Vertical bending vibration analysis of the car body of railway vehicle

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Abstract. The paper features the analysis of the symmetrical vertical bending of the car body railway vehicle, when vehicle is running along on a track with vertical irregularities described in the form of the power spectral density (PSD). The analysis relies on the numerical simulations results, developed based on a rigid-flexible coupled vehicle model, with seven-degree freedom, where the car body is modelled as an Euler-Bernoulli type equivalent beam. The vibration behaviour of the car body is evaluated based on the PSD of the car body acceleration (PSD acceleration), calculated at the car body centre and above the bogies. The main characteristics of the car body bending vibration are pointed in correlation with velocity, the secondary suspension damping and the car body elasticity characteristics.

1. Introduction
The issue of the flexible vibration of the car body in the rail vehicles has gained in importance along with the emergence of the high speed trains. To reach high velocities with lower energy consumption, the vehicle weight had to be reduced. To this purpose, the car body shells have become lighter and structurally simple. However, the lighter the vehicle car body, the more easily excited the flexible vibration modes. At high velocities, the car body vibration can be affected by the flexible vibration, and the ride comfort is becoming worse [1].

In some cases, the car body flexible vibrations represent half of the vibrations felt by the passengers, while the rest is due to the rigid modes of vibration [2]. The flexible vibration also causes car body fatigue which affects the dynamic performances and service life of the vehicle [3, 4].

Car body flexible vibrations are very complex, but the fundamental bending mode of the car body is the most interesting because to its influence upon the ride comfort [5, 6]. The frequency of this vibration mode is commonly within the interval from 6 to 12 Hz, very sensitive for the human body in respect to the vibration.

To identify improvement solutions for the ride comfort in high speed rail vehicles, it is foremost to understand the effect that the car body flexibility has on the vehicle dynamic response. The paper herein examines the characteristics of the car body vertical bending vibration by means of the frequency response of the vehicle calculated for high velocities, the suspension damping and the car body bending module.
2. Railway vehicle mechanical model and motion equations

Railway vehicle is depicted by means of coupled rigid-flexible model type (see figure 1), where the car body is modelled by a free-free beam with constant cross-section and mass per unit length, according to the Euler-Bernoulli hypotheses, whereas the two chassis of bogie and the four wheelsets have considered as rigid [7]. The vehicle suspension stages are modelled through Kelvin-Voigt type systems. The vehicle is running along a rigid track exhibiting vertical geometric defects at the velocity \( V \); the track defects are the inputs of the vehicle model via wheelsets and they are shifted according to the four wheelsets position, \( \eta_{1...4} \).

![Figure 1. The railway vehicle mechanical model.](image)

The vibration vertical rigid modes in the car body are taken into account - bounce \( z_c \), pitch \( \theta_c \), and the car body vertical bending mode – symmetrical bending. The bogies only have rigid vibration modes, namely bounce \( z_b \) and pitch \( \theta_b \). The car body inertia in relation to the vibration rigid modes is represented by mass \( m_c \) and the inertia moment \( J_c \). Each bogie has a mass \( m_b \) and the inertia moment \( J_b \). The car body wheelbase is \( 2a_c \), and the bogie’s is \( 2a_b \). The distances \( l_{1,2} \) settle the supporting points position of the car body on the secondary suspension.

The Euler-Bernoulli beam parameters are: \( L_c \) – beam length; \( \rho_c = m_c/L_c \) – beam mass per length unit; \( \mu \) - damping constant of the car body structure; \( EI \) – bending modulus, where \( E \) is Young’s modulus, and \( I \) is the second area moment of the cross-section.

Two Kelvin-Voigt systems help with the modelling of the secondary suspension associated with a bogie. The parameters for the secondary suspension model are as follows: \( 2k_{zb} \) - the vertical stiffness, \( 2k_{zt} \) – the longitudinal stiffness, \( 2c_{zt} \), and \( 2c_{zb} \) – the corresponding damping coefficients. The car body - bogies connecting system for longitudinal forces is modelled using a Kelvin-Voigt system situated at \( h_c \) bellow the neutral axis of the car body and at \( h_b \) above gravitational centre of the bogie. Also, Kelvin-Voigt system of stiffness \( 2k_{zb} \) and damping coefficient \( 2c_{zb} \) models the primary suspension.

At any car body point, the displacement of the car body \( w_c(x, t) \) is given as a sum of bounce, pitch effect and bending, respectively

\[
w_c(x, t) = z_c(t) + (x - L_c / 2)\theta_c(t) + X_c(x)T_c(t),
\]

where \( T_c(t) \) is the time-dependent function and \( X_c(x) \) is the eigenfunction corresponding to the bending of the car body,
\[ X_c(x) = \sin \beta x + \sinh \beta x - (\sin \beta L_c - \sinh \beta L_c) (\cos \beta x + \cosh \beta x) / (\cos \beta L_c - \cosh \beta L_c), \] 
\[ \beta = 4 \omega_c^2 \rho_c / (EI), \cos \beta L_c \cosh \beta L_c - 1 = 0, \] 
where \( \omega_c \) is the natural angular frequency.

The motion equation of the car body vehicle has the general form as below
\[ EI \frac{\partial^4 w_c(x,t)}{\partial x^4} + \mu J \frac{\partial^5 w_c(x,t)}{\partial x^5} + \rho_c \frac{\partial^2 w_c(x,t)}{\partial t^2} = \sum_{i=1}^{2} F_{cxi} \delta(x-l_i) + \sum_{i=1}^{2} h_c F_{cxi} \frac{d\delta(x-l_i)}{dx}, \] 
where \( \delta(.) \) is Dirac delta function, and \( F_{cxi} \) and \( F_{cxi} \) are the following forces
\[ F_{cxi} = -2K_{xc} \left( \frac{\partial w_c(l_i,t)}{\partial t} - \dot{z}_{bi} \right) - 2K_{zc} \left[ w_c(l_i,t) - z_{bi} \right]; \] 
\[ F_{cxi} = 2K_{xc} \left( h_c \frac{\partial^2 w_c(l_i,t)}{\partial x \partial t} + h_b \dot{\theta}_{bi} \right) + 2K_{zc} \left( h_c \frac{\partial w_c(l_i,t)}{\partial x} + h_b \theta_{bi} \right). \]

To determine the car body equations of bounce, pitch and bending, the modal analysis method is applied, while taking into account the orthogonality property of the eigenfunction of the vertical bending. Therefore, the equation (4) changes into three differential second order equations:
\[ m_c \ddot{z}_{b1} = \sum_{i=1}^{2} F_{xbi} - F_{cxb1}; \quad m_c \ddot{z}_{b2} = \sum_{j=3}^{4} F_{xbj} - F_{cxb2}, \] 
\[ I_b \ddot{\theta}_{b1} = \alpha_b \sum_{j=1}^{2} (-1)^{j+1} F_{xbj} - h_b F_{cxb1}; \quad I_b \ddot{\theta}_{b2} = \alpha_b \sum_{j=3}^{4} (-1)^{j+1} F_{xbj} - h_b F_{cxb2}, \] 
where \( F_{xbj} \) are the following forces,
\[ F_{xb1,2} = -2K_{zb} \left( \dot{z}_{b1} \pm a_b \theta_{b1} - \eta_{1,2} \right) - 2k_{zb} \left( z_{b1} \pm a_b \theta_{b1} - \eta_{1,2} \right), \] 
\[ F_{xb3,4} = -2K_{zb} \left( \dot{z}_{b2} \pm a_b \theta_{b2} - \eta_{3,4} \right) - 2k_{zb} \left( z_{b2} \pm a_b \theta_{b2} - \eta_{3,4} \right), \] 

Hence results a 7-equation system with ordinary derivatives (eq. 7-11), which can be solved numerically using MATLAB code.

3. The frequency response functions of the car body
Further on, the frequency response functions of the acceleration in three car body reference points are calculated. According to figure 1, the reference car body points are located along the car body neutral axis, respectively; the point \( C \) – at the centre of the car body, and the points \( B_{1,2} \) – above the front and rear bogies, respectively.

The PSD acceleration at the three points is
\[ G_c(\omega) = G(\omega) |H_c(\omega)|^2, \quad G_{B_{1,2}}(\omega) = G(\omega) |H_{B_{1,2}}(\omega)|^2, \]
where \( H_c(\omega) \) and \( H_{B_{1,2}}(\omega) \) are given by the equations
\[ H_c(\omega) = \omega^2 \left[ H_{xc}(\omega) + X_c(L_c/2) H_{xf}(\omega) \right], \] 
\[ H_{B_{1,2}}(\omega) = \omega^2 \left[ H_{xc}(\omega) \pm a_c H_{\theta_i}(\omega) + X_c(l_{1,2}) H_{\theta_i}(\omega) \right]. \]
where $\bar{H}_{cc}(\omega)$, $\bar{H}_{bc}(\omega)$, $\bar{H}_{tc}(\omega)$ are the frequency response functions for the three components of the car body displacement. $G(\omega)$ is the PSD of the track irregularities,

$$G(\omega) = \frac{A\Omega^2 V^3}{[\omega^2 + (\Omega c)^2][\omega^2 + (\Omega r)^2]},$$

where the parameters $\Omega_c$, $\Omega_r$ are determined experimentally; the parameter $A$ describes the level of the track quality [8].

4. The results of the numerical simulations

This section shows the outcomes of the numerical calculation concerning the frequency response functions of the car body during the circulation along a track of low quality ($A = 1.080 \times 10^{-6}$ rad/m). Based on these results, the characteristics of the car body vertical bending are examined, in dependence of the velocity, the damping ratio of the secondary suspension and the car body bending module. The vehicle reference parameters are included in table 1. According to these parameters, the bounce, pitch and bending of the car body have the following natural frequencies: 1.17 Hz, 1.46 Hz and 8.20 Hz, respectively.

Table 1. The reference parameters of the numerical model.

| Parameter | Value |
|-----------|-------|
| $m_c$     | 34.0 \times 10^3 kg |
| $k_{zc}$  | 0.60 MN/m |
| $k_{mc}$  | 89.0 MN/m |
| $m_b$     | 3.20 \times 10^3 kg |
| $k_{zc}$  | 2.00MN/m |
| $c_{mc}$  | 53.1 kN/m/s |
| $m_{mc}$  | 35.2 \times 10^3 kg |
| $c_{zc}$  | 17.15 kNs/m |
| $L_c$     | 26.4 m |
| $J_c$     | 1.96 \times 10^6 kg\cdot m^2 |
| $c_{xc}$  | 25.0 kNs/m |
| $h_c$     | 1.30 m |
| $h_b$     | 0.20 m |
| $J_b$     | 2.05 \times 10^5 kg\cdot m^2 |
| $k_{xb}$  | 2.00MN/m |
| $2a_c$    | 19.0 m |
| $2a_b$    | 2.56 m |

Figure 2 shows the PSD acceleration in the three points of the car body mentioned above for different velocities. At a higher velocity, the magnitude of vibrations in all points considered is noticed to amplify at both the resonance frequency of bounce and also of the vertical bending. Similarly, it is obvious that the vibration due to bending is more important at the car body centre as they dominate the car body response at high velocities.

Figure 2. PSD acceleration – influence of the velocity: (a) at the point $C$; (b) at the point $B_1$; (c) at the point $B_2$.

To analyze the influence of the secondary suspension damping upon the car body bending vibration (see figure 3), the damping degree is introduced $\zeta_c = c_{zc}/k_{zc}m_{mc}^{1/2}$. When the damping degree is higher, a reduction in the bounce vibrations of the car body can be observed. On the other hand, an increase in
the damping value triggers amplification in the vibration due the bending of the car body. A more detailed analysis, as seen in figure 4, shows that the level of vibration due to bending decreases along with a growth in damping until reaching a minimum value, in the range of low damping. Beyond that value, the magnitude of the vibration becomes higher as the damping increases.

Figure 3. PSD acceleration at 200 km/h – influence of the damping ratio of the secondary suspension: (a) at the point C; (b) at the point B₁; (c) at the point B₂.

Figure 4. PSD acceleration at the bending resonance frequency – influence of the velocity and the damping ratio of the secondary suspension: (a) at the point C; (b) at the point B₁; (c) at the point B₂.

Figure 5. PSD acceleration – influence of the car body bending module: (a) at the point C; (b) at the point B₁; (c) at the point B₂.
A higher car body bending module brings about an increase in the resonance frequency of the car body bending and a diminution in the magnitude of car body vibration (see figure 5). At the centre of the car body (figure 5, a), the reduction of the vibration level when increasing the bending modulus is much more pronounced than above the bogies. The vibration magnitude due to the bending is noticed to have a significant decrease when the bending frequency rises from 6 to 10 Hz. The continuous increase of the bending frequency at 12 Hz no longer brings important changes.

5. Conclusions
The paper features the analysis regarding the characteristics of the bending vibration in the rail vehicle car body in correlation with velocity, secondary suspension damping and the car body elasticity characteristics. The results derived from numerical simulations concerning the PSD acceleration of the car body have shown that the bending vibrations are higher at the car body centre and at high speeds they dominate the car body response. The level of the bending vibrations increases for high values in the secondary suspension damping. On the other hand, a value of the secondary suspension damping degree that minimizes the car body bending vibrations can be identified. When the car body is rigid, the resonance frequency in the car body bending increases and the magnitude of vibration significantly decreases at the centre of the car body.

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References
[1] Dumitriu M 2017 Vehicle System Dynamics 55(11) 1787-1806
[2] Carlbom P 2000 Car body and passengers in rail vehicle dynamics, Doctoral Thesis, Department of Vehicle Engineering, Royal Institute of Technology, Stockholm, Sweden.
[3] Hui C, Weihua Z and Bingrong M 2015 International Journal of Vehicle Structures & Systems 7(2) 55–60
[4] Yang G, Wang C, Xiang F and Xiao S 2016 Chinese Journal of Mechanical Engineering 29(6) 1120-6
[5] Diana G, Cheli F, Collina A, Corradi R and Melzi S 2002 Vehicle System Dynamics 38(3) 165-83
[6] Dumitriu M and Cruceanu 2017 Archive of Mechanical Engineering 64(2) 119-238
[7] Dumitriu M and Gheți M A 2018 IOP Conf. Series: Materials Science and Engineering 444, 042001.
[8] C 116 1971 Interaction between vehicles and track, RP 1, Power spectral density of track irregularities, Part 1: Definitions, conventions and available data, Utrecht.