The Spacetime Life of a non-BPS D-particle

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Abstract

We investigate the classical geometry generated by a stable non-BPS D-particle. We consider the boundary state of a stable non-BPS D-particle in the covariant formalism in the type IIB theory orbifolded by $(-1)^{F_L} \cdot Z_4$. We calculate the scattering amplitude between two D-particles in the non-compact and compact orbifold and analyse the short and long distance behaviour. At short distances we find no force at order $v^2$ for any radius, and at the critical radius we find a BPS-like behaviour up to $v^4$ corrections for long and short distances. Projecting the boundary state on the massless states of the orbifold closed string spectrum we obtain the large distance behaviour of the classical solution describing this non-BPS D-particle in the non-compact and compact cases. By using the non-BPS D-particle as a probe of the background geometry of another non-BPS D-particle, we recover the no-force condition at the critical radius and the $v^2$ behaviour of the probe. Moreover, assuming that the no-force persists for the complete geometry we derive part of the classical solution for the non-BPS D-particle.
1 Introduction

BPS D-branes enjoy a double life. On the one hand as a conformal field theory described by open strings with Dirichlet boundary conditions [1], and on the other hand as classical solitons of supergravity. For a single D-brane the regimes of validity of these two descriptions are complementary. The conformal field theory description is valid at weak coupling; whereas the supergravity solution corresponds to a strongly interacting gravitational system, which corresponds to strong coupling. On the other hand, for a large number of branes $N$ the system can be well described by a classical solution at weak coupling [4]. The two main properties of the BPS D-branes that assure this consistent dual behaviour are the fact that BPS-branes preserve a fraction of supersymmetry, hence they are stable under variations of the string coupling; and the fact that one can consider a superposition of a large number of D-branes in such a way that they still preserve a fraction of supersymmetry. These properties make possible, for instance, the entropy calculation of black-holes using D-branes [5]. On the other hand, D-branes can be studied in more general situations, for which spacetime supersymmetry is not preserved. This is the case of the non-BPS D-branes [6, 7, 8, 9].

See [2, 3] and references therein.
These branes have an exact conformal field theory description, whose consistency conditions do not rely on spacetime supersymmetry. Having such a precise description for the non-BPS D-branes in the conformal field theory side, it is natural to wonder whether the non-BPS D-branes also enjoy a double life and have a description in terms of a classical solution of a certain effective (super)gravity theory.

It has been recently suggested in [15] that the unstable non-BPS branes of type II theories might have a gravitational counterpart described by a gravitational sphaleron. These are unstable solutions with finite energy which interpolate between two (possibly distinguishable) vacuum configurations. However, these solutions are unstable and probably do not remain valid classically. On the other hand, we expect that consistent classical solutions will be related to non-BPS branes which have some stability properties similar to the BPS D-branes.

In order to find the appropriate conditions for constructing a classical solution for a non-BPS D-brane we must find out which properties of the D-branes are such that (1) they assure the validity of a classical supergravity description and (2) they are not necessarily related to fractional supersymmetry. These properties certainly comprise stability, which ensures that the state survives at strong coupling. Moreover, stability can be based on grounds different from supersymmetry, for instance, being the lightest object carrying a certain quantum number. Another key property is the fact that we can superpose an arbitrary number of parallel D-branes, which is the same as having a no-force condition at all distances. This is a consequence of the BPS property, which can exist independently of supersymmetry. Therefore, it seems natural that in order to find a classical solution for a non-BPS D-brane, this brane should be stable and enjoy a no-force property. Stable non-BPS D-branes have been found in different theories. However, only non-BPS D-branes in a certain orbifold of type II theories are known to enjoy both stability and the no-force property [22]. There it was shown that at a particular critical radius of the compact orbifold the non-BPS branes develop a Bose-Fermi degeneracy at one loop. The critical radius is in fact the value beyond which the non-BPS D-brane becomes unstable and can decay into a pair of BPS D-branes.

In this article we study the particular case of a stable non-BPS D-particle in type IIB string theory orbifolded by $(-1)^{F_L} \cdot \mathcal{I}_4 \cdot \Omega$. This D-particle is a truncated D-brane, is charged electrically under a twisted R-R 1-form field and it is the strong-weak coupling dual of a non-BPS state in the orientifold $\Omega \cdot \mathcal{I}_4$ of type IIB, charged under the $U(1)$ field of the D5-O5 system. The coupling of the non-BPS D-particle to a twisted R-R vector field is the origin of the stability of this non-BPS D-particle. Moreover, at a critical radius of the compact orbifold this D-particle meets all the requirements suggesting the existence of a classical solution. We use the technique...
of the boundary state in the covariant formalism\footnote{For a recent review see\cite{24}}\cite{25} to describe the non-BPS D-particle. We analyse the interaction potential between two non-BPS D-particles in relative motion. Remarkably, we find no force at order $v^2$ as for BPS D-branes. At the critical radius, the static force is moreover vanishing, hence the non-BPS D-particle presents a BPS-like behaviour, up to $v^4$ corrections. Unlike for BPS branes, we find no matching between the $v^4$ terms in the open and closed string description.

Using the boundary state for BPS D-branes, the long distance behaviour of the classical massless fields generated by the D-brane was computed in\cite{27,28}, and it was shown that the asymptotic behaviour of the corresponding classical solution is precisely recovered. A BPS Dp-brane is described by a boundary state with two parts, the NS-NS part and the R-R part. They generate the asymptotic behaviour of NS-NS massless fields (metric and dilaton), and of the R-R massless fields (R-R $(p + 1)$-form potential), respectively. In this paper we implement the same technique to obtain the long distance behaviour of the non-BPS D-particle geometry. This is given by the asymptotic form of a metric and a dilaton propagating in the bulk, and a twisted R-R 1-form potential propagating in the orbifold fixed plane.

One difficulty about recovering the full solution for the non-BPS D-particle from its asymptotic form is that, in principle, there might be many different possible geometries with the same asymptotic structure. In order to restrict these geometries we will assume that the no force condition also takes place when one consider the complete geometry at the critical radius, as it happens for BPS branes\cite{18}. That is, we take into account the extra pieces that one would need to add to the asymptotic behaviour in order to recover the full form of metric. On the other hand, although the Bose-Fermi degeneracy occurs at any distance between the non-BPS D-branes\cite{22}, at short distances, open strings loops might spoil this property. In fact, it has been recently proposed in\cite{29} that even at 1-loop in the open string theory, the no-force is removed in favour of another vacuum configuration in which the branes attract each other. For these reasons, our assumptions will be only acceptable for distances much larger than the string scale, which is also the range of validity of the classical solution for a BPS D-brane\cite{30}.

Using the non-BPS D-particle as a probe in the background of another non-BPS D-particle we recover the no-force behaviour at the critical radius. Moreover, under the assumptions presented above we are able to obtain part of the complete metric, dilaton and twisted R-R 1-form generated by the D-particle. The velocity dependent part of the brane action multiplies a flat metric, agreeing with the vanishing of the $v^2$ in the interaction potential. Extending this property for the complete geometry of the non-BPS D-particle source, we are able to find more properties of the classical geometry. We expect that the solution is consistent classically at the critical radius only, since it is only there where one can consider a superposition of a large number of D-particles $N$.

\footnote{The same results were obtained earlier in\cite{26} using different techniques.}
We find a diagonal metric with $SO(5) \times SO(4)$ symmetry (in Einstein frame):

$$ds_{10,E}^2 = g_{00}(y)dt^2 + g_{mn}(y)dy^m dy^n + g_{ij}(y)dx^i dx^j,$$

where we do the split $\mu = (0, m, i)$ according to the orbifold symmetry, and $\mathcal{I}_4$ acts on the $i$ directions. By using the assumptions mentioned above we are able to find the form of two of the components of the metric at the critical radius:

$$g_{00}(y) = - \left(1 + \frac{\kappa_6 T_0}{2a \Omega_4} (2\pi^2 \alpha')^{-1} \frac{1}{|y|^3} + \ldots\right)^{-\frac{7}{6}a},$$

$$g_{mn}(y) = \left(1 + \frac{\kappa_6 T_0}{2a \Omega_4} (2\pi^2 \alpha')^{-1} \frac{1}{|y|^3} + \ldots\right)^{\frac{1}{6}a} \delta_{mn}.$$

The expressions are given in terms of a parameter $a$, and some possible extra dependence in $|y|^n$, $n < -3$ denoted by $\ldots$ which remain to be determined. We find no expression for $g_{ij}$ since, as will be explained in Section 4, the D-particle cannot probe the precise geometry in these directions. Moreover, the form of the dilaton and twisted R-R potential at the critical radius are found to be:

$$e^\phi = \left(1 + \frac{\kappa_6 T_0}{2a \Omega_4} (2\pi^2 \alpha')^{-1} \frac{1}{|y|^3} + \ldots\right)^a,$$

$$C_0^{(1)} = \left(1 + \frac{\kappa_6 Q_0}{4a \Omega_4} \frac{1}{|y|^3} + \ldots\right)^{-\frac{4}{3}a} - 1.$$

Here $T_0$ is the tension of the D-particle and is related to its charge $Q_0$ by $T_0 = Q_0 \pi^2 \alpha'$, hence at the critical radius the fields above are given in terms of a single function.

This article is organised as follows. In Section 2 we carry out the construction of the covariant boundary state for the non-BPS D-particle in the orbifold of type IIB generated by $(-1)^{F_L} \cdot \mathcal{I}_4$, for the non-compact and compact cases. We also evaluate the amplitude for D-particles in relative motion and analyse the long and short distance behaviour of the interaction. In Section 3 we evaluate the asymptotic behaviour of the massless fields excited by the non-BPS D-particle, for the non-compact and compact cases. In Section 4 we recover the no-force property at the critical radius and derive part of the complete classical solution, by using the assumptions mentioned above. For completeness, we also include the derivation of the zero-mode of the twisted R-R boundary state in Appendix A, and the explicit form of the complete covariant boundary state for the D-particle in Appendix B.

## 2 Stable non-BPS D-particle in IIB/(-1)^{F_L} \cdot \mathcal{I}_4

The stable non-BPS D-particle of type IIB string theory orbifolded by $(-1)^{F_L} \cdot \mathcal{I}_4$ was described in [3, 4, 5]. It corresponds to the S-dual of a massive particle-like state in
the system of a D5-brane on top of an orientifold 5-plane, which is stable but non-BPS \[\text{[3]}\]. Its stability is due to the fact that this particle is the lightest state charged under the \(U(1)\) gauge field of the D5-O5 worldvolume theory. Similarly, the stability of the D-particle is due to its charge under a twisted R-R 1-form, which can be identified with the \(U(1)\) gauge field in the worldvolume of a NS-5 brane on top of the orbifold fixed plane.

Let us consider first the non-compact theory, i.e. type IIB in Minkowski space orbifolded by \((-1)^{F_L} \cdot \mathcal{I}_4\). The operator \(\mathcal{I}_4\) corresponds to a reflection in the directions \(x^i, i = 6, 7, 8, 9\). In the non-compact case, the orbifold contains a fixed plane at \(x^6 = x^7 = x^8 = x^9 = 0\), hence the orbifold breaks the \(SO(1, 9)\) symmetry down to \(SO(1, 5) \times SO(4)\). The operator \(F_L\) is the spacetime fermion number of the left-sector, hence \((-1)^{F_L}\) changes the sign of the of the R-R groundstate, without having any action on the oscillators. The closed string spectrum of the orbifold theory consists of an untwisted sector, given by type IIB states which are invariant under \((-1)^{F_L} \cdot \mathcal{I}_4\), and a twisted sector localised on the orbifold fixed plane. If we split the coordinates as \(X^\mu = (X^\alpha, X^i)\), where \(X^\alpha, \alpha = 0, \ldots, 5\), is longitudinal to the fixed plane, and \(X^i, i = 6, \ldots, 9\), transverse, the oscillators of the twisted sector have the following modding:

\[
\begin{align*}
\text{twisted NS} & : \\
& \left\{ \begin{array}{l}
\alpha_n^\alpha, \ n \in \mathbb{Z} \\
\alpha_r^i, \ r \in \mathbb{Z} + 1/2
\end{array} \right. \\
& \left\{ \begin{array}{l}
\psi_r^\alpha, \ r \in \mathbb{Z} + 1/2 \\
\psi_n^i, \ n \in \mathbb{Z}
\end{array} \right.
\end{align*}
\]

\[
\begin{align*}
\text{twisted R} & : \\
& \left\{ \begin{array}{l}
\alpha_n^\alpha, \ n \in \mathbb{Z} \\
\alpha_r^i, \ r \in \mathbb{Z} + 1/2
\end{array} \right. \\
& \left\{ \begin{array}{l}
\psi_n^\alpha, \ n \in \mathbb{Z} \\
\psi_r^i, \ r \in \mathbb{Z} + 1/2
\end{array} \right.
\end{align*}
\]

There are 4 fermionic zero-modes in the NS sector in the directions \(i = 6, 7, 8, 9\), and 6 fermionic zero-modes in the R-sector in the directions \(\alpha = 0, 1, \ldots, 5\). Since the intercept vanish for both sectors, the twisted groundstate will be given by these zero-modes. The twisted NS-NS sector gives a vector of \(SO(4)\), or equivalently 4 scalars under \(SO(1, 5)\). On the other hand, the twisted R-R sector gives rise to a vector under \(SO(1, 5)\). From the point of view of the fixed plane, the supersymmetries are generated by two Weyl supercharges of different chirality. This corresponds to \((1, 1)\) supersymmetry in 6 dimensions. In fact, the massless twisted sector can be identified with the worldvolume degrees of freedom of a NS-5 brane of type IIB \([7, 31, 32]\).

### 2.1 The D-particle Boundary State

In this section we construct the boundary state for the stable non-BPS D-particle in type IIB orbifolded by \((-1)^{F_L} \cdot \mathcal{I}_4\). This has been carried out in the light-cone gauge in \([3]\). Here we use the covariant formalism for the boundary state, which has not been implemented before for non-BPS D-branes in orbifolds. The D-particle boundary state
is made up of an untwisted NS-NS part and a twisted R-R part\footnote{We use the subindex NS and R in the boundary states for short.}: 

\[ |D0\rangle = |D0\rangle_{NS,U} + |D0\rangle_{R,T}. \]  

(2.2)

The NS-NS boundary state is defined as the GSO-invariant combination of boundary states \( |D0, \eta\rangle_{NS,U} \), with \( \eta = \pm 1 \), which turns out to be \footnote{We use the subindex NS and R in the boundary states for short.}:

\[ |D0\rangle_{NS,U} = P_{GSO,U} |D0, +\rangle_{NS,U} = \frac{1}{2} (|D0, +\rangle_{NS,U} - |D0, -\rangle_{NS,U}), \]  

(2.3)

where the GSO projector in the NS-NS sector is given by

\[ P_{GSO,U} = \frac{1}{4} \left( 1 - (-1)^{F+G} \right) \left( 1 - (-1)^{\tilde{F}+\tilde{G}} \right), \]  

(2.4)

with \( F \) and \( G \) the (worldsheet) fermion and superghost number operators, respectively:

\[ F = \sum_{m=1/2}^{\infty} \psi_{-m} \cdot \psi_{m}, \quad G = - \sum_{m=1/2}^{\infty} (\gamma_{-m} \beta_{m} + \beta_{-m} \gamma_{m}), \]  

(2.5)

and \( \beta, \gamma \) are the superghosts. For the right movers these operators are analogously defined. The state \( |D0, \eta\rangle_{NS,U} \) takes the form\footnote{We use the subindex NS and R in the boundary states for short.}:

\[ |D0, \eta\rangle_{NS,U} = T_{0}^{2} |D0_{X}\rangle |D0_{gh}\rangle |D0_{\psi, \eta}\rangle_{NS} |D0_{sgh, \eta}\rangle_{NS}, \]  

(2.6)

where \( T_{0} \), a constant related to the tension of the D-particle, is to be determined later. There is a bosonic and a fermionic part, \( (|D0_{X}\rangle \text{ and } |D0_{\psi, \eta}\rangle_{NS}) \), and also a ghost and a superghost part \( (|D0_{gh}\rangle \text{ and } |D0_{sgh, \eta}\rangle_{NS}) \). The boundary state \( |D0, \eta\rangle_{NS,U} \) is very similar to the NS-NS boundary state for type II branes. In fact, the only piece modified by the orbifold with respect to the type II case is the zero-mode part of \( |D0_{X}\rangle \). For the bosonic untwisted boundary state the most general conditions invariant under the orbifold symmetry are

\[ \partial_{\tau} X^{0}|_{\tau=0} |D0_{X}\rangle = 0, \]
\[ X^{p}|_{\tau=0} |D0_{X}\rangle = y^{p}, \quad p = 1, \ldots, 5, \]
\[ X^{i}|_{\tau=0} |D0_{X}\rangle = 0, \quad i = 6, \ldots, 9, \]  

(2.7)

from which we can deduce that

\[ |D0_{X}\rangle = \delta^{(5)}(\hat{q}^{p} - y^{p}) \delta^{(4)}(\tilde{q}_{i}) \exp \left[ - \sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n} \cdot S \cdot \tilde{\alpha}_{-n} \right] |k = 0\rangle. \]  

(2.8)

Note that the D-particle position in the directions \( i = 6, \ldots, 9 \) is restricted by the orbifold symmetry to be on the fixed plane. The other pieces of the untwisted boundary state are not modified by the orbifold and are the same as for the type II case. These
are explicitly given in Appendix B. Moreover, since there are nine Dirichlet directions for the case of the D-particle we have $S_{\mu\nu} = -\delta_{\mu\nu}$.

The twisted R-R boundary state that we denote by $|D0\rangle_{R,T}$ is defined as the GSO invariant combination of boundary states $|D0,\eta\rangle_{R,T}$ and is given by $|D0\rangle_{R,T} = \mathcal{P}_{GSO,T}|D0, +\rangle_{R,T} = \frac{1}{2} (|D0, +\rangle_{R,T} + |D0, -\rangle_{R,T})$. (2.9)

In this sector the GSO-operator is given by:

$$\mathcal{P}_{GSO,T} = \frac{1}{4} \left( 1 + (-1)^{F+G} \right) \left( 1 - (-1)^{F+\tilde{G}} \right),$$

with $F$ and $G$ the (worldsheet) fermion and superghost number operators in the twisted R-sector, respectively:

$$(-1)^F = \Psi (-1)^{\sum_{m=1}^{\infty} \eta_{m} \beta_{m} \psi_{m}^{\alpha} (-1)^{\sum_{r=1/2}^{\infty} \delta_{ij} \psi_{r}^{i}}} \right),$$

and similarly for the right movers. Here $\Psi$ and $\tilde{\Psi}$ represent the zero-mode parts of the GSO-projectors, which are explicitly given in Appendix A. The twisted R-R boundary state $|D0, \eta\rangle_{R,T}$ is given by:

$$|D0, \eta\rangle_{R,T} = \frac{Q_0}{2} |D0, x\rangle_{T} |D0_{gh}\rangle |D0_{\psi}, \eta\rangle_{R,T} |D0_{sgh}, \eta\rangle_{R},$$

where $Q_0$ is a normalisation factor related to the charge density of the brane and to be determined below. Notice that since this D-particle is non-BPS, $Q_0 \neq T_0$, unlike the BPS D-branes. The (super)ghosts are not affected by the orbifold, hence the corresponding pieces have the same form as for a type II R-R boundary state. Since in the twisted R-R sector the bosons have integer modding along the orbifold fixed plane and half-integer modding along the orbifolded directions, the state (2.12) have zero-modes along the fixed plane only. Accordingly, the twisted bosonic part takes the form:

$$|D0_{X}\rangle_{T} = \delta^{(5)}(\hat{q}^p - y^p) \exp \left[ \sum_{n=1}^{\infty} \frac{1}{n} \alpha^\alpha_n \delta_{\alpha\beta} \hat{\alpha}_{-n} + \sum_{n=1/2}^{\infty} \frac{1}{n} \delta_{\alpha\beta} \hat{\alpha}_{-n} \right] |k = 0\rangle.$$ (2.13)

The fermions in the twisted R-R sector have integer modding along the fixed plane and half-integer modding along the orbifolded directions. As a consequence (2.12) have fermionic zero-modes on the fixed plane only. The fermionic overlap conditions now read:

$$\left( \psi_m^\alpha - i \eta S^\alpha_\beta \bar{\psi}_m^\beta \right) |D0_{\psi}, \eta\rangle_{R,T} = 0, \quad m \in \mathbb{Z},$$

$$\left( \psi_r^i - i \eta S^i_j \bar{\psi}_{-r}^j \right) |D0_{\psi}, \eta\rangle_{R,T} = 0, \quad r \in \mathbb{Z} + \frac{1}{2}.$$ (2.14)

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We take the Minkowski metric to be mostly plus.
From them we can deduce:

\[
|D_0, \eta\rangle_{R,T} = \exp \left[ -i \eta \sum_{m=1}^{\infty} \psi^\alpha_m \delta_{m} \bar{\psi}^\beta_m - i \eta \sum_{m=1/2}^{\infty} \psi^\alpha_{m+1/2} \delta_{m} \bar{\psi}^\beta_{m+1/2} \right] |D_0, \eta\rangle_{R,T}^{(0)}. \tag{2.15}
\]

The zero-mode part \( |D_0, \eta\rangle_{R,T}^{(0)} \) is determined from the zero mode overlap conditions in (2.14) and is constructed upon the twisted R-R groundstate:

\[
|D_0, \eta\rangle_{R,T}^{(0)} = M_{ab} |a\rangle_{T} \bar{|b\rangle}_{T}, \tag{2.16}
\]

where \( a \) and \( b \) are spinor indices of \( SO(1,5) \). The explicit form of \( M_{ab} \) and its derivation are given in Appendix A.

Having constructed the covariant boundary state for the non-BPS D-particle, we determine next its tension and charge using the open-closed string consistency condition for boundary states. The interaction between two D-particles separated by a distance \( y \) in the fixed plane is given by the amplitude between two boundary states located at a relative distance \( y \) with respect to each other, with the insertion of a closed string propagator

\[
\langle D_0 | \mathcal{D} | D_0 \rangle, \tag{2.17}
\]

with

\[
\mathcal{D}_a = \frac{2\pi}{16} \int_{|z| \leq 1} \frac{d^2 z}{|z|^2} L_0 - a z \bar{L}_0 - a, \tag{2.18}
\]

where \( a = 1/2 \) in the untwisted NS-NS sector and \( a = 0 \) in the twisted R-R sector. Moreover, one needs to take the appropriate moddings in the expression for \( L_0 \) and \( \bar{L}_0 \) in the twisted sector. Also care must be taken in computing the matrix elements involving the zero modes of the superghosts and fermions in the twisted R-R sector. This is because, in general, the superghost zero modes produce infinite number of terms with any superghost number contribution, hence a regularisation is needed. We use the same regularisation as in [33].

Defining \( |D_0, \eta\rangle_{R,T}^{(0)} = |D_0, \eta\rangle_{R,T}^{(0)} \mid D_0_{\text{gh}}, \eta\rangle_{R}^{(0)} \) we give below the regularised result:

\[
\langle D_0, \eta_1 | D_0, \eta_2 \rangle_{R,T}^{(0)} = \lim_{\rho \rightarrow 1} \langle D_0, \eta_1 | \rho^{2F_0 + 2G_0} | D_0, \eta_2 \rangle_{R,T}^{(0)} = -4 \delta_{\eta_1, \eta_2, 0}, \tag{2.19}
\]

where \( F_0 \) and \( G_0 \) are the zero-mode parts of the operators \( F \) and \( G \), implicitly given in (2.11). Making a change of variables according to \( |z| = e^{-\pi \ell} \) and \( d^2 z = -\pi e^{-2\pi \ell} d\ell d\phi \) we find:

\[
\langle D_0 | \mathcal{D} | D_0 \rangle = \frac{V_1 \alpha'}{16} (2\pi^2 \alpha')^{-9/2} \int_0^{\infty} d\ell \ell^{-9/2} e^{-\frac{\pi^2 \ell^2}{2\pi^2 \alpha'}} \times \tag{2.20}
\]

\[
\times \left\{ (T_0)^2 f_3^8(e^{-\pi \ell}) - f_4^8(e^{-\pi \ell}) \right\} 
- (Q_0)^2 (2\pi^2 \alpha')^2 f_2^4(e^{-\pi \ell}) f_3^4(e^{-\pi \ell}) \right\},
\]

where \( V_1 \) is the (infinite) length of the D-particle worldline and \( f_i \) are functions of \( e^{-\pi \ell} \) defined in the usual manner. We can make a worldsheet transformation, \( \ell = 1/\tau \), to
express the above closed string channel result in the open string channel. Note that we have considered both closed string and open string to have same periodicity in the spatial direction of the worldsheet. Open-closed string consistency then requires that the result must be equal to the 1-loop amplitude of the open strings stretched between the two D-particles. However, care must be taken to allow only states invariant under the orbifold projection to propagate in the loop [7].

The open string states on a non-BPS D-particle are labelled by Chan-Paton factors $\mathbb{I}$ and $\sigma_1$. States with Chan-Paton factor $\mathbb{I}$ have usual GSO projection and are even under $I^4$. On the other hand, states with Chan-Paton factor $\sigma_1$ have opposite GSO projection and are odd under $I^4$.

Moreover, the symmetry $(-1)^{F_L}$ does not have any action on the open string oscillators. Accordingly, the open string 1-loop amplitude is given by:

$$A = 2V_1 \int_0^\infty \frac{d\tau}{2\tau} \text{Tr}_{\text{NS-R}} \left\{ \frac{1}{4} \left( 1 - (-1)^{F+G} \right) e^{-2\pi\tau L_0} \right\},$$

where $F$ and $G$ are the (worldsheet) fermion and superghost number operators for the open string. The tachyon is projected out in the trace and this renders the non-BPS D-particle stable [8]. We obtain the following 1-loop amplitude:

$$A = \frac{1}{2} V_1 (8\pi^2\alpha')^{-1/2} \int_0^\infty \frac{d\tau}{\tau^{3/2}} e^{-\frac{\pi^2}{2\pi\alpha'}} \times \left\{ \frac{f_3^8(e^{-\pi\tau}) - f_2^8(e^{-\pi\tau})}{f_3^8(e^{-\pi\tau})} - 4 \frac{f_4^f(e^{-\pi\tau})f_2^4(e^{-\pi\tau})}{f_4^f(e^{-\pi\tau})f_2^4(e^{-\pi\tau})} \right\}.$$

As mentioned above, open-closed string consistency allows us to fix the normalisation of the boundary state:

$$T_0 = 8(\alpha')^{3/2}\pi^{7/2}, \quad Q_0 = 8\pi\sqrt{\pi\alpha'}.$$  

The tension of the non-BPS D-particle is given by $T_0$ and $Q_0$ is related to its electric charge. Using that in ten dimensions $\kappa_{10} = 8\pi^{7/2}g_s(\alpha')^2$, and that for the orbifold $\kappa_{\text{orb}} = \sqrt{2}\kappa_{10}$, we find the mass per unit volume of the D-particle:

$$M_0 = \frac{T_0}{\kappa_{\text{orb}}} = \frac{1}{g\sqrt{2\alpha'}},$$

which agrees with the mass of the D-particle found in [7]. In analogy with the BPS D-branes, we can define an electric charge with respect to the (twisted) R-R field as

\[This\ can\ be\ derived\ from\ a\ D0\ anti-D0\ system\ in\ type\ IIA\ following\ the\ prescription\ given\ in\ [11, 12].\]
\[ \mu_0 = \sqrt{2}Q_0 = 8\pi \sqrt{2} \alpha'. \] It is interesting to observe that the tension \( T_0 \) of this non-BPS D-particle is the same as that of a BPS D-particle of type IIA theory in ten dimensions\(^9\). However, their charges are different. In fact, \( \mu_0 \) is exactly twice the charge of the BPS D-particle of type IIA in six dimensions.

### 2.2 The D-particle in the Compact Orbifold

Let us consider now the boundary state of a non-BPS D-particle in the case of the compact orbifold, i.e. type IIB on \( T^4/(-1)F L \cdot I_4 \). The coordinates \( x^i, i = 6, 7, 8, 9, \) are periodic and there are 16 fixed planes instead of one, located at \( x^i = 0, \pi R_i \). We consider a non-BPS D-particle located on one of the fixed planes, that we choose \( x^i = 0, i = 6, 7, 8, 9, \) for simplicity.

The boundary state is now constructed upon a groundstate with zero winding and zero momentum. The only piece modified in the boundary state with respect to the non-compact case is the bosonic part and the overall normalisation. These two modifications will only affect the NS-NS part of the boundary state. The bosonic part is modified to include Kaluza-Klein modes in the directions \( i = 6, \ldots, 9 \):

\[
|D0_X\rangle = \delta(\vec{q} - y^p) \prod_{i=6}^{9} e^{i \vec{q}_i \cdot \vec{R}_i} \exp \left[ -\sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n} \cdot S \cdot \tilde{\alpha}_{-n} \right] |k = 0, n = 0\rangle. \tag{2.25}
\]

The states \( |n_i\rangle = \exp(i q^i_n |k = 0\rangle |n = 0\rangle \) are normalised as

\[
\langle n_i | n_i' \rangle = \Phi_i \delta_{n_i n_i'}, \tag{2.26}
\]

where \( \Phi_i \) is the self-dual volume [35] which satisfies

\[
\lim_{R_i \to \infty} \Phi_i = 2\pi R_i, \quad \lim_{R_i \to 0} \Phi_i = \frac{2\pi \alpha'}{R_i}. \tag{2.27}
\]

We consider the new overall normalisation factor as a constant times the normalisation for the uncompactified case:

\[
|D0, \eta\rangle_{\text{NS,U}} = \frac{T_0}{2} \mathcal{N} |D0_X\rangle |D0_{\text{gh}}\rangle |D0_{\psi, \eta}\rangle_{\text{NS}} |D0_{\text{gh}, \eta}\rangle_{\text{NS}}, \tag{2.28}
\]

The factor \( \mathcal{N} \) can be obtained by open-closed string consistency and is such that in the decompactification limit \( R_i \to \infty \), we recover the non-compact boundary state [31].

The amplitude between two non-BPS D-particles in the closed string channel is given by:

\[
\langle D0 | D | D0 \rangle = \frac{V_i \alpha' \pi}{16} (2\pi^2 \alpha')^{-5/2} \int_0^{\infty} d\ell \, \ell^{-5/2} e^{-\frac{\alpha''}{2 \pi \alpha'} \ell} \times \tag{2.29}
\]

\(^{9}\)The unstable non-BPS D-particle of type IIB has a tension \( \sqrt{2} \) times bigger than the type IIA BPS D-particle. However, in the orbifold case, there is an extra factor \( 1/2 \) in the open string amplitude coming from the projection operator, such that the \( \sqrt{2} \) factor is compensated.
\[ \times \left\{ (T_0)^2 \mathcal{N}^2 \left( \prod_{i=6}^{9} \Phi_i \right) \prod_{i=6}^{9} \prod_{m_i \in \mathbb{Z}} e^{-\frac{2\pi^2 \alpha'}{m_i}} f_3^8(e^{-\pi \ell}) f_4^8(e^{-\pi \ell}) f_5^8(e^{-\pi \ell}) f_6^8(e^{-\pi \ell}) \right\} \]"}.

Using the worldsheet duality \( \ell = \frac{1}{\alpha'} \) and the Poisson resummation formula
\[ \sum_{m \in \mathbb{Z}} e^{-\frac{2\pi \alpha'}{m^2}} = \sqrt{\frac{2}{\ell \alpha'}} R \sum_{m \in \mathbb{Z}} e^{-\frac{2\pi}{\alpha'} \pi m R^2}, \tag{2.30} \]
one obtains
\[ \langle D0|\mathcal{D}|D0 \rangle = \frac{V_1 \alpha' \pi}{16} (2\pi^2 \alpha')^{-5/2} \int_0^\infty \frac{d\tau}{\tau^{3/2}} e^{-\frac{2\pi^2 \alpha'}{\pi \tau}} \times \tag{2.31} \]
\[ \times \left\{ (T_0)^2 \mathcal{N}^2 \left( \prod_{i=6}^{9} R_i \Phi_i \right) \prod_{i=6}^{9} \prod_{m_i \in \mathbb{Z}} e^{-\frac{2\pi \alpha'}{\alpha'} \pi m_i R_i^2} f_3^8(e^{-\pi \tau}) - f_2^8(e^{-\pi \tau}) \right\} \]
\[- (Q_0)^2 f_2^4(e^{-\pi \tau}) f_3^4(e^{-\pi \tau}) \right\} \].

Open-closed string consistency imposes that this amplitude must be equal to the open string 1-loop amplitude for the compactified case
\[ \mathcal{A} = 2V_1 \int_0^\infty \frac{d\tau}{2\tau} \text{Tr}_{\text{NS-R}} \left\{ \frac{1}{2} \left(1 + (-1)^F I_4\right) e^{-2\pi \tau L_0} \right\} \]
\[ = \frac{1}{2} V_1 (8\pi^2 \alpha')^{-1/2} \int_0^\infty \frac{d\tau}{\tau^{3/2}} e^{-\frac{2\pi^2 \alpha'}{\pi \tau}} \times \tag{2.32} \]
\[ \times \left\{ \prod_{i=6}^{9} \sum_{m_i \in \mathbb{Z}} e^{-\frac{2\pi \alpha'}{\alpha'} \pi m_i R_i^2} f_3^8(e^{-\pi \tau}) - f_2^8(e^{-\pi \tau}) \right\} \]
\[- 4 f_2^4(e^{-\pi \tau}) f_3^4(e^{-\pi \tau}) \right\} \].

This gives the same value for \( Q_0 \) as in the uncompactified case, which is expected since the compactification is done in the transverse directions of the D-particle. This also fixes the normalisation factor \( \mathcal{N} \) to be:
\[ \mathcal{N} = \left( \prod_{i=6}^{9} 2\pi R_i \Phi_i \right)^{-1/2}. \tag{2.33} \]

It is easy to check that with this normalisation and using \( \Phi_i = 2\pi R_i \), in the decompactified limit, we can recover the untwisted boundary state for the non-compact case. Thus the net effect of the compactification is a renormalisation of the tension of the D-particle and as expected, the tension depends on the compactification radii. At this point we can recover the vanishing of the amplitude at the critical radius [22]. The open strings on the D-particle have now winding modes:
\[ M^2 = \sum_{i=6}^{9} \left( \frac{w_i R_i}{\alpha'} \right)^2 + \frac{1}{\alpha'} \left( N - \frac{1}{2} \right). \tag{2.34} \]
The groundstate with zero winding is the tachyon and is projected out by the orbifold symmetry. At a particular critical value of the radii

\[ R_i = \sqrt{\frac{\alpha'}{2}}, \quad i = 6, 7, 8, 9, \quad (2.35) \]

there are four states, for which only one \( w_i \neq 0 \), which become massless [11, 32, 19]. These are the modes of the tachyon field that at the critical radius correspond to the marginal deformation which takes the D-particle to a bound state of a D-string and an anti-D-string [7]. At this critical radius a Bose-Fermi degeneracy takes place, which translates to the fact that the 1-loop amplitude vanishes [22]. In the closed string channel this can be seen by using the relations:

\[ \sum_{n \in \mathbb{Z}} e^{-\pi \ell n^2} = f_1(e^{-\pi \ell})f_2^2(e^{-\pi \ell}) \quad (2.36) \]

\[ f_4(e^{-\pi \ell})f_2(e^{-\pi \ell})f_3(e^{-\pi \ell}) = \sqrt{2}. \]

Moreover, the normalisation factor \( \mathcal{N} \) at the critical radius becomes

\[ \mathcal{N} = \frac{1}{2\pi^2\alpha'} \left( \prod_{i=6}^{9} \Phi_i \right)^{-1/2}. \quad (2.37) \]

Hence we can write the amplitude as

\[
\langle D0|\mathcal{D}|D0 \rangle = \frac{V_1 \alpha' \pi}{16} (2\pi^2\alpha')^{-5/2} \int_0^\infty d\ell \ell^{-5/2} e^{-\frac{\ell^2}{2\pi\alpha'}} \times \\
\times \frac{f_4(e^{-\pi \ell})}{f_1(e^{-\pi \ell})f_2(e^{-\pi \ell})f_3(e^{-\pi \ell})} \left\{ \left( \frac{T_0}{\pi^2\alpha'} \right)^2 \left( f_3^8(e^{-\pi \ell}) - f_4^8(e^{-\pi \ell}) \right) \\
- (Q_0)^2 f_2^8(e^{-\pi \ell}) \right\}. \quad (2.38) 
\]

which vanishes, at any distance \( y \), by the abstruse identity and using the expressions for \( T_0 \) and \( Q_0 \). In the open string channel it is simpler to demonstrate this by using the above identities.

### 2.3 Long and Short Distance Interactions

In this Section we consider the interaction amplitude for non-BPS D-particles in relative motion. This will give a velocity dependent potential which we will compare with the usual BPS case. This amplitude can be obtained by using a boosted boundary state [37] for the non-BPS D-particle. For definiteness we consider a D-particle moving in the direction \( X^1 \) with a velocity \( v \), interacting with another non-BPS D-particle at rest. We consider first the non-compact orbifold. The boosted boundary state include some
v-dependent modifications that we write explicitly below. In the NS-NS untwisted sector, the bosonic part of the boundary state becomes

\[ |D0_X, v⟩ = \sqrt{1 - v^2} \left( \prod_{i=2}^{9} \delta(\hat{q}^i) \right) \delta(\hat{q}^1 + vq^0) \exp \left[ -\sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n} \cdot S(v) \cdot \tilde{\alpha}_{-n} \right] |k = 0⟩, \tag{2.39} \]

where the matrix \( S(v) \) is given by

\[ S_{00}(v) = S_{11}(v) = -\frac{1 + v^2}{1 - v^2}, \quad S_{10}(v) = S_{01}(v) = -\frac{2v}{1 - v^2}, \tag{2.40} \]

and \( S(v) = S \) for the other components. The other pieces in the NS-NS untwisted boundary state have the usual form except for a substitution of the matrix \( S \) by \( S(v) \).

Similarly in the R-R twisted sector, the boosted bosonic boundary state takes the form:

\[ |D0_X, v⟩_T = \sqrt{1 - v^2} \prod_{p=2}^{5} \delta(\hat{q}^p) \delta(\hat{q}^1 + vq^0) \exp \left[ -\sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n} \cdot S(v) \cdot \tilde{\alpha}_{-n} \right] |k = 0, n = 0⟩, \]

where with \( t.m. \) we indicate that one takes the corresponding twisted moddings of the twisted sector. For the other pieces we do the same substitution \( S \rightarrow S(v) \). Finally, for the R-R zero-mode part there is as well a \( v \)-dependent modification:

\[ |D0, η, v⟩_{R,T}^{(0)} = M_{ab}(v)|a⟩_T |\tilde{b}⟩_T, \tag{2.41} \]

and the matrix \( M_{ab}(v) \) is given in Appendix A.

We define the distance between the particles as \( r^2 = b^2 + t^2v^2(1 - v^2)^{-1} \), where \( t \) is the proper time of the moving particle along which we also integrate; \( b \) is the impact parameter: \( b^2 = y_2^2 + \ldots y_5^2. \) For convenience we also define the following variable

\[ u = \frac{1}{2\pi i} \ln \frac{1 - v}{1 + v}. \tag{2.42} \]

Finally, the cylinder amplitude takes the following form:

\[ ⟨D0, v|\mathcal{D}|D0⟩ = (8\pi^2α')^{-1/2} \sin πu \int_{0}^{\infty} d\ell \ell^{-\frac{3}{2}} \int_{-\infty}^{\infty} dt e^{-\frac{t^2}{2π\alpha'}} \times \]

\[ \left\{ \frac{\Theta_3(u, i\ell)f_3^0(e^{-\pi\ell}) - \Theta_4(u, i\ell)f_4^0(e^{-\pi\ell})}{\Theta_1(u, i\ell)f_1^0(e^{-\pi\ell})} - 4\ell^2 \frac{\Theta_2(u, i\ell)f_2^0(e^{-\pi\ell})f_4^0(e^{-\pi\ell})}{\Theta_1(u, i\ell)f_1^0(e^{-\pi\ell})f_1^0(e^{-\pi\ell})} \right\}, \tag{2.43} \]

where \( \Theta_s(u, i\ell) \) are the Jacobi Θ-functions, and we have used the value of the tension and charge of the D-particle. For \( v = 0 \) we recover the amplitude for the static interaction given in (2.20).

In order to study the interaction we define the interacting potential of the scattering \( \mathcal{U} \), in the following way:

\[ ⟨D0, v|\mathcal{D}|D0⟩ = \int_{-\infty}^{\infty} dt \mathcal{U}(v, r(t)). \tag{2.44} \]
We can extract the long range interaction potential taking the limit $\ell \to \infty$ in the integrand of (2.43), and then performing the integral in $\ell$. For slow velocities, we find the following expansion in powers of $v$:

$$U_{\text{closed}} \simeq \frac{(2\pi \alpha')^3}{(4\pi)^{1/2}} 8 \left\{ (1 + \frac{1}{2} v^2 + \frac{1}{24} v^4) \left( \frac{\Gamma(\frac{7}{2})}{r^7} - \frac{1}{(2\pi \alpha')^2} \frac{\Gamma(\frac{3}{2})}{r^3} \right) + \frac{1}{8} v^4 \frac{\Gamma(\frac{7}{2})}{r^7} \right\}, \quad (2.45)$$

to order $v^4$. There is a static force, as expected since the D-particle is non-BPS. Moreover, the velocity dependent corrections start at order $v^2$, as for the potential between two BPS D-branes of different dimensionality [17, 18], and as for the non-BPS D-particle of type I [38]. This is something we could expect from the fact that the particle breaks all supersymmetries.

We can analyse this amplitude for very short distances. In this region the open string description dominates. Using the modular properties of the $\Theta$-functions we can write (2.43) in the open string channel:

$$\langle D_0, v | D | D_0 \rangle = -i (8\pi^2 \alpha')^{-1/2} \sin \pi u \int_0^\infty d\tau \tau^{-1/2} \int_{-\infty}^\infty dt e^{-r^2/2\pi \alpha'} \times$$

$$\left\{ \frac{\Theta_4(-i u \tau, i \tau) f_3^6(e^{-\pi \tau}) - \Theta_2(-i u \tau, i \tau) f_2^6(e^{-\pi \tau})}{\Theta_1(-i u \tau, i \tau) f_1^6(e^{-\pi \tau})} - 4 \frac{\Theta_4(-i u \tau, i \tau) f_3^2(e^{-\pi \tau}) f_3^4(e^{-\pi \tau})}{\Theta_1(-i u \tau, i \tau) f_1^2(e^{-\pi \tau}) f_3^2(e^{-\pi \tau})} \right\} . \quad (2.46)$$

We can derive the interaction potential at short distances by taking the limit $\tau \to \infty$ in the integrand and then performing the integral in $\tau$. Note that the pieces originally from the NS-NS and R-R sectors multiply now in (2.46) the same powers of $\tau$, hence we can expect that the interaction potential will be very different from the closed string case. This time we obtain:

$$U_{\text{open}} = -i (8\pi^2 \alpha')^{-1/2} \sin \pi u \int_0^\infty d\tau \tau^{-1/2} e^{-r^2/2\pi \alpha'} \frac{2 + 2\cos(-2\pi i u \tau) - 8\cos(-i \pi u \tau)}{\sin(-i \pi u \tau)},$$

which for slow velocities and after integrating in $\tau$ becomes (to order $v^4$):

$$U_{\text{open}} = \frac{4}{2\pi \alpha'} r + \frac{(2\pi \alpha')^3 \Gamma(\frac{7}{2})}{(4\pi)^{1/2}} \frac{v^4}{r^7}. \quad (2.47)$$

Surprisingly, there is no $v^2$ correction to the potential, as for BPS D-branes. The first term is the linear repulsive force coming from the stretched strings. Moreover, the $v^4$ correction has precisely the same form as the $v^4$ term for a scattering of two BPS D-particles. For BPS branes the $v^4$ term in the open string channel coincides with the $v^4$ term from the closed string description [30]. This does not occur in the non-compact orbifold. As we will see below for the compact case, even though at the critical radius the static potential and $v^2$ terms will be suppressed, the matching of the $v^4$ terms will not occur either. Hence the BPS-like behaviour will not extend beyond order $v^2$.\[14]
We extend next the analysis of the cylinder amplitude for the case of relative motion of the D-particles in the compact orbifold. We consider again one non-BPS D-particle moving in the direction $X^1$ with a velocity $v$, interacting with another non-BPS D-particle at rest. The boosted boundary state include the $v$-dependent modifications on top of the modifications due to compactification. In the NS-NS untwisted sector, the bosonic part of the boundary state becomes

$$|D_{0X}, v\rangle = \sqrt{1 - v^2} \left( \prod_{p=2}^5 \delta(\hat{q}^p) \right) \delta(\hat{q}^1 + v\hat{q}^0) \times$$

$$\times \left( \prod_{i=6}^9 \sum_{n_i \in \mathbb{Z}} e^{i\hat{q}_{n_i} \hat{q}_i} \right) \exp \left[ - \sum_{n_1=1}^\infty \frac{1}{n} \alpha_{-n} \cdot S(v) \cdot \hat{\alpha}_{-n} \right] |k = 0, n = 0\rangle,$$

where the matrix $S(v)$ is as given before in (2.40). In the R-R twisted sector, the boosted bosonic boundary state and fermionic zero-mode have the same form as for the non-compact case, and the other pieces have the usual form except for a substitution of the matrix $S$ by $S(v)$. Using the same notation as before, the cylinder amplitude takes the following form:

$$\langle D0, v|\mathcal{D}|D0\rangle = \sqrt{\frac{2}{\pi^2\alpha'}} \sin \pi u \int_0^\infty d\ell \ell^{-\frac{3}{2}} \int_0^\infty dt e^{-\frac{r^2}{2\pi\alpha'\ell}} \times$$

$$\left\{ \left( \frac{\alpha'}{4} \right)^2 \prod_{i=6}^9 R_i^{-1} \left( \prod_{i=6}^9 n_i \in \mathbb{Z} \right) e^{-\pi\alpha'(n_i)^2} \right\} \left( \frac{\Theta_3(u, i\ell) f_3^3(e^{-\pi\ell}) - \Theta_4(u, i\ell) f_4^3(e^{-\pi\ell})}{\Theta_4(u, i\ell) f_4^1(e^{-\pi\ell})} \right)$$

$$- \frac{\Theta_2(u, i\ell) f_2^3(e^{-\pi\ell}) f_3^4(e^{-\pi\ell})}{\Theta_1(u, i\ell) f_4^1(e^{-\pi\ell})} \right\}.$$

For $v = 0$ we recover the amplitude for the static interaction given in (2.29). For slow velocities ($u \approx (i\pi)^{-1} v$) and after integration in $\ell$, the long range interaction potential to order $v^4$ is given by

$$U_{\text{closed}} \simeq \frac{4\pi\alpha'}{r^3} \left\{ (1 + \frac{1}{2} v^2 + \frac{1}{6} v^4) \left( \frac{\alpha'^2}{4} \prod_{i=6}^9 R_i^{-1} - 1 \right) - \frac{v^4}{8} \right\}.$$

We see that for generic radii the potential has $v^2$ corrections, as before, which is typical for potentials between two BPS D-branes of different dimensionality [17, 18], and occurs also for the non-BPS D-particle of type I [38]. On the other hand, at the critical radius they start at $v^4$, as for BPS D-branes [17, 39]. Notice that the static and $v^2$ terms of the potential also vanishes for other values of the radii. However, those radii do not make the amplitude (2.29) vanish. Moreover, this would require some of the radii to be below the critical radius, and the D-particle would not be stable.

In order to study the short-distance behaviour we write the scattering amplitude...
in the open channel:

\[ \langle D0, v | \mathcal{D} | D0 \rangle = -i \sqrt{\frac{2}{\pi^2 \alpha'}} \sin \pi u \int_0^\infty d\tau \tau^{-\frac{1}{2}} \int_{-\infty}^\infty dt \, e^{-\frac{t^2}{2\pi \alpha'} \tau} \times (2.53) \]

\[ \left\{ \frac{1}{4} \prod_{i=6}^9 \sum_{m_i \in \mathbb{Z}} e^{-\frac{2\pi^2 (m_i R_i)^2}{9}} \left( \frac{\Theta_3(-iu \tau, i \tau) f_3^6(e^{-\pi \tau}) - \Theta_2(-iu \tau, i \tau) f_2^6(e^{-\pi \tau})}{\Theta_1(-iu \tau, i \tau) f_1^6(e^{-\pi \tau})} \right) \right. \]

\[ \left. - \frac{\Theta_4(-iu \tau, i \tau) f_4^6(e^{-\pi \tau}) f_2^4(e^{-\pi \tau})}{\Theta_1(-iu \tau, i \tau) f_1^4(e^{-\pi \tau}) f_2^4(e^{-\pi \tau})} \right\}. \]

To extract the leading contribution at short distances and low velocities we take the limit \( \tau \to \infty \) in the integrand. This time one must also include contributions with one unit of winding number \( m_i = 1 \), which are relevant very close to the critical radius \( R_i \geq R_c \):

\[ U_{\text{open}} = \sqrt{\frac{2}{\pi^2 \alpha'}} \int_0^\infty d\tau \tau^{-\frac{1}{2}} e^{-\frac{t^2}{2\pi \alpha'} \tau} \frac{\sin \pi u}{2i\sin(-i\pi u \tau)} \times (2.54) \]

\[ \times \left( \cos(2\pi i u \tau) - 4\cos(i \pi u \tau) + 1 + \frac{1}{2} \sum_{i=6}^9 e^{-\frac{2\pi^2 R_i^2}{\pi^2 \alpha'} \tau} \right). \]

At low velocities the expression simplifies. Carrying out the integral in \( \tau \) we find:

\[ U_{\text{open}} = \frac{4}{2\pi \alpha'} \left( r - \frac{1}{4} \sum_{i=6}^9 \sqrt{r^2 + 4\pi^2 (R_i^2 - \frac{\alpha'}{2})} + \frac{(2\pi \alpha')^3 \Gamma(\frac{7}{2})}{(4\pi)^{3/2}} \frac{v^4}{r^7} \right). \]

As for the non-compact case we find no \( v^2 \) corrections, and the \( v^4 \) corrections are again the same as for the BPS D-particle in ten dimensions. Notice also that the \( r \)-dependence of the \( v^4 \) term is as for the uncompactified case. The static part of the potential vanishes when all the radii are equal to the critical radius. Remarkably, at the critical radius \( U_{\text{open}} \) takes a very BPS-like form. However, as we announced before, the \( v^4 \) terms does not match with the closed string result.

From this analysis we conclude that the long and short range interactions of the non-BPS D-particle are quite different for generic radii of the orbifold. In the particular case of the critical radius the static and \( v^2 \) terms of the interaction potential are absent in the open and closed string description. At the critical radius, to order \( v^2 \), we expect an equivalence between the (super)gravity and worldvolume description of the non-BPS D-particle. At order \( v^4 \) the open strings begin to describe the geometry of non-BPS D-particle very differently from the closed strings.

### 3 Spacetime Description

As shown in [27], the boundary state of a D-brane encodes the long distance behaviour of the corresponding classical solution of supergravity. At large distances, this classical solution tends to a flat background configuration. The fluctuations around this
background is exactly reproduced by the boundary state. From the NS-NS part one obtains the asymptotic behaviour of the dilaton and the metric, and from the R-R part one obtains the asymptotic behaviour of the R-R form potential under which the brane is charged. In this section we implement this technique on the stable non-BPS D-particle to obtain the asymptotic form of the solution. We obtain a metric and a dilaton in the bulk, which depend on all the coordinates transverse to the D-particle, and whose dependence is the expected one for a particle in 10 dimensions. On the other hand, we also find a twisted R-R 1-form which is restricted to the fixed plane and whose dependence on the spatial coordinates on the fixed plane is the correct one for a particle in 6 dimensions. We describe first the non-compact case.

3.1 The Non-compact Case

Given a certain massless closed string state \(|\varphi\rangle\), normalised as \(\langle \varphi | \varphi \rangle = \varphi^2\), one can define a projection operator \(\langle P(\varphi) |\) associated to this state, such that \(\langle P(\varphi) | \varphi \rangle = \varphi^{29}\). The asymptotic behaviour of the classical field \(\varphi\), generated by a D-brane is determined by computing the overlap of the boundary state with the state \(\langle P(\varphi) |\) with the insertion of a closed string propagator \([27, 28]\):

\[
\langle P(\varphi) | \mathcal{D} | D \rangle .
\]

Since the D-particle has an untwisted NS-NS part and a twisted R-R part, the D-particle will excite gravitons \(g_{\mu\nu}\) and dilatons \(\phi\) in the untwisted sector and a R-R 1-form field \(C^{(1)}\) in the twisted sector. At large distances, these fields will be of the form of a fluctuation around a flat background\(^{10}\):

\[
g_{\mu\nu} \simeq \eta_{\mu\nu} + 2\kappa_{10} \delta h_{\mu\nu} , \quad \phi \simeq \kappa_{10} \sqrt{2} \delta \phi , \quad C^{(1)} \simeq \kappa_{6} \sqrt{2} \delta C^{(1)} .
\]

The fluctuations \(\delta h_{\mu\nu}, \delta \phi \) and \(\delta C^{(1)}\), are given by \((3.1)\) using the state \(|D0\rangle\) constructed previously. We consider first the case of the uncompactified orbifold. In the untwisted sector, only those which are invariant under the orbifold symmetry will appear in the theory. This implies that string states which depend on the momentum in the orbifolded directions are only invariant if they are symmetric with respect to \(I_4: k^i \rightarrow -k^i, i = 6, 7, 8, 9\). In particular, these will include gravitons that propagate off the orbifold fixed plane which are symmetric in \(k^i\).

The projection operators in the untwisted NS-NS sector are given in the \((-1, -1)\) picture. For simplicity, we use the following notation for the NS-NS groundstate in the \((-1, -1)\) picture:

\[
|k\rangle \equiv |k/2\rangle_{-1}|\bar{k}/2\rangle_{-1} .
\]

The projectors for the dilaton and the metric are given by:

\[
\langle P(\phi) | = \frac{1}{4\sqrt{2}} \left( (k_\perp, k^i) + (k_\perp, -k^i) \right) \psi^\mu_1/2 \psi^\mu_1/2 (\eta_{\mu\nu} - k_\mu \ell_\nu - k_\nu \ell_\mu) ,
\]

\(^{10}\)We use the same normalisation as in [25].
\[ \langle P^{\mu \nu}_{(b)} \rangle = \frac{1}{4} \left( \langle k_\perp, k^\perp \rangle + \langle k_\perp, -k^\perp \rangle \right) \left( \psi_{1/2}^\mu \bar{\psi}_{1/2}^\nu + \psi_{1/2}^\nu \bar{\psi}_{1/2}^\mu \right) \]

\[ -\langle P(\phi) \rangle \frac{1}{2\sqrt{2}} (\eta^{\mu\nu} - k^\mu \ell^\nu - k^\nu \ell^\mu) , \]

where \( k = (k_\perp, k^\perp) \), with \( k_\perp \) the spatial components of the momentum along the fixed plane directions. Moreover, \( \ell \) is such that \( \ell^2 = 0 \) and \( k \cdot \ell = 1 \).

In the R-R sector, the most natural picture is the asymmetric\(^{11} \) picture \((-1/2, -3/2)\), where the R-R vertex operator couples to the potential instead of the field strength \([40, 33]\). In analogy to the type II case described in \([33]\) and using \( a, b \) as the SO(1,5) spinor indices, we write the following quantum state in the \((-1/2, -3/2)\) picture associated to the twisted R-R 1-form field\(^{12} \):

\[ |C^{(1)}\rangle = \frac{1}{\sqrt{2}} C_\alpha \left\{ (C\gamma^a\Pi^+_{ab})_c \cos \gamma_0 \beta_0 + (C\gamma^a\Pi^-_{ab})_c \sin \gamma_0 \beta_0 \right\} |a, k/2\rangle_{-1/2} |b, k/2\rangle_{-3/2} , \]

whose conjugate state is given by:

\[ \langle C^{(1)} | = -C_\alpha \frac{1}{\sqrt{2}} -1/2 \langle b, k/2 \rangle_{-3/2} |a, k/2 \rangle \left\{ (C\gamma^a\Pi^-_{ab})_c \cos \beta_0 \gamma_0 + (C\gamma^a\Pi^+_{ab})_c \sin \beta_0 \gamma_0 \right\} , \]

with

\[ \Pi_\pm = \frac{1}{2} (1 \pm \gamma) , \]

the chirality projector in 6 dimensions, where the gamma-matrix \( \gamma \) is defined in Appendix \([41]\). This state is normalised as \( \langle C^{(1)} | C^{(1)} \rangle = C_\alpha^2 C^{(1)*} \). The corresponding projector is given by

\[ \langle P(C) \rangle | = -\frac{1}{\sqrt{2}} -1/2 \langle b, k/2 \rangle_{-3/2} |a, k/2 \rangle \left\{ (C\gamma^a\Pi^-_{ab})_c \cos \beta_0 \gamma_0 + (C\gamma^a\Pi^+_{ab})_c \sin \beta_0 \gamma_0 \right\} , \]

such that \( \langle P(C) | C^{(1)} \rangle = C_\alpha \). The form of this projector is similar to the case of the type IIA BPS D-particle, with the difference that the twisted R-R groundstate carries spinor indices of SO(1,5).

The NS-NS (R-R) fields have non-zero overlap with the NS-NS (R-R) boundary state only. We find the following results for the asymptotic fields in momentum space:

\[ \delta \phi(k) = \langle P(\phi) | D_{a=1/2} | D0 \rangle_{NS,U} = T_0 \frac{3}{\sqrt{8}} \frac{V_1}{k^2} , \]

\[ \delta h_{\mu\nu}(k) = \langle P(\phi)_{\mu\nu} | D_{a=1/2} | D0 \rangle_{NS,U} = T_0 \frac{V_1}{2} \frac{1}{k^2} \text{diag} \left( \frac{7}{4}, \frac{1}{4}, \ldots, \frac{1}{4} \right) . \]

For the twisted R-R field we only have a contribution along the worldline direction of the D-particle, namely:

\[ \delta C^{(1)}_0 (k_\perp) = \langle P(C) | D_{a=0} | D0 \rangle_{R,T} = -\frac{Q_0}{\sqrt{2}} \frac{V_1}{k_\perp} . \]

\(^{11}\)See \([22]\) for an explanation about this.

\(^{12}\)For simplicity we omit the subindex T of the twisted groundstate.
Note that $\delta C_0^{(1)}$ depends only on the momenta transverse to the D-particle, longitudinal to the fixed plane. We now make use of the following Fourier transformation in order to translate the results into position space. For a generic momentum $K$, with $d - 1$ non-zero components, we have:

$$
\int dt \ d^{(d-1)}x \frac{e^{iK \cdot x}}{(d-3) \Omega_{d-2} |x|^{d-3}} = \frac{V_1}{K^2},
$$

where $x$ are $d - 1$ spatial coordinates transverse to the D-particle and $\Omega_{d-2}$ is the area of a unit sphere surrounding the D-particle. For the NS-NS sector we have $d = 9$, hence we find the following asymptotic behaviour for the metric and dilaton:

$$
\delta \phi(x) = \frac{3T_0}{14\sqrt{2} \Omega_8} \frac{1}{|x|^7},
$$

$$
\delta h_{\mu\nu}(x) = \frac{T_0}{14 \Omega_8} \frac{1}{|x|^7} \text{diag} \left( \frac{7}{4}, \frac{1}{4}, ..., \frac{1}{4} \right).
$$

In the R-R sector, since the momentum dependence is restricted to the fixed plane spatial directions, we have $d = 6$, and the asymptotic form of the R-R 1-form is found to be:

$$
\delta C^{(1)}(y) = -\frac{Q_0}{3\sqrt{2} \Omega_4} \frac{1}{|y|^3},
$$

where now $y$ indicates the spatial directions in the fixed plane only, and $\Omega_4$ is the volume of a unit 4-sphere surrounding the particle inside the fixed plane. Note that the power of $|y|$ is exactly the power expected for a particle in 6 dimensions. We see, however, that the untwisted fields do see the entire spacetime. Thus, although the D-particle is stuck on the fixed plane, the associated metric and dilaton background solution in the uncompactified case will extend also in the orbifolded directions. This is a consequence of the fact that the D-particle can emit massless untwisted fields off the fixed plane which are symmetrised in the momenta along the orbifolded directions.

### 3.2 The Compact Case

In order to consider a spacetime description for a stable non-BPS D-brane, in a regime where it remains valid classically, one should make sense of a superposition of them, which turns out to be possible only in the compact orbifold at the critical radius. In this section we extend the analysis of the metric and dilaton of the D-particle to the case in which the space transverse to the orbifold fixed plane is a 4-torus, i.e. type IIB on $\mathbb{T}^4/(−1)^F \mathcal{L} \cdot \mathcal{I}_4$.

We derive first the asymptotic form of the metric and dilaton in the compactified case using an approximation in the background fields, that we will compare with the boundary state calculation. Consider one of the compact orbifolded directions: $x_9 \sim x_9 + 2\pi R_9$. A D-particle sitting at the origin of this $S^1$ can be seen from the point of
view of the covering space as an infinite array of equally spaced D-particles. Thus we can write:
\[
\frac{1}{|x|^7} \simeq \sum_{n \in \mathbb{Z}} \frac{1}{(r^2 + (x_9 - 2\pi nR_9)^2)^{7/2}},
\]
with \(|x|^2 = r^2 + x_9^2\), and \(r^2 = x_1^2 + \ldots + x_8^2\). We can approximate the sum by an integral assuming that the distance in the non-compact directions is much larger that the size of the compact one, i.e. \(r \gg R_9\). Changing variables we can write
\[
\sum_{n \in \mathbb{Z}} \frac{1}{(r^2 + (x_9 - 2\pi nR_9)^2)^{7/2}} \simeq \frac{1}{2\pi R_9 r^6} \int_{-\infty}^{+\infty} du \frac{1}{(1 + u^2)^{7/2}} = \frac{I_5}{2\pi R_9 r^6},
\]
where we use the notation:
\[
I_n = \int_0^\pi d\theta \sin^n \theta.
\]
Moreover, these integrals satisfy the properties
\[
nI_n = (n-1)I_{n-2}, \quad \text{for } n \geq 2,
\]
\[
\Omega_d = 2I_{d-1}I_{d-2} \cdots I_1 I_0.
\]
If we do this approximation for each of the compactified directions of the orbifold we obtain
\[
\frac{1}{|x|^7} \simeq I_2 I_3 I_5 \prod_{i=6}^9 (2\pi R_i)^{-1} \frac{1}{|y|^3},
\]
with \(y\) now indicating the spatial directions in the fixed plane. Within this approximation we obtain the following asymptotic behaviour for the dilaton and metric for the compactified case:
\[
\delta \phi(y) \simeq \frac{T_0}{2\sqrt{2} \Omega_4} \prod_{i=6}^9 (2\pi R_i)^{-1} \frac{1}{|y|^3},
\]
\[
\delta h_{\mu\nu}(y) \simeq \frac{T_0}{6 \Omega_4} \prod_{i=6}^9 (2\pi R_i)^{-1} \frac{1}{|y|^3} \text{diag} \left( \frac{7}{4}, \frac{1}{4}, \ldots, \frac{1}{4} \right).
\]
We rewrite the fluctuations using the 6-dimensional gravitational constant
\[
g_{\mu\nu} \simeq \eta_{\mu\nu} + 2\kappa_{10} \delta h_{\mu\nu} \equiv \eta_{\mu\nu} + 2\kappa_6 \delta \tilde{h}_{\mu\nu},
\]
\[
\phi \simeq \kappa_{10} \sqrt{2} \delta \phi \equiv \kappa_6 \sqrt{2} \delta \tilde{\phi},
\]
\[
\delta C^{(1)} \equiv \delta \tilde{C}^{(1)}.
\]
Using the relation\footnote{We use that for any dimension \(D\), \(2\kappa_D^2 = 16\pi G_D\), and for any lower dimension \(D - d\), \(G_{D-d} = G_D/V_d\), with \(V_d\) the volume of the compact space.} between \(\kappa_{10}\) and \(\kappa_6\): \(\kappa_{10} = \kappa_6 \prod_{i=6}^9 (2\pi R_i)^{1/2}\), we obtain
\[
\delta \tilde{\phi}(y) \simeq \frac{T_0}{2\sqrt{2} \Omega_4} \prod_{i=6}^9 (2\pi R_i)^{-1/2} \frac{1}{|y|^3},
\]
\[
\delta \tilde{h}_{\mu\nu}(y) \simeq \frac{T_0}{6 \Omega_4} \prod_{i=6}^9 (2\pi R_i)^{-1/2} \frac{1}{|y|^3} \text{diag} \left( \frac{7}{4}, \frac{1}{4}, \ldots, \frac{1}{4} \right).
\]
The asymptotic behaviour of the twisted R-R 1-form stays the same as for the non-compact case (3.11).

We compare now this approximation with the asymptotic behaviour derived from the boundary state. The closed string fields have an infinite number of Kaluza-Klein modes, since the momentum is quantised in the orbifolded directions. The massless closed string states excited by the D-particle correspond to the massless Kaluza-Klein modes. Using the same notation as before and using bars for the objects in the compact case, we can write the projection operators for the massless states as follows:

$$\langle \bar{P}(\phi) | = \frac{1}{2\sqrt{2}} \prod_{i=6}^{9} \Phi_i^{-1/2} \langle k_\perp, n_i = 0 | \psi_{1/2}^{\mu} \tilde{\psi}_{1/2}^{\mu} \left( \eta_{\mu\nu} - (\delta_\mu^\alpha \delta_\nu^\beta + \delta_\mu^\beta \delta_\nu^\alpha) k_\alpha \ell_\beta \right),$$

$$\langle \bar{P}(h) | = \frac{1}{2} \prod_{i=6}^{9} \Phi_i^{-1/2} \langle k_\perp, n_i = 0 | \left( \psi_{1/2}^{\nu} \tilde{\psi}_{1/2}^{\mu} + \psi_{1/2}^{\mu} \tilde{\psi}_{1/2}^{\nu} \right) - \langle \bar{P}(\phi) | \frac{1}{2\sqrt{2}} \left( \eta_{\mu\nu} - (\delta_\mu^\alpha \delta_\nu^\beta + \delta_\mu^\beta \delta_\nu^\alpha) k_\alpha \ell_\beta \right),$$

where now $k_\alpha \eta_{\alpha\beta} \ell_\beta = 1$ and $\ell_\alpha \ell_\beta \eta_{\alpha\beta} = 0$. Instead of splitting the projectors in terms of 6-dimensional fields, we keep the ten-dimensional notation in order to compare it with the non-compact results, and with the approximation made previously. As before, the NS-NS fields will have non-zero overlap with the NS-NS boundary state. The type of calculation we perform in this case is similar to the non-compact case, with the difference that there appear extra normalisation coefficients, and the momentum dependence is only in the spatial directions of the fixed plane. Note that the factors of $\prod_i \Phi_i^{-1/2}$ in the normalisation of the boundary state and the massless states cancel with a factor $\prod_i \Phi_i$ coming from the normalisation (2.26) in the amplitude. We find the following results:

$$\delta \bar{\phi}(k) = \langle \bar{P}(\phi) | D_{a=1/2} | D_0 \rangle_{\text{NS,U}} = \frac{3T_0}{2\sqrt{2}} \prod_i (2\pi R_i)^{-1/2} \frac{V_1}{k_i^2},$$

$$\delta \bar{h}_{\mu\nu}(k) = \langle \bar{P}(h) | D_{a=1/2} | D_0 \rangle_{\text{NS,U}} = \frac{T_0}{2} \prod_i (2\pi R_i)^{-1/2} \frac{V_1}{k_i^2} \text{diag} \left( \frac{7}{4}, \frac{1}{4}, \ldots, \frac{1}{4} \right).$$

We can make use of the Fourier transformation (3.9) in the NS-NS sector to derive the spacetime behaviour:

$$\delta \bar{\phi}(y) = \frac{T_0}{2\sqrt{2} \Omega_4} \prod_i (2\pi R_i)^{-1/2} \frac{1}{|y|^3},$$

$$\delta \bar{h}_{\mu\nu}(y) = \frac{T_0}{6 \Omega_4} \prod_i (2\pi R_i)^{-1/2} \frac{1}{|y|^3} \text{diag} \left( \frac{7}{4}, \frac{1}{4}, \ldots, \frac{1}{4} \right).$$

The twisted R-R sector remains the same as for the uncompactified case (3.11). Note that the asymptotic behaviour derived from the compactified boundary state (3.22)
exactly reproduces the behaviour of the 6-dimensional fluctuations obtained before (3.19) using the approximation given in (3.16). The asymptotics of the dilaton and the graviton are now certainly comparable with that of the twisted R-R 1-form. In particular, at the critical radius there is an accidental Bose-Fermi degeneracy, which as we will see in the next Section, implies a no-force in a brane probe. At the critical radius we can then consider a large number of branes $N$, hence the fluctuations take the following form:

\[
\begin{align*}
    \delta \bar{\phi}(y) &= \frac{T_0 N}{2\sqrt{2}\Omega_4(2\pi^2\alpha')} \frac{1}{|y|^3}, \\
    \delta \bar{h}_{\mu\nu}(y) &= \frac{T_0 N}{6\Omega_4(2\pi^2\alpha')} \frac{1}{|y|^3} \text{diag} \left( \frac{7}{4}, \frac{1}{4}, \ldots, \frac{1}{4} \right), \\
    \delta C^{(1)}(y) &= -\frac{Q_0 N}{3\sqrt{2}\Omega_4} \frac{1}{|y|^3}.
\end{align*}
\] (3.23)

4 No-force Condition at the Critical Radius

A composite of branes preserving the BPS property satisfies a no-force condition between the constituents. This was verified at the level of the effective action and background geometries for BPS branes in [18]. The no-force property is a consequence of the vanishing of the one-loop amplitude between the D-branes. On the other hand, the Bose-Fermi degeneracy for non-BPS D-branes at the critical radius described in [22] represents a remarkable example in which the no-force property takes place without being BPS. In this section we recover the no-force condition using the non-BPS D-particle as a probe of the geometry of another non-BPS D-particle. Firstly, we consider the effective action for the non-BPS D-particle.

4.1 The Action of the D-particle

Following the prescription given in [11] one can construct the action for the D-particle in the orbifold of type IIB we are considering. One starts with the action of a non-BPS D-particle in type IIB, and set to zero all fields which are odd under $(-1)^F L_4$. The worldvolume theory of this D-particle is given by scalars $b^{\alpha}_{-1/2}|0\rangle_{\text{NS}} \otimes \mathbb{I}$, $\alpha = 1, \ldots, 9$, a tachyon $|0\rangle_{\text{NS}} \otimes \sigma_1$, and 16 fermions in the Ramond groundstate of each of the Chan-Paton sectors $\mathbb{I}$ and $\sigma_1$. The orbifold $(-1)^F L_4$ eliminates the scalars in the directions $i = 6, 7, 8, 9$, the tachyon and the fermions with Chan-Paton factor $\sigma_1$ [11]. Accordingly, the non-BPS D-particle is stable and contains 5 physical bosonic degrees of freedom, describing the motion of the D-particle on the orbifold fixed plane, plus 16 fermionic d.o.f. If we label the coordinates as before $X^\mu = (X^\alpha, X^i)$, where $\alpha = 0, 1, \ldots, 5$ labels the orbifold fixed plane directions, we can write the following action for the D-particle:

\[
S_{\text{kin}} = -\frac{T_0}{\kappa_{10}} \int d\tau e^{-\phi} \sqrt{|\partial_\tau X^\alpha \partial_\tau X^\beta G_{\alpha\beta}|}, \quad (4.1)
\]
where $G_{\mu\nu}$ is the background string metric. Furthermore, since the D-particle carries charge under the twisted R-R 1-form $C^{(1)}$, we can include a WZ coupling\(^{14}\), which involves the 1-form potential on the fixed plane:

$$S_{\text{WZ}} = \frac{Q_0}{2\kappa_6} \int \partial_\tau X^\alpha C^{(1)}_\alpha .$$

(4.2)

The couplings of the stable non-BPS D-particle to the background fields given by equations (4.1) and (4.2) can also be derived by projecting the boundary state of the D-particle onto the projectors defined previously, analogously to the BPS case considered in [27]. The coupling to the dilaton is given by

$$J_{(\phi)} = \langle P(\phi)|D0\rangle_{\text{NS, U}} = \frac{3T_0}{2\sqrt{2}} V_1 ,$$

(4.3)

whereas the coupling to the metric is given by

$$J_{(h)} = \eta_{\mu\nu} \langle P_{(h)}^{\mu\nu}|D0\rangle_{\text{NS, U}} = T_0 V_1 h_{00} ,$$

(4.4)

where $h_{\mu\nu}$ is the symmetric and traceless helicity tensor of the graviton. These two results reproduce the couplings of (4.1) in Einstein frame ($g_{\mu\nu} = e^{-\phi/2} G_{\mu\nu}$) after rescaling the dilaton as $\phi = \kappa_{10} \sqrt{2} \phi_1$. Finally, the coupling to the twisted R-R 1-form is given by:

$$J_{(C)} = C^{(1)}_\alpha \langle P_{(C)}^{\alpha}|D0\rangle_{\text{R, T}} = \frac{Q_0}{\sqrt{2}} V_1 C^{(1)}_0 ,$$

(4.5)

which reproduces (4.2), after the rescaling $C^{(1)} = \kappa_6 \sqrt{2} C^{(1)}$.

In the compact case, the coupling of the D-particle to the massless closed string fields can be obtained as above, this time using the compactified boundary state constructed in Section 2.2. The net result is a change of the factor in front of the kinetic term, which amounts to the relation of the 6-dimensional gravitational constant with the 10-dimensional one. The boundary state reproduces the couplings of the following action:

$$S_{\text{kin}} = -\frac{T_0}{\kappa_6} \prod_{i=6}^{9} (2\pi R_i)^{-1/2} \int d\tau e^{-\phi} \sqrt{|\partial_\tau X^\alpha \partial_\tau X^\beta G_{\alpha\beta}|} .$$

(4.6)

Thus although the D-particle does not couple to the components $G_{ij}$ (1.1), it does feel the transverse geometry through the renormalised tension. Moreover, the open strings on the D-particle carry winding modes, and at the critical radius, the winding modes of the tachyon become massless. These extra modes appear as new degrees of freedom, $\chi_i$, in the effective action \[32, 41\]:

$$S_{\text{kin}} = -\frac{T_0}{2\pi^2 \alpha' \kappa_6} \int d\tau e^{-\phi} \sqrt{|\partial_\tau X^\alpha \partial_\tau X^\beta G_{\alpha\beta}|} \left( 1 - G^{\tau\tau} \partial_\tau \chi^i \partial_\tau \chi^i \right) .$$

(4.7)

However, these extra modes will not play any role in our discussion below.

\(^{14}\)Wess-Zumino couplings for stable non-BPS D-branes have been recently considered in \[12\].
4.2 D-particle at the Critical Radius

We consider a non-BPS D-particle probe moving in the background of another non-BPS D-particle in the compactified orbifold. We split the embedding coordinates as $X^\alpha = (X^0, X^m), m = 1, \ldots, 5,$ and choose the static gauge:

$$X^0 = \tau.$$  \hspace{1cm} (4.8)

We identify the embedding scalars with the transverse coordinates of the background geometry and expand the action in derivatives:

$$\sqrt{|\partial_\tau y^\alpha \partial_\tau y^\beta g_{\alpha\beta}|} = \sqrt{|g_{00}|} \left(1 - \frac{1}{2g_{00}} \partial_0 y^m \partial_0 y^n g_{mn} + \ldots\right).$$  \hspace{1cm} (4.9)

Moreover, for a D-particle background, only the zeroth component of the twisted R-R 1-form is turned on:

$$\partial_\tau X^\alpha C^{(1)}_\alpha = C^{(1)}_0.$$  \hspace{1cm} (4.10)

The action can then be approximated by

$$S \simeq \int d\tau V(y) + \frac{T_0}{\kappa_6} \prod_{i=6}^9 (2\pi R_i)^{-1/2} \int d\tau \frac{e^{-\frac{3}{2}\phi}}{2\sqrt{|g_{00}|}} \partial_0 y^m \partial_0 y^n g_{mn},$$  \hspace{1cm} (4.11)

where we have defined the effective static potential $V$:

$$V(y) = -\frac{T_0}{\kappa_6} \prod_{i=6}^9 (2\pi R_i)^{-1/2} e^{-\frac{3}{2}\phi} \sqrt{|g_{00}|} + \frac{Q_0}{2\kappa_6} C^{(1)}_0.$$  \hspace{1cm} (4.12)

If this potential is not constant or zero for a given background $g_{00}(y), \phi(y), C^{(1)}_0(y)$, there will be a force term in the field equations for $y^m$. Plugging into this potential the background geometry generated by another D-particle, one can check whether the Bose-Fermi degeneracy takes place. Since from the boundary state we obtain the asymptotic form of the D-particle solution, we rewrite the potential $V$ in terms of the fluctuations (3.19):

$$V(y) \simeq T_0 \prod_{i=6}^9 (2\pi R_i)^{-1/2} \left(\frac{3}{2\sqrt{2}} \delta\bar{\phi} + \delta\bar{h}_{00}\right) + \frac{Q_0}{\sqrt{2}} \delta C^{(1)}_0.$$  \hspace{1cm} (4.13)

We can insert now the asymptotic form of the solution for the compactified case given by (3.22) and (3.11). The potential then takes the form:

$$V(y) = \frac{2}{3\Omega_4 |y|^3} \left(\frac{T_0^2}{\kappa_6} \prod_{i=6}^9 (2\pi R_i)^{-1} - \frac{Q_0^2}{4}\right)$$

$$= \frac{4\pi\alpha' \Omega_4}{|y|^3} \left(\frac{\alpha'}{2}\right)^2 \prod_{i=6}^9 R_i^{-1} - 1.$$  \hspace{1cm} (4.14)

---

\[15\] We use the Einstein frame since we want to use in the action the results given by the boundary state computation.

\[16\] We neglect the constant parts of the potential, since these would not generate any force.
which coincides with the static potential derived from the cylinder amplitude in (2.52) and vanishes for the critical radius \((R_c = \sqrt{\alpha'}/2)\), hence we recover the no-force at the critical radius using the background geometry generated by the non-BPS D-particles.

According to the interaction potential \(U_{open}\) derived in Section 2.3, the velocity corrections start at order \(v^4\) and there is no \(v^2\) corrections to the potential in for any radius, as happens for BPS branes. From the probe point of view, this translates into the fact that the metric on the moduli space, multiplying the velocity dependent piece of the action (4.11), is flat [18]. In the asymptotic limit, for the non-compact orbifold this metric takes the form:

\[
e^{-\frac{3}{4}\phi} g_{mn} \simeq \delta_{mn} \left( 1 - \frac{3}{2\sqrt{2}} \kappa_{10} \delta \phi(x) + \kappa_{10} \delta h_{00}(x) \right) + 2\kappa_{10} \delta h_{mn}(x). \tag{4.15}
\]

Using the asymptotic fluctuations given in (3.10), it is easy to see that the metric is flat (= \(\delta_{mn}\)). In the case of the compact orbifold, this metric factor takes the form:

\[
e^{-\frac{3}{4}\phi} g_{mn} \simeq \delta_{mn} \left( 1 - \frac{3}{2\sqrt{2}} \kappa_{6} \delta \tilde{\phi}(y) + \kappa_{6} \delta \tilde{h}_{00}(y) \right) + 2\kappa_{6} \delta \tilde{h}_{mn}(y). \tag{4.16}
\]

Substituting the asymptotic fluctuations (3.22), we obtain the same flat metric for any value of the radii. Thus we recover the behaviour given by the open strings in (2.48) and (2.55).

### 4.3 The Classical Geometry of the non-BPS D-particle

In this Section we assume that the no-force condition will persist at the full non-linear level of the field equations. We can then restrict the possible classical geometries by imposing the no-force at the level of equation (4.12). No-force can occur for \(V\) either constant or zero [18]. However, in our case, the no-force condition at the critical radius must enforce the relation \(T_0 = (\pi^2 \alpha') Q_0\), as seen in (2.38). These are in fact the factors of the NS-NS and R-R contributions of the potential (4.12), respectively. Hence we conclude that the classical solution must be such that at the critical radius the following equality holds

\[
e^{-\frac{3}{4}\phi} \sqrt{|g_{00}|} = C_0^{(1)}. \tag{4.17}
\]

We can then deduce part of the form for \(g_{00}\), \(\phi\), and \(C^{(1)}\):

\[
\begin{align*}
g_{00}(y) &= - \left( 1 + \frac{\kappa_6 T_0}{2a \Omega_4} (2\pi^2 \alpha')^{-1} \frac{1}{|y|^3} + \ldots \right)^{-\frac{2}{a}}, \\
e^{\phi}(y) &= \left( 1 + \frac{\kappa_6 T_0}{2a \Omega_4} (2\pi \alpha')^{-1} \frac{1}{|y|^3} + \ldots \right)^{\frac{1}{a}}, \tag{4.18} \\
C_0^{(1)}(y) &= \left( 1 + \frac{\kappa_6 Q_0}{4a \Omega_4} \frac{1}{|y|^3} + \ldots \right)^{-\frac{2}{a}} - 1.
\end{align*}
\]
These functions are given in terms of a parameter $a$ yet to be determined, and are such that asymptotically they become the fluctuations in (3.19) and (3.11), and at the critical radius equation (4.17) holds. The dots indicate other possible contributions with dependence $|y|^n$, $n < -3$, which are subleading in the asymptotic limit ($|y| \to \infty$), but which are relevant when we come closer to the brane. We expect these extra terms to appear since harmonicity, which is a direct consequence of supersymmetry, may not be present in the complete solution.

We can make one further assumption if we consider that the metric factor in the velocity dependent piece of the action (4.11) remains flat for the complete geometry. We impose then the following relation:

$$e^{-\frac{1}{4} \phi} \sqrt{|g_{00}|} g_{mn} = \delta_{mn}. \quad (4.19)$$

Note that the functions describing the dilaton and the metric must be the same at the critical radius. Moreover, it is only at this radius where we actually expect to find a consistent classical geometry. Therefore, we make use of the expressions for $g_{00}(y)$ and $\phi(y)$ given above to obtain:

$$g_{mn}(y) = \left(1 + \frac{\kappa_6 T_0}{2a \Omega_4} (2\pi^2 \alpha')^{-1} \frac{1}{|y|^3} + \ldots \right)^{1/6} \delta_{mn}, \quad (4.20)$$

where the dots represent the same contributions as in (4.18) at the critical radius. Finally, it seems that $g_{ij}$ is out of the reach of the present analysis. A possible way to include it in the analysis is by a coupling to the extra massless states $\chi^i$ in (4.7). On the other hand, although the asymptotic behaviour of the metric in the non-compact case (3.10) presents an $SO(9)$ symmetry, this is certainly broken to $SO(5) \times SO(4)$ by the orbifold when we get closer to the position of the brane. This is in fact already suggested by the asymptotics in the compact case (3.19). Accordingly, the form of $g_{ij}(y)$ is expected to be different from $g_{mn}(y)$.

## 5 Comments

In this paper we have investigated the description of a stable non-BPS D-particle in terms of a classical solution. We have used the technique of the boundary state to obtain the asymptotic form of the classical solution for the non-compact and compact versions of the orbifold. We find a metric and a dilaton propagating in the bulk, and a twisted R-R 1-form propagating in the fixed plane. In the non-compact case the bulk fields have the dependence expected for a particle in ten dimensions, whereas in the compact case they have a dependence typical of a particle in six dimensions. The twisted R-R 1-form has in both cases the same asymptotic form with the usual dependence for a particle in six-dimensions. Using the non-BPS D-particle as a probe in
the background of another non-BPS D-particle, we have recovered the no-force property at the critical radius using the asymptotic behaviour. Moreover, we have calculated the cylinder amplitude for non-BPS D-particles in relative motion. From it we have extracted the long and short distance interactions. For generic radii these contain $v^2$ corrections in the closed string description, but the open string description presents no $v^2$ terms for all radii, like for BPS D-branes. Moreover, the $v^4$ corrections do not match, unlike for BPS branes. On the other hand, at the critical radius they present a BPS-like behaviour, up to the $v^4$ corrections, which do not match in the open and closed descriptions.

We have assumed that the no-force property of a brane probe holds for the full background geometry. This is acceptable for distances much larger than the string scale. This assumption allows us to derive part of the classical solution, which reproduces the asymptotic behaviour and the no-force property. On the other hand, we expect that extra terms may appear in the solution at the classical level. This is due to the fact that the boundary state only gives information about the next-to-leading term in the asymptotic limit, hence subleading terms which become relevant at short distances escape from this analysis. Moreover, the fact that there is a coordinate system in which the metric can be written in terms of harmonic functions is very much related to residual supersymmetry, which does not occur in our case. This fact does not permit to find information about the geometry near the core of the non-BPS D-particle. Moreover, although the asymptotic behaviour of the metric \((3.10)\) presents an $SO(9)$ symmetry, this is expected to be broken to $SO(5) \times SO(4)$ by the orbifold when we get closer to the position of the brane.

The form of the asymptotic behaviour we have found suggests that the classical solution for the non-BPS D-particle will be a solution to the field equations derived from an action involving 10-dimensional bulk fields and 6-dimensional matter fields constrained on an orbifold fixed plane:

\[ S_{\text{total}} = S_{\text{bulk}} + S_{\text{plane}} \]  

The bulk action involves the metric and the dilaton, and other fields of the massless untwisted sector. For the case of the D-particle, only the metric and dilaton are relevant, hence in Einstein frame we have:

\[ S_{\text{bulk}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{|\det(g_{\mu\nu})|} \left( R - \frac{1}{2} (\partial\phi)^2 \right). \]  

Since the twisted sector is provided by a type IIB NS-5 brane hidden in the orbifold fixed plane, we can derive the fixed plane action from the effective action of the type IIB NS-5 brane in Einstein frame:

\[ S_{\text{plane}} = m \int_{x^i = 0} d^6y \sqrt{|\det(\tilde{g}_{\alpha\beta})|} e^{-\frac{4\phi}{3}} \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta}, \]
where the tilde denotes the restriction of the bulk fields to the position of the fixed plane:
\[
\tilde{g}_{\alpha\beta}(y^\alpha) = g_{\alpha\beta}(y^\alpha, x^i = 0), \quad \tilde{\phi}(y^\alpha) = \phi(y^\alpha, x^i = 0),
\]
and \(\alpha, \beta = 0, 1, \ldots, 5\) are the indices along the fixed plane. This particular coupling of the twisted fields with the bulk metric is obtained by expanding the NS-5 brane kinetic term in powers of the worldvolume fields. The twisted R-R 1-form has been identified with the \(U(1)\) gauge field on the NS-5 brane worldvolume \(F\). Here \(m\) is a factor related to the tension of the NS-5 brane. Moreover, the embedding scalars of the NS-5 brane has not been included, since they correspond to the twisted NS-NS sector, to which the non-BPS D-particle does not couple. It is straightforward to see that the asymptotic fields (3.10) and (3.11) are a solution to the weak field limit of the equations of motion of the action \(S_{\text{total}}\). Note that the non-BPS D-particle is not stable below the critical radius, therefore we do not expect it would appear as a solution of this action reduced to six dimensions.

Finally, the behaviour of the stable non-BPS D-particle at the critical radius suggests that it probably saturates a BPS type of bound in the effective theory, without being supersymmetric. This latter property comes from the fact that the particle couples to a 1-form in the wrong supersymmetric multiplet. The analysis carried out here can be extended to other stable non-BPS branes. A study of the classical geometry of other stable non-BPS D-branes will be presented in a future publication [43].

Acknowledgements

We would like to thank E. Bergshoeff and M. de Roo for useful discussions. E.E. is grateful to M. Gaberdiel, T. Dasgupta, N. Lambert, A. Liccardo, M.J. Perry, B. Stefanski, P. Townsend and A. Uranga for useful discussions. E.E. has also enjoyed discussions with C. van der Bruck, P. Vanhove and N. Wyllard. The work of E.E. is supported by the European Community program ’’Human Potential’’ under the contract HPMF-CT-1999-00018. This work is also partially supported by the PPARC grantPPA/G/S/1998/00613.

A Appendix

In this Appendix we present some details about the zero-mode part of the twisted R-R boundary state and its GSO projection. In order to describe the twisted R-R groundstate we make use of the \(8 \times 8\) gamma matrices of \(SO(1,5)\):
\[
\{\gamma^\alpha, \gamma^\beta\} = 2 \mathbb{I}_8 \eta^{\alpha\beta}.
\]
We define
\[
\gamma = -\gamma^0\gamma^1\gamma^2\gamma^3\gamma^4\gamma^5,
\]
where
such that \( \{ \gamma^\alpha, \gamma \} = 0 \) and \( (\gamma)^2 = 1 \). Furthermore, there is a conjugation matrix

\[
\mathcal{C} = \gamma^3 \gamma^5 \gamma^0 ,
\]

such that

\[
\{ \gamma, \mathcal{C} \} = 0 , \quad (\gamma^\alpha)^T = -\mathcal{C} \gamma^\alpha C^{-1} , \quad C^{-1} = \mathcal{C} , \quad C^T = \mathcal{C} .
\]

The twisted R-R groundstate is characterised by left and right spinor indices of \( SO(1, 5) \), and can be constructed from the NS-vacuum by means of spin and twist fields as follows:

\[
\lim_{z, \bar{z} \to 0} S^a(z) \Sigma(z) \tilde{S}^b(\bar{z}) \tilde{\Sigma}(\bar{z}) |0\rangle = |a\rangle_T |\bar{b}\rangle_T .
\]

The zero-mode part of the twisted R-R boundary state satisfies the following overlap equations in the twisted sector:

\[
\left( \psi_0^\alpha - i \eta S^\alpha_\beta \psi_0^\beta \right) |D0_\psi, \eta \rangle_R^{(0)} = 0 ,
\]

where \( \alpha = 0, 1, \ldots 5 \) and \( S^\alpha_\beta \) is the restriction of the matrix \( S^\mu_\nu \) of the D-particle to the 6-dimensional orbifold fixed plane: \( S^\alpha_\beta = \text{diag}(+1, -1, -1, -1, -1, -1) \). We define

\[
|D0_\psi, \eta \rangle_R^{(0)} = M_{ab} |a\rangle_T |\bar{b}\rangle_T .
\]

If we define the action of the fermionic zero-modes in the twisted Ramond sector as

\[
\psi_0^\alpha |a\rangle_T |\bar{b}\rangle_T = \frac{1}{\sqrt{2}} (\gamma^\alpha)^a_c (\gamma^0)^b_d |c\rangle_T |\bar{d}\rangle_T ,
\]

\[
\tilde{\psi}_0^\beta |a\rangle_T |\bar{b}\rangle_T = \frac{1}{\sqrt{2}} (\gamma)^a_c (\gamma^\beta)^b_d |c\rangle_T |\bar{d}\rangle_T ,
\]

the matrix \( M \) must satisfy:

\[
(\gamma^\alpha)^T M - i \eta S^\alpha_\beta \left( \gamma M \gamma^\beta \right) = 0 .
\]

The solution to this equation is given by:

\[
M = \mathcal{C} \gamma^0 \frac{1 + i \eta \gamma}{1 + i \eta} .
\]

The zero-mode R-R boundary state for a non-BPS D-particle moving in a direction \( m \) is given by

\[
|D0_\psi, \eta, v \rangle_R^{(0)} = M_{ab}(v) |a\rangle_T |\bar{b}\rangle_T ,
\]

where the matrix \( M_{ab}(v) \) is given by

\[
M(v) = \frac{1}{\sqrt{1 - v^2}} \mathcal{C} \left( \gamma^0 + v \gamma^m \right) \frac{1 + i \eta \gamma}{1 + i \eta} .
\]
On the other hand, defining
\[
(0)_{\mathbf{R},\mathbf{T}} \langle \mathbf{D}_0 \psi, \eta \rangle = \mathbf{T} \langle a | \mathbf{T} \tilde{b} | \mathbf{N}_{ab} \rangle. \tag{A.13}
\]
and using
\[
\mathbf{T} \langle a | \mathbf{T} \tilde{b} | \psi_0^a \rangle = -\frac{1}{\sqrt{2}} \mathbf{T} \langle c | \mathbf{T} \tilde{d} | (\gamma^0)^a c (\mathbf{1}_8)^b \rangle d
\]
\[
\mathbf{T} \langle a | \mathbf{T} \tilde{b} | \tilde{\psi}_0^a \rangle = \frac{1}{\sqrt{2}} \mathbf{T} \langle c | \mathbf{T} \tilde{d} | (\gamma)^a c (\gamma^b)^b \rangle \tag{A.14}\]
we can write the matrix \( \mathbf{N} \) as follows:
\[
\mathbf{N} = C_0 \gamma^0 \frac{1 + i \eta \gamma}{1 - i \eta}. \tag{A.15}
\]
The boosted version can be written similarly to \((A.12)\):
\[
\mathbf{N}(v) = \frac{1}{\sqrt{1 - v^2}} C \left( \gamma^0 + v \gamma^m \right) \frac{1 + i \eta \gamma}{1 - i \eta}. \tag{A.16}\]
In order to implement the GSO projection on the zero-mode of the boundary state, we define the zero-mode part of the GSO-projector on the twisted R-R sector as:
\[
\Psi = -8 \psi_0^5 \cdots \psi_0^0, \quad \tilde{\Psi} = -8 \tilde{\psi}_0^5 \cdots \tilde{\psi}_0^0. \tag{A.17}\]
Using this definition and \((A.8)\), the action of \( \Psi \) and \( \tilde{\Psi} \) on the twisted R-R groundstate is found to be:
\[
\Psi | \mathbf{a} \rangle_T | \mathbf{b} \rangle_T = (\gamma)^a c | \mathbf{c} \rangle_T | \mathbf{b} \rangle_T, \quad \tilde{\Psi} | \mathbf{a} \rangle_T | \mathbf{b} \rangle_T = (\gamma)^b d | \mathbf{a} \rangle_T | \mathbf{d} \rangle_T. \tag{A.18}\]
Finally, combining \((A.10)\) and \((A.18)\) we find:
\[
\Psi | \mathbf{D}_0 \psi, \eta \rangle_{\mathbf{R},\mathbf{T}}^{(0)} = | \mathbf{D}_0 \psi, -\eta \rangle_{\mathbf{R},\mathbf{T}}^{(0)}, \quad \tilde{\Psi} | \mathbf{D}_0 \psi, \eta \rangle_{\mathbf{R},\mathbf{T}}^{(0)} = | \mathbf{D}_0 \psi, -\eta \rangle_{\mathbf{R},\mathbf{T}}^{(0)}. \tag{A.19}\]

**B Appendix**

For the sake of completeness we include in this Appendix the explicit definitions of the different parts of the boundary state for the non-BPS D-particle. We include as well the conjugate states. Before GSO-projection the NS-NS boundary state is given by:
\[
| \mathbf{D}_0, \eta \rangle_{\mathbf{NS},\mathbf{U}} = \frac{T_0}{2} | \mathbf{D}_0 \rangle | \mathbf{D}_0 \rangle | \mathbf{D}_0 \rangle | \mathbf{D}_0 \rangle | \mathbf{D}_0 \rangle | \mathbf{D}_0 \rangle | \mathbf{D}_0 \rangle | \mathbf{D}_0 \rangle | \mathbf{D}_0 \rangle | \mathbf{D}_0 \rangle. \tag{B.1}\]
The bosonic part is:
\[
| \mathbf{D}_0 \rangle = \delta^{(5)}(\hat{q}^p - y^p) \delta^{(4)}(\hat{q}^i) \exp \left( -\sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n} \cdot S \cdot \bar{\alpha}_{-n} \right) | k = 0 \rangle. \tag{B.2}\]
The ghost part:

$$|D_{0_{gh}}\rangle = \exp \left( \sum_{n=1}^{\infty} (c_{-n} b_{-n} - b_{-n} c_{-n}) \right) \left( \frac{c_0 + \tilde{c}_0}{2} \right) |1\rangle |\tilde{1}\rangle.$$  \hspace{1cm} (B.3)

The fermionic part:

$$|D_{0_{\psi}}, \eta\rangle_{NS} = -i\exp \left( i\eta \sum_{r=1/2}^{\infty} (\psi_{-r} \cdot S \cdot \bar{\psi}_{-r}) \right) |0\rangle,$$  \hspace{1cm} (B.4)

The superghost part:

$$|D_{0_{sgh}}, \eta\rangle_{NS} = \exp \left( i\eta \sum_{r=1/2}^{\infty} (\gamma_{-r} \tilde{\beta}_{-r} - \beta_{-r} \tilde{\gamma}_{-r}) \right) |-1\rangle |\tilde{-1}\rangle.$$  \hspace{1cm} (B.5)

The conjugate states are:

$$\langle D_{0_{X}} | = \langle k = 0 | \delta(5) (\tilde{q}^p - y^p) \delta(4) (\tilde{q}^i) \exp \left( - \sum_{n=1}^{\infty} \frac{1}{n} \alpha_n \cdot S \cdot \tilde{\alpha}_n \right),$$

$$\langle D_{0_{gh}} | = \langle 2 | \tilde{2} | (b_0 - \tilde{b}_0) \exp \left( \sum_{n=1}^{\infty} (b_n c_n - \tilde{c}_n b_n) \right),$$

$$\langle_{NS} D_{0_{\psi}}, \eta | = i \langle 0 | \exp \left( i\eta \sum_{r=1/2}^{\infty} (\psi_{r} \cdot S \cdot \bar{\psi}_{r}) \right),$$

$$\langle_{NS} D_{0_{sgh}}, \eta | = \langle -1 | \langle -\tilde{1} | \exp \left( -i\eta \sum_{r=1/2}^{\infty} (\beta_{r} \tilde{\gamma}_{r} - \gamma_{r} \tilde{\beta}_{r}) \right).$$  \hspace{1cm} (B.6)

For the twisted R-R sector we have (before GSO-projection):

$$|D_{0_{R_{T}}} \rangle = \frac{Q_0}{2} |D_{0_{X}}\rangle_{T} |D_{0_{gh}}\rangle |D_{0_{\psi}}, \eta\rangle_{R_{T}} |D_{0_{sgh}}, \eta\rangle_{R},$$  \hspace{1cm} (B.7)

The bosonic part is given by

$$|D_{0_{X}}\rangle_{T} = \delta^{(5)} (\tilde{q}^p - y^p) \exp \left( - \sum_{t.m.} \frac{1}{n} \alpha_{-n} \cdot S \cdot \tilde{\alpha}_{-n} \right) |k = 0\rangle,$$  \hspace{1cm} (B.8)

where $t.m.$ indicates that the sum is performed according to the twisted moddings of the (twisted) R-R sector given in (2.1). The ghost part is the same as for the NS-NS boundary state. The fermionic part reads:

$$|D_{0_{\psi}}, \eta\rangle_{R_{T}} = -\exp \left( i\eta \sum_{t.m.} (\psi_{-n} \cdot S \cdot \bar{\psi}_{-n}) \right) |D_{0_{\psi}}, \eta\rangle_{R_{T}}^{(0)},$$  \hspace{1cm} (B.9)

where $|D_{0_{\psi}}, \eta\rangle_{R_{T}}^{(0)}$ and is given in Appendix A. Finally, the superghost part:

$$|D_{0_{sgh}}, \eta\rangle_{R} = \exp \left( i\eta \sum_{n=1}^{\infty} (\gamma_{-n} \tilde{\beta}_{-n} - \beta_{-n} \tilde{\gamma}_{-n}) \right) |D_{0_{sgh}}, \eta\rangle_{R}^{(0)},$$  \hspace{1cm} (B.10)

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with the zero-mode given by
\[ |D_{0\text{sgh}}, \eta\rangle^{(0)}_R = e^{i\eta \gamma_0 \tilde{\beta}_0} |-1/2\rangle |-3/2\rangle. \] (B.11)

Finally, the conjugate states are:
\[ T \langle D_0 X \rangle = \langle k = 0 |\delta(5)(\hat{q}^p - y^p) \exp \left(-\sum_{t.m.} \frac{1}{n} \alpha_n \cdot S \cdot \tilde{\alpha}_n \right), \]
\[ R, T \langle D_0 \phi, \eta \rangle = -^{(0)}_R \langle D_0 \phi, \eta \rangle \exp \left(i\eta \sum_{t.m.} \tilde{\psi}_n \cdot S \cdot \hat{\psi}_n \right), \]
\[ \text{NS} \langle D_0_{\text{sgh}}, \eta \rangle = ^{(0)}_R \langle D_0_{\text{sgh}}, \eta \rangle \exp \left(-i\eta \sum_{n=1}^{\infty} (\beta_n \tilde{\gamma}_n - \gamma_n \beta_n) \right), \] (B.12)

where \( ^{(0)}_R \langle D_0 \phi, \eta \rangle \) is given in Appendix A and
\[ ^{(0)}_R \langle D_0_{\text{sgh}}, \eta \rangle = \langle -3/2 \rangle \langle -1/2 \rangle e^{-i\eta \gamma_0 \tilde{\beta}_0}. \] (B.13)

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