ON THE SPECTRAL HARDENING AT $\gtrsim 300$ keV IN SOLAR FLARES

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ABSTRACT

It has long been noted that the spectra of observed continuum emissions in many solar flares are consistent with double power laws with a hardening at energies $\gtrsim 300$ keV. It is now widely believed that at least in electron-dominated events, the hardening in the photon spectrum reflects an intrinsic hardening in the source electron spectrum. In this paper, we point out that a power-law spectrum of electrons with a hardening at high energies can be explained by the diffusive shock acceleration of electrons at a termination shock with a finite width. Our suggestion is based on an early analytical work by Drury et al., where the steady-state transport equation at a shock with a tanh profile was solved for a $p$-independent diffusion coefficient. Numerical simulations with a $p$-dependent diffusion coefficient show hardenings in the accelerated electron spectrum that are comparable with observations.

One necessary condition for our proposed scenario to work is that high-energy electrons resonate with the inertial range of the MHD turbulence and low-energy electrons resonate with the dissipation range of the MHD turbulence at the acceleration site, and the spectrum of the dissipation range $\sim k^{-2.7}$. A $k^{-2.7}$ dissipation range spectrum is consistent with recent solar wind observations.

Key words: acceleration of particles – Sun: flares – Sun: particle emission – Sun: X-rays, gamma rays

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1. INTRODUCTION

Our Sun is an efficient particle accelerator. Ions with energy up to $\sim\text{GeV}$ nucleon$^{-1}$ are detected in situ during large solar energetic particle (SEP) events where both large flares and fast coronal mass ejections (CMEs) often occur together. In flares, electron bremsstrahlung is believed to be the main source of the continuum radiation (e.g., Ramaty et al. 1975; Vestrand 1988).

Continuum emissions provide invaluable information that constrains the underlying acceleration mechanism. These constraints include the energy budget, the total number of electrons, the acceleration timescales, etc. See the reviews by Miller et al. (1997) and Zharkova et al. (2011) for a detailed discussion on various acceleration mechanisms and the implications these constraints have on them.

One observational constraint that received less attention, even though it has been noted for a long time, is the hardening of the continuum spectrum at high energies (often $\gtrsim 300$ keV). In 1975, Suri et al. (1975) examined the X-ray and gamma-ray flux in the 1972 August 4 event and concluded that the X-ray and gamma-ray flux was produced by a single population of electrons with a break in its spectrum, instead of two separate populations acting independently. Later, Yoshimori et al. (1985), using the Hinotori spacecraft, examined the hard X-ray (HXR) spectrum in a broad energy range (20 keV–7 MeV) for four flares that showed significant hardening at energies above $\sim 400$ keV. They confirmed the earlier suggestion of Suri et al. (1975) that the hardening in the continuum reflects an underlying hardening in the source electron spectrum. Spectral hardening also occurs in events where there are clear signatures of gamma-ray lines. The most recent report of such an event is from the Fermi observation (see Ackermann et al. 2012) where spectral hardening was found at above several hundred keV.

Note that hardening in the source electron spectrum is not the only cause for a hardening in the photon spectrum. Various processes, such as electron–electron bremsstrahlung, proton bremsstrahlung, positronium annihilation continuum, and inverse Compton emissions (see Vestrand 1988), may all lead to some hardening of the continuum emission from a straight power-law (without hardening) source electron spectrum. However, for parameters appropriate to a solar flare site, the contributions of these processes are relatively small and the resulting hardening of the spectral index is perhaps $\sim 0.5$ (Kontar et al. 2007). Therefore, these processes cannot explain events where the change of spectral indices is as high as 2. Park et al. (1997) studied photon spectral hardening around 1 MeV for four flares. In their scenario, the emission is a simple sum of the thin target emission from the trapped electrons at the acceleration site near the loop top and the thick target emission from the escaping electrons precipitating on the solar surface. Using the assumption that electrons with lower energies have shorter escape times, and noting that the energy dependence of the bremsstrahlung cross section differs in the nonrelativistic and the relativistic regimes, Park et al. (1997) were able to account for the observed spectral hardening. Note that in the scenario of Park et al. (1997), the energy dependence of the escape time decides the hardening, and the accelerated electron spectrum in the accelerated region does not (need to) have spectral hardening. In situ observations by Moses et al. (1989), however, showed that electron spectral hardening is rather common in short-duration events. Assuming that these in situ electrons are the source electrons escaping from the acceleration site through interchange reconnection, then Moses et al.’s (1989) results also suggest that the accelerated electron population has a hardening at high energies.

High-energy electrons also lead to microwave emissions through gyrosynchrotron radiation (see, e.g., the recent review by White et al. 2011 for a detailed discussion of the relationship between solar radio and HXR emissions). In an early study, using BATSE (HXRs) and the Owens Valley Radio Observatory (microwaves), Silva et al. (2000) examined 27 solar flares with multiple peaks (a total of 57) which were observed at both HXR
and microwave wavelengths. Fitting the HXR spectra by a single power law and the microwave spectra as gyrosynchrotron emissions, Silva et al. (2000) found that in 75% of the bursts, the inferred spectral indices of the electron energy distribution of the microwave-emitting electrons were harder (by 0.5–2.0) than those of the lower energy HXR-emitting electrons. Silva et al. (2000) concluded that there exists a breakup in the energy spectra of the source electrons at around ∼300 keV, in agreement with previous observations of HXR-only spectra of giant flares.

Note, however, that in most events, the HXRs emanate from the footpoints of flare loops and microwaves emanate from the tops of flare loops. Furthermore, there is also a delay between the peak of the microwave emission and the HXR emission. So there are transport effects between electrons generating microwave emissions and those generating HXRs (White et al. 2011). Both the harder HXR spectrum at the footpoints and the delays of the microwave emission could be caused by the magnetic trapping of higher energy electrons near the loop top and the precipitation of lower energy electrons to the footpoints, as first suggested by Melrose & Brown (1976). In a recent study by Kawate et al. (2012), HXR and microwave emissions from 10 flares were analyzed. Although the emissions were at different locations and the spectral indices for the microwave emissions are harder than those of the HXRs, by assuming a spectrum for the accelerated electrons that is consistent with the HXR emissions (but extend to higher energies), Kawate et al. (2012) were able to produce microwave spectra comparable to the observations. The authors concluded that it is a single electron population that is responsible for the HXR and microwave emissions and the hardening of the microwave emission is due to a more efficient trapping of electrons with higher energies. In another study, to minimize the effect of the trapping of high-energy electrons on the resulting spectra of loop-top microwave emissions, Asai et al. (2013) examined both the HXR and microwave spectra prior to the peak emission in 12 flares. They still find a significant hardening of the source electrons for the microwave emissions. These authors suggest that there is an intrinsic spectral hardening for the source electron spectrum around several hundreds of keV and the microwave gyrosynchrotron emission is due to electrons at higher energies (in the harder part of the spectrum).

In this work, we do not consider microwave emissions and focus only on HXRs. Vestrand et al. (1999) identified 258 flare events using Solar Maximum Mission observations. Among these, many are electron-dominated events with no clear signature of gamma-ray emissions (Rieger & Marschhauser 1990; Marschhauser et al. 1994). In these events, the contribution of nuclear gamma-ray lines is minimal and the continuum is mainly due to the bremsstrahlung of the energetic electrons. Spectral hardening can clearly be seen in many of these electron-dominated events. A careful examination of these events based on the mechanism proposed here will be reported elsewhere (Kong et al. 2013).

A hardening in the source electron spectra is hard to explain for any acceleration mechanism. In this paper, to explain the observed spectral hardening, we propose a scenario which is based on diffusive shock acceleration (DSA).

Electron acceleration at a termination shock (TS) in solar flares is not a new idea. Tsuneta & Naito (1998) were the first to consider electron acceleration via DSA at a flare TS. Tsuneta & Naito (1998) pointed out that slow shocks bounding the reconnection X-point can heat up the plasma to perhaps 10–20 MK, providing an abundant seed population which is accelerated to 1 MeV in 0.3–0.6 s at the TS. Noting that the standing TS is quasi-perpendicular in nature, Mann et al. (2009) considered shock drift acceleration (SDA) at a standing TS. In the work of Mann et al. (2009), most of the energy gain of electrons is through a single reflection at the shock front. Therefore, to accelerate electrons to high energies, a stringent requirement of $\theta_{BN}$ (e.g., $\theta_{BN} > 88^\circ$) is needed. However, while the TS on a large scale is quasi-perpendicular, small-scale structures (such as ripples) exist on the shock front. Indeed, the plasma in the reconnection region is unlikely to be homogeneous, so the resulting TS is unlikely to be planar. Recently, Guo & Giacalone (2012) have examined electron acceleration at a flare TS using a hybrid code. In the simulation of Guo & Giacalone (2012), many small-scale ripples were identified along the shock surface. The existence of these ripples suggests that assuming a shock with a $\theta_{BN} > 88^\circ$ across the shock surface may be unrealistic. Furthermore, the existence of these small-scale structures implies that one single field line can intersect the shock surface multiple times. Consequently, the acceleration process will be diffusive in nature. In this work, we follow Tsuneta & Naito (1998) and assume that the electron acceleration at a flare shock can be described by the DSA mechanism. Note that the existence of a TS in a flare site is not trivial. Observational evidence of a flare TS has been reported by Warmuth et al. (2009), who used the dynamic radio spectrum from the Tremsdorf radiospectrograph to show that there were type II radio bursts from a standing TS at ~300 MHz during the impulsive phase of the X1.7 flare of 2001 March 29. Besides the existence of a TS, the area of the TS shock also has to be large enough ($\sim$10$^{20}$ cm$^2$ in large flares) to account for the observed flux of HXRs generated by high-energy electrons. By assuming a 50% contour of the Nancy Radio Heliograph source at 327 MHz (see Figure 2 of Warmuth et al. 2009) as a proxy for the shock, Warmuth et al. (2009) estimated a shock area of $\sim$1.3 $\times$ 10$^{20}$ cm$^2$ in the 2001 March 29 X1.7 flare. We note that these areas are much larger than the areas of HXR sources and are more comparable to active region sizes. Whether or not the size of the TS can be this large remains to be determined. Using the same technique, Warmuth et al. (2009) nevertheless obtained similar areas for other events where TS was observed. Of course, for smaller flares (like M flares), the area of the active region is smaller and we expect that the area of the shock is also smaller.

Besides shock acceleration, models based on stochastic (aka second-order Fermi) acceleration exist. For example, Miller et al. (1996, 1997) assumed the presence of some large-scale turbulence at the flare site and considered the coupled system of the wave cascading and particle acceleration. Miller et al. (1996, 1997) showed that various modes of waves (Alfvénic- and fast-mode waves), as they cascade to small scales, can efficiently accelerate both ions and electrons. Similar processes have also been studied by, for example, Petrovina et al. (1994) and Park et al. (1997). Unlike Miller et al. (1996, 1997), Petrovina et al. (1994) and Park et al. (1997) did not address the cascading of the turbulence and assumed that the wave spectra is given.

In this work, we do not consider stochastic acceleration. However, as in Petrovina et al. (1994), Miller et al. (1996, 1997), and Park et al. (1997), we assume that the diffusion coefficient $\kappa$ is determined by the underlying turbulence power at a flare site.
2. DIFFUSIVE SHOCK ACCELERATION OF ELECTRONS AT A FINITE-WIDTH TERMINATION SHOCK

At a piecewise shock, the standard steady-state DSA predicts a power-law spectrum ∼p−α for energetic particles. The power-law spectral index α is given by 3s/(s − 1), where s = u1/u2 is the compression ratio, and u1 and u2 are the up- and downstream flow speeds in the shock frame. In the case of a shock having a finite width ∼Ldiff, Drury et al. (1982) showed that the spectral index depends on the shock width. Assuming that the background fluid speed is given by a tanh profile,

\[ u(x) = \frac{u_1 + u_2}{2} - \frac{u_1 - u_2}{2} \tanh(x/L_{\text{diff}}), \]

then the spectral index α becomes (Drury et al. 1982)

\[ \alpha = \frac{3s}{s - 1} \left(1 + \frac{1}{\beta_s} \frac{1}{s - 1}\right), \]

where βs is a dimensionless parameter and is related to the diffusion coefficient κ through

\[ \kappa = \beta_s(u - u_1)(u - u_2) \frac{dx}{du} = \beta_s \frac{u_1 - u_2}{2} L_{\text{diff}}. \]

Although Drury et al. (1982) only considered the case of a p-independent κ where analytical solutions can be obtained, one can see from the above that for a κ increasing with p the spectrum will harden at high energies. Because of the factor of 1/βs in Equation (2), the spectral index quickly approaches the limit of 3 when 1/βs is small, however, the second term in the bracket of Equation (2) dominates and the spectrum can be very soft.

Clearly, the momentum dependence of the diffusion coefficient κ decides the shape of the spectrum. At a flare site, the κ of energetic electrons is decided by the turbulence level. At large scales, the turbulence is Alfvénic and particle-wave gyroresonance can accelerate ions to high energies via the stochastic acceleration process (e.g., Miller et al. 1997). For electrons, however, except at very high energies, they do not resonate with Alfvén waves and therefore they interact with other waves, for example, fast mode and/or whistler waves (Miller et al. 1996).

Note that Drury et al. (1982) did not consider the effect of the energetic electrons on the shock. In a more refined and self-consistent analysis, the pressure of the energetic electrons needs to be taken into account and it will affect the shock width. This is similar to the modified shock structure caused by energetic cosmic rays as first examined by Axford et al. (1982). Such a discussion, however, exceeds the scope of this work and we do not consider the back reaction of energetic electrons on the shock structure.

We assume that the turbulence at a flare, as in the solar wind, is described by an inertial range joining to a dissipation range, and the power density I(k) is given by

\[ I(k) = I(k_0) \left(\frac{k}{k_0}\right)^{-\epsilon_i} H(k_b - k) + \left(\frac{k}{k_b}\right)^{-\epsilon_d} H(k - k_b), \]

where \( \epsilon_i \) and \( \epsilon_d \) are the spectral indices in the inertial range and dissipation range, respectively. We assume \( \epsilon_d = 2.7 \) (see below) and consider three cases for \( \epsilon_i: \) 5/3, 1.5, and 1.0. The case of \( \epsilon_i = 5/3 \) corresponds to a Kolmogorov cascading, the case of \( \epsilon_i = 1.5 \) corresponds to a Iroshnikov–Kraichnan (IK) cascading, and the case of \( \epsilon_i = 1.0 \) corresponds to a Bohm-like diffusion (see below). At very small k, the energy containing range sets in and I(k) is bent over. The normalization of I(k) is given by

\[ \int_{-\infty}^{+\infty} I(k) = \langle \delta B^2 \rangle. \]

For a wide range of electron energy, the resonating wavenumber k is in the dissipation range. In the solar wind, one finds a spectrum of ∼k−2.7 to ∼k−3.0 in the dissipation range (Leamon et al. 1998, 1999; Chen et al. 2010; Howes et al. 2011; Alexandrova et al. 2012). Unlike the inertial range, the nature of the turbulence in the dissipation range is still under debate. Two possible scenarios include Landau damping of kinetic Alfvén waves (KAWs) (e.g., Leamon et al. 1999, 2000; Boldyrev & Perez 2012) or whistler waves (e.g., Stawicki et al. 2001; Krishan & Mahajan 2004; Galtier 2006). For KAWs, \( k_{\perp} \gg k_{||} \); electron–wave interaction occurs through the Landau resonance and KAWs can effectively heat electrons. Whistler waves have \( \omega_p < \omega < \Omega \) and electrons can interact with whistler waves through the cyclotron resonance. The resonance condition is

\[ \omega - k_{||} v_{||} = n \Omega, \]

where \( \Omega = eB/(\gamma m_e) \) is the electron cyclotron frequency and \( \gamma \) is the Lorentz factor. For low-frequency waves \( \omega \ll \Omega \), the resonance condition (on taking \( n = 1 \)) yields

\[ \mu v = \Omega/|k_{||}|, \]

where \( \mu \) is the pitch angle of the electron. Note that from Equation (6) one can see that when the energy of an electron is high enough, it can also resonate with Alfvén waves. In this work, we assume that the dissipation range turbulence is whistler-like and thus electrons can resonate with the wave through the cyclotron resonance. As done in Gordon et al. (1999), Rice et al. (2003), and Li et al. (2005), we further simplify the resonance condition by replacing \( k = \Omega/\mu v \) with \( k = \Omega/\mu v \). This corresponds to an extreme resonance broadening. The pitch angle diffusion coefficient, \( D_{\mu\mu} \), from the Quasi-linear Theory (QLT; Jokipii 1966) is

\[ D_{\mu\mu} = \frac{1 - \mu^2 \Omega^2}{|\mu| v B_0^2} I(k = \Omega/\mu v). \]

The diffusion coefficient κ is related to \( D_{\mu\mu} \) through

\[ \kappa = \frac{v^2}{8} \int_{-1}^{+1} \frac{(1 - \mu^2)^2}{D_{\mu\mu}} = \frac{v^3 B_0^2}{16 \Omega^2} I(k = \Omega/\mu v). \]

We make no attempts to estimate the turbulence level at the reconnection site in this work. Instead, we are more interested in the energy dependence of κ. From Equation (9), we have

\[ \kappa = \kappa_0 (p/p_0)^{3-\epsilon_d}, \]

where subscripts i or d denote whether electrons resonate with the inertial or the dissipation range of the turbulence. For electrons resonating with the dissipation range that have a \( \epsilon_d \sim 2.7 \), Equation (10) suggests that κ has a very shallow dependence on electron momentum (energy). In comparison, for
electrons resonating with the inertial range, $\kappa$ increases quickly with particle momentum (energy). In Tsuneta & Naito (1998), the Bohm diffusion approximation was used, in which case $\kappa \sim \nu R_t$, where $R_t$ is the electron’s gyroradius. This corresponds to an $\epsilon_i = 1$.

The fact that $\kappa$ has a very shallow dependence on the electron’s momentum in the dissipation range and a strong dependence in the inertial range is the key to understanding the hardening of the electron spectrum. In Figure 1, we plot $\beta_s$ as defined in Equation (3), where from Equation (10) we have

$$ \beta_s = \beta_0 (p/p_0)^{3-\epsilon_i} \frac{\gamma}{\alpha} . \quad (11) $$

We set $\beta_0 = 0.2$. This value yields an electron spectral index at low energy of $\sim p^{-10}$, which is comparable to flare observations.

Note from Equation (3) that $\beta_s$ also depends on the width of the shock. Simulations by Scholer & Burgess (2006) suggested that the shock width is of the order of the ion inertial scale length $\sim (c/\omega_{pi})$. On the other hand, observations of Earth’s bow shock (at quasi-perpendicular configurations) showed that its ramp width is somewhat smaller than $\sim (c/\omega_{pe})$ (Scudder et al. 1986; Balikhin et al. 1995; Newbury et al. 1998). In particular, Newbury et al. (1998) found considerable fine structures of the order of $\sim (c/\omega_{pe})$. Zank et al. (2001) suggested that these fine structures will help to circumvent the injection problem for Anomalous cosmic rays. In a very recent study, using Clusters observation, Schwartz et al. (2011) showed that at Earth’s bow shock half of the temperature occurred in about $\sim 7c/\omega_{pe}$ or $\sim (1/7)c/\omega_{pi}$. The total width of the shock in Schwartz et al. (2011), which is close to $L_{\text{diff}}$ in our work, however, is another factor of $\sim 6$ (see their Figure 3). Therefore, in this work, we assume that the shock width is given by the ion inertial length scale $L_{\text{diff}} \sim c/\omega_{pi}$.

The break point $p_b$ in Figure 1 is $p_b \sim \gamma m_p \Omega/k_b$, where $k_b$ is the wavenumber separating the inertial range and the dissipation range. The scale at which the inertial range transits into the dissipation range is still a much debated issue. It has been argued that it could be the thermal proton Larmor radius $\sim (\sqrt{k_BT/m_p}/\Omega)$ (Leamon et al. 1998, 1999) or the ion inertial length $\sim (V_A/\Omega)$ with $\Omega_p$ the proton cyclotron frequency (Leamon et al. 2000; Smith et al. 2001). If we consider a typical flare site (Miller et al. 1996; Mann et al. 2009) with a temperature of $T \sim 5 \text{ MK}$, a magnetic field of $B \sim 200 \text{ G}$, and a density of $n_e \sim 10^{6} \text{ cm}^{-3}$, we find an Alfvén speed $V_A \sim 1.38 \times 10^7 \text{ km s}^{-1}$, a thermal proton speed $v_{th} \sim 200 \text{ km s}^{-1}$, and a proton gyrofrequency $\Omega_p = 1.91 \times 10^6 \text{ Hz}$. Consequently, the thermal ion Larmor radius is $\sim 0.10 \text{ m}$ and the ion inertial length is $\sim 7.2 \text{ m}$. If $k_b$ is the exponential of the thermal ion Larmor radius, then $p_b \sim 0.64 \text{ MeV/c}$ and the corresponding kinetic energy is $0.31 \text{ MeV}$. This is in good agreement with the observed continuum emission break locations $\sim 300 \text{ keV}$. On the other hand, if $k_b$ is the reciprocal of the ion inertial length, then $p_b \sim 39 \text{ MeV/c}$, which is much too high for the proposed scenario. Therefore, our proposed scenario favors the suggestion of Leamon et al. (1998, 1999) that the dissipation range sets in at the thermal ion Larmor radius scale. When compared to the width of the shock, which is the ion inertial length scale $7.2 \text{ m}$, the gyroradius of an electron $R_t = \gamma \nu/\Omega_p$ is 0.1 (0.4) m for a kinetic energy of $300 \text{ keV}$ ($2 \text{ MeV}$).

Using a momentum-dependent $\beta_s$ as in Equation (11), we numerically solve the steady-state transport equation. We set $p_0$ to be $32 \text{ keV/c}$, which corresponds to an injection energy of 1 keV. We use the same shock profile as Drury et al. (1982), given by Equation (1), and assume a compression ratio of 3.5 (thus a strong shock). Note that the outflow plasma speed at a reconnection site is $\sim V_A$. Therefore, for a shock with a compression ratio of 3.5, $u_1-u_2$ in Equation (3) is $\sim 10^4 \text{ km s}^{-1}$. We use $\beta_0 = 0.2$. Tsuneta & Naito (1998) have used the Bohm approximation for $\kappa$. If we take the Bohm approximation $\kappa = \sqrt{3} \nu R_t$ and the above values for a typical flare site, then a $300 \text{ keV}$ electron will have $\beta_s = 0.2$ and a $2 \text{ MeV}$ electron will have $\beta_s = 1.0$, suggesting that our choice of $\beta_0 = 0.2$ is reasonable.

Figure 2 plots the steady-state electron spectrum for three cases that have different inertial range turbulence spectra: (1) Kolmogorov-like, (2) IK-like, and (3) Bohm diffusion approximation. In each panel, the two dashed lines are power-law fittings $\sim (p/p_0)^{-\alpha}$, with $\alpha_1$ and $\alpha_2$ being the fitted spectral indices to the spectrum at the low and high energies, respectively, and $p_m$ being the fitted break momentum. We set $p_0$ to be $13 p_0 \sim 0.416 \text{ MeV/c}$ in the simulation, which corresponds to an $E_0$ of 0.15 MeV. Note that $p_m$ is larger than $p_b = \gamma m_p \Omega/k_b$ by about a factor of $\sim 2$. Figure 2 is the most important result of this paper. It shows that DSA at a finite-width TS in solar flares can naturally lead to a hardening of the accelerated electron spectrum.

3. DISCUSSIONS AND CONCLUSIONS

Clearly, hardening requires the following conditions to be met. The first condition is the existence of a TS at a flare site with a finite shock width $L_{\text{diff}} \sim c/\omega_{pi}$. Second, the diffusion coefficient $\kappa$ needs to be close to a constant at low energies and increases with electron energy at high energies. Third, it is necessary that $\kappa < \Delta U/L_{\text{diff}}$ at energies below the break and $\kappa > \Delta U/L_{\text{diff}}$ at energies above the break.

For any given flare, none of these conditions are necessarily satisfied.

Consider the first condition. While it is hard to identify a TS at a flare observationally, there are indirect clues of such shocks. For example, type II radio bursts without frequency drift have
been used by Warmuth et al. (2009) to infer the existence of flare TSs. Further observational evidence of a flare TS, and in particular its size, are welcomed.

For the second condition, if electrons resonate with the dissipation range of the turbulence through cyclotron resonance and the dissipation range has a power spectrum $I(k) \sim k^{-2.7}$, then we find that $\kappa$ is indeed close to a constant at low energies and increases with electron energy at high energies. While in situ solar wind observations do suggest such a $\sim k^{-2.7}$ dissipation range, direct confirmation of such a $k$ dependence in the flare site is impossible.

Satisfying the third condition will place a strong constraint on the turbulence level at the flare, which can vary highly from one to another. Consequently, hardening does not occur in all flares. For example, if for a given flare $\beta_s \sim 1$ instead of $\beta_s \sim 0.2$ at lower energies, then there will be no hardenings, even if both conditions 1 and 2 are satisfied. Since a larger $\beta_s$ implies a larger $\kappa$, and therefore a less efficient acceleration, one implication of our proposed scenario is as follows: events where the continuum emissions do not extend to high energies (inefficient acceleration) likely have harder spectra at low energies than those events that extend to higher energies.

Another consequence of our proposal is the correlation between the low-energy photon spectral index $\gamma_1$ and the break momentum $p_m$. If we consider two flares A and B that are nearly identical, except that flare A has a larger $k_b$ (i.e., the inertial range in flare A extends to a smaller scale), then $\beta_s$ at $p < p_m$ for flare A is smaller. Therefore, $p_m$ and $\alpha_1$ are anti-correlated. Observations do show such an anti-correlation and this is discussed in detail in Kong et al. (2013).

In summary, we offer an explanation for the observed continuum spectral hardening in solar flares that is based on DSA. To our knowledge, no previous works have addressed the hardening of emission spectrum explicitly. Further observational and theoretical studies along the proposed mechanism will be pursued in future works.

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REFERENCES

Ackermann, M., Ajello, M., Allafort, A., et al. 2012, ApJ, 745, 144
Alexandrova, O., Lacombe, C., Mangeney, A., Grappin, R., & Maksimovi, M. 2012, ApJ, 760, 121
Asai, A., Kiyohara, J., Takasaki, H., et al. 2013, ApJ, 763, 87
Axford, W. I., Leer, E., & McKenzie, J. F. 1982, A&A, 111, 317
Balikhin, M. A., Krasnoselskikh, V., & Gelalini, M. 1995, AdSpR, 15, 247
Boldyrev, S., & Perez, J. C. 2012, ApJL, 758, L44
Chen, C. H. K., Horbury, T. S., Schechkhin, A. A., et al. 2010, PhRvL, 104, 255002
Drury, L. O’C., Axford, W. I., & Summers, D. 1982, MNARS, 198, 833
Galtier, S. 2006, JPPPh, 72, 721
Guo, F., & Giacalone, J. 2012, ApJ, 753, 28
Gordon, B. E., Lee, M. A., & M¨obius, E. 1999, JGR, 104, 28263
Howes, G. G., Tenbarge, J. M., & Dorland, W. 2011, PhPl, 18, 102305
Jokipii, J. 1966, ApJ, 146, 480
Kawata, T., Nishizuka, N., Oi, A., Ohyama, M., & Nakajima, H. 2012, ApJ, 747, 131
Kong, X. L., Li, G., & Chen, Y. 2013, ApJ, submitted
Kontar, E. P., Emslie, A. G., Massone, A. M., et al. 2007, ApJ, 670, 857
Krishan, V., & Mahajan, S. M. 2004, JGR, 109, A11105
Leamon, R. J., Matthaeus, W. H., Smith, C. W., et al. 2000, ApJ, 537, 1054
Leamon, R. J., Smith, C. W., Ness, N. F., Matthaeus, W. H., & Wang, H. K. 1998, JGR, 103, 4775
Leamon, R. J., Smith, C. W., Ness, N. F., & Wang, H. K. 1999, JGR, 104, 22331
Li, G., Zank, G. P., & Rice, W. K. M. 2003, JGR, 108, 1369
Leamon, R. J., Matthaeus, W. H., Smith, C. W., et al. 2000, ApJ, 1054
Leamon, R. J., Smith, C. W., Ness, N. F., Matthaeus, W. H., & Wang, H. K. 1998, JGR, 103, 4775
Mann, G., Warmuth, A., & Aurass, H. 2009, A&A, 494, 669
Marschhauser, H., Rieger, E., & Kanbach, G. 1994, in AIP Conf. Proc. 294, High Energy Solar Phenomena—A New Era of Spacecraft Measurements, ed. J. Ryan & W. T. Vestrard (Melville, NY: AIP), 171
Melrose, D. B., & Brown, J. C. 1976, MNARS, 176, 15
Miller, J. A., Cargill, P. J., Emslie, A. G., et al. 1997, JGR, 102, 14631
Miller, J. A., Larosa, T. N., & Moore, R. L. 1996, ApJ, 461, 445
Moses, D., Droge, W., Meyer, P., & Evenson, P. 1989, ApJ, 346, 522
Newbury, J. T., Russell, C. T., & Gedalin, M. 1998, JGR, 103, 29581
Park, B. T., Petrosian, V., & Schwartz, R. A. 1997, ApJ, 489, 358
Petrosian, V., & Arras, H. 2009, A&A, 494, 669
Ramos, H., Rieger, E., & Marschhauser, H. 1994, ApJ, 434, 747
Ramaty, R., Kozlovsky, B., & Lingenfelter, R. E. 1975, SSRv, 18, 341
Rice, W. K. M., Zank, G. P., & Li, G. 2003, JGR, 108, 1369
Rieger, E., & Marschhauser, H. 1990, in Proc. Third Max 91 Workshop, ed. J. Ryan & W. T. Vestrand (Melville, NY: AIP), 171
Scholer, M., & Burgess, D. 2006, PhPl, 13, 062101
Schwartz, S. J., Henley, E., Mitchell, J., & Krasnoselskikh, V. 2011, PrL, 107, 215002

Figure 2. Electron spectrum for three cases: (1) Kolmogorov-like inertial range; (2) Ik-like inertial range; (3) Bohn diffusion approximation. (A color version of this figure is available in the online journal.)
| Reference | Journal | Volume | Page(s) |
|-----------|---------|--------|---------|
| Scudder, J. D., Aggson, T. L., Mangeney, A., Lacombe, C., & Harvey, C. C. | JGR | 91 | 11053 |
| Silva, A. V. R., Wang, H., & Gary, D. E. | ApJ | 545 | 1116 |
| Smith, C. W., Mullan, D. J., Ness, N. F., Skoug, R. M., & Steinberg, J. | JGR | 106 | 18625 |
| Stawicki, O., Gary, S. P., & Li, H. | JGR | 106 | 8273 |
| Suri, A. N., Chupp, E. L., Forrest, D. J., & Reppin, C. | ApJL | 495 | L67 |
| Tsuneta, S., & Naito, T. | ApJ | 495 | L67 |
| Vestrand, W. T. | SoPh | 118 | 95 |
| Vestrand, W. T., Share, G. H., Murphy, R. J., et al. | ApJS | 120 | 409 |
| Warmuth, A., Mann, G., & Aurass, H. | A&A | 494 | 677 |
| White, S. M., Benz, A. O., Christie, S., et al. | SSRv | 159 | 225 |
| Yoshimori, M., Watanabe, H., & Nitta, N. | IPS1 | 54 | 4462 |
| Zank, G. P., Rice, W. K. M., le Roux, J. A., & Matthaeus, W. H. | ApJ | 556 | 494 |
| Zharkova, V. V., Arzner, K., Benz, A. O., et al. | SSRv | 159 | 357 |