Self-interaction spin effects in inspiralling compact binaries

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Gravitational radiation drives compact binaries through an inspiral phase towards a final coalescence. For binaries with spin, mass quadrupole and magnetic dipole moments, various contributions add to this process, which is characterized by the rate of increase $df/dt$ of the gravitational wave frequency and the accumulated number $N$ of gravitational wave cycles. We present here all contributions to $df/dt$ and $N$ up to the second post-Newtonian order. Among them we give for the first time the contributions due to the self-interaction of individual spins. These are shown to be commensurable with the proper spin-spin contributions for the recently discovered J0737-3039 double pulsar, and argued to represent the first corrections to the radiation reaction in the Lense-Thirring approach.

I. INTRODUCTION

Neutron-stars and black-holes forming compact binary systems emit gravitational radiation. The frequency range of this radiation is expected to be in the sensitivity range of the Earth-based interferometric detectors LIGO \textsuperscript{1}, VIRGO \textsuperscript{2}, GEO \textsuperscript{3}, TAMA \textsuperscript{4} and currently observation is under way to capture such signals. Recently, a method of setting upper limits on inspiral event rates for binary neutron stars using interferometer data were established from the first scientific run of LIGO \textsuperscript{5} and limits on gravitational wave emission from selected pulsars using LIGO data were published \textsuperscript{6}. Also analysis methods were specified for inspiral signals from binaries with 3-20 solar masses \textsuperscript{7}. Gravitational radiation from these compact binaries also is expected to be detected by the Laser Interferometer Space Antenna (LISA) \textsuperscript{8}, \textsuperscript{9}. Parameters of the spinning compact binaries can be estimated and alternative theories of gravity can be tested \textsuperscript{10} from these measurements.

The final cataclysmic coalescence of such compact binaries is preceded by a milder inspiral phase, for which the post-Newtonian (PN) approach provides a reliable description. This description is generally considered valid until the system reaches the innermost stable circular orbit (ISCO). For neutron star binaries with realistic equation of state the gravitational wave frequency at ISCO is between $800 - 1230$ Hz, according to \textsuperscript{11, 12, 13}. The upper limit of sensitivity for the LIGO detector is $\sim 1000$ Hz. The simple estimate shows that for a neutron star of 1.4 solar masses the ISCO (at three Schwarzschild radii) is of about 2.5 times the characteristic radius of 10 km of the neutron star. A more sophisticated argument, taking into account various equations of state for the neutron stars \textsuperscript{12} shows $r_{\text{ISCO}}/r_{\text{NS}} \in (1.1, 2.2)$. Buonanno, Chen and Vallisneri have discussed the problem of the failure of the PN expansion during the last stages of inspiral (the intermediate binary black hole, IBBH problem \textsuperscript{13}) both for non-spinning and for spinning black hole binaries on quasicircular orbits \textsuperscript{14, 15}. Reliable results can be achieved by a frequency cutoff, e.g. stopping the integration at the minimum of the energy as function of orbital frequency, at ICO \textsuperscript{18} (called MECO in \textsuperscript{17}). In even latter stages of the inspiral, tidal torques become important \textsuperscript{19, 20}. Then each component can be represented by a Schwarzschild metric corrected with the influence of the Weyl curvature generated by the other component \textsuperscript{21, 22}. The corrected metric was computed to second order in the inverse of the Weyl curvature radius in \textsuperscript{23, 24} and to third order by Poisson \textsuperscript{25}.

In the post-Newtonian regime, the equations of motion were given to 3.5 PN order accuracy, with the inclusion of spin-orbit (SO) effects and their first PN correction in \textsuperscript{26}. Spin-spin (SS) \textsuperscript{27}, quadrupole-monopole (QM) \textsuperscript{28} and magnetic dipole - magnetic dipole (DD) contribution \textsuperscript{29} to the accelerations were also discussed. The backreaction on the orbit of the gravitational waves escaping compact binaries can be characterized by the rate of radiative change of otherwise conserved quantities, like the secular energy loss $\langle dE/dt \rangle$ and secular angular momentum loss $\langle dJ/dt \rangle$. These were computed over the years with great accuracy. The leading order contribution was given by \textsuperscript{30, 31}, its first post-Newtonian (PN) correction in \textsuperscript{32}, while the second post-Newtonian (2PN) correction in \textsuperscript{33} (a correction is expected, see \textsuperscript{34}).

Taking into account the interaction of the spins $S_1$ with the orbit, the total angular momentum $J = L + S_1 + S_2$ is still conserved, however the orbital angular momentum $L$ is not, due to spin precessions. Still, its magnitude $L$ remains a constant of motion, as consequence of the specific functional form of the precession equation \textsuperscript{35}. The radiative changes $\langle dE/dt \rangle$ and $\langle dJ/dt \rangle$ characterize the backreaction on the radial part of the motion. They were computed in \textsuperscript{35} and \textsuperscript{36} (these approaches rely on different spin supplementary conditions and distinct averaging techniques over a radial orbit - the results, however agree). The secular losses $\langle dE/dt \rangle$ and $\langle dL/dt \rangle$ depend on the relative angles $\kappa_i$ of the spins and orbital angular momentum. In order to have a closed system of differential equations their radiative evolution (together with the radiative evolution of the angle $\gamma$, subtended by the two spins) has been given in \textsuperscript{37}. For compact binaries the SO contribution occurs at $\epsilon^{1/2}$ order, $\epsilon$ representing...
one post-Newtonian order.

The SS contribution to \( \langle dE/dt \rangle \) and \( \langle dL/dt \rangle \) and the subsequent evolution of the angles \( \kappa_i \) and \( \gamma \) were derived in [32], [33]. Here a convenient method being kept in the discussion of the QM contribution [39] and the DD contribution [40]. (The magnitude \( L \) of the orbital angular momentum is not conserved due to the spin precessions caused by spin-spin, quadrupole-monopole and magnetic dipole - magnetic dipole interactions.) The SS, QM and DD contributions all arise at \( \epsilon^2 \) order.

Remarkably, the SS contributions in \( \langle dE/dt \rangle \) and \( \langle dL/dt \rangle \) given in [37] contained not only interaction terms between the two spins, but self-interaction spin terms (SS-self) as well. These arose from the terms proportional to \( J_{SO}^{3j} (a_N) J_{SO}^{3j} (a_N) \) in \( dE/dt \) and \( e^{ijk} J_{SO}^{2jl} (a_N) J_{SO}^{3kl} (a_N) \) in \( d\omega/dt \) (here \( J_{SO}^{(n)jl} \) denotes the \( n^{th} \) derivative of the spin-orbit contribution of the velocity quadrupole moment evaluated with the Newtonian acceleration \( a_N \); there is summation understood over the repeated indices \( j, k, l \). These terms are of order \( \epsilon^2 \) as well [41].

Although the SS-self contributions to the closed system of differential equations \( \langle d (E, L, \kappa_i, \gamma) \rangle /dt \) has been given in [35], [38], these terms are notoriously missing from the bookkeeping of different contributions to various results concerning inspiralling compact binaries. In this paper we compute the SS-self contributions to the rate of increase of the gravitational wave frequency \( f \) and to the accumulated number of gravitational wave cycles \( N \). For completeness we enlist all other contributions to \( df/dt \) and \( N \) to order \( \epsilon^2 \) (the PN, SO, SS, QM, DD, 2PN and tail contributions).

On the short run due to the emission of gravitational waves the orbit tends to circularize [30]. Therefore we consider quasicircular orbits, for which the gravitational wave frequency is twice the orbital frequency [42]. In Sec.II first we evaluate the rate of increase of \( f \). This is given by the rate of change of the orbital angular frequency \( \omega = \pi f \) under radiation reaction:

\[
\frac{d\omega}{dt} \bigg| \text{circ} = \left( \frac{dE}{d\omega} \right)_{N}^{-1} \left( \frac{dE}{dt} \right)_{\text{circ}} ,
\]

where the expression \( dE/d\omega \) can be found by differentiating \( E = E(\omega) \). We have verified that the circular orbit limit of the instantaneous energy loss \( \langle dE/dt \rangle_{\text{circ}} \) is the same as the circular orbit limit of the secular energy loss \( \langle dE/dt \rangle_{\text{circ}} \), the latter however is much simpler to compute. Here the various contributions to the secular energy loss

\[
\left\langle \frac{dE}{dt} \right\rangle = \left\langle \frac{dE}{dt} \right\rangle_{N} + \left\langle \frac{dE}{dt} \right\rangle_{PN} + \left\langle \frac{dE}{dt} \right\rangle_{SO+tail} + \left\langle \frac{dE}{dt} \right\rangle_{2PN+(SS-self)+S_{1}S_{2}+QM+DD}.
\]

II. FREQUENCY EVOLUTION AND THE ACCUMULATED NUMBER OF CYCLES

For circular orbits \( \dot{r} = \ddot{r} = 0 \) and \( v^2 = m\omega \) holds. Consequently \( (m\omega)^{2/3} \) is of order \( \epsilon \). The radial projection of the acceleration defines the orbital angular velocity as \( r \cdot a = -r^2 \omega^2 \) [27]. From the explicit form of the acceleration (with various contributions given in [27], [28] and [29]) we then find \( \omega = \omega(r) \) and subsequently \( r = r(\omega) \)
we obtain

\[
\frac{d\omega}{dt} \bigg|_{\text{circ}} = \frac{96\eta m^5/3}{5} \omega^{11/3} \left[ 1 - \frac{743}{336} + \frac{11}{4} \eta \right] (m\omega)^{2/3} \\
+ (4\pi - \beta) m\omega + \frac{34103}{18144} + 13661 \eta \\
+ \frac{59}{18} \eta^2 + \sigma \right] (m\omega)^{4/3} ,
\]

where

\[
\sigma = \sigma_S S + \sigma_{SS-self} + \sigma_{QM} + \sigma_{DD} .
\]

The quantities \(\beta, \sigma_S S, \sigma_{SS-self}, \sigma_{QM}\) and \(\sigma_{DD}\) are the spin-orbit, spin-spin, self-interaction spin, quadrupole-monopole and magnetic dipole-dipole parameters, respectively:

\[
\beta = \frac{1}{12} \sum_{i=1}^{2} \frac{S_i}{m_i^2} \left( 113 \frac{m_i^2}{m^2} + 75\eta \right) \cos \kappa_i ,
\]

\[
\sigma_S S = \frac{S_1 S_2}{48\eta m^4} \left( -247 \cos \gamma + 721 \cos \kappa_1 \cos \kappa_2 \right) ,
\]

\[
\sigma_{SS-self} = \frac{1}{96 m^2} \sum_{i=1}^{2} \left( \frac{S_i}{m_i} \right)^2 (6 + \sin^2 \kappa_i) ,
\]

\[
\sigma_{QM} = \frac{5}{2} \sum_{i=1}^{2} p_i \left( 3 \cos^2 \kappa_i - 1 \right) ,
\]

\[
\sigma_{DD} = -\frac{5}{2} \frac{\eta m}{\eta m^4} d_1 d_2 A_0 ,
\]

The N, PN, SO, SS, 2PN and tail contributions to Eq. (4) were verified to agree with those given in [44], the QM with those in [28] and the DD with those in [29], respectively. We emphasize the SS-self contribution, given for the first time.

Eq. (7) is an ordinary differential equation in \(\omega\), provided all angular variables can be considered constants. To linear order in the perturbations this is true. This is because all angles appear only in the perturbative corrections, therefore they are given with sufficient accuracy to \(\epsilon\) order, in which order they are constants [28]. Thus the time-evolution of the orbital angular frequency \(\omega\) for circular orbits can be easily deduced by an integration over time:

\[
\omega(t) = \frac{\tau^{-3/8}}{8m} \left[ 1 + \left( \frac{743}{2688} + \frac{11}{32} \eta \right) \tau^{-1/4} \\
+ \frac{3}{10} \left( \frac{\beta}{4} - \pi \right) \tau^{-3/8} + \left( \frac{1855099}{14450688} + \frac{56975}{258048} \eta \right) \tau^{-1/2} \right] ,
\]

where the dimensionless time variable \(\tau = \eta(t_c - t)/5m\) is related to the time \((t_c - t)\) left until the final coalescence. A further integration gives the accumulated orbital phase \(\phi_c - \phi\):

\[
\phi_c - \phi = \frac{5m}{\eta} \int \omega(\tau) d\tau .
\]
From here the accumulated number of gravitational wave cycles emerges as

\[
\mathcal{N} = \frac{\phi_e - \phi}{\pi} = \frac{1}{\pi \eta} \left\{ \chi^{5/8} + \left( \frac{3715}{8064} + \frac{55}{96} \right) \chi^{3/8} + \frac{3}{4} \left( \frac{\beta}{4} - \pi \right) \tau^{1/4} + \frac{9275495}{14450688} + \frac{284875}{258048} \eta \right. \\
\left. + \frac{1855}{2048} \left( \frac{15\sigma}{64} \right) \chi^{1/8} \right\} \tag{12}
\]

The tail and 2PN contributions agree with those given in [45], the rest of the terms with Eq. (4.16) of [27]. We also note that the numerical values in Table II of [10] could be recovered from our Eq. (12).

In Table I we enlist all contributions to \( \mathcal{N} \) in terms of \( \beta \) and \( \sigma \), evaluated numerically from our Eq. (12) for several compact binaries. These are two well-known binary neutron star systems, the newly discovered double pulsar J0737-3039 ([46], [47], and for a recent review see [48]), the Hulse-Taylor pulsar B1913+16, one neutron star - stellar mass black hole binary, one binary composed of two stellar mass black holes [49], finally two examples of galactic black hole - galactic black hole binaries [11].

### III. CONCLUDING REMARKS

We have presented the complete set of contributions up to \( \epsilon^2 \) order (PN, SO, SS, QM, DD, tail and 2PN) to the evolution of gravitational wave frequency and to the accumulated number of gravitational wave cycles, with the inclusion of the previously unknown self-interaction spin terms. These results add to the already derived [35], [37]-[40] closed system of first order differential equations governing the secular evolution of radiating compact binaries and represent an important step towards a complete characterization of the orbital evolution.

| PN Order | J0737-3039 | B1913+16 | BDIWW1 | BDIWW2 | BBW1 | BBW2 |
|----------|------------|----------|--------|--------|------|------|
|          | 1.337M⊙  | 1.387M⊙ | 1.4M⊙ | 10M⊙  | 10^4M⊙| 10^5M⊙|
|          | 1.25M⊙  | 1.441M⊙ | 10M⊙  | 10M⊙  | 10^5M⊙| 10^7M⊙|
| \( f_{in}(Hz) \) | 10 | 10 | 10 | 10 | 4.199 x 10^{-4} | 1.073 x 10^{-5} |
| \( f_{fin}(Hz) \) | 1000 | 1000 | 360 | 190 | 3.997 x 10^{-2} | 2.199 x 10^{-3} |
| \( N \) | 18310 | 15772.1 | 3580 | 600 | 21058 | 535 |
| \( PN \) | 475.8 | 435 | 212 | 59 | 677 | 55 |
| \( SO \) | 17.5\( \beta \) | 16.5\( \beta \) | 14\( \beta \) | 4\( \beta \) | 36\( \beta \) | 4\( \beta \) |
| \( SS - self, SS, QM, DD \) | -2.1\( \sigma \) | -2.1\( \sigma \) | -3\( \sigma \) | -\( \sigma \) | -5\( \sigma \) | -\( \sigma \) |
| \( Tail \) | -208 | -206 | -180 | -51 | -450 | -48 |
| 2PN | 9.8 | 9.5 | 10 | 4 | 18 | 4 |
found to be comparable with the proper spin-spin contributions. This happens in spite of the fact that the numerical factors in $\sigma_{SS-self}$ are two orders of magnitude smaller than those in $\sigma_{1,2}$, cf. Eqs. (9b) and (9c). The commensurability of the proper and self SS contributions is due to the fact that one of the spins is two orders of magnitude larger than the other.

Second, consider Solution 2, for which Eq. (9b) gives $\sigma_{1,2} = 0$. (We have checked that even at the limits $\kappa_1 = 80^\circ$ and $100^\circ$ of the allowed domain in Solution 2, the proper spin-spin contribution is still one order of magnitude smaller than the self-interaction contribution.) We conclude, that whenever one of the spins is aligned with the orbital angular momentum and the other spin is perpendicular to it, the proper spin-spin contribution vanishes. This happens to be the case for Solution 2 of the double pulsar J0737-3039. Thus $\sigma_{SS-self}$ is the only spin-spin contribution in this case. We also remark that for Solution 2 the self-interaction spin contribution is only two orders of magnitude smaller than the spin-orbit contribution. This is because only the smaller spin contributes to $\beta$.

The importance of the self-interaction spin parameter is in gravitational wave astronomy, rather than in the traditional electromagnetic observations. In [10] the phasing function $\Psi(f)$ was derived up to 2PN order. The coefficient $\sigma$ there denotes the proper spin-spin parameter. However as demonstrated here, the additional self-spin contributions given by $\sigma_{SS-self}$ is of the same order of magnitude, thus it should be also included (together with $\sigma_{GM}$ and $\sigma_{DD}$, for high mass quadrupole and magnetic dipole moments, respectively).

The self spin-spin contributions derived in this paper are more important whenever one of the spins is negligible compared the other. To leading order in the (dominant) spin and when the mass of the high-spin component also dominates over its companion’s, this is the Lense-Thirring approximation. Its first correction in the gravitational radiation is represented exactly by the SS-self contribution of the higher spin.

| Spin parameter (order) | Solution 1 | Solution 2 |
|------------------------|------------|------------|
| $\beta$                | $-0.166$   | $0.001$    |
| $\sigma_{1,2} (10^{-4})$ | $-0.372$   | $0$        |
| $\sigma_{SS-self} (10^{-4})$ | $0.298$    | $0.345$    |

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