Harmonic Oscillation Characteristic using Visual Basic Application

Zakaria Victor Kareth\textsuperscript{1*}, Khaeriah Dahlan\textsuperscript{2}, Muhammad Akbar\textsuperscript{3}, and Octolia Togibasa\textsuperscript{4}

\textsuperscript{1}Department of Geophysics, Universitas Cenderawasih, Jayapura, 99351, Indonesia, \\
\textsuperscript{2}Department of Physics, Universitas Cenderawasih, Jayapura, 99351, Indonesia, \\
\textsuperscript{3}Department of Geophysics, Universitas Cenderawasih, Jayapura, 99351, Indonesia, \\
\textsuperscript{4}Department of Physics, Universitas Cenderawasih, Jayapura, 99351, Indonesia

zvkareth@gmail.com

Abstract: The Visual Basic Application (VBA) programming application has become a new choice by researchers to work on numerical solutions, since it is accessible to wider user. Although the solution of harmonic oscillation has been done in previous research, however numerical solution using VBA was still limited. In this study we solved and plotted the numerical solution using VBA and obtained a solution of simple harmonic motion that is very close to analytical solutions. It is concluded that the results of harmonic oscillation motion analysis obtained with numerical solutions using VBA provide the same characteristics with analytical solutions, for both Euler and Runge-Kutta second order methods. In this work we have shown how the Euler and Runge-Kutta second order methods were applied, so that it can be exploited by another user.

1. Introduction
A simple harmonic motion, a special type of periodic motion, generally defined as a motion in which the acceleration of a body is directly proportional to its displacement from the equilibrium position but in the opposite direction [1]. A vibrating system that executes simple harmonic motion is called a harmonic oscillator. Harmonic oscillation phenomena can be found in many areas of physics, such as electron motion in atoms, current and voltage behavior in the electrical circuits and planetary orbits. Simple harmonic motion was divided into two types, namely simple linear and angular harmonic motion; however the physics concept of harmonic motion often involves complex and difficult analytical mathematical equations. To overcome these difficulties, numeric solution has been widely used by researchers so that complex motion equations can be done quickly and easily [2].

Computational physics is a mix of analytical work, physics, numerical methods, programming, and visualization [3]. Various widely programming language were used for numeric solution, such as Phyton, Matlab, Borland Delphi, and more. Unfortunately, those languages were rarely used by students and universities in undeveloped region, such as Papua, since they are not available to everyone easily. Previous study introduced an interactive package namely Visual Basic Application (VBA), written in the universally available Microsoft Excel [4]. Recently, VBA language has been favored by researchers in the field of computational physics to work on numerical and simulated solutions [4-6] since the advantage that it is accessible to a much wider user.
The purpose of the present work is to develop the numerical solution of harmonic motion by using VBA from Microsoft Excel. The well known Euler’s and Runge-Kutta methods were chosen for the numerical solution to compare Euler’s accuracy and Runge-Kutta’s time stepping schemes [7]. We consider that it may be useful for both students and researcher. The package has been tested under Excel 2007 and 2010, and runs satisfactorily in both versions.

### 2. Experimental Details

#### 2.1. Euler’s Method

Euler’s method is one of the methods for numerical solution of initial value problem of Ordinary Differential Equations (ODE) of the form given by equations (1) below

\[ \frac{dx}{dt} = f(t,x); \quad x(t_0) = x_0 \]  

(1)

where \( x(t_0) = x_0 \) was the initial condition. The derivative in equation (1) is approximated and expressed in the iterative form so it can be implemented in software like VBA in Microsoft Excel.

\[ x_{n+1} = x_n + \Delta t \cdot f(t,x) \]  

(2)

Experiment steps of implementing Euler’s methods on equations of simple harmonic oscillation and damped oscillation were given as follows: representing the second order ODEs into two couple first order ODE, approximating the derivatives and expressing them in iterative form as follows

\[ x_{n+1} = x_n + \Delta t \cdot v \]  

(3)

\[ v_{n+1} = v_n - \frac{k}{m} x_n \]  

(4)

\[ x_{n+1} = x_n + \Delta t \cdot v_n \]  

(5)

\[ v_{n+1} = v_n - \Delta t \cdot \left( \frac{b}{m} v_n + \frac{k}{m} x_n \right) \]  

(6)

Equations (3) and (4) were the iterative form of simple harmonic equation, whereas equations (5) and (6) were the iterative form of damping oscillation equations.

#### 2.2. Runge-Kutta Second Order Method

Runge-Kutta second order method is one of the methods for numerical solutions of initial value problem of ODE of the form as given in equation (1). Moreover, the iterative forms were given as follow

\[ x_{n+1} = x_n + \frac{\Delta t}{2} \left( k_1 + k_2 \right) \]  

(7)

\[ k_1 = f(x_n, t_n) \]  

(8)

\[ k_2 = f(t_n + \Delta t, x_n + \Delta t \cdot k_1) \]  

(9)

Implementing the Runge-Kutta second order method on equations of simple harmonic motion and damping oscillation, we obtained the iterative forms as follow

\[ x_{n+1} = x_n + \Delta t \cdot v_n \]  

(10)

\[ v_{n+1} = v_n + \frac{\Delta t}{2} \left( k_1 + k_2 \right) \]  

(11)

\[ k_1 = - \frac{k}{m} \]  

(12)

\[ k_2 = - \frac{k}{m} \left( x_n + \Delta t \cdot k_1 \right) \]  

...(13)

\[ x_{n+1} = x_n + \Delta t \cdot v_n \]  

(14)

\[ v_{n+1} = v_n + \frac{b}{m} \left( k_1 + k_2 \right) \]  

(15)

\[ k_1 = - \left( \frac{b}{m} v_n + \frac{k}{m} x_n \right) \]  

(16)

\[ k_2 = - \left( \frac{b}{m} v_n + \frac{k}{m} \left( x_n + \Delta t \cdot k_1 \right) \right) \]  

(17)

Equations (10), (11), (12) and (13) represent iterative form of equation of simple harmonic motion whereas equations (14), (15), (16) and (17) represent iterative form of equation of damping oscillation.
3. Results and Discussions
Figure 1 and 2 show displacement curve of simple harmonic motion and absolute error respectively, for parameters given in table 1, Euler’s method provides better numerical solution with maximum absolute error of lower than of 0.002 whereas that of Runge-Kutta 2\textsuperscript{nd} order method was approximately of 0.007.

![Figure 1. Simple harmonic motion of analytic, Euler and Runge-Kutta 2\textsuperscript{nd} order.](image1)

![Figure 2. Absolute error of Euler and Runge-Kutta 2\textsuperscript{nd} order for simple harmonic motion.](image2)

| Table 1. Computation parameters |
|---------------------------------|
| **Symbols** | **Meaning** | **Value** |
| \(N\) | Iterations | 7000 |
| \(m\) | Mass | 0.68 kg |
| \(k\) | Spring constant | 65 |
| \(\Delta t\) | Time step | 0.0002 |

Figures 3 and 4 shows displacement of critical damping oscillation and its absolute error, both Euler’s and Runge-Kutta 2\textsuperscript{nd} order methods exhibit similar result with maximum error of about 0.057 at early time.
Figure 3. Damped oscillation with damping factor $b = 2\sqrt{k.m}$

Figure 4. Absolute error of Euler and Runge-Kutta 2nd order for critical damping oscillation.

Figure 5 and 6 show displacement of under damping oscillation and its absolute error, Euler method gives better approximation than Runge-Kutta 2nd order method with increasing error gap with time.

Figure 5. Damped oscillation with damping factor $b = 0.15\sqrt{k.m}$
Figure 6. Absolute error of Euler and Runge-Kutta 2nd order for under damping oscillation.

4. Conclusions
In this work, we have shown that the numeric solution of equation of simple harmonic motion and damped oscillation can be performed using visual basic application from Microsoft Excel. Based on parameters used in the computation, Euler's method provides better solutions than Runge-Kutta second order method for simple harmonic motion and under damping oscillation, whereas similar approximations were achieved for critical damping oscillation.

References
[1] DK Jha. Textbook of Simple Harmonic Motion and Wave Theory. Discovery Publishing. 2005 1-8
[2] Durran, D. R. 2013. Numerical Methods for Wave Equations in Geophysical Fluid Dynamics. Volume 32 dari Texts in Applied Mathematics. Springer Science & Business Media.
[3] Harvey Gould, Jan Tobochnik, Dawn C. Meredith, Steven E. Koonin, Susan R. McKay, and Wolfgang Christian. 1996 An Introduction to Computer Simulation Methods: Applications to Physical Systems, 2nd Edition. 10 349-350.
[4] Garry Robinson and Zlatko Jovanoski, 2011 The Use of Microsoft Excel to Illustrate Wave Motion and Fraunhofer Diffraction in First Year Physics Courses. Spreadsheets in Education (eJSIE). 4(3) 1-21.
[5] Dinar Maftukh Fajar, Hari Anggit Cahyo Wibowo and Widya Arisy Putri 2014. Simulasi Asas Torricelli Menggunakan Visual Basic for Application (VBA) pada Microsoft Excel. Prosiding Simposium Nasional Inovasi Pembelajaran dan Sains 2014 (SNIPS 2014). Bandung, Indonesia,
[6] Sri Supatmi. 2010 Simulasi Pengontrolan Lampu Gedung Menggunakan Visual Basic. Prosiding Seminar Nasional Informatika 2010 (semmasiF 2010). UPN "Veteran" Yogyakarta,
[7] Dinshaw S. Balsara, Chad Meyer, Michael Dumbser, Huijing Du, and Zhiliang Xu. 2013 Efficient implementation of ADER schemes for Euler and magnetohydrodynamical flows on structured meshes – Speed comparisons with Runge–Kutta methods. Journal of Computational Physics. 235 934-969