The Multiparty Coherent Channel and its Implementation with Linear Optics

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Abstract.

The coherent nat (conat) channel is a useful resource for coherent communication, producing coherent teleportation and coherent superdense coding. We extend the conat channel to multiparty conat channel by giving a definition of it including the multiparty position-quadrature conat channel and the multiparty momentum-quadrature conat channel. We provide two methods to implement this channel using linear optics. One method is the multiparty version of coherent communication assisted by entanglement and classical communication (CCAECC). The other is multiparty coherent superdense coding. Finally, we discuss the new applications of this channel in controlled teleportation and quantum secret sharing (QSS).

1. Introduction

The coherent bit (cobit) with discrete variables (DV) proposed by Aram Harrow [13] is a powerful resource intermediate between a quantum bit (qubit) and a classical bit (cbit). A cobit communication between two parties with DV performs the following map: $|x\rangle^A \rightarrow |x\rangle^A|x\rangle^B$, $x \in \{0,1\}$. For example, if Alice possesses a arbitrary qubit $|\psi\rangle^A = \alpha|0\rangle^A + \beta|1\rangle^A$ and sends it through a cobit channel, the cobit channel generates the state $|\phi\rangle^{AB} = \alpha|0\rangle^A|0\rangle^B + \beta|1\rangle^A|1\rangle^B$. The cobit channel in two parties maintains the coherent superpositions of Alice’s input state, this is the root of the channel’s name interpreted by Harrow [13]. In [22], Wilde and Brun interpreted and analyzed the differences and connections among classical communication, quantum communication, entanglement and cobit channel.

Recently, Wilde, Krovi, and Brun extended the cobit channel from discrete variables to continuous variables (CV) and introduced the concept of “conat channel” [21] as the CV counterpart of the DV “cobit channel”. Wilde and Brun gave the definitions of the ideal position-quadrature (PQ) conat channel and momentum-qudrature (MQ) conat channel in the Schroedinger-picture, as the following map $|x\rangle^A \rightarrow |x\rangle^A|x\rangle^B$, $|x\rangle$ represents position eigenstate or momentum eigenstate respectively. Nonideal (finitely squeezed) conat channels are also discussed in the Heisenberg picture.
The coherent channel provides for coherent communication. Coherent communication has several useful characteristics and applications. It provides coherent versions of continuous-variable teleportation and continuous-variable superdense coding, moreover they are dual under resource reversal [13, 18]. Also, it can be applied in remote preparation (RSP) consuming less entanglement than standard ones [13]. Coherent communication also can be used in error correcting codes [17, 19].

In this paper, we extend the notion of conat channel to multiple parties that can perform multiparty coherent communication. Also, we construct two methods to implement it with linear optics and we analyze the noise accumulation in different prepared EPR [1] resources. Multiparty conat channel has all the useful properties of the bipartite conat channel, we propose some new applications of multiparty conat channel in controlled communication and quantum secret sharing (QSS). Insights into the conat channel occur by detecting one quadrature of Alice’s modes and replacing Bob’s respective quadrature, finally Bob can extract the transmitted message, which will be highlighted in our protocols later.

Our paper is organized as follows. In Section 2, we provide a general definition of multiparty position-quadrature (PQ) conat channel and multiparty momentum-quadrature (MQ) conat channel in Heisenberg representation. In Section 3, two implementations of the protocol using realistic linear optical devices are outlined. In Section 4, we discuss the applications of our protocol. Finally, a brief conclusion is given in Section 5.

2. Definitions of multiparty conat channel

We provide a general definition of multiparty position-quadrature (PQ) conat channel and multiparty momentum-quadrature (MQ) conat channel in Heisenberg representation. The channel has only one sender Alice and n receivers Alice, Bob, Charlie · · · and Nick. Notice that the sender is also among the receivers. The multiparty PQ conat channel \( \hat{\Delta}_X \) copies the position quadrature of the sender to all the involving receivers with respective noise. The resulting multimode state is similar to multiparty GHZ [2] entanglement. The difference is that the total momentum state of the output modes is close to the original momentum of the sender \( \hat{p}_A \), encoding the transmitted message into all the receivers involved in this channel, while the total momentum of GHZ entanglement is zero. The multiparty MQ conat channel is defined similarly.

Definition 1: The mapping of a multiparty PQ conat channel \( \hat{\Delta}_X \) is presented as follows and the constraints of the channel are also given as well.

Mapping:

\[
[\hat{x}_A \hat{p}_A]^T \rightarrow [\hat{x}_{A'} \hat{p}_{A'} \hat{x}_{B'} \hat{p}_{B'} \cdots \hat{x}_{N'} \hat{p}_{N'}]^T
\]
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Constraints:
\[
\begin{aligned}
\hat{x}_{A'} &= \hat{x}_A \\
\hat{x}_{B'} &= \hat{x}_A + \hat{x}_{\Delta x}^1 \\
\hat{x}_{C'} &= \hat{x}_A + \hat{x}_{\Delta x}^2 \\
\vdots \\
\hat{x}_{N'} &= \hat{x}_A + \hat{x}_{\Delta x}^{n-1} \\
\hat{p}_{A'} &= \hat{p}_A + \hat{p}_{\Delta x} \\
\langle \hat{x}_{\Delta x}^1 \rangle &= \langle \hat{x}_{\Delta x}^2 \rangle = \cdots = \langle \hat{x}_{\Delta x}^{n-1} \rangle = 0 \\
\langle \hat{p}_{\Delta x} + \hat{p}_{B'} + \hat{p}_{C'} + \cdots + \hat{p}_{N'} \rangle &= 0 \\
\langle (\hat{x}_{\Delta x}^1)^2 \rangle &\leq \epsilon_1 \\
\langle (\hat{x}_{\Delta x}^2)^2 \rangle &\leq \epsilon_2 \\
\vdots \\
\langle (\hat{x}_{\Delta x}^{n-1})^2 \rangle &\leq \epsilon_{n-1} \\
\langle (\hat{p}_{\Delta x} + \hat{p}_{B'} + \hat{p}_{C'} + \cdots + \hat{p}_{N'})^2 \rangle &\leq \epsilon_n
\end{aligned}
\] (2)

The canonical commutation relations of the Heisenberg-picture observables are as follows:
\[
[\hat{x}_{A'}, \hat{p}_{A'}] = [\hat{x}_{B'}, \hat{p}_{B'}] = \cdots = [\hat{x}_{N'}, \hat{p}_{N'}] = i
\] (4)

Alice is the sender and possesses mode \( A \), and the number of the receivers is \( n \), from Alice to Nick who possess modes \( A \) through \( N \) respectively. Alice is the sender and also among the receivers. The constraints of position quadratures in (2) and (3) indicate that Alice’s position quadrature remains unchanged, and Alice’s position quadrature copies to the position quadratures of the other receivers with the additional noise \( \hat{x}_{\Delta x}^1, \hat{x}_{\Delta x}^2, \cdots, \hat{x}_{\Delta x}^{n-1} \) respectively. The conditions of momentum quadratures in (2) and (3) ensure that the total momentum is close to Alice’s original momentum with a noise \( \hat{p}_{\Delta x} \). The parameters \( \epsilon_1, \epsilon_2 \cdots \) and \( \epsilon_n \) in (3) determine the performance of channel by bounding the noises presented.

Definition 2: The definition for a multiparty MQ conat channel is similar to definition 1. We present it as follows.

Mapping:
\[
[\hat{x}_A, \hat{p}_A]^T \rightarrow [\hat{x}_{A'}, \hat{p}_{A'} \hat{x}_{B'}, \hat{p}_{B'} \cdots \hat{x}_{N'}, \hat{p}_{N'}]^T
\] (5)

Constraints:
\[
\begin{aligned}
\hat{p}_{A'} &= \hat{p}_A \\
\hat{p}_{B'} &= \hat{p}_A + \hat{p}_{\Delta p}^1 \\
\hat{p}_{C'} &= \hat{p}_A + \hat{p}_{\Delta p}^2 \\
\vdots \\
\hat{p}_{N'} &= \hat{p}_A + \hat{p}_{\Delta p}^{n-1} \\
\hat{x}_{A'} &= \hat{x}_A + \hat{x}_{\Delta p}
\end{aligned}
\] (6)
\[
\begin{aligned}
\langle \hat{p}_{\Delta_p}^1 \rangle &= \langle \hat{p}_{\Delta_p}^2 \rangle = \cdots = \langle \hat{p}_{\Delta_p}^{n-1} \rangle = 0 \\
\langle \hat{x}_{\Delta_p} + \hat{x}_{B'} + \hat{x}_{C'} + \cdots + \hat{x}_{N'} \rangle &= 0 \\
\langle (\hat{p}_{\Delta_p}^1)^2 \rangle &\leq \epsilon_1 \\
\langle (\hat{p}_{\Delta_p}^2)^2 \rangle &\leq \epsilon_2 \\
\vdots \\
\langle (\hat{p}_{\Delta_p}^{n-1})^2 \rangle &\leq \epsilon_{n-1} \\
\langle (\hat{x}_{\Delta_p} + \hat{x}_{B'} + \hat{x}_{C'} + \cdots + \hat{x}_{N'})^2 \rangle &\leq \epsilon_n
\end{aligned}
\]

The canonical commutation relations of the Heisenberg-picture observables are as follows:

\[
[\hat{x}_{A'}, \hat{p}_{A'}] = [\hat{x}_{B'}, \hat{p}_{B'}] = \cdots = [\hat{x}_{N'}, \hat{p}_{N'}] = i
\]

3. Implementations of multiparty conat channel using linear optics

In this section, we outline two different methods to implement the multiparty conat channel. The first way is implemented by the multiparty version of coherent communication assisted by entanglement and classical communication (CCAECC) \cite{22}. The second method is multiparty coherent superdense coding which implements two multiparty coherent channels. The detailed description is given as follows.

**Method 1:**

This method requires the resources including \((n+1)\)-party GHZ entanglement and classical communication. For the purpose of simplicity, we show the three-party PQ conat channel as an example of the general \(n\)-party conat channel. The implementation of multiparty MQ conat channel is similar to multiparty PQ conat channel. We generalize three-party to \(n\)-party later at the end of the description of this method.

We use P. van Loock and S. L. Braunstein’s protocol \cite{7} to generate a four-mode
GHZ entanglement. The correlation equations are given as follows:

\[
\begin{align*}
\hat{x}_{A1} &= \hat{x}_1 = \frac{1}{\sqrt{4}} e^{r_1} \hat{x}_1^{(0)} + \frac{\sqrt{3}}{4} e^{-r_2} \hat{x}_2^{(0)} \\
\hat{p}_{A1} &= \hat{p}_1 = \frac{1}{\sqrt{4}} e^{-r_1} \hat{p}_1^{(0)} + \frac{3}{4} e^{r_2} \hat{p}_2^{(0)} \\
\hat{x}_{A2} &= \hat{x}_2 = \frac{1}{\sqrt{4}} e^{r_1} \hat{x}_1^{(0)} - \frac{\sqrt{12}}{4} e^{-r_2} \hat{x}_2^{(0)} + \frac{\sqrt{2}}{3} e^{-r_3} \hat{x}_3^{(0)} \\
\hat{p}_{A2} &= \hat{p}_2 = \frac{1}{\sqrt{4}} e^{-r_1} \hat{p}_1^{(0)} - \frac{\sqrt{12}}{4} e^{r_2} \hat{p}_2^{(0)} + \frac{\sqrt{2}}{3} e^{r_3} \hat{p}_3^{(0)} \\
\hat{x}_B &= \hat{x}_3 = \frac{1}{\sqrt{4}} e^{r_1} \hat{x}_1^{(0)} - \frac{\sqrt{12}}{4} e^{-r_2} \hat{x}_2^{(0)} - \frac{\sqrt{6}}{4} e^{-r_3} \hat{x}_3^{(0)} + \frac{\sqrt{2}}{\sqrt{2}} e^{-r_4} \hat{x}_4^{(0)} \\
\hat{p}_B &= \hat{p}_3 = \frac{1}{\sqrt{4}} e^{-r_1} \hat{p}_1^{(0)} - \frac{\sqrt{12}}{4} e^{r_2} \hat{p}_2^{(0)} - \frac{\sqrt{6}}{4} e^{r_3} \hat{p}_3^{(0)} + \frac{\sqrt{2}}{\sqrt{2}} e^{r_4} \hat{p}_4^{(0)} \\
\hat{x}_C &= \hat{x}_4 = \frac{1}{\sqrt{4}} e^{r_1} \hat{x}_1^{(0)} - \frac{\sqrt{12}}{4} e^{-r_2} \hat{x}_2^{(0)} - \frac{\sqrt{6}}{4} e^{-r_3} \hat{x}_3^{(0)} - \frac{\sqrt{2}}{\sqrt{2}} e^{-r_4} \hat{x}_4^{(0)} \\
\hat{p}_C &= \hat{p}_4 = \frac{1}{\sqrt{4}} e^{-r_1} \hat{p}_1^{(0)} - \frac{\sqrt{12}}{4} e^{r_2} \hat{p}_2^{(0)} - \frac{\sqrt{6}}{4} e^{r_3} \hat{p}_3^{(0)} - \frac{\sqrt{2}}{\sqrt{2}} e^{r_4} \hat{p}_4^{(0)}
\end{align*}
\] (9)

Figure 1. The three-party position-quadrature (PQ) protocol requires a four-mode GHZ entanglement in which we label the four modes as $A_1, A_2, B, C$. Alice possesses mode $A_1$ and $A_2$, and Bob possesses mode $B$ while Charlie possesses mode $C$. Alice has an input mode $A$ that contains the transmitted message. The elements used in this figure are outlined: $AM$ means an amplitude modulator which displaces the position quadrature of an optical mode [22]. $PM$ means a phase modulator which kicks the momentum quadrature of an optical mode [22]. $BS$ means a beam splitter.
where $\hat{x}_1^{(0)}$, $\hat{p}_1^{(0)}$, $\hat{x}_2^{(0)}$, $\hat{p}_2^{(0)}$, $\hat{x}_3^{(0)}$, $\hat{p}_3^{(0)}$, $\hat{x}_4^{(0)}$ and $\hat{p}_4^{(0)}$ are the quadratures of the four original vacuum modes sent through their protocol. $r_1$, $r_2$, $r_3$ and $r_4$ are the squeezing coefficients of them, and we assume all the four squeezing coefficients are equal to $r$.

After introducing the required resources, we turn to the procedures of this method. We present the detailed transformations of the operators in Heisenberg picture as follows.

**Step 1.** Alice mixes her mode $A$ and $A_1$ locally on a balanced (50%) beam splitter (BS) generating the modes (+) and (-).

$$\hat{x}_\pm = (\hat{x}_A \pm \hat{x}_{A_1})/\sqrt{2}, \quad \hat{p}_\pm = (\hat{p}_A \pm \hat{p}_{A_1})/\sqrt{2} \quad (10)$$

**Step 2.** We denote $\hat{x}_A$, $\hat{x}_{A_1}$, $\hat{x}_B$ and $\hat{x}_{C}$ in terms of $\hat{x}_-$ and $\hat{p}_+$.

$$\begin{align*}
\hat{x}_A & = \hat{x}_A - \hat{x}_{A_1} + \hat{x}_{A_1} - (\hat{x}_A - \hat{x}_{A_1}) = \hat{x}_A - \hat{x}_{A_1} + \hat{x}_{A_2} - \sqrt{2}\hat{x}_-
\hat{p}_A & = \hat{p}_A + (\hat{p}_{A_1} + \hat{p}_{A_2} + \hat{p}_B + \hat{p}_C) - \hat{p}_B - \hat{p}_C - \sqrt{2}\hat{p}_+
\hat{x}_B & = \hat{x}_A - \hat{x}_{A_1} + \hat{x}_B - \sqrt{2}\hat{x}_-
\hat{x}_C & = \hat{x}_A - \hat{x}_{A_1} + \hat{x}_C - \sqrt{2}\hat{x}_-
\end{align*} \quad (11)$$

Then Alice measures $\hat{x}_-$ and $\hat{p}_+$ through the homodyne detection. After the homodyne detection, operator $\hat{x}_-$ and $\hat{p}_+$ collapse to value $x_-$ and $p_+$, then she sends the measurement value $x_-$ to Bob and Charlie.

**Step 3.** Alice displaces the position quadrature of his mode $A_2$ by $\sqrt{2}x_-$ and her momentum quadrature by the value of $\sqrt{2}p_+$, Bob and Charlie displace the position quadrature of their possessed mode $B$ and $C$ respectively by $\sqrt{2}x_-$. After the modulations of three parties, we get

$$\begin{align*}
\hat{x}_A' & = \hat{x}_A - (\hat{x}_A - \hat{x}_{A_2}) - \sqrt{2(1-\eta)/\eta}\hat{x}_1^{(0)}
\hat{p}_A' & = \hat{p}_A + (\hat{p}_{A_1} + \hat{p}_{A_2} + \hat{p}_B + \hat{p}_C) - \hat{p}_B - \hat{p}_C + \sqrt{2(1-\eta)/\eta}\hat{p}_2^{(0)}
\hat{x}_B' & = \hat{x}_B - (\hat{x}_A - \hat{x}_B) - \sqrt{2(1-\eta)/\eta}\hat{x}_2^{(0)}
\hat{x}_C' & = \hat{x}_C - (\hat{x}_A - \hat{x}_C) - \sqrt{2(1-\eta)/\eta}\hat{x}_1^{(0)}
\hat{p}_B' & = \hat{p}_B, \quad \hat{p}_C' = \hat{p}_C
\end{align*} \quad (12)$$

Finally, we can find the results we obtained satisfy the constraints in (2) and (3), we get that

$$\epsilon_1 = \epsilon_2 = 2e^{-2r}, \quad \epsilon_3 = 4e^{-2r} + 2(1-\eta)/\eta \quad (13)$$

In this case , $n$ is equal to 3. As $n$ increases, $\epsilon_1$, $\epsilon_2 \cdots \epsilon_{n-1}$ remain unchanged and $\epsilon_n$ amounts to $(n+1)e^{-2r} + 2(1-\eta)/\eta$. When it comes to the general $n$-party PQ conat channel in this method, we can implement it easily and similarly using the $(n+1)$-party GHZ entanglement. The implementation of multiparty MQ conat channel is similar to multiparty PQ conat channel, viewing the operator $\hat{x}$ as $\hat{p}$ and operator $\hat{p}$ as $\hat{x}$. The required resource is a GHZ-like entanglement state with total position $\hat{x}_A + \hat{x}_A + \hat{x}_B + \cdots + \hat{x}_N \to 0$ and relative momenta equal. So we omit these discussions here.

**Method 2:**
Widle, Krovi and Brun gave the protocol of coherent superdense coding recently [21]. Inspired by their work, we provide a multiparty version of coherent superdense coding. The protocol is equivalent to two multiparty concat channels, a multiparty PQ concat channel and a multiparty MQ concat channel. In this method, the channel has only one sender which has two transported modes, and has \( n \) receivers and each one possesses two modes. Also, \((n-1)\) EPR pairs among the receivers are required.

We also use P. van Loock and S. L. Braunstein’s method to generate a EPR pair. We use the Heisenberg representation to describe it as

\[
\begin{align*}
\hat{x}_1 &= \left( e^{+r}\hat{x}_1^{(0)} + e^{-r}\hat{x}_2^{(0)} \right) / \sqrt{2} \\
\hat{p}_1 &= \left( e^{-r}\hat{p}_1^{(0)} + e^{+r}\hat{p}_2^{(0)} \right) / \sqrt{2} \\
\hat{x}_2 &= \left( e^{+r}\hat{x}_1^{(0)} - e^{-r}\hat{x}_2^{(0)} \right) / \sqrt{2} \\
\hat{p}_2 &= \left( e^{-r}\hat{p}_1^{(0)} - e^{+r}\hat{p}_2^{(0)} \right) / \sqrt{2}
\end{align*}
\]  
(14)

Local quantum nondemolition (QND) interactions are also required. The transformations of QND interaction in Heisenberg picture are given as follows [16]:

\[
\begin{align*}
\hat{x}_{1'} &= \hat{x}_1, \quad \hat{p}_{1'} = \hat{p}_1 - \hat{p}_2 \\
\hat{x}_{2'} &= \hat{x}_1 + \hat{x}_2, \quad \hat{p}_{2'} = \hat{p}_2
\end{align*}
\]  
(15)

The QND interaction with a phase adjust operation means the following transformations [16]:

\[
\begin{align*}
\hat{x}_{1'} &= \hat{x}_1 - \hat{x}_2, \quad \hat{p}_{1'} = \hat{p}_1 \\
\hat{x}_{2'} &= \hat{x}_2, \quad \hat{p}_{2'} = \hat{p}_1 + \hat{p}_2
\end{align*}
\]  
(16)

![Figure 2](image)

**Figure 2.** On the left side of the vertical line, we present all the scenarios of graph which represents the prepared entanglement resources when \( n \) is equal to 3. The number of total scenarios is three. On the right side of the vertical line, the graph indicates the best prepared entanglement resources for \( n \)-party concat channel.

Our protocol requires \((n-1)\) EPR pairs among the receivers. We illustrate these requirements in Figure 2, which gives the scenarios where \( n \) is equal to 3. We use a graph to illustrate the prepared entanglement among the parties involved in the channel. A vertex in the graph represents an individual party in the channel, and the edge between two vertices plays the role of EPR pair between the two parties. When two vertices are the end points of a edge, they are defined as being adjacent. We refer two vertices as connected when a path exists between them. A connected graph is a graph in which any two of the vertices are connected. This method requires that the graph which represents the prepared entanglement resources must be connected. The channel which we want to
implement has \( n \) parties involved. The \((n-1)\) EPR pairs prepared among the \( n \) parties ensure that the representing graph is connected and any two vertices of the graph has only one path.

Without loss of generalization, we discuss two scenarios involved in Figure 2. Scenario 1 corresponds to the first graph in Figure 2. Scenario 2 corresponds to the second graph. Since \( n \) is equal to 3, the channel involves one sender and three receivers, two EPR pairs is also required.

**Scenario 1.** In this scenario, Alice and Bob shares an EPR entanglement labeled as mode 3 in Alice’s possession and mode 4 in Bob’s possession, Bob and Charlie shares an EPR entanglement labeled as mode 5 in Bob’s possession and mode 6 in Charlie’s possession. Alice possesses mode 1 and 2 which is to be transported through the channel. Figure 3 gives the schematic linear optics circuit.

![Schematic linear optics circuit](image)

**Figure 3.** This figure outlines our scheme. The thick red line in this figure represents a quantum channel between two parties. The local operations and modes are enclosed by the dashed with the name of the party. The blue thin rectangle means the phase shifter about \( \pi \).

In step 1, Alice couples her mode 2 and 3 in QND interaction, and then couples mode 1 and 3 in QND phase adjust interaction. In step 2, Alice sends her mode 3 to Bob through a quantum channel, so Bob now possesses three modes, mode 3, 4 and 5. Then he performs a series of QND interactions, first QND interaction in mode 3, 4 then QND interaction in mode 4, 5 finally QND phase adjust interaction in mode 3, 5. In step 3, Bob sends his mode 5 to Charlie through a quantum channel. Charlie couples his two modes in a QND interaction, the we can get the resulting modes which are given
as follows in Heisenberg picture.

\[
\begin{align*}
\hat{x}_1' &= \hat{x}_1 - (\hat{x}_2 + \hat{x}_3), \quad \hat{p}_1' = \hat{p}_1 \\
\hat{x}_2' &= \hat{x}_2, \quad \hat{p}_2' = \hat{p}_2 - \hat{p}_3 \\
\hat{x}_3' &= \hat{x}_2 + \hat{x}_3 - (\hat{x}_2 + \hat{x}_3 - \hat{x}_4 + \hat{x}_5), \quad \hat{p}_3' = \hat{p}_1 + \hat{p}_3 + \hat{p}_4 \\
\hat{x}_4' &= \hat{x}_2 + \hat{x}_3 - \hat{x}_4, \quad \hat{p}_4' = -\hat{p}_4 - \hat{p}_5 \\
\hat{x}_5' &= \hat{x}_2 + \hat{x}_3 - \hat{x}_4 + \hat{x}_5, \quad \hat{p}_5' = \hat{p}_1 + \hat{p}_3 + \hat{p}_4 + \hat{p}_5 + \hat{p}_6 \\
\hat{x}_6' &= \hat{x}_2 + \hat{x}_3 - \hat{x}_4 + \hat{x}_5 - \hat{x}_6, \quad \hat{p}_6' = -\hat{p}_6
\end{align*}
\] (17)

Modes 1’, 3’, 5’ implement a three-party MQ conat channel by satisfying the constraints in definition 2; and modes 2’, 4’, 6’ satisfy definition 1 and performs a three-party PQ conat channel.

**Scenario 2.** In this scenario, Alice possesses four modes in the beginning, mode 1, 2, 3, 5, Bob possesses mode 4 and Charlie possesses mode 6. Mode 1 and 2 is the transmitted modes through the channel while modes 3, 4, 5, 6 are the Ancilla modes. Modes 3, 4 are EPR pair, and modes 5, 6 too. We give Figure 4 to describe this protocol.

![Figure 4](image_url)

**Figure 4.** The thick red line in this figure represents a quantum channel between two parties. The local operations and modes are enclosed by the dashed with the name of the party. The The blue thin rectangle means the phase shifter about $\pi$.

In step 1, Alice performs a series of QND interactions, firstly QND interaction in mode 2, 3, then QND phase adjust interaction in mode 1, 3, then QND interaction in mode 2, 5, finally QND phase adjust interaction in mode 1, 5. In step 2, Alice sends
mode 3 to Bob and mode 5 to Charlie through quantum channels. Then Bob possesses two modes 3, 4 and Charlie possesses modes 5, 6. They perform local QND interactions in their own two modes and then accomplish the channel. The resulting modes are given as follows:

\[
\begin{align*}
\hat{x}'_1 &= \hat{x}_1 - (\hat{x}_2 + \hat{x}_3) - (\hat{x}_2 + \hat{x}_5), \quad \hat{p}'_1 = \hat{p}_1 \\
\hat{x}'_2 &= \hat{x}_2, \quad \hat{p}'_2 = \hat{p}_2 - \hat{p}_3, \quad \hat{p}_5 \\
\hat{x}'_3 &= \hat{x}_2 + \hat{x}_3, \quad \hat{p}'_3 = \hat{p}_1 + \hat{p}_3 + \hat{p}_4 \\
\hat{x}'_4 &= \hat{x}_2 + \hat{x}_3 - \hat{x}_4, \quad \hat{p}'_4 = -\hat{p}_4 \\
\hat{x}'_5 &= \hat{x}_2 + \hat{x}_5, \quad \hat{p}'_5 = \hat{p}_1 + \hat{p}_5 + \hat{p}_6 \\
\hat{x}'_6 &= \hat{x}_2 + \hat{x}_5 - \hat{x}_6, \quad \hat{p}'_6 = -\hat{p}_6
\end{align*}
\] (18)

As in scenario 1, the output modes 1’, 3’, 5’ perform a three-party MQ conat channel and modes 2’, 4’, 6’ perform a a three-party PQ conat channel. So two multiparty conat channels are produced in this protocol.

Then we discuss the noise of the channels in these two scenarios, we assume that the local QND interaction is ideal. So we get that

**In Scenario 1:**

\[
\begin{align*}
PQ\ conat\ channel: & \quad \epsilon_1 = 2e^{-2r}, \quad \epsilon_2 = 4e^{-2r}, \quad \epsilon_3 = 4e^{-2r} \\
MQ\ conat\ channel: & \quad \epsilon_1 = 2e^{-2r}, \quad \epsilon_2 = 4e^{-2r}, \quad \epsilon_3 = 0
\end{align*}
\] (19)

**In Scenario 2:**

\[
\begin{align*}
PQ\ conat\ channel: & \quad \epsilon_1 = 2e^{-2r}, \quad \epsilon_2 = 2e^{-2r}, \quad \epsilon_3 = 4e^{-2r} \\
MQ\ conat\ channel: & \quad \epsilon_1 = 2e^{-2r}, \quad \epsilon_2 = 2e^{-2r}, \quad \epsilon_3 = 0
\end{align*}
\] (20)

Comparing these two scenarios, we can find that the noise of scenario 2 is lower so it is better. We reach a conclusion that if the path between one party and Alice is longer, then the noise this party get is larger so the noise of this channel is larger. As you see in Figure 2, the first graph represents Scenario 1, the second graph represents Scenario 2. We can see that in Scenario 1, the length of the path between Bob and Alice is one, and the length of the path between Charlie and Alice is two. So Charlie get larger noise \(\epsilon_2 = 4e^{-2r}\). The longer the path between one party and Alice, the larger the accumulation of the noise this party gets. In Scenario 2, the length of the path between Alice and any party is one, there is no accumulation of the noise, so Scenario 2 is better. For n-party conat channel, the best prepared entanglement resources is the rightmost graph in Figure 2.

**4. Applications in controlled teleportation and quantum secret sharing**

In this section, we discuss some new and promising applications of our channel, that is, controlled teleportation and quantum secret sharing based on the significant characteristics of our multiparty conat channel.

Controlled teleportation is a very important branch of quantum teleportation. In controlled teleportation, a third party (the supervisor Charlie) is involved, and the
fidelity of the teleportation is controlled by this party. The controlled teleportation of single-qubit is proposed in [3, 10], and the m-qubit version is proposed in [11]. Recently, controlled teleportation of single-mode with continuous variable has been proposed in [15, 24]. Using our multiparty conat channel, we can achieve controlled teleportation of both single-mode and two-mode with continuous variables which we will discuss later.

Quantum secret sharing (QSS) is an important branch of quantum cryptography [5, 9, 12, 20, 23]. Quantum secret sharing was firstly proposed in [4]. QSS has two branches: QSS of classical information [4] and QSS of quantum information [6]. In 2002, Tyc and Sanders proposed continuous-variable secret sharing of quantum information [8].

The output modes of a multiparty PQ conat channel construct a network where not only they are in entanglement but also importantly the original momentum message is divided into $n$ parties involved and is shared by them.

We firstly apply our channel to multiparty-controlled teleportation of an arbitrary coherent state. The input party Alice can teleport her original state to any one (except Alice herself) in the $n$ output parties with the help of all other $(n-1)$ output agents. Without loss of generality, we take a three-party PQ conat channel as an example. If Alice wants to teleport her original state to Bob through the three-party PQ conat channel, Alice and Charlie are controllers. Without the help of Alice and Charlie, Bob can’t obtain the teleported state. Unless Alice and Charlie measure the momentum of their own state and send their measurements to Bob through classical channels, Bob can extract the momentum signal after the local displacement of his momentum quadrature by the sum of the measurement values he gets. Since Bob’s position quadrature is approximately equal to Alice’s original position quadrature, Bob can get the original state that Alice wants to teleport and then the teleportation is accomplished. If Alice wants to teleport her original state to Charlie, the situation is similar. Another point we must mention here, it is senseless for Alice to teleport her state to herself, so we don’t discuss this situation here. Turning to the $n$-party controlled teleportation, a $(n+1)$-party conat channel is required and the operations of measurements, classical communications and displacements are similar, too.

Since the method 2 we presented in section 3 has two input modes possessed by Alice, using it we can accomplish a multiparty-controlled teleportation of arbitrary two-mode coherent states including two-mode entanglement. It might be the coherent-states version of Deng’s work in [14]. In this method, each party possesses two modes. One mode is in the output modes of PQ conat channel and the other is involved in a MQ conat channel. The reconstruction of the two-mode state is same to the individual PQ channel and the individual MQ channel.

Now, let us introduce the process of quantum secret sharing through a multiparty conat channel. As we mentioned earlier in this section, QSS can be used to share both classical information and quantum information. For sharing a unknown one-mode quantum state, it is similar to the controlled teleportation of an arbitrary coherent state, the secret state can be recovered for any agent of the channel only when all other
agents collaborate. For sharing a unknown two-mode quantum state, it is similar to the controlled teleportation of an arbitrary two-mode coherent state. For sharing classical information, the classical information for continuous variable is the measurement value of the operator $\hat{\alpha} = \hat{x} + i\hat{p}$.

5. Conclusion

In this paper, we have extended the notion of continuous-variable coherent (conat) channel to multiparty and given two definitions of it. Then we propose two implementations of multiparty conat channel using linear optics. One method is the multiparty version of coherent communication assisted by entanglement and classical communication (CCAECC). The other is multiparty coherent superdense coding which implements two multiparty coherent channels. We also discuss the noise of the channel in two scenarios when $n$ is equal to 3. Finally, we apply our channel to controlled teleportation and quantum secret sharing (QSS).

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