Damage Detection in Plate Structures Based on Space-time Autoregressive Moving Average Processes

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Abstract. This paper presents a method for damage detection in plate structures based on space-time series analysis. The plate is divided into several spatial sub-regions and a space-time autoregressive moving average process (STARMA) is identified with the vibration responses of certain sub-region. And then the responses of other sub-regions are predicted with the identified model. The predicted errors are used as the criterion to detect and locate the damage by comparing the predicted responses with the practical ones. The validity of the methodology is demonstrated by a numerical model of a cantilever plate with cracks. The results show that this method can effectively detect the location and the size of the cracks even in the absence of the prior knowledge about the undamaged structure.

1. Introduction
Damage detection in structures based on vibration analysis has received a considerable interest over last two decades. A review of the state-of-the-art on vibration-based methods for testing cracked structures alongside an extensive related reference list can be found in [1]. More recently Sabnavis et al. [2] presented a survey on crack detection methods on the cracked shafts. Sekhar [3] summarized the different studies on double/multi-cracks and noted the influences, identification methods in vibration structures.

So far most damage detection methods based on the dynamic characteristics of the structures have been mainly formulated in modal parameter analysis, such as natural frequency, vibration mode, modal curvature, and modal strain energy. The simplest way to conduct damage detection is using the first several natural frequencies of a structure, which can be determined from experiments conveniently. However, they are the global characteristics and thus may remain unaffected by damage, and it is therefore difficult to distinguish the causes of the frequency change in a number of cases.

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Mode shapes are in general more sensitive to damage but they are difficult to measure and/or estimate from measured quantities [1].

Compared with the modal parameters, the dynamic responses are much easier to measure in engineering practice. It suggests purely data-based methods for damage detection that make use of measured responses of structures. More recently data-based damage detection in time domain has been investigated by several researchers[4]-[8]. Although the introduced methods above have achieved encouraging performance, most of them are based on comparing the change between the damaged state and the undamaged or reference state of the structure. Having in mind that the original system parameters in many cases are not easy to obtain, alternative methods independent of the prior knowledge about the undamaged structure become important. Guo and Billings [9] developed a new approach to detect beam crack in the absence of the original system parameters, by using spatio-temporal dynamical system identification techniques. The cracked beam was divided into several spatial regions and a coupled map lattice (CML) model was identified in one of the regions. The model was then used to predict the dynamical behavior of the other regions and in this way to detect the crack.

In the light of the novel idea in [9], a damage detection method for a plate is presented in this study, using space-time autoregressive moving average processes (STARMA). Not by detecting the change between the damaged and the undamaged state, this method is based on the comparison of the responses of the different regions of the plate instead.

Similarly, The STARMA is modeling with the vibration responses of certain region of the plate first, and the responses of other regions are predicted then. If the predicted errors of any region are much larger than that of other regions, it is quite possible that this region has damage. In this way the damage can be located and identified by the relative error value.

The next section will introduce the methods of space-time data analysis. A damage detection steps are presented in Section 3. A numerical example is studied using the proposed methodology in Section 4 and finally conclusions are given in Section 5.

2. Space-time data analysis

2.1. Modeling of space-time series

If the spatial relation is concerned, a structure can be treated as a spatio-temporal dynamical system. Hence, it can be represented using the model that describes the correlation information found in the observations of the system. The technique to describe local correlations actually is space-time series modeling that defines a neighborhood limiting the modeled correlation to a local spatio–temporal region. Of the exiting research, the space-time series are seldom used for damage detection, since it is difficult and costly to obtain data over a region in space in practice. However, the embarrassing situation has been improved with the development of the sensor technology, such as the application of laser-type measurement devices.

A time series is a sequence of data points, measured typically at successive times, spaced at time intervals. Space-time series are the sets of multiple time series that are location-related. Space-time series analyses are developed in many fields ranging from astronomy, geography, economics, finance,
medicine to biology. Lattice based spatio-temporal models such as cellular automata [10] and coupled map lattices [11] are widely used for space-time series modeling. However, these models are restricted to the data sets with a regular lattice structure [12]. The spatio-temporal autoregressive moving average (STARMA) model [13]–[14] avoids the restriction of lattice based models whilst maintaining the neighborhood structure. This motivates their use for real-world system identification, where spatially correlated signals are often gathered, which are regularly sampled in time and irregular in space. STARMA models represent the spatio-temporal process using a set of correlated time series, and an approach which lends itself to systems which are observed at a reasonably small number of observation locations but which are heavily sampled in time.

2.2. Space-time Lag Operator [14]

Assume that \( y_i(t) \) \((i = 1, 2, \cdots N)\) represents the observations of vibration response at each of \( N \) fixed points in a structure over \( T \) time periods. The autoregressive form of the space-time model would express the observation at time \( t \) and site \( i \), \( y_i(t) \) as a linear combination of past observations at zone \( i \) and neighboring zones.

To assist in the formulation of this space-time model, the following definition of the spatial lag operator is needed. Let \( L^{(h)} \), the spatial lag operator of spatial order \( h \), be such that

\[
L^{(0)}y_i = y_i, \quad L^{(1)}y_i = \sum_{j=1}^{N} w_{ij}^{(0)}y_j, \quad \cdots \quad L^{(h)}y_i = \sum_{j=1}^{N} w_{ij}^{(h)}y_j
\]  

(1)

where \( w_{ij}^{(h)} \) are a set of weights with

\[
\sum_{j=1}^{N} w_{ij}^{(h)} = 1
\]  

(2)

If we define:

\[
\sum_{j=1}^{N} w_{ij}^{(h)} = W^{(h)}
\]  

(3)

then

\[
L^{(0)}y_i = I_N y_i, \quad L^{(1)}y_i = W^{(1)}z_j, \quad \cdots \quad L^{(h)}y_i = W^{(h)}y_j
\]  

(4)

For all \( i \), \( w_{ij}^{(h)} \) is nonzero only if sites \( i \) and \( j \) are \( h^{th} \) order neighbors. The matrix representation of the set of weights \( w_{ij}^{(h)} \) is \( W^{(h)} \), an \( N \times N \) square matrix with each row summing to one.

These weights, however, must reflect a hierarchical ordering of spatial neighbors. The first order neighbors are those “closest” to the site of interest. The second order neighbors should be “farther” away than first order neighbors, but “closer” than third order neighbors. Figure 1 shows the first four
spatial order neighbors of a plate with uniform distribution grid. This definition of spatial order represents an ordering in terms of Euclidean distance of all sites surrounding the location of interest.

Figure 1 The first four spatial order neighbors of a plate with uniform distribution grid

When the spatial lag and time lag are taken into account together, spatial-time lag operator can be defined as $B^{(k)}L^{(h)}$, then:

$$y_i(t-1) = B^{(1)}L^{(0)}y_i(t) + \sum_{j=1}^{N} w_{ij} y_j(t-1) = B^{(1)}L^{(1)}y_i(t) + \cdots + \sum_{j=1}^{N} w_{ij} y_j(t-k) = B^{(k)}L^{(h)}y_i(t)$$ (5)

where $k$ represents time lag order and $h$ is spatial lag order.

2.3. STARMA model [14]

For a structure, the vibration responses of any site are influenced not only by the past observations of itself, but also by the responses of its neighbors. It can be expressed by the STARMA model.

$$y_i(t) = \sum_{k=1}^{p} \sum_{h=0}^{m_k} \phi_{kh} L^{(h)} y_i(t-k) - \sum_{l=1}^{q} \sum_{h=0}^{n_l} \theta_{lh} L^{(h)} \varepsilon_i(t-1) + \varepsilon_i(t)$$ (6)

where $p$ is the autoregressive order, $q$ is the moving average order, $m_k$ is the spatial order of the $k^{th}$ autoregressive term, $n_l$ is the spatial order of the $l^{th}$ moving average term, $\phi_{kh}$ and $\theta_{lh}$ are parameters, and $\varepsilon_i(t)$ are random normal errors with:

$$E[\varepsilon_i(t)] = 0$$
$$E[\varepsilon_i(t)\varepsilon_j(t+s)] = \begin{cases} \sigma^2 & i=j, s=0 \\ 0 & \text{otherwise} \end{cases}$$ (7)

The model expressed with spatial-time lag operator is:

$$y_i(t) = \sum_{k=1}^{p} \sum_{h=0}^{m_k} \phi_{kh} B^{(k)}L^{(h)}y_i(t) - \sum_{l=1}^{q} \sum_{h=0}^{n_l} \theta_{lh} B^{(l)}L^{(h)}\varepsilon_i(t) + \varepsilon_i(t)$$

It should be noted that when $q = 0$, only autoregressive terms remain, and hence the model class carries the label space-time autoregressive or STAR model. The model
\[ y_i(t) = \sum_{k=1}^{m} \sum_{h=0}^{m} \phi_{kh} B^{(k)} L^{(h)} y_j(t) + \varepsilon_i(t) \]  

is referred to as a STAR model.

The identification procedure of STARMA models in detail can be found in reference [14].

3. Proposed damage identification method

A dynamical model actually is a kind of description on dynamic characteristics of a system. For the different spatial regions of an anisotropic undamaged plate, the models should have comparability due to the homogeneity. However, when the plate has any damage, such as a crack, the homogeneity is destroyed, and the model of the damaged region will be different from the others. Using the difference, an approach is developed to detect this damage.

The identification procedure for the damage of a plate can be outlined as follows:

Step 1: Divide the spatial domain into several sub-regions and choose one of the regions for identification of space-time model.

Step 2: Choose appropriate model forms, the temporal and spatial orders of the model form.

Step 3: Estimate the model parameters.

Step 4: Do "diagnostic checks" to determine if the model does adequately represent the data.

Step 5: Apply the obtained and validated model to other sub-regions to predict the dynamic responses in these sub-regions.

Step 6: Check the predicted errors for each sub-region to detect the damage.

If necessary, select one of these sub-regions, go back to Step 1 and repeat the above procedure until adequate accuracy is achieved.

This damage identification procedure is only based on the vibration responses. And it is not necessary to know the characteristics of the undamaged plate, which is difficult in practice for many cases. It should be pointed out that the method is based on a hypothesis: the region selected for modeling is undamaged. When there is only single damage and the size of damage is much smaller than the whole plate, the credibility of the hypothesis can be considered as acceptable. Otherwise, we should select two or more regions to model first, and then compare the prediction errors of every model. The region with general error distribution may be the undamaged region.

4. Numerical case study

A model of an elastic, rectangular, thin cantilever plate with a through slant crack is presented in figure 2 and its dimensions and material properties are listed in table 1. The plate is excited by a vertical harmonic force, \( P \sin(\omega t) \), acting on the midpoint of the free edge of the plate.

| Properties           | Values          | Properties          | Values   |
|----------------------|-----------------|---------------------|----------|
| Young’s modulus(E)   | 2.06E11Pa       | coordinate of the crack | x1=0.18m |
| Mass density(\(\rho\)) | 7800kg/m³      |                      | y1=0.06m |
| Poisson's ratio(\(P_{xy}\)) | 0.3           |                      | x2=0.20m |
Length in x direction ($L$) 0.30m  
Length in x direction ($B$) 0.15m 
Thickness ($H$) 0.005m

Figure 2 Thin cantilever plate with a through slant crack

The plate is divided into 30×15 grid. The vibration responses to the harmonic excitation are calculated based on the finite element model. The vibration responses at certain moment are shown in figure 3. The data of the 5×5 lattices points are used for modeling. The neighbors of each point are weighted equally, for the homogeneity of the plate and the uniform lattices. Considering the 1th order neighbors, the four corner points have $w_{ij}^1 = 0.25$ and the remaining nodes on the boundary have $w_{ij}^1 = 1/3$. The time lag is set to be 3 and the space lag is set to be 1. The coefficients of the STAR model are obtained using the identification methods in Ref. [14].

Figure 3 The vibration response of the plate at time $t$.

All the responses of the points are predicted using the identified model. The predicted errors are shown in figure 4, by comparing the predicted responses and the calculated data based on the finite element model. In figure 4, the straight red bold line represents the crack, and the deeper the color, the larger the error of the corresponding position. The figure shows that the predicted error values are obviously larger around the position of the crack, thus indicating the presence of the crack. The range of the region relates to the size of the crack. The result indicates that the location of the crack can be obtained and the size can be estimated with the proposed methodology.
It should be pointed out here that in this case study no iterative procedure mentioned in section 3 is applied, since the grid is dense enough to locate the crack. When the grid is sparse, the iterative procedure should be used to obtain the successive approximation of the exact location of the crack.

A case of double cracks is also studied to verify the validity of the proposed methodology for multiple cracks.

The predicted errors distribution is shown in figure 5. It can also be observed that the predicted error values are larger around the positions of the cracks than other regions. The values of the errors and the range of the right crack are larger than the left one, which is consistent with the size of the two cracks. It can be concluded that the methodology can not only identify the number of cracks but also approximately indicate the size of the cracks.

5. Conclusions
A method is presented in this paper for detecting damage in plate structures based on space-time series analysis. One of the outstanding advantages of the proposed methodology is that the detection process does not need the prior knowledge about the undamaged structure. And it can address a
general situation when there are more than one crack on the plate and the number of cracks is not known in advance.

Two case studies with single and double cracks on a cantilever rectangular plate are considered through computer simulation, to verify the proposed methodology. The proposed methodology successfully identifies the number of cracks and detects the location.

The proposed methodology provides a novel means to detect damage. However, additional research must be carried out before it can be employed in application. First, the robustness of the proposed methodology against noise must be improved. The measurement noise has great influence on the results and even destroyed the detection, if no preprocess is carried out. Second, how to choose the sub-region to identify the model deserves further study, especially for multiple cracks or other damage.

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