OPTIMAL PRICING OF PERISHABLE PRODUCTS WITH REPLENISHMENT POLICY IN THE PRESENCE OF STRATEGIC CONSUMERS

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Abstract. Recognizing that strategic consumers have become increasingly common in the perishable products market, we develop a two-stage pricing model for a monopolistic firm with two classes of inventory strategies: non-replenishment and replenishment. First, the retailer mapping out his pricing policy, and then consumers determining their buying behavior given the retailers policy. Our results indicate that the game equilibrium exists between retailers and consumers in both cases. For a given realized price and inventory policy, the consumer’s space is split into several areas by the optimal threshold functions. Inventory replenishment decisions are affected by market demand and a decline factor of consumers reservation value. The retailers profit loss is not necessarily related to the consumers waiting behavior but results from the ignorance of this behavior when pricing.

1. Introduction. We consider the preannounced pricing strategy of perishable products in the presence of strategic consumers. The perishable product sales period is short, and will be cleared out from the shelves after the sales period. In order to improve their revenue, most retailers choose the strategy to adjust the price when the product is close to the end of sales period rather than adopting a fixed price throughout the perishable product’s lifetime. The preannounced pricing strategy, which a seller commits a deterministic price path for customers at the beginning of the horizon, is adopted by some companies include Filene’s Basement (Bell et al [1] and Elmaghraby et al [23]), Lands overstocks (http://www.landsend.com). At the Filene’s Basement store, most unsold items after two, four, and six weeks will be sold at 25%, 50%, and 75% off the regular price, respectively. Lands End Overstocks put a new group of product “On the Counter” in very limited quantities each Saturday, substantially reduced from the original catalog price. On Monday,
that discount price is reduced another 25%. On Wednesday, 50%. And on Friday, 75%. However, faced with frequent changes in prices, consumers are becoming increasingly rational. There is a type of consumer, called the strategic consumer, who will adjust his/her buying strategies based on price changes. As a result, we can find a constant “cat-and mouse” game that plays out between consumers and retailers. The retailers aim to charge full price for all products, and the consumers behave strategically by putting their purchases to specific product or period with lower prices. This strategic behavior of consumers has received strong criticism. According to the Wall Street Journal, the CEO of BestBuy, a major electronics retailer, labeled those who are strategic as devils and those who purchase high-end gadgets without hesitation as angels [2].

Although the importance of pricing strategy of perishable products is recognized by many, its implications on the joint of replenishment policy and strategic consumers have not been widely studied. Most of the existing researches about strategic consumers focus on the products which are perishable because their values deteriorate, such as a hotel bed, and a movie ticket, or because of technological changes and changes of fashion trends, such as electronic products and fashion products. For this perishable products, such as long production cycle, long-distance transport and short sales period may be restrictions on the ability of restocking. However, the products we focus on are perishable because of physical deterioration, such as dairy products, fruits, fresh vegetable, and meats. For this perishable products, replenishment is feasible and necessary for retailers. However, replenishment policy will result in two substitutable products (namely, complementary products and unsold products) on shelves and more complex strategy choices for customers. A customer will choose when to buy and what to buy between the two products. Conversely, the strategic behavior will influence the retailer’s price strategy for the two products. Thus, we analyze the interaction between the retailers pricing strategies and the customers strategic behavior given this situation.

This paper is organized as follows. In section 2, we review the existing literature. Section 3 describes the basic model setting and notation. In sections 4 and 5, we present the analysis of the two inventory strategy scenarios, non-replenishment and replenishment, respectively. Three reference models are proposed in section 6. In section 7, we present the numerical studies and main results. Finally, Section 7 summarize the key results and suggest topics for future research.

2. Literature review. There is a widespread research on pricing and inventory problems of perishable products. The characteristic of the perishable products we study in this paper is similar to the deteriorating items', which is defined as decay, destruction, corruption, evaporation, out of date, loss of utility of product that induce decreasing usefulness [3]. There exists an extensive literature exploring pricing and inventory policy with respect to deteriorating items [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14].

Coase [15] first considered the strategic behavior of consumers in a study of retailer pricing and indicated that even a monopoly seller must price at marginal costs and earn zero profits for strategic consumers. Later, Stokey [16] and Bulow[17] presented strict proof of this argument. Some scholars have argued for pricing based on these results. For example, Stokey [18], Landsberger and Meilijson [19] and Besanko et al. [20] studied the continuous and discrete price path, but they considered the situation of limitless inventory. Product loss and seasonal changes
cannot be ignored in the sales period of perishable products, which is why a retailer can only order a limited amount of inventory at a time. Therefore, taking into account inventory should be critical especially when pricing perishable products.

The current research is related to the literature on the dynamic pricing of finite inventories. This literature mainly discusses customers behaviors regarding when to buy. Liu, Zhang [21] and Levin et al. [22] studied the pricing competition facing strategic customers. Elmaghraby et al. [23] focused on markdown mechanisms and customers demand for multiple units. Levin et al. [24] introduced a discount factor that simulates consumer strategic behavior under the background of limited inventory. Su [25] divided customers into four categories according to the dimensions of valuation and patience. Mersereau and Mersereau and Zhang [26] considered the markdown pricing strategy in the presence of strategic customer under uncertainty environment. Su and Zhang [27] studied the role of commitment and availability guarantees in attracting consumer demand. Aviv and Pazgal [28] considered stochastic arrivals and time-varying valuation pattern for each consumer. The valuation pattern, which appropriately reflects changes in consumer reservation value when facing perishable products, is easy to match with the characteristics of perishable products. Liang et al. [29] analyzed product rollover strategies when selling to strategic consumers, but their focus was on the innovation of new products rather than the perishability of products. Du et al. [30] studied the pricing and inventory management considering strategic customers’ risk preference and decreasing value. These papers have typically focused on the scenario in which a retailer sells finite inventory and marks down the product in the presence of strategic consumers. However, they have not discussed a retailers willingness to replenish inventory and the cannibalization effect due to restocking. Cachon and Swinney [31, 32] and Yang et al. [33] showed the effect of pricing and quick response when consumers are strategic. Hu et al. [34] analyzed dynamic inventory and markdown decisions for perishable goods. But they didn’t consider the deteriorating characteristic of perishable products and the stochastic process of customer arrivals.

The strategic behaviors of consumers not only affect the decision-making of retailers but also influence other consumers decisions. Some scholars have studied the mutual effects of consumers strategic behaviors [35, 36, 37]. They have considered the interactions between customers in the sense that they compete for the same pool of inventory. With the help of their research, we consider the case in which only the initial inventory can be observed. That is, when a customer determines his/her purchase strategy, he/she must deduce the probability of a product on sale on the basis of the level of inventory.

We contribute to the emerging literature in several ways. First, we seek to develop a game theoretical model to determine optimal price on the joint of replenishment policy and strategic consumers. We show the Nash equilibrium of our model that how customers’ decision respond to retail’s pricing policy and describe customer’s hierarchy structure according to a threshold function. Second, we focus on the deteriorating items which differ from the perishable items in the current literature about strategic consumer and consider a time-varying utility function in the model. It is because of the characteristics of deteriorating items, making the replenishment policy is important. Finally, we compare fixed-price policy and myopic customer model and our model profits, and find some new management insights.
3. Model assumptions and terminology. In this paper, we discuss the two-stage preannounced pricing of limited perishable products with two classes of inventory strategies: non-replenishment and replenishment. In both cases, the retailer behaves as a Stackelberg leader setting the two-stage pricing strategies and the customers following by selecting purchase strategies. Preannounced pricing policy eliminates the uncertainty of the future prices and creates a barrier against strategic waiting. In addition to preannounced pricing, there is a pricing policy, named contingent pricing, is widely used in practice and researched by academics. In this pricing policy, the discounting price depends on the number of unsold items at the end of the first stage and customer behavior. Contingent pricing clearly dominates preannounced pricing when customers are myopic. However preannounced pricing can actually work more effectively than contingent pricing in the presence of strategic customers (see, e.g., Mersereau and Zhang [26], Aviv and Pazgal [28], Dasu and Tong [39]).

The values of products decline over time. Furthermore, the unsold products at the end of the sales season, which are set at $[0, 1]$, for convenience, are salvaged for zero. It is widespread knowledge that the values of products are affected by time in retail industries and the management of hotels and airline tickets. However, these products differ from perishable products in that the former results from exogenous supply and demand, whereas the latter is due to endogenous quality variance. The sales season is divided into two steps, $[0, T]$ and $[T, 1]$, in which the price-cutting point $T$ is preset and the prices are $p_1$ and $p_2$, respectively.

We take advantage of poisson process to describe the reach of consumers with a rate of $\lambda$ arrivals per time unit. According to the assumptions of Bitran and Mondshein [38], the first customer arrivals are related to customer buying habits rather than products prices. Because of the limited inventory, strategic customers must estimate the probability that they must wait to purchase products by analyzing the strategic behaviors of other customers. Although consumers are aware of their valuations of a product, monopolistic retailers do not have this knowledge. Assume that the valuations $V(t)$ are in accord with a uniform distribution of $[0, e^{-\alpha t}]$. We can explain this assumption as follows. On the one hand, the valuations of customers are distributed in the form $[0,1]$, if ignoring the time factor. On the other hand, the valuations are decreased by a known exponential decline factor $\alpha > 0$. This Assumption of utility function being time’s varying is related to the easy corrosion characteristic of perishable goods. Most of the current literature focus on the perishable goods which value deteriorate because of technological changes and changes of fashion trends and so on, and so the decline in the utility of the consumer is for all commodities. But for the deterioration items we study in the paper is different. The customers experience a loss of utility of unsold products rather than complementary products.

In our main model, customers are rational and exhibit strategic behavior, in that they decide their purchase strategies base on the markdown price, the decrease of valuations, and availability of the product. In the case of non-replenishment, strategic customers choose between buying an item at price $p_1$ and waiting until $T$ for a chance of getting the item at price $p_2$. In the case of replenishment, customers exhibit one or both of two possible behaviors, quality-selection and waiting. In the second stage, the complementary products and unsold products are different in the term of quality because of corrosive characteristics. Therefore, strategic customers
who arrive in $[0,T]$ but wait until $T$ will choose between buying a complementary product at price $p_1$ and buying an unsold product at price $p_2$.

Notations

- $\lambda$: Arrival rate of customers, that is, assuming that customers arrive with a Poisson process at an average rate of $\lambda$ arrivals per time unit.
- $p_i (i=1,2)$: Pricing strategies.
- $c$: Cost of product per unit, neglecting the inventory fares for convenience.
- $T$: Price-cutting point, where $0<T<1$.
- $V(t)$: Valuations of customers at point $t$, $V(t)=V e^{-\alpha t}$ in which is in accord with a uniform distribution of $[0,1]$, and the cumulative distribution function is $F(\cdot)$.

4. Scenario 1 the case without inventory replenishment.

4.1. Consumer strategies. In this game, the retailer first announces his pricing policy, and then consumers determine their buying behavior given the retailer’s policy. We can solve this game model using backward induction. First, customers choose to wait or purchase immediately according to the principle of maximum profit under parameters $\{p_1,p_2,\theta_w\}$. Here, $(p_1,p_2)$ is the fixed-price path to which the retailer commits upfront, and $\theta_w$ represents the probability that strategic customer can purchase an unsold product at time $T$. According to Aviv and Pazgal [28], there is a Nash equilibrium in the customers purchasing strategies that is defined by the function $\psi(t)$.

Theorem 4.1. For any given pricing strategy $[p_1,p_2]$, price-cutting point $T$ and variable arrival time $t$. A customer will purchase a unit of product if $V(t) \geq \psi(t)$. Otherwise, if $t<T$, the customer will wait and purchase an available product at time $T$ if $V(T) \geq p_2$ and if $t>T$, the customer will leave immediately. Let.

$$\psi(t) = \begin{cases} \max\{\frac{p_1-\theta_w p_2}{1-\theta_w e^{-\alpha(T-t)}},p_1\}, & 0 \leq t < T \\ p_2, & T \leq t \leq 1 \end{cases}$$

where $\theta_w$ is the solution to

$$\theta_w = \sum_{i=0}^{T-1} Pr(i|m^1) A(I-i|m^2_w)$$

The function $A(K|m) = \sum_{k=0}^{\infty} \frac{K}{\max(k,K)} Pr(k|m)$ represent the probability of a customer obtain a product with parameters $K$ and $m$. The symbol $m$ is the expected number of customers. The symbol $m^1$ is the mean of the Poisson distribution that customers purchase a product at price $p_1$ during $[0,T]$. The symbol $m^2_w$ is the mean of the Poisson distribution that customers purchase during $[T,1]$. It is assumed that, when making purchasing decisions, consumers will take the behaviors of others into account when considering the probability of accessing the product in the future. However, if the inventory is small, we can infer that the probability is lower and, therefore, fewer people will wait.

4.2. Retailers profit model. From the above analyses, consumers are divided into four categories (as show in Figure 1) based on their purchasing strategies.

The first category (denoted by F) is comprised of the consumers who purchase immediately at price $p_1$ during at the first stage $(0<t<T)$. The mean of the Poisson distribution of this category is

$$m^1 = \lambda \int_0^T (1 - F(\psi(t)e^{\alpha t})) dt$$
The second category (denoted by \( W \)) is comprised of strategic consumers who wait until time \( T \) and purchase a product at price \( p_2 \). The mean of the Poisson distribution of this category is
\[
m^2_w = \int_0^T F(max(\psi(t)e^{\alpha t}, p_2 e^{\alpha T})) - F(p_2 e^{\alpha T}) \, dt
\]
The third category (denoted by \( S \)) of consumers contains of those who purchase a product at price \( p_2 \) during \([T, 1]\). The mean of the Poisson distribution of this category is
\[
m^2_l = \int_T^1 (1 - F(p_2 e^{\alpha t})) \, dt
\]
The fourth category (denoted by \( N \)) is comprised of the consumers who do not buy any one product in the sales season. From the above, the retailers goal is to maximize the profit
\[
\pi^*_I = \max_{p_1, p_2} \{ I p_1 [1 - \sum_{j=0}^{I-1} Pr(j|m^1)] + \sum_{j=0}^{I-1} [j p_1 + p_2 N(I - j, m^2_w + m^2_l)] Pr(j|m^1) \} - cI
\]

The function \( Pr(j|m) \) is the density function of sales volume, and \( N(I, m) = \sum_{k=0}^{\infty} \min(I, k) Pr(k|m) \) is the expected value of a Poisson random variable (with mean \( m \)) truncated at \( I \).

Figure 1. Customer types for the case without inventory replenishment

5. Scenario 2 replenish inventory. The existing literature about the pricing of perishable products with strategic consumers seldom considers inventory replenishment. This is because of the short period of product sales, the market uncertainty in the external environment, the attributes of the product itself and other factors that restrict the supplementing of inventory. For example, although airfares are constantly changing, the time to fly is fixed except on major holidays and events.

\[1\text{Notes. F: customers that buy immediately at the first stage, W: wait strategically, S: buy at the second stage, N: do not buy.}\]
Thus, because the number of seats is settled, there is no need to replenish inventories. Retailers are faced with certain risks when replenishing stocks during fashion clothing sales season. First, this situation contributes to the waiting behavior of consumers. Additionally, owing to the effect of season trends, there is a great uncertainty regarding future demand, as the current popular style may not be popular next year. It is unwise to blindly replenish stocks because this can lead to the accumulation of inventories and an increase in costs. However, for easy corrosion products, replenishment is not only feasible but necessary for retailers. Moreover, each order cannot excessively exceed demand, as the value of the product diminishes with time due to its specific characteristics. Because the product is a non-durable good, there is a continuous demand that retailers must meet to satisfy customers needs. Thus, retailers must engage in multi-replenishment. In this paper, we regard this as a scenario of multi-stage sales, a scenario in which retailers sell certain types of easily corrosive or disposable products for a long period of time. Because of market uncertainty, these retailers must provide quantitative products, denoted by “new products”, at a fixed point in time. However, at this time, they hold unsold products, denoted by “old products”, from the previous phase. This gives rise to the following question: how do retailers determine their pricing strategies? As the multi-stage sales situation is similar to the two-stage sales situation, we expand the model of a one-stage sales period to a two-stage sales period model to study the optimal pricing strategy of vendors. For the sake of consistency, the total length of the two-stage sales remains [0, 1], whereas the first stage is [0, \( T \)] and the second stage is \([T, 1]\), assuming that stocks are replenished at \( T \) time. The selling price of new supplement products is \( p_1 \), whereas the rest of the products are cheaper and their prices are \( p_2 \).

5.1. Consumer strategies. The waste that is generated by easy corrosive products is irreversible, and supplementary products in the second stage cannot substitute for remaining products in the first stage. However, the new supplementary products provide new choices to the consumers. Namely, customers wait until the second stage and then have two choices. One choice is to purchase the new products, and the other choice is to buy the old products from the first stage at a lower price. Strategic consumers have high demands regarding the value of commodities. Because the new supplementary commodities can meet their demands, an increasing number of people choose to wait. Consumers will then make an additional decision between new products and old products, which is determined according to the utility offered by the two products. When two products are in stock, consumers select the good that will maximize individual utility. When there is only one product, consumers choose to either leave or purchase it, based on whether the reservation value of the consumer is greater than or equal to the price of the product.

**Theorem 5.1.** For any given supplementary inventory \( Q \), pricing strategy \([p_1,p_2] \) and price-cutting point \( T \), the consumers’ behavior is defined by two threshold functions \( \tilde{\psi}_1(t) \) and \( \tilde{\psi}_2(t) \). In the first stage (\( t \in [0,T] \)), a consumer will purchase a product at price \( p_1 \) if \( V(t) \geq \tilde{\psi}_1(t) \). Otherwise, the consumer will wait until time \( T \) and purchase a new product (the supplementary inventory) at price \( p_1 \) if \( \tilde{\psi}_2(t) \leq V(t) < \tilde{\psi}_1(t) \). Otherwise, if \( V(t) < \tilde{\psi}_2(t) \), the customer will wait until time \( T \) and purchase an old unit (the initial inventory) if \( V(T) \geq p_2 \). In the second stage (\( t \in [T,1] \)), a customer will purchase a new unit if \( \tilde{\psi}_2(t) \leq V(t) < \tilde{\psi}_1(t) \). Otherwise, the customer will purchase an old product if \( p_2 < V(t) < \tilde{\psi}_2(t) \). The threshold functions \( \tilde{\psi}_1(t) \)
and $\tilde{\psi}_2(t)$ are given by

$$\tilde{\psi}_1(t) = \begin{cases} \max\{p_1 - \theta_1p_1, 1 - \theta_1e^{-\alpha t}, p_1\}, & 0 \leq t < \min \left(\frac{1}{\alpha} \ln \frac{1}{\tilde{\theta}}, T\right) \\ 1, & \min \left(\frac{1}{\alpha} \ln \frac{1}{\tilde{\theta}}, T\right) \leq t \leq 1 \end{cases}$$

(3)

$$\tilde{\psi}_2(t) = \begin{cases} \max\{\frac{p_1 - p_2}{e^{\alpha t} - e^{-\alpha (T-t)}}, p_1e^{-\alpha t}\}, & 0 < t < T \\ \max\{\frac{p_1 - p_2}{e^{\alpha (T-t)}}, p_1e^{-\alpha T}\}, & T \leq t \leq 1 \end{cases}$$

(4)

The symbol $\tilde{\theta} \backslash \theta$ represents the probability that a strategic customer can purchase a new \ old product at time $T$. Let $\tilde{\theta} = A(Q|m_3^3)$ and $\theta = \sum_{x=0}^{T-1} Pr(x|m_1^3)A(I-x|m_2^2)$.

5.2. Consumer structure and seller profit. Figure 2 shows the customer types in this situation. Compared with non-replenishment, consumers are divided into six types according to the threshold functions $\tilde{\psi}_1(t)$ and $\tilde{\psi}_2(t)$.

![Figure 2. Customer types for the case with inventory replenishment](image)

The first group (denoted by $F$) includes the customers who purchase a product at price $p_1$ during $[0, T]$. The mean of the Poisson distribution of this category is

$$\bar{m}_1 = \lambda \int_0^{\min(1/\alpha \ln 1/\tilde{\theta}, T)} (1 - F(\tilde{\psi}_1(t)e^{\alpha t})) dt$$

The second category (denoted by WO) is comprised of strategic consumers who wait until time $T$ and purchase old products at price $p_2$ whose reservation value $V(t)$ is lower than $\tilde{\psi}_2(t)$ at time $t$ but is higher than $p_2$ at time $T$ in general. Needing attention, because the inventory is limited, a customer will purchase an old good even if $V(t) > \tilde{\psi}_2(t)$ and $V(t) > p_2$ when the new products sell out. The mean of the Poisson distribution of this category is

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2Notes. F: customers that buy immediately at the first stage, WN: wait strategically for the new product, WO: wait strategically for the old product, SN: buy a new product at the second stage, SO: buy an old product at the second stage, N: do not buy.
\[
\tilde{m}_w^2 = \lambda \int_0^T (F(\max(\widetilde{\psi}_2(t)e^{\alpha t}, p_2 e^{\alpha t}T)) - F(p_2 e^{\alpha t}) + \bar{\theta}(F(\max(\tilde{\psi}_1(t)e^{\alpha t}, \tilde{\psi}_2(t)e^{\alpha t}) - F(\max(\tilde{\psi}_2(t)e^{\alpha t}, p_2 e^{\alpha t}T)))) dt
\]

where \( \tilde{\theta} \) is the probability that the new products are out of stock and \( \bar{\theta} = 1 - \tilde{\theta} \).

The third group (denoted by SO) of customers includes those who purchase the old products arriving after time \( T \). Similar to the above discussion, the mean of the Poisson distribution of this category is

\[
\tilde{m}_s^3 = \lambda \int_0^T (F(\max(\widetilde{\psi}_2(t)e^{\alpha t}, p_2 e^{\alpha t})) - F(p_2 e^{\alpha t}) + \bar{\theta}(F(\max(\tilde{\psi}_1(t)e^{\alpha t}, \tilde{\psi}_2(t)e^{\alpha t}))) dt
\]

The fourth group (denoted by WN) of customers includes those who arrive during \([0, T]\) and wait for new products. The mean of the Poisson distribution of this category is

\[
\tilde{m}_w^3 = \lambda \int_0^{\min(\frac{1}{2}ln\frac{1}{2}, T)} (1 - F(\widetilde{\psi}_2(t)e^{\alpha t})) dt + \lambda \int_{\min(\frac{1}{2}ln\frac{1}{2}, T)}^T (F(\max(\tilde{\psi}_1(t)e^{\alpha t}, \tilde{\psi}_2(t)e^{\alpha t})) - F(\tilde{\psi}_2(t)e^{\alpha t})) dt + \lambda \bar{\theta} \int_0^T (F(\max(\tilde{\psi}_2(t)e^{\alpha t}, p_1)) - F(p_1)) dt
\]

The symbol \( \tilde{\theta} = 1 - \theta \) represents the probability of the old products being out of stock.

The fifth category (denoted by SN) of consumers contain of those who purchase new products immediately during \([T, 1]\). The mean of the Poisson distribution of this category is

\[
\tilde{m}_s^3 = \lambda \int_T^1 (1 - F(\widetilde{\psi}_2(t)e^{\alpha t})) + \bar{\theta}(F(\max(\tilde{\psi}_2(t)e^{\alpha t}, p_1)) - F(p_1)) dt
\]

Finally, a sixth group (denoted by N) includes those who do not buy. Let \( \tilde{\pi}_{Q/S} \) be the revenue of the supplementary inventory, then \( \tilde{\pi}_{Q/S} = cp_1[1 - \sum_{j=1}^{c-1} j p_1 Pr(j|m_3^2 + \tilde{m}_m^3)] + \sum_{j=1}^{c-1} [j p_1 Pr(j|m_3^2 + \tilde{m}_m^3)] + \sum_{j=1}^{c-1} [j p_1 + p_2 N(c - j, \tilde{m}_m^2 + \tilde{m}_m^3)] Pr(j|m_3^2) \]

Thus, a retailer’s goal is to maximize the expected profit

\[
\tilde{\pi}_{I+Q/S}^* = \max_{p_1, p_2} \{\tilde{\pi}_{I/S} + \tilde{\pi}_{Q/S} - c(Q + I)\}
\]

6. Reference models. In this section, three reference models are proposed. Consumers are non-waiting (myopic) and the retailer doesn’t replenish inventory in the first model and replenish inventory in the second model. In the third model, the retailer adopt a fixed-price policy.

The myopic consumers never consider future utility and make their decision only based on the current price and reservation valuation. Specifically, in the first stage \((0 < t < T)\), a consumer will purchase a product if \( V(t) \geq p_1 \), otherwise leaving immediately. Similarly, in the second stage \((t > T)\), a consumer will purchase a product if \( V(t) \geq p_2 \), otherwise leaving immediately. Therefore, in the case of myopic consumers and non-replenishment, we can rewrite formula (2) with

\[
\psi(t) = \begin{cases} 
  p_1, 0 \leq t < T \\
  p_2, T \leq t \leq 1
\end{cases}, m_2^0 = 0
\]

deduce the expected profit of a retailer as follow

\[
\pi_{I/N}^* = \max_{p} \{Ip_1[1 - \sum_{j=0}^{l-1} Pr(j|m^1)] + \sum_{j=0}^{l-1} [j p_1 + p_2 N(I - j, m_2^0)|Pr(j|m^1)] - cI \}
\]

In the case of myopic consumers and replenishment inventory, we rewrite formula (5) with \( \tilde{\theta} = 0 \) and deduce the expected profit of a retailer as follow

\[
\tilde{\pi}_{I+Q/N}^* = \max_{p_1, p_2} \{\tilde{\pi}_{I/N} + \tilde{\pi}_{Q/N} - c(Q + I)\}
\]

where the function \( \tilde{\pi}_{I/N} = cp_1[1 - \sum_{j=0}^{c-1} Pr(j|m^1)] + \sum_{j=0}^{c-1} [j p_1 + p_2 N(c - j, \tilde{m}_m^2) Pr(j|m^1)] \) is the revenue from the initial inventory, and the function \( \tilde{\pi}_{Q/N} = cp_1[1 - \sum_{j=0}^{c-1} \sum_{j=1}^{c-1} j p_1 Pr(j|m_3^2 + \tilde{m}_m^3)] + \sum_{j=1}^{c-1} [j p_1 + p_2 N(c - j, \tilde{m}_m^2) Pr(j|m_3^2)] \)
\[ cp_1[1 - \sum_{j=0}^{c-1} Pr(j|\tilde{m}_1^3)] + \sum_{j=0}^{c-1} [jp_1 Pr(j|\tilde{m}_1^3)] \] is the revenue from the supplementary inventory.

Finally, in the case of a fixed-price policy, the retailer’s goal is to maximize the expected profit

\[ \pi^*_{I/F} = \max_p \{pN(I, \lambda \int_0^1 (1 - F(pe^{\alpha t}))dt)\} \tag{8} \]

7. Discussion.

7.1. Effectiveness of price discrimination: Without inventory limitation.

In the first place, we analyze the effectiveness of price discrimination in the situation that inventory is not limited. In this situation, the unit product cost can be ignored by setting. In this way, we focus on the expected revenue rather than on the profit.

In general, using a two-stage pricing strategy is more useful for the retailer when the consumers are myopic. In the situation of \( \alpha > 0 \), discrimination is made based on the sensitivity of the freshness of the product. In the first stage, the retailer can obtain a good benefit with a higher price and in the second stage, the retailer can try to meet the requirements of consumers with lower valuations through setting a discount at time \( T \).

**Proposition 1.** Assume that \( \alpha > 0 \), \( I/\lambda \to \infty \) and \( 0 < T < 1 \). This case can be expressed as

\[ \pi^*_{I/F} = \frac{\lambda \alpha}{4(e^{\alpha T} - 1)} < \pi^*_{I/N} = \frac{\lambda \alpha}{4} \left[ \frac{T^2}{e^{\alpha T} - 1} + \frac{(1-T)^2}{e^{\alpha T} - e^{\alpha T}} \right] \]

Proposition 1 indicates that the retailer can gain extra income through setting a discount at time \( T \) when the consumers are myopic. For example, in the case \( \{ \alpha = 1, T = 0.5, \lambda = 2 \} \), the two-stage pricing strategy increases by 64% profit than fixed-price strategy. Thus, Proposition 1 provides a powerful explanation as to why retailers use the price strategy for the revenue management of perishable products.

Now that we know the benefits of discounts, we compare the difference between two types of inventory strategy. Although it may seem unnecessary to consider restocking under the condition of large inventory, restocking may yield a different effect when the products are perishable and customers are sensitive to the freshness of the product. Thus, consider the following proposition.

**Proposition 2.** Under the condition of large inventory and \( \alpha > 0 \), the retailer can obtain more benefits by replenishing the inventory, namely, \( \tilde{\pi}^*_{I+Q/N} > \pi^*_{I/N} \) and \( \tilde{\pi}^*_{I+Q/S} > \pi^*_{I/S} \)

This proposition illustrates that replenishment inventory is the optimal strategy for a retailer when facing customers who are sensitive to the freshness of the product under the condition of large inventory. This assessment is specific to the proposition when the customers are non-waiting (myopic). Myopic customers will not consider the expected future surplus; thus, they behave in the same manner in the first stage of the selling season in both cases. However, in the second stage, customers with sufficiently high valuations will purchase the products at price \( p_1 \); thus, the retailer will derive greater benefits through the selling of the new products. Proposition 2 also shows the benefits of replenishment inventory when the customers are strategic. Furthermore, the strategy of replenishment inventory drives some consumers who do not intend to purchase products in the first stage to wait for the new product at time \( T \). In the second stage, we draw the same conclusion through the same analysis as in the situation when customers are myopic.
With respect to the impact of consumer sensitivity to the freshness of the product, we determine that this sensitivity motivates consumer quality choice behavior. Propositions 1 and 2 indicate that price discrimination can achieve benefits and that replenishment inventory is the optimal strategy when consumers are sensitive. By contrast, price discrimination and replenishment inventory lose their effects when consumers are not sensitive to the freshness of the product. We can conclude with the following proposition.

**Proposition 3.** Assume that $\alpha = 1, I/\lambda \to \infty$. This case can be expressed as

$$\pi_{I/F}^* = \pi_{I/S}^* = \pi_{I/N}^* = \pi_{I+Q/N}^* = \pi_{I+Q/S}^* = \frac{1}{2}$$

Proposition 3 indicates the following perspectives: price discrimination can’t segmenting customers to improve retailer’s profit when the consumer’s valuation decline factor approaches to 0 ($\alpha = 0$) and when consumers are myopic, price discrimination is not possible, and because of the disappearance of customer quality choice behavior, the replenishment inventory cannot increase the retailers revenue.

7.2. **Insights from models: Inventory consideration.** We now turn to the case of limited inventory. Due to the complications of equation (2) and equation (5) with respect to this case, it is difficult to reach a precise result. Through numerical experiments, however, we analyze the strategic behavior of consumers and the mutual influence between vendor pricing and inventory pattern.

7.2.1. **The impact of price.** We analyze the impact of pricing strategy on consumer waiting behavior and seller profit. Figure 3 presents graphs of the probability of waiting when $Q=0$ and $Q=1$. As evidenced, the proportion of consumers who choose to wait increases in $p_1$ and decreases in $p_2$. Waiting behavior is more obvious in the restocking situation. Table 1 lists the optimal prices and profits under the cases of inventory non-replenishment and a 1-unit inventory replenishment scenario as well as the corresponding profit distribution ratio. When $Q=0$, by determining the best price, we can effectively inhibit the strategic behavior of consumers, thereby making the profit of waiting consumers only 9.8% of the total. When $Q=1$, the waiting behavior of consumers is difficult to effectively suppress, and the ratio of the profit of the waiting consumer increases to 42.8%. With only 1.1% of the profits obtained in the first stage, most products have had to assume low prices in the second phase. It is also noted that $p_1$ increases rather than decreases and that $p_2$ is reversed. Accordingly, retailers should narrow the gap between $p_1$ and $p_2$ to reduce the waiting expectations of consumers; however, this is not a consistently effective method. Although the supplementary inventory could provide hope for waiting consumers whose valuations are lower, a limited number of stocks cannot meet all of the needs of consumers. Consumers second best choice is to choose old products from the first stage to satisfy their requests for low valuations; however, the price in the second stage must be lower, thus resulting in the decline of $p_2$. Although narrowing the gap between $p_1$ and $p_2$ could mitigate strategic customer behavior, the lower prices will decrease the profits of retailers. In this case, a retailer use price skimming to increase $p_1$ and focus on a high valuations target could be regarded as a good method to quickly increase retailers return on investment profits.

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3Notes. Results are based on $I=10, T=0.5, \lambda =10, \alpha=0.5, c=0.1$. The distribution of expected profits by source corresponds with immediate purchases, strategic waits and late buys in the first row and with immediate purchases, strategic waits for old products, late buys for old products, strategic waits for new products and late buys for new products in the second row, respectively.
Table 1. Pricing Strategy and Retailer Expected Profits Under Both Scenarios

| Q  | \([p_1, p_2]\)     | Expected profits | Distribution of expected profits            |
|----|---------------------|-------------------|---------------------------------------------|
| 0  | [0.419, 0.392]      | 0.909             | (45.4%, 9.8%, 44.8%)                        |
| 1  | [0.548, 0.362]      | 0.790             | (1.1%, 42.8%, 37.5%, 13.4%, 5.1%)           |

Figure 3. Impact of Pricing Strategies on Consumer Waiting Behavior

7.2.2. The impact of inventory. We now analyze the impact of initial inventory and replenishment inventory on retailer profit. When the initial inventory is small, consumers are more concerned with the probability of obtaining the product at a lower price and their behaviors are closer to the myopic consumer. In this case, the retailer profits from a higher price. For example, in the case in which \(\{\lambda = 0, T = 0.5, Q = 0\}\), if \(I = 1\), we get \(\pi_I^* = 0.457(p^* = [0.660, 0.571])\), and if \(I = 10\), we get \(\pi_I^* = 0.909\) \((p^* = [0.419, 0.392])\). As supplementary inventory increases, retailers must cut prices and thereby reduce their profit margins to suppress the strategic behaviors of consumers. Figure 4 shows the case of seller profits with inventory changes; the left graph is the case of non-replenished stocks and the right graph is the case of replenished stock 1. The curves, from top to bottom, represent consumer reservation value reduction factors equaling 0.5, 1 and 1.5. As evidenced from the figure, in both cases, the greater the consumer reservation value reduction factor, the smaller the initial optimal inventory. However, the optimal initial stock in the case of replenishment is less than this case and can, therefore, realize higher profits. When \(\alpha = 0.5\), the optimal initial inventory in the case of non-replenishment is 6 and the corresponding profit is \(\pi_I^* = 1.082\). When the replenished inventory is 1, the optimal initial inventory is 5, with a profit of \(\pi_I^*/Q = 1.123\). Despite the same stocks of the total amount in the two cases, the profits differ. This is because retailers avoid the depletion of the value of perishable products caused by inventory accumulation in a restocking situation and promote an increase in self-interest.

As shown in Table 1, the optimal profit is 0.79 when \(Q = 1\) is less than the case in which the optimal profit is 0.909 when \(Q = 0\). The replenishment of inventory does not necessarily increase the profits of the seller. Considering the cost, overly adding
inventory will inevitably lead to increased costs and consumers increased willingness to wait. Figure 5 describes the impact of restocking $Q$ on a retailer’s optimal profits under different $\lambda$. We find that there is not a simple increment/decrement relationship between $Q$ and optimal profit. When $\lambda$ is smaller, the optimal profit and $Q$ enter into a negative relationship, whereas when $\lambda$ is larger, there is a $Q^*$ maximizing optimal profit. The symbol $\lambda$ represents market demand conditions, that is, market demand is the key to determining whether to add inventory and the number needed to replenish inventory. Under large market demand, restocking results in increasing profits for a retailer, whereas under small market demand, restocking does not result in increasing a retailer’s profits; rather, it decreases profits due to increased costs.

7.2.3. The impact of consumer behavior. This section discusses the impact of customer behavior on the retailer’s profit. We define two types’ behaviors: waiting behavior and quality selection behavior. When inventory is not replenished, the waiting behavior of low retention value consumers is more obvious. However, when the inventory is replenished, some high retention value consumers also choose to wait. The waiting behavior not only boosts the consumption of low retention consumers but also causes the retailer to lose some high retention value consumers.
Table 2. Optimal Pricing and Expected Profit When Customers Are Non-strategic\(^4\)

| \(Q\) | \([p_1, p_2]\) | Expected profits | Distribution of expected profits |
|-------|----------------|------------------|---------------------------------|
| 0     | [0.454, 0.357] | 0.933            | (56.2\%, 43.8\%)               |
| 1     | [0.487, 0.312] | 0.862            | (49.8\%, 38.7\%, 11.5\%)      |

Table 2 shows optimal pricings and profits with respect to two situations, \(Q=0\) and \(Q=1\), without consumer waiting behavior.

Table 2 shows that the number of consumers who purchase at price decline greatly due to consumers waiting behavior, which then leads to the ratio of profit’s decrease in a non-replenishment situation from 56.2\% to 45.4\% and in a replenishment situation from 49.8\% to 1.1\%. At this point, the waiting behavior of consumers accelerates the depreciation of products, and the excessive hoarding of products sold only in the second stage affects the retailer’s overall profit. Comparing Table 1 and Table 2, we find that selling profit with respect to the two situations decrease by 2.61\% and 9.21\%, respectively.

To further examine the impact of customer waiting behavior, this paper conducts two numerical case studies. Case I examines the changes in profit with respect to the non-waiting consumers and the waiting consumers, and Case II considers the optimal pricing in a non-waiting behavior situation as the price of waiting behavior situation, then calculates the retailer’s profit and the change in profit compared with the actual profit. Table 3 lists the results of two cases, each of which considers \(Q \in \{0, 1, 5\}\) and \(\alpha \in \{0.5, 1, 1.5\}\)\(^5\). The first case reflects the impact of customer waiting behavior on the retailer’s profit, and the second case represents the impact on the retailer’s profit after ignoring consumer waiting behavior. For the profit given consumer waiting behavior, the positive figures in Table 3 indicate a decrease in profit, and the negative figures indicate an increase in profit. In the first case, the ratios are negative when \(Q=0\) and 1, indicating that the retailer can obtain high profit with non-waiting consumers. In other words, the waiting behavior of the consumer reduces the retailer’s profit when \(Q=0\) and 1. However, when \(Q=5\), all values are positive. In other words, waiting behavior increases the retailer’s profit when \(Q=5\). Because consumers who are non-waiting will purchase newly replenished products on the basis of the quality selection behavior. Furthermore, the waiting behavior leads some consumers whose price is less than in the first stage to purchase old commodities during the second stage, thereby reducing the loss in profits. With respect to the second case, all figures are negative, thereby indicating that ignoring the waiting behavior significantly lowers profit. As evidenced from this analysis, the waiting behavior of consumers is not necessarily connected with the loss of the retailer’s profit, but pricing strategy that ignores the waiting behavior of consumers will result in losses in seller profits.

\(^4\)Notes. Results are based on \(I=10\), \(T=0.5\), \(\lambda =10\), \(\alpha=0.5, c=0.1\). The distribution of expected profits by source corresponds with immediate purchases and late buys in the first row and immediate purchases, late buys for old products, and late buys for new products in the second row, respectively.

\(^5\)Results for other \(T\) and \(\lambda\) values are similar to those presented herein.

\(^6\)Notes. Results are based on \(I=10\), \(T=0.5\), \(\lambda =10, c=0.1\).
Table 3. Influence of Consumer Waiting Behavior on Seller Profit

| Q | $\alpha = 0.5$ | $\alpha = 1$ | $\alpha = 1.5$ | $\alpha = 0.5$ | $\alpha = 1$ | $\alpha = 1.5$ |
|---|---|---|---|---|---|---|
| 0 | -2.61 | -10.04 | -38.74 | -4.95 | -11.7 | -33.31 |
| 1 | -9.21 | -16.73 | -40.92 | -4.64 | -20.52 | -71.89 |
| 5 | 7.08 | 22.32 | 52.68 | -12.30 | -17.79 | -43.36 |

In the terms of quality selection behavior, this paper introduces a decline factor $\alpha$ that describes the changes of consumer retention value over time. As $\alpha$ increases, the consumers sensitivity to time increases and the retention value decreases. Figure 6 presents the changes of retailer’s profit with respect to $\alpha$ with replenishment inventory $Q \in \{0, 1, 5, 10\}$. When $Q=1$ and 10, the optimal profit is always less or equal to that of $Q=0$. When $Q=5$, then $\alpha > 0.9$ and the optimal profit for a replenishment case is larger than that of a non-replenishment case, thus indicating whether restocking is subject to $\alpha$. A small $\alpha$ is linked to the non-replenishment case, and a large refers to a replenishment case. This is likely because0 when $\alpha$ is small, the quality choice behavior of consumers causes some high reservation value consumers to switch to low-priced products, resulting in the loss of profits after restocking. By contrast, this quality choice behavior is not significant when $\alpha$ is large and, thus, restocking can significantly increase profits. Therefore, correctly estimating the value of $\alpha$ can help the retailer make a better pricing and inventory decision.

8. Conclusion. In the uncertain demand of the perishable products market, customers are increasingly more strategic in arranging reasonable time to buy and choose the best products, which compels the sellers to consider the strategic behavior of consumers when making pricing and inventory decisions. This paper studies consumer purchase decisions and retailer pricing strategy in two separate scenarios, a supplementary inventory scenario and a non-supplementary inventory scenario. First, the retailer announces his pricing and inventory policies, and the consumers determine their purchase behavior in response to the decision of the retailer. Accordingly, the main conclusions are as follows:

1) The equilibrium game exists between the retailer and the consumer. For the announced pricing and inventory strategies, there is an optimal threshold function to divide the types of consumers. Without replenishing inventory, consumers, based on threshold function, decide to buy immediately or to wait. The waiting consumers are divided into two groups, those who wait to buy supplementary products and those who wait to buy surplus products when the inventory is replenished. Furthermore, in the second phase, the threshold function classifies consumers into two types, those who purchase surplus products in the first stage and those who purchase in the second stage.

2) The decision to supplement inventory requires further consideration. In the case of no inventory limit, replenishment inventory is the optimal strategy for a retailer. When considering an inventory limit, the inventory replenishment strategy can avoid the loss of perishable products value caused by inventory accumulation and promote the commodity circulation rate, thereby enhancing retailer’s profit.
However, this strategy promotes, to a certain extent, the waiting behavior of consumers and damages the interests of the seller. A numerical analysis shows that when deciding whether to adopt a replenishment strategy the retailer must consider the market demand and exponential decline factor. When the market demand is greater, inventory restocking derives better results than a supplementary inventory situation. The same is true when is larger, that is, replenishment is better than no replenishment.

(3) Although consumer waiting behavior does not necessarily cause the retailer to suffer profit losses, a pricing strategy that ignores such behavior causes the retailer to suffer profit losses. Consumer waiting behavior accelerates the depreciation of commodities and affects the overall profits of the retailer. On the other hand, the waiting behavior of consumers compels some customers, those whose price is less than , to buy the old products in the second stage, thus reducing the loss of profits. Hence, the interaction of the two results in great uncertainty with respect to profit change, for which the key influencing factor is the added inventory size. The appropriate replenished inventory can enable consumer waiting behavior and promote retailer profits, and vice versa.

This paper considers only pricing and inventory decisions; however, various factors influence consumers to buy more, such as product differentiation, popularity of the Internet and so on. In addition, retailers expect profit for the objectives of this study’s optimal pricing problem. In fact, the risk in an uncertain environment is an important indicator that retailers must consider. Moreover, although consumers are heterogeneous in that they have different sensitivities to the willingness to wait to purchase, etc., we treat them the same. Accordingly, these are our suggestions for further research.

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Appendix A. Proof of theorems.

Proof of Theorem 4.1. First, we consider a consumer arrive at the second stage (namely, t ∈ [T, 1]). It is obvious that the consumer will purchase a product if . That is, when t ∈ [T, 1], when t < T, the conditions for an immediate purchase are V(t) - p1 ≥ 0 and V(t) - p1 ≥ θ(V(t)e^{-α(T-t)} - p2). Through simple mathematical derivation, we have V(t) ≥ max{p1 - p2, p1}. Thus, the theorem is proved.

Proof of Theorem 5.1. Considering from the following three periods:

Period 1: 1 ≥ t ≥ min(\frac{1}{α}ln\frac{1}{\theta}, T)  
Under this condition, we obtain  
V(t) - p1 ≤ \hat{θ}V - p1 ≤ \hat{θ}(V(t)e^{αt} - p1),  
The formula of V(t) - p1 is the surplus gained by purchasing immediately, whereas the formula of \hat{θ}(V(t)e^{αt} - p1) is the expected surplus gained by waiting for purchasing the replenishment products. This inequation indicates that customers will choose to wait and not buy the product immediately if T ≥ t ≥ min(\frac{1}{α}ln\frac{1}{\theta}, T).
Period 2: $t \leq \min \left( \frac{1}{\alpha} \ln \frac{1}{\theta}, T \right)$
In this phase, (a) the conditions for an immediate purchase are

$V(t) - p_1 \geq 0$,
$V(t) - p_1 \geq \tilde{\theta}(V(t)e^{\alpha t} - p_1)$,
$V(t) - p_1 \geq \tilde{\theta}(V(t)e^{-\alpha(T-t)} - p_2)$.

Through simple mathematical derivation, we have

$V(t) \geq \tilde{\psi}_1(t) = \max \left[ \frac{p_1 - \tilde{\theta}p_2}{1 - \theta e^{\alpha t}}, \frac{p_1 - \tilde{\theta}p_2}{1 - \theta e^{-\alpha(T-t)}}, p_1 \right]$.

(b) The conditions for waiting to purchase new products rather than old products are

$V(t)e^{\alpha t} - p_1 \geq 0$,
$V(t) - p_1 \leq \tilde{\theta}(V(t)e^{\alpha t} - p_1)$,
$V(t)e^{\alpha t} - p_1 \leq \tilde{\theta}(V(t)e^{-\alpha(T-t)} - p_2)$,
$V(t)e^{\alpha t} - p_1 \geq V(t)e^{-\alpha(T-t)} - p_2$.

Through simple mathematical derivation, we have

$\psi_1(t) \geq V(t) \geq \tilde{\psi}_2(t) = \max \left\{ \frac{p_1 - \tilde{\theta}p_2}{e^{\alpha t} - e^{-\alpha t}}, p_1 e^{-\alpha T} \right\}$.

(c) The conditions for purchasing old products rather than new products are

$V(t) - p_1 \geq 0$,
$V(t) - p_1 \leq \tilde{\theta}(V(t)e^{\alpha t} - p_1)$,
$V(t) - p_1 \leq \tilde{\theta}(V(t)e^{-\alpha(T-t)} - p_2)$,
$V(T) \geq p_2$.

Through simple mathematical derivation, we have $V(t) \leq \tilde{\psi}_2(t)$ and $V(T) \geq p_2$.

Period 3: $T \leq t \leq 1$
In the second stage, customers purchase new products rather than old products if and only if $V(t)e^{\alpha t} - p_1 \geq 0$ and $V(t)e^{\alpha T} - p_1 \geq V(t) - p_2$, namely, $V(t) \geq \tilde{\psi}_2(t) = \max \left\{ \frac{p_1 - \tilde{\theta}p_2}{e^{\alpha t} - e^{-\alpha t}}, p_1 e^{-\alpha T} \right\}$.

Appendix B. Proof of propositions.

Proof of Proposition 1. When consumers are myopic, the retailer’s goal is to maximize the expected profit

$$\pi^{1/N}_{I/N} = \max_{\lambda \in [0,1]} \left\{ \lambda \pi_1[T - \frac{p_2}{\theta} (e^{\alpha T} - 1)] + \lambda p_2 [1 - T - \frac{p_2}{\theta} (e^\alpha - e^{\alpha T})] \right\}.$$ 

Solving this problem we get $p_1^* = T/2(e^\alpha - 1), p_2^* = (1 - T)\alpha/(e^\alpha - e^{\alpha T})$ and

$$\pi^{1/N}_{I/N} = \frac{\lambda \alpha}{2} \left[ \frac{T}{e^{\alpha T}} + \frac{(1-e^{\alpha T})}{\alpha} \right].$$

When using a fixed-price strategy, the retailer’s goal is to maximize the expected profit:

$$\pi^{1/F}_{I/F} = \max_{\lambda \in [0,1]} \left\{ \lambda \pi_1(1 - \frac{e^\alpha - 1}{\alpha}) \right\}.$$ 

Solving this problem we get $p_1^* = \frac{\alpha}{2(e^{\alpha - 1})}$ and $\pi^{1/F}_{I/F} = \frac{\lambda \alpha}{4(e^{\alpha - 1})}$. Observe that $\lim_{T \to 0} \pi^{1/N}_{I/N} = \lim_{T \to 1} \pi^{1/N}_{I/N} = \pi^{1/F}_{I/F}$ and $\pi^{1/N}_{I/N}$ is convex from 0 to 1 on $T$. Thus, we can draw the conclusion that $\pi^{1/F}_{I/F} < \pi^{1/N}_{I/N}$.

Proof of Proposition 2. It is evident from the proposition that the retailer can receive more benefits with the premium price $p_1$ for replenishment inventory in the second stage of the sales season when the customers are non-waiting (myopic). In the case of strategic customers, the retailer must select the prices by solving the following problems:

$$\pi^{1/S}_{I/S} = p_1 m_1^1 + p_2 (m_2^1 + m_3^1);$$
$$\pi^{1+Q/S}_{I/S} = p_1 (m_1^3 + m_1^Q) + p_2 (m_2^2 + m_3^2).$$
In the above formulas, we have $\theta_w = \bar{\theta} = \hat{\theta} = 0$ and $\tilde{\theta} = \theta = 1$. It is true that $\tilde{m}^3 + \tilde{m}^3 > m^1$ and $\tilde{m}^2 + \tilde{m}^2 + \tilde{m}^3 > m^1 + m^2 + m^2$ under the conditions of Proposition 2. Therefore, we conclude that $\tilde{\pi}_{I+Q/S} > \pi_{I/S}^{\ast}$. \hfill $\square$

**Proof of Proposition 3.** First, in the case of fixed-price strategy, we have $\pi_{I/F}^{\ast} = \lambda p(1 - p)|_{p=1/2} = \frac{1}{4}$.

In the case of two-stage pricing strategy, we have

$$\pi_{I/S}^{\ast} = \max_{p_1, p_2}\{\lambda p_2(m^2_p + m^2_T)\} = \lambda p_2(1 - p_2)|_{p_2=1/2} = \frac{\lambda}{4},$$

$$\pi_{I/N}^{\ast} = \max_{p_1, p_2}\{\lambda p_1 \int_0^T (1 - p_1) dt + \lambda p_2 \int_0^T (1 - p_2) dt\} = \lambda T p_1(1 - p_1)|_{p_1=1/2} + \lambda (1 - T)p_2(1 - p_2)|_{p_2=1/2} = \frac{\lambda}{4}.$$

Under the condition of $\alpha = 0$, consumers are not affected by product freshness, therefore, the replenishment inventory cannot increase retailer revenue whether consumers are strategic or myopic. That is, $\tilde{\pi}_{I+Q/S} > \pi_{I/S}^{\ast} \tilde{\pi}_{I+Q/N} > \pi_{I/N}^{\ast}$. From the above, we can conclude this proposition. \hfill $\square$

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