Intrinsic Dispersion of Correlations among $E_p$, $L_p$, and $E_{iso}$ of Gamma Ray Bursts depends on the quality of Data Set

R. Tsutsui and T. Nakamura

Department of Physics, Kyoto University, Kyoto 606-8502, Japan
tsutsui@tap.scphys.kyoto-u.ac.jp

D. Yonetoku and T. Murakami

Department of Physics, Kanazawa University, Kakuma, Kanazawa, Ishikawa 920-1192, Japan

K. Takahashi

Department of Physics and Astrophysics, Nagoya University, Fro-cho, Chikusa-ku, Nagoya, 464-8602, Japan

ABSTRACT

We reconsider correlations among the spectral peak energy ($E_p$), 1-second peak luminosity ($L_p$) and isotropic energy ($E_{iso}$), using the database constructed by Yonetoku et al. (2010) which consists of 109 Gamma-Ray Bursts (GRBs) whose redshifts are known and $E_p$, $L_p$ and $E_{iso}$ are well determined. We divide the events into two groups by their data quality. One (gold data set) consists of GRBs with peak energies determined by the Band model with four free parameters. On the other hand, GRBs in the other group (bronze data set) have relatively poor energy spectra so that their peak energies were determined by the Band model with fixed spectral index (i.e. three free parameters) or by the Cut-off power law (CPL) model with three free parameters. Using only the gold data set we found the intrinsic dispersion in log $L_p$ ($=\sigma_{int}$) is 0.13 and 0.22 for $E_p$–$T_L$–$L_p$ correlation ($T_L \equiv E_{iso}/L_p$) and $E_p$–$L_p$ correlation, respectively. We also find that GRBs in the bronze data set have systematically larger $E_p$ than expected by the correlations constructed with the gold data set. This means that the intrinsic dispersion of correlations among $E_p$, $L_p$, and $E_{iso}$ of GRBs depends on the quality of data set. At present, using $E_p$–$T_L$–$L_p$ correlation with gold data set, we would be able to determine the luminosity distance with $\sim 16\%$ error, which might be useful to determine the nature of the dark energy at high redshift $z > 3$. 

Not to appear in Nonlearned J., 45.
Subject headings: (cosmology:) distance scale—(stars:) gamma-ray burst: general

1. Introduction

Discoveries of empirical correlations of gamma-ray bursts (GRBs) raised many researches on early universe using GRBs. One of the most well studied correlations is the one between the spectral peak energy ($E_p$) and isotropic equivalent energy ($E_{iso}$) called $E_p$–$E_{iso}$ correlation [Amati et al. 2002; Sakamoto et al. 2004; Lamb et al. 2004; Amati et al. 2006, 2009]. Yonetoku et al. (2004) found a similar but tighter correlation between $E_p$ and 1-second peak luminosity called the $E_p$–$L_p$ correlation. These correlations are tight but they have large dispersions such as $\sigma_{int} = 0.33$ in log $L_p$ and $\sigma_{int} = 0.37$ in log $E_{iso}$ which cannot be explained as statistical errors of $E_p$, $E_{iso}$ and $L_p$ (Yonetoku et al. 2010). Ghirlanda et al. (2004) found that $E_p$ tightly correlates with the collimation-corrected gamma-ray energy ($E_\gamma$). Firmani et al. (2006) proposed that adding the high signal time scale ($T_{0.45}$) to the $E_p$–$L_p$ relation reduces the dispersion of the correlation. This correlation is defined by using only prompt emission properties like $E_p$–$E_{iso}$, $E_p$–$L_p$ correlations so that it seems to be promising tools to constrain the cosmological parameters. However, this correlation is not confirmed by later studies (Rossi et al. 2008; Collazzi & Schaefer 2008). More recently, Tsutsui et al. (2009) found that the luminosity time ($T_L = E_{iso}/L_p$) also improves both the $E_p$–$E_{iso}$ and $E_p$–$L_p$ correlations.

These correlations were used to investigate the star formation history (Yonetoku et al. 2004), the reionization epoch (Murakami et al. 2005), and the cosmological expansion history of the early universe (Takahashi et al. 2003; Oguri & Takahashi 2006; Ghirlanda et al. 2006; Schaefer 2007; Kodama et al. 2008; Liang et al. 2008; Cardone et al. 2009; Tsutsui et al. 2009).

However, in spite of high correlation coefficients, there have been many cautions to use these empirical correlations for cosmology (Nakar & Piran 2004; Band & Preece 2005; Butler et al. 2007; Shahmoradi & Nemiroff 2009). To establish these correlations in GRBs prompt emissions as tools to determine cosmological parameters, we must investigate the origins of systematic errors and the way to remove them. We note that there are many factors to cause systematic errors besides intrinsic dispersions of their prompt emissions. For example the sensitivity of the detectors, the evolution effects of GRBs, the confusion with other sources, the lack of unknown parameters like the jet opening angle $\theta_{jet}$, etc. All of these effects might arise the additional systematic errors over the intrinsic dispersions of GRBs.
Possible selection effects on these correlations are studied by many authors with contrasting results (Butler et al. 2007; Ghirlanda et al. 2008; Nava et al. 2008; Shahmoradi & Nemiroff 2009; Amati et al. 2009; Yonetoku et al. 2010). However previous studies did not consider the difference of spectral models to determine \( E_p \) well. As shown in Kaneko et al. (2006), it often happens that high energy power-law index \( \beta \) for the Band model with four free parameters can not be determined by the data so that the cutoff power-law (CPL) model with three free parameters is used to fit the data. CPL model might be good if the peak energy is close to the high energy end of the detector band width. One can not use Band model but CPL model if the event is so dim that the number of high energy photons is very small. Importantly, simulations in Kaneko et al. (2006) showed that, if the signal-to-noise ratio is relatively low, a true spectrum with the shape of the Band model can be fitted by CPL model with \( E_p^{\text{obs}} \) which is larger than the true value of \( E_p^{\text{obs}} \) up to \( \sim 100 \) keV. Therefore CPL model might overestimate \( E_p \). While, if we fit a true CPL spectrum by the Band model, the estimated value of \( E_p^{\text{obs}} \) is almost equal to the true value since the large value of \(-\beta\) looks like an exponential function. In reality Shahmoradi & Nemiroff (2009) found that \( E_p \) estimated using the CPL model by Kaneko et al. (2006) are systematically harder than \( E_p \) estimated using the Band model by Yonetoku et al. (2004). Although the systematic difference between the peak energies fitted by the Band model and the ones fitted by the CPL model are reported, how this difference affect the spectral-brightness correlations of GRBs has been hardly studied so that we shall study this problem in this paper.

The purpose of this letter is to investigate the effect of uncertainty in using different spectral models which determine \( E_p \) on the \( E_p-T_L-L_p \), \( E_p-L_p \) and \( E_p-E_{\text{iso}} \) correlations, using our database developed in Yonetoku et al. (2010). We examine this model bias by dividing the samples into two data sets as gold and bronze according to the quality of spectral observation. In this paper, we assume, if signal-to-noise ratio is high enough, all of the spectrum of GRBs are well expressed by the Band function.

The structure of this letter is as follows. First we describe our database of 109 GRBs with known redshift and well-determined spectral parameters, 1-second peak luminosity, and isotropic energy in section 2. We construct the \( E_p-T_L-L_p \), \( E_p-L_p \), and \( E_p-E_{\text{iso}} \) correlations with only gold data set in section 3. Finally we will give summary in section 4.

2. Data Description

In Yonetoku et al. (2010), we constructed a database selecting 109 GRBs from GCN Circular Archive (Barthelmy 1997) and GRBlog (Quimby et al. 2003). In this section we briefly describe our database.
Let us begin with $E_p$. In many cases, the prompt gamma-ray spectrum is well fitted with the spectral model of the exponentially-connected broken power-law function suggested by Band et al. (1993). This Band function has four parameters, the low-energy photon index $\alpha$, the high-energy photon index $\beta$, the spectral break energy $E_0$ and the normalization $A$. The peak energy ($E_p$), at which the flux is maximum in the $\nu F_{\nu}$ spectrum, can be calculated as $E_p = (2 + \alpha)E_0$.

However, for some GRBs, the photon index (mostly $\beta$) cannot be determined due to the limited energy range of the detector and/or the lack of the number photons (Pendleton et al. 1997). When the observation of high-energy range is not enough, the spectrum is sometimes fitted with the Cut-off power law (CPL) function. This function has three parameters, the low-energy photon index $\alpha$, the spectral break energy $E_0$ and the normalization $A$. In this case the peak energy can be derived as $E_p = (2 + \alpha)E_0$. Note that even if, for a given GRB spectrum, the reduced chi square value of this model is smaller than that of the Band function, it is difficult to say whether this model reflects the intrinsic property of the GRB or it is just due to the poor statistics in the high-energy range. The reported values of $E_p$ for GRBs which were poorly observed in the high-energy range are based on either the Band function or CPL function, depending on the observation team so that there exists the ambiguity in the definition of $E_p$ from the beginning.

Let us move on to $E_{iso}$, $L_p$ and $T_L$. In Yonetoku et al. (2010), we calculated the bolometric energy and the peak luminosity in the energy range 1-10,000 keV in the rest frame of each GRB by extending the observed spectrum. Here, it should be noted that the integration was performed assuming the Band function even for GRBs whose spectra were not fitted by the Band function and the photon indices were not reported. In these cases we assumed the typical values $\alpha = -1$ and $\beta = -2.25$ to calculate the bolometric fluence ($S_{bol}$) and the bolometric peak flux $F_{p,bol}$. These values are suggested by BATSE observations (Preece et al. 2000) and also supported by Fermi observations of GRB 080916C, 081024B, 090323 and 090428 up to possibly 100 GeV energy range. Then the bolometric isotropic energy ($E_{iso}$) and the 1-second peak luminosity ($L_p$) can be simply calculated as $E_{iso} = 4\pi d_L^2 S_{bol}/(1 + z)$ (erg), and $L_p = 4\pi d_L^2 F_{p,bol}$ (erg s$^{-1}$). Here, $d_L$ is the luminosity distance calculated for the flat universe with the cosmological parameters of $(\Omega_m, \Omega_\Lambda) = (0.3, 0.7)$ and the Hubble parameter of $H_0 = 70$ km s$^{-1}$Mpc$^{-1}$. Further we define the luminosity time as the third parameter of GRB prompt emission as $T_L \equiv E_{iso}/L_p$. The error of the luminosity time is estimated by using error propagation equation. We can neglect the crossterm between $L_p$ and $E_{iso}$ because of the independence of the $E_p - E_{iso}$ and $E_p - L_p$ relation shown in Tsutsui et al. (2009).

Thus, for GRB whose observed photon number is small, there are two possible systematic effects. One comes from the fact that the peak energy $E_p$ is determined by fitting the
spectrum with either the Band function or CPL function. As Kaneko et al. (2006) pointed out that the CPL function tends to overestimate $E_p$ compared to the Band function. This would induce a systematic error in the correlations related to $E_p$. On the other hand, although $L_p$ and $E_{iso}$ are determined in a single straightforward way, the photon indices are set to the typical values if the number of detected photons is small. This would also cause a systematic error.

To estimate these systematic errors, we call a certain GRB belongs to the gold data set if its spectrum is well observed so that it is fitted by the Band function quite well and all four parameters are accurately determined. Other GRBs for which the fixed $\alpha$ and/or $\beta$ are allocated to bronze data set. Here 3 of 109 GRBs in the database of Yonetoku et al. (2010), do not have $E_{iso}$ and then $T_L$ so they are included in neither the gold nor the bronze data sets. As a result, the number of the gold and bronze data sets are 41 and 65 GRBs, respectively.

In the following sections, we construct the $E_p-E_{iso}$, $E_p-L_p$ and $E_p-T_L-L_p$ relations for the gold data set. It is expected that the correlations obtained from the gold data set would suffer from relatively small systematic errors so that we could study real intrinsic dispersions of the correlations.

3. correlations

Here we derive the $E_p-T_L-L_p$, $E_p-L_p$, and $E_p-E_{iso}$ correlations from the gold data set.

First we assume the correlation among $E_p$, $T_L$ and $L_p$ to be of the form, $\log L_p \equiv A + B \log (E_p/440 \text{ keV}) + C \log (T_L/4.70 \text{ s})$, where we take the denominator of the second and third terms as the average value of $E_p$ and $T_L$ in gold data set to minimize the statistical errors for these correlations. For this correlation, we found seven outliers (980425, 980613, 000131, 090328, 091003, 091020, 091127) shown by blue color in top left of Fig.1. The seven outliers deviate from the best-fit relation at more than $3\sigma$ dispersion level. We will give some arguments on this point in section 4. Using 34 gold data set of GRBs, we calculate the best fit function shown by the solid black line in top left of Fig. 1 with red points of the gold data set and $3\sigma$ errors of the $E_p-T_L-L_p$ relation by the yellow color region. The functional form of the best fit function is given by

$$L_p = (52.64 \pm 0.03) \times \left(\frac{E_p}{440\text{keV}}\right)^{1.70\pm0.07} \left(\frac{T_L}{4.70\text{sec}}\right)^{-0.40\pm0.06}. \quad (1)$$

Here, we include not only errors in $L_p$ but also errors in $E_p$ and $T_L$ so that the chi-square function is defined as

$$\chi^2(A, B, C) = \Sigma (\log L_p^{\text{obs}} - A - B \log (E_p/440 \text{ keV}) - C \log (T_L/4.70 \text{ s}))^2/(\sigma_{\text{meas}}^2$$
where the first term of weighting factor is \( \sigma_{\text{meas}}^2 = (1 + 2C)\sigma_{\log L_p}^2 + (B\sigma_{\log E_p})^2 + (C\sigma_{\log T_L})^2 \). The factor 2C in front of \( \sigma_{\log L_p}^2 \) comes from the fact that the definition of \( T_L \) includes \( L_p \). The reduced chi-square is unity with the intrinsic dispersion \( \sigma_{\text{int}} = 0.13 \). This correlation is consistent with our previous study Tsutsui et al. (2009).

Similarly, we can obtain the best-fit function and errors of the \( E_p-L_p \) (top right of Fig.1) and \( E_p-E_{\text{iso}} \) (bottom of Fig.1) correlations for the same 34 gold GRBs,

\[
L_p = (52.63 \pm 0.05) \times \left( \frac{E_p}{440 \text{ keV}} \right)^{1.76\pm0.10},
\]

with the intrinsic dispersion \( \sigma_{\text{int}} = 0.22 \), and

\[
E_{\text{iso}} = (53.31 \pm 0.06) \times \left( \frac{E_p}{440 \text{ keV}} \right)^{1.68\pm0.13},
\]

with the intrinsic dispersion \( \sigma_{\text{int}} = 0.31 \). The best fit values of the \( E_p-L_p \) and \( E_p-E_{\text{iso}} \) correlations are consistent with previous studies but the intrinsic dispersions are tighter than those in Yonetoku et al. (2010) in which both gold and bronze data are used.

The values of \( \sigma_{\text{int}} \) suggest that \( E_p-T_L-L_p \) correlation is tightest among three correlations. In Fig. 1, we show the gold data set (red points) with the best-fit function (solid line) and 3-sigma dispersion region (dotted lines). We see that the \( E_p-T_L-L_p \) correlation is much tighter than the \( E_p-L_p \) and \( E_p-E_{\text{iso}} \) correlations by eye also. The blue points and green points indicate seven outliers of the \( E_p-T_L-L_p \) correlation and bronze data set, respectively. The bronze data set are systematically harder and/or dimmer than the gold data set. This difference causes a large dispersion in addition to the intrinsic dispersion of the correlations if we include the bronze data in the analysis.

4. Summary & Discussion

In this paper, using database constructed by Yonetoku et al. (2010), we examine the model bias, that is, Band or CPL, on \( E_p-T_L-L_p \), \( E_p-L_p \) and \( E_p-E_{\text{iso}} \) correlations. We found that GRBs with the peak energies fitted by the CPL model are distributed in systematically harder and/or dimmer side of the \( E_p-T_L-L_p \), \( E_p-L_p \) and \( E_p-E_{\text{iso}} \) correlations than the ones by the Band function. There might be two interpretations about this result. The first is that these correlations have much larger intrinsic dispersion than that of observed one. If we had the more sensitive detector and could observe dimmer GRBs, the dispersion of the relations would become larger (Butler et al. 2007; Shahmoradi & Nemiroff 2009). Another is that the use of the CPL model to estimate the peak energies causes this systematic difference. As
Fig. 1.— The $E_p - T_L - L_p$ relation (top left), the $E_p - L_p$ correlation (top right) and the $E_p - E_{iso}$ (bottom) correlation with all data set. The solid line and dotted lines indicate the best fit function and 3-σ dispersion region in Eq (1), Eq (2) and Eq (3). The bronze data set seems to be harder and/or dimmer than gold data set. The CPL model or the Band function with fixed power-law index cause this systematic difference.
simulated by Kaneko et al. (2006), the Band function spectrum is well fitted by the CPL model if the detector does not have enough sensitivity to observe the high-energy photons. However the peak energies fitted by the CPL models are always higher than that of the simulated Band function spectrum (see table 3 in Kaneko et al. 2006). Thus, it seems to be natural to conclude that the latter is more acceptable. In short, using only the peak energies determined by the Band function, we would get tighter correlations. If we could have much more GRBs by which we can uniformly analyze the data with the Band function, GRBs would be more powerful tool to constrain cosmological parameters.

We found seven outliers in our gold data set. We classify these outliers in two classes as

(dimmer and/or harder) 980425, 980613, 090328, 091003
(brighter and/or softer) 000131, 091020, 091127

Although we do not know how and why these outliers are different from ordinary GRBs except for the distribution in the \( E_p - T_L - L_p \) space, the effect of eliminating these GRBs is obvious, that is, the correlation becomes tighter. To find the characteristics which distinguish these outliers from ordinary GRBs is urgent. We here point out possible origins of these outliers. Let us assume that if we observe the jet nearly on axis, \( E_p - T_L - L_p \) correlation would be very tight. However if we observe the jet with a certain viewing angle, we might have some dispersions on the observed \( E_p - T_L - L_p \) correlation. In other words, we might not avoid some dispersion in the \( E_p - T_L - L_p \) correlation from viewing angle, especially when the observer locates near the edge of the jet of GRBs. If we will know how to distinguish these outliers from the ordinary gold GRBs, the \( E_p - T_L - L_p \) correlation might be much tighter and very useful in determining the nature of dark energy in redshift larger than \( \sim 3 \). We should note that even in the Period-Luminosity relation of Cepheid variable there are \( \sim 10\% \) outliers (Riess et al. 2009) so it is not surprising that there are \( \sim 20\% \) outliers in the \( E_p - T_L - L_p \) relation.

Butler et al. (2007), using the Bayesian approach to estimate \( E_p \), indicated that dim events close to the detector sensitivity would make large scatter on the \( E_p - E_{iso} \) and \( E_p - L_p \) relations and that there is a significant threshold effect. Thus, they conclude that the \( E_p - E_{iso} \) correlation have larger intrinsic dispersion than observed if we do not suffer from a threshold effect. Recently, Shahmoradi & Nemiroff (2009) argue that using hardness ratio instead of \( E_p \) they also find that \( E_p - \text{Fluence} \) correlation become more wider if we will be able to determine \( E_p \) of dimmer events. However, there is a possible bias by using different method to estimate \( E_p \). Even the difference of \( E_p \) between the Band and CPL models causes the systematic errors so that using the other method to estimate \( E_p \) might cause the additional systematic errors. The smaller the intrinsic dispersion of the relation is, the more the correlation suffers
from these systematic effects. This might be why the $E_p - T_{0.45} - L_p$ relation is not confirmed by later studies (Firmani et al. 2006; Rossi et al. 2008; Collazzi & Schaefer 2008).

Kaneko et al. (2006) suggested the $E_p$ value of CPL function becomes systematically higher than the one of the Band function. If the 65 bronze data previously analyzed by the CPL function are reconsidered by the Band function with the fixed $\beta$ as an average value of $-2.25$ (Preece et al. 2000), they might show the distribution around the best fit line of each correlation estimated with 41 gold data set. They have a good potential to become a "silver" data set. To do so, we need help from each instrument team, and this is a future work.

Finally, we note that there would be many reasons which cause systematic errors on the correlation in addition to intrinsic property of GRBs. These systematic errors must be carefully estimated and removed from the correlation analysis one by one. If we will finish it, GRBs become more powerful and unique standard candles to investigate the nature of the dark energy at high redshift larger than $\sim 3$.

Acknowledgments

This work is supported in part by the Grant-in-Aid from the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan, No.19540283, No.19047004(TN), No.18684007 (DY) and No.21840028(KT), and by the Grant-in-Aid for the global COE program The Next Generation of Physics, Spun from Universality and Emergence at Kyoto University and ”Quest for Fundamental Principles in the Universe: from Particles to the Solar System and the Cosmos” at Nagoya University from MEXT of Japan. RT is supported by a Grant-in-Aid for the Japan Society for the Promotion of Science (JSPS) Fellows and is a research fellow of JSPS.

REFERENCES

Amati, L., et al., 2002, A&A, 390, 81
Amati, L., 2006, MNRAS, 372, 233
Amati, L., Frontera, F., & Guidorzi, C. 2009, A&A, 508, 173
Band, D.L., Matteson, J., Ford, L., et al. 1993, ApJ, 413, 281
Band, D. L., & Preece, R. D. 2005, ApJ, 627, 319
Barthelmy, S., 1997, GCN Circulars Archive, [http://gcn.gsfc.nasa.gov/gcn_main.html](http://gcn.gsfc.nasa.gov/gcn_main.html)

Butler, N. R., Kocevski, D., Bloom, J. S., & Curtis, J. L. 2007, ApJ, 671, 656

Cardone, V. F., Capozziello, S., & Dainotti, M. G. 2009, MNRAS, 400, 775

Collazzi, A. C., & Schaefer, B. E. 2008, ApJ, 688, 456

Cutini, S., Vasileiou, V., & Chiang, J. 2009, GRB Coordinates Network, 9077, 1

Firmani, C., Ghisellini, G., Avila-Reese, V., & Ghirlanda, G. 2006, MNRAS, 370, 185

Ghirlanda, G., Ghisellini, G., & Firmani, C. 2005, MNRAS, 361, L45

Ghirlanda, G., Ghisellini, G., & Firmani, C. 2006, New Journal of Physics, 8, 123

Ghirlanda, G., Nava, L., Ghisellini, G., et al., 2008, MNRAS, 387, 319

Ghirlanda, G., Nava, L., Ghisellini, G., et al., 2009, A&A, 496, 585

Kaneko, Y., Preece, R. D., Briggs, M. S., Paciesas, W. S., Meegan, C. A., & Band, D. L. 2006, ApJS, 166, 298

Kodama, Y. et al., 2008, MNRAS, 391, L1

Krimm, H. A., et al. 2009, ApJ, 704, 1405

Lamb, D. Q. et al., 2004, New Astron. Rev. 48, 423 ([astro-ph/0309462](http://arxiv.org/abs/astro-ph/0309462))

Liang, N., Xiao, W. K., Liu, Y., & Zhang, S. N. 2008, ApJ, 685, 354

Murakami, T., et al., 2005, ApJ, 625, L13

Nakar, E., & Piran, T. 2005, MNRAS, 360, L73

Nava, L., Ghirlanda, G., Ghisellini, G., & Firmani, C. 2008, MNRAS, 391, 639

Oguri, M., & Takahashi, K., Phys. Rev. D, 2006, 73, 123002

Preece, R. D., Briggs, M. S., Mallozzi, G. N., et al. 2000, ApJS, 126, 19

Pendleton, G. N., et al. 1997, ApJ, 489, 175
Quimby, R., McMahon, E., Murphy, J. 2003, GRBlog, arXiv:astro-ph/0312314v1
Rau, A., Connaughton, V., & Briggs, M. 2009, GRB Coordinates Network, 9057, 1
Riess, A. G., et al. 2009, ApJ, 699, 539
Rossi, F., et al. 2008, MNRAS, 388, 1284
Sakamoto, T. et al., 2004, ApJ, 602, 875
Schaefer, B. E., Deng, M. & Band, D. L. 2001 ApJ, 563, L123
Schaefer, B. E. 2007, ApJ, 660, 16
Shahmoradi, A., & Nemiroff, R. J. 2009, arXiv:0904.1464
Takahashi, K., et al., arXiv:astro-ph/0305260
Tsutsui, R., et al., 2009, JCAP, 8, 15
Yamazaki, R., Ioka, K., & Nakamura, T. 2004, ApJL, 606, L33
Yonetoku, D., et al., 2004, ApJ, 609, 935
Yonetoku, D., et al., 2010, PASJ, 62, 1495

This preprint was prepared with the AAS \LaTeX{} macros v5.2.