Topological phase separation in trapped ultracold fermionic gases

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Abstract

We consider harmonically trapped two-dimensional fermionic systems with an effective spin–orbit coupling and intrinsic s-wave superfluidity under the local density approximation. We find that, when the Zeeman field is above a critical value, there is a ubiquitous phenomenon, topological phase separation, in the weakly trapped systems. The topological phase separation is formed between the superfluid phases with different topological numbers in real space. When the Zeeman field is below the critical value, the superfluid phase is topologically trivial and the spatial topological phase separation vanishes.

(Some figures may appear in colour only in the online journal)

Introduction

Recently, topological insulators (TIs) and topological superconductors (TSCs)/superfluid (TS), which are described by topological order [1] instead of the traditional Landau symmetry breaking order, have attracted great attention in condensed matter physics [2–8]. TIs are bulk insulating states, which nevertheless have stable gapless boundary states. As analogues of TIs [9–12], TSCs have gapped superconductive bulk and stable gapless boundary Majorana modes protected by the symmetry of bulk for the bulk–edge correspondence. The topologically distinct phases with the same symmetry cannot be connected to each other without a topological phase transition (TPT). The realizations of TSC/TS have been proposed in large spin–orbit coupling (SOC) semiconductors with a proximity-induced s-wave pairing potential and ultracold atomic gases with intrinsic superfluidity in the presence of a Zeeman field and an effective SOC.

Ultracold atomic gases with remarkable controllability provide an ideal stage to investigate the phenomenon in condensed matter physics [13–20]. The effective SOC, which is essential for the realization of different TIs and TSC/TSs, can be generated by utilizing the spatial varying laser fields [21–23] and has been realized in experiment [24, 25]. The topological properties of the superconductors/superfluids were investigated in [5, 26–28]. The SOC can change the ground state of the boson condensate [29, 30], too.

In this paper, we investigate the topological properties of two-dimensional (2D) s-wave superfluid trapped in a harmonic potential with SOC under the local-density approximation (LDA). The superfluid phase is topologically trivial in the absence of a Zeeman field. An effective Zeeman field, induced by spin imbalance and breaking the time-reverse symmetry, is needed to realize the topologically nontrivial phases. There is a TPT when the Zeeman field crosses the critical value. The topologically nontrivial phases emerge in the trapped region when the Zeeman field is larger than the critical value. In the presence of the harmonic potential, the chemical potential and the pairing gap are functions of the spatial coordinate. The topological properties of the superfluidity are also changing with the coordinate. It has been shown that trapped Fermi gases with spin imbalance and without SOC can exhibit spatial phase separation [15, 20, 31–33] which is the coexistence of the topologically trivial superfluid phase and the normal phase in real space. Analogous to the case without SOC, we find that there is a spatial topological phase separation (TPS) phenomenon, which is the coexistence of superfluid phases with different topological numbers in the trapped region. The gapless Majorana edge states are localized at the boundaries between the topologically superfluid (TS) phases and the normally superfluid (NS) phases in the trapped region.

This paper is organized as follows. In section 2, after introducing the Hamiltonian of the model, we derive the energy gap closing conditions and give the Thouless–Kohmoto–Nightingale–den Nijs (TKNN) number. In section 3, we
investigate the topological properties of the superfluid phases
and find that there exist TPS phenomena in the trapped region.
We also extend the analysis to the continuum model. The
conclusions are given in section 4.

Formalism of the system

We consider spin-singlet superfluidity with the Rashba SOC
on a 2D square optical lattice in the harmonic potential, which
is described by the Hamiltonian ($\hbar = k_B = 1$)

$$H = H_0 + H_{SO} + H_c,$$

(1)

where $H_0$ is the kinetic term with a Zeeman field, $H_{SO}$ is the
spin–orbit interaction term and $H_c$ is the pairing potential term.

They take the forms

$$H_0 = -\frac{i}{2} \sum_{\mathbf{r}_i} \sum_{\sigma} \left( c_{\mathbf{r}_i \sigma}^\dagger \sigma + h.c. \right),$$

$$H_{SO} = -\lambda \sum_{\mathbf{r}_i} \left[ (c_{\mathbf{r}_i \uparrow}^\dagger c_{\mathbf{r}_i \downarrow} - c_{\mathbf{r}_i \downarrow}^\dagger c_{\mathbf{r}_i \uparrow}) + i(c_{\mathbf{r}_i \uparrow}^\dagger c_{\mathbf{r}_i \downarrow} - c_{\mathbf{r}_i \downarrow}^\dagger c_{\mathbf{r}_i \uparrow}) + h.c. \right],$$

$$H_c = -\sum_{\mathbf{r}_i} \Delta_1 (c_{\mathbf{r}_i \uparrow}^\dagger c_{\mathbf{r}_i \downarrow} + h.c.),$$

where $c_{\mathbf{r}_i \sigma}$ ($c_{\mathbf{r}_i \sigma}^\dagger$) denotes the creation (annihilation)
operator of the fermion with spin $\sigma = \uparrow, \downarrow$ at site $\mathbf{r}_i = (i_x, i_y)$, $\lambda$
is the strength of SOC and $\Delta_1$ is the pairing potential of the
superfluid at site $\mathbf{r}_i$, $\mu_i = \mu - V_i$ is the local chemical potential,
with $\mu$ is the global chemical potential and $V_i = m_o \omega^2 r_i^2 / 2$
the harmonic potential at position $r_i$, such that the trapping
centre is located at $\mathbf{r}_c = (i_x = 0, i_y = 0)$. $h$ is a Zeeman
field. Introducing $\Psi_{\mathbf{k}} = (c_{\mathbf{k} \uparrow}^\dagger, c_{\mathbf{k} \downarrow}^\dagger, c_{-\mathbf{k} \uparrow}, c_{-\mathbf{k} \downarrow})$ with $c_{\mathbf{k}} = \sum_i e^{i\mathbf{k} \cdot \mathbf{r}_i}$, the Hamiltonian can be rewritten as

$$H = \frac{1}{2} \sum_{\mathbf{k}} \Psi_{\mathbf{k}}^\dagger \mathcal{H}(\mathbf{k}) \Psi_{\mathbf{k}}.$$

with

$$\mathcal{H}(\mathbf{k}) = \begin{pmatrix}
\epsilon(\mathbf{k}) - h (\sigma_z + \mathbf{g}_K \cdot \sigma) & i \Delta_1 \sigma_y \\
-i \Delta_1^\dagger \sigma_y & -\epsilon(\mathbf{k}) + h (\sigma_z + \mathbf{g}_K \cdot \sigma)^* \end{pmatrix},$$

(6)

where $\sigma = (\sigma_x, \sigma_y)$ are Pauli matrices, $\epsilon(\mathbf{k}) = -2\lambda (\cos k_x + \cos k_y) - \mu_i$ and $\mathbf{g}_K = 2\lambda (\sin k_y, -\sin k_x)$. Diagonalizing the Hamiltonian, we find the eigenvalues $E(\mathbf{k})$ of the Hamiltonian as

$$E_{\pm}(\mathbf{k})^2 = \epsilon(\mathbf{k})^2 + h^2 + |\mathbf{g}_K|^2 + |\Delta_1|^2 \pm 2 \sqrt{h^2 (\epsilon(\mathbf{k})^2 + |\Delta_1|^2) + \epsilon(\mathbf{k})^2 |\mathbf{g}_K|^2}. $$

(7)

The pairing potential and the global chemical potential
can be tuned by the inter-species interaction via the Feshbach
resonance and the filling number. The pairing potential is a
smooth function of the position coordinate. We consider the
Zeeman field $h \geq 0$. In the presence of pairing ($\Delta_1 \neq 0$), the
spectrum of our model is gapless only when $|\mathbf{g}_K| = 0$ at four
momenta $\mathbf{k}_0 = (0, 0), (0, \pi), (\pi, 0), (\pi, \pi)$, and the Zeeman
magnetic field must satisfy $h = \sqrt{\epsilon(\mathbf{k}_0)^2 + |\Delta_1|^2}$ which can be written as

$$\begin{cases}
h_1 = \sqrt{(\lambda + \mu_i)^2 + |\Delta_1|^2}, & \mathbf{k}_0 = (0, 0) \\
h_2 = \mu_i^2 + |\Delta_1|^2, & \mathbf{k}_0 = \mathcal{P}(\pi, 0) \\
h_3 = \sqrt{(\lambda - \mu_i)^2 + |\Delta_1|^2}, & \mathbf{k}_0 = (\pi, \pi),
\end{cases}$$

(8)

where $\mathcal{P}(\pi, 0) = [(\pi, 0), (0, \pi)]$. The TPT boundaries
depend on the SOC strength via the paring potential.

This Bogoliubov–de Gennes (BdG) Hamiltonian has particle–hole symmetry (PHS)

$$\mathcal{H}(\mathbf{k}) = -i_\mathbf{\Gamma} \mathcal{H}^T(\mathbf{k}) i_\mathbf{\Gamma},$$

(9)

where $i_{\mathbf{\Gamma}}$ is the gamma matrix. The classification of time-reversal breaking (TRB) BdG Hamiltonian with PHS is $\mathbb{Z}$
class [6]. The topological numbers that characterize the
topological properties of the superfluid phases are integers. The Hamiltonian has four energy bands, of which only two
can be gapless and others are always gapped. Only the gapless
energy bands contribute to the change of topological numbers
across the phase boundary. We consider the s-wave pairing
with a real pairing gap function $\Delta_1^\dagger = \Delta_1$. Then, we
can deform the Hamiltonian $\mathcal{H}$ into a dual Hamiltonian $\mathcal{H}^D$
by a unitary transformation without closing the energy gap to
investigate the topological properties of the superfluid phase:

$$\mathcal{H}^D(\mathbf{k}) = D \mathcal{H}(\mathbf{k}) D^\dagger,$$

(10)

where the dual Hamiltonian and the unitary matrix $D$ is given as

$$\mathcal{H}^D(\mathbf{k}) = \begin{pmatrix}
\Delta_1 - h \sigma_z & -i \epsilon(\mathbf{k}) \sigma_y - i \mathbf{g}_K \cdot \sigma \sigma_y \\
i \epsilon(\mathbf{k}) \sigma_y + i \mathbf{g}_K \sigma_y \cdot \sigma & \Delta_1 + h \sigma_z
\end{pmatrix},$$

(11)

and

$$D = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \sigma_y \\
i \sigma_y & 1 \end{pmatrix}. $$

(12)

We can adiabatically deform the Hamiltonian by taking $\epsilon(\mathbf{k}) = 0$ without closing the energy gap, such that the dual Hamiltonian is decomposed into the following two $2 \times 2$ parts for dual up-spin and down-spin:

$$\mathcal{H}^D_{\uparrow \uparrow}(\mathbf{k}) = \begin{pmatrix}
\Delta_1 - h & 2\lambda (\sin k_x + i \sin k_y) \\
2\lambda (\sin k_y - i \sin k_x) & -\Delta_1 + h
\end{pmatrix},$$

(13)

$$\mathcal{H}^D_{\downarrow \downarrow}(\mathbf{k}) = \begin{pmatrix}
\Delta_1 + h & 2\lambda (\sin k_x - i \sin k_y) \\
2\lambda (\sin k_y + i \sin k_x) & -\Delta_1 - h
\end{pmatrix}.$$

(14)

When the Zeeman field is positive ($h > 0$), the energy gap
of the dual up-spin can vanish while the down-spin is always
gapped, and when the Zeeman field is negative ($h < 0$), the
converse happens. For the simplification, we consider only the $h > 0$ case and rewrite the Hamiltonian of the dual up-spin as

$$H^{(1)}_{\uparrow\uparrow}(k) = d_\uparrow(k) \cdot \sigma,$$ (15)

with $d_\uparrow(k) = (2\sin k_x, -2\sin k_y, \Delta - h)$. 

Due to the harmonic potential, the topological properties of the superfluid phases change with the distance away from the trapping centre as well as the BdG Hamiltonian. When the harmonic potential is weak, the LDA is reliable. In order to characterize the topological properties of the phases in the trapped region, we consider the TKNN number $I_{\text{TKNN}}$ [3] or the first Chern number

$$I_{\text{TKNN}} = \frac{1}{4\pi} \int dk \hat{d}_\uparrow(k) \cdot \left( \frac{\partial \hat{d}_\uparrow(k)}{\partial k_x} \times \frac{\partial \hat{d}_\uparrow(k)}{\partial k_y} \right),$$ (16)

with $\hat{d}_\uparrow(k) = d_\uparrow(k)/|d_\uparrow(k)|$. For the TRB system, the TKNN number is equal to zero for the NS phase and nonzero for the TS phase. Considering the Hamiltonian with a positive tunable number is equal to zero for the NS phase and nonzero for the PHS; the gapless edge states are chiral Majorana fermion modes. The number of gapless edge states is defined by the TKNN number $I_{\text{TKNN}} = 0$ in the absence of a Zeeman field. So we only need to calculate the change in $I_{\text{TKNN}}(h)$ at critical points which are $h = \Delta_1$ at the momenta $k = (0, 0), P[(\pi, 0), (\pi, \pi)]$. We must keep in mind that the different momenta $k_c$ indicate the different adiabatic deformation $\epsilon(k_c) = 0$, such that the three critical momenta correspond to the three gapless conditions.

Around the critical momentum $k_c = (0, 0)$, the vector $d\uparrow(k)$ has an approximate form $d\uparrow(k) = (2\lambda k_x, -2\lambda k_y, \Delta_1 - h)$. Taking a cutoff $\Lambda < \pi$ and in the limit $|\delta h| < \Delta_1(|\delta h| = h_\text{tr} - \Delta_1)$, the TKNN number expression can be divided into low and high energy parts,

$$I_{\text{TKNN}} = \frac{1}{4\pi} \int dk \frac{d\uparrow(k)}{d_\uparrow(k)} \cdot \left( \frac{\partial \hat{d}_\uparrow(k)}{\partial k_x} \times \frac{\partial \hat{d}_\uparrow(k)}{\partial k_y} \right)$$

$$= -\frac{1}{\pi} \left( \int_{|k| < \Lambda} dk + \int_{|k| > \Lambda} dk \right)$$

$$\times \left( 4\lambda^2 \sin^2 k_x + \sin^2 k_y + \delta h \right)^2$$

$$= I_{\text{TKNN}}^{(1)}(\delta h, \Lambda) + I_{\text{TKNN}}^{(2)}(\delta h, \Lambda).$$ (17)

Since there is no gapless point in the high momentum region $|k| > \Lambda$, the change in $I_{\text{TKNN}}$ only comes from the low energy part $I_{\text{TKNN}}^{(1)}$. The change in $I_{\text{TKNN}}$ can be calculated at critical momentum $k_c = (0, 0)$,

$$\Delta I_{\text{TKNN}}^{(1)}(\delta h = 0^+) = -1.$$ (18)

Similarly, the change in the TKNN number at other critical points can be obtained as

$$\Delta I_{\text{TKNN}}^{(1)}(\delta h = 0^+) = \begin{cases} -1, & k_c = (0, 0), \\ 2, & k_c = P[(0, \pi)], \\ -1, & k_c = (\pi, \pi) \end{cases}.$$ (19)

**Topological phase separation**

In the presence of the harmonically trapping potential, the pairing gap is a function of the distance away from the trapping centre which can be self-consistently determined with the global chemical potential via the minimization of the free energy. By using the change in $I_{\text{TKNN}}$ at the critical points, we can study the topological properties of the superfluid phase in the trapped region. The change in $I_{\text{TKNN}}$ at the boundaries can be listed as below:

$$\Delta I_{\text{TKNN}}^{(1)}(\delta h = 0^+) = \begin{cases} -1, & h = h_1, \\ 2, & h = h_2, \\ -1, & h = h_3 \end{cases}.$$ (20)

In the absence of the Zeeman field, the TKNN number vanishes such that the superfluid phase is topologically trivial in the entire trapped region. As the Zeeman field increases, the TKNN number changes into nonzero at the critical Zeeman field $h_c = \min(h_1, h_2, h_3)$. The topologically nontrivial phase emerges in the trapped region. There is a TPT when the Zeeman field crosses the critical value. When the Zeeman field is larger than the critical value, the topologically trivial and nontrivial superfluid phases coexist in the trapped region. This spatial TPS is very different from the phase separation phenomenon [33] in the absence of the SOC case. The phase separation, which is the coexistence of the topologically trivial and nontrivial superfluid phases, can be described by the traditional Landau symmetry breaking order. The TPS phenomenon, described by the topological order instead of the traditional Landau symmetry breaking order, is the coexistence of superfluid phases with different topological numbers.

The boundaries between the superfluid phases with different topological numbers are localized in the trapped region. From the bulk–edge correspondence, a topologically nontrivial bulk guarantees the existence of topologically stable gapless edge states on the boundaries. This 2D superfluid system breaks the time-reversal symmetry but preserves the PHS; the gapless edge states are chiral Majorana fermion modes. The number of gapless edge states is defined by the TKNN number $I_{\text{TKNN}}$. For the odd $I_{\text{TKNN}}$ phase, there are odd Majorana fermions in each vortex. A Majorana fermion can be seen as half of a Dirac fermion. Therefore, the vortices entangled with each other obey non-Abelian statistics. This phase has applications in topological quantum computation. For the even $I_{\text{TKNN}}$ phase, vortices have even Majorana fermions and are not entangled with each other. There are no non-Abelian anyons.

To capture the various phases in the trap, we investigate the case without trap and determine the phase diagrams in the $\lambda$–$\mu$ plane. A downward vertical line in the $\lambda$–$\mu$ plane represents the trajectory from the trap centre to the edge under the LDA. The global chemical potential is fixed at the starting point of the vertical line. At $T = 0$, the thermodynamic potential of the case without trap can be written as

$$\Omega = \frac{1}{2} \sum_k [2\epsilon(k) - E_+(k) - E_-(k)] + |\Delta|^2/V,$$ (21)

where $\epsilon(k)$ and $E_{\pm}(k)$ can be obtained from equation (7) by replacing $\mu_1$ by $\mu$ and $\Delta_1$ by $\Delta$ and $V$ is the strength of contact interaction between the opposite spins. The thermodynamic potential then satisfies $\Omega(k, \mu) = \Omega(\pi, k, -\mu)$, such that
the superfluid phases in the \(\lambda-\mu\) plane phase diagram have a symmetry through the axis \(\mu = 0\). The change in the TKNN number can be written as equation (20) with \(\mu_i = \mu\) and \(\Delta_i = \Delta\). The phase diagrams can be determined by minimizing the thermodynamic potential, equation (21), in various conditions.

Figures 1(a) and (b) show the phase diagrams in the \(\lambda-\mu\) plane for \(V/h = 4.6\) with different \(t\). Figures 1(c) and (d) show the pairing potential as a function of the chemical potential for figures 1(a) and (b), respectively, with \(\lambda k_0/h = 1.2\). In figures 1(a) and (b), the dashed curves are plotted by \(h = h_1\) and \(h = h_2\) while the dotted curves are plotted by \(h = h_3\) with \(\mu_i = \mu\) and \(\Delta_i = \Delta\). Therefore, the TKNN number changes by 1 along the dashed curves while by 2 along the dotted curves. From figures 1(c) and (d), the sign of \(\delta\mu(\delta h = h - \Delta)\) at the sides of the boundaries in figures 1(a) and (b) can be determined. Therefore, the TKNN number of the topological superfluid phases can be determined. There are three different topological superfluid phases in the phase diagrams. The TKNN numbers of the topological superfluid phases are \(\pm 1\) for the slightly shaded regions and 2 for the darkly shaded region. The blue solid curves represent first-order phase boundaries in which the thermodynamic potential has two degenerate local minima. When the strength of the SOC is small, the superfluid phase is topologically trivial. The topologically nontrivial superfluid phases only exist in the large SOC region. As mentioned above, a downward vertical line in figure 1 reveals the structure of the phase separation in the trapped region. When the SOC is small, the phase separation is topologically trivial (NS–N) in the trapped region. The phase separation can be topologically nontrivial as the SOC increases. When the SOC is large enough, there exist various shell structures of topologically nontrivial phase separation in the trapped region like NS–TS, TS–N, NS–TS–N, NS–TS–NS and so on.

The variation of the pairing potential is smooth in the vicinity of the topological phase boundary in the trapped region. To illustrate the TPS phenomenon visualization, we can also assume the pairing potential to be independent of the distance away from the trapping centre \((\Delta_1 = \Delta_s)\) (this assumption, without changing the topological properties of the superfluid phases, is just to simplify the self-consistent calculation and find the TPS in the trapped region) and the harmonic potential is \(V_1 = m\omega^2 r_1^2/2 = 0.1 r_1^2\), where \(r_1\) is in the unit of the lattice space.

We can obtain the phase diagrams in the \(h-r\) plane with the s-wave pairing gap \(\Delta_s = t\) for the different global chemical potential, shown in figure 2.

From figure 2, we find that the Zeeman field has a critical value at \(h = t\). When the Zeeman field is \(h < t\), the superfluid phase is topologically trivial in the entire trapped region. As the Zeeman field increases, there is a TPT at \(h/t = 1\). When the Zeeman field is \(h > t\), the TS phases emerge and coexist with the NS phases in the trapped region. The superfluid phases are topologically nontrivial for the shaded regions while the others are topologically trivial. When the global chemical potential is \(\mu = -2t\), as shown in figure 2(a), there is a TPS phenomenon in the trapped region for \(h > t\). The TS circle is immersed in the NS disc for the Zeeman fields \(1 < h/t < 2\) and \(h/t > 6\). The phase diagram in the real space is NS–TS–NS as shown in figure 3(a). When the Zeeman field is \(2 < h/t < 6\), the TS disc is surrounded by the NS circle shown in figure 3(b). For the \(\mu = 0\) case, there is a TS disc with \(I_{\text{TKNN}} = 2\) when the Zeeman

Figure 1. The phase diagrams in the \(\lambda k_0/h-\mu/h\) plane with (a) \(t/h = 0.25\), (b) \(t/h = 1\). The pairing potential as a function of the chemical potential for (a) and (b) with \(\lambda k_0/h = 1.2\) is shown in (c) and (d) respectively. The energy is in the unit of the Zeeman field \((h\mu)\), while the unit momentum is \(k_0 = \sqrt{2m\hbar}\). The phases of the shaded regions are TS. The TKNN number minimizing the thermodynamic potential, equation (21), in various conditions.

Figure 2. The phase diagrams in the \(h-r\) plane for the pairing gap \(\Delta_s = t\), and the global chemical potential (a) \(\mu = -2t\), (b) \(\mu = 0\) and (c) \(\mu = 2t\). The labels in these diagrams are the same as those in figure 1. The entire trapped region is the NS phase for \(h/t < 1\). As the Zeeman field increases, there is a TPT at \(h/t = 1\). The TS phases emerge and coexist with the NS for the Zeeman field \(h/t > 1\).
field is $1 < h/t < 4$, as shown in figure 2(b), and the spatial TPS phenomenon occurs only when the Zeeman field is larger than the critical value. Figure 2(c) gives the phase diagram in the $h$–$r$ plane for the global chemical potential $\mu = 2r$. The TS phases emerge and coexist with the NS phase only when $h > t$, too. These results are qualitatively consistent with the results which are derived from the phase diagram in the $\lambda$–$\mu$ plane. The only qualitative difference is that the stability of the normal phase in the trapped region cannot be revealed in figure 2.

In figure 3, we show two types of TPS phenomena in real space. There are two boundaries between the NS and TS in figure 3(a) and one in figure 3(b). The boundaries are Majorana gapless edge states. The TPS is a ubiquitous phenomenon in the ultracold atomic systems with SOC. We extend the above discussion to the continuum model. For the continuum $s$-wave case, the Hamiltonian can be written in the form of equation (5) by replacing $\epsilon(\mathbf{k})$, $\mathbf{g}_a$ and $\Delta_i$ with $\epsilon(\mathbf{k}, r) = \hbar^2 k^2/2m - \mu(r)$, $\mathbf{g}_a = \alpha(k_x, -k_y)$ and $\Delta(\mathbf{k}, r) = \Delta(r)$, respectively. The gapless conditions are $\mathbf{k} = 0$ and $h = \sqrt{\mu(r)^2 + |\Delta(r)|^2}$. There is a TPT at the critical Zeeman field $\left(h_c = \min(\sqrt{\mu(r)^2 + |\Delta(r)|^2})\right)$. The TS emerges and coexists with the NS when $h > h_c$. The topological phase diagrams in the $h$–$r$ plane are shown in figure 4(a) for $\mu = -0.3E_F$ and figure 4(b) for $\mu = 0.3E_F$ with the ansatz $\Delta_i(r) = \Delta_y$. The continuous system only has one gapless condition which indicates that there is only one topological superfluid phase in the trapped region. The shell structure of the TPS for the continuous system is much simpler for the lattice case. We would like to acknowledge that the TPS phenomenon in the continuous case of our work is qualitatively consistent with [34]. The TPS also exists in the uniform system with a Rashba SOC [35]. For the lattice case without a trapping potential, the first-order boundary in figure 1(a) indicates that there is a stable TPS in the region along this curve. This stable TPS can be determined by minimizing the thermodynamic potential which takes account of the possibility of the phase separation [33].

Conclusions

In this paper, we have investigated the spatial topological phase separation (TPS) phenomenon of the trapped 2D fermionic system in an optical lattice with effective SOC and intrinsic $s$-wave superfluidity. We have found that the systems exhibit the spatial TPS between the superfluid phases with different topological numbers in real space when the Zeeman field is above a critical value. Otherwise, the superfluid phase is topologically trivial and the spatial TPS vanishes. We also extend the analysis to the continuum case. We have found that the spatial TPS is a ubiquitous phenomenon in the trapped 2D fermionic systems with SOC and spin imbalance. Finally, for the sake of simplicity, we have mapped out the phase diagrams by replacing the gap function with a constant to visualize the TPS in real space.

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