Ratio of absorption cross section for Dirac fermion to that for scalar in the higher-dimensional black hole background

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(October 25, 2018)

Abstract

The ratio of the low-energy absorption cross section for Dirac fermion to that for minimally coupled scalar is computed when the spacetimes are various types of the higher-dimensional Reissner-Nordström black holes. It is found that the low-energy absorption cross sections for the Dirac fermion always goes to zero in the extremal limit regardless of the detailed geometry of the spacetime. The physical importance of our results is discussed in the context of the brane-world scenarios and string theories.

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Recent brane-world scenarios such as the large extra dimensions [1,2] or warped extra dimensions [3,4] predict an emergence of the TeV-scale gravity. This crucial effect arising due to the extra dimensions opens the possibility to make tiny black holes in the future high-energy colliders [5–7]. In this reason much attention is paid recently to the absorption and emission problems of the black holes in the presence of the extra dimensions.

If the extra dimensions exist, the absorption and emission problems should be analyzed for the brane-localized and the bulk cases separately. It was argued in Ref. [8,9] that the Hawking radiation into the bulk is dominant compared to that on the brane. This argument is mainly originated from the fact that for the tiny black hole\(^1\) the Hawking temperature is much larger than the mass of the light Kaluza-Klein modes. However, different argument was suggested by Emparan, Horowitz and Myers (EHM) [10]. EHM argued that the radiation into the bulk by the light Kaluza-Klein modes is highly suppressed by the geometrical factor, which makes the missing energy much smaller than the visible one. EHM verified their argument explicitly by approximating the black hole as a black body with a critical radius \(r_c\), which is slightly larger than the horizon radius. The detailed calculation on the absorption and emission problems for the \((4 + n)\)-dimensional Schwarzschild black hole was carried out analytically [11–13] and numerically [14], which supports the main result of Ref. [10], i.e. the visible energy via the Hawking radiation is dominant. Especially, Ref. [13] has shown via the analytic computation that the low-energy absorption cross section for the brane-localized massive Dirac fermion over that for the massive scalar is explicitly dependent on \(n\), the number of the toroidally compactified extra dimensions, as \(2^{(n-3)/(n+1)}\). Thus the ratio factor 1/8 is recovered when \(n = 0\), which was derived long ago by Unruh [15].

The absorption and emission problems of the higher-dimensional charged or rotational black holes were also studied [16–21]. For the charged black holes the full absorption and

\(^1\) We assume \(\ell_{\text{fun}} \ll r_0 \ll L\) where \(\ell_{\text{fun}}, r_0,\) and \(L\) are bulk Planck length, black hole radius, and size of the extra dimension.
emission spectra were computed in Ref. [17] by adopting the numerical technique used in Ref. [22,23]. It was shown that the increase of the inner horizon parameter, \( r_- \), in general enhances the absorptivity and suppresses the emission rate compared to the case of the Schwarzschild phase \( (r_- = 0) \) regardless of the brane-localized or bulk case. Also the bulk-to-brane energy emissivity was shown to decrease with increasing \( r_- \). The results of Ref. [17] also support the main result of EHM if \( n \) is not too large. However, for the large \( n \) the emission into the bulk can be dominant. For the rotating black holes it was shown in Ref. [18–21] that the effect of the superradiance is very important for the experimental signature in the future collider. It was argued that the rotating black holes, contrary to the Schwarzschild black hole, radiate mainly into the bulk. More recently, the differences of the tiny black holes arising due to the various extra dimensional scenarios are examined [24]. This may be helpful in the future black hole experiment to determine a type of the extra dimensions.

In this letter we will compute the ratio of the low-energy absorption cross section for Dirac fermion to that for the minimally coupled scalar when the spacetimes are various types of the higher-dimensional Reissner-Nordström(RN) black holes. We start with a brief review of Ref. [25] which examined the universality of the low-energy absorption cross sections. Let us consider the general spherically symmetric metric in \((p+2)\) spacetime dimensions of the form

\[
ds^2 = -f(r)dt^2 + g(r) \left[ dr^2 + r^2 d\Omega^2_{p} \right]
\]

(1)

where \( d\Omega^2_{p} \) is a spherically symmetric angular part in the metric. We also assume

\[
\lim_{r \to \infty} f(r) = \lim_{r \to \infty} g(r) = 1
\]

(2)

for imposing the asymptotically flat. Then it is well-known that the low-energy absorption cross section for the minimally coupled bulk scalar is equal to the horizon area

\[
\sigma_S = \Omega_p R_H^p \equiv A_H
\]

(3)
\[ \Omega_p = \frac{2\pi^{(p+1)/2}}{\Gamma(p + 1/2)}, \quad R_H = r_H \sqrt{g(r_H)}, \quad \text{and} \quad r_H \text{ is a horizon radius usually determined by the largest solution of } f(r) = 0. \]

Furthermore Ref. [25] has shown that the low-energy absorption cross section for the bulk fermion is given by the area measured in the flat spatial metric conformally related to the true metric in the form

\[ \sigma_F = 2\Omega_p r_H^p = 2 (g(r_H))^{-\frac{p}{2}} A_H \] (4)

where the factor 2 comes from the number of spinors. Thus for the bulk fields we can show the ratio factor \( \sigma_F/\sigma_S \) to be

\[ \gamma^{BL} = \frac{\sigma_F}{\sigma_S} = 2 (g(r_H))^{-\frac{p}{2}}. \] (5)

If we assume that the three brane has negligible self-gravity of its own, the induced metric on the brane can be written as

\[ ds^2 = -f(r)dt^2 + g(r) \left[ dr^2 + r^2 d\Omega^2 \right]. \] (6)

Following the same procedure, it is easy to show that the low-energy absorption cross section for the brane-localized scalar is \( \sigma_S = 4\pi R_H^2 \) and the ratio factor is

\[ \gamma^{BR} = \frac{\sigma_F}{\sigma_S} = 2 (g(r_H))^{-1}. \] (7)

Now, we consider the \((4 + n)\)-dimensional charged black hole whose metric is [26]

\[ ds_{RN}^2 = -\left[ 1 - \left( \frac{\tilde{r}_+}{\tilde{r}} \right)^{n+1} \right] \left[ 1 - \left( \frac{\tilde{r}_-}{\tilde{r}} \right)^{n+1} \right] dt^2 + \frac{d\tilde{r}^2}{\left[ 1 - \left( \frac{\tilde{r}_+}{\tilde{r}} \right)^{n+1} \right] \left[ 1 - \left( \frac{\tilde{r}_-}{\tilde{r}} \right)^{n+1} \right]} + \tilde{r}^2 d\Omega^2_{n+2} \] (8)

where \( \tilde{r}_+ \) and \( \tilde{r}_- \) correspond to the external and internal horizons, respectively. It is convenient to introduce a new radial coordinate \( \tilde{r} \) in the form

\[ \tilde{r}^2 = a(\tilde{r})\tilde{r}^2 \] (9)

where
\[ a(\hat{r}) = \left[ 1 + \left( \frac{\hat{r}_1}{\hat{r}} \right)^{n+1} \right]^{\frac{2}{n+1}} \tag{10} \]

and \( \hat{r}_1 \equiv \tilde{r}_- \). Then the metric (8) reduces to the following more tractable form

\[
\begin{align*}
\text{ds}_{\text{RN}}^2 &= -h(\hat{\bar{r}})a^{-n-1}(\hat{\bar{r}})dt^2 + a(\hat{\bar{r}}) \left[ h^{-1}(\hat{\bar{r}})d\hat{\bar{r}}^2 + \hat{\bar{r}}^2 d\Omega_{n+2}^2 \right] \\
\end{align*}
\]

where

\[ h(\hat{r}) = 1 - \left( \frac{\hat{r}_0}{\hat{r}} \right)^{n+1} \tag{12} \]

and \( \hat{r}_0^{n+1} = \tilde{r}_+^{n+1} - \tilde{r}_-^{n+1} \). It is easy to show that the horizon radius \( \tilde{r} = \tilde{r}_+ \) in \( \tilde{r} \)-coordinate corresponds to \( \hat{r} = \hat{r}_0 \) in \( \hat{r} \)-coordinate.

The metric (11) can be written in the form

\[
\begin{align*}
\text{ds}_{\text{RN}}^2 &= -f_{\text{RN}}(r)dt^2 + g_{\text{RN}}(r) \left[ dr^2 + r^2 d\Omega_{n+2}^2 \right] \\
\end{align*}
\]

if \( f_{\text{RN}}(r), g_{\text{RN}}(r) \) and \( r \) satisfy

\[
\begin{align*}
f_{\text{RN}}(r) &= h(\hat{\bar{r}}(r))a^{-n-1}(\hat{\bar{r}}(r)) \\
g_{\text{RN}}(r) &= \frac{\hat{\bar{r}}^2}{r^2}a(\hat{\bar{r}}(r)) \\
\sqrt{g_{\text{RN}}(r)}dr &= \sqrt{a(\hat{\bar{r}})}d\hat{\bar{r}}. \\
\end{align*}
\]

Solving Eq.(14), one can show straightforwardly that the \( r \)-dependence of \( \hat{r} \) is

\[
\hat{r} = \hat{r}_0 \frac{C}{r} \left[ \frac{2}{\left( \frac{\tilde{r}}{\tilde{r}} \right)^{n+1} + 1} \right]^{-\frac{2}{n+1}}. 	ag{15} \]

where \( C \) is an integration constant. Eq.(15) implies

\[
\begin{align*}
h(\hat{r}) &= \left( \frac{\left( \frac{\tilde{r}}{\tilde{r}} \right)^{n+1} - 1}{\left( \frac{\tilde{r}}{\tilde{r}} \right)^{n+1} + 1} \right)^2 \\
a(\hat{\bar{r}}) &= \left[ 1 + \frac{4 \left( \frac{\tilde{r}_0}{\tilde{r}_0} \right)^{n+1} \left( \frac{\tilde{r}}{\tilde{r}} \right)^{n+1}}{\left( \left( \frac{\tilde{r}}{\tilde{r}} \right)^{n+1} + 1 \right)^2} \right]^{\frac{2}{n+1}}. \\
\end{align*}
\]

Combining (14), (15) and (16), one can readily derive
\( f_{RN}(r) = \left( \frac{\left( \frac{r}{\bar{r}} \right)^{n+1} + 1}{\left( \frac{r}{\bar{r}} \right)^{n+1} + 1} \right)^2 \) \( \left( \frac{r}{\bar{r}} \right)^{n+1} \) \( g_{RN}(r) = \left( \frac{C \hat{r}_0}{r^2} \right)^{n+1} \left[ \left( \frac{r}{\bar{r}} \right)^{n+1} + 1 \right]^2 + \left( \frac{\hat{r}_1}{r} \right)^{n+1}\right]^{\frac{2}{n+1}}. \)

The integration constant \( C \) should be \( C = 2 - 2/\left(\hat{r}_0\right) \) due to Eq.(2). Thus \( \hat{r} = \hat{r}_0 \) in Eq.(15) implies

\[ r_H = C = 2 - \frac{2}{\hat{r}_0}. \] (18)

Inserting Eq.(18) into (17) yields

\[ g_{RN}(r_H) = 2^{\frac{4}{n+1}} \left[ \hat{r}_0^{n+1} + \hat{r}_1^{n+1} \right]^{\frac{2}{n+1}} = 2^{\frac{4}{n+1}} \frac{\hat{r}_0^2}{\hat{r}_0^{n+1} - \hat{r}_-^{n+1}} \] (19)

which makes \( R_H = r_H \sqrt{g(r_H)} = \hat{r}_+ \). Thus the low-energy absorption cross section for the bulk scalar should be \( \sigma^{BL}_S = \Omega_{n+2} \hat{r}^{n+2} \) and the ratio factor \( \gamma^{BL} \) given in Eq.(5) becomes

\[ \gamma^{BL} \equiv \frac{\sigma^{BL}_F}{\sigma^{BL}_S} = 2^{-\frac{n+3}{n+1}} \left[ 1 - \left( \frac{\hat{r}_-}{\hat{r}_+} \right)^{n+1} \right]^{\frac{n+2}{n+1}}. \] (20)

Thus increasing \( \hat{r}_- \) decreases \( \gamma^{BL} \) and eventually it goes to zero at the extremal limit \( (\hat{r}_- = \hat{r}_+)^2 \). In the Schwarzschild limit \( (\hat{r}_- = 0) \) \( \gamma^{BL} \) becomes \( 2^{-(n+3)/(n+1)} \) which recovers 1/8 at \( n = 0 \). Contrary to \( \hat{r}_- \), the increase of \( n \) with fixed \( \hat{r}_- \) increases \( \gamma^{BL} \) and eventually is saturated to 0.5 when \( n \) is infinity. For small \( \hat{r}_- \) the \( n \)-dependence of \( \gamma^{BL} \) reaches the saturated value more rapidly compared to the large \( \hat{r}_- \) case.

For the brane-localized scalar the low-energy absorption cross section is \( \sigma^{BR}_S = 4\pi \hat{r}_+^2 \) and the ratio factor reduces to

\[ \gamma^{BR} \equiv \frac{\sigma^{BR}_F}{\sigma^{BR}_S} = \frac{2}{\hat{r}_+} \] (21)

Since the black hole charge is \( Q = \pm(\hat{r}_+ \hat{r}_-)^{(n+1)/2} \sqrt{(n+1)(n+2)/8\pi} \), increase of \( \hat{r}_- \) means increase of the black hole charge. if it is possible to fix the black hole entropy during increase of the black hole charge, one can increase \( \hat{r}_- \) with fixed \( \hat{r}_+ \).
As in the case of the bulk fields $\gamma^{BR}$ approaches to zero at the extremal limit\(^3\). In the Schwarzschild limit Eq.\((21)\) reproduces $\gamma^{BR} = 2^{(n-3)/(n+1)}$, which was derived via the direct matching procedure in Ref.\([13]\). The $n$-dependence of $\gamma^{BR}$ is similar to that of $\gamma^{BL}$, but the saturated value is changed into 2. The increase of $\gamma^{BR}$ with increasing $n$ is consistent with Fig. 3 of Ref.\([14]\).

Next we consider a five-dimensional black hole with three different $U(1)$ charges \([27,28]\)

$$ds^2_{MS} = -h(\tilde{r}) a^{-2/3}(\tilde{r}) dt^2 + a^{1/3}(\tilde{r}) \left[ h^{-1}(\tilde{r}) d\tilde{r}^2 + \tilde{r}^2 d\Omega_3^2 \right]$$

where

$$h(\tilde{r}) = 1 - \frac{\tilde{r}_0^2}{\tilde{r}^2} \quad \quad a(\tilde{r}) = a_1(\tilde{r}) a_5(\tilde{r}) a_K(\tilde{r})$$

$$a_i(\tilde{r}) = 1 + \frac{\tilde{r}_i^2}{\tilde{r}^2} \quad (i = 1, 5, K).$$

The $U(1)$ charges $Q_i = (\tilde{r}_0^2/2) \sinh 2\xi_i$ where $\xi_i = \sinh^{-1}(\tilde{r}_i/\tilde{r}_0)$ are originated from 1D-branes, 5D-branes and Kaluza-Klein charges respectively. The spacetime \((22)\) is extensively used by Maldacena and Strominger in Ref.\([29]\) for the examination of the black hole-D-brane correspondence in the dilute gas region $\tilde{r}_0 << \tilde{r}_K << \tilde{r}_1, \tilde{r}_5$ and by Hawking and Taylor-Robinson \([30]\) in the slightly different region $\tilde{r}_0 << \tilde{r}_1, \tilde{r}_5, \tilde{r}_K$. Note that the spacetime \((22)\) exactly coincides with an usual five-dimensional RN black hole when $\tilde{r}_1 = \tilde{r}_5 = \tilde{r}_K$.

The metric \((22)\) can be re-written in the form

$$ds^2_{MS} = -f_D(r) dt^2 + g_D(r) (dr^2 + r^2 d\Omega_3^2)$$

if $f_D(r)$, $g_D(r)$, and $r$ satisfy

\(^3\)Following Ref.\([13]\), we can derive Eq.\((21)\) by direct matching procedure. Since it is tedious and lengthy procedure, it is not presented in this letter.
\[ f_D(r) = h(\tilde{r})a^{-2/3}(\tilde{r}) \]  
(25)

\[ g_D(r) = \frac{\tilde{r}^2}{r^2}a^{1/3}(\tilde{r}) \]

\[ \sqrt{g_D(r)}dr = \frac{a^{1/6}(\tilde{r})}{\sqrt{h(\tilde{r})}}d\tilde{r}. \]

Solving Eq. (25) yields the following \( r \)-dependence of \( \tilde{r} \);

\[ \tilde{r} = \tilde{r}_0 \frac{C}{2r} \left[ \frac{r^2}{C^2} + 1 \right] \]
(26)

where \( C \) is an integration constant. Eq. (26) implies

\[ h(\tilde{r}) = \left( \frac{\tilde{r}^2 - 1}{\tilde{r}^2 + 1} \right)^2 \]
(27)

\[ a(\tilde{r}) = \left[ 1 + 4 \left( \frac{\tilde{r}_0}{\tilde{r}} \right)^2 \right] \left[ 1 + 4 \left( \frac{\tilde{r}_0}{\tilde{r}} \right)^2 \right] \left[ 1 + 4 \left( \frac{\tilde{r}_0}{\tilde{r}} \right)^2 \right]. \]

Combining (25), (26) and (27), one can derive

\[ f_D(r) = \left( \frac{r^2}{C^2} + 1 \right)^2 \left( \frac{r^2}{C^2} - 1 \right)^2 \left\{ \left( \frac{r^2}{C^2} + 1 \right)^2 + 4 \left( \frac{\tilde{r}_1}{\tilde{r}_0} \right)^2 \left( \frac{r}{C} \right)^2 \right\}^{-2/3} \]
(28)

\[ \times \left\{ \left( \frac{r^2}{C^2} + 1 \right)^2 + 4 \left( \frac{\tilde{r}_1}{\tilde{r}_0} \right)^2 \left( \frac{r}{C} \right)^2 \right\}^{-2/3} \]

\[ g_D(r) = \left[ \frac{C^2r_0^2}{4r^4} \left( \frac{r^2}{C^2} + 1 \right)^2 + \left( \frac{\tilde{r}_1}{r} \right)^2 \right]^{1/3} \left[ \frac{C^2r_0^2}{4r^4} \left( \frac{r^2}{C^2} + 1 \right)^2 + \left( \frac{\tilde{r}_5}{r} \right)^2 \right]^{1/3} \]

\[ \times \left[ \frac{C^2r_0^2}{4r^4} \left( \frac{r^2}{C^2} + 1 \right)^2 + \left( \frac{\tilde{r}_K}{r} \right)^2 \right]^{1/3}. \]

From a condition of the asymptotic flat the integration constant \( C \) is fixed as \( C = \tilde{r}_0/2 \). Since the location of the horizon is \( \tilde{r} = \tilde{r}_0 \) in \( \tilde{r} \)-coordinate, Eq. (26) indicates that the horizon radius in \( r \)-coordinate is

\[ r \equiv \frac{\tilde{r}_0}{2} = r_H. \]
(29)

Inserting Eq. (29) into (28), one can show easily

\[ g_D(r_H) = 4 \left[ 1 + \left( \frac{\tilde{r}_1}{\tilde{r}_0} \right)^2 \right]^{1/3} \left[ 1 + \left( \frac{\tilde{r}_5}{\tilde{r}_0} \right)^2 \right]^{1/3} \left[ 1 + \left( \frac{\tilde{r}_K}{\tilde{r}_0} \right)^2 \right]^{1/3} \]
(30)

\[ R_H \equiv r_H \sqrt{g_D(r_H)} = \left( \tilde{r}_0^2 + \tilde{r}_1^2 \right)^{1/6} \left( \tilde{r}_0^2 + \tilde{r}_5^2 \right)^{1/6} \left( \tilde{r}_0^2 + \tilde{r}_K^2 \right)^{1/6}. \]
Thus Eq.(3) with \( p = 3 \) yields the low-energy absorption cross section for the massless bulk scalar

\[
\sigma_{BL}^{S} = \frac{2}{\pi} R_{H}^{3} = 2\pi^{2}(\tilde{r}_{0}^{2} + \tilde{r}_{1}^{2})^{\frac{1}{2}}(\tilde{r}_{0}^{2} + \tilde{r}_{5}^{2})^{\frac{1}{2}}(\tilde{r}_{0}^{2} + \tilde{r}_{K}^{2})^{\frac{1}{2}}. \tag{31}
\]

which exactly equals to the horizon area.

In the dilute gas region \( (\tilde{r}_{0} \ll \tilde{r}_{K} \ll \tilde{r}_{1, \tilde{r}_{5}}) \) Eq.(31) can be written in the form

\[
\sigma_{BL}^{S} \sim 2\pi^{2}\tilde{r}_{0}\tilde{r}_{1}\tilde{r}_{5}\sqrt{1 + \left(\frac{\tilde{r}_{K}}{\tilde{r}_{0}}\right)^{2}} \sim 2\pi^{2}r_{1}r_{5}^{2}\frac{\pi\omega}{2} \frac{e^{\pi H} - 1}{(e^{2\pi H} - 1)(e^{2\pi R} - 1)} \tag{32}
\]

where \( T_{L} \) and \( T_{R} \), the temperatures of left and right moving oscillations, are related to the Hawking temperature \( T_{H} \) by

\[
\frac{1}{T_{R}} + \frac{1}{T_{L}} = \frac{2}{T_{H}}. \tag{33}
\]

Eq.(5) with \( p = 3 \) gives a ratio factor

\[
\gamma_{BL}^{F} \equiv \frac{\sigma_{F}^{BL}}{\sigma_{S}^{BL}} = \frac{1}{4} \left[ 1 + \left(\frac{\tilde{r}_{1}}{\tilde{r}_{0}}\right)^{2}\right]^{-\frac{1}{2}} \left[ 1 + \left(\frac{\tilde{r}_{5}}{\tilde{r}_{0}}\right)^{2}\right]^{-\frac{1}{2}} \left[ 1 + \left(\frac{\tilde{r}_{K}}{\tilde{r}_{0}}\right)^{2}\right]^{-\frac{1}{2}}. \tag{34}
\]

It is interesting to note that Eq.(34) indicates \( \sigma_{F}^{BL} \) is independent of \( \tilde{r}_{1}, \tilde{r}_{5} \) and \( \tilde{r}_{K} \). If one angle variable in Eq.(22) is regarded as a toroidally compactified extra dimension, one can show using Eq.(7) that the low-energy absorption cross section for the brane-localized scalar is

\[
\sigma_{S}^{BR} = 4\pi^{2}R_{H}^{2} = 4\pi(\tilde{r}_{0}^{2} + \tilde{r}_{1}^{2})^{3}(\tilde{r}_{0}^{2} + \tilde{r}_{5}^{2})^{3}(\tilde{r}_{0}^{2} + \tilde{r}_{K}^{2})^{3} \tag{35}
\]

and the ratio factor is

\[
\gamma_{BR}^{F} \equiv \frac{\sigma_{F}^{BR}}{\sigma_{S}^{BR}} = 2g^{-1}(r_{H}) = \frac{1}{2} \left[ 1 + \left(\frac{\tilde{r}_{1}}{\tilde{r}_{0}}\right)^{2}\right]^{-\frac{1}{2}} \left[ 1 + \left(\frac{\tilde{r}_{5}}{\tilde{r}_{0}}\right)^{2}\right]^{-\frac{1}{2}} \left[ 1 + \left(\frac{\tilde{r}_{K}}{\tilde{r}_{0}}\right)^{2}\right]^{-\frac{1}{2}}. \tag{36}
\]

Eq.(36) also indicates that \( \sigma_{F}^{BR} \) is independent of \( \tilde{r}_{1}, \tilde{r}_{5} \) and \( \tilde{r}_{K} \).
In this letter we calculated the ratio of the low-energy cross section for Dirac fermion to that for scalar when the spacetimes are various types of the higher-dimensional RN black holes. It was found that the low-energy cross section for Dirac fermion always goes to zero in the extremal limit. One may confirm our results (20) and (21) by computing the absorption cross section in the full energy range numerically by adopting the numerical technique used in Ref. [13]. Also it is interesting to check whether or not the D-brane approach [31] derives Eq.(34) and (36). Among them it may be of greatest interest to extend our calculation to the non-spherically symmetric spacetimes such as the Kerr-Newman black holes.

Acknowledgement: This work was supported by the Kyungnam University Research Fund, 2004.
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