We present evidence that the existence of a first order phase transition in compact $U(1)$ with Wilson action is not related to monopole loops wrapping around the toroidal lattice, as has been previously suggested. Our analysis is based on the suppression of such loops by ‘soft boundary conditions’ that correspond to an infinitely large chemical potential for the monopoles on the boundary, during the updating process. It is observed that the double peak structure characteristic for the first order phase transition reappears at sufficiently large lattice sizes and separations from the lattice boundary.

1. Introduction

During this workshop we have heard about plenty of indications that confinement in SU(2) and SU(3) gauge theories can be traced back to an underlying U(1) gauge structure. The dual Meissner effect mechanism suggests monopoles to condense and confine the electric field into a narrow flux tube. It is widely believed—but still debated—that the maximal abelian projection uncovers the relevant U(1) degrees of freedom in this scenario.

In 4d compact U(1) lattice gauge theory with Wilson action, monopoles are defined ab initio as gauge invariant objects, in contrast to non-abelian theories. It was demonstrated clearly in ref. [1] that large monopole loops dominate the confining

*Talk presented by K. Schilling.
heavy quark potential at large distances in 4d compact U(1) gauge theory. This raises
the question about the impact of the dynamics of monopole loops onto the critical
behaviour near the confinement-deconfinement phase transition in U(1).

Despite the ‘simplicity’ of the U(1) gauge theory, the order of its phase transition—
which is relevant to the existence of a continuum limit of the lattice construction—has
been under debate for 15 years by now, as we can see from table 1. Some years ago, it

Table 1. Order of the U(1) phase transition.

| author          | max. lattice size | boundary conditions    | year | order |
|-----------------|-------------------|------------------------|------|-------|
| Lautrup et al.  | $H_4(6)$          | periodic               | 1980 | 2nd   |
| Mütter et al.   | $H_4(8)$          | periodic               | 1982 | 2nd   |
| Jersak et al.   | $H_4(8)$          | periodic               | 1983 | 1st   |
| Bhanot et al.   | $H_4(16)$         | periodic               | 1992 | 1st   |
| Lang et al.     | $SH_4(10)$        | no boundary            | 1993 | 2nd   |
| Baig et al.     | $H_4(16)$         | fixed                  | 1994 | 2nd   |
| Lippert et al.  | $H_4(24)$         | wrapping loop suppressing | 1994 | 1st   |

was argued in ref. [4] that the lattice heuristics of metastabilities is greatly affected by
artifacts from the periodicity on a finite lattice: the wrap-around of large monopole
loops (around the lattice with toroidal topology) might cause the go-slow of any local
updating algorithm and thus might fake first order behaviour on a finite size system.
More recently evidence was presented in support of this view by a study of a lattice
with trivial homotopy group (without such wrap-arounds!) [6, 7]. On a smallish lattice,
equivalent in volume to a periodic $10^4$ lattice, Lang and Neuhaus observed no sign of
first order behaviour.

In this contribution we intend to study the anatomy of this system in a variant
approach. We will be less drastic in changing the lattice topology, remaining on
a 4d hypercubic lattice. We inhibit wrapping monopole loops during the update
by suppression of monopole currents to traverse the boundary. This is achieved by
introduction of an infinitely large chemical potential on the boundary, that confines
monopole currents inside the volume, but allowing spin waves to cross. The work
presented here is an extension of our previous studies to a $32^4$ lattice.

2. Monopole suppressing boundary conditions

Following ref. [11], we start out from the monopole current definition on the lattice,

$$m_\mu(x) = \frac{1}{2} \epsilon_{\mu\rho\sigma} \left[ n_{\rho\sigma}(x + \nu) - n_{\rho\sigma}(x) \right], \quad n_{\rho\sigma} = -2, -1, 0, 1, 2. \quad (1)$$

$n_{\mu\nu}(x)$ counts the number of Dirac sheets,

$$n_{\mu\nu}(x) = \frac{1}{2\pi} \left[ \Phi_{\mu\nu}(x) - \Phi_{\mu\nu}(x) \right]. \quad (2)$$
that is given by the difference between physical flux $\Phi_{\mu\nu}(x)$ and plaquette flux $\Phi_{\mu\nu}(x)$. As the updating procedure is sequential in terms of the links, we need to clarify the interrelation between gauge links and monopole current elements. Let us consider the update of a link $U_\rho$ in a 3d cube sitting on the boundary with normalvector $e_\mu$. We search for the plaquettes associated with the Dirac sheets $n_{\rho\sigma}(x)$, $n_{\rho\mu}(x)$ $n_{\rho,-\sigma}(x)$ and $n_{\rho,-\nu}(x)$ that contribute to the monopole current $m_\mu(x)$ into this direction $e_\mu$. These plaquettes are depicted in Fig. 1 with dotted lines. According to Fig. 1 the update link $U_\rho$ belongs to four 3d boundary cubes.

The monopole suppressing boundary condition is injected into the updating procedure in the following way: if, for any of these cubes, the change in flux is larger than $2\pi$, the proposed link change is rejected in the Monte Carlo. Therefore, the monopole current in $\mu$-direction in the respective 3d cube is a conserved quantity in Monte Carlo time. This allows to enforce configurations with any given number of current elements across the boundary. We initialize the simulation with no such crossings at all, which implies containment of monopole currents in the volume throughout the simulation.

As the number of monopole currents in a boundary cube is a conserved quantity the update rule presented fulfills detailed balance. The procedure of suppressing the monopoles can be viewed as being equivalent to an infinitely large chemical potential for the monopoles on the cube where the monopole current is suppressed: in the equivalent action $S = S_W + S_\lambda$, $S_W$ represents the standard Wilson action and $S_\lambda$ is the additional term which suppresses monopole currents $m_\mu$ at the lattice slice $x_\mu = 0$,

$$S_\lambda = \lambda \cdot \sum_\mu \sum_{x: x_\mu = 0} |m_\mu(x)|, \quad \lambda \to \infty.$$  

Obviously, $S_\lambda$ explicitly breaks translation symmetry.

In Fig. 3a the monopole content of a confined configuration taken from a thermalized ensemble on an $8^4$ lattice at $\beta = 1.0$ is plotted for illustration. Fig. 3a shows the situation on the standard hypertorus and Fig. 3b refers to the implementation with monopole containment. In Fig. 3b, no monopole current can traverse the boundaries of the lattice. The required absence of traversing currents was monitored throughout
Fig. 3. a) Monopole cluster wrapping around the standard hyper-toroidal lattice and b) suppression of currents across the boundaries leading to containment of the monopole clusters.
3. Results

The simulation was carried out on a sequence of lattices, scanning for the location of the phase transition, and in search for first order metastable behaviour. The selected lattice sizes were $8^4$, $16^4$, $24^4$, and $32^4$. We performed about 100,000 sweeps at each $\beta$-value, that were chosen within the suspected metastable region. We used standard Metropolis Monte Carlo updating.

We measured plaquettes and monopole density, with sampling performed on nested shells (see the 3d projection of the shells in Fig. 2), in order to search genuine 4d bulk behaviour. Notice that we have traded the unwanted topological effects due to wrapping monopole loops against surface effects, i.e. penetration phenomena! On a sufficiently large lattice, we would expect with our ‘experimental layout’ to distinguish between first and second order behaviour.

The results on the $8^4$ lattice showed no evidence whatsoever for first order behaviour. On the $16^4$ lattice, we scan in $\beta$ through the phase transition. As be seen from Fig. 1, the plaquette displays little $\beta$-dependence in the outer shells. For the shells with distance greater than 4 from the boundary, however, we discover a rapid transient behaviour. This indicates that the boundary layer thickness close to phase

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$a$In our 4d visualization, see ref. [2], the discrete euclidean time is represented by colors. If an arrow traverses the spatial hyperplanes the colors of the arrow’s head and tip differ. The boundary in time direction is given by the color pink.
transition is not prohibitively large for our method to work. Detailed inspection (see Fig. 6) of the time series on the 8\textsuperscript{4} shell with best signal-to-noise ratio, however, does not reveal indications for metastabilities.

We suspect that the 16\textsuperscript{4} lattice does not suffice to reveal true bulk behaviour and proceed to a 24\textsuperscript{4} lattice. The plaquette is plotted across the phase transition in Fig. 5. It shows a much steeper behaviour on the inner shells. Note the change in scale in Figs. 4 and 5. The region of Fig. 5 transforms to a square drawn in Fig. 4.

Turning now to the time history, we display the Monte Carlo dynamics as seen on the 24\textsuperscript{4} boundary shell (see Fig. 7), the 16\textsuperscript{4} shell (see Fig. 8), the 10\textsuperscript{4} shell (see Fig. 9), and the 4\textsuperscript{4} shell (see Fig. 10), in a $\beta$ scan across the phase transition. From these figures we conclude that $\beta = 1.007125$ is the most promising setting to study the critical dynamics.

Therefore let us take a closer look at $\beta = 1.007125$. Fig. 11 presents the time series of the plaquette from the 24\textsuperscript{4} down to the 4\textsuperscript{4} shell. The fluctuations increase as one proceeds to the inner shells, due to a decrease in self averaging. On top of those fluctuations, however, the 8\textsuperscript{4} shell clearly exhibits long metastabilities of typical length of 10000 Monte Carlo sweeps. On the 4\textsuperscript{4} shell, the fluctuations become so large that the two state metastability signal starts to fade out.

To further improve the situation we extend our simulation to the 32\textsuperscript{4} lattice. The resulting pattern of time histories is similar. But the competing short range fluctuations are reduced and leave the long range flip-flop behaviour more pronounced.
Fig. 8. $\beta$ scan of time series on the $16^4$ shell for the $24^4$-lattice.

Fig. 9. $\beta$ scan of time series on the $10^4$ shell for the $24^4$-lattice.

Fig. 10. $\beta$ scan of time series on the $4^4$ shell for the $24^4$-lattice.

Fig. 11. Scan through different shells of plaquette time series for the $24^4$-lattice at $\beta = 1.007125$. 

In order to reduce the high frequency modes in the time series, we apply a simple filter technique, averaging the series over a sliding window with length $l$, according to $\tilde{P}(t_i) = 1/l \sum_{j=1}^{l} P_{i+j-1}$. The resulting histogram is shown in Fig. 12. It provides definite evidence of a two-peak structure characteristic for a first order phase transition.

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