Coupled plasmon-phonon modes in a two-dimensional electron gas in the presence of Rashba effect

W. Xu\(^1\), M.P. Das\(^1\) and L.B. Lin\(^2\)

\(^1\)Department of Theoretical Physics
Research School of Physical Sciences and Engineering
Australian National University, Canberra, ACT 0200, Australia

\(^2\) Department of Physics, Sichuan University
Chengdu - 610064, Sichuan, People’s Republic of China

PACS: 71.45.Gm, 71.38.+i

Elementary electronic excitation is studied theoretically for a two-dimensional electron gas in the presence of spin orbit (SO) interaction induced by Rashba effect. We find that in such a system, coupled plasmon-phonon excitation can be achieved via intra- and inter-SO electronic transitions. As a result, six branches of the coupled plasmon-phonon oscillations can be observed. The interesting features of these excitation modes are analyzed.
Progress made in realizing spin polarized electronic systems has led to recent proposals of the novel electronic devices, such as spin-transistors [1], spin-waveguides [2], spin-filters [3], quantum computers [4], etc. One important aspect in the field of “spintronics” is to investigate electronic systems with finite spin splitting at zero magnetic field. It is known that in semiconductor-based two-dimensional electron gas (2DEG) systems, the zero-field spin splitting can be realized from inhomogeneous surface electric field induced by the presence of the heterojunction. This feature is known as the Rashba effect [5]. In these systems the strength of the spin-splitting and spin-orbit interaction (SOI) can be altered by applying a gate voltage [6] or varying sample growth parameters [7]. At present most of the published work is focused on electronic and transport properties of the 2DEGs in the presence of SOI. In order to understand these novel material systems more deeply and to explore their further applications to the practical devices, it is necessary for us to examine the roles which electronic many-body effects and phonons can play in a 2DEG with SO coupling.

Here we consider an interacting 2DEG where SOI and electron-phonon (e-p) interaction are present. Our aim is to obtain coupled plasmon-phonon excitation modes for this system. For a typical 2DEG in the xy-plane in narrow gap semiconductors, such as InGaAs/InAlAs quantum wells, the noninteracting Schrödinger equation including the lowest order of SOI can be solved analytically [2]. Applying the electron wavefunctions to the electron-electron (e-e) interaction Hamiltonian induced by the Coulomb potential, the space Fourier transform of the matrix element for bare e-e interaction is written as

\[ V_{\alpha\beta}(k, q) = V_q F_0(q) G_{\alpha\beta}(k, q). \]  

(1a)

Here, \( \alpha = (\sigma', \sigma) \) with \( \sigma = \pm 1 \) referring to different SOs, \( k = (k_x, k_y) \) is the electron wavevector along the 2D-plane, \( q = (q_x, q_y) \) is the change of \( k \) during a scattering event, \( V_q = 2\pi e^2/\epsilon_\infty q \) with \( \epsilon_\infty \) being the high-frequency dielectric constant, and

\[ G_{\alpha\beta}(k, q) = \frac{1 + A_{kq}}{2} \delta_{\alpha, \beta} + i \frac{B_{kq}}{2} (1 - \delta_{\alpha, \beta}) \]  

(1b)

with \( A_{kq} = (k + q \cos \theta)/|k + q|, B_{kq} = q \sin \theta/|k + q| \), and \( \theta \) being the angle between \( k \) and \( q \). Furthermore, the space Fourier transform of the matrix element for bare e-p interaction can be written as

\[ V_{\alpha}^{ph}(k, q; \Omega) = \sum_{Qz} D_0(\omega_Q, \Omega)|U_{\alpha}(k, Q)|^2, \]  

(2)

where \( Q = (q, q_z) \) is the phonon wavevector, \( \omega_Q \) is the phonon frequency, \( D_0(\omega_Q, \Omega) = 2\hbar \omega_Q/[(\hbar \Omega)^2 - (\hbar \omega_Q)^2] \) is the bare phonon propagator, \( |U_{\alpha}(k, q)|^2 = |W_Q|^2 G_0(q_z) A_{\alpha}(k, q) \) is the square of the e-p interaction matrix element, \( A_{\alpha}(k, q) = (1 + \alpha A_{kq})/2 \) is a spin-dependent element, and \( W_Q \) is the e-p coupling coefficient. It should be noted that in contrast to a conventional 2DEG (C2DEG) for which the bare e-e and bare e-p interactions do not depend
on \( \mathbf{k} \) \([8]\), \( V_{\alpha\beta}(\mathbf{k}, \mathbf{q}) \) and \( V_{\alpha}^{ph}(\mathbf{k}, \mathbf{q}; \Omega) \) for a 2DEG with SOI depend not only on \( \mathbf{q} \) but also on \( \mathbf{k} \), because the spin splitting depends explicitly on \( \mathbf{k} \). In the present study, we consider the case of a narrow quantum well in which only one electronic subband is present. Thus, \( F_0(q) = \int dz_1 \int dz_2 |\psi_0(z_1)|^2|\psi_0(z_2)|^2 \exp(-q |z_1 - z_2|) \) and \( G_0(q_z) = |<0|e^{iq_zz}|0>|^2 \) with \( |0> = \psi_0(z) \) being the electron wavefunction along the growth direction.

From electron energy spectrum obtained by solving the Schrödinger equation, we derive the retarded and advanced Green’s functions for electrons. Using these Green’s functions, \( V_{\alpha\beta}(\mathbf{k}, \mathbf{q}) \) and \( V_{\alpha}^{ph}(\mathbf{k}, \mathbf{q}; \Omega) \) in a diagrammatic self-consistent theory \([9]\) (also see Fig. 1), the effective e-e interaction is given by

\[
V_{\text{eff}}(\mathbf{k}, \mathbf{q}; \Omega) = [V_{\alpha\beta}(\mathbf{k}, \mathbf{q}) + V_{\alpha}^{ph}(\mathbf{k}, \mathbf{q}; \Omega)]\epsilon^{-1}_{\alpha\beta}(\mathbf{k}, \mathbf{q}; \Omega). \tag{3}
\]

Here,

\[
\epsilon_{\alpha\beta}(\mathbf{k}, \mathbf{q}; \Omega) = \delta_{\alpha, \beta} \delta(\mathbf{k}) - [V_{\alpha\beta}(\mathbf{k}, \mathbf{q}) + V_{\alpha}^{ph}(\mathbf{k}, \mathbf{q}; \Omega)]\Pi_{\beta}(\mathbf{k}, \mathbf{q}; \Omega) \tag{4}
\]

is the dielectric function matrix element and

\[
\Pi_{\sigma'\sigma}(\mathbf{k}, \mathbf{q}; \Omega) = \frac{f[E_{\sigma'}(\mathbf{k} + \mathbf{q})] - f[E_{\sigma}(\mathbf{k})]}{\hbar \Omega + E_{\sigma'}(\mathbf{k} + \mathbf{q}) - E_{\sigma}(\mathbf{k}) + i\delta} \tag{5}
\]

is the pair bubble in the absence of e-e coupling with \( f(E) \) being the Fermi-Dirac function. In Eq. (5),

\[
E_{\sigma}(\mathbf{k}) = E_{\sigma}(k) = \hbar^2 k^2 / 2m^* + \sigma \alpha_R k \tag{6}
\]

is the energy spectrum of a 2DEG in the presence of SOI, with \( m^* \) being the electron effective mass and \( \alpha_R \) the Rashba parameter which measures the strength of the SOI.

For a 2DEG with SOI, the effective e-e interaction and dielectric function matrix depend not only on \( \mathbf{q} \) but also on \( \mathbf{k} \), in contrast to a C2DEG. After summing \( \epsilon_{\alpha\beta}(\mathbf{k}, \mathbf{q}; \Omega) \) over \( \mathbf{k} \) and setting 1 = (++), 2 = (−−), 3 = (−+) and 4 = (+−), the dielectric function matrix for a 2DEG with Rashba spin splitting in the presence of e-p scattering is obtained as

\[
\epsilon = \begin{pmatrix}
1 + a_1 + b_1 & 0 & 0 & a_4 \\
0 & 1 + a_2 + b_2 & a_3 & 0 \\
a_1 & a_2 & 1 + a_3 + b_3 & 0 \\
0 & 0 & 0 & 1 + a_4 + b_4
\end{pmatrix} \tag{7}
\]

In Eq. (7), \( a_j = -[V_q F_0(q)/2] B_j(\mathbf{q}, \Omega) \) and \( b_j = -\sum_{q_z} D_0(\omega_Q, \Omega)|W_{q}\rangle |G_0(q_z)B_j(\mathbf{q}, \Omega) \) are induced respectively by e-e and e-p interaction. \( B_j(\mathbf{q}, \Omega) = \sum_{\mathbf{k}} (1 \pm A_{\mathbf{kq}}) \Pi_j(\mathbf{k}, \mathbf{q}; \Omega) \), where upper (lower) case refers to \( j = 1 \) or 4 for intra-SO transitions (\( j = 2 \) or 3 for inter-SO transitions). The determinant of the dielectric function matrix is then given by

\[
|\epsilon| = [(1 + a_1 + b_1)(1 + a_4 + b_4) - a_1 a_4][(1 + a_2 + b_2)(1 + a_3 + b_3) - a_2 a_3] \tag{8}
\]
which results from intra- and inter-SO electronic transitions. Thus, the modes of coupled plasmon-phonon excitation are determined by \( \text{Re} |\epsilon| \rightarrow 0 \).

In the present study, we consider an InGaAs-based 2DEG in which electrons interact strongly with longitudinal optical (LO) phonons through the Fröhlich coupling. For electron interaction with LO-phonons, \( \omega Q \rightarrow \omega _{LO} \) the LO-phonon frequency at long-wavelength limit, \( |W_Q|^2 = 2\pi e^2 \hbar \omega _{LO} (\epsilon _{\infty }^{-1} - \epsilon _s^{-1})/Q^2 \) with \( \epsilon _s \) and \( \epsilon _{\infty } \) being respectively the static and high-frequency dielectric constants. At a long-wavelength (i.e., \( q \ll 1 \)) and a low-temperature (i.e., \( T \rightarrow 0 \)) limit, we have

\[
\text{Re} |\epsilon| \simeq \left[ 1 - \frac{\omega _p^2 \Omega _e^2 - \omega _{TO}^2 (1 - \omega _- - \omega _+)}{\Omega^2 } \right] \left[ 1 - \frac{\omega _p^2 \Omega _e^2 - \omega _{TO}^2 (1 - \omega _- - \omega _+)}{\omega _0/2} \right] \ln \left( \frac{\Omega + \omega _- \Omega - \omega _+}{\Omega - \omega _- \Omega + \omega _+} \right). \tag{9}
\]

Here, the first (second) term on the right-hand side is induced by intra-SO (inter-SO) transitions, \( \omega _{TO} = \sqrt{\epsilon _{\infty }/\epsilon _s} \omega _{LO} \) is the TO-phonon frequency, \( \omega _\pm = 4\alpha R \sqrt{\pi n_\pm/e} \) with \( n_\pm \) being the electron density in the \( \pm \) spin channel, \( \omega _0 = 16\pi n_e \hbar /m^* \), and \( \omega _p = (2\pi e^2 n_e q/\epsilon _{\infty } m^*)^{1/2} \) is the plasmon frequency of a 2DEG in the absence of SOI with \( n_e = n_+ + n_- \) being the total electron density of the system. Moreover, it can be shown that at low-temperature limit, electron density in different SOs is

\[
n_\pm = (n_e/2) \mp (k_\alpha /2\pi) \sqrt{2\pi n_e - k_\alpha^2} \tag{10}
\]

for case of \( n_e > k_\alpha^2 /\pi \) with \( k_\alpha = m^* \alpha R /\hbar^2 \). When \( n_e \leq k_\alpha^2 /\pi \), only spin-down states are occupied by electrons and, therefore, \( n_+ = 0 \) and \( n_- = n_e \).

In the presence of SOI, the collective excitation from a 2DEG can be achieved via electron transitions in different spin channels. From Eq. (9), we see that the coupled plasmon-phonon frequency induced by intra-SO excitation is given by

\[
\Omega _+ = \omega _{LO} + \frac{a}{2} \left( 1 - \frac{\epsilon _{\infty }}{\epsilon _s} \right) \frac{\omega _p^2}{\omega _{LO}} \quad \text{and} \quad \Omega _- = \omega _p \sqrt{\frac{a \epsilon _{\infty }}{\epsilon _s}}, \tag{11}
\]

where \( a = 1 - 2(\omega _- - \omega _+)/\omega _0 \); those induced by inter-SO transitions can be obtained by solving

\[
\ln \left( \frac{\Omega + \omega _- \Omega - \omega _+}{\Omega - \omega _- \Omega + \omega _+} \right) = \frac{\omega _0 \Omega _e^2 - \omega _{TO}^2}{\omega _p^2 \Omega^2 - \omega _{TO}^2}. \tag{12}
\]

Thus, two (four) branches of the coupled plasmon-phonon excitation can be observed via intra-SO (inter-SO) electronic transitions.

Now we present the results of our calculations for InGaAs-based quantum well structures. The material parameters are known [10]:

1) the electron effective mass \( m^* = 0.042 m_e \) with \( m_e \) being the rest-electron mass;
2) the high-frequency and static dielectric constants are respectively \( \epsilon _{\infty } = 12.3 \) and \( \epsilon _s = 14.6 \); and
3) the LO-phonon energy $\hbar \omega_{LO} = 30.9$ meV.

The dependence of coupled plasmon-phonon frequencies induced by intra- (in (a)) and inter-SO (in (b)) excitation on $\omega_p = (2\pi e^2 n_e q/\epsilon_\infty)^{1/2}$ or $q$, on Rashba parameter $\alpha_R$ and on total electron density $n_e$ are shown respectively in Figs. 2 - 4. From these results and from Eqs. (11) and (12), it can be found that at a long-wavelength limit, excitations with frequency about $\omega_{LO}$ can be generated via both intra- ($\Omega_+ \sim \omega_{LO}$) and inter-SO ($\Omega_4 \sim \omega_{LO}$) transitions and they depend very weakly on $q$ and sample parameters (such as $\alpha_R$ and $n_e$). Another mode induced by intra-SO transition, $\Omega_- \sim \omega_p \sim q^{1/2}$, is acoustic-like and depends weakly on $\alpha_R$ and $n_e$. The excitation with frequency about $\Omega_3 \sim \omega_{TO}$ can only be generated via inter-SO transitions and its dependance on $q$, $\alpha_R$ and $n_e$ are negligible, which implies that the TO-phonon mode can be excited via inter-SO transitions in an InGaAs-based 2DEG. Although $\Omega_1$ and $\Omega_2$ induced by inter-SO transitions should, in principle, depend on $q$ via $\omega_p$ (see Eq. (12)), the numerical results shown in Fig. 2 suggest that at long-wavelength limit, over a wide regime of $\omega_p$ or $q$, $\Omega_1 \rightarrow \omega_+ = 4\alpha_R \sqrt{\pi n_e}/\hbar$ and $\Omega_2 \rightarrow \omega_- = 4\alpha_R \sqrt{\pi n_e}/\hbar$ depend very little on $q$ and can differ significantly from the phonon frequencies and from $\omega_p$. This indicates that $\Omega_1$ and $\Omega_2$ are optic-like and they rely very strongly on sample parameters such as $\alpha_R$ and $n_e$ (see Figs. 3 and 4).

The important conclusions drawn from this work are, for a 2DEG with SOI,

1) LO-phonon excitation can be achieved via intra- and inter-SO electronic transitions;
2) TO-phonon mode can only be generated via inter-SO excitation; and
3) $\Omega_1$ and $\Omega_2$ induced by inter-SO excitations are optic-like and very sensitive to sample parameters.

There are many investigations on the coupled plasmon-phonon modes in semiconductor-based 2DEG systems without including SOI [9,11]. In comparing these results with those obtained in our present study (where SOI is present), we note some interesting features in the collective excitation modes, especially those excited through inter-SO transitions. It may be noted that the state-of-the-art material engineering and micro- and nano-fabrication techniques have made it possible to achieve an experimentally observable Rashba spin-splitting in, e.g., InGaAs-based 2DEG systems. Very recent experimental results [12] in this system show that the Rashba parameter $\alpha_R$ can reach up to $3 \sim 4 \times 10^{-11}$ eVm. Our results shown in Figs. 2 - 4 indicate that when $\alpha_R \geq 10^{-11}$ eVm, a significant separation between $\Omega_{1,2}$ induced by inter-SO transition can be achieved.

At present, magneto-transport measurement is a powerful and most popularly used experimental technique to identify Rashba spin-splitting in a 2DEG [6,7]. Using this technique to determine the Rashba parameter and electron density in different spin branches, one needs to apply high magnetic fields at low-temperatures so that Shubnikov-de Hass oscillations are observed. From our investigations we suggest that the Rashba spin splitting can be probed by optical measurements. It should be noted that the dispersion of the coupled plasmon-phonon
energies (or frequencies) is of the same order of magnitude as energy splitting in different spin branches. Together with a fact that it is not so easy to measure the dispersion relation of the coupled plasmon-phonon modes in a 2DEG even without inclusion of SOI, we think it may be difficult to measure optically the dispersion relation of the coupled plasmon-phonon mode in a 2DEG with SOI. In the absence of SOI, the dispersion of the coupled plasmon-phonon mode can only be determined by using techniques, such as grating couplers [13]. However, Raman scattering [14] and ultrafast pump-and-probe experiments [15] have been carried out recently to study coupled plasmon-phonon modes in semiconductor-based 2DEG systems without SOI. These optical experiments conveniently measure optic-like excitation modes. Thus, we propose here coupled plasmon-phonon modes in a spin split 2DEG to be detected optically because most of them are optic-like. In particular, if we can measure $\omega_{\pm} = 4\alpha_R \sqrt{\pi n_{\pm}}/\hbar$ (the magnitude of the frequencies and their separation are of the order of THz), we can determine the Rashba parameter and total electron density of the system. We see from Figs. 2 - 4 that when $\alpha_R \geq 10^{-11}$ eVm, $\Omega_2 - \Omega_1$ is much larger than $\Omega_4 - \Omega_3 \simeq \omega_{LO} - \omega_{TO}$. Because even $\omega_{LO}$ and $\omega_{TO}$ in InGaAs can be resolved in, e.g., Raman spectra [10], $\Omega_{1,2}$ induced by inter-SO transition can be more easily resolved in an optical experiment. Finally we suggest these theoretical predictions require experimental verification.

Acknowledgment: One of us (W. X.) is a Research Fellow of the Australian Research Council. This work is also partly supported by the National Natural Science Foundation of China.
References

[1] B. Datta and S. Das, Appl. Phys. Lett. 56, 665 (1990).
[2] X.F. Wang, P. Vasilopoulos, and F.M. Peeters, Phys. Rev. B 65, 165217 (2002).
[3] T. Koga, J. Nitta, H. Takayanagi, and S. Datta, Phys. Rev. Lett. 88, 126601 (2002).
[4] Y. Ohno, D.K. Young, B. Beschoten, F. Matsukura, H. Ohno, and D.D. Awschalom, Nature 402, 790 (1999).
[5] E.I. Rashba and V.I. Sheka, in Landau Level Spectroscopy, edited by G. Landwehr and E.I. Rashba (North-Holland, Amsterdam, 1991).
[6] J. Nitta, T. Akazaki, H. Takayanagi, and T. Enoki, Phys. Rev. Lett. 78, 1335 (1997).
[7] J. Luo, H. Munekata, F.F. Fang, and P.J. Stiles, Phys. Rev. B 41, 7685 (1990).
[8] See, e.g., T. Ando, A.B. Fowler, and F. Stern, Rev. Mod. Phys. 54, 437 (1982).
[9] See, e.g., R. Jalabert and S. Das Sarma, Phys. Rev. B 40, 9723 (1989).
[10] See, e.g., S. Adachi, Physical Properties of III-V Semiconductor Compounds (John Wiley & Sons, NY, 1992).
[11] See, e.g., F.M. Peeters, X.G. Wu, and J.T. Devreese, Phys. Rev. B 36, 7518 (1987); L. Wendler and R. Pechstedt, J. Phys.: Condens. Matter 2, 8881 (1990); W.H. Backes, F.M. Peeters, F. Brosens, and J.T. Devreese, Phys. Rev. B 45, 8437 (1992); M. Reizer, ibid B 61, 40 (2000).
[12] Y. Sato, T. Kita, S. Gozu, and S. Yamada, J. Appl. Phys. 89, 8017 (2001); D. Grundler, Phys. Rev. Lett. 84, 6074 (2000).
[13] See, e.g., M. Voßebürg, H.G. Roskos, F. Wolter, C. Waschke, and H. Kurz, J. Opt. Soc. Am B 13, 1045 (1996).
[14] See, e.g., P. Brockmann, J.F. Young, P. Hawrylak, and H.M. van Driel, Phys. Rev. B 48, 11423 (1993).
[15] See, e.g., T. Dekorsy, A.M.T. Kim, G.C. Cho, H. Kurz, A.V. Kuznetsov, and A. Förster, Phys. Rev. B 53, 1531 (1996).
FIGURE CAPTIONS

Fig. 1. Effective e-e interaction (double-solid-lines) in the presence of phonon scattering. Here, the dashed-line is the bare e-e interaction, the dotted-line is induced by electron-phonon scattering, and the bubble refers to the bare pair-bubble.

Fig. 2. Dependence of the coupled plasmon-phonon frequency induced by intra- (in (a)) and inter-SO (in (b)) excitation on $\omega_p = (2\pi e^2 n_e q/\epsilon_\infty m^*)^{1/2}$ for fixed Rashba parameter $\alpha_R$ and total electron density $n_e$. Here, $\omega_{LO}$ and $\omega_{TO}$ are respectively the LO- and TO-phonon frequencies and $\omega_\pm = 4\alpha_R \sqrt{\pi n_\pm}/\hbar$.

Fig. 3. Coupled plasmon-phonon frequency induced by intra- (in (a)) and inter-SO (in (b)) transitions as a function of Rashba parameter for fixed $\omega_p$ and $n_e$ as indicated.

Fig. 4. Coupled plasmon-phonon frequency caused by intra- (in (a)) and inter-SO (in (b)) excitations versus total electron density for fixed $\alpha_R$ and $\omega_p$. 
W. Xu et. al. Figure 1.
W. Xu et. al. Figure 2.
W. Xu et. al. Figure 3.
W. Xu et al. Figure 4.