Constraining the dark side with observations

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Abstract. The main purpose of this talk is to use the observational evidences pointing out to the existence of a dark side in the universe in order to infer some of the properties of the unseen material. We will work within the Unified Dark Matter models, in which both, Dark Matter and Dark Energy appear as the result of one unknown component. By modeling effectively this component with a classical scalar field minimally coupled to gravity, we will use the observations to constrain the form of the dark action. Using the flat rotation curves of spiral galaxies we will see that we are restringed to the use of purely kinetic actions, previously studied in cosmology by Scherrer. Finally we arrive to a simple action which fits both cosmological and astrophysical observations.

1. Introduction

Much evidence has appeared along the years pointing towards the existence of some dark components in the universe [1]. Basically, we can classify them into two groups: Dark Matter (DM) [2], a clustering component responsible for the formation of large scale structures, and Dark Energy (DE) [3], a non-clustering component permeating all the space which has made the universe to enter into a recent phase of accelerated expansion.

Several evidences in a broad spectrum of scales point to the existence of dark components. Starting with the DM, the first evidence appears at galactic scales. The main sign here is the flat rotation curves in the spiral galaxies [4]. If we consider that the circular orbits of the stars around a galaxy are stable due to the balance of the centrifugal and gravitational forces, the mass profile of the galaxy must be given by $M(r) = \rho v^2(r)/G$. It is observed that the velocity of rotation is approximately an increasing function of the distance until the edge of the visible galaxy, and that remains nearly constant from there on. The existence of some unobserved matter with an energy density profile $\rho \propto r^{-2}$ is then inferred. At larger scales, such as the scales of the galactic clusters, more evidence of unobserved matter appears.

The clusters as equilibrium systems, their age being much longer than their dynamical time-scale, the virial theorem $2 \langle T \rangle + \langle U \rangle = 0$ applies and the mass of the cluster can be evaluated with the result: $M = \alpha \langle v^2 \rangle R/G$. Here $\alpha$ is a constant of order unity which depends on the matter distribution, $M$ is the mass of the cluster and $R$ its radius. By determining the speed of motion of many galaxies within a cluster from the shifts of the spectral lines, Zwicky could infer already back in 1933 that the mass required to maintain the Coma cluster held together was by two orders of magnitude greater than the one observed [5]. Although the observational data in those days was not very accurate, the results have survived over the time. Also the study of gravitational lenses in galaxies [6] and galaxy clusters [7] suggests that there should be more matter than the one obtained by only counting stars. Yet more evidence for the existence of DM comes from cosmological scales.
The theory of cosmological nucleosynthesis and the abundances of light elements [8] restrict the amount of baryonic matter to \( \Omega_b \sim 0.05 \). This, together with the observations of the relative weights of the angular power spectrum peaks of the CMB anisotropies, points to the existence of non-baryonic DM in the universe with \( \Omega_{DM} \sim 0.25 \). Apart from this, non-baryonic DM seems to be an essential ingredient in producing the observed structures in the universe [9], providing the gravitational potential depressions where the galaxies are nourished and formed.

Regarding the DE, the bulk of the evidence comes from cosmological observations. Recent data from the CMB point out that we live in a flat \((k = 0)\), or a nearly flat universe [10]. This agrees with the predictions of the inflationary scenario [11] implying that one should have \( \Omega_{tot} \simeq 1 \). The energy density of the universe, therefore, must be very close to the critical value \( \rho_c \sim 10^{-29} \text{g/cm}^3 \), pointing towards the discrepancy with the observed density of matter (baryonic or dark) and the need of other unknown component(s) to add up the \( \Omega_{unk} \sim 0.70 \) needed for a flat universe. Furthermore, to explain the recent observations of type Ia supernovae [12], which suggest that we live in an accelerated universe, the presence of a new component which behaves like a Cosmological Constant is required.

2. The nature of the Dark Component

All the evidence cited above, obtained from very different observations carried out at very different scales, point to the same values of the dark parameters [13]. But, what could be said about the nature of these unknown components? Many candidates for the DM have been proposed along the years [2]. They range from elementary particles such as WIMPs (neutrinos, axions, neutralino, gravitino, etc.) to the ones in which the DM is formed by compact objects such as primordial black holes or MACHOs (brown dwarfs or Jupiter like objects). If one thing is clear, however, is that, as it was noted above, the DM can not consist of baryonic matter alone. A sensible model for DM should probably solve all the problems stated in the Introduction at the same time, yet no model proposed so far does the job.

Regarding the DE, the Cosmological Constant is probably the most straightforward choice. However, it has several theoretical drawbacks, like the discrepancies between the observed and the theoretical values (fine tuning problem) [3]. Apart from that, the fact that it is nearly equal to the matter density just now (coincidence problem) has perturbed cosmologists along the years. These facts have made them to consider it as a dynamical component, such as in the models of Quintessence [3, 14] or K-essence [15, 16]. Other proposals, in which DE appears as the result of the interaction between the fundamental particles have been also put forward [17].

Until now we have considered DM and DE as different physical objects. After all, we have seen in the Introduction that they behave very differently. But, could we still consider DM and DE as different manifestations of the same component? If we look at the dark sector as described effectively as a condensate whose physical properties depend on the scale, it could behave as DM at high densities \((p = 0 \text{ for } \rho \gg \rho_*)\) and transform into DE at lower densities \((p = -\rho \text{ for } \rho \ll \rho_*)\), where \( \rho_* \) is the critical density for the phase transition. These models in which the dark components appear as different manifestations of the same component are usually known as Unified Dark Matter [18, 19] and the Chaplygin gas \( p = -A/\rho \) [20] is its paradigmatic example.

Here, we consider that the dark sector can be modeled by a condensate effectively described by a classical scalar field. In Section 3 we will analyze some of the observations cited in the Introduction. They restrict the form of the metric tensor and, through the Einstein’s field equations, the form of the energy-momentum tensor for the dark component. Later, in Section 4 we review the most general classical scalar field actions minimally coupled to gravity, usually known as k-fields, which can be constructed from the scalar field itself and its first derivatives. We will consider the dark sector modeled by these actions. In Section 5 we will see how the constraints on the metric tensor obtained in Section 3 will restrict the form of the dark actions.
introduced in Section 4.

3. Observations

As its own name suggests, the dark side of the universe is dark, so obtaining any information about its nature is really complicated. In fact, we can only infer its existence through its gravitational effects on the visible matter, so the Einstein’s field equations will play a key role in the problem. They state that the space-time metric $g_{\mu\nu}$ and the energy-momentum tensor $T_{\mu\nu}$ are related through $G_{\mu\nu} = M_{\text{pl}}^{-2} T_{\mu\nu}$, where $G_{\mu\nu}$ is the Einstein tensor, built from the metric and its first and second derivatives. If we want to obtain any property of the dark component (encoded in $T_{\mu\nu}$), it must be through the “observation” of $g_{\mu\nu}$. Observing $g_{\mu\nu}$, however, is not easy. It would be great if we could probe dark configurations with test particles, as we do for instance in particle physics to study the microphysical properties of the nuclear interactions. However, although astronomy is not an experimental science, we can still use astronomical observations to infer and constrain the form of $g_{\mu\nu}$. Here we will use two different kind of observations to do so: cosmology and the rotation curves of the spiral galaxies.

3.1. Cosmology

One of the possible ways of constraining the dark sector is through cosmological observations. As a first approximation (zeroth order cosmology) the universe can be considered as homogeneous and isotropic (Cosmological Principle), at least at sufficiently large scales [21]. This guarantees that we can choose a suitable set of coordinates, called comoving coordinates, in which the space-time is described by the Friedman-Robertson-Walker (FRW) line-element:

$$ds^2 = (0)g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a(t)^2 \left[\frac{dx^2}{1- k x^2} + x^2 d\Omega^2\right].$$

Here, $k = -1, 0, +1$ for open, flat or closed universes respectively, $a(t)$ is the scale factor, $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$ is the metric of a unit sphere and $x$ is a comoving coordinate. $(0)g_{\mu\nu}$ is known as the Friedman-Robertson-Walker metric. Using the redshift parameter $1 + z = a_0/a(t)$ as time coordinate, where $a_0$ is the current value of the scale factor, the above line-element can be rewritten as:

$$ds^2 = \frac{1}{(1 + z)^2} \left[ -\frac{dz^2}{H^2(z)} + a_0^2 dr^2 \right].$$

Apart from the numerical value of $k$, which is now encoded in the 3 dimensional line-element of the spatial hypersurfaces $dr^2$, the only nontrivial metric function at this order is the Hubble parameter $H = \dot{a}/a$. It measures the rate of expansion of the universe. With the help of a new quantity called the luminosity distance $d_L(z)$ [21], defined from the Hubble parameter $H(z)$ and which can be measured with the use of high redshift supernovae type Ia as cosmological candles, the quantity $H(z)$ can be in principle deduced [12]. Given the current data, the widespread believe is that the universe has recently entered into a phase of accelerated expansion dominated by a Cosmological Constant like material [3].

Now, we can go a little further and consider the more realistic model in which we have small inhomogeneities around the homogeneous and isotropic background (1) (first order cosmology). This theory, also known as the Theory of Cosmological Perturbations [9], can be used to study the origin of the large scale structures observed in the current universe. The space-time line-element is given at this order by:

$$ds^2 = \left[(0)g_{\mu\nu} + \delta g_{\mu\nu}(x^\alpha)\right] dx^\mu dx^\nu,$$
where again \((0)g_{\mu\nu}\) is the FRW metric and \(\delta g_{\mu\nu}\) represents the small inhomogeneities of the space-time. For the linear theory to be applicable, \(|\delta g_{\mu\nu}| \ll |(0)g_{\mu\nu}|\) must be verified in some coordinates. These small inhomogeneities are expected to have been formed during an early phase of accelerated expansion, usually known as inflation [11, 9], after which they have evolved freely under the influence of the gravitational interaction. In this sense these small inhomogeneities generated in the early universe can be viewed as the primordial seeds for all the structures in the universe. The perturbations \(\delta g_{\mu\nu}\) can be decomposed into their scalar, vector and tensor components. In the linear regime all of them evolve independently. The vector modes are not interesting in cosmology, as they quickly dissipate with the expansion. The tensorial modes represent gravitational waves, whose fingerprints are expected to be detectable in the near future observations of the CMB anisotropies. Only the scalar modes contribute to the energy density perturbations and to the formation of large scale structures. Here the angular power spectrum of the CMB anisotropies and the galaxy clustering observations have the key.

3.2. Galactic curves
Apart from cosmology, there are also astrophysical observations we can use to constrain further the dark sector. They are, for instance, the profile of the rotation curves of the spiral galaxies. We consider that a typical galaxy is formed by a thin disk of visible matter immersed in a large halo built of some unseen exotic matter, which can be conveniently described as static and spherically symmetric. This exotic matter would be the main contributor to the dynamics, so that the observed luminous matter can be treated as a test fluid from which information about the physics of the halo can be inferred. One wants to know how much information about the matter making the halo can be figured out from the observed rotation curves. The most general static spherically symmetric metric with flat rotation curves is given by [22, 23]:

\[
ds^2 = -\left(\frac{r}{r_s}\right)^l dt^2 + A(r)dr^2 + r^2d\Omega^2.
\]

Here \(r_s\) is a constant parameter with dimensions of length and \(l = 2(v_c/c)^2\). Therefore, the domain of the parameter \(l\) is restricted to \(0 < l < 2\). To determine the metric function \(A(r)\) one needs to know more about the matter content. It is interesting to point out that the form of the line-element (2), as it stands, is generic in the sense that it does not depend on the metric theory of gravity used, nor it depends on the matter content. To obtain it one only assumes that the rotation curves are flat. In short, the profile of the rotation curves gives us the chance of reproducing completely the 00 component of the metric tensor, but tells nothing about the other independent component - the function \(A(r)\), which is related to the effective gravitational mass \(m(r)\) [22]. If we want to obtain some information about the function \(A(r)\), one must assume either the nature of the matter dominating the configuration, or deduce it from different observations, for example, gravitational lensing [22] etc.

In the observed galaxies the orbiting particles are non-relativistic, therefore \(l\) is a small parameter close to zero \((l < 10^{-5})\). The gravitational field far away from the centre, where super-massive black holes are expected to “hide”, is also small \((2m(r)/r \ll 1\) and \(2\phi(r) \ll 1\). The pressure inside the halo, in the standard models of galaxies, is also usually assumed to be close to zero \((p \ll \rho)\). If these three conditions are met, the Newtonian approximation applies and the results given in the Introduction are correct. Nevertheless, as we will see, depending on the choice of matter the last of the three above assumptions does not always correspond to physical reality.
4. The theoretical model for the Dark component

To proceed any further one should specify the physical properties of the dark component. The importance of scalar fields in modeling the universe is well known. Being an essential ingredient in the inflationary scenario, the scalar fields might drive the initial accelerated expansion. One can basically classify the scalar fields studied in the literature into two main classes: the canonical ones [11, 24], where the action is given by the sum of the standard kinetic and a potential term, and the more general actions which were recently introduced under the colloquial name of K-essence [15]. It is worthwhile mentioning that the non-canonical Lagrangians for the scalar field were studied earlier by Bekenstein and Milgrom [25] in connection with the Modified Newtonian Dynamics (MOND). They also appear naturally in the velocity potential formulation of the Relativistic Hydrodynamics [26].

We will consider the dark component as a condensate which can be modeled effectively by the most general action for a minimally coupled scalar field constructed from the scalar field and its first derivatives:

$$S = \int d^4x \sqrt{-g} \mathcal{L}(\varphi, X).$$

(3)

Here $\mathcal{L}(\varphi, X)$ is the Lagrangian density and $X$ the kinetic scalar defined by $X \equiv -1/2 \, g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$. These kind of actions are usually referred to as K-field, generalized scalar field or, in a cosmological setting, K-essence [15]. Special cases discussed in the literature are the factorisable K-field $\mathcal{L} = K(\varphi) F(X)$ [16] and the purely kinetic K-field $\mathcal{L} = F(X)$ [18]. The canonical scalar field $\mathcal{L} = X - V(\varphi)$ can be always rewritten as a factorisable K-field by a field redefinition.

The energy-momentum tensor for the action (3) is:

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = \partial_\mu \varphi \partial_\nu \varphi + \mathcal{L} g_{\mu\nu}.$$

Here one runs into two qualitatively different situations [27]. If the derivative term $\partial_\mu \varphi$ is timelike ($X > 0$), as it is usual in cosmology, we can identify the energy-momentum tensor with a perfect fluid $T_{\mu\nu} = (\rho + p) u_\mu u_\nu + pg_{\mu\nu}$ by the formal relations:

$$u_\mu = \frac{\partial_\mu \varphi}{\sqrt{2X}}, \quad p = \mathcal{L}, \quad \rho = 2X \frac{\partial \mathcal{L}}{\partial X} - \mathcal{L}.$$

(4)

Here $u_\mu$ is the 4-velocity, $\rho$ the energy density and $p$ the pressure. On the other hand, if the derivative term is spacelike ($X < 0$), this identification is no longer attainable. However, we can still identify the energy-momentum tensor with an anisotropic fluid $T_{\mu\nu} = (\rho + p_\perp) u_\mu u_\nu + pg_{\mu\nu} + (p_\parallel - p_\perp) n_\mu n_\nu$. Now, the formal relations are:

$$n_\mu = \frac{\partial_\mu \varphi}{\sqrt{-2X}}, \quad p_\perp = -\rho = \mathcal{L}, \quad p_\parallel = \mathcal{L} - 2X \frac{\partial \mathcal{L}}{\partial X}.$$

(5)

There are two different pressures in this case, one in the direction parallel to $n_\mu$ ($p_\parallel$) and the other one in an orthogonal direction ($p_\perp$). It is important to stress that the equality $p_\perp = -\rho$ for this sort of matter always holds, implying that the Newtonian approximation is no longer valid.

5. Constraining the dark action with observations

The Einstein’s field equations $G_{\mu\nu} = M_{pl}^{-2}T_{\mu\nu}$ relate the space-time metric with the properties of the dark sector, encoded in the energy-momentum tensor. In Section 3 we have inferred the form of the metric tensor from different astrophysical observations. Later, in Section 4, we have introduced the actions we use to effectively model the dark sector. That means, we have chosen the form of the dark $T_{\mu\nu}$ entering the Einstein’s equations. Now we want to obtain the constraints on these scalar actions (3) due to the observations.
5.1. Cosmology
At zeroth order, the space-time is described by the line-element (1). The homogeneity and isotropy of the universe restricts the scalar field to be of the form \( \phi = \varphi(t) \), which means that the derivative of the field is timelike \((X > 0)\) and the dark condensate can be viewed as a perfect fluid (4). Here we will restrict ourselves to the purely kinetic case \( \mathcal{L} = F(X) \), previously studied in cosmology by Scherrer [18]. As we will see in the following subsection (see also [28]), these actions are the only ones allowed to model the DM of the spiral galaxies. Possible physical motivations for these theories are discussed in [15, 29]. The Einstein’s field equations reduce in this case to

\[
3H^2 = \frac{M_{\text{pl}}^{-2} [2XF'(X) - F(X)]}{X} + 6H c_s^2(X) X = 0.
\]

The first one is the Friedmann equation, which dictates the expansion rate of the universe and the second one, known as the continuity equation, gives the evolution of the kinetic term. For convenience it is useful to define the parameters \( w \) and \( c_s^2 \), the barotropic index and the velocity of sound respectively, given by the following expressions:

\[
w = \frac{F(X)}{2XF'(X) - F(X)}, \quad c_s^2 = \frac{F'(X)}{2XF''(X) + F'(X)}.
\]

The barotropic index \( w \) gives the evolution of the scale factor of the universe (zeroth order cosmology) [21], while the sound speed \( c_s^2 \) dictates the evolution of the first order small perturbations (first order cosmology) [30]. We know that the evolution of the cosmological background interpolates between a DM universe at early times and a DE dominated universe at the late epoch [3]. There is no need to write down the solutions of the Einstein’s equations to see how a particular model behaves. As the universe is expanding \((H > 0)\) and given that the square of the speed of sound \((7)\) must remain always positive, the kinetic scalar \( X \) is a decreasing function of time \((6)\). So the barotropic index should evolve from “dust” \((w = 0)\) at early times \((X \gg X_s)\), to a “cosmological constant” \((w = -1)\) at late times \((X \ll X_s)\). This can be analyzed by looking at the expression for the barotropic index \((7)\) for a concrete \( F(X) \) and studying its behaviour for small and large values of \( X \). A typical example of Unified Dark Matter model is the Chaplygin gas \( F(X) = -\sqrt{1 - X} \).

Recent studies indicate, however, the existence of serious problems related with structure formation in universes dominated by the Chaplygin gas [31] (see however [32]). These problems appear mainly due to a high speed of sound during certain periods of the expansion. They may be alleviated in the so-called kinetic Unified Dark Matter (kUDM) models recently introduced by Scherrer [18]. Here, the problems are solved by working near an extremum of \( F(X) \), which guaranties that the sound speed remains well below the speed of light in the range of applicability of the model.

5.2. Rotation curves
Since we are only interested here in static, spherically symmetric configurations, restrictions on the function \( \varphi(t, r) \) and on the form of the action \( \mathcal{L}(\varphi, X) \) appear. The symmetries imply a diagonal Einstein and therefore a diagonal energy-momentum tensor, forcing the vanishing of the \( T_{tr} \) component. Therefore, either the scalar field is strictly static \( \varphi = \varphi(r) \), or it is strictly homogeneous \( \varphi = \varphi(t) \). The inhomogeneous time dependent scalar fields are not allowed by the symmetries imposed on the halo. Furthermore, the staticity of the spacetime ensures that the energy density and the pressure must be independent of time. This implies that for the case \( \varphi = \varphi(t) \) the action can only depend on the field derivatives \( \mathcal{L}(\varphi, X) = F(X) \), but not on the field itself. Moreover the scalar field \( \varphi \) may only be linear in time, \( \varphi = at \). It is interesting to observe that in the last case the spacetime does not inherit the symmetries of the fundamental field producing the geometry.
5.2.1. The static field $\varphi = \varphi(r)$ In this case the derivative of the field is spacelike ($X < 0$) and the scalar field, therefore, behaves as an anisotropic fluid (5). Solving the Einstein’s field equations, we find for the $rt$ component of the metric tensor:

$$A(r) = \frac{a_i}{1 \pm \left(\frac{R_i}{r}\right)^{b_i}}.$$  

Here $R_i$ is a positive arbitrary integration constant with dimensions of length. We have introduced the subindex $i$ to distinguish between the cases $\varphi = \varphi(r)$ ($i = r$) and $\varphi = \varphi(t)$ ($i = t$) of the following subsection. The strictly positive parameters $a_r$ and $b_r$ are given by:

$$a_r = \frac{4 - l^2}{4}, \quad b_r = 2 + l.$$

Simple analytic solutions can be easily obtained for the special case $R_r = 0$. We do not give the explicit solutions here due to their scarce physical relevance. It is easy to see that the energy density is always negative, therefore these solutions must be considered unphysical. This affects as well the solution presented in [23]. For the general case ($R_r \neq 0$) the situation is more complex and exact analytic expressions for the action can not be easily given. However, it can be seen that negative energy densities or masses are unavoidable [28].

Thus, one may consider this subsection as one leading to a kind of no-go result for the static scalar fields $\varphi = \varphi(r)$. Once we have assumed the form of the line-element suitable for the description of flat rotation curves (2), the minimally coupled generalised static scalar field is found unfit to play the role of the dark matter in the galactic halos.

5.2.2. The homogeneous field $\varphi = \varphi(t)$ The configuration $\varphi = \varphi(t)$, as mentioned above, is only possible if the action (3) is of the purely kinetic form $\mathcal{L}(\varphi, X) = F(X)$ - no potential term. This restricts the type of actions applicable to the rotation curves to those studied by Scherrer in the cosmological setting [18]. In this case, one may interpret the scalar field as an irrotational isentropic perfect fluid in a disguise [33], although the equation of state $p = p(\rho)$ does not have to be of a simple form. The action itself can be thought of as the hydrodynamical action written in terms of the velocity potential [26, 33]. Solving the Einstein’s field equations we obtain again the result (8) for $A(r)$, but now the parameters $a_t$ and $b_t$ are given by:

$$a_t = -\frac{l^2 - 4(l + 1)}{4}, \quad b_t = \frac{l^2 - 4(l + 1)}{l + 2}.$$  

For the special case when the constant $R_t = \infty$, an analytic solution can be easily found where the action is given by $F(X) \propto X^{2/l}$. This solution is analogous to the infinite isothermal sphere.

In the general case ($R_t \neq \infty$) the solution is rather more interesting. For our purposes, we will be interested in the $(-)$ branch of the equation (8). In this case, the complete solution can be derived from a scalar field obeying the following two-parameter family of actions:

$$F(X) = \left(\frac{M_{pl}}{R_t}\right)^2 \left[\rho_1^N \rho_2^N \frac{1}{X^{2/l} + X^{-N/l}}\right].$$  

For these fluids, the energy density and the pressure are:

$$\rho = \rho_1 \left(\frac{M_{pl}}{r}\right)^2 + \rho_2 \left(\frac{M_{pl}}{R_t}\right)^N, \quad p = \rho_1 \left(\frac{M_{pl}}{r}\right)^2 - \rho_2 \left(\frac{M_{pl}}{R_t}\right)^2 \left(\frac{r}{R_t}\right)^N,$$

where $\rho_1, \rho_2, \rho_1, p_2$ and $N$ are positive constants [28]. The solutions are only valid for $r < R_t$ due to the behaviour of the metric component given by the Eq.(8). It is interesting to point out
that it has been possible to obtain a simple action which describes the fluid (9) without referring to an explicit equation of state \( p = p(\rho) \).

The interesting point is that these solutions may be used to describe compact objects of finite size. The expression (10) indicates that the pressure is positive until some value \( r_0 \) is reached, then it vanishes and changes sign. It is thus possible to match the solution with an exterior Schwarzschild vacuum for \( r > r_0 \), defining \( r = r_0 \) as the halo external surface. We will use the subindex 0 to refer to the values evaluated on this surface. The expressions for the radius \( r_0 \) and for the effective gravitational mass \( m_0 \) (\( m_0 = 4\pi \int_0^{r_0} \rho(r) r^2 dr \)) of the compact objects are then given by:

\[
r_0 = \left( \frac{p_1}{p_2} \right)^{\frac{1}{1+\pi}} R_t, \quad m_0 = \frac{1}{2} \left[ \rho_1 + \frac{\rho_2}{3 + N} \frac{p_1}{p_2} \right] \frac{c^2 r_0}{G}.
\]

Since \( r_0 < R_t \), it is always possible to construct these solutions. We are only interested in the behaviour of the fluid within the halo, which in turn restricts the domain of the function \( F(X) \) to \( X \geq 1 \).

The expressions above are exact. However, we are more interested in a rule of thumb to work with, and since the observed rotation velocities in the galaxies are small compared to the speed of light, we have found it convenient to proceed working to first order in \( l \). To this order we propose the simplified action [28]

\[
F(X) = \left( \frac{M_{pl}}{R_t} \right)^2 \left[ X^{2/l} - 1 \right],
\]

which fits amazingly well with (9) within the range \( X \geq 1 \), where the equation (9) applies. This simple action for the homogeneous scalar field (11) can be used to model the DM in the galaxies. It should not be seen as a formal limit of the equation (9), but rather as a suggested, or guessed action describing the matter. This matter approximates the equation (9) within the halo, and therefore reproduces the desired geometry. Moreover, similar actions are used in cosmology under the different names of x-matter [34], wet dark fluid [35] and matter with the generalized linear equation of state [36]. Thus, the action (11) may potentially serve as a unified matter description both for cosmology (\( X < 1 \)), on one hand, and within the galactic halo (\( X > 1 \)), on the other. The two arbitrary constants in the model, \( R_t \) and \( l \), must be determined from the observations. A brief estimate of the order of magnitude of these parameters will be the task of the following section.

6. Fitting the model

6.1. Galaxies

We first adjust the two free parameters of the equation of state to fit a typical galaxy. As we have mentioned in the Subsection 3.2, the parameter \( l \) is directly related to the rotation velocities. We take \( l \sim 10^{-5} \) for a typical galaxy. The size and the mass of a galaxy are then given by the first order expressions:

\[
r_0 \approx \frac{l}{2} R_t, \quad m_0 \approx \frac{l c^2 r_0}{2 G}.
\]

If we take \( R_t \) of the order \( R_t \sim 3000 \) Mpc one obtains a compact object with \( r_0 \sim 15 \) Kpc and \( m_0 \sim 10^{12} M_\odot \), compatible with the size and the mass for a typical galactic halo. Curiously enough we had to assume the constant \( R_t \) of the order of the size of the observable universe to fit the observations. The last two equations can be combined to obtain a relation between the mass of a galaxy and the velocity of the orbiting particles:

\[
m_0 \approx \frac{R_t}{c^2 G} v_c^4.
\]
The equation above is nothing else but the Tully-Fisher relation \( m_0 = \kappa v_c^4 \), where the constant of proportionality \( \kappa \) is determined by the parameter \( R_t \) of the model.

### 6.2. Cosmology

Consider now the action (11) in a cosmological setting. Working still at first order in \( l \), the barotropic index and the velocity of sound (7) are given by the following expressions:

\[
w \simeq \frac{X^{2/l} - 1}{\frac{2}{l} X^{2/l} + 1}, \quad c_{s}^2 \simeq \frac{l}{4}.
\]

Using the analysis given in Subsection 5.1, we see that the barotropic index evolves from “dust” \( (w = 0) \) at early times \( (X \gg 1) \), to a “cosmological constant” \( (w = -1) \) at late times \( (X \ll 1) \). Furthermore, the speed of sound always remains small, \( c_s^2 \ll 1 \). The energy density for the effective value of the “cosmological constant” is determined by the parameter \( R_t \) fixed in the previous subsection against the galactic data, and is given by

\[
\rho_\Lambda = \frac{c_s^2}{8\pi G R_t^2} = 6.5 \cdot 10^{-30} \text{g/cm}^3.
\]

The value of the “cosmological constant” then becomes \( \Lambda_{\text{eff}} = R_t^{-2} \), which gives \( \Lambda_{\text{eff}} \sim 10^{-52}\text{cm}^{-2} \) [3]. We also obtain that there exists a relation between the value of the cosmological constant and the Tully-Fisher proportionality factor \( \kappa \):

\[
\kappa^2 \Lambda_{\text{eff}} = \frac{1}{G^2 c^4},
\]

determined only by the fundamental constants \( G \) and \( c \).

### 7. Conclusions

The main idea of this talk is to show how the observational traces pointing to the existence of a dark sector in the universe can be used to constrain the form of the unseen material. We have worked within the Unified Dark Matter models, in which Dark Matter and Dark Energy appear as different manifestations of the same component. To model this component we have used classical scalar field actions minimally coupled to gravity, usually known as k-fields. The symmetries imposed by the dark halos restrict the action to be of the purely kinetic form, that means, to depend only on the derivatives of the field but not on the field itself. These actions have been analyzed before in a cosmological setting, providing interesting results in the unification of Dark Matter and Dark Energy. Here we have gone a little further giving a simple action which fits not only the cosmological observations, but the rotational curves of the spiral galaxies also. The use of other astrophysical observations can give further constraints on the dark actions.

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