High frequency vortex dynamics and magnetoconductivity of high temperature superconductors

Jan Kolářek
Institute of Physics, ASCR, Cukrovarnická 10, 16200 Prague 6, Czech Republic

Etsuo Kawate
National Research Laboratory of Metrology, 1-1-4, Umezono, Tsukuba, Ibaraki 305, Japan

(November 16, 2017)

The vortex lattice with the superconducting and normal state charge carriers fractions may be regarded as three independent subsystems mutually connected by interactions. The equations of motion for these three subsystems must be solved simultaneously. In this way a new consistent theory of the vortex dynamics is obtained and the magnetoconductivity calculated.

PACS numbers : 74.60.Ge, 74.25.Fy

I. INTRODUCTION

The vortex dynamics attracts great attention from both the theoretical and experimental point of view. Much controversy is concentrated on the question, whether Magnus force is the only transverse force on the vortex as is claimed by Ao and Thouless [1], or if other ones, like Iordanskii force from the normal fluid or Kopnin-Kraftsov force from impurities may also contribute to the total transverse force. This topic is discussed in detail by Sonin [2], where many relevant references can be found.

As the characteristic frequencies of the vortex system are in the far infrared (FIR) region [3] the magnetooptical spectroscopy is a suitable tool to study these problems. Recently results of the FIR magnetotransmission of YBaCuO thin films using elliptically polarized light became available [4,5]. To interpret the measurements, the high frequency conductivity must be calculated. The first such calculation was made by Gittleman-Rosenblum [6]. Later, the dissipation near the flux line depinning threshold using the generalized temperature assisted flux flow (TAFF) model was studied by Yeh [7]. In these papers the reaction of the superconducting (and normal) charge carriers on the vortex movement was neglected. The redistribution of the induced ac current density due to the vortex response is taken into account in the Coffey-Clem [8] calculation of the surface impedance. Other references concerning this problem may be found in a recent review [9].

In principle, our approach is similar to the one of Coffey-Clem model, but the formalism is different. We treat the superconducting fluid, normal fluid and the vortex system as three subsystems mutually connected by interaction. Their equations of motion are solved simultaneously. In this way a new, internally consistent theory of vortex motion is obtained. We use this approach for calculating magnetoconductivity in high temperature superconductor (HTSC) thin films. In this paper we consider the Magnus force as the only interaction between the vortex system and superconducting fluid and Lorentz force to be the main interaction force between the normal fluid and vortex system in HTSC materials. It conforms to the necessary symmetry requirements summarized by Sonin [2] and has the same sign and form as the D,D’ terms derived by Stone [10] for normal particles confined in the vortex core, if the limit of nonlocalized particles is taken. We show, that including this transversal force from normal state charge carriers gives reasonable magnetoconductivity tensor which is compared with recent experimental data [4].

II. THE MODEL

Let us consider the vortex lattice, in which the mean distance between vortices is small in comparison with the penetration depth. In this case the magnetic field in the superconductor is almost homogeneous. This model may be appropriate for HTSC. In the field 5 T the mean distance between vortices is about 20 nm, while in YBCO the London penetration depth $\lambda_L$ is of the order 150 nm. The diameter of the vortex core is approximately equal to the coherence length $\xi$, which in YBCO is about 2 nm. Therefore, it is possible to neglect possible redistribution of the charge and current density in the vortex core and to take it into account only as the source of vortex damping. The damping force $F_D$ is supposed to be proportional to the vortex velocity $v_L$ with frequency independent viscosity coefficient $\eta$, so that
\[ \mathbf{F}_D = -\eta \mathbf{v}_L = -\frac{m_v}{\tau_v} \mathbf{v}_L. \]  

Introducing \( m_v \) as the vortex mass per unit length, we will use the vortex relaxation rate \( 1/\tau_v \) which is given in practical frequency units, rather than the viscosity.

For pinning we use the simplest model of a parabolic well, so that the pinning force \( \mathbf{F}_P \) is proportional to vortex displacement \( \mathbf{r}_L \):

\[ \mathbf{F}_P = -\kappa \mathbf{r}_L = -m_v \alpha^2 \mathbf{r}_L \]  

with \( \kappa \) and \( \alpha \) being the pinning constant and pinning frequency, respectively.

The interaction between superconducting fluid moving with velocity \( \mathbf{v}_s \) and the vortex system is mediated by the Magnus force \( \mathbf{F}_M \) given by

\[ \mathbf{F}_M(v) = \frac{n_s \hbar}{2} (\mathbf{v}_s - \mathbf{v}_L) \times \mathbf{z} = m_v f_s \Omega(\mathbf{v}_s - \mathbf{v}_L) \times \mathbf{z}, \]  

where \( n_s = f_s n \) is the density of superconducting fluid and \( \mathbf{F}_M(v) \) means, that this is the force felt by the vortex. The reaction force \( \mathbf{F}_M(s) \) acting on a superconducting particle is

\[ \mathbf{F}_M(s) = -\frac{n_v}{n_s} \mathbf{F}_M(v) = -m \omega_c (\mathbf{v}_s - \mathbf{v}_L) \times \mathbf{z}, \]  

where \( n_v \) is the vortex density (number of vortices per unit area), \( \omega_c = eB/m = n_v \hbar/2m \) is the cyclotron frequency in the field \( \mathbf{B} = n_v \Phi_0 \mathbf{z} \) caused by the vortex system and \( \Phi_0 \) is the magnetic flux quantum. In \( \{4\} \) we introduced the frequency of the cyclotron vortex motion \( \Omega = n \hbar/2m_v \). It is interesting to note that, using the Hsu’s expression for the vortex mass \( m_v = (\pi^2/4)mk_F^2\xi^2 \) \( \{1\} \), the 2D expression for the Fermi wave vector \( k_F^2 = 2\pi n \) and \( \xi = \hbar v_F/\pi \Delta \) for the coherence length, it is possible to show, that \( \Omega = \Delta^2/E_F \) (\( \Delta \) is the gap and \( E_F \) is the Fermi energy), which is the level separation in the vortex core \( \{3\} \).

The interaction between the vortex system and the normal state fluid may be obtained in the following way. From the Aharonov-Casher Lagrangian \( \{12\} \) it can be shown that if the vortex lattice moves with velocity \( \mathbf{v}_L \), the force imposed by the vortex system on one normal state particle moving with velocity \( \mathbf{v}_n \) is

\[ \mathbf{F}_L(n) = \frac{n_v \hbar}{2} (\mathbf{v}_n - \mathbf{v}_L) \times \mathbf{z} = m \omega_c (\mathbf{v}_n - \mathbf{v}_L) \times \mathbf{z}. \]  

According to the action-reaction law the vortex lattice must feel the same force with opposite direction. If there are \( n_n = f_n n \) normal state particles, the total force per unit length of one vortex is

\[ \mathbf{F}_L(v) = \frac{n_n}{n_v} \mathbf{F}_L(n) = -f_n \frac{\hbar}{2} (\mathbf{v}_n - \mathbf{v}_L) \times \mathbf{z} = -m_v f_n \Omega(\mathbf{v}_n - \mathbf{v}_L) \times \mathbf{z}. \]  

This expression is analogous to the Magnus force formula, but has opposite sign. It satisfies the invariance requirements, according to which only the relative velocity of the particle with respect to the vortex system is decisive. Factor \( f_n \) is justified by the fact that the total force is proportional to the number of particles involved in the interaction.

The questions concerning forces acting on the vortex lattice are still seriously controversial. The Lorentz force \( \{1\} \) following from the Aharonov-Casher Lagrangian is of electrodynamic origin, but similar formula is also used to describe interaction of normal state fluid with vortices in neutral systems (see e.g. \( \{10\} \)). Useful comments and replies regarding the spectral flow force and the Iordanskii force can be also found in \( \{14\} \). Interaction of electric charge with moving vortex and the Aharonov-Casher effect in two-dimensional superconductors was discussed e.g. by Simánek \( \{3\} \). Let us note that it would not be correct to consider the Lorentz force also for the superconducting fluid, which would exactly cancel the Magnus force. The vortex lattice and the accompanying magnetic field are created by superconducting current, so in this case Lorentz force would mean "action on itself".

Having drawn up the interaction forces we will now write the equations of motion for the three subsystems. As in the London model, the superconducting fluid is supposed to move without damping,

\[ m \ddot{\mathbf{v}}_s = e \mathbf{E} + \mathbf{F}_M(s), \]  

while the normal state fluid motion is damped as in the conventional Drude model

\[ m \ddot{\mathbf{v}}_n = e \mathbf{E} + \mathbf{F}_L(n) - \frac{m}{\tau_n} \mathbf{v}_n. \]
Finally, for the vortex system we shall use the Newton type equation of motion (of course the vortex mass and also all the forces are considered per unit length)

\[ m_v \ddot{v}_L = F_P + F_D + F_M(v) + F_L(v). \] (9)

The system of three equations of motion \((9, 10, 11)\) together with expressions for the interaction forces \((9, 12)\), damping and pinning force \((10, 12)\) form a closed set of equations for the unknown \(v_L, v_s, v_n\). Assuming a periodic time dependence \(e^{i \omega t}\), the three differential equations reduce to the set of three linear equations:

\[
\begin{align*}
A_{ss}v_s + A_{sv}v_L &= \frac{e}{m} F_s \\
A_{nn}v_n + A_{nv}v_L &= \frac{e}{m} F_n \\
A_{vs}v_s + A_{vn}v_n + A_{vv}v_L &= 0
\end{align*}
\] (10)

with the coefficients

\[
\begin{align*}
A_{ss} &= i(\omega - \omega_c) & A_{sv} &= i\omega_c \\
A_{nn} &= i(\omega + \omega_c - i/\tau_n) & A_{nn} &= -i\omega_c \\
A_{vs} &= if_n \Omega & A_{vn} &= -if_n \Omega \\
A_{vv} &= i(\omega + (f_n - f_s) \Omega - \alpha^2/\omega - i/\tau_n).
\end{align*}
\] (11)

If the determinant \(D = A_{ss}A_{nn}A_{vv} - A_{nn}A_{sv}A_{vs} - A_{ss}A_{nv}A_{vn}\) is nonzero, the set can be readily solved to get

\[
\begin{align*}
v_s &= g_s \frac{e E}{m} \frac{A_{nn}A_{vv} + A_{nv}A_{ss} - A_{ss}A_{nn} eE}{D} \\
v_n &= g_n \frac{e E}{m} \frac{A_{ss}A_{vv} + A_{nn}A_{vs} - A_{ss}A_{vn} eE}{D} \\
v_L &= g_L \frac{e E}{m} \frac{-A_{ss}A_{vn} - A_{nn}A_{vs} eE}{D} \\
\end{align*}
\] (12)

Now it is straightforward to express the conductivity as

\[
\sigma = \frac{j}{E} \frac{\varepsilon}{e} (n_s v_s + n_n v_n) = \varepsilon_0 \omega_p^2 (f_s g_s + f_n g_n),
\] (13)

where \(\omega_p = \sqrt{ne^2/\varepsilon_0 m}\) is the plasma frequency and the factors \(g_s, g_n\) are defined by eq. \((12)\).

As expected, for physically meaningful parameters \((\tau_n > 0, \tau_n > 0, \omega, \Omega > 0)\) the real part of conductivity is positive and the Kramers-Kronig relation \(\sigma(\omega) = \varepsilon_0 (\omega/\sqrt{i}) \int_{-\infty}^{\infty} \sigma(x)/(x^2 - \omega^2) dx\) as well as the f-sum rule \((1/\pi) \int_{-\infty}^{\infty} \text{Re}(\sigma(\omega)) d\omega = \varepsilon_0 \omega_p^2\) are satisfied. The zero frequency limit of the conductivity \(\sigma_0 = \varepsilon_0 \omega_p^2 \tau_n (f_n + i(\omega_c \tau_n (f_s - f_n) + f_s/\omega_c \tau_n))/(1 + \tau_n^2 \omega_c^2)\) does not have the delta function component, as the pinning constant is supposed to be finite, while pinning range is infinite. Expressing the conductivity tensor components as \(\sigma_{xx}(\omega) = (\sigma(\omega) + \sigma(-\omega))/2\), \(\sigma_{xy}(\omega) = (\sigma(\omega) - \sigma(-\omega))/2\), it is possible to show, that also the Hall sum rule \((1/\pi) \int_{-\infty}^{\infty} \text{Re}(t_H) d\omega = \omega_H\), where \(t_H = \sigma_{xy}/\sigma_{xx}\) and \(\omega_H = \lim_{\omega \to \infty} [-i \omega t_H(\omega)] = \omega_c(f_s - f_n)\) is satisfied. It is necessary to note, that without normal state fraction \((f_n = 0)\) the \(t_H\) function has a pole at zero frequency, so that in this case the Hall sum rule must be modified to \(\omega_H = (\alpha^2 \omega_c/(\Omega \omega_c + \alpha^2)) + (1/\pi) \int_{-\infty}^{\infty} \text{Re}(t_H) d\omega\).

**III. ABSENCE OF NORMAL STATE FLUID**

Usually it is considered that at zero temperature all the charge carriers condense, so that normal state fluid is absent. It is necessary true for all materials, but it is useful to discuss this limit first.

For free vortices (vortices without pinning and damping) the two equations of motion \(m \ddot{v}_s = -m \omega_c (v_s - v_L) \times z\) for the superconducting fluid and \(m \ddot{v}_L = m \omega_c f_s \Omega (v_s - v_L) \times z\) for the vortex system are readily simplified to \(v_L/v_s = \Omega/\Omega - \omega\) and \(v_s/v_n = (\omega_c - \omega)/\omega_c\), respectively. Consequently, two nontrivial solutions exist: for zero frequency \(v_L = v_s\), while for \(\omega = \Omega + \omega_c\) the velocity ratio is \(v_L/v_s = -\Omega/\omega_c\). This means that the superconducting liquid and vortices may move either in parallel with constant velocity (this solution is required by Galilean invariance), or may oscillate with opposite phase, with the inertial center remaining at rest.

In general, with \(f_n = 0\) the coefficient \(A_{vn}\) equals zero and the conductivity formula \((13)\) reduces to
$$\sigma(f_n = 0) = \epsilon \omega_p^2 \frac{A_{sv}}{A_{sv} A_{uv} - A_{sv} A_{uv}}.$$  \hspace{1cm} (14)$$

Let us note that in the limit of zero vortex density ($\omega_c \to 0$), this formula reduces to the London expression for conductivity $\sigma = \epsilon \omega_p^2 / i \omega$, as expected. For zero pinning (but nonzero damping) the explicit expression for conductivity may be written as:

$$\sigma(f_n = 0, \alpha = 0) = \frac{1 + i \tau_e (\omega - \Omega)}{\tau_e \omega (\omega + \Omega - \omega) + i (\omega - \omega_c)}.$$  \hspace{1cm} (15)$$

It is clear that in this case the real part of conductivity is nonzero even at zero frequency $\sigma_1(f_n = 0, \alpha = 0, \omega = 0) = \epsilon \omega_p^2 \tau_e \Omega / \omega_c$. Contrary to it, for nonzero pinning we get $\sigma(f_n = 0, \alpha \neq 0, \omega = 0) = \epsilon \omega_p^2 \tau_e / \omega_c$ with zero real part of conductivity. This result is understandable, if we recall that in our simple model the pinning barrier is infinite, so that the d.c. transport must be nondissipative.

In reality the pinning barrier is not infinite. Depending on frequency, temperature, magnetic field, as well as density and strength of pinning sites, various regimes as flux creep, flux flow, temperature assisted flux flow etc. can be recognized. To keep the discussion simple, we will analyze just two simple limits. In the "full pinning" (FP) limit the driving field is low, so that each vortex is bound to the individual pinning valley, making only small oscillations. In this case the pinning force plays an important role. Contrary to it in the limit of high driving field the amplitude of the vortex oscillation is larger than the distance between the pinning centers, and the averaged pinning force is effectively zero (ZP). In the intermediate state the pinning force is nonzero, but not proportional to the distance from the pinning center which leads to nonlinear effects. We will show that, in some frequency range, nonlinear effects can be expected even at relatively low fields which are commonly used in laboratory experiments.

Let us estimate the realistic values for the parameters of the theory. For coherence length $\xi = 2 \mu$m, and effective mass $m = 4m_e$ using the Hsu’s expression for the vortex mass \[1\], we can estimate $\Omega = 2h/(\pi^2 m \xi^2) = 49 \text{ cm}^{-1}$. The cyclotron frequency in the field $4T$ is $5.9 \text{ cm}^{-1}$. Using the expression \[3\] $\kappa = (0.01 \div 0.05) \mu_0 H^2$ for the pinning coefficient, and the vortex mass estimation \[6\] $m_v = 1.6 \times 10^{10} m_e / m$, the range for pinning frequency $\alpha = 19 \div 95 \text{ cm}^{-1}$ may be obtained. As we did not select any model for the vortex damping, we leave this parameter as free. To make a model calculation we used the following set of parameters: $\omega_c = 5$, $\Omega = 50$, $\alpha = 30$, $\omega_p = 6000$, $1/\tau_e = 10$ (all values are in cm$^{-1}$). The conductivity calculated in FP and ZP limits are displayed in fig.1. The conductivity peaks are expected near the frequencies, where the real or imaginary part of the determinant $D = A_{sv} A_{uv} - A_{sv} A_{uv}$ which appears in the denominator of \[14\] is zero. In ZP limit the expected peak values of conductivity are

$$\sigma_1(\omega = 0) = \epsilon \omega_p^2 \tau_e \Omega / \omega_c$$
$$\sigma_1(\omega = \Omega + \omega_c) = \epsilon \omega_p^2 \tau_e \omega_c / \Omega$$
$$\sigma_1(\omega = \omega_c) = \epsilon \omega_p^2 / \tau_e \Omega \omega_c.$$  \hspace{1cm} (16)$$

In fig.1. only two sharp peaks are present for the ZP limit (dashed line). It is obvious from \[14\] that, while for low vortex damping (large $\tau_e$) the peaks at eigenfrequencies of the system (0 and $\Omega + \omega_c$) are important, for high vortex damping the peak at $\omega_c$ will dominate. This is illustrated in fig.2, where the conductivity for vortex damping 1/$\tau_e$ from 10 to 200 are displayed. It is possible to see, how with increasing vortex damping the peak shifts from zero frequency to the cyclotron frequency $\omega_c$. For FP limit, due to the pinning term $\alpha^2 / \omega$ the order of the determinant $D$ is higher in $\omega$, so one more peak is expected in accord with the model calculation results displayed in fig.1 (solid line).

In fig.3 the relative value of the vortex oscillation amplitude $a_v = |r_L| m/eE$ is shown as a function of $\omega$. It is clear that, while at high frequency the oscillation amplitude is low so that FP limit is appropriate, at lower frequencies the amplitude is high, so that ZP limit must be adopted. In principle, beside the pinning frequency $\alpha$ determining the pinning force at low oscillation amplitude, two characteristic lengths $r_1$ and $r_2$, the amplitudes of vortex oscillation, at which the pinning force declines from the linear law and at which the pinning force is effectively zero, should be introduced. In this way, for a given driving field $E$, two crossover frequencies $\omega_{d1}, \omega_{d2} = \sqrt{eE/r_1 r_2 m}$ are defined. For $a_v < 1/\omega_{d1}^2$ the FP limit is valid, while for $a_v > 1/\omega_{d2}^2$ the ZP must be used. If neither condition is fulfilled, the system is in a nonlinear region, where the conductivity depends on the driving field strength. It is interesting to note, that for some frequencies both conditions $a_v(\text{FP}) < 1/\omega_{d1}^2$ and $a_v(\text{ZP}) > 1/\omega_{d2}^2$ may be fulfilled at one time. This means, that in this frequency region bistability may occur. Depending on the history, at the same experimental conditions two regimes - the low and high vortex oscillation amplitude, corresponding to the low and high resistivity state - may be achieved! All these possibilities are illustrated in fig.3. If we estimate the range of pinning force ($r_d$) to be about 10 nm and if the intensity of radiation used for measurement is 1 mW/mm$^2$ so that the driving field $E$ is of order $1.7 \times 10^4$ V/m, we get $\omega_d = 9 \text{ cm}^{-1}$. For illustration purposes we have chosen $\omega_{d1} = \omega_{d2} = 10 \text{ cm}^{-1}$. It is obvious that, depending on the frequency and intensity of the radiation used for the measurements, many interesting nonlinear effects may be expected.
IV. INFLUENCE OF NORMAL STATE FLUID

For nonzero temperatures there are two contributions to the real conductivity. One is connected with the normal state charge carriers, the other with vortices. As expected, without vortices we get \[\sigma(\omega_c = 0) = \varepsilon_0 \omega^2 p [f_s/i\omega + f_n \tau_n/(1 + i\omega \tau_n)]\], which is the sum of the London and Drude model contributions. The normal state limit \((f_s \to 0)\) does not have much sense, as without superconducting fraction we can not have any vortices. However, if we simulate external magnetic field by making vortices unable to move, we should get the formula for a normal conductor in magnetic field.

Indeed, in the limit of vortices fixed to the lattice \((\alpha \to \infty)\) or of infinite vortex mass \((\Omega = 0)\), we get the expected result \(\sigma(f_s = 0, \Omega = 0) = \sigma(f_s = 0, \alpha \to \infty) = \varepsilon_0 \omega^2 p \tau_n/(1 + i(\omega + \omega_c) \tau_n)\).

The results of model calculations for \(f_n = 0.5\) in FP and ZP limits are displayed in fig.4. To visualize the contribution of vortices, the zero magnetic field conductivity \((\omega_c = 0)\) is also displayed in these graphs. In FP limit, when the amplitude of vortex motion is small, almost all real part of conductivity originates from the normal state charge carriers - except of the very sharp feature near the zero frequency, which is caused by the vortex resonance. On the other hand, in the ZP limit the conductivity is much larger and it is almost completely due to the vortex motion, with the normal state fluid playing only a minor role. However, the sharp vortex resonance peak is absent. It might be somewhat surprising, that the presence of vortices may slightly decrease the real part of conductivity for some frequencies.

It is instructive to see, how increasing the normal state fraction influences the conductivity. For the FP limit it is shown in fig.5. We can see that with increasing \(f_n\), the central peak (connected with the normal state carriers conductivity) gradually develops, while the side peaks diminish. Recently, Lihn at al. \cite{4} measured far infrared magnetoconductivity tensor in YBaCuO thin film. Their data are also displayed in fig.5 (dashed line). The intensity of FIR radiation is usually rather small so the FP limit could be appropriate. It is remarkable, that all the experimentally observed features are quite well simulated by the curve with \(f_n = 0.3\). This seems to indicate the presence of some normal state fraction (probably located on CuO chains) even at the lowest temperature. Alternatively it may be due to the enhanced density of quasiparticles in an applied magnetic field, as predicted for the d-wave superconductor \cite{20,21}. It should be noted, that the sharp vortex resonance peak on the theoretical curve is at lower frequency than the range accessible by FIR spectroscopy, so it could not be observed in the experiment.

V. CONCLUSIONS

Vortex lattice together with the superconducting and normal state fluid form three subsystems mutually connected by interaction. Taking into account reaction forces by which vortices influence superconducting and normal state fluid and solving simultaneously the three equations of motion a new, internally consistent theory of vortex dynamics was developed. It was shown that due to the finite range of the pinning force, at some frequencies nonlinear phenomena may be expected even for relatively low driving fields which are commonly used in laboratory experiments. For comparison with experiment, the knowledge of the power of radiation used for the measurements might be crucial. The presented theory can qualitatively explain recent measurements of far infrared magnetoconductivity tensor made by Lihn at.al \cite{4}. The d.c. conductivity calculated in the framework of this model enables to explain theoretically controversial, but experimentally firmly established Hall voltage sign reversal \cite{22,23}.

ACKNOWLEDGMENTS

The authors are grateful to H.D.Drew and H.Lihn for providing the original data from their magnetooptical measurement, as well as to E.H.Brandt, E.Šimánek and E.Sonin for helpful discussions. This work was supported by grants GACR 202/96/0864 and MŠMT KONTAKT ME 160. One of us (J.K.) thanks the Japanese International Superconductivity Center (ISTEC) and New Energy and Industrial Technology Development Organization (NEDO) for the fellowship, during which part of this work was done.

[1] P.Ao, D.J.Thouless: Phys.Rev.Lett 70, 2158 (1993)
[2] E.B.Sonin Phys.Rev.B 55, 485 (1997)
[3] H.D.Drew, E.Choi, K.Karrai: Physica B, 197, 624 (1994)
[4] H.-T.S.Lihn et.al.: Phys.Rev.Lett, 76 (1996) 3810.
[5] S.Wu et.al.: Phys.Rev.B, 54,13343 (1996)
[6] J.I.Gittleman, B.Rosenblum: Phys.Rev.Lett 16, 734 (1966)
[7] N.-C.Yeh: Phys.Rev.B. 43, 523 (1991)
[8] M.W.Coffey, J.R.Clem: Phys.Rev.Lett. 67, 386 (1991)
[9] M.Golosovsky et.al.: Supercond.Sci.Technol. 9, 1, (1996)
[10] M.Stone: Phys.Rev.B 54, 13222 (1996)
[11] T.C.Hsu: Physica C 213, 305 (1993)
[12] Y.Aharonov, and A.Casher, Phys.Rev.Lett. 53, 319 (1984)
[13] N.B.Kopnin, G.E.Volovik and Ü. Parts, Europhys. Lett. 32, 651 (1995)
[14] P.Ao, Phys.Rev.Lett. 80, 5025 (1998); N.B.Kopnin, and G.E.Volovik, ibid. p.5026
[15] H.E.Hall and J.R.Hook, Phys.Rev.Lett 80, 4356 (1998); C.Wexler et al. ibid. p.4357
[16] E.B.Sonin, Phys.Rev.Lett 81, 4276 (1998); C.Wexler et al. ibid. p.4277
[17] E.Šimánek, Phys.Rev.B 55, 2772 (1997)
[18] H.D.Drew, P.Coleman: Phys.Rev.Lett. 78, 1572 (1997)
[19] G.Blatter et.al.: Rev.Mod.Phys, 66, 1125,(1994)
[20] G.E.Volovik, JETP Lett. 58, 469 (1993)
[21] Y.Wang, A.H.McDonald, Phys.Rev.B 52, 3875 (1995)
[22] J.Koláček, P.Vašek, to be published (see cond-mat-9811222)
[23] J.Koláček, P.Vašek, Int. J. Modern Phys.B 12, 3102 (1998)

FIG. 1. Real part of conductivity for FP (solid line) and ZP (dashed) limits. Parameters of the model are $\omega_c = 5$, $\Omega = 50$, $\alpha = 30$, $\omega_p = 6000$, $1/\tau_v = 10$ (all in cm$^{-1}$).

FIG. 2. Real part of conductivity for ZP limit, for $\omega_c = 5$, $\Omega = 50$, $\alpha = 0$, $\omega_p = 6000$ and $1/\tau_v = 10, 20, 50, 100, 200$ (all in cm$^{-1}$). For increasing vortex damping the conductivity peak shifts from zero frequency to cyclotron frequency.

FIG. 3. The relative vortex oscillation amplitude for FP (solid line) and ZP (dashed) limits calculated using the same parameters as in Fig.1. The pinning range corresponding to crossover frequency $\omega_d = 10$ cm$^{-1}$ is marked by a horizontal line. The frequency regions marked by \__/\_\_ are regions where FP (ZP) limit are appropriate.

FIG. 4. The comparison of real part of conductivity in the FP (solid line) and ZP (dashed) limits with conductivity in zero magnetic field (dotted) in presence of normal state fraction $f_n = 0.5$; $1/\tau_n = 15$. All the other parameters are same as in the Fig.1.

FIG. 5. Real part of conductivity in FP limit for normal state fraction $f_n = 0.1, 0.15, 0.2, 0.25, 0.3$ (the tendencies for increasing $f_n$ are marked by arrows). Other parameters are same as in Fig.1. The experimental data obtained by Lihn et al. [4] are also plotted (dashed line) for comparison.
