PRICING DECISIONS FOR COMPLEMENTARY PRODUCTS IN A FUZZY DUAL-CHANNEL SUPPLY CHAIN

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Abstract. The paper considers the pricing problem of complementary products in a fuzzy dual-channel supply chain environment where there are two manufacturers and one retailer. Four different decision models are established to study this problem: the centralized decision model, MS-Bertrand model, RS-Bertrand model and Nash game model, where the consumer demand and manufacturing cost for each product are characterized as fuzzy variables. A closed form solution has been obtained for each model by using game theory and fuzzy theory. Numerical examples are presented to compare the maximal expected profits and optimal pricing decisions, and to provide additional managerial insights. The finding shows that the decision makers are more likely to choose industries with higher self-price elastic coefficient and lower complementarity in the retail channel to cooperate. We can obtain that consumers can benefit from the cooperation of the two manufacturers because of lower prices. We can also find that it might not be bad for retailer because it can expand demand and obtain more maximal expected profits.

1. Introduction. New research from Forrester reports that the sales through direct channel continues to keep growing at a much faster pace than it through retail channel with the fast developing of the Internet and logistics industry. Companies such as Amway, Tupperware, Dell Computer, Pioneer Electronics and Cisco System sell products directly to the end consumers through their direct channel while continuing to sell through traditional retail channel. According to the United Nations Conference on Trade and Development (UNCTAD) sends the conference “in 2015 Electronic Commerce Development Report” to demonstrate that global...
business-to-consumer (B2C) e-commerce accounted for an estimated $1.2 trillion in 2013. While still considerably smaller than B2B, this segment appears to be growing faster. In developing countries, B2C e-commerce is rapidly expanding, particularly in Asia and Africa. China has already emerged as the largest global market for B2C e-commerce measured both by online buyers and by revenue. The share of the Asia and Oceania region in global B2C e-commerce is expected to surge from 28 to 37 percent between 2013 and 2018, and that of the Middle East and Africa to increase slightly from 2.2 to 2.5 percent.

There have been some studies on price competition or/and coordination in the dual-channel supply chain since 1990s. Various mechanisms for coordinating supply chain channels have been proposed, such as, Revenue Sharing Contract (Xiao et al. [27]; Ding and Liu [8]), Quantity or Price Discount Contract (Shin and Benton [21]; Cai [2]), Information Sharing Contract (Yue and Liu [29]; Erbao [3]) and so on. On the other hand, pricing policy has long been recognized as a significant tool for use in the profit maximization of firms. Regardless of whether it is applied to areas in revenue management or supply chain management, it is used in the daily operations of industries to manipulate demand, and to regulate the production and distribution of products and services (Soon [22]).

The business environment is characterized by high uncertainty and rapid and frequent changes (Wu et al. [26]). Acceleration in the globalization of markets has also increased market uncertainty and risk (Li et al. [12]). One of the important management problems in the dual-channel market is to effectively match demand and supply by dealing with the uncertainty of the quantities of the products in the dual-channel and the uncertainty of the market demand. Traditionally deterministic or probabilistic concepts have been used to model the various parameters among today’s literature published on the dual-channel. These literature provide some general understatement of the behavior of dual-channel supply chain under different assumptions, and their parameters are described as crisp values or having crisp probability distributions, they can’t represent real life problem situations (Wei and Zhao [24]).

Due to the innovation of products or market turbulence, the history data of the customers demand are not always available or reliable. However, these uncertain factors can’t be ignored because they may affect the effectiveness of the supply chain management. The uncertain parameters are approximately forecasted based on manager’s judgements, intuitions and experience such that manufacturing cost may be expressed as “low cost” or “high cost” to make a rough estimate; market base can be expressed as “large market base” or “small market base” to make a rough estimate (Yang and Xiao [28]). However, the probability-based approaches may not always be sufficient to model all uncertainties because some factors, which are often ill-defined and may vary from time to time, do not have known distributions. All of these situations raise challenges for using traditional supply chain models in practice. The fuzzy theory rather than probability theory should be applied to model these kinds of uncertainties (Zimmermann [35]). More recently, a considerable amount of research has adopted fuzzy set theory (Nahmias [17]; Liu [14, 15]) to depict uncertainty in the supply chain problems. More and more researchers have payed attention on studying the supply chain by applying fuzzy theory. Zhou et al. [34] studied the pricing problem of a single product with fuzzy customer demands. Lin and Chang [13] presented a fuzzy approach for order selection and pricing for a manufacturer. Wei et al. [24] solved a pricing problem of
substitutable products by defining vertical and horizontal competition in fuzzy environments. Liu and Xu [16] studied the pricing problem in a fuzzy supply chain that consisted of one manufacturer and two retailers. Li et al. [11] established a multi-period stochastic dynamic programming inventory models of dual channels operated by a vendor. Table 1 illustrates some major papers on pricing strategies in a fuzzy dual-channel supply chain.

**Table 1. Summary of the major literature review. (M: manufacturer; R: retailer)**

| Paper | Supply chain structure | Product status | Dual channel | Fuzzy or not | Decisions |
|-------|------------------------|----------------|--------------|-------------|-----------|
| Chiang et al. [5] | one M one R | single | Yes | No | pricing policy |
| Berger and Weinberg [1] | one firm | single | Yes | No | pricing policy |
| Dan et al. [7] | one M one R | single | Yes | No | pricing policy |
| Zhao et al. [32] | one M one R two Rs | substitutable | No | Yes | pricing policy |
| Zhao and Wang [33] | one M two Rs | single | No | Yes | pricing policy |
| Leila and Mehdi [10] | two Ms one R | substitutable | Yes | No | pricing policy |
| Sang [19] | two Ms one R | substitutable | No | Yes | pricing policy |
| Wang et al. [23] | two Ms one R | complementary | Yes | No | pricing policy |

To the best of our knowledge, there is little work in the literature studying pricing problem of complementary products in a fuzzy dual-channel supply chain environment. The concept of complementary products emerges when customers may have to buy more than one product at the same time to obtain the full utility of the products (Yue et al. [30]). For example, mobile phone and memory card, computer and software, camera and film. The marketing paradigm of complementary goods is different from that of substitutable goods in that the goods benefit from each other’s sales rather than losing sales to the other firm, and the goods can be thought of as a bundle (Yue et al. [30]).

As far as we know, this research, which considers the pricing problem of complementary products in a fuzzy dual-channel supply chain environment, is silent on the extant literature. Our work is intended to bring in a better understanding of the following questions: How do the channel members make their pricing decisions when facing fuzzy dual-channel environments? Whether and how is a complementary product accepted by the supply chain member? Therefore, the proposed model differs from those of prior studies in the two areas: Firstly, although we focus on a traditional pricing problem of dual-channel, we innovatively consider complementary products in fuzzy uncertainty environments. Secondly, it shows that it’s wise to consider the parameters value of products when the decision makers want to introduce the complementary products into the supply chain, and they are more likely to choose industries with higher self-price elastic coefficient and lower complementarity in the retail channel to cooperate.

The rest of the paper is organized as follows: The problem description and assumptions are presented in Section 2. In Section 3, we study the pricing equilibria
under the centralized decision model and three Stackelberg competition models. In Section 4, we give numerical examples to compare the equilibrium solution. Furthermore, we propose some strategic approaches for the manufacturers and retailer. Finally, the conclusion including summary of the main results and some directions for future research are presented in Section 5.

2. Problem description and assumptions. Consider a dual-channel supply chain in a fuzzy environment with two manufacturers, labeled $m_1$ and $m_2$, and one common retailer. The manufacturer $m_1$ sells the product 1 through the outlet of traditional retailer as well as through its own direct E-shop. The manufacturer $m_2$ produces product 2 and wholesales it only through the retail channel. The two products are complementary, and consumers can purchase them via either the retail channel or the E-channel. Under such a scenario, the two manufacturers and common retailer must make their pricing strategies in order to achieve maximal expected profits. In addition, we assume that they have perfect information of the demands and the cost structures of other channel members. For the sake of clarity, the following notations are used to formulate the fuzzy supply chain models.

- $c_i$: unit manufacturing cost of product $i$, which is a fuzzy variable, $c_i > 0, i = 1, 2$;
- $p_0$: unit direct sale price of product 1, which is the manufacturer $m_1$’s decision variable, $p_0 > w_1$ (to prevent arbitrage trading);
- $p_i$: unit retail price of product $i$, which is the retailer’s decision variable;
- $w_i$: unit wholesale price of product $i$ selling to the retailer, which is the manufacturer $m_i$’s decision variable, satisfying $p_i \geq w_i > c_i, i = 1, 2$;
- $D_0$: consumer demand for product 1 through the E-channel, which is a fuzzy variable,
- $D_i$: consumer demand for product $i$ through the retail channel, which is a fuzzy variable, $i = 1, 2$;
- $\Pi_{m_i}$: manufacturer $m_i$’s profit, which is a function of $w_i (i = 1, 2)$ and $p_j (j = 0, 1, 2)$;
- $\Pi_r$: retailer’s profit, which is a function of $w_i (i = 1, 2)$ and $p_j (j = 0, 1, 2)$;
- $\Pi_c$: profit of the whole supply chain system, which is a function of $p_i, i = 0, 1, 2$.

We assume all activity occur within a single period. Moreover, the logistic cost components of the two manufacturers and the retailer (i.e., packing cost, carrying cost and inventory cost, etc.) are ignored for computational convenience.

In defining the consumer demand function, we follow the approach which is similar to Choi [6] and Kurata et al. [9]. The fuzzy linear consumer demands are influenced by the retail prices of two products and two kinds of sale channels, respectively. The corresponding demand functions can be expressed as follows:

\[
D_0 = \Phi_0 - k_0 p_0 + \beta p_1 - \gamma_1 p_2, \quad (1)
\]
\[
D_1 = \Phi_1 - k_1 p_1 + \beta p_0 - \gamma_2 p_2, \quad (2)
\]
\[
D_2 = \Phi_2 - k_2 p_2 - \gamma_1 p_0 - \gamma_2 p_1. \quad (3)
\]

The parameters $\Phi_i$, $k_i$, $\beta$ and $\gamma_j$ are all fuzzy variables and independently nonnegative ($i = 0, 1, 2; j = 1, 2$), where $\Phi_0$ is the primary market base of the product 1 through the E-channel (i.e., potential market base if the goods are free of charge), $\Phi_j$ is the primary market base of the product $j$ through the retail channel. The parameter $k_i$ is self-price elastic coefficient, which measures the effectiveness of the
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Retail prices in stimulating or restraining product’s consumer demand. The cross-price sensitivity \( \beta \) reflects the degree to which the two channels can substitute for each other. The expected values of parameters \( k_0 \) and \( k_1 \) are larger than the expected value of \( \beta \) (\( E[k_0] > E[\beta] \), \( E[k_1] > E[\beta] \)), which signifies that the effect of ownership price is greater than the effect of cross-price. \( \gamma_1 \) and \( \gamma_2 \) denote the degree of complementarity existing between the two products in the E-channel and the retail channel, respectively. The expected values of parameter \( k_i \) is also larger than the expected value of \( \gamma_j \) (\( E[k_i] > E[\gamma_j] \)), because the expected demand for a product should be more sensitive to the changes in its own price than in the other product.

Therefore, the demand function is downward sloping in its own price, increasing with respect to the competitive channel’s price, while, it is decreasing with the complementary product’s retail price. The function follows a tradition of microeconomics analysis and marketing research about brand management and pricing (Raju et al. [18]; Wolfstetter [25]; Sayman et al. [20]; etc.).

According to the problem descriptions and assumptions, the manufacturer \( m_i \) \((i = 1, 2)\) and the retailer’s profit can be expressed as follows

\[
\Pi_{m_1}(p_0, w_1) = (w_1 - c_1)D_1 + (p_0 - c_1)D_0, \quad \text{(4)}
\]

\[
\Pi_{m_2}(w_2) = (w_2 - c_2)D_2, \quad \text{(5)}
\]

\[
\Pi_r(p_1, p_2) = (p_1 - w_1)D_1 + (p_2 - w_2)D_2. \quad \text{(6)}
\]

3. **Main results.** Considering different market power structures between two manufacturers and the retailer, we discussed one centralized model and three decentralized Stackelberg game models in this section, including the MS game, RS game, and Nash game model. In each scenario, each firm makes his decision to maximize his own expected profit. The game-theoretical approach is applied to explore our pricing decision models. First, we consider the dual-channel supply chain as a centralized system where the manufacturer is vertically integrated with the retailer in the dual-channel. Second, we create three decentralized dual-channel supply chains according to the variation in the bargaining power: (a). There are two larger manufacturers and a relatively smaller retailer in the market, so the manufacturers act as the Stackelberg leader; (b). The retailer holds more bargaining power than the two manufacturers which is the Stackelberg leader; (c). Each part in the system has an equal market power, and they simultaneously make their decisions. We will discuss these scenarios in more detail in the following subsections.

3.1. **Centralized decision (CD) model.** With centralized decision making, we assume that there is one entity who aims to optimize the total profit of manufacturer \( m_1 \) and \( m_2 \). The vertically integrated pricing problem of manufacturers can be stated as: determine a pricing policy \((p_0, p_1, p_2)\) for both E-channel and retail channel. The integrated firm tries to maximize his expected profit \( E[\Pi_r(p_0, p_1, p_2)] \), and the centralized decision model can be formulated as:

\[
\begin{aligned}
\max_{(p_0, p_1, p_2)} & \quad E[\Pi_r(p_0, p_1, p_2)] \\
\text{s.t.} & \quad P_{\text{os}}(\{p_1 - c_1 < 0\}) = 0, \\
& \quad P_{\text{os}}(\{p_1 - c_1 < 0\}) = 0, \\
& \quad P_{\text{os}}(\{p_2 - c_2 < 0\}) = 0, \\
& \quad p_i > 0, \quad i = 0, 1, 2
\end{aligned}
\]

Solving the above problem, Proposition 1 gives the results.
Proposition 1. In a centralized dual-channel supply chain, the optimal retail prices $p_0^{C^*}, p_1^{C^*}$ and $p_2^{C^*}$ are given as follows.

$$
\begin{pmatrix}
    \frac{\partial p_0^{C^*}}{\partial E[k_0]} & \frac{\partial p_1^{C^*}}{\partial E[k_1]} & \frac{\partial p_2^{C^*}}{\partial E[k_2]}
\end{pmatrix}
= \begin{pmatrix}
    2E[k_0] & -2E[\beta] & 2E[\gamma_1] \\
    -2E[\beta] & 2E[k_1] & 2E[\gamma_2] \\
    2E[\gamma_1] & 2E[\gamma_2] & 2E[k_2]
\end{pmatrix}^{-1}
\begin{pmatrix}
    J_1 \\
    J_2 \\
    J_3
\end{pmatrix}.
$$

(8)

Under the condition that $\text{Pos} (\Phi_0 - k_0 p_0^{C^*} + \beta p_1^{C^*} - \gamma_1 p_2^{C^*} < 0) = 0$, $\text{Pos} (\Phi_1 - k_1 p_1^{C^*} + \beta p_0^{C^*} - \gamma_2 p_2^{C^*} < 0) = 0$, $\text{Pos} (\Phi_2 - k_2 p_2^{C^*} - \gamma_1 p_0^{C^*} - \gamma_2 p_1^{C^*} < 0) = 0$, $\text{Pos} (p_1^{C^*} - c_1 < 0)$ and $\text{Pos} (p_2^{C^*} - c_2 < 0)$ hold, where $J_1, J_2$ and $J_3$ are constants defined in Appendix B.

The proof of Proposition 1 as well as the proofs of the other propositions is given in Appendix A.

Substituting $p_1^{C^*}(i = 0, 1, 2)$ into $E[\Pi_c(p_0, p_1, p_2)]$, the maximal expected profit of the whole supply chain system, denoted $E[\Pi_c^{C^*}(p_0, p_1, p_2)]$, can be easily obtained.

Corollary 1. (i) $\frac{\partial p_0^{C^*}}{\partial E[k_j]} < 0$, $\frac{\partial p_1^{C^*}}{\partial E[k_j]} < 0$, $\frac{\partial p_2^{C^*}}{\partial E[k_j]} > 0$, $j = 0, 1$

(ii) $\frac{\partial p_0^{C^*}}{\partial E[k_2]} > 0$, $\frac{\partial p_1^{C^*}}{\partial E[k_2]} > 0$, $\frac{\partial p_2^{C^*}}{\partial E[k_2]} < 0$

(iii) $\frac{\partial p_0^{C^*}}{\partial E[\beta]} > 0$, $\frac{\partial p_1^{C^*}}{\partial E[\beta]} > 0$, $\frac{\partial p_2^{C^*}}{\partial E[\beta]} < 0$

Corollary 1 indicates that $p_0^{C^*}$ and $p_1^{C^*}$ decreases with increasing $k_j$. This is straightforward that the decision makers will drop products’ prices of both channel down by larger self-price elastic coefficient $k_j$ because of the channel competition, which is coincident with the findings of Dan et al. [7]. But at the same time, the retailer will raise the complementary product’s price. As self-price elastic coefficient $k_2$ decreases, the demand of complementary products is less affected by price, therefore, the retailer will raise price $p_2$. In order to keep demand steady, the retailer will cut down $p_0$ and $p_1$. As cross-price sensitivity $\beta$ increases, the complementary products’ price will be drop down in order to attract more sales.

3.2. MS-Bertrand model. The two manufacturers hold more market power than the retailer where they act as the Stackelberg leaders and the retailer acts as the Stackelberg follower. Firstly, the two manufacturers announce the wholesale prices of their products, respectively, and the manufacturer $m_1$ determines his direct sale price of product 1. After observing that, the retailer sets his own retail prices that he intends to charge for the two complementary products. The objective of each participant is to maximize his own expected profit. The MS-Bertrand model can be formulated as (9):

$$
\begin{align*}
&\max_{(p_0, w_1)} E[\Pi_{m_1}(p_0, w_1, p_1^*(p_0, w_1, w_2), p_2^*(p_0, w_1, w_2))], \\
&\max_{w_2} E[\Pi_{m_2}(w_2, p_1^*(p_0, w_1, w_2), p_2^*(p_0, w_1, w_2))], \\
&\text{s.t. } \text{Pos} (\{w_1 - c_1 < 0\}) = 0, \\
&\text{Pos} (\{w_2 - c_2 < 0\}) = 0, \\
&\text{Pos} (\{D_i(p_0, p_1, p_2) < 0\}) = 0, \\
&w_j \leq p_j, \quad i = 0, 1, 2; j = 1, 2
\end{align*}
$$

(9)
We first derive the reaction functions of the retailer. Proposition 2 gives the results.

**Proposition 2.** In the MS-Bertrand model, given the wholesale prices \(w_1, w_2\) and direct sale price \(p_0\) made earlier by the two manufacturers, the common retailer’s optimal response functions \(p_1^*(p_0, w_1, w_2)\) and \(p_2^*(p_0, w_1, w_2)\) are:

\[
p_1^*(p_0, w_1, w_2) = \frac{w_1}{2} + \frac{E[\beta]E[k_2] - E[\gamma_1]E[\gamma_2]}{2E[\gamma_2]^2 - 2E[k_1]E[k_2]}p_0 + \frac{E[\Phi_2]E[\gamma_2] - E[\Phi_1]E[k_2]}{2E[\gamma_2]^2 - 2E[k_1]E[k_2]} \tag{10}
\]

\[
p_2^*(p_0, w_1, w_2) = \frac{w_2}{2} + \frac{E[\beta]E[\gamma_2] + E[k_1]E[\gamma_1]}{2E[\gamma_2]^2 - 2E[k_1]E[k_2]}p_0 + \frac{E[\Phi_1]E[\gamma_2] - E[\Phi_2]E[k_1]}{2E[\gamma_2]^2 - 2E[k_1]E[k_2]} \tag{11}
\]

under the condition that \(\text{Pos}(\{\Phi_0 + (A_{11}\beta - A_{21}\gamma_1 - k_0)p_0 + \frac{b_2}{2}w_1 - \frac{b_1}{2}w_2 + A_{12}\beta - A_{22}\gamma_1 < 0\}) = 0\), \(\text{Pos}(\{\Phi_1 + (\beta - A_{11}k_1 - A_{21}\gamma_2)p_0 - \frac{b_1}{2}w_1 - \frac{b_2}{2}w_2 - A_{12}k_1 - A_{22}\gamma_2 < 0\}) = 0\) and \(\text{Pos}(\{\Phi_2 - (\gamma_1 - A_{11}\gamma_2 - A_{21}k_2)p_0 - \frac{b_2}{2}w_1 - \frac{b_1}{2}w_2 - A_{12}\gamma_2 - A_{22}k_2 < 0\}) = 0\) hold, where \(A_{11}, A_{12}, A_{21}\) and \(A_{22}\) are constants defined in Appendix B.

**Corollary 2.** \(\frac{\partial p_1^*(p_0, w_1, w_2)}{\partial p_0} > 0, \frac{\partial p_1^*(p_0, w_1, w_2)}{\partial w_1} > 0, \frac{\partial p_2^*(p_0, w_1, w_2)}{\partial p_0} < 0, \frac{\partial p_2^*(p_0, w_1, w_2)}{\partial w_2} > 0\)

Corollary 2 indicates that the retailer’s best response price \(p_1^*(p_0, w_1, w_2)\) decrease with decreasing \(p_0\) and \(w_1\). It accurately corresponds to what happens in practice that the retailer price is consistent with the change of wholesale price. In order to strive for more sales in channel competition, the decrease of one channel price will invariably lead to the other channel price reductions. Meanwhile, the retailer’s best response price \(p_2^*(p_0, w_1, w_2)\) increase with decreasing \(p_0\) and increasing \(w_2\).

Having the information about the reaction of the retailer, the two manufacturers would use them to maximize their own expected profits by choosing the wholesale prices and the direct sale price. Therefore, the results are given in the following proposition.

**Proposition 3.** In the MS-Bertrand model, the manufacturers’ optimal wholesale prices and direct sale price can be given as:

\[
\begin{pmatrix}
p_0^{M*} \\
w_1^{M*} \\
w_2^{M*}
\end{pmatrix} = \begin{pmatrix}
B_1 & B_2 & E[\gamma_1] \\
B_2 & E[k_1] & E[\gamma_2] \\
E[\gamma_1] & E[k_1] & E[k_2]
\end{pmatrix}^{-1} \begin{pmatrix}
B_3 \\
B_4 \\
B_5
\end{pmatrix}, \tag{12}
\]

where \(B_1, B_2, B_3, B_4\) and \(B_5\) are constants defined in Appendix B.

Together by using Propositions 2 and 3, Proposition 4 can be easily obtained.

**Proposition 4.** In the MS-Bertrand model, the retailer’s optimal retail prices \(p_1^{M*}\) and \(p_2^{M*}\) can be obtained, respectively, as:

\[
p_1^{M*} = \frac{1}{2}w_1^{M*} + \frac{E[\beta]E[k_2] - E[\gamma_1]E[\gamma_2]}{2E[\gamma_2]^2 - 2E[k_1]E[k_2]}p_0^{M*} + \frac{E[\Phi_2]E[\gamma_2] - E[\Phi_1]E[k_2]}{2E[\gamma_2]^2 - 2E[k_1]E[k_2]}, \tag{13}
\]

\[
p_2^{M*} = \frac{1}{2}w_2^{M*} + \frac{E[\beta]E[\gamma_2] + E[k_1]E[\gamma_1]}{2E[\gamma_2]^2 - 2E[k_1]E[k_2]}p_0^{M*} + \frac{E[\Phi_1]E[\gamma_2] - E[\Phi_2]E[k_1]}{2E[\gamma_2]^2 - 2E[k_1]E[k_2]} \tag{14}
\]

3.3 RS-Bertrand model. The RS-Bertrand scenario arises in market where the retailer’s size is larger compared to the two manufacturers. Similar game-theoretic framework as applied in the MS-Bertrand model is implemented to solve this problem. The retailer selects retail prices of the two complementary products in the first step, then, the duopolistic manufacturers decide the wholesale prices and direct sale
price (they are going to charge for two complementary products), simultaneously. The following RS-Bertrand model can be formulated:

$$\begin{align*}
\max_{(p_1,p_2)} & E[\Pi_r(p_1, p_2, p_0^*(p_1, p_2), w_1^*(p_1, p_2), w_2^*(p_1, p_2))] \\
\text{s.t.} & Pos\{\{D_i(p_0, p_1, p_2) < 0\}\} = 0, \\
& w_j \leq p_j, \quad i = 0, 1, 2; j = 1, 2 \\
& p_0^*(p_1, p_2), w_1^*(p_1, p_2), w_2^*(p_1, p_2) \text{ are derived from solving the problem (15)} \\
\end{align*}$$

Given retail prices, we first derive the manufacturer $m_i$’s response functions.

**Proposition 5.** In the RS-Bertrand model, for given retail prices $p_1$ and $p_2$, the manufacturer $m_i$’s response functions are formulated as

$$\begin{align*}
p_0^*(p_1, p_2) &= F_1 p_2 + F_2, \\
w_1^*(p_1, p_2) &= -p_1 + F_3 p_2 + F_4, \\
w_2^*(p_1, p_2) &= F_5 p_1 + F_6 p_2 + F_7,
\end{align*}$$

under the condition $\text{Pos}\{\{-p_1 + F_3 p_2 + F_4 - c_1 < 0\}\} = 0$, and $\text{Pos}\{\{F_5 p_1 + F_6 p_2 + F_7 - c_2 < 0\}\} = 0$ hold, where $F_1, F_2, F_3, F_4, F_5$ and $F_7$ are constants defined in Appendix B.

Obtaining the response functions of the two manufacturers, the retailer would use them to maximize his own expected profit. We have the following results that can be derived as Proposition 6.

**Proposition 6.** In the RS-Bertrand model, if the conditions $\text{Pos}\{\{\Phi_0 - \kappa_0 p_0 + \beta_2 G_1 G_6 - \gamma_1 G_3 G_5 < 0\}\} = 0$, $\text{Pos}\{\{\Phi_1 - \kappa_1 G_2 G_7 - \gamma_1 G_3 G_5 - \beta_2 G_2 G_5 G_6 < 0\}\} = 0$, and $\text{Pos}\{\{G_2 G_5 G_6 - G_3 G_5 < 0\}\} = 0$ hold, where $G_1, G_2, G_3, G_4, G_5$ and $G_6$ are constants defined in Appendix B, the optimal retail prices $p_1^{R*}$ and $p_2^{R*}$ are

$$\begin{align*}
p_1^{R*} &= \frac{G_2 G_6 - G_3 G_5}{G_1 G_5 - G_2 G_4}, \\
p_2^{R*} &= \frac{G_3 G_4 - G_1 G_6}{G_1 G_5 - G_2 G_4}.
\end{align*}$$

By Propositions 5 and 6, the optimal decisions $p_0^{R*}, w_1^{R*}$ and $w_2^{R*}$ of the two manufacturers can be obtained.

**Proposition 7.** In the RS-Bertrand model, the optimal decisions $p_0^{R*}, w_1^{R*}$ and $w_2^{R*}$ of the two manufacturers can be obtained respectively, as:

$$\begin{align*}
p_0^{R*} &= F_1 p_2^{R*} + F_2^{R*}, \\
w_1^{R*} &= -p_1^{R*} + F_3 p_2^{R*} + F_4, \\
w_2^{R*} &= F_5 p_1^{R*} + F_6 p_2^{R*} + F_7.
\end{align*}$$
3.4. Vertical Nash game (NG) model. The two manufacturers and the retailer have the identical market power and thus they determine their strategies independently and simultaneously. So the NG model can be formulated as follows:

\[
\begin{align*}
\max_{(p_0,w_1)} & \quad E[\Pi_{m_1}(p_0,w_1)], \\
\max_{w_2} & \quad E[\Pi_{m_2}(w_2)], \\
\max_{(p_1,p_2)} & \quad E[\Pi_r(p_1,p_2)], \\
\text{s.t.} & \quad \text{Pos}(\{D_i(p_0,p_1,p_2) < 0\}) = 0, \\
& \quad \text{Pos}(\{w_j - c_j < 0\}) = 0, \\
& \quad w_j \leq p_j, \quad i = 0, 1, 2; j = 1, 2
\end{align*}
\]

(24)

From the MS-Bertrand model, the retailer’s decision are obtained in Eqs.(10) and (11), where there are given wholesale rices \(w_1\) and \(w_2\) and direct sale price \(p_0\). From the RS-Bertrand model, the manufacturers’ decisions are obtained in Eqs. (16)-(18), where are given retail prices \(p_1\) and \(p_2\). Solving them simultaneously yields the NG decision solutions as following.

Proposition 8. In the NG model, the optimal direct sale price (denoted as \(p_1^{N*}\)), which is chosen by the manufacturer \(m_1\), the optimal wholesale price (denoted as \(w_1^{N*}\) and \(w_2^{N*}\)), which is chosen by the manufacturer \(m_1\) and \(m_2\) respectively, and the optimal retail prices (denoted as \(p_1^{N*}\) and \(p_2^{N*}\)), which is chosen by the retailer, are

\[
\begin{pmatrix}
  w_1^{N*} \\
  w_2^{N*} \\
  p_1^{N*} \\
  p_2^{N*} \\
  p_0^{N*}
\end{pmatrix} = 
\begin{pmatrix}
  E[k_1] & 0 & E[k_1] & E[\gamma_2] & -2E[\beta] \\
  0 & E[k_2] & E[\gamma_2] & E[k_2] & E[\gamma_1] \\
  E[k_1] & E[\gamma_2] & -2E[k_1] & -2E[\gamma_2] & E[\beta] \\
  E[\gamma_2] & E[k_2] & -2E[\gamma_2] & -2E[k_2] & -E[\gamma_1] \\
  -E[\beta] & 0 & -E[\beta] & E[\gamma_1] & 2E[k_0]
\end{pmatrix}^{-1}
\begin{pmatrix}
  I_1 \\
  I_2 \\
  I_3
\end{pmatrix}
\]

(25)

If the conditions \(\text{Pos}(\{\Phi_0 - \beta_0p_0^{N*} + \beta p_1^{N*} - \gamma_1p_2^{N*} < 0\}) = 0, \text{Pos}(\{\Phi_1 - k_1p_0^{N*} + \beta_1p_1^{N*} - \gamma_0p_2^{N*} < 0\}) = 0, \text{Pos}(\{\Phi_2 - k_2p_0^{N*} - \gamma_1p_1^{N*} < 0\}) = 0, \text{Pos}(\{w_1^{N*} - c_1 < 0\}) = 0\) and \(\text{Pos}(\{w_2^{N*} - c_2 < 0\}) = 0\) hold, where \(I_1, I_2\) and \(I_3\) are constants defined in Appendix B.

4. Numerical examples and insights. In this section, we use some numerical examples to compare the results obtained from the above different decision models and to study the behavior of each firm facing the changing environments, because the optimal pricing strategy derived in this paper has a complicate form containing \(\alpha\)-optimistic value and \(\alpha\)-pessimistic value of fuzzy variable. The data in the numerical example, which have been non-dimensionalized and standardized before being employed, is believed to well represent the real-life condition as closely as possible due to the difficulty of accessing the actual industry data. We will analyze three aspects in the following subsections: (i). Comparing on the optimal price decisions and maximal expected profits of four different models, (ii). An innovative graphic method is proposed for the decision makers to identify the true complementary products for the original product in the supply chain, (iii). Sensitivity analysis of the fuzzy degree of the key parameters on the optimal prices, maximal expected profits and maximal expected demands.

Here the manufacturing costs, primary market bases, self-price elastic coefficients, products’ degree of complementarity and cross-price sensitivity are modeled as fuzzy variables. For the sake of simplicity, we construct triangular fuzzy numbers as the
method presented in Cheng[4] in this paper and assume their relationship with the linguistic expressions are determined by experts’ experiences as shown in Table 2. The set of collected scores is viewed as a sample from the possibility distribution of the grading process, and is used to estimate the parameters of the fuzzy number. When estimating the mode and spreads of a fuzzy number, a weight determination technique to associate a weight with each score is employed(Zhao et al., 2013). Of course, we can also extent some other form of fuzzy number for numerical examples which won’t be covered again here.

### Table 2. Relation between linguistic expression and triangular fuzzy variable.

| Linguistic expression | Triangular fuzzy variable |
|-----------------------|---------------------------|
| Φ₀ Large (about 500)  | (350, 500, 700)           |
| Small (about 200)     | (150, 200, 300)           |
| Φ₁ Large (about 800)  | (600, 800, 900)           |
| Small (about 300)     | (200, 300, 350)           |
| Φ₂ Large (about 1200) | (800, 1200, 1500)         |
| Small (about 600)     | (500, 600, 800)           |
| k₀ Very sensitive     | (4, 5, 6)                 |
| Sensitive             | (1, 2, 2.5)               |
| k₁ Very sensitive     | (5, 8, 10)                |
| Sensitive             | (3, 4, 5)                 |
| k₂ Very sensitive     | (25, 30, 35)              |
| Sensitive             | (10, 20, 25)              |
| β Very sensitive      | (0.3, 0.5, 0.8)           |
| Sensitive             | (0.1, 0.2, 0.3)           |
| γ₁ Very sensitive     | (0.3, 0.4, 0.6)           |
| Sensitive             | (0.1, 0.2, 0.3)           |
| γ₂ Very sensitive     | (0.1, 0.3, 0.5)           |
| Sensitive             | (0.05, 0.1, 0.15)         |
| c₁ High               | (25, 30, 40)              |
| Low                   | (10, 15, 20)              |
| c₂ High               | (4, 5, 6)                 |
| Low                   | (1, 2, 4)                 |

#### 4.1. Comparison of the optimal decisions in different models.

**Example 1.** Considering the case that the manufacturing cost \(c₁\) is in low level, \(c₂\) is in high level (\(c₁\) is about 15, and \(c₂\) about 5), the primary market base \(Φ_j\) (\(j = 0, 1, 2\)) is in large level (\(Φ₀\) is about 500, \(Φ₁\) about 800, and \(Φ₂\) about 1200), self-price elastic coefficients \(k_j\), products’ degree of complementarity \(γ_i\), and cross-price sensitivity \(β\) are in very sensitive level (i.e. \(k₀\) is about 5, \(k₁\) about 8, \(k₂\) about 30, \(γ₁\) about 0.4, \(γ₂\) about 0.3 and \(β\) about 0.5).

By using Table 2, the expected values are \(E[c₁] = \frac{10+2\times15+20}{4} = 15\), \(E[Φ₀] = \frac{350+2\times500+700}{4} = 512.5\), \(E[Φ₁] = \frac{600+2\times800+900}{4} = 775\), \(E[Φ₂] = \frac{800+2\times1200+1500}{4} = 1175\), \(E[k₀] = \frac{4+2\times5+6}{4} = 5\), \(E[k₁] = \frac{5+2\times8+10}{4} = 7.75\), \(E[k₂] = \frac{4+2\times5+6}{4} = 5\).
In Stackelberg decision scenario, the leader usually has the advantage to get
higher profits. Price of product 1 down to expand the demand and obtain the maximal expected
prices. This is because the market leader retailer wants to bring the
profit margins. From Tables 3 and 4, the following results are obtained:

\[ \frac{25 + 2 \times 30 + 35}{4} = 30, E[\beta] = \frac{0.3 + 2 \times 0.5 + 0.8}{4} = 0.525, E[\gamma_1] = \frac{0.3 + 2 \times 0.4 + 0.6}{4} = 0.425, E[\gamma_2] \]

The \( \alpha \)-optimistic value and \( \alpha \)-pessimistic value of parameters \( c_i, \gamma_i \), \( i = 1, 2 \),
\( \Phi_j, k_j \), \( j = 1, 2 \) and \( \beta \) are respectively
\[ c_{1a}^L = 10 + 5\alpha, c_{2a}^L = 4 + \alpha, \Phi_{3a}^L = 350 + 150\alpha, \Phi_{4a}^L = 600 + 200\alpha, \Phi_{5a}^L = 800 + 400\alpha, k_{6a}^L = 4 + \alpha, k_{7a}^L = 5 + 3\alpha, k_{8a}^L = 25 + 5\alpha, \beta_a^L = 0.3 + 0.2\alpha, \gamma_{1a}^L = 0.3 + 0.1\alpha, \gamma_{2a}^L = 0.1 + 0.2\alpha, \]
\[ c_{1a}^U = 20 - 5\alpha, c_{2a}^U = 6 - \alpha, \Phi_{3a}^U = 700 - 200\alpha, \Phi_{4a}^U = 900 - 100\alpha, \Phi_{5a}^U = 1500 - 300\alpha, k_{6a}^U = 6 - \alpha, k_{7a}^U = 10 - 2\alpha, k_{8a}^U = 35 - 5\alpha, \beta_a^U = 0.8 - 0.3\alpha, \gamma_{1a}^U = 0.6 - 0.2\alpha, \gamma_{2a}^U = 0.5 - 0.2\alpha. \]

We can also have \( E[c_1k_0] = 76.67, E[c_1k_1] = 120.42, E[c_1k_2] = 151.67, E[c_1\gamma_1] = 13.67, E[c_1\gamma_2] = 4.83, E[c_2\gamma_1] = 2.18, E[c_2\gamma_2] = 1.57 \) and \( \frac{1}{2} \int_0^1 (c_{1a}^U + c_{2a}^U) \alpha = 7395.8, \frac{1}{2} \int_0^1 (c_{1a}^L + c_{2a}^L) \alpha = 11375, \frac{1}{2} \int_0^1 (c_{2a}^U + c_{2a}^L) \alpha = 5758.3. \) The corresponding results are shown in Tables 3 and 4.

\[ \begin{array}{cccccc}
\text{scenario} & w_1^* & w_2^* & p_1^* & p_2^* & p_1^* - w_1^* & p_2^* - w_2^* \\
\hline
CD & - & - & 63.0097 & 60.8519 & 20.8009 & - \\
MS - Bertrand & 61.17 & 21.36 & 63.3476 & 82.0076 & 29.2987 & 20.8378 \\
RS - Bertrand & 40.02 & 13.20 & 63.3317 & 82.0065 & 29.3002 & 41.9865 \\
NG & 47.14 & 15.97 & 63.4526 & 74.9997 & 26.6042 & 27.8597 \\
\end{array} \]

From Tables 3 and 4, the following results are obtained:

(1-1) As consistent with the findings of Zhao et al. [31, 33], we found that the
optimal wholesale prices are the lowest in the RS-Bertrand model and the highest
in the MS-Bertrand model. The optimal direct sale price are also the lowest in
the RS-Bertrand model but highest in the NG model. The optimal retailer prices
are the highest in the MS-Bertrand model and the lowest in the NG model (i.e.
\( w_i^* > w_i^N > w_i^R, p_0^N > p_0^M, p_0^R, p_i^M > p_i^R, p_i^N > p_i^R (i = 1, 2) \)).

\[ \begin{array}{cccccc}
\text{scenario} & E[D_0^L] & E[D_0^H] & E[D_2^L] & E[D_2^H] & E[\Phi_1^L] & E[\Phi_1^H] \\
\hline
CD & 220.56 & 330.2375 & 505.94 & - & - & 37076 \\
MS - Bertrand & 226.36 & 163.9087 & 244.52 & 19578 & 4173 & 5358 \\
RS - Bertrand & 226.44 & 163.9089 & 244.48 & 16111 & 2180 & 10818 \\
NG & 223.31 & 219.0836 & 327.41 & 18895 & 3761 & 9585 \\
\end{array} \]

(1-2) The optimal retail prices in the CD model are lower than those in the
other models, which is beneficial to the consumers in this scenario. Although,
the optimal retail prices in the RS-Bertrand model are not the highest among all the
models, the profit margins \( p_i^R - w_i^R \) and \( p_2^R - w_2^R \) are larger than those in the
other decision models. This is because the market leader retailer wants to bring the
price of product 1 down to expand the demand and obtain the maximal expected
profits.

(1-3) The maximal expected profits of the whole system is the highest in CD
model. In Stackelberg decision scenario, the leader usually has the advantage to get

the higher expected profit. For example, the manufacturer \( m_1 \)'s expected profit in MS-Bertrand model is the highest, while the retailer has his maximal expected profit in RS-Bertrand model (i.e. \( E[\Pi_{m_1}^{MS}] > E[\Pi_{m_1}^{NR}] > E[\Pi_{m_1}^{M}] \), \( E[\Pi_{r}^{NR}] > E[\Pi_{r}^{M}] \), \( i = 1, 2 \)).

Comparing the manufacturer \( m_1 \)'s profits over the three models, MS-Bertrand model, RS-Bertrand model and NG model, we find they have remarkable difference in the retail channel but they are relatively close in the direct channel. However, the manufacturer \( m_1 \)'s maximal expected profits do not have a considerable difference in the E-channel.

4.2. The influence of different complementary products on profit. The profit is the only criterion for both manufacturer and the retailer in determining whether or not the complementary products should be introduced to the market for the decision-makers. We shall discuss which kinds of complementary products should be selected by decision makers based on their profits. Then, we compare the profits with and without complementary products in the NG decision model (i.e. manufacturers and retailer simultaneously make their decisions), and explore the influence of introducing different complementary products on the profit of each supply chain member. There are five parameters that may affect product 2, i.e. the manufacturing cost \( c_2 \), primary market base \( \Phi_2 \), self-price elastic coefficients \( k_2 \), products’ degree of complementarity in the E-channel \( \gamma_1 \) and in the retail channel \( \gamma_2 \). \( \Phi_2 \), \( k_2 \) and \( \gamma_1 \) are the three most important factors that affect the profit of the supply chain. In the following discussion, “the previous manufacturer” refers to the manufacturer \( m_1 \), “the complementor” refers to the manufacturer \( m_2 \) who provides the complementary products, and “CP” refers to the complementary products for convenience. The superscript “Nocp” signifies the optimal decisions when there is no complementary product in the supply chain in the NG model.

There are some restrictions in selecting the parameters \( \Phi_2 \), \( k_2 \) and \( \gamma_1 \). Without loss of generality, the interval of \( \Phi_2 \), \( k_2 \) and \( \gamma_1 \)'s expected value is restricted to \([0, 20000]\), \([5, 40]\) and \([0.1, 0.9]\), respectively (We can also enlarge parameters to a larger scope with this method). Figure 1 shows the profits with and without CP in the NG decision model.

![Figure 1. The graphic method](image)

The surface I is the boundary between the manufacturer \( m_1 \)'s profit with and without the CPs, i.e. \( \Delta E[\Pi_{m_1}] = E[\Pi_{m_1}^{Nocp}] - E[\Pi_{m_1}^{Nocp}] = 0 \). The surface II is the boundary of retailer’s profits which determines whether the retailer would quit from the previous manufacturer, i.e. \( E[\Pi_{r}^{Nocp}] = 0 \). The space is divided into three parts, denoted by A, B and C. When the value of the CP’s parameters falls into Part
the previous manufacturer cooperating with the complementor will obtain less profit than that without CP, i.e., it is advantageous to retailer, but disadvantageous to previous manufacturer. If the value falls into Part B means that both retailer and manufacturer are profitable. If the value falls into Part C means that it is advantageous to the previous manufacturer, but disadvantageous to retailer. Figure 1 can be used to instruct the previous manufacturer and the retailer’s decision making in whether to adopt the complementary product, intuitively.

The following example validates the above results.

**Example 2.** Suppose that there are 5 kinds of CPs, denoted $CP_1 - CP_5$, for the previous manufacturer’s choice, which are selected randomly from the three Parts (A, B, C) and two surfaces (I, II) (shown as Figure 2). The corresponding triangular fuzzy variables of the CP are shown in Table 5 (the values of other parameters are assumed as follow: $c_1$ and $c_2$ are in high level, $\Phi_0$ and $\Phi_1$ are in large level, $\gamma_0$ and $\gamma_2$ are in very sensitive level, $k_0$ is in very sensitive level, $k_1$ is in sensitive level, and $\beta$ is in very sensitive level).

Table 5 and the results in subsection 3.4 show the comparison of the optimal direct sale price, optimal retail prices, optimal wholesale prices, maximal demands and maximal expected profits with and without CPs. The results are shown in Table 6.

*Note: Horizontal coordinate (the value is +, 0 or -) represents the previous manufacturer cooperating with the complementor will obtain less/more profit than that without CPs. Vertical coordinate (the value is +, 0 or -) shows the retailer’s profit will increase/decrease as compared with no CPs. E.g. $CP_1(-,+)$ means that, when manufacturer 1 cooperates with complementary product 1, his expect profit is less than without it, but the retailer’s expect profit is more than before.*
From Table 6, we can obtain some results and managerial insights as following:

(2-1) It’s better for the previous manufacturer to select CP3, CP4 and CP5, while, better for the retailer to select CP1, CP2 and CP3, which are described as previously. Then, not all complementors want to cooperate with the previous manufacturer, such as CP5 whose retailer’s maximal expected profit decreases. However, the maximal expected profit of the whole supply chain system increases in the market.

(2-2) The optimal direct sale price and the optimal retail price are always decreasing as the previous manufacturer adopts the CPs (i.e. $p^N_0 - p^{nocp}_0 < 0$, $p^N_1 - p^{nocp}_1 < 0$), while, the optimal wholesale price is always increasing (i.e. $w^N_1 - w^{nocp}_1 > 0$). Consumer can benefit from the cooperation of the two manufacturers because of the lower prices. Although the retailer’s marginal profit (i.e. $p^N_0 - w^N_1 < p^{nocp}_0 - w^{nocp}_1$) decreases, it might not always be bad because it would expand demand and obtain more maximal expected profits.

**Table 6. Comparison of the optimal decisions with and without CPs**

|                | CP1   | CP2   | CP3   | CP4   | CP5   |
|----------------|-------|-------|-------|-------|-------|
| $p^N_0 - p^{nocp}_0$ | -36.3415 | -28.0346 | -19.6351 | -20.7345 | -19.284 |
| $p^N_1 - p^{nocp}_1$ | -22.3776 | -18.0985 | -2.4605 | -2.5192 | -0.827 |
| $w^N_1 - w^{nocp}_1$ | 12.2558 | 14.0035 | 18.0239 | 17.8967 | 18.3916 |
| $E[D^N_0] - E[D^{nocp}_0]$ | 8.6757 | 49.2929 | 89.1796 | 83.7494 | 90.7422 |
| $E[D^N_1] - E[D^{nocp}_1]$ | -26.3475 | -23.7178 | -12.0458 | -11.9774 | -10.7595 |
| $E[D^{NG}_0]$ | 640.6542 | 1070.9 | 342.9444 | 319.2612 | 129.3525 |
| $E[\Pi^N_{m1}] - E[\Pi^{nocp}_{m1}]$ | -3141 | 0 | 5177 | 4690 | 5528 |
| $E[\Pi^N_{m2}]$ | 102830 | 143660 | 6082 | 5501 | 611.5339 |
| $E[\Pi^N_{r}] - E[\Pi^{nocp}_{r}]$ | 92082 | 133632 | 417 | 0 | -4373 |
| $E[\Pi^N_{m1+r}] - E[\Pi^{nocp}_{r}]$ | 88941 | 133632 | 5594 | 4690 | 155 |

Synthesizing all the factors, it’s better to choose the complementary product CP3 (whose value of the parameter falls into Part B) to cooperate with than to other products. When the manufacturer wants to introduce the complementary products into the supply chain, it is wise to consider the parameters’ value of complementary products.

4.3. **Sensitivity analysis on some fuzzy parameters.** In this subsection, we change the fuzzy degree of some key parameters and analyze their impact on the optimal prices, maximal expected profits and maximal expected demands in MS-Bertrand and RS-Bertrand models.

Firstly, we discuss the effect of the fuzzy degree of parameter $\beta$ on the equilibrium solutions. By varying the fuzzy degree of $\beta$, the change of the optimal decisions can be obtained, which is shown in Tables 7 and 8. The other parameters are the same as the values in subsection 4.1.

From Tables 7 and 8, we can obtain the following results.

(3-1) Regardless of what kind of power structure, the maximal expected profits of the manufacturer $m_1$ and the whole system decrease slightly with decreasing the fuzzy degree of parameter $\beta$. However, the maximal expected demands will increase, and results in the increase of the manufacturer $m_2$ and retailer’s maximal expected
profits. This is consistent with our intuition that the larger the risk resulted by parameter uncertainty, the more benefit for its own decision, conversely, the more disadvantageous to opponent’s.

(3-2) The optimal wholesale price and retail price of product 2 increase slightly as the fuzzy degree of parameter $\beta$ decreases regardless of what kind of power structure. While, the optimal prices of product 1 (i.e. $p^*_0$, $w^*_1$, $p^*_1$) decrease slightly. In order to reduce the risk resulted by parameter uncertainty, the decision makers would decide a relatively conservative price.

**Table 7.** The change of maximal expected demands and profits with the fuzzy degree of $\beta$

| scenario | $\beta$ | $E[D^*_0]$ | $E[D^*_1]$ | $E[D^*_2]$ | $E[\Pi^*_m]$ | $E[\Pi^*_n_1]$ | $E[\Pi^*_n_2]$ | $E[\Pi^*_r]$ |
|----------|--------|-------------|-------------|-------------|--------------|----------------|----------------|--------------|
| MS-      | (0.2, 0.5, 0.8) | 225.6026 | 163.7939 | 244.5593 | 19452 | 4175.0 | 5354.1 | 28981 |
| Bertrand | (0.4, 0.5, 0.6) | 225.7746 | 163.8772 | 244.5649 | 19403 | 4175.2 | 5357.6 | 28936 |
| RS-      | (0.2, 0.5, 0.8) | 225.6799 | 163.7941 | 244.5216 | 15990 | 2180.7 | 10810 | 28980 |
| Bertrand | (0.4, 0.5, 0.6) | 225.8518 | 163.8774 | 244.5272 | 15938 | 2180.9 | 10817 | 28936 |

**Table 8.** The change of optimal prices with the fuzzy degree of $\beta$

| scenario | $\beta$ | $w^*_1$ | $w^*_2$ | $p^*_0$ | $p^*_1$ | $p^*_2$ | $p^*_1 - w^*_1$ | $p^*_2 - w^*_2$ |
|----------|--------|--------|--------|--------|--------|--------|-------------|-------------|
| MS-      | (0.2, 0.5, 0.8) | 60.9727 | 21.3595 | 63.0687 | 81.7999 | 29.3032 | 20.8272 | 7.9437 |
| Bertrand | (0.4, 0.5, 0.6) | 60.9489 | 21.3599 | 63.0329 | 81.7869 | 29.3037 | 20.8380 | 7.9438 |
| RS-      | (0.2, 0.5, 0.8) | 39.8371 | 13.2063 | 63.0530 | 81.7988 | 29.3034 | 20.8272 | 7.9437 |
| Bertrand | (0.4, 0.5, 0.6) | 39.8025 | 13.2065 | 63.0173 | 81.7858 | 29.3034 | 20.8272 | 7.9437 |

Secondly, we discuss the effect of parameter $k_2$’s fuzzy degree on the equilibrium solutions. By varying the fuzzy degree of $k_2$, the change of the optimal decisions can be obtained, then, the corresponding results are shown in Tables 9 and 10. The other parameters are the same as the values in subsection 4.1.

From Tables 9 and 10, we can obtain the following results.

(4-1) The maximal expected demands will increase, which results in the increase of the manufacturer $m_1$ and retailer’s maximal expected profits regardless of what kind of power structure. However, the maximal expected profit of the whole system will decrease. This is consistent with our intuition that the larger the risk resulted by parameter uncertainty, the larger of the expected profits which is coincident with the findings of Zhao et al.[31].

(4-2) No matter in MS-Bertrand or RS-Bertrand model, the optimal wholesale price and retail price of product 2 decrease slightly as the fuzzy degree of parameter $k_2$ decreases, conversely, the margin of product 2 (i.e. $p^*_2 - w^*_2$) increases. While, the optimal prices of product 1 (i.e. $p^*_0$, $w^*_1$, $p^*_1$) increases.
Table 9. The change of maximal expected demands and profits with the fuzzy degree of $k_2$

| scenario | $k_2$ | $E[D^*_0]$ | $E[D^*_1]$ | $E[D^*_2]$ | $E[\Pi^*_m]$ | $E[\Pi^*_c]$ | $E[\Pi^*_r]$ |
|----------|------|-------------|-------------|-------------|--------------|--------------|--------------|
|          | (10,30,50) | 226.3554 | 163.9024 | 243.2656 | 19576 | 4320.2 | 5338.1 | 29234 |
| $MS-$    | (15,30,45) | 226.3583 | 163.9045 | 243.6821 | 19577 | 4271.3 | 5344.8 | 29192 |
| $Bertrand$ | (20,30,40) | 226.3613 | 163.9066 | 244.0987 | 19577 | 4222.4 | 5351.5 | 29151 |
|          | (25,30,35) | 226.3642 | 163.9087 | 244.5152 | 19578 | 4173.6 | 5358.2 | 29109 |
| $RS-$    | (10,30,50) | 226.4335 | 163.9026 | 243.2279 | 16110 | 2347.0 | 10777 | 29234 |
| $Bertrand$ | (15,30,45) | 226.4364 | 163.9047 | 243.6443 | 16110 | 2291.3 | 10791 | 29192 |
|          | (20,30,40) | 226.4394 | 163.9068 | 244.0608 | 16111 | 2235.7 | 10804 | 29151 |
|          | (25,30,35) | 226.4423 | 163.9089 | 244.4773 | 16111 | 2180.0 | 10818 | 29109 |

Similarly, we can also vary the fuzzy degree of other parameters, such as the primary market base $\Phi_i$ ($i = 0, 1, 2$), self-price elastic coefficients $k_0$ and $k_1$, products’ degree of complementarity $\gamma_1$ and $\gamma_2$, and so on, to discuss the effects of the parameters’ fuzzy degree on the equilibrium solutions, regardless of the complementary product type.

4.4. Managerial insights. According to the above analysis, the following managerial insights are obtained.

Insight 1: When the complementary products are introduced into the system, it had minor difference in the direct channel for the previous manufacturer’s maximal expected profit, whereas it has significant difference in the retail channel. It means that whether the manufacturer $m_1$’s profits increase or not mainly depend on variation of profit in the retail channel.

Insight 2: It would be beneficial for the whole supply chain when the complementary product enters into the market. We can also speculate here that either the retailer or the previous manufacturer may adopt some strategies to cope with the decrease of channel member’s maximal expected profit. Under such circumstance, the retailer may stop doing business with either manufacturer, or the previous manufacturer will take some incentive measurements to urge the retailer to cooperate with him.

Insight 3: When the manufacturer wants to introduce the complementary products into the supply chain, it is wise to consider the parameters value of complementary products. However, it is always beneficial from introducing the complementary products for the whole supply chain in NG model.

Table 10. The change of optimal prices with the fuzzy degree of $k_2$

| scenario | $k_2$ | $w^*_1$ | $w^*_2$ | $p^*_0$ | $p^*_1$ | $p^*_2$ | $p^*_1 - w^*_1$ | $w^*_2 - p^*_2$ |
|----------|------|---------|---------|--------|--------|--------|----------------|----------------|
|          | (10,30,50) | 61.1638 | 21.4399 | 63.3457 | 82.0067 | 29.3404 | 20.8429 | 7.9005 |
| $MS-$    | (15,30,45) | 61.1644 | 21.4121 | 63.3463 | 82.0070 | 29.3265 | 20.8426 | 7.9144 |
| $Bertrand$ | (20,30,40) | 61.1650 | 21.3844 | 63.3469 | 82.0073 | 29.3126 | 20.8423 | 7.9282 |
|          | (25,30,35) | 61.1656 | 21.3566 | 63.3476 | 82.0076 | 29.2987 | 20.8420 | 7.9421 |
| $RS-$    | (10,30,50) | 40.0141 | 13.3298 | 63.3298 | 82.0055 | 29.3418 | 41.9914 | 16.0120 |
| $Bertrand$ | (15,30,45) | 40.0144 | 13.2881 | 63.3305 | 82.0059 | 29.3279 | 41.9915 | 16.0398 |
|          | (20,30,40) | 40.0147 | 13.2465 | 63.3311 | 82.0062 | 29.3141 | 41.9915 | 16.0676 |
|          | (25,30,35) | 40.0150 | 13.2048 | 63.3317 | 82.0065 | 29.3002 | 41.9915 | 16.0954 |
Insight 4: (i) Regardless of power structure, the maximal expected profits of the previous manufacturer and the whole system decrease slightly with decreasing the fuzzy degree of $\beta$. However, the maximal expected demands will increase, and results in the increase of the complementor and retailer’s maximal expected profits. The larger the risk resulted by parameter uncertainty, the more benefit for its own decision, conversely, the more disadvantageous to opponent’s. (ii) The previous manufacturer and retailer’s maximal expected profits increase as the fuzzy degree of $k_2$ decreases, whereas the maximal expected profit of the whole system decrease.

5. Conclusions. In this paper, we demonstrated that competitive equilibrium pricing policies exist under the centralized and Stackelberg competition model in a mixed fuzzy E-channel and traditional retail channel for two complementary products. The consumer demands and manufacturing costs are characterized as fuzzy variables. By using game-theoretic approach and fuzzy theory, the corresponding analytical equilibrium solutions are obtained. Through the numerical examples, we compare the results obtained from the above four different decision scenarios, the centralized decision model, MS-Bertrand model, RS-Bertrand model and Nash game model, and study the behavior of firms facing changing environment. We also discuss the sensitivity analysis of the fuzzy degree of the key parameters on the equilibrium solutions. Finally, we propose an appropriate strategy for the manufacturers to adopt when adding an E-channel and provide a judgement criterion for the decision makers based on the complementary product’s parameters. The finding shows that its wise to consider the parameters value of products when the decision makers want to introduce the complementary products into the supply chain, and they are more likely to choose industries with higher self-price elastic coefficient and lower complementarity in the retail channel to cooperate. Consumers can benefit from the cooperation of the two manufacturers because of lower prices. We can also obtain that it might not be bad for retailer because it can expand demand and obtain more maximal expected profits.

However, there are also certain limitations due to some assumptions. Firstly, our models are based upon simplistic demand function, which are more complex in reality. Further work is desirable to test whether our conclusions extend to other general forms of demand function. Secondly, we assume the two products are complementary. We could further explore the marketing on substitutable products or green products. In addition, We can also consider the supply chain with asymmetric information in the future.

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Appendix A.

Proof of Proposition 1. By Eq. (7), the expected profit $E[\Pi_c(p_0, p_1, p_2)]$ can be given.

The first and second order partial derivatives of $E[\Pi_c(p_0, p_1, p_2)]$ to $(p_0, p_1, p_2)$ can be shown as

$$\frac{\partial E[\Pi_c(p_0, p_1, p_2)]}{\partial p_0} = -2E[k_0]p_0 + 2E[\beta]p_1 - 2E[\gamma_1]p_2 + E[\Phi_0]$$
Following from Eqs. (6), (2), (3) that we can get the first-order and second-order partial derivatives of $E[\Pi_r(p_1, p_2)]$ with respect to $(p_1, p_2)$ are:

\[
\frac{\partial E[\Pi_r(p_1, p_2)]}{\partial p_1} = -2E[k_1]p_1 - 2E[\gamma_2]p_2 + E[\gamma_1]w_1 + E[\gamma_2]w_2 + E[\beta]p_0 + E[\Phi_1],
\]

\[
\frac{\partial E[\Pi_r(p_1, p_2)]}{\partial p_2} = -2E[\gamma_1]p_1 - 2E[k_2]p_2 + E[\gamma_2]w_1 + E[k_2]w_2 - E[\gamma_1]p_0 + E[\Phi_2].
\]
\[
\frac{\partial^2 E[\Pi_r(p_1, p_2)]}{\partial p_1^2} = -2E[k_1] < 0, \quad (31)
\]
\[
\frac{\partial^2 E[\Pi_r(p_1, p_2)]}{\partial p_2^2} = -2E[k_2], \quad (32)
\]
\[
\frac{\partial^2 E[\Pi_r(p_1, p_2)]}{\partial p_1 \partial p_2} = \frac{\partial^2 E[\Pi_r(p_1, p_2)]}{\partial p_2 \partial p_1} = -2E[\gamma_2]. \quad (33)
\]

By Eqs. (31)-(33), the Hessian matrix \( H_2 = \left[ \begin{array}{cc} -2E[k_1] & -2E[\gamma_2] \\ -2E[\gamma_2] & -2E[k_2] \end{array} \right] = 4E[k_1]E[k_2] \). 

By setting Eqs. (29)-(30) equal to zero and solving them with respect to \( p_1 \) and \( p_2 \), the results (10)-(11) can be gained. The following equation is equivalent to the above mentioned conditions: \( Pos\{\{\Phi_0 - k_0p_0 + \beta p_1(p_0, w_1, w_2) - \gamma_1p_2(p_0, w_1, w_2) < 0\}\} = 0, Pos\{\{\Phi_1 - k_1p_1(p_0, w_1, w_2) + \beta p_0 - \gamma_2p_2(p_0, w_1, w_2) < 0\}\} = 0, \) and \( Pos\{\{\Phi_2 - k_2p_2(p_0, w_1, w_2) - \gamma_1p_0 - \gamma_2p_1(p_0, w_1, w_2) < 0\}\} = 0. \) \( \square \)

**Proof of Proposition 3.** By substituting Eqs. (10)-(11) into Eqs. (4) and (5), the expected profits of the manufacturers can be represented.

The first-order and second-order partial derivatives of \( E[\Pi_{m_1}] \) with respect to \((w_1, p_0)\) and \( E[\Pi_{m_2}] \) with respect to \( w_2 \) can also be obtained.

Then, we know that \( E[\Pi_{m_1}] \) is concave in \((w_1, p_0)\), and \( E[\Pi_{m_2}] \) is concave in \( w_2 \).

Setting the first-order partial derivatives of \( E[\Pi_{m_1}] \) with respect to \((w_1, p_0)\) and \( E[\Pi_{m_2}] \) with respect to \( w_2 \) to be zero and solving them, we can easily have (12), so Proposition 3 is proven. \( \square \)

**Proof of Proposition 5.** Let \( t_i (i = 1, 2) \) be the margin of product \( i \) enjoyed by the retailer, i.e. \( p_i = w_i + t_i \). Then, the manufacturer \( m_i \)'s expected profit can be computed by Eqs. (4) and (5). The first-order partial derivatives are given as

\[
\frac{\partial E[\Pi_{m_1}(p_0, w_1)]}{\partial p_0} = -2E[k_0]p_0 + 2E[\beta]w_1 + E[\beta]p_1 - E[\gamma_1]p_2 + E[\Phi_0] + E[c_1k_0] - \frac{1}{2} \int_0^1 (c_{1\alpha}^U \beta^U_\alpha + c_{1\alpha}^U \beta^L_\alpha) d\alpha, \quad (34)
\]
\[
\frac{\partial E[\Pi_{m_1}(p_0, w_1)]}{\partial w_1} = 2E[\beta]p_0 - 2E[k_1]w_1 - E[k_1]p_1 - E[\gamma_2]p_2 + E[\Phi_1] + E[c_1k_1] - \frac{1}{2} \int_0^1 (c_{1\alpha}^L \beta^U_\alpha + c_{1\alpha}^U \beta^L_\alpha) d\alpha, \quad (35)
\]
\[
\frac{\partial E[\Pi_{m_2}(w_2)]}{\partial w_2} = -E[\gamma_1]p_0 - E[k_2]w_2 - E[\gamma_2]p_1 - E[k_2]p_2 + E[\Phi_2] + E[c_2k_2]. \quad (36)
\]

It follows from Eqs.(34)-(36) that the Hessian matrix \( H_3 = 4E[k_0]E[k_1] - 4E[\beta]^2 > 0 \) and \( \frac{\partial^2 E[\Pi_{m_2}(w_2)]}{\partial w_2^2} = -E[k_2] < 0 \), so \( E[\Pi_{m_1}(p_0, w_1)] \) is a concave function with respect to \((p_0, w_1)\) and \( E[\Pi_{m_2}(w_2)] \) is concave function with respect to \( w_2 \). By setting Eqs. (34)-(36) to zero and solving them simultaneously, Eqs. (16)-(18) can be obtained. Thus, Proposition 5 is proved. \( \square \)
Proof of Proposition 6. Substituting Eqs. (16)-(18) into Eq. (6), the first-order partial derivatives of $E[\Pi_r(p_1, p_2, p_0)(p_1, p_2), w_1^r(p_1, p_2), w_2^r(p_1, p_2)]$ with respect to $(p_1, p_2)$ can be established.

The Hessian matrix can be given, which is negative definite according to the assumptions. So $E[\Pi_r(p_1, p_2, p_0)(p_1, p_2), w_1^r(p_1, p_2), w_2^r(p_1, p_2)]$ is a concave function. Let the first-order partial derivatives be equal to zero and solving them, the results (19) and (20) can be gained. □

Appendix B.

$J_1 = E[\Phi_0] + E[c_1 k_0] + E[c_2 \gamma_1] - \frac{1}{2} \int_0^1 (c_{1a}^{L} \beta_{1a}^{U} + c_{1a}^{U} \beta_{1a}^{L}) d\alpha,$

$J_2 = E[\Phi_1] + E[c_1 k_1] + E[c_2 \gamma_2] - \frac{1}{2} \int_0^1 (c_{1a}^{L} \beta_{1a}^{U} + c_{1a}^{U} \beta_{1a}^{L}) d\alpha,$

$J_3 = E[\Phi_2] + E[c_1 \gamma_1] + E[c_2 k_2],$

$A_{11} = \frac{-E[\beta] E[\gamma_1] E[\gamma_2]}{2E[\gamma_1^2] - 2E[k_1] E[k_2]}, A_{12} = \frac{E[\Phi_2] E[\gamma_2] - E[\Phi_1] E[k_2]}{2E[\gamma_1^2] - 2E[k_1] E[k_2]},$

$A_{21} = \frac{E[\beta] E[\gamma_2] + E[k_1] E[\gamma_1]}{2E[\gamma_2^2] - 2E[k_1] E[k_2]}, A_{22} = \frac{E[\Phi_1] E[\gamma_2] - E[\Phi_2] E[k_1]}{2E[\gamma_2^2] - 2E[k_1] E[k_2]},$

$B_1 = -2(E[k_0] + E[\beta] A_{11} - E[\gamma_1] A_{21}),$

$B_2 = -\frac{3E[\beta]}{2} - E[k_1] A_{11} - E[\gamma_2] A_{21},$

$B_3 = E[\Phi_0] + E[\beta] A_{12} - E[\gamma_1] A_{22} + E[c_1 k_1] A_{11} + E[c_1 \gamma_2] A_{21} + E[c_1 \gamma_1] A_{21} + E[c_1 k_0] - \frac{1 + A_{11}}{2} \int_0^1 (c_{1a}^{L} \beta_{1a}^{U} + c_{1a}^{U} \beta_{1a}^{L}) d\alpha,$

$B_4 = E[\Phi_1] - E[k_1] A_{12} - E[\gamma_2] A_{22} + E[c_1 k_1] - \frac{1}{4} \int_0^1 (c_{1a}^{L} \beta_{1a}^{U} + c_{1a}^{U} \beta_{1a}^{L}) d\alpha,$

$B_5 = \frac{E[c_2 k_2]}{2} + E[\Phi_2] - E[\gamma_2] A_{12} - E[k_2] A_{22},$

$F_1 = \frac{-E[k_1] E[\gamma_1] - E[\beta] E[\gamma_2]}{2E[k_0] E[k_1] - 2E[\beta]^2}, F_3 = \frac{-E[\beta] E[\gamma_1] - E[k_0] E[\gamma_2]}{E[k_0] E[k_1] - E[\beta]^2},$

$F_2 = \frac{(E[\Phi_0] - \frac{1}{2} \int_0^1 (c_{1a}^{L} \beta_{1a}^{U} + c_{1a}^{U} \beta_{1a}^{L}) d\alpha)(E[k_1] + E[\beta])}{2E[k_0] E[k_1] - 2E[\beta]^2}
+ \frac{E[c_1 k_0] E[k_1] + E[c_1 k_1] E[\beta]}{2E[k_0] E[k_1] - 2E[\beta]^2},$

$F_4 = \frac{(E[\Phi_0] - \frac{1}{2} \int_0^1 (c_{1a}^{L} \beta_{1a}^{U} + c_{1a}^{U} \beta_{1a}^{L}) d\alpha)(E[k_0] + E[\beta])}{E[k_0] E[k_1] - E[\beta]^2}
+ \frac{E[c_1 k_0] E[\beta] + E[c_1 k_1] E[k_0]}{E[k_0] E[k_1] - E[\beta]^2},$

$F_5 = \frac{E[\gamma_2]}{E[k_2]}, F_6 = -1 - \frac{E[\gamma_1] F_1}{E[k_2]}, F_7 = \frac{E[\Phi_2] + E[c_2 k_2] - E[\gamma_1] F_2}{E[k_2]},$

$G_1 = 2E[\gamma_2] F_5 - 4E[k_1],$

$G_2 = 2E[\beta] F_1 - 3E[\gamma_2] + E[k_1] F_3 + E[k_2] F_5 + E[\gamma_1] F_1 F_5 + E[\gamma_2] F_6,$

$G_3 = 2E[\Phi_1] - E[\Phi_2] F_5 + 2E[\beta] F_2 + E[k_1] F_4 + E[\gamma_1] F_2 F_5 + E[\gamma_2] F_7,$
\[ G_1 = E[k_1]F_3 + 2E[\beta]F_1 - 3E[\gamma_2] + E[\gamma_2]F_6 + E[k_2]F_5 + E[\gamma_1]F_1F_5, \]
\[ G_5 = 2E[\gamma_2]F_6 - E[\beta]F_1F_3 - E[k_2]F_6 + E[\gamma_1]F_1 + E[\gamma_1]F_1F_6, \]
\[ G_6 = E[\Phi_2] - E[\Phi_1]F_3 - E[\Phi_2]F_6 - E[\gamma_1]F_2 + E[\gamma_2]F_4 - E[\beta]F_2F_3 - E[\beta]F_1F_4 + E[\gamma_1]F_2F_6 + E[k_2]F_7 + E[\gamma_1]F_1F_7, \]
\[ I_1 = E[\Phi_1] + E[c_1k_1] - \frac{1}{2} \int_0^1 (c_1^L\beta_1^U + c_1^U\beta_1^L) d\alpha, \]
\[ I_2 = E[\Phi_2] + E[c_2k_2], \]
\[ I_3 = E[\Phi_0] + E[c_1k_0] - \frac{1}{2} \int_0^1 (c_1^L\beta_1^U + c_1^U\beta_1^L) d\alpha. \]

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