Classical tests in brane gravity

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Abstract

The vacuum solutions in brane gravity differ from those in 4D by a number of additional terms and reduce to the familiar Schwarzschild metric at small distances. We study the possible roles that such terms may play in the precession of planetary orbits, bending of light, radar retardation and the anomaly in mean motion of test bodies. Using the available data from solar system experiments, we determine the range of the free parameters associated with the linear term in the metric. The best results come from the anomalies in the mean motion of planets. Such studies should shed some light on the origin of dark energy via the solar system tests.

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1. Introduction

In the past two decades, various attempts have been made to understand the nature of the so-called dark energy and dark matter which would be required to explain many observational data. For a review, see for example [1–4]. Deviation of the galactic rotation curves from Newtonian gravity which occurs at distances larger than the solar system scales [5], the velocity of galaxies in clusters and the bending of light rays from galaxies and clusters [6] are the best evidence for the existence of copious amounts of dark matter. Apart from the efforts made by a number of authors to account for dark matter by considering such elusive objects as massive neutrinos, axions and the weakly interacting massive particles (WIMPS) which is predicted by supersymmetric theories, the nature of dark matter is still unresolved. An interesting attempt to explain the galactic rotation curves was made in [7] where the author considers dark matter as a galactic phenomenon and proposes what is now known as the modified Newtonian dynamics (MOND) at low accelerations. However, since MOND is a non-relativistic theory, it cannot predict any relativistic phenomena such as the bending of light, the precession of planetary orbits, etc. Solar system experiments have been increasingly relied upon to investigate the integrity of the foundation of general relativity in recent years, see for example [8–18].
In the recent past, models incorporating extra dimensions have become the focus of attention for investigating the nature of problems mentioned above. In these theories, one considers a four-dimensional world (brane) embedded in a higher dimensional manifold (bulk) through which only gravity can propagate. Ordinary matter is confined to the brane and cannot propagate through the bulk. The confinement is achieved, through the imposition of $\mathbb{Z}_2$ symmetry and use of the Israel junction conditions which relates the extrinsic curvature of the brane to the energy–momentum of the matter. This method has predominantly been used in theories with one extra dimension. If the number of extra dimensions exceeds one, no reliable method for confining matter to the brane exists. This is so since the requirement to define junction conditions is the existence of a boundary (brane) which cannot be defined if the number of extra dimensions is more than one. For example, a boundary surface in a 3D space is a surface with one less dimension whereas a line in the same space cannot be considered as its boundary. On the back of such concerns, model theories have been proposed where matter is confined to the brane through the action of a confining potential, without the use of any junction condition or $\mathbb{Z}_2$ symmetry [19]. In [20] the authors used the confining potential approach to study a braneworld embedded in an $m$-dimensional bulk. The field equations obtained on the brane contained an extra term which was identified with the X-cold dark matter. The same methodology was used in [21] to find the spherically symmetric vacuum solutions of the field equations on the brane. These solutions were shown to account for the accelerated expansion of the universe and offered an explanation for the galaxy rotation curves.

In this paper, we focus attention on the consequences of the spherically symmetric vacuum solutions mentioned above when considering such questions as the precession of planetary orbits, the deflection of light rays in the solar system, the time delay of signals in the solar system and the mean motion of test bodies. In doing so, we obtain constraints on the free parameters appearing in the metric. This should help us to accounting for the origin of dark energy via solar system tests.

2. The model

Let us start by presenting the model used in our calculations. We only state the results and refer the reader to [20, 21] for a detailed derivation of these results.

As was mentioned in section 1, the braneworld model we invoke here differs from the usual Randall–Sundrum type in that no junction conditions or $\mathbb{Z}_2$ symmetry is used. One thus starts with the usual setup in which a 4D brane is embedded in a five or, in general, $n$-dimensional bulk. Assuming that the brane is devoid of matter with no cosmological constant and that the bulk space has constant curvature, one arrives at the following equations [21, 22]:

$$G_{\mu\nu} = Q_{\mu\nu},$$

(1)

where

$$Q_{\mu\nu} = (K^\rho_{\mu\rho}K_{\rho\nu} - K^{\rho}_{\nu}K_{\mu}^{\rho}) - \frac{1}{2}(K_{\alpha\beta}K_{\mu}^{\alpha\beta} - K^2)g_{\mu\nu}.$$  

(2)

We note that $Q_{\mu\nu}$ is an independently conserved quantity, that is

$$Q_{\mu\nu}^{\mu\nu} = 0,$$  

(3)

so that equation (1) satisfies the covariant conservation law. Equation (1) is the starting point from which a class of solutions was found in [22], representing a black hole. Thus, starting with the metric

$$ds^2 = -e^{\mu(r)}d\tau^2 + e^{\nu(r)}dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2),$$

(4)
the following solution satisfies equations (1) and (3):

\[ e^\mu = e^{-\nu} = 1 - \frac{C}{r} - \alpha^2 r^2 - 2\gamma r - \beta^2. \]  

(5)

This solution represents a black hole which we shall consider in the following section and use to analyze the behavior of a test particle in such a spacetime.

3. The geodesic equations of motion

The induced line element of the vacuum 4D spacetime is

\[ ds^2 = A(r) \, dt^2 - \frac{dr^2}{A(r)} - r^2 \, d\Omega^2, \]  

(6)

where

\[ A(r) = 1 - \frac{2m}{r} - \alpha^2 r^2 - 2\gamma r - \beta^2, \]  

(7)

with \( \gamma \equiv \alpha\beta, m \) is the mass of the central object, \( \alpha \) and \( \beta \) are constants and we have adopted the relativistic units, \( c = G = 1 \). In this spacetime, the geodesic equations of motion for a test particle are the Euler equations resulting from the Lagrangian

\[ L = \frac{1}{2} \left[ A(r) \dot{t}^2 - \frac{\dot{r}^2}{A(r)} - r^2 (\dot{\theta}^2 + \sin^2 \theta \dot{\phi}^2) \right], \]  

(8)

where a dot denotes differentiation with respect to the affine parameter. Without loss of generality, let us consider the equatorial plane, \( \theta = \frac{\pi}{2} \), for a test particle. The Lagrangian then becomes

\[ L = \frac{1}{2} \left[ A(r) i^2 - \frac{\dot{r}^2}{A(r)} - r^2 \dot{\phi}^2 \right]. \]  

(9)

Now, using the Euler–Lagrange equations, we obtain

\[ \dot{i} = \frac{E}{A(r)}, \]  

(10)

\[ \dot{\psi} = \frac{J}{r^2}, \]  

(11)

with \( J \) and \( E \) being constants. Along the orbit for a test particle \( 2L = 1 \) and for photons \( 2L = 0 \). Now, using equations (10) and (11) in equation (9) we obtain

\[ \frac{1}{A(r)} [E^2 - \dot{r}^2] - \frac{J^2}{r^2} = C, \]  

(12)

where \( C = 1 \) and \( C = 0 \) represent the massive and massless particles, respectively.

4. Precession of planetary orbits

To find equation of motion of a massive particle in this spacetime we use equation (12) \( (C = 1) \) and obtain

\[ u^2 + (1 - \beta^2) u^2 = \frac{E^2 + \beta^2 - 1}{J^2} + \frac{2mu}{J^2} + 2\gamma u + 2mu^3 + \frac{\alpha^2}{J^2 u^2} + \frac{2\gamma}{J^2 u} + \alpha^2, \]  

(13)
where \( u = \frac{1}{r} \) and a prime denotes differentiation with respect to \( \varphi \). Now by differentiating equation (13) with respect to \( \varphi \) we find the differential equation of the motion for a massive particle as

\[
\frac{d^2 u}{d\varphi^2} + (1 - \beta^2)u = \frac{m}{J^2} + 3mu^2 + \gamma - \frac{a^2}{J^4u^3} - \frac{\gamma}{J^2u^2}. \tag{14}
\]

In order to solve this equation we consider \( 3mu^2, \frac{u^2}{J^2} \) and \( \frac{\gamma}{J^2} \) as perturbative terms, since these terms are much smaller than \( \frac{m}{J^2} \). This can be seen by noting that, for example, \( \frac{3mu^2}{(m/J^2)} = \left( \frac{3}{r^2} \right) \left( \frac{r^2}{\varphi} \right)^2 \approx 3 \left( \frac{\varphi}{c} \right)^2 \approx 7.7 \times 10^{-8} \) for Mercury.

To first order then, this equation has a solution of the form

\[
u \approx \frac{1}{P} \left[ 1 + e \cos \left( 1 - \frac{1}{1 - \beta^2} \left( \frac{3a^2 P^4}{2J^2} + \frac{\gamma P^3}{J^3} + \frac{3m}{P} \right) \right) (1 - \beta^2)^{1/2} \right], \tag{15}\]

where

\[
P = \frac{1 - \beta^2}{\gamma + \frac{m}{J^2}} \tag{16}\]

and \( e \) is the eccentricity. From equation (15) we find that \( r \) is a periodic function of \( \varphi \) with period

\[
2\pi \left[ (1 - \beta^2)^{1/2} - \frac{1}{(1 - \beta^2)^{1/2}} \left( \frac{3a^2 P^4}{2J^2} + \frac{\gamma P^3}{J^3} + \frac{3m}{P} \right) \right]^{-1} > 2\pi. \tag{17}\]

Since \( \beta^2, \gamma < 1 \), to first-order approximation, the perihelion anomaly \( \Delta \varphi \) of the particle after one revolution is found to be

\[
\Delta \varphi \approx 2\pi \left[ 3m^2 J^{-2} + \gamma \left( \frac{J^4}{m^3} + 3m \right) + \frac{3a^2 \alpha^2}{2m^3} + \left( \frac{1}{2} + \frac{9m^2}{2J^2} \right) \beta^2 \right]. \tag{18}\]

For \( \gamma > 0 \), we find that the perihelion anomaly will increase compared to what one has for the Schwarzschild spacetime [23, 24, 33]. In the case \( \gamma < 0 \), depending on the parameters representing the particle, the perihelion anomaly may increase or decrease compared to what is predicted by the Schwarzschild metric.

5. Deflection of light rays

In this section, we investigate the deflection of light in a spacetime represented by the metric above in two cases, \( \gamma < 0 \) and \( \gamma > 0 \). We assume that both the observer and the light source are far from the compact object whose presence would deflect the light rays. The light emitted from the source approaches the compact object, reaching the observer at infinity. Using equation (12) for light \((C = 0)\) we obtain

\[
u^2 + (1 - \beta^2)u^2 = \frac{E^2}{J^2} + 2mu^3 + a^2 + 2\gamma u. \tag{19}\]

Here, a prime denotes differentiation with respect to \( \varphi \), \( a = \frac{1}{r} \) and \( \varphi \) is the usual angle representing the deflection. If we differentiate the last equation with respect to \( \varphi \) again we find

\[
\frac{d^2 u}{d\varphi^2} + (1 - \beta^2)u = 3mu^2 + \gamma. \tag{20}\]

This equation with \( \beta = 0 \) represents the deflection of light rays in Schwarzschild spacetime. We also note that \( \frac{9m^2}{u^2} = \frac{9E^2}{r^2} \ll \frac{R_s}{R_c} \) is small where \( R_s \) and \( R_c \) are the Schwarzschild and
compact object radii, respectively. For example, for a compact object like the Sun this is of the order of $10^{-6}$. We therefore have, retaining only the first term on the right-hand side of equation (20)

$$u_1 = \frac{3m}{2R^2(1 - \beta^2)} \left[ 1 + \frac{1}{3} \cos(2(1 - \beta^2)^{1/2} \phi) \right]. \quad (21)$$

Equation (20) has also an exact solution if we only retain the second term on the right-hand side. Therefore, the general solution is given by

$$u = \frac{\gamma}{1 - \beta^2} + \frac{1}{R} \sin(2(1 - \beta^2)^{1/2} \phi) + \frac{3m}{2R^2(1 - \beta^2)} \left[ 1 + \frac{1}{3} \cos(2(1 - \beta^2)^{1/2} \phi) \right]. \quad (22)$$

where $R$ is the shortest distance between the object and the undeflected path of the light rays in flat space

$$R = r \sin \phi. \quad (23)$$

At large distances, $r \to \infty$, $\phi \to \varphi_\infty \ll 1$, and from equation (22) we obtain

$$\varphi_\infty = -\frac{R}{(1 - \beta^2)^{3/2}} \left[ \frac{2m}{R^2} + \gamma \right]. \quad (24)$$

As can be seen, the total deflection is twice that of the absolute value of (24)

$$\delta = \frac{1}{1 - \beta^2} - \frac{4m}{R^2} \frac{R}{\gamma} \frac{\gamma}{(1 - \beta^2)^{3/2}}. \quad (25)$$

The first term in (25) with $\beta = 0$ is the deflection angle for the Schwarzschild metric [23, 24, 33] and the second term is the result of the modification of the model. As can be seen, the proper potential increases (decreases) for the bending angle if we choose $\gamma > 0 (\gamma < 0)$, respectively.

6. Radar retardation

We now calculate the time it takes for a photon to travel from one point to another in the gravitational field of a central object. The equation which we shall use here is (12) with $C = 0$. At the point of the closest approach $r = r_0$, we have $dr/dt = 0$. Equation (12) then yields

$$\frac{J^2}{E^2} = \frac{r_0^2}{A(r_0)}, \quad (26)$$

where $A(r)$ is given by equation (7). Using equations (10), (12) and (26) we get the time taken by the photons traversing the distance from $r_0$ to $r$

$$t = \int_{r_0}^r \frac{dr}{A(r) \left[ 1 - \left( \frac{\alpha}{r} \right)^2 \frac{A(r)}{A(r_0)} \right]^{1/2}}. \quad (27)$$

After substituting $A(r)$ in the above integral, we expand the integrand to first order in $\frac{\alpha}{r}$, $\gamma r$ and $\alpha^2 r^2$ which are much less than unity and find

$$t \simeq \int_{r_0}^r \frac{1 + \frac{\alpha}{r} \left( 2 + \frac{\alpha}{r + \alpha} \right) + \alpha^2 \left( r^2 - \frac{1}{2} r_0^2 \right) + \gamma \left( 2r - \frac{\alpha}{r + \alpha} \right)}{\left[ 1 - \left( \frac{\alpha}{r} \right)^2 \right]^{1/2}} \, dr. \quad (28)$$
This integral can be evaluated to give
\[
\begin{align*}
t & \simeq \left[ 1 + \beta^2 + \alpha^2 \left( \frac{r^2 + r_0^2}{3 + 6} \right) \right] \sqrt{r^2 - r_0^2} + 2m \ln \left( \frac{r + \sqrt{r^2 - r_0^2}}{r_0} \right) + m \left( \frac{r - r_0}{r + r_0} \right)^{1/2}.
\end{align*}
\]
Equation (29)

For \( \alpha = \beta = m = 0 \), we get the time traveled by photons in a straight line between \( r \) and \( r_0 \) in a Minkowski spacetime. For \( \alpha = \beta = 0 \) we get the Schwarzschild metric result \([23, 24, 33]\) and for \( \gamma > 0 \) we find an increase in the time delay for photons compared to Minkowski and Schwarzschild spacetimes. However, in the case \( \gamma < 0 \), an increase in the time delay will happen only on scales for which the term containing \( \alpha^2 \) is dominant over the term including \( \gamma \).

7. Limitations on \( \gamma \) from experiment

Let us discuss the possible range and values of the constant \( \gamma \). To do this, we suppose that the value of \( \alpha^2 \) is much less than \( \gamma \) and hence the term including \( \alpha^2 \) will appear only at very large scales such as galactic and cosmological scales. To determine the range of \( \gamma \) we consider some of the tests performed in our solar system.

7.1. Deflection of light rays in the solar system

In solar system tests, the Sun is the central object and we neglect the effects of other planets. The second term in equation (25),
\[
\delta \gamma = \frac{2 \gamma R}{1-(\frac{\beta}{r})^2} \approx 2 \gamma R,
\]
is a simple modification to the standard result, namely \( \delta \text{GR} = \frac{4 GM_{\odot}}{c^2 R} \), with \( G, M_{\odot} \) being the gravitational constant and the Sun’s mass respectively and \( c \) is the speed of light. The deflection of a ray that comes from infinity and grazes the Sun’s limb is \( \delta \text{GR} \approx 1.75 \) arcsec. To date, the best measurements on the deflection of light from the Sun have been obtained using very-long-baseline interferometry data, measuring the deflection of photons emanated from distant compact radio sources. Since no deviation from general relativity has been reported \([25]\), we find that \( |\delta \gamma| \leq 2.11 \times 10^{-10} \) and consequently \( |\gamma| \leq 1.52 \times 10^{-21} \) cm\(^{-1}\). Such experiments constrain the value of \( \gamma \), but give no clue as to the sign.

7.2. Time delay of signals in the solar system

According to equation (29), photons are delayed by the curvature of spacetime characterized by the line element (6). To determine the range of \( \gamma \), in this section, we compare the experimental data of time dilation of signals with predictions afforded by theory. In 1979 \([26]\), the result of Viking relativity experiment confirmed the ‘Shapiro’ time delay in the solar system to an accuracy of 0.1%, but the most recent result is the frequency shift of radio signals to and from the Cassini spacecraft as they passed near the Sun \([27]\). In general relativity, the increase in \( \Delta t \) produced by the gravitational field of the Sun over the time taken for photons to travel the round trip between the ground antenna and the spacecraft at distances \( r_e \) and \( r_s \) respectively from the Sun is \([28]\)
\[
\begin{align*}
\Delta t_{\text{un}} &= 4 \frac{GM_{\odot}}{c^3} \ln \left( \frac{4r_e r_s}{b^2} \right),
\end{align*}
\]
where \( G \) is the gravitational constant, \( M_{\odot} \) is the gravitational mass of the Sun and \( b \) is the impact parameter. It is convenient to use the relative change in the frequency which is caused
by the gravitational time delay [29], because the Doppler shift due to the receiver’s motion has no effect owing to the cancelation at both the receipt and emission of the radio signals [29]. This frequency shift is defined as 

\[ y = -\frac{b}{d(t)} \]

Indeed, the frequency shift was used by the Cassini experiment. For a case of \( b \ll r_e, r_s \), which is valid for the Cassini experiment, the general relativistic contribution is expressed as [28]

\[ y_{\text{GR}} = 4 \frac{M_m}{b^2} \frac{db}{dt}. \]  

(31)

We take the extra term caused by \( \gamma \) in time delay in equation (29) and find the extra frequency shift as

\[ y_{\gamma} = \gamma \left[ \frac{r_0 + 2r_e}{(r_0 + r_e)^2} + \frac{r_0 + 2r_s}{(r_0 + r_s)^2} \right] \frac{db}{dt}, \]

(32)

where we have assumed \( \frac{dr_e}{dr} \approx \frac{dr_s}{dr} \approx \frac{dr_0}{dr} \approx \frac{db}{dr} \) near the solar conjunction \((b \ll r_e, r_s)\). For a spacecraft much farther away from the Sun than the Earth, \( \frac{dr_e}{dr} \) is not very different from the velocity of Earth \( v_e = 30 \text{ km s}^{-1} \). Now we can put constraint on \( \gamma \) from the Doppler tracking of the Cassini spacecraft while it was on its way to Saturn reported in [27]. From [27] we find that \( |y_{\gamma}| \leq 10^{-14} \) and therefore, at \( r_s = 8.43 \text{AU}, h_{\text{min}} = 1.6R_e \), we obtain \( |\gamma| \leq 10^{-28} \text{ cm}^{-1} \). We gain seven order of magnitude more accuracy than the constraint from the bending of light experiments. However, it is still not possible to determine the sign of \( \gamma \).

7.3. Precession of planetary orbits

Now, the measured perihelion shift of Mercury is known accurately; after the perturbing effects of other planets have been accounted for, the excess shift is known to be about 0.1% from radar observations of Mercury between 1966 and 1990 [30, 31]. The prediction of general relativity for perihelion shift of Mercury is \( \Delta \psi_{\text{GR}} = 42.98 \text{ arcsec/century} \) [23, 24] while the observed precession of the perihelion of Mercury is \( \Delta \psi_{\text{obs}} = 43.13 \pm 0.14 \text{ arcsec/century} \) [32]. Therefore the difference \( \delta \psi = \Delta \psi_{\text{obs}} - \Delta \psi_{\text{GR}} = 0.15 \text{ arcsec/century} \) can be used to constrain \( \gamma \). From equation (18) we obtain the contribution of \( \gamma \) term to the perihelion shift

\[ \Delta \psi \approx \frac{2\pi J^2}{m^3}, \]

(33)

where we have assumed that \( 3m \ll \frac{J^2}{m^3} \) and \( \frac{J^2}{m^3} = a(1 - e^2) \), with \( a \) being the semi-major axis and the eccentricity of planet, respectively. By assuming that the difference \( \delta \psi \) is due to the contribution of the \( \gamma \) term in metric (6) (note that \( \alpha^2 \) is much smaller than \( \gamma \)), the observational result imposes the following constraint on \( \gamma \):

\[ |\gamma| \leq \frac{GM_m}{2\pi c^2 a^2 (1 - e^2)} \delta \psi. \]

(34)

Use of the observational data for Mercury, equation (34), then gives \( |\gamma| \leq 1.33 \times 10^{-30} \text{ cm}^{-1} \). This result is two order of magnitude smaller than that given by time delay of signals.

7.4. Mean motion

Due to the extra terms \( 2\gamma r \) and \( \alpha^2 r^2 \) in the metric, the radial motion of a test body around a central object will be affected by an additional acceleration. Let us consider a circular orbit

\[ r \omega^2 = \frac{GM}{r^2} - \gamma c^2 - \alpha^2 c^2 r, \]

(35)
Table 1. Limits on $\gamma$ due to anomalous mean motion of planets in the solar system. $(\gamma)_{\text{lim}}$ is the extreme limit on the parameter $\gamma$.

| Planet      | $\delta a$ (cm) | $(\gamma)_{\text{lim}}$ (cm$^{-1}$) |
|-------------|------------------|---------------------------------------|
| Mercury     | 10.5             | $2.38 \times 10^{-32}$                |
| Venus       | 32.9             | $1.15 \times 10^{-32}$                |
| Earth       | 14.6             | $1.93 \times 10^{-33}$                |
| Mars        | 65.7             | $2.46 \times 10^{-33}$                |
| Jupiter     | 6390.0           | $6.00 \times 10^{-32}$                |
| Saturn      | 4222.0           | $6.35 \times 10^{-32}$                |
| Uranus      | 38 484.0         | 7.15 $\times 10^{-32}$                |
| Neptune     | 478 532.0        | $2.32 \times 10^{-31}$                |
| Pluto       | 3463 309.0       | $7.44 \times 10^{-31}$                |

where $\omega$ is the angular frequency of the orbit. We can rewrite the last equation as

$$r\omega^2 = \frac{GM_{\text{eff}}}{r^2}. \quad (36)$$

Here $M_{\text{eff}}$ is the effective mass of the central object which can affect the motion of the test body. Obviously, a positive $\gamma$ would decrease the mass of the central object, leading to an excess in the orbital semi-major axis of the test body. In other words, the mean motion of the test body $n = \sqrt{\frac{GM}{a}}$ is changed by [36]

$$\frac{\delta n}{n} = -\frac{1}{2r_g} (\alpha^2 r^3 + \gamma r^2), \quad (37)$$

where $r_g = \frac{GM}{\alpha^2}$ is the gravitational radius of the central object.

We can evaluate the statistical error on the semi-major axis of each test body $\delta a = -\frac{2\delta n}{n}$ and interpret it as the uncertainty in the determination of $\gamma$. Bounds on $\gamma$ are shown in Table 1 where $\delta a$ is the statistical error in the orbital semi-major axis of each planet [37]. We see that the value of $\gamma$ is between 3 and 5 order of magnitude smaller than that given by the time delay of signals and about 3 order of magnitude smaller than that of the perihelion shift, but since we are considering a statistical error we have in effect constrained the absolute value of $\gamma$.

8. Conclusions

In this paper, we have studied the classical tests of general relativity in a braneworld scenario in which the localization of matter on the brane is through the action of a confining potential. We have studied the effects of $\alpha^2$ and $\gamma$ parameters on radar retardation, bending of light rays and the change in the mean motion of planets and test bodies. These are similar to the familiar results obtained in a Schwarzschild spacetime but with certain corrections. To determine the value of constant $\gamma$, we assumed that the parameter $\alpha^2$ is much less than $\gamma$ and hence the terms including $\alpha^2$ could be neglected. Using available data from the solar system, we obtained different constraints on $\gamma$. The best constraints come from anomalies in the mean motion of the solar system planets resulting in absolute values for $\gamma$ of $1.93 \times 10^{-33}$ cm$^{-1}$ and $2.46 \times 10^{-33}$ cm$^{-1}$ for the Earth and Mars, respectively.
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