Three-Photon Decay of $J/\psi$ from Lattice QCD

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The rare decay $J/\psi \to 3\gamma$ acts as a probe of higher-order QCD effects.

Experimental difficulties: poor knowledge of matrix element
- Crystal Ball, $B < 5.5 \times 10^{-5}$, PRL 44,712(1980)
- CLEOc, $B = (1.2 \pm 0.3 \pm 0.2) \times 10^{-5}$, PRL 101,101801(2008)
- BESIII, $B = (1.13 \pm 0.18 \pm 0.2) \times 10^{-5}$, PRD 87,032003(2013)

Theoretical difficulties: perturbation fails
- For $\eta_c \to 2\gamma$, both photons are hard with half energies of the charmonium, perturbation is expected to work better;
- For $J/\psi \to 3\gamma$, there exits a soft photon, hindering the perturbation calculation.

We present the first lattice calculation for rare decay $J/\psi \to 3\gamma$.
Y.Meng, C.Liu and K-L.Zhang.(2019).
Decay amplitude on lattice

\[ M(t_f, t; t', t_i) = \epsilon_\mu \epsilon_\nu \epsilon_\rho \epsilon_\alpha M_{\mu \nu \rho \alpha} \]

\[ M_{\mu \nu \rho \alpha} = \frac{e^3}{Z(p) E(p) (t_f - t)} \times \int dt' e^{-\omega_2 |t' - t|} \int dt_i e^{-\omega_1 |t_i - t|} \]

\[ \times \sum_{x, y, z} e^{i(q_3 \cdot z + q_2 \cdot y + q_1 \cdot x)} \langle 0 | \hat{T} \{ O^\alpha_{J/\psi}(0, t_f) j_\rho(z, t) j_\nu(y, t') j_\mu(x, t_i) \} | 0 \rangle \]

- Local current \( j_\mu(x) = Z_V Q_c \bar{c} \gamma_\mu c(x) \);
- Using 'sequential' method to calculate the four-point function;
- The three photons can’t be on-shell simultaneously, with virtualities \( Q_i^2 \).
Decay amplitude $\rightarrow$ decay width

- Conventional: amplitude parameterization. For $\eta_c \rightarrow 2\gamma$:

$$
\mathcal{M}_{\mu\nu} = 2\left(\frac{2}{3}e\right)^2 m_{\eta_c}^{-1} F(Q_1^2, Q_2^2) \epsilon_{\mu\nu\rho\sigma} q_1^\rho q_2^\sigma, \quad \Gamma(\eta_c \rightarrow 2\gamma) = \pi \alpha_e^2 (\frac{16}{81}) m_{\eta_c} |F(0,0)|^2
$$

J.J.Dudek and E.G.Edwards.(2006)

- For three-photon case: $\mathcal{M}_{\mu\nu\rho\alpha}(q_1, q_2, q_3) = \sum_{\text{perm}} \mathcal{M}_{\mu\nu\rho\alpha}(q_1, q_2, q_3)$

$$
\mathcal{M}_{\mu\nu\rho\alpha}(q_1, q_2, q_3) = \mathcal{F}_{123} \frac{1}{q_1 \cdot q_3} \left( \frac{q_3^\mu q_1^\rho}{q_1 \cdot q_3} - g_{\mu\rho} \right) q_1^\alpha \left( \frac{q_3^\nu}{q_2 \cdot q_3} - \frac{q_1^\nu}{q_1 \cdot q_2} \right) \\
+ \mathcal{G}_{123} \left[ \frac{1}{q_2 \cdot q_3} \left( \frac{q_1^\alpha q_3^\mu}{q_1 \cdot q_3} - g^{\alpha\mu} \right) \left( \frac{q_1^\nu q_2^\rho}{q_1 \cdot q_2} - g^{\nu\rho} \right) + \frac{1}{q_1 \cdot q_3} \left( \frac{q_1^\nu}{q_1 \cdot q_2} - \frac{q_3^\nu}{q_2 \cdot q_3} \right) \left( q_1^\rho g^{\alpha\mu} - q_1^\alpha g^{\mu\rho} \right) \right] \\
+ \mathcal{H}_{123} \frac{1}{q_1 \cdot q_3} \left( \frac{q_1^\alpha q_3^\mu}{q_1 \cdot q_3} - g^{\alpha\mu} \right) \left( \frac{q_3^\nu q_2^\rho}{q_2 \cdot q_3} - g^{\nu\rho} \right)
$$

G.S.Adkins.(1996).

- It is a redundant process for lattice simulation.
The three-body decay width:

\[
\Gamma_3 = \frac{1}{3!} \frac{1}{2m} \int \frac{d^3q_1}{(2\pi)^3 2\omega_1} \frac{d^3q_2}{(2\pi)^3 2\omega_2} \frac{d^3q_3}{(2\pi)^3 2\omega_3} (2\pi)^4 \delta(p - q_1 - q_2 - q_3) |M|^2
\]

\[
= \frac{m}{1536\pi^3} \int_0^1 dx \int_{1-x}^1 dy |M|^2, \quad x \equiv 1 - 2q_2 \cdot q_3/m^2, \quad y \equiv 1 - 2q_1 \cdot q_2/m^2
\]

New approach: amplitude summation, define \( \mathcal{T} \) function

\[
\mathcal{T} \equiv |M|^2 = \frac{1}{3} \sum_{\mu\nu\rho\alpha} \sum_{\lambda_1 \lambda_2 \lambda_3 \lambda_0} |\epsilon^\lambda_1_\mu \epsilon^\lambda_2_\nu \epsilon^\lambda_3_\rho \epsilon^\lambda_0_\alpha \mathcal{M}_{\mu\nu\rho\alpha}|^2 = \frac{1}{3} \sum_{\mu\nu\rho\alpha} |\mathcal{M}_{\mu\nu\rho\alpha}|^2
\]

The decay width of \( J/\psi \rightarrow 3\gamma \):

\[
\Gamma(J/\psi \rightarrow 3\gamma) = \frac{m_{J/\psi}}{1536\pi^3} \int_0^1 dx \int_{1-x}^1 dy \mathcal{T}(x, y)
\]
Input parameters

- **Photon momenta:**
  - On-shell as possible: fix photon 1 and 3 on-shell exactly, minimize $Q_2$;
  - The $(x, y)$ cover the physical region as possible, i.e. $x \in [0, 1], y \in [1 - x, 1]$;
  - Fewer momenta to meet above requirements.

| Ensemble | $Q_1^2$ | $Q_3^2$ | $n_1$ | $n_3$ | $n_2$ | $\omega_1$ | $\omega_2$ | $\omega_3$ | $x$ | $y$ | $Q_2^2(GeV^2)$ |
|----------|---------|---------|-------|-------|-------|------------|------------|------------|-----|-----|----------------|
| I        | 0       | 0       | 111   | -1-1-2| 001   | 0.4680     | 0.6525     | 0.2134     | 0.7017| 0.9783| -0.1541        |
|          | 0       | 0       | 111   | -20-1 | 1-10  | 0.4680     | 0.5967     | 0.2692     | 0.7017| 0.8946| -0.4096        |
|          | 0       | 0       | 002   | 11-1  | -1-1-1| 0.5343     | 0.4680     | 0.3316     | 0.8011| 0.7017| -0.6077        |
|          | 0       | 0       | 002   | 11-2  | -1-10 | 0.5343     | 0.6525     | 0.1471     | 0.8011| 0.9783| -0.5690        |
| II       | 0       | 0       | 210   | -1-11 | -10-1 | 0.4257     | 0.3320     | 0.2905     | 0.8123| 0.6335| 0.0932         |
|          | 0       | 0       | 002   | 10-2  | -100  | 0.3810     | 0.4257     | 0.2415     | 0.7269| 0.8123| 0.1857         |
|          | 0       | 0       | 002   | 11-1  | -1-1-1| 0.3810     | 0.3320     | 0.3352     | 0.7269| 0.6335| 0.0187         |

- **On-shell fitting:**

\[
\mathcal{T}(x, y, Q_1^2, Q_2^2, Q_3^2) = \mathcal{T}(x, y) + \text{const} \times \sum_i Q_i^2
\]

- **Twisted Mass Ensembles:**

| Ens | $\beta$ | $a$(fm) | $V/a^4$ | $a\mu_{\text{sea}}$ | $m_\pi$(MeV) | $N_{\text{conf}}$ |
|-----|---------|---------|---------|---------------------|--------------|-----------------|
| I   | 3.9     | 0.085   | $24^3 \times 48$ | 0.004        | 315           | 40              |
| II  | 4.05    | 0.067   | $32^3 \times 64$ | 0.003        | 300           | 20              |
Input parameters

- *xy*-distribution

- Exchange symmetry: \( T(x, y, z) = T(y, x, z) = \ldots \)

- Current renormalization constant: \( Z_{V}^{I,II} = 0.6347(26), 0.6640(27) \).

\[
Z^{(\mu)}_{V} = \frac{p^{\mu}}{E(p)} \frac{1/2 \sum_{k} \Gamma^{(2)}_{\psi_{k}\psi_{k}}(p, t_{\text{source}} = T/2, t_{\text{sink}} = 0)}{\sum_{k} \Gamma^{(3)}_{\psi_{k}\gamma_{\mu}\psi_{k}}(p, t_{\text{source}} = T/2, t_{\text{sink}} = 0, t)}
\]

J.J.Dudek, E.G.Edwards and D.G.Richards.(2006)
Matrix elements

- **Four-point function** $\mathcal{M}_{\mu\nu\rho\alpha}$:

- **On-shell fitting**: $\mathcal{T}(x, y, Q_1^2, Q_2^2, Q_3^2)$
Cubic spline interpolation

- **Decay width:**
  \[ \Gamma(J/\psi \to 3\gamma) = 1.530(15)\text{eV}, 1.715(47)\text{eV} \]

- **Existing problems:**
  - The intermediate contribution \( J/\psi \to \gamma\eta_c \to 3\gamma \) to be removed;
  - Estimate the systematical error caused by cubic spline interpolation, for the region without data covered.
Dalitz analysis

- **Dalitz variables:**

\[
\frac{M(\gamma\gamma)_{l^g/s_m}}{m_{J/\psi}} = \max \left\{ \sqrt{1 - x}, \sqrt{1 - y}, \sqrt{x + y - 1} \right\}
\]

- Dalitz plot is the direct observable for the experiments.
- Bands in Dalitz plot indicate the intermediate two-body states.

**Dalitz plot**

![Dalitz plot](image)
Dalitz analysis

- Removing the $\gamma\eta_c$ contribution by setting the cut $M_{\text{cut}} = m_{\eta_c}$
  
  \begin{align*}
  (a) & : \sqrt{1-x} > M_{\text{cut}}/m_{J/\psi} \\
  (b) & : \sqrt{1-y} > M_{\text{cut}}/m_{J/\psi} \\
  (c) & : \sqrt{x+y-1} > M_{\text{cut}}/m_{J/\psi}
  \end{align*}

  \[ \Rightarrow 0.031\text{eV(Ens.I)}, \quad 0.034\text{eV(Ens.II)} \]

- Regarding the region without \((x, y)\) covered as systematic error, i.e.
  
  \begin{align*}
  (A) & \quad x \in [0.1, 0.3], \; y \in [1-x, 1] \\
  (B) & \quad x \in [1-y, 1], \; x \in [0.1, 0.3]
  \end{align*}

  \[ \Rightarrow 0.243\text{eV(Ens.I)}, \quad 0.274\text{eV(Ens.II)} \]

- The pure decay width:

  \[ \Gamma(J/\psi \rightarrow 3\gamma) = 1.499(15)(243) \text{ eV}; \quad 1.681(47)(274) \text{ eV} \]
$\mathcal{B}(J/\psi \rightarrow 3\gamma) = 2.13(14)(89) \times 10^{-5}$
Dalitz plot in experiments

- BESIII

\[ \mathcal{B} = (1.13 \pm 0.18 \pm 0.2) \times 10^{-5}, \quad N_{J/\psi} = 389. \]
Dalitz plot on lattice

- Normalized $\mathcal{T}$-function distribution:

$$\tilde{T}(x, y) = \frac{T^{int}(x, y)}{\int_0^1 dx \int_{1-x}^1 dy T^{int}(x, y)}$$

- No obvious bands on vertical region for the range $M(\gamma\gamma)_{sm} \in [0.1, 0.16], [0.5, 0.6], [0.9, 1]$, which correspond to the dominant sources $\gamma\pi_0/\eta/\eta'$ in experiments.
Importance of Dalitz analysis

- Providing a direct comparison with the experiments.

- The parametric analytical expression for the $T_{a \rightarrow 0}(x, y)$ could be used as the theoretical input for the matrix element of $J/\psi \rightarrow 3\gamma$ for the experiments.

- The $J/\psi$ events in BESIII are 100 times greater than ever before, a higher precision result of $J/\psi \rightarrow 3\gamma$ could be expected with $T_{a \rightarrow 0}(x, y)$ utilized.

### TABLE III. Summary of the relative systematic uncertainties. $B_{3\gamma}$ and $B_{J\eta\epsilon}$ stand for the measurements of branching fractions $B(J/\psi \rightarrow 3\gamma)$ and $B(J/\psi \rightarrow \gamma\epsilon, \epsilon \rightarrow \gamma\gamma)$, respectively. A dash (–) means the uncertainty is negligible.

| Source                                | $B_{3\gamma}$ | $B_{J\eta\epsilon}$ |
|---------------------------------------|---------------|---------------------|
| Signal model                          | 15            | –                   |
| $\eta\epsilon$ width                  | –             | 5                   |
| $\eta\epsilon$ line shape             | 1             | 1                   |
| Resolution                            | 3             | 9                   |
| $M(\pi^+\pi^-)$ recoil window         | 4             | 4                   |
| $\pi^0, \eta, \eta'$ rejection        | 0.5           | 5                   |
| PWA model                             | 2             | 2                   |
| Photon detection                      | 3             | 3                   |
| Tracking                              | 2             | 2                   |
| Number of good photons                | 0.5           | 0.5                 |
| Kinematic fit and $\chi^2_{4C}$ requirenent | 2             | 2                   |
| Fitting                               | 5             | 5                   |
| Number of $\psi(3686)$                | 0.8           | 0.8                 |
| $B(\psi(3686) \rightarrow \pi^+\pi^- J/\psi)$ | 1.2           | 1.2                 |
| Total                                 | 18            | 14                  |

Taken from BESIII.
New result for $\eta_c \rightarrow 2\gamma$

- Previous results:

| Methods                  | $B \times 10^{-4}$ | $\delta B \times 10^{-4}$ | Refs                  |
|--------------------------|--------------------|--------------------------|-----------------------|
| Quenched Wilson          | 0.83               | 0.50                     | J.J.Dudek et al.(2006)|
| $N_f = 2$ twisted mass    | 0.351              | 0.004                    | CLQCD(2016)           |
| NRQCD                    | 3.1 $\sim$3.2     | -                        | F.Feng(2017)          |
| Exp                      | 1.57               | 0.12                     | PDG(2018)             |

- Amplitude summation:

- $\Gamma : \epsilon_\mu \epsilon_\nu \rightarrow -g_{\mu \nu}$
- $\Gamma_W : \epsilon_\mu \epsilon_\nu \rightarrow -g_{\mu \nu} + (q_\mu^i \bar{q}_\nu^i + \bar{q}_\mu^i q_\nu^i)/2\omega_i^2$

$B(\eta_c \rightarrow 2\gamma) = 1.29(3)(18) \times 10^{-4}$

PRD 102,034502(2020).
Conclusion

- We present the first lattice calculation for $J/\psi \rightarrow 3\gamma$;
- A new method is proposed to calculate multi-photon decay directly, by summing over final and initial state polarizations.
- The Dalitz analysis on lattice is suggested.
- The new method is applied for $\eta_c \rightarrow 2\gamma$, and a most reliable result is obtained.

Outlook

- A new strategy is in progress for $J/\psi \rightarrow 3\gamma$, the large systematic error can be avoided.
- The $T_{a \rightarrow 0}(x, y)$ is our next target, be applied for the experiments to avoid the large systematic uncertainty.
Thank you!