PROBING THE NEUTRON STAR INTERIOR WITH GLITCHES

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Abstract
With the aim of constraining the structural properties of neutron stars and the equation of state of dense matter, we study sudden spin-ups, glitches, occurring in the Vela pulsar and in six other pulsars. We present evidence that glitches represent a self-regulating instability for which the star prepares over a waiting time. The angular momentum requirements of glitches in Vela indicate that ≥ 1.4% of the star’s moment of inertia drives these events. If glitches originate in the liquid of the inner crust, Vela’s ‘radiation radius’ \( R_\infty \) must exceed \( \approx 12 \) km for a mass of \( 1.4 M_\odot \). The isolated neutron star RX J18563-3754 is a promising candidate for a definitive radius measurement, and offers to further our understanding of dense matter and the origin of glitches.

INTRODUCTION
Many isolated pulsars suffer spin jumps, glitches, superimposed upon otherwise gradual spin down under magnetic torque. For example, in the glitch of the Crab pulsar shown in Fig. 1, the star spun up by nearly a part in \( 10^7 \) over several days [1], corresponding to a change in rotational energy of the crust of \( \sim 10^{42} \) ergs. A particularly active glitching pulsar is the Vela pulsar, which has produced more than a dozen glitches since its discovery over 30 years ago. The fractional changes in rotation rate are typically \( \sim 10^{-6} \), occurring every three years on average [2].

Because glitching pulsars are isolated systems, glitches are thought to arise from internal torques exerted by the rotating liquid interior on the crust, whose
spin rate we observe [3]. As the star’s crust is spun down by the magnetic field frozen to it, the interior liquid, which responds to the external torque indirectly through friction with the solid crust, rotates more rapidly. For example, a portion of the liquid could coast between glitches while the solid crust spins down (Fig. 2). Glitches might arise as the consequence of an instability that increases the frictional coupling between the liquid and the solid, causing angular momentum flow to the crust.

\[ \Delta J_i = I_c \Delta \Omega_i, \]  

(1)

The long history of glitches in Vela makes it possible to deduce some of the properties of the interior angular momentum reservoir independent of the details of the instability that triggers these events. Here we discuss the time distribution and average angular momentum transfer rate of Vela’s glitches and present evidence that glitches in Vela represent a self-regulating instability for which the star prepares over a waiting interval. We obtain a lower limit on the fraction of the star’s liquid interior responsible for glitches and discuss how this result can be used to constrain the dense matter equation of state and the structural properties of neutron stars. We conclude with discussion of the nearby isolated neutron star RX J185635-3754, a promising candidate for a robust radius measurement that offers to constrain our understanding of dense matter and the origin of glitches.
where \( I_c \) is the moment of inertia of the solid crust plus any portions of the star tightly coupled to it. Most of the core liquid is expected to couple tightly to the star’s solid component, so that \( I_c \) makes up at least 90% of the star’s total moment of inertia [4]. [Glitches are driven by the portion of the liquid interior that is differentially rotating with respect to the crust]. The cumulative angular momentum imparted to the crust over time is

\[ J(t) = I_c \bar{\Omega} \sum_i \frac{\Delta \Omega_i}{\Omega}, \]

where \( \bar{\Omega} = 70.4 \text{ rad s}^{-1} \) is the average spin rate of the crust over the period of observations. Fig. 3 shows the cumulative dimensionless angular momentum, \( J(t)/I_c \bar{\Omega} \), over \( \sim 30 \) years of glitch observations of the Vela pulsar, with a linear least-squares fit. The average rate of angular momentum transfer associated with glitches is \( I_c \bar{\Omega} A \), where \( A \) is the slope of the straight line in Fig. 3:

\[ A = (6.44 \pm 0.19) \times 10^{-7} \text{ yr}^{-1}. \]

This rate \( A \) is often referred to as the pulsar activity parameter (see, e.g., [5]).

The angular momentum flow is remarkably regular; none of Vela’s 13 glitches caused the cumulative angular momentum curve to deviate from the linear fit by more than 12%. Moreover, glitches occur at fairly regular time intervals; the standard deviation in observed glitch intervals is \( 0.53 \langle \Delta t \rangle \), where \( \langle \Delta t \rangle = 840 \text{ d} \) is the average glitch time interval. The probability of 13 randomly-spaced (Poisson) events having less than the observed standard deviation is only \( \sim 1\% \). These data indicate that Vela’s glitches are not random, but represent a self-regulating process which gives a relatively constant flow of angular momentum to the crust with glitches occurring at fairly regular time intervals.
THE GLITCH RESERVOIR’S MOMENT OF INERTIA

The frequent occurrence of large glitches requires that some fraction of the interior liquid (the reservoir) spins at a higher rate than the crust of the star. The average rate of angular momentum transfer in Vela’s glitches (eq. 3) can be used to constrain the relative moment of inertia of the reservoir. Between glitches, the reservoir acquires excess angular momentum as the rest of the star slows under the magnetic braking torque acting on the crust. Excess angular momentum accumulates at the maximum rate if the reservoir *coasts* between glitches, without spinning down (Fig. 2). Hence, the rate at which the reservoir accumulates angular momentum capable of driving glitches is limited by

\[ \dot{J}_{\text{res}} \leq I_{\text{res}} \dot{\Omega}, \]  

(eq. 4)

where \( \dot{\Omega} \) is the average spin-down rate of the crust, and \( I_{\text{res}} \) is the moment of inertia of the angular momentum reservoir (not necessarily one region of the star). Equating \( \dot{J}_{\text{res}} \) to the average rate of angular momentum transfer to the crust, \( I_c \dot{\Omega} A \), gives the constraint,

\[ \frac{I_{\text{res}}}{I_c} \geq \frac{\dot{\Omega}}{|\dot{\Omega}|} A \equiv G, \]  

(eq. 5)

where the *coupling parameter* \( G \) is the minimum fraction of the star’s moment of inertia that stores angular momentum and imparts it to the crust in glitches. Using the observed value of Vela’s activity parameter \( A \) and \( \dot{\Omega}/|\dot{\Omega}| = 22.6 \) Kyr, we obtain the constraint

\[ \frac{I_{\text{res}}}{I_c} \geq G_{\text{Vela}} = 1.4\%. \]  

(eq. 6)

A similar analysis for six other pulsars yields the results shown in Fig. 4. After Vela, the most significant limit is obtained from PSR 1737-30 which gives \( I_{\text{res}}/I_c \geq G_{1737} = 1\% \).
The similarity of $G$ for the five objects of intermediate age suggests a common physical origin for glitches in these stars. The Crab pulsar and PSR 0525+21, however, appear to be unusual. It may be that the Crab’s interior cannot accumulate significant excess angular momentum between glitches, perhaps as a consequence of rapid thermal creep of superfluid vortices (see, e.g., [6]). The value of $G$ for PSR 0525+21 is not well determined, since only two glitches from this object have been measured.

**IMPLICATIONS FOR THE DENSE MATTER EQUATION OF STATE**

The constraint of $I_{\text{res}}/I_c \geq 1.4\%$ for Vela applies regardless of where in the star glitches originate. A natural candidate for the angular momentum reservoir is the superfluid that coexists with the inner crust lattice [3], where superfluid vortex lines could pin to the nuclei and sustain a velocity difference between the superfluid and the crust. Within this interpretation, the constraint on $I_{\text{res}}/I_c$ can be used to constrain the properties of matter at supranuclear density as we now demonstrate.

The fraction of the star’s moment of inertia contained in the solid crust (and the neutron liquid that coexists with it) is given approximately by [12]:

$$\frac{\Delta I}{I} \simeq \frac{28\pi}{3} \frac{P_t R^4}{GM^2} \left[ 1 + \frac{8P_t}{n_t m_n c^2} \frac{4.5 + (\Lambda - 1)^{-1}}{\Lambda - 1} \right]^{-1}.$$  \hspace{1cm} (7)

Here $n_t$ is the density at the core-crust boundary, $P_t$ is the pressure there, $M$ and $R$ are the stellar mass and radius, $\Lambda \equiv (1 - 2GM/Rc^2)^{-1}$ is the gravitational redshift and $m_n$ is the neutron mass. $\Delta I/I$ is a function of $M$ and $R$ with an additional dependence upon the equation of state (EOS) arising through the values of $P_t$ and $n_t$. $P_t$ is the main EOS parameter as $n_t$ enters chiefly via a
correction term. In general, $P_t$ varies over the range $0.25 < P_t < 0.65$ MeV fm$^{-3}$ for realistic equations of state [13]. Larger values of $P_t$ give larger values for $\Delta I/I$.

\[ R = 3.6 + 3.9M/M_\odot. \] (8)

Stellar models that are compatible with the lower bound on $I_{\text{res}}$ must fall below this line. For models above this line, the inner crust liquid constitutes less than
1.4% of the star’s moment of inertia, incompatible with the observed angular momentum requirments of the Vela pulsar.

**DISCUSSION**

Vela’s mass and radius are unknown, so we cannot say where it falls in Fig. 5. However, mass measurements of radio pulsars in binary systems and of neutron star companions of radio pulsars give neutron star masses that are consistent with a remarkably narrow distribution, \( M = 1.35 \pm 0.04 M_\odot \) [16], indicated by the pair of horizontal dotted lines in Fig. 5. If Vela’s mass is not unusual, eq. [8] constrains \( R \gtrsim 8.9 \) km, under the assumption that glitches arise in the inner crust superfluid. However, the quantity constrained by observations of the stellar luminosity and spectrum is not \( R \) but the larger ‘radiation radius’ \( R_\infty \equiv \Lambda^{1/2} R = (1 - 2GM/Rc^2)^{-1/2} R \). If \( M = 1.35 M_\odot \) for Vela, the above constraint gives \( R_\infty \gtrsim 12 \) km if glitches arise in the inner crust. For comparison, we show in Fig. 5 the mass-radius curves for several representative equations of state (heavy solid lines). Measurement of \( R_\infty \lesssim 13 \) km would be inconsistent with most equations of state if \( M \simeq 1.35 M_\odot \). Stronger constraints could be obtained if improved calculations of nuclear matter properties give \( P_t \) significantly less than 0.65 MeV fm\(^{-3}\). For example, for \( M \simeq 1.35 M_\odot \), \( R_\infty \gtrsim 13 \) km would be required if \( P_t = 0.25 \) MeV fm\(^{-3}\). A measurement of \( R_\infty \lesssim 11 \) km would rule out most equations of state regardless of mass or the angular momentum requirements of glitches, and could indicate that neutron stars are not made of neutrons at all, but of strange quark matter. Explaining glitches in this case would be problematic, as strange stars have very thin crusts \[17\].

A black body fit to the unpulsed component of Vela’s thermal emission gives \( R_\infty = 3 - 4 \) km [18]. This result is difficult to interpret without knowledge of the star’s atmospheric composition and magnetic field strength; atmospheric effects could increase this estimate by a factor of up \( \sim 6 \), but could also decrease it by a factor \( \sim 2 \) [19]. Nevertheless, it would be interesting to check the extent to which our constraints on Vela are obeyed by other neutron stars. A promising candidate for a decisive measurement of a neutron star’s radiation radius is RX J185635-3754, an isolated, non-pulsing neutron star [20]. A black body fit to the X-ray spectrum gives \( R_\infty = 7.3(D/120\,\text{pc}) \) km where \( D \) is the distance (known to be less than 120 pc). Taken at face value, this result would not only be inconsistent with the radius requirements of glitches in Vela - it would rule out all equations of state that do not involve strange matter. [It is possible that

\[\text{If} \ P_t < 0.65 \text{ MeV fm}^{-3}, \ \text{the crust’s moment of inertia would be smaller and the radius constraint more restrictive. For example,} \ P_t = 0.25 \text{ MeV fm}^{-3} \ \text{moves the constraining contour to approximately} \ R = 4.7 + 4.1 M/M_\odot.\]
glitching pulsars are normal neutron stars, while RX J185635-3754 is ‘strange’].
However, either a non-uniform surface temperature or radiative transfer effects
in the stellar atmosphere could raise this estimate significantly [21]. Recent
HST observations of this source by F. Walter should give the proper motion
and parallax, and hence, the distance. Future CHANDRA observations should
yield more detailed spectral data and could establish the composition of the
atmosphere if absorption lines are identified. If lines are present, atmospheric
fits will give $R_\infty$, $R$ and $M$, thus restricting RX J185635-3754 to a region of
Fig. 5. These data will undoubtedly further our understanding of matter at
supranuclear density, and could establish whether neutron stars have properties
consistent with an inner crust explanation of glitches.

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