Relativistic blast wave models of GRB predict the spectrum of the emitted synchrotron radiation. The electrons in the shocked region are heated to a Wien distribution whose “temperature” is 1/3 of the mean electron energy. This energy determines a characteristic (break) frequency of synchrotron radiation. At much lower frequencies a spectrum $F_\nu \propto \nu^{1/3}$ is predicted independently of the details of the emitting region. This is consistent with the observed soft X-ray emission of GRB. It implies low visible and radio intensities, unless there are collective emission processes.

INTRODUCTION

Most of the controversies surrounding GRB involve, directly or indirectly, the shapes of their continuum spectra. Different theoretical models predict different spectral characteristics. For example, if the radiation emerges from a stationary region optically thick to gamma-gamma pair production, there will be a spectral break at an energy $\mathcal{O}(m_e c^2)$. If the source emits as a black body the spectrum will resemble a Planck function, as may be the case for SGR (but not for classical GRB). Electrons with a power-law distribution of energies produce optically thin synchrotron radiation with a power-law spectrum and no characteristic energies or spectral breaks.

The low-frequency extension of the gamma-ray spectrum determines the observability of GRB outside the gamma-ray band. Observations at visible frequencies are widely believed to hold the key to identifying the quiescent counterparts, astronomical sites, and physical mechanisms of GRB. In addition, observations at soft X-ray frequencies may provide a direct measure of the intervening column density of (chiefly) oxygen, while observations of radio dispersion similarly measure the intervening column density of free electrons. These observations may settle the question of Galactic vs. cosmological distances for GRB, as well as measure properties of the intergalactic medium if the distances are cosmological.

HYPOTHESIS

A model of GRB has been developed which unambiguously predicts the shape of the low-frequency part of their spectra. This model involves debris accelerated by a relativistic fireball interacting with a clumpy surrounding interstellar medium; the observed gamma-rays are produced in relativistically shock-heated interstellar matter and fireball debris.

In the more familiar case of relativistic particle acceleration at a nonrelativistic shock (such as in supernova remnants) only a small fraction of the particles are accelerated. The acceleration process provides no characteristic
energy scale for the accelerated particles, so their spectrum is a power law, broken only at the energy at which their gyroradii carry them out of the region of acceleration. This conventional model of shock acceleration is inapplicable to relativistic shocks in GRB.

In the present model of GRB all the charged particles in the shocked matter are accelerated, and the internal energy per particle sets an energy scale. This internal energy is determined by the hydrodynamic jump conditions at the shock, which in turn are set by the velocity of the debris and the densities of the debris and interstellar medium. Because there is a characteristic energy scale (enforced by conservation of energy and number, which are inapplicable to particles accelerated from a thermal reservoir by an imposed flow field) there is no reason to expect a power-law energy distribution of the relativistic particles. They rapidly interact with each other by means of plasma waves (which mediate the collisionless shock), and come to an equilibrium Wien distribution

\[ N_e(E) \propto E^2 \exp(-E/k_B T), \quad (1) \]

where the temperature parameter \( k_B T \) is \( 1/3 \) of the mean energy \( E \) per particle. To obtain an equilibrium distribution it is sufficient that wave-wave and wave-particle interactions be rapid; in this manner all particles are coupled to each other despite the absence of collisions. Estimates show that the plasma wave interaction and acceleration times, typically \( \sim (\omega_g)^{-1} \) in strong turbulence, where \( \omega_g \) is the gyrofrequency, are very much shorter than the other characteristic times in the problem, the hydrodynamic rarefaction and the synchrotron radiation times, so that these latter processes affect \( E \) but not the form (1).

HIGH ENERGY SPECTRA

The observed GRB spectra at high photon energies do not show the exponential cutoff implied by the Wien particle spectrum (1). There are two possible explanations:

1. The radiating electrons interact with each other by means of plasma waves which have a very high brightness temperature (far in excess of the individual particle energies), and which therefore do not constitute a genuine heat bath. As a result, the form (1) is not thermodynamically required, and a power-law spectrum (rather than an exponential cutoff) for \( E > E \) is possible. In order that this high energy tail not dominate the energy content (which would be inconsistent with the definition of \( E \)) the electron energy distribution \( N_e(E) \propto E^{-p} \) must have an index \( p > 2 \) and the spectral index of its synchrotron radiation (defined by \( F_\nu \propto \nu^{-s} \)) \( s = (p-1)/2 > 1/2 \), consistent with the observed\(^2 \) \( s \approx 1 \) at high energies.

2. At any time (and even more so in time-average) the observed radiation is integrated over radiating volumes with a distribution, probably very broad, of values of \( E \), magnetic field, and Doppler shift. As a result, the inferred distribution of energies of radiating particles only shows an exponential cutoff at energies higher than the greatest \( E \) found anywhere in the radiating volume. Observed breaks in the spectrum\(^{11,12} \) reflect a characteristic \( E \) in the radiating region, and their evolution through a burst reflects the evolution of \( E \) as the blast wave progresses through the interstellar medium.
LOW ENERGY SPECTRUM

For any electron energy distribution with power law exponent $p < 1/3$ synchrotron radiation at frequencies below the spectral peak is dominated by the highest energy electrons because the power radiated at a given frequency is $\propto E^{1/3}$. This condition is met by the Wien distribution, for which $p \to -2$ for $E \ll \mathcal{E}$, and by most plausible distributions below their characteristic energy $\mathcal{E}$. The integrated spectrum then has the index $s = -1/3$:

$$F_\nu \propto \nu^{1/3},$$

characteristic of low-frequency synchrotron emission below the spectral peak. This result survives averaging over an emission region with a range of electron energy distributions, $\mathcal{E}$, magnetic field, and Doppler shift, as long as the frequency of observation is everywhere below the characteristic synchrotron frequency (Doppler-shifted to the observer’s frame) for electrons of energy $\mathcal{E}$, and is therefore a robust prediction of relativistic blast wave models.

The predicted spectrum (2) is consistent with data on GRB at X-ray energies below 10 KeV, where the observed photon count rate per unit energy $N_\gamma \propto (h\nu)^{-0.7}$ is equivalent to $s = -0.3$, indistinguishable from $s = -1/3$. This supports the applicability of relativistic blast wave models to GRB. The form (2) also resolves the X-ray paucity problem, which arises in models of GRB emission close to neutron stars.

It is possible to extrapolate (2) to lower frequencies with confidence, once the basic model is accepted. An intense GRB with a flux $10^{-5}$ erg/cm$^2$sec in a soft gamma-ray bandwidth of 400 KeV has a gamma-ray flux of 10 mJy (1 Jy $= 10^{-23}$ erg/cm$^2$ sec Hz) and a visible flux of 0.2 mJy. The total visible power corresponds to a $\approx 18$th magnitude star, difficult to detect as an optical transient.

For the same bright GRB extrapolation to 1 GHz leads to a predicted flux of about 2 $\mu$Jy. The effective flux of a brief transient measured by a broad-band receiver may be further reduced by dispersion. In addition, self-absorption in an incoherent source leads to an independent upper bound:

$$F_\nu < 2\pi \nu^2 m_e \frac{r_s^2}{D^2} \approx 0.5 \mu Jy \left(\frac{\nu}{10^9 \text{ Hz}}\right)^2 \left(\frac{r_s}{2 \times 10^{15} \text{ cm}}\right)^2 \left(\frac{1 \text{ Gpc}}{D}\right)^2,$$

where $r_s$ is the radius of emission and $D$ is the distance to the GRB. The self-absorption bound (3) will exceed the $\mu$Jy level for the brightest GRB, which may be much closer than 1 Gpc, and after the observable gamma-ray emission, when the blast wave expands to $r_s \gg 2 \times 10^{15}$ cm.

As the blast wave expands the number of radiating electrons increases $\propto r^3$. At a given frequency $\nu$ the radiated power density per electron (in the comoving frame in which the electron and field distributions are assumed isotropic) is $\propto (\nu \gamma_e / B)^{1/3}$; $\gamma_e$ and $B$ each vary $\propto r^{-3/2}$. Applying a Lorentz factor $\gamma$ to transform to the observer’s frame multiplies the spectral density by $\gamma^{1+s} = \gamma^{2/3} \approx \gamma_F^{1/3} \propto r^{-1}$, where $\gamma_F$ is the fireball Lorentz factor. Hence the spectral brightness increases $\propto r^2 \propto t^{4/5}$ until a time $t_c \propto \nu^{-5/12}$ characteristic of the frequency of observation is reached, after which the brightness may fall.
exponentially; $t_c$ is about $3 \times 10^4$ times longer at 1 GHz than at 300 KeV, and may be days to weeks. The peak brightness at 1 GHz may be $\sim 10^4$ times brighter ($\sim 20$ mJy) than it was when the spectral peak was at 300 KeV. These numerical results are necessarily rough.

The preceding arguments would have predicted that pulsars should be unobservable at radio frequencies! Fortunately, collective emission processes occur which are not bound by single electron radiation rates or by limits on brightness temperatures. We may hope (with faint reason) that collective processes are similarly effective in GRB.

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