Light Scattering by Low Lying Quasiparticle Excitations in the Fractional Quantum Hall Regime

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The fractional quantum Hall effect (FQHE) occurs at low temperatures in two dimensional (2D) electron systems of very low disorder that are embedded in large perpendicular magnetic fields. Condensation of 2D electron systems into quantum fluids is among the striking manifestations of fundamental interactions in two dimensions. The major sequences of the FQHE occur at electron Landau level (LL) filling factors given by \( \nu = \frac{1}{p} (\phi p \pm 1) \), where \( p \) is an integer that enumerates members of a particular sequence and \( \phi \) is an even integer that labels different sequences.

Excitations of quasiparticles above the FQHE ground state are described as modes of energy \( \omega \) that have wavevector dispersions \( \omega(q) \). Fig. 1 shows schematic renditions of low lying quasiparticle excitations at \( \nu = 1/3 \). Panel (a) is for the charge modes in which there are no changes in spin orientation. The mode at large wavevector \( q \to \infty \), which represents a neutral quasiparticle-quasihole pair at large separation \( \mathbb{1} \), is understood as the energy obtained in activated magnetotransport. The other salient features of the mode dispersions are the \( q \to 0 \) limit and the minima in the dispersion at finite wavevector \( q \sim 1/l_0 \), where \( l_0 = (\hbar c/eB)^{1/2} \) is the magnetic length. The minima, called magnetorotons or simply rotors, are the consequences of excitonic binding interactions between the quasiparticles \( \mathbb{1} \). Panel (b) of Fig. 1 describes the dispersion of spin wave excitations in which the only change is the orientation of quasiparticle spin \( \mathbb{1} \). Here the \( q \to 0 \) limit is at the bare Zeeman energy \( E_z \) and the large wavevector limit \( (q \to \infty) \) is shifted by the interaction energy required to reverse a spin orientation \( E_\uparrow \).

In the composite fermion (CF) picture \( \phi \) vortices of the many-body wavefunction are attached to each electron to construct CF quasiparticles \( \mathbb{1} \). Within this framework vortex attachment takes into account Coulomb interactions and \( p \) is the CF filling factor. CF quasiparticles have spin-split LLs characteristic of charged fermions with spin \( 1/2 \). Chern-Simons gauge fields in-
corporate electron interactions so that CFs in the lowest electron LL ($\nu \leq 1$) experience effective magnetic fields $B^* = B - B_{1/\phi} = \pm B/(\phi p \pm 1)$, where $B$ is the perpendicular component of the external field \[ \text{Fig. 1} \text{a, b, c]. Spin-split CF-LLs are shown schematically in Fig. 2 for } \nu = 1/3 \text{ in panel (a) and } \nu = 2/5 \text{ in panel (b). The spacing between lowest CF levels with same spin is described as a cyclotron frequency } \hbar \omega_{CF} = \frac{eB^*}{cm_{CF}} \tag{1} \]

and $m_{CF}$ is a CF effective mass. In the context of dispersive collective excitations the large wavevector limit ($q \to \infty$) of the lowest charge excitations is at energy $\hbar \omega_{CF}$.

The main sequence of the FQHE corresponds to $\phi = 2$. In this sequence there are $p$ fully occupied CF-LLs, as shown for $p = 1$ and $p = 2$ in Fig. 1. The quasiparticle excitations shown in Fig. 1 are constructed from the neutral quasiparticle-quasihole pairs of the transitions shown in panel (a) of Fig. 1. The CM transitions are for charge modes and the SW transitions for spin wave excitations.

Given that vortex attachment incorporates effects of Coulomb interactions through Chern-Simons gauge fields, CFs have residual interactions that are much weaker than those among electrons. Features of wavevector dispersions of CF excitations such as rotons and many-body spin reversal energies can be regarded as manifestations of residual interactions among CF quasiparticles.

The low lying excitations of CF quasiparticles, with typical energies below 1meV, are being studied by resonant inelastic light scattering methods \[ \text{Fig. 1a, b, c}. \] In this paper we consider recent light scattering studies of low lying quasi- rotator modes at filling factors in the range $2/5 \geq \nu \geq 1/3$. We also consider results obtained in FQHE liquids in higher electron LLs at filling factors in the range $2 > \nu > 1$.

The main features of dispersions of low lying quasiparticle excitations of spin and charge collective modes in the FQHE were reported in resonant inelastic light scattering experiments \[ \text{Fig. 2a, b, c}. \] More recent light scattering results at filling factors $1/3$ and $2/5$ have determined the energies of rotons at $q \sim 1/16$ and of large wavevector ($q \to \infty$) excitations of CFs \[ \text{Fig. 1a, b, c}. \] The light scattering results for spin and charge modes, quantitatively explained within CF theory \[ \text{Fig. 1a, b, c}. \] suggest that the structure of CF-LLs and residual interactions can be explored by light scattering measurements of low lying collective excitations. The residual interactions that manifest in light scattering experiments considered here could lead to formation of higher order quantum liquids such as those probed in recent magnetotransport experiments \[ \text{Fig. 1a, b, c}. \]

FIG. 3: A schematic of the experimental setup and measurement configuration. The inset shows the backscattering geometry.

**EXPERIMENTAL SETUP AND METHODOLOGY**

The high quality 2D electron system studied here is formed in an asymmetrically doped, 330Å-wide GaAs single quantum well (SQW). The sample density is $n = 5.6 \times 10^{10} \text{cm}^{-2}$ and the electron mobility is $\mu \gtrsim 7 \times 10^6 \text{cm}^2/\text{Vsec}$ at $T = 300 \text{mK}$. As shown in Fig. 1, samples are mounted on the cold finger of a dilution refrigerator with a base temperature of 50mK that is inserted into the cold bore of a 17T superconducting magnet. The samples are mounted with the normal to the surface at an angle $\theta$ to the magnetic field $B_T$, making the component of the field perpendicular to the 2D electron layer $B = B_T \cos \theta$.

As shown in the schematic in Fig. 1, light scattering measurements are performed through windows for direct optical access to the sample. The energy of the linearly polarized incident photons $\omega_L$ is tuned close to the fundamental optical transitions of the GaAs well with either a diode or dye laser. The power density is kept below $10^{-4} \text{W/cm}^2$. The samples are measured in a backscattering geometry, making an angle $\theta$ between the incident/scattered photons and the normal to the sample surface. The wavevector transferred from the photons to the 2D system is $k = k^{||}_L - k^{||}_S = (2\omega_L/c)\sin \theta$, where $k^{||}_L$ and $k^{||}_S$ are the components of the incident and scattered photon wavevectors parallel to the 2D plane and the difference between $\omega_L$ and the scattered photon energy $\omega_S$ is small. For typical measurement geometries, $\theta < 60^\circ$ so that $k \lesssim 1.5 \times 10^5 \text{cm}^{-1} \ll 1/|\hbar|$. Conservation of energy in the inelastic scattering processes is expressed as $\omega(q) = \pm(\omega_L - \omega_S)$, where $q$ is the wavevector of the excitation and $+(-)$ corresponds to the Stokes (anti-Stokes) process.

Scattered light is dispersed by a Spex 1404 double Czerny-Turner spectrometer operating in additive mode with holographic master gratings that reduce the stray
At 1/3 have different resonance conditions. For many of these modes, resonances occur when $\omega_L$ or $\omega_S$ overlap transitions seen in photoluminescence [33] that are assigned to recombination of negatively charged excitons $\Delta_{\pm}$. A significant feature of the spectra in Fig. 4(a) is the absence of luminescence. This is characteristic of spectra excited with $\omega_L$ that overlap the lowest optical excitation of the GaAs quantum well [33].

The dispersion curves for the CM [27] and the SW [8] excitations are displayed in Fig. 4(b). The calculations have been scaled by a factor of 0.6 to account for the effects of the finite width of the 2D layer [18, 20]. The main features of the dispersions are at critical points that occur at $q \to 0$, at the $q \to \infty$ limit, and at the roton minimum at $q \sim 1/l_0 \sim 10^6 cm^{-1}$. It is clear that the spectra in panel (a) display the features of the critical points of the wavevector dispersions of the excitation modes that have wavevectors $q \gg k \sim 10^5 cm^{-1}$. The results in Fig. 4 indicate a massive breakdown of wavevector conservation in the resonant inelastic light scattering spectra.

In a translationally invariant system conservation of momentum is equivalent to conservation of wavevector. Because the wavevector transferred to the 2D system by the photons is $k \ll 1/l_0$, light scattering in a translationally invariant system, where $q = k$, can only access excitations with small wavevectors $q \ll 1/l_0$. The original light scattering experiments in the FQHE regime were able to access the $q \to 0$ low energy CM and SW excitations [22], labelled $\Delta_0$ and $SW_0$ respectively in Fig. 4(a).

Breakdown of wavevector conservation allows inelastic light scattering to access the energies of rotons and of large wavevector ($q \to \infty$) excitations of the CM and SW excitations [18, 20]. The peaks labelled $\Delta_R$ and $\Delta_{\infty}$ in Fig. 4(a) have been assigned to the roton and to $q \to \infty$ CM excitations [18, 22, 40]. The mode labelled $SW_{\infty}$ has been assigned to the $q \to \infty$ spin wave excitation [20]. The $SW_{\infty}$ energy is $E^\parallel_\infty = E_z + E^{\perp}_\parallel$ where $E^{\perp}_\parallel$ represents the spin reversal energy due to interactions among quasiparticles [4, 5, 19, 21, 11, 12]. The value of $E^{\perp}_\parallel = 0.054 e^2/l_0$ obtained from the $SW_{\infty}$ energy is consistent with that obtained from the energies of spin flip excitations ($|1 \uparrow\to |0 \downarrow>$) for filling factors $\nu \gtrsim 1/3$ with small populations of the $|1 \uparrow>$ CF-LL [14, 20].

Breakdown of wavevector conservation in resonant inelastic light scattering is attributed to the loss of full translation symmetry in the presence of very weak residual disorder. The breakdown is particularly effective for spectra excited under strongly resonant conditions. Here the higher order terms in a perturbation theory description of light scattering that directly incorporate wavevector relaxation processes are greatly enhanced [43]. Quantum Hall systems of high perfection display extremely sharp optical resonances of widths $< 0.2 meV$ [33], as in Fig. 4. In such systems, light scattering processes that
break wavevector conservation can be extremely strong. For strongly resonant conditions, light scattering can easily access excitation modes with wavevectors far beyond $k$. \cite{15, 22, 23, 24, 40}.

Under conditions of conservation of wavevector, the intensity of light scattering is proportional to the dynamical structure factor $S(q = k, \omega)$ \cite{44}. Phenomenologically, the light scattering intensity with breakdown of wavevector conservation can be represented by a dynamical structure factor defined as \cite{45}

$$S(k, \omega; \alpha) \sim \int dQ f(k, Q; \alpha) S(Q, \omega)$$

(2)

where $\alpha$ is a parameter (or set of parameters) associated with the diverse mechanisms that contribute to breakdown of wavevector conservation and $f(k, Q; \alpha)$ is a function that couples structure factors at different wavevectors. Within this framework, $S(k, \omega; \alpha)$ and therefore the intensity of the scattered light contains contributions from modes with wavevectors which are well beyond the $q = k$ limit.

Given the importance of breakdown of wavevector conservation in resonant inelastic light scattering processes in the integer and fractional quantum Hall regimes it is useful to conjecture about possible mechanisms that contribute to $\alpha$. We may envision two classes of processes. One class arises from processes that contribute to inhomogeneous broadening and lifetimes of optical transitions of the GaAs quantum wells. Fluctuations in quantum well widths typically contribute to such processes. Another important class of contributions is that arising from wavevector relaxation processes responsible for the increase in magnetoresistance that signals the onset of dissipation in the quantum liquid with increasing temperature.

**STATES WITH 2/5 ≥ ν ≥ 1/3 AND ν ≤ 1/3**

The main FQH states linked to CFs that bind two vortices ($\phi = 2$) form the sequence $\nu = p/(2p \pm 1)$, where, as mentioned above, $p$ represents the CF filling factor and is an integer at the main FQHE states. As shown in Fig. 2 at $\nu = 1/3$ there is only one CF-LL populated while at $\nu = 2/5$ the lowest two levels are full. Inelastic light scattering measurements of low lying excitations of the electron liquids in the full range of filling factors $2/5 \geq \nu \geq 1/3$ enable the study of quasiparticles in states with partial population of one CF-LL ($2 \geq p \geq 1$). These experiments uncover low lying excitations at all magnetic fields within this range. \cite{15}. There has been recent interest in the region between major fractions such as $2/5 > \nu > 1/3$ where newly observed FQH states occur at values such as $p = 1 + 1/3$ and $p = 1 + 2/3$. \cite{24, 46, 47, 48} Such liquids may be described by attaching an even number of fluxes to CFs, resulting in quasiparticles with $\phi = 4$.

Figure 5 shows light scattering spectra of low lying excitations ($\omega \lesssim 0.3$ meV) measured at $\nu \lesssim 2/5$. The spectrum at $\nu = 0.388$ shows two structures. The one labelled (SW$_0$) is at $E_z = g \mu_B B_T$. At lower energies, there is a broader structure labelled SF$^-$ that is assigned to an excitation based on transitions similar to SF transitions shown in Fig. 2(b) \cite{15, 20}. The mode evolves from a deep roton minimum of SF excitations at $\nu = 2/5$. As the filling factor is reduced the SF$^-$ evolves into the well-defined peak shown in Fig. 5 for $\nu = 0.375$. At larger magnetic fields $\nu \gtrsim 1/3$ a narrow peak (SF$^+$) appears below the SW$_0$ (not shown in Fig. 5). This peak disappears at $\nu = 1/3$.

Figure 4 summarizes the energies of the low lying spin excitations measured in the range $2/5 \geq \nu \geq 1/3$. The mode energies shown include the SW$_0$ at $E_z$, the SF$^-$, and the SF$^+$. The SF$^-$ mode displays rapid softening to very low energies and then its energy remains constant until it disappears. It is noteworthy that the energy of the SF$^-$ mode does not display a marked magnetic field dependence in the region in which the filling factor of the second CF-LL, given by $p - 1$ varies between $2/3$ $(p = 5/3)$ and $1/3$ $(p = 4/3)$. In this region, where higher order FQH fluids have been observed \cite{20}, the SF$^-$ mode occurs as a well defined peak with energy independent of

![Figure 5: Depolarized inelastic light scattering spectra at filling factors ν = 0.388 (thin line) and 0.375 (thick line) near ν = 2/5. Vertical solid arrows show the positions of roton density of states as presented in Ref. 15. The gray arrow marks the position of the maximum of the peak at ν = 0.375.](image-url)
experiments also show that the crossover from φcitations and higher energy φ= 4 liquids that happens at φ1
FQHE sequences, both near
the remarkable emergence of low lying excitations with
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to FQHE states [49].

Beyond 1/
2
ν,
which can undergo a melting transition
could be related to the appearance of these higher order
FQHE liquids.

A remarkably rich set of phenomena also exists beyond
the primary sequence of FQHE states in the lowest LL.
For ν < 1/, additional FQHE sequences are found for
ν = p/(φp ± 1) with φ = 4, 6, 8 [49]. There has been much
recent interest in the crossover region between different
FQHE sequences, both near ν = 1/3 [21] and 1/5 [42, 50]. Beyond 1/5, the system is thought to condense into a
Wigner solid [50], which can undergo a melting transition
to FQHE states [49].

Light scattering studies of quantum liquids with ν ≤
1/3 is in its infancy. Recent results display evidence of
a surprising coexistence between lower energy φ = 4 ex-
citations and higher energy φ = 2 modes [21]. These
experiments also show that the crossover from φ = 2 to
φ = 4 liquids that happens at ν = 1/3 is associated with
the remarkable emergence of low lying excitations with
energies ω < 0.2meV.

FQHE STATES AROUND ν = 3/2

At lower B, FQHE states are found around ν = 3/2.
These states are also thought to have a CF-LL structure.
Evidence of crossings of these levels is seen in angular-
dependent magnetotransport [51, 52, 53, 54]. These
states are characterized by low lying excitations, mak-
light scattering an ideal probe of their structures.

We have initiated light scattering studies of low lying
excitations of the 2D system in higher electron LLs with
experiments carried out in the filling factor range 2 > ν >
1. Figure 7 shows light scattering spectra from different
filling factors around ν = 3/2. Strong excitations are
seen at FQHE states 4/3 and 8/5. Modes are seen only
near these two FQHE states and are not observed at other
FQHE states that have been seen in previous transport
data [51, 52, 53, 54], consistent with magnetotransport
shown in the lower inset of Fig. 7. These excitations are
sharp (<0.2meV wide) and have resonance enhancement
profiles that are relatively narrow (<0.4meV wide, not
shown). As seen in the upper inset of Fig. 7 the 4/3
state has a very strong temperature dependence. As the
temperature is raised from 63mK, the low energy tail
increases in intensity until the mode disappears above
300mK.

On the basis of particle-hole symmetry, the FQH states
around ν = 3/2 are described in terms of hole states in a
filled spin-split LL with hole filling factor νH = 2 − ν.
Within the CF framework, the CF quasiparticles fill
quasi-LLs that are spaced by an effective cyclotron en-
ergy ωCF with B∗ = 3(B − B3/2), where B3/2 is the value of
B at ν = 3/2, with spin-split partners spaced by an
effective Zeeman energy Ez∗ = g∗μBBy, where g∗ is an

FIG. 6: Energies of the low energy modes in the filling factor
range 2/5 > ν ≥ 1/3 (after [49]). The modes represented are
the SW0, SF+, and SF− described in the text. Two values
are presented for SF− modes: the squares are determined
from the roton density of states [11]; the gray area covers th e
energy range of the maxima of the SF− peak. The dotted line
is a fit of the SW0 energy with Ez = gμBBF and g = 0.44.

FIG. 7: Inelastic light scattering spectra at various filling
factors between 2 > ν > 1. The spectra at FQHE states
ν = 4/3 and ν = 8/5 are highlighted. The upper inset shows
the temperature dependence of the mode at ν = 4/3 and the
lower inset shows transport data identifying various FQHE
states.
enhanced Lande factor including many-body corrections to the spin flip energy. The angular dependent disappearance of minima at various FQHE states is interpreted as an energy level crossing occurring when $E^*_j = j\omega_{CF}$ for integer $j$.

We focus our attention on $\nu = 4/3$, which being equivalent to a $2/3$ state of holes has two CF-LLs populated. For small values of $B_T$, the system is thought to be unpolarized, with the lowest $|\uparrow\rangle$ and $|\downarrow\rangle$ states equally populated. In a non-interacting picture, the lowest energy excitation is from $|\downarrow\rangle \rightarrow |\uparrow\rangle$ and has an energy $\Delta_{4/3} = \omega_{CF} - E^*_\uparrow$. Using this as the form for the activation gap, Du et al. find $m_{CF} = 0.43m_0$ and $g^* \approx 0.76$, where $m_0$ is the free electron mass. Assuming that $m_{CF}$ and $(g^* - g)$ scale with $e^2/\epsilon_0$, we deduce $m_{CF} = 0.30m_0$ and $g^* = 0.66$ for our sample. Using these values, we find $\Delta_{4/3} = 0.15$meV to be in agreement with the peak in the excitation spectrum seen at $4/3$ in Fig. 4.

As seen in the upper inset to Fig. 4, the mode at $4/3$ disappears above $300$K. Similar behavior is also seen for the mode at $8/5$. The temperature dependence is much more sensitive than the energy of $\Delta_{4/3}$. The $4/3$ mode in Fig. 4 also shows significant intensity below $0.1$meV, suggesting the existence of a lower energy roton. This lower energy roton may determine the temperature dependence of the state. Because of the relatively low density, it is difficult to resolve the various features of the excitations described in Fig. 4. Further work in higher density samples may allow us to map the critical points in the dispersions of various modes of the FQHE states.

It is surprising that clear excitation modes are not seen at the FQHE state of $\nu = 5/3$. This is consistent with transport measurements in our sample that indicate that $5/3$ is not as well-defined as the FQHE state at $8/5$, as seen in the lower inset of Fig. 4. Previous reports have indicated that the $5/3$ state is more robust than $8/5$ and $\nu = 1/3$. Recent results, however, have indicated that the $8/5$ state is stronger than the $5/3$ state in very high quality samples at low densities.

**SUMMARY**

Inelastic light scattering accesses the low lying quasiparticle charge and spin excitations at and between fractional quantum Hall states. Breakdown of wavevector conservation allows light scattering to access energies of bound and unbound quasiparticle-quasihole pairs and provides a measure of residual quasiparticle interactions. In the filling factor range $2/5 \geq \nu \geq 1/3$ we find evidence that CF quasiparticles have an electron-like LL scheme. Anomalies in the magnetic field dependence of the SF excitations may be due to the presence of higher order ($\phi = 4$) CFs. In higher LLs, we find excitations at $4/3$ and $8/5$ that are consistent with a spin-unpolarized population of quasi-LLs. Future work will explore higher order FQH states and the even denominator FQH states in higher LLs.

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