Limits on Lorentz Violation from Synchrotron and Inverse Compton Sources

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Abstract

We derive new bounds on Lorentz violations in the electron sector from existing data on high-energy astrophysical sources. Synchrotron and inverse Compton data give precisely complementary constraints. The best bound on a specific combination of electron Lorentz-violating coefficients is at the $6 \times 10^{-20}$ level, and independent bounds are available for all the Lorentz-violating $c$ coefficients at the $2 \times 10^{-14}$ level or better. This represents an improvement in some bounds by fourteen orders of magnitude.

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The possibility that Lorentz invariance may not be exact in nature has been a subject of great interest since the discovery that Lorentz symmetry could be broken spontaneously in string theory \[1\]. Any observed deviations from Lorentz invariance would be powerful clues about the nature of Planck-scale physics. There has been a great deal of work probing the physics of Lorentz violation, both experimental and theoretical. Sensitive tests of Lorentz symmetry have included studies of matter-antimatter asymmetries for trapped charged particles \[2, 3, 4, 5\] and bound state systems \[6, 7\], determinations of muon properties \[8, 9\], analyses of the behavior of spin-polarized matter \[10, 11\], frequency standard comparisons \[12, 13, 14, 15\], Michelson-Morley experiments with cryogenic resonators \[16, 17\], Doppler effect measurements \[18, 19\], measurements of neutral meson oscillations \[20, 21, 22, 23\], polarization measurements on the light from distant galaxies \[24, 25, 26\], and others.

On the theoretical side, a Lorentz- and CPT-violating effective field theory, the standard model extension (SME) has been developed in detail \[27, 28\]. Basic issues regarding this theory, including stability and causality \[29\] and one-loop renormalizability \[30\] have been addressed. The SME contains coefficients that parameterize possible Lorentz violations. Many of these coefficients are tightly constrained by the various experiments, but many others are not.

In this paper, we shall provide some further bounds on an important but relatively poorly constrained sector of the SME. By analyzing data from high-energy astrophysical sources, we can get strong new constraints, the best of which is at the $6 \times 10^{-20}$ level. We shall use data on both synchrotron and inverse Compton (IC) emissions; it turns out that these two types of radiation give complementary bounds.

Synchrotron radiation has previously been used to bound nonrenormalizable Lorentz-violating parameters \[31, 32\]. The Crab nebula shows evidence of synchrotron emission from electrons with Lorentz factors of $\gamma = (1 - \vec{v}^2)^{-1/2} \sim 3 \times 10^9$. The presence of electrons with velocities this large can be used to constrain models with deformed dispersion relations. For a nonrenormalizable Lorentz-violating coefficient with a particular sign, the data show that the coefficient must be at least seven orders of magnitude smaller than $O(E/M_P)$ Planck-level suppression. We shall use a similar technique, but concentrating on the more important renormalizable Lorentz-violating operators. We have previously analyzed synchrotron emission in the presence of these kinds of Lorentz violations and presented bounds based on the data from the Crab nebula \[33\]. There have also been more detailed theoretical analyses of Lorentz-violating synchrotron processes with nonrenormalizable operators \[34\] or noncommutative geometry \[35\].

We shall consider here a carefully chosen subset of the SME coefficients. The Lagrange density for our theory is

$$ L = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi}((\gamma^{\mu} + c^{\mu\nu} \gamma_{\nu})(i \partial_{\mu} - e A_{\mu}) - m)\psi, \tag{1} $$

where $\psi$ is the electron field. $c$ contains nine parameters that contribute to Lorentz-violating physics at leading order. The trace $c^{\mu\nu}$ only affects the overall normalization
of the electron field, and the antisymmetric part of $c$ has no effects at first order, where it is equivalent to a redefinition of the Dirac matrices. We shall not use a symmetric $c$, however, but shall instead set the $c^{\mu}$ coefficients to zero using a field redefinition, so that the theory may be straightforwardly quantized and to simply the coupling to $A^\mu$.

The SME $c$ coefficients modify the kinetic part of the electron Lagrangian, so they grow in importance at high energies. Other kinetic modifications are less important. Most of the operators in question violate the $SU(2)_L$ gauge invariance of the standard model, so they can only arise as vacuum expectation values of higher dimensional (i.e., nonrenormalizable) operators; we therefore expect them to be suppressed. There is also a $d$ interaction, which is gauge invariant, similar in form to $c$ but with an additional $\gamma_5$. However, effects from $d$ should average out, because there should be no net polarization of the electrons in high-energy sources. Mixing between the standard kinetic term and the $c$ term (which have the same basic Dirac structure) also causes the effects of $c$ also grow in importance relative to the effects of the other SME coefficients at high energies [29].

The electromagnetic sector in [11] is conventional. Purely electromagnetic violations of Lorentz invariance fall into two categories. Some terms produce photon birefringence, and all these are very tightly constrained. The other terms can be absorbed into the $c$ coefficients through a coordinate redefinition [36]. Therefore, the terms we consider here are the only ones which should be relevant experimentally.

The $c$ coefficients for nucleons are quite tightly constrained, but those for electrons are not. However, in high-energy astrophysical processes, electrons achieve the largest Lorentz factors and can dominate emissions. The processes we shall consider depend on the ambient electromagnetic field—synchrotron emissions depending primarily on the magnetic field strength and IC emissions on the photon density. As is true conventionally, the electrostatic potential $\Phi = A^0$ is coupled simply to the charge density $e\psi^\dagger \psi$, and the vector potential $A$ couples to $e\psi^\dagger \dot{\vec{x}} \psi$. Radiation from an ultrarelativistic charge is beamed into a narrow pencil of angles centered around the direction of the charge’s velocity. So the radiation we observe from a distant source comes from electrons that are moving in the source-to-Earth direction. The emissions from both processes also show a marked dependence on the relation that connects energy and velocity, and this is the effect we shall utilize in order to constrain $c$.

The velocity $\vec{v}$ of an electron is related to its energy $E$ and momentum $\vec{p}$ according to [37]

$$v_k = \frac{1}{E + c_{0j} \vec{p}_j} \left( \pi_k - c_{kj} \vec{p}_j - c_{jk} \vec{p}_j + c_{jk} c_{jl} \vec{p}_l \right) - c_{0k}.$$  \hspace{1cm} (2)

Neglecting the terms beyond leading order in $c$ (which should be miniscule), the maximum velocity in a particular direction described by a unit vector $\hat{e}$ is $1 - c_{jk} \hat{e}_j \hat{e}_k - c_{0j} \hat{e}_j$. With Lorentz violation, this can be less than one. If a body’s synchrotron spectrum indicates the presence of electrons with Lorentz factors up to at least some value $\gamma_{\text{max}}$, then the
Lorentz-violating coefficients for the electron must be bounded according to
\[c_{jk} \hat{e}_j \hat{e}_k + c_{0j} \hat{e}_j < \frac{1}{2\gamma_{\text{max}}^2},\]
(3)
where \(\hat{e}\) is in the source-to-Earth direction just mentioned. [By the Lorentz factor \(\gamma\), we mean precisely the quantity \(\gamma = (1 - \vec{v}^2)^{-1/2}\).] This is a one-sided limit, so only coefficients with particular signs are restricted. In particular, we can only find upper limits on the diagonal components of \(c\) this way; no negative values of \(c_{jk}\) for \(j = k\) can be excluded. So a complementary measurement is needed if the Lorentz-violating coefficients are to be restricted to a bounded region of the parameter space.

Fortunately, complementary bounds are possible. Just as a positive \(c_{jk} \hat{e}_j \hat{e}_k + c_{0j} \hat{e}_j\) leads to a maximum electron speed that is less than one, a negative value for \(c_{jk} \hat{e}_j \hat{e}_k + c_{0j} \hat{e}_j\) can indicate a maximum electron speed that is greater than one. However, for electrons that are coupled conventionally to the electromagnetic field, new effects would come into play if the speeds actually became superluminal. The validity of the model is questionable under these circumstances, so we cannot make use of the maximum velocity directly. On the other hand, there is instead a maximal energy attainable by subluminal electrons, and we can use this to set further bounds on \(c\).

In IC scattering, an ultrarelativistic electron scatters off a comparatively low-energy photon, transferring a substantial fraction of its energy to the photon. For kinematical reasons, the emitted radiation is (as with synchrotron radiation) beamed into a narrow pencil of angles centered around the direction of the electron’s motion. The resulting radiation can be observed and used to bound the Lorentz violation. The key is that the initial electron must have a greater energy than the observed IC \(\gamma\)-ray. If the emission from a source is well modeled by known processes, we may infer that superluminal electrons (which would produce vacuum Cerenkov radiation and emit synchrotron radiation at a rate which diverges in the approximation of no back reaction) are not the source of the emission. Then the energies of the most energetic IC photons represent lower bounds on the energies of subluminal electrons; and if there are such electrons up to some energy \(E_{\text{max}}\), then the \(c\) coefficients must satisfy
\[-c_{jk} \hat{e}_j \hat{e}_k - c_{0j} \hat{e}_j < \frac{1}{2(E_{\text{max}}/m)^2},\]
(4)
since otherwise, an electron with energy \(E_{\text{max}}\) moving in the Earthward direction would be superluminal.

The similarity between equations (3) and (4) is not coincidental. In each case, the effect we want to measure is a product of how the Lorentz violation affects the relationship between velocity and energy. In the Lorentz-invariant theory, \(\gamma = E/m\), and by measuring \(E\) and \(\gamma\) separately, we can get bounds on any deviations from Lorentz invariance.

There are nine components of \(c\) that can be bounded in this way—the three \(c_{0j}\) and the six-component symmetric part of \(c_{jk}\). If each of these coefficients is to be bounded on both
| Emission source          | $\hat{e}_X$ | $\hat{e}_Y$ | $\hat{e}_Z$ | $\gamma_{\text{max}}$ | $E_{\text{max}}/m$ |
|--------------------------|------------|------------|------------|------------------------|-------------------|
| 3C 273                   | 0.99       | 0.13       | -0.04      | $3 \times 10^7$ [38]   | $2 \times 10^5$ [38] |
| Centaurus A              | 0.68       | 0.27       | 0.68       | $2 \times 10^8$ [39]   | -                 |
| Crab nebula              | -0.10      | -0.92      | -0.37      | $3 \times 10^9$ [40]   | $2 \times 10^8$ [40] |
| G 12.82-0.02             | -0.06      | 0.95       | 0.29       | -                      | $5 \times 10^7$ [41] |
| G 347.3-0.5              | 0.16       | 0.75       | 0.64       | $3 \times 10^7$ [42]   | $2 \times 10^7$ [43] |
| MSH 15-52                | 0.34       | 0.38       | 0.86       | -                      | $8 \times 10^7$ [44] |
| PSR B1259-63             | 0.42       | 0.12       | 0.90       | -                      | $6 \times 10^6$ [45] |
| RCW 86                   | 0.35       | 0.30       | 0.89       | $10^8$ [46]            | -                 |
| SNR 1006 AD              | 0.52       | 0.53       | 0.67       | $2 \times 10^7$ [47]   | $7 \times 10^6$ [47] |
| Vela SNR                 | 0.44       | -0.55      | 0.71       | $3 \times 10^8$ [48]   | $1.3 \times 10^8$ [49] |

Table 1: Parameters for the astrophysical sources that we shall use to constrain $c$. The coordinates $X$, $Y$, and $Z$ are in sun-centered celestial equatorial coordinates [50]. References are given for each value of $\gamma_{\text{max}}$ or $E_{\text{max}}$.

besides, we must obtain at least ten inequalities of the forms (3) or (4), corresponding to emissions from at least nine separate sources. Each of these inequalities generally couples all nine of the coefficients in a nontrivial way, but they may be translated into separate limits on individual coefficients by means of linear programming.

Table 1 lists the parameters for ten astrophysical sources for which useful bounds are available. Most of the sources are supernova remnants, but a few extragalactic sources are included. The radio galaxy Centaurus A and the quasar 3C 273 are among the brightest and best understood objects of their kinds. All the sources can be modeled relatively cleanly, giving fairly secure values of $\gamma_{\text{max}}$ and/or $E_{\text{max}}$. The $\gamma_{\text{max}}$ values must be extracted directly from the models, and so errors in the modeling could affect the validity of the limits; this makes it important that only well-understood sources be used. The $E_{\text{max}}$ limits are less model dependent. IC models of $\gamma$-ray sources typically require maximum electron energies that are several times larger than the highest observed photon energies (because an IC scattering event does not transfer all of the electron’s energy to the photon). However, this model-derived $E_{\text{max}}$ is not the value we have chosen to use in table 1. Instead, we have conservatively identified the highest actually observed photon energy as $E_{\text{max}}$; the only input from a model is that the source’s $\gamma$-ray emission is well described by the IC process. This choice of $E_{\text{max}}$ ensures that Lorentz-violating distortions of the energy-momentum relation at higher than observed energies (which could invalidate the model results) are not a problem.

The bounds derived from the parameters in table 1 (the best of which is at the $6 \times 10^{-20}$ level) are slightly awkward. It is difficult with nine coupled parameters to determine exactly what regions of the parameter space are being excluded. We have therefore extracted, via linear programming, independent limits on each of the nine components of
Table 2: Independent bounds on the components of $c$.

| $c_{\mu\nu}$ | Maximum       | Minimum       |
|----------------|---------------|---------------|
| $c_{XX}$       | $5 \times 10^{-15}$ | $-6 \times 10^{-15}$ |
| $c_{YY}$       | $3 \times 10^{-15}$ | $-3 \times 10^{-15}$ |
| $c_{ZZ}$       | $5 \times 10^{-15}$ | $-3 \times 10^{-15}$ |
| $c_{(XY)}$     | $3 \times 10^{-15}$ | $-3 \times 10^{-15}$ |
| $c_{(YZ)}$     | $3 \times 10^{-15}$ | $-3 \times 10^{-15}$ |
| $c_{0X}$       | $5 \times 10^{-15}$ | $-2 \times 10^{-14}$ |
| $c_{0Y}$       | $2 \times 10^{-15}$ | $-5 \times 10^{-16}$ |
| $c_{0Z}$       | $10^{-16}$     | $-5 \times 10^{-17}$ |

$c$. These bounds represent the absolute maximum and minimum values that are possible for each coefficient. They are therefore not as tight numerically as the raw bounds.

In addition to the astrophysical bounds derived here, we have included some other, roughly comparable bounds in the linear program. Optical resonator tests are usually used to place bounds on the parameters of the SME photon sector. However, these same experiments may be used to place bounds on the electron $c$ coefficients [51]. The key realization is that electronic Lorentz violations will modify the structure of a crystalline resonator, and this effect can be worked out systematically, provided Lorentz violations for nucleons can be safely neglected. The result is further bounds on $|c_{(XY)}|$, $|c_{(YZ)}|$, $|c_{(YZ)}|_0$, and $|c_{XX} - c_{YY}|$, which we shall all conservatively take to be at the $3 \times 10^{-15}$ level. [Here, $c_{(jk)}$ means the symmetric sum $c_{jk} + c_{kj}$.]

The results are given in table 2. The cryogenic resonator bounds are still the best for $|c_{(XY)}|$, $|c_{(YZ)}|$, and $|c_{(YZ)}|_0$, but bounds on almost all the other coefficients are now comparable or better. The bounds on $c_{0Z}$ are especially strong, while in laboratory experiments, boost invariance violation coefficients such as $c_{0j}$ are typically harder to constrain. This shows the advantage of deriving bounds from emissions by relativistic sources. The bounds on the separate coefficients are completely independent; the maximum and minimum values presented are the largest and smallest that a given coefficient can take under any circumstances. Moreover, there are also additional correlations; it is not generally possible for several of the coefficients to take on their extremum values simultaneously.

This method for placing bounds on Lorentz violation could not be used directly to identify actual Lorentz violation. The signature of Lorentz violation in high-energy astrophysical sources would be emission spectra that cannot be modeled by conventional radiation mechanisms. If the calculated bounds on certain Lorentz-violating coefficients were significantly weaker than others, that would be an indication that those coefficients might actually be nonzero. However, since most of the bounds in table 2 are comparable in magnitude, we see no indications that any particular components of $c_{\mu\nu}$ are more likely
to be nonzero than others.

Several of the bounds we have derived are much better than previous constraints. The previous bounds on the electronic $c_{0j}$ coefficients were at the $10^{-2}$ level \([19]\), so the improvements here are by more than eleven orders of magnitude (fourteen orders for $c_{0Z}$). Moreover, all nine of the coefficients that contribute at leading order can be comparably bounded by this method, and as better measurements from a wider variety of astrophysical sources become available, the limits on $c$ will improve as well.

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