Natural phantom dark energy, wiggling Hubble parameter $H(z)$ and direct $H(z)$ data

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Abstract. Recent direct $H(z)$ data indicate that the parameter $H(z)$ may wiggle with respect to $z$. On the other hand, the luminosity distance data of supernovae flatten the wiggles of $H(z)$ because of the integration effect. It is expected that the fitting results can be different in a model permitting a wiggling $H(z)$ because the data of supernovae are degenerate in such a model. As an example the natural phantom dark energy is investigated in this paper. The dynamical property of this model is studied. The model is fitted by the direct $H(z)$ dataset and the SNLS dataset, respectively. The results are rather different, as expected. The quantum stability of this model is also briefly discussed. We find that it is a viable model if we treat it as an effective theory truncated by an upper bound.

Keywords: dark matter, dark energy theory
1. Introduction

The acceleration of the universe is one of the most significant cosmological discoveries over the last decades [1]. The decisive evidence of the present acceleration is shown by supernovae. The principle of this conclusion is based on the fitting of the LCDM (cold dark matter with a cosmological constant) model by the luminosity distances of the supernovae. The cosmological acceleration is of very great and profound interest, for which a large number of models have been proposed besides LCDM, and several of them have been fitted by luminosity distances of supernovae. For a review, see [2].

In a cosmological model, one calculates the luminosity distances as follows:

\[ D_l = \frac{1 + z}{H_0} \int_0^z \frac{H_0}{H(z')} \, dz', \]

where \( H(z') \) denotes the Hubble parameter and \( H_0 \) represents its present value. Then one can constrain the parameters in the model by using the observational data of supernovae through \( \chi^2 \) or other methods. A deficiency of this method is that one obtains the luminosity distance through integrating the Hubble parameter \( H \), and therefore the fine structures, such as wiggles on \( H \), cannot show themselves in such a method. For example, compared with

\[ H(z)/H_0 = 1, \]

\[ H(z)/H_0 = \frac{1}{1 + \sin(nz)}, \]

is surely a different model. But for a large \( n \), they always share the same confidence region in fittings by using luminosity distance data. Also different \( n \) (for large \( n \)) also share the same confidence region. We see that some types of fine structures of \( H \) are degenerate to the luminosity distance data. To break this degeneration one needs the observational data of \( H(z) \), not only an integration of \( H^{-1}(z) \).

Fortunately there is a newly developed scheme to obtain the Hubble parameter directly at different redshifts [3], which is based on a method to estimate the differential...
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Table 1. The direct observational data of $H(z)$ \cite{6}.

| $z$  | 0.09 | 0.17 | 0.27 | 0.40 | 0.88 | 1.30 | 1.43 | 1.53 | 1.75 |
|------|------|------|------|------|------|------|------|------|------|
| $H(z)$ (km s$^{-1}$ Mpc$^{-1}$) | 69 | 83 | 70 | 87 | 117 | 168 | 177 | 140 | 202 |
| 68.3\% confidence interval | $\pm 12$ | $\pm 8.3$ | $\pm 14$ | $\pm 17.4$ | $\pm 23.4$ | $\pm 13.4$ | $\pm 14.2$ | $\pm 14$ | $\pm 40.4$ |

ages of the oldest galaxies \cite{4}. Replacing the scale factor $a$ by the redshift $z$, we can write the Hubble parameter in the form

$$H(z) = -\frac{\dot{z}}{1 + z},$$

where a dot denotes the derivative with respect to the cosmic time $t$. By use of the previously released data \cite{5}, Simon et al obtained a sample of direct $H(z)$ data in the interval $z \in (0, 1.8)$ \cite{6}, almost the same interval as the data of luminosity distances of supernovae. We show this sample in table 1.

The $H(z)$ data in table 1 are obtained from a total of 32 passively evolving galaxies. These galaxies are grouped in the bins with redshift $z = 0.03$, which are small enough to ensure the galaxies have roughly the same age. Then we can calculate age differences for those bins in redshifts that are separated more than $\delta z = 0.1$. This generates a fairly strict result of $\dot{z}$. We note here that the redshift of the galaxies in the sample has an accuracy of $\delta z \sim 10^{-4}$; thus the galaxies will be correctly classified. Also, a nice property of this derivation of $H(z)$ is that differential ages are insensitive to systematic errors, which is different for the absolute ages \cite{8}.

Table 1 displays an unexpected feature of $H(z)$: it decreases with respect to the redshift $z$ at redshift $z \sim 0.3$ and $z \sim 1.5$, which means that the total fluid in the universe behaves as a phantom. This information on the dynamical property of the universe is difficult to draw from the data of supernovae. This feature of $H(z)$ implies that the dark energy component of the cosmic fluid should behave as a phantom sometime, which can be proved by the following argument. In standard general relativity for a spatially flat universe, which is implied either on the theoretical side (inflation in the early universe) or observational side (CMB fluctuations \cite{9}), the Friedmann equation is

$$H^2 = \frac{1}{3\mu^2}(\rho_m + \rho_{de}),$$

where $\rho_m$ denotes the density of dust matter, $\rho_{de}$ stands for the density of dark energy and $\mu$ represents the reduced Planck mass. Differentiating with respect to the redshift $z$, we derive from (5)

$$2H \frac{dH}{dz} = \frac{1}{3\mu^2} \left( \frac{d\rho_m}{dz} + \frac{d\rho_{de}}{dz} \right).$$

Clearly, if $dH/dz < 0$ at some redshift (as shown in table 1), one concludes $d\rho_{de}/dz < 0$ since $d\rho_m/dz > 0$, which means the dark energy behaves as a phantom.

The present (or at very low redshift) phantom behavior of dark energy is also implied by the supernovae data \cite{10}. Generally speaking, a simple phantom field (scalar field with a kinetic term of false sign) is quantum mechanically unstable. However, some
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evidences imply that our present four-dimensional standard model and general relativity is not the final theory. The phantom model can be treated as a reduced theory of a more fundamental theory, in which no field behaves as a phantom [2]. Thus the stability problem may be evaded. Actually, many reduced theories do contain phantoms, such as the ones coming from string/M-theory compactification, or higher-derivative supergravities, or modifications of Einstein gravity itself. For example, such a field may be motivated from S-brane constructions in string theory [11]. Moreover, there exist examples in which an effective phantom description of the late-time universe naturally emerges, even when the starting theory does not clearly show the phantom structure [12]. Therefore it may be reasonable to investigate such models as an effective theory. Phenomenologically, the cosmological models with phantom matter have been investigated extensively [13]. Also, urged by observations, the models with dark energy whose EOS crosses $-1$ have been investigated in [14].

However, the $H(z)$ data in table 1 implies the EOS of total fluid in the universe crosses $-1$, not only the dark energy sector. Moreover, the phase oscillation over the deceleration phase and acceleration phase is clear through the history of the universe. In a fitting in the frame of the LCDM model, the point $z \sim 1.5$ is beyond the $1\sigma$ level [6]. Furthermore, it is shown that the data point near $z \sim 1.5$ drops and stays outside of the best fit of the LCDM, XCDM and $\phi$CDM models studied in [7]. On the other hand, a study shows that the model whose Hubble parameter is directly endowed with an oscillating ansatz by parameterizations fits the data much better than those of LCDM, IntLCDM, XCDM, IntXCDM, VecDE and IntVecDE [15]. Physical dark energy models possessing this oscillating property may be an interesting subject. It may deserve to present a physical model in which the EOS of total fluid crosses $-1$.

In this paper we put forward a model in which a phantom field with natural potential, i.e. the potential of a pseudo Nambu–Goldstone boson (PNGB), drives the universe. We shall show that in such a model all the features of $H$ in table 1 can be realized naturally, and the fitting results of the parameters in this model are rather different according to supernovae and direct $H(z)$ data. PNGB is an important idea in particle physics. It emerges whenever a global symmetry is spontaneously broken. There are two key scales of PNGB generation. One is the scale at which the global symmetry breaks, denoted by $f$, and the other is the scale at which the soft explicit symmetry breaks, denoted by $C$. Under this assumption the potential of PNGB is

$$V = C^4 \left(1 \pm \cos \left(\frac{N\phi}{f}\right)\right).$$

The inflation model driven by a scalar with such a potential was first studied in [16]. Generally speaking, in the context of the inflation model the cosine function is not the potential never completes a cycle. The scalar PNGB can also play the role of dark energy [17]. In this scenario the energy scale of the global symmetry breaking $f$ keeps about the same as in the case of the inflation model, i.e. the Planck scale. In contrast, the scale of explicit symmetry breaking decreases to an extremely low scale, i.e. $10^{-3}$ eV, which is comparable to the neutrino mass yielded by the Mikheyev–Smirnov–Wolfenstein (MSW) mechanism. Phenomenologically, the natural potential has been generated to solve the coincidence problem, in which the cosine function in the potential oscillates many cycles [18], and therefore the densities of dark energy and dust can be comparable.
several times in the history of the universe. But the previous models with PNGB dark energy cannot realize the feature that the EOS of total fluid in the universe crossing $-1$. This feature appears naturally in the present phantom natural dark energy model in this paper.

In the next section we shall present the phantom natural dark energy model and investigate some dynamical properties of it. In section 3, we fit this model by using the SNLS data and direct $H(z)$ data, respectively. In section 4, we discuss the quantum stability of this phantom model. The main conclusions and some discussions appear in section 5.

2. The natural phantom model

We work in the frame of standard four-dimensional general relativity. The phantom is characterized by a false sign of the kinetic term in the Lagrangian

$$\mathcal{L}_p = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi),$$

(8)

where and in the following we take the signature $(-, +, +, +)$. In the present model a phantom field with generalized natural potential plays the role of dark energy. In an FRW universe, $\rho_{de}$ in (5) becomes

$$\rho_\phi = -\frac{1}{2} \dot{\phi}^2 + V(\phi),$$

(9)

and the pressure of the scalar is

$$p_\phi = -\frac{1}{2} \dot{\phi}^2 - V(\phi),$$

(10)

and the equation of motion of $\phi$ is

$$-\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0.$$  

(11)

Based on the former research, we phenomenologically generalize the natural potential to the following form:

$$V(\phi) = V_0 \left(1 + A \cos \left(\frac{\phi}{\mu} \right)\right).$$

(12)

With the new dimensionless variables below:

$$x \equiv \frac{\dot{x}}{\sqrt{6\mu H}},$$

(13)

$$y \equiv \frac{\sqrt{V}}{\sqrt{3\mu H}},$$

(14)

$$l \equiv \frac{\sqrt{\rho_m}}{\sqrt{3\mu H}},$$

(15)

$$b \equiv \frac{\sqrt{V_0}}{\sqrt{3\mu H}},$$

(16)
the dynamics of the universe can be described by the following dynamical system:

\[
x' = \frac{3}{2}x(-2x^2 + l^2) - 3x + \frac{\sqrt{6}}{2}p\sqrt{A^2b^4 - (y^2 - b^2)^2},
\]  
(17)

\[
y' = \frac{3}{2}y(-2x^2 + l^2) - \frac{\sqrt{6}}{2}pxy^{-1}\sqrt{A^2b^4 - (y^2 - b^2)^2},
\]  
(18)

\[l' = \frac{3}{2}l(-2x^2 + l^2) - \frac{3}{2}l,
\]  
(19)

\[b' = \frac{3}{2}b(-2x^2 + l^2),
\]  
(20)

where a prime stands for derivation with respect to \(s \equiv \ln(1 + z)\). Note that the four equations (17), (18), (19) and (20) of this system are not independent. By using the Friedmann constraint, which can be derived from the Friedmann equation

\[-x^2 + y^2 + l^2 = 1,
\]  
(21)

the number of independent equations can be reduced to three. There are four critical points of this system satisfying \(x' = y' = l' = b' = 0\) appearing at

\[x = l = 0, \quad y = 1, \quad b = \pm \sqrt{\frac{1}{1 + A}}; \quad (22)
\]

\[x = l = 0, \quad y = 1, \quad b = \pm \sqrt{\frac{1}{1 - A}}. \quad (23)
\]

All of them satisfy the Friedmann constraint (21). To obtain real values of the variables at the singularities we see that if \(A \geq 1\) only the former two exist, if \(A \leq -1\) only the latter two exist, and only for \(-1 < A < 1\) do all four of the critical points exist. The critical points imply that the universe will enter a pure dark energy phase at last, if the singularity is stationary. To investigate the properties of the dynamical system in the neighborhood of the singularities, we impose a perturbation to the critical points:

\[\delta x' = E_{11}\delta x + E_{12}\delta y + E_{14}\delta b, \quad (24)
\]

\[\delta y' = E_{22}\delta y + E_{24}\delta b, \quad (25)
\]

\[\delta l' = E_{33}\delta l, \quad (26)
\]

\[\delta b' = 0, \quad (27)
\]

where we have used (22) or (23), and the components of the eigenmatrix are

\[E_{11} = -3, \quad (28)
\]

\[E_{12} = -\sqrt{6}py(y^2 - b^2)\alpha^{-1}, \quad (29)
\]

\[E_{14} = \sqrt{6}p[A^2b^3 + (y^2 - b^2)b]\alpha^{-1}, \quad (30)
\]

\[E_{22} = \sqrt{6}pxy^{-1}[A^2b^3 + b(y^2 - b^2)]\alpha^{-1}, \quad (31)
\]

\[E_{24} = -\sqrt{6}pxy^{-1}[A^2b^3 + b(y^2 - b^2)]\alpha^{-1}, \quad (32)
\]

\[E_{33} = -3/2, \quad (33)
\]
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where

$$\alpha \equiv \sqrt{A^2 b^4 - (y^2 - b^2)^2}. \tag{34}$$

The four eigenvalues of this linear system are

$$\lambda_1 = -3, \quad \lambda_2 = \sqrt{6xp(y^2 - b^2)\alpha^{-1}}, \quad \lambda_3 = -3/2, \quad \lambda_4 = 0. \tag{35}$$

The property of $\lambda_2$ is rather complicated around the singularities. For example, it goes to different values along different paths around the singularity $x = 0, z = 0, b = 1/\sqrt{1 + A}$.

The six repeated limits are

$$\lim_{z \to 0} \lim_{b \to 1/\sqrt{1 + A}} \lambda_2 = 0, \tag{36}$$

$$\lim_{z \to 0} \lim_{b \to 1/\sqrt{1 + A}} \lambda_2 = ip\sqrt{\frac{3A}{1 + A}}, \tag{37}$$

$$\lim_{b \to 1/\sqrt{1 + A}} \lim_{z \to 0} \lambda_2 = 0, \tag{38}$$

$$\lim_{b \to 1/\sqrt{1 + A}} \lim_{x \to 0} \lambda_2 = 0, \tag{39}$$

$$\lim_{b \to 1/\sqrt{1 + A}} \lim_{z \to 0} \lambda_2 = 0, \tag{40}$$

$$\lim_{z \to 0} \lim_{b \to 1/\sqrt{1 + A}} \lambda_2 = ip\sqrt{\frac{3A}{1 + A}}. \tag{41}$$

Hence the limit of $\lambda_2$ does not exist at the singularities. However, we see that the real parts of the limits keep zero independent of paths, which means that the system reaches an indifferent equilibrium. In such a de Sitter universe at the critical point the kinetic energy of the phantom and dust matter vanish, but the potential energy can reside at any value, which depends on the initial values of kinetic energy, potential energy, dust density and Hubble parameter.

3. Fitting

As we have pointed out in section 1, the data of supernovae are insensitive to the oscillating behavior of $H(z)$. In this section we show the fitting results by the direct $H(z)$ data and SNLS data by $\chi^2$ statistics, respectively. The $H(z)$ data have been used to constrain models in [7, 15, 19]. Here we adopt SNLS data [20], which are believed to be more consistent with CMB data.

Figure 1 displays the fitting results. We set $A = 1$, which means we adopt the original PNGB potential, $b(z = 0) = 0.616$, $\phi/\mu(z = 0) = 0.022$, $H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [21]. In figure 1 we find an extraordinary property of (a): the 68.3% confidence contour is disconnected. The physical explanation is that the present dataset of direct $H(z)$ is not enough to distinctly illuminate how many ‘wiggles’ inhabit on $H(z)$. New wiggles may hide in the gaps of the dataset, which leads to that a much bigger $p$ lies in the same confidence region as a smaller $p$. (b) clearly shows that the resolution of supernovae data is rather
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![Figure 1](image1.png)

**Figure 1.** The fitting result of the parameters $\Omega_m$ ($t^2_0$) and $p$. (a) The 68.3% confidence contour plot by using the direct $H(z)$ data in table 1. (b) The 68.3% confidence contour plot by using the SNLS data.

![Figure 2](image2.png)

**Figure 2.** The deceleration parameter $q$ with best fit $\Omega_m = 0.397$ and $p = 11.0$.

inefficient for the oscillating behavior of $H(z)$. We show the deceleration parameter $q$ in figure 2 with best fit values of $\Omega_m$ and $p$ by direct $H(z)$ data. Figure 2 illuminates that the universe oscillates between a deceleration phase and acceleration phase.

### 4. Quantum stability

A severe problem of any phantom field is quantum stability. In practice, we do not require that the phantom is fundamentally stable, but quasi-stable, which means its lifetime is larger than the age of the universe. This problem has been discussed in [22, 23]. Here we
Natural phantom dark energy, wiggling Hubble parameter \( H(z) \) and direct \( H(z) \) data follow the investigations in [22]. The simplest interaction between phantom and graviton takes the form

\[
\phi \rightarrow h + \phi_1 + \phi_2, \tag{42}
\]

where \( h \) denotes the gravitational fluctuations on an FRW background, and \( \phi_1 \) and \( \phi_2 \) represent the other two phantom fields. We note here that, though \( h \) will be different if we take a Minkowski background, the difference is tiny and negligible in the spacetime region we considered for this interaction. Here we consider a series expansion around the initial value of the numerical example we studied above, \( \phi(z = 0)/\mu = 0.022 \). Based on the discussion in [22], we set the interaction term

\[
\mathcal{L}_i = \frac{1}{\mu} (\mu h)^{1/3} V'''(\phi_0) \phi^3 = A \lambda_e (\mu h) \phi^3, \tag{43}
\]

where \( \lambda_e \) is defined as

\[
\lambda_e \equiv \frac{1}{6 \mu^4} V_0 \sin \left( \frac{p \phi_0}{\mu} \right) \sim 10^{-118}, \tag{44}
\]

where \( p \) takes the best fit value from the last section, \( p = 15.3 \). Clearly, if we treat a phantom as a fundamental theory, it will be unstable and the reaction rate goes to infinity because the volume of the phase space of \( \phi, \phi_1 \) and \( \phi_2 \) goes to infinity. However, if we treat it as an effective theory which is only valid below some energy scale \( \Lambda \), the reaction rate \( \Gamma \) becomes

\[
\Gamma \sim \lambda_e^2 \frac{\Lambda^2}{m_\phi}, \tag{45}
\]

where the effective mass of the phantom \( m_\phi \) is defined as

\[
m_\phi \equiv (-AV'')^{1/2} \sim p \mu \sqrt{\frac{AV_0}{\mu^4}} = 10^{-60} p \mu A^{1/2}. \tag{46}
\]

Here, and the following, we take \( A = 1 \) without special announcement. The phantom field as an effective field is viable if its reaction rate \( \Gamma \) is smaller than the present Hubble parameter:

\[
\Gamma < H_0 \sim 10^{-60} \mu, \tag{47}
\]

which means

\[
\lambda_e^2 \frac{\Lambda^2}{m_\phi} < H_0, \tag{48}
\]

that is \( \Lambda < 10^{58} \mu \). In fact, any effective theory which is valid only below such a high energy scale, which is far beyond our present lab energy scale, surely can be a perfect effective theory. But, besides the decay channel as shown in (42), we must consider the cases that one phantom decays into several particles. By summing over all these possibilities, one arrives at the total reaction rate [22]

\[
\Gamma_{\text{tot}} = \Gamma \left[ 1 - \left( \frac{\Lambda}{\mu} \right)^2 \right]^{-2}. \tag{49}
\]
We see that, if $\Lambda$ is smaller than $\mu$, $\Gamma_{\text{tot}}$ will keep the same order of $\Gamma$, which is not a stringent constraint.

However, as an effective theory, one must include all possible terms compatible with the symmetry of the Lagrangian up to finite orders to guarantee the renormalizability of the theory. A most famous effective theory is four-fermion interaction theory, as the low effective theory of electro-weak interaction. Expanding the propagator of the gauge boson in electro-weak according to the mass of the $W$ boson, we see that the derivative coupling appears. Hence a derivative coupling in an effective theory is quite reasonable.

We consider the interaction Lagrangian of graviton and phantom, with an approximate global symmetry:

$$L_{\text{ig}} = \frac{\gamma}{\mu^2 \Lambda^4} \left[ \mu h (\partial \phi, \partial \phi) \right]^2,$$

where $\gamma$ is a constant of order 1. The reaction rate becomes

$$\Gamma \sim \frac{\gamma^2 \Lambda^6}{m_\phi \mu^2},$$

which should be smaller than the present Hubble parameter. Therefore we reach

$$\Lambda^6 < \frac{H_0 m_\phi \mu^4}{\gamma^2} \sim 10^{-120} \mu \mu^6.$$  

The key difference between our result and the result in [22] dwells at the effective mass of the phantom field. In fact, for our result the reaction rate $\Gamma$ in (51) is suppressed by a factor $p$. On the observational side, we see that the 1-σ confidence region forms a confidence tower with no clear upper bound of $p$. On the theoretical side, one hardly finds principles to constrain $p$ in the natural potential in this cosmological context. Physically, the effective mass of the phantom in the present model can be notably larger than $H_0 = 10^{-33}$ eV, which is taken as the mass of the phantom in [22]. For example, if $m_\phi = 10^{-27}$ eV, which is quite beyond the present abilities of accelerators, $\Lambda$ will exceed 1 TeV and hence it is also beyond our present lab energy scale. Thus, the present model is promising due to this suppressing mechanism for derivative coupling. Also, we note here that a larger $A$ is helpful to increase the mass of the phantom, which can be seen from (46).

5. Conclusions and discussions

To summarize, this paper illuminates that direct $H(z)$ data are more efficient than the supernovae for the fine structures of the Hubble diagram.

We first put forward a model based on the previous studies on the PNGB. In this model the total fluid in the universe may evolve as a phantom in some stages, which is consistent with the direct $H(z)$ data in table 1. Then we study its dynamical properties and find its critical points. We also investigate the stability about the singularities of this system.

In section 3 we fit our model by using $H(z)$ data and supernovae data, respectively. The results are different, as we expected. Because the sample of $H(z)$ data is too small, the confidence contour is disconnected, which means that we still lack enough information about the oscillations of $H(z)$. We hope future observations offer more data of $H(z)$ so that we can investigate the history of the universe in a more detailed way.
In section 4 we investigate the stability of the present model. Our treatise is to treat the phantom model as an effective model truncated at some energy scale $\Lambda$. As in the previous studies, we find that the coupling to graviton needs a truncated scale much larger than the lab energy scale, if we require the lifetime of the phantom to be longer than that of the universe. Different from the previous studies, we find that the derivative coupling between phantom and graviton is viable due to the special potential of the present model.

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