Universal Scotogenic Fermion Masses
in Left-Right Gauge Model

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Abstract

In the conventional left-right gauge model, if the Higgs scalar sector consists only of an $SU(2)_L$ doublet and an $SU(2)_R$ doublet, fermion masses are zero at tree level. There have been many studies on how they would become massive. With the help of a dark sector with $U(1)_D$ gauge symmetry, it is shown how all standard-model fermions may acquire realistic masses radiatively, including that of the top quark. In this context, the particle content of the model also implies the automatic conservation of baryon number $B$ and lepton number $L$ as in the standard model. Observable anomalous Higgs couplings are predicted.
**Introduction**: In the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge model (SM) of quarks and leptons, the one scalar Higgs doublet serves two purposes. It breaks $SU(2) \times U(1)_Y$ to the electromagnetic gauge symmetry $U(1)_Q$, and renders all fermions massive at tree level, except the left-handed doublet neutrino, unless it has a right-handed singlet counterpart. In the canonical left-right extension to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, left-handed fermions are $SU(2)_L$ doublets, right-handed fermions are $SU(2)_R$ doublets. The breaking of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ to $U(1)_Q$ may be achieved by an $SU(2)_L$ Higgs doublet and an $SU(2)_R$ Higgs doublet, but they do not render the fermions massive at tree level. A scalar bidoublet under $SU(2)_L \times SU(2)_R$ would be needed, but is being withheld on purpose. There have been many studies on how quarks and leptons may acquire masses in this situation \[1,2,3,4,5,6,7\]. They are usually not applicable to the top quark, because $m_t = 173$ GeV is of order the electroweak breaking scale $v = \sqrt{2} \langle \phi^0 \rangle = 246$ GeV, and conventional wisdom would insist that it be accorded a tree-level mass \[8\]. In particular, if a radiative $m_t$ is desired, then it ought to be proportional to $v$, but suppressed by the typical loop factor of $16\pi^2$. Hence very large couplings to new particles are required and perturbative calculations become unreliable. In this work, it will be shown how this objection may be overcome, and all SM fermion masses may be generated radiatively from a dark sector (scotogenic) with a $U(1)_D$ gauge symmetry in the left-right context. Phenomenological consequences will be discussed.

**Outline of Model**: The particle content of the proposed model (similar to those of Refs. \[3,7\]) is listed in Table 1. There are three families of quarks and leptons, as well as $N_{L,R}$. There is only one copy each of the scalars $\Phi_L, \Phi_R, \zeta_{L,R}, \eta_{L,R}, \sigma$. The dark $U(1)_D$ gauge symmetry is broken by three units through the complex singlet scalar $\sigma$. This allows a global $D$ symmetry to remain and prevents $N_{L,R}$ as well as $\nu_{L,R}$ to acquire Majorana masses, as pointed out first in Ref. \[9\] and applied to Dirac neutrinos using $B - L$ in Ref. \[10\]. From the allowed
| fermion/scalar | $SU(3)_C$ | $SU(2)_L$ | $SU(2)_R$ | $U(1)_{B-L}$ | $U(1)_D$ |
|---------------|-----------|-----------|-----------|-------------|---------|
| $(u, d)_L$    | 3         | 2         | 1         | $1/6$       | 0       |
| $(u, d)_R$    | 3         | 1         | 2         | $1/6$       | 0       |
| $(\nu_e, e)_L$ | 1         | 2         | 1         | $-1/2$      | 0       |
| $(\nu_e, e)_R$ | 1         | 1         | 2         | $-1/2$      | 0       |
| $\Phi_L = (\phi^+_L, \phi^0_L)$ | 1         | 2         | 1         | $1/2$       | 0       |
| $\Phi_R = (\phi^+_R, \phi^0_R)$ | 1         | 1         | 2         | $1/2$       | 0       |
| $\zeta_L = (\zeta^{2/3}_L, \zeta^{-1/3}_L)$ | 3         | 2         | 1         | $1/6$       | 1       |
| $\zeta_R = (\zeta^{2/3}_R, \zeta^{-1/3}_R)$ | 3         | 1         | 2         | $1/6$       | 1       |
| $\eta_L = (\eta^0_L, \eta^-_L)$ | 1         | 2         | 1         | $-1/2$      | 1       |
| $\eta_R = (\eta^0_R, \eta^-_R)$ | 1         | 1         | 2         | $-1/2$      | 1       |
| $N_{L,R}$     | 1         | 1         | 1         | 0           | 1       |
| $\sigma$      | 1         | 1         | 1         | 0           | 3       |

Table 1: Fermion and scalar content of left-right model with $U(1)_D$.

Yukawa couplings between the SM fermions and the dark particles, it is easily seen that the conventionally defined baryon number $B = 1/3$ and $L = 1$ for quarks and leptons are automatically transferred to $\zeta_{L,R}$ and $\eta_{L,R}$ with $N_{L,R}$ having $B = L = 0$.

**Origin of Large Radiative Top Mass**: The one-loop diagram for a quark with charge $2/3$ is given in Fig. 1. The scalars $\zeta_{L,R}$ mix to form mass eigenstates $\zeta_1 = \cos \theta \zeta_L - \sin \theta \zeta_R$, $\zeta_2 = \cos \theta \zeta_R + \sin \theta \zeta_L$, with masses $m_{1,2}$. The diagram is then easily calculated to be

$$m_u = \frac{f_L f_R \sin \theta \cos \theta m_N}{16\pi^2} \left[ \frac{m_1^2 \ln(m_2^2/m_N^2) - m_2^2 \ln(m_2^2/m_N^2)}{m_2^2 - m_N^2} \right. - \frac{m_1^2 \ln(m_2^2/m_N^2)}{m_1^2 - m_N^2} \left. \right],$$

(1)
which is of the same form as that of the original scotogenic model [11] for Majorana neutrino mass. The usual assumption is that \( m^2_2 - m^2_1 \) is small compared to \( m^2_0 = (m^2_2 + m^2_1)/2 \) and \( m_0 << m_N \), in which case a seesaw radiative mass is obtained, i.e.

\[
m_u \simeq \frac{f_L f_R \sin \theta \cos \theta (m^2_2 - m^2_1)}{16 \pi^2 m_N} \ln \left[ \frac{m^2_N}{m^2_0} - 1 \right].
\] (2)

This clearly makes \( m_u \) very small. Another choice [12, 13] is \( m_{1,2} >> m_N \), in which case

\[
m_u \simeq \frac{f_L f_R \sin \theta \cos \theta m_N}{16 \pi^2} \ln \left[ \frac{m^2_2}{m^2_1} \right].
\] (3)

This formula was applied [13] to neutrinos where \( m_N \) is of order keV to act as warm dark matter, but it is obvious that \( m_N \) is actually arbitrary and may be chosen large enough to allow \( m_t = 173 \text{ GeV} \) as a radiative effect.

Here the choice \( m^2_2 >> m^2_{N_3} >> m^2_1 \) is made, allowing different choices for \( N_{1,2} \), to be discussed later. Hence

\[
m_t \simeq \frac{f_L f_R \sin \theta \cos \theta m_{N_3}}{16 \pi^2} \ln \left[ \frac{m^2_2}{m^2_{N_3}} \right].
\] (4)

As an example, let \( m_2 = 50 \text{ TeV}, m_{N_3} = 15 \text{ TeV}, m_1 = 1 \text{ TeV} \), then \( m_t = 173 \text{ GeV} \) is obtained for \( f_L f_R \sin \theta \cos \theta = 0.682 \).

The above may be achieved in the SM, but not in a very natural way. First, the tree-level Higgs coupling to fermions must be forbidden by a new symmetry, say \( Z_2 \) under which all right-handed fermions are odd. This is often used for example in models where a small Dirac neutrino mass is desired [14]. Another is to postulate a non-Abelian discrete family symmetry, such as \( A_4 \) [15], and assign left-handed and right-handed fermions differently so that they do not couple to the SM Higgs boson. However, such symmetries must be softly broken appropriately to allow radiative fermion masses to appear [16]. To obtain \( m_t = 173 \text{ GeV} \), this requires the corresponding soft breaking trilinear scalar term to have a coupling of order \( 10^7 \text{ GeV} \). In contrast, it is here naturally derived from the breaking of \( SU(2)_R \) at a high scale.
Dark Scalar Sector: The $2 \times 2$ mass-squared matrix spanning $(\zeta_L, \zeta_R)$ is

$$\mathcal{M}_\zeta^2 = \begin{pmatrix} m_L^2 & \lambda_{LR}v_Lv_R \\ \lambda_{LR}v_Lv_R & m_R^2 \end{pmatrix},$$

where the off-diagonal term comes from the quartic coupling $\lambda_{LR}(\zeta_L\phi^0_L)(\zeta_R\phi^0_R)^*$. Now $m_L^2 = m_1^2 \cos^2 \theta + m_2^2 \sin^2 \theta$, $m_R^2 = m_2^2 \cos^2 \theta + m_1^2 \sin^2 \theta$, and $\lambda_{LR}v_Lv_R = (m_2^2 - m_1^2) \sin \theta \cos \theta$. This means that $v_R$ has to be very large for $m_2 = 50$ TeV, say of order $10^7$ GeV, so that the $SU(2)_R$ gauge bosons are out of reach at the Large Hadron Collider (LHC).

In the $d$ quark sector, the corresponding diagram is given in Fig. 2. Hence the off-diagonal entries of Eq. (5) is replaced by $\lambda'_{LR}$ from the quartic coupling $\lambda'_{LR}(\zeta_L\bar{\phi}^0_L)(\zeta_R\bar{\phi}^0_R)^*$. This means that $m_{1,2}$ and $\theta$ in this sector may be different, allowing $m_b$ to be much smaller than $m_t$.

The lepton masses are obtained in exact analogy with the dark scalars $\eta_{L,R}$ instead of $\zeta_{L,R}$. Whereas $N_{1,2,3}$ are common to quarks and leptons, the $\eta$ masses and their mixing angle in the charged-lepton and neutrino sectors are different. The smallness of the Dirac neutrino masses are then related to the smallness of the quartic coupling $(\eta^0_L\bar{\phi}^0_L)(\eta^0_R\bar{\phi}^0_R)^*$.

Structure of Quark and Lepton Mass Matrices: Let the $N_{1,2,3}$ mass matrix be diagonalized, so that $m_{N_1} = 15$ TeV, $m_{N_2} = 800$ GeV, and $m_{N_3} = 10^8$ GeV. Let the $3 \times 3$ quark mass
matrix linking \((u, c, t)_L\) to \((u, c, t)_R\) be denoted by

\[
\mathcal{M}_u = \begin{pmatrix}
m_{uu} & m_{uc} & m_{ut} \\
m_{cu} & m_{cc} & m_{ct} \\
m_{tu} & m_{tc} & m_{tt}
\end{pmatrix}.
\]

(6)

Whereas only \(N_3\) contributes to \(m_{tt}\) as in Eq. (4), \(N_2\) contributes to the \(2 \times 2\) submatrix spanning \((c, t)\) of the form

\[
m_{(c,t)} \approx \frac{f_L f_R \sin \theta \cos \theta m_{N_2}}{16\pi^2} \left[ \ln \frac{m_{N_2}^2}{m_{N_1}^2} - \frac{m_1^2}{m_1^2 - m_{N_2}^2} \ln \frac{m_{N_2}^2}{m_{N_1}^2} \right],
\]

(7)

and \(N_1\) contributes to the entire \(3 \times 3\) matrix of the form

\[
m_{(u,c,t)} \approx \frac{f_L f_R \sin \theta \cos \theta m_2^2}{16\pi^2 m_{N_1}} \ln \frac{m_{N_1}^2}{m_2^2}.
\]

(8)

It is clear that the \(N_{3,2,1}\) contributions are of decreasing magnitude. Similar structures appear in the \((d, s, b)\), \((e, \mu, \tau)\), and \((\nu_e, \nu_\mu, \nu_\tau)\) mass matrices.

**Dark Matter**: In this model, \(N_2\) is the lightest particle of the dark sector. It interacts mostly with the second family of quarks and leptons. As such, its annihilation through \(\zeta_1\) and \(\eta_1\) to \(c, s\) quarks and \(\mu, \nu_\mu\) leptons may be large enough to have the correct relic abundance, and yet not affect the direct-search constraints which involve the \(u,d\) quarks. The very heavy \(N_1\) serves two purposes. It allows very light masses for the first family of quarks and leptons; and it avoids any serious constraint from direct-search experiments.

The annihilation of \(N_2\bar{N}_2 \to c\bar{c}\) is dominated by \(\zeta_1\), as shown in Fig. 3. The cross section

\[
\text{Figure 3: Diagrams for } N_2\bar{N}_2 \to c\bar{c}.
\]
× relative velocity is given by
\[ \sigma_{\text{ann}} \times v_{\text{rel}} = \frac{m_N^2 (f_L^4 \cos^4 \theta + f_R^4 \sin^4 \theta)}{16\pi (m_N^2 + m_1^2)^2}. \] (9)

This should be multiplied by a factor of 8 to account for the 3 colors of c plus those of s, as well as µ and νµ, assuming these other contributions are the same in magnitude, and set equal to the canonical value of \(3 \times 10^{-26}\) cm³/s. For \(m_N = 800\) GeV and \(m_1 = 1\) TeV,
\[ f_L^4 \cos^4 \theta + f_R^4 \sin^4 \theta = 0.0678 \] (10)
is obtained. As an example, \(\sin \theta = \cos \theta = 1/\sqrt{2}\) yields \((f_L^4 + f_R^4)^{1/4} = 0.72\). Since \(\zeta_1\) is analogous to a scalar quark in supersymmetry and \(N_2\) analogous to a neutralino, they are subject to search limits at the LHC. The updated ATLAS result [17] shows that for \(m_1 = 1\) TeV, \(m_N = 800\) GeV is just at the edge of the allowed region.

**Higgs and Gauge Sectors**: The Higgs sector consists of the scalars \(\Phi_{L,R}\) and \(\sigma\). Their potential is given by
\[ V = -\mu_L^2 \Phi_L^\dagger \Phi_L - \mu_R^2 \Phi_R^\dagger \Phi_R - \mu_2^2 \sigma^* \sigma + \frac{1}{2} \lambda_L (\Phi_L^\dagger \Phi_L)^2 + \frac{1}{2} \lambda_R (\Phi_R^\dagger \Phi_R)^2 + \frac{1}{2} \lambda_\sigma (\sigma^* \sigma)^2 + \lambda_{LR} (\Phi_L^\dagger \Phi_L) (\Phi_R^\dagger \Phi_R) + \lambda_{L\sigma} (\Phi_L^\dagger \Phi_L) (\sigma^* \sigma) + \lambda_{R\sigma} (\Phi_R^\dagger \Phi_R) (\sigma^* \sigma). \] (11)

After the spontaneous breaking of \(SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_D\), the only physical scalars left are the real parts of \(\phi^0_{L,R}\) and \(\sigma\). Let
\[ \Phi_L = \begin{pmatrix} 0 \\ (v_L + h_L)/\sqrt{2} \end{pmatrix}, \quad \Phi_R = \begin{pmatrix} 0 \\ (v_R + h_R)/\sqrt{2} \end{pmatrix}, \quad \sigma = \frac{1}{\sqrt{2}} (v_D + h_D), \] (12)
then the 3 × 3 mass-squared matrix spanning \((h_L, h_R, h_D)\) is
\[ \mathcal{M}_h^2 = \begin{pmatrix} \lambda_L v_L^2 & \lambda_{LR} v_L v_R & \lambda_{L\sigma} v_L v_D \\ \lambda_{LR} v_L v_R & \lambda_R v_R^2 & \lambda_{R\sigma} v_R v_D \\ \lambda_{L\sigma} v_L v_D & \lambda_{R\sigma} v_R v_D & \lambda_\sigma v_D^2 \end{pmatrix}. \] (13)

Since \(v_R \sim 10^7\) GeV, and \(v_D\) should be at least a few TeV, \(h_L\) acts to all intents and purposes as the SM Higgs boson in this model. The heavier scalars \(h_R\) and \(h_D\) decay quickly to \(h_L h_L\) through \(\lambda_{LR}\) and \(\lambda_{L\sigma}\) respectively.
In the gauge sector, the $Z_D$ boson gets a mass equal to $3g_Dv_D$. It should be heavier than $2m_{N_2}$, so it decays at least to $N_2\bar{N}_2$. The charged $W_{L,R}^\pm$ masses are $g_L v_L$ and $g_R v_R$. The $Z, Z'$ mass-squared matrix is

$$M_{Z,Z'}^2 = e^2 \left( \frac{v^2_T}{x(1-x)} \frac{v^2_L}{(1-x)\sqrt{1-2x}} \frac{v^2_R}{x(1-2x) + x v^2_L/(1-x)(1-2x)} \right),$$

where $e^{-2} = g_L^{-2} + g_R^{-2} + g_B^{-2}$, and $g_L = g_R$ with $x = \sin^2\theta_W$. The $Z-Z'$ mixing is then about $x\sqrt{1-2x^2}v_L^2/(1-x)^2v_R^2$, which is of order $10^{-10}$, much less than the experimental bound of about $10^{-4}$ [18].

**Anomalous Higgs Couplings**: In the SM, the Higgs boson $h$ has couplings to fermions fixed at $m_f/v$, where $v = 246$ GeV. Here $h_L$ has anomalous couplings, as first pointed out in Ref. [19]. There are three contributions, the first being the $\lambda_{LR}(\phi_L^0)(\zeta R\phi_R^0)^*$ coupling, the others from $|\phi_L^0|^2|\zeta_L|^2$ and $|\phi_R^0|^2|\zeta_R|^2$. The latter are suppressed by the ratio $v_L/v_R$, and will be neglected. The $h_L$ coupling to $t\bar{t}$ is then given by [19]

$$f_t = \frac{f_L f_R \sin \theta \cos \theta m_N}{16\pi^2 v_L} \left[ (\cos^4 \theta + \sin^4 \theta) \left[ \frac{m_N^2 \ln(m_N^2/m_2^2)}{m_N^2 - m_2^2} - \frac{m_N^2 \ln(m_N^2/m_1^2)}{m_N^2 - m_1^2} \right] + \sin^2 \theta \cos^2 \theta (m_2^2 - m_1^2) \left[ \frac{1}{m_2^2 - m_N^2} - \frac{m_N^2 \ln(m_N^2/m_2^2)}{(m_2^2 - m_N^2)^2} + \frac{1}{m_1^2 - m_N^2} - \frac{m_N^2 \ln(m_N^2/m_1^2)}{(m_1^2 - m_N^2)^2} \right] \right].$$

Using the expression for $m_t$ from Eq. (1),

$$f_t = \frac{m_t}{v_L} \left[ \cos^4 \theta + \sin^4 \theta + \sin^2 \theta \cos^2 \theta (m_2^2 - m_1^2) \left[ \frac{m_N^2 \ln(m_N^2/m_2^2)}{m_N^2 - m_2^2} - \frac{m_N^2 \ln(m_N^2/m_1^2)}{m_N^2 - m_1^2} \right]^{-1} \left[ \frac{1}{m_2^2 - m_N^2} - \frac{m_N^2 \ln(m_N^2/m_2^2)}{(m_2^2 - m_N^2)^2} + \frac{1}{m_1^2 - m_N^2} - \frac{m_N^2 \ln(m_N^2/m_1^2)}{(m_1^2 - m_N^2)^2} \right] \right],$$

where the example of $m_N = 15$ TeV, $m_1 = 1$ TeV, and $m_2 = 50$ TeV has been used as previously. Thus $f_t$ is predicted to be greater than that of the SM, which is a generic result [19]. If $\sin \theta = 0.1$ is assumed, then the ratio of $f_t$ to that of the SM is 1.17. The above may also be applied to $f_b$ with same $m_{L,R}$ but different $\theta$. The present measurements
of Higgs production and decay from collider data [18] are consistent with the predictions of the SM, but there is much room for possible deviations. A comprehensive analysis is left to future work.

**Other Phenomenological Consequences**: Each $3 \times 3$ radiative fermion mass matrix may be diagonalized by unitary matrices $U_L^\dagger$ on the left and $U_R$ on the right. However, they do not diagonalize the corresponding magnetic-moment matrix. This means that on top of the contributions from the dark scalars to the anomalous magnetic moments of quarks and leptons, there are new sources of off-diagonal radiative transitions, such as $b \to s \gamma$, $\mu \to e \gamma$, etc. Constraints from experimental data require these to be small. A comprehensive analysis is left to future work.

Neutrinos are Dirac fermions in this model. They have radiative masses, but are only suppressed relative to those of the charged leptons by making $\sin \theta$ in the neutrino sector very very small. Thus the smallness of neutrino masses requires fine tuning, which is indeed a shortcoming of this model. To remedy this situation, an $SU(2)_R$ Higgs triplet $(\Delta^{++}_R, \Delta^+_R, \Delta^0_R)$ may be added to break $SU(2)_R$ at a high scale, so that $\nu_R$ gets a large Majorana mass. Lepton number $L$ now becomes lepton parity $(-1)^L$ [20]. The usual seesaw mechanism applies and neutrinos obtain naturally small Majorana masses. Note that $\Delta_R$ does not affect the generic radiative mechanism for the Dirac fermion masses.

**Conclusion**: In the context of left-right gauge symmetry, a natural scenario exists where all fermions obtain radiative masses. This is enforced by a very simple Higgs sector, consisting of one $SU(2)_L$ doublet and one $SU(2)_R$ doublet. The absence of a scalar bidoublet means that all quarks and leptons are massless at tree level. A dark sector is then assumed with a gauge $U(1)_D$ symmetry as shown in Table 1. It is spontaneously broken by a complex singlet scalar with $D = 3$. The resulting theory conserves global $D$, as well as global baryon number $B$ and lepton number $L$. 


The $SU(2)_R$ gauge symmetry is broken at a high scale, say of order $10^7$ GeV, allowing $m_t = 173$ GeV to be radiatively generated perturbatively in one loop. This goes against the conventional wisdom that $m_t$ must be a tree-level mass, based on knowing that the electroweak breaking scale is $v_L = 246$ GeV. Anomalous Higgs couplings to all fermions are predicted. This will affect both the production and decay of the observed 125 GeV Higgs boson at the LHC. It accentuates the importance of measuring all Higgs properties precisely in the future.

Dark matter is now intimately related to fermion mass generation. It is a gauge singlet Dirac fermion $N_2$. It couples mainly to the second family of quarks and leptons. It annihilates through dark scalar exchange to $c, s, \mu, \nu_\mu$, but does not interact with nuclei significantly. Because of the dark color scalars which may be produced copiously in pairs at the LHC, $N_2$ may be observed as large missing momentum in their decays.

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References

[1] A. Davidson and K. C. Wali, Phys. Rev. Lett. 59, 393 (1987).

[2] R. N. Mohapatra, Phys. Lett. B201, 517 (1988).

[3] E. Ma, Phys. Rev. Lett. 63, 1042 (1989).

[4] B. Brahmachari, E. Ma, and U. Sarkar, Phys. Rev. Lett. 91, 911801 (2003).

[5] E. Gabrielli, L. Marzola, and M. Raidal, Phys. Rev. D95, 035005 (2017).

[6] E. Ma and U. Sarkar, Phys. Lett. B776, 54 (2018).
[7] E. Ma, Phys. Lett. B811, 135971 (2020).

[8] E. Ma, Phys. Rev. Lett. 64, 2866 (1990).

[9] E. Ma, I. Picek, and B. Radovcic, Phys. Lett. B726, 744 (2013).

[10] E. Ma and R. Srivastava, Phys. Lett. B741, 217 (2015).

[11] E. Ma, Phys. Rev. D73, 077301 (2006).

[12] G. B. Gelmini, E. Osoba, and S. Palomares-Ruiz, Phys. Rev. D81, 063529 (2010).

[13] E. Ma, Phys. Lett. B717, 235 (2012).

[14] E. Ma and O. Popov, Phys. Lett. B764, 142 (2017).

[15] E. Ma and G. Rajasekaran, Phys. Rev. D64, 113012 (2001).

[16] E. Ma, Phys. Rev. Lett. 112, 091801 (2014).

[17] G. Aad et al. (ATLAS Collaboration), arXiv:2010.14293 [hep-ex].

[18] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D98, 030001 (2018).

[19] S. Fraser and E. Ma, Europhys. Lett. 108, 11002 (2014).

[20] E. Ma, Phys. Rev. Lett. 115, 011801 (2015).