New approach to the theory of a moving charge’s radiation in dispersive medium and its application to the case of left-handed materials

S.N. Galyamin, A.V. Tyukhtin
Radiophysics Department, Saint Petersburg State University, Saint Petersburg, 198504, Russia

galiaminsn@yandex.ru, tyukhtin@bk

Abstract. We present a new approach to the analysis of the field of a uniformly moving charge in medium with frequency dispersion. This is based on methods of complex function theory. First, we give the deduction of expressions for the field components directly from the causality principle. Second, the properties of integrands and integrals for the field components in the complex plane are discussed. Further, we use our approach for the case of a “left-handed” medium. The charge’s field is presented as the sum of different components having clear physical interpretation: the “quasi-Coulomb” field, the wave field, and the “plasma trace”. An efficient method for computing the field is developed. Additionally, the case of a boundary between an ordinary medium and a left-handed one is considered. The phenomenon of reversed Cherenkov-transition radiation is described.

1. On using the causality principle for the problem of a point charge moving in a medium

It is well known that solving the field equations calls for the use of one of the radiation principles [1]. The solution of Maxwell equations for the case of the uniformly moving point charge has been obtained primarily by Tamm and Frank [2] with the use of the so-called Sommerfeld’s radiation condition (i.e., requirement that the phase velocity is directed away from the source). Later the absorption principle was used by Fermi for a similar problem [3]. This condition demands the field be exponentially decreased with increasing distance from the source (in lossy medium). However, the Sommerfeld’s condition is false for the “left-handed medium”, and the absorption principle is not true for active medium. Therefore, if we deal with a medium having unusual (“exotic”) properties it is essential to obtain a solution from the most fundamental condition, which is the causality principle [1].

We can use any initial-value problem implying uniform linear motion as a limit case (one of examples is the Tamm’s problem, but this is commonly solved with the use of the Sommerfeld’s condition [4]). In our opinion, the following problem is more convenient for the indicated purpose. Two point particles with charges $q$ and $-q$ are born at a moment $t_1$ at a point $x = y = 0, z = z_1$ and start to move with constant velocities $\vec{v} = V \hat{e}_z$ and $-\vec{v} = -V \hat{e}_z$, correspondingly. In this case the charge density $\rho$ and the current density $\vec{j} = j \hat{e}_z$ have the form

$$\{\rho, j \} = q \{1, V \} \delta(x) \delta(y) [\delta(z - V t) + \delta(z + V (t - 2 t_1))] \Theta(t - t_1), \quad (1)$$
where $\Theta(\xi)$ is the Heaviside step function. For source (1), the causality principle means that the field equals 0 for $t < t_1$.

It is assumed that the medium described by permittivity $\varepsilon(\omega)$ and permeability $\mu(\omega)$ is passive, isotropic, and does not have spatial dispersion. We use potentials $A = A\hat{e}_z$ and $\Phi$ with the Lorenz gauge condition. The solution of the initial-value problem can be obtained by the Fourier method. The result of calculations can be written in the following form:

$$
\begin{align*}
\{A\} & = \frac{-i q}{4\pi V} \int_{-\infty}^{+\infty} dk_p \int_{-\infty}^{+\infty} d\omega \left\{ \frac{\beta \mu(\omega + i0)}{\varepsilon^{-1}(\omega + i0)} \right\} \frac{k_p H_0^{(1)}(k_p \rho)}{k^2 + k_p^2 - (\omega + i0)^2 n^2(\omega + i0)c^2} \\
& \times \left[ (k_z - \omega V^{-1} - i0)^{-1} + (k_z + \omega V^{-1} + i0)^{-1} \right] \exp[i k_z(z - V t_1) + i \omega(t_1 - t)] \\
& \{\Phi\} = \frac{q}{2 V} \int_{-\infty}^{+\infty} d\omega \left\{ \frac{\beta \mu(\omega + i0)}{\varepsilon^{-1}(\omega + i0)} \right\} \int_{-\infty}^{+\infty} dk_p k_p H_0^{(1)}(k_p \rho) \exp\left[i \omega \left( \frac{z}{V} - t \right) \right] \\
& \quad \times \left[ (k_{z0} - \omega V^{-1} - i0)^{-1} + (k_{z0} + \omega V^{-1} + i0)^{-1} \right] \exp[i k_{z0}(z - V t_1) + i \omega(t_1 - t)],
\end{align*}
$$

where $n^2(\omega) = \varepsilon(\omega)\mu(\omega)$, $\rho = \sqrt{x^2 + y^2}$, and $H_0^{(1)}(k_p \rho)$ is the Hankel function. Note that the addend $+i0$ in the square brackets has emerged owing to the Fourier-transform of $\Theta(\xi)$, but the term $+i0$ in the denominator and figure brackets has been added to provide the causality. One can see that this addend shifts the corresponding poles from the real axis into the lower half-plane $\omega$. At the same time, if $t < t_1$ then the integration path (real axis) can be enclosed in the upper half-plane $\omega$. Since there are no poles inside it, $A = \Phi = 0$ for $t < t_1$. Note that if losses are taken into account, the poles lie below the integration path and causality is provided automatically.

To obtain the solution for the infinite particle motion we should let $t_1$ approach $-\infty$ at finite $t$, $\rho$, and $z$. Integration over $k_z$ can be calculated using the residue theorem. As $z - V t_1 > 0$, only the poles in the upper half plane $k_z$ are of importance. One can obtain that $A = A^I + A^II$, $\Phi = \Phi^I + \Phi^II$, where

$$
\begin{align*}
\{A^I\} & = \frac{q}{2 V^2} \int_{-\infty}^{+\infty} d\omega \left\{ \frac{\beta \mu(\omega + i0)}{\varepsilon^{-1}(\omega + i0)} \right\} \int_{-\infty}^{+\infty} dk_p k_p H_0^{(1)}(k_p \rho) \exp\left[i \omega \left( \frac{z}{V} - t \right) \right] \\
& \quad \times \left[ (k_{z0} - \omega V^{-1} - i0)^{-1} + (k_{z0} + \omega V^{-1} + i0)^{-1} \right] \exp[i k_{z0}(z - V t_1) + i \omega(t_1 - t)],
\end{align*}
$$

One can integrate in (3) over $k_p$ utilising the residue theorem once again. Integral (4) is convenient for the asymptotic evaluation with the steepest-descent technique. As a result, $A^II$ and $\Phi^II$ contain the saddle point contribution ($\sim [\rho^2 + (z - V t_1)^2]^{-1/2}$) and possibly the contribution of poles ($\sim \rho^{-1/2}$). Omitting calculations, we write the result for the limit $t_1 \to -\infty$ at finite $t$, $\rho$, and $z$:

$$
\begin{align*}
\{A\} & = \frac{i q}{2 V} \int_{-\infty}^{+\infty} d\omega \left\{ \frac{\beta \mu(\omega + i0)}{\varepsilon^{-1}(\omega + i0)} \right\} \delta(s(\omega + i0)\rho) \exp\left[i \omega \left( \frac{z}{V} - t \right) \right], \\
& \quad s(\omega + i0) = \sqrt{s^2(\omega + i0)}, \\
\Phi & = \frac{q}{V} \int_{-\infty}^{+\infty} d\omega \left\{ \frac{\beta \mu(\omega + i0)}{\varepsilon^{-1}(\omega + i0)} \right\} \delta(s(\omega + i0)\rho) \exp\left[i \omega \left( \frac{z}{V} - t \right) \right], \\
& \quad s(\omega + i0) = \sqrt{s^2(\omega + i0)},
\end{align*}
$$

where $\text{Im} s(\omega + i0) > 0$. Formula (6) represents the field of a uniformly moving charge for any isotropic homogeneous passive medium. The analogous result is obtained with the help of the absorption principle. However, the Sommerfeld’s condition can give false results for some situations (for example, for left-handed media).

### 2. The approach to the analysis of the integrals for the charge field components

Using (6), one can write expressions for the field components in the following form:

$$
\begin{align*}
E_r(\omega) & = \frac{i s(\omega)}{\beta \varepsilon(\omega)} \delta(s(\omega)\rho), \\
E_z(\omega) & = \frac{-c \omega}{\varepsilon(\omega)} s^2(\omega) H_0^{(1)}(s(\omega)\rho), \\
h_\rho(\omega) & = i s(\omega) H_1^{(1)}(s(\omega)\rho),
\end{align*}
$$

where $\beta = \varepsilon(\omega)/\mu(\omega)$, $c$ is the speed of light, and $s(\omega)$ is the skin depth.
where \( \zeta = z - Vt \). First of all, let us extract consequences from the requirement \( \text{Im}\,s(\omega+i0) > 0 \) in the case of a lossless medium. Expanding \( s^2(\omega+i0) \) into a Taylor series in the vicinity of \( \omega \) we get

\[
s(\omega+i0) = \begin{cases} 
|s| \text{sgn } \frac{ds^2}{d\omega} & \text{for } s^2(\omega) > 0, \\
|s| & \text{for } s^2(\omega) < 0,
\end{cases}
\]

\[
\text{sgn } \frac{ds^2}{d\omega} = \text{sgn} \left[ \omega \left( 1 + \frac{\beta^2}{2(n'^2 - 1)} \frac{d\omega^2}{d\omega} \right) \right].
\]

(9)

For the normal dispersion (where \( \omega d\omega^2 / d\omega > 0 \)), one obtains \( \text{sgn } ds^2 / d\omega = \text{sgn } \omega \) and Sommerfeld’s condition is fulfilled, but for anomalous dispersion (\( \omega d\omega^2 / d\omega < 0 \)) \( \text{sgn } ds^2 / d\omega \) can equal both \( \text{sgn } \omega \) and \( -\text{sgn } \omega \). In the latter case Sommerfeld’s condition fails. Note that expression (9) for \( s \) satisfies the so-called Mandelstam’s radiation condition [1] requiring the group velocity \( g = \frac{d\omega}{d\omega} > 0 \).

The essence of our approach is the following. We determine the function \( s \) in the complex plane \( \omega \) in such a way that (9) is fulfilled on the real axis. We draw the cuts in segments where \( \text{Im } s = 0 \), fixing the “physical” sheet of the Riemann surface by the rule \( \text{Im } s > 0 \). Next, the following helpful properties take place (for more detail see [5–7]). If the integration path \( \Gamma \) consists of two parts, with one of them (\( \Gamma_1 \)) lying in the domain \( \Re \omega > 0 \), another (\( \Gamma_2 \)) lying in the domain \( \Re \omega < 0 \), and the total \( \Gamma \) being symmetrical with respect to the imaginary axis, then we get for integrals (7):

\[
\int_{\Gamma} f(\omega) \exp(i\omega \zeta V^{-1}) d\omega = 2 \int_{\Gamma_1} \text{Re} \left[ f(\omega) \exp(i\omega \zeta V^{-1}) d\omega \right], \quad f(\omega) = \{e_p(\omega), e_e(\omega), h_p(\omega)\}. \quad (10)
\]

As we will see later, all our integration paths possess the required symmetry. Thus, we can consider the domain \( \Re \omega > 0 \) only. Next, the asymptote of the steepest-descent path (SDP) for integrands in (7) is determined by the equation

\[
\rho \sqrt{1 - \beta^2} \, \text{Im } \omega = \zeta |\Re \omega| + \text{const}.
\]

(11)

Further manipulations are based on applying methods of complex function theory together with properties (9) – (11) to the calculation of (7).

3. The case of a left-handed medium

Now we apply our approach to the case of a left-handed medium (LHM). LHM has simultaneously negative permittivity \( \varepsilon(\omega) \) and permeability \( \mu(\omega) \). Contrary to the ordinary “right-handed” medium (RHM), vectors \( \vec{E}, \vec{H}, \) and \( \vec{k} \) form a left-handed orthogonal set in LHM. Therefore, the Poyting vector \( \vec{S} = c(4\pi)^{-1} \vec{E} \times \vec{H} \) is opposite to the phase velocity. These peculiarities result in the unusual properties of wave processes in LHM [8,9].

Note that the “left-handed properties” can be realised only for a limited frequency range [8,9]. Therefore, it would be more correct to refer to a “left-handed frequency band” (LHFB) as opposed to a “right-handed frequency band” (RHFB). Nevertheless, we will use the term LHM with the understanding that LHM is a medium possessing both LHFB and RHFB, whereas RHM is a medium with RHFB only.

LHM were realised recently in the gigahertz frequency band by means of artificial metamaterials [10–13]. Cherenkov radiation (CR) in an infinite or semi-infinite LHM was investigated in [14–16]. However, the main attention was given to the analysis of energetic characteristics of the wave field while the field structure was practically not studied.

We use the following typical model for the left-handed metamaterial:

\[
\varepsilon(\omega) = 1 - \omega_{pe}^2 (\omega^2 + 2i\omega_{de} \omega)^{-1}, \quad \mu(\omega) = 1 + \omega_{pm}^2 (\omega_{im}^2 - 2i\omega_{dm} \omega - \omega^2)^{-1},
\]

(12)

where \( \omega_{im}, \omega_{pe}, \omega_{pm}, \omega_{de}, \) and \( \omega_{dm} \) are real positive parameters. Sometimes the function \( F\omega^2 \) \((0 < F < 1)\) is used instead of \( \omega_{pm}^2 \) in the formula for \( \mu \), but it is not important for the description of principal properties. Further analysis is performed for the lossless medium \(( \omega_{de} = \omega_{dm} = 0 )\).
The frequency range of radiation is determined by the condition $s^2(\omega) > 0$. If $\omega_{pe} < \omega_{m}$ then the medium is RHM with the radiation frequency range $\omega_{e} < \omega < \omega_{m}$ (figure 1 (Ia)). In the case of $\omega_{pe} > \omega_{m}$, the medium is LHM with the radiation frequency range $\omega_{m} < \omega < \omega_{e}$ (figure 1 (IIa)). The parameter $\omega_{e}$ is determined by $s(\omega_{e}) = 0$:

$$\omega_{e}^2 = \frac{\omega_{m}^2(1-\beta^2) - (\omega_{pe}^2 + \omega_{m}^2) + [(\omega_{m}^2(1-\beta^2) - (\omega_{pe}^2 + \omega_{m}^2))^2 + 4\beta^2(1-\beta^2)(\omega_{pe}^2(\omega_{m}^2 + \omega_{pe}^2))^{1/2}}{2(1-\beta^2)}. \quad (13)$$

Figure 1 (IIb) shows the cuts (determined by equation $\text{Im} s(\omega) = 0$) and the integration path in the complex plane $\omega$. It is important that the integration path $\gamma_0$ goes on the upper bank of cuts, where $\text{sgn}(s) = \text{sgn}(\omega)$ for RHM and $\text{sgn}(s) = -\text{sgn}(\omega)$ for LHM. This means that the projection of the phase velocity in the radial direction $\rho \beta$ is positive for RHM and negative for LHM (see (9)).

One of the possible analytical manipulations consists of the formation of the closed integration contour and the calculation of contributions of cuts and poles. The contours surrounding these singularities are shown in figure 1 (IIb), (IIIb). As a result, we obtain...
\[ E_\rho = E_{\rho C} + E_{\rho W} + E_{\rho P}, \quad E_z = E_{z C} + E_{z W} + E_{z P}, \quad H_\phi = H_{\phi C} + H_{\phi W}, \quad (14) \]

\[ E_{\rho C} = \frac{q_0}{c} \Theta(-\zeta) \int_{\min(0, \zeta)}^{\max(0, \zeta)} \left[ c^{-1} \mu(\omega) [1 - n^2 (\omega) \beta^{-2}] \omega J_\delta(\rho s(\omega)) \cos(\omega \zeta V^{-1}) -\right. \]
\[ \left. \frac{\varepsilon^{-1} (i\omega) \beta^{-1} s(i\omega) J_\delta(\rho s(i\omega))}{s(i\omega) J_\delta(\rho s(i\omega))} \exp \left( -\frac{\omega |\zeta|^2}{V} \right) \right] d\omega, \quad (15) \]

\[ \begin{aligned}
\left\{ E_{\rho W}, E_{z W}, H_{\phi W} \right\} &= \frac{2q_0}{c} \Theta(-\zeta) \int_{\min(0, \zeta)}^{\max(0, \zeta)} \left[ c^{-1} \mu(\omega) [1 - n^2 (\omega) \beta^{-2}] \omega J_\delta(\rho s(\omega)) \cos(\omega \zeta V^{-1}) \right. \\
&\left. -\frac{\varepsilon^{-1} (\omega) \beta^{-1} s(\omega) J_\delta(\rho s(\omega)) \sin(\omega \zeta V^{-1})}{s(\omega) J_\delta(\rho s(\omega)) \sin(\omega \zeta V^{-1})} \right] d\omega, \\
\left\{ E_{\rho P}, E_{z P} \right\} &= \frac{2q_0 \omega^2}{c^2 \beta^2} \Theta(-\zeta) \left[ K_1(\rho \omega_{pe} V^{-1}) \sin(\omega_{pe} \zeta V^{-1}) \right. \\
&\left. \left. - K_0(\rho \omega_{pe} V^{-1}) \cos(\omega_{pe} \zeta V^{-1}) \right] \right. \quad (17) \]

where \( J_\delta(\xi) \) and \( K_\delta(\xi) \) are the Bessel and modified Hankel functions, correspondingly. Here the index “C” is assigned to the “quasi-Coulomb” part of the field (contributions of cuts along the imaginary axis), the index “W” is assigned to the wave part (contributions of cuts along the real axis), and the index “P” is assigned to the so-called “plasma trace” (contributions of poles \( \omega = \pm \omega_{pe} \)) (see figure 1 (Ib), (IIb)). The “quasi-Coulomb” field exists both behind and in front of the moving charge and quickly decreases with distance from it. The wave field (CR) exists only behind the charge and exponentially decreases with increasing \( \rho \). Note that analogous expressions for a resonant RHM are given in our papers [5,6]. As one can see, representation (14) has a clear physical meaning. Additionally, it possesses certain advantages for computing. One of them is that integrands in (15), (16) are free from Hankel functions, contrary to the initial formulae (7), (8). Therefore, (15), (16) are convenient for computation of the wave and quasi-Coulomb fields for small values of \( \rho \) including \( \rho = 0 \).

Another possible way of investigation is based on certain transformation of the integration path [5–7]. For relatively small values of \( \omega \), it is convenient to transform contour \( \gamma_0 \) in such a way that it bypasses all branch points and poles. For the best convergence at large values of \( \omega \) we can employ the contour being parallel to the SDP asymptote (11). So, for \( \zeta > 0 \), we use the semi-infinite rays parallel to the SDP as the new integration path, while for \( \zeta < 0 \) the trapezoidal contour with the half-infinite part parallel to the SDP asymptote is utilised (contours \( \gamma_+ \) in figure 1 (Ib), (IIb)). This transformation allows the avoidance of the intersection of the integrands’ singularities. The essential advantage of such an approach is that it is possible to choose the most convenient parameters of the new contour for the concrete parameters of the problem.

Some examples of computations of the \( H_\phi \) component are shown in figure 1 (Ic), (IIC). Figure 1 (Id), (IIId) shows corresponding energetic patterns as well. The latter are defined by following the method in [14]. The total energy passing through the unity square is equal to \( W_{\rho} = \int_{-\infty}^{\infty} S_{\rho} d\rho \) (for the square orthogonal to the \( z \) axis) and \( W_z = \int_{-\infty}^{\infty} S_z d\rho \) (for the square parallel to the \( z \) axis). These values can be represented as integrals from the spectral density of energy: \( W_{\rho} = \int_{0}^{\infty} W_{\rho\omega} d\omega, W_z = \int_{0}^{\infty} W_{z\omega} d\omega. \)

For \( |s| \rho >> 1 \) the following approximations take place:

\[ W_{\rho\omega} \approx q^2 (2\pi \rho)^{-1} \text{Re}[|s| s \omega^{-1} \varepsilon^{-1} \exp(-2\rho \text{Im} \omega)], \quad W_{z\omega} \approx q^2 (2\pi \rho)^{-1} \text{Re}[|s| s \omega^{-1} \varepsilon^{-1} \exp(-2\rho \text{Im} \omega)]. \quad (18) \]

The dependence \( W_\delta = \sqrt{W_{\rho\omega}^2 + W_{z\omega}^2} \) on the angle \( \theta_\delta = \text{atan}(W_{\rho\omega}/W_{z\omega}) \) is the energetic pattern of radiation.
Figure 1 (Ic,d) is related to the case of a medium with RHFB only, while figure 1 (IIC,d) is related to the case of a medium with LHFB only. One can see that the pattern in the LHFB case is “reversed”. Additionally, the increase in the field amplitude with $|z|$ in LHM is slower compared with that in RHM. Obviously, this is due to the fact that the wave field in LHM lags more behind the charge in comparison with the RHFM case.

Figure 1 (IIIa–d) is related to a more complex case. Here we use the model of resonant permeability (as in (14)) and resonant permittivity as well: $\varepsilon(\omega) = 1 + \omega_m^2 (\omega_m^2 - 2i\omega_m\omega - \omega^2)^{-1}$. Parameters for the figure 1 (IIIa–d) are selected so that the medium has both RHFB and LHFB. Therefore, the pattern has two lobes corresponding to ordinary and reversed CR (figure 1 (IIIId)). At the magnetic field plot (figure 1 (IIId)) one can see that, for relatively small distance $|z|$, the pattern has two lobes corresponding to ordinary and reversed CR (figure 1 (IIId)).

Additionally, the increase in the field amplitude with $|z|$ is far from any of the lobes mentioned above. In this case, the field behaviour is more complicated. This is explained by the addition of the “left-handed” CR for large enough distances.

### 4. The case of a boundary between RHM and LHM

Here we analyse the electromagnetic field generated by the charge $q$ intersecting the interface (located at $z = 0$) between two passive, homogeneous, isotropic, frequency-dispersive media described by permittivity and permeability: $\varepsilon_1(\omega)$, $\mu_1(\omega)$ for $z < 0$ and $\varepsilon_2(\omega)$, $\mu_2(\omega)$ for $z > 0$. The medium filling the region $z < 0$ is RHM, while the volume $z > 0$ is filled with LHM. The charge motion rule is $z = Vt = c\beta t$, $x = y = 0$.

The electromagnetic field in the present geometry can be described by formulas similar to those (located at 0) between two passive, homogeneous, isotropic, frequency-dispersive media described by permittivity and permeability: $\varepsilon_1(\omega)$, $\mu_1(\omega)$ for $z < 0$ and $\varepsilon_2(\omega)$, $\mu_2(\omega)$ for $z > 0$. The medium filling the region $z < 0$ is RHM, while the volume $z > 0$ is filled with LHM. The charge motion rule is $z = Vt = c\beta t$, $x = y = 0$.

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these angles, the aforementioned approximation represents the spherical wave of transition radiation (TR).

Additionally, the poles $\pm s_2$ contribute to the free field at certain values of $\theta$. These contributions represent the cylindrical waves of reflected or refracted CR. We call these waves the “reversed Cherenkov-transition radiation” (RCTR) because they possess certain features of both CR and TR [20]. As was shown, the refracted RCTR arises if the charge velocity is enclosed between the lower and upper thresholds (see also [21,22]):

$$\beta_{CR} < \beta < \beta_{TIR} \quad , \quad \beta_{CR} = \sqrt{n_2 - 1} \quad , \quad \beta_{TIR} = (n_2^2 - 1)^{1/2} ,$$

where $\beta_{CR}$ is the Cherenkov threshold, while $\beta_{TIR}$ corresponds to the total internal reflection.

In the second way we produce an effective algorithm for numerical computation of both $A^b$, $\Phi^b$ and $A^b$, $\Phi^b$. In the LHF case, the poles $\pm s_2$ are located near the integration path, leading to drastic changes of (21). Our numerical algorithm is adapted for overcoming this difficulty. The free field for the graphical representations shown here was calculated with the use of the second method.

Figure 2 presents the modification in the dependence $E_{\rho \theta}$ (V m$^{-1}$) with increase in $\beta$. The results for the total field (figure 3) demonstrate that RCTR can be the dominant radiation. The medium 1 is supposed to be vacuum, the medium 2 is described by (12). The line marked “full” in figure 3 corresponds to the full field, while the line marked “poles” corresponds to the poles.
contribution (determining RCTR) as if it were alone. One can see that, for \( t > 0.7 \) ns, the total field is very well approximated by the pole contribution. This means that, in this case, RCTR gives the main contribution to the spatial radiation. For \( t < 0.7 \) ns the dominant radiation is TR.

5. Conclusion

The presented approach to the theory of uniformly moving charge radiation has some advantages. One of them consists in obtaining a convenient field decomposition having clear physical meaning. An efficient numerical algorithm for computing the total field is another benefit. We have used the left-handed medium to demonstrate the possibilities of this technique. Additionally, the effect of the reversed Cherenkov-transition radiation occurring at the interface between the ordinary medium and the left-handed one has been analysed both analytically and numerically.

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