U-Spin Tests of the Standard Model and New Physics

Makiko Nagashima\textsuperscript{1}, Alejandro Szynkman\textsuperscript{2} and David London\textsuperscript{3},

\textit{Physique des Particules, Université de Montréal,}
\textit{C.P. 6128, succ. centre-ville, Montréal, QC, Canada H3C 3J7}

(March 26, 2022)

Abstract

Within the standard model, a relation involving branching ratios and direct CP asymmetries holds for the $B$-decay pairs that are related by U-spin. The violation of this relation indicates new physics (NP). In this paper, we assume that the NP affects only the $\Delta S = 1$ decays, and show that the NP operators are generally the same as those appearing in $B \to \pi K$ decays. The fit to the latest $B \to \pi K$ data shows that only one NP operator is sizeable. As a consequence, the relation is expected to be violated for only one decay pair: $B_d^0 \to K^0 \pi^0$ and $B_s^0 \to \bar{K}^0 \pi^0$. 

\textsuperscript{1}makiko@lps.umontreal.ca
\textsuperscript{2}szynkman@lps.umontreal.ca
\textsuperscript{3}london@lps.umontreal.ca
At present, several different experiments are focusing on the measurement of CP violation in various $B$ decays. The principal hope is to find a discrepancy with the predictions of the standard model (SM). This would indicate the presence of physics beyond the SM, which is one of the main aims of these experiments.

U-spin relations between different $B$ decays provide several tests of the SM. This is discussed in detail in Refs. [1, 2]. U-spin is the symmetry that places $d$ and $s$ quarks on an equal footing, and is often given as transposing $d$ and $s$ quarks: $d \leftrightarrow s$. Assuming a perfect U-spin symmetry, the effective Hamiltonian describing a $\Delta S = 0$ transition ($\bar{b} \to \bar{d}$) is equal to that of the corresponding $\Delta S = 1$ transition ($\bar{b} \to \bar{s}$) with $d \leftrightarrow s$ (the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix are changed appropriately). Using the CKM unitarity relation [3],

$$\text{Im}(V_{ub}^* V_{us} V_{cb} V_{cs}^*) = -\text{Im}(V_{ub}^* V_{ud} V_{cb} V_{cd}^*) ,$$  

(1)

this implies that there exists a U-spin relation between the CP-violating rate differences of the $\Delta S = 0$ and $\Delta S = 1$ decays [1, 2]:

$$|A(B \to f)|^2 - |A(\bar{B} \to \bar{f})|^2 = - \left[ |A(UB \to Uf)|^2 - |A(U\bar{B} \to U\bar{f})|^2 \right] ,$$

(2)

in which $U$ is the U-spin operator that transposes $d$ and $s$ quarks. This expression can be written as

$$\frac{-A_{\text{dir}}^{\text{CP}}(\text{decay #1})}{A_{\text{dir}}^{\text{CP}}(\text{decay #2})} = \frac{\text{BR(decay #2)}}{\text{BR(decay #1)}},$$

(3)

where $A_{\text{dir}}^{\text{CP}}$ and $\text{BR}$ refer to the direct CP asymmetry and branching ratio, respectively, and where decays #1,2 are the $\Delta S = 0$ and $\Delta S = 1$ decays, in either order, related by U-spin. Note that if decays #1,2 include $B^0_d$ and $B^0_s$ mesons, there is an additional factor on the right-hand side taking the lifetime difference into account.

The pairs of $B \to PP$ decays ($P$ is a pseudoscalar meson) which are related by U-spin are

1. $B^0_d \to K^+ \pi^-$ and $B^0_s \to \pi^+ K^-$,
2. $B^0_s \to K^+ K^-$ and $B^0_d \to \pi^+ \pi^-$,
3. $B^0_d \to K^0 \pi^0$ and $B^0_s \to \bar{K}^0 \pi^0$,
4. $B^+ \to K^0 \pi^+$ and $B^+ \to \bar{K}^0 K^+$,
5. $B^0_s \to K^0 \bar{K}^0$ and $B^0_d \to K^0 K^0$,
6. $B^0_s \to \pi^+ \pi^-$ and $B^0_d \to K^+ K^-.$

In all cases, the first decay is $\Delta S = 1$; the second is $\Delta S = 0$.

Note that the decays described in pair #6 can only come about if the two quarks in the initial state interact with each other (annihilation). We therefore expect the
branching ratios for these decays to be considerably smaller than those of the other decays. For this reason, we ignore this pair from here on.

Throughout this paper, we refer to a given pair of decays by its number in the list above (#1–#5). If Eq. 3 is not satisfied for any of these pairs of decays, this indicates the presence of physics beyond the SM.

Technically, this is not quite true, as U-spin is only an approximate symmetry. It is a quite common assumption (motivated by factorization in tree-level amplitudes) [4] to take U-spin breaking effects in branching ratios to be given by (known) ratios of decay constants. In this case, the effects total $O(30\%)$. If one includes form factors, QCD sum rules give U-spin breaking effects of $O(80\%)$ [5]. However, both of these estimates are somewhat misleading—they imply that the uncertainty on the U-spin breaking is large, assuming that the decays related by U-spin are unknown. For example, one doesn’t know if the decay-constant breaking effect is $f_K/f_π$ or $f_π/f_K$. However, large U-spin breaking is not a problem for a given decay pair as long as (i) the U-spin breaking is taken into account, and (ii) the uncertainty is not too big.

In this paper, we know which decays are involved. We can thus compute the U-spin breaking effect within naive factorization. The main descriptions of hadronic $B$ decays (soft collinear effective theory )SCET) [6], QCD factorization (QCDf) [7, 8, 9], perturbative QCD (pQCD) [10]) all include an expansion in powers of $1/m_b$. Within QCDf it is proven that factorization corresponds to the leading-order term. Thus naive factorization is a good approximation, and nonfactorizable effects less important, if the higher-order terms in $1/m_b$ are small. One example of a large such term, which would invalidate this approximation, is penguin annihilation [11], proposed to explain the $B \to φK^*$ polarization puzzle. However, to date there is no firm evidence that penguin annihilation is large. Other examples of higher-order terms are exchange and annihilation diagrams, and there is no experimental evidence that these are sizeable. Thus, at present the evidence points to small higher-order terms, and naive factorization is a reasonable approximation. For U-spin breaking we find

1. $\Delta U = (f_K F_{B_d^0 \to π})/(f_π F_{B_d^0 \to K}) = 1.02 \pm 0.11$,

2. $\Delta U = (f_K F_{B_s^0 \to K})/(f_π F_{B_s^0 \to π}) = 1.49 \pm 0.15$,

3. $\Delta U = (f_K F_{B_s^0 \to π})/(f_π F_{B_s^0 \to K}) = 1.02 \pm 0.11$,

4. $\Delta U = F_{B^+ \to π}/F_{B^+ \to K} = 0.79 \pm 0.04$,

5. $\Delta U = F_{B_s^0 \to K}/F_{B_s^0 \to K} = 0.95 \pm 0.11$,

where $f_i$ is the decay constant and $F_i$ is the form factor. In working out the numerical values for the $\Delta U$'s, we take $f_π = 130$ MeV, $f_K = 160$ MeV, $F_{B_s^0 \to K}/F_{B_s^0 \to π} = 1.21^{+0.14}_{-0.11}$ [12], $F_{B^+ \to K}/F_{B^+ \to π} = 1.27 \pm 0.07$ [13]. For the form-factor ratio in #5, we
take $F_{B_s^0 \to \pi}/F_{B_s^0 \to K} = F_{B^+ \to \pi}/F_{B^+ \to K}$ and combine this with $F_{B_s^0 \to K}/F_{B_s^0 \to \pi}$. The calculation of $\Delta U$ for #3 is complicated by the fact that there are several contributing diagrams, with different naive U-spin breaking effects. In order to obtain a numerical result, we assume that $f_K/f_\pi \simeq F_{B_s^0 \to K}/F_{B_s^0 \to \pi}$, which is confirmed by Refs. [12, 13]. Note that the central values give small U-spin breaking for pairs #1, #3 and #5. The uncertainty on the $\Delta U$’s is then due to extent to which the form factors are known. In addition, there are nonfactorizable effects $[O(1/m_b)]$, which we estimate to be $\sim 10\%$. The total error on U-spin breaking is then relatively small, $O(20\%)$.

It is worth mentioning that, in the case of pairs #1 and #3, we expect U-spin breaking effects beyond naive factorization to have a small impact since the final states for both $B_s^0$ and $B_s^0$ decays in each pair are charge conjugate [14].

Eq. 3 must now be modified to include the above U-spin breaking effects. We find that the modification is simple:

$$-A_{CP}^{dir}(\text{decay #1}) A_{CP}^{dir}(\text{decay #2}) = \Delta U^2 \frac{BR(\text{decay #2})}{BR(\text{decay #1})},$$

If the above equation is not satisfied (including the error on $\Delta U$) for any of the U-spin pairs of decays, this indicates the presence of physics beyond the SM.

Taking the U-spin breaking into account, and given that the error is not very large, one will not have a gross violation of Eq. 3. Thus, it is more correct to say that “if Eq. 3 is greatly violated for any of the pairs of decays – such as the direct CP asymmetries having the same sign – this indicates the presence of physics beyond the SM.”

This new physics (NP) can take the form of new contributions to the $\Delta S = 0$ and/or the $\Delta S = 1$ decays. Now, there have been many measurements of quantities in $B$ decays. To date, there have been several hints of discrepancies with the SM. However, none of them lie in the $\Delta S = 0$ sector; they all point to NP in $\bar{b} \to \bar{s}$ transitions. For example, the CP asymmetry in $\bar{b} \to \bar{s}q\bar{q}$ modes ($q = u, d, s$) is found to differ from that in $\bar{b} \to \bar{c}c\bar{s}$ decays by $2.6\sigma$ (they are expected to be approximately equal in the SM) [15, 16, 17]. One also sees a discrepancy with the SM in triple-product asymmetries in $B \to \phi K^*$ [18, 19, 20], and in the polarization measurements of $B \to \phi K^*$ [21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34] and $B \to \rho K^*$ [35, 36, 37, 38]. Finally, some $B \to \pi K$ measurements disagree with SM expectations [39, 40, 41], although it has been argued that the so-called $B \to \pi K$ puzzle [12, 43, 44] has been somewhat reduced [45, 46, 47, 48, 49]. Although none of the discrepancies are statistically significant, together they give an interesting hint of NP. In this paper we follow this indication and assume that the NP appears only in $\bar{b} \to \bar{s}$ decays ($\Delta S = 1$) but does not affect $\bar{b} \to \bar{d}$ decays ($\Delta S = 0$).

There are many NP operators which can contribute to $\Delta S = 1$ decays. However, it was recently shown in Ref. [50] that this number can be reduced considerably. Briefly, the argument is as follows. Following the experimental hints, we assume
that NP contributes significantly to those decays which have large $\bar{b} \to \bar{s}$ penguin amplitudes, and take the NP operators to be roughly the same size as the SM $\bar{b} \to \bar{s}$ penguin operators, so the new effects are sizeable. Each NP matrix element can have its own weak and strong phase. Now, all strong phases arise from rescattering. In the SM, this comes mainly from the $\bar{b} \to \bar{s}c\bar{c}$ tree diagram. The NP strong phases must come from rescattering of the NP operators. However, the tree diagram is quite a bit larger than the $\bar{b} \to \bar{s}$ penguin diagram (the expected size of the NP operator). As a consequence, the generated NP strong phases are correspondingly smaller than those of the SM. That is, the NP strong phases are negligible compared to the SM strong phases. (The appendix of Ref. [51] contains a detailed discussion of the small NP strong phases.)

The idea of small NP strong phases is central to the analysis presented in this paper. As such, it is worthwhile to examine the circumstances under which the idea holds or fails. NP strong phases can be mimicked by rescattering from operators that are higher order in $1/m_b$. Thus, the idea of small NP strong phases can fail if higher-order terms in $1/m_b$ are large, i.e. if the coefficients of such terms are bigger than expected. However, as discussed above in the context of U-spin breaking, there is no evidence that these terms are sizeable. Thus, although the idea of small NP strong phases can fail if higher-order terms in $1/m_b$ are large, at present these terms appear to be small. We therefore conclude that the idea of small NP strong phases is justified.

The neglect of all NP strong phases considerably simplifies the situation. At the quark level, each NP contribution to the decay $B \to f$ takes the form $\langle f | O_{NP}^{ij,q} | B \rangle$, where $O_{NP}^{ij,q} \sim \bar{s}\Gamma_i b \bar{q}\Gamma_j q$ ($q = u, d, s, c$), in which the $\Gamma_{i,j}$ represent Lorentz structures, and colour indices are suppressed. If one neglects all NP strong phases, one can now combine all NP matrix elements into a single NP amplitude, with a single weak phase:

$$\sum \langle f | O_{NP}^{ij,q} | B \rangle = A^q e^{i\Phi_q}.$$  (5)

Thus, all NP effects can be parametrized in terms of a small number of NP quantities. For $\Delta S = 1$ decays, there are two classes of NP operators, differing in their colour structure: $\bar{s}_a \Gamma_i b_a \bar{q}_b \Gamma_j q_b$ and $\bar{s}_a \Gamma_i b\bar{b}_b \bar{q}_b \Gamma_j q_a$. The first class of NP operators contributes with no colour suppression to final states containing $\bar{q}q$ mesons. (The second type of operator can also contribute via Fierz transformations, but there is a suppression factor of $1/N_c$, as well as additional operators involving colour octet currents.) Similarly, for final states with $\bar{s}q$ mesons, the roles of the two classes of operators are reversed. As in Ref. [51], we denote by $A^i_q e^{i\Phi'_{iq}}$ and $A^{ic,q} e^{i\Phi'_{icq}}$ the sum of NP operators which contribute to final states involving $\bar{q}q$ and $\bar{s}q$ mesons, respectively (the primes indicate a $\Delta S = 1$ $\bar{b} \to \bar{s}$ transition). Here, $\Phi'_q$ and $\Phi'^{ic}_q$ are the NP weak phases; the strong phases are zero. We stress that, despite the “colour-suppressed” index $C$, the operators $A^{ic,q} e^{i\Phi'^{ic}_q}$ are not necessarily smaller than the $A^i_q e^{i\Phi'_q}$.

It is now possible to easily compute the effect of the NP operators on the $\Delta S = 1$
decays mentioned in this paper. However, before doing so, we return to $B \rightarrow \pi K$ decays. In Ref. \[52\] a fit was done to the 2006 $B \rightarrow \pi K$ data, shown in Table 1. We summarize the results here. If one defines the ratios

$$ R = \frac{\tau_{B^+} BR[B^0 \rightarrow \pi^- K^+] + BR[B^0 \rightarrow \pi^+ K^-]}{\tau_{B^0} BR[B^0_d \rightarrow \pi^+ K^0] + BR[B^0_d \rightarrow \pi^- K^0]} , $$

$$ R_a = \frac{1}{2} \frac{BR[B^0 \rightarrow \pi^- K^+] + BR[B^0 \rightarrow \pi^+ K^-]}{BR[B^0 \rightarrow \pi^- K^0] + BR[B^0 \rightarrow \pi^+ K^0]} , $$

$$ R_c = \frac{BR[B^0_d \rightarrow \pi^0 K^+] + BR[B^0_d \rightarrow \pi^0 K^-]}{BR[B^0_d \rightarrow \pi^+ K^0] + BR[B^0_d \rightarrow \pi^- K^0]} , $$

one can show that the present values of $R$, $R_a$, and $R_c$ agree with the SM \[53\]. This has led some authors to posit that there is no longer a $B \rightarrow \pi K$ puzzle \[53\] \[54\]. Unfortunately, this analysis is incomplete.

| Mode               | $BR[10^{-6}]$ | $A_{CP}$        | $S_{CP}$       |
|--------------------|---------------|-----------------|----------------|
| $B^+ \rightarrow \pi^+ K^0$ | 23.1 ± 1.0    | 0.009 ± 0.025   |                |
| $B^+ \rightarrow \pi^0 K^+$  | 12.8 ± 0.6    | 0.047 ± 0.026   |                |
| $B^0_d \rightarrow \pi^- K^+$ | 19.7 ± 0.6    | -0.093 ± 0.015  |                |
| $B^0_d \rightarrow \pi^0 K^0$ | 10.0 ± 0.6    | -0.12 ± 0.11    | 0.33 ± 0.21    |

Table 1: Branching ratios, direct CP asymmetries $A_{CP}$, and mixing-induced CP asymmetry $S_{CP}$ (if applicable) for the four $B \rightarrow \pi K$ decay modes. The data is taken from Refs. \[15\] \[55\] \[56\] \[57\] \[58\] \[59\] \[60\] \[61\] \[62\] \[63\].

In Refs. \[64\] \[65\], the relative sizes of the SM $B \rightarrow \pi K$ diagrams were roughly estimated as

$$ 1 : |P_{\ell \ell}'| , \quad O(\lambda) : |T'| , \quad |P_{EW}'| , \quad O(\lambda^2) : |C'| , \quad |P_{\ell \ell}''| , \quad |P_{EW}^{C'}| , $$

where $\lambda \sim 0.2$. These estimates are expected to hold approximately in the SM. Now, one can show that the amplitudes $\sqrt{2}A(B^+ \rightarrow \pi^0 K^+)$ and $A(B^0_d \rightarrow \pi^- K^+)$ are equal, up to a factors of the small diagrams $P_{EW}'$ and $C'$. We therefore expect $A_{CP}(B^+ \rightarrow \pi^0 K^+)$ to be approximately equal to $A_{CP}(B^0_d \rightarrow \pi^- K^+)$ in magnitude (multiplicative factors cancel between the numerator and denominator of asymmetries). However, as can be seen in Table 1, these asymmetries are very different. Thus, the present $B \rightarrow \pi K$ data cannot be explained by the SM. It is only by considering the CP-violating asymmetries that one realizes this.

If one includes all diagrams, a good fit is found \[52\]. This has led some people to argue that there is no discrepancy in $B \rightarrow \pi K$ decays \[60\]. However, this fit

\[\text{footnote}{4\text{According to its website, it appears that the CKMfitter Group has modified its point of view compared with this paper. The } B \rightarrow \pi K \text{ measurements are now not part of the global fit to determine the CKM parameters. In particular, the various sin}2\beta^{\text{eff}} \text{ results from penguin-dominated decays are not included. This suggests that the CKMfitter Group feels that there are some curious results in } \bar{b} \rightarrow \bar{s} \text{ transitions.}}\]
requires $|C'/T'| = 1.6 \pm 0.3$. This value is much larger than the naive estimates of Eq. (7), the NLO pQCD prediction, $|C'/T'| \sim 0.3$ [67], and the maximal SCET (QCDf) prediction, $|C'/T'| \sim 0.6$ [6] [68]. Thus, if one takes these theoretical results seriously, one is led to conclude that the $B \to \pi K$ puzzle is still present, at the level of $\gtrsim 3\sigma$. Indeed, the puzzle is much worse in 2006 than in 2004 [69].

Assuming that the effect is not a statistical fluctuation, one must add NP operators. If one ignores the small $\mathcal{O}(\lambda^2)$ diagrams, the $B \to \pi^i K^j$ amplitudes $(i,j$ are electric charges) can be written [51]

\begin{equation}
A^{\pm 0} = -P'_{tc} + \mathcal{A}^{c,d}_{u,e} e^{i\phi^{c,d}_{u,e}}, \\
\sqrt{2}A^{0+} = P'_{tc} - T' e^{i\gamma} - P'_{EW} + \mathcal{A}^{\text{comb}} e^{i\phi} - \mathcal{A}^{c,u}_{u,e} e^{i\phi^{c,u}_{u,e}}, \\
A^{-+} = P'_{tc} - T' e^{i\gamma} - \mathcal{A}^{c,u}_{u,e} e^{i\phi^{c,u}_{u,e}}, \\
\sqrt{2}A^{00} = -P'_{tc} - P'_{EW} + \mathcal{A}^{\text{comb}} e^{i\phi} + \mathcal{A}^{c,d}_{c,d} e^{i\phi^{c,d}_{c,d}},
\end{equation}

where $\mathcal{A}^{\text{comb}} e^{i\phi'} = -\mathcal{A}^{c,u}_{u,e} e^{i\phi^{c,u}_{u,e}} + \mathcal{A}^{c,d}_{c,d} e^{i\phi^{c,d}_{c,d}}$. It is not possible to distinguish the two component amplitudes in $B \to \pi^i K^j$ decays. $\gamma$ is the SM weak phase.

The NP operators mentioned above correspond to the decay $B \to \pi K$. But the other $\Delta S = 1$ decays in the U-spin list include $B^0 \to K^+ K^-$ and $B^0 \to K^0 \bar{K}^0$. One might think that the NP operators affecting these decays bear no relation to those in $B \to \pi K$. In fact, this is not true: the other $\Delta S = 1$ decays are the same as $B \to \pi K$ in the limit of flavour SU(3) (which treats $u$, $d$ and $s$ quarks identically). The point is that the matrix elements differ only in the quarks involved, which affects their hadronization. If all quarks are identical [flavour SU(3)], then the hadronization is the same (the NP affects this hadronization only at the level of $m_b/M_{NP}$, which is tiny). Thus, the NP operators for all $\Delta S = 1$ decays are the same, up to SU(3)-breaking effects. We will therefore denote all NP operators as $\mathcal{A}^{c,u}_{c,u} e^{i\phi^{c,u}_{c,u}}, \mathcal{A}^{c,d}_{c,d} e^{i\phi^{c,d}_{c,d}}$, and $\mathcal{A}^{\text{comb}} e^{i\phi'}$.

This is the first main result of this paper. Within SU(3), all $\Delta S = 1$ decays receive contributions from the same NP operators. Constraints on most of these operators can be taken from the analysis of $B \to \pi K$ decays.

We can now compute the contribution of NP operators to all $\Delta S = 1$ decays in the U-spin list. $B^0 \to K^+ \pi^-$ (pair #1) and $B^0 \to K^+ K^-$ (pair #2) receive a NP contribution of the form $\mathcal{A}^{c,u}_{c,u} e^{i\phi^{c,u}_{c,u}}$; $B^0 \to K^0 \pi^0$ (pair #3) receives $\mathcal{A}^{\text{comb}} e^{i\phi'} + \mathcal{A}^{c,d}_{c,d} e^{i\phi^{c,d}_{c,d}}$; $B^+ \to K^0 \pi^+$ (pair #4) and $B^0 \to K^0 \bar{K}^0$ (pair #5) receive $\mathcal{A}^{c,d}_{c,d} e^{i\phi^{c,d}_{c,d}}$. Depending on the expected size of the NP operators, not all decays will be equally affected by the NP. In particular, if a given NP operator is expected to be small, Eq. (4) will be satisfied for this pair of decays. Conversely, if Eq. (4) is violated for a particular decay pair, this points to the presence of a specific NP operator.

The three NP operators were then included in the $B \to \pi K$ fit in Ref. [52], one at a time. It was found that the fit remained poor if $\mathcal{A}^{c,u}_{c,u} e^{i\phi^{c,u}_{c,u}}$ or $\mathcal{A}^{c,d}_{c,d} e^{i\phi^{c,d}_{c,d}}$ was added. That is, large values of these NP operators may be allowed, but there is no experimental evidence that these are needed. Below we therefore assume that
The ratio and direct CP asymmetry of $B^{0}_{s} \rightarrow \pi^{+}K^{-}$ have been measured by the CDF experiment \cite{70}: $BR(B^{0}_{s} \rightarrow \pi^{+}K^{-}) = (3.00 \pm 0.75 \pm 1.00) \times 10^{-6}$ and $A^{dir}_{CP} = 0.39 \pm 0.15 \pm 0.08$. Together with the experimental measurements of $B^{0}_{d} \rightarrow \pi^{-}K^{+}$ shown in Table 1, we have

$$-\frac{A^{dir}_{CP}(B^{0}_{s} \rightarrow \pi^{+}K^{-})}{A^{dir}_{CP}(B^{0}_{d} \rightarrow K^{+}\pi^{-})} = 4.2 \pm 2.0 , \quad \frac{BR(B^{0}_{s} \rightarrow K^{+}\pi^{-})}{BR(B^{0}_{s} \rightarrow \pi^{+}K^{-})} = 3.9 \pm 1.0 . \quad (9)$$

We therefore see that, although the errors are still large, the two ratios are equal, and that Eq. 4 is satisfied by the current data. This suggests that $\pi K$ final-state rescattering is in fact small (or that there are cancellations with the NP).

In addition, the SM predicts that $BR(B^{0}_{s} \rightarrow \pi^{+}K^{-}) \sim (3-10) \times 10^{-6}$ \cite{71,72,73}, in agreement with measurement. Moreover, recent theoretical calculations within the SM \cite{74} make predictions for $B^{0}_{s} \rightarrow \pi^{-}K^{+}$ which are consistent with the results shown in Table 1. This indicates that pair #1 shows no sign of NP. In other words, it supports the idea that $A^{C,U}_{C,U}e^{i\Phi^{C,U}_{C,U}}$ is small.

Finally, above we showed that present $B \rightarrow \pi K$ data predicts a disagreement with Eq. 4 only for pair #3 ($B^{0}_{d} \rightarrow K^{0}\pi^{0}$ and $B^{0}_{s} \rightarrow \bar{K}^{0}\pi^{0}$). Obviously, it will be
important to check U-spin violation in all the $B$-decay pairs, but special attention will be paid to this pair, since a nonzero effect is expected. In particular, it will be necessary to measure the branching ratio and direct CP asymmetry in $B_s^0 \to \bar{K}^0\pi^0$. This will be challenging.

In fact, measurements of $B_s^0 \to \bar{K}^0\pi^0$ are quite impracticable at present colliders. Since the $B_s^0$ direction cannot be determined at hadron colliders, the $B_s^0$ decay vertex cannot be reconstructed via $K_s \to \pi^+\pi^-$ ($\pi^0 \to \gamma\gamma$ leaves no track). Instead, with a similar technique to that used for $B_d^0 \to K^0\pi^0$ measurements, the (Super) $B$ factory must be used for measurements of this decay. However, the direct CP asymmetry in $\bar{K}^0\pi^0$ requires measurements of the time-dependent CP asymmetry. For this purpose, Super $B$ must have a much better $\Delta t$ resolution than planned (an improvement of an order of magnitude), so that $\Delta m_{B_s}$ measurements can be performed. It is ironic that, although most $B_s^0$ decays can be well studied by hadron collider experiments, the best machine for a U-spin test turns out to be the Super $B$ factory with a better $\Delta t$ resolution.

In summary, some time ago it was pointed out that, within the standard model (SM), a relation involving branching ratios and direct CP asymmetries [Eq. 4] holds for two $B$ decays that are related by U-spin. This equation includes U-spin breaking. There are five (non-annihilation) decay pairs to which this applies. If this relation is found not to hold for a given pair, this implies the presence of physics beyond the SM in that pair. In this paper, we follow the experimental indications and assume that this new physics (NP) appears only in $b \to s$ decays ($\Delta S = 1$) but does not affect $b \to d$ decays ($\Delta S = 0$). There are only a handful of NP operators that can affect the $\Delta S = 1$ $B$-decay amplitudes. We have shown that, to a good approximation, these operators are the same as the three appearing in $B \to \pi K$ decays. The fit to the latest $B \to \pi K$ data shows that only one NP operator is found to be large. As a result, Eq. [4] is expected to be violated by only one decay pair: $B_d^0 \to K^0\pi^0$ and $B_s^0 \to \bar{K}^0\pi^0$. The measurement of the violation of Eq. [4] in this $B$-decay pair will thus be a test of NP in $B \to \pi K$ decays.

Acknowledgments: We thank Seungwon Baek, Alakabha Datta and Fumihiko Ukegawa for very helpful communications. This work was financially supported by NSERC of Canada.

References

[1] M. Gronau, Phys. Lett. B 492, 297 (2000).

[2] R. Fleischer, Phys. Lett. B 459, 306 (1999).

[3] C. Jarlskog, Phys. Rev. Lett. 55, 1039 (1985).
[4] For example, see C. W. Chiang, M. Gronau, J. L. Rosner and D. A. Suprun, Phys. Rev. D 70, 034020 (2004).

[5] A. Khodjamirian, T. Mannel and M. Melcher, Phys. Rev. D 68, 114007 (2003).

[6] C. W. Bauer, D. Pirjol, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D 70, 054015 (2004).

[7] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999).

[8] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, Nucl. Phys. B 591, 313 (2000).

[9] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, Nucl. Phys. B 606, 245 (2001).

[10] Y. Y. Keum, H. n. Li and A. I. Sanda, Phys. Lett. B 504, 6 (2001).

[11] A. L. Kagan, Phys. Lett. B 601, 151 (2004).

[12] A. Khodjamirian, T. Mannel and M. Melcher, Phys. Rev. D 70, 094002 (2004).

[13] A. Khodjamirian, T. Mannel and N. Offen, Phys. Rev. D 75, 054013 (2007).

[14] H. J. Lipkin, Phys. Lett. B 621, 126 (2005).

[15] Heavy Flavor Averaging Group (HFAG), arXiv:hep-ex/0603003.

[16] For example, see W. S. Hou and M. Nagashima, hep-ph/0602124.

[17] For example, see Y. F. Zhou, Eur. Phys. J. C 46, 713 (2006).

[18] For a study of triple products in the SM and with new physics, see A. Datta and D. London, Int. J. Mod. Phys. A 19, 2505 (2004).

[19] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 93, 231804 (2004).

[20] K. Senyo [Belle Collaboration], hep-ex/0505067.

[21] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 91, 171802 (2003).

[22] K. F. Chen et al. [BELLE Collaboration], Phys. Rev. Lett. 94, 221804 (2005).

[23] C. Dariescu, M. A. Dariescu, N. G. Deshpande and D. K. Ghosh, Phys. Rev. D 69, 112003 (2004).

[24] E. Alvarez, L. N. Epele, D. G. Dumm and A. Szynkman, Phys. Rev. D 70, 115014 (2004).
[25] Y. D. Yang, R. M. Wang and G. R. Lu, Phys. Rev. D 72, 015009 (2005).
[26] P. K. Das and K. C. Yang, Phys. Rev. D 71, 094002 (2005).
[27] C. S. Kim and Y. D. Yang, hep-ph/0412364.
[28] C. H. Chen and C. Q. Geng, Phys. Rev. D 71, 115004 (2005).
[29] C. S. Huang, P. Ko, X. H. Wu and Y. D. Yang, Phys. Rev. D 73, 034026 (2006).
[30] P. Colangelo, F. De Fazio and T. N. Pham, Phys. Lett. B 597, 291 (2004).
[31] M. Ladisa, V. Laporta, G. Nardulli and P. Santorelli, Phys. Rev. D 70, 114025 (2004).
[32] A. L. Kagan, Phys. Lett. B 601, 151 (2004).
[33] W. S. Hou and M. Nagashima, hep-ph/0408007.
[34] H. Y. Cheng, C. K. Chua and A. Soni, Phys. Rev. D 71, 014030 (2005).
[35] S. Eidelman et al., Particle Data Group Collaboration, Phys. Lett. B 592, 1 (2004).
[36] B. Aubert [BABAR Collaboration], hep-ex/0408093.
[37] J. Zhang et al. [BELLE Collaboration], hep-ex/0505039.
[38] S. Baek, A. Datta, P. Hamel, O. F. Hernandez and D. London, Phys. Rev. D 72, 094008 (2005).

[39] For example, see: S. Baek, JHEP 0607, 025 (2006), and references therein.
[40] For example, see: Y. L. Wu, Y. F. Zhou and C. Zhuang, Phys. Rev. D 74, 094007 (2006), and references therein.
[41] J. Matias, Phys. Lett. B 520, 131 (2001).
[42] A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Phys. Rev. Lett. 92, 101804 (2004).
[43] A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Nucl. Phys. B 697, 133 (2004).
[44] A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, PoS HEP2005, 193 (2006).
[45] R. Fleischer, Int. J. Mod. Phys. A 21, 664 (2006).
[46] R. Fleischer, J. Phys. G 32, R71 (2006).
[47] R. Fleischer, arXiv:hep-ph/0608010
[48] Y. L. Wu and Y. F. Zhou, Phys. Rev. D 72, 034037 (2005).
[49] C. W. Chiang and Y. F. Zhou, JHEP 0612, 027 (2006).
[50] A. Datta and D. London, Phys. Lett. B 595, 453 (2004).
[51] A. Datta, M. Imbeault, D. London, V. Page, N. Sinha and R. Sinha, Phys. Rev. D 71, 096002 (2005).
[52] S. Baek and D. London, arXiv:hep-ph/0701181
[53] T. Hurth, arXiv:hep-ph/0612231
[54] For example, see F. Schwab, talk given in http://euroflavour06.ifae.es
[55] W. M. Yao et al. [Particle Data Group], J. Phys. G 33,1 (2006).
[56] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 97, 171805 (2006).
[57] K. Abe et al. [Belle Collaboration], Phys. Rev. Lett. 98, 181804 (2007).
[58] A. Bornheim et al. [CLEO Collaboration], Phys. Rev. D 68, 052002 (2003) [Erratum-ibid. D 75, 119907 (2007)].
[59] B. Aubert et al. [BABAR Collaboration], hep-ex/0607106
[60] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 75, 012008 (2007).
[61] B. Aubert et al. [BABAR Collaboration], hep-ex/0607096
[62] S. Chen et al. [CLEO Collaboration], Phys. Rev. Lett. 85, 525 (2000).
[63] K. Abe et al. [Belle Collaboration], Phys. Rev. D 76, 091103 (2007).
[64] M. Gronau, O. F. Hernandez, D. London and J. L. Rosner, Phys. Rev. D 50, 4529 (1994).
[65] M. Gronau, O. F. Hernandez, D. London and J. L. Rosner, Phys. Rev. D 52, 6374 (1995).
[66] CKMfitter Group, J. Charles et al., Eur. Phys. J. C 41, 1 (2005).
[67] pQCD: H. n. Li, S. Mishima and A. I. Sanda, Phys. Rev. D 72, 114005 (2005).
[68] QCDF: M. Beneke and M. Neubert, Nucl. Phys. B 675 (2003) 333.
[69] S. Baek, P. Hamel, D. London, A. Datta and D. A. Suprun, Phys. Rev. D 71, 057502 (2005).

[70] For recent CDF measurements of the branching ratio and direct CP asymmetry of $B^0_s \rightarrow \pi^+K^-$, see G. Punzi [CDF Collaboration], talk at the 4th Workshop on the CKM Unitarity Triangle, December 2006, Nagoya University, Nagoya, Japan.

[71] M. Beneke and M. Neubert, Nucl. Phys. B 675 (2003) 333.

[72] X. Q. Yu, Y. Li and C. D. Lu, Phys. Rev. D 71, 074026 (2005) [Erratum-ibid. D 72, 119903 (2005)].

[73] A. R. Williamson and J. Zupan, Phys. Rev. D 74, 014003 (2006) [Erratum-ibid. D 74, 03901 (2006)].

[74] See, for example, H. n. Li, S. Mishima and A. I. Sanda, Phys. Rev. D 72, 114005 (2005).