M-theory Lift of Meta-Stable Brane Configuration in Symplectic and Orthogonal Gauge Groups

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Abstract

The M-theory lift for the supersymmetry breaking IIA brane configuration corresponding to the meta-stable state of $\mathcal{N} = 1$ unitary supersymmetric Yang-Mills theory with massive flavors was found by Bena et al.(hep-th/0608157) recently. We extend this to symplectic and orthogonal gauge groups by analyzing the previously known results on M-theory lifts of supersymmetric IIA brane configurations.
1 Introduction

In the “electric theory”, the standard brane configuration \[1, 2\] consists of \(N_c\) D4-branes stretched between NS5-brane and NS5’-brane along the \(x^6\) direction, with \(N_f\) D6-branes to the left of the NS5-brane, each of which is connected to the NS5-brane by a D4-brane. From this, one obtains \(\mathcal{N} = 1\) \(SU(N_c)\) supersymmetric Yang-Mills theory with massless \(N_f\) flavors by taking a string coupling, a string scale and a distance between NS5-brane and NS5’-brane to be zero \(g_s, \ell_s, \Delta L \to 0\) with fixed gauge coupling where \(1/g_{\text{elec}}^2 = |\Delta L|/g_s \ell_s\) (or equivalently by removing all the massive states).

In the “magnetic theory”, the standard brane configuration \[1, 2\] consists of \((N_f - N_c)\) D4-branes stretched between NS5’-brane and NS5-brane along the \(x^6\) direction, with \(N_f\) D6-branes to the left of the NS5’-brane, each of which is connected to the NS5’-brane by a D4-brane. From this, one obtains \(\mathcal{N} = 1\) \(SU(N_f - N_c)\) supersymmetric Yang-Mills theory with massless \(N_f\) flavors by taking \(g_s, \ell_s, \Delta L, L_0 \to 0\) where \(L_0\) is a distance between D6-branes and NS5’-brane with fixed gauge coupling where \(1/g_{\text{mag}}^2 = \Delta L/g_s \ell_s\) and Yukawa coupling \(h = \sqrt{g_s \ell_s / L_0}\) in the notation of \[3\].

We can turn on a mass matrix for the quarks in the “electric theory” by adding a superpotential \(\text{tr} mQ\tilde{Q}\). In the brane configuration, masses correspond to relative displacement of the D6- and D4-branes in the 45 directions and the relation between mass \(m\) and this relative displacement \(\Delta x\) is given by \(m = \Delta x / \ell_s\). The corresponding field theory “decoupling limit” or “SQCD limit” defined in \[3\] can be done in this case.

Similarly, in the “magnetic theory”, one can deform the magnetic superpotential \(Mq\tilde{q}\) by a linear term in \(M\) where \(M\) is magnetic meson fields. This linear term in the deformed superpotential breaks supersymmetry at tree level (and to all orders in perturbation theory) by the rank condition \[4\]. In this magnetic phase, the minimal-energy supersymmetry breaking brane configuration \[5, 6, 3\] is the analogue of the supersymmetry breaking vacuum found in the magnetic dual to SQCD at tree level, although this brane configuration is not related to the meta-stable state of SQCD \[3\].

M-theory lifts of the supersymmetric brane configurations can be described by M5-branes wrapping holomorphic curves in Taub-NUT multiplied by flat two dimensions. Both the electric and magnetic brane configurations at zero mass lift the same holomorphic curves in M-theory. That is, two different brane setups are described by the same M-theory configuration \[7, 8\] and are connected to each other through electric-magnetic duality. However, for nonzero mass, the only one component M-theory curve reduces to the supersymmetric electric brane configuration as \(R \to 0\) where \(R\) is a radius of eleventh direction, as observed in \[3\]. There is
no smooth limit which corresponds to a magnetic brane configuration.

Recently, the lifting of the tree-level supersymmetry breaking brane configuration was studied in [3] by computing the equations of motion for the minimal area nonholomorphic curves in Taub-NUT multiplied by flat two dimensions. It turns out that there is no meta-stable brane configuration in the “D-brane limit” defined in [3] of MQCD when we consider string interactions. The behavior at infinity of this nonsupersymmetric brane configuration is different from that of the standard supersymmetric ground state of MQCD.

In this paper, we generalize the work of [3] to the meta-stable brane configurations in symplectic and orthogonal gauge groups by recalling the known results [9, 10, 11] on M-theory lifts of the supersymmetric configurations and we compute the equations of motion for the minimal area nonholomorphic curves. There exist some relevant works [12]-[20] along the line of [3].

2 M-theory lift of symplectic gauge theory with massive flavors

Let us describe the type IIA brane configuration of minimal energy supersymmetry breaking with symplectic gauge group with massive flavors by taking magnetic theory of supersymmetric brane configuration and moving the D6-branes to the (45) directions: there exist $(N_f - N_c - 2)$ “color” D4-branes stretched between D6-branes and an NS5-brane along the $x^6$ direction and tilted $(N_c + 2)$ “flavor” D4-branes connecting to the remaining D6-branes and NS5’-brane. In order to study symplectic or orthogonal gauge groups, we need to add an additional orientifold 4-plane [9, 10, 11, 21, 2] with worldvolume (01236) directions which is parallel to the above “color” D4-branes and is not of finite extent in $x^6$ direction. The spacetime reflection of this orientifold 4-plane(O4-plane) acts as $(x^4, x^5, x^7, x^8, x^9) \rightarrow (-x^4, -x^5, -x^7, -x^8, -x^9)$ and therefore each object which does not lie at the fixed points must have its mirror image. One can see the change of the number of D4-branes(which determines the rank of magnetic gauge group) during a moving of NS5-brane across the D6-branes and NS5’-brane and there is a contribution from the O4-plane charge. The intersection of two different kinds of D4-branes above arises from the moving of D6-branes to the (45) directions. The various branes and O4-plane are located as follows:

• One NS5-brane(colored by red) with worldvolume (012345) living at a point in the (6789) directions. The positive piece in 45 directions has its mirror image in the negative region in 45 directions.
• One NS5’-brane(colored by blue) with worldvolume (012389) living at a point in the
(4567) directions. The positive piece in 89 directions has its mirror image in the negative region in 89 directions.

- $N_f$ D6-branes (dotted black) with worldvolume (0123789) living at a point in the (456) directions (i.e., positive in 45 directions and are located at $x^6 = 0$) and its mirrors.

- $(N_f - N_c - 2)$ D4-branes (solid black) with worldvolume (01236) living at a point in the (45789) directions (positive in 45 directions) and its mirrors.

- tilted $(N_c + 2)$ D4-branes (solid black) with worldvolume (01236) living at a point in the (45789) directions and its mirrors.

- One O4-plane (colored by green) with worldvolume (01236) living at a point in the (45789) directions. It is located at $x^4 = x^5 = x^8 = x^9 = 0$.

Now we draw this brane configuration which is nothing new and was found in [6] already and we repeat here for this paper to be self-complete as follows.

Since the brane configuration gives only perturbative effect, in order to get the non pertur-
bative effect, we need to lift type IIA brane configuration to M-theory. The M5-brane configuration is in the 11-dimensional spacetime which is a direct product of flat seven-dimensional spacetime(0123789 directions) and the four-dimensional Taub-NUT space(456 and 10 directions). The M5-branes span 0123 directions and wrap on a Riemann surface inside transverse 45689 and 10 directions. The metric by these six-dimensional transverse directions is given by [22]

$$ds_6^2 = G_{AB} dX^A dX^B = V d\tau^2 + V^{-1} \left(dx^{10} + \vec{\omega} \cdot d\vec{r}\right)^2 + (dx^8)^2 + (dx^9)^2$$

where 456 directions are parametrized by $\vec{r}$, the harmonic function $V$ sourced by the coincident $2N_f$ D6-branes located at $x^4 = x^5 = x^6 = 0$ and the vector potential $\vec{\omega}$ are given by

$$V = 1 + \frac{2N_f R}{r}, \quad \nabla \times \vec{\omega} = \nabla V.$$

The compactification radius $R$ along 10th direction is given by a product of a string coupling $g_s$ and a string scale $\ell_s$ via $R = g_s \ell_s$. Once the half of the D6-branes($N_f$ D6-branes) are located at $N_f$ different 45 directions with $x^6 = 0$, in general, then the remaining half of them(other $N_f$ D6-branes) can be located at the other side of 45 directions(reflected $N_f$ points) with $x^6 = 0$ automatically via an orientifold O4-plane whose worldvolume is the same as the one for D4-branes. See Figure 1.

On the other hand, in the holomorphic coordinates [8], the transverse directions to the noncompact 0123 directions are a product of a real line(7th direction) and a three-dimensional complex manifold(4568910 directions). Then the Taub-NUT space is parametrized by two complex variables $(v, y)$ and flat two-dimensions are by a complex variable $w$. The mass dimensions of these variables are given by

$$[v] = 1, \quad [y] = 4(N_c + 1), \quad [w] = 2$$

respectively. It is clear that the mass dimension of $v$ comes from the Seiberg-Witten curve for $\mathcal{N} = 2$ $Sp(N_c)$ gauge theory with $2N_f$ matter fields. For large $v$, since $w$ goes to $\mu v$ where $\mu$ is a mass of the adjoint chiral multiplet, the mass dimension of $w$ is equal to 2. The mass dimension of $y$ corresponding to a variable $\tilde{t}$ in [9] can be determined by the boundary condition near $w = \infty$. See below the classification 2 characterized by NS' asymptotic region.

The explicit relations between the physical coordinates(4568910 directions) and holomorphic coordinates($v, y$ and $w$) are given by

$$v = \frac{x^4 + ix^5}{\ell^2_s}, \quad y = \mu^{4(N_c+1)} e^{\frac{x^6 - \eta + ix^{10}}{2\mu}} \left(\frac{r + x^6}{R}\right)^{N_f}, \quad w = \frac{x^8 + ix^9}{R\ell^2_s}.$$
Note that in the brane configuration of [3], the location of NS5'-brane is given by \( v = 0 \) and \( x^6 = L_0 \). We insert this normalization constant \( L_0 \) above. The power of dimensionful scale \( \mu \) in \( y \) indicates the correct mass dimension above (2.2). The 11-dimensional Planck scale \( \ell_p \) has a relation \( \ell_p^3 = R \ell_s^2 \) and the product of constant terms in \( v \) and \( w \) gives \( \frac{R}{\ell_p^2} \) which is related to the M5-brane tension. The \( N_f \)-dependent term in harmonic function \( V \) induces the \( N_f \)-dependent term in \( y \) and the precise expression \( \left( \frac{x^6 + x^6}{R} \right) \) will appear also after we solve the function \( g(s) \) later using the equations of motion.

The supersymmetric M5-brane configurations for massless matter are described by two holomorphic curves [9] in the IR free magnetic range of \( N_c + 2 < N_f < \frac{3}{2}(N_c + 1) \) [4] as follows:

\[
\begin{align*}
\mathcal{C}_{NS} : & \quad w(z) = 0, \quad v(z) = z, \quad y(z) = \Lambda_{N=1}^{6(N_c+1)-2N_f} z^{2(N_f-N_c-1)} \\
\mathcal{C}_{NS'} : & \quad w(z) = z, \quad v(z) = 0, \quad y(z) = z^{2(N_c+1)}
\end{align*}
\]

where the \( \mathcal{C}_{NS} \) component of the curve describes the \((N_f - N_c - 2)\) “color” D4-branes ending on the NS5-brane and \( \mathcal{C}_{NS'} \) component of the curve describes the \((N_c + 2)\) “flavor” D4-branes ending on the NS5’-brane [6]. Of course, their mirrors, \((N_f - N_c - 2)\) D4-branes and \((N_c + 2)\) D4-branes exist as a reflection via an orientifold O4-plane. See Figure 1. We follow the convention for the orientifold O4-plane as [21] which is different from what used in [6]. One can easily see that the mass dimensions of (2.2) hold for these curves. Since \( v \) has a charge \((2,0)\) under, the rotations in the 45 directions and 89 directions, the \( U(1)_{45} \times U(1)_{89} \) and \( w \) has a charge \((0,2)\) under the \( U(1)_{45} \times U(1)_{89} \), one can read off the charges for \( y \) and \( \Lambda_{N=1}^{6(N_c+1)-2N_f} \). The former has a charge \((0,4N_c+4)\) and the latter has a charge \((4N_c + 4 - 4N_f, 4N_c + 4)\) under the \( U(1)_{45} \times U(1)_{89} \).

When the quarks have equal mass \( m_f \) (i.e., D6-branes are located at one fixed value for \( v = v_{mf} \) and its mirrors at \( v = -v_{mf} \)), the supersymmetric curve for \( \mu \to \infty \) has only one component [9] and is given by

\[
\begin{align*}
\mathcal{C}_{NS} : & \quad w(z) = z, \quad v(z) = 2^{N_f-2N_c} m_f z_0^2, \quad y(z) = z^{2(N_c+1-N_f)} (z^2 - z_0^2)^{N_f} \\
\mathcal{C}_{NS'} : & \quad w(z) = z, \quad v(z) = 0, \quad y(z) = z^{2(N_c+1)}
\end{align*}
\]

where \( z_0^{N_c+1} = 2^{2N_c-N_f} m_f^{N_f-N_c-1} \Lambda_{N=1}^{3(N_c+1)-N_f} \), \( z_0 \) and \( v \) goes to zero for large \( w \) and \( v \) approaches \( \pm 2^{-N_c+1} m_f \) as \( w \) goes to \( \pm z_0 \). One can compute the energy of the supersymmetry breaking vacuum, \( V_0 \), using the effective field theory or the DBI action (i.e., the length of D4-branes). The former can be written as [3, 4]

\[
V_0 = \frac{2(N_c + 2)|\Delta x|^2}{g_s \ell_s^6 L_0} \quad \text{(2.4)}
\]
and the latter can be written, by noting that there exist \((N_f - N_c - 2)\) D4-branes between D6-branes and NS5-brane in Figure 7 of [3] or in Figure 1 and its mirrors and \((N_c + 2)\) D4-branes between D6-branes and NS5’-brane and its mirrors, as

\[
V_{DBI} = \frac{1}{g_s \ell_s^5} \left( 2(N_c + 2) \sqrt{|\Delta x|^2 + L_0^2} + 2(N_f - N_c - 2)(L_0 + |\Delta L|) \right)
\]

(2.5)

by summing up all the contributions when we take the product of D4-brane tension and the length of D4-brane. In the limit of \(|\Delta x| \ll L_0\), the \(|\Delta x|\)-dependent part of (2.4) and (2.5) agrees with each other. The energy difference between the tachyonic state and the vacuum is given by

\[
\Delta V_{DBI} = V_{tach} - V_{DBI} = \frac{1}{g_s \ell_s^5} 2(N_f - N_c - 2) \left( \sqrt{|\Delta x|^2 + L_0^2} - L_0 \right)
\]

which is the same as the energy difference coming from field theory in the limit \(|\Delta x| \ll L_0\) and \(V_{tach} = \frac{1}{g_s \ell_s^5} (2N_f \sqrt{|\Delta x|^2 + L_0^2} + 2(N_f - N_c - 2)|\Delta L|)\) which can be obtained from Figure 6 of [3] as follows: \((N_f - N_c - 2)\) D4-branes between NS5’-brane and NS5-brane and \(N_f\) D4-branes between D6-branes and NS5’-brane as well as their mirrors.

Now one can see the behavior of the supersymmetric curves [9] in various limits as follows:

1. \(v \to \infty\) limit implies

\[
w \to 0, \quad y \to \Lambda^6_{N=1} \Delta^2_{N=1} v^{2(N_f - N_c - 1)} + \ldots \quad \text{NS asymptotic region}
\]

2. \(w \to \infty\) limit implies

\[
v \to m_f, \quad y \to w^{2(N_c + 1)} + \ldots \quad \text{NS’ asymptotic region}
\]

Note that the \(v\) here is different from \(v\) of [9] by a constant shift \(m_f\). Instead of moving D6-branes in the \(v\) direction, the NS5’-brane is moving in the \(v\) direction holding everything else fixed. The mirrors of “color” and “flavor” D4-branes are displaced appropriately. Note that in Figure 1, the origin for 45 directions is located at the position of NS5-brane while in the coordinate we use in the above NS’ asymptotic region, it is located at the position of mirror \(N_f\) D6-branes. Therefore, the location of original \(N_f\) D6-branes is given by \(2m_f\).

3. The map between the holomorphic and physical coordinates requires the condition [3]

\[
y = 0 \quad \text{only if} \quad v = 0.
\]

For the holomorphic massless curve one can see the two brane configurations, one for electric case and the other for magnetic case [9, 11]. In other words, the emergence of the
dual $Sp(N_f - N_c - 2)$ gauge group arises by looking at the same M-theory M5-brane in two different limits. However, for nonzero mass, the only one component M-theory curve (2.3) reduces to the supersymmetric electric brane configuration as $R \to 0$.

The dependence of $x^{10}$ in the curve [3] comes from the phase of $w$ as follows:

$$\arg (x^8 + ix^9) = \frac{x^{10}}{4(N_c + 1)R}.$$ 

Here the $U(1)_{89}$ rotation by $\frac{\pi}{2(N_c+1)}$ leaves the M-theory curve invariant. In particular, the charges of $\Lambda_{N=1}^{6N_c + 6 - 2N_f}$ imply that the groups $U(1)_{45}$ and $U(1)_{89}$ are broken to their discrete subgroups $\mathbb{Z}_{4(N_c+1-N_f)}$ and $\mathbb{Z}_{4(N_c+1)}$ respectively [9, 11]. In the brane configuration, the above $U(1)_R$ is identified with rotations in the 89 plane and the anomaly of the corresponding $U(1)_R$ symmetry group is identified as a shift in $x^{10}$ under rotations in the 89 plane [23].

Now it is ready to consider M5-brane wrapping a non holomorphic minimal area surface in the Taub-NUT multiplied by flat two dimensions satisfying the boundary conditions above [3]. The solution for non-holomorphic curve can be written as

$$x^4 = f(s), \quad x^5 = 0, \quad x^8 + ix^9 = e^{\frac{x^{10}}{4(N_c+1)R}}g(s), \quad x^6 = s. \tag{2.6}$$

We would like to solve the equations of motion for unknown functions $f(s)$ and $g(s)$. The action of a surface parametrized by $x^6$ and $x^{10}$ is given by

$$A = \int d^2z \sqrt{\det G_{AB} \partial_a X^A \partial_b X^B} \tag{2.7}$$

where the six-dimensional metric $G_{AB}$ is given by (2.1). Then the action, by inserting the solution (2.6) and (2.1) into the action (2.7), can be written as

$$A = \int ds \sqrt{\left[V^{-1} + \frac{g^2}{16R^2(N_c+1)^2}\right] \left[V(1 + f'^2) + g'^2\right]}$$

where the harmonic function becomes by realizing that the original $N_f$ D6-branes are located at $v = 2\Delta x/\ell_s^2$(and $x^6 = 0$) and its mirrors are located at $v = 0$(and $x^6 = 0$)

$$V = 1 + \frac{N_f R}{\sqrt{(f - 2\Delta x)^2 + s^2}} + \frac{N_f R}{\sqrt{f^2 + s^2}}. \tag{2.8}$$

Compared with the case for $SU(N_c)$ gauge theory [3], the Lagrangian doesn’t have any big difference in the sense that the color- and flavor-dependent terms are a little bit different. Then, one can follow similar procedure of [3] and arrives at an exact expression for the function $f(s)$ with modified $N_c$- and $N_f$-dependent terms. However, when we look at (A.4) of
Since it does not depend on $N_c$ or $N_f$, the solution for $f(s)$ for symplectic group case is the same as the one in $SU(N_c)$ case. There exists always a straight line solution $f''(s) = 0$ as in [3].

When

$$f(s) = \Delta x$$

which satisfies $f''(s) = 0$, then the equation (A.3) of [3] implies that $g'(s) = \frac{V}{4(N_c+1)R} g(s)$ with $V(s) = 1 + \frac{2N_f R}{\sqrt{(\Delta x)^2 + s^2}}$. So this first order differential equation provides the following solution

$$g(s) = R\ell_s^2 \mu^2 e^{\frac{s-L_0}{4(N_c+1)R}} \left( \frac{s + \sqrt{(\Delta x)^2 + s^2}}{R} \right)^{\frac{N_f}{2(N_c+1)}}. \quad (2.9)$$

The integration constant which is s-independent term is fixed by the boundary condition where $y \to w^{2N_c+2}$ we have discussed before and is given by $R\ell_s^2 \mu^2 e^{-\frac{L_0}{4(N_c+1)R}} \left( \frac{1}{R} \right)^{\frac{N_f}{2(N_c+1)}}$. In other words, this is a simple solution $v = \frac{\Delta x}{2} = m_f$ and $y = w(2N_c+1)$. Even if $\Delta x$ is equal to zero, the function $g(s)$ doesn’t vanish implying that both $w$ and $y$ don’t vanish. Therefore, this doesn’t lead to the classification 3 above: it does not end on D6-brane. Therefore, there is no smooth non-holomorphic M-theory curve.

Instead of imposing the boundary condition at large $s$, we require that M5-branes end on the D6-branes: $f(s)$ at $s = 0$ vanishes. For the case of

$$f(s) = cs$$

which satisfies $f''(s) = 0$, the (A.3) of [3] implies that there exists $g'(s) = \frac{\sqrt{1+f'(s)^2}}{4(N_c+1)R} V g(s)$ and when $L = L_0$ and $c = \frac{\Delta x}{2L_0}$, this straight line solution will lead to the type IIA brane configuration [3] in the $R \to 0$ limit. However, the behavior at infinity is different from the above classification 1 and 2. Therefore, the supersymmetric brane configuration and the supersymmetry breaking brane configuration are vacua of different theories because the boundary conditions at infinity are different.

Also the analysis for a kink quasi-solution [3], in order to understand the absence of metastable vacuum in the “D-brane limit” of MQCD, can be done similarly. For given expression
of (2.9) corresponding to \( x^6 > L \), the magnitude of \( w \) and \( v \) are given by

\[
x^6 < L : \quad |x^8 + ix^9| = R\ell_6^2 \mu^2 e^{-\frac{(L_0-L)}{4(N_c+1)R}} R^{4(N_c+1)} \left( \frac{x^6 + \sqrt{(x^6)^2 + (\Delta x)^2}}{R} \right)^\frac{N_f}{2(N_c+1)}
\]

\[
x^6 > L : \quad |x^8 + ix^9| = R\ell_6^2 \mu^2 e^{\frac{(x^6-L_0)}{4(N_c+1)R}} \left( \frac{x^6 + \sqrt{(x^6)^2 + (\Delta x)^2}}{R} \right)^\frac{N_f}{2(N_c+1)}
\]

\[
x^4 = \frac{\Delta x}{L} x^6,
\]

\[
x^4 = \Delta x.
\]

Plugging these into the action, one gets the potential that has runaway behavior as for \( SU(N_c) \) case [3] which does not depend on \( N_c \) or \( N_f \) and that is proportional to the energy difference \( \Delta V_{DBI} \) with a replacement \( L_0 \) by \( L \) we have considered before.

Let us make some comments on the solution for the differential equation satisfying (A.4) in [3]. Also in our case, we have the same differential equation as (A.4) of [3] because we didn’t put the form of the potential (2.8) in arriving at (A.4). In [3], they have tried to search for the possibility for the other solutions with the right boundary conditions by substituting the explicit form for the potential and have obtained nonlinear differential equation (A.5) of [3]. Miraculously, the exact solution for the \( f(s) \) was found through (A.5)-(A.10) of [3]. However, since our potential has an extra piece in (2.8), the third order nonlinear differential equation for \( f(s) \) cannot be solved exactly. The above solutions \( f(s) = \Delta x \) and \( f(s) = cs \) are particular solutions and, in principle, there could exist a solution having the correct boundary conditions both at infinity and at the D6-brane with \( f''(s) \neq 0 \). It seems to be difficult to construct this solution even if one uses the numerical analysis for the complicated differential equation.

In summary, we have considered the lifting of supersymmetry breaking brane configurations of [4, 6] for symplectic gauge group with massive flavors. The main object was M5-branes wrapping nonholomorphic minimal area curves in Taub-NUT times flat two dimensions. So we have found equations of motion for the ansatz characterized by (2.6) with the same boundary conditions as supersymmetric vacua [9, 11] and it turned out there was no meta-stable brane configuration in the “D-brane limit” of MQCD (at least for the above particular solutions \( f(s) = \Delta x \) and \( f(s) = cs \), like as \( SU(N_c) \) gauge group case [3].
3 M-theory lift of orthogonal gauge theory with massive flavors

As for previous section, the type IIA brane configuration of minimal energy supersymmetry breaking with orthogonal gauge group with massive flavors can be understood similarly: 
\((N_f - \frac{N_c}{2} + 2)\) “color” D4-branes stretched between D6-branes and an NS5-brane along the \(x^6\) direction and tilted \((\frac{N_c}{2} - 2)\) “flavor” D4-branes connecting to the remaining D6-branes and NS5’-brane. We focus on even \(N_c\) case. For odd \(N_c\), an extra single D4-brane is stuck at the O4-plane and this extra D4-brane is not affected by a moving of D6-branes to 45 directions. One can see the change of the number of D4-branes (which determines the rank of magnetic gauge group) during a moving of NS5-brane across the D6-branes and NS5’-brane and there is a contribution from the O4-plane charge which is different from the one for symplectic gauge group. The intersection of two different kinds of D4-branes above arises from the moving of D6-branes to the 45 directions. The various branes and O4-plane are located as follows:

- One NS5-brane (colored by red) with worldvolume (012345) living at a point in the (6789) directions.
- One NS5’-brane (colored by blue) with worldvolume (012389) living at a point in the (4567) directions.
- \(N_f\) D6-branes (dotted black) with worldvolume (0123789) living at a point in the (456) directions (i.e., positive in 45 directions and are located at \(x^6 = 0\)) and its mirrors.
- \((N_f - \frac{N_c}{2} + 2)\) D4-branes (solid black) with worldvolume (01236) living at a point in the (45789) directions (positive in 45 directions) and its mirrors.
- tilted \((\frac{N_c}{2} - 2)\) D4-branes (solid black) with worldvolume (01236) living at a point in the (45789) directions and its mirrors.
- One O4-plane (colored by green) with worldvolume (01236) living at a point in the (45789) directions. It is located at \(x^4 = x^5 = x^8 = x^9 = 0\).

Now we repeat the brane configuration here for this paper to be self-complete as follows.

In this section, we can proceed for orthogonal gauge group based on the previous analysis and let us describe the main facts. The mass dimensions of complex variables are given by

\[
[v] = 1, \quad [y] = 2(N_c - 2), \quad [w] = 2. \tag{3.1}
\]

The mass dimension of \(v\) comes from the Seiberg-Witten curve for \(\mathcal{N} = 2\) \(SO(N_c)\) gauge theory with \(2N_f\) matter fields. For large \(v\), since the \(w\) behaves as \(\mu v\), its mass dimension is equal to 2. The mass dimension of \(y\) corresponding to a variable \(\tilde{t}\) in [10] can be determined by the boundary condition near \(w = \infty\). See below the classification 2 characterized by NS’
Figure 2: The type IIA brane configuration of minimal energy supersymmetry breaking with orthogonal gauge group $SO(2N_f - N_c + 4)$ with $2N_f$ massive flavors. For simplicity, we take equal flavor masses and even number of $N_c$. For odd $N_c$, an extra single D4-brane is stuck at the O4-plane denoted by green dotted line. The negative O4-plane charge appears between NS5'-brane and NS5-brane. The O4-plane charge flips sign whenever one crosses a D6-brane, NS5-brane, or NS5'-brane.

asymptotic region. The holomorphic coordinates $(v, y$ and $w)$ are given by

$$v = \frac{x^4 + ix^5}{\ell_s^2}, \quad y = \mu^{2(N_c-2)} e^{\frac{x^6-x^4+ix^{10}}{2R}} \left( \frac{r + x^6}{R} \right)^{N_f}, \quad w = \frac{x^8 + ix^9}{R\ell_s^2}.$$

The power of dimensionful scale $\mu$ in $y$ indicates the correct mass dimension above (3.1).

The supersymmetric M5-brane configurations for massless matter are described by two holomorphic curves \[10\] in the IR free magnetic range of $N_c - 4 < 2N_f < \frac{3}{2}N_c - 2$ \[4\] as follows:

$$\mathcal{C}_{NS} : w(z) = 0, \quad v(z) = z, \quad y(z) = \Lambda_{N=1}^{3N_c-6-2N_f} z^{2N_f-N_c+2}$$

$$\mathcal{C}_{NS'} : w(z) = z, \quad v(z) = 0, \quad y(z) = z^{N_c-2}$$

where the $\mathcal{C}_{NS}$ component of the curve describes the $(2N_f - N_c + 4)$ D4-branes ending on the NS5-brane and $\mathcal{C}_{NS'}$ component of the curve describes the $(N_c - 4)$ D4-branes ending on the NS5'-brane. Note that these are total number of D4-branes including their mirrors. For
even $N_c$, one can imagine half of the total “color” $(2N_f - N_c + 4)$ D4-branes are located at a fixed value of $v$ above an orientifold O4-plane and its mirrors are located below an orientifold O4-plane. For odd $N_c$, a single D4-brane is stuck at the orientifold O4-plane and half of the remaining D4-branes and its mirrors are located like as for the even $N_c$ case \[21\]. Also this consideration of the locations of D4-branes holds for “flavor” $(N_c - 4)$ D4-branes. See Figure 2. The charges of $y$ and $\lambda_{\sum_{N=1}^{N_c-6-2N_f} N_c - N_f + 4}$ are \( (0, 2N_c - 4) \) and \( (2N_c - 4 - 4N_f, 2N_c - 4) \) under the $U(1)_{45} \times U(1)_{89}$ respectively which can be determined as previous section without any difficulty.

The behavior of the supersymmetric curves \[10\] in various limits can be summarized as follows:

1. $v \to \infty$ limit implies
   
   \[ w \to 0, \quad y \to \lambda_{\sum_{N=1}^{N_c-6-2N_f} N_c - N_f + 2} + \cdots \quad \text{NS asymptotic region} \]

2. $w \to \infty$ limit implies
   
   \[ v \to m_f, \quad y \to w^{N_c-2} + \cdots \quad \text{NS' asymptotic region} \]

The energy of the supersymmetry breaking vacuum $V_0$ from the field theory side can be written as \[4\] with appropriate normalization \[4\]

\[
V_0 = \frac{(N_c - 4)|\Delta x|^2}{g_s \ell_s^5 L_0}
\]

and that from the DBI action can be written as

\[
V_{DBI} = \frac{1}{g_s \ell_s^5} \left( (N_c - 4) \sqrt{\Delta x^2 + L_0^2} + (2N_f - N_c + 4)(L_0 + |\Delta L|) \right)
\]

by realizing that there are \((2N_f - N_c + 4)\) color D4-branes and \((N_c - 4)\) flavor D4-branes altogether in Figure 7 of \[3\] or Figure 2 and summing up all the contributions. In the limit of \(|\Delta x| \ll L_0\), the \(|\Delta x|\)-dependent part for the energies agrees with each other. The energy difference between the tachyonic state and the vacuum is given by

\[
\Delta V_{DBI} = V_{tach} - V_{DBI} = \frac{1}{g_s \ell_s^5} (2N_f - N_c + 4) \left( \sqrt{\Delta x^2 + L_0^2} - L_0 \right)
\]

which is the same as the energy difference coming from field theory in the limit \(|\Delta x| \ll L_0\) and \(V_{tach} = \frac{1}{g_s \ell_s^5} (2N_f \sqrt{\Delta x^2 + L_0^2} + (2N_f - N_c + 4)|\Delta L|)\) where there exist \(2N_f\) flavor D4-branes and \((2N_f - N_c + 4)\) color D4-branes in Figure 6 of \[3\].

3. The map between the holomorphic and physical coordinates requires the condition \[3\]

\[
y = 0 \quad \text{only if} \quad v = 0.
\]
For the holomorphic massless curve one can see the two brane configurations, one for electric case and the other for magnetic case \[10, 11\]. In other words, one sees the emergence of the dual $SO(2N_f - N_c + 4)$ gauge group by looking at the same M-theory M5-brane in two different limits. However, for nonzero mass, the only one component M-theory curve reduces to the supersymmetric electric brane configuration as $R \to 0$.

The dependence of $x^{10}$ in the curve $[3]$ comes from the phase of $w$ as follows:

$$\arg (x^8 + ix^9) = \frac{x^{10}}{2(N_c - 2)R}. $$

In particular, as in previous case, the charges of $\Lambda_{N=1}^{3N_c-6-2N_f}$ imply that the groups $U(1)_{45}$ and $U(1)_{89}$ are broken to their discrete subgroups $\mathbb{Z}_{2(N_c-2-2N_f)}$ and $\mathbb{Z}_{2(N_c-2)}$ respectively $[10]$.

The solution for non-holomorphic curve can be written as

$$x^4 = f(s), \quad x^5 = 0, \quad x^8 + ix^9 = e^{i \frac{x^{10}}{2(N_c-2)R}} g(s), \quad x^6 = s. \quad (3.2)$$

By inserting the solution (3.2) and (2.1) into the action (2.7), the action can be written as

$$A = \int ds \sqrt{V^{-1} + \frac{g'^2}{4R^2(N_c - 2)^2}} [V(1 + f'^2) + g'^2]$$

which is almost the same form as previous section and only the color-dependent term is different. The harmonic function is the same as before and is given by (2.8).

When $f(s) = \Delta x$, the first order differential equation provides the following solution

$$g(s) = R\ell_s^2 \mu^2 e^{\frac{s - L_0}{\Delta x}} \left( s + \sqrt{(\Delta x)^2 + s^2} \right)^{\frac{N_f}{(N_c-2)}}.$$

This is a simple solution $v = \frac{\Delta x}{\ell_s} = m_f$ and $y = w^{N_c-2}$. Even if $\Delta x$ is equal to zero, the function $g(s)$ doesn’t vanish implying that both $w$ and $y$ don’t vanish. There is no smooth non-holomorphic M-theory curve. On the other hand, for the case of $f(s) = cs$, it is easy to see that when $L = L_0$ and $c = \frac{\Delta x}{L_0}$, this straight line solution will lead to the type IIA brane configuration $[6]$ in the $R \to 0$ limit. However, the behavior at infinity is different from the above classification 1 and 2.

Also the analysis for a kink quasi-solution, in order to understand the absence of metastable vacuum in the “D-brane limit” of MQCD, can be done similarly as in previous case.

In summary, we have described the lifting of supersymmetry breaking brane configurations of $[4, 6]$ for orthogonal gauge group with massive flavors. We have found equations of motion for the ansatz characterized by (3.2) with the same boundary conditions as supersymmetric
vacua [10,11]. There was no meta-stable brane configuration in the “D-brane limit” of MQCD, like as $SU(N_c)$ gauge group case [3] or $Sp(N_c)$ gauge group case in previous section.

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