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Fast ignition of precompressed DT fuel placed in an absolutely rigid heat-insulated cylinder

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Abstract. A two-dimensional axisymmetric problem on fast ignition of a cylindrical target of precompressed DT-mixture surrounded by a stationary heat-insulated shell is considered. The target end is ignited with a proton beam, the intensity of which is independent of the radial coordinate. Self-radiation of plasma and $\alpha$-particles of the thermonuclear reaction freely escape out of the fuel through the lateral boundary of the shell. It is shown that the ignition energy threshold for the mixture density 22 and 110 g/cm\textsuperscript{3} about 10 times less than in the case of the known problem with the radius of the beam much less than the radius of fuel. Previously developed quasi-one-dimensional model underestimates the ignition energy threshold by the target radius about 4 times in comparison with the problem under consideration. Estimates for the magnetic field and the shell density at which the corresponding problems are in some sense close to the problem under consideration are presented.

1. Introduction

The concept of fast igniting thermonuclear targets for inertial fusion [1] is based on using two drivers. The first driver compresses the target up to necessary value of density, while the second driver provides for fast rise of temperature. In particular, the proton beam generated by the absorption of laser radiation in a vicinity of the critical density [2, 3].

In the case of spherical targets, if we exclude the converging shock wave [4, 5] as the ignitor, the fast ignition requires a narrow channel for supplying power of the ignition driver to a dense inner core of the target. Keeping in mind such type of ignition, the axisymmetric problem on ignition of flat DT targets of a given density $\rho_0$ by a proton beam with the radius much less than the size of the fuel was considered in a number of papers to determine the ignition energy threshold $E_{ig}$. From this number, we choose a paper of Atzeni [6] as the basis for comparing with our results. As in that paper [6], we consider a beam operating for a given time and having a constant intensity in a circle of a given radius. The beam in [6] consists of some particles characterized by the penetration depth $l$ defined by the equation $l\rho_0 = 0.6$ g/cm\textsuperscript{2} and referred to as optimal for ignition. The results of calculations [6] are generalized by the following formula:

$$E_{ig} = E_*(\rho_*/\rho_0)^{1.85},$$

(1)

$E_* = 140$ kJ, $\rho_* = 100$ g/cm\textsuperscript{3}, in the interval $50 \leq \rho_0 \leq 3000$ g/cm\textsuperscript{3}.

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In the present paper, the beam consists of protons with energy 1 MeV and has the close to [6] penetration depth \( \rho_0 \approx 0.5 \text{ g/cm}^2 \). A possibility to decrease the ignition energy by changing the proton energy is not considered here.

We consider a cylindrical target, which is schematically shown in figure 1. For the first time such a target was proposed in connection with the laser heating [7]. We suppose that it is possible to create a configuration close to that is shown in figure 1 with a cylinder of the fuel precompressed to the necessary density and with a hole for injecting the proton beam.

It is natural to expect that the shell with the density \( \rho_{sh} > \rho_0 \) reduces the lateral expansion of the fuel and, thereby, the ignition energy threshold, in comparison with the formula (1) obtained on the basis of the problem, in which the fuel of the density \( \rho_0 \) played the role of the shell. Assuming a strongly magnetized shell, we can expect a significant decrease in the heat flux between the shell and the fuel [7], which also reduces the ignition energy threshold. In this paper, we consider the problem for the stationary heat-insulated shell (physical conditions of such shell assumption will be discussed in section 4). Suppression of losses by electron thermal conduction under compression of high-temperature plasma by a magnetically driven pusher is verified on experimental facilities of Russia and USA [8].

Unlike the problem with the symmetry conditions on the lateral boundary for all of the governing equations that is equivalent to the one-dimensional problem, \( \alpha \)-particles produced in the DT reaction and self-radiation of plasma are not reflected from the lateral boundary of the cylinder but freely escape out of the fuel. If we neglect a possibility to use strongly magnetized cylindrical targets confining the trajectories of \( \alpha \)-particles and therefore reducing the beam area necessary for the ignition [9], the ignition energy threshold obtained from the problem under consideration can be considered as the lower estimate for this quantity, while formula (1) can be considered as the upper one.

Previously [10], we offered a quasi-one-dimensional model of fast ignition, which enables to estimate the ignition energy threshold using results of one-dimensional calculations. This model is based on the track method for simulation of \( \alpha \)-particle transport corrected for escape of \( \alpha \)-particles from a cylinder of a given radius \( R_\alpha \) that is an additional parameter of the method. If a solution of the 1D problem with a given time-dependent beam intensity \( J(t) \) contains the thermonuclear burn wave, we set \( E_{ig} = \pi R_\alpha^2 \int J(t)dt \), where the integral is taken over the beam lifetime. In this paper we compare the quasi-one-dimensional model with the above 2D problem.

### 2. Problem statement and numerical method

In cylindrical coordinates \((r, \zeta)\), the flow region is bounded by the cylinder surface \( r < R \). At the initial time \( t = 0 \), the part of the cylinder \( \zeta \geq 0 \) is filled by the stationary fuel, which is a mixture of equal amounts of deuterium D and tritium T with the density \( \rho_0 \). The boundary \( \zeta = 0 \) is the free one with the pressure \( p_a = 1 \) bar. The initial pressure of the mixture is determined by the isentrope passing through the point \((\rho_s, p_a)\), where \( \rho_s \approx 0.22 \) g/cm\(^3\) is the fuel density in the solid state at the pressure \( p_a \) and the temperature of 4 K.

The DT-mixture is supposed to be completely ionized. The equations of state for electrons \( p_e(p, T_e) \), \( \varepsilon_e(p, T_e) \) and ions \( p_i(p, T_i) \), \( \varepsilon_i(p, T_i) \) are presented in [10]. To construct them, we use an equation of state of hydrogen based on a semiempirical model [11].
We solve the equations of one-fluid two-temperature hydrodynamics with the electron and ion thermal conduction and its heat by the protons and α-particles:

\[
\frac{d\rho}{dt} = -\rho \text{div} \mathbf{u}, \quad \rho \frac{d\mathbf{u}}{dt} = -\nabla p, \\
\rho \frac{d\varepsilon_e}{dt} = -p_e \text{div} \mathbf{u} - \text{div} q_e + Q_{ei} + D_e + W_e + S, \\
\rho \frac{d\varepsilon_i}{dt} = -p_i \text{div} \mathbf{u} - \text{div} q_i - Q_{ei} + D_i + W_i, 
\]

where \( \rho \) is the density, \( \mathbf{u} \) is the mass velocity, \( d/dt = \partial/\partial t + \mathbf{u} \nabla \) is the Lagrangian derivative with respect to time, \( p_e \) and \( p_i \) are the electron and ion pressures, \( p = p_e + p_i \), \( \varepsilon_e \) and \( \varepsilon_i \) are the electron and ion specific internal energies, \( q_e = -\varepsilon_e \nabla T_e \) and \( q_i = -\varepsilon_i \nabla T_i \) are the electron and ion heat fluxes, \( T_e \) and \( T_i \) are the electron and ion temperatures, \( \varepsilon_e \) and \( \varepsilon_i \) are the electron [12] and ion [13] heat conductivity coefficients. The term \( Q_{ei} = 3n_i k_B (T_i - T_e)/(2\tau_T) \) in equations (2) and (3) defines the energy exchange between electrons and ions, \( n_i = \rho (Am_i)^{-1} \) is the ion number density, \( m_i \) is the atomic mass unit, \( k_B \) is the Boltzmann constant, \( \tau_T \) is the temperature relaxation time [12], \( A = 2.5 \) is the atomic weight. The rest terms in (2) and (3) define the heating of electrons and ions by the protons (\( D_e \) and \( D_i \)) and α-particles (\( W_e \) and \( W_i \)) as well as the energy exchange between electrons and self-radiation of plasma (\( S \)).

The boundary condition \( q_e = q_i = u_r = 0 \), where \( u_r \) is the radial component of the velocity, is given at \( r = R \).

Only the primary fusion reaction between deuteron and triton with α-particle having energy 3.5 MeV and neutron as the reaction products is taken into account. Neutrons are supposed to be escaped from the fuel without interaction. The number of events of the reaction per unit time and unit volume is as follows from [14], \( F = n_D n_T \langle \sigma v \rangle_{DT} \), where \( n_D \) and \( n_T \) are the deuteron and triton number density respectively, \( \langle \sigma v \rangle_{DT} \) is the ion-temperature dependent reaction rate. Burn-up of the mixture nuclei is taken into account by analogy with [10].

The transfer of α-particles is described by the steady-state kinetic equation in Fokker–Planck approximation [15] for the distribution function \( f(\mathbf{r}, v, \Omega) \), which defines \( f(\mathbf{r}, v, \Omega) d\mathbf{vr}d\Omega \) as the number of particles in a unit volume in a vicinity of the point \( \mathbf{r} \) having the velocity in the interval \( dv \) in a vicinity of \( v \) and the direction in the interval of the solid angle \( d\Omega \) in a vicinity of the unit vector \( \Omega \). Besides the function \( F \), there are given the decelerations (negative accelerations) of α-particles by electrons \( a_e(T_e, \rho, v) \) [16] and ions \( a_i(T_i, T_e, \rho, v) \) [17] with the particle charge as a parameter as well as the velocity of particle thermalization \( v^{th}(T_i) \) with the particle mass as a parameter. For brevity, we replace the dependence on the thermodynamic functions by the dependence on \( \mathbf{r} \) assuming that the functions \( a_{e,i}(\mathbf{r}, v) \), \( F(\mathbf{r}) \), \( v^{th}(\mathbf{r}) \) are known.

We suppose that all the produced particles have the same velocity \( v_0 \) and the uniform distribution in the solid angle. Then, in the absence of diffusion of the distribution function in the velocity space, the inhomogeneous kinetic equation is reduced to the following Cauchy problem for the homogeneous equation [15,18]:

\[
v(\Omega \nabla f) + \frac{\partial a f}{\partial v} = 0, \quad f(\mathbf{r}, v_0, \Omega) = -\frac{F(\mathbf{r})}{4\pi a(\mathbf{r}, v_0)}, \quad a = a_e + a_i, \\
v_m(\mathbf{r}, \Omega) = \max\left(v^{th}(\mathbf{r}), v_b(\mathbf{r}, \Omega)\right) \leq v \leq v_0, 
\]

where, in the case of absence of incoming particles, \( v_b(\mathbf{r}, \Omega) \) depends on proximity of the point \( \mathbf{r} \) to the boundary along the ray with the direction \( -\Omega \) [18]. If the point \( \mathbf{r} \) goes to a point on the region boundary, then \( v_b(\mathbf{r}, \Omega) \rightarrow v_0 \) for all of the \( \Omega \) directed inside the region. The
corresponding terms \( W_e \) and \( W_i \) at the right parts of equations (2) and (3) have the form

\[
W_{e,i}(r) = -m_\alpha \int_0^{\nu_i} \int_{(4\pi)^\nu_m(r,\Omega)} f(r, v, \Omega)a_{e,i}(r, v)vd\nu d\Omega, \tag{5}
\]

where \( m_\alpha \) is the particle mass.

The trajectories of protons heating the target rely straight lines \( r = \text{const} \), along which protons are decelerated in accordance with the equation

\[
v_p \frac{dv_p}{dz} = a_e + a_i, \tag{6}
\]

where \( v_p \) is the proton velocity decreasing from the initial velocity \( v_{0p} \) down to the thermalization velocity \( v^{th} \).

The intensity of the incident proton beam \( J_\nu(t) \) is independent of \( r \) and is determined by two parameters, the maximum intensity \( J_0 \) and the beam lifetime \( \Delta t_{pr} \) that in turn determines the initial time interval \( \Delta t_{pr}^0 = 0.02\Delta t_{pr} \) during which \( J_\nu(t) \) increases from 0 to \( J_0 \):

\[
J_\nu(t) = \begin{cases} 
J_0 t/\Delta t_{pr}^0, & t \leq \Delta t_{pr}^0; \\
J_0, & \Delta t_{pr}^0 < t \leq \Delta t_{pr}; \\
0, & t > \Delta t_{pr}.
\end{cases} \tag{7}
\]

The intensity of a monoenergetic proton beam inside the target is determined as

\[
J = n_p v_p \frac{m_p v_p^2}{2},
\]

where \( n_p \) is the proton number density that is assumed to be constant up to the thermalization, \( m_p \) is the proton mass. The value of \( n_p \) is determined by the given initial velocity of protons \( v_{0p} \) and the beam intensity at the entrance to the target \( J_\nu(t) \). The terms in the right parts of equations (2) and (3) have the form

\[
D_e = -\frac{a_e}{a_e + a_i} \frac{\partial J}{\partial z}, \quad D_i = -\frac{a_i}{a_e + a_i} \frac{\partial J}{\partial z}. \tag{8}
\]

Self-radiation of plasma is described by the diffusion for solid angle \( \Omega \) and multigroup for frequency \( \nu \) approximation of the steady-state transfer equation with respect to the radiation intensity \( I(\nu, \Omega, r) \). The equations of the diffusion approximation have the form

\[
\text{div} \ q_\nu = \kappa (B(\nu, T_e) - u_\nu), \quad q_\nu = -\frac{1}{3\kappa} \nabla u_\nu, \tag{9}
\]

where \( q_\nu = \int I\Omega d\Omega, \kappa = \kappa(\nu, T_e, \rho) \) is the bremsstrahlung absorption coefficient with accounting for the induced emission [19], \( B(\nu, T_e) \) is the Planck function, \( u_\nu = \int I d\Omega \).

The multigroup approximation is constructed by dividing the range of \( 0 < \nu < \infty \) into \( N \) groups \( \nu_{l-1} < \nu < \nu_l, l = 1, \ldots, N, \nu_0 = 0, \nu_N = \infty \). At each group, equations (9) are integrated over the corresponding interval using the approximate equalities

\[
\int \kappa u_\nu d\nu \approx \kappa^p u_l, \quad \int \frac{1}{\kappa} \nabla u_\nu d\nu \approx \frac{1}{\kappa^R} \nabla u_l, \quad u_l = \int u_\nu d\nu, \tag{10}
\]

\[
\kappa^p = \frac{K_l}{B d\nu}, \quad K_l = \int \kappa B d\nu, \quad \kappa^R = \frac{\int (\partial B/\partial T_e)d\nu}{\int \kappa^{-1}(\partial B/\partial T_e)d\nu}.
\]
As a result, we obtain the equations

$$\text{div} \mathbf{q}_l = K_l - \kappa_l^P u_l, \quad \int q_\nu d\nu = q_l = -\frac{1}{3\kappa_l^R} \nabla u_l. \quad (11)$$

The coefficients $\kappa_l^P$ and $\kappa_l^R$ are analogous to the single-group Planck and Rosseland coefficients. The term in the right part of equation (2)

$$S = -\sum_{l=1}^{N} (K_l - \kappa_l^P u_l).$$

As in [10], the cooling of electrons due to the inverse Compton effect is taken into account.

The computer code is based on the previously developed code on regular moving grids for two-dimensional axisymmetric flows of plasma and condensed matter [20–22].

To calculate the plasma heating by $\alpha$-particles (4), (5), the inverse track method [18] is used. The rays outgoing from the grid cell center are used to calculate the slowing down of the particles passing through the center rather than escaping from it as in the conventional direct method.

The slowing down of protons (6) is performed on a system of rays $r = \text{const}$ using interpolation of functions $a_e$ and $a_i$ from the centers of the grid cells. The resulting values of intensity $J$ are interpolated on the points of grid lines defined by $r$-coordinates of the cell centers, which enables to approximate the derivative $\partial J/\partial z$ from formula (8) at the cell centers.

Algebraic equations approximating the diffusion equations (11) are solved by the direct method of band matrix decomposition into triangular multipliers [23].

The computations are performed in the domain $0 \leq z \leq Z$, $Z\rho_0 \approx 1.2 \text{ g/cm}^2$. We use a moving regular grid with a clear identification of the free boundary in the form of a grid line. The internal grid nodes are computed by the method [21]. The boundary grid lines are computed as follows.

Number of intervals along the axis of symmetry and the lateral boundary $N_z$ varies from 120 to 240. The arrangement of grid points along the above boundaries satisfies the condition $(z_{i+1} - z_{i})\rho_i = \text{const}$, where $\rho_i$ is the density in a cell adjacent to the interval $z_i < z < z_{i+1}$, which roughly corresponds to the one-dimensional Lagrangian grid. At $t = 0$, $z_{i+1} - z_i = h = Z/N_z$.

At $N_z = 240$, the step $h$ is two times greater than that in 1D computations [10]. In another direction, the grid has $N_r = 20$ intervals with the uniform arrangement of the grid points along the boundary lines. The coarseness of the grid along this direction is due to the fact that the flow considered in this paper turns out to be close to a one-dimensional flow. Check-up computations are performed at $N_r = 40$.

The number of rays outgoing from each grid cell in the track method $N_{tr}$ varies from 16 to 32. The number of groups $N$ for computing self-radiation of plasma varies from 5 to 9; the values $\nu_1$ and $\nu_{N-1}$ are chosen from an analysis of the spectral dependence of radiation in the one-dimensional problem [10].

Numerical results presented in the next section correspond to $N_z = 240$, $N_r = 20$, $N_{tr} = 32$ and $N = 5$.

3. Numerical results

The case of $\rho_0 = 100\rho_s \approx 22 \text{ g/cm}^3$ is studied particularly in the framework of the quasi-one-dimensional model earlier [10]. In that case, for the parameters of the proton beam $\Delta t_{pr} = 50 \text{ ps}$, $J_0 = J_{00} = 10^{19} \text{ W/cm}^2$, the target is heated almost isochorically and reliably ignited starting from the value of $R_\alpha \approx 0.1 \text{ mm}$.

The cylindrical target is not ignited in our present two-dimensional simulation at the above parameters and $R = R_\alpha$. The consecutive increase in $R$ with the step 0.01 mm shows that
the target is ignited starting from the value of \( R = 0.2 \) mm. In figure 2 the profiles of the pressure related to \( \rho_0 \) versus the coordinate \( \rho_0z \) are presented for the target with \( R = 0.2 \) mm at four time instants starting from the beam ending at \( t = \Delta t_{pr} \), along the axis of symmetry (\( r = 0 \)) and the lateral boundary (\( r = R \)), in comparison with the corresponding profiles for the quasi-one-dimensional model with \( R_\alpha = 0.1 \) mm.

It is seen from figure 2 that, at each time instant, all three profiles are close to each other. From this, we can draw two conclusions. First, the solution of the 2D problem weakly depends on \( r \) and thereby is close to a certain one-dimensional flow. Secondly, as compared with the 2D problem, the quasi-one-dimensional model underestimates approximately 2 times the threshold of the target radius, which gives the approximately four-fold underestimation in the ignition energy threshold at the given values of \( \rho_0, J_0 \) and \( \Delta t_{pr} \).

To compare the ignition energy threshold of the cylindrical target under consideration with formula (1), it is necessary to estimate the minimum of the ignition energy not only in the target radius \( R \) but also in the parameters of the proton beam. A set of computations for the given value of \( R = R_\alpha = 0.1 \) mm and with a consecutive increase in \( J_0 \) shows that the target is ignited starting from \( J_0 = 1.5J_{00} \), which gives the threshold ignition energy considerably less than in the previous case \( R = 2R_\alpha, J_0 = J_{00} \) because the beam energy \( E_{pr} \sim J_0R^2 \).

To illustrate the above ignition, the time dependence of the total power of electron heating by \( \alpha \)-particles \( w_e(t) = \int W_e dV \) where the integral is taken over the total volume of the fuel, is shown in figure 3 for the values of \( J_0 = 1.5 \) and \( 1.4J_{00} \). In the first case, the function \( w_e(t) \) increases rapidly at \( t > 300 \) ps indicating the target ignition while in the second case the function decreases indicating lack of the ignition.

Besides the initial density \( \rho_0 = 100\rho_s \) and corresponding values of other parameters \( R = 0.1 \) mm, \( \Delta t_{pr} = 50 \) ps and \( J_0 = 1.5J_{00} \), we consider the case \( \rho_0 = 500\rho_s \approx 110 \) g/cm\(^3\). The condition \( \rho_0R = \text{const} \) gives \( R = 0.02 \) mm. To maintain the isochoric heating at the same value of the beam energy per unit area, one should take (see [6, 10]) \( \Delta t_{pr} = 10 \) ps, \( J_0 = 7.5J_{00} \).
Figure 4. The pressure related to $\rho_0$ as a function of the coordinate $\rho_0 z$ at four time instants $t/\Delta t_{pr} = 1$ (1), 2 (2), 4 (3) and 6 (4) along the axis of symmetry (solid lines) and the lateral boundary (points) for the variants $\rho_0 = 100\rho_s$, $R = 0.1$ mm, $\Delta t_{pr} = 50$ ps and $J_0 = 1.5J_{00}$ (a) and $\rho_0 = 500\rho_s$, $R = 0.02$ mm, $\Delta t_{pr} = 10$ ps and $J_0 = 7.5J_{00}$ (b).

Table 1. The ignition energy thresholds $E_{ig}^{(1)}$ by formula (1) and $E_{ig}^{(12)}$ by formula (12) with the parameters $R$, $J_0$ and $\Delta t_{pr}$ for two values of $\rho_0$.

| $\rho_0$, g/cm$^3$ | $E_{ig}^{(1)}$, kJ | $E_{ig}^{(12)}$, kJ | $R$, mm | $J_0$, $10^{19}$ W/cm$^2$ | $\Delta t_{pr}$, ps |
|-------------------|-------------------|-------------------|-------|-------------------|-------------------|
| 22                | 2300              | 230               | 0.1   | 1.5               | 50                |
| 110               | 120               | 9                 | 0.02  | 7.5               | 10                |

Figure 4 shows the profiles of the relative pressure $p/\rho_0$ against $\rho_0 z$ along $r = 0$ and $r = R$ at four time instants starting from the beam lifetime $\Delta t_{pr}$ for two considered values of $\rho_0$ and the corresponding values of other parameters. One can see that the profiles for $\rho_0 = 100\rho_s$ and $500\rho_s$ are similar to each other. A small decrease in the relative pressure for $\rho_0 = 500\rho_s$ in comparison with the case $\rho_0 = 100\rho_s$ is explained by a small increase in the penetration depth of the proton beam (compare the profiles at $t = \Delta t_{pr}$ in figures 4a and 4b) and by corresponding decrease in the specific internal energy averaged over the region heated by the beam (see [10]). The increase in the peak pressure (close to the detonation wave pressure) on profiles 4 in comparison with profiles 3 indicates the target ignition in the considered cases.

The beam energy for the problem under consideration is defined by the formula

$$E_{pr} = 0.99\pi R^2 J_0 \Delta t_{pr} ,$$

where the factor 0.99 occurs in integration of the function (7) along time.

4. Concluding remarks

If a possible influence of magnetic field on trajectories of alpha particles is ignored, the problem under consideration gives a lower estimate for the energy ignition threshold of cylindrical targets with the proton beam, the intensity of which is defined by two parameters $J_0$ and $\Delta t_{pr}$ according
to (7). This estimate is about 10 times less than the ignition energy threshold for the problem without shell and with the beam radius much less than the radius of the fuel, which is a stimulus for further research of similarly ignited shell targets.

Below, we present approximate estimates for the magnetic field $H$ and the shell density $\rho_{sh}$, at which the corresponding problems are in some sense close to the problem with the heat-insulated absolutely rigid shell.

Available models of magnetized targets for the inertial confinement fusion (see [9,24]; compare with the inertial electrostatic confinement case [25,26]) describe compression of the targets. Here, we deal with the fast ignition and use the simple estimates [7] based on the classic formulas for the electron and ion heat conductivity coefficients in a magnetized plasma.

The first condition $H > H_e$ is defined by the inequality $\omega_e C - \omega_i C > -\omega - \omega_i$ and $\tau_e < \tau_i$ at $H = H_e$, which can be considered as suppression of the electron heat conduction in the radial direction if the magnetic field is directed along the axis of symmetry. The second more strong condition $H > H_i$ defined by the inequality $\omega_i T_i > -\omega - \omega_i$ and $\tau_i$ are the cyclotron frequency and the self-collision time for ions) ensures a considerable decrease in the ion heat conductivity coefficient transversal to the magnetic field in comparison with the case of nonmagnetized plasma.

Dependence of $H_e$ and $H_i$ on thermodynamic parameters (ignoring dependence on their logarithms) has the form $H_e \sim n_e T_e^{-3/2}$, $H_i \sim n_i T_i^{-3/2}$. As $H_e$ and $H_i$ decrease with the temperature increase, the conditions $H > H_e$ or $H > H_i$ are satisfied during the fast ignition provided that the magnetic field $H$ does not fall too fast, if they are satisfied at the preliminary stage of the target compression.

The values of $H_e$ and $H_i$ are presented in Table 2 for two considered initial densities of DT-fuel and for two values of temperature: $T = T_e = T_i = 10$ keV that is approximately the ignition temperature and the much less value $1$ keV corresponding to the moment when the beam energy deposited in the fuel is about $0.1 E_{pr}$. In our computations, we use formulas from [12] for $\tau_e$ and $\tau_i$ including the so-called Debye logarithm instead of the Coulomb one (see, also, [27]).

Table 2. The limiting values of magnetic field $H_e$ and $H_i$ in the estimates [7] at different temperatures and densities of DT mixture.

| $\rho_0$, g/cm$^3$ | $T$, keV | $H_e$, MG | $H_i$, MG |
|---------------------|----------|-----------|-----------|
| 22                  | 10       | 0.15      | 1         |
|                     | 22       | 1         | 2.7       | 19        |
| 110                 | 10       | 0.63      | 4.3       |
| 110                 | 1        | 10        | 70        |

To suppress the electron heat conduction in the radial direction at $T > 10$ keV in the case of $\rho_0 = 22$ g/cm$^3$, it is necessary to have the magnetic field $H > 0.15$ MG. The about 20-fold increase in the magnetic field is necessary to suppress the electron heat conduction at $T > 1$ keV. In the case of $\rho_0 = 110$ g/cm$^3$, the both values of the magnetic field should be increased about four times. To suppress the ion heat conduction, the magnetic field in the all considered cases should be increased about 7 times.

Let the fuel been surrounded by a shell of the finite density $\rho_{sh}$. As a measure of deviation of the flow from the case of the absolutely rigid shell, we choose the relative displacement of the interface between the fuel and the shell $\delta = \Delta r / R$, where $\Delta r$ is the absolute displacement, $R$ is...
the fuel radius. Our consideration is restricted by the stage of the beam heating \( t \leq \Delta t_{\text{pr}} \). To evaluate the interface velocity \( U \) at \( t = \Delta t_{\text{pr}} \), we consider the instantaneous isochoric heating of the fuel up to the pressure \( p_h \) and determine \( U \) from the Riemann problem for the piecewise constant pressure and density (\( p = p_h, \rho = \rho_0 \) in the fuel and \( p = 0, \rho = \rho_{\text{sh}} \) in the shell) and for zero velocity as well as for the perfect gas equation of state \( p = \varepsilon \rho(\gamma - 1) \), \( \gamma = 5/3 \) both in the fuel and in the shell. The solution consists of the rarefaction wave in the fuel and of the strong shock wave in the shell, and can be reduced to the form

\[
U = \varphi(\gamma, \beta) \sqrt{\gamma p_h/\rho_0}, \quad \beta = \sqrt{\rho_{\text{sh}}/\rho_0},
\]

(13)

where the function \( \varphi(\gamma, \beta) \) is defined by a transcendental equation, and in the acoustic approximation for the rarefaction wave takes the form

\[
\varphi(\gamma, \beta) = \frac{2}{\gamma + \beta \sqrt{2\gamma(\gamma + 1)}}.
\]

(14)

In the case of \( \beta = \sqrt{10} \) considered below, the computational error of formula (14) is less than 2%. Supposing the linear growth of the interface velocity in time, we obtain

\[
\delta = 0.5U \Delta t_{\text{pr}}/R.
\]

(15)

Let us now consider two variants of the targets from this paper for \( \rho_0 = 22 \) and 110 g/cm\(^3\). As the pressure \( p_h \), the values of \( 2 \times 10^7 \) GPa for the first variant and of \( 10^8 \) GPa for the second are taken. The both values give the relative pressure \( p_h/\rho_0 \approx 9 \times 10^4 \) km\(^2\)/s\(^2\). As it is seen from the profiles 1 in figures 4a and 4b, the pressure \( p_h \) is close to the maximal pressure at \( t = \Delta t_{\text{pr}} \). The shell density \( \rho_{\text{sh}} = 10\rho_0 \) is taken in the both variants.

One can see from (15), (13) and the values of \( \Delta t_{\text{pr}} \) and \( R \) that the parameter \( \delta \) is the same for the both variants. The computation gives \( \delta \approx 0.055 \).

The smallness of the parameter \( \delta \) allows to hope that the ignition energy for the shell targets with the shell density \( \rho_{\text{sh}} = 10\rho_0 \) will be close to the values from this paper.

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