Two methods to reduce the CPU time needed for the numerical evaluation of cross-sections and similar quantities are discussed.

1 Introduction

The numerical evaluation of cross-sections and similar quantities can be expensive in terms of CPU time, in particular if radiative correction are included, and/or if the model has parameters that need to be scanned over. This contribution discusses new and improved algorithms for phase-space integration and parallelization of parameter scans to speed up the calculation. Both methods have successfully been implemented in version 4 of the package FormCalc.

2 Phase-space integration

The recently completed CUBA library provides four subroutines for multi-dimensional numerical integration. All four have a very similar invocation and can thus be interchanged easily, e.g. for comparison. The flexibility of a general-purpose method is particularly useful in the setting of automatically generated code. The following algorithms are contained in the CUBA library:

- **Vegas** is the classic Monte Carlo algorithm which uses importance sampling for variance reduction. It iteratively builds up a piecewise constant weight function, represented on a rectangular grid. Each iteration consists of a sampling step followed by a refinement of the grid. The present implementation uses Sobol quasi-random numbers for sampling.

- **Suave** is a crossover between Vegas and Miser and combines Vegas-style importance sampling with globally adaptive subdivision: Until the requested accuracy is reached, the region with the largest error is bisected along the axis in which the fluctuations of the integrand are reduced most. In each half the number of new samples is prorated for the fluctuation.

- **Divonne** is a further development of the CERNLIB routine D151. It is intrinsically a Monte Carlo algorithm but has cubature rules built in for comparison, too. The variance-reduction method is stratified sampling. In a first
step, a tessellation of the integration region is constructed in which all subregions have an approximately equal value of the spread, defined as

\[ s(r) = \frac{1}{2} \text{Vol}(r) \left( \max_{\vec{x} \in r} f(\vec{x}) - \min_{\vec{x} \in r} f(\vec{x}) \right). \]  

(1)

Minimum and maximum here are sought using methods from numerical optimization. The subregions are then sampled independently with a number of points extrapolated to reach the required accuracy. For each region, the latterly obtained value is compared to the initial rough estimate and if the two are not compatible within their errors, the region is subdivided or sampled once more. Additions to CERNLib’s D151 are the final comparison phase and the possibility to point out known extrema, to speed up convergence.

*Cuhre* is a new implementation of dcuhre. It is a deterministic algorithm which employs cubature rules of a polynomial degree. Variance reduction is by globally adaptive subdivision: Until the requested accuracy is reached, bisect the region with the largest error along the axis with the largest fourth difference.

Fig. 1 compares the performance of the four algorithms for a real phase-space integration of the process \( e^+ e^- \rightarrow \bar{t} t \gamma \). Above all it is very important to have several independent integration methods to cross-check the results.

3 Parallelization

Calculations in models like the MSSM, where not all input parameters are yet known, often require extensive scans to cover an interesting part of the
parameter space. Such a scan can be a real CPU hog, but on the other hand, the calculation can be performed completely independently for each parameter set and is thus an ideal candidate for parallelization. The real question is thus not how to parallelize the calculation, but how to automate the parallelization.

The method is quite general, but consider FormCalc for a specific instance. The user may specify parameter loops by defining preprocessor variables, e.g.

```
#define LOOP1 do 1 TB = 2, 30
```

These definitions are substituted at compile time into a main loop (see below). The obstacle to automatic parallelization is that the loops are user-defined and in general nested. A serial number is introduced to unroll the loops:

| serial version | parallel version |
|----------------|------------------|
| LOOP1          | serial = 0       |
| LOOP2          | LOOP1            |
|                | LOOP2            |
|                | :                |
|                | calculate cross-section |
| 1              | serial = serial + 1 |
|                | if( serial not in allowed range ) goto 1 |
|                | calculate cross-section |
|                | 1 continue       |

The serial number range can be specified on the command line so that it is quite straightforward to distribute patches of serial numbers on different machines. Most easily this is done in an interleaved manner, since one then does not need to know to which upper limit the serial number runs, i.e. if there are \( N \) machines available, send serial numbers 1, \( N + 1 \), \( 2N + 1 \), etc. on machine 1, send serial numbers 2, \( N + 2 \), \( 2N + 2 \), etc. on machine 2, etc.

This procedure is completely automated in FormCalc: The user once creates a .submitrc file in his home directory and lists there all machines that may be used, one on each line. In the case of multi-processor machines he puts the number of processors after the host name. The executable compiled from FormCalc code, typically called run, is then simply prefixed with submit. For instance, instead of “run uuuu 500,1000” the user invokes “submit run uuuu 500,1000.” The submit script uses ruptime to determine the load of the machines and ssh to log in. Handling of the serial number is invisible to the user.

1. T. Hahn, M. Pérez-Victoria, Comp. Phys. Commun. 118 (1999) 153 [hep-ph/9807565]; T. Hahn, hep-ph/0406288
2. T. Hahn, hep-ph/0404043