IslandFAST: A Semi-numerical Tool for Simulating the Late Epoch of Reionization

Yidong Xu\textsuperscript{1}, Bin Yue\textsuperscript{2}, and Xuelei Chen\textsuperscript{1,3,4}

\textsuperscript{1} Key Laboratory for Computational Astrophysics, National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012, China
\textsuperscript{2} National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012, China
\textsuperscript{3} University of Chinese Academy of Sciences, Beijing 100049, China
\textsuperscript{4} Center for High Energy Physics, Peking University, Beijing 100871, China

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Abstract

We present the algorithm and main results of our semi-numerical simulation, islandFAST, which was developed from 21cmFAST and designed for the late stage of reionization. The islandFAST simulation predicts the evolution and size distribution of the large-scale underdense neutral regions (neutral islands), and we find that the late Epoch of Reionization proceeds very fast, showing a characteristic scale of the neutral islands at each redshift. Using islandFAST, we compare the impact of two types of absorption systems, i.e., the large-scale underdense neutral islands versus small-scale overdense absorbers, in regulating the reionization process. The neutral islands dominate the morphology of the ionization field, while the small-scale absorbers dominate the mean-free path of ionizing photons, and also delay and prolong the reionization process. With our semi-numerical simulation, the evolution of the ionizing background can be derived self-consistently given a model for the small absorbers. The hydrogen ionization rate of the ionizing background is reduced by an order of magnitude in the presence of dense absorbers.

Key words: Cosmology: theory – dark ages, reionization, first stars – intergalactic medium – large-scale structure of universe

1. Introduction

The hydrogen gas in the universe was reionized by the energetic radiation from galaxies and/or quasars. Although the details of this process are still highly uncertain and the nature of the ionizing sources is poorly understood, some knowledge has been obtained in the past decades and was recently updated. The temperature and polarization data of the cosmic microwave background constrain the average redshift of reionization as $z_{\text{reion}} \approx 8$ (Planck Collaboration et al. 2016), while observations of high-redshift quasar (QSO) absorption spectra have marked the completion redshift of the hydrogen reionization as $z \approx 6$ (e.g., Fan et al. 2006). On the other hand, measurements of the kinetic Sunyaev–Zel’dovich (kSZ) effect with the South Pole Telescope and the Atacama Cosmology Telescope, in combination with the Planck data, have given an upper limit on the duration of the reionization, i.e., $\Delta z < 2.8$ (Planck Collaboration et al. 2016). Albeit with the above successes, current measurements of the high-redshift galaxy luminosity function are limited to the bright end (e.g., Schenker et al. 2013; Bouwens et al. 2015), and constraints on the ionization state of the IGM from quasar proximity zone observations (e.g., Bolton et al. 2011; Bosman & Becker 2015) and the Ly$\alpha$ emitting galaxy surveys (e.g., Schenker et al. 2012; Dijkstra et al. 2014) are quite weak and highly model-dependent. Various efforts have been made to explore the 21 cm signatures from the neutral hydrogen present in the IGM during the EoR, and people pin hope on the low frequency radio experiments such as the Precision Array for Probing the Epoch of Reionization (PAPER; Parsons et al. 2010; Ali et al. 2015), the Murchison Widefield Array (MWA; Tingay et al. 2013; Ewall-Wice et al. 2016), the LOw Frequency ARray (van Haarlem et al. 2013), and the Long Wavelength Array (Ellingson et al. 2009), as well as the future Hydrogen Epoch of Reionization Array (DeBoer et al. 2017) and the Square Kilometre Array (Huynh & Lazio 2013). These 21 cm experiments will greatly expand the frontiers of our understanding of reionization.

Theoretically, the “bubble model” (Furlanetto et al. 2004) may be one of the most commonly accepted scenarios for the reionization process. In the inside-out mode of reionization, galaxy formation occurs earlier in regions with higher densities, where the IGM was reionized earlier. Therefore, the large-scale ionization field is closely related to the large-scale fluctuations of the density field. One can associate the ionization redshift, or the ionization status of a given position at a certain redshift, to the local density. This correlation is also confirmed later by numerical simulations (Battaglia et al. 2013b). Based on this idea and the well-established excursion set theory (Bond et al. 1991; Lacey & Cole 1993), the bubble model predicts the growth of the ionized regions (“bubbles”), assuming that ionized bubbles are spherical and isolated. The bubble model provides a reasonable description of the growth of H II regions, showing good agreement with numerical simulation results (Zahn et al. 2007). Furthermore, based on its idea, approximate treatments of the three-dimensional ionization process have been developed, i.e., the so-called semi-numerical simulations (e.g., Mesinger & Furlanetto 2007; Zahn et al. 2007; Choudhury et al. 2009; Mesinger et al. 2011; Zhou et al. 2013).

Strictly speaking, the basic premise of the bubble model is valid only prior to the percolation of H II regions. Once the ionized bubbles start to contact each other, the assumption that the ionized regions are isolated spherical bubbles breaks down (Xu et al. 2014). The inaccuracy of applying the bubble model after percolation was also recognized and studied in detail in Furlanetto & Oh (2016). To generalize the bubble model, the “island model” was developed in order to better describe the evolution of neutral regions that are more isolated during the late stage of reionization (Xu et al. 2014). The island model

\textsuperscript{5} http://reionization.org/
\textsuperscript{6} http://www.skatelescope.org/
takes into account the presence of an ionizing background that should exist in the late EoR, and predicts the distribution and evolution of large-scale neutral regions (“islands”) that are underdense regions.

In this work, we develop a semi-numerical code, island-FAST, to realize the island model in three dimensions, and to simulate the late process of reionization. Besides the local ionizing sources as modeled in the bubble model and its semi-numerical counterpart 21cmFAST (Mesinger et al. 2011), a key ingredient of the island model is the inclusion of an ionizing background, which is inevitable after percolation. Therefore, it is also important to incorporate the ionizing background in the semi-numerical simulation for the late EoR. Given the flux of the ionizing background and the surface area of the region concerned, it is straightforward to compute the ionizations induced by this background.

The flux of the ionizing background depends on the balance of photon production and absorption. To generate the ionizing background self-consistently, we take into account the effect of small-scale underdense absorbers, as well as the large-scale underdense neutral regions— islands (see Alvarez & Abel 2012 who first investigated the effect of these absorbers on the reionization process).

The small-scale dense absorbers, which are believed to be the main contributors to the IGM opacity (Miralda-Escudé et al. 2000; Furlanetto & Oh 2005; Emberson et al. 2013), dominate in regulating the mean-free path (MFP) of the ionizing photons and hence the intensity of the ionizing background (McQuinn et al. 2011; Haardt & Madau 2012). They include dense highly nonlinear structures, such as interstellar medium (ISM) inside galaxies, those mostly ionized, partially self-shielded gas clumps in the ionized IGM outside galaxies, usually referred to as LLSs, as well as minihalos or any other opacity contributors. The effects of ISM inside galaxies are absorbed in the escape fraction \( f_{esc} \) in models. Because the basic idea of the excursion set theory is only valid on large scales, which sets the benchmark of our island model as well as the island-FAST, we adopt an empirical modeling for the small-scale absorbers in the IGM.

As long as the model of the small-scale absorbers is given, the island-FAST simultaneously generate the ionization field as well as the ionizing background for the late EoR. We further use it as a tool to investigate the roles played by the large-scale neutral islands as well as the small-scale absorbers, in regulating the MFP of the ionizing photons and the intensity of the ionizing background, and their effects on the late reionization process.

In the following, we first briefly review the excursion set theory of reionization, i.e., the bubble model and the island model in Section 2. Then we describe the algorithm of our semi-numerical simulation, island-FAST, in Section 3, especially the implementation of an ionizing background. The main results of the simulation are given in Section 4, and we conclude in Section 5. Throughout this paper, we assume the \( \Lambda \)CDM model and adopt the following cosmological parameters: \( \Omega_b = 0.045 \), \( \Omega_c = 0.225 \), \( \Omega_L = 0.73 \), \( H_0 = 70 \text{ km s}^{-1} \text{Mpc}^{-1} \), \( c_s = 0.8 \), and \( n_s = 0.96 \), but the results are not sensitive to these parameters.

2. The Excursion Set Theory and the Island Model

The island model is based on the excursion set theory of halo formation. Here we briefly review the excursion set approach, especially its application to the reionization process, i.e., the bubble model for the early stage of reionization, and the island model for the late stage. We refer the interested readers to Zentner (2007) for a detailed review of the excursion set theory, Furlanetto et al. (2004) and Furlanetto & Oh (2005) for the bubble model, and Xu et al. (2014) for the island model.

In the excursion set theory, the collapse of a region and formation of a halo is determined by its average density exceeding a certain threshold (density barrier), which is a function of redshift and its mass scale. In a random density field, the average density around a given position on different smoothing mass scales corresponds to a random walk trajectory in the overdensity-variance plane, and formation of the halo is identified as the first-up-crossing of the barrier \( \delta_c(M, z) \). By computing the first-up-crossing distribution of random walks with respect to the density barrier for halo formation, the excursion set theory recovers the Press–Schechter formula of halo mass function at any given redshift, and naturally solves the so-called “cloud-in-cloud” problem in the original Press–Schechter model.

The excursion set theory can also be applied to the reionization process with the bubble model. The basic idea of the bubble model is that it asks whether a region has produced a sufficient number of photons to get itself ionized. The number of ionizing photons produced in the region are assumed to be proportional to the total number of baryons in the halos, i.e., total mass of the region times the collapse fraction (Furlanetto et al. 2004). The ionization condition can be written as

\[
\frac{f_{coll}}{\delta_i} > \xi^{-1},
\]

where

\[
\xi = \frac{f_{esc} f_c N_{H_0}}{1 + n_{rec}}
\]

is the ionizing efficiency parameter, in which \( f_{esc} \), \( f_c \), \( N_{H_0} \), and \( n_{rec} \) are the escape fraction, star-formation efficiency, the number of ionizing photons emitted per H atom in stars, and the average number of recombinations per ionized hydrogen atom, respectively. Assuming Gaussian density fluctuations, the collapse fraction of a region with mass scale \( M \) at redshift \( z \) can be written as a function of its mean linear overdensity \( \delta_M \) (Bond et al. 1991; Lacey & Cole 1993):

\[
f_{coll}(\delta_M; M, z) = \text{erfc} \left( \frac{\delta_i(z) - \delta_M}{\sqrt{2[S_{\max} - S(M)]}} \right),
\]

where \( \delta_i(z) \) is the linear critical overdensity for halo collapse at redshift \( z \), \( S(M) = \sigma^2(M) \) is the variance of the density fluctuations smoothed on mass scale \( M \), it decreases with increasing \( M \), and \( S_{\max} = \sigma^2(M_{\text{min}}) \), in which \( M_{\text{min}} \) is the minimum mass of star-forming halos and is usually taken to be the mass corresponding to \( 10^4 \text{K} \) virial temperature, at which point atomic hydrogen line cooling becomes efficient.

With this collapse fraction, the self-ionization condition can be expressed as a random trajectory in the \( S - \delta \) space exceeding the barrier on the density contrast, i.e., the bubble barrier: \( \delta_M > \delta_B(M, z) \), where

\[
\delta_B(M, z) = \delta_i(z) - \sqrt{2[S_{\max} - S(M)]} \text{erfc}^{-1}(\xi^{-1})
\]
Solving for the first-up-crossing probability distribution of random walks with respect to this barrier, \( f(S, z) \), the size distribution of ionized bubbles is obtained.

The bubble model gives a reasonable description of the reionization process before percolation of the ionized regions (Xu et al. 2014; Furlanetto & Oh 2016). After percolation, when the ionized regions are connected with each other and the neutral islands are more isolated, one may use the complementary island model to describe the process. In the island model, we assume that the ionized regions are connected and consider instead isolated neutral regions. A region remains neutral if the number of available ionizing photons is lower than the required number to ionize all hydrogen atoms in the region. A key ingredient of the island model is the inclusion of an ionizing background that is globally produced during the late EoR, and the condition for an island of mass scale \( M \) at redshift \( z \) to keep from being totally ionized is modified accordingly:

\[
\xi_{\text{coll}}(\delta_M; M, z) + \frac{\Omega_m}{\Omega_b} \frac{N_{\text{back}} m_H}{M X_H(1 + \bar{n}_{\text{rec}})} < 1, \tag{5}
\]

where the second term on the LHS accounts for the contribution from the ionizing background, with \( N_{\text{back}} \) being the number of consumed background ionizing photons and \( X_H \) being the mass fraction of the hydrogen in baryons.

Rewriting the condition Equation (5) as a constraint on the density contrast of the region using Equation (3), we derive the island barrier: \( \delta_M < \delta_I(M, z) \),

\[
\delta_I(M, z) \equiv \delta(z) - \sqrt{2[S_{\text{max}} - S(M)] \text{erfc}^{-1}[K(M, z)]}, \tag{6}
\]

where

\[
K(M, z) = \xi^{-1} \left[ 1 - N_{\text{back}} (1 + \bar{n}_{\text{rec}})^{-1} \frac{m_H}{M (\Omega_b / \Omega_m) X_H} \right].
\]

Assuming a spherical shape for the island, and that the number of background ionizing photons consumed by it at any instant is proportional to its surface area, we can derive the total number of background ionizing photons consumed:

\[
N_{\text{back}} = \frac{4\pi}{3} (R_i^3 - R_f^3) \bar{n}_H (1 + \bar{n}_{\text{rec}}), \tag{7}
\]

where \( \bar{n}_H \) is the mean hydrogen number density, and \( R_i \) and \( R_f \) denote the initial and final scale of the island, respectively. The sphere between the scale \( R_i \) and \( R_f \) is ionized by the ionizing background, so that

\[
\Delta R \equiv R_i - R_f = \int_{z}^{z_{\text{back}}} \frac{F(z)}{\bar{n}_H (1 + \bar{n}_{\text{rec}}) \bar{H}(z)} \frac{dz}{H(z)(1 + z)^3}, \tag{8}
\]

where \( F(z) \) is the physical number flux of background ionizing photons, which is related to the comoving photon number density by \( F(z) = n_z(z) (1 + z)^3 c / 4 \). Note that the island barrier has a different shape from the bubble barrier because of the contribution of the ionizing background photons.

In Equation (8), \( z_{\text{back}} \) is the “background onset redshift,” below which we assume that a spatially homogeneous ionizing background flux is established throughout all of the ionized regions. Here we assume this happened when regions with average density (\( \bar{\delta} = 0 \)) is ionized. Using the bubble model, this redshift can be solved from the following equation:

\[
\delta_I(S = 0; z = z_{\text{back}}) = 0. \tag{9}
\]

The solution depends on the parameters of the reionization model. For example, if we take \( \{f_{\text{rec}}, N_{\gamma/H}, n_{\text{rec}}\} = \{0.3, 0.1, 4000, 3\} \), then \( \xi = 30 \) and \( z_{\text{back}} = 7.90 \). The combination of \( \{f_{\text{rec}}, N_{\gamma/H}, n_{\text{rec}}\} = \{0.2, 0.1, 4000, 3\} \) gives \( \xi = 20 \) and \( z_{\text{back}} = 7.10 \), while \( \{f_{\text{rec}}, N_{\gamma/H}, n_{\text{rec}}\} = \{0.2, 0.1, 3000, 3\} \) gives \( \xi = 15 \) and \( z_{\text{back}} = 6.51 \).

Because all trajectories in the excursion set start from the point \( (S, \delta) = (0, 0) \), and islands are identified by down-crossings of the island barrier, which has a negative intercept, and the “island-in-island” problem is solved naturally by considering only the first-down-crossings of the barrier curve. Solving for the first-down-crossing distribution of random trajectories with the island barrier, one obtains the mass distribution and the volume fraction of neutral regions at any given redshift after percolation, or below the background onset redshift.

In addition to the “island-in-island” problem, there is a “bubbles-in-island” effect in the island model because there might also be self-ionized regions inside a large neutral island. We shall call the island including bubbles the host island. The bubbles inside neutral islands are identified in the excursion set framework by considering the trajectories that first-down-crossed the island barrier \( \delta_I \) at \( S_I \), then at a larger \( S_B \) (smaller scale) up-crossed over the bubble barrier \( \delta_B \). The size distribution of bubbles inside an island of scale \( S_I \) and overdensity \( \delta_I \) is characterized by the conditional probability distribution \( f_B(S_B, \delta_B | \delta_I) \), which can be similarly computed with a shifted bubble barrier, i.e., \( \delta_B = \delta_B(S + S_I) - \delta_I(S_I) \), where \( S = S_B - S_I \). Also, the average bubbles-in-island fraction can be calculated by integrating over all possible bubble sizes for a given island.

In order to demarcate the scope of application of both the bubble model and the island model, and to define the bonafide neutral islands, we introduced a percolation threshold \( p_r \) in Xu et al. (2014). The bubble model is considered reliable before the bubble filling factor becomes larger than the percolation threshold \( p_r \), while the island model can make accurate predictions only below a certain redshift after the island filling factor is below \( p_r \). The ionizing background was set up sometime in between, after the ionized bubbles percolated but before the islands were all isolated. The percolation threshold is also applied to the bubbles-in-island fraction. Only those islands with the bubbles-in-island fraction \( q_B < p_r \) are qualified as a whole neutral island, preventing them from being percolated through by the bubbles inside them. This percolation criterion of \( q_B < p_r \) acts as an additional barrier for finding islands, which combines with the basic island barrier to define the host islands. The percolation threshold for a Gaussian random field of \( p_r = 0.16 \) (Klypin & Shandarin 1993) is used, because the ionization field follows the density field (Battaglia et al. 2013a), which is almost Gaussian on large scales (Planck Collaboration et al. 2014). More recently, a percolation threshold of about 0.1 was derived for the reionization process by using the semi-numerical code 21cmFAST (Furlanetto & Oh 2016). However, the basic algorithm of our semi-numerical simulation islandFAST does not depend on this threshold, and the main results shown below are not sensitive to this
threshold. To ease direct comparison with our analytical model predictions, here we set our default value of \( p_c = 0.16 \).

With the combined island barrier taking into account the bubbles-in-island effect, and using an ionizing background model calibrated by the observed ionizing background at redshift \( \sim 6 \), the size distribution of the neutral islands at any given redshift \( z \) when the neutral fraction is below \( p_c \) can be obtained. At a given instant, shortly after the neutral islands become isolated, our model predicts that the size distribution of the islands has a peak of a few Mpc, depending on the model parameters. As the redshift decreases, the small islands disappear rapidly while the large ones shrink, but the characteristic scale of the islands does not change much if we constrain the bubbles-in-island fraction to be lower than \( p_c = 0.16 \). Eventually, all these large-scale neutral islands are swamped by ionization, only compact neutral regions such as galaxies or minihalos remain.

3. The IslandFAST Code

The islandFAST code is a semi-numerical code to reproduce the late stage of the reionization process. It is developed from the 21cmFAST code (Mesinger et al. 2011), which is based on the bubble model, but is extended to treat the late state of reionization by the island model. Compared with the 21cmFAST, the major differences are that (1) islandFAST uses a two-step filtering algorithm in generating the ionization field in order to take the bubbles-in-island effect into account and (2) the effect of absorption systems is taken into account and a self-consistent treatment for the ionizing background is incorporated.

The basic steps of islandFAST are as follows.

1. Create the linear density field with the given power spectrum and the linear velocity field using the standard Zel’dovich approximation (Zel’dovich 1970; Efstathiou et al. 1985; Sirko 2005), just as the first steps in 21cmFAST.
2. Use the 21cmFAST algorithm to generate the ionization field at a redshift slightly higher than the \( z_{\text{back}} \), i.e., update the density field for the redshift using the first order perturbation theory, assuming the baryons trace the dark matter distribution, and filter the bubble field using the bubble barrier. The halo-finding step is bypassed to speed up the computation because we are interested in the large-scale distribution of the neutral islands. We use this ionization field as the initial condition for the following steps.
3. For each redshift step below, use the excursion set approach to generate the host island field with the island barrier Equation (6).
4. Start with the host island field for a specific redshift, apply the bubble barrier within each host island, and generate the bubbles in islands, then we get the final ionization field for this redshift.

Unlike the 21cmFAST, we skip the final step of assigning the partial ionization fraction to each neutral pixel because the collapse fraction computed with the excursion set theory (i.e., Equation (3)) is only accurate on large scales and should not be used on each pixel. When the ionization field is generated for a given redshift, the percolation threshold \( p_c \) can be applied to select those almost neutral and nearly spherical islands, i.e., using the neutral fraction threshold of \( f_{H_1}^c = 1 - p_c \) in quantifying the sizes of the islands with the spherical average method (SAM, McQuinn et al. 2007; Zahn et al. 2007). We may also use lower values of the threshold \( f_{H_1}^c \), and then the neutral regions will be attributed to larger and more sponge-like islands. Comparing between the islands with different values of \( f_{H_1}^c \), one reveals the morphological information of the islands.

The evolution of ionizing background depends on the detailed history of reionization, as it depends both on the photon production rate and the regulation by various absorption systems, which limit the MFP of the photons. Conversely, it also greatly affects how the reionization would proceed. A self-consistent treatment is essential for correct modeling of the reionization process. Besides the large-scale neutral islands that block the propagation of the ionizing photons, the most frequently discussed absorbers are Lyman limit systems, which have large enough H I column density to remain self-shielded (e.g., Miralda-Escudé et al. 2000; Furlanetto & Oh 2005; Bolton & Hafen 2013). Minihalos could also block ionizing photons and contribute to the IGM opacity (Furlanetto & Oh 2005). However, due to their shallow gravitational potential and the complex evaporation process, the contribution from the minihalos is highly uncertain (Barkana & Loeb 1999, 2003; Shapiro et al. 2004; Ilyev et al. 2005; Ciardi et al. 2006; Yue & Chen 2012). Observationally, the post-reionization intensity of the ionizing background has been constrained by the mean transmitted flux in the Ly\( \alpha \) forest (e.g., Wyithe & Bolton 2011; Calverley et al. 2011).

We divide the absorption systems into two categories, the relatively large neutral islands, and the small-scale absorbers, which are not resolved in the simulation. We use a semi-empirical prescription for the contribution to the MFP from small-scale absorbers and take into account the effects of both the large-scale islands and the small-scale absorption systems simultaneously.

Due to the small-scale absorbers as well as the shading of neutral islands, a neutral region (island) will only be illuminated by ionizing photons emitted within a distance that is roughly the MFP of the photon. The comoving number density of background ionizing photons at redshift \( z \) can be modeled as the integration of escaped ionizing photons that are emitted from newly collapsed objects and survived to the distances between the sources and the position under consideration:

\[
 n_\gamma(z) = \int_z dz', \bar{n}_\text{H} \left| \frac{df_{\text{coll}}(z')}{dz'} \right| f_s N_{r/H} f_{\text{esc}} \times \exp \left[ -\frac{l(z, z')}{\lambda_{\text{mp}}(z)} \right] (1 - f_{H_1}^\text{host}) ,
\]

where \( l(z, z') \) is the physical distance between the source at redshift \( z' \) and the redshift \( z \) under consideration, and \( \lambda_{\text{mp}} \) is the mean transmitted flux of the Ly\( \alpha \) forest. The factor \( (1 - f_{H_1}^\text{host}) \) is because only those ionizing photons located outside of the host islands could contribute to the ionizing background.

The treatment of the MFP in this paper differs slightly from the analytic model of Xu et al. (2014), where the MFP was assumed to be from LLS and computed according to the
Miralda-Escudé et al. (2000) model. Now we assume
\[
\lambda^{-1}_{\text{mfp}}(z) = \lambda^{-1}_i(z) + \lambda^{-1}_{\text{abs}}(z),
\]
(11)
where \(\lambda_i\) is the MFP of ionizing photons due to large-scale underdense islands, and \(\lambda_{\text{abs}}\) is the MFP limited by small-scale overdense absorbers, including the effects of Lyman limit systems and minihalos, or other opacity contributions that are not resolved in our simulation. While in principle one could also develop a model including the evolution of the small-scale absorbers, here we adopt a more empirical approach. Songaila & Cowie (2010) provided a fitting formula for the evolution of MFP of ionizing photons based on their observed number density of Lyman limit systems up to redshift 6, which reads
\[
\lambda_{\text{abs}} = 50 \left[ \frac{1 + z}{4.5} \right]^{-4.44} \text{[p Mpc]}.
\]
(12)
We adopt this evolutionary form for the cumulative effective effect of all kinds of small-scale absorbers assuming the LLSs as the main contributor. We further assume that the number density of small-scale absorbers evolves smoothly near the completion of reionization, so the above fitting formula can be extrapolated to the late stage of reionization.

The MFP of ionizing photons and the resultant ionizing background is incorporated in islandFAST in an iterative procedure. We start from a trial value of \(\lambda_i\) for a redshift a bit lower than \(z_{\text{back}}\), and applying the ionizing background model (Equation (10)) and the island barrier (Equation (5)). We generate the host island field, then compute the MFP of ionizing photons directly from the host island field. Practically, we cast lines (a total of \(10^7\) for each realization) with random starting points located in ionized regions and with random directions, and calculate the distances from the starting points and the ending points where phase transitions occur. From the distribution of the distances, we find the MFP by equaling it to the critical value that a fraction \(1/e\) of the total distances are larger than it. This is consistent with the definition of the optical depth, in the sense that only a fraction of \(1/e\) of the ionizing photons can survive to a distance of the MFP. Using the derived \(\lambda_i\), we apply the updated ionizing background again to find the updated host island field. After several iterations, we achieve the converged intensity of the ionizing background and the host island field of this redshift. Then the bubble barrier is applied within each host island to find ionized bubbles in islands, and obtain the ionization field of this snapshot.

Note that the change in the size of a host island is an integration of the changing rate, which is proportional to the redshift-dependent ionizing background \(n_e(z)\) (Equation (8)). We divide the simulated redshift range into small bins, \(\Delta z\), and approximate the \(n_e(z)/\bar{n}_e\) as a constant between \(z\) and \(z + \Delta z\). We use the derived \(\lambda_i\) from the previous redshift as the first trial value for the next redshift. \(\Delta z\) is adaptive, and in each step, we make sure that \(\Delta z\) is small enough, during which period the \(\lambda_i\) does not grow too much, so that the constant approximation for \(n_e(z)/\bar{n}_e\) is valid. This is guaranteed by requiring \(\lambda_i\) to achieve convergence, i.e., the relative error in \(\lambda_i\) is smaller than \(2\%\), within two times of iteration. Therefore, islandFAST has to be run downward from the background onset redshift, and the ionization field for a redshift of interest cannot be obtained without computing the previous redshift steps.

4. Results

In the default run of islandFAST, we take into account the effects of both large-scale islands and small-scale absorbers in regulating the MFP of ionizing photons. Furthermore, we set the box size of \(100\ h^{-1}\text{Mpc}\), and a resolution of \(512^3\) for both the dark matter field and the ionization field. We have made a convergence test for islandFAST by running several simulations of different box scales and resolutions and find that, in terms of the general reionization process and the main results shown below, convergence has arrived for our default simulation.

Taking the ionizing efficiency parameter \(\zeta = 20\), three example boxes of the ionization field at three stages of the late EsR are shown in Figure 1. The black patches are regions of neutral islands, and the white regions are ionized. From left to right, we see the evolution of the ionization field: the large neutral islands shrink, while small islands are being ionized and losing their identity as time goes by. The bubbles-in-island effect is obvious throughout the late EoR. Note that the bubbles-in-island effect does not result in large-scale spherical annuli of neutral hydrogen, though spheres are used when calculating the barriers, because the morphology of the neutral regions is determined by the large-scale structure of the density fields. Instead, this effect creates small ionized dots inside islands, or divides a large island into smaller ones, as seen in the figure. As the mean neutral fraction of the universe decreases, the morphology of the ionization field becomes less and less complex, and the shape of the islands gradually approaches spherical or elliptical. We also find that the late stage of reionization proceeds quite fast; assuming \(\zeta = 20\), the mean neutral fraction drops from \(\sim 0.16\) to \(\sim 0.012\) at \(z = 7.0\) and \(z = 6.425\) in our default run, and the reionization is completed (defined as \(\lambda_{\text{HI}} < 0.01\)) at \(z \sim 6.4\).

We compare slices of the simulation box in Figure 2 for \(\zeta = 15\) (top panels), \(\zeta = 30\) (middle panels), and \(\zeta = 30\) without small-scale absorbers (bottom panels). The simulations are run from the same initial condition. To show the morphology of large islands as complete as possible in this figure, we have shifted the box with the periodic boundary condition, to avoid breaking large islands at boundaries. For each case, we show three slices with a decreasing mean neutral fraction from left to right. The slices are chosen to show the results of the three different cases at about the same mean neutral fraction (\(\sim 0.14, 0.10, 0.013\)), though there are slight differences due to the limitation of simulation step size.

The top panels are from the simulation with \(\zeta = 15\), and the middle panels are from the simulation with \(\zeta = 30\), which is our default run. Comparing the \(\zeta = 15\) case (top panels) with the \(\zeta = 30\) (middle panels), we find that the morphologies of the ionization fields are quite similar at a similar mean neutral fraction, insensitive to the ionizing efficiency parameter \(\zeta\). This can be anticipated because in such models the ionization is determined largely by the density field, though it also has some weak dependence on the reionization history.

To show the relative impact of large-scale islands and small-scale absorbers in regulating the reionization process, we also run a \(\zeta = 30\) simulation without the small-scale absorbers, in which the MFP of the ionizing background photons is limited only by the neutral islands, i.e., \(\lambda_{\text{mfp}} = \lambda_i\). The results are shown in the bottom panels of Figure 2. We find that the morphology of the ionization fields are quite similar between the simulations with or without small-scale absorbers, as long
as they are compared at similar neutral fractions, implying that the large-scale neutral islands are dominant in determining the morphology of the ionization field. However, the reionization process is much faster in the absence of small absorbers. Adopting $\zeta = 30$, the reionization completes at $z_{\text{end}} = 7.61$ (when the mean neutral fraction $x_{\text{HI}} < 0.01$) in the case without small-scale absorbers, compared with $z_{\text{end}} = 7.07$ in the simulation with absorbers. Therefore, the small-scale dense absorbers have only a moderate effect on the morphology of the ionization field at a given global neutral fraction, but could delay or prolong the reionization process significantly.

4.1. Island Size Distribution

In Figure 3, we show the comoving size distributions of neutral islands for various global neutral fractions. The neutral islands are selected using the SAM, with the critical neutral fraction set to $f_{\text{HI}}^c = 0.5$. The resulting size distributions of the neutral islands are shown for various global neutral fractions of the universe with $\zeta = 30$ (thick lines) and $\zeta = 15$ (thin lines). The evolution of the two models characterized by different $\zeta$ values are very similar, which is consistent with our impression from the morphology evolution: because the ionization morphologies are similar at the same neutral fraction, the size distribution should also be similar. In each case, there is a characteristic scale for the peak of the neutral island size distribution at each redshift, this scale decreases as the islands are being ionized, but the change is very slow. Judging from the simulation box, this is perhaps because the large neutral islands only shrink gradually, and as they become smaller they compensate for the disappearance of the smaller islands.

However, we must note that the size distribution of the neutral islands depends on the neutral fraction threshold used to define the islands. Figure 4 shows the size distribution of the
islands selected by different neutral fraction thresholds, when the mean neutral fraction of the universe is fixed at 0.16. The thick lines show the size distributions of host islands, and the thin lines show the size distributions of net neutral islands.

With a lower selection threshold \( f_{\text{H}1} \) value, the evolution of island size is more apparent. The upper and lower panels of Figure 5 show the evolution of the size distribution derived with \( f_{\text{H}1} = 0.5 \) and \( f_{\text{H}1} = 0.3 \) respectively. While the peak size of the neutral islands remains nearly constant when \( f_{\text{H}1} = 0.84 \) is used (shown in the right panel of Figure 6), we find now that the peak size does shrink if \( f_{\text{H}1} = 0.3 \) is used when selecting the islands.

It is also seen from Figure 5 that the difference in the size distributions between the different selection thresholds is more significant at higher mean neutral fractions, or at earlier time. This indicates that the shape of the islands is more complex at an earlier stage of reionization. As the mean neutral fraction decreases, the shape of the islands becomes less and less complex, and the size of an island becomes less dependent on the selection threshold in the SAM. The size dependence on the selection threshold of the neutral fraction also implies that in the future 21 cm observations, the higher sensitivity of a radio array could result in a larger typical size of the neutral islands, and more evident evolution in the size of neutral patches.

There are various ways of quantifying the size of the ionized bubbles or the neutral islands. Lin et al. (2016) argued that compared with the spherical averaged result (SAM), the MFP probability distribution function (PDF) is a more physical description of the bubble size. Here we also try the MFP to derive the size distribution of the neutral islands, shown in the left panel of Figure 6 for three stages of the late EoR. Using the MFP description, we find an almost constant characteristic scale for the neutral islands throughout the late EoR. For comparison, we also plot in the right panel of Figure 6 the SAM size distribution derived with \( f_{\text{H}1}^c = 1 - P_\epsilon = 0.84 \), which shows the size distribution of those almost completely neutral islands as defined in the island model. Interestingly, we find an almost non-varying characteristic scale for the neutral islands in this case. This is consistent with the analytical prediction by the island model. The peak scale of the MFP PDF is about 4 Mpc, a bit larger than the typical scale of \( \sim 3 \) Mpc derived by the SAM with \( f_{\text{H}1} = 0.84 \), which is consistent with the expectation in Lin et al. (2016).

However, this invariance of island size is in conflict with the intuition of shrinking islands as seen from the slice maps. Because of the complex shape of the islands and the bubbles-in-islands effect, the MFP description of the islands underestimate the size of the islands, and tends to predict the size of those almost neutral patches as the SAM with \( f_{\text{H}1} = 0.84 \). This implies that, although the MFP is good at representing the ionized bubble sizes and the MFP of ionizing photons during the early stage of reionization, it may not be the best option for characterizing the neutral island sizes at the late stage of EoR.

To test our semi-numerical algorithm, we compare the island size distribution with the result from a hydrodynamic simulation with radiative transfer made by Battaglia et al. (2013b; BTCL hereafter). The BTCL simulation uses the RadHydro code, which combines a particle–particle–particle–mesh (P³M) N-body code with moving frame hydrodynamics and an adaptive ray-tracing radiative transfer algorithm, to simultaneously trace the evolution of dark matter, baryons, and radiation (Trac & Cen 2007). The simulation has a comoving box size of 100 Mpc \( h^{-1} \) a side with 2048\(^3\) dark matter particles, 2048\(^3\) gas cells, and 17 billion adaptive rays. It resolves dark matter halos down to \( M \sim 10^8 M_\odot \) \( h^{-1} \), and radiation sources are populated using the halo model as in Trac & Cen (2007).

The green solid curves in Figure 6 show the island size distributions from the BTCL simulation, when the mean neutral fraction is 0.15. We find general consistency between the island size distribution from our islandFAST and that from the
BTCL simulation. Although with different treatment of small-scale absorbers, the large-scale morphology of the ionization field is largely determined by the large-scale density field. However, we note that the result of BTCL simulation cannot be regarded as "ground truth" either because it neglected additional clumping and self-shielding of small-scale absorbers, and snapshots at lower redshifts ($x_{\mathrm{H\,I}} < 0.15$) are not available. We plan to make further tests of our algorithm in the future, note, however, that to make a reliable test, the radiative transfer simulation should cover a box region with at least side $\lesssim 100 \, \text{Mpc} \, h^{-1}$ (Iliev et al. 2014), and at the same time resolve small scales down to $\sim 10 \, \text{kpc}$, in order to correctly account for recombinations.

4.2. Ionizing Background

While generating the ionization field, islandFAST simultaneously predicts the evolution of the ionizing background over the redshift range simulated. The solid and dashed curves in Figure 7 show the $\text{H\,I}$ photoionization rate, $\Gamma_{\text{H\,I}}$, as a function of redshift predicted by the simulation with $\zeta = 20$ and $\zeta = 15$ respectively. The curves display rapid increase below the background onset redshift, indicating quick growth of the intensity of the ionizing background during the late EoR. After that, the growth of the ionizing background slows down as the reionization approaching the completion. We find that the intensity of the ionizing background and the timing of its rapid growth depends significantly on the adopted ionizing efficiency parameter $\zeta$. A higher ionizing efficiency would result in a much higher intensity and earlier growth of the ionizing background.

To show the effect of small-scale dense absorbers in regulating the ionizing background, in Figure 7, we also plot with the dotted–dashed line the evolution of $\Gamma_{\text{H\,I}}$ predicted by the simulation without small-scale absorbers for $\zeta = 20$. The intensity of the ionizing background is boosted by an order of magnitude in the absence of small absorbers, and the growth of the ionizing background becomes much faster, which results in the rapid completion of the reionization process. Therefore, we conclude that the small-scale absorbers have played a dominant role in regulating the level of the ionizing background, and they delay and prolong the reionization process significantly.

The solid lines in Figure 8 show the evolution of the MFP of the background ionizing photons derived from islandFAST, with the thick line from the simulation with $\zeta = 20$ and the thin line from the simulation with $\zeta = 15$. The evolution of the MFP shows similar trends as the growth in the intensity of the ionizing background, and the timing of the growth is also
sensitive to the ionization efficiency. To reveal the relative importance of the underdense islands and the overdense absorbers in limiting the MFP of the ionizing photons, we plot the $\lambda_i$ and $\lambda_{\text{abs}}$ separately with the dashed and dotted lines, extrapolated from the fitting formula by Songaila & Cowie (2010). The thick lines are from the simulation with $\zeta = 20$, and the thin lines are from the simulation with $\zeta = 15$.

**Figure 8.** Evolution of the mean-free path of ionizing photons (solid lines). The dashed lines show the mean-free path due to neutral islands, while the dotted-dashed line indicates the mean-free path due to small-scale absorbers only, extrapolated from the fitting formula by Wyithe & Bolton (2017 August 1 Xu, Yue, & Chen 2017). The thick lines are from the simulation with $\zeta = 20$, and the thin lines are from the simulation with $\zeta = 15$.

5. Conclusions and Discussions

In this paper, we present the algorithm and some simulation results from a semi-numerical reionization simulation code, islandFAST, which is designed for the late stage of reionization, after the percolation of ionized bubbles. It is based on the island model, and is an extension of the semi-numerical reionization simulation code based on the bubble model. The simulation incorporates the effect of ionizing background photons on the neutral islands, i.e., underdense regions that are ionized later. As long as a reasonable model for the small-scale absorbers is provided, islandFAST predicts the evolution of the ionization field and simultaneously generates the intensity of the ionizing background as well as the MFP of the ionizing photons.

We find that the late reionization proceeds very fast, showing a characteristic scale of the neutral islands at each redshift. As expected, in the simulation the large islands shrink with time and the small ones are swamped by the ionizing photons as reionization progresses. Using either the SAM with a high neutral fraction threshold ($\zeta_{\text{HI}} = 0.84$) or the MFP PDF, we derive the size distribution of the neutral islands. An interesting result is that these distributions exhibit a relatively robust characteristic scale of a few Mpcs throughout the late EoR. When the SAM is used and a lower threshold of neutral fraction is used for island selection, the islands have larger sizes and the size evolves more evidently.

As expected from the island model, we find that the bubbles-in-island effect is obvious throughout the late EoR. As the mean neutral fraction of the universe decreases, the morphology of the ionization field becomes less and less complex, and the shape of the islands gradually approaches spherical or elliptical.

A number of previous works attempted to improve the treatment of the late stage of reionization in semi-numerical works, though they are somewhat limited by the basic assumption of the bubble model, which becomes inaccurate when the bubbles overlap. For example, Alvarez & Abel (2012) introduced an MFP for the ionizing photons to model the effect of small absorbers, but the ionizing background is not explicitly calculated. Sobacchi & Mesinger (2014) and Kulkarni et al. (2016) calculated the photoionization rate within each local H II region, in order to account for the self-shielding effect, but did not consider the contribution of the global background of ionizing photons, which is inevitable after the percolation of ionized bubbles.

As in Alvarez & Abel (2012), we assumed a uniform ionizing flux, and computed the effect of the absorbers on the progress of reionization. However, instead of taking a constant MFP $\lambda_{\text{abs}}$ and limiting the maximum filter scale for the excursion set random walk to $R_e < \lambda_{\text{abs}}$ throughout the reionization history, we adopt an empirical model of redshift evolution for $\lambda_{\text{abs}}$ fitted to observations and take into account the effect of absorbers by modulating the ionizing background. Sobacchi & Mesinger (2014) implemented a sub-grid recipe of inhomogeneous recombinations inside large-scale, semi-numerical simulations of $21\text{cmFAST}$ to model the growth of H II regions, and studied the role of absorbers. They accounted for reionization feedback effects on both the sources and the sinks and their prediction of MFP in H II regions is consistent with
our input model of absorbers, i.e., empirical model by Songaila & Cowie (2010). Kulkarni et al. (2016) combines high dynamic range cosmological simulations with bubble-model-based algorithm to generate the ionization field during reionization. The hydrodynamical simulations have sufficient resolution to account for the self-shielding of neutral hydrogen in dense regions, and the simulation parameters are calibrated such that they reproduce the observational constraints on the IGM Ly$\alpha$ opacity at $z \leq 6$ as well as the electron-scattering optical depth. In agreement with these previous works, we find that small-scale absorbers are responsible for prolonging and delaying the completion of reionization, and for slowing down the rapid rise of the MFP and the intensity of the ionizing background during the late stage of reionization. However, unlike Alvarez & Abel (2012), we find that the island-model-based islandFAST predicts that the large-scale morphology of the ionization field is dominated by the neutral islands rather than small absorbers.

In summary, islandFAST can be used to investigate the roles played by both the large-scale underdense islands and small-scale overdense absorbers in modulating the ionizing background. Neglecting the small absorbers, we provide a self-consistent model for the evolution of the ionizing background regulated by only the shading effect of large-scale islands. Taking also the small absorbers into account, islandFAST serves as a tool to model the relative contribution of islands and small absorbers to the IGM opacity. We find that while the large-scale islands dominate the morphology of the ionization field, it is the small-scale absorbers that dominate the opacity of the IGM, and play a major role in limiting the MFP of the ionizing photons and determining the intensity of the ionizing background. There is a sudden increase in the intensity of the ionizing background around percolation, and the hydrogen ionization rate of the ionizing background is reduced by an order of magnitude in the presence of dense absorbers. We also confirm the previous claim that the small absorbers delay and prolong the reionization process. However, there is still some quantitative discrepancy in the model prediction with current observation results, so at present the conclusions regarding to the ionizing background should be taken as qualitative.

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