Optomechanically-Based Probing of Spin-Charge Separation in Ultracold Gases

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We propose a new approach to investigate the spin-charge separation in 1D quantum liquids via the optomechanical coupled atom-cavity system. We show that, one can realize an effective two-modes optomechanical model with the spin/charge modes playing the role of mechanical resonators. By tuning the weak probe laser under a pump field, the signal of spin-charge separation could be probed explicitly in the sideband regime via cavity transmissions. Moreover, the spin/charge modes can be addressed separately by designing the probe field configurations, which may be beneficial for future studies of the atom-cavity systems and quantum many-body physics.

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One dimensional quantum liquids have been fascinating for condensed matter physicists for quite a few decades [1]. For a 1D quantum liquid, the low energy behavior of the system lies in a universal class [2] which results in a remarkable phenomenon that a single-particle excitation would fractionizes into a collective charge and spin parts and separates. However, the clear observation of this phenomenon has proven to be challenging in the past decades in solid state materials [3]. Recently, the low dimension quantum fluids have been successfully realized in cold atom context [4]. The unprecedented tunability of interaction and dimensionality make it a powerful tool to explore Luttinger liquid [5] or Tonks gas [6, 7] in 1D. Stimulated by these experimental advances, some authors propose to detect spin-charge separation in ultracold gases by tracking a wave-packet motion or analyzing the spectrum of a single-particle excitation [8–11]. However, because of the small available spatial and limited time scales, the explicit signals of the spin-charge separation have not been observed in cold atom experiments so far.

Very recently, cavity optomechanics with cold atoms [12, 13] or a BEC [14] has acquired remarkable achievements. In such experiments, the low energy collective excitation of cold atoms behaves as a “moving mirror” [15, 16], which can be detected conveniently by cavity transmissions. And therefore, this offers a unique method to probe the low energy excitations of particular quantum phases in ultracold gases [15]. Based on these advances, we propose a new procedure in this Letter to detect spin-charge separation in 1D quantum liquids by considering a 1D two-component ultracold fermionic gases coupled to a polarization-degenerate optical cavity with external pump and probe fields. We show that, by tuning the weak probe field, the explicit signal of spin-charge separation could be probed definitely via transmission spectra within current experimental setups. This technique, which has the advantage of nondemolition measurements [17] and involves no added complications, provides a new practical way to explore the quantum many-body physics in future.

The system under investigation is illustrated in Fig. 1(a), where two-component hyperfine fermionic atoms of mass $M$ with resonant frequency $\omega_a$ are confined in a 1D trap inside an optical cavity of length $L$. The cavity mode is driven by a “Pump” laser of frequency $\omega_L$, and a weak “Probe” laser of frequency $\omega_P$ is added to stimulate the system’s optical response. “Out” is the transmission field. Both the fields are polarization dependent. (b) Internal energy levels of the two-component atoms with detuning $\Delta_a = \omega_L - \omega_a$. Here, $\omega_a$ is the resonant frequency between the ground $|g\rangle$ and excited $|e\rangle$ states.

FIG. 1: (color online). (a) A collection of two-component hyperfine fermionic atoms are confined in an effectively 1D trap inside an optical cavity of length $L$. The cavity mode is driven by a “Pump” laser of frequency $\omega_L$, and a weak “Probe” laser of frequency $\omega_P$ is added to stimulate the system’s optical response. “Out” is the transmission field. Both the fields are polarization dependent. (b) Internal energy levels of the two-component atoms with detuning $\Delta_a = \omega_L - \omega_a$. Here, $\omega_a$ is the resonant frequency between the ground $|g\rangle$ and excited $|e\rangle$ states.

Here, $\hat{\psi}_\sigma(x), \sigma = \uparrow, \downarrow$ is the pseudo-spin atomic field operator and $\hat{\omega}_\sigma = \sqrt{\mu_\sigma} + \frac{\hbar}{2}g_1 \psi^\dagger_\sigma \psi^\dagger_\sigma + \frac{\hbar}{2}\int_0^L dx \hat{\psi}_\sigma^\dagger(x) \hat{\psi}_\sigma(x)$. The system under investigation is illustrated in Fig. 1(a), where two-component hyperfine fermionic atoms of mass $M$ with resonant frequency $\omega_a$ are confined in a 1D trap inside an optical cavity. The cavity mode of frequency $\omega_0$ is driven by a pump laser, and we also add a weak probe field to the cavity, which behaves as a small perturbation to stimulate the fluctuations of the system [18]. To explore the spin-charge separation, both the pump and the probe fields are polarization dependent. The cavity field couples to the atomic internal state (see Fig. 1(b)) and induces a quantized potential on atoms in the far-off resonance limit. Then, in the dipole and rotating-wave approximations, the atomic part of Hamiltonian can be written as

$$\hat{H}_a = \sum_\sigma \int dx \hat{\psi}_\sigma^\dagger(x) \left( \frac{\hbar^2}{2M} \hat{\nabla}_x^2 + \hbar \sqrt{\mu_\sigma} \cos^2(Kx) \hat{c}_\sigma^\dagger \hat{c}_\sigma \right) \hat{\psi}_\sigma(x) + g_{1D} \int dx \hat{\psi}_\uparrow^\dagger(x) \hat{\psi}_\downarrow(x) \hat{\psi}_\uparrow(x) \hat{\psi}_\downarrow(x).$$

Here, $\hat{\psi}_\sigma(x), \sigma = \uparrow, \downarrow$ is the pseudo-spin atomic field operator and $\hat{\omega}_\sigma = \sqrt{\mu_\sigma} + \frac{\hbar}{2}g_1 \psi^\dagger_\sigma \psi^\dagger_\sigma + \frac{\hbar}{2}\int_0^L dx \hat{\psi}_\sigma^\dagger(x) \hat{\psi}_\sigma(x)$. The system under investigation is illustrated in Fig. 1(a), where two-component hyperfine fermionic atoms of mass $M$ with resonant frequency $\omega_a$ are confined in a 1D trap inside an optical cavity. The cavity mode of frequency $\omega_0$ is driven by a pump laser, and we also add a weak probe field to the cavity, which behaves as a small perturbation to stimulate the fluctuations of the system [18]. To explore the spin-charge separation, both the pump and the probe fields are polarization dependent. The cavity field couples to the atomic internal state (see Fig. 1(b)) and induces a quantized potential on atoms in the far-off resonance limit. Then, in the dipole and rotating-wave approximations, the atomic part of Hamiltonian can be written as

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erator for two hyperfine fermionic atoms, $\tilde{c}_\sigma$ is the cavity filed operator for up/down polarization, and $U_0^\sigma = U_0 = g_0^2/\Delta_0$ is the optical dipole potential strength for a single intracavity photon with $K = 2\pi/\lambda_c$ the wave-vector of the cavity mode. $g_{1D} = \frac{\pi \hbar a_s}{N}$ is the strength of contact interaction between fermions with opposite spin, and $a_s$ is the effective 1D low-energy s-wave scattering length, which can be tuned by Feshbach resonance.

First, following the standard procedure, we transform the atomic field operator into momentum representation by $\hat{\Psi}_\sigma(x) = L^{-1/2}\Sigma_k f_{k,\sigma} e^{ikx}$, where $f_{k,\sigma}$ is the fermion annihilation operator for a plane wave with wave-vector $k$. Then, Hamiltonian (1) can be rewritten as

$$\hat{H} = \sum_{k,\sigma} \epsilon(k) \hat{f}^\dagger_{k,\sigma} \hat{f}_{k,\sigma} + g_{1D} \sum_{k_1, k_2, q} \hat{f}^\dagger_{k_1+q, \uparrow} \hat{f}_{k_1, \uparrow} \hat{f}^\dagger_{k_2-q, \downarrow} \hat{f}_{k_2, \downarrow}$$

$$+ \sum_{\sigma} \tilde{c}_{\sigma} \hbar \Delta_{\sigma} + \frac{1}{4} \hbar U_0 \sum_{k} \left( \hat{f}^\dagger_{k+2K} \hat{f}_{k, \sigma} + \text{h.c.} \right), \quad (2)$$

where $\epsilon(k) = \hbar^2 k^2/2M$ is the single particle kinetic energy and $\Delta_{\sigma} = \omega_0 - \omega_L + U_0 N_\sigma/2$ is the effective cavity detuning. Here, we concern the spin-balanced case with $N_\uparrow = N_\downarrow = \Delta$.

We shall work in the low photon numbers limit and consider only the lowest momentum transfer of $2K$ induced by photons. For low temperature and small momentum $K \ll k_F = \pi N/L$, the particle-hole excitations occur around the Fermi surface (Fermi points in 1D). One may then implement the bosonization procedure [1] by introducing the following bosonic operators

$$\hat{b}_{k,\sigma}^\dagger = \sqrt{\frac{2\pi}{Lk}} \tilde{\rho}_{\sigma}^k (-k), \quad \hat{b}_{k,\sigma} = \sqrt{\frac{2\pi}{Lk}} \rho_{\sigma}^k (k > 0). \quad (3)$$

Here, $\tilde{\rho}_{\sigma}^k (k) = \sum_q \hat{f}^\dagger_{k+q, \sigma} \hat{f}_{q, \sigma}$ are density operators for the right and left moving fermions with $\nu = R, L$. By further introducing the charge and spin density bosonic operators $b_{k,\lambda}^\dagger = \frac{1}{\sqrt{2}}(\hat{b}_{k,\lambda}^\dagger \pm \hat{b}_{k,\lambda})$, $\lambda = c, s$, and performing the Bogoliubov transformations $\delta_{k,\lambda}^R = \cos \gamma_{k,\lambda} \hat{b}_{k,\lambda}^R + \sinh \gamma_{k,\lambda} \hat{b}_{k,\downarrow}^R$, $\delta_{k,\lambda}^L = \sin \gamma_{k,\lambda} \hat{b}_{k,\lambda}^L + \cosh \gamma_{k,\lambda} \hat{b}_{k,\uparrow}^L$ with tanh $2\gamma_{k,\lambda} = \frac{\frac{1}{2} \hbar U_n \Delta_{\sigma}}{\frac{1}{2} \hbar U_n \Delta_{\sigma}}$, we derive the effective optomechanical model of the coupled system

$$\hat{H}_{\text{eff}} = \sum_{\nu, \lambda} \frac{\hbar}{2} \omega_{\nu,\lambda} \delta_{\nu,\lambda}^\dagger \delta_{\nu,\lambda} + \sum_{\nu, \lambda} \hbar \tilde{U}_\lambda \tilde{n}_\lambda (\delta_{\nu,\lambda}^\dagger + \delta_{\nu,\lambda})$$

$$+ \sum_{\sigma} \hbar \Delta \tilde{n}_\sigma, \quad (4)$$

where the first term describes the charge-spin fluctuations of the 1D interacting gas, which play the role of mechanical resonators with frequency $\omega_q = \omega_{q=\pm 2K} = 2K u_c$. Here, $u_c = v_F (1 + \frac{\hbar q_{1D}^2}{2m v_F})^2 - (\frac{\hbar q_{1D}^2}{2m v_F})^2$ and $u_s = v_F (1 - \frac{\hbar q_{1D}^2}{2m v_F})^2 - (-\frac{\hbar q_{1D}^2}{2m v_F})^2$ are the sound velocities of the charge-spin excitations for $g_{1D} = \pi v_F \lesssim 1$. The second term is the coupling between the mechanical modes

and cavity fields with $\tilde{U}_\lambda = \frac{\hbar q_{\pi}}{\sqrt{\Delta_\lambda}} (\cosh \gamma_{\lambda} - \sinh \gamma_{\lambda})$ and $\tilde{n}_\sigma = \tilde{n}_\uparrow + \tilde{n}_\downarrow$.

To proceed, we first consider briefly the steady-state behavior of the coupled system. The mean-field solutions of Eqs. (5) are $\tilde{P}_\lambda = 0, \tilde{X}_\lambda = -2\sqrt{2\tilde{U}_\lambda} \tilde{n}_\lambda/\omega_L$, and

$$\tilde{n}_\sigma = \frac{\eta_\sigma}{\Delta^2 - 4(U_0/\omega_c)^2 \tilde{n}_c + \sigma U_0^2 \omega_s^2 \tilde{n}_\sigma)}, \quad (6)$$

where $\kappa$ is the cavity decay rate and $\tilde{s}_m^\sigma = \tilde{s}_\sigma + \delta_\sigma$ denotes the total amplitude of external fields. Here, $s_m \equiv (s_m^\sigma)_0$ represents the pump field, and $\delta_\sigma$ is a small perturbation, which is induced by the weak probe field with $\delta_\sigma \equiv (s_m^\sigma)$. $\theta$ is the tunable coupling parameter.

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$$\tilde{n}_\sigma = \frac{\eta_\sigma}{\Delta^2 - 4(U_0/\omega_c)^2 \tilde{n}_c + \sigma U_0^2 \omega_s^2 \tilde{n}_\sigma)}, \quad (6)$$

where $\eta_\sigma = \sqrt{2\hbar \kappa s_\sigma}$. In Fig. 2, we present the mean-field intracavity photon numbers versus the pump rate and detuning (see below for the parameters used here). It can be shown that the steady-state driven by the pump field exhibits optical multi-stability [20]. This is a characteristic phenomenon of the two-modes optomechanical system, where both the spin and charge modes are strongly coupled with the cavity fields.

While in our proposal, we mainly concern the system’s optical response to the weak probe field perturbation in the presence of a steady state. For a symmetrical pump $\eta_\sigma = \eta_\bar{\sigma}$, the steady solution $\tilde{n}_\sigma = \tilde{n}_\bar{\sigma} = \tilde{n}$ exists [21], which gives rise to $\tilde{n}_c = \tilde{n}_\uparrow + \tilde{n}_\downarrow = 2\tilde{n}$, $\tilde{X}_c$ =
−4\sqrt{2}U_{\rm c}\bar{n}_c/\omega_c and \bar{n}_s = \bar{n}_+ - \bar{n}_- = 0, \  \bar{x}_s = 0. Then, the optical response to the probe field is obtained via a linearization of Eqs. \ref{eq:7} around the steady-state

\begin{align}
\frac{d\delta \hat{X}_\lambda}{dt} &= \omega_\lambda \delta \hat{P}_\lambda, \quad \frac{d\delta \hat{P}_\lambda}{dt} = -\omega_\lambda \delta \hat{X}_\lambda - 2\sqrt{2}U_{\rm c} \sqrt{n} \delta \hat{x}_\lambda, \\
\frac{d\delta \hat{P}_{\lambda, c}}{dt} &= \sqrt{2} \kappa \delta \hat{P}_{\lambda, c} - \Delta \delta \hat{X}_\lambda - \kappa \delta \hat{P}_\lambda - 2\sqrt{2}U_{\rm c} \sqrt{n} \delta \hat{x}_\lambda, \\
\frac{d\delta \hat{X}_\lambda}{dt} &= \sqrt{2} \kappa \delta \hat{X}_{\lambda, c} + \frac{\Delta}{\kappa} \delta \hat{P}_\lambda - \kappa \delta \hat{X}_\lambda, \quad (7)
\end{align}

with \( \Delta = \Delta - 8U_{\rm c}^2\bar{n}_c\omega_c^{-1} \). Here, \( \delta \hat{X}_{\lambda, c} = (\delta \hat{x}_\lambda \pm \delta \hat{x}_c)/\sqrt{2} \), \( \delta \hat{P}_{\lambda, c} = (\delta \hat{p}_\lambda \pm \delta \hat{p}_c)/\sqrt{2} \) represent the cavity-field charge-spin quadratures with \( \delta \hat{x}_\lambda = (\delta \hat{c}_\lambda^+ + \delta \hat{c}_\lambda^-)/\sqrt{2} \), \( \delta \hat{p}_\lambda = i(\delta \hat{c}_\lambda^+ - \delta \hat{c}_\lambda^-)/\sqrt{2} \). And \( \delta \hat{X}_{\lambda, c} = (\delta \hat{x}_\lambda \pm \delta \hat{x}_c)/\sqrt{2} \), \( \delta \hat{P}_{\lambda, c} = (\delta \hat{p}_\lambda \pm \delta \hat{p}_c)/\sqrt{2} \) denote the corresponding probing field terms with \( \delta \hat{x}_\lambda = (s_p^+ - s_p^-)/\sqrt{2} \), \( \delta \hat{p}_\lambda = \frac{\sqrt{2}}{\sqrt{2}} \). For cold-atom system, the damping of the spin-density excitations are much smaller than the \( \omega_c \), and therefore can be neglected.

We note that, although both the two mechanical modes are coupled nonlinearly with cavity field in Eqs. \ref{eq:7}, the fluctuations of the spin and charge modes in the above Eqs. \ref{eq:7} can be excited independently. This is a unique feature of the system, which encodes the explicit signal of spin-charge separation. To see this, we transform Eqs. \ref{eq:7} into frequency space in the rotating frame. Here both the fluctuations of the mechanical and cavity field variables oscillate at frequencies \( \pm \Omega \) around the steady-state, with \( \Omega = \omega_p - \omega_L \) being the frequency difference between the probe and pump fields. Then, the intracavity field amplitude can be derived as \ref{eq:8}

\begin{equation}
A_\lambda(\Omega) = \frac{1 + if_\lambda(\Omega)}{-i(\Omega - \Delta) + \kappa + 2\Delta f_\lambda(\Omega)} \sqrt{2\kappa s_p^\lambda}, \quad (8)
\end{equation}

with

\begin{equation}
f_\lambda(\Omega) = \frac{4U_{\rm c}^2\bar{n}_c\omega_c}{\kappa - i(\Omega + \Delta)} \Omega^2 - \omega_c^2. \quad (9)
\end{equation}

Here, \( s_p^\lambda = (s_p^+ + s_p^-)/2 \) represents the input charge/spin probe field amplitudes.

Before proceeding, we consider the following experimental achievable parameters: \( L \sim 100 \mu m, N \approx 5000 \) alkali metal atoms (\emph{e.g.} \(^{87}\)Rb, \( M = 1.5 \times 10^{-22} \)kg) and \( U_0 = 20 \) kHz, which give rise to \( \omega_L \sim \) several MHz and \( U_\lambda \sim 100 \) KHz. Then cavity dumping \( \kappa \) can be chosen to satisfy \( U_\lambda \sqrt{n} \ll \kappa \ll \omega_L \), which places the system well in the resolved sideband regime \ref{eq:8}. In this regime, there exists normal mode splitting, and the \( -\Omega \) part of cavity fluctuations can be neglected, which may enable further simplification of the solutions of Eq. \ref{eq:8}.

In experiments, one of the most important observable quantity is the polarized intracavity field response to the probe field \( \mathfrak{R}_\lambda(\Omega) \equiv \sqrt{2}\omega_c A_\lambda(\Omega)/s_p^\lambda \), which reads

\begin{equation}
\mathfrak{R}_\lambda(\Omega) = \alpha \mathfrak{R}_s(\Omega) + \sigma \beta \mathfrak{R}_s(\Omega), \quad (10)
\end{equation}

with \( \mathfrak{R}_s(\Omega) = \sqrt{2}\omega_c A_s(\Omega)/s_p^\lambda \) and \( \alpha = s_p^+ / s_p^- \), \( \beta = s_p^- / s_p^+ \) determined by the configuration of probe field. By numerically solving Eq. \ref{eq:8} in the sideband regime, we show

FIG. 3: (color online.) (a) Real and (b) Imaginary parts of the intracavity field response versus \( \Omega \) (in unit of \( \Delta \)). The solid line shows the total response \( \mathfrak{R}_{\text{tot}}(\Omega) \) to the polarized probe field, and the dash-dotted and dashed lines show the responses \( \mathfrak{R}_s/\mathfrak{R}_s \) to the charge/spin modes respectively.

FIG. 4: (color online.) Transmission spectra for different values of detuning \( \Delta \) and three kinds of probe fields with polarized (solid), in-phase (dashed) and out-of-phase (dash-dotted) configurations.
the main results of the intracavity field responses to the polarized probe field in Fig. 5 with $\alpha = \beta = 1/2$ and $\kappa = 2\pi \times 150$ KHz for illustration. A remarkable feature of the spectrum $\mathcal{F}(\omega)$ is that it demonstrates a well-defined double dips centered at the charge/spin modes. The underlying mechanism can be understood as follows: when we input a small probe field to disturb the system around the steady state stabilized by the pump field, the corresponding intracavity field response sets up, which is generally comprised of the response of charge/spin modes. When the frequency difference between the probe and pump fields $\Omega$ is tuned to be one of the mode frequencies, the corresponding collective mode would be excited resonantly, and accordingly the cavity response would drop dramatically. Therefore in this scheme, one doesn’t have to track the motions of spin/charge wave-packets $(\mathbb{R})$, which have different velocities; we only need to detect the resonant frequencies of the collective spin/charge excitations by tuning the probe fields.

To further clarify, we present the probe power transmission $|t_f|^2 = |1 - R_p|^2$ for different probe field configurations (here, the critical pump $\theta = 1/2$ is assumed) in Fig. 4 where the charge/spin modes are also clearly resolved (solid line). We see that, by adjusting the probe field configuration to be in-phase (dashed line) or out-of-phase (dash-dotted line), the charge/spin modes can be addressed separately. In Fig. 4 We also investigate the impact of detuning $\Delta$ on the spectra. It can be seen that although the transmission is generally modified, the peaks of the probe spectra always occur at the charge/spin modes, which are independent of detuning $\Delta$. The frequencies of the charge and spin modes versus interacting parameter $g_{1D}$ for $g_{1D}/\pi v_F < 1$ are shown in Fig. 5 which could also be inspected in future experiments.

Until now, we have mainly focused on the femionic gas. Experimentally, the above scheme could also be realized in two-component Bose gas by implementing the hydrodynamical theory [13]. The velocities of the charge/spin excitations are $u_{c,s} = u_0 \sqrt{1 \pm g_{12}/g}$ with $g$ and $g_{12}$ the intraspecies and interspecies interactions [11]. The effective frequencies of the collective modes are of order MHz in Luttinger liquid regime and lies well in the resolved sideband limit. Further studies will consider trapping potentials, where the frequency of collective modes have to be time-averaged in a period because of the position-dependent velocities of the excitations [8].

In summary, we have shown how to implement the probing of the intriguing spin-charge separation in 1D quantum liquids via the optomechanical coupled atom-cavity system. Such experiments allow us to determine definitely the collective excitations of the 1D strongly correlated system with non-demolition measurements. Furthermore, the two-modes optomechanics itself may be of interests for future studies of quantum physics.

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