Aerodynamic improvement of Darrieus wind turbine

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Abstract. Until recently, vertical-axis wind turbines are less extensively developed in wind energetics. At the same time, there are a number of advantages in turbines of such type like their independence from the change of wind direction, lower levels of aerodynamic and infrasound noises, higher structural reliability (compared to horizontal engines), etc. With these advantages, vertical-axis wind turbines demonstrate promising capacities. Inter alia, the productiveness of such turbines can be refined through the aerodynamic improvement of the structure and comprehensive optimization of the rotor geometry. The main purpose of the presented paper is to aerodynamically improve vertical wind turbine in order to increase the efficiency of wind energy conversion into electricity. Within the framework of the classical theory of impulses, this article presents a study of the effect of variation in Reynolds number on the general energy characteristics of a vertical-axis wind turbine with two blades. The integral approach makes it possible to use a single-disk impulse model to determine the main specific indicators of the system. The power factor was calculated based on the obtained value of the shaft torque factor, which in turn was determined by numerically integrating the total torque generated by the wind turbine. To calculate the test problem, we used the classic NACA airfoils: 0012, 0015, 0018 and 0021. The proposed calculation algorithm makes it possible not to indicate the Reynolds number and corresponding aerodynamic coefficients at the beginning of the calculation, but to recalculate it depending on the relative speed, position of the airfoil and the linear speed of the airfoil around the circumference. Proposed modern design techniques can be helpful for optimization of vertical wind turbines.

1. Introduction
Wind energy is the fastest growing source of clean and renewable energy in the world. Development of this branch of energetics seems to be a priority for Ukraine – the country that is critically dependent on energy imports. Therefore, the development of alternative energy is a top objective of the modern energy industry. It requires ever-higher costs for green technologies of energy generation needed for sustainable development.

The development of the wind energy sector in Ukraine lags far behind the more developed markets. In addition to general economic factors, it should be noted that average annual wind speed in Ukraine is ranging from 4.5 to 5.5 m/s, which makes the use of major part of industrial wind turbines designed to operate at higher nominal wind speeds rather inefficient. The Central Geophysical Observatory of Ukraine expects increasing of average wind speed by 1-2 m/s in the next 30-40 years due to a climate change. Thus, the potential of wind energy industry will grow in future [8].

Wind power generation systems can be variable-speed or fixed-speed. Variable-speed ones are more attractive than fixed-speed ones because of the increased energy output. Until recently, it has been wrongly assumed that vertical-axis wind turbines cannot ensure any high efficiency level of energy
use. However, according to recent studies [1], Darrieus rotors can offer a specific speed of 6:1 and wind efficiency factor comparable to horizontal-axis turbines. At the same time, among the advantages of such turbines, there is also the independence from the change of wind direction, lower level of aerodynamic and infrasound noises, higher structural reliability (compared to horizontal engines) that is beneficial in areas with turbulent wind flow such as rooftops, coastlines and cityscapes. Another advantage is the ability to place the generator directly on the turbine’s foundation that results in a simpler transmission system. Such turbines operate better at a low capacity, which makes them very appealing to standalone and backup energy systems. With these advantages, vertical-axis wind turbines demonstrate promising capacities. Characteristics of such turbines can be improved in particular through aerodynamic improvement of the structure and comprehensive optimization of the rotor geometry. The general scheme of Darrieus rotor is showed in figure 1. The Darrieus rotor is a research subject for many scientists [1–5]. To perfectly calculate the model, one should use the Navier-Stokes equations [7]. Numerical calculation and creation of the model for calculating these equations is a very time consuming process. For the mathematical calculation of Darrieus rotors we used aerodynamic coefficients of classic NACA airfoils: 0012, 0015, 0018 and 0021.

![Figure 1. The scheme of a wind turbine with a vertical axis.](image)

The proposed calculation algorithm, described in this article, uses a single-disk pulse theory with a refinement – taking into account the variation in Reynolds number, which, in turn, gives a more logical result than the calculation of the classical approach with a fixed Reynolds number. The calculations show that the most effective airfoils for Darrieus wind turbine are NACA 0018 and NACA 0021.

2. Mathematical model

2.1 General characteristics of the mathematical model

The mathematical model is based on the classical single-disk impulse theory, the essence of which is that in the wind turbine the position of the "active disk" remains uncertain, but on this disk, the conservation laws in the integral sense are fulfilled. In figure 2 the distribution of wind flow for a single-disk calculation model is shown schematically. In mathematical terms, the implementation of a single-disk impulse theory reduces itself to the following algorithm: forces acting on the blade and traverse, on the one hand, are expressed through aerodynamic coefficients of blade and traverse profiles (calculated by local angles of attack and local relative speed); on the other hand, the same forces are expressed by the momentum theorem. Mathematically, such a balance of forces is formalized in the form of a transcendental equation, the solution of which involves finding the main energy characteristics of the wind turbine – power and torque coefficients [1].
The equation for finding of the value of the unknown speed in the "active disk" depending on the speed \( V_0 \) of the oncoming flow is:

\[
\frac{V_0}{V_1} - 1 = F_1. \tag{1}
\]

In equation (1), the dimensionless quantity \( F_1 \) is defined as follows:

\[
F_1 = \mu_1 \int_0^{2\pi} \left( c_x(Re, \alpha) \cdot \sin(\beta - \psi) - c_y(Re, \alpha) \cos(\beta - \psi) \right) \, d\beta. \tag{2}
\]

The relative velocity \( W_1 \) of the incoming flow on the blade in the position characterized by the azimuthal angle \( \beta \), is:

\[
W_1 = \sqrt{V_1^2 + 2W_{\text{cr}}V_1 \sin \beta + W_{\text{cr}}^2}. \tag{3}
\]

The determination of speed \( W_1 \) is necessary to find the coefficients of torque on the shaft and power of the wind turbine.

**Figure 2.** The scheme of a single-disc model.

**Figure 3.** The scheme of relative velocities and angles of attack of the elements of the blade depending on the position of the azimuthal angle.
In equation (2):

- $\mu_0 = \frac{Nc}{16\pi Rk}$ is a coefficient that takes into account the geometric parameters of the wind turbine;
- $\tilde{W} = \frac{W}{V_1}$ is the dimensionless relative velocity,
- $V_1$ is a previously unknown speed in the active disc;
- $N$ is a number of blades;
- $\beta$ is an azimuthal angle,
- $c$ is a chord length of the airfoil;
- $c_x(Re,\alpha)$ and $c_y(Re,\alpha)$ are aerodynamic coefficients of the selected blade profile depending on the local Reynolds number and the local angle of attack;
- $Re$ is Reynolds number;
- $\alpha = \alpha_0 + \psi$ is a local angle of attack;
- $\psi = \arctg \frac{-V_1 \cos \beta}{Re\omega + V_1 \sin \beta}$ is the angle between the relative velocity $\tilde{W}$ and $\tilde{W}_{cre}$;
- $\tilde{W}_{cre}$ is the linear velocity of the blade in a circumferential direction;
- $\alpha_0$ is the angle of installation of the blade relative to the tangent to the circle (considered positive if the leading edge of the profile deviates inward from the specified circle, and negative - outward).

2.2 The method of calculating the energy characteristics of the wind turbine depending on the variable Reynolds number

Rewriting the equation (1) in the form of:

$$V_1 = \frac{V_0}{F_1 + 1}. \quad (4)$$

The left part of equation (4) depends only on the speed $V_1$, so it is convenient to solve the functional connection in this form:

$$V_1 = f(V_1, V_0, \omega R). \quad (5)$$

Transcendental equation (5) can be solved by the method of successive approximations [10]. By specifying a range of numbers $V_0$, we can get a range of numbers $V_1$.

The total torque relative to the axis of the wind turbine, assuming that the center of pressure of the profile coincides with the middle of the chord, for one blade has the form of [1]:

$$M_{cp} = \frac{1}{2} \rho W^2 c R (c_x(Re, \alpha) \cdot \sin \psi - c_y(Re, \alpha) \cdot \cos \psi). \quad (6)$$

The average value of the total moment has the following form:

$$M_{cp} = \frac{N}{4\pi} \rho c H V_1^2 \int_0^{2\pi} c_y(Re, \alpha) \cdot \sin \psi - c_x(Re, \alpha) \cdot \cos \psi d\beta. \quad (7)$$

The average power that is developed in one rotation is:

$$P_{cp} = M_{cp} \omega. \quad (8)$$

$\omega$ is an angular speed of the wind turbine.

In turn, the coefficients of torque on the shaft and power of the wind turbine are in the formulas:

$$M_{cp} = c_m \frac{1}{2} \rho V_0^2 2R^2 H, \quad (9)$$

$$P_{cp} = c_m \frac{1}{2} \rho V_0^2 2R^2 H \omega = c_p \frac{1}{2} \rho V_0^3 2RH. \quad (10)$$
Using formulas (7), (9) and (10), we obtain the final form of the coefficients of torque on the shaft \( m_c \) and power coefficient \( p_c \):

\[
\begin{align*}
    c_m &= \frac{Nc}{4\pi \left( \frac{V_0 R}{R} \right)^2} \cdot \Phi(V_1, \omega R), \\
    c_p &= c_m \frac{\omega R}{V_0}, \\
    \Phi(V_1, \omega R) &= \int_0^{2\pi} \left[ R\left(R\omega\right)^2 + 2V_0 \omega \sin \beta + V_1 \right] \left( c_x(Re, \alpha) \cdot \sin \psi - c_1(Re, \alpha) \cdot \cos \psi \right) d\beta.
\end{align*}
\]

Figure 4. The dependence of the Reynolds number on the azimuthal angle \( \beta \) and the magnitude of the oncoming flow velocity \( V_0 \) for the airfoil NACA 0018.

The aerodynamic coefficients of the specified profiles for different Reynolds numbers in the range \( Re = 10^3 \) to \( 5 \cdot 10^5 \) [10] were used for the calculation. The dependence of the Reynolds number on the azimuthal angle \( \beta \) and the magnitude of the oncoming flow velocity \( V_0 \) are shown in figure 4 for the airfoil NACA 0018. For other airfoils the graphics are similar. The Reynolds number was calculated from the relative velocity (3) [6]:

\[
Re = \frac{Wc}{V}.
\]

The Reynolds number varies in the range from \( 2 \cdot 10^3 \) to \( 7.9 \cdot 10^5 \) at different speeds \( V_0 \) and a fixed linear velocity of the blade is around \( W_{c,\text{c}} = 5 \text{ m/s} \), and therefore this fact of the variation in Reynolds number is important in the calculation of the overall energy performance of the wind turbine. Finally we get, that according to the method of variable Reynolds numbers, aerodynamic coefficients \( c_x \) and \( c_y \) are calculated depending on the magnitude of value (14) and relative speed (3) and are determined by linear interpolation [10] using the range of found and fixed Reynolds numbers, obtained experimentally for the indicated types of blades [9].

3. Results

To determine the main energy characteristics of the wind turbine based on the proposed mathematical model, an iterative numerical algorithm was built. The software module consists of two parts: in the first part the solution of the transcendental equation (5) is being calculated with the determination of the local Reynolds number (16), in the second – the calculation of the integral (9), with the determination of the torque coefficients on the shaft (13) and power (14). All subsequent graphs are presented depending on the number \( Z_0 \) – the tip-speed factor:
\[ Z_0 = \frac{\omega R}{V_0}. \] (15)

**Figure.** 5. The dependence of the power factor on the tip-speed factor at a wind speed \( V_0 = 5 \text{ m/s} \).

The following geometric dimensions were taken for the calculation of the mathematical model:
- chord length of the blade - \( c = 0.7 \text{ m} \);
- wind turbine radius - \( R = 3.6 \text{ m} \);
- kinematic-viscosity coefficient - \( \nu = 15.06 \cdot 10^{-6} \text{ m}^2/\text{s} \).

Figure. 5 shows that the most optimal airfoils for wind speed \( V_0 = 5 \text{ m/s} \) are NACA 0018 and NACA 0021.

The calculations showed that the values of the power factors in some range of tip-speed factors are negative, which indicates that the rotation unit must consume energy from the outside and its operation is inefficient.

For this mathematical model of the wind turbine the working range of tip-speed factor is \( 0 < Z_0 < 4.2 \).

**4. Conclusion**

The mathematical model for determination of the energy performance of a wind turbine with a vertical axis of rotation, based on a single-disk pulse theory is presented in this article. A numerical algorithm was created and parametric studies of the energy characteristics of the wind turbine model were performed. The proposed calculation algorithm differs from conventional algorithms, which are based on a single-disk theory, in the fact that it allows not to specify the Reynolds numbers before the calculation, but to calculate them directly depending on the relative velocity, the position of the airfoil and the linear velocity of the airfoil around the circumference.

The proposed mathematical model of calculation can be used for different types of Darrieus rotors, and the created software module allows varying the geometric parameters of the wind turbine. The numerical algorithm can be improved by using a two-disk impulse theory taking into account the variation in Reynolds number.

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