The Study on Structural and Photoelectric Properties of Zincblende InGaN via First Principles Calculation

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Abstract: In this paper, the structure and photoelectric characteristics of zincblende InxGa1−xN alloys are systematically calculated and analyzed based on the density functional theory, including the lattice constant, band structure, distribution of electronic states, dielectric function, and absorption coefficient. The calculation results show that with the increase in x, the lattice constants and the supercell volume increase, whereas the bandgap tends to decrease, and InxGa1−xN alloys are direct band gap semiconductor materials. In addition, the imaginary part of the dielectric function and the absorption coefficient are found to redshift with the increase in indium composition, expanding the absorption range of visible light. By analyzing the lattice constants, polarization characteristics, and photoelectric properties of the InxGa1−xN systems, it is observed that zincblende InxGa1−xN can be used as an alternative material to replace the channel layer of wurtzite InxGa1−xN heterojunction high electron mobility transistor (HEMT) devices to achieve the manufacture of HEMT devices with higher power and higher frequency. In addition, it also provides a theoretical reference for the practical application of InxGa1−xN systems in optoelectronic devices.

Keywords: first principles; zincblende InxGa1−xN alloy; electric characteristics; optical characteristic

1. Introduction

GaN, as a representative of third-generation semiconductors with a wide bandgap, has numerous advantages, such as direct bandgap, high-temperature resistance, and easy formation of heterostructures. It has significant military and commercial value for meeting the working requirements of high-power, high-frequency, and anti-radiation semiconductor devices [1]. To further improve the performance of GaN and broaden its applications, an increasing amount of research has been published on the optoelectronic properties of doped GaN structures in experiments and theoretical calculations in recent years [2–5]. Studies have indicated a shift in focus towards InxGa1−xN alloys because they can be used as a candidate material for optoelectronic devices. This was illustrated by Lu, who prepared an ultraviolet detector that showed a good response in the ultraviolet 360–390 nm region, with a peak response rate of 0.15 A/W [6]. Furthermore, the band gap of InxGa1−xN alloys can continuously change.
from 0.7 to 3.4 eV with the change of x, which is almost perfectly matched to the solar spectrum. Taking this into account, In\(_x\)Ga\(_{1-x}\)N alloys have now begun to attract widespread attention as a new type of solar cell material [7–12].

For In\(_x\)Ga\(_{1-x}\)N, the wurtzite structure is more stable than the zincblende structure. Furthermore, it is difficult to achieve high-quality growth of zincblende structure. Thus, current research mainly focuses on the wurtzite structure. However, due to the strong polarization effect of the wurtzite structure, it is difficult to use zincblende In\(_x\)Ga\(_{1-x}\)N to prepare enhanced mode high electron mobility transistor (HEMT) and high-reliability devices compared with the second-generation semiconductor materials, such as III–V compound semiconductors. In addition, the zincblende In\(_x\)Ga\(_{1-x}\)N does not have spontaneous polarization, and its smaller effective mass of electrons at the minimum of the conduction band (CB) is beneficial in enhancing the frequency and power of the device [13–15]. Mullhauer used radio frequency plasma-assisted molecular beam epitaxy to grow zincblende In\(_{0.25}\)Ga\(_{0.75}\)N, of which the band gap was 2.46 eV [16]. Goldhahn studied the refractive index and energy gap of In\(_x\)Ga\(_{1-x}\)N and suggested that the band gap bowing parameter of In\(_x\)Ga\(_{1-x}\)N is different when the x is different [17].

In terms of theoretical research, a growing number of groups have conducted research on the band gap bowing parameter of zincblende In\(_x\)Ga\(_{1-x}\)N [18–24], but few reports have been published that discuss in detail the lattice constant, the change of bandgap, density of state, and optical properties. These physical properties are relevant for In\(_x\)Ga\(_{1-x}\)N alloy-based heterojunction electronic devices and optoelectronic devices. Furthermore, zincblende In\(_x\)Ga\(_{1-x}\)N without the Stark effect helps to improve the luminescence efficiency of optoelectronic devices. Therefore, it is necessary to explore the zincblende In\(_x\)Ga\(_{1-x}\)N.

In this paper, we calculate the basic physical properties of In\(_x\)Ga\(_{1-x}\)N using the first principles, and analyzed the lattice constant, polarization characteristics, the change of bandgap, and optical characteristics. This could provide a theoretical reference for the experimental research of full-spectrum solar cells and HEMT devices with higher frequency and higher power.

2. Method of Calculation

The GaN crystal was confirmed with three stable phases, namely, zincblende, wurtzite, and rock salt structure. The zincblende GaN used in this paper belongs to cubic crystal (F\(\bar{4}3\)M) space group; its lattice constant is \(a = b = c = 0.452\) nm and the crystal angle is \(\alpha = \beta = \gamma = 90^\circ\) [25]. The structure used for calculation is a \(2 \times 1 \times 1\) supercell obtained by extending twice along the a-axis of the zincblende GaN primitive cell, which contains 8 nitrogen and 8 gallium atoms as shown in Figure 1a. Indium atoms are doped into the GaN bulk to replace the positions of gallium atoms to obtain ternary In\(_x\)Ga\(_{1-x}\)N, where x is set as 0, 0.125, 0.25, 0.5, 0.75, and 1. To acquire the lattice mismatch of the close-packed planes of zincblende and wurtzite In\(_x\)Ga\(_{1-x}\)N, we also calculate the lattice constants of wurtzite In\(_x\)Ga\(_{1-x}\)N when \(x = 0, 0.125, 0.25, 0.5, 0.75, \) and 1; the calculated structures are shown in Figure 1b–g.

All calculations in this paper were completed using the Cambridge sequential total energy package (CASTEP) module in material studio 4.0 developed by Accelrys. In this calculation, when both the zincblende and wurtzite In\(_x\)Ga\(_{1-x}\)N alloys were optimized. The Perdew–Burke–Ernzerhof (PBE) was chosen as the exchange-correlation function and the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm was adopted. The optimization parameters of the maximum interaction force between atoms, the convergence criterion of the maximum displacement, the maximum stress acting on each atom, and the self-consistent accuracy were set as: 0.01 eV/Å, 5.0 \times 10^{-5} \text{ nm}, 0.01 \text{ Gpa}, and 5.0 \times 10^{-6} \text{ eV}, respectively. Because the calculation structures of zincblende and wurtzite In\(_x\)Ga\(_{1-x}\)N are different, the setting of their energy cut-off and k-point density are different. For the zincblende In\(_x\)Ga\(_{1-x}\)N, energy cut-off was set as 600 eV and k-point density was 3 \times 6 \times 6. For wurtzite In\(_x\)Ga\(_{1-x}\)N, energy cut-off was set as 600 eV and the k-point densities were set as 4 \times 4 \times 5 for GaN, In\(_{0.125}\)Ga\(_{0.875}\)N, In\(_{0.25}\)Ga\(_{0.75}\)N and InN, and 8 \times 8 \times 2 for In\(_{0.5}\)Ga\(_{0.5}\)N and In\(_{0.75}\)Ga\(_{0.25}\)N.
Figure 1. Supercell structure of In$_x$Ga$_{1-x}$N: (a) zincblende GaN structure of $2 \times 1 \times 1$; (b) GaN structure of $2 \times 2 \times 1$; (c) In$_{0.125}$Ga$_{0.875}$N structure of $2 \times 2 \times 1$; (d) In$_{0.25}$Ga$_{0.75}$N structure of $2 \times 2 \times 1$; (e) In$_{0.5}$Ga$_{0.5}$N structure of $1 \times 1 \times 2$; (f) In$_{0.75}$Ga$_{0.25}$N structure of $1 \times 1 \times 2$; (g) InN structure of $2 \times 2 \times 1$. (From (b) to (g) is wurtzite In$_x$Ga$_{1-x}$N alloys).

3. Results and Discussion

3.1. Lattice Constant

Figure 2 illustrates the optimized lattice constants and supercell volume of the zincblende In$_x$Ga$_{1-x}$N when x is set as 0, 0.125, 0.25, 0.5, 0.75, and 1. The broken line formed by the green triangles in Figure 2 is the supercell volume of In$_x$Ga$_{1-x}$N. It can be observed that the supercell volume of In$_x$Ga$_{1-x}$N increases linearly with the increase in x, which is caused by the indium atomic radius being larger than that of gallium. The broken line formed by the magenta square points shows the lattice constants obtained according to Vegard’s law [26], which can be described via the following equation:

$$a_{\text{In}_x\text{Ga}_{1-x}\text{N}} = xa_{\text{InN}} + (1 + x)a_{\text{GaN}}$$  \hspace{1cm} (1)

where $a_{\text{In}_x\text{Ga}_{1-x}\text{N}}$, $a_{\text{InN}}$, and $a_{\text{GaN}}$ are the lattice constants of In$_x$Ga$_{1-x}$N, InN, and GaN, respectively. The broken line formed by purple dots represents the optimized lattice constant of the zincblende In$_x$Ga$_{1-x}$N systems. The optimized lattice constant of GaN is 0.4548 nm, which has an error of less than 1% from the experimental lattice constant value of GaN of 0.4520 nm [25]. For other In$_x$Ga$_{1-x}$N structures, the errors between the calculated lattice constants of In$_x$Ga$_{1-x}$N and Vegard’s law are less than 2%, which is related to the pseudopotential used in this calculation.

Figure 3 demonstrates the optimized lattice constants of the wurtzite In$_x$Ga$_{1-x}$N. The blue, green, and orange curves are the a, c, and ideal axis $c_0$ ($c_0 = 1.63a$) lattice constants of In$_x$Ga$_{1-x}$N, respectively, and all obviously increase with the increase in indium composition. The calculated lattice constants of wurtzite GaN are $a = 0.323$ nm, $c = 0.525$ nm, and $c/a = 1.626$; the differences between these calculated values and the experimental values are: 1.24%, 1.16%, and 0.062%, respectively [27]. In other indium compositions, the lattice constants that are calculated in this paper are consistent with those of the literature [28–30]. As evident from the illustration, the lattice constant c is different from the ideal lattice constant $c_0$, leading to the spontaneous polarization of the wurtzite In$_x$Ga$_{1-x}$N alloys. Moreover, the spontaneous polarization direction of wurtzite In$_{0.125}$Ga$_{0.875}$N is opposite to that of wurtzite GaN, In$_{0.25}$Ga$_{0.75}$N, In$_{0.75}$Ga$_{0.25}$N, and InN, because when indium composition is 0.5, c is greater than $c_0$, and c is less than $c_0$ for other indium compositions.
wurtzite GaN is calculated and the results are shown in Table 1. As shown, the spontaneous polarization intensity of wurtzite In\(_x\)Ga\(_{1-x}\)N is larger than that of gallium. The broken line formed by the magenta square points shows the lattice constant between wurtzite and zincblende In\(_x\)Ga\(_{1-x}\)N systems. The performance of wurtzite In\(_x\)Ga\(_{1-x}\)N must be considered. Thus, we further calculated the close-packed plane lattice constants of the two structures, as shown in Table 1. The results show a negligible (0.23%) variation in lattice constants under all indium compositions. Hence, theoretically, it can be described as:

\[
P_{SP} = \frac{4}{\sqrt{3}} \left( \frac{1}{4a^2} + \frac{1}{3c^2} \right) (3 \sin \theta - 1)
\]

where \(\theta\) is the angle between the bonds not parallel to \(\mathbf{c}\) in the wurtzite In\(_x\)Ga\(_{1-x}\)N and the plane formed by \(\mathbf{a}_1\) and \(\mathbf{a}_2\). The detailed derivation of spontaneous polarization in In\(_x\)Ga\(_{1-x}\)N is described in the Supporting Information (SI).

According to Equations (2)–(4), the spontaneous polarization intensity \(P_{SP}\) of wurtzite In\(_x\)Ga\(_{1-x}\)N was calculated and the results are shown in Table 1. As shown, the spontaneous polarization intensity of wurtzite GaN is \(-0.030\) C/m\(^2\), which only differs from the value in the literatures by \(-0.001\) C/m\(^2\) [32,33]. The performance of wurtzite In\(_x\)Ga\(_{1-x}\)N heterojunction HEMT devices is limited by the spontaneous polarization in wurtzite In\(_x\)Ga\(_{1-x}\)N. Thus, replacing the wurtzite In\(_x\)Ga\(_{1-x}\)N channel layer with nonpolarized zincblende In\(_x\)Ga\(_{1-x}\)N is a potential choice. However, the difference in lattice constant between wurtzite and zincblende In\(_x\)Ga\(_{1-x}\)N must be considered.
the wurtzite \( \text{In}_x\text{Ga}_{1-x}\text{N} \) channel layer can be replaced with zincblende \( \text{In}_x\text{Ga}_{1-x}\text{N} \) without spontaneous polarization to further improve the performance of HEMT devices.

**Table 1.** Spontaneous polarization intensity of wurtzite \( \text{In}_x\text{Ga}_{1-x}\text{N} \) (\( P_{\text{SP}} \)), close-packed plane lattice constants of wurtzite and zincblende \( \text{In}_x\text{Ga}_{1-x}\text{N} \), and the difference (\( \Delta L \)) in lattice constant of close-packed plane wurtzite and zincblende \( \text{In}_x\text{Ga}_{1-x}\text{N} \) with different indium compositions.

| Indium Compositions | \( P_{\text{SP}} \) / (C/m\(^2\)) | Zincblende (nm) | Wurtzite (nm) | \( \Delta L \) |
|---------------------|---------------------|----------------|--------------|---------|
| 0                   | \(-0.030, -0.029\) [32,33] | 0.3217         | 0.3229       | 0.01%   |
| 0.125               | -0.031              | 0.3262         | 0.3274       | 0.02%   |
| 0.25                | -0.009              | 0.3297         | 0.3309       | 0.02%   |
| 0.5                 | 0.037               | 0.3394         | 0.3401       | 0.00%   |
| 0.75                | -0.011              | 0.3476         | 0.3499       | 0.02%   |
| 1                   | \(-0.029, -0.032\) [32,33] | 0.3573         | 0.3585       | 0.02%   |

### 3.2. Band Structure

#### 3.2.1. Correction of the Energy Gap

Figure 4 demonstrates the bandgap of the zincblende \( \text{In}_x\text{Ga}_{1-x}\text{N} \) alloys with different indium compositions. The green curve in the figure shows the calculated bandgap; the bandgap of GaN is 1.536 eV, which is consistent with the value calculated by Mathieu Cesar et al. using PBE approximation [34]. Note that our calculated band gap of zincblende GaN is different from the 1.69 eV calculated by Poul Georg Moses et al. using PBE approximation [35], this may be due to the structure calculated by Poul Georg Moses et al. is wurtzite GaN structure, while the structure calculated in this paper is zincblende GaN. It is worth mentioning that the calculated band gap of GaN in this paper is smaller than the experimental value of 3.30 eV [12], this is because of the overestimation of the energy of the gallium d state in the calculation, leading to the enhanced interaction between gallium d and nitrogen p orbitals and resulting in broadening of the valence band (VB) [36]. Although this is a common phenomenon in the selection of GGA-PBE exchange-correlation functional calculations, the accurate calculation of the band gap is not important in trend analysis. To be specific, the main topic of this article concerns the same structural system and only changes the incorporation composition of indium [37,38]. Hence, the calculated series of band gaps are still comparable. To make the calculated band gaps of \( \text{In}_x\text{Ga}_{1-x}\text{N} \) alloys closer to the experimental values, the calculated band gaps were corrected based on the experimental values of zincblende GaN and InN [12] using the correction formula as follows [38]:

\[
E_{\text{corr}}^{\text{cal}} = E_{\text{calc}}^{\text{exp}} + \frac{E_{\text{calc}}^{\text{exp}} - E_{\text{calc}}^{\text{exp}}}{} + (1 - \alpha) \left( E_{\text{calc}}^{\text{exp}} - E_{\text{calc}}^{\text{exp}} \right)
\]

where \( E_{\text{calc}}^{\text{exp}} \) is the corrected band gap, \( E_{\text{calc}}^{\text{exp}} \) and \( E_{\text{calc}}^{\text{exp}} \) are the calculated band gaps of \( \text{In}_x\text{Ga}_{1-x}\text{N} \), InN, and GaN in this work, respectively; and \( E_{\text{calc}}^{\text{exp}} \) and \( E_{\text{calc}}^{\text{exp}} \) are the experimental band gaps of InN and GaN, respectively. The corrected results are shown by the curve composed of yellow triangles in Figure 4.

The modified band structures of the \( \text{In}_x\text{Ga}_{1-x}\text{N} \) systems are plotted in Figure 5. The discussion of the energy band diagram is divided into the following points. (1) Type of band gap: According to Figure 5a, we conclude that GaN is a direct band gap semiconductor material; after the indium atoms are doped into GaN, the type of band gap of \( \text{In}_x\text{Ga}_{1-x}\text{N} \) is still a direct gap, and the minimum of the CB and the maximum of the VB are located at the same \( \Gamma \) point in the Brillouin zone (BZ). (2) Degeneracy: Compared with Figure 5a, Figure 5b–f shows an impurity energy level in both the CB and VB, increasing the degeneracy of \( \text{In}_x\text{Ga}_{1-x}\text{N} \). This is mainly due to the contribution of SP\(^3\) hybridization of gallium s/p and indium s/p orbitals. (3) Band gap: It can be clearly seen from Figure 4 that the band gap of \( \text{In}_x\text{Ga}_{1-x}\text{N} \) decrease with the increase in \( x \), which is caused by gallium s/p and indium



\[
\text{Spontaneous polarization intensity of wurtzite } \text{In}_x\text{Ga}_{1-x}\text{N} (P_{\text{SP}}), \text{ close-packed plane lattice constants of wurtzite and zincblende } \text{In}_x\text{Ga}_{1-x}\text{N}, \text{ and the difference (}\Delta L\text{) in lattice constant of close-packed plane wurtzite and zincblende } \text{In}_x\text{Ga}_{1-x}\text{N} \text{ with different indium compositions.}
\]
s/p orbitals in the CB approaching the energy reference point as the indium compositions increase. In addition, it can be seen from Figure 5 that the band gaps of GaN and InN are 3.30 eV and 0.78 eV, respectively; hence, the band gap of In\(_{x}\)Ga\(_{1-x}\)N alloys can continuously vary from 0.78 to 3.30 eV by adjusting the indium compositions. This is almost perfectly matched to the solar energy spectrum, and means that In\(_{x}\)Ga\(_{1-x}\)N can be used to produce photovoltaic devices such as full-spectrum solar cells, by a combination of In\(_{x}\)Ga\(_{1-x}\)N solar cells with various bandgaps.

Figure 4. The bandgap of zincblende In\(_{x}\)Ga\(_{1-x}\)N systems as a function of indium compositions.

Figure 5. Band structure of zincblende In\(_{x}\)Ga\(_{1-x}\)N systems: (a) GaN; (b) In\(_{0.125}\)Ga\(_{0.875}\)N; (c) In\(_{0.25}\)Ga\(_{0.75}\)N; (d) In\(_{0.5}\)Ga\(_{0.5}\)N; (e) In\(_{0.75}\)Ga\(_{0.25}\)N; (f) InN.
3.2.2. Mechanism of Bandgap Reduction and Bandgap Bowing Parameter

From Figures 4 and 5 it can be concluded that the band gap decreases with the increase in indium composition. To explore the cause of this phenomenon, the distribution of electron density difference of the In$_x$Ga$_{1-x}$N systems was calculated, and the results are shown in Figure 6; blue indicates low electron density and red indicates high electron density. It can be seen from Figure 6b that when indium atoms replace the gallium atoms, there is an electron enrichment phenomenon around the indium atoms. This indicates that the indium has a stronger ability to bind electrons, which is due to the electronegativity of indium atoms being greater than that of gallium (according to Pauling’s rule, the electronegativities of gallium and indium are 1.6 and 1.7, respectively). Thus, more covalent bond components of indium–nitrogen and the ionic bond components decrease after indium atoms replace gallium. Furthermore, more covalent bond components of the SP$^3$ hybrid bond of indium–nitrogen results in a smaller bond energy of indium–nitrogen compared with the gallium-nitrogen bond, thus, the band gap decreases with the increase in indium compositions.

![Figure 6](image-url) Distribution of electron density difference on the (111) plane of (a) GaN and (b) In$_x$Ga$_{1-x}$N alloys.

The bandgap bowing parameter is integral in adjusting the energy gap of In$_x$Ga$_{1-x}$N systems [35,39], therefore, here we discuss and analyze the value and origin of the bowing parameter. The relationship between the energy gap of the ternary alloy and the doping compositions can be expressed by the semi-empirical formula [40]:

$$ E_{g,\text{In}_x\text{Ga}_{1-x}N}^{\text{cor}} = xE_{g,\text{InN}}^{\text{exp}} + (1-x)E_{g,\text{GaN}}^{\text{exp}} - bx(1-x) \tag{6} $$

where b is the bandgap bowing parameter. We fitted the modified energy gap of the In$_x$Ga$_{1-x}$N using Equation (3) and obtained $b = 2.1 \pm 0.14$ eV as the average bowing parameter when the indium compositions are 0–1, which is caused by the volume deformation, structural relaxation, and charge exchange after the doping of indium atoms into the GaN system. The band gap bowing parameter obtained in this calculation is slightly different from the result in the literature, which is $b = 1.9 \pm 0.09$ eV [19].

3.3. Density of States

Figure 7 shows the total density of states (TDOS) of zincblende In$_x$Ga$_{1-x}$N and the partial density of states of indium, gallium, and nitrogen. Combining Figure 7b–d, the total density of states of Figure 7a is divided into three parts for discussion, namely: $-10$ to $5$ eV, $-5$ to $0$ eV, and CB. In the range of $-10$ to $5$ eV, its main contribution comes from nitrogen p orbitals and gallium s orbitals. When the indium compositions in the In$_x$Ga$_{1-x}$N systems increase, the electronic states of the gallium s orbitals decrease, while the electronic state of nitrogen p orbitals is almost unchanged, so the TDOS decreases in this range. The density of states in the energy range of $-5$ to $0$ eV is mainly due to the contribution of the nitrogen p orbital and does not change with the increase in doping x. The TDOS in the CB is mainly affected by the SP$^3$ hybridization of gallium s/p orbitals and indium s/p orbitals. Apparently,
with the increase in indium compositions, gallium s/p orbitals and indium s/p orbitals move to lower energy, causing the TDOS of the CB to move to the energy reference point and the In$_x$Ga$_{1-x}$N systems to undergo redshift, which is consistent with the conclusion of the band structure.

**Figure 7.** Density of states of In$_x$Ga$_{1-x}$N alloys: (a) total density of states of In$_x$Ga$_{1-x}$N alloys; (b) density of states of nitrogen s/p orbitals; (c) density of states of gallium s/p orbitals; (d) density of states of indium s/p orbitals.

### 3.4. Optical Properties

Theoretically, the dielectric function can reflect the material’s response to electromagnetic signals and describe other optical parameters of the crystal. In the range of the linear response, the equations of the dielectric function are the following [41]:

\[
\varepsilon(\omega) = \varepsilon_1(\omega) + i\varepsilon_2(\omega) \tag{7}
\]

\[
N(\omega) = n(\omega) + ik(\omega) \tag{8}
\]

\[
\varepsilon_1 = n^2 + k^2 \tag{9}
\]

\[
\varepsilon_2 = 2nk \tag{10}
\]
where \( \varepsilon_1 \) and \( \varepsilon_2 \) are real and imaginary parts of the dielectric function, respectively, and \( n \) and \( k \) are refractive index and extinction coefficient, respectively. In\(_x\)Ga\(_{1-x}\)N alloys are direct bandgap semiconductor materials, therefore, the dielectric function and absorption parameter \( a(\omega) \) can be derived using the definition of the direct transition probability and the Kramers-Kronig relationship [42–44].

\[
\varepsilon_1(\omega) = 1 + \frac{2e}{\varepsilon_0 m^2} \sum_{\mathbf{V}, \mathbf{C}, \mathbf{BZ}} \int \frac{2d\mathbf{K}}{(2\pi)^3} \left| a \cdot M_{\mathbf{V}, \mathbf{C}}(\mathbf{K}) \right|^2 \frac{1}{\left[ E_C(\mathbf{K}) - E_V(\mathbf{K}) \right]^2 / \hbar^2 - a^2} 
\]

\[
\varepsilon_2(\omega) = \frac{\pi}{\varepsilon_0 \lambda_0} \sum_{\mathbf{V}, \mathbf{C}, \mathbf{BZ}} \int \frac{2d\mathbf{K}}{(2\pi)^3} \left| a \cdot M_{\mathbf{V}, \mathbf{C}}(\mathbf{K}) \right|^2 \delta[ E_C(\mathbf{K}) - E_V(\mathbf{K}) - \hbar \omega ] \right) \right) \frac{1}{2} \]  

\[
a(\omega) = \sqrt{2} \omega \left\{ \left[ \varepsilon_2(\omega) - \varepsilon_1(\omega) \right]^2 - \varepsilon_1(\omega) \right\} \right) \frac{1}{2} \]

where \( \varepsilon_2 \) and \( \lambda_0 \) are the dielectric constant and wavelength in vacuum, respectively, footnotes C and V represent the CB and VB, BZ is the Brillouin zone, \( \mathbf{K} \) is the electronic wave vector, \( E_C(\mathbf{K}) \) and \( E_V(\mathbf{K}) \) are the intrinsic energy level of CB and VB, respectively, \( \hbar \) is the Planck constant, \( \mathbf{a} \) is the unit vector of the vector potential \( \mathbf{A} \), and \( M_{\mathbf{V}, \mathbf{C}} \) is transition matrix element.

We calculated the dielectric function of zincblende In\(_x\)Ga\(_{1-x}\)N alloys when the indium compositions were changed to 0, 0.125, 0.25, 0.5, 0.75, and 1. The calculated results of the imaginary part \( \varepsilon_2 \) of the dielectric function are shown in Figure 8. It is evident from the figure that the \( \varepsilon_2 \) curve of GaN has three peaks, namely C1, C2, and C3, which are located near 8.4, 11, and 13 eV, respectively. Absorption peaks C2 and C3 are primarily caused by the transition of electrons in gallium s/p states to the unoccupied states. The absorption peak C1 reaches the maximum value, which is caused by the direct transition. It can be observed in the graph that the \( \varepsilon_2 \) of In\(_x\)Ga\(_{1-x}\)N alloys shift to low energy with the increase in indium atoms. This indicates that the electrons in the In\(_x\)Ga\(_{1-x}\)N can undergo transitions even if they absorb photons with lower energy. Additionally, there is an absorption peak near 4 eV energy, which increases and moves to the lower energy direction with the increase in indium compositions. This may be due to the transition of indium s/p state electrons to the unoccupied state.

![Graph showing the imaginary part \( \varepsilon_2 \) of the dielectric function of the In\(_x\)Ga\(_{1-x}\)N alloys.](https://via.placeholder.com/150)

Figure 9 shows the absorption coefficient of In\(_x\)Ga\(_{1-x}\)N alloys. There are four peaks, \( D_1 \), \( D_2 \), \( D_3 \), and \( D_4 \), in the absorption spectrum of GaN. Absorption peaks \( D_1 \), \( D_2 \), and \( D_3 \) are located near 8, 11, and 13 eV, respectively. This is mainly due to the contribution of gallium s/p orbital electronic states. Therefore, the values of the three absorption peaks all decrease when the indium compositions increase. Another absorption peak \( D_4 \) is located near 26 eV, whose main contribution comes from the gallium s orbital. However, when an indium atom is added, the influence of indium s/p orbitals is greater than
that of the gallium s orbital. Hence, with the increase in indium compositions, the peak value increases. The absorption coefficient curve of In$_x$Ga$_{1-x}$N alloys shifts in the low energy direction as $x$ increases. This indicates that the doping of indium atoms improves the absorption of zincblende GaN for visible light, which is consistent with the result of the imaginary part of the dielectric function.

![Figure 9](image.png)

**Figure 9.** The absorption coefficient of zincblende In$_x$Ga$_{1-x}$N alloys.

4. Conclusions

In this paper, the geometric parameters were optimized and the electronic structure and optical properties of zincblende In$_x$Ga$_{1-x}$N ($x = 0, 0.125, 0.25, 0.5, 0.75, 1$) were calculated based on the density functional theory. The calculated results demonstrate that, after doping indium atoms, the nature of the direct bandgap of zincblende In$_x$Ga$_{1-x}$N does not alter and the indium s/p electronic states are introduced near the energy reference point so that the bandgap decreases with the increase in $x$. By fitting the corrected bandgap, the average energy gap bowing parameter is obtained as $b = 2.10$ eV. Furthermore, as the indium compositions increases, the indium s/p and gallium s/p orbitals of the CB move to the energy reference point, resulting in a narrowing of the energy window in which electronic states cannot exist. In addition, the imaginary part of the dielectric function and the absorption coefficient of the In$_x$Ga$_{1-x}$N structures shifts to low energy with the increase in $x$. This enhances the absorption of visible light and provides a theoretical reference for the application of In$_x$Ga$_{1-x}$N alloys in the field of photovoltaic devices such as solar cells.

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