Light front field theory of relativistic quark matter

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Abstract. Light-front quantization to many-particle systems of finite temperature and density provides a novel approach towards a relativistic description of quark matter and allows us to calculate the perturbative as well as the non-perturbative regime of QCD. Utilizing a Dyson expansion of light-front many-body Green functions we have so far calculated three-quark, quark-quark, and quark-antiquark correlations that lead to the chiral phase transition, the formation of hadrons and color superconductivity in a hot and/or dense environment. Presently, we use an effective zero-range interaction, to compare our results with the more traditional instant form approach where applicable.

Keywords: quark matter, light-front quantisation, finite temperature

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1. Introduction

Light front quantization recognized by Dirac [1] provides a framework to describe the perturbative and the nonperturbative regime of quantum chromodynamics (QCD), see e.g. [2]. A description involving both regimes is necessary, if one is interested in the dynamics driving, e.g., the chiral and the confinement-deconfinement transitions that are discussed during this conference. Recently, light front quantization has been successfully generalized to finite temperature field theories [3, 4, 5, 6, 7, 8]. It has been shown for the NJL model that the basic features such as spontaneous symmetry breaking and chiral symmetry restoration at finite temperature can be achieved [9, 10]. Within the Dyson expansion of many-body Green functions light front quantization enables us to calculate properties of three-quark clusters (viz. nucleons) in hot and dense quark matter [3, 11]. This sets up the framework to treat three-quark correlations in quark matter and investigate the change from quark matter to nuclear matter. There is a perspective to use nonzero temperature light-front field theory to tackle in particular the high density region. Most notable, the sign problem mentioned in a different context may not occur in this Hamiltonian approach and one should be able to develop numerical
techniques (similar to the ones used in lattice QCD) to evaluate expectation values of observables, in particular partition functions.

2. Light-front statistical physics

A formal framework of covariant calculations at finite temperatures in \textit{instant} form has been given in Refs. \cite{12,13,14} in a different context. In light-front quantization the grand canonical partition function can then be written as

\[
Z_G = \text{Tr} \exp \left\{ -\frac{1}{T} \left( P_+ + P_- - \mu N \right) \right\}.
\]

where $T$ is the temperature and $\mu$ the chemical potential, both Lorentz scalars, and $P^\pm$ and $N$ are defined in light front quantization \cite{2}. For a grand canonical ensemble in equilibrium the Fermi distribution functions are given by

\[
f^{\pm}(k^+,\vec{k}_\perp) = \left[ \exp \left\{ \frac{1}{T} \left( \frac{1}{2} k_{\text{on}}^{-} + \frac{1}{2} k^+ \mp \mu \right) \right\} + 1 \right]^{-1}.
\]

where $k_{\text{on}}^{-} = (\vec{k}_\perp^2 + m^2)/k^+ \cite{3}.$

3. Light front cluster many-body Green functions

The many-body Green functions, can all be defined utilizing light front quantization. The causal Green function, e.g., is given by

\[
i G_{\alpha\beta}(x^+, \vec{k}_\perp) = \theta(x^+ - y^+) \langle A_{\alpha}(x^+) A_{\beta}^\dagger(y^+) \rangle \mp \theta(y^+ - x^+) \langle A_{\beta}^\dagger(y^+) A_{\alpha}(x^+) \rangle.
\]

The average $\langle \cdot \cdot \cdot \rangle$ is taken over the exact ground state. The Heisenberg operators $A_{\alpha}(x^+)=e^{iH_{\text{fill}}x^+}A_{\alpha}e^{-iH_{\text{fill}}x^+}$ with $H_{\text{eff}}=P_+ + P_- - \mu N,$ could be build out of any number of field operators (fermions and/or bosons). In equilibrium only one Green function is independent and one may alternatively use the thermodynamic (or imaginary-time) Green function. Dyson equations to disentangle the hierarchy of Green function equations can be established for both forms, cf. \cite{15} for the nonrelativistic case. In the light-front formalism the Dyson equations are given by \cite{3}

\[
\frac{\partial}{\partial x^+} G_{\alpha\beta}(x^+-y^+) = \delta(x^+-y^+) \langle [A_{\alpha}, A_{\beta}]^\dagger_{\pm}(x^+) \rangle + \sum_{\gamma} \int d\bar{x}^+ \mathcal{M}_{\alpha\gamma}(x^+-\bar{x}^+) \mathcal{G}_{\gamma\beta}(\bar{x}^+-y^+).
\]

The mass matrix that appears in \cite{4} is given by

\[
\mathcal{M}_{\alpha\beta}(x^+-y^+) = \delta(x^+-y^+) \mathcal{M}_{0,\alpha\beta}(x^+) + \mathcal{M}_{r,\alpha\beta}(x^+-y^+) \tag{5}
\]
Fig. 1. A: Binding energy of the three-quark bound state (solid) for different temperatures as a function of the chemical potential. The respective continuum lines are indicated as dashed line, \( \Lambda = 8m \). B: Mott line for the three-body system at rest in the medium for \( \Lambda = 4m \) (with di-quark substate) and \( \Lambda = 8m \) (Borromean nucleon) as extreme cases.

\[
(M_0 \mathcal{N})_{\alpha \beta} (x^+) = \langle [[A_\alpha, H](x^+), A_\beta^\dagger(x^+)]_\pm \rangle \tag{6}
\]

\[
(M_1 \mathcal{N})_{\alpha \beta} (x^+ - y^+) = \sum_\gamma \langle T_x^+ [A_\alpha, H](x^+), [A_\beta^\dagger, H](y^+)] \rangle_{\text{irreducible}} \tag{7}
\]

where \( \mathcal{N}_{\alpha \beta}(x^+) = \langle [A_\alpha, A_\beta^\dagger](x^+) \rangle \). The first term in (6) is instantaneous and related to the mean field approximation, the second term is the retardation or memory term. We first solve the mean field problem (neglecting memory). We use a zero range interaction, which can be considered as the lowest order effective interaction appearing in a \( 1/N_c \) expansion of QCD. With the help of (6) it is possible to derive relativistic in-medium few-body equation to describe the formation and properties of clusters, viz. nucleons as three-quark states [3].

4. Results

For the in-medium quark mass we use values of \( m(\mu, T) \) given in [2]. We take \( m = 336 \) MeV for the isolated constituent mass and determine the coupling strength and the cut-off \( \Lambda \) to reproduce the nucleon mass \( m_N = 938 \) MeV. The three-body mass \( M_3(\mu, T) \) as a function of the chemical potential for different temperatures and a cut-off parameter \( \Lambda = 8m \) are shown in Figure 1. The chemical potential where the solid lines intersects the continuum define the Mott transition. The condition is given by \( M_3(\mu_{\text{Mott}}, T_{\text{Mott}}) = 3m(\mu_{\text{Mott}}, T_{\text{Mott}}) \) for the three-quark continuum and \( M_3(\mu_{\text{Mott}}, T_{\text{Mott}}) = m(\mu_{\text{Mott}}, T_{\text{Mott}}) + M_2(\mu_{\text{Mott}}, T_{\text{Mott}}) \) for the quark-diquark continuum. The corresponding Mott lines are shown in Figure 1 along with the chiral phase transition line calculated in the NJL model.
5. Conclusion

We have shown that light front quantization is applicable to finite temperature systems and leads to meaningful results. Future challenges are

- Include three-quark correlations in the distribution function, and the thermodynamical potentials to see how quark matter actually changes to nucleonic matter when the temperature is lowered.

- The parameter values chosen are fixed to the nucleon mass which eventually should give the strength for the two-quark correlation that is important in the studies of color superconductivity.

- The influence of three-quark correlations has so far not been investigated in the context of color superconductivity. The Dyson approach provides the necessary tools to do so.

- An exciting prospect of this approach is to directly use light-front QCD \[2\] to evaluate the partition function given in \[1\]. This would surely need quite some effort, however, might lack the sign problem present in other approaches due to the Hamiltonian formulation.

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