GALAXY CLUSTER ABUNDANCE EVOLUTION
AND COSMOLOGICAL PARAMETERS

PEDRO T. P. VIANA
Centro de Astrofísica da Universidade do Porto, Rua das Estrelas s/n, 4150 Porto, Portugal

ANDREW R. LIDDLE
Astrophysics Group, The Blackett Laboratory, Imperial College, London SW7 2BZ, United Kingdom

ABSTRACT

We use the observed evolution of the galaxy cluster X-ray integral temperature distribution function between $z = 0.05$ and $z = 0.32$ in an attempt to constrain the value of the density parameter, $\Omega_0$, for both open and spatially-flat universes. We conclude that when all the most important sources of possible error, both in the observational data and in the theoretical modelling, are taken into account, an unambiguous determination of $\Omega_0$ is not feasible at present. Nevertheless, we find that values of $\Omega_0$ around 0.75 are most favoured, with $\Omega_0 < 0.3$ excluded with at least 90 per cent confidence. In particular, the $\Omega_0 = 1$ hypothesis is found to be still viable.

1. INTRODUCTION

The number density of rich clusters of galaxies at the present epoch has been used to constrain the amplitude of mass density fluctuations on a scale of $8\, h^{-1}\, \text{Mpc}$ (Evrard 1989; Henry & Arnaud 1991; White, Efstathiou & Frenk 1989; Henry & Arnaud 1991; White, Efstathiou & Frenk 1990; Henry & Arnaud 1991; White, Efstathiou & Frenk 1991). This is usually referred to as $\sigma_8$, where $h$ is the present value of the Hubble parameter, $H_0$, in units of $100\, \text{km\,s}^{-1}\,\text{Mpc}^{-1}$. However, the derived value of $\sigma_8$ depends to a great extent on the present matter density in the Universe, parameterized by $\Omega_0$, and more weakly on the presence of a cosmological constant, $\Lambda$. The clearest way of breaking this degeneracy is to include information on the change in the number density of rich galaxy clusters with redshift (Frenk et al. 1990), the use of X-ray clusters for this purpose having been proposed by Oukbir & Blanchard (1992) and subsequently further investigated (Hattori & Matsuzawa 1995; Oukbir & Blanchard 1997). Several attempts have been made recently, with wildly differing results (Henry 1997; Fan, Bahcall & Cen 1997; Blanchard & Bartlett 1997; Eke et al. 1998; Gross et al. 1998; Reichart et al. 1998; Viana & Liddle 1999).

The best method to find clusters of galaxies is through their X-ray emission, which is much less prone to projection effects than optical identification. Further, the X-ray temperature of a galaxy cluster is at present the most reliable estimator of its virial mass. This can then be used to relate the cluster mass function at different redshifts, calculated for example within the Press–Schechter framework (Press & Schechter 1974; Bond et al. 1991), to the observed cluster X-ray temperature function. We can therefore compare the evolution in the number density of galaxy clusters seen in the data with the theoretical expectation for large-scale structure formation models, which depends significantly only on the assumed values of $\Omega_0$ and $\lambda_0 \equiv \Lambda/3H_0^2$, the latter being the contribution of $\Lambda$ to the total present energy density in the Universe.

We used two complete X-ray samples of galaxy clusters to estimate the evolution in their number density between the redshifts of $z = 0.05$ and $z = 0.32$. The low-redshift X-ray sample is an updated version, kindly provided to us by Pat Henry, of the dataset first presented in Henry & Arnaud (1991). We used it to to estimate $\sigma_8$ as a function of $\Omega_0$ for the cases of of an open universe, where the cosmological constant is zero, and a spatially-flat universe, such that $\lambda_0 = 1 - \Omega_0$. All the dark matter was assumed cold. The calculation was performed within the extended Press–Schechter formalism proposed by Lacey & Cole (1993, 1994), which allows an estimation of the formation times of the dark matter halos associated with the galaxy clusters, and updates the result of Viana & Liddle (1996).

Knowing $\sigma_8$ for each $\Omega_0$ model allows the estimation of the expected number density of galaxy clusters model at any redshift as a function of $\Omega_0$. Hence, comparing with high-redshift data one is then able to find the $\Omega_0$ model which predicts cluster numbers closest to the observed ones.

The high-redshift X-ray sample we considered is made of the 10 clusters with redshifts between 0.3 and 0.4\footnote{Recently, it has been found that one of these clusters, MS1241.5, actually has a redshift of 0.549. Our results do not change significantly.} (the median redshift being 0.32) and with X-ray fluxes in excess of $2.5 \times 10^{-13} \text{\ erg\,cm}^{-2}\,\text{s}^{-1}$, that were identified in the Einstein Medium Sensitivity Survey (EMSS) (Gioia et al. 1990; Henry et al. 1992; Gioia & Luppino 1994; Nichol et al. 1997). For the X-ray fluxes and temperatures of these clusters we used the values published in Henry (1997), which were based on ASCA data.

We chose to concentrate on clusters with X-ray temperatures in excess of 6.2 keV. The value of the integral X-ray temperature function at this point is closest to the mean curves that best represent the integral X-ray temperature functions both at $z = 0.05$ and $z = 0.32$, recovered from the observational data. And considering only the most massive clusters one has several advantages: the underlying density field on the scales associated with these clusters...
still retains to a great extent its assumed initial Gaussianity, which is the main requirement to apply the Press–Schechter ansatz; as these clusters represent the highest peaks in the density field their number density is very sensitive to the normalization of the perturbation spectrum, which in turn means that their number density is expected to change rapidly as we move into the past; the X-ray temperature of these clusters has probably been little affected by a possible pre-heating of the intracluster medium (e.g. Metzler & Evrard 1994; Navarro, Frenk & White 1995; Cavaliere, Menci & Tozzi 1997).

2. MODELS VERSUS DATA

In Viana & Liddle (1999) we describe in detail the steps we took to estimate both the theoretically-expected value for the number density of galaxy clusters with X-ray temperatures in excess of 6.2 keV at $z = 0.32$, and its observational counterpart based on the data in Henry (1997).

2.1. What models predict

The theoretical estimation was performed for different values of $\Omega_0$, and in the two cases of open and spatially-flat models. The input observational data used in this calculation was the the number density of galaxy clusters with X-ray temperatures in excess of 6.2 keV at $z = 0.32$, and its observational counterpart based on the data in Henry (1997).

As a by-product of the calculation of the theoretically-expected value for $N(>6.2\text{ keV}, z = 0.32)$ we obtained a new estimate for $\sigma_8$, as a function of both $\Omega_0$ and $\lambda_0$, so that the corrected observed value for $N(>6.2\text{ keV}, 0.05)$ is reproduced. This supersedes the result obtained in Viana & Liddle (1996). We find that the best-fitting value is given by

$$\sigma_8 = \begin{cases} 0.56 \Omega_0^{-0.34} & \text{Open} \\ 0.56 \Omega_0^{-0.47} & \text{Flat} \end{cases}$$

with an accuracy better than 3 per cent for $\Omega_0$ between 0.1 and 1.

The most important reason why this value is smaller than that quoted in Viana & Liddle (1996) is the decrease in the assumed number density of galaxy clusters at $z = 0.05$. This results from the revision of the Henry & Arnaud dataset and from the correction due to the existence of X-ray temperature measurement errors, which had not been taken into consideration in Viana & Liddle (1996). Also, the cluster X-ray temperature function obtained in Henry & Arnaud (1991) had been slightly overestimated due to a calculational error (Eke, Cole and Frenk 1996).

The overall uncertainty in the value of $\sigma_8$ and therefore also in the theoretically expected value for $N(>6.2\text{ keV}, z = 0.32)$ was calculated in the same way as in Viana & Liddle (1996), through a Monte Carlo procedure where the sources of error, namely the normalization of the cluster X-ray temperature to virial mass relation, the value of $\delta_c$ and the value of $f$, are modeled as being Gaussian distributed, and the observed $\Gamma$ and $N(>6.2\text{ keV}, 0.05)$ as having a lognormal distribution. As in Viana & Liddle (1996) we find that for each $\Omega_0$ between 0.1 and 1 the distribution of $\sigma_8$ can be approximated by a lognormal. For open models, the 95 per cent confidence limits are roughly

$$N(>6.2\text{ keV}, 0.05) = 1.53 \times 10^{+0.16} \times 10^{-7}h^3\text{ Mpc}^{-3}$$

where the errors represent 1-sigma confidence levels. These were obtained through the bootstrap procedure, which allows an estimation of the uncertainty associated with the sampling variance. The uncertainty associated with the counting error (i.e. cosmic variance) was also included by drawing the number of clusters in each bootstrap sample from a Poisson distribution with mean 25.
given by $+20\Omega_0^{0.1\log_{10}\Omega_0}$ per cent and $-18\Omega_0^{0.1\log_{10}\Omega_0}$ per cent, while for flat models we have $+20\Omega_0^{0.2\log_{10}\Omega_0}$ per cent and $-18\Omega_0^{0.2\log_{10}\Omega_0}$.

2.2. What the observations tell us

The observed value for the number density of galaxy clusters with X-ray temperatures in excess of 6.2 keV at $z = 0.32$ was calculated by using the data in Henry (1997). The estimator used was

$$N(> k_B T, z) = \sum_{i=1}^{N} \frac{1}{V_{\text{max},i}}$$

where the sum is over all clusters with $k_B T_i > k_B T$, and $V_{\text{max},i}$ is the maximum volume in which cluster $i$ could have been detected at the 4$\sigma$ level in the EMSS within the redshift shell under consideration (0.3 to 0.4 in our case). The steps which need to be taken in order to calculate these volumes are described in Henry et al. (1992) and Viana & Liddle (1999).

As in the lower redshift case, the calculation of $N(> 6.2 \text{keV}, 0.32)$ using the X-ray temperatures measured for the galaxy clusters found between redshifts 0.3 and 0.4 would lead to the overestimation of the true $N(> 6.2 \text{keV}, 0.32)$ due to the presence of errors in the X-ray temperature determinations. Again, we corrected for this by simulating the repetition of the X-ray temperature measurements through a bootstrap procedure analogous to the one performed for the low-redshift data, though in this case the number of clusters in each sample was now extracted from a Poisson probability distribution with mean 10. When a cluster is selected for a simulated dataset we not only let its X-ray temperature vary within the observational errors, but also its flux, which affects the maximum search volume associated with the cluster. The ratio between the mean value obtained for $N(> 6.2 \text{keV}, 0.32)$ from all the simulated datasets, and the value one gets for $N(> 6.2 \text{keV}, 0.32)$ using the original dataset, then provides an estimate for the expected ratio between the latter and the real value for $N(> 6.2 \text{keV}, 0.32)$ in the Universe. The uncertainty in this value is obtained from the results of the bootstrap procedure, which in practice simulates the repetition a large number of times across the sky of the type of sampling that led to the dataset in Henry (1997).

2.3. The comparison

Let us now assume that in our Universe $N(> 6.2 \text{keV}, 0.32)$ takes some particular overall value, the one theoretically-expected given the $\Omega_0$ model under consideration. We would then expect this value to be the mean of the distribution function assembled with the values that would be measured for $N(> 6.2 \text{keV}, 0.32)$ if the type of sampling that led to the dataset in Henry (1997) was repeated a large number of times across the sky. On the other hand, we would expect that the shape of this distribution would be that obtained through the bootstrap procedure mentioned at the end of the previous subsection. We are therefore now in a position to ask the following question. If $N(> 6.2 \text{keV}, 0.32)$ took such an overall value in the Universe, how probable would it be for an observer to measure the value for $N(> 6.2 \text{keV}, 0.32)$ given by the dataset in Henry (1997), after correcting it for the displacement due to errors in the X-ray temperature measurements. We can then attach, for each value of $\Omega_0$, a probability of the value for $N(> 6.2 \text{keV}, 0.32)$ given by the dataset in Henry (1997) being actually measured. The exclusion level on each value of $\Omega_0$ is obtained simply by subtracting this probability from one.

In summary the following steps were taken, so that an exclusion level can be associated with each $\Omega_0$ based on the X-ray cluster datasets for $z = 0.05$ and $z = 0.32$:

1. The Universe was assumed to be either open or spatially-flat, with $\Omega_0$ taking a value between 0.1 and 1.

2. The best estimate for $N(> 6.2 \text{keV}, 0.32)$ in the Universe, given the dataset in Henry (1997), was calculated taking into consideration the effect of the X-ray temperature measurement errors.

3. A bootstrap procedure analogous to that described for the $z = 0.05$ data was performed in order to determine the expected shape for the distribution function of $N(> 6.2 \text{keV}, 0.32)$, if the type of sampling that led to the dataset in Henry (1997) was repeated a large number of times across the sky. The number of clusters in each sample is now drawn from a Poisson distribution with mean 10, and the input observational errors (most importantly the ASCA X-ray fluxes and temperatures) are modeled as Gaussian distributed.

4. Through the extended Press-Schechter formalism, the theoretically-expected overall value for $N(> 6.2 \text{keV}, 0.32)$ given the assumed $\Omega_0$ was calculated. The normalization $\sigma_8$ of the spectrum was fixed by the low-redshift data.

5. The distribution function for $N(> 6.2 \text{keV}, 0.32)$, determined through the bootstrap procedure, was modified by dividing the values obtained for $N(> 6.2 \text{keV}, 0.32)$ by their mean and multiplying them by the value determined in (iv), so that this value becomes the new mean and the relative shape of the distribution is maintained.

6. We calculated the probability of obtaining a value as high, or as low, as that determined in (ii), given the distribution constructed in (v). The exclusion level on the assumed $\Omega_0$ equals one minus this probability.

However, due to the uncertainties in the estimation of the theoretically-expected overall value for $N(> 6.2 \text{keV}, 0.32)$, the actual calculation of the exclusion level for each $\Omega_0$ was in reality slightly more complicated. So that we could obtain it, we needed to integrate over all possible values for the theoretically-expected $N(> 6.2 \text{keV}, 0.32)$, which we will denote $u$. The overall exclusion is the product of the probability, $P(u, \Omega_0)$, of each $u$ being the correct overall value one would expect for $N(> 6.2 \text{keV}, 0.32)$ in the Universe (given the assumed $\Omega_0$), and the exclusion level $\text{Ex}(u)$ calculated as described above for each assumed $u$, i.e.

$$\text{Exclusion probability of } \Omega_0 = \int_{-\infty}^{+\infty} P(u, \Omega_0) \text{Ex}(u) du$$ (3)
The $P(u, \Omega_0)$ are lognormal distributions with mean equal to the value calculated in (iv), and dispersion obtained through Monte Carlo simulations mentioned at the end of subsection (2.1).

In Figure 1 we show the exclusion levels for $\Omega_0$ obtained in this way. Even for the values of $\Omega_0$ for which it is easiest to reproduce the observations, from 0.7 to about 0.8, the exclusion level is quite high, around 70 per cent. The reason lies with the large uncertainty in the theoretically-expected overall value for $N(> 6.2 \text{ keV}, 0.32)$. Because of it, most theoretically-expected values end far away from the value for $N(> 6.2 \text{ keV}, 0.32)$ obtained from the dataset in Henry (1997). A large uncertainty in the theoretical prediction is clearly no basis to discard models. However, for the high and low $\Omega_0$ we are aiming to constrain, this effect becomes much less important; the high exclusion levels are caused by most of the distribution for the theoretically-expected overall values for $N(> 6.2 \text{ keV}, 0.32)$ being higher (for low $\Omega_0$), or lower (for high $\Omega_0$), than the observations. Note that the exclusion levels are absolute, and not relative as one would obtain from the calculation of a likelihood function.

![Exclusion probability vs. $\Omega_0$](image1.png)

**Fig. 1.** The absolute exclusion levels for different values of $\Omega_0$ in both the open and spatially-flat cases, when a threshold X-ray temperature of 6.2 keV is used.

The calculations we have just described assume that there is no scatter in the relation between cluster X-ray temperature and luminosity. This is not correct and can lead to an increase in the observed value for $N(> k_B T, z)$. This incompleteness problem worsens as the threshold X-ray temperature $k_B T$ is lowered, as one then starts considering clusters with X-ray fluxes dangerously close to the flux detection limit. For the same threshold X-ray temperature, the problem is also potentially much more serious in the case of the $z = 0.32$ data than for the $z = 0.05$ data. The reason is simply that for the same flux detection limit, the faintest clusters that can be detected nearby have X-ray luminosities (and thus temperatures) which are considerably smaller than those of the faintest clusters further away.

While the effect of the scatter in the X-ray cluster temperature–luminosity relation in the calculation of $N(> 6.2 \text{ keV}, 0.05)$ is negligible, as the $z = 0.05$ dataset in Henry & Arnaud (1991) is claimed to be nearly complete down to at least 3 keV, in the case of the $z = 0.32$ data it is not clear whether the presence of scatter in the X-ray cluster temperature–luminosity relation may affect the determination of $N(> 6.2 \text{ keV}, 0.32)$. Due to the scatter, there is a finite probability that some of the 5 EMSS galaxy clusters with X-ray flux below $2.5 \times 10^{-13} \text{ erg cm}^{-2} \text{ s}^{-1}$, that were found in the redshift range from 0.3 to 0.4 (Henry et al. 1992), may not only have an X-ray temperature in excess of the lowest X-ray temperature present in the sub-sample of 10 clusters from Henry (1997), 3.8 keV for MS1512.4, but also in excess of our chosen threshold temperature 6.2 keV. The minimization of this possibility was in fact another reason for our choice of 6.2 keV as the threshold temperature.

We calculated the expected increase in the corrected value of $N(> 6.2 \text{ keV}, 0.32)$ given the dataset in Henry (1997), as a result of the existence of the 5 EMSS galaxy clusters mentioned above, by doing 1000 Monte Carlo simulations where the X-ray temperatures for those clusters were estimated via the X-ray cluster temperature–luminosity relation determined in Eke et al. (1998) using the more recent data for the galaxy clusters in Henry & Arnaud (1991). Though this relation is that observed for galaxy clusters at $z = 0.05$, the recent analyses of Mushotzky & Scharf (1997) and Allen & Fabian (1998) (see also Sadat, Blanchard and Oukbir 1998) seem to imply it holds at least up to $z = 0.4$.

In the end, we found that allowing for the presence of scatter in the X-ray cluster temperature–luminosity relation when calculating $N(> 6.2 \text{ keV}, 0.32)$ has only a small effect, at the few per cent level, on the exclusion levels obtained for different $\Omega_0$, and does not alter our conclusions.

We would also like to draw attention to the fact that in all previous uses of the Press–Schechter framework to calculate the evolution of the number density of rich galaxy clusters with redshift (Oukbir & Blanchard 1992; Eke, Cole & Frenk 1996; Oukbir & Blanchard 1997; Henry 1997; Eke et al. 1998; Markevitch 1998; Reichart et al. 1998), it has been assumed that the redshift of cluster virialization, $z_c$, coincides with that at which the galaxy cluster is observed, $z_{\text{obs}}$. In Figure 2 we compare the value of $N(> 6.2 \text{ keV}, z)$ obtained using the Lacey & Cole (1993, 1994) prescription for the estimation of $z_c$ with the result of the assumption that $z_c = z_{\text{obs}}$. We always require that the observed value for $N(> 6.2 \text{ keV}, 0.05)$ is recovered.

![Redshift evolution](image2.png)

**Fig. 2.** The expected redshift evolution of $N(> 6.2 \text{ keV}, z)$ for $\Omega_0 = 1$ and 0.3. The solid lines show the result obtained using the Lacey & Cole method for estimating $z_c$, and the dashed ones the result obtained assuming that $z_c = z_{\text{obs}}$. Each curve is normalized to reproduce the observed value for $N(> 6.2 \text{ keV}, 0.05)$. Note that the divergence at high $z$ is caused by this renormalization; the absolute correction is largest at the lowest redshift, where $\Omega(z)$ is smallest.

As expected, the difference in the theoretically-predicted
overall value of $N(>6.2\text{ keV}, z)$ resulting from the two distinct assumptions regarding $z_c$ becomes larger for $\Omega_0 = 0.3$, reflecting the fact that as $\Omega_0$ goes down galaxy clusters tend to form increasingly at an earlier epoch than that at which they are observed. We found that neglecting the fact that some clusters of galaxies virialize prior to the epoch at which they are observed leads to an underestimation of the predicted degree of evolution in the value of $N(>k_B T, z)$ for $z > z_{\text{norm}}$, where $z_{\text{norm}}$ is the redshift at which $N(>k_B T, z)$ is normalized through observations, e.g. in our case $z_{\text{norm}} = 0.05$. Taking into account the possibility that $z_c$ may be larger than $z_{\text{obs}}$ therefore requires lower values for $\Omega_0$ in order for the high-redshift data on $N(>k_B T, z)$ to be reproduced.

Allowing for $z_c > z_{\text{obs}}$ means that some galaxy clusters that otherwise would not be massive enough to reach a given threshold temperature $k_B T$ can now be counted when calculating $N(>k_B T, z)$. In principle this would have the effect of increasing the expected value of $N(>k_B T, z)$ for any $z$. However, at the normalization redshift 0.05 the higher value for $N(>k_B T, 0.05)$ means that a less well developed density field at $z = 0.05$ is required, i.e. a lower value of $\sigma_8$ results from introducing the possibility that $z_c > z_{\text{obs}}$. As the number density of virialized objects evolves faster for the same relative change in the value of the dispersion of the density field the smaller this value is, the decrease in the required value for $\sigma_8$ has the effect of enhancing the decrease in the value of $N(>k_B T, z)$ as $z$ gets larger. This effect turns out to be more important than the expected increase in the value of $N(>k_B T, z)$ due to higher cluster X-ray temperatures at fixed cluster mass resulting from the possibility of $z_c > z_{\text{obs}}$.

3. DISCUSSION

From the above analysis, we conclude that at present it is not possible to reliably exclude any interesting value for $\Omega_0$ on the basis of X-ray cluster number density evolution alone, due to the limited statistical significance of the available observational data and to uncertainties in the theoretical modelling of cluster formation and evolution. However, we do find that values of $\Omega_0$ below 0.3 are excluded at least at the 90 per cent confidence level. Values of $\Omega_0$ between 0.7 to 0.8 are those most favoured, though not strongly. These results are basically independent of the presence or not of a cosmological constant.

Our conclusions support those of Colafrancesco, Mazzotta & Vittorio (1997), who tried to estimate the uncertainty involved in the estimation of the cluster X-ray temperature distribution function at different redshifts based on its present-day value. They found this uncertainty, given the still relatively poor quality of the data, to be sufficiently large to preclude the imposition of reliable limits on the value of $\Omega_0$.

Our results disagree with those of Henry (1997) and Eke et al. (1998), as they found the preferred $\Omega_0$ to lie between 0.4 to 0.5, with the $\Omega_0 = 1$ hypothesis strongly excluded. This disagreement is mainly the consequence of our focus on the threshold X-ray temperature of 6.2 keV, while they draw their conclusions based on the analysis of the results obtained for several threshold X-ray temperatures. Other less important contributions to the difference between our results and theirs are the different assumed normalization for the virial mass to X-ray temperature relation, and the corrections in the expected values in the Universe for both $N(>6.2\text{ keV}, 0.05)$ and $N(>6.2\text{ keV}, 0.32)$ due to the uncertainties in the X-ray cluster temperature measurements. Our disagreement with Eke et al. (1998) on the level of exclusion of the $\Omega_0 = 1$ hypothesis is also due to our much larger assumed uncertainty in the theoretically-expected overall value for $N(>6.2\text{ keV}, 0.32)$.

In Figure 3 we show the absolute exclusion levels for different values of $\Omega_0$ in both the open and spatially-flat cases, when a threshold X-ray temperature of 4.8 keV is used. They are substantially different from those we obtained when the threshold X-ray temperature was assumed to be 6.2 keV. We found this to be even more true if a correction for the possibility of any of the 5 clusters with the lowest X-ray fluxes in the $0.3 < z < 0.4$ EMSS subsample having X-ray temperatures in excess of 4.8 keV is included. While the standard analysis without these 5 X-ray clusters prefers a value for $\Omega_0$ between 0.4 to 0.5, when the correction for the scatter in the relation between the cluster X-ray temperature and luminosity is included, in the way described in subsection 2.3, the preferred value for $\Omega_0$ decreases to about 0.3. Now the $\Omega_0 = 1$ hypothesis is excluded at more than the 95 per cent confidence level, with or without the correction. At the 90 per cent confidence level, one finds that $\Omega_0 > 0.8$ is excluded without the correction, being this limit lowered to 0.7 when the correction is included.

One can also estimate the joint probability of some $\Omega_0$ value being excluded on the basis of the results relative to either one or both X-ray temperature thresholds. Assuming the data used in the calculations for the two thresholds is independent, the results then imply that the favoured value for $\Omega_0$ is close to 0.55 (0.50 if the incompleteness correction is included) and the $\Omega_0 = 1$ hypothesis is excluded at the 99 per cent level. This agrees very well with the results of Henry (1997) and Eke et al. (1998), leading us to believe that the main difference between our analysis and theirs was our decision to draw our conclusions solely based on the exclusion levels obtained for the X-ray temperature threshold of 6.2 keV. Nonetheless, we feel that only the data regarding clusters with X-ray temperatures in excess of about 6 keV seems sufficiently free of modelling problems, like sample incompleteness and significant pre-heating of the intracluster medium, so as to be potentially useful in constraining $\Omega_0$, at least as long as these.
issues are not satisfactorily settled. An example of possible sample incompleteness has arisen from recent work by Blanchard, Bartlett and Sadat (1998), who used a sample of 50 galaxy clusters with mean redshift of 0.05, which were identified through the ROSAT satellite, to estimate the cumulative X-ray temperature distribution function at \( z = 0.05 \). They claim the number density of galaxy clusters at \( z = 0.05 \) with X-ray temperatures around 4 keV is being underestimated when the Henry & Arnaud cluster sample is used. Through the X-ray cumulative temperature distribution function at \( z = 0.05 \) they obtain, they then estimate \( \Omega_0 \) using the EMSS cluster abundance in the redshift bin \( 0.3 < z < 0.4 \) and the X-ray temperature data gathered in Henry (1997). They find the favoured value for \( \Omega_0 \) to be 0.75, while \( \Omega_0 < 0.3 \) is excluded at more than the 95 per cent level. These results coincide very well with ours, when only the 6.2 keV threshold X-ray temperature is considered, thus perhaps implying that the discrepancy between the favoured value for \( \Omega_0 \) found when different X-ray temperature thresholds are considered may arise from a underestimation of the cumulative distribution function at \( z = 0.05 \) for X-ray temperatures below about 6 keV.

So at the moment the situation is that, unfortunately, due to uncertainties associated both with the observational measurements and the theoretical modelling of cluster evolution, the presently-available X-ray data on galaxy clusters is not able to strongly discriminate between cosmologies with different values for \( \Omega_0 \). And in any case, the data available is probably not yet statistically significant. More is needed to support or disclaim the preliminary conclusions that can be obtained from it.

4. FUTURE PROSPECTS

Within the next few years, with the launch of the XMM satellite, possibly in early 2000, a significant increase in the quantity and quality of the available data is expected to occur (Romer 1998). It should then be possible to place stronger constraints on \( \Omega_0 \) on the basis of the evolution of the galaxy cluster X-ray temperature function. This would be helped by improvements in the theoretical modelling of cluster evolution, perhaps based on the high-resolution hydrodynamical N-body simulations on cosmological scales expected in the near future.

In Figure 4 we show predictions for the cumulative redshift distribution, both out to and beyond some redshift \( z \), of the number of galaxy clusters per 1000 square degrees in the sky with X-ray temperature in excess of 6 keV. The results were obtained for two models with \( \Omega_0 = 0.3 \), one open and the other spatially-flat, and a model with \( \Omega_0 = 1 \). The solid lines include all clusters with such temperatures, independently of their X-ray flux. The dashed and dotted lines represent only the clusters which have a X-ray flux in the \([2,10]\) keV band in excess of \( 10^{-13} \) and \( 10^{-12} \) erg cm\(^{-2}\) s\(^{-1}\), respectively. The conversion of X-ray temperature to flux was performed using the X-ray temperature to luminosity relation derived by Allen & Fabian (1998) for high-temperature clusters with redshifts up to 0.4. If this relation breaks down for higher redshifts, the cluster abundance numbers presented will change beyond roughly the redshift at which the predicted X-ray flux in the \([2,10]\) keV band for a 6 keV cluster equals the threshold fluxes quoted. This means that the \( 10^{-12} \) erg cm\(^{-2}\) s\(^{-1}\) results would be little affected given that very few clusters above \( z = 0.4 \) are predicted to have such a high X-ray flux, while the \( 10^{-13} \) erg cm\(^{-2}\) s\(^{-1}\) results would only be affected above a redshift of about 1.

**Fig. 4.** — The cumulative redshift distribution, out to redshift \( z \) (upper panel) and beyond redshift \( z \) (lower panel), of the number of galaxy clusters per 1000 square degrees with X-ray temperature in excess of 6 keV. The solid lines show the result for all clusters, while the dashed and dotted lines represent only the clusters which have a X-ray flux in the \([2,10]\) keV band in excess of \( 10^{-13} \) and \( 10^{-12} \) erg cm\(^{-2}\) s\(^{-1}\), respectively. Each curve is normalized to reproduce the observed present abundance of high-temperature galaxy clusters.

It is clear that the predictions of models with different \( \Omega_0 \) start to differ very rapidly as the redshift probed is increased. When \( z \) reaches 0.3, in principle it would be already possible to distinguish between models with substantially different values for \( \Omega_0 \). As one considers data pertaining to increasingly higher redshifts, the error bar on the estimation of \( \Omega_0 \) becomes ever smaller. At redshifts above 1, it starts to be even possible to determine whether in the case of a low \( \Omega_0 \) universe, there is a cosmological constant making it spatially-flat.

Unfortunately, there are two limitations. The first is instrumental noise, as all detectors have a flux threshold below which the signal-to-noise becomes sufficiently small to cast serious doubts over any detection. Further, given that we are interested in surveys where the clusters have their X-ray temperatures measured, in practice the X-ray flux threshold of a cluster catalogue with temperatures will be around an order of magnitude larger than the X-ray flux detection threshold of the survey from which the catalogue was assembled. This problem means that surveys with high X-ray flux thresholds will not include many clusters at high redshifts with their X-ray temperatures measured. In Figure 4, one can see that a cluster catalogue with measured X-ray temperatures assembled by imposing a X-ray...
flux threshold of $10^{-12}$ erg cm$^{-2}$ s$^{-1}$ will most probably not include any cluster with $z > 0.4$, even if the area of the survey covered a large portion of the sky. A survey an order of magnitude more sensitive in flux would include almost all clusters with $kT > 6$ keV up to a redshift of 1, and several beyond if they exist.

The second problem is the uncertainties in the theoretical modelling of the structure and number density evolution of galaxy clusters, and also the still significant error on the present-day cluster abundance. The last issue may become less of a problem once data for several redshift intervals becomes available. These uncertainties were sufficient to make the presently available cluster abundance data for $0 < z < 0.4$ not very useful in the determination of \( \Omega_0 \). Given that a XMM Slew Survey (Lumb 1998; also see Laurence Jones in these proceedings), because of its short exposures, would only be able to measure X-ray temperatures for clusters with fluxes exceeding about $10^{-12}$ erg cm$^{-2}$ s$^{-1}$, almost all such clusters would have $z < 0.4$, and thus such a survey would probably turn out not very useful in the estimation of \( \Omega_0 \). However, a XMM Serendipitous Survey, though it would cover only about one tenth of the sky area that could be covered by a XMM Slew Survey, it would be an order of magnitude more sensitive. All clusters with X-ray fluxes in excess of about $10^{-13}$ erg cm$^{-2}$ s$^{-1}$, which as we mentioned above in practice means almost all clusters with X-ray temperatures in excess of 6 keV up to a redshift of 1, would have their X-ray temperatures measured (Romer 1998). A catalogue built around these clusters would already go deep enough in redshift to allow a good estimate of \( \Omega_0 \), even taking into account the theoretical modelling uncertainties.

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