Measuring Probability of Failure of Thermoelectric Legs through Lognormal and Weibull Distribution

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Abstract. Thermoelectric devices are potential candidates to recover energy from dissipated waste heat and are environmentally friendly. It produces energy through temperature differences. Regarding to its fundamental characters, heat plays a significant role for effective energy conversion. At the same moment, creating high-performance thermoelectric generators at higher temperature with a reliable operational life is one of the main challenges. When the temperature of the device goes up, in order to get maximum output, the device encounters stress at his edges and within volume. This whole phenomenon makes calculation of reliability complex, especially measuring the operational life of thermoelectric device. In this paper TE materials composed of Bismuth Telluride were studied. Stress data was collected from different experiments and analyzed through Lognormal and Weibull distribution in order to calculate probability of failure of TE legs. The simulation was done on MATLAB.

Introduction

One of the potential candidates which converts heat into electricity, is thermoelectric (TE) conversion technology. Since it has no moving part, it has simple operating principle and it has wide range of applications for harvesting energy. TE conversion technology is successfully being used in space technologies [1], remote areas to monitor valuable instruments [2] and industries to harvest energy from waste heat [3]. Different experimental studies [4] have mentioned the influence of thermomechanical stresses on performance and reliability of TE device [5].

To construct a qualitative reliability model for thermoelectric devices, we calculated failure probability of different samples. For the probability distribution we utilized Weibull and lognormal distribution. The Weibull distribution is most widely being used in ceramic and glass industry for developing probabilistic tensile strength of material [6]. At lower temperature, Weibull model is relative, and failure can be defined but becomes less accurate when temperature increases [7].

Change in stress during excessive loading or temperature are subject of fatigue failure and vary from static loading or temperature [8]. As an alternative to Weibull distribution, lognormal is also widely used, especially when the stress varies continuously [9]. We discussed both methods to present adequate probability of failure based on the data. Data is compiled in MATLAB and results are shown in Figure 3 and 4.

Experiment Schemes

To study the strength properties of each material and analyze their role in device’s reliability,
series of experiments were conducted on unsegmented TE leg with the composition of Bi$_2$Se$_3$-Bi$_2$Te$_3$ for n-type leg and Sb$_2$Te$_3$-Bi$_2$Te$_3$ for p-type leg. The dimensions of each p and n-type leg vary during different experiments and are mentioned in each Figure. During the experiments the thermoelectric legs are bent, parallelly and transversely, compressed and stretched to attain results for comprehensive mechanical characterization of each leg.

![Diagram](image1)

**Figure 1.** (a) Transverse and (b) parallel are bending test schemes

We measured proportionality, yield, tensile strength, as well as deformation properties of each material. The deformation properties were measured by considering relative uniform elongation, elongation after rupture and relative narrowing after rupture. Transverse and Parallel loading scheme for bending stress calculation is shown in Figure 1(a) & (b). Bending stress at p-type leg is little bit lower during parallel loading as compare to transverse loading. Consequently, the tensile strength, scheme shown in Figure 2 (a) & (b), of n-type leg is approximately 1.5 times higher than p-type leg. Therefore, we can conclude that the shear stresses arising during transverse load reduces the tensile strength in bending, especially for n-type legs.

![Diagram](image2)

**Figure 2** (a) Compressive and (b) Tensile Stress test scheme
The compression test is carried out at two different temperatures i.e. 20 ºC and 80 ºC, while for tensile stress varying load was applied. The varying temperature during compression test shows us that material’s stability at higher temperature is less as compare to low temperature. Whereas, tensile strength of bismuth telluride during compression was 2 times less than its tensile strength during bending test.

**Probability of Failure**

Probability of failure is calculated through Weibull and Lognormal distribution through MATLAB. Stress was set as main random variable and the values of stresses are obtained from tensile, bending and compression tests.

Figure 3 shows Weibull probability function to estimate failure of different TE leg during tensile, bending and compression tests. The obtained data is distinguished through different lines and colors (blue, red and orange color respectively). Each line and color indicate stress value obtained during experiments. Values (as shown in Figure. 3a) of stress vary from 10 to 50 MPa with shape of 7.4 MPa and scale 25 MPa. Survival and hazard rate graph (3b & 3c) show high probability of survival of leg at tensile stress as compare to bending and compression stress. Whereas the probability plot (3d) shows the probability of failure of device as whole.

![Weibull’s Distribution Graphs](image)

Weibull’s Probability plot shown in figure 3d is relatively taken in regard of constant atmosphere temperature and static loading. In regard to static loading where only extreme load (force) will cause damage, figure 4a and 4b are presented but the differences are very minor observed. By observing figure 4 closely, each graph represents different range for probability. Figure 4a shows failure zone from 50 to 66 MPa, whereas failure zone in figure 4b starts from 45 MPA till 66 MPa. The raise in temperature, especially during compression test, Weibull manifest its effect very slightly. Here we introduced lognormal distribution, shown in figure 4b & 5, in order to see how the plot does vary according to dynamic and static loading. By plotting data at same graph sheet (figure 5), the probability plot presentation becomes much clear. Dynamic load (force) is continuous and varying with temperature. In this regard, Weibull has 50% of failure at 66 MPa, whereas lognormal gives 99% probability of failure at the same stress level.
Figure 4. (a) Probability distribution function of Weibull and (b) Probability distribution function of lognormal

Lognormal presents significantly precise expression of data because during experiments, most of legs encountered damage, crack or deformation at 60 (and above) MPa. In this regard, we noted stress and failure of leg at each level for two different sides i.e. hot and cold side.

The percentile failure of TE legs, shown in Figure 6, is expressed through lognormal distribution function. In this figure we have shown percentage of failure of legs at different temperatures. Since TE legs operate at different temperatures, we took this into account. The blue line shows cold temperature experiments and red shows hot temperature experiments. It was noticed that the legs are more vulnerable to failure at higher temperature as compare to lower temperature. Applicability of lognormal distribution in this phenomenon is valid due to fatigue effect on legs.
Figure 6. Lognormal distribution-based percentage failure of TE modules

Conclusion

The comparison between Weibull and lognormal distribution indicates the effect of stress level on reliability of TE devices. The experimental data shows us that the reliability of any module is proportionally dependent on stress-strength relationship. In general, we observed that the module fails when the stress exceeds the strength at certain temperature.

References

1. Zakrjasek J, Woerner D, and Fleurial J 2018 NASA Special Session: Next-Generation Radioisotope Thermoelectric Generator (RTG) Discussion. NASA, URL: https://rps.nasa.gov/resources/69/next-generation-radioisotope-thermoelectricgenerator-presentation [cited 15].
2. Champier D 2017 Ener. Conver. Manag. 140 167
3. LeBlanc S 2014 Sustain. Mater. Techn. 1 26
4. Barako M, et al. 2013 J Electron. Mater. 42 (3) 372
5. Erturun U, Erermis K, Mossi K 2014 Appl. Thermal Engin. 73 (1) 128
6. Karri N K and Mo C 2018 J Electron. Mater. 47 (10) 6101
7. Seal C, Sherry A 2016 Procedia Struct. Integ. 2 1668.
8. Murty A, Gupta U and Radha A 1995 Inter. J fatigue 17 (2) 85.
9. Nkemnole E B, Samiyu M A 2017 Amse J-Amse Heta 22(1) 77.