More Missing VEV Mechanism in Supersymmetric SO(10) Model

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Abstract

The anomalous gauge $U(1)_A$ symmetry which could emerge in the context of the string theories can be very useful ingredient towards building the complete supersymmetric SO(10) theory. We present an example of the $SO(10) \times U(1)_A$ model which provides the “all order” solution to the doublet-triplet splitting problem via the missing VEV mechanism – the Planck scale corrections only can induce the $\mu$-term naturally of order $1 \text{ TeV}$. An interesting feature of this model is that all relevant GUT scale VEVs are defined by the single dimensional parameter in the Higgs superpotential, so that the $SO(10)$ symmetry should break down to the MSSM practically at one step, without intermediate stages. The colour Higgsino mediated $d = 5$ operators can be naturally suppressed. We also extend the model by implementing $U(1)_A$ as a horizontal symmetry for explaining the fermion mass and mixing pattern, and obtain a predictive texture for fermion masses. This model implies a moderate value of $\tan \beta (\sim 6 - 10)$ and leads to five predictions for the low energy observables. It also leads to the neutrino masses and mixing pattern that could naturally explain both the atmospheric and solar neutrino problems. In addition, a remarkable interplay of the $SO(10)$ and $U(1)_A$ symmetries guarantees an automatic R parity conservation at any order in $M_P^{-1}$, and also suppresses the Planck scale induced $B$ and $L$ violating $d = 5$ operators to the needed level.
1 Introduction

Supersymmetric Grand Unified Theories (SUSY GUT) provide the most plausible possibilities to understand stability of the electroweak scale and the unification of the gauge couplings. It is well known [1] that in the minimal supersymmetric standard model (MSSM) the constants $g_{3,2,1}$ of the gauge group $G_{321} = SU(3) \times SU(2) \times U(1)$ join at energies $M_G \sim 10^{16}$ GeV, at which scale the MSSM can be consistently embedded in $SU(5)$ or some larger group $G$. This suggests a paradigm that may be some SUSY GUT (and not directly MSSM) emerges as a field theory limit of the string “Theory of Everything” which then breaks down to $G_{321}$ at the scale $M_G$.

The main problem which emerges in SUSY GUTs is a problem of the doublet-triplet (D/T) splitting. The MSSM Higgs doublets $H_u,d$ which induce the electroweak symmetry breaking and fermion masses should be light (with mass $\sim M_W$), while their colour-triplet partners in GUT supermultiplets should have masses of order of $M_X$ in order to avoid too fast proton decay. Another puzzle is related to the so-called $\mu$-problem: the theory should provide the superpotential term $\mu H_u H_d$ with $\mu \sim M_W$.

Presently the $SO(10)$ model is a most admired candidate for the grand unification [2]. The supersymmetric $SO(10)$ GUT, with the necessary superfields in representations 16, 10, 45 and 54, could emerge from the string theories at the Kac-Moody level $k_{10} = 2$ or larger [3]. All standard fermion states of one family: $q = (u, d)$, $l = (\nu, e)$, $u^c$, $d^c$, $e^c$, and the ‘right-handed’ neutrino $\nu^c$ fit into one irreducible representation 16 of $SO(10)$. Two MSSM Higgs doublets $H_u$ and $H_d$ are also embedded in one irreducible representation 10. These features provide a great possibility for constructing the predictive ansatzes for the fermion mass matrices [4]. Another virtue of the supersymmetric $SO(10)$ model is that it makes possible to solve the D/T problem via the missing VEV mechanism which was originally suggested by Dimopoulos and Wilczek (DW) [6] and was intensively discussed in many recent papers [7, 8, 9].

In the present paper we try to coherently approach the resistible problems in $SO(10)$ by exploiting a very peculiar possibility suggested by the string theory – an anomalous gauge $U(1)_A$ symmetry. Indeed, the stringy models, in addition to the GUT gauge group itself, can contain also several other gauge group factors. The coupling constants of all gauge groups are determined by the dilaton VEV: $1/g_a^2 = k_a \langle \text{Re}(s) \rangle$, where $k_a$ are the Kac-Moody levels for the corresponding gauge factor $G_a$. One linear combination of the possible gauge $U(1)$ factors can be ‘truly’ anomalous, with nonvanishing trace over the charges of the matter superfields, while the other combinations are rendered traceless. Existence of an anomalous $U(1)_A$ does not imply an anomaly in the original string theory. In the field theory limit it can be understood as a result of truncating the string spectrum to the particle spectrum, and all mixed anomalies of the matter fields can be effectively canceled by the Green-Schwarz mechanism [11], via shift of the axion field $\text{Im}(s)$. This cancellation implies that the $U(1)_A^3$ anomaly coefficient $C_A \propto \frac{1}{3} \text{Tr}(Q^3)$ and the mixed anomaly coefficients of $U(1)_A$ to the other factors $G_a$ ($C_{10} \propto \text{Tr}(QT_a T_a)$) and to gravity ($C_g \propto \text{Tr}Q$) should
be related to the corresponding Kac-Moody levels as \( C_a : C_A : C_g = k_a : k_A : k_g \).

Therefore, in the context of the string theories one can consider a situation when the supersymmetric \( SO(10) \) model is accompanied by the anomalous \( U(1)_A \) symmetry. Then the gauge constants \( g_{10} \) and \( g_A \) of \( SO(10) \) and \( U(1)_A \) should be unified by the condition \( k_{10}g_{10}^2 = k_Ag_A^2 = g^2 \), valid at the string scale \( M_{str} = g_{str}M_P \), where \( M_P \sim 10^{18} \) GeV is a (reduced) Planck scale, while the Green-Schwarz mechanism implies that the mixed \( U(1)_A \) anomaly coefficients are related to the Kac-Moody levels as \( C_{10} : C_A : C_g = k_{10} : k_A : k_g \).

As it was shown in \[12\], D-term of the anomalous \( U(1)_A \) symmetry gets a non-zero Fayet-Iliopoulos term \( \xi \) \[11\]:

\[
D_A = \xi + \sum Q_i|\phi_i|^2, \quad \xi = \frac{C_g}{192\pi^2}M_{str}^2
\]

where the sum runs over all scalar fields \( \phi_i \) present in the theory with \( U(1)_A \) charges \( Q_i \). Therefore, the spontaneous breaking scale of the \( U(1)_A \) symmetry is naturally small as compared to the Planck scale but not too small: \( \sqrt{\xi}/M_P \sim 0.1 \).

In the literature anomalous gauge symmetry \( U(1) \) was applied as a horizontal symmetry for explaining the fermion mass hierarchy, utilizing the fact that the magnitude of \( \sqrt{\xi}/M_P \) is of the order of fermion mass ratios in the neighbouring families \[13\]. Recently the anomalous \( U(1)_A \) symmetry was applied in order to justify the D/T problem solution in the supersymmetric \( SU(6) \) model \[14\], and in the missing doublet \( SU(5) \) model \[15\].

In the present paper we show that the idea of the anomalous \( U(1)_A \) gauge symmetry inspired by the string theory can be useful also for achieving a simple ‘all order’ solution to the DT splitting problem via the missing VEV mechanism (MVM) \[6\] in the supersymmetric \( SO(10) \) theory.

In particular, in section 2 we reproduce the original MVM by arrangement of the \( U(1)_A \) charges of the Higgs superfields, which solution is stable against the Planck scale corrections. Even more, the latter can induce the order 1 TeV \( \mu \)-term and thus contribute in solving the \( \mu \)-problem. We also suggest an improved model where the proton decaying \( d = 5 \) operators \[17\] are strongly suppressed (section 3). In section 4 we implement the anomalous \( U(1)_A \) symmetry as a horizontal symmetry between the fermion generations and study implications of the obtained mass textures for the fermion mass matrices. Interestingly, this model leads to the exact R parity conservation due to \( U(1)_A \) charge content of the superfields as well as to natural suppression of the Planck scale cutoff \( d = 5 \) B and L violating operators (sect. 5). Finally, in sect. 6 we briefly discuss our results.

## 2 The missing VEV \( SO(10) \times U(1)_A \) model

Consider a supersymmetric \( SO(10) \) model containing the fermion superfields \( f_i \sim 16, \ i = 1, 2, 3 \), and the Higgs superfields in the following representations: \( H, H' \sim 10, \)
$S \sim 54$, $A, B, B' \sim 45$, $C, \bar{C} \sim 16, 16$ and two singlets $Z$ and $X$. Let us assume that $X$ has a negative $U(1)_A$ charge which we denote as $Q_X = -2x$. The nonzero charges of the Higgs superfields taken as

$$U(1)_A : \quad Q_X = -2x, \quad Q_H = -x, \quad Q_{H'} = x, \quad Q_C = cx, \quad Q_{\bar{C}} = -cx$$

(2)

while $Z, S, A, B, B'$ have vanishing $U(1)_A$ charges (the value of $c$ will be fixed later from the phenomenological constraints). We also invoke two additional symmetries. First is a discrete $R$-symmetry $\mathcal{R}$ under which all above superfields as well as the superpotential change the sign:

$$\mathcal{R} : \quad X, Z, S, A, B, B', H, H' \rightarrow -X, Z, S, A, B, B', H, H', \quad W \rightarrow -W$$

(3)

Second one is another global or local symmetry $U(1)'$ under which only $B, B'$ and $H$ have nonzero charges:

$$U(1)' : \quad Y_B = 1, \quad Y_{B'} = -1, \quad Y_H = -1$$

(4)

while all other superfields as well as the superpotential are invariant.

The most general renormalizable Higgs superpotential allowed by these symmetries reads as (all $SO(10)$ indices are suppressed):

$$W_{\text{Higgs}} = W_1 + W_2 + W_3 + W_4$$

$$W_1 = M^2Z + Z^3 + ZS^2 + S^3 + SA^2$$

$$W_2 = ZC\bar{C} + AC\bar{C} + ZA^2$$

$$W_3 = ZBB' + SBB' + ABB'$$

$$W_4 = BHH' + XH'^2$$

(5)

where the order one constants are understood in the trilinear terms, and the mass parameter $M$ is of the order of GUT scale $M_G \approx 10^{16}$ GeV. We also assume that charges of the fermion superfields $f_i$ are arranged so that they have the Yukawa

1 Usually the Higgs and fermion superfields are distinguished by introducing the matter parity, positive for Higgses and negative for fermions. In sect. 5 we show that in our model we do not need to introduce ad hoc the matter parity and it can emerge as an automatic consequence of the anomalous $U(1)_A$ charges.

2 In fact, the scale $M$ is the only ad hoc scale in the theory, since the magnitude of the Fayet-Iliopoulos term $\xi$ (II) is essentially determined by the Planck scale modulo the trace over the $U(1)_A$ charges of all superfields. In the context of the stringy GUT it is not easy to motivate a presence of dimensional parameter in the superpotential, and it would be highly desirable to have a realistic mechanism which could naturally explain the presence of the linear term in $W_1$. Perhaps one could think of a situation when there is no linear term in the superpotential (II) (in this case the discrete $\mathcal{R}$ symmetry extends to the continuous $R$-symmetry), and it effectively emerges from the coupling $ZQ^\alpha Q_\alpha$ of singlet $Z$ to some "hidden" matter $Q, \bar{Q}$ from the strongly coupled sector, as a result of the dynamical condensation of $QQ$. 

4
couplings solely to $H$ (see section 4). Note, that the mass term $\mu H^2$ as well as couplings $XH^2$ or $(Z + S)H^2$ are forbidden by the $U(1)_A$ symmetry.

One has to analyze the superpotential (5) together with the D-terms

$$g^{2}_{10} \left( \sum \phi^\dagger_r T^{(r)}_a \phi_r \right)^2 + g^2_A \left( \sum Q_r |\phi_r|^2 - 2x |X|^2 + \xi \right)^2,$$

where under $\phi_r$ we imply the scalar components of all superfields present in the theory besides $X$ with their $U(1)_A$ charges $Q_r$, $T^{(r)}_a$ are the $SO(10)$ generators in the corresponding representations. We assume that the trace Tr$Q$ of the $U(1)_A$ charges over all matter fields is positive (for the concrete model see Table 1 in sect. 4). It is easy to see that the theory has a supersymmetry conserving vacuum (all F- and D-terms vanish) when the scalar $X$ gets a non-zero VEV $\langle X \rangle = \sqrt{\xi/2x}$ entirely from the anomalous D-term (3).

As for the fields $Z, S, A, B, B', C, \bar{C}$, they get nonzero VEVs which induce the $SO(10)$ breaking to the MSSM $SU(3) \times SU(2) \times U(1)$. In particular, $C, \bar{C}$ have the $SU(5)$ conserving VEVs ($\propto |+, +, +, +, +\rangle$ in terms of eigenvalues of the corresponding Cartan subalgebra generators), and their magnitudes should be equal by the vanishing of $D_{10}$: $\langle C \rangle = \langle \bar{C} \rangle = C$. The VEVs of $S$ and $A$ then break $SU(5)$ down to $SU(3) \times SU(2) \times U(1)$. The VEVs of $B$ and $B'$ wind towards the $B - L$ direction, i.e. $(15, 1, 1)$ in terms of the $SU(4) \times SU(2) \times SU(2)'$ subgroup:

$$\langle S \rangle = S \cdot \text{diag}(1, 1, 1, -3/2, -3/2) \otimes 1,$$

$$\langle A \rangle = A \cdot \text{diag}(1, 1, 1, 1 + r, 1 + r) \otimes \sigma,$$

$$\langle B(B') \rangle = B(B') \cdot \text{diag}(1, 1, 1, 0, 0) \otimes \sigma,$$

where

$$1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

The magnitudes of these VEVs all are order $M \sim M_G$, with the accuracy of the $\sim 1$ coupling constants in the Higgs superpotential (3) (note, the VEV of singlet $Z$ in fact plays a role of the mass term for other superfields). Certainly, uncertainties in coupling constants in (3) can allow up to order of magnitude hierarchy between various GUT scales (e.g. $C > S$, in which case $SO(10)$ first breaks down to $SU(5)$ and then to the MSSM). Once again, this feature can justify the one step gauge constant unification in the $SO(10)$ model. Note, the superpotential is arranged in such a way that the $B - L$ direction of $B, B'$ is not affected by the other interaction terms. The presence of the last term in $W_3$ guarantees that no unwanted light modes appear in the theory after the $SO(10)$ symmetry breaking which contribution could spoil the gauge couplings unification.

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3 If the last term in $W_{1}$ would vanish then the VEV of $A$ would be $SU(5)$ invariant: $r = 0$ in eq. (3).
As far as the magnitude of the Fayet-Iliopoulos term $\xi$ is essentially determined by the value of mixed $U(1)_A$ - gravity anomaly $C_g$, the scale $X = \sqrt{\xi/2x}$ in fact cannot arbitrary: modulo the factor $(\text{Tr}Q/2x)^{1/2}$ we have $X \sim 10^{17}$ GeV. In the following the latter value will be used for numerical estimates, and thus for the ratio $\varepsilon = X/M_P$ we take $\varepsilon \sim 1/10 - 1/20$. This estimate implies that Tr$Q$ has a moderate value in units of $Q_x$ which is indeed the case for the model considered below. For the $SO(10)$ symmetry breaking VEVs we adopt the standard value $M_G \sim 10^{16}$ GeV, neglecting possible split between their values and the related threshold corrections. Thus, in estimates we take $\varepsilon_G = M_G/M_P \sim 10^{-2} - 10^{-3}$.

After substituting the relevant VEVs in $W_4$, mass matrices of the doublet and triplet fragments in $H, H'$ respectively get the form:

$$
M_D = \begin{pmatrix} D & D' \\ \bar{D} & \bar{D}' \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & M_{22} \end{pmatrix} \quad \text{and} \quad M_T = \begin{pmatrix} T & T' \\ \bar{T} & \bar{T}' \end{pmatrix} \begin{pmatrix} 0 & M_{12} \\ -M_{12} & M_{22} \end{pmatrix}
$$

with $M_{22} \sim X$ and $M_{12} \sim B \sim M_G$. Therefore, all triplets in $H, H'$ are massive while the doublets $D$ and $\bar{D}$ contained in $H$ remain massless and can be identified with the MSSM Higgses $H_u$ and $H_d$ respectively.

Note, that our mechanism for the $D/T$ splitting is stable against Planck scale corrections. The worst thing the Planck scale cutoff higher dimensional operators could do is to generate the $\mu$-term of the needed size.

Indeed, the lowest order operators which could violate the DW pattern in the $H - H'$ mixing term are the following:

$$
\frac{\text{Tr}(AB)}{M_{Pl}^2} (Z + S + A)HH', \quad \frac{Z + S}{M_{Pl}^2} (BB')(CC)
$$

The first operator directly induces the off-diagonal entries of order $\sim M_G^2/M_{Pl}^2 \sim 10^{10}$ GeV in the matrix $M_D$. The second one (with combinations in brackets taken in the 210-channel of $SO(10)$) gives the same order contribution via the coupling $BHH'$ since it affects the DW structure of $B, B'$ VEVs. Indeed, after substituting the ‘basic’ VEVs it reduces to the coupling containing the $(1,1,3)$ fragment of $B$ (or $B'$) linearly and thus the small VEV will be induced also on the $T_R$ direction. In other words, the VEV of $B$ will change to the form $\langle B \rangle = B \cdot \text{diag}(1,1,1,\kappa,\kappa)$ with $\kappa \sim (M_G/M_{Pl})^2 \sim 10^{-6}$. As a result, the matrix $M_D$ will get off-diagonal entries $m \sim 10^{10}$ GeV, and thus the ‘seesaw’ mixing of doublets $D, \bar{D}$ to the massive ones $D', \bar{D}'$ will generate the $\mu$-term $\mu H_u H_d$ just of the needed order: $\mu \sim m^2/X \sim 1$ TeV.

However, such MVM has a generic problem related to the baryon number violating $d = 5$ operators. Indeed, if $M_{22} \sim X \sim 10^{17}$ GeV, then the triplets $T, \bar{T}$ are too light ($M_T \sim M_G^2/X \sim 10^{15}$ GeV). Therefore, the cutoff scale of the relevant $d = 5$ operators is

$$
(M_T^{-1})_{11} = \frac{M_{22}}{M_{Pl}^2} \sim \frac{X}{M_G}
$$

(10)
and thus they mediate unacceptably fast proton decay \[18\] which is excluded by the present experimental data \[20\], especially for large $\tan \beta$ which is typical for the $SO(10)$ models.

Certainly, one could lower the value of $M_{22}$ to about $10^{14-15}$ GeV by taking very small ($\sim 10^{-2} - 10^{-3}$) coupling constant in the last term in $W_4$ \[3\]. Alternatively, one could replace it by the higher order operator say $(X^3/M_P^2)H'^2$. In this case the proton lifetime could be acceptable. The drawback of this situation would be that in this case the $\mu$ term induced by the couplings \[3\] would also increase by about three orders of magnitude, up to $10^6$ GeV, unless it is suppressed by very small coupling constants in the terms \[3\].

In the next section we propose a more appealing possibility which does not suffer from this problems. It naturally suppresses the proton decay and at the same time naturally leaves the $\mu$ term in the 1 TeV range.

3 Suppressing proton decay

We employ a proposal by Babu and Barr \[4\] to use additional 45-plets $R$ having VEV towards the $T_R$ direction of $SU(4) \times SU(2) \times SU(2)'$.

We assume that $H$ is a 10-plet Higgs having the Yukawa couplings with the fermions $f_i$, while the theory includes also three additional 10-plet Higgses $H_{1,2,3}$. We also introduce two additional 45-plets $R, R'$, and prescribe the following charges to the states:

\[
\begin{align*}
U(1)_A &: \quad Q_X = -2x, \quad Q_H, Q_{H_2} = -x, \quad Q_{H_1}, Q_{H_3} = x, \\
U(1)' &: \quad Y_B, Y_R = 1, \quad Y_{B'}, Y_{R'} = -1, \quad Y_{H_1} = -1
\end{align*}
\]  

(11)

In order to distinguish Higgses $R, R'$ from $B, B'$, we also introduce an additional discrete symmetry $Z_2'$ which changes the sign of $R, R', H_2, H_3$ while other superfields are invariant (for the complete charge content of the theory see below, Table 1).

The Higgs superpotential of the superfields $Z, S, A, B, B'C, \bar{C}$ still has a form \[2\] but now we add also the following terms:

\[
W'_3 = ZRR' + SRR' + ARR'
\]  

(12)

while the last term $W_4$ in \[3\] is modified as follows:

\[
W_4 = BHH_1 + RH_1H_2 + (Z + S + A)H_2H_3 + XH_3^2
\]  

(13)

From the superpotential $W'_3$ one can see that there is a solution when the 45-plets $R, R'$ get the VEVs towards the $T_R$ direction:

\[
\langle R(R') \rangle = R(R') \cdot \text{diag}(0, 0, 1, 1) \otimes \sigma
\]  

(14)

\[\text{footnote}^4\] Another possibility of the stabilizing proton by implementing the 45-plet with VEV towards $T_R$ direction in the Yukawa sector was suggested in refs. \[4\].

\[\text{footnote}^5\] Actually for the consistency of the F-terms minimization one has to introduce another singlet $Z'$ with the same quantum numbers as $Z$ and include it in all terms of superpotential.
One can take into account also the higher order Planck scale cutoff operators. Then the mass matrices for the doublet and triplet fragments in $H$ and $H_{1,2,3}$ gets the form:

$$
\mathcal{M}_D = \begin{pmatrix}
\bar{D} & \bar{D}_1 & \bar{D}_2 & \bar{D}_3 \\
D & 0 & O(m) & 0 \\
D_1 & O(m) & O(m') & R \\
D_2 & 0 & -R & 0 \\
D_3 & O(m) & O(m') & Z + S - A \\
\end{pmatrix}
$$

(15)

$$
\mathcal{M}_T = \begin{pmatrix}
\bar{T} & \bar{T}_1 & \bar{T}_2 & \bar{T}_3 \\
T & 0 & B & 0 \\
T_1 & -B & O(m') & O(m) \\
T_2 & 0 & O(m) & 0 \\
T_3 & O(m) & -O(m') & Z + S - A \\
\end{pmatrix}
$$

(16)

Note, that all zero elements in this expression are “all order” zeros in $X/M_p$, since the $U(1)_A$ charges of these terms are negative. The possible small but non-zero entries behind the order sign $O$ can come form the higher order operators cutoff by the Planck scale. The entries $O(m)$, with $m \sim 10^{10}$ GeV come from the operators (9) considered in the previous section and analogous operators for $R, R'$. Other entries with $m' \sim \varepsilon^2 G_X M_p \sim 10^{11}$ GeV can be induced from the terms like

$$
\frac{R^2 R'}{M_p^2} HH_1, \quad \frac{1}{M_p^2}(RB' + R'B)(Z + A + S)HH_3, \\
\frac{X}{M_p^2}(B^2 + R^2)H_1^2, \quad \frac{Tr(A R)}{M_p^2}(Z + A + S)H_1 H_2, \quad \frac{XR}{M_p^2}(Z + A)H_1 H_3 \\
\frac{1}{M_p^2}(B^2 R^2 + B'^2 R'^2 + BB' RR')(Z + S + A)
$$

(17)

From (14) we see that the Planck scale corrections induce the $\mu$ term for the doublets $H_{u,d}$ contained in $H$, $\mu \sim m^2/X \sim 1$ TeV. One the other hand, now the proton becomes long living. If one neglects the Planck scale induced terms, then $(\mathcal{M}_T)^{-1}$ is vanishing and thus the color Higgsino mediated $d=5$ operators cannot destabilize it. The Planck scale corrections can induce the $d=5$ operators cutoff by the scale:

$$
(\mathcal{M}_T^{-1})_{11} \sim \frac{X}{M_p^2}
$$

(18)

and thus the proton decay is extremely suppressed, by factor $(M_p^2/X M_G)^2 \sim 10^6$ as compared to the standard estimates in the minimal supersymmetric $SU(5)$ model [18], and can be hardly observable even for large $\tan \beta \sim 100$ and low SUSY breaking masses.
Table 1: The superfield transformation properties in the Model 1 \((n = 1)\). With respect to \(\mathcal{R}\) symmetry all superfields change the sign except \(f_2\) which is invariant.

|       | \(X\) | \(Z\) | \(S\) | \(A\) | \(B\) | \(B'\) | \(R\) | \(R'\) | \(C\) | \(C'\) | \(H_3\) | \(H_2\) | \(H_1\) | \(H\) | \(f_i\) | \(F\) |
|-------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| \(SO(10)\) | 1    | 1    | 54   | 45   | 45   | 45   | 45   | 16   | 16   | 10   | 10   | 10   | 10   | 16   | 10   | 10   |
| \(U(1)_A\) | −2   | 0    | 0    | 0    | 0    | 0    | 0    | 0    | \(\frac{2}{3}\) | \(\frac{7}{2}\) | 1    | −1   | 1    | −1   | 1    | 0    |
| \(U(1)'\) | 0    | 0    | 0    | 0    | 1    | −1   | 1    | −1   | 0    | 0    | 0    | 0    | −1   | 0    | 0    | 0    |
| \(Z_2'\) | +    | +    | +    | +    | +    | −    | −    | −    | +    | +    | +    | +    | +    | +    | +    | +    |

4 Incorporating the fermion masses

Here we attempt to incorporate the anomalous \(U(1)_A\) symmetry also as a horizontal symmetry between the fermion generations, in the spirit of the earlier proposals suggested in the framework of the MSSM [13] or supersymmetric \(SU(5)\) model [15]. Let us take as a basis the model considered in the previous section, and prescribe the generation dependent \(U(1)_A\) charges to three fermionic 16-plets \(f_{1,2,3}\) so that only the third family \(f_3\) is allowed to have the renormalizable Yukawa coupling \(f_3f_3H\) to the Higgs \(H\), while the other fermions can get masses from the higher order operators including powers of \(X/M_P\). These operators in the superpotential can be effectively induced after integrating out some heavy states with masses \(\sim M_P\) [19]. Namely, let us take \(Q(f_i) = \frac{1}{2}x + 2(3 - i)x\), and also assume that under the discrete \(\mathcal{R}\) symmetry \(f_i \rightarrow (-1)^if_i\) while with respect to remaining symmetry groups they are invariant. We also introduce an additional fermion state \(F \sim 10\) with \(Q_F = 2nx\) (integer \(n\)) and fix the \(U(1)_A\) charges of \(C, \bar{C}\) in (4) as \(c = -(4n + 1)/2\). The superfield representation and charge content for the case \(n = 1\) (hereafter to be referred as to Model 1) is given in Table 1.

Then the possible Yukawa terms in the superpotential can be expressed as:

\[
W_{\text{Yuk}} = W_1 + W_2 + W_3,
\]

\[
W_1 = g_{ij}Hf_if_j \left(\frac{X}{M_P}\right)^{6-i-j},
\]

\[
W_2 = \gamma_{ij}C\bar{C}f_if_j \left(\frac{X}{M_P}\right)^{2n+7-i-j},
\]

\[
W_3 = h_iCf_iF \left(\frac{X}{M_P}\right)^{3-i} + (hZ + h'S)F^2 \left(\frac{X}{M_P}\right)^{2n},
\]

\(\text{In principle fermions could have non-trivial charges with respect to } U(1)' \text{ symmetry, as well as they could transform non-trivially with respect to } Z_2' \text{ symmetry which also could be extended to } U(1)'' \text{ symmetry related to independent rotations of the } R, R' \text{ states. Then the interplay of these symmetries could fix some interesting textures for the quark and lepton mass matrices, in the spirit of the works [3]. However, in the present paper we take more modest approach extending for fermions only the anomalous } U(1)_A \text{ symmetry.}\)
where \( g_{ij} = g_{ji}, \gamma_{ij} = \gamma_{ji} \) and \( h_i (i = 1, 2, 3) \) are order 1 constants.

There is another relevant coupling which has to be taken into consideration. The symmetries of the theory allow the following term in the Higgs superpotential:

\[
H \hat{C} \hat{C} \left( \frac{X}{\Lambda_{FP}} \right)^{2n}
\]

while no term \( HCC \) is allowed since it has a negative \( U(1)_A \) charge.

Let us use the \( SU(5) \) subgroup language to describe their contributions. The decomposition of the relevant representations is \( 16 = \bar{10} = \bar{10} \) for the mass matrix of the down quarks and charge leptons mass matrices to the mass matrix of the upper quarks. Without lose of generality, one can rotate all \( 16 \)-plets \( f_i \) so that the matrix \( \lambda_{ij} \) is diagonal:

\[
\lambda_{ij} = \text{diag}(\lambda_u, \lambda_c, \lambda_t)
\]

The upper quark masses emerge solely from this term and thus \( m_{ij}^{u} = \lambda_{ij} v_u \) is the mass matrix of the upper quarks. Without lose of generality, one can rotate all 16-plets \( f_i \) so that the matrix \( \lambda_{ij} \) is diagonal:

\[
\lambda_{ij} = \text{diag}(\lambda_u, \lambda_c, \lambda_t)
\]
with angles $\sim \varepsilon$ lead only to irrelevant redefinition of the other parameters without changing their hierarchial pattern.

So, we take the basis where the upper quark mass matrix is diagonal:

$$\hat{m}^u = u_1 \begin{pmatrix} \lambda_u & 0 & 0 \\ 0 & \lambda_c & 0 \\ 0 & 0 & \lambda_t \end{pmatrix} \cdot v_u$$  \hspace{1cm} (24)$$

where the Yukawa eigenvalues scale as $\lambda_t \sim 1$, $\lambda_c \sim \varepsilon^2$ and $\lambda_u \sim \varepsilon^4$, which for $\varepsilon \sim 1/10 - 1/20$ properly fit the up quark mass pattern.

As far as $X$ is a $SO(10)$ singlet, the operators $\mathcal{W}_1$ lead to exactly the same contribution to the charged lepton and the down quark mass Yukawa terms as well as to the neutrino Dirac terms: $\lambda_{ij}(q_i d_j^c H_d + e_i l_j H_d + \nu_i l_j H_u)$. Hence, in the absence of the other contributions we would have $\hat{m}^e = \hat{m}^d = \hat{m}^u/\tan \beta$, and thus $m_{e,\mu,\tau} = m_{d,s,b}$ in the $SU(5)$ limit. It is clear that the additional fermion 10-plet $F$ was introduced in order to remove the degeneracy between the upper quark, down quark and charged lepton states. In the $SU(5)$ fragments $F = 5_F + \bar{5}_F$ are contained only the $d^c$ and $l$ type particles and their conjugates. Moreover, since the 54-plet $S$ contains 24-plet of $SU(5)$, its VEV removes the mass degeneracy between down quark $d^c$ and lepton $l$ states. Therefore, mixing of the corresponding fermion states in 16-plets $f_i$ would induce different Clebsches for the down quark and lepton mass entries and thus can remove the GUT scale $SU(5)$ degeneracy of the physical states $s - \mu$ and $d - e$.

Hereafter we concentrate on the case $n = 1$ (Model 1), leaving another possibilities for the future study (the models with $n = 0$ or 2 also could be of phenomenological interest, but the cases with larger $n$ seem not very appealing). Then the big mass entries which emerge from $\mathcal{W}_3$ read as $d_F(M_3 d_F^c + h_1 \varepsilon^2 C d_1^c + h_2 \varepsilon C d_2^c + h_3 C d_3^c)$ and $l_F(M_3 l_F^c + h_1 \varepsilon^2 C l_1 + h_2 \varepsilon C l_2 + h_3 C l_3)$, where $M_{2,3} \sim \varepsilon^2 M_G$ are respectively the doublet and triplet mass entries in $F$ induced by the last term in (23). Therefore, after decoupling the superheavy $5_F + \bar{5}'$ state where $\bar{5}'$ is dominantly contributed by $\bar{5}_3$, we see that the light states are left in $5'_1 \approx 5_1$, $5'_2 \approx 5_2$ and $5'_3 \approx 5_F$ which contain $\bar{5}_3$ as small admixtures. Notice also that the doublet and triplet fragments in $5_F$ are contained in $\bar{5}'$ with different weights, $s_e = M_2/h_3 C$ and $s_d = M_3/h_3 C$, which will be used for removing the $SU(5)$ mass degeneracy between the down quarks and charged leptons.

Since the superheavy state is formed essentially by $5_F$ and $\bar{5}_3$, the mass matrix of the down quarks and charged leptons should be strongly altered by the rearrangement of the $5$ states. Taking also into account that the MSSM Higgs doublet partially (with the weight $w \sim \varepsilon^2$) resides in $\bar{5}_C$, after decoupling the heavy fermions [19] we obtain
the following mass matrices for the down quarks and charged leptons:

\[
\hat{m}^d = d_1 \begin{pmatrix} d_{1e}^c & d_{1e}^u & d_{1e}^t \\ \lambda_u e^{i\omega} & \varepsilon_1 \varepsilon_2 \lambda & 0 \\ \varepsilon_1 \lambda_c & \varepsilon_2 \lambda & \lambda_c \end{pmatrix}, \quad \hat{m}^e = e_1 \begin{pmatrix} l_1' \\ l_2' \\ l_3' \\ \lambda_u e^{i\omega} & \varepsilon_1 \varepsilon_2 \lambda & 0 \\ \varepsilon_1 \lambda_c & \varepsilon_2 \lambda & \lambda_c \end{pmatrix} \cdot v_d,
\]

and the matrix of the neutrino Dirac masses:

\[
\hat{m}_D = \begin{pmatrix} \nu_1' & \nu_2' & \nu_3' \\ \nu_1 & \nu_2 & \nu_3 \\ \varepsilon_1 \lambda_c & 0 & \lambda_c \\ 0 & \kappa_e - \lambda & \varepsilon_2 \lambda_t \end{pmatrix} \cdot v_u,
\]

where the parameters are the following: \(\varepsilon_1 = \varepsilon h_1/h_2 \sim \varepsilon, \varepsilon_2 = \varepsilon h_2/h_3 \sim \varepsilon\) and \(\lambda = h_3 w \sim \varepsilon^2\). We see that all entries are the same in the matrices \(\hat{m}^{d,e}\) except the 2,3 elements \(\kappa_d = \lambda - \lambda_s \delta_d\) and \(\kappa_e = \lambda - \lambda_s \delta_e\), which are generally different. The impact of the latter is to remove the troblessome \(SU(5)\) degeneracy between the \(s - \mu\) and \(e - d\) mass eigenvalues at the GUT scale. Without lose of generality, all entries in \(\hat{m}^{d,e}\) can be chosen real by redefinition of the fermion phases except the 1,1 entry and 2,3 entries \(\kappa_{d,e}\) which remain complex.

Mass matrices (24), (25) and (26) depend on 12 parameters, consisting of the Yukawa constants \(\lambda_{u,c,t}\), \(\tan \beta = v_u/v_d\) and 8 unknown parameters: \(\varepsilon_1, \varepsilon_2, \lambda\), the phase \(\omega\) and two complex parameters \(\kappa_{e,d}\). Therefore, in general we have to obtain 2 relations between the 14 observables of the MSSM (nine fermion masses, four parameters of the CKM matrix: \(s_{12} = |V_{us}|, s_{23} = |V_{ub}|, s_{13} = |V_{ub}|\) and \(CP\)-phase \(\delta\), and still \(\tan \beta\)). The general analysis of these mass matrices will be presented elsewhere. For the moment we confine ourselves by illustrating the particular case when \(\kappa_{e,d}\) are real and \(\kappa_d = -\kappa_e = \kappa\). In these case the number of parameters are reduced by three and hence one has to obtain five predictions.

Indeed, neglecting the \(O(\varepsilon)\) corrections in diagonalization of the matrices \(\hat{m}^{d,e}\), we obtain the following relations holding with some 10 percent accuracy:

\[
\lambda_b = \lambda_r, \quad \lambda_c = s_{23} \lambda_r = \frac{2}{3} \lambda_\mu, \quad \lambda_s = \frac{1}{3} \lambda_\mu, \quad \lambda_e = \frac{1}{3} \lambda_d + \lambda_u e^{i\omega}, \\
\sin \delta = \frac{\lambda_u}{\lambda_d} s_{12} \sin \omega, \quad s_{13} = \frac{\sqrt{\lambda_d \lambda_s}}{2 \lambda_b} = \frac{1}{4} s_{12} s_{23},
\]

while for the values of the unknown parameters we have:

\[
\varepsilon_2 \lambda = \frac{1}{2} (\lambda_\mu + \lambda_s) = \lambda_c, \quad \kappa = \frac{\lambda_\mu - \lambda_s}{2 \lambda_c} \lambda_r = \frac{1}{2} \lambda_r, \\
\varepsilon_1 \lambda_c = \varepsilon_1 \varepsilon_2 \lambda = \sqrt{\lambda_d \lambda_s} \sim \sqrt{\lambda_c \lambda_\mu}, \quad \varepsilon_2 = \frac{\lambda_t}{\lambda_r} \sim 0.1, \quad \lambda = s_{23} \lambda_t \sim \lambda_r
\]
The first prediction $\lambda_b = \lambda_\tau$ in (27), the famous $b - \tau$ Yukawa unification at the GUT scale, immediately follows from the assumption that the $(3,3)$ element $\varepsilon_2 \lambda_\tau$ is the largest entry in the matrices (24). Parametrically it is indeed $\sim \varepsilon$ while the other entries should be smaller: $\lambda_c, \kappa_{c,d} \sim \varepsilon^2$, etc. The second relation follows from the fact that in our model the $V_{cb}$ element of the CKM matrix is given as $s_{23} = \lambda_c / \lambda_\tau$. Then taking as input the experimental values $s_{23} = 0.04$ and $\lambda_\mu / \lambda_\tau = 0.06$, we obtain $\lambda_c = \frac{2}{3} \lambda_\mu$.

The three next predictions in (27) can be derived by performing the diagonalization of the $1 - 2$ blocks in the matrices (25) and putting the Cabibbo angle to its experimental value $s_{12} = 0.22$. And finally, the last prediction for $s_{13}$ element in the CKM matrix emerges due to the big mixing between the right-handed states $d^c_d$ and $d^c_2$ in $\hat{m}^d$: $s_{23}^c = \kappa / \lambda_b = \frac{1}{2}$. 

One can translate the relations (27) into predictions for the low energy physical observables. As it is well known, the $b - \tau$ Yukawa unification at the GUT scale explains the value of the $b$-quark mass with implication that top mass should be close to its infrared fixed value $M_t \simeq \sin \beta 200$ GeV. The second relation in (27) can be used for deducing the value of $\tan \beta$ using the experimental value of $c$ quark mass. Within the uncertainties related to the experimental values of $\alpha_3(M_Z)$ and $M_t$, we obtain $\tan \beta \simeq 6 - 10$. The next relation fixes the $s$ quark mass: $m_s \simeq 130 - 180$ MeV, in agreement with the ‘current-algebra’ predictions. The fourth relation implies that $m_\mu / m_s \simeq 1/22$, with about 20 percent uncertainty $\sim (3 m_u / m_d \tan \beta)$ related to $\omega$ varying from 0 and $\pi$. The $CP$-violating phase is very small – even for $\omega = \pi/2$, it cannot exceed the value $\delta \sim s_{12} \lambda_u / \lambda_d \sim 0.01$. 

And finally, the last relation in (27) implies that $V_{ub} / V_{cb} \simeq 0.06$, an agreement with the current experimental range [20].

Let us now turn to the neutrino masses. The mass matrix of the physical left handed states $\nu_i$ which results from the ‘seesaw’ decoupling [24] of the heavy Majorana states $\nu'_i$ has a form $\hat{m}_\nu = \hat{m}_D^T \hat{M}^{-1} \hat{M}_D$, where $\hat{m}_D$ is the Dirac mass matrix of eq. (29), and $\hat{M}_{ij} = \gamma_{ij} e^{\delta_{i-j}} M_R$ is the Majorana mass matrix of the $\nu^c$ states, where $M_R \sim \varepsilon^2 \varepsilon^3 M_P \sim 10^{11-12}$ GeV corresponds to the magnitude of its heaviest eigenstate. The matrix $\hat{M}$ cannot be exactly fixed from the theory, but it should have typical structure with the eigenvalues having a hierarchy $\varepsilon^4 : \varepsilon^2 : 1$ and with the rotation angles between the neighbouring families $\sim \varepsilon$.

We see that $\kappa = 1/2 \varepsilon_2 \lambda_4 \sim \varepsilon$, in some contradiction with the parametrical estimate of its value $\kappa \sim \varepsilon^2$. Such an enhancement should not be surprising and can have an accidental origin due to some conspiracies in the parameter space of the model. However, this value of $\kappa$ is still enough small to treat the rotation angle $s_{23}^c$ between the right states $d^c_d$ and $d^c_2$ as small angle, and it and does not affect obtained results more than about 10%. One can see, that for arbitrary complex $\kappa_{c,d}$ their modulus always be more than above value which in general could spoil our approximation and, in particular, the $b - \tau$ Yukawa unification. Therefore, our choice $\kappa_c$ and $\kappa_d$ as both real and opposite maximally corresponds to the spirit of our approximation.

In principle this is no problem since in the context of supersymmetric grand unified theory the $CP$-violation $K^0 - \bar{K}^0$ system, even too strong, can be originated from the supersymmetric contributions to both $\epsilon_K$ and $\tilde{\epsilon}_K$ parameters.
Putting all these together, one can see that the following picture emerges for the neutrino masses and mixing. The neutrino mass eigenvalues exhibit approximately the hierarchy $m_{\nu_\tau} : m_{\nu_\mu} : m_{\nu_e} \sim 1 : \varepsilon^2$, where the mass of the heaviest ($\nu_\tau$) state $m_{\nu_\tau} \sim \lambda_\tau^2 v_\tau^2 / M_R$ can naturally emerge in the range of 0.1 eV. Then the mass of $\nu_\mu$ can be of about $3 \times 10^{-3}$ eV.

On the other hand, there should be the strong mixing between the $\nu_\mu$ and $\nu_\tau$ states. This can be seen by comparing the Dirac mass matrix $\hat{m}_D$ with the parameter values calculated in (28) to the charged lepton mass matrix $\hat{m}_e$ in (25). We see that 2,3 rotation angles needed for the diagonalization of these matrices differ by a quantity $\sim \lambda_\tau / \lambda_\mu \sim 1$. One can hardly imagine that this large angle will be cancelled by contributions from the unknown parameters in $\hat{M}$. Therefore, we expect that $\sin^2 \theta_{\mu\tau} \sim 1$. The mixing between the $\nu_e$ and $\nu_\mu$ states is smaller. By comparing the matrices $\hat{m}_e$ and $\hat{m}_D$, one finds a contribution $\sim \sqrt{\lambda_\mu / \lambda_\tau} \sim \varepsilon$, while the $\sim \varepsilon$ contributions can come also from the structure of $\hat{M}$. Therefore, we expect that $\sin^2 \theta_{e\mu} \sim \varepsilon^2 \sim 10^{-2}$.

These features of the neutrino mass spectrum and mixing provide an appealing possibility to explain simultaneously the atmospheric and solar neutrino problems: deficite of the atmospheric muon neutrinos [22] can be due to the $\nu_\mu - \nu_\tau$ oscillation with $\delta m^2 \sim 10^{-2}$ eV$^2$ and $\sin^2 2\theta \sim 1$, while the solar neutrino problem can be explained by the MSW oscillation $\nu_e - \nu_\mu$ [23] with $\delta m^2 \sim 10^{-5}$ eV$^2$ and $\sin^2 2\theta \sim \varepsilon^2 \sim 10^{-2}$.

5 Automatic R parity

As far as our model includes Higgses in representations $C, \bar{C} \sim 16, \overline{16}$ and the fermions $F \sim 10$, at the first glance the R parity conservation is not automatic anymore, and the low energy theory (MSSM) should include the B and L violating $d = 3$ and $d = 4$ operators:

$$\mu_i l_i H_u + \lambda_{ijk} l_i l_j e_k^c + \lambda'_{ijk} l_i q_j d_k^c + \lambda''_{ijk} u_i^c d_j^c e_k^c$$

(29)

These could emerge from renormalizable couplings like $FH, f_i CH, F f_i f_j$ (recall that 10-plet $F$ is strongly mixed to the physical light fermion states), or nonrenormalizable operators like $\frac{1}{M_{Pl}} f_i f_j f_k C$ or $\frac{1}{M_{Pl}} F^2 f \bar{C}$ after substituting the VEV $C$ unless they are forbidden by an ad hoc matter parity.

Nevertheless, in our theory the R parity conservation occurs to be automatic due to the $U(1)_A$ symmetry: the $SO(10) \times U(1)_A$ invariant terms containing the odd number of the fermion superfields cannot emerge at any order in $M_{Pl}^{-1}$. The simple proof of this statement can be red out the Table 2. Indeed, any $SO(10)$ invariant operator containing the fermion superfields in the odd number can be presented as a product $\psi \cdot \Omega_\psi$, where $\psi = F, f_i C$ or $f_i \bar{C}$, and $\Omega_\psi$ is a complementary operator in the same $SO(10)$ representation as $\psi$, built upon the Higgses and even number of
Table 2: The $D$ and $N$ parities of various $SO(10)$ tensors.

| $SO(10)$: $D$ | $f$ | $fC$ | $f^2n$ | $(f \cdot f)^n$ | $\Omega_{\text{Higgs}}$ |
|--------------|-----|------|--------|----------------|------------------|
| $U(1)_A$: $N$ | +   | +    | +      | + / -          | + / -            |

fermions. More precisely, $\Omega_\psi = \Omega_{\text{ferm}} \cdot \Omega_{\text{Higgs}}$, where a tensor $\Omega_{\text{ferm}}$ combines an even number of fermions and a tensor $\Omega_{\text{Higgs}}$ consists entirely of Higgses.

One can characterize all these tensors by the following two parities:

- the $SO(10)$ parity $D$: negative (positive) for tensors with the odd (even) number of the fundamental (10-plet) indices;
- the $U(1)_A$ parity $N$: negative (positive) for combinations with odd (even) value of the $U(1)_A$ charge in units of $x$.

From Table 1 we see that for $\psi = F, f_iC, f_i\bar{C}$ the $N$ parity is always opposite to the $D$-parity. On the contrary, for any tensor $\Omega_{\text{Higgs}}$ the $D$ and $N$ parities always coincide: Higgses with positive $D$ ($1, 45, 54$ representations, among those combinations $CC \sim 1, 45, 210$) all have positive $N$, and Higgses with negative $D$ (10-plets and combinations $CC \sim 10, 120, 126$) have also negative $N$. Clearly, the same is true for any tensor $\Omega_{\text{ferm}}$ composed upon the even number of fermions, e.g. $\sim F^{2n}$ or $(f \cdot f)^n$.

So, $N$ and $D$ parities of $\Omega_\psi$ always coincide, and for $\psi$ itself these parities are always opposite. Thus, the structure of the matter parity breaking operators can never match both the $SO(10)$ and $U(1)_A$ symmetries: the $SO(10)$ invariant terms containing an odd number of fermions are forbidden by the $U(1)_A$ symmetry, and only the operators with the even number of fermions (like the Yukawa terms in (19)) can be allowed. In other words, the theory has an accidental matter parity $Z_2$ under which the fermion superfields $f_i$ and $F$ change sign while the Higgs superfields are invariant.

Hence, our $SO(10)$ model provides an attractive possibility to understand the Baryon and Lepton number conservation in $d = 3$ and $d = 4$ operators, without imposing matter parity (R-parity) in an ad hoc manner. The exact R-parity conservation emerges as an automatic (accidental) consequence of the $U(1)_A$ charge content of the fields in the theory. Note, the additional abelian $U(1)'$ or discrete $\mathcal{R}$ and $Z_2'$ symmetries play no role in deriving this property.

Let us conclude this section with the following remark. The $d = 5$ B and L violating operators contain the even number of fermions and thus they can emerge in the Planck scale cutoff terms. However, in our model they are naturally suppressed by the by the $U(1)_A$ symmetry. Indeed, the family dependent fermion charges $Q(f_i)$ allow the relevant terms only at the following order:

$$\frac{1}{M_P} f_i f_j f_k f_n \left( \frac{X}{M_{Pl}} \right)^{13-i-j-k-n} \Rightarrow \frac{\varepsilon^{13-i-j-k-n}}{M_P} (q_i q_j q_k l_n + u_i^c u_j^c d_k^c e_n^c)$$ (30)
Consider e.g. the dangerous term $q_1 q_1 q_2 l_2$ leading to the decay $p \rightarrow K^+ \nu_\tau$ (recall that the state $l_3 \subset f_3$ in our model is superheavy and the third generation of leptons actually comes from $f_2$). We see that its constant is suppressed by factor $\sim \epsilon^7$ which for $\epsilon \sim 1/10 - 1/20$ can be enough to rise the proton lifetime above the experimental limits.

6 Discussion

We find that the anomalous gauge $U(1)_A$ symmetry can be of great help for building the complete supersymmetric $SO(10)$ model. It could emerge together with the $SO(10)$ gauge group in the string theory context, and play a key role in solving various SUSY GUT puzzles as are the gauge hierarchy and doublet-triplet splitting problem, problem of fermion mass hierarchy, origin of matter parity (or R parity) conservation and so long lifetime of proton. In particular, we have shown some examples of supersymmetric $SO(10) \times U(1)_A$ models which could provide an "all order" stable solution to the D/T problem via the missing VEV mechanism. We have also extended a picture for the fermion masses by involving $U(1)_A$ as a horizontal symmetry. The fermion mass hierarchy as well as the magnitudes of the CKM mixing angles can be naturally understood in terms of small parameter ($\epsilon \sim 1/10 - 1/20$) with a proper choice of the fermion $U(1)_A$ charges. In addition, the $U(1)_A$ charge content of superfields in the theory can be arranged so that R parity breaking operators will be forbidden at any order in $M_p^{-1}$. In other words, the exact conservation of R parity can be an accidental consequence of the gauge symmetry. The suggested pattern for the neutrino masses and mixing can be of phenomenological interest.

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