One- and Two-proton Transfer Reactions with Vibrational Nuclei

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ABSTRACT

We extend a semiclassical model of transfer reactions to the case in which one of the collision partners is a vibrational nucleus. The model is applied to one- and two-proton stripping reactions in the $^{37}$Cl + $^{98}$Mo system, for which a rapid transition from normal to anomalous slope in the two proton transfer reaction at energies around the Coulomb barrier is experimentally observed. This behavior is satisfactorily reproduced by the present extension of the model.

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I. INTRODUCTION

The standard treatment of subbarrier nuclear transfer reactions considers this process as tunneling of a particle from the potential well created by the donor core to the one of the acceptor core. For a given scattering angle the tunneling is dominated by the contribution from the associated distance of closest approach in the classical Rutherford trajectory, \( D_{\text{Ruth}} \), and the transfer probability is

\[
P_{\text{tr}} \propto \sin(\theta/2) e^{-2\kappa D_{\text{Ruth}}},
\]

with

\[
\kappa = \sqrt{2\mu B_{\text{eff}}/\hbar^2}
\]

where \( \mu \) and \( B_{\text{eff}} \) are the reduced mass and the effective barrier height to be traversed by the transferred particle, respectively. When, as customary, the transfer probability is presented as a function of \( D_{\text{Ruth}} \) in a semilogarithmic plot, this model predicts a straight line with an energy independent slope, being the slope for two-nucleon transfer approximately twice that for one-nucleon transfer. At large distances the experimental slopes are generally in good agreement with the predictions of this model for one-neutron transfer. However, deviations from the expected ratio of two have been observed in two-nucleon transfer reactions, which in the literature are referred to as “slope anomalies.” Furthermore, some experiments show an energy dependence of the slope.

In previous work, we were able to explain available proton and neutron transfer data, including the slope anomaly, by considering the contribution of the two trajectories that lead to a given scattering angle (see Section II). Thus far, we have applied this model exclusively to reactions for which structure effects in the transfer process are assumed unimportant.

In the present work we investigate the transfer process for the case of vibrational nuclei, in particular one- and two- proton stripping reactions, at energies close to the barrier. In Section II we provide a brief review of the semiclassical theory of transfer in the case of structureless spherical nuclei, studied before, and extend it to the vibrational nuclei considered here. The results of the calculations performed with the model are presented in Section III. In the last section we draw the main conclusions and suggestions for additional work.
a. Structureless nuclei

In the framework of this semiclassical model [4, 5], the trajectories of the participant ions are determined taking into account both the Coulomb and the nuclear part of the nucleus-nucleus interaction. This brings, as a consequence, the possibility that more than one trajectory contributes to a given scattering angle of the outgoing particle. Due to absorption usually only two of them contributes to the transfer reactions. For the nuclear optical potential we adopt a Woods-Saxon shape with radius and strength calculated as in Ref. [7]:

\[ R = R_p + R_t + 0.29 \text{ fm} \]  

with

\[ R_i = (1.233A_i^{1/3} - 0.98A_i^{-1/3}) \text{ fm } i = p, t \]  

and

\[ V_0 = 16\pi\gamma\overline{R}a \text{ MeV} \]  

with

\[ \gamma = 0.95 \left[ 1 - 1.8 \left( \frac{N_p - Z_p}{A_p} \right) \left( \frac{N_t - Z_t}{A_t} \right) \right] \text{ MeV fm}^{-2} \]  

and

\[ \overline{R} = \frac{R_p R_t}{R_p + R_t}, \]  

where \( a \) is the diffuseness, and \( A_i, N_i, \) and \( Z_i \) are the mass, neutron, and atomic numbers of the nucleus \( i \) \((i = p \text{ for projectile, } i = t \text{ for target}), \) respectively.

The probability amplitude for survival from absorption due to the imaginary part of the nucleus-nucleus optical potential, \( W(r) \), is calculated by the expression [8]

\[ a_{\text{obs}} = \exp \left( -\frac{1}{\hbar} \int_{-\infty}^{+\infty} W(t)dt \right) \]
in which $W(t) = W(r(t))$.

We denote by $U(r)$ the potential which acts over the transferred particle, $R_B$ the position where this potential barrier reaches its maximum, $U_B = U(R_B)$, and $B.E.$ the binding energy of the particle in the donor nucleus. The probability for tunneling is determined, when $U_B + B.E. > 0$, by the WKB approximation

$$P_{\text{tun}} = |a_{\text{tun}}|^2 = \left(1 + e^S\right)^{-1}$$

in which

$$S = 2 \int_{R_1}^{R_2} \left[\frac{2\mu}{\hbar} (U(r) + B.E.)\right]^{1/2} dr.$$  \hspace{1cm} (10)

In the region $U_B + B.E. < 0$, the potential barrier can be approximated by an inverted parabola, allowing us the use of the analytic expression of Hill and Wheeler \cite{Hill1953}

$$P_{\text{tun}} = \left[1 + \exp\left(\frac{2\pi}{\hbar \omega (U_B + B.E.)}\right)\right]^{-1}$$

with

$$\hbar \omega = \left(-\frac{\hbar^2}{\mu} \frac{d^2 U(R_B)}{dr^2}\right)^{1/2}. \hspace{1cm} (12)$$

We have taken

$$U(r) = U_1(r) + U_2(D - r),$$

$$U_j(r) = U_{C_j}(r) + U_{N_j}(r),$$

where the subscripts 1 and 2 refer to the donor and acceptor cores, respectively, $D$ is the distance of closest approach between them, and $r$ is the spatial coordinate of the transferred particle with respect to the donor core. $U_{C_j}$ is the Coulomb potential and $U_{N_j}$ the nuclear potential generated by the core $j$ over the particle. The Coulomb potential was taken as that generated by a charged sphere of radius $1.25 A_j^{1/3}$ fm acting over a cluster with charge $Z_{cl}$ and the nuclear part as a Saxon-Woods potential with radius parameter $r_0 = 1.2$ fm, diffuseness $a_u = 0.63$ fm and depths
where
\[ U_{0j} = Z_{cl}V_{0j}(+1) + N_{cl}V_{0j}(-1) \]

is the nuclear potential generated by the core \( j \) with mass number \( A_j \), charge \( Z_j \) and neutron number \( N_j = A_j - Z_j \) acting over the neutrons \( \tau_z = +1 \) and the protons \( \tau_z = -1 \) of the transferred cluster with mass number \( A_{cl} \), charge \( Z_{cl} \), and neutron number \( N_{cl} = A_{cl} - Z_{cl} \).

As detailed in Ref. \cite{4}, except in cases in which the measurements are done with high angular resolution, the transfer probability can be approximated by the incoherent sum of contributions by each trajectory leading to a given scattering angle
\[ P_{tr} (\theta) \approx \sum P_{tun} (\theta) \mid a_{abs} (\theta) \mid^2. \]

This expression is employed in the calculations presented in this work.

b. Vibrational nuclei

For a transfer process in which one of the participant ions is a vibrational nucleus (the target nucleus, to fix ideas) the above mentioned model can be extended in the following way. Assuming a quadrupole vibrational mode, we parametrize the radius of the target as
\[ R_t(\alpha_{20}) = R_t(1 + \alpha_{20}Y_{20}(\theta)). \]

The internal Hamiltonian of the target may be written as \cite{10}
\[ H_{int} = \frac{1}{2}(B_2|\dot{\alpha}_{20}|^2 + C_2|\alpha_{20}|^2). \]

from which the zero-point amplitude is
\[ \alpha_{20}^0 = \left( \frac{E_{2+}}{2C_2} \right)^{\frac{1}{2}}. \]

The target radius \( R_t \) is given by Eq. \cite{4}, and \( E_{2+} \) is the transition energy between the first excited and the ground state.
In this kind of reaction the vibration is very slow in comparison to the translational motion of the projectile and the tunneling process is dominated by those trajectories for which the distance between the surfaces of projectile and target at the point of closest approach are smallest. Therefore the most relevant axis of vibration is that directed from this point to the center of the target. In this spirit we substitute the spherical harmonic in Eq. (18) by its maximum value $\sqrt{5/4\pi}$.

With these prescriptions we now calculate the transfer probability as described for structureless nuclei, by substitution of Eqs. (8) and (9) or (14) into Eq. (17). The radius of the target must be replaced by $R_t(\alpha_{20})$, which implies an additional degree of freedom in the calculation. Then, we solve the classical equations of motion using the Hamiltonian

$$H = T_r + H_{\text{int}} + V(r, \alpha_{20})$$

(21)

where $T_r$ is the kinetic energy of the relative motion and $V(r, \alpha_{20}) = V_C + V_N$ includes the potential energy of the relative motion and the interaction between this and the vibrational mode. We approximate

$$V_C = Z_p Z_t e^2 \left( \frac{1}{r} + \frac{3}{5} \frac{R_p R_t(\alpha_{20})}{r^3} \right)$$

(22)

for the Coulomb interaction and write

$$V_N = \frac{V_0}{1 + e^{[r - R_p - R_t(\alpha_{20})]/a}}$$

(23)

for the nuclear part. In the calculation of the absorption, the imaginary part of the nuclear optical potential acting between the incident ions is taken as

$$W_N = \frac{W_0}{1 + e^{[r - R_p - R_t(\alpha_{20})]/a}}$$

(24)

We integrate the resulting coupled differential equations with the initial condition $\alpha(t = -\infty) = \alpha_{20}^0 \cos \phi$, where $\phi$, the initial phase, can take any value in $[0, 2\pi]$ with equal probability. For each phase $\phi$ the distance of closest approach and the value of $\alpha_{20}$ at this time are calculated, in order to calculate the transfer probability for this trajectory. For a given scattering angle the contributions are summed as in Eq. (17) and the final transfer probability is obtained by averaging over all phases.
III. RESULTS AND DISCUSSION

We apply this model to the vibrational nucleus $^{98}$Mo, studied by means of the $^{37}$Cl + $^{98}$Mo reaction. Data for reactions with Mo isotopes, including this system, at energies close to the barrier, were recently measured by the SUNY group [11, 12]. It was found that, in the case of two-proton stripping reactions, there is an abrupt change in slope at approximately the energy corresponding to the Coulomb barrier ($E_{lab} \approx 117$ MeV). A calculation as described in Section II.a, i.e. without considering the vibrational character of the Mo nuclei, yields a value for the energy at which the slope changes higher than the observed one.

In Fig. 1 we show the experimental transfer probabilities divided by $\sin(\theta_{c.m.}/2)$ for one-proton (circles) and two-proton (squares) stripping reactions in the $^{37}$Cl + $^{98}$Mo system [12]. Also shown are the theoretical results calculated as described in Section II.b and normalized to the data (full lines). We used the parameters of Temmer and Heydenburg [13] for $^{98}$Mo, $E_{2^+} = 0.786$ MeV, $C_2 = 70$ MeV. For the optical potential we take a diffuseness $a = 0.7$ fm, and a strength for the imaginary part $W_0 = 70$ MeV. As shown in Fig. 1, a remarkably good agreement between the calculated and the experimental points is obtained at large values of $D_{Ruth}$. In this approach $D_{Ruth}$ is only a parametrization of the scattering angle [4, 5, 6].

We consider particularly interesting that our calculation reproduces the abrupt change in slope observed at bombarding energies around the Coulomb barrier. To understand the origin of this effect we should remember that, for a given deflection angle, there are two trajectories contributing to the transfer probability [5]. One is essentially a Rutherford trajectory while the other feels more strongly the nuclear optical potential. At lower energies the dominant contribution to the transfer probability is the Coulomb trajectory, which gives the energy independent slope of Eq. (1), while at higher energies the nuclear trajectory dominates. As shown in Ref. [6] there is a rapid transition between these two regimes, which, for a structureless nucleus takes place at an energy well above the Coulomb barrier. In the case of a vibrational nucleus, the effective radius increases, which decreases the effective barrier, bringing, as a consequence, the transition in slope to lower energies.

IV. CONCLUSIONS

An extension of a simple semiclassical model including some nuclear struc-
ture effects, quadrupole vibrations to be specific, was proved to be quite suc-
cessful. Applications to other degrees of freedom, such as octupole vibrations,
can be implemented in essentially the same way. One could, in a similar
spirit, include other nuclear properties, such as deformation. The practical
problem with the nuclear deformation case is that the trajectories depend
on the orientation of the deformation axis given by two Euler angles. The
averaging process should then be performed over two parameters, the initial
values of the two Euler angles and, consequently, one would need to consider
a very large number of trajectories contributing to a given scattering angle,
thus making the problem much more difficult to treat.

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Figure Caption

Fig. 1 One- and two-proton transfer probability, divided by \( \sin(\theta_{c.m.}/2) \), as a function of \( D_{Ruth} \) for the \(^{37}\text{Cl} + ^{98}\text{Mo} \) reaction at three laboratory energies. Symbols represent the experimental data of Ref. [12], and lines are the theoretical calculations described in this work, and normalized to the data.
\[ E_{\text{lab}} = 124.9 \text{ MeV} \]

\[ E_{\text{lab}} = 121.6 \text{ MeV} \]

\[ E_{\text{lab}} = 117.9 \text{ MeV} \]

\[ 37\text{Cl} + ^{98}\text{Mo} \]

stripping

- 1p transfer
- 2p transfer

with vibrations

without vibrations