Variations in the stellar CMF and IMF: from bottom to top

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ABSTRACT
We use a recently developed analytic model for the interstellar medium (ISM) structure from scales of giant molecular clouds (GMCs) through star-forming cores to explore how the prestellar core mass function (CMF) and, by extrapolation, stellar initial mass function (IMF) should depend on both local and galactic properties. If the ISM is supersonically turbulent, the statistical properties of the density field follow from the turbulent velocity spectrum, and the excursion set formalism can be applied to analytically calculate the mass function of collapsing cores on the smallest scales on which they are self-gravitating (non-fragmenting). Two parameters determine the model: the disc-scale Mach number $M_h$ (which sets the shape of the CMF) and the absolute velocity/surface density (to assign an absolute scale). We show that, for normal variation in disc properties and gas temperatures in cores in the Milky Way and local galaxies, there is almost no variation in the high-mass behaviour of the CMF/IMF. The slope is always close to Salpeter down to $\lesssim 1 M_\odot$. We predict modest variation in the sub-solar regime, mostly from variation in $M_h$, but this is consistent with the $\sim 1\sigma$ observed scatter in sub-solar IMFs in local regions. For fixed global (galaxy) properties, there is little variation in shape or 'upper mass limit' with parent GMC mass or density. However, in extreme starbursts – ultra-luminous infrared galaxies (ULIRGs) and merging galaxy nuclei – we predict a bottom-heavy CMF. This agrees well with the IMF recently inferred for the centres of Virgo ellipticals, believed to have formed in such a process. The CMF is bottom heavy despite the gas temperature being an order of magnitude larger, because $M_h$ is also much larger. Larger $M_h$ values make the 'parent' cloud mass (the turbulent Jeans mass) larger, but promote fragmentation to smaller scales (set by the sonic mass, not the Jeans mass); this shifts the turnover mass and also steepens the slope of the low-mass CMF. The model may also predict a top-heavy CMF for the in situ star formation in the sub-pc disc around Sgr A*, but the relevant input parameters are uncertain.

Key words: galaxies: active – galaxies: evolution – galaxies: formation – cosmology: theory.

1 INTRODUCTION
The origin of the stellar initial mass function (IMF) is a question of fundamental importance for the study of star formation, stellar evolution and feedback, and galaxy formation. It is an input into a huge range of models of all of these phenomena, and a necessary assumption when deriving physical parameters from many observations. However, despite decades of theoretical study, it remains poorly understood.

A particularly important question which has received great attention is whether the IMF is indeed ‘universal’ or in fact varies in some environments (for a recent review, see Kroupa et al. 2011). A growing body of observations find that the high-mass behaviour of the IMF in the Milky Way (MW) and nearby Small/Large Magellanic Cloud systems is remarkably uniform, with a Salpeter-like ($\alpha = 2.3$ in $dN/dM \propto M^{-\alpha}$) slope consistently appearing in essentially all directly measured systems: young clusters, globulars, disc, bulge, low/high-mass giant molecular clouds (GMCs), etc. (see e.g. Kroupa 2002; Chabrier 2003; Bastian, Covey & Meyer 2010, and references therein). There are suggestions of greater variation – albeit still modest – in how rapidly the IMF turns over in the sub-solar regime ($\lesssim 1 M_\odot$), but the observations remain uncertain at these masses (references above and e.g. Allen et al. 2005; Chabrier 2005; Thies & Kroupa 2007; Covey et al. 2008; Deacon, Nelemans & Hambly 2008; De Marchi, Paresce & Portegies Zwart 2010).

There may be, however, variation in more extreme systems. The IMF in most of the galactic centre is ‘normal’ (Löckmann, Baumgardt & Kroupa 2010), but there is compelling evidence that the pair of eccentric young circum-black hole (BH) nuclear discs at just $\sim 0.1–0.5$ pc from Sgr A* may have a top-heavy IMF (either with a flat slope, $\alpha = 1.7–2$, or a sharp sub-solar cutoff; see *E-mail: phopkins@astro.berkeley.edu

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Paumard et al. 2006; Maness et al. 2007; Bartko et al. 2010). These are believed to have formed in situ, from a circum-BH thin disc (a quasi-accretion disc), as opposed to e.g. galactic centre star clusters (that formed in their own clouds), and so may have had very different formation conditions from ‘normal’ stars. There have been a number of suggestions of top-heavy IMFs in high-star-formation-rate galaxies (e.g. Hoversten & Glazebrook 2008; Gunawardhana et al. 2011; Dabringhausen et al. 2012; Marks et al. 2012) and/or high-redshift galaxies (from studies of galactic evolution; see Hopkins & Beacom 2006; Davé 2008; van Dokkum 2008), but these are indirect, model-dependent constraints and subject to alternative interpretations. Recently, though, van Dokkum & Conroy (2010, 2011) and Spiniello et al. (2012) have attempted to directly constrain the IMF in the centres of Virgo and other nearby elliptical galaxies, and find that the observations strongly favour a bottom-heavy IMF (as steep as \( \alpha \sim 3 \)). This is independently suggested by kinematic and lensing data (see e.g. Auger et al. 2010; Treu et al. 2010; Cappellari et al. 2012; Dutton et al. 2013) as well as spectral modelling (references above and Ferreras et al. 2013). Since these galaxies formed their stars at high redshifts, this is a powerful constraint on any redshift evolution in the IMF. Moreover, it is well established that stars in the central regions of ellipticals are formed via inflows of gas to large densities in extreme (probably merger-induced) nuclear starbursts; this is the process seen ‘in action’ in the centres of local ultra-luminous infrared galaxies (ULIRGs: see Kormendy & Sanders 1992; Mihos & Hernquist 1994; Hibbard & Yun 1999; Rothberg & Joseph 2004; Hopkins et al. 2008, 2009, and references therein). So this should be directly related to the IMF in starburst environments, which many theoretical models have predicted should be top-heavy – the opposite of what is observed – owing to the higher observed gas temperatures and potentially large gas accretion rates (see e.g. Padoan, Nordlund & Jones 1997; Larson 2005).

Explaining the weak IMF variation in ‘normal’ systems has actually been a challenge for theoretical models, which often predict large IMF variations with e.g. the local thermal Jeans mass (although see Jappsen et al. 2005). It is a particularly powerful constraint for any model which attempts to predict IMF variations in systems where it has been suggested. There is no simple analytic model which has been shown to naturally explain both the weak IMF variation in MW and Local Group systems while also predicting large IMF variations in more extreme environments similar to those above.

In this paper, we therefore attempt to develop an analytic understanding of IMF variation and apply it to these observations. It is increasingly clear that the interstellar medium (ISM) is governed by supersonic turbulence over a wide range of scales. Numerical and analytic arguments have shown that the density probability distribution function (PDF) in this regime approaches a lognormal distribution (Vazquez-Semadeni 1994; Padoan et al. 1997; Ostriker, Gammie & Stone 1999). Hennebelle & Chabrier (2008) used this to propose a simple theoretical model for the stellar IMF, similar to the derivation of the halo mass function (MF) in Press & Schechter (1974); if the PDF is lognormal, one can calculate the fraction of gas above a critical density (essentially the Jeans criterion for thermal+turbulent support) on a given scale, and associate this with the MF. However, like the Press & Schechter (1974) argument, this is only approximate as it does not resolve the ‘cloud in cloud’ problem. Moreover, the focus therein was exclusively on small-scale properties, so a number of properties must be simply assumed rather than calculated from first principles and the shape and normalization of the IMF cannot be predicted a priori from global galaxy properties. Recently, Hopkins (2012a, hereafter Paper I) and Hopkins (2012b, hereafter Paper II) generalized this by showing how the mathematical excursion set formalism can be applied to turbulent density fields in galaxy discs, over the entire dynamic range from galactic scales to below the sonic length. This resolves these ambiguities, and in principle allows for a rigorous calculation of the core mass function (CMF), defined as the last-crossing distribution – specifically, the MF of bound objects defined on the smallest scales on which they remain self-gravitating but do not have self-gravitating sub-regions (i.e. are not fragmenting). Paper II showed that the resulting CMF agrees well with CMFs observed, and [provided some plausible mean core-to-stellar mass conversion efficiency as in Matzner & McKee (2000), which we stress is not explicitly part of the model] can explain all of the major features of the canonical MW stellar IMF. Moreover, because the model self-consistently includes the scales of the parent disc (where most of the turbulent power is concentrated), the distribution of parent cloud properties left as free parameters in Hennebelle & Chabrier (2008) can be derived; Paper I showed that the model is completely specified by the assumed turbulent velocity power spectrum and global disc properties (surface density, velocity dispersion).

In this paper, we use the model for the CMF from Paper II to study how the predicted IMF might vary as a function of galactic properties and across different MW regions. In Section 2, we summarize the model. In Section 3, we derive some qualitative scalings for how the resulting CMF should vary with global parameters. In Section 4, we compare the exact model results for different observed properties of the MW, starburst/ULIRG systems and the circum-BH MW stellar disc. In Section 5, we compare the model predictions for more typical variations in properties across the MW disc and in nearby galaxies, and examine how the CMF should vary within the MW in molecular clouds (MCs) of different masses and regions of high/low densities. In Section 6, we summarize our results and discuss the implications of the model.

Before proceeding, we must caution that what the model here predicts directly is the pre-stellar CMF, not the stellar IMF. It is by no means clear that the two can be trivially identified with one another (modulo some mean ‘efficiency’ of core-to-stellar mass conversion). In addition to the turbulent gas structure, the processes of fragmentation as cores contract and form protostars, accretion on to those protostars, outflows expelling material from cores and ejection of protostars before accretion is exhausted can all change the relation between CMF and IMF. It is beyond the scope of this paper to include all of these physics and therefore to make a rigorous absolute prediction of the IMF. Our intention is instead to study how the CMF varies as a function of global galaxy and turbulent properties. Our qualitative expectation is that otherwise identical CMFs, with similar microphysics, should produce a similar stellar IMF, and more bottom/top-heavy CMFs should correspond to more a bottom/top-heavy IMF. These assumptions are valid if the local star formation process depends only on e.g. the global core mass and/or stellar microphysics. But lacking a complete understanding of star formation, it remains possible that these microphysics will serve to ‘regulate’ against variations in the IMF in a manner that offsets some of the predictions here.

2 THE CMF AND IMF

If density fluctuations in supersonic turbulence are lognormal, then the variable \( \delta(x) \equiv \ln \left( \rho(x)/\rho_0 \right) + S/2 \), where \( \rho(x) \) is the density...
at a point $x$, $\rho_0$ is the global mean density and $S$ is the variance in $\ln \rho$, is normally distributed according to the PDF: 

$$P_0(\delta \mid S) = \frac{1}{\sqrt{2\pi S}} \exp \left( -\frac{\delta^2}{2S} \right).$$  (1)

More generally, we can evaluate the field $\delta(x \mid R)$, which is the $\delta(x)$ field averaged around the point $x$ with some window function of characteristic radius $R$; this is also normally distributed, with a variance $S(R)$ that is directly related to the density power spectrum.

Paper II showed that the CMF is equivalent to the last-crossing distribution, given by the numerical solution to the Volterra integral equation:

$$f_l(S) = g_l(S) + \int_S \, dS' \, f_l(S') \, g_2(S, S'),$$  (2)

where

$$g_l(S) = \left[ 2 \frac{d B}{dS} - \frac{B(S)}{S} \right] P_0(B(S) \mid S)$$  (3)

$$g_2(S, S') = \left[ \frac{B(S) - B(S')}{S - S'} + \frac{B(S)}{S} - 2 \frac{d B}{dS} \right] \times P_0(B(S) - B(S') \mid (S') \times (S - S)(S/S'))$$  (4)

and $B(S)$ is the minimum value of the overdensity $\delta(x \mid R)$ which defines objects of interest (here, self-gravitating regions).

In Paper I we derive $S(R)$ and $B(S)$ from simple theoretical considerations for all scales in a galactic disc. For a given turbulent power spectrum, $S(R)$ is determined by summing the contribution from the velocity variance on all scales $R > R$

$$S(R) = \int_0^\infty |W(k, R)|^2 \ln \left[ 1 + \frac{3}{4} \frac{\tilde{v}_2^2(k)}{c_s^4 + k^2} \right] \, dk,$$  (5)

where $W$ is the window function for the smoothing. B(R) is

$$B(R) = \ln \left( \frac{\rho_{\text{crit}}}{\rho_0} \right) + \frac{S(R)}{2}$$  (6)

$$\frac{\rho_{\text{crit}}}{\rho_0} = \frac{Q}{2k} \left( 1 + \frac{h}{R} \right) \left[ \frac{\sigma_2^2(R)}{\sigma_2^2(h)} + \frac{R^2}{h^2} \right],$$  (7)

where $\rho_0$ is the mean mid-plane density of the disc, $Q = \sigma_2[h]/(\pi G \Sigma_{\text{gas}})$ is the standard Toomre $Q$ parameter, $\tilde{k} = k/\Omega = \sqrt{2}$ for a constant-$V_c$ disc, and

$$\sigma_2^2(R) = c_s^4 + \langle \tilde{v}_2^2(R) \rangle.$$

1 The $+S/2$ term in $\delta$ is required so that the integral of $\rho \, P_0(\rho)$ correctly gives $\rho_0$ with $\langle \delta \rangle = 0$.

2 For convenience we take this to be a $k$-space tophat inside $k < 1/R$, which is implicit in our previous derivation, but we show in Paper I and Paper II that this has little effect on our results.

3 In what follows, we will assume $Q \sim 1$ in calculating some values; observed and simulated systems seem to uniformly converge to this value independent of the detailed physics (see e.g. Gammie 2001; Hopkins, Quataert & Murray 2012; but also Marks & Kroupa 2010). We stress, though, that the way it enters the equations here is only as a parameter encapsulating the ratio of the ‘normal’ density to that required for self-gravity (i.e. a virtual-like parameter), not as any statement about whether the global disc is in ‘equilibrium’ (the equations above also apply to a non-steady-state system). As a result, unless $Q$ is very large, the effect on the quantities shown here is primarily only on their normalization (suppressing it with larger $Q \propto \rho_{\text{crit}}$), which we treat as arbitrary.

Here and above $\Sigma_{\text{gas}}$ is the disc gas surface density and $h$ the (exponential) disc scaleheight (which can freely vary with radius). The mapping between radius and mass is

$$M(R) \equiv 4\pi \rho_{\text{crit}} h^3 \left[ \frac{R^2}{2h^2} + \left( 1 + \frac{R}{h} \right) \exp \left( -\frac{R}{h} \right) - 1 \right].$$  (9)

It is easy to see that on small scales, these scalings reduce to the Jeans criterion for a combination of thermal ($c_t$) and turbulent ($v_s$) support, with $M = (4\pi/3)\rho_{\text{crit}} R^3$; on large scales it becomes the Toomre criterion with $M = \pi \Sigma_{\text{gas}} R^2$. The MF is

$$\frac{dn}{dM} = \frac{\rho_0(M)}{M} f_0(M) \left| \frac{dS}{dM} \right|.$$

(10)

There are only two parameters that completely specify the model in dimensionless units. These are the spectral index $p$ of the turbulent velocity spectrum, $E(k) \propto k^{-p}$ (usually $p \approx 5/3$–2), and its normalization, which we define by the Mach number on large scales $M_h^2 \equiv \langle v_s^2(h) \rangle/c_s^2$. The dimensional parameters $h (or c_t)$ and $\rho_0$ simply rescale the predictions to absolute units.

In Hopkins (2013a), we extended the derivation to solve the ‘two-barrier’ problem. This is the solution for $f_l(S \mid \delta_0[S_0])$, i.e. given some value $\delta_0$ on a larger scale $S_0 < S$. We refer there for details, but note that this ultimately reduces to the solution for $f_l(S)$ above with the replacements $B(S) \rightarrow B(S - \delta_0, B(S') \rightarrow B(S') - \delta_0, S \rightarrow S - S_0$ and $S' \rightarrow S - S_0$.

3 Qualitative scalings

For a choice of $p$ and $M_h$, it is straightforward to numerically determine the last-crossing MF (CMF). Unfortunately a closed-form solution is not generally possible. However, we show in Paper II that on sufficiently small scales (nearbelow the sonic length), the ‘run’ in $S(R) \approx S_0(\rho_{\text{crit}})$ becomes small [since most of the power contributing in equation (5) comes from large scales], while $B(S)$ rises rapidly, so $dB/dS \gg B(S)/S \gg 1$. In this limit, the MF can be approximated as

$$\frac{dn}{dM} \sim \frac{\rho_0(M)}{M^2 \sqrt{2\pi S_0}} \left| \frac{dln \rho_0}{dln M} \right| \exp \left[ -\left( \frac{\ln \rho_0 + S_0/2}{2S_0} \right) \right].$$  (11)

Since we are interested in the behaviour on small scales, we drop higher order terms in $h$ and can re-write $\rho_{\text{crit}}$ in terms of the sonic length

$$R_{\text{sonic}} \equiv R[v_s^2 = c_s^2] = h \cdot M_{\text{sonic}}^{2/(p-1)},$$  (12)

which for $p = 2$ becomes just

$$\rho_{\text{crit}} \rho_0 = \frac{M_h^2}{\sqrt{2}} \left( \frac{R_{\text{sonic}}}{R} \right) \left[ 1 + \frac{R_{\text{sonic}}}{R} \right],$$  (13)

with $M \approx (4\pi/3)\rho_{\text{crit}}(R) R^3$. There is clearly a change in behaviour below $R \approx R_{\text{sonic}}$. In Paper II we show that $R \gtrsim R_{\text{sonic}}$ (where $\rho_{\text{crit}} \propto M^{-1/2}$) corresponds to the high-mass end of the CMF/IMF, giving a nearly power-law behaviour, while $R \ll R_{\text{sonic}}$ (where $\rho_{\text{crit}} \propto M^{-2}$) corresponds to the low-mass end, where there is a quasi-lognormal turnover in the CMF and IMF.

The turnover mass is related to $M_{\text{sonic}} \equiv M(R_{\text{sonic}})$.

$$M_{\text{sonic}} = \frac{2}{3} c_{s}^4 R_{\text{sonic}} G = \frac{2\sqrt{2}}{3\pi} \frac{Q^{-1}}{G} \Sigma_{\text{gas}}^4,$$

where in the second equality we use the standard Toomre $Q$ parameter to relate this to global parameters.

Note that this is the global thermal Jeans mass in a disc – not, actually, the local Jeans mass $\propto c_s^3 G^{-3/2} \rho^{-1/2}$, which is
dimensionally the same but differs by powers of $M_{\bullet}$ and $\rho/\rho_0$.

Both however are different from the turbulent Jeans mass

$$M_{\text{turb,Jeans}} \approx \frac{\sigma^2_{\text{turb}}(h)}{\pi G^2 \Sigma_{\text{gas}}},$$

(15)

We showed in Paper I that $M_{\text{turb,Jeans}}$ corresponds to the characteristic mass of MCs/GMCs.

From equation (11), when $R \gg R_{\text{sonic}}$, the high-mass slope of the CMF $(dn/dM \propto M^{-\alpha})$ is approximately

$$\alpha_{\text{high}} \approx \frac{3 (1 + p^{-1})}{2} + \frac{(3 - p)^2}{2 S(M)} \ln \left(\frac{M}{M_0}\right) - \frac{p + 2}{2 S(M)} \ln 2,$$

(16)

where $M_0 = \rho_0 h^3$ and the second term (in $S_0$) is small for the reasonable mass range ($\ll 0.1$), so we find

$$\alpha_{\text{high}} \approx \frac{3}{2} (1 + p^{-1})$$

nearly independent of the Mach number $M_{\bullet}$. The exact solution reproduces this but with an even weaker $p$ dependence (Paper II).

On the other hand, the low-mass slope is

$$\alpha_{\text{low mass}} \approx 3 - \frac{1}{5} (\ln 2 - 4 \ln M_{\bullet} - 2 \ln (M/M_{\text{sonic}}))$$

(18)

which for typical parameters (using the approximate scaling $S_0 \propto \ln M_{\bullet}^2$) reduces to

$$\alpha_{\text{low}} \approx 1.9 + \frac{0.6}{\ln M_{\bullet}} \ln (M/M_{\text{sonic}}).$$

(19)

4 CMF AND IMF VARIATION IN EXTREME SYSTEMS

We now consider an observationally motivated example.

First, we consider canonical MW parameters: a minimum temperature $T \sim 10$ K for the cores, disc gas surface density $\Sigma_{\text{gas}} \sim 10 M_\odot$ pc$^{-2}$ and large-scale turbulent velocity dispersion $\sigma_g(h) \sim 10$ km s$^{-1}$ ($M_{\bullet} \sim 30$). For all cases, we will assume a spectrum $p = 2$, which is what is expected in highly supersonic simulations and suggested by observations of the ISM (Burgers 1973; Larson 1981). As discussed above and in Paper II, our results are not qualitatively changed for $p = 3/5$ instead. Together this is sufficient to completely specify the model.

Next, consider canonical parameters in ULIRG/starburst regions. Observations typically find $T \sim 60$–80 K in the dense molecular gas ($\sim 10^6$–$10^7$ cm$^{-3}$) with turbulent velocity dispersions $\sigma_g(h) \sim 40$–100 km s$^{-1}$ ($M_{\bullet} \sim 50$) (Downes & Solomon 1998; Bryan & Scoville 1999; Westmoquette et al. 2007; Greve et al. 2009) and surface densities $\Sigma_{\text{gas}} \sim 10^3$–$10^4 M_\odot$ pc$^{-2}$ (e.g. Kennicutt 1998).

In Fig. 1, we compare the CMF predicted in each case. Although the broad qualitative behaviour is similar, there are striking differences. As expected from our analysis in Section 3, the characteristic mass in the ULIRG case is somewhat smaller. From equation (14), compare $M_{\text{sonic}} \sim 1.6 M_\odot$ (MW) to $M_{\text{sonic}} \sim 0.4 M_\odot$ (ULIRG).

But even if we ignore the shift in the scale of $M_{\text{sonic}}$, the low-mass turnover occurs much more slowly in the ULIRG case, as expected from equation (19) from the higher $M_{\bullet}$. The high-mass slope is nearly identical in both cases, as expected from equation (17), but there is an intermediate-mass steepening in the ULIRG case that owes to second-order effects from the run in $S(R)$.

An even more extreme example is the pc-scale circum-BH disc in the MW. Here, the initial gas conditions are uncertain, but from the observations, we can take the effective radius $R \approx 0.5$ pc and infer a surface density $\Sigma_{\text{gas}} \sim 0.6 \epsilon^{-1} \times 10^4 M_\odot$ pc$^{-2}$ (where $\epsilon = M_{\bullet}/M_{\text{gas}}$ is the unknown star formation efficiency of the progenitor disc). Simulations of these disc BHs in Nayakshin, Cuadra & Springel (2007) and Hobbs & Nayakshin (2009) suggest an order-unity $\epsilon \sim 0.5$ and $\sigma_g \sim 2$ km s$^{-1}$, which is approximately what is needed to give $Q \sim 1$ (for $M_{\text{BH}} = 4 \times 10^4 M_\odot$, and $\epsilon \approx 0.5$ for the quasi-Keplerian orbit here). We adopt the same typical nuclear gas temperatures as the ULIRG case. The CMF in this case is very different from the previous cases. This is because the system is extremely thin and $\kappa$ large, dominated by the external BH potential, so $M_{\bullet}$ is relatively small, but also our assumptions that $R_{\text{sonic}} \ll h$ and $d\beta/dS \gg 1$ break down and the correction terms in $f_s$ not captured in equation (11) dominate.

5 CMF AND IMF VARIATION IN DIFFERENT LOCAL REGIONS

The above examples are intentionally extreme. Normal galaxies exhibit a much more narrow range of $\Sigma_{\text{gas}}$, $T$ and $\sigma_g$. In Fig. 2 we compare the CMF predicted for a ‘typical’ range in these parameters. In all these cases, the high-mass slope is nearly identical and close to the Salpeter value. Modest changes in $M_{\bullet}$ still lead to differences at the low-mass end, although the shape is quite similar. We compare these variations to the observed range inferred for the IMF at these masses in different systems [or equivalently, at
these masses, the present-day mass function (PDMF)]; the range in ‘turnover speed’ is consistent.4

There is also the question of how the CMF/IMF should vary within different MCs or sub-regions of the Galaxy with fixed global properties. Recall that what we calculated in Figs 1–2 was the CMF integrated over the entire galaxy. One advantage of our approach is that it automatically includes the fact that star formation is clustered (see Hopkins 2013a), with correlated star formation inside of identifiable large-scale bound gas clouds (MCs) with a broad spectrum of properties in good agreement with observations (see Paper I). Indeed, by definition, every last crossing is embedded inside a larger scale first crossing, so in calculating the CMF we have implicitly summed over all ‘clusters’ (defined loosely here as all regions with cores inside of a parent self-gravitating cloud), the same as is done observationally when summing the contributions from different clusters or integrating galaxy-wide (see e.g. Weidner & Kroupa 2006; Kroupa et al. 2011). But it is also interesting to ask how the CMF might vary within those specific regions.

To address this we use the solution to the two-barrier problem, which lets us calculate the CMF for any given overdensity on a larger ‘parent’ scale. In Paper I, we identified self-gravitating regions on larger scales (which contain sub-regions of fragmentation), i.e. crossing the barrier \( B(S_0) \) at smaller \( S_0 < S(M) \), as MCs. We can therefore calculate the CMF for an MC of size/mass scale \( R(S_0) \) or \( M(S_0) \) by taking the two-barrier solution with \( \delta(S_0) = B(S_0) \). We show this in Fig. 3 for a range of ‘parent’ MC scales. We can further arbitrarily vary \( \delta_0 \) on each scale: we consider regions more/less dense than the ‘collapse threshold’ (typical MC) at the same spatial scale. The results are typically less sensitive to parent cloud properties than the other parameters we have considered. Because the global parameters are fixed, parent scale does not change \( M_h \) or the sonic length – it is largely equivalent to shifting the ‘background’ mass normalization. The high-mass end is suppressed in lower mass MCs, when the MC masses are sufficiently low that they cannot contain the highest mass cores.

6 DISCUSSION

We have used the method from Paper I and Paper II, which developed a formal means to calculate the statistical properties of bound objects formed by turbulent density fluctuations in a galactic disc, to study the predicted variation in the pre-stellar CMF, and by implication the stellar IMF, across different environments. Specifically, in Paper I, we showed how the full excursion set formalism can be applied to lognormal density distributions in a galactic disc, and demonstrated that, for a given turbulent power spectrum shape \( \langle \rho(R_{\text{cloud}}) \rangle^{1/2} \) increases as \( R_{\text{cloud}} \) from \( \sim 0.1 \, M_\odot \) to \( \sim 1.5 \, M_\odot \), the CMF changes weakly with parent mass, provided it is large enough to properly sample the mass range. The high-mass end is suppressed in the lowest mass clouds, but mostly because they lack sufficient mass to form many high-mass cores.
velocity dispersions $\sigma_g \sim 5$–15 km s$^{-1}$ and galaxy surface densities $\Sigma_{gas} \sim 1$–20 M$_\odot$ pc$^{-2}$ – there is almost no variation in the high-mass (Salpeter) behaviour. In Paper I, we showed that this is also true if we vary the turbulent power spectrum shape $p = 5/3$–2. The low-mass end is somewhat sensitive to the global Mach number $M_h$, turning over more slowly in systems with higher $M_h$ (per equation 19). The same result was obtained with a more approximation derivation of the CMF in Hennebelle & Chabrier (2008, 2009). The change is not especially large, however, corresponding to a range of slopes $\alpha \sim 0$–1 at $\sim 0.1$ M$_\odot$ (with larger, but much more uncertain, variation below this). We stress that this is the global $M_h$; it does not necessarily imply variation in regions with different local $M_h$ (since their density field has contributions from all scales).

To address that question, in Hopkins (2013a), we extend the excursion set model to derive the solution to the two-barrier last-crossing problem. This corresponds to the solution for the CMF in sub-regions of any specified size/mass scale, within a common parent system of fixed global properties. We apply this here to examine how the CMF should vary in different MC sub-regions within the MW (recall that our galaxy-wide prediction implicitly integrates over the CMF in all individual MCs with the full spectrum of properties over the entire disc). Since in Paper I we show that MCs can be identified as solutions to the first-crossing problem, we can specifically show how the CMF should vary in ‘typical’ MCs of different masses and higher/lower density regions. The variation is minimal in all regions of modest mass (despite systematically different densities and local $M_h$).

At low MC masses the CMF is truncated, but this is basically just because there is not enough collapsing mass to form the most massive cores. Since the mass density in MCs is primarily in high-mass systems and this occurs only in the lowest mass clouds, it has small global effects (but see Weidner & Kroupa 2006; Weidner, Kroupa & Bonnell 2010). The mass scale of the predicted CMF does not vary systematically with parent MC mass.

These predictions agree with the fact that there is remarkably little variation in the high-mass IMF behaviour observed in different MW and Local Group star-forming regions. And the predicted low-mass variation in these regions is in fact quite similar to the scatter inferred between different regions observationally (although some of this may owe to observational uncertainties).

If, however, we consider much more extreme systems, we predict significant variations in the CMF. For the typical conditions in ULIRGs and other dense, starburst regions, we predict a bottom-heavy CMF. There is strong observational and theoretical evidence that the stars in the central $\sim$kpc of essentially all ellipticals must have formed in nuclear (probably merger-induced) starbursts, and the properties of recent ‘ULIRG/starburst relics’ agree well with elliptical/bulge centres (see e.g. Kormendy & Sanders 1992; Hibbard & Yun 1999; Rothberg & Joseph 2004; Hopkins et al. 2008, 2009). In fact, Hopkins & Hernquist (2010) showed that the central densities and mass profile structure of ellipticals imply, if they obeyed any sort of Kennicutt–Schmidt relation in their formation, that they must have been starbursts with densities and velocity dispersions quite similar to what we assume in Fig. 1. This may, therefore, explain recent observational suggestions that the central regions of nearby Virgo and other massive ellipticals exhibit a bottom-heavy IMF similar to that predicted here (Treu et al. 2010; van Dokkum & Conroy 2010, 2011; Cappellari et al. 2012; Ferreras et al. 2013; Spiniello et al. 2012).

The key reason for this is that these regions have much larger global Mach numbers $M_h$ than the typical MW regions, even if we allow for much larger minimum gas temperatures $T \sim 50$–100 K. Mathematically, the predicted bottom-heavy CMF in ULIRGs corresponds to the slow low-mass turnover predicted in equation (19), although there are higher order corrections that only appear in the exact solution (not the approximate CMF derivation in Hennebelle & Chabrier (2008)). Physically, this is a result of turbulent fragmentation. Although the characteristic turbulent Jeans mass – which determines the largest structures – increases (scaling $\propto \sigma^3$), this appears in the first-crossing distribution: i.e. it means larger MCs/GMCs (in size and mass), not larger cores. The MC/first-crossing structures are not directly relevant, since they will be self-gravitating on many smaller scales – i.e. fragment – internally. But the CMF turnover is not exactly set by the local thermal Jeans mass either (recall that we also assume significantly higher minimum temperatures in this regime). Strictly, the mass scale of the last-crossing distribution is set by the sonic mass, so there is some competition between higher $M_h$ (which drives the sonic length down) and higher $c_s$. If the system is globally stable, we can equate this mass scale to purely global parameters, and infer that the rise in surface density $\Sigma_{gas}$ is sufficiently rapid that it ‘wins out’ and lowers the sonic mass. However, even if this mass scale were the same or somewhat higher, the CMF would still be quite bottom-heavy: a higher $M_h$ not only shifts the sonic mass scale, but also imprints larger density fluctuations in the turbulent field on all scales. This promotes more fragmentation over a more broad range of masses, ‘slowing down’ the low-mass turnover or even steepening the slope towards low masses (for simulations demonstrating this, see e.g. Ballesteros-Paredes et al. 2006). The same physics may explain suggestions of more bottom-heavy IMFs at higher metallicities (both galactic and extragalactic; see Kroupa & Gilmore 1994; Kroupa 2001; Marks et al. 2012, and references above), as more efficient cooling can give rise to systematically higher $M_{MC}$.

This is an example where the distinction between the first- and last-crossing distributions is critical: if we ignored it, we would arrive at similar conclusions to Padoan et al. (1997) – essentially, the exact opposite of what we find – where the authors argued that starburst systems should have more top-heavy IMFs. Also, it is clearly critical to account for the different global turbulent conditions, presumably driven on the largest scales in a galactic disc (and therefore not necessarily captured in many idealized simulations of small volumes), since it is the competition between turbulent density fluctuations and thermal support (not just the latter) that determines the shape of the CMF.

Another extreme system is the $\sim$0.5-pc-scale circum-BH disc in the MW, which observations have suggested may have a top-heavy IMF (Pandram et al. 2006; Maness et al. 2007; Bartko et al. 2010). We attempt to extend our estimate to this system as well, although we caution that the characteristic parameters during its formation are quite uncertain and our model may not be applicable at all because of non-linear tidal and eccentricity corrections (Alexander et al. 2008). The predicted CMF is somewhat top-heavy, but it also has an unusual shape which is much more restricted in mass range (truncated by shear and tidal forces). Part of this top-heavy character owes to its having a relatively small $M_h$ if it has $Q < 1$ (for a $Q = 1$ disc near a dominant BH, $\sigma_g/V_c \sim M_{BH}/M_{BH} \sim 10^{-2}$ here). But it is complicated because the disc has such a small scaleheight that the distinction between first and last crossings is blurred. Nevertheless, it is suggestive and agrees with results from numerical hydrodynamic simulations (Nayakshin et al. 2007; Hobbs & Nayakshin 2009). We caution that, as discussed in Hennebelle & Chabrier (2009), the low-mass end is also the trans sonic regime (below the sonic limit of $\sim 1$–30 km s$^{-1}$), where the CMF begins to steepen at $\sigma_g/V_c \sim 0.5$, as noted in Paper I.

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length), so our simple assumption of isothermal gas will break down at some level. Complicated equations of state, magnetic fields, radiative feedback and cosmic ray heating are certainly capable of modifying the relation between CMF and IMF; in simulations, these add to the thermal support in starburst regions and have motivated arguments for a top-heavy IMF (see e.g. Clarke, Bonnell & Hillenbrand 2000; Klessen & Burkert 2000; Kroupa & Bouvier 2003; Bate & Bonnell 2005; Jappsen et al. 2005; Klessen, Spaans & Jappsen 2007; Papadopoulos et al. 2011). Within the context of the present, simplified model, we can only approximately account for the effect of these physics, manifested in their changing the isothermal ‘floor’ temperature (which we freely vary). Preliminary comparison of models with more complicated equations of state in Hopkins (2013b) suggests that this may capture some of the lowest order effects, if the effective temperature is well chosen, but this probably remains the largest uncertainty in our model. On the other hand, simulations can only explore a limited parameter space, and owing to severe resolution and timestep demands very few have considered Mach numbers and densities as extreme as observed in real starbursts. Further development of the models, to allow non-linear changes in the thermal state of the gas especially through the time-dependent fragmentation process of contracting cores, would therefore be of great interest. This is critical to address fundamental questions, such as whether (especially at high masses) single cores can be associated with single stars, and (if not) whether we can statistically map the CMF to the IMF at all. For now, our predictions should not be taken too literally as an absolute prediction for the IMF. Rather, our intention is to motivate some potential observed IMF variations (or lack thereof) as a consequence of the variations in the pre-stellar CMF with global galaxy properties.

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