Scalar Potentials and Accidental Symmetries in Supersymmetric $U(1)'$ Models

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Abstract

We address two closely related problems associated with the singlet scalars’ potential that are often present in supersymmetric $U(1)'$ models, especially those which maintain the gauge unification of the MSSM in a simple way. The first is the possibility of an accidental global symmetry which results in a light Goldstone boson. The second is the problem of generating a vacuum expectation value for more than one field without reintroducing the $\mu$ problem. We give sufficient conditions for addressing both issues and provide a concrete example to generate them.
1 Introduction

Extensions of the standard model (SM) and the Minimal Supersymmetric SM (MSSM) frequently involve additional abelian gauge symmetries, often at the TeV scale (for reviews, see [1, 2, 3]). The primary motivation for considering such scenarios is top-down, i.e., because so many extensions involve larger gauge symmetries which often leave an abelian remnant when they are broken. Another motivation is that many supersymmetric $U(1)'$ models provide an elegant solution of the $\mu$ problem [4], by forbidding an elementary $\mu$ term but allowing a dynamical $\mu$ to be generated by the vacuum expectation value (VEV) of a SM-singlet field charged under the $U(1)'$ [5, 6, 7] (other models with a dynamical $\mu$ are reviewed in [8]). However, like most extensions of the SM or MSSM, the seemingly innocent addition of an abelian group brings with it difficulties and complications in constructing phenomenologically viable models.

Aside from hypercharge, $B-L$ (which does not forbid an elementary $\mu$) is the only abelian family-universal global symmetry of the Standard Model (defined to include three right-handed neutrinos) that is anomaly free. Thus, in order to simultaneously implement the $U(1)'$ solution to the $\mu$ problem and satisfy all the anomaly cancellation conditions (ACC), one is forced to introduce new chiral exotic matter when extending the gauge structure to include an additional abelian group. It is usually assumed to be quasi-chiral, i.e., vector-like with respect to the SM gauge group but chiral under the $U(1)'$. Being charged under the SM gauge groups forbids this exotic matter from being light.

Unless they come in complete $SU(5)$-type multiplets, the exotic matter typically ruins the simple form of unification found in the MSSM [9], although unification can often be restored by adding ad hoc adjoint or vector-like fields at the TeV or intermediate scales. For example, $E_6$-type models [10] can accommodate all the needed exotics and Higgs fields in three 27-plets. However, gauge unification is not respected unless one adds an ad hoc vector pair of $SU(2)$ doublets, e.g., from an incomplete $27 + 27^*$, and generating their masses introduces a new vector doublet version of the $\mu$ problem [11]. Another class of models [12, 13] is consistent with simple gauge unification. This is achieved by starting with the MSSM fields and adding to it additional multiplets that transform like complete $SU(5)$ multiplets under the SM gauge group. However, not all of the fields have the same $U(1)'$ charges, i.e., the assignments do not descend from an underlying $SU(5) \times U(1)'$.

The last ingredient needed are SM singlet fields which are charged under the new abelian group. These singlets are required to break the $U(1)'$ symmetry, give the exotics large enough masses, and satisfy all the linear and cubic ACC for the $U(1)'$ group. Thus, the singlet sector is given several duties and the field content is constrained.

In supersymmetric theories these provisions are further complicated by the holomorphy of the superpotential and the special structure of the resulting scalar potential. Together with charge conservation, the ACC often fix the scalars’ charges almost entirely. This rigidity,
exacerbated by holomorphy, allows few, if any, terms in the superpotential. The resulting scalar potential then suffers from two generic problems. First, accidental symmetries are present which once broken lead to an axion-like boson in the spectrum which is experimentally excluded. Second, the true vacuum is often such that not all the scalars develop a VEV. This is not a difficulty for those scalars which are only needed for the ACC, but is a phenomenological disaster for those which are needed to generate masses for the exotics.

Both of these problems are often easier to resolve once we allow for bilinear terms. However, we would like to avoid reintroducing the usual μ-problem in the singlet sector. There are actually two aspects to the μ problem. The first is to prevent the presence of unacceptably large bilinear terms, e.g., at the string or GUT scale. This is not difficult, as one can always imagine that constraints from some underlying string theory, for example, force the bilinear to vanish initially. More difficult is the problem of introducing the bilinear and its associated soft term at the right scale. What the “right scale” is depends on the problem we are trying to solve. As far the accidental symmetries are concerned, the bilinear needs to be large enough to avoid an axion in the spectrum. This is only a lower bound on the size of the bilinear and does not represent a serious concern. The more serious problem arises when attempting to utilize bilinears to spontaneously break a symmetry. In that case, the relevant “μ” and “Bμ” terms have to be at the same scale as the other soft parameters. In this paper, we refrain from using bilinears in order to achieve spontaneous symmetry breaking. In that strict sense, we do not reintroduce the “μ-problem”.

In this letter we address these issues, the presence of accidental symmetries and the generation of multiple VEVs. We begin with a somewhat general discussion of the points at hand, although it is difficult to produce any rigorous proofs for the most general case. We also provide some specific examples where more definitive statements can be made. Ultimately, our purpose is to draw attention to some aspects of Z’ model building which are often neglected.

2 General Remarks

2.1 Accidental phase symmetries

The problem of accidental symmetries arises whenever we have two or more SM singlet superfields with different nonzero U(1)’ charges, as are required in the models in [13] to generate masses for all of the exotics. To illustrate the problem, assume first that the singlet sector has no superpotential and the scalar potential for the \( N \) singlet scalar components contains only soft mass and D-terms,

\[
V(S_1, \ldots, S_N) = \sum_i m_i^2 |S_i|^2 + \frac{g^2}{2} \left( \sum_i Q_i |S_i|^2 \right)^2 .
\]

\( 3 \)Frequently, the fermionic components of some of the singlets remain massless or very light, even when there are no issues of accidental symmetries or when the scalar does not acquire a VEV. Such fermions are similar to sterile neutrinos, and do not cause any major problems as long as they do not mix with ordinary neutrinos and some non-trivial but not overly stringent conditions are met to avoid astrophysical and cosmological difficulties. Light singlet fermions can even be helpful in allowing sufficiently rapid decays of exotic particles via higher-dimensional operators [15].
Here $g_{\ast'}$ is the $U(1)'$ gauge coupling constant and $Q_i$ and $m_i^2$ are the $U(1)'$ charge and soft mass-squared of the field $S_i$, respectively. This potential has $N-1$ “accidental” global $U(1)$ symmetries. When the scalar fields develop VEVs these symmetries will be broken, resulting in $N-1$ massless Nambu-Goldstone bosons (NGB). These accidental symmetries are generically anomalous with respect to the SM gauge group. Therefore, one linear combination of the NGBs is an axion \cite{16} as it receives a small mass of order $\Lambda_{\text{QCD}}^2/f$, where $f$ is a typical scalar VEV, and the rest are massless. The presence of such massless scalars is excluded by constraints on the existence of a fifth force (see, for example, Refs. \cite{17, 18} for recent reviews). As for the axion state, for $f \sim 100$ TeV the mass would be around 100 eV. Such light scalars are excluded by axion searches, which require that the axion mass should be below 10 meV \cite{19}.

One can attempt to break the symmetry through a higher dimensional operator suppressed by some mass scale, $\Lambda_{\text{cutoff}}$. The axion mass in this case would be of the order $f^2/\Lambda_{\text{cutoff}}$. But even for $\Lambda_{\text{cutoff}} = M_p$, where $M_p$ is the Planck scale, the axion mass would be too large. If, on the other hand, we wish to generate a large mass via this higher dimensional operator, e.g. larger than $\sim$ MeV to avoid astrophysical and laboratory bounds, we must take $\Lambda_{\text{cutoff}}$ to be much smaller than the GUT scale.

To avoid this situation, we need to have $N-1$ linearly independent terms in the superpotential. Since the chiral supermultiplets are charged under $U(1)'$, no linear term can be written. Ideally, one would like to have only cubic terms, but we can allow for bilinear terms as long as they do not reintroduce the $\mu$ problem. Thus, such terms will not be used to achieve any particular vacuum structure, but they can be utilized to break accidental symmetries. A bilinear term of order $F/M_p$, where $F$ is the auxiliary field’s VEV in the hidden sector, can be generated by the Giudice-Masiero \cite{20} mechanism. This can easily give the NGB a large enough mass, e.g. of order MeV, even for a relatively low $F$, such as is found in models of gauge mediation (for a review, see e.g., \cite{21}), thus avoiding any current bounds on light scalars. In contrast, the Giudice-Masiero mechanism is only useful for the original Higgs $\mu$ problem for relatively large $F$, such as is found in supergravity mediation.

Regardless of the mechanism that mediates SUSY breaking, such accidental symmetry breaking terms in the superpotential would radiatively generate $A$ and $B$ terms, which together with the $F$ terms are enough to generate masses for all the would-be-axions.

**Cubic and bilinear terms**

The typical situation one encounters is that we have $k$ singlet fields and we would like to add $l$ singlet fields, none of which has zero $U(1)'$ charge, such that we have $k+l-1$ linearly independent terms in the superpotential. The question one might ask is, can we always find such $l$ fields, for large enough $l$? If we do, can we use only cubic terms? The answer of course will depend on the $U(1)'$ charges of the given $k$ fields. But we can make some fairly general statements, assuming that the charges are “small” rational numbers.

In general the charges of the singlet fields can be divided into equivalence classes according to their congruence modulo 3. The terms in the superpotential can also be written as equations for the charges. The possible cubic terms $S_i^2S_j$ and $S_iS_jS_m$ can be written as $2Q_i + Q_j = 0$ and $Q_i + Q_j + Q_m = 0$, respectively. A possible bilinear term can be written as $Q_i + Q_j = 0$. Equations of the form $2Q_i + Q_j = 0$ can only “connect” charges which are congruent modulo 3. Equations of the type $Q_i + Q_j + Q_m = 0$ connect either charges which are congruent modulo...
3, or connect charges from three different equivalence classes, e.g., $Q_i$ is 0 mod 3, $Q_j$ is 1 mod 3, and $Q_m$ is 2 mod 3. Equations of the form $Q_i + Q_j = 0$ can have solutions only if $Q_i$ is 1 mod 3 and $Q_j$ is 2 mod 3, or if both $Q_i$ and $Q_j$ are 0 mod 3.

It is difficult to break the accidental symmetries with a small number of fields using only the cubic terms if the $U(1)'$ charges of the initial set belong to different equivalence classes. That is because the terms $S_iS_jS_m$ have to connect three different equivalence classes and there must be enough of them. There is no such restriction for bilinears. We will see explicit examples for this phenomena in the next section. On the other hand, if most (or all) of the charges belong to the same equivalence class, we would not expect to have a substantial difference between bilinear and cubic terms and one can probably use only cubic terms.

### 2.2 Multiple scalar condensation

As far as VEVs of the different fields are concerned, cubic terms are often sufficient to ensure that all the relevant scalars develop a VEV. While it is difficult to make any general statements about the minimization of such potentials and the different possible phases encountered, it is fairly easy to understand how one might generate VEVs for multiple fields. Let us first assume that the $U(1)'$ gauge coupling $g_{z'}$ is somewhat larger than any of the cubic couplings, denoted generically by $y$. Then, neglecting the cubic couplings, the scalar potential in (1) minimizes according to,

$$|S_i|\left[m_i^2 + g_{z'}^2 Q_i \left(\sum_j Q_j|S_j|^2\right)\right] = 0,$$

where $i$ and $j$ run over all the scalars involved. At this point, one of two things might happen. The first possibility is that one (and only one) of the fields develops a VEV, provided its mass-squared was driven negative by the renormalization group equations (RGEs). If there are several fields with negative mass-squared, the one for which $|m_i^2/Q_i|$ is the largest, will develop the VEV. This will remain the minimum even after including the $F$ terms generated by the superpotential cubic terms, as long as no flat direction is present. Other fields may then develop VEVs through $A$-terms which add linear terms to the scalar potential (after VEV insertions).

The second possibility is that a flat direction is present. If, for example, any two of the fields $S_i$ and $S_j$ with opposite-sign charges have $|Q_j|m_i^2 + |Q_i|m_j^2 < 0$ then we have a “runaway” direction, i.e., $V \to -\infty$, for $|Q_i||S_i|^2 = |Q_j||S_j|^2 \to \infty$ and $|S_k|^2 = 0, k \neq i, j$. This is desirable, because once we include the $F$-terms in the potential, these fields will be stabilized at finite values\(^4\). The presence of these flat directions lingers in the VEVs being proportional to $1/y^2$. Then again, upon including the $A$-terms, many other fields may develop a VEV as discussed above. Clearly, one cannot count on too many $A$-terms without imposing fairly stringent constraints on the different charge assignments.

\(^4\)This mechanism is utilized in the secluded sector models [22], which allow $M_Z'$ to be considerably above the effective $\mu$. Even if the potential is not stabilized by renormalizable level $F$-terms, it is may be stabilized at intermediate scales because of the running $m^2$ or due to higher dimensional operators [23].
If we relax the condition that \( g_{z'} \) is larger than all of the cubic couplings, we can have two possibilities depending on the presence of \( F \) terms in the scalar potential of the form \( |y|^2 |S_i|^2 |S_j|^2 \) with \( y > g_{z'} \). If such terms are not present, the situation is identical to the \( g_{z'} > y \) case considered above. If such terms are present, we can generate positive effective mass terms for \( S_j \) via VEV insertion, i.e., \( m_j^2 = |y|^2 |S_i|^2 \). Such a positive effective mass term may overcome the negative soft mass term and restabilize the point \( |S_j| = 0 \). We will see examples for both of these cases in the next section.

### 3 An Example: Erler’s Model

As a concrete example, illustrating the challenges and their resolutions discussed above, we consider one of the models presented in Ref. [13], which constructed supersymmetric \( U(1)' \) models which solve the \( \mu \) problem in the MSSM while maintaining simple gauge unification.

The charge assignments in the example we consider, given in Table (1), are slightly generalized [3] from the model in [13]. The free parameters \( x, y \) and \( z \) are determined through the cubic anomaly cancellation condition. The singlet fields \( S, S_D \) and \( S_L \), which are responsible for generating the \( \mu \) term and giving the exotics a mass, are independent of \( x, y \) and \( z \). We will therefore ignore these parameters since they carry no significance in our analysis.

| \( U(1)' \) charge | \( U(1)' \) charge |
|---|---|
| \( Q \) | \( y \) | \( H_u \) | \( x \) |
| \( u^c \) | \( -x - y \) | \( H_d \) | \( -1 - x \) |
| \( d^c \) | \( 1 + x - y \) | \( S_D \) | \( 3/n_{55^*} \) |
| \( L \) | \( 1 - 3y \) | \( D_i \) | \( z \) |
| \( e^+ \) | \( x + 3y \) | \( D_i^c \) | \( -3/n_{55^*} - z \) |
| \( \nu^c \) | \( -1 - x + 3y \) | \( S_L \) | \( 2/n_{55^*} \) |
| \( S \) | \( 1 \) | \( L_i \) | \( \frac{5 - n_{55^*}}{4n_{55^*}} + x + 3y + 3z/2 \) |
| | | \( L_i^c \) | \( -2/n_{55^*} - Q_{L_i} \) |

Table 1: Examples of supersymmetric models consistent with minimal SM gauge unification. \( n_{55^*} \) is the number of pairs of \( 5 + 5^* \). \( Q_S \) is taken to be 1. The free parameters are \( Q_{H_u} \equiv x, Q_Q \equiv y, Q_D \equiv z \).

### 3.1 Singlets’ scalar potential

To generate an effective \( \mu \) term and give masses to the exotics \( D_i, D_i^c \) and \( L_i, L_i^c \), all three singlets, \( S, S_D \) and \( S_L \) need to develop VEVs. Since they all carry positive \( U(1)' \) charge, we cannot write any interaction terms in the superpotential. Assuming SUSY is broken, the scalar potential will involve only the soft mass terms and the \( U(1)' D \)-terms. In this case,
as discussed immediately after Eq. (2), only one of the fields will develop a VEV. Since no $A$-terms are allowed by charge conservation there is no possibility to lift the other two fields when the first develops a VEV. Such a potential also suffers from accidental global symmetries since there are three fields, but no terms in the superpotential. If one hopes to overcome these problems, the addition of extra singlets seems inevitable.

For concreteness we consider the above charge assignment with $n_{55-} = 2$ and identify the singlets $S$ and $S_L$. Adding only one extra singlet $S_1$ will not help resolve the axion problem. To see that, notice that since $Q_S = 1$ and $Q_{S_D} = 3/2$, there is only one term one can add to the superpotential at the renormalizable level ($SS_D S_1$, $SSS_1$, or $S_D S_D S_1$, but not more than one of these terms). Since there are now three fields and one superpotential term, we are still left with one accidental symmetry.

If we add two extra singlet fields (on top of $S$ and $S_D$), the situation is more manageable. To remove all the accidental symmetries we require three independent superpotential terms. While it is impossible to do so with cubic terms alone\(^6\), it is possible to find examples if we allow a bilinear term. One may object to such a construction on the grounds that it defies the original purpose of considering such models as being free of the $\mu$ problem. However, in this case, the bilinear term need not have any particular scale, but is there solely to give the light scalar a large enough mass.

A simple example has two extra singlets with charge assignment, $Q_{S_1} = -1$ and $Q_{S_2} = -1/2$. This allows for the superpotential

$$W \supset \mu SS_1 S_1 + y_1 S_1 S_2 S_D + y_2 S_D S_2.$$  \hspace{1cm} (3)$$

The associated scalar potential is

$$V(S, S_D, S_1, S_2) = \sum_i m_i^2 |S_i|^2 + \frac{g_2^2}{2} \left( \sum_i Q_i |S_i|^2 \right)^2 + \sum_i |F_i|^2,$$  \hspace{1cm} (4)$$

where the sum is over all four fields and the $F$-terms are given by

$$F_S = \mu S S_1 + y_2 S_D S_2$$

$$F_{S_D} = y_1 S_1 S_2$$

$$F_{S_1} = \mu S S_1 + y_1 S_D S_2$$

$$F_{S_2} = y_1 S_D S_1 + 2y_2 S S_2.$$  \hspace{1cm} (5)$$

\(^5\)We are ignoring the Higgs scalars contribution to the potential. This is certainly justified when the $U(1)'$ is broken at a scale much higher than the EW scale. Moreover, since the Higgs fields only couple to $S$, their inclusion will not change any of the conclusions qualitatively.

\(^6\)The smallest example which contains only cubic terms requires the introduction of 4 extra singlet fields $S_1, S_2, S_3, S_4$. The required superpotential contains the following 5 terms: $S_1 S_1 S_2, S_2 S_D S_D, S_1 S_4 S_D, S S_S S_3,$ and $S S S_4$. The charges of the new fields are $Q_{S_1} = 1/2, Q_{S_2} = -1, Q_{S_3} = -1/2, and Q_{S_4} = -2$. (The bilinear terms $S S_2$ and $S_1 S_3$ are allowed but not needed.) As explained in the previous section, the reason we need so many new fields has to do with the fact that $Q_S$ and $Q_{S_D}$ are not congruent modulo 3.
3.2 Multiple Scalar Condensation

Since the singlet fields $S$ and $S_D$ are coupled to the exotic matter fields, it is reasonable to expect their soft masses to run negative at low energies if the Yukawa couplings are sufficiently large. Therefore, it is possible to destabilize the origin for at least one of the fields. However, since both $S$ and $S_D$ have the same sign charge, they cannot both develop a VEV in the absence of the other fields. Suppose $S_D$ has a larger absolute value of mass squared to charge ratio. Then, at least initially, we can set $S = 0$ in everything that follows.

As we argued above, if no flat directions are present in the limit where only the $D$-terms are considered, then $S_D$ alone will develop a VEV, even when $F$-terms are included. Since there are no $A$-terms of the form $S_D S_D S_i$, no other field can develop a VEV. This is phenomenologically unacceptable as no $\mu$ term is generated for the MSSM and some of the exotics remain massless. To avoid that, we must require some flat directions to be present, or in other words, $|Q_{S_i}| m_{S_D}^2 + |Q_{S_D}| m_{S_1}^2 < 0$ and/or $|Q_{S_i}| m_{S_D}^2 + |Q_{S_D}| m_{S_2}^2 < 0$. For simplicity, we will mainly consider examples in which only one is negative.

In this case, we include the $F$-terms to stabilize the potential, considering only the resulting quartics while neglecting the bilinear and $A$ terms. The resulting potential is

$$ V(S, S_D, S_1, S_2) = \sum_i m_i^2 |S_i|^2 + \frac{g_i^2}{2} \left( \sum_i Q_i |S_i|^2 \right)^2 + |y_1|^2 (|S_1|^2 |S_D|^2 + |S_D|^2 |S_2|^2 + |S_2|^2 |S_1|^2) + |y_2|^2 |S_2|^4. \quad (6) $$

The conditions for an extremum, $\partial_{S_i} V = 0$, assuming $S_D \neq 0$, $S_1 \neq 0$, and $S_2 \neq 0$ can be written as

$$ \left( \begin{array}{ccc} \frac{9y_1^2}{2} & -3g_{12}^2 + 2y_1^2 & -\frac{3y_1^2}{2} + 2y_1^2 \\ -3g_{12}^2 + 2y_1^2 & 2g_{12}^2 & g_{12}^2 + 2y_1^2 \\ -\frac{3y_1^2}{2} + 2y_1^2 & g_{12}^2 + 2y_1^2 & \frac{g_{12}^2}{2} + 4y_2^2 \end{array} \right) \left( \begin{array}{c} S_D^2 \\ S_1^2 \\ S_2^2 \end{array} \right) = -2 \left( \begin{array}{c} m_{S_D}^2 \\ m_{S_1}^2 \\ m_{S_2}^2 \end{array} \right). \quad (7) $$

The solution is in general unique, but not necessarily physical, in which case either $S_1 = 0$ or $S_2 = 0$. Instead of presenting the solution, it is somewhat more instructive to consider the following limits.

**$y_1 \ll y_2$, $g_{12}$ limit**

Assuming $|Q_{S_i}| m_{S_D}^2 + |Q_{S_D}| m_{S_1}^2 > 0$, we know that $S_D$ and $S_1$ cannot simultaneously develop a VEV and we therefore set $S_1 = 0$. Solving for $S_D$ and $S_2$ we find

$$ |S_D|^2 = -\frac{4 m_{S_D}^2}{9 g_{12}^2} - \frac{1}{18 y_2^2} \left( 3m_{S_2}^2 + m_{S_D}^2 \right) \quad |S_2|^2 = -\frac{1}{6 y_2^2} \left( 3m_{S_2}^2 + m_{S_D}^2 \right). \quad (8) $$

This is the global minimum as long as $|Q_{S_2}| m_{S_D}^2 + |Q_{S_D}| m_{S_2}^2 < 0$. As mentioned before, the dependence of the VEV on $1/y_2^2$ is a remnant of the flat direction $Q_{S_D} |S_D|^2 + Q_{S_2} |S_2|^2 = 0$. This vacuum is particularly simple, and once the $A$-term for $S_2 S_2 S$ is turned on, $S$ will develop a VEV and an acceptable phenomenology results. Notice that we did not have to assume any
relation between $y_2$ and $g_{z'}$. As explained in the previous section, this is a result of the fact that in this case we only have quartic terms in the scalar potential.

**$y_2 \ll y_1, g_{z'}$ limit**

In this case we have to separate the two parameter regions $y_1 < g_{z'}$ and $g_{z'} < y_1$. If $y_1 < g_{z'}$, $|Q_{S_1}|m_{S_D}^2 + |Q_{S_D}|m_{S_1}^2 > 0$, and $|Q_{S_2}|m_{S_D}^2 + |Q_{S_D}|m_{S_2}^2 > 0$, then we again have a minimum for $S_1 = 0$. Keeping the leading terms in $1/y_1^2$,

$$|S_D|^2 = -\frac{1}{6y_1^2} (3m_{S_2}^2 + m_{S_D}^2),$$

$$|S_2|^2 = -\frac{1}{2y_1^2} (3m_{S_2}^2 + m_{S_D}^2),$$

(9)
similar to the previous limit. Once the $A$-term for $S_2S_2S$ is turned on, $S$ will develop a small VEV.

If $y_1$ is much larger than $g_{z'}$, the flat direction is so strongly lifted that neither $S_1$ nor $S_2$ can develop a VEV. This can be understood qualitatively as follows. After $S_0$ condenses, it can drive the mass of $S_1$ or $S_2$ negative through the $D$ term contribution. However, with large $F$-terms there are additional, strictly positive contributions, which will keep the origin stable if $y_1 \gg g_{z'}$.

Following similar lines as delineated above, it is possible to construct the full phase structure of the scalar potential. In particular, it is straightforward to find solutions for which $S_D, S_1$, and $S_2$ are all nonzero. We will not pursue this course of investigation and simply point out that the phases found above where multiple fields develop a VEV are sufficient and generic.

### 4 Conclusions

In this letter we addressed two issues which are often left unchecked in supersymmetric $U(1)'$ model building: the singlet scalars’ potential must be such that no accidental global symmetries are present, and a sufficient number of fields must develop VEVs.

The former requirement can be satisfied for $N$ fields if $N − 1$ terms can be written in the superpotential which break all the accidental phase rotations. In some cases, this is impossible to achieve without bilinear terms. The inclusion of such bilinears does not necessarily reintroduce the $\mu$ problem of the MSSM since their scale is not needed to achieve a particular vacuum structure and therefore is not tied in with any other scale. Their purpose is simply to remove the degeneracy and give the light boson a large enough mass.

As far as the vacuum structure is concerned, assuming that one scalar’s mass is driven negative by the soft SUSY breaking RGEs, the following prescription emerges. If, in the absence of any $F$-terms, no flat directions are present, then only a single field will develop a VEV. After including $A$-terms one may destabilize the origin for other fields if any tadpoles result from VEV insertions. If flat directions are present, it is possible to generate a VEV for more than one field simultaneously by stabilizing the potential with the $F$-terms. It is then easier to ensure that all the scalars develop a VEV once $A$-terms are included.

We illustrated these points with a particular example constructed by Erler [13]. However, we expect these considerations to be relevant for other $U(1)'$ models whenever there are two or more SM-singlet fields with different nonzero $U(1)'$ charges.
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