Radiogenic power and geoneutrino luminosity of the Earth and other terrestrial bodies through time

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Abstract

We report the Earth’s rate of radiogenic heat production and (anti)neutrino luminosity from geologically relevant short-lived radionuclides (SLR) and long-lived radionuclides (LLR) using decay constants from the geological community, updated nuclear physics parameters, and calculations of the β spectra. We carefully account for all branches in K decay using the updated β energy spectrum from physics and an updated branching ratio from geological studies. We track the time evolution of the radiogenic power and luminosity of the Earth over the last 4.57 billion years, assuming an absolute abundance for the refractory elements in the silicate Earth and key volatile/refractory element ratios (e.g., Fe/Al, K/U, and Rb/Sr) to set the abundance levels for the moderately volatile elements. The relevant decays for the present-day heat production in the Earth (19.9 ± 3.0 TW) are from K, Rb, Sm, Th, U, and U. Given element concentrations in kg-element/kg-rock and density ρ in kg/m, a simplified equation to calculate the heat production in a rock is:

\[ h \ [\mu W/m] = \rho (3.387 \times 10^6 [K] + 0.01139 [Rb] + 0.04607 [Sm] + 26.18 [Th] + 98.29 [U]) \]

The radiogenic heating rate of earth-like material at Solar System formation was some 10 to 100 times greater than present-day values, largely due to decay of Al in the silicate fraction, which was the dominant radiogenic heat source for the first ~10 My. Decay of Fe contributed a non-negligible amount of heating during the first ~15 My after CAI (Calcium Aluminum Inclusion) formation, interestingly within the time frame of core-mantle segregation.
Radiogenic power and geoneutrino luminosity of the Earth and other terrestrial bodies through time

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Key Points:

• Radiogenic heat production and geoneutrino luminosity calculated over the age of the Earth
• Simple formulae proposed for evaluation at arbitrary planetary composition
• Differences in radioactive decay parameters highlighted between nuclear physics and geological communities

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Abstract
We report the Earth’s rate of radiogenic heat production and (anti)neutrino luminosity from geologically relevant short-lived radionuclides (SLR) and long-lived radionuclides (LLR) using decay constants from the geological community, updated nuclear physics parameters, and calculations of the $\beta$ spectra. We carefully account for all branches in $^{40}$K decay using the updated $\beta^-$ energy spectrum from physics and an updated branching ratio from geological studies. We track the time evolution of the radiogenic power and luminosity of the Earth over the last 4.57 billion years, assuming an absolute abundance for the refractory elements in the silicate Earth and key volatile/refractory element ratios (e.g., Fe/Al, K/U, and Rb/Sr) to set the abundance levels for the moderately volatile elements. The relevant decays for the present-day heat production in the Earth (19.9 ± 3.0 TW) are from $^{40}$K, $^{87}$Rb, $^{147}$Sm, $^{232}$Th, $^{235}$U, and $^{238}$U. Given element concentrations in kg-element/kg-rock and density $\rho$ in kg/m$^3$, a simplified equation to calculate the heat production in a rock is:

$$h [\mu W m^{-3}] = \rho \left(3.387 \times 10^{-3} [K] + 0.01139 [Rb] + 0.04607 [Sm] + 26.18 [Th] + 98.29 [U]\right)$$

The radiogenic heating rate of earth-like material at Solar System formation was some $10^3$ to $10^4$ times greater than present-day values, largely due to decay of $^{26}$Al in the silicate fraction, which was the dominant radiogenic heat source for the first ~ 10 My. Decay of $^{60}$Fe contributed a non-negligible amount of heating during the first ~ 15 My after CAI (Calcium Aluminum Inclusion) formation, interestingly within the time frame of core–mantle segregation. Using factors and equations presented here, one can calculate the first-order thermal and (anti)neutrino luminosity history of various size bodies in the solar system and exoplanets.

Plain Language Summary

The decay of radioactive elements in planetary interior’s produces heat that drives the dynamic processes of convection (core and mantle), melting and volcanism in rocky bodies in the solar system and beyond. Uncertainties in the decay constants for elements with $10^5$ to $10^{11}$ half-lives range from 0.2% to ~ 4% absolute and about 1% to 4% relative when comparing data sources in physics and geology. These differences, combined with uncertainties in Q (heat of reaction) values, lead to diverging results for heat production and for predictions of the amount of energy removed from the rocky body by emitted (anti)neutrinos.

1 Introduction

Radioactive decay inside the Earth produces heat, which in turn contributes power to driving the Earth’s dynamic processes (i.e., mantle convection, volcanism, plate tectonics, and potentially the geodynamo). The physics community, using the latest numbers from nuclear physics databases, provide estimates of the radiogenic power and geoneutrino luminosity of the Earth (Dye, 2012; Ruedas, 2017; Usman et al., 2015; Enomoto, 2006a; Fiorentini et al., 2007). These studies include comprehensive reviews of the fundamental physics of these decay schemes, covering both the energy added to the Earth and that removed by the emitted geoneutrino. This note draws attention to differences in decay constants as reported in the geological and physics literature and recommends the former as being more accurate and precise. The absolute accuracy of geological studies is underpinned by the $^{235}$U decay constant (Jaffey et al., 1971) and their relative accuracies are based on multiple cross-calibrations for different decay systems on the same rocks and mineral suites. Improvements in measurement precision comes from repeated chronological experiments.

There are a number of naturally occurring short-lived (relative to the Earth’s age; half-lives $t_{1/2} < 10^3$ years) and long-lived ($t_{1/2} > 10^5$ years) radionuclides; those dis-
cussed here have half-lives between $10^5$ and $10^{11}$ years. The long-lived decay constants are listed in Table 1 along with their decay modes and decay energies. The decay modes include alpha ($\alpha$), beta-minus ($\beta^-$), and electron capture (EC). The beta-plus ($\beta^+$) decay mode is less common, but is seen in the $^{26}$Al system, as well as a few minor branches in the Th and U decay chains and also likely in the $^{40}$K branched decay. Geoneutrinos are naturally occurring electron antineutrinos ($\bar{\nu}_e$) produced during $\beta^-$ decay and electron neutrinos ($\nu_e$) produced during $\varepsilon$ (i.e., $\beta^+$ and EC) decays. The generic versions of these decay schemes are:

$$\text{Alpha (}\alpha\text{)} \quad \frac{1}{2} X \rightarrow \frac{1}{2} X' + \alpha + Q,$$
$$\text{Beta Minus (}\beta^-\text{)} \quad \frac{1}{2} X \rightarrow \frac{1}{2} X' + e^- + \bar{\nu}_e + Q,$$
$$\text{Electron Capture (EC)} \quad \frac{1}{2} X + e^- \rightarrow \frac{1}{2} X' + \nu_e + Q,$$
$$\text{Beta Plus (}\beta^+\text{)} \quad \frac{1}{2} X \rightarrow \frac{1}{2} X' + e^+ + \nu_e + Q,$$

with parent element $X$, daughter element $X'$, mass number $A$, atomic number $Z$, energy of reaction $Q$, electron $e^-$, positron $e^+$, and alpha particle $\alpha$ ($^3$He nucleus).

We report here radiogenic heat production and (anti)neutrino luminosity from geologically relevant short-lived radionuclides (SLR) and long-lived radionuclides (LLR). For the LLR we compare half-lives used in the geological and nuclear physics communities and recommend use of the former. We calculate the heat added to the Earth by these nuclear decays, as well as the energy carried away by (anti)neutrinos that leave the Earth. We calculate estimates of the embedded and removed energy of decay, particularly for the SLR, from $\beta$ decay spectra calculated using Fermi theory and shape factor corrections. We conclude by presenting models for the Earth’s radiogenic power and geoneutrino luminosity for the last 4568 million years, along with simple rules for extrapolating these results to other terrestrial bodies and exoplanets.

2 Contrasting methodologies

In compiling the data needed to calculate all of the observables, we found differences between the decay constants ($\lambda = \ln 2/\tau_{1/2}$) reported by the geological and nuclear physics communities. Values for extant systems are provided in a side-by-side comparison in Table 2. The rightmost column reports the relative difference, in percent, between the decay constants from these communities and for some, the difference can be considerable (more than 30%). An updated physics number for the half-life of $^{190}$Pt reported in Braun et al. (2017) agrees with the numbers obtained by Cook et al. (2004), who presented a detailed study of a suite of well behaved (closed system evolution), 4.5 billion year old, iron meteorites (i.e., group IIAB and IIIAB).

There is a 1.1% difference in the decay constant for $^{40}$K between literature sources, which is a nuclide that provides ~20% of the planet’s present-day radiogenic heat and ~70% of its geoneutrino luminosity (see Table 3). This difference is outside of the uncertainty limits on the half-life of $^{40}$K, recently established by geochronologists (Renne et al., 2011).

Differences in decay constants reported by the geological and nuclear physics communities come from the methods used to establish the absolute and relative half-lives. Physics experiments typically determine a half-life value by measuring the activity $A = -dN/dt = \lambda N$ (N is the number of atoms) of a nuclide over time, whereas geochronology studies empirically compare multiple decay systems for a rock or suite of rocks that demonstrate close system behavior (show no evidence of loss of parent or daughter nuclide). The number of atoms $N$ of parent nuclide evolves according to $N = N_0 e^{-\lambda t}$,
therefore \( \ln N = \ln N_0 - \lambda t \). A plot of \( \ln N \) (ordinate) vs. \( t \) (abscissa) gives a line of slope \(-\lambda\) with y-intercept equal to \( \ln N_0 \).

Direct counting experiments generally involve the isolation of a pure mass of the parent nuclide of interest, knowing exactly the number of parent atoms at the start of the experiment, and then determining the ingrowth of daughter atoms produced at one or more times later (Begemann et al., 2001). Geochronological experiments compare multiple chronometric methods (e.g., U–Pb and K–Ar systems (Renne et al., 2011)) and develop a series of cross calibrations, where the shortcoming of this approach is the anchoring decay system that pins down the accuracy for other chronometers. It is recognized (Begemann et al., 2001; Villa et al., 2015; Ruedas, 2017) that the half-life of \(^{238}\)U (Jaffey et al., 1971) is the most accurately known of the decay constants and thus acts as the anchor in these calculations. Table 2 highlights the differences in half-life values reported in a standard physics reference source NNDC (National Nuclear Data Center, http://www.nndc.bnl.gov) and geology. Relative differences at the \(\sim 1\%\) scale and greater are seen for \(^{40}\)K, \(^{87}\)Rb, \(^{176}\)Lu, \(^{187}\)Re and \(^{190}\)Pt decay systems.

Radioactive decay involves the transition to a lower level energy state of a nuclear shell and the accompanied release of energy, requiring the conservation of energy, linear and angular momenta, charge, and nucleon number. The kinetic energies of emitted alpha particles are discrete and on the order of 4 to 8 MeV, whereas different forms of beta decay show a continuous spectrum with characteristic mean and maximum energies for a given decay and the (anti)neutrino carrying away a complementary part of the energy. The energy of the beta decay process is partitioned between the electron, the antineutrino (or positron and neutrino), and the recoiling nucleus. Differences in heat production per decay reported in different studies are largely due to differences in decay energies (minimal differences), the energy carried off by (anti)neutrinos, and the branching fractions in the case of branched decays (large differences for the latter two). Furthermore, the time rate of heat production is sensitive to the value of the decay constant. This study differs from other recent efforts (Dye, 2012; Ruedas, 2017; Usman et al., 2015; Enomoto, 2006a; Fiorentini et al., 2007) in its input assumptions; we use decay constants and branching fractions from geochronological studies and we calculate the beta decay energy spectrum for most of the SLR and \(^{40}\)K decays. For the remaining LLR decays, we adopt the energy spectra from Enomoto (2006b).

The \(^{40}\)K decay scheme is a good example of where differences in inputs occur. Many naturally occurring decay schemes have a single decay mode, whereas \(^{40}\)K is a branch decay scheme with \(\beta^-\) and \(\varepsilon\) decays (see Figure 1), with emission of an \(\overrightarrow{\nu}_e\) and \(\nu_e\), respectively, removing energy from the Earth. The amount of radioactive heating in the Earth from this branch decay scheme depends on the branching ratio and the energy carried by the \(\nu_e\) and \(\overrightarrow{\nu}_e\). Using only geological data, Naumenko-D`ezes et al. (2018) examined the \(^{40}\)K decay system and report a probability for the \(\beta^-\) branching between 89.25% and 89.62% and for the \(\varepsilon\) branching between 10.38% and 10.75%. They highlight that the errors on these values are non-Gaussian. The physics community reports the branching probabilities of \(\beta^-\) as 89.28(11)% and of \(\varepsilon\) as 10.72(11)% (Chen, 2017). Figure 1 reports the updated \(^{40}\)K decay scheme—the branching fractions, the average energies removed by the antineutrinos and neutrinos, and the energy deposited by these decays.

Beta decay involves transforming a quark state in the nucleus and emission of a pair of fermions \((e^- \overrightarrow{\nu}_e\) or \(e^+ \nu_e\)), where each fermion has an intrinsic angular momentum (or “spin”) of 1/2. The decay satisfies all the relevant conservation principles of particle physics, including the electron-lepton number \((L_e)\) conservation, where \(L_e = 1\) for matter particles \((e^- \overrightarrow{\nu}_e)\) and \(L_e = -1\) for antimatter particles \((e^+ \nu_e)\). The transformation is accompanied by a change in the total angular momentum \((\Delta I)\) which, by conservation of angular momentum, must be reflected in the state of the \(e\nu\) pair, that is, the total orbital angular momentum \((L)\) and the total spin angular momentum \((S^Z)\) of the \(e\nu\) pair. Beta decays can be pure Fermi transitions, pure Gamow-Teller
transitions, or a combination of both. In Fermi transitions the spins of the emitted leptons are anti-parallel, $S_L^L = 0$, and therefore $\Delta I$ is matched only by $\pm L$. In Gamow-Teller transitions the spins of the emitted leptons are aligned, i.e., $S_L^L = 1$, and coupled to the change in nuclear angular momentum state $\Delta I$ together with $L$: $\Delta I = \pm |L\pm 1|$. The so-called “unique” transitions are Gamow-Teller transitions where $L$ and $S_L^L$ are aligned, $\Delta I = \pm |L + 1|$. Typically, transitions with a higher $L$ have a longer half-life ($t_{1/2}$), because of less overlap of the $\nu\nu$ wave functions with the nucleus. Transitions with a non-zero $L$ are called “forbidden” (as opposed to “allowed” for $L = 0$), which really means suppressed decays that involve changes in nuclear spin state; in $n$-th forbidden transition the $\nu\nu$ pair carries $n$ units of orbital angular momentum (Bielajew, 2014). For example, the $^{40}$K decay scheme involves a third unique forbidden transition, whereas the $^{87}$Rb decay scheme involves a third non-unique forbidden transition.

Following Fermi’s theory and working in units $\hbar = m_e = c = 1$, the shape of a $\beta$ spectrum is calculated from

$$\frac{dN}{dw} \propto pwq^2 F(Z, w) S(w)$$

(2)

and normalized to the branching fraction of the specific $\beta$ decay (Enomoto, 2005).

In equation (2) $w = 1 + E$ is the total energy of the $\beta$-particle ($E$ being its kinetic energy), $p = \sqrt{w^2 - 1}$ is the momentum of the $\beta$-particle, $q$ is the total energy of the neutrino (equal to its momentum as the neutrino mass is negligible) satisfying $E + q = E_{end}$, where $E_{end}$ is the endpoint energy of the transition (in the case of a transition to ground state, it is the $Q$-value), and $Z$ is the charge of the daughter nucleus. The left-hand side of equation (2) is the probability of a $\beta$ particle to be created with energy in the $dw$ vicinity of $w$, where $w$ goes from 1 to $1 + E_{end}$. The right-hand side is a product of three factors, the phase space factor $pwq^2$, the Fermi function $F(Z, w)$, and the shape factor $S(w)$.

The Fermi function

$$F(Z, w) \propto (w^2 - 1)^{\gamma - 1} e^{\pi \eta |\Gamma(\gamma + i\eta)|^2},$$

(3)

where

$$\gamma = \frac{\sqrt{1 - (\alpha Z)^2}}{\alpha Z w},$$

$$\eta = \frac{\alpha Z w}{\sqrt{w^2 - 1}},$$

(4)

(5)

$\alpha$ being the fine-structure constant, accounts for the Coulombic interaction between the daughter nucleus and the outgoing $\beta$-particle (Enomoto, 2005). The shape factor $S(w)$, often written at $S(p, q)$, is equal to 1 for allowed transitions and has a more complex energy-dependence in the case of forbidden transitions.

A review of many $\beta^-$ decay energy spectra was recently given by Mougeot (2015), including the shape factors used for the forbidden transitions. We adopt these shape factors in our calculations, but also include additional $\beta$ decays not studied by Mougeot (2015); the shape factors used here are listed in Table 4. We have performed the $\beta$ spectra evaluation and calculated the average energy removed by the $\nu_e$ and $\nu_x$, which are reported as $Q_{\nu}$ (MeV) in Table 3 and can be calculated from $Q = Q_b$ in Table 4.

3 Radiogenic heat and geoneutrino luminosity of the Earth

Using decay constants for short-lived and long-lived radionuclides and $^{40}$K branching ratio from the geological literature we calculate the heat production and geoneutrino
luminosity of the bulk silicate Earth (BSE) based on a model composition (Tables 3 and 4 and references therein). Compositional models differ on the absolute amount of refractory elements (e.g., Ca and Al) in the Earth (see review in W. F. McDonough (2016)), which includes La, Sm, Lu, Re, Pt, Th, and U. The model composition for the BSE fixes the absolute abundances of the refractory elements at 2.75 times that in CI1 chondrites (W. F. McDonough & Sun, 1995). For critical volatile elements, there is a reasonable consensus for ratios with refractory elements. For example, Arevalo Jr. et al. (2009) reported the K/U value for the silicate Earth as 13, 800 ± 1, 300 (1 standard deviation).

Constraints for Rb come from the constancy of the Ba/Rb and and Sr–Nd isotopic system (assumes the BSE has an $^{87}\text{Sr}/^{86}\text{Sr}$ between 0.7040 and 0.7060, based on the mantle array (Hofmann, 2007)) and the Rb/Sr values (Ba and Sr are refractory elements with abundances set at 2.75 times that in CI1 chondrites) for the bulk silicate Earth, leading to a Rb/Sr of 0.032 ± 0.007 (W. F. McDonough et al., 1992).

Heat production and geoneutrino emission data for $^{40}\text{K}$, $^{87}\text{Rb}$, $^{147}\text{Sm}$, $^{232}\text{Th}$, $^{235}\text{U}$, and $^{238}\text{U}$ are reported in Table 3, as these are the most significant present-day producers within the Earth. In fact, 99.5 % of the Earth’s radiogenic heat production comes from $^{40}\text{K}$, $^{232}\text{Th}$, $^{235}\text{U}$, and $^{238}\text{U}$ alone. The fractional contributions to heat production from $^{138}\text{La}$, $^{176}\text{Lu}$, $^{187}\text{Re}$, and $^{190}\text{Pt}$ add up to < $3\times10^{-5}$ of the total radiogenic heat and 1 % of the Earth’s geoneutrino luminosity, with virtually all of this latter minor contributions coming from $^{187}\text{Re}$. Figure 2 illustrates the present day relative contributions of heat production and geoneutrino luminosity from the major radionuclides reported in Table 3.

A simple formula for the present-day radiogenic heating rate $\tilde{h}$ (in nanowatts per kilogram of rock) from long-lived radionuclides is presented in equation (6), where $A$ is elemental concentration as mass fraction (kg-element/kg-rock; e.g., [K] is mass fraction of potassium), and the remaining parameters combine into numerical factors whose values are set ($N_A$ is Avogadro’s number, $X$ is natural molar isotopic fraction, $\mu$ is molar mass of element, $\lambda$ is decay constant, $Q_h$ is radiogenic heat released per decay). Multiplying the mass of the geochemical reservoir of interest $M_{\text{res}}$ (to which the elemental concentrations apply), one gets the total radiogenic power $H$ (in terawatts) in that reservoir as shown in equation (7). Similarly, the natural specific antineutrino and neutrino luminosities $\tilde{l}$ (in number of particles per second per kilogram of rock) are calculated from equations (8) and (9). Multiplication with a reservoir mass gives the total luminosities $L_{\tilde{\ell}}$ and $L_{\tilde{\ell}}$ (equation 10; contributions from individual elements listed in Table 3).

$$\tilde{h} \ [\text{nw} \text{kg}^{-1}] = \sum_{\text{LLRs}} \frac{N_A X \lambda Q_h}{\mu} A = 3.387 [\text{K}] + 11.39 [\text{Rb}] + 46.07 [\text{Sm}] + 26180 [\text{Th}] + 98293 [U] \quad (6)$$

$$H \ [\text{TW}] = \tilde{h} \times M_{\text{res}} \times 10^{-21} \quad (7)$$

$$\tilde{l}_{\ell} \ [\text{s}^{-1} \text{kg}^{-1}] = \sum_{\text{LLRs}} \frac{N_A X \lambda n_{\ell}}{\mu} A = (2.797 [\text{K}] + 86.82 [\text{Rb}] + 1617 [\text{Th}] + 7636 [U]) \times 10^4 \quad (8)$$

$$\tilde{l}_{\ell} \ [\text{s}^{-1} \text{kg}^{-1}] = \sum_{\text{LLRs}} \frac{N_A X \lambda n_{\ell}}{\mu} A = 0.3302 [K] \times 10^4 \quad (9)$$

$$L \ [\text{s}^{-1}] = \tilde{l} \times M_{\text{res}} \quad (10)$$

To understand the evolution of the Earth’s radiogenic heat and geoneutrino luminosity we must understand the initial starting abundances of the SLR in the solar system (listed in Table 4). At 4.57 Ga the local interstellar medium was populated with gas-dust clouds that were likely in secular equilibrium with ambient galactic sources prior to solar system formation. Recent calculations by Wasserburg et al. (2017) demonstrate that the proportional inventory of $^{26}\text{Al}$, $^{60}\text{Fe}$, $^{107}\text{Pd}$, and $^{182}\text{Hf}$ in the early solar system.
is unlikely to be a product from an asymptotic giant branch (AGB) star. Moreover, supernova sources would likely provide abundant $^{26}$Al and $^{60}$Fe, whereas the early solar system content of $^{60}$Fe is equivalent to the mealy ambient galactic supply (Trappitsch et al., 2018). More recent suggestions envisage stellar winds from a massive Wolf-Rayet star injecting $^{26}$Al to complement the local inventory of ambient galactic sources (Young, 2014; Gounelle, M. & Meynet, G., 2012; Dwarkadas et al., 2017). At the same time, the enhanced abundance of $^{53}$Mn and the presence of very short half life isotopes (e.g., $^{41}$Ca $t_{1/2} = 0.1$ Ma) present challenges to be explained by models invoking Wolf-Rayet stars (Vescovi et al., 2018). Thus, the addition of mass and momentum from such a stellar source could cause a gravitational collapse of a molecular gas-dust cloud, which may have triggered our solar system formation and explain the observed proportions of short-lived radionuclides.

The total heat production and geoneutrino luminosity for models of the BSE are plotted with respect to time in Figure 3, which were calculated using results from Tables 3 and 4 and updated values for the BSE (W. F. McDonough & Sun, 1995; Arevalo Jr. et al., 2009; Wipperfurth et al., 2018) and equations (6–10). This figure presents a simple illustrative example of the Earth’s heat production and geoneutrino luminosity that assumes full mass at 1 million years after after solar system initiation.

The uncertainties for the BSE abundances reported in Table 3 are ±10% for K and the refractory lithophile elements (Wipperfurth et al., 2018), with correlations between K, Th and U. Using this Earth model, the present day’s fluxes are 19.9±3.0 TW (terawatts or $10^{12}$ watts) for radiogenic heat and the total geoneutrino luminosity is $(4.91±0.75)\times10^{25} (\bar{\nu}_e + \bar{\nu}_\mu) s^{-1}$. The results shown in Figure 3 are directly scalable for different size planetary bodies with a bulk Earth composition; lowering the mass of a planet by a factor of 10 results in a decease by a factor of 10 in the heat production and (anti)neutrino luminosity. The most important factors are the amount of refractory elements and the volatility curve for the planet. The Earth has an Fe/Al value of 20±2 (W. F. McDonough & Sun, 1995; Allègre et al., 1995), comparable to the chondritic ratio, which is 19±4 (less the 35 value for EH chondrites). The Fe/Al value sets the proportion of refractory elements (Al) to one of the 4 major elements (i.e., O, Fe, Mg and Si) that make up ∼93% of the mass of a terrestrial planet. These latter elements are not in fixed chondritic proportions, as is the case for the refractory elements, thus, the mass proportion of O, Fe, Si and Mg can be approximated as 30:30:20:20 (or 50:15:15:15 for atomic proportions), respectively, with proportional differences leading to variations in the metal/silicate mass fraction and fraction of olivine (Mg$_2$SiO$_4$) to pyroxene (MgSiO$_3$) in the silicate shell. A K/U or K/Th value sets the volatile depletion curve for the planet. Using $h/A$ factors given in Table 4 and equations (6) and (7), one can model the thermal history of various size bodies in the solar system and exoplanets.

We can also compare these results for the present-day radiogenic power and geoneutrino flux versus that reported in the literature. In our comparison and where possible, we used the abundances and masses reported in Table 3 to carry out these comparisons. The calculated BSE radiogenic power in the models of Enomoto (2006a), Dye (2012), and Ruedas (2017) differs from our values by −0.2%, 0.5%, and 0.3%, respectively. The antineutrino luminosity ($\bar{\nu}_e$ from K, Rb, Th and U) of the modeled BSE from this study and that calculated using the numbers in Enomoto (2006a), Dye (2012), and Usman et al. (2015) yields a 68%, 67%, and 25% difference, respectively; we note that Dye (2012), and Usman et al. (2015) did not include $^{87}$Rb in their calculations, which contributes 5% of the flux.

4 Secular variation in the heat and luminosity of the Earth

Secular evolution of the Earth’s heat production reveals that only two of the short-lived radionuclides, $^{26}$Al, and $^{60}$Fe, contribute any significant amount of additional heat-
ing to the accreting Earth above the power coming from the long-lived radionuclides (Figure 4). Formation and growth of the Earth is envisaged as a process that occurred on timescales of 10⁷ years. Planetary growth initiated from planetesimal "seeds" that were 10⁵ km in scale and likely formed contemporaneous with CAI (Calcium Aluminum Inclusion) formation at \( t_{\text{zero}} = t_{\text{CAI}} \) (i.e., oldest known materials in the solar system) or shortly thereafter.

The inner solar system (circa inside of 4 AU), the domain of the terrestrial planets and rocky asteroids, has been characterized as home to the NC meteorites, the non-carbonaceous meteorites (Warren, 2011). Recent findings from various isotope studies of iron meteorites (Kruijer et al., 2017, 2014; Hilton et al., 2019) show that many of these "NC" bodies formed contemporaneous with \( t_{\text{CAI}} \) or up to 3 million years after \( t_{\text{CAI}} \) formation. It is generally concluded that the larger of these early formed planetesimals rapidly grew in a runaway growth phase followed by oligarchic growth to where they reach Mars and Mercury size bodies, however, if the growth process also included pebble accretion it can occur faster (Izidoro & Raymond, 2018; Johansen & Lambrichts, 2017).

The mean timescales \( \tau \) for terrestrial planet formation (corresponding to accretion of \( \sim 63\% \) of the planet’s mass according to planetary mass growth parameterization \( M(t)/M_{\text{final}} = 1-\exp(-t/\tau) \)) are not well constrained. Mars is suggested to have \( \tau \) of \( \sim 2 \) million years after \( t_{\text{CAI}} \), coincident with core formation (Dauphas & Pourmand, 2011; Tang & Dauphas, 2014), meaning it likely formed within the lifetime of the protoplanetary disk. Formation timescales also depend on position in the disk (Johansen & Lambrechts, 2017). Izidoro & Raymond (2018) found, depending on the particular growth regime assumed in a model of oligarchic growth and the role of gravitational focusing, that there is up to two orders of magnitude difference in the timescale of accretion at 1 AU.

The characteristic accretion time for the Earth is recognized as a significant unknown. We calculated a series of plausible growth curves in Figure 4 (inset) assuming the exponential growth function. With \( \tau = 10 \) Myr (red curve), the Earth is virtually fully (\( > 99\% \)) accreted at \( \sim 50 \) million years after \( t_{\text{CAI}} \), approximately at a plausible timing for a putative Giant impact event that lead to Moon formation (Barboni et al., 2017; Hosono et al., 2019).

The calculated radiogenic power of the Earth is plotted as a function of accretion time (Figure 4). The peak radiogenic heating occurs at about 1 to 5 million years after \( t_{\text{CAI}} \), equivalent to the time scale for Mars accretion, when the proto-Earth produces \( 5 \times 10^9 \) to \( 5 \times 10^{14} \) TW of power, mostly from the decay of \( ^{26}\text{Al} \). This power is added on top of the kinetic energy deposited by impacts during accretion.

Some core formation models, particularly those invoking continuous metal-silicate segregation, suggest a mean age of core separation of \( \sim 10 \) million years after \( t_{\text{CAI}} \) (Kleine et al., 2009). At this time the combined heat production from \( ^{26}\text{Al} \) and \( ^{60}\text{Fe} \) accounts for \( \sim 90\% \) of the \( \sim 300 \) TW of radiogenic power in the Earth. Between 10 and 15 million years after \( t_{\text{CAI}} \), heat production from \( ^{60}\text{Fe} \) exceeds that of \( ^{26}\text{Al} \) and the long-lived radionuclides, despite the recent low estimate for the initial \( (^{60}\text{Fe}/^{56}\text{Fe})_0 \), of \( (3.8 \pm 6.9) \times 10^{-5} \) (Trappitsch et al., 2018). These findings leave little doubt as to the early hot start of the Earth and the likely melting temperatures experienced by both the forming Fe-rich core and surrounding silicate mantle. Moreover, isolating \( \geq 90\% \) of the Earth’s iron into the core at this time results in a super-heated condition, given contributions from radiogenic, impact, and differentiation sources.

Figure 5 presents the heat production for two different bulk compositional models of small terrestrial planets (or asteroidal body). One model assume a bulk Earth-like composition (W. McDonough, 2014) with an Fe/Al = 20 and depletions in moderately volatile elements, while the other model assumes the same composition, except with moderately volatile elements set by a CI chondrite K/U value of 69,000 (Barrat et al., 2012).
A small difference in heat production for these two models in the first 15 million years of solar system history is revealed.

The potential of melting of a small body accreted in the first 2 million years of solar system history depends on its specific power $\dot{h}$ in W/kg, where $\dot{h}$ compares to $C_P \Delta T/\Delta t$, giving us the temperature increase of a body due to its radiogenic heating of $\Delta T$ over a time period of $\Delta t$. For a 100 km radius body (i.e., a size commensurate with estimates of some parent bodies of iron meteorites (Goldstein et al., 2009)), a $\tau$ of 0.5 to 1 million years, and 100 nW/kg of average specific radiogenic heating $\dot{h}$ (note the present-day value for bulk Earth in 3 pW/kg) would increase the body’s temperature by some 3000 K ($\Delta T$) in 1 million years ($\Delta t$), according to a simple balance $\dot{h} = C_P \Delta T/\Delta t$, where $C_P$ is specific heat. This temperature increase is sufficient to induce melting and enhance the effectiveness of metal-silicate fractionation, although the actual thermal evolution also depends on the ability of the growing body to get rid of its heat, and therefore its growth curve of accretion (Šrůněk et al., 2012). With a 100 km radius body, one would expect molten core and mantle (Kleine et al., 2009).

5 Geoneutrino flux vs. radiogenic power

There is a positive correlation between the Earth’s radiogenic power and its geoneutrino flux, with the former given in TW and the latter given in number of (anti)neutrinos per cm$^2$ per second (cm$^{-2}$ s$^{-1}$). Measurements of geoneutrino signal can therefore place limits on the amount of Earth’s radiogenic power, or Th and U abundance, and consequently constrain other refractory lithophile element concentrations (e.g., W. F. McDonough, 2017). The Earth’s geoneutrino signal is often reported in units of TNU, which stands for terrestrial neutrino units, and is the number of geoneutrinos counted over a 1-year exposure in an inverse beta decay detector having $10^{32}$ free protons ($\sim$ 1 kiloton detector of liquid scintillation oil) and 100% detection efficiency. The conversion factor between signal in TNU and flux in cm$^{-2}$ $\mu$s$^{-1}$ depends on the Th/U ratio and has a value of 0.11 cm$^{-2}$ $\mu$s$^{-1}$ TNU$^{-1}$ for Earth’s (Th/U)$_{mass}$ of 3.8.

The Borexino experiment reported the geoneutrino signal (Agostini et al., 2015), based on a fixed (Th/U)$_{molar} = 3.9$. We conducted a Monte Carlo simulation to determine the total signal at the Borexino experiment, based on a reference lithospheric model for the local and global contributions to the total flux Wipperfurth et al. (2019). We assumed the following architecture of the BSE: lithosphere underlain by the Depleted Mantle (source of mid-ocean ridge basalt, MORB), with an underlying Enriched Mantle (source of ocean island basalts, OIB) and the volume fraction of Depleted Mantle to Enriched Mantle is 5:1 Arevalo Jr. et al. (2013). Input assumption for the MC simulation include: (1) U abundance in the BSE (6 to 40 ng/g), (2) BSE (Th/U)$_{mass}$ (3.776$^{+0.122}_{-0.075}$; Wipperfurth et al. (2018)) and (K/U)$_{mass}$ (13, 800±1, 300; Arevalo Jr. et al. (2009)), and (3) accept results with abundances of $U_{Depleted\ Mantle} \leq U_{Enriched\ Mantle}$. Figure 6 shows the intersection of the MC model and the measured signal; the ensemble of acceptable BSE models includes the intersection of the best fit line (MC results) and the measurement field determined by the Borexino experiment (i.e., total power of 16 to 38 TW).

Wipperfurth et al. (2019) conducted a similar analysis to that above for the KamLAND signal. The KamLAND experiment recently reported their geoneutrino signal of 34.9$^{+6.0}_{-5.4}$ Watanabe (2016), also based on a fixed (Th/U)$_{molar} = 3.9$. The observed ensemble of acceptable BSE models (i.e., intersection of the best fit line (MC results) and the measurement field) determined by the KamLAND experiment is between 16 and 25 TW. A combined KamLAND and Borexino result favors a medium-H Earth model with moderate radiogenic power.
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Data availability
The calculations performed in this study are openly available in the form of a Jupyter Notebook in 4TU.ResearchData repository at https://doi.org/10.4121/uuid:191c7802-8835-4791-9c2d-31426dcb5d15.

Authors contribution
WFM, SAW, and OŠ proposed and conceived of various portions of this study and also independently conducted heat production and luminosity calculations. OŠ calculated the $\beta$ decay spectra for the SLR and $^{40}\text{K}$. All authors contributed to the interpretation of the results. The manuscript was written by WFM, with edits and additions from OŠ, and SAW. All authors read and approved of the final manuscript.

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molecular clouds and the unexceptional nature of the solar system. *Earth and Planetary Science Letters*, 392, 16–27. doi: 10.1016/j.epsl.2014.02.014
Figure 1. Decay scheme for $^{40}$K. The beta-minus branch directly leads to $^{40}$Ca in the ground state accompanied by the emission of an $\nu_e$, whereas the electron capture branch has the emission of a 44 keV $\nu_e$ and an excited state of $^{40}$Ar*, with the latter undergoing an isomeric transition to the ground state of $^{40}$Ar via the emission of a 1.46 MeV $\gamma$-ray. Minor branches that we account for are electron capture and $\beta^+$ to ground state of $^{40}$Ar. During $\beta^-$ decay the energy is shared between the $e^-$ and $\nu_e$, with the latter particle removing on average 650 keV of energy from the Earth (accounting for branching; the mean $\nu_e$ energy is 727 keV). Data for the branching ratios and the energies are from Chen (2017); Renne et al. (2011); Naumenko-Dèzes et al. (2018) and the antineutrino energy spectrum (with intensity in arbitrary units), shown in the inset, which uses the $\beta^-$ shape factor from Leutz et al. (1965) to account for the correction of the third unique forbidden transition.
Figure 2. The relative contributions to radiogenic heat production and geoneutrino luminosity of the present-day Earth. Note the relative contributions of $\nu_e$ and $\bar{\nu}_e$ from $^{40}\text{K}$ in terms of geoneutrino luminosity. (Antineutrinos emitted by human-made nuclear reactors are not considered.)
Figure 3. The Earth’s radiogenic power (upper panel) and geoneutrino flux (lower panel) over the last 4568 million years. These figures assume an Earth mass of $6 \times 10^{24}$ kg at all times. The power and geoneutrino flux is scalable; if one assumes 1/10 the planetary mass, it has 1/10 the power and luminosity, for an Earth bulk composition.
Accreting Earth’s radiogenic power

Figure 4. A plot of the relative contributions of radiogenic heat to the Earth during accretion over the first 25 million years of Solar system history. The long-lived radionuclides include: $^{40}$K, $^{232}$Th, $^{235}$U, and $^{238}$U. Figure 3 shows that other short-lived radionuclides contribute negligible amounts of power than what is shown here. Inset diagram shows a series of exponential growth curves $M(t)/M_{\text{final}} = 1 - \exp(-t/\tau)$ for planets. Given an age of Mars of between 2 and 5 million years (Dauphas & Pourmand, 2011; Bouvier et al., 2018), its accretion history can be modeled assuming $\tau \leq 5$. For the Earth we assume $\tau = 10$, however the absolute $\tau$ value is not significant, as there is only a 40% reduction in radiogenic power at the peak between a Mars and Earth accretion model.
Figure 5. A plot of the relative contributions of radiogenic heat to a model terrestrial body (i.e., planet or asteroid) during accretion over the first 25 million years of solar system history. [In the bottom-most left corner of the plot is a tiny dark green peak from SRL 126 Sn, which provides a perspective for heat production from all of the remaining contributors.] Two model compositions are shown for K/U = 14,000 (Earth-like, solid lines) and K/U = 69,000 (CI chondrite, dashed lines); both models assume refractory elements at about 2 times that in CI chondrite, which is equivalent to an Earth-like water and CO$_2$ budget. The terrestrial body is modeled as having a $\tau$ value of 2 and a density of 5,000 kg/m$^3$. The left y-axis (radiogenic power) is for a body with a 100 km radius. The radiogenic power scales with the body size (assuming the same composition and density); for example, for a body with 1/10 of radius, hence 1/1000 of volume, the radiogenic power will be 3 orders of magnitude smaller.
Figure 6. The TNU signal (left y-axis) or geoneutrino flux (in cm$^{-2}$ s$^{-1}$; right y-axis) for the Borexino experiment versus the total radiogenic power (bottom x-axis) or only radiogenic power from Th + U (upper x-axis) in TW within the modeled BSE. The sloped array of points are the predictions generated with a Monte Carlo model using a reference lithosphere of the local and global contributions to the total geoneutrino flux for the Borexino location Wipperfurth et al. (2019). The minimum solution (leftmost points) is set by the $8.1^{+2.7}_{-2.0}$ TW continental lithospheric model and negligible radiogenic power in the mantle. Measured data reported by the Borexino experiment (horizontal red band) is from Agostini et al. (2015). The low-H (blue), medium-H (pink), and high-H (purple) bands represent predictions of the BSE heat production. The 1-sigma, mean, and mode are calculated in bins every two TW.
Table 1. Extant long-lived radioactive decay systems

| Decay system       | Mole frac. (%) | Decay mode | $\lambda$ (yr$^{-1}$) | $Q$ (MeV) |
|--------------------|---------------|------------|------------------------|-----------|
| $^{40}$K $\rightarrow$ $^{40}$Ar | 0.01167 | $\varepsilon$ (10.56%) | $5.810 \times 10^{-11}$ | 1.504     |
| $^{40}$K $\rightarrow$ $^{40}$Ca | 0.01167 | $\beta^-$ (89.44%) | $4.910 \times 10^{-10}$ | 1.311     |
| $^{40}$K overall   |              |            | $5.491 \times 10^{-10}$ | (total) 1.331 |
| $^{87}$Rb $\rightarrow$ $^{87}$Sr | 27.83  | $\beta^-$ | $1.397 \times 10^{-11}$ | 0.2823    |
| $^{138}$La $\rightarrow$ $^{138}$Ce | 0.0888 | $\beta^-$ (34.8%) | $2.34 \times 10^{-12}$ | 1.052     |
| $^{138}$La $\rightarrow$ $^{138}$Ba | 0.0888 | EC (65.2%) | $4.39 \times 10^{-12}$ | 1.742     |
| $^{138}$La overall |              |            | $6.73 \times 10^{-12}$ | (total) 1.504 |
| $^{147}$Sm $\rightarrow$ $^{143}$Nd | 14.993 | $\alpha$ | $6.539 \times 10^{-12}$ | 2.311     |
| $^{176}$Lu $\rightarrow$ $^{176}$Hf | 2.598  | $\beta^-$ | $1.867 \times 10^{-11}$ | 1.194     |
| $^{187}$Re $\rightarrow$ $^{187}$Os | 62.60  | $\beta^-$ | $1.666 \times 10^{-11}$ | 0.0025    |
| $^{190}$Pt $\rightarrow$ $^{186}$Os | 0.0136 | $\alpha$ | $1.415 \times 10^{-12}$ | 3.269     |
| $^{232}$Th $\rightarrow$ $^{208}$Pb | 100    | $6\alpha$ and $4\beta^-$ | $4.916 \times 10^{-11}$ | (total) 42.646 |
| $^{235}$U $\rightarrow$ $^{207}$Pb | 0.72033| $7\alpha$ and $4\beta^-$ | $9.8531 \times 10^{-10}$ | (total) 46.397 |
| $^{238}$U $\rightarrow$ $^{206}$Pb | 99.274 | $8\alpha$ and $6\beta^-$ | $1.5513 \times 10^{-10}$ | (total) 51.694 |

Decay energy $Q$ calculated from mass differences between parent and final daughter nuclide mass data from Wang et al. (2017); see Table 2 for details on decay constant $\lambda$. $^{40}$K and $^{138}$La undergo branched decays; $Q$ entries for $^{232}$Th, $^{235}$U, and $^{238}$U account for the decay networks down to Pb nuclides. Mole fraction of U isotopes calculated from $U = \frac{^{238}U}{^{235}U + ^{234}U + ^{238}U}; ^{238}U/^{235}U = 137.818 \pm 0.045$ Hiess et al. (2012); $^{234}U/U = (5.5 \pm 0.1) \times 10^{-5}$ Villa et al. (2016).
Table 2. Comparison of half-life values ($t_{1/2}$, in years) of long-lived radionuclides.

| Nuclide | NNDC $t_{1/2}(\pm)$ | Geochronology $t_{1/2}(\pm)$ | %± | Ref. | NNDC vs. Geo % rel. difference |
|---------|----------------------|-----------------------------|-----|-----|--------------------------------|
| $^{40}$K | $1.248(3)\times10^9$ | $1.262(2)\times10^9$ | 0.2 | Naumenko-Dèzes et al. (2018) | -1.1 |
| $^{87}$Rb | $4.81(9)\times10^{10}$ | $4.961(16)\times10^{10}$ | 0.3 | Villa et al. (2015) | -3.0 |
| $^{135}$La | $1.03(1)\times10^{11}$ | $1.03(2)\times10^{11}$ | 1.9 | Sato & Hirose (1981); Tanimizu (2000) | 0 |
| $^{147}$Sm | $1.07(1)\times10^{11}$ | $1.06(1)\times10^{11}$ | 0.9 | Begemann et al. (2001) | 0 |
| $^{176}$Lu | $3.76(7)\times10^{10}$ | $3.713(16)\times10^{10}$ | 4.3 | Söderlund et al. (2004) | 1.3 |
| $^{187}$Re | $4.33(7)\times10^{10}$ | $4.16(4)\times10^{10}$ | 1.0 | Begemann et al. (2001) | 4.1 |
| $^{190}$Pt | $6.5(3)\times10^{11}$ | $4.899(44)\times10^{11}$ | 0.9 | Cook et al. (2004) | 33 |
| $^{232}$Th | $1.40(1)\times10^{10}$ | $1.41(1)\times10^{10}$ | 1.0 | Farley (1960) | -0.7 |
| $^{235}$U | $7.038(5)\times10^{8}$ | $7.0348(20)\times10^{8}$ | 0.03 | Hiess et al. (2012) | 0.05 |
| $^{238}$U | $4.4683(24)\times10^{9}$ | $4.4683(96)\times10^{9}$ | 0.2 | Villa et al. (2016) | 0 |

(±) values with $t_{1/2}$ represent the absolute uncertainty in the last reported significant figure, %± is the relative uncertainty. The relative difference between NNDC (National Nuclear Data Center: www.nndc.bnl.gov) and geochronology values are listed in the last column.
Table 3. Long-lived radioactive decay systems in the Earth.

| Decay mode          | $^{238}$U | $^{235}$U | $^{232}$Th | $^{40}$K | $^{87}$Rb | $^{147}$Sm |
|---------------------|-----------|-----------|------------|---------|----------|-----------|
| Nuclide mass (g mol$^{-1}$) | 238.0508 | 235.0439 | 232.0381   | 39.9640 | 86.9092  | 146.9149  |
| Atomic mass (g mol$^{-1}$)   | 238.0289 | 238.0289 | 232.038    | 39.998  | 85.468   | 150.362   |
| Decay constant $\lambda$ (10$^{-18}$ s$^{-1}$) | 4.916    | 31.223    | 1.558      | 17.400  | 0.443    | 0.207     |
| Decay constant $\lambda$ (yr$^{-1}$)   | 1.5513 × 10$^{-10}$ | 9.8531 × 10$^{-10}$ | 4.916 × 10$^{-10}$ | 5.491 × 10$^{-10}$ | 1.397 × 10$^{-11}$ | 6.539 × 10$^{-12}$ |
| Half-life $t_{1/2}$ (10$^9$ yr)* | 4.4683   | 0.70348   | 14.1       | 1.262   | 49.61    | 106       |

$Q$ is the energy released per decay, $Q_\nu$ is the energy carried away by the electron antineutrino or neutrino per decay, $Q_h$ is the energy remaining to provide radiogenic heating per decay, “Nuclide mass frac.” and “Element mass frac.” are the abundances in silicate Earth within the reference Earth model (i.e., kg of nuclide or element per kg of rock), $\nu_\alpha$ and $\nu_\beta$ are the specific antineutrino luminosities of pure nuclide or element (i.e., number of $\nu$ per kg of nuclide or element per second), $L_{\nu_\alpha}$ and $L_{\nu_\beta}$ are the antineutrino and neutrino luminosities of the Earth, $h$ and $h'$ are specific heat production rates of pure nuclide or element, $H$ is the radiogenic heat production of the Earth. Mass of $^4$He is 4.002603254 u and conversion of amu to MeV is 931.494. Mass of silicate Earth of 4.042 × 10$^{24}$ kg is used to calculate $L_{\nu_\alpha}$, $L_{\nu_\beta}$, $H$. *values from Table 1; **values from W. F. McDonough & Sun (1995); Arevalo Jr. et al. (2009) and Th/U ratio from Wipperfurth et al. (2018). 1Energy removed from the Earth by the $\bar{\nu}_e$ in the U and Th decay chains was calculated by integrating the anti-neutrino spectrum reported by S. Enomoto: www.awa.tohoku.ac.jp/~sanshiro/research/geoneutrino/spectrum.
| Decay system | Decay mode | Shape factor | $Q$ (MeV) | $Q_h$ (MeV) | $t_{1/2}$ | $(h/A)$ [nW/kg-element] at $t_{\text{rock}}$ (CAI) | Mole frac. (%) parent nucleus | Reference |
|--------------|------------|--------------|-----------|-------------|----------|---------------------------------|-----------------------------|-----------|
| $^{41}$Ca $\rightarrow ^{41}$K | EC | - | 0.4217 | 0 | 9.94(15) $\times 10^4$ | 0 | $(^{41}\text{Ca/}^{40}\text{Ca})_h = (4.2\pm1.9) \times 10^{-5}$ | Liu (2017) |
| $^{99}$Tc $\rightarrow ^{99}$Ru | $\beta^-$ | $0.54q^2 + q$ (M15) | 0.2975 | 0.0957 | 2.11(12) $\times 10^5$ | 46.71 | $(^{99}\text{Tc/}^{100}\text{Ru})_h = < 3.5 \times 10^{-5}$ | Wasserburg et al. (1994) |
| $^{81}$Kr $\rightarrow ^{81}$Br | EC | - | 2.8069 | 0.0908 | 2.25(11) $\times 10^5$ | - | not available | - |
| $^{126}$Sn $\rightarrow ^{126}$Te | $\beta^-$, $\beta^+$ | $1^1$, NuDat | 4.0502 | 2.8597 | 2.35(7) $\times 10^5$ | 3.77 $\times 10^4$ | $(^{126}\text{Sn/}^{124}\text{Sn})_h = < 3 \times 10^{-3}$ | Brennecke et al. (2017) |
| $^{36}$Cl $\rightarrow ^{36}$Ar | $\beta^-$ (88.1%) | M15 | 0.7095 | 0.3343 | $3.01(2) \times 10^5$ | 0.9557 | $(^{36}\text{Cl/}^{36}\text{Cl})_h = (1.9 \pm 0.5) \times 10^{-8}$ | Turner et al. (2013) |
| $^{70}$Se $\rightarrow ^{70}$Br | $\epsilon$ (1.9%) | NuDat | 1.1421 | $3.1(2) \times 10^6$ | 2.011 $\times 10^{-5}$ | - | not available | - |
| $^{26}$Al $\rightarrow ^{26}$Mg | EC (18.3%) | $p^4 + \frac{4}{3}p^2q + q^4$ [th.] | 4.0444 | 0.3610 | $\lambda_{EC}/\lambda = 0.1827$ | 20.57 | $(^{26}\text{Al/}^{26}\text{Al})_h = (5.2 \pm 0.2) \times 10^{-5}$ | Kita et al. (2013) |
| $^{26}$Al $\rightarrow ^{26}$Mg | $\beta^+$ (81.7%) | $p^4 + \frac{4}{3}p^2q + q^4$ [th.] | 2.9824 | 2.7583 | $\lambda_{\beta^+}/\lambda = 0.1873$ | 1.57 $\times 10^4$ | - | - |
| $^{26}$Al $\rightarrow ^{26}$Mg | Overall | 3.1203 | 7.17(24) $\times 10^5$ | 1.777 $\times 10^4$ | - | - | - | - |
| $^{10}$Be $\rightarrow ^{10}$B | $\beta^-$ | $p^4 + \frac{4}{3}p^2q + q^4$ [th.] | 0.5568 | 0.2527 | $1.387(12) \times 10^6$ | 2.270 $\times 10^4$ | $(^{10}\text{Be/}^{10}\text{Be})_h = (5.3 \pm 1.0) \times 10^{-4}$ | Liu et al. (2010) |
| $^{93}$Zr $\rightarrow ^{93}$Nb | $\beta^-$ | $p^4 + q^4$ [th.] | 0.0903 | 0.0456 | $1.61(5) \times 10^6$ | - | not available | - |
| $^{150}$Gd $\rightarrow ^{150}$Sm | $\alpha$ | - | 2.8077 | 2.8077 | $1.78(9) \times 10^6$ | - | not available | - |
| $^{135}$Cs $\rightarrow ^{135}$Ba | $\beta^-$ | $0.10p^2 + q^2$ (M15) | 0.2688 | 0.0615 | $2.3(3) \times 10^6$ | 119.4 | $(^{135}\text{Cs/}^{133}\text{Cs})_h = (2.8 \pm 2) \times 10^{-4}$ | Bermingham et al. (2014) |
| $^{56}$Fe $\rightarrow ^{56}$Ni | $\beta^-$, $\beta^+$ | NuDat | 3.0598 | 2.7077 | $2.62(4) \times 10^6$ | 1.367 | $(^{56}\text{Fe/}^{56}\text{Fe})_h = (3.8 \pm 6.9) \times 10^{-4}$ | Trappisch et al. (2018) |
| $^{154}$Dy $\rightarrow ^{154}$Gd | $\alpha$ | - | 2.3251 | 2.3451 | $3.05(15) \times 10^5$ | - | not available | - |
| $^{53}$Mn $\rightarrow ^{53}$Cr | EC | - | 0.5968 | 0 | $3.74(4) \times 10^6$ | 0 | $(^{53}\text{Mn/}^{55}\text{Mn})_h = (6.5 \pm 1.9) \times 10^{-6}$ | Trinquier et al. (2008) |
| $^{97}$Tc $\rightarrow ^{97}$Ru | $\beta^-$ | $1^1$ | 1.794 | 1.5165 | $4.2(3) \times 10^6$ | 1449 | $(^{97}\text{Tc/}^{99}\text{Ru})_h = < 1.5 \times 10^{-3}$ | Becker & Walker (2003) |
| $^{97}$Tc $\rightarrow ^{97}$Mo | EC | - | 0.3247 | 0 | $4.2(1) \times 10^6$ | 0 | $(^{97}\text{Tc/}^{99}\text{Ru})_h = < 2 \times 10^{-5}$ | Dauphas et al. (2003) |
| $^{107}$Pd $\rightarrow ^{107}$Ag | $\alpha$ | $p^2 + q^2$ [th.] | 0.0341 | 0.0133 | $6.5(3) \times 10^6$ | 0.2768 | $(^{107}\text{Pd/}^{107}\text{Pd})_h = (2.573 \pm 0.07) \times 10^{-5}$ | Mathies et al. (2017) |
| $^{244}$Pu $\rightarrow ^{244}$Th | $\beta^-$, $\beta^+$ | $p^2 + 3.16q^2$ (M15) | 0.1889 | 0.0853 | $1.57(4) \times 10^7$ | 12.71 | $(^{244}\text{Pu/}^{244}\text{Pu})_h = (1.4 \pm 0.1) \times 10^{-4}$ | Knijjer et al. (2014) |
| $^{146}$Sm $\rightarrow ^{146}$Nd | $\alpha$ | - | 2.5288 | 2.5288 | $1.0(5) \times 10^6$ | 77.55 | $(^{146}\text{Sm/}^{144}\text{Sm})_h = (8.28 \pm 0.4) \times 10^{-3}$ | Marks et al. (2014); Meissner et al. (2013); Turner et al. (2014); Iizuka et al. (2016); Marks et al. (2014); Turner et al. (2007); Wasserburg et al. (1994) |

$Q$ is the energy of transition (Q value) not accounting for possible branching; $Q_h$ is the energy that remains in the Earth to provide radiogenic heating per decay, accounting for branching. M15 = shape factor from Mongeon (2015). In some cases, we use shape factors equal to 1 or NuDat-tabulated mean electron energies for forbidden transitions, due to lack of better inputs. $h/A$ is the energy that remains in the Earth to provide radiogenic heating per decay, accounting for branching. $Q_h$ is the energy that remains in the Earth to provide radiogenic heating per decay, accounting for branching. $Q$ is the energy of transition (Q value) not accounting for possible branching; $Q_h$ is the energy that remains in the Earth to provide radiogenic heating per decay, accounting for branching. M15 = shape factor from Mongeon (2015). In some cases, we use shape factors equal to 1 or NuDat-tabulated mean electron energies for forbidden transitions, due to lack of better inputs. $h/A$ is the energy that remains in the Earth to provide radiogenic heating per decay, accounting for branching.