MULTIPLE HARD INTERACTIONS IN
\( \gamma \gamma \) AND \( \gamma p \) PHYSICS AT LEP AND HERA *

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ABSTRACT

At \( e^+ e^- \) and \( ep \) colliders, the large fluxes of almost on-shell photons accompanying the lepton beams lead to the photoproduction of jets. As the centre-of-mass energy is increased, regions of smaller \( x \) in the parton densities are explored and these are regions of high parton density. As a result, the probability for more than one hard partonic scattering occurring in a single \( \gamma \gamma \) or \( \gamma p \) collision can become significant. This effect has been simulated using an eikonal prescription combined with the HERWIG Monte Carlo program. The possible effects of multiple hard interactions on event shapes and jet cross sections have been studied in this framework at a range of energies relevant to HERA and LEPII. The results indicate that the effects could be significant.

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At $e^+e^-$ and $ep$ colliders, the large fluxes of almost on-shell photons accompanying the lepton beams lead to the photoproduction of jets. As the centre-of-mass energy is increased, regions of smaller $x$ in the parton densities are explored and these are regions of high parton density. As a result, the probability for more than one hard partonic scattering occurring in a single $\gamma\gamma$ or $\gamma p$ collision can become significant. This effect has been simulated using an eikonal prescription combined with the HERWIG Monte Carlo program. The possible effects of multiple hard interactions on event shapes and jet cross sections have been studied in this framework at a range of energies relevant to HERA and LEPII. The results indicate that the effects could be significant.

1. Introduction

For both protons and photons, QCD predicts a rapid increase in parton densities at low $x$. In a naive treatment, this rise can lead to a corresponding (but ultimately unphysical) rise with increasing energy of perturbative QCD calculations of the jet contribution to the total cross section. However, the large number of small $x$ partons contributing to jet production can mean that there is a significant probability for more than one hard scatter per $\gamma p$ or $\gamma\gamma$ interaction. The effects of multiple interactions can provide a mechanism for taming the rise in the QCD cross section in accord with unitarity. Clearly there are implications for the hadronic final state and in order to study these effects an eikonal model has been implemented within the hard process generation of HERWIG.

Multiple parton scattering, as shown in Fig. 1, is expected to affect jet rates. The average number of jets per event should be increased when partons from secondary hard scatters are of sufficiently high $p_T$ to give jets in their own right. In addition, lower $p_T$ secondary scatters produce extra transverse energy in the event which contributes to the pedestal energy underneath other jets in the event. Thus multiple scattering can influence jet cross sections even when no parton from the secondary

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scatters is itself of a high enough $p_T$ to produce an observable jet. By boosting the transverse energy of jets in this way, multiple scattering leads to an increase in jet cross sections for jets above a certain $E_T^{jet}$ cut, even though the total cross section is reduced.

In the model implemented in HERWIG, the number of hard scatters is given by the simple probability theory formula,

$$\#\text{events} = \#\text{trials} \times \text{event probability per trial}.$$ 

In the general case of a collision between particles ($A$ and $B$) at an impact parameter $b$, the ‘number of trials’ is the product of the number densities of the partons in the overlapping regions of the parent particles, i.e.

$$\#\text{trials} = \int d^2 b' n_{i/A}(b', x_A)n_{j/B}(b' - b, x_B) = A(b) n_{i/A}(x_A)n_{j/B}(x_B),$$  

(1)

where $n_{i/A}(x_A)$ is the number density of partons of type $i$ within particle $A$ and carrying a light cone momentum fraction, $x_A$. We have assumed that the $x$ and $b$ dependences of the parton densities factorize (we call $A(b)$ the area overlap function). The ‘#events’ is the multiplicity of final state jet pairs produced per interaction. The ‘event probability per trial’ is $d\bar{\sigma}_{ij}/(P_A^{-1}P_B^{-1}\sigma_H(s))$ where $\sigma_H(s)$ is the cross section for the production of one or more jet pairs and $d\bar{\sigma}_{ij}$ is the parton-parton subprocess cross section. $P_A$ is the probability that the particle $A$ actually interacts as a hadron. For hadrons, the factor is unity, whilst for photons it is proportional to $\alpha_{em}$. In fact we take the $\rho$-dominance form $P_A = 4\pi\alpha_{em}/f_{\rho}^2$.

The $b$ dependence of the parton density is assumed to be given by the electromagnetic form factor of the parent particle. By normalizing the integral of the area overlap function to unity we can identify

$$A(b)d^2 b\frac{\sigma_H^{inc}(s)}{P_A P_B} \equiv \chi(b, s)d^2 b$$ 

(2)
with the mean number of hard scatters occurring in the element \(d^2b\) of impact parameter space (per hadronic interaction). This defines the ‘eikonal’ function, \(\chi(b, s)\), in terms of \(\sigma_H^{inc}\), the total inclusive cross section for the production of jets (with \(p_T \geq p_{T_{min}}\)) which is given by,

\[
\sigma_H^{inc}(s) = \int_{p_{T_{min}}^2}^{s/4} dp_T^2 \int_1^{1/4p_T^2/s} dx_A \int_1^{1/4p_T^2/s} dx_B \sum_{ij} f_{i/A}(x_A) f_{j/B}(x_B) \frac{d\hat{\sigma}_{ij}(x_Ax_Bs, p_T)}{dp_T^2},
\]

where the parton distribution functions, \(f_{i/A}(x_A)\), are related to the number densities, \(n_{i/A}(x_A)\), by \(f_{i/A}(x_A) = P_A n_{i/A}(x_A)\). This inclusive cross section includes the mean multiplicity of hard scatters per event, i.e. \(\sigma_H^{inc}(s) = \langle n_H \rangle \sigma_H(s)\). This ensures that \(\sigma_H(s)\) cannot be larger than the total cross section, whereas, in principle, \(\sigma_H^{inc}(s)\) can be. Assuming the successive scatters to be uncorrelated, Poisson statistics then give the probability for \(m\) (and only \(m\)) jet pairs to be produced:

\[
p_m = \frac{[\chi(b, s)]^m}{m!} \exp[-\chi(b, s)].
\]

The HERWIG Monte Carlo generates the required number of hard scatters and the associated initial and final state parton showering. The outgoing partons and remnant jets are then fragmented to the hadronic final state. Two modifications are made to the simple eikonal model in the implementation. Firstly, energy conservation is imposed, i.e. after the backward evolution of all the hard scatters in an event, the energy remaining in the hadronic remnants must be greater than zero. Secondly, if during the backward evolution of the first scatter, the splitting \(q\bar{q} \leftarrow \gamma\) is arrived at before the evolution cut off scale, the event is classified as an ‘anomalous’ event and no multiple interactions are allowed (this is a conservative approach).

2. Multiple Interactions at LEPII

The CM energy at LEPII is sufficiently high that the model predicts a significant proportion of multiple scatter events. This can be seen in table 1, where we show the expected mean number of hard scatters per event at a variety of colliders. In each case, we defined a hard scatter to have \(p_T \geq p_{T_{min}}\) and used the MRS \(D_\perp\) proton and GRV \(H\) photon parton densities. Note that we have generated only ‘twice resolved’ \(\gamma\gamma\) events. For these events, since the cross section falls rapidly with increasing \(p_T\), we expect that most of the multiple interactions will have \(p_T \sim p_{T_{min}}\) and so their main effect will be to increase the mean number of observed jets by boosting the underlying \(E_T\) in the event. Events which contain multiple interactions with high enough \(p_T\) to be observed as jets in their own right are relatively rare, but their observation (they appear as pairs of back-to-back jets) would provide striking evidence in support of the existence of multiple interactions.
Table 1: The mean number of hard interactions per event.

|     | $P_{T_{\text{min}}}$ | $\sqrt{s}$ | Mean no. scatters |
|-----|-----------------------|------------|------------------|
| LEP | 2.0                   | 180        | 1.123            |
| LEP | 2.0                   | 90         | 1.084            |
| TRISTAN | 2.0          | 60         | 1.080            |
| HERA | 3.0                   | 296        | 1.04             |

Jet finding was performed on the hadronic final state using a cone algorithm with cone radius $R = 1$. Jets have $E_T \geq 3$ GeV and pseudorapidity $-3 \leq \eta^{\text{jet}} \leq 3$. The inclusive jet cross section, as shown in Fig. 2a, is only affected by the jet boosting, i.e. for a $p_{T_{\text{min}}} = 2$ GeV, scatters from 2-3 GeV are boosted over the $E_T^{\text{jet}}$ cut. The effect of multiple interactions is most significant in higher jet multiplicity rates, as can be seen in Fig. 2b and c, where the dijet cross section is shown.

A variable found to be useful at HERA for distinguishing between direct and resolved photon events is $x^{\text{obs}}_{\gamma}$. It is defined by

$$x^{\text{obs}}_{\gamma} = \frac{\sum_{\text{jets}} E_T^{\text{jet}} e^{-\eta^{\text{jet}}}}{2E_{\gamma}}$$

and the sum is over the two highest $E_T$ jets in the event. In leading order, this is the fraction of the photon energy which enters the hard scatter, i.e. direct events are peaked at $x^{\text{obs}}_{\gamma} = 1$ whilst resolved events have $x^{\text{obs}}_{\gamma} < 1$. The same variable can be used in $\gamma\gamma$ collisions where twice resolved events populate the lower $x^{\text{obs}}_{\gamma}$ region. In Fig. 2d, the cross section is plotted as a function of $x^{\text{obs}}_{\gamma}$. It can be seen that the effect of multiple scattering is greater in the low $x^{\text{obs}}_{\gamma}$ region. Multiple scatterings are more likely to occur here as it is in this region that the higher parton densities occur; also the energy conservation constraint is less restrictive. We have not included once resolved and direct $\gamma\gamma$ events which will populate the high $x^{\text{obs}}_{\gamma}$ region and should be relatively unaffected by multiple interaction.

3. Multiple Interactions at HERA

For HERA, the minimum transverse momentum of a hard scatter is set to $p_{T_{\text{min}}} = 3$ GeV. A cut on the $\gamma p$ CM energy was made, i.e. $114$ GeV $\leq \sqrt{s} \leq 265$ GeV, similar to those usually made by the experiments. For these choices the mean number of hard scatters per event was found to be 1.04.

Jet finding was again performed with cone radius $R = 1$. Jets have $E_T \geq 6$ GeV and pseudorapidity $-2 \leq \eta^{\text{jet}} \leq 2$. The additional $E_T$ can be seen directly in the jet profile, Fig. 3a, where the $E_T$ in the jets is plotted against $\eta$ relative to the jet axis. The pedestal energy is increased with the inclusion of multiple interactions. The $x^{\text{obs}}_{\gamma}$ cut isolates the resolved interactions, and as expected multiple interactions have no effect
in the direct case, Fig.4b. The extra $E_T$ enhances the inclusive jet rate around $\eta^{jet} = 1$, and an increased sensitivity to multiple scattering can be seen in the $\eta^{jet}$ and $E_T^{jet}$ distributions in dijet events, which are shown in ref.4. An enhancement at high $\eta^{jet}$ and low $E_T^{jet}$ is seen. The $x^{obs}_\gamma$ distribution, Fig.5c, has the direct contribution generated using HERWIG included, hence the rise at $x^{obs}_\gamma \sim 1$. The inclusion of multiple scattering has a significant effect on the lower $x^{obs}_\gamma$ region, as in the $\gamma\gamma$ case.

4. Conclusions

The effect on the hadronically final state of multiple parton scattering in $\gamma\gamma$ and $\gamma p$ interactions has been simulated by interfacing an eikonal model of multiple parton interactions with HERWIG. Multiple scattering must occur at some CM energy, in
Figure 3: The $E_T$ jet profile in $\eta$, for a) $x_{\gamma}^{\text{obs}} < 0.75$ and b) $x_{\gamma}^{\text{obs}} \geq 0.75$. c) $x_{\gamma}^{\text{obs}}$ distribution for dijets, at HERA energies, with direct contribution. Including multiple scattering (solid line) and with multiple scattering turned off (broken line).

order that unitarity is not violated. Our model indicates that the effect of multiple scattering is significant at both HERA and LEPII energies. For reasonable experimental cuts, the inclusion of multiple scattering leads to significant changes in inclusive and dijet cross sections which should be understood before attempting to unfold to parton distribution functions.

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