\(\sigma_L/\sigma_T\) in the \(\rho^0\)-meson diffractive electroproduction.

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Abstract

The background due to the direct diffractive dissociation of the photon into the \(\pi^+\pi^-\)-pair to the "elastic" diffractive \(\rho^0\)-meson production in electron-proton collisions is calculated. At large \(Q^2\) the interference between resonant and non-resonant \(\pi^+\pi^-\) production changes the \(\sigma_L/\sigma_T\) ratio with the mass of the 2\(\pi\) (i.e. \(\rho^0\)-meson) state.

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1 Introduction

It was noted many years ago that the form of the \( \rho \)-meson peak is distorted by the interference between resonant and non-resonant \( \pi^+\pi^- \) production. For the case of ”elastic” \( \rho^0 \) photoproduction the effect was studied by P. Söding in [1] and S. Drell [2] (who considered the possibility to produce the pion beam via the \( \gamma \rightarrow \pi^+\pi^- \) process). At high energies the main (and the only) source of background is the Drell-Hiida-Deck process [3] (see fig. 1). The incoming photon fluctuates into the pion pair and then \( \pi p \)-elastic scattering takes place. Thus the amplitude for the background may be written in terms of the pion-proton cross section. Recently the diffractive elastic production of \( \rho^0 \)-mesons was measured at HERA [4, 5, 6, 7] both for the cases of photoproduction i.e. \( Q^2 = 0 \) and of \( Q^2 \geq 4 \) GeV\(^2 \) (the so called deep inelastic scattering, DIS, regime). It was demonstrated [4, 6] that the interference with some non-resonant background is indeed needed to describe the distribution over the mass - \( M \) of \( \pi^+\pi^- \) pair.

In this paper we present the results of calculation of \( R = \sigma_L/\sigma_T \) with correction the numerical error published in Fig.6 of [8]. In Sect. 2 the formulæ for the \( 2\pi \) background which are valid for the DIS region are presented. The expression differs slightly from the Söding’s one as we take into account the pion form factor and the fact that one pion propagator is off-mass shell. We consider also the absorption correction coming from the diagram where both pions (\( \pi^+ \) and \( \pi^- \)) directly interact with the target proton. In Sect. 3 we compute the ratio \( R = \sigma_L/\sigma_T \) for a pion pair production in DIS. At large \( Q^2 \sim 10 - 30 \) GeV\(^2 \) the background amplitude becomes relatively small, but still not negligible. It changes the ratio of the longitudinal to transverse \( \rho \)-meson production cross section and leads to the decreasing of \( R = \sigma_L/\sigma_T \) value with \( M^2 \).

2 Production amplitudes

The cross section of \( \rho^0 \) photo- and electroproduction may be written as:

\[
\frac{d\sigma^O}{dM^2dt} = \int d\Omega |A_\rho + A_{n.r.}|^2 ,
\]

where \( A_\rho \) and \( A_{n.r.} \) are the resonant and non-resonant parts of the production amplitude, \( D = L, T \) for longitudinal and transverse photons, \( t = -q_t^2 \) is the
momentum transferred to the proton and $d\Omega = d\phi d\cos(\theta)$, where $\phi$ and $\theta$ are the azimutal and polar angles between the $\pi^+$ and the proton direction in the $2\pi$ rest frame.

2.1 Amplitude for resonant production

The dynamics of vector meson photo- and electroproduction was discussed in the framework of QCD in many papers (see, e.g. [9-12]). However here we will use the simple phenomenological parametrization of the production amplitude because our main aim is the discussion of the interference between resonant and non-resonant contributions. So the amplitude for resonant process $\gamma p \rightarrow \rho^0 p$; $\rho^0 \rightarrow \pi^+\pi^-$ reads:

$$A^\rho = \sqrt{\sigma_\rho} e^{-b_\rho q^2_{t}/2} \frac{\sqrt{M_0\Gamma}}{M^2 - M^2_0 + iM_0\Gamma} \frac{H^D(\theta, \phi)}{\sqrt{\pi}}.$$

(2)

To take into account the phase space available for the $\rho \rightarrow \pi^+\pi^-$ decay we use the width $\Gamma = \Gamma_0 \left(\frac{M^2 - 4m^2_\pi}{M^2 - 4m^2_\rho}\right)^{3/2}$ (with $\Gamma_0 = 151$ MeV and $M_0 = 768$ MeV – its mass); $b_\rho$ is the $t$-slope of the ”elastic” $\rho$ production cross section $\sigma_\rho \equiv \frac{d\sigma(\gamma p \rightarrow \rho^0 p)/dt}{(at \; t = 0)}$ and the functions $H^D(\theta, \phi), \; D = T, L$ describe the angular distribution of the pions produced through the $\rho$-meson decay:

$$H^L = \sqrt{\frac{3}{4\pi}} \cos \theta,$$

(3)

$$H^T = \sqrt{\frac{3}{8\pi}} \sin \theta \cdot e^{\pm i\phi}.$$

(4)

Note that for transverse photons with polarization vector $\vec{e}$ one has to replace the last factor $e^{\pm i\phi}$ in eq. (4) by the scalar product $(\vec{e} \cdot \vec{n})$, where $\vec{n}$ is the unit vector in the pion transverse momentum direction.

2.2 Amplitude for non-resonant production

The amplitude for the non-resonant process $\gamma p \rightarrow \pi^+\pi^- p$ is:

$$A_{n.r.} = \sigma_\pi F_\pi(Q^2)e^{bt/2} \frac{\sqrt{\alpha}}{\sqrt{16\pi^3}} B^D \sqrt{z(1-z)} \left| \frac{dz}{dM^2} \right| \left( \frac{M^2}{4} - m^2_\pi \right) |\cos\theta|,$$

(5)
where \( b \) is the \( t \)-slope of the elastic \( \pi p \) cross section, \( F_\pi(Q^2) \) is the pion electromagnetic form factor \((Q^2 = |Q^2_\gamma| > 0 \) is the virtuality of the incoming photon), \( \alpha = 1/137 \) is the electromagnetic coupling constant and \( z \) – the photon momentum fraction carried by the \( \pi^- \)-meson; \( \sigma_{\pi p} \) is the total pion-proton cross section.

The factor \( B^D \) is equal to
\[
B^D = \frac{(e^D_\mu \cdot k^-_\mu) f(k^2_-)}{z(1 - z)Q^2 + m^2_\pi + k^2_-} - \frac{(e^D_\mu \cdot k^+_\mu) f(k^2_+)}{z(1 - z)Q^2 + m^2_\pi + k^2_+} \tag{6}
\]

For longitudinal photons the products \((e^L_\mu \cdot k^\pm_\mu)\) are: \((e^L_\mu \cdot k^-_\mu) = z\sqrt{Q^2}\) and \((e^L_\mu \cdot k^+_\mu) = (1 - z)\sqrt{Q^2}\), while for the transverse photons we may put (after averaging) \(e^T_\mu \cdot e^T_\nu = \frac{1}{2}\delta^T_\mu\nu\).

Expressions (5) and (6) are the result of straightforward calculation of the Feynman diagram fig. 1. The first term in (6) comes from the graph fig. 1 (in which the Pomeron couples to the \( \pi^+ \)) and the second one reflects the contribution originated by the \( \pi^- p \) interaction. The negative sign of \( \pi^- \) electric charge leads to the minus sign of the second term. We omit here the phases of the amplitudes. In fact, the common phase is inessential for the cross section, and we assume that the relative phase between \( A_\rho \) and \( A_{n.r.} \) is small (equal to zero) as in both cases the phase is generated by the same ‘Pomeron’ exchange.

The form factor \( f(k^2) \) is written to account for the virtuality \((k^2 \neq m^2_\pi)\) of the t-channel (vertical in fig. 1) pion. As in fig. 1 we do not deal with pure elastic pion-proton scattering, the amplitude may be slightly suppressed by the fact that the incoming pion is off-mass shell. To estimate this suppression we include the form factor (chosen in the pole form)
\[
f(k^2) = 1/(1 + k^2/m^2) \tag{7}
\]
The same pole form was used for \( F_\pi(Q^2) = 1/(1 + Q^2/m^2_\pi) \). In the last case the parameter \( m_\rho = M_0 \) is the mass of the \( \rho \)-meson – the first resonance on the \( \rho \)-meson (i.e. photon) Regge trajectory, but the value of \( m' \) (in \( f(k^2) \)) is expected to be larger. It should be of the order of mass of the next resonance from the Regge \( \pi \)-meson trajectory; i.e. it should be the mass of \( \pi(1300) \) or \( b_1(1235) \). Thus we put \( m'^2 = 1.5 \text{ GeV}^2 \).

\(^1\)Better to say – ‘vacuum singularity’. 

4
Finally we have to define $k_{\pm}^2$ and $k_{t\pm}$.

\[
\vec{k}_{t-} = -\vec{K}_t + z\vec{q}_t \quad \vec{k}_{t+} = \vec{K}_t + (1-z)\vec{q}_t
\]  

(8)

and

\[
k_{-}^2 = \frac{z(1-z)Q^2 + m_\pi^2 + k_{t-}^2}{z}, \quad k_{+}^2 = \frac{z(1-z)Q^2 + m_\pi^2 + k_{t+}^2}{1-z}.
\]  

(9)

In these notations

\[
M^2 = \frac{K_t^2 + m_\pi^2}{z(1-z)}, \quad dM^2/dz = (2z - 1)\frac{K_t^2 + m_\pi^2}{z^2(1-z)^2}
\]

and $z = \frac{1}{2} \pm \sqrt{1/4 - (K_t^2 + m_\pi^2)/M^2}$ with the pion transverse (with respect to the proton direction) momentum $\vec{K}_t$ (in the $2\pi$ rest frame) given by expression $K_t^2 = (M^2/4 - m_\pi^2)\sin^2\theta$. Note that the positive values of $\cos\theta$ correspond to $z \geq 1/2$ while the negative ones $\cos\theta < 0$ correspond to $z \leq 1/2$.

2.3 Absorptive correction

To account for the screening correction we have to consider the diagram fig. 2, where both pions interact directly with the target. Note that all the rescatterings of one pion (say $\pi^+$ in fig. 1) are already included into the $\pi p$ elastic amplitude. The result may be written in form of eq. (5) with the new factor $\tilde{B}^D$ instead of the old one $B^D = B^D(\vec{K}_t, \vec{q})$:

\[
\tilde{B}^D = B^D(\vec{K}_t, \vec{q}) - \int C \frac{\sigma_{\pi p}e^{-bl^2}}{16\pi^2} B^D(\vec{K}_t - z\vec{l}_t, \vec{q})d^2l_t
\]  

(10)

where the second term is the absorptive correction (fig. 2) and $l_\mu$ is the momentum transferred along the 'Pomeron' loop. The factor $C > 1$ reflects the contribution of the enhancement graphs with the diffractive exitation of the target proton in intermediate state. In accordance with the HERA data [13], where the cross section of "inelastic" (i.e. with the proton diffracted) $\rho$ photoproduction was estimated as $\sigma_{\rho\text{inel}} \simeq 0.5\sigma_{\rho\text{el}}$ we choose $C = 1.5 \pm 0.2$. 

5
3 $\sigma_L/\sigma_T$ ratio in $\pi^+\pi^-$ electroproduction near $\rho$-meson peak

At very large $Q^2$ the background amplitude (5) becomes negligible as, even without the additional form factor (i.e. at $f(k'^2) \equiv 1$), the non-resonance cross section falls down as $1/Q^8$ \footnote{In the amplitude $A_{n,r}$, one factor $1/Q^2$ comes from the electromagnetic form factor $F_\pi(Q^2)$ and another one – from the pion propagator (term $-z(1-z)Q^2$ in the denominator of $B_D$ (see eq.(6)).}, while experimentally \footnote{Unfortunately, the results for $R$ presented in Fig. 6 of our previous paper \cite{8} were wrong due to a misprint in the code.} the $Q^2$ behaviour of the elastic $\rho$ cross sections has been found to be $1/Q^n$, with $n \sim 5(< 8!)$.

Nevertheless, numerically at $Q^2 \sim 10 GeV^2$ the background as well as interference contributions are still important.

The background as well as interference contributions lead to the nontrivial behaviour of the ratio $R = \sigma_L/\sigma_T$ with the two pion mass $M_{2\pi} = M$. In the theoretical formulae which we used the index $D = L, T$ denotes the polarization of the incoming photon. On the other hand experimentally one measures the $\rho$-meson polarization, fitting the angular distribution of decay pions. To reproduce the procedure we take the flows of initial longitudinal and transverse photons to be equal to each other ($\epsilon = N_L/N_T = 1$, which is close to HERA case) and reanalyse the sum of cross sections ($\sigma = \sigma_L + \sigma_T$) in a usual way, selecting the constant and the $\cos^2\theta$ parts.

\begin{align}
I_0 &= \frac{1}{1} \int_{-1}^{1} \sigma(\theta) d\cos\theta ; \\
I_2 &= \frac{1}{L} \int_{-1}^{1} \sigma(\theta) \frac{15}{4} (3\cos^2\theta - 1) d\cos\theta \quad (11)
\end{align}

In these terms the density matrix element $r_{00} = (2I_2 + 3I_0)/9I_0$ and

\begin{align*}
R &= \sigma_L/\sigma_T = \frac{r_{00}}{1-r_{00}} \frac{3I_0 + 2I_2}{6I_0 - 2I_2} \quad (12)
\end{align*}

Namely these last ratios (12) are presented in fig. 3 for different $Q^2$ values \footnote{We have to recall, that the ratio $R$ given by the last expression of eq. (12) strictly speaking is not identical to the ratio of the longitudinal to transverse photon cross sections $\sigma_L/\sigma_T$. Due to the form factor $f(k'^2)$ the background}. We have to recall, that the ratio $R$ given by the last expression of eq. (12) strictly speaking is not identical to the ratio of the longitudinal to transverse photon cross sections $\sigma_L/\sigma_T$. Due to the form factor $f(k'^2)$ the background
pion angular distribution (corresponding to the non-resonant amplitude (5), (6)) are more complicated than the trivial $|d^{01}_{01}(\theta)|^2 = 1/2 \sin^2 \theta$ (for $\sigma^T$) and $|d^{00}_{00}(\theta)|^2 = \cos^2 \theta$ (for $\sigma^L$). For example, even in the case of the transverse photon polarization vector (i.e. $\sigma^T$) at very large $Q^2$ the non-resonant contribution $|A_{n.r.}|^2$ reveals the peaks in the forward and backward ($\cos \theta \approx \pm 1$) directions instead of a pure $\sin^2 \theta$ behaviour. That is why in Fig. 3 we present not the theoretical $\sigma^L/\sigma^T$ ratio, but the values of $R = \frac{3I_0 + 2I_2}{6I_0 - 2I_2}$ which are more close to the ratios measured experimentally. One can see that they depend both on $Q^2$ and $M$.

Note that for the resonant amplitude $A_\rho$ in Fig. 3 we have fixed $R_\rho = \textit{const} = 2$ independently on the mass $M$ and $Q^2$. On the other hand for the non-resonant production ($|A_{n.r.}|^2$), neglecting for simplicity the form factor (i.e. $f(k^2) = 1$) and the absorptive corrections, one has $R_{n.r.} = \sigma^L_{n.r.}/\sigma^T_{n.r.} \approx Q^2/M^2$.

At large $Q^2 \sim 10 \text{ GeV}^2$ it gives $R_{n.r.} \gg 2$. At last we have to take into account the interference, which is positive (constructive) at small $M < M_\rho$ and destructive at $M > M_\rho$. Thus the final curves 3 and 4 fall down with $M$ giving $R \approx 2.5$ at $M = 0.6 \text{ GeV}$ and $R \approx 1.5$ at $M = 1.1 \text{ GeV}$.

Of course at very large $Q^2$ the background amplitude dies out but simultaneously the value of $R_{n.r.} \approx Q^2/M^2$ increases with $Q^2$. Therefore the curves 3 and 4 (which correspond to $Q^2 = 10 \text{ GeV}^2$ and $30 \text{ GeV}^2$) are rather close to each other.

In the region of not too large $Q^2 \sim 1 \text{ GeV}^2$ the non-resonant amplitude is compatible with the resonant one at $M \geq 1 \text{ GeV}$. In particular, for $Q^2 = 1 \text{ GeV}^2$ (curve 1) the destructive interference at $M = 1.2 \text{ GeV}$ cancel the main part of the transverse cross section, leading to a very large value of $R$ ($\sim 7$). This effect depends on the $Q^2$ value. Say, the maximum of the peak of $R$ is at $Q^2 \approx 0.03 \text{ GeV}^2$ for $M = 1.15 \text{ GeV}$ and at $Q^2 \approx 0.3 \text{ GeV}^2$ for $M = 1.25 \text{ GeV}$. However one must not take the presented predictions in Fig. 3 too seriously for large $M$. The interference with the next (heavier) resonances (say, $\rho(1450)$) was not taken into account in Fig. 3 but at $M > 1.1 \text{ GeV}$ their contribution can be significant that can change the results.
4 Conclusion

We presented the results of $R = \sigma^L / \sigma^T$ ratio calculations for $\rho$-meson electroproduction with accounting for the background to ’elastic’ mechanism. The role of $\rho$-meson – background interference decreases with $Q^2$ however it is not negligible even at $Q^2 \sim 10 \text{ GeV}^2$.

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Figure captions

Fig. 1. Feynman diagram for the two pion photo-(electro)production.

Fig. 2. Diagram for the absorbtive correction due to both pions rescattering.

Fig. 3. The ratio $R = \sigma^L/\sigma^T$ in electroproduction process as a function of pion pair mass at $Q^2 = 1$ GeV$^2$ (curve 1), 4 GeV$^2$ (curve 2), 10 GeV$^2$ (curve 3) and 30 GeV$^2$ (curves 4) with (solid curve) and without (dashed curve) form factor.
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