Momentum transfer dependence of the hadron GPDs and Compton form factors

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Abstract.
The generalized parton distributions (GPDs) of the meson and nucleon at small and large values of the momentum transfer were determined on the basis of the comparative analysis of different sets of experimental data of electromagnetic form factors of the proton and neutron, using the different sets of the parton distribution functions (PDFs). As a result, different form factors of the nucleons and meson were calculated. The t-dependence of these form factors was checked up in the description of the nucleon-nucleon and meson-nucleon elastic scattering and the differential cross sections of the real Compton scattering.

1. Introduction
The recent results from the LHC gave plenty of new information about the elastic and dip-inelastic processes which leads to new questions in the study of the structure of hadrons. At first, it is needed to note that the new data of the TOTEM and ATLAS Collaborations on the elastic proton-proton scattering indeed show that none of the old model predictions give the true descriptions of the elastic cross sections at the LHC [1]. The different reactions can be related with the different form factors. The strong hadron-hadron scattering can be proportional to the matter distribution of the hadrons. The Compton scattering is described by the Compton form factors. Now we have more general parton distributions which depend on different variables - $GTMDs(x, \vec{k}, \xi, \Delta)$ - Generalized transverse momentum dependent parton distributions. [2, 3, 4]. They parameterize the unintegrated off-diagonal quark-quark correlator, depending on the three-momentum $\vec{k}$ of the quark and on the four-momentum $\Delta$, which is transferred by the probe to the hadron. Taking $\Delta = 0$, we can obtain the $TMD(x, \vec{k})$ - transverse momentum-dependent parton distribution. In other way, after integration over $\vec{k}$ we obtain $GPDs(x, \xi, \Delta)$ - Generalized parton distributions. The remarkable property of the $GPDs$ is that the integration of the different momentum of GPDs over $x$ gives us the different hadron form factors [5, 6, 7].

The $x$ dependence of $GPDs$ in most part is determined by the standard PDFs, which are obtained by the different Collaborations from the analysis of the dip-inelastic processes. Many different forms of the $t$-dependence of GPDs were proposed. In the quark diquark model [8] the form of GPDs consists of three parts - PDFs, function distribution and Regge-like. In other works (see e.g. [9]), the description of the $t$-dependence of GPDs was developed in a more complicated picture using the polynomial forms with respect to $x$. 

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2. Momentum transfer dependence of GPDs

Let us modify the original Gaussian ansatz and choose the $t$-dependence of GPDs in a simple form $\mathcal{H}(x,t) = q(x) \exp[a_+ f(x) t]$, with $f(x) = (1 - x^2)/x^3$ [10]. The isotopic invariance can be used to relate the proton and neutron GPDs.

The complex analysis of the corresponding description of the electromagnetic form factors of the proton and neutron by the different PDF sets (24 cases) was carried out in [11]. These PDFs include the leading order (LO), next leading order (NLO) and next-next leading order (NNLO) determination of the parton distribution functions. They used the different forms of the $x$ dependence of PDFs. We slightly complicated the form of GPDs in comparison with the equation used in [10], but it is the simplest one as compared to other works (for example [12]).

\[
\mathcal{H}(x,t) = q(x) u e^{2a_H f(x) u} t; \quad \mathcal{H}(x,t) = q(x)^d e^{2a_H f(x) t}; \quad \mathcal{E}(x,t) = q(x) u (1 - x^2) e^{2a_E f(x) u}, \quad \mathcal{E}(x,t) = q(x)^d (1 - x^2) e^{2a_E f(x) t},
\]

where $f_u(x) = (1 - x)^{2 + u_0} / (x_0 + x)^{m_0}$ and $f_d(x) = (1 + e_0)(1 - x)^{1 + s_0} / (x_0 + x)^{m_0}$.

The hadron form factors will be obtained by integration over $x$ in the whole range $0 - 1$. Hence, the obtained form factors will be dependent on the $x$-dependence of the forms of PDF at the ends of the integration region. The different Collaborations determined the PDF sets from the inelastic processes only in some region of $x$, which is only approximated to $x = 0$ and $x = 1$. Some PDFs have the polynomial form of $x$ with different power. Some other have the exponential dependence of $x$. As a result, the behavior of PDFs, when $x \to 0$ or $x \to 1$, can impact the form of the calculated form factors.

On the basis of our GPDs with ABM12 [13] PDFs we calculated the hadron form factors by the numerical integration and then by fitting these integral results by the standard dipole form with some additional parameters $F_1(t) = (4m_p - mt)/(4m_p - t)$, $G_1(t) = 1/(1 + q/a_1 + q^2/a_2^2 + q^3/a_3^3)^2$ which is slightly different from the standard dipole form on two additional terms with small sizes of coefficients. The matter form factor

\[
A(t) = \int_0^1 dx [q_u(x) e^{2a_H f(x) u/t} + q_d(x) e^{2a_H f(x) t}],
\]

is fitted by the simple dipole form $A(t) = \Lambda^4/(A^2 - t^2)$. These form factors will be used in our model of the proton-proton and proton-antiproton elastic scattering.

3. Magnetic transition form factor $G_M^{*}(\gamma \gamma N\Delta)$

To check the momentum dependence of the spin-dependent part of GPDs $E_{u,d}(x, \xi = 0, t)$, we can calculate the magnetic transition form factor which is determined by the difference of $E_u(x, \xi = 0, t)$ and $E_d(x, \xi = 0, t)$. For the magnetic $N \rightarrow \Delta$ transition form factor $G_M^{*}(t)$, in the large $N_c$ limit, the relevant GPD $\Delta N$ can be expressed in terms of the isovector GPD yielding the sum rule [14]

\[
G_M^{*}(t) = \frac{G_M^{*}(t = 0)}{k_v} \int_{-1}^{1} dx(E_u(x, \xi, t) - E_d(x, \xi, t))
\]

where $k_v = k_\mu - k_n = 3.70$.

\[
E_u(x, \xi = 0, t) = d(x) e^{2a_1 (((1 - x) p_1)/(x_0 + x p_2))}; \quad E_d(x, \xi = 0, t) = d(x) e^{2a_1 (((1 - x) p_1)/(x_0 + x p_2) + dx(1 - x))}.
\]

There are two different conventions - Jones-Scadron and Ash convention. They are related as $G_{M,J-S}(Q^2) = G_{M,As} \sqrt{1 + Q^2 / (M + m)^2}$. We will be present the data in the Ash convention.
The results of our calculations are presented in Fig.1 (a). The experimental data exist up to $-t = 8$ GeV$^2$ and our results show a sufficiently good coincidence with experimental data. It is confirmed that the form of the momentum transfer dependence of $E(x, \xi, t)$ determined in our model is right.

4. The Compton cross sections

Now let us calculate the moments of the GPDs with inverse power of $x$. It gives us the Compton form factors. Using the obtained form factors the reaction of the real Compton scattering can be calculating. The differential cross section for that reaction can be written as [12]

$$\frac{d\sigma}{dt} = \frac{\pi\alpha_{em}^2}{s^2} \frac{(s-u)^2}{-us} \left[ R_V^2(t) - \frac{t}{4m^2} R_T^2(t) + \frac{t^2}{(s-u)^2} R_A^2(t) \right],$$

where $R_V(t)$, $R_T(t)$, $R_A(t)$ are the form factors given by the $1/x$ moments of the corresponding GPDs $H^q(x, t)$, $E^q(x, t)$, $H^q(x, t)$. The last is related with the axial form factors. As noted in [12], this factorization, which bears some similarity to the handbag factorization of DVCS, is formulated in a symmetric frame where the skewness $\xi = 0$. For $H^q(x, t)$, $E^q(x, t)$ we used the PDFs obtained from the works [15] with the parameters obtained in our fitting procedure of the description of the proton and neutron electromagnetic form factors in [11].

$$R_i(t) = \sum_q c_q^2 \int_0^1 \frac{dx}{x} \mathcal{F}_{j}(x, \xi = 0, t),$$

where $\mathcal{F}_{j}$ are equal $H_q$, $E_q$ and $\tilde{H}_q$ and give the the form factors $R_V(t)$, $R_T(t)$, $R_A(t)$, respectively.

In the present work for $\tilde{H}^q(x, t)$ we take $\Delta q^+$ in the form [16] for NNLO $Q_0 = 2$ GeV$^2$

$$x\Delta_q(x, Q_0) = N_q q_0 x^{a_q}(1-x)^{b_q}(1+c_q x);$$

Assuming $SU(3)$ flavor symmetry of $\Delta q$ the coefficient $N_q$ is determined as

$$\frac{1}{N_q} = \left(1 + c_q \frac{a_q}{1+a_q+\hat{b}_q} \right) B(a_q, b_q + 1),$$

where $B(a_q, b_q + 1)$ is determined by $B(a, b) = \Gamma(a)\Gamma(b)/\Gamma(a+b) = \int_0^1 t^{a-1}(1-t)^{b-1} dt$. The results of our calculations of the Compton form factors are shown in Fig. 1(b,c). Obviously, $R_V(t)$ and $R_T(t)$ have a similar momentum transfer dependence, but essentially differ in size. On the contrary, the axial form factor $R_A$ has an essentially different $t$ dependence. The results for the cross sections are presented in Fig.2 (a). Except the very large angles at low energy the coincidence with the experimental data is sufficiently good. The calculations of $R_i$ on the whole, correspond to the calculations [12].
5. Hadron form factors and elastic nucleon-nucleon scattering

Both hadron electromagnetic and gravitomagnetic form factors were used in the framework of the high energy generalized structure model of the elastic nucleon-nucleon scattering. This allows us to build the model with minimum fitting parameters [17, 18, 19].

The Born term of the elastic hadron amplitude can now be written as

\[ F_h^{\text{Born}}(s,t) = h_1 G^2(t) F_a(s,t) \left( 1 + \frac{r_1}{s^{0.5}} \right) + h_2 A^2(t) F_b(s,t) \]

\[ \pm h_{\text{odd}} A^2(t) F_b(s,t) \left( 1 + \frac{r_2}{s^{0.5}} \right), \]

where \( F_a(s,t) \) and \( F_b(s,t) \) have the standard Regge form: \( F_a(s,t) = \tilde{s}^{\alpha_L} e^{B(s)/t} \); \( F_b(s,t) = \tilde{s}^{\beta} e^{b_{\text{odd}}/t} \). The intercept \( 1 + \epsilon_1 = 1.11 \) was chosen from the data of the different reactions and was fixed by the same size for all terms of the Born scattering amplitude. The slope of the scattering amplitude has the standard logarithmic dependence on the energy \( B(s) = \alpha' \ln(s) \) with \( \alpha' = 0.24 \text{ GeV}^{-2} \) and with some small additional term [18], which reflects the small non-linear behavior of \( \alpha' \) at small momentum transfer [20]. The final elastic hadron scattering amplitude is obtained after unitarization of the Born term. So, at first, we have to calculate the eikonal phase

\[ \chi(s,b) = -\frac{1}{2\pi} \int d^2 \tilde{q} e^{i\tilde{b} \cdot \tilde{q}} F_h^{\text{Born}}(s,q^2) \]

and then obtain the final hadron scattering amplitude

\[ F_h(s,t) = i s \int b J_0(bq) \Gamma(s,b) \, db; \quad \text{with} \quad \Gamma(s,b) = 1 - \exp[\chi(s,b)]. \]

At large \( t \) our model calculations are extended up to \(-t = 15 \text{ GeV}^2\). We added a small contribution of the energy independent part of the spin flip amplitude in the form similar to the proposed in [23] and analyzed in [24]. \( F_{sf}(s,t) = h_{sf} q^2 F_1^2(t) e^{-B_s t q^2} \). The model is very simple from the viewpoint of the number of fitting parameters and functions. There are no any artificial functions or any cuts which bound the separate parts of the amplitude by some region of momentum transfer. In the framework of the model the description of the experimental data was obtained simultaneously at the large momentum transfer and in the Coulomb-hadron region in the energy region from \( \sqrt{s} = 9 \text{ GeV} \) up to LHC energies. Figure 2(b) represents the description at \( \sqrt{s} = 11.4 \text{ GeV} \) and Figure 2(c) represents the model predictions for \( \sqrt{s} = 13 \text{ TeV} \), which coincide well with the preliminary data presented at the conference BLOIS-17 (2017) [30].
6. Nucleon structure and radii

Let us now consider, as an example, the neutron structure in the impact parameter [25, 26] representation. We are particularly motivated by recent discussion of the definition of charge density of the neutron at small impact parameters corresponding to the ”center” of the neutron [27, 28]. In [28], the charge density of the neutron is related to $F^u_1(t)$ and calculated using the phenomenological representation of $G^u_E(t)$ and $G^u_M(t)$. It differs essentially from the definition of the neutron charge distribution in the Breit frame.

$$\rho_{G_E}(b) = \int dq |F_1(q^2) + \tau F_2(q^2)| e^{i\vec{q}\cdot\vec{b}}$$

$$= \sum_q e_q \int_0^1 dx \int dq |H_q(x, \xi = 0, q^2) + \tau E_q(x, \xi = 0, q^2)| e^{i\vec{q}\cdot\vec{b}}$$

(11)

with, as usual, $\tau = (q/m_p)^2$. Using our model of $t$-dependence of GPDs, we may calculate both forms of the neutron charge distribution in the impact parameter representation and, moreover, determine separate contributions of $u$ and $d$ quarks. The respective separate contributions of $u$ and $d$-quarks are shown in Fig. 3(a). We can see that $u$-quarks have the large negative charge density in the centre of the neutron.

Let us compare the distribution of the electric charge and matter (that is, gravitational charge) in the nucleon with the matter density

$$\rho_{G_r}(b) = \frac{1}{2\pi} \int_0^\infty dq q J_0(qb)A(q^2).$$

(12)

The difference of the matter density and the charge density for proton is shown in Fig.3(b). The radius of the hadron can be determined by the slope of its form factor $\langle r^2_h \rangle = -6/f_h(0)d/dt[f_h(t)]_{t=0}$. For the electromagnetic form factor with our form of $\hat{G}(t)$ we have

$$-\frac{6}{G(0)} \frac{d}{dt}[\hat{G}(0)] = 2a_2t - (2(1/L^2 + a_1/(2\sqrt{t}))/((1 + a_1\sqrt{-t} - t/L^2)^3$$

(13)

As the coefficients $a_1$ and $t$ are very small eq.(13) gives us the simple relations $\langle r^2_h \rangle \approx 1/L^2$. The electromagnetic form factors and the gravitomagnetic form factors are describe almost the same form - the dipole form factor but with different sizes of the $L^2$. Hence we have for the electromagnetic radius the ratio of the radiiuses of the electromagnetic and the gravitomagnetic form factors - $r_E/r_A = (1/L^2)/((1/L^2) \approx 1.5$. It is shown that the matter distribution is concentrated in the center of the hadron and at large distances the charge distribution has a longer tail than the matter distribution.

7. Conclusion

The obtained new momentum transfer dependence of GPDs, based on the analysis of practically all existing experimental data on the electromagnetic form factors of the proton and neutron, shows a good description of the $t$ dependence of a wide circle of different reactions. As a result, the description of various reactions is based on the same representation of the hadron structure. Especially it concerns the real Compton scattering and high energy elastic hadron scattering. The new high energy generalized structure (HEGS) model, based on the electromagnetic and gravitomagnetic form factors, gives a good quantitative description of the existing experimental data of the proton-proton and proton-antiproton elastic scattering in a wide region of the energy scattering and momentum transfer, including the Coulomb-hadron interference region, with the minimum number of the fitting parameters. Their predictions coincides well with the new experimental data obtained at the LHC at $\sqrt{s} = 7$ and $\sqrt{s} = 8$ TeV The predictions of the
model based on the obtained electromagnetic and gravitomagnetic form factors coincides well with the recent preliminary data at $\sqrt{s} = 13$ TeV [30]. The investigation of the nucleon structure shows that the density of the matter is more concentrated than the charge density in the nucleon.

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