Textures and semi-local strings in supersymmetric hybrid inflation

Richard A Battye\textsuperscript{1}, Björn Garbrecht\textsuperscript{2} and Apostolos Pilaftsis\textsuperscript{3}

\textsuperscript{1} Jodrell Bank Centre for Astrophysics, School of Physics and Astronomy, University of Manchester, Oxford Road, Manchester M13 9PL, UK
\textsuperscript{2} Department of Physics, University of Wisconsin-Madison, 1150 University Avenue, Madison, WI 53706, USA
\textsuperscript{3} School of Physics and Astronomy, University of Manchester, Oxford Road, Manchester M13 9PL, UK
E-mail: rbattye@jb.man.ac.uk, bjorn@hep.man.ac.uk and pilaftsi@hep.man.ac.uk

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Abstract. Global topological defects may account for the large cold spot observed in the cosmic microwave background. We explore possibilities for constructing models of supersymmetric $F$-term hybrid inflation, where the waterfall fields are globally $SU(2)$-symmetric. In contrast to the case where $SU(2)$ is gauged, there arise Goldstone bosons and additional moduli, which are lifted only by masses of soft supersymmetry breaking scale. The model predicts the existence of global textures, which can become semi-local strings if the waterfall fields are gauged under $U(1)_X$. Gravitino overproduction can be avoided if reheating proceeds via the light $SU(2)$-modes or right-handed sneutrinos. For values of the inflaton–waterfall coupling $\gtrsim 10^{-4}$, the symmetry breaking scale imposed by normalization of the power spectrum generated from inflation coincides with the energy scale required to explain the most prominent of the cold spots. In this case, the spectrum of density fluctuations is close to scale invariant, which can be reconciled with measurements of the power spectrum by the inclusion of the sub-dominant component due to the topological defects.

Keywords: CMBR theory, inflation, cosmic strings, domain walls, monopoles

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1. Introduction

Recent times have seen cosmology enter a precision era. Measurements of the cosmic microwave background (CMB) have led to estimates of the standard six cosmological parameters (see, for example, the 5 yr results from WMAP [1] and references therein). Of particular interest here are the estimates of the amplitude, $A_s$, and spectral index, $n_s$, of the scalar density fluctuations which are measured to be $n_s = 0.963^{+0.014}_{-0.015}$ and $A_s = (2.41 \pm 0.01) \times 10^{-9}$. It is presumed that the density fluctuations are created during an inflationary era through quantum effects. Constraining the nature of this epoch on the basis of the observations is now the focus of much effort [2]–[4] with the ultimate aim of making contact with particle physics models around the grand unified theory (GUT) scale [5].

In addition to these fluctuations of quantum origin, there may be others created by topological defects formed naturally as a consequence of a phase transition at the end of hybrid inflation [6]. Such fluctuations cannot be the primary source of CMB
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anisotropies, but they could provide a sub-dominant component [7], which can have some interesting effects. In particular, it has been shown that if cosmic strings are formed, then $n_s = 1$ can be made compatible with the data [8]–[10]. This is of interest since many of the models of inflation motivated by fundamental theories predict values for the spectral index larger than the WMAP best-fit value. For example, SUSY $F$- and $D$-term hybrid inflationary models [11]–[13] predict $n_s > 0.98$ [14], while even larger values may be required if one considers gravitino constraints on the reheat temperature or the bounds on the cosmic string tension in $D$-term models. Brane inflation [15] in its simplest form predicts $n_s > 0.97$, but again, more sophisticated implementations may lead to a scale-invariant spectral index [16].

Most work on inflation plus defect scenarios has focused on strings. However, the recent suggestion [17,18] that the previously identified cold spot in the CMB [19] could be due to cosmic textures [20,21] motivates our consideration of hybrid inflation scenarios involving global defects. Such dynamically unstable defects produce a spectrum of anisotropies qualitatively similar to those of strings [22] and are likely to have similar properties as regards the degeneracy with $n_s$. We note, in addition, the other cold spots detected on a smaller scale in the CMB which do not appear to be associated with the Sunyaev–Zel’dovich effect [23].

Textures are most easily implemented in non-supersymmetric models, where they can emerge from a scalar potential of the form [24]

$$V(\Phi) = \frac{1}{4} \lambda (\Phi_i \Phi_i - \phi_0^2)^2,$$

where $\lambda$ is a dimensionless coupling constant and $\phi_0$ is the symmetry breaking scale. The fields $\Phi_i$ are a fundamental representation of a global $O(4)$-symmetry. This potential induces spontaneous symmetry breaking $O(4) \to O(3)$, for which the vacuum manifold is $S^3$, where $S^3$ denotes the three-dimensional sphere. Since $\pi_3(S^3) = \mathbb{Z}$, there is the possibility of non-trivial topological configurations called textures. However, they cannot be stable by Derrick’s theorem and hence, once formed at the phase transition, they unwind. This unwinding process takes place in a self-similar fashion such that the density of textures scales relative to the background. The latter has been shown to be a more general feature of global field models with a broken $O(N)$ symmetry [25]. The process of unwinding creates density fluctuations and CMB anisotropies [21, 26, 27]. The CMB anisotropies result in a distribution of hot and cold spots, which by the central limit theorem, appear as approximately Gaussian on small scales. On larger scales, they may be revealed by more detailed analysis. Taking into account various observational biases, the analysis of Cruz et al [17] has suggested that the large cold spot identified in the CMB would result from such a model with $\phi_0 = (8.7^{+2.1}_{-3.0}) \times 10^{15}$ GeV.

$$\phi_0 = (8.7^{+2.1}_{-3.0}) \times 10^{15} \text{ GeV}. \quad (1.2)$$

In this paper, we study supersymmetric (SUSY) scenarios of $F$-term hybrid inflation that realize the global symmetry $SU(2) \times U(1)_X$. We also consider the possibility that the $U(1)_X$-symmetry is gauged, giving rise to a semi-local model. Even though in the context of SUSY hybrid inflation, models with local symmetries are more commonly discussed, we note that the present paper is not the first one in which the use of global symmetries is considered. In one of the seminal papers on SUSY $F$-term inflation, the use of a spontaneously broken global $U(1)_X$-symmetry is suggested [11], and the possibilities of
larger symmetries and textures are mentioned. To the best of our knowledge, however, an explicit analysis of the particle spectrum has not been performed so far, and the consequences of the higher symmetry have not been investigated in detail either within the framework of $F$-term hybrid inflation.

An important aspect of the model-building is that unless additional $SU(2)$-invariant lifting terms are included, the vacuum manifold is larger than $SU(2) \sim S^3$. This is because the holomorphic nature of the superpotential is consistently maintained by doubling the number of degrees of freedom in the symmetry breaking fields, such as the field $\Phi$ in (1.1). We describe here in detail how these additional degrees of freedom in the vacuum manifold can be lifted such that it is indeed reduced to $S^3$. Furthermore, we include the effect of radiative corrections on the predictions for the initial power spectrum created during the inflationary phase as first performed in [12]. The breaking of a local instead of a global symmetry was first proposed in this work and following this reference most articles on SUSY hybrid inflation consider only local symmetries. This may also be due to the fact that quantum gravitational effects can violate global symmetries. Since no complete model of quantum gravity exists yet, we do not consider this possibility here and take $SU(2)$ to be an exact symmetry. Such an assumption is not entirely unrealistic, since $SU(2)$ is the only group that is self-non-anomalous, independently of the representation of the chiral fermions in the theory.

The organization of the paper is as follows. In section 2 we describe in detail the SUSY hybrid texture model based on the global symmetry $SU(2) \times U(1)_X$. We also calculate its scalar mass spectrum and discuss mechanisms for giving masses to the massless moduli fields present in the model. Formal aspects of the model-building have been relegated to the appendices. In section 3 we analyse possible scenarios for reheating after inflation, including constraints from possible overproduction of gravitinos and big bang nucleosynthesis (BBN). In addition, we estimate the effect that the texture Goldstone bosons may have on stellar cooling. Section 4 discusses a semi-local realization of the SUSY hybrid model, where the aforementioned global $U(1)_X$ symmetry is promoted to a local one. In section 5 we study the inflationary dynamics of the SUSY hybrid model and analyse the possibility of the texture scale $\phi_0$ given in (1.2) being naturally implemented within this model, without being in conflict with other cosmological and astrophysical constraints discussed in section 3. Finally, section 6 summarizes the main conclusions of our study.

2. SUSY hybrid texture model

2.1. Superpotential and scalar potential

The model of SUSY hybrid inflation with global defects is implemented by the superpotential

$$W = \kappa \tilde{S} \left( \tilde{X}_1 \tilde{X}_2 - M^2 \right),$$

(2.1)

with $\tilde{X}_1 = (2)$ being a fundamental, $\tilde{X}_2 = (\overline{2})$ an identical anti-fundamental representation of the global $SU(2)$ and $\tilde{S}$ being the singlet inflaton. We use the freedom of field redefinitions to choose $\kappa$ and $M$ real. In addition to the global $SU(2)$ symmetry,
the superpotential is invariant under an accidental symmetry which we denote as $SU(2)_X$. In particular, $SU(2)$ induces the transformation
\[
\begin{pmatrix} \hat{X}_1^+ \\ \hat{X}_1^- \end{pmatrix} \xrightarrow{SU(2)} e^{i\alpha_1 \tau^1} \begin{pmatrix} \hat{X}_1^+ \\ \hat{X}_1^- \end{pmatrix}, \quad \begin{pmatrix} \hat{X}_2^+ \\ \hat{X}_2^- \end{pmatrix} \xrightarrow{SU(2)} e^{-i\alpha_1 \tau^1} \begin{pmatrix} \hat{X}_2^+ \\ \hat{X}_2^- \end{pmatrix},
\]
while $SU(2)_X$ has the following effect on the $SU(2)$-blocks:
\[
X = \begin{pmatrix} \hat{X}_1 \\ i\tau_2 \hat{X}_2 \end{pmatrix} \xrightarrow{SU(2)_X} e^{i\theta_1 \sigma^1} \otimes \mathbb{I}_2 \begin{pmatrix} \hat{X}_1 \\ i\tau_2 \hat{X}_2 \end{pmatrix}.
\]
Here, $\alpha_i$ and $\theta_i$ ($i = 1, 2, 3$) are real numbers, the $\tau^i$ and $\sigma^i$ denote the Pauli matrices and $\mathbb{I}_2$ is the two-dimensional identity matrix. We employ the notation $\tau^i$ for $SU(2)$-transformations and $\sigma^i$ for $SU(2)_X$. In the following, we assume that $SU(2)$ is an exact symmetry, while $SU(2)_X$ may be explicitly broken through the introduction of new operators in addition to those arising from the superpotential.

We note that the action of a $\theta_3$ rotation of $X$ is equivalent to a $U(1)$-transformation given by
\[
\hat{X}_1 \rightarrow e^{i\theta_3} \hat{X}_1, \quad \hat{X}_2 \rightarrow e^{-i\theta_3} \hat{X}_2.
\]

For definiteness, we identify this particular symmetry with $U(1)_X \subset SU(2)_X$, under which $\hat{X}_1$ and $\hat{X}_2$ transform with opposite charges. More formal aspects of the $SU(2) \times SU(2)_X$ symmetry are discussed in appendix A. In the main part of the paper, we only consider the $SU(2) \times U(1)_X$ symmetry, since the construction of a realistic model leads to the explicit breaking $SU(2)_X \rightarrow U(1)_X$ as discussed in section 2.3.

To this end, we assume that this $SU(2) \times U(1)_X$ symmetry is global. The superpotential immediately leads to the tree-level scalar potential
\[
V_0 = \kappa^2 \left| X_1 X_2 - M^2 \right|^2,
\]
which is minimized for $\langle S \rangle = 0$ and $\langle X_1 X_2 \rangle = M^2$, such that $V_0 = 0$. Here and in what follows, $X_1 X_2$ is understood as $X_1^T X_2$. The scalar fields assume this ground-state configuration at the end of inflation, while inflation takes place for $\langle X_1 \rangle = \langle X_2 \rangle = 0$ and $S > M$.

Since the vacuum expectation values (VEVs) $\langle X_{1,2} \rangle \neq 0$ completely break the $SU(2)$-symmetry, the ground state is degenerate and there will be at least three Goldstone modes\(^4\). In addition, there are extra massless modes, which we call moduli. These acquire masses only when one adds additional $SU(2)$-invariant terms to the potential that break SUSY softly. Finally, there are heavy modes of mass $\sim \kappa M$.

### 2.2. Identification of the scalar particle spectrum

In order to identify these modes and determine the vacuum manifold, we first make use of the symmetries $SU(2)$ and $U(1)_X$ to write the fields as
\[
X_1 = e^{i(\alpha_i \tau^i + \theta \mathbb{I}_2)} \begin{pmatrix} w_1^+ \\ 0 \end{pmatrix} \quad \text{and} \quad X_2 = e^{-i(\alpha_i \tau^i + \theta \mathbb{I}_2)} \begin{pmatrix} w_2^+ e^{i\tau^2} \\ w_2 \end{pmatrix},
\]
\(^4\) The Goldstone modes associated with the broken $SU(2)_X$ may be related to those of the $SU(2)$-modes. For more details, see appendix A.
where \( \mathbb{I} \) is the two-dimensional identity matrix. Note that this corresponds to a completely general parametrization of the eight real degrees of freedom of \( X_1 \) and \( X_2 \) in terms of eight real parameters, \( w_1^+, w_2^+, w_3^+, \varphi_2^+ \), \( \beta \) and \( \alpha_i \) \( i = 1, 2, 3 \).

The \( F \)-term minimization condition,

\[
F_S = \kappa \left( X_1 X_2 - M^2 \right) = 0, \tag{2.7}
\]

then reads

\[
w_1^+ w_2^+ e^{\varphi_2^+} = M^2. \tag{2.8}
\]

This last equality immediately implies that \( \varphi_2^+ = 0 \). The minimization condition (2.7) will persist in the more complicated situations considered in our subsequent discussion.

We therefore define

\[
w = \sqrt{w_1^{+2} + w_2^{+2} + w_2^{-2}}, \tag{2.9}
\]

and note the relation

\[
w \geq \sqrt{w_1^{+2} + w_2^{+2}} \geq \sqrt{2} M, \tag{2.10}
\]

which has to hold true in order to satisfy the condition (2.7) for the \( F_S \)-term.

In order to determine the mass eigenmodes, we use the parametrization (2.6) and, without loss of generality due to the global symmetries, pick a convenient point, e.g. \( \alpha_i = \beta = 0 \), to expand about. In this way, the waterfall doublets \( X_{1,2} \) may be expressed as

\[
X_1 = \left( \begin{array}{c} w_1^+ + \frac{1}{\sqrt{2}} (X_{1R}^+ + iX_{1I}^+) \\ \frac{1}{\sqrt{2}} (X_{1R}^+ - iX_{1I}^+) \end{array} \right) \quad \text{and} \quad X_2 = \left( \begin{array}{c} w_2^+ + \frac{1}{\sqrt{2}} (X_{2R}^+ + iX_{2I}^+) \\ w_2^- + \frac{1}{\sqrt{2}} (X_{2R}^- + iX_{2I}^-) \end{array} \right), \tag{2.11}
\]

where \( w_{1,2}^\pm \) denote the classical VEVs and \( X_{1,2R,I}^\pm \) are the respective quantum excitations.

The massless \( SU(2) \) Goldstone modes are

\[
SU(2): \quad G_{r1} = \frac{1}{w} (w_1^+ X_{1I}^- - w_2^+ X_{2I}^- - w_2^- X_{2I}^+), \tag{2.12a}
\]

\[
G_{r2} = \frac{1}{w} (-w_1^+ X_{1I}^- - w_2^+ X_{2I}^- + w_2^- X_{2I}^+), \tag{2.12b}
\]

\[
G_{r3} = \frac{1}{w} (w_1^+ X_{1I}^+ - w_2^+ X_{2I}^+ + w_2^- X_{2I}^+). \tag{2.12c}
\]

The indices attached to \( G \) indicate the particular generators of \( SU(2) \) that these modes correspond to. For the subsequent discussion, we also note the massless \( U(1)_X \)-direction

\[
U(1)_X: \quad G_{\mathbb{I}} = \frac{1}{w} (w_1^+ X_{1I}^+ - w_2^+ X_{2I}^+ - w_2^- X_{2I}^-). \tag{2.13}
\]

However, it is not orthogonal to the \( SU(2) \)-modes and, therefore, we do not use it here as a local basis vector for the scalar field space. Moreover, for \( w_2^- = 0 \), it is \( G_{\mathbb{I}} = G_{r3} \).

In addition to the Goldstone bosons, there are massless eigenmodes which are not protected by the \( SU(2) \) and \( U(1)_X \) symmetries. We refer to them as moduli, and note that they may be lifted by adding additional \( SU(2) \)- and \( U(1)_X \)-invariant terms to the
We now briefly discuss the possible origins of the three types of mass terms in (2.20) and their physical significance. Categorization of these modes is presented in appendix A.

Unlike the Goldstone bosons (2.12), the moduli fields, \( H_0^R \), \( H_0^I \) and \( H_g \), are not protected by the global symmetry \( SU(2) \times U(1)_X \) and usually acquire non-zero masses if soft SUSY breaking terms which respect the global symmetry are added to the potential. We refer to this mechanism as lifting. Some of the lifted modes turn out to correspond to broken generators of \( SU(2)_X \), which is explicitly broken by the lifting mass terms. A categorization of these modes is presented in appendix A.

The possibility of adding terms to the superpotential of the type

\[
\hat{H}_0 = \frac{1}{w} (w_1^+ X_{1R}^+ - w_1^+ X_{2R}^+ - w_2^+ X_{1R}^+),
\]

and

\[
H_g = \frac{1}{w} (-w_1^+ X_{1R}^+ + w_2^+ X_{2R}^+ - w_2^+ X_{1R}^+).
\]

As well as the massless fields, the scalar particle spectrum contains massive eigenmodes. These are

\[
H^R_\kappa = \frac{1}{w} (w_1^+ X_{2R}^+ + w_2^+ X_{1R}^+ + w_2^+ X_{1R}^-), \quad H^I_\kappa = \frac{1}{w} (w_1^+ X_{2I}^+ + w_2^+ X_{1I}^+ + w_2^+ X_{1I}^-),
\]

\[
S^R = \text{Re} S, \quad S^I = \text{Im} S.
\]

All these states are degenerate, with mass equal to \( \kappa w \). Because of this degeneracy, the mass eigenstates may not necessarily be orthogonal to each other. However, our field basis,

\[
\{ G_+, G_-, G_+, H^R_0, H^I_0, H_g, H^R_\kappa, H^I_\kappa, S^R, S^I \},
\]

has been so chosen to be orthonormal.

2.3. Lifting of the moduli

Unlike the Goldstone bosons (2.12), the moduli fields, \( H_0^R \), \( H_0^I \) and \( H_g \), are not protected by the global symmetry \( SU(2) \times U(1)_X \) and usually acquire non-zero masses if soft SUSY breaking terms which respect the global symmetry are added to the potential. We refer to this mechanism as lifting. Some of the lifted modes turn out to correspond to broken generators of \( SU(2)_X \), which is explicitly broken by the lifting mass terms. A categorization of these modes is presented in appendix A.

The possibility of adding terms to the superpotential of the type

\[
\hat{X}_{1,2} \hat{T} \hat{X}_{1,2}, \quad \hat{X}_{1,2} \hat{T} \hat{X}_{2,1}, \quad \mu_X \hat{X}_1 \hat{X}_2,
\]

where \( \hat{T} \) denotes an \( SU(2) \)-triplet and \( \mu_X \) is a mass of order \( M \), has to be discarded. The \( \mu_X \)-term breaks the \( R \)-symmetry of the superpotential (2.1) explicitly, such that there is no SUSY-preserving minimum. The triplet variant leads to spontaneous \( R \)-symmetry breaking and again, no supersymmetric minimum occurs [28].

The generic form for a potential representing soft SUSY breaking in the inflaton–waterfall sector is

\[
V_{\text{soft}} = m_1^2 |X_1|^2 + m_2^2 |X_2|^2 + b X_1^\dagger X_2 + b^* X_1^\dagger X_2^* + d X_1^\dagger i \tau^2 X_2 - d^* X_2^\dagger i \tau^2 X_1.
\]

We now briefly discuss the possible origins of the three types of mass terms in (2.20) and their physical significance.

(i) The terms \( m_{1,2} \) may directly result from soft SUSY breaking. Alternatively, they could arise by giving a TeV-scale VEV to \( S \), as discussed in [32]–[34], which would also address the \( \mu \)-problem of the minimal supersymmetric standard model (MSSM).
(ii) The $b$-term may be a soft SUSY breaking parameter that could originate from a superpotential term, $\mu_X \hat{X}_1 \hat{X}_2$, as given in (2.19), where $\mu_X$ is of the soft SUSY breaking scale $M_{\text{SUSY}}$. If $b \sim M_{\text{SUSY}}^2$, the $b$-terms lead to a sub-dominant correction to the $F_S = 0$ condition for minimizing the potential. Hence, they are not suitable for lifting the moduli and we do not discuss them here any further.

(iii) The $d$-terms may occur as a result of a non-canonical Kähler potential. Although they break the accidental global $U(1)_X$-symmetry, we show in appendix B, how these $d$-terms can be absorbed in $m_{1}^2$ and $m_{2}^2$ after a field redefinition of $X_1$ and $X_2$. Consequently, we will set both $b = 0$ and $d = 0$ in the subsequent discussion and just allow $m_{1,2}^2$ to vary, that is

$$V_{\text{lift}} = m_{1}^2 |X_1|^2 + m_{2}^2 |X_2|^2.$$  \hspace{1cm} (2.21)

Finally, to keep things at a more general level, we prefer not to specify the soft SUSY breaking masses for the superpartners of the Goldstone bosons and the light moduli. For instance, we could assume that these particles are very light and decouple above the electroweak symmetry breaking scale, such that they constitute additional relativistic species today (see also our discussion in section 3.4).

2.4. Vacuum manifold and texture defects

We now determine the vacuum manifold of the scalar potential, which arises as a combination of (2.5) and (2.21),

$$V_{\text{global}} = V_0 + V_{\text{lift}}.$$ \hspace{1cm} (2.22)

With the parametrization (2.11), we get the potential

$$V_{\text{global}}(w_1^+, w_2^+, w_2^-, \varphi_2^+) = \kappa^2 \left(v_1^+ v_2^+ e^{i \varphi_2^+} - M^2 \right)^2 + m_1^2 v_1^+ v_2^+ + m_2^2 \left(v_2^+ v_2^2 + v_2^2 v_2^2 \right). \hspace{1cm} (2.23)$$

We find its minimum at

$$w_1^+ = \sqrt{\frac{m_2}{m_1} M^2 - \frac{m_2^2}{\kappa}}, \quad w_2^+ = \sqrt{\frac{m_1}{m_2} M^2 - \frac{m_1^2}{\kappa}}, \quad w_2^- = 0, \quad \varphi_2^+ = 0. \hspace{1cm} (2.24)$$

At this point, the moduli are lifted to acquire the masses

$$m^2(H_0^R) = m^2(H_0^I) = m_1^2 + m_2^2, \quad m^2(H_g) = 4 \frac{m_1^2 m_2^2}{m_1^2 + m_2^2}.$$ \hspace{1cm} (2.25)

Recalling that the condition $w_2^- = 0$ implies a degeneracy between the parameters $\alpha_3$ and $\beta$ in the parametrization (2.6), the minimum (2.24) fixes five out of eight degrees of freedom. Hence, after the fixing of the moduli, the vacuum manifold is $S^3$, where the sphere is generated by the three Goldstone modes defined in (2.12). In this model, there are therefore textures of a symmetry breaking scale $\sqrt{2} M$. In appendix A, we present a discussion of the vacuum manifold in the case where $V_{\text{lift}} = 0$. 

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3. Cosmological and astrophysical implications

In this section, we outline possible scenarios for reheating after inflation. In order to do so, we choose to consider couplings to the Higgs fields of the MSSM and the right-handed neutrino sector, such that the inflationary energy can eventually be converted into particles of the standard model (SM). Furthermore, we estimate the effect that the new light degrees of freedom in the waterfall sector have on stellar cooling and on the production of gravitinos after inflation. We also note that the possible existence of the new light fields is not in conflict with bounds on additional relativistic degrees of freedom that arise from BBN.

3.1. Extended superpotential

We extend the superpotential (2.1) by adding the terms

\[ W_{RH} = \lambda \hat{S} \hat{H}_u \hat{H}_d + \frac{\rho_{ij}}{2} \hat{S} \hat{N}_i \hat{N}_j, \]

(3.1)

where \( \hat{H}_u,d \) denote the MSSM Higgs doublets and \( \hat{N}_i (i = 1, 2, 3) \) are right-handed singlet neutrinos, which we assume to be nearly degenerate, \( \rho_{ij} \approx \rho \Pi_3 [29,30] \). Again, we use the freedom of field redefinitions to choose \( \lambda \) and \( \rho \) to be real. The tree-level scalar potential (2.5) now generalizes to

\[ V_0 = \kappa^2 \left| X_1 X_2 + \frac{\lambda}{\kappa} H_u H_d + \frac{\rho_{ij}}{2\kappa} N_i N_j - M^2 \right|^2 + \kappa^2 |S X_1|^2 + \kappa^2 |S X_2|^2 + \lambda^2 |S H_u|^2 + \lambda^2 |S H_d|^2 + |\rho_{ij} S N_j|^2. \]

(3.2)

In appendix C, we present the terms that are quadratic in \( X_{1,2} \), such that possible interactions within this sector and with the Higgs fields and right-handed neutrinos can easily be read off.

3.2. Stellar cooling

It is straightforward to check that the coupling \( \lambda \) does not induce any tree-level mixing between the Higgs bosons, the Goldstone bosons and moduli. Hence, we do not expect related collider phenomenology.

We may now examine whether the same coupling induces effects that are in contradiction with astrophysical observations, especially due to a dangerous increase in the rate of stellar energy loss. To this end, we first observe that because the \( \hat{X}_{1,2} \) are not charged under hypercharge or weak isospin, there is no mixing between these fields and the SM gauge bosons. From (C.3), we see that the light moduli and Goldstone fields are always produced in pairs. Hence, possible bremsstrahlung processes, where a fermion of mass \( m_f \) emits two Goldstone bosons via a virtual Higgs boson exchange, involve a factor

\[ \frac{\kappa \lambda m_f T_S}{2\pi^2 m_H^2}, \]

(3.3)

when compared to the axion Yukawa coupling \( \sim m_f / f_a \), which is relevant for the process of axion bremsstrahlung, e.g. see [31]. Here, \( T_S \) is the temperature of the star, \( m_H \) the
mass of the SM Higgs boson and $f_a$ the axion decay constant. With the supernova bound $f_a \gtrsim 10^{10}$ GeV, $T_S \sim 10^{-1}$ GeV in the core of a supernova and $m_H \sim 10^2$ GeV, it is clear that fast energy loss can be avoided if $\kappa \lambda \lesssim 10^{-4}$, which can easily be achieved for the values of $\kappa$ discussed in section 5.

3.3. Reheating and gravitinos

We sketch two possible ways in which the inflationary vacuum energy can be transferred to the MSSM degrees of freedom through renormalizable interactions. One may alternatively consider ‘gravity-mediated’ reheating through non-renormalizable operators, a possibility which we do not discuss here.

For the first reheat scenario, we assume that $\lambda \neq 0$ and $\rho = 0$. In order to avoid gravitino overproduction mediated by a fast decay of the inflaton into the MSSM sector, a large VEV for $\tilde{t}_R$ or $\tilde{t}_L$ is required during inflation and the initial stages of reheating. The MSSM degrees of freedom only equilibrate after relaxation of the stop VEV. In the second case, we set $\lambda = 0$ and $\rho > \kappa$, such that the $\tilde{N}$ can mediate between the $\tilde{X}_{1,2}$ and the MSSM. Equilibration of the MSSM degrees of freedom is then delayed through small neutrino Yukawa couplings.

3.3.1. Reheating delayed by stop VEV. Considering the scalar potential (2.5) and the interactions arising from (C.3), it is clear that the oscillating fields $S$ and $H^K_R$ can directly decay into Goldstone bosons, light moduli and their superpartner fermions. Since these particles are only weakly coupled to the MSSM degrees of freedom, higher temperatures within this sector are in accordance with gravitino bounds. This is because gravitino production from MSSM degrees of freedom is mediated by non-Abelian and super-gauge interactions, which are absent for the fields $\tilde{X}_{1,2}$. We now estimate the gravitino abundance, which arises from a thermal bath of the light degrees of freedom within these fields.

The decay rate of the oscillating fields is

$$\Gamma_{\kappa} = \frac{3}{32\pi} \kappa^3 w, \quad (3.4)$$

and the reheat temperature within the light components of $X_{1,2}$

$$T_{\kappa} = \left(\frac{45}{4\pi^3 g_*^G}\right)^{1/4} \sqrt{\Gamma_{\kappa} m_{Pl}} = 2.1 \times 10^{16} \text{ GeV} \sqrt{\kappa^{3/2} w^{1/16}} \text{ GeV}, \quad (3.5)$$

where the effective number of degrees of freedom within the Goldstone bosons, light moduli and their superpartners is given by $g_*^G = 11.25$. Considering the scalar interactions arising from (C.3) in combination with the interaction of a gravitino and a chiral multiplet as given for example in [35], we see that gravitinos $\tilde{G}$ may be produced in $2 \rightarrow 3$ processes of the type

$$Y + Y \rightarrow \tilde{G} + Y + \tilde{Y}, \quad (3.6)$$

where $Y$ stands for any of the Goldstone bosons or moduli. There are also $3 \rightarrow 2$ processes of equal importance, which are related to the $2 \rightarrow 3$ by crossing. We estimate the averaged
cross section for the $2 \rightarrow 3$ as
\[
\langle \sigma v \rangle \simeq \frac{6\kappa^2}{8m_{\text{Pl}}^2(2\pi)^2}.
\]  
(3.7)

Compared to the expressions for a $2 \rightarrow 2$ process in [35], we have inserted a phase space factor of $(2\pi)^{-2}$, as well as a factor of 6, which counts the number of light chiral multiplets arising from $\hat{X}_{1,2}$. This estimate should be accurate up to a factor of order 1. The gravitino number can then be calculated as [35]
\[
Y_{\tilde{G}} = \frac{g_s^1 \text{MeV} n(T_{\kappa})}{g_s^0} = 8 \times 10^{-17} \left( \frac{\kappa}{10^{-3}} \right)^{7/2} \sqrt{\frac{w}{10^{16} \text{GeV}}},
\]  
(3.8)

where $n(T_{\kappa}) = \zeta(3)T^3/\pi^2$ and $g_s^1 \text{MeV} = 3.91$. This typically is below the upper bounds $Y_{\tilde{G}} \lesssim 10^{-15}$ for a gravitino of mass $m_{\tilde{G}} \simeq 360$ GeV and $Y_{\tilde{G}} \lesssim 10^{-14}$ for a gravitino of mass $m_{\tilde{G}} \simeq 600$ GeV, as imposed by constraints from BBN [36,37]. Larger gravitino masses relax this bound. Therefore, the Goldstone and light moduli sector does not lead to a harmful amount of gravitino production.

If we alternatively assume that $\lambda \neq 0$ and if the $\tilde{H}_{u,d}$ are massless at reheating, then the inflaton can also directly decay into Higgs bosons and Higgsinos. The condition $\lambda > \kappa$ then leads to a reheat temperature of
\[
T_R = \left( \frac{45}{4\pi^2 g_s} \right)^{1/4} \left( \frac{m_{\text{Pl}}}{32\pi} g_s^1 \Lambda^2 \kappa^2 \right)^{1/2} = 7 \times 10^{15} \text{GeV} \lambda \sqrt{\kappa} \left( \frac{w}{10^{16} \text{GeV}} \right)^{1/2},
\]  
(3.9)

within the MSSM sector, where $g_s = 240$ for the MSSM with right-handed neutrinos. Unless $\kappa$ and $\lambda$ are tuned to be less than about $10^{-5}$, this gives rise to unacceptably high temperatures. On the other hand, these small values would suppress the energy scale of the texture way below the value reported in [17], as we see in section 5. Decays of $S$ and $H^{R,1}_\kappa$ into $\tilde{H}_{u,d}$ can, however, initially be avoided if the MSSM Higgs particles acquire a large mass through a large VEV along a flat direction involving the left- or right-handed stop, such that $\kappa M \ll \langle \tilde{G} \rangle$.

For a large stop VEV, we can also allow for $\lambda < \kappa$, since the $H_{u,d}$ mass eigenvalues remain positive during inflation. To see this, let us denote the stop VEV as $v_{\text{stop}}$ and the top Yukawa coupling as $h$. In the weak basis ($H^*_u, H_d$), the Higgs mass-square matrix may then be written down as
\[
\begin{pmatrix}
\lambda^2 |S|^2 & h^2 v_{\text{stop}}^2 \\
-h^2 v_{\text{stop}}^2 & -\kappa \lambda M^2
\end{pmatrix},
\]  
(3.10)

and the condition for it to be positive definite is
\[
|S|^2 > \frac{h^2 v_{\text{stop}}^2 + \sqrt{4\kappa^2 \lambda^2 M^4 + h^4 v_{\text{stop}}^4}}{2\lambda^2} \approx \frac{\kappa^2 M^4}{h^2 v_{\text{stop}}^2},
\]  
(3.11)

where we have assumed $h^2 v_{\text{stop}}^2 \gg 4\kappa^2 \lambda^2 M^4$ to arrive at the last expression in (3.11). Therefore, during inflation, where $|\langle S \rangle| > M$, the Higgs masses remain positive definite.

If the flat direction is lifted by an operator of TeV scale, its VEV will relax towards zero at temperatures of about $10^{10}$ GeV. Another possibility is the fragmentation of the flat direction into Q-balls [38,39]. To that end, all energy is contained within the
\( \hat{X}_{1,2} \). For the scatterings of the \( X_{1,2} \) into \( H_{u,d} \), we estimate \( \langle \sigma v \rangle n \simeq \kappa^2 \lambda^2 T_R \). The reheat temperature \( T_R \), at which the MSSM degrees of freedom equilibrate, can be estimated as

\[
\langle \sigma v \rangle n \simeq H \simeq \frac{T_R^2}{m_{Pl}}. \tag{3.12}
\]

Thus, up to factors \( \mathcal{O}(1) \), we find that

\[
T_R \simeq 5 \times 10^5 \text{ GeV} \left( \frac{\kappa}{10^{-3}} \right)^2 \left( \frac{\lambda}{10^{-3}} \right)^2. \tag{3.13}
\]

Reheat temperatures which are not in conflict with gravitino overproduction can therefore easily be attained for values of \( \kappa \) which are suggested by the scale of the texture, which we derive in section 5.

3.3.2. Reheating via sneutrinos. If \( \lambda = 0 \) and \( \rho \geq \kappa \), decays into right-handed neutrinos and sneutrinos also take place. The decay rate is

\[
\Gamma_{\kappa \rho} = \frac{3}{32\pi} (\kappa^2 + \rho^2) \kappa w, \tag{3.14}
\]

and the reheat temperature within this sector is given by

\[
T_{\kappa \rho} = \left( \frac{45}{4\pi^3 g^G_*} \right)^{1/4} \sqrt{\Gamma_{\kappa \rho} m_{Pl}} = 1.2 \times 10^{16} \text{ GeV} \sqrt{3\kappa (\kappa^2 + \rho^2)} \sqrt{\frac{w}{10^{16} \text{ GeV}}}, \tag{3.15}
\]

where the effective number of degrees of freedom within the Goldstone bosons, light moduli, right-handed neutrinos and their superpartners is given by \( g^G_* = 22.5 \). By the same argument as given above, there is no gravitino overproduction within this sector.

Potentially dangerous interactions with gravitinos only become important when MSSM degrees of freedom, which have strong and electroweak interactions, are produced in sizable amounts. Let us therefore assume that the largest neutrino Yukawa coupling in a diagonal basis is given by \( h \) and add the superpotential term

\[
h \hat{L} \hat{H}_u \hat{N}. \tag{3.16}
\]

Then, according to the standard seesaw mechanism, the heaviest neutrino mass eigenstate is of mass \( m_\nu \simeq h^2 v^2 / m_N \), where \( v = \langle H_0^u \rangle \) and \( m_N \) the mass of the heavy right-handed neutrinos \( N_i \), which we assume to be of TeV scale. In this picture, all the neutrino Yukawa couplings \( h \) are \( \mathcal{O}(10^{-6}) \), which may be regarded as somewhat unnatural.

The scattering rate of the right-handed neutrinos with left-handed neutrinos and Higgs particles or respective superpartners is given by the averaged cross section \( \langle \sigma v \rangle n \). The reheat temperature \( T_R \), at which the MSSM degrees of freedom equilibrate, can be estimated as

\[
\langle \sigma v \rangle n \simeq h^2 T_R \simeq H \simeq \frac{T_R^2}{m_{Pl}}, \tag{3.17}
\]

up to factors \( \mathcal{O}(1) \). For an order-of-magnitude estimate, we find

\[
T_R \simeq 10^5 \text{ GeV} \left( \frac{m_N}{1 \text{ TeV}} \right) \left( \frac{m_\nu}{50 \text{ meV}} \right) \left( \frac{200 \text{ GeV}}{v} \right)^2. \tag{3.18}
\]

Reheat temperatures of the MSSM which are low enough not to conflict with the gravitino bound but high enough to allow for low-scale resonant leptogenesis [41, 42] or electroweak baryogenesis [43] can be achieved within the suggested model.
3.4. Big bang nucleosynthesis and the new light fields

The number and the temperature of light relativistic species are constrained by considerations from BBN, or even more strongly by a combined analysis of CMB data and observations of abundances of light nuclei; see for example [40]. This is because the results of BBN are sensitive to the effective number of degrees of freedom. We denote by $N_{\nu}$ the number of neutrinos which freeze out before electron–positron annihilation but after the annihilation of any other species, where $N_{\nu} = 3$ in the MSSM and its extension which we discuss in this paper. We consider the case when reheating takes place according to the stop VEV scenario, as outlined in section 3.3.1; the neutrino-reheating scenario will be very similar.

The Goldstone bosons $G_{\tau i}$ will freeze out as soon as the temperature of the Universe drops below the mass of the lightest Higgs boson or Higgsino. If we assume that all other supersymmetric particles and the top quark are heavier than the lightest Higgs boson or Higgsino, we can estimate the effective number of degrees of freedom at this point to be $g_{\text{freeze-out}} \approx 100$. For definiteness, besides the Goldstone bosons, we assume that also six Weyl fermions composed from $\tilde{X}_{1,2}$ are relativistic during BBN. The effective number of degrees of freedom today is then given by

$$g_* = 2 + \frac{7}{8} \times 2 \times N_{\nu} \left( \frac{4}{11} \right)^{4/3} + \left[ (3 + 6 \times \frac{7}{8}) \left( \frac{2}{g_{\text{freeze-out}}} \right)^{4/3} \right].$$

(3.19)

Numerically, the last term is 0.04, while the sum of the first two terms is 3.4. Since the present bounds on $N_{\nu}$ are 1.7–3.2 at 3σ confidence level [40], the contribution from the new light particles of our model to $g_*$ does not endanger the successful predictions of BBN.

4. Semi-local model

It is possible to promote the global symmetry to a semi-local one by assigning opposite $U(1)_X$ gauge charges to $X_1$ and $X_2$. In this case, the scalar gauge Lagrangian is

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - |D_\mu X_1|^2 - |D_\mu X_2|^2 - V_0 - V_{\text{lift}} - \frac{1}{2} D^2.$$  

(4.1)

For the kinetic terms, we use the standard abbreviations

$$D_\mu X_{1,2} = \partial_\mu X_{1,2} \pm i \frac{g}{2} A_\mu X_{1,2}$$

(4.2)

and

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$  

(4.3)

The $D$-term is given by

$$D = \frac{g}{2} (X_1^* X_1 - X_2^* X_2 - m_{F_1}^2).$$

(4.4)

Evidently, the global $SU(2)$ model is the $g = 0$ limit of the semi-local one. Note that if we do not gauge the $U(1)_X$, it still is a global symmetry of the superpotential (2.1). We also allow for a Fayet–Iliopoulos mass term $m_{F_1}$, which we assume to be $m_{F_1} \ll M$.

Here, we only discuss the most important changes that the $U(1)_X$ gauge interaction incurs when compared to the purely global case. Choosing again the
parametrization (2.11) and using $V_{\text{lift}}$, we minimize

$$V_{\text{w}}(w_1^+, w_2^+, w_2^-, \varphi_2^+) = \kappa^2 \left( w_1^+ w_2^+ e^{i\varphi_2^+} - M^2 \right)^2 + m_1^2 w_1^+ w_1^- + m_2^2 w_2^+ w_2^- + \frac{g^2}{8} \left( w_1^+ w_1^- - w_2^+ w_2^- - m_{\text{FI}}^2 \right)^2,$$

and find, assuming $m_{1,2} \ll M$,

$$w_1^+ = M - \frac{2\kappa^2 (m_1^2 - m_2^2 - \frac{g^2}{2} m_{\text{FI}}^2)}{4g^2\kappa^2 M} + O \left( \frac{m_{1,2}^4}{M^3}, \frac{m_{\text{FI}}^4}{M^3} \right), (4.6a)$$

$$w_2^+ = M - \frac{2\kappa^2 (m_2^2 - m_1^2 + \frac{g^2}{2} m_{\text{FI}}^2)}{4g^2\kappa^2 M} + O \left( \frac{m_{1,2}^4}{M^3}, \frac{m_{\text{FI}}^4}{M^3} \right), (4.6b)$$

$$w_2^- = 0, \quad \varphi_2^+ = 0. \quad (4.6c)$$

The field $H_g$ picks up the mass

$$m^2(H_g) = \frac{1}{2} g^2 w^2$$

from the $D$-terms, while the soft SUSY breaking lifting terms give rise to the masses

$$m^2(H_0^R) = m^2(H_Y^I) = m_1^2 + m_2^2. \quad (4.8)$$

Furthermore, from the kinetic terms in (4.1), we see that $G_\| (2.13)$, which due to (4.6c) is identical to $G_{\text{w}3}$, turns into the transverse component of $V_\mu$ according to the Higgs mechanism.

This set-up has similarities to the $F_D$-term hybrid inflationary scenario [34, 44]. However, the mechanism used to circumvent the gravitino problem in that case does not work here. The Higgs state $H_g$, the $U(1)_X$-vector boson $A_\mu$ and their fermionic superpartners all attain the mass $gw/\sqrt{2}$. Because of this, we have called all these states the $g$-sector particles. During the phase transition at the end of inflation, $w$ is evolving rapidly, such that the $g$-sector particles are produced via preheating, which occurs due to their non-adiabatically changing mass term. Even though approximate $D$-parity holds due to the smallness of (C.6), from (C.3) and (C.4), we see that the $g$-sector particles can undergo three-body decays into Goldstone bosons and light moduli. In contrast, this is not possible when $SU(2)$ is fully gauged as in [34, 44], where all particles that have odd $D$-parities acquire the mass $gM/\sqrt{2}$, such that these types of decays are energetically impossible. However, we note that in the semi-local case, we can still resort to the reheating scenarios outlined in section 3.

Note that a possible mixing between the gauged $U(1)_X$ and the $U(1)_Y$ of the SM should be strongly suppressed. Denoting the coupling of $X_{1,2}$ to $U(1)_Y$-hypercharge by $g_X^Y$, we find the constraint

$$g_X^Y g \leq \frac{\Delta \vartheta_w}{\vartheta_w} \frac{v^2}{M^2}. \quad (4.9)$$

where $v$ is the VEV of the SM Higgs boson and $\Delta \vartheta_w/\vartheta_w \approx 7 \times 10^{-4}$ is the uncertainty in Glashow’s weak mixing angle $\vartheta_w$.

This model predicts the formation of semi-local strings [45]. Topology does not protect the stability of semi-local strings. When they unwind, the potential energy is lowered,
while there is an increase in spatial gradient energy. If the spatial gradient energy is larger than the potential energy, the strings are stable. It turns out that this is the case when \[46\]–\[48\]
\[
\kappa / g < 1 / \sqrt{2},
\]
(4.10)
or equivalently \[m^2(H^{R,I}_n) < m^2(H_g)\].

The CMB spectra for semi-local strings will depend on the parameter \(\kappa / g\). It has been speculated that for small \(\kappa / g\), the spectrum will resemble that of Abelian strings, while for large values, it will be closer to that of global textures [49, 50]. A recent simulation for the case of critical coupling, \(\kappa / g = 1 / \sqrt{2}\), yields indeed temperature and polarization spectra for semi-local strings that are very similar, both in terms of amplitude and shape, to those for textures [50]. One will also expect semi-local strings to have a large amount of energy density stored in scalar field gradients, which leads to long-range interactions between the strings. This situation is very similar to the case of textures, such that non-Gaussian hot and cold spots may occur in the CMB.

5. Inflation and impact on the CMB

We now discuss the predictions of our model for the amplitude \(A_s\) of the primordial power spectrum produced during inflation and the scalar spectral index \(n_s\) that can be observed in the CMB. As we discuss below, for given superpotential couplings, the symmetry breaking scale \(M\) can be determined by imposing the measured amplitude of the power spectrum. In addition, the range of values for \(M\) that explains the cold spot due to a texture is given by [17]
\[
M = (6.2^{+1.5}_{-1.2}) \times 10^{15} \text{ GeV}.
\]
(5.1)

Notice that a factor of \(1 / \sqrt{2}\) has to be considered here when we compare \(M\) to the VEV \(\phi_0\) that occurs in (1.1) and (1.2). This factor arises from the different normalizations used for the real fields \(\Phi_i\) and the complex fields \(X_{1,2}\). The fact that the two determinations of \(M\) should be compatible enables us to infer the values of superpotential couplings that lead to a successful explanation of the cold spot in an inflation plus textures scenario. Given the value of \(M\), we can then deduce the value of \(n_s\).

Our aim is now to show that our \(F\)-term hybrid inflationary model can successfully provide quantitative explanations for both the CMB spectrum and the cold spot feature for some values of \(\kappa\). A complete likelihood analysis, yielding precision measurements, would require an exploration of parameter space using Markov chain Monte Carlo techniques. This has been performed for the case of SUSY hybrid inflation and cosmic strings in [8]. Due to the similarity of the contributions of strings and textures to the CMB spectrum, we presume that the central result of [8], that a string contribution close to the maximum allowed value can accommodate \(n_s\) as large as 1, also applies to the case of textures.

Hybrid inflation takes place when \(\langle \tilde{S} \rangle > M\) and all other VEVs are zero. In this configuration, the tree-level potential (3.2) is flat in the inflaton \(\sigma = \sqrt{2} \text{ Re } S\), such that it would not roll towards smaller values. However, the one-loop contribution to the
inflationary potential is (see, for example, [34])

\[ V_{\text{1-loop}} = \frac{1}{32\pi^2} \left\{ 2\kappa^4 \left[ |S|^2 + M^2 \right]^2 \ln \left( \frac{\kappa^2(|S|^2 + M^2)}{Q^2} \right) + |S|^2 - M^2 \right\} \ln \left( \frac{\kappa^2(|S|^2 - M^2)}{Q^2} \right) \]

\[ + 2\lambda^4 \left[ |S|^2 + \frac{\kappa}{\lambda} M^2 \right]^2 \ln \left( \frac{\lambda^2(|S|^2 + (\kappa/\lambda)M^2)}{Q^2} \right) \]

\[ + \frac{3\rho^4}{2} \left[ |S|^2 + \frac{\kappa}{\rho} M^2 \right]^2 \ln \left( \frac{\rho^2(|S|^2 + (\kappa/\rho)M^2)}{Q^2} \right) \]

\[ + \frac{3\rho^4}{2} \left[ |S|^2 - \frac{\kappa}{\rho} M^2 \right]^2 \ln \left( \frac{\rho^2(|S|^2 - (\kappa/\rho)M^2)}{Q^2} \right) \]

\[ - |S|^4 \left[ 2N\kappa^4 \ln \left( \frac{\kappa^2|S|^2}{Q^2} \right) + 4\lambda^4 \ln \left( \frac{\lambda^2|S|^2}{Q^2} \right) + 3\rho^4 \ln \left( \frac{\rho^2|S|^2}{Q^2} \right) \right] \}. \quad (5.2) \]

The logarithmic slope causes the inflaton \( \sigma \) to slowly roll towards smaller VEVs. When \( \langle S \rangle < M \), the combination of waterfall fields,

\[ \frac{1}{\sqrt{2}} (X_1 + X_2), \quad (5.3) \]

develops a negative mass term which induces spontaneous breakdown of \( SU(2) \). In order for this instability to occur first in this direction, rather than in any other direction, which involves the \( N_i \) and \( H_{u,d} \), we either have to impose \( \lambda > \kappa \) and \( \rho > \kappa \), or we just decouple these additional fields from the inflation sector by setting \( \lambda = 0 \) or \( \rho = 0 \). It is at this stage that the textures or semi-local strings form. The condition \( \lambda > \kappa \) can be relaxed if a flat direction involving a top squark has a large VEV during inflation (see the discussion in section 3.3.1). In this case, \( H_u \) acquires a large mass which stabilizes the VEVs of \( H_{u,d} \), even if \( \lambda < \kappa \). In addition, the mass splitting between the \( \tilde{H}_{u,d} \) is suppressed, such that in the expression for the one-loop effective potential, we can effectively set \( \lambda = 0 \), regardless of its actual value.

In addition to the tree-level potential and the one-loop correction, we also allow for a supergravity correction,

\[ V_{\text{SUGRA}} = 32\pi^2\kappa^2 M^4 \frac{S^4}{m_{\text{Pl}}^4}, \quad (5.4) \]

where \( m_{\text{Pl}} = 1.22 \times 10^{19} \) GeV denotes the Planck mass. For this expression, we have assumed that there are only renormalizable contributions to the Kähler potential.

The amplitude of the power spectrum at the scale \( k = 0.002 \) Mpc\(^{-1}\) is given by

\[ A_s = \frac{27\pi}{3m_{\text{Pl}}^6 (\partial V/\partial \sigma)^2} \left. \frac{V^3(\sigma)}{2m_{\text{Pl}}^3} \right|_{\sigma = \sigma_e}, \quad (5.5) \]

where \( \sigma_e \) is the value of the inflaton field when this scale left the horizon. It can be determined from

\[ N_e = \frac{8\pi}{m_{\text{Pl}}^2} \int_{\sigma_e}^{\sigma_0} d\sigma \left( \frac{\partial V}{\partial \sigma} \right)^{-1}, \quad (5.6) \]
where \( N_e \) is the number of e-folds of inflation since horizon exit. Here, we take \( N_e = 55 \), but note that there is a weak dependence of this value on the thermal history of the Universe. This dependence has no significant impact on the results presented in this section and is therefore neglected.

Imposing the observed value \( A_s = 2.35 \times 10^{-9} \), we can determine the symmetry breaking scale \( M \) for a given set of superpotential couplings \( \kappa, \rho \) and \( \lambda \). In other words, we determine the model parameter \( M \) from the observed \( A_s \). The second observable that we consider is the scalar spectral index which is given by

\[
n_s = 1 - 2\frac{m_{\text{Pl}}^2}{8\pi} \left( \frac{\partial^2 V}{\partial \sigma^2} \right) \bigg|_{\sigma = \sigma_e} .
\]

We can now compare the predictions of our \( F \)-term hybrid inflation model with the best-fit value for \( M \) found in [17]. The result is presented in figure 1. We plot the prediction for \( M \) imposed by the normalization of \( A_s \) for a model with \( \lambda = \rho = 0 \) and another model with \( \lambda = 0 \) and \( \rho = \kappa \), according to the two different reheating scenarios discussed in section 3. For both cases, we also show the effect of the supergravity correction, by comparing to the case where this correction is included to the case where it is omitted. To guide the eye, we have also included the best-fit value for the texture scale from [17], along with the 1\( \sigma \) bounds. We see that for the models that we consider, a satisfactory amplitude for the texture perturbation can be achieved for \( 2 \times 10^{-4} \lesssim \kappa \lesssim 5 \times 10^{-2} \).

We also plot the prediction for the scalar spectral index in figure 1. For the values of \( \kappa \) which would explain the cold spot, the scalar spectral index \( n_s \) lies roughly in a range between 0.985 and 1.01. If there were only adiabatic perturbations, these high values would be in tension with the present data [1], which requires a smaller value for \( n_s \). However, in the case where there is a sub-dominant contribution to the CMB spectrum from topological defects, larger values of \( n_s \) are consistent with the data if the defect contribution is close to its maximum allowed value. Since textures have a similar impact on the CMB to strings, we conjecture that a texture contribution close to its maximum allowed value will also allow for larger values of \( n_s \).

Indeed, the maximum allowed value for the texture scale from CMB observations is [50] \( M \leq 7.4 \times 10^{15} \) GeV, such that the value suggested by [17] is close to maximal. Consequently, the high values of \( n_s \) indicated by our analysis do not necessarily constitute a problem for the texture model at present.

6. Conclusions

The current CMB data hint at the presence of topological defects of order the GUT energy scale \( M_{\text{GUT}} \sim 10^{16} \) GeV. First, fits to the CMB spectrum marginally favour models with a 10\% contribution to it that results from defects. Second, it has been suggested that textures associated with GUT energies could provide an explanation of the origin of the non-Gaussian cold spot [17]. In the near future, improved measurements of the CMB spectrum on small scales, for example by Planck, ACT [51,52] and SPT [53], will either severely tighten the bounds on defects or strongly hint at their presence [54]. For instance, detailed estimates of the bounds on the cosmic string tension that are anticipated to be possible using the results of the Planck satellite are given in [10].
In this paper we have presented a realistic model of $F$-term hybrid inflation which can lead to the formation of textures at the waterfall transition. The main results of our study may be summarized as follows:

- The superpotential of the $F$-term hybrid model under consideration realizes a $SU(2) \times SU(2)_X$ global symmetry which is explicitly broken to $SU(2) \times U(1)_X$ by soft SUSY breaking terms. The global symmetry $SU(2) \times U(1)_X$ is spontaneously broken to $U(1)_D$, giving rise to a vacuum manifold that is homeomorphic to $S^3$. Thus, the model can naturally predict the formation of textures at the waterfall transition. If the superpotential coupling $\kappa$ is not too small, i.e. $\kappa \gtrsim 10^{-4}$, the energy scale of the textures produced will be close to the GUT scale, as was suggested in [17].

- The aforementioned $F$-term hybrid model contains a number of flat directions that are lifted by the small soft SUSY breaking masses. The presence of these quasi-flat
directions needs to be taken carefully into account in the reheating process. We have exemplified a viable cosmological scenario, incorporating the transfer of energy from the inflationary to the MSSM sector, while avoiding gravitino overproduction or fast stellar cooling.

- We have also presented a number of alternative scenarios including a semi-local model, where $U(1)_X$ is promoted to a local symmetry. Detailed discussion of explicit and spontaneous breaking patterns due to the presence of the moduli-lifting soft SUSY breaking terms are given in the appendices. Our results may also be useful for building other models with related symmetries.

In conclusion, the analysis presented in this paper has shown that the symmetry breaking scale $\phi_0 \approx 8.7 \times 10^{15}$ GeV required to fit the cold spot can be achieved without field-theoretical, phenomenological or cosmological difficulties. The $F$-term hybrid model under study provides a predictive, realistic framework that can successfully address the recent evidence for texture defects present in the current CMB data and link these to inflation.

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**Appendix A. Symmetries and breaking patterns**

In this appendix, we present more details on the $SU(2) \times SU(2)_X$ symmetry of the superpotential. In particular, we discuss formal aspects of the spontaneous and the explicit breakdown of the symmetry, such as counting of Goldstone modes and homotopy groups of the vacuum manifold.

The combined action of the symmetry transformations (2.2) and (2.3) can be compactly written in terms of a tensor product as

$$X = \begin{pmatrix} \hat{X}_1^+ \\ \hat{X}_2^+ \\ \hat{X}_1^- \\ \hat{X}_2^- \end{pmatrix} \xrightarrow{SU(2)_X \times SU(2)} e^{i\theta_1\sigma_1^i} \otimes e^{i\alpha_1\tau_1^i} \begin{pmatrix} \hat{X}_1^+ \\ \hat{X}_2^+ \\ -\hat{X}_1^- \\ -\hat{X}_2^- \end{pmatrix}. \tag{A.1}$$

To be more explicit, the action of $SU(2)_X$ only is given by

$$\begin{pmatrix} \hat{X}_1^+ \\ \hat{X}_1^- \\ \hat{X}_2^- \\ -\hat{X}_2^+ \end{pmatrix} \xrightarrow{SU(2)_X} \exp \left[ i\theta_1 \begin{pmatrix} \sigma_1^{i_1} & 0 & \sigma_1^{i_2} & 0 \\ 0 & \sigma_1^{i_1} & 0 & \sigma_1^{i_2} \\ \sigma_2^{i_1} & 0 & \sigma_2^{i_2} & 0 \\ 0 & \sigma_2^{i_1} & 0 & \sigma_2^{i_2} \end{pmatrix} \right] \begin{pmatrix} \hat{X}_1^+ \\ \hat{X}_1^- \\ \hat{X}_2^- \\ -\hat{X}_2^+ \end{pmatrix}. \tag{A.2}$$

As an immediate application of this, we may write down the superpotential (2.1) in the manifestly $SU(2) \times SU(2)_X$-invariant form:

$$\hat{X}_1^T \hat{X}_2 = -\frac{1}{2}X^T i\alpha^2 \otimes i\tau^2 X. \tag{A.3}$$
Since we have six symmetries available, and the scalar fields $X_1$ and $X_2$ comprise eight degrees of freedom, we can always reparametrize the VEVs to have the form

$$X_1 = \left( \begin{array}{c} w_1^+ \\ 0 \end{array} \right), \quad X_2 = \left( \begin{array}{c} w_2^+ \\ 0 \end{array} \right).$$  \hfill (A.4)

We continue to denote the $SU(2)_X$ generators by $\sigma^i$ and those of $SU(2)$ by $\tau^i$.

We first discuss the symmetry breaking in the case where there is no additional explicit breaking through the lifting potential $V_{\text{lift}}$ given by (2.21). Using the parametrization of the VEVs (A.4), it is useful to distinguish two cases. An overview of arguments is presented in table A.1.

| Configuration | SSB | Goldstone modes | Vacuum manifold | Topological defects |
|---------------|-----|-----------------|-----------------|---------------------|
| $|X_1| = |X_2|$ | $SU(2) \times SU(2)_X$ \(|X_1|=|X_2|, \ SU(2)_D\) | $G_{\tau^i}$ \((i = 1, 2, 3)\) | $S^3$ (up to non-Goldstone flat directions) | Textures |
| $|X_1| \neq |X_2|$ | $SU(2) \times SU(2)_X$ \(|X_1|=\neq|X_2|, \ U(1)_D\) | $G_{\tau^i}$ \((i = 1, 2, 3)\), $H_{0}^I$, $H_{0}^R$ | $S^3 \times S^2$ (up to non-Goldstone flat direction) | Textures and topologically unstable monopoles |

- When $w_1^+ = w_2^+$ ($|X_1| = |X_2|$ is the reparametrization-invariant form), all three generators of $SU(2)$ and $SU(2)_X$ have the same effect on the VEVs, which means $G_{\tau^i} = G_{\sigma^i}$. Due to this identity, the symmetry is broken to a diagonal group, according to

$$SU(2) \times SU(2)_X \xrightarrow{|X_1|=|X_2|} SU(2)_D. \hfill (A.5)$$

In this case, the additional massless modes $H_{0}^R$ and $H_{0}^I$ cannot be identified as Goldstone bosons. For the third homotopy group associated with the symmetry breaking, we find $\pi_3(SU(2) \times SU(2)_X/SU(2)_D) = \mathbb{Z}$, while all other homotopy groups are equal to the identity. Therefore, there are topologically stable textures and, up to the non-Goldstone flat directions, the vacuum manifold is homeomorphic to the three-dimensional sphere $S^3$.

- When $w_1^+ \neq w_2^+$ (reparametrization invariant: $|X_1| \neq |X_2|$), we only have $G_{\tau^3} = G_{\sigma^3}$, such that only a diagonal Abelian symmetry remains intact according to

$$SU(2) \times SU(2)_X \xrightarrow{|X_1|=\neq|X_2|}, U(1)_D. \hfill (A.6)$$

The five Goldstone bosons are the $G_{\tau^i}$, and $H_{0}^R$. The latter two are linear combinations of $G_{\tau_{1,2}}$ and $G_{\sigma_{1,2}}$ which are orthogonal to $G_{\tau_{1,2}}$. The remaining flat direction $H_{0}^I$ corresponds to the transformation $w_1^+ \rightarrow \zeta w_1^+$, $w_2^+ \rightarrow \zeta^{-1}w_2^+$, which leaves $w_1^+ w_2^+ = M^2$ invariant. The third homotopy group is given by $\pi_3(SU(2) \times SU(2)_X/U(1)_D) = \mathbb{Z}$ and the second by $\pi_2(SU(2) \times SU(2)_X/U(1)_D) = \mathbb{Z}$, such that there may be monopoles besides textures. Indeed, up to the non-Goldstone...
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Table A.2. Symmetry breaking patterns for $V_{\text{lift}} \neq 0$. In the second column of the table, we also exhibit the symmetry breaking steps for the cases $m_1 = m_2$ and $m_1 \neq m_2$ due to the explicit presence of a non-zero $V_{\text{lift}}$.

| Lifting terms | Breaking pattern | Goldstone modes | Pseudo-Goldstone modes | Vacuum manifold and defects |
|---------------|------------------|-----------------|------------------------|----------------------------|
| $m_1 = m_2$   | $SU(2) \times SU(2)_X$ explicit, $SU(2) \times SU(2)_X$ $|X_1| = |X_2|$, $SU(2)_D$ | $G_{\tau i}$ ($i = 1, 2, 3$) | None | $S^3$, textures |
| $m_1 \neq m_2$| $SU(2) \times SU(2)_X$ explicit, $SU(2) \times SU(2)_X$ $|X_1| \neq |X_2|$, $U(1)_D$ | $G_{\tau i}$ ($i = 1, 2, 3$) | $H^{I}_0$, $H^{R}_0$ | $S^3$, textures |

flat direction, the vacuum manifold is homeomorphic to $S^3 \times S^2$. The emergence of the $S^2$ has a simple geometric interpretation. For fixed values of $|X_1|$ and $|X_2|$, the condition $F_S = 0$ implies that $X_1 X_2 = M^2$, which fixes the absolute value of the angle between $X_1$ and $X_2$, when considering each as a four-dimensional real vector. Hence, keeping $|X_1|$ and $|X_2|$ fixed, the vacuum manifold is a direct product of the three-dimensional sphere $S^3$, times an $S^2$ corresponding to rotations that leave this angle invariant. When taking the flat direction $H_g$ into account, the configuration can change to $|X_1| = |X_2|$, such that the monopoles can unwind. Hence, the monopoles are topologically unstable. In other words, even though $\pi_2(SU(2) \times SU(2)_X/U(1)_D) = \mathbb{Z}$, for the full vacuum manifold $M$ including the direction $H_g$, we find $\pi_2(M) = 0$. We have not checked whether the monopoles will nevertheless be dynamically stable, that is, whether there is a barrier in the gradient energy which prevents them from unwinding.

We now turn to the case $V_{\text{lift}} \neq 0$, which is relevant for the main body of the paper. Again, we distinguish two situations, and a summary of these arguments is presented in table A.2.

- Suppose first that in (2.21), we have $m_1 = m_2$. The transformation (A.1) leaves $|X_1|^2 + |X_2|^2$ invariant and hence $V_{\text{lift}}$ is also invariant. For this reason, (2.24) explicitly gives $|X_1| = |X_2|$, and the symmetry breaking pattern is as in (A.5). There are three Goldstone modes $G_{\tau i}$ ($i = 1, 2, 3$) and the moduli $H_g$ and $H^{R}_0$ acquire masses as in (moduli:masses), since they are not associated with a broken symmetry. In this case, the vacuum manifold is an $S^3$ and there may be texture defects.

- If $m_1 \neq m_2$, $V_{\text{lift}}$ is no longer invariant under transformations generated by the $SU(2)_X$-generators $\sigma^{1,2}$. Moreover, we find from (2.24) that $|X_1| \neq |X_2|$. Since $V_{\text{lift}}$ is still invariant under rotations related to the $SU(2)_X$-generator $\sigma^3$, the symmetry
and the moduli masses are modified to be

\[ SU(2) \times SU(2) \xrightarrow{V_{\text{fin}}} SU(2) \times U(1)_X \rightarrow SU(2) \times U(1)_X \text{ with } |X_1| \neq |X_2|, U(1)_D. \]  

(A.7)

In this situation, there are again three Goldstone bosons \( G_i \), \( i = 1, 2, 3 \). The moduli \( H_0^{R,i} \) can become massive, since the breaking effect of \( V_{\text{lift}} \) turns them from Goldstone into pseudo-Goldstone bosons of \( SU(2)_X \). The remaining mode \( H_d \) is not associated with a broken symmetry and can, therefore, acquire a mass. The vacuum manifold is again \( S^3 \), and accordingly, there will be textures.

Finally, we comment on the semi-local model described in section 4. In this case, the \( U(1)_X \) is singled out of \( SU(2)_X \) due to the gauging, which induces gauge kinetic terms as well as \( D \)-terms. These terms explicitly break \( SU(2)_X \). The symmetry breaking pattern is

\[ SU(2) \times SU(2)_X \xrightarrow{\text{gauge}} SU(2) \times U(1)_X \text{ with } |X_1,2| \neq 0, U(1)_D. \]  

(A.8)

Again, there are three Goldstone bosons \( G_i \), \( i = 1, 2, 3 \), out of which \( G_3 \) is absorbed as a transverse degree of freedom of the \( U(1)_X \) gauge boson. The vacuum manifold is \( S^3 \), and there will be semi-local strings.

**Appendix B. Field redefinitions and the lifting potential**

When we include the \( d \)-term in the lifting potential (2.21), it turns out that the minima (2.24) get shifted to

\[ w_1^+ = \sqrt{\frac{m_2^2 M^2}{m_1} \sqrt{1 - (|d|^2/m_1^2 m_2^2)}} - \frac{m_2^2}{\kappa^2}, \]  

(B.1a)

\[ w_2^- = \sqrt{1 - \frac{|d|^2}{m_1^2 m_2^2} \sqrt{\frac{m_1^2 M^2}{m_2} \sqrt{1 - (|d|^2/m_1^2 m_2^2)}} - \frac{m_2^2}{\kappa^2},} \]  

(B.1b)

\[ w_2^- = |d| \sqrt{\frac{M^2}{m_1 m_2} \sqrt{1 - (|d|^2/m_1^2 m_2^2)}} \sqrt{1 - (|d|^2/m_1^2 m_2^2)} - \frac{1}{\kappa^2}, \]  

(B.1c)

\[ \alpha_3 - \beta = \pi + \arg d, \]  

(B.1d)

and the moduli masses are modified to be

\[ m^2(H_0^R) = m^2(H_0^I) = m_1^2 + m_2^2, \quad m^2(H_d) = 4 \frac{m_1^2 m_2^2 - |d|^2}{m_1^2 + m_2^2}. \]  

(B.2)

For a given \( \alpha_3, \beta \) is now fixed by (B.1c), which is due to the fact that the \( d \)-term explicitly breaks the \( U(1)_X \)-symmetry. Therefore, \( \beta \) does not correspond to a massless mode. We recall from (2.24) that in the situation where \( d = 0 \), \( m_1 \neq 0 \) and \( m_2 \neq 0 \), it follows that \( w_2^- = 0 \) and that the parameters \( \alpha_3 \) and \( \beta \) induce the same rotations. Hence, in both situations, \( \beta \) does not correspond to an independent massless degree of freedom.

This is consistent with the fact that when performing a field redefinition, the \( d \)-term can be removed. To see this, we make use of the fact that field redefinitions according to
SU(2)_X have the explicit effect

\[ |X_1|^2 \exp(i \theta_1 \sigma^1) |X_1|^2 \cos^2 \theta_1 + |X_2|^2 \sin^2 \theta_1 + i \sin \theta_1 \cos \theta_1 \left( X_1^1 \tau^2 X_2 + X_2^1 \tau^2 X_1 \right), \quad (B.3a) \]

\[ |X_2|^2 \exp(i \theta_1 \sigma^1) |X_2|^2 \cos^2 \theta_1 + |X_1|^2 \sin^2 \theta_1 - i \sin \theta_1 \cos \theta_1 \left( X_2^1 \tau^2 X_1 + X_1^1 \tau^2 X_2 \right), \quad (B.3b) \]

while a rotation by \( \exp(i \vartheta \sigma^3) \) changes the phase of \( d \) in the lifting potential\(^5\).

Hence, we find that, upon \( SU(2)_X \) field redefinitions, any \( d \)-term can be absorbed into the \( m_{1,2}^2 \)-terms, provided \( |d|^2 < m_{1,2}^2 \). If the latter condition is not fulfilled, we see from (B.2), that the mode \( H_g \) attains a negative mass square and the potential has no stable minimum. We also note that the moduli masses in (B.2) are invariant under this type of reparametrization, as they should be.

**Appendix C. Scalar interaction terms and \( D \)-parities**

In this appendix, we present the expressions (C.3) and (C.4), from which the interactions within the scalar sector can easily be inferred. It turns out that in the situation where \( w_1^+ \approx w_2^+ \), the symmetry of the model is enhanced, which has the consequence that certain interactions are suppressed. These accidental symmetries are important for the \( F_D \)-model, and we adopt the term \( D \)-parities from that context [34].

In the global model, for \( m_1 = m_2, V = V_0 + V_{\text{inh}} \) given by (2.5) and (2.21) is invariant under the discrete accidental symmetries

\[ D_1: X_1 \leftrightarrow X_2, \quad (C.1) \]

\[ D_2: X_1 \leftrightarrow \tau^3 X_1, \quad X_2 \leftrightarrow \tau^3 X_2. \quad (C.2) \]

For the semi-local case, we have to require \( m_{\text{FI}} = 0 \) in addition, for the \( D \)-parities to be respected. In table C.1, we list the \( D \)-parities of the particular mass eigenstates. If \( m_1 = m_2 \), interactions that violate the discrete multiplicative \( D \)-parities are forbidden. On the other hand, allowing for \( m_1 \neq m_2 \) or \( m_{\text{FI}} \neq 0 \), the smallness of \( w_1^+ w_1^+ - w_2^+ w_2^+ \) allows us to control the rate of \( D \)-parity violating interactions, a feature which we make use of when constructing the semi-local model.

In particular, we explicitly see the realization of \( D \)-parity when writing down the expression occurring within the \( F_S \)-term,

\[ X_1 X_2 = w_1^+ w_2^+ + \frac{w}{\sqrt{2}} (H_R^\kappa + i H_I^\kappa) \]

\[ + \frac{w_1^+ w_2^+}{2w^2} (G_{\tau_1}^2 + G_{\tau_2}^2 + G_{\tau_3}^2 - H_g^2 - 2iH_g G_{\tau_3} - H_0^{R^2} - H_0^{I^2} + (H_R^\kappa + i H_I^\kappa)^2) \]

\[ - \frac{i}{2} G_{\tau_1} H_0^R - \frac{i}{2} G_{\tau_2} H_0^I \]

\[ + \frac{w_1^+ w_2^+}{2w^2} (G_{\tau_2} H_0^R - G_{\tau_1} H_0^I - G_{\tau_3}(iH_R^\kappa - H_I^\kappa) + H_g H_R^\kappa + i H_g H_I^\kappa), \quad (C.3) \]

\(^5\) Note that this particular symmetry is identical to the \( U(1)_X \).
where the last term is the $D$-parity violating contribution. Accordingly, the $D$-term (4.4) is

$$D = -\frac{g}{2} m_{\text{FI}}^2 + \frac{g}{2} \frac{w_1^{+2} - w_2^{+2}}{w_1^{+2} + w_2^{+2}} + \frac{g}{\sqrt{2}} w H_g + g \frac{w_1^{+2} - w_2^{+2}}{w_1^{+2} + w_2^{+2}} (H_g H_0^R + H_0^G G_0^1 - H_0^R G_0^2 + H_0^G G_0^3)$$

$$+ \frac{g}{4 w_1^{+2} + w_2^{+2}} (-H_0^{R^2} - H_0^{G^2} - H_0^{R_0^2} - H_0^{G_0^2} + H_0^2 + G_0^1 + G_0^3 + G_{\tau_2}^2 + G_{\tau_3}^2).$$

(C.4)

Note that the third and fourth terms are both $D_1$-odd and $D_2$-even, while the remaining terms are even under both parities. Hence, for $m_{1,2} = 0$ and $m_{\text{FI}} = 0$, the $D$-term has definite $D$-parity, so $D^2$ conserves $D$-parity.

Since we assume $m_{1,2} \ll M$, we may parametrize the $D$-parity violation in the global model by

$$\frac{w_1^{+2} - w_2^{+2}}{w_1^{+2} + w_2^{+2}} \approx \frac{m_2^2 - m_1^2}{2 m_1 m_2},$$

(C.5)

where we have set $d = 0$ and used (2.24). In the semi-local model, using (4.6), we find

$$\frac{w_1^{+2} - w_2^{+2}}{w_1^{+2} + w_2^{+2}} \approx \frac{m_{\text{FI}}^2}{2 M^2} + \frac{m_2^2 - m_1^2}{g^2 M^2}.$$  

(C.6)

Hence, we note that $D$-parity violation in the semi-local case is always suppressed as $m_{1,2}^2 / M^2$ or $m_{\text{FI}}^2 / M^2$, whereas the corresponding expression (C.5) for the global case can be of order 1 in general, unless $m_1 \approx m_2$.

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**Table C.1. $D$-parities of the scalar mass eigenstates.**

| Eigenstate | $D_1$-parity | $D_2$-parity |
|------------|--------------|--------------|
| $H_0^R$, $H_0^G$ | + | + |
| $G_\tau^1$, $H_0^G$ | - | - |
| $G_\tau^2$, $H_0^G$ | + | - |
| $G_\tau^3$, $H_g$ | - | + |
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