Thermodynamic analysis of irreversible closed Brayton engine cycles used in trigeneration systems

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Abstract. The paper presents a comparative thermodynamic analysis of irreversible closed Brayton engine cycles for two working fluids and for four imposed thermodynamic parameters. The chosen fluids are the air and the carbon dioxide. The four imposed thermodynamic constraints are the specific entropy variation during the heating process, the specific input heat, the specific output power and the energy efficiency. The irreversibility is quantified by the isentropic efficiencies of compressor and of gas turbine, and by the pressure drops coefficient during the heating and cooling processes of the working fluid. Each imposed thermodynamic parameter allowed to evaluate all corresponding cycle parameters, i.e. temperatures and pressures, and operational parameters, i.e. energy transfer values, function of pressure compression ratio and irreversibility magnitude. The thermodynamic analysis compares the selected results for the two working fluids.

1. Introduction
The most current researches of open or closed Brayton engine cycles follow to the new energy technologies trends, either new single or combined cycles with new working fluids or hybrid solar assisted engines or outer space applications. For instance, the specific research objectives refer to exhaustive exergy analysis regarding endogenous and exogenous lost exergy [1], exergy analysis following the maximization of power output and genetic algorithm optimization [2, 3], entropy generation minimization [4], performance assessments [5-10], ecological aspects [11] and for instance to specific reviews for solar energy technologies [12]. Recently proposed methods of thermal systems design [13, 14, and 15] apply the Finite Physical Dimensions Thermodynamics (FPDT) to design, optimize or perform sensitivity analysis of various thermal systems. This paper presents a comparative thermodynamic analysis of irreversible closed Brayton engine cycles for two working fluids and for four imposed thermodynamic parameters. The chosen fluids are the air and the carbon dioxide. The four imposed thermodynamic parameters are the specific entropy variation during the heating process, the specific input heat, the specific output power and the energy efficiency. The irreversibility is quantified through the isentropic efficiencies of compressor and of gas turbine, and through the pressure drops coefficient during the heating and cooling processes of the working fluid. Each imposed thermodynamic parameter allowed to evaluate all corresponding cycle parameters, i.e.
temperatures and pressures, and operational parameters, i.e. energy transfer values, function of pressure compression ratio and irreversibility magnitude. The thermodynamic analysis compares the selected results for the two working fluids.

2. Basic mathematical model

The analyzed scheme of the irreversible Brayton cycle is presented in $T – s$ diagram, Figure 1.

2.1. General irreversible energy efficiency depending on the working fluid nature

$$EE_{irr} = 1 - \frac{T_{mq, Airr-5r}}{T_{mq, 2irr-3r}} \left( 1 + \frac{\Delta s_{irr,1-2}}{\Delta s_{irr,4-1}} \right) = 1 - \frac{T_{mq, Airr-5r}}{T_{mq, 2irr-3r}} \cdot N_{irr}$$  \hspace{1cm} (1)

where:

- $T_{mq, Airr-5r} = \frac{T_{5r} - T_{4irr}}{\ln \left( \frac{T_{5r}}{T_{4irr}} \right)} = \frac{T_{1} - T_{4irr}}{\ln \left( \frac{T_{1}}{T_{4irr}} \right)}$ is the mean thermodynamic temperature of the working fluid during the cooling at constant pressure;

- $T_{mq, 2irr-3r} = \frac{T_{3r} - T_{2irr}}{\ln \left( \frac{T_{3r}}{T_{2irr}} \right)} = \frac{T_{3irr} - T_{2irr}}{\ln \left( \frac{T_{3irr}}{T_{2irr}} \right)}$ is the mean thermodynamic temperature of the working fluid during the heating at constant pressure;

- $N_{irr}$ is the number of internal overall irreversibility;

- $s$ and $\Delta s_{irr}$ are the specific entropy and process irreversible additional entropy variations depending on the working fluid nature;

- $T_{3irr} = T_{3r}$ is the maximum temperature on the irreversible cycle;

- $EE_{Carnot} = 1 - \frac{T_{mq, Airr-5r}}{T_{mq, 2irr-3r}}$ is the energy efficiency of Carnot engine cycle operating between $T_{mq, 4irr-5r}$ and $T_{mq, 2irr-3r}$.

2.2. The hypotheses of comparative thermodynamic analysis

The comparative thermodynamic analysis is performed on the basis of below four restrictive requirements.

- Constant specific entropy variation during the reversible heating process:

$$\Delta s_{q} = s_{3r} - s_{2irr} = c_{p} \cdot \ln \left( \frac{T_{3r}}{T_{2irr}} \right) = c_{p} \cdot \ln \left( \frac{T_{3irr}}{T_{2irr}} \right) = c_{p} \ln(2)$$  \hspace{1cm} (2)
Constant specific heat input, $q_H$:

$$q_H = c_p \cdot (T_{3r} - T_{2irr}) = c_p \cdot (T_{3irr} - T_{2irr}) = 500 \frac{kJ}{kg}$$ (3)

Constant specific power output, $w$:

$$w = q_H - |q_C| = c_p \cdot [(T_{3r} - T_{2irr}) - (T_{4irr} - T_{5r})] = c_p \cdot [(T_{3irr} - T_{2irr}) - (T_{4irr} - T_1)] = 100 \frac{kJ}{kg}$$ (4)

Constant energy efficiency, $EE_{irr}$:

$$EE_{irr} = \frac{w}{q_H} = 1 - |q_C|/q_H = 1 - \frac{T_{4irr} - T_{5r}}{T_{3r} - T_{2irr}} = 1 - \frac{T_{3irr} - T_1}{T_{3irr} - T_{2irr}} = 0.3$$ (5)

The irreversibility is quantified by the isentropic efficiencies of compressor and of gas turbine, and by the pressure drops coefficient during the heating and cooling processes of the working fluid.

- The isentropic efficiency of irreversible adiabatic compression $1 - 2_r$:
  $$\eta_{sC} = 0.8 \text{ and } \eta_{sC} = 0.85$$ (6)

- The isentropic efficiency of irreversible adiabatic expansion $3_{irr} - 4_{irr}$:
  $$\eta_{sT} = 0.85 \text{ and } \eta_{sT} = 0.9$$ (7)

- The pressure drop coefficient during the irreversible heating, $2_{irr} - 3_{irr}$ and cooling, $4_{irr} - 1$:
  $$r_p = \frac{p_{3irr}}{p_{2irr}} = 0.975 \text{ and } r_p = \frac{p_{3irr}}{p_{2irr}} = 0.95$$ (8)

The chosen working fluids are the air and the carbon dioxide regarded as perfect gases.

The imposed thermodynamic parameters are:

- $p_1 = 1 \text{ bar and } T_1 = 303 \text{ K}$ (9)

The variable thermodynamic compression ratio:

$$\pi_C = \frac{p_{sC}}{p_1} = \frac{p_{2irr}}{p_1}$$ (10)

2.3. Equations of basic mathematical model

2.3.1. Ending states temperature of consisting processes of the cycle

$$T_{2r} = T_1 \cdot \pi_C^k$$ (11)

$$T_{2irr} = T_1 + \frac{T_{2r} - T_1}{\eta_{sC}}$$ (12)

$$T_{3r} = T_{3irr} = T_1 \cdot e^{c_p \cdot \left(1 + \eta_{sC} \cdot \left(\frac{k-1}{\pi_C^k} - 1\right)\right)} \text{ for imposed } \Delta s_q = c_p \cdot \ln(2)$$ (13)

$$T_{3r} = T_{3irr} = \frac{q_H}{c_p} + T_1 \cdot \left[1 + \eta_{sC} \cdot \left(\frac{k-1}{\pi_C^k} - 1\right)\right] \text{ for imposed } q_H = 500 \frac{kJ}{kg}$$ (14)

$$T_{3r} = T_{3irr} = \frac{c_p \cdot T_1 \cdot \left(\frac{k-1}{\pi_C^k} - 1\right) + \eta_{sC} \cdot w}{c_p \cdot \eta_{sC} \cdot \eta_{sT} \cdot \left[1 - \left(r_p^2 \cdot \pi_C\right)^{1-k}\right]} \text{ for imposed } w = 100 \frac{kJ}{kg}$$ (15)
The maximum temperature on the cycle, the specific output work, and the irreversible energy efficiency are operational parameters used in the design of Brayton engines. The number of internal overall irreversibility puts a figure on overall internal irreversibility depending on the working fluid nature.
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Fig. 2.a. case $\Delta s_q = \text{const.} = c_p \ln(2)$

Fig. 2.b. case $q_H = \text{const.} = 500,000 \, \text{J kg}^{-1}$

Fig. 2.c. case $w = \text{const.} = 100,000 \, \text{J kg}^{-1}$

Fig. 2.d. case $EE_{irr} = \text{const.} = 0.3$

**Figure 2.** The dependences $T_{3irr} = f_1(\pi_C)$

1: air, $\eta_{SC} = 0.85$, $\eta_{ST} = 0.9$, $r_p = 0.975$;  
2: air, $\eta_{SC} = 0.8$ and $\eta_{ST} = 0.85$, $r_p = 0.95$;  
3: CO$_2$, $\eta_{SC} = 0.85$, $\eta_{ST} = 0.9$, $r_p = 0.975$;  
4: CO$_2$, $\eta_{SC} = 0.8$ and $\eta_{ST} = 0.85$, $r_p = 0.95$

The admissible maximum temperature on the cycle, currently from 1100 K to 1400 K, limits the maximum compression ratio, see Figures 2.a. and 2.d. These limitations depend on the fluid nature and on the internal irreversibility intensity. The imposed constraints emphasized different relationships $T_{3irr} = f_1(\pi_C)$. For all constraints a worsen internal irreversibility induces larger temperatures $T_{3irr}$. The constraint $w = \text{const.} = 100 \, \text{J kg}^{-1}$, Figure 2.c., revealed minimum temperatures $T_{3irr}$, the corresponding compression ratios are around five for air, and around eight for CO$_2$.

Fig. 3.a. case $\Delta s_q = \text{const.} = c_p \ln(2)$

Fig. 3.b. case $q_H = \text{const.} = 500,000 \, \text{J kg}^{-1}$

Fig. 3.c. case $EE_{irr} = \text{const.} = 0.3$

**Figure 3.** The dependences $w = f_2(\pi_C)$

1: air, $\eta_{SC} = 0.85$, $\eta_{ST} = 0.9$, $r_p = 0.975$;  
2: air, $\eta_{SC} = 0.8$ and $\eta_{ST} = 0.85$, $r_p = 0.95$;  
3: CO$_2$, $\eta_{SC} = 0.85$, $\eta_{ST} = 0.9$, $r_p = 0.975$;  
4: CO$_2$, $\eta_{SC} = 0.8$ and $\eta_{ST} = 0.85$, $r_p = 0.95$
Figure 3.a. shows that the specific output work is larger for air than for CO$_2$, and also the fact that the increased internal irreversibility diminishes it. Figures 3.b. exhibits smaller differences and the fact that the internal irreversibility has a stronger influence. Figure 3.c. exhibits also smaller differences but the influence of internal irreversibility has an opposite influence.

Figures 4.a. and 4.b. show the energy efficiency is larger for air than for CO$_2$, and also the increased internal irreversibility drastically diminishes it. Figures 4.b. and 4.c. exhibit smaller differences, but the internal irreversibility has also a stronger influence.

Figure 4.a. shows that the energy efficiency is larger for air than for CO$_2$, and also the increased internal irreversibility drastically diminishes it. Figures 4.b. and 4.c. exhibit smaller differences, but the internal irreversibility has also a stronger influence.
Figures 5.a. to 5.d. show numbers of internal overall irreversibility larger for air than for CO₂. Figures 5.a., 5.b. and 5.c. say that as smaller the irreversibility coefficients (isentropic efficiencies and pressure drops ratios) as larger \( N_{irr} \). Figure 5.d. shows an opposite influence of irreversibility coefficients.

4. Conclusions

The design of Brayton engine cycles is performed on the basis of restrictive conditions imposed by the end users. In this paper were analyzed four possible scenarios to design the closed Brayton cycles in order to find out the operational and technological limits regarding:

- the maximum admissible temperature on the cycle, currently from 1100 K to 1400 K, and thus the maximum admissible compression ratio has different limitations depending on the fluid nature and on internal irreversibility;
- the specific output useful work, usually imposed in the designing process;
- the specific input heat, possible in the cases of limited thermal energy resources;
- the irreversible energy efficiency, main criterion in choosing of the convenient decision;
- the internal overall irreversibility evaluation depending on the working fluid nature.

The four scenarios to design Brayton engine closed cycles are:

- constant specific entropy variation of the working fluid during the reversible heating process;
- constant specific input heat on the cycle;
- constant specific output useful work;
- constant energy efficiency.

They were compared two working fluid, air and carbon dioxide, regarded as perfect gases. They were supposed two irreversibility cases. The first considered the best actual irreversibility coefficients: the isentropic efficiency of compressor of 0.85, and the isentropic efficiency of gas turbine of 0.9, and a constant pressure drop coefficient of 0.975, defined as the ratio of final pressure to initial one, for flows to heat exchangers.

The comparative thermodynamic analysis showed:

- for the case of specific constant entropy variation, the admissible maximum temperature \( T_{3r} = T_{3irr} = 1400 \) K imposes a maximum compression ratio of 14 for air, see Figure 2.a. – curves 1 and 2;
- for the case of constant energy efficiency, the admissible maximum temperature \( T_{3r} = T_{3irr} = 1400 \) K imposes the best irreversibility coefficients, see Figure 2.d. – curves 1 and 3;
- for the case of constant specific output work, the curve of maximum temperature \( T_{3r} = T_{3irr} \) has a minimum for all cases, see Figure 2.c.;
- the internal overall irreversibility is smaller for carbon dioxide than for air, and it has similar features for specific constant entropy variation, specific input heat and specific output work, see Figures 5.a., 5.b., 5.c., but different for constant energy efficiency.

This comparative thermodynamic analysis might be completed by an exhaustive sensitivity analysis for all possible irreversibility cases and for all possible working fluids.

References

[1] Mossi I A K and Goni B K 2019 Advanced exergy analysis of a combined Brayton/Brayton power cycle, *Energy*, 166 pp 724-737
[2] Mohammad M N, Said F and Faramarz S 2016 Exergoeconomic multi objective optimization and sensitivity analysis of a regenerative Brayton cycle, *Energy Conversion and Management* 117 pp 95–105
[3] Mohammad M N, Said F and Faramarz S 2017 New exergy analysis of a regenerative closed Brayton cycle, *Energy Conversion and Management* 134 pp 116–124
[4] Haseli Y 2016 Efficiency of irreversible Brayton cycles at minimum entropy generation, *Applied Mathematical Modelling* 40 pp 8366–8376
[5] Xiaoli H and Guoqiang Z 2007 Maximum useful energy-rate analysis of an endoreversible Joule–Brayton cogeneration cycle, *Applied Energy* **84** pp 1092–1101
[6] Wissam S, Bou N, Charbel J M and Maroun G N 2018, Optimization of a Brayton external combustion gas-turbine system for extended range electric vehicles, *Energy* **150** pp 745-758
[7] Mohamed A S E, Khwaja M R and Solimana A M 2017, Performance analysis of different working gases for concentrated solar gas engines: Stirling & Brayton, *Energy Conversion and Management* **150** pp 651–668
[8] Seyed A S et al. 2015 Thermodynamic and thermo-economic analysis and optimization of an irreversible regenerative closed Brayton cycle, *Energy Conversion and Management* **94** pp 124–129
[9] Kunlin C et al. 2014 Performance assessment of a closed-recuperative-Brayton-cycle based integrated system for power generation and engine cooling of hypersonic vehicle, *Aerospace Science and Technology* **87** pp 278–288
[10] Merchán R P, Santos M J, Medina A and Calvo H A 2018 Thermodynamic model of a hybrid Brayton thermosolar plant *Renewable Energy* **128** pp 473-483
[11] Yasin U, Bahri S, Ali K and Ismail H A 2006 Ecological coefficient of performance analysis and optimization of an irreversible regenerative - Brayton heat engine *Applied Energy* **83** pp 558–572
[12] Le Roux W G, Bello-Ochende T and Meyer J P 2013 A review on the thermodynamic optimisation and modelling of the solar thermal Brayton cycle, *Renewable and Sustainable Energy Reviews* **28** pp 677–690
[13] Feidt M 2017 *Finite Physical Dimensions Optimal Thermodynamics 1*, Tuition House, 27-37 St. George’Road – London SW 4EU – UK, 272 p
[14] Feidt M 2017, *Finite Physical Dimensions Optimal Thermodynamics 2*, Tuition House, 27-37 St. George’Road – London SW 4EU – UK, 280 p
[15] Feidt M, Dumitrașcu Ghe, Horbaniue B 2018 Sensitivity analysis of and endoreversible Carnot engine using *Finite Physical Dimensions Optimal Thermodynamics*, Colloque Francophone en Energie, Environnement, Économie et Thermodynamique, June 28 – 29, INSA Strasbourg, France
[16] Dumitrașcu Ghe 2008, The way to optimize the irreversible cycles, *Revista Termotehnica* **2**, 18-22, ISSN 1222-4057, online ISSN 2247-1871

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