A model for pairing in two-dimensional electron gases

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Abstract. If the bare interaction between two electrons is dressed in the two-dimensional (2D) electron gas by the many-body environment, pairing may occur. Here, we study numerically the existence and character of bound states in cases where the basic dressing is described by superimposing normalized holes centered around both electrons. Beyond this modeling, a possible modification of the interaction energy at short range is considered by a repulsive potential term. The effect of the ionic polarizability on pair interaction is approximated by a static dielectric constant. A many-body analysis of pairing, employing the spherical harmonics representation of the Bethe–Salpeter ladder-solution for the two-particle scattering amplitude in 2D, is given as well. Pairing occurs due to attractive spherical harmonics in our interparticle potential.

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1. Introduction and motivation

The study of a strongly correlated two-dimensional (2D) electron system with a charge-compensating rigid background [1] is an active area of research. It is the system that serves as a benchmark for developments of approximate input functionals to density-functional theories in 2D. Moreover, this prototype system offers a rich phenomenology raising a theoretical interest in the role of reduced dimensionality with Coulomb forces between fermions and a practical interest in possible technological applications. Indeed, there are real materials in which the conduction electrons can be considered to form a 2D electron gas and where the residual interparticle interaction plays a determining role in measurable quantities. We emphasize that this residual interaction is different [2, 3] from the one used to describe, e.g. the interaction of the many-body system with particles incident from outside.

In this work, we shall be interested in a possible pairing, i.e. in an intrinsic superconductivity in a homogeneous electron gas without phonons and other intermediaries, by using an instantaneous residual interparticle interaction in 2D. At the simplest, i.e. the Schrödinger equation level for a bound state, the unconventional pairing in a 2D electron gas was discussed earlier by using different [4, 5] static representations for the screening of negative unit charges in order to model a pairing interaction. In the case of a physical electron, these representations [4, 5] rest on essentially different pictures applied to define an independent dressed particle in the many-body system. Ghazali and Gold [4] applied a static dielectric screening to obtain an effective pair potential in real space. A quite strong and monotonically increasing binding efficiency was found by lowering the host density.

We have to note that in a linearized, self-consistent field treatment of the charge polarization generated by an external point charge, the corresponding static screening density around a negative unit charge can seriously overestimate the obviously limited magnitude of the charge depletion at short range. Related to this unphysical aspect of an otherwise properly normalized screening charge at lower host densities, it was pointed out [5] that in 2D a properly normalized but overlocalized screening density around a negative point charge yields an oscillating potential in which the attractive part appears at too short a distance from this charge. This unphysical character of an effective interaction can result in uncontrolled statements on the binding efficiency and its density dependence.

More recently, based on a properly constrained real-space representation of the instantaneous exchange-correlation hole around a mobile electron in an electron gas, the screened-electron–screened-electron [6, 7] electrostatic interaction energy has been used in the Schrödinger equation for relative motion. A non-monotonic, dome-like dependence of the binding energy as a function of the host density was obtained [7]. It is one of the purposes of the
present study to investigate the possible role of additional, physically motivated, modifications in this model which rests on the electrostatic interaction between two symmetrically treated neutral objects. In particular, by viewing our normalized hole as a nonlinear result based on the self-consistent field concept to characterize the screening of a negative unit charge, the effect of an extra induced exchange-correlation-like potential term will be discussed. Furthermore, we shall investigate the influence of a macroscopic dielectric constant that could mimic the ionic polarizability of a real superconductor.

Our treatment, based on an effective Schrödinger equation for the bound state, requires of course an additional many-body analysis on the possibility of pairing. This analysis is the second purpose of our comparative study. In fact, the possibility of an intrinsic superconductivity in a 3D interacting homogeneous electron gas was pointed out by Kohn and Luttinger in 1965. The criterion used for the superconductivity was the occurrence of a pole at $T_c$ in the (vertex-corrected) scattering amplitude for pairs of quasi-particles (modeled by a fixed Thomas–Fermi-type screened potential) of equal and opposite momenta.

This is the so-called Cooper channel with zero total momentum at the Fermi surface, thus with vanishing excitation energy ($\omega$) for a real transition in quasi-particle–quasi-particle scattering. The 2D version of the Kohn–Luttinger mechanism for pairing was investigated earlier by Chubukov and Galitski and Sarma using different approximations for the input interaction $V(q)$. A higher-order perturbation theory with a finite scattering length and an energy-dependent effective interaction with a momentum-independent input interaction to the Bethe–Salpeter equation were used.

In the present work, we apply a similar field-theoretic treatment by considering our screened-electron–screened-electron potential (a residual quasi-particle–quasi-particle interaction) in momentum space as an irreducible scattering amplitude at the first-order Born level. Then, we implement the standard spherical harmonics representation of the Bethe–Salpeter solution for the total two-particle scattering amplitude in a 2D normal Fermi system and investigate the appearance of a pole in this amplitude. We find bound states in this treatment also. Thus, the detailed results obtained suggest a consistent picture of a possible pairing mechanism of electronic origin in dilute 2D electron gases.

The rest of the paper is organized as follows. In the next section, section 2, we outline the potential energies for effective interactions and discuss them in a comparative way. The last part of this section is devoted to a many-body treatment on pairing based on our momentum-space potential as the input interaction. The results obtained from the Schrödinger equation for binding characteristics are presented in section 3. Finally, section 4, contains a short summary.

Hartree atomic units, $\hbar = e^2 = m_e = 1$, are used throughout this work.

### 2. Models

Our model system is an interacting electron gas, i.e. a system of identical fermion particles, not localized in space and in strong interaction with one another. It represents a genuinely important topic in its own right, with its own characteristic methods. For instance, in the Fermi-liquid theory of Landau the quasi-particle–quasi-particle residual interaction involves averaging, screening and correlating effects. This residual interaction is related to the vertex part of the two-particle Green function. Similarly, one is tempted to replace, for both practical computational and fundamental understanding purposes, the quantum-mechanical many-electron
problem by a two-electron problem with a properly dressed, effective electron–electron interparticle interaction.

Following earlier [5]–[7] attempts, the present study is based on the real, coordinate-space representation of a normalized hole around an electron. This hole is a physical quantity. Its significance is in the charge depletion around a system particle, which results in a potential modulation. The direct application of a hole can provide [13, 14] transparent and complementary insight into the nature of multiparticle systems. Due solely to the exclusion principle, i.e. even in the absence of Coulomb interaction, the co-moving holes around each electron have unit norm. Dynamical correlation results in further deepening of a normalized hole.

Considering a point electron, which moves in an electron gas of density \( n_0 = 1/(\pi r_s^2) \), we model the instantaneous hole by a normalized Gaussian charge centered around it

\[
\Delta n(r) = (\beta^2 / \pi) \exp(-\beta^2 r^2).
\]

The test-charge–screened-electron interaction energy is as follows:

\[
V_{te}(R) = \frac{1}{2\pi} \int_0^\infty dq \, J_0(q R) \, V_{te}(q) = \frac{1}{2\pi} \int_0^\infty dq \, J_0(q R) \, \frac{2\pi}{q} [1 - \Delta n(q)],
\]

where \( R \) is the test-charge–screened-electron distance. Here \( J_0(q R) \) is the zeroth-order Bessel function and \( \Delta n(q) = \exp[-q^2/(2\beta)^2] \) is the Fourier transform of the Gaussian screening density. We may fix \( \beta \) as \( \beta = 1/r_s \), using the complete-depletion [5, 7, 15] constraint.

Next, we consider a pair of electrons with antiparallel spins and model their screening by normalized Gaussian charges centered around both electrons. The screened-electron–screened-electron electrostatic interaction energy can be obtained in our symmetric model most easily as a convolution of quantities in the wave-vector space

\[
V_{ee}(R) = \frac{1}{2\pi} \int_0^\infty dq \, J_0(q R) \, V_{ee}(q) = \frac{1}{2\pi} \int_0^\infty dq \, J_0(q R) \, [1 - \Delta n(q)] \frac{2\pi}{q} [1 - \Delta n(q)].
\]

Observe a very important point here: \( V_{ee}(q) \) has its maximum at \( [q/(2\beta)]^2 \simeq 2 \). With \( \beta = 1/r_s \), the maximum appears at \( q = 2k_F \), since \( k_F = \sqrt{2}/r_s \) in 2D.

The above potentials are non-negative in wave-vector \( (q) \) space, and both are zero in the \( q \to 0 \) limit. For comparison, we note that the simplest dielectric (Thomas–Fermi) polarization method gives \( V_{te}^{TF}(q) = 2\pi/(q + 2) \) for a repulsive test charge, which behaves in an opposite manner in \( q \)-space. Our potentials, for any nonzero \( \beta \), belong to the class of circularly symmetric potentials obeying the following constraint:

\[
\int_0^\infty dR \, RV(R) = 0.
\]

This class of potentials was investigated by Simon [16], who proved the theorem that a not-everywhere-non-negative effective potential \( \Delta V(R) \) has a bound state for all \( \Delta \neq 0 \).

The analog with the Fermi-liquid theory [2, 12] becomes deeper with the above observation. The renormalized scattering vertex of that theory, on the Fermi surface \( (\omega = 0) \) and at zero momentum-transfer, is zero [17]. This forward limit characterizes the peculiarity of the scattering process of the quasi-particles at zero energy transfer. Thus, the physically presupposed similarity between our real-space-based construction for an effective interaction

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and a field-theoretic method for the scattering vertex is strengthened further \textit{a posteriori}. Moreover, in the many-body perturbation theory [3] the leading second-order vertex corrections are (with a short-range potential) proportional to $[V(q = 0)V(q = 2k_F)]$ and $[V(q = 0)]^2$.

These corrections are zero with our potentials as input. This \textit{renormalized} behavior is very reasonable, especially for the $V_{ee}(q)$ residual interaction obtained in a symmetric treatment. As we mentioned earlier [7], with a $V(q = 0) = 0$ character for the two-body interaction the kinetic energy-change in the many-body system is zero, at least at the first-order Born level for scattering characteristics. We should note, however, that a possible dynamical (fluctuational) process driven by a spin-flip channel might result in an additional contribution to dressing, according to a linear response [1] theory.

By performing the integrations, we obtain the real-space potentials

\begin{equation}
V_{ee}(R) = \frac{1}{R} - \frac{1}{R} \sqrt{2\pi z} I_0(z) e^{-z},
\end{equation}

\begin{equation}
V_{ee}(R) = \frac{1}{R} - 2 \frac{1}{R} \sqrt{2\pi z} I_0(z) e^{-z} + \frac{1}{R} \sqrt{\pi z} I_0(z/2) e^{-z/2},
\end{equation}

where the shorthand $z = \beta^2 R^2/2$ is introduced and $I_0(x)$ is the modified Bessel function. The potentials are repulsive at short distances. $V_{ee}(R)$ and $V_{ee}(R)$ diminish as $-1/(4\beta^2 R^3)$ and $+9/(16\beta^4 R^5)$ for $R \rightarrow \infty$, respectively. The shielding functions $F_{ee}(z) \equiv RV_{ee}(R)$ and $F_{ee}(z) \equiv RV_{ee}(R)$ are shown in figure 1 for the range $\pi \in [0, 5]$. In order to emphasize the role of dimensionality, we note here that the 3D version of $V_{ee}(R)$ with the corresponding (3D) Gaussian hole is everywhere repulsive, since $RV_{ee}(3D, R) = 1 - \Phi(\beta R)$, where $\Phi(\beta) = (\beta^2 R^3)$ is the standard error function. The interaction energy of a point electron with its surrounding hole is given [5] by $\varepsilon_{\text{int}}(r_s) = -\sqrt{\pi}/(2r_s)$. We shall use this important quantity in an additional analysis below, at around equation (7).

Our $F_{ee}(z)$ has a negative minimum at about $z = 0.5$, which corresponds (with $\beta = 1/r_s$, $R = 1/\beta = r_s$). Furthermore, the potential becomes zero at around $R = 2r_s$, which is an average distance between electrons. Clearly, from geometrical points of view, the present $V_{ee}(R)$ potential shows certain Wigner-crystal-like [18] fingerprints. Thus, our model for the residual interparticle interaction is well-optimized, since the correlated electrons want to avoid each other as much as possible. It is very important to note that in his Eliashberg-type treatment (with retarded interaction) for 3D, Takada discussed [19] the physical origin of superconductivity by emphasizing also the importance of proper modeling close (from below) to the Wigner-crystal limit.

As we mentioned in the introduction, the present screened-electron–screened-electron effective interaction may need some minor modification at short range, due to the fact that for $R < 2r_s$ the overlap of two rigid holes gives some overscreening $[2\Delta n(0) > n_0]$ of the bare Coulomb field in that short range. At the fundamental (cf the physical meaning [13] of a hole) level the most natural modification would be to use $\beta = 1/(\sqrt{2}r_s)$ instead of $\beta = 1/r_s$. In such a way the important (see equation (4) above) behavior, $V_{ee}(q = 0) = 0$, remains. If we view a normalized hole as a nonlinear result based on the charge-polarization concept, we may be tempted to allow the following repulsive and well-localized extra potential term:

\begin{equation}
\Delta \nu(R) = \frac{\alpha}{r_s} \left[1 - (1 - e^{-\beta^2 R^2})^{1/2}\right]
\end{equation}
as a possible modification to \( V_{ee}(R) \). One can motivate this simple form by the induced exchange-correlation potential-form of the local density approximation, used in the Kohn–Sham (KS) scheme to describe \([20]\) the effective potential around an external negative unit charge in 2D. In such a treatment, however, we get \( [V_{ee}(q = 0) + \Delta v(q = 0)] > 0 \). Thus, due to this character, the many-body (leading) vertex corrections (see above) are not zero, and the energy change in the system cannot be optimal. We believe that such an attempt, based on the self-consistent mean-field method to describe an independent electron in the field of an external charge, would need further modifications to make it physically consistent (to avoid double-counting) for the residual interaction of a pair at the Fermi level.

The screening function, defined as \( \Delta F(z) \equiv R \Delta v(R)/\alpha = \sqrt{2z}[1 - (1 - \exp(-2z))^{1/2}] \), of this exchange-correlation-like interaction energy contribution is plotted in figure 2. In our numerical study, we shall treat \( \alpha \) as a convenient parameter, in order to get more information about binding. The difference, \( [V_{ee}(R = 0) - V_{le}(R = 0)] \), between the electric works done in charging-up the single- and two-center objects could suggest, as a conservative physical maximum, the \( \alpha_m = \sqrt{\pi/2}(\sqrt{2} - 1) \approx 0.52 \) value. The sum of \([\varepsilon_{int}(r_s) - \alpha_m/(2r_s)]\) is already very close to a Wigner-like limit \((-1.15/r_s)\) obtained \([1, 18]\) with a disc-like hole.

In order to get a pure mathematical, upper-bound-like estimation on the \( \alpha \) value now we use a standard \([21]\) quasi-classical method and \( \beta = 1/r_s \). In this method, the number of bound states in 2D is proportional to the volume average of \( p_{max}^{\alpha} = 2\mu |V(R)| \), where the space integration is restricted to that interval of \( R \in [R_1, R_2] \) in which the potential is negative (\( \mu = 1/2 \) is the reduced mass). Thus, we apply the following simple constraint:

\[
\int_0^\infty dR \ R \Delta v(R) = \int_{R_1}^{R_2} dR \ R |V(R)|. \tag{8}
\]
Figure 2. The dimensionless function $\Delta F(z)$ defined in the text, as a function of $z = \beta^2 R^2/2$.

The integral on the left-hand side is analytical and gives $I_l(\alpha, r_s) = \alpha r_s (1 - \log 2)$. The right-hand side reduces to

$$I_r(r_s) = r_s \sqrt{2} \int_{z_1}^{z_2} \frac{dz}{\sqrt{z}} |F(z)|.$$  \hspace{1cm} \text{(9)}

In the case of $F_{ee}(z)$, we have $z_1 = 0.13$ and $z_2 = 2.31$, and the numerical evaluation results in $(\sqrt{2}/r_s) I_r(r_s) = 0.37$. Thus an $\alpha = 0.85$ is the quasi-classical (mathematical) limit value to get binding in the presence of the extra repulsive potential term.

As we mentioned above, one gets the $[V_{ve}(q = 0) + \Delta v(q = 0)] > 0$ limit with a local-induced potential $\Delta v(R)$. This signals a violation of Mermin’s constraint \[17\) on a renormalized scattering amplitude at $\omega = 0$ with $q \rightarrow 0$. A possible modification of our $V_{ve}(q) = v(q) - 2v(q)\Delta n(q) + v(q)[\Delta n(q)]^2$, \hspace{1cm} \text{(10)}

interparticle interaction could be a change in $v(q)$ to the second term via a static local-field factor, i.e. $v(q) \Rightarrow v(q)[1 - G(q)]$ in this term. The above terms have clear identifications, similarly to the symmetric model \[22\] of Gravel and Ashcroft for impurity–impurity interaction in an electron gas. The first term describes the electron–electron repulsion, the second one the attractive interactions between electrons and induced holes, and the third term the repulsion between the holes. With a $v(q)[1 - G(q)]$ form in the second term with a simple \[1\] Hubbard-type $G(q) = q/(a + q)$, one could still satisfy the $V_{ve}(q = 0) = 0$ constraint. Such a modeling, where the spin-flip can be important, is left for a future consideration.

3. Results and discussions

Now when we have fixed the electrostatic potential, $V_{ve}(R)$, and a convenient exchange-correlation-like repulsive term $\Delta v(R)$, we turn our attention to the determination of the bound
Figure 3. Binding energies as a function of the density parameter \( r_s \in [4, 30] \), for the case of \( [V_{ee}(R) + \Delta v(R)] \) and \( \mu = \frac{1}{2} \). Solid curve refers to \( \alpha = 0 \), the short-dashed to \( \alpha = 0.25 \), the long-dashed to \( \alpha = 0.5 \), at \( \beta = 1/r_s \). The dotted curve refers to \( \alpha = 0 \) and \( \beta = 1/(\sqrt{2}r_s) \).

state energy \( E_b \) and corresponding wavefunction \( \psi(R) \). We remind readers that we use the \( \beta = 1/r_s \) prescription, which corresponds to a complete (maximal) depletion of the electronic charge at \( R = 0 \). The reduced mass is \( \mu = \frac{1}{2} \) in the case of electron–electron interaction.

Briefly, we solve numerically the Schrödinger equation

\[
\left[ -\frac{1}{2\mu} \nabla^2 + \Lambda V(R) - E_b \right] \psi(R) = 0,
\]

in 2D where the potential energy refers to \( [V_{ee}(R) + \Delta v(R)] \). For further details on the numerics, we refer to our earlier works [5, 7]. A coupling-strength \( \Lambda \) is introduced in the above equation in order to model a static macroscopic polarizability of an ionic environment. Note that Simon’s theorem treats the above-introduced \( \Lambda V(R) \) with \( \alpha = 0 \) only. Thus, the case of \( \alpha > 0 \) is, even from purely mathematical point of view, a delicate problem.

The numerical results for the binding energy (at \( \Lambda = 1 \)) are exhibited in figure 3, in order to show the quantitative role of \( \alpha \) in an additional repulsive \( \Delta v(R) \) term. The solid, short-dashed and long-dashed curves refer, respectively, to \( \alpha = 0 \), \( \alpha = 0.25 \) and \( \alpha = 0.5 \) at fixed \( \beta = 1/r_s \). The dotted curve is based on the reduced \( \beta = 1/(\sqrt{2}r_s) \) at \( \alpha = 0 \), i.e. without an extra potential term. We note that the binding energy with a \( [V_{ee}(R) + \Delta v(R)] \) potential also diminishes gradually when \( \alpha \geq 0 \). This is in agreement with an earlier [20] result obtained at \( r_s = 4 \) by treating antiproton screening in the usual KS scheme for 2D.

Figure 4 contains the details (for \( \beta = 1/r_s \)) of the coupling constant dependence of binding energies with \( \Lambda [V_{ee}(R) + \Delta v(R)] \) and \( \alpha = 0 \). Three values are used, \( \Lambda = 1 \), 0.75 and 0.5. The lowering of binding efficiency is transparent in this figure also. We stress the point that since \( \alpha = 0 \) in this modeling of potential-weakening, the theorem of Simon mentioned at equation (4)
Figure 4. Binding energies as a function of the density parameter $r_s \in [0, 30]$, for the case of $\Lambda [V_{ee}(R) + \Delta v(R)]$ and $\mu = \frac{1}{2}$. Solid curve refers to $\Lambda = 1$, the short-dashed to $\Lambda = 0.75$, and the long-dashed one to $\Lambda = 0.5$. The other parameters are fixed as $\alpha = 0$ and $\beta = 1/r_s$ for all curves.

is still applicable. Note that the solid curve in figures 3 and 4 shows a characteristic $|E_b| \sim n_0^{1/2}$ dependence at low densities.

The radial densities $2\pi R |\psi(R)|^2$ are exhibited in figure 5, at a fixed density parameter ($r_s = 12$, $\alpha = 0$ and $\beta = 1/r_s$) and those $\Lambda$ parameters that were used in figure 4. The lowering of binding efficiency by decreasing $\Lambda$ appears as shape-change in bound-state densities, but they are still fairly well localized at around the negative minima ($R \simeq r_s$) of the potential; the coherence length is, therefore, small. All curves show the role of strong repulsion at small $R$, which result in a p-like character of the bound-state wavefunctions.

The above quantitative treatment is based on the Schrödinger equation for the bound state. In the rest of this section, we shall apply a standard field-theoretic treatment [10, 12] on the Cooper (opposite momenta) channel. In short, in the temperature-dependent formalism of the two-particle Green function for a normal Fermi system, the energy of the bound state of the pair appears as a pole of the total pairing vertex function.

To solve the Cooper problem, one has to expand the irreducible ($\Gamma(q) \sim V_{ee}(q)$, as in vacuum, at the level of the first-order Born approximation) interaction in the eigenfunctions of the angular momentum [10, 12], at the Fermi surface with zero total momentum. Under this procedure, the integral equation for the pairing ($\tilde{\Gamma}$) vertex decouples to a set of algebraic equations for its partial component $\tilde{\Gamma}_l$. In each equation, a partial component of $\tilde{\Gamma}_l$ is coupled to a partial component of $\Gamma_l$ as given [3, 10] below

$$\tilde{\Gamma}_l = \frac{\Gamma_l}{(1 + v_{\Gamma, l})}. \tag{12}$$

In this equation $v = (1/2\pi) \ln (E_F/k_B T)$ at vanishing excitation energy at the Fermi surface [3] of the 2D system. The decoupling in the Cooper channel has [12] a strong consequence; the
liquid state is unstable against pairing if there is an attraction even for a single $\Gamma_l$, at which the denominator of the above equation becomes zero.

With our residual interaction as irreducible vertex, the important $l$-dependence in the above pairing vertex is determined $[\Gamma_l = (1/2k_F)\gamma_l(d)]$ by the integrals

$$\gamma_l(d) = \int_0^{\pi} d\phi \frac{\cos (l \phi)}{\sin(\phi/2)} \left[ 1 - e^{-2d\sin^2(\phi/2)} \right],$$

in which $d = 1$ for complete ($\beta = 1/r_s$) depletion ($d$) and $d = 2$ at reduced (see the discussion at equation (7)) depletion, i.e. with $\beta = 1/(\sqrt{2}r_s)$. At $d = 1$, we obtain from equation (13) $\gamma_0 \simeq 1.4374$, $\gamma_1 \simeq -0.6053$ and $\gamma_2 \simeq -0.1546$. The next term is almost negligible since $\gamma_3 \simeq 0.0064$. At $d = 2$, we have $\gamma_0 \simeq 2.3969$, $\gamma_1 \simeq -0.6681$ and $\gamma_2 \simeq -0.4715$.

If several $\gamma_l < 0$, the transition to a superfluid occurs [10, 12] at that temperature which corresponds to the largest $|\gamma_l|$ value, i.e. the strongest attraction. Since, for the two $d$ values investigated, the leading $\gamma_1$ is quite similar, the similarity of the dotted and solid lines in figure 3 is reasonable. For both $d$ values, the weight of the backward ($\phi = \pi$) limit is very robust in the $l = 1$ term, due to the special $q$-dependence of our $V_{ee}(q)$. For higher $d$ values, i.e. at an even more reduced depletion (which might be governed [23] by the ionic aspects, not treated here), we can have $|\gamma_2| > |\gamma_1|$ where both $\gamma_l$ are negative. In fact, by a simple variation of the $d$ parameter, we can obtain the $\gamma_1 = -0.691$, $\gamma_2 = -0.719$ and $\gamma_3 = -0.720$ limiting values. They are similar for the important $l \in [1, 3]$ momenta.

The critical temperature ($T_c$) is determined, as we mentioned above, by the denominator of equation (12) and it is given ($E_F = 1/r_s^2$ and $k_F = \sqrt{2}/r_s$) by the following expression:

$$k_B T_c = \frac{1}{r_s^2} \exp \left( -\frac{4\sqrt{2}\pi}{r_s|\gamma_1|} \right),$$

Figure 5. Radial densities $2\pi R|\psi(R)|^2$ at $r_s = 12$, and $\Lambda = 1$ (solid curve), $\Lambda = 0.75$ (short-dashed curve), $\Lambda = 0.5$ (long-dashed curve). The other parameter is fixed as $\alpha = 0$ to all curves.
in the investigated $d = 1$ and $d = 2$ cases. The maximum $T_c$ is at around $r_s \approx 13$–15, where the smaller value for the density parameter refers to the $d = 2$ case; cf figure 3. At $r_s = 14$, we obtain from the above equation an about $T_c \approx 180$ K maximum value. Finally, using \[ \Delta = |E_b| \] for the superconducting gap $\Delta$ and our binding energy (solid curve in figure 3) at $r_s = 14$, we obtain an about $2\Delta/(k_B T_c) \approx 9$ ratio. Of course, the precise determination of the gap requires the consideration of Gorkov’s Green function also for the superconducting state \[ 12 \].

A way along this line for the thermodynamics of the system could be the application of methods developed earlier \[ 24 \] in 3D by using the $K$-matrix (reactance matrix) interpretation \[ 25 \] of a renormalized interparticle interaction.

4. Summary

We have discussed the possibility of intrinsic \[ 9 \] pairing of two electrons in a 2D electron gas using two complementary approaches. Namely, the bound-state Schrödinger equation and the Bethe–Salpeter ladder-solution for the scattering amplitude in the Cooper channel are used with the same effective interaction. The basis of this interaction is the physical charge-depletion around an electron. We estimate it in a reasonable way, which is valid also within the important low-density range. However, the most important ingredient, in agreement with the forecast \[ 7,18 \], in the existence of the bound state for the two electrons is the oscillating character of the effective interelectron potential. It is this important character that results in binding in both approaches. The study performed provides a unified picture of the rich phenomenology of strong correlation effects in reduced dimensions. As a future direction, we propose to investigate the spin-flip problem, as a possible channel for additional dressing, in order to incorporate such fluctuations in the present modeling of the residual interaction of quasi-particles of zero total momentum at the Fermi surface.

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References

[1] Giuliani G F and Vignale G 2005 Quantum Theory of the Electron Liquid (Cambridge: Cambridge University Press)
[2] Migdal A B 1977 Qualitative Methods in Quantum Theory (London: Benjamin)
[3] Saraga D S, Altshuler B L, Loss D and Westervelt R M 2005 Phys. Rev. B 71 045338
[4] Ghazali A and Gold A 1995 Phys. Rev. B 52 16634
[5] Nagy I, Puska M J and Zabala N 2006 Phys. Rev. B 74 115411
[6] Corona M, Gori-Giorgi P and Perdew J P 2004 Phys. Rev. B 69 045108
[7] Nagy I, Puska M J and Zabala N 2007 Phys. Rev. B 75 233105
[8] Echenique P M, Nagy I and Arnau A 1989 Int. J. Quantum Chem. 23 521
[9] Kohn W and Luttinger J M 1965 Phys. Rev. Lett. 15 524
[10] Chubukov A V 1993 Phys. Rev. B 48 1097
[11] Galitski V M and Sarma S D 2003 Phys. Rev. B 67 144520
[12] Lifshitz E M and Pitaevskii L P 1996 Statistical Physics (New York: Pergamon) part 2
[13] Kohn W 1999 Rev. Mod. Phys. 71 1253
[14] Becke A D 2003 J. Chem. Phys. 119 2972
[15] Drumond N D and Needs R J 2009 Phys. Rev. B 79 085414
[16] Simon B 1976 Ann. Phys. (NY) 97 279
[17] Mermin N D 1967 Phys. Rev. 159 161
[18] Nagy I 1999 Phys. Rev. B 60 4404
[19] Takada Y 1993 Phys. Rev. B 47 5202
[20] Zaremba E, Nagy I and Echenique P M 2005 Phys. Rev. B 71 125323
[21] Landau L D and Lifshitz L P 1958 Quantum Mechanics (Reading, MA: Addison-Wesley)
[22] Gravel S and Ashcroft N W 2007 Phys. Rev. B 76 144103
[23] Leys F E and March N H 2003 J. Phys. A: Math. Gen. 36 5893
[24] Brueckner K A, Soda T, Anderson P W and Morel P 1960 Phys. Rev. 118 1442
[25] Anderson P W 2007 Science 317 1705