Beam Design and User Scheduling for Non-Orthogonal Multiple Access with Multiple Antennas Based on Pareto-Optimality

Junyeong Seo, Student Member, IEEE, Youngchul Sung†, Senior Member, IEEE

Abstract

In this paper, an efficient transmit beam design and user scheduling method is proposed for multi-user (MU) multiple-input single-output (MISO) non-orthogonal multiple access (NOMA) downlink, based on Pareto-optimality. The proposed beam design and user scheduling method groups simultaneously-served users into multiple clusters with practical two users in each cluster, and then applies spatial zero-forcing (ZF) across clusters to control inter-cluster interference (ICI) and Pareto-optimal beam design with successive interference cancellation (SIC) to two users in each cluster to remove interference to strong users and leverage signal-to-interference-plus-noise ratios (SINRs) of interference-experiencing weak users. The proposed method has flexibility to control the rates of strong and weak users and numerical results show that the proposed method yields good performance.

Index Terms

Non-orthogonal multiple access, multi-user MIMO, scheduling, Pareto-optimal design, SIC

I. INTRODUCTION

NOMA is a promising technology for 5G wireless networks to increase the spectral efficiency [1]. Unlike conventional orthogonal multiple access (OMA) which serves multiple users based on time, frequency and/or spatial domains, NOMA exploits the power domain that results from unequal channel conditions under which users with strong channels are basically limited by degree-of-freedom (DoF) such as bandwidth not by noise but users with weak channels are limited by additive noise [2, P. 239]. In
NOMA with such channel conditions, the base station (BS) uses superposition coding and allocates less power to strong-channel users and more power to weak-channel users. Here, less power to strong users is not so detrimental since strong users are in the DoF-limited regime, but more power to weak users leverages received SINRs of weak users. The strong interference caused from more power assigned to weak users through good channels to strong users is eliminated by SIC to maintain high quality channels for strong users.

Initially, NOMA with a single antenna in both the BS and users was studied [1], [3]–[8]. Recently, there have been efforts to extend NOMA to multiple-antenna systems. Unlike conventional MU multiple-antenna downlink systems which serve as many users as the number of antennas, more users can be served in multiple-antenna NOMA systems. Although the possibility that multiple-antenna NOMA can outperform conventional multiple-antenna OMA was shown [9], [10], many important problems need to be investigated further for multiple-antenna NOMA. In multiple-antenna NOMA systems, typically user grouping is done first by forming clusters as many as the number of transmit antennas and assigning multiple users to each cluster, and then multiple users in each cluster share the spatial dimension and are served in the power domain. Since the performance of multiple-antenna NOMA significantly depends on the channel conditions of users across clusters and within each cluster, effective scheduling and user grouping methods should be devised in order to achieve both MU diversity and the NOMA gain from unequal channel conditions. In addition, the problem of optimal beam design and power allocation compatible to user scheduling should be solved to maximize the performance.

A. Related Works

There have been several studies on multiple-antenna NOMA for cellular downlink especially for MU-MISO downlink which is the main focus of this paper. In [11], the downlink beam design for sum-rate maximization was considered for one given cluster based on minorization-maximization. In [12]–[14], user scheduling and clustering is considered with the assumption that two users are assigned to each cluster. In [12], highly correlated users are chosen as candidates to be clustered, and two users having the largest channel gain difference are assigned to the same cluster. In [13], strong users are selected by the semi-orthogonal user selection (SUS) algorithm [15], and then weak users are selected using the matching user selection algorithm by considering inter-cluster interference. In [14], a fairness-oriented user selection algorithms was proposed by selecting two paired users based on their NOMA data rate. In all the works of [12]–[14], the beam design problem for MU-MISO NOMA was simplified by designing zero-forcing (ZF) beams based on strong users’ channels and allocating the same beam to the weak user.
as the beam of the strong user in each cluster. There also exists some study on multiple-input multiple-output (MIMO) NOMA. For example, in [16] the impact of user pairing is analyzed with fixed power allocation under the assumption that inter-cluster interference is removed by multiple receive antennas. Some part of this work was included in [17].

### B. Contributions

In this paper, we consider MU-MISO NOMA downlink with practical two users in each cluster and solve the aforementioned problem for MU-MISO NOMA downlink. The contributions of this paper are summarized in the below:

- First, we solved the Pareto-optimal beam design and power allocation problem for two-user MISO broadcast channels (BCs) in which superposition coding is used at the transmitter and the interference at the strong user is eliminated by SIC while the interference at the weak user is treated as noise. This work is the basis for successive development in this paper and is valuable as an independent item.

- We proposed an effective user scheduling, beam design and power allocation method for MU-MISO NOMA downlink based on the above two-user Pareto-optimal design result by exploiting both the spatial domain provided by multiple transmit antennas and the power domain provided by SIC. The key advantages of the proposed method compared to the previous methods are that 1) the rates of strong users and weak users can be controlled arbitrarily under Pareto-optimality, which provides great operational flexibility to NOMA networks, and 2) beam design of the proposed method is generalized to include multi-dimensional subspace if available and to yield performance improvement, whereas the same one-dimensional beam is always used by both strong and weak users in the same cluster in the previous methods [12]–[14].

**Notations:** Vectors and matrices are written in boldface with matrices in capitals. All vectors are column vectors. For a matrix $A$, $A^*$, $A^H$ and $A^T$ indicate the complex conjugate, conjugate transpose, and transpose of $A$, respectively, and $\mathcal{L}(A)$ and $\mathcal{L}^\perp(A)$ denotes the linear space spanned by the columns of $A$ and its orthogonal complement, respectively. $\Pi_A$ and $\Pi_A^\perp$ are the projection matrices to $\mathcal{L}(A)$ and $\mathcal{L}^\perp(A)$, respectively. $||x||$ represents the 2-norm of vector $x$. $I$ denotes the identity matrix. $y \sim \mathcal{CN}(\mu, \Sigma)$ mean that random vector $y$ is circularly-symmetric complex Gaussian distributed with mean vector $\mu$ and covariance matrix $\Sigma$. 

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II. System Model

We consider a single-cell MU-MISO NOMA downlink system with a BS equipped with \( N_t \) transmit antennas and \( K \) single-antenna users. We assume that the \( K \) users in the cell are divided into the set \( \mathcal{K}_1 \) of \( K/2 \) strong-channel users and the set \( \mathcal{K}_2 \) of \( K/2 \) weak-channel users. We assume that out of the \( K \) users in the cell, \( 2K_c \) users are selected and simultaneously served for each scheduling interval with \( K_c \leq N_t \) (thus \( 2N_t \) users can be served simultaneously) and this simultaneous service to \( 2K_c \) users is done by forming \( K_c \) clusters with two paired users in each cluster composed of one from the strong user set \( \mathcal{K}_1 \) and the other from the weak user set \( \mathcal{K}_2 \). We assume that the BS uses linear precoding and NOMA is applied to two paired users in each cluster. Under these assumptions, the transmit signal of the BS for one scheduling interval is given by

\[
x = \sum_{k=1}^{K_c} \left( \sqrt{p_1^{(k)}} \tilde{w}_1^{(k)} s_1^{(k)} + \sqrt{p_2^{(k)}} \tilde{w}_2^{(k)} s_2^{(k)} \right),
\]

where \( s_i^{(k)} \) is the transmit symbol for user \( i \) in cluster \( k \) from \( \mathcal{CN}(0,1) \), \( \tilde{w}_i^{(k)} \) is the \( N_t \times 1 \) beamforming vector for user \( i \) in cluster \( k \) out of the feasible beamforming vector set \( \mathcal{W} := \{ \tilde{w} | \| \tilde{w} \|^2 \leq 1 \} \), and \( p_i^{(k)} \) is the power assigned to user \( i \) in cluster \( k \). The total BS transmit power \( P_T \) is equally divided into \( P_T/K_c = P \) for each cluster. Then, the received signals of the two users in cluster \( k \) are given by

\[
y_1^{(k)} = \tilde{h}_1^{(k)H} \left( \sqrt{p_1^{(k)}} \tilde{w}_1^{(k)} s_1^{(k)} + \sqrt{p_2^{(k)}} \tilde{w}_2^{(k)} s_2^{(k)} \right) + w_1^{(k)}
\]

\[
y_2^{(k)} = \tilde{h}_2^{(k)H} \left( \sqrt{p_1^{(k)}} \tilde{w}_1^{(k)} s_1^{(k)} + \sqrt{p_2^{(k)}} \tilde{w}_2^{(k)} s_2^{(k)} \right) + w_2^{(k)}
\]

where \( \tilde{h}_i^{(k)} \) denotes the actual \( N_t \times 1 \) (conjugated) channel vector between the BS and user \( i \) in cluster \( k \), and \( w_i^{(k)} \) is the zero-mean additive white Gaussian noise (AWGN) at user \( i \) in cluster \( k \) from \( \mathcal{CN}(0,[\epsilon_i^{(k)}])^2 \).

In the MU-MISO-NOMA system, we have both the spatial domain and the power domain. We consider the design approach in which two users in each cluster are served in the power domain with SIC while multiple clusters are served based on the spatial domain. Note that the last two terms in each of the right-hand sides (RHSs) of (2) and (3) are the ICI and AWGN. In order to control ICI, we apply spatial ZF across clusters. However, due to lack of spatial dimensions, we cannot remove ICI completely for all
users. Hence, we remove ICI for the strong users with spatial ZF to keep the strong users not interference-limited. With this approach, the beam vector \( \tilde{w}_i^{(k)} \) can be expressed as

\[
\tilde{w}_i^{(k)} = \Pi_{H_k}^{\perp} w_i^{(k)}, \quad i = 1, 2,
\]

for some vector \( w_i^{(k)} \), where

\[
H_k := \{ \hat{h}_i^{(1)}, \hat{h}_i^{(2)}, \ldots, \hat{h}_i^{(k-1)}, \hat{h}_i^{(k+1)}, \ldots, \hat{h}_i^{(K_c)} \}.
\]

Once the ICI is controlled and given, the model (2) - (3) is a two-user MISO BC. Thus, our approach to the overall design is to first investigate the optimal beam design and power allocation for a two-user MISO BC with SIC at the strong user’s receiver in Section III, to derive certain performance properties relevant to selection of two users in a cluster in Section IV, and then to develop an overall user selection and beam design method for all clusters with controlling ICI in Section V.

III. TWO-USER MISO BROADCAST CHANNEL WITH SIC: PARETO-OPTIMAL DESIGN

In this section, we focus on optimal beam vector design and power allocation for a two-user MISO BC with SIC at the strong user’s receiver from the perspective of Pareto-optimality. With the cluster index \( (k) \) omitted, the two-user model (2) - (3) for cluster \( k \) is given by

\[
y_i = h_i^H (\sqrt{p_i} w_i s_i + \sqrt{p_j} w_j s_j) + n_i, \quad i, j \in \{1, 2\}, \quad j \neq i,
\]

where \( p_1 + p_2 \leq P \) with the total power \( P \) allocated to the cluster; \( n_i \sim C \mathcal{N}(0, \sigma_i^2) \) is the sum of ICI and AWGN; and \( h_i \) is the effective channel for user \( i \) (in cluster \( k \)) given by \( h_i = \Pi_{H_k}^{\perp} \hat{h}_i^{(k)} \), \( i = 1, 2 \) from (2), (3) and (4) since \( \hat{h}_i^H w_i = \hat{h}_i^H \Pi_{H_k}^{\perp} w_i = \hat{h}_i^H [\Pi_{H_k}^{\perp}]^H w_i = h_i^H w_i \). The feasible set for \( w_i \) is given by \( W = \{ w \mid \|w\|^2 \leq 1 \} \) since the Pareto-optimal beam \( w_i \) under the model (6) lies in the linear space spanned by \( h_1 = \Pi_{H_k}^{\perp} \hat{h}_1^{(k)} \) and \( h_2 = \Pi_{H_k}^{\perp} \hat{h}_2^{(k)} \) [18], and hence \( \|w_i^{(k)}\| = \|\Pi_{H_k}^{\perp} w_i^{(k)}\| = \|\tilde{w}_i^{(k)}\| \) for Pareto-optimal beams.

Under the NOMA framework, we assume that user 1 is the strong user and user 2 is the weak user, i.e., \( \|h_1\|^2/\sigma_1^2 > \|h_2\|^2/\sigma_2^2 \) and that user 1 decodes the interference from user 2 and subtracts it before decoding its own data while user 2 treats the interference as noise. With this assumption, the rates of the two users are given by

\[
R_1(w_1, p_1) = \log_2 \left( 1 + \frac{s_1(w_1, p_1)}{\sigma_1^2} \right)
\]

\[
R_2(w_1, w_2, p_1, p_2) = \log_2 \left( 1 + \min \left\{ \frac{r_1(w_2, p_2)}{s_1(w_1, p_1) + \sigma_1^2}, \frac{s_2(w_2, p_2)}{r_2(w_1, p_1) + \sigma_2^2} \right\} \right),
\]
where the signal power and the interference power are respectively given by
\[
s_i(w_i,p_i) := p_i |h_i^H w_i|^2 \quad \text{and} \quad r_i(w_j,p_j) := p_j |h_i^H w_j|^2.
\]
(8)

Note in (7) that for the rate of user 1, the interference from user 2 is not incorporated due to SIC and the rate of user 2 is bounded by not only the SINR of user 2 but also the required ‘SINR’ for user 1 to decode the message of user 2 for SIC before decoding its own data. Then, for the given (effective) channel vectors \((h_1,h_2)\), the achievable rate region \(\mathcal{R}\) of the two-user MISO-NOMA BC is defined as the union of the rate-tuples that can be achieved by all feasible beam vectors and power allocation:
\[
\mathcal{R} := \bigcup_{(w_1,w_2) \in \mathcal{W}^2} (R_1(w_1,p_1), R_2(w_1,w_2,p_1,p_2)).
\]
(9)

The Pareto boundary of the rate region \(\mathcal{R}\) is the outer boundary of \(\mathcal{R}\) for which the rate of any one user cannot be increased without decreasing the rate of the other user and Pareto-optimality has been used widely as a general optimal beam design criterion for MU-MISO networks with linear precoding [20]. A pair of beam vectors \((w_1,w_2)\) not achieving a Pareto-boundary point is not optimal since both users’ rates can be increased by a better designed beam pair. Note that the sum-rate optimal point is the point on the Pareto-boundary where the Pareto-boundary and the minus 45\(^\circ\) degree line touch in the \((R_1,R_2)\) plane, and the Pareto-optimality provides a general optimality criterion because we can change the rate operating point arbitrarily and optimally. It is known that the Pareto-boundary can be found by maximizing \(R_2\) for each given feasible \(R_1^*\) [20], i.e.,
\[
\max_{(w_1,w_2) \in \mathcal{W}^2} \quad R_2(w_1,w_2,p_1,p_2)
\]
subject to \(R_1(w_1,p_i) = R_1^*\).
(10)

By exploiting the relationship between the rates and the SINRs in (7), the problem (10) can be rewritten in terms of SINR as
\[
\max_{(w_1,w_2) \in \mathcal{W}^2} \quad \gamma_2 := \min \left\{ \frac{r_1(w_2,p_2)}{s_1(w_1,p_1) + \sigma_1^2}, \frac{s_2(w_2,p_2)}{r_2(w_1,p_1) + \sigma_2^2} \right\}
\]
subject to \(\frac{s_1(w_1,p_1)}{\sigma_1^2} = \gamma_1^*\),
(11)
where \(\gamma_1^*\) is a given feasible target SINR for user 1. An efficient solution to the problem (11) exploits an efficient parameterization of the beam vectors \(w_1\) and \(w_2\). Note that the number of design variables in \(w_1\) and \(w_2\) is \(2N_t\) complex numbers. However, one can realize that it is sufficient that both beam vectors
are linear combinations of \( h_1 \) and \( h_2 \) (equivalently, \( \Pi_{h_2}h_1 \) and \( \Pi_{h_2}h_1 \)). A component in the beam vector not in the span of \( h_1 \) and \( h_2 \) does not affect either the signal power or the interference power and thus does not affect either \( R_1 \) or \( R_2 \) [19]. Thus, it is known from [18] that the Pareto-optimal beam vectors for the problem (11) can be parameterized as [18], [21]

\[
\begin{align*}
\mathbf{w}_1(\alpha_1, \beta_1) &= \alpha_1 \frac{\Pi_{h_2}h_1}{\|\Pi_{h_2}h_1\|} + \beta_1 \frac{\Pi_{h_2}h_1}{\|\Pi_{h_2}h_1\|}, \\
\mathbf{w}_2(\alpha_2) &= \alpha_2 \frac{\Pi_{h_2}h_2}{\|\Pi_{h_2}h_2\|} + \sqrt{1 - \alpha_2^2} \frac{\Pi_{h_2}h_2}{\|\Pi_{h_2}h_2\|},
\end{align*}
\]

where \((\alpha_1, \beta_1) \in \mathcal{F} := \{(\alpha, \beta), \alpha, \beta \geq 0, \alpha^2 + \beta^2 \leq 1\}\) and \(\alpha_2 \in [0, 1]\). Unlike the conventional parametrization without SIC in which both users use full power [20], [22], in the parameterization (12) - (13) user 1 may not use full power whereas user 2 uses full power. This is because full power use of user 2 helps both SIC at user 1 and its own SINR at user 2, but full power use of user 1 is beneficial for its own rate but detrimental to user 2’s rate since user 2 treats interference as noise. Substituting (12) - (13) into (8), we have

\[
\begin{align*}
s_1(\mathbf{w}_1) &= p_1 \left( \alpha_1 \frac{\|\Pi_{h_2}h_1\|}{\|\Pi_{h_2}h_1\|} + \beta_1 \frac{\Pi_{h_2}h_1}{\|\Pi_{h_2}h_1\|} \right)^2 \\
&= p_1 \|\mathbf{h}_1\|^2 (\sqrt{\alpha_1} + \sqrt{1 - \theta \beta_1})^2, \\
r_2(\mathbf{w}_1) &= p_1 \alpha_1^2 \frac{\|\Pi_{h_2}h_1\|^2}{\|\Pi_{h_2}h_1\|^2} = p_1 \|\mathbf{h}_2\|^2 \alpha_1^2, \\
s_2(\mathbf{w}_2) &= p_2 \|\mathbf{h}_2\|^2 (\sqrt{\alpha_2} + \sqrt{1 - \theta \sqrt{1 - \alpha_2^2}})^2, \\
r_1(\mathbf{w}_2) &= p_2 \alpha_2^2 \frac{\|\Pi_{h_2}h_1\|^2}{\|\Pi_{h_2}h_2\|^2} = p_2 \|\mathbf{h}_1\|^2 \alpha_2^2,
\end{align*}
\]

where the angle parameter \(\theta\) between two effective channel vectors \( h_1 \) and \( h_2 \) is defined as

\[
\theta := \frac{\|\mathbf{h}_1\|^2}{\|\mathbf{h}_1\|^2} \in [0, 1].
\]

Substituting (14) - (17) into the problem (11) and taking square-root operation yield

\[
\begin{align*}
\gamma_2 &= \min_{(\alpha_1, \beta_1) \in \mathcal{F}, \alpha_2 \in [0, 1], 0 \leq p_1 \leq P} \left\{ \frac{\sqrt{p_1} \|\mathbf{h}_1\| \alpha_2}{\sqrt{\sigma_1^2 (1 + \gamma_1^*)}}, \frac{\sqrt{p_1} \|\mathbf{h}_2\| (\sqrt{\alpha_2} + \sqrt{1 - \theta \sqrt{1 - \alpha_2^2}})}{\sqrt{p_1} \|\mathbf{h}_2\| \alpha_2^2 + \sigma_2^2} \right\},
\end{align*}
\]

subject to \(\sqrt{p_1} \|\mathbf{h}_1\| (\sqrt{\alpha_2} + \sqrt{1 - \theta \beta_1}) = \sqrt{\gamma_1^* \sigma_1^2}\).

For later use, we define the following channel quality factor \(\lambda_i\) and the normalized target SINR value for user 1, \(\Gamma\):

\[
\lambda_i := \frac{\|\mathbf{h}_i\|^2}{\sigma_i^2}, \quad i = 1, 2, \quad \text{and} \quad \Gamma := \gamma_1^*/\lambda_1.
\]
Here, \( \lambda_i \) indicates the signal-to-noise ratio (SNR) quality of user \( i \)'s channel, whereas \( \theta \) in (18) is a measure of the angle between the two users’ channels. Note that the actual target SINR for user 1 \( \gamma_1^* = \Gamma \lambda_1 \) and the feasible range for \( \Gamma \) is \( \Gamma \in [0, P] \), where the maximum \( P \) occurs when \( w_1 = h_1/||h_1|| \) and \( p_1 = P \) since \( \gamma_1 = s_1(w_1, p_1)/\sigma_1^2 = p_1|H_1^H w_1|^2/\sigma_1^2 \).

The optimization problem (19) - (20) with fixed power allocation \( p_1 = p_2 = 1 \) was solved in [21] under the framework of a two-user MISO interference channel. In the MISO interference channel case, two transmitters in the network neither cooperate nor share transmit power and hence the two transmit power values \( p_1 \) and \( p_2 \) are fixed. On the other hand, in the MISO-BC case the two power values \( p_1 \) and \( p_2 \) can be designed at the BS under the constraints \( p_1, p_2 \geq 0 \) and \( p_1 + p_2 = P \) to maximize the performance. Since our solution to the two-user MISO-NOMA-BC case is based on the result from [21], we briefly introduce the relevant result in [21] for further development in later sections.

A. Background: The fixed power allocation case [21]

In this subsection, we fix \( p_1 = p_2 = 1 \) and follow [21]. First, note that \( \alpha_1 \) appears only in the constraint (20) and the denominator in the second term in the RHS of (19). Thus, optimal \( \alpha_1 \) can be found by solving the following problem [21]:

\[
\min_{(\alpha_1, \beta_1) \in \mathcal{F}} \alpha_1 \quad \text{subject to} \quad ||h_1||(\sqrt{\theta} \alpha_1 + \sqrt{1-\theta} \beta_1) = \sqrt{\gamma_1^* \sigma_1^2}, \tag{22}
\]

This is because the second term in the RHS of (19) decreases monotonically with respect to \( \alpha_1 \) and hence maximizing the second term in the RHS of (19) is equivalent to minimizing \( \alpha_1 \). The problem (22) - (23) can easily be solved based on the relationship between a line segment (23) and the unit-radius ball \( \mathcal{F} \) and the solution is given by [21]

\[
\alpha_1^* = \begin{cases} 
0 & \text{if } \Gamma \leq 1 - \theta \\
\sqrt{\theta \Gamma - \sqrt{(1-\theta)(1-\Gamma)}} & \text{if } \Gamma > 1 - \theta,
\end{cases} \tag{24}
\]

where \( \Gamma \) is defined in (21) and \( \Gamma \in [0, 1] \) for \( p_1 = 1 \). With the optimal \( \alpha_1^* \) in (24), the corresponding optimal \( \beta_1^* \) can be found by the constraint (20), and substituting the optimal \( \alpha_1^* \) into the problem (19) - (20) yields [21]

\[
\max_{\alpha_2 \in [0, 1]} \gamma_2 = \min \left\{ a \alpha_2, b \alpha_2 + c \sqrt{1 - \alpha_2^2} \right\} \tag{25}
\]

where

\[
a := \frac{||h_2||}{\sqrt{\sigma_1^2(1+\gamma_1^*)}}, \quad b := \frac{||h_3||}{\sqrt{||h_2||^2(\alpha_1^*)^2 + \sigma_2^2}}, \quad \text{and} \quad c := \frac{||h_2||}{\sqrt{||h_2||^2(\alpha_1^*)^2 + \sigma_2^2}}.
\]

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There exist three different cases for the solution to (25) depending on the relationship between the two is \( \alpha_2 \) and the optimal solution \( \gamma^* \) to (25) occurs at \( \alpha^*_2 = c/\sqrt{c^2 + (a-b)^2} \). Note that the graph \((\alpha_2, a_2)\) of the first term \( a_2 \) in the minimum in (25) is a straight line plus a quarter circle, as shown in Fig. 1. Note also that \( b_2 + c\sqrt{1 - \alpha_2^2} \) is maximized at \( \alpha^*_2 = b/\sqrt{b^2 + c^2} \) with maximum \( \sqrt{b^2 + c^2} \), and the intersection of \( a_2 \) and \( b_2 + c\sqrt{1 - \alpha_2^2} \) occurs at \( \alpha''_2 = c/\sqrt{c^2 + (a-b)^2} \).

If \( a \leq b \), \( a_2 \) is below \( b_2 + c\sqrt{1 - \alpha^2} \), the minimum of the two is \( a_2 \), and thus optimal \( \alpha^*_2 = 1 \). In the second case that \( a_2 \) and \( b_2 + c\sqrt{1 - \alpha^2} \) intersect between \( \alpha'_2 \) and 1, i.e., \( \alpha'_2 \leq \alpha''_2 \), the solution to (25) occurs at \( \alpha^*_2 = \alpha''_2 = c/\sqrt{c^2 + (a-b)^2} \). Finally, in the third case that \( a_2 \) and \( b_2 + c\sqrt{1 - \alpha^2} \) intersect between 0 and \( \alpha'_2 \), i.e., \( \alpha'_2 > \alpha''_2 \), the solution to (25) occurs at \( \alpha^*_2 = \alpha'_2 = b/\sqrt{b^2 + c^2} \). Since the condition \( \alpha'_2 \leq \alpha''_2 \) can be rewritten as [21]

\[
\frac{b}{\sqrt{b^2 + c^2}} \leq \frac{c}{\sqrt{c^2 + (a-b)^2}} \iff a \leq b + c^2/b, \tag{26}
\]

the optimal solution \( \alpha^*_2 \) to the problem (25) is summarized as [21]

\[
\alpha^*_2 = \begin{cases} 
1 & \text{if } a \leq b \text{ (case 1),} \\
\frac{c}{\sqrt{c^2 + (a-b)^2}} & \text{if } b < a \leq b + c^2/b \text{ (case 2),} \\
\frac{b}{\sqrt{b^2 + c^2}} = \sqrt{\theta} & \text{if } a > b + c^2/b \text{ (case 3),}
\end{cases} \tag{27}
\]

and the corresponding optimal value \( \gamma^*_2 \) is given by [21]

\[
\gamma^*_2 = \begin{cases} 
\gamma^{(1)}_2 = a^2 = \frac{\|b_1\|^2}{\sigma^2_1(1+\gamma^*_1)}, & \text{case 1,} \\
\gamma^{(2)}_2 = a^2(\alpha^*_2)^2 = \frac{\|b_1\|^2}{\sigma^2_1(1+\gamma^*_1)(\alpha^*_2)^2}, & \text{case 2,} \\
\gamma^{(3)}_2 = b^2 + c^2 = \frac{\|b_2\|^2}{\sigma^2_2(1+\gamma^*_2)}, & \text{case 3.}\end{cases} \tag{28}
\]
B. Pareto-optimal design in two-user MISO BC with SIC with power allocation

Now consider the actual problem (19) - (20) of Pareto-optimal beam design and power allocation for the two-user MISO-NOMA BC. We obtain the solution to this problem by exploiting the result in Section III-A. Note that once the power allocation values \( p_1 \) and \( p_2 \) are fixed, the corresponding optimal solution can be obtained from the result in Section III-A. Therefore, we represent the optimal solution as a function of power allocation and then optimize the power allocation. For given \( p_1 \in [0, P] \), the problem (22) - (23) changes to

\[
\begin{align*}
\min_{(\alpha_1, \beta_1) \in F} & \quad \alpha_1 \\
\text{subject to} & \quad \sqrt{p_1} \|h_1\| (\sqrt{\theta \alpha_1} + \sqrt{1 - \theta \beta_1}) = \sqrt{\gamma_1^* \sigma_1^2},
\end{align*}
\]

and the corresponding solution is given by \( \alpha_1^*(p_1) = \)

\[
\begin{cases}
0 & \text{if } \Gamma \leq p_1(1 - \theta), \\
\sqrt{\Gamma / p_1} - \sqrt{(1 - \theta)(1 - \Gamma / p_1)} & \text{if } \Gamma > p_1(1 - \theta).
\end{cases}
\]

Furthermore, the coefficients \( a, b, \) and \( c \) defined in (25) are changed to

\[
\begin{align*}
a(p_1) & := \sqrt{P - p_1} \frac{\|h_1\|}{\sqrt{\sigma_1^2(1 + \gamma_1^*)}} \\
b(p_1) & := \sqrt{P - p_1} \frac{\|h_2\| \sqrt{\gamma}}{\sqrt{p_1} \|h_2\|^2 (\alpha_1^*(p_1))^2 + \sigma_2^2} \\
c(p_1) & := \sqrt{P - p_1} \frac{\|h_2\| \sqrt{\Gamma - \theta}}{\sqrt{p_1} \|h_2\|^2 (\alpha_1^*(p_1))^2 + \sigma_2^2}
\end{align*}
\]

Then, the optimal solution to (19) - (20) can be represented as a function of \( p_1 \):

\[
\alpha_2^*(p_1) = \begin{cases}
1 & \text{if } p_1 \in \mathcal{P}_1, \\
\frac{c(p_1)}{\sqrt{c^2(p_1) + [a(p_1) - b(p_1)]^2}} & \text{if } p_1 \in \mathcal{P}_2, \\
\frac{b(p_1)}{\sqrt{b^2(p_1) + c^2(p_1)}} = \sqrt{\gamma} & \text{if } p_1 \in \mathcal{P}_3,
\end{cases}
\]

where \( \mathcal{P}_1 := \{p_1 | a(p_1) \leq b(p_1)\} \), \( \mathcal{P}_2 := \{p_1 | b(p_1) < a(p_1) \leq b(p_1) + c^2(p_1)/b(p_1)\} \), and \( \mathcal{P}_3 := \{p_1 | a(p_1) > b(p_1) + c^2(p_1)/b(p_1)\} \), and the corresponding SINR for user 2 \( \gamma_2^*(p_1) \) is given by \( \gamma_2^*(p_1) = \)

\[
\begin{align*}
\gamma_2^{*(1)}(p_1) & = (P - p_1) \frac{\|h_1\|^2}{\sigma_1^2(1 + \gamma_1^*)} & \text{if } p_1 \in \mathcal{P}_1, \\
\gamma_2^{*(2)}(p_1) & = (P - p_1) \frac{\|h_1\|^2}{\sigma_1^2(1 + \gamma_1^*) \left[\alpha_2^*(p_1)\right]^2} & \text{if } p_1 \in \mathcal{P}_2, \\
\gamma_2^{*(3)}(p_1) & = (P - p_1) \frac{\|h_2\|^2}{\|h_2\|^2 \left[p_1 \|h_2\| \alpha_2^*(p_1)\right]^2 + \sigma_2^2} & \text{if } p_1 \in \mathcal{P}_3.
\end{align*}
\]

Finally, the original problem (19) - (20) reduces to

\[
\max_{0 \leq p_1 \leq P} \gamma_2^*(p_1),
\]
where $\gamma_2^*(p_1)$ is given by (35). Note that if we knew which $\gamma_2^{s(i)}$ in (35) to use for optimization by directly computing $a(p_1), b(p_1), c(p_1)$ and their relationship, it would be easy to solve the problem (36). However, the parameters $a(p_1), b(p_1)$ and $c(p_1)$ to determine $\gamma_2^{s(i)}$ to use are functions of the design variable $p_1$. Nevertheless, this is possible and the result is given in the following proposition.

**Proposition 1:** For given $h_1, h_2, \sigma_1^2, \sigma_2^2, P$ and $\gamma_1^*$, we have the following regarding the optimal solution $p_1^{opt}$ to the problem (36). If $\theta \Gamma < \tau$ or if $\theta \Gamma \geq \tau \geq 0$ and $P \geq \Gamma + \frac{1}{\theta \Gamma} (\sqrt{\theta \Gamma} - \sqrt{\tau}) \left( \sqrt{\theta \Gamma} + \frac{1}{\lambda_2 \sqrt{\gamma}} \right)$, then $p_1^{opt} \in P_2$. Otherwise, $p_1^{opt} \in P_3$. Here, $\tau := \theta^{-1} \left( \lambda_1^{-1} + \Gamma \right) - \lambda_2^{-1}$, and $\Gamma$ and $\lambda_i$ are defined in (21).

**Proof:** See Appendix.

Due to Proposition 1 we know which of the three cases in (35) is applicable to the given combination of $h_1, h_2, \sigma_1^2, \sigma_2^2, P,$ and $\gamma_1^*$. Once the set $P_i$ to which $p_1^{opt}$ belongs is determined, optimal $p_1^{opt}$ can be found by maximizing the corresponding $\gamma_2^{s(i)}(p_1)$ in (35) with respect to $p_1$. A closed-form solution from solving $d\gamma_2^{s(i)}(p_1) = 0$ seems complicated but the solution can easily be found by a numerical method. The proposed algorithm to design Pareto-optimal beam vectors and power allocation is summarized in Table I. The Pareto-boundary of a two-user MISO-NOMA BC can be computed by sweeping $R_1^* = \log(1 + \gamma_1^*)$ and computing the corresponding maximum $R_2^* = \log(1 + \gamma_2^*)$. An example is shown in Fig. 2. It is seen that power allocation significantly enlarges the achievable rate region over the fixed-power beam-only design and optimal power allocation is crucial for MISO-NOMA BC.

![Pareto-boundary: fixed power versus power allocation](image-url)

Fig. 2: Pareto-boundary: fixed power versus power allocation ($\|h_1\|^2/\sigma_1^2 = 20, \|h_2\|^2/\sigma_2^2 = 3, \theta = 0.5$ and $P = 2$)
TABLE I

Pareto-optimal design for 2-user MISO-BC with SIC

\[
[\sqrt{p_1}w_1, \sqrt{p_2}w_2] = D(h_1, h_2, \sigma_1^2, \sigma_2^2, \gamma_1^*, P)
\]

Input: channel vectors \(h_1, h_2\), noise power \(\sigma_1^2, \sigma_2^2\), target SINR of user 1 \(\gamma_1^*\), and cluster total power \(P\).

Initialization: \(\lambda_1 = \|h_1\|^2/\sigma_1^2, \lambda_2 = \|h_2\|^2/\sigma_2^2, \theta = \frac{|h_1^Hh_2|^2}{\|h_1\|^2\|h_2\|^2}, \Gamma = \gamma_1^*/\lambda_1, \) and \(\tau = \theta^{-1}(\lambda_1^{-1} + \Gamma) - \lambda_2^{-1}\).

if \(\theta \Gamma < \tau\)

obtain \(p_1^{opt}\) maximizing \(\gamma_2^{* (2)}\)

elseif \(\tau \geq 0\) and \(P \geq \Gamma + \frac{1}{1-\theta}(\sqrt{\theta\Gamma} - \sqrt{\tau})\left(\sqrt{\theta\Gamma} + \frac{1}{\lambda_2\sqrt{\tau}}\right)\)

obtain \(p_1^{opt}\) maximizing \(\gamma_2^{* (2)}\)

else

obtain \(p_1^{opt}\) maximizing \(\gamma_2^{* (3)}\)

endif

Obtain \(\alpha_1^*, \beta_1^*\) and \(\alpha_2^*\) using (30), (20) and (34) with \(p_1 = p_1^{opt}\), and obtain \(w_1\) and \(w_2\) from (12) and (13) with \(\alpha_1^*, \beta_1^*\) and \(\alpha_2^*\).

Output: \(\sqrt{p_1^{opt}}w_1\) and \(\sqrt{P - p_1^{opt}}w_2\)

IV. TWO-USER MISO BROADCAST CHANNEL WITH SIC: PERFORMANCE STUDY

In the previous section, we developed a Pareto-optimal beam design and power allocation algorithm for given channel vectors \(h_1\) and \(h_2\). The performance of the Pareto-optimal design is a function of the two (effective) channel vectors \(h_1\) and \(h_2\). In the conventional ZF downlink beamforming with no SIC at the receivers, two users with orthogonal channel vectors are preferred since non-orthogonality between the two channel vectors reduces the effective SINR of ZF beamforming [15]. However, in the considered MISO-NOMA framework in which user 1 intends to decode the interference from user 2 and subtracts it before decoding its own data whereas user 2 treats the interference from user 1 as noise, orthogonality between \(h_1\) and \(h_2\), i.e., \(\theta \approx 0\), and corresponding orthogonal beam vectors \(w_1\) and \(w_2\) (see (12) and (13) with \(h_1 \perp h_2\)) do not necessarily imply high performance. Intuitively, if \(\theta\) is small, user 2 receives less interference from user 1, but user 1 has difficulty in decoding the message of user 2 for SIC under the NOMA framework. In this section, we investigate more on the two-user MISO-NOMA BC before proceeding to overall user scheduling in the next section.

To gain some insight into good channel conditions for two-user MISO-NOMA BCs, let us first consider the fixed power allocation case with \(p_1 = p_2 = 1\) as described in Section III-A and investigate the impact
of the angle $\theta$ between the two channel vectors when the magnitudes are given. For given $||h_1||$, $||h_2||$ and $\gamma_1^*$, the SINR of user 2 $\gamma_2^*$ in (28) can be rewritten as a function of $\theta$ as

$$
\gamma_2^*(\theta) = \begin{cases} 
\gamma_2^{*(1)} (\frac{\lambda_1}{1+\Gamma_1 \lambda_1}) & \text{for case 1}, \\
\gamma_2^{*(2)} (\frac{\lambda_1}{1+\Gamma_1 [\alpha_2^*(\theta)]^2}) & \text{for case 2}, \\
\gamma_2^{*(3)} (\frac{\lambda_2}{\lambda_2[\alpha_2^*(\theta)]^2+1}) & \text{for case 3}, 
\end{cases}
$$

where

$$
\alpha_1^*(\theta) = \begin{cases} 
0 & \text{if } \Gamma \leq 1-\theta \\
\sqrt{\theta \Gamma} - \sqrt{(1-\theta)(1-\Gamma)} & \text{if } \Gamma > 1-\theta 
\end{cases}
$$

and

$$
\alpha_2^*(\theta) = \begin{cases} 
1 & \text{for case 1} \\
\frac{c(\theta)}{\sqrt{c^2(\theta)+(a-b(\theta))^2}} & \text{for case 2} \\
\sqrt{\theta} & \text{for case 3}.
\end{cases}
$$

Regarding optimal $\theta$ that maximizes $\gamma_2^*(\theta)$ in (37), we have the following proposition:

**Proposition 2:** Let $\lambda_1$, $\lambda_2$ and $\Gamma$ be given. If $\Gamma \in [\Gamma_1, \Gamma_2]$, where

$$
\Gamma_1 = \frac{1}{2} \left(1 + \lambda_2^{-1} - \lambda_1^{-1}\right) - \frac{1}{2} \sqrt{(1 + \lambda_1^{-1} + \lambda_2^{-1})^2 - 4\lambda_2^{-1}(1 + 1_2^{-1})},
$$

$$
\Gamma_2 = \frac{1}{2} \left(1 + \lambda_2^{-1} - \lambda_1^{-1}\right) + \frac{1}{2} \sqrt{(1 + \lambda_1^{-1} + \lambda_2^{-1})^2 - 4\lambda_2^{-1}(1 + \lambda_1^{-1})},
$$

then optimal $\theta$ that maximizes $\gamma_2^*(\theta)$ in (37) is given by the region

$$
\left\{ \theta | \theta_0 \leq \theta \leq \frac{z_1 \bar{z}_2 + 2\Gamma(1-\Gamma) + \sqrt{4\Gamma(1-\Gamma)[\Gamma(1-\Gamma) + z_1 \bar{z}_2 - \bar{z}_2^2]}}{z_1^2 + 4\Gamma(1-\Gamma)} \right\}
$$

where $z_1 = \lambda_1^{-1} + 1 - \Gamma$, $z_2 = \lambda_2^{-1} + 1 - \Gamma$, and

$$
\theta_0 = \begin{cases} 
\frac{\lambda_1}{\lambda_1 1+\Gamma_1 \lambda_1}, & \text{if } \frac{\lambda_2}{1+\Gamma_1 \lambda_1} \leq 1 - \Gamma, \\
\frac{z_1 \bar{z}_2 + 2\Gamma(1-\Gamma)-\sqrt{4\Gamma(1-\Gamma)[\Gamma(1-\Gamma) + z_1 \bar{z}_2 - \bar{z}_2^2]}}{z_1^2 + 4\Gamma(1-\Gamma)}, & \text{otherwise}.
\end{cases}
$$

If $\Gamma \notin [\Gamma_1, \Gamma_2]$, optimal $\theta$ is given by the region

$$
\left\{ \theta | \theta_2 = \lambda_2^{-1} \Gamma \lambda_1 \leq \theta \leq 1 - \Gamma \right\} \quad \text{if } \Gamma \leq \frac{\lambda_2^{-1} \Gamma \lambda_1}{1+\lambda_2} \text{ and } \Gamma \notin [\Gamma_1, \Gamma_2],
$$

or \{ $\theta | \frac{\partial \gamma_2^{*(2)}(\theta)}{\partial \theta} = 0$ \} \quad \text{if } \Gamma > \frac{\lambda_2^{-1} \Gamma \lambda_1}{1+\lambda_2} \text{ and } \Gamma \notin [\Gamma_1, \Gamma_2].
$$

**Proof:** See Appendix.

From Proposition 2 we obtain a more insightful corollary as follows:
Corollary 1: Let $\lambda_1$, $\lambda_2$ and $\Gamma$ be given. When $\lambda_1 = \lambda_2$, optimal $\theta$ for the 2-user MISO BC with SIC is given by the set $\{\theta \mid \theta_0 \leq \theta \leq 1\}$ with $\theta_0$ reduced to

$$\theta_0 = \begin{cases} 
\frac{1}{1+\lambda_1} & \text{if } \frac{1}{1+\lambda_1} \leq 1 - \Gamma \\
\frac{z_1^2}{\lambda_1+4(1-\Gamma)} & \text{if } \frac{1}{1+\lambda_1} > 1 - \Gamma 
\end{cases}.$$  

(44)

Proof: See Appendix.

Corollary 1 states that in two-user MISO BCs with SIC with the same channel magnitudes $\lambda_1 = \lambda_2$ and the same power $p_1 = p_2 = 1$, two aligned channel vectors are preferred to two orthogonal channels and channel alignment beyond a certain angle is all optimal.

Although Proposition 2 and Corollary 1 provide some insight into good channel conditions in the SIC BC case, the assumptions for Proposition 2 and Corollary 1 are not valid in the actual NOMA situation in which power allocation is applied. Unfortunately, the optimal power $p_1^{opt}$ was not obtained in closed form in the previous section and this puts difficulty on analysis of the impact of channel angle on the performance. Hence, in the actual case, to enable analysis we derive the SINR $\gamma_2^*$ for user 2 as a function of $\theta$ by assuming the simple power allocation method that assigns minimum power $p_{1,min}(=\gamma_1^*/\lambda_1 = \Gamma)$ to achieve the target SINR $\gamma_1^*$ to user 1 and assigns the rest of power $P$ to user 2. The simple power allocation strategy is based on the assumption that user 1 has a strong channel and is limited by channel’s DoF such as bandwidth, whereas user 2 with a weak channel is limited by noise and needs to receive more power. For this simple power allocation method, the optimal $\gamma_2^*$ is obtained by substituting $p_1 = p_{1,min} = \Gamma$ into (35) and given after some manipulation in closed form as

$$\gamma_2^* = \begin{cases} 
P-\Gamma \lambda_1^{-1} \Gamma^{-1} \theta^{-1} & \text{if } \theta \leq \theta_1 \\
P-\Gamma \lambda_2^{-1} \Gamma^{-1} \theta^{-1} & \text{if } \theta > \theta_1, 
\end{cases}$$  

(45)

where $\theta_1 := \frac{1}{2} \left[ -\lambda_2^{-1} \Gamma^{-1} + \sqrt{\lambda_2^{-2} \Gamma^{-2} + 4(\lambda_1^{-1} \Gamma^{-1} + 1)} \right].$

Examples of $\gamma_2^*$ in (45) as a function of $\theta$ are shown in Fig. 3 together with the optimal $\gamma_2^*$ obtained by running the algorithm in Table I. It is seen that the $\gamma_2^*$ behavior depends on the relative magnitude of $\lambda_1$ and $\lambda_2$ through the two performance limiting factors: the SIC processing at user 1 and the SINR of user 2, as seen in (11). In the case of $\lambda_1 = \lambda_2 \gg 0$, the performance of user 2 is not limited by noise at user 2 but is limited by signal-to-interference ratio (SIR). Thus, in this case it is preferred that channel vectors are aligned and more power is allocated to user 2 under the constraint that the required SINR for user 1 is satisfied. By doing so, SIC at user 1 is easy and SIR at user 2 is high. This behavior is
The optimal power allocation

The simple power allocation

(a) (b) (c)

Fig. 3: $\gamma_2^*$ in (45) as a function of $\theta$: (a) $\lambda_1 = 10, \lambda_2 = 10, \Gamma = 2, P = 10$, (b) $\lambda_1 = 10, \lambda_2 = 1, \Gamma = 2, P = 10$, and (c) $\lambda_1 = 10, \lambda_2 = 0.1, \Gamma = 2, P = 10$.

evident in Fig. 3(a). It is seen that the optimal power control and the simple power allocation strategy yield similar performance in Fig. 3(a). (The behavior for $\lambda_1 = \lambda_2$ with power control seems similar to that stated in Corollary 1.) However, in the medium asymmetric case of $\lambda_1 > \lambda_2$ as in Fig. 3(b), there is a trade-off between the two performance limiting factors. When two channels are orthogonal, SIC at user 1 is difficult. On the other hand, when two channels are aligned, interference from user 1 to user 2 at user 2 is high. Hence, the performance is good when the two user channels are neither too orthogonal nor too aligned. This behavior is evident in Fig. 3(b). Note that in this case, there is a large gap between the optimal power control and the simple power allocation.

In addition to the above simple power allocation result, we have another result exploiting the fact $\lambda_1 \gg \lambda_2$ in actual NOMA, given by the following proposition:

**Proposition 3:** For given $\lambda_1$ and $\gamma_1^*$, if $\theta \neq 0$, as $\lambda_2 \to 0$, the optimal power coefficient $p_{1,\text{opt}} \to p_{1,\text{min}} = \Gamma$; the corresponding $\gamma_2^*$ converges to $\gamma_2^* = \frac{P - \Gamma}{\theta + \lambda_2 \Gamma}$; and the beam vectors converge to $\sqrt{p_1 w_1} = \sqrt{\Gamma h_1 \|h_1\|}$ and $\sqrt{p_2 w_2} = \sqrt{P - \Gamma} \frac{h_2}{\|h_2\|}$. That is, both users use matched-filtering beams.

**Proof:** See Appendix.

Note that $\gamma_2^*$ in Proposition 3 coincides with the second formula in (45). This is because $\theta_1$ in (45) converges to $0^+$ as $\lambda_2 \to 0$ for given $\lambda_1$ and $\Gamma$, and the second formula in (45) is valid in this case. By Proposition 3, if $\lambda_2$ is sufficiently small compared to $\lambda_1$, matched filtering beams for both users with minimum power to user 1 satisfying the target SINR are optimal regardless of the angle between the two channel vectors. This is because if $\lambda_1 \gg \lambda_2$, the limitation for $\gamma_2^*$ results from the SINR of user 2 at user 2. Hence, maximum power should be delivered to user 2 with matched filtering beam $w_2$ by assigning
minimum power to user 1 with matched filtering beam $\mathbf{w}_1$. This behavior is evident in Fig. 3(c). It is seen in Fig. 3(c) that the simple power allocation method almost achieves the optimal performance for all $\theta$.

V. THE PROPOSED SCHEDULING METHOD FOR $K$-USER MISO-NOMA DOWNLINK

Now, we propose our overall user scheduling/pairing and beam design method for the $K$-user MISO-NOMA downlink with $N_t$ BS antennas based on the results in the previous sections. In MISO-NOMA downlink scheduling and beam design, two major aspects should be taken into account simultaneously to guarantee good system performance: One is controlling ICI to reduce interference from other clusters and the other is pairing and beam design for the paired users in each cluster for maximum performance. Recall that the gain of NOMA lies in the case that strong users are in the DoF(such as bandwidth)-limited regime and weak users are limited by noise [2, P. 239]. In MU-MISO NOMA, the beneficial situation for NOMA should be maintained, i.e., the high channel quality for strong users should be maintained by SIC and proper interference control, and low SINRs of weak users should be leveraged by assigning more power to weak users. To be consistent with this design principle, ICI should be eliminated for strong users by proper measures. With these considerations, we propose the following user scheduling, pairing and beam design method for $K$-user MISO-NOMA downlink composed of two steps under the assumption that all channel vector information is available at the BS and the thermal noise variance is known.

Algorithm 1: Overall User Scheduling and Beam Design
**Step 1:** In the first step, we run the semi-orthogonal user selection (SUS) algorithm [15] targeting selection of $N_t$ users from the strong user set $K_1$. Then, the SUS algorithm returns $K_c(\leq N_t)$ users with roughly orthogonal channel vectors out of the $K/2$ users in $K_1$. (Depending on the size $|K_1|$ and the semi-orthogonality parameter, the SUS algorithm may return users less than $N_t$ especially for large $N_t$ although we target selecting $N_t$ users [15].) We set these $K_c$ users returned by the SUS algorithm as the $K_c$ strong users in $K_c$ clusters (one user for each cluster). Let their actual channel vectors be $\tilde{h}_1^{(1)}, \ldots, \tilde{h}_1^{(K_c)}$.

**Step 2:** Weak user selection and overall beam design

**Initialization:** $\Gamma$ is given.

$$K_2 = \{1, \ldots, K/2\} \quad \text{(the original weak user set)},$$
$$S_1 : \text{the set of selected strong users from step 1},$$
$$S_2 \leftarrow \phi \quad \text{(the set of selected weak users)},$$
$$\tilde{\mathbf{W}} = \begin{bmatrix} \Pi_{\tilde{H}_1} \tilde{h}_1^{(1)} \\ \ldots \\ \Pi_{\tilde{H}_{K_c}} \tilde{h}_1^{(K_c)} \end{bmatrix},$$
$$\tilde{\mathbf{W}}_1 = [], \text{ and } \tilde{\mathbf{W}}_2 = [],$$
$$k = 1.$$

**Iteration:**

while $k \leq K_c$ do

S.1: For each user $u \in K_2$, estimate ICI plus AWGN:

$$\tilde{\sigma}_u^2 = \sum_{l<k}(|\tilde{g}_u^H \tilde{W}_1(l)|^2 + |\tilde{g}_u^H \tilde{W}_2(l)|^2) + \sum_{l>k} P|\tilde{g}_u^H \tilde{W}(l)|^2 + \epsilon_u^2,$$

where $\tilde{W}(l), \tilde{W}_1(l)$ and $\tilde{W}_2(l)$ are the $l$-th columns of $\tilde{W}$, $\tilde{W}_1$ and $\tilde{W}_2$ respectively, and $\tilde{g}_u$ is the actual channel vector of user $u$ in $K_2$. With obtained $\tilde{\sigma}_u^2$ and given $\Gamma$, compute the maximum SINR $\gamma_2^*(u)$ of user $u$ when user $u$ is paired with the channel $\tilde{h}_1^{(k)}$ of user 1 of cluster $k$, as described in Section III-B or Section IV by setting $\lambda_1 = \frac{||\Pi_{\tilde{H}_1} \tilde{h}_1^{(k)}||^2}{\tilde{\sigma}_1^2}$, $\lambda_2 = \frac{||\Pi_{\tilde{H}_g} \tilde{g}_u||^2}{\tilde{\sigma}_2^2}$, and

$$\theta = \left| \frac{(\Pi_{\tilde{H}_1} \tilde{h}_1^{(k)})^H (\Pi_{\tilde{H}_1} \tilde{h}_1^{(k)})}{\Pi_{\tilde{H}_g} \tilde{h}_1^{(k)} ||^2 ||\Pi_{\tilde{H}_g} \tilde{g}_u||^2} \right|^2.$$
S.2: Select the weak user of cluster \( k \) as follows:

\[
\begin{align*}
    u^* &= \arg \max_{u \in \mathcal{K}_2} \gamma^2_2(u) \\
    S_2 &\leftarrow S_2 \cup \{u^*\}
\end{align*}
\]

(53)\hspace{3cm}(54)

\[
\tilde{\mathbf{h}}^{(k)}_2 = \tilde{\mathbf{g}}_{u^*}
\]

(55)

and design \([\sqrt{p_1}\tilde{\mathbf{w}}_1^{(k)}, \sqrt{p_2}\tilde{\mathbf{w}}_2^{(k)}] = \mathcal{D}(\Pi_{\tilde{\mathbf{H}}_k}^{\perp} \tilde{\mathbf{h}}_1^{(k)}, \Pi_{\tilde{\mathbf{H}}_k}^{\perp} \tilde{\mathbf{h}}_2^{(k)}, \sigma^2_u, \sigma^2_y, \lambda_1, P)\) by using the two-user Pareto-optimal design algorithm in Table I.

S.3: Store the designed beams, remove \( u^* \) in \( \mathcal{K}_2 \), and repeat until \( k = K_c \):

\[
\begin{align*}
    \tilde{\mathbf{W}}_1 &\leftarrow [\tilde{\mathbf{W}}_1, \sqrt{p_1}\tilde{\mathbf{w}}_1^{(k)}], \quad \tilde{\mathbf{W}}_2 &\leftarrow [\tilde{\mathbf{W}}_2, \sqrt{p_2}\tilde{\mathbf{w}}_2^{(k)}] \\
    \mathcal{K}_2 &\leftarrow \mathcal{K}_2 \setminus \{u^*\} \\
    k &\leftarrow k + 1
\end{align*}
\]

end while

**Remark 1:** Note that the computation of \( \gamma^2_2(u) \) and the Pareto-optimal beam design for two users in each cluster in steps S.1 and S.2 in the while loop of Step 2 of Algorithm 1 is based on the *projected effective channels*. Note that the projected effective channels \( \Pi_{\tilde{\mathbf{H}}_k}^{\perp} \tilde{\mathbf{h}}_1^{(k)} \) and \( \Pi_{\tilde{\mathbf{H}}_k}^{\perp} \tilde{\mathbf{h}}_2^{(k)} \) lie in \( \mathcal{L}^{\perp}(\tilde{\mathbf{H}}_k) \). Thus, the corresponding Pareto-optimal beams \( \mathbf{w}_1^{(k)} \) and \( \mathbf{w}_2^{(k)} \) lie in \( \mathcal{L}^{\perp}(\tilde{\mathbf{H}}_k) \) by the property of Pareto-optimal beams (see (12) and (13))\(^{19}\), and hence \( \mathbf{w}_2^{(k)} = \Pi_{\tilde{\mathbf{H}}_k}^{\perp} \mathbf{w}_1^{(k)} = \tilde{\mathbf{w}}_i^{(k)}, \quad i = 1, 2 \) for (4). Hence, there exists no ICI to all strong users with the proposed beam design.

**Remark 2:** If \( K_c = N_t \), then \( \tilde{\mathbf{H}}_k \) has nullity of one, and \( \Pi_{\tilde{\mathbf{H}}_k}^{\perp} \tilde{\mathbf{h}}_1^{(k)} \) and \( \Pi_{\tilde{\mathbf{H}}_k}^{\perp} \tilde{\mathbf{h}}_2^{(k)} \) are aligned, as shown in Fig. 4(a). In this case, only Pareto-optimal power control is applied for each cluster by the proposed design method. (Note that the algorithm in Table I is still applicable in case of two aligned input channel vectors.) On the other hand, if the number \( K_c \) of the returned users by the SUS algorithm in Step 1 is less than \( N_t \) (which is often true for large \( N_t \) with small \( K \)\(^{15}\)), then \( \tilde{\mathbf{H}}_k \) has nullity larger than or equal to two. In this case, the projected effective channels \( \Pi_{\tilde{\mathbf{H}}_k}^{\perp} \tilde{\mathbf{h}}_1^{(k)} \) and \( \Pi_{\tilde{\mathbf{H}}_k}^{\perp} \tilde{\mathbf{h}}_2^{(k)} \) span a 2-dimensional (2-D) space and the full 2-D Pareto-optimal beam design is applicable for each cluster, as shown in Fig. 4(b). This is another advantage of the proposed method over the previous methods\(^{12}–^{14}\) based on simple spatial ZF beam design ignoring the case of \( K_c < N_t \).

**Remark 3:** In the step (52) of computation of ICI and AWGN for each candidate weak user for cluster \( k \), already designed beam vectors are used up to cluster \( k - 1 \) and the beam estimates \( \sqrt{p_1} \Pi_{\tilde{\mathbf{H}}_k}^{\perp} \tilde{\mathbf{h}}_1^{(k+1)} \), \( \cdots \), \( \sqrt{p_1} \Pi_{\tilde{\mathbf{H}}_{K_c}}^{\perp} \tilde{\mathbf{h}}_1^{(K_c)} \) are used for undesigned clusters \( k + 1, \cdots, K_c \).
Remark 4: Note that there is length reduction from the actual channel $\tilde{h}^{(k)}_1$ to the effective channel $\overline{\Pi_{\tilde{h}_k}} \tilde{h}^{(k)}_1$ of each strong user. This reduction is the typical effective gain loss associated ZF, but the loss is not significant because the strong channels $\tilde{h}^{(1)}_1, \cdots, \tilde{h}^{(K_e)}_1$ are semi-orthogonal by the SUS algorithm [15]. Only the weak users experience ICI whereas ICI to the strong users is completely removed in the proposed method to be consistent with the NOMA design principle. However, the weak users are selected by considering all the factors, i.e., the ICI, projection onto $L^\perp(\tilde{H}_k)$ and the friendliness with the strong users to yield good performance.

VI. NUMERICAL RESULTS

In this section, we provide some numerical results to evaluate the performance of the proposed scheduling, beam design and power allocation method described in Section V for MU-MISO-NOMA downlink.

First, we evaluated the gain of the proposed method over conventional SUS-based MU-MISO scheduling [15] and the results are shown in Figs. 5 and 6. The simulation setup for Figs. 5 and 6 is as follows. The AWGN variance was one for all users. The numbers of transmit antennas were two and four for Fig. 5 and Fig. 6, respectively. Each element of each channel vector in the strong user set $K_1$ with $K/2$ users was randomly and independently generated from $CN(0, \sigma^2_{h,1})$ with $\sigma^2_{h,1} = 1$ and each element of each channel vector in the weak user set $K_2$ with $K/2$ users was randomly and independently generated from $CN(0, \sigma^2_{h,2})$ with $\sigma^2_{h,2} = 0.01$. (Hence, we have $\lambda_1/\lambda_2 = 100 = 20$ dB.) For the conventional SUS-based MU-MISO scheduling we considered two scheduling intervals. At the first interval, user scheduling out of $K_1$ was performed by running the SUS algorithm for MU-MISO with $N_t$ transmit antennas and the scheduled users were served by ZF downlink beamforming [15]. At the second interval, user scheduling out of $K_2$ was performed by running the SUS algorithm for MU-MISO with $N_t$ transmit antennas and the scheduled users were served by ZF downlink beamforming. The average sum rates for $K_1$ and $K_2$ were obtained by averaging the rates of 1000 independent channel realizations. For the overall sum rate of the conventional SUS method, the two rates of $K_1$ and $K_2$ were averaged. For the proposed NOMA method, the same 1000 channel realizations used for the conventional method were used but the proposed NOMA scheduling was performed over the overall user set $K_1 \cup K_2$ in a single scheduling interval. For the SUS algorithm applied separately to $K_1$ and $K_2$ and to Step 1 of the proposed method, the semi-orthogonality parameter $\delta$ should be chosen [15] and we used optimal $\delta$ for each $K/2$ provided from Fig. 2 of [15]. In addition, for the proposed method, $\Gamma$ should be chosen and we set $\Gamma$ appropriately to balance the rates from the two groups $K_1$ and $K_2$. It is seen in Figs. 5 and 6 that the proposed MU-MISO NOMA
Fig. 5: Sum rate ($N_t = 2$) : (a) total sum rate and (b) separate sum rates from $\mathcal{K}_1$ and from $\mathcal{K}_2$

Fig. 6: Sum rate ($N_t = 4$) : (a) total sum rate and (b) separate sum rates from $\mathcal{K}_1$ and from $\mathcal{K}_2$

method outperforms the conventional MU-MISO downlink based on the SUS user scheduling. Note that the sum rates for both groups $\mathcal{K}_1$ and $\mathcal{K}_2$ are better than the conventional scheme. It is also seen that the performance improvement by NOMA reduces as $N_t$ increases from two to four, but there exists non-trivial gain for NOMA for large $K$.

Next, we compared the proposed algorithm with an existing algorithm proposed for MU-MISO NOMA
downlink. For the comparison baseline we considered the NOMA-FOUS algorithm in [14] of which superiority over other methods [12], [13] is shown in [14]. Since the simulation setting is different from that in [14], we slightly modified the NOMA-FOUS algorithm so that the strong user is selected from $\mathcal{K}_1$ and the weak user is selected from $\mathcal{K}_2$, although the original NOMA-FOUS algorithm considers only one set of users. For comparison, we set $\sigma_{h,1}^2 = 1$ and $\sigma_{h,2}^2 = 0.04$, and set $\Gamma$ to make the sum rate of the weak users of the proposed algorithm larger than the sum rate of the weak users of the NOMA-FOUS algorithm. Fig. 7 shows the sum rate performance of the two algorithms. It is seen that the proposed algorithm outperforms the NOMA-FOUS algorithm.

The key feature of our Pareto-optimality-based design is that we have control over the rate operating point. Hence, we finally investigated the rate balancing property between the two groups $\mathcal{K}_1$ and $\mathcal{K}_2$ by controlling the strong-user-target-SINR parameter $\Gamma$ defined in (21) (larger $\Gamma$ means larger rates for strong users), and the result is shown in Fig. 8. The simulation parameters are the same as those for Fig. 6. For reference, the rates of $\mathcal{K}_1$ and $\mathcal{K}_2$ separately obtained by the conventional MU-MISO SUS algorithm are shown. It is seen that by abandoning the improvement for weak users but maintaining the weak-user performance at the level of the conventional SUS method, significant rate gain can be attained for strong users.
Fig. 8: Sum rate versus $\Gamma$ ($N_t = 4$, $K = 2000$ and $P_T = 20$ dB)

VII. CONCLUSION

In this paper, we have considered the problem of transmit beam design and user scheduling for MU-MISO NOMA downlink and proposed an effective beam design and user scheduling method based on Pareto-optimality by exploiting both the spatial and power domains available in MU-MISO NOMA downlink. The proposed method with the ability of rate control between strong and weak users provides great flexibility to NOMA network operation.

APPENDIX

Proof of Proposition 1: The set $\mathcal{P}_i$ to which $p_{1}^{opt}$ belongs is dependent on the relationship among $a(p_1), b(p_1)$ and $d(p_1) := b(p_1) + c^2(p_1)/b(p_1)$, given in terms of $\Gamma$, $\theta$ and $\lambda_i$ by

$$a(p_1) = \sqrt{P - p_1}\sqrt{\frac{\|h_1\|^2}{\sigma_1^2(1 + \gamma_1^1)}} = \sqrt{(P - p_1) \frac{\lambda_1}{1 + \Gamma \lambda_1}}$$  \hspace{0.5cm} (56)

$$b(p_1) = \sqrt{P - p_1}\sqrt{\frac{\|h_2\|^2}{\|h_2\|^2 p_1 [\alpha_1^1(p_1)]^2 + \sigma_2}}$$

$$= \sqrt{(P - p_1) \frac{\lambda_2 \theta}{\lambda_2 p_1 [\alpha_1^1(p_1)]^2 + 1}}$$  \hspace{0.5cm} (57)

$$d(p_1) = \sqrt{P - p_1}\sqrt{\frac{\|h_2\|^2}{\|h_2\|^2 p_1 [\alpha_1^1(p_1)]^2 + \sigma_2}}$$

$$= \sqrt{(P - p_1) \frac{\lambda_2 \theta}{\lambda_2 p_1 [\alpha_1^1(p_1)]^2 + 1}}$$  \hspace{0.5cm} (58)
The three sets \( \mathcal{P}_1, \mathcal{P}_2 \) and \( \mathcal{P}_3 \) can be rewritten by squaring \( a(p_1), b(p_1) \) and \( d(p_1) \) and dropping the common factor \((P - p_1)\) as \( \mathcal{P}_1 = \{ p_1 | \bar{a} \leq b(p_1) \} \), \( \mathcal{P}_2 = \{ p_1 | b(p_1) < \bar{a} \leq \bar{d}(p_1) \} \), and \( \mathcal{P}_3 = \{ p_1 | \bar{a} > \bar{d}(p_1) \} \), where

\[
\bar{a} = \frac{\lambda_1}{1 + \Gamma \lambda_1}, \quad \bar{b}(p_1) = \frac{\lambda_2 \theta}{\lambda_2 p_1 [\alpha_1^*(p_1)]^2 + 1}, \quad \text{and} \quad \bar{d}(p_1) = \frac{\lambda_2 / \theta}{\lambda_2 p_1 [\alpha_1^*(p_1)]^2 + 1}.
\]  

(59) \hspace{1cm} (60)

First, we show \( p_1^{\text{opt}} \notin \mathcal{P}_1 \). Let \( p_{1,\text{min}} \) denote the minimum \( p_1 \) to achieve \( \gamma_1^* \) with \( w_1 = h_1/||h_1|| \). Then, \( p_{1,\text{min}} = \gamma_1^*/\lambda_1 = \Gamma \). Hence, the second condition in (30), i.e., \( \Gamma > p_{1,\text{min}}(1 - \theta) \) or \( \Gamma = p_{1,\text{min}}(1 - \theta) \) is satisfied since \( 0 \leq \theta \leq 1 \), and \( \alpha_1^*(p_{1,\text{min}}) = \sqrt{\theta} \) from (30). Hence, we have

\[
\bar{a} = \frac{\lambda_1}{1 + \Gamma \lambda_1} = \frac{1}{\lambda_1 + \Gamma}
\]

(61)

\[
\bar{b}(p_{1,\text{min}}) = \frac{\lambda_2 \theta}{\lambda_2 \Gamma \theta + 1} = \frac{1}{\theta + \Gamma}.
\]

(62)

By the NOMA condition \( \lambda_1 > \lambda_2 \), we have \( \frac{1}{\lambda_1} < \frac{1}{\theta + \Gamma} \) since \( 0 \leq \theta \leq 1 \) and thus \( \bar{a} > \bar{b}(p_{1,\text{min}}) \). In case of \( \Gamma = p_{1,\text{min}}(1 - \theta) \), we have \( \theta = 0 \) and thus \( \bar{b}(p_{1,\text{min}}) = 0 \) and \( \bar{a} > \bar{b}(p_{1,\text{min}}) \). Hence, \( p_{1,\text{min}} \notin \mathcal{P}_1 \).

Note that \( \bar{a} \) is constant over \( p_1 \). It can be shown from (30) that the term \( p_1 [\alpha_1^*(p_1)]^2 \) in the denominator of \( \bar{b}(p_1) \) in (59) is monotone decreasing with respect to \( p_1 \), and hence \( \bar{b}(p_1) \) is monotone increasing with respect to \( p_1 \). If \( \bar{a} > \bar{b}(p_1) \) for all \( p_1 \), \( \mathcal{P}_1 \) is empty. Otherwise, there exists \( p_1 \), denoted as \( p_{1,a} \), such that \( \bar{a} = \bar{b}(p_1) \), as \( p_1 \) increases, given by

\[
p_{1,a} = \{ p_1 | \bar{a} = \bar{b}(p_1) \} = \{ p_1 | \lambda_1 \frac{\lambda_2}{1 + \Gamma \lambda_1} = \frac{\lambda_2 \theta}{\lambda_2 p_1 [\alpha_1^*(p_1)]^2 + 1} \} = \Gamma + \frac{1}{1 - \theta} \left( \sqrt{\theta \Gamma} - \sqrt{\theta \Gamma + \lambda_1^{-1} \theta - \lambda_2^{-1}} \right)^2.
\]

(63)

At \( p_1 = p_{1,a} \), we have \( \gamma_2^* = \gamma_2^* \) from (35) since \( \alpha_2^*(p_1) = 1 \) at the boundary of \( \mathcal{P}_1 \) (\( p_1 \geq p_{1,a} \) side) and \( \mathcal{P}_2 \) (\( p_1 < p_{1,a} \) side), i.e., \( \bar{a} = \bar{b}(p_1) \). Furthermore, it can be shown that \( \frac{\partial \gamma_2^*(p_1)}{\partial p_1} \bigg|_{p_1 = p_{1,a}} = -\frac{1}{c_1} \), where \( c_1 \) is a non-negative constant with respect to \( p_1 \). Hence, there exists \( p_1 \in \mathcal{P}_2 \) such that \( \gamma_2^*(p_1) > \gamma_2^*(p_{1,a}) \). Since \( \gamma_2^* \) is a monotone decreasing function of \( p_1 \) as seen in (35), optimal \( \gamma_2^* \) does not occur in \( \mathcal{P}_1 \), i.e., \( p_1^{\text{opt}} \notin \mathcal{P}_1 \).

Next, we check the condition that \( \mathcal{P}_3 \) is empty. Since the term \( p_1 [\alpha_1^*(p_1)]^2 \) in the denominator of \( \bar{d}(p_1) \) in (60) is monotone decreasing with respect to \( p_1 \), and thus \( \bar{d}(p_1) \) is monotone increasing with respect to \( p_1 \). Therefore, if \( \bar{a} < \bar{d}(p_{1,\text{min}}) \), then \( \mathcal{P}_3 \) is empty. Since \( \alpha_1^*(p_{1,\text{min}}) = \sqrt{\theta} \) from (30) and \( p_{1,\text{min}} = \Gamma \),
the condition is rewritten from (59) and (60) as
\[
\bar{a} < \bar{d}(p_{1,\min}) \iff \frac{\lambda_1}{1 + \Gamma \lambda_1} < \frac{\lambda_2 / \theta}{\lambda_2 \theta + 1} \quad \iff \quad \theta \Gamma < \frac{1}{\theta} \left( \frac{1}{\alpha_1} + \Gamma \right) - \frac{1}{\lambda_2} =: \tau. \tag{64}
\]
In this case, \( P_3 = \emptyset \) and \( p_{1,\text{opt}} \in P_2 \) since \( p_{1,\text{opt}} \notin P_1 \).

Now assume \( \theta \Gamma \geq \tau \). Then, \( P_3 \) is not empty. Furthermore, we have a sufficient condition for \( \forall \ p_1 \in P_3 \) as follows:
\[
d(p_1) < \bar{a}, \quad \forall p_1 \iff \frac{\lambda_2}{\theta} < \frac{\lambda_1}{1 + \Gamma \lambda_1} \quad \iff \quad \frac{1}{\theta} \left( \frac{1}{\alpha_1} + \Gamma \right) - \frac{1}{\lambda_2} < 0 \quad \iff \quad \tau < 0, \tag{65}
\]
because \( \lambda_2 / \theta \) is an upper bound of \( \bar{d}(p_1) \) (see (60)). In this case, \( p_{1,\text{opt}} \in P_3 \).

Finally, if \( \theta \Gamma \geq \tau \) and \( \tau \geq 0 \), compute \( p_1 \), denoted by \( p_{1,b} \), such that \( \bar{a} = \bar{d}(p_1) \), given by
\[
p_{1,b} = \{ p_1 | a = \bar{d}(p_1) \} = \{ p_1 | \frac{\lambda_1}{1 + \Gamma \lambda_1} = \frac{\lambda_2 / \theta}{\lambda_2 p_1 [\alpha_1^*(p_1)]^2 + 1} \} \tag{66}
\]
\[
= \Gamma + \frac{1}{1 - \theta} \left( \sqrt{\theta \Gamma} - \sqrt{\tau} \right)^2. \tag{67}
\]
If
\[
P < p_{1,b}, \quad \tag{68}
\]
then \( \bar{a} > \bar{d}(p_1) \) for \( p_1 \leq P \) since \( \bar{d}(p_1) \) is a monotone increasing function of \( p_1 \). Hence, in this case, \( \forall p_1 \in P_3 \) and \( p_{1,\text{opt}} \in P_3 \). On the other hand, if \( p_{1,b} \leq P \), we have both nonempty \( P_2 = \{ p_1 \geq p_{1,b} \} \) and \( P_3 = \{ p_1 < p_{1,b} \} \). In this case, we compute the derivatives of \( \gamma_2^{*(2)} \) and \( \gamma_2^{*(3)} \) at point \( p_{1,b} \), which are given by
\[
\left. \frac{\partial \gamma_2^{*(2)}}{\partial p_1} \right|_{p_1 = p_{1,b}} = c_2 \left[ \left( \lambda_2 \sqrt{\tau} \frac{1 - \theta}{\sqrt{\theta \Gamma} - \sqrt{\tau}} \right) P - \left( \lambda_2 \sqrt{\tau} \frac{1 - \theta}{\sqrt{\theta \Gamma} - \sqrt{\tau}} \cdot \Gamma + \lambda_2 \sqrt{\tau} \cdot \sqrt{\theta \Gamma} + 1 \right) \right] \tag{73}
\]
\[
\left. \frac{\partial \gamma_2^{*(3)}}{\partial p_1} \right|_{p_1 = p_{1,b}} = c_3 \left[ \left( \lambda_2 \sqrt{\tau} \frac{1 - \theta}{\sqrt{\theta \Gamma} - \sqrt{\tau}} \right) P - \left( \lambda_2 \sqrt{\tau} \frac{1 - \theta}{\sqrt{\theta \Gamma} - \sqrt{\tau}} \cdot \Gamma + \lambda_2 \sqrt{\tau} \cdot \sqrt{\theta \Gamma} + 1 \right) \right], \tag{74}
\]
where \( c_2 \) and \( c_3 \) are non-negative constants. The two derivatives have the same sign. If the two derivatives are positive, then \( \gamma_2^* \) increases as \( p_1 \) crosses \( p_{1,b} \) from the left to the right and hence \( p_{1,\text{opt}} \in P_2 \). Otherwise,
\( \gamma_2^* \) increases as \( p_1 \) crosses \( p_{1,b} \) from the right to the left and hence \( p_1^{\text{opt}} \in \mathcal{P}_3 \). Equivalently, we have

\[
p_1^{\text{opt}} \in \mathcal{P}_2 \text{ if } P \geq \Gamma + \frac{1}{1 - \theta} (\sqrt{\theta \Gamma} - \sqrt{\tau}) (\sqrt{\theta \Gamma} + \frac{1}{\lambda_2 \sqrt{\tau}}) \tag{75}
\]

\[
p_1^{\text{opt}} \in \mathcal{P}_3 \text{ if } P < \Gamma + \frac{1}{1 - \theta} (\sqrt{\theta \Gamma} - \sqrt{\tau}) (\sqrt{\theta \Gamma} + \frac{1}{\lambda_2 \sqrt{\tau}}). \tag{76}
\]

Since \( p_{1,b} = \Gamma + \frac{1}{1 - \theta} (\sqrt{\theta \Gamma} - \sqrt{\tau})^2 < \Gamma + \frac{1}{1 - \theta} (\sqrt{\theta \Gamma} - \sqrt{\tau}) (\sqrt{\theta \Gamma} + \frac{1}{\lambda_2 \sqrt{\tau}}) \), the set \( \{ P < p_{1,b} \} \) mentioned in (72) is a subset of the set \( \{ P > \Gamma + \frac{1}{1 - \theta} (\sqrt{\theta \Gamma} - \sqrt{\tau}) (\sqrt{\theta \Gamma} + \frac{1}{\lambda_2 \sqrt{\tau}}) \} \). Thus, the case of \( P < p_{1,b} \) is covered by (76). The only two cases for \( p_1^{\text{opt}} \in \mathcal{P}_2 \) are \([\theta \Gamma < \tau] \) or \([\theta \Gamma \geq \tau \geq 0 \) and \( P \geq \Gamma + \frac{1}{1 - \theta} (\sqrt{\theta \Gamma} - \sqrt{\tau}) (\sqrt{\theta \Gamma} + \frac{1}{\lambda_2 \sqrt{\tau}}) \). Hence, the claim follows. \( \square \)

To prove Proposition 2, we introduce the following lemma.

**Lemma 1:** Define

\[
\mathcal{N}_1 := \{ \theta \mid a \leq b(\theta) \} \tag{77}
\]

\[
\mathcal{N}_2 := \{ \theta \mid b(\theta) < a \leq b(\theta) + c^2(\theta)/b(\theta) \} \tag{78}
\]

\[
\mathcal{N}_3 := \{ \theta \mid a > b(\theta) + c^2(\theta)/b(\theta) \}, \tag{79}
\]

where \( a, b(\theta) \) and \( c(\theta) \) are defined just below (25). Then, for given \( \lambda_1, \lambda_2 \) and \( \Gamma \), every \( \theta \) in \( \mathcal{N}_1 \) achieves the same optimal \( \gamma_2^*(\theta) \) in (37), if \( \mathcal{N}_1 \) is not empty.

**Proof:** Let us assume that \( \mathcal{N}_1 \) is not empty. For every \( \theta \in \mathcal{N}_1 \), \( \gamma_2^*(\theta) = \gamma_2^{*(1)} = \frac{\lambda_1}{1 + \lambda_1 \theta} \). From the fact that \( \gamma_2^{*(1)} = \frac{\lambda_1}{1 + \lambda_1 \theta} \) and \( \gamma_2^{*(2)}(\theta) = \frac{\lambda_1}{1 + \lambda_1 \theta} [\alpha_2^2(\theta)]^2 \), and \( 0 \leq \alpha_2^2(\theta) < 1 \) for \( \theta \in \mathcal{N}_2 \), it is obvious that \( \gamma_2^{*(2)}(\theta) < \gamma_2^{*(1)} \) for all \( \theta \in \mathcal{N}_2 \). Hence, \( \gamma_2^*(\theta) \) for \( \theta \in \mathcal{N}_2 \) is less than \( \gamma_2^*(\theta) \) for \( \theta \in \mathcal{N}_1 \). Furthermore, for any \( \theta \in \mathcal{N}_3 \), we have

\[
\gamma_2^{*(3)}(\theta) = \frac{\lambda_1}{\lambda_2 [\alpha_1^2(\theta)]^2 + 1} \leq \frac{\lambda_2 \theta}{\lambda_2 [\alpha_1^2(\theta)]^2 + 1} = \frac{\lambda_2}{\lambda_2 [\alpha_1^2(\theta)]^2 + 1} \leq \frac{\lambda_2}{\lambda_2 [\alpha_1^2(\theta)]^2 + 1} \leq \frac{\lambda_2}{\lambda_2 [\alpha_1^2(\theta)]^2 + 1} = \gamma_2^{*(1)}.
\]

Here, step (a) is by (28), step (b) holds because \( \theta \in [0, 1] \), step (c) is by direction computation based on \( b(\theta) \) and \( c(\theta) \), step (d) holds because \( \theta \in \mathcal{N}_3 \), and step (e) is by (28). Consequently, we have the claim. \( \square \)

**Proof of Proposition 2:** The necessary and sufficient condition for \( \mathcal{N}_1 \) defined in (77) being non-empty is given by

\[
a^2 \leq \max_{0 \leq \theta \leq 1} b^2(\theta), \tag{80}
\]

where

\[
a^2 = \frac{\lambda_1}{1 + \Gamma \lambda_1}, \quad b^2(\theta) = \frac{\lambda_2 \theta}{\lambda_2 \alpha_1^2(\theta)^2 + 1}, \quad c^2(\theta) = \frac{\lambda_2 (1 - \theta)}{\lambda_2 \alpha_1^2(\theta)^2 + 1}. \tag{81}
\]

Since \( b(\theta) \) is maximized at \( \theta = \frac{\lambda_2 (1 - \Gamma)^2 + 1}{\lambda_2 (1 - \Gamma)^2 + \lambda_2 (1 - \Gamma)^2 + 1} \) and the corresponding maximum value is \( \max_{0 \leq \theta \leq 1} b^2(\theta) = \lambda_2 (1 + \lambda_2 \Gamma \lambda_1) \), the condition (80) becomes

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\[
\frac{\lambda_1}{1 + \lambda_1 \Gamma} \leq \lambda_2 (1 + \frac{\lambda_2 \Gamma}{\lambda_2 + 1})
\]
\[
\iff \Gamma_1 := \frac{1}{2} (1 + \lambda_2^{-1} - \lambda_1^{-1}) - \frac{1}{2} \sqrt{(1 + \lambda_1^{-1} + \lambda_2^{-1})^2 - 4 \lambda_2^{-1} (1 + \lambda_2^{-1})} \leq \Gamma
\]
\[
\leq \frac{1}{2} (1 + \lambda_2^{-1} - \lambda_1^{-1}) + \frac{1}{2} \sqrt{(1 + \lambda_1^{-1} + \lambda_2^{-1})^2 - 4 \lambda_2^{-1} (1 + \lambda_2^{-1})} := \Gamma_2.
\]

In the case of non-empty \( \mathcal{N}_1 \), by substituting \( \alpha_1^*(\theta) \) in (24) or (85) into \( b^2(\theta) \), \( b^2(\theta) \) is given by

\[
b^2(\theta) = \begin{cases} 
\lambda_2 \theta & \text{if } \theta \leq 1 - \Gamma \\
\lambda_2 [\sqrt{\theta} - \sqrt{(1-\theta)(1-\Gamma)}]^2 + 1 & \text{if } \theta > 1 - \Gamma
\end{cases}
\] (82)

When \( \theta \leq 1 - \Gamma \), \( b^2(\theta) \) is linear and it can be shown that \( b^2(\theta) \) is quasi-concave function when \( \theta > 1 - \Gamma \). If \( a^2 > \lambda_2 (1 - \Gamma) \), there doesn’t exists \( \theta \) satisfying \( a^2 \leq \lambda_2 \theta \) (for \( \theta \leq 1 - \Gamma \)) and hence the set \( \mathcal{N}_1 = \{ \theta | a^2 \leq b^2(\theta) \} \) is given by

\[
\left\{ \theta \mid \frac{z_1 z_2 + 2 \Gamma (1 - \Gamma) - \sqrt{4 \Gamma (1 - \Gamma) \Gamma (1 - \Gamma) + z_1 z_2 - z_2^2}}{z_1^2 + 4 \Gamma (1 - \Gamma)} \leq \theta \leq \frac{z_1 z_2 + 2 \Gamma (1 - \Gamma) + \sqrt{4 \Gamma (1 - \Gamma) \Gamma (1 - \Gamma) + z_1 z_2 - z_2^2}}{z_1^2 + 4 \Gamma (1 - \Gamma)} \right\}
\] (83)

Otherwise, the minimum of \( \theta \) satisfying \( a^2 \leq b^2(\theta) \) is the point such that \( a^2 = \lambda_2 \theta \) and \( \mathcal{N}_1 \) becomes

\[
\left\{ \theta \mid \frac{\lambda_1}{\lambda_2 1 + \lambda_1 \Gamma} \leq \theta \leq \frac{z_1 z_2 + 2 \Gamma (1 - \Gamma) + \sqrt{4 \Gamma (1 - \Gamma) \Gamma (1 - \Gamma) + z_1 z_2 - z_2^2}}{z_1^2 + 4 \Gamma (1 - \Gamma)} \right\},
\]

where \( z_1 = \lambda_1^{-1} + 1 - \Gamma \) and \( z_2 = \lambda_2^{-1} + 1 - \Gamma \). (83) is obtained by solving \( \frac{\lambda_1}{\lambda_2} \leq \frac{\lambda_2 \theta}{\lambda_2 [\sqrt{\theta} - \sqrt{(1-\theta)(1-\Gamma)}]^2 + 1} \) reducing to a quadratic inequality). In the case of non-empty \( \mathcal{N}_1 \), by Lemma 1, \( \mathcal{N}_1 \) is optimal and we obtain (41).

Next, consider the case that \( \mathcal{N}_1 \) is empty. At \( \theta = 1 \), we have \( a(1) = \sqrt{\frac{1}{\Gamma + \frac{1}{\lambda_1}}} > \sqrt{\frac{1}{\Gamma + \frac{1}{\lambda_2}}} = b(1) + c^2(1)/b(1) \) by the NOMA assumption \( \lambda_1 > \lambda_2 \) and hence \( \theta = 1 \in \mathcal{N}_3 \) by the definition of \( \mathcal{N}_3 \) in (79). We also have

\[
\lim_{\theta \to 0} b(\theta) + c^2(\theta)/b(\theta) = \lim_{\theta \to 0} \sqrt{\frac{\lambda_2 \theta - 1}{\lambda_2 [\alpha_1^*(\theta)]^2 + 1}} = \infty,
\]

(84)

* When \( \theta > 1 - \Gamma \), \( b^2(\theta) \) can be written as \( f^2(\theta)/g(\theta) \), where \( f(\theta) = \sqrt{\lambda_2 \theta} \) and \( g(\theta) = \lambda_2 [\sqrt{\theta} - \sqrt{(1-\theta)(1-\Gamma)}]^2 + 1 \). Since \( f(\theta) \) is the concave function and \( g(\theta) \) is the convex function (it can be proved easily by taking second derivative), we can conclude that \( f^2(\theta)/g(\theta) \) is quasi-concave [23].
which can easily be seen from $\alpha_1^*(0) = \sqrt{1 - \Gamma}$. Thus, $\theta = 0 \in \mathcal{N}_2$. Furthermore, $b(\theta) + c^2(\theta)/b(\theta) = \sqrt{\frac{\lambda_1}{\lambda_2[\alpha_1^*(\theta)]^2 + 1}}$ is a monotone decreasing function of $\theta$ since

$$
\alpha_1^*(\theta) = \begin{cases} 
0 & \text{if } \theta \leq \theta_I := 1 - \Gamma \\
\frac{\sqrt{\theta I}}{\sqrt{(1 - \theta)(1 - \Gamma)}} & \text{if } \theta > \theta_I 
\end{cases} \tag{85}
$$

is a monotone increasing function of $\theta$. Hence, there exists $\theta_a$ such that $a(\theta_a) = b(\theta_a) + c^2(\theta_a)/b(\theta_a)$ to yield $\mathcal{N}_2 = \{\theta | \theta \leq \theta_a\}$ and $\mathcal{N}_3 = \{\theta | \theta > \theta_a\}$.

Now recall that

$$
\gamma_2^*(1) = \frac{\lambda_1}{1 + \lambda_1 \Gamma}, \quad \gamma_2^*(2)(\theta) = \frac{\lambda_1}{1 + \lambda_1 \Gamma} [\alpha_2^*(\theta)]^2, \quad \text{and } \gamma_2^*(3)(\theta) = \frac{\lambda_2}{\lambda_2[\alpha_1^*(\theta)]^2 + 1}. \tag{86}
$$

If $\theta_a \leq \theta_I$, then the optimal $\theta$ set for maximizing $\gamma_2^*$ is given by $\{\theta | \theta_a \leq \theta \leq \theta_I\}$. This is because $\gamma_2^*(2)(\theta_a) = \gamma_2^*(3)(\theta_a)$, because $\gamma_2^*(3)(\theta)$ is monotone decreasing with respect to $\theta$ as seen in (86) since $\alpha_1^*(\theta)$ is a monotone increasing function of $\theta$, and because $\gamma_2^*(2)(\theta)$ is monotone increasing with respect to $\theta$ for $\theta \leq \theta_a$ since $\alpha_2^*(\theta)$ is a monotone increasing function of $\theta$ for $\theta \leq \min\{\theta_a, \theta_I\} = \theta_a$ (this can be shown by substituting $\alpha_1^*(\theta) = 0$ for $\theta \leq \theta_I$ into $\alpha_2^*(\theta)$ and taking derivative of $\alpha_2^*(\theta)$ with respect to $\theta$ and showing the derivative is positive for $\theta \leq \theta_a$). Hence, in this case the optimal $\gamma_2^*$ occurs at $\theta_a$ but for all $\theta$ in $\{\theta | \theta_a \leq \theta \leq \theta_I\}$, $\alpha_1^*(\theta) = 0$ and the corresponding optimal $\gamma_2^* = \gamma_2^*(3) = \lambda_2$ from (86). In this case, from the assumption $\theta_a \leq \theta_I$, $\theta_a$ is computed based on (81) with $\alpha_1^*(\theta) = 0$ as

$$
\theta_a = \theta \quad \text{s.t.} \quad a = b(\theta) + c^2(\theta)/b(\theta) \tag{87}
$$

$$
= \theta \quad \text{s.t.} \quad \frac{\lambda_1}{1 + \lambda_1 \Gamma} = \frac{\lambda_2}{\theta} \tag{88}
$$

$$
= \frac{\lambda_2}{\lambda_1} (1 + \lambda_1 \Gamma) \tag{89}
$$

and the condition $\theta_a \leq \theta_I$ reduces to

$$
\frac{\lambda_2}{\lambda_1} (1 + \lambda_1 \Gamma) \leq 1 - \Gamma \iff \Gamma \leq \frac{\lambda_2^{-1} - \lambda_1^{-1}}{1 + \lambda_2^{-1}}. \tag{90}
$$

On the other hand, if $\theta_a > \theta_I$, i.e., $\Gamma > \frac{\lambda_2^{-1} - \lambda_1^{-1}}{1 + \lambda_2^{-1}}$, then optimal $\theta$ exists between $\theta_I$ and $\theta_a$ because $\gamma_2^*(2)$ is an increasing function for $\theta < \min\{\theta_a, \theta_I\} = \theta_I$ and $\gamma_2^*(3)$ is a decreasing function for $\theta > \theta_a$. Since optimal $\theta$ lies in $\mathcal{N}_2$ in this case, it is obtained by solving

$$
\frac{\partial \gamma_2^*(3)(\theta)}{\partial \theta} = 0. \tag{91}
$$

Therefore, the claim follows.

\textbf{Proof of Corollary 1:} With the assumption of $\lambda_1 = \lambda_2$, we have $\Gamma_1 = 0$ and $\Gamma_2 = 1$ from (40) and hence the condition $\Gamma \in [\Gamma_1, \Gamma_2]$ reduces to $\Gamma \in [0, 1]$ which is always valid for $p_1 =$
\( p_2 = 1 \) (see the definition of \( \Gamma \) in (21)). Furthermore, with the assumption, we have \( z_1 = z_2 \) and \( \sqrt{4\Gamma(1-\Gamma)[\Gamma(1-\Gamma) + z_1z_2 - z_2^2]} \) in (41) is given by \( 2\Gamma(1-\Gamma) \). From (41), the optimal \( \theta \) set is given by \( \{ \theta \mid \theta_0 \leq \theta \leq 1 \} \), where

\[
\theta_0 = \begin{cases} 
\frac{1}{1+\Gamma} & \text{if } \frac{1}{1+\Gamma} \leq 1 - \Gamma \\
\frac{z_1^2}{z_1^2 + 4\Gamma(1-\Gamma)} & \text{if } \frac{1}{1+\Gamma} > 1 - \Gamma 
\end{cases}
\]  

(92)

□

**Proof of Proposition 3:** As \( \lambda_2 \to 0 \), the threshold \( \tau \) in Proposition 1 converges to \(-\infty\) and thus neither of the two conditions for \( p_1^{\text{opt}} \in \mathcal{P}_2 \) in Proposition 1 is satisfied. Hence, by Proposition 1, \( p_1^{\text{opt}} \in \mathcal{P}_3 \). Since \( p_1^{\text{opt}} \in \mathcal{P}_3 \), \( p_1^{\text{opt}} \) can be obtained in closed form by maximizing \( \gamma_2^{(3)}(p_1) \) in (35) and is given by

\[
P_1^{\text{opt}} = -P + 2\Gamma + \frac{\psi_1^2 - \psi_1 \sqrt{\psi_2^2 + 2\lambda_2^{-1}\psi_1 + \lambda_2^{-2}}}{2\theta(1-\theta)\Gamma}
\]

(93)

where \( \psi_1 := \theta\Gamma + (1-\theta)(P - \Gamma) + \lambda_2^{-1} \) and \( \psi_2 := \theta\Gamma - (1-\theta)(P - \Gamma) \). Using L’Hospital’s rule, we can show that \( \lim_{\lambda_2 \to 0} P_1^{\text{opt}} = \Gamma (= p_{1,\text{min}}) \). With \( p_1 = \Gamma \), we have \( \alpha_1^* = \sqrt{\theta} \) from (30) and consequently \( \beta_1^* = \sqrt{1-\theta} \) from the the constraint eq. in (29), and \( \alpha_2^* = \sqrt{\theta} \) from (34), and by substituting these values into \( \gamma_2^{(3)}(p_1) \) in (35) and (12) and (13), we have \( \gamma_2^* = \frac{P - \Gamma}{\theta + \lambda_2^{-1}(1-\Gamma)} \), \( \sqrt{p_1^*w_1} = \sqrt{\Gamma \frac{h_1}{\|h_1\|}} \) and \( \sqrt{p_2w_2} = \sqrt{P - \Gamma \frac{h_2}{\|h_2\|}} \) □

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