Consider two normal leads coupled to a superconductor; the first lead is biased while the second one and the superconductor are grounded. In general, a finite current $I_2(V_1, 0)$ is induced in the grounded lead 2; its magnitude depends on the competition between processes of Andreev and normal quasiparticle transmission from the lead 1 to the lead 2. It is known that in the tunneling limit, when normal leads are weakly coupled to the superconductor, $I_2(V_1, 0) = 0$, if $|V_1| < \Delta$ and the system is in the clean limit. In other words, Andreev and normal tunneling processes compensate each other. We consider the general case: the voltages are below the gap, the system is either dirty or clean. It is shown that $I_2(V_1, 0) = 0$ for general configuration of the normal leads; if the first lead injects spin polarized current then $I_2 = 0$, but spin current in the lead-2 is finite.

XISIN structure, where X is a source of the spin polarized current could be applied as a filter separating spin current from charge current. We do an analytical progress calculating $I_1(V_1, V_2)$, $I_2(V_1, V_2)$.

Hybrid systems consisting of a superconductor (S) and two or more normal metal (N) or ferromagnetic (F) probes recently started to attract a great attention [12]. Among most striking new results is the prediction that NSN (FSF) devices can play the role of entangler producing Einstein-Podosky-Rosen (EPR) pairs [4] having potential applications, for example, in quantum cryptography [6]. Not long ago rather unusual effect was described in normal metal – tunnel barrier – superconductor – tunnel barrier – normal metal (NISIN) junction (see, e.g., Fig.1b) [2].

It was shown that when $N_1$ is biased, $N_2$ and S are grounded there is no current injection from N1 to N2 at subgap biases; main assumptions were: 1) the superconductor is clean, 2) large number of conducting channels are involved in electron tunneling through NS interfaces [2]. In other words, the subgap conductance $G_{12} \equiv \partial I_2(V_1, 0)|_{|V_1|<\Delta} = 0$, where the current $I_1$ flows in $N_1$, $V_1$ is the bias between $N_1$ and S and $V_2$ — between $N_2$ and S. The suppression of $G_{12}$ was attributed to the compensation of the contributions to the current from Andreev and normal quasiparticle tunneling processes between $N_1$ and $N_2$ [2]. It was also noted that $G_{12} \neq 0$ in FISIF junctions: $G_{12}$ decays exponentially as $\exp(-r/\xi)$ with the characteristic distance $r$ between the normal terminals (see, e.g., Fig.1), where $\xi$ is the superconductor coherence length; at small $r/\xi$, $G_{12}$ decays also rather quickly (at atom-scales): as $1/(k_F r)^2$ ($k_F$ in the superconductor) [2]. Thus with clean superconductors a measurement of $G_{12}$ may become difficult.

In this letter we first of all generalize results [2,8] and get rid of the assumption 1) [i.e., S is not restricted to be clean]. We show that when the superconductor is dirty (the mean free path is smaller than $\xi$) Andreev and normal transmission rates [as well as $G_{12}$ in FISIF junctions] slowly decay with the characteristics distance $r$ between the normal (ferromagnetic) terminals (at $r < \xi$) in contrast to the clean regime (see Refs. [2,8]). For example, in FISIF with superconducting layer thinner than $\xi$, see Fig.1), $G_{12} \sim \ln(r/\xi)$; when the superconductor is bulk then $G_{12} \sim \xi/r$ [if $r > \lambda_F$ is supposed]. Measurements of the effects, related to electron tunneling through a superconductor (e.g., $G_{12}$) in the dirty superconductor case can be easier realized experimentally then in the clean case because then $r$ is not restricted to atomic scales but by $\xi \gg \lambda_F$. We show that contributions to the current from Andreev and normal quasiparticle...
particle tunneling processes always compensate each other in NISIN junctions (so, e.g., $I_2(V_1, V_2 = 0) = 0$ for $|V_1| < \Delta$ in first nonvanishing order over the transparencies of the layers I) for any amount of disorder in the S-layer. If one prepares a NISIN junction with layers I having large transparency then normal tunneling start dominating Andreev tunneling (and $I_2(V_1, V_2 = 0) \neq 0$). We also considered FISIN junction, in particular with $V_F \neq 0$ and $V_N = 0$. Then the ferromagnet F plays the role of the spin-polarized current injector. In this case $I_2(V_F, V_N = 0) = 0$ also, but spin current in N is finite: charge component of the current converts into the supercurrent, spin accumulates in N. So XISIN structure: charge component of the current converts into the spin current from charge current. We find Andreev $T_{he}$ and normal transmission probabilities $T_{ee}$ of a NISIN sandwich for subgap energies $|E| < \Delta$ and different angles $\theta$ between incident quasiparticle trajectory and the normal to NS interface. It is shown that the probabilities have resonances where $T_{he} \sim T_{ee}$: averages of $T_{he}$ and $T_{ee}$ over incident channels (over $\theta$) are equal — this is the reason why $I_2(V_1, V_2 = 0)$ is suppressed and the spin current $I_2^{(1)}(V_1, V_2 = 0)$ is finite.

We start investigation of NISIN structures from the sandwich sketched in Fig.1: barriers at NS boundaries provide spectacular reflection; electrons in N and S move ballistically; the number of channels at both NS boundaries is much larger than unity. The transmission probabilities $T_{he}(E, \theta)$ and $T_{ee}(E, \theta)$ [see Fig.2] describe Andreev and normal tunneling of an electron incident on the NS boundary with the angle $\theta$ and the energy $E$ correspondingly into a hole and an electron in the lead 2. Following the Landauer-Büttiker approach [5][9][10]:

$$I_2(V_1, V_2 = 0) = \frac{e}{\hbar} \int dE \sum_{\text{channels}} (T_{ee} - T_{he}) \times$$

$$\times (f^{(1)} - f^{(2)}),$$

where the sum is taken over channels (spin degrees of freedom are included into channel definition); $f^{(1,2)}$ are distribution functions in the leads 1,2; e.g., $f^{(2)} = n_F(E) = 1/[1 + \exp(\beta E)]$, $f^{(1)}$ is not necessary a Fermi-function. We calculate the transmission and reflection probabilities using Boguliubov equations (BdG). The layers I are approximated by $\delta$-barriers. Quasiparticle motion parallel and perpendicular to the NS interfaces can be decoupled [11][12]. Matching appropriate wave functions in the normal region and the superconductor we get $8 \times 8$ linear system of equations for the transmission amplitudes. Analytical progress can be made. It follows that if there is no barrier at NS boundaries (except $\Delta$) $T_{he}/T_{ee} \lesssim (\Delta/E_F)^2$ for any thickness $d$ of the superconducting layer. This result is intuitively quite clear because $\Delta \ll E_F$ can hardly reverse the direction of the quasiparticle momentum being about $k_F$ [11][12][13]. However if there are barriers at NS boundaries in addition to $\Delta$ (e.g., insulating layers I) the situation changes: at certain $\theta$ the transmission probabilities have resonances where $T_{he} \sim T_{ee}$. When the transmission probabilities of the layers I, $T_{NS}^{(1,2)} \ll 1$, the areas under the resonance peaks of $T_{eh}^{(1)}(\theta)$ and $T_{ee}(\theta)$ are nearly same and

$$\langle T_{he} \rangle \approx \langle T_{ee} \rangle,$$
where \( \langle \ldots \rangle = \sum_{\text{channels}} \langle \ldots \rangle / N_{\text{channels}} \approx \int_0^1 \langle \ldots \rangle d \cos^2 \theta \). Eq. 2 is exact in first nonvanishing order over \( T_{NS} \).

The resonances appear at \( k_p d \cos(\theta_n) = \pi n, n = 1, 2, \ldots \), give the leading contributions to \( \langle T_{he} \rangle \), \( \langle T_{ee} \rangle \) and are responsible for Eq. 2. The resonance width \( \Gamma \sim \min\{1, T_{NS}, d/\xi\} \). Typical dependencies of \( T_{he}(\theta) \) and \( T_{ee}(\theta) \) from \( \theta \) and \( T_{NS} \) are illustrated in Fig. 2.

In fact \( \theta \) is discrete variable; its particular value is determined by the channel of the incident particle. Equation (2) is applicable when 1) \( T_{NS}(\theta) \) slightly change when \( \theta \) changes from one channel to an adjacent one and 2) change of \( \theta \) from one channel to another should be smaller than the resonance width. The condition 1) is fulfilled typically when \( T_{NS}(\theta) \ll 1 \), the condition 2) requires \( \lambda_F/\sqrt{A} \ll \min\{1, T_{NS}(0), d/\xi\} \), where \( A \) is the junction surface area.

It follows from Eqs. 1, 2 that subgap charge injection from the lead 1 into the lead 2 in weak coupling regime \( (T_{NS} \ll 1) \) is suppressed: \( I_2(V_1, V_2 = 0) = 0 \), because charge currents of transmitted hole and electron quasiparticles compensate each other in the lead 2; all the electron current converts into Cooper-pair supercurrent in S. However if spin-polarized current is injected from the lead 1 finite spin current appears in the lead 2; transmitted electron and hole quasiparticles contribute the spin current. XISIN structure with \( T^{(1)}_{NS} \ll T^{(2)}_{NS} \ll 1 \) (this condition allows to neglect the contribution to the charge current going in S from Andreev reflection at N1S surface), where X is current “injector”, can play the role of the filter of spin and charge currents, see fig. 4. Equation for the spin current follows from Eq. 1:

\[
I^{(s)}_2(V_1, V_2 = 0) = \frac{e}{2\hbar} \int dE \sum_{\text{channels}} \sigma_1(T_{ee} + T_{he}) \times (f^{(1)} - f^{(2)}),
\]

where \( \sigma_1 = \pm 1 \) labels spin degrees of freedom in X. General feature of transmission probabilities \( T \) and the current — their exponential suppression with \( d/\xi \) when \( d \gg \xi \) (\( \xi \) is the superconductor coherence length).

We show below that all the results discussed above remain true in general NISIN structure with more complicated shape than in Fig. 4, (e.g. like in Fig. 5) no matter dirty or clean.

In general a system of weakly coupled normal (ferromagnetic) and superconducting layers can be described by the Hamiltonian: \( \hat{H} = \hat{H}_1 + \hat{H}_2 + \hat{H}_S + \hat{H}_T \), where \( \hat{H}_{1,2} \) refer to the electrodes N1 and N2, and \( \hat{H}_S \) to the superconductor. The tunnel Hamiltonian \( \hat{H}_T \), which we consider as a perturbation, is given by two terms

\[
\hat{H}_T = \hat{H}^{(1)}_T + \hat{H}^{(2)}_T \text{ corresponding to one-particle tunneling through each tunnel junction:}
\]

\[
\hat{H}^{(i)}_T = \sum_{k,p} \left\{ a^{(i)}_{k,p} \hat{a}^{(i)*}_{k,p} + h.c. \right\},
\]

where indices \( i = 1, 2 \) refer to normal (ferro) electrodes; \( \hat{a}^{(i)}_{k,p} \) is the matrix element for tunneling from the state \( k = (k, \sigma) \) in normal lead N1 to the state \( p = (\mathbf{p}, \sigma') \) in the superconductor. The operators \( \hat{a}^{(i)}_{k,p} \) correspond to quasiparticles in the leads and in the superconductor, respectively.

The current can be expressed through the quasi-particle scattering probabilities within the Landauer-Büttiker approach. It is possible to calculate the scattering probabilities within the tight-binding model 4, but it is more convenient to describe the current on the language of electrons only: Andreev transmission probability \( T_{he} \) is closely related to the crossed Andreev (CA) tunneling rate \( \Gamma^{CA}_{EC} \) which shows how many electron pairs tunnel per second from the leads 1 and 2 into the condensate of the superconductor (each lead gives one electron into a pair) and vice versa correspondingly, see Fig. 5b, and 2. Elastic cotunneling rates \( \Gamma^{EC}_{DA} \) and \( \Gamma^{CA}_{DA} \) describe Andreev reflection in the leads 1 and 2 [see, e.g., Fig. 6]. The current in the lead 2 consists of two contributions: one, \( I^{(i)}_2 \), comes from the electron injection from the lead 1 due to crossed-Andreev and cotunneling processes, the other, \( I^{(j)}_2 \), – from the direct electron-tunneling between the lead and the superconductor. Same applies for the lead 1. Thus \( I^{(j)}_2(V_1, V_2) = I^{(j)}_2(V_1, V_2) + I^{(D)}_2(V_2) \), where

\[
I^{(j)}_2(V_1, V_2) = \Gamma^{(EC)} - \Gamma^{(CA)},
\]

\[
\Gamma^{(EC)} = \Gamma^{EC}_{DA} + \Gamma^{EC}_{DA},
\]

\[
\Gamma^{(CA)} = \Gamma^{CA}_{EC} + \Gamma^{CA}_{DA},
\]

\[
\Gamma^{(D)}_2(V_2) = \Gamma^{DA}_{EC} + \Gamma^{DA}_{EC}.
\]

Using the Fermi Golden rule, the rates can be found...
Along with the classical probability \( \Xi \), the boundary with the momentum directed along \( \hat{p}_2 \) spreading in the superconducting region as follows

\[
\Xi^{(CA)}_{\sigma}(t) = \frac{1}{8\pi^3 e^2\nu_S} \int d\hat{p}_{1,2} \int dx_{1,2} P(X_1, \hat{p}_1; X_2, \hat{p}_2, t) \times \left\{ G^{(1)}(X_1, \hat{p}_1, \sigma)G^{(2)}(X_2, \hat{p}_2, \sigma) \sin^2 \left( \frac{\theta(X_1, X_2)}{2} \right) + G^{(1)}(X_1, \hat{p}_1, \sigma)G^{(2)}(X_2, \hat{p}_2, -\sigma) \cos^2 \left( \frac{\theta(X_1, X_2)}{2} \right) \right\}.
\]

Here the spatial integration is performed over the \( N_1S \) and \( N_2S \) surfaces. We choose the spin quantization axis in the direction of the local magnetization in the terminal \( N_i(2) \). The quasiclassical probabilities \( G^{(i)}(X, \hat{p}, \sigma) \), \( i = 1, 2 \) for the electron with spin polarization \( \sigma \) tunneling from the terminal \( N_i \) to the superconductor are normalized in such way that the junction normal conductance per unit area \( g^{(i)}_N(X) \) and the total normal conductance \( G^{(i)}_N \) are determined as \( \frac{1}{2} \int dX \sum_{\sigma} g^{(i)}_N(X) \).

Then the normal conductance per unit area, discussed above, is defined as \( g^{(i)}_N = G^{(i)}_N / A \), where \( A \) is the surface area of the junction. Symbol \( \theta(X_1, X_2) \) is the angle between the magnetizations of the terminals \( N_1 \) and \( N_2 \) at points \( X_1 \) and \( X_2 \) near the junction surface. If electrons in \( N_1 \) and \( N_2 \) are not polarized then \( \theta = 0 \).

In a similar way:

\[
\Gamma_1^{1/2}(EC) = 4\pi^3 \int d\xi \sum_{\sigma} n^{(1)}_{\sigma}(\xi - V_1)(1 - n^{(2)}_{\sigma}(\xi - V_2)) \times \frac{\Delta^2}{[\Delta^2 - \xi^2]^2} \Xi^{(EC)}_{\sigma}(2\sqrt{\Delta^2 - \xi^2}),
\]

where

\[
\Xi^{(EC)}_{\sigma}(t) = \frac{1}{8\pi^3 e^2\nu_S} \int d\hat{p}_{1,2} \int dx_{1,2} P(X_1, \hat{p}_1; X_2, \hat{p}_2, t) \times \left\{ G^{(1)}(X_1, \hat{p}_1, \sigma)G^{(2)}(X_2, \hat{p}_2, -\sigma) \sin^2 \left( \frac{\theta(X_1, X_2)}{2} \right) + G^{(1)}(X_1, \hat{p}_1, \sigma)G^{(2)}(X_2, \hat{p}_2, \sigma) \cos^2 \left( \frac{\theta(X_1, X_2)}{2} \right) \right\}.
\]

The rate \( \Gamma_1^{1/2}(EC) \) can be obtained from the expression for \( \Gamma_1^{1/2} \) by substitution of \( (1 - n) \) for \( n \). DA-rates are written in [14]. Equations [14] [15] derived here allow to describe transport properties of many types of junctions.
Consider FISIN junction with biased ferromagnet with the respect to the superconductor, the normal metal N has same voltage as S. So the ferromagnet plays the role of a current “injector”; electrons coming from F are distributed with some distribution function \( n^{(1)} \). Electrons in the deep of the terminal N are Fermi-distributed. It follows from Eqs. (5a-7) that contributions to the current from EC and CA processes compensate each other for subgap voltages so \( I_N(V_F \neq 0, 0) = 0 \). However spin current is finite:

\[
P_N^{(\text{spin})}(V_F, 0) = 4\pi^3 \int d\xi \sum_\sigma \sigma [n^{(1)}_{\sigma}(\xi - V_F) - n^{(2)}_{\sigma}(\xi)] \frac{\Delta^2}{(\Delta^2 - \xi^2)^2} \frac{1}{8\pi^2 e^2 \nu_S} \int d\tilde{p}_{1,2} \int dX_{1,2} \times P(X_1, \tilde{p}_1; X_2, \tilde{p}_2, t) G^{(1)}(X_1, \tilde{p}_1, \sigma) G^{(2)}(X_2, \tilde{p}_2).
\]

Finally we consider a FISIF junction. It was shown in [2] that in this junction \( I_2(V_1, 0) \neq 0 \) and \( I_2(V_1, 0) \) changes its sign when the ferromagnetic terminals change their orientation from parallel to antiparallel. Naively it can be supposed that in a FISIN junction where F is a current injector, S, N are grounded spin accumulation at the interface of the normal metal would lead to spin-splitting of the density of states in N and a charge current. However this is not so, this corrections are of higher order over tunneling amplitudes than the processes in Fig. 5 and can be neglected because we assume that tunneling amplitudes are small.

It was also noted in [2] that the cross conductance \( G_{12} \equiv \partial V_1 I_2(V_1, 0)|_{V_1 = 0} \) is suppressed in a FISIF structure as \( 1/(k_Fr)^2 \) when the characteristic distance between the ferromagnets \( r < \xi \). In dirty regime there is no conductance suppression at atomic-scales. Consider, for instance, the layout sketched in Fig. 1b; the width \( d \) of the superconducting film is supposed to be smaller than \( \xi \). According to Eqs. (5a) and (7) the cross-conductance dependence from the distance \( r \) is determined by the Laplace transform \( \tilde{P}(s = 2\sqrt{\Delta}) \) of the probability \( P(r, t) = \exp(-r^2/4D|t|)/4\pi d|t| \), where \( D \) is diffusion constant in the superconductor, \( d < \xi \). When \( \lambda_F < r < \xi, G_{12} \sim \tilde{P} \sim \ln(r/\xi) \) and if \( r \gg \xi, G_{12} \sim \tilde{P} \sim \exp(-r/\xi) \). When the superconductor is bulk \( d > \xi \) similarly we find \( G_{12} \sim \xi/r, \lambda_F < r < \xi \). All considerations above apply also for CA- and EC-rates. Thus it is practically more convenient to measure finite effects related to electron subgap tunneling through a superconductor when it is dirty rather than clean. In dirty case the terminals are not restricted to be as close as \( \lambda_F \) like in clean case but closer then \( \xi \gg \lambda_F \).

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