Spin waves in planar quasicrystal of Penrose tiling

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Abstract

We investigated two-dimensional magnonic structures which are the counterparts of photonic quasicrystals forming Penrose tiling. We considered the slab composed of Ni (or Py) disks embedded in Fe (or Co) matrix. The disks were arranged in quasiperiodic Pernose-like structure. The infinite quasicrystal was approximated by its rectangular section with periodic boundary conditions applied. This approach allowed us to use the plane wave method to find the frequency spectrum of eigenmodes for spin waves and their spatial profiles. The calculated integrated density of states shows more distinctive magnonic gaps for the structure composed of materials of high magnetic contrast (Ni and Fe) and relatively high filling fraction. This proves the impact of quasiperiodic long-range order on the spectrum of spin waves. We also investigated the localization of SW eigenmodes resulting from the quasiperiodicity of the structure.

Keywords: spin waves, magnonics, quasicrystals, Penrose tiling

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1. Introduction

The spectrum of wave excitations reflects the structural properties of the system. It is known, that the long-range order in periodic and quasiperiodic structures can be revealed by the presence of forbidden frequency gaps in the spectrum [1,2]. The quasiperiodic system have more complex band structure resulting from countable set of a Bragg peaks densely filling reciprocal space [3,4] and, connected to them, frequency gaps. Due to this feature, the spectrum of scattered waves from quasiperiodic structures can have fractal structure [5–7] which can be used for advanced signal filtering and processing [8].

In quasiperiodic system, there is a possibility to obtain omnidirectional frequency gap by optimizing the structure with rotational symmetry, which is unique property of quasicrystals [9–12]. It is fundamentally different for periodic structures of the same contrast of constituent materials which can be also useful for application.

The quasiperiodic structures are also interesting because of different localization mechanisms than in periodic systems. The Bloch waves in periodic structures are spatially extended in the absence of defects and surfaces, whereas the eigenmodes in quasiperiodic system can be localized [5,7] in the bulk region of the structure. For the system with translational symmetry every unit cell is equivalent and there is no reason for localization. For self-similar quasicrystals, the system form the hierarchical structure in which the localization can be expected.

The rich spectrum of the gaps and the increase of localization can lead to the strong suppression of group velocity [13]. This affects (deteriorates) the transport properties in quasi-periodic structures. Surprisingly, the introduction into quasi-periodic structure, a particular amount of structural defects (Anderson localization regime) can cause the disorder-enhanced transport [14].

The dynamical properties of quasiperiodic composite structures were investigated for different kind of media [1–12] – electronic [16,17], photonic [1,18,19], plasmonic [20], phononic [12,21] and magnonic systems [22–24]. For all kinds of mentioned media, the two-dimensional (2D) quasiperiodic structures [16–18,25,27] give more possibility to adjust structural parameters and to mold the spectrum of excitation than the one-dimensional (1D) quasiperiodic structures. Therefore, we focused our studies on spin wave (SW) dynamics in 2D magnonic quasicrystals, which is promising but not extensively explored subject.

The most commonly considered 2D quasiperiodic structure is Penrose tiling [28]. SW dynamics in magnonic quasicrystals in the form of Penrose-like structure were, up to now, considered mostly for lattice models, in which a Heisenberg antiferromagnet model was investigated [29,30]. This system, for inhomogeneous Néel-ordered ground state, shows the presence of frequency gaps in the frequency spectrum of SWs [31,32]. The studies of SW excitation in the 2D magnonic quasiperiodic structures, in which the richness of structural and material factors play significant role, are on initial stage.

The subject of SW dynamics in quasiperiodically patterned structures is almost unexplored. However, some interesting reports on magnetic antidotes (quasi-periodic) lattices, forming the P2 Penrose coverage [33] or Ammann tiling [24] were published in the last few years. The structures considered in the studies [24,33] have large filling...
fraction (the small volume fraction of magnetic material). They have form of network of magnetic wires of sizes in the range of single micrometers for one section of the network. In this crossover dipolar-exchange regime, the shape anisotropy results in significant static and dynamic magnetic fields \[34, 35\]. The demagnetizing effects influence significantly both magnetic configuration and SW dynamics. Strongly anisotropic SW dependence on the direction of magnetic field is observed even for in-phase precession of SWs (investigated by ferromagnetic resonance (FMR)). This anisotropy is absent in an exchange regime, in which demagnetizing fields are negligible. For the studies on magnonic quasicrystals presented in \[24, 33\], it is challenging to deduce what is the impact of quasiperiodicity on the SWs dynamics. The reported anisotropy can be related both to the quasiperiodic ordering (resulting in Bloch scattering) and to the shape of large antidots.

The clear signature of coherent wave dynamics on (quasi-)periodicity is Bragg scattering, which can lead to the opening of the frequency gaps. Our aim is to investigate the impact of quasiperiodicity on the spectral characteristics and localization properties of SWs in 2D structures.

We would like to minimize the shape anisotropy effects. Therefore, we will consider the system in which the long-range order can be manifested by the presence of magnonic gaps. In these systems the SW scattering on quasiperiodic lattice can lead to the enhancement of localization. To observe frequency gaps in SW spectrum of 2D quasicrystal nanosucture (in the form of Penrose structure), we considered the bi-component planar magnonic quasicrystals, in which the exchange coupling between magnetic nanoelements is mediated by the matrix. The strong coupling between inclusions and high contrast of magnetic parameters between inclusions and matrix is beneficial for strong SW scattering. It also makes the observation of frequency gaps more feasible. The isotropic shape of inclusions (disks), their small sizes and distances between them, make the exchange interactions overshadowing the dipolar ones and reduce the static shape anisotropy effects.

2. Structure and model

We investigated propagation of SWs in 2D bicomponent magnonic structure in the form of Penrose-like quasicrystal. The structure is based on P3 Penrose coverage \[15\], constructed from two rhombi tiles. Every rhombus has the same lengths of sides, but different acute angle: \(\alpha = \pi/5\) for narrower, \(\alpha = 2\pi/5\) for wider one. For such definition, the ratio between areas of the mentioned two rhombi is \(1 : \tau\), i.e. golden ratio. In the center of every rhombus, the inclusions in a form of a disks are placed \[15, 37\]. These ferromagnetic disks are embedded in the plane of the same thickness made of the different kind of ferromagnetic material. The lengths of the sides of rhombi are \(~10.85\) nm and the radii of disks are \(5.6\) nm, which give us filling fraction equal to \(ff = 0.258\) (calculated for the largest supercell). For magnonic crystals in an exchange dominated regime \[34\], the widest frequency gaps in the SW spectrum occur for \(ff \sim 0.5\). To approach this range, we assumed the filling fraction close to the maximal possible value for considered structure, for which the inclusions do not overlap with each other (see Fig. 1(a)). We premised the thickness equal to \(2\) nm. For the small value of the ratio between thickness and in-plane dimensions of the structure (diameter of inclusions and distances between them), we can treat the system as two-dimensional by avoiding the quantization. To minimize the static dipolar effects, related to the shape anisotropy and to investigate the system in an exchange dominated regime, we assumed relatively small dimensions of the elements composing the structure \[54, 83\]. The gyromagnetic ratio equals \(\gamma = 176\) GHz/T in both materials. For matrix we selected two materials, Fe or Co characterised by a saturation magnetization \(M_s\) and an exchange length \(\lambda_{ex}\), that

![Figure 1: (a), (b) The Penrose P3 tiling in the form of rosette of 5-fold symmetry. The considered magnonic structure consists of Ni (or Py) disks of diameter 5.6 nm embedded in Fe (or Co) slab of the same thickness; 2 nm. The disks are placed in the centers of Penrose tiles: wide (blue) and narrow (green) rhombus of the side \(~10.85\) nm. The orthorhombic cells, marked by the sets of red lines, present the regular grids of periods equal to averaged periods of Penrose tiling \[36\] with two vertices per unit cell in average. The centered orthorhombic unit cells were used to build rectangular supercell for the plane wave method (PWM) calculations](image-url)
equal to $M_{\text{Fe}} = 1.752 \cdot 10^6 \text{ A/m}$, $\lambda_{\text{Fe}} = 3.30 \text{ nm}$, $M_{\text{Co}} = 1.445 \cdot 10^6 \text{ A/m}$, $\lambda_{\text{Co}} = 4.78 \text{ nm}$, accordingly. For inclusions we took Ni or Py where: $M_{\text{Ni}} = 0.484 \cdot 10^6 \text{ A/m}$, $\lambda_{\text{Ni}} = 7.64 \text{ nm}$, $M_{\text{Py}} = 0.860 \cdot 10^6 \text{ A/m}$, $\lambda_{\text{Py}} = 5.29 \text{ nm}$. We combined two pairs of materials: Fe/Ni and Co/Py. We assumed that our sample is saturated by an external field, which value equals to $\mu_0 H_0 = 0.2 \text{ T}$.

The precession of the magnetization vector is described by Landau-Lifshitz equation (LLE):

$$\frac{\partial \mathbf{M}}{\partial t} = \mu_0 \gamma \mathbf{M} \times \mathbf{H}_{\text{eff}},$$

where $\mu_0$ is the permeability of vacuum, $\mathbf{M}$ is saturation magnetization. The effective magnetic field $\mathbf{H}_{\text{eff}}$ is composed of the following terms:

$$\mathbf{H}_{\text{eff}}(\mathbf{r}, t) = \mathbf{H}_0 + \mathbf{H}_{\text{dm}}(\mathbf{r}, t) + \mathbf{H}_{\text{ex}}(\mathbf{r}, t),$$

where $\mathbf{H}_0$ means external field, $\mathbf{H}_{\text{dm}}(\mathbf{r}, t)$ is demagnetizing field and $\mathbf{H}_{\text{ex}}(\mathbf{r}, t)$ is exchange field. We have solved (LLE) using plane wave method (PWM) [39].

The PWM requires periodic structures. Therefore, we use the large supercells with periodic boundary conditions as approximates of the Penrose-like structure. To reduce the impact of artificial structural defect on the edges of the supercells, we constructed the supercells as rectangular arrays of centered orthorhombic cells (see Fig. 1) of the sizes being few averaged periods of the Penrose lattice [36] with:

$$a_{\text{av}}^{\text{PT}} = (3 - \tau) a_r, \quad a_{2\text{av}}^{\text{PT}} = (3 - \tau)^{3/2} a_r,$$

where $a_r$ is the edge length of the rhombus tile (see Fig. 1(a)).

3. Results and discussion

We have approximated the Penrose tiling by the periodic array of rectangular supercells, containing 5x5, 6x6, 7x7, 8x8 small orthorhombic cells [36] for each supercell. Each orthorhombic cell has the sizes being the effective periods of Penrose lattice in two orthogonal directions. From the calculations, we have obtained the eigenvalues (frequencies of SWs) and the corresponding eigenvectors (profiles of SW modes). We have ordered obtained SW modes by increasing eigenfrequencies, and then we calculated the integrated density of states (IDOS) [5]. IDOS spectra (IDOS as a function of frequency), calculated for the supercells of different sizes, are shown in Fig. 2(a). In IDOS spectra we have found a few distinctive frequency gaps. Magnonic frequency gaps can be identified by finding the plateau regions in the IDOS spectrum. For these frequencies, bulk modes do not occur and because of that, IDOS is not increasing (these gaps are marked by grey areas in Fig. 2(a)). Clear identification of frequency gaps in the same frequency ranges for the successive approximates of the Penrose structure (supercells) is an evidence of that the approximates are large enough to reveal the quasiperiodic long-range order. For consecutive approximations, in the same ranges of frequencies, we have also found similar SW mode profiles.

In the Fig. 2(b-g) are presented profiles of SW modes for the system with the supercells of successive sizes. They are grouped accordingly to similar shapes of their spatial distributions. The frequencies of selected modes are marked on the SW spectra (Fig. 2(a)) by blue stripes. The modes of the lowest frequencies have to be discussed separately. By virtue of the fact that on the boundaries of the unit cells the periodic boundary conditions were used, the modes of the lowest frequencies are quantized in the area of the whole supercell. In the Fig. 2(b) are presented profiles of the SW modes of the lowest frequency in the supercells of different sizes. This mode is concentrated mainly in the Ni inclusions and do not have any nodal lines (each Ni inclusion is excited more or less with the same strength). Among the lowest modes we selected also the mode with one nodal line in each of the two perpendicular directions (Fig. 2(c)). The frequency of these modes are dependent on the size of the supercell. It proves that by using the PWM in a supercell approach we cannot rigorously investigate low frequency excitations, which spatial distribution is comparable to the sizes of supercells.

For higher frequencies, the modes of corresponding spatial profiles have very similar frequencies for successive approximates of the Penrose structure. Their amplitudes start to localize at the particular regions of the structure. The larger the supercell is, the better the five-fold symmetry is restored and the lesser is the impact of the rectangular shape of the supercell on the SW spectra. In the Fig. 2(d) are presented modes found beneath the first big band gap. Those modes are concentrated in the strictly chosen inclusions of Ni disks, which which are characterized by the lower FMR frequency then Fe matrix. We can find two or three modes of that kind, which appear at the vertices of the same regular pentagon. For the small supercells, modes are concentrated at the individual/single Ni disks, whereas for the larger supercells we can find the state which concentrates at the four of five available apices of the pentagon. It can be easily deduce that by making bigger Penrose structure and by choosing bigger orthorhombic cell of averaged period, it would be possible to find a mode that is concentrated exactly at all five apices of a mentioned pentagon (at the Ni disks). The next group of the profiles, presented in the Fig. 2(e), refer to the modes of the frequencies just above the biggest frequency gap. In these profiles, the SWs amplitude is localized in few pairs of spots distributed in the vertices of pentagon. Each pair occupy the two, closest to each other, disks (located in the centers of the pairs of narrower rhombi - see Fig. 1(a)). This pentagon is smaller one than the pentagon from Fig. 2(d). Modes from the Fig. 2(f) and Fig. 2(g) appear below the second and above the third, recognized by us, band gap, accordingly. These modes have the frequencies above the FMR of Fe. Therefore, they start
Figure 2: (a) Integrated density of states calculated for SWs in magnonic Penrose-like structure (approximated by supercells of different sizes \textendash composed of 5x5, 6x6, 7x7, and 8x8 orthorhombic cells of averaged periods - see the inset). Gray areas mark the widest magnonic gaps. The magnetic field (0.2 T) was applied along the longer side of supercells. The spatial distribution of the amplitude of out-of-plane component of dynamical magnetization are presented in (b)-(g) for selected eigenmodes.

We investigated the nanostructure in the form of the quasicrystal, composed of matrix with small inclusions placed close to each other. We expect that the static demagnetizing field, dependent on the direction of the applied magnetic field with respect to the structure, will be negligible in an exchange regime. To check this prediction, we calculated IDOS for the two different directions of magnetic field (with respect to the structure), which is presented in Fig. 3(a). We observe only the tiny differences of IDOS in the low frequency range. Positions of the largest frequency gaps and the higher parts of the spectra create the small pentagon, surrounding discussed point. However, the apices of those modes are not concentrated on the Ni disks, but between them, the Ni disks are placed in the middle of the edges of this pentagon.

to concentrate in the material of Fe matrix, in the void areas surrounded by the Ni disks. The modes from Fig. 2(f) are concentrated exactly in the middle of the smallest pentagon - i.e. in the area surrounded by the five Ni disks placed in the centers of the wider rhombuses. Due to the size of the supercell, we have only single location of this kind. The modes appearing above the third big band gap are concentrated also in the template material between the Ni disks. These modes are concentrated in such a way that they also form a pentagon, surrounding the middle of the structure, the center of the main pentagon, in which the modes from Fig. 2(f) are concentrated. It is worth to notice that all modes appear in the vertices of pentagons, which center is located exactly at the point, in which mode from the Fig. 2(f) is concentrated. The modes from Fig. 2(g) create the small pentagon, surrounding discussed point. However, the apices of those modes are not concentrated on the Ni disks, but between them, the Ni disks are placed in the middle of the edges of this pentagon.
are practically unaffected by the change of the direction of the magnetic field. It means that the change of the direction of an applied field do not influences SW dynamics. The independence of IDOS on the direction of external magnetic field do not mean that the SWs propagation is isotropic in quasiperiodic structures. Note that, for fixed magnetic configuration, IDOS collects the contributions of all states for different possible directions of propagation.

In the SW spectrum presented in Fig. 2(a), we can identify a few wide frequency gaps. However, we deliberately selected the values of structural and material parameters which support the opening of frequency gaps. If we choose the constituent materials of lower contrast of magnetic parameters, then the frequency gaps are shrunk and can be too narrow to be clearly identified (see, the IDOS spectrum for Py inclusions in Co matrix in Fig. 3(b)). We chose also the structure of quite large filling fraction $ff$ of Ni inclusions. It is understandable that for the limiting cases, $ff = 0$ and $ff = 1$, we deal with uniform material in which the frequency gaps are not observed. Therefore, the intermediate value of filling fraction is optimal to obtain the widest frequency gaps. For the selected value $ff = 0.258$, the disks are in close proximity. This geometrical constraint makes the further increase of the filling fraction practically impossible. The widths of the gaps, which we obtained for this value of filling fraction, seem to be maximal. We can notice (see Fig. 3(c)) that the decrease of filling fraction below $ff = 0.258$ reduces the size of frequency gaps.

4. Conclusions

We have calculated IDOS spectrum for SWs in Penrose-like planar bi-componet magonic quasicrystal using PWM, in a supercell approach. We used the supercell of the size being the multiplicity of the averaged period of Penrose tiling. This reduces the strength of the defects introduced by the boundaries of supercells, resulting from the presence of void spaces or overlapped inclusions (see Fig. 1(b)) and make the supercell method suitable to get an approximate solutions of quasicrystal.

In the calculated spectrum, we identified frequency gaps, which are visible as plateaus in IDOS spectrum, appearing in the same frequency ranges for the different sizes of supercells. The presence of frequency gaps in the SW spectrum reveals the long range order in the quasicrystal structure. However, the size of the supercell affects the spectrum of modes of the lowest frequencies. For these modes, the spatial changes of the amplitude are much larger than distances between neighboring inclusions (in a quasicrystal structure) and are comparable to the size of a supercell – see Fig. 2(c). The frequency of the mode of a particular symmetry, with respect to the whole supercell, depends on the size of supercell in this regime - see the blue lines in Fig. 2(a), marking the frequencies of the modes with one horizontal and one vertical line, which are presented in Fig. 2(c). The profiles of the higher-frequency modes become more localized and start to reflect structure of the quasicrystal. Their frequencies are converged for larger supercells. However, the profiles of these modes barely preserve a fivefold symmetry, due to rectangular
shape of the supercell. They are localized, with increasing frequency, in: the selected Ni inclusions (the material with lower FMR frequency) (Fig. 2(d)), in the doublets of Ni inclusions (Fig. 2(e)) and within the void spaces of Fe matrix (Fig. 2(g)). For the considered sizes of the structure (diameter of inclusions, thickness), the modes of higher frequencies are in an exchange regime. Their frequencies are quite robust on the changes of the direction of external field. The small anisotropy is observed for the lowest frequency modes (see Fig. 3(a)). The magnonic gaps, which we found, are one of the signatures of the long-range order in the considered planar magnonic quasicrystals. We were able to identify them clearly for the structures with relatively high contrast of magnetic properties of constituent materials and for quite large value of filling fraction, in which the inclusions are in the close proximity (see Fig. 3(b,c)).

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