Tests of CPT

Shmuel Nussinov

Tel Aviv University,
Sackler School Faculty of Sciences,
Tel Aviv 69978, Israel
and
Schmid Science Center,
Chapman University,
Orange, California 92866, USA

Abstract

The ongoing experimental efforts in the high energy and high precision communities keep providing evidence for CPT, a fundamental symmetry holding in any local Lorentz invariant theory. We suggest possible interconnections between different CPT violating parameters. Specifically, the very precise test of CPT in the $K^0 - \bar{K}^0$ system suggests—though definitely does not imply—that CPT violations in other observable parameters (mass, width, charge, magnetic moments, etc.) are much smaller than the directly measured bounds.

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The discrete symmetries of $P$, the reflection of the three-space coordinates, $T$, time reversal, and charge conjugation, $C$, are not separately conserved. Indeed both $C$ and $P$ are maximally violated by the charged current part of the weak interaction Lagrangian. Also our present understanding of CP violations implies that their apparent smallness reflects small CKM mixing rather than any intrinsic, approximate, conservation.

However, CPT symmetry holds in all local, Lorentz invariant, field theories in four dimensions.

The locality and ensuing analyticity of the $n$ point functions in momentum space allow a “Wick” rotation which is an analytic continuation to Euclidean space. $PT$ then becomes the complete inversion, $x_i \rightarrow -x_i$, $i = 1, 2, 3, 4$ in Euclidean four-dimensional space. Unlike for odd dimensions, this inversion is a rotation and not a separate discrete transformation. Hence, $PT$ cannot be violated in a local Lorentz invariant theory of neutral scalar bosons.

The fact that for complex spinors and other charged fields we need also $C$, charge conjugation, and account for the anti-linear nature of time inversion to get the CPT theorem is, however, highly nontrivial.

CPT symmetry implies the equality of masses of particle and anti-particles, and for unstable particles the equality of total widths $\Gamma$ and $\bar{\Gamma}$. Also the equality of electromagnetic and other gauge couplings and ensuing magnetic moments follow from CPT.

Some 40 experimental tests of CPT are listed in the PDG. Let $\delta_p(X)$ be the precision with which the equality of property $p$ (mass, width, charge, magnetic moment, etc.) of a particle $X$ and the anti-particle $\bar{X}$ has been verified:

$$\delta_p(X) = \frac{|p(X) - p(\bar{X})|}{p(X)}$$

The precisions vary over a very wide range and often are not better than $\sim 10^{-5}$. In some cases where special efforts have been made, e.g., the proton–anti-proton mass difference, $\delta_m(p) < 2 \cdot 10^{-9}$, was obtained.

When $\bar{X} - X$ bound positronium-like states are available, some properties of the bound particle and anti-particle have been shown to be equal to within $10^{-12}$.

In one single case involving the $K^0 - \bar{K}^0$ system stunning accuracies of:

$$\delta_m(K^0) < 10^{-18}$$

$$\delta_{\Gamma}(K^0) < 10^{-17}$$
has been achieved, reflecting the very well studied $K_L - K_S$ and $K^0 - \bar{K}^0$ oscillations in this relatively long-lived system. In view of the importance of the CPT theorem this is indeed most gratifying. In passing we note that this system allowed also precise tests of the equality of the gravitational couplings of $K^0$ and $\bar{K}^0$, and a sensitive search for possible deviations from quantum mechanics.

New studies of CPT conservation are underway. One example is measuring the equality of the top ($t$) and $\bar{t}$ quark masses for which a precision of $\delta_m(t) \sim 2.2$ was achieved [2]. This is quite modest compared with the more precise CPT tests. Yet it is worthwhile and may be justified not only by the experimental challenge of finding the best bounds on any $\delta_p(X)$. Some models try to explain the large $m(t)$ by having the top quark experience effects of higher dimensional physics more than the other quarks so that some of the assumptions underlying the proof of the CPT theorem may be slightly violated there. Omitting such subtleties we would like to make the following observation:

Barring unlikely fine-tuned cancelations, the very precise $\delta_m(K^0)$ and $\delta_\Gamma(K^0)$ suggest much stronger upper bounds on CPT violations involving quarks than what can be achieved via direct measurements.

Thus consider first $\delta_m(t)$, $\delta_m(c)$, $\delta_m(u)$ and $\delta_m(W)$. Since weak interactions interconnect different quark (mass eigenstates), any appreciable CPT violation in the $W$ or in the quark masses “trickles down” and affects $\delta_m(K)$, $\delta_\Gamma(K)$.

Thus consider the specific “radiatively-induced” difference of the $s$ and $\bar{s}$ quarks generated via charged current weak interactions by the $t - W$ intermediate in the Feynman diagrams of Fig 1.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{FIG. 1:}
\end{figure}
Unlike each diagram separately, the difference is finite and calculable. It has the form:

\[ \delta_{m}^{1}(s) = C(ts)\alpha_{W}/(2\pi)\ (V_{ts}^{2} \ m(t) - V_{t,s}^{2} \ m_{t}) + C'(t,s)\alpha_{W}/(2\pi)\ (V_{ts}^{2} \ m(W) - V_{t,s}^{2} \ m(\bar{W}) \]

with \( C, C' \) pure numbers depending on mass ratios, the \( V \)'s elements of the CKM matrix and \( \alpha_{W} \), the weak coupling at the relevant scale.

The above difference can be rewritten as:

\[ \delta_{m}^{1}(s) = [\alpha_{W}]/[C[V^{2} - \bar{V}^{2}]](m_{t} + m_{t}) + [V^{2} + \bar{V}^{2}](m_{t} - m_{t}) + [\alpha_{W}]/[C'[V^{2} - \bar{V}^{2}](m_{W} + m_{\bar{W}})] + [V^{2} + \bar{V}^{2}](m_{W} - m_{\bar{W}}) \] (4)

This expression clearly separates between the contributions to \( \delta_{m}(s) \) due to \( t - \bar{t} \) (or \( W^{–} - W^{+} \)) mass asymmetry and those generated by different \( V = V_{t,s} \) and \( \bar{V} = V_{t,s} \) CKM mixings. Similar expressions can be written for the contribution of the \( c-W \) and of \( u-W \) intermediate states, generating altogether 12 terms.

Barring unlikely cancelations between these 12 different terms contributing to the strange quark and the anti-quark masses we can use the small \( \bar{s} - s \) mass difference to bind any of the above terms:

\[ m_{s} - m_{\bar{s}} > V^{2}_{t,s} \alpha_{W}/(2\pi)(m_{t} - m_{t}) \] (5)
\[ m_{s} - m_{\bar{s}} > V^{2}_{c,s} \alpha_{W}/(2\pi)(m_{c} - m_{c}) \] (6)
\[ m_{s} - m_{\bar{s}} > V^{2}_{u,s} \alpha_{W}/(2\pi)(m_{u} - m_{u}) \] (7)

and

\[ m_{s} - m_{\bar{s}} > [V^{2}_{u,s} + V^{2}_{c,s} + V^{2}_{t,s}](m_{W^{–}} - m_{W^{+}}) = m_{W^{–}} - m_{W^{+}} \] (8)

Such considerations apply also to the mass difference of the \( \bar{d} \) and \( d \) quarks yielding the analogs of all the above relations with \( d \leftrightarrow s \) everywhere. Thus we have

\[ m_{d} - m_{\bar{d}} > V^{2}_{t,d} \alpha_{W}/(2\pi)(m_{t} - m_{d}) \], etc. (9)

Similar bounds are obtained for mass weighted asymmetries of the CKM matrix elements for quarks and anti-quarks.

In addition to the above “flavor off-diagonal contribution” to the mass differences of the \( d \) and \( s \) quarks and corresponding anti-quarks, we have the \( Z \), photon and gluon exchange contributions.
The next and last step in connecting with the experiment is to find how a putative $\delta_m(s)$ and/or $\delta_m(d)$ reflects in the measured $\delta_m(K) < 10^{-18}$.

Strictly speaking, this involves nonperturbative QCD and may require lattice calculations of mesonic “$\sigma$ terms”. For a rough estimate we use the expression for the masses of Nambu-Goldstone boson in terms of bare quark masses: 

$$m^2(K^0) = [m^0(s) + m^0(d)] < \bar{q}q > / (f^2(K^0))$$  
$$m^2(\bar{K}^0) = [m^0(\bar{s}) + m^0(\bar{d})] < \bar{q}q > / (f^2(\bar{K}^0))$$  

(10)

By subtracting these two equations we relate $\delta_m(K)$ to $\delta_m(s)$ and $\delta_m(d)$ (and to the difference of decay constants $f(K^0) - f(\bar{K}^0)$). Again assuming no cancelations and using $i\bar{q}q \sim (300 \text{ MeV})^3$, $f(K) \sim 150 \text{ MeV}$, $\alpha_W \sim 1/(20)$ and the values of quark masses and mixing parameters from the PDG we finally obtain:

$$\delta_m(t) = [m_t - m_\bar{t}] / (m_t)$$  
$$= \delta_m(K)[m(K)/(m(t))]2\pi / (\alpha_W) V_{ts}^{-2} \sim 10^{-18} V_{ts}^{-2} = 10^{-15}$$

$$\delta_m(W) = [m_{W^-} - m_{W^+}] / (m_W)$$

$$\delta_m(K) = [m(K)/(m(W))]2\pi / (\alpha_W) / ([V_{ts}^2 + V_{cs}^2 + V_{us}^2]) \sim 10^{-18}$$  

(11)

(See footnote.)

$$\Delta m(c) \sim 10^{-16} / (V_{cs}^2) \sim 10^{-16}$$  

(12)

In the above we implicitly assumed that QCD-gluon exchange interactions are equally strong for the $\bar{s} - d$ and $\bar{d} - s$ systems. Hence using the same line of reasoning and excluding accidental cancelations between CPT violations in SU(3)$_c$ couplings and in bare quark masses we conclude also that:

$$g_{s,d}g_{d} - g_{d,s}g_{s} < 10^{-18}$$  

(13)

All the above exceed by many orders of magnitude the direct bounds for these asymmetries now and in the foreseeable future.

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1 The last inequality may be weaker by two orders of magnitude due to the following: To have non-vanishing radiative corrections to the quark masses, we do need some bare quark masses breaking the chiral invariance. Thus while the $W^+ - W^-$ mass difference is essential for the CPT asymmetry in the above terms, the radiative corrections may involve an extra smaller, say, $c$ quark mass of $\sim 1 \text{ GeV}$ rather than 100 GeV.
Many other bounds, in particular, on differences between mixings of quarks and of anti-
quarks in the CKM and “$\bar{C}KM$ matrix” are suggested by the almost equally stringent bound
on the difference of widths:

$$\delta_{r}(K^0) < 10^{-17} \tag{14}$$

We will not discuss these relations here.

It is useful to elaborate a bit more on the philosophy underlying this note. Altogether
$\delta_{m}(K^0)$ can be expressed as a sum of $\sim 20$ terms depending on quark masses and gauge
coupling, where each term violates CPT. Since there is no separate control of each term, an
almost complete cancelations of relatively large CPT violations in each of these terms in the
overall sum cannot be excluded.

We note, however, that the CPT theorem holds not only in our specific “standard model”
with its $\sim 17$ independent parameters, but in any other local–standard model-like–field
theory. Thus we can vary the strength of the gauge couplings, the mass scales of QCD and
of the weak interactions, and the various diagonal and off-diagonal couplings of the Higgs
particle to quarks and leptons and CPT should hold equally well.

Phrased differently, all observable CPT violations should trace back to CPT asymmetries
in these fundamental, underlying parameters or any other set of parameters in any yet more
fundamental underlying theory. Hopefully such a more fundamental theory will have a
smaller number of independent parameters.

The theory must deviate in some way from purely local Lorentz invariant field theory
so as to allow for violation of the CPT theorem. Most likely all CPT violations will then
trace back just one or very few novel features such as some form of non-locality. In such
a scenario it seems extremely unlikely that the one or two sources of CPT violation will be
large so as to yield large CPT asymmetries in each of the above $\sim 20$ terms and yet conspire
to have the incredible precise cancelations in $\delta_{m}(K^0)$.

In some “landscape approaches” it is believed that we live in one particular string theory
vacuum with many of the specific SM parameters fixed by anthropic considerations. In any
case we cannot perform the above Gedunken experiment of dialing the various parameters
and verify that the measured $\delta_{m}(K)$ is equally small in all cases.

Having no reason to believe that a larger CPT violation in the $K - \bar{K}$ system, say,
$\delta_{m}(K) \sim 10^{-10}$, would prevent intelligent life, it seems unlikely that our vacuum conspired to
minimize $\delta_{m}(K)$. Assuming then that the various terms contributing to $\delta_{m}(K)$ have random
relative signs the probability that their sum will be so small unless these are separately small is truly tiny. Hence we adopted the “No fine-tuned cancelation” hypothesis and looked for its consequences.

Finally we would like to briefly comment on the equality of the electron and (minus) the positron charges. The direct bound quoted in the PDG is:

$$\delta_q(e) = (q_{e^-} + q_{e^+})/(|q_e|) < 4 \cdot 10^{-8}$$  \hspace{1cm} (15)

A much better indirect bound can be obtained if we assume electric charge conservation as follows:

Charge conservation in the annihilation: $e^+ e^- \rightarrow \gamma + \gamma$ implies that $\delta_q(e) = q(\gamma)/(|q_e|)$. The charge of photons is, however, strongly bound, $q(\gamma) < 10^{-33}(|q_e|)$! by using the coherence in the Brown, Hanbury, Twiss effect measuring the relative phase between two paths of a photon.

For a charged particle this phase is the appreciable Aharonov-Bohm phase $\sim$ to the total flux enclosed; hence, the very strong bound.\[4\]

I would like to thank Tom Ferbel for telling me about the recent measurement of top and anti-top masses which inspired this short note.

* Electronic address: nussinov@post.tau.ac.il

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