Single-photon transport in an array of quantum emitters with long-range interactions

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Keywords: Fano resonance, photon transport, long range interaction

Abstract
We investigate photon transport in a waveguide coupled to two-level quantum emitters with long-range interactions. We derive an exact solution for transmission and reflection amplitudes in the framework of real-space Hamiltonian. Our explicit formula allows for a complete description of photon transport in the presence of N quantum emitters with arbitrary non-local interactions. We apply this approach to single-photon transport in an one-dimensional waveguide coupled to two atoms with dipole-dipole interaction (DDI). Our numerical results confirm the Fano spectrum line shapes as a result of strong DDI, and demonstrate how the split of perfect reflection spectrum and the resonance transmission can be achieved by tuning the photon-atom and the atom-atom interactions.

Photon transport in an one-dimensional waveguide coupled to quantum emitters has become an important platform for studying light–matter interaction and its applications to quantum information processing [1–5]. One of the important issues of quantum optics is to investigate and develop novel tools in the control of transport of quantum information at the fundamental level of single quantum elements like photons and atoms. In recent years several quantum optical devices for quantum switch, diode, memory and gate, have been proposed and studied in the framework of waveguide quantum electrodynamics (QED) [6–16]. Manipulation of single photon transport can be realized by incorporating different photon-atom, and atom–atom interaction mechanisms [17–22]. The effects of strong atom–atom couplings in form of the dipole–dipole interaction (DDI) have been discussed in several recent works [23–32]. Most of them are devoted to a simple case where only two quantum emitters with DDI are coupled to a waveguide. More recently single-photon scattering properties in a chain of quantum emitters with DDI has been investigated by using an approach developed in [32], where a set of recursive relations among the transmission and reflection coefficients on each bond that connects two successive quantum emitters are established. Nevertheless, an analytical expression for transmission and reflection amplitudes for general N emitters is still lacking.

In this paper, we derive an exact solution to the problem of photon transport in an array of N quantum emitters with long-range interactions, by using the approach developed in [1] and generalized to multiple emitters in [1, 4]. We are looking for a Bethe-anzatz solution of the Schrödinger equation in terms of the transfer matrix. By reformulating a set of linear equations derived from the Schrödinger equation into a recursive relation for the wave functions at different positions at which the waveguide photon interacts with quantum emitters, we are able to derive a simple closed-form formula for calculating the transmission and the reflection amplitudes, subject to a variety of photon–emitter and emitter–emitter interaction schemes.

To start with, let us consider a one-dimensional waveguide coupled to a chain of atoms with their relative positions defined by \( x_i : i = 1, 2, \ldots, N \), which is illustrated in figure 1. The ground and excited states of the \( j \)-th two-level atom are given by \( |g_j \rangle \) and \( |e_j \rangle \), respectively. The atom transition frequency is denoted by \( \omega_{aj} \) and the resonant frequency of cavity \( j \) by \( \omega_{ck} \). The dispersion relation of the photon is assumed to be \( \omega_k = v k \). Thus, the
Hamiltonian of the system under consideration can be expressed as

$$
H = iv \int dx \left[ c^+_i(x) \frac{\partial c_j(x)}{\partial x} - c^+_j(x) \frac{\partial c_i(x)}{\partial x} \right] 
+ \sum_{j=1}^{N} \omega_j |e_j\rangle \langle e_j| 
+ \sum_{j=1}^{N} \int dx \delta(x-x_j) \left[ V_{Rj} c^+_R(x) + V_{Lj} c^+_L(x) \right] |\sigma_j\rangle + H.c. 
+ \sum_{i<j}^{N} g_{ij} (\sigma_i^+ \sigma_j + \sigma_i^+ \sigma_j),
$$

(1)

where $\sigma_j = |g_j\rangle \langle e_j|$ and $\sigma_j^+ = |e_j\rangle \langle g_j|$ are the atomic raising and lowering operators, respectively. $V_{Rj} (V_{Lj})$ stands for the coupling constant between the atom and the right (left) propagating waveguide photon and $g_{ij}$ is the dipole-dipole interaction strength between the $i$th and $j$th atoms, defined by [30]

$$
g_{ij} = \frac{3 \Gamma_0}{4} \left( 1 - 3 \cos^2(\theta) \right),
$$

(2)

where

$$
\cos \theta = \frac{\mathbf{p} \cdot (\mathbf{r}_i - \mathbf{r}_j)}{||\mathbf{p}|| \mathbf{r}_i - \mathbf{r}_j}, \quad r_{ij} = \frac{\omega_i}{c} ||\mathbf{r}_i - \mathbf{r}_j||.
$$

(3)

Here $\mathbf{r}_j$ and $\mathbf{p}$ are the position vector and the dipole moment of $j$-th emitter. $\Gamma_0$ stands for its free space decay rate. $c^+_R(x)$ is the creation operator for a right-moving photon, and $c^+_L(x)$ for a left-moving photon.

Suppose that an arbitrary state $|\Psi(t)\rangle$ of the Schrödinger equation

$$
i \hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = H |\Psi(t)\rangle,
$$

(4)

can be written in the Bethe ansatz form

$$
|\Psi(t)\rangle = \int dx [\phi_R(x) c^+_R(x) + \phi_L(x) c^+_L(x)] |0, g\rangle + \sum_{j=1}^{N} e_j \sigma_j |0, g\rangle.
$$

(5)

Then the Schrödinger equation turns into a set of differential equations

$$
i \frac{\partial \phi_R}{\partial t} = -iv \frac{\partial}{\partial x} \phi_R(x) + \sum_{j=1}^{N} \delta(x-x_j) V_{Rj} e_j,
$$

(6)

$$
i \frac{\partial \phi_L}{\partial t} = iv \frac{\partial}{\partial x} \phi_L(x) + \sum_{j=1}^{N} \delta(x-x_j) V_{Lj} e_j,
$$

(7)

$$
i \frac{\partial e_i}{\partial t} = \omega_j e_j + V_{Rj} \phi_R(x_j) + V_{Lj} \phi_L(x_j) + \sum_{i=1}^{j-1} e_i g_{ij} + \sum_{i=j+1}^{N} e_i g_{ij}.
$$

(8)

By introducing the left- and the right-propagating wave function as

$$
\psi_R(x) = e^{ikx} [A \theta(x_j - x) + A' \theta(x - x_j)],
$$

$$
\psi_L(x) = e^{-ikx} [B \theta(x_j - x) + B' \theta(x - x_j)].
$$

(9)
and expressing $|\Psi(t)\rangle$ as

$$
\phi_R(x) = f_R(x)e^{ikx}, \quad \phi_L(x) = f_L(x)e^{-ikx},
$$

(10)

with $k = \omega/v_p$, we obtain the following algebraic equations

$$
\begin{align*}
iv_y \frac{\partial f_R(x)}{\partial x} &= e^{-ikx} \sum_{j=1}^{N} \delta(x - x_j)V_{jr}e_j, \\
iv_y \frac{\partial f_L(x)}{\partial x} &= -e^{ikx} \sum_{j=1}^{N} \delta(x - x_j)V_{jl}e_j,
\end{align*}
$$

(11)

$$
\Delta_j e_j = V_{jr} \phi_R(x_j) + V_{jl} \phi_L(x_j) + \sum_{i=1}^{j-1} e_{ij}g_{ij} + \sum_{j<i}^{N} e_{ji}g_{ji},
$$

(12)

where the detuning between the waveguide photon frequency and the transition frequency of $j$-th atom is given by $\Delta_j = \omega_k - \omega_j$. In order to derive a transfer matrix expression from those coupled differential equations, we proceed as the following. First we solve the linear systems (12) to obtain the wave function of quantum emitter $e_j$ in terms of $\phi(x_j)$ ($i = 1, 2, \ldots N$). Then we substitute $\phi(x_j)$ into (11) to get a pair of closed equations for $f_R(x)$ and $f_L(x)$, from which a transfer matrix formalism can be derived. The solution to (12) can be formally written as

$$
e_j = \sum_{l=1}^{N} a_{jl}[V_{jr} \phi_R(x_l) + V_{jl} \phi_L(x_l)], \quad j = 1, 2, \ldots, N
$$

(13)

Now we insert (13) into (11) and integrate (11) from $x_j - 0$ to $x_j + 0$, to eliminate the $\delta$-function at $x = x_j$. With the help of the continuity condition of the wave function at $x_j$, we arrive at the following matrix equation,

$$
\begin{pmatrix}
A'_j \\
B'_j
\end{pmatrix} = \begin{pmatrix}
A_j \\
B_j
\end{pmatrix} - i \sum_{l=1}^{N} a_{jl}L_{jl} \begin{pmatrix}
A_j \\
B_j
\end{pmatrix}
$$

(14)

where

$$
L_{jl} = \begin{pmatrix}
\xi_{RR} e^{-ik(x_j-x_l)} & \xi_{RL} e^{-ik(x_j+x_l)} \\
\xi_{LR} e^{ik(x_j+x_l)} & \xi_{LL} e^{ik(x_j-x_l)}
\end{pmatrix}
$$

(15)

and $\xi_{\alpha\beta} = (V_{\alpha j}V_{\beta j})/v_y (\alpha, \beta = R, L)$. By introducing the free propagator along the edge that connect two adjacent quantum emitters, we have

$$
\begin{pmatrix}
A_{j+1} \\
B_{j+1}
\end{pmatrix} = \Gamma_j \begin{pmatrix}
A'_j \\
B'_j
\end{pmatrix},
$$

(16)

with the free-space propagator defined by

$$
\Gamma_j = \begin{pmatrix}
e^{ik(x_{j+1}-x_j)} & 0 \\
0 & e^{-ik(x_{j+1}-x_j)}
\end{pmatrix}
$$

(17)

Finally together with (14) we obtain a recursive relation

$$
\begin{pmatrix}
A_{j+1} \\
B_{j+1}
\end{pmatrix} = \Gamma_j \begin{pmatrix}
A_j \\
B_j
\end{pmatrix} - \begin{pmatrix}
\sum_{l=1}^{N} a_{jl}L_{jl} \\
\end{pmatrix} \begin{pmatrix}
A_j \\
B_j
\end{pmatrix}, \quad j = 1, 2, \ldots, N - 1.
$$

(18)

The above $N$ coupled linear systems (18) can be solved exactly under the following boundary conditions:

$$
\begin{pmatrix}
A'_N \\
B'_N
\end{pmatrix} = \begin{pmatrix} t \\
0\end{pmatrix}, \quad \begin{pmatrix}
A_{N+1} \\
B_{N+1}
\end{pmatrix} = \begin{pmatrix} 0 \\
1\end{pmatrix},
$$

(19)

where $t$ and $r$ are transmission and reflection amplitudes, respectively. In order to solve (18) we have to express $[A_j, B_j] j = 2, 3, \ldots, N$ in terms of $[A_1, B_1; A'_{N}, B'_N]$. To this end, we reorganize the first $N - 1$ equations in (18) into a $N - 1$ coupled linear system. And then we substitute the solutions of the linear system into the last equation of (18) (for $j = N$) to obtain a relation between the states $[A_1, B_1]$ and $[A'_{N}, B'_N]$. To simplify our derivation we make the following transformation of variables

$$
u_j = \begin{pmatrix}
A_j \\
B_j
\end{pmatrix} = u_j W_j
$$

(20)

where $W_j$ is a $2 \times 2$ matrix, which are determined by the recursive equation (18). To proceed we consider only the first $N - 1$ equations in (18) and reformulate them into the following expressions
The system of N linear matrix equations can be solved by means of Gaussian elimination method. Once the matrix solution is obtained, we set j = N in (14) and employ the boundary condition (19) to get a transfer matrix

\[ M = W_N - i \sum_{l=1}^{N} a_{nl}L_{nl}W_l, \]

where \( W_1 = I \) is an 2 \times 2 unitary matrix by definition.

It follows immediately that the transmission and reflection amplitudes can be calculated from the transfer matrix \( M \), via the following relations

\[ r = -\frac{M_{21}}{M_{22}}, \quad t = \frac{1}{M_{22}} \]

if \( M_{11}M_{22} - M_{12}M_{21} = 1 \). To keep our formalism as general as possible, in the derivation we do not specify the form of non-local interaction between quantum emitters, no do we mention the type of the photon-atom couplings. It is noted that when \( V_{\alpha \beta} \neq V_{\beta \alpha} \), we get that

\[ (t_0) = \begin{pmatrix} A_{2} \\ B_{2} \end{pmatrix} - i\alpha_{23}L_{23} \begin{pmatrix} A_{1} \\ B_{1} \end{pmatrix} + i\beta_{22}L_{22} \begin{pmatrix} A_{2} \\ B_{2} \end{pmatrix} = (I - i\alpha_{22}L_{22}) \begin{pmatrix} A_{2} \\ B_{2} \end{pmatrix} - i\beta_{21}L_{21} \begin{pmatrix} 1 \\ r \end{pmatrix} \]

which is the direct interactions between the atoms or any quantum emitters. The interaction range also can be varied through the link matrices incorporated into our model system, as manifested by the transition frequency of individual atoms. There are several important characteristics in our generic results. (a) Throughout the derivation of the transfer matrix, we have not fixed the exact long-range interaction mechanisms, except that they are assumed to be the direct interactions between the atoms or any quantum emitters. The interaction range also can be varied according to the model system under investigation, from nearest-neighbor to all-to-all interactions. (b) Non-identical coupled atoms or quantum emitters and non-uniform inter-atom coupling constants have been incorporated into our model system, as manifest by the transition frequency of individual atoms and the atom-atom interaction strength \( g_{ij} \). (c) The gain-loss effects of the media can be readily taken into account by adding to the atom transition energy an imaginary part, representing the possible decay or amplification mechanism involved in the photon propagation process. It should be emphasized that our approach is limited to the elastic transport process as indicated by the Bethe ansatz solution. Therefore many other interesting phenomena like the coherent frequency down-conversion processes [28, 29] cannot be analyzed directly. It is nevertheless interesting to generalize our approach to the quantum interference in multi-branch optical waveguide with interacting emitters.

As an illustration of the method derived here, we consider a well-studied case of \( N = 2 \). For simplicity we will not discuss the effects of chiral coupling here, and so we assume that \( V_{\alpha \beta} = V_{j} \) for \( \alpha = R, L \). By setting \( j = N = 2 \) in (14) we get

\[ \begin{pmatrix} t_0 \\ 0 \end{pmatrix} = \begin{pmatrix} A_{2} \\ B_{2} \end{pmatrix} - i\alpha_{23}L_{23} \begin{pmatrix} A_{1} \\ B_{1} \end{pmatrix} + i\beta_{22}L_{22} \begin{pmatrix} A_{2} \\ B_{2} \end{pmatrix} = (I - i\alpha_{22}L_{22}) \begin{pmatrix} A_{2} \\ B_{2} \end{pmatrix} - i\beta_{21}L_{21} \begin{pmatrix} 1 \\ r \end{pmatrix} \]
On the other hand, we get from (18), by letting \( j = 1 \),
\[
\begin{pmatrix}
A_2 \\
B_2
\end{pmatrix} = L_1 \begin{pmatrix}
A_1 \\
B_1
\end{pmatrix} - i L_1 a_{11} L_{12} \begin{pmatrix}
A_1 \\
B_1
\end{pmatrix} - i L_1 a_{12} L_{12} \begin{pmatrix}
A_2 \\
B_2
\end{pmatrix} = (I + i L_1 a_{12} L_{12})^{-1} L_1 (I - i a_{11} L_{12}) \left( \begin{pmatrix}
1 \\
0
\end{pmatrix} \right)
\]
(25)

Inserting (25) into (24) we obtain a transfer matrix,
\[
M = (I - i a_{22} L_{22}) (I + i L_1 a_{12} L_{12})^{-1} L_1 (I - i a_{11} L_{12}) - i a_{21} L_{21}
\]
(26)

To find \( \{ a_j \} \) we resort to (12), when \( N = 2 \) it becomes
\[
\begin{align*}
\Delta_1 c_1 &= V_1 \phi(x_1) + c_2 g_{12} \\
\Delta_1 c_2 &= V_2 \phi(x_2) + c_1 g_{12}
\end{align*}
\]
(27)

For the purpose of demonstration we present only exact solution for identical emitters with symmetric DDI. That is, \( V_1 = V_2 \) and \( \Delta_1 = \Delta_2 \). In all the numerical analysis we take \( V^2 / v_0 \) as a unit of frequency, hence the coefficients \( \{ a_j \} \) can be obtained from the reduced detuning and DDI constant
\[
\begin{align*}
a_{11} &= a_{22} = a = \frac{\Delta}{\Delta^2 - g^2} \\
a_{21} &= a_{12} = b = \frac{g}{\Delta^2 - g^2}
\end{align*}
\]
(28)

Here we have made use of \( \Delta = \Delta / (V^2 / v_0) \) and \( g = g_{12} / (V^2 / v_0) \). Since \( M_{11} M_{22} - M_{12} M_{21} = 1 \), the transmission and reflection amplitude are given by
\[
\begin{align*}
r &= \frac{a(1 + a) - b^2} {a^2 - b^2} e^{2ik_L} + \frac{a(1 - a) + b^2} {a^2 - b^2} e^{-2ik_L} + 2b \\
t &= - \frac{1 - 2ib \sin(2k_L)} {a^2 - b^2} e^{2ik_L} + \frac{[-(1 - a)^2 + b^2]e^{-2ik_L} + 2b} {a^2 - b^2}
\end{align*}
\]
(29)

Before we show how DDI affects the single-photon transport properties, let us first examine the role of the inter-emitter distance \( L \). It is known that even in the absence of the direct interactions between the coupled quantum emitters, the interference between forward- and backward-scattered light may lead to a vanishing or a perfect Bragg reflection at the resonance, depending on the relation between the wavelength of the atomic transition \( \lambda \) and the atom-atom separation \( L \). Since the DDI constant \( g \) decreases as \( 1/ |x_1 - x_2|^3 \). At large values of \( L \), the DDI plays a very limited role in the photon transport. On the other hand, when \( L \) is sufficiently small, the DDI may result in dramatic change of the photon transport properties. In this case the typical effect of DDI is to split or shift the original transmission and reflection peaks. Note that for \( L \approx \lambda / 20 \) with \( k_L \approx 0.3 \pi \), the system is in the strong DDI regime. Therefore in our numerical exploration we take \( k_L \approx 0.25 \pi \) as a reference parameter value \([32]\), and study the impact of DDI on single photon transport in a waveguide coupled to two atoms.

In figure 2 we display some characteristic single-photon transport properties by comparing the reflection coefficient for various values of DDI strength \( g \), as a function of the detuning. Here we assume the Markovian approximation by using \( k_L \approx k_L L \), where \( k_L \) is the quantum emitter wave vector \( k_L = \omega_0 / v_0 \) and \( L \) is the distance between two artificial atoms. We also assume that the frequency of waveguide photon is much larger than the detuning by setting \( \omega_0 = 100 (V^2 / v_0) \) (here we use \( V^2 / v_0 \) as unit of the frequency) and \( \Delta < 0.1 \omega_0 \). Since the emitter-emitter interaction is assumed to be of DDI type, the coupling strength is related to the inter-emitter distance by \( g \propto 1/L^3 \). When \( g \) is sufficiently small one finds an unimodal reflection spectrum with a peak of the perfect reflection \( R = 1 \) at \( \omega_L = \omega_0 \). Numerical analysis reveals that at \( g = 3 \) a symmetric spectrum line shape occurs at \( k_L = 0.25 \pi \), meanwhile asymmetrical Fano resonance line shape emerges for \( k_L > 0.25 \pi \). In order to illustrate the effects of DDI, we take \( L_0 = 0.5462 \pi \) as a critical point, which marks the emergence of the bimodal spectrum. That is, one finds the typical unimodal reflectance spectrum line shape for \( k_L > k_L L \), and the bimodal line shape for \( k_L < k_L L \). In figure 2(a) we depict the reflectance as a function of the detuning, which shows how the unimodal resonance curve splits into a two-peak spectrum, as the dipole-dipole separation \( L \) is decreased such that \( k_L = 0.25 \pi < L = 0.4562 \pi \), the critical value at which the single-peak reflectance line starts to split. In figure 2(b) we compare shows the reflectance for \( k_L = 0.6212 \pi > k_L L \) and \( k_L = 0.425 \pi < k_L L \). Here we find that one may turn the near perfect transmission into the perfect reflection by tuning the DDI constant. Figure 2(c) displays the effect of the strong DDI, it can be seen that the symmetric spectrum line is replaced by an asymmetrical one, and the original reflectance peaks turn into the perfect transmission flat line, indicating a transition from perfect reflection to DDI-induced transparency. Figure 2(d) reveals that the transition displayed in figure 2(c) can be found also for asymmetrical spectrum with \( k_L = 0.1 \pi \). It shows that the reflectance tends to an asymptotic spectrum pattern as the dipole-dipole separation goes to zero.
detuning, see electromagnetically induced transparency at a frequency that is shifted to a critical value determined by the spliting of the retransition frequencies for different atoms may generate quite different transport behaviors. Here we just show how the transport behavior is modiﬁed by varying photon–emitter interaction strength and by tuning the photon–atom interaction constant of one atom, say $V_1$, and tune that of the second atom, i.e., $V_2$. It is

$$\Delta = (\omega_1 - \omega_2)/\omega_1 \approx 0, \quad \text{for instance, } kL \approx 0.25\pi,$$

then the separation of the reflection peaks increases with the detuning, giving rise to a large transparency valley between the two peaks, as illustrated in figure 3(b).

Now we turn to the role played by local photon–atom interaction. For convenience of comparison we ﬁx the photon–atom interaction constant of one atom, say $V_1 = \text{const}$, and tune that of the second atom, i.e., $V_2$. It is

$$\Delta = (\omega_1 - \omega_2)/\omega_1 \approx 0, \quad \text{for instance, } kL \approx 0.25\pi;$$

then the separation of the reflection peaks increases with the detuning, giving rise to a large transparency valley between the two peaks, as illustrated in figure 3(b).

The effect of different photon–atom coupling has been studied in [32]. It is found that DDI may causes splitting of the refection and transmission spectrum. In fact, both the photon–atom couplings and the atomic transition frequencies for different atoms may generate quite different transport behaviors. Here we just show how the transport behavior is modiﬁed by varying photon–emitter interaction strength and by tuning the photon–atom interaction constant of one atom, say $V_1$, and tune that of the second atom, i.e., $V_2$. It is

$$\Delta = (\omega_1 - \omega_2)/\omega_1 \approx 0, \quad \text{for instance, } kL \approx 0.25\pi,$$

then the separation of the reflection peaks increases with the detuning, giving rise to a large transparency valley between the two peaks, as illustrated in figure 3(b).
intriguing to find that when $V_2$ is reduced, the bimodal reflection line shape becomes sharper, leaving out two aislote perfect reflection points and a perfect transmission band in between, as is shown in figure 3(c). Meanwhile a characteristic unimodal spectrum line is approached as $V_2$ is increased, see figure 3(d).

In summary we have studied single-photon transport in a one-dimensional waveguide coupled to a chain of quantum emitters with long-range interaction. By using the real-space Hamiltonian method and the Bethe ansatz solution, we obtain an exact solution for the reflection and transmission amplitudes for a waveguide embedded with $N$ interacting quantum emitters. Our formalism has included explicitly the photon-atom, and atom-atom interactions which allows for a detailed investigation of how the photon transport can be controlled by manipulating those interaction parameters. In particular, when the chiral coupling is considered, the physics underlying the redirection of light in such a waveguide with arbitrary interacting emitters can be readily revealed, at least numerically. As an illustration of our method, we analyse the effects of various coupling schemes on the single-photon transport involving two interacting atoms. In this case the analytical expressions of reflectance and transmittance are obtained. The numerical results validate our approach. Although for general $N$ coupled atoms analytical solutions may become quite involved, our theory provides a feasible and efficient scheme for numerical exploration of transport properties in such a model system with long-range interactions, as one has only to solve a system of linear equations of $2 \times 2$ matrices.

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