Hot nuclear matter in the modified quark-meson coupling model with quark-quark correlations

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Abstract

Short-range quark-quark correlations in hot nuclear matter are examined within the modified quark-meson coupling model (MQMC) by adding repulsive scalar and vector quark-quark interactions. Without these correlations, the bag radius increases with the baryon density. However when the correlations are introduced the bag size shrinks as the bags overlap. Also as the strength of the scalar quark-quark correlation is increased, the decrease of the effective nucleon mass $M_N^*$ with the baryonic density is slowed down and tends to saturate at high densities. Within this model we study the phase transition from the baryon-meson phase to the quark-gluon plasma (QGP) phase with the latter modeled as an ideal gas of quarks and gluons inside a bag. Two models for the QGP bag parameter are considered. In one case, the bag is taken to be medium-independent and the phase transition from the hadron phase to QGP is found to occur at 5-8 times ordinary nuclear matter density for temperatures less than 60 MeV. For lower densities, the transition takes place at higher temperature reaching up to 130 MeV at zero density. In the second case, the QGP bag parameter is considered medium-dependent as in the MQMC model for the hadronic phase. In this case, it is found that the phase transition occurs at much lower densities.

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I. INTRODUCTION

The MQMC model has been recently used to study cold \[1,2\] and hot \[3,4\] nuclear matter. In this model \[5–7\], nucleons are assumed to be nonoverlapping MIT bags interacting through scalar $\sigma$ and vector $\omega$ mean fields coupled to the quarks themselves. In analogy with the nontopological soliton model \[8\], the bag parameter is assumed to decrease when the scalar mean field $\sigma$ increases \[1,2\], which makes the bag parameter medium- or density-dependent. However, as a result of introducing this medium-dependent bag parameter, it is found that as the baryonic density $\rho_B$ increases, the nucleon bag radius increases \[2,9\]. At some value of $\rho_B$, the bags start to overlap \[10\]. Since the MQMC model assumes that the bags do not overlap, the use of the this model has been limited to small and moderate baryonic densities \[2\].

One way to extend the model is to include short-range quark-quark correlations \[10\] which become important when the bags overlap at high baryonic densities. These correlations are introduced by adding extra repulsive scalar and vector contact forces between the quarks in the overlapping bags to reduce the overlapping domain between the nucleons. We will follow Saito et al. \[10\] and introduce these correlations in a simple geometrical way by defining a critical rigid-ball nucleonic radius $R_c$ which, assuming close packing, can be related to the baryonic density by $R_c = (1/4\sqrt{2}\rho_B)^{1/3}$. Hence, for a given nuclear density $\rho_B$, the nucleon bags are assumed to overlap only when the bag radius $R$ is larger than $R_c$. When the nucleon bags overlap, the quarks in the bags correlate with each other by a repulsive potential to reduce the overlapping effect by shrinking the size of the bags. Within this model, we shall study the quark-quark correlations for hot nuclear matter as well as their effects on the phase transition from the hadronic phase to the quark-gluon-plasma (QGP) phase. The QGP is considered as an ideal gas of noninteracting quarks and gluons inside a bubble or bag with bag parameter $B$ \[11\]. It is interesting to study the variation of the phase transition with the strength of the quark-quark correlation and to examine the possibility that the bag parameter for the QGP is also medium-dependent as in the MQMC model.

The outline of the paper is as follows. In Sect. II, the quark-quark correlations for the overlapping bags are generalized to the case of hot nuclear matter. In Sect. III we introduce a simple model for the QGP. Finally, Sect. III is devoted to our results and conclusions.

II. QUARK-QUARK CORRELATIONS

In dense nuclear matter, the nucleons are expected to overlap and the quarks in one nucleon correlate with the quarks in another. This correlation depends basically on how much the nucleons overlap with each other. The probability $P(R_c/R)$ for two nucleons, each of radius $R$, to overlap can be estimated, using a simple geometrical approach \[10,12\], to be

$$P \left( \frac{R_c}{R} \right) = \left[1 - \frac{3}{4} \left( \frac{2R_c}{R} \right) + \frac{1}{16} \left( \frac{2R_c}{R} \right)^3 \right] \theta \left( \frac{R_c}{R} \right) \theta \left( 1 - \frac{R_c}{R} \right).$$

(1)

Since the quark-quark correlations are of short range it is reasonable to approximate them by a contact interaction \[11\]. In the mean-field approximation, the Dirac equation for the quark field inside a nucleon bag is given by
\[
\left[ i \gamma \cdot \partial - (m_q - g_\sigma^q \sigma + f_s^q < \bar{\psi}_q \psi_q >) - (g_\omega^q \omega + f_v^q < \psi_q^\dagger \psi_q >) \beta \right] \psi_q = 0 \quad (2)
\]

where \( m_q \) is the current quark mass, \( f_s^{(v)} \) is the coupling constant for scalar (vector)-type short-range correlations while \( < \bar{\psi}_q \psi_q > \) and \( < \psi_q^\dagger \psi_q > \) are the average values of the quark scalar density and quark density [3]. The latter, following Ref. [10], are approximated by \( < \psi_q^\dagger \psi_q > = m_q^2 \sigma \) and \( < \bar{\psi}_q \psi_q > = 3 \rho_B \). In the present work, as suggested by Ref. [10], the correlation potentials are taken as

\[
f_s^q < \bar{\psi}_q \psi_q > = \alpha P \left( R_c/R \right) \sigma, \quad (3)
\]

and

\[
f_v^q < \psi_q^\dagger \psi_q > = \beta P \left( R_c/R \right) \rho_B, \quad (4)
\]

where \( \alpha \) and \( \beta \) are parameters used to control the strengths of the scalar and vector quark-quark correlations. Note that as defined here \( \alpha \) is a dimensionless parameter, while \( \beta \) has the dimensions of \( 1 \text{/(Energy)}^2 \). The coupling constants \( g_\sigma^q \) and \( g_\omega^q \) for the scalar and vector mean fields are determined by reproducing the properties of normal nuclear matter.

The single-particle quark and antiquark energies in units of \( R^{-1} \) are given as

\[
e_{\pm}^{n\kappa} = \Omega^{n\kappa} \pm \left[ g_\omega^q \omega + f_s^q < \psi_q^\dagger \psi_q > \right] R, \quad (5)
\]

where

\[
\Omega^{n\kappa} = \sqrt{x_{n\kappa}^2 + R^2 m_q^*} \quad (6)
\]

and \( m_q^* = m_q^0 - g_\sigma^q \sigma + f_s^q < \bar{\psi}_q \psi_q > \) is the effective quark mass. The boundary condition at the bag surface is given by

\[
i \gamma \cdot \hat{n} \bar{\psi}_q^{n\kappa} = \psi_q^{n\kappa}, \quad (7)
\]

which determines the quark momentum \( x_{n\kappa} \) in the state characterized by specific values of \( n \) and \( \kappa \). The quark chemical potential \( \mu_q \), assuming that there are three quarks in the nucleon bag, is determined through

\[
n_q = 3 = 3 \sum_{n\kappa} \left[ \frac{1}{e^{(\epsilon_{n\kappa}^+ / R - \mu_q) / T} + 1} - \frac{1}{e^{(\epsilon_{n\kappa}^- / R + \mu_q) / T} + 1} \right]. \quad (8)
\]

The total energy from the quarks and antiquarks is

\[
E_{\text{tot}} = 3 \sum_{n\kappa} \frac{\Omega^{n\kappa}}{R} \left[ \frac{1}{e^{(\epsilon_{n\kappa}^+ / R - \mu_q) / T} + 1} + \frac{1}{e^{(\epsilon_{n\kappa}^- / R + \mu_q) / T} + 1} \right]. \quad (9)
\]

The bag energy is given by

\[
E_{\text{bag}} = E_{\text{tot}} - \frac{Z}{R} + \frac{4\pi}{3} R^3 B(\sigma). \quad (10)
\]
where $B(\sigma)$ is the bag parameter. The medium effects are taken into account for the bag parameter $B$

$$B = B_0 \exp \left( -\frac{4g_B^2 \sigma}{M_N} \right)$$

(11)

where $B_0$ corresponds to a free nucleon and $g_B^2$ is an additional parameter. The spurious center-of-mass momentum of the bag is subtracted to obtain the effective nucleon mass

$$M_N^* = \sqrt{E_{\text{bag}}^2 - <p_{\text{cm}}^2>}$$

(12)

where

$$<p_{\text{cm}}^2> = \frac{<x^2>}{R^2}$$

(13)

and

$$<x^2> = 3 \sum_{nk} x_{nk}^2 \left[ \frac{1}{e^{(e_{nk}^*/(R-\mu_q))/T} + 1} + \frac{1}{e^{(e_{nk}^*/(R+\mu_q))/T} + 1} \right].$$

(14)

The bag radius $R$ is obtained by minimizing the effective nucleon mass with respect to the bag radius

$$\frac{\partial M_N^*}{\partial R} = 0.$$  

(15)

The pressure is given by

$$P = \frac{1}{3} \frac{\gamma}{(2\pi)^3} \int d^3k \frac{k^2}{\epsilon^*} (f_B + \overline{f}_B) + \frac{1}{2} m_\omega^2 \omega^2 - \frac{1}{2} m_\sigma^2 \sigma^2,$$

(16)

where $\gamma = 4$ is the spin-isospin degeneracy factor and $f_B$ and $\overline{f}_B$ are the Fermi-Dirac distribution functions for the nucleons and antinucleons

$$f_B = \frac{1}{e^{(e^*/\mu_B^*)/T} + 1},$$

$$\overline{f}_B = \frac{1}{e^{(e^*/\mu_B^*)/T} + 1},$$

(17)

with $e^* = \sqrt{k^2 + M_N^*}$ and $\mu_B^* = \mu_B - 3 \left[ g_\omega^2 \omega + f_\psi^2 <\psi_q^\dagger \psi_q> \right]$ being the nucleonic effective energy and effective chemical potential, respectively. The chemical potential $\mu_B$ for a given density $\rho_B$ is determined self-consistently by the subsidiary constraint

$$\rho_B = \frac{\gamma}{(2\pi)^3} \int d^3k (f_B - \overline{f}_B)$$

(18)

with

$$\omega = \frac{g_\omega}{m_\omega^2} \rho_B.$$  

(19)

The scalar mean field $\sigma$ is determined through maximizing the pressure with respect to the scalar field. Since the scalar field correlation does not directly involve the $\sigma$ field, the SCC for the $\sigma$ field is identical to that found in our earlier work. The correlations do however affect the $\sigma$ field indirectly through the quark wave functions.
III. THE QUARK-GLUON PLASMA PHASE

In the QGP phase we assume that we have only $u$ and $d$ quarks confined inside a bag with bag parameter $B$. This parameter can be interpreted as the energy per unit volume needed to create a bubble or bag in which the noninteracting quarks and gluons are confined. The total baryonic density is given by \[ \rho_B = \frac{1}{3} \frac{\gamma_Q}{(2\pi)^3} \int d^3k [n_k(T) - \bar{n}_k(T)]. \] (20)

The quark and antiquark distribution functions are given by

\[
\begin{align*}
  n_k &= \frac{1}{\exp[(k - \frac{1}{3}\mu_B)/T] + 1}, \\
  \bar{n}_k &= \frac{1}{\exp[(k + \frac{1}{3}\mu_B)/T] + 1},
\end{align*}
\]
respectively, where $\mu_B$ is the baryon chemical potential, and we have assumed that the quarks have a baryon number of 1/3. At finite $\rho_B$, Eq. (20) is inverted to find the baryon chemical potential $\mu_B$. The pressure of the quark-gluon plasma is given by \[ P = -B + \frac{1}{3} \frac{\gamma_G T^4 \pi^2}{15} + \frac{1}{3} \frac{\gamma_Q}{(2\pi)^3} \int d^3kk [n_k(T) + \bar{n}_k(T)] \] (22)

where $k = |\vec{k}|$ and $\gamma_Q = 12$ for quarks and $\gamma_G = 16$ for gluons. The thermodynamic conditions for phase equilibrium between the baryon-meson phase and the QGP phase are satisfied by assuming mechanical, chemical and thermal equilibrium between the two phases namely, $P_{\text{MQMC}} = P_{\text{QGP}}$ and $\mu_{B\text{MQMC}} = \mu_{B\text{QGP}}$ for a given $T$. These conditions determine the transition line between the hadron-meson phase and the QGP phase.

We consider two cases. In the first case, the bag parameter in the QGP phase is considered as medium-independent and has the same value as that for a free nucleon. This case may be appropriate for most experimental situations aiming at producing the QGP in small chunks of hot nuclear matter produced in heavy ion collisions. In the second case, the bag parameter in the QGP is assumed to be medium-dependent, and is taken to be equal to the bag parameter of the nucleon in the MQMC model at the same density. This corresponds to the idealized case of producing the QGP in a bubble in infinite hot nuclear matter and may approximately apply to the production of the QGP in central collisions between very massive nuclei.

IV. RESULTS AND DISCUSSIONS

We have studied nuclear matter at finite temperature using the MQMC model which takes the medium-dependence of the bag parameter of the nucleon into account. We choose a direct coupling of the bag parameter to the scalar mean field $\sigma$ as given in Eq. (11). The bag parameters are those given in Ref. [1,3,4] where they are chosen to reproduce the free nucleon mass $M_N$ at its experimental value of 939 MeV and a bag radius $R_0 = 0.60$ fm.
For $g_s^2 = 1$, the values of the vector meson coupling and the parameter $g_s^B$, as fitted from the normal saturation properties of nuclear matter [1], are given as $g_s^2/4\pi = 5.24$ and $(g_s^B)^2/4\pi = 3.69$. The current quark mass $m_q$ is taken equal to zero. For the short-range quark-quark correlation strengths we use values comparable to those in Ref. [10] who, in the present notation, use $\alpha \approx 9$ and $\beta \approx 34\text{GeV}^{-2}$. The latter value of $\beta$ is needed to reproduce the empirical value of the energy per nucleon for symmetric nuclear matter $\rho_B/\rho_0 = 2.5 - 4$, where $\rho_0 = 0.17\text{fm}^{-3}$ is normal nuclear density. We carried out calculations for the cases of $\alpha = 0, 5, 7$ and 9 as well as $\beta = 0, 30$ and $60\text{ GeV}^{-2}$. The scalar quark-quark correlations affect the nucleon’s size and mass while the vector quark-quark correlations determine the pressure and energy density of nuclear matter [10].

Fig. 1 displays isotherms of the effective mass $M_N^*$ vs the baryonic density $\rho_B$ for various strengths of the scalar quark-quark correlations. As already mentioned, the vector correlations do not have an effect on the mass. For low values of $\alpha$, $M_N^*$ has the usual trend of decreasing with $\rho_B$. However, as $\alpha$ increases $M_N^*$ tends to saturate and, for still larger values of $\alpha$, the effective mass even starts to increase slightly at high density, especially at the lower temperatures. This novel feature is quite interesting as it is questionable that the monotonic decrease of $M_N^*$ with density can continue unchecked for higher and higher densities.

Fig. 2 displays isotherms of $R/R_0$ vs the baryon density for several values of $\alpha$. Without correlations, i.e. $\alpha = 0$, $R/R_0$ increases monotonically with $\rho_B$. However, when the correlations are introduced, $R/R_0$ starts to decrease, rather abruptly, when the bags start to overlap. This abruptness is due to the simple geometrical way in which the correlations are introduced. As $\alpha$ is increased further, $R/R_0$ decreases more steeply at high densities. The repulsive nature of the quark-quark correlations shrinks the bag size.

Fig. 3 displays the transition line between the baryon-meson phase and the QGP phase in the $(T,\mu_B)$ plane, while Fig. 4 displays it in the $(T,\rho_B)$ plane. This phase transition line is determined by equalizing the pressure $p$ and chemical potential $\mu_B$ in both phases for a given temperature $T$. We have considered two cases for the bag parameter of the QGP bubble. In case I, the bag parameter is taken to be medium-independent and fixed at its free-space value. As the density $\rho_B$ is increased, the nucleon bags start to overlap and the transition line becomes sensitive to the quark-quark correlations for $\mu_B$ larger than about 950 MeV, corresponding to densities larger than about 2.5 times normal nuclear matter density. For temperatures $T < 60\text{ MeV}$, the phase transition takes place at rather large densities and large chemical potentials. The quark-quark correlations are found to move the transition line to lower chemical potentials and lower densities. For cold nuclear matter the correlations can reduce $\mu_B$ from 1850 to 1450 MeV. The corresponding change in the $(T,\rho_B)$ plane is more dramatic as can be seen by inspecting Fig. 4. The phase transition, at low temperatures, takes place at densities as high as $8\rho_0$ without correlations, but this value is reduced to about $5\rho_0$ for the strongest correlations considered. In case II, also shown in Figs. 3 and 4, we have used a medium-dependent bag parameter for the QGP bubble. This bag parameter is identical to the bag parameter $B(\sigma)$ used for the nucleonic bags in the hadronic phase as given in Eq. [10]. This medium-dependence is appropriate for the production of a QGP bubble in infinite nuclear and may be appropriate for its production in the heart of the participant region in central collisions between very massive nuclei. In this case it is found...
that the phase transition from the baryon-meson phase to the QGP phase takes place at much lower densities so that the nucleons do not overlap and the quark-quark correlations do not play a role in determining the transition line. The transition temperature falls rapidly with density and the phase transition at low temperatures occurs at a comparatively low chemical potential $\mu_B = 950$ MeV and a correspondingly low density $\rho_B/\rho_0 = 1.35$. This density is obviously too low for the production of the QGP in heavy ion collisions and is strictly appropriate only for infinite nuclear matter as it does not include any finite size effects. It does however hint at the sizable reduction in the compression needed to produce the QGP in collisions between very heavy systems.

In conclusion we have investigated the effect of short-range quark-quark correlations on the properties of hot nuclear matter and the phase transition to the QGP. We have found that these correlations cure the problem usually encountered in the MQMC model of a very large nucleonic bag radius. They also lead to the saturation of the effective mass at high densities. Moreover, these correlations affect the properties of the phase transition at low temperatures for the case of a medium-independent bag parameter for the QGP bubble in vacuum (case I). Such a situation arises in experimental situations attempting to produce the QGP in small finite hot nuclear systems. In such a case, the present results indicate that the phase transition occurs at very high densities 5-8 times normal nuclear matter density. The only exceptions occur at very high temperatures greater than 100 MeV, in which case the transition occurs at arbitrarily small densities. In case II we have used a medium-dependent bag parameter for the QGP bubble, and the phase transition is found to occur at much lower densities than in Case I. The phase transition occurs before the nucleons overlap and so the quark-quark correlations do not play a role in determining the transition line. This case, strictly speaking, corresponds to producing a QGP bubble in infinite nuclear matter, but may be approximately approached in central collisions involving two very heavy nuclei. The comparatively low compressions required for the phase transition in such collisions would thus offer the best chance of producing the QGP.

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REFERENCES

[1] X. Jin and B. K. Jennings, Phys. Rev. C 54, 1427 (1996); Phys. Lett. B 374, 13 (1996).
[2] X. Jin and B. K. Jennings, Phys. Rev. C 55, 1567 (1997); H. Müller and B. K. Jennings, Nucl. Phys. A 626, 966 (1997); H. Müller, Phys. Rev. C 57, 1974 (1998).
[3] I. Zakout and H. R. Jaqaman, Phys. Rev. C 59, 962 (1999).
[4] I. Zakout and H. R. Jaqaman, Phys. Rev. C 59, 968 (1999).
[5] P. A. M. Guichon, Phys. Lett. B 200, 235 (1988).
[6] P. G. Blunden, G. A. Miller, Phys. Rev. C 54, 359 (1996).
[7] K. Saito and A. W. Thomas, Phys. Lett. B 327, 9 (1994).
[8] M. K. Banerjee, Phys. Rev. C 45, 1357 (1992); V. K. Mishra, Phys. Rev. C 46, 1143 (1992); E. Naar and M. C. Birse, J. Phys. G. 19, 555 (1993).
[9] D. H. Lu, K. Tsushima, A. G. Williams, A. W. Thomas and K. Saito, Nucl. Phys. A 634, 443 (1998).
[10] K. Saito, K. Tsushima and A. W. Thomas, nucl-th/9901084.
[11] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. 16, 1 (1986).
[12] F. E. Close, R. L. Jaffe, R. G. Roberts and G. G. Ross, Phys. Rev. D 31, 1004 (1985).
FIG. 1. Isotherms of the effective nucleon mass $M^*_N$ as a function of the baryonic density $\rho_B$ for different strengths of the scalar quark-quark correlations.
FIG. 2. Isotherms of $\frac{R}{R_0}$ vs the baryonic density $\rho_B$ for different strengths of the scalar quark-quark correlations.
FIG. 3. The phase transition line, in the $(T, \mu_B)$ plane, between the hadronic phase and the QGP phase for different strengths of the scalar and vector quark-quark correlations.
FIG. 4. The onset phase transition line, in the $(T, \rho_B)$ plane, between the hadronic phase and the QGP phase for different strengths of the scalar and vector quark-quark correlations.