Theory of inverse Faraday effect in Rashba system

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Abstract. We theoretically present the theory of photoinduced spin due to the inverse Faraday effect in metals in the presence of Rashba system in the THz regime. We find that the induced spin, $s_\mu$, ($\mu = x, y, z$) is proportional to frequency($\Omega$) and square of Rashba spin-orbit interaction($\alpha$), i.e., $\beta_\alpha \alpha_\nu(E \times E^*)_\nu$ where $E$ represents the electric field. Its spin is derived from the perturbation of spin-orbit coupling and electromagnetic field in THz light regime.

1. Introduction
Photoinduced magnetization induced by the circularly polarized light (inverse Faraday effect) has been studied from the interesting of both fundamental physics and applications for magnetic recording devices. The key interaction for the inverse Faraday effect is the spin-orbit interaction[1, 2, 3]. Two types of the inverse Faraday effect have been studied so far. One is in the case with band splitting due to spin-orbit interaction[1, 2], where the inverse Faraday effect in the optical regime arises linearly with respect with to the spin-orbit interaction. The other is the case with Rashba spin-orbit interaction in semiconductor heterostructures[3]. In this case, the magnetization is proportional to the square of spin-orbit coupling constant $\alpha$, namely, $s_\mu \sim \alpha_\mu \alpha_\nu(E \times E^*)_\nu \Omega^{-3}$, where $s_\mu$ is spin density along $\mu = x, y, z$, $E$ is an electromagnetic field, and $E^*$ is complex conjugate of $E$.

In this paper, we present a theory of the inverse Faraday effect in metal with Rashba interaction material in THz regime. The spin density, $s \propto E \times E^*$, is calculated by using the Keldyshuy-Green function for the perturbation of spin-orbit interaction and electromagnetic field.

2. Hamiltonian
Besides the interaction with the electromagnetic field, $H_{em}$, we consider the impurity scattering of the conduction electrons.

The Hamiltonian is $H = H_0 + H_{em} + H_R$, where

$$H_0 = \sum_k \epsilon_k c_k^{\dagger} c_k + \sum_{k,q} u_{k,q} c_{k+q/2,\omega}^{\dagger} c_{k-q/2,\omega},$$

$$H_{em} = \sum_k \frac{e}{m} k \cdot \mathbf{A}_q^{em} c_{k+q/2,\omega+\Omega}^{\dagger} c_{k-q/2,\omega-\Omega} + \frac{e^2 \hbar}{m} A^{em}_{q-Q} \cdot \mathbf{A}_Q^{em} c_{k+q/2,\omega+\Omega}^{\dagger} c_{k-q/2,\omega-\Omega},$$

$$H_R = \sum_k \alpha \cdot \left[ k \times (c_{k,\omega}^{\dagger} \mathbf{\sigma} c_{k,\omega}) \right] - \frac{e}{\hbar} \sum_{k,q} \alpha \cdot \left[ \mathbf{A}_q^{em} \times (c_{k+q,\omega+\Omega}^{\dagger} \mathbf{\sigma} c_{k,\omega}) \right].$$

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Here, \(c, c^\dagger\) are the conduction electron’s annihilation and creation operators, respectively, and \(u_i\) is the nonmagnetic impurity potential.

\(\mathcal{H}_{\text{em}}\) represents the electromagnetic interaction. The electromagnetic field are represented by \(E_{\text{em}}\) and \(A_{\text{em}}\), respectively. We assume the case of a monochromatic circularly polarized light with frequency \(\Omega\) and wave vector \(q\). In terms of complex amplitudes \(E\) and \(A\),

\[
E_{\text{em}} = \frac{1}{2}(E e^{i(q \cdot x - \Omega t)} + E^* e^{-i(q \cdot x - \Omega t)}) \quad \text{and} \quad A_{\text{em}} = \frac{1}{2}(A e^{i(q \cdot x - \Omega t)} + A^* e^{-i(q \cdot x - \Omega t)}),
\]

where the vector potential and the electric field satisfy \(E_{\text{em}} = -A_{\text{em}}\), and thus \(A = \frac{E}{i\Omega}\). The chirality of light \(E \times E^*\) is proportional to light propagation, \(q\), and the value, \(|E \times E^*|\), is a pure imaginary.

The term \(\mathcal{H}_R\) represents the Rashba spin-orbit interaction in the presence of electromagnetic field (where \(\alpha\) is constant).

We evaluate the magnetization and spin density proportional to the vector product of the electromagnetic field. The spin density is given by \(s^\alpha \equiv \langle c^\dagger \sigma^\alpha c \rangle\), where \(<\ >\) denotes the expectation values \((\alpha = x, y, z\) represents the spin direction\). The magnetization is expressed as \(M^\alpha = -\frac{\mu_B}{2} s^\alpha\), where \(\mu_B\) is the Bohr magneton and \(g\) is the \(g\)-factor. By using lesser Green’s function, \(G^<(x, x; t, t)\), the spin density, \(s^\alpha\), is expressed as

\[
s(x) = -i\hbar \text{Tr} \left[ \sigma \hat{G}^<(x, x; t, t) \right].
\]

![Figure 1](image)

**Figure 1.** The diagram of spin density due to inverse Faraday effect of the first-order (a) and second-order (b) perturbation of spin-orbit interaction. The bold line shows the electron’s Green’s functions \(g\), the dots arrow indicates the electromagnetic field \(E\) and \(E^*\), and the black circle is Rashba spin-orbit interaction \(\alpha\).

### 3. Calculation of spin excitation stimulated by circularly polarized light

The spin density is calculated perturbatively to the first-order and the second-order in \(\mathcal{H}_R^\text{R}\) (Fig. 1). The contribution \(s^{(1)}\) defined by Fig.1(a) is given by

\[
s^{(1)} = -i\hbar \frac{1}{V} \text{Tr} [\sigma^x \sigma^y] \epsilon_{x\beta\gamma} \alpha \zeta (\mathcal{I}(\Omega) - \mathcal{I}(-\Omega)),
\]

where the function \(\mathcal{I}\) is

\[
\mathcal{I}^\eta(\Omega) = \frac{1}{2} \sum_{k,\omega} \left( \frac{\epsilon_\eta}{m_\Xi} \right)^2 \left[ k \cdot E \right] g_{k,\omega} g_{k,\omega + \Omega} g_{k + Q,\omega + \Omega} g_{k + Q,\omega} + \frac{\epsilon_\eta^2}{m_\Xi^2} E^\eta \left[ k \cdot E^* \right] g_{k,\omega} g_{k + Q,\omega + \Omega} g_{k + Q,\omega} + \frac{\epsilon_\eta^2}{m_\Xi^2} \left( E^\eta \right)^* \left[ k \cdot E^* \right] g_{k,\omega} g_{k,\omega + \Omega} g_{k + Q,\omega} + \frac{\epsilon_\eta^2}{m_\Xi^2} \left( E^\eta \right)^* \left[ k \cdot E^* \right] g_{k,\omega} g_{k,\omega + \Omega} g_{k + Q,\omega} \right).
\]

We note that \(\mathcal{I}^\eta(\Omega)\) is odd function with respect to \(k\), and \(\mathcal{I}^\eta(\Omega)\) in spatially isotropic system is zero in any frequency[4].

Next we consider the spin density contribution from the 2nd-order of the Rashba spin-orbit interaction. The contribution, \(s^{(2)}\), described defined by Fig. 1(b) is given by

\[
s^{(2)} = -\frac{2e^2}{V\hbar\Omega^2} \epsilon_{x\beta\gamma}(\alpha \times E)^\zeta \left[ K^\eta(\Omega) - K^\eta(-\Omega) \right],
\]
where a function $K^\eta$ is
\[
K^\eta(\Omega) = \sum_{k, \omega} \left( \frac{2 f_{\omega} k_\mu \epsilon_{\eta \mu \alpha b} E^*_\nu}{\partial \omega} \left( g^a [g^a g^d - c.c] + (g^a)^2 g^d - c.c \right) \right) + f_{\omega}(\alpha \times E^*)^\eta [(g^a)^2 g^d - c.c] + (f_{\omega+\Omega} - f_{\omega}) (\alpha \times E^*)^\eta [(g^a)^2 g^d - c.c].
\]

In low frequency regime, $\Omega \tau \ll 1$, we can expand the Green's functions as $g_{k, \omega + \Omega} = g_{k, \omega}(1 - \hbar \Omega g_{k, \omega} + (\hbar \Omega)^2 g_{k, \omega}^2 - (\hbar \Omega)^3 g_{k, \omega}) + o(\Omega^4)$, and we obtain
\[
K^\eta(\Omega) - K^\eta(-\Omega) = \frac{2}{5} \hbar^2 \Omega^3 \sum_{k, \omega} f''(\alpha \times E^*)^\eta [(g^a)^5 - c.c] + o(\Omega^5).
\]

Here, we have used the result:
\[
\sum_{k, \omega} (f_{\omega+\Omega} - f_{\omega}) + \frac{2 \hbar^2 k_\mu \epsilon_{\eta \mu \alpha b} E^*_\nu}{\partial \omega} [(g^a)^2 (g^d - g_d) - (g^d)^2] = +o(\Omega^4).
\]

The $k$-integral of Eq. (4) is calculated as $\sum_{k, \omega} f''(\alpha \times E^*)^\eta [(g^a)^5 - c.c] = i\frac{\pi \nu e}{128 \pi}$, where $\nu$ is the electron's energy density of state. Therefore we obtain the second order contribution of the spin density as
\[
\chi^{(2)} = i \frac{\pi \nu e^2}{32 \hbar^4 V} \hbar \Omega \alpha_\mu [E \times E^*]_\nu + o(\Omega^3).
\]

The spin density is thus obtained as
\[
\chi^{(2)} = i \chi_{\mu \nu} (E \times E^*)_\nu, \quad \chi_{\mu \nu} = \frac{\pi \nu e^2}{32 \hbar^4 V} \hbar \Omega \alpha_\mu \alpha_\nu,
\]

where $\chi_{\mu \nu}$ is a magnetic-optical susceptibility. The induced spin is proportional to the chirality of light, $E \times E^*$, and to the frequency linearly.

4. Discussion
From Eq. (6) and (7), the induced spin is proportional to the Rashba spin-orbit interaction, and the spin direction is independent on $E \times E^*$. Because the chirality of light, $E \times E^*$, is also parallel to light propagation($q$), the induced spin is also independent on $q$. We discuss the photoinduced magnetization of the inverse Faraday effect in the presence of Rashba spin-orbit interaction from Eq. (6) and (7). The magnetization is given by $M^\alpha = -\frac{\mu m}{2} s^\alpha$, and the magnetization is parallel to $\alpha$ (Fig. 2). For example, when the Rashba spin-orbit vector $\alpha$ is parallel to $z$-axis: $\alpha = (0, 0, \alpha_z)$, the magnetization is $M = (0, 0, M_z)$, where $M_z \sim \Omega \alpha_\nu |E|^2 \sigma^z \cos \theta$, $\sigma^\pm$ is the chirality of light, and $\theta$ is the angle between $z$-axis and light. $\sigma^\pm = \pm 1$ is determined by the left-hand or right-hand circularly polarized light, and then the induced magnetization is parallel and antiparallel for circularly polarization. This magnitude of magnetization is proportional to irradiation angle and frequency.

5. Conclusion
We have theoretically studied the spin density induced by the inverse Faraday effect in the presence of Rashba spin-orbit interaction in metal in the THz frequency regime. The spin is induced by the second order of Rashba spin-orbit interaction, and is proportional to frequency. The spin is parallel to the Rashba spin-orbit interaction and irrespective of the direction of light propagation. The induced magnetization is proportional to frequency and laser intensity.
Figure 2. The induced magnetization of inverse Faraday effect in Rashba system $\mathbf{\alpha} = (0, 0, \alpha_z)$. The magnetization is given by $M_z \sim \Omega \alpha_z^2 |E|^2 \sigma^\mp \cos \theta$. The magnetization is parallel to $\mathbf{\alpha}$ and chirality of light $\sigma^\mp = \pm 1$. The magnitude of magnetization is proportional to laser intensity, frequency, and the angle $\theta$ between $z$ axis and light.

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References
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