MOND and the fundamental plane of elliptical galaxies

Riccardo Scarpa

European Southern Observatory, 3107 Alonso de Cordova, Santiago, Chile

rscarpa@eso.org

ABSTRACT

It is shown that the MOdified Newtonian Dynamics (MOND) explains the tilt of the fundamental plane of elliptical galaxies without the need of non-baryonic dark matter. Results found for elliptical galaxies extends to globular clusters and galaxy clusters, showing that MOND agrees with observations over 7 order of magnitude in acceleration.

1. Introduction

High-surface-brightness (HSB) elliptical galaxies are known to define a “fundamental plane” (FP) when represented in the logarithmic space of their effective radius $r_e$, average surface brightness $\Sigma$ within $r_e$, and central stellar velocity dispersion $\sigma$ (Dressler et al. (1987); Djorgovski and Davis (1987); Fig. 1). If elliptical galaxies had a constant mass-to-light ratio ($M/L$), then the physical base for the FP would be obvious because the virial theorem would imply a link between these three quantities. Real galaxies, however, do not follow the plane defined by the virial theorem (Busarello et al. (1997); Levine & Aguilar (1996); Prugniel and Simien (1997)). Finding an explanation for this unexpected behavior has been and still is one of the major puzzle in modern astrophysics. In a nutshell, both possibilities of a systematic change of $M/L$ with luminosity and departure from homology were fully investigated. In order to preserve the narrowness of the FP, the tilt can be explained fine tuning the amount of dark matter to the galaxy luminosity, or fine tuning the departures from homology to the galaxy size (e.g., Ciotti, Lanzoni and Renzini (1996)). Of the two options the former is the most widely accepted, and the tilt ascribed to $M/L \propto L^{0.25}$ (e.g., Bender, Burstein, and Faber (1993)).

Whatever the reason for the tilt is, both these possibilities face paramount difficulties as soon as low-surface-brightness (LSB) elliptical galaxies are considered, because these galaxies follows an inverse trend $M/L \propto L^{-0.4}$ (Dekel and Silk (1986); Peterson and Caldwell (1993)). Thus, it look like any classical picture conceived to explain the tilt of the FP defined by HSB, is deemed to fail with LSB, and vice versa. The goal of this work is to investigate whether a consistent explanation for the tilt of the FP and the opposite behavior of HSB and LSB galaxies can be ascribed to a completely different physical phenomenon.

In 1983 a modification of Newtonian dynamics known as MOND has been proposed as an alternative to non-baryonic dark matter (Milgrom 1983). Milgrom’s proposal states that gravity
(or inertia) does not follow the prediction of Newtonian dynamics for acceleration smaller than $a_0 = 1.2 \times 10^{-8}$ cm s$^{-2}$ (Begeman et al. 1991). The true acceleration of gravity $g$ being related to the Newtonian acceleration $g_N$ by

$$g_N = g\mu(g/a_0).$$  \hspace{1cm} (1)

The interpolation function $\mu(g/a_0)$ admits the asymptotic behavior $\mu(g/a_0) = 1$ for $g >> a_0$, so to retrieve the Newtonian expression in the strong field regime, and $\mu(g/a_0) = g/a_0$ for $g << a_0$. The constant $a_0$ is meant to be a new constant of physics. The transition from the Newtonian to the MOND regime occurs for values of acceleration that remain undetected within the solar system where the strong field of the sun (of the order of $10^{-4}$ cm s$^{-2}$ at the distance of Pluto) dominates all dynamical processes. As a consequence, we have no direct proof of the validity of Newton’s law in the low acceleration regimes that are typical in galaxies.

Since the seminal papers on MOND (Milgrom 1983a; 1983b,c), a few authors have worked on this subject showing how this simple idea explains many properties of galaxies without the need of non-baryonic dark matter (e.g., Begeman et al. 1991; Milgrom 1994; Sanders 1996; McGaugh & de Block 1998). MOND is also effective in describing the dynamics of galaxy groups and clusters (Milgrom 1998; Sanders 1999) and, with some approximations, gravitational lensing (Qin and Zou 1995; Mortlock and Turner 2001). For a recent review see Sanders and McGaugh (2002).

Clearly, MOND applies to any stellar structure and therefore its effects should be visible also in elliptical galaxies. In this work, I shall study the FP within the contest of MOND, to see if it is possible to explain both the tilt of the FP and the opposite behavior of LSB and HSB galaxies without the needs to invoke non-baryonic dark matter.

### 2. MOND effects on elliptical galaxies

If gravity deviates from Newtonian prediction in the low acceleration regime typical of galaxies, then the logical implication is that the weaker the field, the bigger the deviation. With this in mind, the Kormendy relation (Fig. 2) assumes a new meaning because in elliptical galaxies $\Sigma$ (or equivalently $\mu_e$), is proportional to their internal acceleration of gravity $^1$. Thus the trends followed by HSB and LSB galaxies in the $\mu_e - r_e$ plane can be seen as trends of their internal gravitational field. The opposite dependence of $\mu_e$ on $r_e$ observed in HSB and LSB then suggest these galaxies enter into MOND regime in different ways and, as a result, should follow different FP. In order to use MOND to verify if this is the case, the interpolation function $\mu$ must be specified. Different

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$^1$This derive from the assumption that there is no dark matter. In this case the acceleration at the effective radius is $a = GxM/r_e^2$, where $x$ is the fraction of the total mass within $r_e$. After substituting $M = \tau L$ and $L = 2\pi r_e^2\Sigma$ we have $a = 2\pi Gx\tau\Sigma \propto \Sigma$. 

but functionally equivalent expression for \( \mu \) have been used in the literature (Milgrom 1983b, 1984; Sanders 1996). Currently the most used function, also adopted here, is:

\[
\mu(g/a_0) = \frac{g/a_0}{\sqrt{1 + (g/a_0)^2}},
\]

which works well in describing galactic rotation curves and has the advantage of being simple. Solving for \( g \) the true acceleration of gravity reads

\[
g = g_N \left[ \frac{1}{2} + \frac{1}{2} \sqrt{1 + \left( \frac{2a_0}{g_N} \right)^2} \right]^{1/2}.
\]  \( 3 \)

This expression allows the calculation of the true gravitational acceleration provided the Newtonian one is known.

Before proceeding to compute \( g_N \), it is worth noticing that if \( \tau \) is the true value of M/L of an object, and \( \tau_N \) is the one derived using Newton’s law to convert accelerations to masses, then \( g/g_N \equiv \tau_N/\tau \). Indeed, imagine an acceleration \( g \) is measured at distance \( r \) from an object of luminosity \( L \). In MOND we interpret this \( g \) as due to a mass \( M = gr^2/G \) \( \equiv \tau L \), where the term \( \equiv \) is the factor \( g/g_N \) defined in eq.3. On the other hand, according to Newton’s law we infer a mass \( M_N = gr^2/G = \tau_N L \). Then the ratio \( M_N/M = \tau_N/\tau = \equiv g/g_N \). Thus, any difference between \( g \) and \( g_N \) will appear as a variation of \( \tau \).

To investigate these effects on elliptical galaxies, I shall take as representative of each galaxy the acceleration of gravity \( g_N \) at \( r_e \), computed setting \( \tau \) from the stellar population typical of the galaxies and replacing the gravitational radius with \( r_e \). In absence of dark matter both assumptions are valid. The mass within \( r_e \) is set to be \( x = 0.41 \) of the total mass, as appropriate for galaxies following the \( r^{1/4} \) de Vaucouleurs law (Young 1976). Under these assumptions, the acceleration of gravity at \( r_e \) becomes:

\[
g_N = \frac{GMx}{r_e^2} = 1.406 \times 10^{-11} x \tau L \left[ \text{cm s}^{-2} \right],
\]  \( 4 \)

where the numerical constant is correct for \( \tau \) and \( L \) in solar units, and \( r_e \) in pc. Combining eq. 3 and 4 one can compute \( g/g_N \) for all galaxies.

3. The mass to light ratio

Before proceeding with the calculation of \( g_N \), I have to set the value of \( \tau \). Since I am assuming Newton’s law fails below \( a_0 \), masses derived using any dynamical model based on this law can obviously not be used here.
Stellar evolution theories, which potentially provide the best tools for computing $\tau$, give results that critically depend on the adopted initial mass function (IMF) and the low masses cutoff. These parameters are rather uncertain and imply large differences on the derived $\tau$. For instance, for a single coeval stellar population, assuming a single-slope Salpeter IMF with slope 2.35, $\tau$ changes by a factor 4.5 varying the low masses cutoff from 0.15 to 0.05 $M_\odot$. Note that this problem is only mildly alleviated using a more realistic multi-slope IMF (e.g., Scalo 1986) at low masses (Renzini and Ciotti 1993).

An unbiased way to estimate $\tau$ relies on the analysis of the stars in the solar neighborhood, where stars of all spectral type in all possible evolutionary phases can be studied in great detail. From Binney and Merrifield (1998), the total mass of stars in $10^4$ pc$^3$ is $356M_\odot + 30M_\odot$ of white dwarfs. For this sample it is found $\tau = 0.7$.

This sample of stars can be used to estimate $\tau$ for any stellar population simply integrating the light over all the relevant spectral types, while maintaining the total mass fixed. This will result in an upper limit to the true $\tau$, because more than one generation of stars contributed to it and therefore the number of low mass stars is overrepresented.

In elliptical galaxies the stellar population is composed by stars in: sub-giant branch, red-giant branch, horizontal branch, and main sequence up to spectral type K. In $10^4$ pc$^3$ there are 4 red giants of K type and 0.25 of M type populating the giant branch, which have average luminosity of 57 and 120 solar luminosities, respectively. For each star in the giant branch there are $\sim 21$ stars in the sub-giant branch and 0.75 in the horizontal branch (Renzini 1998), which contribute for 5 and 52 $L_\odot$, respectively. Stars still in the main sequence add another 18 $L_\odot$. From these considerations it follows that $\tau = 386/420 \sim 0.9$. This value is consistent with that derived for globular clusters (Mandushev, Staneva and Spasova 1991; Renzini and Ciotti 1993) that have a similar old stellar population. These objects, however, are slightly different form elliptical galaxies because they don’t contain gas and at least a fraction of the less massive stars, which contributes to the mass but not to the luminosity, have probably left the clusters long ago (e.g. Aguilar, Hut and Ostriker 1988; Smith and Burkert 2002). So, for globular cluster $\tau = 1$ should be fairly correct, while here I shall set $\tau = 2$ for all elliptical galaxies. A fairly conservative assumption that allows for the presence of large amount of gas, particularly relevant in LSB galaxies.

For spiral galaxies, setting $\tau = 1.2$, that is about twice as big as that observed in the solar neighborhood, allows for plenty of gas and the bulge old stellar population. Finally, for cluster of galaxies I assume a mix of spiral and ellipticals, with $\tau \sim 1.6$. I stress that the results reported in this work do not significantly depend on the assumed value of $\tau$ in the range 1-2.

4. **Comparing MOND with observations**

I compute here the quantity $g/g_N$ and investigate its relation with $\sigma$, $r_e$, and $L$ for a sample of elliptical galaxies.
The representative data set for HSB galaxies is taken from the compilation of Bender, Burstein, and Faber (1993), to which I added data for 6 LSB and 11 dwarfs galaxies (Peterson and Caldwell 1993), 6 compact ellipticals (Guhathakurta and van der Marel 2002), and 22 radio galaxies (Bettoni et al. 2001). Throughout the paper, $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ is used.

The acceleration $g_N$ at $r_e$ is found comparable to $a_0$ for HSB galaxies, while being well below this value in LSB. The effect of MOND on objects characterized by different $\Sigma$ is shown in Fig. 3. It is seen that galaxies follow a continuous sequence implying the mass discrepancy, or in other word the dark matter content, is uniquely defined by $\Sigma$. I shall show that this fact has important consequences for understanding why objects as different as globular and galaxy clusters may appear to have the same mass discrepancy.

In Fig. 3, HSB and LSB galaxies are no longer distinguishable because the opposite dependence of $\Sigma$ on $r_e$ (see Fig. 2) disappears. The different behavior of HSB and LSB become evident when plotting $g/g_N$ versus $r_e$ or $L$ (Fig. 4 and 5). It is therefore clear that the effect of modified dynamics is to mimic an increase of $\tau$ as a function of $L$ for HSB galaxies, while doing the opposite for LSB galaxies. Moreover, Fig. 5 shows that with good approximation, and certainly within the spread of the data, the dependence on luminosity is $\tau \propto L^{0.25}$ for HSB galaxies, and $\tau \propto L^{-0.3}$ for LSB galaxies, consistent with the values quoted in the literature (Bender, Burstein, and Faber 1993; Peterson and Caldwell 1993).

The striking different behavior of HSB and LSB galaxies can therefore be easily explained by a unique model assuming a modification of Newton’s law. Under this hypothesis the tilt of the FP is not due to a systematic increase/decrease of the dark matter content of galaxies, but it is the effect of the variation of the strength of their internal gravitational field as compared to $a_0$.

Another way to investigate the effect of MOND on galaxies is shown in Fig. 6, where the acceleration of gravity from eq. 4, is compared to the acceleration derived using the dynamical mass, $M_{dyn} = 3G\sigma^2/r_e$. Though there is significant spread of the data, it is seen that dynamical masses imply no mass discrepancy for acceleration stronger or comparable to $a_0$, while a deviation is apparent when the acceleration goes below it, as prescribed by MOND. Note that the stronger discrepancy occurs for LSB galaxies, because they are in deep MOND regime. While observational data and MOND predictions agree reasonably well, to explain this behavior using Newtonian dynamics requires fine tuning of the total amount of dark matter in galaxies of very different type and morphology.

These considerations are not limited to elliptical galaxies. In fact if MOND is the underlying law regulating gravity in the universe, then all stellar systems should be described by MOND. Though a full discussion of this issue goes beyond the scope of the present work, it is worth to add more data to Fig. 6 to see whether other stellar systems different from elliptical galaxies follow the same trend. In Fig. 7 I plot the data from the data for 1050 objects (51 globular clusters, 300 elliptical galaxies, 513 spiral galaxies, 170 galaxy groups and 16 galaxy clusters) from the compilation of Burstein et al. (1997), are plotted. Data cover 7 order of magnitudes in internal
acceleration, going from globular clusters which are believed to be dark matter free, to the largest cluster which have $\tau \sim 100$. The agreement between MOND and the observed dependence of the mass discrepancy is clear and becomes striking after re-binning the data (Fig. 8), indicating that either the dark-matter content in stellar structure is amazingly well linked to the strength of their internal gravitational field, or that Newton’s law fails below $a_0$.

I stress that under the latter explanation, it is clear that objects like globular clusters are baryons dominated not just for a coincidence but because their internal acceleration of gravity is above $a_0$. And objects as different as a galaxy cluster and a tiny LSB dwarf galaxy have similarly large $\tau$ not for a pure coincidence but because their internal acceleration of gravity is similar.

Finally, I would like to comment on recent result for two globular clusters. Observations of the stellar velocity dispersion of Palomar 13 (Côté et al. 2002) have been interpreted as evidence for large amount of dark matter in this cluster ($\tau \sim 40$). Similarly, my collaborators and I have shown that at large radii the velocity dispersion profile of $\omega$ Centauri remains flat for accelerations below few times $a_0$ (Scarpa, Marconi, and Gilmozzi 2002), as observed in galaxies. This result was interpreted as evidence of a failure of Newton’s law in the low acceleration limit, while the classical scenario would imply large amount of dark matter surrounding the cluster and therefore large mass discrepancy. I show in Fig. 6 that in both cases the clusters uniform to MOND prediction further supporting the proposed scenario.

5. Conclusion

The most important conclusion of this work is that assuming MOND the tilt of the FP can be explained by the different strength, as compared to $a_0$, of the internal gravitational field in galaxies of different type and size. This suffice to show that using Newton’s law of gravity $\tau$ is expected to vary systematically with $r_e$ or, equivalently, with $L$, in a way fully consistent with observations. The opposite behavior of HSB and LSB galaxies, which poses paramount difficulties for Newtonian dynamics, also find a natural explanation in this scenario. The observed systematic deviation extends naturally to other stellar structures over 7 order of magnitude in acceleration, strongly suggesting that a law of physics, rather than dark matter, regulates the behavior of stellar structures in the Universe. It is worth noticing that all this is achieved in MOND without a single free parameter.

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Fig. 1.— Edge on view of the fundamental plane of elliptical galaxies. The quantities $k_1$ and $k_3$, as defined by Bender, Burstein, and Faber (1993), are used. In Newtonian dynamics, $k_1 \propto M$, while $k_3 \propto \tau$. The two dotted lines have slope $-0.4$ and $+0.25$, showing the dependence of $\tau$ on $M$ for LSB and HSB, respectively. Data are from Bender, Burstein, and Faber (1993): Squares= giant ellipticals; Triangles= intermediate ellipticals; Diamonds= dwarf ellipticals; circle= compact ellipticals; Crosses= bulges; Dots= LSB ellipticals; Stars= LSB from Peterson and Caldwell (1993); Pentagons= dwarf elliptical in Virgo from Guhathakurta and van der Marel (2002); Pluses= radio galaxies from Bettoni et al. (2001).
Fig. 2.— Kormendy relation for various types of galaxies. Symbols are as in Fig. 1. HSB galaxies show a decreasing surface brightness as $r_e$ increases, while LSB show the opposite behavior. The solid and dashed lines are an eye fit to HSB and LSB galaxies, respectively, which have the sole purpose of better visualize the effects of MOND in the following figures.
Fig. 3.— Plot of $g/g_n \equiv \tau/\tau_n$ versus average surface brightness $\Sigma$ for elliptical galaxies (symbols are as in Fig. 1). The value of the Newtonian acceleration $g_n$ in units of $a_0$ is also indicated in the upper scale, and the vertical dashed line deviates the Newtonian from the MOND regime areas. The two horizontal lines give the values of $r_e$ (in kpc) covered by HSB (upper line) and LSB (lower line). Note that $r_e$ increases from left to right for LSB and vice versa for HSB. This opposite dependence of $\Sigma$ on $r_e$ and the corresponding variation of the mass-to-light ratio is causing the different tilt of the FP defined by HSB and LSB galaxies.
Fig. 4.— Plot of $g/g_N \equiv \tau/\tau_N$ versus $r_e$. Symbols are as in Fig. 1. The opposite dependence of $\tau$ on $r_e$ shown by LSB and HSB galaxies is evident. The dashed and solid lines are the same drawn in Fig. 2 and are meant to visualize the loci followed by LSB and HSB galaxies.
Fig. 5.— Plot of \( g/g_N \) versus luminosity. It is evident that \( \tau/\tau_0 \) decrease (increase) with luminosity for LSB (HSB) galaxies. The dependence is indicated by two dotted lines, which correspond to \( \tau \propto L^{-0.3} \) in the case of LSB galaxies, and \( \tau \propto L^{0.25} \) for HSB galaxies. In absence of dark matter \( k_1 \propto L \), thus this plot is the equivalent to Fig. 1. MOND therefore predicts the right dependence of \( \tau \) on \( L \) for both HSB and LSB galaxies. The dashed and solid lines are the same drawn in Fig. 2 and are meant to visualize the loci followed by LSB and HSB galaxies.
Fig. 6.— Comparison of the gravitational acceleration at $r_e$ derived using dynamical and luminous masses. The solid line gives the locus of objects with no mass discrepancy, while the dashed line represents the MOND prediction. Note the trend of increasing discrepancy as the internal acceleration decrease. Symbols are as in Fig. 1 except for the two big pentagons representing the globular cluster Palomar 13 (left) and ω Centauri (right) discussed in the text.
Fig. 7.— Same as figure 6 using data for 1050 objects from Burstein et al. (1997). Group and cluster of galaxies (pluses) are mostly on the left very deeply in MOND, spiral galaxies (crosses) and elliptical galaxies (squares) are at the center, and globular cluster (triangles), which are mostly in Newtonian regime, are on the upper right. Luminous masses were computed assuming $\tau = 2$ for elliptical galaxies, $\tau = 1.0$ for globular clusters, $\tau = 1.2$ for spiral galaxies, and $\tau = 1.6$ for group and cluster of galaxies (see text).
Fig. 8.— Same as Fig. 7 after re-binning the data in bins of 0.5 dex. Error bars give 3σ confidence region of the average. In each bin, no distinction is made between objects of different type. The agreement between observations and MOND is striking.