AC gate effects on a nano-electromechanical single electron transistor

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Abstract. AC gate effects on electron transport of a nano electromechanical single electron transistor having a moveable nano particle in the tunnel regime are theoretically studied. The AC gate drives the nano particle vibrations, giving rise to resonance of the vibrations at the natural frequency $\omega_0$ of the confining potential and fractional magnitudes of $\omega_0$. The current and noise show remarkable increase with respect the gate frequency, reflecting the dynamic response of the nano particle vibrations to the gate. Artificial shuttle mechanism is confirmed under certain situation.

1. Introduction
In the last decade, combination of electron transport and dynamics of nano scale structures has developed a variety of possibilities concerning mass and position detection in a quantum limit [1, 2, 3], quantum computation [4], and electron transport associated with the dynamics of nano structure [5, 6]. One of the pioneering studies of such the nano electromechanical systems(NEMS) was given by Gorelik and coworkers [5]. Considering a single electron transistor(SET) having a moveable nano particle, they suggested a transport mechanism termed the shuttle mechanism that spontaneous vibrations of the nano particle carries charges between the electrodes, which is also referred to as shuttling. The shuttle mechanism works when electromechanical instability of the nano particle position takes place for given strength of bias and coupling to heat bath [6, 7]. Unless the shuttle instability occurs, the nano particle remains around the electromechanically stable position, and the only tunneling becomes the dominant transport mechanism like a conventional SET. The former is referred to as the shuttle regime, and the latter the tunnel regime [5, 6]. Because of the distinct difference of the transport mechanisms, the NEM-SET has two different transport characteristics peculiar to each regime. If one can switch the two regimes at will, it becomes possible to exploit the different transport characteristics on the single device, and therefore the NEM-SET will be open to utilization in various ways in nano electronics.

In a previous work [8], it was found that large DC gate voltage stops the nano particle vibrations of the NEM-SET, changing the dominant transport mechanism from shuttling to tunneling. The shuttle mechanism reversibly recovers after turning off the DC gate. The switching is owing to regulation of energy pumping from the bias electric field to the nano particle vibration, therefore, the DC gate cannot excite the nano particle vibrations in the tunnel regime. In contrast to the DC gate, AC gate induces alternately changing electrostatic force acting on the nano particle. Then the AC gate will drive nano particle vibrations in the
tunnel regime. If the forced oscillation reproduces the shuttle mechanism, the current will be controllable by the AC gate since the gate frequency is tunable, and the relevant current noise is expected to reduce [7, 9, 10, 11]. The purpose of the present work is to investigate theoretically AC gate effects on current and noise properties of the NEM-SET in the tunnel regime, aiming for changing the transport mechanism from tunneling to shuttling.

2. Model and Formulation

We consider a NEM-SET consisting of a nano particle of mass \( m \) confined in a harmonic potential, \( \frac{m\omega_0^2 x^2}{2} \) (Fig. 1). The nano particle is linked electrically to the source and drain, and the electron transfer between the electrodes and the nano particle is due to electron tunneling. The tunneling rate depends exponentially on the ratio of nano particle position \( x \) to a typical length \( \lambda \) of tunneling. Energy level separation of electrons in the particle of mass \( m \), where \( e, C \) and \( n \) are the elementary charge, the capacitance of the system, and the excess number of electrons on the nano particle from the electrical neutrality. The bias is fixed so that only the two states with \( n = 0 \) and \( 1 \) are concerned with the transport at \( T = 0 \) K. The AC gate voltage moves the energy levels up and down, which are, however, kept between the Fermi levels of source and drain so that current does not flow against the bias.

Current and noise spectra are formulated by using the probability \( P_N(t) \) that \( N \) electrons transferred to the drain within time interval \( t \) [12]. It is useful for deriving \( P_N \) to introduce the reduced density matrix with respect to \( N \), \( \rho^{(N)}_m = [\text{Tr}\rho]_{n,N} \). In this work, the Wigner function is actually derived instead of the density matrix because the Wigner function is helpful for understanding the results in terms of semi-classical pictures.

The Wigner function corresponding to \( \rho^{(N)}_m \) is defined by \( w_m^{(N)}(x, p) \equiv \frac{1}{\pi \hbar} \int_{-\infty}^{\infty} \langle x - \frac{y}{2}, \rho^{(N)}_m | x + \frac{y}{2} \rangle e^{iyp/\hbar} dy \).

Introducing \( w_m(x, p; z) = \sum_{N=0}^{\infty} z^N w_m^{(N)}(x, p) \), \( w_m(x, p; 1) \) is proved to be the conventional Wigner function \( W_m(x, p) \). The probability \( P_N \) is given by \( P_N = \text{Tr}[w_m^{(N)}(x, p)]_{|N} \). The total charge \( \langle Q(t) \rangle \) transferred to the drain, and time derivative of \( \langle Q(t) \rangle \) and \( \Delta Q^2 \) are expressed in terms of \( W_m \) and \( W_{z,n} = \frac{\partial}{\partial z}w_m^{(N)}|_{z=1} \).

The current \( I \) and the relevant noise power spectrum \( S(0) \) at zero frequency are given by time average of \( J(t) \) and \( \frac{d}{dt}\Delta Q^2 \) as \( I = \langle J(t) \rangle \) and \( S(0) = \frac{d}{dt}\Delta Q^2 \).

Introducing dimensionless position \( \xi \), momentum \( \pi \), time \( \tau \) and electric field \( \kappa \) given by \( x = x_0 (\xi + \frac{\pi}{2}) \), \( p = m\omega_0 x_0 \pi \), \( t = \omega_0^{-1} \tau \), \( \kappa = \frac{eE}{m\omega_0^2 x_0^2} \), where \( x_0 = \left( \frac{\hbar}{m\omega_0} \right)^{1/2} \)

the equation of motion of \( Y_{mn} = (W_{mn}, W_{z,n})^T \) yields

\[
\begin{align*}
\partial_\tau Y_{00} &= M_0 Y_{00} - \frac{\gamma L}{\omega_0} e^{-2\xi x_0/\lambda} \left[ 1 - \frac{1}{2} \left( \frac{x_0}{\lambda} \right)^2 \partial_\pi \right] Y_{00} + \frac{\gamma R}{\omega_0} e^{2\xi x_0/\lambda} \left[ Y_{11} + (0, W_{11})^T \right], \\
\partial_\tau Y_{11} &= M_1 Y_{11} - \frac{\gamma R}{\omega_0} e^{2\xi x_0/\lambda} \left[ 1 - \frac{1}{2} \left( \frac{x_0}{\lambda} \right)^2 \partial_\pi \right] Y_{11} + \frac{\gamma L}{\omega_0} e^{-2\xi x_0/\lambda} Y_{00}
\end{align*}
\]

where \( M_n = -\pi \partial_\xi + \left[ \xi + \left( \frac{1}{2} - n \right) \kappa \right] \partial_\pi + 2\frac{\kappa}{\omega_0} \partial_\pi \pi + \frac{hD_m}{m\omega_0} \partial_\pi^2 \).

The origin of \( \xi \) is set at the

\[\text{Figure 1. Model of a NEM-SET. The nano particle confined in a harmonic potential.} E \text{ is the bias electric field.}\]
midpoint of the two electromechanically stable positions for \( n = 0 \) and 1. Energy dissipation due to coupling with reservoir is represented by the friction \( \gamma_{\infty} \) and diffusion coefficients \( D_{\infty} \) in Eqs. (1) and (2). The diffusion constant \( D_{\infty} \) is \( D_{\infty} = \frac{\gamma_{\infty} m \omega_0}{\hbar} \) at \( T = 0 \) K. The tunneling rates \( \gamma_{R,L} \) at \( \xi = 0 \) are formulated by means of the Golden rule. Given AC gate voltage of frequency \( \omega_G \), \( \gamma_{R,L} = \frac{\gamma_2}{2} \left( 1 \pm U_G \sin \frac{\omega_G \tau}{\omega_0} \right) \), where the dimensionless gate amplitude \( U_G \in [0, 1] \) so that \( \gamma_{R,L} \geq 0 \).

3. Transport Properties
Using the time-advance algorithm called “splitting method” [13] and Cubic Interpolated Propagation (CIP) scheme [14], we numerically solve Eqs. (1) and (2). The used parameters are \( \frac{\lambda}{\omega_0} = 0.5 \), \( \frac{\gamma_{\infty}}{\omega_0} = 0.1 \), \( \frac{\gamma_{L,R}}{\omega_0} = 0.2 \) and \( U_G = 1 \), at \( T = 0 \) K.

3.1. Current
Figure 2 plot the current \( I \), the noise power spectrum at zero frequency \( S(0) \) and the Fano factor \( F \) versus the gate frequency of the semiclassical NEM-SET for \( \kappa = 0.8 \) and 1.0, respectively. For both cases of \( \kappa \), the current increases with increasing \( \omega_G \), and shows a prominent peak at \( \omega_G = \omega_0 \). The frequency dependence of current stems from the resonance of the grain vibrations. The amplitude of the grain vibrations becomes large as \( \omega_G \) approaches \( \omega_0 \). Because of the exponential dependence of the tunneling rates on the grain position, the nearer the wavepacket approaches to the electrodes, the more quickly electron tunneling takes place with high probability. Hence the resonance of grain vibrations enhances remarkably the current, and causes the current peak at the resonance frequency.

3.2. Noise
The current and noise provide important information on the dynamics of the grain. The noise spectrum at zero frequency \( S(0) \) exhibits much richer structures than the current; there is a noticeable peak in \( S(0) \) at \( \frac{1}{3} \omega_0 \) for both \( \kappa = 0.8 \) and 1.0. Another large peak appears at the resonance frequency for \( \kappa = 0.8 \), but the peak splits into two peaks for \( \kappa = 1.0 \). Correspondingly, the peak in the Fano factor changes to a valley with an increase of \( \kappa \) from 0.8 to 1.0.

The Wigner function comprises frequency components of \( \omega_G \) and its harmonics since \( \gamma_{L,R} \) and the dominant part of \( Y_{nn} \) vary at the gate frequency in Eqs. (1) and (2). For the change in sign of the gate voltage and grain position, \( n \) changes its sign to keep Eqs. (1) and (2) unchanged. From the parities, the harmonics are limited to only odd harmonics. When \( \omega_G = \frac{1}{3} \omega_0 \), the third harmonics component induces resonance of the grain vibrations, so that the Wigner function has a finite component of \( \omega_0 \), comparable to that of \( \omega_0 \). Figure 3 shows the Fourier component of \( \langle n \rangle \) and \( \langle \xi \rangle \), together with the limit cycle \( \langle \pi \rangle \) versus \( \langle \zeta \rangle \), evidencing that the Wigner function contains large third harmonics component. These components individually contributes to the
transport since they have different frequencies; an electron is transferred stochastically via either of these components. It is known that such stochastic switching of paths induces extra noise such as random telegraph noise [10]. Hence, we may attribute the noise peak at $\omega_0$ to the stochastic switching between the portions of the wavepacket driven at $\omega_G$ and the third harmonic of $\omega_G$.

Finally, we discuss the remarkable differences in both $S(0)$ and $F$ between the two cases of $\kappa$’s around the resonance frequency. The change in $F$ indicates that random occurrence of tunneling changes to ordered one, in comparison with a conventional SETs ($F = 0.5$), with an increase of $\kappa$. At the resonance for $\kappa = 0.8$, the wavepacket is stretched, and then the uncertainty in the grain position increases. Because the tunneling rate depends on the grain position, the uncertainty in the grain position provides a new source of noise. This causes extra noise, resulting in the peaks in $S(0)$ and $F$. For the increase in $\kappa$ to 1.0, because of the large amplitude of the wavepacket vibration at the resonance, electron tunneling occurs very quickly with high probability close to 1 as the grain approaches the electrodes. $\langle n \rangle$ almost saturates, and the following tunneling events rarely occur. As a result, the current noise is suppressed.

4. Conclusion
The AC gate effects on the current and noise properties of a semi-classical NEM-SET in the tunnel regime were numerically studied. The AC gate induces the nano particle vibrations, and we found that the current and noise properties are governed by the resonance of the nano particle vibrations and fluctuations. The artificial shuttle mechanism, i.e. the current with low noise, is reproduced only at the resonance frequency for $\kappa = 1.0$.

Acknowledgments
This work is supported in part by a grant-in-aid for scientific research from the Ministry of Education, Culture, Science and Technology of Japan (Grant No. 1965106507).

References
[1] Knobel R and Cleland A N 2003 Nature (London) 424 291
[2] LaHaye M D, Buu O, Camarota B and Schwab K C 2004 Science 304 74
[3] Blencowe M P 2004 Phys. Rep. 395 159
[4] Savel’ev S, Hu X and Nori F 2006 New J. Phys. 8 105
[5] Gorelik L Y, Isacsson A, Voinova M V, Kasemo B, Shekhter R I and Jonson M 1998 Phys. Rev. Lett. 80 4526
[6] Isacsson A, Gorelik L Y, Shekhter R I and Jonson M 1998 Physica B 255 150
[7] Novotný T, Donarini A, and Jauho A-P 2003 Phys. Rev.Lett. 90 256801
[8] Nishiguchi N 2002 Phys. Rev. B 65 035403
[9] Pistolesi F 2004 Phys. Rev. B 69 245409
[10] Flindt C, Novotný T and Jauho A-P 2005 Europhys. Lett. 69 475
[11] Fedorets D, Gorelik L Y, Shekhter R I and Jonson M 2004 Phys. Rev. Lett. 92 166801
[12] Elattari B and Gurvitz S A 2002 Physics Letters A 292 289
[13] Cheng C Z and Knorr G 1976 J. Comput. Phys. 22 330
[14] Nakamura T and Yabe T 1999 Comput. Phys. Commun. 120 122