Confrontation of Different Relativistic Descriptions of Nuclear Matter

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In this work we explore different relativistic descriptions of nuclear matter along the lines originally proposed in Refs. [1, 2] where chiral symmetry is incorporated within the Walecka type Relativistic Mean Field (RMF) model as well as the effect of confinement through the nucleon response. The parameters of this model are controlled by fundamental properties, such as the chiral potential, the Lattice-QCD predictions, the quark structure, and two saturation properties (density and energy). The predictions of this chiral+confinement model is compared to two other models: another chiral model - but without confinement effect - and the original RMF model. For these three models, we additionally take care of parameter uncertainties and propagate them to our predictions for dense matter properties employing Bayesian statistics. We show that the combination of chiral potential with nucleon response represents a microscopically motivated and economical way to treat in-medium corrections to the scalar equation of motion, accurately reproducing the other two models which are directly fitted to the empirical properties of nuclear matter. In addition, the order hierarchy in power of the scalar field in the scalar potential is respected, which is not always the case for models where the scalar potential is fitted to empirical data. While these models are calibrated to the same properties at saturation density, they differ in their predictions as the density increases. Interestingly, we also show that, by fixing the $\rho$ coupling constant from the quark structure of the nucleon, these three models reproduce only half of the empirical symmetry energy.

I. INTRODUCTION

The understanding of the origin of nuclear force is a fundamental brick of nuclear physics, which aims at describing atomic-nuclei properties in terms of microscopic interactions and natural degrees of freedom. Nucleons are indeed often considered as fundamental objects since typical nuclear energy scale is of the order of a few MeV's, up to a few tens of MeV per nucleon, which is too low to excite nucleon substructure. By the advent of Quantum Chromo-Dynamics (QCD), it is established that nucleons are however not fundamental, but are composed of colored quarks which interact through the exchange of colored gluons at a resolution scale of the few hundreds of MeV per nucleon. Bridging the fundamental QCD theory with the natural nucleon degrees of freedom has then became a fundamental question in the recent developments of nuclear physics.

In 1935, the Yukawa meson-exchange model [3] was the first theory for the nuclear force, where pions mediate the NN (nn, np, and pp) interaction. This meson-exchange approach, while now known as not being the fundamental theory, has however shaped the global understanding of nuclear physics in terms of nucleons and mesons, producing the first good qualitative results, by fixing the coupling constant and associating a particle exchange to the strong interaction. This first quantum field model for the strong interaction was however not successful to accurately reproduce the properties of the atomic nuclei, e.g. ground-state energy, because of its simplicity compared to more evolved models (e.g. one-boson-exchange potentials or OBEP discussed hereafter) and of the unknown pion dynamics, which we now know is constrained by the chiral symmetry. In the early 1960's, the discovery of heavy mesons helped in the modeling of better one-boson-exchange potentials (OBEP) containing the exchange of well identified vector mesons namely the omega ($\omega$) and rho ($\rho$) mesons. There were however still some problems, e.g., the scalar sigma boson-exchange, now named $f_0(600)$, for which the experimental evidence was polemic as well as its link to the broad $2\pi$ scalar resonance. Nevertheless, high-precision potentials based on meson-exchange picture and including a scalar meson were constructed and successful, see for instance Ref. [4]. Since then, some phenomenological approaches such as the Walecka model [5, 6], have anchored their modeling into the meson-exchange picture in order to reproduce accurately and in a efficient way low-energy nuclear properties. It has also been extended to describe accurately the properties of atomic nuclei, see Ref. [7] for instance.

In this paper we compare different extensions of the initial Relativistic Mean Field (RMF) theory, which differ by the treatment of chiral and confinement properties. For simplicity, we consider the Hartree approximation, also called the classical field case, and the properties of these models are explored in symmetric matter (SM), even though the question of the prediction of the symmetry energy is also addressed at the end of our study. These models are calibrated to reproduce the same properties in SM and the uncertainty in the fitted quantities are propagated towards our prediction for nuclear matter properties using Bayesian statistics. The differences between the models are shown, and these differences while small at low density, appear to be significantly larger as the density increases.

The paper is organized as follows. In Sec. II we present a brief review of chiral relativistic theories and establish
a historical background for this work. The comparison of the models are discussed in Sec. III and beyond. In Sec. III the models are described in more details, including the discussion of the link between the parameters and the fitted data as well as their uncertainties. Note that the parameter adjustment is done in such a way that all models are consistent at saturation density. In Sec. IV the models are compared with a focus on the interpretation of the chiral potential. In Sec. V we show that the predictions of the models do not agree at high density of the Hartree approximation and possible ways to cure those limitations. Finally we concluded our study by underlying the importance of consistently incorporating QCD ingredients such as confinement and chiral symmetry, since it provides a microscopically motivated and an economical way to incorporate in-medium corrections into dense matter.

II. CHIRAL RELATIVISTIC THEORIES

We detail in this section the features of low-energy QCD which are incorporated in chiral relativistic theories.

1. Relativistic mean field approach (RMF) and the Walecka mechanism

The first attempt to go beyond a non relativistic treatment of nuclear matter was the relativistic mean field (RMF) approach initiated by Walecka and collaborators [5, 6], which is based on meson exchange between nucleons whose wave functions are solution of the in-medium Dirac equation. In this framework nucleons move in an attractive (scalar field) and in a repulsive (vector field) backgrounds. This provides both the "Walecka" saturation mechanism and the correct magnitude of the spin-orbit potential. The parameters describing the meson-nucleon couplings are adjusted to the saturation properties of nuclear matter and/or nuclear ground state properties through the nuclear chart (binding energies, charge radii, etc.). Hence there is no explicit or direct connection with the underlying QCD theory but instead this approach describes the nuclear properties in terms of a meson exchange potential renormalized around nuclear saturation density. The link with the bare nucleon-nucleon interaction could nevertheless be performed through the Dirac-Brueckner-Hartree-Fock (DBHF) approach [8, 9], producing a mean-field potential which can guide the parametrization of the in-medium RMF with density-dependent coupling constants [10, 11]. Some recent RMF models accurately reproducing nuclei properties have included this link to DBHF in-medium modified potentials [12, 13]. However, the question of the very nature of these background mesonic fields has still to be elucidated or, said differently, it is highly desirable to clarify their relationship with the low-energy realization of chiral symmetry.

2. Chiral symmetry and color confinement

Chiral symmetry together with color confinement are the most prominent low-energy features of the gauge theory of colored quarks and gluons (QCD) established in the 1970’s. In the low energy/large distance domain only light u and d quarks with masses of a few MeV’s are relevant. In this domain the QCD lagrangian has essentially no dimensional parameter (when neglecting the quark masses). This scale invariance is however broken by quantum fluctuations [14]. The consequence is that, below the characteristic QCD momentum scale \( \Lambda_{\text{QCD}} \approx 200 \text{ MeV} \), the coupling constant of the theory becomes very large, a feature which is supposed to generate color confinement, and consequently renders QCD a non-perturbative theory in the energy range of nuclear physics, often referred to as low-energy. Lattice-QCD is one tool to tackle this problem by bridging the gap between unobserved quarks and gluons degrees of freedom and baryonic or mesonic degrees of freedom [15].

The accidental SU(2f = 3) chiral symmetry of QCD originating from the light quark masses means that in the limit of zero mass (called the chiral limit) non-interacting quarks with opposite helicity are indistinguishable and do not couple to each other. For the description of hyper-nuclei, this concept of chiral symmetry can be extended to the s quark within SU(Nf = 3). In the present study, we however limit ourself to the SU(2) case. At low-energy, i.e. below a few \( \Lambda_{\text{QCD}} \), chiral symmetry is spontaneously broken due to the condensation of quark-antiquark pairs, mixing left handed and right handed quarks in the QCD vacuum: QCD prefers quark-antiquark pairs with negative parity to the quark-quark antiquark pairs with positive parity [16]. The chiral fields associated with the fluctuations of the resulting quark condensate \( \langle \bar{q}q \rangle \) are usually parametrized in a SU(2) matrix \( M \) as:

\[
M = \sigma + i \tau \cdot \vec{\pi} \equiv SU
\]

with \( S = s + f_\pi \) and \( U = e^{-\vec{\phi}/f_\pi} \). The scalar field \( \sigma \) (\( S \)) and pseudoscalar fields \( \vec{\pi} \) (\( \vec{\phi} \)) written in cartesian (polar) coordinates appear as the dynamical degrees of freedom. In particular the sigma field is not vanishing in vacuum since \( \langle \sigma \rangle_{\text{vac}} = \langle S \rangle_{\text{vac}} = f_\pi \propto \sqrt{\langle \bar{q}q \rangle_{\text{vac}}} \), where the last relation is due to the Gellmann-Oakes-Renner relation [15], given hereafter. Note that, at finite density, the values taken by these dynamical degrees of freedom may deviate from their vacuum expectations.

In the chiral limit the potential energy, also called the chiral potential, exhibit a typical mexican hat shape in terms of the scalar and pseudoscalar fields. The bottom is a circle with radius \( f_\pi \) (the chiral circle defined from
\[ \sigma^2 + \bar{\sigma}^2 = f_\pi^2 \]

in a four dimension space (an analogy can be made with the O(4) ferromagnet). The low-energy constant \( f_\pi \) is identified with the pion decay constant. As a result, one moves without cost of energy along the chiral circle and the pseudoscalar modes are massless in the chiral limit. These modes, called "Goldstone bosons", are realized in terms of a few pseudoscalar mesons, e.g. the pions for SU(2) or the kaons for the SU(3) extension.

In addition to the spontaneous chiral symmetry breaking one expects a small but explicit symmetry breaking. For small masses indeed, as it is the case in the physical world, chiral symmetry is explicitly broken and \( u \) and \( d \) quarks weakly interact. In reality the "Goldstone" bosons are quasi-Goldstone bosons with small masses. This explains why the quasi-Goldstone boson, identified as the pion, has a small mass \( \simeq 140 \text{ MeV} \) compared to the other mesons. The Gellmann-Oakes-Renner relation, \( f_\pi^2 m_\pi^2 = -2 m_q \langle \bar{q} q \rangle_{\text{vac}} \), relates the pion mass \( m_\pi \) to \( m_q \), the small but finite mass of the light \( u \) and \( d \) quarks.

A simple realisation of the chiral symmetry breaking mechanism can be studied in the Nambu-Jona-Lasinio (NJL) model, see original Refs. [17, 18] and Ref. [19] for a review. In its simpler version the complex QCD interaction is replaced by a contact 4-quark chirally invariant interaction, where the gluon degrees of freedom are frozen. In addition, quarks acquire a dynamical mass due to the spontaneous breaking of the chiral symmetry. This mass is called the constituent quark mass since it is approximately one third of the nucleon mass \( \simeq 330 \text{ MeV} \). There is another mass, the scalar mass associated to the radial meson mode \( S \), which is related to the curvature of the chiral potential. Since this mass is associated to two constituent quarks, it typically scales as twice the constituent quark mass in the NJL model. For this reason in the following we will take for the mass of the sigma typical values about 600 to 800 MeV.

These chiral properties can be implemented in chiral perturbation theory. In the chiral Effective Field Theory (EFT) initiated by Weinberg [20], the radial mode \( S \) is frozen to its vacuum expectation value, \( f_\pi \), and only low-energy displacements along the chiral circle are allowed. In this approach, the most general Lagrangian is expressed in terms of the matrix \( U \), e.g., pionic degrees of freedom, dictated by symmetries, e.g. chiral symmetry. Nucleons can be introduced as heavy sources coupled to pions, see Ref. [21] for a recent review, while heavy mesons and nucleons resonances are integrated out and replaced by corresponding counter-terms in the Lagrangian, or cut-off, since their mass happens to be much higher than the designed resolution of the model. This effective approach is however unable to fix the parameters of the Lagrangian, which are fitted to data. Within these limitations, the resulting EFT is equivalent to QCD at low energy.

Finally, let us remark that as the density of the medium increases, chiral symmetry is expected to be progressively restored. This is a general expectation: as the energy scale increases – in that case due to the increasing density – the symmetry of the QCD Lagrangian is recovered. It is typically demonstrated for an effective model of QCD such as the NJL model, see Ref. [19] for a review.

3. The "nuclear physics" sigma meson

To bridge the gap between relativistic theories of the Walecka type and approaches based on chiral symmetry, one has to map the "nuclear physics" sigma meson of the Walecka model at the origin of the nuclear binding (let us call it \( \sigma_W \) from now on) with a chiral quantity. For instance, one may be tempted to identify \( \sigma_W \) with \( \sigma \), the chiral partner of the pion. It is however forbidden by chiral constraints. This point has been first addressed by Birse [22]: it would lead to the presence of terms of order \( m_\pi^2 \) in the NN interaction which is not allowed.

It was instead proposed and justified to identify \( \sigma_W \) with the chiral invariant \( s = (S - f_\pi) \) field associated with the radial fluctuation of the chiral condensate \( S \) around the "chiral radius" \( f_\pi \): formally it imposes \( \sigma_W \equiv s \). Equivalently the chiral invariant sigma field and the pion field appearing in the chiral field operator \( M = S U \equiv (s + f_\pi) \equiv (\sigma_W + f_\pi) U \) are promoted to the rank of effective degrees of freedom. This was originally formulated in the framework of the linear sigma model [1] but an explicit construction using a bosonization technique of the chiral effective potential can be done within the NJL model [23], where the linear sigma model potential is recovered through a second order expansion in \( S^2 - f_\pi^2 \) of the constituent quark Dirac sea energy. This proposal, which gives a plausible answer to the long standing problem of the chiral status of Walecka theories, has also the merit of respecting all the desired chiral constraints. In particular the correspondance \( s \equiv \sigma_W \) generates a coupling of the scalar field to the derivatives of the pion field. Hence the radial mode decouples from low-energy pions (as the pion is a quasi-Goldstone boson) whose dynamics is governed by chiral perturbation theory. A detailed discussion of this sometimes subtle topic is given in [1, 24]. From now on, we thus believe that the correct connection to the physical world is to identify \( \sigma_W \equiv s \).

The very origin of this "nuclear physics" sigma meson is nevertheless still a controversial subject since there is no sharp scalar resonance observed in the expected mass range \( \approx 600 \text{ MeV} \). Let us discuss this controversy with some details, by repeating arguments already presented in Ref. [23]. As soon as we start from a model which gives a correct description of chiral symmetry breaking in the QCD vacuum such as the Nambu-Jona-Lasinio model (NJL), the emergence of a scalar field linked to the quark condensate is predicted. This emergence is by construction a low momentum concept which does not imply the existence of a sharp scalar meson if the effect of confinement is taken into account. Indeed it has been demonstrated by Celenza et al [25, 26] that the inclusion of a confining interaction on top of the NJL
model pushes the $q\bar{q}$ scalar state, located originally at twice the constituent quark mass, well above one GeV. The broad resonance, usually refereed as the $f_0(600)$, observed at around 600 MeV, is a $\pi\pi$ resonance which has no direct relation with the background scalar field introduced above. Coming back to this nuclear physics $g_W$, its associated “scalar mass”, which is around twice the constituent quark mass, is a low momentum parameter related to the inverse of the vacuum scalar susceptibility. As reminded in the previous sub-section, it is typically of the order of 600 to 800 MeV.

4. Nucleon response at finite density

There is a well identified problem concerning the nuclear saturation with usual chiral effective theories [27–30]: Independently of the particular chiral model, the solution at finite density is moved away from the vacuum solution, located at the minimum of the chiral potential, into a region of smaller curvature. This single effect (equivalent to the lowering of the sigma mass) destabilizes the ground state solution, creating problems for the applicability of such effective theories in the nuclear context. The effect can be associated with a $s^3$ tadpole diagram generating an attractive three-body force and destroying saturation, even if an effective repulsive three-body force is present - but not strong enough - from the Walecka mechanism.

In the relativistic chiral approach this problem is cured by introducing a nucleon response to the scalar field, $\kappa_{NS}$, which is the central ingredient of the quark-meson coupling model (QMC), introduced in the seminal work of P. Guichon [31] and successfully applied to finite nuclei with an explicit connection to the Skyrme force [32]. The physical motivation to introduce this nucleonic response is the observation that nucleons experience huge fields at finite density, e.g. the scalar field is of the order of a few hundred of MeV at saturation density. Nucleons, being in reality composite objects, will react against the nuclear environment (i.e., the background nuclear scalar fields) through a (self-consistent) modification of the quarks wave functions. This effect may generate a three body force which brings the desired repulsion if confinement dominates spontaneous chiral symmetry breaking, as discussed in Ref. [23] within particular models. In practice this effect generates a non linear coupling of the nucleon to the scalar field, inducing a decrease of the effective scalar coupling constant with increasing density. This is the key ingredient of the saturation mechanism. The attractive chiral $s^3$ tadpole diagram responsible for the instability of the ground state at finite density is counter-balanced by the nucleon response. Anticipating a further discussion, an estimate of the nucleon response parameter $\kappa_{NS}$ can be extracted from lattice-QCD, providing an explicit connection between nuclear saturation mechanism and the QCD itself. One may also notice that a similar mechanism occurs if confinement is simulated by an infrared cut-off in the NJL model as discussed in Ref. [29].

Note that the nucleon response largely contributes as well to the curvature coefficient at saturation – the incompressibility modulus $K_{sat}$. In a set of successive works [2, 23, 33–36] this approach has been applied to the equation of state of nuclear matter and neutron stars as well as to the study of chiral properties of nuclear matter at different levels of approximation in the treatment of the many-body problem (RMF, Relativistic Hartree Fock or RHF, pion loop correlation energy). Note also that, the quark substructure plays also a crucial role for the spin-orbit potential as discussed in a recent paper [37].

In practice the introduction of the nucleon response is done by parametrizing the scalar field $s$ dependence of the nucleon as expressed in Eq. (9) given below. with the exception of Ref. [23] where explicit confinement models have been used to generate it. The parameter $\kappa_{NS}$ can advantageously be replaced by the dimensionless parameter $C = f_0^2 \kappa_{NS}/2M_N \simeq 1.25$ as we will discuss hereafter.

In this paper, this model is taken as our reference model and it will be further detailed in the next section. It will be referred as RMF-CC (as Relativistic Mean Field including Chiral and Confinement effects).

5. Comparison to other chiral approaches

In order to make links with other approaches, we will also compare the RMF-CC model to other models, such as RMF including Chiral potential only (RMF-C) as well as to the original RMF model. Several chiral relativistic theories (RMF-C) have indeed been formulated but without reference to the nucleon response [27, 38–40]. Recently, such a RMF-C model has been used to study the possible mixed phases at the chiral transition in neutron stars [41]. In this approach the chiral potential deviates from the pure linear sigma model potential by terms of first and third order in $S^2 - f_0^2$ (our chiral invariant $S$ field is named $\chi$ in the latter paper) with additional parameters ($a_3, a_4$). This is a legitimate attitude since any microscopic underlying model, including the above mentioned NJL model, will certainly generate such higher order many-body terms at low-energy.

One interesting question is whether this higher order terms may simulate the effect of the nucleon response. One main motivation of the present work is to make a comparison of the two classes of approaches by expanding the chiral effective potentials in power of the scalar field $s$. Since a term in $s^n$ corresponds to a $(n-1)$-body force, one may thus expect that the expansion in the scalar field $s$ is perturbative, at least at low density: the 2-body force is expected to be larger than the 3-body one, itself larger than the 4-body force, and so on. Models violating this ordering will thus be referred as anomalous ones in the following. Anticipating our results, we found that this RMF-C model lead to anomalous chiral potentials.
III. THE MODELS

In relativistic approaches to nuclear matter, the Lagrangian can generically be written as the sum of a kinetic fermionic term,

\[ \mathcal{L}_\psi = \bar{\psi} (i \gamma^\mu - M_N) \partial_\mu \psi , \]

where the field \( \psi \) represents the nucleon spinor, and of meson-nucleon interaction terms,

\[ \mathcal{L}_m = \mathcal{L}_s + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_\delta + \mathcal{L}_\pi , \tag{2} \]

collecting all mesonic contributions considered in a given model. Using notation of Ref. [34] these can be enumerated as,

\[ \mathcal{L}_s = (M_N - M_N(s)) \bar{\psi} \psi - V(s) + \frac{1}{2} \partial_\mu s \partial_\mu s , \]

\[ \mathcal{L}_\omega = -g_\omega \omega_\mu \bar{\psi} \gamma^\mu \psi + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} F^{\mu \nu} F_{\mu \nu} , \]

\[ \mathcal{L}_\rho = -g_\rho \rho_\mu \bar{\psi} \gamma^\mu \tau_a \psi + g_\rho \frac{\rho}{M_N} \partial_\mu \rho_\mu \bar{\psi} \sigma^{\mu \nu} \tau_a \psi + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu - \frac{1}{4} G^{\alpha \beta} G_{\alpha \beta} , \tag{3} \]

\[ \mathcal{L}_\delta = -g_\delta \delta_\mu \bar{\psi} \tau_a \psi - \frac{1}{2} m_\delta \delta_\mu \delta^\mu + \frac{1}{2} \partial_\mu \delta_\mu \partial_\mu \delta + \frac{1}{2} \partial_\mu \psi \bar{\psi} \psi \partial_\mu \delta_a , \]

\[ \mathcal{L}_\pi = \frac{g_A}{2 f_\pi} \partial_\mu \varphi_a \bar{\psi} \gamma^\mu \varphi^a \tau_a \psi - \frac{1}{2} m_\varphi^2 \varphi_a \varphi^a + \frac{1}{2} \partial_\mu \varphi_a \partial_\mu \varphi^a , \]

where the symbols have their usual meaning. In Eq. (3), two quantities are of particular interest to us, the scalar potential \( V(s) \) and the \( s \)-field dependent nucleon mass \( M_N(s) \). Different expressions for these quantities have been employed in the past, depending on the context as we discussed in Sec. II. For instance, the scalar potential \( V(s) \) is associated to the chiral potential in RMF models including chiral symmetry. For the moment we just remind that the chiral potential in RMF-CC is kept as the one breaking the chiral symmetry in the vacuum, while in RMF-C, the parameters of the chiral potential are adjusted to reproduce the saturation properties. The scalar potential \( V(s) \) in RMF is possibly non-linear and have been introduced in a pragmatic way to better reproduce the incompressibility modulus and the effective mass.

In this paper we will restrict our attention to only symmetric matter (SM). The energy density at the Hartree level can be computed from the Lagrangian of Eq. (3) in the usual way [2]. It is expressed as

\[ \varepsilon = \int \frac{4 \pi^3 k}{(2 \pi)^3} \left( \sqrt{k^2 + M_N^2(s)} + g_\omega \omega_0 \right) \Theta(k_F - k) \]

\[ + V(s) - \frac{1}{2} m_\omega^2 \omega_0^2 , \tag{4} \]

where the scalar and vector fields are obtained from the equations of motion given in Appendix A. Note that in relativistic approaches, two densities are defined, the vector density \( \rho \equiv \langle \bar{\psi} \gamma^0 \psi \rangle \) and the scalar density \( \rho_S \equiv \langle \bar{\psi} \psi \rangle \).

In the next subsections, we will start the presentation of the models with RMF-CC since it is the most general one, then we will continue with RMF-C and finally with RMF.

A. Relativistic Mean Field including Chiral potential and Confinement effects (RMF-CC)

We first discuss the relativistic model that incorporates both chiral symmetry as well as the nucleon substructure. The former is enunciated via the chiral effective potential, which is a Mexican hat potential for the scalar field,

\[ V(s) = \frac{m_\sigma^2 - m_\pi^2}{8 f_\pi^2} (\sigma^2 + \pi^2 - v^2)^2 - f_\pi m_\pi^2 \sigma , \tag{5} \]

with

\[ v^2 = f_\pi^2 \frac{m_\sigma^2 - 3m_\pi^2}{m_\sigma^2 - m_\pi^2} . \tag{6} \]

The chiral effective potential (5) corresponds to the original linear sigma model using the "cartesian" coordinates, namely the "chiral partners" \( \sigma \) and \( \pi \).

Employing the polar coordinates and keeping only the leading order mass term for the pion, the chiral potential can also be expressed as

\[ V(s) = \frac{m_\sigma^2}{2} (S^2 - f_\sigma^2) + \frac{m_\pi^2 - m_\sigma^2}{8 f_\pi^2} (S^2 - f_\sigma^2)^2 \]

\[ + \frac{1}{2} m_\pi^2 \phi^2 + \ldots \tag{7} \]

In Eq. (7) the higher order terms generate pion-pion interactions which disappear in the chiral limit.

We finally get the following expression that will be used for this model:

\[ V(s) = \frac{m_\sigma^2}{2} s^2 + \frac{m_\pi^2 - m_\sigma^2}{2 f_\pi} \left( s^3 + \frac{s^4}{4 f_\pi} \right) , \tag{8} \]

### TABLE I. Nuclear Empirical Parameters (NEP) \( (E_{\text{sat}} \text{ and } n_{\text{sat}}) \) as given from Ref. [42] and Lattice parameters \( (a_2 \text{ and } a_4) \) extracted from Ref. [43], used in the fits. For the NEP the mean and standard deviations correspond to a Gaussian distribution, while for the Lattice parameters the standard deviation refers to the width of a uniform distribution.

| Parameter | Mean     | Standard deviation |
|-----------|----------|--------------------|
| \( E_{\text{sat}} \) (MeV) | -15.8    | 0.3                |
| \( n_{\text{sat}} \) (fm\(^{-3}\)) | 0.155    | 0.005              |
| \( a_2 \) (GeV\(^{-1}\)) | 1.533    | 0.136              |
| \( a_4 \) (GeV\(^{-3}\)) | -0.509   | 0.054              |
FIG. 1. Probability Distribution Function (PDF) for the parameters of the RMF-CC model, adjusted to reproduce the NEPs $E_{\text{sat}}$ and $n_{\text{sat}}$ as well as the Lattice parameters $a_2$ and $a_4$, see Table I for more details.

where we only keep the radial fluctuation field $s$ (the field to be identified with the "nuclear physics" sigma meson, $\sigma_W$ of Walecka theories) since at the Hartree mean field level the pion exchange does not contribute. As explained in Sec. II, such a potential does not allow for SM to saturate because of the attractive contribution of the $s^3$ term in Eq. (8), i.e., the tadpole diagram mentioned earlier.

As discussed in Sec. II, this problem can be circumvented by introducing the nucleon response to the scalar field at finite density, driven by the susceptibility $\kappa_{NS}$:

$$M_N(s) = M_N + g_s s + \frac{1}{2} \kappa_{NS} \left( s^2 + \frac{s^3}{3f_\pi} \right). \quad (9)$$

In Eq. (9), the quadratic term leads to a non-zero nucleon susceptibility (NS), and we have added a cubic term as well. The cubic term coupling constant is fixed such that the susceptibility $\kappa_{NS} = d^2 M_N(s)/ds^2$ vanishes at full chiral restoration [2]. In the following, it will be convenient to introduce the dimensionless quantity

$$C = \frac{f_\pi^2}{2M_N} \kappa_{NS}. \quad (10)$$

We will now make the connection with Lattice QCD more clear by following the approach of Refs. [33, 34]. First, from Ref. [43, 44], we express the sigma commutator as

$$m_\pi^2 \frac{d M_N}{d (m_\pi^2)} = a_2 m_\pi^2 + 2a_4 m_\pi^4 + \cdots + m_\pi^2 \frac{d \Sigma_\pi}{d (m_\pi^2)}, \quad (11)$$

where we have separated two contributions: one analytic in $m_\pi^2$ and the other non-analytic piece involving pion dynamics. The latter can be estimated in an essentially model independent way using chiral perturbation theory, with the pion loops suitably regularized. The parameters $a_2$ and $a_4$ are then fit to Lattice QCD results and one obtains a range of values for $a_2$ and $a_4$, given in Table I, due to the ambiguity in the regulator of the pion loops, which are four different functional forms for the regulator: sharp-cutoff, monopole, dipole and Gaussian [44].

The parameters $a_2$ and $a_4$ are related to the parameters of the RMF-CC model, $g_s$, $m_\sigma$ and $C$ as [33, 34]:

$$a_2 = \frac{g_s f_\pi}{m_\sigma^2}, \quad (12)$$

and

$$a_4 = -\frac{f_\pi g_s}{2m_\sigma^2} \left( 3 - 2C \frac{M_N}{f_\pi g_s} \right). \quad (13)$$

Notice that in the expression of $a_4$ the factor $M_N/f_\pi g_s$ was absent in [33, 34] since the nucleon mass was fixed to be $M_N = f_\pi g_s$. 

\[6\]
In our approach we therefore have four fit parameters $a_2$, $a_4$, $m_\sigma$ and $g_\omega$. These are fixed by the analysis of the Lattice results and by two saturation properties, $n_{\text{sat}}$ and $E_{\text{sat}}$ (Table. I). Considering the uncertainties in these parameters, one can also predict the Probability Distribution Functions (PDF) for the parameters: $g_\sigma \equiv g_s$ and $C$ from Eqs. (12) and (13), as well as $K_{\text{sat}}$ and the Dirac mass $M_D^*$. The uncertainties in the quantities to fit (given in Table. I) are explored within a Bayesian method using Markov-Chain Monte-Carlo (MCMC) approach. In this way full exploration of the uncertainties in the Nuclear Empirical Parameters (NEP) and the Lattice parameters are translated into uncertainty in the model parameters. The PDFs obtained from the MCMC sampling over our fit parameters $a_2$, $a_4$, $g_\omega$ and $m_\sigma$ are shown in Fig. 1. The distributions over $a_2$ and $a_4$ are almost flat (as imposed in the prior), and the confrontation against the NEP changes very little. The distributions over $g_\omega$ and $m_\sigma$ are much more peaked. The PDFs of $g_\omega$ is peaked around 6.5. The PDF of $m_\sigma$ is peaked around 820 MeV which is consistent with the commonly assumed value of 800 MeV (see discussion in Section II).

Based on the parameter distributions shown in Fig. 1, we can now analyze the impact of the uncertainties in these parameters on several interesting properties of dense matter, e.g. the Dirac mass at saturation $M_D^*$ and the incompressibility modulus $K_{\text{sat}}$. For completeness, we also show the distribution over the parameters $C$ and $g_\sigma$. These results are shown in Fig. 2. The Dirac mass is peaked around $\approx 0.85 \pm 0.02 M_N$. The predictions for $K_{\text{sat}}$, with a PDF peaked at $\approx 265$ MeV, are slightly larger than the expected empirical value around 230 – 250 MeV [42]. We expect however that quantum corrections, e.g. Fock term or pion cloud [35], could change these quantities and shift them towards lower values. The PDF of $C$ is consistent with the value used in Ref. [34] and $g_\sigma$ is consistent with the canonical value of $M_N / f_\pi \approx 9.98$.

B. Relativistic Mean Field with Chiral Symmetry only (RMF-C)

We now consider an approach where chiral symmetry is incorporated within a chiral potential $V(s)$, but without the effect of confinement in terms of nucleon polarisation. This so-called RMF-C model is inspired from Refs. [39–41].

\footnote{In SM, at the Hartree approximation, the Dirac mass is the same as the s-field dependent nucleon mass, i.e. $M_D^* = M_N(s)$.}
Since the non-trivial scalar response of the nucleon is neglected in this model, the $s$-field dependent nucleon mass is simply given by

$$M_N(s) = M_N + g_s s,$$

as in the linear sigma model [45].

As mentioned in Sec. II, the chiral potential is expressed in a very general way as

$$V(s) = \frac{4}{n!} b_n \frac{(S^2 - f_\pi^2)^n}{2^n} - f_\pi m_\sigma^2.$$  \hfill (15)

Notice that contrary to Eq. (7), this chiral potential contains terms of order $n = 3, 4$ in the $S^2 - f_\pi^2$ expansion.

An important specific point is that, as in the linear sigma model, the scalar coupling parameter is fixed, $g_s = M_N/f_\pi$. This leaves us with 4 unknown parameters $g_\omega$, $b_2$, $b_3$ and $b_4$ that we fit in a consistent way compared to RMF-CC. To do so, we consider the two NEPs in Table I, $E_{\text{sat}}$ and $n_{\text{sat}}$ as in the RMF-CC model and, additionally, we use the predictions of the RMF-CC model for the Dirac effective mass and the incompressibility modulus shown in Fig. 2.

The result of sampling this distribution in the parameter space spanned by $g_\omega$, $b_2$, $b_3$ and $b_4$ is shown in Fig. 3. First let us comment on the marginalized 1-dimensional PDF over $g_\omega$. The distribution is peaked around $\approx 6.25$. This is a rather low value given that the value of $g_\omega$ found in Ref. [41] is 9.47. This discrepancy is due to the fact that we have fitted $g_\omega$ to a Dirac effective mass $M^*_D$ that is $\approx 0.85 M_N$ (see Fig. 2), whereas the value of $M^*_D$ used in Ref. [41] is $0.75 M_N$. We have verified that if $M^*_D = 0.75 M_N$ is used in our approach, we are able to reproduce the $g_\omega$ found in Ref. [41]. Regarding the other parameters $b_2$, $b_3$ and $b_4$, the PDFs are also peaked at values different from the ones found in Ref. [41]. This is due to the fact the NEP used in this work are different from that used in Ref. [41], most notably $K_{\text{sat}}$ but also $E_{\text{sat}}$ and $n_{\text{sat}}$, therefore a precise agreement of our results should not be expected. Indeed for this work what is important is that the three models are parametrized consistently at saturation density. In this way, the differences in the predictions at high density could only be related to the ingredients of the models.

Finally, let us note that the result of $g_\omega$ presented here is consistent with what one obtains in the RMF-CC model, see Fig. 1. We also remark that the PDFs are broad and thus the parameters cannot be very well constrained by empirical knowledge of SM saturation. We will comment more on the values of these parameters later when comparing the scalar potentials of different models.

FIG. 3. Results from the fit of the RMF-C model to the NEPs of Table I and the RMF-CC model's prediction of $M^*_D$ and $K_{\text{sat}}$ shown in Fig. 2.
C. Relativistic Mean Field Theory (RMF)

We now turn to a model in which both the chiral potential and the response of the nucleon is ignored. As in the original Walecka model, the scalar potential is limited to the mass term:

\[ V(\sigma_W) = \frac{1}{2} m_{\sigma}^2 \sigma_W^2 , \]

and for the \(\sigma_W\)-field dependent mass one has:

\[ M_N(\sigma_W) = M_N + g_{\sigma} \sigma_W , \]

where \(g_{\sigma}\) is the scalar coupling constant.

While the saturation mechanism arises from the equilibrium between the scalar and the vector fields and allows a good reproduction of the saturation density and binding energy – at the cost of large coupling constants – other properties of the model, e.g. the compression modulus, the effective nucleon mass and the symmetry energy, are in poor agreement with the empirical values. Boguta and Bodmer [46] have thus suggested an extension of the original Walecka model, whose main purpose is to bring the compression modulus and nucleon effective mass at saturation under control, by introducing self-interactions of the scalar field by modifying the potential (16) as,

\[ V(\sigma_W) = \frac{1}{2} m_{\sigma}^2 \sigma_W^2 + \frac{1}{3} c_2 M_N \sigma_W^3 + \frac{1}{4} c_3 \sigma_W^4 . \]

Such self-interacting scalar field potentials have been largely employed in what is commonly referred as the Relativistic Mean-Field Model (RMF), e.g. NL3 [7] and see also the Glendenning book [47]. Note that other extensions based on density dependent coupling constant will not be considered in the present study. It is interesting to remark that the Euler-Lagrange equation for the scalar field is modified by the self-interaction terms, but the nucleon effective mass remains described by Eq. (17), as in the original Walecka model. This also makes the RMF model qualitatively similar to the RMF-C one, as we will illustrate it in the next section.

We have 4 parameters to fit, \(g_{\sigma}, g_{\omega}, c_2\) and \(c_3\). As with the case of the RMF-C model, we use the two NEPs of Table I and the PDFs of \(M_D^*\) and \(K_{\text{sat}}\) shown in Fig. 2. The results of sampling this distribution is shown in Fig. 4. We see that the PDF of \(g_{\sigma}\) is peaked around 10, which is consistent with what is shown in Fig. 2 for the RMF-CC model. Note that we fix \(m_{\sigma} = 800\text{MeV}\) compatible with the peak predicted by the RMF-CC model. Fixing the value of \(m_{\sigma}\) in RMF is not constraining if \(g_{\sigma}\) is varied: only the ratio \(g_{\sigma}/m_{\sigma}\) matters. We remind that in the RMF-CC model however this degeneracy is broken by
Eqs. (12) and (13). Additionally, the fixing of $m_\sigma$ at a constant value can be seen as analogous to the RMF-C model where the parameter $g_\sigma$ is frozen instead of $m_\sigma$. For $g_\omega$, we again obtain a value $\approx 6.25$ which is again due to the fact that we fit to a large value of $M_D^* \approx 0.85 M_N$. This value of $g_\omega$ is very close to those obtained previously for the RMF-CC and RMF-C models. Finally, we note that the PDF of the $c_2$ parameter has a peak around 10 but with an uncertainty of about 10, making it compatible with 0. The PDF of $c_3$ prefers very large values. We will comment extensively on our results for $c_2$ and $c_3$ later when we compare the scalar potentials of the different models.

Having obtained the values of $g_\sigma(g_\sigma)$ and $g_\omega$, for the three models, we can study how the three models can be separated when the correlation between $g_\sigma$ and $g_\omega$ is analysed. In Fig. 5, this correlation is plotted for the three models in different colors, where the contours represent the 95% confidence level. For the RMF-C model, only a vertical line is shown since $g_\sigma$ is fixed in this case. We see that for the three models, the centroid of $g_\omega$ are very close ($\approx 6.25$). However, the models can be separated along the horizontal coordinate ($g_\sigma$). The RMF-CC model prefers larger values of $g_\sigma$, whereas the RMF-C and RMF models prefer the lower value close to $M_N/f_\pi$.

### IV. CONFRONTATION OF THE MODELS

In the previous section, the three models have been fit to reproduce the same properties at saturation in SM. These properties are the saturation density and energy for all models, and the models RMF-C and RMF are adjusted to reproduce the same Dirac mass and incompressibility modulus as RMF-CC, which, for this model are deduced from fundamental L-QCD properties. The models are therefore treated on an equal footing by ensuring that they agree on the empirical parameters and their uncertainties: $n_{sat}$, $E_{sat}$, $K_{sat}$ and $M_D^*(n = n_{sat})$.

In this section we will show that although the predictions of these three models agree at saturation density, they differ quantitatively at larger densities since they represent different density functionals. Moreover, a detailed analysis of the scalar field properties indicates that RMF-CC represents a microscopically justified and an economical way to incorporate in-medium corrections on top of the chiral potential defined in the vacuum.

#### A. The energy per particle, the self-energies and the effective masses

We first start with an analysis of the energy per particle in SM. In Fig. 6, the results are shown in the left panel. The three models correspond to the three colors. The upper and lower limits represent the 95% CL, allowing a visualization of the uncertainties in the model predictions as a function of density. Recall that these uncertainties originate from our imperfect knowledge of nuclear matter saturation properties and fundamental predictions of LQCD. We see that the three models agree well at densities $n \approx n_{sat}$, since they are constrained to do so. The agreement also appears to be quite good at $n < n_{sat}$. However, at $n > 2n_{sat}$, RMF-CC model predicts the larger values for the energy per particle, while RMF-C model produces the smaller ones and RMF model lies in the intermediate range. Note however, that all model predictions are compatible with each other within the considered 95% confidence levels. Given that $g_\omega$ is similar for all three models, the reason for differences at high densities is most probably related to the scalar interaction, i.e. the scalar potential and/or the scalar coupling to the nucleons. In the next section, we will investigate the former in detail.

In the center and right panels of Fig. 6, the scalar and the vector (time component) self energies, $\Sigma_s$ and $\Sigma_0$ are shown. At the mean-field level in SM, we have

\begin{align}
\Sigma_s &= M_N(s) - M_N \\
\Sigma_0 &= g_\omega^2/m_\omega^2 \rho.
\end{align}

We see again that the three models agree at low densities. At larger densities RMF-CC predicts a slightly larger value for $\Sigma_s$, however there is still significant overlap among the predictions. For $\Sigma_0$, the models still agree at large densities. This is expected since $g_\omega$ is the only parameter that controls the density dependence of $\Sigma_0$, and all three models have similar values of $g_\omega$. Since correlations beyond the mean field level to a more complicated density dependence of $\Sigma_0$ [48, 49], it would be interesting to re-analyse this quantity by including Fock contributions in the future.

Finally, in Fig. 7, the Dirac and Landau masses ($M_D^*$ and $M_L^*$) are shown. The Landau mass has been computed by deriving the Schrödinger equivalent single-
FIG. 6. (left) The energy per particle in SM for the three models considered in this work. The contours show the 95% confidence level. The density dependence of the scalar self energy (center panel) and the time component of the vector self energy (right panel) are also shown.

FIG. 7. The density dependence of the Dirac and Landau masses are shown.

particle potential following Refs. [48, 49]. At the Hartree approximation, it reads

\[ M_L^* = M_N - \Sigma_0. \]  (21)

On the other hand, the Dirac mass is the same as the s-field dependent nucleon mass, i.e. \( M_D^* = M_N(s) \). All the comments made regarding \( \Sigma_s \) and \( \Sigma_0 \) are applicable to the Dirac and Landau masses respectfully, since the relationship between the self energies and the effective masses is quite straightforward in the mean-field level.

In summary, the 3 models presented here, while being calibrated on the same quantities at saturation, lead to slightly different predictions above saturation density: RMF-CC is more repulsive than RMF-C on the average, while RMF lies in between them. In the following, we investigate more closely the properties of the microscopic quantities at the base of the models: the scalar potential and the self-consistent equation for the scalar field.

B. Analysis of the scalar potential \( V(s) \)

The scalar potentials \( V(s) \) have different expressions in the models considered in our analysis. For an easy comparison, we recast the chiral potential \( V(s) \) for RMF-CC and RMF-C into the form of the scalar potential in RMF, see Eq. (18). In doing so, the chiral potential (8) in RMF-CC leads to the following coupling constants,

\[ c_{RMF-CC}^{2} = \frac{3}{2} f_{\pi} M_N \left( m_{\sigma}^2 - m_{\pi}^2 \right) \]  (22)

\[ c_{RMF-CC}^{3} = \frac{1}{2} f_{\pi} \left( m_{\sigma}^2 - m_{\pi}^2 \right) = \frac{M_N}{3 f_{\pi} c_{RMF-CC}^{2}}, \]  (23)

and for RMF-C the chiral potential (15) gives

\[ c_{RMF-C}^{2} = \frac{1}{M_N} \left( \frac{3}{2} b_2 f_{\pi} + \frac{1}{2} b_3 f_{\pi}^3 \right) \]  (24)

\[ c_{RMF-C}^{3} = \frac{1}{2} b_2 + b_3 f_{\pi}^2 + \frac{1}{6} b_4 f_{\pi}^4. \]  (25)

The sign of the parameters \( c_2 \) and \( c_3 \) are important for the interpretation of the scalar potential in terms of a Mexican hat potential. It indeed implies that a positive \( c_2 \) generates an attractive term (since \( s \) is negative) and a positive \( c_3 \) a repulsive term. The magnitude of these parameters is also important in order to interpret the different terms of the potential as an expansion in terms of many-body interactions, since a term in \( s^n \) corresponds to an \( (n - 1) \)-body force. Since these many-body forces are expected to be hierarchically ordered (at least at low densities), truncations at different orders of the scalar potential are expected to evolve smoothly. When this is
FIG. 8. Analysis of the potential of the scalar field. Considering the potential as an expansion in $s$, the number in the legend refers to the order at which this expansion is truncated. The parameters of the scalar potential are taken to be the mean values reported in Table II. In the right panel, for the NL3 parametrization (dashed lines) the orange dashed line is hidden behind the green one.

TABLE II. Coefficients of the scalar potentials expressed as in Eq. (18) for the three models considered here. The quoted uncertainties represent the 95% CL.

| Model   | $c_2$       | $c_3$       |
|---------|-------------|-------------|
| RMF-CC  | $11.3^{+2.2}_{-1.7}$ | $37.5^{+7.3}_{-5.6}$ |
| RMF-C   | $47.2^{+37.4}_{-23.5}$ | $5880^{+3870}_{-2470}$ |
| RMF     | $8.9^{+17.2}_{-12.6}$ | $2010^{+940}_{-820}$ |
| NL3[7]  | -29.89      | -2.19       |

not the case, we will interpret it as an anomaly of the scalar potential.

In Tab. II we compare the parameters $c_2$ and $c_3$ determined for the three models. For all the models considered in this work, except NL3, the centroids of both $c_2$ and $c_3$ are positive, as expected. The parameters $c_2$ and $c_3$ of the RMF model are found to be different from the original NL3 model of Ref. [7], where $c_2$ and $c_3$ are both negative. Since the parameters $c_2$ and $c_3$ are obtained from a fit to the NEPs, their values are determined from the values considered for these NEPs. The values for $E_{\text{sat}}$, $n_{\text{sat}}$, $M_D(n = n_{\text{sat}}) = M_N(s)$ and $K_{\text{sat}}$ are indeed different for NL3 and the RMF case.

Large values of $c_3$ found for RMF and RMF-C indicate that $V(s)$, when considered as an expansion in $s$, might present an anomaly in the order hierarchy. For make it more clear, we show in Fig. 8 the chiral potential truncated at various orders in $s$, starting from order 2. In RMF-CC, the distinction between order 3 and order 4 curves appears only at large $s$, and this 4th order correction is relatively small. In the case of RMF-C however, we see that every addition of a higher order correction drastically changes the behaviour of the scalar potential. Indeed, a truncation at order 3 or 5 would result in an overall change of sign of $V(s)$ at $s \approx 0.5$. Therefore in the case of RMF-C, the correct reproduction of nuclear NEPs in SM is due to a fine tuning between the parameters $b_2$, $b_3$ and $b_4$, rendering difficult the interpretation of $V(s)$ in terms of many-body forces. We thus qualify the chiral potential in RMF-C as presenting an anomaly. We have a similar behaviour for RMF. The 4th order correction to the order 3 curve is very large, which is imposed by the saturation properties of RMF-CC. The scalar potential of RMF is thus also possibly anomalous. This conclusion is of course limited to the explored parameters region considered in our study - and related to the predictions of RMF-CC model - but different parameter sets could lead to a convergent expansion, as illustrated for instance by NL3 (RMF model), see the right panel of Fig. 8. Note that in this case the 4th order correction (dashed green line) is so small that it lies on top of and thus hides the 3rd order term.

In conclusion, we observe that if the three models RMF-CC, RMF-C and RMF are constrained to reproduce the same properties at saturation, the $s$ expansion of the scalar potential $V(s)$ may manifest an anomalous behaviour for RMF-C and RMF, at variance with RMF-CC. In the following section, we show that the origin of this anomaly for RMF-C and RMF can be related to the absence of the scalar nucleon response in their Lagrangian.
C. Analysis of the equation of motion of the scalar field

We now analyze in details the equation of motion (EoM) for the scalar field where the scalar potential plays naturally a crucial role. We will show that the anomaly of the scalar potential observed in the previous subsection for RMF-C and RMF models impacts the solution of the EoM.

The scalar EoM for the scalar field $s$ is, see e.g. Appendix A,

$$V'(s) = -g_s^* \rho_S$$

with

$$g_s^* = \frac{\partial M_N(s)}{\partial s},$$  \hspace{1cm} (26)

where $\rho_S$ is the scalar density. Note that for RMF-C and RMF models, $g_s^* = g_s$ since $M_N(s)$ is simply linear in the field $s$.

In Fig. 9, we represent the graphical solution of the EoM by drawing the two sides of the equation: $V'(s)$ is plotted as the solid blue lines and $-g_s^* \rho_S$ as the solid orange lines for the three models (by columns) and at different densities (by rows). The parameters of the scalar potential are taken to be the mean values reported in Table. II, and similarly we consider the centroid of the PDFs for $g_s$ which are 11.10, 9.98 and 10.08 for RMF-CC, RMF-C and RMF respectively. In case of several solutions, the physical one is the smallest one and it is identified as a black star. We see that as the density increases, the solution for the $s$-field (abscissa of the black star) of the scalar EoM increases. It is interesting to note that for all three models, at a given density, the value of this solution is quite similar. This is due to the fact that the quantity $g_s$ defines the in-medium Dirac mass (except for RMF-CC where the nucleon response is also included) which remains almost identical for the three models, see Fig. 7 and $g_s$ does not differs by more than about 10% between the different models, see Fig. 5. As a consequence the values of the field $s$ are very close between the three models considered here.

The absolute value of the y-coordinate of the solution ($\propto V'(s)$) is however always smaller for RMF-CC compared to the RMF-C and RMF models. Since $V(s)$ is the vacuum chiral potential in the case of RMF-CC, the vertical position of the intersection point informs us about the derivatives of this potential for various values of the field $s$. In other words, for RMF-CC density scans the chiral potential function of $s$ and the in-medium effects are entirely captured by the nucleon response given by $g_s^*$.

The situation is however different for RMF-C and RMF models. These models share two important features: i) they do not incorporate explicitly the nucleon response as in RMF-CC, and ii) the scalar potential is determined from the fit to saturation properties. If the fit imposes a modification of the scalar potential making it different from the vacuum one, it is interpreted as an

![FIG. 9. Analysis of the equation of motion of the scalar field. The different rows correspond to different densities. Dashed lines correspond to the equation of motion written for RMF-CC with the effective potential, see Eq. 27.](image-url)
in-medium correction to the scalar potential. It is interesting to remark that the result of the fit, which is made differently for RMF-C and RMF, is to impose larger absolute values for $V'(s)$ as function of $s$ compared to the vacuum values represented by RMF-CC model. As a consequence, the intersection points in RMF-C and RMF happen at larger absolute values compared to RMF-CC. The vertical intersection point therefore informs us either on the role of the nucleon response in the EoM (26) (for RMF-CC), or on the in-medium modification of the scalar potential (for RMF-C and RMF).

At first sight, the models fitted to saturation and disregarding nucleon response (RMF-C and RMF) suggest large in-medium modification of the scalar potential, while the models considering the vacuum chiral potential complemented with nucleon response (RMF-CC) do not require any in-medium modification of the chiral potential. One may however wonder to which extend two opposite conclusions do not reflect a similar reality suggesting that the nucleon polarization may modify in an effective way the vacuum scalar potential. It may even be the dominant in-medium correction to the scalar EoM.

In order to address this question, we rewrite the EoM for RMF-CC to absorb the effects of the nucleon response in an effective scalar potential $\tilde{V}'(s) = V'(s) + g_S^\star \rho_S - g_S \rho_S$, as

$$\tilde{V}'(s) = -g_S \tilde{\rho}_S. \quad (27)$$

This leaves the standard scalar coupling $g_S$ on the RHS of Eq. (27) (as in the other models). Note that this rearrangement has been done by ensuring that it is still the same self-consistent equation of motion that is being solved for the RMF-CC. Finally, $\tilde{\rho}_S$ denotes that the dependence of the scalar density on $s$ via the nucleon mass $M_N(s)$ is obtained by using a linear relation for $M_N(s)$ (as in RMF-C and RMF) and not the non-linear one used in RMF-CC. In this way, Eq. (27) is formally equivalent to the EoM (26) solved for RMF-C and RMF models.

In Fig. 9, the left column (for RMF-CC) displays dashed blue and dashed orange lines corresponding to the graphical solution of Eq. (27) in terms of the effective potential $\tilde{V}'(s)$. With such a construction, we see that the dashed blue line intersects the dashed orange line for larger absolute values of $\tilde{V}'(s)$, similar to RMF-C and RMF. This clearly demonstrates that the smaller absolute values of $\tilde{V}'(s)$ obtained for RMF-CC is a consequence of the inclusion of the scalar response of the nucleon. In other words, the nucleon polarization captures most of the in-medium correction to the vacuum scalar potential.

While Fig. 9 clearly demonstrates that the nucleon polarization is the dominant in-medium correction to the scalar EoM, a similar conclusion may have been obtained in the previous section based on the values for $c_2$ given in Tab. 5. In the RMF-CC model, the (positive) $c_2$ parameter controls the magnitude of the above mentioned attractive tadpole diagram which destroys saturation. For hierarchically ordered scalar potentials, it has been shown that after an appropriate shift of the scalar field, $\sigma_W = s + (\kappa_{NS}/2q_s)^2$, where the term $\kappa_{NS}$ represents the nucleon response, the Dirac mass of the nucleon becomes $M_N(s) = M_N + g_s \sigma_W$ and the nucleon polarizability renormalizes the cubic term of the scalar potential as $c_2 (1 - 2C) \simeq -2c_2$ if $C \simeq 1.5$ [50]. One sees that this is qualitatively compatible with the values of $c_2$ for RMF-CC and NL3 quoted in Tab. II. One can thus remark that the negative value of $c_2$ in the original NL3 model Ref. [7] simulates in an effective way the nucleon response. Note also that this discussion is not applicable to the RMF model since its scalar potential displays an anomalous behaviour. However it is interesting to note that while having a positive centroid for $c_2$, negative values for $c_2$ are also allowed in the PDF for the RMF model.

In conclusion, we have shown that the in-medium modification of the scalar potential which is captured in RMF-C and RMF models by the fit to saturation properties can also be simulated in RMF-CC by a single in-medium term in the Lagrangian: the nucleon response generated by the coupling of the constituent quarks to the large scalar field at finite density. The nucleon response effect, being characterized by a single coupling constant ($\kappa_{NS}$ or $C$) in the RMF-CC Lagrangian represents therefore a very economical way to capture in-medium correction to the scalar EoM, on top of being well motivated from a microscopic viewpoint. In RMF-CC the chiral potential at finite density is identical to the vacuum one and one could interpret the solution of the scalar EoM as a scan of $\tilde{V}'(s)$ at different values of $s$. This latter point suggests that the solution of the EoM at finite density may be a way to probe the properties of the chiral potential in vacuum.

V. EXCITED STATES IN DENSE MATTER

In the previous section, we have illustrated the equivalence between actual in-medium modification of the scalar potential guided by the fit to saturation properties (as in RMF-C and RMF) and in-medium effect of the nucleon polarization (as in RMF-CC). We have also suggested that the nucleon polarization is a economical way to treat in-medium correction to the scalar EoM. One may however wonder if the effect of the nucleon polarization could influence other properties in medium. It is therefore natural to come to the exploration of the excitation spectrum of dense matter.

We limit ourself to the scalar-isoscalar excitation channel, which is determined by the scalar-isoscalar Landau parameter $F_0$ at low excitation energy (and zero momentum transferred). Following Ref. [2], we have computed the fully relativistic Landau parameter $F_0$ for the three models (see Appendix A for the derivation). The final
The small contribution from the collision of the dense fire-ball produced by relativistic heavy-ion stars. Additionally, it may also modify the properties of scattering and other phenomena in the core of neutron stars. As an example, since it modifies the nuclear response functions, it may have implications for neutrino emission. The fit of all three models to SM properties was performed in a very novel way by simultaneously varying Lattice QCD parameters during the Bayesian fit, establishing a direct link with the underlying QCD theory. The fit of the RMF-CC parameters to SM properties was constrained by ensuring that the three models agree for the three models, as it could be expected because the three models are calibrated such that they reproduce the same incompressibility modulus \( K_{\text{sat}} \), see Eq. (A11). However, at larger densities, the RMF-CC models predict larger values of \( K_{\text{sat}} \) than RMF and then RMF-C. At \( 4n_{\text{sat}} \) the values for \( F_0 \) suggested by RMF-CC are almost twice the ones predicted by RMF-C.

This distinction in the predictions of \( F_0 \) by the three models at large densities may have important phenomenological consequences for dense matter in neutron stars. As an example, since it modifies the nuclear response functions, it may have implications for neutrino scattering and other phenomena in the core of neutron stars. Additionally, it may also modify the properties of the dense fire-ball produced by relativistic heavy-ion collision.

VI. THE SYMMETRY ENERGY

In this paper, we have restricted our many-body treatment to the Hartree approximation (classical fields) and to SM. In the future, we will also include the contribution of the Fock terms and we will explore asymmetric matter. At the Hartree level, the symmetry energy is however only determined by the \( \rho \) vector iso-vector meson. The small contribution from the \( \delta \) scalar-isovector meson is disregarded here, and one obtains

\[
E_{\text{sym}} = \frac{k_F^2}{6\sqrt{k_F^2 + M_N^2(s)}} + \frac{g^2_\rho}{2m_\rho^2} \rho^2,
\]

where \( g_\rho \) and \( m_\rho \) are the coupling constant and the mass of the \( \rho \) meson. If the quark model is assumed, then \( g_\rho = g_\omega/3 \). In this case, the predictions for the symmetry energy at saturation density \( E_{\text{sym}} \) is shown for the three models in the upper panel of Fig. 11. The empirical value is shown as a red band. We see that there is a difference of about 12 – 15 MeV (about half the expected value for \( E_{\text{sym}} \)) between the predicted value and the empirical one.

In the lower panel, in solid lines the value of \( g_\rho \), assuming the quark model (as in the upper panel) is confronted with the dashed lines showing the value of \( g_\rho \) required to reproduce the empirical value for \( E_{\text{sym}} \). There is a factor 2 difference between the solid and the dashed curves.

We interpret the discrepancy between the quark model prediction and the empirical value for \( E_{\text{sym}} \) as originating from the corrections beyond the Hartree approximation. In a future work, we will illustrate this point by adding to the present modeling the contribution of the Fock term, without modification of the fitting procedure. Preliminary results give us confidence in our interpretation.

VII. CONCLUSIONS

In this paper we have analysed the important role of chiral symmetry breaking and scalar nucleon response in the study of nuclear matter. We have done this, for the first time, by systematically comparing a model that takes these features into account (RMF-CC) versus models that neglect some or all of these aspects (RMF-C and RMF).

The systematic analysis was performed by ensuring a democratic treatment of the three models: we constrained the three models to agree with each other in the vicinity of saturation density of SM. In particular, the fit of the RMF-CC parameters to SM properties was performed in a very novel way by simultaneously varying Lattice QCD parameters during the Bayesian fit, establishing a direct link with the underlying QCD theory. The fit of all three models to SM properties was
performed in a consistent Bayesian manner which allows us to properly explore the uncertainties in our empirical knowledge of nuclear matter saturation. 

Examining various aspects of the models and their predictions at different densities we came to the conclusion that the scalar nucleon response is a microscopically justified and an economical way to incorporate in-medium corrections to the scalar EoM. In RMF-CC the modification of the effective potential at high density is driven by a microscopic mechanism, while in RMC-C and RMF approaches the scalar potential already encompasses finite density properties at saturation, that are simply extrapolated. In addition, we have shown that if the nucleon response is neglected, the scalar potential become anomalous since the hierarchy in the orders of \( s \) is not respected as it is expected in a many-body framework. Moreover the ground state and excited states in the scalar-isoscalar channel are predicted to be noticeably different as the density increases (2 to 4 \( n_{\text{sat}} \)).

The RMF-CC approach represents a step in the modeling of matter properties beyond saturation density since the model parameters are mainly given by fundamental properties, e.g., L-QCD predictions or quark model constraints, and only saturation density \( n_{\text{sat}} \) and energy \( E_{\text{sat}} \) have actually been used in the model calibration. All other empirical parameters are predicted, e.g., \( K_{\text{sat}} \) or \( E_{\text{sym}} \). We have shown for instance that the Hartree approximation of the mean field is not sufficient to reproduce the empirical values for \( E_{\text{sym}} \). In our RMF-CC approach \( E_{\text{sym}} \) can thus be used to evaluate the contribution of the correlations beyond the Hartree approximation, as well as of the missing interaction terms, e.g., the pion contribution. This could have important phenomenological consequences in the description of very dense matter.

Finally, phase transitions are expected to occur in the very dense matter found in the core of massive neutron stars. These phase transitions could lead to the appearance of other (non-nucleonic) hadronic degrees of freedom such as pion and kaon condensates as well as hyperons. Other phenomena such as chiral symmetry restoration and transition to deconfined quark matter might also take place. The exploration of these two possibilities will also be very interesting to investigate in the future using the models developed in this work, since they incorporate chiral symmetry and confinement.

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**Appendix A: Compressibility modulus and Landau parameter \( F_0 \) in relativistic theory**

The nuclear matter energy density in the RMF theories discussed in this paper can be written as :

\[
\varepsilon = \int \frac{4d^3k}{(2\pi)^3} \left( \sqrt{k^2 + M_0^2(s)} + g_\omega \omega_0 \right) \Theta(k_F - k) \\
+ V(s) - \frac{1}{2} m_\omega^2 \omega_0^2,
\]

(A1)

where the scalar and vector field are obtained from the equations of motion :

\[
m_\omega^2 \omega_0 = g_\omega \rho
\]

(\( A2 \))

\[
V'(s) = -g_S^s \rho_S \quad \text{with} \quad g_S^s = \frac{\partial M_N(s)}{\partial s}
\]

(A3)

In the following we will make use of the following relations or definitions:

\[
\rho = \frac{1}{3} \frac{2 k_F^3}{\pi^2}, \quad \frac{\partial k_F}{\partial \rho} = \frac{1}{3} \frac{k_F}{\rho} \quad N_{0R} = \frac{2 k_F E_F}{\pi^2},
\]

(A4)
where \( E_F = \sqrt{k_F^2 + M_N^2(s)} \) is the Fermi energy and \( N_{0R} \)
is the density of states on the (relativistic) Fermi surface. The first derivative with respect to the density of the energy density is obtained using equations of motion with the result:

\[
\frac{\partial \varepsilon}{\partial \rho} = E_F + \frac{g_\sigma^2}{m_\sigma^2} \rho \equiv \mu. \tag{A5}
\]

The second derivative is:

\[
\frac{\partial^2 \varepsilon}{\partial \rho^2} = \left[ \frac{k_F}{E_F} + \frac{M_N(s)}{E_F} g_\sigma^2 \frac{\partial \rho}{\partial k_F} \right] \frac{\partial k_F}{\partial \rho} + \frac{g_\sigma^2}{m_\sigma^2} . \tag{A6}
\]

The derivative of \( s \) with respect to the Fermi momentum is obtained by taking the derivative of the equation of motion (Eq. A3):

\[
V''(s) \frac{\partial s}{\partial k_F} = -\kappa_{NS} \frac{\partial s}{\partial k_F} \rho_S + g_\sigma^2 \rho_S \quad \text{with} \quad \kappa_{NS} = \frac{\partial g_\sigma^2}{\partial s}. \tag{A7}
\]

We introduce an effective sigma meson mass, such as \( m_\sigma^2 = m_\sigma^2 + \kappa_{NS} \rho_S \), to obtain:

\[
\frac{\partial s}{\partial k_F} = -\frac{1}{m_\sigma^2} \frac{\partial \rho_S}{\partial k_F} \tag{A8}
\]

and the derivative of the scalar density has the form:

\[
\frac{\partial \rho_S}{\partial k_F} = \frac{\partial}{\partial k_F} \left[ \int \frac{4d^3k}{(2\pi)^3} \frac{M_N(s)}{\sqrt{k^2 + M_N^2(s)}} \Theta(k_F - k) \right]
\]

\[
= \frac{\partial \rho}{\partial k_F} \frac{M_N(s)}{E_F} + I_3(k_F) \frac{\partial s}{\partial k_F} \tag{A9}
\]

with \( I_3 = \int \frac{4d^3k}{(2\pi)^3} \frac{k^2}{(k^2 + M_N^2(s))^{3/2}} \Theta(k_F - k) \).

Combining Eqs. (A8) and (A9), we obtain:

\[
\frac{\partial \rho_S}{\partial \rho} = \frac{\partial \rho_S}{\partial k_F} \frac{\partial k_F}{\partial \rho} = -\frac{g_\sigma^2}{m_\sigma^2} \frac{M_N(s)}{E_F} \left[ 1 + \frac{g_\sigma^2}{m_\sigma^2} I_3(k_F) \right]^{-1}. \tag{A10}
\]

Using the previous results, the compressibility modulus can be written in the following form:

\[
K_{sat} = 9 \rho \frac{\partial^2 \varepsilon}{\partial \rho^2} = \frac{3k_F^2}{E_F} (1 + F_0), \tag{A11}
\]

which depends on the relativistic generalization of the Landau parameter \( F_0 \):

\[
F_0 = N_{0R} \left( \frac{g_\sigma^2}{m_\sigma^2} - \frac{g_\sigma^2}{M_N(s)} \right) \left[ 1 + \frac{g_\sigma^2}{m_\sigma^2} I_3(k_F) \right]^{-1}. \tag{A12}
\]

This result derived in a different manner has been quoted in [2] but omitting the (small) correction arising from the \( I_3 \) integral. Notice that, as demonstrated in Ref. [51], \( g_\sigma^2 I_3(k_F) \) corresponds to the nuclear response associated with \( NN \) excitation. Also notice that our result coincides with the one derived by T. Matsui [52] but with absence of medium modification (i.e., in the absence of the nucleon susceptibility term) of the scalar mass and coupling constant.
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