Controlling irreversibility and directionality of light via atomic motion: optical transistor and quantum velocimeter

To cite this article: C H Raymond Ooi 2008 New J. Phys. 10 123024

View the article online for updates and enhancements.

Related content
- Role of incoherent pumping scheme on gain without population inversion in four-level systems
  Bibhas K Dutta and Prasanta K Mahapatra
- Sharply tunable group velocity in alkali vapors using a single low-power control field
  Pardeep Kumar and Shubhrangshu Dasgupta
- Coherent pump–probe spectroscopy of a system with a close lying excited level
  Niharika Singh, Ayan Ray, R D'Souza et al.

Recent citations
- Controlling Double Quantum Coherence and Electromagnetic Induced Transparency with Plasmonic Metallic Nanoparticle
  C. H. Raymond Ooi and Kai Shuen Tan
- Jin-Jin Li and Ka-Di Zhu
- Electromagnetically induced transparency in nanocells
  A N Litvinov et al.
Controlling irreversibility and directionality of light via atomic motion: optical transistor and quantum velocimeter

C H Raymond Ooi
Department of Physics, Korea University, Anam-dong, Seongbuk-gu, Seoul, 136-713 Republic of Korea
E-mail: bokooi73@yahoo.com

New Journal of Physics 10 (2008) 123024 (17pp)
Received 30 September 2008
Published 18 December 2008
Online at http://www.njp.org/
doi:10.1088/1367-2630/10/12/123024

Abstract. The Doppler effect of moving atoms can create irreversibility of light. We show that the laser field in an electromagnetic induced transparency (EIT) scheme with atomic motion can control the directional propagation of two counter-propagating output probe fields in an atomic gas. Quantum coherence and the Doppler effect enable the system to function like an optical transistor with two outputs that can generate states analogous to the Bell basis. Interference of the two output fields from the gas provides useful features for determining the mean atomic velocity and can be used as a sensitive quantum velocimeter. Some subtle physics of EIT is also discussed. In particular, the sign of the dispersive phase in EIT is found to have a unique property, which helps to explain certain features in the interference.
1. Introduction

In the presence of matter, the reversibility of light can be broken. An example of the broken symmetry is between the absorption and emission processes, due to the presence of spontaneous emission. Atomic motion also breaks the symmetry, particularly between absorption and stimulated emission, which is one of the ingredients for laser cooling [1]. No doubt, irreversibility of light is possible. But how can we control the irreversibility and develop it for useful applications? In a two-level atom, a negative detuned laser would be resonantly absorbed if the atom moves opposite to it and would be transmitted if the atom moves along it, but there is no way to control it.

The absorption of a probe field with frequency $\nu$ (on $b \leftrightarrow a$ transition) by atoms with a three-level $\Lambda$ scheme (see figure 1) can be significantly reduced by applying a control laser (on the $c \leftrightarrow a$ transition) with frequency $\nu_c$ that satisfies two-photon or Raman resonance ($\Delta = \Delta_c$ or $\nu - \nu_c = \omega_c - \omega_b$), where $\omega_b$ and $\omega_c$ are the atomic frequencies of level $b$ and level $c$, respectively. This effect is called EIT and has been widely studied; mainly in the perspectives of nonlinear processes, slow light and quantum information storage [2]. The physics of EIT is mainly due to destructive interference of two absorption pathways, i.e. from level $b$ to two ac Stark split levels of level $a$.

In this paper, we consider the EIT effect in an ensemble of atoms with three-level $\Lambda$ scheme moving with a mean velocity $\vec{u}$, sufficiently large such that the Doppler shift is greater than the linewidth of the excited state, $ku > \Gamma$. In each atom (figure 1(a)), the same transition ($a \leftrightarrow b$) couples to two counter-propagating probe fields with Rabi frequencies $\Omega^+$ and $\Omega^-$, which are typically weaker than the control field $\Omega_c$. The present scheme should be discerned from the polarization gradient cooling scheme [3] and velocity selective coherent population trapping.

New Journal of Physics 10 (2008) 123024 (http://www.njp.org/)
Figure 1. (a) A gas of atoms with mean velocity $\bar{u}$ in a three-level electromagnetic induced transparency (EIT) scheme. The transparency of the two counter-propagating probe fields is controlled by $\Omega_c$. (b) Three major Doppler induced mechanisms (Doppler induced transparency (DIT), Doppler induced resonance (DIR) and Doppler induced detuning (DID)) in EIT with and without probe detuning $\Delta$ are shown for the case of an atom co-propagating with the probe field. The laser frequency is red-shifted (red arrow downward) by the Doppler frequency $\omega_p = ku = kp/M$. A similar diagram can be drawn for a counter-propagating atom, but with blue-shift (blue arrow upward).

(VSCPT) [4] which also has counter-propagating fields, but only one field couples to each transition.

The effect of Doppler broadening (due to moving atoms) on EIT linewidth [5] and slow light [6] has been considered. However, potential applications based on controlling the transmission and irreversibility of two counter-propagating probe fields in moving atoms have not been considered. Under certain controllable conditions, the medium can be made transparent to one of the probe fields while the other field is strongly absorbed. The combined effect of the center of mass (cm) motion and the control field in EIT provides controllable optical directionality or switching of the counter-propagating probe fields\(^1\). Based on this mechanism, we discuss possible applications as a quantum optical transistor and a quantum velocimeter. Further analysis also yields insights into the difference between detuning from a real level and from a level split by the control laser field.

Before we go through the theory, let us refer to figure 1(b) and try to understand the simple physics (for a resonant control field $\Delta_c = 0$). The frequency of a probe field is blue-shifted when absorbed by a counter-propagating atom. Conversely, the frequency is red-shifted when absorbed by a co-propagating atom. The Doppler shift, shown by small arrows and the ac Stark splitting ($2\tilde{\Omega}$) give rise to three limiting classes of internal mechanisms: (a) DIT occurs when the bare frequency is shifted to the midway between the split levels. (b) DIR—the bare frequency is shifted to resonance with one of the split levels. (c) DIT—the bare frequency is shifted farther away from the nearest ac Stark split level.

2. Density matrix approach for EIT with moving atoms

The scheme in figure 1(a) is described by the interaction Hamiltonian $V = -\hbar[|a\rangle\langle c|\Omega_c e^{-i\nu_c t} + |a\rangle\langle b|\Omega_c e^{ikz} + \Omega^+ e^{-i\nu_c t} + \text{adj}]$ with the bare Hamiltonian composed of the kinetic energy

\(^1\) Although ‘optical rectification’ is a better phrase, we refrain from using it to avoid confusion with the optical rectification in nonlinear parametric process. For example, see [7].
\( H_{\text{kin}} = \frac{p^2}{2M} \) and atomic energy \( H_a = \sum_{l=a,b,c} \hbar \omega_l |l\rangle \langle l| \). The control laser is taken to be orthogonal to the atomic beam and has no cm effect along the z-axis. Only the counter-propagating probe lasers give the cm effect. The cm position operator \( \hat{z} \) is quantized and this gives rise naturally to the first-order Doppler shift \( \omega_p = k p / M \) and recoil shift \( \omega_r = \hbar k^2 / 2M \) in the transition frequency.

We have a set of an infinite number of density matrix equations which contain finite coherence between different momentum families \( \rho_{ab}(p_a, p_b) = \langle \alpha, p_a | \hat{\rho} | \beta, p_b \rangle \) that can only be solved numerically. The presence of quantized cm motion makes it impossible to find exact solutions. However, we find that it is possible to obtain analytical results in the weak field limit.

For sufficiently weak probe fields as in the typical EIT case, the population in the excited state is negligible. Thus it is a good approximation to disregard the momentum redistribution as the result of spontaneous emission, which gives rise to integral terms \[ \] in the equations for the populations. Also, the density matrix equations may be solved analytically by truncating the set of equations based on the approximation of neglecting the coherences between two momentum families with \( 2\hbar k \) and larger, i.e. \( \rho_{bc}(p \pm 3\hbar k, p) \ll \rho_{bc}(p \pm \hbar k, p), \rho_{aa}(p \pm 2\hbar k, p) \ll \rho_{aa}(p, p) = \rho_{bb}(p), \rho_{bc(c)c}(p - \hbar k, p + \hbar k) \ll \rho_{bb(c)c}(p). \) We then obtain nine equations in quasi steady state

\[ T_{ab}^\pm A^\pm(p) \approx i B^\pm(p) \Omega_c - i \Omega^+ w_{ab}^\pm(p), \]  \( T_{bc}^\pm B^\pm(p) \approx i \Omega_c^+ A^\pm(p) - i \Omega^+ C(p), \]  \[ T_{ac}^\pm C(p) \approx i \Omega_c^+ w_{ac}(p) - i \Omega^+ B^\pm(p) - i \Omega^\mp B^\mp(p), \]

where the slowly varying coherences are

\[ A^\pm(p, t) = e^{i(\Delta \pm \omega_p + \omega_r)t} \rho_{ab}(p, p \pm \hbar k, t), \]  \[ B^\pm(p, t) = e^{i(\Delta - \omega_p + \omega_r)t} \rho_{cb}(p, p \pm \hbar k, t) \]  \[ C(p, t) = e^{-i(\Delta - \omega_r)t} \rho_{cc}(p, t). \]

The complex decoherences that include the Doppler and recoil shifts are \( T_{ab}^\pm = \gamma_{ab} - i(\Delta \pm \omega_p + \omega_r), T_{bc}^\pm = \gamma_{bc} - i(\delta - (\pm \omega_p + \omega_r)) \) where \( \delta = \Delta_c - \Delta \) and \( T_{ac}^\pm = \gamma_{ac} - i \Delta_c. \) The population inversions are defined as \( w_{ac}(p) = \rho_{aa}(p) - \rho_{cc}(p) \) and \( w_{bb}^\pm(p) = \rho_{aa}(p) - \rho_{bb}(p \pm \hbar k) \) which can be taken to be the initial values for weak probe fields \( \rho_{aa}(p, t) \approx \rho_{aa}(p, 0) = \rho_{aa0} f(p), \rho_{cc}(p, t) \approx \rho_{bb}(p, 0) \) and \( \rho_{bb}(p \pm \hbar k, t) \approx \rho_{bb}(p, t) \approx \rho_{bb0} f(p). \) The populations depend on the momentum distribution of the gas \( f(p) \) with normalization \( \int_{-\infty}^{\infty} f(p) \, dp = 1. \) For a gas with velocity width of \( \Delta p = \sqrt{2 M k_B T} \) and a mean momentum \( \tilde{p} = m \tilde{u} \) (mean velocity towards the right, \( \tilde{u} > 0 \)), we represent \( f(p) \) by a Gaussian function \( \frac{1}{\sqrt{2 \pi} \sigma} \exp[-(p - \tilde{p})^2 / \Delta p^2]. \)

The five coupled equations (1)–(3) can be solved exactly, giving

\[ A^\pm(p, z) = - \frac{\Omega^\pm}{\Gamma} \left[ I^\mp (w_{ab}^\mp T_{ab}^\pm + w_{ab}^\pm I_c) / J^+ J^- + w_{ab}^\mp I^\pm / J^\mp \right. \]
\[ \left. + (w_{ac}^\pm T_{bc}^\pm - w_{ac}^\mp I_c) / J^\mp \right], \]

\textbf{New Journal of Physics 10} (2008) 123024 (http://www.njp.org/)
where

\[ \gamma = \frac{T_{ac}^* + I^* T_{bc}^+}{J^+} + \frac{I^* T_{ab}^-}{J^-}, \]  

(8)

\[ J^\pm = T_{ab}^\pm T_{bc}^\mp + I_c \]  

(9)

and \( I^\pm = |\Omega^\pm|^2, I_c = |\Omega_c|^2. \)

The integration of \( A^\mp \) over the momentum gives the propagation equations

\[ \frac{\partial}{\partial z} \Omega^\pm(z) = i\kappa g \int_{-\infty}^{\infty} A^\mp(p, z) \, dp = G^\pm \Omega^\pm, \]  

(10)

where \( \kappa g = N \frac{\alpha_i |\omega|^2}{2\hbar} \). Please note that we do the momentum integration differently from previous works [5, 6]. The present approach is more general, in that it enables the populations of different levels to take different distributions; for example: \( \rho_{cb}(p) = 0.2 f(p), \rho_{ab}(p) = 0.8 f(p - \hbar k), \rho_{ac}(p) = 0 \) subjected to normalization \( \sum_{x=a,b,c} \int_{-\infty}^{\infty} \rho_{xx}(p) \, dp = 1. \)

Equations (7) and (10) show that \( \Omega^\pm \) is coupled to \( \Omega^- \) due to the nonlinear dependence of equation (7) on \( I^\pm \) and need to be solved numerically when \( I^\pm \) are large. The cross-coupling of the two fields originates from the last two terms in equation (3) and physically corresponds to two probe fields interacting simultaneously with the same atom.

The Doppler effects in nonlinear regime are more complicated and will be reported in the future. Here, we focus on the limit of small probe fields \( I^\pm \ll I_c, (T_{ab}^\pm)^2 \) where the cross-interaction between the fields is weak and negligible. Thus, we have a linear theory

\[ A^\pm(p, z) \approx \frac{-i \Omega^\mp}{J^\pm} \left[ w_{ab}^* T_{bc}^\pm - w_{ac} I_c / T_{ac}^* \right], \]  

(11)

and equation (10) yields the solutions \( \Omega^\pm(z) = \Omega^\pm(0) e^{G^\pm z} \) with the complex ‘gain’ or response functions \( G^\pm \)

\[ G^\pm = \kappa g \int_{-\infty}^{\infty} \frac{w_{ab}^* T_{bc}^\pm - w_{ac} I_c / T_{ac}^*}{J^\pm} \, dp = i \frac{V}{c} (1 + \chi^\pm), \]  

(12)

which are related to the linear susceptibilities \( \chi^\pm = gN \mu_0 \tilde{\rho}_{ab}(p, p \mp \hbar k, t) / \epsilon_0 \Omega^\pm \). If we omit the superscripts \( \pm \), we recover the known [5] relation \( \tilde{\rho}_{ab} \approx -i \Omega_c \frac{w_{ab}^* T_{bc}^\pm - w_{ac} I_c / T_{ac}^*}{(T_{ab}^\mp)^2}. \)

For initialization in level \( b \) (typical EIT) we have

\[ G^\pm = -\kappa g \int \frac{T_{bc}^\mp (T_{ab}^\mp T_{bc}^\pm + I_c) / T_{ac}^*}{Z^\pm} \, dp \]  

(13)

and

\[ Z^\pm = |T_{ab}^\mp T_{bc}^\pm|^2 + I_c (T_{ab}^\mp T_{bc}^\pm + T_{ab}^\pm T_{bc}^\mp) + I_c^2, \]  

(14)

is the real double-peak function that primarily determines the EIT features.

Note that if \( \rho_{cb} \) is neglected (\( B^\pm \rightarrow 0 \)) the probe fields go as (see equation (1)) \( A^\pm \approx -i \Omega^\mp w_{ab}^\pm(p) / T_{ab}^\pm \) that gives the usual absorption with Lorentzian resonance profile as in a two-level system. There would be no EIT (equivalent to setting \( \Omega_c = 0 \)) since \( A^\pm \) does not have a denominator with two resonances.

3. Physics of EIT

The above results can be understood through the following discussions of the physics.
3.1. EIT, ac Stark splitting and detunings

Transparency corresponds to a minimum value of $\text{Re } G^\pm$. For $\Delta_c = 0$ this occurs at $\Delta = 0$. However, for finite $\Delta_c$ there is no simple expression at the minimum. When $\gamma_{bc} = 0$ the minimum is at $\Delta = \Delta_c$.

The beat frequency of emission in EIT is due to ac Stark splitting $2\Omega = \Delta^u - \Delta^l$, where $\Delta^{u/l}$ are the detunings of the upper ($u$) and lower ($l$) split levels that correspond to two resonant peaks in the absorption spectrum. Expressions for $\Delta^{u/l}$ can be found by solving $\frac{d}{d\Delta} (\text{Re } G^\pm) = 0$. For $\Delta_c = 0$ but finite $\gamma_{bc}$ we have

$$\Delta^{u/l} = \pm \sqrt{\frac{\gamma_{bc}}{\gamma_{ab}} + 1} \Omega_c \sqrt{\Omega_c^2 + \gamma_{bc} \lambda_{ab} - \frac{\gamma_{bc}}{\gamma_{ab}} (\Omega_c^2 + \gamma_{bc} \lambda_{ab})},$$

which reduces to

$$\Delta^{u/l} \simeq \sqrt{\Omega_c^2 + \frac{1}{2} \gamma_{bc} (\lambda_{ab} - \gamma_{bc})},$$

when $\gamma_{bc} \lambda_{ab} \ll \Omega_c^2$. Conversely, for $\gamma_{bc} = 0$ but finite $\Delta_c$ we find

$$\Delta^{u/l} = \frac{1}{2} \Delta_c \pm \sqrt{\Omega_c^2 + \frac{1}{4} \Delta_c^2}.$$  

Thus, the frequencies of the resonant peaks $\omega_{ab} + \Delta^{u/l}$ depend on the control field detuning in a nonlinear manner. Equation (17) is very useful for physical interpretation of the double resonance found in figure 2 for the total intensity $I(z) = |\Omega^{\text{tot}}(z)|^2$ with finite detuning $\Delta_c = \Delta$, where

$$\Omega^{\text{tot}}(z) = \Omega^+(z) + \Omega^-(z) = \Omega_0 (e^{G^+z} + e^{G^-z}),$$

is the superposition of the two probe fields.
3.2. Inhomogeneous broadening and Doppler width

The effects of finite velocity width of the gas on EIT have been well studied in optically thick medium [9]. Under the approximation that the Doppler width is much greater than the EIT window, it was found [5] to have little effect on EIT linewidth at low control field, but leads to linewidth narrowing at high field. However, no analytical expression exists for the general case. Although all results are obtained using the Gaussian distribution, here we use Lorentzian distribution \( f(u) = f(p)M = \frac{\Delta \omega/\pi}{(\omega-p)^2 + \Delta \omega^2} \), as in a previous analysis [5]. For simplicity, take \( \omega = k\tilde{u} = 0 \) and from

\[
G^\pm = -\kappa g \int \frac{\gamma_{bc} i(\delta - \omega_r \pm \omega_p)}{(\gamma_{ab} - i(\Delta + \omega_r \mp \omega_p))(\gamma_{bc} + i(\delta - \omega_r \pm \omega_p)) + I_c} \left( \frac{\Delta \omega/\pi}{\omega_p^2 + \Delta \omega^2} \right) d\omega_p, \tag{19}
\]

we can see that \( G^+ = G^- \) by defining the variable as \( \omega_p \to -\omega_p \). The Doppler width is \( \Delta \omega = k\Delta u \).

When \( \Delta + \omega_r = 0, \Delta_c = 0 \), the response function is real, expressible as

\[
G = -\kappa g \int S(\omega_p) \left( \frac{\Delta \omega/\pi}{\omega_p^2 + \Delta \omega^2} \right) d\omega_p, \tag{20}
\]

\[
S(\omega_p) = \frac{\gamma_{bc}(\gamma_{bc}^2 + \omega_p^2) + I_c \gamma_{bc}}{(\gamma_{ab}^2 + \omega_p^2)(\gamma_{bc}^2 + \omega_p^2) + 2I_c(\gamma_{ab}\gamma_{bc} - \omega_p^2) + I_c^2}. \tag{21}
\]

The \( S \) and the \( f \) functions are symmetric upon reflection at \( \omega_p = 0 \). The imaginary part of \( G \) is zero because the imaginary part of the integrand in equation (19) is an antisymmetric function.

In the case of stationary atoms (no Doppler effect), the real part of \( G \) is zero if \( \gamma_{bc} = 0 \) and \( \Delta_c = \Delta \). However, the real part of \( G \) in equation (20) is finite because of the overlap between the integrand \( S(\omega_p) \) and \( f \) due to the finite momentum width, even if \( \gamma_{bc} = 0 \). Thus, the resulting signal depends on the overlap of the transparency window \( W_{EIT} = 2\tilde{\Omega} \) and the Doppler width \( W_D = 2k\Delta p/M = 2k\sqrt{2k_BT/M} \). When the Doppler width \( W_D \) increases, the transparency decreases since more atoms coincide with the absorption peaks. In general, the medium is essentially transparent when \( W_D < W_{EIT} \). Figure 3 shows how the total intensity \( I \) varies with increasing Doppler width. As the temperature increases, \( W_D \) increases and only the regions with sufficiently large control field \( 2\tilde{\Omega} \gtrsim W_D \) show large signal.

3.3. Real and imaginary parts

The real parts of \( G'^\pm \) (corresponding to the imaginary parts of \( \chi^{(1)}\pm \)) give the absorption coefficients (if negative) and gain (if positive). Conversely, the imaginary part of \( G^\pm \) (corresponding to the real parts of \( \chi^{(1)}\pm \)) give the refractive index beyond unity that affects the wavevector. The polar plots in figure 4 illustrate the spatial evolution of the amplitudes \( |\Omega^\pm(z)| = \Omega_0 e^{\text{Re}G^\pm z} \) (radial lengths) and the phases \( \theta^\pm = \text{Im} G^\pm z \) (angles). In the absence of EIT (\( \Delta_c = 0 \)) the resonant (\( \Delta = 0 \)) counter-propagating probe fields are Doppler shifted equally above and below the resonance (neglecting the much smaller recoil shift). Figure 4(a) shows...
Figure 3. Effects of Doppler broadening on the total signal. (a) Normalized distribution $f$ and normalized $S$ the absorptive (real) part of equation (20). Larger control field widens the EIT transparency window, allowing greater proportion of the atomic distribution to fall within the window. (b) The oscillatory feature appears toward higher control field as the Doppler width increases. We use $\Delta = \Delta_c = 0$, $L = 0.1$ m and $\gamma_{bc} = 0.1 \gamma_{ac}$.

Figure 4. (a) The simplest method for making the medium transparent to two output fields without EIT ($\Omega_c = 0$). Doppler shift introduces detuning to the resonant fields ($\Delta = 0$). Note the phases of the counter-propagating fields are opposite. (b) Directional propagation without EIT. For positive detuning $\Delta = k\bar{u}$, the field $\Omega^+$ propagating to the right is red-shifted to resonance and heavily absorbed whereas the left-propagating field $\Omega^-$ is blue-shifted farther from resonance. The splitting of the upper level $a$ is $2\Omega$ ($\bar{\Omega} = \sqrt{\Omega^2_c + \Delta^2_c}/4$ for $\gamma_{bc} = 0$ used here). Other parameters used are: mean velocity $\bar{u} = 300$ ms$^{-1}$ and propagation length $L = 0.1$ m. If two identical probe fields are fed into the medium from both ends, the medium works like an optical diode. If the signal is generated from within the medium, it works like a directional emitter.

that the counter-propagating probe fields $\Omega^\pm$ acquire phases with opposite signs. However, in the presence of EIT, we find a new subtle effect. The polar plot in figure 6(a) shows that the two fields acquire the same phase with the same sign despite being shifted above and below the split level. This is in contrast to the case without EIT (figure 4(a)) where the phases of the fields have opposite signs.
4. Directional flow of light

Several potential applications of quantum coherence effects of EIT in systems with moving atoms are discussed below.

4.1. Directional propagation

We start by analyzing the simplest case—without EIT ($\Omega_c = 0$), as shown in figure 4(b). If the probe is detuned $\Delta = -\omega_p (< 0)$ with $\omega_p = k\bar{u} = k\bar{p}/M$, the field that propagates opposite to the atoms would be absorbed since $G^- \simeq \kappa g w_{ab0} G^L$ is real and negative value where $G^L = \int_\infty^{-\infty} \frac{x^2}{\gamma^2 + x^2} dx$, $x = \omega_p - \omega_p$, whereas the co-propagating field is transmitted since $G^+ \simeq -i\kappa g w_{ab0} \int f(p) dp$ is primarily imaginary for $\gamma \ll \omega_p$. The counter-propagating probe fields undergo optical directionality, i.e. the medium is transparent to one of the probes while opaque to the other probe field. Thus, an appropriate probe field detuning with respect to the mean velocity of a gas jet can serve as an optical diode.

The mechanism may also be used as a directional emitter, if an emitting source like an electrically driven quantum dot is embedded inside the channel of a hollowed waveguide or fiber containing a gas flow. The source emits light in all directions but would be guided only along two opposite directions. The light in one direction propagates with little damping whereas the light in the opposite direction is heavily absorbed.

4.2. Control of directionality

Now, by applying a laser field $\Omega_c$ in EIT configuration, we can control the directional emission by controlling the absorption strength of the two fields. There are basically three control parameters: (a) the field $\Omega_c$ that causes ac Stark splitting of the upper levels, (b) the probe frequency or detuning $\Delta$ and (c) the Doppler shift $\omega_p$. Consider the result with positive detuning $\Delta = k\bar{u}$ shown in figure 5. For $\bar{\Omega} < \omega_p$ (or $2\bar{\Omega} < 2k\bar{u}$ ) in figure 5(a), the co-propagating field (experiences DIT) is damped more rapidly than the counter-propagating field (experiences DID)
although it experiences an EIT. Both fields experience different degrees of transparency, but the transparency due to EIT is less than that of a largely detuned (non-resonant) field. In fact, upon the Doppler shift, the co-propagating frequency becomes closer to one of the split levels than the counter-propagating frequency. We should realize that EIT does not necessarily provide the best transparency. Rather, its essence lies in making a medium more transparent to a resonant probe field that would be otherwise heavily absorbed. The above explanation also applies to the case $\tilde{\Omega} > \omega_p$ (figure 5(b)) which is more obvious, i.e. the medium becomes more transparent to the counter-propagating field through DIT. When $\tilde{\Omega} = \omega_p$, the two fields are equally detuned from the ac Stark shifted upper level, so both fields are equally damped.

5. Proposed applications and insightful analysis

5.1. Optical transistor

In principle, the EIT scheme with stationary atoms works like an optical valve, with the capability to control the direction and intensity of the transmitted probe signal. Typical control parameters are the probe detuning $\Delta$ and the strength of the control field $\Omega_c$. In the absence of atomic motion or Doppler effect, both counter-propagating probe fields are either equally damped or transmitted and there is no rectification. The presence of atomic motion can also make the two probe fields equally transmitted (figure 6(a)) or blocked (absorbed) (figure 6(b)). More importantly, the presence of atomic motion can create rectification of the combined probe signal, by making the medium transparent to one of the counter-propagating fields but opaque to the other field (figures 6(c) and (d)).

The ability to coherently control the resulting direction of the two counter-propagating fields makes the device function like an optical transistor, shown in figure 6(e). Since the probe fields can be weak compared to the control field, the present optical transistor does not involve nonlinear optical processes, but uses quantum interference in atomic dynamics and the Doppler effect as the switching mechanisms. Despite the linear mechanism, the total signal can vary in a nonlinear manner with the control field as shown in figure 6(f) for switching from state $|1\rangle_L |1\rangle_R$ to state $|0\rangle_L |1\rangle_R$ as the Rabi frequency varies from $\tilde{\Omega} = k\tilde{u}$ (case figure 6(a)) to $2k\tilde{u}$ (case figure 6c). The present optical transistor is different from existing optical transistors that use (classical) nonlinear processes as the underlying mechanisms; such as the use of nonlinear refractive index in Fabry–Perot [10] and engineered discontinuity in the electromagnetic density of states using photonic crystals [11]. The recently proposed single photon transistor [12] uses surface plasmons to establish strong nonlinear coupling with an atom for use as a single photon gate.

5.2. Connection with quantum states

The four limiting cases in figures 6(a)–(d) show a possible connection or analogy between the classical directional states with quantum states. For $\tilde{u} = 0$, a resonant field with $\Omega_c = 0$ is damped, giving no-field state $|0\rangle_L |0\rangle_R$. However, a resonant field with finite $\Omega_c$ corresponds to transparency (EIT), thus the state of light is $|1\rangle_L |1\rangle_R$. For positive $\tilde{u}$, when $\Delta = k\tilde{u}$ the co-propagating field experiences transparency whereas the counter-propagating field is absorbed, thus the state is $|0\rangle_L |1\rangle_R$, referred to as ‘right-field’. For $\Delta = -k\tilde{u}$ and $\tilde{\Omega} = 2k\tilde{u}$ or (for negative $\tilde{u}$ and $\Delta = k|\tilde{u}|$) the situation is reversed giving the ‘left-field’ state $|1\rangle_L |0\rangle_R$. The states are subjected to physical factors, namely the driving field and the direction of the fluid.
Figure 6. Spatial variations of the probe fields $|\Omega^+|$ (red solid line) and $|\Omega^-|$ (blue dash line) with polar plots for four limiting cases for the output fields ($|1\rangle$ for transmitted field and $|0\rangle$ for damped field) for finite $\Omega_c$, $\Delta_c$ and $\Delta$. (a) A scheme for producing the output state $|1\rangle_L|1\rangle_R$ with EIT, (b) scheme for damping both fields $|0\rangle_L|0\rangle_R$. Schemes for damping one of the fields: (c) $|1\rangle_L|0\rangle_R$ and (d) $|0\rangle_L|1\rangle_R$. (e) Optical transistor with two output fields can produce four distinct states (shown in (a)–(d)) based on transparency and damping of the fields. In (a)–(d), we use $\Delta_c = 20\gamma_{ac}$. (f) Normalized fields $|\Omega^\pm|$ and total intensity $|\Omega^{\text{tot}}|^2 = |\Omega^+ + \Omega^-|^2$ show an example of transistor operation; switching from state $|1\rangle_L|1\rangle_R$ to $|0\rangle_L|1\rangle_R$ as $\Omega = \Omega_c$ increases from $k\bar{u} = 20\gamma_{ac}$ to $2k\bar{u}$. We use $\gamma_{bc} = \Delta_c = \Delta = 0$ for simplicity. The $|\Omega^{\text{tot}}|^2$ varies nonlinearly with $\Omega_c$. Other parameters used are the same as in figure 5.
If these four states (shown in figure 6(e)) can be used to construct the well-known Bell basis the scheme could be useful in quantum information. For example, the superposition of ‘on’ and ‘off’ control field can be described by a macroscopic entangled state

$$|\Phi^\pm\rangle = |1\rangle_L|1\rangle_R \pm |0\rangle_L|0\rangle_R.$$  

Similarly the superposition of ‘right-field’ and ‘left-field’ states is described by

$$|\Psi^\pm\rangle = |0\rangle_L|1\rangle_R \pm |1\rangle_L|0\rangle_R,$$

which represents an indefinite mean velocity associated with the case of chaotic flow. Thus, the field system can be described by four Bell basis states.

5.3. Quantum velocimeter

An interferometric setup with one arm in the quantum coherence medium has been proposed as magnetometer [13]. The present setup (figure 7(a)) is different as both outputs emerge from the same phaseonium (EIT) medium but their linear responses can be substantially different due to the Doppler effect. The interference of the counter-propagating output fields can be used to detect atomic motion in a gas. The use of the Doppler effect to measure the atomic velocity reminds us of the existing technique of laser Doppler velocimetry [14] which uses crossed laser beams and the Doppler effect to measure the flow velocity, a concept entirely based on classical physics.

Here, we introduce quantum velocimetry that incorporates a different underlying mechanism, based on a quantum coherence effect through the EIT with a laser as a control knob to create a sensitive velocimeter. By combining the two probe fields as shown in figure 7(a), useful results can be obtained from the total field $\Omega^\text{tot}(z) = \Omega_0(e^{G^+z} + e^{G^-z})$. The presence of atomic motion in the gas gives $G^+ \neq G^-$. Figure 7(b) illustrates the physics behind the damped and oscillatory nature of the signal $I(z)$. The real parts of $G^\pm$ determine the degree of absorption or transparency of the fields. The imaginary parts superimpose to give oscillations or beating in $I(z) = |\Omega^\text{tot}(z)|^2$, as shown in figure 7(c). The key point is that gross atomic motion or mean velocity $\bar{u}$ can be detected through the presence of oscillations in $I(z)$ versus $z$ or $\Omega_c$. There are no oscillations when $\bar{u} = 0$. A small change in $\bar{u}$ can be detected from rapid oscillations of $I(z)$ versus $\bar{u}$ (see figure 8) and will be discussed in the subsequent section.

The existence of the oscillatory feature in figure 7(c) at $\Omega_c = 0$ might suggest that a velocimeter could even be contrived in the absence of EIT. However, it is the exponentially decaying region around $\Omega_c \approx \omega_D = k\bar{u}$ (due to EIT resonance) in figure 7(c) that provides information about $\bar{u}$. From figure 4(a), one can immediately deduce that a velocimeter without EIT may not be feasible since it would produce very weak signal for small $\bar{u}$ because the Doppler induced detuning is small. This conclusion is supported by figure 8 for $\Delta = 0$ and small $L$. For large $L$, the propagation effect amplifies the small phase difference between the two Doppler shifted probe fields, giving rise to small oscillations at $\Omega_c = 0$.

The region around $\Omega_c \approx k\bar{u}$ corresponds to the EIT resonance which arises from DIR as shown by the level schematic (on the right) of figure 7(c). Here, the two fields are shifted to resonance and the imaginary part of $G^\pm$ essentially vanishes, so $I(z)$ shows no oscillations. The probe fields add as $(e^{-bz} + e^{-bz})^2 \sim e^{-2bz}$. The channel divides the profile into two regions with oscillations. The oscillations in $\Omega_c < k\bar{u}$ are more rapid than the region $\Omega_c > k\bar{u}$. The fields in these regions interfere as $(e^{iaz}e^{-bz} + e^{-iaz}e^{-bz})^2 \sim e^{-2bz}\cos^2az$. 

http://www.njp.org/
Figure 7. (a) Interferometric setup as a quantum velocimeter detects the intensity $I = |\Omega_{\text{tot}}(z)|^2$ of the total signal from the two probe fields. M—mirror, BS—beam splitter. (b) The real and imaginary parts of $G^\pm$ show the absorptive and dispersive features for two counter-propagating probe fields in typical EIT, each displaced by the Doppler shift. Identical dispersion (dash-dot line) and different dispersion (dashed line) give total intensity with exponential decay and oscillations, respectively. Variations of the interference with $z$ and $\Omega_c$ for (c) $\Delta = 0$ and (d) $\Delta = k\bar{u}$, where $\bar{u} = 100 \text{ms}^{-1}$ corresponds to $\omega_D/\gamma_{ac} = 6.67$. (e) Sensitivity of the device is found from the (normalized) gradient $dR/d\bar{u}$ in equation (28). Other parameters used are $\bar{u} = 100 \text{ms}^{-1}$, $T = 1 \text{K}$, $M = 87 \text{amu}$ (for Rb), $\gamma_{bc} = 0.1 \gamma_{ac}$. 

New Journal of Physics 10 (2008) 123024 (http://www.njp.org/)
Figure 8. Variations of the signal with $\bar{\nu}$ and $\Omega_c$ for different propagation lengths $L$ and detunings. The oscillations become more rapid as $L$ increases. Finite detuning shifts the resonant features and introduces additional ridged features. Other parameters used are $T = 1$ K, $M = 87$ amu, $\gamma_{bc} = 0.1 \gamma_{ac}$.

For $\Delta \neq 0$ (finite detuning), figure 7(d) shows the ridge at $\Omega_c \simeq \omega_D$ corresponding to equal detuning (but opposite signs) of both probe fields from the ac Stark shifted level. One field experiences EIT whereas the other is normal detuning. We learn from figure 6(a) that their phases would be of the same sign. So the fields interfere as $|e^{-b\bar{\nu}e^{i\alpha_2z} + e^{-b\bar{\nu}e^{i\alpha_2z}}}|^2 \rightarrow e^{-2b\bar{\nu}}$, which explains the exponential decay. However, at $\Omega_c \simeq 2\omega_D$, we have one EIT and one resonant with the shifted upper level. Here, the fields interfere as

$$|e^{-b\bar{\nu}e^{i\alpha_2z} + e^{-(b+c)\bar{\nu}z}}|^2 = e^{-2b\bar{\nu}} \left[ 4e^{-cz} \cos^2 \frac{\alpha_2}{2} + (e^{-cz} - 1)^2 \right],$$  \hspace{1cm} (24)$$

where $c > 0$ which means that the damping due to absorption is larger than the damping in EIT. The rate of oscillations is less rapid by half. The rising ridge can be explained as due to the term $(e^{-cz} - 1)^2$ in equation (24).

The effect of Doppler broadening on the total signal has been shown in figure 3 and section 3.3. At $T = 1$ K, the Doppler width is $W_D \simeq 0.7 \gamma_{ac}$. Thus, the effect of Doppler broadening is important only at the field around $\Omega_c \sim \gamma_{ac}$ and not visible in figure 7. Effects of other noises such as finite control laser linewidth and dephasings would be similar to the effects of increasing the Doppler width and decoherences, respectively. Typical semiconductor laser linewidth of MHz corresponds to a width of less than 1 K, so it would not affect the above results. Detailed theory using the quantum noise approach will be reported elsewhere.

5.4. Sensitivity to mean velocity

In the presence of EIT, the quantum velocimetry can be a sensitive device. There is no unique way to determine the mean velocity. The sensitivity depends on the method used. Consider that the total detected intensity $I$ is converted to the detection current $\bar{j}$. Then, in analogy to [13]

$$I \propto \bar{j} = n_m \delta \Omega,$$

where $\delta \Omega$ is the width of the absorptive channel.

New Journal of Physics 10 (2008) 123024 (http://www.njp.org/)
where $n_{in} = P \tau / \hbar \nu$ is the number of input probe photons, $P$ is input probe power, $\tau$ is the measurement duration and $\Re = I / I_{max}$ is a normalized function plotted in figures 7(c) and (d).

We want to find the minimum detectable change of the mean velocity $\delta \bar{u}_{min} = \delta \bar{u}_{min}/ \frac{dj}{dt}$ with $\frac{dj}{dt} = n_{in} \frac{d\Re}{du}$. The minimum detectable change in the current is [13]

$$\delta j_{min} = n_{in} \Delta \Re \sim \sqrt{\frac{1}{2} (\kappa^{(+)2} + \kappa^{(-)2}) n_{in}}, \tag{26}$$

where $\kappa^{(\pm)}(z)$ are the transmission factors ($\exp\{-\frac{3}{8\pi} \lambda^2 \gamma_{bc} \gamma N z / |\Omega_c^2|\}$ in the case of no Doppler effect). Thus, we obtain

$$\Delta \bar{u}_{min} \sim \sqrt{\frac{1}{2n_{in}} (\kappa^{(+)2} + \kappa^{(-)2}) \left(\frac{d\Re}{du}\right)^{-1}}. \tag{27}$$

Note that $\frac{d\Re}{du}$ can be obtained analytically as

$$\frac{d\Re}{du} = \frac{d}{du} \left[ \tilde{f}^+ + \tilde{f}^- + \exp(G^+G^-)z + \exp(G^+G^-)z \right],$$

$$= 2z \left[ \frac{d(Re G^+)}{du} \tilde{f}^+ + \frac{d(Re G^-)}{du} \tilde{f}^- + \text{Re} \left( \frac{d(G^+ + G^-)}{du} \exp(G^+G^-)z \right) \right]. \tag{28}$$

where $\tilde{f}^\pm = \exp(G^\pm e^{G^\pm}z)$ are the normalized intensities. The $\frac{d\Re}{du}$ plotted in figures 7(e) and (f) is obtained numerically, for example using

$$\frac{d(Re G^\pm)}{du} = -\kappa g \text{Re} \int \frac{T_{bc}^\pm (T_{ab}^\pm + T_{bc}^\pm + I_c) \, df(p)}{Z^\pm} \, dp. \tag{29}$$

Equation (28) shows the importance of propagation in enhancing the resolution $\delta \bar{u}_{min}$, i.e. $\frac{d\Re}{du}$ varies linearly with $z$. This effect agrees with the results in figure 8 which show the increasing oscillations for larger $z$.

Using $\lambda = 0.5 \mu m$, $P = 0.1 \text{ mW}$, $\tau = 1 \text{ s}$ we have $n_{in} = 10^{14}$. For a gas at $T = 1 \text{ K}$, $\tilde{u} = 300 \text{ ms}^{-1}$ and resonant condition $\Delta = \Delta_c = 0$ with $\gamma_{bc} = 0$, figures 7(e) and (f) show that in the region of large $z$ and $\Omega_c > \tilde{kR}$ the $\frac{d\Re}{du}$ is as large as 0.2. Taking $\kappa^{(\pm)} \sim 0.5$ we estimate that at the short noise limit, the quantum limit for detectable change in the mean velocity is $\delta \bar{u}_{min} \sim 0.2 \mu \text{ms}^{-1}$.

Another way to estimate the sensitivity is by measuring the change in the control field using $\delta \tilde{u} = \frac{du}{dt} \delta \Omega_c$. In order to estimate $\delta \Omega_c$ we use $\delta n_c = \frac{dn_c}{dt} \delta \Omega_c$ with the well-known variance [15] $(\delta n_c)^2 = \bar{n}_c$ the mean photon number for the control laser above threshold. Using $\bar{n}_c = P_c \tau_c / \hbar \nu_c$, $P_c = U A_c$ and $U = \nu_0 E^2/2 = \nu_0 \hbar^2 \Omega_c^2/2 \phi_{ac}^2$ we obtain

$$\delta \Omega_c = \sqrt{\bar{n}_c} \frac{2 \pi \phi_{ac}}{\lambda_c \tau_c} \sqrt{\frac{c}{2\nu_0 P_c A}}, \tag{30}$$

where $A$ is the laser cross-section, $\lambda_c$ is the control laser wavelength and $\phi_{ac}$ is the transition matrix element. By using a diode laser with typical power $P_c = 1 \text{ mW}$, $\tau_c = 1 \text{ s}$ we find $\bar{n}_c = 10^{15}$. From the slope of the ridges in the three-dimensional (3D) plot against $\tilde{u}$ and $\Omega_c$ (figure 8), we estimate $\frac{du}{dt} \sim 15/\gamma_{ac}$ for $L = 0.1 \text{ m}$. Thus, the minimum detectable mean velocity variation.

New Journal of Physics 10 (2008) 123024 (http://www.njp.org/)
is $\delta \bar{u} \sim 10^{-7} \text{ms}^{-1}$, which is comparable to the sensitivity of the former method. We have used typical values: $\gamma_{ac} = 10^{7} \text{s}^{-1}$, $A = 10^{-5}$ and $\lambda_c = 500 \text{nm}$.

Although the sensitivity is higher ($\frac{d\bar{u}}{d\Omega_{1c}}$ is smaller) for smaller $\Omega_{1c}$ (which is compatible with the narrow EIT lines), the signal is much weaker and less detectable. The gradient $\frac{d\bar{u}}{d\Omega_{1c}}$ of the features in figure 8 decreases with $L$, in agreement with the conclusion of equation (28).

6. Conclusions

We have presented the physics behind EIT for moving particles and discussed potential applications of controlled irreversibility and directional flow of light. In particular, we explore the mechanisms for controlled switching of the probe fields that can function as a two-output optical transistor and as the basis for quantum states. We also showed that the interference of the two counter-propagating fields produces oscillations that can be used to detect a small change in the mean velocity, a quantum version of laser Doppler velocimetry.

Finally, we note that the full potentials of optical directional control and quantum velocimetry could be realized through miniaturization and integration into microoptical systems. Since EIT has been shown in liquid [16] containing active components, it would be possible to integrate the quantum optical transistor and velocimeter concepts into existing microfluidic technology [17], creating a new class of quantum optical-microfluidic sensors for chemical and biomedical sensing. The study of this exciting possibility would involve fluid dynamics with large inhomogeneous broadening and decoherence rates. It is beyond the present scope and will be the subject of future publications.

Acknowledgments

I thank the Korean Research Foundation for the grant funded by the Korean Government (KRF-2008-331-C00120). I am grateful to the Institute of Mathematical Sciences, National University of Singapore for supporting the MHQP program during the summer.

References

[1] Cohen-Tannoudji C 1998 Rev. Mod. Phys. 70 707
  Phillips W D 1998 Rev. Mod. Phys. 70 721
  Chu S 1998 Rev. Mod. Phys. 70 685
  Hansch T and Schawlow A 1975 Opt. Commun. 13 68
  Letokhov V 1968 JETP Lett. 7 272
[2] Fleischhauer M, Imamoglu A and Dowling J P 2005 Rev. Mod. Phys. 77 633
[3] Dalibard J and Cohen-Tannoudji C 1989 J. Opt. Soc. Am. 6 2023
[4] Prudnikov O N and Arimondo E 2003 J. Opt. Soc. Am. B 20 909
  Aspect A et al 1988 Phys. Rev. Lett. 61 826
[5] Javan A, Kocharovskaya O, Lee H and Scully M O 2002 Phys. Rev. A 66 013805
[6] Kocharovskaya O, Rostovtsev Y and Scully M O 2001 Phys. Rev. Lett. 86 628
[7] Mills D L 1998 Nonlinear Optics (Berlin: Springer)
[8] Ooi C H R, Marzlin K-P and Audretsch J 2002 Phys. Rev. A 66 063413
[9] Silva F, Mompart J, Ahufinger V and Corbalán R 2000 Europhys. Lett. 5 286
  Mikhailov E E et al 2006 Phys. Rev. A 74 013807

New Journal of Physics 10 (2008) 123024 (http://www.njp.org/)
[10] Vigoureux J M and Raba F 1991 J. Mod. Opt. 38 2521
[11] John S and Floreescu M 2001 J. Opt. A: Pure Appl. Opt. 3 S103
[12] Chang D E, Sorensen A S, Demler E A and Lukin M D 2007 Nat. Phys. 3 807
[13] Scully M O and Fleischhauer M 1992 Phys. Rev. Lett. 69 1360
       Fleischhauer M and Scully M O 1994 Phys. Rev. A 49 1973
[14] Fansler T D and French D T 1993 Appl. Opt. 32 3846
       Drain L E 1980 The Laser Doppler Technique (Chichester: Wiley)
[15] Haken H 1986 Light: Laser Light Dynamics (Amsterdam: North-Holland)
[16] Fernandes L E et al 2007 Concepts Magn. Reson. A 30 236
[17] Squires T M and Quake S R 2005 Rev. Mod. Phys. 77 977