A total least squares solution to a 3D coordinate transformation parameters of large Euler angles with closure constrain

Jian Qin¹, Jinyun Guo²*, Xiaofei Xu²,³

¹Tianjin Survey and Design Institute for Water Transport Engineering, 2618 Xingang Second Road, Tianjin 300456, China
²College of Geodesy and Geomatics, Shandong University of Science and Technology, Qingdao 266590, China
³College of Oceanography and Space Informatics, China University of Petroleum, Qingdao 266580, China

*Corresponding to: jinyunguo1@126.com

Abstract: A new method for calculating three-dimensional (3D) coordinate transformation parameters of large Euler angles with closure constrain is presented. This method obtains the initial value by traditional seven-parameter model. Then we derive the closure constraint condition and linearize it. The constrained total least-squares (CTLS) approach is adopted to obtain corrected values under the closure constraint condition. A practical case study is presented to illustrate the effectiveness of this new model.

1. Introduction

The three-dimensional (3D) coordinate transformation has been frequently used in the traditional engineering survey, geodesy, photogrammetry, and geographical information science[9]. With the development of surveying and mapping, the coordinate transformation is also present in the 3D laser scanning, attitude measurement and precision industrial measurement. One 3D coordinate transformation is made to convert common points from a source coordinate system to a target coordinate system. The higher the accuracy, the more accurate the transformation parameters in the 3D transformation model.

In recent decades, the research of coordinate transformation methods mainly concentrated in 3D coordinate transformation with big rotation angles between two systems. The non-linear adjustment model, Lodrigues matrix method, Procrustes analysis method, and the unit quaternion-based approach are used to compute 3D coordinate transformation parameters. The non-linear adjustment model was developed in the three-dimensional coordinate transformation [3, 11, 30, 35]. Directly calculating the transformation parameters is investigated by using Lodrigues matrix method [15, 31]. The Procrustes analysis method and the unit quaternion-based approach were developed for photogrammetry [4, 22] and later used in the geodetic datum transformation [12, 20, 26]. It can be seen that the above studies are limited to two coordinate systems, and do not take the closure constraint between multiple independent stations into account.
In the traditional precision industrial measurement, the total station is often used to collect 3D coordinates of interesting points. In precision control of large industrial objects, like aircraft and ship, one station can’t complete the measurement of large components. Therefore, several independent instrument stations should be set up for measurement. The suitable location could be chosen to set up the instrumental station, and then the observations can be transformed into one unified coordinate frame through many common points, which can yield constraint conditions. Pan et al. [21] carried out the surveying adjustment with the moving station method in the measurement of ship. If one building scanned with the 3D laser scanning technique is very large, it is common to use a multi-station strategy to collect point clouds of the building. The point clouds scanned by adjacent stations must have a certain degree of overlap to ensure the precision of stitching points. The point clouds obtained on each station are in the independent coordinate system. We should make all the scanned point clouds into one unified coordinate systems with the 3D coordinate transformation with large Euler angles. In the process of parameter estimation, there will be closure constraint due to the overlap of the point clouds. Yan et al. [29] proposed a rigorous registration method of multi-view point clouds and analyzed the accuracy and reliability. But the Euler rotation angles are limited to small angles in the above method.

Total least squares (TLS) method is widely used in the fields of science and engineering since Gloub and van Loan [10] put forward this algorithm and proved that the solution is existed [36]. In geodesy, the TLS method is studied in 2D affine transformation [19, 28, 34] and later development for 3D coordinate transformation [3, 8, 23-25]. Obviously, there is no study on the closure constraint in multiple independent coordinate systems. All of the above studies are limited to the coordinate transformation between two systems. It is necessary to study the closure constraint between several independent coordinate systems by using TLS method. The purpose is to present a new model of the 3D coordinate transformation with large Euler angles based on the total least squares method under condition of closure constrain in the precise geographical data process. The manuscript is organized as follow: the closure constraint in the 3D coordinate transformation is proposed and the nonlinear constraint is linearized by Taylor’s expansion in section 2; the TLS with constraints method is used to estimate the transformation parameters in section 3; a practical case is studied by the proposed model in section 4; section 5 contains conclusions and predictability about the new transformation model.

2. Coordinate transformation model under condition of closure constrain

![Coordinate transformation model](image)

**Fig. 1.** Coordinate transformation. (a) multiple independent coordinate systems; (b) two coordinate systems.

Figure 1(a) shows one the case of coordinate transformation between multiple coordinate systems. There are \( m \) common points between two adjacent coordinate systems. One large object cannot be completely measured on only one station so that it is necessary to set up several independent stations to complete the data collecting task. The independent coordinate system is used in the measurement of free instrumental station and the points measured on all independent stations are finally converted into one unified coordinate system. At least 3 common points are needed between the two adjacent stations when the 3D coordinate transformation is performed. As shown in figure 1(a), there are common
points between every two adjacent stations starting from the first station. The closure constraint can be formed according to the common points between the first and the terminal stations. In order to improve the accuracy of the transformation parameters between the independent stations, the closure constraint should be taken into consideration. Then the total least squares with the constraint is used to solve the 3D coordinate transformation model with large Euler angles.

2.1. Seven-parameter model with large Euler angles

3D coordinates in one coordinate system can be converted into these in another 3D coordinate system by the axis rotation, origin translation and scale zoom [7, 13]. The seven-parameter model of 3D coordinate transformation commonly used is

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_T = \begin{bmatrix}
\Delta X \\
\Delta Y \\
\Delta Z
\end{bmatrix} + kR \begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_S
\]

(1)

Where \([X \ Y \ Z]_S^T\) represents the coordinate of one point in the source coordinate system, \([X \ Y \ Z]_T^T\) represents the coordinates of same point in the target coordinate system, \([\Delta X \ \Delta Y \ \Delta Z]^T\) is the origin translation, \(k\) is the scale factor, and \(R\) is the rotation matrix, that is

\[
R = R_x R_y R_z = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \varphi & \sin \varphi \\
0 & -\sin \varphi & \cos \varphi
\end{bmatrix}
\]

(2)

in which \(R_z, R_x\) and \(R_y\) present the rotation matrices around axis \(z, x\) and \(y\) axes, respectively. \(R\) is formed by these three proper rotations. Equation (1) can be rewritten as

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_T = \begin{bmatrix}
\Delta X \\
\Delta Y \\
\Delta Z
\end{bmatrix} + kR_x R_y R_z \begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_S
\]

(3)

In the traditional Burse-Wolf conversion model [27], the trigonometric function is approximated by Taylor expansion so that it is restricted to small rotation angles in practical applications. Equation (3) would be linearized by Taylor series expansion with respective to 7 parameters to process the 3D coordinate transformation with large Euler angles [32], that is

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_T = \begin{bmatrix}
\Delta X^0 \\
\Delta Y^0 \\
\Delta Z^0
\end{bmatrix} + k^0 R^0 \begin{bmatrix}
d\Delta X \\
d\Delta Y \\
d\Delta Z
\end{bmatrix} + R^0 \begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_S + dk + k^0 dR \begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_S
\]

(4)

where \(\Delta X^0, \Delta Y^0, \Delta Z^0, k^0, R^0\) represent approximate values of parameters, and \(d\Delta X, d\Delta Y, d\Delta Z, dk, dR\) represent corrected values of parameters.

The error equation can be expressed as

\[
\begin{bmatrix}
V_x \\
V_y \\
V_z
\end{bmatrix} = k^0 R^0 \begin{bmatrix}
d\Delta X \\
d\Delta Y \\
d\Delta Z
\end{bmatrix} + R^0 \begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_S + dk + k^0 dR \begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_S
\]

(5)

Equation (5) can be expressed in the matrix form

\[
V_{3 \times 1} = A \begin{bmatrix}
x \\
l + X_T^T
\end{bmatrix}_{3 \times 1}
\]

(6)

where \(V = [V_x \ V_y \ V_z]^T\),

\(x = [d\Delta X \ d\Delta Y \ d\Delta Z \ d\varphi \ d\psi \ dk]^T\),

\(l = -[\Delta X \ \Delta Y \ \Delta Z]^T - k^0 R^0 \begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_S^T\)

and \(X_T = [X \ Y \ Z]^T\) represents the coordinate in the target coordinate system.

Equation (6) can be solved by the iterative procedure as follow:
Step 1: Set the initial value \( k=1 \), and the other 6 parameters to be 0, which are expressed as \( X^0 \).

\[
X^1 = (A^TPA)^{-1}AP(l + X_T)\quad X^1 = X^0 + x^1
\]

(where \( x \) denotes corrected value, and \( X \) denotes the estimation of unknown parameters, \( P \) is the weight matrix of the observation vector);

Step 2: Update the initial value.

\[
x^i = (A^TPA)^{-1}AP(l + X_T),\quad X^i = X^{i-1} + x^i
\]

Step 3: The iteration ends until the calculated correction value is less than the given threshold; otherwise, return to Step 2 to continue the iteration.

2.2. Closure constraint

In traditional industrial measurement several stations have been set up so that multiple coordinate transformations are existed. There are \( m \) common points between every two adjacent stations shown in Figure 2. The 3D coordinates are estimated in two independent coordinate systems. There are common points between any two stations, and observations can be converted into one unified coordinate system. That is to say, the common points of the initial station that transform back to the initial station coordinate system should be equal to the original coordinate of these points after several coordinate transformations. Without considering the closure constraint, the cumulative error will be accumulated.

In the post-processing of laser point clouds, the point cloud registration (also called point cloud splicing) is one of the most important data processing. In order to obtain the complete information of the object surface, the measurement of the external industry when using a 3D laser scanner to get the data of an object, usually require multi stations and multi angle scanning, and a unified station data into the same coordinate system. Simply point cloud matching is constrained to feature information between two adjacent station two, calculate the transition between station parameters, two site cloud unified into the same coordinate system. However, when many coordinate systems are converted, the conversion parameters between two stations can be considered, and the accuracy of point cloud matching can be reduced. Thus, closure constraints can be formed by using these common points.

In the coordinate transformation among independent coordinate systems, the closure constraint can be expressed as

\[
\Delta_n + k_n R_n \Delta_{n-1} + \cdots + k_2 R_2 \Delta_1 + k_n \cdots k_2 R_n R_2 \cdots R_1 \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}
\]

(7)

Where \( k_i, \Delta_i, R_i \) denote the scale factor, translation parameters, and rotation matrix, respectively.

To ensure equation (7), we can get

\[
k_n \cdots k_2 R_n R_2 \cdots R_1 \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}
\]

(8)
\[ \Delta_n + k_n R_n \Delta_{n-1} + \cdots + k_n \cdots k_2 R_n \cdots R_2 \Delta_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \]  
(9)

Equations (8) and (9) are linearized by Taylor series expansion at the initial values of all parameters. So we can get

\[
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = k^0_n \cdots k^0_1 R^0_n \cdots R^0_1 \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + k^0_n \cdots k^0_1 R^0_n \cdots R^0_1 d\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} dk_1 + \\
k^0_n \cdots k^0_1 R^0_n \cdots R^0_1 \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \cdots + k^0_n \cdots k^0_1 R^0_n \cdots R^0_1 dR_1 \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}
\]
(10)

\[
\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \Delta^0_n + k^0_n R^0_n \Delta^0_{n-1} + \cdots + k^0_n \cdots k^0_2 R^0_n \cdots R^0_2 \Delta^0_1 + \cdots + k^0_n \cdots k^0_1 R^0_n d\Delta^0_{n-1} + \cdots + k^0_n \cdots k^0_1 R^0_n d\Delta^0_1
\]

Where

\[
dR = \frac{dR}{d\theta} + \frac{dR}{d\phi} + \frac{dR}{d\psi}
\]
(12)

\[
d\Delta = [d\Delta X \quad d\Delta Y \quad d\Delta Z]^T
\]
(13)

We can get the simplified expression in matrix as

\[
K_x + W = 0
\]
(14)
in which

\[
W = (\Delta^0_n + k^0_n R^0_n \Delta^0_{n-1} + \cdots + k^0_n \cdots k^0_2 R^0_n \cdots R^0_2 \Delta^0_1) + k^0_n \cdots k^0_1 R^0_n \cdots R^1 \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} - \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}
\]
(15)

\[
K = \begin{pmatrix} K_1 & K_2 & \cdots & K_n \end{pmatrix}
\]
(16)

\[
K_i = \left[ \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} R^0_n \cdots dR_i \cdots R^0_1 \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} R^0_n \cdots dR_i \cdots R^0_1 \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \right] R^0_n \cdots dR_i \cdots R^0_1 \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}
\]
(17)

Where \( I \) denotes the identity matrix.

3. Total least squares solution with closure constraint

In 3D coordinate transformation the coefficient matrix also has errors which should be considered to improve the accuracy of the transformation. Taking the EIV model definition into account [14, 17, 18, 37], the error equation with the closure constraint by combining equations(6) and (14) can be expressed as

\[
\begin{cases} 
(A + E_A) \hat{x} = L + e_i \\
K \hat{x} + W = 0
\end{cases}
\]
(18)

where \( L \) is observational data, \( \hat{x} \) is a vector of unknown parameter, \( A \) is a coefficient matrix, \( E_A \) is a matrix of random errors in coefficient matrix, and \( e_i \) is vector of observational errors.

The objective function of total least squares [2, 8, 33] is

\[
e_i^T e_i + e_A^T e_A = \min
\]
(19)

Where \( e_A = vec(E_A) \).

We can firstly obtain the approximate values of the transformation parameters according to the above-mentioned 3D transformation model in section 2.1. Next, we can solve the model according to
the TLS with linear constraints.

According to Euler-Lagrange method, we can obtain the following objective function [16, 23]

$$\Phi(e_l, e_A, \lambda, \mu, \hat{x}) = e_l^T e_l + e_A^T e_A + 2\lambda^T (L - A\hat{x} + e_l - E_A\hat{x}) - 2\mu^T (W + K\hat{x})$$  \hspace{0.5cm} (20)

Where \( \lambda \) and \( \mu \) denote vectors of Lagrange multipliers. Equation (20) leads to the necessary Euler-Lagrange equations from which the solution can be finally derived

$$\frac{1}{2} \frac{\partial \Phi}{\partial e_l} = e_l + \lambda = 0$$  \hspace{0.5cm} (21a)

$$\frac{1}{2} \frac{\partial \Phi}{\partial e_A} = vec(E_A + \lambda \hat{x}^T) = 0$$  \hspace{0.5cm} (21b)

$$\frac{1}{2} \frac{\partial \Phi}{\partial \lambda} = -A^T \hat{x} + E_A^T \lambda - K^T \mu = 0$$  \hspace{0.5cm} (21c)

$$\frac{1}{2} \frac{\partial \Phi}{\partial \mu} = L - A\hat{x} + e_l - E_A\hat{x} = 0$$  \hspace{0.5cm} (21d)

$$\frac{1}{2} \frac{\partial \Phi}{\partial \mu} = -(W - K\hat{x}) = 0$$  \hspace{0.5cm} (21e)

So we can obtain

$$\hat{\nu} = (L - A\hat{x})^T (L - A\hat{x}) / (1 + \hat{x}^T \hat{x})$$  \hspace{0.5cm} (22)

Then, we should take the closure constraint into consideration, and get the following equation

$$N = \begin{bmatrix} A^T A & K \\ K & 0 \end{bmatrix}^{-1}$$  \hspace{0.5cm} (23)

$$\begin{bmatrix} \hat{x} \\ \hat{\mu} \end{bmatrix} = N \cdot \begin{bmatrix} A^T L + \hat{x} \cdot \hat{\nu} \\ W \end{bmatrix}$$  \hspace{0.5cm} (24)

By equation (22), the increment of the parameter can be calculated

$$\bar{\nu}^{l-1} = (L - A\tilde{x}^{l})^T (L - A\tilde{x}^{l}) / [1 + (\tilde{x}^{l})^T \tilde{x}^{l}]$$  \hspace{0.5cm} (25)

$$\begin{bmatrix} \tilde{x}^{l} \\ \tilde{\mu}^{l} \end{bmatrix} = N \cdot \begin{bmatrix} A^T L + \tilde{x}^{l-1} \cdot \bar{\nu}^{l} \\ W \end{bmatrix}$$  \hspace{0.5cm} (26)

Iterating over the above equations (25) and (26), until

$$\|\tilde{x}^{l} - \tilde{x}^{l-1}\| < \varepsilon$$  \hspace{0.5cm} (27)

$$\|\bar{\nu}^{l} - (L^T L - L^T A\tilde{x}^{l} - W^T \tilde{\mu}^{l})\| < \delta$$  \hspace{0.5cm} (28)

where \( \varepsilon \) and \( \delta \) are the threshold, respectively.

4. Case study and analysis

![Spatial distribution of points](image)

Fig. 3. Spatial distribution of point

In order to verify the feasibility and effectiveness of this new model, the transformation of four independent coordinate systems is studied using the actual measurement data. An experiment was performed in the square of SDUST. As shown in the fellow figure, we set up 4 independent stations. In the actual observation, there are 3 common points between two adjacent instrumental stations. Similarly, the first station and the last station also have 3 common points, which constitute a closure constraint to improve the transformation parameters.
3 common points participated in solving the transformation parameters between adjacent coordinates with closure constraint. We count the closure difference of point 1, 2, 3, 13. Table 1 lists the measured data in each independent coordinate system.

| Station 1 | Station 2 |
|-----------|-----------|
| Point     | X   | Y   | Z   | Point | X   | Y   | Z   |
| 1         | 15.0765 | 10.0988 | 2.2536 | 4     | 34.7882 | 18.5078 | 2.2566 |
| 2         | 20.0050 | 9.8654  | 2.2527 | 5     | 34.3840 | 21.8186 | 2.2601 |
| 3         | 25.6955 | 9.9965  | 2.2545 | 6     | 34.7412 | 27.8081 | 2.2523 |
| 4         | 30.0005 | 15.6985 | 2.2513 | 7     | 30.8485 | 33.0919 | 2.2632 |
| 5         | 29.5620 | 19.0065 | 2.2595 | 8     | 25.5493 | 32.8651 | 2.2493 |
| 6         | 29.8602 | 25.0001 | 2.2520 | 9     | 20.0038 | 32.9829 | 2.2511 |
| 13        | 29.8765 | 10.5230 | 2.2555 |        |        |        |      |

According to the measured data, the initial values of the parameters are obtained by using the traditional seven-parameter model that the parameters are estimated by the least squares method. Then the model proposed in this paper is used for the iterative solution. The threshold is set during the iteration, and the iteration withdraws when the corrected value is less than the given threshold. The solution strategy is shown in Figure 4. In order to illustrate the effectiveness of the new model, we can get the calculated values of points 1, 2, 3, and 13 in station 1, which uses results of traditional seven-parameters model. We also can obtain the calculated values of these four points by using the results of the new model. Then difference between the observed and calculated values should be obtained respectively, as listed in Table 2.
According to Table 2, we can see from the comparison of the coordinate difference that there are accumulations of errors when using the traditional model to deal with the transformation between multiple independent coordinate systems. Therefore, the difference of the coordinate will be relatively large. Using the model proposed in the paper can reduce the accumulation of errors, and calculated coordinate difference is reduced obviously. We can find that the RMS of the transformation parameter is 0.12 mm by using the model proposed in this paper and 0.20 mm by using the traditional model. It can be seen that the new model has more advantages.

The new model proposed in this paper considers the closure constraints between multiple independent stations and the errors in the coefficient matrix, and uses the total least squares with constraints to solve the model. The above results also validate the feasibility of the proposed model, and the reliability of the calculation accuracy.

5. Conclusions
A new model has been presented to solve the 3D coordinate transformation with closure constraint. It was employed to estimate the parameters of a practical case. The traditional transformation model
doesn’t take the closure condition into consideration. Using the new model, we can obtain an improved accuracy since closure condition and coefficient matrix errors had been introduced. The above results also validate the feasibility of the proposed model. In some practical applications, the transformation parameters are constrained. It can be encountered in rigorous registration of multi-view point clouds and the precision industrial measurements using new models.

Acknowledgements: This research was supported by the National Natural Science Foundation of China (grant No. 41774001 &41374009), the Basic Science and Technology Project of China (grant No. 2015FY310200), and the SDUST Research Fund (grant No. 2014TDJH101).

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