Inverse bremsstrahlung absorption with full electron-electron collisions operator

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Abstract. A two-dimensional Fokker-Planck program is developed, in which full e-e collisions are taken into account self-consistently with arbitrary anisotropy of electron distribution function (EDF), to investigate inverse bremsstrahlung and the evolution of EDF. The numerical results show that e-e collisions will enhance inverse bremsstrahlung absorption. The evolution of EDF can be divided into two stages distinguished by different absorption rates. During the first stage, an initially Maxwellian EDF transforms rapidly into an anisotropic one with two temperatures. With the increase of anisotropy, the absorption rate decreases dramatically while the contribution ratio of e-e collisions increases rapidly. In particular, we find that with the increase of ion charge state $Z_i$ the contribution ratio of e-e collisions increases in the high laser frequency regime, while it decreases in the low frequency regime.

1. Introduction

Collisional absorption by inverse bremsstrahlung plays an important role in laser-plasma interactions, and it has been extensively investigated for many decades. The full description of inverse bremsstrahlung requires information of EDF. However, the accurate description of EDF in laser-embedded plasmas remains an open problem.

An important parameter in the modification of EDF is $\alpha = Z_i v_{\text{max}}^2 / v_e^2$, where $Z_i$ is ion charge state, $v_{\text{max}}$ is the peak velocity of electrons oscillating in laser field, $v_e$ is the electron thermal velocity. When $\alpha \leq 1$ EDF is close to the Maxwellian. For $\alpha \geq 1$ inverse bremsstrahlung absorption results in a so-called self-similar state distribution $\ln f \approx -v^5$, for which the absorption rate is reduced dramatically [1, 2]. Numerical calculations and theoretical analysis have shown that EDF can also assume a self-similar state when e-e collisions are taken into account [3, 4]. However, these treatments rely on an assumption of small anisotropy of EDF which is valid only if $v_{\text{max}} \ll v_e$; here $v_e$ is the initial electron thermal velocity. Porshnev et al. [5] removed this assumption and obtained EDF with arbitrary anisotropy. They found that an initially isotropic EDF transforms rapidly into an anisotropic one with two temperatures. The same results were deduced by theoretical analysis using Legendre polynomial expansion in an oscillating frame [6]. However, in these cases they ignored e-e collisions.

In fact, e-e collisions with large anisotropy are much more violent than those with small anisotropy. As a result, the accurate analysis of EDF with any degree of anisotropy should take e-e collisions into account self-consistently. In order to treat EDF and inverse bremsstrahlung more accurately, we numerically solve a Fokker-Planck equation with the e-e collisions included.
2. Master Equation and Numerical Solution

The evolution of EDF in a homogeneous laser-embedded plasma can be described by the well-known Fokker-Planck equation [7]

$$\frac{\partial f_e}{\partial t} + \frac{e\vec{E}}{m} \cdot \nabla \theta f_e = C_{ei}(f) + C_{ee}(f),$$

where $\vec{E}$ is the electric field, $C_{ei}(f)$ and $C_{ee}(f)$ are $e-i$ and $e-e$ collisions terms respectively. In this paper we just consider the linearly polarized laser field. Then we can choose a spherical coordinate system and assume that the laser field is along the z direction. In this coordinate system all functions are independent of the azimuth angle $\phi$, so that we simplify the original problem into a two-dimensional problem. In this case, the final normalized Fokker-Planck equation can be written as [7]

$$\frac{\partial f_e}{\partial t} = \nabla \cdot [(\nabla \nabla \theta G^e + \frac{1}{2\nu} Z_i \bar{e}_e \bar{e}_\theta) \cdot \nabla \theta f_e]
- (\nabla \theta H^e + 2\pi v_L v_{max} \cos(2\pi v_L t)(\cos \theta \bar{e}_e - \sin \theta \bar{e}_\theta)) f_e,$$  

where $v_L$ is the laser frequency, $G^e$ and $H^e$ are Rosenbluth potentials of EDF which can be calculated efficiently using Legendre polynomial expansion [7].

It is useful to define following quantities to evaluate the inverse bremsstrahlung absorption rate and the evolution of EDF: (a) The average inverse bremsstrahlung absorption rate over a laser period $R_n = \frac{1}{2\nu} (E_{n+1} - E_{n-1})$, where $\tau_L$ is laser period and $E_n$ is the average electron thermal energy over the n-th laser period. (b) The contribution ratio of e-e collisions effect will be enhanced while the distribution departs from the Maxwellian; therefore the contribution ratio $\gamma_n$ increases fast with the time in the first 50 laser periods. After the 50-th laser period, the shape of $f_0^e(v)$ will not change significantly, neither do the parameters $R_n$ and $\gamma_n$.

In order to solve equation (2), we develop a two-dimensional Fokker-Planck program, with a scheme similar to Ref. [7], together with some special numerical methods and techniques [8, 9, 10, 11] to enforce stability and particle conservation. In the following, we discuss results of four representational cases: (i) $Z_i = 10, \nu_L = 10\nu_{ee}$; (ii) $Z_i = 1, \nu_L = 10\nu_{ee}$; (iii) $Z_i = 10, \nu_L = 1.25\nu_{ee}$; (iv) $Z_i = 1, \nu_L = 1.25\nu_{ee}$, with an invariable parameter $v_{max} = 0.5v_{e0}$, where $v_{ee}$ is e-e collision frequency.

In case (i), $Z_i v_{max}^2/v_{e0}^2 = 2.5 > 1$ and $\nu_{ee}\tau_L = 0.1 \ll 1$, so that inverse bremsstrahlung absorption is sufficient and e-e collisions is slow. And e-e collisions are not rapid or powerful enough to reestablish a Maxwellian distribution, but a self-similar state of EDF is formed. We find that $f_0^e(v)$ departs from the Maxwellian and almost achieves a flat-topped distribution in the first 50 laser periods (Fig. 2). Meanwhile the absorption rate $R_n$ decreases acutely with time (Fig. 1(a)). This means that this flat-topped distribution will reduce the absorption rate dramatically, which is in agreement with Ref [1, 5] very well. On the other hand, e-e collisions effect will be enhanced while the distribution departs from the Maxwellian; therefore the contribution ratio $\gamma_n$ increases fast with the time in the first 50 laser periods. After the 50-th laser period, the shape of $f_0^e(v)$ will not change significantly, neither do the parameters $R_n$ and $\gamma_n$.

Because absorption rate is proportional to ion state $Z_i$, $R_n$ in case (ii) is reduced by an order of magnitude comparing with those in the case (i). And $f_0^e(v)$ in Fig. 2(ii) departs from the Maxwellian much smaller than that in Fig. 2(i). Meanwhile the main part of e-e collisions
will decrease much significantly, while 1(c) and Fig. 2). Consequently, decreasing energy will be absorbed more efficiently in this regime, and the self-similar state will be achieved with the increase of $C$ Legendre coefficient of EDF. The energy is in units of the initial thermal energy. The time is in units of laser period.

$C(f_0^e, f_0^i)$ sensitively depends on the degree of $f_0^i(v)$ departing from the Maxwellian. Therefore the e-e collision effect $C_{ee}(f)$ for $Z_i=10$ will be larger than that for $Z_i=1$. And it is obvious that the main part of i-e collisions $C_{ei}(Z_i, f_0^i)$ will increase with $Z_i$, but the first anisotropic Legendre coefficient of EDF $f_0^i$ decreases significantly with increasing $Z_i$ as illustrated by the degree of anisotropy $\kappa_n$ shown in Fig. 1(c). Therefore, $C_{ei}(Z_i, f_0^i)$ may not increase as fast as $C(f_0^e, f_0^i)$ with $Z_i$. As a result, the contribution ratio of e-e collisions $\gamma_n$ may increase slightly with the increase of $Z_i$.

Our code can be applied to the low frequency regime such as case (iii) and (iv). The laser energy will be absorbed more efficiently in this regime, and the self-similar state will be achieved in a significantly shorter time (Fig. 2). With the decrease of $\nu_L$, the degree of anisotropy $\kappa_n$ will decrease much significantly, while $f_0^i(v)$ approaches to the Maxwellian not so obviously [Fig. 1(c) and Fig. 2]. Consequently, $C_{ei}(Z_i, f_0^i)$ will not increase as obviously as $C(f_0^e, f_0^i)$ does with decreasing $\nu_L$. Finally, the contribution ratio $\gamma_n$ will increase with decreasing $\nu_L$.

Similar to the high frequency regime, we observe that the absorption rate in the case (iv) is reduced almost by an order of magnitude compared with the case (iii) in the low laser frequency regime [Fig. 1(a)]. From Fig. 1(b), we see that the contribution ratio $\gamma_n$ increases from 0.16 to 0.23 when ion state $Z_i$ decreases from 10 to 1 in low frequency regime. This seems to be contrary to the result in the high laser frequency regime. But if we notice that the absorption with a low $\nu_L$ is efficient [Fig. 1(a)] and $f_0^i(v)$ departs from the Maxwellian seriously even if $Z_i = 1$ (Fig. 2), so there is not much space remained for $Z_i = 10$ departing from the Maxwellian. On the other hand, the anisotropy is very small for any $Z_i$ when $\nu_L$ is small [Fig. 1(c)]. Therefore, with the increase of $Z_i$, $C(f_0^e, f_0^i)$ will increase more slowly than $C_{ei}(Z_i, f_0^i)$ does even if $f_0^i$ decreases. As a result, the contribution ratio of e-e collisions $\gamma_n$ decreases with the increase of $Z_i$ in the
low frequency regime.

3. Summary
Numerical results show that e-e collisions tend to enhance inverse bremsstrahlung. The contribution ratio of e-e collisions $\gamma_n$ varies from 0.07 to 0.23 in reported cases. Similar to previous works, we observe that the evolution of EDF can be divided into two stages distinguished by different absorption rates. During the first stage, an initially Maxwellian EDF transforms rapidly into an anisotropic two-temperature EDF. The anisotropic degree $\kappa_n$ is a monotone-increasing function of $\upsilon_L$ but a monotone-decreasing function of $Z_i$. With the increase of anisotropy, the absorption rate $R_n$ decreases dramatically and the contribution ratio $\gamma_n$ increases rapidly. In particular, we find that with the increase of $Z_i$ the contribution ratio $\gamma_n$ increases in the high laser frequency regime, while it decreases in the low frequency regime.

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