Effect of correlated noise channels on quantum speed limit

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We study the effect of correlated Markovian noise channels on the quantum speed limit of an open system. This is done for correlated dephasing and amplitude damping channels for a two qubit atomic model. Our model serves as a platform for a detailed study of speed of quantum evolution in correlated open systems.

I. INTRODUCTION

There is arguably no perfect isolation of a system from its surroundings. When the system interacts with its environment, the quantum mechanical nature of the system may get lost as the system “mixes” with the environment over time. Generally, we designate a system in contact with an environment as an open system [1]. The theory of an open quantum system leads to dissipation in the form of decoherence [2]. The concept of memory arises when the interaction of system with its environment is non-Markovian.

Many theoretical approaches [5–12] have been introduced to retrieve information about the state which was lost due to environment effect. The concept of memory as an open system [1] is done for correlated dephasing and amplitude damping channels for a two qubit atomic model. Our model serves as a platform for a detailed study of speed of quantum evolution in correlated open systems.

In this paper, we aim to study the quantum speed of evolution for correlated Markov noise, which has been proposed by Macchiavello and Palma [13]. Bound to the speed of evolution derives from a time-energy uncertainty relation for a system undergoing completely positive trace preserving (CPTP) evolution. Two different kinds of Markov noise have been taken into consideration: one is amplitude damping channel and another one is phase damping channel. We attempt to connect the concept of correlated noise to that of quantum speed limit. As is well-known, there are many varied applications of quantum speed limit, including quantum metrology [15], computational limits of physical systems [16], and development of quantum optimal control algorithms [17].

We generalise the master equation for a two qubit atomic system [18], with the environment modelled as a thermal radiation field, interacting as a global environment. The dynamics of global system-environment interaction characterize the decay rate of non-Markovian noise in terms of bound of speed of evolution. We are using non-Markovian noise model in generalizing global master equation for system-environment interaction.

The organization of the paper is as follows In Sec. II, we introduce the dynamics of correlated Markov Noise. In Sec III, we discuss the dynamics of quantum speed limit. In Sec. IV, we study the effect of quantum speed limit under Markov noise. In Sec. V, we discuss the quantum speed limit for a master equation. In Sec. VI we give our conclusions.

II. DYNAMICS OF CORRELATED MARKOV NOISE

We begin with a brief discussion of Markov noise channels; subsequent use of N channels generates some correlation, so that $e^N \neq e^{N}$. Such kind of channel is called correlated Markov noise channel. Suppose we have an input state $\rho$. Then a completely positive trace preserving map can be defined as

$$\mathcal{E}(\rho) = \sum_i E_i \rho E_i^\dagger$$

where $E_i$ are Kraus operators of channels which satisfy completely positive trace preserving map. Based on the Kraus operator approach, for an initial state $\rho$, the final state under noise is given by [19]

$$\mathcal{E}(\rho) = (1 - \mu) \sum_{ij} E_{ij} \rho E_{ij}^\dagger + \mu \sum_k E_{k\mu} \rho E_{k\mu}^\dagger$$

In the above expression the probability is $\mu$ for the operation to remain correlated, and the probability is $1 - \mu$ for the operation to remain uncorrelated.

We consider a well-established model for quantum channels with Markov noise. The time-dependent Hamiltonian [20] of a qubit is described as $H(t) = \hbar \Gamma(t) \sigma_z$, where $\Gamma(t)$ is an independent random variable. In order to establish a model for Markov noise channel we are using time dependent Kraus operators governed by time dependent Hamiltonian. The dynamics can be described by the following Kraus operator [20]

$$K_1(\nu) = \frac{\sqrt{1 + \phi(\nu)}}{2} I$$

$$K_2(\nu) = \frac{\sqrt{1 - \phi(\nu)}}{2} \sigma_3$$

where we have $\phi(\nu) = e^{-\nu} [\cos(\nu) + \frac{\sin(\nu)}{u}]$ and $u = \sqrt{4\tau^2 - 1}$ with $\nu = \frac{\tau}{\tau}$ being the scale time. The calculation for Kraus operator is done for two-qubit channel and can be written as $A_1 = K_1 \otimes K_1, A_2 = K_1 \otimes K_2, A_3 = K_2 \otimes K_1$ and $A_4 = K_2 \otimes K_2$. Similarly, calculation is done for correlated channels.

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III. QUANTUM SPEED LIMIT FOR NOISY DYNAMICS

The evolution of closed system follows a unitary map. There is a limiting speed for dynamical evolution. The evolution of a quantum state dictates the speed of quantum computation. Quantum physics imposes limit on the speed of evolution of state: this is the quantum speed limit (QSL) [21]. The average value [22, 23] of the generator of such dynamical map gives the maximum evolution speed of quantum system, which gives rise to the concept of limiting speed of dynamical evolution. Quantum speed limit also arises when there is finite energy exchange between system and environment and the state of system may evolve according to slow dynamics. In this work, we present a new framework of QSL for noisy dynamics. Our framework is based on bound on the speed of evolution under Markov noisy channels. As is known, for a closed system quantum evolution follows unitary map. The minimum time for evolution is given by [24]

$$\tau \geq \frac{\pi \hbar}{2 \Delta E} \tag{5}$$

$\Delta H$ is the energy variance defined as $\sqrt{\langle \phi | H^2 | \phi \rangle - \langle \phi | H | \phi \rangle^2}$. This inequality is known as the Mandelstam-Tamm bound [25]. A bound can be derived for the map represented in terms of time-independent Kraus operator [24]

$$\tau_\theta \geq \frac{2\theta^2}{\pi^2} \sqrt{\text{tr}[ho_0^2]}$$  

The time evolution of a quantum $\rho_0$ can be written in form of $\rho_t = \Sigma_a K_a(t, 0) \rho_0 K_a^\dagger(t, 0)$. When $\rho_0^{SE}$ is extended in form of $\rho_0 \otimes \rho_E^0$, then such a dynamical map is said to be universal. Let the map be governed by the evolution

$$f(t) = \frac{\text{tr}[\rho_0 \rho_t]}{\text{tr}[\rho_0]} = \frac{1}{\text{tr}[\rho_0]} \sum_a \text{tr}[\rho_0 K_a \rho_0 K_a^\dagger] \tag{7}$$

For compactness let us denote $K_a = K_a(t, 0)$; then

$$f(t) = \frac{1}{\text{tr}[\rho_0]} \sum_a \text{tr}[\rho_0 (K_a \rho_0 K_a^\dagger + (K_a \rho_0 K_a^\dagger))] \tag{8}$$

On solving the above equation and using the Cauchy-Schwarz inequality, a bound can be derived. Parametrizing $f(t) = \cos \theta$ we have,

$$\tau_\theta \geq \frac{|\cos \theta - 1|}{2} \frac{\sqrt{\text{tr}[\rho_0^2]}}{\sum_a \sqrt{\text{tr}[\rho_0 K_a^\dagger \rho_0 K_a]}} \tag{9}$$

In eq (9) we have used $|\cos \theta - 1| \geq \frac{2\theta^2}{\pi^2}$. Here, $\rho_0$ is the initial state, which we have considered as the Bell state $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

We explicitly compute and plot QSL for the following cases.

IV. EFFECT OF CORRELATED CHANNEL ON QSL

Dynamics with Amplitude noises: Consider the dynamics of amplitude damping channel. The Kraus operators for a two-qubit system are as follows [21]

$$A_1 = \begin{pmatrix} \sqrt{1+\phi(\tau, \tau)} & 0 \\ 0 & \sqrt{1-\phi(\tau, \tau)} \end{pmatrix} \tag{10}$$

and

$$A_2 = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{11}$$

where, $\phi(\tau) = e^{-\tau}[\cos(\nu \tau) + \frac{\sin(\nu \tau)}{\nu \tau}]$, $\nu = \frac{1}{2\tau}$ and $\tau = \sqrt{(4\tau)^2 - 1}$ and $\tau$ refers to degree of non-Markovianity [26]. In this section, we establish a link between the ratio $R$ of speed limit of uncorrelated and correlated channel, and the degree of non-Markovianity. After straightforward calculations, using (6), we find that the ratio of quantum speed limit decreases with increase in $\tau$. Fig 1., demonstrates the decay of speed of evolution for a two qubit amplitude Markov noise. The ratio of quantum speed limit gradually decreases with increase in $\tau$.

![Figure 1](image.png)

FIG. 1. Ratio $R$ of quantum speed limit time for correlated and uncorrelated two-qubit amplitude damping channel, as a function of $\tau$, controlling the degree of Markovianity. Its maximum value is fixed to $\frac{1}{4}$. From this figure it is clear that ratio of speed of evolution decreases with increase in $\tau$.

Dynamics with Phase damping noise: Phase damping channel experiences a quantum noise with loss of quantum phase information. The Kraus operator for a single qubit can be represented in terms of Pauli operators $\sigma_0 = I$ and $\sigma_x$. The Kraus operator for two-qubit system can be represented as [27].

$$E_{i,j} = \sqrt{P_i P_j} \sigma_i \otimes \sigma_j \tag{12}$$

and the Kraus operator of correlated channels are

$$E_{k,k} = \sqrt{P_k} \sigma_k \otimes \sigma_k \tag{13}$$
where \(i, j, k = 0.3\) and \(P_0 = 1 - p\) and \(P_3 = p\) where \(p = \phi(v)\) and \(\phi(v) = e^{-v}[(\cos(vu) + \frac{\sin(vu)}{v})].\) Using Eq. 11 and Eq. 12 we derive the expression for ratio of quantum speed limit for two-qubits passing through phase damping channel. From Fig 2, we see that the evolution of ratio of speed limit for a two-qubit system increases with increase in \(\tau.\)

\[L_{cor} = \sum_{i=1,2} \gamma_i N{(\sigma_i^+ \rho \sigma_i^- - \frac{1}{2}((\sigma_i^+ \sigma_i^- \rho + \rho \sigma_i^- \sigma_i^+))}\]

and

\[L_{un} = \sum_{i=1,2} \gamma_i (N + 1)(\sigma_i^+ \rho \sigma_i^- - \frac{1}{2}((\sigma_i^+ \sigma_i^- \rho + \rho \sigma_i^- \sigma_i^+))\]

Let us consider the master equation constructed for a global system-bath interaction. Consider a given system with state \(\rho_0\) coupled to an environment in state \(\rho_0^E.\) The global state can be approximated as \(\rho_0 \otimes \rho_0^E.\) The global reversible dynamics is governed by unitary evolution and reduced dynamics of system is given by \(\rho_E = [U_t \rho_0 \otimes \rho_0^E U_t^\dagger]\). One can assume Markovian dynamics when the time scale of environment is much smaller than that of system. Maps for Markovian dynamics form quantum dynamical semigroup and can be represented in terms of Markovian master equation [28]

\[
\frac{d\rho_t}{dt} = L\rho_t
\]

In this section, we compute the quantum speed limit (QSL) using master equation for two qubit atomic system [29]:

\[
\frac{d\rho}{dt} = L_{un}(\rho) + L_{cor}(\rho)
\]

Here \(L_{un}\) represents uncorrelated Lindbladian operator and \(L_{cor}\) the correlated operator:

\[L_{un} = \sum_{i=1,2} \gamma_i (N + 1)(\sigma_i^+ \rho \sigma_i^- - \frac{1}{2}((\sigma_i^+ \sigma_i^- \rho + \rho \sigma_i^- \sigma_i^+))\]

\[L_{cor} = \sum_{i=1,2} \gamma_i N{(\sigma_i^+ \rho \sigma_i^- - \frac{1}{2}((\sigma_i^+ \sigma_i^- \rho + \rho \sigma_i^- \sigma_i^+))\]

is the Planck distribution function and

\[\sigma_i^+ = \sigma^- \otimes I; \sigma_i^- = \sigma^+ \otimes I,\]

where \(\gamma_i\) are decay parameters. The bound for the speed of evolution is calculated for the correlated channel having generator in the form of Eq. 14. Using Eq. 6, we determine ratio of uncorrelated-correlated bound on speed of evolution for this model. We establish a link between ratio of bound of evolution as a function of \(a.\) Here, \(a\) is a measure of the degree of non-Markovianity, like \(\tau\) and has been defined in Eqn. (2.15) of [29]. The generalization of time-dependent Lindbladian for uncorrelated channel can be calculated in a straightforward way using Eq.6. Similar calculation can be done for the sum of correlated-uncorrelated noise. The interaction with the environment is described by the model given in Eqn. (2.10) of [29]. The coupling depends on the qubit position \(r_n,\) and the interaction Hamiltonian is proportional to \(\sqrt{\gamma_{ij}},\) where \(\gamma_{ij} = \sqrt{\gamma_j \gamma_d a(k_0 r_j),}\) \(\gamma_i\) is the spontaneous emission rate, and \(r_{12} = r_1 - r_2\) is the relative position vector. \(k_0\) is the magnitude of the wave vector. We studied this model under two cases.

Case 1: Consider the case when \(\gamma_1 = \gamma_2 = \gamma;\) we generalise Lindbladian form of master equation for uncorrelated channel which can be written in the form

\[
L_{un} = \begin{pmatrix}
-(1 + N)\gamma & 0 & 0 & -\frac{\gamma(1 + 2N)}{2} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\frac{-\gamma(1 + 2N)}{2} & 0 & 0 & -N\gamma
\end{pmatrix}
\]

Case 2: Consider the case when \(\gamma_{12} = \gamma_{21} = \gamma a(k_0 r_{12});\) we generalise the Lindbladian form of master equation for correlated channel which can be written in the form

\[
L_{cor} = \begin{pmatrix}
0 & aN(1+2\gamma) & 0 & 0 \\
0 & 0 & 0 & 0 \\
aN(1+2\gamma) & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

where \(\gamma_i_j = \sqrt{\gamma_j \gamma_d a(k_0 r_j)};\) \(\gamma_i \neq \gamma_j,\) is the multi-qubit interaction of composite system with bath. For a two-qubit system for identical parties we have \(\gamma_{12} = \gamma_{21} = \gamma_1 = \gamma_2 = \gamma,\) also \(k_0 = \frac{2\pi}{\nu},\) is the resonant wave vector. Applying triangle inequality, we determine the bound of evolution as upper and lower bound.
**Triangle inequality:** Triangle inequality states that for any triangle the sum of length of any two side must be greater than or equal to the length of remaining side. The triangle inequality is a theorem about vector distances and is written using vector and vector lengths.

$$||A + B|| \leq ||A|| + ||B||$$ (17)

Speed limit for uncorrelated channel can be written in the form $\tau_{un} = \frac{4\theta^2}{\pi ||L^\dagger_{un}(\rho_0)||}$. Similarly, speed limit for correlated channel can be expressed as $\tau_{cor} = \frac{4\theta^2}{\pi ||L^\dagger_{un}(\rho_0) + L^\dagger_{cor}(\rho_0)||}$. Therefore, their ratio can be expressed as

$$\frac{\tau_{cor}}{\tau_{un}} = \frac{L^\dagger_{un}(\rho_0)}{L^\dagger_{un}(\rho_0) + L^\dagger_{cor}(\rho_0)}$$ (18)

Now,

$$\frac{\tau_{cor}}{\tau_{un}} \geq \frac{L^\dagger_{un}(\rho_0)}{L^\dagger_{un}(\rho_0) + L^\dagger_{cor}(\rho_0)}$$ (19)

On solving, we get

$$\frac{\tau_{cor}}{\tau_{un}} \leq \frac{L^\dagger_{un}(\rho_0)}{|1 - \frac{L^\dagger_{cor}(\rho_0)}{L^\dagger_{un}(\rho_0)}|}$$ (20)

This we have upper and lower bound

$$\frac{1}{1 + x} \leq \frac{\tau_{cor}}{\tau_{un}} \leq \frac{1}{|1 - x|}$$ (21)

where $x = \frac{||L^\dagger_{cor}(\rho_0)||}{||L^\dagger_{un}(\rho_0)||}$.

From Fig 3., the ratio of uncorrelated-correlated noise lies in between upper and lower bound. The speed of evolution increases for upper bound, and decreases for lower bound as the value of $\alpha$ increases. Whereas $U$ and $L$ determine the ratio of upper bound as well as lower bound, $R$ is the ratio of speed of time limit for correlated- uncorrelated noise.

**VI. CONCLUSIONS**

In conclusion, we have proposed a scheme for detailed study of correlated channel under Markov noise. Different types of Markov noise channel have been taken into account, such as amplitude damping channel and phase damping channel. The effect of Markov noise on correlated channels has been discussed in detail. We summarize the results as follows. Firstly, the ratio of QSL for correlated/uncorrelated channels generated for amplitude damping channel and phase damping channel was calculated. The speed of evolution for correlated channel under Markov noise decreases for amplitude damping channel and increases for phase damping channel. Secondly, global system environment interaction is taken into consideration. We considered a two qubit atomic model and the master equation for a two qubit atomic system consists of Lindbladian operator for correlated and uncorrelated noise.
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