On energy transfer by detection of a tunneling atom

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We are in the process of building an experiment to study the tunneling of laser-cooled Rubidium atoms through an optical barrier. A particularly thorny set of questions arises when one considers the possibility of observing a tunneling particle while it is in the “forbidden” region. In earlier work, we have discussed how one might probe a tunneling atom “weakly,” so as to prevent collapse. Here we make some observations about the implications of a more traditional quantum measurement. Considerations of energy conservation suggest that attempts to observe tunneling atoms will enhance inelastic scattering, but not in a way which can be directly observed. It is possible that attempts to make such measurements may lead to experimentally realizable “observationally assisted barrier penetration.”

I. INTRODUCTION

Tunneling is one of the most striking predictions of quantum mechanics, and continues to provoke some of its most heated controversies. Despite the appearance of tunneling phenomena in numerous physical and technological areas (and in first-year physics courses), certain aspects of the effect remain poorly understood. This is seen most clearly in the debate over how long a particle takes to traverse a tunnel barrier, and in particular, whether or not it can do so faster than light [1,2].

The confusion over these issues can be traced to certain common elements of quantum “paradoxes.” For one, definite trajectories cannot be assigned to particles in general, and in this sense it is not even clear how to rigorously phrase a question about how much time a transmitted particle spent in a forbidden region— in fact, it may not even be necessary that a particle “traverse” a region in order to be found on the far side.

Of course, at one level quantum mechanics is merely a wave theory, and quite thoroughly understood. In many physical situations, more controversial, interpretational, issues (related to “collapse” or other alternatives) may easily be skirted without loss of predictive power. In tunneling, however, it is quite natural to look for a description of transmitted particles, as distinct from reflected ones (or from the ensemble as a whole) [3]. But such a description is impossible without an attempt to model the detection itself, because without the detection event, transmitted and reflected packets necessarily coexist. Detection naturally raises other interesting questions. What is the nature of a detection process which occurs inside a forbidden region (cf. [4,5])? According to the collapse postulate, if a particle is found to be in the barrier region, it is subsequently described by a new wave function, confined to that region. Any such wave function has $E > V_0$, and suddenly, the problem is no longer one of tunneling.

Our plans to observe tunneling of laser-cooled Rubidium atoms, and to perform “weak measurements” [6,7] in order to study the behaviour of a transmitted subensemble, have been presented at length elsewhere [8,2,9]. Here we repeat only the essential elements, to provide context for the present discussion.

II. TUNNELING IN ATOM OPTICS

Laser-cooled atoms offer a unique tool for studying quantum phenomena such as tunneling through spatial barriers. They can routinely be cooled into the quantum regime, where their de Broglie wavelengths are on the order of microns, and their time evolution takes place in the millisecond regime. They can be directly imaged, and if they are made to impinge on a laser-induced tunnel barrier, transmitted and reflected clouds should be spatially resolvable. With various internal degrees of freedom (hyperfine structure as well as Zeeman sublevels), they offer a great deal of flexibility for studying the various interaction times and nonlocality-related issues. In addition, extensions to dissipative interactions and questions related to irreversible measurements and the quantum-classical boundary are easy to envision.

In our work, we prepare a sample of laser-cooled Rubidium atoms in a MOT, and cool them in optical molasses to approximately $6 \mu K$. As explained below, further cooling techniques are under investigation for achieving yet lower temperatures.
We plan to use a tightly focussed beam of intense light detuned far to the blue of the D2 line to create a dipole-force potential for the atoms.\(^1\)\(^2\). In this intense beam, the atom becomes polarised, and the polarisation lags the field by 90° when the light frequency exceeds that of the atomic resonance. This polarisation out of phase with the local electric field constitutes an effective repulsive potential, proportional to the intensity of the perturbing light beam. It can also be thought of in terms of the new (position-dependent) energy levels of the atom dressed by the intense laser field. Using a 500 mW laser at 770 nm, we will be able to make repulsive potentials with maxima on the order of the Doppler temperature of the Rubidium vapour. Acousto-optical modulation of the beam will let us shape these potentials with nearly total freedom, such that we can have the atoms impinge on a thin plane of repulsive light, whose width would be on the order of the cold atoms’ de Broglie wavelength. This is because the beam may be focussed down to a spot several microns across (somewhat larger than the wavelength of atoms in a MOT, but of the order of that of atoms just below the recoil temperature, and hence accessible by a combination of cooling and selection techniques). This focus may be rapidly displaced by using acousto-optic modulators and motorized mirrors. As the atomic motion is in the mm/sec range, the atoms respond only to the time-averaged intensity, which can be arranged to have a nearly arbitrary profile.

As a second stage of cooling, we follow the MOT and optical molasses with an improved variant of a proposal termed “delta-kick cooling”\(^3\). In our version, the millimetre-sized cloud is allowed to expand for about ten milliseconds, to several times its initial size. This allows individual atoms’ positions \(x_i\) to become strongly correlated with atomic velocity, \(v_i \approx v_{i,\text{free}}\). Magnetic field coils are then used in either a quadrupole or a harmonic configuration to provide a position-dependent restoring force for a short period of time. By proper choice of this impulse, one can greatly reduce the rms velocity of the atoms. So far, we have achieved a one-dimensional temperature of about 700 nK, corresponding to a de Broglie wavelength of about half a micron. We are currently working on improving this temperature by producing stronger, more harmonic potentials, and simultaneously providing an antigravity potential in order to increase the interaction time.

However, the tunneling probability through a 5-micron focus will still be negligible at these temperatures. Furthermore, the exponential dependence of the tunneling rate on barrier height will be difficult to distinguish from the exponential tail of a thermal distribution at high energies. We will therefore follow the delta-kick with a velocity-selection phase\(^4\). By using the same beam which is to form a tunnel barrier, but increasing the width to many microns, we will be able to “sweep” the lowest-energy atoms from the center of the magnetic trap off to the side, leaving the hotter atoms behind. Our simulations suggest that we will be able to to transfer about 7% of the atoms into the one-dimensional ground state of this auxiliary trap. This new, smaller sample will have a thermal de Broglie wavelength of approximately 3.5 \(\mu m\), leading to a significant tunneling probability through a 10-micron barrier. We expect rates on the order of 1% per secular period, causing the auxiliary trap to decay via tunneling on a timescale of the order of 100 ms.

### III. MEASURING TUNNELING ATOMS

A weak measurement is one which does not significantly disturb the particle being studied (nor, consequently, does it provide much information on any single occasion). Why not perform a strong measurement? Simply because if one can tell with certainty that a particle is in a given region, one has also determined that the particle has enough energy to be in that region; one is no longer studying tunneling. The measurement has too strongly disturbed the unitary evolution of the wave function.

At the 6th Symposium on Laser Spectroscopy in Taejon, I made the above glib assertion as I had frequently done in the past, and went on to discuss weak measurements. Afterwards, however, Bill Phillips raised the question of where exactly the energy comes from to turn a forbidden region into an allowed one. The imaging of an atom involves a small transfer of momentum, and typically the only energy exchange is the more-or-less negligible recoil shift. But in this scenario, an atom observed under an arbitrarily high tunnel barrier must—merely by being observed—acquire enough energy to ride on top of the barrier. Why should the effect of a weak probe beam (in fact, the interaction with a single resonant photon) scale with the completely unrelated height of a potential barrier?

The situation envisioned is shown in schematic form in Fig. 1. The wave function of the atom decays exponentially into the barrier region over a characteristic length \(1/\kappa\). If this length is greater than the resolution of the imaging lens, then it is possible for the appearance of a spot of focussed fluorescence on an appropriate point on the screen to indicate that an atom is in the barrier region. This atom, having scattered perhaps only a single photon, must according to quantum mechanics have acquired an energy of at least \(V_0\) to exist confined to the barrier region. This energy depends not on the wavelength or intensity of the imaging light, but only on the height of the barrier created by the dipole-force beam, which may greatly exceed the recoil energy associated with the momentum transfer involved...
in elastic scattering of a photon. An interesting point is that it is unnecessary to actually focus and detect the photon in question. The very possibility that some future observer could use the scattered light to determine that an atom was in the forbidden region is sufficient to decohere spatially separated portions of the atomic wavefunction, causing some fraction of the atoms to “collapse” (if you will) into the barrier region.

At first, one might think that the energy comes from the interaction between the atom and the dipole-force beam. A little thought suffices, however, to demonstrate that this cannot be the solution. Even in the absence of a potential, an imaging beam may localize a previously unlocalized particle, increasing its momentum uncertainty and hence its energy. The energy must come from the imaging beam. Why, then, does the quantity of energy transferred depend on the barrier height?

A partial answer comes from carefully considering the energy levels of the atom. Inside the barrier region, the presence of the dipole beam couples the atomic eigenstates, creating an AC Stark shift (which is the effective repulsive potential). An atom which makes a transition between a state primarily outside the barrier (of energy $E_g + P^2/2m$) to a state localized in the barrier is simultaneously making a transition to a new, higher-energy electronic state ($E_g + V_0 + P'^2/2m$). Energy-conservation will be enforced by the time-integral in perturbation theory, causing the amplitudes for this process to interfere destructively unless the scattered photon energy plus the final energy of the atom equals the initial photon energy plus the initial energy of the atom. In other words, the presence of the dipole beam makes possible inelastic (Raman) transitions between different atomic states. When an elastic scattering event occurs, the atom is left in the original state, and cannot be localized to the barrier. Only when an inelastic collision occurs can the atom be transferred to the state dressed by the dipole field, and localized in the formerly forbidden region.

Can one then determine that an atom is in the barrier without imaging, by merely measuring the energy of the scattered photon? Unfortunately, no. Recall that this argument hinges on an imaging resolution

$$\delta l < 1/\kappa ,$$  

where

$$\hbar^2 \kappa^2 = 2m(V_0 - E) .$$  

A particle localized to within $\delta l$ has a momentum uncertainty

$$\Delta P \geq \hbar/2\delta l .$$

This means that it will only remain within the resolution volume for a time

$$t \leq \frac{\delta l}{\Delta P/m} = 2m\delta l^2/\hbar .$$

This in turn implies

$$t < 2m/\hbar \kappa^2 .$$

Unless the imaging light is time-resolved to better than this limit, it is impossible to maintain the spatial resolution necessary to conclude with certainty that the particle is in the barrier. (Strictly speaking, it would suffice to image to better than the barrier width. However, a particle in the barrier is most likely to be within the first exponential decay length $1/\kappa$. While with lower resolution, one might still conclude that the particle was deeper in the barrier, the likelihood will be exponentially suppressed. Thus on some occasions, the photon energy will be shifted by an amount greater than its rms spectrum, but the low amplitude of this frequency component will be matched by the low probability of finding the atom so deep in the barrier, and the present arguments may easily be generalized.) This implies that the energy uncertainty of the scattered photon must be

$$\Delta E \geq \hbar/t > \hbar^2 \kappa^2/2m .$$

But this is $V_0 - E$, just the energy required to excite the tunneling atom above the barrier. So the only way to image an atom in the forbidden region is to use light with sufficient energy uncertainty that it can boost the atom above the barrier without a significant change in its own spectrum.
Just as these issues were beginning to make themselves clear to us, Terry Rudolph suggested an even more con-founding extension. His idea is outlined in Figure 2. Suppose one decides to determine the location of the tunneling atom in a more indirect manner. Specifically, suppose a nearly ideal imaging system is devised (relying, for example, on \( \pi \)-pulses of probe light), but that a beam stop is imaged onto the barrier region. In this way, any atom in the classically allowed region will be imaged, but an atom which finds itself in the forbidden region will be out of the reach of probe light, and no photon will be scattered. When no scattered photon is observed, we can conclude with near certainty that the atom is in the forbidden region, and has therefore made a transition to a higher-energy dressed state. But now where did the energy come from? After all, it appears that the “detected” atom became localized without ever undergoing an interaction.

This picture is ill-founded, however. The atom cannot be considered in isolation; it is in fact the entire system, composed of atom, dipole-force beams, and imaging photons, which undergoes a transition and must conserve energy. Under the influence of a probe pulse, the atom’s quantum state becomes entangled with the state of the imaging light. There is some amplitude for a photon to travel along its original path, unscattered, but this amplitude is correlated with an atomic state localized to the barrier region. For the time-integral of this amplitude to lead to a real probability for detecting an unscattered photon, the detected photon will necessarily lose enough energy to boost the atom above the barrier, just as in the case previously discussed.

Once more, the situation becomes less startling when we observe that (1) there is indeed a mechanism for energy-exchange between the (unscattered) imaging beam and the atom; and (2) this energy exchange never exceeds the intrinsic uncertainty in the initial photon energy. The interaction between imaging light and an atom comprises not only the possibility of scattering, but also the real part of the atomic polarizability, which is to say the index of refraction experienced by the light due to the presence of the atom. For a near-resonant photon with a probability \( \eta \) of being scattered by an atom, the extra optical path introduced by the presence of the atom, \( \int [n(z) - 1] dz \), is of the order of an optical wavelength times \( \eta \), corresponding to an optical phase shift approximately equal to \( \eta \). If an atom is found to have appeared in the dark region enclosing the barrier, this implies that it left the region of interaction with the probe light, causing the light to experience a time-varying index of refraction. If the atom’s departure from the illuminated region is known to have occurred within a time \( t \), then the phase of the light was modulated by an amount \( \eta \) in a time smaller than \( t \), producing a frequency shift of the order \( \eta / t \). Each photon’s energy can in this way be altered by the “disappearance” of the atom, by the quantity \( \hbar \eta / t \). Since on the order of \( 1/\eta \) photons are necessary to detect atoms with near-unit probability in such a scenario, this phase-modulation effect is automatically sufficient to produce an energy exchange of up to \( \hbar / t \) between the moving atom and the probe beam, even when no photons are scattered.

As in the original discussion of bright imaging of the barrier region, we know that \( \hbar / t > \hbar^2 \kappa^2 / 2m \), and this energy exchange is enough to propel the particle above the barrier. Furthermore, the same argument concerning the duration of a probe pulse remains intact. If the pulse lasts long enough that even a particle localized to the barrier would have time to escape while the light was on, then one will never completely avoid fluorescence, and thus never be able to conclude with certainty that the particle is in the barrier region. One might instead envision a CW probe but time-gated photodetection; in this case, the argument is similar, but it is the detected photon whose energy can no longer be determined precisely enough to be certain that energy exchange has taken place on any individual occasion. Nevertheless, by studying an ensemble of particles, one should be able to build up enough statistics to confirm the shift in the mean photon frequency.

V. CONCLUSION

We see that tunneling is just one more prototypical example of the way in which observation may disturb a quantum system. It is instructive to consider the mechanisms which allow the necessary energy transfer to take place, along with the requisite uncertainties behind which this transfer hides. Ultracold atoms in Bose condensates, and at temperatures achievable through related laser-cooling techniques as well, have long enough de Broglie wavelengths that tunneling effects should soon be observed in a regime where these questions become more than purely academic. Particularly intriguing is the possibility of modifying the barrier-traversal rate by the application of a probe beam which could in principle be used to image an atom in the forbidden region. Even if no attempt is made to actually perform the imaging, the simple possibility that one could do so should be enough to turn the quantum amplitude for an atom to be within about \( 1/\kappa \) of the edge of the barrier into an actual probability, in the sense of a real fraction of atoms
localized into that region of the barrier. These atoms have enough energy to traverse the barrier classically in either direction, and may therefore be observed on the far side.

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**Figure Captions** 1. In this setup, a tunneling atom is illuminated by a plane wave, and the scattered fluorescence may be imaged on a screen to determine whether or not the particle was in the barrier region.

2. Here, a beam block is imaged onto the barrier region, in such a way that an image may be observed on a screen *unless* the particle is in the process of tunneling.
atom → probe light

repulsive dipole-force barrier

image

screen

imaging lens
A diagram illustrating the interaction of a probe light with an atom. The light interacts with the atom, creating a repulsive dipole force barrier. The light is then directed through an imaging lens, creating an image on a screen. The imaging process involves the light interacting with a beam block.