Theoretical and Experimental Study on Normal Contact Stiffness of Plane Joint Surfaces With Surface Texturing

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ABSTRACT
Effects of surface texturing on normal contact stiffness have been investigated by experiments in previous research studies; however, there are relatively few theoretical models in this regard. This article aims to investigate the influence of surface texturing on the normal contact stiffness in theory and experiment. In the present research, a rough surface with surface texturing is divided into two parts, the textured zone and the remaining zone, and their theoretical models of normal contact stiffness are established respectively, considering surface morphology and material properties. For the textured zone, microtextures are modeled theoretically based on the three-dimensional topographic data obtained via a laser profilometer. For the remaining zone, the surface topography is characterized based on fractal theory, and a calculation model is established considering the elastoplastic deformation of surface asperities. In the experiment, the normal contact stiffness of specimens is obtained through the self-developed experimental platform. The proposed theoretical model is appropriate through the comparison of test results and theoretical predictions. The research results show that the effect of surface texture on the contact stiffness is related to the surface roughness, Sa. For the joint surfaces with Sa > 2.69 μm, the normal contact stiffness can be effectively increased through proper surface texturing.

Introduction
Plane joint surfaces are widely used in mechanical structures, such as aero-engines and flanges. For the overall contact characteristics of mechanical systems under combined loads, contact stiffness is very important, as one of the important parameters among contact properties. The enhancement of normal contact stiffness can reduce the deformation, thereby improving the machining quality of machine tool and optimizing the dynamic connection properties of gear systems (1). However, the prediction of contact stiffness has always been very difficult. For the development of the machine dynamics modeling and analysis, it is very critical to improve the accuracy of the contact stiffness model (2, 3).

Scholars have been working to propose a more reasonable theoretical model of contact stiffness. Greenwood and Williamson (4) established a statistical model to calculate contact stiffness for the first time, which was called the G-W model for short. This research is very meaningful for the study of contact mechanisms of rough surface. On the other hand, Bhushan and Majumbar (5) proposed a theoretical model (the M-B model) from the perspective of fractal geometry, which considered the contact characteristics of joint surfaces, and it also solved the problem of scale dependence in statistical models. Based on the work just described, the research on the contact characteristics was further improved, such as considering surface asperity interaction (6), elastic–plastic deformation characteristics (7), dynamic contact force (8), surface finish (9), lubrication (10), and so on. Also, the research on normal contact stiffness was gradually further developed. Ciavarella et al. (11) improved the G-W model, and modified the relationship between load and deformation through considering the interaction of asperities. Belhadjamor et al. (12) proposed a numerical model of contact stiffness by calculating the actual area and the number of contact asperities, considering the influence of asymmetry and peak value on rough contact performance. Based on the elastoplastic deformation mechanism of contact asperities, Zhang et al. (13) proposed a contact stiffness model, in which the fractal theory is used for describing the surface morphology. Essentially, the contact properties analysis of joint surfaces is closely relevant to the micromorphology. The morphology of rough surfaces greatly affects the physical properties of joint surfaces (14, 15). Therefore, it is significant to propose an accurate normal contact stiffness model from the perspective of micromorphology.

In recent years, laser micromachining technique has gradually become a research hotspot due to its good surface modification function (16, 17). Contact properties of joint surfaces can be improved or modified by generating microtextures using laser micromachining technique, which can...
change the micromorphology of a rough surface significantly. Microtextures in the form of circular dimples can effectively reduce friction and wear due to the improvement of surface hardness and the storage capacity of texture for abrasive particles (18). An increase of surface hardness can enhance the deformation resistance, which is the main factor for the textured sample. Microtexture is essentially a microstructure with regular arrangement on the surface of structures through active design and manufacturing. Because the surface texturing changes the morphology and material properties of rough surface, the normal contact stiffness of textured surface is significantly affected.

Scholars have been working on the contact stiffness of the textured surface. For the sealing ring with surface texturing, Arghir et al. (19) proposed an approach for the analysis of normal contact stiffness through simulation. By comparing the theoretical predictions and the experimental data, the estimation model was proved to be appropriate. The accuracy of the model predictions was improved through using a modified bulk-flow model. Research by Medina (20) showed that the density of surface asperities can be controlled by altering the distance between the rough surfaces, so as to control the normal contact stiffness. Fuadi et al. (21) machined the microtexture like a pyramid with different areal density on the surfaces of specimens, and measured tangent contact stiffness. Perris et al. (15) fabricated a square micropattern on the sample surface via a microfabrication process. The normal contact stiffness of structured surfaces is tailored and repeatable, which is useful for applications requiring specified interface stiffness.

The previous research mainly studied the contact stiffness of textured surface through simulations and experiments. However, there are few theoretical models, and the impact mechanism of surface texturing is still unclear and needs further investigation. Considering the changes in micromorphology and material properties of rough surfaces with surface texturing, the research suggests a theoretical calculation method of normal contact stiffness. The theoretical model is then verified by experiments.

**Nomenclature**

| Symbol | Description |
|--------|-------------|
| $a_c$  | critical contact area |
| $a_l$  | actual contact area of the largest asperity |
| $A$    | contact area of textured zone |
| $A_n$  | apparent contact area of the specimens |
| $b$    | ratio of the remaining zone area to the apparent contact area |
| $c$    | variation coefficient of elastic modulus in textured zone |
| $G$    | fractal roughness coefficient |
| $D$    | fractal dimension of rough surface profile |
| $E$    | equivalent elastic modulus |
| $E_0$  | elastic modulus of the original material |
| $E_{1,2}$ | elastic moduli of the two rough surfaces |
| $f_r$  | certain eigenmode frequency of the test system |
| $H$    | height of the texture |
| $H_a$  | height of the asperity |
| $H_T$  | surface hardness of textured surface |
| $K$    | normal contact stiffness of the entire joint surfaces |
| $K_0$  | normal contact stiffness before processing the surface texture |
| $K_1$  | normal contact stiffness of the remaining zone |
| $K_2$  | normal contact stiffness of the textured zone |
| $K_m$  | normal contact stiffness of the specimens |
| $m$    | mass of the upper and lower specimens |
| $r$    | curvature radius of the top |
| $R_1$  | external diameter at the bottom of texture |
| $R_2$  | internal diameter at the top of texture |
| $\nu_1$, $\nu_2$ | Poisson ratios of the two rough surfaces |
| $\delta$ | deformation of the texture |
| $\delta_1$ | deformation of the textured plane |
| $\psi$ | domain expansion factor |

**Figure 1.** Equivalent of the rough joint surfaces.

**Theory model**

**Effect of surface texturing on normal contact stiffness**

**Surface topography**

Surface topography plays a critical role on contact characteristics of joint surfaces. According to previous studies (22), the contact between rough surfaces can be equivalent to that of a rigid smooth surface and a rough surface, so the same can be done for the contact between rough surface and textured surface. A schematic diagram of the rough joint surfaces is shown in Fig. 1.

The microtextures only change the morphology of the textured zone, but have almost no effect on the morphology of the remaining zone. Therefore, the surface morphology of the remaining zone is similar to that of specimens before fabricating the surface texture. Regardless of the differences among microtextures, it is assumed that the three-dimensional morphology of each texture is analogous, as shown in Fig. 2.
Microstructure

The microstructure of a cross section of a textured specimen was obtained by wire cutting and etched with 4% nital, as shown in Fig. 3. In the textured zone, laser processing causes surface materials to melt rapidly. The absorbed laser energy decreases gradually along the depth direction, raising the temperature. The temperature exceeds the phase transition temperature in the region near the texture, and martensite and troostite are produced \(^\text{(23, 24)}\). Otherwise, the temperature does not reach the phase transition temperature and there is no phase transformation, but the grain size is refined and the dislocation density is increased. The heat-affected zone is formed with a thickness of about 100 \(\mu\text{m}\). The elastic modulus \(E_2\) of material in the textured zone is regarded as an invariant constant.

The surface hardness is measured using a Vickers hardness tester. The testing positions of the sample surface and cross section are shown in Fig. 3 and Fig. 4, respectively. According to the roughness of the test surface, the test force was 0.254 N, and the holding time is 15 s. The distance between the testing points is 30 \(\mu\text{m}\). The surface hardness \(H_T\) of textured surface is about 220 HV\(_{0.025}\). For the sample cross section, the results are the averages of five detection points at different positions on the unified horizontal line. Figure 4 shows that the hardness \(H_C\) of the material decreases gradually along the depth direction of the microtexture, and the maximum value...
is 290 HV$_{0.025}$, When the depth reaches 100 $\mu$m, the hardness is stable at about 200 HV$_{0.025}$, which is consistent with the surface hardness before processing the surface texture.

Similar to the hardness test, the elastic modulus is measured using an IMicro Nanoindenter. The continuous stiffness measurement method is adopted, and the target load is set to 0.25 N. The experimental results show that the elastic modulus of the nontextured surface is 198.3 GPa and that of the textured surface is 225.8 GPa.

**Normal contact stiffness of the remaining zone**

Through introducing the fractal dimension $D$ of rough surface profiles, the interaction between asperities is considered for the contact stiffness model on the basis of KE model (25). Based on the fractal model, the normal contact stiffness $K_0$ of joint surfaces can be given by (26, 27)

$$K_0 = \frac{\sqrt{6ED}}{4(1-D)} \psi^\frac{1}{D} a_0^\frac{1}{D} (a_l^\frac{1}{D} - a_c^\frac{1}{D})$$

where $E$ is the equivalent elastic modulus, $D$ is the fractal dimension of the rough surface profile, $\psi$ is the domain expansion factor of the contact size distribution of asperities, $a_l$ is the actual contact area of the largest asperity, and $a_c$ is the critical contact area.

According to Hertz contact theory, the equivalent elastic modulus $E$ can be given by (28)

$$E = \left(1 - \nu_a^2 \frac{E_u}{E} + 1 - \nu_l^2 \frac{E_l}{E}\right)^{-1}$$

where $\nu_a$ and $\nu_l$ are the Poisson’s ratio of the rough surfaces, and $E_u$ and $E_l$ are the elastic modules of the rough surfaces, respectively.

According to fractal contact theory, the critical contact area of elastoplastic deformation can be expressed as (29)

$$a_c = G^2 \left(\frac{2E}{HT}\right)^{\frac{1}{2/D}}$$

where $H_T$ is the surface hardness of the textured surface, and $G$ is the fractal roughness coefficient.

The relationship between $\psi$ and $D$ can be expressed as (30)

$$\psi^\frac{1}{D} - \left(1 + \psi^\frac{1}{D}\right) \frac{\nu}{D} = \frac{2 - D}{D}$$

For the given surface, the relationship between the critical contact area $a_c$ and the actual contact area $a_l$ of the largest asperity can be expressed as follows:

$$a_l^\frac{1}{D} = \lambda a_c^\frac{1}{D}$$

$\lambda = 100^{(1-D)/2}$

Therefore, Eq. [2] can be expressed as

$$K_0 = \frac{3\sqrt{3} (\lambda - 1) ED}{1 - D} \psi^\frac{1}{D} a_c^\frac{1}{D}$$

The normal contact stiffness is closely related to the micro-surface morphology and material properties. For the remaining zone, the surface morphology is almost not affected by surface texturing, and the surface hardness and elastic modulus remain unchanged. As a result, the normal contact stiffness $K_0$ before texturing can be obtained from the experiment, then the normal contact stiffness $K_1$ of the remaining zone can be expressed as follows:

$$K_1 = bA_rK_0$$

where $b$ is the area ratio coefficient of the remaining zone to apparent contact.

In order to obtain the ratio coefficient $b$, it is necessary to accurately obtain the area of textured zone. Nine sampled textures are selected by a cross-sampling method, as shown in Fig. 5. The sampled textures are selected with equal spacing in two directions perpendicular to each other. According to the data of surface topography, the bottom diameter of each texture can be obtained. It is assumed that the average value is the bottom diameter of all the textures, and the area of the textured zone can then be calculated, as well as the coefficient $b$.

**Fractal parameters $D$ and $G$**

According to fractal geometry theory, the rough surfaces of the specimens can be considered isotropic. The structure function method is used to calculate the fractal dimension of rough surface profiles. Incremental variance of a specific rough surface contour curve is defined as the structure function of the curve (31),

$$S(\tau) = \langle [Z(x + \tau) - Z(x)]^2 \rangle$$

| $S_0(\mu m)$ | 2.69 | 1.86 | 0.39 |
| $D$ | 3.22 | 1.34 | 1.49 |
| $G$ | 8.84e-9 | 2.68e-7 | 3.42e-6 |
where $Z(x)$ represents the height values of the contour, and $\tau$ is any interval of the sampling interval.

Using double logarithmic coordinates, the structure function $S(\tau)$ of the W-M function can be expressed as follows:

$$\lg S(\tau) = \lg \left( \frac{\Gamma(2D-3)\sin[(2D-3)\pi/2]}{(4-2D)\ln \gamma} \right) + 2(D-1)\lg G + 2(2-D)\lg \tau$$

Due to the equivalent contact of joint surfaces, the contour features of two rough surfaces are converted to the elastic rough surface. The equivalent fractal parameters $D$ and $G$ can be calculated by superimposing the structure functions of the contacted surfaces (27). Table 1 lists the equivalent fractal parameters $D$ and $G$.

**Normal contact stiffness of the textured zone**

Surface profiles of specimens are measured using a laser profilerometer, which can be used for three-dimensional measurement and quantitative analysis of rough surface profile at micrometer and submicrometer scale. The instrument uses a laser source and white-light source to obtain the height information and color information of specimens, respectively. According to the material, shape, and measurement range of the specimen, three different scanning principles, including laser confocal scanning, white light interference, and focusing change, can be selected for high-precision measurement. Then the morphology of each texture can be obtained. The three-dimensional model of texture is established based on the morphology of sample textures, as shown in Fig. 6.

As shown in Fig. 6(c), the texture can be regarded as the melting layer on the outside and the hump on the inside. For the melting layer, it is assumed that the height is $H$, the external radius and internal radius is $R_1$ and $R_2$ respectively,
and the curvature radius of the top is \( r \). For the hump, it is assumed that the height is \( h \), the diameter at the bottom is \( R_3 \), the curvature radius of the top is \( r_1 \), and the deformation of the texture is \( \delta \). Then the contact area \( A \) of textured zone can be given by

\[
A = \begin{cases} 
\pi \left( \frac{R_1 + R_2}{2} + \sqrt{2r \delta - \delta^2} \right)^2 - \pi \left( \frac{R_1 + R_2}{2} - \sqrt{2r \delta - \delta^2} \right)^2 & \delta \leq r \\
\pi \left( \frac{R_1 + R_2}{2} + l_1 \right)^2 - \pi \left( \frac{R_1 + R_2}{2} - l_1 \right)^2 & r \leq \delta \leq H - h \\
\pi \left( \frac{R_1 + R_2}{2} + l_1 \right)^2 - \pi \left( \frac{R_1 + R_2}{2} - l_1 \right)^2 + \pi R_3 \left[ \delta - (H - h) \right] & H - h \leq \delta \leq H - h + r_1 \\
\pi \left( R_1^2 - R_2^2 + R_3^2 \right) & \delta \geq H \\
\end{cases}
\]

where \( l_1 \) is the ring width of a melting layer and \( l_2 \) is the diameter of a hump.

\[
l_1 = 2r + \frac{2(\delta - r)}{h - r} \left( \frac{R_1 - R_2}{2} - r \right) \\
l_2 = 2r_1 + \frac{2(R_3 - r)}{h - r} \left( \delta - H + h + r_1 \right)
\]

The normal contact stiffness \( K_2 \) of textured zone can be expressed in the form

\[
K_2 = \frac{E_2 A}{A_n} 
\]

where \( E_2 \) is the elastic modulus of textured zone, \( A_n \) is the apparent contact area.

During processing the surface texturing, the material of the textured zone is melted and the material properties are changed. The elastic modulus \( E_2 \) of material in the textured zone can be given by

\[
E_2 = cE_0 
\]

where \( c \) is the variation coefficient of elastic modulus of material in the textured zone and \( E_0 \) is the elastic modulus of the remaining zone.

The normal contact stiffness \( K \) of the entire joint surfaces is synthesized by that of the remaining zone \( K_1 \) and textured zone \( K_2 \), which are juxtaposed. The textured zone is the first contact region under the normal load, and then the remaining zone. As a result, the normal contact stiffness \( K \) can be expressed as follows:

\[
K = \begin{cases} 
K_2 & \delta \leq H - H_a \\
K_1 + K_2 & \delta > H - H_a \\
\end{cases}
\]

where \( H \) is the height of the textures, \( \delta \) is deformation of texture, and \( H_a \) is the height of surface asperities.

Figure 7. Test specimens (unit, mm): (a) upper specimen, (b) lower specimen.
**Experiment**

**Specimens preparation**

The material of the specimens is R683/IC45e steel, $E = 198.3 \text{ GPa}$, $\nu = 0.3$, and dry joint surfaces are considered. Considering the previous research (32) and experimental conditions, the sample is designed, as shown in Fig. 7. The investigated joint surfaces are fabricated by milling. The apparent contact area $A_n$ is $412.3 \text{ mm}^2$, and the mass $m$ of the upper and lower specimens is $1.05 \text{ kg}$ in total. Considering the different processing accuracy, three kinds of specimens were processed. The surface roughness values $Sa$ are $2.69 \mu\text{m}$, $1.86 \mu\text{m}$, and $0.39 \mu\text{m}$, respectively. $Sa$ is the arithmetic mean or geometric mean of the distance between the points in the contour surface and the central plane. Before processing the surface texture, the surface of the lower specimen is polished in order to eliminate the surface defects, including burr, flash, and plowing. After that, the polished specimens are cleaned in an ultrasonic cleaner to remove the grinding debris. Then the microtextures in the form of circular...
Dimples are created on the surface of the lower specimen using a laser marking machine. The diameter of the microtextures is 100 μm and the surface density is 30%, which can effectively reduce wear (18).

During texturing, under the effect of recoil, some liquid and gaseous material will be washed out of the molten pool. Then part of the material falls onto the surface of the specimens or into the texture, which cools rapidly and forms slag. Since the slag seriously affects the contact characteristics, it should be removed. The textured surface is polished.

Table 2. The $R_{\text{New}}$ of specimens.

| $S_a$ (μm) | 0.39 | 1.68 | 2.69 |
|---|---|---|---|
| $R_{\text{New}}$ | 0.89 | 0.90 | 0.91 |

Figure 10. Comparison of theoretical predictions and experimental data for textured joint surface: (a) the textured sample with $S_a = 0.39$ μm, (b) the textured sample with $S_a = 1.68$ μm, and (c) the textured sample with $S_a = 2.69$ μm.

Figure 11. Experimental values of the textured specimen and original specimen: (a) the textured sample with $S_a = 2.69$ μm, (b) the textured sample with $S_a = 1.68$ μm, and (c) the textured sample with $S_a = 0.39$ μm.
with W10 metallographic sandpaper. There are still wear debris and metal slag on the surfaces of the specimens and inside the textures. Consequently, ultrasonic cleaning is used to remove these debris and slag. After that, liquid on the surfaces of the specimens is blown dry. Finally, the specimens are numbered and sealed in a clean container.

**Experimental setup**

The normal contact stiffness of samples is measured using a modal frequency method \((33)\). The test rig is shown in Fig. 8. The normal preload 50 N is applied using a nut and spring, with a loading range of 0–900 N. The steel balls can make the normal static load evenly distributed on the joint surface. The piezoelectric accelerometers are fixed on the upper specimen symmetrically, with a resolution of \(5 \times 10^{-4}\) g and an axial sensitivity of 10.67 mV/(m·s\(^{-2}\)). The impact hammer can provides about 800 N exciting force on the surface of upper specimen, with a range of 0–5000 N and a sensitivity of 4.24 pC/N. A schematic view of experiment principle is shown in the Fig. 9. The acceleration signal and force signal are input into a computer for processing using the Dynamic Analysis System. Based on the frequency response function of the joint surface can be obtained. The tests were repeated based on the frequency response function of the joint surface can be obtained. The tests were repeated three times, and the average value was taken as the final result. Before the experiment, the experimental system needs to be calibrated with standard samples. According to Eq. [14], the normal contact stiffness \(K_m\) of specimens can be given by

\[
K_m = m(2\pi f_n)^2/A_n
\]

where \(m\) is the mass of the upper and lower specimens in total, and \(f_n\) is the certain eigenmode frequency of the joint surface.

**Results and discussion**

According to Eqs. [7], [14], and [17], the function ExpDec1 was used to fit the experimental data of normal contact stiffness, as shown in Fig. 10. The values of \(R\) represent the correlation between experimental data and fitting curve, describing the fitting quality. As shown in Fig. 10, the parameters \(R\) are greater than 0.94, and the fitting function ExpDec1 is appropriate. For the textured specimens, the consistency between the theoretical predictions and the experimental data is judged using the parameter \(R_{New}\). \(R_{New}\) can be expressed as follows:

\[
R_{New} = 1 - (Q/\sum y^2)^{l/2}
\]

\[
Q = \sum (y - y^*)^2
\]

where \(y\) is the experimental value and \(y^*\) is the theoretical prediction value, and \(Q\) is the residual sum of squares.

The values of the parameter \(R_{New}\) for the specimens according to Eq. [18] are recorded in Table 2. Table 2 shows that the values of the parameter \(R_{New}\) are 0.89, 0.90, and 0.91, respectively. Therefore, the predicted values of model are consistent with the experimental results.

For the textured specimens with different surface roughness \(S_a\), the relationships of predictions of the proposed model and experimental data are shown in Fig. 10. As surface roughness decreases, the normal contact stiffness gradually increases for a fixed normal load. The reason is that the real contact area gradually increases as the roughness of rough surface decreases for the rough surfaces contact. The normal contact stiffness is closely related to the real contact area.

For the textured sample with \(S_a = 2.69\ \mu m\), when the normal load is less than 1.3 MPa, the surface texturing reduces the normal contact stiffness. Figure 11(a) shows that the difference first increases and then decreases with the increase of normal load. However, when the load is greater than 1.3 MPa, the surface texturing increases the normal contact stiffness. As the load increases, the increasing difference gets larger. The normal contact stiffness is increased by 15% when the load is 2.4 MPa, as shown in Fig. 11(a).

For the textured sample with \(S_a = 1.86\ \mu m\), when the normal load is less than 1.8 MPa, the surface texturing reduces the normal contact stiffness of the samples. It is similar to that of the textured sample with \(S_a = 2.69\ \mu m\). Then, when normal load continues to increase, the surface texturing on the contrary increases the normal contact stiffness. Compared with the textured sample with \(S_a = 2.69\ \mu m\), the growth difference is relatively slow. The normal contact stiffness is increased by 8% when the normal load is 2.4 MPa, as shown in Fig. 11(b).

For the textured sample with \(S_a = 0.39\ \mu m\), the surface texturing obviously reduces the normal contact stiffness. With increases of the normal load, the difference gradually increases. When the load is greater than 1.2 MPa, the normal contact stiffness is reduced by 20%, as shown in Fig. 11(c).

Essentially, the effect of surface texturing on the normal contact stiffness of the joint surfaces is that part of the original contact area is replaced by the textured zone. The average height of the textures is larger than that of the surface asperities. The textures first yield contact deformation under a normal load. As the amount of deformation gradually increases, the surface asperities also begin to yield contact deformation. Therefore, when the normal load is small, the textured zone is mainly involved in the contact deformation, which is equivalent to the textured zone replacing the entire joint surfaces. Thus, compared with the original specimens, the normal contact stiffness of textured specimens is smaller in the initial stage of contact deformation.

For the different joint surfaces, the real contact area gradually decreases with the increase of \(S_a\). However, for the textured zone, the contact area is constant. When \(S_a > 1.86\ \mu m\), the normal contact stiffness of the original sample is relatively small. For the textured zone, the normal contact stiffness is greater than that of the replaced original joint surfaces. Therefore, the overall contact stiffness is increased. Otherwise, when \(S_a\) is relatively small, the overall normal contact stiffness is reduced.
Conclusions
Considering the surface morphology and material properties, this research proposed the theoretical model for normal contact stiffness of the textured surfaces. An experimental platform was built to measure the normal contact stiffness. The conclusions are summarized as follows:

1. The theoretical predictions are consistent with experimental data through the parameter $R_{New}$. The research result shows that the theoretical model for normal contact stiffness of the textured surfaces is appropriate.

2. The effect of surface texturing on the normal contact stiffness is closely related to the surface roughness $Sa$. For joint surfaces with $Sa = 2.69 \, \mu m$, the normal contact stiffness is effectively increased through proper surface texturing. Measurements were only reported for three different roughness cases, and no measurements results are presented using roughness higher than 2.69 $\mu m$.

3. Compared with the original specimen, the normal contact stiffness of textured specimens is smaller in the initial stage of contact deformation.

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