Symmetry breaking in annular jets with different blockage ratio

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Abstract. We perform direct numerical simulations of the annular laminar jet flows for two inner-to-outer diameter (blockage) ratios $br = d / D = 0.5, 0.9$ in the range of the Reynolds number based on the bulk velocity inside the nozzle and its outer diameter $Re = 200 – 400$. In spite of the axisymmetric formulation, asymmetric states are observed for moderate and high $Re$. A gradual increase of the Reynolds number allow building a regime map. The symmetry-asymmetry transition Reynolds number decreases with the increase of $br$, while the steady-unsteady transition Reynolds numbers are close for both $br$. The degree of asymmetry increases with the increase of $Re$, leading to a shift of the global stagnation point from the axis of symmetry.

1. Introduction

An annular jet serves as a basis for a widespread type of industrial burners. The burners with stabilizing bluff-body are of practical interest because of highly strained and recirculated turbulent flows produced by these burners. A fluid or gas is forced to pass through an annular gap forming a separated flow whereby a lowered pressure region is formed. Strong recirculation is established in this area increasing the mixing rate and stabilizing the flame. The more stable flame allows one to use a leaner fuel-air mixture which results in decreasing NOx emission [1, 2, 3, 4].

In spite of a fully axisymmetric problem (geometries and boundary conditions) asymmetric flows have been observed in numerous experimental works and simulations including different symmetric problem formulations: sudden expansion [5, 6] or constriction [7] flows, two coaxial interacting jets [8] and confined jets [9, 10]. Theoretically asymmetric states of a flow are possible solutions of the Navier-Stokes equations due to nonlinearity. The asymmetry resulting from a fully symmetric formulation is referred to as a spontaneous break of symmetry in e.g. [11, 12] while an asymmetric formulation results in a forced symmetry breaking.

For turbulent flows a possible explanation of the asymmetry is very low-frequency oscillations observed in experimental works [13, 14, 15] as well as in numerical simulations [4, 12]. However, the asymmetric states were also observed in laminar symmetric configurations including jets [11], flow over a disk [16, 17], a bullet-like obstacle [18] and a rectangular Ahmed bluff body [19, 20]. A possible mechanism of bifurcation leading to transition between symmetry-asymmetry states was proposed by Del Taglia [11]. Perturbations originating in the boundary layer near an obstacle are displaced downstream the flow to the stagnation point and cause its shift. Whereas appearance of these perturbations is of random nature, an appropriate time-averaged interval must provide axisymmetric flow fields, but the asymmetric state is observed for time-converged statistics.
In this paper we perform direct numerical simulations of three-dimensional steady and unsteady regimes of laminar annular jet flows. To solve Navier–Stokes equations we use Nek5000 code [22] based on the Spectral element method for spatial discretization [21]. We show the instantaneous velocity fields and streamlines for each $br$ corresponding to the first asymmetric $Re$. The results are summarized in a map of regimes depending on the $br$ and $Re$.

2. Problem formulation and computational details

![Figure 1. The main computational domain.](image)

We perform direct numerical simulations (DNS) of the annular laminar jet outflow of incompressible fluid into an infinite domain. The Reynolds number based on the bulk velocity inside the jet nozzle and its outer diameter is in the range of $200 - 400$. The cylindrical computational domain is $7.5D \times 7D$ in size with supplying nozzle of $2D$ in length, see Figure 1. We use the cylindrical coordinates system $(x, r, \phi)$ with the origin at the center of the supplying pipe. The outflow boundary conditions are simulated using the Neumann condition. We consider two values of the supplying pipe blockage ratio $br = d / D = 0.5$ and 0.9. The inflow axial velocity profile is set using the exact solution of the Navier–Stokes equation for the annular pipe flow: where $u_0$ is chosen according to the value of the Reynolds number. The total number of the spectral elements is about 65000 that with the use of polynomial order $N = 5$ (216 nodes for each elements) provides 14 million computational nodes in total. The numerical accuracy is verified by a mesh convergence based on a series of computations for two different mesh resolutions (with $N = 5, 6$). The first moments are equal for two simulations with different polynomial order.

To perform DNS we use Nek5000 [22] which is based on the Spectral–element method (SEM) [21]. The Navier–Stokes equations are discretized in space with the use of the Galerkin approximation with $N$th-order Lagrange polynomial interpolants based on the Gauss–Lobatto–Legendre points for both the velocity and pressure field ($P_N–P_N$ formulation). The time discretization is performed by means of the
semi-implicit third-order accuracy scheme. The accuracy of Nek5000 was previously validated for a number of configurations, including the channel flow and non-swirling annular jet [24,25].

3. Results
We conduct a series of simulations for two blockage ratios \( br = 0.5, 0.9 \) in the range of Reynolds numbers \( Re = 200, 250, 300, 350, 400 \). The results are summarized in a regime diagram depending on the value of \( br \) and \( Re \) for steady–unsteady and symmetry–asymmetry transitions of the jet flows as described below (see Table 1).

**Table 1. Map of regimes**

| \( Re \) | \( br = 0.5 \) | \( br = 0.9 \) |
|---|---|---|
| 200 | Steady, symmetric | Steady, symmetric |
| 250 | Steady, symmetric | Steady, asymmetric |
| 300 | Steady, asymmetric | Steady, asymmetric |
| 350 | Unsteady, asymmetric | Unsteady, asymmetric |
| 400 | Unsteady, asymmetric | Unsteady, asymmetric |

Figure 2. shows instantaneous velocity fields with streamlines in \( r-\phi \) cross section for \( x/D = 0.25 \).
For better understanding of the flow topology we plot instantaneous velocity components with streamlines for first asymmetric Reynolds number $Re = 250$ and 300 for $br = 0.9$ and 0.5, respectively. Figure 2 shows the velocity in the $r$-$\phi$ cross-section corresponding to the $x/D = 0.25$.

Figure 3. Isocontour of instantaneous axial velocity for $br = 0.9$ and $Re = 250$ in two perspectives (plots a and b). The black and white dots indicate the axis of symmetry and “global” stagnation point. Plots c and d show the main planes.

Figure 4. Velocity distribution in the asymmetric plane with streamlines.
Then in Figure 3 we plot isocontour of instantaneous axial velocity for $br = 0.9$ only since the asymmetry for this case is more pronounced. The black and the white dots on the plot 3b are the axis of symmetry and the global stagnation point. We define two cross-sections in the flows that are referred to as the “asymmetric” and “symmetric” planes. The first one passes through the axis of symmetry and “global” stagnation point (Fig. 3c) while the second one is orthogonal to the “asymmetric” plane (Fig. 3d). Figure 4 (for asymmetric) and 5 (for symmetric) show the velocity distribution with streamlines in these main planes demonstrating the breaking of symmetry.

![Figure 5. Velocity distribution in the symmetric plane with streamlines.](image)

**Conclusion**

We have performed direct numerical simulation of the laminar annular jet flows using Nek5000 for two blockage ratios ($br = 0.5, 0.9$) in the range of small Reynolds number ($Re = 200 - 400$). The symmetry–asymmetry and steady-unsteady transitions were observed and summarized in the regimes map. The symmetry-asymmetry transition Reynolds number decreases with increasing blockage ratio while the steady-unsteady transition $Re$ is the same for both $br$. The global stagnation point displaces from the jet axis with the increase of the diameter ratio, and the angle between the axis and the direction of velocity streamlines increases. Further increase in the Reynolds number for both $br$ leads to an unsteady behavior, moreover the asymmetry state persists while the Reynolds number increases up to 8900 [26].
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