The Role of 2D Fast Fourier Transform and High Pass Filter in Regional and Residual Anomaly Separation in Gravity

I R Palupi, W Raharjo, S Kiswanti
Geophysical Engineering, UPN “Veteran” Yogyakarta, Indonesia
email: inden13101986@gmail.com

Abstract. Regional and residual Separation anomaly is one thing that must do in gravity processing data. It is important before calculating the depth of anomaly by power spectrum. There are several ways to do this, one of them is using 2D Fast Fourier Transform (FFT). 2D FFT will calculate the two-dimensional power of the gravity map (Bouger anomaly) to change the spatial domain into the wavenumber domain. 2D FFT result has no unit because it works in the wavenumber domain. Power spectrum do in wavenumber domain map. Besides that, to make the wavenumber map in the frequency domain, it should be convolved with some filter (high-pass filter) and then inverse to separate the regional and the residual map. The design of the filter matrix depends on the number of the data and the location of anomalies will be enhanced. It will influence the separation result. The best result gets from the trial and error process. 2D FFT is act like Upward Continuation or Polynomial Fitting in the gravity method with the simple process. In this paper, the process fully done in Python. Python is an effective and simple language programming because it has many modules to support the processing and covering the big data. It also gives the flexibility to the researcher to determine the specific location that will be enhanced

Keywords: Gravity, Regional and residual separation, 2D FFT, high pass filter, Python

1. Introduction
Regional and residual separation is an important thing in Gravity processing data to get a good interpretation in geology like determine the depth of the anomaly. To do this, upward continuation is chosen as a mathematical method based on Jacobsen in [1]. The concept of upward continuation is to transform a point assumed Q located in mean sea level with the distance R to the higher position, assumed P, with the distance –h (negative sign interpreted as an up direction).
The other way to separate regional and residual anomaly of gravity data can use the trend surface by Menke in [1]. This is a filtering procedure that uses the polynomial surface fitting concept by applied least square techniques. In other words, the trend surface tries to solve the nonlinear problem (gravity) by using a linear approximation. Besides the two methods mentioned before, Zahra and Oweis applied the 2D Fast Fourier Transform (FFT) to define the residual source by changing the spatial domain into the wavenumber first [2]. After that, a high pass filter is applied by convolving it with the 2D FFT map result to enhance data characteristics and separating the regional and the residual map anomalies as the final model.

This research’s purpose to explain the role of 2D Fast Fourier Transform and High Pass Filter in the separation of the regional and residual anomalies in Gravity. The Data that used in this research is gotten from topex and assumed as Bouger Anomaly. We do the process by making the program in Python. It gives us the flexibility to express our ideas unusually in Gravity data processing. Python is chosen because it is easy to understand and has a lot of modules supported in processing and covering big data.

2. Methods

Fast Fourier Transform (FFT) that usually used in seismic to convert or change the data from time to frequency domain, can also be applied to gravity data. It calculates the spectrum energy and changed the spatial data in the gravity map into the wavenumber domain before estimates residual and regional anomalies and to be used in calculating the depth of anomaly. In gravity, 2D FFT will calculate the two-dimensional power spectrum before operated with a filter [3]. The example of the power spectrum to calculate the depth of the anomaly is in Figure 2.
Generally, FFT is the development product of Discrete Fourier Transform (DFT). DFT considered is not effective to solve some problem data related to periodicity, symmetries, and orthogonality. The basic FFT formulation is defined from the DFT equation in equation (1) [4],

\[
C(k) = \sum_{n=0}^{N-1} x(n)W_N^{nk}
\]  

(1)

\[
W_N^{nk} = e^{-j2\pi nk/N}
\]  

(2)

where \(n\) and \(k\) are integers, \(j\) is the imaginer sign and \(N\) is the numbers of data. However, when FFT is applied to the continued problem, equation (2) will be modified like in equation (3)

\[
F(k) = \int_{-N/2}^{N/2} f(n)e^{-j2\pi nk/N} \, dn
\]  

(3)

FFT uses complex number parameters in its formula. It includes real and imaginary parts. When both of them calculate together, magnitude or power spectrum is produced. FFT in equation (3) is 1D FFT and works in one dimension only (\(n\)). 2D FFT works in two dimensions spatial like in equation (4)

\[
F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)e^{-j2\pi xu+vy} \, dx \, dy
\]  

(4)

where \(u\) and \(v\) assumed works in \(x\) and \(y\).

2D FFT in gravity works in the spatial domain. When it is applied, the result of 2D FFT has no unit. To make it to the frequency domain, the result of 2D FFT should be convolved with some filters (low-pass, band-pass, high-pass, or Butterworth) [5]. After that, inverse 2D FFT should be applied to restore the map and separate the regional and residual anomaly, with equation (5).

\[
f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v)e^{j2\pi xu+vy} \, du \, dv
\]  

(5)
3. Result and Discussion

Data used in this research is downloaded from topex and assumed as Bouger Anomaly. The focus of this research is about the role of 2D FFT and high pass filter and how to processed them in Python. It just two modules needed to support the process, they are NumPy and Matplotlib. NumPy is used to load and calculate the FFT of the gravity map. While Matplotlib is used to display the map. The Anomaly Bouger can be seen in Figure 3.

![Figure 3. The Example of Anomaly Bouger data](image)

When 2D FFT is applied on the Anomaly Bouger map, Fourier Transform works in each pair of x and y data in each grid of map like in equation (5). The result will have real and imaginary parts of each grid, to get the magnitude of them, it is needed to calculate the root square like in Figure 4. In Figure 4 also there is an illustration digitizing step before calculating the depth of anomaly by power spectrum method.

![Figure 4. Illustration of anomaly Bouger anomaly map after 2D FFT with the section (yellow line)](image)
Power spectrum analysis is done to get the depth of anomaly can be applied in Figure 4. It is done by digitizing the map in Figure 4 based on the research area. Because the map has no unit, the y axis information is assumed as the amplitude. Then, the 2D FFT map is convolved with the high pass filter and inverse to get regional and residual separation. The example of the high-pass filter can be seen in Figure 5.

![Image of high-pass filter](image1.png)

**Figure 5.** Illustration of high-pass filter

A high-pass filter is a matrix arrangement of some number. In this research, the numbers are 0 (blue color) and 1 (red color). Location and width of red color are gotten from the trial and error process. If both of them are not precise, the result will not maximal. As a comparison, in Figure 6 there is a display of other illustrations of high-pass filters and the result can be seen in Figure 7 and Figure 8.

![Image of another high-pass filter](image2.png)

**Figure 6.** The other illustration of high pass filter

The next process is convolving the result of the 2D FFT map in Figure 4 with a high-pass filter in Figure 5 and Figure 6. In this process, the convolve will calculate each pair of x and y in a 2D FFT map with a high-pass filter map. So, the size of the matrix in 2D FFT and high-pass filter maps should be the same. Then, after convolve, the map should be inverse of 2D FFT to get the separation of the regional and residual map. Then, inverse the map to separate regional and residual anomalies.
Figure 7. Regional and residual map as a result of inverse result of the convolve between 2D FFT map and high pass filter in Figure 5

Figure 8. Regional and residual map as a result of inverse result of the convolve between 2D FFT map and high pass filter in Figure 6

The result in Figure 7 is more appropriate than in Figure 8. Figure 7 is more relatable with the original data or Anomaly Bouger map. The high anomaly of the Bouger Anomaly map is suitable with the result in Figure 7. It means that placement and determination are two parameters that should be considered. In Figure 7, the regional and residual anomaly is separated better than in Figure 8. In Figure 7, two high anomalies of gravity data are good at corresponding with Bouger anomaly, but in Figure 8, the arrangement of high anomaly scattered in the south part of the map and not corresponding with Bouger anomaly. 2D FFT and the filtering process give a chance to the geoscientist to determine the specific location will be enhanced.

Conclusion

There are several ways to separate regional and residual anomalies in the gravity method. One of them is 2D FFT that will be convolved with some filters (high-pass filter) and finally inverse to separate the
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regional and residual map. 2D FFT calculates the power spectrum of each pair x and y in a map to define the source of the residual data by changing spatial domain into wavenumber. Besides acting like input data before it convolves with the high pass filter matrix, the 2D FFT result also is basic data to do the power spectrum method to get the depth of anomaly. However, it means that 2D FFT and high pass filter are the alternative way to gravity data processing besides the usual method like the upward continuation. It can be easily done by making the program of ourselves (with Python or others) so the researcher has the flexibility to determine the location will be processed.

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