Spin torque switching of a nanostructured ferromagnet has been extensively studied because the switching probability in a thermally activated region provides us important information about spintronics devices such as the thermal stability of magnetic random access memory (MRAM). \(^1\)-\(^9\) Since the retention time of MRAM depends strongly on its thermal stability, an accurate estimation of the thermal stability is required. However, there is some controversy regarding the theoretical formula of the switching probability of an in-plane magnetized system. \(^10\)-\(^18\) The issue is the validity of the linear scaling of the current dependence of the switching barrier. Here, the switching barrier \(\Delta\) relates to the switching probability \(P\) through the switching rate \(v = e^{-\Delta}\) as \(P = 1 - e^{-vt}\), in which \(v\) and \(t\) are the attempt frequency and current pulse duration time, respectively. The thermal stability can be defined as the switching barrier in the absence of a current. In the analyses of the experiments, the switching barrier is assumed to be

\[
\Delta = \Delta_0 \left(1 - \frac{I}{I_0^c}\right)^b,
\]

where the thermal stability \(\Delta_0 = MH_{K}V/(2k_BT)\) depends on the magnetization \(M\), the uniaxially anisotropic field along the in-plane easy axis \(H_{K}\), the volume of the free layer \(V\), and the temperature \(T\). The current is denoted as \(I\) while \(I_0^c\) is the spin torque switching current at zero temperature, which the thermally activated region is defined as \(I < I_0^c\). The important point is that the switching exponent \(b\) in previous works \(^10\)-\(^14\) was assumed to be unity. Since those publications, linear scaling \((b = 1)\) has been widely used to analyze the spin torque switching experiments. \(^3\)-\(^9\) On the other hand, Refs. \(^16\) and \(^17\) argued that the switching exponent \(b\) depends on the current. The value of \(b\) is larger than unity in the relatively low-current region \(I \ll I_0^c\), and reaches almost square in the relatively high-current region, \(I \approx I_0^c\). It is important to clarify the value of the switching exponent \(b\) used to analyze the experiments because the value of \(b\) strongly affects the evaluation of the thermal stability. \(^19\)

In this letter, we studied the reason why the linear scaling \((b = 1)\) seemed to work well to analyze the spin torque switching experiments. First, we showed that when the current is small, Eq. (1) can be exactly rewritten as the switching barrier with linear scaling by introducing another scaling current \(\tilde{I}_0\). Second, we calculated the current dependence of the switching probability, and investigated the temperature and current pulse duration time regions in which linear scaling is valid. A comparison of the calculated values with the previous experiment is also discussed.

The current dependence of the switching barrier for spin torque switching of an in-plane magnetized ferromagnet was studied. Two scaling currents, \(I\) and \(I_0^c\) \((> I_0^c)\), were introduced to distinguish the magnetization stability. In the low-current region \(I < I_0^c\), the switching barrier is linear to the current with another scaling current \(\tilde{I}_0\), while such linear scaling does not hold in the high-current region \(I_0^c \leq I < I_0^c\). The linear scaling is valid for the high temperature and the long current pulse duration time. © 2013 The Japan Society of Applied Physics

![Fig. 1. Schematic view of an in-plane magnetized system. The z-axis is parallel to the in-plane easy axis of the free layer while the x-axis is normal to the film plane.](image-url)
average of the LLG equation. The explicit form of the switching barrier is

$$\Delta = \frac{V}{k_B T} \int_{E_s}^{E_c} dE \left(1 - \frac{\mathcal{M}_s}{\alpha \mathcal{M}_a}\right),$$

(4)

where $\mathcal{M}_s = \gamma^2 H_s \int d\mathbf{n} \cdot \mathbf{H}$ and $\mathcal{M}_a = \gamma^2 \int d\mathbf{H} \cdot (\mathbf{m} \times \mathbf{H})^2$ are the functions of the energy density $E$ of the free layer, and are proportional to the work done by spin torque and the energy dissipation due to the damping during a precession on the constant energy line, respectively. The upper boundary of the integral, $E_c$, corresponds to the saddle point ($\mathbf{m} = \pm \mathbf{e}_z$). On the other hand, the lower boundary of the integral, $E_s$, corresponds to the energy density at which the spin torque balances the damping. To discuss this point, the following two characteristic currents must be introduced:17,18)

$$I_c = \frac{2aeMV}{\hbar \eta} \left(\frac{H_K + 4\pi M}{2}\right),$$

(5)

$$I_s^* = \frac{4aeMV}{\pi \hbar \eta} \sqrt{4\pi M(H_K + 4\pi M)}.$$  

(6)

In a thin-film geometry ($H_K \ll 4\pi M$), $I_c^* \simeq 1.27I_c$. The physical meanings of $I_c$ and $I_s^*$ are that for $I > I_c$ the initial equilibrium state ($\mathbf{m} = \mathbf{e}_z$) becomes unstable while for $I > I_s^*$ the magnetization can switch its direction without the thermal fluctuation.17) In the following, we call the current regions $I < I_c$ and $I_c < I < I_s^*$ the low- and high-current regions, respectively. In the low-current region $I < I_c$, the damping overcomes the spin torque at the equilibrium. In this case, $E^*$ is the minimum of the energy density, $-M_HK/2$, corresponding to $\mathbf{m} = \pm \mathbf{e}_z$. Then, Eq. (4) can be rewritten as

$$\Delta(I < I_c) = \Delta_0 \left(1 - \frac{I}{I_c^*}\right).$$

(7)

Here, the scaling current $I_c^*$ is defined as

$$\frac{I}{I_c^*} = \frac{1}{M_HK/2} \int_{-M_HK/2}^{0} dE \frac{\mathcal{M}_s}{\alpha \mathcal{M}_a}. $$

(8)

Using the explicit forms of $\mathcal{M}_s$ and $\alpha \mathcal{M}_a$ given in Refs. 17 and 18 (see also Ref. 20), the explicit form of $I_c^*$ is given by

$$I_c^* = \frac{2aeMV}{\hbar \eta} \frac{4\pi M}{S^*}, $$

(9)

where the dimensionless quantity $S$ is given by

$$S = \int_{-\sqrt{k/2}}^{0} \frac{dE}{\sqrt{1 + k(1 - 2\sqrt{k}k_\varepsilon + k\varepsilon k_\varepsilon)}} + 2\sqrt{k},$$

(10)

where $k = H_K/4\pi M$ and $k_\varepsilon = \sqrt{(k + 2\varepsilon)/(k(1 - 2\varepsilon))}$. The first and second kinds of complete elliptic integrals are denoted as $K(k)$ and $E(k)$, respectively. The current $I_c$ satisfies $I_c < I_c^* < I_s^*$. On the other hand, for $I_c < I < I_s^*$, $E^*$ satisfies $\mathcal{M}_a(E^*) = \alpha \mathcal{M}_a(E^*)$, and depends on the current. In that case, Eq. (4) depends on the current nonlinearly.

Equation (7) means that by replacing $I_s^*$ with $I_c^*$, the switching exponent in the low-current region becomes exactly unity, i.e., $b$ in Eq. (1) is

$$b(I < I_c) = \log\left(\frac{1 - I/I_c^*}{1 - I/I_c}\right),$$

(11)

which satisfies $b > 1$ and $\lim_{I \rightarrow 0} b = I_s^*/I_c^*$. We emphasize that Eq. (7) is valid only in the low-current region while Eq. (1) is applicable to the entire range of the thermally activated region. If the switching in the experiments occurs in the low-current region, the linear scaling of the switching barrier works well to analyze the experimental results. On the other hand, in the high-current region, we cannot introduce another scaling current that makes $b$ of Eq. (1) unity. If the switching occurs in the high-current region, linear scaling is not applicable. Another important point indicated by Eq. (7) is that, even if the switching occurs in the low-current region, the scaling current estimated using the linear fit of the experimentally observed $\Delta(I)$ is $I_c^*$, not $I_c$ nor $I_s^*$. Since $I_c < I_c^*$, the linear fit leads to an underestimation of the switching current at zero temperature.

To study whether switching occurs in the low-current region, we calculated the switching probability in the low-current region for several temperatures $T$ and current pulse duration times $t$. The switching probability is given by

$$P(I) = 1 - \exp\left[-\gamma t s \exp(-\Delta I)\right],$$

where $\Delta$ is the low-current region is given by Eq. (7). The attempt frequency in the low-current region is

$$f(I < I_c) = \frac{2\alpha \Delta_0 \gamma \sqrt{4\pi M(H_K + 4\pi M)}}{\pi} \left[1 - \left(\frac{I}{I_c^*}\right)^2\right].$$

(12)

Figure 2 shows examples of the current dependences of the switching probability at 50 and 300 K, in which the current pulse duration time is 1 μs. The values of the other parameters are $M = 1000$ emu/c.c., $H_K = 200$ Oe, $V = \pi \times 80 \times 35 \times 2.5$ nm$^3$, $\gamma = 17.64$ MHz/Oe, $\alpha = 0.01$, and $\eta = 0.8$, respectively, by which ($I_c^*$, $I_s^*$) = (0.54, 0.58, 0.67) mA.23) These are typical material parameters of an in-plane magnetized MTJ consisting of CoFeB ferromagnets and MgO barrier.5,6,21,22) As shown, the switching probability at 300 K reaches almost 100% in the low-current region, $I < I_c$. On the other hand, the switching probability at 50 K is much smaller than 100%, which indicates that the switching at 50 K mainly occurs for $I_c < I < I_s^*$. Experimentally, the thermal stability $\Delta_0$, as well as the switching current $I_c^*$, have been evaluated from the switch-
The switching probability in the low-current region is larger than 99%. The linear scaling of the switching barrier, \( b = 1 \), is applicable in the \((t, T)\) region above this line.

The temperature is high or the current pulse duration time is long, the evolution of the switching probability from 0 to 100% mainly occurs in the low-current region. In this case, the linear scaling of the switching barrier, Eq. (7), can be used to analyze the experimentally observed switching probability. The switching probability in the high-current region already reaches 100%, and does not affect the evaluation of the thermal stability. On the other hand, when the temperature is low or the current pulse duration time is short, a large current \( \sim I_c \) is required to saturate the switching probability 100%. In this case, the evolution of the switching probability mainly occurs in the high-current region, where the linear scaling of the switching barrier is not applicable.

Figure 3 shows the relation between the current pulse duration time and the temperature \( T \) above which the switching probability in the low-current region is larger than 99%. This means that the above line in Fig. 3, the switching probability reaches almost 100% in the low-current region, where linear scaling of the switching barrier can be used to evaluate the thermal stability. On the other hand, below the line in Fig. 3, the switching in the high-current region is not negligible, and linear scaling is not applicable. It should be noted that the range of the current pulse duration time in Fig. 3 is from 40 ns to 1 s. For \( t < 40 \) ns, because the temperature \( T \) is very high, the switching probability becomes larger than 1% at zero current, which means that the initial state deviates from the easy axis significantly due to the large thermal fluctuation. Therefore, we neglect the region \( t < 40 \) ns. For such very short \( t \), linear scaling is no longer applicable.

Now let us discuss the validity of the linear scaling of the switching barrier used in the analyses of the experiments. For example, in Ref. 5, the current dependences of the switching barrier, \( \Delta(I) \), for several current pulse duration times, \( 5 \mu s \leq t \leq 1 \) ms, were measured at room temperature. The material parameters are similar to those used in Figs. 2 and 3. According to Fig. 3, \( T \) in this range of \( t \) is much lower than room temperature, which means that the switching in Ref. 5 occurs in the low-current region. Therefore, the linear scaling of the switching barrier is applicable to fit the experimental results of Ref. 5. However, experimentally, a short current pulse less than 1 \( \mu s \) is also practicable. For such short \( t \), \( T \) is much higher than room temperature, as shown in Fig. 3, and therefore, linear scaling is not applicable. In this region, the numerical calculation of the switching rate from the Fokker–Planck equation is valid for an accurate evaluation of the thermal stability.

In summary, we showed a theoretical formula for the switching barrier of an in-plane magnetized ferromagnet, and pointed out that the switching barrier in the low-current region showed linear dependence on the current with a new scaling current \( I_c \). The temperature \( T \) and current pulse duration time \( t \) regions in which linear scaling of the switching barrier is applicable were obtained. We also pointed out that previous experimental analyses underestimated the spin torque switching current at zero temperature.

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23) Because it is very difficult to obtain the analytical expression of \( I_c \), the relation between \( I_c \) and \( I_v \), or \( I_s \), cannot be obtained analytically, contrary to \( I_s/I_v \approx 1.27 \) for \( H_{\varphi} \ll 4\pi M_{\varphi} \). Instead, we numerically evaluated the ratios, \( I_c/I_v \), which varies from 1.08 to 1.05 and (from 0.85 to 0.88) for 100 \( \leq H_{\varphi} \leq 500 \) Oe.
24) Private communication with A. Fukushima.
25) In other words, \( I \) can be defined as \( \lim_{t \to \infty} P \approx 1 \). Since, strictly speaking, the theoretical switching probability \( P = 1 - e^{-\alpha t} \) is always less than 99%, we define the switching as the probability larger than 99%. It should also be noted that Eq. (12) is zero at \( I = I_c \), due to limitation of the validity of the approximated solution for \( I \). Therefore, we considered the current region satisfying \( dP/dI > 0 \) for \( I < I_c \).