Radiative capture of proton by $^{13}$C at low energy

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Abstract Radiative capture $p + ^{13}$C → $^{14}$N + γ at energies bearing astrophysical consequences is one of the important processes in the CNO cycle. We focus on the possibility of describing the main contribution to the total cross section of the radiative capture process in the framework of the single-particle potential model without separation into direct and resonant transitions. In case where the single-particle potential model fails to describe other partial components, we use the R-matrix approach. The partial components of the astrophysical S-factor are calculated for all possible electric dipole transitions. The calculated value of the total S-factor at zero energy is in good agreement with earlier reported values. Based on the value of total astrophysical S-factor depending on the collision energy, we calculate the nuclear reaction rates for $p + ^{13}$C → $^{14}$N + γ.

Keywords CNO cycle · Potential model · Cross section · Astrophysical S-factor · Electric dipole transitions · Rate of radiative capture

1 Introduction

In stars massive than the Sun, hydrogen burning can proceed by the CNO cycle. The $^{13}$C($p$,γ)$^{14}$N is the third step of cold CNO cycle. It takes place in different astrophysical sites e.g. the red giants and asymptotic giant branch (AGB) stars. The $^{13}$C($p$,γ)$^{14}$N reaction is important for calculation of the $^{12}$C/$^{13}$C ratio, which is important to estimate the composition of different layers of a star. This reaction is seed for $^{14}$N($p$,γ)$^{15}$O reaction which determines the rate of energy production in stars (Burbidge et al. 1957; Rolfs and Rodney 1988). The $^{13}$C is not only astrophysically important for the nucleosynthesis but also acts as a source of neutron in the intermediate mass stars (Busso et al. 2001).

It is well known that the fusion reaction at low energy, related to nuclear physics, proceeds via formation of a compound nucleus containing a large number of resonances, provided the nucleon capture may occur by a heavy nucleus. The radiative capture reactions relevant to nuclear astrophysics go either through non-resonant or resonant reactions at low energy especially for light nuclei. Usually, the total reaction cross section is represented as the sum of the resonant and non-resonant contributions. At times terms corresponding to interference between resonant and non-resonant processes are also included. The non-resonant partial cross section is calculated in the single-particle potential approximation whereas the cross section of the resonant reaction is analyzed in the R-matrix approach (King et al. 1994; Angulo et al. 1999). However, such a recipe in the region of low energies of the incident proton may lead to an overestimation or underestimation of the capture cross section depending on the sign of the interference term. In the case of the reaction of radiative capture of a nucleon by light nuclei with completely filled shells, it is possible to calculate the capture cross section in the framework of a completely single-particle potential model. This holds true even in the presence of resonance in the considered energy region. The description of the radiative capture data of experiments may be completely described by the R-matrix theory.
Half of 13C

Two resonance transitions mostly determine the cross section for the astrophysical S-factor at low energies due to its width. These transitions are at the energy cross section, and one must take into account the presence of energy states including ground state. Its effect on the total cross section of the proton capture process is made by the electric E1 transition and will only be considered in this work. Among all the resonances given on website (http://www.tn1l.duke.edu/nucladata), the E1 electric transitions from the Jπ = 1−, T = 1 state with excitation energy Eπ = 8.062 MeV ± 1.0 keV and width Γ = 23 ± 1 keV to the three low-lying states of the 14N nucleus make the biggest contribution to the total cross section at the astrophysically important region (King et al. 1994). The next resonance, which should be taken into account, is a wide resonance at an energy of Eπ = 8.776 MeV ± 7 keV with Γ = 410 ± 20.0 keV (Jπ = 0− and T = 1). It decays to the low-lying states including ground state. Its effect on the capture cross section is significant at an incident proton energy of ≈1 MeV. However, it alters the value of the low-energy cross section, and one must take into account the astrophysical S-factor at low energies due to its width. These two resonance transitions mostly determine the cross section of 13C(p, γ)14N at low energy up to 1.2 MeV. Another resonant transition occurs from the state Jπ = 2−, T = 0 located at Eπ = 7.966 MeV, with l = 2. This resonance is very narrow, therefore the transition from this state gives negligible contribution to the total cross section at the important astrophysical energy region. Finally, at energy Eπ = 8.618 MeV, there is a narrow resonance Jπ = 0+, T = 1, from which the E1 transition occurs to the low-lying bound state Jπ = 1− with a binding energy of 1.8591 MeV.

Many authors have worked on this reaction and it still attracts the attention of many researchers in this field. Hebbard and Vogl measured the elastic scattering cross section for 13C(p, γ)14N within 140–750 keV proton energy (Hebbard and Vogl 1960). King et al. studied this reaction within 120–950 keV proton energy and measured the proton capture cross section for the first six excited state including the ground state (King et al. 1994). Mukhamedzhanov et al. analyzed 13C(p, γ)14N reaction by using the R-matrix approach for the calculation of S-factor. They used ANC from the experimental analysis of 13C(14N, 13C)14N and 13C(3He, d)14N (Mukhamedzhanov et al. 2003). Genard et al. measured experimentally the S-factor within 225–561 keV (proton energy). They procure dominant resonance at 511 keV (Genard et al. 2010). Dubovichenko used the potential cluster model and calculated the astrophysical S-factor considering the E1 transition from the continuous s-state to the ground state only. He obtained the resonant transition at the proton energy of 551 keV in the laboratory frame (Dubovichenko 2012). However, the following important question arises: why does the difference in depth of potentials describing the ground and the resonance states differ by as much as a factor of nine? Li et al. (2012) proposed an independent study of the existing results of 13C(p, γ)14N reaction based on the hybrid method and presented the values of the astrophysical factor at zero energy and fitted the parameters of the reaction rate. Chakraborty et al. (2015) used the multichannel, multilevel R-matrix code AZURE2 for the calculation of the values of the partial S-factors. Moghadasi et al. (2018) calculated wave function for the ground state of 14N by Faddeev’s method and later used the obtained results for calculation of the direct capture cross section and Breit–Wigner formulae of 13C(p, γ)14N reaction. Lastly we mention the papers (Artemov et al. 2008) and (Chakraborty et al. 2019). The calculations in Artemov et al. (2008) Δl based on the R-matrix approach and confirmed the results of King et al. (1994), Mukhamedzhanov et al. (2003) while the authors of Chakraborty et al. (2019) used the hybrid model description reconfirming the results of King et al. (1994), Mukhamedzhanov et al. (2003).

In this article, we calculate the S-factor and nuclear reaction rates for 13C(p, γ)14N capture by using the potential model (PM). We consider transitions from continuous Jπ = 1−, T = 1 and few other states to the bound states, without separating the transitions into resonant and non-resonant terms. We apply the R-matrix approach whenever the result of the calculation by PM does not give satisfactory agreement with the available experimental data. The bound state wave functions of 14N are calculated by modifying the Woods-Saxon potential to achieve the correct behavior of the bound state wave functions in the ground and excited states. Our calculations for the astrophysical S-factor and nuclear rates reproduce well the experimental results within the energy range (0.01–1.2) MeV.

In the next section we introduce the formalism of our calculation. Results and discussion are presented in Sect. 3. We draw conclusions in the last section.

2 Model for calculation

The calculation of nuclear reaction rate directly depends on the reaction cross sections within the Gamow window.
The Coulomb barrier for nuclear interaction is much higher than the stellar energy. Therefore, the reaction cross section for such an interaction is too small to be measured in laboratory. It is therefore necessary to provide some estimate using a sound theoretical model. The important reactions in nuclear astrophysics are the radiative capture reactions, in which the nucleons fuse with light, intermediate and heavy nucleus through electromagnetic interaction (Descouvemont 2003). Some models, e.g. R-matrix approaches are based on fitting parameters (Descouvemont 2003; Descouvemont and Baye 2010) and ab initio calculations (Navratil and Quaglioni 2012). The PM method is used mostly for calculation of astrophysical S-factor by the PM we take the 13C nucleus.

For calculation of astrophysical S-factor by the PM we take the total potential of the interaction of colliding particles as

$$V(r) = V_N(r) + V_C(r),$$

(1)

where in Eq. (1) $V_N(r)$ and $V_C(r)$ are the nuclear and Coulomb potentials, respectively. The nuclear part of Eq. (1) is represented by the modified Woods-Saxon potential to obtain the correct behavior of the bound state wave functions. For calculation of the continuous state wave function the modification factor of the potential is selected as unity.

$$V_N(r) = -\left[ V_0 - V_{LS}(L \cdot S) \frac{1}{m_r^2 r} \frac{d}{dr} \right] \frac{1}{1 + q \left( \exp \frac{r-R_N}{\alpha} \right)},$$

(2)

where $V_0$, $V_{LS}$ and $a$ represent the central potential depth, coupling strength of the spin-orbit potential and the diffuseness parameter, respectively; the term $(L \cdot S)$ is the product of orbital and spin operators; $q$ is the modification parameter. The nuclear radius $R_N$ is defined by $R_N = r_0 \times A^{1/3}$, where $A$ is mass number and $r_0$ is a parameter which is varied within the range (1.2–1.3) fm and $m_r = 0.684$ fm$^{-1}$ is the pion mass in $h = c = 1$ unit system. The potential defined by Eq. (2) describes correctly the asymptotic behavior of the wave functions of both bound and continues states of the interacting nuclei. The potential parameters are selected in such a way as to best describe the energies of bound and resonance states, as well as the known experimental data on the capture from states of continuous spectrum to bound low-lying states. The corresponding spectroscopic factors are chosen in such a way as to match the astrophysical S-factor calculated using the PM with the available experimental data on the radiative capture of the proton by the 13C nucleus.

For the uniform charge distribution the Coulomb potential is defined as

$$V_C(r) = \begin{cases} \frac{\hbar e}{2} \frac{Z_1 Z_2 a}{R_c} (3 - \frac{r^2}{R_c^2}) & \text{if } r \leq R_c, \\ \frac{\hbar e}{2} \frac{Z_1 Z_2 a}{R_c} & \text{if } r \geq R_c, \end{cases}$$

(3)

where $R_c$ is the Coulomb radius, $Z_1$, $Z_2$ are charges of colliding particles and $a$ is the fine structure constant. To simplify the notation, we omit the quantum numbers of states, excluding the orbital moment $L$.

For the $p$–13C interaction the function $\phi_L(r)$ is the radial wave function which satisfies the radial part of the Schrödinger equation in both the initial and final states:

$$\frac{d^2}{dr^2} \phi_L(r) + \frac{2\mu}{\hbar^2} \left[ E - V(r) - \frac{\hbar^2 L(L+1)}{2\mu r^2} \right] \phi_L(r) = 0,$$

(4)

where $V(r)$ is defined by Eq. (1), $E$ is the energy of relative motion of the interacting particles in the center of mass (CM) system and $\mu$ is the reduced mass. The asymptotic behavior of the continuous radial wave function is given by

$$\phi_L(r) \xrightarrow{r \to \infty} \cos \delta_L F_L(kr) + \sin \delta_L G_L(kr),$$

(5)

where $k$ is the wavenumber corresponding to the interacting nuclei, $\delta_L$ is the elastic scattering phase shift, $F_L$ and $G_L$ are the regular and irregular Coulomb functions, respectively. The asymptotic of the radial bound state wave function is defined as

$$\phi_L(r) \xrightarrow{r \to \infty} C_w W_{-\eta_0, L+\frac{1}{2}}(2\kappa_0 r),$$

(6)

where $C_w$ is the single-particle asymptotic normalization coefficient (Plattner and Viollier 1980), $W_{-\eta_0, L+1/2}(r)$ represents the Whittaker function, $L$ is the bound state orbital momentum, and $\eta_0$ is the Coulomb parameter ($\eta_0 = Z_1 Z_2 e^2 \mu / \kappa_0$), $\kappa_0$ is the wavenumber for bound state.

### 2.1 Potential and wave function

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$$\frac{d^2}{dr^2} \phi_L(r) + \frac{2\mu}{\hbar^2} \left[ E - V(r) - \frac{\hbar^2 L(L+1)}{2\mu r^2} \right] \phi_L(r) = 0,$$

(4)

where $V(r)$ is defined by Eq. (1), $E$ is the energy of relative motion of the interacting particles in the center of mass (CM) system and $\mu$ is the reduced mass. The asymptotic behavior of the continuous radial wave function is given by

$$\phi_L(r) \xrightarrow{r \to \infty} \cos \delta_L F_L(kr) + \sin \delta_L G_L(kr),$$

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where $k$ is the wavenumber corresponding to the interacting nuclei, $\delta_L$ is the elastic scattering phase shift, $F_L$ and $G_L$ are the regular and irregular Coulomb functions, respectively. The asymptotic of the radial bound state wave function is defined as

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### 2.2 Potential model for astrophysical S-factor and nuclear cross section

For interaction of charged particles the astrophysical S-factor is defined as

$$S(E) = \sigma(E) E \exp(2\pi\eta),$$

(7)
where \(S(E)\) and \(\sigma(E)\) are the astrophysical S-factor and total cross section of the reaction, respectively; \(E\) is the collision energy in CM frame; \(\eta\) is Sommerfeld parameter defined as

\[
\eta = Z_1 Z_2 e^2 / \hbar \nu,
\]

where \(Z_1\) and \(Z_2\) are the charge numbers of a projectile and target nuclei, respectively; \(\nu\) is their relative velocity; \(\hbar\) is the Plank’s constant. The total capture cross section is a sum of the radiation capture reaction is parameterized by the equation

\[
\sigma_r(E) = \frac{\pi}{k^2} \omega \frac{\Gamma_p(E) \Gamma_\gamma(E)}{(E - E_R)^2 + \Gamma(E)^2/4},
\]

where \(k\) is the wave number of incoming particle in the CM system, \(\Gamma_p(E)\), and \(\Gamma_\gamma(E)\) are the proton partial width, radiative partial width and total width, respectively; \(\omega\) is the statistical term.

The Kronecker delta \(\delta_{ij}\) accounts for the identity of interacting particles, \(I_1\), \(I_2\) and \(I\) are the spins of interacting particles and total spin in resonance state (Rolfs and Rodney 1988; Fowler et al. 1967).

The energy dependent proton width \(\Gamma_p(E)\) and radiative capture gamma width \(\Gamma_\gamma(E)\) are defined as

\[
\Gamma_p(E) = \frac{P_l(E)}{P_l(ER)} \Gamma_p(ER),
\]

and

\[
\Gamma_\gamma(E) = \left( \frac{E + \varepsilon_f}{E_R + \varepsilon_f} \right)^{2\lambda + 1} \Gamma(ER),
\]

where \(\Gamma_p(ER)\) and \(\Gamma_\gamma(ER)\) are the experimental particle and radiative widths, respectively; \(l, \lambda, \varepsilon_f\) represents the orbital angular momentum, electric multi-polarity and final bound state energy of compound nucleus, respectively. The penetrability \(P_l(E)\) is defined as

\[
P_l(E) = \frac{kb}{F_l^2(k,b) + G_l^2(k,b)},
\]

where \(b, F_l(k,b)\) and \(G_l(k,b)\) are the channel radius, regular and irregular Coulomb functions.

### 2.4 Nuclear reaction rates

Nuclear reactions rates are important for the description of stellar burn models. They are strongly dependent on the presence of resonance position in the low energy region. The nuclear reaction rate for the \(p + ^{13}\text{C}\rightarrow ^{14}\text{N} + \gamma\) process is defined below (Fowler et al. 1967),

\[
N_A(\sigma v) = N_A \left( \frac{8}{\pi \mu (k_B T)^2} \right)^{1/2} \times \int_0^{\infty} \sigma(E) E \exp(-E/k_B T) dE,
\]

where \(N_A\) represents Avogadro number, \(\mu = m_p m_{^1\text{C}} / (m_p + m_{^1\text{C}})\) is the reduced mass, \(T\) is the core temperature of star, \(k_B\) is the Boltzmann constant, \(\sigma(E)\) is reaction cross section, \(v\) is the relative velocity calculated in center of mass (CM) frame and \(E\) is the collision energy.
3 Results and discussion

We calculated the astrophysical S-factors for all possible E1 transitions from five different continuous states, of the incident proton relative to $^{13}$C in the energy range less than 1.2 MeV, leading to the formation of resonance states of $^{14}$N at energies indicated in Table 1. In this table $E_R$ is the resonance energy defined above the $p-^{13}$C threshold, the $J^\pi$, $\Gamma$ and $l_i$ are the initial total angular momentum, total width of resonance and orbital angular momentum of the initial states, respectively. The $J^\pi_f$, $\epsilon_f$, and $l_f$ are the total angular momentum, binding energy of the bound state and orbital angular momentum of the final state, respectively.

The resonance presented in the first row of Table 1 is very narrow with $l_i = 2$. Taking this resonance into account does not make a large contribution to the total cross section for radiation capture in the energy region under consideration, especially at zero energies, since the resonance width is extremely narrow ($\Gamma = 2.5 \pm 0.7$ eV http://www.tunl.duke.edu/nucldata/). For this reason, we use the PM describing the wave function of the proton in the initial state as the regular Coulomb function in the $d$-state. However, such description of the transition also leads to an insignificant contribution to the total cross section. The resonance shown in the third row of Table 1 has energy greater than 1 MeV and it is narrow too. From this state, the E1 transition can occur in a low-lying bound state with an orbital moment of 0 and binding energy $\epsilon = 1.8591$ MeV. Due to the small width and large value of the excitation energy $E_x = 8.618$ MeV, we have not been able to reproduce the resonance width in the single-particle PM. Most likely, this is owing to the fact that the single-particle approximation is not applicable in the description of this state. Therefore, we calculated the cross sections for radiation capture in the R-matrix approach using Eq. (11) and using the data from Table 1 as well as a radiative width of 0.69 eV (Ajzenberg-Selove 1981). Since the capture cross section calculated by the R-matrix method turned out to be too small in the zero-energy region, we used the PM for the direct transition to calculate in the energy region of our interest by choosing the regular Coulomb function in the $p$-state as the proton wave function in the initial state. The contribution of the result of the PM calculation to the cross section is also weak, albeit larger than R-matrix calculation result. The same situation with the description of the wide resonance $0^- (E_R = 1.07$ MeV) is presented in the fourth line. We can not describe this resonance by the PM. So, the proton capture cross section from this continuous state is also described by the R-matrix approach. For the numerical calculation of the resonant reaction cross sections, experimental widths shown in Table 1 are used as well as radiative widths equaling to 46.0 and 0.46 eV (King et al. 1994) corresponding to transitions to the ground and second excited states, respectively. The cross section of transition to the ground state is important and it is much greater than cross section of transition to the second excited state of $^{14}$N. For all the resonant transitions considered, we have used the value of the channel radius $b = 5.0$ fm.

We also take into account direct transitions of proton from the corresponding states of the continuous spectrum in the zero-energy region to the bound states of the proton in $0^- (\epsilon = 2.6354$ MeV) and $1^- (\epsilon = 1.8591$ MeV). The wave functions of the initial state are regular Coulomb functions with an orbital moment of $l_i = 1$. These transitions make a very small contribution to the total value of the astrophysical S-factor.

The final states of proton capture at the E1 transition through the resonance state $J^\pi = 1^-, T = 1$, corresponding to the excitation energy $E_x = 8.062$ MeV, are low-lying states of $^{14}$N nucleus, presented in Table 2.

Only transitions, occurring through the resonant state $J^\pi = 1^-, T = 1$, corresponding to the excitation energy $E_x = 8.062$ MeV, presented in the second row of Table 1, to the three underlying bound states can be performed in the approach of the single-particle PM describing the initial

| Resonance state | Bound state |
|----------------|-------------|
| $J^\pi$ | $E_R$ (MeV) | $\Gamma(E_R)$ (keV) | $l_i$ | $J^\pi_f$ | $\epsilon_f$ (MeV) | $l_f$ |
| 2$^-$ | 0.416 | 0.0025 | $d$ | 1$^+$ | 7.5505 | $p$ |
| 1$^-$ | 0.511 | 23.0 | $s$ | 1$^+$ | 7.5505 | $p$ |
| 0$^+$ | 1.07 | 3.80 | $p$ | 1$^-$ | 1.8591 | $s$ |
| 0$^-$ | 1.23 | 410.0 | $s$ | 1$^+$ | 7.5505 | $p$ |
Table 2  Low-lying bound states of $^{14}$N nucleus. The first, second, and third columns represent the state, single-particle binding energy, and proton orbital, respectively.

| State  | Binding energy | Proton orbital |
|--------|----------------|----------------|
| $J^p_f$ | $\epsilon_f$ (MeV) | $l_f$ |
| $1^+$  | 7.5505         | $p_{1/2}$     |
| $0^+$  | 5.2377         | $p_{1/2}$     |
| $1^+$  | 3.6024         | $p_{1/2}$     |
| $0^-$  | 2.6354         | $s_{1/2}$     |
| $2^-$  | 2.4447         | $d_{5/2}$     |
| $1^-$  | 1.8591         | $s_{1/2}$     |
| $3^-$  | 1.7163         | $d_{5/2}$     |

and bound state wave functions received from the solution of the Schrödinger equations. Transitions from this continuous state are crucial in the radiative capture of a proton by the nucleus of $^{13}$C, especially for transition to the ground state. All cross sections have resonant behavior at $E_R = 0.511$ MeV. The results of the S-factor for these transitions are shown in Figs. 2, 3, and 4 and compared with the experimental data. The values of spectroscopic factors for these transitions are 0.45, 0.15 and 0.24, respectively. These values are obtained by comparison of our calculated values of S-factor to the measured data of Ref. (King et al. 1994). The most dominant transition is the capture to the ground state.

The levels of the bound states of $^{14}$N nucleus, which as usually are taken into account to calculate the total astrophysical S-factor, are presented in Table 2. Note, we consider the radiative transitions of a proton to final states with orbital moments 0 and 1, since final states with a greater moment are not so significant when calculating the total cross section in the considered energy range. We calculate the bound state wave functions of $^{14}$N in the single-particle approach using the modified Woods-Saxon potential to achieve good agreement with the experimental astrophysical S-factors. To calculate the continuous wave function of the proton on the nucleus of $^{13}$C, the proton energy varied in the range from 0 to 1 MeV, and the system $^{13}$C−$p$ is in the state $1^-$. We remind that the corresponding parameters of the potentials are chosen in such a way as to obtain the correct values of the resonance energy and width close to the experimental results. The parameters of the potentials for the bound and continuous states are shown in Table 3. The ground state wave function of $^{14}$N($^{12}$C+$n$+$p$) was calculated in Ref. (Moghadasi et al. 2018) by using the Faddeev’s method and the $^{12}$C+$p$ configuration radial wave function was defined. The comparison of our calculated ground state wave function with earlier calculated result, given in Ref. (Moghadasi et al. 2018), is shown in Fig. 1. We see that the calculated single-particle wave function of $^{14}$N wave function is in good agreement with the $^{13}$C−$p$ wave function recovered from solving the Faddeev’s equation of the system $2^{N-1}$C.

We do not calculate transitions to the fourth ($\epsilon_f = 2.4447$ MeV) and sixth ($\epsilon_f = 1.7163$ MeV) excited states with $l_f = 2$ (Table 2), since the contribution of proton capture to these two states is much smaller than the contribution from ground and other excited states. This is a valid assumption also confirmed by previous calculations (Mukhamedzhanov et al. 2003; Li et al. 2012).

Summing up the results of calculation, describing by the PM, of the values of astrophysical S-factors at zero energy, we may conclude that contribution to the total S(0) of the transitions from continuous state $1^-$ to the three low-lying bound states is 83%, out of which 69.4% is due to transition from $1^-$ (continuous state) to $1^+$ (ground state).
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The distribution of the transition from the broad resonance state $0^-$, described by the R-matrix model to the total S(0), is approximately 11%. The remaining contribution to the total S(0) of roughly 6% is determined by direct transitions described by the regular Coulomb function of the initial state. It should also be noted that, the contribution of the $1^- \rightarrow 1^+$ (ground state) increases with increase in energy and reaches 83.7% at resonance energy of $E_R = 0.511$ MeV.

The sum of all partial S-factors for transition to the ground state of $^{14}$N within energy range (0–1) MeV is presented in Fig. 5, which also confirms that $1^- \rightarrow 1^+$ (ground state) is decisive. The result coincides well with ex-

![Figure 3](image_url) Solid line shows our calculated S-factor for the $E1$ transition from $J^\pi = 1^-$, $T = 1$ to the first excited state using the PM. Open squares (King et al. 1994) represent the experimental data with error bars.

![Figure 4](image_url) Solid line shows our calculated S-factor for the $E1$ transition from $J^\pi = 1^-$, $T = 1$ to the second excited state using the PM. Open squares (King et al. 1994) represent the experimental data with error bars.

**Table 3** The parameters of potentials Eqs. (2) and (3). The first column represents the states, the succeeding columns represent the binding energy, depth of central potential, spin-orbit potential, diffuseness parameter, the nuclear and Coulomb radii, while the last column represents the modification parameter in Woods-Saxon potential.

| State   | $\varepsilon_f$ (MeV) | $V_0$ (MeV) | $V_{LS}$ (MeV) | $a$ (fm) | $R_N$ (fm) | $R_C$ (fm) | $q$ |
|---------|-----------------------|-------------|----------------|----------|------------|------------|-----|
| Bound state |
| $1^+$   | 7.550                 | 102.74      | 14.0           | 0.5      | 2.586      | 3.056      | 4.0 |
| $0^+$   | 5.238                 | 110.50      | 14.0           | 0.5      | 2.586      | 3.056      | 5.4 |
| $1^+$   | 3.602                 | 93.60       | 14.0           | 0.5      | 2.586      | 3.056      | 4.0 |
| $0^-$   | 2.635                 | 64.27       | 14.0           | 0.7      | 2.939      | 2.939      | 1.0 |
| $1^-$   | 1.859                 | 51.72       | 14.0           | 0.5      | 2.821      | 2.821      | 0.3 |
| Continuous state |
| $1^-$   | 70.113                | 0           | 0.5            | 2.609    | 2.633      | 1          |

**Table 4** Comparison of S-factor at zero energy with previous reported works. The comparison is made with Li.12 (Li et al. 2012), Mu.13 (Mukhamedzhanov et al. 2003), Ch.15 (Chakraborty et al. 2015), Ar.08 (Artemov et al. 2008), Mo.18 (Moghadasi et al. 2018), Ge.10 (Genard et al. 2010), Be.10 (Bertulani and Guimarães 2010), Ki.94 (King et al. 1994). The $\varepsilon_f$ is given in units of MeV.

| State of $^{14}$N $\varepsilon_f$ | Li.12 | Mu.13 | Ch.15 | Ar.08 | Mo.18 | Ge.10 | Be.10 | Ki.94 | This work |
|-----------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-----------|
| $1^+$                             | 5.78 ± 0.48 | 5.16 ± 0.72 | 4.72 ± 0.86 | 5.06  | 5.8 ± 0.7 | 3.94 ± 0.59 | 6.216 | 5.25 ± 0.15 | 6.08  |
| $0^+$                             | 0.42 ± 0.06 | 0.32 ± 0.08 | 0.34 ± 0.15 | 0.35  |
| $1^+$                             | 0.90 ± 0.06 | 0.90 ± 0.13 | 0.66 ± 0.19 | 0.79  |
| $0^-$                             | 0.20 ± 0.01 | 0.33 ± 0.07 | 0.45 ± 0.10 | 0.17  |
| $1^-$                             | 0.56 ± 0.04 | 0.77 ± 0.09 | 0.60 ± 0.31 | 0.50  |
Experimental results below and above the resonance energies 0.511 MeV and 1.22 MeV, respectively.

The calculated $S(0)$ values corresponding to transitions to the ground and excited states as well as their comparison with the previous reported data are listed in Table 4. The results for the total S-factor depending on the energy are shown in Table 5. The value of the total S-factor at zero energy is $S(0) = 7.59$ keV b. This value is in good agreement with the reported data of Refs. (King et al. 1994; Mukhamedzhanov et al. 2003). The polynomial fit between (0–0.20) MeV is calculated by using the following expression.

$$S(E) = S(0) + S'(0)E + \frac{1}{2} S''(0)E^2.$$  (17)

We obtained $S(0) = 7.59$ keV b, $S'(0) = 1.10 \times 10^{-2}$ b and $S''(0) = 2.27 \times 10^{-4}$ b/keV. Our calculated results are comparable with the results of Refs. (King et al. 1994; Mukhamedzhanov et al. 2003).

The nuclear reaction rate for the $p + ^{13}\text{C} \rightarrow ^{14}\text{N} + \gamma$ process is calculated by using Eq. (16). The results for rates are compared in Table 6 with NACRE II data from Ref. (Xu et al. 2013). The ratio of our calculated results to the adopted

![Fig. 5](image-url) Solid line shows the sum of the S-factors of the radiative capture to the ground state. The dot-dash line are calculated results of transition from $J^\pi = 1^-$, $T = 1$ to the ground state using the PM. Filled lozenges (Hester and Lamb 1961), filled circle (Woodbury and Fowler 1952), filled triangles (Hebbard and Vogl 1960), open squares (King et al. 1994), open circles (Genard et al. 2010) and filled stars (Zeps et al. 1995) represent the experimental data with error bars. The dashed line are calculated results of transition from $J^\pi = 0^-$, $T = 1$ to the ground state using Eq. (11). The dotted line are the calculation of Ref. (Li et al. 2012)

![Fig. 6](image-url) Solid line shows our calculation of rates for $p + ^{13}\text{C} \rightarrow ^{14}\text{N} + \gamma$ interaction. Dashed, dotted and short dotted are the low, adopted and high rates from the Ref. (Xu et al. 2013), respectively. The inset of this figure shows the ratio of our calculated results to the adopted rates of Ref. (Xu et al. 2013)

### Table 5: The total astrophysical S-factor depending on the energy for all considered E1 transitions

| $E$ (MeV) | S-factor (MeV b) | $E$ (MeV) | S-factor (MeV b) | $E$ (MeV) | S-factor (MeV b) | $E$ (MeV) | S-factor (MeV b) |
|----------|-----------------|----------|-----------------|----------|-----------------|----------|-----------------|
| 0.01     | 0.000762        | 0.15     | 0.01170         | 0.29     | 0.02419         | 0.43     | 0.12915         |
| 0.02     | 0.000782        | 0.16     | 0.01216         | 0.30     | 0.02604         | 0.44     | 0.15974         |
| 0.03     | 0.000802        | 0.17     | 0.01266         | 0.31     | 0.02816         | 0.45     | 0.20205         |
| 0.04     | 0.000824        | 0.18     | 0.01320         | 0.32     | 0.03059         | 0.46     | 0.26182         |
| 0.05     | 0.000847        | 0.19     | 0.01379         | 0.33     | 0.03341         | 0.47     | 0.34723         |
| 0.06     | 0.000871        | 0.20     | 0.01443         | 0.34     | 0.03671         | 0.48     | 0.46735         |
| 0.07     | 0.000897        | 0.21     | 0.01512         | 0.35     | 0.04059         | 0.49     | 0.62205         |
| 0.08     | 0.000924        | 0.22     | 0.01589         | 0.36     | 0.04520         | 0.50     | 0.77422         |
| 0.09     | 0.000953        | 0.23     | 0.01673         | 0.37     | 0.05075         | 0.51     | 0.83377         |
| 0.10     | 0.000983        | 0.24     | 0.01766         | 0.38     | 0.05749         | 0.52     | 0.74758         |
| 0.11     | 0.001016        | 0.25     | 0.01869         | 0.39     | 0.06580         | 0.53     | 0.58397         |
| 0.12     | 0.001051        | 0.26     | 0.01984         | 0.40     | 0.07620         | 0.54     | 0.42944         |
| 0.13     | 0.001088        | 0.27     | 0.02112         | 0.41     | 0.08941         | 0.55     | 0.31346         |
| 0.14     | 0.001128        | 0.28     | 0.02256         | 0.42     | 0.10653         | 0.56     | 0.23258         |
results of NACRE II are present in Fig. 6. The average ratio of our calculated results to NACRE II adopted calculation is 1.05, where the ratio of our calculated results to adopted rates of Ref. (Angulo et al. 1999) is 1.09 within the temperature range $T_0 = (0.007 - 1)$ presented in Table 6. Here $T_0$ is temperature in unit of $10^9$ K.

| $T_0$ | Results from Ref. Xu et al. (2013) | This work |
|-------|-----------------------------------|-----------|
|       | Low Adopted High                  | Adopted   |
| 0.007 | $2.00 \times 10^{-22}$ $2.34 \times 10^{-22}$ $2.68 \times 10^{-22}$ | $2.57 \times 10^{-22}$ |
| 0.008 | $4.15 \times 10^{-21}$ $4.86 \times 10^{-21}$ $5.57 \times 10^{-21}$ | $5.37 \times 10^{-21}$ |
| 0.009 | $5.39 \times 10^{-20}$ $6.32 \times 10^{-20}$ $5.57 \times 10^{-20}$ | $6.96 \times 10^{-20}$ |
| 0.010 | $4.92 \times 10^{-19}$ $5.76 \times 10^{-19}$ $6.60 \times 10^{-19}$ | $6.31 \times 10^{-19}$ |
| 0.011 | $3.40 \times 10^{-18}$ $3.98 \times 10^{-18}$ $4.57 \times 10^{-18}$ | $4.34 \times 10^{-18}$ |
| 0.012 | $1.88 \times 10^{-17}$ $2.20 \times 10^{-17}$ $2.52 \times 10^{-17}$ | $2.38 \times 10^{-17}$ |
| 0.013 | $8.68 \times 10^{-17}$ $1.02 \times 10^{-16}$ $1.16 \times 10^{-16}$ | $1.09 \times 10^{-16}$ |
| 0.014 | $4.34 \times 10^{-17}$ $4.02 \times 10^{-16}$ $4.61 \times 10^{-16}$ | $4.33 \times 10^{-16}$ |
| 0.015 | $1.20 \times 10^{-15}$ $1.41 \times 10^{-15}$ $1.61 \times 10^{-15}$ | $1.50 \times 10^{-15}$ |
| 0.016 | $3.76 \times 10^{-15}$ $4.41 \times 10^{-15}$ $5.05 \times 10^{-15}$ | $4.72 \times 10^{-15}$ |
| 0.018 | $2.84 \times 10^{-14}$ $3.32 \times 10^{-14}$ $3.81 \times 10^{-14}$ | $3.55 \times 10^{-14}$ |
| 0.020 | $1.62 \times 10^{-13}$ $1.89 \times 10^{-13}$ $2.17 \times 10^{-13}$ | $2.02 \times 10^{-13}$ |
| 0.025 | $5.27 \times 10^{-12}$ $6.17 \times 10^{-12}$ $7.07 \times 10^{-12}$ | $6.56 \times 10^{-12}$ |
| 0.03  | $7.50 \times 10^{-11}$ $8.77 \times 10^{-11}$ $1.01 \times 10^{-11}$ | $9.31 \times 10^{-11}$ |
| 0.04  | $3.57 \times 10^{-09}$ $4.18 \times 10^{-09}$ $4.79 \times 10^{-09}$ | $4.42 \times 10^{-09}$ |
| 0.05  | $5.55 \times 10^{-08}$ $6.49 \times 10^{-08}$ $7.44 \times 10^{-08}$ | $6.88 \times 10^{-08}$ |
| 0.06  | $4.49 \times 10^{-07}$ $5.25 \times 10^{-07}$ $6.01 \times 10^{-07}$ | $5.57 \times 10^{-07}$ |
| 0.07  | $2.38 \times 10^{-06}$ $2.78 \times 10^{-06}$ $3.19 \times 10^{-06}$ | $2.96 \times 10^{-06}$ |
| 0.08  | $4.42 \times 10^{-06}$ $1.10 \times 10^{-05}$ $1.26 \times 10^{-05}$ | $1.17 \times 10^{-05}$ |
| 0.09  | $3.01 \times 10^{-05}$ $3.52 \times 10^{-05}$ $4.04 \times 10^{-05}$ | $3.78 \times 10^{-05}$ |
| 0.10  | $8.20 \times 10^{-05}$ $9.60 \times 10^{-05}$ $1.10 \times 10^{-04}$ | $1.03 \times 10^{-04}$ |
| 0.11  | $1.97 \times 10^{-04}$ $2.31 \times 10^{-04}$ $2.64 \times 10^{-04}$ | $2.49 \times 10^{-04}$ |
| 0.12  | $4.28 \times 10^{-04}$ $5.01 \times 10^{-04}$ $5.74 \times 10^{-04}$ | $5.43 \times 10^{-04}$ |
| 0.13  | $8.58 \times 10^{-04}$ $1.00 \times 10^{-03}$ $1.15 \times 10^{-03}$ | $1.09 \times 10^{-03}$ |
| 0.14  | $1.61 \times 10^{-03}$ $1.88 \times 10^{-03}$ $2.15 \times 10^{-03}$ | $2.05 \times 10^{-03}$ |
| 0.15  | $2.84 \times 10^{-03}$ $3.33 \times 10^{-03}$ $3.81 \times 10^{-03}$ | $3.65 \times 10^{-03}$ |
| 0.16  | $4.80 \times 10^{-03}$ $5.61 \times 10^{-03}$ $6.43 \times 10^{-03}$ | $6.19 \times 10^{-03}$ |
| 0.18  | $1.21 \times 10^{-02}$ $1.42 \times 10^{-02}$ $1.63 \times 10^{-02}$ | $1.58 \times 10^{-02}$ |
| 0.20  | $2.71 \times 10^{-02}$ $3.18 \times 10^{-02}$ $3.64 \times 10^{-02}$ | $3.57 \times 10^{-02}$ |
| 0.25  | $1.40 \times 10^{-01}$ $1.63 \times 10^{-01}$ $1.87 \times 10^{-01}$ | $1.88 \times 10^{-01}$ |
| 0.30  | $5.21 \times 10^{-01}$ $6.10 \times 10^{-01}$ $6.99 \times 10^{-01}$ | $7.15 \times 10^{-01}$ |
| 0.35  | $1.69 \times 10^{00}$ $1.98 \times 10^{00}$ $2.26 \times 10^{00}$ | $2.28 \times 10^{00}$ |
| 0.40  | $5.09 \times 10^{00}$ $5.97 \times 10^{00}$ $6.86 \times 10^{00}$ | $6.52 \times 10^{00}$ |
| 0.45  | $1.41 \times 10^{01}$ $1.66 \times 10^{01}$ $1.91 \times 10^{01}$ | $1.68 \times 10^{01}$ |
| 0.50  | $3.52 \times 10^{01}$ $4.14 \times 10^{01}$ $4.76 \times 10^{01}$ | $3.90 \times 10^{01}$ |
| 0.60  | $1.53 \times 10^{02}$ $1.80 \times 10^{02}$ $2.07 \times 10^{02}$ | $1.53 \times 10^{02}$ |
| 0.70  | $4.52 \times 10^{02}$ $5.32 \times 10^{02}$ $6.13 \times 10^{02}$ | $4.26 \times 10^{02}$ |
| 0.80  | $1.02 \times 10^{03}$ $1.20 \times 10^{03}$ $1.38 \times 10^{03}$ | $9.25 \times 10^{02}$ |
| 0.90  | $1.90 \times 10^{02}$ $2.24 \times 10^{03}$ $2.59 \times 10^{03}$ | $1.68 \times 10^{03}$ |
| 1.00  | $3.11 \times 10^{03}$ $3.66 \times 10^{03}$ $4.22 \times 10^{03}$ | $2.70 \times 10^{03}$ |

4 Conclusion

In this article, we employed the PM and R-matrix approach for calculation of the partial astrophysical S-factors for both resonant and non-resonant transitions. We modified the Woods-Saxon potential for calculation of the bound
state wave functions. The modification was represented by $q$-values in the Woods-Saxon potential. We kept $q = 1$ for calculation of the continuous state wave function. The $E1$ radiative capture of a proton, from continuous spectrum state of a $p-^{13}\text{C}$ system with $J^\pi = 1^-$, $T = 1$ into three low-lying bound states of $^{14}\text{N}$, may be described in the range of proton energy up to 1 MeV with sufficient accuracy using the PM. Our defined parameters of the Woods-Saxon potential correctly determine the binding energies and energy of $1^-$ resonance.

The transition from $J^\pi = 0^-$, $T = 1$, having resonance at $E_x = 8.776$ MeV, was calculated using the R-matrix approach. This was done because the PM failed due to the broad width of this resonance. The calculated result for the total S-factor is in a good agreement with the previously reported results (King et al. 1994; Mukhamedzhanov et al. 2003). We also took into account direct transition processes which also made a small contribution to the total value of the astrophysical S-factor.

Based on the calculated values of the total S-factor, the computed radiative capture rates showed a better comparison with the results of Ref. Xu et al. (2013).

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