Self–similar and charged spheres
in the free–streaming approximation

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Abstract

We evolve nonadiabatic charged spherical distributions of matter. Dissipation is described by the free–streaming approximation. We match a self–similar interior solution with the Reissner–Nordström–Vaidya exterior solution. The transport mechanism is decisive to the fate of the gravitational collapse. Almost a half of the total initial mass is radiated away. The transport mechanism determines the way in which the electric charge is redistributed.

Key words: Characteristic Evolution, Einstein–Maxwell system

1 Introduction

There is a renewed interest in the study of self–gravitating spherically symmetric charged fluid distributions ([1], [2] and references therein). In self-gravitating systems the electric charge is believed to be constrained by the fact that the resulting electric field should not exceed the critical field for pair creation, $10^{16}$ V cm$^{-1}$ [3]. This restriction in the critical field has been questioned [4]–[7] and does not apply to phases of intense dynamical activity with time scales of the order of (or even smaller than) the hydrostatic time scale, and for which the quasistatic approximation [8] is clearly not reliable as in the collapse of very massive stars or the quick collapse phase preceding neutron star formation.

Electric charge has been studied mostly under static conditions [9]–[11]. Of recent interest are charged quasi–black holes [12] and the electrically charged extension to quasispherical realization [13]. Distributions electrically charged can be considered in practice as anisotropic [14], [15]. Some authors combine anisotropy and electric charge [11], [10], [17] but not as a single entity by means of an equation of state.

The electric field has been postulated to be very high in strange stars with quark matter [18], [19], although other authors suggest that strange stars would not need a large electrical field [20]. The effects of dissipation, in both limiting cases of radiative transport, within the context of the quasistatic approximation, have been studied in [21]. Using this approximation is very sensible because the

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hydrostatic time scale is very small, compared with stellar lifetimes, for many phases of the life of a star. It is of the order of 27 minutes for the sun, 4.5 seconds for a white dwarf, and 104 seconds for a neutron star of one solar mass and 10 km radius [22], [23]. However, such an approximation does not apply to the very dynamic phases mentioned before. In those cases it is mandatory to take into account terms which describe departure from equilibrium, i.e., a full dynamic description has to be used [24].

We are concerned in this paper with configurations out of (static) equilibrium with intense dynamical activity. We use the approach of considering the electrically charged matter distribution as an anisotropic fluid. It is well known that different energy–momentum tensors can lead to the same spacetime [25]–[29]. Under spherical symmetry, electric charge can be considered as a special case of anisotropy [14], [2]. To obtain a dynamical model we consider the free–streaming approximation as the transport mechanism, and a self–similar spacetime for the inner region. We explore the fate of the gravitational collapse. Since dissipation is introduced by the free–streaming of radiation and local anisotropy by the electric charge, it is not necessary to consider in our context any hyperbolic theory of dissipation as for heat flow or viscosity. In many circumstances the mean free path of particles transporting energy may be large enough to justify the free–streaming approximation. We do a comparison with previous work [14] in which the transport mechanism was diffusive.

Is it well known that the Einstein field equations admit homothetic motion [30]–[32]. Applications of self–similarity range from modeling black holes to producing counterexamples to the cosmic censorship conjecture [33]–[44]. It is well established that in the critical gravitational collapse of an scalar field the spacetime can be self–similar [45]–[47]. We have applied characteristic methods to study the self–similar collapse of spherical matter and charged distributions [48]–[50], [14]. The assumption of self–similarity reduces the problem to a system of ODE’s, subject to boundary conditions determined by matching to an exterior Reissner–Nordström–Vaidya solution. Heat flow in the internal fluid is balanced at the surface by the Vaidya radiation. One simulation [50] illustrates how a nonzero total charge can halt gravitational collapse and produce a final stable equilibrium. It is interesting that the pressure vanishes in the final equilibrium state so that hydrostatic support is completely supplied by Coulomb repulsion. Another possible final state is extremely compact and oscillatory with non zero pressure [14]. In this last case electric charge redistribution in the fluid is possible. We explore here if these results depend on the mechanism of transport.

This work is organized as follows. In Sec. 2 we write the field equations for electrically charged interior fluids as seen by Bondian observers. We also show in this section the junction conditions with the exterior spacetime and sketch the general procedure to get physical variables. In Sec. 3 the set of surface equations are presented for parametrized self–similar solutions. Modeling is performed in section 4 to discuss results and conclude in section 5.
2 Field equations

We proceed now to describe the matter distribution, the inner and outer line elements and the field equations.

2.1 Interior spacetime

Our starting point is the Bondi approach to study the evolution of gravitating spheres [51]. Let us consider a nonstatic distribution of matter which is spherically symmetric. In radiation coordinates [52] the metric takes the form

\[ ds^2 = e^{2\beta}[(V/r)du^2 + 2dudr] - r^2(d\theta^2 + \sin^2\theta d\phi^2), \]  

(1)

where \( \beta \) and \( V \) are functions of \( u \) and \( r \). Here, \( u \equiv x^0 \) is the Bondi time at \( I^+ \), with \( u = \text{const.} \) on the outgoing cones. \( r \equiv x^1 \) is the surface-area distance on these null cones. \( \theta \equiv x^2 \) and \( \phi \equiv x^3 \) is the usual polar coordinates. We use geometrized units (\( c = G = 1 \)). To get physical input we use Bondian observers [53], introducing the Minkowski coordinates \( (T, X, Y, Z) \) by

\[ dT = e^\beta [(V/r)^{1/2}du + (r/V)^{1/2}dr], \]  

(2)

\[ dX = e^\beta (r/V)^{1/2}dr, \]  

(3)

\[ dY = r d\theta, \]  

(4)

\[ dZ = r \sin \theta d\phi. \]  

(5)

Next one assumes that for an observer moving relative to these coordinates with velocity \( \omega \) in the radial direction, the space contains a charged fluid with energy density \( \hat{\rho} \), pressure \( \hat{p} \), electric energy density \( \hat{\mu} \) and unpolarized energy density \( \hat{\epsilon} \) traveling in the radial direction. For this comoving observer, the covariant energy tensor is

\[
\begin{pmatrix}
\hat{\rho} + \hat{\mu} + \hat{\epsilon} & -\hat{\epsilon} & 0 & 0 \\
-\hat{\epsilon} & \hat{p} - \hat{\mu} + \hat{\epsilon} & 0 & 0 \\
0 & 0 & \hat{p} + \hat{\mu} & 0 \\
0 & 0 & 0 & \hat{p} + \hat{\mu}
\end{pmatrix},
\]  

(6)

where \( \hat{\mu} = E^2/8\pi \) and \( E = Q/r^2 \) is the local electric field being \( Q(u, r) \) interpreted naturally as the charge within the radius \( r \) at time \( u \), which satisfies the conservation equation

\[ DQ \equiv Q_{,u} + \frac{dr}{du}Q_{,r} = 0, \]  

(7)

where the \( a \) comma represents partial derivative respect to the indicated coordinate, and the matter velocity is given by

\[ \frac{dr}{du} = \frac{V}{r} \frac{\omega}{1 - \omega}. \]  

(8)
Observe that the adiabatic fluid has a diagonal covariant energy tensor \((\rho, p_r, p_t, p_l)\) with 
\[ \rho = \tilde{\rho} + \tilde{\mu}, \quad p_r = \tilde{p} - \tilde{\mu} \quad \text{and} \quad p_t = \tilde{p} + \tilde{\mu}, \]
which is exactly the same as for an anisotropic fluid [54]. Clearly the electric charge produces local anisotropy, contributing to the matter energy density and pressure. If electric charge is zero we recover the Pascalian character of neutral matter, that is, isotropy.

Now, we define the mass function as
\[ \tilde{m} = \frac{1}{2}(r - V e^{-2\beta}), \quad (9) \]
related with the usual total mass by means of [3], [55]
\[ m_T = \tilde{m} + \frac{Q^2}{2r}, \quad (10) \]
which is the generalization of the Misner–Sharp mass for the charged case [1]. Thus, the field equations can be written as [14], [56]
\[ \frac{\rho + p_r \omega^2}{1 - \omega^2} + \varepsilon \frac{(1 + \omega)}{(1 - \omega)} = -\frac{e^{-2\beta} \tilde{m}_u}{4\pi r(r - 2\tilde{m})} + \frac{\tilde{m}_r}{4\pi r^2}, \quad (11) \]
\[ \tilde{\rho} = \frac{\tilde{m}_r}{4\pi r^2}, \quad (12) \]
\[ \tilde{\rho} + \tilde{p} = \frac{\beta_r}{2\pi r^2}(r - 2\tilde{m}), \quad (13) \]
\[ p_t = -\frac{1}{4\pi} \beta_{ur} e^{-2\beta} + \frac{1}{8\pi} (1 - 2\tilde{m}/r)(2\beta_{rr} + 4\beta_r^2 - \beta_r/r) \]
\[ + \frac{1}{8\pi r}[3\beta_r(1 - 2\tilde{m}_r) - \tilde{m}_rrr], \quad (14) \]
where
\[ \tilde{\rho} = \frac{\rho - \omega \rho}{1 + \omega}, \quad (15) \]
and
\[ \tilde{\rho} = \frac{\rho - \omega \rho_r}{1 + \omega}. \quad (16) \]
Observe that the field equations (11)–(14) are exactly the same as for anisotropic matter [55], [61], that is, electric charge can be interpreted as local anisotropy of the fluid. That is possible because of the mass definition given by (27), otherwise electric charge has to be viewed as part of the metric. From this point of view, electric charge is formally an additional physical variable which contributes clearly to the matter energy density and pressure. Also this interpretation is better understood physically due to the Bondi point of view about the comoving reference system. This procedure, except for the mass definition, was the same followed to interpret viscosity as anisotropy [56].
2.2 The exterior spacetime and junction conditions

We consider that the spherically symmetric distribution of collapsing charged fluid is bounded by the surface Σ. Outside Σ we have the Reissner–Nordström–Vaidya spacetime, that is, all outgoing radiation is massless, described by

\[ ds^2 = \left(1 - \frac{2M(u)}{r} + \frac{q^2}{r^2}\right)du^2 + 2dudr - r^2(d\theta^2 + \sin^2\theta d\phi^2), \]  

where \( M(u) \) and \( q \) denote the total mass and charge, respectively.

It can be shown that the junction conditions for the smooth matching of (1) and (17) on Σ implies [62]

\[ \hat{p} \Sigma = 0, \quad Q \Sigma = q, \quad m(u, r) \Sigma = M(u), \quad \beta \Sigma = 0, \]  

where Σ means that both sides of equation are evaluated on Σ. It is remarkable that the zero pressure on Σ is equivalent to the continuity of the second differential form equation

\[ -\beta_u e^{-2\beta} + \left(1 - \frac{2\tilde{m}}{r}\right)\beta_r - \frac{\tilde{m}_r}{2r} + \frac{Q^2}{4r^3} \Sigma = 0. \]  

Up to this point, within the Bondi framework and spherical symmetry, all the written equations are general. We have five physical variables (\( \rho, p, \omega, \epsilon, Q \)) and two metric functions (\( \tilde{m}, \beta \)), for which we have the five equations (7), (11)–(14). Thus, additional assumptions are necessary to solve the characteristic initial value problem. In the next section we suppose that the spacetime is self-similar to illustrate how the approach works and explore the influence of the transport mechanism. But before any assumption to get the metric functions we sketch the general procedure to obtain physical variables.

2.3 Getting physical variables

The algorithm to calculate the physical variables once we get the dynamics at Σ and the metric functions everywhere is as follows:

1. Specifying an initial charge distribution (any) we calculate all physical variables (\( \hat{p}, \hat{\rho}, \omega, \epsilon \)) at any piece of material;
2. We integrate numerically equation (7) to advance \( Q \) in time;
3. Once again we get all physical variables up to the integrate time at any piece of material.
3 Self–similarity and surface equations

Self–similarity is invariably defined by the existence of a homothetic Killing vector field \([30]\). A homothetic vector field on the manifold is one that satisfies \(\mathcal{L}_\xi g = 2ng\) on a local chart, where \(n\) is a constant on the manifold and \(\mathcal{L}\) denotes the Lie derivative operator. If \(n \neq 0\) we have a proper homothetic vector field and it can always be scaled to have \(n = 1\); if \(n = 0\) then \(\xi\) is a Killing vector on the manifold \([57, 58, 59]\). So, for a constant rescaling, \(\xi\) satisfies \(\mathcal{L}_\xi g = 2g\) and has the form \(\xi = \Lambda(u, r)\partial_u + \lambda(u, r)\partial_r\). If the matter field is a perfect fluid, the only equation of state consistent with \(\mathcal{L}_\xi g = 2g\) is a barotropic one \([30]\).

The homothetic equations reduce to \(\mathcal{L}_\xi X = 0\), \(\mathcal{L}_\xi Y = 0\), \(\lambda = \rho\) and \(\Lambda = \Lambda(u, r)\). Therefore, \(X = X(\zeta)\) and \(Y = Y(\zeta)\) are solutions if the self–similar variable is defined as \(\zeta \equiv r \exp(-\int du/\Lambda)\). Here we assume that \(X = C_1 \zeta^k\) and that \(Y = C_2 \zeta^l\), where \(C_1, C_2, k\) and \(l\) are constants.

This power–law dependence on \(\zeta\) is based on the fact that any function of \(\zeta\) is solution of \(\mathcal{L}_\xi g = 2g\). Demanding continuity of the first fundamental form we get the following metric solutions \([60], [14]\):

\[
\dot{\tilde{m}} = \dot{\tilde{m}}_\Sigma \left( \frac{r}{r_\Sigma} \right)^{k+1},
\]

and

\[
e^{2\beta} = \left( \frac{r}{r_\Sigma} \right)^{l+1},
\]

where \(k\) and \(l\) are constants; the subscript indicates that the quantity is evaluated at the surface \(\Sigma\). Thus \(r_\Sigma(u)\) represents the radius of the distribution. Clearly the metric functions and consequently the matter variables and electric charge are determined up to the time dependent functions \(\dot{\tilde{m}}_\Sigma\) and \(r_\Sigma\) for which we have the surface equations obtained from \([11]\) and \([8]\) evaluated at \(\Sigma\):

\[
\dot{\tilde{m}}_\Sigma = -4\pi r_\Sigma^2 \epsilon_\Sigma \left( 1 - \frac{2\tilde{m}_\Sigma}{r_\Sigma} \right) + \dot{r}_\Sigma \frac{q^2}{2r_\Sigma^2} \tag{22}
\]

and

\[
\dot{r}_\Sigma = \left( 1 - \frac{2\tilde{m}_\Sigma}{r_\Sigma} \right) \frac{\omega_\Sigma}{1 - \omega_\Sigma}, \tag{23}
\]

where \(\epsilon_\Sigma = \dot{\epsilon}_\Sigma (1 + \omega_\Sigma)/(1 - \omega_\Sigma)\). Using the dimensionless variables

\[
M = \frac{\tilde{m}_\Sigma}{\tilde{m}_\Sigma(0)}; \quad R = \frac{r_\Sigma}{\tilde{m}_\Sigma(0)},
\]

including the dimensionless time and electric total charge

\[
U = \frac{u}{\tilde{m}_\Sigma(0)}; \quad C = \frac{q}{\tilde{m}_\Sigma(0)},
\]

the last two equations can be written as

\[
\frac{dM}{dU} = -L \left( 1 - \frac{2M}{R} \right) + \frac{dR}{dU} \frac{C^2}{2R^2}, \tag{24}
\]

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Figure 1: Evolution of the radius $R$ for $k = 0.35$, $l = 0.5$, $C = 0.292$ and $R(0) = 2.923$. 
Figure 2: Evolution of the dimensionless energy density $\tilde{\rho} = \tilde{m}_\Sigma(0)^2 \hat{\rho}$ (multiplied by $10^2$) for $k = 0.35$, $l = 0.5$, $C = 0.292$ and $R(0) = 2.923$. 
Figure 3: Evolution of the dimensionless radial pressure $\bar{p} = \hat{\bar{m}} \Sigma(0)^2 \hat{\bar{p}}$ (multiplied by $10^2$) for $k = 0.35$, $l = 0.5$, $C = 0.292$ and $R(0) = 2.923$. 
Figure 4: Evolution of the dimensionless energy flux $\bar{\epsilon} = \tilde{m}_\Sigma(0)^2 \bar{\epsilon}$ (multiplied by $10^2$) for $k = 0.35$, $l = 0.5$, $C = 0.292$ and $R(0) = 2.923$. 
Figure 5: Evolution of the radial velocity $\omega$ for $k = 0.35$, $l = 0.5$, $C = 0.292$ and $R(0) = 2.923$. 

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Figure 6: Evolution of the dimensionless charge function $\bar{Q} = \tilde{m}_\Sigma(0)Q$ for $k = 0.35, l = 0.5, C = 0.292$ and $R(0) = 2.923$. 
and
\[ \frac{dR}{dU} = \left( 1 - \frac{2M}{R} \right) \frac{\omega_\Sigma}{1 - \omega_\Sigma}, \]
(25)
where \( L = 4\pi r_\Sigma^2 \epsilon_\Sigma \). From (19) we obtain
\[ \omega_\Sigma = 1 - \frac{2(l + 1)(1 - 2M/R)}{2(k + 1)M/R - C^2/R^2}. \]
(26)
Now, from the field equation (14) evaluated at \( \Sigma \) we obtain \( M \) as a function of \( R \),
\[ M = \frac{1}{\psi} \left( \xi R - \frac{C^2}{R} \right), \]
(27)
where \( \xi = (l + 1)^2 \) and \( \psi = 2\xi + k(4 + 3l + k) \) and consequently we get \( L \) from (22) and (23)
\[ L = \frac{C^2}{R^2} \left[ \frac{1}{2} + \frac{1}{\psi} \right] \frac{\omega_\Sigma}{1 - \omega_\Sigma}. \]
(28)
Thus, we have one independent surface equation to integrate numerically if we specify initially in some way \( M \) or \( R \). We have preference for (23), taking into account (26) and (27).

We do not have, \textit{a priori}, any restriction on values of \( k, l \) and \( C \). Only physical criteria and expectations on the foregoing models determine our choices, as we illustrate in the following Section. For instance, any choice of values for \( k, l \) and \( C \) have to be consistent with \( -1 < \omega < 1 \), \( \bar{\rho} \geq 0 \), \( \hat{\rho} > 0 \), \( \hat{\rho} > \tilde{\rho} \) for any light cone.

4 Modeling

As a first step for modeling we want to know the minimal radius for a given set of \((k, l, C)\) imposing the conditions on the surface of the distribution. It is easy to show that \( \hat{\rho}_\Sigma \geq 0 \) imposes the minimum radius:
\[ R^2 \geq \frac{C^2}{\xi} \left\{ \frac{\psi}{2(k + 1)} + 1 \right\}, \]
if \( \omega_\Sigma > -1 \). It is also important to note that any model based on self–similar solutions (20)–(21) is singular at \( r = 0 \). We specify an initial profile for the dimensionless electric charge function as
\[ \tilde{Q}(0) = C \left( \frac{r}{r_\Sigma} \right)^P, \]
(29)
where \( P \) is a free parameter.

We do a general survey for \( k, l \) and \( C \). There are values for which the distribution apparently is static. For instance, for \( k = 0.35, l \approx 0.575 \) and
\[ C = 10^{-3}, \text{ the sphere has } \omega_\Sigma \approx 1.192 \times 10^{-7} \text{ and } L \approx -4.172 \times 10^{-8}, \text{ and stays there indefinitely. The situation remains the same for other values of total electric charge.} \]

We opt for a general model with

\[ k = 0.35; \quad l = 0.5; \quad C = 0.292; \quad R(0) = 2.923. \]

In this case \( M(0) \approx 1 \) and \( \omega_\Sigma(0) \approx -0.037 \). Thus, we integrate numerically the surface equation (23) using the fourth order Runge–Kutta method, constrained by (26) and (27). Choosing the free parameter \( P = 2 \) we integrate numerically the equation (7) using finite differences. The procedure is straightforward and standard (see [2] and references therein). It basically consists of using the Lax method (with the appropriate Courant–Friedrichs–Levy (CFL) condition). The conservation equation dynamics is restricted by the surface evolution. Once the charge function is advanced in time we can get any other physical variable. Thus the evolution of the whole distribution proceed up to the final time. For other choice of parameters and initial conditions the procedure is the same and the results displayed in figures 1–6 are representative.

### 5 Discussion

In this paper we consider electrically charged matter as anisotropic matter. We explore a dynamical model under the free streaming approximation and self-similarity within the source. The example we have shown is representative of many others varying the initial condition for the radius \( R \), the self-similar parameters \( k, l \), the total charge \( C \). Only regions \( 0.6 \leq r/r_\Sigma \leq 1 \) satisfy physical conditions pointed out by end of Section 3.

The main motivation for this work were previous results using the same system and solutions but a different transport mechanism, that is, the diffusion limit [14]. Heat flow makes the distribution evolve in a very different way: i) the electric charge halts the collapse; ii) the distribution becomes dust asymptotically in one special case; iii) the final state is oscillatory; iv) the electric charge is redistributed.

In the present case the electric charge does not change the fate of the gravitational collapse. The distribution evolves radiating a huge quantity of its mass (\( \approx 45\% \)) reaching relativistic velocities of collapse. In fact, inner regions are out of the physical domain. Although the evolution looks catastrophic no evidence of black hole formation appears during the monitored evolution.

The only common feature for both transport mechanisms is the electric charge redistribution, but in an opposite manner as we explain below.

We can read from figure 6 that for any time the interior electric charge, for any comoving space marker \( r/r_\Sigma \), is always less than the total electric charge enclosed by the boundary surface. Therefore, the electric charge for inner regions can in fact increase (or decrease) by means a redistribution, conserving the total
Figure 7: Dimensionless charge function $\bar{Q} = \tilde{m}_\Sigma(0)Q$ for $k = 0.35$, $l = 0.5$, $C = 0.292$ and $R(0) = 2.923$, as a function of comoving markers $r/r_\Sigma$ for initial and final time $U$. The electric charge redistributes in the interior space with time.
electric charge. In general, the electric charge grows from zero to the maximum value (the total) in all cases and, in the present case, as \((r/r_\Sigma)^2\) initially and near \((r/r_\Sigma)^{1/5}\) later, conserving partially and totally the electric charge, as dictated by Eq. (7). It is also important to consider that redistribution occurs while the sphere is contracting. Figure 7 shows clearly these issues. In reference \([2]\) an analogous behavior is shown in other context, using other method of solution but with the same transport mechanism (see figures 15 and 16 there).

Why the differences found in the redistribution of charge depend on the transport mechanism? Clearly, the profiles of the flux energy density for the present case (figure 4) and the heat flux as reported in \([14]\) (figure 4 there), are responsible for opposite radial velocity profiles (figure 5 in \([14]\) and figure 5 here) and in consequence opposite electric charge redistribution (figure 6 in \([14]\) and figure 6 here), conserving in all cases the electric charge. We mean by opposite that if the free streaming (heat flow) is lesser (greater) at the interior, the radial velocity is greater (negatively) at the interior, showing the opposite behavior for the diffusion mechanism, that is, lesser (negatively), all this for time–windowing where comparison applies. Thus, opposite electric charge redistribution means that between fixed extremes the inner electric charge per shell increases with evolution for the streaming out and diminishes for the diffusion transport mechanism.

Generic numerical solutions for homothetic motion are currently under consideration and will be reported elsewhere.

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