Empirical Analysis of Knowledge Distillation Technique for Optimization of Quantized Deep Neural Networks

Sungho Shin, Yoonho Boo, and Wonyong Sung

1Department of Electrical and Computer Engineering
Seoul National University
Seoul, 08826 Korea
sungho.develop@gmail.com, wysung@snu.ac.kr

Abstract

Knowledge distillation (KD) is a very popular method for model size reduction. Recently, the technique is exploited for quantized deep neural networks (QDNNs) training as a way to restore the performance sacrificed by word-length reduction. KD, however, employs additional hyper-parameters, such as temperature, coefficient, and the size of teacher network for QDNN training. We analyze the effect of these hyper-parameters for QDNN optimization with KD. We find that these hyper-parameters are inter-related, and also introduce a simple and effective technique that reduces coefficient during training. With KD employing the proposed hyper-parameters, we achieve the test accuracy of 92.7% and 67.0% on Resnet20 with 2-bit ternary weights for CIFAR-10 and CIFAR-100 data sets, respectively.

1 Introduction

Deep neural networks (DNNs) are extremely important in various applications including computer vision (He et al. 2016a), speech recognition (Graves, Mohamed, and Hinton 2013), and natural language processing (Vaswani et al. 2017). However, DNNs require a lot of parameters, it is necessary to reduce the size of the model by using quantization to operate the network in an embedded system rather than a server environment. Quantized deep neural networks (QDNNs) do not degrade performance when quantized to a high bit, such as 8 bits, but the memory compression rate is low. Quantize to 1- or 2-bit can increase the memory compression rate, but severe quantization will cause huge performance degradation. To solve this problem, many QDNN papers suggest various types of quantizers or complex training algorithms (Hubara et al. 2017; Hwang and Sung 2014; Xu et al. 2018; Zhou et al. 2016; Zhou et al. 2017).

Knowledge distillation (KD) is widely used as a teacher-student training method that trains small networks using larger networks with better performance (Hinton, Vinyals, and Dean 2015). Leveraging the knowledge contained in previously trained networks has attracted a lot of attention in a variety of applications for model compression (Tang, Wang, and Zhang 2016; Song et al. 2018; Asami et al. 2017; Wang et al. 2019).

Recently, a combination of quantization and knowledge distillation has emerged as a popular solution to restore the performance of QDNN, which has fallen due to quantization errors (Mishra and Marr 2018; Polino, Pascanu, and Alistarh 2018). These papers mainly studied the macroscopic method for training QDNN with KD. A student quantized network is trained from scratch with soft loss produced by a good teacher network for QDNN training with KD (Polino, Pascanu, and Alistarh 2018). (Mishra and Marr 2018) focused on the initialization of the teacher and student models when training QDNNs with KD. More specifically, they analyzed teacher network and student network should be trained simultaneously or independently. Their results showed that the best performance was achieved when the teacher network was trained independently and the student network was fine-tuned with the soft labels produced by the teacher network.

Most of the previous studies have been conducted from the macro perspective for combining QDNN and KD. However, micro perspective research has not been conducted on how the important hyper-parameters of knowledge distillation such as temperature, coefficient and size of the teacher network affect the performance of QDNN. In this paper, we analyze how KD’s hyper-parameters can be operated to achieve good performance while using macroscopic training mechanism (e.g. pretraining teacher and student networks respectively and fine-tuning student network only using KD). Based on the analysis results, we propose the coefficient reducing (CR) technique. CR is a simple training technique for KD training to improve the performance of QDNNs dramatically. The contributions of this paper are as follows:

- We analyze how KD’s hyper-parameters influence each other and how QDNN’s final performance is affected.
- We show how well-used hyper-parameters can improve QDNN performance dramatically.
- Based on the analysis results, we proposed a simple training technique, coefficient reducing, for KD and obtained higher performance than the previous studies.

This paper is organized as follows. We firstly introduce related works in Section 2. Section 3 describes how the QDNN can be trained with KD and simultaneously explain why the
hyper-parameters of KD are important in QDNN training. Section I show the experimental results and we conclude the paper in Section II.

2 Related Works

In this section, we discuss related literature in knowledge distillation and quantized neural network.

2.1 Quantization of Deep Neural Networks

QDNN has been researched for a long time. Hwang and Sung (2014) and Courbariaux, Bengio, and David (2015) are suggested training methods in quantization domain to restore the performance that reduced by the quantization error. The gradient is usually smaller than the quantization scale factor, they maintain full-precision weights to accumulate the gradients while the quantized weights are used for computing forward and backward propagation (Hung and Sung 2014). Courbariaux, Bengio, and David (2015) trained jointly the same size of the full-precision teacher network and quantized student network simultaneously. They defined guidance loss that minimizes the $l_2$ norm between the same located hidden layer in teacher and student network (Mishra and Marr 2018). Zhuang et al. (2016) Zhou et al. (2017) Hubara et al. (2017) Xu et al. (2018).

Several techniques have been introduced to optimize QDNNs, by adopting elaborate quantization techniques, which include data distribution (Zhou et al. 2017), stochastic rounding (Gupta et al. 2015), weight cluster (Park, Ahn, and Yoo 2017), trainable quantization (Zhang et al. 2018), online quantization function (Car et al. 2017), power-termination (Ott et al. 2016), parameter-dependent adaptive threshold (He et al. 2016), increasing the size of the neural network (Kapur, Mishra, and Marr 2017), or quantization interval learning (Jung et al. 2019).

2.2 Knowledge Distillation

Knowledge distillation is popular model compression method that transfers the knowledge from accurate large teacher models to small student model (Hinton, Vinyals, and Dean 2015). Bucilu, Caruana, and Niculescu-Mizil (2006). The promising performance improvement of the knowledge distillation technique, it utilizes in variety of applications (Chebotar and Waters 2016). Dai and Van Gool (2018) Chen et al. (2017) Oord et al. (2017) and learning algorithms (Romero et al. 2014). Kulkarni, Patil, and Karande (2017) Park et al. (2019) Yim et al. (2017).

2.3 Knowledge Distillation with QDNN

Recently, several papers have begun to employ knowledge distillation to restore the performance loss of quantized deep neural networks. Zhang et al. (2017) Polino, Pascaru, and Alistarh (2018) Mishra and Marr (2018). Zhang et al. (2018) trained jointly the same size of the full-precision teacher network and quantized student network simultaneously. They defined guidance loss that minimizes the $l_2$ norm between the same located hidden layer in teacher and student model. Mishra and Marr (2018) proposed three methods to find out how to train QDNN effectively with KD. The first scheme is training the teacher and student network jointly. Second scheme trains only the quantized student network with pretrained teacher networks using KD. The last scheme is both networks are pretrained independently, and only fine-tunes the student network using KD. Polino, Pascaru, and Alistarh (2018) also suggests two methods to combine QDNN and KD which include quantized distillation and differentiable quantization.

The previous literature mainly concentrates on the way to combine QDNN training and Knowledge distillation. Unlike the previous studies, however, we focus on improving QDNN’s performance using KD with a microscopic point of view. We concentrate on analyze the hyper-parameters of KD that affects the accuracy of QDNN significantly.

3 Quantized Deep Neural Network Training Using Knowledge Distillation

In this section, we first briefly describe the conventional neural network quantization method and also explain how QDNN training can be combined with KD. We also present the hyper-parameters of KD and state why they are very important in QDNN training.

3.1 Quantization of Deep Neural Network & Knowledge Distillation

The deep neural network parameter vector, $w$, can be expressed in $2^b$ level when quantized in $b$-bit. This can be generalized to Equation (1) and Equation (2) for the case of $b = 1$ and $b > 1$ through the symmetric uniform quantization function $Q^b(\cdot)$ as follows:

$$Q^b(w) = \text{Binarize}(w) = \Delta \cdot \text{sign}(w)$$

$$Q^b(w) = \text{sign}(w) \cdot \Delta \cdot \min\left\{\left\lfloor \frac{|w|}{\Delta} + 0.5 \right\rfloor, \frac{(M - 1)}{2}\right\}$$

where $M$ is quantization level $2^b - 1$ and $\Delta$ represents quantization step size. $\Delta$ can be computed by L2-error minimization between floating and fixed-point weights or by the standard deviation of the weight vector (Hwang and Sung 2014). Rastegari et al. (2016). Zhou et al. (2016).

Severe quantization such as 1- or 2-bit causes huge performance degradation. Retraining on quantization domain is very important to recover the performance degradation (Sung, Shin, and Hwang 2015). When retraining the student network on quantization domain, forward, backward, and gradient computations should be computed using quantized weights but the computed gradients must be added to full-precision weights (Hubara et al. 2017). Hwang and Sung (2014). Xu et al. (2018). Zhou et al. (2016) Zhou et al. (2017).

In many cases, deep neural networks generate probabilities with the softmax layer. Logit, $z$, is fed into the softmax layer and generates the probability of each class, $p$, using $p_i = \frac{\exp(z_i/\tau)}{\sum_j \exp(z_j/\tau)}$. $\tau$ is a hyper-parameter of KD as known as 'temperature'. A high value of $\tau$ generates soften probability distribution. KD employs the probability generated by the teacher network as a soft label to train the student network so that the following loss function is minimized during training.
Therefore, QDNN is to be more sensitive to the same number of parameters as the network with full precision of a quantized network is much smaller, even with the representation level of the weight parameter, the capacity of a network is too small to simulate the incoming information. This phenomenon is because the model capacity of student networks becomes too large.

Table 1: Training results of full-precision and 2-bit quantized ResNet20 on CIFAR-10 and CIFAR-100 dataset in terms of accuracy (%). The models are trained with hard label only.

|                | CIFAR-10 | CIFAR-100 |
|----------------|----------|-----------|
|                | Train acc. | Test acc. | Train acc. | Test acc. |
| Full-precision | 99.62     | 92.63     | 90.12     | 68.43     |
| 2-bit quantized| 98.92     | 91.48     | 98.92     | 91.48     |

\[
\mathcal{L}(w_S) = (1 - \lambda)\mathcal{H}(y, p^S) + \lambda \mathcal{H}(p^T, p^S) \tag{3}
\]

\(\mathcal{H}(\cdot)\) denotes a loss function, \(y\) is the ground truth hard label, \(w_S\) is weight vector of student network, \(p^T\) and \(p^S\) are the probability of teacher and student network, and \(\lambda\) is a coefficient for adjusting the ratio of soft and hard target.

A recent paper [Mirzadeh et al. (2019)] reports that the performance is gradually decreased when the size difference between the teacher and student networks becomes too large. This phenomenon is because the model capacity of student network is too small to simulate the incoming information (i.e., soft target from teacher network). Since QDNN limits the representation level of the weight parameter, the capacity of a quantized network is much smaller, even with the same number of parameters as the network with full precision. Therefore, QDNN is to be more sensitive to the size of the teacher network.

We consider the three hyper-parameters described above (temperature \((\tau)\), coefficient \((\lambda)\), and size of teacher network) as the hyper-parameters that have a significant impact on the performance on QDNN training with KD. Algorithm 1 shows how these three hyper-parameters play a role in QDNN training using KD.

### 3.2 Discussion on Hyper-parameters of KD

Previous papers that trained QDNNs using KD mainly focused on finding a macroscopic method of how to apply KDs to QDNNs [Mishra and Marr (2018); Polino, Pascanu, and Alistarh (2018)]. At present, the best known QDNN training with KD method is that firstly train full-precision teacher and a full-precision student network independently, and apply KD when fine-tuning the student network in quantization domain. We agree that the above method is the best way to train a QDNN with KD, so we also proceed with the QDNN training in the same way for all experiments. However, it is still not fully studied how the hyper-parameters of KD should be applied to QDNN training.

As we mentioned in Section 3.1, the hyper-parameters temperature \((\tau)\), coefficient \((\lambda)\), and size of teacher network can significantly impact on QDNN performance. Existing papers usually fixed these hyperparameters when training QDNN with KD. For example, [Mishra and Marr (2018)] always fixes \(\tau\) to 1, and [Polino, Pascanu, and Alistarh (2018)] holds it to 1 or 5 depending on the dataset. However, these three parameters are closely inter-related. For example, [Mirzadeh et al. (2019)] points out that when the teacher model is very large compared to the student model, the softer labels produced by the teacher network become sharper, making it difficult to transfer the knowledge of the teacher network to the student. However, even in this case, fine-control of temperature may be able to transfer the knowledge. Therefore, when the value of one hyper-parameter is changed the others are also needed to fine-tune carefully.

We can also employ KD to obtain a better pretrained full-precision student network before applying to retrain in quantization domain with KD. In general, if the pretrained full-precision model has high accuracy, the quantized model ob-

### Algorithm 1: QDNN training with KD

**Initialization**: \(w_T\): Pretrained teacher model, \(w_S\): Pretrained student model, \(\lambda\): Coefficient, \(\tau\): Temperature

**Output**: \(w_S^q\): Quantized student model

**while not converged**

\[ w_S^q = \text{Quant}(w_S) \]

Run forward teacher (\(w_T\)) and student model (\(w_S^q\))

Compute distillation loss \(L(w_S^q)\)

Run backward and compute gradients \(\frac{\partial L(w_S^q)}{\partial w_S^q}\)

\[ w_S = w_S - \eta \cdot \nabla L(w_S) \]

**end**

**Return** \(w_S^q\)
Table 4: Results of QDNN training with KD on ResNet-20 for CIFAR-10 and CIFAR-100 dataset.

| CIFAR-10  | Teacher (full-precision) | Student (2-bit) | 
|-----------|--------------------------|-----------------| 
|           | # of params (M) (network name) | Test acc. (%) | # of params (M) (network name) | Test acc. (%) | τ | λ | 
| QDistill  | 5.3 (small network) | 89.7 | 0.1 (student model 1) | 67.0 | 5 | 0.5 | 
|           |                       |               | 0.3 (student model 2) | 74.2 | 5 | 0.5 | 
|           |                       |               | 1.0 (student model 3) | 82.4 | 5 | 0.5 | 
|           |                       |               | 5.8 (Deeper student) | 89.3 | 5 | 0.5 | 
| Apprentice | 145 (WideResNet28*20) | 95.7 | 82.7 (WideResNet22*16) | 94.23 | 5 | 0.5 | 
| Ours      | 0.? (WideResNet20*1.5) | 93.5 | 0.27 (ResNet20) | 92.52 | 10 | 0.5 | 
|           | 0.? (WideResNet20*1.2) | 92.9 | 0.27 (ResNet20) 1-bit | 91.3 | 3 | 0.5 | 
| CIFAR-100 | Teacher (full-precision) | Student (2-bit) | 
|           | # of params (M) (network name) | Test acc. (%) | # of params (M) (network name) | Test acc. (%) | τ | λ | 
| QDistill  | 36.5 (WideResNet28*10) | 77.2 | 17.2 (WideResNet22*8) | 49.3 | 5 | 0.5 | 
| Guided    | 22.0 (AlexNet) | 65.4 | 22.0 (AlexNet) | 64.6 | - | - | 
| Ours      | 0.? (WideResNet20*1.2) | 69.64 | 0.28 (ResNet20) | 66.6 | 2 | 0.5 | 
|           | 0.? (WideResNet20*1.7) | 72.17 | 0.28 (ResNet20) | 67.0 | 3 | CR | 

4 Experimental Results

4.1 Experimental Setup

Dataset: To analyze QDNN training with KD we employ CIFAR-10 and CIFAR-100 dataset. CIFAR-10 and CIFAR-100 consist of 10 and 100 classes, respectively. Both datasets contain 50K training images and 10K testing images. Thus the CIFAR-10 has 5000 images per class and CIFAR-100 include 500 images per class. The size of each image is 32x32 with RGB channels.

Model Configuration & Training Hyper-parameter: To analyze the impact of hyper-parameters on QDNN training, we train WideResNet20xN [Zagoruyko and Komodakis 2016] as the teacher networks. We set N to 1, 1.2, 1.5, 1.7, 2, 3, 4, 5, and 10. It should be noted that when N is 1, the network structure is the same with ResNet20 [He et al. 2016a]. All the train and test accuracies of teacher networks on CIFAR-10 and CIFAR-100 datasets are reported in Table 2 and Table 3, respectively. We employ ResNet20 as the student network for both the CIFAR-10 and CIFAR-100 datasets. If the network size is large enough
against the amount of the dataset, the accuracy drop due to the quantization error is reduced. So any quantization method seems to be worked well. Therefore, to evaluate the performance of the quantization algorithm, the best choice might be employing a small network which is located in the under-parameterized region (Sung, Shin, and Hwang 2015). Full-precision ResNet20 model is located in the over-parameterized region and the model turns down to the under-parameterized region when employing severe quantization on CIFAR-10 dataset. Likewise, on the CIFAR-100 dataset, both the full-precision and the quantized model are located in the under-parameterized area, so it is the good network configuration to evaluate the effect of KD QDNN training. It should be noted that one way to determine that the network is lying at under- or over-parametrized is check the train accuracy. If the train accuracy is not reached almost 100%, the model may be located in under-parameterized region. We reports the train and test accuracy for ResNet20 for CIFAR-10 and CIFAR-100 in Table 1.

4.2 Results

We firstly show that the importance of the hyper-parameters of KD when training QDNN in Table 1. The compared QDNNs model that is trained with KD include QDistill (Polino, Pascanu, and Alistarh 2018), Teacher (Mishra and Marr 2018), and Guided (Zhuang et al. 2018). We achieve the results that significantly exceed those of previous QDNNs trained with KDs. For QDistill, comparing our model (0.27M) with ‘student model 2’ that includes 0.3M parameters achieve an 18.32% of difference gap in test accuracy. Also, it was about 1% better than the ResNet20 results reported by Apprentice and even achieved the same performance with their ResNet32 result. Even when quantized to 1-bit, the performance was 91.3%, which is almost the same as Apprentice’s ResNet20 2-bit model. Even in the case of CIFAR-100, the numbers of parameters of QDistil and Guided student models are 17.2M and 22.0M, only 0.28M parameters of our model can achieve higher than 17.3% and 2.4% test accuracy, respectively. This huge performance difference shows how important the hyper-parameter is in QDNN training with KD.

4.3 Model Size & Temperature

We reports the results of 2-bit ResNet20 using KD on the CIFAR-10 dataset in Figure 1(a). To demonstrate the effect of temperature for QDNN training, we train 2-bit resnet20 student model with varying the size of the teacher network from ‘WideResNet20x1’ to ‘WideResNet20x5’. Each experiment is repeated for three τ values (small, medium, and large). It should be noted that WideResNet20xN means that the number of channel maps is increased by N times. When τ is small (blue line in the figure), its performance increases and decreases rapidly as following the x-axis. This steep slope becomes soft as the value of τ increase to medium (orange line) or large (blue line). The reason for this is related to the accuracy (red line) of the teacher model. The larger the teacher model, the higher the confidence for the correct answer. In other words, for the same input image, the small teacher model can be sure that the answer has 80% of the confidence, while the large teacher model can be sure it has 99.9% of the confidence. That is, a shape of the soft label may become similar to a ground truth hard label. In this case, even though employ KD, the results that are not very different from training with hard labels. Therefore, if τ is 1, the performance decreases to 91.9% when the teacher network becomes larger than WideResNet20 * 2. This is similar to 91.48 % of the performance of a 2-bit ResNet20 trained on hard labels. Increasing τ larger value such as 5 or 10, the sharp soft labels produced by the large teacher network can become softened, and it aids to improve the QDNN performance very much. This phenomenon can occur when training a full-precision model with KD, but it is more important when training QDNN considering the model capacity of the student network, which is lowered due to quantization. Therefore, when training QDNN with KD, it should be considered carefully the relationship between size of teacher model and temperature.
Figure 2: Results of 2-bit ResNet20 models that trained by various size of teacher networks and temperature ($\tau$) on CIFAR-100. “HT-KD” represents the student is pretrained using by hard target and retrained using by KD. “CR” means coefficient reducing technique. In (c), the black horizontal line represents the test accuracy when quantize the network without KD.

4.4 Network Pretraining Methods

We applied KD to QDNN as suggested by Mishra and Marr (2018). The method is first train teacher and student network in full-precision independently, and then fine-tune the student network in low-bit using with the soft label generated by the teacher network. Thus, there are two options for pre-training the full-precision student network. The first is training with a hard target which is a conventional way to train deep neural networks and the second is also employing KD for full-precision student network training. The results are reported in Figure 2(a) and (b). Figure 2(a) represents that the full-precision student network is pretrained by the hard target and the quantized student network is retrained using by KD. Figure 2(b) shows both the full-precision and quantized student networks are trained using by KD. We run both experiments with varying temperature ($\tau$) from 1 to 50 and the size of teacher network ($N$) from ResNet20 to WideResNet20x10. The results clearly show that employing KD for both full-precision and quantized student networks can increase the performance very much.

4.5 Coefficient Reducing

Throughout the paper, we have discussed the effects of temperature and size of teacher network on QDNN training with KD. Since the two hyper-parameters are inter-correlated with each other, careful fine-tuning is required, which can be challenging when training the QDNN with KD. Therefore, it requires a way to alleviate this exhaustive search. In general, when a well-trained teacher network gives hints (soft label) to the student network, the performance of the student network can be able to increase. Strictly speaking, however, soft labels are the answer expected by the teacher network, and it might not be the absolute answer. Therefore, the following method can be considered. At the beginning of the training, where the gradient changes a lot, use the soft label and the hard label half and half and gradually reduce the amount of hints provided by the teacher network as the training progresses. We name this simple method as coefficient reducing (CR) technique and use it for QDNN training with KD. To evaluate the effectiveness of the CR in QDNN training with KD, we applied the same experiment as shown in Figure 2(b) with CR and report the results in Figure 2(c). The results clearly show that CR greatly aids to improve the performance of the QDNN training with KD. In almost of the hyper-parameters setup, the results with CR improved the performance significantly. The important point is that in case of performance that is not working well according to too large size of teacher network or temperature in KD-KD, the performance also improves or at least ties with the results that trained without KD (black horizontal line in Figure 2(c)). This is because coefficient ($\lambda$) is a hyper-parameter that can control the applying degree of size of teacher network and temperature as shown in Equation eqref3. Therefore, CR is a promising technique that can prevent performance degradation (at least as similar to the results that trained without KD) even if an inappropriate hyper-parameters are selected.

4.6 Ensemble of Multiple Teacher Networks

Many papers that related to knowledge distillation are often used to train a student network by averaged the soft label
Table 5: Test accuracy of QDNN training with KD on ResNet-20 for CIFAR-10 dataset. ‘Mix’ represents that all the teacher models are employed to generate the ensemble of the soft targets. For the number of teacher 1 and 5, we use the WideResNet20x1.5 which shows the best results in our QDNN training with KD.

| Teacher       | 1 | 5 | Mix |
|---------------|---|---|-----|
| $\tau = 1$    | 91.78 | 92.24 | 91.9 |
| $\tau = 2$    | 92.08 | 92.10 | 92.06 |
| $\tau = 3$    | 92.56 | 92.40 | 92.27 |
| $\tau = 5$    | 92.05 | 92.69 | 92.61 |
| $\tau = 10$   | 92.67 | 92.52 | 92.63 |
| $\tau = 20$   | 92.41 | 92.53 | 92.46 |

from multiple cumbersome models ( ). We can also consider the ensemble of multiple teachers in QDNN training with KD. The performance of the teacher networks used in the ensemble experiment is reported in Table 2. As the model gradually widened, the performance is continually increasing and saturated at 95.24% on WideResNet20x5. Quantizing the student network by using the ensemble of multiple teacher networks is shown in Table 5. In the case of training with one Teacher model, the highest performance is 92.67% with a temperature of 10 and the ensemble of five teachers shows 92.69% with a temperature of 5. Even with the ensemble of multiple teacher models, the best performance is similar to the result that trained with single teacher model. It means that if $\tau$ is adjusted properly, the number of teacher model may not be that important. We also employ the multiple teacher models that have different size each other, but the student performance is not that much different. Therefore, considering computational efficiency, it is better to use only one teacher network to train the quantized student network with careful temperature selection.

## 5 Concluding Remarks

In this study, we investigate the impact of the hyper-parameters in quantized deep neural networks training with knowledge distillation. We found that the three hyper-parameters (temperature, coefficient, and size of the teacher network) are closely inter-related. When the size of the teacher network is growing, increasing temperature aids to boost performance. However, if the temperature is increased too much, the knowledge from the teacher network can be disappeared. We also introduce a simple training technique, coefficient reducing (CR), for quantized deep neural networks training with KD. At the beginning of the training, CR keeps the rate of the hard target and the soft target equally, but gradually reduces the rate of the soft target so that the KD loss become the conventional loss function that employing only hard target at the end of the training. With careful hyper-parameter selection and coefficient reducing technique, we achieve far exceed performances than the previous studies for the 2-bit quantized deep neural networks on CIFAR-10 and CIFAR-100.

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