SCATTERING OF COSMIC RAYS BY MAGNETOHYDRODYNAMIC INTERSTELLAR TURBULENCE

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Recent advances in understanding of magnetohydrodynamic (MHD) turbulence call for substantial revisions in our understanding of cosmic ray transport. In this paper we use gyroresonance recently obtained scaling laws for MHD modes to calculate the scattering frequency for cosmic rays in the ISM. We consider gyroresonance with MHD modes (Alfvénic, slow and fast) and transit-time damping (TTD) by fast modes. We conclude that the gyroresonance with fast modes is the dominant contribution to cosmic ray scattering for the typical interstellar conditions. In contrast to earlier studies, we find that Alfvénic and slow modes are inefficient because they are far from isotropy usually assumed.

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I. INTRODUCTION

The propagation of cosmic rays (CRs) is affected by their interaction with magnetic field. This field is turbulent and therefore, the resonant interaction of cosmic rays with MHD turbulence has been discussed by many authors as the principal mechanism to scatter and isotropize cosmic rays (1). Although cosmic ray diffusion can happen while cosmic rays follow wandering magnetic fields (2), the acceleration of cosmic rays requires efficient scattering.

While most investigations are restricted to Alfvén waves propagating along an external magnetic field (the so-called slab model of Alfvénic turbulence), obliquely propagating MHD waves have been included in (3) and later studies (4, 5). The problem, however, is that the Alfvénic turbulence considered in their studies is isotropic turbulence, and this is contrary to the modern understanding of MHD turbulence (6), see (7) for a review and references therein).

A recent study (6) found a strong dependence of scattering on turbulence anisotropy. Therefore the calculations of CR scattering must be done using a realistic MHD turbulence model. An important attempt in this direction was carried out in (8, 9). There Alfvén modes were treated in the spirit of Goldreich-Shridhar (10) (hereafter GS95) model of incompressible turbulence and marginal scattering was obtained. However, a more accurate description is now available (11) and thus there is a need to revisit the problem. Moreover, (6) did not consider compressible modes, while we show below that these modes provide the dominant contribution to the scattering.

II. MHD STATISTICS

MHD perturbations can be decomposed into Alfvénic, slow and fast modes (see (12)). Alfvénic turbulence is considered by many authors as the default model of interstellar magnetic turbulence. This is partially motivated by the fact that unlike compressible modes, the Alfvén ones are essentially free of damping in fully ionized medium (see (13, 12)).

Unlike hydrodynamic turbulence, Alfvénic turbulence is anisotropic, with eddies elongated along the magnetic field. This happens because it is easier to mix the magnetic field lines perpendicular to the direction of the magnetic field rather than to bend them. The GS95 model describes incompressible Alfvénic turbulence, which formally means that plasma $\beta = P_{gas}/P_{mag} = 2C_s^2/V_A^2$ is infinity. It was first conjectured in (8, 9) that GS95 scaling should be approximately true for moderately compressible plasma. For low $\beta$ plasma Cho & Lazarian (14) (henceforth CL02) showed that the coupling of Alfvénic and compressible modes is weak and that the Alfvénic modes follow the GS95 spectrum (15). This is consistent with the analysis of observational data (16, 17). In what follows, we consider both Alfvén modes and compressible modes and use the description of those modes obtained in CL02 to study CR scattering by MHD turbulence in a medium with energy injection scale $L = 100pc$, density $n = 10^{-4}cm^{-3}$, temperature $T = 2 \times 10^6K$. Recent observations (18) suggest that matter in the galactic halos is magnetic-dominant, corresponding to low $\beta$ medium, here we choose $\beta \approx 0.1$. The injection length scale is important as Alfvénic turbulence exhibits scale-dependent anisotropy that increases with the decrease of the scale.

We describe MHD turbulence statistics by correlation functions. Using the notations from (8, 9, 10), we get the expressions for the correlation tensors in Fourier space

$$< B_i(k)B_j^*(k') > / B_0^2 = \delta(k-k')M_{ij}(k),$$
$$< v_i(k)B_j^*(k') > / V_A B_0 = \delta(k-k')C_{ij}(k),$$
$$< v_i(k)v_j^*(k') > / V_A^2 = \delta(k-k')K_{ij}(k),$$

where $B_{\alpha,\beta}$ is the magnetic field fluctuations.

The isotropic tensor usually used in the literature is

$$K_{ij}(k) = C_0\{\delta_{ij} - k_i k_j / k^2\}k^{-11/3},$$

where $k_i = \delta_{ij} - k_i k_j / k^2$. The anisotropy of the correlation tensor is described by the scalar $\epsilon_{ij}$, which is the ratio of the anisotropic part of the correlation tensor to the isotropic part. This scalar can be expressed as a function of the anisotropy parameter $\epsilon_{ii}$, which is the ratio of the anisotropic part of the correlation tensor to the isotropic part. In the limit of weak turbulence, $\epsilon_{ii} \approx 0$, and the anisotropy of the correlation tensor can be approximated by $\epsilon_{ii}$. However, in the case of strong turbulence, $\epsilon_{ii} \approx 1$, and the anisotropy of the correlation tensor cannot be approximated by $\epsilon_{ii}$.
The normalization constant $C_0$ can be obtained if the energy input at the scale $L$ is defined. Assuming equipartition, the kinetic energy density $\epsilon_k = \int d^3k \sum_{i,j} K_{ij} v_i^2/2 \sim B_0^2/8\pi$, we get $C_0 = L^{-2/3}/12$. The analytical fit to the anisotropic tensor for Alfvén modes, obtained in [13] is,

$$K_{ij}(k) = C_a I_{ij} k_+^{10/3} \exp(-L^{1/3} k_+/k_z^{2/3}),$$  \hspace{1cm} (3)

where $I_{ij} = \{\delta_{ij} - k_i k_j / k_z^2\}$ is a 2D matrix in x-y plane, $k_\parallel$ is the wave vector along the local mean magnetic field (see Fig. 1), $k_z$ is the wave vector perpendicular to the magnetic field and the normalization constant $C_a = L^{-1/3}/6\pi$. The tensors in [13] used step function instead of the exponent. We assume that for the Alfvén modes $M_{ij} = K_{ij}$, $C_{ij} = \sigma M_{ij}$ where the fractional helicity $-1 < \sigma < 1$ is independent of $k$ (Fig. 1).

Numerical calculations in CL02 demonstrated that slow modes in low frequency of relativistic particle, the corresponding resonant scales are much smaller than the injection scale. Therefore the fluctuation on the resonant scale $\delta B \ll B_0$ even if they are comparable at the injection scale. QLT disregards diffusion of cosmic rays that follow wandering magnetic field lines (Fig. 2) and this diffusion should be accounted separately. Obtained by applying the QLT to the collisionless Boltzmann-Vlasov equation, the Fokker-Planck equation is generally used to describe the involvement of the gyrophase-average distribution function $f$,

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial \mu} \left( D_{\mu \nu} \frac{\partial f}{\partial \mu} + D_{\mu \nu} \frac{\partial f}{\partial p} \right) + \frac{1}{p^2} \frac{\partial}{\partial p} \left[ p^2 \left( D_{\mu \nu} \frac{\partial f}{\partial \mu} + D_{\mu \nu} \frac{\partial f}{\partial p} \right) \right],$$

where $p$ is the particle momentum. The Fokker-Planck coefficients $D_{\mu \nu}, D_{\mu \nu}$ are the fundamental physical parameter for measuring the stochastic interactions, which are determined by the electromagnetic fluctuations [23]:

$$< B_i(k) B_j^*(k') >= \delta(k-k') P_{ij}(k),$$
$$< B_i(k) E_j^*(k') >= \delta(k-k') T_{ij}(k),$$
$$< E_i(k) B_j^*(k') >= \delta(k-k') Q_{ij}(k),$$
$$< E_i(k) E_j^*(k') >= \delta(k-k') R_{ij}(k).$$  \hspace{1cm} (6)

From Ohm’s Law $E(k) = -(1/c)v(k) \times B_0$, we can express the electromagnetic fluctuations $T_{ij}, R_{ij}$ in terms of correlation tensors $C_{ij}, K_{ij}$. Adopting the approach in [23], we can get the Fokker-Planck coefficients in the lowest order approximation of $V_A/c$,

$$\begin{bmatrix} D_{\mu \mu} \\ D_{\mu \nu} \\ D_{\nu \nu} \end{bmatrix} = \frac{\Omega^2 (1 - \mu^2)}{2 B_0^2} \begin{bmatrix} 1 \\ m^2 c^2 \end{bmatrix} Re \sum_{n=-\infty}^{n=\infty} \int_{k_{min}}^{k_{max}} dk^3$$

$$\int_0^\infty dt e^{-i(k_+ v_1 - \omega - \Omega t)} \begin{bmatrix} j \frac{k_+ v_1}{\Omega} \\ -P_{RL}(k) \\ -T_{RL}(k) \end{bmatrix} + J_{n+1}(\frac{k_+ v_1}{\Omega}) J_{n-1}(\frac{k_+ v_1}{\Omega})$$

$$\begin{bmatrix} e^{i2\phi} \frac{-P_{RL}(k)}{T_{RL}(k)} \\ -T_{RL}(k) \\ R_{RL}(k) \end{bmatrix} + e^{-i2\phi} \begin{bmatrix} -P_{RL}(k) \\ T_{RL}(k) \\ R_{RL}(k) \end{bmatrix}.$$

\hspace{1cm} (7)

### III. SCATTERING BY ALFVÉNIC TURBULENCE

Particles get into resonance with MHD perturbations propagating along the magnetic field if the resonant condition is fulfilled, namely, $\omega = k_+ v_1 + \Omega n (n = \pm1, 2,...)$ where $\omega$ is the wave frequency, $\Omega = \Omega_0 / \gamma$ is the gyrofrequency of relativistic particle, $\mu = \cos \alpha$, where $\alpha$ is the pitch angle of particles. In other words, resonant interaction between a particle and the transverse electric field of a wave occurs when the Doppler shifted frequency of the wave in the particle’s guiding center rest frame $\omega = \omega - k_+ v_1$ is a multiple of the particle gyrofrequency. For high energy particles, the resonance happens for both positive and negative $n$. We employ quasi-linear theory (QLT) to obtain our estimates. QLT has been proved to be a useful tool in spite of its intrinsic limitations ([9, 21, 22]). For moderate energy cosmic rays, the corresponding resonant scales are much smaller than the injection scale. Therefore the fluctuation on the resonant scale $\delta B \ll B_0$ even if they are comparable at the injection scale. QLT disregards diffusion of cosmic rays that follow wandering magnetic field lines (Fig. 2) and this diffusion should be accounted separately. Obtained by applying the QLT to the collisionless Boltzmann-Vlasov equation, the Fokker-Planck equation is generally used to describe the involvement of the gyrophase-average distribution function $f$,
where $k_{\text{min}} = L^{-1}$, $k_{\text{max}} = \Omega_0/v_{th}$ corresponds to the dissipation scale, $m = \gamma m_H$ is the relativistic mass of the proton, $v_{\perp}$ is the particle’s velocity component perpendicular to $B_0$, $\phi = \arctan(k_y/k_x)$, $L, R = (x \pm iy)/\sqrt{2}$ represent left and right hand polarization.

The integration over time gives us a delta function $\delta(k_{||} - \omega + n\Omega)$ corresponding to static magnetic perturbations [14, 27]. For cosmic rays, $k_{||}v_{||} \gg \omega = k_{||}V_A$ so that the resonant condition is just $k_{||}v_{\perp} = n\Omega = 0$. From this resonance condition, we know that the most important interaction occurs at $k_{||} = k_{\text{res}} = \Omega/v_{||}$.

Noticing that the integrand for small $k_{\perp}$ is substantially suppressed by the exponent in the anisotropic tensor (see Eq. (3)) so that the large scale contribution is not important, we can simply use the asymptotic form of Bessel function for large argument. Then if the pitch angle $\alpha$ not close to 0, we can derive the analytical result for anisotropic turbulence,

$$
\begin{align*}
D_{\mu\nu} & = \frac{v_{\perp}^2 \cos \alpha^{5.5}}{20.5 L^{2.5} \sin \alpha^{1/2}} \Gamma[6.5, k_{\text{max}} k_{\text{res}} L^{1/2}] \left[ 1 + \frac{m_H^2 V_A^2}{4 m^2 V_A^4} \right], \\
D_{pp} & = 2 \frac{m_H^2 V_A^2}{4 m^2 V_A^4}
\end{align*}
$$

where $\Gamma[a, z]$ is the incomplete gamma function. The presence of this gamma function in our solution makes our results orders of magnitude larger than those in [14] for the most of energies considered (see Fig.1a). However, the scattering frequency $\nu = 2D_{\mu\nu}/(1 - \mu^2)$ are much smaller than the estimates for isotropic model. Unless we consider very high energy CRs ($\geq 10^5$ GeV) with the corresponding Larmor radius comparable to the turbulence injection scale, we can neglect scattering by the Alfvenic turbulence. What is the alternative way to scatter cosmic rays?

IV. SCATTERING BY FAST MODES

Our result that anisotropic turbulence is inefficient in CR scattering agrees well with the conclusions reached in [14] and [3]. The contribution from slow modes is not larger than that by Alfven modes since the slow modes have the similar anisotropies and scalings. More promising are fast modes, which are isotropic [14]). For fast modes we discuss two types of resonant interaction: gyroresonance and transit-time damping; the latter requires longitudinal motions. However, fast modes are subject to collisionless damping which suppresses scattering[27].

The damping rate $\gamma_d = \tau_d^{-1}$ for the low $\beta$ case (11) is

$$
\gamma_d = \frac{\sqrt{\beta}}{4} V_A \frac{m_e}{m_H} \frac{\sin^2 \theta}{\cos \theta} \times \left( \frac{m_e}{m_H} \exp(-\frac{m_e}{m_H \beta \cos^2 \theta}) + 5 \exp(-\frac{m_e}{2 \beta \cos^2 \theta}) \right),
$$

where $m_e$ is the electron mass. We see that the damping increases with $\beta$. According to CL02, fast modes cascade over time scales $\tau_{fk} = \tau_{k} V_A/v_k = (k \times k_{\text{min}})^{-1/2} \times V_A/V^2$, where $\tau_k = (k_{||} v_k)^{-1}$ is the eddy turn-over time, $V$ is the turbulence velocity at the injection scale.

Consider gyroresonance scattering in the presence of collisionless damping. The cutoff of fast modes corresponds to the scale where $\tau_{fk} \gamma_d \approx 1$ and this defines the cutoff scale $k_{c}^{-1}$. As we see from Eq.(8), the damping increases with $\theta$ unless $\theta$ is close to $\pi/2$.

Using the tensors given in Eq.(4) we obtain the corresponding $D_{\mu\nu}$ for the CRs interacting with fast modes by integrating Eq.(7) from $k_{\text{min}}$ to $k_{c}$ (see Fig.1b)). When $k_{c}^{-1}$ is less than $r_L$, the results of integration for damped and undamped turbulence coincides. Since the $k_{c}$ decreases with $\beta$, the scattering frequency decreases with $\beta$.

Adopting the tensors given in Eq.(5), it is possible to calculate the scattering frequency of CRs in high $\beta$ medium. For instance, for density $n = 0.5 c^{-3}$, temperature $T = 8000$ K, magnetic field $B_0 = 1$ G, the mean free path is smaller than the resonant wavelength for the particles with energy larger than 0.1 GeV, therefore collisional damping rather than Landau damping should be taken into account. Nevertheless, our results show that the fast modes still dominate the CRs’ scattering in spite of the viscous damping.

Apart from the gyroresonance, fast modes potentially can scatter CRs by transit-time damping (TTD) [12]. TTD happens due to the resonant interaction with parallel magnetic mirror force $-nv_{\perp}^2/2B\nabla_n B$. For small amplitude waves, particles should be in phase with the wave so as to have a secular interaction with wave. This gives the Cherenkov resonant condition $\omega - k_{||} v_{||} \sim 0$, corresponding to the $n = 0$ term in Eq.(7). From the condition, we see that the contribution is mostly from nearly perpendicular propagating waves ($\cos \theta \sim 0$). According to Eq.(4), we see that the corresponding correlation tensor for the magnetic fluctuations $M_{ij}$ are very

FIG. 1: The scattering frequency $\nu$ vs. the kinetic energy $E_k$ of cosmic rays (a) by Alfvénic turbulence, (b) by fast modes. In (a), the dash-dot line refers to the scattering frequency for isotropic turbulence. The ‘x’ represents our numerical result for anisotropic turbulence, the solid line is our analytical result from Eq.(8). Also plotted (dashed line) is the previous result for anisotropic turbulence in [14]. In (b), the dashed line represents the scattering by fast modes immune from damping, the solid and dashdot line are the results taking into account collisionless damping.
small, so the contribution from TTD to scattering is not important.

Self-confinement due to the streaming instability has been discussed by different authors ([9, 28, 29]) as an effective alternative to scatter CRs and essential for CR acceleration by shocks. However, we will discuss in our next paper that in the presence of the turbulence the streaming instability will be partially suppressed owing to the nonlinear interaction with the background turbulence.

Thus the gyroresonance with the fast modes is the principle mechanism for scattering cosmic rays. This demands a substantial revision of cosmic ray acceleration/propagation theories, and many related problems may need to be revisited. For instance, our results may be relevant to the problems of the Boron to Carbon abundances ratio. We shall discuss the implications of the new emerging picture elsewhere.

V. SUMMARY

In the paper above we have shown that

1. Scattering by fast modes is the dominant scattering process provided that turbulent energy is injected at large scales.

2. Gyroresonance is the most important for pitch angle scattering. Transit-time damping (TTD) of the resonant waves is subdominant because the corresponding magnetic fluctuations are nearly perpendicular to the mean magnetic field.

3. The scattering frequency by fast modes depends on collisionless damping for viscous damping, therefore it varies with plasma $\beta$.

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[1] R. Schlickeiser, Cosmic Ray Astrophysics (Springer-Verlag Berlin Heidelberg, 2002)
[2] J.R. Jokipii, Astrophys. J., 146, 480 (1966)
[3] L.A. Fisk, M.L. Goldstein, & G. Sandri, Astrophys. J., 190, 417 (1974)
[4] J.W. Bieber, C.W. Smith, & W.H. Matthaeus, Astrophys. J., 334, 470 (1988)
[5] J.M. Pryadko & V. Petrosian, Astrophys. J., 515, 873 (1999)
[6] P. Goldreich & H. Sridhar, Astrophys. J., 438, 763 (1995)
[7] J. Cho, A. Lazarian, & H. Yan, ASP, in press, astro-ph/0112366 (2002)
[8] I. Lerche & R. Schlickeiser, A&A, 378, 279 (2001)
[9] B. Chandran, Phys. Rev. Lett., 85(22), 4656 (2000)
[10] J. Cho, A. Lazarian, & E.T. Vishniac, Astrophys. J., 564, 291 (2002)
[11] V.L. Ginzburg, Propagation of Electromagnetic Waves in Plasma (New York: Gordon & Breach, 1961)
[12] R. Kulsrud & W.P. Pearce, Astrophys. J., 156,445 (1969)
[13] Y. Lithwick and P. Goldreich, Astrophys. J. 562, 279 (2001)
[14] J. Cho & A. Lazarian, Phys. Rev. Lett., 88(24), 5001 (2002)
[15] Simulations by Cho & Lazarian in preparation show that this is also true in high $\beta$ medium.
[16] A. Lazarian & D. Pogosyan, Astrophys. J., 537, 720 (2000)
[17] S. Stanimirovic & A. Lazarian, Astrophys. J., 551, L53 (2001)
[18] R. Beck, Space Sci. Rev., 99, 243 (2001)
[19] Apparently $M_{ij}, C_{ij}$ are 3D matrixes. However, the third dimension is not needed for our calculations.
[20] These tensors are different from that in [22]. The fact that the fluctuations $\delta B$ in fast modes are in the $k-B$ plane place another constraint on the tensor so that the term $\delta_{ij}$ doesn’t exist.
[21] J.A. Miller, Astrophys. J., 491, 939 (1997)
[22] R. Schlickeiser & J.A. Miller, Astrophys. J., 492, 352 (1998)
[23] R. Schlickeiser & U. Achatz, J. Plasma Phy. 49, 63 (1993)
[24] For $D_{\mu\nu}$, the expression is only true for Alfvén modes. There are additional compressional terms for compressible modes.
[25] Cosmic rays have such high velocities that the slow variation of the magnetic field with time can be neglected.
[26] We counted only the resonant term in [28]. The nonresonant term is spurious as noted by [28].
[27] Since the turbulence injection scale is less than the mean free path, only collisionless effects are involved.
[28] C. Cesarsky, Annu. Rev. Astr. Ap., 18, 289 (1980)
[29] M.S. Longair, High Energy Astrophysics ( Cambridge University Press, 1997)