A PROPOSED ANALYTICAL SOLUTION OF CYLINDER SHELL CONTAINING A CIRCUMFERENTIAL PART-THROUGH FISSURE

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Abstract

This study proposes an analytical solution method for investigating vibrational characteristics for a tubular cylindrical shell of a finite-length and bares a circumferential part-through fissure. The effect of different parameters i.e. length, depth and the fissure's location, on the vibrational characteristics, were also investigated. The equations for motion, that are founded on the classical shell theory for the fissured shell were transformed into simpler equations via Donell–Mushtari–Vlasov (DMV) hypothesis. The equivalent bending stiffness of the shell (D) was calculated by an exponential function while taking into consideration the effect of the fissure. The analytical approach gave us results for a structure with simply supported (S-S) at both ends boundary conditions. The natural frequencies were obtained by solving the general equations on a program built for "MATLAB" SOFTWARE. The results that were obtained from the suggested modal were confirmed by the use of a modal created by ANSYS APDL ver.15 in addition to the results that were attained from literature. There was a passable agreement between the results of the analytical and FE model. The results set forth that as fissure's parameters, length & depth, increasing them reduces the natural frequency. In addition to this, the natural frequency will also decrease if the fissure is located in the middle of the shell is larger than if it were in other locations.

Keywords: Cylindrical shell, vibration characteristics, Part-Through fissure, natural frequency.

I. Introduction

Shell structures have existed in numerous different fields i.e.: (mechanical, aeronautical, and civil engineering). A cylindrical tubular shell is one an important kind of the shell that has received obtained attention and is used in numerous fields,
for example: cooling towers, pipelines, aeronautical, automobile industries, large dams, shell roofs, naval structures, and liquid-retentive constructions. Due to the growing requirements for tubular shells and their extensive area of uses, So there is a necessary need for analyzing their dynamic features theoretically. These constructions commonly employ in complex conditions susceptible to various types of loads like (dynamic and fatigue loads) lead to damaging it's by presenting fissures and other flaws deteriorating operational parameters of constructions. Cracks that are on the surface are the most usual flaws in any type of plates and shells, cracks in tubular shell constructions may have an effect on the drop shell's stiffness and strength, of which in turn would lower the shell's natural frequencies. Keeping this in mind, the reflection of defects that are pre-formed is an important prodigy when evaluating tubular shell in free vibration. The vibration features' analysis for tubular shells are more difficult than other structures like (beams and plates structures). This is due to that mainly the motion's equations for tabular shells jointly with boundary conditions are more complicated. Conversely, the case of free vibration for fissured tubular shells has been investigated widely in recent decades, so as to evade undesired accidents which may produce unstainess and tearing.

Scholars have aided considerably to the vibration investigation for shell constructions so as to create analytic relations between the quantity and position of the fissure and variant in the dynamic features of the shell as a result from the fissure. Nikpour, 1990 [X] presented a method for solving symmetrical vibrations for the tubular shell that have thin laminated anisotropic involving a circumferential type fissure to reveal the depth and position of the fissure which pre-existed on the shell. C. Wang and J.C.S. Lai ,2000 [IV] introduced the approach to predict the natural frequencies based on Love's equations of finite length cylindrical shells with different boundary conditions without simplifying the equations of motion. Roytman and Titova, 2002 [VII] enhanced numerous mathematical Techniques for elastic vibrations of a tubular shell with a surface-closed type fissures.  Ip and Tse, 2002 [VIII] created the fissure detecting method in tubular composite shells built on the natural frequencies and the shapes of the modes at mode shape according to information at certain places. Javidruzi et al.,2004 [XVII] investigated the dynamic behavior of tabular l shells containing different types of fissures fissure subjected to fissure with set supports and lay open to an in-plane (compressive/tensile cyclic edge load). Vaziri and Estekanchi, 2006 [III] found that the existence of fissures in tabular shells could harshly affect the buckling performance of shell constructions by decreasing their capability for carrying-loads as well as the insertion of buckling locally at the fissure zone. Xin and Wang, 2011 [VII], investigated how the performance of free vibration and buckling of a tabular shell are effected by the length of a fissure, orientation, constant rotating speed and the length diameter ratio. In addition to this the stability features of the fissured shell as well as how the fissure's length. Its orientation, basic speed of rotation, steady load factor, the factor for dynamic load and the damping ratio were also investigated.

At 2012, M. J. Jweeg et al., [XV], presented an analytical solution by solving the general equation of motion to calculate the natural frequency of a plate structure with various crack depth and location effect. Yin and Lam, 2013 [XXI] proposed a
A new analytical method founded on the bending stiffness of a shell is presented to analyze the vibration behavior for a cylindrical shell involving a circumferential fissure. The fissure's effect on the natural frequencies and mode shape are analyzed. The analytical results were verified with the results obtained from FEM and revealed good agreement.

II. Mathematical Model

shell is supposed to be made up of a material that is isotropic, linear and has a thickness \( h \), which is small when compared to its other geometrical dimensions. (\( R \) is the radius of the cylindrical shell; \( l \) is the length of the shell), presume that there is a fissure on a circumferential surface for a tubular shell having a (length \( l \), mean radius \( R \) and thickness \( h \)). A circumferential fissure has a (finite-length of \( a \) and a uniform depth of \( h_c \) ) is lying at a distance \( x_c \) from one end on the external surface of the shell and in parallel to its length.
The equations that dominate the cylindrical shell’s motion without fissure in terms of the displaced components $u, v$ and are to be derived based on the classical shell theory. [XX]

\[
\frac{\partial N_{xx}}{\partial x} + \frac{1}{R} \frac{\partial N_{x\theta}}{\partial \theta} + f_x = \rho
\]

(1)

\[
\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R} \frac{\partial N_{xx}}{\partial \theta} + f_\theta = \rho \ddot{v} \rho
\]

(2)

\[
\frac{\partial \theta_x}{\partial x} + \frac{1}{R} \frac{\partial \theta_{x\theta}}{\partial \theta} - \frac{n_{x\theta}}{R} + f_z = \rho \ddot{w}
\]

(3)

Where the density of the material for the shell is $\rho$, $(N_{xx}, N_{x\theta}, N_{x\theta}, N_{\theta\theta})$ denote the resultant forces and transverse shear forces acting on the mid-surface, respectively. $(f_x, f_\theta, f_z)$ represent the external forces along the $x, \theta$ and $z$ directions, respectively.

Load and moment components in shell displacement terminology can be stated as [XX]:

\[
N_{xx} = C \left( \frac{\partial v}{\partial x} + \frac{v}{R} \frac{\partial v}{\partial \theta} + \frac{v}{R} w \right)
\]

(4)

\[
N_{x\theta} = N_{x\theta} = C \left( \frac{1}{R} \frac{\partial v}{\partial \theta} + \frac{\partial u}{\partial x} \right)
\]

(5)

\[
N_{\theta\theta} = C \left( \frac{1}{R} \frac{\partial v}{\partial \theta} + \frac{w}{R} + \frac{v}{R} \frac{\partial u}{\partial x} \right)
\]

(6)

Where:

\[
C = \frac{Eh}{1 - \nu^2} = \frac{12D}{R^2}
\]

(7)

\[
M_{xx} = D \left( - \frac{\partial^2 w}{\partial x^2} + \frac{v}{R^2} \frac{\partial v}{\partial \theta} - \frac{v}{R} \frac{\partial^2 w}{\partial x^2} \right)
\]

(8)

\[
M_{x\theta} = D \left( \frac{1}{R^2} \frac{\partial v}{\partial \theta} - \frac{1}{R^2} \frac{\partial^2 w}{\partial \theta^2} - \frac{v}{R} \frac{\partial^2 w}{\partial x^2} \right)
\]

(9)

\[
M_{x\theta} = M_{x\theta} = D \left( \frac{1 - \nu}{2} \left( \frac{1}{R} \frac{\partial v}{\partial x} + \frac{2}{R} \frac{\partial^2 w}{\partial x \partial \theta} \right) \right)
\]

(10)

Fig. 1: Coordinate system of a thin tubular cylindrical shell.
Finally, using equations (4-6) and (11,12), the equations for motion, equations (1-3) can be stated in the form of the components of displacement \( u, v \) and was: [XX]

\[
\mathcal{C} \left( \frac{1-v \partial^2 u}{\partial x^2} + \frac{1}{2R^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{v \partial w}{R \partial x} + \frac{1+v \partial^2 v}{2R} \frac{\partial^2 v}{\partial x \partial \theta} \right) + f_x = \rho h \ddot{u}
\]

(13)

\[
\mathcal{C} \left( \frac{1-v \partial^2 u}{\partial x^2} + \frac{1}{2R^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{v \partial w}{R \partial x} + \frac{1+v \partial^2 v}{2R} \frac{\partial^2 v}{\partial x \partial \theta} \right) + D \left( \frac{1-v \partial^2 v}{\partial x^2} + \frac{1}{R^2} \frac{\partial^2 v}{\partial \theta^2} - \frac{\partial^3 w}{R^2 \partial \theta^2} \right) + f_y = \rho h \ddot{v}
\]

(14)

The motion equations for a cylindrical shell, (13-15) can be made simpler with the aid of the Donnell–Mushtari–Vlasov (DMV) theory. When vibration is taken as the frame of reference, the following assumptions are made: 1. neglecting the terms of in-plane displacements \( u, v \) and to the bending moment resultant (8-10). 2. neglecting the influence term of the shear \((1/R) Q_{\partial z} \) in the equation of motion corresponding to \( v \) Eq. (2). This is equivalent to neglecting the term involving \( D \) in Eq. (14). The equations of motion corresponding to the DMV theory can be expressed as follows: [XX]

\[
\frac{\partial^2 u}{\partial x^2} + \frac{1-v \partial^2 u}{2R^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{v \partial w}{R \partial x} + \frac{1+v \partial^2 v}{2R} \frac{\partial^2 v}{\partial x \partial \theta} = \left(1-v^3\right) \rho \frac{\partial^2 u}{\partial t^2}
\]

(16)

\[
\frac{1-v \partial^2 v}{2R^2} \frac{\partial^2 v}{\partial \theta^2} + \frac{v \partial w}{R \partial x} + \frac{1+v \partial^2 u}{2R} \frac{\partial^2 u}{\partial x \partial \theta} = \left(1-v^3\right) \rho \frac{\partial^2 v}{\partial t^2}
\]

(17)

\[
-h^2 \left( \frac{\partial^2 w}{\partial x^2} + \frac{2 \partial^2 w}{R^2 \partial x^2 \partial \theta^2} + \frac{\partial^3 w}{R^2 \partial \theta^2 \partial \phi^2} \right) - \left(1-v^3\right) \rho \frac{\partial^2 w}{\partial t^2}
\]

(18)

The effect of a circumferential fissure on a cylindrical shell is obtained by calculating the bending stiffness \( D \) by an exponential function as given by: [XIV]

\[
D = \frac{D_o}{1+e^{-2(\frac{h-c}{h})}} = D_o \cdot f(x)
\]

(19)

Where:

\[
D_o = \frac{Eh^3}{12(1-v^2)}
\]

(19-A)

\[
f(x) = \frac{1}{1+e^{-2(\frac{h-c}{h})}}
\]

(19-B)

\[
S = \frac{I_o}{l_c}
\]

(19-C)

\[
l_o = \frac{2}{3} (R_o^2 - R_i^2)
\]

(19-D)
\[ I_c = \frac{1}{2} (R_0^2 - R_1^2) - \frac{1}{4} (R_0^4 - R_1^4) + \frac{a}{2x} \]  
(19-E)

\[ \alpha = -0.02391 + 0.027616 \frac{x}{t} + 0.002666 \frac{h_s}{h} + 0.00415 \frac{a}{2\pi R} \]  
(19-F)

\( I_o, I_c \) are the second moment of areas of the un-fissured and fissured cylindrical shell, respectively. \( R_0, R_1 \) are the outer and inner radius of the shell, respectively. \( R_c \) is the radius of cracked shell, \( h \) is the thickness of shell, \( x \) is the position along the shell, \( x_c \) is the position of the fissure, and \( \alpha \) is a non-dimensional value which is determined by multiple linear regression method and using numerical results.

Fig [2] shows the boundary conditions, which are simply supported, for the fissured cylindrical shell at \( x=0 \& x=l \) and are represented by the following equations [XX]:

\[ v(x, \theta, t) = 0 \]  
(20)

\[ w(x, \theta, t) = 0 \]  
(21)

\[ N_{xx}(x, \theta, t) = C \left( \frac{\partial u}{\partial x} + \frac{v}{R} \frac{\partial v}{\partial \theta} + \frac{w}{R} \right)(x, \theta, t) = 0 \]  
(22)

\[ M_{xx}(x, \theta, t) = D \left( -\frac{\partial^2 w}{\partial x^2} + \frac{v}{R} \frac{\partial^2 v}{\partial \theta^2} - \frac{v}{R^2} \frac{\partial^2 w}{\partial \theta^2} \right)(x, \theta, t) = 0 \]  
(23)

\[ v(l, \theta, t) = 0 \]  
(24)

\[ w(l, \theta, t) = 0 \]  
(25)

\[ N_{xx}(l, \theta, t) = C \left( \frac{\partial u}{\partial x} + \frac{v}{R} \frac{\partial v}{\partial \theta} + \frac{w}{R} \right)(l, \theta, t) = 0 \]  
(26)

\[ M_{xx}(l, \theta, t) = D \left( -\frac{\partial^2 w}{\partial x^2} + \frac{v}{R} \frac{\partial^2 v}{\partial \theta^2} - \frac{v}{R^2} \frac{\partial^2 w}{\partial \theta^2} \right)(l, \theta, t) = 0 \]  
(27)

\[ f(0) = A_1 \]  
(28)

\[ f(l) = A_2 \]  
(29)

\[ f'(0) = B_1 \]  
(30)

\[ f'(l) = B_2 \]  
(31)

\[ f''(0) = E_1 \]  
(32)

\[ f''(l) = E_2 \]  
(33)

**Fig. 2:** simply supported shell. [XX]

The following are the assumed forms for solving the equations of motion forms that are consistent with the DMV theory: [XX]

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Where $\omega$ is the circular frequency, and $A_{mn}$, $B_{mn}$ and $C_{mn}$ are arbitrary coefficients, $m$ is the number of displaced half-waves along the length of the shell known as "Longitudinal Half-Waves"; whereas $n$ is the number of displaced circumferential half-waves "Circumferential Waves" [see Fig. (3)].

![Fig. 3: Circumferential and longitudinal modes of cylindrical shell][1]

The assumed solution [Eqs. (34-36)] fulfills the boundary conditions [Eqs. (20-33)]. After compensating an equation of the bending stiffness $D$ for a cracked cylindrical shell in eqs. (16-18), Substituting eqs. (34-36) then eqs. (20-33) in the fissured shell's equations for motion and obtain:

$$
- A_1 (\lambda^2 + a_1 n^2) + \Omega]A_{mn} + [A_1 (a_2 \lambda n)]B_{mn} + [A_1 (\nu \lambda)]C_{mn} = 0 \quad (37)
$$

$$
- B_1 , R (a_1 n)]A_{mn} + [B_1, R(a_1 \lambda)]B_{mn} = 0 \quad (38)
$$

$$
[B_1, R(-2\lambda \mu^2 - 2R \lambda n^2 \mu)]C_{mn} = 0 \quad (39)
$$

Where $\lambda = \frac{m \pi R}{l}$, $\mu = \frac{h^2}{12}$, $\Omega = \frac{(1-\nu)R^2 \rho}{E} \omega^2$, $a_1 = \frac{1-\nu}{2}$, $a_2 = \frac{1+\nu}{2}$.

For a significant solution of the coefficients, $A_{mn}$, $B_{mn}$ and $C_{mn}$, the determinant for their coefficient matrix in Eqs. (37-39) must be zero. This produces:

$$
\begin{bmatrix}
- A_1 (\lambda^2 + a_1 n^2) + \Omega & A_1 (a_2 \lambda n) & A_1 (\nu \lambda) \\
- B_1, R (a_1 n) & B_1, R(a_1 \lambda) & 0 \\
0 & 0 & -B_1, R(2\lambda \mu^2 + 2R \lambda n^2 \mu)
\end{bmatrix}
\begin{bmatrix}
A_{mn} \\
B_{mn} \\
C_{mn}
\end{bmatrix}
= 0
\quad (40)
$$

The expansion of Eq. (40) produces the frequency equation:

$$
e_1 \Omega + e_2 = 0 \quad (41)$$
Where:

\[
e_1 = -\left(2a_1 \lambda^2 B R^2 + 2 R^3 a_1 \lambda^2 n^2 \mu B \right)
\]

\[
e_2 = \left(2A_1 \lambda^4 B R^2 a_1 \mu^3 + 2 A_1 a_1^2 n^2 B R^2 \lambda^2 \mu^3 + 2A_1 \lambda^4 B R^2 a_1 n^2 \mu + 2A_1 a_1^2 \mu^3 B R^3 \lambda^2 \mu \right.
\]

\[
- 2A_1 B R^2 a_1 a_2 \lambda^2 n^2 \mu^3 - 2A_1 B R^3 a_1 a_2 \lambda^2 n^3 \mu \right)
\]

\[
\omega^2 = \frac{\Omega E}{(1 - \nu^2) R^2 \rho}
\]

\[
= \frac{E e_2}{(1 - \nu^2) R^2 \rho e_1}
\]

From eq. (44) we are able to obtain the natural frequencies and corresponding mode shapes for the fissured cylindrical shell construction.

With the aid of the MATLAB program the analytical solution can be obtained in addition to obtaining the natural frequency of the circumferential fissure for the cylindrical shell.

**III. Numerical Model**

The numerical analysis is performed using the Finite Element Software, ANSYS Mechanical APDL ver.15 is used to model the cylindrical shells containing a circumferential surface fissure for linear isotropic material. element type SHELL 93 is suitable to modeling curved shells.

The Validation of the finite element models for perfect cylindrical shells is verified, by the free vibration analysis of a perfect cylindrical shell is performed with radius (R = 242.3 mm), length (l = 609.6 mm), thickness (h = 0.648 mm), Young’s modulus (E = 70 MPa), density (ρ = 2700 Kg/m³) and Poisson’s ratio (ν = 0.35). The simply supported shell (S-S) type boundary conditions are applied to both ends of the shell for the analysis of the tubular shell as shown in (fig.4).

![Fig. 4: mesh of perfect cylindrical shell.](image)

Table 1 shows that the results (analytical, experimental and numerical) obtained from the literature are united agreement with the present results (analytical and numerical) of the fundamental frequencies.
Table 1: Comparison the results obtained from the literature with the present work.

| No. | References                                    | Frequency (Hz) |
|-----|-----------------------------------------------|----------------|
| 1   | Experimental (Cook, 1981)[XVIII]              | 163 and 169    |
| 2   | Analytical (Bolotin 1964)[XXII]               | 168.13         |
| 3   | Numerical (Sewall and Naumann, 1968)[V1]      | 166.22         |
| 4   | Numerical (Javidruzi et al., 2004)[XVII]      | 166.40         |
| 5   | Numerical (Xin and Wang 2011)[VII]           | 168.73         |
| 6   | Present analytical analysis                   | 168.47         |
| 7   | Present numerical analysis                    | 165.39         |

IV. Results and Discussions

The vibration characteristics of a S–S type cylindrical shells with and without a circumferential surface fissure are studied analytically and numerically. The free vibration analysis of a cylindrical tubular shell is performed, the effect of parameters, such as (fissure length, fissure depth and location of crack) are investigated. The dimensions of the cracked shell as follows: \((R) = 500\text{mm}, (l) = 1000\text{mm},\) and \((h) = 1\text{mm}\).

![Fig. 1: (a) mesh of shell with middle fissure location. (b) mesh of shell with side fissure location.](image)

IV.i. The Effect of Fissure Length on the Natural Frequency

A circumferential fissured cylindrical shell with specific depth and location of the fissure is studied considering the effect of fissure length. (Fig. 6) shows the influence of circumferential fissure length on the fundamental natural frequency of free vibration. The presence of the fissure reduces the first vibration mode; this lowering is at first progressively as the fissure length increases but being very prompt when the fissure length overtake about 10% of the circumference of the shell.

(fig. 8) indicate that the vibration of the first mode is localized at the fissure zone, and the local phenomenon becomes more and more serious as the fissure length increases. It is also found that vibration exists only in a very small fissure region when the fissure size reaches 0.2 of the circumference of the shell. vibration is very week.
IV.ii. The Effect of Fissure Depth on the Natural Frequency

The natural frequency decreasing with increasing fissure depth, this is because of the variant in stiffness of cylindrical shell. (Fig. 6) shows that the analytical, and numerical results obtained of the first order natural frequency for different depth and length of the circumferential crack.

IV.iii. The effect of location of the fissure on the natural frequency

Fig.(7 and 9) shows the analytical, and numerical results obtained of the fundamental natural frequency with 0.25 (2πR) fissure length and depth fissure (hc = 0.75h) for different location of the circumferential crack. The natural frequency decreases when the fissure in the middle of the shell over any location of the crack, the effect of fissure when it reaches the middle is higher than when it’s in the other places.

Fig. 6: Effect of circumferential fissure size (length, depth) on the fundamental natural frequency

Fig. 7: effect of different fissure locations on the fundamental natural frequency

(a) Vibration modes with a / 2πR = 0.1 , and hc/h= 0.75
(b) Vibration modes with $a/2\pi R = 0.2$, and $hc/h = 0.75$

(c) Vibration modes with $a/2\pi R = 0.1$, $a/2\pi R = 0.1$, and $hc/h = 0.95$

(d) Vibration modes with $a/2\pi R = 0.2$, $a/2\pi R = 0.1$, and $hc/h = 0.95$

Fig. 8: Vibration modes of cracked shell with different crack lengths and depth locate at 0.5l
V. Conclusion

An analytical modeling has been advanced to study the characteristics of dynamic vibration of a simply supported circular cylindrical shells involving circumferential fissure. The influence, of the fissure's features i.e. : length, depth and location of the fissure on the natural frequencies as well as the tubular cylinder shaped shell's mode shapes, were investigated. Results from both were intact and fissured cylindrical tubular shells that were obtained diagnostically and verified versus literature and finite element models results, respectively. The results can be summarized as follows:

1. The presence of fissure affects the dynamic vibration characteristics of the cylindrical shell as expected, a reducing in the natural frequencies.
2. The fissure reduces the fundamental natural frequency of vibration of cylindrical tubular shells gradually as the fissure length increases.
3. A fissure has a significant effect on the shell's natural frequency as well as its mode shapes and stiffness. We found that the shell's stiffness will decrease as a reaction the an increase in the shells depth and this will lead to a reduction of the shell's natural frequency.
4. The tabular cylindrical shell's natural frequency and its stiffness are greatly affected by the fissure's location. When the fissure is situated at its longitudinal middle then the frequency is at its lowest as to when is at the edges.

Fig. 9: Vibration modes of cracked cylindrical shell with different crack location.

(a) Vibration modes with crack locate ($x_c$) at alimit

(b) Vibration modes with crack locate ($x_c$) at $0.3l$

(c) Vibration modes with crack locate ($x_c$) at $0.5l$
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