Constraining a class of $B - L$ extended models from vacuum stability and perturbativity

Joydeep Chakrabortty, Partha Konar and Tanmoy Mondal

1Department of Physics, Indian Institute of Technology, Kanpur-208016, India
2Physical Research Laboratory, Ahmedabad-380009, India

Abstract

The precise knowledge of the Standard Model (SM) Higgs boson and top–quark masses and couplings are crucial to understand the physics beyond it. An SM–like Higgs boson having a mass in the range of 123–127 GeV squeezes the parameters for physics beyond the Standard Model. In recent the LHC era many TeV–scale neutrino mass models have earned much attention as they pose many interesting phenomenological aspects. We have contemplated $B - L$ extended models which are theoretically well motivated and phenomenologically interesting, and they successfully explain neutrino mass generation. In this article we analyze the detailed structures of the scalar potentials for such models. We compute the criteria which guarantee that the vacuum is bounded from below in all directions. In addition perturbativity (triviality) bounds are also necessitated. Incorporating all such effects we constrain the parameters of such models by performing their renormalization group evolutions.

PACS numbers: 11.10.Hi, 14.60.Pq, 14.60.St, 14.80.Ec.
Keywords: Beyond Standard Model, Renormalization Group, Vacuum Stability

*Electronic address: konar@prl.res.in
†Electronic address: tanmoym@prl.res.in
I. INTRODUCTION

The recent announcements from both ATLAS \cite{1} and CMS \cite{2} have revealed the existence of a new boson having a mass in the range 123–127 GeV. The data so far indicates a close resemblance to one having some of the measured properties of the Standard Model (SM) Higgs. However, it has yet to confirm firmly whether this boson is the SM Higgs or a beyond the Standard Model artifact. This long awaited quest will only be examined more vigorously in the near future with the help of more data.

If the newly discovered particle is indeed the SM Higgs boson then its mass can carry a signature of new physics which embeds SM at low energy. The Higgs mass can be recast solely in terms of the Higgs quartic coupling, $\lambda_h$. The stability of the electroweak (EW) vacuum demands a positive $\lambda_h$. Now if the SM is the only existing theory in nature then this condition, $\lambda_h > 0^1$, must be maintained at each scale of its evolution up to the Planck scale ($M_{Pl}$). The evolution of $\lambda_h$ with the renormalization (mass) scale limits two boundary values – one at the EW scale for which we have $\lambda_h(M_{Pl}) = \pi$, and one at the Planck scale for which we have 0 – from the demands of perturbativity of the coupling (triviality) and the stability of the vacuum (vacuum stability) respectively. It has been noted in Refs. \cite{3–5} that the SM electroweak vacuum is not stable up-to the Planck scale for most of the SM parameters (top-quark mass, Higgs mass and strong coupling $\alpha_s$). Thus it indicates that some new physics might be there before the SM vacuum stability gets raptured. Thus the physics beyond Standard Model is expected to take care of stability of the vacuum of the full scalar potential along with the electroweak ones. In brief, the present range of the SM-like Higgs mass entertains the presence of new physics solely from the vacuum stability point of view.

Apart from this, we already have hints of new physics beyond the Standard Model from the neutrino sector. Many experimental observations, like neutrino oscillations, confirm that neutrinos have tiny nonzero masses which cannot be accommodated naturally within the SM. Thus we must have physics beyond the Standard Model to explain this feature. Among the neutrino mass generation procedures the seesaw mechanism \cite{6–15} is very popular. In usual (natural) seesaw models light neutrino masses are $\sim m_D^2/M$ where the Dirac-type mass

$^1$ This is the necessary condition but not the sufficient to confirm the sole existence of the SM till the Planck scale.
$m_D \sim 100$ GeV and $M$ is the Majorana mass of heavy fermion which gets integrated out during the process. The mass of this heavy fermion, $M$, determines the scale of the seesaw models which needs to be very high ($\sim 10^{11}$ GeV) to avoid any fine tuning in $m_D$. As the natural scale of the seesaw is very high these models are suffer from a lack of testability. But it is also possible to construct low scale ($\sim$ TeV) models either importing some new fields [16] or incorporating higher-dimensional operators [17–21]. These models not only generate the correct order of neutrino masses and mixing, but are also phenomenologically interesting as the scale of these theories are well within the reach of present experiments like the LHC. These models are extended by some extra gauge symmetry and(or) new particles. The presence of these new fields might affect the evolution of the SM couplings, like gauge, Higgs quartic, and top Yukawa couplings if they couple to the SM particles. Hence it is necessary to examine the status of the SM vacuum once these new physics models come into play. Thus by using knowledge of the SM parameters and from the demand of vacuum stability the new parameters involved in the theory might be severely constrained. In the literature the stability of the vacua was discussed in several scenarios considering beyond Standard Models (BSMs). These models are extended by the extra gauge symmetry and (or) addition new particles. Quantum corrections of the quartic couplings depend on the spin of the particles belonging to a particular model. The fermion loop contributions contain a relative minus ‘-' sign compared to for the bosonic fields. Thus the Yukawa couplings tend to spoil the stability unlike the gauge and other scalar self-couplings. Vacuum stability in different variants of see-saw models has been adjudged in Refs. [22–28] which has richer particle spectrum compared to the SM. In the context of gauge extensions, vacuum stability for the alternative left-right Symmetric Model has been discussed in Ref. [29].

In a theory involving multiple scalar fields the structure of the potential is complicated. The vacuum stability criteria depend on some combinations of the scalar quartic couplings. Moreover, the perturbativity (triviality) bounds also play crucial roles in finding a consistent parameter space compatible with the choice of new physics scales. Non tachyonic scalar masses are guaranteed with these constraints. It has been noted that some of the quartic

---

2 In this paper we are considering stability up to the Planck scale. We are not considering the metastability which does not require the vacuum to be bounded from below. If the decay life time of the vacuum is larger than the life time of the universe then that vacuum is metastable. But as our procedure concerns only boundedness of the scalar potential it fails to pin down the existence of the metastable vacuum.
couplings can be recast in terms of the heavy scalar masses and thus can be constrained from phenomenological point of view. On the contrary, few of them do not have that much impact on scalar masses rather they determine the splitting among the narrowly spaced massive scalar modes. Our present collider experiments still not sensitive to address that fine splittings thus those quartic couplings are beyond the reach of any experimental verification. But those couplings can be constrained through vacuum stability, perturbativity (triviality) depending on the choice of scale of new physics.

In our study we have concentrated on Left-Right $U(1)_{B-L}$ extended models which are classified into two categories: $SM \otimes U(1)_{B-L}$ or left-right (LR) symmetry. We have adopted two variants of the LR symmetric models containing (i) two $SU(2)$ triplet scalars $\Delta_{L(R)}$, and (ii) two $SU(2)$ doublet scalars, $H_{L(R)}$. In section III we introduce the basic structures of these models. Then we include the renormalization group evolutions of all the necessary couplings and show how the vacuum stability, perturbativity (triviality) bounds constrain the parameter space of each models in section III. We have analysed the structure of the potentials in detail and computed the criteria for vacuum stability using the formalism shown in reference [30]. All vacuum stability conditions corresponding to different models are listed in appendix B.

II. MODELS

The Standard Model symmetry group is expressed as $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. It has been noted in [31] that an extra $U(1)$ gauge symmetry along with the SM can provide solutions to some of the unaddressed issues in the Standard Model. These extra Abelian symmetry groups can, in general, originate from different high scale Grand Unified Theories (GUTs), like $SO(10)$, $E(6)$. These larger groups contain $U(1)_{B-L}$ as a part of the intermediate gauge symmetries. In nonsupersymmetric GUT models the $U(1)_{B-L}$ breaking scale can be lowered as few TeV$^3$ [32], which is consistent with unification pictures. In our present study we concentrate on TeV scale $U(1)_{B-L}$ extended models where neutrino mass generation can be explained. However, any high scale root of these models are not considered and kept for future work.

$^3$ This is also true for supersymmetric GUT models, see [32].
A. \( U(1)_{B-L} \)

The gauge group under consideration is \( SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L} \). This minimal model contains an extra complex singlet scalar field \( S \) and this extra \( B-L \) symmetry is broken once it acquires vacuum expectation value \( (vev) \) \[33\]. Thus the \( vev \) determines the symmetry-breaking scale of this symmetry and also the mass of the extra neutral gauge boson \( Z_{B-L} \). For the purpose of our study we will focus only on the relevant part of the Lagrangian, namely the scalar kinetic, and potential terms and the lepton Yukawa couplings. The scalar kinetic term is:

\[
\mathcal{L}_s = (D^\mu \Phi)^\dagger (D_\mu \Phi) + (D^\mu S)^\dagger (D_\mu S) - V(\Phi, S). \tag{1}
\]

Here the potential \( V(\Phi, S) \) is given as:

\[
V(\Phi, S) = m^2 \Phi^\dagger \Phi + \mu^2 |S|^2 + \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 |S|^4 + \lambda_3 \Phi^\dagger \Phi |S|^2, \tag{2}
\]

where \( \Phi \) and \( S \) are the complex scalar doublet and singlet fields respectively. After gauging away the extra modes and acquiring the \( vevs \) these fields are redefined as:

\[
\Phi \equiv \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + \phi) \end{pmatrix}, \quad S \equiv \frac{1}{\sqrt{2}}(v_{B-L} + s), \tag{3}
\]

where, EW symmetry breaking \( vev, v \) and \( B-L \) breaking \( vev, v_{B-L} \) are real and positive.

We also find the scalar mass matrix in the following form:

\[
\mathcal{M} = \begin{pmatrix} \lambda_1 v^2 & \frac{\lambda_3 v_{B-L} v}{2} \\ \frac{\lambda_3 v_{B-L} v}{2} & \lambda_2 v_{B-L}^2 \end{pmatrix} = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{pmatrix}. \tag{4}
\]

After diagonalising this mass matrix we construct two physical scalar states, a light \( h \) and a heavy \( H \), having masses \( M_h \) and \( M_H \), respectively,

\[
M_{H,h}^2 = \frac{1}{2} \left[ \mathcal{M}_{11} + \mathcal{M}_{22} \pm \sqrt{(\mathcal{M}_{11} - \mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2} \right]. \tag{5}
\]

The scalar mixing angle, \( \alpha \) can be expressed as:

\[
\tan(2\alpha) = \frac{2\mathcal{M}_{12}}{\mathcal{M}_{11} - \mathcal{M}_{22}} = \frac{\lambda_3 v v_{B-L}}{\lambda_1 v^2 - \lambda_2 v_{B-L}^2}. \tag{6}
\]
Using eqs. 5 and 6 the quartic coupling constants $\lambda_1$, $\lambda_2$, and $\lambda_3$ can be recast in the following forms:

$$\lambda_1 = \frac{1}{4v^2} \left\{ (M_H^2 + M_h^2) - \cos 2\alpha (M_H^2 - M_h^2) \right\},$$

$$\lambda_2 = \frac{1}{4v^2_{B-L}} \left\{ (M_H^2 + M_h^2) + \cos 2\alpha (M_H^2 - M_h^2) \right\},$$

$$\lambda_3 = \frac{1}{2vv_{B-L}} \left\{ \sin 2\alpha (M_H^2 - M_h^2) \right\}. \tag{7}$$

It can be noted from the last equation in eq. 7 that we would get a duplicate set of solutions with inverted signs for both $\alpha$ and $\lambda_3$. Hence one choice of positive $\alpha$ suffices as presented at section III A.

Due to the presence of an extra $U(1)_{B-L}$ gauge theory the SM gauge kinetic terms is modified by

$$\mathcal{L}_{B-L}^{KE} = -\frac{1}{4} F'_{\mu\nu} F'_{\mu\nu}, \tag{8}$$

where,

$$F'_{\mu\nu} = \partial_\mu B'_\nu - \partial_\nu B'_\mu. \tag{9}$$

The covariant derivative for $SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L}$ sector in this model is modified as

$$D_\mu \equiv \partial_\mu + ig_2 T^a W^{a}_\mu + ig_1 Y B_\mu + i(\tilde{g}Y + g_{B-L} Y_{B-L}) B'_\mu. \tag{10}$$

The SM gauge bosons $B_\mu$ and $W_\mu^3$ will mix with the new gauge boson $B'_\mu$ to create two massive physical fields $Z$ and $Z_{B-L}$ and one massless photon field $A$. Assuming there is no kinetic mixing at tree level, i.e., $\tilde{g} = 0$ at the EW scale, the physical gauge-boson masses are given as

$$M_Z^2 = \frac{1}{4} \left( g_i^2 + g_2^2 \right) v^2, \quad \tag{11}$$

$$M_{Z_{B-L}}^2 = 4g_{B-L}^2 v_{B-L}^2. \quad \tag{12}$$

Along with the Standard Model particles, three right-handed neutrinos ($\nu_R$) are introduced$^4$. The relevant term of the Lagrangian of the Yukawa interactions can be written as

$$-\mathcal{L}_Y = y^l_{ij} \overline{iL} \Phi \nu_j^R + y^h_{ij} \overline{(\nu_R)^i} \nu_j^R S + h.c. \tag{13}$$

$^4$ One right-handed neutrino ($Q_{B-L} = -1$) for each generation is required for the sake of gauge anomaly cancellation.
where $\Phi = i\sigma_2\Phi^*$ with $\sigma_2$ being the Pauli matrix. The second term of the above equation is the Majorana mass term. Note from the eq. [13] that conservation of $B - L$ charge requires that the singlet scalar field, $S$, must have $Q_{B-L} = -2$. When the SM Higgs and singlet scalar $S$ acquire vevs the neutrino mass matrix takes the form

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & m_R \end{pmatrix},$$

(14)

where $m_D = y^l v / \sqrt{2}$ and $m_R = \sqrt{2} y^h v_{B-L}$. The light ($m_{\nu_l}$) and heavy ($m_{\nu_h}$) neutrino masses are

$$m_{\nu_l} = -m_D^T m_R^{-1} m_D,$$

(15)

$$m_{\nu_h} = m_R.$$  

(16)

In this model heavy neutrino mass $m_R$ is also generated through the Yukawa terms unlike the gauge-invariant Majorana mass term in type-I seesaw. It can be noted that with $m_R \sim O(\text{TeV})$, $y^l$ needs to be very small to generate light neutrino masses $\sim O(\text{eV})$. But $y^h$ can be large $\sim O(1)$ as $v_{B-L}$ is around TeV scale. Thus successful light neutrino mass generation does not constrain $y^h$. But as the heavy neutrino is also coupled to the SM-like Higgs, $y^h$ affects the vacuum stability of the scalar potential in this model and gets constrained. The gauge coupling $g_{B-L}$, and, vev of $B - L$ breaking scale are also free parameters. In the following section we have shown how these parameters are constrained from vacuum stability of the scalar potential and also from perturbativity (triviality) of the couplings.

**B. Left-Right Symmetry**

The full LR symmetric gauge group is written as $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$. The $SU(2)_R \otimes U(1)_{B-L}$ is broken to $U(1)_Y$ at a scale higher than the EW symmetry breaking one. Thus the hypercharge generator is a linear combination of $SU(2)_R$ and $U(1)_{B-L}$ generators. In this model, hypercharge, $Y$, can be reconstructed from the $SU(2)_R$ and $U(1)_{B-L}$ quantum numbers as:

$$Y = T_{3R} + (B - L)/2,$$

(17)

$T_{3R}$ being $3^{rd}$ component of $SU(2)_R$ isospin.

Here we briefly present two variants of Minimal left-right Symmetric Models (MLRSMs):
• The scalar sector consists of a bidoublet (Φ), one left-handed triplet (Δ_L), and one right-handed triplet (Δ_R) [36–39].

• Scalar sector consists of a bidoublet (Φ), one left-handed doublet (H_L), and one right-handed doublet (H_R) [40–42].

1. LR Model with Triplet Scalars

The most generic scalar potential of this model with bidoublet and triplet scalars (Φ, Δ_L,R) is given in appendix A 2. The explicit structures of the scalars can be presented in the following form

\[ \Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}, \quad \Delta_{L,R} = \begin{pmatrix} \delta_{L,R}^+/\sqrt{2} & \delta_{L,R}^{++} \\ \delta_{L,R}^0 & -\delta_{L,R}^+/\sqrt{2} \end{pmatrix}. \]

These fields transform under \( SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \) gauge groups in the following manners:

\[ \Phi \equiv (2, 2, 0), \quad \Delta_R \equiv (1, 3, 2), \quad \Delta_L \equiv (3, 1, 2). \] (18)

Once neutral components of these scalars acquire vacuum expectation values, they can be written in the following form

\[ \langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 e^{i\theta} \end{pmatrix}, \quad \langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}, \quad \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \] (19)

where, for simplicity we have chosen \( v_2 = 0 \) without loss of generality. With these structures of the vacuum expectation values, symmetry breaking occurs in two stages. The symmetry group \( SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \) breaks down to \( SU(2)_L \otimes U(1)_Y \) by \( v_R \) at high scale. Consequently, the vacuum expectation value \( v_1 \) of bidoublet breaks \( SU(2)_L \otimes U(1)_Y \) to \( U(1)_{EM} \). So total number of Goldstone bosons will be six. Now the Higgs sector has 20 degrees of freedom (eight real field for the bidoublet and six each for triplet fields). Hence, the remaining 14 fields will be massive scalars and they are as follows:

1. Two doubly charged scalars (\( H_1^{\pm \pm}, H_2^{\pm \pm} \)),

2. Two singly charged scalars (\( H_1^{\pm}, H_2^{\pm} \)),

10
3. Four neutral \( CP - even \) scalars (\( H_0^0, H_1^0, H_2^0, H_3^0 \)),

4. Two neutral \( (CP - odd) \) pseudoscalars (\( A_0^0, A_1^0 \)).

Since already mentioned that the scale \( v_R \) is much higher than the vev of electroweak breaking \( v_1 \), the scalar masses can be expressed in leading-order terms\(^5\) \cite{43, 44}

\[
M_{H_0^0}^2 \simeq 2\lambda_1 v_1^2, \\
M_{H_1^0}^2 \simeq \frac{1}{2}\lambda_{12} v_R^2, \\
M_{H_2^0}^2 \simeq M_{A_0^0}^2 \simeq M_{H_2^\pm}^2 \simeq 2\lambda_5 v_R^2, \\
M_{H_3^0}^2 \simeq M_{A_0^2}^2 \simeq M_{H_1^\pm}^2 \simeq \frac{1}{2}(\lambda_7 - 2\lambda_5) v_R^2, \\
M_{H_2^{\pm\pm}}^2 \simeq 2\lambda_6 v_R^2. \tag{20}
\]

\( M_{H_0^0} \) is the Standard Model Higgs boson and denoted as \( M_h \) from here onwards. For simplicity and to reduce the number of free parameters, we consider degenerate heavy scalars at the \( v_R \) scale, i.e., \( M_{H_0^0} = M_{H_2^0} = M_{H_3^0} = M_{H_1^\pm} = M_H \). It is important to note that the remaining quartic couplings only contribute in the scalar masses as subleading terms and they are proportional to the \( v_1^2 \) at the electroweak symmetry-breaking scale (EWSB) scale. Hence, \( \lambda_2, \lambda_3, \lambda_4, \lambda_8, \lambda_9, \lambda_{10}, \) and \( \lambda_{11} \) induce only the relative mass splittings among these heavy scalars which are almost phenomenologically unaccessible at present experiments.

The kinetic term of scalar part can be written as

\[
\mathcal{L}_{kin} = \text{Tr}\left[(D_\mu \Phi)^\dagger (D^\mu \Phi)\right] + \text{Tr}\left[(D_\mu \Delta_L)^\dagger (D^\mu \Delta_L)\right] + \text{Tr}\left[(D_\mu \Delta_R)^\dagger (D^\mu \Delta_R)\right], \tag{21}
\]

where,

\[
D_\mu \Phi = \partial_\mu \Phi - ig_{2L} T^a W^a_{\mu L} \Phi + ig_{2R} \Phi T^a W^a_{\mu R}, \tag{22}
\]

\[
D_\mu \Delta_{(L/R)} = \partial_\mu \Delta_{(L/R)} - ig_{(2L/2R)} \left[T^a W^a_{(L/R)\mu}, \Delta_{(L/R)}\right] - ig_{B_{L/R}} B_\mu \Delta_{(L/R)}. \]

We choose the gauge couplings \( g_{2L} \) and \( g_{2R} \) for the \( SU(2)_L \) and \( SU(2)_R \) gauge groups respectively to be same for the sake of minimality of the model in terms of number of

\(^5\) These leading order terms match exactly with the masses of the heavy scalars at scale \( v_R \), i.e., before electroweak symmetry breaking (EWSB). After the EWSB, some correction terms are generated which are proportional to the \( v_1^2 \). But as \( v_R >> v_1 \), the splitting among the masses of these heavy scalars are negligible compared to their relative masses. It is important to note that this ‘\( \simeq \)’ will be replaced by ‘\( = \)’ in eq.\(^{20}\) when these masses are given at \( v_R \) scale.
parameters. After spontaneous breaking of LR and EW symmetries, two charged $W^\pm_{L/R}$ and two neutral $Z_{L/R}$ gauge bosons become massive, while photon $A$ remains massless:

$$M^2_{W^\pm_L} = \frac{1}{4} g_2^2 v_1^2, \quad M^2_{W^\pm_R} = \frac{1}{4} g_2^2 \left(v_1^2 + 2 v_R^2\right), \quad (23)$$

$$M^2_{Z_{L,R}} = \frac{1}{4} \left[ \left(g_2^2 v_1^2 + 2 v_R^2 (g_2^2 + g_{B-L}^2)\right) + \sqrt{\left(g_2^2 v_1^2 + 2 v_R^2 (g_2^2 + g_{B-L}^2)\right)^2 - 4 g_2^2 (g_2^2 + 2 g_{B-L}^2) v_1^2 v_R^2}\right].$$

Under the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ quarks and leptons are doublets,

$$L_{i(L/R)} = \begin{pmatrix} \nu_i \\ l_i \end{pmatrix}_{(L/R)}, \quad Q_{i(L/R)} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_{(L/R)}. \quad (24)$$

The most general lepton Yukawa Lagrangian can be written as,

$$- \mathcal{L}_Y = \overline{L_L} \left( y^l \Phi + \tilde{y}^l \tilde{\Phi} \right) L_R + h.c. + y^h_R \overline{L_L} \tilde{\Delta} L_L + y^h_L \overline{L_L} \tilde{\Delta} R R, \quad (25)$$

here, $\tilde{\Phi} = i \sigma_2 \Phi^*$ and $\tilde{\Delta}_{L/R} = i \sigma_2 \Delta_{L/R}$. Here we have considered that the Yukawa matrices are diagonal\(^6\). The neutral fermion masses are generated once the $\Phi$ and $\Delta$ acquire vev.

The neutral fermion mass matrix is given as

$$M_\nu = \begin{pmatrix} m_{11}^H & m_D \\ m_D^T & m_R \end{pmatrix}, \quad m_D = \frac{1}{\sqrt{2}} y_L^i v_1, \quad m_R = \sqrt{2} y_R^i v_R, \quad m_{11}^H = \sqrt{2} y^h L, \quad (26)$$

here, $y_R^h = y_R^h = y^h$ because of left-right symmetry. Thus the light neutrino mass

$$m_{\nu_i} = m_{11}^H - m_D^T m_R^{-1} m_D, \quad (27)$$

is generated through type-II (first term) and type-I (second term) seesaw mechanisms.

As the vev of the left-handed triplet scalar is constrained from $\rho$ parameter of the SM it cannot be larger than $\sim \mathcal{O}(\text{few GeV})$. Thus it is indeed possible to generate light neutrino masses $\sim \text{eV}$ with $v_L \sim \text{eV}$ while the neutrino Yukawa coupling can be $\sim \mathcal{O}(1)$. In our

\(^6\) There exist two different discrete symmetries which can relate Left and Right handed fields\(^45\). Yukawa matrices are diagonal as we have considered the parity operation as defined in \(^43\) to relate $L$ and $R$ fields.
further analysis we consider $v_L=0$, thus type-II seesaw is absent here. The heavy neutrino mass $m_R$ is also generated through the Yukawa terms and proportional to $v_R$. It can be noted that with $m_R \sim O(\text{TeV})$, the Dirac term $m_D$ needs to be very small to generate light neutrino masses $\sim O(\text{eV})$. But $y^h$ can be as large as $\sim O(1)$ even when $v_R$ is around TeV scale. Thus successful light neutrino mass generation is still possible keeping $y^h$ as large as $\sim O(1)$. But $y^h$ affects the vacuum stability of the scalar potential in this model as the heavy neutrino is also coupled to the SM like Higgs. In the following section we have shown how these parameters are constrained due to vacuum stability and perturbativity (triviality).

It has been noted that the minimal left-right symmetric model is constrained by flavour-changing neutral currents (FCNCs) \cite{46–49}. The model we have worked with contains the bidoublet whose one of the vev is zero. Thus there is no FCNC problem in this model. There are also constraints from neutral kaon mixing, i.e., the kaon mass difference. Our choice of $v_R$ scale and the masses for the heavy neutral scalars takes care of those bounds. As the vevs and the Yukawa couplings in our scenario are real there is neither a source of nor spontaneous or explicit CP-violation. But since we have considered the Yukawa matrices to be diagonal we will boil down to the trivial, i.e., identity CKM and PMNS matrices. To fit all the masses and mixings we need to go for the non-minimal extension of this model and that certainly modify the set of RGEs that we have used here.

2. LR Model with Doublet Scalars

In this case the scalar sector consists of a bidoublet ($\Phi$), one left-handed doublet ($H_L$), and one right-handed doublet ($H_R$). The scalar potential is depicted in appendix \ref{appendix}. In terms of $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ gauge group these fields can be written as,

$$
\Phi \equiv (2, 2, 0), \quad H_L \equiv (2, 1, 1), \quad \text{and,} \quad H_R \equiv (1, 2, 1).
$$

The structure of $H_{L/R}$ is written as,

$$
H_{L/R} = \begin{pmatrix}
            h^0_{L/R} \\
            h^+_{L/R}
        \end{pmatrix}. \quad (29)
$$
The neutral components of $\Phi$ and $H_{L/R}$ acquire the vacuum expectation values:

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 e^{i\theta} \end{pmatrix}, \quad \langle H_L \rangle = \begin{pmatrix} 0 \\ v_L \end{pmatrix}, \quad \langle H_R \rangle = \begin{pmatrix} 0 \\ v_R \end{pmatrix}. \quad (30)$$

As before, we put $v_2 = 0$. The scalar sector consists of sixteen real scalar fields out of which six will be Goldstone bosons. Finally we will have four CP-even scalars and two CP-odd scalars and two charged scalars. Among the CP-even scalars one is Standard Model Higgs boson with mass $M_h$ and other three are taken as degenerate heavy scalars having mass $M_H$.

The parameters in the Higgs potential can be recast in terms of the masses of the neutral and charged scalars. The details about the scalar sector have been discussed in Ref. [50].

The gauge sector is similar to the previous case, i.e. the LR model with triplet scalars.

In the limit $v_R \gg v_1$ and assuming all the heavy scalars are degenerate, we have

$$f_1 = (M_H/v_R)^2 = \kappa_1 = -\kappa_2, \quad (31)$$

whereas, minimisation of the potential requires:

$$\frac{v_1^2}{v_R^2} = \frac{f_1 - 2\beta_1}{4\lambda_1}.$$

The structure of the covariant derivative in this model is very similar to that for the triplet scenario, see eq. (22)

$$D_\mu \Phi = \partial_\mu - ig_2L T^a W^a_{L\mu} \Phi + ig_2R \Phi T^a W^a_{R\mu},$$

$$D_\mu H_{(L/R)} = \partial_\mu H_{(L/R)} - ig_{(2L/2R)} T^a W^a_{(L/R)\mu} H_{(L/R)} - ig_{B-L} B_{\mu} H_{(L/R)}.$$

Following the previous convention we also set $g_{2L} = g_{2R} = g_2$. After spontaneous breaking of $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ symmetry, two charged $W^{\pm}_{L/R}$ and two neutral $Z_{L/R}$ gauge bosons become massive, while photon $A$ remains massless.

$$M^2_{W^\pm_L} = \frac{1}{4} g_2^2 v_1^2, \quad M^2_{W^\pm_R} = \frac{1}{4} g_2^2 \left( v_1^2 + v_R^2 \right),$$

$$M^2_{Z_{L/R}} = \frac{1}{8} \left( 2g_2^2 v_1^2 + v_R^2 (g_2^2 + g_{B-L}^2) \right) \mp \sqrt{4g_2^4 v_1^4 + (g_2^2 + g_{B-L}^2)v_R^4 - 4g_2^2g_{B-L}^2v_1^2v_R^2}.$$

In left-right symmetric model with doublet scalar leptonic part of the Yukawa interaction can be written as

$$- \mathcal{L} = \bar{L}_L \left( y_1 \Phi + y_2 \tilde{\Phi} \right) L_R + h.c. \quad (34)$$
where $SU(2)_L \otimes SU(2)_R$ quantum numbers of $L_L$ and $L_R$ are (2,1) and (1,2) respectively. So from this Lagrangian the Dirac mass term for the neutrinos can be written as

$$m_D = y_1 v_1.$$  \hfill (35)

Here, it is not possible to write the renormalizable Majorana mass term for the light and heavy neutrinos. But we can add non-renormalizable effective terms as

$$\mathcal{L}_{\text{eff}} = \frac{\eta_L}{M} L_L L_L H_L H_L + \frac{\eta_R}{M} L_R L_R H_R H_R,$$  \hfill (36)

where, $M$ is some very high scale and $\eta$’s are dimensionless parameters denote the strength of these non-renormalizable couplings. Once $H_R$ acquires the $vev$ the right-handed neutrino mass is generated as

$$m_R \simeq \frac{\eta_R v_R^2}{M}.$$  

Here we consider that $\langle H_L \rangle = v_L = 0$, thus this effective term does not contribute to the light neutrino mass. The neutrino mass matrix in ($\nu_l, \nu_h$) basis reads as

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & m_R \end{pmatrix},$$  \hfill (37)

and the light neutrino mass can be written as

$$m_{\nu_l} = -m_D^T m_R^{-1} m_D,$$  \hfill (38)

which is a variant of the type-I seesaw mechanism.

In the left-right symmetric model associated with two doublet scalars, neutrino masses cannot be generated through type-II seesaw mechanism due to the lack of left-handed triplet scalar\footnote{Although, through an effective operator the Majorana mass term for light neutrino can be generated, see eq. 36. But this contribution is absent here as we have set $v_L = 0$.}. Thus the type-I seesaw mechanism is the natural choice in this case. But the right-handed neutrino masses are generated through an effective operator suppressed by a heavy scale. This may provide a possible explanation how the right-handed neutrinos can be lowered to TeV scale. Here, the correct order of light neutrino masses are generated if the Dirac-type neutrino Yukawa coupling needs to be very small unless one considers the special textures for the Dirac Yukawa couplings. Then vacuum stability is automatically satisfied.
as these Dirac Yukawa couplings are much smaller. Thus here only the quartic couplings get constrained through the vacuum stability, perturbativity (triviality) of the couplings. Within a framework very similar to this it is indeed possible to generate light neutrino masses of correct order without lowering the Yukawa coupling as the light neutrino masses are independent of \( v_R \) but suppressed by some high scale \([51, 52]\). On that case the vacuum stability constraints cannot be avoided and play the most crucial role in constraining the Yukawa couplings and other parameters.

### III. VACUUM STABILITY

The presence of new physics introduces exotic non-SM particles in the theory and if they couple to the SM fields then the renormalization group evolutions (RGEs) of the Higgs quartic coupling (\( \lambda_h \)) will be modified. Moreover, additional quartic interactions of extra scalar fields should also be introduced. Extended gauge interactions from the larger gauge groups as well as Yukawa interactions would contribute to these evolution equations. Now the question arises of whether or not the vacuum is stable in the presence of the new physics. In particular, when we have narrowed down a preferred range of the Higgs mass between 123-127 GeV, the new physics could be constrained by the vacuum stability criteria. To adjudge the stability of these models we have considered the one loop RGEs of all the required parameters. In passing we would like to mention that the allowed parameter space in our analysis is the minimal set which will be extended once one includes the higher order renormalization group (RG) effects. The RGEs for SM and each of the \( B - L \) models which are used in our calculation are given in appendix. Since we are dealing with the TeV scale models, all the SM RGEs will be modified once the new physics effects are switched on. Thus from EW scale to TeV (specific values are dictated in plots) the RGEs will be SM like and from the TeV scale to the Planck scale they will be the modified ones, and during the process proper matching conditions are incorporated at the TeV scale.

#### A. \( U(1)_{B-L} \) Model

It is clear from the structure of the potential as shown in eq. [2] for the \( U(1)_{B-L} \) model, that the vacuum stability conditions are different from that for the SM due to the presence
of extra singlet scalar. If all the quartic couplings are positive, the potential will be trivially bounded from below, i.e., vacuum is stable and these stability conditions read simply as $\lambda_{1,2,3} > 0$. But it is indeed possible to allow $\lambda_3$ to be negative and still have the vacuum be stable. Thus vacuum stability conditions beyond the trivial ones allow larger parameter space and need to be accommodated in these conditions. We find the non-trivial vacuum stability criteria using the proposal dictated in [30] and shown in appendix B1,

\[ 4\lambda_1\lambda_2 - \lambda_3^2 > 0, \]
\[ \lambda_1 > 0, \quad \lambda_2 > 0. \]  

(39)

Together with these we have also incorporated perturbativity constraints on quartic couplings by demanding upper limit, i.e., $|\lambda_i| < 1$ ($i = 1, 2, 3$).

Noting down from eqs. 5 and 6 that the physical Higgs field is an admixture of two scalar fields $\phi$ and $s$, in our study the scalar mixing angle $\alpha$ is considered to be a free parameter instead of the quartic couplings $\lambda_i (i = 1, 2, 3)$. This model consists of two different scales in the theory, those are EW scale and $B - L$ symmetry breaking scale. Thus two RGEs are invoked for the analysis. As we have two Abelian couplings in this model, there might be mixing between them [53, 54]. To simplify the situation, and off course without hampering any other conclusions, we impose no mixing between the $Z_{B-L}$ and $Z$ gauge bosons at the tree-level. This is followed from the condition $\tilde{g}(Q_{EW}) = 0$ as already discussed above eq. 11.

As a consequence $B - L$ breaking vev $v_{B-L}$ relates to the new $Z_{B-L}$ boson mass given as in eq. 12. For demonstration, we have picked the perturbative value of this additional gauge coupling at breaking scale as, $g_{B-L} = 0.1$. For simplicity we further assume heavy neutrinos are degenerate and fixed at $m_{v_h}^{1,2,3} \equiv m_{v_h} \simeq 200$ GeV, which are within the allowed values. We have used central value of light Higgs mass ($M_h$) at 125 GeV, top quark mass at 173.2 GeV and strong coupling constant $\alpha_s$ at 0.1184. Thus remaining free parameters in our study are $M_H$, $\alpha$ and $v_{B-L}$. We have explored the correlated constraints on these parameters from vacuum stability.

The set of RGEs of different couplings that we have used in our analysis are encoded in appendix C2 [35]. The parameter space consistent with vacuum stability in heavy Higgs mass ($M_H$) and scalar mixing angle ($\alpha$) plane is depicted in figure 1. All the couplings are perturbative through out their evolutions. The grey region is the domain of allowed input parameters. The red, green, and black sub-parameter spaces show the domain of $M_H$ and
FIG. 1: The allowed parameter space in heavy Higgs mass ($M_H$) and scalar mixing angle ($\alpha$) plane, consistent with vacuum stability and perturbativity bounds are shown. The grey region is the domain of allowed input parameters. The red, green, and black sub-parameter spaces show the domain of $M_H$ and $\alpha$ for which this $B-L$ theory is valid till $10^7$, $10^{10}$ and $10^{19}$ GeV respectively. The Majorana neutrino mass is fixed at 200 GeV and $B-L$ breaking vev ($v_{B-L}$) is set at 7.5 TeV. The $U(1)_{B-L}$ gauge coupling is taken to be 0.1 which implies $M_{Z_{B-L}}=1.5$ TeV. The shaded region satisfy $\lambda_3 < 0$ (as well as $\alpha < 0$ from eq. 7). Thus the non-trivial vacuum stability conditions are being satisfied in this region. These conditions are stringent than the trivial one that applied in the positive $\alpha$ region. Although the pattern of the allowed parameter space is very similar for both positive and negative $\alpha$ region, the $\alpha > 0$ region covers larger parameter space.
the perturbativity of the couplings and thus not affected by the vacuum stability conditions which are different for different signs of $\lambda_3$. However, the lower boundaries are outcome of the demand to satisfy the criteria of vacuum stability. Allowed parameters in the yellow shaded region (which represents $\lambda_3 < 0$) in figure 1 are reflected by the non-trivial vacuum stability condition in eq. [39] which sequentially plays a role in determining the lower boundaries in the allowed parameters. Thus expectedly in the positive $\alpha$ region the allowed parameter space is larger than that for negative $\alpha$. Also, note that $\alpha = 0$ leads to the decoupling limit when the heavy scalar will not affect the vacuum stability. The parameter space has also shrunk as the validity of the model must be closer to the Planck scale as can be inferred from the figure 1.

To study the dependence of different parameters as shown in figure 1, we plot the allowed parameter space in $M_H - \alpha$ plane which remains consistent with vacuum stability and where all the couplings are perturbative till the Planck Scale. In figure 2(a) Majorana neutrino Yukawa coupling $y^h$ is varied keeping $v_{B-L}$ and $g_{B-L}$ fixed. As the $y^h$ increases, the allowed parameter space is shrunk since the Yukawa coupling affects the quartic couplings negatively in their RG evolutions. Thus larger Yukawa couplings spoil the vacuum stability. In figure 2(b) shows the dependence on $B - L$ breaking vev for fixed $g_{B-L}$ and $y^h$. $v_{B-L}$ determines the scale of new physics beyond the Standard Model, i.e., from where the RGEs are being modified due to the presence of new particles. The larger $v_{B-L}$ implies that new set of RGEs come to play later. In $B - L$ extended model $\lambda_3$ is inversely proportional to $v_{B-L}$ at EW scale (see eq. 7). Thus for same set of values of $M_H$ and $\alpha$, $\lambda_3$ is smaller for larger $v_{B-L}$ at 15 TeV. The RGE of $\lambda_3$ is such that for our choice of parameters it grows with mass scale. Thus there is a possibility of generating large $\lambda_3$ such that vacuum stability and perturbativity conditions are not validated at some higher scale. This plot therefore shows that it is possible to have larger allowed parameter space for larger $v_{B-L}$. Finally in figure 2(c), $g_{B-L}$ varies where $v_{B-L}$ and $Y_\nu$ are kept constant. As the larger values of the gauge couplings affect the RGEs of the quartic couplings positively, the vacuum stability is improved. Thus with the larger value of gauge coupling the larger parameter space is allowed. But the $U(1)$ couplings increases with the mass scale. Hence the couplings with much larger values at low scale might be non-perturbative in the high scale. In our analysis, when $v_{B-L}$ is at 7.5 TeV, any value of $g_{B-L}$ more than 0.34 are disallowed as the coupling becomes non-perturbative before Planck scale.
FIG. 2: Allowed parameter space in $M_H - \alpha$ plane, with $\alpha$ varying between $[0, -\pi/2]$, consistent with vacuum stability and perturbativity (triviality) bounds up to the Planck scale. Figure (a): The Majorana neutrino Yukawa coupling $y^h$ is varied keeping $v_{B-L}$ and $g_{B-L}$ fixed. Figure (b): Two different set of $B - L$ breaking vev, $v_{B-L}$ are chosen keeping $g_{B-L}$ and $y^h$ fixed. Figure (c): In this plot $g_{B-L}$ varies where $v_{B-L}$ and $y^h$ are kept constant. In our analysis any value of $g_{B-L}$ for $v_{B-L} = 7.5$ TeV more than 0.34 are disallowed as the coupling becomes non-perturbative before Planck scale. Corresponding regions for positive $\alpha$ are not shown here, as they remain unaffected and are the same as those given in blue strip in figure 1 owing to the trivial conditions.

B. left-right Symmetry

1. LR Model with Triplet Scalars

In this model the scalar potential for the left-right Symmetric model with triplet scalar as shown in the appendix A2 contains many quartic couplings. To find the condition of vacuum stability we have considered all two-fields, three-fields and four-fields directions and find their stability criteria. Detailed field directions corresponding to the potential together
with calculated stability conditions are listed in appendix B. Finally, the effective non-trivial vacuum stability conditions which are necessary and sufficient are

\[
\begin{align*}
\lambda_1 &> 0, \\
\lambda_5 &> 0, \\
\lambda_5 + \lambda_6 &> 0, \\
\lambda_5 + 2\lambda_6 &> 0, \\
\lambda_{12} - 2\sqrt{\lambda_1\lambda_5} &< 0.
\end{align*}
\] (40)

Along with the above conditions, we find an additional condition \(\lambda_{12} > 0\) from eq. (20).

The renormalization group evolutions that we have considered in our analysis are depicted in appendix C. In figure 3 we show the constraints on universal quartic coupling \(\lambda_u\) (\(\equiv \lambda_2, \lambda_3, \lambda_4, \lambda_8, \lambda_9, \lambda_{10}, \lambda_{11}\)) for LR model with triplet scalars in low \(v_R\) region. Yellow Shaded region is disallowed from low energy data \((M_{W_R} > 3.5 \text{ TeV})\) and green shaded region is excluded from direct search at LHC \((M_{W_R} > 2.5 \text{ TeV})\). These limits can be extracted using the eq. (33). In our analysis we also set Majorana Yukawa, \(y^h\) at 0.25. We note that, for any particular heavy scalar mass \((M_H)\), universal quartic coupling \(\lambda_u\) is disallowed above the corresponding line shown in the figure. For example, as seen from the plot, maximum allowed value of the universal quartic coupling is 0.024 if one consider LR breaking scale at 10 TeV and heavy scalar mass at 1 TeV. Allowed maximum quartic coupling is lowered for heavier scalar which can be understood from vacuum stability and
FIG. 4: Compatibility for stable vacuum in $v_R$ and heavy scalar $M_H$ allowed region in LR model with triplet scalar. Each color represents a particular set of light Higgs mass ($M_h$) and top mass ($M_t$) in respective plot. In figure (a) Higgs mass is fixed at 125 GeV for different top quark mass where as, in figure (b) top quark mass is fixed at 173.2 GeV and Higgs mass is varying. Upper-left region (shaded with light blue) above the line $M_H = v_R$ is disallowed since quartic couplings are non-perturbative in this domain. Lower-right region (shaded with light pink) quartic coupling related with heavy scalar mass becomes extremely small ($\leq O(10^{-7})$). We choose universal quartic coupling $\lambda_u$ fixed at 0.03. Inset to both figures show the higher $v_R$ scale where color patches terminate, representing the very scale where in fact Standard Model breaks down for a particular Higgs mass or top quark mass at one loop.

In figure 4 we check the compatibility for the stable vacuum in left-right symmetric breaking scale $v_R$ and heavy scalar $M_H$ allowed region in LR model with triplet scalar. Each color represents a particular set of light Higgs mass ($M_h$) and top mass ($M_t$) in respective plot. In figure 4(a) Higgs mass is fixed at 125 GeV and top quark mass is varying from 170 GeV to 175 GeV where as, in figure 4(b) top quark mass is fixed at 173.2 GeV and Higgs mass is varying from 122 GeV to 127 GeV. Upper-left region (shaded with light blue) above the line $M_H = v_R$ is disallowed since quartic couplings are non-perturbative in this domain. The blank (white) strip is also ruled out as the value of the couplings in this region is such that they become non-perturbative before reaching the Planck scale. Lower-right region (shaded with light pink) quartic coupling related with heavy Higgs mass becomes extremely small ($\leq O(10^{-7})$). We choose universal quartic coupling $\lambda_2$, $\lambda_3$, $\lambda_4$, $\lambda_8$, $\lambda_9$, $\lambda_{10}$, $\lambda_{11} = \lambda_u$ fixed at 0.03. This choice of $\lambda_u$ allows only $v_R \geq 100$ TeV which can be inferred...
FIG. 5: Constraints on universal quartic coupling $\lambda_u \equiv \lambda_2, -\lambda_3$ for LR model with doublet scalars in low $v_R$ region for different set of heavy scalar masses $M_H$. Yellow Shaded region is disallowed from low energy data ($M_{W_R} > 3.5$ TeV) and green shaded region is excluded from direct search at LHC ($M_{W_R} > 2.5$ TeV).

from figure 3. Inset to both figures shows the higher $v_R$ scale where color patches terminate, representing the very scale where in fact Standard Model breaks down for a particular Higgs mass or top quark mass at one loop.

2. LR Model with Doublet Scalars

Using the similar technique used in previous section we depicted all the multiple field directions of the potential and the corresponding stability criteria in appendix B 3. We find the non-trivial vacuum stability conditions which read as

$$\lambda_1 > 0, \quad 2\beta_1 + f_1 > 0, \quad 2\beta_1 - f_1 > 0. \quad (41)$$

We have also noted the required RGEs for our analysis in appendix C[4]. In figure 5 we constrain universal quartic coupling $\lambda_u \equiv \lambda_2, -\lambda_3$ for LR model with doublet scalars in low $v_R$ region for different set of heavy scalar masses $M_H$. Similar to the previous case, yellow shaded region in the plot is disallowed from low energy data ($M_{W_R} > 3.5$ TeV) and green shaded region is excluded from direct search at LHC ($M_{W_R} > 2.5$ TeV).

As we noticed at figure 5, for any particular heavy scalar mass ($M_H$), universal quartic
FIG. 6: Compatibility for stable vacuum in $v_R$ and heavy Higgs $M_H$ allowed region in LR model with doublet scalars. Each color represents a particular set of light Higgs mass ($M_h$) and top mass ($M_t$) in respective plot. In figure (a) Higgs mass is fixed at 125 GeV and top quark mass is varying, where as, in figure (b) top quark mass is fixed at 173.2 GeV and Higgs mass is varying. Upper-left region (shaded with light blue) above the line $M_H = v_R$ is disallowed since quartic couplings are non-perturbative at the low scale itself in this domain. Lower-right region (shaded with light pink) quartic coupling related with heavy Higgs mass becomes extremely small ($\leq O(10^{-7})$). We choose universal quartic coupling $\lambda_u$ fixed at 0.04. Inset to both figures shows the higher $v_R$ scale where color patches terminate, representing the very scale where in fact Standard Model breaks down for a particular Higgs mass or top quark mass at one loop.

coupling $\lambda_u$ is disallowed above the corresponding line. For example, as seen from the plot, maximum allowed value of the universal quartic coupling is 0.033 if one consider LR breaking scale at 10 TeV and heavy scalar mass at 1 TeV. As before, allowed maximum quartic coupling is lowered for heavier scalar.

In figure 6 we check the compatibility for stable vacuum in $v_R$ and heavy scalar $M_H$ allowed region in LR model with doublet scalars. Each color represents a particular set of light Higgs mass ($M_h$) and top mass ($M_t$) in respective plot. In figure 6(a) Higgs mass is fixed at 125 GeV and top quark mass is varying from 170 GeV to 175 GeV where as, in figure 6(b) top quark mass is fixed at 173.2 GeV and Higgs mass is varying from 122 GeV to 127 GeV. Upper-left region (shaded with light blue) above the line $M_H = v_R$ is disallowed since quartic couplings are non-perturbative at the low scale itself in this domain. The blank (white) strip is also ruled out as the value of the couplings in this region is such that they become non-perturbative before reaching the Planck scale. Lower-right region (shaded
with light pink) quartic coupling related with heavy Higgs mass becomes extremely small ($\leq O(10^{-7})$). We choose universal quartic coupling $\lambda_1 = -\lambda_2 = \lambda_u$ fixed at 0.04. Here, the choice of $\lambda_u$ allows only $v_R \geq 100 \text{ TeV}$. Inset to both figures show the higher $v_R$ scale where color patches terminate, representing the very scale where in fact Standard Model breaks down for a particular Higgs mass or top quark mass at one loop.

IV. CONCLUSIONS

We have noted that one needs to study the scalar potential to understand the structure of the vacuum and its compatibility with successful spontaneous symmetry breaking. In addition, the perturbativity (triviality) of the couplings also plays a crucial role. We have analysed the structure of the scalar potentials of $B - L$ extended models – namely, $\text{SM} \otimes U(1)_{B-L}$ and left-right symmetry – with different scalar representations. We have computed the criteria for the potential to be bounded from below, i.e., the conditions for vacuum stability. We also performed the renormalization group evolutions of the parameters (couplings) of these models at the one loop level with proper matching conditions. We have shown how the phenomenologically unaccessible couplings can be constrained for different choices of scales of new physics. They in turn also affect the RGEs of the other couplings. We have noted that the new physics effects must be switched on before the SM vacuum face the instability. This helps the vacuum stability of the full scalar potential and achieve a consistent spontaneous symmetry breaking. We have analyzed these aspects by varying the Higgs and top quark mass over their allowed ranges. In summary, it is meaningful to mention that more precise knowledge of the SM parameters, like Higgs mass, top quark mass and strong coupling will constrain the parameters (couplings, masses, scales) of new physics and that might direct us towards the correct theory for beyond standard model physics. In principle one can study the left-right symmetric models including the radiative correction in the scalar potential and use the Coleman-Weinberg mechanism, e.g., [50] has considered the scenario and calculated the flat directions using one-loop effective potential. This will certainly change the correlations among the parameters of the scalar potential leading to stable vacuum. While submitting our paper there appeared [65] where the vacuum stability for $\text{SM} \otimes U(1)_{B-L}$ has been discussed. The view points of our analysis is quite different from this work.
Acknowledgements

Work of JC is supported by Department of Science & Technology, Government of INDIA under the Grant Agreement number IFA12-PH-34 (INSPIRE Faculty Award). Authors want to thank Srubabati Goswami and Namit Mahajan for useful discussions.
Appendix A: Scalar Potential for Different Models

1. $U(1)_{B-L}$ Model

$$V(\Phi, S) = m^2 \Phi^\dagger \Phi + \mu^2 | S |^2 + \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 | S |^4 + \lambda_3 \Phi^\dagger \Phi | S |^2. \quad (A1)$$

2. LR model with triplet scalars

Most general form of the scalar potential can be written as in [64]

$$V_{LRT}(\Phi, \Delta_L, \Delta_R) =$$

\[-\mu_1^2 \left\{ \text{Tr} [\Phi^\dagger \Phi] \right\} - \mu_2^2 \left\{ \text{Tr} [\Phi^\dagger \Phi] + \text{Tr} [\Phi^\dagger \Phi] \right\} - \mu_3^2 \left\{ \text{Tr} [\Delta_L^\dagger \Delta_L] + \text{Tr} [\Delta_R^\dagger \Delta_R] \right\}
\]

\[+ \lambda_1 \left\{ \left( \text{Tr} [\Phi^\dagger \Phi] \right)^2 \right\} + \lambda_2 \left\{ \left( \text{Tr} [\Phi^\dagger \Phi] \right)^2 + \left( \text{Tr} [\Phi^\dagger \Phi] \right)^2 \right\} + \lambda_3 \left\{ \text{Tr} [\Phi^\dagger \Phi] \text{Tr} [\Phi^\dagger \Phi] \right\}
\]

\[+ \lambda_4 \left\{ \text{Tr} [\Phi^\dagger \Phi] \left( \text{Tr} [\Phi^\dagger \Phi] + \text{Tr} [\Phi^\dagger \Phi] \right) \right\} + \lambda_5 \left\{ \left( \text{Tr} [\Delta_L^\dagger \Delta_L] \right)^2 + \left( \Delta_R^\dagger \Delta_R \right)^2 \right\}
\]

\[+ \lambda_6 \left\{ \text{Tr} [\Delta_L^\dagger \Delta_L] \text{Tr} [\Delta_L^\dagger \Delta_L] + \text{Tr} [\Delta_R^\dagger \Delta_R] \text{Tr} [\Delta_R^\dagger \Delta_R] \right\} + \lambda_7 \left\{ \text{Tr} [\Delta_L^\dagger \Delta_L] \text{Tr} [\Delta_R^\dagger \Delta_R] \right\}
\]

\[+ \lambda_8 \left\{ \text{Tr} [\Delta_L^\dagger \Delta_L] \text{Tr} [\Delta_L^\dagger \Delta_L] \right\} + \lambda_9 \left\{ \text{Tr} [\Phi^\dagger \Phi] \left( \text{Tr} [\Delta_L^\dagger \Delta_L] + \text{Tr} [\Delta_R^\dagger \Delta_R] \right) \right\}
\]

\[+ (\lambda_{10} + i \lambda_{11}) \left\{ \text{Tr} [\Phi^\dagger \Phi] \text{Tr} [\Delta_L^\dagger \Delta_L] + \text{Tr} [\Phi^\dagger \Phi] \text{Tr} [\Delta_R^\dagger \Delta_R] \right\}
\]

\[+ (\lambda_{10} - i \lambda_{11}) \left\{ \text{Tr} [\Phi^\dagger \Phi] \text{Tr} [\Delta_R^\dagger \Delta_L] + \text{Tr} [\Phi^\dagger \Phi] \text{Tr} [\Delta_L^\dagger \Delta_R] \right\}
\]

\[+ \lambda_{12} \left\{ \text{Tr} [\Phi^\dagger \Phi \Delta_L^\dagger \Delta_L] + \text{Tr} [\Phi^\dagger \Phi \Delta_R^\dagger \Delta_R] \right\} + \lambda_{13} \left\{ \text{Tr} [\Phi \Delta_R \Phi^\dagger \Delta_R] + \text{Tr} [\Phi \Delta_L \Phi^\dagger \Delta_L] \right\}
\]

\[+ \lambda_{14} \left\{ \text{Tr} [\Phi \Delta_R \Phi^\dagger \Delta_L] + \text{Tr} [\Phi^\dagger \Delta_L \Phi^\dagger \Delta_R] \right\} + \lambda_{15} \left\{ \text{Tr} [\Phi \Delta_R \Phi^\dagger \Delta_R] + \text{Tr} [\Phi^\dagger \Delta_L \Phi^\dagger \Delta_L] \right\},
\]

where all the coupling constants are real.
3. LR model with doublet scalars

Scalar potential for LR model with doublet scalars can be written as:

\[
V_{LRD}(\Phi, H_L, H_R) = 4\lambda_1(\text{Tr}[\Phi^\dagger\Phi])^2 + 4\lambda_2(\text{Tr}[\Phi^\dagger\tilde{\Phi}] + \text{Tr}[\tilde{\Phi}\Phi^\dagger])^2 + 4\lambda_3(\text{Tr}[\Phi^\dagger\tilde{\Phi}] - \text{Tr}[\tilde{\Phi}\Phi^\dagger])^2
\]

\[
+ \frac{\kappa_1}{2}(H_L^\dagger H_L + H_R^\dagger H_R)^2 + \frac{\kappa_2}{2}(H_L^\dagger H_L - H_R^\dagger H_R)^2
\]

\[
+ \beta_1(\text{Tr}[\Phi^\dagger\tilde{\Phi}] + \text{Tr}[\tilde{\Phi}\Phi^\dagger])(H_L^\dagger H_L + H_R^\dagger H_R)
\]

\[
+ f_1(H_L^\dagger(\tilde{\Phi}\Phi^\dagger - \Phi\tilde{\Phi})H_L - H_R^\dagger(\Phi^\dagger\Phi - \tilde{\Phi}\tilde{\Phi})H_R).
\]

Appendix B: Calculation of non-trivial Vacuum Stability Conditions

Here we have gathered the structure of the scalar potential in 2, 3 and 4-field directions. We have calculated vacuum stability conditions from these fields direction keeping in mind that the conditions should cover most of the parameter space spanned by the quartic couplings.

1. \(U(1)_{B-L}\) Model

For \(U(1)_{B-L}\) model the potential has a simple structure and the stability conditions can be calculated easily. The quartic potential has the form

\[
\lambda_1 |\Psi|^4 + \lambda_2 |S|^4 + \lambda_3 |\Phi|^2 |S|^2,
\]

and we can easily write this potential as

\[
\left(\sqrt{\lambda_1} |\Psi|^2 + \frac{\lambda_3}{2\sqrt{\lambda_1}} |S|^2\right)^2 + \left(\lambda_2 - \frac{\lambda_3^2}{4\lambda_1}\right) |S|^4.
\]

Clearly the above equation is positive definite if

\[
\lambda_1 > 0, \quad \lambda_2 > 0,
\]

\[
4\lambda_1\lambda_2 - \lambda_3^2 > 0.
\]

These are the non-trivial vacuum stability conditions with \(\lambda_3 < 0\). The trivial boundary conditions are when all the \(\lambda_{1,2,3} > 0\).
2. LR Model with Triplet Scalars

Absence of any tachyonic pseudoscalar modes imposes the condition $\lambda_{12} > 0$.

a. 2 Field Directions and Stability Conditions

$$2F V_1(\phi_1^0, \phi_1^+) = \lambda_1 (\phi_1^0 + \phi_1^+)^2$$
$$2F V_2(\phi_1^0, \delta^0) = \lambda_5 \delta^{04} + \lambda_1 \phi_1^{04}$$
$$2F V_3(\phi_1^0, \delta^+) = (\lambda_5 + \lambda_6) \delta^{++} + \lambda_1 \phi_1^{04} + \frac{1}{2}(\lambda_{12} + 2\lambda_9) \delta^{+2} \phi_1^{02}$$
$$2F V_4(\phi_1^0, \delta^{++}) = \lambda_5 \delta^{++} + \lambda_1 \phi_1^{04} + \lambda_{12} \delta^{+2} \phi_1^{02}$$
$$2F V_5(\phi_1^+, \delta^0) = \lambda_5 \delta^{04} + \lambda_1 \phi_1^{+4} + \lambda_{12} \delta^{02} \phi_1^{+2}$$
$$2F V_6(\phi_1^+, \delta^+) = (\lambda_5 + \lambda_6) \delta^{+4} + \lambda_1 \phi_1^{+4} + \frac{1}{2}(\lambda_{12} + 2\lambda_9) \delta^{+2} \phi_1^{+2}$$
$$2F V_7(\phi_1^+, \delta^{++}) = \lambda_5 \delta^{++} + \lambda_1 \phi_1^{+4}$$
$$2F V_8(\delta^0, \delta^+) = \lambda_5 (\delta^{02} + \delta^{+2})^2 + \lambda_6 \delta^{+4}$$
$$2F V_9(\delta^0, \delta^{++}) = \lambda_5 (\delta^{02} + \delta^{+2})^2 + 4\lambda_6 \delta^{+2} \delta^{02}$$
$$2F V_{10}(\delta^+, \delta^{++}) = \lambda_5 (\delta^{+2} + \delta^{++2})^2 + \lambda_6 \delta^{+4}$$

**Stability conditions**

$$2F V_1 \rightarrow \lambda_1 > 0$$
$$2F V_2, 2F V_4, 2F V_5, 2F V_7 \rightarrow \lambda_1 > 0; \lambda_5 > 0$$
$$2F V_3, 2F V_6 \rightarrow \lambda_1 > 0; \lambda_5 + \lambda_6 > 0$$
$$2F V_8, 2F V_9, 2F V_{10} \rightarrow \lambda_5 > 0; \lambda_5 + \lambda_6 > 0$$
b. 3 Field Directions and Stability Conditions

\[ 3F V_1(\phi_1^0, \phi_1^+, \delta^0) = \lambda_1 (\phi_1^0 + \phi_1^+)^2 + \lambda_5 \delta^0 + \lambda_{12} \delta^0 \phi_1^2 \]
\[ 3F V_2(\phi_1^0, \phi_1^+, \delta^+) = \lambda_1 (\phi_1^0 + \phi_1^+)^2 + (\lambda_5 + \lambda_6) \delta^+ + \frac{1}{2}(\lambda_{12} + 2\lambda_9) (\phi_1^0 + \phi_1^+) \delta^+ \]
\[ 3F V_3(\phi_1^0, \phi_1^+, \delta^{++}) = \lambda_1 (\phi_1^0 + \phi_1^+)^2 + \lambda_5 \delta^{++} + \lambda_{12} \phi_1^0 \delta^{++} \]
\[ 3F V_4(\phi_1^0, \delta^0, \delta^+) = \lambda_1 \phi_1^0 + \lambda_5 (\delta^0 + \delta^+)^2 + \lambda_6 \delta^+ + \frac{1}{2}(\lambda_{12} + 2\lambda_9) \phi_1^0 \delta^+ \]
\[ 3F V_5(\phi_1^0, \delta^0, \delta^{++}) = \lambda_1 \phi_1^0 + \lambda_5 (\delta^{++} + \delta^+)^2 + \lambda_6 \delta^+ + \frac{1}{2}(\lambda_{12} + 2\lambda_9) \phi_1^0 \delta^{++} \]
\[ 3F V_6(\phi_1^0, \delta^+, \delta^{++}) = \lambda_1 \phi_1^0 + \lambda_5 (\delta^{++} + \delta^+)^2 + \lambda_6 \delta^+ + \frac{1}{2}(\lambda_{12} + 2\lambda_9) \phi_1^0 \delta^{++} \]
\[ 3F V_7(\phi_1^+, \delta^0, \delta^+) = \lambda_1 \phi_1^+ + \lambda_5 (\delta^0 + \delta^+)^2 + \lambda_6 \delta^+ + \frac{1}{2}(\lambda_{12} + 2\lambda_9) \phi_1^+ \delta^+ \]
\[ 3F V_8(\phi_1^+, \delta^0, \delta^{++}) = \lambda_1 \phi_1^+ + \lambda_5 (\delta^{++} + \delta^+)^2 + \lambda_6 \delta^+ + \frac{1}{2}(\lambda_{12} + 2\lambda_9) \phi_1^+ \delta^{++} \]
\[ 3F V_9(\phi_1^+, \delta^+, \delta^{++}) = \lambda_1 \phi_1^+ + \lambda_5 (\delta^+ + \delta^{++})^2 + \lambda_6 \delta^+ + \frac{1}{2}(\lambda_{12} + 2\lambda_9) \phi_1^+ \delta^+ \]
\[ 3F V_{10}(\delta^0, \delta^+, \delta^{++}) = \lambda_5 (\delta^0 + \delta^+ + \delta^{++})^2 + \lambda_6 (\delta^+ + 2\delta^0 \delta^{++})^2 \]

**Stability conditions**

\[ 3F V_1, 3F V_3 \rightarrow \lambda_1 > 0; \ \lambda_5 > 0 \]
\[ 3F V_2 \rightarrow \lambda_1 > 0; \ \lambda_5 + \lambda_6 > 0 \]
\[ 3F V_4, 3F V_6, 3F V_7, 3F V_9 \rightarrow \lambda_1 > 0; \ \lambda_5 > 0; \ \lambda_5 + \lambda_6 > 0 \]
\[ 3F V_5, 3F V_8 \rightarrow \lambda_1 > 0; \ \lambda_5 > 0; \ \lambda_5 + 2\lambda_6 > 0 \]
\[ 3F V_{10} \rightarrow \lambda_5 > 0; \ \lambda_5 + \lambda_6 > 0; \ \lambda_5 + 2\lambda_6 > 0 \]
c. 4 Field Directions and Stability Conditions

\[ 4F V_1 (\phi_1^0, \phi_1^+, \delta^0, \delta^+) = \lambda_5 \left( \delta^{02} + \delta^{+2} \right)^2 + \lambda_6 \delta^{++} + \lambda_1 \left( \phi_1^{02} + \phi_1^{+2} \right)^2 + \frac{1}{2} \lambda_{12} \left( 2\delta^{02} \phi_1^{+2} + 2\sqrt{2} \phi_1^0 \phi_1^+ \delta^{0} \delta^{+} + \delta^{+2} \left( \phi_1^{02} + \phi_1^{+2} \right) \right) + \lambda_9 \delta^{+2} \left( \phi_1^{02} + \phi_1^{+2} \right) \]

\[ 4F V_2 (\phi_1^0, \phi_1^+, \delta^0, \delta^{++}) = \lambda_5 \left( \delta^{02} + \delta^{++2} \right)^2 + 4\lambda_6 \delta^{02} \delta^{++2} + \lambda_1 \left( \phi_1^{02} + \phi_1^{+2} \right)^2 + \lambda_{12} \left( \delta^{++2} \phi_1^{02} + \phi_1^{02} \phi_1^{+2} + 2 \lambda_9 \delta^{02} \delta^{++} \left( \phi_1^{02} + \phi_1^{+2} \right) \right) \]

\[ 4F V_3 (\phi_1^0, \phi_1^+, \delta^+, \delta^{++}) = \lambda_5 \left( \delta^{+2} + \delta^{++2} \right)^2 + \lambda_6 \delta^{++} + \lambda_1 \left( \phi_1^{02} + \phi_1^{+2} \right)^2 + \frac{1}{2} \lambda_{12} \left( 2\delta^{+2} \phi_1^{02} + 2\sqrt{2} \phi_1^0 \phi_1^+ \delta^{+} \delta^{++} + \delta^{+2} \left( \phi_1^{02} + \phi_1^{+2} \right) \right) + \lambda_9 \delta^{+2} \left( \phi_1^{02} + \phi_1^{+2} \right) \]

\[ 4F V_4 (\phi_1^0, \delta^0, \delta^+, \delta^{++}) = \lambda_5 \left( \delta^{02} + \delta^{+2} + \delta^{++2} \right)^2 + \lambda_6 \left( \delta^{+2} + 2\delta^0 \delta^{++} \right)^2 + \lambda_1 \phi_1^{04} + \frac{1}{2} \lambda_{12} \phi_1^{02} \left( \delta^{02} + \delta^{+2} \right) + \lambda_9 \phi_1^{02} \left( \delta^{+2} + 2\delta^0 \delta^{++} \right) \]

\[ 4F V_5 (\phi_1^+, \delta^0, \delta^+, \delta^{++}) = \lambda_5 \left( \delta^{02} + \delta^{+2} + \delta^{++2} \right)^2 + \lambda_6 \left( \delta^{+2} + 2\delta^0 \delta^{++} \right)^2 + \lambda_1 \phi_1^{+4} + \frac{1}{2} \lambda_{12} \phi_1^{+2} \left( \delta^{02} + \delta^{+2} \right) + \lambda_9 \phi_1^{+2} \left( \delta^{+2} + 2\delta^0 \delta^{++} \right) \]

**Stability conditions**

\[ 4F V_1 \rightarrow \lambda_1 > 0; \ \lambda_5 > 0; \ \lambda_5 + 2\lambda_6 > 0 \]

\[ 4F V_2 \rightarrow \lambda_1 > 0; \ \lambda_5 > 0; \ \lambda_5 + \lambda_6 > 0 \]

\[ 4F V_3 \rightarrow \lambda_1 > 0; \ \lambda_5 > 0; \ \lambda_5 + \lambda_6 > 0; \ \lambda_{12} - 2\sqrt{2} \lambda_1 \lambda_5 < 0 \]

\[ 4F V_4, 4F V_5 \rightarrow \lambda_1 > 0; \ \lambda_5 > 0; \ \lambda_5 + \lambda_6 > 0; \ \lambda_5 + 2\lambda_6 > 0 \]
3. LR Model with Doublet Scalars

a. 2 Field Directions and Stability Conditions

\[ 2F_{V1}(\phi^0_1, \phi^+_1) = \lambda_1 \left( \phi^{02}_1 + \phi^{+2}_1 \right)^2 \]
\[ 2F_{V2}(\phi^+_1, h^+_R) = \lambda_1 \phi^{+4}_1 + \frac{2\beta_1 + f_1}{2} h^+_R \phi^{+2}_1 \]
\[ 2F_{V3}(\phi^0_1, h^+_R) = \lambda_1 \phi^{04}_1 + \frac{2\beta_1 - f_1}{2} h^+_R \phi^{02}_1 \]
\[ 2F_{V4}(\phi^+_1, h^+_0) = \lambda_1 \phi^{+4}_1 + \frac{2\beta_1 - f_1}{2} h^+_0 \phi^{+2}_1 \]
\[ 2F_{V5}(\phi^0_1, h^+_0) = \lambda_1 \phi^{04}_1 + \frac{2\beta_1 + f_1}{2} h^+_0 \phi^{02}_1 \]

Stability conditions

\[ 2F_{V1} \rightarrow \lambda_1 > 0 \]
\[ 2F_{V2}, 2F_{V5} \rightarrow \lambda_1 > 0; \ 2\beta_1 + f_1 > 0 \]
\[ 2F_{V3}, 2F_{V4} \rightarrow \lambda_1 > 0; \ 2\beta_1 - f_1 > 0 \]

b. 3 Field Directions and Stability Conditions

\[ 3F_{V1}(\phi^0_1, \phi^+_1, h^+_R) = \lambda_1 \left( \phi^{02}_1 + \phi^{+2}_1 \right)^2 + h^+_R \left( \beta_1 (\phi^{02}_1 + \phi^{+2}_1) + \frac{1}{2} f_1 (\phi^{02}_1 - \phi^{+2}_1) \right) \]
\[ 3F_{V2}(\phi^0_1, \phi^+_1, h^+_R) = \lambda_1 \left( \phi^{02}_1 + \phi^{+2}_1 \right)^2 + h^+_R \left( \beta_1 (\phi^{02}_1 + \phi^{+2}_1) + \frac{1}{2} f_1 (\phi^{+2}_1 - \phi^{02}_1) \right) \]
\[ 3F_{V3}(\phi^0_1, h^+_0, h^+_R) = \frac{1}{2} \phi^{02}_1 \left( f_1 \left( h^+_R - h^+_0 \right) + 2\beta_1 \left( h^+_0 \phi^{02}_1 + h^+_R \phi^{+2}_1 \right) + 2\lambda_1 \phi^{02}_1 \right) \]
\[ 3F_{V3}(\phi^+_1, h^+_0, h^+_R) = \frac{1}{2} \phi^{+2}_1 \left( f_1 \left( h^+_R - h^+_0 \right) + 2\beta_1 \left( h^+_0 \phi^{+2}_1 + h^+_R \phi^{02}_1 \right) + 2\lambda_1 \phi^{+2}_1 \right) \]

Stability conditions

\[ 3F_{V1}, 3F_{V2}, 3F_{V3}, 3F_{V4} \rightarrow \lambda_1 > 0; \ 2\beta_1 + f_1 > 0; \ 2\beta_1 - f_1 > 0 \]
c. 4 Field Directions and Stability Conditions

\[ 4FV_1(\phi_1^0, \phi_1^+, h_R^0, h_R^+) = \frac{1}{2} \left( f_1 \left( h_R^+(\phi_1^0 - \phi_1^+) + h_R^0(\phi_1^0 + \phi_1^+) \right) \left( h_R^0(\phi_1^0 - \phi_1^+) - h_R^+(\phi_1^0 + \phi_1^+) \right) \right. \]
\[ \left. + 2(\phi_1^{0^2} + \phi_1^{+2}) \left( (h_R^{0^2} + h_R^{+2})\beta_1 + \lambda_1(\phi_1^{0^2} + \phi_1^{+2}) \right) \right) \]

Stability conditions

\[ 4FV_1 \rightarrow \lambda_1 > 0 ; \ 2\beta_1 + f_1 > 0 ; \ 2\beta_1 - f_1 > 0 \]

Appendix C: Renormalization Group Evolution Equations

1. Standard Model RGEs

For Standard Model we have used renormalization group evolution equations from [66] with matching conditions for top Yukawa and Higgs quartic coupling at their pole masses.

2. $U(1)_{B-L}$ Model

Gauge RG Equations

Renormalization group equations for $SU(3)_C$ and $SU(2)_L$ gauge couplings $g_3$ and $g_2$:

\[ 16\pi^2 \frac{d}{dt} g_3 = - \frac{1}{3} n_g \frac{g_3^3}{16\pi^2} \left[ -1 + 4 \frac{\beta_1}{3} \right] \]
\[ 16\pi^2 \frac{d}{dt} g_2 = - \frac{22}{3} n_g + \frac{1}{6} \frac{g_2^3}{16\pi^2} \left[ -1 + \frac{19}{6} \right] \]

where $n_g$ is number of generations.

Renormalization group equations for Abelian gauge couplings $g_1$, $g_{B-L}$ and $\tilde{g}$:

\[ 16\pi^2 \frac{d}{dt} g_1 = \left[ \frac{41}{6} g_1^3 \right] \]
\[ 16\pi^2 \frac{d}{dt} g_{B-L} = \left[ 12 \left( g_{B-L}^3 + \frac{32}{3} g_{B-L} \tilde{g} + \frac{41}{6} g_{B-L} \tilde{g}^2 \right) \right] \]
\[ 16\pi^2 \frac{d}{dt} \tilde{g} = \left[ \frac{41}{6} \tilde{g}(\tilde{g}^2 + 2g_1^2) + \frac{32}{3} g_{B-L}(\tilde{g}^2 + g_1^2) + 12 g_{B-L} \tilde{g} \right] \]
Fermion RG Equations

RG evolution equation for top quark Yukawa coupling $Y_t$

$$16\pi^2 \frac{d}{dt} Y_t = Y_t \left[ \frac{9}{2} Y_t^2 - 8 g_3^2 - \frac{9}{4} g_2^2 - \frac{17}{12} g_1^2 - \frac{17}{12} \tilde{g}^2 - \frac{2}{3} g_{B-L}^2 - \frac{5}{3} \tilde{g}_{B-L} \right]$$

In case of RH neutrinos RGEs we are considering degenerate RH neutrino Yukawa coupling and we are in a basis where these couplings are diagonal, then we have:

$$16\pi^2 \frac{d}{dt} y_i^h = y_i^h \left[ 4(y_i^h)^2 + 2 \text{Tr}[(y^h)^2] - 6 g_{B-L}^2 \right]$$

Scalar RG Equations

RGEs for the scalar couplings $\lambda_1$, $\lambda_2$ and $\lambda_3$ are:

$$16\pi^2 \frac{d}{dt} \lambda_1 = \left[ 24\lambda_1^2 + \lambda_3^2 - 6Y_t^4 + \frac{9}{8} g_3^4 + \frac{3}{8} g_2^4 + \frac{3}{4} g_2^2 g_1^2 + \frac{3}{4} g_2^2 \tilde{g}^2 + \frac{3}{4} g_1^2 \tilde{g}^2 + \frac{3}{8} g_4^4 + 12\lambda_1 Y_t^2 - 9\lambda_1 g_2^2 - 3\lambda_1 g_1^2 - 3\lambda_1 \tilde{g}_L^2 \right]$$

$$8\pi^2 \frac{d}{dt} \lambda_2 = \left[ 10\lambda_2^2 + \lambda_3^2 - \frac{1}{2} \text{Tr}[(y^h)^4] + 48 g_{B-L}^4 + 4\lambda_2 \text{Tr}[(y^h)^2] - 24\lambda_2 g_{B-L}^2 \right]$$

$$8\pi^2 \frac{d}{dt} \lambda_3 = \lambda_3 \left[ 6\lambda_1 + 4\lambda_2 + 2\lambda_3 + 3Y_t^2 - \frac{3}{4}(3g_2^2 - g_1^2 - \tilde{g}^2) + 2 \text{Tr}[(y^h)^2] - 12 g_{B-L}^2 \right] + 6g_2^2 g_{B-L}^2$$

3. LR Model with Triplet Scalars

Gauge RG Equations

$$16\pi^2 \frac{d}{dt} g_3 = g_3^3 \left( -\frac{7}{3} \right)$$

$$16\pi^2 \frac{d}{dt} g_2 = g_2^3 \left( -\frac{15}{6} \right)$$

$$16\pi^2 \frac{d}{dt} g_{B-L} = g_{B-L}^3 \left( \frac{28}{9} \right)$$

Note that in our case $g_{2R} = g_{2L} = g_2$. 

Fermion RG Equations

\[ 16\pi^2 \frac{d}{dt} Y_t = \left[ 8Y_t^3 - Y_t \left( \frac{2}{3}g_t^2 - \frac{9}{2}g_t^2 - 8g_t^2 \right) \right] \]

\[ 16\pi^2 \frac{d}{dt} Y^M_i = \left[ 2Y_i^M \left( - \frac{3}{4}g_t^2 - \frac{9}{4}g_t^2 \right) + 2Y_i^M \text{Tr}[(Y^M)^2] + 6(Y_i^M)^3 \right] \]

Scalar RG Equations

To write down scalar RG equations, we classified 15 scalar couplings into three categories depending on how they coupled with scalar fields.

- **Coefficients with \( \Phi^4 \)**

\[ 16\pi^2 \frac{d}{dt} \lambda_1 = 32\lambda_1^2 + \frac{5}{3}\lambda_{12}^2 + \frac{1}{2}\lambda_{13}^2 + 2\lambda_{14}^2 + 64\lambda_2^2 + 16\lambda_1\lambda_3 + 16\lambda_3^2 \]
\[ + 48\lambda_4^2 + 6\lambda_{12}\lambda_9 + 6\lambda_9^2 + 12\lambda_1Y_t^2 - 6Y_t^4 - 18\lambda_1g_t^2 + 3g_2^4 \]

\[ 16\pi^2 \frac{d}{dt} \lambda_2 = 6(\lambda_{10}^2 - \lambda_{11}^2) + \frac{3}{2}\lambda_{14}\lambda_{15} + 24\lambda_1\lambda_2 + 48\lambda_2\lambda_3 \]
\[ + 12\lambda_4^2 + 12\lambda_2Y_t^2 - 18\lambda_2g_2^2 \]

\[ 16\pi^2 \frac{d}{dt} \lambda_3 = 12(\lambda_{10}^2 + \lambda_{11}^2) - (\lambda_{12}^2 - \lambda_{13}^2) - \frac{1}{2}(\lambda_{14}^2 + \lambda_{15}^2) + 128\lambda_2^2 \]
\[ + 24\lambda_1\lambda_3 + 16\lambda_3^2 + 24\lambda_5^2 + 12\lambda_3Y_t^2 + 3Y_t^4 - 18\lambda_3g_2^2 + \frac{3}{2}g_2^2 \]

\[ 16\pi^2 \frac{d}{dt} \lambda_4 = 48\lambda_4(\lambda_1 + 2\lambda_2 + \lambda_3) + 6\lambda_{10}(2\lambda_9 + \lambda_{12}) \]
\[ + \frac{3}{2}\lambda_{13}(\lambda_{14} + \lambda_{15}) + 12\lambda_4Y_t^2 - 18\lambda_4g_2^2 \]
• Coefficients with $\Delta^4$

\[
16\pi^2 \frac{d}{dt} \lambda_5 = 28\lambda_5^2 + 16\lambda_6(\lambda_5 + \lambda_6) + 16(\lambda_{10}^2 + \lambda_{11}^2) + 2\lambda_{12}^2 + 3\lambda_7^2 \\
+ 4\lambda_9(\lambda_9 + \lambda_{12}) + 2\lambda_6 Y_t^2 - 16Y_t^4 - 12\lambda_5 g_{B-L}^2 \\
+ 6g_{B-L}^4 + 12g_{B-L}^2 g_2^2 - 24\lambda_5 g_2^2 + 9g_2^4
\]

\[
16\pi^2 \frac{d}{dt} \lambda_6 = 12\lambda_6(\lambda_6 + 2\lambda_5 - g_{B-L}^2 - 2g_2^2) + 12\lambda_8^2 \\
- \lambda_{12}^2 + 8Y_t^4 + 8\lambda_6 Y_t^2 - 12g_{B-L}^2 g_2^2 + 3g_2^4
\]

\[
16\pi^2 \frac{d}{dt} \lambda_7 = 4\lambda_7^2 + 16\lambda_7(2\lambda_5 + \lambda_6) + 32(\lambda_{10}^2 - \lambda_{11}^2) + 2(\lambda_{12}^2 + \lambda_{13}^2) \\
+ 4(\lambda_{14}^2 + \lambda_{15}^2) + 32\lambda_8^2 + 8\lambda_{12} \lambda_9 + \lambda_9^2 \\
+ 8\lambda_7 Y_t^2 - 12\lambda_7(g_{B-L}^2 + g_2^2) + 12g_{B-L}^4
\]

\[
16\pi^2 \frac{d}{dt} \lambda_8 = \lambda_{13}^2 + 4\lambda_{14} \lambda_{15} + 8\lambda_8(\lambda_5 + 5\lambda_6 + \lambda_7 + Y_t^2) - 12\lambda_8(2g_{B-L}^2 + g_2^2)
\]
• Coefficients with $\Phi^2 \Delta^2$

\[
16\pi^2 \frac{d}{dt} \lambda_9 = \lambda_9 \left( 20\lambda_1 + 8\lambda_3 + 16\lambda_5 + 8\lambda_6 + 6\lambda_7 + 4\lambda_9 + 6Y_\lambda^2 + 4\text{Tr}[(Y^M)^2] - 6g_{B-L}^2 - 21g_2^2 \right) \\
+ 6g_2^4 + 16(\lambda_1^2 + \lambda_2^2) + \lambda_2(8\lambda_1 + \lambda_2) + 3\lambda_{12}^2 + 12\lambda_{14}^2 \\
+ 8\lambda_{12}\lambda_3 + 48\lambda_{10}\lambda_4 + \lambda_2(6\lambda_5 + 8\lambda_6 + 3\lambda_7)
\]

\[
16\pi^2 \frac{d}{dt} \lambda_{10} = \lambda_{10} \left( 4\lambda_1 + 4\lambda_2 + 48\lambda_2 + 16\lambda_3 + 16\lambda_4 + 16\lambda_5 + 8\lambda_6 + 6\lambda_7 + 8\lambda_9 \\
+ 6Y_\lambda^2 + 4\text{Tr}[(Y^M)^2] - 6g_{B-L}^2 - 21g_2^2 \right) - 3\lambda_{13}(\lambda_{14} + \lambda_{15}) + 12\lambda_4\lambda_9
\]

\[
16\pi^2 \frac{d}{dt} \lambda_{11} = \lambda_{11} \left( 4\lambda_1 + 4\lambda_2 - 48\lambda_2 + 16\lambda_3 + 16\lambda_5 + 8\lambda_6 - 6\lambda_7 \\
+ 8\lambda_9 + 6Y_\lambda^2 + 4\text{Tr}[(Y^M)^2] - 6g_{B-L}^2 - 21g_2^2 \right)
\]

\[
16\pi^2 \frac{d}{dt} \lambda_{12} = \lambda_{12} \left( 4\lambda_1 + 4\lambda_2 - 8\lambda_3 + 4\lambda_5 - 8\lambda_6 + 8\lambda_9 + 6Y_\lambda^2 \\
+ 4\text{Tr}[(Y^M)^2] - 6g_{B-L}^2 - 21g_2^2 \right) - 12(\lambda_{14}^2 - \lambda_{15}^2)
\]

\[
16\pi^2 \frac{d}{dt} \lambda_{13} = \lambda_{13} \left( 4\lambda_1 + 4\lambda_2 + 8\lambda_3 + 2\lambda_7 + 8\lambda_8 + 8\lambda_9 + 3Y_\lambda^2 + \text{Tr}[(Y^M)^2] \\
- 6g_{B-L}^2 - 21g_2^2 \right) + (8\lambda_4 + 16\lambda_{10})(\lambda_{14} + \lambda_{15})
\]

\[
16\pi^2 \frac{d}{dt} \lambda_{14} = \lambda_{14} \left( 4\lambda_1 - 4\lambda_2 + 2\lambda_7 + 8\lambda_9 + 6Y_\lambda^2 + 4\text{Tr}[(Y^M)^2] - 6g_{B-L}^2 - 21g_2^2 \right) \\
+ 4\lambda_{13}(\lambda_4 + 2\lambda_{10}) + 8\lambda_{15}(2\lambda_2 + \lambda_8)
\]

\[
16\pi^2 \frac{d}{dt} \lambda_{15} = \lambda_{15} \left( 4\lambda_1 + 12\lambda_2 + 2\lambda_7 + 8\lambda_9 + 6Y_\lambda^2 + 4\text{Tr}[(Y^M)^2] - 6g_{B-L}^2 - 21g_2^2 \right) \\
+ 4\lambda_{13}(\lambda_4 + 4\lambda_{10}) + 8\lambda_{14}(2\lambda_2 + \lambda_8)
\]

4. LR Model with Doublet Scalars

**Gauge RG Equations**

\[
16\pi^2 \frac{d}{dt} g_3 = g_3^3 \left( -7 \right)
\]

\[
16\pi^2 \frac{d}{dt} g_2 = g_2^3 \left( -\frac{17}{6} \right)
\]

\[
16\pi^2 \frac{d}{dt} g_{B-L} = g_{B-L}^3 \left( 3 \right)
\]

Note that in our case $g_{2L} = g_{2R} = g_2$. 
Fermion RG Equations

\[ 64\pi^2 \frac{d}{dt} Y_t = \left( -\frac{2}{9} g_{B-L}^2 - 9 g_2^2 - 32 g_3^2 \right) Y_t + 7 Y_t^3 \]

Scalar RG Equations

- **Coefficients with \( \Phi^4 \)**

\[
128\pi^2 \frac{d}{dt} \lambda_1 = \lambda_1 \left( -72 g_2^2 + 256 (\lambda_1 + \lambda_2 - \lambda_3) + 24 Y_t^2 \right) \\
+ \quad 1024 (\lambda_1^2 + \lambda_2^2) + 32 \beta_1^2 + 8 f_1^2 + 9 g_2^4 - 12 Y - Y_t^4 \\
512\pi^2 \frac{d}{dt} \lambda_2 = \lambda_2 \left( -288 g_2^2 + 768 \lambda_1 + 3072 \lambda_2 + 1024 \lambda_3 + 96 Y_t^2 \right) - 8 f_1^2 + 3 g_2^4 - 3 Y_t^4 \\
256\pi^2 \frac{d}{dt} \lambda_3 = \lambda_3 \left( -144 g_2^2 - 384 \lambda_1 - 512 \lambda_2 - 1536 \lambda_3 + 48 Y_t^2 \right) + 4 f_1^2 - 3 g_2^4 - 3 Y_t^4
\]

- **Coefficients with \( H_L^4 \)**

\[
512\pi^2 \frac{d}{dt} \kappa_1 = \kappa_1 \left( -96 g_{B-L}^2 - 144 g_2^2 + 576 \kappa_1 + 384 \kappa_2 \right) \\
+ \quad 192 \kappa_2^2 + 256 \beta_1^2 + 128 f_1^2 + 24 g_{B-L}^2 + 12 g_{B-L}^2 g_2^2 + 9 g_2^4 \\
512\pi^2 \frac{d}{dt} \kappa_2 = \kappa_2 \left( -96 g_{B-L}^2 - 144 g_2^2 + 512 \kappa_1 + 384 \kappa_2 \right) + 128 f_1^2 + 12 g_{B-L}^2 g_2^2 + 9 g_2^4
\]

- **Coefficients with \( \Phi^2 H_L^2 \)**

\[
256\pi^2 \frac{d}{dt} \beta_1 = -4 \beta_1 \left[ -8 \beta_1 + 6 g_{B-L}^2 + 27 g_2^2 - 2 (20 \kappa_1 + 4 \kappa_2 + 40 \lambda_1 + 32 \lambda_2 - 32 \lambda_3 + 3 Y_t^2) \right] \\
+ 24 f_1^2 + 9 g_2^4 \\
256\pi^2 \frac{d}{dt} f_1 = f_1 \left( 16 \beta_1 - 6 g_{B-L}^2 - 27 g_2^2 + 8 (\kappa_1 + \kappa_2) + 16 (\lambda_1 - 4 \lambda_2) + 64 \lambda_3 + 6 Y_t^2 \right)
\]

[1] G. Aad et al. (ATLAS Collaboration), Phys.Lett. **B716**, 1 (2012), 1207.7214.
[2] S. Chatrchyan et al. (CMS Collaboration), Phys.Lett. B716, 30 (2012), 1207.7235.
[3] G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, et al., JHEP 1208, 098 (2012), 1205.6497.
[4] S. Alekhin, A. Djouadi, and S. Moch, Phys.Lett. B716, 214 (2012), 1207.0980.
[5] I. Masina, Phys.Rev. D87, 053001 (2013), 1209.0393.
[6] P. Minkowski, Phys.Lett. B67, 421 (1977).
[7] T. Yanagida, Conf.Proc. C7902131, 95 (1979).
[8] M. Gell-Mann, P. Ramond, and R. Slansky, Conf.Proc. C790927, 315 (1979), 1306.4669.
[9] S. Glashow, NATO Adv.Study Inst.Ser.B Phys. 59, 687 (1980).
[10] R. N. Mohapatra and G. Senjanovic, Phys.Rev.Lett. 44, 912 (1980).
[11] M. Magg and C. Wetterich, Phys.Lett. B94, 61 (1980).
[12] G. Lazarides, Q. Shafi, and C. Wetterich, Nucl.Phys. B181, 287 (1981).
[13] R. N. Mohapatra and G. Senjanovic, Phys.Rev. D23, 165 (1981).
[14] J. Schechter and J. Valle, Phys.Rev. D25, 774 (1982).
[15] R. Foot, H. Lew, X. He, and G. C. Joshi, Z.Phys. C44, 441 (1989).
[16] R. Adhikari and A. Raychaudhuri, Phys.Rev. D84, 033002 (2011), 1004.5111.
[17] K. Babu, S. Nandi, and Z. Tavartkiladze, Phys.Rev. D80, 071702 (2009), 0905.2710.
[18] F. Bonnet, D. Hernandez, T. Ota, and W. Winter, JHEP 0910, 076 (2009), 0907.3143.
[19] S. Kanemura and T. Ota, Phys.Lett. B694, 233 (2010), 1009.3845.
[20] G. Bambhaniya, J. Chakrabortty, S. Goswami, and P. Konar (2013), 1305.2795.
[21] F. del Aguila, M. Chala, A. Santamaria, and J. Wudka (2013), 1305.3904.
[22] C.-S. Chen and Y. Tang, JHEP 1204, 019 (2012), 1202.5717.
[23] W. Rodejohann and H. Zhang, JHEP 1206, 022 (2012), 1203.3825.
[24] J. Chakrabortty, M. Das, and S. Mohanty, Mod.Phys.Lett. A28, 1350032 (2013), 1207.2027.
[25] E. J. Chun, H. M. Lee, and P. Sharma, JHEP 1211, 106 (2012), 1209.1303.
[26] S. Khan, S. Goswami, and S. Roy (2012), 1212.3694.
[27] W. Chao, J.-H. Zhang, and Y. Zhang, JHEP 1306, 039 (2013), 1212.6272.
[28] W. Chao, M. Gonderinger, and M. J. Ramsey-Musolf, Phys.Rev. D86, 113017 (2012), 1210.0491.
[29] A. Kobakhidze and A. Spencer-Smith, JHEP 1308, 036 (2013), 1305.7283.
[30] A. Arhrib, R. Benbrik, M. Chabab, G. Moultaqa, M. Peyranere, et al., Phys.Rev. D84, 095005
(2011), 1105.1925.

[31] R. Marshak and R. N. Mohapatra, Phys.Lett. B91, 222 (1980).
[32] J. Chakrabortty and A. Raychaudhuri, Phys.Rev. D81, 055004 (2010), 0909.3905.
[33] S. Iso, N. Okada, and Y. Orikasa, Phys.Rev. D80, 115007 (2009), 0909.0128.
[34] S. Khalil, Phys.Rev. D82, 077702 (2010), 1004.0013.
[35] J. Chakrabortty and A. Raychaudhuri, Phys.Rev. D81, 055018 (2010), 1004.3039.
[36] J. C. Pati and A. Salam, Phys.Rev. D10, 275 (1974).
[37] R. N. Mohapatra and J. C. Pati, Phys.Rev. D11, 566 (1975).
[38] R. Mohapatra and J. C. Pati, Phys.Rev. D11, 2558 (1975).
[39] G. Senjanovic and R. N. Mohapatra, Phys.Rev. D12, 1502 (1975).
[40] G. Senjanovic, Nucl.Phys. B153, 334 (1979).
[41] B. Brahmachari, E. Ma, and U. Sarkar, Phys.Rev.Lett. 91, 011801 (2003), hep-ph/0301041.
[42] U. Sarkar, Phys.Lett. B594, 308 (2004), hep-ph/0403276.
[43] P. Duka, J. Gluza, and M. Zralek, Annals Phys. 280, 336 (2000), hep-ph/9910279.
[44] M. Czakon, M. Zralek, and J. Gluza, Nucl.Phys. B573, 57 (2000), hep-ph/9906356.
[45] A. Maiezza, M. Nemevsek, F. Nesti, and G. Senjanovic, Phys.Rev. D82, 055022 (2010), 1005.5160.
[46] N. Deshpande, J. Gunion, B. Kayser, and F. I. Olness, Phys.Rev. D44, 837 (1991).
[47] P. Ball, J. Frere, and J. Matias, Nucl.Phys. B572, 3 (2000), hep-ph/9910211.
[48] G. Barenboim, M. Gorbahn, U. Nierste, and M. Raidal, Phys.Rev. D65, 095003 (2002), hep-ph/0107121.
[49] J.-Y. Liu, L.-M. Wang, Y.-L. Wu, and Y.-F. Zhou, Phys.Rev. D86, 015007 (2012), 1205.5676.
[50] M. Holthausen, M. Lindner, and M. A. Schmidt, Phys.Rev. D82, 055002 (2010), 0911.0710.
[51] M. Malinsky, J. Romao, and J. Valle, Phys.Rev.Lett. 95, 161801 (2005), hep-ph/0506296.
[52] J. Chakrabortty, Phys.Lett. B690, 382 (2010), 1005.1377.
[53] B. Holdom, Phys.Lett. B166, 196 (1986).
[54] F. del Aguila, G. Coughlan, and M. Quiros, Nucl.Phys. B307, 633 (1988).
[55] G. Aad et al. (ATLAS Collaboration), Phys.Rev.Lett. 107, 272002 (2011), 1108.1582.
[56] G. Beall, M. Bander, and A. Soni, Phys.Rev.Lett. 48, 848 (1982).
[57] P. Langacker and S. U. Sankar, Phys.Rev. D40, 1569 (1989).
[58] M. Czakon, J. Gluza, and J. Hejczyk, Nucl.Phys. B642, 157 (2002), hep-ph/0205303.
[59] J. Chakrabortty, J. Gluza, R. Sevillano, and R. Szafron, JHEP 1207, 038 (2012), 1204.0736.
[60] M. Nemevsek, F. Nesti, G. Senjanovic, and Y. Zhang, Phys.Rev. D83, 115014 (2011), 1103.1627.
[61] A. Ferrari, J. Collot, M.-L. Andrieux, B. Belhorma, P. de Saintignon, et al., Phys.Rev. D62, 013001 (2000).
[62] S. Chatrchyan et al. (CMS Collaboration), Phys.Lett. B720, 63 (2013), 1212.6175.
[63] G. Aad et al. (ATLAS Collaboration), JHEP 1211, 138 (2012), 1209.2535.
[64] I. Rothstein, Nucl.Phys. B358, 181 (1991).
[65] A. Datta, A. Elsayed, S. Khalil, and A. Moursy (2013), 1308.0816.
[66] M. Holthausen, K. S. Lim, and M. Lindner, JHEP 1202, 037 (2012), 1112.2415.