On the anomalous thermal evolution of the low-temperature, normal-state specific heat of various nonmagnetic intermetallic compounds.

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Abstract

The low-temperature normal-state specific heat and resistivity curves of various nonmagnetic intermetallic compounds manifest an anomalous thermal evolution. Such an anomaly is exhibited as a break in the slope of the linearized \( C/T \) versus \( T^2 \) curve and as a drop in the \( R \) versus \( T \) curve, both at the same \( T_{\beta\gamma} \). It is related, not to a thermodynamic phase transition, but to a Kohn-type anomaly in the density of states curves of the phonon or electron subsystems. On representing these two anomalies as additional Dirac-type delta functions, situated respectively at \( k_B\theta_L \) and \( k_B\theta_E \), an analytical expression for the total specific heat can be obtained. A least-square fit of this expression to experimental specific heat curves of various compounds reproduced satisfactorily all the features of the anomalous thermal evolution. The obtained fit parameters (in particular the Sommerfeld constant \( \gamma_0 \) and Debye temperatures \( \theta_D \)) compare favorably with the reported values. Furthermore, the analysis shows that (i) \( T_{\beta\gamma}/\theta_L = 0.2 (1 \pm 1/\sqrt{6}) \) and (ii) \( \gamma_0 \propto \theta_{D}^{2} \); both relations are in reasonable agreement with the experiments. Finally, our analysis (based on the above arguments) justifies the often-used analysis that treats the above anomaly in terms of either a thermal variation of \( \theta_D \) or an additional Einstein mode.

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I. INTRODUCTION

The low-temperature, normal-state specific heat of a nonmagnetic intermetallic is usually described in term of a sum of lattice, \( C_L \), and electronic, \( C_E \), contributions. The former is commonly approximated by the Debye model \( (C_L = \beta T^3 \text{ for } T << \theta_D \text{; } \theta_D \text{ is the Debye temperature}) \) while the latter by the Sommerfeld model \( (C_E = \gamma T \text{ for } T << \theta_F ; \theta_F \text{ is the Fermi temperature}) \).

Surprisingly, this very \( \beta\gamma \)-change is evident as a break in the slope of the normal-state \( C/T \) versus \( T^2 \) curve, a break that separates two distinct linearized sections; these linearly extrapolated upper and lower sections intersect at \( T_{\beta\gamma} \) which is taken to be a measure of the energy of this event.

Varieties of materials exhibit this \( \beta\gamma \)-change: examples include group V transition metals \( \text{V, Nb and Ta} \) (Fig. 1). The Chevrel phases \( \text{PhMo}_{6}X_8 \) \( \text{(X=S, Se)} \), the \( \text{A12-type Re}_{3}W_{8} \), the \( \text{A15-type Nb}_{3}Sn \) and \( \text{V}_{3}Si \) (Fig. 2), the layered \( \text{NbSe}_{2} \), the perovskite \( \text{MgCNi}_{3} \), the \( \text{borocarbides } \text{RNi}_{2}B_{2}C \) \( \text{(R=Y, La, Lu)} \), the \( \text{Li}_{2}(\text{Pd}_{1-x}\text{Pt}_{x})_{3}B \) \( \text{(x=0, 0.5, 1)} \) (Fig. 3) and \( \text{Li}_{2}\text{RhB}_{15} \) \( \text{(x=0.8, 1.0, 1.2)} \). In particular, for \( \text{Nb}_{3}Sn \), this \( \beta\gamma \)-change was shown to be required by the condition of entropy balance.

Generally, the analysis of the normal-state specific heat is undertaken so as to obtain \( \theta_D \) from \( \beta \) and \( N(E_F) \) from \( \gamma \). \( \theta_D \) is the density of states at the Fermi level, \( E_F \), and it is not surprising that such a \( \beta\gamma \)-anomaly did not attract much attention; rather it is considered as an undesirable complication that hinders the precise evaluation of \( \theta_D \) and \( N(E_F) \). In spite of this back-ground, some dedicated investigations were carried out even though no common agreements on the interpretation were reached. Nevertheless, there was a consensus about the following common features.

First, the manifestation of a \( \beta\gamma \)-change is evident in various compounds such as normal intermetallics as well as the normal-state of type II superconductors: each compound differs strongly from the others in its crystal structure, chemical composition, electronic properties, and the type of superconductivity (whether conventional or unconventional). As a result, there are strong differences in \( T_{\beta\gamma} \), in the strength of the event, and in

\[
\begin{align*}
\text{(a) V} & \quad T_c \quad T_{\beta\gamma} \quad T_{\gamma}
\text{(b) Nb} & \quad T_c \quad T_{\beta\gamma} \quad T_{\gamma}
\text{(c) Ta} & \quad T_c \quad T_{\beta\gamma} \quad T_{\gamma}
\end{align*}
\]

FIG. 1: \( C/T \) versus \( T^2 \) curves of (a) \( \text{V} \); (b) \( \text{Nb} \) \( (H = 1.15 \text{ } T) \); (c) \( \text{Ta} \). Lines are fits to Eq. 8 with parameters as given in Table II. For (b) the solid line is a fit to the data (circles) of Da Silva et al. while the dashed line to those (stars) of Leupold et al.
TABLE I: Normal-state parameters of various compounds as obtained from the least-squares fit of Eq. 8 to the data of Figs. 1-5. \( T_c \) is the superconducting critical temperature at zero field; \( sc (pc) \) denotes a single-crystal (polycrystalline) sample. Other symbols are explained in the text.

| Compound     | \( T_c \) | \( T_{\beta\gamma} \) | \( \theta_D \) | \( \theta_L \) | \( \gamma_0 \) | \( b \) | \( T_{\beta\gamma}/\theta_L \) | Ref |
|--------------|-----------|-----------------|-------------|-------------|------------|------|-----------------|-----|
| Nb           | 9.28      | 3.11            | 275         | 31.3        | 7.8        | 0.0017 | 0.099           | 4,15 |
| V            | 5.4       | 7.5             | 397         | 76.0        | 9.7        | 0.0056 | 0.099           | 5,11 |
| Ta           | 4.5       | 7.2             | 238         | 69.0        | 5.4        | 0.0107 | 0.104           | 17  |
| V\(_3\)Si    | 16.8      | 12.6            | 435         | 32.5        | 52.3       | 0.0030 | 0.388           | 9,10 |
| Nb\(_3\)Sn  | 17.8      | 13.7            | 254.3       | 60.1        | 33         | 0.0267 | 0.228           | 9,10 |
| Re\(_3\)W   | 9         | 8.3             | 300.1       | 88.7        | 16.4       | 0.0556 | 0.094           | 8   |
| MgC\(_N\)\(_3\) | 6.4      | 7.5             | 301.0       | 77.5        | 37.4       | 0.0242 | 0.097           | 12  |
| Nb\(_3\)Se\(_2\) | 7.3      | 5.4             | 235.5       | 49.2        | 19.3       | 0.0081 | 0.110           | 14,15|
| Nb\(_0.8\)Ta\(_0.2\)Se\(_2\) | 5.1    | 4.5             | 219.6       | 48.0        | 14.6       | 0.0048 | 0.094           | 14,15|
| YCo\(_2\)B\(_2\) | 0       | 8.9             | 575.9       | 116.2       | 6.7        | 0.0091 | 0.077           | 12  |
| La\(_3\)Ni\(_2\)B\(_3\)C | 0      | 8.2             | 443.4       | 82.1        | 8.0        | 0.0098 | 0.100           | 12  |
| La\(_3\)(Pd\(_{1-x}\)Pt\(_x\))\(_2\)B\(_2\)C | 10.7   | 5.6             | 260.7       | 54.4        | 7.1        | 0.0077 | 0.103           | 13  |
| Y(Pd\(_{1-x}\)Ni\(_x\))\(_2\)B\(_2\)C | 12.1   | 10              | 461.7       | 106.0       | 14.6       | 0.0258 | 0.094           | 14,15|
| YNi\(_2\)B\(_2\)C \((sc)\) | 15.4   | 12.8            | 481.7       | 136.8       | 20.6       | 0.0410 | 0.094           | 14,16|
| YNi\(_2\)B\(_2\)C \((pc)\) | 14.3   | 13              | 463.1       | 164.0       | 17.9       | 0.0709 | 0.079           | 13  |
| LuNi\(_2\)B\(_2\)C \((pc)\) | 16.1   | 10.7            | 402.3       | 103.9       | 19.2       | 0.0546 | 0.103           | 13  |
| LuNi\(_2\)B\(_2\)C \((sc)\) | 16.8   | 9.2             | 358.1       | 95.4        | 17.6       | 0.0279 | 0.096           | 14,15|
| Li\(_2\)Pt\(_3\)B | 2.56   | 3.1             | 231.7       | 35.4        | 9.2        | 0.0380 | 0.088           | 17  |
| Li\(_2\)Pd\(_1.5\)Pt\(_1.5\)B | 3.9    | 3.0             | 242.4       | 32.2        | 9.8        | 0.0030 | 0.093           | 17  |
| Li\(_2\)Pd\(_3\)B | 6.95   | 2.0             | 226.2       | 28.9        | 9.4        | 0.0030 | 0.069           | 17  |

Third, none of the \( C/T \) versus \( T^2 \) curves exhibits a discontinuity or a hysteresis effect at \( T_{\beta\gamma} \).

Fourth, in spite of the above-mentioned differences, there are, at least, two ingredients common to most of the studied materials: a relatively strong electron-phonon coupling and a lifting of spin-degeneracy at \( E_F \), say, by an applied magnetic field (as in, e.g., conventional superconductors) or by an anisotropic spin orbit coupling ASOC interaction (as in, e.g., non-centrosymmetric \( \text{Li}_2\text{RhB}_1\text{B}_1\text{B}_1 \) superconductors). Curiously, although \( H > H_{c2} \) is necessary for the quench of the superconductivity and for the lift of spin degeneracy, it has no influence on \( T_{\beta\gamma}, \beta \) or \( \gamma \). Third, none of the \( C/T \) versus \( T^2 \) curves exhibits a discontinuity or a hysteresis effect at \( T_{\beta\gamma} \).

FIG. 2: Isofield \( C/T \) versus \( T^2 \) curves of the normal-state of (a) \( \text{Re}_3\text{W} \) at \( H=7 \text{T} \), (b) \( \text{V}_3\text{Si} \) at \( H=18 \text{T} \), (c) \( \text{Nb}_3\text{Sn} \) at \( H=12.5 \text{T} \). Lines are fits to Eq. 8 with parameters as given in Table I. The \( \beta\gamma \)-change of \( \text{Nb}_3\text{Sn} \) and \( \text{V}_3\text{Si} \) is strongly related to the martensitic transformation, which reduces the cubic symmetry of the structure into a low-symmetry one.

Second, \( \beta \) and \( \gamma \) are strongly correlated (the electronic and phonic degrees of freedom are strongly coupled) and that the trend of this correlation is not arbitrary: apparently, for the two linearized sections, if \( \gamma_{\text{low}} < \gamma_{\text{high}} \) then \( \beta_{\text{low}} > \beta_{\text{high}} \) and vice versa.

Third, none of the \( C/T \) versus \( T^2 \) curves exhibits a discontinuity or a hysteresis effect at \( T_{\beta\gamma} \).
II. THEORETICAL BACKGROUND

A. Lattice specific heat

An anomaly, such as a Kohn type, in the phonon DOS can be represented, as done in earlier investigations, by a Dirac delta function. Then

$$D(E) = a.3N_L.E^2 + b.3N_L.δ(E - E_L)$$  \hspace{1cm} (1)

where $N_L = rN_A$ is the number of total atoms, $N_A$ is the Avogadro number, and $E_L = k_BT_L$ is the energy at which the Dirac-type anomaly is situated. $a$ and $b$ represent the fractional weights and are related by the

abrupt change in the lifetimes of phonons with energies less than $2\Delta_s(T)$. The net result is that $C_L + C_e$ would be modified. Second, Moore and Paul as well as Leupold et al. assumed that the $\beta\gamma$-deviation in group V transition metals is related to a Kohn-type anomaly in the phonon (or electron) spectra. They demonstrated that such a deviation can be satisfactorily reproduced if one represents the DOS of such an anomaly by a Dirac delta function and calculates analytically the total specific heat.

Although these two approaches addressed adequately some features of the $\beta\gamma$-deviation, however serious ques-
normalization condition \[ \int_0^{E_D} D(E) \, dE = 3rN_A \] which gives

\[ C_L(T) = \int_0^{E_D} E \cdot D(E) \frac{1}{\exp\left(\frac{E}{k_BT}\right) - 1} \, dE = \sum_{\nu=1}^{E_D} \int_0^{E_D} E \cdot D(E) \cdot \exp\left(-\frac{\nu E}{k_BT}\right) \, dE = \frac{12}{5} \pi^4 N_L k_B \left(\frac{T}{\theta_D}\right)^3 \xi = \beta_0 T^3 \xi \quad (2) \]

\[ \xi = (1 - b) \left\{ 1 - \frac{15}{4\pi^4} \left(\frac{\theta_D}{T}\right)^4 L_0(Z_D) - \frac{15}{\pi^4} \left(\frac{\theta_D}{T}\right)^3 L_1(Z_D) - \frac{45}{\pi^4} \left(\frac{\theta_D}{T}\right)^2 L_2(Z_D) - \frac{90}{\pi^4} \left(\frac{\theta_D}{T}\right) L_3(Z_D) - \frac{90}{\pi^4} L_4(Z_D) \right\} + b \left\{ \frac{5}{4\pi^4} \left(\frac{\theta_D}{T}\right)^3 \left(\frac{\theta_L}{T}\right)^2 L_{-1}(Z_L) \right\} \]

where \( L_i(Z) \) is the polylogarithm function; \( Z_D = \exp(-\theta_D/T); \) \( Z_L = \exp(-\theta_L/T) \). The expression with \( b = 0 \) is the usual Debye contribution (for numerical calculation, this form is much better than the one given in solid-state text books). The Debye \( T^3 \) expression is obtained when \( T \to 0 \) \([\xi \text{ acts as a } T\text{-dependent correction factor, see Fig. } 6(a)]\), while the Dulong-Petit value is reached when \( T > \theta_D \). Eq. [2] justifies the often-used procedure of treating the \( \beta\gamma \)-deviation as a thermal variation of an effective temperature-dependent \( \theta_D' = \theta_D/\xi^{1/3} \): indeed Fig. [5] (b) reproduces the often-observed thermal variation of \( \theta_D' \).

The low-temperature limit of Eq. [2] reproduces the expressions of Moore and Paul \([\text{as well as that of Leupold et al.} \text{]}) Evidently, the introduction of that anomaly leads to both a Debye-type contribution with weight factor \((1 - b)\) and an Einstein-type contribution with frequency \( \omega_L \) and weight factor \( b \). The latter conclusion justifies the often-used practice of adding an Einstein contribution to the specific heat of such intermetallic systems.\(^{20}\)

**B. Electronic specific heat**

Along similar lines, a DOS curve of an electron subsystem with an additional Dirac delta function can be represented as\(^{5}\)

\[ N(E) = c \cdot N_E \cdot E^{1/2} - d \cdot N_E \cdot \delta(E - E_E) \quad (4) \]

where \( N_E = xN_A \) is the total number of conduction electrons and \( E_E = k_B \theta_E \) denotes the position of the Dirac-type anomaly. \( c \) and \( d \) are fractional weights that are related by the normalization condition

\[ \int_0^{\infty} dE \cdot N(E) / \left( \exp\left(\frac{E - E_E}{k_B T}\right) + 1 \right) = xN_A. \quad (5) \]

This gives

\[ c \cong 1 + d / \left[ 1 + \exp((\theta_E - \theta_F)/T) \right] \]

\[ - \frac{2}{\pi^3} \frac{E_F^{3/2}}{\pi T} \left[ 1 + (\pi T / \sqrt{8\theta_F})^2 \right]. \]

As the electron number is independent of temperature, then the thermal rate of the chemical potential is \(-\pi^2 k_B^2 T / 6 E_F\) as \( T \to 0 \).

Using Sommerfeld expansion in the expression of the electronic energy and taking the derivative with respect to temperature, one obtains

\[ C_E(T) \cong \frac{\pi^2 N_E k_B T}{2 \theta_F} \eta = \gamma_0 T \eta \]

\[ \eta = 1 + \frac{d}{1 + \exp((\theta_E - \theta_F)/T)} - \frac{1}{2\pi^2 T^3} \theta_E \theta_F (\theta_E - \theta_F) \sech^2((\theta_E - \theta_F)/2T) \]

\( \eta \) is a \( T \)-dependent correction factor, see Fig. 6(c). Eq. [6] is a sum of three contributions: the first is the usual Sommerfeld expression \((d = 0)\) while the second and third terms are related to the Dirac delta function.
C. Total specific heat (Lattice plus electron)

From Eqs. 2 and 6 the total $C/T$ versus $T^2$ can be presented in the familiar form

$$C/T = \gamma_0 + \beta_0 \xi T^2 = \gamma + \beta T^2$$  (8)

For $T < \theta_D << \theta_F$, the thermal evolution of $\gamma$ and $\beta$ is determined by the thermal expansion, and the thermal conductivity of Nb: $\gamma \sim 1 + \frac{4 \beta}{\pi^2} \xi^2 T^2$. The above arguments, in particular Fig. 6, emphasize that, the slope break (whether smooth or sharp) is not a phase transition, rather it is a consequence of the thermal evolution of $\eta$ and $\xi$. The slope break is observed as a consequence of the thermal evolution of $\eta$ and $\xi$ (consequently that of $\gamma$ and $\beta$), which leads to the experimentally observed two limiting behaviors. Analytically, the low-temperature ($\theta_E, \theta_L > T_{\beta\gamma} > T \rightarrow 0$) limit is

$$\gamma_{\text{low}} = \gamma_0 (1 + d)$$  (9)
$$\beta_{\text{low}} = \beta_0 (1 - b)$$  (10)

while the high-temperature ($\theta_E, \theta_L > T > T_{\beta\gamma}$) limit is

$$\gamma_{\text{high}} \approx \gamma_0 \left[ 1 + d \left( 1 + \frac{4 \beta}{\pi^2} \xi^2 \right) \right]$$

$$\beta_{\text{high}} \approx \beta_0 \left[ 1 - b \left( 1 - \frac{4 \beta}{\pi^2} \xi^2 \right) \right]$$

These two calculated ratios have a small discrepancy. The slope break is not reported for the isomorphous Ta and V suggesting that it might be unique to Nb.

Equations [8][14] predict a positive curvature and $\beta_{\text{high}} > \beta_{\text{low}}$ whenever $b > 0$: this is consistent with the features of all analyzed curves in Figs. 1[8][13] except those of $V_3Si$ and $Nb_3Sn$: these indicate that $b < 0$.

D. The correlation of $\theta_D$ ($\beta$) and $\gamma$

A correlation between $\beta$ and $\gamma$ can be obtained if we consider the low-temperature specific heat of these intermetallics as being due to Sommerfeld-type electrons and longitudinal acoustic Debye-type phonons. As the total dielectric function (electrons plus ions) of these longitudinal modes must be zero, then the sound velocity, as $k \rightarrow 0$, would be $v_s = \sqrt{\frac{\pi}{m}}$ M, where $v_F$, $m$, and $M$ denote, respectively, the Fermi velocity, the electronic mass, and ionic mass. Inserting this $v_s$ into $\theta_D$ (as obtained from the Debye model) and replacing $v_F$ by $\gamma$ (as obtained from Sommerfeld model) one gets the correlation of $\theta_D$ and $\beta_D$:

$$\gamma = \left( \frac{3 \pi^2 \hbar^2 N_A}{2} \right)^{1/3} \frac{1 + d}{M} \frac{1}{V_m} \frac{x^3 T^2}{1 + \frac{1}{\theta_D^2}}$$  (16)

where $V_m$ is the molar volume. For $N_F = N_L$ (or $x = r$) and $\theta_D$ as obtained from Table 1, Eq. 16 gives $\gamma_{\text{cal}}$ having the same order of magnitude as the experimentally determined $\gamma$.

E. The $T_{\beta\gamma}$-event as a resistivity drop

The $T_{\beta\gamma}$-events in $Nb_3$ Fig. 1 and its $Nb_1-xY_x$ ($Y = Ti, W$) alloys were confirmed by the manifestation of a resistivity drop, at the same $T_{\beta\gamma}$. Based on the above-mentioned arguments, this drop can be interpreted along the following two lines: (i) The electronic concentration $n$ in the neighborhood of $T_{\beta\gamma}$ varies as $n_0/\eta^{3/2}$ ($n_0$ is the electronic concentration at $d = 0$). Then an increase in $n$ below $T_{\beta\gamma}$ would lead to such a resistivity drop. (ii) The temperature-dependent resistivity is usually approximated by the Bloch–Grüneisen expression:

$$\rho(T) - \rho_0 = (4\pi)^2 (\lambda \omega_p^2) \omega_D \left( \frac{2T}{\theta_D} \right)^5 \int_0^{\infty} \frac{x^5}{\sinh(x)\sqrt{x}} dx$$  (17)

where $\rho_0$ is the temperature-independent contribution, $\lambda$ is the electron-phonon coupling, and $\omega_p$ is the Drude
plasma frequency \(\omega_p^2 = 4\pi e^2n/m\); factors have their usual meaning). Then an increase in \(\theta_D\) (a drop in \(\beta\)) below \(T_{\beta\gamma}\) would be manifested as a decrease in the resistivity.

**III. ANALYSIS AND DISCUSSION**

All the curves reported in Figs. 1-5 were least-squares analyzed with Eq. 8 assuming \(\theta_L = \theta_L^0\) and \(d = b\); these reasonable simplifications would not influence any of the conclusions drawn from this analysis since, as mentioned above, the dominant thermal variation is due to that of \(\xi\).

As can be observed in Figs. 1-5 there is a satisfactory agreement between theory (solid lines) and experiment (symbol). The fit parameters are shown in Table I:

![FIG. 6: (Color online) The evolution of (a) \(\xi\), (b) the effective \(\theta_D = \theta_D/\xi^{1/3}\), (c) \(\eta\), and (d) \(\xi T^2 = C_L/\beta_0 T^2\) versus \(T^2\) for various values of \(b\) and \(d\). The following values were used (typical for Nb:\(\theta_B = 275\) K, \(\theta_L = 33\) K, and \(\gamma_0 = 7.8\) ml/moleK \(^2\) (see Table I). For \(T < \theta_D\), the thermal variation of \(\xi\) is nearly constant while \(\xi\) varies sharply above \(T_{\beta\gamma}\). Evidently the slope break at \(T_{\beta\gamma}\) is due to the thermal variation of \(\xi T^2 = C_L/\beta_0 T = (C_{tot} - \gamma_0 (1 + d)T) / \beta_0 T\) (\(\beta_0\) and \(\gamma_0\) are \(T\)-independent).](image)

The following values were used:

- For Nb: \(\theta_B = 275\) K, \(\theta_L = 33\) K, and \(\gamma_0 = 7.8\) ml/moleK \(^2\):
  - \(C_{tot} = C_{cal}\): for lower ASOC, \(T_{\beta\gamma} < T_c\); while for higher ASOC, \(T_{\beta\gamma} > T_c\).

Equations 12 and 14 each being related to an independent subsystem, suggest an absence of appreciable correlation between \(\beta_{low}\) and \(\beta_{high}\): in fact the former depends on \(b\) and \(\theta_B^0\) while the latter on \(d\) and \(\theta_B^0\); only for the \(T \rightarrow 0\) limit, \(\beta_{low}\) is \(T\)-independent but there is no correlation since it tends to 1. On the other hand, there is a strong correlation between the experimentally determined \(\gamma_{low}\) and \(\gamma_{high}\) (obtained from the two linearized sections). This apparent inconsistency can be clarified if we note that (i) \(\beta_{low} = \theta_{low}/\xi_{cal}\) and \(\gamma_{low} = \gamma_{cal}\): for the cases where \(T_{\beta\gamma} < T_c\), the zero-field superconducting state masks the \(T_{\beta\gamma}\)-event, however, the latter can be recovered if the superconductivity is quenched with, say, \(H > H_{c2}\). In general, the \(T_{\beta\gamma}\)-event, if dominated by phonon contributions, is
$H$-independent but can be influenced by substitution. Two type of substitution-induced influences can be identified: in the first, an increase in doping leads to a decrease in $T^{\beta\gamma}$ such as in the case of $Y(Ni_{1-x}Pt_x)2B_2C$, $Nb_{1-x}V_x$ and $Nb_0xTa_05Sc_{025}$ while in the second, $T^{\beta\gamma}$ is increased on doping such as in the case of $NbW_{x}\beta Lix\beta Pb_{1.5}$. 

Finally, an anomaly in the electron DOS, such as an opening of a pseudogap, would be coupled by a relatively strong electron-phonon interaction to the phonon excitations in the same way as was described above for the strong electron-phonon interaction to the phonon excitations. 

IV. CONCLUSIONS

Variety of compounds exhibit a $\beta\gamma$-change; some are conventional type-II superconductors, some are unconventional superconductors, while others are normal intermetallics. In cases where both superconductivity and $\beta\gamma$-anomalies are manifested, some compounds exhibit $T^{\beta\gamma}<T_c$ while others $T^{\beta\gamma}>T_c$. The onset of $\beta\gamma$-change can be sharp or smooth depending on material properties however such an event is not related to a thermodynamic phase transition. The strength, character, and trend of this $\beta\gamma$-change vary widely, nonetheless, there is a systematic correlation between $\gamma$ and $\beta$. Furthermore, this $\beta\gamma$-change can be influenced by perturbations such as ASOC interaction and substitution but hardly by a variation in $N(E_F)$ or a magnetic field.

It was shown that this $\beta\gamma$-change is related to anomalies within the phonon or electron dispersion relation. Assuming a Dirac-type anomaly in the phonon and electron DOS curves, an analytical expression for the thermal evolution of the total specific heat of the electron and phonon quasiparticles was derived and was found to compare favorably with the studied experimental curves. The term expressing the lattice contribution can be interpreted either as a sum of a Debye and an Einstein mode or else as a Debye term with an effective $T$-dependent $\theta_D$. The overall features of the resulting $C/T$ versus $T^2$ curve indicate (i) a manifestation of a break in the slope at $T^{\beta\gamma} = 0.2 (1 \pm 1/\sqrt{6}) \theta_L$, (ii) that the slope break is mostly determined by the phonon anomaly. The correlation between $\theta_D$ and $\gamma$ is traced down to the influence of the dielectric properties on the sound velocity of the low-temperature acoustic phonons. Finally, the drop in the resistivity curve at $T^{\beta\gamma}$ is shown to be caused by the same mechanism that gives rise to the slope break in the $C/T$ versus $T^2$ curve.

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