Wrapping Branes in Space and Time

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Abstract

Branes may be approximated semi-classically by solutions of supergravity theories with event and Cauchy horizons. I suggest that if one wishes to avoid singularities and to capture accurately some of the properties of branes then these classical spacetimes must be identified so as to render them periodic in time.

1 Introduction

Great progress in string and M-theory has been achieved in recent years by considering classical solutions of their low energy supergravity limits. The solutions of greatest interest typically represent electrically or magnetically charged extreme p-branes with degenerate (i.e zero-temperature) event horizons. These \((p + q + 4)\) dimensional metrics may be expressed in isotropic form as

\[
ds^2 = H^{-\frac{1}{p+1}} \left\{-dt^2 + d\mathbf{x}_p.d\mathbf{x}_p\right\} + H^{\frac{1}{q+1}} d\mathbf{y}_{q+3}.d\mathbf{y}_{q+3}
\]

where \(H\) is a harmonic function on the transverse euclidean space \(\mathbb{R}^{p+3}\) with coordinates \(\mathbf{y}_{q+3}\). In addition one has a \(p + 2\) form or \(q + 1\) form field strength and one may have a dilaton and other moduli fields. One of the simplifications of M-theory is that the dilaton and other scalars are absent. In what follows I shall mainly restrict myself to the M-theory case for which \(p = 2\) (the membrane) and \(p = 5\), the five-brane.

As explained in a recent paper [1], the coordinates \((t, \mathbf{x}_p, \mathbf{y}_{q+3})\) provide a harmonic map of the portion of full spacetime exterior to the brane into \(\mathbb{R}^{p+q+4}\). The horizons then get mapped into distributional sources. That description is very close to one based on elementary or fundamental branes moving in flat
or almost flat spacetime but it leaves out all the subtle geometrical details connected with the the horizons and their interiors. In this paper by contrast it is precisely some aspects of the horizon geometry that I wish to explore. To some extent one is dealing with some complementary features of branes. The electric or magnetic charges carried by the $p + 2$ form or $q + 2$ form respectively are central charges and saturate the supergravity version of the familiar Bogomol'nyi bound of magnetic monopole theory. This implies that the solutions are supersymmetric and one anticipates, at least in those cases when only one half of the maximum supersymmetries are unbroken, that some classical properties may be extrapolated to the fully quantum regime.

These self-gravitating solutions are similar in many ways to their analogues in theories without gravity. For example they may be considered to interpolate spatially between different maximally supersymmetric vacua. For the $M$-2-brane the near horizon state corresponds to the $AdS_4 \times S^7$. For the $M$-5-brane it corresponds to its dual: $AdS_7 \times S^4$.

The electrical solutions resemble the Coulomb solutions (for which $E^a = D\Phi^a$), of non-Abelian Yang-Mills theory have singularities inside the event horizon. One expects them to provide a good approximation to the field generated by many ‘fundamental’ or ‘elementary’$p$-branes sitting on top of one another. By contrast the magnetically charged solutions resemble the solitons (for which $B^a = D\Phi^a$) of non-Abelian Yang-Mills theory. In some cases, such as the 5-brane of M-theory, they are completely non-singular.

On the horizon $H^{-1}$ vanishes and thus if $p$ or is greater than zero the area of the event horizons vanish. In other words fundamental and solitonic extreme branes carry no Bekenstein-Hawking entropy. This is what one usually expects of fundamental objects and solitons whose fundamental dynamics is non-dissipative. Of course 0-branes, even extreme ones, do have entropy but this is associated with the “intersection” of branes.

However there remain some difficulties with this interpretation. They include:

- The solutions above represent infinite p-branes. One wants to be able to wrap them over tori, but this cannot be done naively because, as we shall see shortly, translations of the world volume coordinates $x_p$ do not act freely on the horizon.
- M-theory seems to admit 2-branes whose world volumes are not space-orientable. Five-brane world volumes however must be orientable. How do we construct them?
- The existence of horizons, albeit extreme, would seem on the face it to lead semi-classically to “information loss” and dissipative dynamics.
- The existence of Cauchy horizons allows passage through to an infinite chain of other universes. This is surely rather a superfluous if one wants to describe a single brane.
Of course one might argue that these are just symptoms of a breakdown of the semi-classical approximation. However since M-theory has no dimensionless parameter to expand in this is a rather dangerous argument since a failure to capture the essential qualitative features of M-theory, such as toroidal and possibly non-orientable world volumes in the classical limit would cast doubt on the relevance of the classical solutions at all. Moreover recent work in M-theory has revealed that a surprisingly large amount of the structure is reflected in the classical solutions see e.g. [6]. Therefore in this paper I shall take a different approach. I will explore a possible resolution by considering how one can identify the classical solutions by isometries without fixed points and thus avoid the introduction of extra singularities. This leads to the suggestion that the solutions should be identified periodically in time. It has previously been argued that an approximation to the world volume theories may be obtained by considering the singleton and doubleton representations of the relevant Anti-De-Sitter groups [2]. I shall argue that my suggestion is consistent with that viewpoint.

2 Mapping into \( CAdS_{p+2} \times S^{q+2} \)

A convenient way of exhibiting the global structure of the single extreme \( p \)-brane spacetimes is to map them onto a portion of the universal covering space of Anti-de-Sitter spacetime times the \((q+2)\)-sphere \( CAdS_{p+2} \times S^{q+2} \). For multi-\( p \)-brane solutions many such patches will be required. We begin by reviewing some standard material on \( AdS_{p+2} \).

2.1 Some properties of \( AdS_{p+2} \) and \( CAdS_{p+2} \)

One usually defines \( AdS_{p+2} \) as the hyperboloid in \( E^{p+1,2} \) whose coordinates are \( X^A, A = 0, \ldots, p + 3 \) and which is given by

\[
(X^0)^2 - (X^i)^2 + (X^{p+2})^2 = 1,
\]

with its induced lorentzian metric, where \( i = 1, \ldots, p + 1 \). Note that \( X^0 \) and \( X^{p+2} \) are timelike coordinates and \( X^i \) are spacelike coordinates. \( AdS_{p+2} \) is not simply connected, it has topology \( S^1 \times \mathbb{R}^{p+1} \). The simply connected universal covering space has topology \( \mathbb{R}^{p+2} \) and is traditionally called \( CAdS_{p+2} \). We introduce coordinates \((\tau, \chi, n)\) making manifest the maximal compact subgroup \( SO(p + 1) \times SO(2) \subset SO(p + 2, 2) \) by

\[
X^0 = \cos \tau \cosh \chi, \\
X^{p+2} = \sin \tau \cosh \chi, \\
X^i = \sinh \chi n^i.
\]
where \( n^i \) is a unit vector in \( \mathbb{E}^{p+1} \) defining \( S^p \). In these coordinates the metric takes the globally static form:

\[
ds^2 = -\cosh^2 \chi d\tau^2 + d\chi^2 + \sinh \chi d\Omega^2_p
\]

where \( d\Omega^2_p \) is the standard round metric on \( S^p \). We take \( 0 \leq \tau \leq 2\pi \) to get \( \text{AdS}_{p+2} \) but allow it to take all real values to obtain \( \text{CAdS}_{p+2} \). Both \( \text{AdS}_{p+2} \) and \( \text{CAdS}_{p+2} \) are space and time orientable. Evidently \( \text{AdS}_{p+2} \) is periodic in time and has closed timelike curves. In fact every timelike geodesic is periodic with proper-time period \( 2\pi \). The universal covering space \( \text{CAdS}_{p+2} \) is not periodic in time and has no closed timelike curves.

The antipodal map \( J \) lies in the centre of \( O(p+2,2) \) and acts on \( \text{AdS}_{p+2} \) by

\[
J : X^A \to -X^A.
\]

In global, static coordinates this becomes \((\tau, \chi, n) \to (\tau + \pi, \chi, -n)\). The antipodal map preserves time orientation but, since it induces the antipodal map on \( S^p \), it reverses space orientation or preserves it depending upon whether \( p \) is even or odd respectively. In fact \( J \) lies in the identity component of \( \text{SO}(p+1,2) \) if \( p \) is odd. On \( \text{AdS}_{p+1} \) \( J \) acts an involution : \( J^2 = \text{id} \). We may extend \( J \) to an action of the integers on \( \text{CAdS}_{p+2} \) by defining

\[
J^n : (\tau, \chi, n) \to (\tau + n\pi, \chi, (-1)^n n).
\]

Clearly we can take the quotient of \( \text{CAdS}_{p+2} \) by any integer power of \( J \). If we take \( J^{2k} \), \( k \in \mathbb{Z} \) we get the \( k \)-fold cover of \( \text{AdS}_{p+2} \). Of course it is now a \( k \)-fold cover of \( \text{SO}(p+1,2) \) which acts.

**2.2 Horospheric coordinates**

For brane purposes it is convenient to introduce another set of coordinates for \( \text{AdS}_{p+2} \). They are \((t, x_p, z)\) where \( x_p \) has \( p \) components and make manifest the action of the Poincare subgroup of \( \text{SO}(p+1,2) \). One sets

\[
X^0 = \frac{t}{z},
\]

\[
X^a = \frac{x^a}{z},
\]

\[
X^{p+2} - X^{p+1} = \frac{1}{z},
\]

and

\[
X^{p+2} + X^{p+1} = z + \frac{(x_p, x_p - t^2)}{z}.
\]

The metric takes the form

\[
ds^2 = \frac{1}{z^2} \left\{-dt^2 + dx_p dx_p + dz^2\right\}.
\]
These coordinates provide a foliation of $\text{AdS}_{p+2}$ by flat timelike hypersurfaces $z = \text{constant}$. These hypersurfaces are the intersections of the null hyperplane with the hyperboloid. In fact, exact horosphere is a solution of the Dirac action for a test p-brane and so we see very vividly that the foliation by horospheres corresponds to the idea of a supergravity p-brane consisting of a very large number of fundamental p-branes.

Horospheric coordinates cover only one half of $\text{AdS}_{p+1}$. They break down at $z = \infty$ which is the intersection of the null hyperplane which passes through the origin,

$$X^{p+2} - X^{p+1} = 0,$$

with the hyperboloid. The antipodal map $J$ simply reverses the sign of $z$. If one were to identify $\text{AdS}_{p+2}$ under the action of $J$ a single horospheric patch would suffice. If one passes to the universal covering space $\mathbb{C}\text{AdS}_{p+2}$ one needs infinitely many patches. One may regard $z = 0$ as a degenerate Killing horizon associated to the everywhere non-spacelike time translation Killing vector field $\partial / \partial t$. The Killing vector $\partial / \partial t$ generates the action $t \to t + c$ and these act freely on $\text{AdS}_{p+2}$. To see this one may use the embedding coordinates. Time translations act as

$$X^0 \to x^0 + c(X^{p+2} - X^{p+1}),$$

$$X^i \to X^i,$$

$$X^{p+2} - X^{p+1} \to X^{p+2} - X^{p+1},$$

and

$$X^{p+2} - X^{p+1} \to X^{p+2} + X^{p+1} + 2cX^0 + c^2(X^{p+2} - X^{p+1}).$$

Any possible fixed points must satisfy

$$X^{p+2} - X^{p+1} = 0,$$

and

$$X^0 = 0.$$

But this implies that

$$-X^iX^i = 1$$

which is impossible.

By contrast the spatial translations $x_p \to x_p + a_p$ do not act freely. The action on the embedding coordinates is

$$X^0 \to X^0,$$

$$X^i \to X^i + a^i(X^{p+2} - X^{p+1}),$$

$$X^{p+2} - X^{p+1} \to X^{p+2} - X^{p+1}$$

and

$$X^{p+2} + X^{p+1} \to X^{p+2} + X^{p+1} - 2a^iX^i - a^2(X^{p+2} - X^{p+1}).$$
The fixed points lie on the horizon

\[ X^{p+2} - X^{p+1} = 0 \]

and must also satisfy

\[ a^i X^i = 0. \]

It is easy to satisfy this condition.

It follows form this that one may, if one wishes, identify \( AdS_{p+2} \) under a discrete time translation and obtain a non-singular quotient but one cannot identify under a discrete space translation and obtain a non-singular quotient. It is also clear that spatial parity \( \Omega_p \), i.e, \( x_p \rightarrow -x \) does not act freely. In
the brane context there is no good motivation for identifying under world sheet time translations but if the world sheet is toroidal one must identify under a discrete group \( \Lambda_p \) of spatial translations and if one considers orientifolds, or if one wishes to consider non-orientable world sheets one is interested in identifying under spatial parity \( \Omega_p \). Note that as a general principle one should not identify under time orientation reversing isometries, either in spacetime or on the world volume since that leads to difficulties with quantization. One would be forced into real quantum mechanics.

### 2.3 Maximal extensions of \( p \)-branes.

Having developed the necessary material about \( AdS_{p+2} \) we are in a position to discuss the maximal extensions of the \( p \)-brane solutions. The basic idea is to map them into the universal cover, \( CAdS_{p+2} \), pushing forward the metric. Note that the metric will not be conformal to the \( AdS_{p+2} \) metric.

We may take, by choosing the unit of length appropriately,

\[ H = \left( 1 + \frac{1}{\rho^{q+1}} \right) = \frac{1}{(1 - \frac{1}{p+1})}, \]

where \( \rho = \sqrt{y^2} \) is an isotropic radial coordinate and \( r \) is a Schwarzschild radial coordinate. The \( (q + 1) \)-volume of a transverse \( q + 1 \)-sphere is thus \( r^{q+1} \omega_{q+1} \) where \( \omega_{q+1} \) is the volume of a unit \( (q + 1) \)-sphere. The required mapping is obtained by setting

\[ \frac{p+1}{z} = H^{-\frac{1}{p+1}} \]

The horizon, at which \( H \) diverges, is mapped to the degenerate horizon at \( z = \infty \) and in the neighbourhood of the horizon we have the asymptotic form of the metric

\[ ds^2 \approx \left( \frac{p+1}{z} \right)^2 \left\{ -dt^2 + dx_p dx_p + dz^2 \right\} + d\Omega_{q+2}^2. \]

The exact form of the metric is obtained by expressing \( \rho \) in terms of \( z \) and substituting. Note that the \( AdS_{p+2} \) factor is scaled up by a factor \( p + 1 \). The maximal extension is obtained by lifting this metric to the universal cover \( CAdS_{p+2} \).
The result depends upon the parity of $p$. If $p$ is even the spacetime is like extreme Reissner-Nordstrom: there is an infinite chain of asymptotically flat regions joined via degenerate horizons to an infinite chain of internal regions containing timelike singularities at $r = 0$ which are inside the horizons. By contrast if $p$ is odd the solution is symmetrical and completely non-singular: the extension consists of two infinite chains of asymptotically flat regions joined by degenerate horizons.

We may express this in terms of the map $J$ which continues to act freely on the spacetime manifold but not necessarily as an isometry. Thus if $p$ is even $J$ takes exterior regions to interior regions but is not an isometry of the $p$-brane metric. However if $p$ is odd $J$ is an isometry of the $p$-brane metric and takes one exterior region to an adjacent one. Of course $J^2$ always acts by isometries and takes one along the chains of exterior or interior regions.

It follows from this that one may identify the $p$-brane metric by any even integer power of $J$ if $p$ is odd even and any integer power of $J^2$ if $p$ is even without introducing any more singularities than were originally present. The resulting spacetimes will be time orientable but contain closed timelike curves. Taking the smallest possible case this means that for the 2-brane one has just one external region and one internal region. A timelike curve falling through the future horizon from the outside may either hit the singularity or leave the internal region and return to the external region through the past horizon before it left it. In the case of the 5-brane the curve would never leave the external region since just as it crossed the future horizon it would re-appear entering the exterior region through the past horizon. More elaborate delay phenomena may can take place if one identifies under higher powers of $J$. In many ways the nicest case is the minimal one because in that case all but one infinite region is eliminated. This means that there is no possibility of information being lost from one infinite region into another. Any possible information loss could only be into the singularity. However that is where the semi-classical approximation breaks down and so one might argue that one should not be concerned about it since it might be just an artefact of an inapplicable approximation. By contrast the loss of information to other infinite region is is potentially more worrying because the loss is via regions near the horizon in which the curvature need not be especially high.

2.4 Toroidal p-branes.

It is clear from our discussion of $AdS_p$ that one cannot identify under the action of the discrete group of $p$ spatial translations $\Lambda$ to obtain a non-singular quotient since the lattice $\Lambda_p$ (in the mathematical sense of a finite freely generated abelian group) does not act freely. For the same reason we cannot identify the $p$-brane spacetimes under the action of the lattice $\Lambda_p$ to obtain a regular solution with a toroidal world volume. In other words we cannot obtain a regular non-singular classical solution representing a $p$-brane wrapped around a toroidal cycle. Since
much of current work on D-branes assumes that this is possible this is rather disturbing. There is, however, an obvious way out. One may compose the lattice $\Lambda$ with any power of the antipodal map $J^n$. In other words the map $J^n \cdot \Lambda$ does act freely on $CAdS_{p+2}$. Thus as long as one wraps the $p$-brane in time direction as well as in space one will obtain a smooth quotient. The integer $n$ should be multiple of 2 for the 2-brane but it could be any integer for the 5-brane. We shall see in the next section that singleton considerations suggest that we should take $n = 4$ for the 2-brane and $n = 1$ for the 5-brane.

2.5 Orientifolding and non-orientable world volumes.

The previous remarks can be immediately extended to cover world volumes which are not space-orientable. World volume parity $\Omega_p$ we take to be

$$\Omega_p : x_p \rightarrow -x_p.$$  

in terms of the embedding coordinates this takes

$$X^i \rightarrow -X^i$$  

and leaves the other coordinates unchanged. Clearly $\Omega_p$ does not act freely but we could compose with with any integer power of $J$ to get something with no fixed points.

Suppose we wanted a Klein Bottle world volume. The Klein bottle may obtained by identifying $E^2$ in say the 1-direction direction by a translation and in the 2-direction by a glide reflection, in other words other by the composition of a translation in the 2-direction and a reflection in the 2-axis. As before this does not act freely at the horizon but composing with an appropriate integer power of $J$ we can obtain a free action. Thus again we are led to wrap in time as well as in space.

2.6 Intersecting branes and black holes

There exist solutions of M-theory depending upon more than one harmonic function which represent ‘intersecting ’branes. Not all such solutions are non-singular but some are. Thus there is a solution depending on three harmonic functions $H_1, H_2, H_3$ on $E^4$ with coordinates $y_4$ which may be interpreted as three 2-branes ‘intersecting ‘on a point.

The metric takes the form

$$ds^2 = -(H_1H_2H_3)^{-\frac{1}{2}} dt^2 + (H_1H_2H_3)^{\frac{1}{2}} dy_4 \cdot dy_4$$

$$+ \left(\frac{H_2H_3}{H_1^2}\right)^{\frac{1}{2}}(dx_1^2 + dx_4^2) + \left(\frac{H_3H_1}{H_2^2}\right)^{\frac{1}{2}}(dx_2^2 + dx_5^2) + \left(\frac{H_1H_2}{H_3^2}\right)^{\frac{1}{2}}(dx_3^2 + dx_6^2).$$
The 6 Killing fields $\frac{\partial}{\partial x^i}$, $i = 1, 2, 3, 4, 5, 6$ never vanish and so one may identify the coordinates $x_i$, $i = 1, 2, 3, 4, 5, 6$ without introducing any quotient singularities. With this identification the metrics admit a free action of the torus group $T^6$. They may therefore be dimensionally reduced to $4 + 1$ spacetime dimensions. The resulting solution represents a black hole. We may further reduce to $4 + 1$ dimensional solution representing an extreme black hole.

Near the throat the geometry behaves like $AdS_2 \times S^3$. The causal structure is Reissner-Nordstrom-like. The maximal extension is obtained using the same procedure as described above. It follows that one may extend the action of the antipodal map for two-dimensional Anti-de-Sitter space $J$ to the maximal extension such that $J^2$ acts isometrically. Thus we may again identify periodically, just as we may for the spacetime of a single 2-brane.

Another non-singular solution representing the intersection of three 5-branes over a string and depending upon three harmonic function on $E^3$ has the metric

$$ds^2 = (H_1 H_2 H_3)^{-\frac{1}{3}} (-dt^2 + dx_7^2) + (H_1 H_2 H_3)^{\frac{1}{3}} dy_3 dy_3$$

$$+ \left( \frac{H_1^2}{H_2 H_3} \right)^{\frac{1}{3}} (dx_1^2 + dx_4^2) + \left( \frac{H_2^2}{H_3 H_1} \right)^{\frac{1}{3}} (dx_2^2 + dx_5^2) + \left( \frac{H_3^2}{H_1 H_2} \right)^{\frac{1}{3}} (dx_3^2 + dx_6^2).$$

Again there is a free $T^6$ action and one may dimensionally reduce to get a solution in $4 + 1$ dimensions. The solution is independent of the coordinate $x_7$ and thus represents an extreme string. Near the throat it looks like $AdS_3 \times S^2$. However the Killing field $\frac{\partial}{\partial x_7}$ does not generate a free action and further reduction dimensions, to get $3 + 1$ dimensional black hole solutions is problematical. Let us therefore remain in $4 + 1$ dimensions and maximally extend. In the simple case in which we assume that $H_1 = H_2 = H_3 = 1 + \frac{1}{\rho}$ may apply the results of [2] to discover we find that maximal extension is symmetric and singularity free like the 5-branes whose intersection it is. The extension with three distinct harmonic functions $H_i$ each of the form:

$$H_i = 1 + \frac{\mu_i}{\rho},$$

is only slightly more difficult. One sets

$$H_1 H_2 H_3 = \mu_1 \mu_2 \mu_3 \left( \frac{z^2}{2} \right)^3.$$

It is clear that the metric functions will be even functions of $z$ and hence the antipodal map $J$ will act isometrically.

Now if we wish to the string coordinate identify $x_7$ we may do so as long as we compose with $J$ as well.
3 Singletons and Doubletons

Some extra evidence in favour of the identifications I am proposing is provided by considering the singleton and doubleton representations of $SO(3,2)$ and $SO(6,2)$ and their super-extensions. These have no analogues among the representations of the Poincare groups of 4 or 7 dimensional Minkowski spacetime. They do not, therefore, survive in the limit as the curvature of $AdS_4$ or $AdS_7$ is sent to zero. Even if the curvature is non-vanishing they cannot be realized as quantum fields propagating in the bulk in $AdS_4$ or $AdS_7$, respectively. However they can be realized as conformally-invariant supersymmetric quantum field theories on $S^1 \times S^2$ or $S^1 \times S^5$. By defining a coordinate $\lambda$ by

$$\sin \lambda = \tanh \chi$$

one may conformally embed $CAdS_{p+2}$ into the static Einstein universe. The metric becomes

$$ds^2 = \frac{1}{\cos^2 \lambda} \left\{ -d\tau^2 + d\lambda^2 + \sin^2 \rho d\Omega^2_p \right\}.$$ 

The conformal boundary of $AdS_{p+2}$ is given by $\lambda = \frac{\pi}{2}$. Thus one may regard $S^1 \times S^p$ as the conformal boundary of $AdS_{p+2}$ and the isometry group $SO(p+1,2)$ of $AdS_{p+2}$ acts by conformal transformations on $S^1 \times S^p$. If $p = 2$ or $p = 5$ these quantum field theories have the same degrees of freedom as the world volume fields of $p$-branes and it was suggested that the lowest scalar component, call it $X$, may be regarded as the transverse oscillation of the $p$-brane which is localized in the vicinity of the horizon. Note that this interpretation is similar to, but not exactly the same as, the idea of the ‘membrane at the end of the universe’ since we are now regarding a small timelike tube around the horizon as the location of the 2-brane or 5-brane and claiming that the membrane oscillations can be described group-theoretically in terms of singletons. Presumably there are fluctuations of the spacetime geometry carrying the singleton representations.

In the linearized approximation we are considering here $X$ satisfies the conformally invariant wave equation on $S^1 \times S^p$

$$-\nabla^2 X + \frac{p-1}{4p} R X = 0,$$

where $R$ is the Ricci scalar of $S^1 \times S^p$ which equals the Ricci scalar of $S^p$. On a unit $p$-sphere $R = p(p-1)$, and the eigenvalues of the spherical harmonics $Y_l(n)$ on $S^p$ are $l(l+p-1)$, where $l = 0, 1, \ldots$. Thus the modes of $X$ behave like

$$X_l = \exp -i(l + \frac{p-1}{2}) \tau Y_l(n).$$

Thus

$$X(t + 2\pi) = (-1)^{p-1} X(t).$$
It follows that if $p$ is odd, as it is for the 5-brane, then $X$ is periodic with the basic Anti-de-Sitter period. If however $p$ is even, as it is for the 2-brane, then $X$ is periodic with half the basic Anti-de-Sitter period. In other words the theory is really defined not on $S^1 \times S^p$ but its double cover.

We may express this in terms of the antipodal map $J$ which acts on the boundary of $AdS_{p+2}$ as

$$J : (\tau, n) \rightarrow (t + \pi, -n).$$

In other words we advance $t$ by half a period and compose with the antipodal map on $S^p$. As before we may extend $J$ to act in the obvious way on the universal covering space $\mathbb{R} \times S^p$. Now the spherical harmonics satisfy

$$Y_l(-n) = (-1)^l Y_l(n).$$

Thus

$$X \cdot J = i^{p-1}X.$$

The singletons are therefore invariant under $J$ if $p = 5$ and under $J^4$ if $p = 2$. Thus indeed the singletons indicate that one may identify the inside with the outside of the 5-brane. In the case of the 2-brane however they indicate that one needs at least two exterior and two interior regions.

The above analysis of singletons would also apply in the case of strings, i.e. $p = 1$. It is striking that we get the same result as for the 5-brane. Particularly so when we recall that we may construct a string as a triple intersection of 5-branes.

## 4 Conclusion

In this paper I have studied the global structure of the classical spacetimes of branes and their non-singular intersection in M-theory. I have found that if one wraps the branes in space it is also necessary to wrap them in time in order to avoid quotient singularities. This procedure is consistent with properties of the relevant singleton representations and may help resolve some of the puzzles connected with the possible loss of information in spacetimes with horizons and eliminates the infinite chain of other universes.

An initial draft of the present paper was written over a year ago, long before the current rise in the number of papers relating p-branes and singletons. It has been revised recently but since the ideas expressed here, while being relevant to that activity are independent of it, I have made no attempt to include them in the bibliography.

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References

[1] G W Gibbons, Branes as BIons, hep-th/9803203

[2] G W Gibbons and P K Townsend, Vacuum Interpolation in Supergravity via Super p-branes, Phys Rev Lett 23 (1993) 3754-3753 hep-th/9307043

[3] M J Duff, G W Gibbons and P K Townsend, Macroscopic Superstrings as Interpolating Solitons, Phys Lett B332(1994) 321-328 hep-th/9405124

[4] G W Gibbons, G T Horowitz and P K Townsend, Higher Dimensional Resolution of Dilatonic Black Hole Singularities, Class Quant Grav 12 (1995) 297-318 hep-th/9410073

[5] A A Tseytlin, Harmonic superpositions of M-branes, Nucl Phys B276 (1996) 149 hep-th/9604035

[6] G W Gibbons, Born-Infeld particles and Dirichlet p-branes, Nucl Phys B in press hep-th/9709027