WHY DO GOVERNMENTS END UP WITH DEBT? SHORT-RUN EFFECTS MATTER

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This paper reconsiders the impact of public debt in an economy with heterogeneous households and incomplete markets to emphasize the short-run effects of an increase in public debt. As compared to models that rest on steady-state analysis, we show that the welfare gains of a public debt increase are substantially higher when transitional dynamics are accounted for. The additional debt issue allows for a temporary reduction in the income tax rate, which stimulates labor supply and generates an overshooting of the interest rate. The short-run gains create a temptation to deviate toward higher levels of debt. Debt increases continue to generate welfare gains even when debt is considerably higher than its long-run optimal level. (JEL E60, H60)

I. INTRODUCTION

The last decades have been characterized by a quick accumulation of public debt. In the Organization for Economic Cooperation and Development (OECD) area, the ratio of public debt over GDP was equal to 59% in 1989 (OECD 2009). In 2008, this ratio was much higher: it amounted to 79% (OECD 2009). Yet, the optimal taxation literature predicts that governments should accumulate assets rather than issue debt. How can theory be reconciled with the observation that governments issue large amounts of debt? This paper sheds light on the liquidity-constraint loosening effect of public debt to explain why governments have a bias toward the accumulation of debt.

The seminal contribution of Ricardo (1951a, 1951b) has given rise to a rich theoretical literature on public debt. According to Barro (1979), tax smoothing, under certain circumstances, implies that the optimal tax rate follows a random walk. Consequently, the level of public debt is irrelevant for current debt issue: the government should issue debt only when government expenditures depart from their average value. Lucas and Stokey (1983) analyze a Ramsey model where public debt is state-contingent. As in the study by Barro (1979), the role of public debt is to smooth tax distortions over time. Contrary to Barro (1979), the optimal tax rate does not follow a random walk, but it inherits the statistical properties of government spending. Moreover, the current level of public debt depends on its initial level, which is exogenous. If the public debt is initially high (resp. low), the public debt will continue to be high (resp. low). Therefore, it does not enable to understand why governments tend to accumulate public debt. Chari, Christiano, and Kehoe (1994) revisit the result of Lucas and Stokey (1983) in a model augmented to allow for capital accumulation and business cycles. Chari, Christiano, and Kehoe (1994) show that the result of Lucas and Stokey (1983) continues to hold. The analysis of Aiyagari et al. (2002) reconsiders the result of Lucas and Stokey (1983) in a model where the government cannot fully insure against aggregate shocks because of market incompleteness. When the government has only access to one period risk-free bonds, the long-run public debt is negative. The behavior of the government parallels that of households, as it may find it worth to save for a precautionary issue.

ABBREVIATIONS

GDP: Gross Domestic Product
OECD: Organization for Economic Cooperation and Development
motive when facing expenditure shocks. Krusell, Martin, and Rios-Rull (2006) relax the commitment assumption in the Lucas and Stokey (1983) model. In the absence of aggregate uncertainty, they find no bias in favor of debt accumulation. It appears that these different theories are not able to explain the high and increasing level of public debt of the recent decades.

However, these different analyses rest on the assumption that agents are perfectly insured against idiosyncratic income risk. This assumption has, since then, been challenged (Aiyagari 1994b). Aiyagari and McGrattan (1998) depart from the representative agent growth model by assuming that agents face income shocks. In this setup, they show that the optimal quantity of public debt is positive. A higher public debt enhances the liquidity by providing an additional means of smoothing consumption and loosens the borrowing constraint. That is why public debt is welfare enhancing. However, the welfare gains of a higher public debt are very low: the role for public debt is of minor importance. Moreover, Floden (2001) emphasizes that positive public debt effects vanish if transfers are used optimally. Here, we argue that the analysis provided by Aiyagari and McGrattan (1998) and Floden (2001) may underestimate the role of public debt. Their analyses do not take into account the liquidity-constraint loosening effect of public debt that operates in the short run. At the date of the debt increase, the additional resources obtained from the issue can, for instance, be redistributed through tax reductions. As some households in the economy would like to borrow, but are unable to do so, they are likely to increase their consumption in response to the temporary increase in disposable income (Daniel 1993; Heathcote 2005).

In this paper, Floden’s (2001) economy is extended to take into account the short-run effects of public debt that we capture by modeling explicitly the transition from one steady state toward another. Financial markets are incomplete and there is a large number of ex ante identical infinitely lived agents. Agents differ in ability and face idiosyncratic labor income shocks. Labor supply is assumed to be endogenous. Households are subject to a proportional income tax. As private insurance markets are incomplete, agents, who cannot borrow, save for a precautionary motive (Aiyagari 1994a). We recall one of Floden’s (2001) results: when comparing steady states, the consumption gain of being at the optimal public debt-to-GDP (gross domestic product) ratio (150%), instead of its benchmark level (2/3), amounts to 0.123%. We show that the consumption gain is much higher when the short-run impact of public debt is taken into account. The consumption gain here amounts to 0.9%. Moreover, the analysis shows that the temporary reduction in the income tax rate implied by the additional debt issue is not the only mechanism which explains why consumption gains are higher. Indeed, the temporary reduction in the income tax rate leads to a temporary increase in labor supply and a temporary interest rate overshooting which contribute to raise the consumption gain. These results lead us to question the relevance of the long-run optimal level of public debt. The existence of short-run gains creates an incentive to deviate toward higher levels of public debt and could make increases above the long-run optimum beneficial. We search for the ratio of debt to GDP such that no further upward deviation would be welfare improving and find that it is as high as 560%.

Needless to say, this result should be handled carefully, as the fiscal environment considered here is highly stylized. In particular, it could be that shocks on transfers would do a better job than the adjustment of a unique tax rate. In addition, all policy shocks, which are considered here, occur as pure surprise, and the issue of the predictability of the implemented policies is not addressed. Consequently, our result ought to be regarded as an analysis of the effectiveness of unexpected debt increases in a constrained fiscal environment.

The paper is organized as follows. Section II presents the model and the transitional dynamics definition. We discuss the calibration in Section III. In Section IV, the optimal quantity of public debt and its distributional characteristics are determined. In Section V, we characterize the transitional path and compute the welfare gains of a higher ratio of public debt to GDP when short-run effects of public debt are taken into account. Section VI concludes.

II. THE MODEL

The economy is populated by a continuum of households, a representative firm, and the government.

A. The Representative Firm

Each period, one good is produced by a representative firm which has a Cobb-Douglas
production function. There is no aggregate risk and the production function writes:

$$F(K_t, Z_t, N_t) = K_t^\alpha (Z_t, N_t)^{1-\alpha}$$

where \(t\), \(K_t\), \(N_t\), and \(Z_t\), respectively, denote the date, the aggregate stock of physical capital, the detrended aggregate labor supply in efficiency units, and the labor productivity. \(Z_t\) grows at the exogenous rate \(g\), so we will write \(Z_t = (1 + g)^t\), given that the initial level of labor productivity is set to unity. Factor markets are competitive, so each factor earns its marginal efficiency units, and the labor productivity.\( \bar{z} \)

Given the homogeneity of degree 1 of the production function, a balanced growth path is possible, where \(N_t\) and \(r_t\) are both constant, and where \(K_t\) and \(w_t\) grow at the rate \(g\).

**B. Households**

There is a continuum of infinitely lived agents of unit mass. Each period, households receive capital, bond, and labor income. Households are born with permanently different abilities and face an idiosyncratic risk on their individual labor productivity. This shock \(e_t \in E\) is governed by a Markov chain. Agents cannot borrow and there is no insurance market against this risk. In order to smooth consumption, agents hold are born with permanently different abilities. This shock \(\tilde{w}_t\), \(r_t\), \(\tau_t\), and \(\bar{T}r_t\) are all constants, so that the time dependence can be dropped. The decision rules will be denoted \(\tilde{c}_t\), \(\tilde{a}_t\), \(\tilde{l}_t\), and \(\tilde{a}_{t+1}\) in the dynamic version (resp. in the stationary one).

**C. The Government**

The government issues public debt, taxes labor, and capital income and operates transfers. Its expenses—government spending, transfers and the interest payment of the current debt—match its revenues—tax collection and the net public debt issue:

$$G_t + \bar{T}r_t + r_t B_t = B_{t+1} - B_t + T_t$$

where \(G_t\), \(\bar{T}r_t\), \(B_t\), and \(T_t\) are, respectively, public consumption, lump-sum transfers, the current stock of public debt, and tax revenues, defined as follows:

$$T_t = \tau_t (N_t w_t + r_t A_t) = \tau_t (N_t w_t + r_t (K_t + B_t))$$

We assume that the policy instruments which the government chooses to impose are the following:

$$G_t / Y_t = \gamma_t, T_r / Y_t = \chi_t, B_t / Y_t = b_t.$$  

1. The time subscript for the stock of public debt refers to the date when debt will be paid back and not to the date of issue.  
2. The ratios are time-dependent because we will implement the transition from one equilibrium, characterized by certain values for these instruments, to another one. This notation then allows for any adjustment of the policy over time.
The detrended version of the government budget constraint writes:
\[ \hat{G}_t + \hat{T}_t + r_t \hat{B}_t = (1 + g) \hat{B}_{t+1} - \hat{B}_t + \hat{T}_t. \]

**D. Equilibrium Definition**

In this section, we define both the stationary economy and the transitional dynamics. Only the latter makes it possible to capture the short-run effects of a higher ratio of public debt to GDP.

**Stationary Equilibrium.** In this economy, we will denote stationary equilibrium, an equilibrium where detrended variables are constant over time. Given a policy \( \{ \chi, \gamma, b \} \), the stationary equilibrium consists of the vector \( \{ \hat{c}(\hat{a}, e), \hat{a}(\hat{a}, e), l(\hat{a}, e), \Lambda(\hat{a}, e), \hat{K}, N, \hat{Y}, r, \hat{w}, \tau \} \) where \( \Lambda(\hat{a}, e) \) is the probability measure of agents over the state space, or equivalently, the cross-sectional distribution of agents. The stationary equilibrium is attained when this vector is such that:

- Given \( r, \hat{w}, \tau \) and \( \hat{T}_t = \chi \hat{Y}_t \), the decision rules \( \{ \hat{c}(\hat{a}, e), \hat{a}(\hat{a}, e), l(\hat{a}, e) \} \) are solutions of the stationary version of program (1).
- \( \Lambda(\hat{a}, e) \) is the unique stationary distribution consistent with the previous decision rules.
- The labor and the capital markets clear:
  \[ \hat{K} + \hat{B} = \sum_{e \in E} \int \hat{a} \Lambda(\hat{a}, e) d\hat{a} \]
  \[ N = \sum_{e \in E} \int el(\hat{a}, e) \Lambda(\hat{a}, e) d\hat{a}. \]
- Factor prices verify:
  \[ r = F'_K(\hat{K}, N) - \delta \]
  \[ \hat{w} = (F'_N(K, ZN)) / (Z) = F'_N(\hat{K}, N). \]
- The government budget is balanced:
  \[ (r - g) \hat{B} = \hat{T} - (\hat{G} + \hat{T}_t) \iff (r - g) b = \tau (1 - \delta \hat{K} / \hat{Y} + rb) - (\gamma + \chi). \]

**Transitional Dynamics.** As we will compute the transition from one steady state to another, we need a theoretical characterization of more complex dynamics. Basically, we need, as a starting point, initial conditions, which fully describe the state of the economy—that is, the state of all the heterogeneous agents, and that of the government—at date \( t = 0 \). Given these initial conditions, the transition consists of a path of all relevant variables, namely that of \( r_t, \hat{w}_t, \hat{Y}_t, N_t, \hat{K}_t, \hat{B}_t, \hat{T}_t, \) and \( \tau_t \). The transition rests on these paths being consistent with the agents’ decision rules, which are simply the time-varying solutions of program (1)—note that this program is not only relevant for stationary environments, but also for time-varying ones. In practice, not all of the above variables are mutually independent. The core of the transition can be summarized by only two vectors, \( \{ r_t \}_{t \geq 0}, \{ N_t \}_{t \geq 0} \). The paths of \( \hat{w}_t, \hat{K}_t, \hat{Y}_t, \hat{B}_t, \) and \( \hat{T}_t \) can then be obtained. This leads us to the following definition of a transitional path:

Given initial conditions \( \{ \Lambda_0(\ldots), \hat{B}_0 \} \), and given the time vector of exogenous policy instruments \( \{ b_t, \chi_t, \gamma_t \} \), a dynamic rational expectation equilibrium consists of \( \{ \hat{c}_t(\hat{a}, e), \hat{a}_{t+1}(\hat{a}, e), l_t(\hat{a}, e), \Lambda_t(\hat{a}, e), r_t, \hat{w}_t, \tau_t, \hat{Y}_t \} \) such that:

- At any date \( T, \) given the vectors \( \{ (r_t)_{t \geq T}, (\hat{w}_t)_{t \geq T}, (\tau_t)_{t \geq T}, (\hat{T}_t)_{T \leq t \leq T+1} \}, \)
  \( \hat{c}_t(\hat{a}, e), \hat{a}_{t+1}(\hat{a}, e), l_t(\hat{a}, e) \) are the decision rules obtained from program (1),
- At any date \( T, \) \( \Lambda_{T+1} \) is derived from \( \Lambda_T \) and the above decision rules,
- At any date \( T, \) the labor and the capital markets clear, that is:
  \[ N_T = \sum_{e \in E} \int el(\hat{a}, e) \Lambda_T(\hat{a}, e) d\hat{a} \]
  \[ \hat{K}_T + \hat{B}_T = \sum_{e \in E} \int \hat{a} \Lambda_T(\hat{a}, e) d\hat{a}. \]
- At any date \( T, \) factor prices verify:
  \[ r_T = F'_K(\hat{K}_T, N_T) - \delta \]
  \[ \hat{w}_T = F'_N(\hat{K}_T, N_T). \]
- At any date \( T, \) the law of motion of the stock of public debt is:
  \[ (\gamma_T + \chi_T) \hat{Y}_T + r_T \hat{B}_T = (1 + g) \hat{B}_{T+1} - \hat{B}_T + \tau_T (\hat{Y}_T - \delta \hat{K}_T + r_T \hat{B}_T). \]

**III. Calibration**

The model period is the year. Because the calibration of the model is identical to Floden’s (2001), we will only stress the major points. The productivity process is approximated with Tauchen’s (1986) procedure. Wages are made up of two components, one permanent ability level, \( \mu \), and one temporary component, \( q. \mu \).
is \(iid\) with mean zero and variance \(\sigma^2_e\). The productivity process is defined as follows:

\[
\log(q_t) = \psi \log(q_{t-1}) + \varepsilon_t
\]

where \(\psi\) is the degree of persistence of shocks and \(\varepsilon \sim N(0, \sigma_e)\). Table 1 summarizes Floden’s (2001) calibration.

According to this calibration, there are 14 different productivity levels, divided into two subsets \(E_1 = \{e^1, e^2, e^3, e^4, e^5, e^7\}\) and \(E_2 = \{e^8, e^9, e^{10}, e^{11}, e^{12}, e^{13}, e^{14}\}\). Half of the agents belongs to subset \(E_1\), and the other half to \(E_2\), and no transition between the two subsets is possible. Each subset is ergodic. The productivity levels are:

|   | \(e^1\) | \(e^2\) | \(e^3\) | \(e^4\) | \(e^5\) | \(e^6\) | \(e^7\) |
|---|---|---|---|---|---|---|---|
| \(e^8\) | 0.135288 | 0.219024 | 0.354587 | 0.574056 | 0.929363 | 1.504585 | 2.435837 |
| \(e^9\) | 0.267042 | 0.432326 | 0.699911 | 1.133116 | 1.834449 | 2.969867 | 4.808044 |

**IV. STEADY-STATE OPTIMALITY**

In this section, we briefly present the results from stationary equilibrium simulations. Our baseline corresponds to a public debt-to-GDP ratio equal to \(2/3\). In the simulations, a single policy parameter is modified, namely the ratio \(b = B/Y\). The welfare is computed using the utilitarian criterion, as follows:

\[
W = \sum_{e \in E} \int_{\tilde{a} \in \Lambda} \tilde{V} (\tilde{a}, e) \Lambda (\tilde{a}, e) \, d\tilde{a}.
\]

For the gains or losses to be interpretable, we compute the percentage of consumption \(x\) that agents in the baseline would need in order to be indifferent with the policy change.\(^3\)

As in the study by Floden (2001), the optimal ratio of public debt to GDP is 150% and the welfare gains amount to 0.123%. The welfare differs according to the agents’ classes. We sort agents by their intertemporal welfare and focus on the lowest 5% and highest 5% percentiles of this distribution. We will call the former the poorest 5%, and the latter the richest 5%.\(^4\)

While the richest 5% of agents experience a high increase in consumption which amounts to 3.68%, the poorest 5% of agents undergo a decrease in consumption of 2.31%.

The distributional properties of public debt are reported in Figure 1. It plots the individual consumption-equivalent variation in percentage points, when comparing the baseline (\(b = 2/3\)) with the optimal steady state (\(b = 1.5\)). Each individual state is characterized by the current level of financial wealth—that is, the horizontal axis—and by the level of productivity—one curve for each level. These figures reveal that the welfare changes are increasing in both the stock of assets and the productivity shock. The increasing pattern of the curve derives from the fact that the gain of a higher interest rate depends positively on the current stock of wealth. The welfare variations are positive only for asset stocks above a threshold (between 3.29 and 7.58, depending on the productivity level considered). Moreover, it turns

\[
\hat{V} (\tilde{a}, e),
\]

this implies that:

\[
\hat{V} (\tilde{a}, e) = E_0 \sum_{t \geq 0} \left[ \beta (1 + g)^{(1-\rho)} \right]^t u (\tilde{c}_t, \ell_t)
\]

\[
\Rightarrow E_0 \sum_{t \geq 0} \left[ \beta (1 + g)^{(1-\rho)} \right]^t u ((1 + x) \tilde{c}_t, \ell_t)
\]

\[
= (1 + x)^{(1-\rho)} \hat{V} (\tilde{a}, e)
\]

\(^3\) We assume that the relative change in consumption leaves the leisure choice unchanged as Aiyagari and McGrattan (1998). For a given agent, with an expected utility on the lowest 5% and highest 5% percentiles of this distribution. We will call the former the poorest 5%, and the latter the richest 5%.\(^4\)

\(^4\) Our classification, in terms of intertemporal welfare, does not correspond exactly to the distribution of asset holdings. Indeed, the welfare also depends on the current productivity level of the agent. Calling the lowest 5% “the poorest 5%” is therefore not fully correct, but we use this expression for an obvious reason of simplicity.
out that 88.8% of the households are located below these thresholds. This is not inconsistent with the fact that the higher level of public debt yields a higher welfare; indeed, agents are richer with a higher public debt. This composition effect accounts for the apparent paradox.

Public debt affects the individual welfare levels through three channels: (1) it increases the income tax rate, (2) it reduces the wage, and (3) it increases the interest rate. The first two effects act negatively on the individual well-being, whereas the third one acts positively. Agents, whose productivity shock is low and/or assets holdings are low, are likely to suffer from increasing public debt. Agents, whose productivity shock and/or assets holdings are high, are likely to experience large gains from increased public debt.

V. OPTIMALITY IN THE TRANSITION

We here simulate the transition of the economy from a steady state, characterized by a given set of policy instruments, to another one. Our main motivation is to shed light on the liquidity-constraint relaxing effect of public debt in the short run. If public debt is increased at date \( t = 0 \), it allows for an income tax rate reduction which should benefit liquidity constrained agents. Consequently, the analyses of Aiyagari and McGrattan (1998) and Floden (2001) do not capture the short-run gains associated with a higher ratio of public debt to GDP. The model simulations are intended to answer the following questions:

1. What are the order of magnitude of the short-run effects, omitted by Aiyagari and McGrattan (1998) and Floden (2001)?
2. What are the distributional characteristics of public debt on the transitional path, as compared to their steady-state counterpart?

3. Do the short-run welfare effects considerably alter the steady-state message regarding the optimal level of public debt?

A. Simulations of the Transitional Dynamics

The pre-reform (resp. the post-reform) public debt-to-GDP ratio will be denoted $b_{\text{init}}$ (resp. $b$). Although the model itself has already been fully described in the second section, some assumptions regarding the timing of the policy change are necessary. Precisely, we assume that the change pertains to the ratio of public debt to GDP, and is operated in $T_{\text{policy}}$ periods, from date $t = 0$ to date $t = T_{\text{policy}} - 1$. We assume that the public debt-to-GDP ratio is chosen to evolve linearly toward its final level:

\begin{equation}
(2) \quad b_t = b_{\text{init}} + \left(\frac{t}{T_{\text{policy}}}\right) (b - b_{\text{init}}),
\end{equation}
for $t < T_{\text{policy}}$

\begin{equation}
(3) \quad b_t = b, \quad \text{for } t \geq T_{\text{policy}}.
\end{equation}

The ratios $\gamma_t$ and $\chi_t$ are kept unchanged throughout the transitional path. We assume that the income tax rate adjusts to guarantee a balanced budget constraint. It therefore satisfies the following equations:

\begin{equation}
\tau_0 = \left((\gamma_0 + \chi_0)\hat{Y}_0 + (1 + r_0)\hat{B}_0\right)
- (1 + g) b_1\hat{Y}_1/(\hat{Y}_0 - \delta K_0 + r_0\hat{B}_0)
\end{equation}

\begin{equation}
\tau_t = \left((\gamma_t + \chi_t)\hat{Y}_t + (1 + r_t) b_t\hat{Y}_t\right)
- (1 + g) b_{t+1}\hat{Y}_{t+1}/(\hat{Y}_t - \delta K_t + r_t\hat{B}_t)
\end{equation}
for $t \geq 1$.

The difference between the two equations derives from the fact that, for $t \geq 1$, the level of debt is given by $\hat{B}_t = b_t\hat{Y}_t$. It is however not the case at date $t = 0$; indeed, the level of debt represents the fraction $b_{\text{init}}$ of the initial GDP $\hat{Y}_{-1}$, but the GDP is not predetermined (because of the endogenous labor supply), which means that $\hat{Y}_0 \neq \hat{Y}_{-1}$.

The date 0 welfare change, which takes full account of the short-run effects of public debt and which will be denoted as the transition-adjusted welfare change, is here again presented in percentage points of consumption. The baseline economy ($b = 2/3$) is chosen to be the initial steady state and the welfare of the whole population on the transitional path is measured at date $t = 0$. Because this utilitarian criterion hides the diversity of the individual welfare adjustments, we also include the proportion of agents for whom the welfare is increased by the policy change in Table 2.

As the public debt-to-GDP ratio is switched from its initial level ($b_{\text{init}} = 2/3$) to its long-run optimal one ($b = 1.5$), we have set $T_{\text{policy}}$ at different values, ranging from 8 to 20. Table 2 presents the results.

The welfare gains are much larger when short-run effects are taken into account. The poorest 5% of agents do not undergo a decrease in consumption anymore. It can be noticed that the wealthiest agents are better off, the lower $T_{\text{policy}}$, while the reverse applies for the poorest ones. This follows from the nature of the gains for these different agents. For the richest 5%, the gains are driven both by their high level of asset holdings and by their high current wage. Because of the Markovian structure of the individual productivity shocks, which is characterized by the regression-toward-the-mean property, these agents are likely to be hit by a negative productivity shock in the near future, bringing their wage closer to the average one. It is in their interest that the income tax rate reduction be as high and as short-lived as possible. Conversely, as $T_{\text{policy}}$ increases, the expected gains from a public debt increase would then be lower, as the reduction of the income tax rate would be of a lesser magnitude, and over a longer time spell. Regarding the poorest 5%, their current wage is very low, and they hardly own any financial assets. The income tax rate reduction would not be so beneficial at date 0, precisely because their current labor supply is rather low.

The next section addresses the second question. A particular transition is more thoroughly analyzed, and the distributional properties of a public debt increase are brought to light.

5. That is, those for whom the individual percentage change of consumption is positive.

6. We have not calculated the proportion of agents favoring the policy shock through steady-state comparisons because different steady states imply different distributions of agents. For example, the poorest 5% for $b_{\text{init}} = 2/3$ are not the same as the poorest 5% when $b = 1.5$. Moreover, comparing steady states means that we only consider long-run equilibria, and in the long run, all agents are ex ante alike.
TABLE 2
Transition-Adjusted Welfare Change When $b$ Is Switched from $2/3$ to 1.5

| $b_{\text{init}} = 2/3 \rightarrow b = 1.5$ | Steady State     | Transition          |
|---------------------------------------------|------------------|---------------------|
|                                             | Average          | Highest 5%          | Lowest 5%          | % of Agents |
| 0.123%                                      | 3.68%            | −2.31%              | —                  |

### Transition

| $T_{\text{policy}}$ | Average | Top 5% | Lowest 5% | % of Agents |
|---------------------|---------|--------|-----------|-------------|
| 8                   | 0.909%  | 2.971% | 0.196%    | 92.37%      |
| 12                  | 0.915%  | 2.694% | 0.345%    | 97.60%      |
| 16                  | 0.904%  | 2.472% | 0.425%    | 98.74%      |
| 20                  | 0.885%  | 2.283% | 0.471%    | 99.55%      |

*Note: The increase in $b$ is operated in $T_{\text{policy}}$ periods.*

B. Zooming in on a Particular Transitional Path

The previous results have shown that a non-negligible gain, in terms of utilitarian criterion, is to be expected from a public debt increase. We intend to better understand the macroeconomic adjustment in the short run and the distributional impact of the policy shock. In particular, we wish to single out the liquidity-constraint relaxing effect, and possibly other short-run effects which were not thought of in the first place.

We here present more detailed results for the transitional path characterized by an initial public debt-to-GDP ratio $b_{\text{init}} = 2/3$, and a final public debt-to-GDP ratio $b_t = 0.8$ for $t \geq 1$, with $T_{\text{policy}} = 1$. Transition-adjusted welfare gains and welfare gains based on steady-state comparison are reported in Table 3.

Transition-adjusted welfare gains are 4.5 times higher than welfare gains based on steady-state comparison. The poorest 5% do not experience welfare loss anymore. The welfare gain for the richest 5% is roughly equivalent, whether the transition is taken into account or not.

TABLE 3
Transition-Adjusted Welfare Change When $b$ Is Switched from $2/3$ to 0.8

| $b_{\text{init}} = 2/3 \rightarrow b = 0.8$ | Steady State | Transition |
|---------------------------------------------|--------------|------------|
| Average                                     | 0.0384%      | 0.176%     |
| Top 5%                                      | 0.610%       | 0.605%     |
| Lowest 5%                                   | −0.420%      | 0.022%     |
| % of agents                                 | —            | 89.20%     |

Figure 2 plots the paths of the stock of capital, the interest rate, labor supply, consumption, GDP, the wage and the tax rate.

Labor supply instantly jumps from 0.3153 to 0.3408. The intertemporal substitution effect accounts for the large increase in labor supply at date $t = 0$. As hours worked are less heavily taxed, households want to work more. From $t = 1$ onward, labor supply is brought back to a considerably lower level, close to its final value. This increase in labor supply explains the nonmonotonic path of the interest rate. It first increases substantially at date $t = 0$ (a 14.91% increase, or equivalently, a 0.67 percentage point increase), then falls sharply at date $t = 1$. From $t = 1$ onward, it progressively re-increases to reach its final steady-state level and the main determinant of the interest rate is then the household capital supply. The increase in labor supply accounts for the considerable increase in GDP (5.60%) at date $t = 0$. Therefore, the increase in savings at the end of date $t = 0$ is not driven only by the temporary reduction in income tax rate (which amounts to 40.51% or 15.2 percentage point), but also by the increase in gross income. It then explains why the interest rate drops so much at date 1 (the decrease amounts to 15.11% or 0.78 percentage point): labor supply is no longer above its initial level, while the stock of capital is

7. Note that the increase in public debt is only effective at the end of date 0 (with our convention, it refers to date $t = 1$). Therefore, both the beginning-of-period financial wealth of households, and the public debt $\tilde{B}_0$, are predetermined. With the convention adopted, all income flows are end-of-period, which means that the capital supply is totally predetermined at date $t = 0$. The increase in the interest rate is then fully driven by the increase in the labor supply.
considerably increased. Capital increases from $t = 0$ to $t = 1$, by 0.964%, and then reverts back to its new long-run level. Consumption at date $t = 0$ instantly increases and then diminishes. It can be observed that consumption does not seem fully continuous between dates 0 and 1 (although the deviation from a continuous graph is very modest). This is due to the fact that consumption and labor supply choices are mutually dependent. The higher the labor supply, the higher the marginal utility of consumption, as the following equation shows:

$$u_{c_t} = c_t^{-\rho} \exp[-(1 - \rho)\xi_t^{1+\eta}] = \partial u_{c_t}/\partial l_t$$

$$= -(1+\eta)(1-\rho)l_t^{-\rho} \exp[-(1 - \rho)\xi_t^{1+\eta}].$$

This property rests on the relative risk aversion $\rho$ being greater than unity. In such a case, the consumption and labor supply choices seem complementary. The more agents work, the more they are willing to consume. As labor supply is much higher at date $t = 0$ than at any other date, it follows that there is an additional motive to consume.\(^8\)

The differentiated impact of the policy change on the heterogeneous households is presented on Figure 3. The latter plots the welfare change, measured at date $t = 0$, as a function of the current asset holdings, for the 14 different levels of productivity. The asset axis has been scaled so that its maximum corresponds to the richest agents in the initial stationary distribution. Yet, half (resp. 75%) of the households hold less than 0.78 (resp. 2) units of assets. Unlike the analysis based on the steady-state comparison, the welfare gains are characterized by a U-shape, for all productivity levels. For productivity shocks $e_2, e_3, e_8, \text{ and } e_9$ (which correspond to low productivity shocks) the welfare variation is first positive for those whose financial assets level is respectively less than 0.0088, 0.0753, 0.0496, and 0.1359, then negative, and positive again (for those whose financial assets level is respectively higher than 1.7771, 1.1788, 8. Note that, as we have illustrated just above, this does not prevent households from saving considerably at the end of date $t = 0$.\)
of 3.2067, and 2.6292). The other productivity shocks experience a welfare gain regardless of the level of financial assets. It was not the case when the calculation of the welfare gains did not take the short-run effects into account.

The time path of the income tax rate, of labor supply, of the interest rate, and of the wage account for the differentiated welfare impact of the reform. Part of the gain is derived from the income tax rate reduction at date $t = 0$. Other things being equal, the decrease in the tax rate at date $t = 0$ should benefit more:

(i) households who are close to the liquidity constraint as it increases their disposable income;

(ii) households who are currently hit by a high productivity shock as it increases the after-tax wage; and

(iii) households who hold a large amount of financial assets. The larger the financial wealth, the larger the after-tax capital income for any given productivity level.

The pure liquidity-relaxing effect (channel (i)) can easily be singled out from these graphs. Indeed, all curves are initially downward sloping, implying that the gain, measured in consumption equivalent, is higher, the lower current asset holdings. Households with no wealth and the lowest productivity undergo a welfare loss of 0.413% when short-run effects are not taken into account. The welfare loss is significantly reduced when short-run effects are taken into account. The welfare loss is barely negative, equal to 0.008%. Channel (ii) appears very clearly in the graph. For any given level of assets, the higher the productivity, the higher the welfare gain curve. Finally, Channel (iii) explains why all curves are upward sloping, for high levels of assets.

C. Transfer Adjustment at Date 0

For anyone who has in mind the potentially powerful liquidity-relaxing effect of public debt, the previous results may not be so compelling and may well be attributable to the type of fiscal adjustment chosen at date $t = 0$. Instead of reducing the income tax rate, the budgetary surplus can also be redistributed to agents in many other ways. Among them,
lump-sum transfers should operate a significant redistribution. Therefore, we here consider the case where the initial income tax rate $\tau_0$ remains equal to its pre-reform value, and where initial transfers endogenously adjust. The government budget constraint at date 0 then implies:

$$\tilde{T}r_0 = (1 + g) \hat{Y}_1 - (1 + r_0) \hat{B}_0$$

$$+ \tau_0 (\hat{Y}_0 - \delta \hat{K}_0 + r_0 \hat{B}_0) - \gamma \hat{Y}_0.$$

The transition bears on a public debt-to-GDP ratio increase from $b_{\text{init}} = 2/3$ to $b = 0.8$, as was the case in the previous section. Figure 4 plots the time profile of capital, the interest rate, aggregate hours, consumption, GDP, and the wage, when the initial adjustment is operated through taxes (baseline) and when it is operated through transfers (TR).

The time profiles are qualitatively quite different from their baseline counterparts. The path of capital is almost smooth. Labor supply slightly diminishes at date $t = 0$ and remains smooth afterward. This accounts for the slight decrease in the interest rate at date $t = 0$. From date $t = 1$ onward, the path of the interest rate is smooth. Consumption increases at date $t = 0$ as a result of the temporary increase in the lump-sum transfer. However, the increase at date $t = 0$ is much lower than in the baseline case because labor supply does not increase initially. Moreover, consumption remains lower during the first 40 periods in the lump-sum transfer setting. This result illustrates the high and persistent effect of the initial increase in labor supply in the case of the temporary income tax rate reduction. The transition-adjusted welfare changes are presented in Table 4 when the

**TABLE 4**

Transition-Adjusted Welfare Change When There Is a Lump-Sum Transfer

| $b_{\text{init}} = 2/3 \rightarrow b = 0.8$ | Steady State | Transition | Transition (%) |
|------------------------------------------|--------------|------------|----------------|
| Average | 0.0384 | 0.176 | 0.381 |
| Top 5% | 0.610 | 0.605 | 0.263 |
| Lowest 5% | -0.420 | 0.022 | 1.154 |
| % of agents | -- | 89.20 | 100 |

FIGURE 4
The Dynamics of the Economy When a Lump-Sum Transfer Is Operated
budgetary surplus is redistributed in the form of a lump-sum transfer.

The welfare gains are different when the budgetary surplus is redistributed through lump-sum transfers. As we expected, the gains are much higher for the poorest agents and lower for the richest ones. What may seem less obvious is that the utilitarian criterion rises considerably more here: its increase amounts to 0.381%. The welfare gains are twice as high as those in the baseline case (0.176%). As compared to the steady-state calculation, the gain is now 10 times higher. Besides, all agents gain from this policy shock, while a significant minority of agents opposed the income tax rate reduction.

The differentiated welfare effects are shown in Figures 5 and 6. The consumption variations (labeled Tr) are compared with the transition-adjusted consumption variations of the baseline model (labeled baseline). We also report the corresponding consumption variations computed from steady-state (ss) comparisons. For the productivity levels $e^1, e^2, e^3, e^5, e^8, e^9,$ and $e^{10}$, that is, the low productivity levels, the consumption variation curve displays a “U” pattern. The decreasing part of the curve captures the welfare gain generated by the liquidity-relaxing effect of the temporary increase in the lump-sum transfer. It is higher than in the baseline case. When asset holdings are higher, the welfare gain becomes smaller than in the baseline case. Indeed, rich agents do benefit from a capital income tax rate reduction in the baseline case, while they do not in this scenario.

For productivity levels $e^6, e^7, e^{11}, e^{12}, e^{13},$ and $e^{14}$, the welfare gain is lower than in the

**FIGURE 5**
Consumption-Equivalent Variation by Productivity Level (From Level 1 to Level 7) When a Lump-Sum Transfer Is Operated
baseline case, regardless of the financial asset level. Indeed, their disposable income would raise more significantly if an income tax rate reduction were implemented.

This comparison corroborates the intuition that the type of fiscal adjustment at date $t = 0$ has a considerable impact on how poor agents might, or might not, benefit from the public debt increase.

**D. Transition-Adjusted Optimality and Equilibrium**

The previous simulations have shown that (1) a progressive transition toward the long-run optimal level yields significant welfare gains and that (2) considerable transitional adjustments operate in the short run. We could then wonder whether the short-run gains, arising when debt is increased, could not make further increases in the public debt-to-GDP ratio, above its long-run optimal level, beneficial. It is clear that long-run costs will emerge, but it is much less clear whether they should necessarily dominate the short-run gains.

To perform this exercise, we have simulated a 10% increase, starting with steady states associated with various public debt-to-GDP ratios. Whenever the policy shock generates gains according to the utilitarian criterion, the initial public debt-to-GDP ratio does not correspond to the highest utilitarian criterion. Table 5 presents, for various initial public debt-to-GDP ratios $b_{init}$, the welfare change implied by a 10 percentage
The purpose of this study is to highlight the liquidity-constraint loosening effect of public debt, which contributes to explain why governments end up with debt. We extend Floden’s economy (2001) to describe explicitly the short-run effects resulting from a public debt increase. It turns out that consumption gains are considerably higher when short-run effects of public debt

9. The U.S. ratio of public debt to GDP amounted to 75% in 2008 (OECD 2009).

10. In the case of a 10 percentage point decrease in a single time period, we have not found an initial debt/GDP ratio such that a reduction is welfare increasing. For smaller debt adjustments, a modest inaction zone can be revealed. When the adjustment in the debt/GDP ratio amounts to 2 percentage points, the inaction zone is [644%;686%].
are taken into account. Moreover, the analysis suggests that the higher consumption gains are not only driven by the temporary decrease in the income tax rate. The increase in the ratio of public debt to GDP leads to an increase in labor supply which, in turn, is responsible for an interest rate overshooting.

The existence of short-run gains creates a temptation to deviate from the long-run optimal debt-to-GDP ratio. It appears that a benevolent government, following the utilitarian criterion, is tempted to increase the ratio of public debt to GDP up to 560%. This figure is higher than its empirical counterpart. This analysis does not take the tax smoothing motive of the government into account, as government spending is assumed to be constant. Moreover, this exercise ignores important issues related to the definition of the optimal level of public debt: why should the government implement a once-and-for-all policy change, and why not consider more sophisticated time paths for the debt-to-GDP ratio? How could a truly time-consistent behavior of the government alter this message? The goal of this study is to stress the importance of the liquidity-constraint loosening effect of public debt, so that these questions are left for future research.

APPENDIX

Numerical Strategy

The state space consists of the individual asset holdings and the current productivity shock. We have used a nonuniform grid with 7000 points to discretize the interval \([0; a_{\text{max}}]\). With \(a_{\text{max}} = 51.3847\), no agent comes as near as two-thirds of this bound. The productivity process is approximated by a 14-value Markov chain.

The steady-state equilibrium computation requires to find the equilibrium interest rate, and the average labor supply, that enter the agents’ program as inputs. We therefore look for values of these two endogenous variables that guarantee the equilibrium conditions, simply by initiating the computation with an initial guess, and then updating the guess by a relaxation method.

The individual decision rules are computed by finding the zero of the Euler equation. The asset accumulation is not restricted to belong to the grid, and is computed by a simple linear interpolation between the two consecutive grid points for which the Euler equation residual switches from a negative value to a positive one (if this switch occurs; otherwise, the agent is liquidity constrained).

For the transitional dynamics, the algorithm rests on a fixed point in the paths of the interest rate and labor supply (expressed in efficiency units). We used 140 periods, to ensure that the model has converged to its final steady state. The initial condition corresponds to the steady-state distribution for initial policy parameters. The terminal condition consists of the derivatives of the value functions (the marginal utilities) for the final steady state. Therefore, before computing the transition itself, we need to find these two stationary equilibria. To compute the transition, we proceed as follows. First, we simply guess a path for the interest rate and the aggregate labor supply, and compute its associated decision rules on the transition by backward induction, starting from the terminal condition. Then, we simulate the dynamics of the economy by applying the time-varying decision rules on the distribution of agents (forward computation), and we obtain the ex post paths for labor supply and the interest rate. We update these two paths by a relaxation method. The convergence criterion corresponds to the maximal relative difference between the ex ante and ex post values for the interest rate and the aggregate labor supply, and is equal to \(10^{-5}\%\).

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