$D$ leptonic and semileptonic decays

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Based on 2.92 fb$^{-1}$ data taken at the center-of-mass energy $\sqrt{s} = 3.773$ GeV with the BESIII detector, we report recent results on the decay constant $f_{D^+}$, the hadronic form factors, as well as the quark mixing matrix elements $|V_{cs(d)}|$, which are extracted from analyses of the leptonic decay $D^+ \rightarrow \mu^+\nu_\mu$ and the semileptonic decays $D^0 \rightarrow K(\pi)^-e^+\nu_e$, $D^+ \rightarrow K^0_L e^+\nu_e$, $D^+ \rightarrow K^-\pi^+e^+\nu_e$ and $D^+ \rightarrow \omega(\phi)e^+\nu_e$ at BESIII.

PRESENTED AT

The 7th International Workshop on Charm Physics
(CHARM 2015)
Detroit, MI, 18-22 May, 2015
1 Introduction

In the Standard Model, the $D^+$ mesons decay into $\ell \nu_\ell$ via a virtual $W^+$ boson. The decay rate of the leptonic decays $D^+ \rightarrow \ell^+ \nu_\ell$ can be parameterized by the $D^+$ decay constant $f_{D^+}$ via

$$\Gamma(D^+ \rightarrow \ell^+ \nu_\ell) = \frac{G_F^2}{8\pi} |V_{cd}|^2 f_{D^+} m_\ell^2 m_{D^+} (1 - \frac{m_\ell^2}{m_{D^+}^2}),$$

(1)

where $G_F$ is the Fermi coupling constant, $|V_{cd}|$ is the quark mixing matrix element between the two quarks $c$ and $d$, $m_\ell$ and $m_{D^+}$ are the lepton and $D^+$ masses.

On the other hand, the $D$ semileptonic decays can be parameterized by the quark mixing matrix element and the form factor of hadronic weak current simply, thus providing an ideal window to probe for the weak and strong effects. For example, the differential decay rates of $D \rightarrow K(\pi)e^+\nu_e$ can be simply written as

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3} |V_{cs(d)}|^2 |p_{K(\pi)}|^2 f^K_+(q^2)^2,$$

(2)

where $G_F$ is the Fermi coupling constant, $|V_{cs(d)}|$ is the quark mixing matrix element between the two quarks $c$ and $d$, $p_{K(\pi)}$ is the kaon(pion) momentum in the $D^0$ rest frame, $f^K_+(q^2)$ is the form factor of hadronic weak current depending on the square of the four momentum transfer $q = p_D - p_{K(\pi)}$.

In 2010 and 2011, BESIII [1] accumulated 2.92 fb$^{-1}$ data at $\sqrt{s} = 3.773$ GeV [2], where $e^+e^- \rightarrow \psi(3770) \rightarrow D^0\bar{D}^0$ or $D^+D^-$ is produced predominantly. Based on the studies of the leptonic and semileptonic decays of $D^0$ and $D^+$ mesons, the $D^+$ decay constant, the hadronic form factors or the quark mixing matrix elements $|V_{cs(d)}|$ can be extracted accurately. These will validate the LQCD calculations of the $D^+$ decay constant and the hadronic form factors or test the unitarity of the quark mixing matrix at higher accuracies. They are also helpful to improve the measurement precisions in the experimental studies of the leptonic and semileptonic decays of $B$ mesons indirectly. Herein, we report recent results on the studies of the leptonic decay $D^+ \rightarrow \mu^+\nu_\mu$ and the semileptonic decays $D^0 \rightarrow K(\pi)^-e^+\nu_e$, $D^+ \rightarrow K^0 e^+\nu_e$, $D^+ \rightarrow K^-\pi^+e^+\nu_e$ and $D^+ \rightarrow \omega(\phi)e^+\nu_e$ at BESIII. Throughout the proceeding, charge conjugate is implied.

2 Leptonic decay [3]

To investigate the leptonic decay $D^+ \rightarrow \mu^+\nu_\mu$, we reconstruct the singly tagged $D^-$ mesons using 9 hadronic decays. Figure [1] (left side) shows the fits to the beam-energy-constrained mass ($M_{BC}$) spectra of the (a) $K^+\pi^-\pi^-$, (b) $K^0_S\pi^-$, (c) $K^0_SK^-$,
which has the best precision in the world to date. By using the measured $B$ (the signal region is marked by the pair of arrows in each sub-figure). (right side) $M_{\text{miss}}^2$ distribution for $D^+ \to \mu^+\nu_\mu$ candidates.

(d) $K^+K^-\pi^-$, (e) $K^+\pi^-\pi^-\pi^0$, (f) $\pi^-\pi^-\pi^+$, (g) $K_S^0\pi^-\pi^0$, (h) $K^+\pi^-\pi^-\pi^-$ and (i) $K_S^0\pi^-\pi^-\pi^+$ combinations, which yield $(170.31 \pm 0.34) \times 10^4$ singly tagged $D^-$ mesons.

Figure 1 (right side) shows the $M_{\text{miss}}^2$ distribution for (left side) Fits to the $M_{BC}$ spectra for the singly tagged $D^-$ candidates (the signal region is marked by the pair of arrows in each sub-figure). (right side) $M_{\text{miss}}^2$ distribution for $D^+ \to \mu^+\nu_\mu$ candidates.

$B(D^+ \to \mu^+\nu_\mu) = (3.71 \pm 0.19_{\text{stat}} \pm 0.06_{\text{sys}}) \times 10^{-4}$.

Using the measured $B(D^+ \to \mu^+\nu_\mu)$ and the quark mixing matrix element $|V_{cd}|$ from a global Standard Model fit [4], we determine the $D^+$ decay constant

$$f_{D^+} = 203.2 \pm 5.3_{\text{stat}} \pm 1.8_{\text{sys}} \text{ MeV}.$$  

The $B(D^+ \to \mu^+\nu_\mu)$ and $f_{D^+}$ measured at BESIII are consistent within errors with those measured at BESI [3], BESII [6] and CLEO-c [7], but with the best precision. Figure 2 compares the $f_{D^+}$ measured at BESIII and CLEO-c as well as those calculated by recent theories.

So far, the quark mixing matrix element $|V_{cd}|$ has been measured though experimental studies of the semileptonic decay $D \to \pi \ell^+\nu_\ell$ or measurement of charm production cross section of $\nu\Sigma$ interaction, among which the best measurement precision is 4.8% [4]. By using the measured $B(D^+ \to \mu^+\nu_\mu)$ and the Lattice QCD calculation on $f_{D^+}$ [8], we determine

$$|V_{cd}| = 0.2210 \pm 0.058_{\text{stat}} \pm 0.047_{\text{sys}},$$

which has the best precision in the world to date.
Figure 2: Comparison of the $D^+$ decay constant.

3 Semileptonic decays

3.1 $D^0$ semileptonic decays [9]

To investigate the semileptonic decays $D^0 \rightarrow K(\pi)^-e^+\nu_e$, we reconstruct the singly tagged $D^0$ mesons using 5 hadronic decays. Figure 3 (left side) shows the fits to the $M_{BC}$ spectra of the (a) $K^+\pi^-$, (b) $K^+\pi^-\pi^0$, (c) $K^+\pi^-\pi^+\pi^+$, (d) $K^+\pi^-\pi^+\pi^0$ and (e) $K^+\pi^-\pi^0\pi^0$ combinations. $(279.33 \pm 0.37) \times 10^4$ singly tagged $D^0$ mesons are accumulated.

Figure 3: (left side) Fits to the $M_{BC}$ spectra for the singly tagged $D^0$ candidates. (right side) Fits to the $U_{\text{miss}}$ distributions for (a) $D^0 \rightarrow K^-e^+\nu_e$ and (b) $D^0 \rightarrow \pi^-e^+\nu_e$ candidates.

Figure 3 (right side) shows the fits to the $U_{\text{miss}}$ distributions of the candidates for $D^0 \rightarrow K^-e^+\nu_e$ and $D^0 \rightarrow \pi^-e^+\nu_e$, which are selected in the systems against the singly tagged $D^-$ mesons. From the fits, we obtain $70727 \pm 278$ and $6297 \pm 87$ signals of $D^0 \rightarrow K^-e^+\nu_e$ and $D^0 \rightarrow \pi^-e^+\nu_e$. Based on these, we determine the branching
fractions

\[ B(D^0 \to K^- e^+ \nu_e) = (3.505 \pm 0.014_{\text{stat.}} \pm 0.033_{\text{sys.}})\% \]

and

\[ B(D^0 \to \pi^- e^+ \nu_e) = (0.2950 \pm 0.0041_{\text{stat.}} \pm 0.0026_{\text{sys.}})\% \]

respectively. The \( B(D^0 \to K^- e^+ \nu_e) \) and \( B(D^0 \to \pi^- e^+ \nu_e) \) measured at BESIII are consistent within errors with those measured at BESII \([10]\), CLEO-c \([11]\), BELLE \([12]\) and BABAR \([13, 14]\), but with the best precision.

Figure 4 shows the fits to the partial widths and the projections on the form factors of \( D^0 \to K^- e^+ \nu_e \) and \( D^0 \to \pi^- e^+ \nu_e \) using the Simple Pole model \([15]\), the Modified Pole model \([15]\), the ISGW2 model \([16]\), the two-parameter series expansion (Series.2.Par.) \([17]\) and the three-parameter series expansion (Series.3.Par.) \([17]\). From the fits, we obtain the extracted parameters of different models, which are summarized in Table 1.

![Figure 4](image-url)  

**Figure 4:** The fits to the partial widths and the projections on the form factors of \( D^0 \to K^- e^+ \nu_e \) (left side) and \( D^0 \to \pi^- e^+ \nu_e \) (right side).

With the extracted \( f_+^{K(\pi)}(0) |V_{cs(d)}| \) and the expected \( f_+^{K(\pi)}(0) \) by LQCD \([18]\), we determine the quark mixing matrix element \( |V_{cs(d)}| \). Figure 5 compares the \( |V_{cs(d)}| \) extracted at BESIII with the ones from other experiments.

### 3.2 \( D^+ \) Semileptonic decays

To study the semileptonic decays \( D^+ \to K^0 e^+ \nu_e \), \( D^+ \to K^- \pi^+ e^+ \nu_e \) and \( D^+ \to \omega(\phi) e^+ \nu_e \), we reconstruct the singly tagged \( D^- \) mesons using 6 hadronic decays of
Table 1: Summary of the extracted parameters from the fits to the partial widths, where the first errors are statistical and the second systematic.

| Model          | $D^+ \rightarrow K^- e^+ \nu_e$ | $D^+ \rightarrow \pi^- e^+ \nu_e$ |
|----------------|---------------------------------|----------------------------------|
| Simple Pole    | $f^S_+(0)|V_{cs}|$               | $f^S_+(0)|V_{cd}|$               |
| $f^S_+(0)|V_{cs}|$ | $0.7209 \pm 0.0022 \pm 0.0033$ | $1.9207 \pm 0.0103 \pm 0.0069$ |
| $f^S_+(0)|V_{cd}|$ | $1.015 \pm 0.0015 \pm 0.0109$ | $1.041 \pm 0.0014 \pm 0.0089$ |
| Modified Pole  | $f^M_+(0)|V_{cs}|$               | $f^M_+(0)|V_{cd}|$               |
| $f^M_+(0)|V_{cs}|$ | $0.7163 \pm 0.0024 \pm 0.0034$ | $1.031 \pm 0.0015 \pm 0.0119$ |
| $f^M_+(0)|V_{cd}|$ | $1.012 \pm 0.0014 \pm 0.0089$ | $1.031 \pm 0.0015 \pm 0.0119$ |
| ISGW2          | $f^G_+(0)|V_{cs}|$               | $f^G_+(0)|V_{cd}|$               |
| $f^G_+(0)|V_{cs}|$ | $0.7139 \pm 0.0023 \pm 0.0034$ | $1.6900 \pm 0.0141 \pm 0.0091$ |
| $f^G_+(0)|V_{cd}|$ | $0.978 \pm 0.002 \pm 0.007$ | $2.0688 \pm 0.0394 \pm 0.0124$ |
| Series 2. Par. | $f^P_+(0)|V_{cs}|$               | $f^P_+(0)|V_{cd}|$               |
| $f^P_+(0)|V_{cs}|$ | $0.7172 \pm 0.0025 \pm 0.0035$ | $-2.2278 \pm 0.0864 \pm 0.0575$ |
| $f^P_+(0)|V_{cd}|$ | $0.961 \pm 0.002 \pm 0.007$ | $-0.2365 \pm 0.0807 \pm 0.0260$ |
| Series 3. Par. | $f^P_+(0)|V_{cs}|$               | $f^P_+(0)|V_{cd}|$               |
| $f^P_+(0)|V_{cs}|$ | $0.7196 \pm 0.0035 \pm 0.0041$ | $-2.3331 \pm 0.1587 \pm 0.0804$ |
| $f^P_+(0)|V_{cd}|$ | $0.961 \pm 0.002 \pm 0.007$ | $-1.8434 \pm 0.2212 \pm 0.0690$ |

Figure 5: Comparison of the extracted $|V_{cd(d)}|$. 

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**Figure 5**: Comparison of the extracted $|V_{cd(d)}|$. 

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**Table 1**: Summary of the extracted parameters from the fits to the partial widths, where the first errors are statistical and the second systematic.
$K^+\pi^-\pi^-, K^+\pi^-\pi^-\pi^0, K_S^0\pi^-\pi^0, K^0_S\pi^-\pi^-\pi^+$ and $K^+K^-\pi^-$. About 1.6 millions of singly tagged $D^-$ mesons are accumulated \cite{19}. Based on these, we study the semileptonic decays $D^+ \to K^0_L e^+\nu_e$, $D^+ \to K^- e^+\nu_e$ and $D^+ \to \omega(\phi)e^+\nu_e$.

3.2.1 Analysis of $D^+ \to K^0_L e^+\nu_e$

Although $K^0_L$ flights long distance, it interacts with the Electron Magnetic Cluster of BESIII and deposits a portion of energy, thus leaving some position information. So, after reconstructing all other particles in the final states, the $K^0_L$ mesons can be inferred with the position information and constraining the $U_{\text{miss}}$ of the candidates to zero. We obtain about 24 thousands of signals of $D^+ \to K^0_L e^+\nu_e$, based on which we determine the branching fraction

$$B(D^+ \to K^0_L e^+\nu_e) = (4.482 \pm 0.027_{\text{stat}} \pm 0.103_{\text{sys}})\%$$

and the CP asymmetry

$$A_{CP}^{D^+ \to K^0_L e^+\nu_e} = (-0.59 \pm 0.60_{\text{stat}} \pm 1.50_{\text{sys}})\%,$$

supporting that there is no CP asymmetry in this decay. In addition, simultaneous fit to the event density $I(q^2)$ for different tag modes with the two-parameter series expansion is performed, as shown in Fig. 6 which yields the product of $f_+(0)|V_{CS}| = 0.728 \pm 0.006_{\text{stat}} \pm 0.011_{\text{sys}}$. These are made for the first time.

![Figure 6: Simultaneous fit to the event density $I(q^2)$ for different tag modes, where the points with error bars are data and blue curves are the fits. The violet, yellow, green and black curves refer to the different background sources.](image)
### 3.2.2 $D^+ \to K^-\pi^+e^+\nu_e$

Based on 18262 signals of $D^+ \to K^-\pi^+e^+\nu_e$, we determine the branching fraction

$$B(D^+ \to K^-\pi^+e^+\nu_e) = (3.71 \pm 0.03 \pm 0.08)\%.$$ 

A partial wave analysis (PWA) is performed on the selected candidates, with results shown in Fig. 7. The PWA results show that the dominant $\bar{K}^{0}$ component is accompanied by an $S$-wave contribution accounting for $(6.05 \pm 0.22 \pm 0.18)\%$ of the total rate, and other components can be negligible. We obtain the mass and width of $\bar{K}^{0}(892) M_{\bar{K}^{0}(892)} = (894.60 \pm 0.25 \pm 0.08)$ MeV/$c^2$ and $\Gamma_{\bar{K}^{0}(892)} = (46.42 \pm 0.56 \pm 0.15)$ MeV/$c^2$, the Blatt-Weisskopf parameter $r_{BW} = 3.07 \pm 0.26 \pm 0.11$ (GeV/$c$)$^{-1}$, as well as the parameters of the hadronic form factors $r_V = \frac{V(0)}{A_1(0)} = 1.411 \pm 0.058 \pm 0.007$, $r_2 = \frac{A_2(0)}{A_1(0)} = 0.788 \pm 0.042 \pm 0.008$, $m_V = (1.81^{+0.25}_{-0.17} \pm 0.02)$ MeV/$c^2$, $m_A = (2.61^{+0.22}_{-0.17} \pm 0.03)$ MeV/$c^2$, $A_1(0) = 0.585 \pm 0.011 \pm 0.017$. Here, the first errors are statistical and the second systematic.

![Figure 7: Projections of the kinematic variables of PWA for $D^+ \to K^-\pi^+e^+\nu_e$, where $m_{K\pi}$ is the $K\pi$ mass, $q^2$ is the $e\nu_e$ mass square, $\theta_K$ is the angle between $\pi$ and $D$ momenta in the $K\pi$ rest frame, $\theta_e$ is the angle between $\nu_e$ and $D$ momenta in the $e\nu_e$ rest frame and $\chi$ is the angle between the two decay planes. The dots with error bars are data, the blue curves are the weighted signal MC and the hatched histograms are the simulated backgrounds.](image)

In the above PWA process, the phase of the non-resonant background $\delta_S(m_{K\pi})$ is factorized by the LASS parameterizations, and the helicity form factors $H_+(q^2, m_{K\pi})$, $H_-(q^2, m_{K\pi})$ and $H_0(q^2, m_{K\pi})$ are parameterized by the spectroscopic pole dominance (SPD) model. We also make model-independent measurements of the $\delta_S(m_{K\pi})$, and the helicity form factors, respectively. The results are consistent with the expectations of the corresponding models and previous measurements.
3.2.3 \( D^+ \rightarrow \omega(\phi)e^+\nu_e \) \[19\]

Based on 491 \( \pm \) 32 signals of \( D^+ \rightarrow \omega e^+\nu_e \), we determine the branching fraction

\[
B(D^+ \rightarrow \omega e^+\nu_e) = (1.63 \pm 0.11_{\text{stat}} \pm 0.08_{\text{sys}}) \times 10^{-3},
\]

which is consistent with previous measurements but with better precision. We perform amplitude analysis of the selected candidates, with results shown in Fig. 8. We obtain the ratios of the hadronic form factors to be

\[
r_V = \frac{V(0)}{A_1(0)} = 1.24 \pm 0.09_{\text{stat}} \pm 0.06_{\text{sys}}
\]

and

\[
r_2 = \frac{A_2(0)}{A_1(0)} = 1.05 \pm 0.15_{\text{stat}} \pm 0.05_{\text{sys}}.
\]

Figure 8: Projections of the kinematic variables of amplitude analysis for \( D^+ \rightarrow \omega e^+\nu_e \), where the dots with error bars are data, the histograms are the fitted results and the hatched histograms are the simulated backgrounds.

Also, we search for \( D^+ \rightarrow \phi e^+\nu_e \), but do not find obvious signal. So, we set the upper limit on the branching fraction for \( D^+ \rightarrow \phi e^+\nu_e \) to be \( 1.3 \times 10^{-5} \) at 90\% Confidence Level, which is significantly better than previous searches.

4 Summary

By analyses of the leptonic decay \( D^+ \rightarrow \mu^+\nu_\mu \) and the semileptonic decays \( D^0 \rightarrow K(\pi)^{-}e^+\nu_e \), \( D^+ \rightarrow K_L^0 e^+\nu_e \), \( D^+ \rightarrow K^-\pi^+e^+\nu_e \) and \( D^+ \rightarrow \omega(\phi)e^+\nu_e \) from 2.92 fb\(^{-1}\) data taken at \( \sqrt{s} = 3.773 \) GeV with the BESIII detector, we extract the \( D^+ \) decay constant, the hadronic form factors and the quark mixing matrix elements \( |V_{cs(d)}| \). These provide key experimental data to validate the LQCD calculations of the \( D^+ \) decay constant and the hadronic form factors and to test the unitarity of the quark mixing matrix at higher accuracies.
ACKNOWLEDGEMENTS

I would like to thank for the support of the National Natural Science Foundation of China (NSFC) under Contracts No. 10935007 and No. 11305180, and the Ministry of Science and Technology of China (973 by MOST) under Contracts No. 2009CB825200 and No. 2015CB856700.

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