Research Article

Multifuzzy Cubic Sets and Their Correlation Coefficients for Multicriteria Group Decision-Making

Jun Ye, Shigui Du, and Rui Yong

School of Civil and Environmental Engineering, Ningbo University, Ningbo 315211, China

Correspondence should be addressed to Jun Ye; yejun1@nbu.edu.cn

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The notion of multifuzzy sets (MFSs) or multi-interval-valued fuzzy sets (MIVFSs) provides a new method to represent some problems with a sequence of the different and/or same fuzzy/interval-valued fuzzy membership values of an element to the set. Then, a fuzzy cubic set (FCS) consists of a certain part (a fuzzy value) and an uncertain part (an interval-valued fuzzy value) but cannot represent hybrid information of both MFS and MIVFS. To adequately depict the opinion of several experts/decision-makers by using a union/sequence of the different and/or same fuzzy cubic values for an object assessed in group decision-making (GDM) problems, this paper proposes a multifuzzy cubic set (MFCS) notion as the conceptual extension of FCS to express the hybrid information of both MFS and MIVFS in the fuzzy setting of both uncertainty and certainty. Then, we propose three correlation coefficients of MFCSs and then introduce correlation coefficients of MFSs and MIVFSs as special cases of the three correlation coefficients of MFCSs. Further, the multicriteria GDM methods using three weighted correlation coefficients of MFCSs are developed under the environment of MFCSs, which contains the MFS and MIVFS GDM methods. Lastly, these multicriteria GDM methods are applied in an illustrative example on the selection problem of equipment suppliers; then their decision results and comparative analysis indicate that the developed GDM methods are more practicable and effective and reflect that either different correlation coefficients or different information expressions can also impact on the ranking of alternatives. Therefore, this study indicates the main contribution of the multifuzzy cubic information expression, correlation coefficients, and GDM methods in the multifuzzy setting of both uncertainty and certainty.

1. Introduction

Fuzzy set (FS) [1] is usually depicted by a membership degree in the interval [0, 1], where only indicate one occurrence of each element. Hence, FSs have been wildly applied in various areas [2–10] regarding vague and incomplete real problems. As the extension of FSs, Yager [11] and Sebastian and Ramakrishnan [12] presented a fuzzy multiset/bag/multifuzzy set (MFS), which provides a better method to represent the numbers of copies of an element to the set by the different and/or same membership values. Thus, fuzzy multisets/MFSs have been utilized for various applications [13–17]. By extending the fuzzy multiset/MFS to the interval-valued fuzzy set (IVFS) [18], Kreinovich and Sriboonchitta [19] defined the notion of a multi-interval-valued fuzzy set (MIVFS), where each element can be repeated more than once with the different and/or same interval-valued membership values to adequately describe a union/sequence of interval-valued fuzzy values (IVFVs).

Then, there exist the certainty and uncertainty of human judgments regarding complicated real-world problems. Hence, Jun et al. [20] proposed a notion of a fuzzy cubic set (FCS), which can depict the hybrid fuzzy information of the partial certainty (fuzzy value) and partial uncertainty (IVFV). Afterward, some researchers [21–24] also introduced some theory and applications of FCSs since FCS is a very useful tool for expressing potentially uncertain and certainty information provided by the decision-makers. Based on the hybrid notion of FCS and a hesitant FS [25], some researchers proposed hesitant FCSs and their decision-making methods [26, 27] due to the hesitancy of human judgments in the setting of FCSs; then some researchers
presented a cubic hesitant fuzzy set (CHFS) by means of a hybrid form of both an IVFV and several hesitant fuzzy values and after that developed similarity measures of CHFSs for medical diagnosis/assessments [28, 29] and decision-making [30]. More recently, Fu and Ye [31] proposed the notion of cubic hesitant neutrosophic numbers (CHNNs) by using a hybrid form of both an IVFV and several neutrosophic numbers and then developed the in-determinate parameter-based similarity measure of CHNNs for the risk grade assessment of prostate cancer patients. Next, Ye et al. [32] further proposed a notion of fuzzy credibility cubic numbers (FCCNs) by a hybrid form of both a cubic credibility degree and a fuzzy cubic value (FCV) and then developed two weighted aggregation operators of FCCNs and used them in the decision-making problem of slope design schemes so as to enhance the credibility degree/level of fuzzy decision-making problems. Based on the mixed form of an intuitionistic fuzzy set (IFS) and an interval-valued IFS (IVIFS), some researchers [33–35] presented cubic intuitionistic fuzzy sets (CIFSs) and their applications in decision-making problems. Although extension forms of various fuzzy information, such as Pythagorean FSs [36], interval-valued Pythagorean FSs [37], and neutrosophic sets [38], were introduced in recent years and applied in GDM problems, they lack corresponding cubic information expression forms.

However, the hesitant FS concept can only express a set of several different cubic membership values of each element to the set in the hesitant environment, but it cannot reflect the number of copies of an element to the set by a union/sequence of the different and/or same membership values so as to lose some useful information in the group decision-making (GDM) process. Generally, the aforementioned various fuzzy cubic concepts cannot also reflect the number of copies of an element to the set by a union/sequence of the different and/or same FCVs. Then, MFS or MIVFS can reflect a union of the different and/or same fuzzy values or IVFVs of an element to the set, but MFS or MIVFS cannot express the mixed information of both MFS and MIVFS. To suitably describe the union/sequence of the different and/or same FCVs, this study presents the concept of MFCS based on the hybrid information of MFS and MIVFS. The MFCS notion can provide a new expression form to overcome the insufficiency of the hesitant FCS and to extend the notion of FCS. Thus, MFCS can describe the opinions that several decision-makers propose by using a union/sequence of the different and/or same FCVs in GDM problems. Next, we propose three correlation coefficients of MFCSs and then introduce the correlation coefficients of MFSs and MIVFSs as special cases of the three correlation coefficients of MFCSs. Section 5 develops multicriteria GDM methods using the proposed weighted correlation coefficients of MFCSs in the MFCS setting. In Section 6, the developed GDM methods are used for an illustrative example on the selection problem of equipment suppliers, and then the comparative analysis of the developed multicriteria GDM methods and the related methods are investigated to illustrate the practicability and suitability of the developed GDM methods in the setting of MFCSs. Conclusions and further work are included in Section 7.

2. Some Notions of MFSs, MIVFSs, and FCSs

Set \( U = \{ u_1, u_2, \ldots, u_n \} \) as a universe set. Then, a MFS \( S \) on \( U \) is defined as the following form [11, 12]:

\[
S = \left\{ \left( u_j, M_S(u_j) \right) \mid u_j \in U \right\},
\]

(1)

where \( M_S(u_j) \) is the membership sequence denoted by \( (m_{S1}(u_j), m_{S2}(u_j), \ldots, m_{Sk}(u_j)) \) for \( m_{Sk}(u_j) \in [0, 1] \) \((k = 1, 2, \ldots, q(j); j = 1, 2, \ldots, n) \) and \( u_j \in U \), which is named a MFS/FMS. In the MFS \( S \), each element \( u_i \) in \( U \) may occur more than once with the different and/or same membership values.

To depict the fuzzy uncertain information in real-world problems, Kreinovich and Sibroonochnittha [19] put forward a MIVFS \( T \) on a universe set \( U = \{ u_1, u_2, \ldots, u_n \} \) and defined the following form:

\[
T = \left\{ \left( u_j, M_T(u_j) \right) \mid u_j \in U \right\},
\]

(2)

where \( M_T(u_j) \) is a union/sequence of the interval-valued membership values defined by \( (m_{T1}(u_j), m_{T2}(u_j), \ldots, m_{Tk}(u_j)) \) for \( m_{Tk}(u_j) \in [0, 1] \) \((k = 1, 2, \ldots, q(j); j = 1, 2, \ldots, n) \) and

In the original study, the main contributions are summarized in the following:

(i) The proposed MFCS can solve the hybrid information expression problem of both MFS and MIVFS and provide an adequate expression form to depict the opinion of several experts/decision-makers by using a union/sequence of several different and/or the same FCVs in multicriteria GDM problems

(ii) The proposed correlation coefficients of MFCSs can provide effective mathematical models for the multicriteria GDM methods in the environments of MFCSs, MFSs, and MIVFSs

(iii) The developed multifuzzy cubic GDM methods can solve GDM problems under the environment of MFCSs and show their superiority and usability in the setting of MFCSs.

For this study, this paper is composed of the following structures. Section 2 introduces some notions of MFSs, MIVFSs, and FCs. In Section 3, we present the concept of MFCS based on the hybrid information of MFS and MIVFS and define the relations of multifuzzy cubic values (MFCVs). Section 4 proposes three correlation coefficients of MFCSs and then introduces the correlation coefficients of MFSs and MIVFSs as special cases of the three correlation coefficients of MFCSs. Section 5 develops multicriteria GDM methods using the proposed weighted correlation coefficients of MFCSs in the MFCS setting. In Section 6, the developed GDM methods are used for an illustrative example on the selection problem of equipment suppliers, and then the comparative analysis of the developed multicriteria GDM methods and the related methods are investigated to illustrate the practicability and suitability of the developed GDM methods in the setting of MFCSs. Conclusions and further work are included in Section 7.
$u_j \in U$, which is named a MIVFS. In the MIVFS $T$, each element $u_j$ in $U$ may occur more than once with the different and/or same interval-value membership values.

To represent the fuzzy hybrid information of the uncertainty and the certainty in real-world problems, Jun et al. [20] put forward a FCS on a universe set $U = \{u_1, u_2, \ldots, u_n\}$, which is denoted by

$$C = \{\langle u_j, A_C(u_j), a_C(u_j) \rangle | u_j \in U\}$$

where $A_C(u_j) : U \rightarrow [0, 1]$ and $a_C(u_j) : U \rightarrow [0, 1]$ are an IFVF and a fuzzy value, respectively, such that $A_C(u_j) = \{a(u_j), b(u_j)\} \subseteq [0, 1]$ and $a_C(u_j) \in [0, 1]$ for $u_j \in U (j = 1, 2, \ldots, n)$. In the FCS $C$, each element $u_j$ in $U$ only occurs once with the hybrid information of both an interval-value membership value and a single-value membership value.

For the simplified expression, each element $\langle u_j, A_C(u_j), a_C(u_j) \rangle (j = 1, 2, \ldots, n)$ in $C$ is simply denoted as $c_j = \langle a_{uj}, b_{uj} \rangle$, $a_{uj}$ and $b_{uj}$ named FCV. If $a_{uj} \leq a_{uj} \leq b_{uj}$, then $c_j = \langle a_{uj}, b_{uj} \rangle$, and $\alpha_{uj}$ is named an internal FCV; if $a_{uj} \notin \{a_{uj}, b_{uj}\}$, then $c_j = \langle a_{uj}, b_{uj} \rangle$, and $\alpha_{uj}$ is named an external FCV.

For any two FCVs $c_{ij} = \langle a_{ij}, b_{ij}, a_{ij} \rangle$ and $c_{ij} = \langle a_{ij}, b_{ij}, a_{ij} \rangle$, their relations are introduced as follows [20]:

1. $c_{ij} \geq c_{ij} \Leftrightarrow [a_{ij}, b_{ij}] \subseteq [a_{ij}, b_{ij}]$ and $a_{ij} \geq a_{ij}$
2. $c_{ij} = c_{ij} \Leftrightarrow c_{ij} \geq c_{ij}$ and $c_{ij} \geq c_{ij}$
3. $c_{ij} \wedge c_{ij} = \langle [a_{ij} \wedge a_{ij} \wedge b_{ij} \vee a_{ij} \wedge a_{ij}] \rangle$
4. $c_{ij} \cap c_{ij} = \langle [a_{ij} \wedge a_{ij} \wedge a_{ij} \wedge a_{ij}] \rangle$
5. $c_{ij} = \langle \{1 - b_{ij}, 1 - a_{ij}, 1 - a_{ij}\} \rangle$ (complement of $c_{ij}$)

3. Multifuzzy Cubic Sets (MFCSs)

Based on the hybrid concept of MFS and MIVFS, this section presents the concept of MFCS as an extension of the FCS notion and defines the relations of MFCSs.

**Definition 1.** Set $U = \{u_1, u_2, \ldots, u_n\}$ as a universe set. A MFCS $M$ over $U$ is defined by the following mathematical expression:

$$M = \bigg\{ \begin{array}{c}
\langle [a_{M1}(u_j), b_{M1}(u_j)], a_{M1}(u_j) \rangle, \\
\langle [a_{M2}(u_j), b_{M2}(u_j)], a_{M2}(u_j) \rangle, \\
\vdots \\
\langle [a_{Mq}(u_j), b_{Mq}(u_j)], a_{Mq}(u_j) \rangle
\end{array} \bigg\} | u_j \in U$$

where $\{[a_{M1}(u_j), b_{M1}(u_j)], a_{M1}(u_j)\}, \{[a_{M2}(u_j), b_{M2}(u_j)], a_{M2}(u_j)\}, \ldots, \{[a_{Mq}(u_j), b_{Mq}(u_j)], a_{Mq}(u_j)\}$ is a membership sequence of q copies of an element $u_j$ to the set $M$, such that IFVFs $[a_{Mk}(u_j), b_{Mk}(u_j)] \subseteq [0, 1]$ and fuzzy values $\alpha_{Mk}(u_j) \in [0, 1] (j = 1, 2, \ldots, n; k = 1, 2, \ldots, q)$ for $u_j \in U$. In the MFCS $M$, each element $u_j$ may occur more than once with the different and/or same FCVs.

For example, there is a MFCS $M_1 = \{u_1, \{0.7, 0.9, 0.8\}, \{0.7, 0.9, 0.8\}, \{0.5, 0.7, 0.6\}, \{0.7, 0.8, 0.7\}, \{0.6, 0.7, 0.6\}\}$. If $a_{jk} = [a_{jk}, b_{jk}] (j = 1, 2, \ldots, n)$, $M_1$ is named the internal MFCS; in $M_1$, each element $u_{jk}$ may occur more than once with the different and/or same FCVs.

Each MFCV $\langle u_j, [a_{M1}(u_j), b_{M1}(u_j)], a_{M1}(u_j) \rangle$, $\langle [a_{M2}(u_j), b_{M2}(u_j)], a_{M2}(u_j) \rangle, \ldots, \{[a_{Mq}(u_j), b_{Mq}(u_j)], a_{Mq}(u_j)\}$ is named the external MFCS.

**Definition 2.** Set any two MFCSs as $m_{ij} = \{\langle a_{ij}, b_{ij}, a_{ij} \rangle, \langle [a_{ij}, b_{ij}, a_{ij}] \rangle, \langle [a_{ij}, b_{ij}, a_{ij}] \rangle, \ldots, \{[a_{ij}, b_{ij}, a_{ij}] \rangle \}$ and

$m_{ij} = \{\langle [a_{ij}, b_{ij}, a_{ij}] \rangle, \langle [a_{ij}, b_{ij}, a_{ij}] \rangle, \langle [a_{ij}, b_{ij}, a_{ij}] \rangle, \ldots, \{[a_{ij}, b_{ij}, a_{ij}] \rangle \}$ (j = 1, 2, \ldots, n). Then, their relations are defined as follows:

1. $m_{ij} \leq m_{ij} \Leftrightarrow [a_{ij}, b_{ij}, a_{ij}] \subseteq [a_{ij}, b_{ij}, a_{ij}]$ and $a_{ij} \leq a_{ij}$
2. $m_{ij} = m_{ij} \Leftrightarrow [a_{ij}, b_{ij}, a_{ij}] \subseteq [a_{ij}, b_{ij}, a_{ij}]$ and $a_{ij} = a_{ij}$
3. $m_{ij} \wedge m_{ij} = \langle [a_{ij} \wedge a_{ij} \wedge b_{ij} \vee a_{ij} \wedge a_{ij}] \rangle$
4. $m_{ij} \cap m_{ij} = \langle [a_{ij} \wedge a_{ij} \wedge a_{ij} \wedge a_{ij}] \rangle$
5. $m_{ij} = \langle \{1 - b_{ij}, 1 - a_{ij}, 1 - a_{ij}\} \rangle$ (complement of $m_{ij}$)

4. Correlation Coefficients of MFCSs

Three correlation coefficients of MFCSs and their special cases are proposed in this section under environments of MFCSs, MFSs, and MIVFSs.

**Definition 3.** Set $M_1 = \{m_{11}, m_{12}, \ldots, m_{1l}\}$ and $M_2 = \{m_{21}, m_{22}, \ldots, m_{2l}\}$ as two MFCSs, where $m_{ij} = \langle [a_{ij}, b_{ij}, a_{ij}] \rangle, \langle a_{ij} \wedge a_{ij} \wedge a_{ij} \wedge a_{ij} \rangle$ and $m_{ij} = \langle [a_{ij}, b_{ij}, a_{ij} \wedge a_{ij}], [a_{ij}, b_{ij}, a_{ij}], [a_{ij}, b_{ij}, a_{ij}], [a_{ij}, b_{ij}, a_{ij}] \rangle$.

Then, correlation coefficients between $M_1$ and $M_2$ are proposed as follows:
where

\[ N(M_1, M_2) = \sum_{j=1}^{n} \frac{q(j)}{m_1(j)} \frac{m_2(j)}{N(M_1, M_1)} \]

and

\[ P_2 \quad C_p(M_1, M_2) = C_p(M_n, M_n) \]

\[ P_3 \quad C_p(M_1, M_2) \in [0, 1] \]

**Proof.** It is obvious that the verification of the properties (P1) and (P2) in Proposition 1 is straightforward. Hence, one only verifies the property (P3).

Based on the Cauchy–Schwarz inequality \((\sum_{j=1}^{n} y_j^2) \times (\sum_{j=1}^{n} z_j^2) \geq (\sum_{j=1}^{n} y_j z_j)^2\) for any positive real numbers \(y_j\) and \(z_j\) \((j = 1, 2, \ldots, n)\), there is the inequality \(\sqrt{\sum_{j=1}^{n} y_j^2} \times \sqrt{\sum_{j=1}^{n} z_j^2} \geq \sum_{j=1}^{n} y_j z_j\). Hence, \(C_1(M_1, M_2) \in [0, 1]\) can hold based on the inequality. Then, it is obvious that there exist the following inequalities:

\[ C_2(M_1, M_2) \in [0, 1] \text{ and } C_3(M_1, M_2) \in [0, 1] \text{ can exist based on the above inequalities.} \]

Therefore, the verification of Proposition 1 is completed. If we consider the importance of MFCVs \(m_1\) and \(m_2\) \((j = 1, 2, \ldots, n)\) in \(M_1\) and \(M_2\), one can give the weight \(\beta_j \in [0, 1]\) of \(m_{ij}\)

\[ C_w(M_1, M_2) = \frac{N_w(M_1, M_2)}{\sqrt{N_w(M_1, M_1) N_w(M_2, M_2)}} = \frac{\sum_{j=1}^{n} \beta_j (a_{1jk} a_{2jk} + b_{1jk} b_{2jk} + \alpha_{1jk} \alpha_{2jk})}{\sqrt{\sum_{j=1}^{n} \beta_j (a_{1jk}^2 + b_{1jk}^2 + \alpha_{1jk}^2) \sqrt{\sum_{j=1}^{n} \beta_j (a_{2jk}^2 + b_{2jk}^2 + \alpha_{2jk}^2)}} \]

\[ C_w(M_1, M_2) = \frac{N_w(M_1, M_2)}{\max(N_w(M_1, M_1), N_w(M_2, M_2))} = \frac{\sum_{j=1}^{n} \beta_j (a_{1jk} a_{2jk} + b_{1jk} b_{2jk} + \alpha_{1jk} \alpha_{2jk})}{\max(\sum_{j=1}^{n} \beta_j (a_{1jk}^2 + b_{1jk}^2 + \alpha_{1jk}^2), \sum_{j=1}^{n} \beta_j (a_{2jk}^2 + b_{2jk}^2 + \alpha_{2jk}^2))} \]

\[ C_w(M_1, M_2) = \frac{\sum_{j=1}^{n} \beta_j (m_{ij} \wedge m_{ij})}{\sum_{j=1}^{n} \beta_j (m_{ij} \vee m_{ij})} = \frac{\sum_{j=1}^{n} \beta_j (a_{1jk} \wedge a_{2jk} + b_{1jk} \wedge b_{2jk} + \alpha_{1jk} \wedge \alpha_{2jk})}{\sum_{j=1}^{n} \beta_j (a_{1jk} \vee a_{2jk} + b_{1jk} \vee b_{2jk} + \alpha_{1jk} \vee \alpha_{2jk})} \]
Similarly, the weighted correlation coefficients $C_{wp}(M_1, M_2)$ ($p = 1, 2, 3$) also indicate the following proposition. 

**Proposition 2.** The weighted correlation coefficients $C_{wp}(M_1, M_2)$ ($p = 1, 2, 3$) contain the following properties:

1. **(P1)** $C_{wp}(M_1, M_2) = 1$ for $M_1 = M_2$
2. **(P2)** $C_{wp}(M_1, M_2) = C_{wp}(M_2, M_1)$
3. **(P3)** $C_{wp}(M_1, M_2) \in [0, 1]$

Similar to the verification of Proposition 1, the verification of Proposition 2 is straightforward. Hence, it is not repeated here.

**Remark 1**

(i) When there exist all $[a_{ijk}, b_{ijk}] = [a_{2jk}, b_{2jk}] = [0, 0]$ ($k = 1, 2, \ldots, q(j)$; $j = 1, 2, \ldots, n$) in $M_1$ and $M_2$, the weighted correlation coefficients of equations (7)–(9) are reduced to the following weighted correlation coefficients of MFSs:

(ii) When there exist all $\alpha_{ijk} = \alpha_{2jk} = 0$ ($k = 1, 2, \ldots, q(j)$; $j = 1, 2, \ldots, n$), the weighted correlation coefficients of equations (7)–(9) are reduced to the following weighted correlation coefficients of MIFVs:

(iii) Since the above-weighted correlation coefficients of MFSs and MFCs are special cases of the weighted correlation coefficients of MFCs, the three weighted correlation coefficients of MFCs are more general and more useful in actual applications.

5. **GDM Methods Using the Weighted Correlation Coefficients of MFCs**

This section develops multicriteria GDM methods using the weighted correlation coefficients in the environment of MFSs.

In the GDM process, there usually exists a set of alternatives $M = \{M_1, M_2, \ldots, M_r\}$ to be assessed by a set of criteria $E = \{e_1, e_2, \ldots, e_n\}$ regarding a multicriteria GDM problem. Then, a weight vector $\beta = (\beta_1, \beta_2, \ldots, \beta_n)$ is specified to consider the importance weights of various criteria $e_j$ ($j = 1, 2, \ldots, n$) in $E$. Due to certain and uncertain situations of decision-makers’ cognition regarding the different criteria, the satisfactory assessment values of the alternatives $M_j$ ($j = 1, 2, \ldots, s$) over the criteria $e_j$ ($j = 1, 2, \ldots, n$) are given by MFCVs $m_{ij} = <(a_{ij1}, a_{ij2}), (a_{ij2}, a_{ij3}), (a_{ij3}, a_{ij4}), \ldots, (a_{ijq(j)}, a_{ijq(j)})>$ ($j = 1, 2, \ldots, s$; $j = 1, 2, \ldots, n$) for $a_{ijk}, a_{2jk} \in [0, 1]$, and $a_{ijk} \in [0, 1]$ ($k = 1, 2, \ldots, q(j)$). Thus, all the assessed MFCVs can be constructed as the decision matrix of MFCs $M = (m_{ij})_{n \times s}$.

In the multicriteria GDM problem with the information of MFCs, the decision algorithm of the multicriteria GDM methods is presented by the following steps:
Step 1: regarding the decision matrix of MFCSs
\( M = (m_{ij})_{n \times m} \) one first yields an ideal alternative \( M^* = \{m^*_1, m^*_2, \ldots, m^*_n\} \) by the following formula:

\[
m^*_j = \left( \left( \left( a^*_{i1}, b^*_{i1} \right), a^*_{i2}, \ldots, \left( a^*_{ij}, b^*_{ij} \right), \ldots, \left( a^*_{ij(q)}, b^*_{ij(q)} \right) \right), \right) \times \left( \max \left( \max \left( a_{i1}, a_{i2}, \ldots, \max (a_{ij}), \ldots, \max (a_{ij(q)}) \right) \right), \right)
\]

\[
= \left( \left( \max \left( \max \left( a_{i1}, a_{i2}, \ldots, \max (a_{ij}), \ldots, \max (a_{ij(q)}) \right) \right), \right) \times \left( \max \left( \max \left( a_{i1}, a_{i2}, \ldots, \max (a_{ij}), \ldots, \max (a_{ij(q)}) \right) \right), \right)
\]

\[
\ldots
\]

\[
= \left( \max \left( \max \left( a_{i1}, a_{i2}, \ldots, \max (a_{ij}), \ldots, \max (a_{ij(q)}) \right) \right), \right) \times \left( \max \left( \max \left( a_{i1}, a_{i2}, \ldots, \max (a_{ij}), \ldots, \max (a_{ij(q)}) \right) \right), \right)
\]

(16)

Step 2: the values of the weighted correlation coefficient are obtained by using one of the three weighted

\[
C_w(M, M^*) = \frac{\sum^n_{j=1} \beta_j \sum^{q(j)}_{k=1} (a_{ijk}a_{jik}^* + b_{ijk} + a_{jk}a_{jik}^*)}{\sqrt{\sum^n_{j=1} \beta_j \sum^{q(j)}_{k=1} (a_{ijk}^*)^2 + (b_{ijk})^2 + (a_{jk}^*)^2}}
\]

(17)

\[
C_w(M, M^*) = \frac{\sum^n_{j=1} \beta_j \sum^{q(j)}_{k=1} (a_{ijk}a_{jik}^* + b_{ijk} + a_{jk}a_{jik}^*)}{\sqrt{\sum^n_{j=1} \beta_j \sum^{q(j)}_{k=1} (a_{ijk})^2 + (b_{ijk})^2 + (a_{jk})^2}}
\]

(18)

\[
C_w(M, M^*) = \frac{\sum^n_{j=1} \beta_j \sum^{q(j)}_{k=1} (a_{ijk} + b_{ijk} + a_{ijk})}{\sum^n_{j=1} \beta_j \sum^{q(j)}_{k=1} (a_{ijk})^2 + (b_{ijk})^2 + (a_{ijk})^2}
\]

(19)

Step 3: ranking of the alternatives is given in descending order of the values of the weighted correlation coefficient and the best one is selected.

Step 4: end.

6. Illustrative Example and Comparative Analysis

6.1. GDM Example on the Selection Problem of Equipment Suppliers. A manufacturing company wants to enhance the competitiveness of products and to enlarge the production capacity, the company plans to replace an existing equipment supplier to improve the product quality. Thus, the decision department has to select four potential suppliers preliminarily as a set of four alternatives \( M = \{M_1, M_2, M_3, M_4\} \). Then, the company must make a decision by the suitable assessment of

the four alternatives over the requirements of three criteria: cost \((c_1)\), quality \((c_2)\), and supplier’s reputation \((c_3)\). The importance of the three criteria is taken into account by the weight vector \( \beta = (0.35, 0.4, 0.25) \). Thus, the company invites three experts/decision-makers to give the assessment values of the four alternatives over the three criteria based on the partial certain and partial uncertain judgments of their cognition, which are presented in the form of MFCSs \( m_{ij} = \langle[a_{ij1}, b_{ij1}, a_{ij2}, b_{ij2}, a_{ij3}, b_{ij3}] \rangle \) for \( [a_{ijk}, b_{ijk}] \subseteq [0, 1] \) and \( a_{ijk} \in [0, 1] \) (\( i = 1, 2, 3; \ j = 1, 2, 3, 4 \)). Thus, all the assessed MFCSs can be constructed as the decision matrix of MFCSs \( M = (m_{ij})_{4 \times 3} \) in Table 1.

By applying equation (16) for the decision matrix of Table 1, we first give the ideal alternative:

\( M^* = \{m^*_1, m^*_2, m^*_3, m^*_4\} = \langle((0.8, 0.9), 0.9), ((0.8, 0.9), 0.9), ((0.8, 0.9), 0.9), \rangle \)
two ideal alternatives are yielded in the following:

\[ C_{\text{MIFS}}(\mathbf{M}_i, \mathbf{M}_j) \text{ and } C_{\text{MIVFS}}(\mathbf{M}_i, \mathbf{M}_j) \] are identical in the GDM example, if there exist either all \( a_{ijk} = 0 \) or all \( b_{ijk} = 0 \) (i = 1, 2, 3 and j = 1, 2, 3, 4). In the decision matrix of MAFSs, the decision matrix \( \mathbf{M} = (m_{ijk})_{4 \times 3} \) is reduced to the decision matrices of MFSs or MIVFSs:

\[
\mathbf{M}_{\text{MIFS}} = \begin{bmatrix}
M_{\text{MIFS}_1} \\
M_{\text{MIFS}_2} \\
M_{\text{MIFS}_3} \\
M_{\text{MIFS}_4}
\end{bmatrix} = \begin{bmatrix}
(0.7, 0.7, 0.6) & (0.8, 0.7, 0.7) & (0.9, 0.8, 0.7) \\
(0.8, 0.8, 0.7) & (0.9, 0.8, 0.7) & (0.7, 0.7, 0.7) \\
(0.7, 0.7, 0.6) & (0.75, 0.7, 0.7) & (0.8, 0.7, 0.7) \\
(0.9, 0.8, 0.7) & (0.8, 0.7, 0.6) & (0.9, 0.8, 0.8)
\end{bmatrix}
\]

\[
\mathbf{M}_{\text{MIVFS}} = \begin{bmatrix}
M_{\text{MIVFS}_1} \\
M_{\text{MIVFS}_2} \\
M_{\text{MIVFS}_3} \\
M_{\text{MIVFS}_4}
\end{bmatrix} = \begin{bmatrix}
(0.7, 0.8, 0.7, 0.6) & (0.7, 0.9) & (0.7, 0.8) & (0.8, 0.9), (0.8, 0.9), (0.7, 0.8) \\
(0.8, 0.9, 0.7, 0.8) & (0.8, 0.9, 0.8, 0.9) & (0.7, 0.9) & (0.7, 0.8, 0.7, 0.7) \\
(0.7, 0.8, 0.6, 0.7) & (0.7, 0.85, 0.7, 0.8) & (0.6, 0.8) & (0.7, 0.7, 0.8, 0.7) \\
(0.7, 0.8, 0.9, 0.7, 0.8) & (0.7, 0.8, 0.6, 0.8) & (0.5, 0.7) & (0.8, 0.95), (0.8, 0.9), (0.7, 0.9)
\end{bmatrix}
\]

Based on the decision matrices of MFSs and MIVFSs, the two ideal alternatives are yielded in the following:

\[
M^*_1 = \{m^*_1, m^*_2, m^*_3\} = \{(0.9, 0.9, 0.9), (0.9, 0.9, 0.9), (0.9, 0.9, 0.9)\}
\]

\[
M^*_2 = \{m^*_1, m^*_2, m^*_3\} = \{(0.8, 0.9, 0.8, 0.9), (0.8, 0.9, 0.8, 0.9), (0.8, 0.9, 0.8, 0.9), (0.8, 0.9, 0.8, 0.9)\}
\]

Thus, by applying equations (10)–(15), we obtain all decision results of the weighted correlation coefficients of MFSs and MIVFSs, which are shown in Table 3.

In the decision results of Table 3, the ranking orders of the four alternatives based on different weighted correlation coefficients under different environments of MFSs and MIVFSs indicate different ranking results, and then the best alternative is \( M_2 \) or \( M_3 \) or \( M_4 \).

Regarding the decision results of Tables 2 and 3, it is clear that the developed multifuzzy cubic GDM methods contain the multifuzzy and multi-interval-valued fuzzy GDM methods under the specific environments of MFSs and MIVFSs. However, their ranking orders and the best ones show the partial same and partial difference based on different weighted correlation coefficients under different environments of MFCs, MFSs, and MIVFSs, which show that either different weighted correlation coefficients or different information environments can impact on the ranking orders.
of alternatives in the GDM example. Since MFCs consist of both MFSs and MIVFSs, the multifuzzy and multi-interval-valued GDM methods introduced in this study are only special cases of the multifuzzy cubic DM methods. Therefore, the developed multifuzzy cubic DM methods are more general and more reasonable in the information expressions and decision usability, which show their highlighting advantages in the setting of MFCs.

Since hesitant cubic FSs [26, 27], CHFSs [28–30], CHNNs [31], FCCNs [32], and cubic IFSs [33–35] cannot express the MFS information, their decision-making methods [26–35] cannot also carry out the GDM problems in the setting of MFCs. In the situations, it is obvious that the developed multifuzzy cubic GDM methods also indicate their superiority and usability in the setting of MFCs.

### 7. Conclusion

Regarding the hybrid expression problem of MFS and MIVFS in the multifuzzy setting of uncertainty and certainty, this paper presented a MFCs concept as the extension of FCS in order to adequately depict the opinion of several experts/decision-makers by using a union/sequence of the same and/or different FCVs. Then, we proposed three correlation coefficients of MFCs and after that introduced the correlation coefficients of MFSs and MIVFS as the special cases of the three correlation coefficients of MFCs. Further, multicriteria GDM methods using the three weighted correlation coefficients of MFCs were developed under the environment of MFCs, which contain the multifuzzy and multi-interval-valued fuzzy GDM methods. Lastly, these multicriteria GDM methods were applied in an illustrative example on the selection problem of equipment suppliers; then their decision results and comparative analysis indicated that the developed GDM methods are more practicable and effective and reflected that either different correlation coefficients or different information expressions can impact on the ranking orders of alternatives.

As future work, we shall extend this original study to clustering analysis, pattern recognition, and image processing under the environment of MFCs and further develop new similarity measures of MFCs for GDM, medical diagnosis, pattern recognition, and so on.

### Data Availability

No data were used to support the results in this study.

### Conflicts of Interest

The authors declare no conflicts of interest.

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