Testing Lorentz-symmetry violation via electroweak decays

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In this work we introduce CPT-odd non-minimal Lorentz-symmetry violating couplings to the electroweak sector modifying the interaction between leptons and gauge bosons. The vertex rules allow us to calculate tree-level processes modified by the presence of the novel dimension-five operators. For definitiveness, we investigate the $W$ decay into a lepton-neutrino pair, the $Z$ decay into pairs of charged and neutral leptons, as well as the decay of the muon. By comparing the experimental measurements on these processes to our results we are able to place upper bounds on combinations of the background 4-vectors of up to $\sim 10^{-4}$ GeV$^{-1}$.

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I. INTRODUCTION

The Standard Model (SM) is based on gauge and Lorentz symmetries, and most of its predictions have been experimentally confirmed, including the 2012 discovery of the long-sought Higgs boson. Nonetheless, it is believed that the SM must be an effective theory corresponding to the low-energy limit of some broader one. In many such beyond the SM scenarios it is possible that Lorentz symmetry is broken at very high energies, what could affect their low-energy limit (that is, the SM) by generating static tensors coupled to dynamic fields that would break Lorentz invariance [1–4]. These terms, which are generally suppressed by inverse powers of some large mass or energy scale (e.g., the Planck scale), could generate small physical effects potentially accessible at current or future experiments [5].

V.A. Kostelecký and D. Colladay have systematically collected the possible low-energy terms arising from Lorentz-symmetry violation (LSV) into the so-called Standard Model Extension (SME) [5, 6], which complements the usual SM by introducing novel LSV interactions in all its sectors, from quantum chromodynamics to gravitation. Diverse experimental tests, ranging from atomic spectroscopy to astrophysical observations, have placed bounds on many of the possible LSV coefficients [7–9].

Since experimental tests of standard quantum electrodynamics (QED) have attained exquisitely high precision levels, the LSV coefficients in the corresponding sector of the SME are strongly constrained. An example is the Carroll-Field-Jackiw correction to the photon propagator [10], whose most stringent upper limit – based on the non-observation of the rotation in the polarization of radiation from astrophysical sources – is in the level of $\lesssim 10^{-43}$ GeV [11, 12]. Here, though complementary, laboratory-based tests are not entirely competitive [13]. The electroweak sector of the SME, on the other hand, has not been studied to the same extent (see Table D35 in ref. [7] and references therein).

In the SM electroweak processes are generally harder to detect than pure electromagnetic ones due to the presence of inverse powers of the large mass of the mediating bosons in the scattering amplitudes. In well-measured processes, such as Bhabha scattering [14, 15], QED effects are responsible for the leading contributions, whereas electroweak effects amount to only a few percent at energies already close to the $Z$ pole [16]. Since at low energies $W$- and $Z$-mediated processes are strongly suppressed relative to photon-mediated ones, we are going to focus on purely electroweak interactions.

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Lorentz-symmetry violation may be incorporated into the electroweak sector by considering couplings analogous to those in the QED sector. A possibility is to introduce a dimension-3 Chern-Simons-like operator which generalizes the Carroll-Field-Jackiw case \[10\], but with the interesting feature of a term directly coupling the photon and Z boson, which leads to photon-Z mixing \[17\]. Another interesting possibility is to directly modify the propagator of the intermediate bosons, thus affecting any W- or Z-mediated process such as muon decay \[15\], neutron β decay \[19\] and nuclear processes \[20\–22\]. A modification of the vertex in the context of meson decay was proposed in ref. \[23\].

In this paper, we propose LSV non-minimal couplings that modify the SU\(_L\)(2) × U(1)\(_Y\) covariant derivative, thereby introducing novel fermion-gauge interaction terms that could have observable effects in electroweak processes. These non-abelian LSV couplings are generalization of Abelian couplings discussed in many different contexts within QED, such as the spectrum of the hydrogen atom \[24\], magnetic and electric dipole moments of charged leptons \[25\–27\], scattering processes \[28\–29\], and topological effects \[30\–31\].

This modification generates new interaction terms involving the gauge bosons and the leptons via space-time-independent LSV background 4-vectors. These novel terms produce modifications already at tree level and in this context we shall address the effects of LSV in W decay \(W^-\to\nu\ell\), the Z decay \(Z\to\ell\bar{\ell}\), where \(f\) is any SM lepton or neutrino, as well as to muon decay, a purely leptonic process of historical and practical importance in tests of the SM (see e.g. ref. \[32\] and references therein). By using the data presented in the latest edition of the Particle Data Group \[33\], we are able to constrain combinations of the LSV coefficients (cf. eqs. \[17\], \[18\], \[19\], \[30\] and fig. 2).

This paper is organized as follows: in section \[II\] we present the LSV-modified covariant derivative that will be applied in section \[III\] to explicitly construct the Lagrangian for the lepton-gauge sector with LSV interactions. In section \[IV\] we apply the LSV-modified Feynman rules to a few processes at tree level to obtain upper bounds on the LSV parameters. Finally, in section \[V\] we summarize our results and present our concluding remarks. In our calculations we employed the Package-X \[34\] to automatically evaluate the traces and contractions involving spinors and Dirac gamma matrices. We use natural units \((c = \hbar = 1)\) throughout.

II. THE LSV-MODIFIED COVARIANT DERIVATIVE

Lorentz violation is generally characterized by the presence of non-dynamical background tensors that select some direction in space-time, thus breaking the invariance under Lorentz transformations. One way to introduce LSV is by means of a non-minimal coupling, i.e., a modification to the minimal derivative through the inclusion of a term containing the field-strength tensor. This approach has been implemented in many different contexts, predominantly in connection with QED \[24\–31\].

We extend the class of Abelian couplings discussed in the aforementioned references to the SM group SU\(_L\)(2) × U(1)\(_Y\). This can be done by coupling the field-strength tensors of the U(1)\(_Y\) and SU\(_L\)(2) gauge fields, \(B_{\mu\nu} = \partial_\mu B_{\nu} - \partial_\nu B_{\mu}\) and \(W^a_{\mu\nu} = \partial_\mu W^a_{\nu} - \partial_\nu W^a_{\mu} + g\varepsilon^{abc} W^b_{\mu} W^c_{\nu}\) \((a = 1, 2, 3)\), respectively, to two real LSV 4-vectors \(\xi^\mu\) and \(\rho^\mu\), so that the covariant derivative now reads

\[
D_\mu = \partial_\mu - ig' Y B_\mu - ig W^a_\mu \frac{\sigma^a}{2} + \frac{i}{2} \xi^\nu B_{\mu\nu} + i \rho^\nu W^a_{\mu\nu} \frac{\sigma^a}{2},
\]

where \(g'\) and \(g\) are the respective U(1)\(_Y\) and SU\(_L\)(2) coupling constants, \(Y\) is the weak hypercharge and \(\{\sigma^a\}\) are the usual Pauli matrices. The charge operator is still given by the Gell-Mann-Nishijima formula \(Q_{em} = Y + I^3\), where \(I^3 = \sigma^3 / 2\), so that the charge assignments of the matter fields are the same as in the SM: \(\psi_{eL} = (\nu_e\ e)\frac{\tau}{2} \sim (2, -1/2)\) and \(\ell_R \sim (1, -1)\), in which \(\ell = \{e, \mu, \tau\}\). Here, \(\psi_{R,L} = P_{R,L}\psi\), where \(P_{R,L} = \frac{1}{2} (1 \pm \gamma_5)\) are the usual right- and left-handed projection operators. Right-handed neutrinos are singlets under SU\(_L\)(2) × U(1)\(_Y\) and are a priori not contained in the SM apart from issues related to neutrino masses and mixing, which are not going to be of consequence here, since we shall treat neutrinos as massless.

A few comments are in order at this point. Firstly, the LSV background 4-vectors coupled to the field-strength tensors preserve gauge invariance. They also introduce interaction terms which display extra momentum-dependent contributions, which are a typical signature of LSV non-minimal couplings (see e.g. refs. \[28\–29\]). Furthermore, while the second and third terms of eq. \(1\) explicitly depend on
the hypercharge and isospin quantum numbers of the fields on which $D_\mu$ is applied, the LSV terms are in principle insensitive to those. Similar approaches have been followed in refs. [34, 35, 36].

Since in our model the LSV couplings appear exclusively in the covariant derivative, the kinetic part of the Lagrangian of the lepton sector will be modified. The lepton-gauge Lagrangian is

$$\mathcal{L}_{\ell\ell} = i \overline{\psi}_\ell \gamma^\mu D_\mu \psi_\ell + i \overline{\ell}_R \gamma^\mu D_\mu \ell_R$$

and now the SM covariant derivative is replaced by eq. (1). The mass terms stemming from the Yukawa interactions are omitted, since the LSV terms do not influence them at tree level. This means that the equations of motion for the free leptons are unchanged and the propagators (and associated Feynman rules) will be the same as in the SM.

As a final remark, we would like to mention that the LSV couplings have negative canonical dimension, meaning that the LSV terms are non-renormalizable. This is a general feature of such non-minimal couplings and indicates that the associated Lagrangian are, in fact, only low-energy effective theories. This will not disturb us here, since we are only dealing with relatively low-energy processes – exclusively at tree level –, so no divergences are expected.

III. THE LSV LEPTON-GAUGE INTERACTIONS

We are now able to dissect eq. (2) further and determine the Feynman rules governing the lepton-boson interactions. We may decompose the lepton-gauge Lagrangian into $\mathcal{L}_{\ell\ell} = \mathcal{L}_{\ell\ell}^{SM} + \mathcal{L}_{\ell\ell}^{LSV}$; where the first term contains the SM contributions to processes involving neutral and charged currents and the second contains only LSV terms.

The mechanism of spontaneous symmetry breaking is the same as in the SM, so we apply the standard Weinberg rotation to write $B_\mu$ and $W_\mu^3$ in terms of $A_\mu$ and $Z_\mu$ [37]. The physical fields $W^\pm$ and $Z$ have masses $m_W = 80$ GeV and $m_Z = 91$ GeV, respectively, and $A$ represents the massless photon. The field-strength tensors of the photon and the Z boson are defined as $F_\mu^\nu = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $Z_\mu \nu = \partial_\mu Z_\nu - \partial_\nu Z_\mu$, while $W_\mu^3$ and $F_\mu^\nu$ (with $F_\mu^\nu$ being its complex conjugate and $W_\mu^\pm \equiv \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm$) are given by

$$W_\mu^3 = \cos \theta_W Z_\mu^\nu + \sin \theta_W F_\mu^\nu - ig \left( W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+ \right),$$

$$F_\mu^\nu = W_\mu^+ + ig \cos \theta_W \left( W_\mu^3 Z_\nu - Z_\mu W_\nu^3 \right) + ig \sin \theta_W \left( W_\mu^+ A_\nu - W_\nu^+ A_\mu \right).$$

With the relations above we may write the LSV piece as

$$\mathcal{L}_{\ell\ell}^{LSV} = \frac{1}{2} \epsilon_\mu^\nu \left( \overline{\ell}_R \gamma^\nu \ell_L + \bar{\ell}_R \gamma^\nu \ell_L \right) \left( \cos \theta_W F_\mu^\nu - \sin \theta_W Z_\mu^\nu \right) + \rho^\mu \bar{\psi}_L \gamma^\nu \left( F_\mu^\nu \sigma^\nu_{2} + F_{\mu \nu} \sigma^\nu_{1} + W_\mu^3 \sigma^3_{2} \right) \psi_L,$$

where $\sqrt{2} \sigma^\pm = \sigma^1 \pm i \sigma^2$. The Lagrangian involving only left-handed leptons reads then

$$\mathcal{L}_{\ell\ell,L}^{LSV} = \frac{1}{2} v_1^\mu \bar{\ell}_L \gamma^\mu \ell_L F_\mu + \frac{1}{2} v_2^\mu \bar{\ell}_L \gamma^\mu \nu_L F_\mu + \frac{1}{2} v_3^\mu \bar{\ell}_L \gamma^\mu \ell_L Z_\mu + \frac{1}{2} v_4^\mu \bar{\ell}_L \gamma^\mu \nu_L Z_\mu + \frac{i g}{2} \rho^\mu \left( \bar{\ell}_L \gamma^\nu \ell_L - \bar{\nu}_L \gamma^\nu \nu_L \right) W_\mu^+ W_\nu^- + \text{H.c},$$

where we defined $A_{[\mu} B_{\nu]} \equiv A_\mu B_\nu - B_\mu A_\nu$. The coupling constants $g$ and $g'$ are connected via $e = g \sin \theta_W = g' \cos \theta_W$, where $e \approx 4 \pi / 128 \approx 0.31$ is the fundamental electric charge and $\theta_W$ is the Weinberg angle satisfying $\sin^2 \theta_W = 0.23$ [34]. For simplicity, we have defined the rotated vectors

$$v_1^\mu = \cos \theta_W \xi_\mu - \sin \theta_W \rho_\mu,$$

$$v_2^\mu = \cos \theta_W \xi_\mu + \sin \theta_W \rho_\mu,$$

$$v_3^\mu = - \sin \theta_W \xi_\mu - \cos \theta_W \rho_\mu,$$

$$v_4^\mu = - \sin \theta_W \xi_\mu + \cos \theta_W \rho_\mu.$$
In the previous section we developed the LSV interaction Lagrangian (cf. eq. (2)) and obtained the Feynman rules for the vertices with which we can now construct the amplitudes for decay widths and scattering processes. Our goal is to calculate observable quantities that have been experimentally measured and, under the – so far justified – assumption that the SM appropriately describes the central value of the experimental results, use the quoted uncertainties to extract upper limits on the LSV coefficients.

IV. APPLICATION TO SELECTED ELECTROWEAK PROCESSES

In the previous section we developed the LSV Lagrangian (cf. eq. (2)) and obtained the Feynman rules for the vertices with which we can now construct the amplitudes for decay widths and scattering processes. Our goal is to calculate observable quantities that have been experimentally measured and, under the – so far justified – assumption that the SM appropriately describes the central value of the experimental results, use the quoted uncertainties to extract upper limits on the LSV coefficients.
A. The W decay width

Let us first consider the decay of the $W^-$ boson into a lepton and its anti-neutrino. The $W^-$ boson starts with 4-momentum $k^\mu$ and polarization vector $\epsilon_\mu(k, \lambda)$, whereas the decay products have 4-momenta $q^\mu$ (lepton with spin $s$) and $q'^\mu$ (anti-neutrino with spin $s'$). The tree-level amplitude for this process is

$$iM_{\lambda ss'}(W^- \to \ell \nu \bar{\nu'}) = \epsilon_\mu(k, \lambda) \bar{u}_\ell(q, s) V_{\ell \nu \bar{\nu}'}^\mu(k) u_{\nu'}(q', s'),$$

(11)

where $\bar{u}_\ell$ and $u_{\nu'}$ are the Dirac spinors for the lepton and anti-neutrino, respectively. The relevant vertex, including the charged-current interaction from the SM and the LSV contribution (cf. table I), is

$$V_{\ell \nu \bar{\nu}'}^\mu(k) = -i \frac{g}{2\sqrt{2}} \gamma^\mu (1 - \gamma_5) + \frac{1}{2\sqrt{2}} (\rho' \gamma^\mu - \rho^\mu \gamma') (1 - \gamma_5) k_\nu.$$  

(12)

Since we are interested in the unpolarized decay rate, we need to average the squared amplitude over initial polarizations and sum over final spins. Using the fact that $(1 - \gamma_5) \gamma^\mu (1 + \gamma_5) = 2\gamma^\mu (1 + \gamma_5)$, the spin-averaged square amplitude is given by

$$\langle |M_{\lambda ss'}|^2 \rangle = \frac{1}{12} \left( -\eta_{\mu\lambda} + \frac{k_\mu k_\lambda}{m_W^2} \right) q'_\alpha q_{\beta} \text{Tr} \left[ \Gamma_+^{\mu} \gamma^{\alpha} (1 + \gamma_5) \Gamma_-^{\lambda} \gamma^{\beta} \right],$$

(13)

where the $\Gamma_\pm^\mu$ matrices are defined by

$$\Gamma_\pm^\mu = g \gamma^\mu \pm i (\rho' \gamma^\mu - \rho^\mu \gamma') k_\nu.$$  

(14)

It is now convenient to move to the rest frame of the $W^-$ boson, where $k^\mu = (m_W, 0)$. Since $m_W \gg m_\ell, m_\nu$, we may ignore the smaller masses, so that eq. (13) is the sum of the following partial amplitudes:

$$\langle |M_{\lambda ss'}|^2 \rangle_{SM} = \frac{g^2}{3m_W^2} \left[ 2(k \cdot q) (k \cdot q') + m_W^2 q \cdot q' \right],$$

(15)

$$\langle |M_{\lambda ss'}|^2 \rangle_{LSV}^{(1)} = -\frac{2g}{3} k^\mu q'^\nu q^\alpha \rho^\beta \epsilon_{\mu\alpha\beta},$$

(16)

$$\langle |M_{\lambda ss'}|^2 \rangle_{LSV}^{(2)} = \frac{2m_W \rho_0}{3} \left( (k \cdot q) (\rho \cdot q') + (k \cdot q') (\rho \cdot q) + \frac{\rho^2}{2m_W \rho_0} \left[ 2(k \cdot q) (k \cdot q') - m_W^2 q \cdot q' \right] \right).$$

(17)

In the rest frame of the $W^-$ boson we may use momentum conservation to show that $q'^\mu = \frac{m_W}{2} (1, -\hat{u})$, and $q^\mu = \frac{m_W}{2} (1, -\hat{u})$, where $\hat{u}$ is a unitary vector in the direction of the 3-momentum of the outgoing lepton. Furthermore, at the vertex we have $k = q + q'$, which makes eq. (16) identically zero. Incorporating all this in the equations above finally gives us

$$\langle |M_{\lambda ss'}|^2 \rangle = \frac{g^2 m_W^2}{3} \left( 1 + \frac{m_W^2 \rho_0^2}{g^2} \right),$$

(18)

which shows no first-order LSV contribution. It is also worthwhile noticing that the LSV piece depends only on the isotropic time component $\rho_0$.

The general expression for the unpolarized two-body decay rate of the $W^-$ boson is

$$\Gamma(W^- \to \ell \nu) = \frac{1}{32\pi^2 m_W} \int \frac{d^3 q}{E_\ell E_{\nu'}} \langle |M_{\lambda ss'}|^2 \rangle \delta^{(4)} (k - q - q'),$$

(19)

and, given that both the SM and LSV contributions contain no angular factors, we are able to perform the phase-space integrals in the same way as in the SM. Dividing this by the full $W$ width $\Gamma_W$ gives us the branching ratio for the channel $W^- \to \ell \nu \bar{\nu}$, so that, using $G_F = \sqrt{2}g^2/8\pi m_W$, we have

$$\text{BR}(W^- \to \ell \nu) = \frac{G_F m_W^3}{6\sqrt{2} \sqrt{\pi} \Gamma_W} \left( 1 + \frac{\rho_0^2}{2G_F^2} \right),$$

(20)
The first term in eq. (20) is the well-known result from the SM, whereas the second is a small deviation arising from the non-minimal coupling introduced in eq. (1). The branching ratio for the channel is measured to be
\[
\text{BR}(W^- \to \ell \bar{\nu}_\ell)_{\text{exp}} = (10.86 \pm 0.09) \%. 
\] (21)
The measurements are well fitted by the first term in eq. (20), so we may assume that the LSV effects are hidden within the experimental uncertainty and demand the second term in eq. (20) to be smaller than the relative experimental error in eq. (21), which is \(\sim 8 \times 10^{-3}\). Doing so we obtain the following upper bound (at 1\(\sigma\))
\[
|\rho^0| \lesssim 8 \times 10^{-4} \text{ GeV}^{-1}.
\] (22)

**B. The \(Z\) decay width**

As a second application we calculate the correction to the decay width of the \(Z\) boson into a lepton/anti-lepton and a neutrino/anti-neutrino pair. The tree-level amplitude for \(Z \to f\bar{f}\), with \(f\) being either a lepton or a neutrino, is
\[
iM_{\lambda ss}(Z \to f\bar{f}) = \epsilon_{\mu} (\lambda, k) \bar{\pi}_f(q, s) V_{\mu f}(k) v_f(q', s'),
\] (23)
with the vertex factor \(V_{\mu ff}(k)\) (including the SM and LSV terms)
\[
V_{\mu ff}(k) = -\frac{ig}{4 \cos \theta_W} \gamma^\mu (g_V - \gamma_5) + \delta_{\ell f} \left( c_{\ell f}^{[\mu} \gamma_{\nu]} + c_{\ell f}^{[\mu} \gamma_{\nu]} \gamma_5 \right) k_\nu + \frac{1}{4} \left( 1 - \delta_{\ell f} \right) c_{\ell f}^{[\mu} \gamma_{\nu]} (1 - \gamma_5) k_\nu,
\] (24)
where we have briefly introduced the Kronecker delta \(\delta_{\ell f}\), which is one if \(f\) is a lepton (\(f = \ell\)) and zero otherwise (\(f = \nu\)). As in the SM, \(g_V = 1 - 4 \sin^2 \theta_W\) for \(f = \ell\) and \(g_V = 1\) for \(f = \nu\).

1. \(Z\) decay into charged leptons

Let us start with the \(Z\) decaying into a lepton/anti-lepton pair. The calculation is very similar to the one leading to eq. (18) and the unpolarized tree-level amplitude for the process is
\[
\langle |M_{\lambda ss}|^2 \rangle = \frac{g^2 m_Z^2}{12 \cos^2 \theta_W} \left[ 1 + \frac{16 \cos^2 \theta_W m_Z^2}{g^2 (1 + g_V^2)} \left( c_{30}^2 + c_{40}^2 \right) \right],
\] (25)
and we notice that, again, no SM-LSV interference term is left, so that the first non-zero correction is of second order in the LSV parameters. Plugging this fully isotropic amplitude into eq. (19) (with the adequate changes) and dividing by the full \(Z\) width \(\Gamma_Z\) gives us the branching ratio (using \(m_W = \cos \theta_W m_Z\))
\[
\text{BR}(Z \to \ell \bar{\ell}) = \frac{G_F m_Z^3 (1 + g_V^2)}{24 \sqrt{2} \pi \Gamma_Z} \left[ 1 + \frac{2 \sqrt{2}}{1 + g_V^2} \left( c_{30}^2 + c_{40}^2 \frac{c_{30}^2 + c_{40}^2}{G_F} \right) \right].
\] (26)

The decay rate of the \(Z\) boson into lepton/anti-lepton pair has been experimentally determined and reads
\[
\text{BR}(Z \to \ell \bar{\ell})_{\text{exp}} = (3.3658 \pm 0.0023) \%,
\] (27)
so that, assuming again that the LSV effects are buried under the experimental errors (\(\sim 7 \times 10^{-4}\)), we find the following upper bound (at 1\(\sigma\))
\[
\sqrt{c_{30}^2 + c_{40}^2} \lesssim 5 \times 10^{-5} \text{ GeV}^{-1}.
\] (28)
2. Z decay into neutrinos

Next we consider the Z boson decaying into a neutrino/anti-neutrino pair. For this process we find that the unpolarized amplitude is (using $g_V = 1$)

$$\langle |M_{\lambda s's'}|^2 \rangle = \frac{g^2 m_Z^2}{6 \cos^2 \theta_W} \left( 1 + \frac{\cos^2 \theta_W m_Z^2}{g^2 v^2} \right),$$

so that the corresponding branching ratio is

$$\text{BR}(Z \to \nu_\ell \bar{\nu}_\ell) = \frac{G_F m_Z^3}{12 \sqrt{2} \pi \Gamma_Z} \left( 1 + \frac{v^2}{4 \sqrt{2} G_F} \right).$$

In collision experiments neutrinos are not directly detected due to their feeble interactions with matter. Detectors in high-energy experiments usually measure the tracks of electrons, muons and photons, which are typical final products of the heavier particles emerging in energetic collisions. From these tracks it is possible to reconstruct the energy and momentum of the original products of the collision and, given that energy and momentum are conserved, it is then possible to infer how much energy and momentum are missing and these are attributed to the so-called invisible products. In the SM, the three neutrino families are able to successfully account for the partial width into invisible final states [37].

The inferred branching ratio of the Z boson into invisible products is measured to be [33, 38]

$$\text{BR}(Z \to \text{invisible})_{\text{exp}} = (20.000 \pm 0.055) \%,$$

which is well fitted by the SM assuming lepton universality, i.e., essentially using the first term in eq. (30) multiplied by three to account for the neutrino families. From eq. (31), we see that the relative uncertainty is $\sim 3 \times 10^{-3}$, so we obtain the following 1σ upper bound

$$|v_{40}| \lesssim 4 \times 10^{-4} \text{ GeV}^{-1}.$$  

C. Muon decay

Now we analyse the process $\mu^- \to \nu_\mu e^- \bar{\nu}_e$, which accounts to practically 100% of the branching ratio for muon decay (other channels are responsible for < 1% of the total decay rate and will be ignored) [33]. This is also a purely leptonic process and we analyse it at tree level, where the amplitude is

$$M_{s_1s_2s_3s_4}(\mu^- \to \nu_\mu e^- \bar{\nu}_e) = \frac{\eta_{\mu e}}{m_W^2} \bar{u}_\mu(p_1, s_1) V_{W\ell\nu}(p_1 - p_3) u_{\nu_e}(p_3, s_3) \bar{u}_e(p_4, s_4) V_{W\ell\nu}^*(p_2 + p_4) v_{\nu_e}(p_2, s_2),$$

with the LSV-modified interaction vertex given by eq. (12). Here it is important to observe that the vertex is defined with the momentum transfer $k$ flowing into the vertex, so that the LSV part of $V_{W\ell\nu}^*(k)$ has opposite signs in the two vertices.

Taking care of the sign of the momentum transfer and averaging over the initial spin, we obtain the following amplitude squared

$$\langle |M_{s_1s_2s_3s_4}|^2 \rangle = \frac{1}{128 m_W^2} \text{Tr} \left[ \Gamma^\mu_+ (1 - \gamma_5) \left( p_1 + m_\mu \right) (1 + \gamma_5) \Gamma^\nu_\mu \right] \times \text{Tr} \left[ \Gamma^-_\mu (1 - \gamma_5) \left( p_2 + m_\mu \right) (1 + \gamma_5) \Gamma^\nu_\mu \right],$$

where the matrix operators are defined in eq. (14). As usual, we have neglected the mass of the electron.
relative to that of the muon. The squared amplitude is then the sum of the following partial amplitudes:

\[
\langle |M_{s_1s_2s_3s_4}|^2 \rangle_{\text{SM}} = \frac{g^4 m_\mu^2}{m_W^4} E_2 (m_\mu - 2E_2),
\]

\[
\langle |M_{s_1s_2s_3s_4}|^2 \rangle_{\text{LSV}}^{(1)} = \frac{g^3 m_\mu^2}{m_W^2} \varepsilon_{\mu \rho} e^{\mu \rho \varepsilon} \left( p_2 \cdot p_3 + p_4 \cdot p_3 \right) e^{\mu \rho \varepsilon},
\]

\[
\langle |M_{s_1s_2s_3s_4}|^2 \rangle_{\text{LSV}}^{(2)} = \frac{g^2 m_\mu^2}{4m_W^2} \left\{ m_\mu^2 \rho^2 E_3 (2E_3 - m_\mu) + 4E_2 (m_\mu - 2E_2) \left[ (p_2 \cdot \rho)^2 + (p_4 \cdot \rho)^2 \right] \\
+ 2 (m_\mu \rho_0 - p_3 \cdot \rho) \left[ 2E_2 m_\mu \rho_0 (m_\mu - 2E_2) + p_3 \cdot \rho (4E_2 - 2E_2 + E_3) m_\mu + m_\mu^2 \right] \\
+ 2 p_2 \cdot \rho \left[ m_\mu (2E_2 - m_\mu) (m_\mu \rho_0 - p_3 \cdot \rho) - 2p_4 \cdot \rho (4E_2 - 2E_2 m_\mu - E_3 m_\mu) \right] \\
- 2 m_\mu p_4 \cdot \rho (m_\mu - 2E_4) (m_\mu \rho_0 - p_3 \cdot \rho) \right\}.
\]

The decay rate of the muon is the generalization of eq. [19] to the case of a three-body decay, that is

\[
\Gamma (\mu^- \to \nu_\mu e^- \bar{\nu}_e) = \frac{1}{16(2\pi)^5 m_\mu} \int \frac{d^3 p_2}{E_2 E_3 E_4} \frac{d^3 p_3}{E_3} \frac{d^3 p_4}{E_4} \langle |M_{s_1s_2s_3s_4}|^2 \rangle^{(4)} \delta (p_1 - p_2 - p_3 - p_4),
\]

which can be simplified using \( \delta (p_1 - p_2 - p_3 - p_4) = \delta (m_\mu - E_2 - E_3 - E_4) \delta (p_2 + p_3 + p_4) \), thus making \( p_3 = -(p_2 + p_4) \) and \( E_3 = |p_2 + p_4| \) upon integration over \( p_3 \). This leaves us with

\[
\Gamma (\mu^- \to \nu_\mu e^- \bar{\nu}_e) = \frac{1}{16(2\pi)^5 m_\mu} \int \frac{d^3 p_2}{E_2^2 E_3 E_4} \frac{d^3 p_4}{E_4^2} \langle |M_{s_1s_2s_3s_4}|^2 \rangle \delta (m_\mu - E_2 - E_3 - E_4).
\]

We must now integrate over \( p_2 \), the 3-momentum of the electron neutrino. Here we may set the \( z \) axis along \( p_4 \), which is constant at this point, so that \( d^3 p_2 = E_2^2 dE_2 \sin \theta_2 d\theta_2 d\phi_2 \). The \( \theta_2 \) integral may be approached using \( E_3 = \sqrt{E_2^2 + E_4^2 + 2E_2 E_4 \cos \theta_2} \). With this substitution we are able to integrate over \( \theta_2 \) and eq. [40] reduces to

\[
\Gamma (\mu^- \to \nu_\mu e^- \bar{\nu}_e) = \frac{1}{16(2\pi)^5 m_\mu} \int \frac{d^3 p_4}{E_4^2} \int d\phi_2 dE_2 \langle |M_{s_1s_2s_3s_4}|^2 \rangle,
\]

where the \( \phi_2 \) and \( E_2 \) integrals must be evaluated under the following conditions: \( p_4 = E_4 \hat{z}, E_3 = m_\mu - E_2 - E_4 \) and \( \cos \theta_2 = (E_2^2 - E_3^2 - E_4^2) / 2E_2 E_4 \).

Unlike the SM case, where at this point only \( p_2 \) and \( p_4 \) must be considered, we have also the background \( \rho^\mu \) contracted to the outgoing momenta. It is then convenient to write \( \rho = |\rho| (\sin \theta_2, \cos \phi_2, \sin \theta_2, \sin \phi_2, \cos \theta_2) \) and \( p_2 = E_3 (\sin \theta_2 \cos \phi_2, \sin \theta_2 \sin \phi_2, \cos \theta_2) \), so that the integral over \( \phi_2 \) may be performed. We will not quote this intermediate result explicitly, but we remark that, due to the totally anti-symmetric contractions, the first-order amplitude (cf. eq. [39]) vanishes identically.

The limit of the \( E_2 \) integral is determined by the kinematics to be \( E_2 = \left[ m_\mu^2 / 2, m_\mu^2 / 2 - E_4 \right] \). After performing this integral, the only dynamic variable is \( p_4 \), the 4-momentum of the electron, which appears in combination with the LSV background. The same trick as above may be employed here, i.e., we let \( \rho = |\rho| \hat{z} \) and write \( p_4 = E_4 (\sin \phi_4, \cos \phi_4, \sin \phi_4, \cos \phi_4) \), so that \( d^3 p_4 = E_4^2 dE_4 d\Omega_4 \). After integrating over \( \Omega_4 \) we obtain the energy spectrum of the emitted electrons with the LSV correction (making \( t \equiv E_4 / m_\mu \))

\[
\frac{d\Gamma}{dE_4} = \frac{g^4 m_\mu^4 t^4 (3 - 4t)}{384\pi^3 m_W^8} \left\{ 1 - \left( \frac{8t^3 - 50t^2 + 65t - 15}{10t^2 (3 - 4t)} \right) \frac{m_\mu^2 |\rho|^2}{20 t g^2 (3 - 4t)} \right\},
\]

which is shown in fig. [1] for different values of the LSV parameters.
Equation (41) displays a few interesting features. For the SM (at tree level) the peak energy of the emitted electron is $E^\text{max}_4 = m_\mu/2$, which is also a kinematical threshold imposed by momentum conservation. The inclusion of LSV disturbs the general shape of the spectrum as shown in fig. 1, where we see that a purely time-like background would suppress the spectrum, whereas a purely space-like $\rho$ would enhance it. The peak energy also recedes from its LSV-free value at different rates for purely time- or space-like components. All these effects could potentially be searched for in sensitive experiments, specially if time-stamped data are taken (see discussion in section V and in appendix A).

Finally, we may integrate eq. (41) in the range $E_4 = [0, m_\mu/2]$ to obtain the decay rate of the muon

$$\Gamma(\mu^- \to \nu_\mu e^- \bar{\nu}_e) = \frac{G_F^2 m_\mu^5}{192 \pi^3} \left[ 1 + \frac{113}{15360} m_\mu^2 m_W^2 G_F \left( \rho_0 + \frac{55|\rho|^2}{226} \right) \right], \quad (42)$$

whose first term is the result from the SM. The experimentally measured lifetime of the muon is

$$\tau_\mu = \Gamma^{-1}_\mu = (2.1969811 \pm 0.0000022) \times 10^{-6} \text{s}, \quad (43)$$

and, by demanding that the second term in eq. (42) be smaller than the relative uncertainty from eq. (43) ($\sim 10^{-6}$), we find the following bound at the $1\sigma$ level

$$\sqrt{\rho_0^2 + \frac{55|\rho|^2}{226}} \lesssim 3 \times 10^{-2} \text{GeV}^{-1}. \quad (44)$$

V. CONCLUDING REMARKS

We studied a modification to the Glashow-Salam-Weinberg electroweak model through non-minimal couplings in the non-Abelian and Abelian sectors of the covariant derivative. These couplings introduce LSV via two real 4-vectors that break and isotropy of space-time. Our results show that such LSV interactions would lead to modifications in the branching ratios of the $W$ and $Z$ bosons, as well as to the lifetime of the muon.
The respective amplitudes have been evaluated at tree level and we found that, for all processes considered, the LSV parameters only contribute to second order. The SM-LSV interference terms drop out of the amplitudes for $W^- \to \ell \overline{\nu}_\ell$ and $Z \to \ell \overline{\nu}_\ell \nu_\ell$ due to anti-symmetry (cf. eq. (16)) or, in the case of $Z \to \ell \ell$ and $\mu^- \to \nu_\mu e^- \overline{\nu}_e$, they automatically cancel in the squared amplitude. LSV effects often show up only in second order in scattering and decay processes (see e.g. refs. [28, 29, 39–41]).

Using recent experimental results we were able to constrain the magnitude of combinations of the LSV parameters (cf. eqs. (22), (28), (32) and (44)). It is important to remember that the bounds quoted above were obtained in the rest frame of the decaying particles, but LSV parameters are not static as seen from the particle’s own rest frame, not to mention from Earth’s rotating reference frame. Therefore, we need to introduce a reference frame in which the LSV tensors are (approximately) static and a convenient option is the so-called Sun-centered frame (SCF), which is discussed in appendix A.

The measurements determining the $W$ and $Z$ widths (and branching ratios) have a center-of-mass energy $\sim 100$ GeV, which is the same order of magnitude of their masses [42, 43], so that the respective Lorentz factors $\gamma_{\text{rest}}$ are very close to unity ($\beta \ll 1$). The MuLan experiment [44] used muons created through pion decay with momenta $\sim 30$ MeV, which also amounts to very small Lorentz factors. Therefore, the components of a generic LSV 4-vector $V^\mu$ in the laboratory frame (LAB) are approximately equal to those in the rest frame, i.e. $V^\mu_{\text{LAB}} \approx V^\mu_{\text{rest}}$, where factors proportional to $\gamma_{\text{rest}} \beta$ may be neglected. With $\gamma_{\text{rest}} \approx 1$ and using eqs. (A10) and (A11) (integrating over $T_{\odot}$) we have

$$\left|V^\mu_{\text{rest}}\right|^2 \approx \left(V^T_{\text{SCF}}\right)^2,$$

$$\left|V_{\text{rest}}\right|^2 \approx \left(1 + s^2_X\right) \left(V^X_{\text{SCF}}\right)^2 + \left(1 + c^2_X\right) \left(V^Y_{\text{SCF}}\right)^2 + \left(V^Z_{\text{SCF}}\right)^2 - 2 c_X s_X \left(V^X_{\text{SCF}}\right) \left(V^Y_{\text{SCF}}\right).$$

Now we can translate the bounds in eqs. (22), (28), (32) and (44) – obtained in the rest frame of the decaying particles – into the SCF. In this frame the bounds for time-like components read

$$|\rho^T_{\text{SCF}}| \lesssim 8 \times 10^{-4} \text{ GeV}^{-1},$$

$$\sqrt{(\rho^T_{\text{SCF}})^2 + 0.6 (\xi^T_{\text{SCF}})^2 + (\epsilon^T_{\text{SCF}})^2} \lesssim 2 \times 10^{-4} \text{ GeV}^{-1},$$

$$|\rho^T_{\text{SCF}} - 0.6 \xi^T_{\text{SCF}}| \lesssim 5 \times 10^{-4} \text{ GeV}^{-1},$$

$$|\xi^T_{\text{SCF}}| \lesssim 5 \times 10^{-4} \text{ GeV}^{-1},$$

$$|\epsilon^T_{\text{SCF}}| \lesssim 10^{-4} \text{ GeV}^{-1}.$$
whereas from eqs. (44) and (46) we find

\[ \sqrt{(\rho_T^\text{SCF})^2 + 0.23 (\rho_X^\text{SCF})^2 + 0.25 (\rho_Y^\text{SCF})^2 + 0.24 (\rho_Z^\text{SCF})^2 - 0.24 (\rho_X^\text{SCF})(\rho_Y^\text{SCF})} \lesssim 3 \times 10^{-2} \text{GeV}^{-1}, \]  

(50)

where we used \( \sin^2 \theta_W = 0.23 \) and \( \chi \approx 43^\circ \) for the co-latitude of the MuLan experiment in Villigen, Switzerland [44]. We note in passing that the limits above do not constrain the spatial components of the 4-vector \( \xi \) in either a rest frame or in the SCF. In fig. 2 we show the allowed regions for the time components of the LSV backgrounds from eqs. (47), (48) and (49).

The couplings listed in table I include terms that are present in the SM (left panel) and terms that are originally not possible at tree level (right panel). Of particular interest is the \( \gamma \nu \bar{\nu} \) term, which would endow the neutrino with a tree-level electromagnetic interaction that could be detected as a magnetic (or electric) dipole moment – both only possible in the SM at loop level and including non-zero neutrino masses [45, 46]. Also the quartic couplings in table I would provide distinctive signatures of LSV in collider experiments, especially \( W^+ W^- \ell \bar{\ell} \) and \( W^+ W^- \nu \bar{\nu} \ell \bar{\nu} \), which represent LSV-induced vector-boson fusion as contact interactions, thus strongly contrasting with the analogous, loop-mediated, SM processes [47]. These topics are currently being investigated and will be reported elsewhere.

We would also like to point out that we did not consider the effects of the Higgs sector in our results. The covariant derivative (cf. eq. (1)) is in principle also introduced in the Higgs sector, but the spontaneous symmetry breaking (SSB) mechanism is not altered. However, after SSB takes place new LSV-dependent terms emerge in the gauge sector, which may modify the propagators of gauge bosons. These effects have not been taken into account here, but they would induce changes to the dispersion relations that could affect causality, unitarity and stability [48, 49].

As a closing remark we note that analogous LSV non-minimal couplings have been proposed in ref. [50], but the authors report a first-order LSV correction to the amplitudes. The discrepancies have been clarified and a corrigendum to their original paper will be submitted soon.

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Appendix A: Sun-centered frame for LSV

In LSV models Lorentz symmetry is broken through 4-vectors – or tensors, in general – that transform differently under observer- and particle-Lorentz transformations and that are fixed in space-time, i.e., they are static backgrounds. This means that there is a reference frame where the LSV 4-vectors are fixed, but the physical observables that we have discussed are measured in Earth-bound reference frames and as such cannot be taken as static. For this reason we must look for a convenient reference frame where the aforementioned coefficients are fixed.

It is clear that a frame fixed to Earth’s surface will not suffice, as it is a non-inertial reference frame, so we cannot expect an external background to be fixed from our point of view – in fact we would see it rotating. The next – and perhaps most convenient – possibility is to use a reference frame fixed relative to the Sun. This is a good choice for a few reasons: it is approximately inertial over the time scale of most experiments (its motion around the galaxy has a period of \( \sim 200 \) million years), it is experimentally accessible, and may have its axes conveniently oriented relative to the Earth.

We will then adopt the Sun-centered frame (SCF) as a standard reference frame where the LSV coefficients are constant, i.e., time-independent [7]. Therefore, relative to an observer fixed on Earth, the background will seem to rotate, so that experimental signals affected by LSV effects should generally present time modulations, specially with sidereal frequencies. Also important is to note that even isotropic backgrounds in the SCF will appear to be anisotropic in our frame because of both rotational and
translational motions of the Earth relative to itself and to the Sun, respectively, which produce boosts. In this sense, rotation violations are a key signal for Lorentz violations in Earth-bound experiments (also in space-based tests [51]).

According to refs. [7, 51], the axes in the SCF are defined such that the Z axis is directed north (parallel to Earth’s rotational axis), X points from the Sun to the vernal equinox, while Y completes a right-handed system. The origin of time $T$ is at the 2000 vernal equinox. Regarding the standard Earth-bound frame for a point in the northern hemisphere, the $z$ axis is vertical from the surface (points to the local zenith), $x$ points south and $y$ points east. The local time $T_{\oplus}$ is defined to be the time measured in the SCF from one of the moments when $y$ lies along $Y$.

To see how we can make the passage from the LSV coefficients in the laboratory frame (LAB), where they are in general time dependent, to the SCF, where they are fixed, we use a generic 4-vector background $V^\mu$. The components of this vector in the two frames are connected via

$$V^\mu_{\text{LAB}} = \Lambda^\mu_\nu V^\nu_{\text{SCF}},$$

with $\Lambda^\mu_\nu$ representing an observer Lorentz transformation between Earth and the SCF. From now on, we represent the components of $V^\mu$ in the LAB frame by $V^\mu_{\text{LAB}}$ and those in the SCF by $V^\mu_{\text{SCF}}$.

The explicit form of the (time-dependent) Lorentz transformation $\Lambda^\mu_\nu$ is

$$\Lambda^0_0 = 1, \quad \Lambda^0_1 = -\beta^t, \quad \Lambda^0_T = -(R \cdot \beta)^t, \quad \Lambda^t_T = R^{tL},$$

where $\beta$ is the velocity ($v/c$ in natural units) of the LAB relative to the SCF and $R^{tL}$ is a spatial rotation. Notice that the Lorentz factor $\gamma = 1/\sqrt{1 - \beta^2}$ is essentially unity due to the smallness of the relative speed of Earth relative to the Sun. The boost components are given by ($\eta \approx 23.4^\circ$ is the inclination of Earth’s axis relative to the orbital plane)

$$\beta^X = \beta_\oplus \sin(\Omega_\oplus T) - \beta_L \sin(\omega_\oplus T_{\oplus}),$$

$$\beta^Y = -\beta_\oplus \cos \eta \cos(\Omega_\oplus T) + \beta_L \cos(\omega_\oplus T_{\oplus}),$$

$$\beta^Z = -\beta_\oplus \sin \eta \cos(\Omega_\oplus T),$$

and, defining $\sin \chi \equiv s_\chi$, $\cos \chi \equiv c_\chi$; $\sin(\omega_\oplus T_{\oplus}) \equiv s_\oplus$, $\cos(\omega_\oplus T_{\oplus}) \equiv c_\oplus$, the matrix $R^{tL}$ is given by

$$R^{tL} = \begin{pmatrix} c_\chi & c_\chi & -s_\chi \\ s_\chi & c_\chi & 0 \\ -s_\chi & s_\chi & c_\chi \end{pmatrix}. \quad (A6)$$

The $\Lambda^i_T = -(R \cdot \beta)^i$ read

$$\Lambda^x_T = -c_\chi c_\oplus \beta^X - c_\chi s_\oplus \beta^Y + s_\chi \beta^Z,$$

$$\Lambda^y_T = s_\chi \beta^X - c_\chi \beta^Y,$$

$$\Lambda^z_T = -s_\chi c_\oplus \beta^X - s_\chi s_\oplus \beta^Y - c_\chi \beta^Z,$$

where the numerical values of the parameters appearing above are

$$\beta_\oplus \approx 10^{-4}, \quad \text{Earth’s orbital velocity}$$

$$\beta_L = r_\oplus \omega_\oplus \sin \chi < 10^{-6}, \quad \text{Earth’s rotational velocity}$$

$$\omega_\oplus = 2\pi/\text{day} \approx 7 \times 10^{-5} \text{s}^{-1}, \quad \text{Earth’s rotational angular velocity}$$

$$\Omega_\oplus = 2\pi/\text{year} \approx 2 \times 10^{-7} \text{s}^{-1}, \quad \text{Earth’s orbital angular velocity}$$

$$\chi = \text{experiment’s co-latitude}.$$

From the values above we see that $\Lambda^0_1 = -\beta^t$ and $\Lambda^t_T = -(R \cdot \beta)^t$ are strongly suppressed due to the smallness of the boost factors, so we may safely ignore them. Applying this to our generic vector we find that its components are translated from the LAB frame to the SCF as

$$V^\mu_{\text{LAB}} = V^\nu_{\text{SCF}} + \mathcal{O}(\beta),$$

$$V^i_{\text{LAB}} = R^{iL} V^l_{\text{SCF}} + \mathcal{O}(\beta),$$

where $l = 0,1,2,3$ are states for the LAB to the SCF.
which means that, up to very small contributions proportional to boost factors, time and space components of $V_{LAB}$ and $V_{SCF}$ do not mix. We are therefore able to separately analyse LSV background 4-vectors that have either purely time or spatial components in the SCF.

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