Do intradot electron-electron interactions induce dephasing?

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We investigate the degree of coherence of electronic transport through a quantum dot (QD) in the presence of an intradot electron-electron interaction. By using an open multi-terminal Aharonov-Bohm (AB) setup, we find that the intradot interaction does not induce any dephasing effect and the electron transport through the QD is fully coherent. We also observe that the asymmetric amplitude of the AB oscillation in the conductance through the two-terminal AB setup originates from the interplay between the confined structure and the electron-electron interaction. Thus, one can not associate a dephasing process with this asymmetric amplitude, as has been done in previous studies.

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How electron-electron (e-e) interactions influence the phase coherence of electronic transport through mesoscopic systems, e.g. quantum dots (QD), has been one of the most significant and challenging issues from fundamental physics point of view as well as for realization of quantum devices. By embedding a QD in one arm of an Aharonov-Bohm (AB) interferometer, it has been experimentally demonstrated that the transport through the QD is at least partially coherent despite the existence of a strong intradot e-e interaction.1,2,3 Naturally, one may ask further about the coherence rate, i.e. how much coherence is maintained in such a tunneling process? Recent theoretical studies4,5,6 addressed this question and arrived at the conclusion that an intradot e-e interaction will induce partial dephasing. According to these studies, the simple reason for the partial incoherence is from the spin-flip process.4,5,6 In this process, for instance, a spin-up electron enters the QD and a spin-down electron exits, whereas the spin in the QD gets flipped. In other words, the traversing electron has left a trace in the QD (i.e. the “environment”), causing dephasing. However, this simple intuitive spin-flip picture is not quite transparent, as mentioned by the same authors4,5,6. For instance, one possible drawback in the argument given above is that only one electron in the leads is involved, i.e. neglecting of many-body features in the leads. Another shortcoming is that one has to artificially divide the successive tunneling process into a series of second-order processes. The tunneling process is a continuous one and it is not clear that such a division is a proper procedure. In order to further clarify the question of the dephase by the e-e interaction, König and Gefen2,6 have analyzed the transport behavior of the two-terminal AB interferometer embedded with a QD. They found that the amplitude of AB oscillations in the conductance is suppressed compared with the non-interacting case. Furthermore, they predicted that, as a consequence of dephasing, an asymmetry would appear around resonant peaks. This prediction has been observed in a recent experiment,7 lending a strong support to the belief of interaction induced dephasing.

However, we question whether the two-terminal AB setup is a proper geometry to study the dephasing effect. Since in this confined and closed structure, the repeated reflection and tunneling processes are plentiful. Is it possible that it is this confinement, not the dephasing that suppresses the amplitude of AB oscillations and makes them asymmetric? It is known that, due to the confinement, the two-terminal AB setup has the phase locking effect1,2,8,9 and the phase of the transmission amplitude can not be determined. In this Letter, for the first time, we investigate the coherence issue in an open multi-terminal AB setup. We find that the intradot e-e interaction does not induce any dephasing effect and the electronic transport through the QD maintains fully coherence. To a larger extent, our results will shed light on the issue of low temperature saturation of phase coherence time10 observed in many solid-state samples11.

We consider an open multi-terminal AB setup (see Fig.1), mimic experimental configurations1,2,3,7. Four extra leads are attached to four sites and a QD is embedded in the lower arm. In general, the extra leads may introduce dephasing effects into the system as demonstrated by Buttiker.12 In order to avoid the dephasing by the extra leads, we keep their voltages to be the same as that for the drain (i.e. \( V_i = V_d = 0 \), with \( i = 1, 2, 3, \) and 4). Electrons will only inject from the source into the AB ring, and leave from the drain and the other four extra leads. In this way, due to the current bypass effect, the processes of repeated reflection and tunneling are strongly suppressed, and the first-order tunneling process dominates.

In order to quantitatively describe the phase coherence, we introduce the coherence rate parameter \( r_T(\epsilon) \):
\(r_T(\epsilon) \equiv \frac{T_1(\epsilon)}{2\sqrt{T_{tr}T_{te}(\epsilon)}}\), where \(T_1(\epsilon)\) is the first-order AB oscillation amplitude of the transmission probability \(T(\epsilon)\) from source to drain, \(T_{tr} = |t_{tr}|^2\) and \(T_{td}\) are transmission probabilities through the reference and the QD, respectively. \(T_d = T_{co} + T_{in}\), where \(T_{co} = |t_{co}|^2\) and \(T_{in}\) are the coherent and incoherent components of the transmission probability through the QD. If we assume only the first-order tunneling process exist, then \(T_1 = 2|t_{tr}t_{co}|\) and \(r_T\) reduces to:

\[
r_T = \frac{|t_{co}|}{\sqrt{|t_{co}|^2 + T_{in}}}. \tag{1}
\]

Obviously, in this case the value of \(r_T\) directly reflects the degree of coherence. If \(r_T = 1\), it is fully coherent; on the other hand, if \(r_T = 0\), it is completely incoherent. Of course, when the higher-order tunneling processes are not negligible, Eq.(1) is no longer valid. In this case, \(r_T\) does not reflect the degree of coherence. In this work we design a system (i.e. open multi-terminal AB setup) in which the first-order tunneling process dominates, and we carry out a study of the coherence rate \(r_T\) in such a system.

The entire system of the AB interferometer considered here (see Fig.1) is modelled by the Hamiltonian

\[
H = H_0 + H_R + H_T. \tag{2}
\]

Here \(H_0 = \sum_{\alpha,k,\sigma} \epsilon_{\alpha k} c_{\alpha k}^+ c_{\alpha k}\sigma\) describes the non-interacting electrons in the source, drain, and four extra leads with \(\alpha = s, d, \) and \(1, \cdots, 4\), respectively. The Hamiltonian of the isolated AB ring, composed of the QD and four sites, is represented by:

\[
H_R = \sum_{\beta,\sigma} \epsilon_{\beta \sigma} d_{\beta \sigma}^+ d_{\beta \sigma} + U n_{d\uparrow} n_{d\downarrow} + \sum_{\sigma} (t_e \epsilon_{\uparrow} d_{2\uparrow}^+ d_{3\sigma} + t_1 d_{1\sigma}^+ d_{2\sigma} + t_4 d_{4\downarrow}^+ d_{3\sigma} + H.c.))\]

where \(d_{\beta \sigma}\) are the creation (annihilation) operators in the four sites \((\beta = 1, \cdots, 4)\), or in the QD \((\beta = 3)\). The QD includes a single energy level \(\epsilon_{d\sigma}\) having spin index \(\sigma\) and an intradot e-e interaction \(U\). To account for the system threaded by a magnetic flux \(\Phi\), a phase factor \(e^{i\phi}\) with \(\phi = 2\pi \Phi / \Phi_0\) is attached to the hopping matrix element \(t_r\) through the reference arm. The last term in Eq. (2),

\[
H_T = (\sum_{\alpha,\beta,k,\sigma} t_{\alpha \beta} c_{\alpha k\sigma}^+ d_{\beta \sigma} + \sum_{\gamma,k,\sigma} t_{\gamma \gamma} c_{\gamma k\sigma}^+ d_{\gamma \sigma} + H.c.),
\]

describes the tunneling between the AB ring and the leads. Here \(t_{\alpha \beta}\) represents the coupling matrix elements between lead \(\alpha = s (d)\) and site \(\beta = 1, 2, 3, 4\), while \(t_{\gamma \gamma}\) denotes that between the \(\gamma\)th extra lead and the \(\gamma\)th site with \(\gamma = 1, \cdots, 4\).

By using the standard Keldysh non-equilibrium Green’s function method,[14] the conductance of the drain current \(I_D\) versus the source voltage \(V_s\) can be derived as:

\[
G \equiv \frac{dI_D}{dV_s} = -\frac{e}{h} \sum_{n,m,\sigma} \text{Im} \int \frac{d\epsilon}{2\pi} \Gamma_{nm}^d d\epsilon [G_{mn}^{\text{<}} + 2f_{d}(\epsilon)G_{mn}^{\text{r}}], \tag{3}
\]

where the coupling strength \(\Gamma_{nm}^d = 2\pi \sum_k t_{dn}^* t_{dm} \delta(\epsilon - \epsilon_{dk})\), and \(f_{d}(\epsilon)\) is the Fermi distribution function of the drain/source. \(G_{mn}^{\text{<}}(\epsilon)\) and \(G_{mn}^{\text{r}}(\epsilon)\) are the standard retarded and Keldysh Green’s functions.[13] They are 5×5 matrices and the index \(n, m = 1, \cdots, 4\) for the corresponding sites and \(n, m = 5\) for the QD. We solve the Green’s functions by the following procedures. First, the isolated QD Green’s functions is exactly obtained: \(G_{55} = [\epsilon - \epsilon_{d\sigma} - U + Un] / [(\epsilon - \epsilon_{d\sigma}) (\epsilon - \epsilon_{d\sigma} - U)]\). Second, using the Dyson equation \(G' = g' + G' \Sigma' g'\) and the Keldysh equation \(G^\text{<} = G^\text{r} \Sigma^\text{<} G^\text{r}\), the Green’s functions \(G^\text{<}\) and \(G^\text{r}\) of the whole system can be derived. [16] As the last step, \(n_{\sigma}\), the intradot electron occupation number for spin state \(\sigma\), is solved self-consistently with the self-consistent equation \(n_{\sigma} = -i \int \frac{d\epsilon}{2\pi} G_{55}(\epsilon)\).

In order to study the degree of coherence of electronic transport through an interacting QD, we numerically study the linear conductance and the coherence rate. In the numerical calculations, we choose a very weak \(t_r = 0.001\) and low temperature \(k_B T = 0.01\). The four sites’ energy levels are chosen to be \(\epsilon_1 = -\epsilon_2 = \epsilon_3 = -\epsilon_4 = 2\). [17] We also set \(\Gamma_{11} = \Gamma_{22} = \Gamma_{33} = \Gamma_{44} = \Gamma = 10\) as the energy unit, and \(\Gamma_{11} = \Gamma_{22} = \Gamma_{33} = \Gamma_{44} = \Gamma\).

Here \(\Gamma_{11} = \sum_k |t_{\gamma \gamma}|^2 \delta(\epsilon - \epsilon_{r_k})\) describe the coupling strength between the extra leads and the corresponding sites. A larger \(\Gamma\) gives a stronger coupling, and enhances the current bypass effect, thus, the first-order tunneling process dominates. In the limit case of \(\Gamma \rightarrow \infty\), only first-order tunneling process survives.

We first investigate the spin-degenerate case with no magnetic field in the QD. However, we are still under AB configuration with non-zero magnetic flux \(\Phi\) passing through the AB ring. The total linear conductance \(G\) versus the QD’s level \(\epsilon_d\) for the open AB setup exhibits two Coulomb oscillation peaks at \(\epsilon_d = 0\) and \(-U\) (see Fig.2a). At a fixed \(\epsilon_d\), \(G\) versus the magnetic flux \(\Phi\) shows periodic oscillations with a period of \(2\pi\) (see Fig.2b). Due to this periodic oscillations, \(G(\Phi)\) can be expanded in a Fourier series: \(G(\Phi) = G_0 + G_1 \cos(\phi + \varphi_1) + G_2 \cos(2\phi + \varphi_2) + \cdots\), where \(G_1\) is the first-order amplitude of AB oscillations. Since \(t_r\) is chosen as a small parameter, \(G_1 \propto t_r\). When the AB setup is decoupled with the reference (or the QD), i.e. when \(t_r = 0\) (or \(t_1 = t_4 = 0\)), the conductance \(G_1\) (or \(G_{\text{ref}}\)) through the QD (or the reference arm) can also be obtained. This enable us to define an experimental measurable conductance coherence rate \(r_G(\epsilon_d) = G_1 / 2 \sqrt{G_{\text{ref}} G_d}\). In the low temperature limit \(T \rightarrow 0\), \(r_G\) is equivalent to \(r_T(\epsilon_f)\) since \(G_1/\epsilon_d = \int \frac{d\epsilon}{2\pi} T_{1/\epsilon_d} (\epsilon_f - \epsilon_f)\). We keep the temperature sufficiently low \((k_B T\) much smaller than the width of the intradot level), so \(r_G\) is very close to \(r_T\).

The coherence rate \(r_{T_G}\), the conductance \(G_d\), and the first-order amplitude \(G_1\) versus the QD’s level \(\epsilon_d\) for different values of \(U\) are shown in Fig.2(c),(e). \(G_0\) or \(G_1\) exhibits a peak at \(\epsilon_d = 0\) regardless of the value of \(U\).

When \(U = 0\), this peak is symmetric, but at \(U \neq 0\), it is slightly asymmetric. Now we focus on the study of coherence rate \(r_{G_1}\). First, far away from the resonance
peak, i.e. in the co-tunneling regime, $r_G$ is almost equal to 1 for both $U = 0$ and $U \neq 0$. This result implies that the traversing electron keeps coherence in this regime. Second, in the proximity of the peak, i.e. in the resonant tunneling regime, $r_G > 1$. This shows that the higher-order reflecting and tunneling processes still exist and are not negligible in the resonance regime despite of the open AB setup with large rate value of $\Gamma/\Gamma^{sd} = 5$. Under the circumstance, $r_T$ (or $r_G$) can not be used to describe the degree of coherence as mentioned before.

Therefore the degree of coherence in the resonant regime has to be further studied. In the following we investigate this question by using two methods. (i) With increasing $\Gamma/\Gamma^{sd}$, the bypass effect is enhanced. As $\Gamma/\Gamma^{sd} \to \infty$, the higher-order processes completely disappear, left with only the first-order process. In this limit, we find that $r_T \to 1$, regardless of $U$ and $\epsilon_d$ (see Fig.3a,b). (ii) At finite value of $\Gamma/\Gamma^{sd}$, although the higher-order and the first-order processes all exist, we can distinguish them in our calculation, because that the first-order process has the factor $\Gamma^1$, while the higher-order processes has the factors $(\Gamma^*)^2\Gamma^d$, $\Gamma^e(\Gamma^d)^2$, and so on. So we can withdraw the part of the contribution of the first-order process in the transmission probability (or the conductance). And they are: $T_1 = 2[\Gamma_{12}^d\Gamma_{34}^d\Gamma_{13}^d\Gamma_{24}^d]$, $T_{ref} = \Gamma_{22}^d\Gamma_{33}^d|G_{d2}^d|^2$, and $T_d = \Gamma_{11}^d\Gamma_{44}^d|G_{d4}^d|^2$, where $G$ is the Green’s function when $\Gamma_{mn}^d = 0$. Following the coherent rate parameter of only the contribution of the first-order process, $r_T = T_1/2\sqrt{T_{ref}T_d}$ is exact 1 regardless of the values for $U$, $\epsilon_d$ and $\Gamma$. Any one of the results of (i) and (ii) clearly demonstrates that the electron transport through the QD is fully coherent, and intradot e-e interaction does not induce any incoherent effect! In order to check the reliability of the conclusion reached above, we have purposely introduced a dephasing source (e.g. virtual Buttiker’s voltage lead) to the QD, we find that $r_T$ for both (i) and (ii) are indeed less than 1 due to the dephasing effect.

Next, we turn to investigate the crossover of $r_G$ in going from the open multi-terminal AB interferometer to the closed two-terminal setup (see Fig.3a,b). As $\Gamma/\Gamma^{sd} \to \infty$, the setup is completely open and $r_T = 1$. With decreasing of $\Gamma/\Gamma^{sd}$, a peak emerges at $\epsilon_d = 0$. This peak goes up initially and goes down as $\Gamma/\Gamma^{sd}$ is further reduced. Eventually a valley emerges at $\epsilon_d = 0$, and the bottom of the valley reaches to zero as $\Gamma/\Gamma^{sd} \to 0$ (i.e. a completely closed AB setup). We emphasize that $r_G < 1$ (or even $r_G \approx 0$) for a closed two-terminal setup does not imply the occurrence of incoherence. For example, for $U = 0$, it is well known that the electron transport through the QD is fully coherent, but $r_G \approx 0$ still in the vicinity of $\epsilon_d = 0$. This clearly means that the reduction of $r_G$ is from the existence of the higher-order reflecting and tunneling processes due to the constraint of the two-terminal interferometer. For the case of $U = 0$, $r_G$ has a similar behavior as for the $U = 0$ case. The only difference between them is that the valley is asymmetric for $U = \infty$ and symmetric for $U = 0$. Similarly, one can also not conclude the appearance of incoherence at $U \neq 0$ from $r_G < 1$ or asymmetry in $r_G$. It only shows that the closed two-terminal AB interferometer is not a suitable setup to quantitatively study the dephasing effect due to the constraint structure, just as it is not a suitable geometry to study the phase of the transmission amplitude.

The same conclusion is reached by studying the amplitude $G_1$, in the two-terminal AB system. Fig.3c and 3d show our results of $G_1$ versus $\epsilon_d$ for $U = 0$ and $U = \infty$. $G_1$ exhibits two peaks around $\epsilon_d = 0$. Those two peaks are symmetric for $U = 0$, and asymmetric for $U = \infty$ (or a finite $U$). Those behavior are consistent with the previous theoretical and experimental findings. However, we emphasize again that those results (including the asymmetric peaks at $U \neq 0$) are the consequences of the confined structures.

Finally, let us turn to the case of applying a magnetic field $B$ to the QD. Now $\epsilon_d \neq \epsilon_d$, due to the Zeeman splitting. Fig.2f shows the results of $G_{d1}$, $G_{d1}$, and $r_G$ for $U = \infty$ at $\Delta \epsilon_d = \epsilon_d - \epsilon_d = 4$. Both $G_{d1}$ and $G_{d1}$ have similar results with the zero magnetic field case, except that the peaks are slightly suppressed. However, $r_G$ is obviously smaller than 1 (see Fig.2f). Even in the limit $\Gamma/\Gamma^{sd} \to \infty$, $r_G$ still is less than 1. Does this mean that the electron transport through the QD is partially dephased by $U$ and $B$? To address this question, we first consider the simple model with $U = 0$ and $\Gamma/\Gamma^{sd} \to \infty$. While $U = 0$, the electron transport through the QD is fully coherent, and let $t_{d1}$ and $t_{d1}$ describe the transmission amplitudes for up and down spins. To assume that the transport through the reference arm is spin independent, then the coherence rate $r_T$ reduces to:

$$r_T = \frac{T_1}{2\sqrt{T_{d1}T_{ref}}} = \frac{|t_{d1} + t_{d1}|}{\sqrt{2(|t_{d1}|^2 + |t_{d1}|^2)}}.$$  \hspace{1cm} (4)

While $B \neq 0$, $t_{d1}$ is generally not equal to $t_{d1}$, then Eq.(4) clearly shows that $r_T < 1$ despite of fully coherent transport. For the same reason, $r_G < 1$ as shown in Fig.2f for $U = \infty$ is similar as the above case of $U = 0$. Thus, one can not judge incoherence from $r_G < 1$ when $B \neq 0$. One has to further study the spin-resolved coherence rates $r_{G \uparrow}$ and $r_{G \downarrow}$ ($r_{G \uparrow} = 2(r_{G \uparrow})^2|G_{d1}^\uparrow|^{-2}$ for spin-up and spin-down components (see Fig.2f)). Both of them are very close to 1. In particularly, in the limit of $\Gamma/\Gamma^{sd} \to \infty$, both $r_{T \uparrow}$ and $r_{T \downarrow}$ approach to 1 independent of $U$, $B$, and $\epsilon_d$. Therefore, we conclude that the electron transport through the QD is fully coherent in the presence of both $B$ and $U$.

In conclusion, by using an open multi-terminal AB interferometer we investigate the degree of coherence of the electron transport through an interacting QD. We demonstrate that the intradot e-e interaction does not in-
duce any dephasing effects. Furthermore, we clarify that the asymmetric amplitude in the AB oscillation of the linear conductance in the two-terminal AB setup originates from the constraint of this closed setup, and does not reflect partial dephasing.

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Fig. 2
Fig. 3