Abstract. This paper presents the results from collaboration between the National Physical Laboratory (NPL) and the University of Sheffield on an ongoing research project at NPL. A 50 year old reinforced concrete footbridge has been converted to a full scale structural health monitoring (SHM) demonstrator. The structure is monitored using a variety of techniques; however, interrelating results and converting data to knowledge are not possible without a reliable numerical model. During the first stage of the project, the work concentrated on static loading and an FE model of the undamaged bridge was created, and updated, under specified static loading and temperature conditions. This model was found to accurately represent the response under static loading and it was used to identify locations for sensor installation. The next stage involves the evaluation of repair/strengthening patches under both static and dynamic loading. Therefore, before deliberately introducing significant damage, the first set of dynamic tests was conducted and modal properties were estimated. The measured modal properties did not match the modal analysis from the statically updated FE model; it was clear that the existing model required updating. This paper introduces the results of the dynamic testing and model updating. It is shown that the structure exhibits large non-linear, amplitude dependant characteristics. This creates a difficult updating process, but we attempt to produce the best linear representation of the structure. A sensitivity analysis is performed to determine the most sensitive locations for planned damage/repair scenarios and is used to decide whether additional sensors will be necessary.

1. Introduction

The footbridge transformed to SHM demonstrator is at the heart of the series of projects run at NPL. The bridge was in use for over 40 years at NPL. During the demolition of the buildings it connected in 2007, the bridge was moved to its current location suitable for destructive testing. It has become a truly multidisciplinary collaborative project with more that 40 companies and academic institutions involved and working together over the last two years. The main aims of the project are to:

- provide test facilities for long-term all year around performance of the sensors covering 18 different technologies employed for continuous monitoring and short-term testing
- demonstrate various advanced real-life monitoring technologies for asset holders and maintenance companies
- contribute to understanding how to assess the state of the structure and the ways of extending safe usage of ageing infrastructure.

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More details about the project can be found in [1-2] and summary of technologies shown in figure 1.

| Technology                          | Monitoring | Measurand                      | contributors                        |
|-------------------------------------|------------|--------------------------------|-------------------------------------|
| 1 Digital Image Correlation         | Survey     | Full field strain, displacement| NPL                                 |
| 2 Laser scan                        | Survey     | Shape                          | John Moore University               |
| 3 Optical strain gauge technique    | Tests only  | Displacement                   | Imetrum                             |
| 4 Optical Fiber Bragg Grating       | Continuous | Strain, RH                      | SmartFibers, City University         |
| 5 Acoustic Emission                 | Continuous | Cracking                       | Physical Acoustics, Cardiff          |
| 6 Distributed crack sensor          | Continuous | Cracking                       | Strainall                           |
| 7 Wireless crack sensor             | Continuous | Crack opening,                 | SPPS                                |
| 8 Digital leveling                  | Survey     | Shape                          | SPPS                                |
| 9 Wireless accelerometer sensor     | Continuous | Tilt                           | SPPS                                |
| 10 Resistance strain gauges         | Continuous | Strain                          | Vishay/NPL                          |
| 11 Vibrating wire sensors           | Continuous | Strain                          | Soil Instruments                    |
| 12 Electrolevel tilt sensors        | Continuous | Tilt                           | Soil Instruments                    |
| 13 LVDT                              | Continuous | Displacement                   | NPL                                 |
| 14 Corrosion mapping                | Survey     | Change in el/magnetic field    | University of Keele, SciSite         |
| 15 Hydralign                        | Continuous | Vertical displacement          | Pruftechnik                         |
| 16 Accelerometers                   | Dynamic tests | Vibration, mode shape calculations | Sheffield University, Full scale dynamics |
| 17 RFID sensor                      | Inspection | Rebar strain                    | Technimeasure                       |
| 18 Wireless system                  | Continuous | New transmission prorochol      | The IMC group                       |

**Figure 1.** A list of technologies used in the bridge project.

This paper presents a case study focusing on updating the FE model for the relatively undamaged bridge using dynamic vibration tests (modal analysis). A lot of SHM and damage detection approaches are based on information obtained from dynamic response to identify the existence, location and magnitude of damage. An accurate and reliable model is important for assessment and prediction of the performance of the bridge at different stages of damage. Our bridge has been monitored extensively during the testing program and between the tests. All observed types of damage, both artificially introduced during tests and naturally occurring due to gradual degradation of the structure and materials have been recorded. A well-documented history of damage introduced to the footbridge gives the opportunity to assess the performance of various methods of damage detection. An experimental program for 2011 will include damage/repair tests with different levels of damage introduced. The data collected during this period will be used to estimate the sensitivity and accuracy of various damage detection methods.
2. **Experiment**

2.1 *Bridge description*

The footbridge-demonstrator, a 15 tonne, 5 metre high, 20 metre long concrete reinforced bridge, is one of the largest specimens used at NPL. It was built in the mid 1960s using concrete and reinforcement typical for that time and aged naturally in service for 40 years. Therefore, it has become a very good example for studying the ageing infrastructure and of the 1950-70s period in particular. It is a simple construction with a deck supported by two A-frame columns as shown in figure 2.

![Figure 2. The bridge in its current location before installation of any sensors.](image2)

2.2 *Dynamic vibration tests and modal parameter evaluation*

The Vibration Engineering Section (VES), part of the Civil and Structural Engineering Department at the University of Sheffield, performed an experimental modal analysis (EMA) of the bridge. The testing consisted of forced vibration testing (FVT) using an electro-dynamic shaker in the vertical, lateral and longitudinal directions. The test grid used is shown in figure 3. Each point has an accelerometer measuring either vertically or laterally with the shaker at position 10. A photo of the setup form the bridge deck is shown in figure 4.

![Figure 3. EMA test grid.](image3)

![Figure 4. EMA setup.](image4)
The FRFs were measured with 50 averages using an 80 Hz sampling rate with a 20 second window. A sample of the vertical FRFs is shown in figure 5, with a number of clear peaks in the data identifying well separated modes.

![Figure 5](image)

**Figure 5.** A sample of the vertical measured FRFs.

From curve fitting the FRFs the modal properties were estimated. Figure 6 shows the first 6 vertical modal parameters ranging from approximately 5 Hz to 32 Hz. The FRFs were very clean and easily used to estimate the modal properties. The modal mass of mode 5 is much larger than the other modal masses, indicating that this is a vertical component of a predominantly lateral mode. Although not so evident in the mode shape plots, but in animated mode shapes, there is clear movement of the foundations.

| Mode | Frequency (Hz) | Damping (%) | Modal Mass (kg) |
|------|---------------|-------------|-----------------|
| 1    | 5.1           | 1.89        | 5900            |
| 2    | 8.9           | 1.59        | 3090            |
| 3    | 12.7          | 2.08        | 1080            |
| 4    | 15.5          | 1.85        | 1390            |
| 5    | 30.4          | 3.2         | 14000           |
| 6    | 31.9          | 5.1         | 2070            |

![Table](table)

**Figure 6.** First 6 vertical modal properties from the EMA.

During the test some non-linear amplitude dependant behaviour was detected and this was investigated further. A shaker shutdown test was conducted, which consists of running the shaker at a resonant frequency and estimating the frequency and damping on a cycle-by-cycle basis of the free decay of the signal. A number of different frequencies were attempted, but only the excitations at 8.4 Hz and 15 Hz produced a good free decay fit.

Figure 7 shows how frequency and damping varies for each of two frequencies with respect to the amplitude of acceleration. There is a small change in damping, but the change in frequency is quite substantial with a large frequency drop with larger amplitudes of vibration.
3. FEA model

An initial three-dimensional linear elastic finite element model of the bridge was constructed where the geometry of the bridge was created in ABAQUS CAE version 6.9-1. The geometry was meshed using continuum linear brick elements. As suggested in the ABAQUS manual the C3D8R element type was chosen for static bending loading (see comparison with Bernoulli-Euler theory prediction in [3]). The model was validated under static loading and results compared reasonably well with experiments conducted during static loading tests in 2008-2009. A discussion of the results can be found in [4].

When vibration analysis was considered, the suitability of the elements was reassessed. Three types of elements were selected and a comparison of the results using the same material parameters and boundary conditions showed a significant difference in natural frequencies, see table 1. After an investigation of different element types the difference was attributed to an effect called shear locking or parasitic shear [5]. The incompatible mode element C3D8I was chosen as the most suitable for this work. Elements of the type C3D8I are enhanced by incompatible modes to improve the bending behaviour to eliminate the parasitic shear stresses and artificial stiffening in bending due to Poisson's effect [3]. Although a discussion of the full integration, reduced integration, and incompatible elements is not presented here more details of the effect of different element types and mesh density on the frequency analysis can be found in [5].
The mesh created had 8 elements across the width of the deck, 4 elements through the thickness and approximately 160 elements along the length. The bases of the foundations of the bridge were fully constrained. Material properties were applied to the whole bridge to represent concrete plus steel reinforcement as one single set of properties. The Young’s modulus was 38 kN/mm$^2$, Poisson’s ratio was 0.2, density was 2.4E-9 tonnes/mm$^3$ (2400 kg/m$^3$) and thermal expansion was 12E-6/°C.

Various loading conditions were analysed. Firstly, a gravity load was applied to the bridge. In some analyses, the bridge was loaded in the middle of the main span, in other cases cantilever loading was applied at one end. Secondly, several temperature analyses were performed. In the first scenario, all nodes in the model were given an initial temperature e.g. 0°C. In the first step of the analysis the temperature of all nodes was increased to a new value e.g. 20°C. The second scenario consisted of applying a linear temperature gradient along the width of the bridge deck. Finally, to investigate the modes and natural frequencies of the bridge, a modal analysis perturbation step was added after either gravity or thermal loading as a pre-stress loading. The lanczos eigensolver was used with the analysis outputting the first 10 vibration modes of the bridge.

4. Model updating
This case of FE model updating is a first step in our investigation to improve the existing model of the bridge whilst the bridge remains well within serviceability limits. The next step will be focused on damage detection after set of damage/repair experiments are conducted.

The strong amplitude dependency of natural frequencies and damping was observed during the tests indicate that the frequencies will be not possible to match exactly. A small movement of foundations were also visible in the EMA mode shapes animation. However, the results obtained from the FE analysis before updating and EMA agreed reasonably well: the mode shapes looked similar but there was a small variation in frequencies. Although it is not possible to model amplitude dependant non-linear variation in modal properties with a linear analysis of a finite element model we have attempted to use a best linear fit approach discussed below.

4.1 Correlation analysis.
The natural frequencies and modes obtained from the experimental analysis and FE model were compared. The discussion here will be limited to the 5 vertical modes shown in the tables 2 and 3. Table 2 shows EMA and FE frequencies, their differences and modal assurance criterion (MAC) values. Table 3 is a comparison of mode shapes where FE shows all degrees of freedom of the vertical mode shape, but for EMA only vertical components are presented, thus the columns of the FE model are clearly moving in the x direction, whereas EMA are not.
Table 2. Comparison of natural frequencies of experimental and FE vertical modes

|   | Frequencies EMA | Frequencies FE | Differences in frequencies, % | MAC values |
|---|-----------------|----------------|-------------------------------|------------|
| V1 | 5.09            | 4.71           | -7.47                         | 0.986      |
| V2 | 8.92            | 8.41           | -5.72                         | 0.995      |
| V3 | 12.70           | 11.97          | -5.75                         | 0.991      |
| V4 | 15.50           | 16.7           | 7.74                          | 0.978      |
| V6 | 31.90           | 32.68          | 2.45                          | 0.971      |

Table 3. Comparison of experimental and FE vertical modes.
4.2 Sensitivity analysis

Modes obtained from the original FE model (updated by static loading experiments) were close enough to EMA experimental results to justify a narrow range of input parameters for sensitivity analysis. Four types of parameters were included in the sensitivity analysis: Young’s modulus, concrete density, temperature and boundary conditions.

As it is assumed that there is no significant localised damage, material properties such as the Young’s modulus and concrete density can be varied as global parameters in the model and only the variations in modal frequencies need to be examined.

The effect of temperature was included because it was found that the temperature underwent substantial daily changes and the variations had a significant influence on the deflection of the bridge during the static tests (details can be found [4]). Here this effect was studied by comparing the results of vibration analysis on its own to a model subjected first to temperature changes and then vibration analysis. Firstly, uniform temperature loading of 20 degrees (equivalent to the daily temperature variation at the bridge’s location) was applied to the whole structure. Secondly, a linear gradient of 10 degrees, representative of the temperature variation between the open sunny side and shady side of the bridge near the parkland, was applied across the deck. As the cantilevers of the bridge are not constrained, the shape of the deck was changed as shown in figure 10. An example of a sensitivity plot for fixed boundary conditions is shown in figure 11, where D is concrete density, E is global Young’s modulus and T is temperature. The figure shows that the applied temperature gradient, although sufficient to change the shape of the deck, has little effect on the frequencies of the vertical modes.

![Figure 8. Change in shape of the bridge deck due to the temperature gradient of 10 degrees applied across the deck from front to back.](image)

![Figure 9. An example of sensitivity plot for fixed boundary conditions where x-axis is the vertical mode number, y-axis shows model parameter, and z-axis is a relative sensitivity scale.](image)
4.3 Model updating

The model was updated using a standard optimisation algorithm to solve a constrained least-squares problem. The function to be minimised is shown in (1).

\[
F = \sum_{i=1}^{2} [f_{EMA} - f_{FE}(D, E, k, T)]^2
\]

where \( f_{EMA} \) is a frequency obtained experimentally, \( D \) is global concrete density, \( E \) is global Young’s modulus, \( k \) is stiffness coefficient, \( T \) is temperature and \( f_{FE} \) is the calculated frequency with parameters between 2400 kg/m³ < \( D \) < 2700 kg/m³, 35 N/mm² < \( E \) < 45 N/mm² and 0.35 N/m < \( k \) < 6*10¹⁰ N/m.

5. Results and discussion

5.1 Model updating results

The updated FE frequencies are shown in table 4. The results are also in Figure 5 where FE mode frequencies for 5 vertical modes plotted against experimental EMA mode frequencies. This is a straight line if the frequencies match. The set of parameters used are: \( D = 2530 \) kg/m³, \( E = 38 \) N/mm², one of the foundations is free in y direction and fixed in x and z ones.

|       | Frequencies EMA | Frequencies FE | Differences in frequencies, % |
|-------|----------------|----------------|------------------------------|
| V1    | 5.09           | 4.71           | -7.47                        |
| V2    | 8.92           | 8.41           | -5.72                        |
| V3    | 12.70          | 11.97          | -5.75                        |
| V4    | 15.50          | 16.7           | 7.74                         |
| V6    | 31.90          | 32.68          | 2.45                         |

Table 4. Comparison of natural frequencies of experimental and FE vertical modes.

This work represents a first attempt to use the dynamic testing for the NPL bridge project and a comparison between experimental and calculated frequencies is very encouraging at this early stage. It is also important to note that the analysis presented here is semiautomatic. The changes in boundary conditions (i.e. how free foundations can move) alter the number of modes significantly and non-linearly and therefore the correlation (section 4.1 above) and sensitivity (section 4.2) analyses must be repeated for each boundary condition. To overcome this limitation a full three dimensional modal analysis is required that has sufficient number of modes for optimisation even for the relatively straightforward geometry of the footbridge.

5.2 Non-linear effects.

Although the structure is simple, the updating task is not trivial, as discussed earlier. Modal analysis is linear but there are a number of significant non-linearities in the model. The effects of concern are:

- the ground deformation is non-linear, this is currently modelled with a single linear spring – a more complex method maybe required
- the handrail, although not structural, may add sufficient stiffness to contribute to the overall stiffness, but due to weak connections, the degree is unknown.
- during the tests we clearly identified a very large amplitude dependant behaviour of the modes. This is non-linear and hard to model.

To address this situation the dynamic response of the foundations and columns themselves need to be investigated experimentally in more detail. It is clear from the analysis that the movement of the
foundations should be assessed in all three directions. The amount of movement is small and may be experimentally challenging to monitor accurately. New and more sophisticated optimisation methods will need to be considered.

5.1 Dynamic testing to improve sensor locations for future damage detection
Determining best sensor locations is an optimisation task on its own and depends on the objectives of monitoring with different types of constraints, for example access or cost etc. In the current project, we are mostly interested in getting as much information as possible form the most sensitive locations. After examining the mode shapes and the positions of nodes it was clear that most of the areas of interest have already been covered but a new location was identified. Although there are sensors measuring strain on the foundations on the bottom of the columns, it will be important to use accelerometers or other sensors to determine the dynamic stiffness of the foundations. Monitoring any changes during damage accumulation will be important for interpreting the data.

6. Concluding remarks and further work
The results of dynamic tests and modal analysis showed that it is potentially a very powerful tool for structural assessment, with the modal properties being sensitive to a number of structural parameters. The FE model updating does not only improve our understanding of the overall behaviour of the footbridge, but also ranks the sensitivity to different parameters such as material parameters, temperature and boundary conditions. It will help in preparing for the next stage of the project, including concrete removal during damage/repair cycles, assessing existing sensor locations and prediction the variation of modal properties with respect to the induced damage.

Based on this study two new areas will be included in the work plan:
- identify and add more sensors to monitor foundations and columns in more details
- develop more generic and sophisticated optimisation routine to include several substructures and vary properties of each substructure and boundary conditions.

7. References
[1] Barton E and Zhang B 2010 Proc. Int. Conf Structural Faults & Repair-2010, (Edinburgh) p 20
[2] Barton E and Zhang B 2010 NDE/NDT for Highways and Bridges Structural Materials Technology (SMT) (New York ASNT)
[3] ABAQUS/CAE User’s Manual Version 6.9.
[4] NPL SHM Footbridge Website http://www.npl.co.uk/advanced-materials/materials-areas/structural-health-monitoring/footbridge-monitoring-project-(shm)
[5] Brown J 1997 MSC Aerospace Users’ Conference (MSC Software Corporation)