Supersymmetric Solutions to the Strong CP Problem

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Abstract

It is argued that in the context of supersymmetry, the Strong CP Problem is most naturally seen as an aspect (particularly severe) of the whole complex of flavor-violating and CP-violating problems of supersymmetry. It is shown that certain approaches to solving these flavor problems also allow simple solutions to the Strong CP Problem. The idea of “flavor alignment” suggested by Nir and Seiberg allows not only flavor violation to be controlled but supersymmetric contributions to the theta parameter to be made acceptably small. Another approach to the flavor-violation problem, namely low-energy supersymmetry breaking, allows another class of solutions to the Strong CP Problem to be viable.
1 Introduction

An alternative to the Peccei-Quinn solution to the Strong CP Problem is the idea that $\theta$ is small due to an approximate or spontaneously broken CP invariance of the Lagrangian. There are a number of facts that commend this approach.

On the negative side is the fact that axions have not yet been observed, and that experimental constraints leave only a relatively narrow window in parameter space where the axion might live. Moreover, the axion solution requires the existence of a continuous global symmetry that is exact (except for the QCD anomaly) to a remarkably high degree. This is problematic from the point of view of both quantum gravity and superstrings.

On the positive side there are facts which lend credibility to the idea that CP may be a spontaneously broken symmetry. First, theories with few parameters sometimes conserve CP automatically. This appears to be the case with superstrings, where it has been argued that four-dimensional CP invariance is actually a local symmetry. Second, in theories with low energy supersymmetry, the soft supersymmetry-breaking terms have to approximately conserve CP in order not to give excessive electric dipole moments.

Indeed, in supersymmetry there is a whole cluster of problems related to the smallness of flavor-changing and CP-violating effects that are unsolved. It would seem natural to regard the Strong CP Problem as being among them and to seek a common approach to all these problems, rather than treating the Strong CP Problem in isolation as the Peccei-Quinn approach does.

The attractive idea that a (spontaneously broken) CP invariance is responsible for the smallness of $\theta$ can be implemented in two kinds of models, which we shall call Type I and Type II. In Type I models the quark mass matrix has large CP-violating phases in it, which give rise to a Kobayashi-Maskawa phase, but has a determinant that is real at tree level because of some flavor symmetry. In Type II models the quark mass matrix itself is real at tree level, so that the CP violation seen in the Kaon system must be accounted for by some milliweak or superweak force.

In this paper we are interested in models of Type I. The first models of this type generally had CP invariance broken at the weak scale by the relative phase of the VEVs of two or more $SU(2)$-doublet Higgs. It is easy to arrange by some flavor symmetry a pattern of Yukawa couplings so that this phase
appears in the quark mass matrix, but not (at tree level) in its determinant. An example is the following triangular form for the up and down quark mass matrices, which can be the result of a simple family symmetry:

\[
M_d = \begin{pmatrix}
  \lambda_{11}v_0 & \lambda_{12}v_1 & \lambda_{13}v_2 \\
  0 & \lambda_{22}v_0 & \lambda_{23}v_1 \\
  0 & 0 & \lambda_{33}v_0
\end{pmatrix},
M_u = \begin{pmatrix}
  \lambda'_{11}v_0^* & 0 & 0 \\
  \lambda'_{21}v_1^* & \lambda'_{22}v_0^* & 0 \\
  \lambda'_{31}v_2^* & \lambda'_{32}v_1^* & \lambda'_{33}v_0^*
\end{pmatrix}.
\]

(1)

Note that the determinant of the full quark mass matrix \(\text{diag}(M_d, M_u)\) only depends on \(|v_0|^2\) and does not see the relative phases of \(v_0\), \(v_1\), and \(v_2\). Since, by the assumed CP invariance of the lagrangian, the \(\lambda_{ij}\) and \(\lambda'_{ij}\) are real, \(\theta = 0\) at tree level. However, the relative phases of the \(v_n\) does lead to a non-trivial KM phase, \(\delta\).

There are two problems with this type of model. The first, is that there is more than one doublet contributing to \(M_d\) (or \(M_u\)) which leads to flavor-changing processes mediated by scalar exchange.\(^{10}\) The second is that CP is broken spontaneously at the weak scale, which leads to unacceptable cosmological domain walls.

Another approach was suggested by A. Nelson in reference 3 and further developed in Ref. 4. In this class of models, which we will call Nelson models, there are, in addition to the three families of quarks, a vectorlike set of quarks and mirror quarks that have superlarge \(SU(2)_L\)-singlet masses. These mirror quarks mix with the three families through superlarge and complex VEVs, which also break CP spontaneously. Because these complex VEVs appear in the off-diagonal block which couples the usual families to the vectorlike quarks, the determinant of the complete quark mass matrix (including both light and superheavy states) remains real, but when the superheavy quarks are integrated out the resulting light-quark mass matrices have a KM phase.

This kind of model has neither of the problems that characterized the earlier models. There is only one Higgs doublet (or in supersymmetric versions only \(H_u\) and \(H_d\)), and CP is broken at superlarge scales, so that domain walls can be inflated away. This approach can also be implemented in supersymmetry.\(^{9,11}\) However, it was pointed out by Dine, Leigh, and Kagan\(^{12}\) that unless “universality” of the soft supersymmetry-breaking terms is satisfied to a high degree of exactness — that is, unless the squark masses are highly degenerate and the “A terms” are very nearly proportional to the Yukawa terms — the down-quark mass matrix and the gluino mass will pick
up unacceptably large phases at one loop from the diagrams shown in Fig. 1. But, as emphasized by the same authors, there is no \textit{a priori} reason to expect such exact or nearly exact “universality” to hold. Indeed, in general it would violate the ’tHooft criterion of naturalness, and in fact does not seem to hold in superstring models.

The upshot is that the same non-universality that is at the root of all the well-known flavor and CP problems of supersymmetry also creates a problem for the $\theta$ parameter. In this sense, it is natural to regard the Strong CP Problem as another (and particularly severe) aspect of the general problems of flavor changing and CP violation in supersymmetry$^8$ and to seek a common approach to solving all of them.

In this paper we point out that certain approaches to solving the general flavor-changing problems of supersymmetry proposed in recent years also allow one to construct acceptable Type I models for solving the strong CP Problem.

One possible solution to the flavor-changing problems of supersymmetry, suggested by Nir and Seiberg$^{13}$, and Nir and Rattazzi$^{14}$ is that the squark and quark mass matrices are “aligned” because of an abelian family symmetry. In section 2 we show that this idea allows acceptable Type I models to be constructed. If the fields that spontaneously break the family symmetry also break CP, the CP violation in the squark and quark mass matrices can also be aligned in such a way that it cancels in the lowest-order contributions to $\theta$. These models have a strong similarity to the older Type I models of Ref. 2, except that they have a minimal number of Higgs doublets and have CP broken at large scales as in the Nelson models. In section 3, we observe that Nelson models avoid the problems pointed out by Dine, Leigh, and Kagan if supersymmetry is broken at low scales.

## 2 Flavor Alignment Models

The flavor-alignment approach to the Strong CP Problem in supersymmetry is similar in spirit to the type of model$^2$ we illustrated in Eq. 1. Indeed, we will consider the following toy model for simplicity: the effective down and up quark mass matrices have the forms
\[ M_d = \frac{1}{M} \begin{pmatrix}
\lambda_{11} v'(S_0) & \lambda_{12} v'(S_1) & \lambda_{13} v'(S_2) \\
0 & \lambda_{22} v'(S_0) & \lambda_{23} v'(S_1) \\
0 & 0 & \lambda_{33} v'(S_0)
\end{pmatrix}, \tag{2} \]

and

\[ M_u = \frac{1}{M} \begin{pmatrix}
\lambda'_{11} v(S_0) & 0 & 0 \\
\lambda'_{21} v(S_1) & \lambda'_{22} v(S_0) & 0 \\
\lambda'_{31} v(S_2) & \lambda'_{32} v(S_1) & \lambda'_{33} v(S_0)
\end{pmatrix}, \tag{3} \]

where \( S_0, S_1, \) and \( S_2 \) (and their barred counterparts) are singlets under the Standard Model gauge group, but carry some abelian family quantum number. The VEVs of the \( S_n \) are roughly of the same order as the scale \( M \), assumed to be large compared to the weak scale, that appears in the expressions for \( M_d \) and \( M_u \). The family symmetry is responsible for the triangular form of the mass matrices. These singlet scalars, \( S_n \), not only break the family symmetry, but are assumed to break CP spontaneously as well because of a non-trivial relative phase among their VEVs.

As in the model described in Eq. 1, the determinant of the full quark mass matrix \( \text{diag}(M_d, M_u) \) does not “see”, at tree level, the relative phases of the \( \langle S_n \rangle \), so that \( \theta_{\text{tree}} = 0 \), whereas a non-trivial KM phase does result. But, unlike the model of Eq. 1, there is only one doublet contributing to \( M_d \) and one to \( M_u \), so that that FCNC from Higgs exchange is avoided.\(^{10}\) Moreover, CP is broken not by the doublets at the Weak scale but by the singlets at a very high scale, so that the resulting cosmological domain walls may be inflated away. One sees, then, that this kind of model combines certain features of the older models of Ref. 2 and the Nelson models of Ref. 3 and 4. We shall see shortly how this kind of model solves the problems posed by the diagrams in Fig. 1 that were pointed out by Dine, Leigh, and Kagan, but first we must go into more detail.

The matrices given in Eqs. 2 and 3 involve non-renormalizable operators. These are conceived to arise from integrating out heavy vectorlike states in a way now to be described.

Consider that in addition to the ordinary down-type quarks, \( d_n \), and \( d^c_n \), \( n = 1, 2, 3 \), a vectorlike set of down-quarks, \( D_n \) and \( D^c_n \), \( n = 1, 2, 3 \) exists. These new quarks are singlets under \( SU(2)_L \). In addition, let there be a \( U(1)_H \times Z_3 \) family symmetry. The first, second, and third generation quarks
(both of the $d$ and of the $D$) have $U(1)_H$ charges $-1$, $0$, and $+1$ respectively. The $d^n$ all pick up a phase $e^{2\pi i/3}$ under the $Z_3$ symmetry, while the other down quarks are invariant under it. The down quark masses come from the following set of Higgs: an $H_d$ which is an $SU(2)_L$ doublet and neutral under the family symmetries; a singlet $T$, which gets a superlarge VEV and is neutral under both the standard model gauge interactions and the family symmetries; and three singlets, $S_0$, $S_1$, $S_2$, which have charges $0$, $1$, and $2$ under $U(1)_H$ and pick up phase $e^{-2\pi i/3}$ under $Z_3$. The VEVs of the $S_n$, which are assumed to be of the same order (roughly) as that of $T$, not only break the abelian family symmetries, but their relative phases are assumed to spontaneously break CP as well.

With this set of fields the down-type quarks have a $6 \times 6$ mass matrix of the form

$$
\begin{pmatrix}
0 & 0 & 0 \\
\lambda_{12} \langle S_1 \rangle & \lambda_{13} \langle S_2 \rangle & \lambda_{14} \langle H_d \rangle \\
\lambda_{15} \langle S_0 \rangle & \lambda_{16} \langle S_1 \rangle & \lambda_{17} \langle H_d \rangle \\
\lambda_{18} \langle S_0 \rangle & \lambda_{19} \langle S_1 \rangle & \lambda_{20} \langle H_d \rangle \\
0 & 0 & 0 \\
\lambda_{22} \langle S_0 \rangle & \lambda_{23} \langle S_1 \rangle & \lambda_{24} \langle T \rangle \\
\lambda_{25} \langle S_0 \rangle & \lambda_{26} \langle S_1 \rangle & \lambda_{27} \langle T \rangle \\
0 & 0 & 0 \\
\lambda_{32} \langle S_0 \rangle & \lambda_{33} \langle S_1 \rangle & \lambda_{34} \langle T \rangle \\
\lambda_{35} \langle S_0 \rangle & \lambda_{36} \langle S_1 \rangle & \lambda_{37} \langle T \rangle \\
\end{pmatrix}
$$

which we shall abbreviate as

$$
(d^c | D) \begin{pmatrix}
0 \\
\lambda_1 \langle H_d \rangle \\
\lambda_2 \langle H_d \rangle \\
\lambda_3 \langle H_d \rangle \\
\lambda_4 \langle T \rangle \\
\lambda_5 \langle T \rangle \\
\lambda_6 \langle T \rangle \\
\end{pmatrix} \begin{pmatrix}
(d^c) \\
(D) \\
\end{pmatrix}.
$$

There is an entirely analogous structure for up quarks, with in addition to $u_n$ and $u^c_n$ a set of $U_n$ and $U^c_n$. The $U(1)_H$ charges are again $-1$, $0$ and $+1$ for the three families, and the $u^c$ are assumed to pick up a phase $e^{-2\pi i/3}$ under $Z_3$. There are in addition to the above named Higgs, an $H_u$, which is a doublet neutral under family symmetry, and a set of $S_n$, $n = 0, 1, 2$, whose quantum numbers are opposite to those of $S_n$. The $6 \times 6$ mass matrix of the up quarks will have a form analogous to that in Eq. 4. It is clear that upon integrating out the heavy states one is left with an effective $3 \times 3$ mass matrix for the light down quarks that is given by the familiar “see-saw” formula $M_d = -H T^{-1} S$, and similarly for the up mass matrix. These have just the forms given in Eqs. 2 and 3.
As already stated, CP violation is supposed to arise spontaneously as a result of non-trivial relative phases among the VEVs of the \( S_n \) and \( S_n \). For concreteness, we can imagine that the superpotential of the Higgs that get superlarge VEVs has the \( U(1)_H \times Z_3 \)-invariant form

\[
W = (T^2 - M_T^2) X_T + (S_0 \bar{S}_0 - M_0^2) X_0
+ (S_1 \bar{S}_1 - M_1^2) X_1 + (S_2 \bar{S}_2 + M_2^2) X_2
+ (S_3^2 + \bar{S}_3^2)
+ (S_2 S_0^2/M^2 + Z^2) Y + (\bar{S}_2 S_1^2 \bar{S}_0^2/M^2 + Z^2) \bar{Y}.
\]

This is not the only possibility, but is simple and illustrates the essential idea. One finds upon minimizing the (supersymmetric) scalar potential that arises from this that

\[
\langle T \rangle = M_T, \quad \langle S_0 \rangle = \langle \bar{S}_0 \rangle = M_0, \quad \langle S_1 \rangle = \langle \bar{S}_1 \rangle = M_1,
\]

and

\[
\langle S_2 \rangle = \langle \bar{S}_2 \rangle = i M_2,
\]

where the \( M \)'s are all real parameters by the CP invariance of the lagrangian. The phase in the VEV of \( S_2 \) comes from the plus sign in the term involving \( X_2 \) in \( W \). We shall discuss corrections to these phases later. Notice also that when the scalar component of \( W \) is evaluated at the minimum it is real, as is necessary if the A parameters are to be real at tree level in supergravity theories.

The physical, CP-violating order parameter must be invariant under the family symmetry \( U(1)_H \times Z_3 \). The lowest-dimension such operator is \( S_2 \bar{S}_1 S_0 \) or equivalently \( S_2 S_1^2 S_0 \). This is the combination that in fact comes into the expression for the leading contribution to \( \bar{\theta} \). Because it involves (as can be seen from Eq. 2) several small flavor-changing Yukawa interactions, these contributions will end up being quite suppressed, as will now be shown.

As noted, the mass matrix of Eq. 4 has three superheavy eigenstates and three light eigenstates, with the effective mass matrix of the three light states being given by the “see-saw” formula

\[
M_d = -H T^{-1} S,
\]

where the notation is defined in Eq. 5. It is convenient to define the combination \( s \equiv T^{-1} S = -H^{-1} M_d \). Note that \( s, S, \) and \( M_d \) all have the same triangular form. If we assume for simplicity that all the (non-zero) elements of \( H \) are of order \( \langle H_d \rangle \equiv v' \), then \( s_{mn} \sim (M_d)_{mn}/v' \). Using the Wolfenstein parameter, \( \lambda \), one can then write
\[s_{33} \sim m_b/v',
\]
\[s_{23} \sim s_{22} \sim \lambda^2 m_b/v',
\]
\[s_{13} \sim s_{12} \sim \lambda^3 m_b/v',
\]
\[s_{11} \sim \lambda^4 m_b/v'.\]

(8)

One is now in a position to estimate the contributions to \(\bar{\theta}\) coming from the dangerous diagrams in Fig. 1. From what has already been said, it is obvious that they will be proportional to \(\text{Im}(s_{13}s_{21}^*s_{22}s_{12}^*)\), which is of order \(\lambda^{10}\). Because of small denominators the leading contribution to \(\bar{\theta}\) will be lower order in \(\lambda\) than this, as we shall see.

The contribution to the down-quark mass matrix coming from Fig. 1 (a) is proportional to the \(A\) parameter and to the gluino mass, so that one may ignore the part coming from superheavy states circulating in the loop as these will be suppressed by \((m_{\text{SUSY}}/M_{\text{GUT}})^2\). Let us therefore block-diagonalize the \(6 \times 6\) matrix given in Eq. 4, which we will call \(\mathcal{M}\), to separate the light and superheavy states. This is done by \(\mathcal{M} \rightarrow U_L \mathcal{M} U_R^\dagger\), where the unitary matrices are given by

\[
U_L \simeq \begin{pmatrix} I & -H(I + ss^\dagger)^{-1}T^{-1} \\ T^{-1}(I + ss^\dagger)^{-1}H & I \end{pmatrix},
\]

\[
U_R = \begin{pmatrix} (I + s^\dagger s)^{-\frac{1}{2}} & 0 \\ 0 & (T(I + ss^\dagger)T)^{-\frac{1}{2}} \end{pmatrix} \begin{pmatrix} I & -s^\dagger \\ s & I \end{pmatrix}.
\]

(9)

This gives, when applied to the matrix in Eq. 5, \(M_d = -Hs\), as it should.

The same transformations applied to the squark fields will also separate the light and superheavy squarks, since the superheavy elements of the squark mass matrices are just given by the SUSY-invariant terms. In the original basis, the SUSY-breaking \(LL\) and \(RR\) squark mass terms are just diagonal matrices whose elements are all of order \(m_0^2\). They are diagonal because of the abelian family symmetry. Of course, loop effects and gravity effects can induce family-changing elements of these matrices, but these will involve insertions of the VEVs of the \(S_n\) divided by either the Planck scale or the scale of \(\langle T \rangle\). Thus the contributions to \(\bar{\theta}\) so produced are smaller than the ones we shall consider.
The $LR$ squark mass term has, in the original basis, the same form as the mass term given in Eq. 5, namely

$$m^2_{LR} = A_0 \begin{pmatrix} 0 & \tilde{H} \\ \tilde{S} & \tilde{T} \end{pmatrix}. \quad (10)$$

Here, $\tilde{H}$, $\tilde{S}$, and $\tilde{T}$ are matrices which have the same structure as $H$, $S$, and $T$, because of family symmetry, namely $\tilde{H}$ is diagonal and proportional to $\langle H_d \rangle$, $\tilde{T}$ is diagonal and proportional to $\langle T \rangle$, and $\tilde{S}$ is triangular and proportional to $\langle S_n \rangle$. We can define by analogy to $s$ a dimensionless matrix $\tilde{s} \equiv \tilde{T}^{-1} \tilde{S}$. In a model in which the inter-family hierarchy was completely explained by family symmetry, one would expect that $\tilde{s}$ would exhibit the same hierarchy as $s$, so that $\tilde{s}_{mn}$ would be of the same order in the Wolfenstein parameter as $s_{mn}$. In the toy model we are discussing, there is not enough family symmetry to completely explain the hierarchy. The smallness of the non-vanishing off-diagonal elements of $s$ can be explained by supposing that $\langle S_1 \rangle$ and $\langle S_2 \rangle$ are small compared to $\langle S_0 \rangle$. Then one expects that the non-vanishing off-diagonal elements of $\tilde{s}$ are also small and of the same order in $\lambda$. However, in this toy model the hierarchy among the diagonal elements of $s$ is not explained, since they all come from the VEV of $S_0$. No symmetry principle thus demands that the diagonal elements of $\tilde{s}$ must have a similar hierarchy. Actually, this will not turn out to matter, since even if only the off-diagonal elements of $\tilde{s}$ are suppressed, a sufficient suppression of $\tilde{\theta}$ will result. But we will assume, since it is simpler, and since one expects it to be true in a theory where family symmetry explains the fermion mass hierarchy, that $\tilde{s}_{mn} \sim s_{mn}$.

After applying the transformations $U_L$ and $U_R$ to the squark-mass matrices one finds that they have the forms

$$m^2_{LL,\text{light}} = m^2_{LL(d)},$$

$$m^2_{RR,\text{light}} = (I + s^\dagger s)^{-\frac{1}{2}} m^2_{RR(d^c)} (I + s^\dagger s)^{-\frac{1}{2}}$$

$$\quad + (I + s^\dagger s)^{-\frac{1}{2}} s^\dagger m^2_{RR(D^c)} s(I + s^\dagger s)^{-\frac{1}{2}}, \quad (11)$$

$$m^2_{LR,\text{light}} = (-\tilde{H}s - H (I + ss^\dagger)^{-1} T^{-1} \tilde{T}[\tilde{s} - s])(I + s^\dagger s)^{-\frac{1}{2}}.$$

If we take the lowest order in the small quantities $s$ and $\tilde{s}$, $m^2_{LL,\text{light}}$ and
\(m^2_{RR,\text{light}}\) are diagonal matrices, and the matrix \(m^2_{LR,\text{light}}\) is equal to \((-\tilde{H}s - HT^{-1}\tilde{T}[\tilde{s} - s])\), which has the same triangular form as \(s\) and \(\tilde{s}\). Moreover, in the supersymmetric basis in which we are working, the gluino couplings are flavor-diagonal. Thus, at lowest order in \(s\), the result of doing the loop in Fig. 1 (a) is just to give a contribution to the light down-quark mass matrix which (like the tree level term) has the same triangular form as \(s\). Hence, the determinant of the quark mass matrix is still real at one loop if we ignore terms of higher order in \(s\) and \(\tilde{s}\). Of course, this trivially had to be the case, since the CP-violating invariant, as noted above, is \(s_{31}^* s_{32} s_{22}^* s_{21}\). But if one now keeps higher order terms in \(s\) and \(\tilde{s}\), one finds a number of contributions to \(\theta\) all of which are of order

\[
\delta\theta \sim \frac{8}{34\pi} \frac{A_0 m_g}{m_0^2} \left( \frac{s_{21} s_{31} s_{32}}{s_{22}} \right) \sim 2 \times 10^{-2} \lambda^6 \left( \frac{A_0 m_g}{m_0^2} \right) \left( \frac{m_b}{v'} \right)^2, \tag{12}
\]

or

\[
\bar{\theta} \sim 10^{-9} \left( \frac{A_0 m_g}{m_0^2} \right) \tan^2 \beta. \tag{13}
\]

We have assumed that the phases in \(s\) are of order unity, as is necessary if the KM phase is to be of order unity.

The contribution to \(\bar{\theta}\) that comes from Fig. 1 (b) is smaller than that just given. It is typically of order \(\frac{A_0 m_g}{m_0^2} \text{Im}(s_{31} s_{32}^* s_{22} s_{21}^*) \sim \lambda^{10}\).

In addition, one must investigate the phase of \(\langle S_0 \rangle\) and \(\langle \tilde{S}_0 \rangle\). First, there are possible gravitational effects. For example, there can be a term in the superpotential for the \(S_n\) that has the form \(\epsilon_G S_2 S_1^2 S_0 X_0 / M_{Pl}^2\), where \(\epsilon_G\) is some suppression factor that depends on the nature of the Planck-scale physics and is presently uncalculable. The \(F_{X_0} = 0\) equation then gives a phase to \(\langle S_0 \rangle\) that is of order \(\epsilon_G \langle S_n \rangle^2 / M_{Pl}^2\). The phase of \(\langle S_0 \rangle\) contributes directly at tree level to \(\bar{\theta}\). Thus one requires that the family symmetries and CP are broken at scales less than or of order \(10^{14.5}\text{GeV} \epsilon_G^{-\frac{1}{2}}\). This is perfectly consistent with a GUT-scale breaking, especially since \(\epsilon_G\) may be small.

Another contribution to the phase of \(\langle S_0 \rangle\) comes from supersymmetry breaking. In particular, there will be a supersymmetry-breaking term of the form \(A(s_2 S_1^2 S_0 Y) / M^2\) in the scalar potential for \(S_0\) in addition to the term \(|S_0^2 - M_0^2|^2\). In the supersymmetric limit \(Y\) has vanishing VEV, so that this
term will not matter. However, when supersymmetry is broken $Y$ gets a VEV of order $A$. Therefore there will be induced a linear term for $S_0$ that contains a CP-violating phase. The contribution to the phase of $\langle S_0 \rangle \langle S_0 \rangle$ (which appears in the determinant of the full quark mass matrix and thus in $\theta$) is of order $m_{SUSY}^2 / M^2$, which is easily smaller than $10^{-9}$.

The model we have presented above is far from being unique. The triangular form has been chosen for simplicity, but there are many other forms that would admit the same kind of solution of the strong CP problem. For example, by having the non-zero elements of the $s$ matrix be the 11, 22, 33, 12, 13, and 32, with the phase being in the 13 element, the leading contribution to $\theta$ is suppressed by order $\lambda^8$. There are patterns that would allow a suppression by $\lambda^{10}$ but these seem less realistic. (For example, having the non-zero elements being 11, 33, 31, 13, 32, and 23.) In any event, it would seem to be generally the case that $\theta$ in this kind of model should be not far below $10^{-11}$.

A challenge for any completely satisfactory theory of flavor is to find a family symmetry that aligns the quarks and squarks in such a way that both the Strong CP Problem and the other flavor problems of supersymmetry are solved at the same time.

3 Nelson Models and Low-energy Supersymmetry Breaking

The basic idea of the models proposed in Refs. 4 and 5 is very simple. Let $d_i + d_i^c$ be the usual three families of down quark, and $D_I + D_I^c$ be a set of mirror down quarks which are $SU(2)_L$ singlets. (Such a set of particles can arise in $SU(5)$, for example from having three sets of $10 + \mathbf{5}$ and some additional sets of $\mathbf{5} + \mathbf{5}$.) The Yukawa interactions of the down quarks are assumed to have the following form

$$
\mathcal{L}_{\text{down mass}} = (d_i, D_I) \left( \begin{array}{cc}
\lambda_{ij} \langle H \rangle & 0 \\
\lambda_{ij} \langle S \rangle & \lambda_{ij} \langle T \rangle
\end{array} \right) \left( \begin{array}{c}
d^c_i \\
D^c_I
\end{array} \right).
$$

(14)

$H$ is just the Higgs doublet of the Standard Model. (For the moment we are describing a non-supersymmetric model.) Its vacuum expectation value does not have a physically meaningful phase. (It can be changed by a global
weak hypercharge rotation.) $S_m$ and $T$ are singlets with VEVs of the same order and much larger than the Weak scale. (It is natural but not necessary to take this to be near the unification scale.) $\langle T \rangle$ is assumed not to break CP, while $\langle S_m \rangle$ have relative phases that do violate CP spontaneously. By the CP invariance of the lagrangian, the Yukawa couplings $\lambda$ are real. Here it should be noted, in contrast to the models discussed in the last section, no family symmetry need distinguish among the $d_i$ or among the $S_n$, but some symmetry does distinguish the $d$ from the $D$.

The up quark mass matrix has the simple $3 \times 3$ form $u_iu_j^c \langle H^* \rangle$. Thus it is easy to see that the complete quark mass matrix, including the heavy states, has determinant proportional to $|\langle H \rangle|^6 \langle T \rangle^n$ (where there are $n$ mirror quarks). Since this is real, and $\theta_{QCD}$ is real by the CP invariance of the Lagrangian, $\theta$ vanishes at tree level. On the other hand, when the superheavy states are integrated out, one is left with three families of light quarks, which have a non-trivial KM phase (coming from the VEVs of the $S_m$). The model is thus indistinguishable from the KM model at low energy, but has no Strong CP Problem.$^{3,4}$

The severe difficulty pointed out by Dine, Leigh, and Kagan$^{12}$ is that the diagrams in Fig. 1 are murderous here if there are order unity violations of “universality” in the supersymmetry-breaking terms. In fact, $\theta$ is expected to be of order $\alpha_s/4\pi$ in that case.

This is not the case, however, if SUSY is broken at low scales.$^{15}$ For simplicity, consider an $SU(5)$ model where supersymmetry breaking is communicated from some hidden sector by a singlet field, $S$, to a “messenger sector” consisting of a $\mathbf{5} + \overline{\mathbf{5}}$, which have masses of order 100TeV. From the messenger sector, the supersymmetry breaking is communicated to the known particles and their superpartners by $SU(3) \times SU(2) \times U(1)$ gauge interactions. In addition, let there be, as discussed above, a set of mirror fields, $D_I + D_I^c$, that are contained in some other set of $\mathbf{5} + \overline{\mathbf{5}}$ representations, denoted $\mathbf{5}_I + \overline{\mathbf{5}}_I$. These are assumed to have superlarge masses. That the down-quark mass matrix have the form given in Eq. 14, in particular that there be no $d_i D_I^c H_d$ coupling, requires that some symmetry distinguish the $D_I^c$ from the $d_I^c$. (They have the same $SU(3) \times SU(2) \times U(1)$ quantum numbers, however.)

It is easy to see that in such a model, the diagrams in Figs. 1 and 2 are not dangerous. Simply integrating out the superheavy mirror quarks, $\mathbf{5}_I + \overline{\mathbf{5}}_I$, leads to a low energy theory that is nothing but the MSSM together with the
low-energy SUSY breaking sector and messenger sector. This effective low-energy theory has a KM phase (from the phases of $\langle S_m \rangle$ as discussed above), but no tree-level $\vec{\theta}$. The non-universality of the soft SUSY breaking terms is then small enough to render the one-loop contributions in Fig. 1 harmless. In particular, since the flavor symmetry of the Standard Model gauge interactions is broken only by Yukawa interactions, the non-universality of the soft supersymmetry-breaking terms involving the ordinary quarks is suppressed by powers of $(m_q/\Lambda)^2$, where $m_q$ is a light quark mass and $\Lambda \sim 100$TeV is the scale of SUSY breaking.

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Figure Captions

**Fig. 1:** In supersymmetric models where $\theta$ vanishes at tree level due to a spontaneously broken CP invariance, in general diagram (a) gives too large a phase to the mass of the gluino, and diagram (b) gives too large a contribution to the phase of $\text{det } M_q$. 
Fig. 1 (a).

Fig. 1 (b).