Heavy-light meson spectroscopy and Regge trajectories in the relativistic quark model

D. Ebert¹, R. N. Faustov¹,² and V. O. Galkin¹,²

¹ Institut für Physik, Humboldt–Universität zu Berlin, Newtonstr. 15, D-12489 Berlin, Germany
² Dorodnicyn Computing Centre, Russian Academy of Sciences, Vavilov Str. 40, 119991 Moscow, Russia

Masses of the ground, orbitally and radially excited states of heavy-light mesons are calculated within the framework of the QCD-motivated relativistic quark model based on the quasipotential approach. Both light ($q = u, d, s$) and heavy ($Q = c, b$) quarks are treated fully relativistically without application of the heavy quark $1/m_Q$ expansion. The Regge trajectories in the $(M^2,J)$ and $(M^2, n_r)$ planes are investigated and their parameters are obtained. The results are in good agreement with available experimental data except for the masses of the anomalous $D_s^*(2317)$, $Ds_1(2460)$ and $Ds_f^*(2860)$ states.

PACS numbers: 14.40.Lb, 14.40.Nd, 12.39.Ki

I. INTRODUCTION

Recently significant experimental progress has been achieved in studying the spectroscopy of mesons with one heavy ($Q = c, b$) and one light ($q = u, d, s$) quarks [1]. Several new excited states of heavy-light mesons were discovered, some of which have rather unexpected properties [2, 3].

The most investigated and intriguing issue is the charmed-strange meson sector, where masses of nine mesons have been measured [1, 2, 4, 5]. Even six years after the discovery of $D_{s0}^*(2317)$ and $Ds_1(2460)$ mesons their nature remains controversial in the literature. The abnormally light masses of these mesons put them below $DK$ and $D^*K$ thresholds thus making these states narrow since the only allowed decays violate isospin. The peculiar feature of these mesons is that they have masses almost equal or even lower than the masses of their charmed counterparts $D_{0}^*(2400)$ and $D_{1}(2427)$ [1, 2, 3]. Most of the theoretical approaches including lattice QCD [6], QCD sum rule [7] and different quark model [8, 9] calculations give masses of the $0^+$ and $1^+ P$-wave $c\bar{s}$ states significantly heavier (by 100-200 MeV) than the measured ones. Different theoretical solutions of this problem were proposed [10] including consideration of these mesons as chiral partners of $0^-$ and $1^-$ states [11], $c\bar{s}$ states which are strongly influenced by the nearby $DK$ thresholds [12], $DK$ or $Ds\pi$ molecules [13], a mixture of $c\bar{s}$ and tetraquark states [14]. However the universal understanding of their nature is still missing. Therefore it is very important to observe their bottom counterparts. The unquenched lattice calculations of their masses can be found in Ref. [15].

Very recently three new charmed-strange mesons $Ds_1(2710)$, $Ds_f^*(2860)$ and $Ds_f(3040)$ were observed [4, 5]. These states are considered to be candidates for the $2S$, $1D$ and $2P$
states, respectively. Therefore it is important to have theoretical predictions not only for the lowest orbital and radial excitations of heavy-light mesons but also for the highly excited states.

In Refs. [8] we calculated the masses of ground and first orbitally and radially excited states of heavy-light mesons on the basis of a three-dimensional relativistic wave equation with a QCD-motivated potential. The heavy quark $1/m_Q$ expansion was used to simplify calculations, while the dynamics of light quark was treated fully relativistically. It was found that the heavy quark $1P$ multiplets with total angular momenta of light quark $j = 1/2$ ($0^+$, $1^+$) and $j = 3/2$ ($1^+$, $2^+$) are inverted in the infinitely heavy quark limit. The account of the first order $1/m_Q$ corrections results in splittings and shifts of the levels in these multiplets, which begin to overlap. As a result a very complicated pattern of $P$-level structure emerges.

During the last few years we further developed our model for the treatment of mesons composed from light quarks [16, 17]. For this purpose an approach which allows to consider the highly relativistic dynamics of light quarks without either the $v/c$ or $1/m_q$ expansion was developed. The consistent relativistic treatment of the light quark dynamics resulted in a nonlinear dependence of the bound state equation on the meson mass which allowed to get correct values of pion and kaon masses in the model [16] with explicitly broken chiral symmetry. The obtained wave functions of the pion and kaon were successfully applied to the relativistic calculation of their decay constants and electromagnetic form factors [16]. Such approach allowed us to get masses of highly excited light mesons and on this basis to check the linearity and parallelism of arising Regge trajectories [17]. Good overall agreement of the obtained predictions and experimental data was found.

Here we improve and extend our study of heavy-light meson spectroscopy by using the fully relativistic approach without the heavy quark $1/m_Q$ expansion. We calculate the masses of highly orbitally and radially excited states and investigate the Regge trajectories both in the $(M^2, J)$ and $(M^2, n_r)$ planes ($M$ is the mass, $J$ is the spin and $n_r$ is the radial quantum number of the meson state). Such analysis is important for elucidating the nature of current and future experimentally observed heavy-light mesons.

II. RELATIVISTIC QUARK MODEL

In the relativistic quark model based on the quasipotential approach a meson is described by the wave function of the bound quark-antiquark state, which satisfies the quasipotential equation of the Schrödinger type [18]

$$
\left(\frac{b^2(M)}{2\mu_R} - \frac{p^2}{2\mu_R}\right)\Psi_M(p) = \int \frac{d^3q}{(2\pi)^3} V(p, q; M)\Psi_M(q),
$$

where the relativistic reduced mass is

$$
\mu_R = \frac{E_1E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3},
$$

and $E_1, E_2$ are given by

$$
E_1 = \frac{M^2 - m_1^2 + m_2^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}.
$$
Here $M = E_1 + E_2$ is the meson mass, $m_{1,2}$ are the quark masses, and $\mathbf{p}$ is their relative momentum. In the center-of-mass system the relative momentum squared on mass shell reads

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}.$$  \hfill (4)

The kernel $V(\mathbf{p}, \mathbf{q}; M)$ in Eq. (1) is the quasipotential operator of the quark-antiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. Constructing the quasipotential of the quark-antiquark interaction, we have assumed that the effective interaction is the sum of the usual one-gluon exchange term with the mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli interaction. The quasipotential is then defined by

$$V(\mathbf{p}, \mathbf{q}; M) = \bar{u}_1(p)\bar{u}_2(-p)V(\mathbf{p}, \mathbf{q}; M)u_1(q)u_2(-q),$$  \hfill (5)

with

$$V(\mathbf{p}, \mathbf{q}; M) = \frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma_\mu\gamma_\nu + V^V_{\text{conf}}(\mathbf{k})\Gamma^\mu_{12}\gamma_\mu + V^S_{\text{conf}}(\mathbf{k}),$$

where $\alpha_s$ is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge, and $\mathbf{k} = p - q$; $\gamma_\mu$ and $u(p)$ are the Dirac matrices and spinors.

The effective long-range vector vertex is given by

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{ik}{2m}\sigma_{\mu\nu}k^\nu,$$  \hfill (6)

where $\kappa$ is the Pauli interaction constant characterizing the anomalous chromomagnetic moment of quarks. Vector and scalar confining potentials in the nonrelativistic limit reduce to

$$V^V_{\text{conf}}(r) = (1 - \varepsilon)(Ar + B),$$
$$V^S_{\text{conf}}(r) = \varepsilon(Ar + B),$$  \hfill (7)

reproducing

$$V_{\text{conf}}(r) = V^S_{\text{conf}}(r) + V^V_{\text{conf}}(r) = Ar + B,$$  \hfill (8)

where $\varepsilon$ is the mixing coefficient.

All the model parameters have the same values as in our previous papers [8, 18]. The constituent quark masses $m_u = m_d = 0.33$ GeV, $m_s = 0.5$ GeV, $m_c = 1.55$ GeV, $m_b = 4.88$ GeV and the parameters of the linear potential $A = 0.18$ GeV$^2$ and $B = -0.3$ GeV have the usual values of quark models. The value of the mixing coefficient of vector and scalar confining potentials $\varepsilon = -1$ has been determined from the consideration of charmonium radiative decays [18] and matching heavy quark effective theory (HQET). Finally, the universal Pauli interaction constant $\kappa = -1$ has been fixed from the analysis of the fine splitting of heavy quarkonia $^3P_J$-states [18]. In this case, the long-range chromomagnetic interaction of quarks, which is proportional to $(1 + \kappa)$, vanishes in accordance with the flux-tube model.

The quasipotential (5) can in principal be used for arbitrary quark masses. The substitution of the Dirac spinors into (5) results in an extremely nonlocal potential in the configuration space. Clearly, it is very hard to deal with such potentials without any additional approximations. In order to simplify the relativistic $q\bar{q}$ potential, we make the following replacement in the Dirac spinors:

$$\epsilon_{1,2}(p) = \sqrt{m_{1,2}^2 + p^2} \rightarrow E_{1,2}$$  \hfill (9)
(see the discussion of this point in [8, 16]). This substitution makes the Fourier transformation of the potential local.

The resulting $Q\bar{q}$ potential then reads

$$V(r) = V_{SI}(r) + V_{SD}(r),$$

where the explicit expression for the spin-independent $V_{SI}(r)$ can be found in Ref. [17]. The structure of the spin-dependent potential is given by

$$V_{SD}(r) = a_1 L S_1 + a_2 L S_2 + b \left[ -S_1 S_2 + \frac{3}{r^2} (S_1 r) (S_2 r) \right] + c S_1 S_2 + d (L S_1) (L S_2),$$

where $L$ is the orbital angular momentum, $S_i$ is the quark spin. The coefficients $a_1$, $a_2$, $b$, $c$ and $d$ are expressed through the corresponding derivatives of the Coulomb and confining potentials. Their explicit expressions are given in Ref. [17].

Since we deal with mesons containing light quarks we adopt for the QCD coupling constant $\alpha_s(\mu^2)$ the simplest model with freezing [19], namely

$$\alpha_s(\mu^2) = \frac{4\pi}{\beta_0 \ln \frac{\mu^2}{M_B^2}}, \quad \beta_0 = 11 - \frac{2}{3} n_f,$$

where the scale is taken as $\mu = 2m_1 m_2 / (m_1 + m_2)$, the background mass is $M_B = 2.24\sqrt{A} = 0.95$ GeV [19], and $\Lambda = 413$ MeV was fixed from fitting the $\rho$ mass [17]. Note that the other popular parametrization of $\alpha_s$ with freezing [20] leads to close values.

The resulting quasipotential equation with the complete kernel (10) is solved numerically without any approximations.

### III. RESULTS AND DISCUSSION

The calculated masses of heavy-light $D$, $D_s$, $B$ and $B_s$ mesons are given in Tables I and II ($n = n_r + 1$, $L$ is the orbital momentum and $S$ is the total spin). They are confronted with available experimental data from PDG [1].

The heavy-light meson states with $J = L$, given in Tables I [1], are mixtures of spin-triplet $|^3L_L\rangle$ and spin-singlet $|^1L_L\rangle$ states:

$$|\Psi_J\rangle = |^1L_L\rangle \cos \varphi + |^3L_L\rangle \sin \varphi,$$

$$|\Psi'_J\rangle = -|^1L_L\rangle \sin \varphi + |^3L_L\rangle \cos \varphi, \quad J = L = 1, 2, 3 \ldots$$

where $\varphi$ is a mixing angle and the primed state has the heavier mass. Such mixing occurs due to the nondiagonal spin-orbit and tensor terms in Eq. (11). The masses of physical states were obtained by diagonalizing the mixing terms. The found values of mixing angle $\varphi$ are given in Table III.

In the heavy quark limit heavy-light mesons are usually described in the $|J, j\rangle$ basis, where $j = L + s_q$ is the total angular momentum of the light quark. The relation between the $|J, j\rangle$ and $|J, S\rangle$ basises is given by

$$|J; j\rangle = \sum_s (-1)^{J + L + 1} \sqrt{(2S + 1)(2j + 1)} \begin{pmatrix} 1/2 & 1/2 & S \\ L & J & j \end{pmatrix} |J; S\rangle,$$

where

$$\sqrt{(2S + 1)(2j + 1)} \begin{pmatrix} 1/2 & 1/2 & S \\ L & J & j \end{pmatrix} = \begin{pmatrix} \sqrt{(2S + 1)(2j + 1)} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$
where \( |J; S\rangle \) corresponds to the \( |^{2S+1}L_J\rangle \) state. The following relations for the states with \( J = L \) than follow

\[
|J = L; j = L + \frac{1}{2}\rangle = \sqrt{\frac{L + 1}{2L + 1}} |J = L; 0\rangle + \sqrt{\frac{L}{2L + 1}} |J = L; 1\rangle,
\]
\[ J = L; j = L - \frac{1}{2} \] = \sqrt{\frac{L}{2L+1}} \left[ J = L; 0 \right] + \sqrt{\frac{L + 1}{2L+1}} \left[ J = L; 1 \right]. \quad (15)

In the \( m_Q \to \infty \) limit \( |\Psi_J\rangle \) and \( |\Psi'_J\rangle \) turn into \( |J; j = L + \frac{1}{2} \rangle \) and \( |J; j = L - \frac{1}{2} \rangle \) states, respectively. Comparing Eqs. (13) and (15) it is easy to obtain the infinitely heavy quark
TABLE III: Mixing angles $\varphi$ for heavy-light mesons (in °).

| State | $D$  | $D_s$ | $B$  | $B_s$ | $m_Q \to \infty$ |
|-------|------|-------|------|-------|------------------|
| $1P$  | 35.5 | 34.5  | 35.0 | 36.0  | 35.3             |
| $2P$  | 37.5 | 37.6  | 37.3 | 34.0  | 35.3             |
| $3P$  | 38.4 | 38.2  | 34.1 | 36.7  | 35.3             |
| $1D$  | 40.7 | 39.2  | 38.0 | 38.1  | 39.2             |
| $2D$  | 39.0 | 41.2  | 41.6 | 41.1  | 39.2             |
| $1F$  | 39.6 | 40.5  | 39.5 | 40.1  | 40.9             |
| $1G$  | 40.2 | 40.3  | 40.4 | 41.9  | 41.8             |

It is clearly seen from Table III that the found values of mixing angles $\varphi$ are very close to $\varphi_{m_Q \to \infty}$. This means that the physical $|\Psi_J\rangle$ and $|\Psi'_J\rangle$ states in our model are almost pure $|J; j = L + \frac{1}{2}\rangle$ and $|J; j = L - \frac{1}{2}\rangle$ HQET states, respectively. Since $|J; j = L + \frac{1}{2}\rangle$ states have higher value of the light quark angular momentum $j$ than $|J; j = L - \frac{1}{2}\rangle$ ones they should decay to the pair of ground state heavy-light and light mesons in a higher wave and therefore are expected to be significantly narrower than the partner states with $j = L - 1/2$. For example, the $P$-wave mesons with $j = 1/2$ can decay in an $S$-wave and, therefore, are expected to be broad, while those with $j = 3/2$ can decay in a $D$-wave and should be narrow. The found values of the mixing angle $\varphi$ in our model indicate that there is only a small admixture of broad states to the narrow ones and therefore they should remain narrow which is in accord with available experimental data.

In our analysis we calculated masses of both orbitally and radially excited heavy-light mesons up to rather high excitation numbers ($L = 4$ and $n_r = 4$). This makes it possible to construct the heavy-light meson Regge trajectories in the $(J, M^2)$ and $(n_r, M^2)$ planes. We use the following definitions.

a) The $(J, M^2)$ Regge trajectory:

$$J = \alpha M^2 + \alpha_0;$$  \hspace{1cm} (17)

b) The $(n_r, M^2)$ Regge trajectory:

$$n_r = \beta M^2 + \beta_0,$$  \hspace{1cm} (18)

where $\alpha$, $\beta$ are the slopes and $\alpha_0$, $\beta_0$ are intercepts. The relations (17) and (18) arise in most models of quark confinement, but with different values of the slopes.

In Figs. 11-18 we plot the Regge trajectories in the $(J, M^2)$ plane for mesons with natural ($P = (-1)^J$) and unnatural ($P = (-1)^{J-1}$) parity. The Regge trajectories in the $(n_r, M^2)$ plane are presented in Figs. 9-12. The masses calculated in our model are shown by diamonds. Available experimental data are given by dots with error bars and corresponding meson names. Straight lines were obtained by a $\chi^2$ fit of the calculated values. The fitted
FIG. 1: Parent and daughter \((J, M^2)\) Regge trajectories for charmed mesons with natural parity. Diamonds are predicted masses. Available experimental data are given by dots with particle names; \(M^2\) is in GeV\(^2\).

slopes and intercepts of the Regge trajectories are given in Tables IV and V. We see that the calculated heavy-light meson masses fit nicely to the linear trajectories in both planes. These trajectories are almost parallel and equidistant.

From the comparison of the slopes in Tables IV, V we see that the \(\alpha\) values are systematically larger than the \(\beta\) ones. The ratio of their mean values is about 1.4 both for the charmed and bottom mesons. This value of the ratio is slightly larger than the one obtained in our recent [17] calculations of the light meson masses, where \(\alpha/\beta\) was found to be in average about 1.3.

We can combine the results of our current calculation performed without using the heavy quark \(1/m_Q\) expansion with our previous analysis [8] which was based on such an expansion in order to analyze the pattern of \(P\)-levels. As a result we get the following picture. In the heavy quark limit \(m_Q \rightarrow \infty\) the \(P\)-wave mesons form two heavy quark spin multiplets with light-quark total angular momentum \(j = 1/2\) (0\(^+\), 1\(^+\)) and \(j = 3/2\) (1\(^+\), 2\(^+\)). Masses of the levels with \(j = 1/2\) are heavier than of the ones with \(j = 3/2\). Therefore we have inversion of \(P\)-levels in the infinitely heavy quark limit. When we switch on the \(1/m_Q\) corrections we get spin splittings in these multiplets and mixing of the 1\(^+\) states. Moreover the levels from these multiplets begin to overlap. This tendency is further strengthened when the nonperturbative approach in \(1/m_Q\) is used. We see from Tables I, II that there are significant overlaps of the levels resulting from these multiplets, especially in the charm sector. However this more sophisticated approach confirms our previous conclusion that the remnants of the inversion remain in both bottom and charmed meson spectra. For all considered heavy-light mesons it is found that the heavier \(P_1\) state, which has the main contribution from the \(j = 1/2\) multiplet (see above), has the heaviest mass, which is even higher (by a few MeV for charmed mesons and by almost 30 MeV for bottom mesons) than
the mass of the $^3P_2$ state from the $j = 3/2$ multiplet.

Experimentally complete sets of $1P$-wave meson candidates are known in the charm sector. In the bottom sector masses of only narrow states originating from the $j = 3/2$ heavy quark spin multiplet are known reliably. There are some indications of the broad $j = 1/2$ states both of bottom ($0^+$) and bottom-strange ($1^+$) mesons, but additional confirmation is needed. We find good agreement of our predictions for $1P$ wave states with available
FIG. 4: Same as in Fig. 1 for bottom-strange mesons with natural parity.

FIG. 5: Same as in Fig. 1 for charmed mesons with unnatural parity.

data except for the masses of $D^*_{s0}(2317)$ and $D_{s1}(2460)$ mesons. These two charmed-strange meson states have anomalously low masses which are even lower than the experimentally observed masses of the corresponding charmed $D_0^*(2400)$ and $D_1(2427)$ mesons. Our model predictions for the masses of the $1P$-wave $0^+$ and $1^+$ states are almost 200 MeV and 110 MeV higher than the measured masses of $D^*_{s0}(2317)$ and $D_{s1}(2460)$ mesons. Such phenomenon is very hard to understand within the quark-antiquark picture for these states. Most of the
explanations available in the literature are based on some very specific fine tuning of the model parameters. The influence of such tuning on the spectroscopy of other mesons, which are well described in the framework of the conventional approach, is not well understood. It is probable that these mesons could have an exotic nature and the genuine quark-antiquark $P$-wave charmed-strange $0^+$ and $1^+$ states have higher masses above the $D^0K$ and $D^*K$ thresholds and are, therefore, broad. We find that the $D_{s0}^*(2317)$ and $D_{s1}^*(2460)$ mesons
FIG. 8: Same as in Fig. 4 for bottom-strange mesons with unnatural parity.

FIG. 9: The \((n_r, M^2)\) Regge trajectories for pseudoscalar, vector and tensor charmed mesons (from bottom to top). Notations are the same as in Fig. 4.

Our model suggests that \(D_{s1}(2700)\) and \(D^*(2637)\) mesons are the first radial excitations \((2^3S_1)\) of the vector charmed-strange and charmed mesons. Figures 1, 2 and 8, 10 show that they do not lie on the corresponding Regge trajectories. This can be an additional indication of their anomalous nature. All other experimentally observed \(1P\)-wave states match well their trajectories.

These do not lie on the corresponding Regge trajectories. This can be an additional indication of their anomalous nature. All other experimentally observed \(1P\)-wave states match well their trajectories.
Recent experimental observation \[5\] that $D_{sJ}^*(2860)$ decays to both $DK$ and $D^*K$ indicates that this state should have natural parity. In our model natural parity states $1^- (1^3D_1)$ and $3^- (1^3D_3)$ have masses which exceed the experimental value by about 50 and 100 MeV, respectively. In Ref. \[21\] it was argued that from the point of view of decay rates the $3^-$ assignment is favored. However the measurement of the branching ratios of the $D_{sJ}^*(2860)$ decay into $D^*K$ to the branching ratio of the decay into $DK$ differs from the theoretical expectations \[21\] by three standard deviations \[5\]. From Fig. \[2\] we see that this state does not fit well to the corresponding Regge trajectory.
On the other hand, the state $D_{sJ}(3040)$, recently observed by BaBar [5] in the $D^*K$ mass spectrum, has a mass coinciding within errors with the mass of the $1^+ (2P_1)$ state predicted by our model (see Table I). This state nicely fits to the daughter Regge trajectory in Fig. 6.

IV. CONCLUSIONS

The mass spectra of charmed and bottom mesons were calculated in the framework of the QCD-motivated relativistic quark model. The dynamics of both light ($q = u, d, s$) and heavy ($Q = c, b$) quarks was treated fully relativistically without application of either nonrelativistic $v/c$ or heavy quark $1/m_Q$ expansions. The results found in the nonperturbative in $1/m_Q$ approach confirm the conclusion, previously obtained within the heavy quark expansion up to the first order in Ref. [8], that the remnants of the inversion of the $1P$-levels remain. The final level ordering is rather complicated, but the higher $1^+$ state is always heavier than the $2^+$ state.

We calculated the masses of ground, orbitally and radially excited heavy-light mesons up to rather high excitations. This allowed us to construct the Regge trajectories both in $(J, M^2)$ and $(n_r, M^2)$ planes. It was found that they are almost linear, parallel and equidistant. Most of the available experimental data nicely fit to them. Exceptions are the anomalously light $D_{s0}^*(2317)$, $D_{s1}(2460)$ and $D_{sJ}^*(2860)$ mesons, which masses are 100-200 MeV lower than various model predictions. The masses of the charmed-strange $D_{s0}^*(2317)$, $D_{s1}(2460)$ mesons almost coincide or are even lower than the masses of the partner charmed $D_0^*(2400)$ and $D_1(2427)$ mesons. These states thus could have an exotic origin. It will be very important to find the bottom counterparts of these states in order to reveal their nature.
TABLE IV: Fitted parameters of the \((J, M^2)\) parent and daughter Regge trajectories for heavy-light mesons with natural and unnatural parity \((q = u, d)\).

| Trajectory | natural parity | unnatural parity |
|------------|----------------|------------------|
|            | \(\alpha\) (GeV\(^{-2}\)) | \(\alpha_0\) | \(\alpha\) (GeV\(^{-2}\)) | \(\alpha_0\) |
| \(c\bar{q}\) | \(D^*\) | \(-1.003 \pm 0.040\) | \(0.494 \pm 0.005\) | \(-1.776 \pm 0.115\) |
| parent     | \(0.499 \pm 0.009\) | \(-2.495 \pm 0.091\) | \(0.513 \pm 0.006\) | \(-3.424 \pm 0.063\) |
| daughter   | \(0.527 \pm 0.003\) | \(-4.489 \pm 0.003\) | \(0.557 \pm 0.005\) | \(-4.084 \pm 0.047\) |
| \(c\bar{s}\) | \(D_s^*\) | \(-1.102 \pm 0.035\) | \(0.469 \pm 0.004\) | \(-1.824 \pm 0.114\) |
| parent     | \(0.463 \pm 0.008\) | \(-2.522 \pm 0.097\) | \(0.470 \pm 0.005\) | \(-3.427 \pm 0.052\) |
| daughter   | \(0.497 \pm 0.013\) | \(-3.161 \pm 0.119\) | \(0.482 \pm 0.007\) | \(-2.114 \pm 0.065\) |
| \(b\bar{q}\) | \(B^*\) | \(-6.302 \pm 0.357\) | \(0.254 \pm 0.010\) | \(-6.960 \pm 0.572\) |
| parent     | \(0.282 \pm 0.012\) | \(-8.961 \pm 0.497\) | \(0.280 \pm 0.017\) | \(-9.918 \pm 0.726\) |
| daughter   | \(0.263 \pm 0.012\) | \(-8.774 \pm 0.474\) | \(0.262 \pm 0.013\) | \(-7.651 \pm 0.506\) |
| \(b\bar{s}\) | \(B_{s0}^*\) | \(-11.232 \pm 0.590\) | \(0.288 \pm 0.013\) | \(-10.135 \pm 0.575\) |
| parent     | \(0.249 \pm 0.011\) | \(-6.429 \pm 0.419\) | \(0.241 \pm 0.016\) | \(-7.111 \pm 0.621\) |
| daughter   | \(0.277 \pm 0.013\) | \(-9.087 \pm 0.565\) | \(0.272 \pm 0.012\) | \(-9.836 \pm 0.508\) |
| \(b\bar{s}\) | \(B_{s1}^*\) | \(-8.869 \pm 0.512\) | \(0.259 \pm 0.013\) | \(-8.258 \pm 0.495\) |
| parent     | \(0.285 \pm 0.013\) | \(-11.455 \pm 0.576\) | \(0.290 \pm 0.010\) | \(-10.695 \pm 0.468\) |

Acknowledgments

The authors are grateful to V. Matveev, M. Müller-Preussker, V. Savrin, D. Shirkov, P. Uwer and M. Wagner for support and discussions. This work was supported in part by the Deutsche Forschungsgemeinschaft under contract Eb 139/4-1, the Russian Science Support Foundation (V.O.G.) and the Russian Foundation for Basic Research (RFBR), grant No.08-02-00582 (R.N.F. and V.O.G.).

[1] C. Amsler et al. (Particle Data Group), Phys. Lett. B667, 1 (2008).
[2] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 90, 242001 (2003); D. Besson et al. [CLEO Collaboration], Phys. Rev. D 68, 032002 (2003) [Erratum-ibid. D 75, 119908 (2007)]
TABLE V: Fitted parameters of the \((n_r, M^2)\) Regge trajectories for heavy-light mesons.

| Meson | \(\beta\) (GeV\(^{-2}\)) | \(\beta_0\) | Meson | \(\beta\) (GeV\(^{-2}\)) | \(\beta_0\) |
|-------|-----------------|---------|-------|-----------------|---------|
| \(c\bar{q}\) | 0.362 ± 0.011 | -1.322 ± 0.090 | \(D_s\) | 0.320 ± 0.006 | -1.273 ± 0.053 |
| \(D^*\) | 0.375 ± 0.007 | -1.550 ± 0.058 | \(D_s^*\) | 0.334 ± 0.002 | -1.499 ± 0.016 |
| \(D_0^*\) | 0.369 ± 0.002 | -2.141 ± 0.018 | \(D_{s0}^*\) | 0.331 ± 0.001 | -2.082 ± 0.005 |
| \(D_1\) | 0.339 ± 0.006 | -2.072 ± 0.051 | \(D_{s1}\) | 0.309 ± 0.005 | -0.204 ± 0.046 |
| \(D_2\) | 0.378 ± 0.006 | -2.234 ± 0.051 | \(D_{s2}\) | 0.336 ± 0.001 | -2.161 ± 0.001 |
| \(b\bar{q}\) | 0.345 ± 0.009 | -2.101 ± 0.082 | \(b\bar{s}\) | 0.313 ± 0.004 | -2.077 ± 0.042 |
| \(B\) | 0.173 ± 0.007 | -4.913 ± 0.269 | \(B_s\) | 0.171 ± 0.006 | -4.978 ± 0.249 |
| \(B^*\) | 0.176 ± 0.006 | -5.082 ± 0.243 | \(B_s^*\) | 0.172 ± 0.006 | -5.124 ± 0.224 |
| \(B_{0}^*\) | 0.183 ± 0.004 | -6.069 ± 0.151 | \(B_{s0}^*\) | 0.177 ± 0.004 | -6.031 ± 0.179 |
| \(B_{2}\) | 0.172 ± 0.007 | -5.665 ± 0.267 | \(B_{s2}\) | 0.169 ± 0.006 | -5.765 ± 0.252 |

K. Abe et al., Phys. Rev. Lett. 92, 012002 (2004); B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 69, 031101 (2004).

[3] K. Abe et al. [Belle Collaboration], Phys. Rev. D 69, 112002 (2004); J. M. Link et al. [FOCUS Collaboration], Phys. Lett. B 586, 11 (2004); S. Anderson et al. [CLEO Collaboration], Nucl. Phys. A 663, 647 (2000); I. V. Gorelov [CDF Collaboration], J. Phys. Conf. Ser. 9, 67 (2005).

[4] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 97, 222001 (2006); J. Brodzicka et al. [Belle Collaboration], Phys. Rev. Lett. 100, 092001 (2008).

[5] B. Aubert et al. [BABAR Collaboration], arXiv:0908.0806 [hep-ex].

[6] R. Lewis and R. M. Woloshyn, Phys. Rev. D 62, 114507 (2000); G. S. Bali, Phys. Rev. D 68, 071501 (2003); A. Dougal, R. D. Kenway, C. M. Maynard and C. McNeile [UKQCD Collaboration], Phys. Lett. B 569, 41 (2003).

[7] Y. B. Dai, C. S. Huang, C. Liu and S. L. Zhu, Phys. Rev. D 68, 114011 (2003); S. Narison, Phys. Lett. B 605, 319 (2005).

[8] D. Ebert, V. O. Galkin and R. N. Faustov, Phys. Rev. D 57, 5663 (1998) [Erratum-ibid. D 59, 019902 (1999)].

[9] S. Godfrey and N. Isgur, Phys. Rev. D 32, 189 (1985); M. Di Pierro and E. Eichten, Phys. Rev. D 64, 114004 (2001) Yu. S. Kalashnikova, A. V. Nefediev and Yu. A. Simonov, Phys. Rev. D 64, 014037 (2001); S. Godfrey, Phys. Rev. D 72, 054029 (2005).

[10] For a recent review see E. S. Swanson, Phys. Rept. 429, 243 (2006).

[11] W. A. Bardeen, E. J. Eichten and C. T. Hill, Phys. Rev. D 68, 054024 (2003); M. A. Nowak, M. Rho and I. Zahed, Phys. Rev. D 48, 4370 (1993); D. Ebert, T. Feldmann, R. Friedrich and H. Reinhardt, Nucl. Phys. B 434, 619 (1995); D. Ebert, T. Feldmann and H. Reinhardt, Phys. Lett. B 388, 154 (1996).

[12] E. van Beveren and G. Rupp, Phys. Rev. Lett. 91, 012003 (2003); D. S. Hwang and D. W. Kim, Phys. Lett. B 601, 137 (2004); A. M. Badalian, Yu. A. Simonov and M. A. Trusov, Phys. Rev. D 77, 074017 (2008).

[13] T. Barnes, F. E. Close and H. J. Lipkin, Phys. Rev. D 68, 054006 (2003); A. P. Szczepaniak, Phys. Lett. B 567, 23 (2003); A. Faessler, T. Gutsche, V. E. Lyubovitskij and Y. L. Ma, Phys.
Rev. D 76, 114008 (2007).
[14] K. Terasaki, Phys. Rev. D 68, 011501 (2003); L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D 71, 014028 (2005); M. E. Bracco, A. Lozea, R. D. Matheus, F. S. Navarra and M. Nielsen, Phys. Lett. B 624, 217 (2005); J. Vijande, F. Fernandez and A. Valcarce, Phys. Rev. D 73, 034002 (2006) [Erratum-ibid. D 74, 059903 (2006)]; M. V. Carlucci, F. Gianinnuzzi, G. Nardulli, M. Pellicoro and S. Stramaglia, Eur. Phys. J. C 57, 569 (2008).
[15] K. Jansen, C. Michael, A. Shindler and M. Wagner [ETM Collaboration], JHEP 0812, 058 (2008).
[16] D. Ebert, R. N. Faustov and V. O. Galkin, Mod. Phys. Lett. A 20, 1887 (2005); Eur. Phys. J. C 47, 745 (2006).
[17] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D 79, 114029 (2009).
[18] D. Ebert, R.N. Faustov and V.O. Galkin, Phys. Rev. D 67, 014027 (2003).
[19] A.M. Badalian, A.I. Veselov and B.L.G. Bakker, Phys. Rev. D 70, 016007 (2004); Yu.A. Simonov, Phys. Atom. Nucl. 58, 107 (1995).
[20] D. Shirkov, [arXiv:0807.1404 [hep-ph]]; D. V. Shirkov and I. L. Solovtsov, Phys. Rev. Lett. 79, 1209 (1997).
[21] P. Colangelo, F. De Fazio and S. Nicotri, Phys. Lett. B 642, 48 (2006).