Hunting $\eta_b$ through radiative decay into $J/\psi$

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Abstract

We propose that the radiative decay process, $\eta_b \rightarrow J/\psi \gamma$, may serve as a clean searching mode for $\eta_b$ in hadron collision facilities. By a perturbative QCD calculation, we estimate the corresponding branching ratio to be of order $10^{-7}$. Though very suppressed, this radiative decay channel in fact has larger branching ratio than the hadronic decay process $\eta_b \rightarrow J/\psi J/\psi$, which was previously hoped to be a viable mode to search for $\eta_b$ in Tevatron Run 2. The discovery potential of $\eta_b$ through this channel seems promising in the forthcoming LHC experiments, and maybe even in Tevatron Run 2, thanks to the huge statistics of $\eta_b$ to be accumulated in these experiments. The same calculational scheme is also used to estimate the branching ratios for the processes $\eta_b (\eta_c) \rightarrow \phi \gamma$.

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The existence of $\eta_b$, the pseudo-scalar partner of $\Upsilon(1S)$, is a firm prediction of QCD, about which nobody would seriously challenge. It is rather unsatisfactory that although three decades have elapsed since the discovery of $\Upsilon(1S)$, this particle eludes intensive experimental endeavors and still eagerly awaits to be established.

On the theoretical side, many work have been devoted to uncovering various properties of $\eta_b$, such as its mass, inclusive hadronic and electromagnetic widths, transition rates and production cross sections in different collider programs [1]. Among all the observables, its mass is believed to be the simplest and most unambiguous to predict. Recent estimates for $\Upsilon - \eta_b$ mass splitting span the 40–60 MeV range [2, 3, 4, 5]. An eventual definite sighting of $\eta_b$ and precise measurement of its mass will critically differentiate varieties of theoretical approaches, consequently sharpening our understanding towards the $b\bar{b}$ ground state.

The $\eta_b$ has recently been sought from $\gamma\gamma$ collisions in the full LEP 2 data sample, where approximately few hundreds of $\eta_b$ are expected to be produced. ALEPH has one candidate event with the reconstructed mass of $9.30 \pm 0.03$ GeV, but consistent to be a background event [6]. ALEPH, L3, DELPHI have also set upper limits on the branching fractions for $\eta_b$ decays into 4, 6, 8 charged particles [6, 7, 8]. Based on the 2.4 fb$^{-1}$ data taken at the $\Upsilon(2S)$ and $\Upsilon(3S)$ resonances, CLEO has searched distinctive single photons from hindered $M1$ transitions $\Upsilon(2S), \Upsilon(3S) \to \eta_b\gamma$, and from the cascade decay $\Upsilon(3S) \to h_b\pi^0, h_b\pi^+\pi^-$ followed by $E1$ transition $h_b \to \eta_b\gamma$, but no signals have been found [9].

Hadron collider experiments provide an alternative means to search for $\eta_b$. Unlike the $e^+e^-$ machines which are limited by the low yield of $\eta_b$, hadron colliders generally possess a much larger $\eta_b$ production rate, which in turn allows for triggering it through some relatively rare decay modes yet with clean signature. However, one caveat is also worth being emphasized. The corresponding background events are in general copious in hadron machines, so the virtue of this sort of decay modes may be seriously discounted (Such an example is the electromagnetic decay $\eta_b \to \gamma\gamma$, with an expected branching fraction $\sim 10^{-4}$, which is nevertheless overshadowed by the ubiquitous $\gamma$ events originating from $\pi^0$ decay).

Several years ago, Braaten, Fleming and Leibovich suggested that the hadronic decay $\eta_b \to J/\psi J/\psi$, followed by both $J/\psi$ decays to muon pairs, can be used as a very clean trigger to search for $\eta_b$ at Tevatron Run 2 [10]. By some simple scaling assumption, they
estimate the branching ratio of the double $J/\psi$ mode to be $7 \times 10^{-4\pm1}$, and conclude that the prospect of observing the $\eta_b \to 4\mu$ channel at Tevatron Run 2 is rather promising. CDF in fact has followed this suggestion and looked at the full Run 1 data for the $4\mu$ events in the expected $\eta_b$ mass window \cite{11}.

However, some objection has been recently put forward by Maltoni and Polosa, who argue that the estimate of the branching ratio of double $J/\psi$ mode by Braaten $et$ $al$ might be too optimistic \cite{12}. They instead advocate that $\eta_b \to D^*D^{(*)}$, with the estimated decay ratios lying in the $10^{-3} - 10^{-2}$ range, may serve as better searching modes for $\eta_b$ in Run 2.

Very recently, one of us (Y. J.) has surveyed the discovery potential of various hadronic decay channels of $\eta_b$ \cite{13}. The explicit perturbative QCD (pQCD) calculation predicts $\text{Br}[\eta_b \to J/\psi J/\psi]$ to be only of order $10^{-8}$. If this is the case, the chance of observing this decay mode in Run 2 then becomes rather gloomy, whereas the observation possibility at LHC may still remain. Another noteworthy assertion of \cite{13} is that, by some rough but physical considerations, one expects $\text{Br}[\eta_b \to D^*D] \sim 10^{-5}$ and $\text{Br}[\eta_b \to D^*D^*] \sim 10^{-8}$, which are much smaller than the optimistic estimates by Maltoni and Polosa. Taking the low reconstruction efficiency of $D$ mesons further into consideration, these double charmful decay modes may not be as attractive as naively thought.

In this paper, we try to propose another decay process, $\eta_b \to J/\psi\gamma$, as a viable discovery channel for $\eta_b$ in hadron collider experiments. At first sight, this mode, being a radiative decay process, may not look very economical due to the large total width of $\eta_b$. In fact, our explicit pQCD calculation reveals that the corresponding branching ratio is indeed very suppressed, only about $10^{-7}$. Nevertheless, it is worth emphasizing that this is already larger than that of the double $J/\psi$ mode. The usual way of reconstructing $J/\psi$ is via the dimuon mode, with a branching fraction about 6%. In the light of this, the advantage of this radiative decay process over the double $J/\psi$ mode will be further amplified, since only one $J/\psi$ needs to be reconstructed in our case. Unlike the open-charm decay processes $\eta_b \to D^*D^{(*)}$, in which the final decay products such as $K$, $\pi$ in general suffer severe contamination from the combinatorial background, the presence of $J/\psi$ in the final state renders our radiative decay channel much cleaner to look for experimentally.

Thanks to the huge amount of $\eta_b$ to be accumulated in high energy hadron collision facilities, we expect that this radiative decay channel, albeit being a rare decay mode, may still have bright prospect to be observed in Run 2 and in the forthcoming LHC program.
FIG. 1: Lowest-order diagrams that contribute to $\eta_b \rightarrow J/\psi \gamma$. Diagrams (1), (2), (3) stand for QCD-initiated process, while (4) represents the dominant QED process governed by photon fragmentation. Crossed diagrams are implicitly implied.

Not surprisingly, one should be aware that the major QCD background events, i.e. the associated $J/\psi + \gamma$ production [14, 15, 16], may exceedingly outnumber our signal events. To make this mode practically viable, one has to ensure that those background events can be significantly depressed by judiciously adjusting the kinematical cuts.

The remainder of the paper is distributed as follows. In Sec. II we present the pQCD calculation for the radiative decay process $\eta_b \rightarrow J/\psi \gamma$, treating heavy quarkonium states in NRQCD approach which, to our purpose, is equivalent to the color-singlet model. In Sec. III we present the numerical prediction to the corresponding branching ratio, and explore the observation potential of this decay mode in Tevatron Run 2 and in the coming LHC experiment. We subsequently apply the same formalism to estimate the branching ratios for analogous processes $\eta_b (\eta_c) \rightarrow \phi\gamma$, which is equivalent to work in the context of constituent quark model. We summarize in Sec. IV.

II. COLOR-SINGLET MODEL CALCULATION

In this section, we present a pQCD calculation for the decay rate of $\eta_b \rightarrow J/\psi \gamma$. This decay process can be initiated by either strong or electromagnetic interaction, with the
corresponding lowest-order diagrams shown in Fig. 1. Since the annihilation of the $b\bar{b}$ pair, the creation of the $c\bar{c}$ pair, as well as the emission of the hard photon, are all dictated by short-distance physics, with the hard scales set by the heavy quark masses, it is appropriate to utilize the pQCD scheme to tackle this exclusive process.

Nowadays it has become standard to employ the NRQCD factorization framework to cope with hard processes involving heavy quarkonia [17]. One of the major advantages of this model-independent EFT approach over the old color-singlet model is that it can in principle incorporate the contribution of higher Fock components of quarkonium (color-octet effect) in a systematic fashion. This is exemplified by the systematic treatment of inclusive annihilation decay of various quarkonium states within this framework [17, 18]. In contrast, the inclusion of color-octet effect in exclusive processes has not been developed to a comparable level within this context. To date, there are only few cases where this effect has been investigated in the model-independent language. One example is the magnetic dipole transition process such as $J/\psi \rightarrow \eta_c \gamma$, where the leading color-octet contribution vanishes [19]. Another example is the quarkonium radiative decay to light mesons, such as $\Upsilon \rightarrow f_2(1270)\gamma$, where the color-octet effect turns out to be quite insignificant [20].

If we take the lessons from the aforementioned radiative decay processes, it seems persuasive to assume that the color-octet effect may also be unimportant in our case. We will completely ignore this effect in this work. In this regard, there will be no difference between the NRQCD approach and the color-singlet model, consequently these two terms will be used interchangeably.

Before launching into the actual calculation, it is worth mentioning that, Guberina and Kuhn has studied the analogous radiative decay process $\Upsilon \rightarrow \eta_c \gamma$ in NRQCD approach more than two decades ago [21]. Our calculation will intimately resemble theirs.

It is useful to take notice of a simple trait of this process, that the photon can only be transversely polarized, so is the recoiling $J/\psi$ by angular momentum conservation. This property holds true irrespective of whether this process is initiated by strong or electromagnetic interaction. In fact, parity and Lorentz invariance constrains the decay amplitude to have the following unique tensor structure:

$$M(\lambda_1, \lambda_2) = A \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\mu}_{J/\psi}(\lambda_1) \epsilon^{*\nu}_{\gamma}(\lambda_2) Q^\alpha k^\beta.$$  \hspace{1cm} (1)

We use $Q$, $P$ and $k$ to signify the momenta of $\eta_c$, $J/\psi$ and $\gamma$, respectively, and use $\lambda_1$
and $\lambda_2$ to label the helicities of $J/\psi$ and $\gamma$ which are viewed in the $\eta_b$ rest frame. It is straightforward to infer from (1), that the only physically allowed helicity configurations are $(\lambda_1, \lambda_2) = (\pm 1, \pm 1)$. All the dynamics is encoded in the coefficient $A$, which we call reduced amplitude. Our task then is to find out its explicit form.

In the color-singlet model, it is customary to start with the parton process $b(p_b) \bar{b}(p_b) \rightarrow c(p_c) \bar{c}(p_c) + \gamma(k)$, then project this matrix element onto the corresponding color-singlet quarkonium Fock states. For reactions involving heavy quarkonium, it is conventional to organize the amplitude in powers of the relative momentum between its constituents, to accommodate the NRQCD ansatz. This work is intended for the leading order accuracy in relativistic expansion only, it is then legitimate to neglect the relative momentum inside both $\eta_b$ and $J/\psi$. We thus set $p_b = p_{\tilde{b}} = Q/2$ and $p_c = p_{\tilde{c}} = P/2$. For the $b\bar{b}$ pair to form $\eta_b$, it is necessarily in a spin-singlet and color-singlet state, and one can replace the product of the Dirac and color spinors for $b$ and $\bar{b}$ in the initial state with the projector

$$u(p_b) \overline{v}(p_b) \rightarrow \frac{1}{2\sqrt{2}} (Q + 2m_b) i\gamma_5 \times \left( \frac{1}{\sqrt{m_b}} \psi_{\eta_b}(0) \right) \otimes \left( \frac{1}{\sqrt{N_c}} \right).$$

(2)

For the outgoing $J/\psi$, one can employ the following projection operator:

$$v(p_c) \overline{u}(p_c) \rightarrow \frac{1}{2\sqrt{2}} \varepsilon_{J/\psi}^\mu (P + 2m_c) \times \left( \frac{1}{\sqrt{m_c}} \psi_{J/\psi}(0) \right) \otimes \left( \frac{1}{\sqrt{N_c}} \right),$$

(3)

where $\varepsilon_{J/\psi}^\mu$ is the polarization vector of $J/\psi$ satisfying $\varepsilon_{J/\psi}^\mu (\lambda) \cdot \varepsilon_{J/\psi}^\nu (\lambda') = -\delta^{\lambda \lambda'}$ and $P \cdot \varepsilon = 0$. $N_c = 3$, and $1_c$ stands for the unit color matrix. The nonperturbative parameters, $\psi_{\eta_b}(0)$ and $\psi_{J/\psi}(0)$, are Schrödinger wave functions at the origin for $\eta_b$ and $J/\psi$, which can be either inferred from phenomenological potential models or directly extracted from experiments. By writing (2) and (3) the way as they are, it is understood that $M_{\eta_b} = 2m_b$ and $M_{J/\psi} = 2m_c$ have been assumed.

We commence with the strong decay amplitude. As can be seen in Fig. 1, the lowest order QCD-initiated contribution starts already at one loop. Note that the photon can only be emitted form the $c$ quark line, because the even $C$-parity of $\eta_b$ forbids the $\gamma$ to attach to its constituents. Using the projection operators in (2) and (3), it is straightforward to write
down the Feynman rules for the strong decay amplitude:

\[
M_{\text{str}} = -C_{\text{str}} e e g_s^4 \left( \psi_{\omega_b}(0) \psi_{J/\psi}(0) \right) \frac{1}{8 \sqrt{m_b m_c}} \int \frac{d^4k_1}{(2\pi)^4} \frac{1}{k_1^2 k_2^2} \times \left\{ \right.
\]

\[
\left. \begin{array}{c}
\times \left[ \text{tr}[Q + 2m_b \gamma_5 \gamma_\mu (p_b - k_1 + m_b) \gamma_\mu] + (\mu \leftrightarrow \nu, \ k_1 \leftrightarrow k_2) \right] \\
\times \left[ \text{tr}[\gamma^\mu (Q + 2m_c) \gamma_\mu (p_c - k_1 + m_c) \gamma_\mu (k_2 - p_c + m_c) \gamma_\nu] \right] \\
+ \left[ \text{tr}[\gamma^\mu (Q + 2m_c) \gamma_\mu (p_c - k_1 + m_c) \gamma_\mu (k_2 - p_c + m_c) \gamma_\nu] \right] \\
+ \left[ \text{tr}[\gamma^\mu (Q + 2m_c) \gamma_\mu (p_c - k_1 + m_c) \gamma_\mu (k_2 - p_c + m_c) \gamma_\nu] \right] \\
\left. \right\},
\]

\[ (4) \]

where the corresponding color factor \( C_{\text{str}} = N_c^{-1} \text{tr}(T^a T^b) \text{tr}(T^a T^b) = \frac{2}{3} \). The momenta carried by two internal gluons are labeled by \( k_1, k_2 \), respectively, which are subject to the constraint \( k_1 + k_2 = Q \). Notice that the box diagrams in Fig. 1 are related to one-loop four- or five-point functions, and the corresponding loop integrals are ultraviolet finite. Moreover, the occurrence of heavy \( b \) and \( c \) masses ameliorate the infrared behavior of the loop integral so that the result turns out to be simultaneously infrared finite. Since there is no need for regularization, we have directly put the spacetime dimension to four.

After completing the Dirac trace in (4), we end up with terms which do not immediately possess the desired tensor structure of (1), instead with one index of Levi-Civita tensor contracted to the loop momentum variable. Of course, when everything is finally worked out, all these terms must conspire to arrive at the desired Lorentz structure. Conversely, one may exploit this knowledge to simplify the algebra prior to performing the loop integral [21, 22].

First pull out the partial amplitude \( M_{\mu\nu} \) through \( M_{\text{str}} = M_{\mu\nu} \varepsilon^\mu_{J/\psi} \varepsilon^\nu_{J/\psi} \). Eq. (1) then demands

\[
M_{\mu\nu} = A_{\text{str}} \varepsilon_{\mu\nu\alpha\beta} Q^\alpha k^\beta,
\]

which is compatible with the expectation that \( M_{\mu\nu} \) would vanish unless \( \mu, \nu \) are in transverse directions. Contracting both sides of (5) with \( \varepsilon^{\mu\nu\rho\sigma} Q_\rho k_\sigma \), we can express the reduced amplitude as

\[
A_{\text{str}} = \frac{1}{2 (k \cdot Q)^2} M_{\mu\nu} \varepsilon^{\mu\nu\rho\sigma} Q_\rho k_\sigma.
\]

After this manipulation is done, it is convenient to adopt a new loop momentum variable \( q \), which is related to the old one via \( k_1 = (Q + q)/2 \) and \( k_2 = (Q - q)/2 \). We end in a
concise expression:

\[ A_{\text{str}} = -\frac{e_c e g_s^4}{6 \pi^2} \sqrt{\frac{m_b}{m_c}} \frac{\psi_{b}(0) \psi_{J/\psi}(0)}{(m_b^2 - m_c^2)^2} f \left( \frac{m_b^2}{m_c^2} \right), \]  

(7)

where

\[ f \left( \frac{m_b^2}{m_c^2} \right) = \frac{8}{i\pi^2} \int d^4q \frac{(k \cdot Q) q^2 - (k \cdot q) (Q \cdot q)}{(q + Q)^2 (q - Q)^2 (q^2 + 2k \cdot q - P^2) (q^2 - 2k \cdot q - P^2)}. \]  

(8)

Since \( f \) is dimensionless, it can depend upon \( m_b \) and \( m_c \) only through their dimensionless combination. It is interesting to note that the \( b \) quark propagator has been canceled in this expression. The loop integral can be performed analytically by the standard method, and the result is

\[
\begin{align*}
\text{Re} f(u) &= \frac{2(1 - u)}{2 - u} \ln \left[ \frac{u}{2(1 - u)} \right] - \frac{2}{1 + u} \left\{ \ln^2 2 + \frac{1}{2} \ln^2 u + \ln(2 - u) \ln \left[ \frac{u}{2(1 - u)} \right] \right. \\
&\quad + \ln u \ln \left[ \frac{2}{1 - u} \right] - u \ln \left[ \frac{u}{2 - u} \right] \ln \left[ \frac{u}{2(1 - u)} \right] + 2 \text{Li}_2[-u] + \text{Li}_2 \left[ \frac{u - 1}{2u} \right] \\
&\quad + 2 \text{Li}_2 \left[ \frac{u}{2} \right] - \text{Li}_2 \left[ \frac{u^2 - u}{2} \right] - u \text{Li}_2 \left[ \frac{2 - 2u}{2} \right] \right\}, \\
\text{Im} f(u) &= 2\pi \left\{ \frac{1 - u}{2 - u} + \frac{u \ln u}{1 + u} - \ln(2 - u) \right\},
\end{align*}
\]

(9)

where \( \text{Li}_2 \) denotes the dilogarithm (Spence) function. We will expound the derivation of this result in the Appendix. Note \( f \) has an absorptive part, which reflects that the intermediate gluons are kinematically permissible to stay on shell. One can apply Cutkosky’s cutting rule to verify (10). The real and imaginary parts of \( f \) as function of \( m_c^2/m_b^2 \) are displayed in Fig. 2.

It is instructive to know the asymptotic behavior of \( f \) in the \( u \to 0 \) limit, which can be readily read out from (9) and (10):

\[
\begin{align*}
\text{Re} f(u) &\to (1 - 2 \ln 2) \ln u + \frac{\pi^2}{3} - \ln 2 + \ln^2 2, \\
\text{Im} f(u) &\to \pi (1 - 2 \ln 2).
\end{align*}
\]

(11)

Note as \( u \) approaches 0, the imaginary part remains finite, whereas the real part blows up logarithmically. This trend can be clearly visualized in Fig. 2. The logarithmic divergence in the \( m_c \to 0 \) limit is obviously of infrared origin. Nevertheless, this does not pose any practical problem, since a non-relativistic description for a zero-mass bound state, as well as the resulting predictions, should not be trusted anyway. It is interesting to note that, the
FIG. 2: Real and imaginary parts of $f(u)$.

analogous radiative decay process, $\Upsilon \to \eta_c \gamma$, has a different asymptotic behavior, in which both the real and imaginary parts of the QCD amplitude admits a finite limit [21].

We next turn to the pure QED contribution to this radiative decay process. One may naively expect that this contribution is much less important than the QCD contribution. However, it turns out that this expectation is not true and the QED contribution must be retained. Due to the neutral color charge of photon, this radiative decay can arise at tree level from the photon fragmentation, as shown in Fig. 1 (4). Obviously, the fragmentation type contribution is much more dominant over other types of QED diagrams. To calculate this contribution, a necessary input is the $\gamma - J/\psi$ coupling, which is characterized by the $J/\psi$ decay constant$^1$:

$$\langle J/\psi(\lambda) | \bar{c} \gamma^\mu c | 0 \rangle = -g_{J/\psi} \epsilon^\mu(\lambda).$$

In the non-relativistic limit, $g_{J/\psi}$ is linked to $\psi_{J/\psi}(0)$ through the relation $g_{J/\psi} = 2^{3/2} N_c^{1/2} m_c^{1/2} \psi_{J/\psi}(0)$, which can be derived from (3). The calculation is much easier than

$^1$ In conformity with the sign convention adopted in (3), a minus sign is compulsory to put in here, which takes into account the Grassmann nature of the quark field operator.
its QCD counterpart, and the reduced QED amplitude is

\[ A_{\text{em}} = e_c e_b^2 e^3 N_c \sqrt{ \frac{m_c}{m_b} } \frac{\psi_{\eta_b}(0) \psi_{J/\psi}(0)}{m_b^2 (m_b^2 - m_c^2)} . \] (14)

Substituting (7) and (14) into the formula

\[ \Gamma[\eta_b \to J/\psi\gamma] = \frac{|k|^3}{4 \pi} |A_{\text{str}} + A_{\text{em}}|^2 , \] (15)

we then obtain the desired partial width. Here $|k| = (m_b^2 - m_c^2)/m_b$ is the photon momentum in the $\eta_b$ rest frame. This formula already takes into account the sum over transverse polarizations of both $J/\psi$ and $\gamma$.

For phenomenological purpose, it is instead more convenient to have an expression for the branching ratio, where $\psi_{\eta_b}(0)$ drops out:

\[ \text{Br}[\eta_b \to J/\psi\gamma] = \frac{8 e_c^2 \alpha_s \alpha^2}{3 \pi} \frac{m_c}{m_b} \frac{\psi_{J/\psi}(0)}{m_b^2 (m_b^2 - m_c^2)} \left| f\left(\frac{m_c^2}{m_b^2}\right) - g\left(\frac{m_c^2}{m_b^2}\right)\right|^2 , \] (16)

where

\[ g(u) = \frac{9 \pi e_b^2 \alpha_s}{2 \alpha_s^2} \frac{1 - u}{u} \] (17)

encodes the electromagnetic contribution. In deriving this, we have approximated the total width of $\eta_b$ by its gluonic width:

\[ \Gamma_{\text{tot}}[\eta_b] \approx \Gamma[\eta_b \to gg] = \frac{8 \pi \alpha_s^2}{3 m_b^2} \psi_{\eta_b}(0) , \] (18)

where the LO expression in $\alpha_s$ and $v_b$ is used for simplicity.

Eq. (16) constitutes the key formula of this work. This equation conveys that, despite the adversity caused by the suppression $\alpha/\alpha_s$, the QED fragmentation contribution nevertheless enjoys the kinematic enhancement of $m_b^2/m_c^2$ relative to the QCD amplitude. For the physical masses of $b$ and $c$, these two competing effects have comparable magnitudes. In the asymptotic regime as $m_b/m_c \gg 1$, the QED amplitude will eventually dominate over its QCD counterpart, because the enhancement factor $m_b^2/m_c^2$ of the former is much more eminent than the mild $\log(m_b^2/m_c^2)$ rising of the latter. At any rate, it is imperative to include the QED fragmentation contribution. Moreover, it is important to recognize that the interference between these two amplitudes is destructive. This is opposite to what occurs in the analogous decay process $\Upsilon \to \eta_c \gamma$[21] and in the decay $\eta_b \to J/\psi J/\psi$[13], where the interference is constructive.
III. PHENOMENOLOGY

A. Observation Potential of $\eta_b \to J/\psi\gamma$ in Tevatron and LHC

It is now the time to explore the phenomenological implication of (16). The input parameters are $m_b, m_c, \alpha, \alpha_s$ and $\psi_{J/\psi}(0)$, all of which can be inferred from other independent sources. The wave function at the origin for $J/\psi$ can be extracted from its electronic width:

$$
\Gamma[J/\psi \to e^+e^-] = \frac{4\pi e^2 \alpha^2}{m_c^2} \psi_{J/\psi}^2(0),
$$

(19)

where the LO formula in $\alpha_s$ and $v_c^2$ is used for simplicity. Using the measured dielectron width 5.55 keV [23], we obtain $\psi_{J/\psi}(0) = 0.205$ GeV$^{3/2}$ for $m_c = 1.5$ GeV. Taking $m_b = M_{\eta_b}/2 \approx 4.7$ GeV, $m_c = 1.5$ GeV, $\alpha = 1/137$ and $\alpha_s(m_b) = 0.22$, we then find

$$
\text{Br}[\eta_b \to J/\psi \gamma] = (1.5 \pm 0.8) \times 10^{-7}.
$$

(20)

The uncertainty is estimated by varying $\alpha_s(\mu)$ between 0.18 and 0.26 (which corresponds to slide the scale from $\mu = 2m_b$ to $2m_c$), as well as taking into account the errors in the measured $\Gamma_{e^+e^-}$ (of $\pm 0.14$ keV). The destructive interference between electromagnetic and strong amplitudes has pronounced effect. For the central values of input parameters, omitting the QED contribution will result in a prediction of $3.5 \times 10^{-7}$, which is more than twice larger than the actual value. To develop a concrete perception, we enumerate the values of $f$ and $g$ evaluated at $u = m_c^2/m_b^2 = 0.10$ and $\alpha_s = 0.22$:

$$
f = 3.5 - 2.4i, \quad g = 2.1.
$$

(21)

Clearly, $\text{Re} f$, $\text{Im} f$ and $g$ all have comparable magnitudes. As a result, the destructive interference effect is particularly important.

The numerical prediction presented in (20) is obtained by only using the tree-level matching coefficients for the total $\eta_b$ width and leptonic width of $J/\psi$. One may worry that this oversimplified procedure will induce some error because it is known that the next-to-leading perturbative corrections to both quantities, especially to the $J/\psi$ leptonic width, are large. Let us assess their effects now. To the NLO accuracy in $\alpha_s$, one needs to multiply Eq. (18) by

$$
1 + \left(\frac{53}{2} - 31 \pi^2/24 - 8 n_f/9\right) \alpha_s(2m_b)/\pi \quad (n_f \text{ stands for the number of active light flavors}),
$$

as well as multiply Eq. (19) by $(1 - 8\alpha_s(2m_c)/3\pi)^2$ [17]. Incorporating these corrections
amounts to multiplying (20) by a factor

\[ 1 + \frac{10.2 \alpha_s(2m_b)}{\pi} \]^{-1} \left( 1 - \frac{8\alpha_s(2m_c)}{3\pi} \right)^{-2} = 1.04, \tag{22} \]

where \( n_f = 4 \) has been taken. Since including the perturbative radiative corrections has negligible net impact, we will keep using (20) in the following phenomenological analysis.

It is enlightening to compare (20) with the NRQCD prediction to the branching ratio for \( \eta_b \) decays to double \( J/\psi \) [13]:

\[ \text{Br}[\eta_b \to J/\psi J/\psi] = 2.4^{+4.2}_{-1.9} \times 10^{-8}. \tag{23} \]

Notice that the branching ratio of our radiative decay process is almost one order-of-magnitude larger than that of this hadronic decay process! The reason can be traced as follows. The double \( J/\psi \) decay mode, though being a hadronic one, has maximally violated the helicity selection rule of pQCD. As a result, the branching ratio gets severely suppressed, \( \propto \alpha_s^2 v_c^{10} (m_c/m_b)^8 \) [13]. In contrast, if we count \( f \sim O(1) \) 3, Eq. (16) then implies that our radiative decay process admits a scaling behavior \( \text{Br} \sim \alpha \alpha_s^2 v_c^3 (m_c/m_b)^4 \), which is much more mildly suppressed by powers of \( 1/m_b^2 \) and \( v_c \) relative to \( \eta_b \to J/\psi J/\psi \), hence is more favorable.

Experimentally \( J/\psi \) can be cleanly reconstructed by decays to lepton pairs. Multiplying (20) by the branching ratios of 12% for \( J/\psi \) decays to \( \mu^+ \mu^- \) and \( e^+e^- \), we obtain \( \text{Br}[\eta_b \to J/\psi \gamma \to l^+l^-\gamma] = (0.8 - 2.8) \times 10^{-8} \). The total cross section for producing \( \eta_b \) at Tevatron has been estimated to be about 2.5 \( \mu b \) [12]. The production cross section for the \( l^+l^-\gamma \) events is thus about 0.02 – 0.07 pb. For the full Run 1 data of 100 pb\(^{-1} \), we then obtain between 2 and 7 produced events. Because the kinematical cuts, as well as taking into account the acceptance and efficiency for detecting leptons, will further cut down this number, it seems not so fruitful to assiduously seek the \( \eta_b \) through this radiative decay mode in Run 1 data sample.

Tevatron Run 2 aims to achieve an integrated luminosity of 8.5 fb\(^{-1} \) by 2009. Assuming equal \( \sigma(p\bar{p} \to \eta_b + X) \) at \( \sqrt{s} = 1.96 \) and 1.8 TeV, we then estimate there are about 200-600

\(^2\) It should be kept in mind that the relativistic correction to leptonic decay of \( J/\psi \) is also large. We have not considered this complication.

\(^3\) The slow rising \( \ln(m_b^2/m_c^2) \) term in \( \text{Ref} \) is of no concern here for physical \( b \) and \( c \) masses. Also we neglect the pure QED contribution for the lucidity of the argument.
produced events. The product of acceptance and efficiency for detecting $J/\psi$ decay to muon pair is estimated to be $\epsilon \approx 0.1$ [10]. It may sound reasonable to assume the corresponding factor for the electron pair also of the same magnitude. Multiplying the number of the produced events by $\epsilon$, we expect between 20 and 60 observed events in the full Run 2 period. This is quite encouraging, but we should be cautious about the fact that the major QCD background events, the associated $J/\psi + \gamma$ production, could also be copious. At the Tevatron and LHC colliders, the dominant production mechanism for these events is through $gg$ fusion [14, 15, 16]. A naive analysis indicates that the direct $J/\psi + \gamma$ production with an invariant mass near 9.4 GeV preponderates over that from the $\eta_b$ decays.

It is important to keep in mind that the $J/\psi$ stemming from the radiative $\eta_b$ decay must be transversely polarized. This characteristic may be used to discriminate the signal events from the background events, because $J/\psi$ from the latter processes can also be longitudinally polarized. Furthermore, the detailed kinematical distributions of background processes need to be thoroughly studied, in order to guide experimentalists to choose the optimal kinematical cuts to oppress as many background as possible, and in the meanwhile without significantly sacrificing the signal events. This is somewhat beyond the scope of this work and needs further independent studies.

The forthcoming LHC experiments will greatly increase the number of the produced $\eta_b \to l^+l^-\gamma$ events. To assess the discovery potential of this mode at LHC, we need first know the inclusive production rate for $\eta_b$. There are rough estimates for the $\chi_{b0,2}$ cross sections at LHC, which are about 6 times larger than the corresponding ones for producing them at Tevatron [24]. Assuming the same scaling also holds for $\eta_b$, we then obtain the cross section for $\eta_b$ at LHC to be about 15 $\mu$b, subsequently the production cross section for the $l^+l^-\gamma$ events to be about 0.1-0.4 pb. For 300 fb$^{-1}$ data, which is expected to be collected in one year run at LHC design luminosity, the number of produced events may reach about $3 \times 10^4 - 1 \times 10^5$. Multiplying the number of the produced events by $\epsilon$, we expect between $3 \times 10^3$ and $1 \times 10^4$ observed events per year.

Based on this analysis, we are tempted to conclude that, the chance of observing $\eta_b$ at LHC through this mode is very promising. With such a large amount of signal events, it is possible to measure the leptonic angular distribution to pin down the polarization of $J/\psi$. As has been stressed, in order to effectively selected the signal events out of the abundant background events, one needs to develop a thorough understanding towards the
QCD background.

B. Radiative decay of $\eta_b$ ($\eta_c$) into $\phi$

When coping with light mesons in hard exclusive processes, the most natural treatment for them is using light-cone wave functions. Indeed, heavy quarkonium radiative decays to light mesons, such as $\Upsilon \to f_2(1270)\gamma$ [20, 25] (Ref. [20] has heavily exploited the EFT machinery, i.e. NRQCD combined with SCET), $J/\psi(\Upsilon) \to \eta(\eta')\gamma$ [26, 27], have been studied along this line. On the other hand, the constituent quark model, which treats the light meson as a non-relativistic bound state, is also frequently invoked as an alternative method for a quick order-of-magnitude estimate. Various radiative decay processes, e.g. $J/\psi$ decays into light pseudo-scalar and $P$-wave mesons, as well as $\chi_{cJ}$ into light vector mesons, have already been studied in this context [22, 28].

We may apply the same strategy to analyze the radiative decay $\eta_b(\eta_c) \to V\gamma$. We take $\eta_b \to \phi\gamma$ as a representative. By regarding $\phi$ as a strangenium, we can directly use (16), only with some trivial changes of input parameters. We take the $m_s = M_\phi/2 \approx 0.5$ GeV. The wave function at the origin of $\phi$, $\psi_\phi(0)$, can be extracted through (19) from its measured electronic width of $1.27 \pm 0.04$ keV [23]. Taking $m_b = 4.7$ GeV, varying the strong coupling constant between $\alpha_s(2m_b) = 0.18$ and $\alpha_s(2m_s) = 0.51$, and including the experimental uncertainty in $\Gamma_{e^+e^-}$, we obtain

$$\text{Br}[\eta_b \to \phi\gamma] = (0.3 - 6.9) \times 10^{-8}.$$  \hfill (24)

The QED contribution dominates in this case due to the larger ratio of $m_b$ to $m_s$. To see this concretely, we list the values of $f$ and $g$ evaluated at $u = m_s^2/m_b^2 \approx 0.01$ and $\alpha_s = 0.22$:

$$f = 4.9 - 1.5 i, \quad g = 23.5.$$  \hfill (25)

The dominance of $g$ over $|f|$ is apparent. Neglecting QED contribution for this $\alpha_s$ value will decrease the branching fraction by one order of magnitude. For the absence of clean signature for $\phi$, such a rare decay mode will be extremely difficult to observe in hadron collision experiments such as LHC. It may be interesting to compare this radiative decay channel with the following hadronic mode [13]:

$$\text{Br}[\eta_b \to \phi\phi] = (0.9 - 1.4) \times 10^{-9}.$$  \hfill (26)
This $2\phi$ mode is even smaller, and hopeless to be seen experimentally.

One can proceed to consider the analogous decay process $\eta_c \rightarrow \phi \gamma$. Parallel to the preceding analysis, taking $m_c = 1.5$ GeV, and varying $\alpha_s(\mu)$ from $\alpha_s(2m_c) = 0.26$ to $\alpha_s(2m_s) = 0.51$, we obtain

$$\text{Br}[\eta_c \rightarrow \phi \gamma] = (2.1 - 8.6) \times 10^{-7}. \quad (27)$$

The destructive interference pattern is similar to that in $\eta_b \rightarrow J/\psi \gamma$. This decay mode is still too much suppressed to be observed in the functioning charmonium factory like BES II, perhaps also in the forthcoming BES III experiment. It seems also rather difficult to observe this decay mode in the current and future hadron collision facilities.

IV. SUMMARY

The motif of this work is to suggest one viable way to ferret out $\eta_b$ in the functioning and forthcoming hadron collider facilities, that is, through its radiative decay into $J/\psi$. This decay mode owns the advantage that both $J/\psi$ and photon can be tagged cleanly. The presence of $J/\psi$ is particularly helpful to reduce the combinatorial background. In this regard, this mode is practically much more useful than the purely electromagnetic decay $\eta_b \rightarrow \gamma \gamma$.

By an explicit pQCD calculation based on NRQCD approach, we infer the branching ratio of this process to be of order $10^{-7}$. Although the absolute value of this ratio is small, it is already larger than that of the hadronic decay mode $\eta_b \rightarrow J/\psi J/\psi$, which was previously thought of as a golden mode for searching $\eta_b$. Our analysis indicates that the chance of observing $\eta_b$ through this decay channel, followed by $J/\psi$ decay to a lepton pair, with a corresponding branching ratio about $10^{-8}$, seems still open in Tevatron Run 2, and is quite promising in the forthcoming LHC experiment. However, one should bear in mind that the major QCD background events are expected to greatly outnumber the desired signal events. A thorough study of the associated $J/\psi + \gamma$ production is welcome, in order to help experimentalists to impose optimal kinematical cuts to singlet out the signal events from the abundant background events. The transverse polarization of $J/\psi$ in the signal events should be employed to effectively veto the background.

While this paper is being written, we are informed of the same $\eta_b \rightarrow J/\psi \gamma$ process
being also considered by Gao, Zhang and Chao [30]. These authors evaluated loop integrals numerically. It has been checked that once the same input parameters are assumed, our numerical prediction for the branching ratio is compatible with theirs. These authors have also studied various other radiative decay channels of bottomonium to charmonium.

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APPENDIX A: DERIVING ANALYTICAL EXPRESSION FOR $f$

In this Appendix we illustrate how to reduce the one-loop four-point function in (8) to the sum of much simpler two- and three-point scalar integrals. We start with the dimensionless integral $f$

$$f \left( \frac{m_c^2}{m_b^2} \right) = \frac{8}{i \pi^2} \int d^4q \frac{(k \cdot Q) q^2 - (k \cdot q) (Q \cdot q)}{(q + Q)^2 (q - Q)^2 (q^2 + 2k \cdot q - P^2) (q^2 - 2k \cdot q - P^2)},$$

where $Q^2 = 4m_b^2$ and $P^2 = 4m_c^2$. The $+i\varepsilon$ prescription in the propagators is implicitly implied.

First note the second term in the integrand can be simplified by using the identity of fractional sum:

$$\int d^4q \frac{(k \cdot q) (Q \cdot q)}{(q + Q)^2 (q - Q)^2 (q^2 + 2k \cdot q - P^2) (q^2 - 2k \cdot q - P^2)} = \frac{1}{16} \int d^4q \left[ \frac{1}{(q - Q)^2} - \frac{1}{(q + Q)^2} \right] \left[ \frac{1}{q^2 - 2k \cdot q - P^2} - \frac{1}{q^2 + 2k \cdot q - P^2} \right],$$

which is nothing but the scalar 2-point functions, thus can be trivially worked out.

Disentangling the first term in (A1) needs slightly more labor. Since the integrand is an even function of $q$, we are free to add terms linear in $q$ in the numerator, without influencing
one can readily reproduce the analytic results shown in (9) and (10).

where the result:

\[
\int d^4q \frac{Q^2}{(q + Q)^2 (q - Q)^2 (q^2 + 2k \cdot q - P^2) (q^2 - 2k \cdot q - P^2)} = \frac{1}{Q^2 + P^2} \int d^4q \frac{(Q^2 + P^2) q^2 + 2Q^2 k \cdot q + 2P^2 Q \cdot q + Q^2 P^2 - Q^2 P^2}{(q + Q)^2 (q - Q)^2 (q^2 + 2k \cdot q - P^2) (q^2 - 2k \cdot q - P^2)}
\]

which consists of two independent 3-point scalar integrals. They can also be worked out in closed form, following the method outlined in Ref. [29].

With the aid of \((A2)\) and \((A3)\), we can decompose the original \(f\) into three pieces:

\[
f(u) = f_1(u) + f_2(u) + f_3(u), \tag{A4}
\]

where \(u \equiv P^2/Q^2 = m_\ell^2/m_\pi^2\), and

\[
f_1(u) = \frac{i}{\pi^2} \int d^4q \frac{1}{(q - Q)^2} \left[ \frac{1}{q^2 - 2k \cdot q - P^2} - \frac{1}{q^2 + 2k \cdot q - P^2} \right]
\]

\[
= \frac{2(1 - u)}{2 - u} \ln \left[ \frac{u}{2(1 - u)} \right] + 2\pi i \left( \frac{1 - u}{2 - u} \right), \tag{A5}
\]

\[
f_2(u) = \frac{8Q^2 k \cdot Q}{i\pi^2(Q^2 + P^2)} \int d^4q \frac{1}{(q + Q)^2 (q - Q)^2 (q^2 - 2k \cdot q - P^2)}
\]

\[
= -\frac{2}{1 + u} \left\{ \ln^2 2 + \frac{1}{2} \ln^2 u + \ln[2 - u] \ln \left[ \frac{u}{2(1 - u)} \right] + \ln u \ln \left[ \frac{2}{1 - u} \right] \right\} + 2 \operatorname{Li}_2[-u] + \operatorname{Li}_2 \left[ \frac{u - 1}{2u} \right] + 2 \operatorname{Li}_2 \left[ \frac{u}{2} \right] - \operatorname{Li}_2 \left[ \frac{u^2 - u}{2} \right] - \frac{2\pi i}{1 + u} \ln[2 - u], \tag{A6}
\]

\[
f_3(u) = \frac{8P^2 k \cdot Q}{i\pi^2(Q^2 + P^2)} \int d^4q \frac{1}{(q - Q)^2 (q^2 + 2k \cdot q - P^2) (q^2 - 2k \cdot q - P^2)}
\]

\[
= \frac{2u}{1 + u} \left\{ \ln \left[ \frac{u}{2 - u} \right] + \ln \left[ \frac{u}{2(1 - u)} \right] + \operatorname{Li}_2 \left[ \frac{2 - 2}{u} \right] \right\}
\]

\[
+ 2\pi i \left( \frac{u}{1 + u} \right) \ln \left[ \frac{u}{2 - u} \right]. \tag{A7}
\]

One can then readily reproduce the analytic results shown in (9) and (10).

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