Performance Investigation of $H_\infty$ Controller for Quarter Car Semi-active Suspension System using Simulink

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Abstract
This paper affords the design and improvement of a semi-active suspension system for an automobile. The main idea is to increase the semi-active suspension system damping vibration of the automobile body even as crossing the bump and sine pavement on the road. This system is modelled for 1/4 car system after which the entire system has been simulated using Matlab/Simulink. It is used to physically simulate the quarter vehicle system of the automobile and have a look at the time domain response to the road disturbances. $H_\infty$ controllers is used to govern the damping properties of the semi-active suspension system mechanically. The system is designed in contrast to the most of the available suspension systems using a third order hydraulic actuator. The proposed system is compared with $H_2$ optimal controller to test the performance of the system for the control targets suspension deflection, body acceleration and body travel for the bump and sine road disturbances. The simulation result of this study reveals the efficiency of the advanced $H_\infty$ controller for the quarter car semi-active suspension system.

Keywords: Quarter Car Semi-active Suspension System, $H_\infty$ Controller, $H_2$ optimal Controller
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I.INTRODUCTION
Suspension systems are labeled into three types: Passive, Semi Active and Active suspension systems. Passive suspension system includes an electricity dissipating detail, that's the damper, and an energy-storing detail, which is the spring. Since these two factors cannot add electricity to the system this form of suspension structures are called passive.

Sensors constantly display the running situations of the automobile body. Based at the alerts received via the sensors and prescribed manipulate strategy the pressure inside the actuator is modulated to obtain progressed experience and coping with. It need to be cited, that an active suspension system calls for outside power to function, and that there may be additionally a large penalty in complexity, reliability, cost and weight.

Semi-active suspension is a type of automotive suspension systems that controls the damping pressure of the shock absorber in reaction to input from the constantly varying road surfaces. It is intended to approximately implement the active suspension with a damping force adjustable shock absorber.

II.Mathematical MODELS
A. Semi-active Suspension System Mathematical Model
The design of the semi-active suspension block diagram is shown in Figure 1 below.

\[
K_s (Z_u - Z_s) - B_s (\ddot{Z}_u - \ddot{Z}_s) = M_s \dddot{Z}_s \tag{1}
\]

\[
-K_s (Z_u - Z_s) - B_s (\ddot{Z}_u - \ddot{Z}_s) + K_u (Z_r - Z_u) = M_u \dddot{Z}_u \tag{2}
\]
Where Zs is the position of the sprung mass, Zu is the position of the unsprung mass and Zr is the road displacement. These equations are solved numerically using MATLAB’s dynamic system simulation software, SIMULINK.

**B. Hydraulic System Transfer Function**
The hydraulic actuator system is a third order system with transfer function of the form:

\[
\text{Hydraulic Actuator} = \frac{s + 5}{s^3 + 4s^2 + 25}
\]  

(3)

**III. ROAD PROFILES**
Four types of road disturbance input are used to simulate the semi-active suspension system road conditions.

**A. Bump Road Disturbance:**
The bump input road disturbance is shown in Figure 2.

![Fig. 2. Bump road disturbance](image1.png)

**B. Sine Pavement Road Disturbance:**
The sine wave input road disturbance is shown in Figure 3.

![Fig. 3. Sine Input pavement road disturbance](image2.png)

**IV. THE PROPOSED CONTROLLER DESIGN**

**A. H∞ Controller Design**
A control system is robust if it remains stable and achieves certain performance criteria in the presence of possible uncertainties. The robust design is to find a controller, for a given system, such that the closed-loop system is robust.

The objective is to find a stabilizing controller K to minimize the output z, in the sense of energy, over all w with energy less than or equal to 1. Thus, it is equivalent to minimizing the H infinity-norm of the transfer function from w to z as shown in Figure 4.
B. **H2 Optimal Controller Design**

There are many ways in which feedback design problems can be cast as H2 optimization problems. It is very useful therefore to have a standard problem formulation into which any particular problem may be manipulated. Such a general formulation is afforded by the general configuration shown in Figure 5.

![Fig. 5. General control configuration of H2 optimal controller](image)

V. **Result and Discussion**

The semi-active suspension system parameter is shown in Table I

| Model parameters            | symbol | symbol Values |
|-----------------------------|--------|---------------|
| Vehicle body mass           | ms     | 390 Kg        |
| Wheel assembly mass         | mu     | 63 Kg         |
| Suspension stiffness        | ks     | 10,000 N/m    |
| Tire stiffness              | ku     | 110,000 N/m   |
| Suspension damping          | Bs     | 1890 N-s/m    |

A. **Simulation of the Proposed Controllers**

The semi-active suspension system with H infinity and H2 optimal controllers are simulated using Matlab/Simulink for the control targets suspension deflection, body acceleration and body travel using bump and sine pavement road disturbances.

B. **Simulation of a Bump Road Disturbance:**

The Simulink model for a bump input road disturbance is shown in Figure 6.

![Fig. 6. Simulink model of the proposed controllers for bump road profile](image)

The H infinity and H2 optimal controller subsystem is shown in Figure 7 and Figure 8 respectively.
The simulation results for the control targets suspension deflection, body acceleration and body travel using bump road disturbances is shown in Figure 9, Figure 10 and Figure 11 respectively.
C. Simulation of a Sine Input Pavement Road Disturbance:
The Simulink model for a sine pavement input road disturbance is shown in Figure 12.

Fig. 12. Simulink model for a Sine input pavement road disturbance
The simulation results for the control targets suspension deflection, body acceleration and body travel using sine pavement road disturbances is shown in Figure 13, Figure 14 and Figure 15 respectively.

Fig. 13. Suspension deflection for Sine input pavement road disturbance
Fig. 14. Body acceleration for Sine input pavement road disturbance
D. Numerical Comparison for Bump Road Disturbance:
The amplitude of the semi-active suspension system with H infinity and H2 optimal controllers for bump road disturbance is shown in Table II.

| Parameters       | $H_2$        | $H_\infty$ |
|------------------|--------------|------------|
| Suspension Deflection | 0.144 m     | 0.1 m      |
| Body Acceleration    | $2.8 \times 10^{-9} \frac{m}{s^2}$ | $2 \times 10^{-9} \frac{m}{s^2}$ |
| Body Travel         | 0.27 m       | 0.15 m     |

E. Comparison for Sine Pavement Road Disturbance:
The amplitude of the semi-active suspension system with H infinity and H2 optimal controllers for sine pavement road disturbance is shown in Table III.

| Parameters       | $H_2$        | $H_\infty$ |
|------------------|--------------|------------|
| Suspension Deflection | 0.15 m     | 0.1 m      |
| Body Acceleration    | $5 \times 10^{-9} \frac{m}{s^2}$ | $3.5 \times 10^{-9} \frac{m}{s^2}$ |
| Body Travel         | 0.27 m       | 0.15 m     |

VI. Conclusion
The H infinity controller successfully controlled the semi active suspension. When compared to the semi-active suspension system with H2 optimal controller, semi-active suspension system with H infinity controller substantially decreased the sprung mass displacement and therefore increased ride comfort of the automobile. Finally the simulation results prove the effectiveness of the semi-active suspension system with H infinity controller.

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