Model of Electro-Weak Interaction

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The $U_L(2) \otimes U_R(2)$ gauge model for the unified theory of the electromagnetic and weak interactions which is free from a priori self-interaction scalar field, is developed. Due to breaking the initial symmetry the $SU_L(2) \otimes U_R(1)$ Lagrangian is derived. The obtained $SU_L(2) \otimes U_R(1)$ Lagrangian contains the whole of terms corresponding both to free boson and fermion fields and to interaction between them, as it takes place in the Standard Model (SM). We show that all boson fields, including the Higgs one, directly arise due to breaking the initial symmetry, and are generated by the initial gauge fields in contrary to the Standard Model consideration. The Higgs fields are studied in detail. A broad spectrum of states of the Higgs bosons is found. The masses of the Higgs particle in such states are calculated.

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I. INTRODUCTION

The first attempt to unify the electromagnetism and weak interaction have been made by H.Yukawa[1] and O.Klein[2] where the boson-exchange model for the charge-changing weak interactions was proposed. J.Schwinger has studied this problem in the 1950s[3]. The hitch in the unification of the electromagnetic and weak interaction has been made due to the papers by S.Glashow[4], S.Weinberg[5], A. Salam[6]. The developed Gleshow-Weinberg-Salam theory appeared to be in a good agree with the neutral currents experiments[7–9]. Since the papers[4–6] have been issued the unified theory of weak and electromagnetic interactions is one of the most important and discussable problem in the physics of particles and fields.

The observations of $W^\pm$ and $Z^{(0)}$ bosons by the UA1 and UA2 Collaborations in CERN in 1983[10], which supported the Glashow-Weinberg-Salam model, have additionally increased interest to the Standard Model of electro-weak interaction due to the problem of the Higgs boson[11] which has been still undiscovered.

Searches for the Higgs boson in the Standard Model and beyond it have still remained the main goals in investigations of both the Tevatron (CDF and D0 Collaborations) and LHC (ATLAS and CMS Collaborations). In this way, the present LEP, Tevatron and LHC data have already placed strong direct bounds on the possible mass of the Higgs particle. According to the result obtained them the allowed range of the Higgs mass is between 114 GeV and 145 GeV, and above 450 GeV[12–15].

Studying the Higgs boson strongly stimulates developments of various theoretical models beyond SM which can be, generally, verified due to the Tevatron and LHC machines. The LHC results have been recently considered in the context of SUSY singlet extensions[16], in more general 2HDM scenarios[17], and also for dark Higgs models[18] (where the SM Higgs sector is enlarged with the SM singlet). In the past years there has been extensive work in extensions of the Higgs sector in terms of the Minimal Supersymmetric Standard Model (MSSM) by higher-dimension operators[19] and due to models beyond MSSM (BMSSM)[20].

In the present paper unification of the electromagnetic and weak interaction is studied in terms of the $U_L(2) \otimes U_R(2)$ gauge symmetry without a priory self-interaction scalar field. Due to braking the initial symmetry the $SU_L(2) \otimes U_R(1)$ Lagrangian is derived. The obtained $SU_L(2) \otimes U_R(1)$ Lagrangian is found to take into account correctly of both the charged and neutral interaction currents as it takes place in the SM[4–6]. In this way, all massive boson fields (including the Higgs one), the massive fermion field as well as electromagnetic field naturally arise as the superposition of various modes of the initial gauge fields without any additional self-interacting scalar field as compared with the situation taking place in the SM consideration[4–6]. The structure of the Higgs field is studied in detail. On a basis of the experimental data for the fine structure coupling constant, Fermi coupling constants, and $W$ boson mass, the coupling constant for the interaction current as well as the masses of the Higgs and $Z^{(0)}$ bosons are calculated.

It is shown that there is a broad spectrum of the mass states of the Higgs boson which are formed by various modes of the initial gauge fields. The masses of such states are calculated. The Higgs boson mass is found to be in the interval $M_H = 92.8 \div 231.7$ GeV.

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The paper is organized as follows. The second section contains the statement of the problem. Breaking the initial symmetry and deriving the $SU_L(2) \otimes SU_R(1)$ Lagrangian are in Section III. The qualitative calculations of the coupling constants and masses are in the section IV. The last section is Conclusion.

II. STATEMENT OF THE PROBLEM

We start from the $U_L(2) \otimes U_R(2)$ gauge invariant Lagrangian:

\[ \mathcal{L} = \mathcal{L}_c + \mathcal{L}_\mu + \mathcal{L}_\tau; \]

\[ \mathcal{L}_c = Tr \left\{ \frac{i}{2} \left( \bar{\Psi}^{(e)}(x) \gamma^\mu \partial_\mu \Psi^{(e)}(x) - \bar{\Psi}^{(e)}(x) \gamma^\mu \partial_\mu \Psi^{(e)}(x) \right) + g_1 \bar{\Psi}^{(e)}(x) \gamma^\mu A_\mu^a(x) T_a \Psi^{(e)}(x) - g_2 \bar{\Psi}_R^{(e)}(x) \gamma^\mu B_\mu^a(x) T_a \Psi_R^{(e)}(x) \right\} - \frac{1}{4} F_{\mu\nu}^a(x) F_{\mu\nu}^a(x) + G_{\mu\nu}^a(x) G_{\mu\nu}^a(x) \equiv \mathcal{L}^{(free)} - \mathcal{L}^{(int)}, \]

where the indexes $(e), (\mu), (\tau)$ correspond to the electronic, muonic and $\tau$-leptonic parts of the Lagrangian. The Lagrangians $\mathcal{L}_c$ and $\mathcal{L}_\tau$ are obtained from $\mathcal{L}_c$ due to the substitutions of the index $(e)$ by $(\mu)$ and $(\tau)$, respectively [24]. The superscripts mean the part of the Lagrangian which correspond to free particles and interactions between them, respectively.

The "left" $\Psi_L^{(e)}(x)$ and "right" $\Psi_R^{(e)}(x)$ doublets of the massless fermion field $\Psi^{(e)}(x)$ are given by the formulae:

\[ \Psi_L^{(e)}(x) = \frac{1 + \gamma^5}{2} \Psi^{(e)}(x); \quad \Psi_R^{(e)}(x) = \frac{1 - \gamma^5}{2} \Psi^{(e)}(x); \quad \Psi^{(e)}(x) = \begin{pmatrix} \nu^{(e)}(x) \\ e^{(e)}(x) \end{pmatrix}, \]

where $\nu^{(e)}(x); e^{(e)}(x)$ are neutrino and massless electron fields, respectively; $\gamma^k$ are the Dirac matrices.

The tensors of the gauge fields are determined by the standard way [21]:

\[ F_{\mu\nu}^a(x) = \partial_\mu A_\nu^a(x) - \partial_\nu A_\mu^a(x) + g_1 f^a_{bc} A_\mu^b(x) A_\nu^c(x) \equiv A_\mu^a(x) A_\nu^a(x) + g_1 f^a_{bc} A_\mu^b(x) A_\nu^c(x); \]

\[ G_{\mu\nu}^a(x) = \partial_\mu B_\nu^a(x) - \partial_\nu B_\mu^a(x) + g_2 f^a_{bc} B_\mu^b(x) B_\nu^c(x) \equiv B_\mu^a(x) B_\nu^a(x) + g_1 f^a_{bc} B_\mu^b(x) B_\nu^c(x) \]

where $g_1$ and $g_2$ are the coupling constants. The symbols $f^a_{bc}$ are the structure of constants of the $U(2)$ group which govern the commutative relations between the generators $T_a$:

\[ T_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \quad T_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad T_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \quad T_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \]

\[ [T_a, T_b] = T_a T_b - T_b T_a = i f_{abc} T_c; \quad Tr (T_a T_b) = \frac{1}{2} \delta_{ab} \left( 1 + \frac{3(\delta_{a0} + \delta_{b0})}{2} \right) \]

The constant $f^a_{bc}$ are

\[ f_{abc} = \pm 1; \quad \text{when} \quad a \neq b \neq c; \quad a, b, c \neq 0; \quad f_{123} = +1. \]

In this way, $f^a_{bc} = 0$, when either any two indexes are the same, or some index is equal to zero.

In the formulae (1)-(4) we take that $x \equiv x^\mu = (x^0; \vec{x})$ is a vector in the Minkowski space-time; $\partial_\mu = (\partial/\partial t; \vec{\nabla})$. We use the signature $diag (g^{\mu\nu}) = (1; -1; -1; -1)$ for the metric tensor $g^{\mu\nu}$. The line over $\Psi$ means the Dirac conjugation.
III. BREAKING THE INITIAL SYMMETRY

We break the $U(2) \otimes U(2)$ symmetry by shifting the gauge fields $A_c^a(x)$ and $B_c^a(x)$ according to the formula:

$$A_c^a(x) = a_c^a + A_c^a(x); \quad B_c^a(x) = b_c^a + B_c^a(x);$$

where $a_c^a$ and $b_c^a$ are the constant vectors such that

$$a_c^a a_c^b = -\frac{1}{16} g_{\mu}^\nu \delta_c^b a^2; \quad b_c^a b_c^b = -\frac{1}{16} g_{\mu}^\nu \delta_c^b a^2, \quad a, b = 0, 1, 2, 3;$$

where $a$ and $b$ are some real constants.

We also simultaneously change the "left" and "right" components of the fermion fields by the following unitarian transformation in the Dirac space:

$$\Psi_L^{(\epsilon)}(x) = \{(T_{l(X;x)}(x) \exp\left\{ i g_1 a_{\mu} T_a (x^\mu - X^\mu) + i g_1 T_3 \int x^\mu a_{\mu} + i g_1 \frac{1}{2} (3 T_0 - T_3) \int x^\mu b_{\mu} \right\} \right\}. \quad (8)$$

$$\Psi_R^{(\epsilon)}(x) = \{(T_{l(X;x)}(x) \exp\left\{ -i g_2 b_{\mu} T_a (x^\mu - X^\mu) + i g_2 \int x^\mu a_{\mu} + i g_2 \frac{1}{2} (3 T_0 - T_3) \int x^\mu b_{\mu} \right\} \right\}. \quad (9)$$

where $G_{(\epsilon)}$ is some coupling constant. The symbol $\{(T_{l(X;x)}(x) \exp\}$ is the chronological exponent which means integration along the line $l(X;x)$ in the Minkowski space-time so that the points $X$ is always before the point $x$.

The parameters $a$ and $b$ introduced in Eqs.(6) and (7) as well as the coupling constant $G_{(\epsilon)}$ can be expressed in terms of the observable masses and coupling constants as it will be shown below (see Discussion).

We substitute the $\Psi_L^{(\epsilon)}(x)$ and $\Psi_R^{(\epsilon)}(x)$ transformed according to Eqs.(8), (9) into the Lagrangian (1). After that, following the Schwinger’s idea [22] we calculate a limit:

$$\mathcal{L}[\bar{\Psi}(x); \ \Psi(x)] = \lim_{x' \to x} \mathcal{L}[\bar{\Psi}(x'); \ \Psi(x)]. \quad (10)$$

As a result, the kinematic term of the fermion part of the Lagrangian (1) takes the form:
\[ \text{lim}_{x' \to x} \left\{ \text{Tr} \left[ \tilde{\psi}^{(e)}(x) \gamma^\mu \partial_\mu \psi^{(e)}(x) - \tilde{\psi}^{(e)}(x) \gamma^\mu \partial_\mu \psi^{(e)}(x) \right] \right\} \] 

where the differentiation \( \partial_\mu \) with respect to the unprimed variable \( x \) is assumed in the last formula. The symbols \( e_\nu^a \) are the constant vectors which is governed by the expressions:

\[ e_\nu^a = e_\nu^a + e_\nu^{\prime a}; \quad e_\nu^a e_\nu^b = -\frac{1}{12} \delta^a_b; \quad e_\nu^a e_\nu^{\prime b} = -\frac{1}{12} \delta^a_b; \quad e_\nu^a e_\nu^b = 0; \quad a, b = 1, 2, 3. \]

We calculate the limit in Eq. (11) when \( x' \) goes to \( x \). After that, substituting the transformations (6), (8), (9) into the initial Lagrangian we derive:

\[ \mathcal{L}^{(\text{free})} = \text{Tr} \left\{ \tilde{\psi}^{(e)}(x) \gamma^\mu \partial_\mu \psi^{(e)}(x) - \tilde{\psi}^{(e)}(x) \gamma^\mu \partial_\mu \psi^{(e)}(x) \right\} - \]

\[ \frac{1}{4} \left\{ A^a_{\mu\nu}(x) A^a_{\mu\nu}(x) + B^a_{\mu\nu}(x) B^a_{\mu\nu}(x) - \frac{3}{4} g_1^2 \bar{\psi}_L(x) A^a_{\mu}(x) A^a_{\mu}(x) - \frac{3}{4} g_2^2 \bar{\psi}_R(x) A^a_{\mu}(x) A^a_{\mu}(x) \right\} + \mathcal{L}(e)[A^4; B^4] \]

\[ -\mathcal{L}_{(e)}^{(\text{int})} = \text{Tr} \left\{ \tilde{\psi}^{(e)}(x) \gamma^\mu (T_1 A^a_{\mu} + T_2 A^a_{\mu} \psi^{(e)}(x) + g_1 \bar{\psi}_L(x) \gamma^\mu (T_0 + T_3) A^a_{\mu} \psi^{(e)}(x) - \right. \]

\[ \frac{g_1}{2} \bar{\psi}_L(x) \gamma^\mu (T_0 - T_3) B^a_{\mu} \psi^{(e)}(x) + g_2 \bar{\psi}_R(x) \gamma^\mu (T_3 - T_0) B^a_{\mu} \psi^{(e)}(x) \} - \]

\[ \text{Tr} \left\{ \frac{g_1}{2} \bar{\psi}_L(x) \gamma^\mu (T_0 - T_3) B^a_{\mu} \psi^{(e)}(x) + g_2 \bar{\psi}_R(x) \gamma^\mu (T_3 - T_0) B^a_{\mu} \psi^{(e)}(x) \} - \]

where \( \mathcal{L}^{(\text{free})} \), \( \mathcal{L}_{(e)}^{(\text{int})} \) are the Lagrangians of free field and interaction between them, respectively; \( \mathcal{L}(e)[A^4; B^4] \) is the part of the Lagrangian which contains the gauge field in the 4th power. We keep the old notations \( e(x) \) for the electronic field in Eq. (14), although the transformations (9), (10) have been already made.

Following the standard way we introduce the charged boson fields \( W^\pm_\nu(x) \) as well as the electromagnetic \( A_\nu(x) \) and neutral boson field \( Z^{(0)}_\nu(x) \):

\[ A^1_\nu = \frac{W^-_\nu + W^+_\nu}{\sqrt{2}}; \quad A^2_\nu = \frac{W^-_\nu - W^+_\nu}{i\sqrt{2}}; \quad A^0_\nu = \frac{-g_2 A_\nu + g_1 Z_\nu}{\sqrt{g_1^2 + g_2^2}}; \quad B^0_\nu = \frac{g_1 A_\nu + g_2 Z_\nu}{\sqrt{g_1^2 + g_2^2}}; \quad B^3_\nu = \frac{\sqrt{g_1^2 + g_2^2}}{2g_1} Z_\nu; \quad Z_\nu = \left( \frac{4g_1^2}{5g_1^2 + g_2^2} \right)^\frac{1}{2} Z^{(0)}_\nu \]
As to the components $B^1_\nu$, $B^2_\nu$ and $A^0_\nu$ we determine them by means of the realtions:

\[ B^1_\nu = (\sigma(1)(x)e^1_\nu + \sigma_0 e^1_\nu); \quad B^2_\nu = (\sigma(2)(x)e^2_\nu + \sigma_0 e^2_\nu); \quad A^0_\nu = (\sigma(3)(x)e^3_\nu + \sigma_0 e^3_\nu), \]

where $\sigma(i)$ are scalar functions while the constant $\sigma_0$ is related to the electron mass $m_e$ and the coupling constant $G_e$ (see Eqs.(8), (9)) so that:

\[ m_e = -\frac{G_e \sigma_0}{2}. \]

Substituting $A^0_\nu$ and $B^\mu_\nu$ in the form given by Eqs.(15)-(17) into the formulae (13), (14) we derive:

\[ \mathcal{L}^{(free)}(e) = \frac{1}{2} \left( \bar{e}(x)\gamma^\mu \partial_\mu e(x) - \bar{e}(x)\gamma^\mu \gamma^5 \partial_\mu e(x) \right) + \frac{i}{2} \left( \bar{\nu}(x)\gamma^\mu \partial_\mu \nu(x) - \bar{\nu}(x)\gamma^\mu \gamma^5 \partial_\mu \nu(x) \right) - m_e \bar{e}(x) e(x) - \frac{1}{2} W^{-\mu \nu} W^{\mu \nu} + \frac{3g_1^2a^2}{8} W^{-\mu \nu} W^{\mu \nu} - \frac{1}{4} Z^{(0)\mu \nu} Z^{(0)\mu} + \frac{3g_2^2b^2}{16} g_1^2 + \frac{g_2^2}{g_1^2 + g_2^2} Z^{(0)\mu} Z^{(0)\mu} - \frac{1}{4} A^{\mu \nu} A_{\mu \nu} + \frac{1}{8} \sum_1^3 (\partial_\mu \sigma(i)(x))(\partial^\mu \sigma(i)(x)) - \frac{1}{16} (g_1^2 a^2 \sigma(1)(x) + g_2^2 b^2 \sigma(1)(x) + \sigma(2)(x)) + \mathcal{L}(c)[A^4; B^4] \]

\[ - \mathcal{L}^{(int)}(c) = \frac{g_1}{2\sqrt{2}} (\bar{\nu}(x)\gamma^\mu (1 + \gamma^5) \nu(x) W^+_{\mu} + \bar{e}(x)\gamma^\mu (1 + \gamma^5) \nu^{(c)}(x) W^-_{\mu} - \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \bar{e}(x)\gamma^\mu e(x) A_{\mu} - \frac{g_1^2 + g_2^2}{2\sqrt{g_1^2 + g_2^2}} \bar{\nu}(x)(1 + \gamma^5) \nu^{(c)}(x) Z^{(0)\mu} - 2 \bar{e}(x)\gamma^\mu \left( \frac{g_1^2 - g_2^2}{g_1^2 + g_2^2} + \gamma^5 \right) e(x) Z^{(0)\mu} - \frac{G_e^2}{6} \sum_1^3 \bar{e}(x)\sigma(i)(x)e(x). \]

In the last formula we kept the old notations for the neutrino $\nu^{(c)}(x)$ and electron $e(x)$ fields although they have been already transformed by means of Eqs.(8), (9).

Let us introduce the masses of the intermediate charged $W^{\pm}$ and neutral $Z^{(0)}$ bosons according to the formulae:

\[ M_{W^\pm}^2 = \frac{3}{8} g_1^2 a^2; \quad M_Z^2 = \frac{3}{8} g_2^2 b^2 \frac{g_1^2 + g_2^2}{5g_1^2 + g_2^2}; \quad b^2 = a^2 \frac{5g_1^2 + g_2^2}{g_2^2}. \]

It follows from Eqs.(17), (20) that the parameters $a$, $b$ and $G_e$ are expressed via the observable coupling constants and masses as it has been already mentioned before.

Then, the Lagrangian of free particles can be written as follows:

\[ \mathcal{L}^{(free)}(e) = \frac{1}{2} \left( \bar{e}(x)\gamma^\mu \partial_\mu e(x) - \bar{e}(x)\gamma^\mu \gamma^5 \partial_\mu e(x) \right) + \frac{i}{2} \left( \bar{\nu}(x)\gamma^\mu \partial_\mu \nu(x) - \bar{\nu}(x)\gamma^\mu \gamma^5 \partial_\mu \nu(x) \right) - m_e \bar{e}(x) e(x) - \frac{1}{2} W^{-\mu \nu} W^{\mu \nu} + M_W^2 W^{-\mu \nu} W^{\mu \nu} - \frac{1}{4} Z^{(0)\mu \nu} q Z^{(0)\mu} + \frac{1}{2} M_Z^2 Z^{(0)\mu \nu} Z^{(0)\mu} - \frac{1}{4} A^{\mu \nu} A_{\mu \nu} + \frac{1}{8} \sum_1^3 (\partial_\mu \sigma(i)(x))(\partial^\mu \sigma(i)(x)) - \frac{1}{6} \left( M_W^2 \sigma(3)(x) + \sigma(1)(x) + \sigma(2)(x) \right) + \mathcal{L}(c)[A^4; B^4] \equiv \mathcal{L}^{(free)}(lepton) + \mathcal{L}^{(free)}(W) + \mathcal{L}^{(free)}(Z) + \mathcal{L}^{(free)}(Higgs) + \mathcal{L}(c)[A^4; B^4], \]

where $W^{\pm}$ and $Z^{(0)}_\mu$ are the massive vector fields, $A_\mu$ is the electromagnetic field. The functions $\sigma(i)(x)$ form the field of a scalar boson.

Eqs. (19), (21) consist of the electro weak Lagrangian which is $SU_L(2) \otimes U_R(1)$ invariant. Although the Lagrangian given by Eqs.(19), (21) is very similar in its structure to the one derived by S.Weinberg and A.Salam [5, 6], it dramatically differ from the Weinberg-Salam Lagrangian since all terms in the formulae (18), (19) are governed by the initial gauge fields without any additional scalar field interacting with fermion and gauge fields [5, 6].
IV. DISCUSSION

Following the standard way we take

\[
\sqrt{\alpha} = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}; \quad \frac{G_F}{\sqrt{2}} = \frac{g_1^2}{8M_W^2}; \quad \sin \theta_W = \frac{\sqrt{\alpha}}{g_1} = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}.
\]

(22)

where \( \theta_W \) is the Weinberg angle; \( \alpha \) and \( G_F \) are the fine structure coupling constant and the Fermi coupling constants, respectively. Provided that \( M_W \) and \( \theta_W \) are known due to experiments, the formulae (20), (22) establish relation between \( M_W \) and \( M_Z \):

\[
M_Z^2 = \frac{M_W^2 \cos^2 \theta_W}{\cos^2 \theta_W}.
\]

(23)

Setting \( \sin^2 \theta_W = 0.23 \) we obtain the well-known result for the \( Z^0 \) boson mass\( 23 \).

Taking into account of the experimental results \( 23 \), we obtain that the constants \( g_1 \) and \( g_2 \) are:

\[
g_1 \approx 0.65, \quad g_2 \approx 0.36
\]

(24)

A. Interaction Lagrangian

Comparing the interaction Lagrangian (19) with the corresponding terms in the Standard Model calculations\( 5, 6 \) we have found that the Lagrangians slightly differ by the coupling constant \( C(\text{neutral}) \) for a neutral current. They are connected by the relation:

\[
C(\text{neutral}) = C_{SM}(\text{neutral}) \cdot \frac{2}{\sqrt{5 + g_2^2/g_1^2}} \approx 0.87 C_{SM}(\text{neutral}),
\]

(25)

where \( C_{SM}(\text{neutral}) \) is the coupling constants corresponding to the neutral currents in the Standard Model\( 5, 6 \).

B. Higgs bosons

The terms in Eq.(21) which depend on the scalar functions \( \sigma_i(x) \) contain the main differences from the SM results. First, they as well as the whole of the Lagrangian (19), (21) are derived in terms of the gauge fields \( A_\mu^a \) and \( B_\mu^a \) without any additional scalar field\( 5, 6 \). Besides that, various superpositions of \( \sigma_i(x) \) appear to form the different mass states of the scalar Higgs mode. The structure of the Lagrangian (21) dictates the following mass states to the Higgs boson. It is obvious that there are possible either a singlet or doublet or singlet and doublet together. Let us study mass states of the scalar Higgs mode.

a) When all fields \( \sigma_i(x) \) are different and not equal to zero, so that \( \sigma(1) \neq \sigma(2) \neq \sigma(3) \), the Lagrangian \( \mathcal{L}_{(e)}^{(free)}(Higgs) \) can be presented in the form:

\[
\mathcal{L}_{(e)}^{(free)}(Higgs) = \frac{1}{2} (\partial_\mu H_S)(\partial^\mu H_S) - \frac{1}{2} M_S^2(H) H_S^2 + \frac{1}{2} \partial_\mu \left( \begin{array}{c} H_D(1) \\ H_D(2) \end{array} \right) \cdot \partial^\mu \left( \begin{array}{c} H_D(1) \\ H_D(2) \end{array} \right) - \frac{1}{2} M_D^2(H) \left( \begin{array}{c} H_D(1) \\ H_D(2) \end{array} \right) \cdot \left( \begin{array}{c} H_D(1) \\ H_D(2) \end{array} \right)
\]

\[
H_S = \frac{\sigma(3)(x)}{2}; \quad H_D(1) = \frac{\sigma(1)(x)}{2}; \quad H_D(2) = \frac{\sigma(2)(x)}{2}
\]

(26)

where \( M_D(H) \) and \( M_S(H) \) are the masses of a doublet and singlet, respectively, which are
\[ M_D(H) = M_W \sqrt{\frac{20}{3} + \frac{4g_2^2}{3g_1^2}} \approx 213.7\text{GeV}; \quad M_S(H) = M_W \sqrt{\frac{4}{3}} \approx 92.8\text{GeV}. \] (27)

b) If \( \sigma_3(x) = 0 \) but \( \sigma_1(x) \neq \sigma_2(x) \) the Lagrangian has the form:

\[
\mathcal{L}^{(\text{free})}_{(Higgs)} = \frac{1}{2} \partial_{\mu} \left( \begin{array}{c} H_D(1) \\ \sigma_D(2) \end{array} \right)^\dagger \cdot \partial^\mu \left( \begin{array}{c} H_D(1) \\ \sigma_D(2) \end{array} \right) - \frac{1}{2} M_D^2(H) \left( \begin{array}{c} H_D(1) \\ \sigma_D(2) \end{array} \right)^\dagger \cdot \left( \begin{array}{c} H_D(1) \\ \sigma_D(2) \end{array} \right),
\]

such that the doublet mass \( M_D(H) \) is:

\[
M_D(H) = M_W \sqrt{\frac{20}{3} + \frac{4g_2^2}{3g_1^2}} \approx 213.7\text{GeV};
\] (29)

c) All remaining cases correspond to the singlet states \( H_S(i) \) of the Higgs boson. Both the mass of the states and explicit form of them depend strongly on the gauge of the initial gauge fields \( A^a_\mu(x) \) and \( B^a_\mu(x) \). The Lagrangian governing such states is:

\[
\mathcal{L}^{(\text{free})}_{(Higgs)} = \frac{1}{2} (\partial_{\mu} H_S(i)) (\partial^\mu H_S(i)) - \frac{1}{2} M_S^2(H) H_S^2(i),
\] (30)

where \( H_S(i) \) is the field of the Higgs mode which is directly expressed via the functions \( \sigma_i(x) \).

When \( \sigma_1(x) = \sigma_3(x) = 0; \quad \sigma_2(x) \neq 0, \) or \( \sigma_2(x) = \sigma_3(x) = 0; \quad \sigma_1(x) \neq 0, \) or \( \sigma_1(x) = \sigma_2(x) = 0; \quad \sigma_3(x) \neq 0, \) the Higgs mass is

\[
M_S(H) = M_W \sqrt{\frac{44}{9} + \frac{8g_2^2}{9g_1^2}} \approx 182.6\text{GeV}.
\] (31)

In the case \( \sigma_1(x) = \sigma_2(x) = 0; \quad \sigma_3(x) \neq 0 \) the boson mass is

\[
M_H(S) = M_W \sqrt{\frac{4}{3}} = 92.8\text{GeV},
\] (32)

Provided that \( \sigma \equiv \sigma_1(x) = \sigma_2(x) = \sigma_3(x) \neq 0, \) the singlet mass is equal to

\[
M_S(H) = M_W \sqrt{\frac{44}{9} + \frac{8g_2^2}{9g_1^2}} \approx 182.6\text{GeV}.
\] (33)

When \( \sigma \equiv \sigma_1(x) = \sigma_3(x) \neq 0; \quad \sigma_2(x) = 0, \) or \( \sigma \equiv \sigma_2(x) = \sigma_3(x) \neq 0; \quad \sigma_1(x) = 0, \) the mass is

\[
M_S(H) = M_W \sqrt{\frac{4}{3} + \frac{2g_2^2}{9g_1^2}} \approx 164.8\text{GeV}.
\] (34)

The situations \( \sigma_2(x) \neq \sigma_1(x) = \sigma_3(x) \neq 0; \sigma_2(x) \neq 0, \) and \( \sigma_1(x) \neq \sigma_2(x) = \sigma_3(x) \neq 0; \sigma_1(x) \neq 0, \) lead to arising two singlets which masses are given by the formulae (31), (33). Two singlets also arise when \( \sigma_3(x) \neq \sigma_2(x) \neq \sigma_1(x) \neq 0, \) \( \sigma_2(x) \neq \sigma_3(x) \neq 0, \) \( \sigma_3(x) \neq 0, \) \( \sigma_1(x) \neq 0. \) In such case the Higgs mass is given by Eqs. (31), (32).

Finally, it is obviously possible the situation when \( \sigma_1(x) = \sigma_2(x) = \sigma_3(x) = 0. \) It means that the Higgs boson mode appears to be unexcited in such case.

Thus, there is the spectrum of the mass states of the Higgs bosons which also include the situation when the Higgs degrees of freedom appear to be unexcited. Since the source of the Higgs bosons in the developed model is gauge fields, what case takes place can be only experimentally revealed. In this way, as soon as any gauge of the fields \( A^a_\mu(x) \) and \( B^a_\mu(x) \) is realizable the different mass states can generally arise in different experiments that can create additional problems in Higgs identification.
V. CONCLUSION

The unified theory of the electromagnetic and weak interactions is developed in the paper. On a basis of violation of the initial \( U_L(2) \otimes U_R(2) \) gauge symmetry, and in the absence of a priori self-interaction scalar field the \( SU_L(2) \otimes U_R(1) \) Lagrangian is derived. The derived Lagrangian is found to take into account correctly interactions between particles via both the charged and neutral currents as it takes place in the SM\[4–6\]. In this way, all massive boson field (including the Higgs boson), massive fermion fields as well as electromagnetic field naturally arise as the superposition of the different modes of the initial gauge fields. On a basis of the experimental data for the fine structure coupling constant, Fermi constant and \( W \) boson mass, the coupling constant \( g_1 \) and \( g_2 \) of the developed consideration as well as the mass of the \( Z^{(0)} \) boson are calculated. The structure of the Higgs field is studied in detail.

It is shown that the different gauges of the initial fields \( A_\alpha(x) \) and \( B_\alpha(x) \) lead to the different states of the Higgs bosons which include the situation when the Higgs degrees of freedom appear to be unexcited. In the cases when the Higgs mode arises the masse of the Higgs boson is found to be in the interval \( M_H = 92.8 \pm 231.7 \) GeV.

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