Halo-independent upper limits on the dark matter scattering cross section with nucleons

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Abstract. We present a novel method that allows to derive an upper limit on the scattering cross section of dark matter with nucleons which is independent of the velocity distribution. To this end, we combine null results from direct detection experiments and neutrino telescopes, and use the fact that taken together, these classes of experiments probe the whole range of possible dark matter velocities. The resulting halo-independent upper limits on the dark matter scattering cross section are remarkably strong, and can be used to robustly rule out models of dark matter, without the need to invoke specific assumptions about the local velocity distribution.

1. Introduction
Weakly interacting massive particles (WIMPs) are one of the prime candidates for the particle nature of dark matter (DM). Searches for the hypothetical population of WIMPs inside the Solar System are under way, specifically in the form of direct detection experiments, which aim at the observation of the nuclear recoil induced by the elastic scattering of a DM particle with a nucleus of the detector, as well as by searching for an high-energy neutrino flux correlated with the direction of the Sun. These neutrinos could be produced by the annihilations of DM particles which have previously been captured in the gravitational potential of the Sun, via the energy loss induced by scatterings with the solar matter.

In order to directly deduce particle physics properties of DM from the results of these experiments, one needs to specify the local DM velocity distribution \( f(\vec{v}) \). As this distribution is not known from data, one typically invokes simplifying assumptions about the form of \( f(\vec{v}) \), most commonly adopting a Maxwell-Boltzmann distribution (defined in the galactic rest frame) with a velocity dispersion \( v_0 \simeq 230 \text{ km/s} \). However, various N-body simulations clearly suggest deviations from this simple picture, and predict velocity distributions that can not be described by a pure Maxwell-Boltzmann distribution. Furthermore, in addition to the smooth halo component described by the simulations, local effects as e.g. tidal streams, but also the possible presence of a dark disk could significantly alter \( f(\vec{v}) \). Any conclusion drawn from the null results of direct detection experiments or neutrino telescopes that explicitly assumes the Maxwell-Boltzmann distribution then suffers significantly from this uncertainty. In view of this situation, several halo-independent methods have been proposed (see e.g. [1, 2, 3] for early
works), which use the experimental data without making explicit assumptions about the form of $f(\vec{v})$.

In these proceedings, we present a novel halo-independent method suggested recently in [4], based on the combination of data from direct detection experiments with searches for neutrinos from DM annihilations inside the Sun, and which, for the first time, allows to derive an upper limit on the scattering cross section of DM with nucleons which is independent of the velocity distribution.

2. Dark matter in the Solar System as a superposition of streams

The halo-independent approach discussed in this work is based on the decomposition of the unknown velocity distribution $f(\vec{v})$ (defined in the rest frame of the Sun) in terms of infinitely many streams with fixed velocities $\vec{v}_0$:  

$$f(\vec{v}) = \int_{|\vec{v}_0| \leq v_{max}} d^3 v_0 \delta^{(3)}(\vec{v} - \vec{v}_0) f(\vec{v}_0). \tag{1}$$

Here we have introduced $v_{max}$ as the maximal velocity of DM particles, which can be set to $v_{max} \simeq 777$ km/s, assuming that all DM particles in the Galaxy are gravitationally bound [4]. Furthermore, we define $R$ to be the number of expected scattering events in a given direct detection experiment, and $C$ to be the capture rate of DM in the Sun. As both of these rates are linear in the velocity distribution $f(\vec{v})$, one can use the expansion given in eq. (1) to write

$$R = \int_{|\vec{v}_0| \leq v_{max}} d^3 v_0 f(\vec{v}_0) R_{\vec{v}_0}, \quad C = \int_{|\vec{v}_0| \leq v_{max}} d^3 v_0 f(\vec{v}_0) C_{\vec{v}_0}, \tag{2}$$

where $R_{\vec{v}_0}$ ($C_{\vec{v}_0}$) is the number of scattering events (the capture rate) assuming a hypothetical, pure DM stream with fixed velocity $\vec{v}_0$. For a given $\vec{v}_0$, we calculate $R_{\vec{v}_0}$ and $C_{\vec{v}_0}$ using the standard techniques for computing direct detection and capture rates; details can be found in [4].

Assuming for the moment that all DM particles have the same velocity $\vec{v}_0$ in the rest frame of the Sun, one can infer an upper limit on the scattering cross section from a specific direct detection experiment by requiring that $R_{\vec{v}_0} \leq R_{max}$. Here, $R_{max}$ is the upper limit on the number of expected scattering events derived from the null-result of the corresponding experiment, which is given in [4] for XENON100 [8], SuperCDMS [9], SIMPLE [10], and COUPP [11]. As an example, we show in Fig. 1 the upper limit on the scattering cross section as a function of the speed $v_0 \equiv |\vec{v}_0|$ of the stream, for the exemplary case of spin-independent interactions with equal couplings to protons and neutrons, assuming $m_{DM} = 100$ GeV and for concreteness employing the XENON100 experiment. The various dashed red lines correspond to different angles between the stream and the velocity of the Earth, while the red solid line shows $\sigma_{DD}^{max}(v_0)$, which is defined to be the weakest among all the limits corresponding to the different angles, separately for every speed $v_0$. Then, by construction one has $R_{\vec{v}_0}(\sigma) \geq R_{max}$ for $\sigma \geq \sigma_{DD}^{max}(v_0)$.

Furthermore, the non-observation of an excess of high-energy neutrinos from the direction of the Sun can be used to set an upper limit $C_{max}$ on the capture rate, assuming equilibrium between capture and annihilation. With that, analogously to the discussion for direct detection in the previous paragraph, one can define $\sigma_{NT}^{max}(v_0)$ to be the upper limit on the DM scattering cross section, under the assumption that all DM particles have the same velocity $\vec{v}_0$. By definition, one then has $C_{\vec{v}_0}(\sigma) \geq C_{max}$ for $\sigma \geq \sigma_{NT}^{max}(v_0)$. Assuming again spin-independent interactions and $m_{DM} = 100$ GeV, $\sigma_{NT}^{max}(v_0)$ is shown as a solid blue curve in Fig. 1, for the case of DM annihilating into $W^+W^-$, and using the upper limit on the capture rate $C_{max}$ following from the IceCube data [12].

1 See also [5, 6, 7], where a finite number of streams is used to fit positive signals in direct detection experiments.
Figure 1. Upper limits on the spin-independent dark matter-proton scattering cross section, for a velocity distribution corresponding to a pure stream, \( f_{\tilde{v}_0}(\tilde{v}) = \delta^3(\tilde{v} - \tilde{v}_0) \). See text for details.

Finally, using eq. (2) as well as the definitions of \( \sigma_{\text{DD}}^\star(v_0) \) and \( \sigma_{\text{NT}}^\star(v_0) \), the upper limit on the scattering cross section for a given, but arbitrary velocity distribution \( f(\tilde{v}) \) inferred from either the direct detection experiment (DD) or neutrino telescope (NT) at hand can be written as

\[
\text{DD: } \sigma \leq \left( \int_{|\tilde{v}| \leq v_{\text{max}}} \frac{d^3v_0}{\sigma_{\text{DD}}^\star(v_0)} \right)^{-1} f(\tilde{v}_0) \quad \text{NT: } \sigma \leq \left( \int_{|\tilde{v}| \leq v_{\text{max}}} \frac{d^3v_0}{\sigma_{\text{NT}}^\star(v_0)} \right)^{-1} f(\tilde{v}_0),
\]

(3)

In other words, for a given velocity distribution \( f(\tilde{v}) \), the corresponding upper limit on the scattering cross section derived from a direct detection experiment or a neutrino telescope can be written as a superposition of the upper limits \( \sigma_{\text{DD}}^\star(v_0) \) or \( \sigma_{\text{NT}}^\star(v_0) \) obtained under the assumption of a pure DM stream with velocity \( \tilde{v}_0 \), with the weights of the superposition being precisely the velocity distribution \( f(\tilde{v}_0) \).

3. A halo-independent upper limit on the scattering cross section

The formalism developed in the previous section directly visualizes the complementarity of direct detection experiments and neutrino telescopes with respect to the velocity distribution. As it can be seen in Fig. 1, direct detection experiments are sensitive to large velocities (due to the finite threshold of the experiment), while neutrino telescopes are most sensitive to slow moving WIMPs, because these can more easily be captured in the Sun. Taken together, both approaches probe the complete range of DM velocities between 0 and \( v_{\text{max}} \). In the following, we use this observation in order to derive an upper limit on the scattering cross section which is independent of the velocity distribution. To this end, we define \( \sigma_\star \) to be the largest value of the scattering cross section which is allowed by a given direct detection experiment and a given neutrino telescope for all stream-like velocity distributions with speeds between 0 and \( v_{\text{max}} \), i.e. \( \sigma_\star \equiv \max \{ \sigma_{\text{DD}}(\tilde{v}), \sigma_{\text{DD}}(\tilde{v}) \} \), with \( \tilde{v} \) being defined as the speed for which \( \sigma_{\text{DD}}(\tilde{v}) = \sigma_{\text{DD}}^\star(v_0) \). This construction is illustrated in Fig. 1 for the given exemplary choice of DM mass, annihilation channel, and set of experiments. By construction, one has

\[
\sigma_{\text{DD}}(v_0) \leq \sigma_\star \quad \text{for } \tilde{v} \leq v_0 \leq v_{\text{max}},
\]

(4)

\[
\sigma_{\text{NT}}^\star(v_0) \leq \sigma_\star \quad \text{for } 0 \leq v_0 \leq \tilde{v}.
\]

(5)
Defining $\delta f \equiv \int_{\vec{v}_0(v_0) \leq v_{\text{max}}} d^3 v_0 f(\vec{v}_0)$, an upper limit on the scattering cross section $\sigma$ for the given velocity distribution $f(\vec{v})$ can then be obtained separately from the direct detection experiment and neutrino telescope, using eqs. (3)–(5):

$$\sigma \leq \left[ \int_{0 \leq v_0 \leq v_{\text{max}}} d^3 v_0 \frac{f(\vec{v}_0)}{\sigma_{\text{DD max}}(v_0)} \right]^{-1} \leq \left[ \int_{\tilde{v} \leq v_0 \leq v_{\text{max}}} d^3 v_0 \frac{f(\vec{v}_0)}{\sigma_{\text{ND max}}(v_0)} \right]^{-1} \leq \frac{\sigma_{\star}}{\delta f},$$

$$\sigma \leq \left[ \int_{0 \leq v_0 \leq v_{\text{max}}} d^3 v_0 \frac{f(\vec{v}_0)}{\sigma_{\text{ND max}}(v_0)} \right]^{-1} \leq \left[ \int_{0 \leq v_0 \leq \tilde{v}} d^3 v_0 \frac{f(\vec{v}_0)}{\sigma_{\text{ND max}}(v_0)} \right]^{-1} \leq \frac{\sigma_{\star}}{1 - \delta f},$$

where in the last step the normalization condition $\int_{0 \leq v_0 \leq v_{\text{max}}} d^3 v_0 f(\vec{v}_0) = 1$ has been used. Finally, combining the inequalities (6) and (7) gives rise to the halo-independent upper limit

$$\sigma \leq 2\sigma_{\star}.$$  

This is the central result of this work: by combining the upper limit on the recoil rate derived from a direct detection experiment with the upper limit on the capture rate inferred from a neutrino telescope, one can deduce an upper limit on the scattering cross section of DM, which is completely independent of the velocity distribution $f(\vec{v})$.

We then construct $\sigma_{\star}$, and correspondingly the halo-independent upper limit $2\sigma_{\star}$, for various DM masses $m_{\text{DM}}$, for both the annihilation channels $W^+W^-$ ($\tau^+\tau^-$ for $m_{\text{DM}} \leq m_W$) and $bb$, as well as for spin-independent scattering with equal coupling to protons and neutrons and spin-dependent scattering with pure coupling to protons. Furthermore, we consider upper limits from the direct detection experiments XENON100 [8], SuperCDMS [9], SIMPLE [10], and COUPP [11], as well as the results from the neutrino telescopes IceCube [12] and SuperKamiokande [13]. We apply the procedure outlined above for every combination of direct detection experiment and neutrino telescope, and choose the most constraining result as our final upper limit. Fig. 2 shows the resulting halo-independent upper limit for the case of spin-independent scattering (spin-dependent scattering) in the upper (lower) panel, and for annihilation into $W^+W^-/\tau^+\tau^-$ ($bb$) in the left (right) panel. Here, the red solid lines correspond to the choice $v_{\text{max}} = 777$ km/s (see section 2), while the red dashed lines refer to the more conservative value $v_{\text{max}} = 0.05c$. For comparison, we furthermore show in the plots the upper limits derived under the standard assumption of a Maxwell-Boltzmann distribution with a velocity dispersion $v_0 = 230$ km/s and a galactic escape velocity $v_{\text{esc}} = 533$ km/s (black solid and black dashed lines). As it can be seen from the plot, the halo-independent upper limit can be remarkably strong, reaching $\sigma_{\star}^b \leq 10^{-43} (10^{-42})$ cm$^2$ and $\sigma_{\star}^p \leq 10^{-37} (3 \times 10^{-37})$ cm$^2$, assuming annihilations into $W^+W^- (bb)$ at $m_{\text{DM}} = 1$ TeV.

4. Conclusions

We have presented a novel method that allows to draw halo-independent conclusions from the combination of direct detection and neutrino telescope data. In particular, we have shown that for a given annihilation channel of DM, it is possible to derive a fully robust upper limit on the scattering cross section of DM, which does not depend on the concrete choice of the local velocity distribution $f(\vec{v})$. To this end, we decomposed the unknown velocity distribution in terms of infinitely many streams, and from the upper limits on the scattering cross section derived for pure streams, we were able to deduce an upper limit valid for an arbitrary (normalized) velocity distribution.\footnote{Our approach can also be used to set a lower limit on the scattering cross section in case of a positive signal in a future direct detection experiment. See [4] for details.} The resulting halo-independent upper limits turn out to be remarkably strong, and for some cases they can be comparable to the constraints derived under the assumption of the standard Maxwell-Boltzmann distribution.
Limits on $\sigma^{\text{SD}}_{\text{SD}}$, for ann. into $W^+W^−$ ($\tau^+\tau^−$ for $m_{\text{DM}} < m_W$)

Limits on $\sigma^{\text{SI}}_{\text{SI}}$, for ann. into $b\bar{b}$

Figure 2. Red curves: halo-independent upper limits on the dark matter scattering cross section, for spin-independent (upper panel) and spin-dependent scattering (lower panel), and for annihilation into $W^+W^−/\tau^+\tau^−$ (left panel) and $b\bar{b}$ (right panel). Black curves: upper limits assuming the standard Maxwell-Boltzmann velocity distribution. See text for details.

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