, J. R. Anderson, and F. C. Wellstood, Quantum Logic Gates for Coupled Superconducting Phase Qubits, Phys. Rev. Lett. 91, 167005 (2003).
[12] L. DiCarlo, J. M. Chow, J. M. Gambetta, L. S. Bishop, B. R. Johnson, D. I. Schuster, J. Majer, A. Blais, L. Frunzio, S. M. Girvin, and J. R. Schoelkopf, Demonstration of two-qubit algorithms with a superconducting quantum processor, Nature(London) 460, 240 (2009).
[13] J. Ku, X. Xu, M. Brink, D. C. McKay, J. B. Hertzberg, M. H. Ansari, and B.L.T. Plourde, Suppression of Unwanted ZZ Interactions in a Hybrid Two-Qubit System, arXiv:2003.02775 (2020).
[14] P. Zhao, P. Xu, D. Lan, X. Tan, H. Yu, and Y. Yu, High-contrast ZZ interaction using multi-type superconducting qubits, arXiv:2002.07560 (2020).
[15] H. C. J. Gan, G. Maslennikov, Ko-Wei Tseng, C. Nguyen, and D. Matsukevich, Hybrid quantum computation gate with trapped ion system, arXiv:1908.10117 (2019).
[16] N. J. S. Loft, M. Kjaergaard, L. B. Kristensen, C. K. Andersen, T. W. Larsen, S. Gustavsson, W. D. Oliver, and N. T. Zinner, Quantum interference device for controlled two-qubit operations, arXiv:1809.09049 (2018).
[17] F. P. Liebermann, P-L. Dailleire-Demers, F. K. Wilhelm, Implementation of the iFREDKIN gate in scalable superconducting architecture for the quantum simulation of Fermionic systems, arXiv:1701.07870 (2017).
[18] S. E. Rasmussen, and N. T. Zinner, Simple implementation of high fidelity controlled-iSWAP gates and quantum circuit exponentiation of non-Hermitian gates, arXiv:2002.11728 (2020).
[19] J. Koch, T. M. Yu, J. Gambetta, A. A. Houck, D. I. Schuster, J. Majer, A. Blais, M. H. Devoret, S. M. Girvin, and J. R. Schoelkopf, Charge-insensitive qubit design derived from the cooper pair box, Phys. Rev. A 76, 042319 (2007).
[20] J. Q. You, X. Hu, S. Ashhab, and F. Nori, Low-decoherence flux qubit, Phys. Rev. B 75, 140515(R) (2007).
[21] M. Steffen, S. Kumar, D. P. DiVincenzo, J. R. Rozen, G. A. Keefe, M. B. Rothwell, and M. B. Ketchen, High-Coherence Hybrid Superconducting Qubit, Phys. Rev. Lett. 105, 100502 (2010).
[22] F. Yan, S. Gustavsson, A. Kamal, J. Birenbaum, A. P. Sears, D. Hover, T. J. Gudmundsen, D. Rosenberg, G. Samach, S. Weber, J. L. Yoder, T. P. Orlando, J. Clarke, A. J. Kerman, and W. D. Oliver, The flux qubit revisited to enhance coherence and reproducibility, Nat. Commun. 7, 12964 (2016).
[23] S. Bravyi, D. P. DiVincenzo, and D. Loss, Schrieffer-Wolff transformation for quantum many-body systems, Ann. Phys. 326, 2793 (2011).
[24] S. Poletto, Jay M. Gambetta, Seth T. Merkel, John A. Smolin, Jerry M. Chow, A. D. Córcoles, George A. Keefe, Mary B. Rothwell, J. R. Rozen, D. W. Abraham, Chad Rigetti, and M. Steffen, Entanglement of Two Superconducting Qubits in a Waveguide Cavity via Monochromatic Two-Photon Excitation, Phys. Rev. Lett. 109, 240502 (2015).
[25] See Supplemental Material.
[26] J. Ghosh, A. Galiautdinov, Z. Zhou, A. N. Korotkov, J. M. Martinis, and M. R. Geller, High-fidelity controlled-σ z gate for resonator-based superconducting quantum computers, Phys. Rev. A 87, 022309 (2013).
[27] R. Barends et al., Diabatic Gates for Frequency-Tunable Superconducting Qubits, Phys. Rev. Lett. 123, 210501 (2009).
[28] L. H. Pedersen, N. M. Møller, and K. Mølmer, Fidelity of quantum operations, Phys. Lett. A 367, 47 (2007).
[29] E. Zahedinejad, J. Ghosh, and B. C. Sanders, Designing High-Fidelity Single-Shot Three-Qubit Gates: A Machine-Learning Approach, Phys. Rev. Applied 6, 054005 (2016).
[30] E. Barnes, C. Arenz, A. Pitchford, and S. E. Economou, Fast microwave-driven three-qubit gates for cavity-coupled superconducting qubits, Phys. Rev. B 96, 024504 (2017).
[31] J. Kelly, R. Barends, A. G. Fowler, A. Megrant, E. Jeffrey, T. C. White, D. Sank, J. Y. Mutus, B. Campbell, Yu Chen, Z. Chen, B. Chiaro, A. Dunsworth, I.-C. Hoi, C. Neill, P. O’Malley, C. Quintana, P. Roushan, A. Vainsencher, J. Wenner, A. N. Cleland, and John M. Martinis, State preservation by repetitive error detection in a superconducting quantum circuit, Nature 519, 66 (2015).
[32] J. M. Gambetta, A. D. Córcoles, S. T. Merkel, B. R. Johnson, J. A. Smolin, J. M. Chow, C. A. Ryan, C. Rigetti, S. Poletto, T. A. Ohki, M. B. Ketchen, and M. Steffen, Characterization of Addressability by Simultaneous Randomized Benchmarking, Phys. Rev. Lett. 109, 240504 (2012).
[33] Yu Chen, C. Neill, P. Roushan, N. Leung, M. Fang, R. Barends, J. Kelly, B. Campbell, Z. Chen, B. Chiaro, A. Dunsworth, E. Jeffrey, A. Megrant, J. Y. Mutus, P. J. J. O’Malley, C. M. Quintana, D. Sank, A. Vainsencher, J. Wenner, T. C. White, M. R. Geller, A. N. Cleland, and J. M. Martinis, Qubit
architecture with high coherence and fast tunable coupling, Phys. Rev. Lett. 113, 220502 (2014).

[34] C. Neill. A path towards quantum supremacy with superconducting qubits. PhD thesis, University of California Santa Barbara, Dec 2017.

[35] P. S. Mundada, G. Zhang, T. Hazard, and A. A. Houck, Suppression of Qubit Crosstalk in a Tunable Coupling Superconducting Circuit, Phys. Rev. Applied 12, 054023 (2019).

[36] B. Foxen et al., Demonstrating a Continuous Set of Two-qubit Gates for Near-term Quantum Algorithms, arXiv:2001.08343 (2020).

[37] F. Arute, K. Arya, R. Babbush, D. Bacon, J. C. Bardin, R. Barends, R. Biswas, S. Boixo, F. G. Brandao, D. A. Buell, et al., Quantum supremacy using a programmable superconducting processor, Nature 574, 505 (2019).

[38] J. D. Whitfield, J. Biamonte, and A. Aspuru-Guzik, Simulation of Electronic Structure Hamiltonians Using Quantum Computers, Mol. Phys. 109, 735 (2011).

[39] P.-L. Dallaire-Demers and F. K. Wilhelm, Quantum gates and architecture for the quantum simulation of the Fermi-Hubbard model, Phys. Rev. A 94, 062304 (2016).