Coherence Factors and Quantum Interferences in
Excitonic Condensation of Ta$_2$NiSe$_5$

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Abstract. In order to elucidate whether Ta$_2$NiSe$_5$ is in an excitonic condensation state or not, we study macroscopic quantum interferences in ultrasonic attenuation rate and nuclear magnetic resonance relaxation rate. Using the three-chain model describing the excitonic condensation of Ta$_2$NiSe$_5$, we demonstrate analytically that the ultrasonic attenuation rate shows a characteristic peak just below the transition temperature of the excitonic condensation, while the nuclear magnetic resonance relaxation rate shows a rapid drop. In particular, we find that the constructive interference originates from the hybridization between the conduction and valence bands induced by an external field.

1. Introduction

It is known that, in narrow-gap semiconductors or semimetals, pairs of electrons and holes (excitons) are spontaneously formed and go into a condensed state with macroscopic phase coherence. This state is referred to as an excitonic condensation [1], and it entails the flattening of the band edges in the case of semiconductors, and the opening of the band gap in the case of semimetals. It was predicted half a century ago, but actual materials of this phase are not known. However, thanks to the recent development of angle-resolved photoemission spectroscopy experiments that enable us to observe band dispersions directly, the realizations of excitonic condensation in some materials have been suggested [2, 3, 4, 5, 6]. Especially, Ta$_2$NiSe$_5$, which is known as a direct-gap semiconductor [7, 8], is regarded as one of the most promising candidates of excitonic condensation [2, 3, 9].

A measurement of transformation of the band dispersion is, however, merely an indirect evidence for excitonic condensation. In this respect, there have been some proposals that the ultrasonic attenuation rate shows a coherence peak in a simple electron-gas model [10, 11] although the connection to an actual material lacks. In order to elucidate whether Ta$_2$NiSe$_5$ is actually in the excitonic phase or not, we have recently proposed that measurements of the ultrasonic attenuation rate and nuclear magnetic resonance (NMR) relaxation rate can lead to a direct evidence for excitonic condensation in Ta$_2$NiSe$_5$ [12]. In our proposal, we predicted that the ultrasonic attenuation rate shows a characteristic peak just below the transition temperature, while the NMR relaxation rate shows a rapid drop just below it. These behaviors originate from the quantum interferences due to the superposition of the wave functions of electrons and holes in the excitonic condensation state.
The purpose of the present paper is then to provide an analysis of the coherence factors just below the transition temperature from an analytical (rather than numerical) point of view, which has not been given in Ref. [12]. The origins of the peak in the ultrasonic attenuation rate and the drop in the NMR relaxation rate are thus elucidated. In Sect. 2, we will introduce the three-chain model describing Ta$_2$NiSe$_5$ and apply the BCS-like mean-field approximation to obtain the Bogoliubov transformation for excitonic condensation. In Sect. 3, we will calculate the coherence factors appearing in the ultrasonic attenuation rate and NMR relaxation rate and discuss the origins of the peak and drop analytically. In Sect. 4, we will summarize our results.

2. Three-chain model and mean-field approximation

In order to describe the electronic properties in the excitonic condensation state of Ta$_2$NiSe$_5$, we make use of the three-chain model that we have previously introduced [9, 12]. There are doubly degenerate conduction bands (CBs) of two Ta chains (labeled $\alpha = 1, 2$) and a valence band (VB) of one Ni chain in our model. The Hamiltonian is written as $H = H_0 + H_{\text{e-e}} + H_{\text{lat}}$, where $H_0$ is a kinetic term, $H_{\text{e-e}}$ is an electron-electron interaction term, and $H_{\text{lat}}$ is a lattice-distortion term relating to the structural phase transition in Ta$_2$NiSe$_5$. The creation (annihilation) operator of a CB electron at site $j$ with spin $\sigma$ on the chain $\alpha$ is defined as $c_{j,\alpha,\sigma}^\dagger$ ($c_{j,\alpha,\sigma}$), and the creation (annihilation) operator of a VB electron at site $j$ with spin $\sigma$ is defined as $f_{j,\sigma}^\dagger$ ($f_{j,\sigma}$). Also, the number operators of the CB and VB electrons are defined as $\hat{n}_{j,\alpha,\sigma} = c_{j,\alpha,\sigma}^\dagger c_{j,\alpha,\sigma}$ and $\hat{n}_{j,\sigma} = f_{j,\sigma}^\dagger f_{j,\sigma}$, respectively. The kinetic term is written as

$$H_0 = t_c \sum_{j,\alpha,\sigma} \sum_{\delta = \pm 1} c_{j+\delta,\alpha,\sigma}^\dagger c_{j,\alpha,\sigma} + \varepsilon_c \sum_{j,\alpha,\sigma} \hat{n}_{j,\alpha,\sigma} + t_l \sum_{\delta = \pm 1} \sum_{j,\sigma} f_{j+\delta,\sigma}^\dagger f_{j,\sigma} + \varepsilon_l \sum_{j,\sigma} \hat{n}_{j,\sigma},$$

where $t_c$ and $\varepsilon_c$ are the hopping integrals and the on-site energies of the CB (VB) electrons, respectively. The electron-electron interaction term is expressed as $H_{\text{e-e}} = H_{U_c} + H_{U_l} + H_V$ with

$$H_{U_c} = U_c \sum_{j,\alpha} \hat{n}_{j,\alpha,\uparrow} \hat{n}_{j,\alpha,\downarrow}, \quad H_{U_l} = U_l \sum_j \hat{n}_{j,\uparrow} \hat{n}_{j,\downarrow},$$

and

$$H_V = V \sum_{j,\sigma,\sigma',\alpha} \hat{n}_{j,\sigma,\alpha} \hat{n}_{j,\sigma',\alpha'} + V \sum_{j,\sigma,\sigma',\alpha} (\hat{n}_{j+1,1,\sigma} + \hat{n}_{j-1,2,\sigma}) \hat{n}_{j,\sigma',\alpha},$$

where $U_{U_c(\alpha)}$ is the on-site Coulomb interaction $U_{U_c(\alpha)}$ of the CB (VB) electrons, and $H_V$ is the intersite interaction $V$ between the CB and the VB electrons. The lattice-distortion term consists of a term of harmonic oscillators for Ta atoms and a hybridization term between CB and VB electrons caused by the lattice distortion, and is written as

$$H_{\text{lat}} = \sum_{j,\alpha} \left\{ \frac{K}{2} X_{j,\alpha}^2 - \gamma_\alpha X_{j,\alpha} \sum_\sigma \left( c_{j,\alpha,\sigma}^\dagger f_{j,\sigma} + f_{j,\sigma}^\dagger c_{j,\alpha,\sigma} \right) \right\},$$

where $X_{j,\alpha}$ is the displacement of a Ta atom at site $j$ in the $\alpha$-th chain measured from its equilibrium position, and $K$ is a spring constant of the harmonic oscillators. $\gamma_\alpha$ is the strength of hybridization between the CB and VB electrons caused by the lattice distortion. We assume $\gamma_1 = \gamma$ and $\gamma_2 = -\gamma$.

The mixed order parameters of excitonic condenstation and lattice distortion are defined as

$$\hat{\Delta}_1(k) = V \left\langle c_{j,1,1,\sigma}^\dagger f_{j,\sigma} \right\rangle + V \left\langle c_{j+1,1,1,\sigma}^\dagger f_{j,\sigma} \right\rangle e^{ik_\alpha} + \gamma_1 X_{j,1},$$

$$\hat{\Delta}_2(k) = V \left\langle c_{j,2,\sigma}^\dagger f_{j,\sigma} \right\rangle + V \left\langle c_{j-1,2,\sigma}^\dagger f_{j,\sigma} \right\rangle e^{-ik_\alpha} + \gamma_2 X_{j,2},$$

where $\alpha = 1, 2$. The mixed order parameters $\hat{\Delta}_1(k)$ and $\hat{\Delta}_2(k)$ are eigenstates of the Hamiltonian $H$.
where $a$ is the lattice constant, which is set to be unity hereafter. We assume that the system is homogeneous, and thus the order parameters are independent of the lattice site. In the mean-field approximation, the Hamiltonian in $k$ space without a constant term reads

$$H^{\text{MF}} = \sum_{k,\alpha,\sigma} \tilde{\varepsilon}_c(k) c_{k,\alpha,\sigma}^\dagger c_{k,\alpha,\sigma} + \sum_{k,\sigma} \tilde{\varepsilon}_i(k) f_{k,\sigma}^\dagger f_{k,\sigma} - \sum_{k,\sigma,\alpha} \left\{ \tilde{\Delta}_\alpha(k) f_{k,\sigma}^\dagger c_{k,\alpha,\sigma} + \tilde{\Delta}_\alpha^*(k) c_{k,\alpha,\sigma}^\dagger f_{k,\sigma} \right\}, \quad (7)$$

where $\tilde{\varepsilon}_{c(i)}(k)$ is the same as that in the Supplemental Material of our previous work [12]. Diagonalizing the mean-field Hamiltonian given in Eq. (7), we obtain

$$H^{\text{MF}} = \sum_{k,\sigma} \sum_{\epsilon = c, i} E_{k,\epsilon} \gamma_{k,\epsilon,\sigma} \gamma_{k,\epsilon,\sigma}^\dagger,$$

where

$$E_{k,\epsilon} = \tilde{\varepsilon}_c(k), \quad E_{k,\pm} = \eta_k \pm E_k$$

with

$$\eta_k = \frac{\tilde{\varepsilon}_c(k) + \tilde{\varepsilon}_i(k)}{2}, \quad \xi_k = \frac{\tilde{\varepsilon}_c(k) - \tilde{\varepsilon}_i(k)}{2}, \quad E_k = \sqrt{\xi_k^2 + |\tilde{\Delta}_1(k)|^2 + |\tilde{\Delta}_2(k)|^2}. \quad (10)$$

$\gamma_{k,\epsilon,\sigma}$ ($\gamma_{k,\epsilon,\sigma}^\dagger$) is the annihilation (creation) operator of the quasiparticle. They satisfy the relation

$$c_{k,\mu,\sigma} = \sum_\epsilon \psi_{k,\sigma,\mu,\epsilon} \gamma_{k,\epsilon,\sigma}, \quad \text{where} \quad \psi_{k,\sigma,\mu,\epsilon} \text{ is the Bogoliubov transformation coefficient and } f_{k,\sigma} = c_{k,3,\sigma}.$$  

The detailed studies of this model are given in our previous papers [9, 12].

Figure 1 shows the band dispersions of this model. In the excitonic phase, one of the doubly degenerate CBs ($\epsilon = +$) hybridizes with the VB ($\epsilon = -$), leading to the flattening of these bands and the lifting of the degeneracy in the CBs. The other CB ($\epsilon = c$) sustains its dispersion even in the excitonic phase.

In this paper, we are interested in a behavior just below the transition temperature, where the order parameters $|\tilde{\Delta}_\alpha(k)|$ are sufficiently small compared to the typical energy scales such as $\xi_k$. To the second order of $|\tilde{\Delta}_\alpha(k)|$, the Bogoliubov transformation in Eq. (11) becomes

$$\left( \begin{array}{c} c_{k,1,\sigma} \\ c_{k,2,\sigma} \\ f_{k,\sigma} \end{array} \right) \approx \left( \begin{array}{ccc} \frac{|\Delta_1(k)|}{|\Delta_1(k)|} & -\frac{|\Delta_2(k)|}{|\Delta_1(k)|} & 0 \\ \frac{|\Delta_2(k)|}{|\Delta_2(k)|} & -\frac{|\Delta_1(k)|}{|\Delta_2(k)|} & 0 \\ 0 & 0 & \frac{|\Delta_1(k)|}{2|\xi_k|} \end{array} \right) \left( \begin{array}{c} \frac{|\Delta_1(k)|}{2|\xi_k|} \\ \frac{|\Delta_2(k)|}{2|\xi_k|} \\ 1 - \frac{|\Delta_1(k)|^2}{8|\xi_k|^2} \end{array} \right),$$

where $|\tilde{\Delta}(k)|^2 = |\tilde{\Delta}_1(k)|^2 + |\tilde{\Delta}_2(k)|^2$. 

(13)
Figure 1. (Left) The band dispersions in the normal phase. Hopping integrals of CB and VB are $t_c = -0.8 \text{ eV}$ and $t_f = 0.4 \text{ eV}$, respectively. The energy gap is set to be $0.2 \text{ eV}$. Blue line indicates the doubly degenerate CBs and red line indicates the VB. (Right) The band dispersions in the excitonic phase. We set $V = 0.8 \text{ eV}$, $U_c = U_f = 4 \text{ eV}$, and $\lambda = 0.01 \text{ eV}$, where $V$ is larger than that of our previous work ($V = 0.6 \text{ eV}$) [12] in order to emphasize the flattening of the bands. Green, blue, and red lines indicate the bands $c$, $+$, and $-$, respectively.

3. Coherence factors as the response to external fields

3.1. Ultrasonic attenuation rate

We define the operator of the phonon of Ta atoms on $\alpha$-th chain as $A_{q,\alpha} = a_{q,\alpha} + a_{-q,\alpha}$. The electron-phonon interaction Hamiltonian by the ultrasonic wave is then written as

$$H' = \sum_{k,q,\sigma,\alpha} \left\{ M_{c}^{cc} A_{q,\alpha} \frac{c_{k+1,\omega}^{\dagger} c_{k+\alpha,\sigma} - M_{c}^{cf} A_{q,\alpha} \left( f_{k+1,\omega}^{\dagger} f_{k+\alpha,\sigma} + f_{k+1,\omega}^{\dagger} c_{k+\alpha,\sigma} \right)}{2} \right\}, \quad (14)$$

where $k_{\pm} = k \pm \frac{q}{2}$, $M_{c}^{cc}$ term is the coupling between the phonon and electron-charge density, and $M_{c}^{cf}$ term is the hybridization of CB and VB electrons due to the phonon oscillation. For simplicity, we neglect the phonon of Ni atoms. In the shear-mode ultrasonic wave, the phonon does not couple to the charge density, and the $M_{c}^{cf}$ term may be the dominant contribution to the electron scattering. The ultrasonic attenuation rate $\alpha_{q=0,\alpha}$ then reads [12]

$$\alpha_{q=0,\alpha} = 2\pi \omega_{\text{ph}} \sum_{k,\sigma} \sum_{\epsilon_1,\epsilon_2} \frac{\beta}{4 \cosh^2 \left( \frac{\beta}{2} \left( E_{k,\epsilon_1} - E_{k,\epsilon_2} \right) \right)} \delta (E_{k,\epsilon_1} - E_{k,\epsilon_2})$$

$$\times \sum_{\mu,\nu=1}^{2} \left\{ \frac{1}{4} (M_{0}^{cc})^2 \delta_{\alpha,\mu} \delta_{\alpha,\nu} + \left( M_{0}^{cf} \right)^2 \delta_{\mu,3} \delta_{\nu,3} - M_{0}^{cc} M_{0}^{cf} \delta_{\alpha,\mu} \delta_{\nu,3} \right\}$$

$$\times 2 \text{Re} \left\{ \Psi_{\alpha,\mu,\epsilon_1}(k,\sigma) \Psi_{\alpha,\nu,\epsilon_2}(k,\sigma) + \Psi_{\alpha,\alpha,\epsilon_1}(k,\sigma) \Psi_{\mu,\nu,\epsilon_2}(k,\sigma) \right\}, \quad (15)$$

where $\Psi_{\mu,\nu,\epsilon}(k,\sigma) = \psi_{k,\sigma,\nu}^{\epsilon} \psi_{k,\sigma,\mu,\epsilon}^{*}$ and $\omega_{\text{ph}}$ is the phonon frequency. The interband contribution to the ultrasonic attenuation rate is prohibited since the energy of the phonon is too small to scatter an electron from one band to another, and thus we can assume $\epsilon_1 = \epsilon_2$ in the summation.

There are three contributions to the ultrasonic attenuation rate; the charge-density term (proportional to $(M_{0}^{cc})^2$), the CB-VB hybridization term (proportional to $(M_{0}^{cf})^2$), and the...
cross term (proportional to $M_0^c M_0^d$). We examine the coherence factors up to the second order of the order parameters. The coherence factors are written as products of four wave functions given in Eq. (13). We focus on the attenuation of phonons in the Ta $\alpha = 1$ chain below.

3.1.1. Charge-density term  The orbitals are restricted to $\alpha = \mu = \nu = 1$ in this case. For $\epsilon = c$ and $\epsilon = -$, we have

$$
\Psi_{1,1;c}(k, \sigma)\Psi_{1,1;c}(k, \sigma) = \left| \frac{\Delta_2(k)}{\Delta(k)} \right|^4,
$$

(16)

$$
\Psi_{1,1;-(k, \sigma)}\Psi_{1,1;-(k, \sigma)} = O \left( \frac{\Delta_1(k)}{|\xi_k|^4} \right),
$$

(17)

respectively, and they are almost irrelevant to the quantum interference. However, for $\epsilon = +$, we have

$$
\Psi_{1,1;+}(k, \sigma)\Psi_{1,1;+}(k, \sigma) \simeq \left| \frac{\Delta_1(k)}{\Delta(k)} \right|^4 \left( 1 - \frac{2}{\xi_k^2} \right),
$$

(18)

where the second term leads to the destructive interference. As a result, the coupling between the phonons and charge density leads to the drop in the attenuation rate.

3.1.2. CB-VB hybridization term  The orbitals are restricted to $\alpha = 1$ and $\mu = \nu = 3$ in this case. For $\epsilon = c$, we have

$$
\Psi_{3,1;c}(k, \sigma)\Psi_{3,1;c}(k, \sigma) = \Psi_{1,1;c}(k, \sigma)\Psi_{3,3;c}(k, \sigma) = 0,
$$

(19)

which is completely irrelevant to the attenuation rate. On the other hand, for $\epsilon = +$ and $\epsilon = -$, we have

$$
\Psi_{3,1;+}(k, \sigma)\Psi_{3,1;+}(k, \sigma) \simeq \frac{\Delta_1(k)^2}{4|\xi_k|^2},
$$

$$
\Psi_{1,1;+}(k, \sigma)\Psi_{3,3;+}(k, \sigma) \simeq \frac{|\Delta_1(k)|^2}{4|\xi_k|^2},
$$

(20)

$$
\Psi_{3,1;-(k, \sigma)}\Psi_{3,1;-(k, \sigma)} \simeq \frac{\Delta_1(k)^2}{4|\xi_k|^2},
$$

$$
\Psi_{1,1;-(k, \sigma)}\Psi_{3,3;-(k, \sigma)} \simeq \frac{|\Delta_1(k)|^2}{4|\xi_k|^2},
$$

(21)

all of which lead to the constructive interferences. This fact indicates that the peak appearing in ultrasonic attenuation rate just below the transition temperature originates from the CB-VB hybridization by the phonons.

3.1.3. Cross term  The orbitals are restricted to $\alpha = \mu = 1$ and $\nu = 3$. As in the case of the charge-density term, we have

$$
-\Psi_{1,1;c}(k, \sigma)\Psi_{1,3;c}(k, \sigma) = 0
$$

(22)

$$
-\Psi_{1,1;-(k, \sigma)}\Psi_{1,3;-(k, \sigma)} = O \left( \frac{\Delta_1(k)^3}{|\xi_k|^3} \right)
$$

(23)
for $\epsilon = c$ and $\epsilon = -$, respectively, which are also irrelevant to the quantum interference. For $\epsilon = +$, however, we have

$$-\Psi_{1,1;+}(k, \sigma)\Psi_{1,3;+}(k, \sigma) \simeq -\frac{|\tilde{\Delta}_1(k)|^2 |\tilde{\Delta}_1'(k)|}{2|\xi_k||\Delta(k)|^2},$$

which leads to the destructive interference.

3.2. NMR relaxation rate

The NMR relaxation rate $1/T_1$ for $\mu$-th chain atom is obtained from the spin-transverse correlation function. It is written as [12]

$$\frac{1}{T_1} \propto \sum_{k,q} \sum_{\epsilon_1,\epsilon_2} \Psi_{\mu,\mu;\epsilon_1}(k_-, \uparrow)\Psi_{\mu,\mu;\epsilon_2}(k_+, \downarrow) \frac{1}{4 \cosh^2 (\beta (E_{k_-, \epsilon_1} - \mu) / 2)} \delta(E_{k_-, \epsilon_1} - E_{k_+, \epsilon_2}).$$

There appears a summation over the wave number $q$, which is different from the case of the ultrasonic attenuation rate where the wave vector of the phonon is fixed. The interband scattering can therefore occur. Note that, as shown in Fig. 1, the $\epsilon = c$ band and $\epsilon = +$ band are above the Fermi level, while $\epsilon = -$ band is below the Fermi level. Thus, only the scattering between $c$ band and $+$ band is allowed.

3.2.1. For Ta atom NMR

We consider the Ta $\mu = 1$ chain here. In the case of the intraband scattering $\epsilon_1 = \epsilon_2$, we have

$$\Psi_{1,1;\epsilon}(k_-, \uparrow)\Psi_{1,1;\epsilon}(k_+, \downarrow) = \frac{|\tilde{\Delta}_2(k_-)|^2 |\tilde{\Delta}_2'(k_+)|^2}{|\Delta(k_-)|^2 |\Delta(k_+)|^2}$$

and

$$\Psi_{1,1;+}(k_-, \uparrow)\Psi_{1,1;+}(k_+, \downarrow) \simeq \frac{|\tilde{\Delta}_1(k_-)|^2 |\tilde{\Delta}_1'(k_+)|^2}{|\Delta(k_-)|^2 |\Delta(k_+)|^2} \left(1 - \frac{|\tilde{\Delta}_2(k_+)|^2}{\xi_{k_+}^2} - \frac{|\tilde{\Delta}(k_-)|^2}{\xi_{k_-}^2}\right).$$

This implies that the intraband scattering of $+$ band leads to the destructive interference.

In the case of the interband scattering $\epsilon_1 = c$ and $\epsilon_2 = +$, we have

$$\Psi_{1,1;c}(k_-, \uparrow)\Psi_{1,1;+}(k_+, \downarrow) \simeq \frac{|\tilde{\Delta}_2(k_-)|^2 |\tilde{\Delta}_1'(k_+)|^2}{|\Delta(k_-)|^2 |\Delta(k_+)|^2} \left(1 - \frac{|\tilde{\Delta}_2(k_+)|^2}{\xi_{k_+}^2}\right),$$

which also leads to the destructive one.
3.2.2. For Ni atom NMR $\mu = 3$ indicates the Ni chain. In this case, there is no scattering in the band relating to $\epsilon = c$ since the coherence factors in this case are necessarily zero.

In the case of $\epsilon_1 = \epsilon_2 = +$, we have

$$\Psi_{3,3;+}(k_-, \uparrow)\Psi_{3,3;+}(k_+, \downarrow) = O\left(\frac{\Delta(k_\pm)}{\xi_{k_\pm}^4}\right)^4,$$

which is negligible. On the other hand, in the case of $\epsilon_1 = \epsilon_2 = -$, we have

$$\Psi_{3,3;-}(k_-, \uparrow)\Psi_{3,3;-}(k_+, \downarrow) \simeq 1 - \frac{\Delta(k_-)^2}{4\xi_{k_-}^2} - \frac{\Delta(k_+)^2}{4\xi_{k_+}^2},$$

which leads to the destructive interference.

4. Summary

Introducing the three-chain model describing the excitonic condensation in Ta$_2$NiSe$_5$, we have obtained the Bogoliubov transformation for the excitonic condensation in the mean-field approximation. From the transformation coefficients, we have calculated the coherence factors in the ultrasonic attenuation rate and the NMR relaxation rate and have expanded them to the second order of the order parameters. The results indicate that the CB-VB hybridization term in the ultrasonic attenuation rate leads to the constructive interference, giving rise to the coherence peak just below the transition temperature. On the other hand, the charge-density term in the ultrasonic attenuation rate leads to the destructive interference, resulting in the drop of the rate below the transition temperature. In the NMR relaxation rate, only the destructive interference terms appear, and therefore the rate inevitably decreases just below the transition temperature. These results imply that the coherence peak can be observed only if the external field hybridizes the CB electrons and VB electrons in excitonic condensation.

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References

[1] Jerome D, Rice T M and Kohn W 1967 Phys. Rev. 158 462
[2] Wakisaka Y et al. 2009 Phys. Rev. Lett. 103 026402
[3] Seki K et al. 2014 Phys. Rev. B 90, 155116
[4] Pillo T, Hayoz J, Berger H, Levy F, Schlappbach L and Aebi P, 2000 Phys. Rev. B 61, 16213
[5] Kidd T E, Miller T, Chou M Y and Chiang T C 2002 Phys. Rev. Lett. 88, 226402
[6] Cercellier H et al. 2007 Phys. Rev. Lett. 99, 146403
[7] Sunshine S A and Ibers J A 1985 Inorg. Chem. 24, 3611
[8] Di Salvo F J, Chen C H, Fleming R M, Waszczak J V, Dunn R G, Sunshine S A and Ibers J A 1986 J. Less Common Met. 116 51
[9] Kaneko T, Toriyama T, Konishi T and Ohta Y 2013 Phys. Rev. B 87 035121
[10] Maki K and Nakaniishi K 1971 J. Low Temp. Phys. 5 55
[11] Amritkar R E and Kumar N 1978 Solid State Commun. 26 627
[12] Sugimoto K, Kaneko T and Ohta Y 2016 Phys. Rev. B 93 041105(R)