Gauge–Higgs grand unification

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The discovery of the Higgs boson at LHC supports the current scenario of the unification of electromagnetic and weak forces. The electroweak (EW) gauge symmetry, $SU(2)_L \times U(1)_Y$, is spontaneously broken to $U(1)_{EM}$ by the VEV (vacuum expectation value) of the Higgs scalar field. All experimental data so far are consistent with the standard model (SM) of electroweak and strong interactions. Yet it is not clear whether the observed Higgs boson is precisely what the SM predicts. Detailed study of the interactions among the Higgs boson and other SM particles in the forthcoming experiments is desperately needed.

There remain uneasy features in the Higgs boson sector in the SM. Unlike such gauge bosons as the photon, $W$ boson, $Z$ boson, and gluons, whose dynamics is governed by the gauge principle, the Higgs boson is an elementary scalar field for which there lacks an underlying fundamental principle. The Higgs couplings of quarks and leptons as well as the Higgs self-couplings are not regulated by any principle. At the quantum level, there arise huge corrections to the Higgs boson mass, which have to be canceled and tuned by hand to obtain the observed 125 GeV mass. One way to achieve natural stabilization of the Higgs boson mass against quantum corrections is to invoke supersymmetry, and many investigations have been made along this line. In this paper, we focus on an alternative approach, the gauge–Higgs unification [1–5].

The Higgs boson is unified with gauge bosons in the gauge–Higgs unification, which is formulated as a gauge theory in five or more dimensions. When the extra-dimensional space is not simply connected, an Aharonov–Bohm (AB) phase in the extra-dimensional space plays the role of the Higgs boson, breaking part of the non-Abelian gauge symmetry. The 4D fluctuation mode of the AB phase appears as a Higgs boson in four dimensions at low energies. In other words, the Higgs boson is part of the extra-dimensional component of gauge potentials, whose dynamics is controlled by the gauge principle. The gauge invariance guarantees the periodic nature of physics associated with the AB phase in the extra dimension, which we denote as $\theta_H$. 
The value of $\theta_H$ is determined dynamically, from the location of the global minimum of the effective potential $V_{\text{eff}}(\theta_H)$. At the classical (tree) level, $V_{\text{eff}}(\theta_H)$ is completely flat, as $\theta_H$ is an AB phase yielding vanishing field strengths. At the quantum level, $V_{\text{eff}}(\theta_H)$ becomes nontrivial as the particle spectra and their interactions depend on $\theta_H$. It has been shown that the $\theta_H$-dependent part of $V_{\text{eff}}(\theta_H)$ is finite at the one-loop level, free from ultraviolet divergence even in five or more dimensions as a consequence of the gauge invariance. Nontrivial minimum $\theta_H^{\text{min}}$ induces gauge symmetry breaking in general. The mass of the corresponding 4D Higgs boson, proportional to the second derivative of $V_{\text{eff}}(\theta_H)$ at the minimum, becomes finite irrespective of the cutoff scale in a theory, giving a way to solve the gauge hierarchy problem. This mechanism of dynamical gauge symmetry breaking is called the Hosotani mechanism.

Gauge–Higgs unification models of electroweak interactions have been constructed [6–13]. The orbifold structure of the extra-dimensional space is vital to have chiral fermions, and natural realization of dynamical EW symmetry breaking is achieved in the 5D Randall–Sundrum (RS) warped spacetime. The most promising is the $SO(5) \times U(1)_X$ gauge–Higgs unification in RS, which is consistent with the observation at low energies provided its AB phase $\theta_H \lesssim 0.1$. The model accommodates the custodial symmetry, and gives almost the same couplings in the gauge sector as the SM. It has been shown that one-loop corrections to the Higgs-boson decay to $\gamma\gamma$ due to the running of an infinite number of Kaluza–Klein (KK) excitation modes of the $W$ boson and top quark turn out to be finite and very small, being consistent with the present LHC data [10]. The model predicts Kaluza–Klein excitations of the $Z$ boson and photon as $Z'$ events with broad widths in the mass range 5–8 TeV; a dark-matter candidate (dark fermion) of a mass 2–3 TeV and other signals such as anomalous Higgs couplings are predicted as well [12–17].

With the gauge–Higgs EW unification model at hand, the next step is to incorporate strong interactions to achieve gauge–Higgs grand unification [18–26]. There are models of gauge–Higgs grand unification in five dimensions with gauge group $SU(6)$, which breaks down to $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ by the orbifold boundary condition on $S^1/Z_2$. Burdman and Nomura [20] showed that the EW Higgs doublet emerges. Haba et al. [21,22] and Lim and Maru [23] showed that dynamical EW symmetry is achieved with extra matter fields, though they yield exotic particles at low energies. Kojima et al. [24] have proposed an alternative model with $SU(5) \times SU(5)$ symmetry. Grand unification in the composite Higgs scenario has been discussed by Frigerio et al. [25]. Yamamoto [26] has attempted to dynamically derive orbifold boundary conditions in gauge–Higgs unification models.

In this paper, we propose a new model of gauge–Higgs grand unification in RS with gauge symmetry $SO(11)$ that carries over the good features of $SO(5) \times U(1)_X$ gauge–Higgs EW unification. We show that the EW symmetry breaking is induced even in the pure gauge theory by the Hosotani mechanism, in sharp contrast to other models. Quarks and leptons are implemented in a minimal set of fermion multiplets. Proton decay is naturally suppressed by the conservation of a new fermion number.

The model is defined in the RS spacetime with metric $ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^{\mu} dx^{\nu} + dy^2$, where $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$, $\sigma(y) = \sigma(-y) = \sigma(y + 2L)$, and $\sigma(y) = ky$ for $0 \leq y \leq L$. $z_L = e^{kL} \gg 1$ is called the warp factor. The anti-de Sitter (AdS) space with a cosmological constant $\Lambda = -6k^2$ in $0 < y < L$ is sandwiched by the Planck brane at $y = 0$ and the TeV brane at $y = L$. 


The $SO(11)$ gauge potential, $A_M$, expressed as an $11 \times 11$ antisymmetric Hermitian matrix, satisfies the orbifold boundary condition (BC) given by
\[
\begin{pmatrix}
A_\mu \\
A_y
\end{pmatrix}(x, y_j - y) = P_j \begin{pmatrix}
A_\mu \\
-A_y
\end{pmatrix}(x, y_j + y) P_j^{-1}, \quad (y_0, y_1) = (0, L),
\]

\[P_0 = \text{diag}(I_{10}, -I_1), \quad P_1 = \text{diag}(I_4, -I_7).\]  

(1)

$P_0$ and $P_1$ break $SO(11)$ to $SO(10)$ and $SO(4) \times SO(7)$, respectively. In all, the symmetry is broken to $SO(4) \times SO(6) \simeq SU(2)_L \times SU(2)_R \times SU(4)$. Write $A_M = 2^{-1/2} \sum_{1 \leq j < k \leq 11} A_{jk}^{(j,k)} T_{jk}$, where the $SO(11)$ generators satisfy $[T_{ij}, T_{kl}] = i \left( \delta_{ik} T_{jl} - \delta_{il} T_{jk} + \delta_{jl} T_{ki} - \delta_{jk} T_{li} \right)$. Zero modes of $A_y$ exist only for $A_{y_{(j,11)}}^j (j = 1-4)$, which become an $SO(4)$ vector or $SU(2)_L$ doublet Higgs field in four dimensions.

Fermions are introduced in the bulk in 32 and 11 of $SO(11)$, $\Psi_{32}$ and $\Psi_{11}$. We introduce a scalar field in 16 of $SO(10)$, $\Phi_{16}$, on the Planck brane. To make the matter content in $\Psi_{32}$ and $\Phi_{16}$ transparent, let us adopt the following representation of $SO(11)$ Clifford algebra $\{\Gamma_j, \Gamma_k\} = 2\delta_{jk} I_{32}$ ($j, k = 1-11$):

\[\Gamma_{1,2,3} = \sigma^{1,2,3} \otimes \sigma^1 \otimes \sigma^1 \otimes \sigma^1 \otimes \sigma^1,\]
\[\Gamma_{4,5} = \sigma^0 \otimes \sigma^{2,3} \otimes \sigma^1 \otimes \sigma^1 \otimes \sigma^1,\]
\[\Gamma_{6,7} = \sigma^0 \otimes \sigma^0 \otimes \sigma^{2,3} \otimes \sigma^1 \otimes \sigma^1,\]
\[\Gamma_{8,9} = \sigma^0 \otimes \sigma^0 \otimes \sigma^0 \otimes \sigma^{2,3} \otimes \sigma^1,\]
\[\Gamma_{10,11} = \sigma^0 \otimes \sigma^0 \otimes \sigma^0 \otimes \sigma^0 \otimes \sigma^{2,3}.\]  

(2)

Here $\sigma^0 = I_2$ and $\sigma^{1,2,3}$ are Pauli matrices. Note that $\Gamma_{11} = -i \Gamma_1 \cdots \Gamma_{10}$. The $SO(11)$ generators in the spinorial representation are given by $T_{jk}^{sp} = -\frac{1}{2} i \Gamma_j \Gamma_k$. In this representation, the upper and lower half-components of $\Psi_{32}$ correspond to 16 and 16 of $SO(10)$. $\Psi_{32}$ and $\Psi_{11}$ satisfy

\[\Psi_{32}(x, y_j - y) = -P_{32}^{sp} \gamma^5 \Psi_{32}(x, y_j + y),\]
\[\Psi_{11}(x, y_j - y) = \eta_j^{11} P_j^{sp} \gamma^5 \Psi_{11}(x, y_j + y),\]
\[P_0^{sp} = \Gamma_{11}, \quad P_1^{sp} = -\Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4 = I_2 \otimes \sigma^3 \otimes I_8.\]  

(3)

Here $\gamma^5 = \pm 1$ correspond to right- and left-handed Lorentz spinors, and $\eta_j^{11} = \pm 1$. The action in the bulk is given by

\[S_{\text{bulk}}^{c}\! = \! \int d^5 x \sqrt{-\det G} \left\{ -\frac{1}{4} \text{Tr} F_{MN} F^{MN} + \mathcal{L}_{g.f.} + \mathcal{L}_{gh} + \bar{\Psi}_{32} D(c_{32}) \Psi_{32} + \bar{\Psi}_{11} D(c_{11}) \Psi_{11} \right\}\]  

(4)

where $\mathcal{L}_{g.f.}$ and $\mathcal{L}_{gh}$ are gauge fixing and ghost terms. Here $F_{MN} = \partial_M A_N - \partial_A M - ig[A_M, A_N]$, $D(c) = \gamma^A e_A M D - c \sigma^A(y)$, and $D_M = \partial_M + \frac{1}{2} \omega_{MBC} [\gamma^B, \gamma^C] - ig A_M$.

The action for $\Phi_{16}$ is given by

\[S_{\Phi_{16}}^{\text{brane}} = \int d^5 x \sqrt{-\det G} \delta(y) \left\{ -(D_\mu \Phi_{16})^\dagger D^\mu \Phi_{16} - \lambda \Phi_{16} \left( \Phi_{16}^\dagger \Phi_{16} - w^2 \right)^2 \right\},\]  

(5)

where $D_\mu \Phi_{16} = (\partial_\mu - ig A_\mu^{SO(10)}) \Phi_{16}$ and $A_\mu^{SO(10)} = 2^{-1/2} \sum_{1 \leq j < k \leq 10} A_{jk}^{(j,k)} T_{jk}$. $\Phi_{16}$ develops VEV. Without loss of generality, we suppose that the 12th component of $\Phi_{16}$ develops VEV,
Table 1. The fermion content of $\Psi_{32}$ in the representation (2) of $\Gamma_j$ matrices. Each $SU(2)_L$ or $SU(2)_R$ doublet, from top to bottom in $\Psi_{32}$, is listed from left to right in the table. Fields with hats have opposite electric charges to the corresponding fields without hats. Zero modes resulting from the BC in (3) are shown. The $SO(10)$ and $SU(5)$ content of each field is also indicated.

| Name     | $\nu_e$ | $d_1$ | $u_3$ | $d_2$ | $u_1$ | $\bar{\nu}_e$ | $\bar{d}_1$ | $\bar{u}_3$ | $\bar{d}_2$ | $\bar{u}_1$ | $c$ | $\bar{c}$ | $v'$ | $\bar{v}'$ |
|----------|---------|-------|-------|-------|-------|---------------|-------------|-------------|-------------|-------------|----|------------|-----|-----------|
| Zero mode | $v_L$   | $u_{3L}$ | $u_{1L}$ | $u_{2L}$ | $u_{2R}$ | $u_{1R}$ | $u_{3R}$ | $v_R$ | $w_L$ | $d_{3L}$ | $d_{2L}$ | $d_{2R}$ | $d_{1R}$ | $d_{3R}$ | $e_L$ | $e_R$ |
| $SO(10)$ | $\mathbf{5}$ | $\bar{\mathbf{5}}$ | $\mathbf{10}$ | $\bar{\mathbf{10}}$ | $\mathbf{10}$ | $\bar{\mathbf{10}}$ | $\mathbf{1}$ | $\bar{\mathbf{1}}$ | $\mathbf{1}$ | $\bar{\mathbf{1}}$ | $\mathbf{1}$ | $\bar{\mathbf{1}}$ | $\mathbf{1}$ | $\bar{\mathbf{1}}$ |
| $SU(5)$  | $\mathbf{5}$ | $\mathbf{5}$ | $\mathbf{10}$ | $\bar{\mathbf{5}}$ | $\bar{\mathbf{5}}$ | $\mathbf{10}$ | $\bar{\mathbf{10}}$ | $\mathbf{10}$ | $\bar{\mathbf{10}}$ | $\mathbf{10}$ | $\bar{\mathbf{10}}$ | $\mathbf{10}$ | $\bar{\mathbf{10}}$ | $\mathbf{1}$ | $\bar{\mathbf{1}}$ |

$\langle \Phi_{16}^{12} \rangle = w \neq 0$, which reduces $SO(4) \times SO(6)$ to $SU(2)_L \times SU(3)_C \times U(1)_Y$. The generators of $SU(2)_L$ and $SU(2)_R$ are $T_{L/R} = \frac{1}{4}(4\epsilon_{abc}T_{bc} \pm T_{4a})$. In the spinorial representation

$$\left[ T_{L}^a, T_{R}^a \right] = \frac{1}{2} \sigma^a \otimes \left[ \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right] \otimes I_8, \quad (6)$$

$\langle \Phi_{16}^{11} \rangle$, $\langle \Phi_{16}^{12} \rangle$ is $(1, 2)$ of $SU(2)_L \times SU(2)_R$.

One finds later that the zero mode of $A_\nu$ (4D Higgs field) develops nonvanishing VEV by the Hosotani mechanism, and $SU(2)_L \times U(1)_Y$ breaks down to $U(1)_{EM}$. Without loss of generality, we suppose that $\langle A_{\nu}^{(4, 11)} \rangle \neq 0$. The $U(1)_{EM}$ charge in units of $e$ is given by

$$Q_{EM} = T_{12} - \frac{1}{3}(T_{56} + T_{78} + T_{9, 10}) \quad (7)$$

The content of $\Psi_{32}$ is easily determined by examining $Q_{EM}$ in the representation (2) with BC (3). The result is summarized in Table 1. BC at $y = 0$ with $P_0^{sp}$ admits parity-even left-handed (right-handed) modes only for $\mathbf{16}$ ($\bar{\mathbf{16}}$) of $SO(10)$, whereas BC at $y = L$ with $P_1^{sp}$ admits parity-even left-handed (right-handed) modes only for $SU(2)_L$ ($SU(2)_R$) doublets. In Table 1, a field with a hat has an opposite charge to the corresponding one without a hat. For instance, $u_j$ and $\hat{u}_j$ have $Q_{EM} = +\frac{2}{3}$ and $-\frac{2}{3}$, respectively. Notice that all leptons and quarks in SM, but nothing additional, appear as zero modes in $\Psi_{32}$. In the $SU(5)$ grand unified theory (GUT) in 4D, the $\mathbf{5}$ ($\bar{\mathbf{5}}$) multiplets contain $e_L^c$ ($\bar{e}_L^c$, $\bar{u}_L^c$, $d_L^c$, etc) so that gauge interactions alter the quark/lepton number and $u_L \rightarrow u_L^c$, $d_L \rightarrow e_L$, etc transitions are induced. In the present case, these processes do not occur and proton decay is suppressed. Indeed, proton decay is forbidden to all orders, provided that the $\Psi_{32}$, $\Psi_{11}$ fermion number $N_q$ is conserved. All of $u$, $d$, $e^-$ have $N_q = +1$ in the current model. Although the fermion number current is anomalous, its effect on proton decay is expected to be negligible, as in the case of baryon number nonconservation in SM.

In the gauge-field sector, BC (1) alone leads to zero modes (4D massless gauge fields) in $SO(4) \times SO(6)$, some of which become massive due to $\langle \Phi_{16} \rangle \neq 0$, leaving only $SU(2)_L \times SU(3)_C \times U(1)_Y$ invariance. Indeed, $-g^2 |A_{\mu}^{SO(10)} (\Phi_{16})|^2$ on the Planck brane generates mass terms of the form $L_{mass}^{gauge} = -\delta(y) \frac{1}{2} g^2 w^2 (A_{\mu}^a)^2$. We assume that $gw/\sqrt{L}$ is much larger than the KK mass scale $m_{KK} = \pi k z^{-1}_L$. In this case, even if $A_{\mu}^a$ is parity even at $y = 0$, the boundary condition effectively becomes a Dirichlet condition and the lowest KK mode acquires a mass of $O(m_{KK})$ [7,9]. It is straightforward to check that all gauge fields in $SO(4) \times SO(6)/SU(2)_L \times SU(3)_C \times U(1)_Y$
become massive. In particular, among the $SU(5)$ diagonal and $SU(3)_{C}$ neutral components
\[
A^{3L}_{\mu} = \frac{1}{\sqrt{2}} (A^{12}_{\mu} + A^{34}_{\mu}),
\]
\[
B^{y}_{\mu} = \sqrt{\frac{3}{10}} (A^{12}_{\mu} - A^{34}_{\mu}) - \sqrt{\frac{2}{15}} (A^{56}_{\mu} + A^{78}_{\mu} + A^{9.10}_{\mu}),
\]
\[
C_{\mu} = \sqrt{\frac{1}{5}} (A^{12}_{\mu} - A^{34}_{\mu} + A^{56}_{\mu} + A^{78}_{\mu} + A^{9.10}_{\mu}),
\]
(8)

$C_{\mu}$ becomes massive due to $\langle \Phi_{16} \rangle \neq 0$. $B^{y}_{\mu}$ is a gauge field of $U(1)_{y}$. After EW symmetry breaking by the Hosotani mechanism, $A^{34}_{\mu}$ mixes with $A_{\mu}^{4,11}$. The photon is given by
\[
A^{EM}_{\mu} = \frac{\sqrt{3}}{2} A^{12}_{\mu} - \frac{1}{2\sqrt{3}} (A^{56}_{\mu} + A^{78}_{\mu} + A^{9.10}_{\mu}).
\]
(9)

In terms of the $SU(2)_{L}$ coupling $g_{w} = g/\sqrt{L}$ in 4D, the $U(1)_{EM}$ and $U(1)_{Y}$ couplings are $e = (3/8)^{1/2} g_{w}$ and $g_{Y} = (3/5)^{1/2} g_{w}$. The Weinberg angle is given by $\sin^{2} \theta_{W} = 3/8$.

$\langle \Phi_{16} \rangle \neq 0$ breaks $SO(10)$ to $SU(5)$ on the Planck brane. We add a comment that there appear 21 would-be Nambu–Goldstone (NG) bosons associated with this symmetry breaking, 9 of which are eaten by gauge fields in $SO(4) \times SO(6)/SU(2)_{L} \times SU(3)_{C} \times U(1)_{Y}$. There remain 12 uneaten NG modes corresponding to a complex scalar field with the same SM quantum numbers $(3, \overline{2})_{1/6}$ as a quark doublet. They are massless at the tree level, but would acquire masses at the quantum level. Further, they are color-confined. It is expected that these colored scalars and quarks form color-singlet bound states, whose dynamics can be explored by collider experiments. Evaluation of the masses of these new bound states, as well as deriving their experimental consequences, is reserved for future investigation. We note that $\langle \Phi_{16} \rangle \neq 0$ also gives large brane mass terms for gauge fields in $SO(10)/SU(5)$, which effectively alters the Neumann BC at $y = 0$ to the Dirichlet BC for their low-lying modes ($m_{n} \ll g_{w}/\sqrt{L}$).

The extra-dimensional component of gauge fields, $A^{a,11}_{y}$ ($a = 1, \ldots, 4$), admits a zero mode, and yields a nonvanishing Aharonov–Bohm (AB) phase playing the role of 4D Higgs fields. AB phases are defined as phases of eigenvalues of $\hat{W} = P \exp \{ i g \int_{-L}^{L} dy \ A_{y} \} \cdot P_{1} P_{0}$, which are invariant under gauge transformations preserving the orbifold BC [2,21]. We expand $A^{4,11}_{y}(x, y)$ as
\[
A^{4,11}_{y}(x, y) = \{ \theta_{H} f_{H} + H(x) \} u_{H}(y) + \cdots,
\]
(10)
where $f_{H} = (2/g) \sqrt{k/\left(z_{L}^{2} - 1\right)}$, $u_{H}(y) = \sqrt{2k/\left(z_{L}^{2} - 1\right)} e^{2ky}$ ($0 \leq y \leq L$), and $u_{H}(-y) = u_{H}(y + 2L)$. $H(x)$ is identified with the neutral Higgs boson in four dimensions. Insertion of (10) into $\hat{W}$ shows that $\theta_{H}$ is the AB phase. A gauge transformation generated by
\[
\Omega(y; \beta) = \exp \left\{ -i \beta \frac{z_{L}^{2} - e^{2ky}}{z_{L}^{2} - 1} T_{4,11} \right\},
\]
(11)
shifts $\theta_{H}$ to $\theta_{H} + \beta$, and changes BC matrices to $P_{0}' = e^{-2i\beta T_{4,11}} P_{0}$ and $P_{1}' = P_{1}$. Note that $T_{4,11} = \sigma^{2}$ in the 4-11 subspace in the vectorial representation, and $T_{4,11}^{sp} = -\frac{1}{2} \sigma^{0} \otimes \sigma^{2} \otimes \sigma^{1} \otimes \sigma^{2}$ in the spinorial representation. The boundary conditions in (1) and (3) are preserved provided $\beta = 2\pi n$ ($n$: an integer). The gauge invariance guarantees the periodicity in $\theta_{H}$ for physical quantities.

The value of $\theta_{H}$ is determined by the location of the global minimum of the effective potential $V_{\text{eff}}(\theta_{H})$, which is flat at the tree level but becomes nontrivial at the one-loop level. To find the
mass spectra for $\theta_H \neq 0$ and evaluate $V_{\text{eff}}(\theta_H)$, it is most convenient to move to the twisted gauge generated by $\Omega(y; -\theta_H)$. In this gauge, the background $\tilde{A}_v$ vanishes and $\tilde{\theta}_H = 0$. (Quantities with tildes denote those in the twisted gauge.) The boundary condition matrices become

$$
\tilde{P}_0 = \begin{pmatrix}
\cos 2\theta_H & -\sin 2\theta_H \\
-\sin 2\theta_H & -\cos 2\theta_H
\end{pmatrix}
$$

(12)

in the 4-11 subspace, and $\tilde{P}_1 = P_1$. For $\tilde{\Psi}_{32}$,

$$
\tilde{P}_0^{sp} = \begin{pmatrix}
\cos \theta_H & -i \sin \theta_H \\
i \sin \theta_H & -\cos \theta_H
\end{pmatrix}
$$

(13)

for pairs $= (v, v'), (e, e'), (u_j, u'_j), (d_j, d'_j)$, whereas, for pairs $(\tilde{v}, \tilde{v}'), (\tilde{e}, \tilde{e}'), (\tilde{u}_j, \tilde{u}'_j), (\tilde{d}_j, \tilde{d}'_j), \theta_H \rightarrow -\theta_H$ in (13). $\tilde{P}_1^{sp} = P_1^{sp}$.

$\tilde{\theta}_H = 0$ in the twisted gauge so that all fields satisfy free equations in the bulk to the leading order and obey the original boundary conditions at $y = L$. It is convenient to do the analysis in the conformal coordinate $z = e^{k y} \ (1 \leq z \leq z_L)$. Mode functions are expressed in terms of Bessel functions. Base functions are tabulated in Appendix A of Ref. [12]. For instance, $C(z; \lambda) = \frac{1}{2} \pi \lambda \cos \theta_L F_0(\lambda z, \lambda z_L)$ and $S(z; \lambda) = -\frac{i}{2} \pi \lambda \sin \theta_L F_1(\lambda z, \lambda z_L)$ where $F_{\alpha, \beta}(u, v) = J_{\alpha}(u)Y_{\beta}(v) - Y_{\alpha}(u)J_{\beta}(v)$.

Only particle spectra depending on $\theta_H$ affect the $\theta_H$-dependent part of $V_{\text{eff}}$ at 1-loop. In the gauge-field sector, $\tilde{A}_{\mu 0}^L, \tilde{A}_{\mu 0}^R, \tilde{A}_{\mu 11}^L$ (with $\mu = 1, 2$) mix with each other. Their mass spectra ($m_n = k \lambda_n$) are determined by zeros of $C(2SC' + \lambda \sin^2 \theta_H)|_{z=1} = 0$ where $C' = dC/dz$:

$$
W \text{ tower: } 2S(1; \lambda_n)C'(1; \lambda_n) + \lambda_n \sin^2 \theta_H = 0,
$$

$$
W_R \text{ tower: } C(1; \lambda_n) = 0.
$$

(14)

Similarly, $\tilde{A}_{\mu 0}^L, \tilde{B}_Y^L, \tilde{C}_\mu$, and $\tilde{A}_{\mu 11}^{3L}$ mix with each other; their spectra are given by

$$
\gamma \text{ tower: } C'(1; \lambda_n) = 0,
$$

$$
Z \text{ tower: } 5S(1; \lambda_n)C'(1; \lambda_n) + 4\lambda_n \sin^2 \theta_H = 0,
$$

$$
Z_R \text{ tower: } C(1; \lambda_n) = 0.
$$

(15)

The $Y$-boson part, $\tilde{A}_{\mu 1}^L$ ($\mu = 3, 4, 11$) also yields $\theta_H$-dependent spectra. It decomposes into 6 sets of $\{(a, j)\} = \{(3, 5), (4, 6), (11, 6)\}, \{(3, 6), (4, 5), (11, 5)\}$, etc. In each set,

$$
\hat{Y} \text{ tower: } 2S(1; \lambda_n)C'(1; \lambda_n) + \lambda_n \left(1 + \cos^2 \theta_H\right) = 0,
$$

$$
\hat{Y} \text{ tower: } S(1; \lambda_n) = 0.
$$

(16)

Other components of $\tilde{A}_\mu$ have $\theta_H$-independent spectra. The spectra of $\tilde{A}_z = (kz)^{-1} \tilde{A}_Y$ are simpler, as $\Phi_{16}$ does not couple to $\tilde{A}_z$. The spectra of $[\tilde{A}_z^{a4}, \tilde{A}_z^{a11}]$ ($a = 1-3, 5-10$) are given by

$$
S(1; \lambda_n)C'(1; \lambda_n) + \lambda_n \left(\frac{\sin^2 \theta_H}{\cos^2 \theta_H}\right) = 0 \quad \text{for } a = \begin{cases} 1-3, \\ 5-10. \end{cases}
$$

(17)

Other components of $\tilde{A}_z$ have $\theta_H$-independent spectra. It follows from Eqs. (14) and (15) that $m_W \sim (k/L)(1/2)z_L^{-1} \sin \theta_W$ and $m_Z \sim m_W / \cos \theta_W$ for $z_L \gg 1$. The wave functions of $W$ and $Z$ are the same as in the $SO(5) \times U(1)_X$ theory with $\sin^2 \theta_W = 3/8$. 

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Fig. 1. \( V_{\text{eff}}(\theta_H) \) in pure gauge theory. \( U = (4\pi)^2(kz_L^{-1})^{-4}V_{\text{eff}} \) is plotted in the \( \xi = 0 \) gauge. The shape of the potential in the \( \xi = 1 \) gauge is almost the same as depicted. The global minimum is located at \( \theta_H = \pm \frac{1}{2}\pi \). \( V_{\text{eff}}(\theta_H) \) with a minimum at \( 0 < \theta_H < \frac{1}{2}\pi \) is achieved with the inclusion of fermions and brane interactions.

The spectrum of \( \tilde{\Psi}_{32} \) is determined by the boundary condition (3) with \( \tilde{P}_0^{sp} \) in (13). The spectrum is given, in the absence of the brane interactions discussed below, by

\[
S_L (1; \lambda_n, c_{32}) S_R (1; \lambda_n, c_{32}) \left( \frac{\sin^2 \frac{1}{2} \theta_H}{\cos^2 \frac{1}{2} \theta_H} \right) = 0 \tag{18}
\]

where the upper component is for pairs \( (v, v'), (e, e'), (u_j, u'_j), (d_j, d'_j) \) and the lower component for pairs \( (\tilde{v}, \tilde{v}'), (\tilde{e}, \tilde{e}'), (\tilde{u}_j, \tilde{u}'_j), (\tilde{d}_j, \tilde{d}'_j) \). Here \( S_{L/R}(v; \lambda, c) = \mp \frac{1}{2} \pi \lambda \sqrt{z_L z} L F_{c \pm \frac{1}{2}, c \pm \frac{1}{2}}(\lambda z, \lambda z_L) \). For \( \tilde{\Psi}_{11} \), the 4th and 11th components mix through \( \tilde{P}_0 \) in (12), and their spectrum is given by

\[
S_L(1; \lambda_n, c_{11}) S_R(1; \lambda_n, c_{11}) \left( \frac{\sin^2 \theta_H}{\cos^2 \theta_H} \right) = 0 \tag{19}
\]

for \( \eta_0^{11} \eta_1^{11} = \pm 1 \). Other components have \( \theta_H \)-independent spectra. We note that the spectrum of \( \Psi_{32} \) is periodic in \( \theta_H \) with a period \( 2\pi \), whereas that of gauge fields and \( \Psi_{11} \) has a period \( \pi \).

With the mass spectrum at hand, one can evaluate \( V_{\text{eff}}(\theta_H) \) at 1-loop in the standard method [7,10]. There is a distinct feature in the spectrum in the gauge-field sector. In the gauge–Higgs grand unification, there are six \( Y \) towers with the spectrum (16) where the lowest modes have the smallest mass for \( \cos \theta_H = 0 \). This leads to an important consequence that even in pure gauge theory the EW symmetry is spontaneously broken by the Hosotani mechanism. \( V_{\text{eff}}(\theta_H) \) evaluated with (14)–(17) has the global minimum at \( \theta_H = \pm \frac{1}{2}\pi \); see Fig. 1. This has never happened in the gauge–Higgs EW unification models. \( \Psi_{32} \) does not affect this behavior very much in the absence of brane interactions. Contributions from particles with the upper spectrum in (18) and those with the lower spectrum almost cancel numerically in \( V_{\text{eff}}(\theta_H) \) for \( z_L \gg 1 \). \( \Psi_{11} \) with \( \eta_0^{11} \eta_1^{11} = 1 (-1) \) in (19) strengthens (weakens) the EW symmetry breaking.

At this stage, however, quarks and leptons have degenerate masses. The degeneracy is lifted by interactions on the Planck brane (at \( y = 0 \)) that must respect \( SO(10) \) invariance. Let us decompose \( \Psi_{32} \) into \( 16 \) and \( \bar{16} \) of \( SO(10) \): \( \Psi_{16}^{sp}, \Psi_{\bar{16}}^{sp} \). Similarly, we decompose \( \Psi_{11} \) into \( \Psi_{10}^{vec}, \Psi_{\bar{16}}^{vec} \). In terms of these fields with \( \Phi_{16} \), various \( SO(10) \)-invariant brane interactions such as \( \overline{\Psi}_{10}^{sp} \Psi_{\bar{16}}^{vec} \Phi_{16} \) and \( \overline{\Psi}_{\bar{16}}^{sp} \Psi_{10}^{vec} \Phi_{16} \) are allowed on the Planck brane, with which a more realistic fermion spectrum can be achieved. One may introduce terms like \( \overline{\Psi}_{1}^{vec} \Psi_{1}^{vec,c} \), which, in combination with mixing of neutral components in \( \Psi_{32} \), may induce Majorana masses for neutrinos. However, it has to be kept in mind that such terms may lead to proton decay at higher loops. As mentioned above, \( V_{\text{eff}}(\theta_H) \) is minimized
at $\theta_H = \pm \frac{1}{2}\pi$ in pure gauge theory. $\theta_H = \pm \frac{1}{2}\pi$, however, leads to a stable Higgs boson due to the $H$ parity \cite{27,28}, which is excluded phenomenologically. A desirable value of $\theta_H$ can be achieved by an appropriate choice of $\eta^{11}_1\eta^{11}_1$ and inclusion of brane interactions for $\Psi_{32}$ and $\Psi_{11}$. Alternatively, one may introduce fermions ($\Psi_{55}, \Psi_{11}, \Psi_{32}$) such that quarks and leptons are dominantly contained in ($\Psi_{55}, \Psi_{11}$).

In this paper, we have presented the $SO(11)$ gauge–Higgs grand unification model that generalizes the $SO(5) \times U(1)_X$ gauge–Higgs EW unification. The orbifold boundary condition and brane scalar $\Phi_{16}$ reduce the $SO(11)$ symmetry directly to the SM symmetry. The 4D Higgs doublet appears as the extra-dimensional component of the gauge potentials with custodial symmetry. The EW symmetry is spontaneously broken by the Hosotani mechanism, even in the pure gauge theory. We have presented a model with $\Psi_{32}$ and $\Psi_{11}$ for quarks and leptons. Proton decay is suppressed by the fermion number $N_F$ conservation in the absence of Majorana masses. The effect of the fermion number current anomaly for proton decay is expected to be small. Although neutrino Majorana masses lead to proton decay at higher loops, the contribution will be suppressed by large Majorana masses and the loop effect. There remains a task to pin down the parameters of the model to reproduce the observed Higgs boson mass and quark–lepton spectrum, and derive phenomenological predictions. Further, the masses of the colored would-be NG bosons from $\Phi_{16}$ and color-singlet bound states need to be clarified and the consistency with experimental results at LHC needs to be examined. We will come back to these issues in forthcoming papers.

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