A Constraint on Electromagnetic Acceleration of Highest Energy Cosmic Rays

Mikhail V. Medvedev

Department of Physics and Astronomy, University of Kansas, Lawrence, KS 66045

The energetics of electromagnetic acceleration of ultra-high-energy cosmic rays (UHECRs) is constrained both by confinement of a particle within an acceleration site and by radiative energy losses of the particle in the confining magnetic fields. We demonstrate that the detection of $3 \times 10^{20}$ eV events is inconsistent with the hypothesis that compact cosmic accelerators with high magnetic fields can be the sources of UHECRs. This rules out the most popular candidates, namely spinning neutron stars, active galactic nuclei (AGNs), and γ-ray burst blast waves. Galaxy clusters and, perhaps, AGN radio lobes remain the only possible (although not very strong) candidates for UHECR acceleration sites. Our analysis places no limit on linear accelerators. With the data from the future Auger experiment one should be able to answer whether a conventional theory works or some new physics is required to explain the origin of UHECRs.

PACS numbers: 41.60.-m, 96.40.-z

I. INTRODUCTION

The detection of ultra-high-energy cosmic rays (UHECRs) with energies above $10^{20}$ eV has posed a challenge to the understanding of their origin and nature. At present, 17 such events were reported by the AGASA group and 2 events were observed by HiRes. An energy of the one of them is estimated to be $3 \times 10^{20}$ eV, close to the energy of the largest ($3.2 \times 10^{20}$ eV) event observed with the Fly's Eye detector. Nearly isotropic distribution on the sky and the absence of large-scale clustering suggests the cosmological origin of UHECRs. This is in apparent conflict with the observed energy spectrum which lacks the Greisen-Zatsepin-Kuzmin (GZK) cutoff at energy $\sim 5 \times 10^{19}$ eV [1,2] indicating that UHECRs have traveled the distance smaller than $60$ Mpc ($\sim 20$ Mpc for protons with $E \gtrsim 3 \times 10^{20}$ eV).

Cosmic rays are accelerated in astrophysical sources either by repeated scattering off macroscopic flows, such as shocks, winds and outflows, turbulent flows, or directly by an induced electric field around a magnetized rotating object. For the acceleration to operate, a particle must remain confined within the acceleration region: the gyro-radius of the particle should not exceed the size of the system, $R$. This sets the maximum energy of the accelerated particle,

$$E_{\text{acc}} = Z e B R \simeq 9.3 \times 10^{23} Z B R_{\text{kpc}} \text{ eV}, \quad (1)$$

where $Ze$ is the charge of a particle, $B$ is the characteristic magnetic field strength in the acceleration region (in gauss), and $R_{\text{kpc}}$ is the size in kiloparsecs. If the whole medium is moving relativistically with the Lorentz factor $\Gamma$ towards an observer, $E_{\text{acc}}$ is boosted to $\Gamma E_{\text{acc}}$, with $B$ and $R$ being measured in the co-moving frame. Specifying the particle energy, say $E_{\text{acc}} = 3 \times 10^{20}$ eV (the largest observed), and assuming $Z \sim 1$, one obtains the magnetic field–size relation [3,4], which is plotted in Figure 1 with the long-dashed line. Any astrophysical object to the right from this line can accelerate protons to energies $\geq 3 \times 10^{20}$ eV.

Various energy losses, such as collisional and inverse Compton scattering in a radiation field, limit the UHECR energy. But even in the absence of any such processes, an energetic particle will still lose energy radiatively while moving through magnetic and electric fields which confine and accelerate this particle. The radiative (synchrotron) losses have been considered by various authors [5–8], who usually estimated the synchrotron cooling time, but didn’t take into account the finite size of the source self-consistently [9].

In this paper we re-analyze the energetics of electromagnetic acceleration to elucidate its inherent and inevitable limitations. The Hillas criterion, equation (1), is shown to be often inaccurate and even misleading. We consider the most idealized models that carry only the most robust properties of cosmic accelerators, any details of the electromagnetic acceleration process are not important for us. Thanks to the simplicity of the models, analytical solutions for the particle energy, which account for a source size self-consistently, were obtained. These results represent the most relaxed (and, hence, unavoidable) constraints on the maximum energy of an electromagnetically accelerated particle. This implies that the most favorable sources of UHECRs, such as neutron stars (NSs), active galactic nuclei (AGNs) and gamma-ray bursts (GRBs), cannot accelerate protons to the energy $\sim 3 \times 10^{20}$ eV and, hence, must be ruled out. However, our criterion does not apply to linear accelerators (e.g., axial jets) [10]. The Auger experiment may be capable of ruling out the remaining candidates, radio lobes of AGNs and galaxy clusters, and if it does, the whole conventional astrophysical picture of UHECRs as accelerated particles must be revisited.

* Also at the Institute for Nuclear Fusion, Russian Research Center “Kurchatov Institute”, Moscow 123182, Russia; Electronic address: medvedev@ku.edu
II. INEFFICIENT ACCELERATION

Let us consider diffusive acceleration first. This type of acceleration operates in shocks of $\gamma$-ray bursters, galaxy clusters, jets from AGNs interacting with the intergalactic medium and producing radio lobes, and, perhaps, in AGN cores near the base of a jet. Let us consider a shock propagating through a magnetized medium (intercluster medium [ICM], for instance), as shown in Figure 2. Magnetic field may be inhomogeneous on a scale $\sim R$. An accelerated particle gains energy by repeated scattering off a shock or a flow. After every scattering, the particle travels a great distance along the Larmor orbit until it returns and gets another kick. As long as the particle moves freely in the magnetic field it radiates and slows down. We will see that the maximum terminal energy of a particle (i.e., when the particle escapes the orbit until it returns and gets another kick. As long as the particle travels a great distance along the Larmor orbit until it returns and gets another kick. As long as the particle moves freely in the magnetic field it radiates and slows down. We will see that the maximum terminal energy of a particle (i.e., when the particle escapes the system), $E$, is determined by these radiative losses, but is insensitive to how large the energy, $E_0$, of the particle at the shock front is. Therefore, we refer to this regime as “inefficient acceleration”.

Let us consider a particle with some initial energy $E_0$ propagating through a region of a size $R$ filled with a magnetic field $B$. The energy of the particle gradually decreases according to the equation (see [3]):

$$\frac{dE}{dx} = F_{\text{Rad}} = - \frac{2}{3} \left( \frac{Ze}{Am_p c^2} \right)^4 B^2(x) E^2,$$

where $x$ is the distance along the particle trajectory, $F_{\text{Rad}}$ denotes the radiation friction force, and $Am_p$ is the particle mass. In [3] only the transverse field component enters; we assumed that $B_\perp \sim B$. The solution of this equation is

$$E^{-1} = E_0^{-1} + E_{cr}^{-1},$$

where $E_0$ is the initial energy of the particle and

$$E_{cr} = \frac{3}{2} \left( \frac{Am_p c^2}{Ze} \right)^4 \left( \int_0^R B^2(x) dx \right)^{-1},$$

$$\simeq 2.9 \times 10^{16} \frac{(A/Z)^4}{B^2 R_{\text{kpc}}} \text{ eV}.\quad (4)$$

For simplicity, it is assumed here that $B(x) \sim constant$ within the system. It must be clear now that no matter how energetic the particle is ($E_0 \to \infty$), after traveling through a region with a magnetic field its energy will not exceed the critical energy $E_{cr}$. Specifying the energy $E_{cr} = 3 \times 10^{20}$ eV and assuming $A \sim 1$, one obtains another $B$ vs. $R$ constraint, shown in Figure 4 with the short-dashed line. All astrophysical sources located above this line have $E_{cr} \leq 3 \times 10^{20}$ eV and hence cannot accelerate UHECRs.

It should be mentioned that the classical expression [2] for the radiation friction force is valid if the wavelength of the emitted radiation is larger than the “classical radius” of a charge, $c/\omega_B \gg (Ze)^2/(Am_p c^2)$ (where $\omega_B = Ze B/Am_p c$), that is for the field strengths not exceeding $(Am_p)^2 c^4/(Ze)^3$ in the rest frame of a particle. This yields the condition for the particle energy

$$E \ll \left( \frac{Am_p c^2}{Ze} \right)^3 \frac{1}{B} \simeq 1.9 \times 10^{31} (A/Z)^3 B^{-1} \text{ eV}, \quad (5)$$

which is satisfied for $E \lesssim 10^{21}$ eV for practically all sources.

III. EFFICIENT ACCELERATION

Let us now consider “cosmic inductors” where particles are accelerated by electric fields induced by rapid rotation of a magnetized object. Acceleration of this type should occur in neutron star magnetospheres and around accreting supermassive black holes in the centers of AGNs. We naturally assume that the magnetic field has a dipolar (or a multipolar) structure, hence field lines are bent on a scale $\sim R$, as shown in Figure 2. Because of rapid rotation, there is an induced electric field $E_{\text{ind}} \simeq |\mathbf{B}|/c$ which accelerates a particle. The maximum value of $E_{\text{ind}}$ is achieved near the light cylinder $\Omega^2$ is close where it is equal to, at most, $E_{\text{ind}} \simeq B$. The particle in such a system gains energy rapidly, within one passage through the system. Hence one must retain the electromagnetic accelerating force $F_{\text{EM}} = Ze E_{\text{ind}} \simeq Ze B$ in the energy equation (2). Then it reads

$$\frac{dE}{dx} \simeq Ze B - \frac{2}{3} \left( \frac{Ze}{Am_p c^2} \right)^4 B^2 E^2.\quad (6)$$

For a small initial energy of an accelerated particle ($E_0 \ll E_{\text{acc}}, E_{cr}$), the solution of this equation takes a simple and elegant form:

$$E = \sqrt{E_{\text{acc}} E_{cr}} \tanh \sqrt{E_{\text{acc}}/E_{cr}},$$

where $E$ is the terminal energy of the particle. The solution has two obvious asymptotics. If $E_{\text{acc}} \ll E_{cr}$ one recovers the Hillas constrain [3] $E \simeq E_{\text{acc}}$ (equation [1]), whereas in the opposite limit one has

$$E_{\text{max}} \simeq \sqrt{E_{\text{acc}} E_{cr}} \simeq 1.3 \times 10^{20} A^2 Z^{-3/2} B^{-1/2} \text{ eV}.\quad (8)$$

This equation, in fact, follows from the balance between the acceleration and radiative losses, $F_{\text{EM}} = F_{\text{Rad}}$, in equation (6). Hence, we refer to this regime as “efficient acceleration”. Acceleration above this energy is impossible because at larger energies radiative friction begins to dominate over electromagnetic acceleration. This constraint for $E_{\text{max}} = 3 \times 10^{20}$ eV is plotted in Figure 4 with the dot-dashed line. No objects above this line can accelerate cosmic rays to this energy.
IV. DISCUSSION

The conventional astronomical picture for the origin of these cosmic rays is the acceleration of charged particles, e.g., protons or heavier atomic nuclei, in extragalactic objects [4, 8]. There are only few types of such objects. The most favorable are: spinning neutron stars (NSs) and magnetars central regions of active galactic nuclei (AGNs) AGN radio lobes and γ-ray burst (GRB) shocks. It is unlikely that shocks in galaxy clusters are the UHECR sources because they are too far, beyond the GZK distance, and they are not able to accelerate protons to energies above few times $10^{19}$ eV [10].

In the previous sections we demonstrated that the energy of a particle confined by magnetic fields within an acceleration site is determined by its radiative losses. We now discuss the constraints on B and R given by equations (1), (4), and (8) and plotted in Figure 1.

First, the constraint (8) shown with the dot-dashed line is the most stringent. It tells that protons cannot be accelerated electromagnetically at the sources which lie above this line. Equation (8) also holds for heavier (e.g., iron) nuclei. Hence, compact stars, AGN cores, and GRB shocks (except during the late afterglow phase) are readily ruled out from the list of possible sources of ultra-high-energy protons and nuclei.

Second, the remaining candidates, i.e., the radio lobes and galaxy clusters, may accelerate particles only via the diffusive mechanism, hence equation (4) is appropriate. This equation together with (1) specifies the allowed B–R parameter region. In the figure the dotted line corresponds to iron nuclei with the energy $3 \times 10^{20}$ eV and the solid lines correspond to protons with three energies: $3 \times 10^{20}, 10^{22},$ and $3 \times 10^{23}$ eV. One can see that the AGN radio lobes are at most marginally consistent with being the sources of the highest energy cosmic rays. Moreover, only a handful of such sources are relatively nearby (e.g., M87, Cen A, NCG 315) but their angular distribution is completely uncorrelated with the nearly isotropic distribution arrival directions of UHECRs. Shocks in galaxy clusters can be such sources, according to Figure 1. It has been argued from a more detailed analysis of shock acceleration, however, that they are not able to accelerate protons above $\sim 6 \times 10^{19}$ eV [10]. Overall, large objects are more preferable candidates for the highest energy cosmic ray sources, both due to the larger terminal energy of an accelerated particle and the larger energy reservoir available, see Fig. 5 in the paper by Kronberg [11].

Equation (1) together with (4) or (8) puts a lower bound on the size of the accelerating source:

$$ R \gtrsim 6.0 \times 10^{-5} E_{20}^2 Z^2 A^{-4} \text{kpc}, $$

where $E_{20} = E/10^{20}$ eV. This is a quite remarkable result since it sets the absolute limit on R, independent of the field strength. Now, two special cases follow. First, the size $R$ exceeds the GZK distance $\sim 20$ Mpc when the energy of the proton is larger than $E_{GZK} \sim 7 \times 10^{22}$ eV. That is, such an energetic proton will lose its energy through the interaction with the 2.7 K background radiation right at the acceleration site. Hence, it is unlikely that protons may be accelerated to the energies above $E_{GZK}$. Second, above the energy $E_{\text{Hor}} \sim 4 \times 10^{23}$ eV, the size of the accelerator exceeds the size of the Universe. Thus, $E_{\text{Hor}}$ is the ultimate upper bound on the energy of electromagnetically accelerated protons. Note that this energy limit is below the energy of primary protons ($\sim 10^{24}$ eV) in the most popular (among other models [12, 13]) Z-burst model.

To conclude, we arrived at an interesting result. Practically all known astronomical sources are not able to produce cosmic rays with energies near few times $10^{20}$ eV. There is not too much room left for the conventional electromagnetic (in a broad sense) acceleration. Pushing observations up in energy by about an order of magnitude should clarify this situation. During ten years of operation of the Auger cosmic ray observatory about 300 events above $10^{20}$ eV are expected to be recorded. Extrapolating the present AGASA spectrum one can then expect a few events near or above $10^{21}$ eV to be recorded, which may be enough to exclude radio lobes from UHECR sources. Should this happen, a considerable revision of the current astrophysical picture will be inevitable.

Acknowledgements

We thank Andrei Beloborodov for a discussion and an anonymous referee for very careful reading and many suggestions greatly improving the manuscript.

[1] Greisen, K., Phys. Rev. Lett., 16, 748 (1966)
[2] Zatsepin, G. T., & Kuzmin, V. A., Sov. Phys. JETP Lett., 4, 78 (1966)
[3] Schluter, A. & Biermann, L., Z. Naturforsch. 5A, 237 (1950)
[4] Hillas, A. M., Annual Rev. Astron. & Astrophys., 22, 425 (1984)
[5] Aharonian, F. A., et al., astro-ph/0202229 (2002)
[6] Kazanas, D., & Ellison, D. C., Nature (London), 319, 380 (1986)
[7] Lovelace, R. V. E., Nature (London), 262, 649 (1976)
[8] Pomeranchuk, I., J. Phys., 2, 65 (1940)
[9] Hillas, A. M., Nature (London), 395, 15 (1998)
[10] Kang, H., Ryu, D., & Jones, T. W., Astrophys. J., 456, 422 (1996)
[11] Kronberg, P. P., Phys. Today, 55-12, 40 (2002)
[12] Sarkar, S., hep-ph/0202013 (2002)
[13] Watson, A. A., astro-ph/0112474 (2001)
FIG. 1: The $B$ vs. $R$ diagram for UHECR sources. The long-dashed line is the original Hillas relation, eqn. (1), for a proton of energy $3 \times 10^{20}$ eV. The short-dashed and dot-dashed lines represent the radiative cooling constraints for the diffusive and inductive acceleration, given by eqns. (4) and (8), for the same proton energy. Note that above the dot-dashed line the force of radiative friction dominates over any electromagnetic forces. The dotted and solid lines represent the boundaries of the allowed parameter regions for $3 \times 10^{20}$ eV iron nuclei and for protons of energies $3 \times 10^{20}$, $10^{22}$, and $3 \times 10^{23}$ eV, respectively. Only those astronomical objects which fall inside the “wedges” are, in principle, capable of accelerating the particles to such energies. The gray vertical lines mark two characteristic scales: the GZK attenuation distance ($\sim 20$ Mpc) and the Hubble horizon size ($\sim 4$ Gpc). For GRBs we took into account that the Lorentz boost changes with radius.

[14] The paper by \cite{footnote14}, appeared when the present paper was in preparation, accounts for the finite source size but still not self-consistently.

[15] That is at a cylindrical radius $R_{lc}$ at which the linear velocity of magnetospheric field lines is close to the speed of light, $\Omega R_{lc} \simeq c$, where $\Omega$ is the angular velocity of a magnetized object, e.g., a pulsar.

[16] The above criteria are applicable only to cosmic accelerators which involve acceleration on curved paths; no constraints on linear cosmic accelerators (if any) are placed.
FIG. 2: Cartoons representing a typical system with inefficient (diffusive) acceleration (a) and with efficient (inductive) acceleration (b).