Prospects for Higgs Searches with the Tri-bottom Channel in Unified SUSY Models

Howard Baer, Chung Kao, and Joshua Sayre

Homer L. Dodge Department of Physics and Astronomy
University of Oklahoma
Norman, Oklahoma 73019, USA

(Dated: December 30, 2011)

Abstract

We investigate the prospects for the discovery of a neutral Higgs boson produced in association with a $b$ quark, followed by the Higgs decay into a pair of bottom quarks, $pp \rightarrow b\phi^0 \rightarrow b\bar{b}b + X$, at the CERN Large Hadron Collider (LHC) within the framework of unified supersymmetric models. The Higgs boson $\phi^0$ can be a heavy scalar $H^0$ or a pseudoscalar $A^0$. Furthermore, this direct discovery channel is compared with the indirect Higgs searches in the rare decay $B_s \rightarrow \mu^+\mu^-$ at hadron colliders. Promising results are found for the minimal supergravity (mSUGRA) model, the anomaly mediated supersymmetry breaking (AMSB) model, and the gauge mediated supersymmetry breaking (GMSB) model. We find that the indirect search for $B(B_s \rightarrow \mu^+\mu^-) \geq 5 \times 10^{-9}$ is complementary to the direct search for $b\phi^0 \rightarrow b\bar{b}b$ with $\sqrt{s} = 14$ TeV and an integrated luminosity ($L$) of 300 fb$^{-1}$. In the AMSB and GMSB models, $b\phi^0 \rightarrow b\bar{b}b$ with $L = 300$ fb$^{-1}$ covers a larger area in the parameter space than $B(B_s \rightarrow \mu^+\mu^-) \geq 5 \times 10^{-9}$. In addition, we present constraints from $b \rightarrow s\gamma$ and muon anomalous dipole moment ($\Delta a_\mu$) on the parameter space.
I. INTRODUCTION

Although the Standard Model (SM) remains an incredibly successful description of most of the phenomena studied by high energy physics, it is expected to be replaced by a more complete model in order to address a number of theoretical problems. There are also several experimental signatures for which the Standard Model does not seem to account. Perhaps the most pressing theoretical question is the stabilization of the hierarchy between the weak and Planck scales. Supersymmetry (SUSY) provides the most popular solution to this problem and, due to the large number of exotic particles introduced, also suggests solutions to a variety of other Standard Model anomalies. These include the origin of dark matter and deviations in flavor and CP physics. On the other hand, SUSY must be a broken symmetry at low scales and in the absence of a detailed theory of spontaneous breaking we are left with a large number of free parameters in the form of soft supersymmetry breaking terms. To ameliorate this profusion of parameters, it is useful to adopt one of several unified frameworks which reduce the number of new variables.

For phenomenological reasons, it is assumed that supersymmetry breaking is driven by the dynamics of a hidden sector separate from the Standard Model, and this breaking is communicated to the Standard Model particles and superpartners by a messenger sector which couples to both. Gravity will act as a natural candidate for this messenger, leading to supergravity theories (SUGRA). However, it is possible that the gravity-induced SUSY breaking terms are not dominant and that the leading effects are generated either by messenger fields which carry Standard Model gauge numbers (gauge-mediated supersymmetry breaking, or GMSB), or by terms arising from the superconformal anomaly (anomaly-mediated supersymmetry breaking, or AMSB).

It is also a striking fact that adding the minimal particle content required to incorporate the SM fields in a supersymmetric theory changes the running of the gauge couplings such that they unify at a high scale (approximately $10^{16}$ GeV). This is a much better unification than is seen for the SM fields alone and is consistent with the Grand Unified Theory (GUT) hypothesis that the three Standard Model gauge groups are remnants of a larger symmetry which is restored at high scales.

With the assumptions of grand unification and the messenger sector for SUSY breaking, the theory can be characterized by measured Standard Model inputs and a small number of additional unification scale parameters. This is enough to determine all the masses and couplings of all the superpartners, allowing us to predict the potential for direct discovery and to calculate indirect effects, which appear as corrections to precision measurements relative to their Standard Model values.

The LHC is rapidly accumulating data, currently with $\sqrt{s} = 7$ TeV, and already setting new limits on observables sensitive to new physics. It is planned to run at this energy through 2012, followed by a long shutdown for refurbishing and then a new run in 2014 aiming at the design energy of $\sqrt{s} = 14$ TeV. With high energy and luminosity, the LHC should be able to discover both the Higgs boson and supersymmetry if they exist in the $100 - 1000$ GeV range as anticipated by theorists.

One of the basic predictions of supersymmetry is the existence of (at least) two Higgs doublets, in contrast with the Standard Model. This leads to five physical states at low energy: two neutral scalars $h^0$ (lighter) and $H^0$ (heavier), a neutral pseudoscalar $A^0$, and a pair of charged scalars $H^\pm$. Thus a key signature of supersymmetry would be the detection of this extended Higgs sector. In this paper, we focus on the prospects for detecting the
heavy neutral Higgs scalar and the Higgs pseudoscalar. We investigate the decay channel
\( \phi^0 \rightarrow b \overline{b}, \phi^0 = H^0, A^0 \), which becomes important for large values of the ratio of vacuum
equation values (VEVs) \( (\tan \beta \equiv v_2/v_1) \).

We have considered this channel in a previous paper in the context of the minimal supersymmetric standard model (MSSM) \([1]\). In this study, we choose instead to work in the
three unifying frameworks mentioned above. We plot the estimated detection contour in the
space of the unification scale parameters for mSUGRA, mGMSB, and mAMSB respectively. These models also make predictions for rare decay rates and precision parameters which are
currently measured or constrained. In particular, we compare the LHC discovery potential of
the \( 3b \) channel with \( B_s \rightarrow \mu^+ \mu^- \) at hadron colliders, which is especially enhanced by a large
value of \( \tan \beta \). Furthermore, we consider constraints from \( b \rightarrow s \gamma \) and the anomalous
magnetic moment of the muon, \( (g - 2)_\mu \).

In our mSUGRA and mAMSB plots, the lightest neutralino \( \tilde{\chi} \) is always required to be the
lightest SUSY particle, or LSP.\(^1\) If \( R \)-parity is conserved, then the \( \tilde{\chi} \) will be absolutely stable
and may comprise all or some of the cold dark matter (CDM) in the universe. Within the
confines of the MSSM, a neutralino as CDM requires rather high fine-tuning\(^2\) and also very
low reheat temperature \( T_R < 10^5 \text{ GeV} \) in order to avoid bounds on late-decaying gravitinos
from BBN\(^3\). If we invoke late-decaying scalar fields, such as TeV-scale moduli, into our
theory, then these can either increase or decrease the standard neutralino abundance\(^4\).
Alternatively, invoking the Peccei-Quinn solution to the strong CP problem then requires
either mixed axion/axino\(^5\) or mixed axion/neutralino CDM\(^6\), for which again the relic
density predictions can be quite different. Here, we will simply refrain from implementing
any such constraints.

II. HIGGS PRODUCTION IN ASSOCIATION WITH B QUARKS

The Minimal Supersymmetric extension of the Standard Model (MSSM) requires two
Higgs doublets with opposite hypercharge to account for fermion masses since the superpo-
tential must be analytic:

\[
\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \phi_2^0 \\ \phi_2^- \end{pmatrix},
\]

Each doublet acquires a VEV in its neutral component, \( v_1 \) and \( v_2 \). The model is char-
acterized by the ratio of these VEVs \( \tan \beta \equiv v_2/v_1 \). After symmetry breaking three of the
degrees of freedom between the doublets are eaten by the massive weak vector bosons and
the remaining five become the massive physical eigenstates \( h^0, H^0, A^0 \) and \( H^\pm \) which are a
mixture of the original doublet fields. By convention, \( h^0 \) is lighter than \( H^0 \). We will use \( \phi^0 \)
to refer generically to any of the neutral Higgs bosons.

At the LHC, \( \phi^0 \) is predominantly produced by gluon fusion \((gg \rightarrow \phi^0)\) for \( \tan \beta \lesssim 5 \).
However, in a 2HDM the coupling of down-type quarks to the Higgs is proportional to the
quark mass divided by \( \cos \beta \), which can be quite large compared to the Standard Model value.
Hence for \( \tan \beta \gtrsim 7 \) the leading production of \( \phi^0 \) comes from \( b \overline{b} \) fusion \([7,11]\). Similarly, the
branching fraction for \( \phi^0 \rightarrow \overline{b}b \) can become nearly 90%.

\(^1\) In mGMSB, the gravitino \( \tilde{G} \) is taken as LSP.
In the case of high tan β, the simplest channel to look for neutral Higgs would seem to be $b\bar{b} \to \phi^0 \to b\bar{b}$. However, this signal is swamped by large QCD dijet backgrounds. The next step is to take advantage of b-tagging capabilities by looking for $\phi^0$ in association with one or more additional b-jets. Here we have the option of requiring four b-tagged jets with high $p_T$; $pp \to \phi^0 b\bar{b} \to bbb\bar{b} + X$, which will greatly reduce the background compared to the dijet case but which suffers from a small signal in turn [12, 14]. In this paper, we will pursue the intermediate case with 3 high-$p_T$ tagged b-jets in the final state. It has been argued that this channel is more promising at the LHC [15].

We treat this channel in a 5-flavor quark Parton Distribution Function (PDF) scheme. In a 4-flavor scheme, the leading order process of interest would be $gg \to \phi^0 b\bar{b}$, where the Higgs subsequently decays back to b-jets. One would then need to sum over all configurations with 3 high-$p_T$ quarks in the final state. This requires careful treatment of higher-order corrections since the 4th quark, at low $p_T$, gives rise to large leading-log corrections. In the 5-flavor scheme the b-quark is treated as an initial parton and the large corrections are absorbed by the b-quark PDF. For 5 flavors the leading order process is $bg \to b\phi^0 \to b\bar{b}$. (As well as the conjugate process with $b\phi^0$. Henceforth we will use $b$ to refer to both $b$ and anti-$b$ quarks.) The process $bg \to b\phi^0$ has two Feynman graphs and by choosing the factorization and renormalization scale $\mu_F = \mu_R = M_H/4$, NLO corrections can be made relatively small [10, 15].

The decay of the Higgs boson $\phi^0 \to b\bar{b}$ depends at first order on the size of the Yukawa coupling to bottom quarks, which scales with tan β. Here, we take into account several higher order effects since we wish the effective size of this vertex to correspond with the Higgs decay width to $b\bar{b}$. The decay width at NLO can be written [16]

$$\Gamma(\phi^0 \to b\bar{b}) = \frac{3G_F M_\phi^2}{4\sqrt{2}\pi} \frac{m_b^2 (M_\phi)}{[1 + \Delta_{QCD} + \Delta_t (g_\phi)^2]} \tag{2}$$

where

$$\Delta_{QCD} = 5.67 \frac{\alpha_s(M_\phi)}{\pi} + (35.94 - 1.36N_F)(\frac{\alpha_s(M_\phi)}{\pi})^2 \tag{3}$$

$$\Delta_t^b = \cot^2 \beta [3.83 - \ln(\frac{M_A^2}{M_t^2}) + \frac{1}{6} \ln(\frac{M_A^2}{M_t^2})^2](\frac{\alpha_s(M_\phi)}{\pi})^2 \tag{4}$$

$$\Delta_t^H = -\cot \alpha \cot \beta [1.57 - \frac{2}{3} \ln(\frac{M_H^2}{M_t^2}) + \frac{1}{9} \ln(\frac{M_H^2}{M_t^2})^2](\frac{\alpha_s(M_\phi)}{\pi})^2 \tag{5}$$

$$\Delta_t^A = -\tan \alpha \cot \beta [1.57 - \frac{2}{3} \ln(\frac{M_A^2}{M_t^2}) + \frac{1}{9} \ln(\frac{M_A^2}{M_t^2})^2](\frac{\alpha_s(M_\phi)}{\pi})^2 \tag{6}$$

$$g_b^b = \frac{-\sin \alpha}{\cos \beta (1 + \Delta_b)} \left(1 - \frac{\Delta_b}{\tan \alpha \tan \beta}\right) \tag{7}$$

$$g_b^H = \frac{\cos \alpha}{\cos \beta (1 + \Delta_b)} \left(1 + \frac{\Delta_b}{\cot \alpha \tan \beta}\right) \tag{8}$$

$$g_b^A = \frac{\tan \beta}{1 + \Delta_b} \left(1 - \frac{\Delta_b}{\tan^2 \beta}\right). \tag{9}$$

The quantity $\Delta_b$ is an effective shift in the b-quark mass induced by SUSY QCD.
corrections\cite{17\textendash}20. It is computed from the sum of two terms, $\Delta_b = \Delta^b_b + \Delta^t_b$.

$$\Delta^b_b = \frac{2\alpha_s}{3\pi} m_\beta \mu \tan \beta I(m_{\tilde{b}_1}, m_{\tilde{t}_2}, m_\beta)$$

(10)

$$\Delta^t_b = \frac{\alpha_t}{4\pi} A_t \mu \tan \beta I(m_{\tilde{t}_1}, m_{\tilde{t}_2}, \mu)$$

(11)

$$I(a, b, c) = \frac{-1}{(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)} (a^2 b^2 \ln \frac{a^2}{b^2} + b^2 c^2 \ln \frac{b^2}{c^2} + c^2 a^2 \ln \frac{c^2}{a^2})$$

(12)

In the equations above $\alpha_t \equiv \lambda_t/(4\pi)$, where $\lambda_t = \sqrt{2} m_t / v_2$ is the top Yukawa coupling, and $A_t$ is the trilinear stop coupling. $\Delta_b$ can be a significant correction to the decay rate. One can see that $\Delta^b_b$ decreases the rate for a positive sign of $\mu$ and increases the rate for negative $\mu$. The $\Delta^t_b$ term may be comparable in size or quite small, with a variable sign depending on the trilinear coupling. In addition to the Higgs decay vertex, we include a $\Delta_b$ correction factor at the production vertex.

In order to evaluate the expressions above, as well as the Higgs mass itself, we use the program ISAJET, which implements one of several available SUSY-breaking schemes in terms of GUT scale input parameters, then computes the low scale SUSY masses based on running of the parameters according to the renormalization group equations\cite{21}. We use the full width of the Higgs as computed by ISAJET, modified by the corrected $b\bar{b}$ decay width as indicated above. In general, the full width of the Higgs is small enough in comparison to the Higgs mass to use the Narrow Width Approximation (NWA)\cite{1}. For large masses and high tan $\beta$ the NWA becomes slightly worse. In practice we use a full Breit-Wigner calculation for $bg \to b\phi^0 \to bbb$ with the coupling of the two outgoing $bs$ from Higgs decay set to an effective value which reproduces the $b\bar{b}$ decay width.

We simulate the signal using MadGraph4\cite{22\textendash}24 to generate our matrix elements ($bg \to b\bar{b}$) and then evaluate the total cross-section with a Monte Carlo program. We smear the momenta of the outgoing $b$-quarks with a Gaussian distribution parameterized by

$$\frac{\Delta E}{E} = \frac{0.60}{\sqrt{E}} \oplus 0.03$$

(13)

based on ATLAS estimates of detector effects\cite{25}. We then impose the following cuts on our signal:

- $p_T > 70$ for all three jets,
- $|\eta| < 2.5$ for all three jets,
- $\Delta R_{ij} > 0.7$ for each pair of jets (i,j),
- Missing $E_T < 40$ GeV,
- $|M_{ij} - M_\phi| < 0.15 M_\phi$ for at least one pair of jets.

In the list above $\eta$ is the pseudorapidity and $\Delta R \equiv \sqrt{\Delta \eta^2 + \Delta \phi^2}$ is a measure of jet separation. We choose the $p_T$, $|\eta|$, and $\Delta R$ cuts to provide for good reconstruction of three jets in the b-tagging region. The $p_T$ cut is also chosen in accord with the CMS Level 1 Trigger for 3 jets\cite{26}. $M_{ij}$ is the invariant mass constructed from two of the three jets; we require that it be within a 15% window of the true pseudoscalar mass $M_A$ for at least one
pair of jets. We found in a previous work that allowing only the two highest $p_T$ jets for this requirement produces a slight improvement with respect to background at high masses [1].

The backgrounds to our signal are dominated by pure QCD processes which produce three jets. The irreducible background is $bg \rightarrow bbb$. We also include the reducible backgrounds: $pp \rightarrow cbb + X$, $pp \rightarrow gbb + X$, and $pp \rightarrow qbb + X$ where $q = u, d, s$ along with $pp \rightarrow t\bar{t} \rightarrow bbjjl\nu + X$ and $pp \rightarrow t\bar{t} \rightarrow bbjjjj + X$. These involve one mis-tagged non-b jet and can potentially be reduced with improved b-tagging. For both the signal and the background we assume an effective b-tag rate of $\epsilon_b = 0.6$ with a mis-tag rate $\epsilon_c = 0.14$ for c-quark jets and $\epsilon_j = 0.01$ for light jets ($g, u, d, s$). With these efficiencies, the pure QCD backgrounds ($bbb, cbb, gbb, qbb$) are all of comparable size, with $bbb$ or $gbb$ being the largest, while the backgrounds with intermediate t-quarks are negligible in comparison. We include them nonetheless for completeness. The double mis-tagged background $ccb$ should be roughly $\frac{1}{16}$ the size of the $bbb$ background after tagging and thus only a $1−2\%$ correction to the total background.

For simulation, backgrounds are generated using MadGraph amplitudes with the renormalization and factorization scales set to $p_T(1)/2$, half the transverse momentum of the leading jet, for the pure QCD backgrounds, and $\sqrt{s}$ for the $t\bar{t}$ backgrounds. Cuts are the same as used for the signal, except that we also apply a veto to $t\bar{t}$ events with 4 jets having $p_T > 15$ GeV. We assume a K factor of two for each background while keeping the K factor for the signal at one [12, 27].

Once we have both signal and background we are able to compute a statistical significance, for which we use

$$N_{SS} \equiv \frac{N_S}{\sqrt{N_S + N_B}}$$

where $N_S$ and $N_B$ are the number of expected signal and background events respectively. We set $N_{SS} = 5$ as the discovery limit. In practice, we use the following process to find the discovery contour: First we generate a set of background estimates over a range of masses $M_A$. The background cross-sections only depend on the mass through the location and size of the invariant dijet mass cut as given above. Next we choose a point in the GUT-scale parameter space for one of the SUSY-breaking models and use Isajet to calculate the relevant weak scale parameters, namely, the Higgs mass and decay width. We modify the $\phi^0 \rightarrow b\bar{b}$ width with the corrections listed above and feed these parameters into our MadGraph/Monte Carlo program which calculates our signal for that point in parameter space. The background at that point is determined by cubic spline interpolation from our array of $M_A$ dependent backgrounds and a significance is calculated. Then by scanning over one of the GUT-scale parameters with the others fixed we can locate the discovery contour where $N_{SS} = 5$.

### III. EXPERIMENTAL CONSTRAINTS

In addition to the estimated range of sensitivity in the $3b$ channel, it is interesting to consider other effects of the high tan $\beta$ scenarios and model assumptions. There are a number of experimentally measured or constrained quantities which are quite sensitive to deviations from the Standard Model. Especially important at high tan $\beta$ is the rare decay $B_s \rightarrow \mu^+\mu^-$. In the Standard Model this decay is expected to have the low branching fraction $BF(B_s \rightarrow \mu^+\mu^-) = (3.6 \pm 0.37) \times 10^{-9}$ [28]. However, diagrams involving $H$ and $A$
in a 2HDM are proportional to \((\tan \beta)^3\), meaning that their contributions to the decay rate scale as \((\tan \beta)^6\). This can change the predicted decay rate by orders of magnitude and the current results from several experiments put us in a very exciting time frame.

At the Tevatron, D0 and CDF set limits on the observed branching fraction of \(BF(B_s \to \mu^+\mu^-) < 5.1 \times 10^{-8}\) and \(BF(B_s \to \mu^+\mu^-) < 4. \times 10^{-8}\). Meanwhile, early results from the LHC have lowered this limit to \(1.9 \times 10^{-8}\) (CMS) and \(1.5 \times 10^{-8}\) (LHCb)\([29, 30]\), using approximately 300 pb\(^{-1}\) of data. A combined analysis using CMS and LHCb data puts the limit at \(1.1 \times 10^{-8}\)\([31]\). Thus we are already constrained to limit the effects of heavy Higgs bosons with large \(\tan \beta\), while still allowing for an enhanced branching fraction up to 3 times the Standard Model rate. Interestingly, CDF has reported a weak excess corresponding to a signal at \(1.8^{+1.1}_{-0.5} \times 10^{-8}\) with 98% probability of exceeding the SM rate\([32]\). LHCb and CMS have not seen evidence of this signal but have also not strongly ruled it out. LHCb in particular should cover the remaining space down to the Standard Model limit in the near future. It was anticipated to reach the SM limit with \(\sim 2\) fb\(^{-1}\) of data at 14 TeV running\([33]\). Thus we expect it to show evidence of an excess signal or to put stringent limits on the allowed branching fraction in the next year or two.

Supersymmetric models in general are constrained by electroweak precision data and searches for new flavor and CP violation. Two constraints of particular interest to us are the measured values of \(g_\mu - 2\) and \(b \to s\gamma\). Both quantities are sensitive to \(\tan \beta\) (though not so strongly as \(B_s \to \mu^+\mu^-\)) and to the effects of the SM superpartners.

The anomalous magnetic moment of the muon, \(g_\mu - 2\), is one of the more precisely calculated and measured quantities of quantum field theory. The experimental value is found to be \(a_\mu \equiv (g-2)/2 = (116592089.0 \pm 6.3) \times 10^{-10}\)\([34]\). A recent calculation puts the theoretical value for the Standard Model at \(a_\mu = (11659182.8 \pm 4.9) \times 10^{-10}\), leading to a discrepancy \(\Delta a_\mu = (26.1 \pm 8.1) \times 10^{-10}\), i.e. a 3.3\(\sigma\) excess in experiment\([35]\). Other groups find similar results. It is tempting to attribute this excess to new physics and supersymmetric models can easily account for it if they have the correct set of masses and parameters.

At high \(\tan \beta\) the dominant new contribution to \(a_\mu\) is proportional to \(\tan \beta\) with the same sign as \(\mu M_2\)\([36]\). (Unless \(|M_2|, M_E \ll |M_1|, M_L\), where \(M_L, M_E\) are the left- and right-handed slepton soft SUSY-breaking masses, respectively.) Thus, for the minimal models we consider below, one requires a positive sign for \(\mu\) if MSSM contributions are to account for the observed excess of \(a_\mu\).

A third sensitive probe is the flavor changing decay \(b \to s\gamma\), observed through \(B \to X_s\gamma\). Experimentally, this is measured at \((3.55 \pm 0.26) \times 10^{-4}\)\([34]\). Theoretical predictions in the Standard Model put the value at \((3.15 \pm 0.23) \times 10^{-4}\)\([37]\). In supersymmetric theories, loops involving the charged Higgs boson, as well as those involving charginos and squarks, can make large contributions to \(b \to s\gamma\).

There are, of course, other precision constraints which one may take in to account. As we shall see in more detail, however, between \(B_s \to \mu^+\mu^-\), \(\Delta a_\mu\), and \(b \to s\gamma\), the models we consider are already strongly constrained if we wish to use them to fit the experimental data within reasonable error estimates. We use the IsaTools\([38, 41]\) set of subroutines incorporated with ISAJET to calculate our estimates of these observables.

IV. MINIMAL SUPERGRAVITY

The first SUSY-breaking model we consider is minimal Supergravity mediation (mSUGRA)\([42, 40]\). In the absence of other effects, gravity should act as a messenger
between the hidden sector where Supersymmetry is spontaneously broken and the Standard Model sector. That is, the scale of the messenger interactions \( M_{\text{mes}} \) is approximately the Planck mass \( M_{\text{Pl}} \), otherwise referred to as high-scale SUSY breaking. This leads to a gravitino/goldstino with mass on the order of a few TeV. Gravitational interactions induce SUSY-breaking terms at the high scale.

In the minimal model, the GUT scale parameters are chosen to be a common scalar mass \( M_0 \), a common fermion mass \( M_{1/2} \), a common trilinear coupling \( A_0 \), and the value of \( \tan \beta \). All other parameters are fixed except the sign of \( \mu \). We will consider only the \( \mu > 0 \) case, since otherwise we will have a larger than 3σ discrepancy in \( \Delta a_\mu \) for the decoupling limit and worse for detectable superpartners. The GUT scale parameters are run down to the weak scale, resulting in a typical ratio of gaugino masses \( M_1 : M_2 : M_3 \approx 1 : 2 : 6 \). The lightest supersymmetric partner is typically a neutralino.

In Fig. 1 we show 5σ discovery contours (solid red) in the \( M_0,M_{1/2} \) plane for four choices of \( \tan \beta \) in the mSUGRA model. We set \( A_0 = 0 \) and \( \mu > 0 \). We present two contours, corresponding to 30 fb\(^{-1} \) of data running at 14 TeV and to 300 fb\(^{-1} \) at that energy. For the lower luminosity we apply the cuts as described above. For the high luminosity figure, we assume more restrictive triggers will be required to reduce the total recorded event rate to manageable levels. For \( \tan \beta = 30 \) and higher, we raise the \( p_T \) cuts to 150 GeV and apply a reduced b-tagging efficiency \( \epsilon_b = 0.5 \). We also modify the invariant mass selection so that only the leading two jets in \( p_T \) are considered as candidates for the Higgs mass peak. In our previous work we found that this strategy can improve the statistical significance and the signal to background ratio for high neutral higgs masses. For \( \tan \beta = 20 \), these cuts are very restrictive and would offer little improvement over the 30 fb\(^{-1} \) results due to the relatively low masses accessible. We include a 300 fb\(^{-1} \) contour in the \( \tan \beta = 20 \) frame using a \( p_T > 75 \) GeV cut and \( \epsilon_b = 0.5 \) with any pair of jets considered as a candidate for the Higgs decay.

The dark gray regions are excluded for theoretical reasons such as tachyonic masses at the weak scale or lack of electroweak symmetry breaking. The solid blue region at low \( M_{1/2} \) indicates charginos with mass \( M_{\chi^+} < 103 \) GeV, which have been excluded by experiment except in the case where \( M_2 \gtrsim 1 \) TeV or sneutrino masses are less than \( \sim 200 \) GeV [34]. A more general bound including these cases can be put at \( M_{\chi^+} > 92 \) GeV. For high values of \( M_{1/2} \) and relatively low \( M_0 \), the lightest slepton, a stau, becomes the LSP. This region is indicated by the solid light-gray area on the plot.

The experimental value for \( \Delta a_\mu \) is shown by the solid cyan line. The shaded cyan (forward slant hatched) region around it indicates a \( \pm 2\sigma \) error around it. We represent the experimental value and a 2σ error for \( b \to s\gamma \) with a green dash-dot-dot line and yellow (backward slant hatched) shading.

Note that for this choice of model the measured value (solid yellow) does not appear; the edge of the shaded region shows the lower 2σ limit. This is because SUSY contributions generate a negative correction to the SM value, while the experimentally measured value is slightly above the SM prediction. For \( B_s \to \mu^+\mu^- \) we have drawn dashed magenta contours for \( BF(B_s \to \mu^+\mu^-) = 1 \times 10^{-8} \) and \( 5 \times 10^{-9} \). The higher value, corresponding to the smaller area on the plot, is approximately the current exclusion limit, set by CMS and LHCb. It should be noted that this is also roughly the value which provides the best fit for CDF’s reported excess. The outermost line is not yet excluded by any experiment but should be reached by LHCb in the near future. LHCb should be able to

---

2 We take the 1σ error to be the sum in quadrature of the experimental and theoretical errors quoted above.
approach the SM limit with a few fb\(^{-1}\) of data, which in principle would push the excluded region out to arbitrarily high SUSY masses, depending on the errors.

Current LHC data already strongly constrains some areas of parameter space. We include an exclusion bound based on LHC searches for SUSY particles in events with jets and missing energy \([48]\), this limit includes 1.1 fb\(^{-1}\) of data. It appears as as a blue, dash-dot line on the figure. However, the available bound was calculated based on a scenario with tan\(\beta = 10\) and so should not be taken as definitive here. We also include a current exclusion bound for the mass of \(A_0\) as a function of tan\(\beta\), based on LHC searches for the decay \(H \rightarrow \tau^+\tau^-\). \([49]\) This bound is shown by the dotted black line on the plot. At low tan\(\beta\) it does not significantly extend the excluded region beyond chargino searches, but at higher values a significant region corresponding to lighter pseudoscalar masses is already ruled out. It was calculated in an MSSM framework using the \(M_H^\text{Max}\) scenario and so may not exactly reflect the bounds in the specific SUSY-breaking models we consider.

Several comments are in order. First, we see that the discovery curve is roughly a quarter ellipse in \(M_0\) and \(M_{1/2}\). This approximately tracks the shape of a contour of constant \(M_A\). An increase in \(M_0\) or \(M_{1/2}\) tends to increase \(M_A\) due to differential running of the Higgs parameters. Recall that in SUSY theories, the mass of \(A^0\) is given to first order by

\[
M_A^2 = \frac{M_{H^2}^2 - M_{H_1}^2 - M_Z^2 \cos 2\beta}{2 \cos 2\beta}.
\]  

(15)

\(M_{H^2}^2\) and \(M_{H_1}^2\) are equal (set by \(M_0\)) at the GUT scale but differ at the weak scale due to different Yukawa and trilinear couplings that affect their running. \(M_0\) and \(M_{1/2}\) contribute to the size of these differences and a higher initial scale \(M_0\) can lead to larger differences after running. As we increase tan\(\beta\) we can detect heavier Higgs through the increased Yukawa coupling to \(b\)s and the discovery contour moves to higher \(M_0\) and \(M_{1/2}\). For tan\(\beta \simeq 20\), this range is up to \(M_0, M_{1/2} \sim 250 - 300\) GeV with 30 fb\(^{-1}\) of integrated luminosity. For tan\(\beta \simeq 50\), this extends \(M_{1/2}\) out to \(\sim 750\) GeV and \(M_0 \sim 1600\) GeV. With high luminosity running this range is extended to \(M_0, M_{1/2} \sim 350 - 400\) at low tan\(\beta\) and up to \(M_{1/2} \sim 1000\) GeV or \(M_0 \sim 2100\) GeV at tan\(\beta \simeq 50\).

As expected, the regions excluded by or soon to be explored by \(B_s \rightarrow \mu^+\mu^-\) also grow rapidly with tan\(\beta\). For comparison one should bear in mind that the \(B_s \rightarrow \mu^+\mu^-\) contours shown should be reached with only a few fb\(^{-1}\) of data. The 30 fb\(^{-1}\) reach for the 3\(b\) signal significantly exceeds the current bounds from \(B_s\) for all cases shown. However, at high tan\(\beta\) the \(B_s\) search will rapidly begin to outperform the 3\(b\) range at high \(M_{1/2}\) and relatively low \(M_0\). Conversely, \(B_s \rightarrow \mu^+\mu^-\) performance is limited for high \(M_0\) and low \(M_{1/2}\), even at high tan\(\beta\). The leading order terms in tan\(\beta\) are proportional to \(M_{\chi^\pm}/M_H^\text{Max}\) which becomes small in this region \([51]\).

The 2\(\sigma\) band around the measured value of \(\Delta a_\mu\) prefers relatively light masses since we wish to generate a significant non-zero value. This range gradually moves to higher mass values as we increase tan\(\beta\). For tan\(\beta \gtrsim 40\) the preferred range is almost entirely covered by a 3\(b\) Higgs search, while for lower values the lower 2\(\sigma\) region eludes us. The measured value is covered by the 3\(b\) search for all cases shown.

A significant tension in these models is generated by the \(b \rightarrow s\gamma\) prediction. The measured value is slightly in excess of the SM prediction, while mSUGRA generically predicts a negative contribution to the decay rate. Thus only the lower 2\(\sigma\) bound appears on the plots and satisfying this constraint favors the decoupling limit with very high masses. The new physics contributions to \(b \rightarrow s\gamma\) include Higgs-quark loops, which give a positive contribu-
FIG. 1: Discovery contours (solid red) for the 3 $b$-quark Higgs signal in mSUGRA with 30 (LL) and 300 fb$^{-1}$ (HL). We have chosen $A_0 = 0$ and $\mu > 0$. Dark gray regions are excluded by theory. Light gray indicates a stau LSP. The dark blue area is ruled out by current chargino search limits. The experimental values with 2$\sigma$ errors are shown for $\Delta a_\mu$ (cyan, forward-slant hatched) and $b \to s\gamma$ (yellow, backward-slant hatched; central value in green dash-dot-dot). $B_s \to \mu^+\mu^-$ limits of $1 \times 10^{-8}$ (current, lighter) and $5 \times 10^{-9}$ (darker) are indicated by dashed magenta lines. Current LHC exclusion limits are shown for $\phi^0 \to \tau^+\tau^-$ (dotted black) and SUSY searches in jets plus missing $E_T$ (dash-dot blue).
tion, and chargino-stop loops which unfortunately give us a larger negative contribution in this case.

The result is that if we wish to satisfy the measured $b \to s\gamma$ and $\Delta a_\mu$ values within $2\sigma$ errors on both then we are pushed to a small region of parameter space in mSUGRA with $A_0$ and $\text{sgn}(\mu)$ as chosen. This overlap lies along the stau LSP region with $M_{1/2} > 500$ GeV and low $M_0$. It is well known that the stau coannihilation region, where the lightest neutralino and the stau are nearly degenerate, provides one of the viable explanations for the observed dark matter relic density. Statistical fit analyses $^{52,54}$ over the parameter space, taking into account a large number of constraints including dark matter density and searches, favor a similar region as indicated on our plots. In general, for this model, the $\Delta a_\mu$ constraint favors lighter SUSY mass parameters, while LHC data pulls us towards higher values. Large values of $\tan \beta$ are favored because they partially ameliorate this tension.

The preferred area is covered by the $3b$ search with $300 \text{ fb}^{-1}$ for $\tan \beta \gtrsim 40$ but becomes more difficult to cover with lower $\tan \beta$. It should, however, be well-explored by $B_s$ decay in the near future for moderate to high $\tan \beta$. Inclusive direct searches for SUSY particles also have the potential to rule out or favor this region with accumulating LHC data $^{55,56}$. With $100 \text{ fb}^{-1}$, the LHC is expected to probe mSUGRA space up to $M_{1/2} \sim 1400$ GeV at low $M_0$ and $M_{1/2} \sim 700$ GeV at high $M_0$.

V. ANOMALY MEDIATION

If tree-level soft SUSY-breaking terms arising from supergravity are suppressed, there remain loop-level contributions arising from the superconformal anomaly $^{57,58}$. Such suppression can happen in extra-dimensional models where SUSY breaking does not occur on the brane which includes the SM sector. These anomalies generate mass terms which depend on the renormalization group beta functions and a mass scale set by the gravitino, $M_{3/2}$.

\begin{equation}
M_{\lambda_i} = \frac{\beta_i}{g_i} M_{3/2} \quad (16)
\end{equation}

\begin{equation}
M_\phi^2 = -\frac{1}{4} \left( \frac{\partial \gamma}{\partial g} \beta_g + \frac{\partial \gamma}{\partial y} \beta_y \right) M_{3/2}^2 \quad (17)
\end{equation}

\begin{equation}
A_y = -\frac{\beta_y}{y} M_{3/2} \quad (18)
\end{equation}

The resulting SUSY spectrum at low scales is significantly distinct from that found in mSUGRA. In particular, the gaugino masses have the ratio $M_1 : M_2 : M_3 \simeq 2.7 : 1 : 7.1$ where the gluino mass term has the opposite sign compared to the other two. This results in the lightest neutralino being primarily a wino, with the lightest chargino and neutralino nearly degenerate. This has important consequences for decay phenomenology, with a long lived chargino. The purely anomaly generated terms are renormalization group invariant. However, this leads to an important problem: it predicts tachyonic masses for the sleptons. To ameliorate this problem several solutions have been proposed which can generate positive mass contributions from, e.g., bulk terms, gauge-mediated terms, new Yukawa terms, or higher order effects $^{57,59,60}$.

The minimal anomaly mediated model, mAMSB, is a simple phenomenological model which assumes a universal additional mass term $M_0$, which is added to the anomaly generated
scalar terms at the GUT scale [62, 63]. The addition of this new non-anomaly mediated term breaks the RG invariance of the soft-SUSY masses. The deviation from the pure-anomaly case for scalars depends at first order on the Yukawa couplings. Hence, for first and second generation scalars the mass terms can remain close to their GUT-scale, diagonal relations and this represents a possible solution to the SUSY flavor problem. The complete set of GUT-scale parameters for the mAMSB model can be given by \( M_0, M_{3/2}, \tan \beta, sgn(\mu) \).

In Figure 2 we show the 3\( b \) discovery curves in the \( M_0, M_{3/2} \) plane for the four values of \( \tan \beta \), as in the mSUGRA case. The color and pattern coding are the same as used before. The tachyonic region occurs where \( M_0 \) is not large enough to offset the negative contributions to scalar mass coming from \( M_{3/2} \). This region grows as \( \tan \beta \) is increased. As before, the discovery curves roughly parallel curves of constant \( M_A \), which exhibit interesting behavior for large \( \tan \beta \). The mass depends on the splitting between \( M_{H1} \) and \( M_{H2} \) at the weak scale and for low to moderate \( \tan \beta \) the curve behaves similarly to the mSUGRA case. Increasing \( M_0 \) or \( M_{3/2} \) tends to increase \( M_A \) due to the differential running and the starting point at the GUT scale. However, as can be seen in the plot for \( \tan \beta = 40 \), the situation becomes more complicated as the down-type Yukawa couplings increase.

This is because, for values of \( \tan \beta \) in this range, we approach an approximate Yukawa unification where the couplings of the up and down-type quarks are very similar at the GUT scale. This does not occur with mSUGRA assumptions even for high \( \tan \beta \) where the weak scale coupling become similar. Threshold corrections at the SUSY scale are generically large for the \( b \)-quark, whereas they are required to be relatively small for good Yukawa unification [64]. Following Wells and Tobe, the leading finite threshold corrections for the \( b \)-quark go as

\[
\delta_b \equiv \frac{y_b^{MSSM} - y_b^{SM}}{y_b^{SM}} \frac{\sin \beta}{\sin \beta} \sim -\frac{g_3^2 \mu M_3 \tan \beta}{12\pi^2 M_b^2} + \frac{y_t A_t \mu \tan \beta}{32\pi^2 M_t^2}. \tag{19}
\]

These corrections should be roughly 5% or less for good unification, but for TeV squarks each term can be \( \sim 50\% \) in general. For the mSUGRA plots shown above \( A_t = 0 \) at the GUT scale and becomes negative at the weak scale, thus both terms above are of the same sign and the correction is generically large enough to spoil any unification. In mAMSB on the other hand, \( A_t \) is generated by the beta function as shown above and is positive at the GUT and weak scales. It’s size is also large enough at the weak scale to be comparable to the gluino mass. The result is that it can significantly offset the corrections proportional to \( g_3^2 \), rather than add to them. This cancellation is not necessarily so precise as to guarantee exact Yukawa unification, but the difference between \( y_t \) and \( y_b \) at the GUT scale is \( \sim 10\% \) rather than \( \sim 100\% \) as in mSUGRA.

The result of this is that the running of \( M_{H1} \) and \( M_{H2} \) can become very similar in mAMSB for high \( \tan \beta \) and secondary effects become important. At low \( M_{3/2} \), increasing \( M_{3/2} \) tends to make the Yukawa unification better, generating less difference between the Higgs mass terms during the running from the GUT to the weak scale and thus producing a lighter \( A_0 \). However, at the GUT scale the mass terms have a common positive contribution from \( M_0 \) and a negative contribution which scales as \( M_{3/2} \). Due to inexact Yukawa unification and contributions to \( M_{H1} \) from the tau coupling, the beta functions for \( M_{H1} \) and \( M_{H2} \) retain some difference and larger \( M_{3/2} \) increases this initial difference. Hence, even with very similar running the initial splitting becomes large enough that \( M_{3/2} \) begins to increase \( M_A \). When \( M_{3/2} \) is large and Yukawa couplings are similar, \( M_A \) is not particularly sensitive to \( M_0 \) since it does not contribute to any initial splitting at the high scale. One can see that...
FIG. 2: Discovery contours (solid red) for the 3 $b$-quark Higgs signal in mAMSB with 30 (LL) and 300 fb$^{-1}$ (HL). We set $\mu > 0$. Dark gray regions are excluded by theory. The dark blue area is ruled out by current chargino search limits. The experimental values with 2$\sigma$ errors are shown for $\Delta a_\mu$ (cyan, forward-slant hatched) and $b \to s\gamma$ (yellow, backward-slant hatched; central value in green dash-dot-dot). $B_s \to \mu^+\mu^-$ limits of $1. \times 10^{-8}$ (current, lighter) and $5. \times 10^{-9}$ (darker) are indicated by dashed magenta lines. Current LHC exclusion limits are shown for $\phi^0 \to \tau^+\tau^-$ (dotted black).
at the right edge of the curve $M_A$ actually increases as $M_0$ is lowered toward the tachyonic bound. This is because the lepton contributions to the initial splitting and the differential running can become important. For high $M_{3/2}$ they can be negative and their magnitude increases as the offset $M_0$ decreases.

For $\tan\beta = 50$, a detectable $M_A$ can correspond to very high values of both $M_0$ and $M_{3/2}$ because of the effects discussed above. On the other hand, as can be seen in the figure, the tachyonic region becomes very large, while another disallowed region, one without EW symmetry breaking, grows for high $M_0$ and lower $M_{3/2}$. This leaves only a narrow band of theoretically allowed parameter space.

As in mSUGRA, $\Delta a_\mu$ prefers relatively light superpartners which sit in a band at low $M_0$ and $M_{3/2}$. This band is only moderately dependent on $\tan\beta$. The regions probed by $B_s \to \mu^+\mu^-$ grow quickly with $\tan\beta$ and current exclusion limits begin to become important for $\tan\beta > 30$. The shape of the $B_s$ contour echoes that of the mass and discovery curves. Similar behavior is also seen for the $b \to s\gamma$ allowed region. The situation with respect to $b \to s\gamma$ is, however, notably different than in mSUGRA models. For mAMSB, the chargino-squark loops give a net positive contribution to the decay rate. The additional Higgs contributions remain positive as before so the total correction relative to the Standard Model is the right sign to account for the observed value.

Qualitatively, this change in sign can be understood as a result of the $A_t$ term. If we consider $b$ to $s$ transitions with each quark coupling to one side of a chargino-squark loop, the dominant terms come from the Higgsino-like coupling for the bottom quark paired with the Wino or Higgsino coupling of the strange quark. Before mass diagonalization, a left handed quark couples to a left squark via the Wino or to a right squark via the Higgsino. Thus we have one term proportional to the Higgsino-Higgsino term in the chargino mass matrix times the left-right coupling in the squark mixing matrix. The other term is proportional to the Higgsino-Wino mixing times the left-left or right-right coupling of the squarks. Most of these remain qualitatively similar between the mSUGRA and mAMSB scenarios, however the off diagonal stop mixing term can change signs between the two. Recall that this term is generically given by

$$M_{LR} \propto A_t - \mu \cot\beta.$$  \hfill (20)

For mSUGRA as shown above, we assumed $A_0 = 0$ at the GUT scale and renormalization effects drive it to negative values at the weak scale, so the mixing term is always negative for positive $\mu$. In mAMSB, on the other hand, the $A_t$ term as given by Eq. (20) is positive at the GUT scale and remains positive and large enough at the weak scale to make $M_{LR}$ positive. The sign of the loop involving the Higgsino-Higgsino coupling is therefore changed while the other loops remain the same. As mentioned before, a generic feature of AMSB is a nearly pure Wino for the lightest chargino and, by extension, a pure Higgsino heavy chargino. Thus the loops which have changed sign tend to dominate over those which do not: the latter depend on the chargino mixing angle which is typically small. The end result is that the stop-chargino terms can give a moderate positive contribution to $b \to s\gamma$, unlike mSUGRA. Examining the figure we see that $b \to s\gamma$ now disfavors superpartners and Higgs which are too light because they would give too much positive correction to the Standard Model. For $\tan\beta \simeq 20 - 30$ we do somewhat better than mSUGRA. With relatively low $M_0$ and high $M_{3/2}$ we can satisfy both $\Delta a_\mu$ and $b \to s\gamma$ within one or two $\sigma$ error on either measurement. As we push to higher $\tan\beta$, the tension increases as $b \to s\gamma$ requires higher masses while $\Delta a_\mu$ still favors lower values of the GUT scale parameters. Only two small
wedges remain at $\tan \beta = 40$ where both predictions can satisfy the experimental numbers with $2\sigma$ errors on each. These wedges can be excluded by $B_s \rightarrow \mu^+\mu^-$ in the near future and are well within reach of the $3b$ search.

Based on current LHC data, exclusion bounds for mAMSB have been extrapolated in Ref. [65]. The authors find that mAMSB is excluded for $M_{3/2} \lesssim 40$ TeV when $M_0 \lesssim 1$ TeV, for $\tan \beta = 10$.

At $\tan \beta \approx 20$ even the 300 fb$^{-1}$ search does not cover a significant region beyond that excluded by chargino mass bounds. However the detectable space grows quickly with $\tan \beta$. If $\tan \beta \approx 30$ the $3b$ search probes a portion of the favored area with 300 fb$^{-1}$, while $B_s$ decay does not currently put any new constraints on the parameter space. As $\tan \beta$ increases 30 fb$^{-1}$ quickly becomes sufficient to explore the experimentally preferred areas. For $\tan \beta = 50$ our search can probe into the theoretically allowed band, but this solution is strongly disfavored by $\Delta a_\mu$. Direct searches for gluino and squark cascade decays at LHC with 100 fb$^{-1}$ are expected to probe to $M_{3/2} \sim 140$ TeV for low $M_0$ and to $M_{3/2} \sim 100$ TeV for high $M_0$ [66].

VI. GAUGE MEDIATION

A third proposal for SUSY-breaking involves additional matter which is charged under the SM gauge interactions [67–69]. If this matter also interacts with the hidden sector then it can act as the messenger for breaking. The minimal model, mGMSB, assumes $N$ pairs of fundamental $SU(5)$ GUT representations, $5 + \overline{5}$. In general, the gravitino mass is given by

$$M_\tilde{G} = \frac{F}{\sqrt{3}M_{Pl}},$$

(21)

where $F$ is the characteristic scale of SUSY breaking in the hidden sector. Since the superpartner masses are given schematically by $\tilde{M} \sim F/M_{mes}$, the gravitino will be the LSP if the messenger mass, $M_{mes}$ is much below the Planck scale. This is the default assumption for mGMSB. The mass of the messenger 5-plets is an input to mGMSB models, but the superpartner masses only depend logarithmically on it. The observable SUSY masses are parametrized by $\Lambda \equiv \frac{F}{M_{mes}}$, where $F = F/C_G$. $C_G$ is introduced since the scale of SUSY breaking in the visible sector may be lower than that in the hidden sector. The complete set of high scale parameters is then $\Lambda, M_{mes}, N, C_G, \tan \beta$ and $\text{sgn}(\mu)$. The scalar and gaugino masses are given by

$$M_\phi^2 = 2N\Lambda^2\left(\frac{5}{3}\left(\frac{Y}{2}\right)^2\left(\frac{\alpha_1}{4\pi}\right)^2 + C_2\left(\frac{\alpha_2}{4\pi}\right)^2 + C_3\left(\frac{\alpha_3}{4\pi}\right)^2\right)$$

(22)

and

$$M_\lambda = k_a\Lambda\frac{\alpha_a}{4\pi},$$

(23)

where $k_1 = 5/3$, $k_2 = k_3 = 1$, and $Y$ is the hypercharge. $C_2 = 3/4$ for weak doublets and zero for singlets, $C_3 = 4/3$ for squarks and zero otherwise. Changing the number of messenger fields $N$ introduces a relative splitting between the gauginos and sparticles since the former scale as $N$ and the latter as $\sqrt{N}$. Since the messenger scale can be well below the Planck scale and the presumed scale of flavor physics, and since the initial masses depend only on gauge couplings, GMSB is a potential solution to the SUSY flavor problem.

In the figures below we set $C_G = N = 1$ and plot in $\Lambda, M_{mes}$. The dark gray excluded region on the left comes from the requirement that $M_{mes} > \Lambda$. As expected, varying $M_{mes}$
has only a weak effect on most of the lines. For fixed \( \Lambda \), increasing \( M_{\text{mes}} \) has little effect on the overall scale of SUSY partners. This is seen in the contours for \( \Delta a_\mu \), which are nearly flat. The SUSY corrections to \( a_\mu \) come from chargino-sneutrino loops and neutralino-smuon slepton loops. They scale as \( \sim \tan \beta M_\mu^2/M_{3\text{SUSY}}^2 \) and are thus insensitive to changing \( M_{\text{mes}} \) \cite{71}. Increasing \( M_{\text{mes}} \) does gradually tend to increase the pseudoscalar Higgs mass due to increased running effects.

The curve for \( b \to s \gamma \) exhibits some interesting behavior which requires a bit of explanation. For mGMSB, the predicted corrections to \( b \to s \gamma \) are generically positive but not for the same reasons as in mAMSB. The \( A_t \) terms are negative at the weak scale as in mSUGRA, so the contributions from chargino-squark loops are negative. However, they are comparatively small in mGMSB, so that the positive contributions from charged Higgs loops can dominate and the total correction is a small net positive. The chargino-stop terms are suppressed due to the super-GIM mechanism. That is, they depend on the squark mixing matrix which is unitary and in the limit of degenerate squark masses the various terms cancel as required for unitarity. For low scale SUSY breaking, running effects are small and the degeneracy of scalars with the same quantum numbers at the messenger scale is only mildly broken. As seen in the figure, increasing \( M_{\text{mes}} \) at low scales decreases the contribution to \( b \to s \gamma \) due to the increased Higgs masses and the spoiling of the squark degeneracy. Notably, although the overall correction to the SM value can become negative for large \( M_{\text{mes}} \), it does not exceed the lower 2\( \sigma \) bound on the measured \( b \to s \gamma \) rate.

For reasonably low \( M_{\text{mes}} \) we can comfortably reconcile \( \Delta a_\mu \) and \( b \to s \gamma \) over a wide range of \( \tan \beta \). \( B_s \to \mu^+\mu^- \) limits do not currently put a strong constraint on these favored regions although at high \( \tan \beta \) they should begin to cover this region with enough luminosity. The preferred region also indicates excellent prospects for a 3\( \ell \) detection of the heavy neutral Higgs if \( \tan \beta \gtrsim 30 \). Much of the space would still be covered by this search even for somewhat lower \( \tan \beta \). For a discussion of current mass limits see Ref. \cite{72}. The exclusion depends on the mass of the NLSP, which for our choice of parameters may be a neutralino or, at high \( \tan \beta \) a stau. This puts a bound on the gluino mass of approximately 650 GeV, which corresponds to \( \Lambda \sim 80 \) TeV. Direct searches for sparticle pair production at LHC with 100 fb\(^{-1} \text{in the mGMSB model are expected to probe to} \Lambda \sim 600 \text{ TeV in the} \gamma \gamma + E_T^{\text{miss}} \text{channel.} \)

VII. MSUGRA REVISITED

As discussed above, minimal supergravity appears to only marginally satisfy experimental constraints in a few limited regions of parameter space. This is due to the conflicting pull of the \( g - 2 \) excess, which favors lighter SUSY partners, and the measured \( b \to s \gamma \) branching fraction, which receives the wrong sign contribution from chargino loops in mSUGRA and therefore favors heavier masses. However, we have seen that mAMSB models can change the sign of the chargino contribution and that this can be traced back to the significant positive sign of \( A_t \) at the weak scale. This suggests that by choosing a large, positive \( A_0 \) in mSUGRA we can induce a similar effect which may improve the prospects for this model.

In the figure below we plot results for mSUGRA as before but with \( A_0 = 1.5 \) TeV as our initial condition. This has several interesting effects on the weak scale observables. First we see that, as intended, the overall correction to \( b \to s \gamma \) can be made positive and the experimental central value is predicted (solid yellow line) for \( M_{1/2} \sim 200 \) GeV with a weak dependence on \( M_0 \). The contours for discovery and for \( \Delta a_\mu \) are not dramatically effected.
FIG. 3: Discovery contours (solid red) for the 3 b-quark Higgs signal in mGMSB with 30 (LL) and 300 fb$^{-1}$ (HL). We set $N = C_G = 1$ and $\mu > 0$. Dark gray regions are excluded by theory. The dark blue area is ruled out by current chargino search limits. The experimental values with 2$\sigma$ errors are shown for $\Delta a_\mu$ (cyan, forward-slant hatched) and $b \to s\gamma$ (yellow, backward-slant hatched; central value in green dash-dot-dot). $B_s \to \mu^+\mu^-$ limits of $1. \times 10^{-8}$ (current, lighter) and $5. \times 10^{-9}$ (darker) are indicated by dashed magenta lines. Current LHC exclusion limits are shown for $\phi^0 \to \tau^+\tau^-$ (dotted black).
On the other hand, the tachyonic region for lower values of $M_0$ and $M_{1/2}$ is notably larger than for the $A_0 = 0$ case. The stau LSP region also grows somewhat. The result is that it becomes possible to satisfy the $b \to s\gamma$ measured value while keeping the predicted $\Delta a_\mu$ within the 2$\sigma$ lower bound.

The region for $\Delta a_\mu$ exceeding the measured value is largely excluded by the requirement that we avoid tachyons. However, for $\tan \beta = 50$ a small area where $\Delta a_\mu$ falls exactly on the measured value remains. Moreover, the preferred value for $b \to s\gamma$ runs through this patch, so it is possible to have both predictions very close to the measured numbers for $M_0 \simeq 700$ GeV and $M_{1/2} \simeq 200$ GeV. One should note though, that the region within 2$\sigma$ error for $b \to s\gamma$ extends over the entire plot. It also appears that the area of exact overlap for $b \to s\gamma$ and $\Delta a_\mu$ as currently measured may already be excluded by the $\phi \to \tau^+\tau^-$ searches. As $M_{1/2}$ grows the chargino-squark corrections can again become negative but the SUSY effects diminish on the whole so we do not fall below the lower bound.

Another significant effect of positive $A_0$ is seen in the contours for $B_s \to \mu^+\mu^-$. They are significantly reduced, such that only at $\tan \beta = 50$ does a limit of $BF < 5 \times 10^{-9}$ begin to cover parameter space that isn’t already excluded by theoretical requirements. Additionally, the shape of the contours is slightly more complicated than the single ellipsoid seen with $A_0 = 0$. For $\tan \beta = 50$ the contours include a region at low $M_{1/2}$ which quickly fall offs for $M_{1/2} \sim 300$ GeV, then grows again to cover an ellipsoidal patch from $M_{1/2} \sim 500 – 1400$ GeV. This can be qualitatively understood as follows: The ISAJET calculation of the branching fraction is based on an effective Lagrangian for flavor changing couplings between strange and bottom quarks [41]:

$$-\mathcal{L}_{\text{eff}} = \overline{D}_R f_D Q_1 H_d + \overline{D}_R f_D [a_g M_g + a_u M_u f_u^* f_u + a_w M_w] Q_L H_u^*$$  \hspace{1cm} (24)

where

$$a_g = -\frac{2\alpha_s}{3} \mu M_g, \quad a_u = -\frac{1}{16\pi^2} \mu A_t, \quad a_w = \frac{g^2}{16\pi^2} \mu M_2.$$  \hspace{1cm} (25)

$M_g, M_u$ and $M_w$ are diagonal mass matrices that depend on loop functions arising from gluino-down squark, stop-higgsino, and wino-up squark graphs respectively. The effective FCNC interactions are proportional to a function $\chi_{FC}$ which depends on a sum over these three terms with appropriate mixing coefficients. (See reference for details.) For our regions of interest the wino-squark loops are not as important as the other contributions. At low $M_{1/2}$ both the gluino-squark and higgsino-stop loops are of the same sign, corresponding to a positive $A_t$ at the weak scale and generating a significant enhancement of $B_s \to \mu^+\mu^-$. As $M_{1/2}$ increases, the sign of $A_t$ at the weak scale changes due to running effects. For $M_{1/2} \sim 1000$ GeV the higgsino-stop term dominates as $A_t$ becomes large and negative, leading to a detectable excess in the branching fraction until the relevant particles become too heavy. However, for intermediate values of $M_{1/2}$ the $A_t$ terms are of the right order to cancel the other contributions, giving us the trough seen in the graph.

In general, the discovery contours in this high $A_0$ scenario are similar to the $A_0 = 0$ case. One can see that they cover most of the experimentally preferred region for $\tan \beta \gtrsim 40$. This scenario is quite interesting to us, since it shows the possibility, particularly at high $\tan \beta$, to satisfy both the $b \to s\gamma$ and $\Delta a_\mu$ measurements in a region which is not particularly sensitive to $B_s \to \mu^+\mu^-$ but should be well within the range of $3b$ (and related) searches.
FIG. 4: Discovery contours (solid red) for the 3 $b$-quark Higgs signal in mSUGRA with 30 (LL) and 300 fb$^{-1}$ (HL). We have chosen $A_0 = 1.5$ TeV and $\mu > 0$. Dark gray regions are excluded by theory. Light gray indicates a stau LSP. The dark blue area is ruled out by current chargino search limits. The experimental values with 2$\sigma$ errors are shown for $\Delta a_\mu$ (cyan, forward-slant hatched) and $b \to s\gamma$ (yellow, backward-slant hatched; central value in green dash-dot-dot). $B_s \to \mu^+\mu^-$ limits of $1 \times 10^{-8}$ (current, lighter) and $5 \times 10^{-9}$ (darker) are indicated by dashed magenta lines. Current LHC exclusion limits are shown for $\phi^0 \to \tau^+\tau^-$ (dotted black).
VIII. CONCLUSIONS

The search for supersymmetry and the details of electroweak symmetry breaking is currently in an exciting phase with the LHC quickly accumulating data at 7 TeV and expected to increase energy in 2014. Updated constraints on precision measurements limit the parameter space in many models and some will rapidly improve with LHC data. A key signature of supersymmetry is the extended Higgs sector with two additional neutral particles and a pair of charged Higgs, unlike the Standard Model.

To limit the rather large number of parameters in the MSSM, we have considered three simple models of SUSY breaking: mSUGRA, mAMSB, and mGMSB. We have delineated the parameters space regions of these models where a 3 b-quark signature arising from $bA_0^0$, $bH_0^0$ production should be visible at LHC with 30 or 300 fb$^{-1}$ of integrated luminosity. Since the Higgs to b-quark couplings become large at high tan $\beta$ (a scenario favored by Yukawa coupling unification), prospects for such a Higgs search are most promising at large tan $\beta$ values. The $3b$ signal will be complementary to direct sparticle searches, and will provide information on the heavy Higgs sector of the model.

The rare decay $B_s \to \mu^+\mu^-$ is also highly sensitive to tan $\beta$ and already strongly constrains new physics at very high tan $\beta$ values. Limits from this decay mode are expected to improve quickly, potentially excluding important regions of parameter space. At the same time, CDF has seen a weak signal near the current limit which would strongly suggest new physics if it were to be confirmed.

Meanwhile, the measured 3$\sigma$ excess of $a_\mu$ suggests a positive sign for $\mu$ and relatively light SUSY particles. For mSUGRA, this is in tension with the experimental value of $b \to s\gamma$, which is somewhat above the SM prediction, while the theoretical prediction receives negative corrections from chargino-squark loops. This pulls one toward higher masses to minimize the corrections and only a small region of parameter space is left where both numbers can fall within 2$\sigma$ deviations. It is possible that a large, positive value for $A_0$ at the GUT scale can ameliorate this tension. For moderate to high tan $\beta$, the $3b$ search will probe into the allowed region, as will $B_s \to \mu^+\mu^-$ in the near future.

Anomaly mediated models provide one way to improve the predictions, since they give a positive contribution to $b \to s\gamma$ for a positive sign of $\mu$. This solution favors moderate ($\lesssim 40$) values of tan $\beta$ so as not to increase $b \to s\gamma$ by too much and to avoid the rapidly growing constraints from $B_s \to \mu^+\mu^-$. The $3b$ search will cover most of the preferred space for scenarios with tan $\beta > 30$ but will require a very high integrated luminosity to probe lower values.

Perhaps the most natural fit to experiment, among the models we consider, is mGMSB. In this case, relatively light messenger fields lead to degenerate squarks which results in a cancellation of the negative chargino-stop terms in $b \to s\gamma$, leaving an overall small positive contribution. This allows one to fit both the $\Delta a_\mu$ and $b \to s\gamma$ measurements easily. If this is indeed a hint at the nature of SUSY-breaking then the $3b$ search is quite promising and $A_0^0/H_0^0$ should be discoverable at the LHC for a large range of moderate to high tan $\beta$.
[1] C. Kao, S. Sachithanandam, J. Sayre and Y. Wang, Phys. Lett. B 682, 291 (2009).
[2] J. R. Ellis and K. A. Olive, Phys. Lett. B 514, 114 (2001); H. Baer and A. D. Box, Eur. Phys. J. C 68, 523 (2010).
[3] R. H. Cyburt, J. Ellis, B. D. Fields and K. A. Olive, Phys. Rev. D 67 (2003) 103521; R. H. Cyburt, J. Ellis, B. D. Fields, F. Luo, K. Olive and V. Spanos, JCAP 0910 (2009) 021; M. Kawasaki, K. Kohri and T. Moroi, Phys. Lett. B 625 (2005) 7, and Phys. Rev. D 71 (2005) 083502; K. Kohri, T. Moroi and A. Yotsuyanagi, Phys. Rev. D 73 (2006) 123511; for an update, see M. Kawasaki, K. Kohri, T. Moroi and A. Yotsuyanagi, Phys. Rev. D 78 (2008) 065011; K. Jedamzik, Phys. Rev. D 70 (2004) 063524, and Phys. Rev. D 74 (2006) 103509.
[4] T. Moroi and L. Randall, Nucl. Phys. B 570 (2000) 455; G. Gelmini and P. Gondolo, Phys. Rev. D 74 (2006) 023510; G. Gelmini, P. Gondolo, A. Soldatenko and C. Yaguna, Phys. Rev. D 74 (2006) 085011; K. Jedamzik, Phys. Rev. D 70 (2004) 063524, and Phys. Rev. D 71 (2005) 083502.
[5] H. Baer, A. Box and H. Summy, J. High Energy Phys. 0908 (2009) 080.
[6] K-Y. Choi, J. E. Kim, H. M. Lee and O. Seto, Phys. Rev. D 77 (2008) 123501; H. Baer, A. Lessa, S. Rajagopalan and W. Sreethawong, JCAP 1106 (2011) 031; H. Baer, A. Lessa and W. Sreethawong, arXiv:1110.2491.
[7] D. A. Dicus and S. Willenbrock, Phys. Rev. D 39, 751 (1989).
[8] D. Dicus, T. Stelzer, Z. Sullivan and S. Willenbrock, Phys. Rev. D 59, 094016 (1999).
[9] C. Balazs, H. J. He and C. P. Yuan, Phys. Rev. D 60, 114001 (1999).
[10] F. Maltoni, Z. Sullivan and S. Willenbrock, Phys. Rev. D 67, 093005 (2003).
[11] R. V. Harlander and W. B. Kilgore, Phys. Rev. D 68, 013001 (2003).
[12] J. Dai, J. F. Gunion and R. Vega, Phys. Lett. B 345, 29 (1995); Phys. Lett. B 387, 801 (1996).
[13] J. L. Diaz-Cruz, H. J. He, T. M. P. Tait and C. P. Yuan, Phys. Rev. Lett. 80, 4641 (1998); C. Balazs, J. L. Diaz-Cruz, H. J. He, T. M. P. Tait and C. P. Yuan, Phys. Rev. D 59, 055016 (1999).
[14] M. S. Carena, S. Mrenna and C. E. M. Wagner, Phys. Rev. D 60, 075010 (1999).
[15] J. Campbell, R. K. Ellis, F. Maltoni and S. Willenbrock, Phys. Rev. D 67, 095002 (2003).
[16] J. Guasch, P. Hafliger and M. Spira, Phys. Rev. D 68, 115001 (2003).
[17] L. J. Hall, R. Rattazzi and U. Sarid, Phys. Rev. D 50, 7048 (1994).
[18] M. S. Carena, M. Olechowski, S. Pokorski and C. E. M. Wagner, Nucl. Phys. B 426, 269 (1994).
[19] D. M. Pierce, J. A. Bagger, K. T. Matchev and R. j. Zhang, Nucl. Phys. B 491, 3 (1997).
[20] M. S. Carena, S. Heinemeyer, C. E. M. Wagner and G. Weiglein, Eur. Phys. J. C 45, 797 (2006).
[21] F. E. Paige, S. D. Protopopescu, H. Baer, X. Tata, arXiv:0312045[hep-ph] (2003).
[22] MADGRAPH, by T. Stelzer and W.F. Long, Comput. Phys. Commun. 81, 357 (1994).
[23] F. Maltoni and T. Stelzer, JHEP 0302, 027 (2003).
[24] HELAS, by H. Murayama, I. Watanabe and K. Hagiwara, KEK report KEK-91-11 (1992).
[25] G. Aad et al. [The ATLAS Collaboration], “Expected Performance of the ATLAS Experiment - Detector, Trigger and Physics,” arXiv:0901.0512[hep-ex] (2009).
[26] G. L. Bayatian et al. [CMS Collaboration], J. Phys. G 34, 995 (2007).
[27] R. Bonciani, S. Catani, M. L. Mangano and P. Nason, Nucl. Phys. B 529, 424 (1998); P. Nason,
S. Dawson and R. K. Ellis, Nucl. Phys. B 303, 607 (1988).

[28] A. J. Buras, M. V. Carlucci, S. Gori, G. Isidori, JHEP 1010, 009 (2010).

[29] S. Chatrchyan et al. [CMS Collaboration], arXiv:1107.5834 [hep-ex] (2011).

[30] The LHCB Collaboration, CERN-LHCB-CONF-2011-037 (2011).

[31] The LHCB Collaboration, CERN-LHCB-CONF-2011-047 (2011).

[32] T. Aaltonen et al. [CDF Collaboration], arXiv:1107.2304 [hep-ex] (2011).

[33] S. Chatrchyan et al. [CMS Collaboration], arXiv:0912.4179 [Unknown] (2009).

[34] K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010).

[35] K. Hagiwara, R. Liao, A. D. Martin, D. Nomura, T. Teubner, J. Phys. G G38, 085003 (2011).

[36] The LHCb Collaboration, CERN-LHCb-CONF-2011-037 (2011).

[37] The LHCb Collaboration, CERN-LHCb-CONF-2011-047 (2011).

[38] H. Baer, M. Brhlik, D. Castano and X. Tata, Phys. Rev. D 58, 015007 (1998).

[39] H. Baer and M. Brhlik, Phys. Rev. D 55, 3201 (1997).

[40] H. Baer, C. Balazs, J. Ferrandis and X. Tata, Phys. Rev. D 64, 035004 (2001).

[41] J. K. Mizukoshi, X. Tata and Y. Wang, Phys. Rev. D 66, 115003 (2002).

[42] A. H. Chamseddine, R. L. Arnowitt and P. Nath, Phys. Rev. Lett. 49, 970 (1982).

[43] L. E. Ibanez and G. G. Ross, Phys. Lett. B 110, 215 (1982).

[44] R. Barbieri, S. Ferrara and C. A. Savoy, Phys. Lett. B 119, 343 (1982).

[45] L. J. Hall, J. D. Lykken and S. Weinberg, Phys. Rev. D 27, 2359 (1983).

[46] N. Ohta, Prog. Theor. Phys. 70, 542 (1983).

[47] K. Mahboubi, “ATLAS level-1 jet trigger rates and study of the ATLAS discovery potential of the neutral MSSM Higgs bosons in b-jet decay channels,” PhD Thesis, Heidelberg U. (2001).

[48] S. Chatrchyan et al. [CMS Collaboration], Phys. Rev. Lett. 107, 221804 (2011).

[49] The ATLAS Collaboration, ATLAS-CONF-2011-132 (2011).

[50] T. Becher, M. Neubert, Phys. Rev. Lett. 98, 022003 (2007).

[51] C. Bobeth, T. Ewerth, F. Kruger, J. Urban, Phys. Rev. D64, 074014 (2001).

[52] O. Buchmueller, R. Cavanaugh, M. J. Dolan, J. R. Ellis, H. Flacher, S. Heinemeyer and G. Isidori et al., arXiv:1110.3568 [hep-ph] (2011).

[53] A. Fowlie, A. Kalinowski, M. Kazana, L. Roszkowski and Y. L. S. Tsai, arXiv:1111.6098 [hep-ph] (2011).

[54] G. Bertone, D. G. Cerdeno, M. Fornasa, R. R. de Austri, C. Strege and R. Trotta, arXiv:1107.1715 [hep-ph] (2011).

[55] H. Baer, V. Barger, A. Lessa and X. Tata, JHEP 0909, 063 (2009).

[56] H. Baer, C. Balazs, A. Belyaev, T. Krupnovickas and X. Tata, JHEP 0306, 054 (2003).

[57] L. Randall, R. Sundrum, Nucl. Phys. B557, 79 (1999).

[58] G. F. Giudice, M. A. Luty, H. Murayama, R. Rattazzi, JHEP 9812, 027 (1998).

[59] A. Pomarol and R. Rattazzi, JHEP 9905 (1999) 013.

[60] Z. Chacko, M. A. Luty, I. Maksymyk and E. Ponton, JHEP 0004 (2000) 001.

[61] E. Katz, Y. Shadmi and Y. Shirman, JHEP 9908, 015 (1999).

[62] T. Gherghetta, G. F. Giudice and J. D. Wells, Nucl. Phys. B 559, 27 (1999).

[63] J. L. Feng and T. Moroi, Phys. Rev. D 61, 095004 (2000).

[64] K. Tobe, J. D. Wells, Nucl. Phys. B663, 123-140 (2003).

[65] B. C. Allanach, T. J. Khoo and K. Sakurai, arXiv:1110.1119 [hep-ph] (2011).

[66] H. Baer, J. K. Mizukoshi and X. Tata, Phys. Lett. B 488 (2000) 367; A. J. Barr, C. G. Lester, M. A. Parker, B. C. Allanach and P. Richardson, JHEP 0303 (2003) 045.

[67] M. Dine, W. Fischler, and M. Srednicki, Nucl. Phys. B189 (1981) ; S. Dimopoulos and S.
Raby, Nucl. Phys. B192 (1981); M. Dine and W. Fischler, Phys. Lett. B110 (1982); M. Dine and M. Srednicki, Nucl. Phys. B202 (1982); L. Alvarez-Gaumé, M. Claudson, and M. Wise, Nucl. Phys. B207 (1982); C. Nappi and B. Ovrut, Phys. Lett. B113 (1982)

[68] M. Dine and W. Fischler, Nucl. Phys. B204 (1982); S. Dimopoulos and S. Raby, Nucl. Phys. B219 (1983).

[69] M. Dine and A. Nelson, Phys. Rev. D48 (1993); M. Dine, A. Nelson, and Y. Shirman, Phys. Rev. D51 (1995); M. Dine, A. Nelson, Y. Nir, and Y. Shirman, Phys. Rev. D53 (1996).

[70] H. Baer, P. G. Mercadante, F. Paige, X. Tata and Y. Wang, Phys. Lett. B 435 (1998) 109; H. Baer, P. G. Mercadante, X. Tata and Y. l. Wang, Phys. Rev. D 62 (2000) 095007.

[71] M. Endo, T. Moroi, Phys. Lett. B525, 121 (2002).

[72] Y. Kats, P. Meade, M. Reece and D. Shih, arXiv:1110.6444 [hep-ph] (2011).