Two-Point Velocity Correlations in Fully Developed Turbulent Pipe Flow at Shear Reynolds Number of $Re_\tau \approx 4000$

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The aim of this study is to report about the large-scale pipe flow structures using the two-point velocity correlations at a radial location of $y/R = 0.5$ and 41 azimuthal positions for shear Reynolds number $Re_\tau \approx 4000$. The streamwise velocity component was measured, utilizing two single-wire probes, simultaneously, based on various azimuthal probe separations. One dimensional pre-multiplied energy spectra, and two-point joint statistics were adopted. Both the low and the high wavenumber motions were highlighted using the pre-multiplied cross-spectral density. The cross-power spectral analysis for all the 41 measurement points indicates strong correlation between the hot-wire signals with small azimuthal separation, on contrary, the peak of the spectrum is getting damped as the azimuthal distance increases. For present shear Reynolds number, the large and the very large scale motions showed wavelengths of $\approx 3R$ and $14R$, respectively, at half of the pipe radius.

1 Introduction

Fully developed turbulent pipe flow represents an axisymmetric flow case to often study experimentally utilizing thermal, and/or optical measuring techniques, improving our physical understanding of flow structures in pipe facilities. It is worth reminding the readers here that flow structures were defined by Robinson (1991) as “three-dimensional region of the flow over which at least one fundamental flow variable (velocity component, density, temperature, etc.) exhibits significant correlation with itself or with another variable over a range of space and/or time that is significantly larger than the smallest local scales of the flow,” motivating further structure studies via velocity correlations of the streamwise velocity component. The two-point correlations either space-time or two-point spatial correlations, and the cross correlations are of fundamental interest for statistical theories of turbulence, and data analysis of the turbulent pipe flows. The two-point, i.e. velocity-velocity, space correlations of the velocity field is, therefore, often used to characterize the organized/coherent motions/structures in turbulent flows in terms of their sizes and wave numbers. The two-point velocity correlation coefficient, $R_{uu}$, is defined as follows:

$$R_{uu}(\Delta s) = \frac{\langle u'(s,t)u'(s+\Delta s,t) \rangle}{\sqrt{\langle u'^2(s,t) \rangle \langle u'^2(s+\Delta s,t) \rangle}}$$

(1)

where $\langle \cdot \rangle$ denotes the time averaging, $\Delta s$ is the azimuthal space separation, and $u'$ is the streamwise velocity fluctuations as a function of space $(s)$ and time $(t)$. Using the two-point velocity correlations, recently, Bailey et al. (2008), and Monty et al. (2007) addressed the azimuthal structures of turbulence in pipe flow, as well as in turbulent pipe and channel flows, respectively. However, considerable discrepancies among experimental results still exist; such as the wide scatter in estimating sizes of turbulent structures, see e.g. [4-7]. Concrete definition of the origin, nature, and evolutions of such structures are still under debate, in particular, at high Reynolds numbers, see e.g. Jiménez (2018), motivating further investigations about the dynamics of the large scale motions.

The present work is undertaken with the aim to obtain consistent pipe flow turbulence data, utilizing the CoLa-Pipe in fully developed turbulent flow regime. The experimental results reported here represent a continuation of contributions from the Cottbus large pipe facility to the priority program “Turbulent Super-Structures” DFG SPP (1881).

2 Experimental Setup

Experiments were carried out, utilizing the large pipe facility (CoLa-Pipe) at Brandenburg University of Technology (BTU-Cottbus-Senftenberg), Germany. The pipe facility, see Fig.1(left), is a closed return facility equipped with a water cooler to provide air with $80\text{m/s}$ maximum velocity at the pipe inlet section, having inner pipe diameters of $190\pm0.35\text{mm}$, and the very large scale motions showed wavelengths of $\approx 3R$ and $14R$, respectively, at half of the pipe radius.

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respectively. Hot-wire anemometry (HWA), laser-Doppler velocimetry (LDV), and Particle Image Velocimetry (PIV) are representing the major instrumentation being under use to carry out measurements in the facility.

For present study, two hot-wire probe module is being utilized to measure the streamwise-velocity component at different radial and azimuthal positions. The pipe section that hosts the two hot-wire probes is photographed, see Fig. 1(right). It consists of one probe that moves radially, while the other probe moves in radial and in azimuthal directions via a rotating pipe section, Fig. 1(top right). Both probe holders have been engraved with mm scale, and the rotational pipe section as well. Hence, the angular separation (\(\Delta \theta\)) desired can be easily calculated based on the circumferential displacement (\(\Delta S\)) and the pipe radius relation \(\Delta S/R = (\Delta \theta) \times 2\pi/360\). For our present contribution, all two-point correlations were calculated based on data with the radial probe kept fixed at half of the pipe radius while the radial-azimuthal probe, located at the same wall-normal distance, was displaced 41 times azimuthally between 10\(^\circ\) and 210\(^\circ\) minimum and maximum separation with 5\(^\circ\) step with respect to the fixed/radial probe. Measurements were performed using Dantec Streamline 9091N0102 Constant Temperature Anemometry (CTA) with commercial Dantec probes, having sensing elements of measures \(\ell \times d = 1200\mu \times 5\mu\).

The new CTA system allows sampling frequency up to 450 kHz per channel, providing highly time resolved velocity data.

![Fig. 1: The CoLa-Pipe facility (left) and the two hot-wire probes test section (right).](image)

### 3 Results and Analysis

This section starts with the analysis of the uncertainty introduced in the streamwise velocity component due to the blockage of the hot-wire holders and probable eccentricity of the hot-wire probes. Variations of the mean and fluctuations of the streamwise velocity component obtained from the two probes were examined at half of the pipe radius, i.e. \(R/2\), and various azimuthal positions (\(\Delta S/R\)). For each measure, both probes were located at the same wall-normal distance and therefore both are expected to measure the same time-averaged statistics for all azimuthal positions. Figure 2(left) illustrates an uncertainty (\(\Delta U/U\)) within ±0.5% for the local mean velocity obtained either by the fixed or the movable probes. Figure 2(right) shows the turbulence intensity level (\(\sqrt{u'^2}/U\)) from both probes with average values of \(\approx 5.7\)% and \(\approx 5.5\)% from the fixed and the movable probes, respectively. This level of local fluctuating velocity at \(y/R = 0.5\) is expected, however, small discrepancy obtained between the two probes might be attributed to the blockage and/or the eccentricity effects.

To further check the blockage and/or the eccentricity consequences on the pipe flow statistics, the pre-multiplied energy spectra measured by the two hot-wire probes were estimated and presented in Fig. 3 using outer scaling at the the minimum probe separation \(\Delta S/R = 0.087\), Fig. 3(left), and the maximum probe separation \(\Delta S/R = 1.83\), Fig. 3(right), for \(Re_x \approx 4000\) and at wall-normal location \(y/R = 0.5\). One might observe from the the figure that at fixed wall-normal location the energy spectra does not depend on the azimuthal position. A satisfactory collapse in pre-multiplied spectra between the two signals was observed at the minimum same as well as the maximum separation distance between the two probes. Hardly noticeable difference in amplitude of the energy spectra between both probes might be observed that could be attributed to the small effects of the blockage and eccentricity. Observable two peaks in spectra at low and moderate wavenumbers are clear, representing a footprint of the large scale motions associated with the VLSM, and LSM, respectively. Based on earlier spectral analysis by Zanoun et al. (2019), sizes of the large-scale structures were estimated. For the present shear Reynolds number \(Re_x \approx 4000\) at half of the pipe radius, i.e. outside the log layer, the very large scale motion (VLSM) shows a wavelength of \(\approx 14R\) while the large scale motion (LSM) was estimated to have a wavelength of \(\approx 3R\). The two observable peaks located at \(k_xR \approx 0.3\) and \(k_xR \approx 2\) are attributed to the distribution of VLSMs and LSMs respectively, in good agreement.

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with Kim & Adrian (1999). One might also observe that the LSM peak for the present wall-normal location \( y/R = 0.5 \), i.e. outside the logarithmic layer, is more prominent than the VLSM peak in good agreement with similar observation made by Bailey et al. (2008).

To further highlight, qualitatively, the characteristics of the pipe flow structure, the two-point velocity correlation coefficient \( (R_{uu}) \) defined in section (1) was calculated. Figure 4 presents one azimuthal profile of the velocity correlation coefficient obtained from probes located at \( y/R = 0.5 \) and 41 azimuthal positions and \( Re_x \approx 4000 \). The profile shows clearly a region of negative correlation within the range \( \Delta S/R \approx 0.35 \) and 1.8 in good agreements with Bailey et al. (2008); Monty et al. (2007). The negative values received in \( R_{uu} \) is attributed to the contribution of the large-scale motions to coherent structure as reported by Bailey et al. (2008) for the pipe flow.

Being in alignment with Bailey et al. (2008), the data of the two-point velocity correlations shown in Fig. 4 comprises contributions from flow motions over variety of turbulence scales. It was claimed by Bailey et al. (2008) that the averaging process could obscure the azimuthal scale of the large and the very large scale motions. Hence, to further study the dependence of \( R_{uu} \) on the streamwise wavenumber \( k_x \), and the azimuthal separation \( \Delta s \), the cross-spectral density, \( G(\Delta s, k_x) \), proposed by Bailey et al. (2008) is calculated, and its real component which defined as:

\[
\Psi(\Delta s, k_x) = \text{Re}[G(\Delta s, k_x) \sqrt{(u'^2(s,t))} \sqrt{(u'^2(s + \Delta s,t))}]
\]

(2)

is estimated and presented in Fig. 5, where \( G \) is calculated using Welch’s averaged method, and \( \text{Re} \) is the real component of the cross-spectral density. Figure 5 illustrates the contours of the pre-multiplied cross-spectral density, \( k_x R \Psi^{++} \), at \( y/R = 0.5 \) wall-normal location, and 41 azimuthal positions for \( Re_x \approx 4000 \) in outer scaling. Corresponding to the region of the negative correlations shown in Fig. 4 between \( \Delta S/R \approx 0.35 \) and 1.8, the contribution of the LSMs to the coherent structure was
claimed to be connected with observed negative region in iso-contours of the pre-multiplied cross-spectral density presented in Fig. 5, see Bailey et al. (2008). The figure also shows that $\Delta S/R$ at which negative correlations appeared decreases with increasing the wavenumber.

![Graph showing $\Delta S/R$ vs. $R_{uu}$](image)

**Fig. 4:** The measured two-point correlations of the streamwise velocity component at $y/R = 0.5$ wall-normal location and $Re_\tau \approx 4000$.

![Graph showing Contours of the pre-multiplied cross-spectral density](image)

**Fig. 5:** Contours of the pre-multiplied cross-spectral density at $y/R = 0.5$ wall-normal location for $Re_\tau \approx 4000$, the color bar represents $k_x R \Psi / u_\tau u_\tau$, and the contour levels are separated by 0.015.

### 4 Conclusions and Final Remarks

An experimental study about the pipe flow large-scale structures utilizing the two-point velocity correlations has been carried out in CoLa-Pipe where flow was assured to be fully developed for shear Reynolds number $Re_\tau \approx 4000$. The streamwise velocity component was measured at two points, simultaneously, based on various azimuthal probe separations, utilizing two single-wire probes. Both the low and the high wavenumber motions were highlighted using both the pre-multiplied spectral energy and the pre-multiplied cross-spectral density. The cross-power spectral analysis for all the 41 measurement points indicates strong correlation between the hot-wire signals with small azimuthal separation, while the peak of the spectrum is getting damped as the azimuthal distance increases. For $Re_\tau \approx 4000$, the large and the very large scale motions showed wavelengths of $\approx 3R$ and $14R$, respectively, at half of the pipe radius. Further experiments for various Reynolds numbers and at 4 different wall-normal locations are under progress. Flow spectra examination using the one-dimensional energy spectra and the cross-spectral density analysis are being under further use to better understand the dynamical characteristics of the pipe flow structures over a wide range of Reynolds number.

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