Model investigation on the probability of QGP formation in relativistic heavy ion collisions *

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The formation probability of quark-gluon plasma in relativistic heavy ion collisions for colliding nuclei of different sizes is investigated in the framework of a bond percolation model. The results show that nuclei with sizes smaller than that of Pb or Au produce QGP with probability less than unity even at very high collision energies. The dependence of the QGP-formation probability on different nuclear sizes and on various centralities of Au-Au collision are presented.

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I. INTRODUCTION

It is believed that a medium with deconfined quarks and gluons as constituents, referred to as quark-gluon plasma (QGP), could be created in high energy nucleus-nucleus collisions. Up to now, abundant experimental data have been obtained through the collisions of Si, O, Al, Cu, Au and Pb et al. at center-of-mass energies varying from about 2 to 200 AGeV [1, 2, 3, 4, 5, 6, 7, 8]. The recent data analyses on RHIC Au-Au collisions at 200 AGeV show strong evidences for liberated quark degree of freedom over nuclear volume [5, 6, 7, 8].

The widely accepted definition for QGP is: a (locally) thermally equilibrated state of matter in which quarks and gluons are deconfined from hadrons, so that color degrees of freedom become manifest over nuclear, rather than merely nucleonic, volumes [2]. From this definition, formation of QGP requires quark deconfinement in a large volume at least larger than that of a nucleon. Therefore, proton-proton collision could not form QGP even at very high energy. Then it is natural to raise a question: what is the dependence of QGP-formation probability on the size of colliding nuclei?

The above question could be understood qualitatively from the point of view of energy density deposited in the collision region. People believe that QGP could be created if the energy density is above ten times that of the normal nuclear matter [3, 10, 11]. The energy deposition in collision region is due to multiple scattering between nucleons from two incident nuclei. In the Glauber model [12], which describes the multiple scattering of nucleus-nucleus collision, the probability of $n$ multi-scattering in the collision of two nuclei at a given impact parameter is a binomial distribution. For two head-on nuclei with identical nucleon number $A$, the mean number of multi-scattering is proportional to $A^{4/3}$ [13]. For small nuclei the multi-scattering number $n$ may be large enough for QGP formation only at the tail of $n$-distribution. So it could be inferred that if the colliding nuclei are not large enough, the probability for producing QGP in their collision will be much less than 100%.

In this paper we suggest to investigate QGP-formation probability in heavy ion collisions by using a bond percolation model [14]. Application of percolation theory to quark deconfinement was first suggested by G. Baym [15] and further extended by H. Satz et al. [16, 17, 18, 19, 20]. In their work, they use site percolation model and discuss the critical nucleon density of phase transition. In Ref. [14] the bond percolation model is applied to discuss the cluster formation in analytic crossover between hadronic gas and QGP. We are now applying this model to study the probability of QGP formation.

Firstly, let us have an intuitive look at the process that occurs in heavy ion collisions. When two nuclei collide with high velocity, due to Lorentz contraction the scale in the longitudinal direction is much less than that in the transverse plane, and the two nuclei can be described as two discs without thickness. During the collision, the nucleons in the two discs interact with each other and the potential barriers between them decrease as the increase of the center-of-mass energies. That is to say, the wave-function of nucleons will be distorted at high center-of-mass energies, and the infinitely high confinement-potential between neighboring nucleons might be reduced to a finite-height potential barrier, cf. the central sub-figure in Fig. 1(a). The higher the center-of-mass energy is, the more distorted is the nucleon wave function and the lower are the potential barriers. Because of quantum tunneling effect, quarks in nucleons are able to delocalize from single nucleon and the nearer the two nucleons the larger probability of delocalization. Let us use $S$ to denote the distance between the neighboring nucleons that have quark-delocalization, cf. the central sub-figure of Fig 1(a). At fixed energy, there exists a maximum distance $S_{\text{max}}$, such that quark delocalization is possible when $S \leq S_{\text{max}}$, but is impossible when $S > S_{\text{max}}$. As the increase of energy, the maximum delocalization

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The bond or site occupation probability in the geometrical percolation model, the control parameter is the maximum delocalization distance \( S_{\text{max}} \) [21]. When \( p \) equals a critical value, an infinite cluster appears and the system turns from disconnected phase to connected phase. In our bond percolation model, the control parameter is the maximum delocalization distance \( S_{\text{max}} \), which depends on the collision energy of the two nuclei. If two nucleons depart with distance less than \( S_{\text{max}} \), there could be a bond formed between them, representing the tunneling of quarks through potential barrier. The nucleons (cells) aggregate through bond-connection and form clusters. The appearance of infinite cluster changes the medium from color insulator to color conductor, or from hadron phase to quark-gluon phase.

II. THE CONSTRUCTION OF THE BOND PERCOLATION MODEL

In the bond percolation model, two head-on colliding nuclei with nucleon number \( A \) are simplified as two overlapped discs of radius \( R = 1.2A^{1/3} \) and with 2\( A \) nucleons randomly distributed inside the overlapped region. Each nucleon has a hard-core radius \( r_e = 0.1 \) fm. A nucleon, or cell after cluster formation, departs from the center of the big disc farther than \( R - r_e \) is referred to as a boundary cell.

The percolation procedure is as follows:

1. Randomly select a cell \( \alpha \) as a mother cell.
2. Find the cells that can form bonds with the mother cell, which will be referred to as bond-candidate cells and are defined as those cells with \( |r - r_\alpha| \leq S \). It is assumed that each cell can be connected by 3 bonds at most since each nucleon has 3 constituent quarks. So we randomly select 3 cells from the bond-candidate cells to form 3 bonds connected to the mother cell \( \alpha \). These are referred to as daughters. If the number of candidate cells is less than 3, then the number of daughters is equal to the candidate number.
3. Find the bond-candidate cells for the daughters of cell \( \alpha \). For every daughter find her bond-candidate cells from the remaining unbounded cells, and randomly select 2 bond-candidate cells to form bonds. The cells connected to daughters are called granddaughters.
4. Repeat the procedure to granddaughters and granddaughters’ daughters ..., we will get a cluster, which grows until no bond-candidate cell can be found anymore.
5. Then choose another cell \( \beta \) from the left unbounded cells as another mother cell, and repeat the procedure starting from step 2.

In this way, every cell is assigned to a cluster. In every cluster, find the boundary cells if any, calculate the distance between every two boundary cells, and denote the maximum distance by \( d \). A cluster with \( d > \sqrt{2}R \) is called an infinite cluster. The probability \( P_\infty \) for the appearance of infinite cluster is defined as:

\[
P_\infty = \frac{\mathcal{N}_\infty}{\mathcal{N}},
\]

where \( \mathcal{N}_\infty \) is the number of events with infinite cluster, \( \mathcal{N} \) is the total number of events in the sample. In this model, \( P_\infty \) is the formation probability of QGP and will, therefore, be denoted in the following by \( P_{\text{QGP}} \).

III. THE DEPENDENCE OF THE PROBABILITY OF QGP FORMATION ON THE SYSTEM SIZE AND CENTRALITY

The bond percolation simulation is done for nucleus-nucleus collisions of different nuclei. The variation of \( P_{\text{QGP}} \) as a function of \( S_{\text{max}} \) is shown in Fig. 2 for nuclei with different nucleon number \( A \). It can be seen that for each kind of nucleus as the increase of \( S_{\text{max}} \), \( P_{\text{QGP}} \) gradually increases from 0 to a saturation value. This is typical for finite size percolation model, while for an infinite system \( P_{\text{QGP}}(S_{\text{max}}) \) would be a step function and the point where \( P_{\text{QGP}} \) starts to be greater than 0 would...
be the threshold \(S_c\). In our case the system is of finite size, so we use a function \[22\]

\[ P_{QGP}(S_{\text{max}}) = a [1 + \tanh(b(S_{\text{max}} - c))] \]

(2)
to fit the shape of \(P_{QGP}(S_{\text{max}})\) in Fig. 2 where \(a, b, c\) are parameters. It turns out that the fits are very good, cf. the curves in Fig. 2.

The inflexion point \(c\) of the fitting curve can be used as an evaluation of the threshold of \(S_{\text{max}}\): \(S_c = c\), and the saturation value of \(P_{QGP}\) is \(P_{QGP}^s = 2a\). The results obtained from the parameters of the fits are listed in Table I. It can be seen from Table I that comparing with small nuclei, the bigger ones have higher saturation values \(P_{QGP}^s\) and smaller \(S_c\).

**TABLE I:** The saturation value \(P_{QGP}^s\) of \(P_{QGP}\) and the critical percolation distance \(S_c\) for different size of nuclei.

| \(A\) | \(U\) | \(Pb\) | \(Au\) | \(Sn\) | \(Cu\) | \(S\) | \(Si\) | \(O\) | \(C\) |
|------|------|------|------|------|------|------|------|------|------|
| \(P_{QGP}^s\) (%) | 100 | 99.8 | 99.8 | 98.7 | 91.4 | 71.6 | 66.6 | 45.4 | 35.6 |
| \(S_c\) (fm) | 0.67 | 0.69 | 0.70 | 0.78 | 0.90 | 1.05 | 1.08 | 1.22 | 1.29 |

The \(P_{QGP}^s\) is the maximum (saturation) QGP-formation probability and the \(S_c\) is the point where the QGP-formation probability is half of the saturation value. Both of them depend on the function \(P_{QGP}(S_{\text{max}})\), where \(S_{\text{max}}\) is the maximum delocalization distance. As the increase of center-of-mass energy \(\sqrt{s_{\text{NN}}}\), the delocalization will happen in an increasing range and \(S_{\text{max}}\) increases. From Table I we see that larger nuclei have smaller \(S_c\), which means that the energy threshold to form QGP for larger nuclei is lower than that of the smaller ones. The maximum QGP-formation probability \(P_{QGP}^s\) for smaller nuclei are lower than those of the bigger ones. For small nuclei the QGP-formation probability is less than 100% even at very large \(S_{\text{max}}\), or equivalently at very high center-of-mass energy \(\sqrt{s_{\text{NN}}}\), i.e. the QGP-formation probability gets saturated. It can be seen from Table I that the saturation values of \(P_{QGP}\) for U-U, Pb-Pb and Au-Au collisions are about 100%, while those of smaller nuclei do not reach 100%. For example, Cu-Cu collisions only have about 91% events to form QGP at very large \(S_{\text{max}}\) (very high \(\sqrt{s_{\text{NN}}}\)). From the variation of \(P_{QGP}\) on nuclear size \(A\) shown in Fig. 3 we see that the saturation value of \(P_{QGP}\) decreases quickly when the nuclear size is less than that of copper.

We further transform the dependence of maximum (saturation) QGP formation probability on nuclear size to that on the centrality of Au-Au collisions at \(\sqrt{s_{\text{NN}}} = 200\) GeV by using the function between centrality and number of participants calculated from the Glauber model \[23\]. The dependence of QGP-formation probability on centrality in Au-Au collisions is also shown in Fig. 3. We see that for Au-Au collision at \(\sqrt{s_{\text{NN}}} = 200\) GeV, almost all the events with 0–40% centralities could produce QGP, while in the most peripheral collisions (80–100% centrality) there are only about 40% events that can form QGP.

**IV. CONCLUSION AND DISCUSSION**

It is argued that since the number \(n\) of multi-scattering in high energy nucleus-nucleus collisions has a binomial distribution with the average value of \(n\) proportional to \(A^{2/3}\) of the colliding nuclei, when the colliding nuclei are small the energy density of the medium produced in each collision may not be high enough to form quark-gluon plasma. Therefore, the probability \(P_{QGP}\) for QGP formation in high energy heavy ion collisions may not be 100% in all cases.

The bond percolation describes the quark delocalization and transition from hadronic matter to quark-gluon plasma reasonably. Assuming that the production probability of infinite clusters in the percolation model is the...
formation probability of quark-gluon plasma, we found from the percolation simulation that the maximum (saturation) QGP-formation probability depends on the nuclear size and collision centrality. Due to this dependence, when comparing experimental results of not very large nuclei and/or not very high centralities with theoretical predictions, an efficiency correction would be needed, since the data are a mixing of events with QGP and those without QGP.

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