Quantum computation with coupled-quantum-dots embedded in optical microcavities

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Based on an idea that spatial separation of charge states can enhance quantum coherence, we propose a scheme for quantum computation with quantum bit (qubit) constructed from two coupled quantum dots. Quantum information is stored in electron-hole pair state with the electron and hole locating in different dots, which enables the qubit state being very long-lived. Universal quantum gates involving any pair of qubits are realized by coupling the quantum dots through cavity photon which is a hopeful candidate to transfer long-range information. Operation analysis is carried out by estimating the gate time versus the decoherence time.

To build a practical quantum computer is a challenging task, since the computational quantum objects (the qubits) must be sufficiently isolated from the dissipative environment, precisely and conditionally manipulated, efficiently read-out and initialized, and most importantly scalable. To date, a variety of quantum computation (QC) schemes based on some unique systems have been proposed. 

(i) In each qubit (two coupled QDs), one and only one excess electron is required in the conduction band. We showed that the spatial separation of the logic states can efficiently reduce the qubit decoherence. Nevertheless, several shortcomings exist there and in some of the aforementioned QC schemes based on QDs: (i) In each qubit (two coupled QDs), one and only one excess electron is required in the conduction band. This is a challenging task within current technology. (ii) The intersubband transition with THz lasers is currently not a mature technology. (iii) The coupling between qubits is mediated by Coulomb interactions, which makes it very difficult to perform conditional gate operation between any pair of qubits.

In this paper, still based on the idea by constructing a single qubit from two coupled QDs to reduce decoherence, we propose an alternative scheme to remove all of these shortcomings. In this newly proposed scheme, no excess electron is required in the qubit, and quantum information is stored in electron-hole pair state.

The physical system we are concerned with for quantum computation is similar as that proposed by Imamoglu et al, i.e., many QDs are located in an optical microcavity. Both the QDs and cavity are threedimensionally (3D) confined; however, the cavity has a size and thus the fundamental wavelength much larger than the individual QD. In our structure, we suggest to use two weakly coupled QDs to construct a qubit as shown in Fig. 1. We assume a relatively large distance between neighboring qubits such that the qubits can be selectively addressed by lasers, and the Coulomb correlations between them can be neglected. As have mentioned above, in our structure no excess single electron is required in the conduction band. The quantum information is stored in electron-hole pair state: the qubit logic states $|0\rangle$ and $|1\rangle$ correspond to the ground state and an electron-hole pair state, respectively. (Here the symbol \(\tilde{\ldots}\) is used to distinguish the notation of logic states.
Here $\Omega_{1}$ is the Rabi frequency, and $\phi$ is the laser phase. With the use of this interaction, arbitrary single-qubit operations can be performed.

Next, consider the cavity-photon involved transition. This is an essential ingredient to realize the two-qubit gate, in which the cavity photon plays a role of data-bus to control the two-qubit state evolution. Switching on a laser action with frequency $\omega_{2} = E_{e} - E_{v} - \omega_{c}$, a resonant transition from $|v\rangle$ to $|e\rangle$ takes place by involving two photons, namely, the $\omega_{2}$ laser photon and the $\omega_{c}$ cavity photon. A simple perturbation theory gives rise to

$$H_{I}^{(2)} = \Omega_{\text{eff}} |e\rangle \langle v|a e^{i\phi} + \text{H.c.}|$$

with $\Omega_{\text{eff}} = \Omega_{2} \Omega_{c}/\delta$. Here $\Omega_{2}$ is the optical coupling strength between $|\tilde{e}\rangle$ and $|v\rangle$ associating with the laser field, and $\Omega_{c}$ is the coupling strength between $|e\rangle$ and $|\tilde{e}\rangle$ due to the cavity field. $\delta$ is the detuning between the laser frequency and the transition energy from $|v\rangle$ and $|e\rangle$, i.e., $\delta = \omega_{2} - (E_{2} - E_{v})$. Note that this second-order process is mediated via the intermediate state $|\tilde{e}\rangle$.

However, due to the off-resonance of the laser frequency with the transition $|v\rangle \rightarrow |\tilde{e}\rangle$, there is no real occupation on state $|\tilde{e}\rangle$, accordingly its relatively strong decoherence resulting from its radiative recombination with the intra-dot hole $|v\rangle$ is avoided. In the latter part of this paper, we will show that owing to the spatial separation of $|e\rangle$ from the states in the larger dot, the coherence of qubit state $|e\rangle$ can be essentially improved.

To realize the conditional two-bit gate such as the control NOT (CNOT), typical methods include the Cirac-Zoller (CZ) protocol or the pulse technique developed in the spin QC model. In the proposals based on QDs in cavity, both of these two gating techniques have been employed. Very recently, an improved gating technique for the ion-trap QC was developed where only two electronic states are required, and the third auxiliary state in the CZ protocol is not needed. In the following, we employ this technique in our scheme with certain modification due to the only red-band pulse in our case. To make the description more transparent, we introduce the following notations: the states of the control qubit (the jth one) and the target qubit (the kth one) together with the cavity photon are denoted as $|a_{j}\rangle|b_{k}\rangle|p\rangle = |a_{j}\rangle|b_{k}\rangle|p\rangle$; $a, b, p = 0, 1$, where $0$ and $1$ correspond to either the qubit state (i.e. logic $|0\rangle \equiv |v\rangle$ and $|1\rangle \equiv |e\rangle$), or the cavity field state with zero and one photon. Below we outline how to realize the CNOT gate.

(i) First, swap the control qubit state to the cavity photon state:

$$R_{j}(\pi, \phi) |\theta_{j}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0_{j}0_{k}\rangle \\ 1_{j}0_{k}\rangle \\ 0_{j}1_{k}\rangle \\ 1_{j}1_{k}\rangle \end{pmatrix} = |\tilde{0}_{j}\rangle \begin{pmatrix} e^{-i\theta}0_{k}\rangle \\ 0_{k}\rangle \\ e^{-i\theta}1_{k}\rangle \\ 1_{k}\rangle \end{pmatrix} |\tilde{1}_{j}\rangle \begin{pmatrix} e^{-i\theta}0_{k}\rangle \\ 0_{k}\rangle \\ e^{-i\theta}1_{k}\rangle \\ 1_{k}\rangle \end{pmatrix} \begin{pmatrix} 0_{j}\rangle \\ 1_{j}\rangle \\ 0_{j}\rangle \\ 1_{j}\rangle \end{pmatrix} .$$

Hereafter the evolution operator $R_{j}(k)(\theta, \phi)$ is determined by the interaction Hamiltonian $H_{I}^{(2)}$ in terms of
$R(\theta, \phi) = \exp\left[i\frac{\theta}{2} (|e\rangle\langle e| + H.c.) \right]$, where $\theta = 2\Omega_{\text{eff}}T$ with $T$ the duration time of the laser pulse.

(ii) Now, the cavity photon can play a role of control qubit, which controls the evolution of the target qubit. A series of pulse operations on the $k$th qubit by involving the participation of the cavity photon yield

$$G_k = \begin{bmatrix}
|\tilde{0}_k; 0\rangle & |\tilde{1}_k; 0\rangle \\
|\tilde{0}_k; 1\rangle & |\tilde{1}_k; 1\rangle
\end{bmatrix} = \begin{bmatrix}
\delta |\tilde{0}_k; 0\rangle & \delta |\tilde{1}_k; 0\rangle \\
-\delta^* |\tilde{1}_k; 1\rangle & -\delta^* |\tilde{0}_k; 1\rangle
\end{bmatrix},$$

where $\delta = e^{i\phi_0}$, and $\phi_0 = \pi/2\sqrt{2}$. We see that the cavity photon has played a control role in the conditional evolution of the $k$th qubit. In Eq. (4), $G_k$ constitutes a series of operations on the $k$th qubit, $G_k \equiv H_k P_k Z_k(\phi_0) H_k$. Here $H_k$ and $Z_k(\phi_0)$ are the single-qubit Hadamard and phase transformations

$$H_k = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad Z_k(\phi_0) = \begin{bmatrix} e^{i\phi_0} & 0 \\ 0 & e^{-i\phi_0} \end{bmatrix}.$$  \hspace{1cm} (5)

In the subspace $\{|\tilde{0}_k; 0\rangle, |\tilde{0}_k; 1\rangle, |\tilde{1}_k; 0\rangle, |\tilde{1}_k; 1\rangle\}$, the operator $P_k$ has a diagonal form

$$P_k = R_k(-\pi/2, 0) R_k(\sqrt{\pi}, -\pi/2) R_k(\pi/2, 0) = \text{diag}(1, e^{-i\pi/\sqrt{2}}, e^{i\pi/\sqrt{2}}, -1).$$ \hspace{1cm} (6)

(iii) Finally, the cavity photon state is swapped back to the qubit state by performing operation $R_j(\pi, 0)|\tilde{0}_j; 1\rangle = e^{i\phi}|1_j; 0\rangle$, on the $j$th-qubit. After a phase gate $Z_j(\phi_0)$ on the $j$th-qubit, the standard CNOT gate is realized

$$U_{jk} = \begin{bmatrix}
|\tilde{0}_j\tilde{0}_k\rangle & |\tilde{0}_j\tilde{1}_k\rangle & |\tilde{1}_j\tilde{0}_k\rangle & |\tilde{1}_j\tilde{1}_k\rangle
\end{bmatrix} = \begin{bmatrix}
|\tilde{0}_j\tilde{0}_k\rangle & |\tilde{0}_j\tilde{1}_k\rangle & |\tilde{1}_j\tilde{0}_k\rangle & |\tilde{1}_j\tilde{1}_k\rangle
\end{bmatrix},$$

where $U_{jk} = Z_j(\phi_0) R_j(\pi, 0) G_k R_j(\pi, 0)\phi_0$.  \hspace{1cm} (7)

In the remainder part of this paper, we present an analysis for the QC operation. In Ref. \[8\] based on a model GaAs system and disk geometry for the QDs, we have demonstrated by detailed numerical calculations that the ratio $\rho = \tau_d/\tau_G$ can be enhanced remarkably by the spatial separation of the qubit states, where $\tau_d$ and $\tau_G$ are the qubit decoherence and gating time, respectively. In what follows we provide an alternative way to understand this issue in general, not specifying the concrete QD material and geometry.

In the approximation of two-level model, $|e\rangle$ and $|\tilde{e}\rangle$ come from the coupling of two isolated dot states $|d\rangle$ and $|\tilde{d}\rangle$ with coupling strength $t$ and energy separation $\Delta = E_d - E_{\tilde{d}}$. (For the highest two valence band state, similar treatment can be done). As a result, the eigenstates $|e\rangle$ and $|\tilde{e}\rangle$ have eigenenergies $E_{\pm} = \frac{1}{2}[(E_d + E_{\tilde{d}}) \pm \sqrt{\Delta^2 + 4t^2}]$, and wavefunctions

$$|e\rangle = \sqrt{1 - \gamma}|d\rangle + \sqrt{\gamma}|\tilde{d}\rangle$$

$$|\tilde{e}\rangle = \sqrt{1 - \gamma}|\tilde{d}\rangle - \sqrt{\gamma}|d\rangle.$$  \hspace{1cm} (8)

where $\gamma = t^2/(\Delta^2 + t^2)$. With this state nature in mind, we below estimate various decoherence time and operation time in order.

The decoherence time of the qubit state is characterized by the relaxation time of $|e\rangle$. In our structure, the main intrinsic decoherence mechanisms come from the radiative relaxation and electron-phonon scattering. For both mechanisms, the relaxation rate can be expressed on the basis of Fermi golden rule as $W^{(j)} = \frac{2\pi}{\hbar} \sum_q |M^{(j)}_q| |\delta(E_e - E_f - \hbar\omega_q)|$, where $\omega_q$ is the emitted photon (phonon) frequency, and $M^{(j)}_q$ is the perturbative matrix element $M^{(j)}_q = \langle f|H^{(j)}_{\text{ph}}(q)|e\rangle$. Here the index $j = 1, 2$ and 3, together with the final state $|f\rangle$, denote three relaxation channels, namely, the spontaneous radiation from $|e\rangle$ to $|\tilde{e}\rangle$ and $|\tilde{d}\rangle$, and the phonon-scattering induced relaxation from $|e\rangle$ to $|\tilde{d}\rangle$. Due to the spatial separate nature of the electronic states shown in Eq. (6), we roughly estimate that the relaxation rate of each channel would be reduced by a factor $\gamma$, in comparison with the relaxation rate in a single dot. As a consequence, the decoherence time can be considerably enhanced by $\tau_d \approx \tau_d/\gamma$, where $\tau_d$ is the intra-dot decoherence time.

The operation time is limited by the optical coupling between $|e\rangle$ and $|\tilde{d}\rangle$ via the external laser field, and between $|e\rangle$ and $|\tilde{e}\rangle$ via the cavity photon. For both cases, the coupling strengths can be expressed in terms of $\Omega_{1e}(\gamma) = |e\rangle H^{(1,e)}_{\text{ph}}(\gamma)|\tilde{e}\rangle$. Similarly as above, due to the spatial separation of state $|e\rangle$ from $|\tilde{e}\rangle$ and $|\tilde{d}\rangle$ as shown in Eq. (6), $\Omega_{1}$ and $\Omega_{c}$ will be reduced approximately by a factor $\sqrt{\gamma}$ in comparison with the corresponding intra-dot coupling strengths. From Eq. (6) and (2), the logic state flipping time (between $|0\rangle$ and $|1\rangle$) is $\pi/\Omega_1$ or $\pi/\Omega_{\text{eff}}$ corresponding to the cavity photon involved or non-involved transition. As a consequence, the gate ratio $\rho = \tau_d/\tau_G$ will be enhanced by a factor $\approx 1/\sqrt{\gamma}$ due to the spatial separation of the qubit states. Note that $\gamma = t^2/(\Delta^2 + t^2)$, which can be a considerably small factor by reducing $t$ and increasing the energy-level separation $\Delta$. Similar conclusion has been quantitatively demonstrated by numerical calculation in Ref. \[8\].

We now parameterize the gate operation and decoherence times. Generally, consider each qubit consisting of two weakly coupled quantum dots with coupling strength between $|e\rangle$ and $|\tilde{e}\rangle$ (see Fig. 1) as, for example, $t = 0.01$ meV, and energy difference $\Delta = E_{\tilde{d}} - E_d = 10$ meV due to the distinct dot sizes. With these parameters, the spatial separation factor $\gamma = 10^{-6}$. Concerning the optical coupling with the electronic states, for the intra-dot interband transition due to the laser pulse, we assume a coupling strength $\Omega_2 = 0.1$ meV; and for the intra-dot state coupling with the cavity photon, a typical value of $\Omega_c = 300$ MHz is adopted here. To avoid a real occupation on the state $|\tilde{e}\rangle$, a detuning $\delta = 1$ meV is assumed between the laser frequency and the energy difference between $|\tilde{e}\rangle$ and $|e\rangle$. As an approximate estimate, for the cavity-photon involved transition from $|\tilde{e}\rangle$ to $|e\rangle$,
the effective Rabi frequency $\Omega_{\text{eff}} = \Omega_2 \omega_c / \delta \simeq 30$ KHz; and for the same transition in the absence of cavity photon, the Rabi frequency $\Omega_1 \sim \sqrt{\gamma \Omega_2} = 10^{-4}$ meV. As a consequence, for single qubit operations, the operation time is of the order of hundreds nanosecond, whereas for two-qubit conditional operations, the characteristic time is determined by the Rabi frequency $\Omega_{\text{eff}}$ in terms of $\tau_\Omega = \pi / \Omega_{\text{eff}} \simeq 10^{-4}$ sec.

For the decoherence time, we note that with current technology the quantum dot is available with energy level spacing larger than 10 meV, thus we assume no other electronic states between $|e\rangle$ and $|\tilde{e}\rangle$ in our structure. As a result, the intrinsic decoherence channels would be the radiative relaxation and electron-phonon scattering from $|e\rangle$ to $|\tilde{e}\rangle$ and $|v\rangle$. For the spontaneous emission, if the quantum dot has certain geometric symmetry, it has been shown that the so called dark states can have radiative lifetime longer than microsecond. Moreover, due to the CQED effect resulting from three dimensional cavity with high finesse, the spontaneous emission lifetime can be further suppressed. With these considerations, the CQED effect resulting from three dimensional cavities with extremely low loss.

Our calculations showed that the confined LO phonons in quantum dots have similar lifetime as in bulk materials, with the order of magnitude of picosecond. As a consequence, the LO phonon can induce electron relaxation even under the off-resonance condition. In the weak coupling limit as discussed here due to the spatial separation of qubit states, it can be shown that the LO phonon induced relaxation time is proportional to $g^{-2}$, with $g$ being the coupling strength between electron and LO phonons. Since this effect is still a topic in debate in the context of phonon bottleneck in quantum dots, we are not quite sure whether it is a severe decoherence resource in the above proposed QC scheme.

To further improve the phonon scattering induced de- coherence, a slightly modified qubit structure can be designed as follows. Similar as shown in Fig. 1, each qubit still constitutes two quantum dots. We suggest here to use two identical quantum dots, and to apply constantly a static electric field that results in an energy level structure as depicted in Fig. 2. In this qubit structure, the information is also stored in the states $|v\rangle$ and $|e\rangle$, and $|\tilde{e}\rangle$ plays a role of mediating transition within only virtual occupation on it. The gate operations based on this structure can be performed similarly as in the previous one, only noticing that the cavity-photon involved interaction Hamiltonian is now in the form of $H_j^{(2)} = \Omega_{\text{eff}} |e\rangle \langle v| a e^{i\phi} + H.c. |$, instead of Eq. (2).

As a result, the swap operation corresponds to generating a cavity photon via the transition from $|v\rangle$ to $|e\rangle$, and annihilating a cavity photon vice versa. The main advantage of this scheme is that the phonon scattering from $|e\rangle$ to $|\tilde{e}\rangle$ can be almost completely suppressed in the low temperature limit. In particular, there would be no LO phonon

\[ \text{FIG. 2: An alternative configuration for qubit construction from two coupled identical QDs in the presence of external electric field. This structure is expected to be able to suppress phonon scattering from } |e\rangle \text{ to } |\tilde{e}\rangle \text{ in low temperature regime. The price paid here is the use of external electric field which might cause additional dephasing.} \]
excitations. Another merit is that the electric field can be conveniently used to tune the level spacing between $|e\rangle$ and $|\tilde{e}\rangle$ in near resonance with the cavity photon energy. The price paid here is the constant presence of an external electric field, whose thermal fluctuations (the Johnson noise) may cause additional dephasing. Fortunately, in our scheme the electric field is not varied to perform the logic operations. Thus the electrodes which generate the electric field can be connected to a superconducting ground, which can remove the thermal fluctuations since there is no dissipation.

In summary, we proposed a scheme based on coupled QDs embedded in optical microcavity to implement quantum computation. The proposed qubit constructed from two weakly coupled QDs is expected to have long decoherence time due to the spatial separation of the logic states. The recent progress of gating technique in ion-trap QC enables us to realize the universal quantum gate in our structure based on certain simple electronic state configuration, namely, the LUMO and HOMO states. From the consideration on the tradeoff between phonon scattering and fluctuations of electrostatic field, we suggested two possible qubit configurations for practical choice. The most challenging aspect in the proposed QC scheme is to locate QDs in optical cavity with high finesse. Modification to the proposed gating scheme is possible by using the cavity state only as a virtual state, which can in certain sense relax the requirement to the cavity finesse.

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