Discussion on Construction of OCDMA PON Address Code-(F,K,2)

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Abstract. Optical code division multiple access (OCDMA) passive optical network (PON) can find wide applications in the next optical access network. One of its key techniques of is construction of address code. Aiming at the facts that \((F,K,1)\) optical orthogonal code (OOC) possesses good performance but capacity is small, and number of users in OCDMA PON is not very big thereafter OOC auto-correlation or cross-correlation may not be very strict, \((F,K,2)\) OOC can be used as address codes for OCDMA PON. In this paper, the method of constructing \((F,K,2)\) OOC based on block design is discussed. The algorithm of construction of OOC from block design is presented and simulated; several groups of \((F,K,2)\) OOC are gained. The results show that the algorithm has good astringency and simplicity. It can construct \((F,K,2)\) OOC effectively. It is feasible.

Introduction

Since optical code division multiple access (OCDMA) has the features of asynchronous access, soft-capacity, good security and so on, it is an alternative for next generation passive optical networks (PON) [1,2,3]. An \((F,K,1)\) optical orthogonal code (OOC) is a family of \((0,1)\)-sequences with desired auto- and cross-correlation properties, it is considered as optimal address code for OCDMA. Since the introduction of OOC, there has been continuous and extensive efforts for devising efficient algorithms to construct \((F,K,1)\) OOC [4, 5, 6, 7], particularly optimal \((F,K,1)\) OOC because of its limit capacity. Although there exists some methods which can construct optimal \((F,K,1)\) OOC, but they all exist some restricts, including only the codes with small code length or with special parameters can be constructed. So, construction of big capacity OOC remains an urgent issue to be solved.

Since there are at most several hundreds of users in an OCDMA PON, if the auto- or cross-correlation restricts of OOC is relaxed to 2, that is \((F,K,2)\) OOC, bit error probability can also meet the demand of PON and the capacity of OOC and be improved \(\frac{(F-2)}{(K-2)}\) times. This kind of address OOC will meet the requirement of OCDMA PON better. In this paper, construction of \((F,K,2)\) OOC based on block design is discussed, concrete algorithm is presented and simulations are done. Some code words are gained. The results show that the method has the features of simplicity and feasibility.

OOC And Block Design

OOC. OOC is a family of “0”, “1” sequences with good auto-correlation and co-correlation properties. A \((F,K,\lambda_0,\lambda_c)\) OOC C is a set of “0”, “1” sequences with length \(F\) and weight \(K\), \(\lambda_0\) and \(\lambda_c\) denote auto-correlation and co-correlation restricts. When \(\lambda_0=\lambda_c=\lambda\), \((F,K,\lambda_0,\lambda_c)\) is denoted \((F,K,\lambda)\). \((F,K,\lambda)\) OOC C satisfies the following two properties: [8]:

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Optical Orthogonal Codes

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1) Auto-correlation:

\[ \theta_a (\tau) = \sum_{i=0}^{\lambda_i} x_i \cdot x_{i+\tau} = \begin{cases} K & \tau = 0 \\ \leq \lambda_i & 1 \leq \tau \leq n-1 \end{cases} \]

(1)

Where \( x \in C \), \( \tau \) is cyclic shift, \( \oplus \) denotes module \( F \) operation.

2) Co-correlation:

\[ \theta_c (\tau) = \sum_{i=0}^{\lambda_i} x_i \cdot y_{i+\tau} \quad 0 \leq \tau \leq n-1 \]

(2)

Where \( x \neq y \in C \), \( \tau \) is cyclic shift, \( \oplus \) denotes module \( F \) operation.

A \((F, K, \lambda)\) OOC C can be regarded as a family of \( K \)-set of module \( F \). Each \( K \)-set is corresponding to a code word; the integers in \( K \)-set represent the non-zero bit positions in codeword. In fact, the smaller \( K \) is, the bigger capacity is. But if \( K \) is too small, the difference between auto-correlation peak value and side lobe is not obvious. So code weight and correlation value must be chosen appropriately. According to Johnson bound of unequal-weight code, \( (F, K, \lambda) \) capacity \( \Phi \) satisfies the following inequality [9]:

\[ \phi(F, k, \lambda) \leq \frac{\{(F - 1)(F - 2)\cdots(F - \lambda)\}}{K(K - 1)\cdots(K - \lambda)} \]

(3)

It can be seen from upper expression; capacity is proportional to code length and inverse proportional to code weight. But code length can not be too long and code weight cannot be too small. So under the condition that code length and weight is stable, capacity is related to \( \lambda \). Since code length is far bigger than code weight, increasing \( \lambda \) can increase capacity. For example, let \( \lambda \) be 2, the capacity will be \( t \) times bigger than the case \( \lambda \) be 1, \( t = \frac{F - 2}{K - 2} \).

**Block design.** Let \( S = \{1, 2, \cdots, F\} \) be a basal set including \( F \) different elements, \( B = \{B_1, B_2, \cdots, B_b\} \) be \( K \)-set of \( S \), \( r \) be the number of \( K \)-set which includes certain random elements, then \((S, B)\) constructs a block design[10] and

1) For any pair of elements \( i, j \ (i, j = 1, 2, \cdots, F, i \neq j) \), \( \lambda_s \) blocks include them simultaneously; for any pair of blocks \( i, j \ (i, j = 1, 2, \cdots, b, i \neq j) \), \( \lambda_b \) elements are the same, and \( \lambda_b = \lambda_c = \lambda \), then it is called a symmetric balance incomplete block design (SBIBD), written in a simple form as SBIBD \((F, b, r, K, \lambda)\).

2) If in SBIBD, elements in some block \( B_i \) is \( \{a_{i1}, a_{i2}, \cdots\} \), then elements in \( B_{ni} \) must be \( \{a_{i1} + 1, a_{i2} + 1, \cdots\} (mod F) \), then this SBIBD which possesses cyclic performance is called a cyclic balanced incomplete block design (CSBIBD). According to the terms in block design in SBIBD, there exist \( r = k \) and \( N = b \), so SBIBD can be written as SBIBD \((F, K, \lambda)\) and CSBIBD as CSBIBD \((F, K, \lambda)\) [10].

**Cyclic difference sets and difference sets.** Cyclic deference sets combined by \( K \) difference integers, which have different remarks calculated modulo \( F \), can be expressed as:

If for each \( d \neq 0 (mod F) \), there are \( \lambda \) pairs of sequence \( \{a_i, a_j\} \) making \( d = (a_i - a_j)(mod F) \), then it is called a \((F, K, \lambda)\) cyclic difference sets and written as \( D(F, K, \lambda) \).

In above \( D \), if only for some special \( d \neq 0 (mod F) \), there are \( \lambda \) pairs of sequence \( \{a_i, a_j\} \) making \( d = (a_i - a_j)(mod F) \), then it is called a \((F, K, \lambda)\) difference sets and written as \( D_i(F, K, \lambda) \).
CONSTRUCTION METHOD

Algorithm idea. In block design, \((F, K, \lambda)\) OOC is corresponding to CSBIBD \((F, K, \lambda)\). If CSBIBD \((F, K, \lambda)\) can meet auto-correlation and co-correlation condition, it will be an OOC. So construction OOC is transformed into constructing CSBIBD which satisfies auto-correlation and co-correlation terms. Whereas CSBIBD \((F, K, \lambda)\) is corresponding to cyclic difference set, construction \((F, K, \lambda)\) OOC is transformed into constructing cyclic difference set. But cyclic difference set can not construct more than one OOC, non-cyclic difference set need to be constructed first, that is difference set[8].

\((F, K, \lambda)\) OOC is a special case of \((F, K, \lambda)\) at \(\lambda = 1\). There can not appear equal code distance at the case \(\lambda = 1\), and there can not exist same elements in difference matrix of code words. The case \(\lambda = 2\) is equivalent to that the correlation restrict can be 1 or 2. At this situation, the distance between codes can be one same value at most, and difference matrix between code words can appear two same elements at most.

So, the algorithm of constructing \((F, K, 2)\) OOC can be following process. Introducing code word auto-producing procedure, transforming difference set, performing auto-correlation operation to examine if there be equal code distance; then transforming difference set to OOC, performing co-correlation operation to judge maximal number of same elements in difference matrix, at last gaining \((F, K, 2)\) OOC.

Constructing \((F, K, 2)\) OOC procedures.

- Producing difference sets: Produce \(K\)-set \(L\) by \(F\) number, \(l_i (i = 1, 2, \cdots, K)\), \(l_1 = 1\) is its elements;

  Let \(l'_i = l_i - l_{i-1} (\text{module } F)\) \((i = 1, 2, \cdots, K; \text{and } i = 1, l_{i-1} = l_K, )\)

  Produce \(L'\) from \(L\), element \(l'_i\) represents code distance.

- Perform auto-correlation operation on difference sets; Solve maximal auto-correlation value in \(L'\), i.e. maximal number of \(l'_i\) of equal code distance. Compare \(l'_i\) with given auto-correlation value “2”, if \(l'_i > 2\), delete the code word, others store it.

- Produce new \(K\)-sets \(L\) by above difference sets: perform \(l_i = l'_i + l_{i-1} (\text{module } F)\) operation on the stored code words \((l_1 = 1, \text{while } i = 1, l_{i-1} = l_K)\).

- Perform co-correlation operation on new \(K\)-sets \(L\): First, choose one code word, and then choose another code word. If the number of \(l_{i, j}\) of same elements between the two code words is smaller or equal to 2, the latter code word can be chosen as the second code word. So does the third code word and some other. The checking method is difference matrix. The definition of it is:

  \[ A_{x,y} = \{C_{y,j} - C_{x,j}\} (\text{module } F) \]  

  Where \(0 \leq i \leq F-1, 0 \leq j \leq F-1\). If maximal number \(l_{i, j}\) of same elements in \(A_{x,y}\) is bigger than 2, it does not satisfy the co-correlation term; otherwise it does. Then code word can be gained.

Diagram block of the algorithm. The diagram block of the algorithm is shown at Fig.1.

SIMULATION RESULTS

The simulation results are as Table 1, with \(F = 40, 100\) and \(160, K=3\) and \(4, \lambda = 1, 2\). Since the algorithm produce a lot of code words, only part of them are listed below. The algorithm is applicable to arbitrary code length and weight.
Produce $K$-set $L$ by $F$ number, $l_i (i=1,2,\ldots,K), l_1$ is its elements;

$\lambda_i = l_i - l_1$ (module $F$)

Produce $L$ from $L$

Solve maximal number of $\lambda_i$ of equal code distance

$\lambda_i > 2$

Produce new $K$-sets $L_S$ by upper difference sets $l_i = l_1 + l_i$ (module $F$)

Begin

Given $F, K, 2$

Choose one code word

Choose another code word

Judge maximal number of same elements in difference matrix $\lambda_i$

$\lambda_i > 2$

N

Store

Output OOC and capacity

End

Figure 1. Diagram block of construction $(F, K, 2)$ OOC

TABLE 1 PRODUCED OOC CODE WORDS

| $F$ | $K$ | $\lambda$ | Capacity | Code words |
|-----|-----|-----------|----------|------------|
| 40  | 3   | 1         | 5        | {1 2 4} {1 5 10} {1 7 14} {1 9 19} {1 12 24} |
| 100 | 3   | 1         | 13       | {1 2 4} {1 5 10} {1 7 14} {1 9 19} {1 12 24} |
|     |     |           |          | {1 15 30} {1 17 34} {1 20 40} {1 22 44} {1 25 50} {1 27 54} {1 29 59} {1 32 64} |
| 160 | 3   | 1         | 21       | {1 2 4} {1 5 10} {1 7 14} {1 9 19} {1 12 24} {1 15 30} {1 17 34} {1 19 38} {1 22 44} {1 25 50} {1 27 54} {1 29 59} {1 32 64} |
|     |     |           |          | {1 47 94} {1 49 99} {1 52 104} |
| 40  | 3   | 2         | 247      | {1 2 3} {1 2 4} {1 2 5} {1 2 6} {1 2 7} {1 2 8} {1 12 29} {1 13 25} {1 13 26} |
|     |     |           |          | {1 13 27} {1 13 28} {1 14 27} |
| 100 | 3   | 2         | 1617     | {1 2 3} {1 2 4} {1 2 5} {1 2 6} {1 2 7} {1 2 8} {1 12 29} {1 13 25} {1 13 26} |
|     |     |           |          | {1 2 14 27} {1 2 14 28} {1 2 14 29} |
| 160 | 3   | 2         | 4187     | {1 2 3} {1 2 4} {1 2 5} {1 2 6} {1 2 7} {1 2 8} {1 12 29} {1 13 25} {1 13 26} |
|     |     |           |          | {1 15 30} {1 17 34} {1 20 40} {1 22 44} {1 25 50} {1 27 54} {1 29 59} {1 32 64} |
|     |     |           |          | {1 47 94} {1 49 99} {1 52 104} |
| 40  | 4   | 1         | 2        | {1 2 4 8} {1 6 14 23} |
| 100 | 4   | 1         | 5        | {1 2 4 8} {1 6 14 23} {1 11 22 34} {1 15 30 46} |
| 160 | 4   | 1         | 9        | {1 2 4 8} {1 6 14 23} {1 11 22 34} {1 15 30 46} |
|     |     |           |          | {1 31 65 100} {1 37 75 115} |
| 40  | 4   | 2         | 50       | {1 2 3 5} {1 2 6 7} {1 2 8 9} {1 2 10 11} {1 2 12 13} |
| 100 | 4   | 2         | 347      | {1 2 3 5} {1 2 6 7} {1 2 8 9} {1 2 10 11} {1 2 12 13} {1 2 14 15} {1 2 16 17} |
| 160 | 4   | 2         | 916      | {1 2 3 5} {1 2 6 7} {1 2 8 9} {1 2 10 11} {1 2 12 13} {1 2 14 15} {1 2 16 17} |

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Simulations show that the number of code words at $\lambda = 2$ is far more than that of the case $\lambda = 1$. The method can construct big capacity address codes, and the operating time is only several seconds to several minutes, it exhibits good astringency.

**CONCLUSIONS**

The construction method of $(F, K, 2)$OOC based on block design is discussed. The algorithm follows the procedure of constructing $K$-set, producing difference set, transforming difference sets to OOC, performing correlation operation according to correlation value “2” to the above OOC. So the gained OOCs satisfy auto-correlation and co-correlation terms.

The algorithm is simple and general. Arbitrary code length and weight OOC can be constructed. The code capacity of $(F, K, 2)$OOC is bigger; it can meet the demands of OCDMA PON. The algorithm possesses the features of good astringency and fast operating speed. So, it is feasible and can find broad application prospects.

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**REFERENCES**

[1] M. Massoud Karbassian, Hooshang Ghafouri-Shiraz: IEEE Journal of Lightwave Technology, Vol. 27(2009),p. 3896-3903
[2] Jawad A. Salehi: Journal of Optical Network, Vol. 6(2007), p. 1138-1178
[3] S. Sahuguede, M. Morelle, A. Julien-Vergonjanne: IEEE CCNC’2008 proceedings, 2008 p.5437-5442
[4] Mohammad M. Alem-Karladani, J. A. Salehi: IEEE Transaction Inform Theory, Vol. 56 (2010), p. 4659-4667
[5] Haitao Cao and Ruizhong Wei: IEEE Transaction Inform Theory, Vol. 55(2009), p. 1387-1394
[6] Ryoh Fuji-Hara and Ying Miao: IEEE Transaction Inform Theory, Vol. 46,(2000),p.2396-2405
[7] Li xiaobin: Acta photonica sinica,vol.35(2006), pp.1899-1902
[8] F. R. K. Chung, J. A. Salehi, V. K. Wei.: IEEE Transaction Inform Theory, vol.35(1989), p.594-604
[9] Li chuanqi, Li xiaobin, *Optical fiber communication OCDMA system*, Science Publishing House, 2008.
[10]Jin fan, Chen zhi, *Combinatory Coding principle and application*, Shanghai Science Publishing House, 1995.