Rogue wave management in an inhomogeneous Nonlinear Fibre with higher order effects

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We consider an inhomogeneous Hirota equation with variable dispersion and nonlinearity. We introduce a novel transformation which maps this equation to a constant coefficient Hirota equation. By employing this transformation we construct the rogue wave solution of the inhomogeneous Hirota equation. Furthermore, we demonstrate that one can control the rogue wave dynamics by suitably choosing the dispersion and the nonlinearity. These results suggest an efficient approach for controlling the basic features of the relevant rogue wave and may have practical implications for the management of the rogue waves in nonlinear optical systems.

Keywords: Rogue wave; Soliton; inhomogeneous Nonlinear Fibre; Hirota equation.

2010 Mathematics Subject Classifications: 35Q55, 35Q60, 37K40

1. Introduction

Communication technology in the 21st century has reached a remarkably high level. This is mainly due to the achievement of astonishing bit rate transmissions by utilizing different effects of non-linearity [2]. From 1974 onwards, several developments in the field of nonlinear fibre optics in the context of propagation of optical solitons have helped scientists to attain increasingly higher bit-rates. Soliton-type non-dispersive pulse propagation through optical fibres has been realized by balancing the group velocity dispersion, which broadens the pulse, and the self-phase modulation, which contracts the pulse, thereby obtaining a pulse whose profile is similar with the initial profile. The ability of conducting both laboratory and field experiments has also increased the interest in the potential applications of optical solitons, including bright solitons, dark solitons and dispersion
managed solitons. These soliton bits could play an important role in achieving high-speed and low-loss communication, which has been confirmed through several field trials in different Labs [1, 35]. The nonlinear Schrödinger (NLS) equation describes the idealized soliton transmission in a nonlinear dispersive optical fibre. Several experimental studies, mainly in the femtosecond regime, have shown that in addition to the linear dispersion and the Kerr effect, there are several other important higher order effects should be included in the model. These effects, including third order dispersion, stimulated Raman effect and Kerr dispersion, influence the soliton propagation in the femtosecond regime. Taking these effects into consideration, Kodama and Hasegawa derived in [28, 29] a higher order NLS equation which is integrable [58] only for certain choices of parameters. For example, considering the femtosecond pulse propagation through an inhomogeneous fibre, with suitable choice of the linear and nonlinear coefficients, one obtains (see [58]) an inhomogeneous Hirota equation of the form

\[ q_z = \alpha_1(z)(iq_{tt} + \frac{1}{3}q_{ttt}) + \alpha_4(z)(i\delta q|q|^2 + |q|^2q_t) + \alpha_6(z)q, \quad (1.1) \]

where \( \alpha_6 = \frac{\alpha_1\alpha_4 - \alpha_1\alpha_4}{2\alpha_1\alpha_4} \), \( q(z,t) \) is the complex field envelope, \( z \) denotes the normalized propagation distance and \( t \) denotes the normalized retarded time. Here \( \alpha_1(z) \) and \( \alpha_4(z) \) denote the contribution of the dispersion and nonlinearity respectively and \( \alpha_6 \) denotes the amplification (or absorption) coefficient.

In real systems, the refractive index and/or the core diameter of the optical fibre are functions of the axial coordinate, which means that the fibre is axially inhomogeneous, and hence the governing equation is also inhomogeneous. Under these circumstances, the corresponding nonlinear evolution equations are called variable coefficient soliton equations [15, 23, 30, 32, 38, 41, 43]. The solutions of these equations contain several arbitrary functions and hence soliton management can be realized by a suitable choice of these different control functions [23].

An interesting question is to analyze how the behaviour of solitons is affected by these arbitrary functions and also to find out whether the associated variable coefficient soliton equations are integrable or not. Soliton management, including dispersion management and nonlinearity management, provides a useful approach for reducing several detrimental effects, so that optical soliton with high speed and stability can be transmitted in practical all-optical networks [21, 34]. Soliton energy control, soliton tunnelling, and pulse compression, are some of the new important developments in the area of variable coefficient soliton equations. It has been found that inhomogeneity in the nonlinearity can split the incoming fundamental soliton into a symmetric pair of separate small amplitude solitons. We recall that inhomogeneous nonlinear equations have also been widely used in Bose-Einstein condensates, in plasma physics, in hydrodynamics and so on [1].

Considering several recent developments in soliton theory, the study of modulational instability has also been widely used to explain why experiments involving white coherent light supercontinuum generation (SCG), admit a triangular spectrum. Such universal triangular spectra can be well-described by the analytical expressions for the spectra of Akhmediev breather solutions at the point of extreme compression. In the context of the NLS equation, Peregrine already in [37] had identified the role of modulational instability in the formation of patterns resembling freak waves or rogue waves; these theoretical results were later supported by several experiments. Rogue waves in ocean are localized large amplitude waves on a rough background, which have two remarkable characteristics: a) “appear from nowhere and disappear without a trace” [3], b) exhibit one dominant peak.
Rogue waves have recently appeared in several areas of science. In particular, in photonic crystal fibre rogue waves have been well established in connection with SCG [45]. This actually has stimulated research for rogue waves in other physical systems and has paved the way for many important applications, including the control of rogue waves by means of SCG [17, 46], as well as studies in superfluid Helium [18], in Bose-Einstein condensates [9], in plasmas [36,40], in microwave [26], in capillary phenomena [44], in telecommunication data streams [50], in inhomogeneous media [7], in water experiments [13, 14], and so on. More recently, Kibler et al. [27] using their elegant experimental apparatus in optical fibres were able to generate femtosecond pulses with strong temporal and spatial localization and near-ideal temporal Peregrine soliton characteristics.

In addition to the NLS equation, for the past few years, several nonlinear evolution equations in different branches of physics have been shown to admit rogue waves. For example, the Hirota equation [5, 33, 49], the first type derivative NLS equation [55], the third type Gerdjikov-Ivanov equation [56], the NLS-Maxwell-Bloch equations [24], the non-integrable and integrable discrete NLS equations [6, 10], the two-component NLS equations [8, 11, 20], the variable coefficient NLS [4, 48, 52, 53, 60], the variable coefficient derivative NLS [57], and the variable coefficient higher order NLS [16] are few of the equations which admit Rogue waves. From the above studies, it is clear that one of the possible generating mechanisms [25] for rogue waves is the creation of degenerate breather solutions associated with a special eigenvalue of the underlying equation.

In this paper, we present an explicit formulation of rogue wave management with higher order effects, which possesses two arbitrary functions, so that both dispersion management and nonlinear management are possible. In particular, the associated self-similar evolution implies that the wave profile remains invariant, whereas its amplitude and width scale with the modulation of the system parameters. We also derive different backgrounds and profiles of the relevant rogue wave, by adjusting the dispersion and nonlinear coefficients so that required properties of rogue waves can be achieved.

2. The General Method

First, we shall study the possibility of nonlinearity and dispersion management of the relevant rogue waves. Using similar techniques with those used in [23, 57], we have constructed a transformation which maps the inhomogeneous Hirota equation to the following Hirota equation [5, 49]

\[ iu_Z + \alpha (2|u|^2 u + u_{TT}) + i\beta (u_{TTT} + 6|u|^2 u_T) = 0. \]  

(2.1)

Specifically, let \( Z = Z(z), T = T(z,t) \) where

\[ Z = -\frac{\sqrt{2\delta}}{12\beta} \int \alpha_1 dz, \]  

(2.2)

\[ T = \frac{\sqrt{2\delta}}{2} t - \frac{\sqrt{2\delta}(\alpha^2 - 18\delta \beta^2)}{36\beta^2} \int \alpha_1 dz. \]  

(2.3)

Define \( q \) by

\[ q = f(z)u(Z,T)e^{ig(z,t)}, \]  

(2.4)
another solution of the inhomogeneous Hirota equation by choosing suitable forms for it is straightforward to construct a solution of the inhomogeneous Hirota equation. For example, where arbitrary functions \( q \) and \( s \) are two monotonic functions which are unbounded as \((t,z)\) approaches infinity. In our recent work, we have systematically constructed the breather and rogue wave solutions of the Hirota equation [49]. For instance, the first order rogue wave solution of the Hirota equation in two variables \((x,t)\) is obtained in the form (see eq. (29) in [49])

\[
 q^{[1]}_{\text{rougwave}} = e^{ibt} \sqrt{\frac{b}{2\alpha_1}} \frac{(-2b\alpha^2\xi^2 + 12b^2\alpha\beta xt - 18b^3\beta^2 z^2 - 4b^2\alpha^3 t^2 + 8ib\alpha^3 t + 3\alpha^3)}{4b^2\alpha^2 t^2 + 2b\alpha^2 z^2 - 12b^2\alpha\beta xt + 18b^3\beta^2 t^2 + \alpha^3} e^{i(g(x,t) + bZ)},
\]

where \( Z, T \) and \( g(x,t) \) are given by (2.2), (2.3) and (2.5) respectively. Similarly, if \( \alpha = 0 \), we get another solution of the inhomogeneous Hirota equation

\[
 q = -\sqrt{\frac{(b + 8\beta \xi^3)\alpha_1}{12\beta \xi \alpha_4}} \left( 1 + \frac{ih_1 \int \alpha_1 dz - 4h_2}{h_1 + h_2} \right) e^{i\phi}.
\]

Here

\[
 h_1 = s_1 t^2 + s_2(\int \alpha_1 dz)t + s_3(\int \alpha_1 dz)^2, \quad h_2 = 364\beta^3 \xi^3,
\]

\[
 s_1 = 576\beta^2 \xi^3 b\sqrt{2\delta} + 4608\beta^3 \xi^6 \sqrt{2\delta}, \quad s_2 = 1152\beta^3 \xi^5 \delta + 144\beta^2 \xi^2 \delta b,
\]

\[
 s_3 = -3072\beta^3 \xi^7 \delta + 2304\beta^3 \xi^5 \delta^2 + 288\beta^2 \xi^2 \delta^2 b + 24b\xi \delta b^2 - 192\beta^2 \xi^4 \delta b - 2592\beta^3 \xi^3,
\]

\[
 \phi = - (\delta + \sqrt{2\delta} \xi^2 t) - \frac{1}{12\beta} (8\beta \xi^2 + b\sqrt{2\delta} + 12\beta \xi \delta \sqrt{2\delta}) \int \alpha_1 dz.
\]

Eq.(2.7) and Eq.(2.8) involve the four arbitrary positive constants \( \alpha(\xi), \beta, b, \delta \), and the two arbitrary functions \( \alpha_1 \) and \( \alpha_4 \). Eq.(2.7) and Eq.(2.8) provide two explicit expressions which can be used to control the dispersion management and nonlinear management effects. The validity of \( q \) and \( \tilde{q} \) has been verified by a simple symbolic computation with a computer.
3. The Dispersion Management and Nonlinear Management of the Rogue Wave

We next investigate the possibility of controlling the background of $q$. If $Z$ and $T$ tend monotonically to infinity, then $q$ satisfies asymptotically $|q|^2 = |u|^2 f^2 = \frac{b^2 f}{2}$. The dispersion management and nonlinear management profiles drastically affect the background and the structure of $q$ by means of $f$.

In the following three cases (Case 1, 2 and 3), $Z, T$ and $f$, satisfy the conditions for the generation of a rogue wave and hence the with the background of $b/(2\alpha_f)^2 = 0.3 f^2$.

- **Case 1:** Let $\alpha_1 = 1 + \varepsilon_1 z^2$ and $\alpha_4 = 1 + \varepsilon_2 z^2$, $\varepsilon_1$ and $\varepsilon_2$ are real constants. The profiles of the rogue wave solution $|q|^2$ are plotted in Figure 1. Fig.1(a) represents a single-dent background with $\varepsilon_1 = 1$ and $\varepsilon_2 = 0.1$ at height 3 because $f^2$ tends to 10. While Fig.1(b) represents a single-lump background with $\varepsilon_1 = 1$ and $\varepsilon_2 = 0.1$ at height 0.03 because $f^2$ tends to 0.1.

- **Case 2:** [41, 58]: Now choose $\alpha_1 = d_2 [1 + \varepsilon_1 \sin(\sigma_1 z)]$ and $\alpha_4 = \frac{d}{2} [1 + \varepsilon_2 \sin(\sigma_2 z)]$, $\varepsilon_1 < 1$ and $\varepsilon_2 < 1$. The profiles of rogue wave $|q|^2$ are plotted in Fig.2, which are rogue waves for $|z| \gg 0$ between $[0.18, 0.5]$ with a periodic background $\frac{0.3(1+0.25\sin(2z))}{1+0.25\sin(0.25z)}$ for Fig.2(a) and a periodic background $\frac{0.3(1+0.25\sin(3z))}{1+0.25\sin(0.25z)}$ for Fig.2(b).

- **Case 3:** Let $\alpha_1 = 1 + \tanh(\sigma_1 z)\text{sech}(\sigma_2 z)$ and $\alpha_2 = 1$, where $\sigma_1$ and $\sigma_2$ are real constants; the rogue wave $|q|^2$ profiles are displayed in Figure 3. Fig.3(a) is a rogue wave on a dent-lump background with the height of $q$ tending to 0.3 by setting $\sigma_1 = \sigma_2 = 1$. Fig.3(b) is a rogue wave on a lump-dent background when $\sigma_1 = \sigma_2 = -1$.

In cases 4-6 below we show that it is possible to use dispersion management and nonlinear management in order to control the rogue wave profiles $q$ and to prevent the appearance of a rogue wave in inhomogeneous fibre.
Fig. 2. (Color online) The RW $q$ on a periodic background with $\alpha_1 = d \left[ 1 + \varepsilon_1 \sin(\sigma_1 z) \right], \alpha_4 = \frac{r}{\delta} \left[ 1 + \varepsilon_2 \sin(\sigma_2 z) \right]$ and $\alpha = 5, \beta = 1, b = 3, \delta = 1, d = 1, r = 1, \varepsilon_1 = \varepsilon_2 = 0.25$. (a) Lower frequency with $\sigma_1 = 2$ and $\sigma_2 = 0.25$; (b) Higher frequency with $\sigma_1 = 3$ and $\sigma_2 = 0.25$.

Fig. 3. (Color online) The RW $q$ with $\alpha_1 = 1 + \tanh(\sigma_1 z) \text{sech}(\sigma_2 z), \alpha_4 = 1$ and $\alpha = 5, \beta = 1, b = 3, \delta = 1$. (a) A dent-lump background with $\sigma_1 = \sigma_2 = 1$; (b) A lump-dent background with $\sigma_1 = \sigma_2 = -1$.

- Case 4: [22]: Let $\alpha_1 = \exp(\sigma_1 z)$ and $\alpha_4 = \exp(\sigma_2 z)$; the corresponding profiles of $|q|^2$ are plotted in Figure 4 with $\sigma_1$ and $\sigma_2$ real constants. In Fig.4(a), $T = \frac{\sqrt{3}}{2} t - \frac{7}{36} \sqrt{2} e^z, Z = -\frac{1}{12} e^z, f = 1$, so $|q|^2 = |u|^2$ is local for any $t$ only for $z > 0$ (recall that $u$ admits one dominant lump along the line $t = 0$ for $z < 0$). We call this solution a half W-shape soliton [31]. The height of the lump is found to be 2.7 and the height of the plane is 0.3. In Fig.4b, $T = \frac{\sqrt{3}}{2} t - \frac{175}{177} \sqrt{2} e^{0.13z}, Z = -\frac{41}{17} \sqrt{2} e^{0.13z}, f = e^{0.015z}$, which resembles like L-shape rogue
wave on an exponentially growing background. The function $|q|^2 = f^2 |u|^2$ is local for any $t$ for $z < 0$, and the background is $0.3e^{0.03z}$.

- Case 5: \([51]\): Let $\alpha_1 = \exp(\sigma_1 z), \alpha_4 = \exp(\sigma_2 z)$ and $\alpha = 5, \beta = 1, b = 3, \delta = 1$. The corresponding profiles of $|q|^2$ are shown in Figure 5. For Fig.5(a), $T = \frac{\sqrt{2}}{2} t - \frac{\sqrt{2}}{18} e^z - \frac{2\sqrt{2}}{30} (\frac{1}{2} e^z \cos(z) + \frac{1}{2} e^z \sin(z))$, $Z = -\frac{\sqrt{2}}{6} e^z \frac{\sqrt{2}}{12} (\frac{1}{2} e^z \cos(z) + \frac{1}{2} e^z \sin(z))$, then $|u(Z, T)|^2$ is local for any $t$ only

Fig. 4. (Color online) Solution $q$ with $\alpha_1 = \exp(\sigma_1 z), \alpha_4 = \exp(\sigma_2 z)$ and $\alpha = 5, \beta = 1, b = 3, \delta = 1$. (a) Half-space W-shape soliton with $\sigma_1 = 1$ and $\sigma_2 = 1$; (b) The L-shape rogue wave on an exponential growing background with $\sigma_1 = 0.13$ and $\sigma_2 = 0.1$.

Fig. 5. (Color online) Quasi space-breather $q$ on a periodic background with $\alpha_1 = (2 + \cos(\sigma_1 z)) \exp(\sigma_2 z)$ and $\alpha_4 = \exp(\sigma_2 z); \alpha = 5, \beta = 1, b = 3, \delta = 1, \sigma_1 = 1$. (a) Higher frequency with $\sigma_1 = 1$; (b) Lower frequency with $\sigma_1 = 0.75$. 

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413
for $z > 0$, which implies $|q|^2$ has a quasi-space-breather along $t = 0$. This quasi space-breather is not exactly a periodic function of $z$ because it involves $e^z$, but it asymptotes to the space-breather $\frac{2^7}{3^7} + \frac{2^7}{10^7}\cos(z)$ for $z \ll 0$. The background is given by $\frac{3}{3} + \frac{3}{10}\cos(z)$ for $|z| \gg 0$. There exist asymptotic synchronization of the quasi-breather with the background; the height of the former is found to be nine times of the latter. This is one of the novel nonlinear results of this paper, which is possible only in the case of inhomogeneous nonlinear fibre media. A visual difference between Fig5.(a) and Fig5(b) is the different frequency of the quasi-breather because $\sigma_1 = 1$ and $\sigma_1 = 0.75$, respectively.

- Case 6: Let $\alpha_1 = z$ and $\alpha_4 = \frac{1}{2}z$; the corresponding profiles of $|q|^2$ are depicted in Figure 6. The ridges of $|q|^2$ are adjustable by choosing different values of the parameter $\alpha$. Equivalently, this can also be achieved by changing the parameters $\beta$ or $b$. These deformable properties, clearly show that it is possible to control the rogue wave in an inhomogeneous system. Note that for these three cases, $|q|^2$ is local in both $z$ and $t$, and $q$ satisfies the two basic characteristics of a rogue wave. By plotting three dimensional profiles and by plotting the contour line at height $b/(2\alpha)$ (the asymptotic plane) one clearly sees that $q$ is a rogue wave. For $\alpha = 0$, the profile of the rogue wave can also be controlled for the nonlinear effects associated with the parameters $\alpha_1$ and $\alpha_4$. We present below three additional examples with different profiles.

- Case 7: Let $\alpha_1 = z$ and $\alpha_4 = \frac{1}{2}z$; the corresponding profile of $|q|^2$ is depicted in Figure 7.

- Case 8: Let $\alpha_1 = d_1 [1 + \cos(\sigma_1 z)]$, $\alpha_4 = d_2 [1 + \cos(\sigma_2 z)]$; the corresponding profile $|\tilde{q}|^2$ is plotted in Fig.8. There exist a twist of the profile along the t-direction of the rogue wave; however the rogue wave is still well localization.

- Case 9: Let $\alpha_1 = d_1 [1 + \sin(\sigma_1 z)]$, $\alpha_4 = d_2 [1 + \sin(\sigma_2 z)]$; the corresponding profile $|\tilde{q}|^2$ is plotted in Fig.9. The explicit expression of $\tilde{q}$, implies that the profile is localized in both the $z$ and $t$ directions, but the decay along the $z$ direction is very slow. This gives rise to an approximate W-shape soliton [31].

![Fig. 6. (Color online)Rogue wave $q$ with $\alpha_1 = z$, $\alpha_4 = \frac{1}{2}z$ and $\beta = 1$, $b = 5$, $\delta = 1$. (Left panel) $\alpha = 1$, (Middle panel) $\alpha = 5$, (Right panel) $\alpha = 9$.](image)
Cases 1-3 illustrate the possible detrimental effects of rogue waves. These effects can be attributed to the associated self-similar structure of the soliton so that even when the wave profile remains invariant, its amplitude and width grow with the modulation parameters of the system. Several backgrounds and profiles responsible for the generation of rogue waves are discussed in detail in cases 1-3. Cases 4-6 illustrate that these detrimental effects can be effectively controlled by resorting to a special class of soliton dispersion management, known as W-shaped soliton dispersion management. Soliton dispersion management has been used effectively in a variety of situations [19, 39, 42, 51, 54, 58, 59]. Also, recently Sysoliatin et al. have experimentally realized the
Fig. 9. (Color online) The approximate W-shape soliton $\tilde{q}$ with $b = 0.1, \sigma_1 = 1, \beta = 0.1, \delta = 0.1, \xi = 0.1, d_1 = 0.01, d_2 = 0.01, \sigma_2 = 1$ (left panel). The right panel for the one dimensional profiles at $z=-150$ (yellow, left), $z=0$ (green, middle), $z=150$ (red, right), which shows the slight decay of height at different positions.

concept of soliton dispersion management in a fiber with a sine-wave variation of the core diameter along the longitudinal direction of propagation [12, 47].

4. conclusion

We have presented an analytical rogue wave solution of the inhomogeneous Hirota equation with variable dispersion and nonlinearity profiles. The pulse width, shape, height and direction of this rogue wave, are variable and depend strongly on both the dispersion and nonlinear profiles. We have clearly demonstrated how to control and optimize the rogue wave properties by changing the above profiles. In this way, the rogue wave can be completely controlled and manipulated to a required shape and width. It is interesting to note that inhomogeneous systems allow one to control the rogue wave in a more realistic way than their homogeneous analogues. Regarding possible applications of our results, we note that frequency tuning in broadband SCG can be managed through controlling the rogue wave profiles by suitably tuning the dispersion as well as nonlinear profiles. Our investigations provide a model study for the optimization and control of a rogue wave, which is relevant for the realization of broadband SCG sources in many applications. It should be possible to extend our results to include distributed dispersion and nonlinearity with Raman effect. It is also possible to control higher order rogue wave of the inhomogeneous Hirota equation by our method, which will be given a separate paper in the near future.

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