Acceleration field of a Universe modeled as a mixture of scalar and matter fields

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Abstract

A model of the Universe as a mixture of a scalar (inflaton or rolling tachyon from the string theory) and a matter field (classical particles) is analyzed. The particles are created at the expense of the gravitational energy through an irreversible process whereas the scalar field is supposed to interact only with itself and to be minimally coupled with the gravitational field. The irreversible processes of particle creation are related to the non-equilibrium pressure within the framework of the extended (causal or second-order) thermodynamic theory. The scalar field (inflaton or tachyon) is described by an exponential potential density added by a parameter which represents its asymptotic value and can be interpreted as the vacuum energy. This model can simulate three phases of the acceleration field of the Universe, namely, (a) an inflationary epoch with a positive acceleration followed by a decrease of the acceleration field towards zero, (b) a past decelerated period where the acceleration field decreases to a maximum negative value followed by an increase towards zero, and (c) a present accelerated epoch. For the energy densities there exist also three distinct epochs which begin with a scalar field dominated period followed by a matter field dominated epoch and coming back to a scalar field dominated phase.

1 Introduction

The solution of the problems of flatness, horizon and unwanted relics by the inflationary theory on the one hand and the recent measurements of the anisotropy of the cosmic microwave background and of the type Ia supernova SN 1997ff red-shift indicating that the Universe is flat with a present positive acceleration and a past decelerating period on the other hand, allows us to classify the evolution of the acceleration field of the Universe according to three major epochs, beginning with an accelerated inflationary period following a past decelerated epoch and leading back to a present accelerated phase.

The early and present accelerated periods are dominated by a scalar field whereas the past decelerated phase is dominated by a matter field. In inflationary cosmology the scalar field is normally represented by the inflaton (see, e.g., the works [1, 2, 3]) but recently a rolling tachyon – which has arisen from string theory [4, 5] – was also considered as another candidate for the description of the accelerated phases of the Universe (see, e.g., the works [6, 7, 8, 9],). The inflationary period comes to an end when the scalar field has rolled to its potential minimum and begins to oscillate about it. By this time, the Universe is very cold and void. Some mechanism is then required to account for particle creation, that will fill and heat the Universe to allow for the standard hot Big-Bang evolution to take place. The most common mechanism adopted is the decay of the inflaton field in other particles, and it is known as reheating process. It goes without

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1Recently a model for the accelerated phase of the Universe was analyzed in the works [10, 11] where the van der Waals equation of state plays the role of the scalar field.
saying that this requires the coupling of the inflaton to other fields. However, the inflaton needs
to be weakly coupled in order to inflation to work well. In view of that, in the present work
we will make another approach and ignore the coupling of the scalar field to other fields. It is
known from quantum field theory in curved space-time (see e.g. [12]), that matter constituents
may be produced quantum-mechanically in the framework of Einstein’s equations. The energy of
the produced particles is extracted from the gravitational field. Our procedure will then be to
take this fact into account by a phenomenological description – first proposed, to the best of our
knowledge, by Prigogine et al [13] – in which the particles are created during the evolution of the
Universe at the expense of the gravitational energy through an irreversible process (see also the
works [14, 15, 16, 17]).

In the present work we have considered that the irreversible processes of particle production
are related to a non-equilibrium pressure with an evolution equation coming from the extended
(causal or second-order) thermodynamic theory (see e.g. [15, 16, 17, 18, 19, 20, 21, 22]). Moreover,
since the scalar field potential does not require a minimum about which the inflaton will oscillate
and decay in other particles, we shall adopt an exponential potential density added by a parameter
which represents its asymptotic value and can be interpreted as the vacuum energy [23, 24, 25, 26].

We have shown, among other results, that a mixture of a scalar field (inflaton or tachyon) and a
matter field can simulate the three phases of the Universe related to its acceleration field, namely,
(a) an inflationary epoch with a positive acceleration followed by a decrease of the acceleration field
towards zero, (b) a past decelerated period where the acceleration field decreases to a maximum
negative value followed by an increase towards zero, and (c) a present accelerated epoch. For the
energy densities there exist also three distinct epochs which begin with a scalar field dominated
period followed by a matter field dominated epoch and coming back to a scalar field dominated
phase. These results can also be obtained by choosing other types of potential densities for the
scalar field (inflaton or tachyon).

The work is organized as follows. In section 2 a system of three coupled differential equations
for the scalar (inflaton or tachyon), acceleration and non-equilibrium pressure fields is determined.
The solutions of the systems of coupled differential equations obtained in section 2 are found in
section 3 for given initial conditions and for a given potential energy density of the scalar field. We
close the work with a discussion of the results obtained in section 3. Units have been chosen so
that $c = \hbar = k = 1$.

2 Field Equations

Let us consider a homogeneous, isotropic and spatially flat Universe modeled as a mixture of a
scalar field and a matter field. The scalar field (inflaton or tachyon) represents a hypothetical
particle while the matter field refers to the classical particles which are created at the expense of
the gravitational energy.

For this model of the Universe the energy-momentum tensor of the mixture is written as

$$T^{\mu\nu} = (\rho + p + \varpi)U^\mu U^\nu - (p + \varpi)g^{\mu\nu},$$

where the pressure and the energy density of the mixture are given in terms of the corresponding
quantities for its constituents by $p = p_s + p_m$ and $\rho = \rho_s + \rho_m$, with the indexes $s$ and $m$ denoting
the scalar and the matter fields, respectively. We shall adopt the following convention: (a) $s = \phi$
for the inflaton field and (b) $s = \phi$ for the tachyon field. Furthermore, $U^\mu$ (such that $U^\mu U_\mu = 1$) is
the four-velocity, $g_{\mu\nu}$ denotes the metric tensor with signature (+ − − −) whereas $\varpi$ refers to the
non-equilibrium pressure. The non-equilibrium pressure is responsible for the irreversible processes
of particle production [14, 15] during the evolution of the Universe.

The conservation law of the energy-momentum tensor $T^{\mu\nu}_{\;\mu\nu} = 0$ follows from Einstein’s field
equations and the Bianchi identities. In a comoving frame described by the Robertson-Walker
metric it leads to the balance equation for the energy density of the mixture, i.e.,

$$\dot{\rho} + 3H(\rho + p + \varpi) = 0,$$
where the quantity $H = \dot{a}(t)/a(t)$ denotes the Hubble parameter, $a(t)$ is the cosmic scale factor and the over-dot refers to differentiation with respect to time $t$.

The equation which connects the Hubble parameter with the energy density of the mixture is the Friedmann equation which – in a spatially flat Universe described by the Robertson-Walker metric – is written as

$$H^2 = \frac{8\pi G}{3} \rho,$$

(3)

where $G$ is the gravitational constant.

2.1 Inflaton field

First we shall analyze the case where the scalar field is represented by the inflaton field $\phi(x^\mu)$, which is described by the Lagrangian density

$$\mathcal{L}_\phi = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi),$$

(4)

where $V(\phi)$ denotes the potential density of the inflaton field. The identification of the inflaton with a perfect fluid – i.e., by requiring that the inflaton interacts only with itself and it is minimally coupled with the gravitational field – allows us to write its energy-momentum tensor as

$$T^\mu_\nu = (\rho_\phi + p_\phi)U^\mu U^\nu - p_\phi g^\mu\nu = \partial_\mu \phi \partial_\nu \phi - \mathcal{L}_\phi g^\mu\nu.$$

(5)

The last equality above is a consequence of Noether’s theorem.

By considering a homogeneous inflaton field and a comoving frame one can obtain from equations (4) and (5) the relationships

$$\begin{align*}
\rho_\phi &= \frac{1}{2} \dot{\phi}^2 + V(\phi), \\
p_\phi &= \frac{1}{2} \dot{\phi}^2 - V(\phi),
\end{align*}$$

(6)

which connect the energy density $\rho_\phi$ and the pressure $p_\phi$ of the inflaton to its kinetic and potential energies.

The time evolution equation of the inflaton field follows from the Euler-Lagrange equation which, in the homogeneous case, reads

$$\ddot{\phi} + 3H \dot{\phi} + V'(\phi) = 0.$$ 

(7)

Above, the prime denotes differentiation with respect to $\phi$.

The evolution equation of the energy density of the inflaton field decouples from that of the matter field, since the differentiation of (6) with respect to time by taking into account the equation (7) leads to

$$\dot{\rho_\phi} + 3H(\rho_\phi + p_\phi) = 0.$$ 

(8)

Hence the evolution equation of the energy density of the matter field can be written as

$$\dot{\rho}_m + 3H(\rho_m + p_m) = -3H \varpi.$$ 

(9)

thanks to (2) and (8). We can interpret the term $-3H \varpi$ in the above equation as the energy production rate of the matter field (see e.g., [13, 16, 17]). In a previous work [15] one of the authors have calculated the energy-momentum pseudo-tensor of the gravitational field and found $T^0_0 = -3H^2/(8\pi G)$. If we identify $T^0_0$ with the energy density of the gravitational field $\rho_G$, we can regard the term $3H \varpi$ as the energy production rate of the gravitational field:

$$\dot{\rho}_G + 3H(\rho_G - p_\phi - p_m) = 3H \varpi,$$ 

(10)

that is, there is an irreversible energy flow from the gravitational field to matter creation.

The matter field is supposed to obey a barotropic equation of state $p_m = w_m \rho_m$ where the coefficient $w_m$ may assume values in the range between $0 \leq w_m \leq 1$. Some values for this coefficient
are: (a) $w_m = 0$ for dust or pressure-less fluid; (b) $w_m = 1/3$ for radiation; (c) $w_m = 2/3$ for non-relativistic matter and (d) $w_m = 1$ for stiff matter or Zel’dovich fluid.

The equation which gives the time evolution of the cosmic scale factor can be obtained from differentiation of the Friedmann equation \(^\text{(3)}\) with respect to time and elimination of $\dot{\rho}$ from the resulting equation, by using the balance equation for the energy density of the mixture \(^\text{(2)}\). Hence, it follows

$$
\dot{H} + \frac{3}{2}(w_m + 1)H^2 = 4\pi G \left[ (w_m - 1) \frac{\dot{\phi}^2}{2} + (w_m + 1)V(\phi) - \bar{\omega} \right],
$$

\(^\text{(11)}\)

thanks to \(^\text{(6)}\), \(^\text{(3)}\) and to the barotropic equation of state of the matter field.

Equation \(^\text{(11)}\) refers to a differential equation for the cosmic scale factor $a(t)$ that depends on the non-equilibrium pressure $\bar{\omega}(t)$ and on the potential density of the inflaton field $V(\phi)$. If we know a relationship between $\bar{\omega}(t)$ and $a(t)$ for a given $V(\phi)$ it would be possible to find a solution of the system of differential equations \(^\text{(7)}\) and \(^\text{(11)}\) for the time evolution of the cosmic scale factor and for the inflaton field. Here we shall consider that the non-equilibrium pressure obeys – within the framework of extended (causal or second-order) thermodynamic theory – the linearized evolution equation \(^2\)

$$
\bar{\omega} + \tau \ddot{\bar{\omega}} = -3\eta H.
$$

\(^\text{(12)}\)

Above, the coefficient of bulk viscosity $\eta$ and the characteristic time $\tau$ are considered as functions of the energy density of the mixture $\rho$, i.e., $\eta = \alpha \rho$, and $\tau = \eta/\rho$ where $\alpha$ is a constant (see e.g. \cite{15,16,17,18}).

Now we have a system of three differential equations \(^\text{(7)}, \text{(11)}\) and \(^\text{(12)}\), and in order to solve it we introduce the dimensionless quantities

\begin{equation}
\begin{cases}
H \equiv H/H_0, & t \equiv tH_0, & \bar{\omega} \equiv \bar{\omega}[8\pi G/(3H_0^2)], \\
\alpha \equiv \alpha H_0, & V \equiv V[8\pi G/(3H_0^2)], & \phi \equiv \phi\sqrt{8\pi G/3}.
\end{cases}
\end{equation}

\(^\text{(13)}\)

Above, the Hubble parameter $H_0$ at $t = 0$ (by adjusting clocks) is related to the energy density $\rho_0^\phi$ of the inflaton field at $t = 0$ by $H_0 = \sqrt{8\pi G\rho_0^\phi/3}$, since we have assumed that at $t = 0$ the energy density of the matter field vanishes ($\rho_0^m = 0$) and $\dot{\phi}(0) = 0$.

With respect to the dimensionless quantities \(^\text{(13)}\) the system of differential equations \(^\text{(4)}, \text{(11)}\) and \(^\text{(12)}\) reads

\begin{equation}
\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0,
\end{equation}

\(^\text{(14)}\)

\begin{equation}
\dot{H} + \frac{3}{2}(w_m + 1)H^2 = \frac{3}{2} \left[ (w_m - 1) \frac{\dot{\phi}^2}{2} + (w_m + 1)V(\phi) - \bar{\omega} \right],
\end{equation}

\(^\text{(15)}\)

\begin{equation}
\bar{\omega} + \alpha \ddot{\bar{\omega}} = -3\alpha H^3.
\end{equation}

\(^\text{(16)}\)

The time evolution of the inflaton field $\phi(t)$, of the cosmic scale factor $a(t)$ and of the non-equilibrium pressure $\bar{\omega}(t)$ can be determined from the system of differential equations \(^\text{(14)}\) through \(^\text{(16)}\) once: (a) a potential density of the inflaton field $V(\phi)$ is chosen; (b) initial conditions at $t = 0$ (by adjusting clocks) are given for the cosmic scale factor $a(0)$ and its derivative $\dot{a}(0)$, for the inflaton field $\phi(0)$ and its derivative $\dot{\phi}(0)$ and for the non-equilibrium pressure $\bar{\omega}(0);$ (c) values for coefficient $w_m$ – related to the barotropic equation of state of the matter field – and for the coefficient $\alpha$ – related to the irreversible processes of particle production – are selected.

From the knowledge of the fields $\phi(t)$, $a(t)$ and $\bar{\omega}(t)$ it is possible to determine the time evolution of the energy densities of the inflaton and matter fields from

\begin{equation}
\frac{\dot{\rho}_\phi}{\rho_0^\phi} = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad \frac{\rho_m}{\rho_0^\phi} = H^2 - \left[ \frac{1}{2} \dot{\phi}^2 + V(\phi) \right],
\end{equation}

\(^\text{(17)}\)

thanks to \(^\text{(6)}, \text{(1)}\) and \(^\text{(8)}\).

\(^2\)One is referred to \cite{24} for a derivation of the evolution equation \(^\text{(12)}\) within the framework of the Boltzmann equation.
2.2 Tachyon field

Recently, Sen [4, 5] has shown from a string theory that a rolling tachyon can be described by the Lagrangian density

\[ \mathcal{L}_\phi = -V(\phi)\sqrt{1 - \partial_\mu \varphi \partial^\mu \varphi}, \]  

where \( V(\phi) \) denotes the potential density of the tachyon field \( \varphi(x^\mu) \). The corresponding Born-Infeld action for the tachyon field reads

\[ S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} - V(\phi)\sqrt{1 - \partial_\mu \varphi \partial^\mu \varphi} \right]. \]  

In the above equation \( R \) represents the curvature scalar.

From the action (19) it follows the energy-momentum tensor of the tachyon field

\[ T^\mu_\nu = (\rho_\phi + p_\phi)U^\mu U^\nu - p_\phi g^\mu_\nu, \]  

where the energy density \( \rho_\phi \), the pressure \( p_\phi \) and the four-velocity \( U^\mu \) are identified as

\begin{align*}
\rho_\phi &= \frac{V(\phi)}{\sqrt{1 - \dot{\varphi}^2}}, \\
p_\phi &= -\frac{V(\phi)}{\sqrt{1 - \dot{\varphi}^2}}, \\
U^\mu &= \frac{\partial^\mu \varphi}{\sqrt{1 - \dot{\varphi}^2}}. \tag{21}
\end{align*}

The field equation for the tachyon is obtained from the Euler-Lagrange equation and reads

\[ (\partial_\mu \varphi)_{,\nu} \left[ g^{\mu\nu} + \frac{\partial^\mu \varphi \partial^\nu \varphi}{1 - \partial_\sigma \varphi \partial^\sigma \varphi} \right] + \frac{V'(\varphi)}{V(\varphi)} = 0. \]  

For a homogeneous tachyon field in a spatially flat Robertson-Walker metric, equations (21) and (22) reduce to

\[ \rho_\phi = \frac{V(\varphi)}{\sqrt{1 - \dot{\varphi}^2}}, \quad p_\phi = -\frac{V(\varphi)}{\sqrt{1 - \dot{\varphi}^2}}, \]  

\[ \dot{\varphi}^2 + 3H\dot{\varphi} + \frac{V'(\varphi)}{V(\varphi)} = 0, \]  

respectively, whereas the four-velocity (21) becomes that of a comoving frame.

The differentiation of equation (23) with respect to time leads to the evolution equation of the energy density of the tachyon field, i.e.,

\[ \dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 0, \]  

thanks to (25) and (24). Hence, the energy density of the matter field decouples from the corresponding equation for the tachyon field, has the same expression as that given by (25) and can be interpreted in the same manner as in the previous section.

Furthermore, from the differentiation of the Friedmann equation (26) with respect to time and by taking into account the barotropic equation of state for the matter field, it follows the dimensionless equation

\[ \dot{H} + \frac{3}{2}(w_m + 1)H^2 = \frac{3}{2} \left[ \frac{V(\varphi)}{\sqrt{1 - \dot{\varphi}^2}}(w_m + 1 - \dot{\varphi}^2) - \omega \right], \]  

which relates the time evolution of the cosmic scale factor with the non-equilibrium pressure and the potential density of the tachyon field. The dimensionless quantities in this case are

\begin{align*}
\{ H \equiv H/H_0, \quad t \equiv tH_0, \quad \omega \equiv \omega[8\pi G/(3H_0^2)], \\
\alpha \equiv \alpha H_0, \quad V \equiv V[8\pi G/(3H_0^2)], \quad \varphi \equiv \varphi H_0, \} \tag{27}
\end{align*}
where the Hubble parameter $H_0$ at $t = 0$ (by adjusting clocks) is connected with the energy density $\rho_0^\phi$ of the tachyon field at $t = 0$ by $H_0 = \sqrt{8\pi G \rho_0^\phi / 3}$. Here we have also assumed that at $t = 0$ the energy density of the matter field vanishes ($\rho_0^m = 0$) and $\dot{\varphi}(0) = 0$.

The system of differential equations we have to solve now consists of the evolution equations: (a) for the non-equilibrium pressure (16); (b) for the tachyon field (24) and (c) for the cosmic scale factor (26). Once the time evolution of these fields is known one can determine the time evolution of the energy densities of the tachyon and matter fields from

$$\frac{\rho_\phi}{\rho_0^\phi} = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}, \quad \frac{\rho_m}{\rho_0^\phi} = H^2 - \frac{V(\varphi)}{\sqrt{1 - \dot{\varphi}^2}}.$$  \hspace{1cm} (28)

### 3 Results and Discussions

In order to solve the two systems of coupled differential equations for:

- (a) the non-interacting inflaton field [eqs. (14), (15) and (16)],
- (b) the non-interacting tachyon field [eqs. (16), (24) and (26)],

we have to choose at first the initial conditions at $t = 0$ (by adjusting clocks). Here we have assumed:

(i) $a(0) = 1$ for the dimensionless cosmic scale factor,

(ii) $H(0) = 1$ for the dimensionless Hubble parameter,

(iii) the inflaton field $\phi(0)$ and the tachyon field $\varphi(0)$ were chosen in such a manner that the initial values of the corresponding potential densities were given by $V(\phi(0)) = V(\varphi(0)) = 1$,

(iv) $\dot{\phi}(0) = 0$ and $\dot{\varphi}(0) = 0$ so that the initial values of the energy densities of the inflaton and tachyon fields refer only to the corresponding potential densities, and

(v) $\varpi(0) = 0$ so that the irreversible processes of particle production begin just after the time $t = 0$.

Among several models for the potential density of the scalar field (see, for example, Liddle and Lyth [28]) we fix our attention to the exponential potential

$$\begin{align*}
V(\phi) &= \exp[-\mu \phi(t)] + \lambda, & \text{for the inflaton field,} \\
V(\varphi) &= \exp[-\mu \varphi(t)] + \lambda, & \text{for the tachyon field,}
\end{align*}$$  \hspace{1cm} (29)

where $\mu > 0$ and $0 < \lambda \ll 1$ are free parameters, the first one is connected with the slope of the potential density and has influence on the velocity of the field which rolls down toward the potential minimum, whereas the second one represents the asymptotic value of potential density $V$ and can be interpreted as the vacuum energy [23, 24, 25, 26].

Apart from the two free parameters $\lambda$ and $\mu$ there still remains much freedom to find the solutions of the four systems of coupled differential equations, since they do depend on the parameters: (a) $w_m$ which is related to the barotropic equation of state of the matter field, and (b) $\alpha$ which is connected to the transfer of energy from the gravitational field to the matter field.

Before discussing the results in detail, let us briefly describe the evolution of the system. In the beginning all the energy is in the form of potential energy of the scalar field, which has negative pressure and therefore leads to an inflationary expansion of the Universe. As the scalar field begins to roll down its potential $V$, its kinetic energy increases and its potential energy decreases, raising its pressure and making its total energy decay. At the same time, the non-equilibrium pressure $\varpi$ increases in module and the irreversible processes of particle production begins. Therefore, the increasing production of matter and the decay of the potential energy of the scalar field contribute to
slow down the expansion of the Universe and the decelerated period begins. During this transition process the non-equilibrium pressure had started to decrease in module, and a little time after deceleration begins \( \varpi \) was already driven towards zero. That means that the irreversible processes are only important during the early Universe, and there is no matter creation during the decelerated phase and thereafter, so that the matter fields evolve as usual. Moreover, the roll of the scalar field has been damped by the friction term \( 3H\dot{\phi} \) or \( 3H\dot{\varphi} \), such that its kinetic energy tends to zero and its potential energy tends to \( \lambda \). As long as \( \rho_m \gg \lambda \) the Universe evolves as in the standard hot Big-Bang model and the role of the scalar field is negligible. But when \( \rho_m \) became comparable to \( \lambda \) (the vacuum energy of the scalar field), a transition period takes place and the Universe begins to accelerate.

The time evolution of the acceleration field \( \ddot{a} \) is plotted in figure 1 whereas in figure 2 are plotted the time evolution of the energy densities of inflaton \( \rho_\phi \), tachyon \( \rho_\varphi \) and matter \( \rho_m \) fields. In these figures we have chosen the following values for the parameters: \( \lambda = 0.005 \), \( \mu = 6.5 \), \( w_m = 1/3 \), and \( \alpha = 0.2 \). Below we shall comment how the changes of these parameters affect the solutions of the differential equations.

We infer from figure 1 that there exist three distinct periods for the acceleration field, namely, (a) an inflationary epoch with an exponential growth to a maximum value followed by a decrease of the acceleration field towards zero, (b) a past decelerated period where the acceleration field decreases to a maximum negative value followed by an increase towards zero, and (c) a present accelerated epoch. We note from figure 1 that the acceleration field of the inflaton begins its decelerated period earlier than the corresponding one for the tachyon. This behavior is a consequence of the fact that the pressure of the inflaton field \( p_\phi = \dot{\phi}^2/2 - V(\phi) \) may assume positive values, and in fact it does, due to the increase in the kinetic energy of the inflaton and the decrease in its potential energy. Hence, the inflaton field behaves as matter with positive pressure for a while, leading to a precocious deceleration. By contrast, the pressure of the tachyon field \( p_\varphi = -V(\varphi)\sqrt{1 - \dot{\varphi}^2} \) cannot assume positive values (for \( V(\varphi) > 0 \)). Its contribution for the deceleration is not as intense as in the case of the inflaton field, since it can only come close to dust-like behavior \( (p_\varphi \sim 0) \). That prolongs the first accelerated period.

We observe from figure 2 that for the energy densities there exist three distinct epochs which begins with a scalar field dominated period followed by a matter field dominated epoch and coming back to a scalar field dominated phase. The following conclusions for the energy densities of the inflaton and tachyon fields can be obtained also from figure 2: (a) in the earliest times the energy densities of the inflaton and tachyon fields coincide, since \( \dot{\phi}^2 \ll V(\phi) \) and \( \dot{\varphi}^2 \ll 1 \) so that \( \rho_\phi \approx V(\phi) \approx V(\varphi) \approx \rho_\varphi \), (b) during the first accelerated phase the energy density of the tachyon field decays more slowly than that of the inflaton field, for two reasons: first, the influence of the downhill roll of the tachyon field (the increase in \( \dot{\varphi} \)) on its respective energy density is stronger
than that of the inflaton field, since the former is proportional to \(1/\sqrt{1 - \dot{\phi}^2}\), while the latter is proportional to \(\dot{\phi}^2\); second, the equation of motion of the tachyon field, \(\ddot{\phi} = -(1 - \dot{\phi}^2)(3H\dot{\phi} + V'/V)\), tells us that the change in \(\dot{\phi}\) tends to zero as it approaches the value 1, therefore \(\dot{\phi}\) grows more than \(\dot{\phi}\) and stands for a longer period nearby the value 1, and (c) in the transition from the past decelerated epoch to the present accelerated period the energy density of the tachyon becomes smaller than that of the inflaton, because the tachyon has rolled faster and further than the inflaton, implying that \(V(\phi) < V(\phi)\) and therefore \(\rho_{\phi} < \rho_{\phi}\) – since in this period the potential energies dominate the kinetic energies.

The behavior of the matter field can be interpreted as follows: the decrease in module of the non-equilibrium pressure \(\varpi\) for the tachyon field is slower – since its positive acceleration is more prolonged – and therefore more energy is transfered to the matter field than the corresponding case for the inflaton field.

We shall comment upon the coefficient that is responsible for particle creation, namely \(\alpha\). By increasing the value of \(\alpha\): (a) the initial accelerated period grows due to the increase in module of the non-equilibrium pressure, and (b) the energy density of the matter field increases so that the matter dominated period becomes larger and the present accelerated period begins at later times.

Let us now comment on the coefficient \(w_m\) which refers to the barotropic equation of state of state of the matter field. By decreasing its value the positive pressure of the matter field decreases, and that implies: (a) a less pronounced deceleration, and (b) a slower decay of the energy density of the matter.

If we decrease the value of the parameter \(\lambda\) – which is related to the asymptotic value of the potential density \(V\) – we infer that: (a) there exists a longer matter dominated period since the vacuum energy \(\lambda\) – which is the responsible for the present accelerated period – will dominate at later times, and (b) if \(\lambda \to 0\) there exists no period of present acceleration, since \(\rho_{\phi} \approx V(\phi) \to 0\) and \(\rho_{\phi} \approx V(\phi) \to 0\). Hence, \(\lambda\) is responsible for the present acceleration and can be interpreted as the dark energy.

As was previously remarked the parameter \(\mu\) is related with the slope of the potential density of the scalar field. By decreasing the value of \(\mu\) the scalar fields \(\phi\) and \(\varphi\) roll more slowly so that the decay of their corresponding energy densities \(\rho_{\phi}\) and \(\rho_{\varphi}\) is slower and the initial accelerated period becomes larger. Moreover, if \(\mu\) is very small the decelerated period for the inflaton may not exist due to the fact that the inflaton rolls more slowly when the slope of its potential density decreases. This behavior is not followed by the tachyon, since as was previously commented \(\dot{\phi}\) is pulled toward 1 and \(\varphi\) rolls more rapidly than \(\phi\). Hence, even for small values of \(\mu\) there exists a decelerated period for the tachyon field in this model.
As a final remark, we note that the same general behavior of the acceleration and of the energy densities fields may be obtained by choosing other usual types of potential densities for the scalar field found in the literature (see e.g. [28]).

References

[1] Guth, A. H. (1981). *Phys. Rev.* D **23** 347.
[2] Linde, A. (1982). *Phys. Lett.* B **108** 389.
[3] Albrecht, A. and Steinhardt, P. J. (1982). *Phys. Rev. Lett.* **48** 1220.
[4] Sen, A. (2002). *J. High Energy Phys.* **07** 065.
[5] Sen, A. (2002). *Mod. Phys. Lett.* A **17** 1797.
[6] Bagla, J. S., Jassal, H. K. and Padmanabhan, T. (2003). *Phys. Rev.* D **67** 063504. Padmanabhan, T. (2002). *Phys. Rev.* D **66**, 021301, 081301.
[7] Gibbons, G. W. (2003). *Class. Quant. Grav.* **20** S321.
[8] Bento, M. C., Bertolami O. and Sen, A. A. (2003). *Phys. Rev.* D **67** 063511.
[9] Fairbairn, M. and Tytgat, M. H. (2002). *Phys. Lett.* B **546** 1.
[10] Capozziello, S., De Martino, S. and Falanga, M. (2002). *Phys. Lett.* A **299** 494.
[11] Kremer, G. M. (2003). *Phys. Rev.* D **68** 123507.
[12] Parker, L. (1969). *Phys. Rev.* **183** 1057.
[13] Prigogine, I., Gheenian, J., Gunzig, E. and Nardone, P. (1989). *Gen. Relat. Grav.* **21** 767.
[14] Zimdahl, W. (1998). *Phys. Rev.* D **57** 2245.
[15] Kremer, G. M. and Devecchi, F. P. (2002). *Phys. Rev.* D **66** 063503.
[16] Kremer, G. M. and Devecchi, F. P. (2003). *Phys. Rev.* D **67** 047301.
[17] Kremer, G. M. (2003). *Gen. Relat. Grav.* **35** 1459.
[18] Belinskiĭ, V. A., Nikomarov, E. S. and Khalatnikov, I. M. (1979). *Sov. Phys. JETP* **50** 213.
[19] Romano, V. and Pavón, D. (1993). *Phys. Rev.* D **47** 1396.
[20] Chimento, L. P. and Jakubi, A. S. (1993). *Class. Quant. Grav.* **10** 2047.
[21] Coley, A. A. and van den Hoogen, R. J. (1995). *Class. Quant. Grav.* **12** 1977.
[22] Zimdahl, W. (2000). *Phys. Rev.* D **61** 083511.
[23] Sahni, V. and Starobinsky, A. (2000). *Int. J. Mod. Phys.* D **9** 373.
[24] Carroll, S. M. (2001). *Living Rev. Relativity* **4** 1.
[25] Peebles, P. J. E. and Ratra, B. (2003). *Rev. Mod. Phys.* **75** 599.
[26] Padmanabhan, T. (2003). *Phys. Rept.* **380** 235.
[27] Cercignani, C. and Kremer, G. M. (2002). *The Relativistic Boltzmann Equation: Theory and Applications* (Basel, Birkhäuser).
[28] Liddle, A. R. and Lyth, D. H. (2000). *Cosmological Inflation and Large-Scale Structure* (Cambridge, Cambridge University Press).