A NEW METHOD TO CALIBRATE THE MAGNITUDES OF TYPE Ia SUPERNOVAE AT MAXIMUM LIGHT

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ABSTRACT

We present a new empirical method for fitting multicolor light curves of Type Ia supernovae (SNe). Our method combines elements from two widely used techniques in the literature: the \( \Delta m_{15} \) template fitting method (Phillips et al.) and the Multicolor Light-Curve Shape method (MLCS; Riess et al.). An advantage of our technique is the ease of adding new colors, templates, or parameters to the fitting procedure. We use a large sample of published light curves to calibrate the relations between the absolute magnitudes at maximum and the postmaximum decline rate \( \Delta m_{15} \) in \( BVRI \) filters. If we perform a cut in reddening and compare an unreddened with a reddened sample, we find that the two samples produce relations that are marginally consistent with each other. We find that individual subsamples from a given survey or publication have significantly tighter relationships between light-curve shape and luminosity than the relationship derived from the sum of all the samples, pointing to uncorrected systematic errors in the photometry, mainly in \( B \) filters. Using our method, we calculate luminosity distances and host galaxy reddening to 89 SNe in the Hubble flow and construct a low-\( z \) Hubble diagram. The dispersion of the SNe in the Hubble diagram is \( \sigma = 0.20 \) mag, or an error of \( \sim 9\% \) in distance to a single SN. Our technique produces similar or smaller dispersion in the low-\( z \) Hubble diagram than other techniques in the literature.

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1. INTRODUCTION

Type Ia SNe, the thermonuclear explosions of accreting white dwarfs (Arnett 1982), are the most precise distance indicators at cosmological distances. Even though they are not standard candles (Pskovskii 1977; Branch 1987; Phillips 1993), their absolute magnitudes at maximum light are closely correlated with the shape of their light curves (Phillips 1993). As precise standardizable candles they have been used to trace the expansion of the universe as a function of redshift (see Leibundgut 2000), leading to the discovery of the acceleration of the expansion (Riess et al. 1998; Branch et al. 1999), and the transition from deceleration to acceleration at early times to acceleration in the present (Tonry et al. 2003; Riess et al. 2004).

All the information of the luminosity distance of a Type Ia SN is in the multicolor light curves. Different empirical methods have been presented in the literature to fit Type Ia SNe light curves (LCs). Among the most developed are \( \Delta m_{15} \), MLCS, and the "stretch" method.

The \( \Delta m_{15} \) method (Hamuy et al. 1996a; Phillips et al. 1999) uses six \( BVI \) templates of nearby SNe (Hamuy et al. 1996c) covering a wide range of light-curve shapes that are parameterized by a single discrete value: the 15 day postmaximum decline in magnitude in the \( B \) band \( \Delta m_{15}(B) \). Germany et al. (2004) provided a modified version of the method by including the \( R \) band in the fits, new estimations of the \( K \)-corrections, and new templates from well-observed nearby SNe.

In the MLCS method (Riess et al. 1996, 1998) a continuous parameter characterizes different light-curve shapes. This parameter is the difference between the absolute magnitude at maximum of a SN and a fiducial value, constructed from a set of well-observed SNe. The latest version of the method (Jha et al. 2006a) has added the \( U \) filter to \( BVRI \) and better extinction estimators.

The stretch method (Perlmutter et al. 1997; Goldhaber et al. 2001) measures the light-curve shape by simply adjusting the scale on the time axis by a multiplicative factor. This is an elegant solution, since the stretch of the LC is the combination of the cosmological redshift effect (1 + \( z \) factor) and the intrinsic light-curve shape. Different LCs are stretched in the time domain to fit a template (Leibundgut 1988). This method has been used mainly in \( UBV \) filters because of the homologous nature of the LCs across all light-curve shapes; \( \Delta m_{15} \) and MLCS are more robust to nonhomologous variations as a function of light-curve shape in the redder filters.

Other methods have been proposed (Tripp 1998; Tripp & Branch 1999; Tonry et al. 2003; Wang et al. 2003, 2005). Wang et al. (2005) found a tight correlation between peak luminosities and the intrinsic colors at \( \sim 12 \) days after \( B \) maximum. Using this correlation but with a small sample of SNe, they obtain very low dispersion Hubble diagrams.

The new technique presented in this paper is a combination of \( \Delta m_{15} \) and MLCS methods. We implement what we consider the advantages from each method: the direct use of actual (not interpolated) SNe LC templates characterized by different \( \Delta m_{15} \) values in the fitting method, and a proper statistical model of the errors in the fitting parameters. We use a simple mathematical framework to account for the nonuniform distribution in the values of the \( \Delta m_{15} \) parameter. Our method allows us to calculate a
TABLE 1

| SN     | $\Delta m_{15}$ | $B(V-V)_{\text{rest}}$ | $\Delta m_B$ | $\Delta m_V$ | $\Delta m_I$ | $\Delta m_R$ | Reference |
|--------|-----------------|------------------------|--------------|--------------|--------------|--------------|-----------|
| (1)    | (2)             | (3)                    | (4)          | (5)          | (6)          | (7)          |
| 1991T  | 0.94            | 0.022                  | -0.02        | -0.01        | 0.00         | 1, 2         |           |
| 1991bg | 1.93            | 0.040                  | -0.05        | -0.14        | -0.10        | 1, 2, 3, 4  |           |
| 1992A  | 1.47            | 0.017                  | -0.03        | -0.03        | -0.05        | 1, 5         |           |
| 1992d  | 1.11            | 0.034                  | 0.00         | -0.02        | -0.01        | 1, 6         |           |
| 1992be | 0.87            | 0.022                  | 0.00         | 0.00         | -0.02        | 1, 6         |           |
| 1992bo | 1.69            | 0.027                  | 0.00         | 0.00         | -0.02        | 1, 6         |           |
| 1994D  | 1.34            | 0.022                  | 0.00         | 0.00         | -0.10        | 7            |           |
| 1994ae | 0.94            | 0.031                  | -0.01        | 0.00         | 0.00         | 8            |           |
| 1995D  | 1.05            | 0.058                  | 0.00         | 0.00         | 0.00         | 8            |           |
| 1995al | 0.89            | 0.014                  | -0.01        | -0.01        | -0.01        | 8            |           |
| 1996X  | 1.25            | 0.069                  | -0.02        | 0.00         | -0.07        | 9            |           |
| 1996bu | 1.05            | 0.025                  | 0.00         | -0.01        | -0.05        | 10, 11       |           |
| 1999aa | 0.83            | 0.015                  | -0.02        | -0.02        | -0.03        | 12, 13       |           |
| 2001el | 1.12            | 0.014                  | -0.03        | -0.01        | -0.06        | 14           |           |

Notes.—Col. (1): Name of the SN. Col. (2): $\Delta m_{15}(B)$ from the $B$ light-curve template; Col. (3): $V_{\text{max}} = V_{\text{obs}}$, Col. (4): $R_{\text{max}} = R_{\text{obs}}$, Col. (5): $I_{\text{max}} = I_{\text{obs}}$, Col. (6):References containing the data with the LCs used to construct the templates.

References.—(1) Hamuy et al. 1996c; (2) Lira et al. 1998; (3) Filipenek et al. 1992; (4) Leibundgut et al. 1993; (5) N. B. Suntzeff et al. 2004, unpublished; (6) Hamuy et al. 1996d; (7) R. C. Smith et al. 2004, unpublished; (8) Riess et al. 1999b; (9) Covarriabiasi et al. 2004, unpublished; (10) Jha et al. 1999; (11) Suntzeff et al. 1999; (12) Krisciunas et al. 2000; (13) Jha et al. 2000b; (14) Krisciunas et al. 2003.

2. LIGHT-CURVE TEMPLATES

The template set, listed in Table 1, is composed of the six $BVRI$ templates of Hamuy et al. (1996c) augmented with the $R$-filter templates of Hamuy et al. (1996c) and eight new $BVRI$ templates constructed from published and unpublished data of well-observed nearby Type Ia SNe. The SNe were selected to cover a wide range in observed light-curve shapes. We recalculated the $\Delta m_{15}$ from the spline interpolation of the $B$-band data.

We used the same recipe as Hamuy et al. (1996c) to construct the LC templates. A cubic spline interpolation was applied to the data obtaining the time and magnitude at maximum in each filter, with typical uncertainties of $\sim 0.2$–0.5 days and $\sim 0.02$–0.05 mag, respectively. The time axis in all the filters was parameterized by the rest-frame time relative to $B$ maximum. In the cases in which the redshift in the cosmic microwave background (CMB) frame is $z_{\text{CMB}} \geq 0.01$, the time axis was corrected for cosmological time dilation dividing by $1 + z$, and the magnitudes were $K$-corrected to the rest frame.

We calculated the $K$-corrections in $BVRI$ filters using the recipe of Hamuy et al. (1993) and a set of spectrophotometry from different epochs and SNe: SN 1972E, SN 1990N, SN 1991T, SN 1991bg, SN 1992A, and SN 1994D. We fitted the results with a cubic spline to provide values of $K$-corrections in different epochs, with a typical error $\sim 0.02$ mag.

Finally, the magnitudes at $B$ maximum were subtracted in all the filters to have $B = V = R = I = 0$ at $t_0(B)$. The final templates have interpolated rest-frame magnitudes in the range $-5 \leq r - t_0(B)$ (days) $\leq 80$, with a sampling of 1 day.

In Table 1 we present basic information of the templates and the references of the photometric data. All the LC templates are plotted in Figure 1, ordered by increasing values of $\Delta m_{15}(B)$ from slow decliners to fast decliners. From Figure 1 we can observe the main characteristics of the $BVRI$ LCs: a principal maximum, followed by a change in the curvature of the LC $\sim 10$–30 days after maximum and a linear decay at greater than 30–40 days. The $RI$ LCs also have a secondary hump fainter than the first peak at $\sim 20$–30 days. A small inflection in the $V$ LC can be seen during the same epoch as the $I$ secondary maximum. Different morphological properties of the LCs, such as the time of the secondary peak in the $RI$ filters and the time of inflection, are correlated with the postmaximum decline rate (Hamuy et al. 1996c). The rise times in the $BV$ filters are generally correlated with the postmaximum decline rate (Riess et al. 1999a): slow risers are slow decliners, and fast risers are fast decliners.

The dashed lines around each solid curve in Figure 1 represent the approximate $\pm 1 \sigma$ statistical uncertainties in the photometry. These curves were obtained doing a linear interpolation of the errors in the original data points. Typical uncertainties in the photometry from photon statistics and errors in the standards are $\sim 0.02$ mag around maximum and $\geq 0.04$ mag at late times. Note, however, that the systematic errors that arise from doing photometry on a nebular (as opposed to stellar) spectral
energy distribution (SED) of late-time SNe are much larger, at the ~0.1 mag level (Suntzeff 2000).

3. THE METHOD

The principal idea of the method is a formalism that allows us to construct a linear combination of the discrete template set, composed by the observed templates, into a template parameterized by \( \Delta m_{15} \). The \( \Delta m_{15} \) parameter can then be included in the \( \chi^2 \) function when fitting a new multicolor LC, allowing us to calculate a covariance matrix and the estimation of statistical uncertainties in the best-fitting parameters.

3.1. Interpolation between Light-Curve Templates

We interpolate between different templates using a weighting function. It assigns different weights to each template in the template set, in order to obtain

1. the \( \Delta m_{15} \) of the constructed template; and
2. the constructed template as a function of time in different filters, parameterized by \( \Delta m_{15} \).

We have engineered the weighting function to allow us to use the information in all the templates with \( \Delta m_{15} \) values near the real value, while ignoring the templates with light-curve shapes very different from the LC we are trying to fit. This weighting scheme will allow us to use real light-curve templates to fit the observed data, and at the same time allows us to avoid relying on any one particular light-curve template. With this philosophy, we can drop in or remove any template without any need to regenerate a series of interpolated (in \( \Delta m_{15} \)) templates. In this sense, our fitting technique tries to stay as close to the observed template data as possible.

The shape of the weighting function should be selected according to the template sample and their \( \Delta m_{15} \). In general we want to have \( \sim 2-4 \) observed templates in the interpolation to obtain a constructed template, without introducing uncertainties in the interpolation. For example: if we want a constructed template with \( \Delta m_{15} = 1.20 \) mag, the weight assigned to the template of SN 1991bg, the fastest declining LC in the template set, should be very small or 0 because their LC shapes are very different.

We choose here a triangle as the weighting function (see Fig. 2). The weight assigned to each observed template, \( w_i \), is equal to the value of the triangle function at \( \Delta m_{15}(B) \) of the template \( i \) in the template set:

\[
 w_i = g[\Delta m_0, \Delta m_{15}(B)],
\]

where \( g \) is the weighting function as a function of \( \Delta m_0, \Delta m_{15}(B) \).

Note that the final values of the weights are normalized so that their total sum is 1. When the best-fit template is constructed, the center of the triangle function, \( \Delta m_0 \), moves along the \( \Delta m_{15} \) axis until the required value of the \( \Delta m_{15} \) is obtained. This value is calculated directly by weighting the values of the observed templates:

\[
 \Delta m_{15} = \sum_{i=1}^{n} w_i \Delta m_{15}(B)_i.
\]

The best-fit template \( N \) in the filter \( X \) is constructed by adding the weighted observed templates \( T \) in the template set:

\[
 N^X(\Delta m_{15}) = \sum_{i=1}^{n} w_i T^X_i.
\]

In this case \( n = 14 \) (see Table 1) is the number of discrete templates in the template set. The vectors represent the time dependence of the templates in different filters, \( X = BVRI \), measured in the rest frame with respect to the time of \( B \) maximum.

In Figure 3 we show the value of \( \delta \), the width of the triangle function as a function of \( \Delta m_{15} \). The effect of this weighting scheme is that the discrete templates outside of the triangle function are given zero weight. The larger values of \( \delta \) for \( \Delta m_{15} > 1.40 \) mag were selected to overcome the poor sampling and small number of fast-declining SNe in the template set (see Table 1).
In Figure 4 we present constructed templates with different values of $\Delta m_{15}$ as an example of the interpolation scheme. We have extrapolated the initial templates to earlier times ($-15 \leq t - t_0(B)\text{[days]} \leq -5$) assuming a quadratic dependence with time. The correlation between $\Delta m_{15}$, the postmaximum decline rate, and the rise time is very clear in the $BV$ templates: slow risers are slow decliners, and fast risers are fast decliners (Riess et al. 1999a). In the $RI$ templates this correlation is not clearly present, but it is observed that the secondary maximum occurs later for the slow-declining LCs.

The final range of fitted $\Delta m_{15}$ values is restricted to the range in the input template set. For the template set presented in our work: $0.83 \leq \Delta m_{15}\text{[mag]} \leq 1.93$ (Table 1).

3.2. $\chi^2$ Fitting

In the original method (Phillips 1993; Hamuy et al. 1996a, 1996b; Phillips et al. 1999) and the modified version of Germany et al. (2004), $\Delta m_{15}$ is not fitted directly in one step. When a new LC is fitted, the total reduced $\chi^2$ is calculated for each template summed across all colors. The $\Delta m_{15}$ of the SN is derived with a second-order polynomial fit to the curve $\Delta m_{15}$ versus $\chi^2$, selecting the value where $\chi^2$ reaches a minimum as the best-fitting parameter. This technique is justified by the fact that $\chi^2$ function changes quadratically with the parameters near the minimum (Bevington & Robinson 1992; Press et al. 1988). This approach has two problems:

1. the time of $B$ maximum, $BVRI$ magnitudes at maximum, and $\Delta m_{15}$ are correlated parameters. This correlation is not considered in the quadratic fit to $\chi^2$; and
2. the statistical uncertainties in the best-fitting value of $\Delta m_{15}$ are difficult to estimate in a consistent way.

The linear combination of templates weighted as described above allows us to include $\Delta m_{15}$ directly in the $\chi^2$ function. In this way, the general version of the empirical model for a new LC in the observed filter $Y$ is

$$\mathcal{M}^Y = N^X(\Delta m_{15}) + M^X_{\text{max}}(\Delta m_{15}) + \mu_0 + K_{XY}[\Delta m_{15}, E(B-V)] + R_X E(B-V)_{\text{host}} + R_Y E(B-V)_{\text{Gal}}.$$  

(4)

where $M^X_{\text{max}}(\Delta m_{15})$ are the relations between the absolute magnitude at maximum in the rest-frame filter $X = BVRI$ and $\Delta m_{15}$ (see § 4 for the derivation of this relation); $\mu_0$ is the reddening-corrected distance modulus of the SN; $K_{XY}[\Delta m_{15}, E(B-V)]$ are the cross-filter $K$-corrections (Kim et al. 1996; Schmidt et al. 1998; Nugent et al. 2002; Germany et al. 2004), which depends on the colors of the SN and total extinction; $R_X$ and $R_Y$ are the ratio of total to selective extinction in the host galaxy and in the Milky Way, respectively; and $E(B-V)_{\text{host}}$ and $E(B-V)_{\text{Gal}}$ are the color excesses in the host galaxy and in the Milky Way (Schlegel et al. 1998).

When fitting a new SN LC we use this empirical model to construct a multidimensional $\chi^2$ function,

$$\chi^2[\mu_0, \Delta m_{15}, E(B-V)_{\text{host}}, \mu_0] = [\mathcal{M}^Y - m^Y]_Y(C^Y)^{-1}[\mathcal{M}^Y - m^Y],$$

(5)

where $(\mathcal{M}^Y - m^Y)$ are the residuals of the fit; $m^Y$ are the observed magnitudes in the $Y$ filter (light-curve data); $C^Y$ is the correlation matrix (Riess et al. 1996). This symmetric matrix contains the variances (squared statistical uncertainties) of the data for a given date (diagonal terms) and the correlation between data of different days in the same filter (nondiagonal terms). We use a diagonal matrix for $C^Y$, setting the nondiagonal elements to zero, where each element takes into account the uncertainties in the data, the model, and the $K$-corrections: $\sigma^2_{\text{data}} + \sigma^2_{\text{model}} + \sigma^2_{\text{corr}}$. The uncertainties in the model, $\sigma_{\text{model}}$, are calculated by weighting the uncertainties in the observed templates with the same weights calculated from equation (1).

The $\chi^2$ function of equation (5) is minimized when a new SN LC is fitted. This multidimensional (i.e., multicolor) minimization gives the best values for the parameters of the empirical model of equation (4). The fitting parameters of the model are:

- time of $B$ maximum, $\Delta m_{15}$, color excess of the host galaxy, and the reddening-corrected distance modulus.

Several works in the literature suggest that the average values of selective to total extinction of host galaxies are smaller than the Galactic values (Riess et al. 1996; Tripp & Branch 1999; Phillips et al. 1999; Wang et al. 2003; Altavilla et al. 2004; Riendl et al. 2005). Also, $R$ changes in time because of the fast evolution of the SED of Type Ia SNe (Leibundgut 1988; Phillips et al. 1999; Nugent et al. 2002; Jha et al. 2006a), producing a dependence of $\Delta m_{15}$ with the total extinction toward the SN. We are restricted in this work to constant values of $R$, equal to the Galactic reddening law (Cardelli et al. 1989): $R_B = 4.20$, $R_Y = 3.10$, $R_R = 2.54$, and $R_I = 1.85$. In a future paper we will explore fitting the extinction law as part of the generalized $\chi^2$ fit.

To summarize, equations (3)–(5) allow us to introduce a linear combination of templates directly into the $\chi^2$ fitting procedure. We do not have to output an intermediate table of interpolated templates (such as those shown in Fig. 4).
framework allows us to change the weighting, the template set, and even allows the parameters to be fit in an elegant way.

3.3. Uncertainties in the Fitting Parameters

We calculate the uncertainties in the parameters of the best-fitted LC model by constructing the covariance matrix. If we have \( \chi^2 (a_i) \), with \( a_i \) as the fitting parameters, the elements of the covariance matrix \( \alpha \) are (Bevington & Robinson 1992; Press et al. 1988)

\[
\alpha_{ij} = \frac{1}{2} \frac{\partial^2 \chi^2}{\partial a_i \partial a_j}. \tag{6}
\]

The partial derivatives are evaluated in the best-fit parameters \( a_i^{\text{best}} \). The inverse of the covariance matrix, the error matrix \( \alpha \), is the value of the minimum \( \chi^2 \) divided by the number of degrees of freedom of the fit \( n = N - m \), with \( N \) as the total number of data points fitted and \( m \) as the number of fitting parameters \( (m = 4 \) in current implementation).

4. CALIBRATION OF THE RELATIONS BETWEEN \( M_{\text{max}} \) AND \( \Delta m_{15} \)

The measurement of distances to Type Ia SNe using the \( \Delta m_{15} \) method is based on the observed correlations between the absolute magnitudes at maximum light \( (M_{\text{max}}) \) and the rate of evolution away from maximum light as parameterized originally with \( \Delta m_{15} \) (Phillips 1993; Hamuy et al. 1996a; Phillips et al. 1999; Germany et al. 2004) established with a low-redshift sample of SNe in different rest-frame filters (second term in the right-hand side of eq. [4]). Different versions of this relation have been proposed in the literature: linear relations (Phillips 1993; Hamuy et al. 1996a), quadratic polynomials (Phillips et al. 1999; Germany et al. 2004), both restricted to \( \Delta m_{15} \leq 1.70 \) mag, and an exponential that includes the fast-declining LCs (Garnavich et al. 2004). Finally, a relation that includes both light-curve shape and color (as a second parameter) has been published by Parodi et al. (2000).

We used a modified version of the analytical model of equation (4) to fit the multicolor LCs of a large sample of Type Ia SNe at \( z < 0.12 \) (see Table 2) in order to study the relations between \( M_{\text{max}} \) and \( \Delta m_{15} \) in different filters. The parameters of this multicolor fits are: rest-frame apparent \( BVRI \) magnitudes at maximum corrected by Galactic and host reddening, \( E(B-V)_{\text{host}} \), \( \Delta m_{15} \), and time of maximum in \( B \). This methodology is similar to the one applied by Germany et al. (2004; their § 4.4.4) in their modified \( \Delta m_{15} \) method. The main difference is that they used the unreddened sample of Type Ia SNe defined by Phillips et al. (1999), whereas we use a larger sample of SNe in a self consistent way analyzed with the analytical technique introduced in § 3 to calculate \( \Delta m_{15} \) and \( E(B-V)_{\text{host}} \) simultaneously from the multicolor LCs.

Since the magnitudes at maximum light and the host galaxy reddening are highly correlated parameters in the analytical model, priors are needed to soften this degeneracy. We applied them directly in the \( \chi^2 \) function as in Jha et al. (2006a). First, negative values of \( E(B-V)_{\text{host}} \) were handled using a Bayesian filter (Riess et al. 1996), assuming a one-sided Gaussian a priori distribution of \( A_g \) with maximum at \( A_g = 0 \) and \( \sigma (A_g) = 0.3 \) mag (Phillips et al. 1999). Second, Jha et al. (2006a) studied the intrinsic colors of a large sample of Type Ia SNe and found that their colors at 35 days after \( B \) maximum are very homogeneous and well described by a Gaussian distribution with \( (B-V)_{35} = 1.055 \) and \( \sigma = 0.055 \) mag. We applied this as a prior in the reddening-corrected \( B-V \) colors, which is very similar to the “Lira law” (Lira 1995). Because it is outside the scope of this paper, we leave as an open question how the host galaxy reddening obtained from the fits depends on the priors.

We assumed a concordance cosmology with \( (h, \Omega_M, \Omega_k) = (0.72, 0.3, 0.7) \) to transform the rest-frame apparent magnitudes.

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### References

- Bevington & Robinson 1992
- Press et al. 1988
- Jha et al. 2006a
- Phillips et al. 1996
- Phillips et al. 1999
- Germany et al. 2004
- Lira 1995

### Notes

- Col. (1): Name of the data sample. Col. (2): SN in the sample. Col. (3): References.
to absolute magnitudes for SNe in the Hubble flow at redshifts $z > 0.01$ (Calán/Tololo, CfA1, Krisciunas et al., and CfAII samples in Table 2). For the nearby sample we used the distance moduli to the host galaxies obtained with the surface brightness fluctuation method (Ajhar et al. 2001), matching the derived distances in $H_0 = 75$ km s$^{-1}$ Mpc$^{-1}$ scale to $H_0 = 72$ km s$^{-1}$ Mpc$^{-1}$. The final error budget in the absolute magnitudes includes statistical uncertainties in the reddening-corrected apparent magnitudes from the covariance matrix and a Hubble flow “noise” of 600 km s$^{-1}$ to crudely model the effects of peculiar velocities (Marzke et al. 1995). For the nearby sample we only included the error in the distance modulus of the host galaxies.

In Figure 5 we plot the results of the fits to the complete sample of Type Ia SNe listed Table 2. For the SNe that are in common between Krisciunas et al. and CfAII sample (four SNe), we have selected the results of the fits with smaller common between Krisciunas et al. and CfAII sample (four SNe), sample of Type Ia SNe listed Table 2. For the SNe that are in the complete sample. The lines are linear fits performed in the range 0 $< \Delta m_{15}(\text{mag}) < 1.70$ to the complete sample (solid lines) of Table 2, unreddened sample with $E(B-V)_{\text{host}} < 0.06$ mag (dot-dashed lines), and the reddened sample with $E(B-V)_{\text{host}} > 0.06$ mag (dashed lines). [See the electronic edition of the Journal for a color version of this figure.]

![Figure 5](image-url)

**TABLE 3**

| Filter | $a$ | $b$ | $\sigma$ | $\chi^2$ | $N$ |
|--------|-----|-----|----------|-----------|-----|
|        | (1) | (2) | (3)      | (4)       | (5) |
| All    |     |     |          |           |     |
| $B$    | $-19.319(022)$ | $0.634(077)$ | 0.17 | 0.73 | 94 |
| $I$    | $-19.246(019)$ | $0.606(069)$ | 0.14 | 0.68 | 94 |
| $R$    | $-19.248(025)$ | $0.566(101)$ | 0.13 | 0.45 | 73 |
| $I$    | $-18.981(020)$ | $0.524(079)$ | 0.15 | 0.90 | 87 |

Notes.—The assumed linear relations are of the form $M_{\text{max}} = a + b(\Delta m_{15} - 1.1)$ in the range 0.80 $\leq \Delta m_{15} \leq 1.70$. We also give the linear fits using a cut in host galaxy color excess at $E(B-V)_{\text{host}} = 0.06$ mag. Col. (1): Filter. Col. (2): Zero point of the linear fits with uncertainties in parentheses, in units of 0.001 mag. Col. (3): Slope of the linear fits with uncertainties in parentheses, in units of 0.001 mag. Col. (4): The rms scatter of each sample around the linear fits. Col. (5): $\chi^2$ per degree of freedom of the fits. Col. (6): Number of SNe.

There are several important points to note about Figure 5. The dispersion around the linear fits does not correlate with wavelength, although the dispersion in $B$ is slightly bigger than in $I$ filters. This is probably due to a few SNe that deviate substantially from the whole sample, as is evident in the figure. This could be caused by a different reddening law in the host galaxies of this SNe, but varying the reddening law was outside the scope of the analysis presented here.

The zero points of the linear relations are consistent, within the uncertainties, when we split the sample by host galaxy reddening. The slopes are marginally inconsistent, mainly in $R$ and $I$ filters, and steeper in the high-reddening sample. We think this is produced by the smaller number of fast-declining SNe (1.5 times higher) in the range $0.80 \leq \Delta m_{15} \leq 1.70$ in the sample. The relations obtained here are consistent with previous works in the literature that include corrections for host galaxy reddening (Phillips et al. 1999; Germany et al. 2004).

In almost all the cases $\chi^2 < 1.0$ in the linear fits, which could be produced by an overestimation of uncertainties in the absolute magnitudes; $\chi^2$ depends strongly on the adopted noise in the Hubble flow due to peculiar velocities, which has values between 250 and 600 km s$^{-1}$ in the literature (Baker et al. 2000; Landy 2002; Zehavi et al. 2002; Hawkins et al. 2003). If we had chosen 400 km s$^{-1}$ for instance, the values of $\chi^2$ would be $\sim 1.5$ times higher.

The dispersion around the linear fits seems to be larger at the lower end for the slowly declining SNe. This could be an indication of a higher intrinsic dispersion in the slow decliners, as noted by other authors. However, it is possible that the $\Delta m_{15}$ values of some of these slow SNe are smaller than the minimum $\Delta m_{15}$ of SNe in the template set. More well-sampled LCs of slow decliners are needed to extend and improve the template set in order to study the apparent larger natural dispersion.
TABLE 4
Best-Fit Linear Relations $M_{\text{max}}$ versus $\Delta m_{15}$ in $BVRI$ Filters for Different Subsamples of Table 2

| Filter | $a$   | $b$   | $\sigma$ | $\chi^2$ | $N$   |
|--------|-------|-------|----------|----------|-------|
| Calán/Tololo |       |       |          |          |       |
| $B$     | $-19.376(037)$ | $0.814(118)$ | $0.11$ | $0.80$  | $27$  |
| $V'$    | $-19.295(032)$ | $0.775(105)$ | $0.11$ | $0.97$  | $27$  |
| $R$     | $-19.249(062)$ | $1.003(364)$ | $0.13$ | $0.56$  | $9$   |
| $I$     | $-18.974(032)$ | $0.559(130)$ | $0.14$ | $1.49$  | $23$  |
| CFAI   |       |       |          |          |       |
| $B$     | $-19.224(055)$ | $0.880(210)$ | $0.19$ | $0.56$  | $15$  |
| $V'$    | $-19.167(049)$ | $0.659(187)$ | $0.16$ | $0.61$  | $15$  |
| $R$     | $-19.260(059)$ | $0.721(205)$ | $0.11$ | $0.33$  | $14$  |
| $I$     | $-19.049(053)$ | $0.787(193)$ | $0.16$ | $0.98$  | $14$  |
| Krisicuñas et al. |       |       |          |          |       |
| $B$     | $-19.279(084)$ | $1.224(570)$ | $0.21$ | $0.51$  | $13$  |
| $V'$    | $-19.218(082)$ | $1.159(551)$ | $0.16$ | $0.32$  | $13$  |
| $R$     | $-19.192(091)$ | $0.868(546)$ | $0.12$ | $0.21$  | $12$  |
| $I$     | $-18.958(078)$ | $0.406(520)$ | $0.12$ | $0.21$  | $13$  |
| Nearby |       |       |          |          |       |
| $B$     | $-19.343(051)$ | $0.166(284)$ | $0.14$ | $0.20$  | $13$  |
| $V'$    | $-19.265(046)$ | $0.398(262)$ | $0.12$ | $0.34$  | $16$  |
| $R$     | $-19.259(046)$ | $0.515(263)$ | $0.12$ | $0.37$  | $15$  |
| $I$     | $-18.986(044)$ | $0.478(242)$ | $0.13$ | $0.47$  | $14$  |

Notes.—Col. (1): Filter. Col. (2): Zero point of the linear fits with uncertainties in parentheses, in units of 0.001 mag. Col. (3): Slope of the linear fits with uncertainties in parentheses, in units of 0.001 mag. Col. (4): $\chi^2$ per degree of freedom of the linear fits. Col. (5): Number of SNe.

In Table 4 we present the linear relations obtained for different subsamples in Table 2. The zero points of the linear relations are consistent, within the 1 $\sigma$ uncertainties, in $RI$ filters. This is not the case in $BV$ filters where there is a clear difference (up to $\sim 0.17$ mag in $B$) between CFAl, Krisciunas et al., and CFAlII, which have consistent values, and Calán/Tololo, which is brighter. The nearby sample is consistent with Calán/Tololo. This zero-point difference roughly represents the magnitude difference at $\Delta m_{15} = 1.1$ mag.

This same difference persists if we take the weighted average difference in the range $1.0 \leq \Delta m_{15} \leq 1.2$. This difference is $\sim 0.10 \pm 0.03$ mag in $BV$ filters, with Calán/Tololo magnitudes again being brighter.

It is well known, but poorly studied, that photometry of SNe is sensitive to the throughpout system of the telescope/filter/detector due to the nonstellar nature of SN spectra (Suntzeff 2000). Riess et al. (1999b) showed (their Table 1) that there are systematic differences in the observed photometry of SN1994B between the Fred Lawrence Whipple Observatory (CFAl and CFAlII surveys) and CTIO data (Calán/Tololo survey). These differences are larger in the $B$ filter but always $< 0.04$ mag, which are significantly less than the differences seen in absolute magnitudes at $\Delta m_{15} = 1.1$. However, the comparison of the absolute magnitudes in different data sets is not as simple because the absolute magnitudes at maximum are obtained from the fitting, which involves $K$-corrections and extinction corrections.

The systematic differences could arise from the poorly measured telescope/filter/detector transmission functions that define the natural photometric system. With accurate transmission functions and accurate atlas of spectrophotometry of SNe, we could calculate the $S$-corrections (Stritzinger et al. 2002) needed to bring the photometry of nonstellar SEDs onto a photometric standard system.

Another effect that could be introducing zero-point shifts between subsamples is the existence of a Hubble bubble (Zehavi et al. 1998), a systematically lower expansion rate of $\sim 7\%$ measured for SNe Ia at $cz \geq 7400$ km s$^{-1}$ (Jha et al. 2006a). If we assume that the Hubble bubble alone is producing the systematic differences, the difference in zero points between Calán/Tololo and CFAl–II subsamples would be $0.06$–$0.09$ mag. This effect does not fully explain the differences that we find.

The slopes of the linear fits in different subsamples, listed in Table 4, correct the SN peak magnitudes to a standard candle value. The slopes are consistent within the errors for Calán/Tololo, CFAl, and Krisciunas et al. The shallower slopes of CFAlII and nearby samples are produced by the SNe at high values of $\Delta m_{15}$. Removing the SNe with $\Delta m_{15} > 1.5$ mag brings the slope of the CFAlII sample into agreement with the fits to the other samples.

The dispersion around the linear fits are different between subsamples. For instance, the average dispersions in Calán/Tololo sample are smaller than the other subsamples (see Table 4). While the dispersion of Calán/Tololo for different filters is between $\sigma = 0.11$ and 0.14 mag, CFAl and CFAlII are in the range $\sigma = 0.11$–$0.19$ and $0.15$–$0.17$.

Part of the difference in dispersions could be due to the fact that the subsamples have different redshift coverage. We can compare the intrinsic dispersions subtracting off the effect of the random peculiar velocity of galaxies at the mean redshift of each subsample from the dispersions in Table 4; this is $\sigma_{\text{int}}^2 = \sigma^2 - \sigma_{p}^2$. The mean redshift of the Calán/Tololo sample is $\bar{z} \approx 0.05$, while for CFAl–II it is $\bar{z} \approx 0.02$–$0.03$. If we assume a random peculiar velocity of $\sigma_{p} = 400$ km s$^{-1}$, the intrinsic dispersions in the $B$ filter of Calán/Tololo, CFAl, and CFAlII are consistent within 0.03 mag, with $\sigma_{\text{int}} = 0.09$–$0.12$ mag. This result is strongly dependent on the value of the peculiar velocity used.

The different dispersions and the different absolute magnitudes at $\Delta m_{15} = 1.1$ among the data subsamples point out the urgency of producing a new set of uniform SN LCs, taken preferably on a single telescope with a well-calibrated telescope/filter/detector system. However, a more detailed analysis is needed to properly model the effects of a Hubble bubble in the zero points and the peculiar velocity field of galaxies, coupled with the real redshift distributions of subsamples; this could explain part the differences observed.

5. HUBBLE DIAGRAM OF LOW-z TYPE Ia SNe

With the relations for $M_{\text{max}}(\Delta m_{15})$ in different filters (right-hand side of eq. [4]), we can now apply our fitting technique minimizing the $\chi^2$ function in equation $(5)$ to obtain the distance modulus of a large sample of SNe in the Hubble flow. We use the complete sample of 89 SNe in Table 2 with $z > 0.01$ and $\Delta m_{15} < 1.70$ mag. The results of the fits are presented in Table 5, where we give the best-fit parameters $\Delta m_{15}, E(B-V)_{\text{host}}, \mu_{0}$ including their statistical errors from the covariance matrix. We plot all the LCs and best-fit models in Figures 7–16. We obtained a median value of $\chi^2 = 1.3$ for all the fits.
### Table 5

| SN  | $z$  | $\mu_0$ | $E(B-V)_{\text{R}^{\odot}}$ | $\Delta m_{15}$ |
|-----|-----|--------|-----------------|----------------|
| 1999X | 0.0257 | 35.256(0.062) | 0.079(0.029) | 0.958(0.027) |
| 1999aa | 0.0157 | 34.215(0.023) | 0.000(0.007) | 0.837(0.020) |
| 1999ac | 0.0098 | 33.090(0.026) | 0.081(0.009) | 1.067(0.010) |
| 1999cc | 0.0316 | 35.718(0.037) | 0.035(0.012) | 1.408(0.026) |
| 1999cp | 0.0104 | 33.452(0.034) | 0.002(0.007) | 0.838(0.030) |
| 1999cw | 0.0113 | 33.349(0.124) | 0.010(0.040) | 0.830(0.020) |
| 1999dd | 0.0141 | 34.157(0.056) | 0.000(0.020) | 0.830(0.010) |
| 1999dq | 0.0136 | 33.390(0.16) | 0.092(0.036) | 0.941(0.010) |
| 1999ee | 0.0104 | 33.349(0.015) | 0.230(0.005) | 0.830(0.010) |
| 1999ef | 0.0380 | 36.417(0.044) | 0.000(0.010) | 1.065(0.015) |
| 1999eg | 0.0128 | 34.197(0.043) | 0.051(0.017) | 1.534(0.029) |
| 1999ek | 0.0176 | 34.268(0.070) | 0.157(0.027) | 1.058(0.034) |
| 1999ed | 0.0193 | 34.451(0.074) | 0.411(0.022) | 1.180(0.025) |
| 2000B | 0.0260 | 35.280(0.032) | 0.034(0.010) | 0.832(0.010) |
| 2000Bb | 0.0193 | 34.470(0.038) | 0.075(0.015) | 1.300(0.016) |
| 2000bb | 0.0240 | 35.118(0.021) | 0.001(0.003) | 1.129(0.009) |
| 2000bk | 0.0266 | 35.267(0.048) | 0.146(0.020) | 1.700(0.010) |
| 2000ca | 0.0245 | 35.092(0.017) | 0.000(0.005) | 0.858(0.010) |
| 2000cc | 0.0164 | 34.170(0.037) | 0.513(0.016) | 0.998(0.031) |
| 2000cf | 0.0360 | 36.207(0.031) | 0.023(0.013) | 1.157(0.019) |
| 2000cn | 0.0233 | 35.023(0.036) | 0.099(0.013) | 1.700(0.010) |
| 2000dk | 0.0164 | 34.246(0.013) | 0.004(0.006) | 1.690(0.010) |
| 2000fe | 0.0218 | 34.925(0.029) | 0.063(0.011) | 1.049(0.016) |
| 2001ba | 0.0305 | 35.590(0.013) | 0.000(0.004) | 1.054(0.010) |
| 2001bt | 0.0144 | 33.616(0.036) | 0.214(0.011) | 1.170(0.014) |
| 2001cn | 0.0155 | 33.901(0.019) | 0.128(0.006) | 1.152(0.010) |
| 2001cz | 0.0163 | 34.025(0.032) | 0.096(0.009) | 0.977(0.018) |

**Notes.**—Col. (1): Name of the SN. Col. (2): Redshift in the CMB frame. Col. (3): Best-fit distance modulus ($h = 0.72$); statistical errors in parentheses. Col. (4): Best-fit color excess $E(B-V)_{\text{R}^{\odot}}$; statistical errors in parentheses. Col. (5): Best-fit $\Delta m_{15}$; statistical errors in parentheses.
Fig. 7.—Results of the fits to the multicolor LCs. The different filters (symbols) are: $B$ (squares), $V$ (triangles), $R$ (circles), and $I$ (diamonds). To avoid overlap between different filters, we applied shifts to the relative magnitudes: $\Delta B, \Delta V-1, \Delta R-2,$ and $\Delta I-3$.

Fig. 8.—Same as Fig 7, but for SN 1992ae–SN 1992bk.

Fig. 9.—Same as Fig 7, but for SN 1992bl–SN 1993ac.

Fig. 10.—Same as Fig 7, but for SN 1993ae–SN 1995ac.
Fig. 11.—Same as Fig 7, but for SN 1995ak–SN 1997Y.

Fig. 12.—Same as Fig 7, but for SN 1997bp–SN 1998ab.

Fig. 13.—Same as Fig 7, but for SN 1998bp–SN 1998X.

Fig. 14.—Same as Fig 7, but for SN 1999aa–SN 1999ef.
In Figure 6 we plot the Hubble diagram constructed with results of the fits to the 89 SNe in the Hubble flow. The final error in the distance modulus is obtained by adding in quadrature the statistical uncertainties from the covariance matrix and 0.17 mag, which is the maximum natural dispersion in the Hubble diagram.

In Table 6, we compare the dispersion in the Hubble diagram with other results in the literature, using the same SNe as in the original papers. In general we find similar or smaller values for the dispersion, with the Calán/Tololo subsample giving the fit with the lowest dispersion. The dispersion varies in the range $\sigma = 0.14$–0.21 mag with our technique and between $\sigma = 0.14$ and 0.24 mag for results in the literature, using identical samples of SNe. This further demonstrates that our technique performs equally well or better than other existing techniques in the literature. To reduce the dispersion in the Hubble diagram further will require new uniform data sets, realistic priors that allow different reddening laws for the host galaxies, and careful attention to the calibration of the telescope/filter/detector system required by the S-corrections.

6. CONCLUSIONS

We have presented an empirical method to fit multicolor light curves of Type Ia supernovae and estimate their luminosity distances. This technique combines what we think are the advantages from two widely used methods in the literature: the $\Delta m_{15}$ template fitting method (Phillips et al. 1999) and MLCS (Riess et al. 1996).

Our basic fitting algorithm uses a set of 14 $BVRI$ light-curve templates with different values of $\Delta m_{15}(B)$ and a simple triangle weighting function. A linear combination of the templates, weighted by this function, is introduced directly into the $\chi^2$ fitting, avoiding the construction of a secondary grid of interpolated templates. This allows us to add or change templates trivially and also allows us to introduce other parameters to be fit in the $\chi^2$ minimization. The $\chi^2$ fit returns the 1 $\sigma$ uncertainties in the best-fit parameters from the covariance matrix.

From a sample of 94 nearby Type Ia SNe ($z \lesssim 0.1$) we established linear relations between the absolute magnitudes at maximum $M_{\text{max}}$ and $\Delta m_{15}$ in $BVRI$ filters. These relations are valid in the range $0.80 < \Delta m_{15}(\text{mag}) < 1.70$, and they are consistent with other results presented in the literature (Phillips et al. 1999; Garnavich et al. 2004; Germany et al. 2004). We studied the relations using different subsamples of SNe, associated with different surveys and publications in the literature: Calán/Tololo, CFAI, Krisciunas et al., CFALL, and a nearby sample ($z \lesssim 0.01$) with distances obtained from the SBF method. The results from different subsamples are consistent in almost all the cases when we compare the slopes and zero points of the linear relations in the same filter, but disturbing differences do exist mainly in $B$ and $V$ absolute magnitudes at $\Delta m_{15} = 1.1$. Further progress will require new light-curve data sets, realistic priors that allow different reddening laws, and careful attention to the calibration of the telescope/filter/detector system required by the S-corrections.

We have constructed a Hubble diagram with low-$z$ SNe fitting the LCs of 89 SNe in the Hubble flow. The dispersion of the Hubble diagram is $\sigma = 0.20$ mag or an error of $\sim 9\%$ in distance to single objects, consistent with other fitting techniques (Jha...
et al. 2006a; Germany et al. 2004). We compared the dispersion in the Hubble diagram using our technique with other results in the literature. In general our technique gives similar or smaller dispersions than published results when the comparison is made using identical samples of SNe.

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