Proof of the multi-Regge form of QCD amplitudes with gluon exchanges in the NLA *

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Abstract

The multi–Regge form of QCD amplitudes with gluon exchanges is proved in the next-to-leading approximation. The proof is based on the bootstrap relations, which are required for the compatibility of this form with the s-channel unitarity. We show that the fulfillment of all these relations ensures the Reggeized form of energy dependent radiative corrections order by order in perturbation theory. Then we prove that all these relations are fulfilled if several bootstrap conditions on the Reggeon vertices and trajectory hold true. Now all these conditions are checked and proved to be satisfied.

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1 Introduction

Reggeization of gluons as well as quarks [1]-[7] is one of remarkable properties of Quantum Chromodynamics (QCD). The gluon Reggeization is especially important since cross sections non-vanishing in the high energy limit are related to gluon exchanges in cross channels. A primary Reggeon in QCD turns out to be the Reggeized gluon.

The gluon Reggeization gives the most common basis for the description of high energy processes. In particular, the famous BFKL equation [3] was derived supposing the Reggeization. The most general approach to the unitarization problem is the reformulation of QCD in terms of a gauge-invariant effective field theory for the Reggeized gluon interactions [8].

Let us emphasize that we use the term “Reggeization” in a much stronger sense than the existence of the Reggeon with gluon quantum numbers and trajectory $j(t) = 1 + \omega(t)$ with $\omega(0) = 0$. We use it as the statement that contributions of solely this Reggeon determine the high energy behaviour of all QCD amplitudes in the multi-Regge kinematics (including, of course, the Regge kinematics as a particular case).

The gluon Reggeization was proved [7] in the leading logarithmic approximation (LLA), i.e. in the case of summation of the terms $(\alpha_s \ln s)^n$ in cross-sections of processes at energy $\sqrt{s}$ in the c.m.s., but till now remains a hypothesis in the next-to-leading approximation (NLA), when the terms $\alpha_s (\alpha_s \ln s)^n$ are also kept. Now the BFKL approach, based on the gluon Reggeization, is intensively developed in the NLA; in particular, the BFKL kernel is known now both for forward [9] and non-forward [10] scattering. Moreover, some effective Reggeon-particle vertices, which can be used for the development of the next-to-next-to-leading approximation, are also calculated (see, for instance, Ref. [11]). Meanwhile, there is the statement [12] that the Regge form of QCD amplitudes is violated at the three-loop level in the next-to-leading order (NLO), that means, in our terminology, absence of the gluon Reggeization in the NLA. It makes extremely important the problem of proving or rejecting the Reggeization hypothesis in this approximation.

A possible way of solution of this problem was outlined in Ref. [13]. It is based on the “bootstrap” relations, which are required for the compatibility of the gluon Reggeization with the $s$-channel unitarity. In this paper we present the proof of the gluon Reggeization in the NLA which is obtained in this way. First we show that the fulfillment of the bootstrap relations guarantees the multi–Regge form of QCD amplitudes. Then we demonstrate that an infinite set of these bootstrap relations are fulfilled if several conditions imposed on the Reggeon vertices and the trajectory (bootstrap conditions) hold true. Now all these conditions are proved to be satisfied, and this means that the gluon Reggeization is true, contrary to the statement of Ref. [12].

To be definite, we have to say that our consideration is limited by QCD perturbation theory. Our particles are actually partons — quark and gluons. Moreover, we confine ourselves in the framework of the NLA, and all our assertions and equations given below must be taken with this accuracy.
2 Multi-Regge form of QCD amplitudes

Objects of our investigation are QCD amplitudes in the multi-Regge kinematics (MRK). We call MRK the kinematics where all particles have limited (not growing with $s$) transverse momenta and are combined into jets with limited invariant mass of each jet and large (growing with $s$) invariant masses of any pair of the jets. The MRK gives dominant contributions to cross sections of QCD processes at high energy $\sqrt{s}$. At that in the LLA only gluons are produced and each jet is actually a gluon. In the NLA one of jets can contain a couple of particles (two gluons or quark-antiquark pair). Such kinematics is called also quasi multi-Regge kinematics (QMRK). We use the notion of jets and extend the notion of MRK, so that it includes the QMRK, in order to unify considerations.

Let us consider the amplitude $A_{2\to n+2}$ of the process $A + B \to A' + J_1 + \ldots + J_n + B'$ in the MRK. We will use light-cone momenta $n_1$ and $n_2$, with $n_1^2 = n_2^2 = 0$, $(n_1, n_2) = 1$, and denote $(p_{m2}) \equiv p^+, (p_{m1}) \equiv p^-$. We assume that initial momenta $p_A$ and $p_B$ have predominant components $p_A^+$ and $p_B^-$. For generality it is not assumed that the components $p_{A\perp}$, $p_{B\perp}$ transverse to the $(n_1, n_2)$ plane are zero. Moreover, $A$ and $B$, as well as $A'$ and $B'$, can represent jets. We suppose that rapidities of final jets $k_i$; $y_i = \frac{1}{2} \ln \left( k_i^+ / k_i^- \right)$ decrease with $i$: $y_0 > y_1 > \ldots > y_n > y_{n+1}$; as for $y_0$ and $y_{n+1}$, it is convenient to define them as $y_0 = y_A \equiv \ln \left( \sqrt{2p_A^2} / |q_{\perp}| \right)$ and $y_{n+1} = y_B \equiv \ln \left( |q_{n+1}^\perp| / \sqrt{2p_B^2} \right)$. Notice that $q_i$ indicate the Reggeon momenta and $q_1 = p_{A'} - p_A \equiv q_A$, $q_{n+1} = p_B - p_{B'} \equiv q_B$.

Our aim is to prove that in the NLA the real part of the amplitude $A_{2\to n+2}$ has the multi-Regge form

$$A_{2\to n+2}^R = \tilde{\Gamma}_{A'A}^{R_i} \prod_{i=1}^{n} \frac{e^\omega(q_i)(y_{i-1} - y_i)}{q_i^2} \gamma_{R_iR_{i+1}} \frac{e^\omega(q_{n+1})(y_n - y_{n+1})}{q_{n+1}^2} \tilde{\Gamma}_{B'B}^{-R_{n+1}}.$$  \hspace{1cm} (1)

Here $\omega(q)$ is called gluon Regge trajectory, $\tilde{\Gamma}_{B'B}^R$ and $\tilde{\Gamma}_{A'A}^R$ are the scattering vertices, i.e. the effective vertices for $B \to B'$ and $A \to A'$ transitions due to interaction with Reggeized gluons $R$; $\gamma_{R_iR_{i+1}}$ are the production vertices, i.e. the effective vertices for production of jets $J_i$ with momenta $k_i = q_{i+1} - q_i$ in $R_{i+1} \to R_i$ transition of Reggeons with momenta $q_{i+1}$ and $q_i$. We use for particles and Reggeons notations which accumulate all their quantum numbers. All Reggeon vertices, as well as the gluon trajectory, are known now with the required accuracy (see Ref. \[11\] and references therein; the scattering vertices $\tilde{\Gamma}_{B'B}^R$ and $\tilde{\Gamma}_{A'A}^R$ in Eq. (1) differ by the factors $2p_B^+$ and $2p_A^-$ correspondingly from the analogous quantities used there).

Remind that as compared with ordinary particles Reggeons possess an additional quantum number, the signature, which is negative for the Reggeized gluon. In each order of perturbation theory amplitudes with negative signature do dominate, owing to the cancellation of leading logarithmic terms in amplitudes with positive signature which become pure imaginary in the leading order for them (which coincides with the next-to-leading for negative signature). We emphasize that only the real parts of the amplitudes have the representation (1). Only these parts have such a simple form, and only these parts are given by the Reggeized gluon contributions. As for imaginary parts, they come into amplitudes both from the parts with positive and negative signatures. They can be calculated using the unitarity relations and the amplitudes (1). It is well known from the BFKL equation for the
Pomeron exchange that they are complicated even for elastic amplitudes.

Let us show that the amplitudes \( \Pi \) have negative signatures in all \( q_i \)–channels. In order to construct amplitudes with definite signatures one needs to perform the “signaturization”. In general the signaturization is not a simple task. It requires partial-wave decomposition of amplitudes in cross-channels with subsequent symmetrization (anti-symmetrization) in “scattering angles” and analytical continuation into the \( s \)–channel. The procedure is relatively simple only in the case of elastic scattering of spin-zero particles. At that, generally speaking, even in this case the amplitudes with definite signatures cannot be expressed in terms of physical amplitudes related by crossing. Fortunately, at high energy the signaturization can be easily done not only for elastic, but in the MRK also for inelastic amplitudes, for particles with spin as well as for the spin-zero ones. The signaturization (as well as crossing relations) is naturally formulated for “truncated” amplitudes, i.e. for amplitudes with omitted wave functions (polarization vectors and Dirac spinors). The crucial points are that in the MRK all energy invariants \( s_{i,j} = (k_i + k_j)^2 \) are large and that they are determined only by the longitudinal components of momenta \( s_{i,j} = 2k_i^+ k_j^- \). Due to largeness of \( s_{i,j} \) signaturization in the \( q_l \)–channel means symmetrization (anti-symmetrization) with respect to the substitution \( s_{i,j} \leftrightarrow -s_{i,j}, i < l \leq j \). Since \( s_{i,j} \) are determined by longitudinal components, this substitution is equivalent to the replacement \( k_i^+ \leftrightarrow -k_i^+, i < l \), \( p_A^+ \leftrightarrow -p_A^- \) (or, equivalently, \( k_j^+ \leftrightarrow -k_j^+, j \geq l \), \( p_B^+ \leftrightarrow -p_B^- \)) in the truncated amplitudes without change of transverse components. Note that such substitution does not violate the total momentum conservation due to the strong ordering of the longitudinal components. At that all particles remain on their mass shell, so that the substitution is equivalent to the transition into the cross-channel. Note that the limitation by the real parts in the form \( \Pi \) means that the Regge factors remain unchanged under the crossing.

In order to understand the behaviour of the amplitudes \( \Pi \) under the signaturization it is convenient to take the production vertices \( \gamma^J_{R_iR_{i+1}} \) in the physical light–cone gauges with gauge–fixing vectors \( n_2 \) or \( n_1 \). Then it becomes evident that these vertices do not depend on longitudinal components of momenta and remain unchanged under the crossing, whereas the scattering vertices entering in the form \( \Pi \) change their signs due to the discussed factors \( p_A^+ \) and \( p_B^- \). It ensures negative signature in all \( q_i \)–channels.

The factorized form of QCD amplitudes in the MRK was proved at the Born level using the \( t \)–channel unitarity and analyticity \( [3] \). Their Reggeization was first derived in the LLA on the basis of the direct calculations at the three-loop level for elastic amplitudes and the one-loop level for one-gluon production amplitudes. Later it was proved \( [7] \) in the LLA for all amplitudes at arbitrary number of loops with the help of bootstrap relations. At NLA the Reggeization remained a hypothesis till now.

The hypothesis is extremely powerful since an infinite number of amplitudes is expressed in terms of the gluon Regge trajectory and several Reggeon vertices.
3 Bootstrap relations

The proof of the form (1) is based on the s-channel unitarity, which provides us with the discontinuities \( \text{disc}_{s_{i,j}} \) of the signaturized amplitude \( A^S_{2 \rightarrow n+2} \) in the channels \( s_{i,j} = (k_i + k_j)^2 \). Note that generally speaking these discontinuities are not pure imaginary in the NLA, since a discontinuity in one of the channels can have, in turn, a discontinuity in another channel. But it is clear that these double discontinuities are sub-sub-leading, so that we will neglect them in the following.

For elastic amplitudes the connection of real parts of the amplitudes and their discontinuities is well known. Unfortunately, it is quite not so for inelastic amplitudes. Analytical properties of the production amplitudes are very complicated even in the MRK [15]. But, fortunately, if in the MRK we confine ourselves to the NLA, these properties are greatly simplified and allow us [13] to express partial derivatives \( \partial/\partial y_j \) of the amplitudes, considered as functions of rapidities \( y_j \) (\( j = 0, \ldots, n+1 \)) and transverse momenta, in terms of the discontinuities of the signaturized amplitudes:

\[
\frac{1}{\pi i} \left( \sum_{l=j+1}^{n+1} \text{disc}_{s_{j,l}} - \sum_{l=0}^{j-1} \text{disc}_{s_{l,j}} \right) A^S_{2 \rightarrow n+2}/(p_A^+p_B^-) = \frac{\partial}{\partial y_j} A^S_{2 \rightarrow n+2}(y_i)/(p_A^+p_B^-) \quad (2)
\]

Note that taking the sum of the equations (2) over \( j \) from 0 to \( n+1 \) it is easy to see from Eq. (2) that \( A^S_{2 \rightarrow n+2} \) depends only on differences of the rapidities \( y_i \), as it must be. The division by \( (p_A^+p_B^-) \) is performed in Eq. (2) in order to differentiate the rapidity dependence of radiative corrections only.

Equalities (2) can be easily proved using the Steinmann theorem [16], or, more definitely, the statement [13] that the amplitudes can be presented as a sum of contributions corresponding to various sets of the \( n+1 \) non-overlapping channels \( s_{i,j} \), \( i < j \), \( k = 1, \ldots, n+1 \); at that each of the contributions can be written as a signaturized series in logarithms of the energy variables \( s_{i,j} \) with coefficients which are a real function of transverse momenta. Remind that two channels \( s_{i,j_1} \) and \( s_{i_2,j_2} \) are called overlapping if either \( i_1 < i_2 \leq j_1 < j_2 \), or \( i_2 < i_1 \leq j_2 < j_1 \). Since scattering amplitudes enter in the relations (2) linearly and uniformly, it is sufficient to prove these relations separately for the contribution of one of the sets. Now two observations are important: first, we need not to consider the coefficients depending on transverse momenta neither calculating the discontinuities, nor calculating the derivatives over \( y_j \) in Eq. (2); and second, the energy variables \( s_{i,j_k} \) entering in each set are independent, i.e. there are no relations between the differences \( y_{i_k} - y_{j_k} \) for non-overlapping channels \( s_{i,j_k} \); this means, in particular, that we need to consider only leading and next-to leading orders in logarithms of these variables.

Therefore, it is sufficient to prove the equalities (2) with the NLO accuracy for the symmetrized products

\[
SP = \hat{S} \prod_{i<j=1}^{n+1} \left( \frac{s_{ij}}{|k_{i\perp}| |k_{j\perp}|} \right)^{\alpha_{ij}} \quad (3)
\]

instead of \( A^S_{2 \rightarrow n+2}/(p_A^+p_B^-) \). Here the exponents \( \alpha_{ij} \sim \alpha_S \) are different from zero only for some set of non-overlapping channels and are arbitrary in all other respects; \( \hat{S} \) means sym-
metrization with respect to simultaneous change of signs of all $s_{i,j}$ with $i < k \leq j$, performed independently for each $k = 1, \ldots, n + 1$. Indeed, due to the above mentioned arbitrariness of $\alpha_{ij}$ the fulfilment of the equalities (2) for $SP$ guarantees it for any logarithmic series.

Since we consider only real parts of discontinuities in the invariants $s_{i,j}$, calculating the discontinuity of $SP$ in one of $s_{i,j}$ at real $\alpha_{ij} \sim \alpha_S$ we can neglect signs of the other invariants not only in the leading, but in the NLO, so that we have

$$\frac{1}{-\pi i} \left( \sum_{l=j+1}^{n+1} \text{disc}_{s_{j,l}} - \sum_{l=0}^{j-1} \text{disc}_{s_{l,j}} \right) SP = \left( \sum_{l=j+1}^{n+1} \alpha_{jl} - \sum_{l=0}^{j-1} \alpha_{lj} \right) SP. \quad (4)$$

On the other hand, taking into account that

$$\left( \frac{s_{i,j}}{|k_{i\perp}||k_{j\perp}|} \right)^{\alpha_{ij}} = e^{\alpha_{ij}(y_i-y_j)}, \quad (5)$$

we have for the real part

$$SP = e^{\sum_{i<j}^{n+1} \alpha_{ij}(y_i-y_j)} \left( 1 + O(\alpha^2) \right), \quad (6)$$

so that, with the NLO accuracy

$$\frac{\partial}{\partial y_j} SP = \left( \sum_{l=j+1}^{n+1} \alpha_{jl} - \sum_{l=0}^{j-1} \alpha_{lj} \right) SP. \quad (7)$$

It is clear from Eqs. (4) and (7) that the equalities (2) are fulfilled.

The important point is that the relations (2) give a possibility to find in the NLA all MRK amplitudes in all orders of coupling constant, if they are known (for all $n$) in the one-loop approximation. Indeed, these relations express all partial derivatives of the real parts at some number of loops through the discontinuities, which can be calculated using the $s$-channel unitarity in terms of amplitudes with a smaller number of loops; moreover in the NLA only the MRK is important and only real parts of the amplitudes do contribute in the unitarity relations. To find $A_{2\to n+2}^S$ besides the derivatives determined by Eq. (2) suitable initial conditions are required; but since they can be taken at fixed $y_i$, in the NLA they are necessary only with one-loop accuracy. Therefore the relations (2) together with the one-loop approximation for the MRK amplitudes unambiguously determine all $A_{2\to n+2}^S$.

Thus, in order to prove the multi-Regge form (1) in the NLA it is sufficient to know that it is valid in the one-loop approximation and satisfies the equalities (2), where the discontinuities are calculated using this form in the unitarity relations.

Substituting Eq. (1) in the R.H.S. of Eq. (2), we obtain the relations

$$\frac{1}{-\pi i} \left( \sum_{l=j+1}^{n+1} \text{disc}_{s_{j,l}} - \sum_{l=0}^{j-1} \text{disc}_{s_{l,j}} \right) A_{2\to n+2}^S = (\omega(t_{j+1}) - \omega(t_j)) A_{2\to n+2}^R, \quad (8)$$

which are called bootstrap relations. The discontinuities in these relations must be calculated using the $s$-channel unitarity and the multi-Regge form of the amplitudes (1). Evidently,
there is an infinite number of the bootstrap relations, because there is an infinite number of amplitudes $A_{2\to n+2}^R$. At the first sight, it seems a miracle to satisfy all of them, since all these amplitudes are expressed through several Reggeon vertices and the gluon Regge trajectory. Moreover, it is quite nontrivial to satisfy even some definite bootstrap relation for a definite amplitude, because it connects two infinite series in powers of $y_i$, and therefore it leads to an infinite number of equalities between coefficients of these series.

In fact, two miracles must occur in order to satisfy all the bootstrap relations: first, for each particular amplitude $A_{2\to n+2}^R$ it must be possible to reduce the bootstrap relation to a limited number of restrictions (bootstrap conditions) on the gluon trajectory and the Reggeon vertices, and secondly, starting from some $n = n_0$ these bootstrap conditions must be the same as obtained for amplitudes with $n < n_0$. Finally, all bootstrap conditions must be satisfied by the known expressions for the trajectory and the vertices.

It is necessary to add here that the amplitude in the R.H.S. of Eq. (8) contains only colour octets in each of the $q_{i}$–channel. The discontinuities in the L.H.S., taken separately, along with the colour octet hold other representations of the colour group, which cancel in the sum.

4 Calculation of the discontinuities

Each of the $s_{i,j}$–channel discontinuity, being expressed with the help of the $s$–channel unitarity through the product of amplitudes of the multi-Regge type $|\Pi\rangle$, contains two Reggeons in the channels $q_i$ at $i < l \leq j$. As an example, the $s_{j,n+1}$–channel discontinuity is presented schematically in Fig. 1. Large blobs there stand for account of the signaturization. In order to present the discontinuities in a compact way it is convenient to use operator notations in the transverse momentum and colour space. We will use also notations which accumulate all quantum numbers. Thus, $\langle G_i G_j |$ and $|G_i G_j \rangle$ are bra– and ket–vectors for the $t$–channel states of two Reggeized gluons with transverse momenta $r_{i\perp}$ and $r_{j\perp}$ and colour indices $c_i$ and $c_j$ correspondingly. It is convenient to distinguish the states $|G_i G_j \rangle$ and $|G_j G_i \rangle$. We will associate the first of them with the case when the Reggeon $G_i$ is contained in the amplitude with initial particles (in the lower part of Fig.1 for the example depicted there), and the second with the case when it is contained in the amplitude with final particles (in the upper part of Fig.1). It is convenient to introduce the scalar product

$$\langle G_i G_j |G'_i G'_j \rangle = r_{i\perp}^2 r_{j\perp}^2 \delta(r_{i\perp} - r'_{i\perp})\delta(r_{j\perp} - r'_{j\perp}) \delta_{c ic'_i} \delta_{cjc'_j}. \quad (9)$$

These states are complete, and with the scalar product (9) the completeness means

$$\langle \Psi | \Phi \rangle = \int \frac{d^{D-2}r_{1\perp} d^{D-2}r_{2\perp}}{r_{1\perp}^2 r_{2\perp}^2} \langle \Psi | G_1 G_2 \rangle \langle G_1 G_2 | \Phi \rangle. \quad (10)$$

In the following we will also use the letters $G_i$ instead of $c_i$.

Let us discuss the calculation of the discontinuities. Our goal is to write them as matrix elements of operator expressions, consisting of the operators $\hat{J}_i$ for jet-$J_i$ production and of
the operator $\hat{K}$ for the Reggeon-Reggeon interaction kernel, between bra- and ket-vectors, describing either particle-particle or Reggeon-particle transitions (actually “particle” here can denote a jet, as it was already mentioned) due to interaction with Reggeized gluons ($R_j \rightarrow J_j$ and $B \rightarrow B'$ transitions in Fig.1). We will call these states particle-particle or Reggeon-particle impact-factors.

To calculate the discontinuity we need to convolute Reggeon vertices with account of the signaturization and to integrate over momenta of particles in intermediate states. Since the convolutions of the Reggeon vertices depend on the transverse components of momenta only, the signaturization is reduced to anti-symmetrization with respect to the attached Reggeon lines. In order to escape double counting in the NLA we introduce an auxiliary parameter $\Delta \gg 1$ which constrains the difference in rapidities of particles belonging to one jet. Note that the largeness of $\Delta$ is numerical, but not parametrical (related to $s$), so that terms of order $\alpha_S \Delta$ are considered as sub-leading. Of course, the final answer must not depend on $\Delta$.

We denote the momenta of the intermediate jets $l_\alpha$, their rapidities $z_\alpha$, with $z_\alpha = \frac{1}{2} \ln \left( \frac{r_\perp}{l_\alpha} \right)$. Rapidities of the intermediate jets not related neither to jet-jet or Reggeon-jet transitions, nor to the final jet production (in the example depicted in Fig.1 not contained in the blobs) are confined in the intervals $[y_{k+1} + \Delta, y_k - \Delta]$, where for the $s_{ij}$-channel discontinuity $k$ takes values from $i$ to $j - 1$. In each interval we need to perform integration over rapidities and summation over number of jets from 0 to $\infty$. Denoting $r_{1\perp}$ and $r_{2\perp}$ the momenta of Reggeons between the jets $l_\alpha$ and $l_{\alpha+1}$ we write all corresponding Regge
factors in the same form \( e^{(\omega(r_{1\perp}) + \omega(r_{2\perp})) (z_n - z_{n+1})} \). Instead, the Regge factors for the Reggeons interacting either with the scattering particles or Reggeons, or with the produced particles (in Fig.1 attached to the blobs) are not uniform. In order to unify them we include uniformity violating multipliers in the definitions of jet-production operators and impact-factors for particle-particle and Reggeon-particle transitions. After that the two-Reggeon exchange in the \( q_J \)-channel is represented by the operator

\[
\hat{G}(Y_{j})^{\Delta} = \sum_{n=0}^{\infty} \int_{y_{j}+n\Delta}^{y_{j+1}-\Delta} e^{\hat{\Omega}(y_{j+1}-z_{1})} \hat{K}_{r}^{\Delta} dz_{1} \int_{y_{j}+(n-1)\Delta}^{z_{1}-\Delta} d\phi_{1} e^{\hat{\Omega}(z_{1}-z_{2})} \hat{K}_{r}^{\Delta} \cdots \int_{y_{j}+\Delta}^{z_{n+1}-\Delta} d\phi_{n} e^{\hat{\Omega}(z_{n}-y_{j})} \hat{K}_{r}^{\Delta},
\]

(11)

where the term for \( n = 0 \) is equal to \( e^{\hat{\Omega}Y_{j}} \), with \( Y_{j} = y_{j-1} - y_{j} \), \( \hat{\Omega} = \omega(\hat{r}_{1}) + \omega(\hat{r}_{2}) \).

The operator \( \hat{K}_{r}^{\Delta} \) takes into account production of the intermediate jets \( J \) with intervals of particle rapidities \( \Delta_{j} \) in them less than \( \Delta \):

\[
\langle G_{1}G_{2}|\hat{K}_{r}^{\Delta}|G_{1}'G_{2}' \rangle = \delta(r_{1\perp} + r_{2\perp} - r_{1\perp}' - r_{2\perp}') \sum_{J} \int \gamma_{1}^{J} \gamma_{2}^{J} \gamma_{J}^{G_{2}G_{2}'} \frac{d\phi_{J}}{2(2\pi)^{D-1}} \theta(\Delta - \Delta_{J}).
\]

(12)

Here the sum is taken over all discrete quantum numbers; \( \gamma_{J}^{G_{2}G_{2}'} \) is the effective vertex for absorption of the jet \( J \) in the Reggeon transition \( G_{2}' \to G_{2} \). It is related to \( \gamma_{J}^{G_{2}G_{2}'} \) by the crossing described above (i.e. by the change of signs of longitudinal momenta and the corresponding change of wave functions). Then, we have

\[
d\phi_{J} = \frac{dk_{J}^{2}}{2\pi} (2\pi)^{D} \delta(k_{J} - \sum p_{i}) \prod_{i} \frac{d^{D-1}p_{i}}{(2\pi)^{D-1} 2\epsilon_{i}}
\]

(13)

for a jet \( J \) with total momentum \( k_{J} \) consisting of particles with momenta \( p_{i} \). The integration limits in Eq. (11) correspond to the limitation on the intervals of particle rapidities in Eq. (12).

As it was already mentioned, terms of order \( \alpha_{s}\Delta \) are sub-leading, therefore we need to retain in Eq. (11) only terms linear in \( \Delta \) with coefficients of order \( \alpha_{s} \). With this accuracy we can write \( \hat{G}(Y_{j})^{\Delta} = (1 - \hat{K}_{r}^{B\Delta})\hat{G}(Y_{j})^{\Delta} \hat{K}_{r}^{B\Delta} \); the superscript \( B \) here and below denotes leading order, so that \( \hat{K}_{r}^{B} \) is given by \( O(\alpha_{s}) \) terms in Eq. (12), and \( \hat{G}(Y_{j}) \) is obtained from Eq. (11) by the omission of \( \Delta \) in the integration limit and the replacement \( \hat{K}_{r}^{\Delta} \to \hat{K}_{r} \), where

\[
\hat{K}_{r} = \hat{K}_{r}^{\Delta} - \hat{K}_{r}^{B}\hat{K}_{r}^{B\Delta}.
\]

(14)

We include the multipliers \( (1 - \hat{K}_{r}^{B\Delta}) \) in definitions of jet-production operators and impact-factors for particle-particle and Reggeon-particle transitions. Then the two-Reggeon exchange in the \( q_J \)-channel is represented by the operator \( \hat{G}(Y_{j}) \). It is easy to see that it obeys the equation \( d\hat{G}(Y)/dY = \hat{K}\hat{G}(Y) \), where

\[
\hat{K} = \omega(\hat{r}_{1}) + \omega(\hat{r}_{2}) + \hat{K}_{r}.
\]

(15)

Using the initial condition \( \hat{G}(0) = 1 \) we obtain

\[
\hat{G}(Y) = e^{\hat{K}Y}.
\]

(16)
With account of the terms discussed before Eq. (11) and after Eq. (14) the impact-factor for the \( B \to B' \) transition is defined as

\[
|\bar{B}'B\rangle = |\bar{B}'B\rangle^\Delta - \left( \omega^B(\hat{r}_1) \ln \frac{|\hat{r}_1\perp|}{q_B\perp} + \omega^B(\hat{r}_2) \ln \frac{|\hat{r}_2\perp|}{q_B\perp} + \hat{K}^B \Delta \right) |\bar{B}'B\rangle^A,
\]

(17)

where

\[
\langle G_1G_2|\bar{B}'B\rangle^\Delta = \delta(q_B\perp - r_{1\perp} - r_{2\perp}) \frac{1}{2p_B} \sum_{B'} \int \left( \Gamma^G_{BB'} \Gamma^G_{B'B} - \Gamma^G_{BB'} \Gamma^G_{B'B} \right) d\phi_{B'} \prod_l \theta(\Delta - (z_l - y_B)).
\]

(18)

Here \( q_B = p_B - p_{B'} \) and \( z_l \) are the rapidities of particles in intermediate jets. The terms with \( \omega^B \) in Eq. (17) takes into account the difference of the Regge factors related to the Reggeons interacting with the particles \( B \) and \( B' \) and the “uniform” factors used in the series (11) for \( \hat{G}(Y_{n+1})^\Delta \). The term with \( \hat{K}^B \) in Eq. (17) comes from the relation between \( \hat{G}(Y_{n+1})^\Delta \) and \( \hat{G}(Y_{n+1}) \). Note that in the case when \( B \) or \( B' \) is a two-particle jet, only the first term must be kept in Eq. (17); moreover, only the Born approximation for this term must be taken in Eq. (18).

It is clear that for the impact-factor of the \( A \to A' \) transition we have

\[
\langle A'\bar{A} \rangle = \langle A'\bar{A} \rangle^\Delta - \langle A'\bar{A} \rangle^B \left( \omega^B(\hat{r}_1) \ln \frac{|\hat{r}_1\perp|}{q_A\perp} + \omega^B(\hat{r}_2) \ln \frac{|\hat{r}_2\perp|}{q_A\perp} + \hat{K}^B \Delta \right),
\]

(19)

\[
\langle A'\bar{A}|G_1G_2\rangle^\Delta = \delta(q_A\perp - r_{1\perp} - r_{2\perp}) \frac{1}{2p_A} \sum_{\bar{A}} \int \left( \Gamma^G_{\bar{A}A'} \Gamma^G_{\bar{A}A} - \Gamma^G_{\bar{A}A'} \Gamma^G_{\bar{A}A} \right) d\phi_{\bar{A}} \prod_l \theta(\Delta - (y_A - z_l)),
\]

(20)

where \( q_A = p_{A'} - p_A \).

The anti-symmetrization with respect to the permutation \( G_1 \leftrightarrow G_2 \) in Eqs. (18) and (20) takes into account the signaturization. The important fact is that due to the signaturization only the antisymmetric colour octet survives from all possible colour states of the two Reggeons \( G_1 \) and \( G_2 \). For quark and gluon impact-factors it follows from results of Ref. [17]. For the case when some state is a two-particle it can be seen from results presented in Ref. [14].

Accordingly, the Reggeon-particle impact-factors are defined as

\[
|\bar{J}_iR_{i+1}\rangle = |\bar{J}_iR_{i+1}\rangle^\Delta - \left( (\omega(q_{i+1}) - \omega(\hat{r}_1)) \ln \frac{|k_i\perp|}{(q_{i+1}\perp - \hat{r}_1\perp)} \right) - \omega(\hat{r}_2) \ln \frac{|k_i\perp|}{r_2\perp} + \hat{K}^B \Delta \right) |\bar{J}_iR_{i+1}\rangle^B,
\]

(21)

\[
\langle G_1G_2|\bar{J}_iR_{i+1}\rangle^\Delta = \delta(q_{(i+1)\perp} - k_i\perp - r_{1\perp} - r_{2\perp}) \frac{1}{2k_j} \sum_j \int \left( \gamma^j_{G_1R_{i+1}} \Gamma^G_{j,J} - \gamma^j_{G_2R_{i+1}} \Gamma^G_{j,J} \right) d\phi_j \prod_l \theta(\Delta - (z_l - y_i)).
\]

(22)
and
\[
\langle J_i R_i | = \langle J_i R_i | \Delta - \langle J_i R_i | \bigg( \omega(q_i) - \omega(\hat{r}_i) \bigg) \ln \left| \frac{k_{i\perp}}{(q_{i\perp} - \hat{r}_{i\perp})} \right| \\
- \omega(\hat{r}_2) \ln \left| \frac{k_{i\perp}}{\hat{r}_{2\perp}} \right| + \hat{K}_{r} \Delta \bigg),
\]
(23)

\[
\langle J_i R_i | G_i G_2 | \Delta = \delta(r_{1\perp} + r_{2\perp} - q_{i\perp} - k_{i\perp}) \\
\times \frac{1}{2k_{\perp j}^{+}} \sum \int \left( \gamma_{R_i G_1} G_{G_2} G_{J, j} - \gamma_{R_i G_2} G_{G_1} G_{J, j} \right) \frac{d\phi_j}{\prod i} \theta(\Delta - (y_i - z_i)).
\]
(24)

At last, the operators \( \hat{J}_i \) for production of jets \( J_i \) are defined as
\[
\hat{J}_i = \hat{J}_i \Delta - (\hat{K}_{r} \hat{J}_i + \hat{J}_i \hat{K}_{r}) \Delta, \quad \langle G_i G_2 | \hat{J}_i \Delta | G_i G_2 \rangle =
\]
\[
\delta(r_{1\perp} + r_{2\perp} - k_{i\perp} - r_{1\perp} - r_{2\perp}) \left[ \gamma_{J_i G_i} G_{J, i} \delta(r_{1\perp} - r_{1\perp}) \delta(r_{2\perp} - r_{2\perp}) + \gamma_{J_i G_i} G_{J, i} \delta(r_{1\perp} - r_{1\perp}) r_{1\perp} \delta(r_{2\perp} - r_{2\perp}) \right.
\]
\[
+ \sum G \int \frac{dG}{y_{i, - \Delta}} \frac{\delta_{y_{i, \Delta}}}{\Gamma_{y_{i, \Delta}}} \left( \gamma_{J_i G_i} G_{J, i} \delta(r_{1\perp} - r_{2\perp}) + \gamma_{J_i G_i} G_{J, i} \delta(r_{1\perp} - r_{2\perp}) \right). \quad (25)
\]

Here the last term appears only in the case when \( J_i \equiv G_i \) is a single gluon, the sum in this term goes over quantum numbers of the intermediate gluon \( G \) and the vertices must be taken in the Born approximation. At that \( \gamma_{J_i G_i} \) is the vertex for production of the jet consisting of the gluons \( G_i \) and \( G \), \( \gamma_{G_i G_i} G_{G_i} \) is the vertex for absorption of the gluon \( G \) and production of the gluon \( G_i \) at the \( G_2 \rightarrow G_2 \) transition; it can be obtained from \( \gamma_{G_i G_i} G_{G_i} \) by crossing with respect to the gluon \( G \).

With the definitions given above we obtain
\[
-4i(2\pi)^{D-2} \delta(q_{i+1} - q_{i-1} - \sum_{l=1}^{l=j} k_{l\perp}) \text{disc}_{s_{l=1}^{j}} A_{2 \rightarrow n+2}^S = \Gamma_{R_i A} R_i e^{\omega(q_i)(y_{i-1} - y_i)} q_{i\perp}^{2} \times
\]
\[
\times \left( \prod_{l=2}^{i} \gamma_{R_{l-1} R_l} \right) e^{\omega(q_i)(y_{i-1} - y_i)} \langle J_i R_i | \left( \prod_{l=1}^{j} e^{\hat{\mathcal{K}}(y_{l-1} - y_l)} \hat{J}_l \right) e^{\hat{\mathcal{K}}(y_j - y_i)} | J_j R_{j+1} \rangle \times
\]
\[
\times \left( \prod_{l=j+1}^{n} e^{\omega(q_i)(y_{i-1} - y_i)} q_{l\perp}^{2} \gamma_{R_l R_{l+1}} \right) \frac{e^{\omega(q_{n+1})(y_{n-1} - y_{n+1})} q_{n+1\perp}^{2}}{\Gamma_{R_n B} B' B}. \quad (26)
\]

If \( i = 0 \) we must omit all factors on the left from \( \langle J_0 R_0 | \) and substitute \( \langle J_0 R_0 | \) with \( \langle A' A | \), \( q_0 + k_0 \) with \( p_{A'} - p_A \); in the case \( j = n+1 \) we must omit all factors on the right from \( | J_{n+1} R_{n+2} \rangle \) and perform the substitutions \( | J_{n+1} R_{n+2} \rangle \rightarrow | \vec{B}' B \rangle, \ q_{n+2} - k_{n+1} \rightarrow p_B - p_{B'} \).
5 Bootstrap conditions

Let us prove, using the representation (26) for the discontinuities, that an infinite number of the bootstrap relations (8) are satisfied if the following bootstrap conditions are fulfilled: the impact-factors for scattering particles satisfy equations

\[ |\bar{B}'B\rangle = g\Gamma^{R_n}_{B'B} |R_\omega(q_{B\perp})\rangle, \quad \langle A'\bar{A}| = g\Gamma^{R_{A'}}_{A'A} \langle R_\omega(q_{A\perp})|, \]

where \(|R_\omega(q_{\perp})|\) and \(|R_\omega(q_{\perp})\rangle\) are the bra– and ket– vectors of the universal (process independent) eigenstate of the kernel \(\hat{K}\) with the eigenvalue \(\omega(q_{\perp})\),

\[ \hat{K}|R_\omega(q_{\perp})\rangle = \omega(q_{\perp})|R_\omega(q_{\perp})\rangle, \quad \langle R_\omega(q_{\perp})| = \langle R_\omega(q_{\perp})|\omega(q_{\perp}), \]

and the normalization is fixed through the scalar product

\[ \frac{g^2 q^2_{\perp}}{2(2\pi)^{D-1}} \langle R_\omega(q_{\perp})|R_\omega(q_{\perp})\rangle = -\delta(q_{\perp} - q'_{\perp})\omega(q_{\perp}); \]

the Reggeon-gluon impact-factors and the gluon production vertices satisfy the equations

\[ \hat{J}_i|R_\omega(q_{(i+1)\perp})\rangle g q^2_{(i+1)\perp} + |\bar{J}_iR_{i+1}\rangle = |R_\omega(q_i\perp)\rangle g \gamma^{J_i}_{R_iR_{i+1}}, \]

\[ g q^2_{\perp} \langle R_\omega(q_{(i+1)\perp})|\hat{J}_i + \langle J_iR_i\rangle = g \gamma^{J_i}_{R_iR_{i+1}} \langle R_\omega(q_{(i+1)\perp})|, \]

where \(q_{(i+1)\perp} = q_{\perp} + k_{i\perp}\). Actually the second of Eqs. (27), (28) and (30) are not independent since bra– and ket–vectors are related with each other by the change of + and − momenta components.

The bootstrap conditions (27) and (28) are known since a long time [18]–[20] and have been proved to be satisfied [17]–[23]. The bootstrap relations for elastic amplitudes require only a weak form of the conditions (27) and (28), namely only the projection of these conditions on \(|R_\omega\rangle\). It was recognized [13] that the bootstrap relations for one-gluon production amplitudes besides the conditions (27) and (28) require also a weak form of the condition (30). Thus, the bootstrap relations for one-gluon production amplitudes play a twofold role: they strengthen the conditions imposed by the elastic bootstrap and give a new one. One could expect that the history will repeat itself upon addition of each next gluon in the final state. If it were so, we would have to consider the bootstrap relations for production of an arbitrary number of gluons and would obtain an infinite number of bootstrap conditions. Fortunately, the history is repeated only partly: it was shown [24] that already the bootstrap relations for two-gluon production require the strong form of the last condition (i.e. Eq. (30)) and do not require new conditions.

The bootstrap conditions with two-particle jets are required in the NLA only with the Reggeon vertices taken in the Born approximation. They were checked and proved to be satisfied in Refs. [25] and [14]. After that only the condition (30) remained not evident. Its fulfilment was proved recently [26]. Thus, now it is shown that all bootstrap conditions are fulfilled.

To prove that the bootstrap conditions (27), (30) secure the fulfilment of all infinite set of the bootstrap relations (8), consider first the terms with \(l = n\) and \(l = n + 1\) in the
representation (8). Using this last representation for the discontinuities and applying the bootstrap conditions (27) and (28) to the \( s_{k,n+1} \)-channel discontinuity, we obtain that the sum of the discontinuities in the channels \( s_{k,n} \) and \( s_{k,n+1} \) contains

\[
g \hat{J}_n |R_\omega(q_{(n+1)\perp})\rangle + |\bar{J}_n R_{n+1}\rangle \frac{1}{q_{(n+1)\perp}^2} = |R_\omega(q_{n\perp})\rangle g \gamma_{R_n R_{n+1}}^{J_n} \frac{1}{q_{(n+1)\perp}^2}. \tag{31}
\]

The equality here follows from the bootstrap condition (30). Now the procedure can be repeated: we can apply to this sum the bootstrap condition (28), and to the sum of the obtained result with the \( s_{k,n-1} \)-channel discontinuity again Eq. (30). Thus all sum over \( l \) from \( j+1 \) to \( n+1 \) in the representation (8) is reduced to one term. A quite analogous procedure (with the use of the bootstrap conditions for bra-vectors) can be applied to the sum over \( l \) from 0 to \( j-1 \). As a result we have that the left part of the representation (8) with the coefficient \(-2(2\pi)^{D-1}\delta(q_{(j+1)\perp} - q_{j\perp} - k_{j\perp})\), where \( q_{(j+1)\perp} = p_{B\perp} - p_{B'\perp} - \sum_{l=j-1}^{n} k_{l\perp} \) and \( q_{j\perp} = p_{A\perp} - p_{A\perp} + \sum_{l=1}^{j-1} k_{l\perp} \), can be obtained from the R.H.S. of the multi-Regge form (11) by the replacement

\[
\gamma_{R_j R_{j+1}}^{J_j} \rightarrow \langle J_j R_j | R_\omega(q_{(j+1)\perp})\rangle g \bar{q}_{J_j}^2 \langle R_\omega(q_{j\perp})\rangle |\bar{J}_j R_{j+1}\rangle. \tag{32}
\]

Taking the difference of the first equality in the condition (30) for \( i = j \) multiplied by \( g \bar{q}_{j\perp}^2 \langle R_\omega(q_{j\perp})\rangle \) and the second equality multiplied by \( |R_\omega(q_{(j+1)\perp})\rangle g \bar{q}_{J_{j+1}}^2 \) and using the normalization (29) we obtain

\[
\langle J_j R_j | R_\omega(q_{j+1})\rangle g \bar{q}_{J_{j+1}}^2 \langle R_\omega(q_j)\rangle |\bar{J}_j R_{j+1}\rangle = -2(2\pi)^{D-1}\delta(q_{(j+1)\perp} - q_{j\perp} - k_{j\perp})
\]

\[
\times \langle \omega(q_{j+1}) - \omega(q_j)\rangle \gamma_{R_j R_{j+1}}^{J_j}. \tag{33}
\]

That concludes the proof.

Thus, the fulfilment of the bootstrap conditions (27)–(30) guarantees the implementation of all the infinite set of the bootstrap relations (8).

### 6 Summary

We presented the basic steps of the proof that in the multi-Regge kinematics real parts of QCD amplitudes for processes with gluon exchanges have the simple multi-Regge form depicted in Eq. (11), with the accuracy up to next-to-leading logarithms. This statement is extremely powerful. An infinite number of QCD processes is described by several Reggeon vertices and the gluon Regge trajectory. This remarkable property of QCD amplitudes is extremely important for the description of high energy processes. In particular, it appears as the basis of the BFKL approach.

The proof is based on the bootstrap relations required by the compatibility of the multi-Regge form (11) of inelastic QCD amplitudes with the \( s \)-channel unitarity. It consists of several steps. First, we proved in Section 3 that the multi-Regge form (11) is guaranteed in all orders of perturbation theory if it is valid in the one-loop approximation and if the set of
the bootstrap relations (8) holds true. These relations contain the s–channel discontinuities of inelastic amplitudes which must be calculated using the unitarity relations and the multi–Regge form (11). Then, to find a representation for the discontinuities we developed the operator formalism introduced in Ref. [20] for taking into consideration inelastic amplitudes. This permitted us to find in Section 4 the closed expressions (26) for the discontinuities in terms of the Reggeon vertices and the gluon Regge trajectory. The last step, performed in Section 5, concerns the proof that the bootstrap relations (8) are fulfilled if the vertices and trajectories submit to the bootstrap conditions (27)–(30). It is extremely nontrivial that an infinite set of the bootstrap relations is reduced to several conditions on the Reggeon vertices and the gluon Regge trajectory. All these vertices, as well as the gluon Regge trajectory are known now in the next-to-leading order. The bootstrap conditions were examined for a long time in a series of papers with increase of understanding of their role (see for instance [23] and references therein). On the parton level (for quarks and gluons) only the condition (30) remained unchecked till recently. Now the fulfilment of this condition is proved [26].

To be rigorous we have to say that strictly speaking the form (11) in the one-loop approximation was actually derived only for one-gluon production [27]. Although there are general arguments that it is correct for any \( n \), a strong evidence is absent.

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