$X(3872), \ I^G(J^{PC}) = 0^+(1^{++}), \ \text{as the } \chi_{1c}(2P) \ \text{charmonium}$

N.N. Achasov$^a$ and E.V. Rogozina$^{a,b}$

$^a$Laboratory of Theoretical Physics, Sobolev Institute for Mathematics, 630090, Novosibirsk, Russia

$^b$Novosibirsk State University, 630090, Novosibirsk, Russia

(Dated: January 16, 2015)

Abstract

Reasons are given that $X(3872), \ I^G(J^{PC}) = 0^+(1^{++}), \ \text{is the } \chi_{1c}(2P) \ \text{charmonium}$. Possibility of verification of this assumption is discussed.

PACS numbers: 13.75.Lb, 11.15.Pg, 11.80.Et, 12.39.Fe
The two dramatic discoveries have generated a stream of the $D^{*0}D^0 + D^0D^{*0}$ molecular interpretations of the $X(3872)$ resonance.

The mass of the $X(3872)$ resonance is 50 MeV lower than predictions of the most lucky naive potential models for the mass of the $\chi_{c1}(2P)$ resonance,

$$m_X - m_{\chi_{c1}(2P)} = -\Delta \approx -50 \text{MeV},$$

and the relation between the branching ratios

$$BR(X \rightarrow \pi^+\pi^-\pi^0J/\psi(1S)) \sim BR(X \rightarrow \pi^+\pi^-J/\psi(1S)),$$

that is interpreted as a strong violation of isotopic symmetry.

But the bounding energy is small, $\epsilon_B \lesssim (1 \div 3) \text{MeV}$. That is, the radius of the molecule is large, $r_{X(3872)} \simeq (3 \div 5) \text{fm} = (3 \div 5) \cdot 10^{-13} \text{cm}$. As for the charmonium, its radius is less one fermi, $r_{\chi_{c1}(2P)} \approx 0.5 \text{fm} = 0.5 \cdot 10^{-13} \text{cm}$. That is, the molecule volume is 100 ÷ 1000 times as large as the charmonium volume, $V_{X(3872)}/V_{\chi_{c1}(2P)} \simeq 100 \div 1000$.

How to explain sufficiently abundant inclusive production of the rather extended molecule $X(3872)$ in a hard process $pp \rightarrow X(3872) + \text{anything}$ with rapidity in the range 2.5 - 4.5 and transverse momentum in the range 5-20 GeV \[1\]? Really,

$$\sigma(pp \rightarrow X(3872) + \text{anything})BR(X(3872) \rightarrow \pi^+\pi^-J/\psi) = 5.4 \text{nb}$$

(3)

and

$$\sigma(pp \rightarrow \psi(2S) + \text{anything})BR(\psi(2S) \rightarrow \pi^+\pi^-J/\psi) = 38 \text{nb}.$$  

(4)

But, according to Ref. \[2\],

$$BR(\psi(2S) \rightarrow \pi^+\pi^-J/\psi) = 0.34$$

(5)

while

$$0.023 < BR(X(3872) \rightarrow \pi^+\pi^-J/\psi) < 0.066$$

(6)

according to Ref. \[3\]. So,

$$0.74 < \frac{\sigma(pp \rightarrow X(3872) + \text{anything})}{\sigma(pp \rightarrow \psi(2S) + \text{anything})} < 2.1.$$  

(7)

The extended molecule is produced in the hard process as intensively as the compact charmonium. It’s a miracle.
FIG. 1: The contribution of the $\bar{D}^0D^0$ and $D^-D^{**}$ loops into the self energy of the $X(3872)$ resonance.

As for the problem of the mass shift, Eq. (1), the contribution of the $D^-D^{**}$ and $\bar{D}^0D^0$ loops, see Fig. 1, into the self energy of the $X(3872)$ resonance, $\Pi_X(s)$, solves it easily.

$$\Pi_X(s) = \Pi_X^{D^0\bar{D}^0}(s) + \Pi_X^{D^-D^{**}}(s) = \frac{g_A^2}{8\pi^2} \left( I^{D^0\bar{D}^0}(s) + I^{D^-D^{**}}(s) \right),$$  \hspace{1cm} (8)

where

$$I^{D\bar{D}^*}(s) = \int_{m^2_+}^{\Lambda^2} \frac{\sqrt{(s' - m_+^2)(s' - m^2)}}{s'(s' - s)} ds' \approx 2\ln \frac{2\Lambda}{m_+} - 2\sqrt{\frac{m_+^2 - s}{s}} \arctan \sqrt{\frac{s}{m_+^2 - s}},$$  \hspace{1cm} (9)

where

$$m_+ = m_{D^*} + m_D, \quad m_- = m_{D^*} - m_D, \quad s < m_+^2, \quad \Lambda^2 \gg m_+^2.$$  \hspace{1cm} (10)

For the calculations we use the Lagrangian

$$L(x) = g_A X^\mu \left( D_\mu(x)\bar{D}(x) + \bar{D}_\mu(x)D(x) \right)$$
$$= g_A X^\mu \left( D^\mu_0(x)\bar{D}^0(x) + \bar{D}^\mu_0(x)D^0(x) + D^+_\mu(x)D^-(x) + D^+_\mu(x)\bar{D}^-(x) \right).$$  \hspace{1cm} (11)

The width of the $X \rightarrow D^{*0}\bar{D}^0 + c.c.$ decay

$$\Gamma(X \rightarrow D^{*0}\bar{D}^0 + c.c., s) \approx (g_A^2/8\pi)(2|\vec{k}|/s).$$  \hspace{1cm} (12)

The inverse propagator of the $X(3872)$ resonance

$$D_X(s) = m_{X\chi(2P)}^2 - s - \Pi_X(s) - i m_X \Gamma,$$  \hspace{1cm} (13)

where $\Gamma = \Sigma_i \Gamma_i$ is the total width of the $X(3872)$ decays into all $\{i\}$ non-$D^{*0}\bar{D}^0 + c.c.$ channels. According to Refs. [4] and [5] $\Gamma < 1.2$ MeV!
The renormalization of mass \[6\]

\[
m_{\chi c(2P)}^2 - m_X^2 - \Pi_X(m_{\chi}^2) = 0
\] (14)

results in

\[
\Delta (2m_X + \Delta) = \Pi_X(m_{\chi}^2) \approx \left(\frac{g_A^2}{8\pi^2}\right)4\ln(2L/m_+). \tag{15}
\]

The renormalized propagator has the form \[7\]

\[
D_X(s) = m_X^2 - s + \Pi_X'(m_{\chi}^2) - \Pi_X'(s) - m_X\Gamma.
\] (16)

If \(\Delta = m_{\chi c(2P)} - m_X \approx 50\) MeV, see Eq. (11), then \(g_A^2/8\pi \approx 0.2\) GeV\(^2\) for \(L = 10\) GeV and \(g_A^2/8\pi \approx 0.1\) GeV\(^2\) for \(L = 100\) GeV. According to Ref. [5] such \(g_A^2/8\pi\) results in \(BR(X \to D^0\bar{D}^{*0} + D^0D^{*0}) \approx 0.3\) and \(BR(X \to D^0\bar{D}^{*0} + \bar{D}^0D^{*0}) \approx 0.2\), respectively.

Thus, we expect that unknown decays of \(X(3872)\) into non-\(D^{*0}\bar{D}^0 + c.c.\) states are considerable or dominant. For details see Ref. [3]. The discovery of these decays would be the strong (if not decisive) confirmation of our scenario.

As for \(BR(X \to \omega J/\psi) \sim BR(X \to \rho J/\psi)\), Eq. (2), this could be a result of dynamics. In our scenario the \(\omega J/\psi\) state is produced via the three gluons, see Fig. 2(a). As for the \(\rho J/\psi\) state, it is produced both via the one photon, see Fig. 2(b), and via the three gluons (via the contribution \(\sim m_u - m_d\)), see Fig. 2(a).

\[\begin{align*}
\text{(a)} & \quad \text{The three gluon production of the } \omega \text{ and } \rho \text{ mesons (via the contribution } \sim m_u - m_d), \\
\text{(b)} & \quad \text{The one photon production of the } \rho \text{ meson. All possible permutations of gluons (and photon) are assumed.}
\end{align*}\]
Close to our scenario is an example of the $J/\psi \rightarrow \rho \eta'$ and $J/\psi \rightarrow \omega \eta'$ decays. According to Ref. [2]

$$BR(J/\psi \rightarrow \rho \eta') = (1.05 \pm 0.18) \cdot 10^{-4} \quad \text{and} \quad BR(J/\psi \rightarrow \omega \eta') = (1.82 \pm 0.21) \cdot 10^{-4}.$$ (17)

It is well known that the physics of charmonium ($c\bar{c}$) and bottomonium ($b\bar{b}$) is similar. Let us compare the already known features of $X(3872)$ with the ones of $\Upsilon_{b1}(2P)$.

Recently, the LHCb Collaboration published a landmark result [8]

$$\frac{BR(X \rightarrow \gamma \psi(2S))}{BR(X \rightarrow \gamma J/\psi)} = C_X \left(\frac{\omega_{\psi(2S)}}{\omega_{J/\psi}}\right)^3 = 2.46 \pm 0.7,$$ (18)

where $\omega_{\psi(2S)}$ and $\omega_{J/\psi}$ are the energies of the photons in the $X \rightarrow \gamma \psi(2S)$ and $BR(X \rightarrow \gamma J/\psi)$ decays, respectively.

On the other hand, it is known [2] that

$$\frac{BR(\chi_{b1}(2P) \rightarrow \gamma \Upsilon(2S))}{BR(\chi_{b1}(2P) \rightarrow \gamma \Upsilon(1S))} = C_{\chi_{b1}(2P)} \left(\frac{\omega_{\Upsilon(2S)}}{\omega_{\Upsilon(1S)}}\right)^3 = 2.16 \pm 0.28,$$ (19)

where $\omega_{\Upsilon(2S)}$ and $\omega_{\Upsilon(1S)}$ are the energies of the photons in the $\chi_{b1}(2P) \rightarrow \gamma \Upsilon(2S)$ and $\chi_{b1}(2P) \rightarrow \gamma \Upsilon(1S)$ decays, respectively.

Consequently,

$$C_X = 136.78 \pm 38.89$$ (20)

and

$$C_{\chi_{b1}(2P)} = 80 \pm 10.37$$ (21)

as all most lucky versions of the potential model predict for the quarkonia, $C_{\chi_{c1}(2P)} \gg 1$ and $C_{\chi_{b1}(2P)} \gg 1$.

According to Ref. [2]

$$BR(\chi_{b1}(2P) \rightarrow \omega \Upsilon(1S)) = \left(1.63 \pm 0.4\right)^\%.$$ (22)

If the one photon mechanism dominates in the $X(3872) \rightarrow \rho J/\psi$ decay, see Fig.2(b), then one should expect

$$BR(\chi_{b1}(2P) \rightarrow \rho \Upsilon(1S)) \sim (e_b/e_c)^2 \cdot 1.6 \% = (1/4) \cdot 1.6 \% = 0.4\%,$$ (23)

where $e_c$ and $e_b$ are the charges of the $c$ and $b$ quarks, respectively.
If the three gluon mechanism (its part \( \sim m_u - m_d \)) dominates in the \( X(3872) \rightarrow \rho J/\psi \) decay, see Fig.2(a), then one should expect

\[
\text{BR}(\chi_{b1}(2P) \to \rho \Upsilon(1S)) \sim 1.6\%.
\] (24)

We believe that discovery of a significant number unknown decays of \( X(3872) \) into non-\( D^* \bar{D}^0 + c.c. \) states and discovery of the \( \chi_{b1}(2P) \rightarrow \rho \Upsilon(1S) \) decay could decide destiny of \( X(3872) \).

This work was supported in part by RFBR, Grant No 13-02-00039, and Interdisciplinary project No 102 of Siberian division of RAS.

[1] R. Aaij et al. (LHCb Collaboration), Eur. Phys. J. C. 72, 1972 (2012).
[2] K.A. Olive et al. (Particle Data Group), Chin. Phys. C 38, 090001 (2014).
[3] C.-Z. Yuan (Belle Collaboration), arXiv: 0910.3138 [hep-ex], Proceedings of the XXIX PHYSICS IN COLLISION, 2009, Kobe, Japan.
[4] S.K. Choi et al. (Belle Collaboration), Phys. Rev. D 84, 052004 (2011).
[5] N.N. Achasov and E.V. Rogozina, Pis’ma v ZhETF 100, 252 (2014) [JETP Lettrs 100, 227 (2014)].
[6] When \( (m_{D_0} + m_{D^*})^2 < s < (m_{D^+} + m_{D^{*-}})^2 \), the renormalization of mass has the form
\[
m_{\chi_{b1}(2P)}^2 - m_X^2 = -\text{Re}(\Pi_{D^0 D^{*0}}^D(m_X^2)) - \Pi_{D^+ D^{*-}}^D(m_X^2) = 0.
\]
[7] The exact formulae of \( \text{Re} \Pi_X^D(m_X^2) - \Pi_X^D(s) \) in all regions of \( s \) can be found in Ref. [5].
[8] R. Aaij et al. (LHCb Collaboration), Nucl. Phys. B 886, 665 (2014).