 Conditional generation of multiphoton-subtracted squeezed vacuum states: loss consideration and operator description

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Abstract
In terms of the characteristic functions of the quantum states, we present a complete operator description of a lossy photon-subtraction scheme. Feeding a single-mode squeezed vacuum into a variable beam splitter and counting the photons in one of the output channels, a broad class of multiphoton-subtracted squeezed vacuum states (MSSVSs) can be generated in other channel. Here, the losses are considered in the beginning and the end channels in the circuit. Indeed, this scheme has been discussed in Ref. [Phys. Rev. A 100, 022341 (2019)]. However, different from the above work, we give all the details of the optical fields in all stages. In addition, we present the analytical expressions and numerical simulations for the success probability, the quadrature and number squeezing effect, photon number distribution, and Wigner function of the MSSVSs. Some interesting results effected by the losses are obtained.

Keywords Quantum state engineering · Photon-subtracted Operation · Conditional measurement · Loss channel · Characteristic function

1 Introduction
A key requirement for many quantum protocols is to use specific quantum states of light as a resource for information processing [1]. These quantum states can be divided into Gaussian and non-Gaussian cases [2]. For example, the coherent state

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and the single-mode squeezed vacuum state are the typical Gaussian states, which are applied in many tasks [3]. However, various important protocols for quantum-enhanced information processing cannot be performed when restricted to Gaussian states [4]. Thus, it is necessary to introduce non-Gaussianity into an optical system. In recent years, many non-Gaussian quantum states have been used as resources for useful quantum information processing tasks [5,6]. Therefore, a crucial goal for experimental quantum optics is to prepare high-quality non-Gaussian quantum states.

Photon-subtraction operation is just a useful way to conditionally manipulate a non-Gaussian state of the optical field, which has been shown to enhance entanglement [7,8] and teleportation fidelity [9]. Indeed, some quantum states after applying photon-subtraction operation or other non-Gaussian operations may produce higher degrees of nonclassicality and entanglement [10–13]. Theoretically, subtraction of $m$ photons from a single-mode quantum state $|\psi\rangle$ can be expressed as $a^m |\psi\rangle$, where $a$ is the photon annihilation operator [14]. Experimentally, such photon-subtracted state can be implemented by transmitting $|\psi\rangle$ through a beam splitter (BS) and detecting the output of the beam splitter with photon number resolving detector [15]. Studies have shown that photon subtraction on a single-mode squeezed vacuum state yields optical coherent-state superposition [16] or Schrodinger-catlike states [17].

In real life, when a signal (a quantum state) passes through a quantum channel, the losses of the radiation field are not inevitable. Often, these losses can be described by the interaction between the field and its reservoir with a large number of degrees of freedom. We need to analyze and control the effect of loss in the quantum protocols [18,19]. From the perspective of mathematical physics, reservoir can be modeled by BS. Therefore, we can abstract the loss of optical field into an equivalent BS model. Very recently, Quesada et al. considered some schemes of preparing conditional non-Gaussian states in the presence of photon loss. Among them, the photon-subtraction scheme has more attracted our attention [20]. In the present paper, we shall give a description for the same scheme in terms of the characteristic function (CF) of the quantum states involved. The CF of density operator $\rho$ can be defined as $\chi_\rho (\alpha) = \text{Tr}[\rho D (\alpha)]$, i.e., the expectation value of the Weyl displacement operator $D (\alpha) = \exp (a a^\dagger - a^* a)$ [21]. One main tool in dealing with optical field is the Weyl expansion of the density operator, that is,

$$\rho = \int \frac{d^2\alpha}{\pi} \chi_\rho (\alpha) D (-\alpha),$$

which means that the function $\chi_\rho (\alpha)$ uniquely determines the density operator $\rho$ [22,23]. Another tool in deriving input–output relation of loss channel $L (\eta)$ (with loss factor $\eta \in [0, 1]$) is that the output density operator $\rho_{\text{out}}$ can be expressed as the integration form of the input CF $\chi_{\rho_{\text{in}}} (\alpha)$, i.e.,

$$\rho_{\text{out}} = \int \frac{d^2\alpha}{\pi} \chi_{\rho_{\text{in}}} (\sqrt{1 - \eta} \alpha) D (-\alpha) e^{-\frac{1}{2} \eta |\alpha|^2},$$

which describes that $\rho_{\text{in}}$ evolves into $\rho_{\text{out}}$ through loss channel. This equation has been derived in our previous work [24].
The paper is organized as follows. In Sect. 2, we introduce the density operator description of generating such state. Here, we shall give the conceptual scheme and decompose the whole circuit into five stages, whose density operators are derived. Then, in Sects. 3–5, we study the analytical and numerical results of the properties related to our generated states, including quadrature squeezing effect, photon number distribution and Wigner function, and explore how the photon subtraction and loss factors affect these nonclassicalities. Finally, a summary is given in Sect. 6.

2 Density operator description of theoretical scheme

Figure 1 shows the conceptual scheme of generating multiphoton-subtracted squeezed vacuum states (MSSVSs). The whole circuit can be decomposed into five stages, that is,

\[ \rho^{(1)}_{ab} \rightarrow \rho^{(2)}_{ab} \rightarrow \rho^{(3)}_{ab} \rightarrow \rho^{(4)}_{ab} \rightarrow \rho. \]  

(3)

In this scheme, red line and blue line are corresponding to mode a and mode b, respectively. \( \rho^{(1)}_{ab} \) is the direct product of the single-mode squeezed vacuum state (SVS) \( S(r)|0\rangle \) and the vacuum \( |0\rangle \). Here, \( S(r) = \exp[\frac{r}{2}(a^\dagger - a)^2] \) is the single-mode squeezed operator with the real squeezing parameter \( r \) [25]. After \( \rho^{(1)}_{ab} \) passing through channel \( L(\eta_1) \) with loss factor \( \eta_1 \), \( \rho^{(2)}_{ab} \) is obtained. Injecting \( \rho^{(2)}_{ab} \) into a variable beam-splitter \( B(\theta) \), \( \rho^{(3)}_{ab} \) is obtained. Then, after \( \rho^{(3)}_{ab} \) passing through channel \( L(\eta_2) \) with loss factor \( \eta_2 \), \( \rho^{(4)}_{ab} \) is obtained. In the last stage, the MSSVS \( \rho \) is generated heraldedly by performing a \( m \)-photon detection.

State in stage 1: The total density operator in this stage can be written as

\[ \rho^{(1)}_{ab} = (\rho_{SV})_a \otimes (|0\rangle \langle 0|)_b \]  

(4)

with \( \rho_{SV} = S(r)|0\rangle \langle 0| S^\dagger(r) \), where the SVS can be further expressed as \( S(r)|0\rangle = (1 - \lambda^2)^{1/4} e^{\frac{r}{2}a^\dagger a^2}|0\rangle \) with \( \lambda = \tanh r \). Therefore, \( \rho^{(1)}_{ab} \) can be given by

\[ \rho^{(1)}_{ab} = \int \frac{d^2\alpha_1 d^2\beta_1}{\pi^2} \chi^{(1)}_{ab}(\alpha_1, \beta_1) D_a(-\alpha_1) D_b(-\beta_1), \]  

(5)
where $\chi_{\rho_{ab}}^{(1)}(\alpha_1, \beta_1)$ is the CF of $\rho_{ab}^{(1)}$ satisfying
\[
\chi_{\rho_{ab}}^{(1)}(\alpha_1, \beta_1) = \text{Tr}[\rho_{ab}^{(1)} D_a (\alpha_1) D_b (\beta_1)] \\
= e^{-\frac{(1+\lambda^2)|\alpha_1|^2}{2(1-\lambda^2)} + \frac{\lambda^2(\alpha_1^2 + \beta_1^2)}{2(1-\lambda^2)} - \frac{|\beta_1|^2}{2}},
\]
and both $D_a (\alpha_1)$ and $D_b (\beta_1)$ are the displacement operators in mode $a$ and mode $b$, respectively.

**State in stage 2:** After $\rho_{ab}^{(1)}$ passing through $L (\eta_1)$, we obtain
\[
\rho_{ab}^{(2)} = \int \frac{d^2\alpha_1 d^2\beta_1}{\pi^2} \chi_{\rho_{ab}}^{(1)}(\sqrt{1-\eta_1}\alpha_1, \beta_1) \\
\times e^{-\frac{\eta_1|\alpha_1|^2}{2}} D_a (-\alpha_1) D_b (-\beta_1),
\]
where we have considered the input–output formula in Eq. (2). It is noted that we can still use the CF in stage 1 and it is not necessary to calculate the CF in stage 2.

**State in stage 3:** Injecting $\rho_{ab}^{(2)}$ into a variable beam-splitter, we have
\[
\rho_{ab}^{(3)} = B (\theta) \rho_{ab}^{(2)} B^\dagger (\theta),
\]
where the BS is described by $B (\theta) = \exp[\theta (a^\dagger b - ab^\dagger)]$ with the transmissivity $T = \cos^2 \theta$, satisfying $B (\theta) a B^\dagger (\theta) = \sqrt{T} a + \sqrt{1-T} b$ and $B (\theta) b B^\dagger (\theta) = -\sqrt{T} a + \sqrt{1-T} b$ [26]. After making the detailed derivation, $\rho_{ab}^{(3)}$ can be expressed as
\[
\rho_{ab}^{(3)} = \int \frac{d^2\alpha_1 d^2\beta_1}{\pi^2} \chi_{\rho_{ab}}^{(1)}(\sqrt{1-\eta_1}\alpha_1, \beta_1) \\
\times e^{(-\eta_1)|\alpha_1|^2 + \frac{|\beta_1|^2}{2}} \\
\times e^{a(\sqrt{T} a_1^* - \beta_1^* \sqrt{1-T})} e^{-a^\dagger(\sqrt{T} a_1 - \beta_1 \sqrt{1-T})} \\
\times e^{b(\alpha_1^* \sqrt{1-T} + \sqrt{T} \beta_1^*)} e^{-b^\dagger(\alpha_1 \sqrt{1-T} + \sqrt{T} \beta_1)}. \]

Since the CF of $\rho_{ab}^{(3)}$ is useful to calculate $\rho_{ab}^{(4)}$, we must obtain its analytic expression as follows:
\[
\chi_{\rho_{ab}}^{(3)}(\alpha_3, \beta_3) = \text{Tr}[\rho_{ab}^{(3)} D_a (\alpha_3) D_b (\beta_3)] \\
= e^{-\frac{1}{2} \lambda^2 (\alpha_3^2 + \alpha_3^2)} \\
\times e^{-\frac{1}{2} \lambda^2 \tau_2 (\beta_3^2 + \beta_3^2)} \\
\times e^{\lambda \tau_3 (\alpha_3 \beta_3 + \alpha_3^2 \beta_3^*) - \lambda^2 \tau_3 (\alpha_3 \beta_3^* + \beta_3 \alpha_3^*)},
\]
where
\[ \tau_1 = (1 - \eta_1) \frac{T}{(1 - \lambda^2)}, \]
\[ \tau_2 = (1 - \eta_1) (1 - T) / (1 - \lambda^2), \]
\[ \tau_3 = (1 - \eta_1) \sqrt{T (1 - T)/ (1 - \lambda^2)}. \] (11)

**State in stage 4:** After \( \rho^{(3)}_{ab} \) passing through \( L (\eta_2) \), similarly using Eq. (2) we also may obtain \( \rho^{(4)}_{ab} \),
\[ \rho^{(4)}_{ab} = \int \frac{d^2 \alpha_3 d^2 \beta_3}{\pi^2} \chi^{(3)}_{\rho_{ab}} (\alpha_3, \sqrt{1 - \eta_2 \beta_3}) \times e^{-\eta_2 |\beta_3|^2} \frac{1}{\mathcal{D}_a (-\alpha_3) \mathcal{D}_b (-\beta_3)}. \] (12)

**State in stage 5:** At the last stage, making a \( m \)-photon detection, the MSSVS can be obtained
\[ \rho = \frac{1}{p_d} \langle m | \rho^{(4)}_{ab} | m \rangle, \] (13)
where \( p_d \) is the success probability of producing such state. Substituting \( \langle m | = 1 / \sqrt{m!} d^m_{\mu} e^{\mu b} |_\mu = 0 \) and \( | m \rangle = 1 / \sqrt{m!} d^m_{\nu} e^{\nu b \dagger} |_\nu = 0 \), as well as Eq. (12) into Eq. (13), we obtain the density operator
\[ \rho = \frac{1}{m! p_d} \frac{d^{2m}}{d\mu^m d\nu^m} \exp (\mu \nu) \int \frac{d^2 \alpha_3 d^2 \beta_3}{\pi^2} \chi^{(3)}_{\rho_{ab}} (\alpha_3, \sqrt{1 - \eta_2 \beta_3}) \times e^{-\frac{(1 + \eta_2) |\beta_3|^2}{2}} -\mu \beta_3 + \nu \beta_3 \mathcal{D}_a (-\alpha_3) |_\mu = \nu = 0, \] (14)
and its corresponding success probability
\[ p_d = \frac{1}{m! \sqrt{\epsilon_4}} \frac{d^{2m}}{d\mu^m d\nu^m} e^{(1 - \frac{\epsilon_4}{4}) \mu \nu + \frac{\epsilon_4}{4} (\mu^2 + \nu^2)} |_\mu = \nu = 0, \] (15)

with \( \epsilon_1 = 1 + \lambda^2 \tau_2 (1 - \eta_2), \) \( \epsilon_2 = \frac{1}{2} \lambda \tau_2 (1 - \eta_2), \) and \( \epsilon_4 = \epsilon_1^2 - 4 \epsilon_2^2. \) Thus, by varying all the interaction parameters, involving the input squeezing parameter \( r, \) the loss factors \( \eta_1, \eta_2, \) the BS transmissivity \( T \) and the detecting photon number \( m, \) a broad class of MSSVSs can be obtained.

Success probability is an important character in the conditional quantum state engineering. In Fig. 2, according to Eq. (15) we plot several distributions of success probability in different parameter spaces. We note that the probability is limited to zero if the loss factors \( \eta_1, \eta_2 \) are equal to 1. Next, we shall discuss the nonclassical properties of the MSSVSs in terms of quadrature squeezing effect, photon number.
Fig. 2  Success probabilities in different parameter space at fixed other parameters for several different $m = 1, 2, 3$, where in $(\eta_1, \eta_2)$ space with $r = 0.5$ and $T = 0.97$ (Row 1), in $(r, T)$ space with $\eta_1 = \eta_2 = 0.02$ (Row 2), and in $(\eta, T)$ space with $r = 0.5$ and $\eta_1 = \eta_2 = \eta$ (Row 3). Column 1, Column 2, and Column 3 correspond to $m = 1$, $m = 2$, and $m = 3$, respectively.

distribution, and Wigner function and explore how the photon subtraction and loss factors affect these nonclassicalities.

### 3 Quantum squeezing effect

No doubt, the prominent character of the SVS is the squeezing effect. But compared with the original SVS, how the squeezing effect for the MSSVSs will change? Now, we explore the quadrature squeezing and photon number squeezing [12] of the MSSVSs.

#### 3.1 Quadrature squeezing

Variances of coordinate operator $X = (a + a^\dagger)/\sqrt{2}$ and momentum operator $P = (a - a^\dagger)/(\sqrt{2}i)$ can be expressed as follows [27]:

$$\Delta^2 X = \langle a^\dagger a \rangle - \left| \langle a^\dagger \rangle \right|^2 + \text{Re}(\langle a^\dagger a \rangle - \langle a^\dagger \rangle^2) + \frac{1}{2},$$
\[ \Delta^2 P = \langle a^+ a \rangle - \langle (a^+)^2 \rangle - \text{Re}(\langle a^{+2} \rangle - \langle a^+ \rangle^2) + \frac{1}{2}, \]  

which obeys the uncertainty relation \( \Delta^2 X \Delta^2 P \geq 1/4 \). Generally, a quantum state is called quadrature squeezing if \( \Delta^2 X < 1/2 \) or \( \Delta^2 P < 1/2 \).

### 3.2 Photon number squeezing

As we know, \( \hat{n} = a^+ a \) is the photon number operator. Its mean value is \( \langle \hat{n} \rangle \) and variance is \( (\Delta \hat{n})^2 = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2 \). If the variance is narrower than mean value, i.e., \( (\Delta \hat{n})^2 < \langle \hat{n} \rangle \), then a quantum state is called photon number squeezing. Here, we define the Mandel parameter

\[ Q_M = \frac{(\Delta \hat{n})^2 - \langle \hat{n} \rangle}{\langle \hat{n} \rangle} = \frac{\langle a^{+2} a^2 \rangle}{\langle a^+ a \rangle} - \langle a^+ a \rangle. \]  

For states with \( Q_M = 0 \), the statistics are Poissonian, while \( Q_M > 0 \) and \( Q_M < 0 \) correspond to the cases of super-Poissonian and sub-Poissonian statistics, respectively. Furthermore, we can judge whether a quantum state is a number squeezing by seeing \( Q_M < 0 \).

In order to study the squeezing effect, one can firstly calculate the general expected value \( \langle a^{+k} a^l \rangle = \text{Tr} \left( a^{+k} a^l \rho \right) \) with different integers \( k, l \) for quantum state \( \rho \). By choosing proper integers \( k, l \), one can obtain any expected values one needed. In the process of calculation, one can resort to the techniques \( a^{+k} = \frac{d^k}{df \epsilon f a^l \rho} \bigg|_{f=0} \) and \( a^l = \frac{d^l}{dg \epsilon g a^k} \bigg|_{g=0} \).

For example, the SVS has

\[ \langle a^{+k} a^l \rangle_{\rho_{SV}} = \frac{d^{k+l}}{df^k dg^l} e^{\frac{1}{2} (f^2 + g^2) + \frac{\lambda^2}{12} (f^2 + g^2)} \bigg|_{f=g=0}. \]  

When \( k = l = 1 \), it leads to \( \langle a^+ a \rangle_{\rho_{SV}} = \lambda^2 / (1 - \lambda^2) \). If \( k = 1, l = 0 \) and \( k = 2, l = 0 \), we have \( \langle a^+ a \rangle = 0 \) and \( \langle a^{+2} \rangle = \lambda / (1 - \lambda^2) \), respectively.

For the MSSVSs, we have

\[ \langle a^{+k} a^l \rangle = \frac{1}{m^l \mu \sqrt{\epsilon_4}} \frac{d^{2m+k+l}}{d \mu^m d \nu^n d \xi^k d \eta^l} \]

\[ \times \left[ \frac{\xi_{k+l}}{\epsilon_4} (\mu f + g v) + \frac{\xi_{k+l}}{\epsilon_4} (v f + \mu g) \right] \]

\[ \times e^{(\lambda^2 t_1 + \frac{\lambda^2}{\epsilon_4}) f g + (\frac{1}{2} \lambda t_1 + \frac{\lambda}{\epsilon_4}) (f^2 + g^2)} \]

\[ \times e^{(1 - \frac{\lambda}{\epsilon_4}) \mu v + \frac{\lambda}{\epsilon_4} (\mu^2 + v^2)} \bigg|_{f=g=\mu=\nu=0} \]  

with \( \epsilon_3 = \lambda \tau_3 \sqrt{1 - \eta_2} \), \( \epsilon_5 = 4 \lambda \epsilon_2 \epsilon_3^2 - (1 + \lambda^2) \epsilon_1 \epsilon_3^2 \), \( \epsilon_6 = (1 + \lambda^2) \epsilon_2 \epsilon_3^2 - \lambda \epsilon_1 \epsilon_3^2 \), \( \epsilon_7 = \lambda \epsilon_1 - 2 \epsilon_2 \), and \( \epsilon_8 = \epsilon_1 - 2 \lambda \epsilon_2 \).

Obviously, we have \( \Delta^2 X = \frac{1}{4} e^{2r} \) and \( \Delta^2 P = \frac{1}{4} e^{-2r} \) for the SVS, which shows that the SVS has quadrature squeezing effect for any nonzero \( r \). Compared with the
SVS, how will the squeezing parameter change for the MSSVS? In Fig. 3, we plot their variations of $\Delta^2 P$ and $\Delta^2 X$ versus $r$ for several cases. It is clearly seen that the variance of $\Delta^2 P$ monotonically decreases as $r$ increases for a given $m$ and there exists squeezing in $P$ quadrature component within a certain range of parameter $r$. For the case of $m = 1$, only when the squeezing parameter $r$ is bigger than a threshold value $r_c$, depending on the loss factor, the MSSVS may have the possibility of the squeezing effect (noticing $r_c = 0.626381$ for $\eta_1 = \eta_2 = 0$ and $r_c = 0.609918$ for $\eta_1 = \eta_2 = 0.1$). For the case of $m = 2$, the MSSVS always presents the squeezing effect for any nonzero $r$, but slightly different from the SVS. For the case of $m = 3$, only when the squeezing parameter $r$ is bigger than a threshold value $r_c$, the MSSVS may have the possibility of the squeezing effect, where $r_c = 0.396049$ for $\eta_1 = \eta_2 = 0$ and $r_c = 0.387008$ for $\eta_1 = \eta_2 = 0.1$, respectively. On the other hand, we have $Q_M = \cosh 2r \geq 1$ for the SVS, which shows that the SVS has no number squeezing effect for any $r$. Numerically, we plot $Q_M$ for the MSSVSs in Fig. 4. The results show that MSSVSs may present number squeezing only for odd $m$ and small initial squeezing $r < r_c$. A state exhibiting sub-Poissonian statistics ($Q < 0$) is said to possess the number squeezing and shows nonclassical behavior.

4 Photon number distribution

Photon number distribution (PND) is defined by $P (n) = \langle n | \rho | n \rangle$, which means the probability of detecting $n$ photons in the field $\rho$. Using the technique $\langle n | = \frac{1}{\sqrt{n!}} \frac{d^n}{ds^n} (0) e^{ts} | s = 0 \rangle$ and $| n \rangle = \frac{1}{\sqrt{n!}} \frac{d^n}{dt^n} e^{t \alpha^\dagger} | 0 \rangle | t = 0$, one can easily obtain the PNDs for the given optical field $\rho$.

For the SVS $\rho_{SV}$, we have

$$P_{\rho_{SV}} (n) = \frac{(1 - \lambda^2)^{1/2}}{n!} \frac{d^{2n}}{ds^n dt^n} e^2 (s^2 + t^2) | s = t = 0$$

$$= \begin{cases} 
\frac{n! \lambda^n (1 - \lambda^2)^{1/2}}{2^n (\lambda/2)^n (\lambda/2)!}, & \text{even}, \\
0, & \text{odd}, 
\end{cases} \quad (20)$$

which implies that the SVS contains only even-photon components, as shown in Fig. 5. This is one of the key characteristics of the SVS.

But what will happen to the PNDs for the MSSVSs? Noticing Eq. (14), we obtain the PND of the MSSVSs expressed as

$$P (n) = \frac{1}{n! m! \eta_d \eta_0 \sqrt{\epsilon_4 \kappa_4}} \frac{d^{2m+2n}}{d \mu^m d \nu^m d s^n d t^n}$$

$$e^{[1 - \frac{\epsilon_4}{\epsilon_4} + \frac{s^2}{4 \kappa_4} (\nu_1 \kappa_5 + \nu_2 \kappa_6)] \mu \nu + (1 - \frac{s^2}{4 \kappa_4}) s t}$$

$$\times e^{[\frac{s^2}{4 \kappa_4} - \frac{s^2}{4 \kappa_4} (\nu_1 \kappa_6 + \nu_2 \kappa_5)] (\mu^2 + \nu^2) + \frac{s^2}{4 \kappa_4} (s^2 + t^2)}$$

$$\times e^{\frac{s^2}{4 \kappa_4} [\kappa_7 (\kappa_7 + s^2) + \kappa_8 (\kappa_8 + t^2)]} | _{\mu = \nu = s = t = 0}, \quad (21)$$
Fig. 3 $\Delta^2 X$ (dashed lines) and $\Delta^2 P$ (solid lines) versus $r$ for the SVS (red lines) and the MSSVSs (black lines) at fixed $T = 0.9$, where $m = 1$ (Row 1), $m = 2$ (Row 2), $m = 3$ (Row 3) and $\eta_1 = \eta_1 = 0$ (left column), $\eta_1 = \eta_1 = 0.1$ (right column) (Color figure online)

Fig. 4 $Q_M$ versus $r$ for the SVS (red solid lines) and the MSSVSs (black dashed lines) at fixed $T = 0.9$, where $m = 1$ (Row 1), $m = 2$ (Row 2), $m = 3$ (Row 3) and $\eta_1 = \eta_1 = 0$ (left column), $\eta_1 = \eta_1 = 0.1$ (right column) (Color figure online)
where \( \kappa_1 = 1 + \lambda^2 \tau_1 + \frac{\epsilon_5}{\epsilon_4}, \kappa_2 = \frac{1}{2} \lambda \tau_1 + \frac{\epsilon_6}{\epsilon_4}, \kappa_3 = \frac{\epsilon_1}{\epsilon_4}, \kappa_4 = \kappa_1^2 - 4\kappa_2^2, \kappa_5 = 8\lambda \epsilon_1 \epsilon_2 - (1 + \lambda^2) (\epsilon_1^2 + 4\epsilon_2^2), \kappa_6 = \lambda (\epsilon_1^2 + 4\epsilon_2^2) - 2 (1 + \lambda^2) \epsilon_1 \epsilon_2, \kappa_7 = \kappa_1 \epsilon_7 - 2\kappa_2 \epsilon_8, \) and \( \kappa_8 = \kappa_1 \epsilon_8 - 2\kappa_2 \epsilon_7. \)

In order to analyze the effect of loss on the PNDs of the MSSVSs, we depict the PNDs in Figs. 6 and 7. From Fig. 6 without loss \( (\eta_1 = 0, \eta_2 = 0) \), we see that the MSSVSs contain only even-photon (odd-photon) components if \( m \) is even (odd), which agrees with the results of Ref. [14]. However, we observe that, surprisingly, if there is a loss, the MSSVSs contain all-photon components (Fig. 7). Moreover, the ratio between even-component and odd-component can be adjusted by the value of \( m \) and the loss factors \( \eta_1, \eta_2 \).

### 5 Wigner function

Wigner function \( W(\beta) \) for quantum state \( \rho \) can be defined by[28–30]

\[
W(\beta) = \frac{2}{\pi} \text{Tr}[\rho D(\beta) (-1)^{a^\dagger a} D^\dagger(\beta)],
\]  
(22)
where $D(\beta) = \exp(\beta a^\dagger - \beta^* a)$ is the usual displacement operator with $\beta = (x + iy)/\sqrt{2}$.

For the SVS, we know
\[
W_{SV}(\beta) = \frac{2}{\pi} e^{-\frac{2(1+\frac{\kappa_1}{\kappa_9})|\beta|^2 + \frac{2\kappa_2}{\kappa_9} (\beta^2 + \beta^* 2^2)}},
\]
which is Gaussian and implies that the SVSs are Gaussian states (Fig. 8), while for the MSSVSs, we have

\[
W(\beta) = \frac{1}{\pi m! p_{d} \sqrt{\epsilon \kappa_9}} e^{-\left(\frac{\kappa_1}{\kappa_9} - \frac{1}{2\kappa_9}\right)|\beta|^2 + \frac{\kappa_2}{\kappa_9} (\beta^2 + \beta^* 2^2)}
\]
\[
\times \frac{d^{2m}}{d\mu^m d\nu^m} \left[ 1 - \frac{\kappa_1}{\kappa_9} + \frac{\kappa_2^2}{\kappa_9} (4\kappa_2\kappa_6 + \kappa_1\kappa_5 - \frac{1}{2}\kappa_9) \right] \mu \nu
\]
\[
\times e^{\left[ \frac{\kappa_1}{\kappa_9} - \frac{1}{2\kappa_9} \kappa_5 \epsilon - \frac{2\kappa_2}{\kappa_9} \kappa_5 \epsilon (v\beta + \mu \beta^*) \right]}
\]
\[
\times e^{\left[ \frac{\kappa_1}{\kappa_9} - \frac{1}{2\kappa_9} \kappa_5 \epsilon - \frac{2\kappa_2}{\kappa_9} \kappa_5 \epsilon (\mu \beta + v \beta^*) \right]} |_{\mu = v = 0},
\]
where $\kappa_9 = (\kappa_1 - \frac{1}{2})^2 - 4\kappa_2^2$.

According to Eq. (24), we plot the Wigner functions of the MSSVSs in Fig. 9 without loss and in Fig. 10 with loss in phase space. Clearly, the Wigner functions of the MSSVSs are non-Gaussian in phase space. Compared with the surfaces in Fig. 9 without loss, the surfaces in Fig. 10 with loss have more peaks and valleys. As an evidence of the nonclassicality of the state [31], there are some negative regions
Fig. 8 Wigner functions of the SVS, where \( r = 0.5, 0.7, \) and 1 from left to right, respectively.

Fig. 9 Wigner functions of the MSSVSs without loss (\( \eta_1 = 0, \eta_2 = 0 \)) at fixed \( T = 0.9 \) for several different cases, where \( m = 1 \) (Row 1), \( m = 2 \) (Row 2), and \( m = 3 \) (Row 3). Column 1, Column 2, and Column 3 correspond to \( r = 0.5, r = 0.7, \) and \( r = 1 \), respectively.

of the Wigner function in phase space (see Figs. 9 and 10), compared with Fig. 8. Moreover, Wigner functions of the MSSVSs can reflect non-Gaussianity of quantum states [32,33].
6 Conclusion

To summarize, we present a conditional scheme of generating the MSSVSs in the presence of pure-loss channels. By adjusting the relevant interaction parameters (including $r$, $\eta_1$, $\eta_2$, $T$, and $m$), a broad class of MSSVSs with figure of merit can be obtained. For the theoretical model, we have given the complete description of density operator of the optical fields in terms of CF. Analytical derivation and numerical simulation for the properties of the MSSVSs are explored in detail.

In Sect. 3, we have analyzed the squeezing effects (quadrature squeezing in Fig. 3 and number squeezing in Fig. 4). It is found that the MSSVS may not have better quadrature but have better number squeezing than the SVS to some extent. Section 4 consists of the study of photon number distribution for both SVS (Fig. 5) and MSSVS in the absence (Fig. 6) and presence (Fig. 7) of loss. We have found that in the absence of lossy medium the photon number distributions of MSSVSs resemble to those of the Schrödinger cat states, which means either the even or odd photons exist in the distributions, whereas, in the presence of lossy channels, both even and odd photons

Fig. 10 Wigner functions of the MSSVSs with loss ($\eta_1 = 0.1$, $\eta_2 = 0.1$) at fixed $T = 0.9$ for several different cases, where $m = 1$ (Row 1), $m = 2$ (Row 2), and $m = 3$ (Row 3). Column 1, Column 2, and Column 3 correspond to $r = 0.5$, $r = 0.7$, and $r = 1$, respectively.
exist in the distributions. Thus, the photon number distribution of the resulting states clearly does not resemble that of the cat states. So the MSSVSs with loss are not catlike. Similar studies also appear in Figs. 8 and 9, albeit for the study of Wigner distribution functions in Sect. 5. Therefore, in order to prepare the ideal catlike state, we must try our best to reduce the influence of loss.

Moreover, some interesting results effected by the loss are summarized as follows: (1) The losses change a threshold value of initial squeezing parameter, which may present the appearance of squeezing effect; (2) the losses will let the MSSVSs contain all-photon components (including odd-photon and even-photon); (3) the losses make the Wigner function more complex. By the way, we would like to mention about a shortcoming of this article. Indeed, adding an $\eta_3$ term to the detection channel would be a novel item that would make this work more attractive. In principle, loss in the detection arm is not an issue for low squeezing, while loss will result in errors on conditionally detected photon numbers for larger squeezing, as these photon numbers originated from higher photon number states. However, due to the method of characteristic function, the calculation will become very complicated and the result will be very tedious. Therefore, we did not continue to consider the loss in the last channel.

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