GAUGE INVARIANCE FOR THE MASSIVE AXION

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ABSTRACT

A massive gauge invariant formulation for scalar ($\phi$) and antisymmetric ($C_{mnp}$) fields with a topological coupling, which provides a mass for the axion field, is considered. The dual and local equivalence with the non-gauge invariant proposal is established, but on manifolds with non-trivial topological structure both formulations are not globally equivalent.

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1 INTRODUCTION

In four dimensions a massless (pseudo)scalar field: the axion, is dual to the antisymmetric field \( B_{mn} \) (only if derivative couplings are considered, therefore massive terms are excluded) as a particular case of the general duality between \( p \) and \( D - p - 2 \) forms in \( D \) dimensions. Since non-perturbative effects break the local Peccei-Quinn symmetry for the axion field, in order to give mass to the axion, the duality between a massive axion and an antisymmetric field was considered an enigma until two independent approaches \([1],[2]\) were developed recently. Now, is understood that take into account non-perturbative effects, the usual duality between a massless scalar field and an antisymmetric field \( B_{mn} \) is not broken, but replaced by the duality between a massive scalar \( \phi \) and an massive antisymmetric field \( C_{mnp} \). An early attempt to understand this duality was considered in the reference \([3]\). A characteristic feature of this duality is the lost of abelian gauge invariance for the antisymmetric field. In this article, we will show that a gauge invariant theory, which involved a topological coupling and considered several years ago \([4]\) in the context of the \( U(1) \) problem, is locally equivalent to the non gauge invariant proposal. This equivalence is similar to what happen in three dimensions for massive topologically and self-dual theories \([5]\) and the Proca and massive topologically gauge invariant theories in four dimensions \([6]\). We will study the equivalence through the existence of a master action from which local and global considerations are established.

2 THE GAUGE INVARIANT MODEL

An illustrative model for the massive axion is given by the following master action\([7]\)
\[
I = \langle -\frac{1}{2} v_m v^m + \phi \partial_m v^m + \frac{1}{2} m^2 \phi^2 \rangle , \tag{1}
\]
where \( v_m \) is a vector field and \( \phi \) is a scalar field (\( \langle \rangle \) denotes integration in four dimensions). Eliminating the field \( v_m \) through its equation of motion: \( v_m = \partial_m \phi \), the action for a massive scalar is obtained, while using the equation of motion, obtained by varying the scalar field \( \phi \) (\( \phi = \frac{1}{m^2} \partial_m v^m \)), we have
\[
I_v = \frac{1}{2} \langle v_m v^m + \frac{1}{m^2} (\partial_m v^m)^2 \rangle \tag{2}
\]
and the propagator corresponding to the field $v_m$ is $\eta_{mn} - \frac{k_m k_n}{k^2 + m^2}$, which is just equal to those discussed in [1]. A simple way to show the duality, rely on introducing the dual of the vector field $v^m = \frac{1}{3!} m \epsilon^{mpq} C_{npq}$ in the action $I_v$, yielding the master action

$$I_{M1} = \langle -\frac{1}{2.3!} m^2 C_{mpn} C_{mpn} + \frac{1}{3!} m \epsilon^{mpq} \dot{\phi} \partial_m C_{npq} - \frac{1}{2} m^2 \phi^2 > . \quad (3)$$

from which the duality is easily inferred. In fact, eliminating the scalar field $C_{mnq}$ (or $\phi$) through its equation of motion, the action for a massive scalar field (or the massive antisymmetric field $C_{mnq}$) is obtained. In anycase, the gauge invariance is spoiled. Now, we can ask whether there really exist an invariant gauge theory compatible with a massive term for the axion field. The answer is positive. We will show that the following action

$$I_{M2} = \langle -\frac{1}{2} \partial_m \phi \partial^m \phi - \frac{1}{2.4!} G_{mpq} G_{mpq} - \frac{m}{6} \epsilon^{mpq} C_{mpn} \partial_q \phi > , \quad (4)$$

where $G_{mpq} = \partial_m C_{npq} - \partial_n C_{mpq} + \partial_p C_{mnq} - \partial_q C_{mnp}$ is the field strenght associated to the antisymmetric field $C_{mnq}$, is locally equivalent to $I_{M1}$, describing the propagation of a massive scalar excitation: a massive axion. Note that the coupling term is an extension of the usual $BF$ term and the action is invariant under the abelian gauge transformations

$$\delta_\xi C_{mpq} = \partial_m \xi_{np} + \partial_n \xi_{pm} + \partial_p \xi_{mn}, \quad \delta_\xi \phi = 0. \quad (5)$$

This action was considered previously in ref [4], as a generalization to four dimensions of the Schwinger model in two dimensions.

Let us see, how this action is related to the propagation of a massive axion and why the equivalence with the non gauge invariant action must hold. Rewritten down the action (eq. (4)) by introducing $F_{mpn} = \epsilon^{mpq} F_q$ as the dual tensor of $F_m = \partial_m \phi$, we can eliminate $F_{mpn}$ through its equation of motion: $F_{mpn} = -m C_{mpn}$ and substituing, the action for the massive antisymmetric field $C_{mpn}$ appears. Going on an additional step, the dual of the antisymmetric field $C_{mpn} = \frac{1}{m} \epsilon_{mpn} v_q$ is introduced, and the action for the vector field $v_m$, eq. (2), is obtained. On the other hand, if we introduce $\lambda = -\frac{1}{4} \epsilon^{mpq} G_{mpq}$ as the dual of the strenght field $G_{mpq}$ into the action (4), we observe that $\lambda$ plays the role of an auxiliary field, whose elimination through its equation of motion ($\lambda = -m \phi$) lead to the action of a massive scalar field.
It is worth recalling, since the action is expressed only in derivatives of the scalar field, that the dual theory can be achieved, reemplacing \( \partial_m \phi \) by \( \frac{1}{2l_m} \) and add a BF term: \( \frac{1}{4}l_m \epsilon^{mnpq} \partial_n B_{pq} \). The dual action is:

\[
I_d = \langle - \frac{1}{2.4!} G_{mnpq} G^{mnpq} - \frac{1}{2.3!} (mC_{mnp} - H_{mnp})(mC^{mnp} - H^{mnp}) \rangle, \tag{6}
\]

where \( H_{mnp} = \partial_m B_{np} + \partial_n B_{pm} + \partial_p B_{mn} \) is the field strength of the antisymmetric field \( B_{mn} \), which was introduced in the BF term. This action just describes the interaction of open membranes whose boundaries are closed strings and is invariant under the following gauge transformations:

\[
\delta C_{mnp} = \partial_m \xi_{np} + \partial_n \xi_{pm} + \partial_p \xi_{mn}, \quad \delta B_{mn} = \partial_m \lambda_n - \partial_n \lambda_m - m \xi_{mn}. \tag{7}
\]

The \( \xi \) gauge transformation allows us gauged away the antisymmetric field \( B_{mn} \), leading to the massive antisymmetric field \( C_{mnp} \) action.

### 3 THE EQUIVALENCE

Now, we are going on to show the equivalence. Let us take the following master action

\[
I_M = \langle - \frac{1}{3!} m^2 a_{mnp} a^{mnp} - \frac{1}{2!} m^2 \psi^2 + \frac{1}{4!} m \epsilon^{mnpq} \psi G_{mnpq} + \frac{1}{3!} m \epsilon^{mnpq} (a_{mnp} - C_{mnp}) \partial_q \phi \rangle. \tag{8}
\]

Independent variations in \( a_{mnp}, \psi, C_{mnp} \) and \( \phi \) lead to the following equations of motion

\[
a^{mnp} = \frac{1}{m} \epsilon^{mnpq} \partial_q \phi, \tag{9}
\]

\[
\psi = \frac{1}{4!m} \epsilon^{mnpq} G_{mnpq}, \tag{10}
\]

\[
\epsilon^{mnpq} \partial_m (\psi - \phi) = 0, \tag{11}
\]

and

\[
\epsilon^{mnpq} \partial_q (a_{mnp} - C_{mnp}) = 0. \tag{12}
\]

Replacing the expressions for \( a_{mnp} \) and \( \psi \) given by eqs. (9) and (10) into \( I_M \), the gauge invariant action \( I_{M2} \) is obtained. On the other hand, the solutions of the equations of motion (11) and (12) are

\[
\phi - \psi = \omega, \quad C_{mnp} - a_{mnp} = \Omega_{mnp}. \tag{13}
\]
where $\omega$ and $\Omega_{mnp}$ are 0 and 3-closed forms, respectively. Locally, we can set

$$\omega = \text{constant}, \quad \Omega_{mnp} \equiv L_{mnp} = \partial_m l_{np} + \partial_n l_{pm} + \partial_p l_{mn},$$

and substituting into $I_M$, we obtain the following "Stuckelberg" action

$$I_s = \left< -\frac{1}{3!} m^2 (C_{mnp} - L_{mnp})(C_{mnp} - L_{mnp}) - \frac{1}{2} m^2 (\phi - \omega)^2 + \frac{1}{4!} m \epsilon_{mnpq} (\phi - \omega) G_{mnpq} \right>.$$  

This action is invariant under

$$\delta_\xi C_{mnp} = \partial_m \xi_{np} + \partial_n \xi_{pm} + \partial_p \xi_{mn}, \quad \delta_\xi l_{mn} = \xi_{mn},$$

which allow us gauged away the $l_{mn}$ field and recover $I_{M1}$ (we have redefined $\phi - \omega$ as $\phi$ since $\omega$ is a constant). In this way, the local equivalence is stated. This local equivalence can also be established from a Hamiltonian point of view and will be reported elsewhere \[10\]. In an ample sense, we must consider $\psi = \phi - \omega$ and $a_{mnp} = C_{mnp} - \Omega_{mnp}$ as the general solutions and $I_{M2}$ is locally and globally equivalent to

$$\bar{I}_M = \left< -\frac{1}{3!} m^2 (C_{mnp} - \Omega_{mnp})(C_{mnp} - \Omega_{mnp}) - \frac{1}{2} m^2 (\phi - \omega)^2 \right.\
\left. + \frac{1}{4!} m \epsilon_{mnpq} (\phi - \omega) G_{mnpq} - \frac{1}{3!} m \epsilon_{mnpq} \Omega_{mnp} \partial_q \phi \right>,$$

which is an adequate extension of $I_{M1}$.

Finally, we can eliminate $\phi$ and $C_{mnp}$ to achieve

$$\bar{I}_M = I_{M1[a, \psi]} - I_{top[\omega, \Omega]},$$

where

$$I_{top[\omega, \Omega]} = \left< \frac{1}{3!} m \epsilon_{mnpq} \Omega_{mnp} \partial_q \omega \right>.$$  

is the extension of the BF term for the topological coupling between 0 and 3-forms in four dimensions. From this result, we have that the partition functions of $I_{M1}$ and $I_{M2}$ differ by a topological factor.

$$Z_{M2} = Z_{top} Z_{M1}$$

In general, on manifolds with non trivial topological structure $Z_{top} \neq 1$. Only when the manifold has a trivial structure, we must have $Z_{top} \equiv 1$, reflecting the local equivalence.
Sumarizing, we have seen that a gauge invariant description for massive axions is possible which is (locally) equivalent to the non-gauge invariant proposal. Several aspects of this proposal are under consideration: a detailed hamiltonian description for both proposal of generating mass for the axion and a complete BRST analysis of the gauge invariant model considered in this paper\cite{10}.

4 REFERENCES

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