The effects of tidally induced disc structure on white dwarf accretion in intermediate polars

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ABSTRACT

We investigate the effects of tidally induced asymmetric disc structure on accretion onto the white dwarf in intermediate polars. Using numerical simulation, we show that it is possible for tidally induced spiral waves to propagate sufficiently far into the disc of an intermediate polar that accretion onto the central white dwarf could be modulated as a result. We suggest that accretion from the resulting asymmetric inner disc may contribute to the observed X-ray and optical periodicities in the light curves of these systems. In contrast to the stream-fed accretion model for these periodicities, the tidal picture predicts that modulation can exist even for systems with weaker magnetic fields where the magnetospheric radius is smaller than the radius of periastron of the mass transfer stream. We also predict that additional periodic components should exist in the emission from low mass ratio intermediate polars displaying superhumps.

Key words: accretion, accretion discs — instabilities — hydrodynamics — magnetic fields — binaries: close — novae, cataclysmic variables.

1 INTRODUCTION

In this paper we investigate the effects of tidally induced non-axisymmetric disc structure on white dwarf accretion in intermediate polars.

The largest tidal response in a close binary accretion disc is to the $m = 2$ component of the binary potential. The response takes the form of a pair of trailing spiral density waves. Excited at the outer edge of the disc where tidal forces are strongest, these waves generally steepen to become shocks as they extend inwards through the disc, and lead to a net outwards flux of angular momentum (Goldreich & Tremaine 1979; Sawada, Matsuda & Hachisu 1986). However in the cool discs of cataclysmic variables, where the Mach number of the Keplerian flow is probably $M = v_\phi/c \gtrsim 25$, a variety of analytic and numerical arguments suggest that the efficiency of angular momentum transport is low, with a Shakura-Sunyaev (1973) $\alpha \sim 0.01$ or lower (Spruit 1987; Papaloizou & Lin 1994; Papaloizou & Lin 1995 and references therein). In contrast, modelling of dwarf novae outbursts suggests that high mass transfer cataclysmic variables have an $\alpha$ twenty to thirty times higher than this (Cannizzo 1993). The observation of a two armed spiral in the disc of the outbursting dwarf nova IP Peg (Steeghs, Harlaftis & Horne 1997) does not appear to alter this general conclusion (Armitage & Murray 1998).

In this paper, we suggest that spiral waves in close binary could have interesting observational consequences even if they are unable to account for the bulk of the disc angular momentum transport. The presence of spiral waves breaks the axisymmetry of the inner disc and tells the accreting star the orbital phase of its companion, resulting in an additional orbital period dependent variation in the accretion rate onto the white dwarf. The amplitude of such variations will depend on the strength of the spiral waves. In the presence of turbulent fluid motions or vertical temperature gradients in the disc, we expect that waves will be damped (Lubow & Ogilvie 1998), perhaps strongly. Intermediate polars, where the inner disc radius is large and the disc’s thermal structure is modified by irradiation, may therefore be the best place to search for these effects.

For systems with mass ratios $q \gtrsim 0.25$, which encompasses most intermediate polars, the tidally induced spiral structure is fixed in the binary frame. In binaries with a more extreme mass ratio, $q \lesssim 0.25$, an additional complication is the possibility of resonance between the binary orbit and gas orbits in the outer disc. This can force the disc into an eccentric precessing state (Whitehurst 1988, Hirose & Osaki 1990, Lubow 1991, Murray 1998), observationally identified with the superhumps seen in SU UMa stars in superoutburst, and in some short period novalikes. We also investigate the effects of this resonant tidal disc structure on accretion in intermediate polars.

In section 2 we briefly summarise the periodicities that
are found in the optical and X-ray light curves of intermediate polars, and comment on the strengths and weaknesses of the stream overflow model. In section 3 we describe the details of our numerical method. Simulations of tidally induced spiral waves are shown in section 4. A simulation of superhumps in an intermediate polar is outlined in section 5, and our conclusions are presented in section 6.

2 PERIODICITIES IN THE LIGHT CURVES OF INTERMEDIATE POLARS

In intermediate polar systems, the magnetic field of the white dwarf is sufficiently strong as to truncate or even completely disrupt the surrounding accretion disc, but not so strong as to synchronise the spin of the white dwarf with the rotation of the binary. In those systems where it is not completely disrupted, the accretion disc is typically thought to extend down to radii \( r_{\text{in}} \approx 8 - 12 \ r_{\text{wd}} \). Inwards of this radius, the motion of the gas is dominated by the magnetic field.

For comprehensive reviews of the state of knowledge regarding intermediate polars, the reader is referred to Patterson (1994) and Warner (1995). Of particular interest here is the range of periodicities that are apparent in both the optical and X-ray emission from these systems. For example, Buckley & Tuohy (1989) measured the orbital period and white dwarf spin period of TX Columbae to be 5.72 ± 0.07 hours and 1911 ± 2 seconds respectively. They found the optical emission to be modulated at frequencies \( \omega_{\text{spin}} - \Omega_{\text{orb}} \), \( \omega_{\text{spin}} - 2 \Omega_{\text{orb}} \), and \( \omega_{\text{spin}} + \Omega_{\text{orb}} \), and the X-ray emission to be modulated at frequencies \( \omega_{\text{spin}} \) and \( \omega_{\text{spin}} - \Omega_{\text{orb}} \). Norton et al. (1997) found the relative strengths of the X-ray periodicities changed over a time scale of one year. In 1994 they observed TX Col with ASCA and found the strongest modulation of the X-ray emission occurred at \( \omega_{\text{spin}} \) and \( \Omega_{\text{orb}} \).

Using ROSAT one year later, they found the system also displayed modulation at \( \omega_{\text{spin}} - \Omega_{\text{orb}} \) and \( \omega_{\text{spin}} - 2 \Omega_{\text{orb}} \). The X-ray light curve of FO Aquarui is similarly variable (Beardmore et al. 1998).

The presence of these different frequencies in the light curve provides clues as to the mode of accretion. Modulation at a frequency of \( \omega_{\text{spin}} \) implies that accretion onto the white dwarf occurs at a roughly fixed location on the surface, most likely due to misaligned spin and magnetic axes, while the presence of a frequency \( \Omega_{\text{orb}} \) implies structure fixed in the rotating binary frame. The mixed frequencies such as \( \omega_{\text{spin}} - \Omega_{\text{orb}} \) then suggest that there is asymmetry in the inner disc, where the flow is becoming magnetically dominated by the white dwarf field, that is aware of the phase of the binary companion.

The stream-fed accretion model provides one way of creating these conditions. In this model there are two components to the accretion onto the white dwarf; flow from the inner edge of an axisymmetric disc, and direct accretion from an overflowing stream that carries some fraction of the mass transfer from the inner Lagrange point. The stream trajectory is essentially ballistic and fixed in the binary frame, so the accretion rate from this component, \( \dot{m}_s \), will be modulated at the beat between the spin period of the white dwarf and the orbital period of the binary. Norton et al. (1997) proposed such a model to explain their observations of TX Col, and similar ideas have been suggested both for EX Hya in outburst (Hellier et al. 1989), and to explain the empirically based 10 : 1 ratio between orbital and spin periods (Warner & Wickramasinghe 1991, King & Lasota 1991). There are convincing observational (Marsh & Horne 1989), analytical (Lubow 1989) and numerical (Armitage & Livio 1998; Blondin 1998) arguments supporting the basic idea that a significant fraction of the gas stream from the secondary skims over the inner edge of the disc only to impact upon the disc plane at smaller radii.

Despite this, the circumstances in which stream overflow can give rise to modulated accretion are limited. In this model, the radius of periastron, \( r_{\text{per}} \), achieved by a ballistic particle emerging from the inner Lagrange point, is the minimum radius to which an overflowing stream can penetrate. Any encounter with the disc, either at the hot spot or along the disc surface, will result in gas from the stream being entrained in the disc flow at radii \( r > r_{\text{per}} \). Therefore if the magnetospheric radius \( r_{\text{m}} < r_{\text{per}} \), accretion is expected to occur from an axisymmetric inner disc. Conversely, if \( r_{\text{m}} \geq r_{\text{per}} \), then it is unclear whether a disc could have formed in the first place, though once formed (perhaps when the accretion rate was high and \( r_{\text{m}} \) was smaller) it might well be able to survive. The long term behaviour of systems in this regime is, frankly, extremely murky, especially when the possibility of significant torques applied at the inner boundary is included (King & Lasota 1991). However, assuming that there is a combination of disc and stream fed accretion, the clear prediction of the stream fed model is that the modulation should be strongest when the accretion rate is low. In this state the magnetospheric radius is largest, and the fraction of material that overflows in the stream is probably enhanced (Armitage & Livio 1998), both of which favour a larger pulsed fraction of the emission at the orbital side band period \( \omega - \Omega_{\text{orb}} \). At least in EX Hya, the observed periodicity was most apparent when the system was in outburst (Hellier et al. 1989) and \( r_{\text{m}} \) would likely be at a minimum.

In summary then, it appears necessary for the adequate explanation of the observed modulations in intermediate polar light curves, that accretion onto the white dwarf be modulated by the rotational motion of the white dwarf \textit{with respect to the binary frame}. Simple orbital variation of the net system luminosity due to asymmetries in the reprocessed radiation from sites such as the secondary, and the disc bright spot, cannot explain all the frequencies seen in these systems. Although direct accretion from the \( L_1 \) stream would naturally give a modulated rate of accretion, there are arguments to suggest that this is not the whole picture.

3 NUMERICAL METHOD

The two and three dimensional simulations described in this paper were completed using a Smooth Particle Hydrodynamics (SPH) code that has been described in detail elsewhere (Murray 1996a). Other calculations made using the code can be found in Murray (1998), Armitage & Murray (1998), and Murray & Armitage (1998). The SPH technique is comprehensively reviewed in Monaghan (1992).

Murray (1996b), and Yukawa, Boffin & Sawada (1997) presented SPH calculations of spiral shocks in close binary systems.
accretion discs. Previously it had been thought that the artificial shock technique used for improving shock capture in SPH simulations was too dispersive for meaningful results to be obtained in shearing discs. In fact, previous SPH simulations that had failed to show nonlinear spiral waves (e.g. Murray 1996a), had done so because high values for the Mach number $M$ were used, and the $m = 2$ tidal response of the disc was tightly wrapped and rapidly damped. Simulations shown in the next section will reinforce this point.

SPH codes incorporate a variety of different viscous terms in the equations of motion. These terms are generally used both to ensure stable resolution of shocks, and to model the anomalously high shear viscosity that occurs in discs. The simulations described here follow Meglicki, Wickramasinghe & Bicknell (1993) and Murray (1996a), and use a dissipation term based upon the linear artificial viscosity term described in Monaghan (1992). However in the shearing flow of an accretion disc it is appropriate that viscous forces be enabled for receding particles as well as for approaching particles, so the viscous switch is disabled. In the continuum limit, the introduction of such a term is equivalent to introducing shear and bulk viscosity, in a fixed ratio, into the fluid equation of motion (see equation 3, Murray 1998).

Flebbe et al. (1994) and Watkins et al. (1996) described terms that introduce independent shear and bulk viscosities into the equations of motion. Kunze, Speith & Riffert (1997) describe several two dimensional simulations made with the Flebbe et al. formulation. As has been previously pointed out, the forces these terms introduce between particles are not antisymmetric and along the particles’ line of centres, so angular momentum is not conserved at the particle level. In fact, Riffert et al. (1995) show that, with the Flebbe et al. term, total angular angular momentum is only approximately conserved (i.e. to second order in the smoothing length).

In their disc simulations Yukawa et al. incorporated both the linear ($\alpha$) and nonlinear ($\beta$) SPH artificial viscosity terms, exactly as described in Monaghan (1992). Tests show that the nonlinear term is often necessary to prevent particle interpenetration in high Mach number shocks, particularly with an isothermal equation of state (e.g. Tate 1995). We have found that in disc situations, the linear term is sufficient for adequate shock capture. Furthermore, in simulations we completed with a non-zero $\beta$, many particles from the outer regions of the disc were flung to very large radii. As a result the calculations slowed down enormously, and large numbers of particles either escaped the system or were returned to the secondary. We found that the use of the nonlinear viscous term lead to a degraded treatment of disc boundaries, and to less reliable estimates of accretion rates on to the central object.

In these simulations, masses are scaled to the total system mass, $M = M_1 + M_2$, lengths are scaled to the interstellar separation, $d$, and times are scaled to the reciprocal of the orbital angular velocity, $\Omega_{\text{orb}}^{-1}$. We choose the centre of mass of the binary to be the origin of our inertial coordinate system.

4 SPIRAL WAVE SIMULATIONS

In this section we describe simulations that show that it is possible for tidally induced spiral waves to propagate sufficiently far into the disc of an intermediate polar that the white dwarf accretion may be modulated on the beat period between the white dwarf spin and binary orbital periods.

As we are interested in the propagation of spiral structure to small radii, we elected to complete several high resolution two dimensional simulations as opposed to lower resolution calculations in three dimensions. We use a constant smoothing length $h = 0.005 d$, where $d$ is the stellar separation.

A Keplerian disc with a surface density profile $\Sigma = \text{constant}$ was constructed by laying down particles on concentric rings $\frac{1}{3} h$ apart, out to a radius $r = 0.395 d$. Particle spacing within each ring was also $\frac{1}{3} h$. Each simulation began with 74445 particles. Particles were then added at the $L_1$ point in the manner described in Murray (1998), at a rate of one particle per 0.05 $\Omega_{\text{orb}}^{-1}$. We take a representative value for the binary mass ratio of $q = 0.5$.

Present theoretical uncertainty precludes any attempt to model the details of the interaction between the inner disc and the magnetic field of the white dwarf. We simply use an open boundary condition, so that at every time step particles with a radius $r < r_\text{in}$ are removed from the calculation (as are particles that either return to the secondary or that are further from the primary than is the $L_1$ point). This boundary condition neglects the influence of magnetic torques on the disc exterior to $r_\text{in}$, which may be important in some systems (Wynn & King 1995).

An $r^{-3/4}$ temperature profile is imposed upon the disc by requiring that the sound speed

$$c = c_0 \left( \frac{r}{r_0} \right)^{-3/8},$$

(1)

where $c_0$ is the sound speed at some arbitrary reference radius $r_0$, which we take as $r_0 = 0.5 d$.

With the parameters given above, the Mach number of the Keplerian flow,

$$M(r) \approx 1.06 \, c_0^{-1} \, r^{-1/8}.$$  

(2)

We complete simulations for $c_0 = 0.02, 0.05, 0.10 \, d \Omega_{\text{orb}}$, giving Mach numbers of 60, 24 and 12 respectively at the approximate outer edge of the disc, $r = 0.4 d$. For a Shakura-Sunyaev disc solution (Shakura & Sunyaev 1973), $M \approx 25$ is roughly appropriate for a CV disc. We note that the Yukawa et al. calculations assumed a hotter disc, with $M \approx 5$.

We have previously shown (Murray 1996a) that the SPH linear artificial viscosity term generates a shear viscosity equivalent to that of a disc with Shakura-Sunyaev viscosity parameter

$$\alpha(r) = \frac{1}{8} \, \zeta \, \Omega(r) \, c^{-1} \, h.$$  

(3)

Here $\zeta$ is the SPH artificial viscosity parameter. Although the discs described here are noticeably asymmetric, the velocity divergence is only significant in the outer regions ($r \gtrsim 0.3 d$). In these outer regions the SPH artificial viscosity also generates a bulk viscosity which cannot be represented in terms of an $\alpha$ parameter. With $\zeta = 1.0$, and the parameter values listed above,
Figure 1. The dependence of spiral structure upon disc temperature and upon the radius of the inner boundary \( r_{in} \). Grey scale density maps for six disc calculations at time \( t = 20.00 \, \Omega_{orb}^{-1} \). The same density scale is used in all frames. The SPH artificial viscosity parameter \( \zeta = 1.0 \) in all simulations. \( M_1 = 1.02M_\odot \), \( M_2 = 0.5M_\odot \), \( r_{in} = 0.05 \, d \) in the simulations on the left hand side, and \( r_{in} = 0.10 \, d \) in the right hand calculations. The sound speed is \( c_0 (2r)^{-3/8} \), where \( c_0 = 0.02 \, d \Omega_{orb} \) in the top two simulations, \( c_0 = 0.05 \, d \Omega_{orb} \) in the middle two panels and \( c_0 = 0.10 \, d \Omega_{orb} \) for the bottom two calculations. The Roche lobe of the primary is shown as a solid line in each panel. Coordinates (marked on both axes) are centred on the binary centre of mass and are scaled to the interstellar separation \( d \).
Figure 2. For each of the six discs shown in Fig. 1, the surface density along the line $y = 0$ (the binary axis) at time $t = 20.00 \Omega_{\text{orb}}^{-1}$ is plotted. The same length (horizontal axis) and density (vertical axis) scalings are used in each panel. The position is measured with respect to the centre of mass of the white dwarf ($x_{\text{wd}}$).
\[ \alpha(r) \approx 0.13 \frac{\bar{h}}{c_0} r^{-9/8}. \]  

(4)

For sound speed \( c_0 = 0.05 \, d\Omega_{orb} \), \( \alpha(0.4) \approx 0.04 \) and \( \alpha(0.1) \approx 0.2 \).

As we have already mentioned, mass transfer from the secondary stars was included in the simulations. However the computational time required for these calculations is so large that it was not feasible to evolve each disc to steady state. Instead we ran each calculation to a point where we were certain that the discs’ tidal structure was stationary i.e. until the location and pitch angle of the spiral arms had stabilised. This was largely the case by \( t = 20 \, \Omega_{orb}^{-1} \), although most simulations were actually run for 60 or 100 \( \Omega_{orb}^{-1} \).

The surface density of six two dimensional simulations at time \( t = 20 \, \Omega_{orb}^{-1} \) are shown in Fig. 1. Each panel shows the disc and the primary’s Roche lobe in a frame that co-rotates with the binary such that the origin is at the binary’s centre of mass, and the primary lies along the positive x-axis. The grey scale maps economically demonstrate a point that is already well known; that in cool viscous discs (top two panels) the spiral response is weak and restricted to the outermost regions. For discs of intermediate Mach number (and value of \( \alpha \), tightly wrapped spiral waves extend down to radii \( r < 0.2d \) before being completely damped. For yet higher temperatures (\( c_0 = 0.10 \, d\Omega_{orb} \)), the spiral waves can easily penetrate all the way to the inner edge of the disc.

Simulations were completed with \( r_{in} = 0.05 \, d \) (left hand panels), and with \( r_{in} = 0.10 \, d \). Fig. 2 clearly shows that the position of the inner boundary has minimal effect upon the disc’s tidal structure.

Fig. 3 shows the surface density along the line \( y = 0 \) for each of the density maps shown in Fig. 1. We caution that although the tidal structure of each disc was stationary, the disc masses have not reached stable values. Rather more accretion and radial spreading has occurred in the hottest discs, and so the density in these simulations is reduced with respect to the cooler calculations. However it is clear that except for the coolest disc, the spiral waves maintain significant overdensities down to close to the inner boundary.

We also wish to compare the two spiral waves within a given simulation. Should the two waves differ significantly in strength in the inner disc, then we would expect matter to be accreted preferentially from the stronger of the two waves. The relative strengths of the resulting periodic luminosity variations would be different than if the arms had equal strength. Although Fig. 2 reveals the two arms to be significantly different in the outer disc, in the inner disc the arms are very similar in strength and in the location at which they intersect the x-axis. In other words, these simulations do not reveal any significant differences between the two spiral waves in the inner regions of the disc. However more detailed simulations may be required to definitively answer this question.

The open inner boundary strongly influences the simulations in an annulus \( r_{in} < r < r_{in} + 0.05 \, d \). The effective inner boundary of the calculations should be taken at \( r \simeq r_{in} + 0.05 \, d \). Fig. 3 shows again, that whilst tidal structure is restricted to the outer regions in the coolest discs, both the intermediate temperature and hot disc simulations show spiral structure extending down to radii \( r \simeq 0.20 \, d \). The key result is that, in contrast to stream overflow, there is no minimum radius below which spiral structure is forbidden.

The degree to which spiral waves affect flow in the inner regions depends upon the temperature, and upon the rate at which the waves are damped. We will return to this point in the discussion section.

5 SUPERHUMPS

The majority of intermediate polars lie above the period gap, implying their mass ratios are too large for the eccentric resonance to be populated by disc material. There are some exceptions however. Most notably EX Hya has an orbital period \( P_{orb} = 0.068234 \) days, and an estimated mass ratio \( q \leq 1/6 \) (Hellier et al. 1987). One would expect disc precession to occur in this system. Superhumps have in fact been observed in the intermediate polar candidate, VZ Pyxidis (Kato & Nogami, 1997). Most recently, the X-ray source RX J0757.0+6306 has been identified as an intermediate polar with orbital and white dwarf spin periods of \( 81.5 \pm 5 \) and \( 8.52 \pm 0.15 \) minutes respectively (Tovmassian et al. 1998).

To investigate the properties of superhumps in intermediate polars we completed a three dimensional SPH simulation of a disc in a binary with mass ratio \( q = 3/17 \). To set up an eccentric precessing disc we began the calculation from zero disc mass, and added single particles at regular time intervals \( (\Delta t = 0.01 \, \Omega_{orb}^{-1}) \), in circular orbit at the circularisation radius. The smoothing length was kept fixed at \( h = 0.02 \, d \), and the sound speed \( c = 0.02 \, d\Omega_{orb} \). In fact with these values, the smoothing length was everywhere greater than the pressure scale height \( H \), so that the vertical structure was not resolved and the calculation was in effect two dimensional. Neighbour numbers averaged 160 for the calculation.

As in the previous section, an open inner boundary condition was used. Initially we set the radius of the inner boundary \( r_{in} = 0.02 \, d \). We then followed the simulation until the disc had encountered the resonance and subsequently reached an eccentric equilibrium state. The disc precession period agreed with the results of previous two dimensional simulations (Murray 1996a, 1998).

We set our SPH artificial viscosity parameter \( \zeta = 1.0 \). For the three dimensional SPH code,

\[ \nu = \frac{1}{10} \bar{c} \, \bar{h}, \]  

(5)

so for this isothermal run the disc had a shear viscosity \( \nu \) that is constant with radius, i.e. the Shakura-Sunyaev parameter \( \alpha \propto r^{-3/2} \).

At time \( t = 644.00 \, \Omega_{orb}^{-1} \), the radius of the inner boundary was adjusted to a value more appropriate to an intermediate polar. Particles lying interior to the new inner boundary, \( r_{in} = 0.10 \, d \), were immediately rejected from the simulation. The calculation restarted with 18094 particles in the disc. At this stage we changed the mode of mass addition. Particles were now added (at the same rate as before) at the L1 point, rather than at \( r_{circ} \). In the initial calculation mass was added at \( r_{circ} \) so that the disc could rapidly reach an equilibrium with the resonance. But adding mass in the inner disc also artificially symmetrises the flow there. Any dependance on the disc precession would thus be washed out of the accretion from the disc’s inner edge. The run was then continued for a further 366.00 \( \Omega_{orb}^{-1} \).

Fig. 4 shows the disc mass as a function of time for
the “intermediate polar” section of the simulation. The disc took approximately 100 \( \Omega_{\text{orb}}^{-1} \) to accommodate the new inner boundary condition and the new mode of mass addition. A minimum of 16799 particles was reached at time \( t = 771.95 \) \( \Omega_{\text{orb}}^{-1} \). Thereafter the disc slowly accumulated mass, so that by the end of the calculation it contained 16955 particles.

No particles were removed from the simulation as a result of either escaping to large radii or returning to the secondary, and mass addition occurred at a constant rate. Therefore the rate of accretion onto the white dwarf can be directly determined from the change in the disc mass.

After removing the trend and the shortest time scale variability, we took the Fourier transform of the disc mass time series to reveal an underlying periodicity in the white dwarf accretion rate. The power spectrum (Fig. 3) has only one significant peak, at 45 cycles, which corresponds to a period \( P_{\text{acc}} = 1.08 \pm 0.02 \) \( P_{\text{orb}} \). When folded on this period, the disc mass shows a noisy but highly significant periodicity (Fig. 5). The disc precession period obtained from the simulation light curve \( P_{\text{sh}} = 1.07 \pm 0.02 \) \( P_{\text{orb}} \). Thus we found accretion from the inner edge of the truncated disc was modulated on the period of the disc’s motion as measured in the binary frame. In the eccentric disc model for superhumps this is the superhump period, \( P_{\text{sh}} \).

What are the observational consequences of this result? In Murray (1998) it was demonstrated that the modulated component of a precessing disc’s luminosity emerged from a disc region that was fixed in the binary frame. On this basis we similarly expect that the modulated component of the accretion comes from a point in the disc that is fixed in the binary frame. Any emission resultant from the accretion onto the white dwarf will be modulated on the beat between the white dwarf spin as measured in the binary frame, and the superhump period. Thus we expect to see modulations in the X-ray emission from such systems at frequencies

\[
(\omega_{\text{spin}} - \Omega_{\text{orb}}) - \omega_{\text{sh}} \simeq \omega_{\text{spin}}(1 - \delta),
\]

and

\[
(\omega_{\text{spin}} - \Omega_{\text{orb}}) + \omega_{\text{sh}} \simeq \omega_{\text{spin}}(1 - \delta),
\]

where \( \delta \ll 1 \).

6 DISCUSSION

In this paper we have presented simulations of spiral structure in the accretion discs of intermediate polars. Our results suggest that the observed periodic emission in intermediate polars could be caused by the interaction of the white dwarf magnetic field with asymmetric structure in the accretion disc induced by the tidal field of the binary. In contrast to the stream overflow model for these periodicities, the tidal model predicts that modulation can occur even if \( r_m < r_{\text{per}} \).
Table 1. Methods of introducing various frequencies in the light curves of magnetic cataclysmic variables.

| Mechanism                              | Expected Frequency          |
|----------------------------------------|----------------------------|
| Accretion in Magnetic CVs with Discs   |                            |
| from two spiral arms to one pole       | $\omega_{\text{spin}} - 2 \Omega_{\text{orb}}$ |
| from two spiral arms to two poles      | $2 \omega_{\text{spin}} - 2 \Omega_{\text{orb}}$ |
| from eccentric disc onto one pole      | $\omega_{\text{spin}} - \Omega_{\text{orb}} \pm \Omega_{\text{sph}}$ |
| Accretion in Discless Magnetic CVs     |                            |
| from stream to one pole                | $\omega_{\text{spin}} - \Omega_{\text{orb}}$ |
| from stream to two poles               | $2 \omega_{\text{spin}} - \Omega_{\text{orb}}$ |
| Radiation from one pole reprocessed    |                            |
| at disc hot spot                       | $\omega_{\text{spin}} - \Omega_{\text{orb}}$ |
| by spiral arms                         | $\omega_{\text{spin}} - 2 \Omega_{\text{orb}}$ |

and that frequencies related to the superhump period should be present in low mass ratio systems ($q \lesssim 0.25$) displaying superhumps.

In Table 1 we have summarised the frequencies that each mechanism would be expected to introduce into a magnetic cataclysmic variable’s light curve. In addition, accretion from a disc is expected at some level to lead to modulation at the spin period $\omega_{\text{spin}}$. Wynn & King (1992) have demonstrated that the X-ray power spectrum generated by stream accretion is strongly dependent upon the configuration of the white dwarf’s magnetic field, and upon the observer’s inclination to the binary orbital plane. Therefore we caution against using this table in isolation to determine the mode of accretion occurring in a particular system. However, consider the recently discovered intermediate polar RX J1238-38. Optical photometry by Buckley et al. (1998) revealed periodicities at 1860 and 2147 seconds, “with similar amplitudes of $\approx 8\%$”. Spectroscopy showed emission lines varied on the longer of the two periods, indicating that the white dwarf spin period was 2147 s. Further analysis of the emission lines revealed evidence of a longer period at $5300^{+1260}_{-850}$ s. Buckley et al. found the 1860 second period was consistent with a frequency $2(\omega_{\text{spin}} - \Omega_{\text{orb}})$. Now in the stream-accretion picture such a frequency is not expected to feature prominently in the power spectrum. On the other hand, accretion from a tidally distorted disc would naturally produce this periodicity.

Of necessity, we have had to gloss over the uncertain details of the coupling between the white dwarf magnetic field and the inner disc. This could be an important omission, especially for white dwarfs whose spin rate and magnetic field are such that the coupling exerts a positive torque on the inner disc immediately outside the magnetospheric radius. By analogy with the results of Artymowicz & Lubow (1996) for circumbinary discs (where the torques are gravitational), such a situation could lead to an additional source of asymmetry in the inner disc, and associated modulation of the accretion rate.

We chose to simplify the thermodynamics, by prescribing the radial temperature profile of each disc. Whilst this assumption influences the detail of the simulations (e.g. larger density contrasts can occur across our essentially isothermal shocks than if we had used an adiabatic equation of state), it does not take away from the general result that the spiral modes are stronger in hotter discs. Our results are consistent with the numerical and analytic work of other authors (see e.g. the simulations using a polytropic equation of state completed by Różycka and Spruit 1989), though the possible effects of vertical temperature gradients (Lubow & Ogilvie 1998) have not been addressed systematically in any simulations to date.

Fig. 1 shows that an increase in the disc temperature results in a strengthening of the spiral modes. This would suggest an increase in the modulated component of the accretion onto the white dwarf. On the other hand, an increase in disc temperature implies an increase in the mass flux through the disc $M_\text{d}$. Under these circumstances we would expect $r_\text{in}$ to decrease which in turn would make it more difficult for the spiral waves to penetrate right to the disc’s inner edge. Worse, the damping due to turbulence in the disc is also likely to vary with the accretion rate. Assessing the relative importance of these effects is difficult. However it is natural to expect the modulated fraction to change. Thus changes in disc temperature, as may be caused by changes in the mass flux from the secondary, could explain the finding of Norton et al. (1997) that the relative strengths of the periodicities in the light curve of TX Col changed over the time scale of one year.

Theoretical work suggests that the development of observable spiral structure that extends over a large range in radii depends on the disc Mach number, the presence of vertical temperature gradients, and the strength of any turbulent viscosity (Savonije, Papaloizou & Lin 1994; Godon 1997; Lubow & Ogilvie 1998). Doppler tomography of a variety of systems can potentially help in understanding these mechanisms. For example, spiral waves that were present in the inner disc of quiescent dwarf novae, but not present in outburst, would suggest that damping due to turbulence was the dominant process, the converse would imply that the Mach number was of greater import. Further calculations are also required in order to quantify the relative importance of these effects.

Recently, Steeghs et al. (1997) used Doppler tomographic techniques to identify spiral structure in the accretion disc of IP Peg down to radii $r \approx 0.28d$. Similar observational identification of spiral structure in intermediate polar systems would provide strong support for the scenario presented here.

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