Verification of flow velocity measurements using micrometer-order thermometers

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In flow velocity measurements, resolution, miniaturization, and accuracy of measuring devices are important issues because the measuring devices significantly affect the flow in the micro-space, sonic flow, and turbulent flow. We studied recovery temperature anemometry (RTA) using micrometer-order thermometers and evaluated its validity in two velocity ranges (40–90 and 315–420 m/s) by conducting two experiments and a numerical simulation. The results confirmed that the difference between the reference velocity and RTA was within 5% in the velocity range 60–90 m/s for both the thermocouple and platinum thermometer given the same recovery temperature coefficient of 0.83. It is a valuable finding that velocity measurement by RTA is independent of the type of thermometer used. This suggests that the accuracy of about 5% can be guaranteed even without calibration by giving the recovery temperature coefficient according to the thermometer geometry, which is an excellent advantage not found in other anemometers. Furthermore, the supersonic flow measured using RTA agrees well with the simulation results and theoretical trends. Our findings ensure that the micrometer-order point measurement of flow velocity, which is difficult with existing anemometers, using RTA is possible over a wide range of flow velocities.

Airflow measurement is extremely important for clarifying natural phenomena and for engineering applications. In the industrial field, there is a need for the measurement of a wide range of flow velocities (from low-speed flow in a clean room to supersonic flow in a rocket engine nozzle). Furthermore, there has been a high demand for flow velocity measurements in a minute space (approximately micrometers to millimeters) due to miniaturization of manufacturing in recent years. To this end, a variety of flow velocity measurement methods have already been proposed, such as pitot tubes1, hot wire anemometry2–4, particle image velocimetry (PIV)5–7, and laser Doppler velocimetry (LDV)8,9. Although hot wire anemometers are widely used for turbulence measurements, two or more probes supporting the thin wire often affect flow, and the insertion angle of the probes is an issue that needs to be considered10,11. Furthermore, the measured flow velocity is a line average (line measurement) because the hot wire length ranges from several hundred micrometers to several millimeters in general. Optical methods such as PIV and LDV are more expensive than pitot tubes and hot wire anemometers, thereby hindering their introduction. Additionally, it is difficult to apply these methods to practical situations, such as flow velocity measurement in pipes in plants and for outdoor flow measurement because of the need to insert tracer particles and laser beams.

Ishibashi12,13 proposed recovery temperature anemometry (RTA), which is based on the difference between the recovery temperature and stagnation temperature. Figure 1 illustrates a jet impacting a wall and then flowing over the wall surface. In a gas flow with a Prandtl number (kinematic viscosity/thermal diffusivity) less than 1.0, the wall temperature of an object that does not block the flow is smaller than the stagnation temperature $T_\infty$ due to the large effect of thermal diffusion. This temperature is called the recovery temperature $T_r$.

$T_0 = T_\infty + \gamma - 1 \frac{1}{2} M_\infty^2 T_\infty = T_\infty + \frac{U_\infty^2}{2C_p}$

$T_r = T_\infty + r \gamma - 1 \frac{1}{2} M_\infty^2 T_\infty = T_\infty + r \frac{U_\infty^2}{2C_p}$

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where $T_\infty, M_\infty, U_\infty, \gamma, C_p, r$ denote the mainstream temperature, mainstream Mach number, mainstream flow velocity, specific heat ratio, specific heat of constant pressure, and recovery temperature coefficient respectively. It is possible to measure the velocity distribution based on the temperature field because the temperature difference between $T_0$ and $T_r$ is proportional to the square of the main flow velocity.

It is possible to realize a micrometer-order point measurement of flow velocity using a small temperature sensor to measure $T_r$, which is difficult with existing anemometers. Ishibashi\cite{12,13} measured the flow in a sonic nozzle using thermocouples and captured the shock waves generated in the nozzle. However, the accuracy of RTA and the applicability of other thermometers are yet to be verified. Schmirler\cite{14} proposed new RTA that calculates the flow velocity from two recovery temperature coefficients of two thermometers located close to each other. However, the validated flow velocity range is limited to 120–260 m/s, and only one type of thermometer with a relatively large sensor diameter of 1.5 mm was used.

In this study, the validity of RTA was verified in two velocity ranges (40–90 and 315–420 m/s) through two experiments and a numerical simulation. In the velocity ranges, the accuracy of RTA has not been verified so far. Furthermore, two types of probe temperature sensors were verified to extend the applicability of RTA. A thermocouple (contact diameter: 300 μm) and platinum thermometer (sensor diameter: 300 μm) shown in Fig. 2a were used to measure the recovery temperature. The two probe temperature sensors were selected to be as small as possible, and the same value of recovery temperature coefficient $r$ of 0.83 was used for both sensors. The accuracy of RTA using these thermometers was evaluated using the reference velocity (range: approximately 40–90 m/s) with an expanded uncertainty of 0.63% (coverage factor: 2)\cite{15} owned by the National Metrology Institute of Japan. In addition, RTA was applied to the measurement of flow velocity in a small sonic nozzle with a throat diameter of 13.4 mm; the accuracy of the measurement in sonic flow was evaluated by comparing it with the results of the 3D numerical simulations. Therefore, we evaluated the validity of RTA from various viewpoints, such as comparison experiments with the national standard, theory, and numerical simulation.

The results of this study confirmed that the difference between RTA and the reference velocity was within 5% in the velocity range of 60–90 m/s, regardless of the type of thermometer. This suggests that the accuracy of about 5% can be guaranteed even without calibration by giving the recovery temperature coefficient according to the thermometer geometry, which is an excellent advantage not found in other anemometers. These results and the presentation of the benefits have not been described in other papers. Furthermore, the sonic flow in the nozzle measured by RTA was in good agreement with the simulation results and theoretical trends. Various existing inexpensive thermometers have enabled the micrometer-order point measurement of flow velocities over a wide range of flow velocities. It is confirmed in this study that RTA can be used in point velocity measurement, which was earlier difficult with existing anemometers, over a wide range of flow velocities.

**Figure 1.** Conceptual diagram of stagnation temperature $T_0$ and recovery temperature $T_r$. Both $T_0$ and $T_r$ are points on the wall, and therefore, the flow velocity is zero. However, $T_r$ is lower than $T_0$ because of the effect of the temperature gradient in the main flow. The flow velocity in the mainstream is proportional to the square of the difference between $T_0$ and $T_r$. Where $T_\infty, M_\infty, U_\infty, \gamma, C_p, r$ denote the mainstream temperature, mainstream Mach number, mainstream flow velocity, specific heat ratio, specific heat of constant pressure, and recovery temperature coefficient respectively.
Results

Comparison experiment with reference velocity. A comparison experiment with a national standard of flow velocity was conducted to evaluate the measurement accuracy of RTA. The reference velocity was derived from a flow rate reference based on reference nozzles. As depicted in Fig. 2b, the reference nozzles are installed in parallel at the upstream of the test section using the cross-sectional area of the wind tunnel outlet. The reference nozzles can generate a constant flow rate because the flow velocity at the throat with the smallest cross-sectional area becomes equal to the speed of sound when a differential pressure of a certain level or more is applied by the blower. The thermocouple or platinum thermometer used to measure the flow velocity (recovery temperature) is attached to the traverse apparatus at the downstream side of the wind tunnel; the measured value by RTA is compared with the value of the reference velocity.

![Diagram of the comparison experiment with reference velocity](image)

Figure 2. Conceptual diagram of the comparison experiment with reference velocity. (a) Thermometers for measuring the recovery temperature. The contact diameter of the thermocouple is approximately 300 μm. The sensor diameter of the platinum thermometer is approximately 300 μm, and sensor length is approximately 1.5 mm. (b) Conceptual diagram of test section. The reference velocity was derived by dividing the flow rate of the reference nozzles installed upstream of the test section using the cross-sectional area of the wind tunnel outlet. The reference nozzles can generate a constant flow rate because the flow velocity at the throat with the smallest cross-sectional area becomes equal to the speed of sound when a differential pressure of a certain level or more is applied by the blower. The thermocouple or platinum thermometer used to measure the flow velocity (recovery temperature) is attached to the traverse apparatus at the downstream side of the wind tunnel; the measured value by RTA is compared with the value of the reference velocity.
Figure 3a,b illustrate the relationship between the reference velocity $v_{\text{REF}}$ and the value measured by RTA $v_{\text{RTA}}$ based on absolute values and ratios, respectively. Figure 3b shows that the maximum difference between $v_{\text{REF}}$ and $v_{\text{RTA}}$ is approximately 30% when the flow velocity is approximately 40 m/s. The standard error of $v_{\text{RTA}}$ is also larger than that of the other velocity points. It is considered that the accuracy of temperature measurement affects the measurement accuracy of RTA, and the difference between $T_0$ and $T_r$ is as small as approximately 0.13 °C at approximately 40 m/s. In the velocity range of 60–90 m/s, the temperature difference between $T_0$ and $T_r$ increases to approximately 0.75 °C, and the difference between $v_{\text{REF}}$ and $v_{\text{RTA}}$ is within 5% for both the thermocouple and platinum thermometer. Therefore, in the range of large flow velocities where the temperature difference between $T_0$ and $T_r$ increases, a high accuracy can be expected regardless of the type of thermometers. The standard error of $v_{\text{RTA}}$ for both the thermocouple and the platinum thermometer increased for $v_{\text{REF}} = 90$ m/s compared to that for $v_{\text{REF}} = 75$ m/s. This can be attributed...
to the increase in temperature fluctuation in the test section when the maximum flow rate of 1000 m³/h is generated using the reference nozzles and blower. Figure 3c shows the time variation of \( T_r \) and \( T_e \) measured using platinum thermometers, and Fig. 3d depicts the relationship between the temperature difference between \( T_r \) and \( T_e \) and the flow velocity calculated using Eq. (3).

**Velocity measurement and numerical simulation of flow in a sonic nozzle.** After verifying the validity of the flow velocity measurement by RTA through comparison with the reference velocity, RTA using the thermocouple or platinum thermometer was applied to the flow velocity measurement inside a sonic nozzle. Unlike the experiment with the reference velocity described above, there is no reference value at each spatial position inside the nozzle, and therefore, the experimental results are compared with the results of the numerical simulations. Figure 4a presents an overview of the experiment. A 90° sonic nozzle (throat diameter: 13.4 mm) without a diffuser is installed upstream of the traverse apparatus, and the critical back pressure ratio of the nozzle is approximately 0.53. A vacuum pump and control valve were used to provide sufficient and constant differential pressures to the nozzle. The approximate back pressure ratio during the experiments was obtained from pressure gauges installed upstream and downstream of the nozzle. The \( T_r \) was measured using a thermocouple or platinum thermometer fixed to the traverse apparatus, and the \( T_e \) of the fluid in the streamline was measured using a platinum thermometer on the upstream side of the nozzle. The following three velocities were verified: (1) flow velocity at the center of the throat, (2) flow velocity in the axial direction, and (3) flow velocity in the radial direction 1 mm downstream of the throat.

Numerical simulations were performed using OpenFOAM (Open source Field Operation And Manipulation)\(^\text{21}\), which is a fluid analysis software, to perform unsteady three-dimensional simulations. For the simulation region shown in Fig. 4b, the shape inside the 90° nozzle was reproduced with the same dimensions as in the experiment. However, the spaces upstream and downstream of the nozzle did not have the same shape and volume as in the experiment. Therefore, a rectifying section was installed downstream of the nozzle to equalize the flow velocity at the outlet boundary and decrease the simulation area. In the simulations, the inlet boundary and internal pressure of the nozzle were set to 101 kPa, and a differential pressure greater than the critical back pressure ratio was applied to the outlet boundary as in the experiment. The mass flow rate at the throat after 0.005 s was stable, and the flow field after 0.005 s was provided as the calculation result. Figure 4c shows the velocity contours as an example of the calculation results.

**Comparison with the theoretical value at the center of the nozzle throat.** Figure 5 depicts the comparison of the results of RTA and numerical simulations at each back pressure ratio with the theoretical values for the flow velocity at the center of the nozzle throat. The theoretical value of the speed of sound \( c \) is derived from the following equation.

\[
c = \sqrt{\gamma R T_{th}} = \sqrt{\frac{\gamma R}{\gamma + 1} T_0}
\]  

where \( R \) denotes the gas constant \((J/K \cdot kg^{-1})\), and \( T_{th} \) represents the temperature at the nozzle throat. The theoretical value of the speed of sound calculated using this equation is approximately 315 m/s. For each back pressure ratio (0.5, 0.45, 0.40 and 0.37) less than the critical back pressure ratio of 0.53, the flow velocity at the throat measured by RTA is approximately constant despite the different pressure differences. This effectively captures the qualitative characteristics of the sonic nozzle. The velocities obtained by RTA and numerical simulation are up to approximately 8% lower than the theoretical value of 315 m/s because the theoretical equation for the speed of sound does not consider the effect of the curvature of the nozzle inlet which reduces the flow velocity at the center of the throat\(^\text{21-26}\). RTA and numerical simulations show a trend in harmony with the previous studies\(^\text{21-26}\). Considering the back pressure ratio of 0.45 as an example, the flow velocities measured by RTA and numerical simulations agree within a range of approximately 3%, which indicate that the measurement accuracy is high. For reference, when the back pressure ratio is less than 0.5, the difference between \( T_r \) and \( T_e \) measured at the center of the throat was approximately 8 °C or more. These results confirm that RTA can be applied to sonic flow in small-diameter nozzles because low-disturbance measurement can be achieved using a temperature sensor on the order of micrometers.

**Traverse measurement in axial and radial directions.** Figure 6 depicts the traverse measurement of flow velocity in the sonic nozzle in the axial and radial directions. Each result is obtained under the condition that the back pressure ratio is fixed at 0.4. In RTA and the numerical simulation, a supersonic flow is confirmed downstream of the throat, and in Fig. 4c, an oblique shock wave is generated downstream of the throat in the simulation. In RTA, the flow velocity and Mach number increase to approximately 420 m/s and 1.3, respectively, at \( x_{axial} = 7 \) mm, which indicates that RTA is valid for supersonic flow. The flow velocity decreases after \( x_{axial} = 7 \) mm, and the shock wave phenomenon in the nozzle can be captured. Although experimental and numerical simulation show good agreement in terms of flow velocity at the center of the throat, a difference of up to 10% is observed downstream of the nozzle throat. This is attributed to the fact that the experimental data does not fully reproduce the inside space inside the traverse apparatus downstream of the nozzle. Although the pressure ratio between the inflow and outflow boundaries was set to 0.4, the pressure field at the nozzle exit in the simulation may be different from that in the experiment, and this effect is expected to occur downstream of the nozzle throat.
Figure 6b depicts the results of the radial traverse measurements at 1 mm downstream of the throat. The measurement points were set at intervals of 1 mm at \(x_{\text{radial}} = 0 - 9\) mm, and \(x_{\text{radial}} = 0\) represents the center of the nozzle throat. Each result was obtained under the constant back pressure ratio of 0.4. At \(x_{\text{radial}} = 0 - 6\) mm, the flow velocity increases as \(x_{\text{radial}}\) increases because of the influence of the inlet curvature described above\(^{22-26}\) and RTA, and the simulation results show extremely good agreement. The throat radius of the 90° nozzle is 6.7 mm, and it is presumed that the region of \(x_{\text{radial}} = 7 - 9\) mm is outside the jet of supersonic flow; therefore, a decrease in flow velocity is confirmed. However, there is a large difference between RTA and the numerical simulation in terms of the reduction rate of flow velocity in the region outside the jet, and the experimental results indicate that the flow velocity is approximately 250 m/s even in the region considered to be outside the jet. Normally, \(T_0\)
**Figure 5.** Comparison of RTA and simulation results with theoretical values for flow velocity at the center of the nozzle throat. The experiments were performed three times under the same conditions. The plots show the mean values and the error bars show the standard errors. The critical back pressure ratio of the 90° sonic nozzle is approximately 0.53.

**Figure 6.** Traverse measurement of flow velocity in a sonic nozzle in the axial and radial directions. The experiments were performed three times under the same conditions. The plots show the mean values and the error bars show the standard errors. The experimental and simulation results were obtained at a back pressure ratio of 0.4. (a) Axial velocity distribution from the center of the throat to 10 mm downstream. (c) Radial velocity distribution at 1 mm downstream of the throat. The throat radius of the nozzle is approximately 6.7 mm.
and \( T_r \) at the same point have the same value when the flow velocity is close to zero. \( T_0 \) and \( T_r \) at the same point are required to accurately measure the flow velocity outside the jet, and in this experiment, \( T_0 \) in the streamline was calculated from the temperature measured upstream of the nozzle. Therefore, \( T_r \) outside the jet (outside the streamline) did not match \( T_{in} \) even when the flow velocity was near zero. Low-disturbance measurements of \( T_0 \) and \( T_r \) at the same point will be considered in the future.

**Discussion**

In this study, a velocity measurement method (RTA) based on \( T_0 \) and \( T_r \) was verified. The validity of RTA based on the flow velocity measurement traceable to the national standard was uniquely verified. When RTA was applied to the flow in the sonic nozzle, the experimental and simulation results were in good agreement, which suggest the possibility of measuring the supersonic flow using RTA. Furthermore, similar results were confirmed with a thermocouple and platinum thermometer given the same recovery temperature coefficient, and it was found that the flow velocity measurement by RTA does not depend on the type of thermometer. These results indicate the possibility of realizing flow velocity measurement without calibration using Eq. (3) by giving a recovery temperature coefficient suitable for the thermometer shape (e.g., 0.83 for a probe sensor). The main feature of RTA is the simplicity and ease of micrometer-order point measurement using existing inexpensive thermometers. In comparison with existing anemometers, for hot wire anemometry requires a wire length of more than 100 times the wire diameter for achieving a uniform temperature distribution at the hot wire center, and the output results are line averaged rather than point. Optical methods such as PIV and LDV are not inexpensive and easy to use due to the bottleneck of using expensive equipment and lasers. The thermometers used in RTA are commercial products, and as mentioned above, RTA can measure the flow velocity without calibration by using the appropriate recovery temperature coefficient. Therefore, RTA can be easily used by engineers.

The issues of RTA at this stage are described below. In the region of small flow velocity, the difference between \( T_0 \) and \( T_r \) is also small, which may make increase the difficulty of application of RTA in terms of the accuracy of temperature measurement. In this study, the difference between \( T_0 \) and \( T_r \) is approximately 0.13 °C at approximately 40 m/s. To expand the application range and improve the measurement accuracy of RTA, measurements of \( T_0 \) and \( T_r \) at the same point are necessary. As a condition for the application of RTA, the Prandtl number of the fluid must be less than 1.0 in terms of the difference between \( T_0 \) and \( T_{in} \) for instance, RTA cannot be applied to water at 20 °C. (Prandtl number: approximately 7.0). Several prospects for measurements using RTA exist in the future. For instance, the measurement range of thermocouples extends to high temperatures, and therefore, RTA is applicable to high-enthalpy flow, characteristic of the aerospace field. Moreover, the heat capacity of small temperature sensors is small, and therefore, RTA is expected to demonstrate a high response to velocity fluctuations and can be applied to turbulence research.

**Methods**

**Experimental procedures and equipment.** In the comparison experiment with the reference velocity and the velocity measurement experiment in the sonic nozzle, the flow was generated by sucking air with a relative humidity of less than 50% from the atmosphere. The thermocouple measurement system comprised a sensor (TPK-01, Mother tool) and a logger (TM-947SD, Mother tool). The platinum thermometer measurement system also comprised a sensor (special order, Netsushin) and a logger (Multimeter 2001, Keithley). \( T_0 \) and \( T_r \) are measured at intervals of about 10 and 1 s, respectively, and are the average values obtained over 100 s of measurement. \( T_0 \) was derived using

\[
T_0 = \left( 1 + \frac{\gamma - 1}{2} \frac{M_0^2}{\gamma} \right) T_p
\]

where \( M_0 \) denotes the Mach number in the pipe, and \( T_p \) denotes the temperature in the pipe. If the flow velocity through the pipe is sufficiently low, the measured temperature can be treated as \( T_{in} \). For the measurement accuracy of each system in the comparison experiment with the reference velocity, all reference nozzles were calibrated with a constant volume tank, which is the national standard for the gas flow rate; furthermore, the expanded uncertainty of the calibration is approximately 0.17% (coverage factor: 2)\(^{27}\). The flow rate fluctuation of 5–1000 m³/h generated by the reference nozzles and blower is controlled to ±0.1%/10 min by the buffer tank and temperature controller. The expanded uncertainty of the flow rate is approximately 0.28% (coverage factor: 2)\(^{28}\). The expanded uncertainty of the reference velocity based on the flow rate reference is estimated to be approximately 0.63% (coverage factor: 2)\(^{15}\). The traverse apparatus (Siguma Koki) shown in Fig. 4a has a movement accuracy of 3 μm and can be operated by an actuator from outside the traverse apparatus.

**Numerical simulation.** Three-dimensional unsteady numerical simulations of a compressible fluid with an ideal gas were performed using OpenFOAM to verify the flow velocity inside the nozzle measured by RTA. The finite volume method was used to solve the governing equations (continuity, momentum, and energy equations) given by

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0
\]
The data that support the findings of this study are available from the corresponding authors upon reasonable request.

Data availability
The data that support the findings of this study are available from the corresponding authors upon reasonable request.

Received: 11 October 2021; Accepted: 23 November 2021
Published online: 10 December 2021

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Table 1. Conditions of the numerical simulations.

| Conditions of the numerical simulations | Value |
|----------------------------------------|-------|
| Total number of cells                  | 1,691,062 |
| Solver                                 | rhoPimpleFoam |
| Turbulence model                       | k–ω shear stress transport |
| Boundary condition for velocity        | Inlet: zero gradient |
|                                       | Nozzle wall: 0 m/s |
|                                       | Outlet: InletOutlet (zero gradient with backflow prevention) |
| Boundary condition for pressure        | Inlet: 101 kPa |
|                                       | Nozzle wall: zero gradient |
|                                       | Outlet: 50.5, 45.45, 40.4, or 35.35 kPa |
| Boundary condition for temperature     | Inlet: 20 °C |
|                                       | Nozzle wall: zero gradient |
|                                       | Outlet: zero gradient |
| Courant number                         | 0.7 |
| Simulation time                        | 0.005 s |

Where \( \rho \), \( t \), \( u \), \( p \), \( \mu \), \( E \), \( k \), and \( T \) denote the density, time, velocity vector, pressure, viscosity coefficient, total energy per unit mass, thermal conductivity, and temperature, respectively. \( \mu \) was calculated using Sutherland’s equation, and \( \kappa \) was calculated from the viscosity coefficient and the constant volume specific heat. Values of 1005 and 717.9 \((J/K·kg^{-1})\) were used for the constant pressure specific heat and constant volume specific heat, respectively. The pressure and velocity were coupled using the rhoPimpleFoam solver, which is a combination of PISO (pressure implicit with splitting operators) and SIMPLE (Semi-Implicit Method for Pressure). The \( k–ω \) shear stress transport was adopted as the turbulence model. A second-order accurate linear upward difference, linear Upwind limited, was used as the advection term for the velocity vector, internal energy \( e \), turbulent energy \( k \), and energy dissipation rate \( ω \). The total number of cells was approximately 1.69 million, and the mesh in the axial direction \((x)\) was approximately 1.3 times longer than that in the radial direction \((y, z)\). In the detailed mesh region inside the nozzle, the length in the radial directions was approximately 0.24 mm and was approximately 0.96 mm in other regions. Boundary layer meshes with a mesh height of about 0.2 times the normal mesh height were inserted on the wall surface for three layers. Ryzen Threadripper 3990X (Advanced Micro Devices) was used as the CPU, and the number of parallelisms was set to 118. A variable time width was set for the calculation, where the Courant number did not exceed 0.7. The conditions for the numerical simulations are listed in Table 1.
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Acknowledgements
This work was supported by a Grant-in-Aid for Early-Career Scientists from the Japan Society for the Promotion of Science (Grant No. 20K14657).

Author contributions
T.K. conducted the experiments and numerical simulations and drafted the manuscript. I.M. and I.A. contributed to the experimental setup and data interpretation. H.N. and M.T. supervised the research and contributed to reviewing the manuscript. All authors discussed the results and commented on the manuscript.

Funding
This article was funded by Japan Society for the Promotion of Science (Grant no. 20K14657).

Competing interests
The authors declare no competing interests.

Additional information

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