TeV-scale black hole lifetimes in extra-dimensional Lovelock gravity†

Thomas G Rizzo

Stanford Linear Accelerator Center, 2575 Sand Hill Rd, Menlo Park, CA 94025, USA

E-mail: rizzo@slac.stanford.edu

Received 10 April 2006, in final form 24 April 2006
Published 5 June 2006
Online at stacks.iop.org/CQG/23/4263

Abstract

We examine the mass loss rates and lifetimes of TeV-scale extra-dimensional black holes (BH) in Arkani-Hamed, Dimopoulos and Dvali-like models with Lovelock higher-curvature terms present in the action. In particular, we focus on the predicted differences between the canonical and microcanonical ensemble statistical mechanics descriptions of the Hawking radiation that result in the decay of these BH. In even numbers of extra dimensions, the employment of the microcanonical approach is shown to generally lead to a significant increase in the BH lifetime as in the case of the Einstein–Hilbert action. For odd numbers of extra dimensions, stable BH remnants occur when employing either description provided the highest order allowed Lovelock invariant is present. However, in this case, the time dependence of the mass loss rates obtained employing the two approaches will be different. These effects are in principle measurable at future colliders.

PACS numbers: 04.50.+h, 11.10.Kk, 04.80.Cc

(Some figures in this article are in colour only in the electronic version)

1. Introduction and background

The large extra dimensions picture of Arkani-Hamed, Dimopoulos and Dvali (ADD) [1] suggests that the fundamental scale of gravity, $M_*$, may not be far above the weak scale $\sim$TeV. In this scenario, gravity propagates in the $D = 4 + n$-dimensional bulk while the standard model (SM) is confined to a three-dimensional ‘brane’, which is assumed to be flat. In such a scenario, one finds that $M_*$ is related to the usual 4D (reduced) Planck scale, $M_{\text{Pl}}$, via the expression

$$M_{\text{Pl}}^2 = V_n M_*^{n+2},$$

(1)

† Work supported in part by the Department of Energy, contract DE-AC02-76SF00515.
where \( V_n \) is the volume of the compactified extra dimensions. Assuming for simplicity that they form an \( n \)-dimensional torus, if all compactification radii, \( R_c \), are the same, then
\[
V_n = (2\pi R_c)^n.
\]
This basic ADD picture leads to three essential predictions [2]: (i) the emission of graviton Kaluza–Klein (KK) states during the collision of SM particles leading to signatures with apparent missing energy [3–5], (ii) the exchange of graviton KK excitations between SM fields leading to dimension-8 contact interaction-like operators with distinctive spin-2 properties [3, 4, 6] and (iii) the production of black holes (BH) at colliders and in cosmic rays with geometric cross sections,
\[
\approx \pi R_s^2,
\]
with \( R_s \) being the BH Schwarzschild radius, once collision energies greater than \( \sim M_* \) are exceeded [7–10].1 It has been noted that while (i) and (ii) are the results of an expansion of the \( D \)-dimensional Einstein–Hilbert (EH) action to leading order in the gravitational field and are in some sense perturbative, (iii) on the other hand relies upon the full non-perturbative content of the EH action. Thus, the TeV-scale BH production is actually testing the \( D \)-dimensional general relativity and not just the ADD picture. This is important as many other alternative theories of gravity in extra dimensions can lead to the same leading order graviton interactions. Within the ADD scenario, future collider measurements of the (i) and (ii) type processes should be able to tell us the values of both the quantities \( n \) and \( M_* \) [2] rather precisely.

Of course, ADD is at best an effective theory that operates at energies below the scale \( M_* \). It is reasonable to expect that at least some aspects of the full UV theory may leak down into these collider tests and lead to potentially significant quantitative and/or qualitative modifications of simple ADD expectations that can be probed experimentally. We have recently begun an examination of the effect of the presence of higher-curvature invariants in the \( D \)-dimensional action of ADD-like models [11] as well as in models with a warped metric [13]. We note that since the ADD bulk is flat and the SM fields are confined to a brane, the predictions for (i) and (ii) above are not influenced by the addition of such extra terms in the action [12] as the analogous predictions would be in the case of warped extra dimensions. Motivated by the string theory [14–16], we examined a very special class of such invariants with interesting properties first described by Lovelock [17], called Lovelock invariants.

Lovelock invariants come in fixed order, \( m \), which we denote as \( \mathcal{L}_m \), that describes the number of powers of the curvature tensor out of which they are constructed. We can express \( \mathcal{L}_m \) as
\[
\mathcal{L}_m \sim \delta_{A_1 B_1 \ldots A_m B_m} R_{A_1 C_1 D_1 \ldots C_m D_m},
\]
where \( \delta_{A_1 B_1 \ldots A_m B_m} \) is the totally antisymmetric product of Kronecker deltas and \( R_{A B C D} \) is the \( D \)-dimensional curvature tensor. For a space with an even number of dimensions, the \( D = 2m \) Lovelock invariant is topological and leads to a total derivative, i.e. a surface term, in the action. All of the higher order invariants, \( D \leq 2m - 1 \), can then be shown to vanish identically by using curvature tensor index symmetry properties. On the other hand, for the cases with \( D \geq 2m + 1 \), \( \mathcal{L}_m \) are truly dynamical objects such that once added, the action can significantly alter the field equations usually associated with the EH term. However, it can be shown that the addition of any or all of \( \mathcal{L}_m \) to the EH action still results in a theory with only second-order equations of motion as in ordinary Einstein gravity. In particular, variation of the action leads to modifications of Einstein’s equations by the addition of new terms which are second-rank symmetric tensors with vanishing covariant derivatives, depending only on the metric and its first and second derivatives, i.e. they have the same general properties as the Einstein tensor itself but are of higher order in the curvature. These are quite special

1 Note that in the simplest picture the BH production threshold is just a simple step function.

2 This result holds in other higher-curvature scenarios where the action is taken to be an arbitrary function of the Ricci scalar provided the masses of the SM fields can be neglected in comparison to the fundamental scale.
properties not possessed by arbitrary invariant structures which usually lead to equations of motion of higher order, i.e. more co-ordinate derivatives of the metric tensor and graviton field, e.g. terms with quartic derivatives. Such theories would, in general, have serious problems in the presence of tachyons and ghosts as well as with perturbative unitarity [14]. The Lovelock invariants are constructed in such a way as to produce an action which is free of these problems. In addition, as might be expected, the introduction of Lovelock terms into the action does not modify the number of degrees of freedom encountered by studying the EH action.

In our earlier work [11] we showed that the presence of Lovelock invariants in the action can lead to TeV-scale BH in ADD-like models with thermodynamical properties that can significantly differ from the usual EH expectations. This includes the possibility that BH may be stable in n-odd dimensions and have production cross sections with calculable mass thresholds. In a more general context, BH in theories with Lovelock invariants have been discussed by a large number of authors [15]. The usual thermodynamical description of the Hawking radiation produced by TeV-scale BH decays is via the canonical ensemble (CE) [10] which has been employed in most analyses in the literature (in particular, our previous analysis of ADD-like BH). However, as pointed out by several groups [19], though certainly applicable to very massive BH, this approach does not strictly apply when $M_{\text{BH}}/M_*$ is not much greater than $O(1)$ or when the emitted particles carry an energy comparable to the BH mass itself due to the back-reaction of the emitted particles on the properties of the BH. This certainly happens when the resulting overall BH Hawking radiation multiplicity is low. In the decay of TeV-scale BH that can be made at a collider, the energy of the emitted particles is generally comparable to both $M_*$ and the mass of the BH itself thus requiring the microcanonical ensemble (MCE) treatment. In the CE approach, the BH is treated as a large heat bath whose temperature is not significantly influenced by the emission of an individual particle. While this is a very good approximation for reasonably heavy BH, it becomes worse as the BH mass approaches the $M_*$ scale as it does for the case we will consider below. Furthermore, the BH in an asymptotically flat space (which we can assume here since the BH Schwarzschild radii, $R_*$, are far smaller than $R_c$) cannot be in equilibrium with its Hawking radiation.

It has been suggested [19] that all these issues can be dealt with simultaneously if we instead employ the correct, i.e. the MCE approach in the statistical mechanics treatment for BH decay. As $M_{\text{BH}}/M_*$ grows larger, $\gtrsim 10^{-20}$, the predictions of these two treatments will be found to agree, but they differ in the region which is of most interest to us since at colliders we are close to the BH production threshold, where $M_{\text{BH}}/M_*$ is not far above unity. Within the framework of the EH action, it has been emphasized [19] that TeV-scale BH lifetimes will be increased by many orders of magnitude when the MCE approach is employed in comparison to the conventional CE expectations. This is not due to modifications in the thermodynamical quantities, such as the temperature, themselves, but how they enter the expressions for the rate of mass loss in the decay of the BH. Here, we will address the issue of how these two statistical descriptions may differ in the BH mass range of interest to us when the additional higher-curvature Lovelock terms are present in the action. In particular, we need to address what the combination of Lovelock terms and the MCE description does to the BH mass loss rates and lifetimes. In the case of even $n$, we will show that BH lifetimes are significantly increased as was found in the EH case. In the case of odd $n$, the Lovelock BH will of course be found to produce stable remnants using either prescription for the same set of parameters as we will see below.

3 As is well known, the addition of arbitrary curvature invariants to the EH action can lead to new propagating degrees of freedom in the resulting equations of motion.
The outline of the paper is as follows. In section 2 we will present the basic ingredients associated with the Lovelock invariant extended action and the altered expectations for the Schwarzschild radius, temperature and entropy of TeV-scale BH in ADD-like models, where the bulk is essentially flat. We will then provide a brief overview of the general formalism for calculating BH lifetimes using the MCE contrasting with the conventional CE approach. In section 3 we will perform a numerical comparison of the predictions for the BH mass loss rate and lifetime in both the CE and MCE frameworks. For purposes of comparison, we first analyse the results when only the EH term is present in the action. We also address the issue as to whether the dominance of BH decays to brane fields is influenced by the choice of thermodynamic description. Section 4 contains a discussion and our conclusions.

2. Formalism

Based on the above discussion and the definition of $\mathcal{L}_m$, we see that the most general action with Lovelock invariants in 4D is just EH plus a cosmological constant, i.e. ordinary general relativity. In 5D, all of $\mathcal{L}_{n>3}$ still vanishes as in 4D but $\mathcal{L}_2$, which is the familiar Gauss–Bonnet (GB) invariant, is no longer a total derivative and its presence will modify the results obtained from Einstein gravity. The generalization is clear; for $D = 5, 6$ only $\mathcal{L}_{0,2}$ can be present in the action. For $D = 7, 8$ only $\mathcal{L}_{0,3}$ can be present while for $D = 9, 10$ only $\mathcal{L}_{0,4}$. Since ADD assumes that the compactified space is flat, the coefficient of $\mathcal{L}_0$ is taken to be zero in the present framework and, to reproduce the correct limit, the coefficient of $\mathcal{L}_1$ is normalized so that it can be identified with the usual EH term. Thus, in the Lovelock–ADD picture for $D \leq 10(n \leq 6)$, there are at most three new pieces to add to the EH action and so the general form relevant for the extended ADD model we consider is given by

$$S = \frac{M_s^{n+2}}{2} \int d^{4+n} x \sqrt{-g} \left[ R + \frac{\alpha}{M_s^2} \mathcal{L}_2 + \frac{\beta}{M_s^4} \mathcal{L}_3 + \frac{\gamma}{M_s^6} \mathcal{L}_4 \right],$$

where $\alpha, \beta$ and $\gamma$ are dimensionless coefficients which we take to be positive in the discussion below. (If we consider $D > 10$, it is quite easy to extend this parameterization by including potential $\mathcal{L}_5$ contributions.) If we expect this expansion to be the result of some sort of perturbation theory, some algebra suggests $\alpha D^2 \beta D^4$ and $\gamma D^6 \leq 1$ which yields the (only) suggestive values $\alpha \sim 10^{-2}, \beta \sim 10^{-3} - 10^{-4}$ and $\gamma \sim 10^{-5}$. \(^4\)

In order to calculate BH lifetimes we will need the relationships between the BH mass, the Schwarzschild radius, $R_s$, temperature, $T$, and entropy, $S$. When Lovelock terms are present in the action, these relations can be significantly modified from their conventional EH expectations. Here, we simply summarize the results from our earlier work. (For details, see [11]). The $M_{\text{BH}} - R_s$ relationship is given by

$$m(x) = c[x^{n+1} + \alpha n(n+1)x^{n-1} + \beta n(n+1)(n-1)(n-2)x^{n-3} + \gamma n(n+1)(n-1)(n-2)(n-3)(n-4)x^{n-5}],$$

where

\(^4\) It is important to note that the fundamental mass parameter, $M_s$, appearing in the above action is the same as the one appearing in the ADD $M_s - M_5$ relationship, equation (1). It is also the parameter appearing in the 5D coupling of the graviton to matter fields. $M_s$ can be directly related to several other mass parameters used in the literature. The fundamental scale employed by Dimopoulos and Landsberg [8] is given by $M_{DL} = (8\pi)^{(1/6-2)}M_s$ while that of Giddings and Thomas [9] by $M_{GT} = (2\pi)^{(1/6-2)}M_s$; moreover, Giudice et al [3] employ a different scale, $M_{\text{GRW}} = (2\pi)^{(1/6-2)}M_s$. $M_s$ is thus correspondingly smaller than all these other parameters with consequently far weaker experimental bounds [2]. For example, if $n = 2(6)$ and $M_{\text{GRW}} > 1.5$ TeV, then $M_s > 0.60(0.38)$ TeV; the existing direct bounds on $M_s$ are thus well below 1 TeV at present.
where \( m = M_{\text{BH}}/M_{\ast} \), \( x = M_{\ast} R_{\ast} \), and the numerical constant \( c \) is given by

\[
c = \frac{(n + 2)\pi^{(n+3)/2}}{\Gamma\left(\frac{n+3}{2}\right)}.
\]  

(5)

Since what we really want to know is \( x(m) \) and not \( m(x) \) as given above, we must find the roots of this polynomial equation; this must be done numerically in general except for some special cases. Fortunately, for the range of parameters of interest to us and with \( \alpha, \beta, \gamma \geq 0 \), we find that this polynomial has only one distinct real positive root. Similarly, the BH temperature is found to be given by

\[
T = \frac{(n + 1) U(x)}{4\pi V(x)},
\]

(6)

where \( T = T_{\text{BH}}/M_{\ast} \), and

\[
U(x) = x^6 + \alpha n(n-1)x^4 + \beta n(n-1)(n-2)(n-3)x^2
\]

\[
+ \gamma n(n-1)(n-2)(n-3)(n-4),
\]

\[
V(x) = x[x^6 + 2n\alpha(n+1)x^4 + 3\beta n(n+1)(n-1)(n-2)x^2
\]

\[
+ 4\gamma n(n+1)(n-1)(n-2)(n-3)(n-4)].
\]

(7)

The corresponding BH entropy can then be calculated using the familiar thermodynamical relation

\[
S = \int_{0}^{x} dx T^{-1} \frac{\partial m}{\partial x},
\]

(8)

which yields

\[
S = \frac{4\pi c}{n+2} \left[ x^{n+2} + 2\alpha n(n+1)(n+2)x^n + 3\beta n(n+1)(n+2)(n-1)x^{n-2}
\right.
\]

\[
\left. + 4\gamma n(n+1)(n-1)(n-2)(n-3)x^{n-4} \right].
\]

(9)

Note that here we required that the entropy vanishes for a zero-horizon size.

In order to calculate the BH mass loss rates, we follow the formalism in [19]; to simplify our presentation and to focus on the differences between thermodynamical treatments, we will ignore the effects due to grey-body factors [10] in the present analysis\(^5\). In this approximation, the rate of BH mass loss (time here being measured in units of \( M_{\ast}^{-1} \)) into bulk fields employing the MCE approach is given by

\[
\left[ \frac{dm}{dt} \right]_{\text{bulk}} = \frac{2\pi}{(2\pi)^{d+3}} \zeta(d+4) x^{d+2} \sum_{i} N_{i} \int_{m_{\text{min}}}^{m} dy (m - y)^{d+3}[e^{S(m) - S(y)} + s_i]^{-1},
\]

(10)

where \( x = M_{\ast} R_{\ast} \) as above; \( S \) is the corresponding entropy of the BH, \( d \) labels the number of bulk dimensions into which the particles are emitted, \( d \leq n \), \( i \) labels various particle species with \( N_{i} \) degrees of freedom which live in the bulk and obey Fermi–Dirac (Boltzmann, Bose–Einstein) statistics, corresponding to the choices of \( s_i = 1(0, -1) \) and \( \zeta \) is the familiar Riemann zeta function and as usual

\[
\Omega_{d+3} = \frac{2\pi^{(d+3)/2}}{\Gamma((d + 3)/2)}.
\]

(11)

\(^{5}\) It is interesting to note that the presence of Lovelock invariants in the action can alter the usual values obtained for scalar, fermion and gauge grey-body factors by terms of order unity. However, for the range of parameters of interest to us, it has recently been shown that \( \alpha \neq 0 \) does not lead to any significant change in the bulk or brane grey-body factors from those obtained from EH [20]. Our expectation is that similar results will hold when \( \beta, \gamma \neq 0 \) contributions are included, but this needs to be explored directly. These results need, however, to be fully completed for the graviton modes.
Figure 1. A comparison of the BH mass loss rates to brane fields following the MCE prescription assuming final states which, from top to bottom in each set of curves, are purely Fermi–Dirac, Boltzmann or Bose–Einstein. The solid (dashed) set corresponds to \( n = 5, \alpha = 0.005, \beta = 0.0003, \gamma = 1.14 \times 10^{-5} \) \( n = 3, \alpha = 0.01, \beta = 0.005 \).
appearing in the exponential factor above are considered nearly the same since back-reaction is neglected, i.e. taking in the MCE expression above $m \rightarrow \infty$ (the no-recoil limit) and $S(m) - S(m - \omega) \simeq \omega \delta_{m} S = \omega / T$, then integrating over $\omega$ yields the usual CE result. In particular, allowing for the different possible particle statistics in the CE case as well, we obtain for bulk decay

$$\frac{dm}{dt}_{\text{bulk}} = \sum_{i} N_{i} Q_{i} \frac{\Omega_{u}^{2}}{(2\pi)^{d+7}} \xi (d + 4) x^{d+2} \Gamma (d + 4) T^{d+4},$$

(12)

where $N_{i}$ is again just the appropriate number of degrees of freedom for each statistics type and $Q_{i}$ takes the value $\pi^{d}/90(1, 7\pi^{d}/720)$ for BE (B, FD) statistics. The corresponding expression for brane decays in the CE case is obtained straightforwardly by setting $d = 0$. Note that the values of $Q_{i}$ differ from each other by less than $\sim 10\%$ resulting in only small differences in lifetimes. It is interesting to note that since the Hawking radiation emitted from BH is generally softer in the MCE approach in comparison to the CE one, the average multiplicity for a fixed initial BH mass and number of dimensions are found to be somewhat larger in the MCE case. This difference should be observable at future colliders.

Given the large number of SM brane fields, it is well known that for the EH action the brane modes tend to dominate over bulk modes by a factor of order $\sim 100$ or more, a result which seems to continue to hold when GB terms (and the corresponding brane field grey-body factors for scalar, fermion and gauge emissions) are included [20]. It is to be noted that these results were obtained without the inclusion of grey-body factors for bulk graviton emission. The first question to be addressed is whether these results remain valid when further Lovelock terms are present in the action. As we will see in section 3, in the absence of grey-body factors, while the bulk to the brane ratio increases with $D$ it never exceeds unity when Lovelock terms are present in either MCE or CE descriptions6. A full analysis including all grey-body effects is clearly needed.

### 3. Analysis

In order to address the above question of how the MCE versus CE choice might influence the ratio of bulk to brane BH mass loss rates in the absence of grey-body factors, we construct the ratio

$$R = \left[ \frac{dm}{dt} \right]_{\text{bulk}} \left/ \left[ \frac{dm}{dt} \right]_{\text{brane}} \right..$$

(13)

Using $s_{i} = 0$, we will assume $N_{\text{brane}} = 60$ and $N_{\text{bulk}} = 1$ in what follows and thus we might naively expect that $R \sim 10^{-3}$–$10^{-2}$ if EH provides a reasonable estimate over most of the parameter range. Does this estimate remain valid when Lovelock terms are present and does the result depend on whether one chooses to follow the MCE or the CE approach? To be specific in addressing these issues, we consider the cases of $n = 3, 5$ and examine the ratio $R$ as a function of $m$ for different values of the Lovelock parameters. The results of this analysis are shown in figures 2 and 3 where we see that $R$ is a strong and increasing function of $m$ and that $R$ also increases with $n$. For large $m$, indeed $R \sim 10^{-3}$ but the precise value depends in detail upon the value of $n$ as well as the Lovelock parameters in a rather sensitive fashion. As the BH decays and $m$ becomes smaller, $R$ rapidly goes to zero in all cases implying a very strong suppression of the bulk modes. While the CE and MCE approach results do differ in

---

6 See, however, Cardoso, Cavaglia and Gualtieri in [19] where graviton grey-body factors are included for the first time indicating bulk decay dominance for large $D$ in the case of the EH action. How Lovelock terms may modify these results is at present unknown.
The ratio $R$ as a function of $m = M_{\text{BH}}/M_*$ assuming $n = 3$ for $\alpha = 0$ to $\alpha = 0.025$ from top to bottom in steps of 0.005 assuming $\beta = 0.0005$. The top (bottom) panel is the CE (MCE) result.

Before addressing the more complex case of the extended action with Lovelock terms, let us briefly review the numerical differences between the CE and MCE treatments of BH decay in the case of the EH action by setting $\alpha = \beta = \gamma = 0$; the results are shown in figure 4. The first thing to examine is the mass loss rate of BH; to this end, we consider the dimensionless quantity $M_*^{-2}dM/dt = dm/dt$, with time measured in Planck units, which is shown in the upper panel in the figure for $n = 3, 5$ for both the CE and MCE cases. For larger values of $m$, both the MCE and CE analyses yield identical results as expected but differ significantly once $m \lesssim 4$. In the CE case, the rates grow rapidly as the mass of the BH decreases below this range.
Figure 3. Same as in the previous figure but now for $n = 5$ assuming $\gamma = 1.14 \times 10^{-5}$. From the top to the bottom on the right-hand side of the figure, the curves correspond to $\alpha = \beta = 0, \alpha = 0.005$ with $\beta = \beta = 0.0003$ with $\alpha = 0$ and $\alpha = 0.005$ with $\beta = 0.0003$, respectively.

whereas in the MCE case the rate drops quite dramatically. The influence on the lifetimes of these significantly different behaviours is shown in the lower panel for a BH which begins life with $m = 5$, a reasonable value for production at the LHC. Here, what is specifically shown is the time taken (in units of $M^{-1}_* t$) for an initial $m = 5$ BH to decay to a BH with $m < 5$ via the Hawking radiation for various values of $n$. Note that while the state of complete BH evaporation, $m = 0$, is reached in the CE case for $0.03 \lesssim M_* t \lesssim 1$, for the MCE analysis one obtains values which are larger than this by many (6 to 8 or more) orders of magnitude. This shows, assuming the EH action, that the slowing of the BH evaporation rate in the MCE approach leads to a dramatic increase in the lifetime of the BH that can be produced in TeV collisions as has been emphasized by several authors [19].

We now turn to the case where Lovelock invariants are present in the action. In order to analyse a scenario where the number of extra dimensions is even, we consider a specific example with $n = 2$ so that only the quadratic GB term can be present, i.e. only $\alpha$ can be
Figure 4. (Top) rate of change of the BH mass through the Hawking radiation on the brane assuming the EH action as discussed in the text; the CE (top) and MCE (bottom) results are represented by the two sets of curves with the case $n = 5(3)$ being the upper (lower) curve. (Bottom) decay times for a BH with an initial $m = M_{BH}/M^* = 5$; the rising (flat) set of curves at low $m$ corresponds to the MCE (CE) case. In each set of curves, $n$ ranges from 2 to 7 going from top to bottom.

non-zero. Note that as $n$ is even, the GB invariant cannot lead to a BH mass threshold. Figure 5 shows the results of the calculations which are analogous to those displayed in figure 4. Here, in the upper panel we see that for both the MCE and CE analyses, the presence of the GB term leads to a suppression of the BH decay rate, $M_{*}^{-2}dM/dt$, which is active for all values of $m$ but is somewhat magnified as $m$ gets smaller. For larger $m$, the CE and MCE analyses are seen to agree just as in the case of the EH action above. For the overall BH lifetime, $M_{*}t$, we see in the bottom panel that a non-zero $\alpha$ for the case of $n = 2$ can increase the value of this quantity up to two orders of magnitude in the CE analysis. Using the MCE combined with the non-zero values of $\alpha$ leads to a further increase in the BH lifetime by two more orders of magnitude, i.e. the presence of the GB term is seen to further augment the BH lifetime obtained by employing the MCE analysis in comparison to the standard EH picture employing the CE. However, this relative enhancement is far smaller than that found in the EH case since
the common enhancement arising from the Lovelock terms present in both cases is already large. We thus conclude that for even \( n \) the presence of Lovelock invariants together with the use of the MCE will significantly increase the BH lifetimes, but only by a few orders of magnitude.

What happens when \( n \) is odd and a BH remnant can form? In this case, we consider typical non-zero values for all of the allowed Lovelock parameters for the fixed number of extra dimensions considered. Figures 6 and 7 show the values of \( M^2 \frac{dM}{dt} \) for \( n = 3 \) and \( n = 5 \) comparing the expectations of the CE and MCE approaches. For the case of \( n = 3 \), we hold \( \beta = 0.0005 \) fixed and vary the values of \( \alpha \); for \( n = 5 \), we hold fixed \( \gamma = 1.14 \times 10^{-5} \) and vary the values of both \( \alpha \) and \( \beta \). In both figures, we see that the general patterns associated with a reduced rate of the Hawking radiation employing the MCE analysis observed for \( n = 2 \) are repeated. However, unlike for even values of \( n \), for \( n \) odd we see that \( M^2 \frac{dM}{dt} \rightarrow 0 \) for both the MCE and CE approaches as \( m \rightarrow m_{\text{crit}} \). In addition, for large values of \( m \), \( M^2 \frac{dM}{dt} \) is

Figure 5. As figure 4, but now assuming \( n = 2 \) for \( \alpha \neq 0 \). In the top panel, for both sets of curves, \( \alpha \) goes from 0 to 0.025 in steps of 0.005 going from top to bottom. In the bottom panel, \( \alpha \) goes over the same range but in the opposite order.
generally seen to be larger when the CE technique is employed than when one instead uses the MCE.

In order to access the influence on the BH decay time of the CE versus MCE choice in the presence of higher-curvature terms, we show in figures 8 and 9 the integrated decay time for the above studied cases of $n = 3$ and $n = 5$. As expected from the $n = 2$ analysis, here we again see that the BH described by the MCE has a decay time for $m > m_{\text{crit}}$, which is somewhat longer than the corresponding CE result. This enhancement in the decay time to a fixed mass final state, which can be up to a couple of orders of magnitude, is observed to be substantially less than that obtained in the pure EH scenario found above. We thus conclude, for $n$ odd, that while the MCE approach always leads to an enhancement in the BH decay time relative to that obtained in the CE approach, the effect of the higher Lovelock terms is to reduce the degree of this enhancement in comparison to that obtained in the case of the pure EH action. Note that in either approach the resulting total BH lifetime is infinite, as the BH decay still results in a stable remnant as was found in our earlier work.
It is clear from this analysis that the Lovelock extended action leads to significant modifications in the mass loss rates and lifetimes of BH and that the choice of MCE versus CE is critical. Theoretical arguments support the use of the MCE description, but experiments will be able to distinguish these two approaches at colliders.

4. Discussion and conclusions

Higher-curvature invariants of the Lovelock type could be present in the extra-dimensional effective gravitational action and would make their presence known at energies of order $M_*$ and above. If the higher-dimensional bulk is flat (as in ADD-like models but not in RS-like models), there are few ways to directly probe the existence of these additional terms experimentally. The reason for this is that the conventional ‘perturbative’ graviton-related ADD signatures are found to be quite insensitive to the existence of the Lovelock terms. On the other hand, since they are non-perturbative structures, the properties of the TeV-scale
black holes in extra-dimensional models are potentially very sensitive to these new interactions which can be probed at future particle colliders.

In this paper we have considered how the existence of Lovelock invariant extensions to the Einstein–Hilbert action will modify the mass loss rates and lifetimes of TeV-scale BH. In particular, we examined the sensitivity of both these quantities to the choice made in the statistical mechanics treatment of BH. It had been shown, and we verified those results here, that in the case of the EH action, BH lifetimes are significantly enhanced by many orders of magnitude when the microcanonical ensemble description is employed in comparison to the more conventional canonical ensemble approach. There are several reasonable arguments in the literature as to why BH in the mass range of interest to us are in fact best described by the MCE.

Within this context, when Lovelock terms were present in the case of ADD-like flat extra dimensions, we demonstrated in the present paper that (i) BH decays to SM fields on the brane remain dominant over those to graviton bulk fields employing either the MCE or CE.
descriptions when Lovelock terms are present. However, in all the cases the bulk/brane ratio was shown to grow as the number of extra dimensions increased. (ii) Unlike in the case of a single warped extra dimension, the BH decay rates and lifetimes for ADD-like extra dimensions are found to be insensitive to the statistics ‘mix’ of the particles on the brane. (iii) For even numbers of extra dimensions, the lifetimes of BH described by the MCE are up to a few orders of magnitude larger than those obtained employing the CE. While this significant enhancement is large, it is many orders of magnitude smaller than that obtained employing only the EH action. (iv) For odd numbers of extra dimensions, with the highest order allowed Lovelock term present, BH are found to decay to stable relics independent of the MCE/CE choice. However, the functional dependence of the mass loss rate in the two cases can be somewhat different, but the details are sensitive to the particular values of the model parameters. It is interesting to note that the existence of a remnant and a BH mass threshold in models with the Lovelock invariants in the action is not an uncommon feature of models which probe beyond the EH action; such phenomena may happen for a 4D BH when a
renormalization group running of Newton’s constant is employed [21] in order to approximate leading quantum corrections. Such a remnant scenario can also be seen to occur in theories with a minimum length [22], in loop quantum gravity [23] and in resumed quantum gravity [24]. In, e.g. the case of a minimum length, stable remnants occur for all numbers of extra dimensions. It is interesting to note that these phenomena occur in all these models where one tries to incorporate quantum corrections in some way; though the quantitative nature of such remnants differ in detail in each of these models, it would be interesting to learn whether or not this is a general qualitative feature of all such approaches.

Black holes observed at future colliders may open an exciting window to the fundamental theory of gravity in extra dimensions.

Acknowledgment

The author would like to thank J Hewett and B Lillie for discussions related to this work.

References

[1] Arkani-Hamed N, Dimopoulos S and Dvali G R 1999 Phys. Rev. D 59 086004 (Preprint hep-ph/9807344)
Arkani-Hamed N, Dimopoulos S and Dvali G R 1998 Phys. Lett. B 429 263 (Preprint hep-ph/9803315)
Antoniadis I, Arkani-Hamed N, Dimopoulos S and Dvali G R 1998 Phys. Lett. B 436 257 (Preprint hep-ph/9804398)
[2] Hewett J and Spiropulu M 2002 Ann. Rev. Nucl. Part. Sci. 52 397 (Preprint hep-ph/0205106)
[3] Giudice G F, Rattazzi R and Wells J D 1999 Nucl. Phys. B 544 3 (Preprint hep-ph/9811291)
[4] Han T, Lykken J D and Zhang R J 1999 Phys. Rev. D 59 066004 (Preprint hep-ph/9810321)
[5] Mirabelli E A, Perelstein M and Peskin M E 1999 Phys. Rev. D 60 015005 (Preprint hep-ph/9811219)
[6] For a recent review, see Kanti P 2004 Phys. Rev. D 69 044016 (Preprint hep-ph/0312200)
See also Stojkovic D 2004 Preprint hep-ph/0409124
[7] Banks T and Fischler W 1999 Preprint hep-th/9906038
[8] Antoniadis I, Arkani-Hamed N, Dimopoulos S and Dvali G R 1998 Phys. Lett. B 436 257 (Preprint hep-ph/9804398)
[9] Giddings S B and Thomas S 2002 Phys. Rev. D 65 056010 (Preprint hep-ph/0106219)
[10] For a recent review, see Kanti P 2004 Phys. Rev. D 69 044016 (Preprint hep-ph/0312200)
See also Stojkovic D 2004 Preprint hep-ph/0409124
[11] For a detailed discussion and further references, see Rizzo T G 2005 J. High Energy Phys. JHEP06(2005)079
(Preprint hep-ph/0503163)
Rizzo T G 2005 J. High Energy Phys. JHEP01(2005)028 (Preprint hep-ph/0412087)
Rizzo T G 2005 J. High Energy Phys. JHEP01(2005)028 (Preprint hep-ph/0510420)
[12] Demir D A and Tanyildizi S H 2005 Preprint hep-th/0512078
[13] Randall L and Sundrum R 1999 Phys. Rev. Lett. 83 3370 (Preprint hep-ph/9905221)
[14] Zweibach B 1985 Phys. Lett. B 156 315
See also Boulware D G and Deser S 1985 Phys. Rev. Lett. 55 2656
Zumino B 1986 Phys. Rept. 137 109
[15] There is a vast literature on this subject. For a far from exhaustive list, see for example Cai R G 2002 Phys. Rev. D 65 084014 (Preprint hep-th/0109133)
Cai R G 2004 Phys. Lett. B 582 237 (Preprint hep-th/0311240)
Cai R G 2001 Phys. Rev. D 63 124018 (Preprint hep-th/0102113)
Cai R G and Guo Q 2004 Phys. Rev. D 69 104025 (Preprint hep-th/0311020)
Myers R C and Simon J Z 1988 Phys. Rev. D 38 2434
Nojiri S, Odintsov S D and Ogushi S 2002 Phys. Rev. D 65 023521 (Preprint hep-th/0108172)
Cho Y M and Neupane I P 2003 Int. J. Mod. Phys. A 18 2703 (Preprint hep-th/0112227)
Cho Y M and Neupane I P 2002 Phys. Rev. D 66 024044 (Preprint hep-th/0202140)
Clunan T, Ross S F and Smith D J 2004 Class. Quantum Grav. 21 3447 (Preprint gr-qc/0402044)
Cai R G and Guo Q 2004 Phys. Rev. D 69 104025 (Preprint hep-th/0311020)
Okuyama N and Koga J-I 2005 Preprint hep-th/0501044
Crisostomo J, Troncoso R and Zanelli J 2000 Phys. Rev. D 62 084013 (Preprint hep-th/0003271)
Cvetic M, Nojiri S and Odintsov S D 2002 Nucl. Phys. B 628 295 (Preprint hep-th/0112045)
Aros R, Troncoso R and Zanelli J 2001 Phys. Rev. D 63 084015 (Preprint hep-th/0011097)
Abdalla E and Correa-Borbonet L A 2002 Phys. Rev. D 65 124011 (Preprint hep-th/0109129)
Jacobson T, Kang G and Myers R C 1995 Phys. Rev. D 52 3518 (Preprint gr-qc/9503020)
Jacobson T, Kang G and Myers R C 1994 Phys. Rev. D 49 6587 (Preprint gr-qc/9312023)
Jacobson T and Myers R C 1993 Phys. Rev. Lett. 70 3684 (Preprint hep-th/9305016)
Deser S and Tekin B 2003 Phys. Rev. D 67 084009 (Preprint hep-th/0212292)
Barrau A, Grain J and Alexeyev S O 2004 Phys. Lett. B 584 114 (Preprint hep-ph/0311238)
Konoplya R 2005 Phys. Rev. D 71 024038 (Preprint hep-th/0410057)

[16] Mavromatos N E and Rizos J 2000 Phys. Rev. D 62 124004 (Preprint hep-th/0008074)
[17] Lovelock D 1971 J. Math. Phys. 12 498
See also Lanczos C 1932 Z. Phys. 73 147
Lanczos C 1938 Ann. Math. 39 642

[18] See for example Yoshino H and Rychkov V S 2005 Improved analysis of black hole formation in high-energy
particle collisions Phys. Rev. D 71 104028 (Preprint hep-th/0503171)
Cardoso V, Berti E and Cavaglia M 2005 Class. Quantum Grav. 22 L61 (Preprint hep-ph/0505125)
Yoshino H, Shiromizu T and Shibata M 2005 Preprint gr-qc/0508063

[19] See for example Casadio R and Harms B 2001 Phys. Rev. D 64 024016 (Preprint hep-th/0101154)
Casadio R and Harms B 2000 Phys. Lett. B 487 209 (Preprint hep-th/0004004)
Casadio R and Harms B 2002 Int. J. Mod. Phys. A 17 4635 (Preprint hep-th/0110255)
Kraus P and Wilczek F 1995 Nucl. Phys. B 437 231 (Preprint hep-th/9411219)
Kraus P and Wilczek F 1995 Nucl. Phys. B 433 403 (Preprint gr-qc/9408003)
Parikh M K and Wilczek F 2000 Phys. Rev. Lett. 85 5042 (Preprint hep-th/9907001)
Page D N 1976 Phys. Rev. D 13 198
Keski-Vakkuri E and Kraus P 1997 Nucl. Phys. B 491 249 (Preprint hep-th/9610045)
Massar S and Parentani R 2000 Nucl. Phys. B 575 333 (Preprint gr-qc/9903027)
Jacobson T and Parentani R 2003 Found. Phys. 33 323 (Preprint gr-qc/0302099)
Hossenfelder S, Koch B and Bleicher M 2005 Preprint hep-ph/0507140
Koch B, Bleicher M and Hossenfelder S 2005 Preprint hep-ph/0507138
Hossenfelder S 2004 Preprint hep-ph/0412265
Hossenfelder S, Hofmann S, Bleicher M and Stoecker H 2002 Phys. Rev. D 66 101502 (Preprint hep-ph/0109085)
Cardoso V, Cavaglia M and Gualtieri L 2005 Preprint hep-th/0512116
Cardoso V, Cavaglia M and Gualtieri L 2005 Preprint hep-th/0512002

[20] Grain J, Barrau A and Kantz P 2005 Preprint hep-th/0509128
[21] Bonanno A and Reuter M 2000 Phys. Rev. D 62 043008 (Preprint hep-th/0002196)
[22] See Cavaglia M, Das S and Maartens R 2003 Class. Quantum Grav. 20 L205 (Preprint hep-ph/0305223) and
references therein
See also Hossenfelder S 2004 Phys. Lett. B 598 92 (Preprint hep-th/0404232)
[23] Bojowald M, Goswami R, Maartens R and Singh P 2005 Preprint gr-qc/0503041
[24] Ward B F L 2006 Preprint hep-ph/0605054