Analytic approximations, perturbation theory, effective field theory methods and their applications

Vitor Cardoso\textsuperscript{1,2} and Rafael A. Porto\textsuperscript{3,4}

\textsuperscript{1} CENTRA, Departamento de Física, Instituto Superior Técnico, Universidade de Lisboa, Avenida Rovisco Pais 1, 1049 Lisboa, Portugal.

\textsuperscript{2} Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada.

\textsuperscript{3} School of Natural Sciences, Institute for Advanced Study, Olden Lane, Princeton, NJ 08540, USA.

\textsuperscript{4} Deutsches Elektronen-Synchrotron DESY, Theory Group, D-22603 Hamburg, Germany.

Abstract

We summarize the parallel session B4: ‘Analytic approximations, perturbation theory effective field theory methods and their applications’ and the joint session B2/B4: ‘Approximate solutions to Einstein equations: Methods and Applications’, of the GR20 & Amaldi10 conference in Warsaw, July 2013. The contributed talks reported significant advances on various areas of research in gravity.
1 Summary

The GR20 & Amaldi10 meetings in Warsaw are the last ones of the series before the centennial of Einstein’s theory of gravitation. After nearly one hundred years since the theory of General Relativity was discovered, Kerr’s metric describing vacuum (neutral) rotating bodies, are among the few known exact solutions in four-dimensional, asymptotically flat space-times. (However, see [1] for a comprehensive review.) The lack of (generic) solutions to the $N$-body problem in General Relativity highlights the importance of numerical techniques, perturbative methods such as the Post-Newtonian (PN) approximation for comparable masses or black hole perturbation theory for extreme-mass-ratio inspirals (EMRIs), as invaluable tools to solve for gravitational dynamics.

These techniques are of paramount importance in light of the programme to directly observe Gravitational Waves (GWs) on Earth- and space-based detectors, which includes the computation of GW observables for compact binary systems, such as the phase and amplitude of the GWs produced as the system inspirals towards merger, at the moment still awaiting detection by a network of laser interferometers including (advanced) LIGO/Virgo, and the next generation of GW observatories. The payoff of GW science will rely upon the use of accurate signal models (aka templates) from the most promising sources, and therefore analytic and/or combined numerical/analytic efforts for the study of binary dynamics are essential to extract the most information from the data, like measurements of masses and spins to high precision. The inspiral and merger of compact objects are also a natural laboratory to test gravity in the strong-field regime to an unprecedented level. Although not expected to prevent GW detection, the lack of sufficiently accurate templates and/or putative modifications of gravity in this regime may hinder parameter estimation and the ability to correctly map the contents throughout the universe, if not properly modeled.

Finally, with all the above as motivation, the combination of perturbation techniques with numerical methods is providing novel insight into the structure of the non-linear field equations in the strong-field regime, including cosmic censorship violations and interesting connections with high-energy and particle physics.

1.1 Dynamics in General Relativity: Post-Newtonian, Self-force, EFT & EOB methods

For comparable mass non-rotating compact bodies the dynamics of the binary and GW phase have been derived up to 3.5PN order [2]. This includes the computation of the binding energy (and equations of motion) of the orbit, and radiative multipole moments (including tail effects) to next-to-next-to-next-to leading order (NNNLO). (See [3] for a recent review.) The conservative part of the dynamics has been tackled independently by several groups using different techniques, including the computation of the ADM Hamiltonian [4], the equations of motion in harmonic coordinates [5], and the two-body Lagrangian [6] using the Effective Field Theory (EFT) framework introduced in [7]. (The EFT constructed in [7] was coined NRGR due to similarities with EFTs in heavy quark physics.) The analytic computation of the binary dynamics to 4PN order is underway, and progress has been reported during the conference by two independent groups, the NRGR [8] and ADM [9] teams, represented by R. Sturani and P. Jaranowski respectively.
All these derivations are obtained using a point-particle approximation for the constituents of the binary, which entails regularizing divergences that appear due to the uses of δ-like sources. The decoupling of internal structure in the dynamics of the binary is often assumed to hold to 5PN order for non-spinning objects. This is the so-called Effacement Theorem. The proof has been discussed in different approaches, and it is most clearly understood within the EFT formalism, where divergences of the point-particle approximation are tackled by dimensional regularization and renormalized by the introduction of higher derivatives (non-minimal) couplings in the worldline action of the body, which account for their finite sizes. Using the standard power counting rules of the EFT one can show the first such term appears at $O(v^{10})$ once incorporated in the dynamics of the binary with spinless bodies. Moreover, for the case of black holes it has been argued that many of these new terms (electric-type), including the one at 5PN, have vanishing (renormalized) coefficients in four space-time dimensions, but in general do not vanish in higher dimensions. This result was presented by M. Smolkin, and implies that finite size effects for black hole binaries must enter at higher orders (magnetic-type). For other type of objects, such as neutron stars, these terms (aka Love numbers) do not vanish and instead encode the corrections due to the short distance physics that enters in the equation of state, formally starting at 5PN order, although expected to be numerically enhanced. This means GW observations will probe the inner structure of neutron stars. Progress towards a complete description of these effects in the worldline approach of NRGR, and obtaining the (Wilson) coefficients of these new terms form the physics of neutron stars, was reported by J. Steinhoff.

The recent observations (e.g. [18]) which suggest compact objects in binary systems as well as supermassive black holes may be rapidly rotating, and the exciting possibility to test the most twisted properties of General Relativity, has motivated the study of spin effects in the GW waveforms. The parameter estimation from GW waveform including spin was discussed by A. Nielsen [19], and other aspects of GW detection was presented by A. Gupta [20].

Allowing the bodies to rotate complicates matters significantly, and different approaches have been pursued to study spin effects in General Relativity; most notably the extensions of NRGR and the ADM canonical formalism to spinning bodies [21, 22], as well as computations in harmonic gauge. The leading order effects in the dynamics linear in the spin at 1.5PN order were first obtained in the 70’s [23]. The NLO terms were (much) later computed in [24, 25] and re-derived in [26] and [27, 28]. The NNLO equations of motion linear in spin have been computed independently in harmonic [29] and the ADM formalism [30], the details of the former were reported by S. Marsat.

\footnote{It is still possible these terms may be needed as pure counter-terms to regularize divergences.}

\footnote{One somewhat intriguing aspect of these computations is the fact that the (dissipative) imaginary part of the response function for the mass multipole moments of a non-rotating black hole (in four space-time dimensions) to a gravitational external perturbation does not vanish, unlike its real (conservative) part. This implies the traditional (unsubtracted) dispersion relation connecting real and imaginary parts of Green’s functions is not valid. (Something similar occurs in describing dissipative effects in the EFT of fluids using the methods developed in [14, 15], which may not be unrelated in light of the Membrane Paradigm.)}
At quadratic order in the spin not only one encounters spin-spin interactions between the constituents of the binary, but also spin$^2$ terms that encode finite size effects, such as the intrinsic quadrupole moment of a spinning black hole. Unlike spinless bodies, there is no effacement of internal structure for spinning objects since spin$^2$ terms already appear at 2PN, namely at the same order as the leading spin-spin interaction, computed in [23] (see also [31]). Finite size effects due to spin can be readily incorporated in the EFT framework developed in [21] where a worldline effective action approach for spinning bodies in General Relativity was constructed, including higher derivative terms encoding the extendedness of the compact objects. Using the power counting rules of NRGR it is simple to show that one and only one new term is required to 3PN order, whose Wilson coefficient can be determined by matching the metric of an isolated rotating compact object in the full theory and EFT sides. This highlights some of the simplifications behind the EFT approach, rather than working at the level of the field equations, where many (a priori independent) contributions to the stress-energy tensor of a spinning extended object can be shown to derive from the same (finite size) term in the effective action, a scalar under the symmetries. Using the (Feynman) rules derived in [21] the NLO spin-spin and spin$^2$ gravitational potentials for spinning compact bodies were obtained in [32, 33, 34], as well as the equivalent ADM Hamiltonians computed in [35, 36, 37]. Full agreement between these results has been reported [38]. To date the NNLO spin-spin potential/Hamiltonian has been computed in NRGR [39] and ADM [40] frameworks.

The computation of the GW amplitude and phase require obtaining the energy loss in GW emission, which in turn is decomposed in multipole moments. (See [3] and [41] for a derivation in the more traditional and EFT frameworks.) The radiative multipole moments necessary to account for effects linear in spin in the waveforms to NLO were obtained in [42], and [43] in NRGR, and to NNLO in [41]. The details of the latter were reported by A. Bohe. To date, only the NRGR formalism has succeeded in computing the radiative multipole moments necessary to include spin-spin and spin$^2$ terms to 3PN order [43], as well as higher order spin dependent multipoles for the amplitude, up to 2.5PN order [45]. Progress towards obtaining the GW waveforms including all spin effects to 3PN order was the subject of A. Ross presentation.

One important contribution to the GW emission is the so-called tail effects, or scattering of the emitted GW off the binaries background geometry. For spinning bodies the leading order tail contribution linear in spin enters at 3PN [46], and the NLO contribution has been computed in [47] and also reported by A. Marsat. The tail effect introduces novel features, such as the renormalization of the multipole moments and subsequent renormalization group structure of NRGR [48], which allows to resum certain logarithmic UV (short distance) corrections. More generally, the renormalization group structure of the terms in the long-distance effective action allows one not only to resum UV logs, but also to identify logarithmic contribution to the conservative sector, such as the leading logarithmic term to the binding energy at 4PN for spinless bodies [49], derived in [50] together with the NLO contribution at 5PN.
Recently (analytic and numerical) computations of the self-force have received significant attention in the PN community after some higher order PN corrections have been shown to derive from the former at leading order in the mass ratio [50, 51, 52, 53, 54]. This was part of the presentation of J. L. Friedman, who reported on the status of self-force computations for EMRIs. Remarkably, a conservative 5.5PN term has been found in the expansion of the binding energy of the binary [55], and higher order effects have already been computed [56] (see also [57]). As discussed by G. Faye at the conference, this term derives from the ‘tail-of-tail’ contribution to the stress-energy tensor [58]. This correspond to a higher order term in the analysis of [49]. In general, conservative and dissipative terms appear at nPN order with n even and odd respectively for spinless bodies. (Spin changes the parity properties of the equations.) This is related to the time-reversal properties of the different terms. The appearance of a 5.5PN term demonstrates the subtleties of the problem, which it is ultimately dissipative in nature. As discussed in [49] an emitted GW can scatter back off the geometry of the binary which means, for long distance observables, there is a renormalization of the mass/energy of the system. This can therefore accommodate odd PN effects into the conservative quantities found in [50, 51, 52, 53, 54, 55], defined in this way.

In a priori completely different regime, the self-force computations for EMRIs is expected to provide templates for space-based GW observatories operating at much lower frequencies than LIGO/Virgo. Many of the subtleties of computing the waveforms for EMRIs and other aspects of black hole perturbation theory were discussed during the conference, with talks by C. Merlin [59], A. Heffernan [60] and B. Nolan [61], on the regularization issues of the self-force, and T. Hinderer [62] and R. Cole on the importance of resonances.

Analytic computations of the gravitational self-force are difficult, and up to date only second order effects in the mass ratio, i.e. $O(q^2)$ with $q \equiv m/M \ll 1$, are (formally) known [63, 64]. In principle to compare with PN effects the self-force computations referred above should be applied to EMRIs in weak field configurations in non-relativistic motion. In order to relate self-force with comparable mass PN or numerical results, the following replacement is often performed $q \to \nu \equiv \frac{m_1 m_2}{(m_1 + m_2)^2}$, which is accurate to leading order in the mass ratio for EMRIs. Many of these comparisons, most notably with numerical results, are obtained using the self-force to $O(\nu)$. In his talk A. Le Tiec showed that a remarkable agreement is found already at leading order even for comparable mass inspirals, when $\nu \simeq 1/4$ [65, 66, 67]. This may suggest another peculiar feature of gravitational dynamics, for instance for the expansion of the binding energy [66] (and other observables such as the total angular momentum, periastron advance, etc.) in terms of the relative velocity and symmetric mass ratio, where effects naively down by a factor of $\nu \simeq 1/4$ may be further suppressed. It is known $O(\nu^2)$ terms start at 2PN order, and therefore kick in for relativistic motion. However, the agreement remains surprisingly faithful even in the strong gravitational regimes, for $\nu \simeq 1/2$ [66]. This may be related to the ‘Unreasonable Effectiveness of Post-Newtonian Theory’ [68], and might entail cancelations, perhaps of the same type encountered in other computations in gravity [69], see also [70]. (It may also be related to the vanishing of the electric-type finite size effects for binary black holes [10, 12, 13].)
Obtaining higher order self-force effects is therefore of great relevance. One example in which simplifications arise is the ultra-relativistic limit of the self-force problem. As discussed by C. Galley at the conference one can re-organize the standard perturbative expansion in the mass ratio $q$ within the EFT formalism applied to EMRIs [71] in powers of $\lambda = N\epsilon$, where $N = 1/\gamma^2$ and $\epsilon = \gamma q$ with $\gamma$ the boost factor [72]. Using the power counting rules developed in [72] one can show the large $N$ limit reduces the number of terms significantly, similarly to what occurs in gauge theories [73], and higher order effects can be readily obtained at $\mathcal{O}(1/N)$. For instance, in [72] the computation of the self-force was carried up to fourth order in $\lambda$ at leading order in $1/N$, while the regularization of the divergences of the point-particle approximation become trivial in dimensional regularization\footnote{Scaleless integrals are set to zero in dimensional regularization.} since higher order terms in the effective action are suppressed in the large $N$ limit. The computation of the self-force in the EFT approach entails a subtle use of the \textit{in-in} formalism in a classical setting, as discussed by C. Galley [74]. This was also the topic of B. Kol’s talk [76].

Understanding different corners of the perturbative expansion is essential to unravel the underlying features of gravitational dynamics. An attempt to produce analytical waveforms that can be applied to different regimes, used to scan the different parameters of the problem and perhaps shed some light on these matters, including the strong gravitational realm, is the so-called Effective One Body (EOB) approach [77]. Different aspect of the EOB paradigm to describe the inspiral, merger and ringdown phases were discussed during the conference, most notably for spinning binary systems, with talks by Y. Pan and A. Taracchini [78, 79]. EOB waveforms are calibrated with numerical counterparts. Numerical techniques have matured into a very successful area of research (see the proceedings for session B2). As part of the study of binary systems, although without yet a full control of all the cycles, numerical templates are useful not only to describe the merger but also to match models for the waveforms. Hence the meeting also incorporated a joint session B2/B4 which had (among others described above) reports on the status of numerical methods in General Relativity and hybrid approaches. In particular the talks by H. Pfeiffer [80] and S. Husa [81] on numerical simulations for binary black holes, G. Lovelace [82] on simulations for compact binaries with nearly extremal black-hole spins, and S. Kahn on the structure of ringdown modes [83].

1.2 Modified Theories of Gravity and Fundamental Issues

One hundred years of Einstein’s gravity have introduced a potentially dangerous bias towards a theory which may breakdown somewhere between the six-orders of magnitude difference in gravitational potential at the surface of the Sun and at the surface of a neutron star or black hole. Thus, a considerable amount of intellectual effort is being channelled into understanding the consequences of modified theories of gravity and how they may affect gravitational dynamics. One of the most popular models to modify the field equations includes scalar-tensor theories, which give rise to novel effects in the presence of matter [84, 85, 86], but can also
affect vacuum spacetimes. Breakdown of no-hair theorems and scalar-emission by black hole binaries in these theories was discussed by L. Gualtieri [87]. Other examples of quadratic theories are Gauss-Bonnet [88] and Chern-Simons gravity [89, 90]. Limits on the latter coming from GW and pulsar observation probes were discussed by K. Yagi and L. Stein [91, 92]. (For other type of –more fundamental– constraints see [93].) On the other hand, P. Pani discussed perturbations of slowly rotating black holes [94] as well as extensions of General Relativity including minimally coupled massive fields, and how these well-motivated theories can yield interesting smoking-gun effects in strong-field gravity. A particularly interesting consequence is the resulting competitive bounds on the photon mass from observations of supermassive black holes [95].

The sessions were completed with the application of perturbation theory to other fundamental issues in gravity. It was recently conjectured that the event horizon of black holes (and cosmic censorship) could be destroyed by throwing point particles at charged or spinning black holes [96, 97]. J. Rocha discussed the extension of these results to higher dimensional and asymptotically anti-de Sitter spacetimes [98, 99]. Such processes neglect conservative self-force effects, which have been conjectured to prevent destruction of the horizon and therefore to preserve cosmic censorship [100]. M. Colleoni described on-going efforts to analyse rigorously self-force effects in such challenging spacetimes and on the possibility that self-force prevents overspinning a Kerr black hole. J. Camps discussed an important perturbation-theory result concerning the Gregory-Laflamme instability [101]; N. Warburton discussed iso-frequency pairing of geodesic orbits for Kerr black holes [102] and O. Moreschi talked about properties of a ‘Particle Model’ to describe compact objects in the null gauge [103]. Finally, the computations reported in [72] in the ultra-relativistic limit may also be relevant to study the high-energy behavior of scattering amplitudes, and the ‘S-matrix’ for gravity [104]. Features of trans-Planckian gravitational scattering were also discussed by D. Gal’tsov [105].

Acknowledgments

We thank all the participants of B4 and B2/B4 at GR20 & Amaldi 10, as well as the organizers of such a wonderful conference, especially B. Iyer for inviting us to chair these sessions. V. C. acknowledges financial support provided under the European Union’s FP7 ERC Starting Grant “The dynamics of black holes: testing the limits of Einstein’s theory” grant agreement no. DyBHo–256667. R.A.P. acknowledges support by the NSF grant AST-0807444 and the DOE grant DE-FG02-90ER40542 at the IAS, and by the German Science Foundation within the Collaborative Research Center 676 ‘Particles, Strings and the Early Universe,’ at DESY.
References

[1] H. Stephani, D. Kramer, M. A. H. MacCallum, C. Hoenselaers and E. Herlt, “Exact solutions of Einstein’s field equations,” Cambridge, UK: Univ. Pr. (2003) 701 P.

[2] L. Blanchet, T. Damour, G. Esposito-Farese and B. R. Iyer, “Gravitational radiation from inspiralling compact binaries completed at the third post-Newtonian order,” Phys. Rev. Lett. 93, 091101 (2004) [gr-qc/0406012].

[3] L. Blanchet, “Gravitational Radiation from Post-Newtonian Sources and Inspiralling Compact Binaries,” arXiv:1310.1528 [gr-qc].

[4] Piotr Jaranowski and Gerhard Schaefer, “Third post-Newtonian higher order ADM Hamilton dynamics for two-body point mass systems,” Phys. Rev. D 57, 7274 (1998) [Erratum-ibid. D 63, 029902 (2001)] [gr-qc/9712075].

[5] L. Blanchet and G. Faye, “General relativistic dynamics of compact binaries at the third post-Newtonian order,” Phys. Rev. D 63, 062005 (2001) [gr-qc/0007051].

[6] S. Foffa and R. Sturani, “Effective field theory calculation of conservative binary dynamics at third post-Newtonian order,” Phys. Rev. D 84, 044031 (2011) [arXiv:1104.1122].

[7] W. D. Goldberger and I. Z. Rothstein, “An Effective field theory of gravity for extended objects,” Phys. Rev. D 73, 104029 (2006) [hep-th/0409156].

[8] S. Foffa and R. Sturani, “Dynamics of the gravitational two-body problem at fourth post-Newtonian order and at quadratic order in the Newton constant,” Phys. Rev. D 87, no. 6, 064011 (2013) [arXiv:1206.7087 [gr-qc]]. “Effective field theory methods to model compact binaries,” arXiv:1309.3474 [gr-qc].

[9] P. Jaranowski and G. Schafer, “Towards the 4th post-Newtonian Hamiltonian for two-point-mass systems,” Phys. Rev. D 86, 061503 (2012) [arXiv:1207.5448 [gr-qc]]. “Dimensional regularization of local singularities in the 4th post-Newtonian two-point-mass Hamiltonian,” Phys. Rev. D 87, 081503 (2013) [arXiv:1303.3225 [gr-qc]].

[10] B. Kol and M. Smolkin, “Black hole stereotyping: Induced gravito-static polarization,” JHEP 1202, 010 (2012) [arXiv:1110.3764 [hep-th]].

[11] E. E. Flanagan and T. Hinderer, “Constraining neutron star tidal Love numbers with gravitational wave detectors,” Phys. Rev. D 77, 021502 (2008) [arXiv:0709.1915].

[12] T. Damour and A. Nagar, “Relativistic tidal properties of neutron stars,” Phys. Rev. D 80, 084035 (2009) [arXiv:0906.0096 [gr-qc]].

[13] T. Binnington and E. Poisson, “Relativistic theory of tidal Love numbers,” Phys. Rev. D 80, 084018 (2009) [arXiv:0906.1366 [gr-qc]].
[14] W. D. Goldberger and I. Z. Rothstein, “Dissipative effects in the worldline approach to black hole dynamics,” Phys. Rev. D 73, 104030 (2006) [hep-th/0511133].

[15] R. A. Porto, “Absorption effects due to spin in the worldline approach to black hole dynamics,” Phys. Rev. D 77, 064026 (2008) [arXiv:0710.5150 [hep-th]].

[16] S. Chakrabarti, Tr. Delsate and J. Steinhoff, “Effective action and linear response of compact objects in Newtonian gravity,” Phys. Rev. D 88, 084038 (2013) [arXiv:1306.5820 [gr-qc]].

[17] S. Endlich, A. Nicolis, R. A. Porto and J. Wang, “Dissipation in the effective field theory for hydrodynamics: First order effects,” Phys. Rev. D 88, 105001 (2013) [arXiv:1211.6461 [hep-th]].

[18] J. E. McClintock, R. Shafee, R. Narayan, R. A. Remillard, S. W. Davis and L.-X. Li, “The Spin of the Near-Extreme Kerr Black Hole GRS 1915+105,” Astrophys. J. 652, 518 (2006) [astro-ph/0606076].

[19] A. B. Nielsen, “Compact binary coalescence parameter estimations for 2.5 post-Newtonian aligned spinning waveforms,” Class. Quant. Grav. 30, 075023 (2013) [arXiv:1203.6603 [gr-qc]].

[20] A. Gupta and A. Gopakumar, “Time-domain inspiral templates for spinning compact binaries in quasi-circular orbits described by their orbital angular momenta,” arXiv:1308.1315 [gr-qc].

[21] R. A. Porto, “Post-Newtonian corrections to the motion of spinning bodies in NRGR,” Phys. Rev. D 73, 104031 (2006) [gr-qc/0511061].

[22] J. Steinhoff and G. Schaefer, “Canonical formulation of self-gravitating spinning-object systems,” Europhys. Lett. 87, 50004 (2009) [arXiv:0907.1967 [gr-qc]].

[23] B. M. Barker and R. F. O’Connell, “Derivation of the equations of motion of a gyroscope from the quantum theory of gravitation,” Phys. Rev. D 2, 1428 (1970).

[24] H. Tagoshi, A. Ohashi and B. J. Owen, “Gravitational field and equations of motion of spinning compact binaries to 2.5 postNewtonian order,” Phys. Rev. D 63, 044006 (2001) [gr-qc/0010014].

[25] G. Faye, L. Blanchet and A. Buonanno, “Higher-order spin effects in the dynamics of compact binaries. I. Equations of motion,” Phys. Rev. D 74, 104033 (2006) [gr-qc/0605139].

[26] T. Damour, P. Jaranowski and G. Schaefer, “Hamiltonian of two spinning compact bodies with next-to-leading order gravitational spin-orbit coupling,” Phys. Rev. D 77, 064032 (2008) [arXiv:0711.1048 [gr-qc]].
[27] R. A. Porto, “Next to leading order spin-orbit effects in the motion of inspiralling compact binaries,” Class. Quant. Grav. 27, 205001 (2010) [arXiv:1005.5730 [gr-qc]].

[28] M. Levi, “Next to Leading Order gravitational Spin-Orbit coupling in an Effective Field Theory approach,” Phys. Rev. D 82, 104004 (2010) [arXiv:1006.4139 [gr-qc]].

[29] S. Marsat, A. Bohe, G. Faye and L. Blanchet, “Next-to-next-to-leading order spin-orbit effects in the equations of motion of compact binary systems,” Class. Quantum Grav. 30, 055007 (2013) [arXiv:1210.4143 [gr-qc]].

[30] J. Hartung, J. Steinhoff and G. Schaefer, “Next-to-next-to-leading order post-Newtonian linear-in-spin binary Hamiltonians,” Annalen Phys. 525, 359 (2013) [arXiv:1302.6723].

[31] R. M. Wald, “Gravitational spin interaction,” Phys. Rev. D 6, 406 (1972).

[32] R. A. Porto and I. Z. Rothstein, “The Hyperfine Einstein-Infeld-Hoffmann potential,” Phys. Rev. Lett. 97, 021101 (2006) [gr-qc/0604099].

[33] R. A. Porto and I. Z. Rothstein, “Spin(1)Spin(2) Effects in the Motion of Inspiralling Compact Binaries at Third Order in the Post-Newtonian Expansion,” Phys. Rev. D 78, 044012 (2008) [Erratum-ibid. D 81, 029904 (2010)] [arXiv:0802.0720 [gr-qc]].

[34] R. A. Porto and I. Z. Rothstein, “Next to Leading Order Spin(1)Spin(1) Effects in the Motion of Inspiralling Compact Binaries,” Phys. Rev. D 78, 044013 (2008) [Erratum-ibid. D 81, 029905 (2010)] [arXiv:0804.0260 [gr-qc]].

[35] J. Steinhoff, S. Herdt and G. Schaefer, “On the next-to-leading order gravitational spin(1)-spin(2) dynamics,” Phys. Rev. D 77, 081501 (2008) [arXiv:0712.1716 [gr-qc]].

[36] J. Steinhoff, S. Herdt and G. Schaefer, “Spin-squared Hamiltonian of next-to-leading order gravitational interaction,” Phys. Rev. D 78, 101503 (2008) [arXiv:0809.2200].

[37] S. Herdt, J. Steinhoff and G. Schaefer, “Reduced Hamiltonian for next-to-leading order Spin-Squared Dynamics of General Compact Binaries,” Class. Quant. Grav. 27, 135007 (2010) [arXiv:1002.2093 [gr-qc]].

[38] S. Herdt, J. Steinhoff and G. Schaefer, “On the comparison of results regarding the post-Newtonian approximate treatment of the dynamics of extended spinning compact binaries,” arXiv:1205.4530 [gr-qc].

[39] M. Levi, “Binary dynamics from spin1-spin2 coupling at fourth post-Newtonian order,” Phys. Rev. D 85, 064043 (2012) [arXiv:1107.4322 [gr-qc]].

[40] J. Hartung and J. Steinhoff, “Next-to-next-to-leading order post-Newtonian spin(1)-spin(2) Hamiltonian for self-gravitating binaries,” Annalen Phys. 523, 919 (2011) [arXiv:1107.4294 [gr-qc]].
[41] A. Ross, “Multipole expansion at the level of the action,” Phys. Rev. D 85, 125033 (2012) [arXiv:1202.4750 [gr-qc]].

[42] L. Blanchet, A. Buonanno and G. Faye, “Higher-order spin effects in the dynamics of compact binaries. II. Radiation field,” Phys. Rev. D 74, 104034 (2006) [Erratum-ibid. D 75, 049903 (2007)] [Erratum-ibid. D 81, 089901 (2010)] [gr-qc/0605140].

[43] R. A. Porto, A. Ross and I. Z. Rothstein, “Spin induced multipole moments for the gravitational wave flux from binary inspirals to third Post-Newtonian order,” JCAP 1103, 009 (2011) [arXiv:1007.1312 [gr-qc]].

[44] A. Bohe, S. Marsat and L. Blanchet, “Next-to-next-to-leading order spinorbit effects in the gravitational wave flux and orbital phasing of compact binaries,” Class. Quant. Grav. 30, 135009 (2013) [arXiv:1303.7412 [gr-qc]].

[45] R. A. Porto, A. Ross and I. Z. Rothstein, “Spin induced multipole moments for the gravitational wave amplitude from binary inspirals to 2.5 Post-Newtonian order,” JCAP 1209, 028 (2012) [arXiv:1203.2962 [gr-qc]].

[46] L. Blanchet, A. Buonanno and G. Faye, “Tail-induced spin-orbit effect in the gravitational radiation of compact binaries,” Phys. Rev. D 84, 064041 (2011) [arXiv:1104.5659 [gr-qc]].

[47] S. Marsat, A. Bohe, L. Blanchet and A. Buonanno, “Next-to-leading tail-induced spin-orbit effects in the gravitational radiation flux of compact binaries,” Class. Quant. Grav. 31, 025023 (2014) [arXiv:1307.6793 [gr-qc]].

[48] W. D. Goldberger and A. Ross, “Gravitational radiative corrections from effective field theory,” Phys. Rev. D 81, 124015 (2010) [arXiv:0912.4254 [gr-qc]].

[49] W. D. Goldberger, A. Ross and I. Z. Rothstein, “Black hole mass dynamics and renormalization group evolution,” arXiv:1211.6095 [hep-th].

[50] L. Blanchet, S. L. Detweiler, A. Le Tiec and B. F. Whiting, “High-Order Post-Newtonian Fit of the Gravitational Self-Force for Circular Orbits in the Schwarzschild Geometry,” Phys. Rev. D 81, 084033 (2010) [arXiv:1002.0726 [gr-qc]].

[51] S. L. Detweiler, “A Consequence of the gravitational self-force for circular orbits of the Schwarzschild geometry,” Phys. Rev. D 77, 124026 (2008) [arXiv:0804.3529 [gr-qc]].

[52] L. Blanchet, S. L. Detweiler, A. Le Tiec and B. F. Whiting, “Post-Newtonian and Numerical Calculations of the Gravitational Self-Force for Circular Orbits in the Schwarzschild Geometry,” Phys. Rev. D 81, 064004 (2010) [arXiv:0910.0207 [gr-qc]].

[53] L. Blanchet, S. Detweiler, A. Le Tiec and B. F. Whiting, “High-Accuracy Comparison Between the Post-Newtonian and Self-Force Dynamics of Black-Hole Binaries,” Fund. Theor. Phys. 162, 415 (2011).
[54] A. Le Tiec, L. Blanchet and B. F. Whiting, “The First Law of Binary Black Hole Mechanics in General Relativity and Post-Newtonian Theory,” Phys. Rev. D 85, 064039 (2012) [arXiv:1111.5378 [gr-qc]].

[55] A. G. Shah, J. L. Friedman and B. F. Whiting, “Finding high-order analytic post-Newtonian parameters from a high-precision numerical self-force calculation,” arXiv:1312.1952 [gr-qc].

[56] D. Bini and T. Damour, “High-order post-Newtonian contributions to the two-body gravitational interaction potential from analytical gravitational self-force calculations,” arXiv:1312.2503 [gr-qc].

[57] S. Foffa, “Gravitating binaries at 5PN in the post-Minkowskian approximation,” arXiv:1309.3956 [gr-qc].

[58] L. Blanchet, G. Faye and B. F. Whiting, “Half-integral conservative post-Newtonian approximations in the redshift factor of black hole binaries,” arXiv:1312.2975 [gr-qc].

[59] A. Pound, C. Merlin and L. Barack, “Gravitational self-force from radiation-gauge metric perturbations,” arXiv:1310.1513 [gr-qc].

[60] A. Heffernan, A. Ottewill and B. Wardell, “High-order expansions of the Detweiler-Whiting singular field in Kerr spacetime,” arXiv:1211.6446 [gr-qc].

[61] M. Casals and B. C. Nolan, “A Kirchhoff integral approach to the calculation of Green’s functions beyond the normal neighbourhood,” Phys. Rev. D 86, 024038 (2012) [arXiv:1204.0407 [gr-qc]].

[62] J. Brink, M. Geyer and T. Hinderer, “Orbital resonances around Black holes,” arXiv:1304.0330 [gr-qc].

[63] A. Pound, “Second-order gravitational self-force,” Phys. Rev. Lett. 109, 051101 (2012) [arXiv:1201.5089 [gr-qc]].

[64] S. E. Gralla, “Second Order Gravitational Self Force,” Phys. Rev. D 85, 124011 (2012) [arXiv:1203.3189 [gr-qc]].

[65] A. Le Tiec, A. H. Mroue, L. Barack, A. Buonanno, H. P. Pfeiffer, N. Sago and A. Taracchini, “Periastron Advance in Black Hole Binaries,” Phys. Rev. Lett. 107, 141101 (2011) [arXiv:1106.3278 [gr-qc]].

[66] A. Le Tiec, E. Barausse and A. Buonanno, “Gravitational Self-Force Correction to the Binding Energy of Compact Binary Systems,” Phys. Rev. Lett. 108, 131103 (2012) [arXiv:1111.5609 [gr-qc]].
[67] A. L. Tiec, A. Buonanno, A. H. Mroue, H. P. Pfeiffer, D. A. Hemberger and G. Lovelace, “Periastron Advance in Spinning Black Hole Binaries: Gravitational Self-Force from Numerical Relativity,” Phys. Rev. D 88, 124027 (2013) [arXiv:1309.0541 [gr-qc]].

[68] See the somewhat related discussion by K. S. Thorne and C. Will in http://kersten.uchicago.edu/event_video/chandrasekhar_symposium/chandrasekhar_symposium.html

[69] Z. Bern and H. Ita, “Harmony of scattering amplitudes: From QCD to gravity,” Nucl. Phys. Proc. Suppl. 216, 2 (2011).

[70] D. Neill and I. Z. Rothstein, “Classical Space-Times from the S Matrix,” Nucl. Phys. B 877, 177 (2013) [arXiv:1304.7263 [hep-th]].

[71] C. R. Galley and B. L. Hu, “Self-force on extreme mass ratio inspirals via curved spacetime effective field theory,” Phys. Rev. D 79, 064002 (2009) [arXiv:0801.0900 [gr-qc]].

[72] C. R. Galley and R. A. Porto, “Gravitational self-force in the ultra-relativistic limit: the ”large-N” expansion,” JHEP 1311, 096 (2013) [arXiv:1302.4486 [gr-qc]].

[73] A. Zee, “Quantum field theory in a nutshell,” Princeton, UK: Princeton Univ. Pr. (2010).

[74] C. R. Galley and A. K. Leibovich, “Radiation reaction at 3.5 post-Newtonian order in effective field theory,” Phys. Rev. D 86, 044029 (2012) [arXiv:1205.3842 [gr-qc]].

[75] C. R. Galley, “The classical mechanics of non-conservative systems,” Phys. Rev. Lett. 110, 174301 (2013) [arXiv:1210.2745 [gr-qc]].

[76] O. Birnholtz, S. Hadar and B. Kol, “Theory of post-Newtonian radiation and reaction,” Phys. Rev. D 88, 104037 (2013) [arXiv:1305.6930 [hep-th]].

[77] For a review see: T. Damour, “The general relativistic two body problem,” arXiv:1312.3505 [gr-qc], and references therein.

[78] Y. Pan, A. Buonanno, A. Taracchini, L. E. Kidder, A. H. Mroue, H. P. Pfeiffer, M. A. Scheel and B. Szilagyi, “Inspiral-merger-ringdown waveforms of spinning, precessing black-hole binaries in the effective-one-body formalism,” arXiv:1307.6232 [gr-qc].

[79] A. Taracchini, A. Buonanno, Y. Pan, T. Hinderer, M. Boyle, D. A. Hemberger, L. E. Kidder and G. Lovelace et al., “Effective-one-body model for black-hole binaries with generic mass ratios and spins,” arXiv:1311.2544 [gr-qc].

[80] P. Kumar, I. MacDonald, D. A. Brown, H. P. Pfeiffer, K. Cannon, M. Boyle, L. E. Kidder and A. H. Mroue et al., “Template Banks for Binary black hole searches with Numerical Relativity waveforms,” arXiv:1310.7949 [gr-qc].
[81] I. Hinder, A. Buonanno, M. Boyle, Z. B. Etienne, J. Healy, N. K. Johnson-McDaniel, A. Nagar and H. Nakano et al., “Error-analysis and comparison to analytical models of numerical waveforms produced by the NRAR Collaboration,” Class. Quant. Grav. 31, 025012 (2013) [arXiv:1307.5307 [gr-qc]].

[82] G. Lovelace, M. Boyle, M. A. Scheel and B. Szilagyi, “Accurate gravitational waveforms for binary-black-hole mergers with nearly extremal spins,” Class. Quant. Grav. 29, 045003 (2012) [arXiv:1110.2229 [gr-qc]].

[83] I. Kamaretsos, M. Hannam and B. Sathyaprakash “Is black-hole ringdown a memory of its progenitor?,” Phys. Rev. Lett. 109, 141102 (2012) [arXiv:1207.0399 [gr-qc]].

[84] T. Damour and G. Esposito-Farese, “Tensor multiscalar theories of gravitation,” Class. Quant. Grav. 9, 2093 (1992).

[85] V. Cardoso, S. Chakrabarti, P. Pani, E. Berti and L. Gualtieri, “Floating and sinking: The Imprint of massive scalars around rotating black holes,” Phys. Rev. Lett. 107, 241101 (2011) [arXiv:1109.602].

[86] V. Cardoso, I. P. Carucci, P. Pani and T. P. Sotiriou, “Matter around Kerr black holes in scalar-tensor theories: scalarization and superradiant instability,” [arXiv:1305.6936].

[87] E. Berti, V. Cardoso, L. Gualtieri, M. Horbatsch and U. Sperhake, “Numerical simulations of single and binary black holes in scalar-tensor theories: circumventing the no-hair theorem,” Phys. Rev. D 87, 124020 (2013) [arXiv:1304.2836 [gr-qc]].

[88] P. Pani and V. Cardoso, “Are black holes in alternative theories serious astrophysical candidates? The Case for Einstein-Dilaton-Gauss-Bonnet black holes,” Phys. Rev. D 79, 084031 (2009) [arXiv:0902.1569 [gr-qc]].

[89] R. Jackiw and S. Y. Pi, “Chern-Simons modification of general relativity,” Phys. Rev. D 68, 104012 (2003) [gr-qc/0308071].

[90] S. Alexander and N. Yunes, “Chern-Simons Modified General Relativity,” Phys. Rept. 480, 1 (2009) [arXiv:0907.2562 [hep-th]].

[91] K. Yagi, N. Yunes and T. Tanaka, “Gravitational Waves from Quasi-Circular Black Hole Binaries in Dynamical Chern-Simons Gravity,” Phys. Rev. Lett. 109, 251105 (2012) [arXiv:1208.5102 [gr-qc]].

[92] K. Yagi, L. C. Stein, N. Yunes and T. Tanaka, “Isolated and Binary Neutron Stars in Dynamical Chern-Simons Gravity,” Phys. Rev. D 87, 084058 (2013) [arXiv:1302.1918].

[93] S. Dyda, E. E. Flanagan and M. Kamionkowski, “Vacuum Instability in Chern-Simons Gravity,” Phys. Rev. D 86, 124031 (2012) [arXiv:1208.4871 [gr-qc]].
[94] P. Pani, E. Berti and L. Gualtieri, “Scalar, Electromagnetic and Gravitational Perturbations of Kerr-Newman Black Holes in the Slow-Rotation Limit,” Phys. Rev. D 88, 064048 (2013) [arXiv:1307.7315 [gr-qc]].

[95] P. Pani, V. Cardoso, L. Gualtieri, E. Berti and A. Ishibashi, “Black hole bombs and photon mass bounds,” Phys. Rev. Lett. 109, 131102 (2012) [arXiv:1209.0465 [gr-qc]].

[96] V. E. Hubeny, “Overcharging a black hole and cosmic censorship,” Phys. Rev. D 59, 064013 (1999) [gr-qc/9808043].

[97] T. Jacobson and T. P. Sotiriou, “Over-spinning a black hole with a test body,” Phys. Rev. Lett. 103, 141101 (2009) [Erratum-ibid. 103, 209903 (2009)] [arXiv:0907.4146 [gr-qc]].

[98] J. V. Rocha and V. Cardoso, “Gravitational perturbation of the BTZ black hole induced by test particles and weak cosmic censorship in AdS spacetime,” Phys. Rev. D 83, 104037 (2011) [arXiv:1102.4352 [gr-qc]].

[99] M. Bouhmadi-Lopez, V. Cardoso, A. Nerozzi and J. V. Rocha, “Over spinning a black hole?,” J. Phys. Conf. Ser. 314, 012064 (2011).

[100] E. Barausse, V. Cardoso and G. Khanna, “Test bodies and naked singularities: Is the self-force the cosmic censor?,” Phys. Rev. Lett. 105, 261102 (2010) [arXiv:1008.5159].

[101] M. M. Caldarelli, J. Camps, B. Goutraux and K. Skenderis, “AdS/Ricci-flat correspondence and the Gregory-Laflamme instability,” Phys. Rev. D 87, no. 6, 061502 (2013) [arXiv:1211.2815 [hep-th]].

[102] N. Warburton, L. Barack and N. Sago, “Isofrequency pairing of geodesic orbits in Kerr geometry,” Phys. Rev. D 87, 084012 (2013) [arXiv:1301.3918 [gr-qc]].

[103] E. Gallo and O. M. Moreschi, ‘Approximation Method for the Relaxed Covariant Form of the Gravitational Field Equations for Particles’, Journal of Modern Physics 3 (2012b).

[104] S. B. Giddings and R. A. Porto, “The Gravitational S-matrix,” Phys. Rev. D 81, 025002 (2010) [arXiv:0908.0004 [hep-th]].

[105] D. Gal’tsov, P. Spirin and T. N. Tomaras, “Gravitational bremsstrahlung in ultra-planckian collisions,” JHEP 1301, 087 (2013) [arXiv:1210.6976 [hep-th]].