Precise positioning of mechanisms based on dynamics

S Y Orekhov, V M Masyuk, V S Kuznetsov, S P Dolgolenko and K G Kislov
Kaluga Branch of the Bauman Moscow State Technical University, Bazhenova street 4, Kaluga, 248004, Russia

kuznetsov850@ya.ru

Abstract. The purpose of this work is to study the factors that affect the positioning accuracy and find a way to determine the positioning accuracy of a mechatronic system. The paper investigates a model of a hexapod manipulator with six degrees of freedom. A mathematical model of the manipulator has been built. The study of the dependence of the positioning accuracy on the influence of external forces was carried out. It is concluded that manipulators of a parallel structure are applicable for light loads.

For today, mechatronic systems are widely used in such areas as: mechanical engineering, instrumentation, nanotechnology, medicine, photo and video, etc. This list is only expanding every day. In any production, considerable attention is paid to the quality of details. Complex automated systems are used to comply with all norms and obtain sufficient quality. A manipulator occupies the main place in these systems. High demands are placed on the accuracy of these mechatronic systems.

While working on a given program, it is difficult to achieve the required accuracy in the absence of corrective actions. The large mass of the structure, in particular, the working link, does not allow obtaining the required accuracy due to the forces and torques arising in the system.

Figure 1. Block diagram of the manipulator (a) and model in CAD Solidworks (b).
The positioning accuracy study will be based on a six-degree-of-motion controlled hexapod with actuators attached to a lower platform, with motors in parallel (figure 1). The construction of this model was based on the works [1], [2]. According to the Somov-Malyshev formula we made the choice of kinematic pairs and the type of hinge joints. When using the mode with six degrees of mobility, the robot has an RRTS structure (rotational-rotational-translational-spherical), in which two rotational pairs are used at the base, equivalent to a gimbal drive, a prismatic pair, a spherical gearbox. The rotary joints are passive, with only six linear actuators operating. The robot has 9 kinematic pairs of the fifth class (12 turning pairs and 6 rectilinear sliding pairs) and 6 kinematic pairs of the third class (6 spheres). The number of degrees of freedom according to the Somov-Malyshev formula is 6 [1].

This hexapod model describes the dynamic processes in the unloaded state and the method for finding the error in the displacement of the output link under the action of external forces [3].

To study this manipulator, it is necessary to solve the direct dynamic problem. Consider the forces that act on the entire system: the force of gravity acting on the movable platform, and the force of gravity acting on each of the rods, the force applied to the points and directed along the rods [4].

Having compiled the expression of work and substituting the values of the acting forces, we find the values of the generalized forces:

\[
Q_j = \sum_{i=1}^{6} \left( F_{ix} \frac{\partial x}{\partial q_i} + F_{iy} \frac{\partial y}{\partial q_i} + F_{iz} \frac{\partial z}{\partial q_i} \right), j = 1, 2, 3, 4, 5, 6. \quad (1)
\]

Solving this system of differential equations, which describes the dynamics of a mechanical system we can find the law of variation of the generalized coordinates \( q_i \), according to the given values of the control forces \( F_i \) [5].

The rigidity of a parallel manipulator determines its technical characteristics, such as accuracy, reliability, productivity, durability. The work performed by an external force and three forces in the rods are forces multiplied by the corresponding displacements [4]. Let's write the matrix equation.

\[
F_p^T \times \Delta P = f^T \times \Delta q, \quad (2)
\]

where \( F_p^T \) – resultant of system of forces;

\( \Delta P \) – vector of infinitesimal displacement of the output link arising under the action of the applied force;

\( f^T \) – vector of forces in the rods;

\( \Delta q \) – vector of infinitesimal deformations arising under the action of forces in the rods.

The vector of infinitesimal displacement of the output link can be represented as the product of the direct Jacobi matrix (\( J \)) and the error vector of the rod drives [2]. After transformation, we get the expression:

\[
F_p = J^{-T} \cdot f. \quad (3)
\]

The stiffness of the translational and rotational mechanisms does not depend on each other, since the structure of this manipulator with a parallel structure has a kinematic decoupling of the actuators. According to Hooke's law:

\[
\varepsilon = \frac{\Delta q_i}{q_i} = \frac{\sigma}{E} = \frac{f_i}{A \cdot E}, \quad (4)
\]

where \( \varepsilon \) – the relative deformation;

\( \Delta q_i \) – change in the length of the rod under the action of the applied force;
$q_i$ – rod length;
$E$ – elastic modulus in tension and compression;
$A$ – the cross-sectional area of the rod;
$f_i$ – strength in the rod;

$\frac{A \cdot E}{q_i}$ – the stiffness of the rod in tension and compression.

We express from equation (4) the force arising in the rod and substitute it into equation (2):

$$F_p = J^{-T} \cdot K_s \cdot J^{-1} \cdot \Delta P = K_c \cdot \Delta P,$$

where $K_c = J^{-T} \cdot K_s \cdot J^{-1}$ – the spatial matrix of the rigidity of the mechanism in the orthogonal coordinates system of the base.

Let us express from equation (4) the error of displacement of the centre of the platform under the action of an external force:

$$\Delta P = K_c^{-1} \cdot F_p,$$

where $K_c^{-1}$ – the matrix of static stiffness of the parallel robot;

$F_p$ – applied force.

The algorithm for finding the stiffness and displacement error of the output link under the action of an external force is implemented in the Matlab environment and is shown in figure 2. Based on the data obtained, dependencies were built (figure 3).

![Algorithm for calculating the error.](image)

**Figure 2.** Algorithm for calculating the error.
Figure 3. Dependences of the linear and angular displacements of the output link on the component of the applied force.

The experiment will be carried out on a controlled robot-hexapod with actuators fixed on the lower platform. The measurements will be taken from a platform that is acted upon only by gravity. The obtained measurements will be displayed on graphs in the Matlab environment.

As can be seen from the graphs, the offset error of the output link increases linearly with increasing force. It follows from this that manipulators of a parallel structure should be used at low dynamic loads. Also, from the graph of the angular displacement it can be seen that their values are so small that they can be ignored.

The data obtained are of a theoretical nature, the measurements were obtained on the basis of experiment modeling in SolidWorks and Matlab environments. The experiment was carried out according to the well-known methods [6]. At the moment, the choice of technical and hardware means is being made for the actual assessment of these parameters for the mechanism of the parallel structure and verification of the obtained theoretical basis.
As a result of the research, the influence of external forces on the positioning accuracy of mechanisms of a parallel platform-type structure with six degrees of freedom was determined. It was determined that the positioning accuracy of mechanisms of a parallel structure, arising from the error of drives, requires additional study and finding new methods to improve accuracy. The dependences of linear and angular displacements on external forces were experimentally determined.

References
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