On T-duality for open strings in general abelian and nonabelian gauge field backgrounds

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Abstract

We discuss T-duality for open strings in general background fields both in the functional integral formulation as well as in the language of canonical transformations. The Dirichlet boundary condition in the dual theory has to be treated as a constraint on the functional integration. Furthermore, we give meaning to the notion of matrix valued string end point position in the presence of nonabelian gauge field background.
1 Introduction

Target space duality and its implications for string theory have been studied to a great extent and in many details, see [1, 2] and references therein. All these investigations refer to closed strings. However, due to the crucial observation [3] that Dirichlet branes [4] play a prominent role in the general duality pattern of string theory there is now a considerable activity on these D-branes including aspects of their T-duality relations to open strings.

Generalized $\sigma$-models on 2D manifolds with boundary describing open strings in the background of target space fields corresponding to excitations of both closed and open strings are the natural framework to proceed from strings in flat (and in some directions compactified) target space to the more general situation of nontrivial backgrounds and to derive the open string extension of Buscher’s formulae [5]. Besides the early paper [6] this question has been addressed only very recently [7].

Although the basic new effect of T-duality in the open string case, the mapping of Neumann to mixed Neumann and Dirichlet boundary conditions follows immediately from the closed string duality transformation $\partial_m X \rightarrow \epsilon_{mn} \partial_n X$, a more careful analysis is necessary to uncover the precise meaning of the boundary condition in the quantized 2D world sheet field theory. At the classical level it makes no difference whether the boundary condition is treated as an external constraint on the fields competing in the action principle or whether these fields are varied freely on the boundary and the boundary condition arises as part of the stationarity condition of the action on the same footing as the equation of motion in the bulk. However, in the quantized theory both procedures may yield different theories.

Our aim is to contribute to a systematic study of the 2D field theoretical aspects of the boundary in T-duality. We will investigate a model containing target space fields corresponding to the metric $G$, the antisymmetric tensor $B$, the dilaton $\Phi$, a gauge field $A$ and a technically motivated field $V$ [6]. The model can be used to deal with the open oriented bosonic string ($B \neq 0$, gauge group $U(N)$) or with the type I superstring in nonvanishing bosonic backgrounds ($B = 0$, gauge group $SO(N)$). Only local world sheet effects will be considered.

In section 2 we treat the case of an abelian gauge field via functional integral manipulations. Section 3 presents a formalism to handle nonabelian gauge fields thereby giving meaning to the notation of open strings bound with their end points to a matrix D-brane. Section 4 supplements the functional integral treatment by a look at the implications of the presence of a boundary on the interpretation of T-duality as a canonical transformation.

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2 A preliminary version of our paper has been reported by H.D. at the workshop “Physics beyond the standard model”, Bad Honnef, March 11-14. While preparing this manuscript ref. [7] appeared with partial overlap in the abelian case.
2 Functional integral treatment of T-duality

Our starting point is a generalized $\sigma$-model action describing a string moving in background fields corresponding to massless excitations of both closed ($\psi = (G_{\mu\nu}, B_{\mu\nu}, \Phi)$) and open string states (gauge field $A_\mu$, for simplicity first of abelian type)

$$
S[\psi, A, V; X] = \frac{1}{4\pi\alpha'} \int_M d^2z \sqrt{-g} \left( \partial_m X^\mu \partial_n X^\nu E_{\mu\nu}^{mn} + \alpha' R^{(2)}(\Phi)(X) \right) + \int_{\partial M} \left( A_\mu(X(z(s))) \partial_n X^\mu(z(s)) \dot{z}^n + V_\mu \partial_m X^\mu \frac{\epsilon_{mn}}{\sqrt{-g}} \dot{z}_n(s) - \frac{1}{2\pi k(s)} \Phi \right) ds ,
$$

$$
E_{\mu\nu}^{mn} = g_{mn} G_{\mu\nu}(X) + \frac{\epsilon_{mn}}{\sqrt{-g}} B_{\mu\nu}(X) .
$$

$R^{(2)}$ is the 2D curvature scalar, $k(s)$ the geodesic curvature on the boundary $\partial M$. We added a boundary coupling of the normal derivative of the world sheet position to a vector field $V$. This field does not correspond to any string excitation. It is added for technical reasons. Even starting with $V = 0$ there appear counter terms proportional to the normal derivative $[8]$. Although for free string ends they can be included in the $A$ renormalization by using the boundary condition $[9]$ they get independent physical meaning in the presence of Dirichlet boundary conditions $[6]$. In the rest of the paper we take flat 2D metrics and disregard the dilaton field $\Phi$ whose transformation under T-duality should arise from some functional Jacobian like in the closed string case $[5]$.

We assume the existence of one Killing vector field $k^\mu(X)$ and invariance of our action under a diffeomorphism in the direction of $k$

$$
\mathcal{L}_k \psi = 0, \mathcal{L}_k V = 0, \mathcal{L}_k F = 0 .
$$

The condition on the field strength $F$ implies

$$
\mathcal{L}_k A_\mu = \partial_\mu a .
$$

Choosing suitable adapted coordinates we have

$$
X^\mu = (X^0, X^M), \quad k^\mu = (1, 0), \quad \mathcal{L}_k = \partial_0 .
$$

Starting from a generic allowed gauge field fulfilling $\partial_0 A_\mu = \partial_\mu a$ one can reach

$$
\partial_0 A_\mu = 0
$$

via a gauge transformation. We take $[3]$ as a gauge condition. Altogether now all our target space fields in $[4]$ are independent of $X^0$. For later use we classify the residual gauge transformations respecting $[5]$. The obvious condition $\partial_0 \partial_\mu \Lambda = 0$ implies $\Lambda(X^\mu) = \lambda(X^M) + c \cdot X^0$, i.e. the still allowed gauge transformations are

$$
A_0 \rightarrow A_0 + c, \quad A_M \rightarrow A_M + \partial_M \lambda(X^M) .
$$

Having in mind the nonabelian generalization we ignore the possibility $\mathcal{L}_k B = -\mathcal{L}_k F \neq 0$. 

3
To start the dualization procedure we rewrite the partition function of our $\sigma$ model in standard way with the help of a Lagrange multiplier field $\tilde{X}^0$:

$$Z[\psi, A, V] = \int DX^\mu e^{iS[\psi, A, V; X^\mu]} = \int DX^M Dym DX^0 e^{i\tilde{S}[\psi, A, V; X^M, y_m, \tilde{X}^0]} ,$$

with

$$\tilde{S}[\psi, A, V; X^M, y_m, \tilde{X}^0] = \frac{1}{4\pi \alpha'} \int_M d^2z (\partial_m X^M \partial_n X^N E_{MN}^{m n} + 2\partial_m X^m y_n E_{M0}^{n n}$$

$$+ y_m y_n E_{00}^{m n} + 2\tilde{X}^0 \epsilon^{m n} \partial_m y_n)$$

$$+ \int_{\partial M} (A_M \partial_n X^M + A_0 y_n + (V_M \partial_m X^M + V_0 y_m) \epsilon^m n) \tilde{z}^n (s) ds .$$

The dualized version will be obtained by performing the $y_m$ functional integration. In the closed string case this integral decouples after a shift read off from a suitable quadratic completion. Now the presence of the boundary induces a crucial subtlety:

$$\tilde{S} = \frac{1}{4\pi \alpha'} \int_M d^2z \left( g^{mn} g_{00} v_m v_n + E_{MN}^{m n} \partial_m X^M \partial_n X^N - \frac{g^{m n} g_{00}}{G_{00}} K_m K_n \right)$$

$$+ \int_{\partial M} (A_M \partial_n X^M + A_0 y_n + (V_M \partial_m X^M + V_0 y_m) \epsilon^m n) \tilde{z}^n ds + \int_{\partial M} \left( v_m L^m - \frac{1}{G_{00}} K_m L^m \right) ds ,$$

with

$$K^m = E_{00}^{m n} \partial_n X^M + \epsilon^{m n} \partial_n \tilde{X}^0 ,$$

$$L^m = A_0 \tilde{z}^m + V_0 \epsilon^m n \tilde{z}^n + \frac{1}{2\pi \alpha'} \tilde{X}^0 \tilde{z}^m , \quad v_m = y_m + \frac{K_m}{G_{00}} .$$

Due to the dependence of $L^m$ on $X^M$, $\tilde{X}^0$ the $v$ integral even after the rescaling $v \to \sqrt{G_{00}} v$ does not decouple.

Since the $v$ integral is ultralocal we can use

$$\int_M Dv_m e^{i\tilde{S}} = \int_M Dv_m \exp \left( i\tilde{S} - i \int_{\partial M} v_m L^m ds \right) \cdot \int_{\partial M} Dv_m \exp \left( i \int_{\partial M} v_m L^m ds \right) .$$

Then the $v_m$ integral on the boundary $\partial M$ produces a functional $\delta$ function which imposes for the remaining functional integral the constraints $L^m(s) = 0, \ m = 1, 2$, i.e.

$$V_0(X^M) = 0 ,$$

$$2\pi \alpha' A_0 (X^M) + \tilde{X}^0 = 0 \quad \text{on} \ \partial M .$$

Including the remaining bulk $v_m$ integral into the overall normalization we finally arrive at

$$Z[\psi, A, V] = \int_{(12) \Box (13)} D\tilde{X}^\mu \exp(iS[\tilde{\psi}, \tilde{A}, \tilde{V}; \tilde{X}]) ,$$

4 To identify the dual target space fields one can restrict oneselfs to the partition function. A more complete study of the 2D field theory would require the inclusion of source terms for $X^\mu$. 3
with
\[ \tilde{X}^\mu = (\tilde{X}^0, X^M), \]  
\[ \tilde{A}_\mu = (0, A_M), \quad \tilde{V}_\mu = (0, V_M), \]  
\[ \tilde{G}_{00} = \frac{1}{G_{00}}, \quad \tilde{G}_{0M} = \frac{B_{0M}}{G_{00}}, \quad \tilde{G}_{MN} = G_{MN} - \frac{G_{M0}G_{0N} + B_{M0}B_{0N}}{G_{00}}, \]  
\[ \tilde{B}_{0M} = \frac{G_{0M}}{G_{00}}, \quad \tilde{B}_{MN} = B_{MN} - \frac{G_{M0}B_{0N} + B_{M0}G_{0N}}{G_{00}}. \]  

The formulas (17) coincide with that from the closed string case [5]. The notation in (14) means that the integrand of the functional integral has to respect the boundary conditions (12) and (13). In general the two boundary conditions are not compatible and we have to put \( V \equiv 0 \) [6]. Our model in its dual version describes an open string bound with its end points to the D-brane \( \tilde{X}^0 = -2\pi\alpha' A_0(\tilde{X}^M) \). The boundary condition arising in the course of T-dualization appears as an external condition on the quantum theory, it is not only a consequence of the stationarity condition of the action. Due to (8) gauge invariance in the original theory is mapped into gauge invariance for the gauge fields parallel to the D-brane and to translational invariance of the D-brane position in the dual theory.

3 Open strings with nonabelian Chan-Paton factors

We begin with the partition function for the generalized \( \sigma \)-model describing an open string in nontrivial metric, antisymmetric tensor, dilaton and nonabelian gauge field background [10, 11, 8]
\[ Z[\psi, A] = \int DX \, e^{iS[\psi, A]} e^{i\int \partial M A_\mu dX^\mu}. \]  

This expression no longer has the standard form of a functional integral over the exponential of a local action. We can achieve such a structure by the introduction of a one-dimensional auxiliary field \( \zeta_a(s) \) living on the boundary \( \partial M \). It has the propagator \( \langle \zeta_a(s_1)\zeta_b(s_2) \rangle_0 = \delta_{ab}\Theta(s_2 - s_1) \) and a coupling to \( X^\mu \) via the interaction term [12]
\[ i\zeta A_\mu(X(z(s)))\zeta(s)\partial_m X^\mu z^m(s) \]  
\[ (z(s) \text{ parametrizes } \partial M, \ 0 \leq s \leq 1) \]  
\[ Z = \int DX \, D\bar{\zeta} \, D\zeta \, \bar{\zeta}_a(0)\zeta_a(1) \, e^{iS_0[\bar{\zeta}, \zeta]} \exp(i\tilde{S}[\psi, \bar{\zeta}_a A_\mu \zeta; X]). \]  

For general gauge groups there is an additional factor induced by the vacuum graphs of the \( \zeta \)-theory. The form presented in [13] is valid for unimodular Wilson loops only, e.g.
elements of SU(n) or SO(n). In this case the choice \( \xi \) fermionic or bosonic is irrelevant. To condense the notation we introduced an action \( \hat{S} \) depending on the target space fields \( \psi = (G, B, \Phi) \) the string world sheet position \( X^\mu(z) \) and a vector field \( C_\mu(X^M, s) \) with possibly explicit dependence on \( s \) by

\[
\hat{S}[\psi, C; X] = \frac{1}{4\pi \alpha'} \int_M d^2z \sqrt{-g} \left( \partial_\mu X^\nu \partial_\nu X^\mu E^{\mu\nu}(X(z)) + \alpha' R^{(2)}(z)\Phi(X(z)) \right) + \int_{\partial M} \left( C_\mu(X(z(s)), s) \dot{X}^\mu - \frac{1}{2\pi} k(s)\Phi(X(z(s))) \right) ds .
\]

(20)

Under the \( \zeta \)-path integral we now can repeat the dualization procedure performed above for the case of abelian \( A \). Due to the presence of \( \zeta(s) \) in \( C_\mu \) the resulting Dirichlet condition, besides its implicit dependence on \( s \) via the embedding of the world sheet boundary in the target space, also depends on \( s \) explicitly. Introducing

\[
\mathcal{F}[\psi, C; f] = \int_{X^0(z(s)) = f(X^M(z(s)), s)} DX^\mu \exp(i\hat{S}[\psi, C; X])
\]

(21)

we can write our partition function \( Z \) as

\[
Z[\psi, A] = \int D\zeta D\tilde{\zeta} \tilde{\zeta}_a(0)\tilde{\zeta}_a(1)e^{iS_0[\zeta, \tilde{\zeta}]} \mathcal{F}[\tilde{\psi}, \tilde{\zeta}\tilde{A}\zeta] - 2\pi \alpha' \tilde{\zeta} A_0 \zeta .
\]

(22)

The \( \zeta \)-integration results in ordering the matrices sandwiched between \( \tilde{\zeta}(s) \) and \( \zeta(s) \) with respect to increasing \( s \). But unfortunately after performing the functional integral over the world surfaces \( X^\mu(z) \) there is no longer any correlation between \( s \) and the target space coordinate at which the gauge field has to be taken. The arguments of \( \mathcal{F} \) are \( \tilde{\psi}(x^M), C(x^M, s) = \zeta(s)A(x^M)\zeta(s) \) and \( f(x^M, s) = \tilde{\zeta}(s)A_0(x^M)\zeta(s) \). Therefore, we cannot further simplify (22) by manipulations with the general formal expression.

However, usually all calculations in the \( \sigma \)-model framework are done within the background field method. We assume that its justification along the line of arguments of \[17\] can be extended to the case of manifolds with boundary and the type of boundary conditions involved in our above discussion. Then the background field method variant of \[21,22\] with (\( \xi \) denoting Riemann normal coordinates in the target space)

\[
X^\mu(z) = X^\mu_{cl}(z) + \delta X^\mu(\xi(z))
\]

(23)

is

\[
\mathcal{F}_{bm}[\psi(X_{cl}), C(X_{cl}, s); X_{cl}, f] = \int_{(s)} DX^\mu \exp(i\hat{S}[\psi(X_{cl} + \delta X(\xi)), C(X_{cl} + \delta X(\xi), s); X_{cl} + \delta X(\xi)])
\]

\[\star\]

(24)

\[\xi\] Assuming again independence of \( X^0 \) for all target space fields involved.

\[\mu\] We use \( x^\mu \) to denote the target space coordinates in writing \( \tilde{\psi} \) and \( \tilde{A} \) as target space functions. A way to disentangle \( x^\mu \) and the functional integration variable \( X^\mu \) is to write e.g. \( \tilde{\psi}(X) = \int dx \tilde{\psi}(x)\delta(x - X(z)) \)[16].
and

\[ Z_{bm}[\psi(X_{cl}), A(X_{cl}); X_{cl}] = \int D\bar{\zeta} D\zeta \zeta_\alpha(0)\zeta_\alpha(1)e^{iS_0[\bar{\zeta}, \zeta]} F_{bm}[\bar{\psi}, \bar{\zeta}A; \bar{X}_{cl}; X_{cl}] - 2\pi\alpha'\bar{\zeta}A_0\zeta]. \] (25)

The functional \( F_{bm} \) depending on the Dirichlet boundary function \( f(s) \) at first sight introduces a considerable generalization if explicit \( s \)-dependence is allowed. Then the target space interpretation of a D-brane with attached open string breaks down. However, we have to remember that afterwards we need the special case \( f(s) = -2\pi\alpha'\bar{\zeta}A_0\zeta \) only.

In the perturbative evaluation of the \( \zeta \) path integral the \( \zeta \)'s combine to propagators which are nearly constant, either 0 or 1 in a way just realizing path ordering of the accompanying \( A \)-matrices. Although the original definition of \( F_{bm} \) in (24) makes sense for scalar (not matrix valued) \( f(s) \) only, at least if \( F_{bm} \) has a well defined functional Taylor expansion in \( f \) one can put into this expansion also a matrix-valued \( f \). The statement that the \( \zeta \)-integral results in nothing else but the path ordering can be proven by the following step (until the final formula we suppress a factor \( 2\pi\alpha' \))

\[ Z_{bm} = F_{bm}[\bar{\psi}, \delta J; \bar{X}_{cl}; \delta K] = \int D\bar{\zeta} D\zeta \zeta_\alpha(0)\zeta_\alpha(1)e^{iS_0 + i(\bar{\zeta}A_0\zeta)ds}] \bigg|_{J=0, K=0} \]

\[ = F_{bm}[\bar{\psi}, \delta J; \bar{X}_{cl}; \delta K]\text{tr} Pe^{\int (\bar{A}_M J^M - A_0 K)ds}] \bigg|_{J=0, K=0}. \] (26)

Since under the path ordering differentiation acts as if all quantities would commute we arrive at (note (15,16))

\[ Z_{bm}[\psi, A; X_{cl}] = \text{tr} P(F_{bm}[\bar{\psi}, \bar{A}; \bar{X}_{cl}] - 2\pi\alpha' A_0)]. \] (27)

Together with eq.(24) this yields an explicit realization of the statement that in the nonabelian case the end point position of the open string in the dual prescription becomes a matrix \([14, 15]\).

Before any further interpretation we should have a closer look on the consequences of gauge invariance. As in the abelian case in a generic situation the Lie derivative of the gauge field can be nonzero. \( \mathcal{L}_k A_\mu = D_\mu a \) ensures \( \mathcal{L}_k F_{\mu\nu} = -i[F_{\mu\nu}, a] \) and henceforth the invariance of gauge invariant quantities like \( \text{tr}(F_{\mu\nu}F^{\mu\nu}) \). However, by a suitable gauge transformation again one can reach \( \mathcal{L}_k A_\mu = 0 \). Hence we are allowed to enforce in adapted coordinates \([4]\) the gauge condition \([3]\). The condition constraining the allowed residual (infinitesimal) gauge transformations is \( D_\mu \partial_0 \Lambda = 0 \). If a Lie algebra valued field \( B(X) \) is covariantly constant in a whole neighborhood of a point \( X \) its value at any point \( Y \) in this neighborhood must fulfill the equation

\[ B(Y) = P \exp \left( i \int_C A_\mu dX^\mu \right) B(X) P \exp \left( i \int_{C-1} A_\mu dX^\mu \right) \] (28)

for arbitrary curves \( C \) connecting \( X \) and \( Y \). Comparing (28) for two different curves we conclude that \( B(X) \) has to commute with all path ordered phase factors for closed
curves starting and ending at $X$, i.e. with all elements of the holonomy group. The most restrictive situation arises if this group coincides with or is an irreducible subgroup of the gauge group. Then $B$ must be a multiple of unity. As a consequence $B(X)$ commutes with $A_\mu(X)$ and therefore covariant constancy implies constancy. Applying this to $\partial_0 A$ we get $\Lambda = cX^0 \cdot 1 + \lambda(X^M)$ and for the residual gauge transformations the nonabelian version of (6)

$$
A_0 \rightarrow A_0 + c \cdot 1 - i[A_0, \lambda(X^M)] , \quad A_M \rightarrow A_M + D_M \lambda(X^M) .
$$

(29)

Of course for less restrictive holonomy groups there are more general gauge transformations left. Translational invariance of the D-brane position as a consequence of gauge invariance in the original model one has only for gauge groups where $c \cdot 1$ is an element of the corresponding Lie algebra, e.g. for $U(N)$. For $SO(N)$ $c$ has to vanish. This breaking of translational invariance is in accord with the orientifold construction \[15\].

Furthermore, the transformation (29) can always be used to bring the matrix $A_0$ into the simplest possible form: diagonal for $U(N)$ and block diagonal (with 2x2 blocks) for $SO(N)$.

Discussing string endpoints with definite Chan-Paton indices corresponds to the situation before the $\zeta$-integral is performed. Our partition function can be understood as an off shell extension of the generating functional of S-matrix elements for the elementary string excitations. Therefore, the boundary of our world sheet $\partial M$ carries two Chan-Paton indices related to the two indices associated with the endpoints of the would be emitted open string state. Choosing $\bar{\zeta}_a = \delta_{ab}$, $\zeta_a = \delta_{ac}$ one has $\bar{\zeta}A_0\zeta = (A_0)_{bc}$ and the boundary point under consideration has to be on the hypersurface defined by $\bar{X}_0 = -2\pi\alpha'(A_0)_{bc}$. Specializing to the case of diagonal $A_0$ we reproduce the brane pattern obtained in the presence of Wilson lines \[13\].

4 T-duality as a canonical transformation

For a general 2D field theory on a manifold with boundary with the action

$$
S[\phi] = \int_M d^2z L(\phi, \partial \phi) + \int_{\partial M} ds l(\phi, \partial \phi)
$$

stationarity under free variation of $\phi$ both inside $M$ and on $\partial M$ implies besides the standard bulk equation of motion

$$
\frac{\partial L}{\partial \partial_a \phi} \epsilon_{ab} z^b + \frac{\partial l}{\partial \phi} - \partial_t \frac{\partial l}{\partial \phi} = 0 ,
$$

$$
\frac{\partial l}{\partial \partial_n \phi} = 0 .
$$

(31)

The indices $t$ and $n$ denote tangential and normal directions, respectively. In general it may happen that the two boundary conditions arising in (31) are not compatible.
For the transition to the Hamilton formulation we denote the 1D space coordinate by \( \sigma \) and the time coordinate by \( \tau \). Let us assume that the boundary consists of two parts \( \partial M|_i \) and \( \partial M|_f \) which are identified in the infinite past and future. To avoid distributional formulas we do not include the boundary action in the definition of canonical momenta \( \pi(\sigma) = \partial L/\partial \dot{\phi}(\sigma) \). Then the Hamilton function is of the Routh type in so far as the boundary degrees of freedom are kept in Lagrangian form. We skip writing down the resulting equations and proceed directly to the canonical transformations keeping them form invariant. For a generating function of the type \( F(\phi, \tilde{\phi}) \) we get

\[
\tilde{\pi} = -\frac{\partial F}{\partial \tilde{\phi}} + \partial_\sigma \frac{\partial F}{\partial \phi'}, \quad \pi = \frac{\partial F}{\partial \phi} - \partial_\sigma \frac{\partial F}{\partial \phi'}, \quad \tilde{H} = H + \frac{\partial F}{\partial \tau},
\]

(32)

The \( \pm \) alternative refers to the two boundary components corresponding to the two string end points. The dot and prime denote differentiation with respect to \( \tau \) and \( \sigma \), respectively.

We now apply this general framework to our \( \sigma \)-model with one abelian target space isometry. The generating function

\[
F = \frac{1}{4\pi\alpha'} \left( X'^{0}\tilde{X}^{0} - X^{0}\tilde{X}'^{0} \right)
\]

(33)

known already from the closed string case \([2]\) leads with (32) to

\[
\dot{X}^{0} = -\frac{\tilde{X}^{0} + G_{0N} \dot{X}^{N} + B_{0N}X'^{N}}{G_{00}}
\]

(34)

and

\[
\pm \tilde{l} = A_M \dot{X}^M - \left( A_0 + \frac{\tilde{X}^{0}}{2\pi\alpha'} \right) \frac{\tilde{X}'^{0} + G_{0N} \tilde{X}^{N} + B_{0N}X'^{N}}{G_{00}}.
\]

(35)

Although we started in the original theory with \( V \equiv 0 \) the dualized version contains a boundary interaction of the \( V \)-type, i.e. proportional to the normal derivative of \( \tilde{X}^{0} \).

The boundary condition arising from \( \partial \tilde{l}/\partial X'^{0} = 0 \) is given by (13). Using this the other conditions from (33) applied to the dual theory yield Neumann conditions for the remaining target space coordinates \( \tilde{X}^M = X^M \). Since Neumann and Dirichlet refer to different coordinates there is no compatibility problem.

5 Conclusions

We have sketched how the well known techniques to describe T-duality for closed strings in general background fields can be extended to the case of open strings. In the simplest case of one abelian isometry the dual theory corresponds to strings bound with their ends

\footnote{Note that we have choosen the \( \sigma-\tau \) coordinates in such a way that on the boundary \( \partial/\partial \sigma \) is the normal and \( \partial/\partial \tau \) the tangential derivative.}
to the hypersurface whose position in target space is given by setting the coordinate in the direction of the Killing vector equal to the corresponding component of the gauge potential. In the language of canonical transformations we see no simple possibility to decide whether the boundary condition is a dynamical one, resulting as part of the stationarity condition for the action, or whether it is an external constraint. On the other hand the functional integral treatment clearly shows that the Dirichlet condition has to be handled as a constraint on the integrand of the functional integral.

Using the ζ auxiliary field formalism we were able to proceed from abelian gauge fields to nonabelian ones. Here at a first step one has to calculate the partition function for explicitly boundary parameter dependent Dirichlet conditions and then, in a second step, to insert in the arising functional the matrix valued gauge potential in a path ordered manner.

Obviously, a lot of interesting problems remain open for further investigations. One should clarify the structure of residual gauge transformations in the case of holonomy groups not as restrictive as assumed in (29). Furthermore, the relation between β-functions in the original and dual theory and their role in deriving equations of motion for the D-brane discussed in ref. [6] for the abelian case has to be extended to nonabelian gauge fields. Concerning more general aspects the inclusion of fermions, questions of supersymmetry and global issues [7] require further study.

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References

[1] A. Giveon, M. Poratti, E. Rabinovici, Phys. Rep. 244 (1994) 77
[2] E. Alvarez, L. Alvarez-Gaumé, Y. Lozano, Nucl. Phys. B41, Proc. Suppl. (1995) 1
[3] J. Polchinski, Phys. Rev. Lett. 75 (1995) 4724
[4] J. Dai, R.G. Leigh, J. Polchinski, Mod. Phys. Lett. A4 (1989) 2073
  P. Horava, Phys. Lett. B231 (1989) 251
  M.B. Green, Phys. Lett. B266 (1991) 325
[5] T.H. Buscher, Phys. Lett. B194 (1987) 51, Phys. Lett. B201 (1988) 466
[6] R.G. Leigh, Mod. Phys. Lett. A4 (1989) 2767
[7] E. Alvarez, J.L.F. Barbon, J. Borlaf, T-duality for open strings preprint hep-th/9603089
[8] H. Dorn, H.-J. Otto, Zeitschr. f. Phys. C32 (1986) 599
[9] C.G. Callan, C. Lovelace, C.R. Nappi, S.A. Yost, *Nucl. Phys.* B288 (1987) 525

[10] E.S. Fradkin, A.A. Tseytlin, *Nucl. Phys.* B261 (1985) 1
    A.A. Tseytlin, *Nucl. Phys.* B276 (1986) 391

[11] C.G. Callan, E. Martinec, M.J. Perry, D. Friedan, *Nucl. Phys.* B262 (1985) 593

[12] S. Samuel, *Nucl. Phys.* B149 (1979) 517
    R.A. Brandt, F. Neri, D. Zwanziger, *Phys. Rev.* D19 (1979) 1153
    J.L. Gervais, A. Neveu, *Phys. Rev.* B163 (1980) 189
    I. Ya. Arefyeva, *Phys. Lett.* 93B (1980) 347
    H. Dorn, *Fortschr. d. Phys.* 34 (1986) 11

[13] N. Marcus, A. Sagnotti, *Phys. Lett.* 188B (1987) 58

[14] E. Witten, *Nucl. Phys.* B460 (1996) 335

[15] J. Polchinski, S. Chaudhuri, C.V. Johnson, preprint *Notes on D-Branes*, hep-th/9602052

[16] A.A. Tseytlin, *Nucl. Phys.* B294 (1987) 383

[17] P.S. Howe, G. Papadopoulos, K.S. Stelle, *Nucl. Phys.* B296 (1987) 26