The Casimir effect \[1\] is the tiny force between two neutral macroscopic polarizable bodies, that originates from quantum and thermal fluctuations of the electromagnetic (em) field in the region of space bounded by the surfaces of the two bodies. This is one of the rare manifestations of quantum mechanics at the macroscopic scale, like superconductivity or superfluidity, and it has attracted increasing interest in recent years in view of its potential applications to nanotechnology, condensed matter physics, gravitation and cosmology. Several experiments have now probed the Casimir effect for surfaces made of metals, dielectrics, semiconductors, etc., in vacuum as well as in liquids. Very recently, Casimir experiments with ferromagnetic Ni surfaces have been carried out \[4, 5\]. Numerous geometries of the surfaces have been probed, and it has been demonstrated \[6, 7\] that the magnitude of the Casimir force can be controlled by fabricating surfaces with nanoscale corrugations, a result which opens the way to novel nanotechnological applications of the Casimir effect. For a review see \[2, 3\].

In his original seminal work Casimir computed the (unit-area) force \(F_C\) between two discharged perfectly conducting plane parallel plates of area \(A\) at a distance \(d\) in vacuum, by carefully summing the zero-point energies of the em modes that can propagate in the empty space between the plates, obtaining the famous result:

\[
F_C = \frac{\pi^2 \hbar c}{240} \frac{A}{d^4}.
\]  

(1)

The presence of Planck's constant clearly indicates the quantum origin of the Casimir force. When dealing with real surfaces, it is of course necessary to take into account a number of factors that significantly affect the magnitude of the Casimir force, as compared to the idealized case of two perfectly conducting plates. These include the finite skin depth of em fields in real materials, the temperature of the plates, as well as possible imperfections of the surfaces like roughness, absorbed impurities and especially patch potentials and/or stray charges on the plates surfaces.

An important unresolved problem that has been a subject of intense theoretical debate in the last ten years, regards the influence of the plates temperature on the magnitude of the Casimir force. For finite temperatures, the modes of the em field propagating in the region of space between the plates get populated according to the Bose-Einstein distribution, thus modifying the free-energy of the system. The resulting correction to the Casimir force is quite easy to compute for two perfectly conducting parallel plates. One finds that for separations \(d\) smaller than the thermal length \(\lambda_T = \hbar c/(2\pi k_BT)\) (\(\lambda_T = 1.2\mu m\) at room temperature) the thermal correction scales like \((T/\lambda_T)^4\) and is negligibly small, while for \(d \gg \lambda_T\) it is linear in the temperature \(F_T^{(d,\infty)} = k_B T \zeta(3)/(4\pi d^3)\) and it represents the dominant contribution to the Casimir force. It thus came as a big surprise \[10\] when the thermal force was computed for two plates of large but finite conductivity, as real metals have, because the result turned out to be sharply different from the perfect-conductor case: the thermal correction was found to scale linearly in the temperature also for small separations, and it was orders of magnitude larger than the one for an ideal metal (even though it still represented a small correction to the total force, of the order of a few percent for separations around 400 nm at room temperature). Moreover its limiting value for large separations was only one-half the perfect-conductor limit \(F_T^{(d,\infty)}\).

A perplexing feature of the real metal result was that the thermal force could not be reconciled with the perfect conductor limit by taking the conductivity to infinity. The conundrum worsened when it was realized that the finite conductivity result,
while consistent at large separations with the Bohr-van Leeuwen theorem of classical statistical physics \[11\] [12], was shown to violate Nernst third law of thermodynamics in the idealized limit of two plates with a perfect crystal structure \[13\] [14]. It was soon realized that the large thermal correction, linear in \(T\), and the troubles with Nernst theorem both originated from the absence of a contribution to the Casimir force from the zero-frequency transverse-electrical TE mode (TE \(\omega = 0\)), that follows from the plausible assumption (since then dubbed as Drude prescription) that in the limit of small frequencies the permittivity of a ohmic conductor has a \(1/\omega\) singularity as in the familiar Drude model. A much smaller thermal correction, proportional to \(T^3\) at small distances, satisfying Nernst theorem could however be obtained \[13\] [14] if the contribution to the Casimir force from the TE \(\omega = 0\) was calculated in accordance with the plasma model of infrared optics \(\epsilon_{\text{plasma}} = 1 - \omega_p^2/\omega^2\), with \(\omega_p\) the plasma frequency (this has since been dubbed as plasma prescription). Several small-distance Casimir experiments have been interpreted as being in accordance with the plasma prescription, and to rule out the Drude prescription (for a review of several of these experiments see \[17\]). After ten years of investigations, it is fair to say that the thermal problem is still waiting for a satisfying theoretical solution. For a recent review, see \[15\].

Te view is widely shared that there is an acute need for experiments probing the thermal Casimir force. So far, there is only one experiment by the Yale group \[10\] which claims to have observed the thermal force between a large sphere and a plate both covered with gold, in the wide range of separations form 0.7 to 7.3 \(\mu m\). The results have been interpreted by the authors as being in accordance with the Drude prescription. This experiment has been criticized in the literature \[17\], because the thermal Casimir force was in fact obtained only after subtracting from the total measured force a much larger electrostatic force originating from large patches on the gold surfaces. The subtraction was perfomed by making a fit of the total observed force, based on a two-parameter analytical model of the electrostatic force, and not by a direct and independent measurement of the electrostatic force, as it would have been desirable.

As of now there is no reported observation of the thermal Casimir force at separations less than 0.7 \(\mu m\), which is the range in which the Casimir force has been measured most accurately in several experiments during the last ten years. The reason is that, as note before, the thermal Casimir force represents a small correction at sub micron separations, and therefore tight experimental demands must be fulfilled to observe it: these include a very accurate electrostatic calibration of the apparatus \[13\] [25], a detailed knowledge of the optical properties of the involved surfaces over a wide frequency range \[24\] [26] and a precise determination of the separation between the two surfaces. These factors represent sources of systematic errors that are difficult to quantify, thus complicating the interpretation of the experiments and often stimulating heated controversies. In this Letter we propose a novel setup which is immune of all these complications.

The setup, schematically shown in Fig. 1, consists of two perfectly aligned uniaxially corrugated Ni surfaces \[44\]. The key feature of the setup is the gold film with a flat exposed surface (shown in yellow), that covers one of the two corrugations (the lower one in Fig. 1). The quantity to measure is the phase-dependent modulation of the Casimir (unit-area) force \(F(\phi)\)

\[
\Delta F(\phi) = F(\phi) - \frac{1}{2\pi} \int_0^{2\pi} d\phi F(\phi) ,
\]

where \(\phi\) the relative phase between the corrugations \[49\]. The basic idea behind the measurement scheme can be explained easily. One notes that the main contribution to the \(T = 0\) Casimir force for two plates at distance \(d\) is from photons having characteristic frequency \(\omega_c = c/(2d) = 3.7 \times 10^{14}\) rad/sec for \(d = 400\) nm. On the contrary, the thermal Casimir force is a low-frequency phenomenon, and it has been estimated \[27\] [28] that the main contribution is from TE thermal photons with the much lower characteristic frequencies \(\omega_T = \gamma (\omega_c/\omega_p)^2 = 10^{10}\) rad/s for gold (\(\gamma\) is the relaxation frequency). Recalling that the penetration depth of a photon in a metal is \(\delta = c/\sqrt{2\pi \mu \sigma \omega}\) (for gold \(\sigma = 3 \times 10^{17} \text{ s}^{-1}, \mu = 1\)), we see that the two sets of photons have the widely different penetration depths \(\delta_c \approx 10\) nm and \(\delta_T \approx 1\mu m\) respectively. If the minimum thickness \(w\) of the gold layer in Fig. 1 is chosen such that

\[
\delta_c \ll w \ll \delta_T .
\]

it is clear that photons with frequencies of order \(\omega_c\) cannot reach the lower Ni corrugation. One concludes from this that the dominant \(T = 0\) contribution to the Casimir force shows no phase modulation, because as far as it is concerned, everything goes as if the upper corrugation were placed in front of a flat gold layer \[46\]. On the contrary, the low frequency TE photons easily make it through the gold layer, and are strongly scattered by the Ni plate, because Ni is a magnetic material with a large \(\mu\) that couples strongly to low-frequency photons with TE polarization. The conclusion is that the thermal part of the Casimir force does depend on the phase \(\phi\), and we estimate below that the resulting phase-modulation of the force is measurable with current apparatuses. This scenario is valid within the so-called Drude prescription. If the plasma prescription is considered, this scenario changes dramatically, and we predict an unmeasurably small modulation. This is so because when gold is modeled as a collisionless plasma, it heavily screens low frequency TE photons as well, and therefore neither set of photons reaches the lower Ni corrugation. We thus see
that with our setup a clean discrimination between the two models should be possible.

The present setup presents a number of distinct advantages. The first, and perhaps the most important one, is that the electrostatic calibration is unnecessary, because patches and/or stray charges that may exist on the surfaces of the flat gold layer and of the upper Ni corrugation do not contribute to the phase modulation, and therefore they are automatically subtracted out from the signal \( \Delta F(\phi) \). The second advantage is that the optical properties of Ni or Au are irrelevant because, as we show below, all we need to compute the modulation of the Casimir force is the \textit{static} magnetic permeability \( \mu \). The latter property of Ni or Au is irrelevant because, as we shall assume in what follows that the amplitudes \( A_1 \) and \( A_2 \) of the corrugations and the mean distance \( a \) (see Fig. 1) are all much smaller than the period \( \Lambda \). We let \( H_1(x) = a + A_1 \cos[2\pi(x-L)/\Lambda] \) \((0 < A_1 < a)\) and \( H_2(x) = -w - A_2[1 - \cos(2\pi x/\Lambda)] \) \((0 < A_2)\) the profiles of the two uniaxial corrugations, and we assume that the region \( H_2(x) \leq z \leq 0 \) is filled with gold. For \( A_1, A_2, a \ll \Lambda \), in the neighborhood of the point \( x \) our system looks locally like a planar cavity consisting of Ni slab at distance \( d(x) = H_1(x) \) from a planar two-layer slab consisting of a layer of gold of thickness \( s = -H_2(x) \) on top of a Ni substrate. If we let \( F_{pp}(T, d; s) \) the unit-area Casimir force for such a planar system, the PFA posits that the unit-area Casimir force between two gently corrugated plates is:

\[
F_{\text{PFA}} = \frac{1}{\Lambda} \int_0^\Lambda dx F_{pp}(T, H_1(x); -H_2(x)) .
\]

According to \([32, 33]\), the Casimir force \( F_{pp} \) (per unit area) between two magnetodielectric possibly layered plane-parallel slabs \( S_j, j = 1, 2 \) at distance \( d \) in vacuum can be represented by the formula (positive forces indicate attraction):

\[
F_{pp}(T, d) = \frac{k_B T}{\pi} \sum_{l=0}^\infty \left( 1 - 2 \delta_0 \right) \int_0^\infty dk_\perp k_\perp q_l \times 
\sum_{\alpha = \text{TE}, \text{TM}} \left[ \frac{e^{2\delta_0}}{R_\alpha^{(1)}(i\xi_l, k_\perp) R_\alpha^{(2)}(i\xi_l, k_\perp)} - 1 \right]^{-1} ,
\]

where \( k_B \) is the Boltzmann constant, \( \xi_l = 2\pi l k_B T/\hbar \) are the (imaginary) Matsubara frequencies, \( k_\perp \) is the modulus of the in-plane wave-vector, \( q_l = \sqrt{k_\perp^2 + k_{\perp l}^2} \), and \( R_\alpha^{(1)}(i\xi_l, k_\perp) \) is the reflectivity coefficient of slab \( j \) for polarization \( \alpha \). The Casimir force \( F_{pp}(T, d; s) \) for a planar Ni-Au-Ni system, can be obtained from Eq. \([3]\) by substituting \( R_\alpha^{(1)} \) by the Fresnel reflection coefficient \( r_\alpha^{(0\text{Ni})} \) of a Ni slab (given in Eqs.\([7]\) and \([8]\) below, with \( a = 0, b = \text{Ni} \)), and \( R_\alpha^{(2)} \) by the reflection coefficient \( r_\alpha^{(0\text{AuNi})} \) of a two-layer planar slab consisting of a layer of thickness \( s \) of gold on a Ni substrate. The latter reflection coefficient has the expression:

\[
r_\alpha^{(0\text{AuNi})}(i\xi_l, k_\perp; s) = \frac{r_\alpha^{(0\text{Au})} + e^{-2s k_{l}^{(\text{Au})}} r_\alpha^{(\text{AuNi})} r_\alpha^{(\text{Au})}}{1 + e^{-2s k_{l}^{(\text{Au})}} r_\alpha^{(\text{Au})} r_\alpha^{(\text{AuNi})}} .
\]
Here $r_{ab}^{(a)}$ are the Fresnel reflection coefficients for a planar interface separating medium a from medium b:

$$r_{TE}^{(ab)} = \frac{\mu_b(i\xi i) k^{(a)}_l - \mu_a(i\xi i) k^{(b)}_l}{\mu_a(i\xi i) k^{(a)}_l + \mu_a(i\xi i) k^{(b)}_l}, \quad (7)$$

$$r_{TM}^{(ab)} = \frac{\epsilon_b(i\xi i) k^{(a)}_l - \epsilon_a(i\xi i) k^{(b)}_l}{\epsilon_a(i\xi i) k^{(a)}_l + \epsilon_a(i\xi i) k^{(b)}_l}, \quad (8)$$

where $k^{(a)}_l = \sqrt{\epsilon_a(i\xi i)\mu_a(i\xi i)\xi^2_k + k^2_{\perp}}$, $\epsilon_a$ and $\mu_a$ denote the electric and magnetic permittivities of medium a, and we define $\epsilon_0 = \mu_0 = 1$. In our computations, we modeled the electric permittivities of gold and Ni by a simple Drude model

$$\epsilon_a(\omega) = 1 - \frac{\Omega_a^2}{\omega(\omega + i\gamma_a)}, \quad (9)$$

where $\omega_a$ is the plasma frequency and $\gamma_a$ is the relaxation frequency. The magnetic permeability of Ni was modeled by the Debye formula

$$\mu(\omega) = 1 + \frac{\mu(0) - 1}{1 - i\omega/\omega_m}, \quad (10)$$

where $\mu(0)$ is the static magnetic permeability and $\omega_m$ is a characteristic frequency. Since for typical magnetic materials $\omega_m$ is much smaller than the frequency of the first Matsubara mode $\xi_1 \sim 10^{14}$ Hz at room temperature, we can safely set $\mu = 1$ in all $l > 0$ terms of Eq. (5) and just substitute the static permeability $\mu(0) = 110$ in the $l = 0$ TE mode. For the plasma frequencies and the relaxation frequencies of Au and Ni, we used the values:\[4, 5\]: $\omega_{Au} = 9$ eV/h, $\gamma_{Au} = 0.035$ eV/h, $\omega_{Ni} = 4.89$ eV/h and $\gamma_{Ni} = 0.0436$ eV/h.

It is useful to separate the contribution $\Delta F_{0}^{TE}$ of the TE $\omega = 0$ mode from the combined contributions $\Delta F = \Delta F_{0}^{TE} + \sum_{a,T>0} \Delta F_a^{(a)}$ of the TM $\omega = 0$ plus the non-vanishing Matsubara modes:

$$\Delta F = \Delta F_{0}^{TE} + \Delta F. \quad (11)$$

In Fig. 2 we show (blue line) the modulation $\Delta F$ of the unit-area Casimir force (in mPa) versus the relative phase $\phi = 2\pi\delta/\Lambda$ of the Ni corrugations in the range $0 \leq \phi \leq \pi$, for $T = 300$ K, $a = 140$ nm, $A_1 = 85$ nm, $A_2 = 100$ nm and $w = 70$ nm. The dashed line represents the contribution $\Delta F_{0}^{TE}$ of the TE zero-frequency Matsubara mode. Both the blue and the dashed line were computed using the Drude prescription. The red line represents the modulation that obtains if the plasma prescription is used. The Figure clearly displays the main features of our setup. As it was anticipated earlier, we see that within the Drude prescription there is a periodic modulation of the Casimir force, which is entirely due to the thermal TE $\omega = 0$ contribution. It is useful to write down this contribution explicitly. According to Eqs. (4) and (5):

$$F_{TE|PFA}^{0} = \frac{k_B T}{2\pi} \int_{-\Lambda}^{\Lambda} dx \frac{1}{(H_1(x) - H_2(x))^3} \int_0^\infty dy^2 \left[ \epsilon_2^{-1} \left( \frac{\mu(0) + 1}{\mu(0) - 1} \right) - 1 \right]^{-1}. \quad (12)$$

This formula shows that $F_{TE|PFA}^{0}$ represents a purely thermal effect proportional to $T$, and that it is fully determined by the static magnetic permeability of Ni. A modulation of the size shown in Fig. 2 is expected to be detectable.\[4, 5, 42, 43\].

From the red curve in Fig. 2, we see that there is essentially no modulation of the force within the plasma prescription. This is obvious, if one realizes that within the plasma prescription the TE $\omega = 0$ mode gets also screened out by the gold layer, since with the plasma prescription at $\omega = 0$ everything goes as if gold were a strongly diamagnetic substance, that expels from its interior static magnetic fields. By comparing the blue and red lines in Fig. 2, we thus see that our setup should allow for a clean discrimination between the Drude and the plasma prescriptions for the thermal Casimir effect.

In conclusion, we have shown that a Casimir setup consisting of a pair of uniaxial periodically corrugated aligned Ni plates one of which is covered by a thin layer of gold with a flat exposed surface, should allow for a clean observation of the thermal Casimir effect in the sub micron range, with currently available Casimir apparatuses. We proved that the phase modulation of the Casimir force is a purely thermal effect, that depends only on the static magnetic permeability of the Ni plates.
Distinctive advantages of the proposed scheme, over conventional Casimir setups, is that no electrostatic calibration of the apparatus is needed, nor any detailed knowledge of the optical properties of the involved materials. An interesting alternative to the scheme considered here consists in the replacement of Ni by a superconducting material \cite{11,12}. We expect though that observation of the force modulation should be harder in this case, because the amplitude of the modulation would be suppressed by a factor of order $T_c/T_{\text{room}}$ with respect to the room temperature Ni setup.

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\bibitem{44} In a real setup the Ni surfaces would consist of Ni films with a thickness of a few hundred nanometers, as in \cite{11,12}, deposited on some substrate. As far as the Casimir force is concerned Ni films of this thickness are undistinguishable from an infinitely thick substrate.
\bibitem{45} The magnetic force originating from the interaction between magnetic domains on the Ni plates, which is also expected to exhibit a periodic modulation, has been shown to be completely negligible in comparison to the Casimir force, for thin films of Ni like those considered in this work, in Refs. \cite{11,12}.
\bibitem{46} It is sufficient to consider the range $0 \leq \varphi \leq \pi$ because the modulation is symmetric under a change of sign of $\varphi$.\end{thebibliography}