Electron–phonon coupling in a two-dimensional inhomogeneous electron gas: consequences for surface spectral properties

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Abstract
We investigate the coupling of an inhomogeneous electron system to phonons. The properties of an electronic system composed of a mixture of microscopic ordered and disordered islands are changed fundamentally by a phonon mode. In high-$T_c$ cuprates, such a phase separation scenario is supported by recent STM and nuclear quadrupole resonance studies. We show that even a weak or moderate electron–phonon coupling can be sufficient to produce dramatic changes in the electronic state of the inhomogeneous electron gas. The spectral properties calculated in our approach provide a natural explanation of the low-energy nodal ARPES features and exhibit a novel non-Fermi-liquid state stabilized through electron–phonon coupling.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

An eminent feature observed in the angle-resolved photoemission spectroscopy (ARPES) studies of high-$T_c$ cuprates is the kink-like change of electron velocity, interpreted in terms of the coupling to oxygen phonon modes \cite{1, 2}. These kinks are associated with a distinct break-up of the spectral weight into a high-intensity part which develops near the Fermi surface, and a broad structure of less intensity at higher energies below the Fermi level. Despite the structural differences between various types of cuprates, the peculiarities in the dispersion at 50–80 meV were detected in Bi$_2$Sr$_2$CaCu$_2$O$_{8+x}$ (Bi2212), La$_{2-x}$Sr$_x$CuO$_4$ (LSCO), YBa$_2$Cu$_3$O$_{6+x}$ (YBCO) and other related systems \cite{1–6}, and are believed to shed light on the microscopic mechanism of high-$T_c$ superconductivity. Although significant efforts have been directed towards a theoretical investigation of the influence of many-body interactions on the one-electron properties of Hubbard and $t$–$J$ models, typically used for cuprates \cite{7}, the origin of the nodal ARPES features and of their unusual doping and temperature behavior still remains an open question.

Many-body effects are known to produce changes in the electronic dispersion, as deduced from electronic photoemission spectra. One mechanism, which was shown to cause low-energy kinks, is controlled by a strong electron–phonon coupling in the free electron gas \cite{8, 9}. This coupling leads to a renormalization of the electron effective mass in the energy range close to the Fermi energy $E_F$, below a characteristic phonon frequency $\omega_{\text{ph}}$: $E - E_F < \omega_{\text{ph}}$. On the other hand, when the electrons in a metal are strongly correlated, a purely electronic mechanism can also lead to dispersion kinks, which have been related to a crossover between Fermi- and non-Fermi-liquid behavior \cite{10}.

In the analysis of ARPES intensities, one should always consider the fact that photoemission experiments analyze electronic surface and subsurface states, which brings the near-surface correlations into the focus. Due to the strong interaction of these electrons with subsurface phonon modes and virtual charge transfer excitations, the effect of such collective modes on the electronic subsystem is crucial \cite{11, 12}. A direct consequence of these interactions is a significant reduction of the local Hubbard repulsion to values well below the electronic bandwidth $8t$, where $t$ is...
the electron hopping. This finding allows us to suggest that mechanisms different from the purely electronic could play a role in the appearance of the dispersion kinks in cuprates. As the coupling with magnons cannot satisfactorily explain the doping and temperature behavior of such kinks, we will focus on the analysis of the interaction with phonons.

In the high-$T_c$ cuprates another important factor, which has to be addressed in studies of their electronic properties, is charge inhomogeneity. In the pseudogap state and in regimes with suppressed superconductivity, scanning tunneling microscopy (STM) experiments indicate a local electron order [13–16, 21–24]. This, together with nuclear quadrupole resonance (NQR) and resonant soft x-ray scattering studies [25–28], provides strong support for a state of electronic phase separation as one of the widely discussed scenarios for under- and optimally doped cuprates [13, 15, 16, 25–27]. Such a state exhibits a mixture of microscopic charge ordered (characterized as ‘more insulating’ with suppressed local density of states) and charge disordered uniform (metallic) domains with a dominance of the metallic phase at higher doping levels [16]. The ARPES intensities obtained for the electronic phase separated state inevitably contain the contributions from both types of domains. This fact can be easily understood since the electrons collected on the ARPES analyzer can be emitted from the disordered as well as from the ordered surface islands. Due to the quantum nature of the collected electrons, they cannot be described in terms of pure ‘ordered’ or ‘disordered’ electron wavefunctions, but rather as a superposition of both states. Therefore, in order to understand ARPES intensities within such an inhomogeneous scenario, we need to analyze the consequences of the electron–phonon coupling not only for the disordered, but also for the ordered sections of the surface.

In the hole-doped cuprates, the recent ab initio studies have demonstrated a weakness of the electron–phonon coupling which should result in a negligibly small contribution to the formation of the dispersion kinks [17, 18]. Despite the different approaches, these works have focused on the electronic homogeneous state, without any serious attention to a possible electronic charge order. As a consequence of such a predominant consideration of the uniform electronic state, a general view about the relative unimportance of the electron–phonon coupling for the electronic properties of the copper oxide planes became widely accepted in the literature. In an alternative approach, strongly inhomogeneous interactions of the electrons with several optical phonon modes (buckling and breathing) have been proposed [19], which was not really successful in the explanation of real physical mechanisms of spectral anomalies in the cuprates.

In the present work, we analyze the electron–phonon coupling in the charge ordered state. We show that, in contrast to the uniform electron gas, in the ordered system even a weak or moderate coupling to an optical phonon mode produces dramatic changes in the electronic properties and leads to formation of new electronic state which cannot be described by a standard Fermi-liquid theory. Due to significant advances achieved in the recent STM studies of the cuprates [13, 20], the existence of local electron order in these systems becomes a fairly well established fact. To make a step towards an interpretation of the spectral features in the context of the inhomogeneous state observed in the STM studies, the inhomogeneous surface in our work is described as a mixture of the electronically uniform and ordered states. We obtain that the superposition of these states produces several distinct features and a characteristic intensity break-up in the electronic spectral maps. In our work, we connect this break-up with the spectral anomalies observed in the cuprates. Consequently, the proposed approach is expected to shed new light on the mechanisms of kinks and on the origin of non-Fermi-liquid behavior detected in these systems.

2. Electronic spectral properties in the charge ordered state

To gain deeper insight into the ordered surface state, we provide a comparison of a disordered two-dimensional electron gas, characterized by a free tight-binding dispersion $\varepsilon_0(k)$, with an ordered electron system, both coupled to an optical phonon mode with a frequency $\omega_{\text{ph}} = \omega_0$. In our studies, we take into account the Coulomb interaction $V$ between nearest neighbors

$$ H_{\text{el}} = \sum_{k,\sigma} \varepsilon_0(k) n_{k\sigma} + V \sum_{ij} n_i n_j $$

where $n_{k\sigma}$ are the electron number operators and $n_i = \sum_\sigma n_{i\sigma}$. The electron–phonon interaction is considered in terms of a Holstein approach

$$ H_{\text{el-ph}} = -g \sum_i n_i (b_i^† + b_i) + \omega_0 \sum_i b_i^† b_i, $$

where the phonon operators refer to a vibration mode of frequency $\omega_0$. The parameter $g = \sqrt{\omega_0 E_p} (E_p$ is the polaron-binding energy) refers to the coupling of the holes in the copper oxide planes to the motion of apical oxygens in the top surface planes of the samples. In our calculations, the low-energy phonon frequency $\omega_0 = (0.05–0.1)\mu$ reflects the softening of the surface optical phonon modes suggested in [12]. For the free electronic dispersion $\varepsilon_0(k) = -2t\eta_k^+ - \xi_k$ (where $\eta_k^+ = \cos k_x + \cos k_y, \xi_k = \mu + 4t^2 \cos k_x \cos k_y$, and $\mu$ is the chemical potential) we choose $t = 0.18$ eV, $t' = -0.4r$, which is in the range of typical values found from fitting ARPES data for Bi2212 and Tl2201 [29, 30].

In our analysis we consider a checkerboard electronic ordering which introduces a doubling of the square unit cell shown in figure 1. This ordering is parametrized in

![Figure 1. Schematic view of checkerboard ordering in a two-dimensional electron gas.](image-url)
characterized by a transformation of the local maximum $E_k$ disordered uniform states, where the dotted lines refer to structure with vanishing phonon coupling in the ordered and charge regions $\Delta_1/\Sigma_1$. The correction free dispersion $\epsilon_0(k)$ is generated by their splitting through the charge-order gap $G_k$ from intersublattice electron correlations, introduced through $\Sigma_{\text{HF}}(k,\omega)$ calculated in the ordered state with $\omega_0 = 0.05t$ for different doping levels. Here $k$ is located in the nodal region close to the point N = ($\pi/2, \pi/2$) of the Brillouin zone; $|k| = 0.9, kT/\tau = 0.03$ and $E_p = 1.2t$. (b) Real part of $\Sigma_{\text{HF}}(k,\omega)$ calculated in the ordered state with $\omega_0 = 0.05t$ for $x = 0.11$ and $E_0 = 0.02t$. The transition at $E_p = E_p^* \approx 0.2t$ is characterized by a transformation of the local maximum $E_\Sigma(k = N)$ into a hat-shaped structure and by the buildup of a singular $E_S(k)$ in the regions $n_1$ and $n_2$ with singular electron velocity $v$. The regions $k \sim k_B$ are separated by a gap $\Delta_x$ from $k \sim N$ with hole-like excitations. The Fermi level is indicated by dots.

Figure 2. Electronic band structure and self-energy for a charge ordered system coupled to phonons where $V/\tau = 1.3$. (a) Band structure with vanishing phonon coupling in the ordered and charge disordered uniform states, where the dotted lines refer to $\pm \omega_0 = 0.5t$. Here $kT/\tau = 0.03$ and $x = 0.11$. (b) Real part of $\Sigma_{\text{HF}}(k,\omega)$ calculated in the ordered state with $\omega_0 = 0.05t$ for different doping levels. Here $k$ is located in the nodal region close to the point N = ($\pi/2, \pi/2$) of the Brillouin zone; $|k| = 0.9, kT/\tau = 0.03$ and $E_p = 1.2t$. (c) Evolution of the renormalized low-energy band structure in the ordered state with increasing $E_p$ for $x = 0.11$ and $\omega_0 = 0.05t$. The transition at $E_p = E_p^* \approx 0.2t$ is characterized by a transformation of the local maximum $E_\Sigma(k = N)$ into a hat-shaped structure and by the buildup of a singular $E_S(k)$ in the regions $n_1$ and $n_2$ with singular electron velocity $v$. The regions $k \sim k_B$ are separated by a gap $\Delta_x$ from $k \sim N$ with hole-like excitations. The Fermi level is indicated by dots.

Figure 3. Electronic contour plots of $\epsilon_0(k_x, k_y)$ and $\epsilon_1(k_x, k_y)$ in the disordered ($V/\tau = 0.2$) and ordered ($V/\tau = 1.3$) state where the local extremal points are indicated by $\Gamma$, M, Y, N, and S. Here $kT/\tau = 0.03$, $x = 0.11$, $E_0/\tau = 1.2$.

terms of sublattice electronic occupancies $n_\pm$ with order parameter $\delta = n_+ - n_-$. The values of $\delta$ are obtained from the minimization of the mean-field-type free energy. In such a charge ordered state, the electronic band structure is characterized by two subbands $\epsilon_j(k)$ which are obtained by the splitting through the charge-order gap $\Delta_\delta = 4V\delta$, figure 2(a). Here, in contrast to the disordered free dispersion $\epsilon_0(k)$, the emergence of the gap $\Delta_\delta$ results in the formation of new local extrema of the ordered dispersions $\epsilon_j(k)$. Figure 3 shows a comparison of the detailed $(k_x, k_y)$ map of the low-energy ordered subband $\epsilon_1(k)$ with the corresponding map of the free electron dispersion $\epsilon_0(k)$. One can immediately see that the electron order produces dramatic changes in the topology of the electronic structure which include (i) the formation of a new maximum in the central nodal point N = ($\pi/2, \pi/2$) of the Brillouin zone and (ii) the appearance of new saddle points S = $(k_x^S, k_y^S)$. The saddle point S is located close to $\Gamma$ (the second symmetric saddle point is close to Y) in the nodal direction so that $k_x^S = k_y^S = k_\delta$ where the coordinate $k_\delta$ is given by

$$\cos k_\delta = \frac{\sqrt{(4t + 2VI_{-\delta})^2 - (16t'V\delta)^2}}{8t'|4t + 2VI_{-\delta}|},$$

where the quantity $I_{-\delta}$ parametrizes the Hartree–Fock self-energy $\Sigma_{-\delta}(k) = -(V/2)\eta_\delta k_\delta$. For the chosen values of the model parameters, the maximal values of $I_{-\delta}$ are of the order 0.5. The expression (3) clearly shows that the saddle point S is controlled by the charge-order gap and disappears for weaker Coulomb repulsion $V$, i.e., it is absent in the disordered uniform state with $\delta \rightarrow 0$. We also note that the nodal point N indicated in the top map of the band $\epsilon_0(k)$ in figure 3 corresponds in fact to the intersection point of the two branches of $\epsilon_0(k)$ (see figure 2(a)). These branches are generated with the convolution to the first Brillouin zone and do not show an extremum at N. In contrast to $\epsilon_0(k)$, the new local extrema N

$$\Delta_\delta = 4V\delta,$$
and $S$ of $\varepsilon_2(k)$ are well defined and should be considered as a generic feature of the band structure in the electron-ordered state.

When the flat low-energy subband $\varepsilon_2(k)$ is coupled to the phonon mode, the local maximum at $N$ and the saddle point at $S$ lead to van Hove singularities in the sublattice contributions to the electronic self-energies $\Sigma_{\alpha}^{\text{ph}} = -g^2 \int dq \sum_{\alpha_0} G_{\alpha_0}^{\text{HF}}(q) D(q, \omega_0)$. Here $D(q, \omega_0) = 2\omega_0/((\omega_0^2 - \omega_q^2)$ is the unperturbed phonon Green function and the one-electron sublattice propagators $G_{\alpha_0}^{\text{HF}}(\alpha = \{+, -\})$ are calculated in the ordered electron state in the self-consistent Hartree–Fock approximation. Furthermore, the phonon scattering term can be decomposed as $\Sigma_{\alpha}^{\text{ph}} = \Sigma_{\alpha}^{\text{ph}} + \Delta \Sigma_{\alpha}^{\text{ph}}$, where the part $\Delta \Sigma_{\alpha}^{\text{ph}} \sim V_\delta$ disappears in the charge disordered state. With this form of $\Sigma_{\alpha}^{\text{ph}}$, the renormalized electronic propagators can be conveniently presented as a combination of the band Green functions $g_j = 1/(i\omega_n + \xi_k - \Sigma_j(k, \omega))$: $G_{\alpha 0} = G_0 \mp \Delta \Sigma_j$, where $G_0 = (g_1 + g_2)/2$ and $\Delta \Sigma_j = (2\omega_0^2 + \Delta \Sigma_0^0)/(g_1 - g_2)/(\Sigma_1 - \Sigma_2)$. In the band propagators $g_j$, the effective self-energy parts $\Sigma_j = \Sigma_0^{\text{ph}} + 4V_\delta n \pm \sqrt{(\Delta \Sigma_0^{\text{ph}} + 2V_\delta)^2 + (2\Omega_0^2 - \Sigma_0^{\text{HF}})^2}$ contain information about both charge density fluctuations and electron–phonon scattering processes. We note that $\Sigma_3$ and $g_2$, which determine the low-energy quasiparticle excitations, are of prime importance.

In figure 2(b), we show the frequency-dependent real part of $\Sigma_{\alpha}^{\text{ph}}(k, \omega)$ at different doping levels $x$. The kinks appearing in $\Sigma_{\alpha}^{\text{ph}}$ at $\omega = \omega_i$ (for $i = 1, \ldots, 3$) correspond to the van Hove singularities in $\Sigma_{\alpha}^{\text{ph}}$. In the charge disordered state, the typical van Hove singularities in $\Sigma_0^{\text{ph}}$ are caused by the local extrema in the electron dispersion $\varepsilon_0(k)$ which appear at the high-symmetry points $\Gamma$, $Y$ and $M$ of the Brillouin zone (figure 2(a)). In the charge ordered state, the flattening of $\varepsilon_2(k)$ produces the additional nodal maximum of $\varepsilon_2(k = N)$ and the saddle point at $k = S$, which lead to the appearance of new van Hove singularities in $\Sigma_{\alpha}^{\text{ph}}$ at small $\omega$. These low-energy singularities correspond to the peaks in $\Im \Sigma_{\alpha}^{\text{ph}}$ (see figure 4) and are related to the kinks in $\Re \Sigma_{\alpha}^{\text{ph}}$ (figure 2(b)). In the ordered state, the energies of the low-frequency singularities are determined by the equations $\omega = \varepsilon_1(k_i) + \omega_0 = 0$ where $k_i = \{\Gamma, N, S\}$. Specifically, while the maximum at $k = N$ produces a jump of $\Sigma_{\alpha}^{\text{ph}}$ described by $\Im \Sigma_0^{\text{ph}} \sim \Theta(\Omega^2 - \omega)$ at small $\Omega^2 = \varepsilon_2(k = N) + \omega_0 > 0$, the saddle point $S$ leads to a distinct logarithmic singular behavior of $\Sigma_{\alpha}^{\text{ph}}$:

$$
\Im \Sigma_0^{\text{ph}} \sim g^2 [(1 + b_0 - f(\omega - \omega_0)) \log |(\Omega_3^2 - \omega)/|t|] + (b_0 f(\omega + \omega_0)) \log |(\Omega_2^2 - \omega)/|t|],
$$

where $b_0 = b(\omega_0)$ and $f(\omega)$ are the Bose and Fermi distribution functions, and the energies $\Omega_3^{2/3} = \varepsilon_2(k \pm N) \mp \omega_0$ are located close to the Fermi level: $\omega_3 < \Omega_2^2 < \omega_2, \omega_2 < \Omega_1^2 < 0$.

As the chemical potential directly affects $\varepsilon_2$ in these equations, the kinks of $\varepsilon_2(k, \omega)$ at $\omega = \omega_i$ which result from the new van Hove singularities are strongly doping dependent. It is noteworthy that these low-frequency kinks at $\omega = \omega_i$ (for $i = 1, \ldots, 3$) shown in figure 2(b) will inevitably change the one-electron spectral properties near the Fermi level. To see this, we present in figure 2(c) the low-energy dispersion $E_2(k)$ of the underdoped system ($x = 0.11$), now calculated from the equation $\omega + \xi_k - \Sigma_2(k, \omega) = 0$ for the poles of the renormalized electron propagators for different values of electron–phonon coupling. A central property resulting from the new van Hove singularities is a topological reconstruction of $E_2(k)$ close to the N-point. In figure 2(c), (case $E_p = 0.8r$), this reconstruction corresponds to a transformation of the maximum $E_2(k = N)$ into a new singular hat-shaped band region, which appears above a critical value $E_p^* \approx 0.2t$.

The novel nodal dispersion exhibits a nonanalytic character in the symmetric regions $n_1$ and $n_2$ near the Fermi level. In these regions, $E_2(k)$ is a multi-valued function of $k$. The two extra sub-branches of $E_2$ emerge from the additional poles of the one-electron Green function and are caused by the van Hove kinks in $\Sigma_0^{\text{ph}}$. At the singular points $S_1$ and $S_2$ of the regions $n_1$ and $n_2$ which connect monotonically decreasing and increasing branches of $E_2(k)$, the nodal electron velocity $v = |V_k E_2(k)|$ approaches infinite values. The range $k$ close to $k_p$ with singular $v$ is separated from the region $k \approx N$ (characterized by hole-like excitations) through a gap $\Delta N/t \approx 0.2$. These topological anomalies are especially significant in the under- and optimally doped range and for polaron energies above $E_p^*$. The nodal properties of the new electronic state, stabilized at $E_p = E_p^*$, cannot be classified in terms of Fermi-liquid theory.

In our analysis, the nodal non-Fermi-liquid (NFL) behavior is also evidenced by an anomalously high scattering.
rate $\Gamma_{kr} = Z_k r |\text{Im} \Sigma^0_k| (Z_k r$ is the calculated quasiparticle residue). In Fermi-liquid theory of the free electron gas, one always finds $|\text{Im} \Sigma^0_k| \ll E_2(k)$ sufficiently close to the Fermi level, which signifies the existence of coherent quasiparticles with a long lifetime $\tau = 1/\Gamma_k$. At low $T$ and very close to the Fermi level, the interactions of the free electrons with an Einstein phonon lead to an exponential decrease of $\text{Im} \Sigma^0_k \sim \exp(-\beta \omega_0)$ as $T \to 0$. This means that for small excitation energies $\varepsilon$ close to the Fermi level there always exists a low-temperature range for which $|\text{Im} \Sigma^0_k| \ll \varepsilon$, which is consistent with the concept of Landau quasiparticles [31]. In contrast to this, in the ordered system the nodal electron–phonon scattering near the Fermi level produces a substantial $\text{Im} \Sigma^{ph}_0 \sim g^2 \log |\Omega^2_k/\omega| + \log |\Omega^2_k/\omega + \eta N(t, t', V)| \exp(-\beta \omega_0)$, where the function $\eta N(t, t', V)$ results from a quadratic expansion of $\varepsilon^2(k)$ in the vicinity of the nodal van Hove singularities. Due to the large dominant contributions of these van Hove singularities, in the temperature range $kT/\tau \ll 0.03$ which is of relevance for the cuprates, we obtain $|\text{Im} \Sigma^{ph}_0| > \varepsilon$, where the excitation energies $\varepsilon \ll kT$ are located close to the Fermi level. The comparison of the corresponding $\text{Im} \Sigma^0_k$ in the free and in the ordered electron gas is presented in figure 4. As a consequence, the strong increase of $\text{Im} \Sigma^0_k$ and of the electron scattering rate $\Gamma_k$ leads to a violation of the condition for well defined quasiparticles in the electron-ordered system. We emphasize that the ultimate reason for such a high electron–phonon scattering rate is the anomalously high density of the nodal electronic states related to the flat electron dispersion.

The incoherent NFL nodal state which develops for $E_p > E_0^x$ is in striking contrast to the Fermi-liquid state with its distinctive hole-type quasiparticle excitations for $E_p < E_0^x$ (dispersion similar to $\varepsilon_2(k)$ in figure 2(a)). The existence of the singular multipole structure of the one-electron Green functions and of the anomalously high scattering rates has been confirmed by more elaborate calculations, which include higher-order vertex corrections in the electron–phonon contribution to the electronic self-energy. As follows from these calculations, the inclusion of vertex corrections leads to slight shifts of the low-energy van Hove singularities in $\text{Im} \Sigma^{ph}_0$, without qualitative changes in the nature of the NFL state. In fact, evidence for an NFL behavior along the nodal region, which is even unaffected by the onset of superconductivity, has been found for optimally doped BSCCO [1], in agreement with our findings.

3. Electronic phase separation state

As a state of electronic phase separation would imply a superposition of the ordered and disordered electronic states on the analyzer, the resulting ARPES intensity contains a combination of the disordered ($A^d(k, \omega)$) and the ordered ($A^o(k, \omega)$) spectral functions: $I(k, \omega) = (c_{ord} A^o(k, \omega) + c_{dis} A^d(k, \omega)) f(\omega)$. Here the coefficients $c_{ord}$ and $c_{dis}$ refer to the ordered and disordered surface fractions. We assume that the ratio $c_{dis}/c_{ord}$ depends linearly on doping, so that $c_{dis}/c_{ord} \sim x$. The consequential dominance of the charge disordered contribution for larger $x$ is consistent with the...
expansion of metallic disordered regions at higher doping levels as observed in [16].

Figure 5 shows the spectral intensity map in the nodal direction calculated for $x = 0.06$. One can clearly distinguish two structures on this map. The needle-shaped high-intensity structure (1) is related to the free electronic band $\varepsilon_0$. Moreover, a symmetric broad feature (2) appears, originating from the ordered state. Structure (2) is located on the plot in the range between $\omega_o^2 \approx -60$ meV and $\omega_o^1 \approx -10$ meV. The high spectral intensity of structure (2) appears due to the singular character of $E_2(k)$ in the vicinity of the point $S_1$ of the nodal $n_1$-region in figure 2(c). Near the Fermi level, the NFL region $n_1$ forms due to an additional van Hove singularity at $\omega = \omega_3$ (see figure 2(b)), which results in multiple poles of the electronic propagator $g_2$ and in high spectral intensity in the region between $\omega_o^2$ and $\omega_o^1$. In the intensity $I(k, \omega)$, the ordering-induced feature (2) is superimposed with the free band (1), which produces a break-up of the intensity at $\omega \approx -50$ meV, shown in more detail in the bottom panels of figure 5. This break-up of the spectral weight into a quasiparticle peak along the needle (1) and a broad high-intensity structure (2) is typical for the nodal ARPES measurements observed in a wide variety of cuprate compounds. As the break-up is produced by the charge-order gap, it should be considered as a direct manifestation of the local electron order in the cuprates.

We note that the broad low-energy feature (marked as part (2) in figure 5) extends in the $k$-direction, which is in contrast to the experimental observations. The reason for such a wide spread of this charge-order-induced feature (2) is a simplified approximation for the characteristic phonon frequency $\omega_0 = \text{const}$. Within an Einstein approximation the electron–phonon contribution to the electronic self-energy $\Sigma_{1ph}(\omega)$ does not depend on the momentum vector $k$, which in turn leads to a wide spread of the self-energy part $\Sigma_2$ and of the corresponding high spectral intensity region in the $k$-space. To obtain better agreement with experiment, where the high-intensity structure is localized in $k$-space, one needs to consider a realistic description for the phonon dispersion, which is beyond the scope of our current studies.

Due to the doping dependence of $\Sigma_2(k, \omega)$, the high-intensity features also depend strongly on doping, which is demonstrated in figure 6. The high-intensity structure, at about
−50 meV for \( x = 0.04 \), becomes smoother with increasing \( x \) up to \( x = 0.11 \) and in the overdoped regime (\( x = 0.2 \) in figure 6). Moreover, as the local order is destroyed with increasing \( T \), the contribution of ordered domains to \( I(k, \omega) \) will decrease, which can explain the fact of vanishing kinks for higher \( T \) discussed in [5, 32].

The interpretation of ARPES intensities in terms of electronic inhomogeneities is not only a possible scenario for cuprates, but can also be applied for manganite compounds. In manganites, a significant electron–phonon coupling and an electronic phase separation of ferromagnetic metallic and charge ordered states is strongly supported by numerous experimental and theoretical studies [33–35]. Consequently, the nodal ARPES features reported for \( \text{La}_{1-x}\text{Sr}_x\text{Mn}_2\text{O}_7 \) [36] can also be explained by the proposed approach for inhomogeneous states.

In conclusion, we show that coupling to phonon modes leads to different spectral properties in ordered and disordered electronic states. It appears to be the key mechanism responsible for the main features detected in ARPES experiments of high-\( T_c \) cuprates. The broad character of the ARPES features can be related to the incoherent nature of the nodal non-Fermi-liquid state, which forms essentially due to the electron–phonon coupling. In this context, the implications of electron–phonon coupling in charge ordered systems demonstrate in a novel way how collective modes can qualitatively change the fundamental properties of the electron liquid.

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