Wavefunction of polariton condensates in a tunable acoustic lattice

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\textbf{Abstract.} We study the spatial coherence of polariton condensates subjected to coherent modulation by a one-dimensional tunable acoustic potential. We use an interferometric technique to measure the amplitude and phase of the macroscopic condensate wavefunction. By increasing the acoustic modulation amplitude, we track the transition from the extended wavefunction of the unperturbed condensate to a regime where the wavefunction is spatially modulated and then to a fully confined regime, where independent condensates form at the minima of the potential with negligible particle tunneling between adjacent sites.

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1. Introduction

The phase transition of an incoherent ensemble of bosonic particles to a collective coherent state is characterized by the formation of a macroscopic order parameter \([1, 2]\). In the solid state, such collective phases, or condensates, have recently been observed for microcavity exciton–polaritons, which are light–matter quasi-particles resulting from the strong coupling between photons and quantum well (QW) excitons. Polaritons have very low masses \((10^{-5}–10^{-4}m_e\text{, where } m_e\text{ is the electron mass})\) and, consequently, long de Broglie wavelengths (a few microns) due to their photonic character. As a consequence of the low mass, the transition to the condensed phase can be achieved at a low critical density \(N_c\). They also display repulsive nonlinear interactions mediated by the electric dipoles of their excitonic component. Polariton condensates are non-equilibrium phases, where new particles have to be continuously introduced in order to compensate for the losses due to photon escape from the microcavity. Despite that, macroscopic off-diagonal long-range order can be established \([2]\). The order parameter used to characterize the polariton condensates is their wavefunction, which, depending on the excitation conditions, can extend to tens of micrometers. Polariton condensates can be optically generated by incoherent particle scattering through non-resonant excitation \([3]\) or by a nonlinear parametric scattering mechanism through resonant excitation \([4]\). The macroscopic wavefunction is independent of the way the condensate is generated \([2]\) and therefore constitutes a fundamental feature of these quantum phases.

In this work, we study the wavefunction of a polariton condensate subjected to a variable acoustic periodic potential. The condensates were formed via an optical parametric oscillation (OPO) process by resonant excitation with a laser and then subjected to the modulation by a surface acoustic wave (SAW) propagating along the surface of the microcavity. The latter produces a tunable array of potentials for polaritons consisting of wire-like minima aligned with the SAW wavefronts. Using an interference contrast imaging (ICI) technique \([3]\), we have imaged the amplitude and phase of the condensate emission for different SAW amplitudes, which correspond to different degrees of confinement in the acoustic potential. As expected, the ICI of the unperturbed condensate (i.e. in the absence of an SAW) is uniform with a spatial coherence length \(L\) comparable to the size of the laser excitation spot. In a former work \([5]\), we have shown that the coherence length of the condensates decreases under acoustic...
modulation. We show here that the ICI can directly map the components of the polariton condensate wavefunction in different spatial sites. In particular, we report on the observation of ICIs at intermediate potential heights, which are characterized by a cigar-shaped central lobe oriented along the SAW wavefronts with satellite lobes located at a distance equal to $\lambda_{\text{SAW}}$. These lobes turn out to be essential for understanding the physics of the system under intermediate modulation amplitude since they give information about how the wavefunction is modified by a change in the tunneling rate through the potential barrier between neighboring wires. We discuss two possible scenarios for the formation of the satellite lobes. In the first one, the wire condensate wavefunction corresponds to a bonding (s-like) state which can tunnel to the adjacent wires. In the second one, condensation takes place at the borders of the folded Brillouin zone induced by periodic modulation. The satellites disappear for high acoustic amplitudes, thus indicating that the wavefunction cannot penetrate the barriers, and therefore that the condensate has become fragmented into an array of decoupled wires. In contrast to other works where synchronization/disynchronization phenomena of spatially separated condensates under random photonic disorder have been studied [6, 7], in our work the tunable acoustic modulation provides a uniform periodic potential independent of the photonic noise where the periodicity plays a fundamental role in the determination of the polariton condensate wavefunction.

2. Experiment

The experiments were carried out on an (Al,Ga)As microcavity with three pairs of 15 nm thick QWs placed at the three central maxima of a confined electromagnetic field. The exciton–photon coupling energy (i.e. Rabi splitting) is 6.8 meV. The detuning of the bare photonic resonance with respect to the bare excitonic energy is 1 meV, thus indicating that the polaritons have excitonic and photonic components in the ratio 58%/42%. The polaritons were excited using a single-mode laser of variable wavelength ($\lambda_\ell$) and incidence angle ($\theta_\ell$, see figure 1(a)) to give the incoming pump photons the adequate planar-momentum $k_\ell = (2\pi/\lambda_\ell)\sin\theta_\ell$ to trigger the OPO process illustrated in figure 1(b). The photoluminescence (PL) emitted by the microcavity was spatially and angularly resolved in order to yield real and reciprocal space spectral images, respectively. The interference measurements were carried out using a Michelson interferometer following the procedure described in [3]. The periodic lattice was created by a Rayleigh SAW with a fixed wavelength $\lambda_{\text{SAW}} = 8 \mu m$ determined by the interdigital transducers (IDTs) deposited on the microcavity surface used to generate it [5]. As illustrated in figure 1(a), the SAW propagates along a non-piezoelectric surface [100] direction, which avoids the detrimental dissociation of excitons by the piezoelectric field [5]. The SAW modulates the polariton energy by changing the band gap of the QWs as well as the optical resonance energy of the microcavity [10, 11], the potential modulation amplitude for polaritons ($\Phi_{\text{SAW}}$) being proportional to the square root of the nominal rf power ($P_{\text{rf}}$) applied to the IDT.

3. Linear and condensed regimes

At low optical excitation, an uncorrelated gas of polaritons with broad energy and momentum distribution forms. The lower branch polariton dispersion is shown in figure 1(c). The line is a parabolic fit from which an effective polariton mass of $1.2 \times 10^{-4}m_e$ ($m_e$ is the electron mass)
Figure 1. (a) Periodic potential for polaritons created by an SAW propagating along a non-piezoelectric (⟨100⟩) surface direction of a microcavity. The wavelength of the SAW is \( \lambda_{\text{SAW}} = 8 \mu\text{m} \). (b) Energy versus in-plane wave vector \( k \) dispersion of uncondensed polaritons (i.e. for low optical excitation powers). The red dots indicate the signal, pump and probe condensates formed during the OPO process. The energy and momentum of the pump polaritons are determined by those from the pump laser. (c) Measured dispersion of uncondensed polaritons at low excitation power. The parabolic fit yields a mass equal to \( 1.2 \times 10^{-4} \text{m}_e \). (d) Corresponding polariton dispersion under the SAW potential with applied radio-frequency (rf) power \( P_{\text{rf}} = 22 \text{ mW} \). The flat dispersion levels indicate that the polaritons are well confined in the minima of the potential in panel (a).

was extracted. Under the periodic SAW potential, the dispersion folds, forming mini-Brillouin zones (MBZ) of magnitude \( k_{\text{SAW}} = 2\pi/\lambda_{\text{SAW}} = 0.78 \mu\text{m}^{-1} \) (see figure 1(d)). The flatness of the lowest-energy dispersion branches indicates that these states are spatially confined close to the minima of the potential in figure 1(a). The energy gap \( \Delta E \sim 0.5 \text{ meV} \) at the border of the Brillouin zone is proportional to the confinement potential \( \Phi_{\text{SAW}} \) [11, 12]. Increasing the optical power and tuning the laser close to the dispersion (energy 1.5357 and \( k_\ell = 1.1 \mu\text{m}^{-1} \) in this case) allows the critical density \( N_c \) to be achieved and the phase-matching requirements to be
Figure 2. (a) Spatially resolved spectrum of a polariton condensate in the absence of acoustic excitation. (b) Oscillations of the intensity of the interference between two points located at \( r = (\pm 6, 0) \) \( \mu m \) on the condensate in panel (a) obtained by scanning the relative phase between the normal and retro-reflected images. (c) Fourier transform of the oscillation pattern in panel (b), from where the amplitude of the Fourier coefficients of the oscillating part and the average contrast amplitude can be extracted to obtain the interference contrast. (d–g) Contrast spatial maps of the condensate under SAWs excited by acoustic powers \( P_{rf} = 0, 0.6, 1.4 \) and 22 mW, respectively.

fulfilled, triggering the OPO. The stimulated scattering of pairs of polaritons in the pump (with the laser energy and wave vector \( k_{\ell} \)) sustains the population of the condensates at the conjugate final signal (at lower energy and \( k_{||} = 0 \)) and idler (at higher energy and \( k_{||} = 2k_{\ell} \)) states. The real space emission image of the signal condensate is shown in figure 2(a). The emission has the distinctive features of condensation; namely, it is single-mode, orders of magnitude more intense than for the uncorrelated polariton gas, and blueshifted with respect to the bottom of the dispersion (see figure 1(c)). The blueshift arises from repulsive polariton–polariton interactions [13].

4. Condensate spatial coherence

4.1. Interference technique

The photons emitted from the microcavity carry information about the quantum state of the condensate—the spatial extension of the condensate wavefunction can then be determined by measuring the coherence length \( L \) of the spatially resolved PL emission. The phase coherence between the PL at positions \( r \) and \(-r\) is quantified by the first-order correlation function \( g^{(1)}(r, -r) \). \( g^{(1)}(r, -r) \) was measured using a Michelson interferometer with a retro-reflector in one of its arms in order to interfere an image of the condensate with its retro-reflection [3].
these conditions, every image point \( r \) is superposed on its spatial conjugate at \(-r\). By changing the relative phase \( \phi \) between the images by varying the length of one of the arms, the intensity \( I_{PL}(r, \phi) \) detected at each image point \( r \) oscillates with period \( \phi \). As an example, figure 2(b) displays the interference oscillations recorded at the point \( r = \pm 6 \ \mu m \) of the condensate of figure 2(a) (the center of the condensate is the origin \( r = 0 \)). \( g^{(1)}(r) \) is proportional to the contrast ratio defined as \( C(r) = [I_{PL, max}(r) - I_{PL, min}(r)]/[I_{PL, max}(r) + I_{PL, min}(r)] \), where \( I_{PL, max}(r) \) and \( I_{PL, min}(r) \) are the maximum and minimum \( I_{PL}(r, \phi) \) values recorded during a phase scan. In order to calculate the contrast, we applied a discrete Fourier transformation to determine the coefficients \( c_i \) (i.e. periodic component amplitudes) of \( I_{PL}(r, \phi) \). Figure 2(c) displays such a transformation for the scan in figure 2(b), where the peaks \( c_i \) close to \( i = 9 \) correspond to the periodicity of the phase modulation. The contrast is then given by \( C(r) = (\Sigma_i c_i(r))/2c_1(r) \), where the sum is over the coefficients adjacent to \( c_9 \) (i.e. \( i = 6–12 \) in this case) and \( c_1 \) is the zero frequency component, which determines the average contrast amplitude.

4.2. Coherence under acoustic modulation

Figure 2(d) displays the ICI for the unperturbed condensate. As expected, the contrast (i.e. degree of spatial correlation) is high all over the condensation region, yielding a coherence length \( L \sim 20 \ \mu m \) comparable to the size of the laser spot (30 x 50 \( \mu m \) elliptical shape; the minor axis is along the x-direction). Figures 2(e)–(g) show ICIs of the condensate under SAWs propagating along the y-direction with increasing amplitude (specified in terms of the nominal rf power \( P_{rf} \) applied to the transducer). In contrast to the spatially uniform contrast of the unperturbed condensate, the contrast of the modulated condensate is characterized by a cigar-shaped central lobe at \( y = 0 \) with satellite lobes at positions \( y = \pm \lambda_{SAW} = 8 \ \mu m \). As will be discussed in detail below, the cigar-shaped lobes correspond to the wire-like potential minima created by the SAW along its wave fronts. The satellite lobes result from the interference of wavefunction components at neighboring wires. The intensity of the satellite lobes decreases at high acoustic powers (figure 2(g)), as the tunneling probability between neighboring minima reduces due to the increasing barrier heights between them.

Figures 3(a) and (b) show the full dependence of the contrast profiles across and along the wires on applied acoustic power \( P_{rf} \) obtained by taking a cross section of the contrast images along the y- and x-directions, respectively. Figure 3(a) clearly tracks the gradual formation of the satellites at \( y = \pm \lambda_{SAW} \) by the spatial modulation of the condensate wavefunction and their subsequent disappearance, when the potential barriers become large enough to prevent particle tunneling within the acoustic lattice. Figure 3(c) displays the intensity of the central (squares) and satellite lobes (dots) as a function of \( P_{rf} \). The transition to isolated wire condensates occurs around \( P_{rf} \sim 2 \ \text{mW} \), where the intensity of the satellite lobes drops below the noise level (dashed line in figure 3(c)). The contrast of the central wire decreases slightly (within the experimental error) for higher powers, thus attesting to the coherent nature of the SAW modulation process. This reduction can be attributed to dimensionality effects [5], as discussed in detail below. At the maximum applied \( P_{rf} \), the central lobe has a width along \( y \) of 5 \( \mu m \), which is slightly larger than the size of the confinement potential \( \lambda_{SAW}/2 = 4 \ \mu m \). The larger value for the measured width is probably due to the limited spatial resolution of the experiment \( \sim 3 \ \mu m \).

The contrast profiles for the unconfined condensate (upper plots in figures 3(a) and (b)) have different shapes along the x- and y-directions (i.e. along and across the wires, respectively). This difference is attributed to the asymmetric shape of the exciting laser spot, which is actually

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ellipsoidal with the larger axis oriented along the $x$ (i.e. along the wire). The coherence length along $x$ also decreases with increasing acoustic power. The reduction, however, is much smaller than the dramatic one observed along $y$. This behavior is not yet fully understood, but could be due to a change in the dimensionality, which makes it less robust to incoherent processes \cite{11}.

5. Discussion

The contrast images in figures 2(d)–(g) clearly demonstrate that the SAW potential modulates and segments the condensate wavefunction. In this section, we discuss in more detail the dependence of the ICIs on the SAW modulation amplitude.

A special feature of the interference measurements under an SAW is that the modulation potential *moves* with the SAW propagation velocity $v_{\text{SAW}} \sim 3 \, \mu \text{m ns}^{-1}$, with the normal and retro-reflected images moving in opposite directions\footnote{We note that the coherence time of the polariton condensates, which is 150–200 ps, is much smaller than the period of the SAW (2.6 ns), thus restricting the coherent transport of the condensate to very short distances ($\sim 0.5 \, \mu \text{m}$). The potential can be considered, therefore, as almost static for the condensates generated.}. The position of the wire minima at $t = 0$ (defined arbitrarily as the reference time) and $t = t_{\text{SAW}}/2$ (where $t_{\text{SAW}}$ is the SAW period) are illustrated in figures 4(a) and (c), respectively. The unperturbed condensates are 20 $\mu \text{m}$ wide.
Figure 4. (a) Schematic diagram illustrating the superposition of the normal and its retro-reflection of the SAW modulation at a time $t = 0$ (left panel) and $t = \frac{\tau_{SAW}}{2}$ (right panel). We assume that the area of the unconfined condensate (green circle) fits up to three wires. (b) Wavefunction $\Psi_1(r, t)$ of the condensate at $t = 0$ (left panel) and $t = \frac{\tau_{SAW}}{2}$ (right panel). The wavefunction is represented as a superposition of components $\Psi_i(r, t) (i = 0, \pm 1)$ localized at neighboring wires. (c) Corresponding profiles for the retro-reflected wavefunctions.

and can therefore accommodate at most three SAW minima. In the interference experiments, the normal condensate image is superimposed on its retro-reflection to access the correlation between points located symmetrically with respect to the center of the image, as indicated by circles in figure 4(a). In order to understand the effects of the movement, we will assume the wavefunction $\Psi(r, t)$ (and its retro-reflected counterpart $\Psi(-r, t)$) to have the shapes indicated in figures 4(b) and (d) at these two times. For a perfectly periodic array, $\Psi(r, t)$ should have the same amplitude and phase in all lattice sites. In the presence of potential fluctuations, however, the wavefunction becomes localized with a localization length given by the phase coherence length $L$. Note that figures 4(b) and (d) display only schematic representations. A proper determination of $\Psi(r, t)$ requires a numerical solution of the non-equilibrium Gross–Pitaevskii equations [14] under the SAW potential by taking into account the partial screening due to...
polariton–polariton interactions \[11\], which is beyond the scope of this work. This simple scheme, however, is very useful in understanding the main features observed in the experiments.

In a time-integrated interferometric measurement, the maximum contrast is expected to be generated at the time instants \(t = 0\) and \(t = \tau_{\text{SAW}}/2\), when the wire-minima of the normal and retro-reflected images superimpose on each other. We first consider the simplest situation where the SAW introduces a density fluctuation with spatial periodicity \(\lambda_{\text{SAW}}\) and amplitude \(\delta \rho\) on an unperturbed condensate with particle density \(\rho_0\). If we assume that the phase coherence between two points only depends on the distance between them, we obtain the following equation for the time-averaged contrast ratio under an SAW:

\[
C(y) = e^{-2|y|/L} \left[ 1 + \frac{1}{2} \frac{\delta \rho}{\rho_0} \cos \left( 2k_{\text{SAW}} y \right) \right],
\]

(1)

where \(k_{\text{SAW}} = 2\pi/\lambda_{\text{SAW}}\). In general, the phase coherence length \(L\) should depend on the amplitude of the acoustic lattice. For small coherence lengths (\(L < \lambda_{\text{SAW}}\)), the contrast vanishes except close to \(y = 0\). Under these conditions, the high potential barriers block wavefunction tunneling between adjacent sites, leading to its localization in single wires. This corresponds to the situation for high acoustic powers illustrated in figure 3(a).

For large coherence lengths (i.e. \(L \gg \lambda_{\text{SAW}}\)), the contrast images should display a series of lobes of the same amplitude and separated by \(\lambda_{\text{SAW}}/2\). The experimental data for moderate acoustic powers in figure 3(a) (or in figures 2(e) and (f)) only show lobes at \(y = \pm \lambda_{\text{SAW}}\), thus indicating that the coherence length for these modulation amplitudes exceeds \(\lambda_{\text{SAW}}\). The intensity of the satellite lobes relative to the central one yields directly the relative amplitude of the two wavefunction components. Interestingly, the satellites predicted by equation (1) for \(y = \pm \lambda_{\text{SAW}}/2\), which correspond to the interference processes indicated in figure 4(c), could not be observed in the data for moderate rf powers of figure 3(a). Different mechanisms can be invoked to explain the absence of these lobes. First, they may be buried into the wide signal from the central lobe \(y = 0\). In addition, the separation between the \(y = \lambda_{\text{SAW}}/2\) lobes and the central one at \(y = 0\) is comparable to the spatial resolution of the setup, which further hinders their detection. Finally, due to the Gaussian profile of the exciting laser, the polariton density in the wires of figure 4(c) is expected to be approximately 30% lower than in the central wire of figure 3(a). The reduced density may also reduce \(L\), leading to a lower contrast signal for the wires at \(y = \pm \lambda_{\text{SAW}}/2\).

In the previous paragraph, we assumed that condensation takes place at the \(\Gamma\) point of the Brillouin zone (\(k = 0\)). A second possibility for the formation of lobes separated by \(\lambda_{\text{SAW}}\) is that condensation takes place at the border of the MBZ (X point, \(k = k_{\text{SAW}}/2\)). The wavefunction of the extended state has in this case a periodicity equal to \(2\lambda_{\text{SAW}}\). The interference process described in figure 4 would then take place when the maxima of the wavefunction is located at the \(y = 0\) or \(y = \pm \lambda_{\text{SAW}}\) positions in panel (b). There would be no interference in the situation described in panel (d), since the maximum would overlap with its minimum in the retro-reflected image. This would be consistent with the experimental observations reported in \[5\] and \[9\], where a strong emission was reported at the border of the MBZ.

Under the present experimental conditions it is not possible to fully discriminate between the two cases. More detailed experiments with larger spot sizes and full \(k\)-space mappings, however, could help us to extract information on which type of condensation occurs under which exact conditions.

Finally, we briefly discuss possible mechanisms leading to the localization of the wavefunction and, therefore, to the reduction of the coherence length under the acoustic
modulation. In unconfined condensates, the particle density can spread along the QW plane and, in that way, screen potential fluctuations over the two-dimensional (2D) condensation area. An SAW beam produces a uniform and perfectly periodic potential lattice of width comparable to the width of the beam (of approx. 170 µm). The 1D confinement restricts particle motion along its propagation direction, thus making the confined condensates more sensitive to potential fluctuations than extended ones. The reduction in dimensionality can explain the slight reduction of the contrast of the central lobe in figure 3(c) at high $P_T$ values. When condensates are excited using a laser with a Gaussian intensity profile, more polaritons are generated in the center of the beam than at its boundaries. In unconfined condensates, polaritons can be expelled from the regions of a high generation rate in order to keep the same energy all over the condensate area [15]. In the presence of an acoustic lattice, however, this process becomes partially blocked, leading to different polariton densities within the different wires. Due to nonlinear interactions, the different wires will have different energy renormalizations $\Delta E_{BS} = gN$ [16] roughly proportional to the polariton density $N$, where $g$ is the polariton–polariton interaction constant with a value estimated to be of the order of 1–5 µeV µm$^{-1}$ [17]. For the condensate wires to interfere with each other, the energy difference between them must be smaller than their linewidth, which was determined to be of the order of $\sim 4$ µeV from independent time coherence experiments. The energetic shift due to a difference in density of 1–10 polaritons per µm is therefore large enough to hinder interference. Due to this strong dependence it is not surprising that the coherence length perpendicular to the wires decreases so dramatically even for low applied acoustic powers, as shown in figure 3(a). Long coherence lengths in the 1D polariton system thus require very tight control of the optical excitation conditions.

6. Conclusions

We have demonstrated the modulation and fragmentation of an extended condensate wavefunction by the tunable potential modulation generated by an SAW. The spatial coherence of the wavefunction was directly measured using interferometry. The ICIs track the transition from an extended wavefunction, which tunnels between the barriers separating two neighboring potential minima, to the full confinement regime, where the wavefunction becomes localized in a single minimum. We present a simple scheme to explain the main features of the observed contrast patterns and hope to motivate efforts towards a full theoretical description within the community. This work opens a venue for the study of polariton condensation under periodic potentials. Through the fine control of the correlation among spatially separated condensates, we show how the complex dynamics of this nonlinear system change the wavefunction of the condensate, which is essential for the study of phase transitions [8]. 2D SAW potentials with smaller SAW wavelengths, for example, should allow reaching the quantum blockade regime where a single particle is trapped per site (i.e. Mott insulator phase [18]). Under these conditions, massive single-photon emission is expected [19]. Efficient approaches for single-photon generation have also been proposed for coupled quantum boxes [20]. Also, generation of branch entangled states in a scheme similar to that proposed for a confined system in [21] or squeezed states [22] can be implemented by the controlled engineering of the confinement. Finally, the periodicity provides a rich workbench for the study of condensate phases at higher orbitals, for example, which are relevant for the description/simulation of quantum many-body phenomena [9].
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