On the Effectiveness of Sparsification for Detecting the Deep Unknowns

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ABSTRACT
Detecting out-of-distribution (OOD) inputs is a central challenge for safely deploying machine learning models in the real world. Previous methods commonly rely on an OOD score derived from the overparameterized weight space, while largely overlooking the role of sparsification. In this paper, we reveal important insights that reliance on unimportant weights and units can directly attribute to the brittleness of OOD detection. To mitigate the issue, we propose a sparsification-based OOD detection framework termed DICE. Our key idea is to rank weights based on a measure of contribution, and selectively use the most salient weights to derive the output for OOD detection. We provide both empirical and theoretical insights, characterizing and explaining the mechanism by which DICE improves OOD detection. By pruning away noisy signals, DICE provably reduces the output variance for OOD data, resulting in a sharper output distribution and stronger separability from ID data. DICE establishes superior performance, reducing the FPR95 by up to 24.69% compared to the previous best method.

1 Introduction
Deep neural networks deployed in real-world systems often encounter out-of-distribution (OOD) inputs—samples from unknown classes that the network has not been exposed to during training, and therefore should not be predicted by the model in testing. Being able to estimate and mitigate OOD uncertainty is paramount for safety-critical applications such as medical diagnosis [48, 41] and autonomous driving [8]. For example, an autonomous vehicle may fail to recognize objects on the road that do not appear in its detection model’s training set, potentially leading to a crash. This gives rise to the importance of OOD detection, which allows the learner to express ignorance and take precautions in the presence of OOD data.

The main challenge in OOD detection stems from the fact that modern deep neural networks can easily produce overconfident predictions on OOD inputs, making the separation between in-distribution (ID) and OOD data a non-trivial task. The vulnerability of machine learning to OOD data can be hard-wired in high-capacity models used in practice. In particular, modern deep neural networks can overfit observed patterns in the training data [55], and worse, activate features on unfamiliar inputs [37]. To date, existing OOD detection methods commonly derive OOD statistics using overparameterized weights, while largely overlooking the role of sparsification. This paper aims to bridge the gap.

In this paper, we start by revealing key insights that reliance on unimportant units and weights can directly attribute to the brittleness of OOD detection. Empirically on a network trained with CIFAR-10, we show that an OOD image can activate a non-negligible fraction of units in the penultimate layer (see Figure 1, right). Each point on the horizontal axis corresponds to a single unit. The y-axis measures the unit contribution (i.e., weight × activation) to the class output, with the solid line and the shaded area indicating the mean and variance, respectively. Noticeably, for OOD data (gray), we observe a non-negligible fraction of “noisy” units that display high variances of contribution, which is then aggregated to the model’s output through summation. As a result, such noisy signals can undesirably manifest in model output—increasing the variance of output distribution and reducing the separability from ID data.

The above observation inspires a simple and effective method, Directed Sparsification (DICE), for OOD detection. In particular, we show that utilizing a sparse subset of weights with a significant contribution to the final logit

1Code is available at: https://github.com/deeplearning-wisc/dice.git
Figure 1: Illustration of unit contribution (i.e., weight × activation) to the class output. For class c, the output \( f_c(x) \) is the summation of unit contribution from the penultimate feature layer of a neural network. Units are sorted in the same order, based on the expectation of ID data’s contribution (averaged over many CIFAR-10 samples) on the \( x \)-axis. Shades indicate the variance for each unit. Left: For in-distribution data (CIFAR-10, airplane), only a subset of units contributes to the model output. Right: In contrast, out-of-distribution (OOD) data can trigger a non-negligible fraction of units with noisy signals, as indicated by the variances.

output can better differentiate between ID and OOD data. DICE leverages the observation that a model’s prediction for an ID class depends on only a subset of important units (and corresponding weights), as evidenced in Figure 1 (left). To exploit this, our key idea is to rank weights based on the measure of contribution, and selectively use the most significant weights to derive the output for OOD detection. Importantly, DICE can be conveniently used by \textit{post hoc} weight masking on a pre-trained network and therefore can preserve the ID classification accuracy. Orthogonal to existing works on sparsification for accelerating computation, our novel contribution is to explore the sparsification approach for improved OOD detection performance.

We provide both empirical and theoretical insights characterizing and explaining the mechanism by which DICE improves OOD detection. We perform extensive evaluations and establish superior performance on a suite of common OOD detection benchmarks, including CIFAR-10, CIFAR-100 [28], and a large-scale ImageNet dataset [6]. DICE reduces the FPR95 by 30.13% compared to the same scoring function without sparsification on ImageNet. Moreover, we perform ablation using various sparsification techniques and demonstrate the superiority of directed sparsification for OOD detection. Theoretically, by pruning away noisy signals from unimportant units and weights, DICE \textit{provably reduces the output variance} and results in a sharper output distribution (see Section 6). The sharper distributions lead to a stronger separability between ID and OOD data and overall improved OOD detection performance (\textit{c.f.} Figure 3).

Our \textbf{key results and contributions} are:

- (Methodology) We introduce DICE, a simple and effective approach for OOD detection utilizing \textit{post hoc} weight sparsification. To the best of our knowledge, DICE is the first to explore and demonstrate the effectiveness of sparsification for OOD detection.

- (Experiments) We extensively evaluate DICE on a suite of OOD detection tasks and establish superior performance. DICE outperforms the previous best baseline by a large margin, reducing the FPR95 by 24.69% on a challenging ImageNet evaluation task. We show DICE can generalize effectively to different network architectures, achieving improved OOD detection while preserving the classification accuracy.

- (Theory and ablations) We provide ablation and theoretical analysis that improves understanding of a sparsification-based method for OOD detection. Our analysis reveals an important variance reduction effect, which provably explains the effectiveness of DICE. We hope our insights inspire future research on utilizing weight sparsification for OOD detection.

2 Preliminaries and Analysis

We start by recalling the general setting of the supervised learning problem. We denote by \( \mathcal{X} = \mathbb{R}^d \) the input space and \( \mathcal{Y} = \{1, 2, ..., C\} \) the output space. A learner is given access to a set of training data \( D = \{(x_i, y_i)\}_{i=1}^N \) drawn from an unknown joint data distribution \( P \) defined on \( \mathcal{X} \times \mathcal{Y} \). Furthermore, let \( P_X \) denote the marginal probability distribution on \( \mathcal{X} \).
Problem statement OOD detection is a binary classification problem to distinguish between in- vs. out-of-distribution data. Given a classifier \( f \) learned on in-distribution \( P_X \), the goal is to design a scoring function, 
\[
g: x \rightarrow \{\text{in, out}\},
\]
that classifies whether a sample \( x \in X \) is from \( P_X \) or not.

Why noisy signals are undesirable for OOD detection? A conceptual example We consider a toy example in Figure 2, where the model’s output \( f \) is the summation of two input variables \( v_1 \) and \( v_2 \), each representing a unit. For simplicity, we assume \( v_1 \) and \( v_2 \) are independent. For OOD samples, we assume two inputs are Gaussian distributed noise signals, with \( v_1 \sim \mathcal{N}(-1, 1) \) and \( v_2 \sim \mathcal{N}(1, 1) \). The output \( f = v_1 + v_2 \) has a variance of 2 despite a zero mean:
\[
f \sim \mathcal{N}(0, 2).
\]
Importantly, the variance increases significantly with more units, which is \( \sigma_1^2 + \sigma_2^2 + \ldots + \sigma_m^2 \) under \( m \) independent variables. The larger variance can result in less separability from ID data’s output distribution (see Figure 3), and therefore potentially hinders OOD detection performance.

![Figure 2: A toy example of summation of two independent Gaussian variables has increased variance.](image)

3 Method

Our OOD uncertainty estimation framework with Directed Sparsification (DICE) is illustrated in Figure 3. In what follows, we provide a method overview and then describe directed sparsification in detail (Section 3.1). We propose a new OOD detection method with sparsification in Section 3.2.

Method overview As aforementioned, OOD detection performance can suffer from the noisy signals from the high-dimensional inputs. To mitigate this issue, our key idea is to selectively use a subset of important weights to derive the output for OOD detection. By utilizing sparsification, the network prevents adding irrelevant variables to the output. We illustrate our idea in Figure 3. Without DICE (left), the final output is a summation of weighted activations across all units, which can have a high variance for OOD data (colored in gray). In contrast, with DICE (right), the variance of output can be significantly reduced, which improves separability from ID data. We proceed with describing the mechanism that achieves our novel idea.

3.1 DICE: Directed Sparsification

We consider a deep feature extractor parameterized by \( \theta \), which encodes an input \( x \in \mathbb{R}^d \) to a feature space with dimension \( m \). We denote by \( h(x) \in \mathbb{R}^m \) the feature vector from the penultimate layer of the network. A weight matrix \( W \in \mathbb{R}^{m \times C} \) connects the feature \( h(x) \) to the output \( f(x) \).

Contribution matrix We perform a directed sparsification based on a measure of contribution, and preserve the most important weights in \( W \). To measure the contribution, we define a contribution matrix \( V \in \mathbb{R}^{m \times C} \), where each column \( v_c \in \mathbb{R}^m \) is given by:
\[
v_c = E_{x \sim D} [w_c \odot h(x)],
\]
where \( \odot \) indicates the element-wise multiplication, and \( w_c \) indicates weight vector for class \( c \). Each element in \( v_c \in \mathbb{R}^m \) intuitively measures the corresponding unit’s average contribution to class \( c \), estimated on in-distribution data \( D \). A larger value indicates a higher contribution to the output \( f_c(x) \) of class \( c \). The vector \( v_c \) is derived for all classes \( c \in \{1, 2, \ldots, C\} \), forming the contribution matrix \( V \). Each element \( v^i_c \in V \) measures the average contribution (weight \( \times \) activation) from a unit \( i \) to the output class \( c \in \{1, 2, \ldots, C\} \).

We can now select the top-k weights based on the \( k \)-largest elements in \( V \). In particular, we define a masking matrix \( M \in \mathbb{R}^{m \times C} \), which returns a matrix by setting 1 for entries corresponding to the \( k \) largest elements in \( V \) and setting other elements to zeros. The model output under contribution-directed sparsification is given by
\[
f^{\text{DICE}}(x; \theta) = (M \odot W)^\top h(x) + b,
\]
Figure 3: Illustration of out-of-distribution detection using Directed Sparsification (DICE). We consider a pre-trained neural network, which encodes an input $x$ to a feature vector $h(x) \in \mathbb{R}^m$. **Left:** The logit output $f_c(x)$ of class $c$ is a linear combination of activation from all units in the preceding layer, weighted by $w_i$. The full connection results in a high variance for OOD data’s output, as depicted in the gray. **Right:** Our proposed approach leverages a selective subset of weights, which effectively reduces the output variance for OOD data, resulting in a sharper score distribution and stronger separability from ID data. The output distributions are based on CIFAR-10 trained network, with ID class label “frog” and SVHN as OOD.

where $b \in \mathbb{R}^C$ is the bias vector. The procedure described above essentially accounts for inputs from the most relevant units in the penultimate layer. Importantly, the sparsification can be conveniently imposed by post hoc weight masking on a pre-trained network, without changing any parameterizing of the neural network. Therefore one can improve OOD detection while preserving the ID classification accuracy.

**Sparsity parameter $p$** To align with the convention in literature, we use the sparsity parameter $p = 1 - \frac{k}{m \cdot C}$ in the remainder paper. A higher $p$ indicates a larger fraction of weights dropped. When $p = 0$, the output becomes equivalent to the original output $f(x; \theta)$ using dense transformation, where $f(x; \theta) = W^\top h(x) + b$. We provide ablations on the sparsity parameter later in Section 5.

### 3.2 OOD Detection with DICE

Our method DICE in Section 3.1 can be flexibly leveraged by the downstream OOD scoring function:

$$g_\lambda(x) = \begin{cases} \text{in} & S_{\theta}(x) \geq \lambda \\ \text{out} & S_{\theta}(x) < \lambda \\ \end{cases}$$

(4)

where a thresholding mechanism is exercised to distinguish between ID and OOD during test time. The threshold $\lambda$ is typically chosen so that a high fraction of ID data (e.g., 95%) is correctly classified. Following recent work [32], we derive an energy score using the logit output $f_{\text{DICE}}(x)$ from contribution-directed sparsification. The function maps the logit outputs $f_{\text{DICE}}(x)$ to a scalar $E_{\theta}(x) \in \mathbb{R}$, which is relatively lower for ID data:

$$S_{\theta}(x) = -E_{\theta}(x) = \log \sum_{c=1}^{C} \exp(f_{\text{DICE}}(x)),$$

(5)

Note that DICE can also be compatible with alternative scoring function such as maximum softmax probability (MSP) [17], though the performance of MSP is less competitive (see Appendix G). Later in Section 6, we formally characterize and explain why DICE improves the separability of the scores between ID and OOD data.

### 4 Experiments

In this section, we evaluate our method on a suite of OOD detection tasks. In Section 4.1, we begin with a large-scale image classification network trained on ImageNet [6]. We continue with the CIFAR benchmarks [28] that are routinely used in literature (Section 4.2). A reproducibility statement is in Appendix.
Table 1: Main results. Comparison with competitive post hoc out-of-distribution detection methods. All methods are based on a discriminative model trained on ImageNet, without using any auxiliary outlier data. ↑ indicates larger values are better and ↓ indicates smaller values are better. All values are percentages. Bold numbers are superior results. § indicates model retraining using a different loss function is required. ⋆ indicates a separate binary classifier needs to be trained for OOD detection. We report standard deviations estimated across 5 independent runs.

| Methods       | OOD Datasets                  | Average |
|---------------|------------------------------|---------|
|               | INaturalist                  | SUN     | Places | Textures |
|               | FPR95 | AUROC | FPR95 | AUROC | FPR95 | AUROC | FPR95 | AUROC | FPR95 | AUROC |
| MSP [73]      | 69.59 | 81.59 | 78.98 | 78.34 | 81.44 | 78.06 | 82.73 | 74.43 | 76.96 | 79.29 |
| ODIN [31]     | 62.60 | 89.36 | 71.67 | 83.92 | 76.27 | 80.67 | 81.31 | 76.30 | 72.99 | 82.56 |
| Mahalanobis* [29] | 96.34 | 46.33 | 88.43 | 65.20 | 89.75 | 64.46 | 52.23 | 72.10 | 81.69 | 62.02 |
| Energy [32]   | 64.91 | 88.48 | 65.33 | 85.32 | 73.02 | 81.37 | 80.87 | 75.79 | 71.03 | 82.74 |
| G-ODIN [21]   | 66.36 | 84.68 | 65.93 | 84.70 | 65.39 | 85.40 | 64.68 | 80.50 | 65.59 | 83.82 |
| DICE (ours)   | 33.78 | 90.97 | 93.70 | 96.67 | 93.61 | 90.65 | 50.59 | 81.86 | 89.35 | 90.78 |

4.1 Evaluation on ImageNet

Dataset We first evaluate DICE on a large-scale ImageNet classification model. Following [23], we use four OOD test datasets from (subsets of) Places365 [56], Textures [5], iNaturalist [20], and SUN [50] with non-overlapping categories w.r.t ImageNet. The evaluations span a diverse range of domains including fine-grained images, scene images, and textural images. OOD detection for the ImageNet model is more challenging due to both a larger feature space \((m = 2,048)\) as well as a larger label space \((C = 1,000)\). In particular, the large-scale evaluation can be more relevant to real-world applications, where the deployed models often operate on images that have high resolution and contain more class labels. Moreover, as the number of feature dimensions increases, noisy signals may increase accordingly, which can corrupt the output more and make OOD detection more challenging.

Experimental details We use publicly available Google BiT-S models [27] for all methods (including baselines). This ensures the reproducibility of our results reported. The models are trained on ImageNet-1k, using ResNet-v2 architecture [15] with depth 101. At test time, all images are resized to 480 × 480. We use the entire training dataset to estimate the contribution matrix and mask M. We use a validation set of Gaussian noise images, which are generated by sampling from \(N(0, 1)\) for each pixel location. The optimal \(p\) is selected from \([0.1, 0.3, 0.5, 0.7, 0.9, 0.99]\), which is 0.9 for all datasets. We use Gaussian noise for its simplicity, easy to generate and use, and the fact that it is a clear OOD w.r.t ID data. We also show in Figure 5 using Gaussian noise can already find the near-optimal one on all three ID datasets considered. The hardware used for experiments is specified in Appendix D.

DICE achieves superior performance. In Table 1, we compare DICE with competitive OOD detection methods, where DICE outperforms the best baseline by a large margin. We report performance for each OOD test dataset as well as the average of the four. For a fair comparison, all the methods derive the OOD score from the same ImageNet pre-trained model. In particular, we compare with Maximum Softmax Probability [17], ODIN [31], Mahalanobis distance [29], Generalized ODIN [21], and Energy score [32]. Note that the only exception is Generalized ODIN [21], which requires model retraining under the DeConf-C loss. For readers’ convenience, a brief introduction of baselines and hyperparameters is provided in Appendix A.

Noticeably, DICE reduces the FPR95 by 24.69% compared to the best baseline [21]. Moreover, we contrast energy score [32] and DICE, which allows us to see the direct benefit of using sparsification under the same scoring function \(S(x)\). DICE reduces the FPR95 drastically from 71.03% to 40.90%, a 30.13% improvement using sparsification. Lastly, we observe that Mahalanobis displays limiting performance with FPR95 40.79% lower than DICE. This is likely due to the increased size of label space makes the class-conditional Gaussian density estimation less viable. In contrast, DICE is easier to use in practice, and can be implemented through simple post hoc weight masking.

ID classification accuracy Given the post hoc nature of DICE, once the input image is marked as ID, one can always use the original fully connected layer, which is guaranteed to give identical classification accuracy. This incurs minimal overhead and results in the optimal performance for both classification and OOD detection. We also measure the classification accuracy under different sparsification parameter \(p\). Full results are available in Table 7 in Appendix C.
4.2 Evaluation on Common Benchmarks

Experimental details We use CIFAR-10 [28], and CIFAR-100 [28] datasets as in-distribution data. We use the standard split with 50,000 training images and 10,000 test images. We evaluate the model on six common OOD benchmark datasets: Textures [5], SVHN [36], Places365 [56], LSUN-Crop [34], LSUN-Resize [54], and iSUN [52]. We use DenseNet-101 architecture [22] and train on in-distribution datasets. The feature dimension of the penultimate layer is 342. For both CIFAR-10 and CIFAR-100, the model is trained for 100 epochs with batch size 64, weight decay 0.0001 and momentum 0.9. The start learning rate is 0.1 and decays by a factor of 10 at epochs 50, 75, and 90.

DICE vs. discriminative-based approaches We show the comparison in Table 2. Due to the space limit, all the numbers reported are averaged over six OOD test datasets described above. The full results for each evaluation dataset are provided in Appendix H. On CIFAR-100, we show that using a sparse connection reduces the average FPR95 by 18.73% compared to the counterpart without sparsification [32]. In practice, DICE can be implemented through a simple post hoc weight masking, and therefore can improve OOD detection while ensuring the ID classification accuracy. We show that DICE is effective on a different architecture such as ResNet-101 (results in Supplementary, Section B).

| Method       | CIFAR-10 FPR95 | AUROC | CIFAR-100 FPR95 | AUROC |
|--------------|---------------|-------|----------------|-------|
| MSP          | 48.73         | 92.46 | 80.13          | 74.36 |
| ODIN         | 24.57         | 93.71 | 58.14          | 84.49 |
| Mahalanobis  | 31.42         | 89.15 | 50.14          | 85.03 |
| Energy       | 26.55         | 94.57 | 68.45          | 81.19 |
| Generalized ODIN | 34.25 | 90.61 | 52.87          | 85.24 |
| **DICE (ours)** | **20.83±1.58** | **95.24±0.24** | **49.72±1.69** | **87.23±0.73** |

Table 2: Comparison with competitive post hoc out-of-distribution detection method on CIFAR benchmarks. All values are percentages and are averaged over 6 OOD test datasets. We report standard deviations estimated across 5 independent runs.

5 Discussion and Ablations

Ablation on sparsity parameter $p$ We now characterize the effect of sparsity parameter $p$. In Figure 5, we summarize the OOD detection performance for DenseNet trained on CIFAR-100, where we vary $p = \{0.1, 0.3, 0.5, 0.7, 0.9, 0.99\}$. Interestingly, we observe the performance improves as with mild sparsity parameter $p$. A significant improvement can be observed from $p = 0.0$ (no sparsity) to $p = 0.1$. As we will theoretically later in Section 6, this is because the leftmost part of units being pruned have larger variances for OOD data (gray shade). Units in the middle part have small variances and contributions for both ID and OOD, therefore leading to...
similar performance as $p$ increases mildly. This ablation confirms that over-parameterization does compromise the OOD detection ability, and DICE can effectively alleviate the problem. In the extreme case when $p$ is too large (e.g., $p = 0.99$), the OOD performance starts to degrade. A similar trend holds on other ID datasets including CIFAR-10 and ImageNet, with full details in Appendix C.

Effect of variance reduction for output distribution Figure 4 shows that DICE has an interesting variance reduction effect on the output distribution for OOD data, and at the same time preserves the information for the ID data (CIFAR-10, class “frog”). The output distribution without any sparsity ($p = 0$) appears to have a larger variance, resulting in less separability from ID data (see left of Figure 4). In contrast, sparsification with DICE results in a sharper distribution, which benefits OOD detection. In Figure 5, we also measure the standard deviation of energy score for OOD data (normalized by the mean of ID data’s OOD scores in each setting). By way of sparsification, DICE can reduce the output variance. In Section 6, we formally characterize this and provide a theoretical explanation.

Ablation on unit selection We have shown that choosing a subset of weight parameters (with top-$k$ unit contribution) significantly improves the OOD detection performance. In this ablation, we also analyze those “lower contribution units” for OOD detection. Specifically, we consider: (1) Bottom-$k$ which only includes $k$ unit contribution with least contribution values, (2) top+bottom-$k$ which includes $k$ unit contribution with largest and smallest contribution values, (3) random-$k$ which randomly includes $k$ unit contribution and (4) top-$k$ which is equivalent to DICE method. In Table 3, we show that DICE outperforms all variants.

| Method                  | CIFAR-10↑ | CIFAR-100 ↓ | ImageNet ↓ |
|-------------------------|-----------|-------------|------------|
| Bottom-$k$              | 91.87     | 99.70       | 99.10      |
| (Top+Bottom)-$k$        | 24.25     | 59.93       | 57.22      |
| Random-$k$              | 62.12     | 77.48       | 70.77      |
| Top-$k$ (DICE)          | **21.76** | **50.60**   | **42.17**  |

Table 3: Ablation on different strategies of choosing a subset of units. Values are FPR95 (averaged over multiple test datasets).

Ablation on pruning methods In this ablation, we evaluate OOD detection performance under the most common post hoc sparsification methods. Here we primarily consider post hoc sparsification strategy which operates conveniently on a pre-trained network, instead of training with sparse regularization or architecture modification. The property is especially important for the adoption of OOD detection methods in real-world production environments, where the overhead cost of retraining can be sometimes prohibitive. Orthogonal to existing works on sparsification, our primary goal is to explore the role of sparsification for improved OOD detection performance, rather than establishing a generic sparsification algorithm. We consider the most common strategies, covering both unit-based and weight-based sparsification methods: (1) unit dropout [43] which randomly drops a fraction of units, (2) unit pruning [30] which drops units with the smallest $L_2$ norm of the corresponding weight vectors, (3) weight dropout [47] which randomly drops weights in the fully connected layer, and (4) weight pruning [14] drops weights with the smallest entries under the $L_1$ norm. For consistency, we use the same OOD scoring function and the same sparsity parameter $p = 0.9$ for all methods.
### Table 4: Ablation results.

| Method          | FPR95 | AUROC |
|-----------------|-------|-------|
| Weight-Dropout  | 70.99 | 79.27 |
| Unit-Dropout    | 91.66 | 53.96 |
| Weight-Pruning  | 57.22 | 87.70 |
| Unit-Pruning    | 79.96 | 76.07 |
| DICE            | 40.90 | 91.22 |
| No sparsification [32] | 71.03 | 82.74 |

All sparsification methods are based on the same OOD scoring function [32], with sparsity parameter \( p = 0.9 \). \( \uparrow \) indicates larger values are better and \( \downarrow \) indicates smaller values are better. All values are percentages and are averaged over multiple OOD test datasets.

Our ablation reveals several important insights shown in Table 4. First, in contrasting weight dropout vs. DICE, a salient performance gap of 30.09% (FPR95) is observed under the same sparsity. This suggests the importance of dropping weights *directedly* rather than *randomly*. Second, DICE outperforms a popular \( L_1 \)-norm-based pruning method [14] by up to 16.32% (FPR95). While it prunes weights with low magnitude, negative weights with large \( L_1 \)-norm can be kept. The negative weights can undesirably corrupt the output with noisy signals (as shown in Figure 1). The performance gain of DICE over [14] attributes to our contribution-directed sparsification, which is better suited for OOD detection. Results for CIFAR-10 and CIFAR-100 are available in Appendix H.

### 6 Why does DICE improve OOD detection?

In this section, we formally explain the mechanism by which reliance on irrelevant units hurts OOD detection and how DICE effectively mitigates the issue. Our analysis highlights that DICE reduces the output variance for both ID and OOD data. Below we provide details.

**Setup**

For a class \( c \), we consider the unit contribution vector \( \mathbf{v} \), the element-wise multiplication between the feature vector \( \mathbf{h}(x) \) and corresponding weight vector \( \mathbf{w} \). We contrast the two outputs with and without sparsity:

\[
f_c = \sum_{i=1}^{m} v_i \quad \text{(w.o sparsity)} \tag{6}
\]

\[
f_{c}^{\text{DICE}} = \sum_{i \in \text{top units}} v_i \quad \text{(w. sparsity),} \tag{7}
\]

where \( f_c \) is the output using the summation of all units’ contribution, and \( f_{c}^{\text{DICE}} \) takes the input from the top units (ranked based on the average contribution on ID data, see bottom of Figure 6).

![Figure 6: Units in the penultimate layer are sorted based on the average contribution to a CIFAR-10 class (“airplane”). OOD data (SVHN) can trigger a non-negligible fraction of units with noisy signals on the CIFAR-10 trained model. Units are sorted in the same order, based on the ID contribution.](image)
DICE reduces the output variance  We consider the unit contribution vector for OOD data \( \mathbf{v} \in \mathbb{R}^m \), where each element is a random variable \( v_i \) with mean \( E[v_i] = \mu_i \) and variance \( \text{Var}[v_i] = \sigma_i^2 \).

For simplicity, we assume each component is independent, but our theory can be extended to correlated variables (see Remark 1). Importantly, indices in \( \mathbf{v} \) are sorted based on the same order of unit contribution on ID data. By using units on the rightmost side, we now show the key result that DICE reduces the output variance.

Proposition 1. Let \( v_i \) and \( v_j \) be two independent random variables. Denote the summation \( r = v_i + v_j \), we have \( E[r] = E[v_i] + E[v_j] \) and \( \text{Var}[r] = \text{Var}[v_i] + \text{Var}[v_j] \).

Lemma 2. When taking the top \( m - t \) units, the output variable \( f_c^{\text{DICE}} \) under sparsification has reduced variance:

\[
\text{Var}[f_c] - \text{Var}[f_c^{\text{DICE}}] = \sum_{i=1}^t \sigma_i^2
\]

Proof. The proof directly follows Proposition 1.

Remark 1 (Extension to correlated variables)  We can show in a more general case with correlated variables, the variance reduction is:

\[
\sum_{i=1}^t \sigma_i^2 + 2 \sum_{1 \leq i < j \leq m} \text{Cov}(v_i, v_j) - 2 \sum_{t<i<j \leq m} \text{Cov}(v_i, v_j),
\]

where \( \text{Cov}(\cdot, \cdot) \) is the covariance. Our analysis shows that the covariance matrix primarily consists of 0, which indicates the independence of variables. Moreover, the summation of non-zero entries in the full matrix (i.e., the second term) is greater than that of the submatrix with top units (i.e., the third term), resulting in a larger variance reduction than in Lemma 2. See complete proof in Appendix F.

Remark 2  Energy score is directly compatible with DICE since \( \log \text{sumexp} \) operation is a smooth approximation of maximum logit, i.e., \( \log \sum_j e^{f_j(x)} \approx \max_x f_j(x) \). Our theoretical analysis above shows that DICE reduces the variance of each logit \( f_j(x) \). This means that for detection scores such as energy score, the gap between OOD and ID score will be enlarged after applying DICE, which makes thresholding more capable of separating OOD and ID inputs and benefit OOD detection.

Remark 3 (Mean of output)  Beyond variance, we further show in Table 5 the effect of sparsity on the mean of output: \( E_{\text{in}}[\max_c f_c^{\text{DICE}}] \) and \( E_{\text{out}}[\max_c f_c^{\text{DICE}}] \). The gap between the two directly translates into the OOD score separability. We show that DICE maintains similar (or even enlarges) differences in terms of mean as sparsity \( p \) increases. Therefore, DICE overall benefits OOD detection due to both reduced output variances and increased differences of mean—the combination of both effects lead to stronger separability between ID and OOD.

| Sparsity | \( p = 0.9 \) | \( p = 0.7 \) | \( p = 0.5 \) | \( p = 0.3 \) | \( p = 0.1 \) | \( p = 0 \) |
|----------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( \Delta \) | 7.92 | 7.28 | 7.99 | 8.04 | 7.36 | 6.67 |

Table 5: Difference between the mean of ID’s output and OOD’s output. Here we use CIFAR-100 as ID data and \( \Delta = E_{\text{in}}[\max_c f_c^{\text{DICE}}] - E_{\text{out}}[\max_c f_c^{\text{DICE}}] \) is averaged over six common OOD benchmark datasets described in Section 4.

Remark 4 (Variance reduction on ID data)  Note that we can also show the effect of variance reduction for ID data in a similar way. Importantly, DICE effectively preserves the most important information akin to the ID data, while reducing noisy signals that are harmful for OOD detection. Overall the variance reduction effect on both ID and OOD data leads to stronger separability, as evidenced in Figure 4.

7 Related Work

OOD detection for discriminative models  The problem of classification with rejection can date back to early works on abstention \([4, 9]\), which considered simple model families such as SVMs. The phenomenon of neural networks’ overconfidence in out-of-distribution data is first revealed by Nguyen et al. \([37]\). Existing works attempted to improve the OOD uncertainty estimation by using OpenMax score \([3]\), ODIN \([21, 31]\), Mahalanobis distance \([29]\), and the energy score \([32]\). However, previous methods primarily derive OOD scores using overparameterized weights. In contrast, our work is motivated by a novel analysis of unit contribution, and shows that sparsification is a surprisingly effective approach for OOD detection. A separate line of methods uses an auxiliary OOD dataset for model regularization during training \([12, 18, 39, 34]\). In contrast, our method does
not assume the availability of any auxiliary data. Post-hoc methods have the advantages of easy to use and general applicability without modifying the training procedure and objective. The latter property is especially important for the adoption of OOD detection methods in real-world production environments, where the overhead cost of retraining can be sometimes prohibitive. A comprehensive survey on OOD detection can be found in [53].

**OOD detection with generative models** Alternative approaches for detecting OOD inputs resort to generative models that directly estimate in-distribution density [7, 24, 45, 46]. Intriguingly, Nalisnick et al. [35] showed that deep generative models can assign a high likelihood to OOD data. Several methods improve OOD detection using generative models, including likelihood ratios [40], input complexity score [42], and likelihood regret [51]. However, generative models can be prohibitively challenging to train and optimize [19], and the performance can often lag behind the discriminative counterpart [25, 49].

**Pruning and sparsification** A great number of effort has been put towards improving *post hoc* pruning and training time regularization for deep neural networks [33, 10, 2, 13, 14, 30, 1]. Many works obtain a sparse model by training with sparse regularization [2, 1, 33, 13] or architecture modification [10, 30], while our work primarily considers *post hoc* sparsification strategy which operates conveniently on a pre-trained network. On this line, two popular Bernoulli dropout techniques include unit dropout [43] and weight dropout [43]. *Post hoc* pruning strategies truncate weights with low magnitude [14], or drop units with low weight norms [30]. Orthogonal to existing works, our goal is to improve the OOD detection performance rather than accelerate computation. In this paper, we first demonstrate that sparsification can be useful for OOD detection. An in-depth discussion and comparison of these methods are presented in Section 5.

**Distributional shifts.** Distributional shifts have attracted increasing research interests [26]. It is important to recognize and differentiate various types of distributional shift problems. Literature in OOD detection is commonly concerned about model reliability and detection of label-space shifts, where the OOD inputs have disjoint labels with ID data and therefore should not be predicted by the model. This is fundamentally different from the OOD generalization or domain adaption task whose goal is to provide accurate prediction on OOD images with the distributional shift. For example, some works considered covariate shift in the input space [16, 44, 38, 21], where the model is is expected to generalize to the OOD data.

8 Conclusion

This paper provides a simple sparsification strategy termed DICE, which ranks weights based on a contribution measure and then uses the most significant weights to derive the output for OOD detection. We provide both empirical and theoretical insights characterizing and explaining the mechanism by which DICE improves OOD detection. By exploiting the most important weights, DICE provably reduces the output variance for OOD data, resulting in a sharper output distribution and stronger separability from ID data. Extensive experiments show DICE can significantly improve the performance of OOD detection for over-parameterized networks. We hope our research can raise more attention to the importance of weight sparsification for OOD detection.

9 Social Impact

This paper aims to improve the reliability and safety of modern neural networks. Our study can lead to direct benefits and societal impacts when deploying machine learning models in the real world. Our work does not involve any human subjects or violation of legal compliance. We do not anticipate any potentially harmful consequences. Through our study and releasing our code, we hope to raise stronger research and societal awareness towards the problem of out-of-distribution detection in real-world settings.

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A Details of Baselines

For the reader’s convenience, we summarize in detail a few common techniques for defining OOD scores that measure the degree of ID-ness on a given input. By convention, a higher (lower) score is indicative of being in-distribution (out-of-distribution).

**MSP [17]** This method proposes to use the maximum softmax score as the OOD score.

**ODIN [31]** This method improves OOD detection with temperature scaling and input perturbation. In all experiments, we set the temperature scaling parameter $T = 1000$.

For ImageNet, we found the input perturbation does not further improve the OOD detection performance and hence we set $\epsilon = 0$. Following the setting in [31], we set $\epsilon$ to be 0.004 for CIFAR-10 and CIFAR-100.

**Mahalanobis [29]** This method uses multivariate Gaussian distributions to model class-conditional distributions of softmax neural classifiers and use Mahalanobis distance-based scores for OOD detection. We use 500 examples randomly selected from ID datasets and an auxiliary tuning dataset to train the logistic regression model and tune the perturbation magnitude $\epsilon$. The tuning dataset consists of adversarial examples generated by FGSM [11] with a perturbation size of 0.05. The selected $\epsilon$’s are 0.001, 0.0, and 0.0 for ImageNet-1k, CIFAR-10, and CIFAR-100, respectively.

**Generalized ODIN [21]** This method proposes a specialized network to learn temperature scaling and a novel strategy to choose perturbation magnitude, in order to replace manually-set hyperparameters. Our training configurations strictly follow the original paper, where we train the DeConf-C network for 200 epochs without applying the weight decay in the final layer of the Deconf classifier (notated as $h_i(x)$ in [21]). The other settings such as learning rate, momentum and training batch size are the same as ours. Note that G-ODIN has a slight advantage due to a longer training time than ours (100 epochs). We choose the best perturbation magnitude $\epsilon$ by maximizing the confidence scores on 1,000 examples randomly selected from ID datasets. The selected $\epsilon$ value is 0.02 for all (ImageNet-1k, CIFAR-10, and CIFAR-100).

**Energy [32]** This method proposes using energy score for OOD detection. The energy function maps the logit outputs to a scalar $E(x; f) \in \mathbb{R}$, which is relatively lower for ID data. Note that [32] used the negative energy score for OOD detection, in order to align with the convention that $S(x)$ is higher (lower) for ID (OOD) data. Energy score does not require hyperparameter tuning.

B Performance on Different Architecture

In the main paper, we have shown that DICE is competitive compared to other discriminative-based OOD detection methods on DenseNet. In this section, we show in Table 6 that DICE is also competitive on other network architecture including ResNet-101 [15]. For a fair comparison, all the methods use pre-trained networks post hoc, without regularizing with additional data. The model is trained on the in-distribution dataset CIFAR-100. All the numbers reported are averaged over six OOD test datasets described in Section 4.2. Our proposed method DICE outperforms baselines.

| Method                  | FPR95 | AUROC | In-dist acc. |
|-------------------------|-------|-------|--------------|
|                         | ↓     | ↑     | ↑            |
| MSP [17]                | 81.80 | 74.74 | 76.38        |
| ODIN [31]               | 68.82 | 79.18 | 76.38        |
| Mahalanobis [29]        | 88.57 | 67.77 | 76.38        |
| Generalized-ODIN [21]   | 76.28 | 75.24 | 77.63        |
| Energy score [32]       | 72.38 | 79.34 | 76.38        |
| DICE (ours)             | 64.55 | 80.55 | 76.38        |

Table 6: Main comparison results with ResNet-101. Comparison with competitive post hoc out-of-distribution detection methods. All methods are based on a discriminative model trained on ID data only, without using any auxiliary outlier data. ↑ indicates larger values are better and ↓ indicates smaller values are better. All values are percentages and are averaged over six OOD test datasets.
C More results on effect of Sparsity Parameter $p$

We characterize the effect of sparsity parameter $p$ on other ID datasets. In Table 7, we summarize the OOD detection performance and classification performance for DenseNet trained on CIFAR-100, CIFAR-10 and ImageNet, where we vary $p = \{0.1, 0.3, 0.5, 0.7, 0.9, 0.99\}$. A similar trend is observed as discussed in the main paper.

| Sparsity | CIFAR-10 | ImageNet |
|----------|----------|----------|
|          | FPR95 | AUROC | Acc. | FPR95 | AUROC | Acc. |
| $p = 0.99$ | 57.57 | 84.29 | 60.81 | 59.64 | 83.57 | 63.28 |
| $p = 0.9$  | 21.76 | 94.91 | 94.38 | 41.91 | 91.10 | 73.36 |
| $p = 0.7$  | 21.76 | 94.91 | 94.35 | 41.88 | 91.21 | 73.82 |
| $p = 0.5$  | 21.76 | 94.91 | 94.35 | 41.83 | 91.21 | 73.80 |
| $p = 0.3$  | 21.75 | 94.91 | 94.35 | 41.20 | 91.22 | 73.57 |
| $p = 0.1$  | 21.92 | 94.90 | 94.33 | 43.97 | 89.87 | 73.38 |
| $p = 0.099$ | 26.55 | 94.57 | 94.50 | 71.03 | 82.74 | 75.20 |

Table 7: Effect of varying sparsity parameter $p$. Results are averaged on the test datasets described in Section 4.

D Hardware

We conduct all the experiments on NVIDIA GeForce RTX 2080Ti GPUs.

E Reproducibility Statement

Authors of the paper recognize the importance and value of reproducible research. We summarize our efforts below to facilitate reproducible results:

1. **Dataset.** We use publicly available datasets, which are described in detail in Section 4.1 and Section 4.2.
2. **Assumption and proof.** The complete proof of our theoretical contribution is provided in Appendix F, which supports our theoretical claims made in Section 6.
3. **Baselines.** The description and hyperparameters of baseline methods are specified in Appendix A.
4. **Model.** Our main results on ImageNet are based on Google’s BiT pre-trained model, which has been publicly released [https://github.com/google-research/big_transfer](https://github.com/google-research/big_transfer). Due to the post hoc nature of our method, this allows the research community to reproduce our numbers provided with the same model and evaluation datasets.
5. **Implementation.** The simplicity of the DICE eases the reproducibility, as it only requires a few lines of code modification in the PyTorch model specification. Specifically, one can replace the weight matrix in the penultimate layer of deep networks using the following code:

```python
threshold = numpy.percentile(V, p)
M = V > threshold
W_new = W * M
```

6. **Open Source.** The codebase and the dataset is released for reproducible research.

F Variance Reduction with Correlated Variables

**Extension of Lemma 2.** We can show variance reduction in a more general case with correlated variables. The variance of output $f_c$ without sparsification is:

$$\text{Var}[f_c] = \sum_{i=1}^{m} \sigma_i^2 + 2 \sum_{1 \leq i < j \leq m} \text{Cov}(v_i, v_j),$$

where $\text{Cov}(\cdot, \cdot)$ is the covariance. The expression states that the variance is the sum of the diagonal of covariance matrix plus two times the sum of its upper triangular elements.
Similarly, the variance of output with directed sparsification (by taking the top units) is:

$$\text{Var}[f_{\text{DICE}}] = \sum_{i=t+1}^{m} \sigma_i^2 + 2 \sum_{t<i<j \leq m} \text{Cov}(v_i, v_j).$$

Therefore, the variance reduction is given by:

$$\sum_{i=1}^{t} \sigma_i^2 + 2 \sum_{1 \leq i < j \leq m} \text{Cov}(v_i, v_j) - 2 \sum_{t<i<j \leq m} \text{Cov}(v_i, v_j),$$

Figure 7: Covariance matrix of unit contribution estimated on the OOD dataset SVHN. Model is trained on ID dataset CIFAR-10. The unit indices are sorted from low to high, based on the expectation value of ID’s unit contribution (airplane class, same as in Figure 1). The matrix primarily consists of elements with 0 value.

We show in Fig. 7 that the covariance matrix of unit contribution $v$ primarily consists of elements of 0, which indicates the independence of variables by large. The covariance matrix is estimated on the CIFAR-10 model with DenseNet-101, which is consistent with our main results in Table 1.

Moreover, the summation of non-zero entries in the full matrix (i.e., the second term) is greater than that of the submatrix with top units (i.e., the third term), resulting in a larger variance reduction than in Lemma 2. In the case of OOD data (SVHN), we empirically measure the variance reduction, where

$$\sum_{i=1}^{t} \sigma_i^2 + 2 \sum_{1 \leq i < j \leq m} \text{Cov}(v_i, v_j)$$

equals to 6.8 and

$$2 \sum_{t<i<j \leq m} \text{Cov}(v_i, v_j)$$

equals to 2.2. Therefore, DICE leads to a significant variance reduction effect.

G Effect of DICE on MSP

Our theory shows the variance reduction effect directly in the logit output space, which is more compatible with the energy score. As a further investigation in Table 8, we find empirically that using DICE for MSP can improve the performance for MSP though it does not yield better performance than our existing results.

| Method      | SVHN | LSUN-c | LSUN-r | iSUN | Texture | places365 | Average |
|-------------|------|--------|--------|------|---------|-----------|---------|
| MSP [17]    | 48.25| 33.80  | 42.37  | 41.42| 63.99   | 62.57     | 48.73   |
| DICE+MSP    | 45.94| 24.36  | 35.68  | 34.60| 62.06   | 59.40     | 43.67   |

Table 8: Effect of applying DICE with MSP on DenseNet101 pretrained on CIFAR-10. The number is reported in FPR95.

H Detailed OOD Detection Performance for CIFAR

We report the detailed performance for all six test OOD dataset for models trained on CIFAR10 and CIFAR-100 respectively in Table 9 and Table 10.
| Method Type | Method       | SVHN         | LSUN-c       | LSUN-r       | iSUN      | Textures    | Places365 | Average |
|-------------|--------------|--------------|--------------|--------------|-----------|-------------|-----------|---------|
|             | FPR95 | AUROC | FPR95 | AUROC | FPR95 | AUROC | FPR95 | AUROC | FPR95 | AUROC | FPR95 | AUROC | FPR95 | AUROC | FPR95 | AUROC | FPR95 | AUROC |
| Non-Sparse  | MSP     | 47.24  | 93.48  | 33.57  | 95.54  | 42.10  | 94.51  | 42.31  | 94.52  | 64.15  | 88.15  | 63.02  | 88.57  | 48.73  | 92.46  | 48.73  | 92.46  | 48.73  | 92.46  |
|             | ODIN    | 25.29  | 94.57  | 4.70   | 98.86  | 3.09   | 99.02  | 3.98   | 98.90  | 57.50  | 82.38  | 52.35  | 88.55  | 24.57  | 93.71  | 24.57  | 93.71  | 24.57  | 93.71  |
|             | Mahalanobis | 6.42  | 98.31  | 56.55  | 86.96  | 9.14   | 97.09  | 9.78   | 97.25  | 21.51  | 92.15  | 85.14  | 63.15  | 31.42  | 89.15  | 31.42  | 89.15  | 31.42  | 89.15  |
|             | Energy  | 40.61  | 93.99  | 3.81   | 99.15  | 9.28   | 98.12  | 10.07  | 98.07  | 36.12  | 86.43  | 39.40  | 91.64  | 26.55  | 94.57  | 26.55  | 94.57  | 26.55  | 94.57  |
|             | Generalized ODIN | 6.68  | 98.32  | 17.58  | 95.09  | 36.56  | 92.09  | 36.44  | 91.75  | 35.18  | 89.24  | 73.06  | 77.18  | 34.25  | 90.61  | 34.25  | 90.61  |
| Sparse      | Unit-Dropout | 89.16  | 60.96  | 72.97  | 81.33  | 87.03  | 68.78  | 87.29  | 68.07  | 88.53  | 60.10  | 94.82  | 59.18  | 86.63  | 66.40  | 86.63  | 66.40  | 86.63  |
|             | Weight-Dropout | 81.34  | 80.03  | 21.06  | 96.15  | 54.70  | 90.33  | 58.88  | 89.80  | 83.34  | 73.31  | 73.42  | 81.10  | 62.12  | 85.12  | 62.12  | 85.12  |
|             | Unit-Pruning | 40.56  | 93.99  | 3.81   | 99.15  | 9.28   | 98.12  | 10.07  | 98.07  | 56.1   | 86.43  | 39.47  | 91.64  | 26.55  | 94.57  | 26.55  | 94.57  |
|             | Weight-Pruning | 28.61  | 95.40  | 3.01   | 99.30  | 8.58   | 98.19  | 9.08   | 98.16  | 49.45  | 88.20  | 46.78  | 89.77  | 24.25  | 94.84  | 24.25  | 94.84  |
|             | DICE (ours) | 25.99±5.10 | 95.90±1.08 | 0.26±0.11 | 99.92±0.02 | 3.91±0.56 | 99.20±0.15 | 4.36±0.71 | 99.14±0.15 | 41.90±1.41 | 88.18±1.90 | 48.59±1.53 | 89.13±0.31 | 20.83±1.58 | 95.24±0.24 | 20.83±1.58 | 95.24±0.24 |

Table 9: Detailed results on six common OOD benchmark datasets: Textures [5], SVHN [36], Places365 [56], LSUN-Crop [54], LSUN-Resize [54], and iSUN [52]. For each ID dataset, we use the same DenseNet pretrained on CIFAR-10. ↑ indicates larger values are better and ↓ indicates smaller values are better.
| Method Type | Method       | SVHN| LSUN-c | LSUN-r | iSUN | Textures | Places365 | Average |
|-------------|--------------|-----|--------|--------|------|----------|-----------|---------|
|             |              | FPR95 | AUROC  | FPR95  | AUROC | FPR95   | AUROC  | FPR95  | AUROC |
| Non-Sparse  | MSP          | 81.70 | 75.40  | 60.49  | 85.60 | 85.24   | 69.18   | 85.99  | 70.17 | 84.79 | 71.48 | 82.55 | 74.31 | 80.13 | 74.36 |
|             | ODIN         | 41.35 | 92.65  | 10.54  | 97.93 | 65.22   | 84.22   | 67.05  | 83.84 | 82.34 | 71.48 | 82.32 | 76.84 | 58.14 | 84.49 |
|             | Mahalanobis  | 22.44 | 95.67  | 68.90  | 86.30 | 94.20   | 31.38   | 93.21  | 82.39 | 79.39 | 92.66 | 61.39 | 50.14 | 85.03 |
|             | Energy       | 87.46 | 81.85  | 14.72  | 97.43 | 70.65   | 80.14   | 74.54  | 78.95 | 84.15 | 71.03 | 79.20 | 77.72 | 68.45 | 81.19 |
|             | Generalized ODIN | 36.74 | 93.51  | 43.15  | 89.55 | 40.31   | 92.61   | 37.41  | 93.05 | 64.26 | 76.72 | 95.33 | 65.97 | 52.87 | 85.24 |
| Sparse      | Unit-Dropout | 91.43 | 54.71  | 56.24  | 85.25 | 91.06   | 57.79   | 90.88  | 57.90 | 89.59 | 54.57 | 94.15 | 56.15 | 85.56 | 61.06 |
|             | Weight-Dropout | 92.97 | 64.39  | 18.96  | 95.62 | 88.57   | 69.54   | 87.12  | 67.82 | 88.45 | 64.38 | 88.69 | 71.87 | 77.48 | 71.99 |
|             | Unit-Pruning | 87.52 | 81.83  | 14.73  | 97.43 | 70.62   | 80.18   | 74.46  | 79.00 | 84.20 | 71.02 | 79.32 | 77.70 | 68.48 | 81.19 |
|             | Weight-Pruning | 77.99 | 84.14  | 5.17   | 99.05 | 59.42   | 87.13   | 61.80  | 86.09 | 72.68 | 73.85 | 82.53 | 75.06 | 59.93 | 84.22 |
|             | DICE (ours)  | 54.65 | 88.84  | 0.39   | 99.74 | 10.01   | 49.45   | 68.55  | 89.39 | 90.08 | 13.36 | 65.04 | 9.06   | 76.42 | 55.35 | 79.58 |

Table 10: Detailed results on six common OOD benchmark datasets: Textures [5], SVHN [36], Places365 [56], LSUN-Crop [54], LSUN-Resize [54], and iSUN [52]. For each ID dataset, we use the same DenseNet pretrained on CIFAR-100. ↑ indicates larger values are better and ↓ indicates smaller values are better.