Scalar form-factors $f_{\pi\pi}(Q^2)$ and $f_{KK}(Q^2)$ with light-cone QCD sum rules

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Abstract

In this article, we calculate the scalar form-factors $f_{\pi\pi}(Q^2)$ and $f_{KK}(Q^2)$ in the framework of the light-cone QCD sum rules approach. The numerical value of the $f_{\pi\pi}(Q^2)$ changes quickly with variation of $Q^2$ near zero momentum transfer, while the $f_{KK}(Q^2)$ has rather good behavior at small momentum transfer. The value $f_{KK}(0) = 2.21_{-0.19}^{+0.35}$ GeV is compatible with the result from the leading order chiral perturbation theory. At large momentum transfer with $Q^2 > 6$ GeV$^2$, the form-factor $f_{\pi\pi}(Q^2)$ takes up the asymptotic behavior $1/Q^2$ approximately, while the $f_{KK}(Q^2)$ decreases more quickly than $1/Q^2$.

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1 Introduction

In the standard model, the gauge symmetry $SU(2) \times U(1)$ is spontaneously broken down by the nonvanishing vacuum expectation value $v$ of the Higgs field, the fermions obtain their masses through Yukawa couplings with the Higgs field. For the light mass Higgs, the main decay channels maybe $\pi\pi$ and $\mu\mu$, although the Yukawa coupling is very small $\sim 1/v$, the scalar form-factor $f_{\pi\pi}(t)$ enters the process $H \rightarrow \pi\pi$. However, the scalar form-factor $f_{\pi\pi}(t)$ is a highly nonperturbative quantity, not a directly measurable quantity. Omnes representation and Watson theorem can relate it with $\pi\pi$ and $KK$ scattering data in the spin $J = 0$ and isospin $I = 0$ channel [2]. It is not unexpected, in the timelike region,

$$\langle \pi\pi|\bar{u}u + \bar{d}d|0\rangle = \langle \pi\pi|\bar{u}u + \bar{d}d|0\rangle + \langle \pi\pi|\mathcal{T}|\pi\pi\rangle \langle \pi\pi|\bar{u}u + \bar{d}d|0\rangle + \cdots.$$  

(1)

The scattering matrix elements $\mathcal{T}$ have copious information and can be confronted with the experimental data. The scalar form-factor $f_{\pi\pi}(t)$ has been calculated with the chiral perturbation theory up to two-loop order now [2]. In the limit $t = 0$, $\langle \pi(p)|m_u\bar{u}u + m_d\bar{d}d|\pi(p)\rangle (= m^2_\pi)$ and $\langle K(p)|m_u\bar{u}u + m_s\bar{s}s|K(p)\rangle (= m^2_K)$ are often referred to as $\sigma$ terms of the mesons $\pi$ and $K$. Just like $\sigma$ terms of the nucleons [3], they can put a severe constraint on the scalar form-factors at zero momentum transfer.

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Semileptonic decays $K \to \pi \ell \nu$ ($K_{\ell 3}$) provide the most precise determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{us}|$ [4]. The experimental input parameters are the semileptonic decay widths and the vector form-factors $f_{K\pi}^+(q^2)$ and $f_{K\pi}^-(q^2)$, which are necessary in calculating the phase space integrals. The main uncertainty in the quantity $|V_{us} f_{K\pi}^+(0)|$ comes from the unknown shape of the hadronic form-factor $f_{K\pi}^+(q^2)$, which is measurable at $m_{\ell}^2 < q^2 < (m_K - m_\pi)^2$ in $K_{\ell 3}$ decays or at $(m_K + m_\pi)^2 < q^2 < m_\tau^2$ in $\tau \to K\pi\nu$ decays. The experimental data can be fitted to the functions with either pole models or series expansions; though systematic errors are introduced due to the different parameterizations.

In the limit $t = 0$, the scalar form-factor $f_{K\pi}(0)$ has the value $f_{K\pi}(0) = \frac{m_K^2 - m_\pi^2}{m_s - m_u} f_{K\pi}^0(0) = \frac{m_K^2 - m_\pi^2}{m_s - m_u} f_{K\pi}^+(0)$. Conservation of the vector current implies $f_{K\pi}(0) = 1$ at zero momentum transfer [5]. If the $SU(3)$ symmetry breaking effects in the scalar channels are small, the scalar form-factors $f_{\pi\pi}(0)$ and $f_{KK}(0)$ would have the value about $1.7 GeV$, which is also expected from the leading order chiral perturbation theory [2].

In this article, we calculate the value of the scalar form-factor $f_{\pi\pi}(Q^2)$ (and $f_{KK}(Q^2)$ as byproduct) in the framework of the light-cone QCD sum rules approach. In previous works, the scalar form-factors of the nucleons, which relate with the $\sigma$ terms of the nucleons, have been calculated with the light-cone QCD sum rules approach [6]. The light-cone QCD sum rules approach carries out operator product expansion near the light-cone, $x^2 \approx 0$, instead of the short distance, $x \approx 0$, while the nonperturbative matrix elements are parameterized by the light-cone distribution amplitudes (which are classified according to their twists) instead of the vacuum condensates [7, 8]. The nonperturbative parameters in the light-cone distribution amplitudes are calculated by the conventional QCD sum rules and the values are universal [9].

The article is arranged as: in Section 2, we derive the scalar form-factors $f_{\pi\pi}(Q^2)$ and $f_{KK}(Q^2)$ with the light-cone QCD sum rules approach; in Section 3, the numerical results and discussions; and in Section 4, conclusions.

## 2 Scalar form-factors $f_{\pi\pi}(Q^2)$ and $f_{KK}(Q^2)$ with light-cone QCD sum rules

In the following, we write down the definitions for the scalar form-factors $f_{\pi\pi}(q^2)$ and $f_{KK}(q^2)$:

\[
\begin{align*}
\langle \pi(q + p)|\bar{u}(0)u(0) + \bar{d}(0)d(0)|\pi(p)\rangle &= 2f_{\pi\pi}(q^2), \\
\langle K(q + p)|\bar{s}(0)s(0)|K(p)\rangle &= f_{KK}(q^2).
\end{align*}
\] 

(2)
We study the scalar form-factors $f_{\pi\pi}(q^2)$ and $f_{KK}(q^2)$ with the two-point correlation functions $\Pi_{\mu}^\pi(p, q)$ and $\Pi_{\mu}^K(p, q)$ respectively,

$$\Pi_{\mu}^\pi(p, q) = i \int d^4x e^{-iq\cdot x} \langle 0| T \{ J_{\mu}^\pi(0) J_d(x) \} |\pi(p)\rangle, \quad (3)$$

$$\Pi_{\mu}^K(p, q) = i \int d^4x e^{-iq\cdot x} \langle 0| T \{ J_{\mu}^K(0) J_s(x) \} |K(p)\rangle, \quad (4)$$

where the axial-vector currents $J_{\mu}^\pi(x)$ and $J_{\mu}^K(x)$ interpolate the $\pi$ and $K$ mesons respectively. The correlation functions $\Pi_{\mu}^P(p, q)$ (thereafter the $P$ denotes the pseudoscalar mesons $K$ and $\pi$) can be decomposed as follows:

$$\Pi_{\mu}^P(p, q) = i \Pi^P_p(q^2, (q + p)^2) p_{\mu} + i \Pi^P_q(q^2, (q + p)^2) q_{\mu}, \quad (6)$$
due to Lorentz covariance. In this article, we derive the sum rules with the tensor structures $p_{\mu}$ and $q_{\mu}$ respectively.

According to the basic assumption of current-hadron duality in the QCD sum rules approach [9], we can insert a complete series of intermediate states with the same quantum numbers as the current operators $J_{\mu}^P(x)$ into the correlation functions $\Pi_{\mu}^P(p, q)$ to obtain the hadronic representation. After isolating the ground state contributions from the pole terms of the $\pi$ and $K$ mesons, the correlation functions $\Pi_{\mu}^P(p, q)$ can be expressed in the following form:

$$\Pi_{\mu}^P(p, q) = \frac{if_P}{m_P^2 - (q + p)^2} \left\{ f_{PP}^P(q^2)p_{\mu} + f_{PP}^q(q^2)q_{\mu} \right\} + \cdots, \quad (7)$$

where we have not shown the contributions from the high resonances and continuum states explicitly, they are suppressed after Borel transformation and subtraction. We introduce up-indexes $p$ and $q$ to denote the form-factors with the tensor structures $p_{\mu}$ and $q_{\mu}$ respectively. We use the standard definitions for the weak decay constants $f_P$,

$$\langle 0| J_{\mu}^P(0)|P(p)\rangle = if_P p_{\mu}. \quad (8)$$

In the following, we briefly outline operator product expansion for the correlation functions $\Pi_{\mu}^P(p, q)$ in perturbative QCD theory. The calculations are performed at large spacelike momentum regions $P^2 = -(q + p)^2 \gg 0$ and $Q^2 = -q^2 \gg 0$, which correspond to small light-cone distance $x^2 \approx 0$ required by validity of the operator product expansion approach. We write down the propagator of a massive quark in
the external gluon field in Fock-Schwinger gauge first \[10\]:

\[
\langle 0 | T \{ q_i(x_1) \bar{q}_j(x_2) \} | 0 \rangle = i \int \frac{d^4k}{(2\pi)^4} e^{-ik(x_1-x_2)}
\]

\[
\left\{ \frac{k + m}{k^2 - m^2} \delta_{ij} - \int_0^1 dv g_s G_{ij}^{\mu\nu}(vx_1 + (1-v)x_2) \right. \\
\left. \left[ \frac{1}{2} \frac{k + m}{k^2 - m^2} \gamma_{\mu
u} - \frac{1}{k^2 - m^2} v(x_1 - x_2)_{\mu\nu} \right] \right\}, \tag{8}
\]

where \( G_{\mu\nu} \) is the gluonic field strength, \( g_s \) denotes the strong coupling constant. Substituting the above \( d, s \) quark propagators and the corresponding \( \pi, K \) mesons light-cone distribution amplitudes into the correlation functions \( \Pi^\pi_\mu \) and \( \Pi^K_\mu \) in Eqs.(3-4), and completing the integrals over the variables \( x \) and \( k \), finally we obtain the representations at the level of quark-gluon degrees of freedom. The explicit expressions are given in the appendix.

In calculation, we have used the two-particle and three-particle \( K \) and \( \pi \) mesons light-cone distribution amplitudes \[7, 8, 10\]. The explicit expressions of the \( K \) meson light-cone distribution amplitudes are presented in the appendix, the corresponding ones for the \( \pi \) meson can be obtained by simple substitution of the nonperturbative parameters. The parameters in the light-cone distribution amplitudes are scale dependent and can be estimated with the QCD sum rules approach \[7, 8, 10\]. In this article, the energy scale \( \mu \) is chosen to be \( \mu = 1\text{GeV} \).

We take Borel transformation with respect to the variable \( P^2 = -(q + p)^2 \) for the correlation functions \( \Pi^p_\mu \) and \( \Pi^P_\mu \), and obtain the analytical expressions for those invariant functions. After matching with the hadronic representations below the thresholds, we obtain the following four sum rules for the scalar form-factors \( f^p_{PP}(q^2) \) and \( f^q_{PP}(q^2) \):
\[ f_{\pi\pi}^p(q^2) = \frac{m_{\pi}^2}{m_u + m_d} \int_{\Delta_{\pi}}^1 du \phi_p(u) e^{-DD} \]

\[ -m_dm_{\pi}^2 \int_{\Delta_{\pi}}^1 du \int_0^u dt \frac{B(t)}{uM^2} e^{-DD} \]

\[ -\frac{1}{6} m_{\pi}^2 \int_{\Delta_{\pi}}^1 du \phi_{\sigma}(u) \left\{ \left[ 1 - u \frac{d}{du} \right] \frac{1}{u} + \frac{2m_d^2}{u^2M^2} \right\} e^{-DD} \]

\[ +m_d \int_{\Delta_{\pi}}^1 du \left\{ \frac{\phi_{\pi}(u)}{u} - \frac{m_{\pi}^2 m_d^2 A(u)}{4u^3 M^4} \right\} e^{-DD} \]

\[ +\frac{f_{3\pi}}{f_\pi} \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_d \varphi_{3\pi}(\alpha_u, \alpha_g, \alpha_d) \Theta(u - \Delta_{\pi}) \]

\[ \left\{ \frac{(2v - 3)m_{\pi}^2}{uM^2} + 2v \frac{d}{du} \frac{1}{u} \right\} e^{-DD} \bigg|_{u=(1-v)\alpha_g + \alpha_d} \]

\[ -2m_dm_{\pi}^4 \int_0^1 dv \int_0^1 d\alpha_g \int_0^{\alpha_g} d\beta \int_0^{1-\beta} d\alpha \]

\[ \Phi(1 - \alpha - \beta, \beta, \alpha) \Theta(u - \Delta_{\pi}) e^{-DD} \bigg|_{u=(1-v)\alpha_g + \alpha_d} \]

\[ +2m_dm_{\pi}^4 \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_d \int_0^{\alpha_d} d\alpha \]

\[ \Phi(1 - \alpha - \alpha_g, \alpha_g, \alpha) \Theta(u - \Delta_{\pi}) e^{-DD} \bigg|_{u=(1-v)\alpha_g + \alpha_d} \]

\[ +m_dm_{\pi}^2 \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_d \]

\[ \Psi(\alpha_u, \alpha_g, \alpha_d) \Theta(u - \Delta_{\pi}) e^{-DD} \bigg|_{u=(1-v)\alpha_g + \alpha_d} , \]

(9)
\[
\frac{f^q_{\pi\pi}(q^2)}{m_{\pi}^2} = \frac{m_{\pi}^2}{m_u + m_d} \int_{\Delta \pi}^{1} du \frac{\phi_p(u)}{u} e^{-DD} \\
- m_d m_{\pi}^2 \int_{\Delta \pi}^{1} du \int_{0}^{u} dt \frac{B(t)}{u^2 M^2} e^{-DD} \\
+ \frac{1}{6} \frac{m_{\pi}^2}{m_u + m_d} \int_{\Delta \pi}^{1} du \phi_\sigma(u) \frac{d}{du} \frac{1}{u} e^{-DD} \\
+ \frac{3 \pi}{m_{\pi}} \int_{0}^{1} dv \int_{0}^{1} d\alpha_g \int_{0}^{1-\alpha_g} d\alpha_d \Phi(\alpha_u, \alpha_g, \alpha_d) \Theta(u - \Delta \pi) \\
\Theta(u - \Delta \pi) \frac{2v - 3}{u^2 M^2} e^{-DD} \bigg|_{u=(1-v)\alpha_g + \alpha_d} \\
- 2m_d m_{\pi}^4 \int_{0}^{1} dv \int_{0}^{1} d\alpha_g \int_{0}^{1-\alpha_g} d\beta \int_{0}^{1-\beta} d\alpha \\
\Phi(1 - \alpha - \beta, \beta, \alpha) \Theta(u - \Delta \pi) \frac{1}{u^3 M^4} e^{-DD} \bigg|_{u=(1-v)\alpha_g + \alpha_d} \\
+ 2m_d m_{\pi}^4 \int_{0}^{1} dv \int_{0}^{1} d\alpha_g \int_{0}^{1-\alpha_g} d\alpha_d \int_{0}^{\alpha_d} d\alpha \\
\Phi(1 - \alpha - \alpha_g, \alpha_g, \alpha) \Theta(u - \Delta \pi) \frac{1}{u^3 M^4} e^{-DD} \bigg|_{u=(1-v)\alpha_g + \alpha_d} , \tag{10}
\]
\[ f_{KK}(q^2) = \frac{m_K^2}{m_u + m_s} \int_{\Delta_K}^1 du \phi_p(u) e^{-EE} \]

\[ -m_s m_K^2 \int_{\Delta_K}^1 du \int_0^u dt \frac{B(t)}{uM^2} e^{-EE} \]

\[ -\frac{1}{6} \frac{m_K^2}{m_u + m_s} \int_{\Delta_K}^1 du \phi_s(u) \left\{ \left[ 1 - u \frac{d}{du} \right] \frac{1}{u} + \frac{2m_s^2}{u^2M^2} \right\} e^{-EE} \]

\[ +m_s \int_{\Delta_K}^1 du \left\{ \frac{\phi_K(u)}{u} - \frac{m_K^2 m_s^2 A(u)}{4w^3M^4} \right\} e^{-EE} \]

\[ +\frac{f_{3K}}{f_K} \int_0^1 dv \int_0^1 \alpha_g \int_0^{1-\alpha_g} d\alpha_s \varphi_{3K}(\alpha_u, \alpha_g, \alpha_s) \Theta(u - \Delta_K) \]

\[ \left\{ \frac{(2v - 3)m_K^2}{uM^2} + 2v \frac{d}{du} \frac{1}{u^2M^2} \right\} e^{-EE} |_{u=(1-v)\alpha_g+\alpha_s} \]

\[ -2m_s m_K^4 \int_0^1 dv \int_0^1 \int_0^{1-\alpha_g} d\alpha_g \int_0^{1-\beta} d\beta \int_0^{1-\alpha} d\alpha \]

\[ \Phi(1 - \alpha - \beta, \beta, \alpha) \Theta(u - \Delta_K) e^{-EE} |_{u=1-\nu_g} \]

\[ +2m_s m_K^4 \int_0^1 dv \int_0^1 \int_0^{1-\alpha_g} d\alpha_g \int_0^{1-\alpha_g} d\alpha_s \int_0^{1-\alpha} d\alpha \]

\[ \Phi(1 - \alpha - \alpha_g, \alpha_g, \alpha) \Theta(u - \Delta_K) e^{-EE} |_{u=(1-v)\alpha_g+\alpha_s} \]

\[ +m_s m_s^2 \int_0^1 dv \int_0^1 \int_0^{1-\alpha_g} d\alpha_g \int_0^{1-\alpha_g} d\alpha_s \]

\[ \Psi(\alpha_u, \alpha_g, \alpha_s) \Theta(u - \Delta_K) e^{-EE} |_{u=(1-v)\alpha_g+\alpha_s}, \]

(11)
where

\[
\sigma = \frac{m_K^2}{m_s} \int_{\Delta_K}^1 du \frac{\phi_p(u)}{u} e^{-EE} \\
- m_s m_K^2 \int_{\Delta_K}^1 du \int_0^u dt \frac{B(t)}{u^2 M^2} e^{-EE} \\
+ \frac{1}{6 m_u + m_s} \int_{\Delta_K}^1 du \phi_u(u) \frac{d}{du} e^{-EE} \\
+ \frac{f_{3K}}{f_K} m_K^2 \int_{\Delta_K}^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_s \varphi_{3K}(\alpha_u, \alpha_g, \alpha_s) \\
\Theta(u - \Delta_K) \frac{2v - 3}{u^2 M^2} e^{-EE} \bigg|_{u=(1-v)\alpha_g + \alpha_s} \\
- 2 m_s m_K^4 \int_{\Delta_K}^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\beta \int_0^{1-\beta} d\alpha \\
\Phi(1 - \alpha - \beta, \beta, \alpha) \Theta(u - \Delta_K) e^{-EE} \bigg|_{u=(1-v)\alpha_g + \alpha_s},
\]

(12)

where

\[
DD = \frac{m_\pi^2 - u^2 m_\pi^2 - (1 - u)q^2}{u M^2}, \\
EE = \frac{m_K^2 - u^2 m_K^2 - (1 - u)q^2}{u M^2}, \\
\Delta_\pi = \frac{m_\pi^2 - q^2}{s_\pi^0 - q^2}, \\
\Delta_K = \frac{m_K^2 - q^2}{s_K^0 - q^2}.
\]

(13)

Where the \(s_\pi^0\) and \(s_K^0\) are threshold parameters for the interpolating currents \(J_{\mu}^\pi(x)\) and \(J_{\mu}^K(x)\), respectively.

### 3 Numerical results and discussions

The input parameters of the light-cone distribution amplitudes are taken as \(\lambda_3 = 1.6 \pm 0.4\), \(f_{3K} = (0.45 \pm 0.15) \times 10^{-2}\) GeV\(^2\), \(\omega_3 = -1.2 \pm 0.7\), \(\omega_4 = 0.2 \pm 0.1\), \(a_2 = 0.25 \pm 0.15\), \(a_1 = 0.06 \pm 0.03\), \(\eta_1 = 0.6 \pm 0.2\) for the \(K\) meson; \(\lambda_3 = 0.0\), \(f_{3\pi} = (0.45 \pm 0.15) \times 10^{-2}\) GeV\(^2\), \(\omega_3 = -1.5 \pm 0.7\), \(\omega_4 = 0.2 \pm 0.1\), \(a_2 = 0.25 \pm 0.15\), \(a_1 = 0.0\), \(\eta_4 = 10.0 \pm 3.0\) for the \(\pi\) meson \([7, 8, 10]\); and \(m_s = (137 \pm 27)\) MeV.
\(m_u = m_d = (5.6 \pm 1.6) MeV\), \(f_K = 0.16 GeV\), \(f_\pi = 0.13 GeV\), \(m_K = 498 MeV\), \(m_\pi = 138 MeV\). The threshold parameters are chosen to be \(s^0_K = 1.1 GeV^2\) and \(s^0_0 = 0.8 GeV^2\), which can reproduce the values of the decay constants \(f_K = 0.16 GeV\) and \(f_\pi = 0.13 GeV\) in the QCD sum rules approach.

The Borel parameters in the four sum rules are taken as \(M^2 = (0.6 - 2.0) GeV^2\), in this region, the values of the form-factors \(f_{\pi\pi}(Q^2)\) and \(f_{KK}(Q^2)\) are rather stable. In this article, we take the special values \(M^2 = 1.2 GeV^2\) for the \(f_{\pi\pi}(Q^2)\) and \(M^2 = 1.5 GeV^2\) for the \(f_{KK}(Q^2)\) in numerical calculations. Although such a definite Borel parameter cannot take into account some uncertainties, the predictive power cannot be impaired qualitatively.

From the four sum rules, we observe that the main contributions come from the two-particle twist-3 light-cone distribution amplitudes, not the twist-2 light-cone distribution amplitudes. The contributions from the twist-2 light-cone distribution amplitudes are suppressed by the extra factors of small masses \(m_d\) or \(m_s\). For the heavy-light form-factors \(B \to \pi, K\), the contributions from the twist-2 light-cone distribution amplitudes are enhanced by the extra factor of large mass \(m_u\), we can take the chiral limit for the masses of the mesons \(K\) and \(\pi\); the contributions from the two-particle twist-3 light-cone distribution amplitudes are very small and can be neglected safely.

The uncertainties of the seven parameters \(f_{3K}(f_{3\pi})\), \(a_2\), \(a_1\), \(\lambda_3\), \(\omega_3\), \(\omega_4\) and \(\eta_4\) can only result in small uncertainties for the numerical values. The main uncertainties come from the two parameters \(m_s\) and \(m_q(m_u, m_d)\); the variations of those parameters can lead to large changes for the numerical values, we should refine the input parameters \(m_q\) (for \(f_{\pi\pi}(Q^2)\)) and \(m_s\) (for \(f_{KK}(Q^2)\)), especially the \(m_q\) to improve the predictive ability. It is a difficult work.

Taking into account all the uncertainties, finally we obtain the numerical values of the scalar form-factors \(f_{\pi\pi}(Q^2)\) and \(f_{KK}(Q^2)\), which are shown in Figs.1-3. At zero momentum transfer,

\[
\begin{align*}
  f_{3\pi}^p(0) & = -0.56^{+0.21}_{-0.20} GeV, \\
  f_{3\pi}^q(0) & = 0.81^{+0.51}_{-0.52} GeV, \\
  f_{KK}^p(0) & = 0.32^{+0.24}_{-0.23} GeV, \\
  f_{KK}^q(0) & = 2.21^{+0.35}_{-0.39} GeV. \\
\end{align*}
\]

In the light-cone QCD sum rules approach, we carry out operator product expansion near the light-cone \(x^2 \approx 0\), which corresponds to \(Q^2 \gg 0\) and \(P^2 \gg 0\). The four sum rules \(f_{PP}^p(Q^2)\) and \(f_{PP}^q(Q^2)\) can be taken as some functions that model the scalar form-factors \(f_{PP}(Q^2)\) at large momentum transfer, we extrapolate them to zero momentum transfer with an analytical continuation.

We can borrow some ideas from the electromagnetic \(\pi\)-photon form-factor \(f_{\gamma\pi\gamma}(Q^2)\). The value of \(f_{\gamma\pi\gamma}(0)\) is fixed by partial conservation of the axial current and the effective anomaly lagrangian, \(f_{\gamma\pi\gamma}(0) = \frac{1}{\pi f_\pi^2}\). In the limit of large-\(Q^2\), perturbative QCD predicts that \(f_{\gamma\pi\gamma}(Q^2) = \frac{4\pi f_\pi^2}{Q^2}\). The Brodsky-Lepage interpolation formula
Figure 1: $f^{p}_{\pi\pi}(Q^2)(A)$, $f^{q}_{\pi\pi}(Q^2)(B)$, $f^{p}_{KK}(Q^2)(C)$ and $f^{q}_{KK}(Q^2)(D)$ at the range $Q^2 = (1 - 6)\text{GeV}^2$. 
Figure 2: $f_{\pi\pi}(Q^2)(A)$, $f_{\pi\pi}(Q^2)(B)$, $f_{KK}(Q^2)(C)$ and $f_{KK}(Q^2)(D)$ at the range $Q^2 = (0 - 1) GeV^2$. 
Figure 3: \( f_{\pi\pi}(Q^2) \) at the range \( Q^2 = (0 - 0.03) GeV^2 \).

\[ f_{\gamma^*\pi^0}(Q^2) = \frac{1}{\pi f_\pi [1 + Q^2/(4\pi^2 f_\pi^2)]} = \frac{1}{\pi f_\pi (1 + Q^2/s_0)} \]

can reproduce both the value at \( Q^2 = 0 \) and the behavior at large-\( Q^2 \). The energy scale \( s_0 \) (\( s_0 = 4\pi^2 f_\pi^2 \approx 0.67 GeV^2 \)) is numerically close to the squared mass of the \( \rho \) meson, \( m_\rho^2 \approx 0.6 GeV^2 \). The Brodsky-Lepage interpolation formula is similar to the result of the vector meson dominance approach, \( f_{\gamma^*\pi^0}(Q^2) = 1/\{\pi f_\pi (1 + Q^2/m_\rho^2)\} \). In the latter case, the calculation is performed at the timelike energy scale \( q^2 < 1 GeV^2 \) and the electromagnetic current is saturated by the vector meson \( \rho \), where the mass \( m_\rho \) serves as a parameter determining the pion charge radius. With a slight modification of the mass parameter, \( m_\rho = \Lambda_\pi = 776 MeV \), the experimental data can be well described by the single-pole formula at the interval \( Q^2 = (0 - 10) GeV^2 \).

In Ref.\[13\], the four form-factors of \( \Sigma \rightarrow n \) have satisfactory behaviors at large \( Q^2 \), which are expected by naive power counting rules, and they have finite values at \( Q^2 = 0 \). The analytical expressions of the four form-factors \( f_1(Q^2), f_2(Q^2), g_1(Q^2) \) and \( g_2(Q^2) \) are taken as Brodsky-Lepage type of interpolation formulae, although they are calculated at rather large \( Q^2 \), the extrapolation to lower energy transfer has no solid theoretical foundation. The numerical values of \( f_1(0), f_2(0), g_1(0) \) and \( g_2(0) \) are compatible with the experimental data and theoretical calculations (in magnitude).

In Ref.\[13\], the vector form-factors \( f_{K\pi}^{+}(Q^2) \) and \( f_{K\pi}^{-}(Q^2) \) are also taken as Brodsky-Lepage type of interpolation formulae, the behaviors of low momentum...
transfer are rather good in some channels.

In this article, we take the scalar form-factors $f_{PP}^p(Q^2)$ and $f_{PP}^q(Q^2)$ as Brodsky-Lepage type of interpolation formulae, unfortunate, the low energy behaviors (for $Q^2 < 0.03 GeV^2$) of the $f_{PP}^p(Q^2)$ and $f_{PP}^q(Q^2)$ are rather bad.

It is obvious that the model functions $f_{PP}^p(Q^2)$ and $f_{PP}^q(Q^2)$ may have good or bad low-$Q^2$ behaviors, although they have solid theoretical foundation at large momentum transfer. We extrapolate the model functions tentatively to zero momentum transfer, systematic errors maybe very large and the results maybe unreliable. The predictions merely indicate the possible values of the light-cone QCD sum rules approach, they should be confronted with the experimental data or other theoretical approaches.

In the limit $Q^2 = 0$, $\Delta_K \approx 0.017$ and $\Delta_\pi \approx 0.00004$. Although the terms proportional to $\frac{1}{u^2} \exp\{−DD\}$ and $\frac{1}{u^3} \exp\{−EE\}$ have finite values, the contributions from the end-point are greatly enhanced. Comparing with the $f_{PP}^p(Q^2)$, the $f_{PP}^q(Q^2)$ have more terms with the extra factor of $\frac{1}{u^2}$. It is not unexpected that the $f_{PP}^q(Q^2)$ have larger values than the corresponding $f_{PP}^p(Q^2)$ at small momentum transfer, which are shown in Fig.2.

If we take the value $Q^2 = (0.01 − 0.02) GeV^2$, $\Delta_\pi \approx 0.012 − 0.024$ and $\Delta_K \approx 0.026 − 0.035$. The $f_{\pi\pi}^q(Q^2)$ has larger contributions from the end-point of the light-cone distribution amplitudes than the $f_{KK}^q(Q^2)$. Without nice cancelation among the end-point dominating terms, such an infrared behavior can result in that the $f_{\pi\pi}^q(Q^2)$ changes quickly with variation of $Q^2$ at $Q^2 < 0.03 GeV^2$, which is shown explicitly in Fig.3 (also Fig.2).

One can adjust the input parameters to cancel the infrared enhancement, however, the input parameters are calculated with the QCD sum rules approach [7,8,10], they are not free parameters. We should introduce extra phenomenological form-factors (for example, the Sudakov factor [16]) to suppress the contribution from the end-point. It is somewhat of fine-tuning.

The vector form-factor $f_{K\pi}^+(q^2)$ and scalar form-factor $f_{K\pi}^0(q^2)$ (with the relation $f_{K\pi}^0(q^2) = f_{K\pi}^+(q^2) + \frac{q^2}{m_K^2 - m_{\pi}^2} J_{K\pi}(q^2)$) are measured in $K_{e3}$ decays with the squared momentum $q^2$ transfer to the leptons, where $q^2 > m_l^2$. The curves (or shapes) of the form-factors are always parameterized by the linear model, quadratic model and pole models to carry out the integrals in the phase space. The normalization is always chosen to be $f_{K\pi}^+(0)$, i.e. $f_{K\pi}^0(q^2) = f_{K\pi}^+(0) \left\{1 + \lambda_1 q^2 + \lambda_2 q^4 + \cdots\right\}$, etc, the parameters $\lambda_1, \lambda_2, \cdots$ can be fitted by $\chi^2$, etc [18].

In the limit $q^2 = 0$, $f_{K\pi}^0(0) = f_{K\pi}^+(0) \approx 1$. The vector form-factor $f_{K\pi}^+(Q^2)$ has been calculated by the ChPT [20], lattice QCD [19], (light-cone) QCD sum rules [13,21], etc. If the $SU(3)$ symmetry works well in the scalar channels, the values of the $f_{\pi\pi}(0)$ and $f_{KK}(0)$ would not differ from the value of the scalar form-factor $f_{K\pi}(0)$ greatly, $f_{K\pi}(0) = \frac{m_K^2 - m_{\pi}^2}{m_s^2 - m_{\pi}^2} J_{K\pi}(0) \approx 1.7 GeV$. The leading order chiral form-factors are always parameterized by the linear model, quadratic model and pole models to carry out the integrals in the phase space. The normalization is always chosen to be $f_{K\pi}^+(0)$, i.e. $f_{K\pi}^0(q^2) = f_{K\pi}^+(0) \left\{1 + \lambda_1 q^2 + \lambda_2 q^4 + \cdots\right\}$, etc, the parameters $\lambda_1, \lambda_2, \cdots$ can be fitted by $\chi^2$, etc [18].

Current algebra predicts the value of the scalar form-factor $f_{K\pi}^0(\Delta)$ be $f_{K\pi}^0(\Delta) = -f_{K\pi}/f_{\pi}$ at Callan-Treiman point $\Delta = m_K^2 - m_{\pi}^2$ [17].
perturbation theory also predicts that \( f_{\pi\pi}(0) = f_{KK}(0) = f_{\pi\pi}(0) \approx 1.7 GeV \) [2]. The numerical value \( f_{KK}^q(0) = 2.21^{+0.35}_{-0.19} GeV \) makes sense, not very bad.

In Fig.4, we plot the form-factors \( f_{PP}^p(Q^2) \) and \( f_{PP}^q(Q^2) \) at the range \( Q^2 = (0 - 15) GeV^2 \). From Fig.4, we can see that the curves (or shapes) of \( Q^2 f_{\pi\pi}^p(Q^2) \) and \( Q^2 f_{\pi\pi}^q(Q^2) \) are rather flat at \( Q^2 > 6 GeV^2 \), which means that at large momentum transfer, the scalar form-factor \( f_{\pi\pi}(Q^2) \) takes up the asymptotic behavior \( f_{\pi\pi}(Q^2) \sim \frac{1}{Q^2} \) approximately. It is expected from naive power counting rules [22], the terms proportional to \( \frac{1}{Q^{2n}} \) with \( n \geq 2 \) are canceled out approximately with each other. The scalar form-factor, axial form-factor and induced pseudoscalar form-factor of the nucleons show the behavior \( \frac{1}{Q^2} \) at large \( Q^2 \) [6, 14, 23], which is also expected from naive power counting rules [22]. The curves (or shapes) of \( Q^2 f_{KK}^p(Q^2) \) and \( Q^2 f_{KK}^q(Q^2) \) at \( Q^2 < 6 GeV^2 \) are analogous to the electromagnetic form-factors of the \( K \) and \( \pi \) mesons. At large momentum transfer with \( Q^2 > 6 GeV^2 \), the terms of the \( f_{KK}(Q^2) \) proportional to \( \frac{1}{Q^{2n}} \) with \( n \geq 2 \) manifest themselves, which results in the curves (or shapes) of \( Q^2 f_{KK}(Q^2) \) decreasing with increasing \( Q^2 \).
The scalar form-factors are complex functions of the input parameters, in principle, they can be expanded in terms of Taylor series of $\frac{1}{Q^2}$. At large momentum transfer, for example, $Q^2 = (6 - 15) GeV^2$, the central values of the four form-factors can be fitted numerically as

$$f_{\pi\pi}^p(Q^2) = \frac{1.9}{Q^2} - \frac{2.0}{Q^4},$$
$$f_{\pi\pi}^q(Q^2) = \frac{2.0}{Q^2} - \frac{2.1}{Q^4},$$
$$f_{KK}^p(Q^2) = \frac{1.1}{Q^2} + \frac{4.3}{Q^4},$$
$$f_{KK}^q(Q^2) = \frac{1.2}{Q^2} + \frac{4.5}{Q^4}. \quad (15)$$

The form-factor $f_{\pi\pi}(Q^2)$ has larger $\frac{1}{Q^2}$ dependence and smaller $\frac{1}{Q^4}$ dependence than the $f_{KK}(Q^2)$. Although the analytical expressions of $f_{PP}^p(Q^2)$ (or $f_{PP}^q(Q^2)$) have the same type of $Q^2$ dependence, the coefficients are quantitatively different from each other due to the $SU(3)$ symmetry breaking effects for the mesons $\pi$ and $K$. At large momentum transfer, $\Delta_{\pi} = \frac{m_{\pi}^2 + Q^2}{2 m_{\pi}^2} \approx 1$ and $\Delta_{K} = \frac{m_{K}^2 + Q^2}{2 m_{K}^2} \approx 1$, the extra factor of $\frac{1}{m}$ in the scalar form-factors $f_{PP}^q(Q^2)$ will not play any significant roles, the form-factor $f_{PP}^p(Q^2)$ and the corresponding $f_{PP}^q(Q^2)$ approach almost the same form of $Q^2$ dependence.

4 Conclusions

In this article, we calculate the scalar form-factors $f_{\pi\pi}(Q^2)$ and $f_{KK}(Q^2)$ in the framework of the light-cone QCD sum rules approach. The scalar form-factor $f_{\pi\pi}(t)$ enters the light Higgs decay $H \rightarrow \pi\pi$, and it is not a directly measurable quantity. The scalar form-factors $f_{\pi\pi}(0)$ and $f_{KK}(0)$ relate with the $\sigma$ terms of the $\pi$ and $K$ mesons, respectively. Just like the $\sigma$ terms of the nucleons, they are highly nonperturbative quantities. The numerical values of the $f_{\pi\pi}^p(Q^2)$ and $f_{\pi\pi}^q(Q^2)$ change quickly with the variation of $Q^2$ near zero momentum transfer, while the $f_{KK}^p(Q^2)$ and $f_{KK}^q(Q^2)$ have rather good behaviors at small momentum transfer. The value $f_{KK}^q(0) = 2.21^{+0.35}_{-0.19} GeV$ is compatible with the result from the leading order chiral perturbation theory. At large momentum transfer with $Q^2 > 6 GeV^2$, the form-factor $f_{\pi\pi}(Q^2)$ takes up the asymptotic behavior of $\frac{1}{Q^2}$ approximately, while the $f_{KK}(Q^2)$ decreases more quickly than $\frac{1}{Q^2}$.
Appendix

The light-cone distribution amplitudes of the $K$ meson are defined as follows:

\[
\langle 0| \bar{u}(0) \gamma_\mu \gamma_5 s(x) | K(p) \rangle = i f_K p_\mu \int_0^1 du e^{-i p \cdot u} \left\{ \phi_K(u) + \frac{m_K^2 x^2}{16} A(u) \right\}
+ i f_K m_K^2 \frac{x_\mu}{2 p \cdot x} \int_0^1 du e^{-i p \cdot u} B(u),
\]

\[
\langle 0| \bar{u}(0) i \gamma_5 s(x) | K(p) \rangle = \frac{f_K m_K^2}{m_s + m_u} \int_0^1 du e^{-i p \cdot u} \phi_\sigma(u),
\]

\[
\langle 0| \bar{u}(0) \sigma_{\mu \nu} \gamma_5 g_s G_{\alpha \beta}(vx) s(x) | K(p) \rangle = f_K \left\{ (p_\mu p_\nu g_{\nu \beta} - p_\nu p_\alpha g_{\mu \beta}) - (p_\mu p_\beta g_{\nu \alpha} - p_\nu p_\alpha g_{\mu \beta}) \right\} \int D \alpha_i \varphi 3K(\alpha_i) e^{-ip \cdot (\alpha_s + v \alpha_s)},
\]

\[
\langle 0| \bar{u}(0) \gamma_\mu \gamma_5 g_s G_{\alpha \beta}(vx) s(x) | K(p) \rangle = \frac{p_\mu}{p \cdot x} f_K m_K^2 \int D \alpha_i A_{\perp}(\alpha_i) e^{-ip \cdot (\alpha_s + v \alpha_s)} + f_K m_K^2 (p_\beta g_{\alpha \mu} - p_\alpha g_{\beta \mu}) \int D \alpha_i A_{\parallel}(\alpha_i) e^{-ip \cdot (\alpha_s + v \alpha_s)},
\]

\[
\langle 0| \bar{u}(0) \gamma_\mu \gamma_5 g_s G_{\alpha \beta}(vx) s(x) | K(p) \rangle = \frac{p_\mu}{p \cdot x} f_K m_K^2 \int D \alpha_i V_{\perp}(\alpha_i) e^{-ip \cdot (\alpha_s + v \alpha_s)} + f_K m_K^2 (p_\beta g_{\alpha \mu} - p_\alpha g_{\beta \mu}) \int D \alpha_i V_{\parallel}(\alpha_i) e^{-ip \cdot (\alpha_s + v \alpha_s)},
\]

where the operator $\tilde{G}_{\alpha \beta}$ is the dual of $G_{\alpha \beta}$, $\tilde{G}_{\alpha \beta} = \frac{1}{2} \epsilon_{\alpha \beta \mu \nu} G^{\mu \nu}$, $D \alpha_i$ is defined as $D \alpha_i = d \alpha_1 d \alpha_2 d \alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$. The light-cone distribution amplitudes are
parameterized as follows:

\[
\begin{align*}
\phi_K(u, \mu) &= 6u(1-u) \left\{ 1 + a_1C_1^3(2u - 1) + a_2C_2^3(2u - 1) + a_4C_4^3(2u - 1) \right\}, \\
\phi_p(u, \mu) &= 1 + \left\{ 30\eta_3 - \frac{5}{2} \rho^2 \right\} C_2^3(2u - 1) \\
&\quad + \left\{ -3\eta_3\omega_3 - \frac{27}{20} \rho^2 - \frac{81}{10} \rho^2 a_2 \right\} C_4^3(2u - 1), \\
\phi_\sigma(u, \mu) &= 6u(1-u) \left\{ 1 + \left[ 5\eta_3 - \frac{1}{2} \eta_3\omega_3 - \frac{7}{20} \rho^2 - \frac{3}{5} \rho^2 a_2 \right] C_2^3(2u - 1) \right\}, \\
\varphi_{3K}(\alpha_i, \mu) &= 360\alpha_u\alpha_s\alpha_g^2 \left\{ 1 + \lambda_3(\alpha_u - \alpha_s) + \omega_3 \frac{1}{2}(7\alpha_g - 3) \right\}, \\
V_\parallel(\alpha_i, \mu) &= 120\alpha_u\alpha_s\alpha_g (v_{00} + v_{10}(3\alpha_g - 1)), \\
A_\parallel(\alpha_i, \mu) &= 120\alpha_u\alpha_s\alpha_g a_{10}(\alpha_g - \alpha_u), \\
V_\perp(\alpha_i, \mu) &= -30\alpha_g^2 \left\{ h_{00}(1 - \alpha_g) + h_{01}[\alpha_g(1 - \alpha_g) - 6\alpha_u\alpha_s] \\
&\quad + h_{10} \left\{ \alpha_g(1 - \alpha_g) - \frac{3}{2}(\alpha_u^2 + \alpha_s^2) \right\} \right\}, \\
A_\perp(\alpha_i, \mu) &= 30\alpha_g^2(\alpha_u - \alpha_s) \left\{ h_{00} + h_{01}\alpha_g + \frac{1}{2} h_{10}(5\alpha_g - 3) \right\}, \\
A(u, \mu) &= 6u(1-u) \left\{ \frac{16}{15} + \frac{24}{35} a_2 + 20\eta_3 + \frac{20}{9} \eta_4 \\
&\quad + \left\{ -\frac{1}{15} + \frac{1}{16} - \frac{7}{27} \eta_3\omega_3 - \frac{10}{27} \eta_4 \right\} C_2^3(2u - 1) \\
&\quad + \left\{ -\frac{11}{210} a_2 - \frac{4}{135} \eta_3\omega_3 \right\} C_4^3(2u - 1) \right\} + \left\{ -\frac{18}{5} a_2 + 21\eta_4\omega_4 \right\} \\
&\quad \{ 2u^3(10 - 15u + 6u^2) \log u + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2) \log \bar{u} + u\bar{u}(2 + 13u\bar{u}) \right\}, \\
g_K(u, \mu) &= 1 + g_2C_2^3(2u - 1) + g_4C_4^3(2u - 1), \\
B(u, \mu) &= g_K(u, \mu) - \phi_K(u, \mu),
\end{align*}
\]
where
\[ h_{00} = v_{00} = -\frac{\eta_4}{3}, \]
\[ a_{10} = \frac{21}{8} \eta_4 + \frac{9}{20} a_2, \]
\[ v_{10} = \frac{21}{8} \eta_4, \]
\[ h_{01} = \frac{7}{4} \eta_4 \pm \frac{3}{20} a_2, \]
\[ h_{10} = \frac{7}{2} \eta_4 + \frac{3}{20} a_2, \]
\[ g_2 = 1 + \frac{18}{7} a_2 + 60 \eta_3 + \frac{20}{3} \eta_4, \]
\[ g_4 = -\frac{9}{28} a_2 - 6 \eta_3 \omega_3, \] (18)

here \( C_2^\pi(\xi), C_4^\pi(\xi), C_2^\tau(\xi), C_2^\pi(\xi) \) and \( C_4^\pi(\xi) \) are Gegenbauer polynomials, \( \eta_3 = \frac{f_{4K} m_u + m_d}{m_K} \) and \( \rho^2 = \frac{(m_u + m_d)^2}{m_K^2} \) [7, 8, 10].

The explicit expressions of the correlation functions at the level of quark-gluon degrees of freedom:

\[
\Pi_\pi = \frac{f_\pi m^2_\pi}{m_u + m_d} \int_0^1 du \frac{u \phi_\pi(u)}{m^2_d - (q + up)^2} - m_d f_\pi m^2_\pi \int_0^1 du \int_0^u dt \frac{u B(t)}{m^2_d - (q + up)^2} \]
\[ - \frac{1}{6} \frac{f_\pi m^2_\pi}{m_u + m_d} \int_0^1 du \phi_\sigma(u) \left\{ \left[ 1 - \frac{d}{du} \right] \frac{1}{m^2_d - (q + up)^2} + \frac{2m^2_d}{(m^2_d - (q + up)^2)^2} \right\} \]
\[ + m_d f_\pi \int_0^1 du \left\{ \frac{\phi_\pi(u)}{m^2_d - (q + up)^2} - \frac{m^2_d m^2_\pi}{2} \frac{A(u)}{m^2_d - (q + up)^2} \right\} \]
\[ + f_3 \int_0^1 dv \int_0^1 du \int_0^{1-\alpha_g} d\alpha_3 \int_0^{\alpha_g} d\alpha_2 \varphi_3(\alpha_u, \alpha_g, \alpha_d) \]
\[ \left\{ \frac{(2v - 3) u m^2_\pi}{(m^2_d - (q + up)^2)^2} + 2v \frac{d}{du} \frac{1}{m^2_d - (q + up)^2} \right\} \big|_{u=(1-v)\alpha_g+\alpha_d} \]
\[ - 4m_d f_\pi m^4_\pi \int dv \int_0^1 du \int_0^{\alpha_g} d\alpha_3 \int_0^{1-\beta} d\beta \int_0^{\alpha_g} d\alpha_2 \frac{u \Phi(1 - \alpha - \beta, \beta, \alpha)}{(m^2_d - (q + up)^2)^3} \big|_{u=(1-v)\alpha_g+\alpha_d} \]
\[ + 4m_d f_\pi m^4_\pi \int dv \int_0^1 du \int_0^{\alpha_g} d\alpha_3 \int_0^{1-\alpha_g} d\alpha_2 \int_0^{\alpha_g} d\alpha_1 \frac{u \Phi(1 - \alpha - \alpha_g, \alpha_g, \alpha)}{(m^2_d - (q + up)^2)^3} \big|_{u=(1-v)\alpha_g+\alpha_d} \]
\[ + m_d f_\pi m^2_\pi \int dv \int_0^1 du \int_0^{\alpha_g} d\alpha_3 \int_0^{1-\alpha_g} d\alpha_2 \frac{\Psi(\alpha_u, \alpha_g, \alpha_d)}{(m^2_d - (q + up)^2)^2} \big|_{u=(1-v)\alpha_g+\alpha_d}, \] (19)
\[ \Pi^K_p = \frac{f_K m^2_K}{m_u + m_s} \int_0^1 du \frac{1}{m^2_u - (q + up)^2} - m_s f_K m^2_K \int_0^1 du \int_0^u \frac{u B(t)}{\{m^2_u - (q + up)^2\}^2} \]
- \frac{1}{6} f_K m^2_K \int_0^1 du \frac{d}{du} \left\{ \left[ 1 - u \frac{d}{du} \right] \frac{1}{m^2_u - (q + up)^2} + \frac{2m^2_s}{[m^2_u - (q + up)^2]^3} \right\}
+ m_s f_K \int_0^1 du \left\{ \frac{\phi_K(u)}{m^2_s - (q + up)^2} - \frac{m^2_K m^2_s}{2 \{m^2_u - (q + up)^2\}^3} \right\}
+ f_3 K \int_0^1 dv \int_0^1 da \int_0^{1 - \alpha_s} d\alpha_s \Psi_3 K(\alpha_u, \alpha_g, \alpha_s)
\left\{ \frac{(2v - 3) u m^2_K}{m^2_s - (q + up)^2} + 2v \frac{d}{du} \frac{1}{m^2_s - (q + up)^2} \right\} \bigg|_{u = (1 - v) \alpha_g + \alpha_s}
- 4 m_s f_K m^4_K \int_0^1 dv \int_0^1 da \int_0^{1 - \alpha_s} d\alpha_s \Psi_3 K(\alpha_u, \alpha_g, \alpha_s) \bigg|_{u = (1 - v) \alpha_g + \alpha_s}
+ m_s f_K m^2_K \int_0^1 dv \int_0^1 da \int_0^{1 - \alpha_s} d\alpha_s \left\{ \frac{\phi_K(u)}{m^2_u - (q + up)^2} - \frac{m^2_K m^2_s}{2 \{m^2_u - (q + up)^2\}^3} \right\}
\bigg|_{u = (1 - v) \alpha_g + \alpha_s}, \quad (20) \]

\[ \Pi^K_q = \frac{f_{\pi} m^2_{\pi}}{m_u + m_d} \int_0^1 du \frac{\phi_p(u)}{m^2_u - (q + up)^2} - m_d f_{\pi} m^2_{\pi} \int_0^1 du \int_0^u \frac{B(t)}{\{m^2_d - (q + up)^2\}^2} \]
+ \frac{1}{6} f_{\pi} m^2_{\pi} \int_0^1 du \frac{d}{du} \left\{ \frac{1}{m^2_u - (q + up)^2} \right\}
+ f_{3 \pi} m^2_{\pi} \int_0^1 dv \int_0^1 da \int_0^{1 - \alpha_g} d\alpha_s \left\{ \frac{2v - 3}{m^2_u - (q + up)^2} \right\} \bigg|_{u = (1 - v) \alpha_g + \alpha_d}
- 4 m_d f_{\pi} m^4_{\pi} \int_0^1 dv \int_0^1 da \int_0^{1 - \alpha_g} d\alpha_s \left\{ \frac{\phi_{1 - \beta, \beta, \alpha}}{m^2_u - (q + up)^2} \right\} \bigg|_{u = (1 - v) \alpha_g + \alpha_d}
+ 4 m_d f_{\pi} m^4_{\pi} \int_0^1 dv \int_0^1 da \int_0^{1 - \alpha_g} d\alpha_s \int_0^{1 - \alpha_d} d\alpha_3 \left\{ \frac{\phi_{1 - \beta, \beta, \alpha}}{m^2_d - (q + up)^2} \right\} \bigg|_{u = (1 - v) \alpha_g + \alpha_d}, \quad (21) \]
\[ \Pi^K_q = \frac{f_K m^2_K}{m_u + m_s} \int_0^1 du \frac{\phi_p(u)}{m^2_s - (q + up)^2} - m_s f_K m^2_K \int_0^1 du \int_0^u dt \frac{B(t)}{\{m^2_s - (q + up)^2\}^2} \]

\[ + \frac{1}{6} \frac{f_K m^2_K}{m_u + m_s} \int_0^1 du \phi_p(u) \frac{d}{du} \frac{1}{m^2_s - (q + up)^2} \]

\[ + f_3 K m^2_K \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_s \Phi_3 K(\alpha_u, \alpha_g, \alpha_s) \frac{2v - 3}{\{m^2_s - (q + up)^2\}^2} \bigg|_{u=(1-v)\alpha_g+\alpha_s} \]

\[ - 4m_s f_K m^4_K \int_0^1 dv \int_0^1 d\alpha_g \int_0^{\alpha_g} d\beta \int_0^{1-\beta} d\alpha \frac{\Phi(1-\alpha - \beta, \beta, \alpha)}{\{m^2_s - (q + up)^2\}^3} \bigg|_{u=(1-v)\alpha_g+\alpha_s} \]

\[ + 4m_s f_K m^4_K \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_s \int_0^{\alpha_s} d\alpha \frac{\Phi(1-\alpha - \alpha_g, \alpha_g, \alpha)}{\{m^2_s - (q + up)^2\}^3} \bigg|_{u=(1-v)\alpha_g+\alpha_s} , \]

(22)

where \( \Psi = A_\parallel - V_\parallel - 2A_\perp + 2V_\perp \) and \( \Phi = A_\perp + A_\parallel - V_\perp - V_\parallel \).

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