Acousto-optic properties of metamaterials

N I Petrov and V I Pustovoit
Scientific and Technological Center of Unique Instrumentation of the Russian Academy of Sciences, 15 Butlerova str., Moscow, 117342 Russia

e-mail: petrovni@mail.ru

Abstract. The possibility of efficiently using metamaterials in acousto-optics has been demonstrated. Diffraction of light in heterogeneous medium with non-uniform spatial distribution of dielectric nanoparticles taking into account absorption of light is investigated. It is shown that by changing the concentration of dielectric nanoparticles in the medium, complete elimination of side oscillations and suppression of the “tails” of the diffraction reflection curve can be achieved. The possibility of controlling the hardware function of acousto-optic devices by changing the material, concentration, size, shape and spatial orientation of the inclusions, as well as the polarization of the incident radiation is shown.

1. Introduction
Periodic structures created in crystals by an acoustic wave have a number of advantages compared to conventional devices based on diffraction gratings. They allow changing the period of a volume grating, modulating its characteristics, and high spectral resolution to be achieved.

There is a limited list of crystalline materials that can be used in the creation of elements and devices of acousto-optics. Lack of necessary crystalline materials is especially felt for the infrared region of the spectrum. In this regard, the ideas on the basis of which it would be possible to overcome this difficulty and significantly expand the list of materials by using different heterogeneous media [1-3] are of great interest.

In this paper we investigate the possibility of using metamaterials for the purposes of acoustooptics. The theoretical analysis of light diffraction in heterogeneous medium with different dielectric inclusions taking into account light absorption is carried out. The possibility of controlling the reflection and transmission function by changing the concentration of dielectric inclusions in the medium, their size, shape, spatial orientation and polarization of the incident radiation is shown. The mechanism of such a change in the dielectric permeability (DP) under the action of a sound wave is as follows. If a longitudinal sound wave propagates in the medium, in which compression and rarefaction regions are formed, it is obvious that due to local changes in the concentration of nanoparticles, a change in the DP and the refractive index of the medium will occur [4].

2. Problem formulation

As a heterogeneous medium, we consider a medium formed by an optically transparent material with DP $\varepsilon_m$, in which dielectric inclusions with the complex permeability $\varepsilon_p$ are introduced in the form of ellipsoids of rotation. It is believed that the size of the ellipsoids of rotation is significantly less than the wavelength of light and they are all located randomly, but their main axes of rotation are directed in the same direction.
In the framework of the Maxwell-Garnett model, such a medium is described by the averaged DP medium $\varepsilon$, which satisfies the relation [2, 5]

$$\frac{\varepsilon - \varepsilon_m}{\gamma (\varepsilon - \varepsilon_m) + \varepsilon_m} - \eta \frac{\varepsilon_p - \varepsilon_m}{\varepsilon_m + \gamma (\varepsilon_p - \varepsilon_m)} = 0,$$

(1)

where $\eta(x, t)$ is the volume fraction of inclusions - nano-ellipsoids of rotation, and $\gamma$ is the depolarization factor, which can be expressed in terms of the ratio of the half-axes $\xi = a / b$ of ellipsoids for different directions of the external field [2].

Note that the model of the Maxwell-Garnett for isotropic effective medium agrees well with the experimental data provided that the particle sizes are less than the wavelength of the radiation and the distance between the particles is greater than their radii. The advantage of this model is that for the analysis of radiation propagation in a heterogeneous medium there is no need to solve Maxwell's equations at each point of space and of accounting for scattering by the particles which compose the heterogeneous medium and interference of scattered waves [2, 3].

Solving equation (1) with respect to $\varepsilon$ we find the explicit form of DP of the heterogeneous medium for the electric field directed along and across the polar axis of the ellipsoid of rotation, respectively [4]. In the chosen coordinate system, the $z$ axis coincides with the direction of the main (polar) axis of the spheroid, and the $x$ and $y$ axes lie in a perpendicular plane.

The change in the dielectric constant of heterogeneous medium in the direction of wave propagation $x$ is described by the expression

$$\varepsilon(x) = \varepsilon_0 + \Delta \varepsilon_{\text{eff}}(x) \cos(\Omega t - qx)$$

(2)

where $\varepsilon_0$ is the constant component of the dielectric constant, $\Omega$ and $q$ are the frequency and wavevector of a sound wave, respectively, $\Delta \varepsilon_{\text{eff}}(x)$ is the amplitude of the change in the dielectric constant, which is a function of the volume fraction of inclusions, and $\Delta \varepsilon_{\text{eff}} < < \varepsilon_0$.

The diffraction of light on a periodic structure (2) is described by reduced equations derived from Maxwell's equations [6, 7]:

$$\frac{dE_1}{dx} = -ik_0\Delta \varepsilon_{\text{eff}}(x)e^{i\Delta k x}E_2,$$

$$\frac{dE_2}{dx} = ik_0\Delta \varepsilon_{\text{eff}}(x)e^{-i\Delta k x}E_1,$$

(3)

where $E_1$ is the amplitude of the incident wave, $E_2$ is the amplitude of the diffracted wave, $k_0 = 2\pi / \lambda$ is the wavenumber of the radiation, $\Delta k = 2k_0 - q$ is the wave detuning, which specifies the deviation from the condition of the Bragg synchronism.

Solutions of equations (3) will be sought under the following boundary conditions: $E_2(L) = 0$, $E_1(0) = E_0$, where $L$ is the length of the crystal.

3. Simulation results
The amplitude of the diffracted wave and the reflection coefficient for the medium with the constant value $\Delta \varepsilon_{\text{eff}}$ along the crystal length are determined analytically [6, 7]. It is followed from the simulations that in this case the reflection spectra demonstrate strong side oscillations. An essential requirement for any broadband filter is that they should have weak side oscillations. Below we show that these oscillations can be suppressed by the amplitude modulation (apodization) of the volume grating. In case of arbitrary dependence of $\Delta \varepsilon_{\text{eff}}(x)$, the system of equations (3) has no analytical solution and it is necessary to use numerical methods to calculate the reflection and transmission.
coefficients. To solve the equations, the method of recurrent relations is used, which is widely used in the physics of X-ray diffraction. According to this method, the crystal is divided into \( N \) layers, and the reflection amplitude from the first \( k \) layers is connected with the reflection amplitude from the previous \( k-1 \) layers [8, 9]. The method of recurrence relations used for numerical calculations is effective for the study of structures with an arbitrary spatial change in the refractive index. In the case of periodic structures where analytical solutions of the wave equation are known (in particular, for the constant function and the exponential apodization function), the numerical results coincide with the exact analytical solutions. Note, that the simple analytical solutions can be found also for \( \sin \) apodization function, when exact Bragg condition is fulfilled, i.e. at \( \Delta k = 0 \).

In [10, 11] the diffraction curves of reflection and transmission were investigated taking into account light absorption in the medium. It was shown that the spectral resolution of \( \Delta \lambda \approx 10^{-2} \) pm, which is three orders of magnitude higher than the resolution of existing acousto-optic filters, can be achieved using known technologies for producing high-purity materials. However, spatial modulation of the DP of medium by changing the sound intensity along the crystal length is technically difficult to implement. Here we consider the change in the dielectric permeability of the medium by introducing dielectric nanoparticles with known parameters, which will allow us to observe the effects under consideration experimentally. Therefore, the use of heterogeneous media in acousto-optics is important from both theoretical and practical point of view.

Below we consider the longitudinal sound wave propagating along the minor optical axis \( x \) of the ellipsoid of revolution when the light wave with the polarization along the \( z \) axis propagates in the same direction. In Fig. 1 the reflection curves for heterogeneous medium with elongated (a) and flattened (b) ellipsoids made of LasF9 material are presented for the dielectric constant of the medium, which is apodized according to the law \( \Delta \varepsilon(x) = \Delta \varepsilon_0 \sin(2\pi x / L) \). The polarization of the electromagnetic wave is directed parallel to the axis of revolution of the ellipsoid. A narrow dip is observed on the reflectance curve for flattened ellipsoids at \( \Delta k = 0 \) (Fig. 1b). This indicates that in this area of the spectrum the considered system does not reflect, but transmits radiation. The width of narrow transmission line is limited by the absorption of light in a heterogeneous medium.

![Reflection coefficients for heterogeneous medium with elongated (a) and flattened (b) ellipsoids](image)

**Figure 1.** Reflection coefficients for heterogeneous medium with elongated (a) and flattened (b) ellipsoids: \( \Delta \varepsilon_0 = 0.5 \cdot 10^{-5} \), \( L = 3 \) cm, \( \lambda = 630 \) nm, \( \varepsilon'_m = 2.123 \), \( \varepsilon''_m = 8.74 \cdot 10^{-11} \), \( \eta_0 = 10^{-3} \). Semi-axes of ellipsoids: \( a = 5 \) nm, \( b = 10 \) nm (a), \( a = 10 \) nm, \( b = 5 \) nm (b).

In Fig. 2 the coefficients of reflection from the meta-medium depending on the deviation from the Bragg condition in the infrared region (IR) of the spectrum are presented. The polarization of the electromagnetic wave is directed perpendicular to the axis of revolution of the ellipsoid. As the matrix medium, the quartz is considered with absorption coefficient \( \alpha = 3.46 \cdot 10^{-7} \) cm\(^{-1} \) (\( \varepsilon'_m = 2.095 \); \( \varepsilon''_m = 2.49 \cdot 10^{-11} \) [12]), nano-ellipsoids are made of a material LasF9 with the dielectric constant \( \varepsilon'_p = 3.2893 \); \( \varepsilon''_p = 2.4891 \cdot 10^{-7} \) and absorption coefficient \( \alpha = 5.638 \cdot 10^{-3} \) cm\(^{-1} \) at a wavelength \( \lambda = 1.53 \) \( \mu \)m. The reflection coefficients depend on the polarization of the incident radiation. The width of
the frequency domain with a high reflection coefficient is greater for the case of the field directed along the axis of rotation of the spheroid. The width of the reflection band increases with the size and concentration of embedded particles.

Figure 2. Reflection coefficients for heterogeneous medium with elongated (a) and flattened (b) ellipsoids in IR spectral region. \( \Delta \varepsilon(x) = \Delta \varepsilon_0 \sin(\pi x / L) \), \( \Delta \varepsilon_0 = 0.5 \cdot 10^{-5} \), \( L = 3 \text{ cm} \), \( \lambda = 1.53 \mu\text{m} \), \( \eta_0 = 3 \cdot 10^{-3} \). Semi-axes of ellipsoids: \( a = 5 \text{ nm} \), \( b = 10 \text{ nm} \) (a), \( a = 10 \text{ nm} \), \( b = 5 \text{ nm} \) (b).

4. Conclusions
Thus, the possibility to expand the spectral band of reflection of the incident radiation by changing both the parameters of the particles, their concentration and the polarization of the incident radiation is shown. The inhomogeneous distribution of nanoparticles in the matrix medium makes it possible to implement special conditions for collinear diffraction of light on sound and, as a consequence, to create acousto-optic devices and devices with unique properties, such as tunable optical radiation filters without side transmission maxima, to control the transfer function of acousto-optic devices.

The introduction of nanoparticles into the matrix medium, the dielectric permeability of which differs from the DP of the matrix medium, leads to the appearance of photoelastic constants for longitudinal sound waves in the heterogeneous medium. The mechanism of occurrence of photoelastic constants is a local change in the concentration of nanoparticles.

Wide opportunities for the creation of qualitatively new acousto-optic (AO) devices are opened with the use of inclusions of nanoparticles (metamaterials) with a complex shape and with special magnetic, dielectric and conductive properties. Acousto-optics based on heterogeneous media opens up opportunities for the creation of devices of infrared (IR) and terahertz technology, inaccessible to conventional crystals.

The obtained results are of practical interest and can be used to create acousto-optic devices and devices with unique properties, such as tunable optical filters without side reflection and transmission maxima, modulators and AO spectrometers.

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