Quantum Stochastic Neural Network and Sentence Classification

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Computers has been endowed with a part of human-like intelligence owing to the rapid development of the artificial intelligence technology represented by the neural networks. Facing the challenge to make machines more imaginative, we consider a quantum stochastic neural network (QSNN), and propose a learning algorithm to update the parameters governing the network evolution. The QSNN can be applied to a class of classification problems, we investigate its performance in sentence classification and find that the coherent part of the quantum evolution can accelerate training, and improve the accuracy of verses recognition which can be deemed as a quantum enhanced associative memory. In addition, the coherent QSNN is found more robust against both label noise and device noise so that it is a more adaptive option for practical implementation.

INTRODUCTION

Propelled by rapid advances in computer science, neural networks via machine learning (ML) technique [1, 2], regarded as the heart of artificial intelligence (AI), have found numerous applications in both image processing [3, 4], natural language processing [5–8], bioinformatics [9], etc. However, a bottleneck of classical computation may be right around the corner as it is too expensive to get a significant performance boost in integration of chip components and the classical physical rules do not apply when the scale is getting smaller and smaller. The quantum computation [10–12], which has displayed superiority in some specific problems such as factorization of large numbers [13], search problems [14] and solving linear systems of equations [15], combined with ML brings forth a new potential future era of quantum AI [16, 17].

Some quantum generalizations of classical feedforward [18–32] and feedback [33–39] neural networks called quantum neural networks (QNNs) have been proposed, which show remarkable ability in quantum tasks driving the classical analogues to the wall, such as quantum automatic encoder [27, 30, 40], quantum-state classification [23, 28, 31], learning unknown unitaries [29, 30], the simulation of many-body systems [38, 39] and so on [32]. A number of exploration in its availability for classical task [18–26, 34–36] also show the potential to substitute for classical neural networks one day.

In addition to QNNs for fundamental tasks mentioned above, it is gratifying to note that some quantum-inspired formulations, not for quantum computation, have been proposed to explain the inherent behavior of the macro-world problems such as the quantum language model [41–45]. In related works, these have been shown perform efficiently in question answering [41, 42], ad-hoc retrieval [43], speech recognition [44] and sentiment analysis [45], which inspire us to bridge the gap between the quantum computation and the language model by QNNs and search for its quantum advantage further.

In this paper, we construct a universal quantum stochastic neural network (QSNN) based on continuous-time quantum walk, which can be applied to a class of classical classification problems, taking discriminative language models that focus on identifying if a word sequence a sentence as an elementary example, and propose a learning algorithm to optimize its performance. We compare the training and test performances of the QSNNs and a classical model, and find the dynamic evolution of the QSNNs, especially the coherent one, exhibits delightful advantages to accelerate training and enhance a certain associative memory. In the end, we investigate the robustness of the QSNNs against both label noise and device noise, and the coherent QSNN displays the appealing performance in both cases.

QUANTUM STochastic NEURAL NETWORK

Our quantum stochastic neural network (QSNN) is based on a continuous-time quantum walk. The state of a QSNN of $N$ neurons is represented by a density operator $\rho$ on a $N$-dimension Hilbert space $\mathcal{H}$, with each basis state $|i\rangle$ ($i=1,2,\cdots,N$) corresponding to a neuron (vertex). The state of the QSNN is initialized according to the particular task, and evolves according to the master equation in the Lindblad form [46]

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_{k} \left(L_{k}\rho L_{k}^{\dagger} - \frac{1}{2} \{L_{k}^{\dagger}L_{k},\rho\}\right)$$

(1)

where the first term on the right side determined by the Hamiltonian $H$ represents the coherent part of the dynamic (a factor $1/\hbar$ is absorbed in the Hamiltonian), while the second term determined by the Lindblad operators $L_{k}$ represents the coupling of the QSNN to its environment or a measuring device. In order to build an oracle-like classical neural network with an input and output, we choose

$$L_{k} = L_{i\rightarrow j} = \gamma_{k}|j\rangle\langle i|$$

(2)
to simulate the information transferring layer by layer and

$$H = \sum_{ij} h_{ij}(|i\rangle\langle j| + |j\rangle\langle i|) = \sum_k h_k H_k \tag{3}$$

to characterize the transmission between some particular neurons, like a feedback process of Hopfield networks, in a completely quantum way. The coefficients $h_k, \gamma_k \in \mathbb{R}$ characterize the coherent strength between the neuron $|i\rangle$ and $|j\rangle$ and the dissipation rate from the neuron $|i\rangle$ to $|j\rangle$, respectively. We donate the strengths of interaction as $\vec{h} = (h_1, h_2, \ldots, h_m) \in \mathbb{R}^m$ and $\vec{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_n) \in \mathbb{R}^n$, where $m$ and $n$ are the numbers of $h_k$ and $\gamma_k$. According to the Choi-Jamiolkowski isomorphism, we can rewrite Eq. (1) as

$$\frac{d|\rho\rangle}{dt} = \mathcal{L}|\rho\rangle \tag{4}$$

where $|\rho\rangle$ is a ket in $\mathcal{H} \otimes \mathcal{H}$, and the Liouvillian superoperator $\mathcal{L}$ is given by

$$\mathcal{L} = i(H \otimes I - I \otimes H) + \sum_k \left[ \hat{L}_k \otimes L_k - \frac{1}{2} I \otimes (L_k^\dagger L_k) - \frac{1}{2}(L_k^\dagger \otimes I) \right] \tag{5}$$

with $\bar{\Omega}$ denotes the complex conjugate of $\Omega$.

The above QSNN can be used as a discriminative language model. We consider at least three layers of neurons, the first layer contains a single neuron and the network is initialized in this local state. The second layer contains a certain number of neurons with each neuron corresponding to a word we possibly use. The single neuron in the first layer is connected to each neuron in the second layer by a Lindblad operator (a one-way transmission) that is turned on according to the order of the words in the sequence. The neurons in the second layer are coherently connected with each other via parameters in the Hamiltonian. A more complex QSNN can contain many hidden layers. The output layer contains a certain number of neurons required to express the answer. Two neurons from two adjacent layers are typically connected by a one-way Lindblad operator. The time evolution of the QSNN is described by Eq. (1).

For an example, which can be generalised easily, if there are only 2 different words in our corpora, the QSNN can be constructed by 5 neurons as shown in Fig. 1. In this case, the classical training set is donated as $S = \{ (se_s, l_s) | s = 1, 2 \}$, where the pair $se_s = (w_1, w_2)_{i \neq j \in \{1, 2\}}$ is labeled $l_s \in \{Yes, No\}$. The QSNN evolves as follows for a training pair $(se_1 = (w_1, w_2), l_1)$:

1. **Input**: At time $t = 0$, the network is initialized in the state $\rho_0 = |0\rangle\langle 0|$ and the channel controlled by $L^0_i = \gamma_1|1\rangle\langle 0|$ is turned on. At $t = \frac{T_{in}}{2}$, the channel controlled by $L^0_2 = \gamma_2|2\rangle\langle 0|$ is turned on. After this input process, the state of the network is given as

$$\rho_{in} = \omega|0\rangle\langle 0| + \alpha|1\rangle\langle 1| + \beta|2\rangle\langle 2| \tag{6}$$

2. **Unitary evolution**: $\Lambda^U(\vec{h})$, a coherent evolution for a duration $T^U$, gives

$$|\rho_U\rangle = \Lambda^U(\vec{h})|\rho_{in}\rangle = e^{\mathcal{L}_H(\vec{h})T^U}|\rho_{in}\rangle, \tag{7}$$

where $\mathcal{L}_H$ is the Liouvillian superoperator with $H = h_i(|1\rangle\langle 2| + |2\rangle\langle 1|)$ and all Lindblad operators zero.

3. **Output**: $\Lambda^D(\vec{\gamma})$, describing a completely dissipative process for the binary classification for a duration $T^D$ with $\{L^D_k\}_{(k = 1, 2, 3, 4)}$ connecting between the hidden layer and output layer, gives the final state

$$|\rho_{out}\rangle = \Lambda^D(\vec{\gamma})|\rho_U\rangle = \Lambda^D(\vec{\gamma})\Lambda^U(\vec{h})|\rho_{in}\rangle = e^{\mathcal{L}_D(\vec{\gamma})T^D} e^{\mathcal{L}_H(\vec{h})T^U}|\rho_{in}\rangle. \tag{8}$$

A projective measurement onto the output layer gives Yes (the input is a grammatical sentence) with a probability $p_{Yes} = \langle Yes|\rho_{out}|Yes\rangle$, and No (the input is not a grammatical sentence) with a probability $p_{No} = \langle No|\rho_{out}|No\rangle$, as well as an undetermined result with probability $1 - p_{Yes} - p_{No}$ that is negligible for a sufficiently large $T^D$ and $T^D$.

In Sec. I of Supplemental materials, we demonstrate that an optimal evolution can always be found for the QSNN to classify input sequences in an arbitrary set approximatively, as long as the QSNN contains the incoherent process (1) input and (3) output.
LEARNING ALGORITHM

We train the QSNN to learn basic syntax by updating the parameters ($\hat{h}$ and $\vec{\gamma}$) governing the evolution processes with a learning algorithm based on gradient descent. We perform a measurement of $\Omega^s = |l_s\rangle\langle l_s|$ on the final state $\rho^s_{\text{out}}$ for the input training pair $(s_0, l_s)$ and define the loss function as

$$\text{Loss} = 1 - \frac{1}{N} \sum_{s=1}^{N} \text{Tr}(\Omega^s \rho^s_{\text{out}}), \quad (9)$$

where $N$ is the number of training pairs in the training set. We aim to minimize Loss to get the maximal success probability of correct classification.

The gradient of the loss function with respect to any parameter $\theta$ can be written as

$$\frac{\partial \text{Loss}}{\partial \theta} = -\frac{1}{N} \sum_{s=1}^{N} \text{Tr}(\Omega^s \frac{\partial \rho^s_{\text{out}}}{\partial \theta}). \quad (10)$$

Then the parameter can be updated according to

$$\theta' = \theta - \eta \frac{\partial \text{Loss}}{\partial \theta}. \quad (11)$$

The gradients described in Eq. (10) are calculated in Sec. II of Supplemental materials, where $\eta$ is an adjustable non-negative parameter called learning rate. After numberable iterations, with the gradient descent one can approach the minimum of the loss function arbitrarily close.

We show that the vanishing gradient problem [47–49] is more tolerable by our QSNN than the circuit-model-based quantum neural networks [50] for practical application in Sec. III of Supplemental materials.

QUANTUM ADVANTAGES

According to whether the Hamiltonian in Eq. (1) vanishes, we can divide our QSNN into two categories: the coherent QSNN that has coherent evolution governed by a nonzero Hamiltonian, and the incoherent QSNN that has a zero Hamiltonian and only a dissipation evolution governed by the Lindblad operators. Next we compare the training and test performances of three types of neural networks, namely, the coherent QSNN, the incoherent QSNN, and the classical analogous named classical NN. We investigate what role is played by the coherent dynamic governed by the nonzero Hamiltonian, and find the quantum advantages. We find that the coherent dynamic can accelerate the training, enhance a certain associative memory, and improve the robustness of the network against certain noise.

Accelerate training

Fast training is an important desired property of a good neural network model, as we always pursue the training efficiency in practical applications.

We consider the 5-neuron model shown in FIG. 1 with the training set $S = \{(s_1, w_1, \text{Yes}), (s_2, w_2, \text{No})\}$. We take 1000 random $\vec{\gamma} \in \mathbb{R}^n$ samples, train the QSNN from these 1000 different initializations, and average over the samples.

![FIG. 2. We plot the sample mean Loss after each iteration for the classical NN (grey), the incoherent QSNN (red), and the coherent QSNN with initial $h = 0.1$ (blue). The inset contains more initial choices of $h$ for the coherent QSNN. The coherent QSNN has a mean Loss going down faster than the incoherent QSNN, which again faster than the classical NN. Therefore, the coherent QSNN always completes learning at the fastest speed. The error bars are drawn as 95% confidence intervals. More simulation details are given in Sec. IV.A of Supplemental materials.](image-url)

At first, from FIG. 2 we find the QSNNs, both coherent and incoherent, perform better than the classical NN. The classical NN for comparison here is a typical feed-forward NN with 3 layers and 5 neurons as in FIG. 1, neurons in the same layer has no connection, and it uses a softmax function as the output function. The output $\vec{y}_s(\vec{w}, \vec{b}) = (y_{\text{No}}, y_{\text{Yes}})$ gives the probabilities that the processed input $(\alpha, \beta)$ is a sentence $(y_{\text{Yes}})$ or not $(y_{\text{No}})$. The loss function of a classical NN is defined as

$$\text{Loss}_C = 1 - \frac{1}{N} \sum_{s=1}^{N} \vec{l}_s \cdot \vec{y}_s,$$

where $\vec{l}_s \in \{\vec{l}_{\text{No}} = (1, 0), \vec{l}_{\text{Yes}} = (0, 1)\}$ is a classical label.

Next, we compare the training performances between two kinds of QSNNs. FIG. 2 shows the mean Loss always goes down faster when the model contains coherent evolution. So we are pleasantly surprised to find that the
coherent evolution by a nonzero Hamiltonian can accelerate training further!

In addition, we can further accelerate the training of the coherent QSNN by choosing an optimal initial value for the coherence strength $h$ rather than a random value just at a small cost. Details about the strategy are given in Sec. IV.A of Supplemental materials.

Therefore, the training of the QSNN is always more efficient than classical NN in terms of step complexity and the coherent QSNN always completes learning at the fastest speed. This clearly shows the quantum advantages of QSNNs.

Verse recognition

Everyone meets the need of understanding poetry, a treasure of culture, in varying degrees of demand, which is a motivation for teaching the computer to recognize verses. However, it seems rather challenging because of the complex syntax of verses, and an advanced intelligence named association is a prerequisite. Here, we give some evidence that the quantum property, especially the coherent connection between the word neurons, can equip the network with the associative memory used to improve the accuracy of verse recognition.

To display the ability for sentence recognition concretely, we train the QSNNs with 8 neurons $\{|w_1\rangle, |w_2\rangle, \cdots, |w_8\rangle\}$ in the hidden layer and 12 labeled training sequences without stopwords. The test set consists of 2 verses and 2 normal sentences, whose probabilities of being recognized as a sentence (Yes) (i.e. the accuracy) will be given to evaluate the performance of the trained QSNNs (both coherent and incoherent) and classical NN. More details about the training and test are given in Sec. IV.B of Supplemental materials.

Just as we expect from intuition, the recognition accuracy of these two verses are lower than the normal sentences, shown in FIG. 3(b-f), apparently because their unusual word order has never been met by the network. However, the verse recognition accuracy of the incoherent QSNN shows a significant improvement compared with the classical NN. And the verse recognition accuracy is improved further when a coherent dynamic is present in the QSNN. Intuitively, this can be understood as the coherent dynamic can actually be used to train certain relations between the words. Meanwhile, the QSNN wins only a little in the accuracy for recognizing 2 normal sentences (see e and f) since they are close to 1 in each case. The error bars are drawn as the sample variances.

FIG. 3. As introduced above, the quantum property, especially the coherent dynamic, accelerates the training of the networks(see a). The coherent QSNN performs best for the test set (see b), the main advantages come from recognizing verse1 (see c) and verse2 (see d). Meanwhile, the QSNN wins only a little in the accuracy for recognizing 2 normal sentences (see e and f) since they are close to 1 in each case. The error bars are drawn as the sample variances.

ROBUSTNESS

Label noise

Our proposal QSNN aiming at accurate classification is based on the supervised learning solution, which has implicit assumption that annotators are experts and they would provide perfectly labeled training data. It is however rarely met in real-world scenarios and constantly evolving grammar also brings about changes of the correspondences between sequences and labels in the matter of our concern, which increases mislabeled data damaging effects for the model’s performance inevitably. For saving computing resources, the ideal model should perform better than the alternatives in re-training the partially trained QSNN after a correction of the training set, i.e., it should reach a new convergence faster.

To hide the significant coherence superiority with the random initialization condition described before and study this re-training ability in isolation, we train the
QSNN with the same structure and training set as in the verse recognition from some uniform initializations samples \( \{\gamma_i = 0.1, 0.3, 0.5, 0.7 | i = 1, 2, \ldots, n \} \). We can see the performances are indistinguishable when corrupted data are learned at first, but the coherent QSNN shows the better convergence property after the errors are corrected in the 100th iteration in FIG. 4. In terms of these results, the coherent QSNN is more adaptive to deal with the constantly developing grammar.

![Graph](image)

**FIG. 4.** With some uniformly initialized samples \( \{\gamma_i = 0.1, 0.3, 0.5, 0.7 | i = 1, 2, \ldots, n \} \), we train and compare the incoherent QSNN and a coherent QSNN with initial \( h = 0.1 \). The sample mean Loss for both networks are indistinguishably at first, but the coherent QSNN shows a better convergence after the errors are corrected in the 100th iteration ((a), (b) and (c) correspond to three different corrupted sets). More training details are given in Sec. IV.C of Supplemental materials. The error bar is drawn as the sample variance.

**Device noise**

For practical implementation, a robust training is necessary as small perturbations of parameters may affect the performance of trained QSNN. The derivatives \( \frac{\partial \text{Tr}(\Omega \rho^s)}{\partial \gamma_i} \) represent the sensitivity of the successful classification probability to a perturbation \( \delta \gamma_i \), and the robustness is defined as

\[
\text{Robustness} = 1 - \frac{1}{N} \frac{1}{n} \sum_{s=1}^{N} \sum_{i=1}^{n} \left| \frac{\partial \text{Tr}(\Omega \rho^s)}{\partial \gamma_i} \right|^2. \tag{12}
\]

We monitor the robustness performance after each iteration in the training shown by FIG. 2, and find that the coherence-induced quantum advantages described before are not at the cost of robustness, as shown in FIG. 5. In fact, the coherent QSNN is more strikingly robust against device noise.

**CONCLUSION AND OUTLOOK**

We construct a universal quantum stochastic neural network (QSNN) and introduce a learning algorithm to find appropriate parameters which achieve the maximal success probability of correct classification. Our numerical simulation shows that the quantum dynamic can not only accelerate training, but also obtain an associative memory that is very useful for recognizing new types of inputs like the verses. And We interestingly find that the coherent QSNN performs best in the training and test for sentence classification and in the robustness test.

Some recent theoretical proposals sticking to quantum neural networks using universal quantum gates [23, 24, 27, 29, 40] are experimentally demanding so far. But some recent experimental advances of quantum walks [37, 51–53] make our QSNN more promising for implementation. Sentence classification as an elementary example has showed quantum superiority of the QSNN versatilely. More tasks that classical networks can complete will be mapped into quantum cases with similar QSNNs in the future.

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