Symmetry restoration by pricing in a duopoly of perishable goods

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Abstract. Competition is a main tenet of economics, and the reason is that a perfectly competitive equilibrium is Pareto efficient in the absence of externalities and public goods. Whether a product is selected in a market crucially relates to its competitiveness, but the selection in turn affects the landscape of competition. Such a feedback mechanism has been illustrated in a duopoly model by Lambert \textit{et al} in which a buyer’s satisfaction is updated depending on the freshness of a purchased product. The probability for buyer \( n \) to select seller \( i \) is assumed to be \( p_{n,i} \propto e^{S_{n,i}/T} \), where \( S_{n,i} \) is the buyer’s satisfaction and \( T \) is an effective temperature to introduce stochasticity. If \( T \) decreases below a critical point \( T_c \), the system undergoes a transition from a symmetric phase to an asymmetric one, in which only one of the two sellers is selected. In this work, we extend the model by incorporating a simple price system. By considering a greed factor \( g \) to control how the satisfaction depends on the price, we argue the existence of an oscillatory phase in addition to the symmetric and asymmetric ones in the \((T, g)\) plane, and estimate the phase boundaries through mean-field approximations. The analytic results show that the market preserves the inherent symmetry between the sellers for lower \( T \) in the presence of the price system, which is confirmed by our numerical simulations.

Keywords: applications to game theory and mathematical economics, critical phenomena of socio-economic systems
1. Introduction

Socio-economic systems have attracted the attention of physicists due to their inherent dynamic complexity [1–3]. Simple interactions between social agents are known to give rise to nontrivial collective patterns, e.g. segregation [4–6], opinion [7–9] or language dynamics [10], and crowd behavior [11, 12]. Focusing more specifically on economic aspects, the interaction of sellers and buyers in a market can be viewed as a dynamical process. For instance, taking space into account and including transportation costs in the agents’ utility leads to the well known Hotelling model [13, 14], in which stores try to find the optimal location to maximize their profit.

Here we consider a different problem, neglecting spatial aspects but taking into account a finite lifetime of products, leading to a potentially rich dynamics. Buyers and sellers change their respective states upon every purchase: namely, each buyer evaluates the sellers based on the purchased products, and each buyer also updates the list of products in stock. Under certain condition, their interaction may form a positive feedback loop in such a way that a seller, if selected by a buyer, becomes more likely to be selected in the future. Such a mechanism has been proposed and analyzed in detail by Lambert et al [15]: they have considered two sellers dealing with perishable goods, so that a seller can replace existing products with fresh ones when buyers continuously make purchases from the seller. As a result of this positive feedback, the symmetry among the sellers is broken spontaneously, leading to a virtual monopoly. Lambert et al have analytically identified a critical threshold for this phenomenon, when the control parameter is the degree of randomness in buyers’ choices.
However, their duopoly model in [15] does not contain any price system, and it is an interesting question whether the market can restore the symmetry when the sellers are allowed to charge different prices. If the price declines as time goes by, and if buyers are sensitive to the price as well as freshness, it is indeed plausible that the market can better resist the tendency toward a monopoly. We thus extend the model of [15] by including a price mechanism, which defines the price of a product as a function of its freshness, and check to what extent this prescription stabilizes the market.

In this paper, we show that the symmetry is recovered at a lower degree of randomness in buyers by including a price system coupled to the freshness of perishable goods. The threshold is estimated on a mean-field level in the sense that we neglect fluctuations among buyers. We also find the possibility of a third phase, in which buyers seesaw between the two sellers. The period of this seesaw motion is estimated approximately by assuming homogeneity of the products in stock. These analytic results are consistent with a phase diagram obtained numerically.

This work is organized as follows. We explain our model in section 2. In section 3.1, we analyze its time evolution under the assumption that buyers remember the past for a long time. The threshold for symmetry breaking is given by examining stationary states in section 3.2. Section 3.3 explains the origin of the oscillatory phase and gives an approximate estimate of the period. We briefly discuss implications of our findings in section 5 and then conclude this work.

2. Model

Suppose that two sellers are competing to sell products of the same kind. Each seller has $N_p$ products in stock, and each of them has its own age $\tau$. As soon as a product is sold, it is replaced by a new product with $\tau = 0$ to conserve the number of products on the market all the time. As in [15], our assumption is that the products are perishable, so that the freshness changes with $\tau$ in the following functional way:

$$h(\tau) = e^{-\tau/\eta}, \quad (1)$$

where $\eta$ is the characteristic time scale for aging. It is a common marketing strategy to lower the prices of shop-soiled products. We may therefore assume that each seller can choose a price policy, parametrized by $h_c$: this parameter means the characteristic freshness for a markdown. If $h_c$ is low, the price will not drop down until the product becomes very old. Specifically, we set

$$x(h(\tau)) = x(\tau) = 1 - \exp \left[ -\frac{h(\tau)}{h_c} \right], \quad (2)$$

which is a decreasing function of $\tau$, bounded between zero and $1 - e^{-h_c^{-1}}$. On the other hand, we have $N_b$ buyers. Let $n \in \{1, \ldots, N_b\}$ be an index to denote each buyer. If buyer $n$’s satisfaction from the $i$th seller is denoted by $S_{n,i}$, the probability to choose this seller is given as

$$p_{n,i} = Z_n^{-1} e^{S_{n,i}/T}, \quad (3)$$
where $T$ is an effective temperature to control the degree of stochasticity in making the decision, and $Z_n = e^{S_{n,1}/T} + e^{S_{n,2}/T}$ is a normalization factor. This formalism is often called the logistic selection model or logit rule [16], and it bears some analogy with the Glauber rate in physical systems, if we assume that energy plays the role of negative satisfaction. Suppose that buyer $n$ chooses the $i$th seller to purchase a product of freshness $h_{n,i}$ for price $x_{n,i}$ at time $t$. This product updates his or her satisfaction from this seller and affects the buyer’s next purchase at $t' > t$. We assume that the satisfaction at $t'$ can be described as

$$S_{n,i}(t') = \alpha S_{n,i}(t) + (1 - \alpha)\{ g(1 - x_{n,i}) + (1 - g)h_{n,i} \},$$

(4)

where $\alpha$ characterizes the buyer’s memory and $g$ is a greed factor to determine the sensitivity to the price. The superscript $t$ means that the price and freshness are measured at time $t$.

3. Analysis

3.1. Time evolution of probabilities

Let us assume that the buyers can be represented by a single average buyer. In other words, we will neglect fluctuations in the buyers’ satisfaction as well as their behavior to drop the buyer index $n$. Then, the probability to choose the $i$th seller becomes

$$p_i = Z^{-1}e^{S_i/T}$$

with $Z = \sum_i e^{S_i/T}$. At time $t$, our average buyer pays $\bar{x}_i$ to purchase a product of freshness $\bar{h}_i$ from the $i$th seller. According to equation (4), the buyer’s satisfaction evolves as

$$S_i(t + \tau_0) = \alpha S_i(t) + (1 - \alpha)\{ g(1 - \bar{x}_i) + (1 - g)\bar{h}_i \},$$

(5)

where $\tau_0$ denotes the average time interval to make a purchase. If the memory decays very slowly, i.e. $0 < 1 - \alpha \ll 1$, we may regard $S_i$ as a smooth function of $t$ to obtain

$$\dot{S}_i = \frac{1 - \alpha}{\tau_0} [g(1 - \bar{x}_i) + (1 - g)\bar{h}_i - S_i],$$

(6)

where the dot means the time derivative. Note also that we have suppressed the superscript $t$, because equation (6) is not a difference equation between $t$ and $t + \tau_0$, but a differential equation at time $t$. The time evolution of $p_i$ can also be expressed as

$$p_i = \frac{1}{T}\dot{S}_i e^{S_i/T} - \left( \sum_j \frac{1}{T}\dot{S}_j e^{S_j/T} \right) \frac{e^{S_i/T}}{Z^2}$$

$$= \frac{1}{T}p_i \left( \dot{S}_i - \frac{1}{Z} \sum_j \dot{S}_j e^{S_j/T} \right)$$

$$= \frac{1}{T}p_i (\dot{S}_i - \langle \dot{S} \rangle),$$

(7)
where $\langle X \rangle = \sum_j p_j X_j$ means the average over the sellers. Equation (7) can be understood as the replicator equation \[17–19\], where equation (6) plays the role of fitness. By substituting equation (6) into equation (7), we find
\[\dot{p}_i = \frac{1 - \alpha}{\bar{h}_i T} p_i \{ [g(1 - \bar{x}) + (1 - g) \bar{h}_i - S_i] - \langle g(1 - \bar{x}) + (1 - g) \bar{h} - S \rangle \} = \frac{1 - \alpha}{\bar{h}_i T} p_i \{ [g(1 - \bar{x}) + (1 - g) \bar{h}_i - T \ln p_i] - (g(1 - \bar{x}) + (1 - g) \bar{h} - T \ln p) \}, \]
where we have used $S_i = T(\ln p_i + \ln Z)$.\[
\]
### 3.2. Stationary state
We first consider a stationary state in which all the $p_i$ are constant. The average fraction of products purchased from the $i$th seller during a time interval $(t, t + dt)$ is expressed as
\[\lambda_i(t) dt = \frac{N_i p_i}{N p_0} dt, \]
because the average number of buyers visiting $i$ per unit time is $N_i p_i / p_0$. The distribution of product ages $\tau$ at the $i$th seller is denoted as $\phi_i(\tau, t)$ with a normalization condition $\int_0^\infty d\tau \phi_i(\tau, t) = 1$. The buyer randomly chooses a product, regardless of its age. The time evolution of $\phi_i(\tau, t)$ is thus written as
\[\phi_i(\tau + dt, t + dt) = \phi_i(\tau, t) - [\lambda_i(t) dt] \phi_i(\tau, t). \]
By expanding equation (10) to the linear order of $dt$, we find that
\[\frac{\partial \phi_i}{\partial \tau}(\tau, t) + \frac{\partial \phi_i}{\partial t}(\tau, t) \approx -\lambda_i(\tau) \phi_i(\tau, t). \]
This equation has a stationary solution $\phi_i^{st}(\tau) = \lambda e^{-\lambda \tau}$, from which $\bar{h}_i$ for the stationary state is obtained as $\bar{h}_i = \int_0^\infty d\tau \phi_i^{st}(\tau) h(\tau) = 1 / [1 + (\lambda_i/n)^{-1}]$. If we introduce $R \equiv \frac{N_i}{N p_0} = \lambda_i n / p_0$, this result can also be written as
\[\bar{h}_i = \left(1 + \frac{1}{R p_i} \right)^{-1}. \]
In addition, the value of $\bar{x}_i \equiv \int_0^\infty d\tau \phi_i^{st}(\tau) x(\tau)$ can be computed from the stationary distribution as follows:
\[\bar{x}_i = \int_0^\infty d\tau \lambda e^{-\lambda \tau} \left[ 1 - \exp \left( -\frac{e^{-\tau / h_c}}{h_c} \right) \right]. \]
Let us introduce another variable, $y \equiv e^{-\tau / h_c}$. The change of variables then leads to
\[ \bar{x}_i = 1 - p_i R h_c^\gamma(p_i R, h_c^{-1}), \]  
\[ \gamma(s, x) \] is the lower incomplete gamma function \[ \gamma(s, x) \equiv \int_0^x dy e^{-y} y^{s-1}. \]

A possible stationary state is such that each seller is chosen with equal probability, i.e. \( p_i = p_0 = 1/2 \). We will check the stability of such a state under a small perturbation in the following way: first, let us rearrange equation (8) as
\[
\dot{p}_i = \frac{1 - \alpha}{\tau_0} \left( g [ p_i (1 - \bar{x}_i) - p_i (1 - \bar{x})] + (1 - g)[p_i \bar{h}_i - p_i \langle \bar{h} \rangle] - T [ p_i \ln p_i - p_i \langle \ln p \rangle] \right). 
\]  
(15)

By adding a small perturbation, we change the probabilities to \( p_i = p_0 + \epsilon_i \) with \( \sum_j \epsilon_j = 0 \). The expressions inside the square brackets on the right-hand side (RHS) of equation (15) can then be expanded as
\[
p_i (1 - \bar{x}_i) - p_i (1 - \bar{x}) = \mathcal{F}(R, p_0, h_c) \epsilon_i + \mathcal{O}(\epsilon_i^2),
\]  
(16)
\[
p_i \bar{h}_i - p_i \langle \bar{h} \rangle = \frac{R p_0}{(1 + R p_0)^2} \epsilon_i + \mathcal{O}(\epsilon_i^2),
\]  
(17)
\[
p_i \ln p_i - p_i \langle \ln p \rangle = \epsilon_i + \mathcal{O}(\epsilon_i^2),
\]  
(18)
with
\[
\mathcal{F}(R, p_0, h_c) = \frac{\partial}{\partial \epsilon} \left. \{ p_0 (p_0 + \epsilon) R h_c^{(p_0 + \epsilon) R} \gamma [(p_0 + \epsilon) R, h_c^{-1}] \} \right|_{\epsilon = 0}.
\]  
(19)

We note that the zeroth order of \( \epsilon_i \) does not exist, so equation (15) can be rewritten to the linear order of \( \epsilon_i \) as
\[
\dot{\epsilon}_i \approx \frac{1 - \alpha}{\tau_0} \left( g \mathcal{F}(R, p_0, h_c) + (1 - g) \frac{R p_0}{(1 + R p_0)^2} - T \right) \epsilon_i.
\]  
(20)

The unperturbed stationary state is stable when the prefactor of \( \epsilon_i \) on the RHS is negative. In other words, we can observe a symmetric phase, in which \( p_i = 1/2 \) for every \( i \), provided that
\[
T > T_c(g) \equiv g \mathcal{F}(R, p_0, h_c) + (1 - g) \frac{R p_0}{(1 + R p_0)^2},
\]  
(21)

for given \( R \) and \( g \). One should note that multiple stable states can coexist in some range of \( R \) [15]. If \( R \) is small enough, however, we may say that the symmetric state is the only possibility when \( T > T_c \). The question is what happens when \( T \) becomes lower than \( T_c \). When the greed factor \( g \) approaches zero, the above analysis reduces to the result of [15], in which an asymmetric phase emerges below \( T_c(g = 0) \). The origin of the asymmetric phase can be explained as follows: if the buyer’s choice is slightly biased against a certain seller by chance, the products of this seller get older, which in turn lowers the probability to choose this seller, forming a positive feedback loop. In the end, only a single seller occupies the whole market, which is a stable stationary state at
even if \( g \) has a small finite value, it is reasonable to expect the same spontaneous symmetry-breaking phenomenon. When \( g \) becomes high enough, however, the price system comes into play in a nontrivial way, as will be explored below.

### 3.3. Oscillatory phase

Let us rewrite equation (6) as

\[
\dot{S}_i = \frac{1 - \alpha}{\tau_0} \left[ -S_i + Q_i(\tau) \right],
\]

by defining \( Q_i(\tau) \equiv g[1 - \bar{x}_i(\tau)] + (1 - g)\bar{h}_i(\tau) \). This implies that we may expect \( S_i \approx Q_i(\tau) \) when the system is evolving slowly. Suppose that only one seller, say, \( i = 1 \), is selected due to the symmetry-breaking mechanism explained above. This seller is always equipped with new products, so the satisfaction from this seller will be \( Q_1(\tau) \approx Q_0(\tau) \). On the other hand, \( \tau \) increases for the other seller as time \( t \) goes by, so \( S_2(t) \) will follow the trajectory of \( Q_2(\tau) \). Figure 1 shows two possible cases of \( Q_i(\tau) \), i.e. with small and large greed factors, respectively. When \( g \) is small, \( Q_2(\tau > 0) \) stays below \( Q_0(0) \), so the asymmetric phase remains stable (figure 1(a)). On the other hand, if \( g \) is large enough, the low price compensates for the freshness reduction, and \( Q_i(\tau) \) can actually be greater than \( Q_0(0) \) when \( \tau \) exceeds a certain threshold (figure 1(b)). Now, the second seller begins to be selected, and the situation is less advantageous to the first seller: the value of \( Q_1 \) decreases, because some products are not sold, which lowers the freshness further. On the other hand, \( Q_2 \) does not decrease, because old products make positive contributions to \( Q_1 \) due to their low prices while refreshed products also attract buyers. The result is that the second seller is preferred until the prices of the other seller become low enough. To sum up, they take turns in playing the role of the champion and the challenger, which is the origin of the oscillating behavior.

The above argument leads to some quantitative predictions: assume that the behavior can be approximated as that of the asymmetric phase on a short time scale. We therefore suppose that \( p_i = 1 \) so that only seller \( i \) is being selected. In this case, \( \bar{h} \) and \( \bar{x} \) are derived in equations (12) and (14), respectively, whereby we obtain an explicit expression for \( Q_i \) at \( p_i = 1 \). This is denoted as \( Q_0 \) and represented as dotted horizontal lines in figure 1. Note that \( Q_0 \approx Q_1(\tau = 0) \), because, by assumption, this sellers’ products are being replaced with fresh ones. However, it takes some finite time to sell the products, so \( Q_0 \) is slightly lower than \( Q(\tau = 0) \). As a result, \( Q(\tau) \) has a trivial crossing at \( \tau \approx O(N_0\tau_0/N_0) \) (figure 1). For old products to be as competitive as fresh ones, \( Q(\tau) \) should have another crossing with \( Q_0 \) at some \( \tau = \tau^* \gg O(N_0\tau_0/N_0) \). The shape of the curves in figure 1 suggests that this condition can be rephrased as \( Q(\tau \to \infty) = g > Q_0 \). It is clear that \( Q(\tau) \) approaches \( g \) as \( \tau \) grows, so that the critical value \( g_c \) is a solution of the following equation:

\[
g[1 - Rh_c^2\gamma(R, 1/h_c)] + (1 - g) \left( 1 + \frac{1}{R} \right)^{-1} = g,
\]

where the left-hand side express \( Q_0 \). We may expect oscillatory behavior when \( g \) exceeds \( g_c \). At the same time, we stress that this calculation assumes deterministic behavior when it comes to buyers, so the prediction will be the most accurate at \( T = 0 \).
Furthermore, the crossing point $\tau^*$ between $Q_i(\tau)$ and $Q_0$ provides an estimate of the half-period of oscillation. It is obtained by numerically solving the following equation:

$$g \exp(-h_c^{-1} e^{-\tau^*/\gamma}) + (1 - g)e^{-\tau^*/\gamma} = Q_0.$$  \hfill (24)

### 4. Numerical results

Our numerical simulation shows the existence of the symmetric, oscillatory, and asymmetric phases as predicted above (figure 2). In this simulation, we set the number of products at each seller as $N_p = 5 \times 10^3$ and the number of buyers as $N_b = 10^2$. The typical time interval of a purchase is given as $\tau_0 = 10^{-1}$, and freshness is assumed to decay with a characteristic time scale $\tau_1 = 20$. The price policy is parametrized by a freshness threshold $h_c = 5 \times 10^{-2}$, and the memory factor is set as $\alpha = 0.99$. With all these parameters fixed, we change $g$ and $T$ to locate the phase boundaries. Recall that the boundary of the symmetric phase is predicted by equation (21), whereas the boundary between the asymmetric and oscillatory phases is predicted to be $g = g_c$, obtained by solving equation (23).

Let us define some numerical quantities to distinguish the three phases. First, the symmetry breaking can be detected by the following:

$$m_A = |\langle p_1(t) - p_2(t)\rangle|,$$  \hfill (25)

where $p_i(t) = N_a^{-1} \sum p_{n,i}$ is the probability to choose seller $i$ at time $t$ and $\langle \ldots \rangle_h$ means the average over time. Clearly, this quantity will be nonzero only in the asymmetric phase. To make a distinction between the symmetric and oscillatory phases, we also measure the following:

$$m_O = |\langle |p_1(t) - p_2(t)|\rangle - |\langle p_1(t) - p_2(t)\rangle|,$$  \hfill (26)

which will be nonzero only in the oscillatory phase. If we combine these two into a single parameter $m = m_A - m_O$, its value will be positive (negative) in the asymmetric (oscillatory) phase, and close to zero in the symmetric phase. Figure 3 shows
the results: the solid lines are the phase boundaries estimated by equations (21) and (23). The boundary between the symmetric and asymmetric phases shows an excellent agreement, especially for $g \lesssim 0.8$. The division between the oscillatory and asymmetric phases is also consistent with the estimation of $g$, when $T$ is low. The region $\Lambda'$ is characterized by asymmetric oscillation, which has not been captured by our analysis. Such a phase can exist at sufficiently high $T$, because the unfavored seller sells its old products too early due to buyers’ stochastic choices.

Finally, we compare $\tau^*$ with the half-period in the oscillatory phase in figure 4. We choose very low temperature, $T = 10^{-3}$, because the estimation of $\tau^*$ can easily be disturbed by noise. As shown in figure 4, the agreement is striking for large $g$. The failure at small $g$ is related to one of our fundamental assumptions: recall that we have assumed that all the buyers are homogeneous by using a single average buyer in section 3.1. When $g$ is close to $g_0 \approx 0.44$, this assumption breaks down and the buyers have different probabilities to choose the sellers. This inhomogeneity dramatically reduces the period of oscillation.

5. Discussion

The phase diagram in figure 3 shows that the price system indeed restores the symmetry in a broader range of $T$ as compared to the case without price formally obtained as $g = 0$. However, the asymmetric phase can exhibit oscillations when buyers are sensitive to the price. One might point out that our model bears similarity to the Bertrand
duopoly, in which two sellers sell homogeneous products without any possibility of collusion [20]. The sellers in the Bertrand duopoly compete by setting prices simultaneously, and the one with a lower price occupies the whole market. If the two sellers charge the same price, they will evenly share the market. The pure-strategy Nash equilibrium turns out to be such that both of the sellers set the price equal to marginal cost of production, which means that they earn nothing from their business [21]. In the Bertrand duopoly, therefore, the existence of two sellers is already enough to actualize the lowest possible price. However, one should note its underlying assumption that a single seller can cover the whole market. If this is not the case, e.g. due to limited capacity of production, we encounter the Edgeworth paradox, which means that this game has no pure-strategy Nash equilibrium, hence no stationary price. For example, one of the two firms can make profits by deviating from the marginal cost, because the
other firm alone cannot meet the demand. Although a mixed strategy can constitute a Nash equilibrium \[22, 23\], it is hardly feasible in practice and one would instead observe the Edgeworth price cycle (\[20\] also points out that the capacity constraint may not be a necessary condition for the cycle). The Edgeworth paradox suggests that the price mechanism can introduce instabilities. However, the cycle is actually a pseudo-dynamic process in the sense that it originates from an equilibrium concept. In contrast, our oscillatory phase is born out of dynamical rules without any consideration of strategic equilibrium.

To consider some strategic aspects in our model, let us suppose that the variables \(T\) and \(g\) characterizing buyers are just given parameters when viewed by the sellers. The sellers can instead decide \(h_c\) to control the price policy, and the symmetry between the sellers suggests that they will end up with the same \(h_c\). The critical temperature \(T_c\) in equation (21) depends on \(h_c\) unless \(g = 0\). For example, if \(h_c \to 0\), the price will become insensitive to freshness. This implies that \(\mathcal{F}(R, p_0, h_c)\) vanishes in equation (21) so that the phase boundary of the symmetric phase in figure 3 will converge to a line connecting \((T_c(0), 0)\) and \((0, 1)\) on the \((T, g)\) plane. In a price war, the sellers will increase \(h_c\) to lower the average price, but the possible range of \(h_c\) must be bounded due to the cost of production. We may regard our \(h_c\) in the previous sections as the highest possible one determined by such a competitive process. In contrast to the Bertrand duopoly, however, the sellers do not always divide the market half and half, when the mechanism of Lambert et al is at work \[15\]: the market of perishable goods tends to become monopolistic, especially when the purchasing behavior is deterministic with low \(T\). Our point is that this tendency is nevertheless weakened by the price sensitivity, either by extending the symmetric phase, or by restoring the symmetry over a long period in the oscillatory phase.

6. Summary

To summarize, we have incorporated a simple price system coupled with freshness into the duopoly model suggested by Lambert et al. In addition to the effective temperature \(T\) to control the randomness in purchasing behavior, we have introduced the greed parameter \(g\) to determine the sensitivity of satisfaction to the price. We have identified the symmetric, asymmetric, and oscillatory phases and estimated their boundaries in the \((T, g)\) plane. Our numerical simulations show nice agreements with our analytic results. Based on our analysis, we conclude that the price system resists the tendency to a monopoly: on one hand, it lowers the critical temperature \(T_c\) below which the symmetric phase becomes unstable. On the other hand, the market with high \(g\) can keep oscillating without settling on a single seller, preserving the symmetry in a time-averaged sense.

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