Cold nuclear matter

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The behavior of nuclear matter is studied at low densities and temperatures using classical molecular dynamics with three different sets of potentials with different compressibility. Nuclear matter is found to arrange in crystalline structures around the saturation density and in non-homogeneous (i.e. pasta-like) structures at lower densities. Similar results were obtained with a simple Lennard-Jones potential. Finite size effects are analysed and the existence of the non-homogeneous structures is shown to be inherent to the use of periodic boundary conditions and the finitude of the system. For large enough systems the non-homogeneous structures are limited to one sphere, one rod or one slab per simulation cell, which are shown to be minimal surface structures under cubic periodic boundary conditions at the corresponding volume fraction. The relevance of these findings to the simulations of neutron star and supernovae matter is discussed.

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I. INTRODUCTION

The study of cold nuclear matter at subsaturation densities is important for a variety of topics which include the equation of state of nuclear matter, phenomena related to the study of heavy ion reactions, the structure of neutron star crusts, etc. Pioneering studies based on the compressible liquid drop model [1, 2], Hartree-Fock method [3], and energy minimization techniques [4, 5] showed that transitions from spherical nuclei to shapes such as rods, slabs, tubes, spherical bubbles, are to be expected in cold nuclear matter. More recent investigations [4–16] have used dynamical methods to study such transitions in neutron star crust environments.

Coulomb interaction has been thought of as an essential ingredient for the formation of the rich “pasta” like structures. Koonin [6], for instance, explained the transitions between different topologies in terms of a competition between a short-range nuclear surface energy which becomes minimized through aggregation, and a long-range Coulomb energy which gets reduced by an opposite dispersion. More recent studies of Horowitz and coworkers [7] re-examine this assertion as an example of frustration, a phenomenon that emerges from the impossibility to simultaneously minimize all interactions, and yields a large number of low-energy configurations. Indeed in all of the previous studies listed, the Coulomb interactions have been approximated either by electrically grounded surfaces [4], uniform electric charge densities [5], Thomas-Fermi screened Coulomb potential (see, e.g. [18]) or by an Ewald summation [19].

The importance the Coulomb repulsion has on the formation of the pasta-like structures prompted a previous study [20] which dissected the role of this interaction as a function of density, temperature and isotopic content through the use of classical molecular dynamics simulations at fixed volume and number of particles. In an unexpected outcome, however, such study, which varied the strength of the electric interaction from full to none, observed that the pasta structures, namely, “gnocchi”, “spaghetti”, “lasagna” and their anti-structures (i.e. those obtained by replacing particles by holes and vice versa), existed even in the absence of Coulomb interaction. Although the structures appeared modified somewhat in their overall scales, topology and location in the density-temperature plane, the nuclear potential seemed to be sufficient to give rise to a rich pasta-like structure in nuclear systems at subsaturation densities and low temperatures under periodic boundary conditions. Nuclear matter at subsaturation densities and high temperatures decomposes into a mix of liquid and gaseous phases [21], at lower temperatures ($T \lesssim 1 \text{ MeV}$), however, it seems to self-assemble into pasta-like objects even without Coulomb interaction. This gives rise to very interesting questions that are addressed in the present work.

The approach to be followed is a combination of classical molecular dynamics of large (but finite) systems under periodic boundary conditions, topological analysis tools to study the structure of infinite nuclear matter at subsaturation densities and very cold temperatures, and schematic geometrical considerations. In the next section the classical molecular dynamics model used will be briefly reviewed for completeness. Section III presents a detailed study of the structure of symmetric nuclear matter with medium compressibility, followed in Section IV by similar studies with other potentials that have been used in the past to study nuclear systems. The role of finite size effects is briefly discussed in Section V and a summary of the main results is presented in Section VI along with some concluding remarks. Visual representations of simulated systems where made using VMD [22].
II. CLASSICAL MOLECULAR DYNAMICS

This work uses a classical molecular dynamics (CMD) model to study infinite nuclear matter at low temperatures and subsaturation densities; the use of molecular dynamics to study nuclear reactions was pioneered by Willets and coworkers [23] and advanced by Pandharipande [24] and others [23,24,28,29]. Recently, classical molecular dynamics models have been used to study cold nuclear matter in neutron star crust environments [17,27–29]; in particular the CMD model, which was developed to study nuclear reactions was pioneered by Pandharipande et al. [24] and has been adapted to study infinite nuclear systems under such conditions [15,16,20].

In this study, the trajectories of the nucleons are governed by classical equations of motion dictated by forces produced by the Pandharipande [24] potentials:

\[
V_{np}(r) = V_r \left( \exp(-\mu_r r)/r - \exp(-\mu_r r_c)/r_c \right) - V_a \left( \exp(-\mu_a r)/r - \exp(-\mu_a r_c)/r_c \right),
\]

\[
V_{NN}(r) = V_0 \left( \exp(-\mu_0 r)/r - \exp(-\mu_0 r_c)/r_c \right),
\]

where the attractive potential between a neutron and a proton is \(V_{np}\), and the repulsive interaction between similar nucleons (\(nn\) or \(pp\)) is \(V_{NN}\); they both use a cutoff radius of \(r_c = 5.4\ fm\) after which the potentials are set to zero. The Yukawa parameters \(\mu_r\), \(\mu_a\), and \(\mu_0\) were phenomenologically adjusted by Pandharipande to yield a saturation density of \(\rho_0 = 0.16\ fm^{-3}\), a binding energy \(E(\rho_0) = 16\ MeV/\text{nucleon}\) and a compressibility (actually, bulk modulus) listed in [24] as 250 MeV for the “Medium” model, and 535 MeV for the “Stiff” [24].

The trajectories of all nucleons are obtained by solving the classical equations of motion using a symplectic Verlet algorithm with energy conservation of \(O(0.01\%)\). To mimic an infinite system \(A = 1728\) to \(A = 13824\) nucleons were placed in cubic cells under periodic boundary conditions. We focus on isospin symmetric systems \(x = z/A = 0.5\). The number densities were enforced by placing a fixed number of nucleons in cubic boxes with sizes selected to adjust the density. To study systems at different temperatures, the nuclear matter is force-heated or cooled using the Andersen thermostat [10] to control the temperature. Previous studies already presented samples of structures obtained through this method [15,16]. In this study of cold nuclear matter the range of densities is selected to be \(0.01\ fm^{-3} \leq \rho \leq 1.25\rho_0\), and that of densities is \(0.001 \lesssim T \lesssim 1.0\ MeV\).

The procedure we follow is twofold. To study the uniform phase exactly at zero temperature, a crystalline structure at a given density is constructed and its energy per nucleon calculated by direct summation between all nucleons. The dependence of the binding energy on the density is explored by changing the lattice parameter different, the energy versus density curve is shown in Fig. 4. This procedure, identical to the one used by Pandharipande [24], produces the characteristic “U”, with a minimum signalling the saturation or normal nuclear density.

Pandharipande et al. assumed that the nucleons in the ground state were arranged as a simple cubic lattice for the whole range of densities, but that is not necessarily the case as we shall see in Section IV. Given that nuclear matter is composed of two kinds of particles (neutrons and protons), the crystal geometries they adopt are similar to those formed by binary alloys. As we will see, the relevant crystal structures for the present case are the B1 (a simple cubic lattice in which every first neighbor of a proton is a neutron and viceversa, used by Pandharipande et al.), the B2 (a BCC lattice), and the B3 (a diamond lattice with nucleons arranged so that every first neighbor of a protons is a neutron and viceversa).

The second method uses CMD starting from a random positioning of a fixed number of nucleons in a central cubic cell under periodic boundary conditions, with a Maxwell-Boltzmann velocity distribution corresponding to a given initial temperature. The system is then equilibrated at a high temperature \((T \gtrsim 2\ MeV)\) and then brought down to the final desired temperature using the Andersen thermostat procedure in small temperature steps. \(T = 0.001\ MeV\) is taken as zero temperature for the CMD calculations, other values explored are in the range \(0.001\ MeV \leq T \leq 1\ MeV\). After reaching equilibrium, the analysis tools described in [15,16] are used to visualize and characterize the produced structures.

It is with these tools that cold nuclear matter will be studied under a variety of conditions and for several potentials.
FIG. 2. (Color online) Structures corresponding to the labelled points of Figure 1. Point A corresponds to a formation in the regular \((B_1)\) lattice, while the rest of the points are non-homogeneous structures.

FIG. 3. Energy (full lines with squares), pressure (dashed line) and compressibility (dotted line) for the uniform \(B_1\) lattice of the Medium potential. The arrow points at the density at which the system departs from homogeneity.

III. MEDIUM COMPRESSIBILITY MATTER

Figure 1 shows the near zero temperature results for symmetric \((x = 0.5)\) matter interacting through the Pandharipande medium potential (see figure caption for details). As can be seen, CMD reproduces the simple cubic \((B_1)\) lattice calculations up to a density of \(\rho \approx 0.13 fm^{-3}\), while for lower densities the system breaks into pasta-like objects of different shapes. Also shown in Figure 1 are the energies expected for a uniform body centered cubic \((B_2)\) and diamond \((B_3)\) lattice structures which, being higher in energy, do not correspond to the \(T = 0\) case. Figure 2 shows the structures corresponding to the four densities labelled from “A” to “D” in Figure 1.

The structures of Figure 2 resemble those obtained by Williams and Koonin in 1985 with a static mean field model \(\ddagger\). Placing nuclear matter in a periodic simple cubic lattice at a given number density of nucleons, the system was allowed to relax freely until a local energy minimum was achieved. Although the method did not take into account the possibility of having free nucleons and used a fixed geometry by construction, it yielded results comparable to those obtained by the dynamical method used in this study. For instance, structure “C” in Figure 2, which corresponds to what in \(\ddagger\) was dubbed as “lasagna” – was also found by Koonin \(\ddagger\) (“alternating slabs of matter and vacuum”) and Ravenhall \(\ddagger\). A significant difference, however, is that both Ravenhall and Koonin included the Coulomb interaction as a main ingredient, which we do not do; this will be addressed in Section V.

As an aside, the \(B_1\) curve of Figure 1 can be used to perform a polynomial fit around its minimum to extract an analytic expression, which can then be used to calculate the compressibility at around the saturation density through \(K = 9\rho_0^2\left[\frac{d^2(E/A)}{d\rho^2}\right]_{\rho_0}\). In the case of the medium Pandharipande potential the value of the bulk modulus is found to be 283 \(MeV\), comparable with the value of 250 \(MeV\) quoted by its creators.

In Figure 3 we plot the energy, pressure and compressibility for the homogeneous \(B_1\) lattice of Pandharipande’s Medium potential. The pasta-like structures are found in a mechanically unstable (negative pressure) density region, but well above the divergence in compressibility.

Continuing with the study at higher temperatures, Fig-

FIG. 4. Binding energy per nucleon for systems obtained with the Pandharipande medium potential at the listed temperatures.
FIG. 5. (Color online) Structures corresponding to the labelled points of Figure 4 obtained with the Pandharipande medium potential at $T = 1.0 \text{ MeV}$.

FIG. 6. Curvature - Euler coordinates of the structures of Figure 4. The lines connect points with the same densities but temperatures varying from $T = 0.001 \text{ MeV}$ to $1.0 \text{ MeV}$.

TABLE I. Classification Curvature - Euler

| Euler | Curvature $<$ 0 | Curvature $\sim$ 0 | Curvature $>$ 0 |
|-------|----------------|-------------------|----------------|
| > 0   | Anti-Gnocchi   | Gnocchi           |                |
| $\sim$ 0 | Anti-Spaghetti | Lasagna           | Spaghetti     |
| < 0   | Anti-Jungle Gym| Jungle Gym        |                |

IV. HIGHER COMPRESSIONIBILITY POTENTIALS

As the comportment presented in the previous section is bound to be potential-dependent, it is instructive to repeat the study using other nuclear interactions to extract generalities of the behavior of nuclear matter at low temperature and subsaturation densities. In particular, the study uses potentials with higher values of the compressibility that have been used in the past to study nuclear matter.
A. A simple semiclassical potential

A higher compressibility set of potentials that has been used for a variety of studies of nuclear matter [27–29] is the one described by its creators [17] as a “simple semiclassical potentials” (SSP). In summary, the SSP is composed of:

\[ V_{np}(r) = ae^{-r^2/\Lambda} + [b - c]e^{-r^2/2\Lambda}, \]
\[ V_{NN}(r) = ae^{-r^2/\Lambda} + [b + c]e^{-r^2/2\Lambda}, \]

where, again, the potential between a neutron and a proton is attractive, and that between like particles is repulsive. The parameters \(a, b, c,\) and \(\Lambda\) have been adjusted to have the proper energy and density scales to mimic nuclear matter. Notice that, at a difference from the Pandharipande potentials, the SSP potentials do not have repulsive hard cores.

Figure 7 shows the binding energies per nucleon obtained from symmetric systems constructed with the SSP in crystalline lattices with \(B1, B2\) and \(B3\) crystal geometries. When used in our molecular dynamics code at \(T = 0.001\ MeV\) — and without the Coulomb potential — the SSP produces structures with the energies labelled in Figure 7 as CMD. Interestingly, at saturation density this potential produces a \(B2\) crystal instead of the \(B1\) produced by the lower compressibility model used in Section III.

Noticeably, the “normal” density point of this potential is at the correct value (i.e., at \(\rho = 0.16\ fm^{-3}\)) but at the lower binding energy of about \(-20.3\ MeV\). Again, using a polynomial fit to the bottom part of the “U” of the \(B2\) curve of Figure 7 allows us to extract a value of the compressibility of the order of \(418\ MeV\), much higher than the value of the Pandharipande medium potential.

As in the case of the medium Pandharipande potential, at densities \(\rho \gtrsim 0.12\ fm^{-3}\) the CMD results agree with the lowest-energy crystalline structure, which in this case is the \(B2\), and at lower densities the systems form pasta-like objects. The structures corresponding to the four
labeled points (“A” through “D”) are shown in Figure 8. Again, point A corresponds to formations in the regular lattice, while points B through D are pasta structures.

Except for minor differences, the scenario that the SSP presents is very similar to that obtained with the Pandharipande medium potential. Namely, it is possible to obtain rich pasta-like structures in an SSP medium at cold temperatures and subsaturation densities without the modulating effect of the Coulomb interaction.

B. Stiff Pandharipande potential

The same calculations are now repeated for the stiff version of the Pandharipande potential, see [24]. This interaction produces the $E/A$ versus $\rho$ shown in Figure 9 for symmetric nuclear matter and zero temperature in $B1$ and $B3$ crystals; the $B2$ structure produced energies higher than the scale of the figure. Also presented are the binding energies of the structures obtained with CMD at $T = 0.001$ MeV.

In this case the saturation point for cold matter occurs at $\rho = 0.15$ fm$^{-3}$, with an energy of 16.5 MeV and for a $B3$ structure. The authors of the potential did not realize that the lowest energy geometry was a $B3$ lattice and assumed a $B1$ structure in their estimation of the compressibility which led to an incorrect value. Using a polynomial fit of the $B3$ curve to estimate the compressibility at the saturation point, as done with the other two potentials, yields a value of about 494 MeV, somewhat smaller than the value of 535 MeV that Pandharipande obtained using a $B1$ lattice [24].

Furthermore, at a density of about 0.17 fm$^{-3}$, this potential presents a solid-solid phase transition between $B1$ and $B3$ structures. The CMD simulations correctly show that at $\rho \gtrsim 0.17$ MeV the preferred crystalline structure is the $B1$, while for lower densities in the range $0.13 \lesssim \rho \lesssim 0.17$ fm$^{-3}$ it is $B3$. The departure from the homogeneous phase into the pasta structures starts at densities smaller than $\rho \approx 0.13$ fm$^{-3}$; the structures corresponding to the four labelled points (“A” through “D”) are shown in Figure 10.

Again, except for minute differences, stiff nuclear matter also produces pasta formations at cold temperatures and subsaturation densities without any Coulomb interaction.

V. PSEUDO PASTA

Historically, the existence of pasta phases in nuclear matter has been attributed to the competition between surface and Coulomb energies. While the short-ranged attractive nuclear interaction drives the system to a minimum surface configuration, the long-ranged repulsive Coulomb interaction drives protons as far away from each other as possible producing the non-homogeneous pasta structures. However, as we have seen in the previous sections and in previous studies [20], pasta-like structures can be found in molecular dynamics simulations with attractive-repulsive nuclear potentials without Coulomb interaction.

To elucidate on the origin of these pasta-like structures, this section will first demonstrate that such structures can be generated in single-component systems interacting through a single potential. Since this proves that the attractive-repulsive interplay of forces cannot be responsible for the formation of these pasta-like structures, we then proceed to study some geometrical aspects of the simulations which appear to bear a large share of the responsibility for the formation of these structures.

A. Lennard Jones pasta

Pasta, as expected to exist in neutron star crusts, originates from the competition between short-range attractive nuclear interaction and long-range Coulomb repulsion. Due to such proposed origin, pasta structures were not expected to exist in pure nuclear matter but, as seen in the previous sections, they indeed appear in molecular dynamics simulations. In nuclear matter, presumably, one could blame the formation of the pasta structures on the interplay between the attractive and repulsive parts of the nuclear interactions; since such potentials are of the same range, that would rule out the phenomenon of frustration (i.e. the impossibility of obtaining a minimization of all forces at the same time) as the origin of the pasta. Motivated by this startling discovery, we decided to simplify the systems treated down to a one-
component one-potential case to see the structures that could be obtained.

We adapted CMD to perform simulations using a simple Lennard-Jones (LJ) potential under the same conditions of number density and system sizes as those for the nuclear case. The interaction potential between any two “nucleons” is given by

\[ V_{\text{LJ}}(r) = 4\epsilon \left( \frac{\sigma}{r} \right)^{12} - \left( \frac{\sigma}{r} \right)^{6} \]

with \( \sigma \) and \( \epsilon \) chosen to have the minimum energy in \(-16\) in arbitrary units and at \( \rho = 0.16 \) in a close-packed lattice: conditions similar to those of the Pandharipande medium potential of Section III.

Figure 11 shows some of structures obtained with the LJ potential, the resemblance to the nuclear matter structures is striking; as it turns out, the formation of pasta-like structures had been observed in grand canonical Monte Carlo simulations of LJ pasta-like structures \[11\], which found only one spherical drop, one rod, one slab, one cylindrical tube or one spherical bubble per cell, except in densities during transitions. To examine this intriguing result we look in turn at the volume and surface of structures obtained at zero temperature with the medium compressibility potential of Section III.

In our simulations at zero temperature and with \( x = 0.5 \) the number of particles is fixed and the cell volume is adjusted to yield the desired density. As it can be seen in Fig. 11 the binding energies per nucleon maintains a relatively constant value, which implies that the local densities of matter bound in pasta-like structures correspond roughly to the saturation density as conjectured by Ravenhall in \[1\]. Then, if the number of particles is kept fixed, the volume occupied by all nucleons in all pasta-like structures will be approximately the same irrespective of the value of the number density.

Such effect can be corroborated by estimating the total volume occupied by the nucleons (though the digitalization process described in \[16\]) and comparing it to the total volume of the cells, which increases with decreasing density. Figure 12 shows that the total volume occupied by nucleons in the non-homogeneous configurations varies by less than 10% while the cell’s volume increases almost ten times. In other words, as the pasta-like structures change from drops to rods, slabs, etc. the total volume occupied by nucleons remains practically constant. This indicates that the structures do not to minimize their bulk (volume) energy when they change shapes, and points to the surface energy as the critical factor in determining which specific pasta-like structure is the most appropriate for a given density.

To quantify the variation of the surface area of the structures in terms of the cell size we perform a simple geometric exercise. We calculate the surface area of simple shapes (spherical drop, cylindrical rod, slab and their bubble counterparts) and plot it as a function of the fraction of the cubic cell’s volume they occupy, \( u \).

Each pasta-like shape can be characterized by its volume and surface area, which in turn can be expressed in terms of the cubic cell’s side \( L \) and a characteristic length \( a \), namely the radius of a spherical drop, cylindrical rod, the width of a lasagna slab, etc.

It must be remembered that for finite systems with
periodic boundary conditions the simulation cell imposes some constrains. Because of the periodic boundary conditions, both slabs and cylindrical rods (or bubbles) have some faces attached to the surfaces of the cubic cell in which they are inscribed. These lateral faces produce artificial surfaces and should not be taken into account.

In this way, the effective surface area of each shape can be written in terms of $a$ and the cubic cell’s length $L$ and its variation with the volume fraction $u$ can be studied. The surface area of a single shape per cell as a function of $L$ and $u$ is found to be

$$S_{sphere} = 4\pi \left(\frac{3}{4\pi}\right)^{\frac{2}{3}} \times u^{\frac{2}{3}} \times L^2$$
$$S_{rod} = 2(\pi)^{\frac{3}{2}} \times u^{\frac{1}{2}} \times L^2$$
$$S_{slab} = 2 \times L^2.$$

The bubble counterparts have similar expressions with $u$ replaced by $(1 - u)$.

Since the surface areas of all shapes studied scale as $L^2$, there will not be a specific shape that will have a minimum surface area for a given cell length $L$. Consequently, the shape structures that will be dominant at a given density will be selected entirely by their volume fraction $u$. Figure 13 shows these surfaces as a function of the volume fraction.

In order of increasing volume fraction, the preferred shapes (minimum surfaces) go from spherical drop to cylindrical rod, to slab, to cylindrical bubble and finally to spherical bubble; basically the same ordering found in almost every study of nuclear pasta.

This schematic result should be exact for large enough systems, i.e. for cell sizes much larger than the range of the interaction potential where interfacial and curvature effects can be neglected. Of course, in order to determine the most stable configuration a figure such as 13 should be constructed using every possible surface. The question then is, how small is large enough? To address this question we now turn to a study of the punctured slab.

Figure 14 shows configurations found with Pandharipande’s medium potential at $T = 0.1$ MeV and $\rho = 0.05 fm^{-3}$ and $0.08 fm^{-3}$ for three system sizes: $A = 1728$, $4096$ and $13824$. It is clear that only one structure per cell is found, independently of system size. For density $\rho = 0.08 fm^{-3}$, the same structure (cylindrical bubble) is observed for the three sizes, but for density $\rho = 0.05 fm^{-3}$, however, the smallest system forms a single punctured slab (Pslab) with a single hole, whereas the
larger systems form a single regular slab. Such structure appears to be the minimum energy configuration between a “rod” and a “slab” over a very small range of densities, becoming a slab by adiabatically increasing the density in a larger cell. A suggestively similar situation was observed by Williams and Koonin who reported a “slab with regular holes”.

To compare the surface area of this structure to that of the slab it is necessary to use the width of the slab \( w \), and the radius of the holes \( b \) as independent parameters. In terms of the volume fraction \( u \) it is

\[
b = L \sqrt{1 - \frac{L \times u}{a}}
\]

\[
S_{P\text{slab}} = 2L^2 \times \frac{L \times u}{a} + 2\pi b \times a
\]

As shown in Figure 15, the punctured slab is never the minimum surface shape among those structures considered. Furthermore, the total surface area of the punctured slab does not scale as \( L^2 \) as all the others. The leading term actually scales as \( L^3/a \) which is always larger than \( L^2 \), so for larger values of \( L \) this shape will always have a larger surface than the other shapes considered; yet our simulations yielded one. This happens because small cells yield thin slabs in which the particles at one surface could interact with those at the opposite surface; such effect would occur in slab with thicknesses comparable to twice the range of the interaction potential.

In such case higher order surface terms become relevant. Indeed, the punctured slab with \( A = 1728 \) from Fig. 14 has a maximum width of \( \sim 20 \text{ fm} \) and a minimum of \( \sim 10 \text{ fm} \), while the potential has a range of \( r_c = 5.4 \text{ fm} \). The regular slab with \( A = 4096 \) has a constant width of \( \sim 16\text{ fm} \). For \( A = 1728 \) and \( \rho = 0.05 \text{ fm}^{-3} \), the cubic cell’s length is \( 32.6 \text{ fm} \), while the range of the potential is \( r_c = 5.4 \text{ fm} \). That is, the cell is only 6 times larger than the range of the potential, hence interfacial or curvature effects might be of the same order than surface effects.

This same interplay of geometrical parameters might be responsible for the more exotic structures shown in Figures 2, 5, 8 and 10. At any rate, \( A = 1728 \) is not large enough to shake off finite size effects, due only to the nuclear interaction. The scenario is expected to be even worse with a long range interaction such as Coulomb.

For the sake of argument, we show in Fig. 16 results obtained with \( A = 13824 \) and \( A = 46656 \) particles. At density \( \rho = 0.05 \text{ fm}^{-3} \) we found slab-like structures for all system sizes equal or larger than \( A = 4096 \). For density \( \rho = 0.08 \text{ fm}^{-3} \), with \( A = 13824 \) almost cylindrical holes are observed, with some modulation. That modulation might be due to fast cooling. Incidentally, the slab for \( A = 46656 \) is stable at higher temperatures than for smaller systems. Compare, for example, fig. 2 and fig. 5. For \( A = 1728 \) the \( T \sim 0\text{MeV} \) solution is a slab, but it becomes distorted at higher temperatures.

A rule of thumb to distinguish pseudo-pasta – i.e. that due to periodic boundary conditions– and “true” pasta (that arises from a balance between nuclear and Coulomb interactions) is that the scale of the pseudo-pasta structures is set exclusively by the size of the cell. This is evidenced by the fact that, without Coulomb inter-
teration, there is systematically a single structure per cell. Any model that includes Coulomb interaction and aims to produce "true" pasta should, at least, be able to produce more than one structure per simulation cell.

VI. CONCLUDING REMARKS

Nuclear matter was studied at subsaturation densities and low temperatures using molecular dynamics using three different pairs of two-body potentials corresponding to different values of the compressibility. The average binding energy per nucleon as a function of the density indicated that, around saturation density, crystalline arrangements of the type simple cubic (B1), body centered cubic (B2) and diamond (B3) crystal lattices were found in simulations using the potentials with compressibility of 283 MeV, 418 MeV and 494 MeV. These lattice structures were observed in the temperature range of 0.001 MeV ≲ T ≲ 1.0 MeV. A departure from the crystalline structure into non-homogeneous structures occurred very consistently whenever the density decreased below ρ ≈ 0.13 fm⁻³; this effect was observed at all temperatures studied and for the three potentials.

The structures formed correspond to the usual pasta-like structures that have been observed in previous studies. The fact that these pasta shapes were formed in systems without the Coulomb interaction (i.e. not through nuclear-Coulomb frustration) prompted a study of the role of the size cell and periodic boundary conditions on the formation of the structures. After demonstrating that pasta-like structures can be obtained in a one-component Lennard-Jones system and that the volume energy played a minor role in the selection of the shapes, surface effects were investigated in turn. A simple geometric calculation showed that the unavoidable use of periodic boundary conditions combined with the finite size of the system favors the creation of pasta-like structures. These surface effects are not exclusive to this study nor to a particular interaction potential, in our case we found them to be present in systems as large as 13824 particles. We believe that these effects might still play a significant role in the formation of pasta even with a Coulomb interaction; these and other related issues will be tackled in an upcoming contribution.

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