Curvilinear Stress-Strain Relationship for Concrete of EN-2 Regulation in the ZI Method and the Calculation of Beam Strength

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1. Introduction

The present article may be considered to be another new supplement to the monographs [1, 2]. This supplement deals with the resolution of issues of practical application of the ZI method that was developed by the author and presented in his monographs. The ZI method is a uniform (general) method and is applicable for any stage of load operation when bending moments and/or axial forces are acting. At structural members’ cross sections, it is possible to make theoretical calculations of each individual actual value of stress-strain state parameters (crack, compression and tensile zone height, member layer strain and tension at cracks and between cracks). The method directly takes into account the actual properties of the materials. A very important and complicated problem is resolved, namely that of theoretical estimation of the actual position of the neutral axis. Previously, its position used to be determined either very roughly or through costly experimental equations. The ZI method is suitable for the calculation of variously reinforced structural members (with or without-tensioned reinforcement, with tensioned reinforcement, with mixed reinforcement, with not necessarily metallic reinforcement located at any height of the structural member; where there can be any number of rows of reinforcement) of different materials (concrete, reinforced concrete, metal, wood, plastic, etc.), also for members with any kind of cross-section. Stress-strain relationships can be described by various equations, i.e., the stress diagrams may be curvilinear, rectangular, triangular, etc. It should be noted that for the calculations we only need to have stress-strain diagrams. The method enables estimation of the deviation of strain from the flat section. More information on this may be found in monographs [1 and 2] and in articles [3, 4, etc.]

The most complex reinforced concrete member is reinforced concrete since members made of reinforced concrete may have cracks in the tension zone. This article focuses on the calculation of strength of reinforced concrete beams based on curvilinear stress-strain relationship $\sigma_c - \varepsilon_c$ for concrete presented in regulations [5–7]. But here still remains one unresolved issue. This relationship is presented for the case where the reliability is 50 %. It is suitable for the analysis of results of reinforced concrete member tests. There is no option for the calculation of the SLS – serviceability limit states – reliability 95 %) or the ULS – ultimate limit states reliability – ~100 %).

To calculate strength of reinforced concrete beams, articles [4, 8] and supplements B and C to the monographs [1, 2] used the eurocode $\sigma_c - \varepsilon_c$ diagram described by the ZI method. The reliability was raised from 50 % to ~100 % not through the increase of the reliability of concrete strength, as it is usually done in the ultimate limit states method, but rather by dividing the beam compression zone strength $F_{cm}$ by factor $\gamma_k = 1.95$. The calculation yielded good results – see Table 2.

In order to retain the integrity of the ultimate limit states method, this article offers relationships $\sigma_c - \varepsilon_c$ analogous to the ones used in EN-2 regulations. Their reliability is not 50 %, but 95 % and ~100 %. They are described by polynomials by the ZI method. The latter relationship with reliability of ~100 % is used in the present article to calculate the strength of beam, and the relationship with reliability of 95 % will be used in the next article to examine the serviceability limit states.

One of the aims of this article is to improve the method for calculating the strength of heavily reinforced structural members.

2. Stress-strain diagrams for concrete offered by EN-2 and proposed by the present article

Regulations for reinforced concrete [5-7] present stress-strain relationships as shown in Table 1.

![Fig. 1 Stress-strain relationship for concrete as presented in regulation EN-2](image)

The diagram parameters are expressed by the following equations:

$$\frac{\sigma_c}{f_{cm}} = \frac{k\eta - \eta^2}{1 + (k - 2)\eta}$$  \hspace{1cm} (1)

$$\eta = \frac{\varepsilon_c - \varepsilon_{c1}}{\varepsilon_{c1}}$$  \hspace{1cm} (2)
The ZI methodology takes into account 3rd grade stress-strain relationship for concrete

In order to have reliability of 95\%, which is used for the calculation of the serviceability limit states (SLS states), the author of the present article suggests in Eqs. (5 and 6) (rather than using mean values of parameters) assuming characteristic values – index \( k \). Then in Fig. 2 \( E_{cm} = \tan \alpha \). \( \sigma_{ci} = f_{ck} \), \( \varepsilon_{ci} = 0.7 f_{ck}^{0.31} \leq 2.8 \). This option is planned to be further analysed in the author’s next article.

When reliability of \(~\sim 100\%\) is required, the author of the present article proposes when calculating the ultimate limit states (ULS states) in Eqs. (5 and 6) (rather than assuming mean values of parameters) to assume the calculated values index \( d \). Then, in Fig. 2 \( E_{ci} = \tan \beta \), \( \sigma_{ci} = f_{cd} \), \( \varepsilon_{cd} = 0.7 f_{cd}^{0.31} \leq 2.8 \). This option is analysed in greater detail in this article. We obtain good results – Tables 1 and 2.

3. The main equations of the ZI method when a 3rd degree polynomial is used

When the function of Fig. 2 is expressed by the ZI method’s 3rd degree polynomial, the following simple equations are used:

\[
\sigma_c = E_c \varepsilon_c \left( 1 + c_1 \eta + c_2 \eta^2 \right) = \nu_v E_c \varepsilon_c = \nu_v \sigma_{vc},
\]

\[
\nu_v = 1 + c_1 \eta + c_2 \eta^2 = 1 + (3 \nu_v - 2) \eta + (1 - 2 \nu_v) \eta^2.
\]

\[
\nu_v = \frac{\sigma_{vc}}{\sigma_{ci}} = \frac{f_{cm}}{E_{cm} \varepsilon_{ci}}.
\]

\[
\eta = \varepsilon_c / \varepsilon_{ci}.
\]

\[
F_c = \int_0^\varepsilon_c \sigma_c b(\delta x_c) = \omega_{cc} \varepsilon_c E_c b x_c^c,
\]

\[
M_c = \int_0^\varepsilon_c \sigma_c b x_c (\delta x_c) = \omega_{mc} \varepsilon_c E_c b x_c^c.
\]

\[
\varepsilon_w = \frac{M_c}{F_c} = \frac{\omega_{mc}}{\omega_{cc}} x_w,
\]

\[
\omega_{cc} = \frac{1}{2} + \frac{c_1}{3} \eta + \frac{c_2}{4} \eta^2,
\]

\[
\omega_{mc} = \frac{1}{3} + \frac{c_1}{4} \eta + \frac{c_2}{5} \eta^2.
\]

\( \varepsilon_w \) is maximum (edge) strain of the compression zone of the beam.

For more information, see the examples.

4. Assumptions and calculations made in the present article

4.1. Assumptions

1. For the concrete of beam compression zone, a curvilinear EN-2 stress diagram \( \sigma_c - \varepsilon_c \) is assumed that is shown in Fig. 1, which is described by the ZI method’s 3rd degree polynomial as shown in Fig. 2.

2. For the reinforcement of tensile zone of beams analysed in the article, the diagram \( \sigma_r - \varepsilon_r \) as shown in Fig. 3 is used.

3. The strength of a beam is considered to be the state when the stress of the concrete compression zone’s stresses
\[ \sigma_c = \sigma_{c1} \] and strain \( \varepsilon_c = \varepsilon_{c1} \) or when the strain of the reinforcement in tension \( \varepsilon_s = \varepsilon_{su} \).

4. Hypothesis of plane sections (Bernoulli) is applied.
5. The impact of the tensioned concrete over the crack is disregarded.

Fig. 3 Reinforcement stress-strain diagram is assumed in the article

4.2. Calculations made in the article

**Scope of the research.** Four reinforced concrete beam's tensile zone's reinforcement variants – little (reinforcement factor \( \rho_1 \approx 0.44\% \)), average (\( \rho_1 \approx 1.07\% \) and \( \rho_1 = 1.60\% \)) and large (\( \rho_1 = 2.13\% \)). All the [5–7] strength classes of regular concrete as presented in the regulations: \( f_{ck} \) from 08 to 90 MPa. Beam cross-section parameters: \( b = 0.20 \text{ m} \), \( h = 0.50 \text{ m} \), \( d = 0.46 \text{ m} \) (Fig. 4).

4.3. Research results

The results of the calculations are presented in Table 2.

Further in the text we supply examples of the calculation and explanations of the results presented in the tables.

Examples of calculations of \( M_{Rd,EN-2} \) by EN-2 equations and calculation of \( M_{Rd,ZI} \) by the ZI method when \( f_{c1} = 1.95 \) coefficient is used are presented in example to monographs [1, 2]. Results of the calculations are presented in Table 2.

The article presents examples of variants of calculation of \( M_{Rd,ZI} \) by the ZI method, when the calculated expression is used of the diagram \( \sigma_c - \varepsilon_c \) presented in the EN-2 regulation described by the ZI method. There are three possible cases: Case 1, where \( \sigma_c = f_{cd} \) and \( \varepsilon_{cd} \leq \varepsilon_s \leq \varepsilon_{su} \) – these are regularly (economically) reinforced beams; Case 2, where \( \sigma_c = f_{cd} \) and \( \varepsilon_s < \varepsilon_{cd} \), – these are abundantly reinforced beams and Case 3, where \( \sigma_c = f_{cd} \) and \( \varepsilon_s \geq \varepsilon_{su} \) economical reinforcement – when the calculation is made on the assumption that \( \sigma_c = f_{cd} \) and \( \varepsilon_s = \varepsilon_{su} \). In all cases, the calculated version of the curvilinear EN-2 diagram described by the ZI method is used, i.e. partial material strength factors are used that are accepted in the limit state method. In the examples provided for the simplification of the calculations, the impact of tensile concrete over the crack is disregarded, as in most cases it is insignificant. If necessary, it is possible to factor in also the impact of the tensioned concrete over the crack, the impact of the axial force, and the impact of the flanges.

In examples 1, 2 and 3, \( M_{Rd,ZI} \) is calculated by the ZI method by applying to the compression zone concrete partial factor \( \gamma_c = 1.5 \).

**Example 1. Regularly (economically) reinforced beam**

When \( \sigma_c = f_{cd} \) and \( \varepsilon_{cd} \leq \varepsilon_s \leq \varepsilon_{su} \), strength of regularly (economically) reinforced concrete beams with rectangular cross-section is calculated by the method of ultimate limit states of both zones (tension zone reinforcement and compression zone concrete). \( f_{ck} = 25 \) MPa.

\[ f_{cd} = f_{ck} / 1.5 = 25/1.5 = 1.6667 \text{ MPa}. \]

2Ø16 mm \( \rightarrow A_i = 4.02 \text{ cm}^2 \).

\[ \frac{f_{cd}}{\gamma_c} = \frac{400}{1.1} = 363.636 \approx 364 \text{ MPa}, \]

\[ F_{cd} = f_{cd} A_i = 366.6 \cdot 10^{-6} \cdot 4.02 \cdot 10^{-4} = 146.167 \text{ kN}. \]

\[ x_{nl} = \frac{f_{cd} A_i + P}{\omega_{nl} \varepsilon_{cd} E_{c} b} = \frac{146.167 \cdot 10^{-4}}{0.061352} = 0.27743 \cdot 42.9375 \cdot 10^{-6} \cdot 0.20 = 0.061352 \text{ m}. \]

\[ \varepsilon_s = \varepsilon_{cd} \left( d - x_{nl} \right) = \frac{1.6744}{6.1352} = 0.27743 \cdot 42.9375 \cdot 10^{-6} = 10.8798 \% > \]

\[ > \varepsilon_{cd} = \frac{f_{cd}}{E_{c}} = \frac{363.6 \cdot 10^{-6}}{200 \cdot 10^{-6}} = 1.818 \cdot 10^{-3} = 1.818 \% \).

For instance, if \( \varepsilon_{sa} = 35 \% \), then \( \varepsilon_s = \varepsilon_{su} \). Control:

\[ F_{cd} = \omega_{nl} \varepsilon_{cd} E_{c} b x_{nl} = \]

\[ = 0.27743 \cdot 1.6744 \cdot 10^{-6} \cdot 25.6435 \cdot 10^{-6} \cdot 0.20 \cdot 0.061352 = \]

\[ = 0.146167 \text{ MN} = 146.167 \text{ KN} = F_{cd}. \]

Distance between \( F_{c1} \) and \( F_{c2} \):

\[ z_s = z_{cd1} = z_{cd2} = d - \left( 1 - \omega_{nl} / \omega_{c} \right) x_{nl} = \]

\[ = 0.46 - \left( 1 - 0.16920 / 0.27743 \right) = 0.061352 = 0.43507 \text{ m}. \]

\[ M_{Ru} = F_{cd} z_{nl} = 146.167 \cdot 0.43507 = 63.739 \text{ kNm}. \]

If we factor in the impact of tensioned concrete above the crack, then we get \( M_{Ru} = 63.747 \text{ kN-m}, i.e. almost the same result. \]

If we factor in not only the tension zone reinforcement force \( F_{cd} \) and compression zone concrete force \( F_{cd} \), but also tension zone over the crack concrete force and the forces, values of which do not depend on \( x_{nl} \) (axial and tension forces \( N \) and \( P \), compression zone reinforcement \( F_{sc} = f_{cd} A_{sc} \) and flanges \( F_{fl} = \eta f_{cd} (b_f - b) \), forces (Fig. 5), then \( x_{nl} = \frac{N_{const}}{(\omega_{nl} \varepsilon_{cd} - \eta f_{cd} E_{c} b ) E_{cd} b} \), \( N_{const} = N + P + F_{cf} - F_{sc} - F_{cf} \).

**Example 2. \( \varepsilon_s < \varepsilon_{cd} \), i.e. case of abundant reinforcement**

\( f_{ck} = 20 \text{ MPa}, 4\varnothing 25 \text{ mm} \rightarrow A_i = 19.63 \text{ cm}^2, \)

\( \rho_r = 19.63 \cdot 100 / (20 \cdot 46) = 2.1337 \approx 2.134 \% \).
This is a rather...
If in Example 3 instead of $\varepsilon_s = \varepsilon_{u_s} = 35\%$, we take $\varepsilon_s = \varepsilon_{u_s} = 41.25936\%$, then we get the same result as the result of the calculation according to both zones ULS method of Example 1, i.e.

$\varepsilon_s = \varepsilon_{u_s} = 41.25936\%$ and $M_{RU} = 65.932$ kNm.

This confirms the correctness of the method of Example 3.

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 若在示例3中，我们取$\varepsilon_s = \varepsilon_{u_s} = 35\%$，则我们得到与根据两个区域ULS方法计算得出的结果相同，即$\varepsilon_s = \varepsilon_{u_s} = 41.25936\%$和$M_{RU} = 65.932$ kNm。

这证实了方法的正确性。
### Table 1

**Strength and deformation characteristics for concrete**

| Parameters | Strength classes for concrete |
|------------|------------------------------|
| $f_{ck}$, MPa | $\gamma_{fi}$ |
| | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 |
| | 8 | 12 | 16 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 |
| $f_{cm} = f_{ck} + 8$ (MPa) | 16 | 20 | 24 | 28 | 33 | 38 | 43 | 48 | 53 | 58 | 63 | 68 | 73 | 78 | 83 |
| $\beta_{cm} = \frac{25}{f_{cm}^{0.8}}$ | 25.33 | 27.08 | 28.60 | 29.96 | 31.47 | 32.86 | 34.07 | 35.22 | 36.23 | 37.79 | 38.21 | 39.09 | 40.28 | 42.24 | 43.63 |
| $\sigma_{cm} = \beta_{cm} \cdot \gamma_{fi}$ (GPa) | 26.597 | 28.43 | 30.03 | 31.46 | 33.05 | 34.48 | 35.78 | 36.98 | 38.09 | 39.16 | 40.14 | 41.03 | 42.79 | 44.37 | 45.82 |
| $\varepsilon_{cm} = 0.7 \frac{f_{cm}^{0.8}}{f_{cm}} \leq 2.8$ (%) | 1.653 | 1.771 | 1.847 | 1.960 | 2.063 | 2.163 | 2.246 | 2.342 | 2.396 | 2.464 | 2.527 | 2.583 | 2.701 | 2.8 | 2.8 |
| $\sigma_{cm}$, MPa | 3.5 %, when $f_{ck} < 50$ MPa; $\sigma_{cm} = 2.8 + 0.027 \left[ \left( f_{cm} - 100 \right)^{0.5} \right]$ %, when $f_{ck} \geq 50$ MPa |
| | 3.5 | 3.491 | 3.205 | 3.018 | 2.843 | 2.802 | 2.8 |
| $\sigma_{cm}$, MPa | 43.97 | 50.38 | 56.31 | 61.86 | 65.93 | 74.35 | 80.37 | 85.95 | 91.31 | 96.47 | 101.47 | 106.31 | 115.58 | 124.19 | 128.27 |
| $V_{cm} = \beta_{cm} \left( 10^{0.5} \sigma_{cm} \right)$ | 0.3618 | 0.396 | 0.426 | 0.452 | 0.482 | 0.509 | 0.535 | 0.558 | 0.584 | 0.603 | 0.621 | 0.639 | 0.675 | 0.709 | 0.740 |
| $e_{cm} = 3 \varepsilon_{cm}$ | -0.909 | -0.809 | -0.721 | -0.614 | -0.525 | -0.470 | -0.395 | -0.325 | -0.258 | -0.196 | -0.137 | -0.089 | 0.024 | 0.128 | 0.292 |
| $e_{cm} = \left( 1 - 2 \gamma_{fi} \right)$ | 0.272 | 0.206 | 0.147 | 0.098 | 0.050 | 0.019 | 0.001 | 0.003 | 0.008 | 0.016 | 0.032 | 0.065 | 0.118 | 0.204 | 0.428 |
| **Design values for the calculation of the ultimate limit states (ULS) – reliability = 100%** |
| $f_{ck}$, MPa | 20.73 | 24.3 | 28.05 | 31.72 | 33.87 | 35.62 | 37.38 | 39.13 | 40.87 | 42.62 | 44.36 | 46.08 | 47.72 | 49.34 | 50.90 |
| $E_{ck} = 25 \left( f_{ck} / 10^{0.5} \right)$ (GPa) | 18.21 | 20.78 | 22.86 | 25.01 | 26.48 | 28.06 | 29.59 | 31.15 | 32.71 | 34.28 | 35.85 | 37.42 | 38.99 | 40.55 | 42.11 |
| $e_{cm} = 0.7 \frac{f_{cm}^{0.8}}{f_{cm}} \leq 2.8$ (%) | 1.762 | 1.333 | 1.458 | 1.562 | 1.674 | 1.778 | 1.885 | 1.937 | 2.009 | 2.075 | 2.138 | 2.196 | 2.304 | 2.401 | 2.490 |
| $a_{ck}$, MPa | 3.5 %, when $f_{ck} < 50$ MPa; $a_{ck} = 2.8 + 0.035 \left[ \left( f_{cm} - 100 \right)^{0.5} \right]$ %, when $f_{ck} \geq 50$ MPa |
| | 3.5 | 3.491 | 3.205 | 3.018 | 2.843 | 2.802 | 2.8 |
| $a_{ck}$, MPa | 21.429 | 27.145 | 32.075 | 37.473 | 42.937 | 47.896 | 52.724 | 57.969 | 61.456 | 65.539 | 69.456 | 73.240 | 78.066 | 82.745 | 89.370 |
| $V_{ck} = f_{ck} \cdot \beta_{ck} \cdot \gamma_{fi}$ | 0.249 | 0.289 | 0.326 | 0.358 | 0.388 | 0.416 | 0.442 | 0.466 | 0.482 | 0.508 | 0.532 | 0.541 | 0.580 | 0.611 | 0.639 |
| $e_{ck} = 3 \varepsilon_{ck}$ | -1.253 | -1.125 | -1.021 | -0.931 | -0.835 | -0.747 | -0.672 | -0.603 | -0.535 | -0.471 | -0.413 | -0.367 | -0.300 | -0.167 | -0.081 |
| $e_{ck} = 1 - 2 \gamma_{fi}$ | 0.502 | 0.416 | 0.347 | 0.281 | 0.227 | 0.165 | 0.114 | 0.067 | 0.023 | -0.017 | -0.058 | -0.092 | -0.159 | -0.219 | -0.279 |

When $e_{ck} - \gamma_{fi} = -1$

- $\omega_{ck} = \frac{1}{2} \frac{e_{ck} + \varepsilon_{ck}}{2}$
- $\omega_{ck} = \frac{1}{3} \frac{e_{ck} + \varepsilon_{ck}}{2}$
- $\omega_{ck} = \frac{1}{4} \frac{e_{ck} + \varepsilon_{ck}}{2}$
- $\omega_{ck} = 0.5 \frac{e_{ck} + \varepsilon_{ck}}{2}$
- $\omega_{ck} = 0.7970 \cdot 0.5900 \cdot 0.5902 \cdot 0.40430 \cdot 0.6198 \cdot 0.6130 \cdot 0.6210 \cdot 0.6234 \cdot 0.6286 \cdot 0.6298 \cdot 0.6330 \cdot 0.6357 \cdot 0.6399$

$x_{ult} = e_{ck} \left( f_{ck} + \varepsilon_{ck} \right)$
Continuation of Table 1

| $f_{cd}$, MPa | 8   | 12  | 16  | 20  | 25  | 30  | 35  | 40  | 45  | 50  | 55  | 60  | 70  | 80  | 90  |
|---------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $x_{W1}$, cm  | 16.4139 | 11.8224 | 9.0694 | 7.4657 | 6.1352 | 5.2203 | 4.5510 | 3.9847 | 3.6445 | 3.3029 | 3.0298 | 2.7998 | 2.4229 | 2.1531 | 1.9327 |
| $\sigma_x = \sigma_{cd}(d - x_{W1})/x_{W1}$, % | 7.1201 | 3.9436 | 5.9374 | 8.0648 | 10.8748 | 13.8410 | 16.9254 | 20.4249 | 23.3446 | 26.8844 | 30.8747 | 33.8099 | 41.7954 | 48.9047 | 56.7903 |
| $M_{ku} = F_{cd}L_2 = F_{cd}L_2$, kN/m | 57.153 | 60.285 | 61.915 | 62.919 | 63.739 | 64.294 | 64.695 | 65.029 | 65.232 | 65.431 | 65.590 | 66.090 | 66.809 | 68.214 |

Below for all types classes of concrete when $2\phi6mm \rightarrow A_e = 4.02cm^2$, $\rho_e = 0.42 \text{ to } 0.46 \times 0.1370 \%$, $N_{ud} = f_{cd}A_e = 363.6 \times 10^{-4} = 146.672 \times 10^{-4} \text{ kN}$. $N_{ud} = 146.157 \text{ kN}$
### Table 2

Comparison of the calculations of strength of beams $M_{Rd}$ by ZI method and EN-2 method (in case of abundant reinforcement, the figures in the table are highlighted)

| $f_{ck}$ (MPa) | 08 | 12 | 16 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 70 | 80 | 90 |
|---------------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| $M_{Rd,ZI}$   | 61.547 | 62.458 | 63.103 | 63.688 | 64.163 | 64.500 | 64.767 | 65.000 | 65.176 | 65.248 | 65.478 | 65.695 | 65.869 | 66.007 |
| $M_{Rd,EN-2}$| 56.106 | 59.819 | 61.675 | 62.786 | 63.678 | 64.270 | 64.994 | 65.008 | 65.256 | 65.454 | 65.578 | 65.673 | 65.826 | 65.929 | 66.002 |
| $M_{Rd,ZI} / M_{Rd,EN-2}$ | 1.0259 | 1.0127 | 1.0030 | 1.0002 | 0.9985 | 0.9960 | 0.9963 | 0.9961 | 0.9970 | 0.9980 | 0.9991 | 1.0001 | 1.0001 | 1.0001 | 1.0001 |
| $M_{Rd,ZI}$ | 57.153 | 60.285 | 62.918 | 62.919 | 63.739 | 64.294 | 64.892 | 65.029 | 65.232 | 65.431 | 65.590 | 65.722 | 65.932 | 66.090 | 66.214 |
| $M_{Rd,EN-2}$ | 1.0187 | 1.0078 | 1.0039 | 1.0021 | 1.0001 | 1.0000 | 1.0000 | 1.0000 | 0.9996 | 0.9996 | 1.0002 | 1.0022 | 1.0016 | 1.0024 | 1.0032 |

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**Reinforcement ratio for tensile zone of reinforced concrete beams**

| $M_{Rd,ZI}$ | 107.506 | 135.728 | 139.578 | 143.069 | 145.901 | 147.916 | 149.305 | 150.897 | 151.927 | 152.974 | 153.748 | 154.377 | 156.682 | 156.947 |
| $M_{Rd,EN-2}$ | 78.798 | 118.197 | 131.039 | 137.650 | 143.001 | 145.535 | 149.070 | 150.943 | 152.445 | 153.628 | 154.337 | 154.926 | 155.819 | 156.426 | 156.872 |
| $M_{Rd,ZI} / M_{Rd,EN-2}$ | 0.9095 | 1.0358 | 1.0138 | 1.0005 | 0.9987 | 0.9992 | 0.9993 | 0.9998 | 0.9990 | 0.9992 | 0.9997 | 1.0002 | 1.0002 | 1.0002 | 1.0002 |
| $M_{Rd,ZI}$ | 76.588 | 103.204 | 127.403 | 138.480 | 143.368 | 146.635 | 149.078 | 151.074 | 152.323 | 153.469 | 154.415 | 155.203 | 156.438 | 157.404 | 158.143 |
| $M_{Rd,EN-2}$ | 0.9720 | 0.8732 | 0.9732 | 1.0038 | 1.0056 | 1.0010 | 1.0000 | 0.9991 | 0.9990 | 1.0005 | 1.0018 | 1.0041 | 1.0063 | 1.0081 |

**Reinforcement ratio for tensile zone of reinforced concrete beams**

| $M_{Rd,ZI}$ | 107.506 | 129.953 | 151.588 | 198.321 | 204.344 | 209.652 | 212.604 | 215.714 | 218.017 | 220.356 | 222.061 | 224.890 | 227.302 | 229.147 |
| $M_{Rd,EN-2}$ | 78.798 | 118.197 | 157.597 | 186.189 | 198.077 | 203.978 | 211.668 | 215.878 | 219.156 | 221.799 | 223.403 | 224.733 | 226.700 | 228.099 | 229.035 |
| $M_{Rd,ZI} / M_{Rd,EN-2}$ | 0.9095 | 0.9245 | 0.9150 | 1.0007 | 0.9931 | 0.9878 | 0.9849 | 0.9843 | 0.9830 | 0.9864 | 0.9883 | 0.9925 | 0.9986 | 1.0004 |
| $M_{Rd,ZI}$ | 83.489 | 113.578 | 141.413 | 166.040 | 197.263 | 205.280 | 211.655 | 216.097 | 218.988 | 221.452 | 222.564 | 223.532 | 228.130 | 230.193 | 231.894 |
| $M_{Rd,EN-2}$ | 1.0595 | 0.9689 | 0.8956 | 0.89656 | 0.9961 | 1.0015 | 1.0001 | 1.0010 | 0.9988 | 0.9983 | 1.0007 | 1.0027 | 1.0063 | 1.0092 | 1.01242 |

**Reinforcement ratio for tensile zone of reinforced concrete beams**

| $M_{Rd,ZI}$ | 78.798 | 118.197 | 157.597 | 196.996 | 243.423 | 257.556 | 267.691 | 275.257 | 281.145 | 285.856 | 288.746 | 291.102 | 294.625 | 297.097 | 298.846 |
| $M_{Rd,EN-2}$ | 88.020 | 120.579 | 158.287 | 178.779 | 212.122 | 243.767 | 257.729 | 275.683 | 280.082 | 285.266 | 289.044 | 292.264 | 297.207 | 300.983 | 303.933 |
| $M_{Rd,ZI} / M_{Rd,EN-2}$ | 1.1710 | 1.0202 | 0.9555 | 0.9076 | 0.8714 | 0.9465 | 1.0001 | 1.0016 | 0.9984 | 0.9979 | 1.0010 | 1.0038 | 1.0087 | 1.0131 | 1.0486 |

The moments $M_{Rd,ZI}$ marked by asterisk were calculated using the concrete strength partial factor $\gamma_{fc} = 1.5$.

The partial factor for the beam concrete compression zone $\gamma_{pc} = 1.95$.

$M_{Rd,ZI}$ and $M_{Rd,EN-2}$ were calculated by the method proposed by the article assuming partial factor $\gamma_{pc} = 1.95$.

$M_{Rd,ZI}$ was calculated by the method proposed by the article assuming partial factor $\gamma_{pc} = 1.5$ - reliability -100%.

$M_{Rd,ZI}$ and $M_{Rd,EN-2}$ were calculated by the method proposed by the article assuming partial factor $\gamma_{pc} = 1.5$.
Conclusions

1. Description of non-linear diagrams $\sigma_e - \varepsilon_e$ for concrete as presented in regulations EN-2 with reliability of 50%, by ZI method provides a possibility to have a reliability not only of 50%, but also of 95% and ~100%. The calculations made in the article confirm that the proposal is realistic.

2. When calculating the strength of reinforced concrete beams using the ZI method, there is no need to have a limit value for the thickness of the concrete layer of the compression zone, which at present is traditionally calculated either from empirical equations or theoretically very roughly.

3. Calculation of the strength of reinforced concrete beams using the ZI method presented in the article is logical and gives actual values of normal and abundantly reinforced beam stress-strain state at the crack. No empirical equations are required for these calculations. This is especially important for the calculation of abundantly reinforced beams, as their calculation that is used at present is either complex or, alternatively, the simplified calculation that is made is imprecise.

4. The data presented in Table 1 makes it possible to simplify the calculations. The data in Table 1 show that the proposals are realistic. In addition to that data in Table 2 shows that the ULS required reliability can be achieved in two ways: (1) through the use of diagram $\sigma_e - \varepsilon_e$ with reliability of 50% and reinforced concrete compression zone concrete force factor $\gamma_{FC} = 1.95$ or (2) through the use of a diagram $\sigma_e - \varepsilon_e$ with reliability of ~100%; reinforced concrete compression zone concrete (not force) factor $\gamma_e = 1.5$ is used. The second option is in line with the limit states methods that are currently used.

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CURVILINEAR STRESS-STRAIN RELATIONSHIP FOR CONCRETE OF EN-2 REGULATION IN THE ZI METHOD AND THE CALCULATION OF BEAM STRENGTH

Summary

The article illustrates the possibilities of the practical application of the ZI method [1 and 2] when calculating the strength of reinforced concrete beams. The article presents variants of description of the EN-2 regulation curvilinear diagram for concrete $\sigma_e - \varepsilon_e$ with reliability of 50% by the ZI method with reliability of 50%, 95% and ~100%. The article demonstrates how, when calculating the strength of normally and abundantly reinforced concrete beams by the ZI method, it is possible to do without the calculation of the load limit value of the thickness of the concrete layer of the beam compression zone. This is important in the case of the calculation of the strength of abundantly reinforced beams. The method for calculating the strength of abundantly reinforced beams has been improved. When calculating strength, we also obtain actual values of stress-strain parameters at the crack. The tables provide data supporting the proposed innovations and facilitating calculations.

Keywords: ZI method, reliability of curvilinear diagram, reinforced concrete beam strength, abundantly reinforced beams, limit value compression zone.

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