SOLAR NEUTRINOS

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ABSTRACT

The status of solar neutrino experiments and their implications for both nonstandard astrophysics (e.g., cool sun models) and nonstandard neutrino properties (e.g., MSW conversions) are discussed. Assuming that all of the experiments are correct, the relative rates observed by Kamiokande and Homestake are hard to account for by a purely astrophysical solution, while MSW conversions can describe all of the data. Assuming the standard solar model, there are two allowed regions for MSW conversions into $\nu_\mu$ or $\nu_\tau$, with the non-adiabatic solution giving a better fit than the large angle. For conversions into sterile neutrinos there is only a nonadiabatic solution. Allowing both MSW conversions and nonstandard astrophysics, the data simultaneously determine the temperature of the core of the sun to within five percent, consistent with the standard solar model prediction. The implications of the atmospheric $\nu_\mu/\nu_e$ ratio and of a hot component of the dark matter are briefly discussed, and the expectations of theoretical models motivated by grand unification are summarized.

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1. Introduction

We now have a good idea of the number of neutrinos. From the $Z$ width $\Gamma_Z$ measured at LEP \[1\] one has $N_\nu = 3.04 \pm 0.04$, where $N_\nu$ is the number of active (normal) neutrinos of mass less than $M_Z/2$. There is a complementary limit $N'_\nu < 3.3$ from nucleosynthesis. This applies only to neutrinos of masses less than about 20 MeV \[2\], but includes sterile neutrinos (neutrinos that do not have interactions except for mixing) for a wide range of masses and mixings \[3\].

We still do not know whether any of the neutrinos have mass. There is no compelling evidence for nonzero masses from laboratory experiments. However, most new physics beyond the standard model predicts $m_\nu \neq 0$ at some level. Usually, $m_\nu \sim v^2/M$ where $v$ is the weak interaction scale and $M$ is the scale of new physics \[4\]. Therefore, small neutrino masses are a probe of physics at very high scales.

On the other hand, there are indications of possible neutrino masses from non-laboratory experiments. The deficit of solar neutrinos as observed by Homestake \[5\], Kamiokande \[6\], GALLEX \[7\], and SAGE \[8\] suggests that either there is nonstandard astrophysics or that there are new properties of the neutrinos, such as nonzero masses. An overall deficit could be accounted for by a lower temperature of the core of sun than is usually predicted. However, the relative rates of the Homestake and Kamiokande experiments strongly disfavor an astrophysical solution, suggesting that there are new properties of the neutrinos (assuming that the experiments are correct within the stated uncertainties).

Similarly, the deficit in the ratio of atmospheric $\nu_\mu/\nu_e$ as observed by the Kamiokande \[9\] and IMB \[10\] collaborations in their contained events suggests the possibility of neutrino oscillations \[11\]. No anomaly is observed in the stopping or through-going upward muons. However, these are more sensitive to absolute theoretical calculations of the fluxes than is the $\nu_\mu/\nu_e$ ratio.

Finally, there is still the possibility of a hot component of the dark matter of the universe. For $m_{\nu_\tau} \sim (1 \sim 28)$ eV the tau neutrino would be important. Smaller masses would be cosmologically irrelevant, while larger ones would over-close the universe. Hot dark matter has long been out of favor as the only component of dark matter because it is difficult to explain how small structures such as galaxies could have formed in the time available. One would therefore need to combine the hot dark matter with other ingredients such as cos-
Figure 1: Neutrino oscillation parameters excluded or suggested by various observations. The combined MSW solar neutrino solution and the region suggested by the atmospheric neutrinos are indicated. The predictions of various theoretical models are also shown.

| Theory         | SSM (BP)     | SSM (TC)     |
|----------------|--------------|--------------|
| Homestake (Cl) | $8 \pm 1$ SNU | $6.4 \pm 1.3$ SNU |
| Kamiokande     | $1 \pm 0.14$ (arb units) | $0.77 \pm 0.20$ |
| gallium        | $132 \pm 7$ SNU | $125 \pm 5$ SNU |

Table 1: Predictions of Bahcall-Pinsonneault (BP) and Turck-Chièze (TC) for the solar neutrino fluxes. All uncertainties are at one standard deviation, which for BP is defined simply as (total range)/3.

mic strings, cold dark matter, or decaying neutrinos to seed galaxies. Such a cocktail has been suggested by a number of authors as one explanation of the COBE and galaxy distribution data.

2. Solar Neutrinos

The predictions for the fluxes in the Homestake, Kamiokande, and gallium experiments from two recent theoretical studies are shown in Table 1. There is reasonable agreement between them, especially for the gallium experiments. However, the Bahcall-Pinsonneault (PB) calculation predicts a somewhat higher $^{8}B$ flux than that of Turck-Chièze (TC)

The current experimental results are compared with the theoretical predictions in Table 2. In each case there is a significant reduction from the predictions of both standard solar models.
Table 2: The observed rates, and the rates relative to the standard solar model (SSM) calculations of BP and TC.

|                  | Rate            | Rate/SSM (BP) | Rate/SSM (TC) |
|------------------|-----------------|---------------|---------------|
| Homestake        | 2.1 ± 0.3 SNU   | 0.26 ± 0.04   | 0.33 ± 0.05   |
| Kam-II (1040 days) | 0.47 ± 0.05 ± 0.06 | 0.56 ± 0.07 ± 0.06 | 0.65 ± 0.09   |
| Kam-III (395 days) | 0.50 ± 0.07     | 0.67 ± 0.15   |
| Kam-II + III (prelim syst.) | 83 ± 19 ± 8 SNU | 0.63 ± 0.14   | 0.67 ± 0.15   |
| GALLEX           | 58 ± 17 ± 14 SNU | 0.44 ± 0.19   | 0.47 ± 0.20   |
| SAGE (90 + 91)   | 71 ± 15 SNU     | 0.54 ± 0.11   | 0.57 ± 0.12   |
| GALLEX + SAGE    | 71 ± 15 SNU     | 0.54 ± 0.11   | 0.57 ± 0.12   |

There are several resolutions of the discrepancy. The first is that something is wrong with the astrophysics. Here one must distinguish between the standard solar models, which are in excellent agreement with all other data but which are far from the observed neutrino fluxes, and nonstandard solar models which involve totally new ingredients [15, 16, 17]. Another possibility is that there is something wrong with our understanding of the neutrinos. To my mind the simplest and most likely possibility is that of MSW conversion [18], which can be considered both within the SSM and within nonstandard solar models [11, 17, 13, 20, 21]. One can also consider MSW oscillations with three flavors [22], vacuum oscillations [23, 24, 21], or more exotic possibilities, such as the interesting suggestion of gravitational effects associated with a violation of the principle of equivalence [23, 26], neutrino decay [27], or large magnetic moments [28]. The latter would probably be needed if there is indeed a time dependence, but sufficiently large moments are apparently in conflict with limits from red giants [29]. A final possibility is that one or more of the experiments is incorrect or has significantly underestimated its uncertainties. For example, if one discounted the Homestake result astrophysical solutions or larger ranges for $\Delta m^2$ would be allowed. However, I have no reason to doubt any of the experiments, so I will restrict my comments to astrophysical solutions and to the simple MSW solution.

3. Astrophysical Solution

The standard model of Bahcall and Pinsonneault [13] incor-
porates helium diffusion, new $S_{17}$ values, is in good agreement with other calculations when the same parameters are used, and it is in reasonable agreement with helioseismology and other observations. The uncertainties in the SSM have been estimated using Monte Carlo methods by Bahcall and Ulrich [30], who considered uncertainties in the parameters relevant to the opacities, nuclear cross-sections, etc. As can be seen in Table 1, the standard solar model is not in agreement with the data for any reasonable range of the uncertainties, and is therefore excluded.

Still possible, however, is some type of nonstandard solar model (NSSM), which may differ from the SSM by entirely new physics inputs such as weakly interacting massive particles (WIMPs), a large core magnetic field, core rotation, etc. Most of these models affect the solar neutrinos by leading to a lower temperature of the core of the sun, i.e., $T_c < 1$, where $T_c = 1$ corresponds to the standard solar model. Many of these models are rather ad hoc. More important, all reasonable models lead to a larger suppression of the Kamiokande counting rate, which is essentially all $^8B$, than that of Homestake, which has in addition a nontrivial component of $^7Be$ neutrinos. This is in contrast to the experimental data, and therefore almost all nonstandard solar models are excluded unless one or more of the experiments is incorrect.

This can be argued quantitatively, following the treatment of Bludman, Hata, Kennedy and myself [16]. For this, and also for the following discussion of the astrophysical uncertainties within the MSW solution, I will use a simplified treatment of the uncertainties. The many astrophysical effects are parameterized by an arbitrary core temperature $T_c$, with the standard solar model corresponding to $T_c = 1 \pm 0.006$. Nuclear physics uncertainties in the production and detection cross-sections are also included. This simplified error treatment reproduces quite well the uncertainties in the SSM and generalizes to the NSSM.

3.1. Cool Sun Models

In cool sun models we assume that because of some new physics input the core temperature can be considerably below the range $T_c = 1 \pm 0.006$ of the standard solar model. Following Bahcall and Ulrich [30] the temperature dependence of the dominant flux components is $\varphi(^8B) \sim T_c^{18}$, $\varphi(^7Be) \sim T_c^8$, and $\varphi(pp) \sim T_c^{-1.2}$. Actually, these are derived assuming small variations around the standard solar model.
If one allows for much larger deviations, as will be needed to account for the observed deficits, these exponents can only be regarded as qualitative. The most questionable is the $pp$ rate. The exponent of $-1.2$ is not a good approximation when one gets very far away from the SSM. It is more realistic to assume that the $pp$ flux is reduced by a factor $f(pp)$ which is chosen so that the total solar luminosity remains constant.

The expected counting rates $R$ for each experiment relative to the expectations of the standard solar model are then

$$
R_{cl} = 0.26 \pm 0.04 = (1 \pm 0.033)[0.775(1 \pm 0.10)T_e^{18} + 0.150(1 \pm 0.036)T_e^8 + \text{small}] \\
R_{Kam} = 0.50 \pm 0.07 = (1 \pm 0.10)T_e^{18} \\
R_{Ga} = 0.54 \pm 0.11 = (1 \pm 0.04)[0.538(1 \pm 0.0022)f(pp) + 0.271(1 \pm 0.036)T_e^8 + 0.105(1 \pm 0.10)T_e^{18} + \text{small}]$$

In (1) the temperature dependence of the individual flux components is displayed. The uncertainties which multiply the overall rates are from the nuclear detection cross-sections, and those which multiply the individual flux components are cross-section uncertainties for the relevant reactions in the sun. The latter must, of course, be correlated from experiment to experiment.

The best fit is for $T_e = 0.92 \pm 0.01$. This requires an enormous deviation from the SSM. Even more disturbing is that it is a terrible fit: $\chi^2 = 20.6$ for 2 d.f., which is statistically excluded at the 99.9% cl. If we accept the experimental values, a cool sun model simply cannot account for the data, because one expects $R_{Kam} < R_{cl}$, contrary to what is seen [16].

This conclusion is much more general than the specific exponents assumed above. For example, if one simply assumes that $\varphi(B) \sim T_e^{n_B}$ and $\varphi(Be) \sim T_e^{n_{Be}}$, then for any reasonable values of the exponents (e.g., $n_B = 27$, $n_{Be} = 15$, or even arbitrary exponents satisfying $n_{Be} \leq n_B$) one again finds that the data cannot be fit. We have also found [16] that even if one doubles all of the uncertainties in the nuclear cross-sections one is not able to describe the data, because the cross-section uncertainties are strongly correlated from experiment to experiment. Other NSSMs, such as models with an ad hoc reduction of the $^8B$ flux [13] or explicit WIMP and low $Z$ models, also fail for similar reasons. The conclusion is that as long as the stated experimental uncertainties are taken seriously the nonstandard solar models
are not a likely explanation of the observations. They would, however, be possible if the true flux that should be observed in the Homestake experiment were $\geq 3$ SNU (as opposed to $2.1 \pm 0.3$), but only if one significantly stretches the astrophysical uncertainties [17].

4. MSW Conversions

4.1. Standard Solar Model

There have been a number of recent studies of the MSW [18] solution [16, 17, 19, 20, 21]. Here I will follow the work in [16], which uses the Bahcall-Pisonneault [13] production distribution $\phi_i(r)$ (for the $i$th flux component as a function of the distance from the center), electron density $n_e(r)$ (needed to calculate the MSW conversion), and neutron density $n_n(r)$ (which enters for oscillations $\nu_e \rightarrow \nu_s$ into sterile neutrinos). We have carefully considered theoretical uncertainties; in particular, we have parameterized the SSM uncertainties using the core temperature $T_c = 1 \pm 0.006$ relative to the SSM, which is due namely to uncertainties in opacities, plus the production and detection cross-section uncertainties as described earlier. These agree well with the stated uncertainties calculated by Bahcall and Pisonneault for the standard solar model. There are additional theoretical uncertainties involved in the MSW conversion itself, including the uncertainties in $\phi_i(r)$ and $n_e(r)$. We have made various estimates of these, which turn out to be small.

For Kamiokande the detector resolution, threshold corrections, and neutral current scattering (for the case of conversion into $\nu_\mu$ or $\nu_\tau$) have been included. The effect of reconversion in the earth has not yet been incorporated. This will slightly affect the shape of the allowed large-angle solution. The theory errors are significant but not dominant. It should be emphasized that one must include the correlation between the theoretical uncertainties in the various experiments [3]. For example, whatever value $T_c$ takes, it is the same for all experiments.

Another technical comment concerns how one estimates confidence level contours. We have used the $\chi^2$ method. However, the $\chi^2$ method can only be rigorously justified for error regions that are Gaussian in the parameters, and it is difficult to interpret when there is more than one allowed region. For this reason the interpretation in terms of confidence levels should only be considered qualitative. An-

\footnote{A recent calculation [33] which ignored the correlations obtained unrealistically large theoretical uncertainties.}
Figure 2: Allowed regions for MSW conversions of $\nu_e \rightarrow \nu_\mu$ or $\nu_\tau$, from [10]. The 90\% c.l. ($\Delta \chi^2 = 4.6$) regions allowed by the Homestake, Kamiokande, and gallium experiments and by the combined fit are shown. The astrophysical and nuclear uncertainties are included. The corresponding allowed region for oscillations into sterile neutrinos has a similar small-angle non-adiabatic solution, but no large-angle solution.

other possibility is to simply overlap the allowed regions (at a given confidence level) for the different experiments [21]. This method has two drawbacks. In some circumstances (e.g., when two allowed bands cross) it underestimates the true uncertainties. On the other hand, it sometimes admits solutions which are only marginally compatible with each of several experiments and which are therefore really quite bad fits.

For oscillations into active neutrinos ($\nu_\mu$ or $\nu_\tau$) there are two solutions allowed by all the data, the non-adiabatic (small mixing angle) and the large-angle solutions, as can be seen in Figure 2. The non-adiabatic solution gives a much better fit. In this region there is more suppression of the intermediate energy $^7$Be neutrinos, accounting for the larger suppression seen by Homestake. The large-angle fit is much poorer, corresponding to $\chi^2 = 3.8$ for 1 df, because there the survival probability varies slowly with neutrino energy. One can also consider the possibility that the $\nu_e$ is oscillating into a sterile neutrino. In this case the oscillation probabilities are changed slightly due to the (small) neutron density in the sun. A much more significant effect is that the sterile neutrino cannot undergo neutral current interactions in the Kamiokande experiment, aggravating the difference between Homestake and Kamiokande. The result is that there is no large-

\footnote{The large-angle solution would extend down to $\Delta m^2 \sim 10^{-7}$ eV$^2$ if one used the overlap method [21], but the $\chi^2$ in that region is very large.}
angle solution at 90% C.L. (it only enters at 95% confidence level). There is a non-adiabatic solution but even that yields a relatively poor fit $\chi^2 = 3.6$ for 1 df.

The conclusion is that MSW oscillations, combined with the SSM and corresponding uncertainties, give an excellent description of the data, especially the non-adiabatic solution for active neutrinos. The general range of $\Delta m^2$ is consistent with the expectations of seesaw models motivated by grand unification. However, the specific regions of leptonic mixings are not in agreement with the naive expectation $V_{\text{lepton}} = V_{\text{CKM}}$, which is predicted by some of the simplest models.

4.2. MSW and Non-Standard Solar Model

It is also interesting to consider MSW oscillations for an arbitrary core temperature $T_c$, that is for NSSM. (We will assume the same nuclear physics uncertainties as in the SSM case.) One now has three parameters, $T_c$, $\sin^2 2\theta$, and $\Delta m^2$. There are sufficient constraints to determine all three [16]. There is an expanded non-adiabatic solution that corresponds to that in the SSM. In addition, the core temperature is determined; one finds $T_c = 1.02^{+0.03}_{-0.05}$ at 90% C.L. Similarly, there is a large-angle solution with $T_c = 1.04^{+0.03}_{-0.04}$. Thus the core temperature is measured by the solar neutrino experiments, even allowing for the complication of MSW oscillations. In particular, it is consistent with the standard solar model prediction $T_c = 1 \pm 0.0057$, although at present the solar neutrino data only allows a precision of $\sim 5\%$. Allowing for an arbitrary $T_c$ the MSW parameter range broadens [16]. For lower $T_c$ one obtains smaller $\sin^2 2\theta$ for the non-adiabatic solution, while the large angle solution disappears. For larger temperatures the regions move closer to each other, and the two regions are even connected by a narrow neck at the 95% C.L.

4.3. Future

In the future one will want to verify the MSW, uniquely determine which solution is relevant, distinguish between conversions into active or sterile neutrinos, etc. To do this one needs better statistics in the gallium experiments (as well as calibrations), and new experiments with high event rates. The SNO experiment should observe spectral distortions if the non-adiabatic solution is correct, measure

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5 The large angle solution for sterile neutrinos is also most likely excluded by nucleosynthesis arguments, while non-adiabatic parameters are allowed.

6 Of course, one of the original motivations for the solar neutrino experiments was to probe the core of the sun.
the ratio of neutral current to charged current (NC/CC) events, and
could even observe oscillations into $\bar{\nu}_e$, which is not predicted by the
SSM or MSW. The SuperKamiokande experiment should also be able
to observe spectral distortions, while Borexino should be sensitive
to the $^7\text{Be}$ line and possible time dependence. There may also be
an ICARUS experiment sensitive to the spectrum and some NC/CC.
The implications of these are that a spectral distortion could essen-
tially unambiguously identify non-adiabatic MSW conversions \[32\].
Observations of the $^7\text{Be}$ line are sensitive to nonstandard neutrino
properties. The NC/CC ratio distinguishes active neutrinos from both
sterile neutrinos and astrophysical solutions, and for active neutrinos
the NC rate would give an independent measure of $T_c$. It is important
to look for seasonal variations (expected from vacuum oscillations),
day/night effects (relevant to some parameter regions of MSW), and
correlations with the sunspot cycle (characteristic of magnetic
moments). It is also useful to look for $\bar{\nu}_e$, which would be indicative of
neutrino decay or spin-flavor oscillations.

5. Atmospheric Neutrinos

The predicted fluxes $\nu_\mu$ and $\nu_e$ produced by the interactions of
cosmic rays in the atmosphere are uncertain by around 20%. However,
the ratio $\nu_\mu/\nu_e$ is believed to be accurate to $\sim 5\%$ \[1\]. There are additional uncertainties associated with interaction cross-sections, particle
identification, etc. The Kamiokande and IMB groups have observed a
deficit in the ratio of contained muon and electron events

$$\frac{(\mu/e)_{\text{data}}}{(\mu/e)_{\text{theory}}} = \begin{cases} 0.65 \pm 0.08 \pm 0.06, & \text{Kamiokande \cite{9]} } \\ 0.54 \pm 0.05 \pm 0.12, & \text{IMB \cite{10}} \end{cases}$$

(2)

This effect, if real, suggests the possibility of $\nu_\mu \rightarrow \nu_\tau$ or possibly
$\nu_\mu \rightarrow \nu_e$. It probably is not compatible with sterile neutrino oscillations
$\nu_\mu \rightarrow \nu_s$, because for the relevant parameter range one would have a
clear violation of the nucleosynthesis bound \[3\]. From Figure \[1\] one
sees that the oscillation hypothesis \[7\] requires a mass range $\Delta m^2 \sim $(10$^{-3}$–1) eV$^2$, larger than that relevant to the solar neutrinos, and
large mixing angles such as $\sin^2 2\theta \sim 0.5$.

On the other hand, the IMB collaboration \[10\] has measured
the flux of upward-going muons, and has concluded from the nonob-
servation of any deficit that most of the range suggested by the con-

\footnote{An interesting alternative involves an enhancement in the number of positrons due to proton
decay $p \rightarrow e^+\nu\bar{\nu}$ in the detector \[33\]. However, this is disfavored by the observed spectrum.}
tained events is excluded. However, the upward-going rate is sensitive to the absolute calculation of the flux. Moreover, it is sensitive to the deep inelastic cross-section needed to produce the muon, and recently it has been argued [11] that the Owens cross-sections [34], which give a better fit to the accelerator data than the EHLQ [34] (used by IMB), lead to a weaker constraint compatible with the contained events. IMB also studied the ratio of upward through-going to stopping muons, which is more reliable than the absolute calculation; this excludes the low $\Delta m^2$ part of the region suggested by the contained events, but allows most of the range above $\Delta m^2 > 10^{-2}$ eV$^2$. The situation is clearly confused. Oscillations are suggested but certainly not proved, and there are still many uncertainties associated with the fluxes, cross-sections, and particle identification.

6. Implications

It is hard to make concrete predictions for neutrino masses in most models. However, the $\Delta m^2$ range suggested by the solar neutrinos is compatible with the general range expected in quadratic up-type seesaw models, such as in grand unified theories [16], for which $m_{\nu_i} \sim m_{u_i}^2/M_N$, where $u_i = u, c, t$ and $M_N$ is the heavy neutrino mass. For $M_N \sim 10^{11} - 10^{16}$ GeV one obtains the appropriate mass range, for oscillations into $\nu_\tau$ for $M_N \sim 10^{16}$ GeV, and into $\nu_\mu$ for $M_N \sim 10^{11}$ GeV. However, the simplest models predict equal lepton and quark mixing angles, $V_{\text{lepton}} = V_{\text{CKM}}$, which is not satisfied by the data unless $T_c$ is far from the SSM [16].

The various hints suggest two general scenarios. One could have $\nu_e \to \nu_\mu$ in the sun for $m_{\nu_e} \ll m_{\nu_\mu} \sim 3 \times 10^{-3}$ eV, with $\nu_\tau$ a component of the dark matter ($m_{\nu_\tau} \sim \text{few eV}$). This pattern is compatible with the mass predictions of GUT-type seesaws. However, in this scenario there is no room for oscillations to account for the atmospheric neutrinos. A separate possibility is that again $\nu_e \to \nu_\mu$ in the sun, with $\nu_\mu \to \nu_\tau$ oscillations with $m_{\nu_\tau} \sim (0.1 - 0.6)$ eV for the atmospheric neutrinos. In this case there would be no room for hot dark matter. This second solution requires large leptonic mixings, $\sin^2 2\theta_{\nu_\mu\nu_\tau} \sim 0.5$. This is certainly allowed, but is not what one would expect in most models: if there is a large hierarchy of masses one typically expects $\sin^2 2\theta \ll 1$, while $\sin^2 2\theta \simeq 1$ if the neutrinos are nearly degenerate.

Given the constraints from LEP, nucleosynthesis, etc., it is hard

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8There may be a broader class of solutions if one allows for three-neutrino oscillations [22].
to find a reasonable scenario which would combine both solar and atmospheric neutrino oscillations with HDM. This is mainly because there are just not enough neutrinos. About the only viable scenario is to invoke non-adiabatic oscillations into a sterile neutrino in the sun, and to assume \( m_{\nu_s} \sim m_{\nu_s} \sim \text{few eV} \) to provide hot dark matter. One would need nearly degenerate neutrinos so that \( \Delta m^2 = m_{\nu_s}^2 - m_{\nu_s}^2 \sim 10^{-1}\text{eV}^2 \) for the atmospheric neutrino oscillations. This pattern is possible but seems very contrived. Of course, if one relaxes some of the experimental constraints the range of possibilities increases. For example, if one ignored the Homestake experiment one could have \( \nu_e \leftrightarrow \nu_\mu \) for both solar and atmospheric neutrinos, with \( \Delta m^2 \sim 10^{-2}\text{eV}^2 \). However, in this case much of the rationale for a nonastrophysical solar solution would also disappear.

7. Viable Models Inspired by Coupling Constant Unification

There has been a recent revival of interest in grand unification, inspired by the success of the unification of the coupling constants \[35\] in the supersymmetric extension of the standard model. The various models have implications for neutrino mass, and predict the right general mass range for the MSW solution. However, the simplest models do not agree with the data in detail, at least not assuming the SSM.

Let us first consider the old supersymmetric grand unified theories \[16\]. These are the type of models that were popular ten years ago, which allowed large and complicated Higgs representations, such as \( SO_{10} \) models with both 10 and 126-dimensional Higgs multiplets. If only the \( \varphi_{10} \) generates the masses for the quarks and charged leptons, then one has the simple relation \( m_D = m_u, m_e = m_d \) for the mass matrices at the large scale, where \( m_D \) is the matrix of Dirac neutrino masses. If one also introduces a \( \varphi_{126} \) whose only role is to generate a large majorana mass for the new right-handed neutrinos, then one obtains typical see-saw predictions

\[
\begin{align*}
    m_{\nu_i} &\sim c_i \frac{m_u^2}{M_{N_i}} \\
    m_{N_i} &\sim (10^{-2} - 1) M_X,
\end{align*}
\]

where \( c_i \sim 0.05 - 0.1 \) is a radiative correction and \( M_X \sim 10^{16} \text{GeV} \) is the unification scale \[16, 35\]. For these assumptions

\[
m_{\nu_e} \leq 10^{-11} \text{eV}
\]
This is the correct mass range for $\nu_e \rightarrow \nu_\tau$ in the sun. However, for a wide range of parameters one also expects

$$V_{\text{lepton}} \sim V_{\text{CKM}},$$

implying a smaller mixing angle than is favored by the data. A more serious problem is that the same models predict $m_d/m_s = m_e/m_\mu$, which fails by an order of magnitude [30].

Within the same class of theories there have been more sophisticated recent models in which the $\varphi_{126}$ not only generates $M_N$, but also contributes to the quark, charged lepton, and (possibly) Dirac neutrino masses. For example, Dimopoulos, Hall, and Raby [37] have revived an interesting model of Georgi and Jarlskog [38], in which both $\varphi_{10}$ and $\varphi_{126}$ contribute and for which, in addition, there are many discrete symmetries. These are arranged so that there are a number of zeros in the mass matrices, and so that each non-zero element is generated by a unique representation. One can then fix up the problematic mass relation, replacing it by the acceptable $9m_e/m_\mu = m_d/m_s$.

The model leads to interesting predictions for the CKM matrix and $m_t$. The neutrino mixing angle predictions are somewhat modified with respect to (5), but are still not in perfect agreement with the data unless $T_c > 1$. Other interesting models and calculations are in [39, 40, 41, 42].

One can also consider the new supersymmetric grand unified theories inspired by superstrings. These differ in that most superstring compactifications do not lead to large Higgs representations. In particular, in $SO_{10}$ there is no $\varphi_{126}$. In such models there is no way to generate $m_N$, at tree-level and it is not obvious how to even have a seesaw. However, the relevant right-handed neutrino masses could be generated by effective non-renormalizable operators left over from the underlying theory [44]. For example, one may have an effective operator [45]

$$L_{\text{eff}} = \frac{C}{m_C} \bar{N}_L N_R \Phi \Phi,$$

where $m_C \sim 10^{18}$ GeV is the compactification scale and $\Phi$ is a 10-plet of $SO_{10}$. For $\langle \Phi \rangle \sim M_X \sim 10^{16}$ GeV (the unification scale),

$$M_N \sim \frac{C |\langle \Phi \rangle|^2}{m_C} \sim 10^{-2} C |\langle \Phi \rangle|.$$
An interesting example are large-radius Calabi-Yau spaces, for which
\[ C \sim e^{-R^2/\alpha'}, \quad \frac{M_X}{m_C} \sim \frac{1}{\sqrt{2R^2/\alpha'}} \]  \hspace{1cm} (8)
where \( R' \) is the radius and \( \alpha' \) is the string tension, yielding an intermediate scale \( M_N \sim 10^{11} \) GeV, and
\[ m_{\nu_e} \sim 10^{-7} \text{ eV} \quad m_{\nu_\mu} \sim 10^{-3} \text{ eV} \quad m_{\nu_\tau} \sim 10 \text{ eV}. \] \hspace{1cm} (9)
One could then have \( \nu_e \to \nu_\mu \) in the sun and the \( \nu_\tau \) would be a candidate for the HDM. There is no quantitative prediction for the leptonic mixing angles.

The crucial feature of this type of seesaw is the intermediate scale \( m_N \sim 10^{11} \) GeV, considerably lower than a GUT scale. This may also come about in other ways. For example, in ordinary (non-supersymmetric) grand unified theories it is possible that there are intermediate scales associated with stages of the symmetry breaking. For an intermediate scale in this range one would typically reproduce the above predictions for the neutrino mass \[16\]. However, in the simplest versions one might again have trouble with the prediction \( V_{\text{lepton}} \sim V_{\text{CKM}} \).

8. Summary/Conclusions

The relative rates for the various solar neutrino experiments are more significant than the fact that they are all smaller than the standard solar model prediction, and strongly suggest that astrophysics is not the solution to the solar neutrino problem (unless, of course, one of the experiments is wrong). There are various MSW solutions that agree with the data. Assuming the standard solar model for the initial production, there are two MSW solutions, non-adiabatic and large-angle for oscillations into active neutrinos, with the non-adiabatic favored. For oscillations into sterile neutrinos there is only a non-adiabatic solution. One can also allow the possibility of nonstandard solar models. In a three-parameter fit one obtains \( T_C \sim 1.02^{+0.03}_{-0.05}, \) \textit{i.e.,} the core temperature is determined to within 5\% even allowing for MSW oscillations, and agrees with the predictions of the standard model.

The deficit observed in the ratio \( \nu_\mu/\nu_e \) as measured in contained atmospheric neutrino events suggests \( \nu_\mu \to \nu_\tau \) oscillations, or possibly \( \nu_\mu \to \nu_e \), but not oscillations into sterile neutrinos. However, the in-
interpretation is still clouded by uncertainties in particle ID, flux, and cross-sections.

The $\Delta m^2$ range suggested by the solar neutrinos is compatible with the GUT seesaw. However, $V_{\text{lepton}} \neq V_{\text{CKM}}$ unless the temperature $T_c$ is far from the standard solar model. There is still room for the $\nu_\tau$ to play the role of hot dark matter. One can have simple solutions involving $\nu_e \rightarrow \nu_\mu$ in the sun and $\nu_\tau$ as the HDM, or, alternately, $\nu_e \rightarrow \nu_\mu$ in the sun and $\nu_\mu \rightarrow \nu_\tau$ to account for atmospheric neutrinos. However, it is hard to combine all of these given other constraints, except in a complicated scenario which seems rather contrived.

Clearly, we need more and better solar neutrino experiments, with better statistics, measurements of the spectrum, $NC/CC$ ratio, and the $^7\text{Be}$ line, searches for $\bar{\nu}_e$, and a resolution of whether there is any time dependence. Finally, it is important to continue the program of laboratory experiments. In particular, experiments searching for $\nu_\mu \rightarrow \nu_\tau$ for small $\sin^2 2\theta$ are strongly indicated by the hot dark matter scenario, while $\nu_\mu \rightarrow \nu_\tau$ and $\nu_e \rightarrow \nu_\tau$ searches for small $\Delta m^2$ and large angles are motivated by the atmospheric neutrinos.

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