I. INTRODUCTION

The quest for the unification of inflation, dark matter, and dark energy, in different combinations, by a single field, has been studied in Refs. [1–15]. The main motivation behind all proposals is that we do not yet understand the nature of the components responsible for the three phenomena, but we do know that their special properties are beyond the realm of the ordinary matter described by the standard model of particle physics.

An extreme, most economical, possibility is that all three phenomena can be explained by the existence of one single field. As was first put forward in Ref. [8], the simplest option at hand is a scalar field one single field. As was first put forward in Ref. [8], the three phenomena can be explained by the existence of a scalar field, has been studied in Refs. [1–15]. The main motivation behind all proposals is that we do not yet understand the nature of the components responsible for the three phenomena, but we do know that their special properties are beyond the realm of the ordinary matter described by the standard model of particle physics.

The viability of some phenomenological models of reheating for unification schemes was explored in Refs. [2, 8]. Some other, more detailed proposals have been explored in the literature, from extra periods of inflation at low energy scales [8], to the more popular gravitational particle production at the end of inflation [e.g. 1, 11, 16–18].

This reheating mechanism is largely ineffective, however, because the relative proportion of radiation to scalar field \( \rho_{rel}/\rho_\phi \) evolves at the same rate as the proportion of gravitational waves to scalar field \( \rho_{GW}/\rho_\phi \). In the case of steep potentials, inflation is followed by a kination era (a possibility first explored in the context of electroweak baryogenesis [20]). If the kinetic energy of the scalar field dominates for some time, the relative density of gravitational waves created during inflation can grow significantly and violate the observational bounds. Specifically, the process of Big Bang Nucleosynthesis (BBN) imposes a condition on the energy density of gravitational waves \( \rho_{GW} \). BBN requiring that

\[
\rho_{GW}(t_{BBN}) < 0.2 \rho_{rel}(t_{BBN}).
\]

In fact, the BBN bound in Eq. (1) has been used to constrain and exclude some models of braneworld inflation and other unification models [22].

In this paper, we explore an alternative mechanism to reheat the Universe, exploiting the evaporation of primordial black holes (PBHs) produced at the end of the inflationary phase [23, 24], and study its usefulness in unification models. A small initial population of tiny black holes at the right time would produce the required amount of radiation more effectively than, for instance, the process of quantum particle production in an expanding space-time.

In the PBH reheating scenario, the black hole component effectively reduces the relative density of gravitational waves as

\[
\frac{\rho_{GW}}{\rho_{PBH}} \propto \left( \frac{a}{a_{end}} \right)^{-1}. 
\]

Reheating takes place when the energy density \( \rho_{PBH} \) is transformed into relativistic particles. Therefore, even if the densities \( \rho_{GW} \) and \( \rho_{PBH} \) were equivalent at the end of inflation, we would only require 3 e-folds of kinetic
energy domination to meet the nucleosynthesis requirements. PBH reheating therefore solves the problem described above. Another advantage of reheating through the evaporation of PBHs is that the unification field does not need to be coupled to other matter fields, hence it can later behave as a totally uncoupled dark matter component. PBH reheating for unification purposes was first considered in Ref. [2]; see also Ref. [13].

Our aim here is to determine the general conditions under which the unification of inflation with either, or both, of the dark fields can achieve the appropriate reheating with the aid of PBHs and set up the required conditions before BBN.

The featured model of PBH reheating is described in detail in Sec. III. Together with this description, we establish the conditions that the scalar field potential must fulfill in order to undergo a successful PBH reheating. A particular example of our proposal is illustrated in Sec. III. Finally, in Sec. IV we frame our proposal in the context of unification models and draw some concluding remarks.

II. PBH REHEATING

There are three basic requirements for a successful PBH reheating:

i) a significant number of PBHs produced at the end of inflation,

ii) that PBHs become the dominant component in the Universe, and

iii) the evaporation of the PBHs after they become the dominant component and before the time of BBN.

The sequence described above introduces some tuning. We require the formation of black holes massive enough to last for a few e-folds while the Universe is dominated by the scalar field kinetic energy. Additionally, the same PBHs must evaporate before BBN, or before any prior well-tested process at higher energies in the standard Big Bang model. Each one of these conditions can be used to constrain the free parameters of the reheating process.

A. Production of PBH and domination

In recent years, several works have demonstrated the compatibility of the observed power spectrum of matter fluctuations at cosmological scales with the formation of a significant number of PBHs at the end of inflation (\( t_{\text{end}} \)), see e.g. Ref. [27], or indeed due to preheating [26]. Moreover, the relative growth of \( \rho_{\text{PBH}} \) with respect to other components is largest for PBHs formed at \( t_{\text{end}} \). This justifies considering the black holes formed right after the end of inflation as those responsible for reheating. PBHs could also be originated by features in the inflationary potential. In the inflationary phase, these features break the slow-roll conditions momentarily, and may enhance the amplitude of fluctuations and black hole production at a specific scale after the end of inflation [e.g. 24]. This presents an alternative to the production mechanism considered here.

To quantify the number of black holes at the end of inflation, it is customary to define with \( \beta \) the initial mass fraction of PBHs at the time of formation \( t_{\text{end}} \). The Press–Schechter formalism of structure formation prescribes that \( \beta \) is a function of the variance \( \sigma \) of matter fluctuations. If we define \( \delta \) as the Gaussian probability distribution of \( \delta \), with \( \sigma \) its variance at that particular scale, then

\[
\beta(\sigma) = f_M \int_{\delta_{\text{min}}}^{\infty} P(\delta, \sigma) d\delta ,
\]

where \( \delta_{\text{min}} \simeq w_{\text{end}} [28] \), and \( w_{\text{end}} = w_{\phi}(t_{\text{end}}) \) is the equation of state at the end of inflation, which coincides with that of the inflaton field which is by then the dominant matter component. Here, \( f_M \approx 3^{-3/2} \) is the fraction of the horizon mass which constitutes the black hole at the time of formation [29]. The amplitude of threshold inhomogeneities \( \delta_{\text{min}} \) is determined by the equation of state of the dominant energy component at the end of inflation, \( w_{\text{end}} \).

The energy density of the produced PBHs evolves as pressureless matter (i.e. dust, with \( w = 0 \)), so that after time \( t_{\text{end}} \), the ratio of the PBHs to the scalar field energy densities is

\[
\frac{\rho_{\text{PBH}}}{\rho_{\phi}} = \frac{\beta(\sigma)}{1 - \beta(\sigma)} \frac{a(t)}{a_{\text{end}}} \frac{\Delta(N)}{N} = \frac{\beta(\sigma)}{1 - \beta(\sigma)} e^{\Delta(N)} ,
\]

\[
\Delta(N) = 3 \int_0^N w_{\phi}(N') dN' ,
\]

where \( N \) is the number of e-folds elapsed from \( t_{\text{end}} \), \( \sigma \) is the mean amplitude of matter fluctuations, and \( \Delta(N) \) is the (exponential) growth factor of the PBHs. Eq. [1] takes into account the fact that typically the scalar field equation of state does not remain constant after the end of inflation.

The mass fraction of PBHs will always increase as long as \( \Delta(N) > 0 \), that is:

\[
w_{\phi} > 0 , \quad \text{for some } t_{\text{end}} < t < t_{\text{kin}} ,
\]

where \( t_{\text{kin}} \) is the time when the kination era of the scalar field ends and it starts to behave like dark matter or dark energy. For definiteness, we take the equation of state at the end of the kination era to be \( w_{\text{kin}} := w_{\phi}(t_{\text{kin}}) = 0 \).

1 Note that we evaluate \( w_{\text{end}} \) at times shortly after the end of inflation, where \( w \) can have a value between \( w_{\text{inf}} = -1/3 \) right at the end of inflation, and the maximum at the kinetic energy domination \( w = 1 \). Also, the precise value of \( \delta_{\text{min}} \) is dependent of the shape of the initial configuration [25].
and the total growth factor is $\Delta(N_{\text{kin}})$. So far, we are assuming that, as in most cases, the dark field-like behaviour of the inflationary field appears at the onset of oscillations of the field around the minimum of the scalar potential. This need not necessarily be the case, but the final results are insensitive to the precise details of the dark matter/dark energy transition.

It is also clear, from Eq. (4), that for PBH domination we require

$$\ln \beta(\sigma) > -\Delta(N_{\text{kin}}). \quad (6)$$

Whether this condition is satisfied will depend upon the scalar field model and the exact evolution of its equation of state in the post-inflationary era. One can foresee, though, that some models may fail to satisfy condition (6).

Finally, we note that in the PBH reheating process we do not expect a significant PBH merging rate at the phase of domination. This is because the value of $\beta$ is initially much smaller than 1, and the density of PBHs grows predominantly because of the rapidly diluting scalar field energy density $\rho_{\Phi}$. The dominant radiation at the start of the Big Bang is generated by the evaporation products of PBHs. De-}

**B. The inflationary energy scale**

The dominant radiation at the start of the Big Bang is generated by the evaporation products of PBHs. Depending on the initial mass of the black holes, the time of evaporation, $t_{\text{ev}}$, can be shorter than, equal to, or larger than the time at which $\phi$ starts behaving as dark matter/dark energy, $t_{\text{kin}}$ (the relation between these times can be derived directly from the characteristics of the scalar field model under consideration).

For the sake of clarity, let us consider the case in which $t_{\text{ev}} = t_{\text{kin}}$. The evaporation time of PBHs is determined by their mass, which is directly related to the energy scale at the end of inflation, $\rho_{\text{end}} \simeq V_{\text{end}}$.

$$M_{\text{PBH}} = f_M \frac{4}{3} \pi \rho_{\text{end}} H_{\text{end}}^{-3} = 4 \pi \sqrt{3} f_M \left( \frac{\rho_{\text{end}}}{m_{\text{pl}}} \right)^{-1/2} m_{\text{pl}}. \quad (7)$$

where $m_{\text{pl}}$ is the Planck mass. The time of evaporation is given by $\rho_{\text{end}} \simeq V_{\text{end}}$.

$$t_{\text{ev}} = 6.3 \times 10^{-41} \Phi^{-1}(M_{\text{PBH}}) \left( \frac{M_{\text{PBH}}}{m_{\text{pl}}} \right)^3 \text{ sec.} \quad (8)$$

Here $\Phi(M_{\text{PBH}})$ is a function of the directly-emitted species of particles and takes a value of order 10 for the PBH masses of interest [e.g. 33]. Combining Eqs. (7) and (8), we find that

$$t_{\text{ev}} = 2.3 \times 10^{-41} \Phi^{-1}(M_{\text{PBH}}) \left( \frac{\rho_{\text{end}}}{m_{\text{pl}}} \right)^{-6} \text{ sec.} \quad (9)$$

This time must also be the beginning of the standard Hot Big Bang (HBB), at some point before BBN. The last requirement imposes a bound on the initial mass of the black holes and, consequently, on $\rho_{\text{end}}$. Indeed, in view of Eq. (9), the condition $t_{\text{ev}} \leq t_{\text{BBN}} = 1 \text{ sec.}$ implies

$$V_{\text{end}}^{1/4} \geq 1.6 \times 10^{-7} \Phi^{1/6}(M_{\text{PBH}}) m_{\text{pl}}. \quad (10)$$

**C. The right amount of radiation**

On the other hand, we can determine whether radiation domination is achieved after the evaporation of PBHs. Assuming again that the standard HBB is recovered at the time of evaporation, we can write a simple expression for the ratio of PBH to scalar field densities as

$$\left( \frac{\rho_{\text{PBH}}}{\rho_{\Phi}} \right)_{\text{ev}} = \left( \frac{\rho_{\text{rel}}}{\rho_{\text{dm}}} \right)_{\text{ev}} \simeq \left( \frac{g_{\text{ev}} f_{\text{eq}}}{g_{0,\text{eq}} f_{\text{ev}}} \right)^{1/2} \simeq 3 \times 10^{26} \left( \frac{V_{\text{end}}^{1/4}}{m_{\text{pl}}} \right)^3, \quad (11)$$

where we have considered that from $t_{\text{ev}}$ onwards the Universe is radiation dominated, and $g_{\text{ev}}$ ($g_{0,\text{eq}}$) are the relativistic energy degrees of freedom at that time of evaporation (matter–radiation equality), respectively.

Radiation domination is ensured as long as the evolution of the scalar field equation of state allows the matching between Eqs. (4) and (11) at the time of PBH evaporation. We will see that this is possible even when the constraints on the products of PBH evaporation are considered.

**D. Additional constraints**

There are two important bounds on the density of the products of evaporation of PBHs. These can be written in terms of the energy density at the time of black hole formation and, therefore, introduce bounds to the unification inflationary potential. The first constraint comes from the consideration that a PBH could leave behind a Planck mass relic after evaporation. Such relics must not exceed in density the dark matter component [21, 34], i.e., we require

$$\frac{\rho_{\text{rel}}}{\rho_{DM}} < 1. \quad (12)$$

From Eq. (7) we can read the proportion between the PBH mass and the Planck mass of its relic. Therefore the energy density ratio of PBHs with relics is

$$\left( \frac{\rho_{\text{PBH}}}{\rho_{\text{rel}}} \right)_{\text{ev}} = \left( \frac{\rho_{\text{rel}}}{\rho_{\text{DM}}} \right)_{\text{ev}} = 4 \pi \sqrt{3} f_M \left( \frac{m_{\text{pl}}}{V_{\text{end}}^{1/4}} \right)^2. \quad (13)$$

Combining Eqs. (11) and (13), the condition (12) is met when

$$V_{\text{end}}^{1/4} \lesssim 6.8 \times 10^{-6} m_{\text{pl}}. \quad (14)$$
If the condition \( \sigma_{\text{ev}} \) is met after evaporation, then the same ratio of relics to dark matter is maintained at subsequent times.

The combination of Eqs. (10) and (14) leaves but a narrow window for the range of energies at which inflation must end if we want the PBHs to reheat the Universe. The tight constraint can however be relaxed if no Planck mass relics are left over after evaporation.

This is a crucial point for standard (slow-roll) inflationary scenarios, for which the end of inflation takes place at energies \( V_{\text{end}}^{1/4} \sim 10^{-3} m_{\text{pl}} \). PBH reheating in this case is only possible if no Planck relics are left over after evaporation, or if the black holes responsible for reheating are created at energies below \( \rho_{\text{end}}^{1/4} \).

A second constraint comes into play in the form of gravitational waves. When black holes evaporate, a considerable fraction of energy is emitted in the form of gravitons. Paradoxically for the initial motivation of this paper, it appears that in the case of a dominant energy density component of PBHs, a significant amount of gravitational waves is produced at evaporation. This could violate the very constraint we intend to alleviate.

When PBHs dominate and then evaporate, the amount of gravitational radiation produced is \( \Omega_{GW} = 0.36/g_{\text{ev}} \),

\[ \Omega_{GW} = 0.36/g_{\text{ev}}, \quad (15) \]

We are interested, however, in times of evaporation prior to nucleosynthesis, when \( g_{\text{ev}} \geq 10.75 \). Consequently,

\[ \left( \frac{\rho_{GW}}{\rho_{\text{rel}}} \right)_{ev} \sim \Omega_{GW}(t_{ev}) \lesssim 0.03, \quad (16) \]

which readily satisfies the gravitational waves bound in Eq. (1).

III. EXAMPLES

To illustrate our proposal, let us look at some scenarios where PBH reheating can take place. In the model considered in Ref. [2], which intended a unification into a single scalar field of inflation and dark matter only, as in many other braneworld models, inflation ends at an energy scale of \( V_{\text{end}}^{1/4} \approx 10^{-6} m_{\text{pl}} \), which is within the allowed values of constraints (10) and (14). The expansion of the Universe between the end of inflation and the onset of oscillations is such that

\[ \frac{a_{\text{kin}}}{a_{\text{end}}} = 60 \approx e^4, \quad (17) \]

and the scalar field enters a kination phase in between, during which \( w_{\phi} \approx 1 \). This means that the growth factor in Eq. (1) is \( \Delta(N_{\text{kin}}) \approx 12 \). In consequence, Eq. (11) tells us that to reach the domination of PBHs before \( \phi \) starts oscillating, we must have \( \beta > 10^{-6} \). This is in agreement with the results in Ref. [2].

Another example is the quintessential inflation model in [1], for the unification of inflation and dark energy. After inflation, the field goes into a kination era for which the Universe expands \( a_{\text{kin}}/a_{\text{end}} = 10^8 \approx e^{18} \). Thus, the quintessential model can be more easily implemented, as it only requires \( \beta > 10^{-24} \) to generate enough radiation from PBHs. Moreover, if the inflation ends due to an instability of the hybrid inflation kind, then the smaller-scale fluctuations would show an enhanced amplitude [30], rendering a larger probability of PBH formation.

In Fig. 1 we present graphically the valid parameter values for a successful PBH reheating. The curves in both figures indicate the mean amplitude of matter fluctuations, \( \sigma \) in Eq. (15), required for a given energy density (or time) at which the scalar field starts oscillating.

The figures show the importance of the difference between the equation of state right after the end of in-
flation \(w_{\text{end}}\) and at the subsequent kination epoch \(w_{\phi}\). For simplicity, we have assumed that \(w_{\phi} = \text{const. during the kination phase of the scalar field. This is not a strong restriction, as the total growth factor can be written in terms of a mean equation of state as } \Delta(N_{\text{kin}}) = 3w_{\text{mean}}N_{\text{kin}}, \text{ and then the plots in Fig. 4 can be interpreted in terms of } w_{\text{mean}} \text{ too.}

In particular, we see that the small value \(w_{\text{end}} \leq 0.1\) allows the formation of large populations of PBHs with a relatively small variance. Note that the potential of the figure at the bottom violates the constraint imposed by Planck relics, a constraint that, as mentioned above, need not necessarily apply.

By considering a range of values for \(\rho_{\text{kin}}\) above (below) the evaporation scale \(\rho_{\text{ev}}\) in the top (bottom) figure, we are showing that the conditions required for PBH reheating when \(t_{\text{ev}} \neq t_{\text{kin}}\) are not too different from those in the case of equality (described in the previous section).

For the sake of generality we are not computing here the amplitude of fluctuations for specific models at the end of inflation. It suffices to mention that for some models with \(w_{\text{end}} \approx 0\), and under suitable conditions, the production of a considerable amount of PBHs is possible at horizon and subhorizon scales [37, 38].

IV. UNIFICATION MODELS AND FINAL REMARKS

If the unification of inflation with dark matter or dark energy, in any given combination, is to be achieved by a single field, then an efficient reheating process is needed which does not heavily rely upon the decay of the unification field.

We have seen that, in the presence of a large enough population of PBHs, unification models can be possible if the unification field enters a long enough kinetic-dominated era, so that the PBHs come to dominate the matter budget. PBH domination is easier for long kination periods and stiffer values of the effective equation of state \(w_\phi\), but the latter also requires a larger value of the mean amplitude of primordial perturbations \(\sigma\) at the end of inflation.

In this article we have not included an explicit mechanism for the PBH formation, as would be required for a fully self-contained model. Most economical, if possible, would be for the unification field itself to feature perturbations rapidly growing at late times, as may happen if its second derivative becomes large as inflation ends. Alternatively, a phase transition (for instance of hybrid inflation form) may lead to enhanced perturbations as it takes place [e.g. 24, 36, 39]. In any case, it is worth mentioning that the mechanism required for PBH formation need not spoil or interfere with the dynamics followed by a unification model in the dark matter or dark energy eras.

Because PBHs form right after the end of inflation, most of the observational restrictions can be written in terms of the energy scale at the end of inflation. In the simplest PBH scenario, the constraints are satisfied only if this energy scale is a few orders of magnitude below the Planck value. One should bear in mind that the possibility of production of Planck relics renders PBH reheating incompatible with standard inflationary models, unless the energy scale at the end of inflation happens to be of the order of \(10^{-6} m_{\text{Pl}}\).

The strongest constraint on the featured mechanism comes from the evolution of the scalar field equation of state after inflation. The unification model should have a long enough kination period to allow PBH domination before evaporation. Typical examples are braneworld inflation with an exponential potential [2, 10] and quintessential inflation [1]. However, even if the Planck relic constraint could be set aside, this is a condition that many inflationary models will not be able to surpass.

Our results rely heavily on the assumption that reheating proceeds from PBHs formed right at the end of inflation. The results can change if PBHs are formed after the end of inflation, or if we consider the kination period to happen beyond BBN (a possibility explored in Ref. [11]). Therefore, the parameters of the theory can be flexible beyond the constraints presented here.

ACKNOWLEDGMENTS

J.C.H. gratefully acknowledges sponsorship from DGAPA-UNAM and the support of PAPIIT-UNAM (grant IN116210-3). A.R.L. was supported by the Science and Technology Facilities Council [grant number ST/1000976/1]. L.A.U.-L. thanks the Berkeley Center for Cosmological Physics (BCCP) for its kind hospitality, and the joint support of the Academia Mexicana de Ciencias and the United States-Mexico Foundation for Science for a summer research stay at BCCP. This work was partially supported by PROMEP, DAIP-UG, and by CONACyT México under grants 56946, 167335 and 10101/131/07 C-234/07 of the Instituto Avanzado de Cosmología (IAC) collaboration.

[1] P. J. E. Peebles and A. Vilenkin, Phys.Rev. D59, 063505 (1999), arXiv:astro-ph/9810509
[2] J. E. Lidsey, T. Matos, and L. A. Urena-Lopez, Phys. Rev. D66, 023514 (2002), arXiv:astro-ph/0111292
[3] T. Padmanabhan and T. R. Choudhury, Phys.Rev. D66, 081301 (2002), arXiv:hep-th/0203055
[4] R. J. Scherrer, Phys.Rev.Lett. 93, 011301 (2004), arXiv:astro-ph/0402316
