The conditional mean acceleration of fluid particle in developed turbulence

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Using the random intensity of noise (RIN) approach to the one-dimensional Laval-Dubrulle-Nazarenko type model for the Lagrangian acceleration in developed turbulence [cond-mat/0305459] we study the probability density function and mean acceleration conditional on velocity fluctuations. The additive noise intensity and the cross correlation between the additive and multiplicative noises are assumed to be dependent on velocity fluctuations in an exponential way. The obtained fit results are found to be in a good qualitative agreement with the recent experimental data on the conditional acceleration statistics by Mordant, Crawford, and Bodenschatz. The fit to the observed conditional mean acceleration is of pure illustrative character which is performed to study influence of variation of the cross correlation parameter on the shape of conditional acceleration distribution and conditional acceleration variance. The conditional mean acceleration should be zero for homogeneous isotropic turbulence. The observed conditional mean acceleration increases for bigger velocity fluctuation amplitude and is associated to anisotropy of the studied flow.

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I. INTRODUCTION

The nonextensive statistics developed by Tsallis [1] has motivated a phenomenological approach [2, 3, 4] which was recently used [5, 6] to describe Lagrangian acceleration of fluid particle in developed three-dimensional turbulent flow within the framework of Langevin type equation; see also [7, 8, 9, 10]. Some recent stochastic particle models and refinements of this technique [11, 12, 13] were reviewed in Ref. [14]. The one-dimensional Langevin toy models of Lagrangian turbulence lacks of physical interpretation, e.g., of short term dynamics, or small-scale and large-scale contributions, in the context of three-dimensional incompressible Navier-Stokes equation, which is generally assumed to be capable to describe fully developed turbulence. A deep physical analysis of this problem has been made by Gotoh and Kraichnan [15].

Recently [14, 16] we have shown that the one-dimensional Laval-Dubrulle-Nazarenko (LDN) type toy model [17, 18] of the acceleration dynamics with the model turbulent viscosity $\nu_t$ and coupled delta-correlated Gaussian multiplicative and additive noises is in a good agreement with the recent high-precision experimental data on acceleration statistics [14, 20, 21]. Particularly, we have demonstrated that the predicted contribution to fourth order moment of acceleration does peak at the same values as the experimental curve, in contrast to predictions of most of the other stochastic particle models [5, 6, 10, 12]. We note however that characteristic parts of the acceleration distribution are the core and tails, while the intermediate range of accelerations which is responsible for positions of the peaks may be not robust.

Below we discuss at some length on foundations of the present model, and focus on modeling acceleration statistics conditional on velocity fluctuations.

The original one-dimensional LDN model was formulated both in the Lagrangian and Eulerian frameworks for small-scale velocity increments. It is based on a stochastic kind of Batchelor-Proudman rapid distortion theory (RDT) approach to the three-dimensional Navier-Stokes equation and Gabor transformation [17]. Small scales are separated and assumed to be stochastically distorted by much larger scales. Such nonlocal interactions which can be viewed as elongated triads are featured by the three-dimensional LDN model, to which the one-dimensional model makes an approximation, particularly via mimicking relationship between stretching and vorticity. Local interactions are modeled by the turbulent viscosity which is taken due to the renormalization group approach.

In the present paper, we use the Lagrangian formulation of the one-dimensional LDN model, which is characterized by a simple structure, as an ansatz to formulate Langevin type equation for the component of fluid particle acceleration in statistically homogeneous and isotropic developed turbulence.

In general the Langevin type equation which we employ here contains wellknown terms, namely, time derivative of the variable, an additive noise, and a nonlinear drift term with a multiplicative noise; see Eq. (1) below. The noises represent large scales and are treated independent on small scales. In the Lagrangian frame, they are taken in the simplest realization, Gaussian white-in-time with zero mean, and are assumed to be delta-correlated to each other. Such choice of the noises correspond to the zero-time approximation of small time-scale correlations viewed along the Lagrangian trajectory of fluid

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particle. Direct account for finite time correlations in a Langevin type equation is important but this is beyond the scope of the present study, in which we concentrate on the longtime behavior and qualitative analysis. Long-time dynamics of large scales is ignored in this particular case of the LDN model.

In the conventional approach, wandering of the tracer particle can be described within the framework of generalized Brownian like motion, with the acceleration chosen to be the random variable depending on time \( t \) (a stochastic process). The statistical isotropy allows one to decompose the acceleration into the ‘transverse’ and ‘longitudinal’ components with respect to the corresponding velocity that reduces formally the analysis to effective two-dimensional consideration of the essentially three-dimensional system. The isotropy is assumed to be local, in the neighbor of fluid particle. In one-dimensional models one considers one of the two components not coupled to each other. Joint dynamics of the components of acceleration is of much interest and can be considered elsewhere.

It should be emphasized here that the usual Eulerian framework (fixed probe) represents the structural viewpoint which is extensively studied in the literature while the Lagrangian framework is associated to stochastic dynamical consideration, a statistical viewpoint. These two basic approaches are different both in their theoretical formulations and experimental technique, and compliment each other. In the Eulerian frame, the acceleration, \( a_i = dv_i/dt = \partial v_i + v_k \partial_k v_i \), is expressed in terms of the velocity field and its temporal and spatial derivatives, while in the Lagrangian frame the acceleration is \( a_i = \partial v_i \), in terms of the Lagrangian velocity, \( v_i = \partial x_i \), of fluid particle with the coordinates \( x_i(x_i(0), t); i, k = 1, 2, 3 \). In the present paper we will use the Lagrangian framework.

Measurement of the time series \( x_i(t) \) for individual tracer particle in a turbulent flow to a high resolution requires very high speed imaging sensors, and allows one to compute its instantaneous Lagrangian velocity and Lagrangian acceleration. Experimental data are collected for such individual particles moving in the approximately isotropic and homogeneous turbulent flow domain.

With delta-correlated noises the Langevin type model of one of the two components of acceleration is Markovian (no memory effects) so that the well established Fokker-Planck approximation can be used to link the dynamical framework to a statistical (PDF) approach. Thus, one can study stationary solution of the associated Fokker-Planck equation for the acceleration one-point probability density function \( P(a, t) \) under the assumption of balance between the energy injected by driving forces at large scales and the energy dissipated by viscous processes at small scales (a statistically steady state).

In the case when stationary probability distribution can be found exactly one can make further analysis without referring to dynamics. The resulting stationary probability density function of the component of acceleration, \( P(a) \), contains some free parameters inherited from the Langevin type equation. This PDF is the main prediction of the model which can be directly fitted to experimental data or direct numerical simulation (DNS) of the Navier-Stokes equation.

The Random Intensity of Noise (RIN) approach makes a simple extension of the above Langevin type model of the Lagrangian acceleration. The main idea of this approach is to use the presence of two well separated characteristic time scales (Kolmogorov time scale and the integral time scale) of the system and assume that parameters entering the resulting acceleration PDF, such as intensities of the noises, are not constant but fluctuate at large time scale and depend on Lagrangian velocity fluctuations.

Effectively, we approximate the time evolution of acceleration by separating fast and slow time varying parts, and thus account for large time scales which are usually ignored. This improves the delta-correlation approximation of the noises adopted in the one-dimensional LDN model mentioned above, provided that characteristic longtime fluctuations along the particle trajectory occur mainly at time scales much bigger than the dissipative time scale, typically at the integral time scale.

In general, one can assume hierarchy of a number of characteristic time scales associated to the discrete cascade picture with characteristic times of the flow modes of descending eddies. However, in the present paper we simplify the consideration by extending the large time scale up to the Lagrangian integral time scale in order to make it more analytically tractable, in accord to the presence of two basic characteristic scales in the Kolmogorov 1941 (K41) picture of fully developed turbulence, and the recent Lagrangian experimental data.

In the context of stochastic equation the delta-correlated multiplicative noise provides short-time acceleration bursts the origin of which is thought to be due to the presence of very intense vortical structures relatively slowly varying in time. These structures make an essential contribution to the acceleration statistics, namely, to the tails of acceleration probability density function, making it highly non-Gaussian. Accounting for longtime fluctuations of the parameters corresponds to an accounting for fluctuations of intensity of the vortical structures, intensity of noisy incoherent background, and their cross correlation. The key point is then to identify distributions of the parameters. We will discuss it below and further in Sec. III.

It is worthwhile to mention that accounting for the fluctuating parameter \( \beta \) in the chi-square Langevin model characterized by the sole delta-correlated Gaussian additive noise was recently found to yield a stationary probability density function of the same power law form as in simple Langevin model characterized by the delta-correlated Gaussian additive and multiplicative noises with constant parameters and linear drift term. In other words, the chi-square distributed fluctuations of \( \beta \)
appears to be mimicking the presence of the multiplicative noise in this particular case.

In the first approximation the velocity statistics in the Lagrangian frame is taken as usually stationary and Gaussian, partially because one can easily deal with it analytically. Due to the recent Lagrangian experiments the distribution of Lagrangian velocity is approximately Gaussian for both the $x$ and $z$ components (the flatness is 3.2 and 2.8 respectively as compared with 3 for a Gaussian) and their characteristic time variation is of the order of Kolmogorov time, yet velocity fluctuations exceed root-mean square (rms) velocity that is usually associated to large scales. This can be understood as a manifestation of the nonlocal (inter-scale) interactions when considering small scales. It is important to note here that the Lagrangian velocity autocorrelation, as well as the autocorrelation of absolute value of velocity increments in time, was found to cross zero at the integral time scale, while the full signed velocity increments in time decay at the Kolmogorov time scale.

Under the assumption that the parameters are given independent random variables (large scales are weakly affected by small scales in a three-dimensional high-Reynolds-number flow since the former are local in the wave number space) characterized by stationary statistics, the stationary probability density function obtained from the Fokker-Planck equation makes a sense and is treated as the distribution conditional on small scale velocity fluctuations through assumed dependencies of the parameters on $u$.

By this way the RIN approach enables one to study acceleration statistics conditional on velocity fluctuations, $P(a|u) = P(a|\text{Parameters}(u))$. The velocity fluctuations (large scales) are assumed to be decoupled from the acceleration (small scales) at high Taylor microscale Reynolds numbers, $R_\lambda > 500$.

It should be noted that such an approach is in agreement with the Heisenberg-Yaglom picture of developed turbulence which relates statistics of the fluid particle acceleration to statistics of velocity fluctuations on the basis of K41 scaling theory, with pressure gradient contribution to the acceleration variance strongly dominating over that of viscous forces, in the inertial range. The long-standing Heisenberg-Yaglom scaling, $\langle a^2 \rangle \sim \bar{u}^3/2$, where $\bar{u}$ is rms velocity, was recently confirmed to a high accuracy for seven orders of magnitude in $\langle a^2 \rangle$, and was found to be broken for $R_\lambda < 500$ due to increasing coupling of the acceleration to large scales of the flow.

Also, it should be mentioned that a similar approach, with the variance of intermittent variable viewed phenomenologically as a parameter which follows log-normal distribution, was considered by Castaing, Gagne, and Hopfinger. From this point of view RIN models can be referred to as Castaing type models. However, the Castaing model does not refer to a stochastic dynamical approach, which is an important ingredient of the Lagrangian modeling, and does not relate the fluctuating parameter to velocity fluctuations.

In the RIN approach the marginal (i.e., unconditional) stationary probability density function $P(a)$ is found simply by integrating out $u$ in the conditional distribution $P(a|u)$ with (Gaussian) distributed $u$. This procedure requires prior determination of dependencies of the parameters on velocity fluctuations $u$.

Particularly, in the RIN approach to the LDN type model the assumption that the additive noise intensity $\alpha$ depends on absolute value of velocity fluctuations $u$ in an exponential way, $\alpha \sim e^{u|a|}$, was found to imply a set of conditional probability density functions $P(a|u)$ and the conditional acceleration variance $\langle a^2 | u \rangle$ which are in a good qualitative agreement with the recent experimental data on the conditional statistics of the $x$ component of acceleration obtained by Mordant, Crawford, and Bodenschatz, for the normalized velocity fluctuations $|u|/\langle u \rangle^{1/2}$ ranging from 0 to 3. This issue will be discussed further in Sec. III.

The effect of nonzero cross correlation $\lambda$ between the additive and multiplicative noises (which models the relationship between small-scale stretching and vorticity and can be seen as a small skewness of the probability density function of the ‘longitudinal’, i.e., pointed along the Lagrangian velocity component of acceleration) has been studied in Ref. 16 for an illustrative purpose. The experimental unconditional acceleration probability density function reveals small skewness that has been fitted by using the value $\lambda = -0.005$.

The observed very small skewness of the probability density functions for the $x$ component of acceleration can be assigned to anisotropy of the studied $R_\lambda = 690$ flow rather than to the effect of correlation between stretching and vorticity. Particularly, it was found that, in addition to the dependence $\alpha = \alpha(u)$, the parameter $\lambda$ should also depend on velocity fluctuations to meet the experimentally observed appreciable increase of the conditional mean acceleration, $\langle a|u \rangle$, with the increase of $|u|$. The form of functional dependence $\lambda(u)$ was assumed to produce a weak effect but it was not specified.

In the present paper we fill this gap and demonstrate that (rather strong) exponential dependence, $\lambda \sim e^{u|a|}$, is in a good qualitative agreement with the experimental data. This fit is of a pure illustrative character since the observed nonzero conditional mean acceleration is due to the anisotropy of the studied flow. We perform this fit to study influence of variation of the cross correlation parameter on the shape of conditional acceleration distribution and conditional acceleration variance.

The layout of the paper is as follows. In Sec. III we outline results of the one-dimensional LDN type model of the component of acceleration. In Sec. IV we study the conditional mean acceleration using the RIN extension of the LDN type model, with the additive noise intensity $\alpha \sim e^{u|a|}$, the cross correlation parameter $\lambda \sim e^{u|a|}$, and the other parameters of the model fixed. We summarize the obtained results in Sec. IX.
II. THE LDN TYPE LANGEVIN MODEL

In this Section, we give only a brief sketch of the LDN model and refer the reader to Refs. \[14, 17\] for more details; see also recent paper \[22\] for the stochastic RDT approach.

We use the exact result for probability density function of the LDN type model obtained as a stationary solution of the Fokker-Planck equation associated to the one-dimensional Langevin equation for the component of acceleration \[14, 17\],

\[
\partial_t a = (\xi - \nu k^2)a + \sigma_\perp. \tag{1}
\]

This equation is a Lagrangian description in the scale space, in the reference frame comoving with the wave number packet. This toy model can also be viewed as a passive scalar in a compressible one-dimensional flow \[17\]. Here, \(\xi\) and \(\sigma_\perp\) model stochastic forces in the Lagrangian frame and are chosen to be Gaussian white-in-time noises,

\[
\langle \xi(t) \rangle = 0, \quad \langle \xi(t)\xi(t') \rangle = 2D\delta(t-t'),
\]
\[
\langle \sigma_\perp(t) \rangle = 0, \quad \langle \sigma_\perp(t)\sigma_\perp(t') \rangle = 2A\delta(t-t'), \tag{2}
\]

where the averaging is over ensemble realizations. The probability density function was calculated exactly \[14\] and appeared to be of rather complicated form despite the simplicity of the starting equation \[11-2\]. It is given by

\[
P(a) = \frac{C \exp[-\nu k^2/D + F(c) + F(-c)]}{(Da-2\lambda a+\alpha)^{1/2}(2Bka+\nu k^2)2Bk\nu/D}, \tag{3}
\]

where we have denoted

\[
F(c) = \frac{c_1 k^2}{2c_2 D c} \ln\left[\frac{2D^3}{c_1 c_2 (c - Da + \lambda)}\right], \tag{4}
\]

\[
c = -i\sqrt{\frac{D\alpha - \lambda^2}{}}; \quad \nu_k = \sqrt{\nu_0^2 + B^2 a^2/k^2}, \tag{5}
\]

\[
c_1 = B^2(4\lambda^2 + 4\alpha^2 - 3D\alpha \lambda - cD\alpha) + D^2(c + \lambda)\nu_0^2 k^2, \tag{6}
\]

\[
c_2 = \sqrt{B^2(2\lambda^2 + 2\alpha^2 - D\alpha)k^2 + D^2\nu_0^2 k^4}, \tag{7}
\]

and \(C\) is normalization constant.

Without loss of generality one can put, in a numerical study, \(k = 1\) and the additive noise intensity parameter \(\alpha = 1\) by appropriate rescaling of the multiplicative noise intensity \(D > 0\), the turbulent viscosity parameter \(B > 0\), the kinematic viscosity \(\nu_0 > 0\), and the cross-correlation parameter \(\lambda\) \[14\], and make a fit of \(P(a)\) to the experimental data.

The particular cases \(B = 0\) and \(\nu_0 = 0\) with \(\lambda\) put to zero were studied in Ref. \[14\]. As one can see from \[3\], the real parameter \(\lambda\) is responsible for an asymmetry of the distribution with respect to \(a \rightarrow -a\). For \(\lambda = 0\) the distribution is symmetrical. Note that \(Da = \lambda^2\) makes a special case, and \(P(a)\) was found to be well defined for \(|a|/(a^2)^{1/2} \leq 60\) in the practically interesting case of negative \(\lambda\) with \(|\lambda| \ll \alpha\) and \(|\lambda| \ll D\). One observes also a rather nontrivial dependence of \(P(a)\) on \(a\), \(\lambda\) and other parameters through \(F(c)\) given by Eq. \[4\].

III. THE CONDITIONAL MEAN ACCELERATION

In this Section, we use the exact distribution \[4\] as a starting point.

Since the experimental unconditional distribution \(P(a)\) and the experimental conditional distribution \(P(a|u)\) at \(u = 0\) are approximately of the same stretched exponential form \[21\] we use the result of our fit \[14\] of the probability density function \[4\] to the unconditional distribution \(P(a)\) \[21\]. This implied the following set of values of the real parameters:

\[
k = 1, \quad \alpha = 1, \quad D = 1.100, \quad B = 0.155, \quad \nu_0 = 2.910, \quad \lambda = -0.005, \quad C = 3.230. \tag{8}
\]

It should be stressed that a fit to the experimental conditional distribution \(P(a|u)\) at \(u = 0\) would yield a different particular set of values of the parameters. The above fit is however justified as a first step, as we are mainly interested in a qualitative analysis. Also, the reason that we do not use a fit to the experimental \(P(a|u)\) at \(u = 0\) presented in Ref. \[21\] is an illustrative character of the curve and that the shown range, \(|a|/(a^2)^{1/2} < 15\), is too small to capture information encoded in the long tails that is essential for an accurate determination of the fit parameters.

Following the RIN approach, we will assume that the parameters \(\alpha\) and \(\lambda\) entering \[3\] are stochastic and depend on velocity fluctuations \(u\). In the present paper, the remaining parameters, \(k\), \(D\), \(B\), and \(\nu_0\), are taken to be fixed at the fitted values given in \[3\].

The exponential form of \(\sigma(u)\) has been studied in Ref. \[14\] and was found to be relevant from both the theoretical and experimental points of view. Namely, for Gaussian distributed \(u\) the positive parameter \(\alpha\) is log-normally distributed variable that corresponds to Kolmogorov 1962 (K62) refined theory which assumes log-normal distribution of the stochastic energy dissipation rate per unit mass, \(\varepsilon\), to which we relate \(\alpha\) due to the K62 universality hypothesis, in the inertial range.

The following remark is in order. More precisely, instead of the stochastic energy dissipation rate it seems reasonable to use here the stochastic energy flux through the surface of the domain of a given spatial scale. The fluctuating energy flux is related to the nonlinear term in the Navier-Stokes equation which dominates in the inertial range for high-Reynolds-number flow, while the fluctuating energy dissipation rate is associated specifically to the dissipative scale. Under the stationarity condition (the mean energy dissipation \(\bar{\varepsilon}\) is equal to the injected...
energy), the time variation of the energy contained in
the given scale is defined by the fluctuating energy flux
minus fluctuating part of the energy dissipation rate at
the given scale.

From the experimental point of view, the exponential
form of $\alpha(u)$ leading to the log-normal RIN model
yields the acceleration probability density function with
one free parameter which is in a good agreement with
tails of the experimental acceleration distribution.

Clearly, this approach builds first approximation since
the stochastic energy dissipation rate is known to fol-
low log-normal distribution only approximately. Also, we
note that the stochastic energy dissipation rate is posi-
tive while velocity fluctuations $u$ are not. Hence one is
motivated to seek for specific model forms of the depen-
dence between two stochastic variables (monotonic Borel
function), among which we choose an exponential form
for the function $\alpha(u)$ that means the same form for the
function $\varepsilon(u)$ as we will see below.

Statistical properties of the stochastic energy dissipa-
tion rate and of the velocity fluctuations are not the same.
However, they are related to each other, particularly due
to the wellknown K62 similarity hypothesis. We assume
the relationship $\ln \alpha \sim \ln \varepsilon$ in a statistical sense which
is evidently insensitive to the details related to a power
law functional dependence of $\alpha$ on $\varepsilon$, i.e., insensitive
to this particular type of nonlinearity. This can be
viewed a manifestation of the universality. The choice
of an exponential dependence for $\alpha(u)$ and a power law
dependence for $\alpha(\varepsilon)$ implies the relationship $\varepsilon \sim e^u$, by
which we model the nonlinear relationship between $\varepsilon$ and
$u$. It should be stresses that only absolute value of $u$
contributes the acceleration probability density function,
for normally distributed $u$. The framework for dealing
with more general situation has been recently proposed
in Ref.

From the phenomenological point of view, the expo-
nential form, $\alpha(u) \sim e^{[u]}$, was found to provide appre-
ciable increase of the conditional acceleration variance
$\langle a^2 | u \rangle$ with increasing $|u|$ that meets the experimental
data.

Guided by the above observations the simplest choice
is to try an exponential dependence for $\lambda(u)$, in a phe-
nomenological way. Particularly, in the present paper we take

$$\alpha(u) = e^{[u]}, \quad \lambda(u) = -0.005 e^{3[u]},$$

which recover the values given in Eq. at $|u| = 0$. The procedure
is to refine the guess on $\lambda(u)$ depending on the result.

Of course, the determination of functional forms for
both $\alpha(u)$ and $\lambda(u)$ is essentially (K62) phenomenological
but in general this approach is justified from the turbu-
lence dynamics and allows us to deal with the conditional
statistics of the fluid particle acceleration, which exhibits
a quite nontrivial behavior. It should be emphasized that
the very model is justified by the Navier-Stokes equation
based LDN model, and the very dependence of the
additive noise intensity $\alpha$ (and the cross correlation $\lambda$
) on $u$ is specific to the LDN model, in which the additive
noise was found to depend on small-scale velocities
coupled to large-scale velocities.

The normalization constant $C$ in will be calculated
for each value of $u$, while $C = 3.230$ corresponds to the
case $u = 0$. The conditional probability density function
is thus given by treated in the form $P(a|\alpha(u), \lambda(u))$
with the parameters defined by and 9.

We are now in a position to investigate numerically
how the variation of $u$ affects the conditional statistical
properties of the component of acceleration of fluid part.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{fig1}
\caption{The conditional acceleration probability density function
$P(a|\alpha(u), \lambda(u))$ given by for $k = 1$, $\alpha = e^{[u]}$,
$D = 1.100$, $B = 0.155$, $v_u = 2.910$, $\lambda = -0.005 e^{3[u]}$;
$u = 0, 0.25, 0.50, 0.75, 1.00, 1.19$ in the rms units. The top
curve is conditional on $u = 0$ while the bottom curve is con-
ditional on $u = 1.19$. $x = a/(a^2)^{1/2}$ denotes normalized ac-
celeration.}
\end{figure}
For the normalized velocity fluctuations values $u/(u^2)^{1/2} = 0, 0.25, 0.5, 0.75, 1.0, 1.19$ the distributions $P(a|\alpha(u), \lambda(u))$ and mean accelerations $\langle a|u\rangle/(a^2)^{1/2}$ are shown in Figs. 1 and 2. For convenience the curves in Fig. 1 were shifted by using repeated factor 0.1.

One observes a good qualitative correspondence of the obtained conditional distributions plotted in Fig. 1 with the experimental curves (Fig. 6a in [21]). Note that both the variance and skewness of the distribution $P(a|u) = P(a|\alpha(u), \lambda(u))$ increase for bigger velocity fluctuations $|u|$. While the increase of the variance [related to the increase of $\alpha(u)$] is readily seen, the increase of the skewness [related to the increase of $\lambda(u)$] results in rather small change of the shape of distribution despite the fact that $|\lambda|$ increases by about two orders of magnitude, from $|\lambda|=0.005$ to 0.18. This change could be however readily seen in a plot of the contribution to fourth order conditional moment, $a^4P(a|u)$, as it is, e.g., illustrated in Fig. 4 of Ref. [10]. Notice that the tails of the predicted conditional acceleration distributions in Fig. 1 remain almost the same while the central peaks become weaker for bigger amplitudes of velocity $u$.

The obtained conditional mean acceleration $\langle a|u\rangle/(a^2)^{1/2}$ plotted in Fig. 2 is also in a good qualitative agreement with the experimental dependence $\langle a|u\rangle/(a^2)^{1/2}$ (Fig. 6b in [21]). The mean acceleration is evidently zero for symmetrical distribution ($\lambda = 0$). One observes an appreciable increase of the mean acceleration for bigger velocity fluctuations $|u|$. We note that the experimental dependence of $\langle a|u\rangle/(a^2)^{1/2}$ on $u$ exhibits some asymmetry with respect to $u \to -u$. This feature is not captured by the model since we have chosen $\lambda \approx e^{|u|}$ which is a symmetric function.

It should be stressed that the obtained fit for the mean acceleration is of purely illustrative character. It demonstrates effects produced by nonzero parameter $\lambda$ which depends on velocity fluctuations $u$, on a qualitative level. Our primary purpose was to quantify to which extent variation of $\lambda$ affects the conditional variance of acceleration. This effect has been found ignorably small. Statistically homogeneous and isotropic turbulence is characterized by zero mean acceleration for any component in the laboratory frame of reference. The observed nonzero conditional mean acceleration was claimed to reflect anisotropy of the studied von Karman flow [21]. It is however interesting to note that DNS also reveals slight departures from zero. Probably this is due to approximate character of the isotropy used in the DNS.

The reason that we make this illustration in the present paper is that for the components of acceleration which are aligned to trajectory of fluid particle, some skewness of the acceleration distributions may be present. Namely, for the component pointed transverse to the velocity, $\lambda$ is zero by construction while for the component pointed along the velocity it is nonzero. We remind that the acceleration is calculated due to Lagrangian longitudinal velocity increments, $u(t + \tau) - u(t) = \tau a(t)$, in the dissipative time-scale $\tau$. It is known that Lagrangian longitudinal velocity structure functions of odd order are small but not zero for inertial time-scales so that small skewness of the velocity increments PDF should be observed. We expect that this skewness persists in the dissipative range of time-scales that implies (small) skewness of the acceleration distribution for the corresponding component.

Below we make some remarks regarding numerics.

(i) We have restricted the present numerical study by the upper value $|u|/(u^2)^{1/2} = 1.19$ because the distribution $P(a|\alpha(u),\lambda(u))$ given by (3) turns out to be ill-defined (discontinuous drop appears at some positive value of $a$) for bigger normalized velocity fluctuation values. If this is not due to a failure of the used numerical procedure at big $|\lambda|$ in intermediate calculations, it may mean that some adjustment of the parameters $k$, $D$, $B$, or $\nu_0$ is required to get well-defined $P(a|\alpha(u),\lambda(u))$. Note that it is not presumably well-defined in the entire domain of allowed parameters values due to the presence of logarithm in (4) and imaginary terms. Particularly, we have found that certain increase of the value of $B$ or $\nu_0$ implies well-defined conditional distributions for all $|u|/(u^2)^{1/2}$ up to 1.5 but again these become not continuous for bigger values. Formally, the appearance of discontinuous drop of $P(a|u)$ at some positive $a$ for big $|u|$ leads to a steeper increase (saturation) of the conditional mean acceleration $\langle a|u\rangle$ for big $|u|$ which is however not observed up to $|u|/(u^2)^{1/2} = 2.5$, and the experimental conditional distributions do not exhibit such a behavior up to $|u|/(u^2)^{1/2} = 3$. In the physical context, the condition that one should avoid the emerging ill-defined character can be understood as that in addition to $\alpha(u)$ and $\lambda(u)$ some of the other parameters should depend on $u$ in certain way to provide well-defined distributions for any $u$. This is important in various aspects, e.g., to provide the integration over $u \in [-\infty, +\infty]$ to get the marginal.
distribution \( P(a) \). This issue is of much importance to the present formalism but it is beyond the scope of the present paper and can be considered elsewhere. Another possible reason of the ill-defined character is that the conditional mean acceleration should be very small or zero that requires much steeper increase of the parameter \( \lambda \) for bigger \( |u| \).

(ii) In Fig. 2 we have used \( \langle a^2 | 0 \rangle^{1/2} \) instead of \( \langle a^2 \rangle^{1/2} \) for normalization of the conditional mean acceleration \( \langle a | u \rangle \). This makes a change in the overall constant factor which is obviously not of much importance in the qualitative analysis made in the present paper. For example, smaller value of \( \langle a^2 | 0 \rangle^{1/2} \) would shift the whole curve (triangles) in Fig. 2 up in the vertical direction.

A direct numerical calculation of the mean square, \( \langle a^2 \rangle = \int_{-\infty}^{\infty} |u|^2 \int_{-\infty}^{\infty} P(a | u), \lambda(u) | g(u) | du | \), where \( g(u) \) is Gaussian distribution, with the above set up reveals slow divergence which is associated to the ill-defined character of \( P(a | \alpha(u), \lambda(u)) \) at big \( |u| \) mentioned above.

IV. SUMMARY

(i) Using the RIN approach to the LDN type one-dimensional Langevin model of fluid particle acceleration in developed turbulent flow, we have shown that when the cross correlation parameter is taken in the exponential form, \( \lambda \simeq e^{\epsilon |u|} \), is in a good qualitative agreement with the observed behavior of the experimental mean acceleration conditional on velocity fluctuations \( u \), as shown in Fig. 2. This fit is of purely illustrative character performed with the only purpose to investigate effects produced by nonzero cross correlation parameter \( \lambda(u) \). We stress that the observed conditional mean acceleration is related to the flow anisotropy rather than to the correlation between stretching and vorcity controlled by \( \lambda \). Our primary purpose was to quantify to which extent variation of \( \lambda \) affects the conditional variance of acceleration \( \langle a^2 | u \rangle \). This effect has been found ignorably small as the result of the increase of additive noise intensity \( \alpha(u) \) despite the fact that \( |\lambda(u)| \) increases by about two orders of magnitude. We encountered ill-defined character of the distribution for big values of \( |u| \) that indicates either failure of the used numerical procedure or inappropriateness of this illustrative fit. For homogeneous isotropic case the conditional mean acceleration should be zero in contrast to the experimental data shown in Fig. 2. For \( \lambda = 0 \), the conditional acceleration distribution is well defined for any value of \( u \).

(ii) The additive noise intensity was taken to be \( \alpha \simeq e^{\epsilon |u|} \) that together with \( \lambda \simeq e^{\epsilon |u|} \) have implied the variation of the shape of the conditional probability distribution function \( P(a | \alpha(u), \lambda(u)) \) which qualitatively agrees with the shapes of the experimental \( P(a | u) \) of the transverse component of acceleration at various values of \( u \), as shown in Fig. 4. The increase of \( \alpha \) tends to symmetrize the acceleration distribution.

(iii) Variation of \( |u| \) beyond certain value was found to imply ill-defined conditional distribution \( P(a | u) = P(a | \alpha(u), \lambda(u)) \) probably because \( |\lambda(u)| \) becomes comparable to the additive and multiplicative noise intensities which case requires a more detailed study of the nontrivial dependency of the distribution on \( \lambda \) on the parameters. If this is not due to a failure of the used numerical procedure at big \( |\lambda| \) in intermediate calculations, this problem can be cured by the assumption that some of the other parameters of the model depend on \( u \) in certain way as well, to keep \( P(a | u) \) well-defined for any \( u \).

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[1] C. Tsallis, J. Stat. Phys. 52, 479 (1988).
[2] R. Johal, An interpretation of Tsallis statistics based on polydispersity, cond-mat/9909389 (1999).
[3] A.K. Aringazin and M.I. Mazhitov, Physica A 325, 409 (2003); Quasicanonical Gibbs distribution and Tsallis nonextensive statistics, cond-mat/0204359 (2002).
[4] C. Beck and E.G.D. Cohen, Superstatistics, cond-mat/0205097 (2002).
[5] C. Beck, Lagrangian acceleration statistics in turbulent flow, cond-mat/0212566 (2002).
[6] C. Beck, Phys. Rev. Lett. 87, 180601 (2001); Generalized statistical mechanics and fully developed turbulence, cond-mat/0110075 (2001).
[7] B.L. Sawford, Phys. Fluids A 3, 1577 (1991). S.B. Pope, Phys. Fluids 14, 2360 (2002).
[8] G. Wilk and Z. Wlodarczyk, Phys. Rev. Lett. 84, 2770 (2000).
[9] C. Beck, Physica A 277, 115 (2000); Phys. Lett. A 287, 240 (2001); Europhys. Lett. 57, 329 (2002); Nonadditivity of Tsallis entropies and fluctuations of temperature, cond-mat/0105371 (2001).
[10] A.M. Reynolds, Phys. Fluids 15, L1 (2003).
[11] A.K. Aringazin and M.I. Mazhitov, Phenomenological Gaussian screening in the nonextensive statistics approach to fully developed turbulence, cond-mat/0212462 (2002).
[12] A.K. Aringazin and M.I. Mazhitov, Gaussian factor in the distribution arising from the nonextensive statistics approach to fully developed turbulence, cond-mat/0301040 (2003).
[13] A.K. Aringazin and M.I. Mazhitov, Phys. Lett. A 313, 284 (2003); The PDF of fluid particle acceleration in turbulent flow with underlying normal distribution of velocity fluctuations, cond-mat/0301245 (2003).
[14] A.K. Aringazin and M.I. Mazhitov, One-dimensional Langevin models of fluid particle acceleration in developed turbulence, cond-mat/0305186 (2003), to appear in Phys. Rev. E.
[15] T. Gotoh and R.H. Kraichnan, *Turbulence and Tsallis statistics*, nlin.CD/0305040 (2003).

[16] A.K. Aringazin, *Skewness of probability density functions of fluid particle acceleration in developed turbulence*, cond-mat/0305459 (2003).

[17] J.-P. Laval, B. Dubrulle, and S. Nazarenko, Phys. Fluids **13**, 1995 (2001); *Non-locality and intermittency in 3D turbulence*, physics/0101036 (2001).

[18] J.-P. Laval, B. Dubrulle, and J.C. McWilliams, Phys. Fluids **15**, 1327 (2003).

[19] A. La Porta, G.A. Voth, A.M. Crawford, J. Alexander, and E. Bodenschatz, Nature **409**, 1017 (2001). G.A. Voth, A. La Porta, A.M. Crawford, E. Bodenschatz, J. Alexander, J. Fluid Mech. **469**, 121 (2002); *Measurement of particle accelerations in fully developed turbulence*, physics/0110027 (2001).

[20] A.M. Crawford, N. Mordant, E. Bodenschatz, and A.M. Reynolds, *Comment on "Dynamical foundations of nonextensive statistical mechanics"*, physics/0212080 (2002), submitted to Phys. Rev. Lett.

[21] N. Mordant, A.M. Crawford, and E. Bodenschatz, *Experimental Lagrangian acceleration probability density function measurement*, physics/0303003 (2003).

[22] N. Mordant, P. Metz, O. Michel, and J.-F. Pinton, Phys. Rev. Lett. **87**, 214501 (2001); *Measurement of Lagrangian velocity in fully developed turbulence*, physics/0103084 (2001).

[23] B. Castaing, Y. Gagne, and E.J. Hopfinger, Physica D **46**, 177 (1990).

[24] B. Dubrulle, J.-P. Laval, S. Nazarenko, and O. Zaboronski, *A model for rapid stochastic distortions of small-scale turbulence*, physics/0304035 (2003).