Two-neutron drip lines of a few single lambda hypernuclei

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Abstract

Two-neutron drip lines of a few single lambda hypernuclei are studied through a phenomenological binding energy model. This model, which is built from the Bethe-Weizsäcker formula, explicitly takes into account hyperon mass and strangeness. For the hypernuclear isotopic chains of the elements C—Mn, the heaviest isotope in each chain that is stable with respect to two-neutron decay is located.

1. Introduction

Delimitation of the nuclear landscape is one of the topical problems in contemporary nuclear physics. The table of Nuclides, which is used to represent the nuclear landscape, is a two-dimensional chart of number of neutrons (N) plotted against number of protons (Z). Every (Z, N) point represents a unique atomic nucleus i.e. a nuclide. Some of these nuclides are already known while the remaining points are hypothetical nuclides whose existence is still to be determined theoretically or experimentally. Neutron and proton drip lines delimit the boundaries of the nuclear landscape. Neutron drip lines are usually located far away from the line of beta stability, making them more challenging to observe than proton drip lines. Consequently, one-neutron drip lines are only known experimentally for the first ten elements on the Mendeleev Periodic table i.e. hydrogen to neon [1]. Locating neutron drip lines is a challenging problem because of the technicalities involved in producing nuclides with very high neutron-to-proton ratios. \(^{8}\)He (N = 6) is still the nuclide with the highest neutron-to-proton ratio (3:1) of all currently known nuclides.

In a hypernucleus, one or more hyperons are also present, in addition to neutrons and protons. Hyperons are baryons that contain at least one strange quark and have a strangeness quantum number (S) of \(S = −1, −2\) or \(−3\). They include \(Λ, Σ, Ξ\) and \(Ω\) hyperons. A hyperon carries a charge of 0, just like a neutron. However, \(Λ\) is more massive than a neutron, and has a strangeness of \(S = −1\) whereas a neutron has a strangeness of \(S = 0\). Hyperons are known to significantly modify nuclear structure. For example, hypernuclear bound states are known to exist where the non-strange system with an identical baryon number is particle unbound e.g. \(^{6}\)Li is bound [2] whereas \(^{10}\)Li is unbound. Hyperon states inside the nucleus are not restricted by the Pauli principle between protons and between neutrons; therefore, they can penetrate deep into the nucleus, attracting protons and neutrons with them. This results in a reduction of nuclear size in some hypernuclei, as evidenced by reduced probabilities of electric quadrupole (E2) transitions [3]. Since the hyperon’s mass, strangeness and nuclear size-reduction effect cause changes in the nucleus, it is plausible to assume that neutron and proton drip lines in hypernuclei (S = \(−1, −2\) or \(−3\)) will differ from those in non-strange nuclei (S = 0). The effect of a lambda hyperon on one-neutron and one-proton drip lines for single \(Λ\) hypernuclei was reported through a phenomenological binding energy model in [4, 5] and through microscopic methods in [6-9]. A similar study was carried out for \(He\) and \(Li\) hypernuclei using the no-core–shell method [10].

The goal of this study is to locate two-neutron drip lines in the hypernuclear isotopic series of C—Mn. Two-neutron separation energies are computed within a binding energy model which is an extension of the Bethe-Weizsäcker formula [4, 5].
Results and discussion

Two-neutron separation energies for lambda hypernuclei \( S_{2n}(A, Z) \) are computed using the following relation, which is based on energy conservation:

\[
S_{2n}(A, Z)_\Lambda = B(A, Z)_\Lambda - B(A - 2, Z)_\Lambda,
\]

where \( A \) is the baryon number and \( Z \) is the total charge in the nucleus. The binding energy, \( B(A, Z)_\Lambda \), is obtained using the model in equation (2), which is an extension of the Bethe-Weizsäcker formula [4, 5].

\[
B(A, Z)_\Lambda = 15.777A - 18.34A^{2/3} - 0.71Z(Z - 1)A^{-1/3} - \frac{23.21(N - Z)^2}{(1 + e^{-A/30})A} + (1 - e^{-A/30})\delta + n_\Lambda \left(0.0335m_\Lambda - \alpha - \frac{48.7[S]}{A^{2/3}}\right),
\]

where \( \alpha = 27.8, Z \) is the proton number of the non-strange core, \( n_\Lambda \) the number of lambda hyperons, \( m_\Lambda = 1115.683 \text{ MeV} \) the mass of lambda hyperon and \( S = -1 \) the strangeness of a lambda hyperon. In the pairing term \( \delta = +12A^{-1/2} \) for \( N, Z \) being even–even, \( \delta = -12A^{-1/2} \) for \( N, Z \) being odd–odd and \( \delta = 0 \) for \( N, Z \) being odd–odd or odd–even. The baryon number \( A \) is given by \( A = N + Z + n_\Lambda \) while the total charge in the nucleus \( Z \) is given by \( Z = Z_\text{c} + n_\Lambda q_\Lambda = Z \). Since the charge, \( q_\Lambda \), of a lambda hyperon is 0. In this paper, the binding energy model is used with the parameter \( \alpha \) modified from \( \alpha = 26.7 \) to \( \alpha = 27.8 \), based on updated nuclear data in [11, 12]. The first three terms in this binding energy model are the volume energy, surface energy and Coulomb repulsion energy. These three terms all arise within the Liquid Drop Model of the nucleus. The fourth term is the asymmetry energy, a quantum-mechanical term that is due to the Pauli principle, modified with an extra factor. The fifth term is the pairing energy, also modified with an extra factor. The sixth term, which is not part of the Bethe-Weizsäcker formula, explicitly takes into account the number of hyperons, the hyperon mass and hyperon strangeness.

3. Results and discussion

Binding energies were computed for the single lambda isotopic series of the elements C—Mn, using Equation (2). From these binding energies, two-neutron separation energies \( S_{2n} \) were obtained through equation (1). For a particle-bound nucleus, \( S_{2n} \) is positive while it is negative for an unbound case. Therefore, in order to locate the drip line the change in the sign of \( S_{2n} \) as it crosses \( S_{2n} \approx 0 \) from positive to negative is used as a marker. In this paper, the two-neutron separation energies computed for each isotopic series were analysed to find each drip line. The drip lines so obtained are displayed in Table 1, alongside their respective \( S_{2n} \).

| Element | 2n drip line /\( S_{2n}(\text{MeV}) \) | 1n drip line /\( S_{2n}(\text{MeV}) \) |
|---------|----------------------------------|----------------------------------|
| C       | \( ^2\text{C} (N = 14) / 1.61 \) | \( ^3\text{C} (N = 14) / 1.63 \) |
| N       | \( ^2\text{N} (N = 16) / 1.20 \) | \( ^4\text{N} (N = 16) / 1.50 \) |
| O       | \( ^2\text{O} (N = 18) / 0.88 \) | \( ^3\text{O} (N = 18) / 1.40 \) |
| F       | \( ^2\text{F} (N = 20) / 0.59 \) | \( ^3\text{F} (N = 22) / 0.01 \) |
| Ne      | \( ^2\text{Ne} (N = 22) / 0.37 \) | \( ^3\text{Ne} (N = 24) / 0.04 \) |
| Na      | \( ^2\text{Na} (N = 24) / 0.20 \) | \( ^3\text{Na} (N = 26) / 0.08 \) |
| Mg      | \( ^2\text{Mg} (N = 26) / 0.09 \) | \( ^3\text{Mg} (N = 28) / 0.13 \) |
| Al      | \( ^2\text{Al} (N = 28) / 0.02 \) | \( ^3\text{Al} (N = 30) / 0.18 \) |
| Si      | \( ^2\text{Si} (N = 29) / 0.99 \) | \( ^3\text{Si} (N = 31) / 0.24 \) |
| P       | \( ^2\text{P} (N = 31) / 0.92 \) | \( ^3\text{P} (N = 34) / 0.31 \) |
| S       | \( ^2\text{S} (N = 33) / 0.88 \) | \( ^3\text{S} (N = 36) / 0.37 \) |
| Cl      | \( ^2\text{Cl} (N = 36) / 0.04 \) | \( ^3\text{Cl} (N = 38) / 0.44 \) |
| Ar      | \( ^2\text{Ar} (N = 38) / 0.09 \) | \( ^3\text{Ar} (N = 40) / 0.51 \) |
| K       | \( ^2\text{K} (N = 40) / 0.17 \) | \( ^3\text{K} (N = 42) / 0.58 \) |
| Ca      | \( ^2\text{Ca} (N = 42) / 0.23 \) | \( ^3\text{Ca} (N = 46) / 0.07 \) |
| Sc      | \( ^2\text{Sc} (N = 44) / 0.33 \) | \( ^3\text{Sc} (N = 48) / 0.17 \) |
| Ti      | \( ^2\text{Ti} (N = 46) / 0.41 \) | \( ^3\text{Ti} (N = 50) / 0.25 \) |
| V       | \( ^2\text{V} (N = 48) / 0.51 \) | \( ^3\text{V} (N = 52) / 0.34 \) |
| Cr      | \( ^2\text{Cr} (N = 51) / 0.05 \) | \( ^3\text{Cr} (N = 54) / 0.41 \) |
| Mn      | \( ^2\text{Mn} (N = 53) / 0.18 \) | \( ^3\text{Mn} (N = 58) / 0.05 \) |
Due to neutron pair correlations, odd–even oscillations are known to occur in the one-neutron separation energy ($S_n$) as one progresses through a given isotopic series. Whereas $S_n$ oscillates, $S_{2n}$ decreases monotonically, giving rise to cases where $S_{2n}$ reaches a negative value while $S_n$ is still positive. As a result, the two-neutron drip line is usually expected to occur before the one-neutron drip line. From table 1, it is observed that this is the case for the $S = −1$ series of F—Mn. However, for the $S = −1$ isotopic series of C, N and O it is observed that the two-neutron drip lines coincide with their respective one-neutron drip line. The fact that the $S = −1$ isotopic series of C, N and O have coinciding one-neutron and two-neutron drip lines may be an indication that $\lambda$ C ($N = 16$), $\Lambda$ N ($N = 18$) and $\Lambda$ O ($N = 20$) are two-neutron emitters, provided there is a strong neutron pair correlation. Di-neutron correlation, the strong localisation of valence neutron pairs, is the main determinant of two-neutron decay in neutron-rich nuclei beyond the drip lines [13]. It was shown in [10] using the no-core–shell method, that the one-neutron drip line of the $S = 0$ isotopic series of He and Li have the same number of neutrons as their respective $S = −1$ drip lines.

One can also observe from the two-neutron drip lines in table 1 that for each new element obtained by adding a proton, two neutrons are added to the drip line hypernucleus, except in the cases of S—Cl and V—Cr where three neutrons are added and for Al—Si where just one neutron is added. For the one-neutron drip lines, two neutrons are added for each extra proton, except for the cases of O—F, K—Ca and Cr—Mn where 4 neutrons are added. In the $S = 0$ sector, 6 neutrons are added at the drip line from O—F [14, 15], a situation that is usually described as the oxygen anomaly. The range of $S = −1$ drip lines observed in this paper does not reveal any such anomaly as only 1, 2 or 3 neutrons are added.

4. Conclusion

Computations were carried out to determine the maximum number of neutrons a hypernucleus can hold and still be stable with respect to two-neutron decay. The single-lambda hypernuclear isotopic series of C—Mn were considered. A phenomenological model, which is an extension of the BetheWeizsäcker formula, was used in computing binding energies for each of the isotopic series. Two-neutron drip lines were located from these binding energies. It was observed that the two-neutron drip line of C, N and O coincide with their respective one-neutron drip line while the two-neutron drip line of F—Mn occurs a few neutrons before their respective one-neutron drip line. As one progresses through the elements, one or two neutrons are added to the two-neutron drip line, except in the cases of S—Cl and V—Cr where three neutrons are added and for Al—Si where one neutron is added. Unlike the case of one-neutron driplines of non-strange nuclei where the oxygen anomaly is observed, no anomaly in the number of neutrons at the drip line is observed for the range of hypernuclei studied in this paper. The impact of hyperon mass, strangeness and glue effect on one- and two-neutron drip lines can only be fully appreciated through microscopic computations, using accurate lambda-nucleon and nucleon-nucleon forces. The binding energy model employed here provides valuable insights at minimal computational cost and it can be extrapolated to mass regions that are out of reach of current experimental techniques.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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