GAUGE–STRING DUALITIES AND SOME APPLICATIONS

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Abstract

The first part of these lectures contains an introductory review of the AdS/CFT duality and of its tests. Applications to thermal gauge theory are also discussed briefly. The second part is devoted to a review of gauge-string dualities based on various warped conifold backgrounds, and to their cosmological applications.

1. Introduction

String theory is well known to be the leading prospect for quantizing gravity and unifying it with other interactions. One may also take a broader view of string theory as a description of string-like excitations that arise in many different physical systems, such as the superconducting flux tubes or the chromo-electric flux tubes in non-Abelian gauge theories. From the point of view of quantum field theories describing the physical systems where these string-like objects arise, they are “emergent” rather than fundamental. However, thanks to the AdS/CFT correspondence [1–3] and its extensions, we now know that at least some field theories have dual formulations in terms of string theories in curved backgrounds. In these examples, the strings that are “emergent” from the field theory point of view are dual to fundamental or D-strings in the string theoretic approach. Besides being of great theoretical interest, such dualities are becoming a useful tool for studying strongly coupled gauge theories. These ideas also have far-reaching implications for building connections between string theory and the real world.

These notes, based on five lectures delivered by I.R.K. at Les Houches in July 2007, are not meant to be a comprehensive review of what has become a vast field. Instead, they aim to present a particular path through it, which begins with old and well-known concepts, and eventually leads to some recent developments. The first part of these lectures is based on an earlier brief review [4]. It begins with a bit of history and basic facts about string theory and its connections with strong interactions. Comparisons of stacks of Dirichlet branes with curved backgrounds produced by them are used to motivate the AdS/CFT correspondence between superconformal gauge theory and string theory on a product of Anti-de Sitter space and a compact manifold. The ensuing duality between semi-classical spinning strings and gauge theory operators carrying large charges is briefly reviewed. We go on to describe recent tests of the AdS/CFT correspondence using the Wilson loop cusp anomaly as a function of the coupling, which also enters dimensions of high-spin operators. Strongly coupled thermal SYM theory is explored via a black hole in
5-dimensional AdS space, which leads to explicit results for its entropy and shear viscosity.

The second part of these lectures (sections 7–11) focuses on the gauge-string dualities that arise from studying D-branes on the conifold geometry. The $AdS_5 \times T^{1,1}$ background appears as the type IIB dual of an $SU(N) \times SU(N)$ superconformal gauge theory with bi-fundamental fields. A warped resolved conifold is then presented as a description of holographic RG flow from this theory to the $\mathcal{N} = 4$ supersymmetric gauge theory produced by giving a classical value to one of the bi-fundamentals. The warped deformed conifold is instead the dual of the cascading $SU(kM) \times SU((k + 1)M)$ gauge theory which confines in the infrared. Some features of the bound state spectrum are discussed, and the supergravity dual of the baryonic branch, the resolved warped deformed conifold, is reviewed. We end with a brief update on possible cosmological applications of these backgrounds.

2. Strings and QCD

String theory was born out of attempts to understand the strong interactions. Empirical evidence for a string-like structure of hadrons comes from arranging mesons and baryons into approximately linear Regge trajectories. Studies of $\pi N$ scattering prompted Dolen, Horn and Schmid [5] to make a duality conjecture stating that the sum over s-channel exchanges equals the sum over t-channel ones. This posed the problem of finding the analytic form of such dual amplitudes. Veneziano [6] found the first, and very simple, expression for a manifestly dual 4-point amplitude:

$$A(s, t) \sim \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s) - \alpha(t))}$$

(2.1)

with an exactly linear Regge trajectory $\alpha(s) = \alpha(0) + \alpha' s$. Soon after, its open string interpretation was proposed [7–9] (for detailed reviews of these early developments, see [10]). In the early 70’s this led to an explosion of interest in string theory as a description of strongly interacting particles. The basic idea is to think of a meson as an open string with a quark at one end-point and an anti-quark at the other. Then various meson states arise as rotational and vibrational excitations of such an open string. The splitting of such a string describes the decay of a meson into two mesons.

The string world sheet dynamics is governed by the Nambu-Goto area action

$$S_{NG} = -T \int d\sigma d\tau \sqrt{-\det \partial_a X^\mu \partial_b X_\mu},$$

(2.2)

where the indices $a, b$ take two values ranging over the $\sigma$ and $\tau$ directions on the world sheet. The string tension is related to the Regge slope through $T^{-1} = 2\pi\alpha'$. The quantum consistency of the Veneziano model requires that the Regge intercept is $\alpha(0) = 1$, so that the spin 1 state is massless but the spin 0 is a tachyon. But the $\rho$ meson is certainly not massless, and the presence of a tachyon in the spectrum
indicates an instability. This is how the string theory of strong interactions started to run into problems.

Calculation of the string zero-point energy gives \( \alpha(0) = \frac{(d-2)}{24} \). Hence the model has to be defined in 26 space-time dimensions. Consistent supersymmetric string theories were discovered in 10 dimensions, but their relation to strong interactions in 4 dimensions was initially completely unclear. Most importantly, the Asymptotic Freedom of strong interactions was discovered [11], singling out Quantum Chromodynamics (QCD) as the exact field theory of strong interactions. At this point most physicists gave up on strings as a description of strong interactions. Instead, since the graviton appears naturally in the closed string spectrum, string theory emerged as the leading hope for unifying quantum gravity with other forces [12, 13].

Now that we know that a non-Abelian gauge theory is an exact description of strong interactions, is there any room left for string theory in this field? Luckily, the answer is positive. At short distances, much smaller than 1 fermi, the quark anti-quark potential is approximately Coulombic due to the Asymptotic Freedom. At large distances the potential should be linear due to formation of a confining flux tube [14]. When these tubes are much longer than their thickness, one can hope to describe them, at least approximately, by semi-classical Nambu strings [15]. This appears to explain the existence of approximately linear Regge trajectories. For the leading trajectory, a linear relation between angular momentum and mass-squared

\[
J = \alpha' m^2 + \alpha(0),
\]

is provided by a semi-classical spinning relativistic string with massless quark and anti-quark at its endpoints. In case of baryons, one finds a di-quark instead of an anti-quark at one of the endpoints. A semi-classical string approach to the QCD flux tubes is widely used, for example, in jet hadronization algorithms based on the Lund String Model [16].

Semi-classical quantization around a long straight Nambu string predicts the quark anti-quark potential [17]

\[
V(r) = Tr + \mu + \frac{2\gamma}{r} + O(1/r^2).
\]

The coefficient \( \gamma \) of the universal Lüscher term depends only on the space-time dimension \( d \) and is proportional to the Regge intercept: \( \gamma = -\pi(d-2)/24 \). Lattice calculations of the force vs. distance for probe quarks and anti-quarks [18] produce good agreement with this value in \( d = 3 \) and \( d = 4 \) for \( r > 0.7 \) fm. Thus, long QCD strings appear to be well described by the Nambu-Goto area action. But quantization of short, highly quantum QCD strings, that could lead to a calculation of light meson and glueball spectra, is a much harder problem.

The connection of gauge theory with string theory is strengthened by 't Hooft’s generalization of QCD from 3 colors (\( SU(3) \) gauge group) to \( N \) colors (\( SU(N) \) gauge group) [19]. The idea is to make \( N \) large, while keeping the 't Hooft coupling \( \lambda = g^2 N / M \) fixed. In this limit each Feynman graph carries a topological factor \( N^\chi \), where \( \chi \) is the Euler characteristic of the graph. Thus, the sum over graphs
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of a given topology can perhaps be thought of as a sum over world sheets of a hypothetical “QCD string.” Since the spheres (string tree diagrams) are weighted by \( N^2 \), the tori (string one-loop diagrams) by \( N^0 \) etc., we find that the closed string coupling constant is of order \( N^{-1} \). Thus, the advantage of taking \( N \) to be large is that we find a weakly coupled string theory. In the large \( N \) limit the gauge theory simplifies in that only the planar diagrams contribute. But directly summing even this subclass of diagrams seems to be an impossible task. From the dual QCD string point of view, it is not clear how to describe this string theory in elementary terms.

Because of the difficulty of these problems, between the late 70’s and the mid-90’s many theorists gave up hope of finding an exact gauge-string duality. One notable exception is Polyakov who in 1981 proposed that the string theory dual to a 4-d gauge theory should have a 5-th hidden dimension [20]. In later work [21] he refined this proposal, suggesting that the 5-d metric must be “warped.”

3. The Geometry of Dirichlet Branes

In the mid-90’s Dirichlet branes, or D-branes for short, brought string theory back to gauge theory. D-branes are soliton-like “membranes” of various internal dimensionalities contained in theories of closed superstrings [22]. A Dirichlet \( p \)-brane (or \( D_p \)-brane) is a \( p + 1 \) dimensional hyperplane in \( 9 + 1 \) dimensional space-time where strings are allowed to end. A D-brane is much like a topological defect: upon touching a D-brane, a closed string can open up and turn into an open string whose ends are free to move along the D-brane. For the end-points of such a string the \( p + 1 \) longitudinal coordinates satisfy the conventional free (Neumann) boundary conditions, while the \( 9 - p \) coordinates transverse to the \( D_p \)-brane have the fixed (Dirichlet) boundary conditions; hence the origin of the term “Dirichlet brane.” In a seminal paper [22] Polchinski showed that a \( D_p \)-brane preserves \( 1/2 \) of the bulk supersymmetries and carries an elementary unit of charge with respect to the \( p + 1 \) form gauge potential from the Ramond-Ramond sector of type II superstring.

For our purposes, the most important property of D-branes is that they realize gauge theories on their world volume. The massless spectrum of open strings living on a \( D_p \)-brane is that of a maximally supersymmetric \( U(1) \) gauge theory in \( p + 1 \) dimensions. The \( 9 - p \) massless scalar fields present in this supermultiplet are the expected Goldstone modes associated with the transverse oscillations of the \( D_p \)-brane, while the photons and fermions provide the unique supersymmetric completion. If we consider \( N \) parallel D-branes, then there are \( N^2 \) different species of open strings because they can begin and end on any of the D-branes. \( N^2 \) is the dimension of the adjoint representation of \( U(N) \), and indeed we find the maximally supersymmetric \( U(N) \) gauge theory in this setting.

The relative separations of the \( D_p \)-branes in the \( 9 - p \) transverse dimensions are determined by the expectation values of the scalar fields. We will be interested in the case where all scalar expectation values vanish, so that the \( N \) \( D_p \)-branes are stacked on top of each other. If \( N \) is large, then this stack is a heavy object
embedded into a theory of closed strings which contains gravity. Naturally, this macroscopic object will curve space: it may be described by some classical metric and other background fields. Thus, we have two very different descriptions of the stack of D$p$-branes: one in terms of the $U(N)$ supersymmetric gauge theory on its world volume, and the other in terms of the classical charged $p$-brane background of the type II closed superstring theory. The relation between these two descriptions is at the heart of the connections between gauge fields and strings that are the subject of these lectures.

Parallel D3-branes realize a 3 + 1 dimensional $U(N)$ gauge theory, which is a maximally supersymmetric “cousin” of QCD. Let us compare a stack of coincident D3-branes with the Ramond-Ramond charged black 3-brane classical solution whose metric assumes the form [23]:

\[ ds^2 = h^{-1/2}(r) \left[ -f(r)(dx^0)^2 + \sum_{i=1}^{3} (dx^i)^2 \right] + h^{1/2}(r) \left[ f^{-1}(r)dr^2 + r^2d\Omega_5^2 \right], \tag{3.1} \]

where

\[ h(r) = 1 + \frac{L^4}{r^4}, \quad f(r) = 1 - \frac{r_0^4}{r^4}. \]

Here $d\Omega_5^2$ is the metric of a unit 5 dimensional sphere, $S^5$.

In general, a $d$-dimensional sphere of radius $L$ may be defined by a constraint

\[ \sum_{i=1}^{d+1} (X^i)^2 = L^2 \tag{3.2} \]

on $d + 1$ real coordinates $X^i$. It is a positively curved maximally symmetric space with symmetry group $SO(d+1)$. Similarly, the $d$-dimensional Anti-de Sitter space, $AdS_d$, is defined by a constraint

\[ (X^0)^2 + (X^d)^2 - \sum_{i=1}^{d-1} (X^i)^2 = L^2, \tag{3.3} \]

where $L$ is its curvature radius. $AdS_d$ is a negatively curved maximally symmetric space with symmetry group $SO(2, d - 2)$. There exists a subspace of $AdS_d$ called the Poincaré wedge, with the metric

\[ ds^2 = \frac{L^2}{z^2} \left( dz^2 - (dx^0)^2 + \sum_{i=1}^{d-2} (dx^i)^2 \right), \tag{3.4} \]

where $z \in [0, \infty)$. In these coordinates the boundary of $AdS_d$ is at $z = 0$.

The event horizon of the black 3-brane metric [21] is located at $r = r_0$. In the extremal limit $r_0 \to 0$ the 3-brane metric becomes

\[ ds^2 = h^{-1/2}(r)\eta_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(r) \left( dr^2 + r^2d\Omega_5^2 \right), \tag{3.5} \]

where $\eta_{\mu\nu}$ is the 3+1 dimensional Minkowski metric. Just like the stack of parallel, ground state D3-branes, the extremal solution preserves 16 of the 32 supersymmetries present in the type IIB theory. Introducing $z = L^2/r$, one notes that the
limiting form of (3.5) as $r \to 0$ factorizes into the direct product of two smooth spaces, the Poincaré wedge (3.4) of $AdS_5$, and $S^5$, with equal radii of curvature $L$:

$$ds^2 = \frac{L^2}{z^2} \left( dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu \right) + L^2 d\Omega_5^2. \quad (3.6)$$

The 3-brane geometry may thus be viewed as a semi-infinite throat of radius $L$ which for $r \gg L$ opens up into flat $9 + 1$ dimensional space. Thus, for $L$ much larger than the string length scale, $\sqrt{\alpha'}$, the entire 3-brane geometry has small curvatures everywhere and is appropriately described by the supergravity approximation to type IIB string theory.

The relation between $L$ and $\sqrt{\alpha'}$ may be found by equating the gravitational tension of the extremal 3-brane classical solution to $N$ times the tension of a single D3-brane, and one finds

$$L^4 = g_{YM}^2 N \alpha'^2. \quad (3.7)$$

Thus, the size of the throat in string units is $\lambda^{1/4}$. This remarkable emergence of the 't Hooft coupling from gravitational considerations is at the heart of the success of the AdS/CFT correspondence. Moreover, the requirement $L \gg \sqrt{\alpha'}$ translates into $\lambda \gg 1$: the gravitational approach is valid when the 't Hooft coupling is very strong and the perturbative field theoretic methods are not applicable.

4. The AdS/CFT Correspondence

Studies of massless particle absorption by the 3-branes [24] indicate that, in the low-energy limit, the $AdS_5 \times S^5$ throat region ($r \ll L$) decouples from the asymptotically flat large $r$ region. Similarly, the $N = 4$ supersymmetric $SU(N)$ gauge theory on the stack of $N$ D3-branes decouples in the low-energy limit from the bulk closed string theory. Such considerations prompted Maldacena [1] to make the seminal conjecture that type IIB string theory on $AdS_5 \times S^5$, of radius $L$ given in (3.7), is dual to the $N = 4$ SYM theory. The number of colors in the gauge theory, $N$, is dual to the number of flux units of the 5-form Ramond-Ramond field strength.

It was further conjectured in [2, 3] that there exists a one-to-one map between gauge invariant operators in the CFT and fields (or extended objects) in AdS$_5$. The dimension $\Delta$ of an operator is determined by the mass of the dual field $\varphi$ in AdS$_5$. For example, for scalar operators one finds that $\Delta(\Delta - 4) = m^2 L^2$. For the fields in AdS$_5$ that come from the type IIB supergravity modes, including the Kaluza-Klein excitations on the 5-sphere, the masses are of order $1/L$. Hence, it is consistent to assume that their operator dimensions are independent of $L$, and therefore independent of $\lambda$. This is due to the fact that such operators commute with some of the supercharges and are therefore protected by supersymmetry. Perhaps the simplest such operators are the chiral primaries which are traceless symmetric polynomials made out of the six scalar fields $\Phi^i$: $\text{tr} \Phi^{(i_1} \ldots \Phi^{i_k)}$. These operators are dual to spherical harmonics on $S^5$ which mix the graviton and RR 4-form fluctuations.
Their masses are $m_k^2 = k(k-4)/L^2$, where $k = 2, 3, \ldots$. These masses reproduce the operator dimensions $\Delta = k$ which are the same as in the free theory. The situation is completely different for operators dual to the massive string modes: $m_n^2 = 4n/\alpha'$. In this case the AdS/CFT correspondence predicts that the operator dimension grows at strong coupling as $2n^{1/2}\lambda^{1/4}$.

Precise methods for calculating correlation functions of various operators in a CFT using its dual formulation were formulated in [2, 3] where a gauge theory quantity, $W$, was identified with a string theory quantity, $Z_{\text{string}}$:

$$W[\varphi_0(\vec{x})] = Z_{\text{string}}[\varphi_0(\vec{x})].$$

(4.1)

$W$ generates the connected Euclidean Green’s functions of a gauge theory operator $O$,

$$W[\varphi_0(\vec{x})] = \langle \exp \int d^4 x \varphi_0 O \rangle.$$  

(4.2)

$Z_{\text{string}}$ is the string theory path integral calculated as a functional of $\varphi_0$, the boundary condition on the field $\varphi$ related to $O$ by the AdS/CFT duality. In the large $N$ limit the string theory becomes classical, which implies

$$Z_{\text{string}} \sim e^{-I[\varphi_0(\vec{x})]},$$

(4.3)

where $I[\varphi_0(\vec{x})]$ is the extremum of the classical string action calculated as a functional of $\varphi_0$. If we are further interested in correlation functions at very large ‘t Hooft coupling, then the problem of extremizing the classical string action reduces to solving the equations of motion in type IIB supergravity whose form is known explicitly.

If the number of colors $N$ is sent to infinity while $g_{YM}^2 N$ is held fixed and large, then there are small string scale corrections to the supergravity limit [1–3] which proceed in powers of $\alpha'/L^2 = \lambda^{-1/2}$. If we wish to study finite $N$, then there are also string loop corrections in powers of $c^2/L^8 \sim N^{-2}$. As expected, taking $N$ to infinity enables us to take the classical limit of the string theory on $AdS_5 \times S^5$.

Immediate support for the AdS/CFT correspondence comes from symmetry considerations [1]. The isometry group of $AdS_5$ is $SO(2,4)$, and this is also the conformal group in $3+1$ dimensions. In addition we have the isometries of $S^5$ which form $SU(4) \sim SO(6)$. This group is identical to the $R$-symmetry of the $\mathcal{N} = 4$ SYM theory. After including the fermionic generators required by supersymmetry, the full isometry supergroup of the $AdS_5 \times S^5$ background is $SU(2,2|4)$, which is identical to the $\mathcal{N} = 4$ superconformal symmetry.

The fact that after the compactification on $Y_5$ the string theory is 5-dimensional supports earlier ideas on the necessity of the 5-th dimension to describe 4-d gauge theories [20]. The $z$-direction is dual to the energy scale of the gauge theory: small $z$ corresponds to the UV domain of the gauge theory, while large $z$ to the IR.

In the AdS/CFT duality, type IIB strings are dual to the chromo-electric flux lines in the gauge theory, providing a string theoretic set-up for calculating the quark anti-quark potential [25]. The quark and anti-quark are placed near the boundary of Anti-de Sitter space ($z = 0$), and the fundamental string connecting them is required to obey the equations of motion following from the Nambu
The string bends into the interior \((z > 0)\), and the maximum value of the \(z\)-coordinate increases with the separation \(r\) between quarks. An explicit calculation of the string action gives an attractive \(q\overline{q}\) potential [25]:

\[
V(r) = -\frac{4\pi^2 \sqrt{\lambda}}{\Gamma\left(\frac{1}{4}\right)^4 r}.
\] (4.4)

Its Coulombic \(1/r\) dependence is required by the conformal invariance of the theory. Historically, a dual string description was hoped for mainly in the cases of confining gauge theories, where long confining flux tubes have string-like properties. In a pleasant surprise, we now see that a string description applies to non-confining theories too, due to the presence of extra dimensions with a warped metric.

5. Semiclassical Spinning Strings vs. Highly Charged Operators

A few years ago it was noted that the AdS/CFT duality becomes particularly powerful when applied to operators with large quantum numbers. One class of such single-trace “long operators” are the BMN operators [26] that carry a large \(R\)-charge in the SYM theory and contain a finite number of impurity insertions. The \(R\)-charge is dual to a string angular momentum on the compact space \(Y_5\). So, in the BMN limit the relevant states are short closed strings with a large angular momentum, and a small amount of vibrational excitation [27]. Furthermore, by increasing the number of impurities the string can be turned into a large semi-classical object moving in \(AdS_5 \times Y_5\). Comparing such objects with their dual long operators has become a very fruitful area of research [28]. Work in this direction has also produced a great deal of evidence that the \(\mathcal{N} = 4\) SYM theory is exactly integrable (see [29] for reviews).

Another familiar example of an operator with a large quantum number is the twist-2 operator carrying a large spin \(S\),

\[
\text{Tr} \, F_{\mu \nu} D_+^{S-2} F_+^\mu .
\] (5.1)

In QCD, such operators play an important role in studies of deep inelastic scattering [30]. In general, the anomalous dimension of a twist-2 operator grows logarithmically [31] for large spin \(S\):

\[
\Delta - S = f(g) \ln S + O(S^0),
\] (5.2)

where \(g = \sqrt{g_{YM}^2 / 4\pi}\). This was demonstrated early on at 1-loop order [30] and at two loops [32], where a cancellation of \(\ln^2 S\) terms occurs. There are solid arguments that (5.2) holds to all orders in perturbation theory [31,33]. The function of coupling \(f(g)\) also measures the anomalous dimension of a cusp in a light-like Wilson loop [31], and is of definite physical interest in QCD.

There has been significant interest in determining \(f(g)\) in the \(\mathcal{N} = 4\) SYM theory, in which case we can consider operators in the \(SL(2)\) sector, of the form

\[
\text{Tr} \, D_+^S Z^\dagger + \ldots ,
\] (5.3)
where $Z$ is one of the complex scalar fields, the R-charge $J$ is the twist, and the dots serve as a reminder that the operator is a linear combination of such terms with the covariant derivatives acting on the scalars in all possible ways. The object dual to such a high-spin twist-2 operator is a folded string [27] spinning around the center of $AdS_5$; its generalization to large $J$ was found in [34]. The result (5.2) is generally applicable when $J$ is held fixed while $S$ is sent to infinity [35]. While the function $f(g)$ for $\mathcal{N} = 4$ SYM is not the same as the cusp anomalous dimension for QCD, its perturbative expansion is in fact related to that in QCD by the conjectured “transcendentality principle” [36].

The perturbative expansion of $f(g)$ at small $g$ can be obtained from gauge theory, but calculations in the planar $\mathcal{N} = 4$ SYM theory are quite formidable, and until recently were available only up to 3-loop order [36, 37]:

$$f(g) = 8g^2 - \frac{8}{3}\pi^2 g^4 + \frac{88}{45}\pi^4 g^6 + O(g^8).$$

This $\mathcal{N} = 4$ answer can be extracted [36] from the corresponding QCD calculation [38] using the transcendentality principle, which states that each expansion coefficient has terms of definite degree of transcendentality (namely the exponent of $g$ minus two), and that the QCD result contains the same terms (in addition to others which have lower degree of transcendentality).

The AdS/CFT correspondence relates the large $g$ behavior of $f(g)$ to the energy of a folded string spinning around the center of a weakly curved $AdS_5$ space [27]. This gives the prediction that $f(g) \to 4g$ at strong coupling. The same result was obtained from studying the cusp anomaly using string theory methods [39]. Furthermore, the semi-classical expansion for the spinning string energy predicts the following correction [34]:

$$f(g) = 4g - \frac{3\ln 2}{\pi} + O(1/g).$$

An interesting problem is to smoothly match these explicit predictions of string theory for large $g$ to those of gauge theory at small $g$. During the past few years methods of integrability in AdS/CFT [40] (for reviews and more complete references, see [29]) have led to major progress in addressing this question. In an impressive series of papers [41–43] an integral equation that determines $f(g)$ was proposed. This equation was obtained from the asymptotic Bethe ansatz for the $SL(2)$ sector by considering the limit $S \to \infty$ with $J$ finite, and extracting the piece proportional to $\ln S$, which is manifestly independent of $J$. Taking the spin to infinity, the discrete Bethe equations can be rewritten as an integral equation for the density of Bethe roots in rapidity space (though the resulting equation is most concisely expressed in terms of the variable $t$ that arises from performing a Fourier-transform).

The cusp anomalous dimension $f(g)$ is related to value of the fluctuation density $\hat{\sigma}(t)$ at $t = 0$ [41, 43, 44]:

$$f(g) = 16g^2 \hat{\sigma}(0),$$

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where $\sigma(t)$ is determined by the integral equation

$$\sigma(t) = \frac{t}{e^t - 1} \left( K(2gt, 0) - 4g^3 \int_0^\infty dt' K(2gt, 2gt') \sigma(t') \right).$$  \hspace{1cm} (5.7)$$

The kernel $K(t, t') = K^{(m)}(t, t') + K^{(d)}(t, t')$ is the sum of the so-called main scattering kernel $K^{(m)}(t, t') = K_0(t, t') + K_1(t, t')$ and the "dressing kernel"

$$K^{(d)}(t, t') = 8g^2 \int_0^\infty dt'' K_1(t, 2gt'') \frac{t''}{e^{t''} - 1} K_0(2gt'', t').$$  \hspace{1cm} (5.8)$$

Here $K_0$ and $K_1$ can be expressed as the following sums of Bessel functions, which are even and odd, respectively, under change of sign of both $t$ and $t'$:

$$K_0(t, t') = \frac{2}{tt'} \sum_{n=1}^\infty (2n - 1) J_{2n-1}(t) J_{2n-1}(t'),$$  \hspace{1cm} (5.9)

$$K_1(t, t') = \frac{2}{tt'} \sum_{n=1}^\infty (2n) J_{2n}(t) J_{2n}(t').$$  \hspace{1cm} (5.10)$$

Including the dressing kernel [43] is of crucial importance since it takes into account the dressing phase in the asymptotic two-particle world-sheet S-matrix, which is the only function of rapidities appearing in the S-matrix that is not fixed a priori by the $PSU(2, 2|4)$ symmetry of $\mathcal{N} = 4$ SYM theory, and whose general form was deduced in [45, 46]. The perturbative expansion of the phase starts at the 4-loop order, and at strong coupling coincides with the earlier results from string theory [42, 45, 47–49]. The important requirement of crossing symmetry [50] is satisfied by this phase, and it also obeys the transcendentality principle of [36]. Thus, there is strong evidence that this phase describes the exact magnon S-matrix at any coupling, which constitutes important progress in the understanding of the $\mathcal{N} = 4$ SYM theory, and the AdS/CFT correspondence.

An immediate check of the validity of the integral equation, which gives the expansion

$$f(g) = 8g^2 - \frac{8}{3} \pi^2 g^4 + \frac{88}{45} \pi^4 g^6 - 16 \left( \frac{73}{630} \pi^6 + 4 \zeta(3)^2 \right) g^8 + O(g^{10}),$$  \hspace{1cm} (5.11)$$

was provided by the gauge theory calculation of the 4-loop, $O(g^8)$ term. In a remarkable paper [51], which independently arrived at the same conjecture for the all-order expansion of $f(g)$ as [43], the 4-loop coefficient was calculated numerically by on-shell unitarity methods in SYM theory, and was used to conjecture the analytic result of (5.11). Subsequently, the numerical precision was improved to produce agreement with the analytic value within 0.001 percent accuracy [52].

In fact the perturbative expansion of $f(g)$ can be obtained from the integral equation (5.7) to arbitrary order. Although the expansion has a finite radius of convergence, as is customary in planar theories, it is expected to determine the function completely. But solving (5.7) for $\sigma(t)$ at values of the coupling constant
$g$ beyond the perturbative regime is not an easy task. By expanding the fluctuation density as a Neumann series of Bessel functions,

$$
\hat{\sigma}(t) = \frac{t}{e^t - 1} \sum_{n \geq 1} s_n \frac{J_n(2gt)}{2gt}
$$

(5.12)

one can reduce the problem to an (infinite-dimensional) matrix problem, which can be consistently truncated and is thus amenable to numerical solution [53]. This was shown to give $f(g)$ with high accuracy up to rather large values of $g$, and the function was found to be monotonically increasing, smooth, and in excellent agreement with the linear asymptotics predicted by string theory (5.5). The crossover from perturbative to the linear behavior takes place at around $g \sim 1/2$, which is comparable to the radius of convergence $|g| = 1/4$.

Subsequently, the leading term in the strong coupling asymptotic expansion of the fluctuation density $\hat{\sigma}(t)$ was derived analytically [54,55] from the integral equation. More recently, the complete asymptotic expansion of $f(g)$ was determined in an impressive paper [56] (for further work, see [57]). This expansion obeys its own transcendentality principle; in particular, the coefficient of the $1/g$ term in (5.5) is

$$
-\frac{K}{4\pi^2} \approx -0.0232
$$

(5.13)

in agreement with the numerical work of [53] ($K$ is the Catalan constant). As a further check, this coefficient was reproduced analytically from a two-loop calculation in string sigma-model perturbation theory [58].

Thus, the cusp anomaly $f(g)$ is an example of a non-trivial interpolation function for an observable not protected by supersymmetry that smoothly connects weak and strong coupling regimes, and tests the AdS/CFT correspondence at a very deep level. The final form of $f(g)$ was arrived at using inputs from string theory, perturbative gauge theory, and the conjectured exact integrability of planar $\mathcal{N} = 4$ SYM. Further tests of this proposal may be carried out perturbatively: in fact, the planar expansion (5.11) makes an infinite number of explicit analytic predictions for higher loop coefficients. This shows how implications of string theory may sometimes be tested via perturbative gauge theory.

6. Thermal Gauge Theory from Near-Extremal D3-Branes

Entropy

An important black hole observable is the Bekenstein-Hawking (BH) entropy, which is proportional to the area of the event horizon, $S_{BH} = A_h/(4G)$. For the 3-brane solution (3.1), the horizon is located at $r = r_0$. For $r_0 > 0$ the 3-brane carries some excess energy $E$ above its extremal value, and the BH entropy is also non-vanishing. The Hawking temperature is then defined by $T^{-1} = \partial S_{BH}/\partial E$.

Setting $r_0 \ll L$ in (3.1), we obtain a near-extremal 3-brane geometry, whose Hawking temperature is found to be $T = r_0/(\pi L^2)$. The small $r$ limit of this
geometry is $S^5$ times a certain black hole in $AdS_5$. The 8-dimensional “area” of the event horizon is $A_h = \pi^6 L^8 T^3 V_3$, where $V_3$ is the spatial volume of the D3-brane (i.e. the volume of the $x^1, x^2, x^3$ coordinates). Therefore, the BH entropy is [59]

$$S_{BH} = \frac{\pi^2}{2} N^2 V_3 T^3. \tag{6.1}$$

This gravitational entropy of a near-extremal 3-brane of Hawking temperature $T$ is to be identified with the entropy of $\mathcal{N} = 4$ supersymmetric $U(N)$ gauge theory (which lives on $N$ coincident D3-branes) heated up to the same temperature.

The entropy of a free $U(N)\mathcal{N} = 4$ supermultiplet, which consists of the gauge field, $6N^2$ massless scalars and $4N^2$ Weyl fermions, can be calculated using the standard statistical mechanics of a massless gas (the black body problem), and the answer is

$$S_0 = \frac{2\pi^2}{3} N^2 V_3 T^3. \tag{6.2}$$

It is remarkable that the 3-brane geometry captures the $T^3$ scaling characteristic of a conformal field theory (in a CFT this scaling is guaranteed by the extensivity of the entropy and the absence of dimensionful parameters). Also, the $N^2$ scaling indicates the presence of $O(N^2)$ unconfined degrees of freedom, which is exactly what we expect in the $\mathcal{N} = 4$ supersymmetric $U(N)$ gauge theory. But what is the explanation of the relative factor of $3/4$ between $S_{BH}$ and $S_0$? In fact, this factor is not a contradiction but rather a prediction about the strongly coupled $\mathcal{N} = 4$ SYM theory at finite temperature. As we argued above, the supergravity calculation of the BH entropy, (6.1), is relevant to the $\lambda \to \infty$ limit of the $\mathcal{N} = 4$ $SU(N)$ gauge theory, while the free field calculation, (6.2), applies to the $\lambda \to 0$ limit. Thus, the relative factor of $3/4$ is not a discrepancy: it relates two different limits of the theory. Indeed, on general field theoretic grounds, in the ’t Hooft large $N$ limit the entropy is given by [60]

$$S = \frac{2\pi^2}{3} N^2 f_e(\lambda) V_3 T^3. \tag{6.3}$$

The function $f_e$ is certainly not constant: Feynman graph calculations valid for small $\lambda = g_\text{YM}^2 N$ give [61]

$$f_e(\lambda) = 1 - \frac{3}{2\pi^2} \lambda + \frac{3 + \sqrt{2}}{\pi^2} \lambda^{3/2} + \ldots \tag{6.4}$$

The BH entropy in supergravity, (6.1), is translated into the prediction that

$$\lim_{\lambda \to \infty} f_e(\lambda) = \frac{3}{4}. \tag{6.5}$$

A string theoretic calculation of the leading correction at large $\lambda$ gives [60]

$$f_e(\lambda) = \frac{3}{4} + \frac{45}{32} \frac{(3)}{3} \lambda^{-3/2} + \ldots \tag{6.6}$$

These results are consistent with a monotonic function $f_e(\lambda)$ which decreases from 1 to 3/4 as $\lambda$ is increased from 0 to $\infty$. The 1/4 deficit compared to the free field value is a strong coupling effect predicted by the AdS/CFT correspondence.
It is interesting that similar deficits have been observed in lattice simulations of deconfined non-supersymmetric gauge theories [62–64]. The ratio of entropy to its free field value, calculated as a function of the temperature, is found to level off at values around 0.8 for $T$ beyond 3 times the deconfinement temperature $T_c$. This is often interpreted as the effect of a sizable coupling. Indeed, for $T = 3T_c$, the lattice estimates [63] indicate that $g_Y^2 N \approx 7$. This challenges an old prejudice that the quark-gluon plasma is inherently weakly coupled. We now turn to calculations of the shear viscosity where strong coupling effects are more pronounced.

Shear Viscosity

The shear viscosity $\eta$ may be read off from the form of the stress-energy tensor in the local rest frame of the fluid where $T_{0i} = 0$:

$$T_{ij} = p\delta_{ij} - \eta(\partial_i u_j + \partial_j u_i - \frac{2}{3} \delta_{ij} \partial_k u_k),$$

where $u_i$ is the 3-velocity field. The viscosity can be also determined [65] through the Kubo formula

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int d^3x e^{i\omega t} \langle [T_{xy}(t, \vec{x}), T_{zy}(0, 0)] \rangle$$

For the $\mathcal{N} = 4$ supersymmetric YM theory this 2-point function may be computed from absorption of a low-energy graviton $h_{xy}$ by the 3-brane metric [24]. Using this method, it was found [65] that at very strong coupling

$$\eta = \frac{\pi}{8} N^2 T^3,$$

which implies

$$\frac{\eta}{s} = \frac{h}{4\pi}$$

after $h$ is restored in the calculation (here $s = S/V$ is the entropy density). This is much smaller than what perturbative estimates imply. Indeed, at weak coupling $\eta/s$ is very large, $\sim h\lambda^{-2}/\ln(1/\lambda)$ [66], and there is evidence that it decreases monotonically as the coupling is increased [67].

The saturation of $\eta/s$ at some value of order $h$ is reasonable on general physical grounds [68]. The shear viscosity $\eta$ is of order the energy density times quasi-particle mean free time $\tau$. So $\eta/s$ is of order of the energy of a quasi-particle times its mean free time, which is bounded from below by the uncertainty principle to be some constant times $h$. These considerations prompted a suggestion [68] that $h/(4\pi)$ is the lower bound on $\eta/s$. However, the generality of this bound was called into question in [69]. Recently it was shown [70] that for special large $\mathcal{N}$ gauge theories whose AdS duals contain D7-branes, the leading $1/\mathcal{N}$ correction to (6.10) is negative; thus, the bound can be violated even for theories that have a holographic description.

Nevertheless, (6.10) applies to the strong coupling limit of a large class of gauge theories that have gravity duals. Its important physical implication is that a low value of $\eta/s$ is a consequence of strong coupling. For known fluids (e.g. helium,
nitrogen, water) $\eta/s$ is considerably higher than $6.10$. On the other hand, the quark-gluon plasma produced at RHIC is believed to have a very low $\eta/s$ [71, 72], with some recent estimates [73] suggesting that it is below $6.10$. Lattice studies of pure glue gauge theory [74]) also lead to low values of $\eta/s$. This suggests that, at least for $T$ near $T_c$, the theory is sufficiently strongly coupled. Indeed, a new term, sQGP, which stands for “strongly coupled quark-gluon plasma,” has been coined to describe the deconfined state observed at RHIC [75, 76] (a somewhat different term, “non-perturbative quark-gluon plasma”, was advocated in [77]). As we have reviewed, the gauge-string duality is a theoretical laboratory which allows one to study some extreme examples of such a new state of matter. These include the thermal $\mathcal{N} = 4$ SYM theory at very strong ‘t Hooft coupling, as well as other gauge theories which have less supersymmetry and exhibit confinement at low temperature. An example of such a gauge theory, whose dual is the warped deformed conifold, will be discussed in section 8.

Lattice calculations indicate that the deconfinement temperature $T_c$ is around 175 MeV, and the energy density is $\approx 0.7$ GeV/fm$^3$, around 6 times the nuclear energy density. RHIC has reached energy densities around 14 GeV/fm$^3$, corresponding to $T \approx 2T_c$. Furthermore, heavy ion collisions at the LHC are expected to reach temperatures up to $5T_c$. Thus, RHIC and LHC should provide a great deal of new information about the sQGP, which can be compared with calculations based on gauge-string duality. For a more detailed discussion of the recent theoretical work in this direction, see U. Wiedemann’s lectures in this volume.

7. Warped Conifold and its Resolution

To formulate the AdS/CFT duality at zero temperature, but with a reduced amount of supersymmetry, we may place the stack of D3-branes at the tip of a 6-dimensional Ricci flat cone, whose base is a 5-dimensional compact Einstein space $Y_5$. The metric of such a cone is $dr^2 + r^2 ds^2_{Y_5}$; therefore, the 10-d metric produced by the D3-branes is obtained from (3.5) by replacing $d\Omega^2_{S^5}$, the metric on $S^5$, by $ds^2_{Y_5}$, the metric on $Y_5$. In the $r \to 0$ limit we then find the space $AdS_5 \times Y_5$ as the candidate dual of the CFT on the D3-branes placed at the tip of the cone. The isometry group of $Y_5$ is smaller than $SO(6)$, but $AdS_5$ is the “universal” factor present in the dual description of any large $N$ CFT, making the $SO(2, 4)$ conformal symmetry geometric.

To obtain gauge theories with $\mathcal{N} = 1$ superconformal symmetry the Ricci flat cone must be a Calabi-Yau 3-fold [80, 81] whose base $Y_5$ is called a Sasaki-Einstein space. Among the simplest examples of these is $Y_5 = T^{1,1}$. The corresponding Calabi-Yau cone is called the conifold. Much of the work on gauge-string dualities originating from D-branes on the conifold is reviewed in the 2001 Les Houches lectures [82]. Here we will emphasize the progress that has taken place since then, in particular on issues related to breaking of the $U(1)$ baryon number symmetry.
The conifold is a singular non-compact Calabi-Yau three-fold [83]. Its importance arises from the fact that the generic singularity in a Calabi-Yau three-fold locally looks like the conifold, described by the quadratic equation in $\mathbb{C}^4$:
\begin{equation}
z_1^2 + z_2^2 + z_3^2 + z_4^2 = 0.
\end{equation}
This homogeneous equation defines a real cone over a 5-dimensional manifold. For the cone to be Ricci-flat the 5d base must be an Einstein manifold ($R_{ab} = 4g_{ab}$). For the conifold [83], the topology of the base can be shown to be $S^2 \times S^3$ and it is called $T^{1,1}$, with the following Einstein metric [84]:
\begin{equation}
d\Omega^2_{T^{1,1}} = \frac{1}{9} \left( d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2 \right)^2 + \frac{1}{6} \left( d\theta_1 + \sin^2 \theta_1 d\phi_1^2 \right) + \frac{1}{6} \left( d\theta_2 + \sin^2 \theta_2 d\phi_2^2 \right).
\end{equation}
$T^{1,1}$ is a homogeneous space, being the coset $SU(2) \times SU(2)/U(1)$. The metric on the cone is then $ds^2 = dr^2 + r^2 d\Omega^2_{T^{1,1}}$.

We may introduce two other types of complex coordinates on the conifold, $w_a$ and $a_i, b_j$, as follows:
\begin{equation}
Z = \begin{pmatrix} z_3 + iz_4 & z_1 - iz_2 \\ z_1 + iz_2 & z_3 - iz_4 \end{pmatrix} = \begin{pmatrix} w_1 & w_3 \\ w_4 & w_2 \end{pmatrix} = \begin{pmatrix} a_1b_1 & a_1b_2 \\ a_2b_1 & a_2b_2 \end{pmatrix}
\end{equation}
\begin{equation}
= r^{1/2} \begin{pmatrix} -c_1s_2 & e^{\frac{i}{2}(\psi + \phi_1 - \phi_2)}c_1c_2 & e^{\frac{i}{2}(\psi + \phi_1 + \phi_2)}c_1s_2 \\ -s_1s_2 & e^{\frac{i}{2}(\psi - \phi_1 - \phi_2)}c_1c_2 & e^{\frac{i}{2}(\psi - \phi_1 + \phi_2)}s_1s_2 \end{pmatrix},
\end{equation}
where $c_i = \cos \frac{\theta_i}{2}$, $s_i = \sin \frac{\theta_i}{2}$ (see [83] for other details on the $w, z$ and angular coordinates). The equation defining the conifold is now $\det Z = 0$.

The $a, b$ coordinates above will be of particular interest to us because the symmetries of the conifold are most apparent in this basis. The conifold equation has $SU(2) \times SU(2) \times U(1)$ symmetry since under these symmetry transformations,
\begin{equation}
\det LZR = \det e^{i\alpha}Z = 0.
\end{equation}
This is also a symmetry of the metric presented above where each $SU(2)$ acts on $\theta_i, \phi_i, \psi$ (thought of as Euler angles on $S^3$) while the $U(1)$ acts by shifting $\psi$. This symmetry can be identified with $U(1)_R$, the R-symmetry of the dual gauge theory, in the conformal case. The action of the $SU(2) \times SU(2) \times U(1)_R$ symmetry on $a_i, b_j$ defined in (7.3) is given by:
\begin{equation}
SU(2) \times SU(2) \text{ symmetry : } \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \rightarrow L \begin{pmatrix} a_1 \\ a_2 \end{pmatrix},
\end{equation}
\begin{equation}
\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \rightarrow R \begin{pmatrix} b_1 \\ b_2 \end{pmatrix},
\end{equation}
\begin{equation}
R\text{-symmetry : } (a_i, b_j) \rightarrow e^{i\frac{\theta_i}{2}}(a_i, b_j),
\end{equation}
i.e. $a$ and $b$ transform as $(1/2, 0)$ and $(0, 1/2)$ under $SU(2) \times SU(2)$ with $R$-charge 1/2 each. We can thus describe the singular conifold as the manifold parametrized...
by \(a, b\), but from (7.3), we see that there is some redundancy in the \(a, b\) coordinates. Namely, the transformation

\[
a_i \rightarrow \lambda a_i, \quad b_j \rightarrow \frac{1}{\lambda} b_j, \quad (\lambda \in \mathbb{C}),
\]

(7.8)
give the same \(z, w\) in (7.3). We impose the constraint \(|a_1|^2 + |a_2|^2 - |b_1|^2 - |b_2|^2 = 0\) to fix the magnitude in the above transformation. To account for the remaining constraint above, we describe the singular conifold as the quotient of the 6-space with the above constraint by the relation \(a \sim e^{i\alpha} a, b \sim e^{-i\alpha} b\).

The importance of the coordinates \(a_i, b_j\) is that in the gauge theory on D3-branes at the tip of the conifold they are promoted to chiral superfields. The low-energy gauge theory on \(N\) D3-branes is a \(\mathcal{N} = 1\) supersymmetric \(SU(N) \times SU(N)\) gauge theory with bi-fundamental chiral superfields \(A_i, B_j\) \((i, j = 1, 2)\) in the \((\mathbf{N}, \mathbf{N})\) and \((\mathbf{\overline{N}}, \mathbf{\overline{N}})\) representations of the gauge groups, respectively [80, 81]. The superpotential for this gauge theory is

\[
W \sim \text{Tr} \det A_i B_j = \text{Tr} (A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1).
\]

(7.9)
The continuous global symmetries of this theory are \(SU(2) \times SU(2) \times U(1)_B \times U(1)_R\), where the \(SU(2)\) factors act on \(A_i\) and \(B_j\), respectively, \(U(1)_B\) is a baryonic symmetry under which the \(A_i\) and \(B_j\) have opposite charges, and \(U(1)_R\) is the R-symmetry with charges of the same sign \(R_A = R_B = \frac{1}{2}\). This assignment ensures that \(W\) is marginal, and one can also show that the gauge couplings do not run. Hence this theory is superconformal for all values of gauge couplings and superpotential coupling [80, 81].

A simple way to understand the resolution of the conifold is to deform the modulus constraint above into

\[
|b_1|^2 + |b_2|^2 - |a_1|^2 - |a_2|^2 = u^2,
\]

(7.10)
where \(u\) is a real parameter which controls the resolution. The resolution corresponds to a blow up of the \(S^2\) at the bottom of the conifold. In the dual gauge theory turning on \(u\) corresponds to a particular choice of vacuum [85]. After promoting the \(a, b\) fields into the bi-fundamental chiral superfields of the dual gauge theory, we can define the operator \(\mathcal{U}\) as

\[
\mathcal{U} = \frac{1}{N} \text{Tr}(B_1^2 B_1 + B_2^2 B_2 - A_1^2 A_1 - A_2^2 A_2).
\]

(7.11)
Thus, the warped singular conifolds correspond to gauge theory vacua where \(\langle \mathcal{U} \rangle = 0\), while the warped resolved conifolds correspond to vacua where \(\langle \mathcal{U} \rangle \neq 0\). In the latter case, some VEVs for the bi-fundamental fields \(A_i, B_j\) must be present. Since these fields are charged under the \(U(1)_B\) symmetry, the warped resolved conifolds correspond to vacua where this symmetry is broken [85].

A particularly simple choice is to give a diagonal VEV to only one of the scalar fields, say, \(B_2\). As seen in [86], this choice breaks the \(SU(2) \times SU(2) \times U(1)_B \times U(1)_R\) symmetry of the CFT down to \(SU(2) \times U(1) \times U(1)\). The string dual is given by a warped resolved conifold

\[
ds^2 = h^{-1/2} \eta_{\mu\nu} dx^\mu dx^\nu + h^{1/2} ds_6^2.
\]

(7.12)
The explicit form of the Calabi-Yau metric of the resolved conifold is given by [87]
\[ ds_6^2 = K^{-1} dr^2 + \frac{1}{9} K r^2 \left( d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2 \right)^2 \]
\[ + \frac{1}{6} r^2 (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) + \frac{1}{6} (r^2 + 6u^2)(d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2) , \] (7.13)
where
\[ K = \frac{r^2 + 9u^2}{r^2 + 6u^2} . \] (7.14)
The $N$ D3-branes sourcing the warp factor are located at the north pole of the finite $S^2$, i.e. at $r = 0, \theta_2 = 0$. The warp factor is the Green function on the resolved conifold with this source [86]:
\[ h(r, \theta_2) = L^4 \sum_{l=0}^{\infty} (2l + 1) H_l(r) P_l(\cos \theta_2) . \] (7.15)
Here $L^4 = \frac{27\pi g_s N}{4 \alpha'}$, $P_l(\cos \theta)$ is the $l$-th Legendre polynomial, and
\[ H_l = \frac{2C_{\beta}}{9u^2 r^{2+2\beta}} \, 2F_1 \left( \beta, 1 + \beta, 1 + 2\beta; -\frac{9u^2}{r^2} \right) , \] (7.16)
with the coefficients $C_{\beta}$ and $\beta$ given by
\[ C_{\beta} = \frac{(3u)^{2\beta} \Gamma(1 + \beta)^2}{\Gamma(1 + 2\beta)} , \quad \beta = \sqrt{1 + \frac{3}{2}(l+1)} . \] (7.17)
Far in the IR the gauge theory flows to the $\mathcal{N} = 4$ $SU(N)$ SYM theory, as evidenced by the appearance of an $AdS_5 \times S^5$ throat near the location of the stack of the D3-branes. This may be verified in the gauge theory as follows. The condensate $B_2 = uN \times N$ breaks the $SU(N) \times SU(N)$ gauge group down to $SU(N)$, all the chiral fields now transforming in the adjoint of this diagonal group. After substituting this classical value for $B_2$, the quartic superpotential (7.9) reduces to the cubic $\mathcal{N} = 4$ form,
\[ W \sim \text{Tr}(A_1 [B_1, A_2]) . \] (7.18)
This confirms that the gauge theory flows to the $\mathcal{N} = 4$ $SU(N)$ SYM theory. The gauge theory also contains an interesting additional sector coupled to this infrared CFT; in particular, it contains global strings due to the breaking of the $U(1)_B$ symmetry [88].

**Baryonic Condensates and Euclidean D3-branes**

The gauge invariant order parameter for the breaking of $U(1)_B$ is $\text{det} B_2$. Let us review the calculation of this baryonic VEV using the dual string theory on the warped resolved conifold background [86].
The objects in $AdS_5 \times T^{1,1}$ that are dual to baryonic operators are D3-branes wrapping 3-cycles in $T^{1,1}$ [89]. Classically, the 3-cycles dual to the baryons made out of the $B$’s are located at fixed $\theta_2$ and $\phi_2$, while quantum mechanically one
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has to carry out collective coordinate quantization and finds wave functions of spin $N/2$ on the 2-sphere.

To calculate VEVs of such baryonic operators we need to consider the action of a Euclidean D3-brane whose world volume ends at large $r$ on the 3-sphere at fixed $\theta_2$ and $\phi_2$. The D3-brane action should be integrated up to a radial cut-off $r$, and we identify $e^{-S(r)}$ with the field $\varphi(r)$ dual to the baryonic operator. Close to the boundary, a field $\varphi$ dual to an operator of dimension $\Delta$ in the AdS/CFT correspondence behaves as

$$\varphi(r) \to \varphi_0 r^{\Delta-4} + A_\varphi r^{-\Delta},$$  \hspace{1cm} (7.19)

Here $A_\varphi$ is the operator expectation value [90], and $\varphi_0$ is the source for it. There are no sources added for baryonic operators, hence we will find that $\varphi_0 = 0$, but the term scaling as $r^{-\Delta}$ is indeed present in $e^{-S(r)}$.

The Born-Infeld action of the D3-brane is given by

$$S_{BI} = T_3 \int d^4\xi \sqrt{g},$$  \hspace{1cm} (7.20)

where $g_{\mu\nu}$ is the metric induced on the D3 world-volume. The smooth 4-chain which solves the BI equations of motion subject to our boundary conditions is located at fixed $\theta_2$ and $\phi_2$, and spans the $r$, $\theta_1$, $\phi_1$ and $\psi$ directions. Using the D3-brane tension

$$T_3 = \frac{1}{g_s (2\pi)^3 (\alpha')^2},$$  \hspace{1cm} (7.21)

we find

$$S_{BI} = \frac{3N}{4} \int_0^r d\tilde{r} \tilde{r}^3 h(\tilde{r}, \theta_2).$$  \hspace{1cm} (7.22)

Using the expansion (7.15), we note that the $l = 0$ term needs to be evaluated separately since it contains a logarithmic divergence:

$$\int_0^r d\tilde{r} \tilde{r}^3 H_0(r) = \frac{1}{4} + \frac{1}{2} \ln \left(1 + \frac{r^2}{9u^2}\right).$$  \hspace{1cm} (7.23)

For the $l > 0$ terms the integral converges and we find the simple result [86]

$$\int_0^r d\tilde{r} \tilde{r}^2 \sum_{l=1}^{\infty} H_l(\tilde{r}) P_l(\cos \theta_2) = \frac{2}{3} (-1 - 2 \ln[\sin(\theta_2/2)]).$$  \hspace{1cm} (7.24)

This expression is recognized as the Green’s function on a sphere. Combining the results, and taking $r \gg u$, we find

$$e^{-S_{BI}} = \left(\frac{3N^{5/4}}{2}\right)^{3N/4} \sin^N(\theta_2/2).$$  \hspace{1cm} (7.25)

In [89] it was argued that the wave functions of $\theta_2, \phi_2$, which arise through the collective coordinate quantization of the D3-branes wrapped over the 3-cycle $(\psi, \theta_1, \phi_1)$, correspond to eigenstates of a charged particle on $S^2$ in the presence of a charge $N$ magnetic monopole. Taking the gauge potential $A_\phi = N(1 + \cos \theta)/2$, $A_\theta = 0$ we find that the ground state wave function $\sim \sin^N(\theta_2/2)$
Gauge–String Dualities and Some Applications

These are the SU(2) quantum numbers of \( \det B_2 \). Therefore, the angular dependence of \( e^{-S} \) identifies \( \det B_2 \) as the only operator that acquires a VEV, in agreement with the gauge theory. The power of \( r \) indicates that the operator dimension is \( \Delta = 3N/4 \), which is indeed the exact dimension of the baryonic operators [89]. The VEV depends on the parameter \( u \) as \( \sim u^{3N/4} \). This is not the same as the classical scaling that would give \( \det B_2 \sim u^N \). The classical scaling is not obeyed because we are dealing with a strongly interacting gauge theory where the baryonic operator acquires an anomalous dimension.

Goldstone Bosons and Global Strings

Since the \( U(1) \) symmetry is broken spontaneously, the spectrum of the theory must include a Goldstone boson. Its gravity dual is a normalizable RR 4-form fluctuation around the warped resolved conifold [88]:

\[
\delta F^{(5)} = (1 + \ast)d(a_2(x) \wedge W). \tag{7.26}
\]

Here \( W \) is a closed 2-form inside the warped resolved conifold,

\[
W = \sin \theta_2 d\theta_2 \wedge d\phi_2 + d(f_1 g^5 + f_2 \sin \theta_2 d\varphi_2), \tag{7.27}
\]

where \( f_1, f_2 \) are functions of \( r, \theta_2 \). The equations of motion reduce to

\[
d \ast_4 da_2 = 0, \tag{7.28}
\]

provided \( W \) satisfies

\[
d(h \ast_6 W) = 0, \tag{7.29}
\]

where \( \ast_4, \ast_6 \) are the Hodge duals with respect to the unwarped Minkowski and resolved conifold metrics, respectively. This gives coupled PDE’s for \( f_1, f_2 \). Their solution can be obtained by minimizing a positive definite functional subject to certain boundary conditions [88]. Introducing the Goldstone boson field \( p(x) \) through \( \ast_4 da_2 = dp \), we note that the fluctuation in the 5-form field strength reads

\[
\delta F^{(5)} = da_2 \wedge W + dp \wedge h \ast_6 W. \tag{7.30}
\]

The corresponding fluctuation of the 4-form potential is

\[
\delta C^{(4)} = a_2(x) \wedge W + p \wedge h \ast_6 W. \tag{7.31}
\]

In addition to the existence of a Goldstone boson, a hallmark of a broken \( U(1) \) symmetry is the appearance of “global” strings. The Goldstone boson has a non-trivial monodromy around such a string, thus giving it a logarithmically divergent energy density. On the string side of the duality these global strings are \( D3 \)-branes wrapping the 2-sphere at the bottom of the warped resolved conifold [88]. The Goldstone bosons and the global strings interact with the \( \mathcal{N} = 4 \) SYM theory that appears far in the infrared. The coupling of such an extra sector and an infrared CFT is an interesting fact reminiscent of the unparticle physics scenarios [91].

\[1\] In a different gauge this wave function would acquire a phase. In the string calculation the phase comes from the purely imaginary Chern-Simons term in the Euclidean D3-brane action.
8. Deformation of the Conifold

We have seen above that the singularity of the cone over $T^{1,1}$ can be replaced by an $S^2$ through resolving the conifold (7.1) as in (7.10). An alternative supersymmetric blow-up, which replaces the singularity by an $S^3$, is the deformed conifold [83]

$$z_1^2 + z_2^2 + z_3^2 + z_4^2 = \varepsilon^2 . \tag{8.1}$$

To achieve the deformation, one needs to turn on $M$ units of RR 3-form flux. This modifies the dual gauge theory to $\mathcal{N} = 1$ supersymmetric $SU(N) \times SU(N + M)$ theory coupled to chiral superfields $A_1, A_2$ in the $(N, \bar{N} + \bar{M})$ representation, and $B_1, B_2$ in the $(\bar{N}, N + M)$ representation. Indeed, in type IIB string theory D5-branes source the 7-form field strength from the Ramond-Ramond sector, which is Hodge dual to the 3-form field strength. Therefore, the $M$ wrapped D5-branes create $M$ flux units of this field strength through the 3-cycle in the conifold; this number is dual to the difference between the numbers of colors in the two gauge groups. Thus, unlike the resolution, the deformation cannot be achieved in the context of the $SU(N) \times SU(N)$ gauge theory.

The 10-d metric takes the following form [92]:

$$ds_{10}^2 = h^{-1/2}(\tau) \eta_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(\tau) ds_6^2 , \tag{8.2}$$

where $ds_6^2$ is the Calabi-Yau metric of the deformed conifold:

$$ds_6^2 = \frac{\varepsilon^{4/3}}{2} K(\tau) \left[ \sinh^2 \left( \frac{\tau}{2} \right) \left[ (\tilde{g}_1^1)^2 + (\tilde{g}_2^1)^2 \right] + \cosh^2 \left( \frac{\tau}{2} \right) \left[ (\tilde{g}_3^1)^2 + (\tilde{g}_4^1)^2 \right] + \frac{1}{3K'(\tau)} \left[ d\tau^2 + (\tilde{g}_5^1)^2 \right] \right] , \tag{8.3}$$

where

$$K(\tau) = \frac{(\sinh \tau \cosh \tau - \tau)^{1/3}}{\sinh \tau} . \tag{8.4}$$

For $\tau \gg 1$ we may introduce another radial coordinate $r$ defined by

$$r^2 = \frac{3}{2\varepsilon^{4/3}} \varepsilon^{2r/3} , \tag{8.5}$$

and in terms of this coordinate we find $ds_6^2 \to dr^2 + r^2 ds_{1,1}^2$.

The basis one-forms $g^i$ in terms of which this metric is diagonal are defined by

$$g^1 \equiv \frac{e_2 - e_1}{\sqrt{2}}, \quad g^3 \equiv \frac{e_1 - e_1}{\sqrt{2}} , \tag{8.6}$$

$$g^3 \equiv \frac{e_2 + e_2}{\sqrt{2}}, \quad g^4 \equiv \frac{e_1 + e_1}{\sqrt{2}} , \tag{8.7}$$

$$g^5 \equiv e_3 + \cos \theta_1 d\phi_1 , \tag{8.8}$$

where the $e_i$ are one-forms on $S^2$

$$e_1 \equiv d\theta_1, \quad e_2 \equiv -\sin \theta_1 d\phi_1 , \tag{8.9}$$
and the $\epsilon_i$ a set of one-forms on $S^3$

\[ \epsilon_1 \equiv \sin \psi \sin \theta_2 d\phi_2 + \cos \psi d\theta_2, \]  
\[ \epsilon_2 \equiv \cos \psi \sin \theta_2 d\phi_2 - \sin \psi d\theta_2, \]  
\[ \epsilon_3 \equiv d\psi + \cos \theta_2 d\phi_2. \]  

The NSNS two-form is given by

\[ B^{(2)} = \frac{g_s M_{\alpha'}^2}{2} \left[ \frac{\coth \tau}{\sinh \tau} \right] \sinh \left( \frac{\tau}{2} \right) g^1 \wedge g^2 + \cosh \left( \frac{\tau}{2} \right) g^3 \wedge g^4 \]  

and the RR fluxes are most compactly written as

\[ F^{(3)} = \frac{M_{\alpha'}^2}{2} \left[ g^1 \wedge g^4 \wedge g^5 + \frac{\sinh \tau}{2 \sinh \tau} \left( g^1 \wedge g^3 + g^2 \wedge g^4 \right) \right], \]  
\[ \tilde{F}^{(5)} = dC^{(4)} + B^{(2)} \wedge F^{(3)} = (1 + \ast) \left( B^{(2)} \wedge F^{(3)} \right). \]  

Note that the complex three-form field of this BPS supergravity solution is imaginary self dual:

\[ \ast G_3 = i G_3, \quad G_3 = F_3 - i g_s H_3, \]  

where $\ast$ again denotes the Hodge dual with respect to the unwarped metric $ds^2_6$.

This guarantees that the dilaton is constant, and we set $\phi = 0$.

The above expressions for the NSNS- and RR-forms follow by making a simple ansatz consistent with the symmetries of the problem, and solving a system of differential equations, which owing to the supersymmetry of the problem are only first order [92]. The warp factor is then found to be completely determined up to an additive constant, which is fixed by demanding that it go to zero at large $\tau$:

\[ h(\tau) = \left( g_s M_{\alpha'}^2 \right)^{2/3} e^{-8/3 I(\tau)}, \]  
\[ I(\tau) \equiv 2^{1/3} \int_0^\tau dx \frac{x \coth x - 1}{\sinh^2 x} (\sinh x \cosh x - x)^{1/3}. \]  

For small $\tau$ the warp factor approaches a finite constant since $I(0) \approx 0.71805$. This implies confinement because the chromo-electric flux tube, described by a fundamental string at $\tau = 0$, has tension

\[ T_s = \frac{1}{2\pi \alpha' \sqrt{h(0)}}. \]  

The KS solution [92] is $SU(2) \times SU(2)$ symmetric and the expressions above can be written in an explicitly $SO(4)$ invariant way. It also possesses a $\mathbb{Z}_2$ symmetry $I$, which exchanges $(\theta_1, \phi_1)$ with $(\theta_2, \phi_2)$ accompanied by the action of $-I$ of SL(2, $\mathbb{Z}$), changing the signs of the three-form fields.

Examining the metric $ds^2_6$ for $\tau = 0$ we see that it degenerates into

\[ d\Omega_3^2 = \frac{1}{2} e^{4\tau/3} (2/3)^{1/3} \left[ \frac{1}{2} (g^5)^2 + (g^3)^2 + (g^4)^2 \right], \]
which is the metric of a round $S^3$, while the $S^2$ spanned by the other two angular coordinates, and fibered over the $S^3$, shrinks to zero size. In the ten-dimensional metric (8.2) this appears multiplied by a factor of $h^{1/2}(\tau)$, and thus the radius squared of the three-sphere at the tip of the conifold is of order $g_s M_\alpha'$. Hence for $g_s M$ large, the curvature of the $S^3$, and in fact everywhere in this manifold, is small and the supergravity approximation reliable.

The field theoretic interpretation of the KS solution is unconventional. After a finite distance along the RG flow, the $SU(N + M)$ group undergoes a Seiberg duality transformation [93]. After this transformation, and an interchange of the two gauge groups, the new gauge theory is $SU(\tilde{N}) \times SU(\tilde{N} + M)$ with the same bi-fundamental field content and superpotential, and with $\tilde{N} = N - M$. The self-similar structure of the gauge theory under the Seiberg duality is the crucial fact that allows this pattern to repeat many times. For a careful field theoretic discussion of this quasi-periodic RG flow, see [94]. If $N = (k + 1)M$, where $k$ is an integer, then the duality cascade stops after $k$ steps, and we find a $SU(M) \times SU(2M)$ gauge theory. This IR gauge theory exhibits a multitude of interesting effects visible in the dual supergravity background, which include the confinement and chiral symmetry breaking.

9. Normal Modes of the Warped Throat

As we shall see below, the $U(1)$ baryonic symmetry of the warped deformed conifold is in fact spontaneously broken, since baryonic operators acquire expectation values. The corresponding Goldstone boson is a massless pseudoscalar supergravity fluctuation which has non-trivial monodromy around D-strings at the bottom of the warped deformed conifold [95]. Like fundamental strings they fall to the bottom of the conifold (corresponding to the IR of the field theory), where they have non-vanishing tension. But while F-strings are dual to confining strings, D-strings are interpreted as global solitonic strings in the dual cascading $SU(M(k + 1)) \times SU(Mk)$ gauge theory.

Thus the warped deformed conifold naturally incorporates a supergravity description of the supersymmetric Goldstone mechanism. Below we review the supergravity dual of a pseudoscalar Goldstone boson, as well as its superpartner, a massless scalar glueball [95]. In the gauge theory they correspond to fluctuations in the phase and magnitude of the baryonic condensates, respectively.

The Goldstone mode

A D1-brane couples to the three-form field strength $F_3$, and therefore we expect a four-dimensional pseudo-scalar $p(x)$, defined so that $*_4 dp = \delta F_3$, to experience monodromy around the D-string.

The following ansatz for a linear perturbation of the KS solution

$$\delta F^{(3)} = *_4 dp + f_2(\tau) \, dp \wedge dg^5 + f'_2(\tau) \, dp \wedge d\tau \wedge g^5.$$ (9.1)
The normalizable solution of this equation is given by \[95\]

where \(f\) is discussed by Papadopoulos and Tseytlin \[96\]. The metric, dilaton and \(\epsilon\) vacua remain unchanged. This can be shown to satisfy the linearized supergravity equations \[95\], provided that \(\delta \star_4 dp = 0\), i.e. \(p(x)\) is massless, and \(f(\tau)\) satisfies

\[
\frac{d}{d\tau} \left[ K^4 \sinh^2 \tau f_2^2 \right] + \frac{8}{9K^2} f_2 = \left( \frac{g_4 M^4}{3\epsilon^{4/3}} \right) (\tau \coth \tau - 1) \left( \cosh \tau - \frac{\tau}{\sinh^2 \tau} \right).
\]

The normalizable solution of this equation is given by \[95\]

\[
f_2(\tau) = -\frac{2c}{K^2 \sinh^2 \tau} \int_0^\tau dx h(x) \sinh^2 x,
\]

where \(c \sim \epsilon^{4/3}\). We find that \(f_2 \sim \tau\) for small \(\tau\), and \(f_2 \sim \tau e^{-2\tau/3}\) for large \(\tau\).

As we have remarked above, the \(U(1)\) baryon number symmetry acts as \(A_i \rightarrow e^{i\alpha} A_i, \quad B_j \rightarrow e^{-i\alpha} B_j\). The massless gauge field in \(AdS_5\) dual to the baryon number current originates from the RR 4-form potential \[80,97\]:

\[
\delta C(4) \sim \omega_3 \wedge \hat{A}.
\]

The zero-mass pseudoscalar glueball arises from the spontaneous breaking of the global \(U(1)_B\) symmetry \[98\], as seen from the form of \(\delta F_5\) in \[9.1\], which contains a term \(\sim \omega_3 \wedge dp \wedge d\tau\) that leads us to identify \(\hat{A} \sim dp\).

If \(N\) is an integer multiple of \(M\), the last step of the cascade leads to a \(SU(2M) \times SU(M)\) gauge theory coupled to bifundamental fields \(A_i, B_j\) (with \(i,j = 1,2\)). If the \(SU(M)\) gauge coupling were turned off, then we would find an \(SU(2M)\) gauge theory coupled to \(2M\) flavors. In this \(N_f = N_c\) case, in addition to the usual mesonic branch there exists a baryonic branch of the quantum moduli space \[99\]. This is important for the gauge theory interpretation of the KS background \[92,98\]. Indeed, in addition to mesonic operators \((N_i)_{ij}^2 \sim (A_i B_j)_{ij}^2\), the IR gauge theory has baryonic operators invariant under the \(SU(2M) \times SU(M)\) gauge symmetry, as well as the \(SU(2) \times SU(2)\) global symmetry rotating \(A_i, B_j\):

\[
\mathcal{A} \sim \epsilon_{\alpha_1 \cdots \alpha_{2M}} (A_1)^{\alpha_1} (A_2)^{\alpha_2} \cdots (A_M)^{\alpha_M} (A_1)^{\alpha_{M+1}} (A_2)^{\alpha_{M+2}} \cdots (A_M)^{\alpha_{2M}},
\]

\[
\mathcal{B} \sim \epsilon_{\alpha_1 \cdots \alpha_{2M}} (B_1)^{\alpha_1} (B_2)^{\alpha_2} \cdots (B_M)^{\alpha_M} (B_1)^{\alpha_{M+1}} (B_2)^{\alpha_{M+2}} \cdots (B_M)^{\alpha_{2M}}.
\]

These operators contribute an additional term to the usual mesonic superpotential:

\[
W = \lambda (N_{ij})_{ij}^2 (N_{ij}) + X \left( \det((N_{ij}))^2 - \Lambda_2^{AB} - \Lambda_2^{AM}\right),
\]

where \(X\) can be understood as a Lagrange multiplier. The supersymmetry-preserving vacua include the baryonic branch:

\[
X = 0; \quad N_{ij} = 0; \quad \Lambda_2^{AM} = -\Lambda_2^{AM}.
\]

\[
\delta F^{(5)} = (1 + \star) i F_3 \wedge B_2 = (\star_4 dp - \frac{c^{4/3}}{6K^2} h(\tau) dp \wedge d\tau \wedge g^3) \wedge B_2,
\]

where \(f_2 = df_2/d\tau\), falls within the general class of supergravity backgrounds discussed by Papadopoulos and Tseytlin \[96\]. The metric, dilaton and \(B^{(2)}\)-field remain unchanged. This can be shown to satisfy the linearized supergravity equations \[95\], provided that \(\delta \star_4 dp = 0\), i.e. \(p(x)\) is massless, and \(f_2(\tau)\) satisfies

\[
\frac{d}{d\tau} \left[ K^4 \sinh^2 \tau f_2^2 \right] + \frac{8}{9K^2} f_2 = \left( \frac{g_4 M^4}{3\epsilon^{4/3}} \right) (\tau \coth \tau - 1) \left( \cosh \tau - \frac{\tau}{\sinh^2 \tau} \right).
\]

The normalizable solution of this equation is given by \[95\]
where the $SO(4)$ global symmetry rotating $A_i, B_j$ is unbroken. In contrast, this global symmetry is broken along the mesonic branch $N_{ij} \neq 0$. Since the supergravity background of [92] is $SO(4)$ symmetric, it is natural to assume that the dual of this background lies on the baryonic branch of the cascading theory. The expectation values of the baryonic operators spontaneously break the $U(1)$ baryon number symmetry $A_k \rightarrow e^{i\alpha}A_k, B_j \rightarrow e^{-i\alpha}B_j$. The KS background corresponds to a vacuum where $|A| = |B| = \Lambda_{2M}^2$, which is invariant under the exchange of the $A$’s with the $B$’s accompanied by charge conjugation in both gauge groups. This gives a field theory interpretation to the $I$-symmetry of the warped deformed conifold background. As noted in [98], the baryonic branch has complex dimension one, and it can be parametrized by $\xi$ as follows

$$A = i\xi \Lambda_{2M}^2, \quad B = \frac{i}{\xi} \Lambda_{2M}^2. \quad (9.8)$$

The pseudo-scalar Goldstone mode must correspond to changing $\xi$ by a phase, since this is precisely what a $U(1)_B$ symmetry transformation does.

Thus the non-compact warped deformed conifold exhibits a supergravity dual of the Goldstone mechanism due to breaking of the global $U(1)_B$ symmetry [95, 98]. On the other hand, if one considered a warped deformed conifold throat embedded in a flux compactification, $U(1)_B$ would be gauged, the Goldstone boson $p(x)$ would combine with the $U(1)$ gauge field to form a massive vector, and therefore in this situation we would find a manifestation of the supersymmetric Higgs mechanism [95].

The Scalar Zero-Mode

By supersymmetry the massless pseudoscalar is part of a massless $N = 1$ chiral multiplet, and therefore there must also be a massless scalar mode and corresponding Weyl fermion, with the scalar corresponding to changing $\xi$ by a positive real factor. This scalar zero-mode comes from a metric perturbation that mixes with the NSNS 2-form potential.

The warped deformed conifold preserves the $\mathbb{Z}_2$ interchange symmetry $I$. However, the pseudo-scalar mode we found breaks this symmetry: from the form of the perturbations (9.1), we see that $\delta F^{(5)}$ is even under the interchange of $(\theta_1, \phi_1)$ with $(\theta_2, \phi_2)$, while $F^{(5)}$ is odd; similarly $\delta F^{(5)}$ is odd while $F^{(5)}$ is even. Therefore, the scalar mode must also break the $I$ symmetry because in the field theory it breaks the symmetry between expectation values of $|A|$ and of $|B|$. The necessary translationally invariant perturbation that preserves the $SO(4)$ but breaks the $I$ symmetry is given by the following variation of the NSNS 2-form and the metric:

$$\delta B_2 = \chi(\tau) \, dg^5, \quad \delta G_{13} = \delta G_{24} = \lambda(\tau), \quad (9.9)$$

where, for example $\delta G_{13} = \lambda(\tau)$ means adding $2\lambda(\tau) g^{13} g^{31}$ to $d\ell^2_{10}$. To see that these components of the metric break the $I$ symmetry, we note that

$$(e_1)^2 + (e_2)^2 - (e_1)^2 - (e_2)^2 = g^1 g^3 + g^3 g^1 + g^2 g^4 + g^4 g^2. \quad (9.10)$$
Defining $\lambda(\tau) = h^{1/2} K \sinh(\tau) z(\tau)$ one finds [95] that all the linearized supergravity equations are satisfied provided that

$$\frac{((K \sinh(\tau))^2 z')'}{(K \sinh(\tau))^2} = \left(2 + \frac{8}{9} \frac{1}{K^6} - \frac{4}{3} \frac{\cosh(\tau)}{K^3}\right) \frac{z}{\sinh(\tau)^2},$$

(9.11)

and

$$\chi' = \frac{1}{2} g_s M z(\tau) \frac{\sinh(2\tau) - 2\tau}{\sinh^2 \tau}.$$  (9.12)

The solution of (9.11) for the zero-mode profile is remarkably simple:

$$z(\tau) = s \left(\frac{\tau \coth(\tau) - 1}{\sinh(2\tau) - 2\tau} \right)^{1/3},$$

(9.13)

with $s$ a constant. Like the pseudo-scalar perturbation, the large $\tau$ asymptotic is again $z \sim \tau e^{-2\tau/3}$. We note that the metric perturbation has the simple form $\delta G_{13} \sim h^{1/2} [\tau \coth(\tau) - 1]$. The perturbed metric $d\tilde{s}^2_6$ differs from the metric of the deformed conifold (8.3) by

$$\sim (\tau \coth \tau - 1)(g^1 g^3 + g^2 g^4 + g^2 g^3 + g^4 g^3),$$

(9.14)

which grows as $\ln r$ in the asymptotic radial variable $r$.

The scalar zero-mode is actually an exact modulus: there is a one-parameter family of supersymmetric solutions which break the $I$ symmetry but preserve the $SO(4)$ (an ansatz with these properties was found in [96], and its linearization agrees with (9.9)). These backgrounds, the resolved warped deformed conifolds, will be reviewed in the next section. We add the word resolved because both the resolution of the conifold, which is a Kähler deformation, and these resolved warped deformed conifolds break the $I$ symmetry. In the dual gauge theory turning on the $I$ breaking corresponds to the transformation $A \to (1 + s)A$, $B \to (1 + s)^{-1}B$ on the baryonic branch. Therefore, $s$ is dual to the $I$ breaking parameter of the resolved warped deformed conifold.

The presence of these massless modes is a further indication that the infrared dynamics of the cascading $SU(M(k + 1)) \times SU(Mk)$ gauge theory, whose supergravity dual is the warped deformed conifold, is richer than that of the pure glue $N = 1$ supersymmetric $SU(M)$ theory. The former incorporates a Goldstone supermultiplet, which appears due to the $U(1)_B$ symmetry breaking, as well as solitonic strings dual to the D-strings placed at $\tau = 0$ in the supergravity background.

**Massive Glueballs**

Let us comment on massive normal modes of the warped deformed conifold. These can be found by studying linearized perturbations with four-dimensional momentum $k_\mu$, and looking for the eigenvalues of $-k_\mu^2 = m^2_1$ at which the resulting equations of motion admit normalizable solutions. Some early results on the massive glueball spectra were obtained in [100, 101]. More recently, several families of such massive radial excitations which arise from a subset of the deformations...
contained in the PT ansatz were discussed in [102]. Interestingly, in all cases the mass-squared grows quadratically with the mode number \(n\):

\[
m^2_n = A n^2 + \mathcal{O}(n).
\] (9.15)

The quadratic dependence on \(n\), which is characteristic of Kaluza-Klein theory, is a general feature of strongly coupled gauge theories that have weakly curved 10-d gravity duals (see [103] for a discussion). It was also observed that the coefficient \(A\) of the \(n^2\) term is approximately universal in the sense that the values it takes for different towers of excitations are numerically very close to each other [102].

The towers of massive glueball states based on the pseudoscalar Goldstone boson and its scalar superpartner were recently studied in [104]. The necessary generalization of the scalar ansatz (9.9) to non-zero 4-d momentum is

\[
\delta B^{(2)} = \chi(x, \tau) \, dg_5 + \partial_\mu \sigma(x, \tau) \, dx^\mu \wedge g_5^4 ,
\] (9.16)

\[
\delta G_{13} = \delta G_{24} = \lambda(x, \tau).
\] (9.17)

A gauge equivalent way of writing (9.16) is

\[
\delta B^{(2)} = (\chi - \sigma) \, dg_5^4 - \sigma' \, d\tau \wedge g_5^4.
\] (9.18)

A gauge equivalent way of writing (9.16) is

\[
\delta B^{(2)} = (\chi - \sigma) \, dg_5^4 - \sigma' \, d\tau \wedge g_5^4.
\] (9.18)

Such an ansatz is more general than the generalized PT ansatz used in [102] in that it contains an extra function, \(\sigma\). After some transformations we find that the linearized supergravity equations of motion reduce to two coupled equations that determine the glueball masses:

\[
\ddot{z}'' + \frac{2}{\sinh^2 \tau} \, \dot{z} + \tilde{m}^2 \frac{I(\tau)}{K^2(\tau)} \, \dot{z} = \tilde{m}^2 \frac{9}{4 \cdot 2^{2/3}} K(\tau) \, \tilde{w} ,
\] (9.19)

\[
\ddot{w}'' - \frac{\cosh^2 \tau + 1}{\sinh^2 \tau} \, \dot{w} + \tilde{m}^2 \frac{I(\tau)}{K^2(\tau)} \, \dot{w} = \frac{16}{9} K(\tau) \, \ddot{z} ,
\] (9.20)

where

\[
\dot{z} = h^{-1/2} \lambda(\tau) , \quad \dot{w} = \frac{\epsilon^{4/3}}{g_s M \alpha'} K^5 \sinh(\tau)^2 \sigma' ,
\] (9.21)

and \(\chi\) is determined by the solution for \(\sigma\). Here the dimensionless eigenvalue \(\tilde{m}^2\) is related to the mass-squared through

\[
\tilde{m}^2 = m_4^2 \frac{2^{2/3} (g_s M \alpha')^2}{6 \epsilon^{4/3}} .
\] (9.22)

These coupled equations were solved numerically in [104] yielding radial excitation spectra of the asymptotic form (9.15) with the coefficients of quadratic terms close to those found in [102].

Note that the scale of these glueball mass-squared \(m_4^2\), calculated in the limit \(g_s M \gg 1\), is parametrically lower than the confining string tension (8.19). Using (9.22) and (8.17), we find that the coefficient \(A\) of the \(n^2\) term is

\[
A \sim \frac{T_s}{g_s M} .
\] (9.23)

Thus, for radial excitation numbers \(n \ll \sqrt{g_s M}\), these modes are much lighter than the string tension scale, and therefore much lighter than all glueballs with spin.
2. Such anomalously light bound states appear to be special to gauge theories that stay very strongly coupled in the UV, such as the cascading gauge theory; they do not appear in asymptotically free gauge theories. Therefore, the anomalously light low-spin glueballs could perhaps be used as a special “signature” of gauge theories with weakly curved gravity duals, if they are realized in nature.

10. The Baryonic Branch

Since the global baryon number symmetry $U(1)_B$ is broken by expectation values of baryonic operators, the spectrum contains the Goldstone boson found above. The zero-momentum mode of the scalar superpartner of the Goldstone mode leads to a Lorentz-invariant deformation of the background which describes a small motion along the baryonic branch. In this section we shall extend the discussion from linearized perturbations around the warped deformed conifold solution to finite deformations, and describe the supergravity backgrounds dual to the complete baryonic branch. These are the resolved warped deformed conifolds, which preserve the $SO(4)$ global symmetry but break the discrete $I$ symmetry of the warped deformed conifold.

The full set of first-order equations necessary to describe the entire moduli space of supergravity backgrounds dual to the baryonic branch, also called the resolved warped deformed conifolds, was derived and solved numerically in [105] (for a further discussion, see [106]). This continuous family of supergravity solutions is parameterized by the modulus of $\xi$ (the phase of $\xi$ is not manifest in these backgrounds). The corresponding metric can be written in the form of the Papadopoulos-Tseytlin ansatz [96] in the string frame:

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + e^x ds_{M^2}^2 = h^{-1/2} dx_{1,3}^2 + \sum_{i=1}^{6} G_i^2,$$

where

$$G_1 \equiv e^{(x+g)/2} e_1,$$

$$G_2 \equiv \frac{\cosh \tau + a}{\sinh \tau} e^{(x+g)/2} e_2 + \frac{e^g}{\sinh \tau} e^{(x-g)/2} (\epsilon_2 - ae_2),$$

$$G_3 \equiv e^{(x-g)/2} (\epsilon_1 - ae_1),$$

$$G_4 \equiv \frac{e^g}{\sinh \tau} e^{(x+g)/2} e_2 - \frac{\cosh \tau + a}{\sinh \tau} e^{(x-g)/2} (\epsilon_2 - ae_2),$$

$$G_5 \equiv e^{x/2} v^{-1/2} d\tau,$$

$$G_6 \equiv e^{x/2} v^{-1/2} g^5.$$

These one-forms describe a basis that rotates as we move along the radial direction, and are particularly convenient since they allow us to write down very simple expressions for the holomorphic $(3,0)$ form

$$\Omega = (G_1 + iG_2) \wedge (G_3 + iG_4) \wedge (G_5 + iG_6).$$
and the fundamental (1, 1) form

\[
J = \frac{i}{2} \left[ (G_1 + iG_2) \wedge (G_1 - iG_2) + (G_3 + iG_4) \wedge (G_3 - iG_4) + (G_5 + iG_6) \wedge (G_5 - iG_6) \right].
\]  

(10.7)

While in the warped deformed conifold case there was a single warp factor \(h(\tau)\), now we find several additional functions \(x(\tau), g(\tau), a(\tau), v(\tau)\). The warp factor \(h(\tau)\) is deformed away from (8.17) when \(|\xi| \neq 1\).

The background also contains the fluxes

\[
F^{(2)} = h_1 (e_1 \wedge e_2 + e_1 \wedge e_2) + \chi (e_1 \wedge e_2 - e_1 \wedge e_2) + h_2 (e_1 \wedge e_2 - e_1 \wedge e_1),
\]

(10.8)

\[
F^{(3)} = -\frac{1}{2} g_5 \wedge \left[e_1 \wedge e_2 + e_1 \wedge e_2 - h (e_1 \wedge e_2 - e_1 \wedge e_1)\right] - \frac{1}{2} d\tau \wedge \left[b' (e_1 \wedge e_1 + e_2 \wedge e_2)\right],
\]

(10.9)

\[
\tilde{F}^{(5)} = \tilde{F}^{(5)} + *_{10} F^{(5)},
\]

(10.10)

\[
\tilde{F}^{(5)} = - (h_1 + h_2) e_1 \wedge e_2 \wedge e_1 \wedge e_2 \wedge e_3,
\]

(10.11)

parameterized by functions \(h_1(\tau), h_2(\tau), b(\tau)\) and \(\chi(\tau)\). In addition, since the 3-form flux is not imaginary self-dual for \(|\xi| \neq 1\) (i.e. \(*_6 G_3 \neq iG_3\)), the dilaton \(\phi\) now also depends on the radial coordinate \(\tau\).

The functions \(a\) and \(v\) satisfy a system of coupled first order differential equations [105] whose solutions are known in closed form only in the warped deformed conifold and the Chamseddine-Volkov-Maldacena-Nunez (CVMN) [108] limits. All other functions \(h, x, g, h_1, h_2, b, \chi, \phi\) are unambiguously determined by \(a(\tau)\) and \(v(\tau)\) through the relations

\[
h = \gamma U^{-2} \left(e^{-2\phi} - 1\right), \quad \gamma = 2^{10/3} (g, M a')^2 e^{-8/3},
\]

(10.12)

\[
e^{2x} = \frac{(bC - 1)^2}{4(aC - 1)^2} e^{2\phi + 2\phi} (1 - e^{2\phi}) ,
\]

(10.13)

\[
e^{2g} = -1 - a^2 + 2a C , \quad b = \frac{a}{S},
\]

(10.14)

\[
h_2 = \frac{e^{2\phi} (bC - 1)}{2S} , \quad h_1 = -h_2 C,
\]

(10.15)

\[
\chi' = a(b - C)(aC - 1) e^{2(\phi - g)},
\]

(10.16)

\[
\phi' = \frac{(C - b) (aC - 1)^2}{(bC - 1) S} e^{-2g} ,
\]

(10.17)

where \(C \equiv -\cosh \tau, S \equiv -\sinh \tau\), and we require \(\phi(\infty) = 0\). In writing these equations we have specialized to the baryonic branch of the cascading gauge theory by imposing appropriate boundary conditions at infinity [106], which guarantee that the background asymptotes to the warped conifold solution [107]. The full two parameter family of SU(3) structure backgrounds discussed in [105] also includes the
Gauge–String Dualities and Some Applications

CVMN solution [108], which however is characterized by linear dilaton asymptotics that are qualitatively different from the backgrounds discussed here. The baryonic branch family of supergravity solutions is labelled by one real “resolution parameter” $U$ [106]. While the leading asymptotics of all supergravity backgrounds dual to the baryonic branch are identical to those of the warped deformed conifold, terms subleading at large $\tau$ depend on $U$. As required, this family of supergravity solutions preserves the $SU(2) \times SU(2)$ symmetry, but for $U \neq 0$ breaks the $\mathbb{Z}_2$ symmetry $I$.

On the baryonic branch we can consider a transformation that takes $\xi$ into $\xi^{-1}$, or equivalently $U$ into $-U$. This transformation leaves $v(\tau)$ invariant and changes $a(\tau)$ as follows

$$a \rightarrow -\frac{a}{1 + 2a \cosh \tau}.$$  \hfill (10.18)

It is straightforward to check that $ae^{-g}$ is invariant while $(1 + a \cosh \tau)e^{-g}$ changes sign. This transformation also exchanges $e^0 + a^2 e^{-g}$ with $e^{-g}$ and therefore it is equivalent to the exchange of $(\theta_1, \phi_1)$ and $(\theta_2, \phi_2)$ involved in the $I$-symmetry.

The baryonic condensates have been calculated on the string theory side of the duality [109] by identifying the Euclidean D5-branes wrapped over the deformed conifold, with appropriate gauge fields turned on, as the appropriate object dual to the baryonic operators in the sense of gauge/string duality [98]. Similarly to the case of baryonic operators on the warped resolved conifold discussed in section 7, the field corresponding to a baryon is recognized as the (semi-classical) equivalent of $e^{-S_{D5}(r)}$, where $S_{D5}(r)$ is the action of a Euclidean D5-brane wrapping the Calabi-Yau coordinates up to the radial coordinate cut-off $r$. The different baryon operators $A, B$, and their conjugates $A, B$, are distinguished by the two possible D5-brane orientations, and the two possible $\kappa$-symmetric choices for the world volume gauge field that has to be turned on inside the D5-brane.

This identification is legitimate since in the cascading theory, which is near-AdS in the UV, equation (7.19) holds modulo powers of $\ln r$ [110, 111]. Due to the absence of sources for baryonic operators we again have $\varphi_0 = 0$, and $e^{-S_{D5}(r)}$ gives the dimensions of the baryon operators, and the values of their condensates.

In contrast to the simpler case of the warped resolved conifold, a world-volume gauge bundle $F^{(2)} = dA^{(1)}$ is required by $\kappa$-symmetry in this case, which leads to the conditions [112] that $\mathcal{F} \equiv F^{(2)} + B^{(2)}$ be a $(1, 1)$-form, and that

$$\frac{1}{2!} J \wedge J \wedge \mathcal{F} - \frac{1}{3!} \mathcal{F} \wedge \mathcal{F} = g \left( \frac{1}{3!} J \wedge J \wedge J - \frac{1}{2!} J \wedge \mathcal{F} \wedge \mathcal{F} \right).$$  \hfill (10.19)

Here $g$ would simply be a constant if the internal manifold were Calabi-Yau, but since we are dealing with a generalized Calabi-Yau with fluxes, $g$ becomes coordinate dependent, a function of $\tau$ in our case.

The $SU(2) \times SU(2)$ invariant ansatz for the gauge potential is given by

$$A^{(1)} = \zeta(\tau) g^5,$$  \hfill (10.20)
which together with (10.19) implies that $\zeta$ has to satisfy the differential equation

$$
\zeta' = \frac{e^z (ga + b)}{v(a - gb)},
$$

(10.21)

where we have defined

$$
a(\zeta, \tau) \equiv e^{-2x} [e^{2x} + h^2 \sinh^2(\tau) - (\zeta + \chi)^2],
b(\zeta, \tau) \equiv 2e^{-x/a} \sinh(\tau) [a(\zeta + \chi) - h(1 + acosh(\tau))].
$$

(10.22)

Using the explicitly known Killing spinors of the baryonic branch backgrounds (or from an equivalent argument starting from the Dirac-Born-Infeld equations of motion) one can show [109] that

$$
g = -e^{-x} + \frac{h}{2} \sinh(\tau) = e^\phi \sqrt{1 - e^{2\phi}},
$$

(10.23)

which determines $\zeta$ and thus the action of the Euclidean D5-brane:

$$
S_{D5} \sim \int d\tau e^{-\phi} \sqrt{\det G + F} = \int d\tau \frac{e^\phi e^{3x} \sqrt{1 + g^2 (a^2 + b^2)}}{v(a - gb)}. 
$$

(10.24)

The dimension of the baryon operators in the KS background can be extracted from the divergent terms of the D5-brane action as a function of the radial cut-off $r$, which leads to

$$
\Delta(r) = r \frac{dS_{D5}(r)}{dr} = \frac{27g_s^2 M^3}{16\pi^2} (\ln(r))^2 + \mathcal{O}(\ln(r)).
$$

(10.25)

To compare this with the cascading gauge theory, we use the fact that the baryon operators of the $SU(M(k + 1)) \times SU(Mk)$ theory have the schematic form $A \sim (A_1 A_2)^{(k+1)M/2}$ and $B \sim (B_1 B_2)^{(k+1)M/2}$, with appropriate contractions described in [98]. For large $k$, their dimensions are $\Delta(k) \approx 3Mk(k+1)/4$. If we remember that the radius at which the $k$-th Seiberg duality is performed is given by

$$
r(k) \sim e^{2/3} \exp \left( \frac{2\pi k}{3g_s M} \right),
$$

(10.26)

we find that the leading term in the operator dimension agrees with (10.25).

The baryon expectation values as a function of $U$ can be computed by evaluating the finite terms in the action (10.24). The baryonic condensates calculated in this fashion [109] satisfy the important condition that $\langle A \rangle \langle B \rangle = \text{const.}$ along the whole baryonic branch. This leads to a precise relation between the baryonic branch modulus $|\xi|$ in the gauge theory [98] and the modulus $U$ in the dual supergravity description.

Furthermore, pseudoscalar perturbations around the warped deformed conifold background are seen explicitly to shift the phase of the baryon expectation value, through the Chern-Simons term in the D5-brane action, as required for consistency.
11. Cosmology in the Throat

A promising framework for realizing cosmological inflation [113] in string theory is D-brane inflation (for reviews and more complete references, see [114]). The original proposal [115] was to consider a D3-brane and a $\overline{\text{D3}}$-brane separated by some distance along the compactified dimensions, and with their world volumes spanning the 4 observable coordinates $x^\mu$. From the 4-d point of view, the distance $r$ between the branes is a scalar field that is identified with the inflaton. However, in flat space the Coulomb potential $\sim 1/r^4$ is typically too steep to support slow-roll inflation. An ingenious proposal to circumvent this problem [116] is to place the brane-antibrane pair in a warped throat region of a flux compactification, of which the warped deformed conifold is an explicitly known and ubiquitous [117] example.

The $\overline{\text{D3}}$-brane breaks supersymmetry and experiences potential $\frac{2T_3}{h(0)} \frac{\delta h(0, r)}{h(0)^2}$ attracting it to the bottom of the conifold, $\tau = 0$. Its energy density there, $\frac{2T_3}{h(0)} \frac{\delta h(0, r)}{h(0)^2}$, plays the important role of “uplifting” the negative 4-d cosmological constant to a positive value in the KKLT model for moduli stabilization [118]. When a mobile D3-brane is added, it perturbs the background warp factor. The energy density of the D3-brane becomes

$$V(r) = \frac{2T_3}{h(0)} + \frac{\delta h(0, r)}{h(0)} \approx \frac{2T_3}{h(0)} - 2T_3 \frac{\delta h(0, r)}{h(0)^2}, \quad (11.1)$$

where $\delta h(0, r)$ is the perturbation of the warped factor at the position of the $\overline{\text{D3}}$-brane $r = 0$, caused by the D3-brane at radial coordinate $r$. For a D3-brane far from the tip of the throat, with radial coordinate $r \gg \frac{\varepsilon^2}{3}$, $\delta h(0, r) \approx 27/(32\pi^2 T_3 r^4)$ [119]. Thus, the potential assumes the form [116, 120] (note that the definition of $r^2$ here differs by a factor of $3/2$ from that in [119])

$$V(r) = \frac{2T_3}{h(0)} - \frac{27}{16\pi^3 h(0)^2} r^2. \quad (11.2)$$

Thus, the force on the D3-brane is suppressed by a small factor $h(0)^{-2}$ compared to the force in unwarped space used in the original model [115]. We recall that

$$h(0) = a_0 (g_s M \alpha')^2 2^{2/3} 3^{-8/3} e^{-8/3}, \quad a_0 \approx 0.71805. \quad (11.3)$$

In flux compactifications containing a long KS throat, $h(0)$ is of order $e^{8K/3g_s M}$ [121]. The flattening of the brane-antibrane potential by exponential warping is an important factor in constructing realistic brane inflation scenarios.

There are possibilities other than a $\overline{\text{D3}}$-brane at the bottom of the throat for creating a slow-varying potential for the mobile D3-brane. As pointed out in [106], if the throat is taken to be a resolved warped deformed conifold, which was reviewed in section 10 then the potential experienced by the D3-brane is

$$V(\tau) = T_3 h^{-1}(\tau) (e^{-\phi(\tau)} - 1). \quad (11.4)$$

The first term comes from the Born-Infeld term and has a factor of $e^{-\phi(\tau)}$; the second term, originating from the interaction with the background 4-form $C_{0123}$, does not have this factor. For the KS solution ($U = 0$), $\phi(\tau) = 0$; therefore, the
potential vanishes and the D3-brane may be located at any point on the deformed conifold. For $U \neq 0$ we may use (10.12) to write

$$V(\tau) = \frac{T_3}{\gamma} \frac{U^2}{e^{-\phi(\tau)} + 1}.$$  \hspace{1cm} (11.5)

Since $\phi(\tau)$ is a monotonically increasing function, the D3-brane is attracted to $\tau = 0$.

For large enough $\tau$, the potential becomes

$$V(\tau) = \frac{T_3}{\gamma} \left[ \frac{1}{2} \left( \frac{3U^4}{256} (4\tau - 1) e^{-4\tau/3} + \ldots \right) \right].$$  \hspace{1cm} (11.6)

Since $e^{-4\tau/3} \sim \epsilon^{8/3} \tau^{-4}$, this is rather similar to the brane-antibrane potential (11.2), and has the additional feature that the resolution parameter $U$ may be varied.

In a complete treatment, $U$ should be determined by the details of the compactification.

**Cosmic Strings**

In addition to suppressing the D3-brane potential, the large value of $h(0)$ is responsible for the viability of cosmic strings in flux compactifications containing long warped throats. Cosmic strings with Planckian tensions are ruled out by the CMB spectrum and other astrophysical observations (for a review, see [122]). The current constraints on cosmic string tension $\mu$ suggest $G\mu \ll 10^{-7}$, and they continue to improve. In traditional models where the string scale is not far from the 4-d Planck scale, the tension of a fundamental string far from the warped throat, $1/(2\pi\alpha')$, badly violates this constraint. However, at the bottom of the throat the tension is reduced to $1/(2\pi\alpha' \sqrt{h(0)})$ which can be consistent with the constraint [123]. It is remarkable that such a cosmic string has a dual description as a confining string in the cascading gauge theory dual to the throat. This shows that the phenomenon of color confinement may have implications reaching far beyond the physics of hadrons.

Type IIB flux compactifications with long warped throats may contain a variety of species of cosmic strings. $q$ fundamental strings at the bottom of the throat may form a bound state, which is described by a D3-brane wrapping a 2-sphere within the 3-sphere [124]. A D-string at the bottom of the throat is dual to a certain solitonic string in the gauge theory [95]. Furthermore, $p$ D-strings and $q$ F-strings can bind into a $(p, q)$ string [125]. Networks of such $(p, q)$ cosmic strings have various characteristic features in their evolution and interaction probabilities which could distinguish them from other cosmic string models. Importantly, such strings are copiously produced during the brane-antibrane annihilation that follows the brane inflation [126]. However, in models where some D3-branes remain at the bottom of the throat after inflation (for example, [106]), long cosmic strings cannot exist because they break on the D-branes [122]. Thus, non-observation of cosmic strings would not rule out D-brane inflation.
Compactification Effects

The potentials (11.2), (11.6) are very flat for large \( r \), approaching a constant that arises due to D-term breaking of supersymmetry. However, additional contributions to the potential that arise due to moduli stabilization effects tend to destroy this flatness and generally render slow-roll inflation impossible. Indeed, in the compactified setting, the contribution of the brane-antibrane interaction to the 4-d “Einstein frame” potential is

\[
V_D(\rho, r) = \frac{V(r)}{U^2(\rho, r)} ,
\]

The extra factor comes from the DeWolfe-Giddings Kähler potential [127] which depends both on the volume modulus, \( \rho \), and the D3-brane position \( z_\alpha, \alpha = 1, 2, 3 \):

\[
\kappa^2 K(\rho, \bar{\rho}, z_\alpha, \bar{z}_\alpha) = -3 \log [\rho + \bar{\rho} - \beta k(z_\alpha, \bar{z}_\alpha)] \equiv -3 \log U .
\]

Here \( k(z_\alpha, \bar{z}_\alpha) \) denotes the Kähler potential of the Calabi-Yau manifold, which in the throat reduces to

\[
k = \frac{3}{2} \left( \sum_{i=1}^{4} |z_i|^2 \right)^{2/3} = r^2 ,
\]

where we ignored the deformation \( \varepsilon \). The normalization constant \( \beta \) in (11.8) may be expressed as

\[
\beta \equiv \frac{\sigma_0}{3 M_P} ,
\]

where \( 2\sigma_0 \equiv 2\sigma_0(0) = \rho_*(0) + \bar{\rho}_*(0) \) is the stabilized value of the Kähler modulus when the D3-brane is near the tip of the throat. For a D3-brane far from the tip, we may ignore the deformation of the conifold and use

\[
U(\rho, r) = \rho + \bar{\rho} - \beta r^2 .
\]

The \( r \)-dependence from the factor \( U^{-2}(\rho, r) \) spoils the flatness of the potential even in the region where \( V(r) \) is very flat.

The complete inflaton potential

\[
V_{\text{tot}} = V_F(\rho, z_\alpha) + V_D(\rho, r)
\]

also includes the F-term contribution whose standard expression in \( \mathcal{N} = 1 \) supergravity is

\[
V_F = e^{\kappa K} \left[ D_{\Sigma} W \kappa^{2(\Sigma)} D_{\Sigma} W - 3 \kappa^2 W W \right] , \quad \kappa^2 = M_P^{-2} \equiv 8\pi G ,
\]

where \( \{ Z^\Sigma \} \equiv \{ \rho, z_\alpha; \alpha = 1, 2, 3 \} \) and \( D_{\Sigma} W = \partial_{\Sigma} W + \kappa^2 (\partial_{\Sigma} K) W \). The superpotential \( W \) has the structure

\[
W(\rho, z_\alpha) = W_0 + A(z_\alpha)e^{-a\rho} , \quad a \equiv \frac{2\pi}{n} .
\]

where the second, nonperturbative term arises either from strong gauge dynamics on a stack of \( n > 1 \) D7-branes or from Euclidean D3-branes (with \( n = 1 \) ).
We assume that either sort of brane supersymmetrically wraps a four-cycle in the warped throat that is specified by a holomorphic embedding equation \( f(z_{\alpha}) = 0 \). The warped volume of the four-cycle governs the magnitude of the nonperturbative effect, by affecting the gauge coupling on the D7-branes (equivalently, the action of Euclidean D3-branes) wrapping this four-cycle. The presence of a D3-brane gives rise to a perturbation to the warp factor, and this leads to a correction to the warped four-cycle volume. This correction depends on the D3-brane position and is responsible for the prefactor \( A(z_{\alpha}) \) \([128]\). In \[119\], D3-brane backreaction on the warped four-cycle volume was calculated leading to a simple formula

\[
A(z_{\alpha}) = A_0 \left( \frac{f(z_{\alpha})}{f(0)} \right)^{1/n}.
\]

The canonical inflaton \( \phi \) is proportional to \( r \), the radial location of the D3-brane. Using \((11.12)\) to compute the slow-roll parameter

\[
\eta \equiv M_P^2 \frac{V_{,\varphi\varphi}}{V},
\]

one finds

\[
\eta = \frac{2}{3} + \Delta \eta(\phi),
\]

where the \( 2/3 \) arises from the Kähler potential \((11.8), (11.11)\), and \( \Delta \eta \) from the variation of \( A(z_{\alpha}) \). A simple model studied in \[120\] was based on the Kuperstein embedding of a stack of D7-branes

\[
f(z_1) = \mu - z_1 = 0.
\]

Explicit calculation of the full inflaton potential in this model \[120,129\] shows that it is possible to fine-tune the parameters to achieve an inflection point in the vicinity of which slow-roll inflation is possible.

Other supergravity and string theory constructions where such Inflection Point Inflation may be achieved were proposed in \[130–133\]. While such models are fine-tuned, and the initial conditions have to be chosen carefully to prevent the field from running through the inflection point with high speed (see, however, \[132,134\] for possible ways to circumvent this problem) the model is fairly predictive. In particular, the spectral index \( n_s \) is around 0.93 in the limit that the total number of e-folds is large during inflation. This fact may render this model distinguishable from others by more precise observations of the CMB.

12. Summary

Throughout its history, string theory has been intertwined with the theory of strong interactions. The AdS/CFT correspondence \[1–3\] has succeeded in making precise connections between conformal 4-dimensional gauge theories and superstring theories in 10 dimensions. This duality leads to a multitude of dynamical predictions about strongly coupled gauge theories. While many of these predictions are
difficult to check, recent applications of methods of exact integrability to planar $\mathcal{N} = 4$ SYM theory have produced some impressive tests of the correspondence for operators with high spin. When extended to theories at finite temperature, the correspondence serves as a theoretical laboratory for studying a novel state of matter: a gluonic plasma at very strong coupling. This appears to have surprising connections to the new state of matter, sQGP, which was observed at RHIC and will be further studied at the LHC.

Breaking symmetries in the AdS/CFT correspondence is important for bringing it closer to the real world. Some of the supersymmetry may be broken by considering D3-branes at conical singularities; the case of the conifold is discussed in detail in these lectures. In this set-up, breaking of gauge symmetry typically leads to a resolution of the singularity. The associated breaking of global symmetry leads to the appearance of Goldstone bosons and global strings. Extensions of the gauge-string duality to confining gauge theories provide new geometrical viewpoints on such important phenomena as chiral symmetry breaking and dimensional transmutation, which are encoded in the dual smooth warped throat background. Embedding of the throat into flux compactifications of string theory allows for an interesting interplay between gauge-string duality and models of particle physics and cosmology. For example, D3-branes rolling in the throat might model inflation while various strings attracted to the bottom of the throat may describe cosmic strings. All of this raises hopes that the new window into strongly coupled gauge theory opened by the discovery of gauge-string dualities will one day lead to new striking connections between string theory and the real world.

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