CP violation flavor asymmetries in slepton pair production at
leptonic colliders from broken R parity *

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Abstract

We examine the effect of the R parity odd, lepton number violating, renormalizable
interactions on flavor non-diagonal rates and CP asymmetries in the production of slepton
pairs, $e^− + e^+ \rightarrow \tilde{e}_{HJ} + \tilde{e}^*_{H,J'}$, $[J \neq J']$, $[H, H' = (L, R)]$ at leptonic colliders. The R parity
odd coupling constants are assumed to incorporate CP odd complex phases. The flavor
changing rates are controlled by tree level amplitudes and quadratic products of different R
parity violating coupling constants and the CP violating asymmetries by interference terms
between tree and loop level amplitudes and quartic products. The consideration of loop
amplitudes is restricted to the photon and Z-boson vertex corrections. We present numerical
results using a family and (quarks and leptons) species independent mass parameter, $\tilde{m}$,
for all the scalar superpartners and making simple assumptions for the family dependence
of the R parity odd coupling constants. The flavor non-diagonal rates, $\sigma_{JJ'}$, vary in the
range, $(\lambda_0/0.1)^4 2 − 20$ fbarns, for sleptons masses $\tilde{m} < 400$ GeV, as one spans the interval
of center of mass energies from the Z-boson pole up to 1000 GeV. For sleptons masses,$\tilde{m} > 150$ GeV, these observables could be of use at NLC energies to set useful bounds on
the R parity odd coupling constants. The predicted asymmetries are in order of magnitude,
$A_{JJ'} = \frac{\sigma_{JJ'} - \sigma_{J'J}}{\sigma_{JJ'} + \sigma_{J'J}} \simeq 10^{-2} − 10^{-3}$.

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1 Introduction

On side of the familiar low energy tests of CP symmetry non-conservation, a large number of tests have been developed over the years for high energy colliders [1, 2, 3]. The existing proposals have dealt with different types of CP odd observables (quark and leptons flavor asymmetries [4], spin polarization asymmetries [3, 4, 7], heavy quarks or leptons electric dipole moments [8], ...) and covered a wide variety of physical processes, ranging from decay reactions ($Z$, $W^\pm$ gauge bosons [1, 9], Higgs bosons [10, 11] or top-quarks [12]) to production reactions (leptons-antileptons and light quarks-antiquarks pairs [1], single top-quarks [13], top-antitop-quark pairs [14, 15, 16], or superpartners pairs, $\tilde{\chi}^+\tilde{\chi}^-$, $\tilde{q}\tilde{\bar{q}}$, and $\tilde{l}^+\tilde{l}^-$ [19-20]). For lack of space, we have referred to those works from which one could hopefully trace the extensive published literature.

One of the primary motivations for these high energy tests is the search for physics beyond the standard model. The supersymmetry option is especially attractive in this respect since any slight generalization of the minimal model, allowing, say, for some generational non-universality in the soft supersymmetry breaking parameters or for an approximate R parity symmetry, would introduce several new parameters, with a non trivial structure on quarks and leptons flavors which could accommodate extra CP violating phases. As is known, high energy supercolliders are expected to provide for precision determinations of these supersymmetry parameters. Regarding the much studied sleptons pair production reaction [21, 22, 23], one can define a simple spin-independent CP asymmetry observable in terms of the difference of integrated rates, $(\sigma_{JJ'} - \sigma_{J'J})$, with $\sigma_{JJ'} = \sigma(e^- + e^+ \to \tilde{\ell}_J^- + \tilde{\ell}_{J'}^+)$, for the case of sleptons pairs of different flavors, $J \neq J'$. Recent works, based on the mechanism of sleptons flavor oscillations, have examined for correlated slepton pairs production, the flavor non-diagonal rates [24, 25] and the CP-odd flavor asymmetries, defined as, $A_{JJ'} = \frac{\sigma_{JJ'} - \sigma_{J'J}}{\sigma_{JJ'} + \sigma_{J'J}}$ [13, 20]. Encouraging values of order, $A_{JJ'} \approx 10^{-3}$ were predicted at the next linear colliders (NLC) energies [13, 20]. While the rates, $\sigma_{JJ'}$, depend on pairwise non-degeneracies in the sleptons mass spectra, the asymmetries, $A_{JJ'}$, entail the much stricter conditions that both non-degeneracies and mixing angles between all slepton flavors, as well as the CP odd phase, must not vanish.

Our main observation in this work is that the R parity odd interactions could provide an alternative mechanism for explaining flavor non-diagonal CP asymmetries through possible complex CP odd phases incorporated in the relevant dimensionless coupling constants. While these interactions can contribute to flavor changing changing processes already at tree level, their contributions to CP asymmetries involve interference terms between tree and loop amplitudes. Two important questions then are, first, whether the contributions from the RPV (R parity violating) interactions, given the known bounds on the R parity odd coupling constants, could lead to observable production rates; second, whether the CP asymmetries could reach observable levels. We shall present in this work a study of the contributions to the CP asymmetries, in the reactions, $e^- + e^+ \to \tilde{\ell}_{HJ}^- + \tilde{\ell}_{HJ'}^+$, $[H = L, R, J \neq J']$, at the high energy leptonic colliders, for
center of mass energies from the Z-pole up to 1000 GeV. The RPV lepton number violating interactions are defined by the familiar superpotential, \[ W_{R-odd} = \sum_{ijkl} \frac{1}{2} \lambda_{ijkl} L_i L_j E_k^+ + \lambda_{ijkl} Q_i L_j D_k^+ . \]

A comparison with the oscillations mechanism should enhance the impact of future experimental measurements of these observables at the future high energy colliders.

The contents are organized into 3 sections. In Section 2, we develop the basic formalism for describing the scattering amplitudes at tree and one-loop levels for the production of slepton pairs, \( \tilde{e}_L \tilde{e}_L^+ \) and \( \tilde{e}_R \tilde{e}_L^+ \). In Section 3, we present and discuss our numerical results for the integrated cross sections and the CP asymmetries.

## 2 Production of charged sleptons pairs

### 2.1 General formalism

The evaluation of spin-independent CP asymmetries in the production of a pair of sleptons, \( e^-(k) + e^+(k') \rightarrow \tilde{e}_H(p) + \tilde{e}_{H'}(p') \), of different flavors, \( J \neq J' \), with chiralities, \( H = (L, R) \), \( H' = (L, R) \), involves both tree and loop amplitudes. Let us start with the case of two left-chirality sleptons, \( H = H' = L \). At tree level, the R parity odd couplings, \( \lambda_{ijk} \), give a non-vanishing contribution which is described by a neutrino, \( \nu_i \), \( t \)-channel exchange Feynman diagram, as displayed in (a) of Fig. 1. The associated flavor non-diagonal amplitude reads:

\[
M_{\text{tree}}^{JJ'}(\tilde{e}_L) = -\frac{\lambda_{ijL}^* \lambda_{jR} \bar{v}(k') P_L(\not{k} - \not{p}) P_R u(\not{p})}{t - m_{\nu_i}^2}. \tag{1}
\]

Under our working assumption that flavor changing effects are absent from the supersymmetry breaking interactions, no other tree level contributions arise, since the gauge interactions can contribute, through the familiar neutralinos \( t \)-channel and gauge bosons \( s \)-channel exchanges, to flavor diagonal amplitudes, \( J = J' \), only.

At one-loop level, there occurs \( \gamma - \) and \( Z - \) boson exchange amplitudes with dressed \( \gamma \vec{f} \vec{f}' \) and \( Z \vec{f} \vec{f}' \) vertices involving three-point vertex correction loop diagrams, as well as box diagrams, of the type depicted schematically in (b) and (c) of Fig. 1. We shall restrict consideration to the one-loop triangle diagrams contributions in the gauge bosons exchange amplitude only.

Defining the dressed vertex functions for the Z-boson coupling to sleptons of chirality \( Z_\mu(P) \rightarrow \vec{f}_H^f(p) + \vec{f}_H^{f'}(p') \), \( [H = L, R] \), by the effective Lagrangian,

\[
L = -\frac{g}{2 \cos \theta_W} Z_\mu Z^\mu (p, p'), \quad \Gamma_i^Z(\not{p}, \not{p'}) = (p - p')^i \mu_a H_{H}(\vec{f}_H) \delta_{JJ'} + A_{H}^{JJ'}(\vec{f}, s + i\epsilon), \tag{2}
\]

where, \( a(H) = a(f_H) = a_H(f) = 2T^{H}_3(f) - 2Q(f)x_W \), \( [x_W = \sin^2 \theta_W] \) such that, \( a(e_L) = -1 + 2x_W \), \( a(e_R) = 2x_W \), we can express the one-loop Z-boson exchange amplitude as:

\[
M_{\text{loop}}^{JJ'}(\tilde{e}_H) = \left( \frac{g}{2 \cos \theta_W} \right)^2 \bar{v}(k') \gamma^\mu \left[ a(e_L) P_L + a(e_R) P_R \right] u(k) \frac{1}{s - m_Z^2 + i m_Z \Gamma_Z} \times (p - p')^\mu_a [\hat{e}_H \delta_{JJ'} + A_{H}^{JJ'}(\hat{e}, s + i\epsilon)], \tag{3}
\]
Figure 1: Flavor non-diagonal process of $e^- e^+$ production of a sfermion-antisfermion pairs, $e^-(k) + e^+(k') \rightarrow \tilde{e}_J^-(p) + \tilde{e}_J^+(p')$. The tree level diagram in (a) represents a neutrino, $f = \nu$, $t$-channel exchange amplitude. The loop level diagram in (b) represents $\gamma$– and $Z$– boson exchange amplitudes with dressed vertices and that in (c) box amplitudes.

where the shifted complex argument, $s+i\epsilon$, is incorporated to remind us that the vertex functions are complex functions in the complex plane of the Z-boson virtual mass squared, $s = (k+k')^2 = (p+p')^2$, to be evaluated at the upper lip of the cut along the positive real axis. In the dressed vertex function describing the coupling, $Z\tilde{f}\tilde{f}^*$, eq.(2), we have omitted the Lorentz covariant proportional to, $P_\mu = (p+p')_\mu = (k+k')_\mu$, since this will give negligibly small lepton mass terms upon contraction in the total Z-boson exchange amplitude, eq.(3), with the initial state leptons vertex covariant. It is most convenient to describe the initial leptons polarizations in the helicity eigenvalue basis. In the limit of vanishing initial leptons masses, only the two helicity flip configurations, $e^- e^+_R, e^- e^+_L$, are non vanishing. While the gauge bosons $s$-channel exchange contributes to both of these configurations, the R parity violating neutrino $s$-channel exchange contributes only to the first. The summed tree and loop amplitude, $M^{JJ'}(\tilde{e}_L) = M_{\text{tree}}^{JJ'}(\tilde{e}_L) + M_{\text{loop}}^{JJ'}(\tilde{e}_L)$, in the relevant configuration, namely, $e^- e^+_R = e^- (h = -1/2) + e^+ (\tilde{h} = 1/2)$, reads:

$$M^{JJ'}(\tilde{e}_L) = M(e^- e^+_L \rightarrow \tilde{e}_L^J + \tilde{e}^+_L) = \frac{1}{2} \beta s \sin \theta \left[ \frac{\lambda^{J1}_{JJ'} \lambda^{i_1 p_1}_{iL}}{t - m^2_{\nu_s}} \right] + 2 \left( \frac{g}{2 \cos \theta_W} \right)^2 \frac{a(e_R) A^{JJ'}_{L,R}(\tilde{e}, s + i\epsilon)}{s - m^2_Z + i m \Gamma_Z}. \quad (4)$$

The Z-boson exchange contribution to the other helicity flip configuration, $e^- e^+_L$, is simply obtained by the substitution, $a(e_R) \rightarrow a(e_L)$. We also note that the $\gamma$ exchange contribution has the same formal structure as that of the Z-boson exchange, and can be easily incorporated by adding to the above amplitudes the terms obtained by the replacements, $\frac{g}{2 \cos \theta_W} \rightarrow \frac{g \sin \theta_W}{2}$, $a_{L,R}(f) \rightarrow 2Q(f)$, $(s - m^2_Z + i m \Gamma_Z)^{-1} \rightarrow s^{-1}$, along with the substitution of Z-boson by photon vertex functions, $A^{JJ'}_{L,R}(\tilde{e}, s + i\epsilon) \rightarrow A^{JJ'}_{L,R}(\tilde{e}, s + i\epsilon)$. The kinematical notations
here refer to the center of mass system, where \( \beta = \frac{p}{k} = \frac{2a_p}{\sqrt{s}} \). \( \theta \) is the scattering angle and the differential cross section for unpolarized initial leptons reads: 
\[
d\sigma/d\cos \theta = \frac{p}{12\pi s} \sum_{pol} |M^{JJ'}|^2. 
\]
(For unpolarized beams, one must remove the polarization sums and multiply by a factor of 4. Our results agree with those quoted in [22].) Denoting the amplitude for the charge conjugate process, \( e^- + e^+ \rightarrow \tilde{e}_HJ' + \tilde{e}_HJ, \) \( [H = L, R] \), by \( \tilde{M}^{JJ'}(\tilde{e}_H) \) and using the simple relationship, \( \tilde{M}^{JJ'}(\tilde{e}_H) = M^{JJ'}(\tilde{e}_H) \), one can describe the decomposition into tree and loop components for the pair of CP conjugate processes as,
\[
M^{JJ'}(\tilde{e}_H) = a_0^{JJ'} + \sum_{\alpha} a_{\alpha}^{JJ'} F_{\alpha}^{JJ'} (s + i\epsilon), \quad \tilde{M}^{JJ'}(\tilde{e}_H) = a_0^{JJ*'} + \sum_{\alpha} a_{\alpha}^{JJ*'} F_{\alpha}^{JJ'} (s + i\epsilon). \tag{5}
\]

A spin-independent CP asymmetry can be defined in the familiar way as the normalized difference of rates,
\[
A_{JJ'}(\tilde{e}_H) = \frac{|M^{JJ'}(\tilde{e}_H)|^2 - |\tilde{M}^{JJ'}(\tilde{e}_H)|^2}{|M^{JJ'}(\tilde{e}_H)|^2 + |\tilde{M}^{JJ'}(\tilde{e}_H)|^2} \approx \frac{2}{|a_0|^2} \sum_{\alpha} Im(a_0 a_{\alpha}^*) Im(F_{\alpha} (s + i\epsilon)), \tag{6}
\]
where we have assumed in the second step that the tree level flavor non-diagonal amplitude, \( a_0 \), dominates over the loop level amplitude, \( a_{\alpha}F_{\alpha} \), and used the index \( \alpha \) to label the internal states running inside the loop.

### 2.2 Loop amplitudes

The one-loop triangle diagrams, describing the dressed vertex functions, \( Z\bar{f}_L\bar{f}_L' \), arise in two distinct charge configurations, shown in Fig. 2 by the diagrams (a) and (b), which involve the d- and u-quark Z-boson currents, respectively. The associated vertex functions read:
\[
\Gamma_{\mu}(p, p')_a = -i N_c \lambda_{\alpha \beta} L_{jk} L_{j'k'} \times \frac{\int Q \Gamma_{\mu}(Q + m_{d_k} + a(d_R)P_L + a(d_R)P_R)(-P + Q + m_{d_k})P_L(Q - \phi + m_{u_j})}{(Q^2 + m_{d_k}^2)(-(Q - p')^2 + m_{u_j}^2)} , \\

\Gamma_{\mu}(p, p')_b = -i N_c \lambda_{\alpha \beta} L_{jk} L_{j'k'} \times \frac{\int Q \Gamma_{\mu}(Q + \phi + m_{d_k} + a(u_L)P_L + a(u_R)P_R)(Q + m_{u_j})}{(Q^2 + m_{u_j}^2)(-(Q + p')^2 + m_{u_j}^2)} . \tag{7}
\]

Applying the formalism of Passarino-Veltman [26], the vertex function from diagram (a) can be expressed in the form:
\[
A_{\mu L}^{JJ'}|_a = \frac{\lambda_{\alpha \beta} L_{jk} L_{j'k'}}{2(4\pi)^2} N_c \left[ 2a(d_L)m_{d_k}^2(C_0 + C_{11} - C_{12}) + a(d_R) \left( B_0^{(2)} + B_0^{(3)} + 2P \cdot p(C_{11} - C_{12}) \right) \\
+ P^2 C_0 + 2m_{\tilde{e}_j}^2(-C_{11} + C_{12}) - 2m_{\tilde{e}_j}^2 C_0 + 2m_{\tilde{e}_j}^2(C_{11} - C_{12}) \right] . \tag{8}
\]

The conventions of ref. [24] are used for the two-point and three-point integral functions, \( B_X [X = 0, 1] \) and \( C_X [X = 0, 11, 12, 21, 22, 23, 24] \). For notational convenience, we have introduced the following abbreviations for the dependence on argument variables: \( B_X^{(1)} = B_X(-p' - p', m_d, m_d), \) \( B_X^{(2)} = B_X(-p, m_d, m_u) \), \( B_X^{(3)} = B_X(-p', m_u, m_d) \) and \( C_X(-p, -p', m_d, m_u, m_d) \). The amplitude from
The equations for the scalar field $\phi$ are most conveniently calculated through the consideration of the scalar fields renormalization factor $s$ substitutions, $\Pi(p, m)$. The total amplitudes read as, $A_{\phi}(p) = f(Q) + f(Q') \to f(p) + f(p')$.

Figure 2: One-loop diagrams for the dressed $\bar{Z} \bar{f} f^*$ vertex. The flow of four-momenta for the intermediate fermions is denoted as, $Z(P = k + k') \to f(Q) + f(Q') \to f(p) + f(p')$.

Diagram (b) can be obtained from that of diagram (a) by performing the following substitutions: $m_u \to m_u$, $p \to p'$, $P_R \to P_H, a(d_H)P_H \to a(u_H)P_H$, $[H = L, R]$. The self-energy contributions, which are represented by the diagrams (c) in Fig. 2, with a single configuration only for the $d$- and $u$-quarks which propagate inside the loop, are most conveniently calculated through a consideration of the scalar fields renormalization factors $Z_{\phi, \phi}$. Starting from the schematic equations for the scalar field $\phi$ bare Lagrangian density, $L = \phi^*(p^2 - m^2 + \Pi(p))\phi$, where, $\Pi(p) = \Pi_1 p^2 - m^2 \Pi_0 + \cdots$, one transfers from bare to renormalized quantities by applying the substitutions, $\phi \to \phi/(1 + \Pi_1)$, $m^2 \to m^2(1 + \Pi_1)/(1 + \Pi_0)$, such that the renormalization equations for the fields and mass parameters read, $\phi_J = Z_{\phi, \phi} \phi_J^{ren}$, $m_{\phi, \phi}^{2} = Z_{\phi, \phi}^{ren}$, with $Z = (1 + \Pi_1)^{-1}$, $Z^{m} = (1 + \Pi_0)(1 + \Pi_1)^{-1}$, using a matrix notation for the flavor dependence. The self-energy contribution in the vertex function becomes then,

$$A_{\phi}^{J'}|_{SE} = [(Z_{\phi} Z^{*}_{\phi})^{1/2} - 1] \Gamma_{\mu}^{Z} = 2 N_c \frac{\chi^j_{i,j,k,k}^{ij} \chi^j_{i,j,k,k}}{(4\pi)^2} a_L(\bar{e}) B_1^{(2)}.$$

Grouping together the self-energy and the fermionic triangle diagram contributions, such that the total amplitudes read as, $A_{\phi}^{J'}(\bar{e}) = A_{\phi}^{J'}(\bar{e})_a + A_{\phi}^{J'}(\bar{e})_b$, yields the final formulas:

$$A_{\phi}^{J'}(\bar{e})_a = \frac{N_c}{2} \chi^{ij}_{j,k,k} \chi^{j,k}_{i,j,k} \left[ \frac{2a(d_L)m_d^2(C_0+C_{11}-C_{12})+a(d_R)(B_0^{(2)}+B_0^{(3)})+2P \cdot p(C_{11}-C_{12})}{(4\pi)^2} \right],$$

$$A_{\phi}^{J'}(\bar{e})_b = -\frac{N_c}{2} \chi^{ij}_{j,k,k} \chi^{j,k}_{i,j,k} \left[ \frac{2a(u_R)m_u^2(C_0+C_{11}-C_{12})+a(u_L)(B_0^{(2)}+B_0^{(3)})+2P \cdot p(C_{11}-C_{12})}{(4\pi)^2} \right].$$

For notational convenience, we have split the self-energy contribution into two equal parts that we absorbed within the above two amplitudes, distinguished by the suffices $a$ and $b$. Note that the arguments in the $B$- and $C$-integrals for the amplitude $b$ are deduced from those of the
amplitude $a$ by replacing, $d_k \rightarrow u_j$. To obtain these results we have used the mathematica routine package “Tracer” \cite{27} and, for a cross-check, “FeynCalc” \cite{28}. A very useful check concerns the cancellation of the ultraviolet divergencies. We indeed find that the familiar \cite{26} logarithmically divergent term, $\Delta$, enters with the factors, $+a(\bar{e}_L) - 2a(d_R)$ (amplitude $a$) and $a(\bar{e}_L) + 2a(u_L)$ (amplitude $b$), whose total sum vanishes identically.

The interactions associated with the coupling constants, $\lambda_{ijk}$, can also contribute at one-loop order. Exploiting the formal similarity between the $\lambda$ and $\lambda'$ interaction terms in the Lagrangian density, namely, $L = -\lambda'_{ijk} \bar{e}_i L d_k R u_j L - \lambda_{ijk} \bar{e}_i L \bar{e}_k R \nu_j L + \cdots$, dispenses us from performing a new calculation. The results can be derived from those in eq.\cite{11} by substituting for the internal lines, $d_k \rightarrow e_k$, $u_j \rightarrow \nu_j$, and for the parameters, $a_H(u) \rightarrow a_H(\nu)$, $a_H(d) \rightarrow a_H(e)$, $\lambda'_{ijk} \lambda_{ijk} \rightarrow \lambda'_{ijk} \lambda'_{ijk}$.

Let us now turn to the production of right-chirality sleptons where analogous results can be derived. The tree level amplitude is related to that in eq.\cite{11} by a simple chirality change,

$$M_{\text{tree}}(\tilde{e}_R) = -\frac{\lambda'_{ijk} \lambda_{ijk}}{t - m_{\nu_i}^2} \bar{\nu}(k')P_R(\not{k} - \not{p})P_L u(k).$$

There occurs only one non-vanishing helicity flip configuration for the initial leptons, namely, $e^+_L \nu^+_R$, in which the neutrinos $t$-channel and the gauge bosons $s$-channel contributions interfere. The amplitude is given by a formula similar to eq.\cite{11}, except for the substitution in the second term, $a_R(e)A_{L}^{J'}(\bar{e}, s + i\epsilon) \rightarrow a_L(e)A_{R}^{J'}(\bar{e}, s + i\epsilon)$. Concerning the one-loop contribution to the vertex function $A_{R}^{J'}(\tilde{f})$, we find that the RPV interactions with the coupling constants $\lambda_{ijk}$ can only contribute, while those with $\lambda'_{ijk}$ vanish identically. Diagram (a) in Fig. 3 refers to an $e_j$ current and diagram (b) to a $\nu_f$ current. The results can be derived by inspection from eq.\cite{11} by substituting, $\lambda'_{ijk} \lambda'_{ijk} \rightarrow \lambda_{ijj} \lambda_{ijk}$, $d_j R \rightarrow e_j L$, $u_j L \rightarrow \nu_j R$, $\bar{e}_L \rightarrow \bar{e}_R$ and, accordingly, $a(d_H) \rightarrow a(e_H)$, $a(u_H) \rightarrow a(\nu_H)$, $[H = L, R]$, $a(\tilde{e}_L) \rightarrow a(\tilde{e}_R)$. For definiteness, we quote the explicit formulas:

$$A_{R}^{J'}(\tilde{e}) = \frac{N_c \lambda_{ijj} \lambda_{ijk}}{(4\pi)^2} \left[2a(e_R)m^2_e(C_0 + C_{11} - C_{12}) + a(e_L)\left(B_0^{(2)} + B_0^{(3)} + 2P \cdot p(C_{11} - C_{12}) \right) + P^2 C_0 + 2m^2_e(-C_{11} + C_{12}) - 2m^2_e C_0 + 2m^2_e(C_{11} - C_{12}) \right] + 2a(\bar{e}_R)B_1^{(2)},$$

$$A_{R}^{J'}(\tilde{e}) = \frac{N_c \lambda_{ijj} \lambda_{ijk}}{(4\pi)^2} \left[2a(\nu^+_L)m^2_\nu(C_0 + C_{11} - C_{12}) + a(\nu^+_R)\left(B_0^{(2)} + B_0^{(3)} + 2P \cdot p(C_{11} - C_{12}) \right) + P^2 C_0 + 2m^2_\nu(-C_{11} + C_{12}) - 2m^2_\nu C_0 + 2m^2_\nu(C_{11} - C_{12}) \right] - 2a(\bar{e}_R)B_1^{(2)}.$$
Finally, let us add here a general comment concerning the photon vertex functions, $A_{L,R}^{\gamma JJ'}$, which are given by formulas similar to those in eqs. (10) or (12) with the appropriate replacements, $a_{L,R}(f) \rightarrow 2Q(f)$. Therefore, to incorporate the $\gamma$-exchange contributions in the total amplitudes (eq. (11) and related equations) one needs to substitute,

$$a_{R,L}(e)A_{L,R}^{\gamma JJ'} \rightarrow a_{R,L}(e)\sum_f a(f)C_f + 2Q(e)\sin^2\theta W \cos^2\theta W [(s - m_Z^2 + imZ\Gamma_Z)/s] \sum_f 2Q(f)C_f,$$

where we have used the schematic representation, $A_{L,R}^{\gamma JJ'} = \sum_f a(f)C_f$.

## 3 Results and discussion

Let us first comment briefly on the experimental observability of flavor non-diagonal sleptons pair production. One convenient non degraded signal here is that which corresponds to lepton pair final states, $e^-e^+$, which are produced through the two-body decay channels for sleptons, $\tilde{e}_{[L,J]} \rightarrow e_{[L,J]} + \tilde{\chi}_1^0$. Of course, in the broken R parity case, the produced lightest neutralinos are unstable and could conceivably be reconstructed through their dominant decay channels which involve two leptons, or two jets, together with missing energy. We shall not elaborate further on this issue, except to note that the efficiency factors at NLC energies for the flavor diagonal rates, assuming a stable $\tilde{\chi}_1^0$, and including rough detection cuts, such that the physical rates for the fermion pairs channels is, $\sigma_{JJ'}\epsilon$, are typically set at $\epsilon \approx 30\% \ [25]$.

Proceeding to the predictions, we observe that the main source of uncertainties concerns the RPV coupling constants. The sfermion mass eigenvalues are not known, but these parameters appear explicitly through the kinematics. We shall neglect mass splittings and mixings between L- and R-sleptons. A unique sleptons mass parameter, $\tilde{m}$, will be used and varied in the interval, $60 < \tilde{m} < 400$ GeV. Regarding the RPV coupling constants, it is useful here to catalog the family configurations and intermediate states entering the calculations. Examining the structure of the flavor non-diagonal tree amplitudes, we note that these involve a onefold summation over leptons families weighted by the factors, $t_{JJ'}^{iJ} = \lambda_{iJ}^*\lambda_{Ji}$, for L-sleptons and $t_{JJ'}^{iJ} = \lambda_{iJ}^*\lambda_{Ji}$, for R-sleptons. The loop amplitudes involve a twofold summation over leptons families of form, $\sum_{jk} t_{JJ'}^{iJ} F^{jk}(m_j,m_k,s + i\epsilon)$, where $t_{JJ'}^{iJ}$ depend quadratically on the RPV coupling constants while the loop integrals, $F^{jk}$, have a non-trivial dependence on the fermions masses, as exhibited on the formulas derived in Section 2 [see, e.g., eq. (10)]. The relevant coupling constants, the species and family configurations for the internal fermions are for L-sleptons, $t_{JJ'}^{iJ} = \lambda_{iJ}^*\lambda_{Ji}[d_k, u_j]$; and for R-sleptons, $t_{JJ'}^{iJ} = \lambda_{iJ}^*\lambda_{Ji}[e_j, \nu_j]$. The dependence of rates on the RPV coupling constants has the schematic structure, $\sigma_{JJ'} \approx \sum |t_{JJ'}^{iJ}|^2$, and that of CP asymmetries, $A_{JJ'} \approx \sum_{ijk} I m(t_{JJ'}^{iJ})/\sum_i |t_{JJ'}^{iJ}|^2$ for L-sleptons and $A_{JJ'} \approx \sum_{ijk} I m(t_{JJ'}^{iJ})/\sum_i |t_{JJ'}^{iJ}|^2$ for R-sleptons. Therefore, rates (asymmetries) are controlled by two (four) RPV coupling constants in different family configurations. Note the expected invariance of asymmetries under phase redefinitions of the fields.
While the dependence on the mass of the exchanged neutrino family index in \( t_{j,j'} \) can be clearly ignored, that on the pair of indices \((i,j)\) in \( t_{j,j'}^{i} \), which involves the ratios of the masses of the appropriate internal fermions, \( m_{i,j} \), to the external scale associated with the center of mass energy, \( \sqrt{s} \), can be ignored as long as, \( \sqrt{s} >> m_{i,j} \). Therefore, at the energies of interest, the only relevant fermion mass parameter is that of the top-quark. Instead of listing the various distinct family configurations for the quadratic (tree) or quartic (loop) products of the RPV coupling constants, we shall consider a set of specific assumptions concerning the family dependence. First, for the cases involving \([e_j, \nu_i']\) or \([e_k, \nu_j]\) internal states, neglecting neutrino masses, we need only account for the masses of charged leptons. For the case with \([d_k, u_j]\) internal states, we restrict consideration to the diagonal family configuration, namely, \( k = j \). Second, we include a CP odd phase, \( \psi \), between \( t_{j,j'}^{i} \) and all of the \( t_{j,j'}^{i} \) or \( t_{j,j'}^{i} \), as the case may be. Finally, we consider the following four discrete choices for the variation intervals on which run the internal fermion indices indices, \( j = k \) or \( i, j \). Case I: \{1\}; Case II \{2\}; Case III \{3\}; Case IV \{1, 2, 3\}. In all these four cases, we set the relevant coupling constants at the reference values, \( t_{j,j'}^{i} = t_{j,j'}^{i} = 10^{-2} \), \( t_{j,j'}^{i} = 10^{-2} \) and use a maximal CP odd phase, \( \text{arg}(t_{j,j'}^{i}t_{j,j'}^{i}) = \psi = \pi/2 \).

Because of the proportionality of asymmetries to the imaginary part of the phase factor, the requisite dependence may be simply reinstated by inserting a factor, \( \sin \psi \). To illustrate the dependence of asymmetries on the internal fermions families and on the \( \lambda' \) or \( \lambda \) interaction types, we display in Table II a set of representative results obtained for selected subsets of Cases I, II, III, IV. The reason is that the results for Cases I, II (light families) are identical in all cases, while those for Case III (heavy families) differ only for cases involving up-quarks. As one sees on Table II, the interference between photon and Z-boson exchange contributions has a significant effect on the results. The strongly reduced values for the L-sleptons asymmetries found in Cases I for the \( \lambda'\lambda^* \) interactions and in all Cases for the \( \lambda \lambda^* \) interactions, arise from the existence of a strong cancellation between the amplitudes termed \((a)\) and \((b)\) for nearly massless internal quarks or leptons. Case III with the \( \lambda'\lambda^* \) interactions is relatively enhanced thanks to the top-quark contribution (configuration \( \bar{t}b \)). That the above cancellation is not generic to the RPV contributions is verified on the results for R-sleptons production, where all three families of leptons give nearly equal, unsuppressed contributions to loop amplitudes.

In the currently favored situation where the RPV coupling constants are assumed to exhibit a strong hierarchical structure, the peculiar rational dependence of CP asymmetries on ratios of quartic products of the coupling constants, might lead to strong enhancement factors. We recall the schematical structure of this dependence, \( A_{j,j'} \propto |\sum_{i,j,k} \text{Im}(\lambda_{ijk}\lambda'_{j'jk}\lambda'_{l1l1}^\ast)|/\sum_{l} |\lambda_{l1l1}|^2 \), and note that the coupling constants involving third family indices are amongst those that are the least strongly constrained. Therefore, assuming that the coupling constants take the values given by the current bounds from low energy constraints, \( 29 \), one would obtain,

\[
A_{j=3,j'=2} \approx |\text{Im}(\lambda'_{333}^\ast \lambda'_{323}^\ast \lambda'_{331}^\ast \lambda'_{321}^\ast)|/|\lambda_{131}\lambda_{121}^\ast|^2 \approx 90 \sin \psi.
\]

The dependence of rates and asymmetries on center of mass energy and sleptons masses are
Table 1: CP asymmetries, $A_{JJ'}$, in sleptons pair production at two values of the center of mass energy, $s^{1/2} = 200, 500$ GeV and for values of the sleptons mass parameter, $\tilde{m} = 60, 100, 200$ GeV, appearing in the column fields. For each case, the first line $(Z)$ is associated with the gauge Z-boson exchange contribution and the second line $(\gamma + Z)$ with both photon and Z-boson exchanges added in together. The contributions to left-chirality ($\tilde{e}_L \tilde{e}_L$) and right-chirality ($\tilde{e}_R \tilde{e}_R$) sleptons, induced by the $\lambda'_{ijk}$ and $\lambda_{ijk}$ interactions, are distinguished by the labels, $\lambda'\lambda'^*$, $\lambda\lambda^*$, respectively. Cases I, III correspond to internal fermions belonging to the first and third families, respectively. The notation $nd - x$ stands for $n \times 10^{-x}$.

|                | $s^{1/2} = 200$ GeV | $s^{1/2} = 500$ GeV |
|----------------|---------------------|---------------------|
|                | $\tilde{m} = 60$    | $\tilde{m} = 60$    | $\tilde{m} = 100$ | $\tilde{m} = 200$ |
| $\tilde{e}_L \tilde{e}_L$ |                    |                     |                    |                     |
| $\lambda'\lambda'^*$ |                    |                     |                    |                     |
| I $Z$           | $-2.1d - 5$         | $-3.3d - 6$         | $-2.6d - 6$        | $-2.4d - 6$         |
| $\gamma + Z$   | $-7.7d - 5$         | $-1.39d - 5$        | $-1.09d - 5$       | $-1.03d - 5$        |
| III $Z$        | $+2.6d - 4$         | $-1.6d - 3$         | $-1.8d - 3$        | $-2.3d - 3$         |
| $\gamma + Z$   | $-1.01d - 3$        | $+5.1d - 3$         | $+5.3d - 3$        | $+8.1d - 3$         |
| $\lambda\lambda^*$ |                    |                     |                    |                     |
| I $Z$           | $-2.1d - 5$         | $-3.3d - 6$         | $-2.6d - 6$        | $-2.4d - 6$         |
| $\gamma + Z$   | $-7.69d - 5$        | $-1.39d - 5$        | $-1.09d - 5$       | $-1.03d - 5$        |
| III $Z$        | $-2.4d - 5$         | $-5.5d - 6$         | $-3.4d - 6$        | $-2.7d - 6$         |
| $\gamma + Z$   | $-6.39d - 5$        | $+2.59d - 6$        | $-5.06d - 6$       | $-8.32d - 6$        |
| $\tilde{e}_R \tilde{e}_R$ |                    |                     |                    |                     |
| $\lambda\lambda^*$ |                    |                     |                    |                     |
| I $Z$           | $-7.2d - 3$         | $-5.5d - 3$         | $-5.4d - 3$        | $-7.2d - 3$         |
| $\gamma + Z$   | $-2.1d - 2$         | $-1.83d - 2$        | $-1.80d - 2$       | $-2.40d - 2$        |
displayed for Case IV in Figure 3. Regarding the variation with energy (figure (a)), after a rapid rise at threshold (with the expected $\beta^3 p$-wave like behavior) the rates settle, roughly as $\tilde{m}^2/s$, to constant values with growing energy, and vary inside the range, $(\lambda^{\lambda}_{\, 0.01})^2 20 - 2$ fbarns, as one sweeps through the interval, $\tilde{m} \in [60, 400]$ GeV. The variation with $\tilde{m}$ (figure (b)) is rather smooth. For the envisaged integrated luminosities, $L \approx 50 - 100$ fbarns$^{-1}$/yr, these results indicate that reasonably sized samples of order 100 events could be collected at NLC. Noting that the dependence of rates on energy rapidly saturates for $\sqrt{s} > \tilde{m}$, we conclude that the relevant bounds that could be inferred on quadratic products of different the RPV coupling constants, should, for increasing sleptons masses, become competitive with those deduced from low energy constraints, which scale typically as, $[\lambda \lambda, \lambda' \lambda'] < 0.1 (100 \text{GeV}/\tilde{m})^2$. The results in Fig.3 (c,d,e,f) for the CP asymmetries, $A_{JJ'}$, indicate the existence of a wide, nearly one order of magnitude, gap between L-sleptons with $\lambda' \lambda$ interactions and R-sleptons with $\lambda \lambda'$ interactions, with values that lie at a few times $10^{-3}$ and $10^{-2}$, respectively.

In our prescription of using equal numerical values for the RPV coupling constants ($t_{JJ'}$ and $l_{JJ'}$) which control tree and loop contributions, the asymmetries are independent of the specific reference values chosen. In the event that the rates would be dominated by some alternative mechanism, say, lepton flavor oscillations, whereas RPV effects would remain significant in asymmetries, these would then scale as, $Im(t_{JJ'}l_{JJ'})$. It is instructive in view of such a possibility to compare with predictions found in the flavors oscillation approach. Scanning over wide intervals of values for the relevant parameters, $[\cos 2\theta_R, \Delta \tilde{m}/\Gamma]$, associated with the common values for all three mixing angles and ratios of families mass differences to the total sleptons decay widths, respectively, the authors of [25] found flavor non-diagonal rates which ranged between 250 and 0.1 fbarns for $\sqrt{s} = 190$ GeV and 100 and 0.01 fbarns for $\sqrt{s} = 500$ GeV. Our predictions, $\sigma_{JJ'} \simeq (\lambda^{\lambda}_{\, 0.1})^4 2 - 20$ fbarns, which hold approximately for energies, $\sqrt{s} > \tilde{m}$, lie roughly in between these extreme values. On the other hand, the authors of [19] found CP asymmetry rates, $S_{JJ'} = \sigma_{JJ'} - \sigma_{J'J} \approx 3 - 16$ fbarns. For comparison, our predicted asymmetry rates for the same quantity, namely, $S_{JJ'} = 2\sigma_{JJ'} A_{JJ'} \approx 10^0 - 10^{-1}$ fbarns, lie around one order of magnitude below these values. It should be said, however, that the flavor oscillation contributions could have a stronger model dependence than the variation range exhibited by the above predictions, and that these predictions were obtained subject to assumptions that tend to maximize CP violation effects. The existing constraints, [30] which are mostly derived from low energy phenomenology, constrain only a small subset of the parameters describing the scalar superpartners mass spectra and generational mixings.

To summarize, we have shown that moderately small contributions to flavor non-diagonal rates and CP violating spin-independent asymmetries in sleptons pair production could arise from the RPV interactions. These contributions seem to be of smaller size than those currently associated with flavor oscillations, although the model dependence of predictions in the flavor oscillation approach is far from being under control. An experimental observation of the non-diagonal slepton production rates would give information on quadratic products of different
coupling constants, $\lambda\lambda^*$. Owing to the smooth dependence of rates on the slepton masses, already for masses, $\tilde{m} > 100$ GeV, it should be possible here to deduce stronger bounds than the current ones inferred from low energy constraints. The observation of CP violating asymmetries requires the presence of non vanishing CP odd phases in quartic products of the coupling constants, $Im(\lambda^*_{jk}\lambda_{jk}\lambda_{ij1}\lambda^*_{ij1})$, (and similarly with $\lambda \rightarrow \lambda'$) which remain largely unconstrained so far. The peculiar rational dependence, $Im(\lambda\lambda^*\lambda\lambda^*)/\lambda^4$, leaves room for possible strong enhancement factors.

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Figure 3: Integrated flavor non-diagonal cross sections and CP asymmetries in the production of slepton-antislepton pairs of left-chirality (L) (interactions $\lambda'_{ijk}$ only) and of right-chirality (R) (interactions $\lambda_{ijk}$). The three windows on the left-hand side ((a), (c), (e)) show the variation with center of mass energy, $s^{1/2}$, for three choices of the scalar superpartners mass parameter, $\tilde{m}$: $60\,\text{GeV}$ (continuous lines), $100\,\text{GeV}$ (dashed-dotted lines), $150\,\text{GeV}$ (dashed lines). The three windows on the right-hand side ((b), (d), (f)) show the variation with scalar superpartner mass, $\tilde{m}$, for three choices of the center of mass energy $s^{1/2}=200\,\text{GeV}$ (continuous lines), $500\,\text{GeV}$ (dashed-dotted lines), $1000\,\text{GeV}$ (dashed lines). The tree level amplitude includes the t-channel exchange contribution. The one-loop amplitudes (with both photon and Z-boson exchanges) correspond to Case IV which includes the contributions from all three internal fermions generations.