Simulated Symmetry in a Mental Model of Kinematics of Special Relativity in Liquid Environment

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Abstract. A mental model that in symmetrical relativistic form describes the behaviour of slow-moving objects such as watercrafts and groups of watercrafts in a water environment is constructed. The watercrafts are equipped with hardware components and perform metrological operations. The modelled objects conduct themselves in accordance with the formal laws similar to those of the special theory of relativity. Simulation of the symmetry in the proposed model makes it possible to simulate all known kinematic effects (including the constancy of the speed of light) of special theory of relativity in the water medium and to obtain Lorentz transforms.

1. Introduction

The presence of the arbitrary assumptions in the special theory of relativity allows modeling it in a liquid environment using elementary methods of classical mechanics. In particular special theory of relativity can be modeled based on the example of watercrafts that travel at the usual speeds in water.

At first glance, it is impossible to model the STR using the medium, if only for the reason that the processes on one of two identical bodies in motion relative to one another occur in the theory of relativity symmetrically. In models tied to the environment, however, if physical processes in the body A in motion occur more slowly then in the body B at rest, than the physical processes in the body B at rest occur more fast then in the body A i.e. the processes simulated in the medium are not symmetric.

Most of the work that deals with simulating the special relativity theory refers either to visualization (usually computer visualization), or to simulation of any particular effect of this theory [1-4]. Our goal was to simulate the special theory of relativity in such a way that the simulation would encompass all kinematic phenomena. The goal was achieved, but in our earlier works this was made unnecessarily confusing. In this paper, we tried to avoid the previous shortcomings.

2. Nonsymmetrical simulation of processes in rapidly moving bodies observed from single reference system

Consider individual ships and groups of ships located on the surface of a flat-bottomed water body with a depth of h, filled with slack water. The ships and the groups serve as the objects of our conceptual observation. The ships are equipped with hardware components that perform metrological operations. The hardware components have at their disposal sources of sonar signals. The latter travel between the ships and between the ships and the bottom. The velocity of the sonar signals equals V and is unapproachable for the other ships, i.e., the velocity of the ships, v, correspond to the inequality
$v < V$. We assume that we can monitor the behavior of the ships using optical instruments, but the hardware components of the ships do not have the means of communication with the speed of information transfer more than $V$.

Each ship is equipped with a clock, whose pendulum function performs a sonar signal that continuously travels along a plumb line (relative to this ship) between the ship and the bottom. Each sonar signal travel to the bottom and back requires a time of $\Delta t(1) = 2h/V_2$, where $V_2$ is the speed of submersion and surfaced of the signal. If the ship is at rest then $V_2 = V$. The sonar clock “mechanism” controls not only the clock’s hands, but also all the ship hardware components, thereby ensuring the proportionality of their working speed to the clock’s working speed.

Let’s simulate a solid body, presenting it in the form of a group of ships which are at the constant distance from each other. The procedure for maintaining distance between ships consists of the following.

A sonar signal is dispatched from each ship to the neighboring ship, and upon reaching it, the signal goes back again. The ship’s hardware components, using their clocks, measure the signal’s time of movement to the neighboring ship and back, then approach or move away from the neighboring ship as necessary in order to maintain this time and the immutability of the sonar “distance”.

Consider a group of ships located at the points of intersection of an imaginary coordinate grid of the $K'$ coordinate system. The group is in motion at a velocity of $v$ in the direction of the $X'$ axis (this axis lies on the water surface). As the ships move at a velocity of $v$, the $V_2$ submersion and surfaced speed of a signal traveling between a ship and the bottom along the hypotenuses of right-angled triangles is equal to $V \sqrt{1-(v/V)^2}$ [5-7]. Time on the moving ships, which we named simulated time $t'$, flows more slowly than time $t$ on ships at rest by $1/\sqrt{1-(v/V)^2}$ times [5-7].

Suppose that the method of maintaining the distance between the ships remains the same as in the resting groups.

It is easy to show that in this instance the lateral dimensions of the group are retained in this instance, however, the longitudinal dimensions (in the direction of the $X'$ axis) of the ship group in motion are contracted by $1/\sqrt{1-(v/V)^2}$ times.

Indeed, let us assume that a signal is sent out from and returns to a ship, $r'_{o'}$, that is moving within the $R'$ group and that is located at the origin of coordinates of the $K'$ system. The signal travels along the $Y'$ axis to a point with a coordinate of $y'$ where the neighboring ship, $r'_{y'}$, is moving in this same group. If the $Y'$ axis is located on the water surface perpendicular to the $X'$ axis, the signal then moves over the water surface along the hypotenuses of right-angled triangles at a velocity of $V$. This corresponds to the movement of a signal along the $Y'$ axis at a velocity of $V_Y$ in our time scales and with a length equaling $V \sqrt{1-(v/V)^2}$. Since the $t'$ time equals $t \sqrt{1-(v/V)^2}$, the simulated time of movement of the signal from ship $r'_{o'}$ to ship $r'_{y'}$ and back, $\Delta t'$, is independent of the speed of movement of the $R'$ group, as well as the distance between ships $r'_{o'}$ and $r'_{y'}$, and the hardware components perceive the group as unchanged when the velocity changes.

However, the longitudinal dimensions (in the direction of the $X'$ axis) of the ship group in motion are contracted for the following reason.

In negotiating the distance, $l_{o'y'}$, between ship $r'_{o'}$, which is located at the origin of coordinates, $O'$, and ship $r'_{y'}$, which is located on the $X'$ axis at a point with a coordinate of $x'$, the signal needs a $\Delta t_1$ time that equals $l_{o'y'}(V - v)$ in order to move from ship $r'_{o'}$ to ship $r'_{x'}$, and a $\Delta t_2$ time that equals $l_{o'y'}(V + v)$ for the trip back. The total time of movement, $\Delta t_1 + \Delta t_2$, from ship $r'_{o'}$ to ship $r'_{y'}$ and back comes to $2l_{o'y'}V(V^2 - v^2)$. According to the ship’s slow clocks, the $\Delta t'_{1} + \Delta t'_{2}$ time is $1/\sqrt{1-(v/V)^2}$ times shorter and comes to $2l_{o'y'}/V \sqrt{1-(v/V)^2}$. To save time as before the hardware components reduce the distance by $1/\sqrt{1-(v/V)^2}$ times.
Let’s now transit to an examination of the synchronization of the clocks of two groups of ships – group \( R \) and group \( R' \) – and the related coordinate systems, \( K \) and \( K' \). The \( R \) group and the \( K \) system are at rest on the water, while the \( R' \) group and the \( K' \) system are in motion on the water and relative to the \( R \) group at a velocity of \( v \).

Imagine that at a certain moment in time when the origin of coordinates and the axes of the \( K \) and \( K' \) coordinate systems coincide, the readings on all the ships of the ship groups at rest and in motion were reset to zero. From that moment in time forward, the synchronous change in the readings on all the ships of the ship group in motion occurs more slowly than the synchronous change in the readings on the ships of the group at rest.

If the hardware components on the ships of the group at rest, \( R \), track the clock of ship \( r' \) of the group in motion, \( R' \), which is moving past them, they then record the slowness of the clock rate on moving ship \( r' \). If the hardware components on the ships of the group in motion, \( R' \), track the clock on ship \( r \) of the \( R \) group, which moving past them, but is at rest relative to water, they then record the fastness of the clock rate on ship \( r \). There is no symmetry of any kind. What we have here is the lack of symmetry of the clock movement speeds on the ships at rest and in motion. The simulated time readings of the group in motion are linked to the time readings of the group at rest by the transformations \( t' = t \sqrt{1 - (v/V)^2} \) and \( t = t'/\sqrt{1 - (v/V)^2} \). The coordinate transformations take the form of \( x' = (x - vt)/\sqrt{1 - (v/V)^2} \), \( x = (x'+v't')\sqrt{1 - (v'/V')^2} \), and \( y' = y \), where the primed quantities are expressed in the distance and time scales of the ship group in motion.

It is clear that if the hardware components of the group in motion, \( R' \), now measure the speed of movement of a signal from one of the ships in their group to another ship in this same group using the synchronously running clocks on these ships, they will then find that the signal’s speeds of movement in the ship group’s direction of movement, which we see from the outside, and opposite its direction of movement, are different.

### 3. Simulation of symmetry of relativistic processes

Assume now that the hardware components on the \( R \) and \( R' \) group ships are not in contact with the water and have no information concerning their motion or rest relative to the water. Not finding a basis for synchronization during which the velocity of a signal back and forth is assumed to be different, the hardware components resynchronize the clocks within the group of ships in motion, \( R' \), so that the signal’s speed of movement there becomes identical to the signal’s speed of movement back. In this instance, the time after resynchronization, \( t'' \) – we named this \( t'' \) time the double simulated time – is linked to the simulated time, \( t' \), by the correlation \( t'' = t' - x'v/V^2 \), while the coordinates and the clock readings are linked by the transformations \( x' = (x - vt)/\sqrt{1 - (v/V)^2} \), \( y' = y \), and \( t'' = (t - xv/V^2)/\sqrt{1 - (v/V)^2} \), as well as \( x = (x'+v''t'')/\sqrt{1 - (v'/V')^2} \), \( y = y' \), and \( t = (t''+x''v'/V'')/\sqrt{1 - (v'/V')^2} \) [5-7]. Here, the quantities with double primes are expressed through the double simulated time. The transformations obtained are consistent with the direct and inverse Lorentz transformations to notational accuracy. In particular, this results in the fact that by tracking the clock rate of ship \( r \) at rest in the water, which is motionless relative to the water, but moves past a ship of the group in motion, the hardware components on the group of ships in motion, \( R' \), detect the slowness of the clock rate on ship \( r \). The results of the measurements made by the hardware components on the ships of the group in motion and at rest become symmetrical. The same thing is true of the distances.

### 4. Simulated “Space-Time”

In deriving the Lorentz transformations, we did, as a matter of fact, also simulate pseudo-Euclidean space-time, since these transformations ensure the invariance of the space-time interval in the \( K \) and \( K' \)
systems, as well as in any other systems associated with groups of ships in motion at different velocities. It is clear that this “space-time” has nothing at all to do with enigmatic multidimensional worlds and is an elementary mathematical construct that pertains more to the formalization of the measurement errors caused by the failure to take the presence of an aquatic environment into account than to the behavioral features of the ships on the water surface.

5. Conclusion
The special theory of relativity is closely linked to philosophical suppositions, and many of the problems that arise over the course of interpreting its physical content are philosophical in nature.

The special theory of relativity is very simple and does not involve any problems other than the problems of its interpretation. It can be described in the simplest language and using the simplest examples from our everyday life. There is nothing in it that is unintelligible or that signifies the four-dimensional world as inaccessible to human imagination.

The simulation presented in this paper demonstrates the simplicity of the special theory of relativity and its “earthiness”. It is not difficult to see that the possibility of using a “four-dimensional formalism” that does not differ from Minkowski formalism in this model issues from a simulation that yields Lorentz transformations. I.e., a most primitive model results in the possibility of describing the “operating principle” of this model in four-dimensional space-time.

A more detailed examination of four-dimensional space-time led us to the conclusion that space-time is a space of errors and must be relegated to the error theory.

References

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