Gyrokinetic simulations of collisionless tearing modes

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Abstract. We investigate the effects of temperature and density gradients on collisionless magnetic reconnection by means of PIC-$\delta f$ simulations with standard gyrokinetic equations using gyrokinetic ions and drift kinetic electrons. In a slab model ion and electron temperature gradients act stabilising on the tearing mode instability in the cases that have been considered. An additional ion and electron density gradient completely stabilises the mode below a critical $\eta$ (ratio of temperature gradient to density gradient). The critical $\eta$ becomes larger with decreasing equilibrium current.
Nonlinear single-mode simulations show that the island width saturates at the level predicted by [1], if the linear instability growth rate is sufficiently small. Including ion and electron temperature and density gradients in nonlinear calculations the saturated island width vanishes below a critical $\eta$.

1. Introduction
Magnetic reconnection is an important subject in astrophysics and magnetic confinement physics [2]. For instance solar flares are thought to be caused by reconnection events strongly accelerating charged particles [3]. The complicated structure of the Sun’s surface can be simplified with locally antiparallel magnetic field line structures, which are unstable with respect to tearing modes. In these high-temperature plasmas resistivity can play a significant role, as described e. g. by the resistive MHD [2, 4]. However magnetic reconnection in Earth’s magnetotail could not be explained this way since the conductivity in Ohms law almost vanishes completely in this collisionless plasma. Additional inertia terms could solve the problem of triggering reconnection [4, 5]. It was also suggested that nongyrotropic effects could influence reconnection [6].
Here we investigate laboratory aspects of magnetic reconnection, where a strong magnetic guiding field and a small sheared magnetic field in the reconnection plane are present. We can employ standard gyrokinetics assuming that the Larmor radius is much smaller than the equilibrium and gradient scales and the growth rates are much smaller than the gyrofrequencies. Up to date there are only very few particle-in-cell (PIC) simulations of tearing modes which include density and temperature gradients [7]. Although previous gyrofluid approaches show that a complete stabilisation must occur [8], a comparison with PIC results was not available.
Another aspect are nonlinear simulations of tearing modes. There are numerical results of saturated island widths resulting from kinetic theories [9], but they have not been compared with other PIC or gyrofluid codes in an extended parameter space. Kinetic nonlinear tearing theories [1] predict saturated one-mode behaviour and saturated island widths depending on the shear length \( l_s \) and the perpendicular wave vector \( k \). Here we compare our PIC simulations with this theory for saturated island widths, which could be important for toroidal systems in future. Moreover, there are only very few simulations using nonlinear gyrokinetics that include temperature and density gradients [7]. It has been suggested that gradients should influence the saturated island width due to diamagnetic effects [8]. This is generally of importance since for most confined plasmas a sheared magnetic field is accompanied by gradients due to the equilibrium pressure condition.

2. The model
We use the PIC-\( \delta f \) code EUTERPE, where the distribution function \( f_s \) for each species \( s \) (i: ions, e: electrons) is split into a background distribution function \( f_{0,s} \) and a perturbed part \( \delta f_s \). Here the background is assumed to be a Maxwellian with a bulk velocity \( u_{0,e} \) for the electrons in the rest frame of the ions (\( u_{0,i} = 0 \)). EUTERPE solves the gyrokinetic electromagnetic Vlasov-Maxwell-system globally in toroidal 3D equilibria. Here the tearing mode has been simulated in a slab geometry. The guiding field \( B_{z,0} \) is along the periodic \( z \)-direction. The \( y \)-direction is also treated periodically, whereas the equilibrium quantities only depend on the spatial \( x \)-coordinate ranging from \( 0 \) to \( L_x \). In the \( x \)-direction Dirichlet boundary conditions are employed for the vector potential \( A_\parallel \) and the electrostatic field \( \Phi \). We restrict our studies to two spatial dimensions by setting \( \partial/\partial z = 0 \). The local approach for the gradients of the temperature and densities has been used, but not for the current \( i.e., \) the gradients are constant over the whole simulation domain as well as the temperature and density itself. We work in normalised units, \( i.e., \) \( r_\ast = \sqrt{T_\ast m_p/e B_\ast} \), \( v_\ast = \sqrt{T_\ast/m_p} \), \( \Omega_\ast = v_\ast/r_\ast \) with \( B_\ast \) the guiding field \( B_{z,0} \) and \( T_\ast \) the normalising electron temperature \( T_{0,e} \). Charge and mass are normalised to the proton charge \( q_i = e \) and mass \( m_p \). Under these assumptions the gyrokinetic equations in the \( p_\parallel \)-formulation simplify to:

- Trajectories: \( \dot{R}_s = v_\parallel \tilde{b} - \frac{q_s}{m_s} \tilde{b} \langle A_\parallel \rangle + \frac{1}{B} \times \nabla \langle \Psi \rangle = v_\parallel \tilde{b} + \dot{R}_1 \) \hspace{1cm} (1)

- \( \delta f \) equation: \( \dot{\delta f}_s = - f_{0,s} S_s, \) \( \kappa_s = \kappa_{n,s} - \left( \frac{3}{2} - \frac{v_\parallel^2}{2v_{th,s}^2} \right) \kappa_{T,s} + \kappa_{u_{0,s}} \)

- Quasineutrality (long wave length limit):

\[
\sum_{s=i,e} q_s n_s = 0 \hspace{1cm} (5)
\]

for ions and drift kinetic electrons

\[
n_i = \langle n_i \rangle + \frac{m_i}{q_i} \nabla \cdot \left( \frac{n_{0,i}}{B^2} \nabla_\perp \Phi \right) \hspace{1cm} (6)
\]

(7)
• Ampère’s law:

$$-\frac{1}{\beta} \nabla_\perp^2 A_\parallel + \sum_{s=i,e} n_0 s \frac{q_s}{m_s} A_\parallel = \sum_{s=i,e} (j_{s\parallel})$$

(8)

with the gyroaveraging procedure $$\langle \Phi \rangle = \frac{1}{2\pi} \int \Phi(\vec{R} + \vec{\rho}) \, d\alpha$$ and the gradients for temperature, density and current

$$\kappa_{T,s} = -T_0,s' \frac{T_0,s}{T_0,s}, \quad \kappa_{n,s} = -n_0,s' \frac{n_0,s}{n_0,s}, \quad \kappa_{u_\parallel,s} = \left( v_\parallel - u_{0,s} \right) \frac{u_{0,s}}{v_{th,s}}$$

(9)

$$B$$ is the modulus of the equilibrium magnetic field, $$v_{th,s}$$ thermal speed of each species, $$v$$ the modulus of the velocity and a prime denotes $$d/dx$$.

Here we only are interested in configurations with $$\beta \ll 1$$; thus only the parallel vector potential $$A_\parallel$$ in Ampère’s law is used. Because of the parallel momentum formulation, a skin term in Ampère’s law appears. This formulation has the advantage that no time derivative of the vector potential enters the equations but necessitates the use of an adaptive control variate scheme [10].

3. Simulating the tearing mode

The magnetic field in the $$y$$-direction depends only on the $$x$$-coordinate:

$$B_{y,0}(x) = \frac{\beta}{2} \sqrt{\mu} C_1 a \sqrt{\pi} \text{erf} \left( \frac{x - Lx}{2a} \right)$$

(10)

with the resonant surface at $$Lx/2$$. $$a$$ is the half width of $$B_{y,0}(x)$$, $$C_1$$ is a free parameter which controls the strength of $$B_{y,0}(x)$$, and $$\mu = 1836$$ is the proton to electron mass ratio.

First EUTERPE has to be benchmarked for the tearing mode. Therefore we focus first on the simplest model of tearing, where we neglect electrostatic potential fluctuations $$\Phi$$, and the ions only serve as a neutralising fixed background. The electrons are the only kinetic species (electron tearing mode). Later we include gyrokinetic ions as well together with the electrostatic potential $$\Phi$$. If not otherwise stated, this will be the standard case (tearing mode).

To get the linear dispersion relation of the electron tearing mode, one applies the Fourier transform $$A_\parallel(x,y,t) = e^{i(ky-\omega t)} A_\parallel(x)$$. Then the problem of finding the dispersion relation $$\omega(k)$$ with $$k$$ as the wave vector in $$y$$-direction, reduces to an eigenvalue problem:

$$\frac{d^2 A_\parallel}{dx^2} = -q(x,\omega) A_\parallel$$

(11)

$$q(x,\omega) = -k^2 + \frac{\beta}{k_\parallel} \left( u_{0,e} \left[ \omega + \frac{k}{\mu} u_{0,e} u_{0,e} \right] \langle V^1 \rangle \right. + \left. \left[ \omega + 2 \frac{k}{\mu} u_{0,e} u_{0,e} \right] \langle V^2 \rangle + \frac{k}{\mu} u_{0,e} \langle V^3 \rangle \right).$$

(12)

Here $$k_\parallel(x) = k B_{y,0}/B_{z,0}$$ and $$\langle V^n \rangle(x,\omega)$$ are the $$n$$-th moments of the plasma dispersion function :
This eigenvalue problem is solved numerically using a shooting method. The shooting method reformulates the problem as a Riccati problem and integrates from both sides to a certain matching point. If the electrostatic potential is included together with drift kinetic ions the eigenvalue problem is of fourth order and can also be solved with the Riccati method.

The densities $n_{0,i} = n_{0,e}$ and temperatures $T_{0,i} = T_{0,e}$ are first assumed to be constant for the case of the electron tearing mode. The simulation domain in the perpendicular plane is set to $L_x \times L_y = 10 \times 10$ throughout the paper if not otherwise stated. When using noise as the initial condition for the fields, the instability evolves within about $t = 10$. The mode develops a very thin current layer of width smaller than the ion Larmor radius at the resonant surface $L_x/2$ in which magnetic energy is converted into thermal energy of the particles. Therefore the mode structure is localised around $L_x/2$. The equilibrium scale $L_x$ is much larger than the current channel and therefore the boundary conditions in the $x$-direction do not play an important role.

Figure (1) (left, green) shows the solution of the eigenvalue problem for the electron tearing mode with $\beta = 10^{-3}$ and $\gamma$ the imaginary part of the eigenvalue $\omega$. Varying the slab parameter $a$ and fixing the wave vector $k$, the range of instability of the mode is depicted. The agreement between the two methods is better than two percent.

The more general case including gyrokinetic ions and the electrostatic potential $\Phi$ (the tearing mode case) is shown in red in the left panel of Fig. (1). The dispersion relations $\omega(k)$ show a pure growing mode without real frequency. It turns out that here finite Larmor radius (FLR) effects have no influence, but the additional electrostatic potential slightly destabilises the mode. In our units, the Larmor radius is $\rho_i = 1$, while the electron skin depth $k_0^{-1} = 1/\sqrt{\beta \mu} \sim 0.7$. So the skin depth is somewhat smaller than the gyroradius $\rho_i$ and therefore the influence of the electrostatic potential $\Phi$ is expected to be small [9]. It also turns out that this small difference of the growth rate between including and neglecting the electrostatic potential $\Phi$ vanishes as $C_1$ goes to zero. Since $C_1 = -1$ is a rather large value so that the bulk velocity $u_{0,e}$ is in the range of $v_{th,e}$, this parameter set gives rather benchmarks than realistic scenarios. Furthermore because the growth rate of the instability is proportional to $v_{th,e}$, the runs do not need much computing time. For these benchmarks we have used about $5 \times 10^6$ markers for each kinetic species. Using 8 grid points in the $y$-direction, 256 points in the $x$-direction and the time step $\Delta t = 0.1$ these runs need about 20 CPUh.

![Figure 1.](image-url) Benchmarks of EUTERPE against shooting method match very well. Left: electron tearing and tearing mode including the electrostatic potential. Right: electron tearing mode with temperature gradient. The insert shows the real frequency of the mode.
4. Influence of density and temperature gradients

A natural generalisation, which is more realistic, includes gradients of temperature and density for each kinetic species, i.e. diamagnetic effects. In this case the electron tearing instability acquires a drift frequency, which also has been benchmarked together with the growth rate and is shown in Fig. (1) (right) for $\kappa_{T,e} = 1$, $\kappa_{n,e} = 0$ and $\beta = 10^{-3}$. The eigenvalue problem eq. (11) can also be extended to include a temperature gradient $\kappa_{T,e}$ in the electron tearing mode case. The benchmark shows that the mode becomes slightly more stable if a temperature gradient is applied; the inserted figure shows the real part of the eigenvalue. Again the solution of the eigenvalue problem and the EUTERPE results agree very well.

If both temperature and density gradients are present in the tearing mode case, the mode can be completely stabilised. Figure (2) (left) shows the growth rate obtained with EUTERPE changing the density gradient $\kappa_{n,s}$ while fixing the temperature gradient at $\kappa_{T,s} = -0.05$, $\beta = 10^{-3}$ and $a = 0.5$. Here $\eta_s = \kappa_{T,s}/\kappa_{n,s}$ and $\eta_i = \kappa_{e} = \eta$.

**Figure 2.** Critical behaviour of the tearing mode. Left: complete stabilisation below a critical $\eta$ (points EUTERPE results, curves from eq. (14)) Right: Critical $\eta$ depending on $\beta$ for electron tearing mode.

For various $C_1$ in Fig. (2) (left) one observes the same critical behaviour: below a certain $\eta_{cr}$ the mode is completely stabilised. If $C_1$ is decreased $\eta$ reduces to somewhat more realistic values in the range of $\eta \sim 1$. In this $\eta$-range the simulation time increases rapidly since the growth rates are very small. Especially for $C_1 = -0.002$ one needs at least $2 \times 10^4$ CPUh for each point. This is the first time such calculations have been performed within the PIC framework using standard gyrokinetics.

Previous works have pointed out that diamagnetic effects change the growth rates completely [8, 11]. There the linear gyrofluid growth rate has been estimated as

$$\gamma^2 = \gamma_0^2 - \beta \left[ \frac{k_{T,e}}{2 \eta} (1 + \tau) \right]^2 \quad \text{(14)}$$

This result has been applied for the runs in Fig. (2) (left) with $\gamma_0$ as the growth rate with $\kappa_{T,s} = 0$, $\kappa_{n,s} = 0$ and $\tau = T_{0,i}/T_{0,e}$. The simulation results agree very well with the predictions. As mentioned in [8], the analytical results are valid when $\rho_i \gg k_{0,e}^{-1}$. Here only $\rho_i > k_{0,e}^{-1}$ holds, but since the growth rate only depends weakly on $\tau$ one could increase $\tau$ and thereby $\rho_i$ without changing $\gamma_0$ very much. Then this case here corresponds approximately to the large $\rho_i$-limit. As can be seen in the left panel of Fig. (2), $C_1$ acts as a free parameter, which measures just the strength of the $B_{0,0}$-field. It does not introduce new physical effects and only shifts $\eta_{cr}$.

In Fig. (2) (left) $\eta$ has low values down to $\sim 10^{-2}$, but the characteristic behaviour is the same
for all $C_1$.

Another interesting point concerns the $\beta$-dependence of the critical $\eta$. A scan is shown in Fig. (2) (right) for the electron tearing mode for $C_1 = -1$ and $a = 0.5$. Figure (2) (right) shows that $\eta_{cr}$ decreases with higher $\beta$-values. Since the thermal velocity and density stay the same, increasing $\beta$ means here that the guiding field changes its values towards lower strengths. So the ratio between $B_{y,0}$ and guiding field strength becomes larger. This corresponds to the case when fixing $\beta$ and increasing $C_1$, which was shown in Fig. (2) (left). This means that for realistic $\beta$ values and temperature and density gradients the tearing mode in a slab is always unstable.

Our results show that the kinetic tearing theory in [12] for linear electron tearing mode cannot adequately account for the influence of the density and temperature gradients on the growth rate and drift frequency. It predicts, for instance, that the growth rate is not affected by diamagnetic effects $\gamma \approx \gamma_0$.

5. Nonlinear simulations of the tearing mode

This section describes for nonlinear one-mode simulations of the collisionless tearing mode. The mode of interest is selected by applying a Fourier filter. We simulate one-mode runs to compare the saturated island widths $w$ with nonlinear kinetic theories for collisionless electron tearing modes [1].

After the linear stage of the simulation the mode enters the nonlinear regime and saturates when the island width $w(t)$ approaches the width of the perturbed current layer. In the fluid picture, third-order force terms become important, which stop the plasma inflow towards the current layer and reconnection is inhibited [13]. Then the field energy of the mode oscillates about a mean value with a characteristic frequency $\omega_B$. According to kinetic theory electrons bounce within the islands with this bounce frequency $\omega_B$ and exchange energy with the wave and vice versa.

The saturated island width $w$ has been predicted for the electron tearing mode for a single $k$ [1]. In these calculations the constant $\Psi$-approximation has been used. To compare with this theory, we have used the perturbed vector potential $A_\parallel$ and the equilibrium vector potential $A_{\parallel,0}$ producing $B_{y,0}$, to calculate the island width geometrically at each time solving the equation:

$$A_\parallel(x = 0, y = 0, t) = A_{\parallel,0}(x = w) + A_\parallel(x = w, y = \frac{\pi}{k}, t)$$ (15)

and in the constant $\Psi$-approximation:

$$w = 2 \sqrt{A_\parallel l_s}$$ (16)

which only needs the time-averaged perturbed vector potential at $L_x/2$ for $t \gg \gamma^{-1}$. $l_s$ is the shear length defined by $l_s = B_{z,0}/B_{y,0}'$ at $L_x/2$. It is important to note, that the energy conservation of the runs is fulfilled better than a few percent. It turns out that the constant $\Psi$-approximation is valid since the relative deviation from the results of eq. (15) and (16) is less than two percent.

In Fig. (3) (left) $w$ is shown as a function of the growth rate $\gamma$ of the electron tearing mode with given wave vector $k$. The growth rate was increased by decreasing $a$ starting with $a = 0.7$. This is similar to Fig. (1) (left) when increasing the small $\gamma$ around $ka \sim 0.8$ to the maximum of the dispersion relation by decreasing $a$. The point at $\gamma = 0.5$ corresponds to a very small value $ka$ left to the maximum of the dispersion relation. The simulation domain is $L_x \times L_y = 2.5 \times 2\pi$.

The points represent the saturated mean island width $w$ and show a shows linear dependence on the growth rate if $\gamma$ is small enough. These values correspond to large $a$: the $B_{y,0}$ field approaches a slab configuration in which the bulk velocity $u_{0,e}$ is a constant over the whole simulation domain. In this case $B_{y,0} \sim x/l_s$ resulting in the prediction:
Figure 3. Left: theoretical prediction of saturated island width is confirmed for low growth rates. Comparison in between kinetic theory [9, 1] and EUTERPE. Right: saturated island widths $w$ depending on $\eta$. Below a finite threshold $w$ vanishes.

$$w = \frac{\gamma l_s}{\sqrt{\mu k G}}$$

with $G \sim 0.41$ (straight line in Fig. (3) (left)) [1]. This analytical result contains only information about the mode structure and mode dynamics around the resonant surface $L_x/2$ and not explicitly the boundary conditions. A variation of the simulation domain $L_x$ would influence the growth rate and therefore the saturated island width. The growth rate already contains the outer behaviour and boundary conditions of $A_{\parallel}$. So the boundary conditions do not appear explicitly in the analytical expression of the island width. For larger growth rates $\gamma$ the magnetic configuration deviates from the $B_{y,0} \sim x / l_s$ behaviour and the saturated island widths deviate from this theory. Additionally the point $w = 0.597$ was benchmarked with the one result given in [9] as well as the scaled field energy of the mode.

The next step of generalisation is to include gradients in the full tearing mode framework. Both electrons and gyrokinetic ions enter the simulation and the temperature gradient $\kappa_{T,e} = \kappa_{T,i} = -0.05$ was fixed. Then, varying the density gradient $\kappa_{n,e} = \kappa_{n,i}$ the saturated island widths are shown in Fig. (3) (right). The island width of the most unstable mode with $a = 0.5$ and $\beta = 10^{-3}$ has been calculated using the constant-$\Psi$ approximation according to eq. (16). Since the linear growth rate drops to zero when $\eta \rightarrow 0$ the island width has to vanish. Also the qualitative behaviour of the growth rate at finite $\eta > \eta_{cr}$ is the same as for the island width.

6. Conclusions
First EUTERPE was benchmarked against a shooting method for electromagnetic tearing mode simulations. In case of the electron tearing mode the results of EUTERPE agree better then two percent with results from the shooting method. Calculations with and without temperature gradient also in the tearing mode case with drift kinetic ions the shooting method deliver the same results as EUTERPE.

Introducing both density and temperature gradients in linear simulations a critical behaviour regarding $\eta$ has been observed. Analytical predictions of the growth rate have been shown to be in a good agreement [8]. The kinetic theory of [12] is not able to predict this behaviour. Nonlinear one-mode simulations have been carried out and have been benchmarked to the only published result from a PIC-\(\delta f\) code given in [9]. These simulations were extended to give a comparison between the predicted saturated island width [1] and our numerical results. For low
growth rates the predictions can be recovered. Temperature and density gradients have also been applied to the nonlinear evolution of the magnetic island. It was shown that reconnection is also inhibited in the nonlinear regime when $\eta$ is small enough. A parameter scan of $C_1$ has shown the dependence on the strength of the equilibrium current. If the bulk velocity $u_{0,e}$ is much smaller than the thermal speed the critical $\eta$ is in the range of unity. It would be interesting to find out analytically whether the linear connection $w \sim \gamma$ also holds in the case when gradients are present.

A rich physics needs to be explored in the nonlinear regime, e. g. study the behaviour of higher $k$ modes and increasing the filter size. Up to now, all nonlinear simulations assume that there is almost no coupling between the Fourier modes with different $k$. This issue of mode coupling has not been simulated yet and further simulations are currently under progress.

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