Inclusive Quarkonium Decays

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I review recent progress in the calculation of inclusive quarkonium decays in the framework of QCD nonrelativistic effective field theories and in relation to the experimental measurements.

1. INTRODUCTION

Inclusive quarkonium decays, more precisely inclusive annihilation decay rates of heavy quarkonium states into light hadrons (hadronic decays) and photons and lepton pairs (electromagnetic decays), were among the earliest calculations of perturbative QCD. There, it was assumed that the decay rate of the quarkonium state factored into a short distance part, calculated as the annihilation rate of the heavy quark and antiquark and given in terms of $\alpha_s(m)$, and a long distance nonperturbative part given in terms of the quarkonium wave function (or its derivatives) evaluated at the origin. Explicit calculations at next to leading order in $\alpha_s$ in perturbation theory for $S$- and $P$-wave decays supported the factorization assumption which could not however be proved on general grounds for higher orders of perturbation theory. Indeed, in the case of $P$-wave decays into light hadrons, it turned out that at order $\alpha_s^3$ the factorization was spoiled by logarithmic infrared divergences [1]. The same problem appeared in relativistic corrections to the annihilation decays of $S$-wave states [1].

This problem has been solved [2,3] and new predictions have been obtained [4,5] with the introduction of non-relativistic effective field theories (EFTs) of QCD, that has put our description of these systems on the solid ground of QCD.

The reason for which the EFT approach is so successful for heavy quarkonium is the following. Heavy quarkonium, being a non-relativistic bound state, is characterized by a hierarchy of energy scales $m, mv$ and $mv^2$, where $m$ is the heavy-quark mass and $v \ll 1$ the relative heavy-quark velocity. A hierarchy of EFTs may be constructed by systematically integrating out modes associated to these energy scales in a matching procedure that enforces the complete equivalence between QCD and the EFT at a given order of the expansion in $v$ and $\alpha_s$.

Integrating out degrees of freedom of energy $m$, which for heavy quarks can be done perturbatively, leads to non-relativistic QCD (NRQCD)[2,3]. This EFT still contains the lower energy scales as dynamical degrees of freedom. In the last years, the problem of integrating out the remaining dynamical scales of NRQCD has been addressed by several groups and has reached a good level of understanding (a list of references can be found in [6]). The EFT obtained by subsequent matchings from QCD, where only the lightest degrees of freedom of energy $mv^2$ are left dynamical, is called potential NRQCD, pNRQCD [4,5]. This EFT is close to a quantum-mechanical description of the bound system and, therefore, as simple. It has been systematically explored in the dynamical regime $\Lambda_{\text{QCD}} \lesssim mv^2$ in [5,7,8] and in the regime $mv^2 \ll \Lambda_{\text{QCD}} \lesssim mv$ in [5,9,10]. The quantity $\Lambda_{\text{QCD}}$ stands for the generic scale of non-perturbative physics.

The EFT approach made it possible, in the case of several observables, among which the inclusive decay widths, to achieve a rigorous factorization between the high-energy dynamics encoded into matching coefficients calculable in perturbation theory and the non-perturbative QCD dynamics encoded into few well-defined matrix elements. Systematic improvements are possible, either by
calculating higher-order corrections in the coupling constant or by adding higher-order operators. In this way the prediction of inclusive decays are put in direct relation to QCD and the theoretical uncertainties may be consistently estimated.

From the experimental side new data have recently been produced for heavy-quarkonium observables. Measurements relevant to the determination of heavy-quarkonium inclusive decay widths have come from Fermilab (E835)[11], BES[12], CLEO[13,14] and Belle[15]. and demand accurate QCD theoretical predictions. On the other hand the inclusive decays of heavy quarkonium may provide competitive information on $\alpha_s$ at the scale $m$ once the theoretical and experimental errors are under control.

In the following I will review recent progress in our theoretical understanding of inclusive and electromagnetic heavy-quarkonium decays. I will recall the NRQCD factorization results in Sec. 2 and the further simplification achieved within the pNRQCD factorization in Sec. 3. The presented pNRQCD formulas apply to quarkonia that fulfil $m v \sim \Lambda_{QCD} \gg m v^2$.

2. NRQCD

NRQCD is the EFT obtained by integrating out the hard scale $m$. This $m$ being larger than the scale of non-perturbative physics, $\Lambda_{QCD}$, the matching to NRQCD can be done order by order in $\alpha_s$. Hence, the NRQCD Lagrangian can be written as a sum of terms like $f_n O_n^{(d_n)} / m^{d_n-4}$, ordered in powers of $\alpha_s$ and $v$. More specifically, the Wilson coefficients $f_n$ are series in $\alpha_s(m)$ and encode the ultraviolet physics that has been integrated out from QCD. The operators $O_n^{(d_n)}$ of dimension $d_n$ describe the low-energy dynamics and are counted in powers of $v$. Heavy quarkonium inclusive decays are controlled by the imaginary part of the NRQCD Hamiltonian, i.e. the imaginary part of the Wilson coefficients of the 4-fermion operators $O_n^{(d_n)} = \psi K_n \chi K_n^* \psi$ in the NRQCD Lagrangian. The NRQCD factorization formula for quarkonium ($H$) inclusive decay widths into light hadrons (LH) reads [3]

$$\Gamma(H \to LH) = \sum_n \frac{2 \text{Im} f_n}{m^{d_n-4}} \langle H | \psi K_n \chi K_n^* \psi | LH \rangle.$$

The 4-fermion operators are classified with respect to their rotational and spin symmetry (e.g. $O^{(2S+1)J_{1/2}}$, $O^{(2S+1)J_{3/2}}$, ...) and of their colour content (octet, $O_8$, and singlet, $O_1$, operators). Singlet operator expectation values may be easily related to the square of the quarkonium wave functions (or derivatives of it) at the origin. These are unknown non-perturbative parameters.

Let us make a concrete example by considering the P-wave inclusive decays.

In NRQCD the P-wave inclusive decay width for the $S = 0$ ($h$) and $S = 1$ ($\chi$) quarkonium states is given at leading (non-vanishing) order in $v$ (which is $m v^5$) by [3]:

$$\Gamma(h \to LH) = \frac{9 \text{Im} f_1 (1P)}{\pi m^4} |R_P|^2 \langle h | O_8 (1S_0) | h \rangle,$$

$$\Gamma(\chi J \to LH) = \frac{9 \text{Im} f_1 (3P)}{\pi m^4} |R_P|^2 \langle h | O_8 (1S_0) | h \rangle,$$

where $R_P$ is the derivative of the radial P-wave function at the origin. We stress that, according to the power counting of NRQCD, the octet contribution $\langle h | O_8 (1S_0) | h \rangle$ is as relevant as the singlet contribution [3]. This octet contribution reabsorbs the dependence on the infrared cut-off $\mu$ of the Wilson coefficients $\text{Im} f_1 (P)$ solving the problem mentioned in the introduction. From the above equations, we see that in NRQCD the 8 P-wave bottomonium states ($1P$, $2P$) and the 4 P-wave charmonium states ($1P$), which lie under threshold, depend at leading order in the velocity expansion on 6 non-perturbative parameters (3 wave functions + 3 octet matrix elements).

The inclusive decays of the heavy quarkonium (either hadronic or electromagnetic) are usually considered up to, and including, NRQCD matrix elements of 4-fermion operators of dimension 8. This means to consider the $O(1/m^2, 1/m^3)$ local 4-fermion operators of the NRQCD Lagrangian.

\footnote{For updates see also [25].}
and the decay rate up to order $mv^5$ in the $v$ counting. If we consider that in the bottomonium system in principle 14 $S$- and $P$-wave states lie below threshold ($\Upsilon(nS)$ and $\eta_c(nS)$ with $n = 1, 2, 3$; $h_b(nP)$ and $\chi_{bJ}(nP)$ with $n = 1, 2$ and $J = 0, 1, 2$) and that in the charmronium system this is the case for 8 states ($\psi(nS)$ and $\eta_c(nS)$ with $n = 1, 2$; $h_c(1P)$ and $\chi_{cJ}(1P)$ with $J = 0, 1, 2$), all the bottomonium and charmonium $S$- and $P$-wave decays into light hadrons and into photons or $e^+e^-$ are then described in NRQCD up to $v^5$ by 46 unknown NRQCD matrix elements (40 for the $S$-wave decays and 6 for the $P$-wave decays), where we have already used spin symmetry and vacuum saturation [3,10]. These matrix elements have to be fixed either by lattice simulations [16] or by fitting the data [17]. Only in the specific case of matrix elements of singlet operators does NRQCD allow an interpretation in terms of quarkonium wave functions and one can resort to the leading order. This is not so within pNRQCD, assuming the counting $\Lambda_{QCD} \sim mv$, they would only be $O(v^2)$-suppressed [10]. This is potentially relevant for $\Gamma(V \rightarrow LH)$ since $\text{Im} f_1(^3S_1)$ is $O(\alpha_s(m))$-suppressed with respect to $\text{Im} f_0(S)$ [10]. In other words, the octet matrix element effects could potentially be much more important than usually thought for these decays.

In the next section we will show that in the framework of pNRQCD it is possible to achieve a noticeable reduction in the number of the nonperturbative parameters and thus to formulate new predictions with respect to NRQCD.

3. pNRQCD

Pushing further the EFT programme for non-relativistic bound states, further simplifications occur if we integrate out also soft degrees of freedom. pNRQCD is the resulting EFT. We will consider pNRQCD under the condition $\Lambda_{QCD} \gg mv^2$. Then, two situations are possible. First, the situation when $mv \ll \Lambda_{QCD} \gg mv^2$. In this case the soft scale $mv$ can be integrated out perturbatively. This leads to an intermediate EFT that contains singlet and octet quarkonium fields and ultrasoft gluons as dynamical degrees of freedom. The octet quarkonium field and the ultrasoft gluons are eventually integrated out by the (nonperturbative) matching to pNRQCD [5]. Second, the situation when $\Lambda_{QCD} \sim mv$. In this case the (nonperturbative) matching to pNRQCD has to be done in one single step [9]. Under the circumstances that other degrees of freedom develop a mass gap of order $\Lambda_{QCD}$ the quarkonium singlet field $S$ remains as the only dynamical degree of freedom in the pNRQCD Lagrangian, which reads [5,9,10] $\mathcal{L}_{pNRQCD} = \text{Tr} \left\{ S \left( i \partial_0 - \mathcal{H} \right) S \right\}$, $\mathcal{H}$ being the pNRQCD Hamiltonian, to be determined by matching pNRQCD to NRQCD. The inclusive quarkonium decay width into light hadrons is given by

$$\Gamma(H \rightarrow LH) = -2 \text{Im} \langle n, L, S, J|\mathcal{H}|n, L, S, J \rangle,$$  

where $|n, L, S, J \rangle$ is an eigenstate of $\mathcal{H}$ with the quantum numbers of the quarkonium state H.
3.1. Matching in a $1/m$ expansion

We consider first the case in which the matching between NRQCD and pNRQCD is made within a $1/m$ expansion. In this case from the matching we obtain schematically:

$$\text{Im} \, H = \delta^3(r) \sum_n \frac{\text{Im} f_n}{m^{d_n-4}} A_n + \{\delta^3(r), \Delta\}$$

$$\times \sum_n \frac{\text{Im} f_n}{m^{d_n-4}} \mathcal{B}_n + \nabla^i \delta^3(r) \nabla^j \sum_n \frac{\text{Im} f_n}{m^{d_n-4}} C_{ij}^n + \ldots,$$

where the imaginary part of $f_n$ are inherited from the 4-heavy-fermion NRQCD matching coefficients, and $A_n$, $B_n$, ... are nonperturbative operators, which are universal in the sense that they do not depend either on the heavy-quark flavour or on the specific quantum numbers of the considered heavy-quarkonium state. Inserting Eq. (5) into (4) and comparing with Eq. (1), we see [10] that all NRQCD matrix elements, including the octet ones, can be expressed through pNRQCD as products of universal nonperturbative factors by the squares of the quarkonium wave functions (or derivatives of it) at the origin. In [10] the inclusive decay widths into light hadrons, photons and lepton pairs of all $S$-wave and $P$-wave states (under threshold) have been calculated up to $\mathcal{O}(mv^3 \times (\Lambda_{QCD}^2/m^2, E/m))$ and $\mathcal{O}(mv^5)$. A large reduction in the number of unknown nonperturbative parameters is achieved and, therefore, new model-independent QCD predictions may be obtained. The universal nonperturbative parameters are all expressed only in terms of gluonic field-strength correlators, which may be fixed by experimental data or by lattice simulations. Thus at the same level of accuracy discussed before in NRQCD, $S$- and $P$-wave bottomonium and charmonium decays are described in pNRQCD, under the dynamical assumption $\Lambda_{QCD} \gg mv^2$ and within a $1/m$ expansion matching, by only 19 nonperturbative parameters. These are the 13 wave functions and 6 universal nonperturbative parameters.

This same approach may be useful also for quarkonium production.

3.2. Contributions from the scale $\sqrt{m \Lambda_{QCD}}$

Once the methodology to compute the potentials (real and imaginary contributions) and from these the inclusive decays, within a $1/m$ expansion in the matching has been developed, the next question that appears naturally is to which extent one can compute the full potential within a $1/m$ expansion in the case $\Lambda_{QCD} \gg mv^2$. It has been recently shown [21] that new non-analytic terms arise due to the three-momentum scale $\sqrt{m \Lambda_{QCD}}$. These terms can be incorporated into local potentials ($\delta^3(r)$ and derivatives of it) and scale as half-integer powers of $1/m$. Moreover, it is possible to factorize these effects in a model independent way and compute them within a systematic expansion in some small parameters.

These terms are due to the existence of degrees of freedom, namely the quark-antiquark pair, with relative three-momentum of order $\sqrt{m \Lambda_{QCD}}$. The on-shell energy of these degrees of freedom is of $\mathcal{O}(\Lambda_{QCD})$, i.e. the same energy scale that is integrated out when computing the standard $1/m$ potentials, which corresponds to integrating out (off-shell) quark-antiquark pairs of three momentum of order $\Lambda_{QCD}$. Therefore both degrees of freedom should be integrated out at the same time. Note that the scale $\sqrt{m \Lambda_{QCD}}$ fulfils $\sqrt{m \Lambda_{QCD}} \gg \Lambda_{QCD}$, from which it follows that at this scale we always are in the perturbative regime. The matching may be performed in two different ways depending on the two situations $mv \gg \Lambda_{QCD}$ or $\Lambda_{QCD} \lesssim mv$ [21].

The result for the inclusive decays is the following. In general, the size of the contributions coming from the scale $\sqrt{m \Lambda_{QCD}}$ depends on the size of $\alpha_s(\sqrt{m \Lambda_{QCD}})$ [21]. For $P$ decays the leading effect turns out to be $\mathcal{O}(\alpha_s/\sqrt{m \Lambda_{QCD}})$ suppressed with respect to the leading contribution of order $mv^5$ (we assume $\alpha_s \ll 1/\sqrt{m \Lambda_{QCD}}$). For the $S$-wave decay widths the leading effect is $\mathcal{O}(\Lambda_{QCD}/m \alpha_s/\sqrt{m \Lambda_{QCD}})$ suppressed with respect to the leading contribution of order $mv^5$. All the results fulfil the same factorization properties as those obtained in [10] and mentioned in Section 3.1, and thus the nonperturbative parameters are still encoded into few, only glue dependent, operators.
3.3. Applications to $P$ decays

Here we show how the reduction obtained by pNRQCD in the numbers of unknown nonperturbative factors makes new theoretical predictions possible in the case of $P$-wave inclusive quarkonium decays into light hadrons \[10, ?\].

In pNRQCD the $P$-wave inclusive decay widths are given at leading order in $v$ (which is $v^5$) by \[10\]:

\[
\Gamma(h \rightarrow LH) = \frac{R^2_P}{\pi m^4} \left[ 9 \Im f_1(1P_1) + \frac{\Im f_5(1S_0)}{9} \right],
\]

\[
\Gamma(\chi_J \rightarrow LH) = \frac{R^2_P}{\pi m^4} \left[ 9 \Im f_1(3P_J) + \frac{\Im f_5(3S_1)}{9} \right],
\]

where $v = \langle \mu \rangle / 1$ is the universal nonperturbative (only glue dependent) parameter that describes $P$-wave quarkonium decays in pNRQCD.

By comparing Eqs. (2) and (3) with Eqs. (6) and (7) we get at leading order in $v$ the relation between the octet matrix element of NRQCD and $\mathcal{E}$: $\langle h|O_{8}(1S_0)|h\rangle = |R_P|^2 \mathcal{E}/(18\pi m^2)$. The quarkonium-state dependence factorizes in the pNRQCD formulas. This allows some new predictions with respect to NRQCD, which are synthetized by the formula (valid at LO in $v$):

\[
\frac{\Gamma(H(2S+1nP_J) \rightarrow LH)}{\Gamma(H(2S+1nP_J') \rightarrow LH)} = \frac{81 \Im f_1(2S+1P_J) + \Im f_5(2S+1S_J) \mathcal{E}}{81 \Im f_1(2S+1P_J') + \Im f_5(2S+1S_J') \mathcal{E}},
\]

where the left-hand side is a ratio between inclusive decay widths of $P$-wave quarkonia with the same principal quantum number $n$ and the right-hand side no longer depends on $n$ and has the whole flavour dependence encoded in the Wilson coefficients, which are known quantities.

In practice, the 12 $P$-wave quarkonium states, which lie under threshold, depend only, in pNRQCD at leading (non-vanishing) order in the velocity expansion, on 4 nonperturbative parameters (3 wave functions + 1 chromoelectric correlator $\mathcal{E}$). The reduction by 2 in the number of unknown nonperturbative parameters with respect to NRQCD, allows us to formulate two new statements. In particular we can use the charmonium data to extract a determination of $\mathcal{E}$, which in turn can be used to produce two new predictions for bottomonium, (at NLO):

\[
\frac{\Gamma(\chi_{b0}(1P) \rightarrow LH)}{\Gamma(\chi_{b1}(1P) \rightarrow LH)} = \frac{\Gamma(\chi_{b0}(2P) \rightarrow LH)}{\Gamma(\chi_{b1}(2P) \rightarrow LH)} = 8.0 \pm 1.3,
\]

or alternatively

\[
\frac{\Gamma(\chi_{b1}(1P) \rightarrow LH)}{\Gamma(\chi_{b2}(1P) \rightarrow LH)} = \frac{\Gamma(\chi_{b1}(2P) \rightarrow LH)}{\Gamma(\chi_{b2}(2P) \rightarrow LH)} = 0.50 \pm 0.06.\]

The errors here refer only to the experimental errors in the charmonium inclusive decays data (that in turn produce an error on $\mathcal{E}$) while no attempt has been done up to now to include also the theoretical error. Recently preliminary determinations of these two ratios have been produced by CLEO-III\[14\]: $19.3 \pm 9.8$ for the first ratio and $0.29 \pm 0.06$ for the second one.

In Fig. 1 we display a plot of the first ratio of decay widths as a function of the factorization scale $\mu$ and at leading and next-to-leading order in the matching coefficients. As it should be, the result is stable in $\mu$. The obtained bands compare well with the first CLEO-III determination, published after the completion of our work, $\Gamma_{\chi_{b0}\rightarrow LH}/\Gamma_{\chi_{b1}\rightarrow LH} = 19.3 \pm 9.8$. However, both theoretical and experimental determinations are still affected by large uncertainties and the large correction in the NLO of the matching coefficients should be put under control (e.g. via renormalon resummation) or at least considered in the errors.

4. CONCLUSIONS AND OUTLOOK

The progress in our understanding of non-relativistic effective field theories makes it possible to move beyond ad hoc phenomenological models and have a unified description of the different heavy-quarkonium observables, so that the same quantities determined from a set of data may be used in order to describe other sets. Moreover, predictions based on non-relativistic EFTs are conceptually solid, and systematically improvable. In the framework of pNRQCD, for
Figure 1. The ratio $\frac{\Gamma_{\chi^0 \rightarrow LH}}{\Gamma_{\chi^1 \rightarrow LH}}$ plotted vs. $\mu$ (the figure is taken from [23]).

physical states that satisfy $\Lambda_{QCD} \gg mv^2$, octet matrix elements may be expressed in terms of the wave function in the origin and some universal non-local gluon-field correlators obtaining a significant reduction of the nonperturbative parameters. The same nonperturbative correlators enter also in the expression of the masses of some heavy-quarkonium states [7,24]. Difficulties still exist at the level of the control of higher-order corrections in the velocity and $\alpha_s$ expansion. In principle, the tools to overcome these difficulties already exist, so we expect relevant progress in the field from the coordinated effort of the heavy-quarkonium community in the near future[25].

Acknowledgments The support of the Alexander Von Humboldt foundation is gratefully acknowledged. I would like to thank the Jefferson Lab theory group and José Goity for hospitality during the writing up.

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