Pseudorandom Phase Ensembles and Non-locality

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Abstract

In this paper, we introduce a new concept of a pseudorandom phase ensemble to simulate a quantum ensemble. A pseudorandom sequence is inseparability and integral that are demonstrated only for a whole sequence, not for a single phase unit, which is similar to that of quantum ensembles and a quantum particle. Using the ensemble concept, we demonstrate non-locality properties for classical fields similar to quantum entanglement.

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Introduction

Quantum entanglement is a physical phenomenon of multiple particles such that the quantum state of each particle cannot be described independently—instead, a quantum state must be described for the system as a whole [1]. Non-locality, which is related to the phenomenon of quantum entanglement, means that the states of two entangled quantum particles are interdependent no matter how far the particles are separated from each other. This interdependence is demonstrated in the correlation measurement that is obtained under the quantum ensemble framework [2].

In quantum mechanics, the classical exclusivity of wave and particle models has given rise to the puzzle of quantum wave–particle duality for a long time [3]. Indeed, it is difficult to understand wave characteristics such as superposition and non-locality using only the particle model. In turn, it is also difficult to describe the indivisibility of a particle using the wave model. Recently, several researches have proposed a new concept of classical entanglement based on classical optical fields by introducing a new degree of freedom to realize the tensor product in quantum entanglement [4–8]. Further, [9, 15] proposed that phase modulation by orthogonal pseudo-random sequences is able to simulate quantum entanglement effectively. In that scheme, these classical fields with an increased degree of freedom not only realized tensor product structure but also simulated the non-local property of quantum entanglement using the properties of orthogonal pseudo-random sequences such as orthogonality, balance, and closure. More importantly, a pseudo-random sequence provides inseparability and integral for a classical field similar to a particle and randomness of measurement similar to a quantum ensemble. In the classical field, the phase freedom is unique because a phase can keep original when the field is propagating and operated on within the coherent length.

Different from other freedoms, a pseudorandom phase sequence is generated using a linear feedback shift register (LFSR) method, which satisfies orthogonal, closure, and balance properties [10]. It has been widely applied to code division multiple access (CDMA) communication technology as a way to distinguish different users [13, 14]. In this paper, we use these properties of the pseudorandom sequence to simulate a quantum particle. A pseudorandom sequence is inseparability and integral that is demonstrated only for a whole sequence, not for a single phase unit, which is similar to that of quantum ensembles and a quantum particle. Therefore, we define a new concept of a pseudorandom phase ensemble
similar to a quantum ensemble in this paper. Further, using this ensemble concept, we
demonstrate non-locality properties for classical fields similar to quantum entanglement.

ENSEMBLE MODEL LABELED BY PSEUDORANDOM PHASE SEQUENCES

In [9, 15], an effective simulation of quantum entanglement using classical fields modulated
with pseudorandom phase sequences (PPSs) was presented. In the current paper, we will
promote this proposal further. To simulate a quantum ensemble, we need to define a new
concept of a pseudorandom phase ensemble similar to a quantum ensemble.

Definition 1 A pseudorandom phase ensemble is defined as a large number of similar
classical fields modulated by different pseudorandom phase sequences, which are labeled by
phase units $\theta_k$.

The PPSs in our proposal derive from orthogonal pseudorandom sequences, which have
been widely applied to CDMA communication technology as a way to distinguish different
users [13, 14]. A set of pseudorandom sequences is generated using a shift register guided
by a Galois field $GF(p)$ that satisfies orthogonal, closure, and balance properties [13, 14].

Definition 2 A pseudorandom phase ensemble is discrete if the phase unit $\theta_k$ is a uniformly
distributed discrete value within $[0, 2\pi]$.

In this paper, we consider an m-sequence (or M-sequence) of period $M-1$ (where $M = p^s$)
generated by a primitive polynomial of degree s over $GF(p)$ and apply it to $p$-ary phase shift
modulation. A scheme is proposed to generate a PPS set $\Xi = \{\lambda^{(0)}, \lambda^{(1)}, \ldots, \lambda^{(M-1)}\}$ over
$GF(p)$. $\lambda^{(0)}$ is an all-0 sequence. Other sequences can be generated using the following
method:

1. Given a primitive polynomial of degree s over $GF(p)$, a base sequence of length $p^s - 1$
is generated using a linear feedback shift register [10];

2. Other sequences are obtained by cyclic shifting of the base sequence;

3. By adding zeroes to the sequences, the occurrence of any element can be set equal to
   $p^s - 1$;

4. Mapping to the phase element $\theta_k$ in $[0, 2\pi]$; hence 0 mapping to 0, 1 mapping to $2\pi/p$,
   $\ldots$, and $p - 1$ mapping to $2(p - 1)\pi/p$.

Further, we define a map $f : \lambda \rightarrow e^{i\lambda}$ on the set of $\Xi$ and obtain a new sequence
set $\Omega = \{\varphi^{(j)} \mid \varphi^{(j)} = e^{i\lambda^{(j)}}, j = 0, \ldots, M - 1\}$. In fact, the map $f$ corresponds to phase
modulations of PPSs of $\Omega$ on the classical field.

**Definition 3** A phase ensemble is complete if finite phase units are ergodic.

According to the characteristics of PPSs, obviously the phase ensemble is discrete and complete. Clearly, we can obtain the following lemma:

**Lemma 1** Classical fields modulated by PPSs constitute a complete discrete phase ensemble.

**Proof.** According to m-sequence theory, the occurrence of each value in the sequence should be the same. The phase ensemble is obviously ergodic in finite length.

**Definition 4** An ensemble average is defined as weighted average of each phase unit $\varphi_k^{(i)}$ within a sequence period as follows:

$$\bar{A} = \frac{1}{M} \sum_{k=1}^{M} \varphi_k^{(i)} A.$$ (1)

**Definition 5** A normalized correlation for two sequences $\varphi^{(i)}$ and $\varphi^{(j)}$ is defined as

$$E(\varphi^{(i)}, \varphi^{(j)}) = \frac{1}{M} \sum_{k=1}^{M} \varphi_k^{(i)} \varphi_k^{(j)*}.$$ (2)

According to the properties of an m-sequence [9], we can obtain the following properties of the set $\Omega$:

(1) the closure property: the product of any sequence in the set remains in the set;

(2) the balance property: with the exception of $\varphi^{(0)}$, any sequence of the set $\Omega$ satisfies

$$\sum_{k=1}^{M} e^{i\theta} \varphi_k^{(j)} = \sum_{k=1}^{M} e^{i(\theta+\lambda_k^{(j)})} = 0, \forall \theta \in R,$$ (3)

(3) the orthogonal property: any two sequences satisfy the following normalized correlation:

$$E(\varphi^{(i)}, \varphi^{(j)}) = \frac{1}{M} \sum_{k=1}^{M} \varphi_k^{(i)} \varphi_k^{(j)*} = \begin{cases} 1, i = j, \\ 0, i \neq j. \end{cases}$$ (4)

According to the properties above, the classical fields modulated with different PPSs become independent and distinguishable. The PPS in our proposal represents a sort of additional degree of freedom, which not only allows us to render the distinct features of different classical fields but also provides a remarkably rich tensor product structure [15].
HILBERT SPACE AND QUBIT BASIS STATES IN THE PSEUDORANDOM PHASE ENSEMBLE MODEL

There are two orthogonal modes (polarization or transverse) of a classical field, which are denoted by $|0\rangle$ and $|1\rangle$, respectively. Thus, a qubit state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ can be expressed by the mode superposition, where $|\alpha|^2 + |\beta|^2 = 1, (\alpha, \beta \in C)$. Obviously, all the mode superposition states span a Hilbert space. Choosing any $N$ PPSs from the set $\Xi$ to modulate $N$ classical fields, we can obtain the states expressed as follows:

$$|\psi_1\rangle = e^{i\lambda_1^{(1)}} (\alpha_1|0\rangle + \beta_1|1\rangle),$$

$$\vdots$$

$$|\psi_N\rangle = e^{i\lambda_k^{(N)}} (\alpha_N|0\rangle + \beta_N|1\rangle).$$

According to the properties of PPSs and Hilbert space, we can define the inner product of any two fields $|\psi_a\rangle$ and $|\psi_b\rangle$. We obtain the orthogonal property in our simulation,

$$\langle \psi_a | \psi_b \rangle = \frac{1}{M} \sum_{k=1}^{M} e^{i(\lambda_k^{(a)} - \lambda_k^{(b)}) (\alpha_a^* \alpha_b + \beta_a^* \beta_b)} = \begin{cases} 1, & a = b, \\ 0, & a \neq b, \end{cases}$$

where $\lambda_k^{(a)}$, $\lambda_k^{(b)}$ are the $k$-th units of $\lambda^{(a)}$ and $\lambda^{(b)}$, respectively. The orthogonal property supports the construction of the tensor product structure of the multiple states.

**Definition 6** A formal product state $|\Psi\rangle$ for the $N$ classical fields is defined as being a direct product of $|\psi_n\rangle$,

$$|\Psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_N\rangle.$$

According to the definition, $N$ classical fields of Eq. (5) can be expressed as the following states:

$$|\psi_n\rangle = \sum_{i=1}^{N} \alpha_i^{(n)} e^{i\lambda_i^{(n)}} |0\rangle + \sum_{j=1}^{N} \beta_j^{(n)} e^{i\lambda_j^{(n)}} |1\rangle.$$

As mentioned in [15], a general form of $|\Psi\rangle$ for $N$ fields can be constructed from Eq. (5) using a gate array model,

$$|\Psi\rangle = \left( \sum_{i=1}^{N} \alpha_i^{(i)} e^{i\lambda_i^{(i)}} |0\rangle + \sum_{j=1}^{N} \beta_j^{(j)} e^{i\lambda_j^{(j)}} |1\rangle \right) \otimes \cdots \otimes \left( \sum_{i=1}^{N} \alpha_i^{(N)} e^{i\lambda_i^{(N)}} |0\rangle + \sum_{j=1}^{N} \beta_j^{(N)} e^{i\lambda_j^{(N)}} |1\rangle \right).$$
Further, we can obtain each item of the superposition of $|\Psi\rangle$ as follows:

$$
C_{00\cdots0}|00\cdots0\rangle = \left[\left(\sum_{i=1}^{N} \alpha^1_{i} e^{i\lambda_{i}}\right) \left(\sum_{i=1}^{N} \alpha^2_{i} e^{i\lambda_{i}}\right) \cdots \left(\sum_{i=1}^{N} \alpha^{N}_{i} e^{i\lambda_{i}}\right)\right]|00\cdots0\rangle,
$$

$$
C_{00\cdots1}|00\cdots1\rangle = \left[\left(\sum_{i=1}^{N} \alpha^1_{i} e^{i\lambda_{i}}\right) \left(\sum_{i=1}^{N} \alpha^2_{i} e^{i\lambda_{i}}\right) \cdots \left(\sum_{j=1}^{N} \beta^{j}_{N} e^{i\lambda_{j}}\right)\right]|00\cdots1\rangle,
$$

$$
\vdots
$$

$$
C_{11\cdots1}|11\cdots1\rangle = \left[\left(\sum_{j=1}^{N} \beta^1_{j} e^{i\lambda_{j}}\right) \left(\sum_{j=1}^{N} \beta^2_{j} e^{i\lambda_{j}}\right) \cdots \left(\sum_{j=1}^{N} \beta^{N}_{N} e^{i\lambda_{j}}\right)\right]|11\cdots1\rangle.
$$

\begin{equation}
(11)
\end{equation}

According to the closure property, the phase sequences of $C_{i_1i_2\cdots i_N}$ remain in the set $\Omega$, which means $C_{i_1i_2\cdots i_N} = \sum_{j=1}^{M} C_{i_1i_2\cdots i_N}^{(j)} e^{i\lambda_{(j)}}$. Therefore, we obtain the following theorem:

Theorem 1 The formal product state $|\Psi\rangle$ spans a Hilbert space with the basis

$$
\left\{e^{i\lambda_{(j)}} |i_1 i_2 \cdots i_N\rangle \middle| e^{i\lambda_{(j)}} \in \Omega, j = 1 \cdots M, i_n = 0or1\right\}
$$

and can be expressed as follows:

$$
|\Psi\rangle = \sum_{i_1=0}^{1} \cdots \sum_{i_N=0}^{1} \sum_{j=1}^{M} C_{i_1i_2\cdots i_N}^{(j)} e^{i\lambda_{(j)}} |i_1 i_2 \cdots i_N\rangle,
$$

\begin{equation}
(12)
\end{equation}

where $C_{i_1i_2\cdots i_N}^{(j)}$ denotes a total of $M2^N$ coefficients.

**NON-LOCALITY IN THE PSEUDORANDOM PHASE ENSEMBLE MODEL**

A single measurement result of a quantum particle cannot provide the probability distribution predicted by a wave function that requires many measurements based on the concept of a quantum ensemble. The non-locality correlation demonstrated in quantum entanglement also depends on the ensemble summaries of many measurement results. Similarly, we examine the non-locality correlation demonstrated in classical fields under the pseudorandom phase ensemble framework. In order to demonstrate the tensor product structure, we classify the form product states using the phase sequence.

Definition 7 A consensus PPS sub-state (CPSS) is defined as being items with the same PPS in the formal product state $|\Psi\rangle$.

Definition 8 A single PPS sub-state (SPSS) is defined as being each of the items, except all consensus PPS sub-states, in the formal product state $|\Psi\rangle$.

For simplicity, we assume that $N_1$ CPSS $\left\{|S^{(1)}_1\rangle, |S^{(1)}_2\rangle, \cdots, |S^{(1)}_{N_1}\rangle\right\}$ corresponding to the same PPS $\lambda^{(s_1)}$, $\ldots$, $N_m$ CPSS $\left\{|S^{(m)}_1\rangle, |S^{(m)}_2\rangle, \cdots, |S^{(m)}_{N_m}\rangle\right\}$ corresponding to the same
PPS $\lambda^{(s_m)}$, other $N'$ SPSS in the formal product state $|\Psi\rangle$. Thus $|\Psi\rangle$ can be expressed as

$$|\Psi\rangle = e^{i\lambda^{(s_1)}} \sum_{i=1}^{N_1} C_i \left| S_1^{(1)} \right\rangle + \cdots + e^{i\lambda^{(s_m)}} \sum_{j=1}^{N_m} C_j \left| S_j^{(m)} \right\rangle + \sum_{k=1}^{N'} C_k e^{i\lambda^{(k)}} |x_k\rangle,$$

(13)

where $\lambda^{(s_1)}, \ldots, \lambda^{(s_m)}$, and $\lambda^{(k)}$ are non-redundant PPSs.

Further, we introduce the definition of the density matrix $\rho$:

$$\rho = \frac{2^N}{M} \sum_{n=1}^{2^N} |C_n|^2 |x_n\rangle \langle x_n| + \sum_{k=1}^{N_k} \sum_{i \neq i'} \left( C_i^* C_{i'} \left| S_i^{(k)} \right\rangle \langle S_{i'}^{(k)} | + C_{i'}^* C_i \left| S_{i'}^{(k)} \right\rangle \langle S_i^{(k)} | \right)$$

$$+ \sum_{k \neq l=1}^{N_k} \sum_{i=1}^{N_i} \sum_{j=1}^{N_j} \left( C_j^* C_l \left| S_j^{(l)} \right\rangle \langle S_l^{(l)} | e^{i(\lambda^{(s_j)} - \lambda^{(s_l)})} + C_l^* C_j \left| S_l^{(l)} \right\rangle \langle S_j^{(l)} | e^{i(\lambda^{(s_j)} - \lambda^{(s_l)})} \right)$$

$$+ \sum_{l=1}^{m} \sum_{i=1}^{N_i} \sum_{k=1}^{N_k'} \left( C_i^* C_k \left| x_k \right\rangle \langle x_k | e^{i\lambda^{(s_i)} - \lambda^{(s_k)}} + C_k^* C_i \left| x_k \right\rangle \langle x_k | e^{i\lambda^{(s_i)} - \lambda^{(s_k)}} \right).$$

(15)

By applying phase ensemble averaging $\frac{1}{M} \sum_{k=1}^{M} e^{i\lambda^{(k,m)}} = 0$, the ensemble-averaged density matrix is defined by

$$\bar{\rho} \equiv \frac{1}{M} \sum_{k=1}^{M} \rho = \frac{2^N}{M} \sum_{n=1}^{2^N} |C_n|^2 |x_n\rangle \langle x_n| + \sum_{k=1}^{N_k} \sum_{i \neq i'} \left( C_i^* C_i \left| S_i^{(k)} \right\rangle \langle S_{i'}^{(k)} | + C_{i'}^* C_i \left| S_{i'}^{(k)} \right\rangle \langle S_i^{(k)} | \right).$$

(16)

Note that all off-diagonal elements of the density matrix are contributed from the CPSS after ensemble averaging. Also, it shows that the ensemble-averaged reduced density matrix might not be expressed in terms of a direct product of the states $|x_n\rangle$, identical to the case of quantum entanglement states.

In the phase ensemble, an expectation value can be obtained for an arbitrary operator $\hat{P}$ in any formal product state:

$$\bar{\hat{P}} \equiv \frac{1}{M} \sum_{k=1}^{M} tr \left( \rho \hat{P} \right).$$

(17)
Further, using the exchange of summation and matrix trace can be exchanged, the expectation value can be simplified to

$$\bar{P} = tr \left[ \left( \frac{1}{M} \sum_{k=1}^{M} \rho \right) \hat{P} \right] = tr \left( \tilde{\rho} \hat{P} \right)$$

$$= \sum_{n=1}^{2^N} |C_n|^2 \langle x_n | \hat{P} | x_n \rangle + \sum_{k=1}^{m} \sum_{i \neq i' = 1}^{N_k} \left( C_i^* C_i \left\langle S_i^{(k)} | \hat{P} | S_i^{(k)} \right\rangle + C_i^* C_i' \left\langle S_i^{(k)} | \hat{P} | S_i'^{(k)} \right\rangle \right)$$

(18)

It is worth noting that non-local (off-diagonal items) is contributed from CPSSs in the correlation measurement. Therefore, we can obtain the following theorem:

**Theorem 2** In a pseudorandom phase ensemble, non-local correlations originate from the CPSSs.

In conclusion, a quantum state simulated by classical fields formally agrees with CPSSs in the formal product state under the pseudorandom phase ensemble framework.

**MINIMUM COMPLETE PHASE ENSEMBLE**

In a pseudorandom phase ensemble model, we are interested in the simplest model that requires minimal resources to be constructed. This model is the minimum complete phase ensemble and is defined as follows:

**Definition 9** The minimum complete phase ensemble is defined as that ensemble, which is ergodic with the least classical fields.

**Definition 10** The minimum complete state is defined as that state, which only has a CPSS within the minimum complete ensemble.

By definition, the state in Eq. (5) is a type of minimal complete state.

**Lemma 2** The CPSS of a minimum complete state has one and only one PPS $\lambda^{(S)} = \sum_{n=1}^{N} \lambda^{(n)}$, which is the sum of all phase sequences.

**Proof.** According to the definition of a minimum complete phase ensemble, $N$ fields must correspond to $N$ different phase sequences. The simplest case is as following,

$$\left( e^{i\lambda^{(1)} |i_1\rangle} \otimes e^{i\lambda^{(2)} |i_2\rangle} \otimes \cdots \otimes e^{i\lambda^{(N)} |i_N\rangle} \right) = e^{i \sum_{n=1}^{N} \lambda^{(n)} |i_1 i_2 \cdots i_N\rangle} = e^{i \lambda^{(S)} |i_1 i_2 \cdots i_N\rangle}.$$  

(19)

Considering all possible combinations, we can obtain $N!$ combinations of classical fields and phase sequences. All combinations can be expressed in the same form, $e^{i\lambda^{(S)} |i_1 i_2 \cdots i_N\rangle}$, in the formal product state $|\Psi\rangle$ and with the same PPS $e^{i\lambda^{(S)}}$. 
Further, we can obtain the following theorem:

**Theorem 3** In the formal product state $|\Psi\rangle$, a CPSS $e^{i\lambda^{(s)}} |i_1i_2 \cdots i_N\rangle$ has $N!$ equivalent direct product decompositions, corresponding to $N!$ combinations of $N$ classical fields and phase sequences $\lambda^{(1)}, \ldots, \lambda^{(N)}$.

Now, we can express a minimum complete state $|\Psi\rangle$ as follows:

$$|\Psi\rangle = e^{i\lambda^{(s)}} \sum_{i=1}^{N'} C_i |x_i\rangle + \sum_{i=1}^{N''} C_k e^{i\lambda^{(k)} |x_k\rangle = e^{i\lambda^{(s)}} \left( \sum_{i=1}^{N'} C_i |x_i\rangle + \sum_{i=1}^{N''} C_k e^{i(\lambda^{(k)} - \lambda^{(s)})} |x_k\rangle \right) ,$$

where $e^{i\lambda^{(k)}} |x_k\rangle$ corresponds to all SPSSs. According to the analysis in the last section, the ensemble-averaged density matrix can be obtained:

$$\tilde{\rho} = \sum_{n=1}^{2^N} |C_n|^2 |x_n\rangle \langle x_n| + \sum_{i\neq i'=1}^{N'} \left( C_{i'}^* C_i |x_i\rangle \langle x_{i'}| + C_i^* C_{i'} |x_{i'}\rangle \langle x_i| \right) .$$

In conclusion, a minimum complete state satisfies the necessary conditions for the simulation of quantum states.

In [15], using a gate array model, classical fields are transformed from initial states as given in Eq. (5) to final states as given in Eq. (9). In order to simulate certain quantum states, a sequential cycle permutation mechanism based on quadrature demodulation is proposed in [15]. The sequential cycle permutation is shown as follows:

$$R_1 = \{\lambda^{(1)}, \lambda^{(2)}, \ldots, \lambda^{(N)}\} ,$$
$$R_2 = \{\lambda^{(2)}, \lambda^{(3)}, \ldots, \lambda^{(1)}\} ,$$
$$\vdots$$
$$R_N = \{\lambda^{(N)}, \lambda^{(1)}, \ldots, \lambda^{(N-1)}\} .$$

It is clear that the sequential cycle permutation is a subset of the full sequential permutation. According to Theorem 3, the simulation obtained in [15] is a minimum complete state. Therefore, we can state the following proposition:

**Proposition 1** A sequential cycle permutation method can be used to obtain minimum complete states.
**Proof.** According to [15], each state corresponds to a sequential cycle permutation,

\[
R_1 : \left( e^{i\lambda_1} |i_1\rangle \right) \otimes \left( e^{i\lambda_2} |i_2\rangle \right) \cdots \otimes \left( e^{i\lambda_N} |i_N\rangle \right) = e^{i\lambda} |i_1i_2\cdots i_N\rangle,
\]

(23)

\[
R_2 : \left( e^{i\lambda_2} |i_1\rangle \right) \otimes \left( e^{i\lambda_3} |i_2\rangle \right) \cdots \otimes \left( e^{i\lambda_1} |i_N\rangle \right) = e^{i\lambda} |i_1i_2\cdots i_N\rangle,
\]

\[\vdots\]

\[
R_N : \left( e^{i\lambda_N} |i_1\rangle \right) \otimes \left( e^{i\lambda_1} |i_2\rangle \right) \cdots \otimes \left( e^{i\lambda_{N-1}} |i_N\rangle \right) = e^{i\lambda} |i_1i_2\cdots i_N\rangle.
\]

Hence, each sequential cycle permutation provides a subset of minimum complete states.

**DISCUSSION AND CONCLUSION**

After introducing the concept of a pseudorandom phase ensemble, classical fields can demonstrate non-locality similar to quantum states. We consider that this kind of non-locality is derived from classical entanglement [4–9]. This concept might reveal a possible origin of quantum entanglement in quantum mechanics, which may be a kind of phase mechanism [16]. Further research will focus on simulation and reconstruction of arbitrary quantum states, which might be dependent on some sequential mechanism which in turn corresponds to a gate array model. These will be discussed in a future paper.

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