Anomalous Couplings in $W$ Pair Production

Jochen Biebel

Deutsches Elektronen-Synchrotron DESY Zeuthen,
Platanenallee 6, D-15738 Zeuthen, Germany

Abstract

I present a short overview over $W$ pair production and studies of angular differential cross-sections with and without initial state radiation applying semi-analytical methods and using the Fortran program GENTLE. The influence of anomalous couplings to this process is also discussed.

1Talk given at Zeuthen Workshop on Loops and Legs in Gauge Theories, 19–24 April 1998, Rheinsberg, Germany
1 Introduction

Since the formulation of the standard model of electroweak interactions [1], more and more precision tests have confirmed its validity. However, for a central part of the theory, the non-abelian structure of the gauge couplings, we have only poor experimental information. With the results of LEP2 [2] and potential future linear colliders [3] the situation will change. In processes with $W$ production triple and quartic gauge boson vertices appear and may be measured. Of special interest in $e^+e^-$ annihilation is the process

$$e^+e^- \rightarrow W^+W^-,$$  \hfill (1)

with contributions from $\gamma W^+W^-$ and $ZW^+W^-$ vertices.

The current limits for anomalous couplings from combined results of LEP2 and D0 are [4]:

$$\alpha_{W\phi} = -0.03^{+0.06}_{-0.06}, \hfill (2)$$
$$\alpha_W = -0.03^{+0.08}_{-0.08}, \hfill (3)$$
$$\alpha_{B\phi} = -0.05^{+0.22}_{-0.20}. \hfill (4)$$

where the $\alpha$’s are given by the identities:

$$\alpha_{W\phi} = c_W s_W \delta_Z, \hfill (5)$$
$$\alpha_W = y_\gamma = \frac{s_W}{c_W} y_Z, \hfill (6)$$
$$\alpha_{B\phi} = x_\gamma - c_W s_W \delta_Z = -\frac{c_W}{s_W} \left( x_Z + s_Z^2 \delta_Z \right). \hfill (7)$$

The anomalous parameters $\delta_Z$, $x_\gamma$, $x_Z$, $y_\gamma$, $y_Z$, and $z_Z$ are defined by the Lagrangian in eq. (18).

In section 2 I give an overview on $W$ pair production and the studies of the differential cross-sections performed with GENTLE version 2 [5]. I discuss the influence of potential anomalous three gauge boson couplings to $W$ pair production in section 3.

2 $W$ Pair Production

The first calculations in the standard model for the process in (1) were done in the narrow width approximation [6], e.g. neglecting the finite width of the $W$ bosons. At this time it was known, that the decay width $\Gamma_W$ of the $W$ will give rise to large corrections if the $W$ is much heavier than the proton [7]. As a consequence the finite
$W$ width must be considered. This can be done by convoluting the cross-section with Breit-Wigner factors [8]:

$$\sigma(s) = \int_0^s ds_1 \rho(s_1) \int_0^{(\sqrt{s} - \sqrt{s_1})^2} ds_2 \rho(s_2) \sigma_0(s, s_1, s_2)$$  \hspace{1cm} (8)

with

$$\rho(s_i) = \frac{1}{\pi} \frac{\sqrt{s_i} \Gamma_W(s_i)}{(s_i - m_W^2)^2 + m_W^4 \Gamma_W^2(s_i)} \times B(f),$$  \hspace{1cm} (9)

where $s$ is the center-of-mass energy squared and $B(f)$ the branching fraction for the $W$ decaying in the fermion doublet $f$. The invariant masses of the decay products of the $W$ bosons are denoted by $s_1$ and $s_2$.

Since the produced $W$ bosons decay almost immediately, the production of 4 fermions

$$e^+e^- \rightarrow W^+W^- \rightarrow 4f$$  \hspace{1cm} (10)

is observed. Additional diagrams, so called background diagrams, contribute to the same final states and should be taken into account, too [9]. The number of contributing diagrams depends on the final state fermions. Here, I will concentrate on the CC11 class, since it includes the semi-leptonic final states. These final states are important in the measurement of the gauge couplings, because they offer the most complete kinematical information. The CC11 class is defined by having two different weak doublets and no electrons nor neutrinos as final state fermions. Depending on the number of produced neutrinos, there are 9, 10, or 11 Feynman diagrams.

For the CC11 class the $\sigma_0$ of (8) can be written as a sum over all interferences and combinations of coupling constants. For the differential cross-section one gets:

$$\frac{d\sigma_0}{d \cos \theta} = \frac{\sqrt{\lambda(s, s_1, s_2)}}{\pi s^2} \sum_k C_k \cdot G_k(s; s_1, s_2, \cos \theta)$$  \hspace{1cm} (11)

The coefficient functions $C_k$ are rather trivial and contain the coupling constants of the particles and the $s$-channel propagators. The kinematical functions $G_k$ are more complicated and describe the non-trivial dependencies of $s$, $s_1$, and $s_2$ and other variables like the scattering angle $\cos \theta$. To express the total cross-section a smaller set of $C$ and $G$ functions as in eq. (11) is needed, since the parity violating contributions disappear after the integration over the scattering angle.

As a simple example I give the expressions of the $C$ and $G$ functions for the differential cross-section for the square of the $t$-channel diagram [10]:

$$C^t = \frac{(G_tm_W^2)^2}{s_1 s_2} \rho_W(s_1) \rho_W(s_2),$$  \hspace{1cm} (12)
and
\[ G' = \frac{1}{8} \left[ 2s(s_1 + s_2) + \frac{\lambda}{4} \sin^2 \theta + \frac{\lambda s_1 s_2 \sin^2 \theta}{t^2_{\nu}} \right], \]  
(13)

where \( \lambda \) is the Källen-function
\[ \lambda \equiv \lambda(s, s_1, s_2) = s^2 + s_1^2 + s_2^2 - 2ss_1 - 2ss_2 - 2s_1s_2, \]  
(14)

and \( t_{\nu} \) is the neutrino propagator
\[ t_{\nu} = \frac{1}{2} \left( s - s_1 - s_2 - \sqrt{\lambda} \cos \theta \right). \]  
(15)

The interferences between signal diagrams and background diagrams are more complicated and I give only the kinematical function for the interference between the \( t \)-channel diagram and the \( u_1 \)-diagram as an example:
\[ G_{\text{tu}}(s, s_1, s_2) = \]
\[ \frac{-1}{\lambda} \left\{ \frac{3 \cos \theta}{4} \sqrt{\lambda} s^2 s_1^2 \sin^2 \theta - 2 \right\} \left[ \frac{1}{t_{\nu}} (s + s_1 - s_2) + 2s\mathcal{L}(s_1; s_2, s) \right] \]
\[ + \lambda \left[ \frac{\sin^2 \theta}{8t_{\nu}} [2s_1 s_2 (s_2 - s_1) - 6s^2 s_2 (s_1 + s_2) \mathcal{L}(s_1; s_2, s) - 3ss_2 (s + s_2)] \right. \]
\[ + \frac{3s^2 s_2}{16} [(s - s_1)^2 - s_2^2] + \frac{ss_1 s_2}{2} \left[ - \frac{3}{4} ss_2 \mathcal{L}(s_1; s_2, s) (5s \sin^4 \theta \right. \]
\[ + 4(s_1 + s_2) - \frac{1}{8} (3s_2^2 - 2ss_1 + 4s_1 s_2 - 7s_1^2 + 30ss_2 + 9s_2^2) \sin^2 \theta \]
\[ - \frac{1}{2} (3s_2^2 - 2s_1^2 - s_1 s_2 + 2ss_1)] \]
\[ + \frac{3s^2 s_2}{4} \mathcal{L}(s_1; s_2, s) [(4s_1 s_2 + s_1^2 + s_2^2 - s(s_1 + s_2)] \sin^2 \theta \]
\[ - 4[s_1 s_2 + s_1^2 + s_2^2 - s(s_1 + s_2))] + \frac{ss_2 \sin^2 \theta}{8} (2s_1 s_2 - 5s_1^2 + 3s_2^2 \]
\[ - 14ss_1 - 3s^2 \right) + \frac{s}{2} (5s_1 s_2 - 2s_1 s_2^2 - 3s_2^3 + 5ss_1 s_2 + 3s^2 s_2) \right\}, \]  
(16)

with
\[ \mathcal{L}(s; s_1, s_2) = \frac{1}{\sqrt{\lambda}} \ln \frac{s - s_1 - s_2 + \sqrt{\lambda}}{s - s_1 - s_2 - \sqrt{\lambda}}, \]  
(17)

The remaining coefficient and kinematical functions are presented in [11].

To make precise predictions for the cross-section, radiative corrections have to be taken into account [12]. To demonstrate the size of initial state radiation (ISR), I show in fig. 1 the difference between the ISR corrected cross-section and the Born
cross-section in the case of signal diagrams (CC03) and for the complete semi-leptonic process (CC10). While the difference peaks in the region \( \cos \theta > 0.8 \) it is almost constant in the other parts. Especially in the region of \( \cos \theta \to -1 \) this leads to important effects, because the differential cross-section drops here significantly and the relative corrections amount to 30\% for \( \cos \theta = -1 \) [11].

Figure 1: Differences between Born cross-section and cross-section with initial state radiation.

The numerical results of fig. 1 were produced with GENTLE and radiative corrections were treated as described in [5]. A short overview over GENTLE is given in appendix A.

3 Anomalous Couplings

The most general form for the \( \gamma WW \) and \( ZWW \) vertices compatible with Lorentz invariance was first considered in [13], where 9 parameters were introduced for each vertex. In [14] it was shown that these parameters were not independent and the number could be reduced to 7.

The number of parameters can be further reduced by using a restricted set of anomalous couplings, which is invariant under \( CP \) transformations. The anomalous
couplings are defined by the Lagrangian:

\[ \mathcal{L} = -ie \left[ A_\mu \left( W^{-\mu \nu} W^\nu_\nu - W^{+\mu \nu} W^-_\nu \right) + F_{\mu \nu} W^{+\mu W^-_\nu} \right] 
- ie \left( \cot \Theta_W + \delta_Z \right) \left[ Z_\mu \left( W^{-\mu \nu} W^\nu_\nu - W^{+\mu \nu} W^-_\nu \right) + Z_{\mu \nu} W^{+\mu W^-_\nu} \right] 
- ie x_\gamma F_{\mu \nu} W^{+\mu W^-_\nu} - ie x_Z Z_{\mu \nu} W^{+\mu W^-_\nu} 
+ ie \frac{y_\gamma}{M_W^2} F^{\nu \lambda} W^{\lambda \nu}_{\lambda \nu} W^{+\mu} + ie \frac{y_Z}{M_W^2} Z^{\nu \lambda} W^{\lambda \nu}_{\lambda \nu} W^{+\mu} 
+ \frac{e z Z}{M_W^2} \partial_\alpha \tilde{Z}_{\mu \sigma} \left( \partial^\sigma W^{-\sigma W^+ -} - \partial^\sigma W^{-\sigma W^+ +} \right) 
+ \partial^\sigma W^{+\sigma W^- -} - \partial^\sigma W^{+\sigma W^- +} \right). \] (18)

In the standard model the anomalous parameters \( \delta_Z, x_\gamma, x_Z, y_\gamma, y_Z, \) and \( z_Z \) are zero. The parameter \( z_Z \) violates both \( C \) and \( P \) symmetry, but is invariant under the product \( CP \). The parameters \( x_\gamma \) and \( y_\gamma \) contribute to the magnetic dipole moment \( \mu_W \) and the electromagnetic quadrupole moment \( q_W \) of the \( W \) boson [15]:

\[ \mu_W = \frac{e}{2 m_W^2} \left( 2 + x_\gamma + y_\gamma \right), \] (19)
\[ q_W = -\frac{e}{m_W^2} \left( 1 + x_\gamma - y_\gamma \right). \] (20)

With these additional parameters the cross-section for \( W \) pair production can be written as:

\[ \sigma^{\text{ano}} = \sigma^{\text{SM}} + x_\gamma \sigma^{x_\gamma} + x_Z \sigma^{x_Z} + \ldots 
+ x_\gamma^2 \sigma^{x_\gamma x_\gamma} + x_\gamma x_Z \sigma^{x_\gamma x_Z} + x_Z^2 \sigma^{x_Z x_Z} + \ldots, \] (21)

where the anomalous parameters appear at most bilinearly.

If one considers all anomalous couplings of eq. (18) 28 coefficients are needed to calculate \( \sigma^{\text{ano}} \). Eq. (21) can also be applied to multi-differential cross-sections. In the search for anomalous couplings multi-differential cross-sections are used, since they contain more kinematical information [16].

**GENTLE** can be used to calculate the coefficients in eq. (21). This can be done for the differential cross-section and in the CC03 process also for the bin-wise integrated differential cross-section. By setting the number of bins to 2, one gets predictions for forward (\( \cos \theta > 0 \)) and backward (\( \cos \theta < 0 \)) scattering. A study of the sensitivity of the forward-backward asymmetry to pairs of anomalous couplings was performed for the pairs \( (x_\gamma, \delta_Z) \) and \( (x_\gamma, z_Z) \) in [11] and for \( (x_\gamma, x_Z) \) and \( (x_Z, z_Z) \) in [17]. The forward-backward asymmetry proved to be useful for studies which include the parity violating parameter \( z_Z \).
4 Conclusions

I gave a short report over the present state of GENTLE and the studies of differential cross-sections and anomalous couplings with it. It was shown that radiative corrections (initial state radiation) give sizeable effects to the differential cross-section at a center-of-mass energy of 500 GeV. This is especially important in the region of backward scattering, where the cross-section is small and the corrections are about 30%.

Acknowledgement

I would like to thank the organizers of the conference for their hospitality and the very pleasant atmosphere they provided. Further, I am especially grateful to T. Riemann for the collaboration on this topic.

A GENTLE

GENTLE version 2 is a Fortran package to calculate cross-sections for 4 fermion production processes for charged currents (CC) and neutral currents (NC) with the semi-analytical method. Table 1 gives an overview over the different branches of GENTLE and the publications they are based on.

|      | QED ISR total cross-section | [18] |
|------|-----------------------------|------|
| CC   | Background total cross-section | [10] |
|      | Anomalous couplings          | [11] |
|      | Differential cross-section   | [11] |
| NC   | QED ISR total cross-section  | [19] |
|      | Background total cross-section | [20] |

Table 1: Overview over the different branches of GENTLE.

While in GENTLE version 2 the calculation of the differential cross-section and the effects of anomalous couplings were only available for the signal diagrams, in the newer version 2.01 these features were extended to the complete CC11 class.

References

[1] S. L. Glashow, *Nucl. Phys.* 22 (1961) 579;
    S. Weinberg, *Phys. Rev. Lett.* 19 (1967) 1264;
    A. Salam, “Weak and Electromagnetic Interactions”, in *Proc. of the Nobel Symposium,*
1968, Lerum, Sweden (N. Svartholm, ed.), pp. 367–377, Almqvist and Wiksell, Stockholm, 1968.

[2] G. Altarelli, T. Sjöstrand, and F. Zwirner (eds.), “Physics at LEP2”, CERN report CERN 96–01 (1996).

[3] ECFA/DESY LC Physics Working Group Collaboration, E. Accomando et al., Phys. Rept. 299 (1998) 1.

[4] R. Clare, “LEP Electroweak Physics Results”, these proceedings.

[5] D. Bardin, J. Biebel, D. Lehner, A. Leike, A. Olchevski, and T. Riemann, Comput. Phys. Commun. 104 (1997) 161.

GENTLE is available at http://www.ifh.de/theory/publist.html.

[6] V. Flambaum, I. Khriplovich, and O. Sushkov, Sov. J. Nucl. Phys. 20 (1975) 537–540; W. Alles, C. Boyer, and A. J. Buras, Nucl. Phys. B119 (1977) 125.

[7] Y.-S. Tsai and A. C. Hearn, Phys. Rev. 140 (1965) B721–B729.

[8] T. Muta, R. Najima, and S. Wakaizumi, Mod. Phys. Lett. A1 (1986) 203.

[9] F. A. Berends, R. Pittau, and R. Kleiss, Nucl. Phys. B424 (1994) 308–342;
D. Bardin, M. Bilenky, D. Lehner, A. Olchevski, and T. Riemann, Nucl. Phys. (Proc. Suppl.) 37B (1994) 148–157.

[10] D. Bardin and T. Riemann, Nucl. Phys. B462 (1996) 3–28.

[11] J. Biebel and T. Riemann, “Off-shell W Pair Production with Anomalous Couplings: The CC11 Process”, preprint DESY 98–047 (1998), hep-ph/9805355.

[12] W. Beenakker et al., “WW cross-sections and distributions”, in Physics at LEP2, CERN 96–01 (1996) (G. Altarelli, T. Sjöstrand, and F. Zwirner, eds.), pp. 79–139, and references therein;
W. Beenakker and A. Denner, these proceedings;
A. Vicini, these proceedings.

[13] K. J. F. Gaemers and G. J. Gounaris, Z. Phys. C1 (1979) 259.

[14] K. Hagiwara, R. D. Peccei, D. Zeppenfeld, and K. Hikasa, Nucl. Phys. B282 (1987) 253.

[15] H. Aronson, Phys. Rev. 186 (1969) 1434–1441.

[16] G. Gounaris et al., “Triple gauge boson couplings”, in Physics at LEP2, CERN 96–01 (1996) (G. Altarelli, T. Sjöstrand, and F. Zwirner, eds.), pp. 525–576.
[17] J. Biebel and T. Riemann, “Semianalytic predictions for W pair production at 500 GeV”. Contribution to: R. Settles (ed.), Proc. of ECFA/DESY Study on Physics and Detectors for the Linear Collider, DESY 97-123E, pp. 139–142, hep-ph/9709207; J. Biebel, “Four fermion production with anomalous couplings at LEP-2 and NLC”, talk at XIIth International Workshop on High Energy Physics and Quantum Field Theory, 4–10 Sep 1997, Samara, Russia, DESY 97-219 (1997), hep-ph/9711439.

[18] D. Bardin, A. Olshevsky, M. Bilenky, and T. Riemann, Phys. Lett. B308 (1993) 403–410. E: ibid., B357 (1995) 725, hep-ph/9507277.

[19] D. Bardin, D. Lehner, and T. Riemann, Nucl. Phys. B477 (1996) 27–58.

[20] D. Bardin, A. Leike, and T. Riemann, Phys. Lett. B344 (1995) 383–390.