Josephson Plasma Resonance as a Structural Probe of Vortex Liquid

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Recent developments of the Josephson plasma resonance and transport c-axis measurements in layered high T\textsubscript{c} superconductors allow to probe Josephson coupling in a wide range of the vortex phase diagram. We derive a relation between the field dependent Josephson coupling energy and the density correlation function of the vortex liquid. This relation provides a unique opportunity to extract the density correlation function of pancake vortices from the dependence of the plasma resonance on the ab-component of the magnetic field at a fixed c-axis component.

Thermal suppression of Josephson interlayer coupling by vortex fluctuations \cite{1} is the key factor, which enhances vortex mobility and limits useful applications of Bi- and Tl-based high T\textsubscript{c} superconductors (HTSC). Recent discovery of the theoretically predicted \cite{2} Josephson plasma resonance (JPR) effect in the mixed state of highly anisotropic layered high temperature and organic superconductors \cite{3} provides a new and unique opportunity to probe the Josephson coupling of layers in a wide range of magnetic fields and temperatures. An alternative way to probe Josephson coupling in the vortex state is direct c-axis transport measurements \cite{4}.

A necessary minimum condition for the existence of a narrow JPR line is short-time/short-range phase coherence between the superconducting layers. In the London regime this coherence is quantitatively characterized by the spatial and temporal behavior of the “local coherence parameter” \( C_n(r,t) = \cos[\varphi_{n,n+1}(r,t)] \), with \( \varphi_{n,n+1} = \varphi_{n+1} - \varphi_n - (2\pi s/\Phi_0)A_z \) being the gauge-invariant phase difference between layers \( n \) and \( n+1 \). Here \( r = (x, y) \) and \( z \) are coordinates in the \( ab \) plane and along the c axis, \( s \) is the interlayer spacing.

The first important condition is that the typical time of variation of this factor (phase slip time) must be much larger than the period of plasma oscillations. Their ratio was estimated \cite{4} as \( \approx 10^9 \) at \( T = 45 \) K and magnetic field \( B_z = 0.1 \) T. In this case the plasma mode probes a snapshot of the instantaneous phase distribution.

In the decoupled liquid state the cosine factor \( C_n(r,t) \) also rapidly oscillates in space because correlations between pancake positions in neighboring layers are almost absent. The possibility of a sharp resonance in the situation when fluctuations of local Josephson coupling are much stronger than its average is an interesting and nontrivial issue. JPR resonance in such situation occurs because small phase oscillations induced by the external electric field change slowly in space and average out these rapid variations \cite{2}.

In this Letter, using the high temperature expansion with respect to the Josephson coupling, we establish a relation connecting the plasma frequency with the density correlation function of the pancake liquid. A similar relation was derived in Ref. \cite{5} assuming the Gaussian distribution for the interlayer phase fluctuations. We demonstrate that the pair distribution function of the liquid is unambiguously connected with the dependence of the plasma frequency on the in-plane component of the magnetic field for fixed c-axis component. Therefore, this dependence can be used to extract quantitative information about the structure of the liquid phase.

The essential physics of JPR is captured by a simplified equation for small oscillations of the phase difference \( \varphi_{n,n+1}(r,\omega) \) induced by the external microwave electric field with the amplitude \( D_z \) and the frequency \( \omega \) (see Ref. \cite{2})

\[
\left[ \left( \frac{\omega^2}{\omega_0^2} + \lambda_J^2 \nabla^2 - C_n(r) \right) \varphi_{n,n+1} = -\frac{\hbar \omega D_z}{8\pi \epsilon E_{J}} \right],
\]

where \( \omega_0(T) = c/\sqrt{\epsilon_c \lambda_c(T)} \) is the zero field plasma frequency, \( \epsilon_c \) is the high frequency dielectric constant, \( \lambda_c \) and \( \lambda_{ab} \) are the components of the London penetration depth, \( E_J = E_{J0}/\lambda_J^2 \) is the Josephson energy per unit area, \( E_{J0} = s \Phi_0^2/16\pi^3 \lambda_{ab}^2 \), \( \lambda_J = \gamma s \) is the Josephson length, and \( \gamma = \lambda_c/\lambda_{ab} \) is the anisotropy ratio. The inductive matrix \( \hat{L} \) is defined as \( \hat{L} A_n = \sum_{m} L_{nm} A_m \) with \( L_{nm} \approx (\lambda_{ab}/2s) \exp(-|n-m|s/\lambda_{ab}) \). We neglect in Eq. \cite{2} time variations of \( C_n(r,t) \) assuming them to be small during the time \( 1/\omega \). Eq. \cite{2} also does not take into account charging effects \cite{6} and quasiparticle dissipation. These effects don’t influence much the position of the resonance but may modify its lineshape.

Static configurations \( \varphi_{n,n+1}(r) \) are mainly determined by thermal fluctuations of pancake vortices and quantitatively characterized by the average value of the cosine factor, \( \langle \cos \varphi_{n,n+1}(r) \rangle \)

and its static correlation function \( \langle \cos[\varphi_{n,n+1}(r) - \varphi_{n,n+1}(0)] \rangle \), where \( \langle \ldots \rangle \) denotes the thermal average. In this Letter we consider the pancake liquid state where the correlations between pancake arrangements in different layers
are almost absent. In such situation $C_n(r)$ rapidly oscillates in space, so that $S(r, B, T)$ drops at distances of the order of the intervortex spacing $a = (\Phi_0/B_x)^{1/2}$, and $C(B, T) \ll 1$.

An important observation is that in spite of rapid variations of $C_n(r)$ the oscillating phase $\varphi' \equiv \varphi^{(r)}_{n,n+1}(r)$ varies smoothly in space. The typical length scale $L_\varphi$ of $\varphi'$ variations can be estimated by balancing the typical kinetic energy of supercurrents, $E_0(\varphi')^2/L_x^2$, with the typical Josephson energy, $E_J(\varphi')^2/a/L_x$, in the vortex state which is strongly disordered along the $c$ axis. This gives $L_\varphi = \lambda_J^2/a$. Smoothly varying $\varphi'$ effectively averages rapid variation of $C_n(r)$ and the plasma frequency is simply determined by $C(B, T)$ (see Refs. [3] [4])

$$\omega_p^2(B, T) = \omega_0^2(T)C(B, T)/C(T), \quad (4)$$

Here the factor $C(T) = C(0, T)$ takes into account the suppression of zero field plasma frequency by phase fluctuations (see below). Fluctuations of $C_n(r)$ smoothed over the area $L_\varphi^2$, $C(L_\varphi) = L_\varphi^2 \sum_{r} dC_n(r)$, produce inhomogeneous broadening of the JPR line. Calculating the mean squared fluctuation of $C(L_\varphi)$, $\langle C(L_\varphi) - C^2 \rangle \approx a^2/L_x^2 = a^2/\lambda_J^2$, we obtain the estimate for the inhomogeneous line width

$$\delta(\omega_p^2) \approx \omega_0^2(T)a^2/\lambda_J^2. \quad (5)$$

Detailed calculations of the intrinsic lineshape due to this mechanism will be published elsewhere [3].

We establish now a relation between plasma frequency and the vortex liquid structure. In general, the Josephson coupled system in the London regime, $B_x \ll H_c2$, may be described in terms of vortex coordinates $r_{nu}$ (index $\nu$ labels vortices in the layer $n$) and by the regular (spin-wave type) phase difference $\varphi_{n,n+1}^{(r)}(r)$. The free energy functional in terms of these variables is [12]

$$\mathcal{F}\{r_{n\nu}, \varphi_{n,n+1}^{(r)}\} = \mathcal{F}_\varphi(r_{n\nu}) + \mathcal{F}_\varphi\{\varphi_{n,n+1}^{(r)}\} + \mathcal{F}_J\{r_{n\nu}, \varphi_{n,n+1}^{(r)}\}, \quad (6)$$

Here $\mathcal{F}_\varphi(r_{n\nu})$ accounts for the two-dimensional energy of pancake and also their electromagnetic interaction in different layers [6]. The second term, $\mathcal{F}_\varphi\{\varphi_{n,n+1}^{(r)}\} = E_0/2 \sum_n \int dr \nabla \varphi_{n,n+1}^{(r)} \nabla \varphi_{n,n+1}^{(r)}$, (7)

is the energy of inralateral currents associated with regular phase fluctuations, and the third term,

$$\mathcal{F}_J\{r_{n\nu}, \varphi_{n,n+1}^{(r)}\} = E_J \sum_n \int dr \left[ 1 - \cos \left( \varphi_{n,n+1}^{(r)} + \varphi_{n,n+1}^{(r)} - \frac{2\pi s}{\Phi_0} B_x y \right) \right], \quad (8)$$

is the Josephson energy. We split the phase difference $\varphi_{n,n+1}^{(r)}$ into vortex part, regular spin-wave part, and contribution coming from the in-plane magnetic field $B_x$, $\varphi_{n,n+1}^{(r)}(r) = \varphi_{n,n+1}^{(v)}(r) + \varphi_{n,n+1}^{(r)}(r) - (2\pi s/\Phi_0) B_x y$. (9)

The phase $\varphi_{n,n+1}^{(v)}(r; r_{n\nu}, r_{n+1\nu})$ is the singular part of the phase difference induced by vortices at positions $r_{n\nu}, r_{n+1\nu}$ when Josephson coupling is absent $(E_J = 0)$:

$$\varphi_{n,n+1}^{(v)}(r; r_{n\nu}, r_{n+1\nu}) = \sum_{\nu} [\varphi_{\nu}(r - r_{n\nu}) - \varphi_{\nu}(r - r_{n+1\nu})], \quad (10)$$

where $\varphi_{\nu}(r)$ is the polar angle of the point $r$.

We calculate $C(B, T)$ with the functional (4) using high temperature expansion (1) with respect to $F_J$ as:

$$C(B, T) \approx \frac{E_J}{2T} \int dr S(r, B, T) = f(B, T) \frac{E_0 B_J}{2T B_x}, \quad (11)$$

Here $f(B, T) = a^{-2} \int dr S(r, B, T)$ is a universal function and $B_J = \Phi_0/\lambda_J^2$. Eq. (11) is valid until $C(B, T) \ll 1$ which corresponds to the field and temperature range $TB_z \gg E_0 B_J$. Comparing Eqs. (11) and (12) with Eq. (14) we obtain an estimate for the relative line width in the liquid state due to the inhomogeneous broadening, $\delta \omega_p/\omega_p \approx T/E_0 \ll 1$.

In the lowest order in $E_J$ spin-wave and vortex degrees of freedom do not interact:

$$S(r, B) = S_v(r, B_z)S_{\phi}(r) \cos (2\pi s B_x y/\Phi_0), \quad (12)$$

where the correlation function $S_v(r)$ is determined by the functional $\mathcal{F}_\varphi$, while $S_{\phi}(r)$ is determined by $\mathcal{F}_\varphi$. In the pancake liquid regime, which we consider in this Letter, $B_z$ penetrates almost freely into the sample and has no influence on the phase fluctuations $\varphi_{n,n+1}^{(v)}$ and $\varphi_{n,n+1}^{(r)}$. An external spatial dependence of $\varphi_{n,n+1}^{(v)}$ induced by $B_z$ gives possibility to probe phase correlations at given $B_z$ by measuring $B_z$-dependence of the plasma frequency.

We now express the vortex phase correlation function $S_v(r)$ via the density correlation function of pancake vortices induced by $B_z$. The difference $\Phi_v(r) = \varphi_{n,n+1}^{(v)}(r) - \varphi_{n,n+1}^{(v)}(0)$ can be connected with pancake densities $\rho_n(R) = \sum_{\nu} \delta(R - r_{n\nu})$ in the layers as

$$\Phi_v(r) = \int dR [\rho_n(R) - \rho_{n+1}(R)] \beta_r(R, R). \quad (13)$$

Here $\beta(r, R) = \varphi_{\nu}(r/2 - R) - \varphi_{\nu}(-r/2 - R)$ is the angle at which the segment connecting points $-r/2$ and $r/2$ is seen from the point $R$,

$$\cos \beta(r, R) = (R^2 - r^2/4)((R^2 + r^2/4)^2 - (Rr)^2)^{-1/2}. \quad (14)$$

The function $\beta(r, R)$ has a jump of $2\pi$ when point $R$ intersects segment $-r/2, r/2$. Using the Gaussian approximation for $\Phi_v(r)$ we obtain for vortex phase correlation function $S_v(r) \equiv \langle \cos[\Phi_v(r)] \rangle$

$$S_v(r) = \exp[-F_v(r)] \quad (14)$$

2
with

\[ F_v(r) = \langle |\phi_v(r)|^2 \rangle / 2. \]  

(15)

Using Eqs. (13) and (15) we connect \( F_v(r) \) with the pair distribution function \( h(r) \) defined by the relation

\[ n_v^2 h(r) = \langle \rho_n(0) \rho_n(r) \rangle - \langle \rho_n(0) \rho_{n+1}(r) \rangle - n_v \delta(r) \]  

(16)
as

\[ F_v(r) = -n_v^2 \int dRdr' h(r') \left[ \beta \left( r, R + r' \right) - \beta \left( r, R - r' \right) \right]^2. \]  

(17)

For this identity \( n_v \int dr h(r) = -1 \) has been used, \( n_v = \langle \rho_n(0) \rangle = a^{-2} \). Performing integration with respect to \( R \) and averaging over directions \( r' \) we finally obtain

\[ F_v(r) = -\pi n_v^2 \int r'dr h(r') J(r, r'), \]  

(18)

where the universal function \( J(r, r') \) is given by

\[ J(r, r') = 2\pi^2 \left[ \ln(r'/r) + 2 \right], \quad r' > r, \]  

\[ J(r, r') = 4\pi \left[ 2rr' - r^2 - (r^2/2) \ln(r/r') \right], \quad r' < r. \]  

Eqs. (14), (18) and (19) represent general relations connecting phase fluctuations with vortex density fluctuations. The function \( F_v(r) \) has the following asymptotics

\[ F_v(r) \approx -8\pi^2 \frac{r}{a} \int_0^\infty dx x^2 h(x), \quad r \gg a, \]  

\[ F_v(r) \approx \frac{\pi r^2}{a^2} \left( \ln \frac{a}{r} + 2 \right) \int_0^\infty dx x h(x) \ln x, \quad r \ll a. \]  

Linear increase of \( F_v(r) \) at large \( r \) indicates exponential decay of phase correlations at large distances.

The problem of calculating of \( S_v(r) \) is now reduced to calculation of the integral (18) with the density correlation function of the liquid. This function is not available analytically. It can be calculated from Monte Carlo simulations or (approximately) using the density functional theory [17]. In the decoupled pancake liquid regime the correlations between two dimensional pancake liquids in different layers are very weak because the energy of the magnetic interlayer interaction of disordered pancakes has an additional small parameter \( s^2/\lambda_{ab}^2 \) in comparison with the intralayer energy. If these correlations are neglected then the vortex system is equivalent to the one-component two-dimensional Coulomb plasma which was studied extensively [13]. The pair distribution function \( h_{2D}(r, B_z, T, E_0) \) within this approximation has an exact scaling property \( h_{2D}(r, B_z, T, E_0) = h_{2D}(r/a, T/E_0) \), i.e., it depends on magnetic field only through the spatial scale. As follows from Eq. (18) this scaling property is also transferred to the function \( F_v(r) \), \( F_v(r) = F_v(r/a, T/E_0) \). If, in addition, fluctuations of the regular phase are neglected then at \( B_z = 0 \) the function \( f(T/E_0) \) in the expression (11) for \( C(B, T) \) is field independent. Therefore, the scaling property \( C(B_z, T) \propto 1/B_z \) is a consequence of the approximation of completely decoupled pancake liquids in pinning-free layers when spinwave phase fluctuations are negligible.

We generated the pair distribution functions \( h(x) \) with \( x = r/a \) for the two-dimensional pancake liquid using Langevin dynamics simulations. The model and the algorithm have been described in Ref. [19]. Fig. 1 shows an example of the pair distribution function \( h(x) \) for temperature \( T = 0.012\pi E_0 \approx 1.7 T_m \) and the corresponding function \( F_v(x) \) obtained by numerical integration of Eq. (18). To show a weak waving of \( F_v(x) \) due to liquid correlations we also plot the derivative of \( F_v(x) \).

As temperature approaches \( T_c \) the role of fluctuations of the regular phase progressively increases. The spinwave phase correlations decay algebraically

\[ S_v(r) = (\xi_{ab}/r)^{2\alpha}, \quad \alpha = T/2\pi E_0(T), \]  

(20)

where \( \xi_{ab}(T) \) is the superconducting correlation length \( 1/\xi_{ab} \) determines the upper cut-off for momenta characterizing spatial variations of \( \varphi^{(r)}_{n,n+1}(r) \) in Eq. (18). As follows from Eq. (14) the contribution from regular phase fluctuations becomes essential if \( S_v(r) \gg 1 \). This gives the condition \( T_c > E_0(T) \ln(H_{c2}/B) \).

To relate \( \omega_p(B, T) \) with the plasma frequency at zero field and the Josephson coupling \( E_J \propto \lambda_c^{-2} \) we have to take into account that these quantities are also renormalized by the phase fluctuations. Their suppression is determined by the cosine factor at \( B = 0 \), \( C(T) \), which was estimated in Ref. [24] as

\[ C(T) = (\xi_{ab}/\lambda)^{\alpha}. \]  

(21)

Finally using Eqs. (1), (12), (14), (20), and (21) we obtain an expression for plasma frequency which incorporates effects of vortices and of regular phase fluctuations at all temperatures :

\[ \frac{\omega_p^2(B, T)}{\omega_p^2(0)} = \frac{E_0(T)}{2T} \left( \frac{B_J}{B_z} \right)^{1-\alpha} f_s \left( \frac{2\pi s B_z}{\sqrt{\pi} \xi_{ab} B_z} \right), \]  

(22)

\[ f_s(b) = 2\pi \int_0^\infty dx x^{1-2\alpha} \exp[-F_v(x)] J_0(bx), \]  

(23)

where \( J_0(x) \) is the Bessel function. Note that regular phase fluctuations reduce the power index in the \( B_z \) dependence of \( C(B_z, T) \) at \( B_z = 0 \) and make it temperature dependent. Using the typical parameters for optimally doped \( Bi_2Sr_2CaCu_2O_{8-x} \) \( \lambda_{ab}(0) = 2000 \ \text{Å}, \ T_c = 90 \ \text{K} \) we obtain that the index \( \alpha \) increases from \( \alpha \approx 0.05 \) at \( T = 50 \ \text{K} \) to \( \alpha \approx 0.18 \) at \( T = 75 \ \text{K} \).

Measurements of \( B_z \)-dependence of the resonance frequency \( \omega_p \) at fixed \( B_z \) and \( T \) allow one to obtain, in principle, the pair distribution function \( h(x, T) \), i.e., to extract quantitative information about correlations in the
vortex liquid. Using Eq. (22) the function \( f(b) \) can be found from the dependence \( \omega_p(B_x, B_z) \) as:

\[
fs \left( \frac{2\pi s B_z}{\sqrt{\Phi_0 B_x}} \right) = \frac{\omega_p^2(B_x, B_z)}{\omega_p^2(0, B_z)} f_s(0).
\]  
(24)

With \( f_s(b) \) known, the function \( F_v(x) \) may be found by the reverse Fourier transform from Eq. (23):

\[
F_v(x) = -\ln \left[ x^{2\alpha} \int_0^\infty b df_s(b) J_0(bx) \right].
\]  
(25)

Finally, the pair distribution function \( h(x) \) can be expressed through \( F_v(x) \) using Eqs. (18) and (19):

\[
h(x) = -\frac{1}{8\pi x^3} \frac{d}{dx} \left\{ x^3 \frac{d}{dx} \left[ x \frac{d^2 F_v(x)}{dx^2} \right] \right\}.
\]  
(26)

Unfortunately, high order differentiations in the last equation complicate practical realization of the procedure. As it is shown in Fig. 1, oscillating behavior of liquid correlations is hardly seen in \( F_v(x) \) but the first derivative, \( F_v'(x) \), may expose it quite clearly. The experimental angular dependence of the plasma frequency [5] was found to be in a good qualitative agreement with Eq. (22). However an accuracy of existing measurements is not sufficient to perform the described quantitative analyses.

In conclusion, we obtain a general expression connecting the plasma frequency with the density correlation function of pancake liquid. We demonstrate that the dependence of the plasma frequency on the magnetic field component parallel to the layers \( B_x \) at fixed perpendicular component \( B_z \) contains full structural information about the vortex liquid at given \( B_z \) and temperature.

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![FIG. 1. (a) Pair distribution function \( h(x) \) obtained by Langevin dynamics simulations of two dimensional vortex liquid for \( T/\pi E_0 = 0.012 \); (b) Plot of mean squared phase difference between points separated by distance \( r = xa \), \( F_v(x) \), [see Eq. (18)] obtained by numerical integration of Eq. (18) with plotted above \( h(x) \). Derivative of \( F_v(x) \) is also plotted to enhance weak oscillations due to liquid correlations.](image-url)