Small x physics and the initial conditions in heavy ion collisions

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At very high energies, the high parton densities (characterized by a semi-hard saturation scale $\Lambda_s$) ensure that parton distributions can be described by a classical effective field theory with remarkable properties analogous to those of spin glass systems. This color glass condensate (CGC) of gluons also provides the initial conditions for multi-particle production in high energy nuclear collisions. In this talk, we briefly summarize recent theoretical and phenomenological progress in the CGC approach to small x physics. In particular, we discuss recent numerical work on the real time gluodynamics of partons after a nuclear collision. The implications of this work for the theoretical study of thermalization in nuclear collisions and on the phenomenological interpretation of results of the recent RHIC experiments are also discussed.

1. Introduction

Why is small $x$ physics relevant to heavy ion collisions? The canonical picture of how matter is produced and evolves in a high energy nuclear collision was articulated by Bjorken \cite{Bjorken}. The “valence” components of the nuclear wavefunction (partons carrying a large fraction, per nucleon, of the nuclear momentum) interact weakly and populate primarily the fragmentation regions at large projectile and target rapidities. It is the “wee” virtual excitations of the nuclear ground state, gluons (mostly) and sea quarks, carrying a small fraction of the nuclear longitudinal momentum, that populate the central rapidity region, eventually producing the $\sim 1000$ hadrons that were measured in one unit of pseudorapidity at RHIC. If the nucleus is viewed as a superposition of Fock states, these wee partons belong to a state with a large number of gluons and sea quarks—each of these can therefore have only a small fraction of the nuclear momentum \cite{Kharzeev}. The higher the energy of the nucleus, the larger the number of wee partons. In a nuclear collision, this Fock state is “destroyed” and the wee partons go on–shell creating the large energy densities in the relatively large space–time volumes associated with quark gluon plasma formation. Understanding the possible formation and dynamical evolution of this partonic state therefore requires understanding small $x$ physics at a deep level.

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In the perturbative QCD approach to small $x$ physics, the “cloud” of gluons is generated by Bremsstrahlung of a ladder of soft gluons off harder “valence” partons. The parton density grows rapidly with decreasing $x$ at a rate which, if extrapolated to $x \to 0$, would violate the Froissart (unitarity) bound. This seems unlikely because at sufficiently small $x$ the Bremsstrahlung approach must break down. When the density of partons becomes large, parton recombination and screening effects become important, and may perturbatively “saturate” the rapid growth of parton distributions at small $x$.

Since the density of partons is large, one can formulate the small $x$ problem as a classical effective field theory (EFT). The small $x$ partons described by the EFT constitute a color glass condensate (CGC). They are a glass because their properties are analogous to a spin glass. They form a condensate because they belong to a state with high occupation number of order $1/\alpha_S$.

The CGC contains a dimensionful scale, the saturation scale $\Lambda_s(x, A)$. It is proportional to the gluon density per unit area and grows as $\Lambda_s^2 \sim A^{1/3}/x^\delta$, where $\delta \approx 0.2 - 0.3$ is the power that controls the rise in parton distributions in a nucleon. If $\Lambda_s \gg \Lambda_{QCD}$, weak coupling methods are applicable and one can compute the parton distributions in the classical EFT. At RHIC, one can estimate $\Lambda_s = 2$ GeV, while at LHC, $\Lambda_s = 4$ GeV—weak coupling methods are therefore only marginally applicable at the former.

This talk is organized as follows. In section 2, we discuss the color field of a single nucleus. We will advertise the impressive recent theoretical developments in this approach. We will also discuss the striking phenomenological success of saturation models in describing the HERA data. In the following section, we will discuss the application of these ideas to nuclear scattering. Results from non-perturbative numerical simulations of the initial stages of nuclear collisions will be presented. In section 4, we will briefly confront saturation models against the recent RHIC data. We end in section 5 with an outlook describing relevant projects in progress.

2. The color field of a nucleus: QCD is a color glass condensate at high energies

The classical EFT approach to small $x$ physics has been discussed extensively in the literature. It contains a dimensionful scale $\Lambda_s$ equal to the color charge squared per unit area of the valence sources. The EFT, formulated in the infinite momentum frame, in light cone gauge, is a (nearly) two-dimensional theory with a structure mathematically analogous to that of a disordered system of Ising spins in a random magnetic field. That’s where the color glass comes from. Further, since the occupation number of the gluonic fields is large—the field strength squared $F_{\mu\nu}^2 \sim 1/\alpha_S$—what we have is a color glass condensate (CGC). A virtual photon (from an energetic leptonic probe) would therefore indeed “see” a colored glass on the time scales of its interaction.

The classical theory is solvable and classical correlation functions have been computed. The gluon number has the Weizsäcker–Williams distribution $\Lambda_s^2/k_t^2/\alpha_S$ for $k_t \gg \Lambda_s$ and (due to an infinite resummation in $\Lambda_s/k_t$) saturates as the distribution

\footnote{The approach described here is by no means the only approach. For a review of the recent status of different approaches to small $x$ physics, see Ref. 5.}

\footnote{Unlike the original model of Parisi and Sourlas, which fails in describing real spin glasses, this theory, on account of its peculiar quantum structure, may indeed be a glass. This remains to be proven though.}
\[ \ln(\Lambda_s/k_t)/\alpha_s. \] This suggests that the typical momentum of the gluons is peaked around \( \Lambda_s \). Recently, Lam and Mahlon have done a lot of work elucidating the nature of the classical distributions when strict color neutrality is imposed [13].

When quantum corrections are included, a Wilsonian renormalization group (RG) picture emerges. The structure of the classical field remains intact while the scale \( \Lambda_s \) grows. This is because including hard gluons (harder than the \( x \) scale of interest) in the source increases the typical size of color fluctuations. The scale \( \Lambda_s(x) \) is then given by the gluon density per unit area,

\[ \Lambda_s^2(x) = \frac{A^{1/3}}{x^\delta}. \] (1)

Here \( \delta(\Lambda_s) \) denotes the power of the rise in the nucleon gluon distribution. This equation must be solved self-consistently for \( \Lambda_s \) at each \( x \).

In particular, the weight function that gluonic correlation functions have to be averaged over satisfies a non-linear RG equation [11]. There has been very important work recently in understanding and solving this RG equation [8]. (The correlation functions obtained from this functional equation are identical to those obtained independently in different approaches by Balitsky and Kovchegov [14]. For numerical solutions of these, see Ref. [15].) The equations have been solved in a mean field approximation—for large transverse momenta \( (k_t >> \Lambda_s) \) the structure of the source is the simple Gaussian one of the McLerran–Venugopalan model, while at small transverse momenta, \( (k_t << \Lambda_s) \) it is entirely different. Much work remains to be done to uncover the complex structure of these equations—their phenomenological implications for heavy ion collisions and for deeply inelastic scattering [16] are tremendous.

Simple saturation models which incorporate the scale \( \Lambda_s \) have been applied to study the HERA data. With only three parameters, they fit the entire range of the HERA inclusive data (for \( x < 0.01 \) and for all \( Q^2 \) upto 450 GeV\(^2\)). The model, with the same parameters, reproduces the HERA diffractive and (with some caveats) vector meson production data [17].

3. Classical approach to nuclear collisions: shattering the color glass condensate

In a nuclear collision, the color glass shatters producing a large number of gluons. The full quantum description of how this happens is a very difficult problem which has no solution yet. The standard pQCD factorization ("mini-jets") approach to particle production is not of much help here because it is most unreliable at small \( x \) and small \( Q^2 \). Inevitably, the better pQCD based models of nuclear scattering include saturation ideas in some form [18].

However, one can make quite a bit of headway in the classical EFT approach. Since the classical fields of the two nuclei are known, they provide the initial conditions for the evolution of the gluon fields produced in the nuclear collision. The small \( x \) fields are described by the classical Yang-Mills equations

\[ D_\mu F_{\mu \nu} = J_\nu. \] (2)
with the random sources on the two light cones: \( J_\nu = \sum_{1,2} \delta_{\nu,\pm} \delta(x_\mp) \rho_{1,2}(r_1) \). The two signs correspond to two possible directions of motion along the beam axis \( z \). As shown by Kovner, McLerran and Weigert (KMW) \[19\], low-\( x \) fields in the central region of the collision obey sourceless Yang-Mills equations (this region is in the forward light cone of both nuclei) with the initial conditions in the \( A_\tau = 0 \) gauge given by

\[
A^i = A^i_1 + A^i_2; \quad A^\pm = \pm \frac{ig}{2} x^\pm [A^i_1, A^i_2]. \tag{3}
\]

Here the pure gauge fields \( A^i_{1,2} \) are solutions of (2) for each of the two nuclei in the absence of the other nucleus.

In order to obtain the resulting gluon field configuration at late proper times, one needs to solve (2) with the initial condition (3). Since the latter depends on the random color source, averages over realizations of the source must be performed. KMW showed that in perturbation theory the gluon number distribution by transverse momentum (per unit rapidity) suffers from an infrared divergence. A reliable way to go beyond perturbation theory is to re-formulate the EFT on a lattice by discretizing the transverse plane. (Boost invariance in pseudorapidity is assumed, which reduces the theory to a 2 + 1–dimensional theory.) The resulting lattice theory can then be solved numerically \[4\]. We shall not dwell here on the details of the lattice formulation, which is described in detail in Ref. \[22\].

We have simplified the problem considerably since \( \Lambda_s \) and the linear size \( L \) of the nucleus are the only parameters in the problem. We can write any dimensionful physical quantity \( q \) as \( \Lambda^d_s f_q(\Lambda_s L) \), where \( d \) is the dimension of \( q \). All the non-trivial physical information is contained in the dimensionless function \( f_q(\Lambda_s L) \) \[7\]. We can estimate the values of the product \( \Lambda_s L \) which correspond to key collider experiments. Assuming Au-Au collisions, we take \( L = 11.6 \) fm (for a square nucleus!) and estimate the standard deviation \( \Lambda_s \) to be 2 GeV for RHIC and 4 GeV for LHC \[1\]. Also, we have approximately \( g = 2 \) for energies of interest. The rough estimate is then \( \Lambda_s L \approx 120 \) for RHIC and \( \Lambda_s L \approx 240 \) for LHC.

Our simulations were performed for the SU(2) gauge group—all results for physical quantities are based on extrapolations to SU(3). A straightforward computation is of the initial energy per unit transverse area per unit rapidity, deposited in the central region by the colliding nuclei \[23\]. At late times, this is given by the non–perturbative formula

\[
\frac{1}{\pi R^2} \frac{dE_i}{dt} \bigg|_{t=0} = \frac{1}{g^2} f_E \Lambda^3_s, \tag{4}
\]

where \( f_E \equiv f_E(\Lambda_s R) = 0.21–0.26 \). Using this formula, and assuming, in accordance with Ref. \[24\], the \((N_c^2 - 1)/N_c \) dependence of the energy on the number of colors \( N_c \), we arrive at the values of 2700 GeV and of 25000 GeV for the initial transverse energy per unit rapidity at RHIC and at LHC, respectively \[23\]. We can also estimate a formation time \( \tau_f \) in our picture. For RHIC (LHC), we have \( \tau_f = 0.3 \) fm (0.13 fm). This then enables one to estimate the initial energy density to be 66.5 GeV/fm\(^3\) (1.3 TeV/fm\(^3\)) at RHIC (LHC).

The number and distribution of produced gluons are of considerable interest as initial data for possible evolution of the gluon gas towards thermal equilibrium \[25–27\]. Strictly

\[\text{For alternative analytical approaches, see the work of Balitsky \[20\] and of Kovchegov \[21\].}\]
speaking, a particle number is only well-defined in a free-field theory, and there is no unique extension of this notion to a general interacting case. For this reason, we use two different generalizations of the particle number to an interacting theory, each having the correct free-field limit. We verify that the two definitions agree in the weak-coupling regime corresponding to late proper times in the central region [24]. Our first definition is straightforward. We impose the Coulomb gauge condition in the transverse plane: \( \vec{\nabla}_\perp \cdot \vec{A}_\perp = 0 \) and determine the momentum components of the resulting field configuration. Our second definition is based on the behavior of a free-field theory under relaxation. This "cooling" technique allows a gauge invariant determination of the total particle number. Unfortunately, it cannot be used to determine the momentum distributions leaving the Coulomb gauge determination as the only one available.

We obtain
\[
\frac{1}{\pi R^2} \frac{dN}{d\eta} \bigg|_{\eta=0} = \frac{1}{g^2} f_N \Lambda_s^2, \tag{5}
\]
where \( f_N \equiv f_N(\Lambda_s L) = 0.13 \) (0.15) for RHIC (LHC) energies. The agreement between the cooling and Coulomb techniques at the larger values of \( \Lambda_s L \) relevant for RHIC and LHC is excellent. It is not as good at the smaller values: in general, the cooling number is more reliable. Naively extrapolating our results to SU(3), we find for Au-Au central collisions at RHIC energies, \( dN/d\eta \sim 950 \) for RHIC and \( dN/d\eta \sim 4300 \) for LHC energies.

Our results for the number distribution, computed in Coulomb gauge, are as follows. We have verified that for large \( k_\perp \) our numerically obtained multiplicity agrees with the lattice analogue of the perturbative expression. At smaller \( k_\perp \), the distribution softens and converges to a constant value, unlike its perturbative counterpart. Notably, this qualitative change of the distribution occurs at \( k_\perp \sim \Lambda_s \). We tried to quantify this non-perturbative behavior by fitting the distribution to a variety of physically motivated functional forms. Surprisingly, we find that the shape of the distribution is closely reproduced by the two dimensional Bose-Einstein form \( n(k) = A/(\exp(\beta \omega_k) - 1) \), with the inverse temperature \( \beta \) of the order of 1 in units of \( \Lambda_s \), and with \( \omega_k \) corresponding to a free massive dispersion relation, with the mass of the order of 0.1\( \Lambda_s \). This is a remarkable result for a purely classical theory, whose meaning, beyond providing us with a convenient parametrization, is not yet clear.

We have studied the number distribution at different proper time slices. Our numerical results show clearly that the size of the mass gap decreases as the square root of the proper time—this is what one would expect for instance for a screening mass. Further, though the peak of the distribution softens, its slope remains unchanged—the inverse two dimensional “temperature” \( \beta \) is a constant as a function of proper time. Thus the particle number is essentially unchanged—it increases slightly due to the “decay” of the mass gap.

Interestingly, all of these results (as well as the \( \Lambda_s/k_t \) form of the distribution for \( k_t < \Lambda_s \)) are consistent with the theoretical analysis of Baier et al. [26] as communicated to us by Son [28]. Baier et al. have studied the importance of \( 2 \to 3 \) processes in driving the system to equilibrium. The particular feature of their work, distinguishing it from previous ones, is the inclusion of the Landau–Pomeranchuk–Migdal (LPM) effect that results in the suppression of hard gluon radiation in QCD [29]. In their kinetic theory analysis, for proper times \( \tau < 1/\alpha_s^{3/2} \), the occupation number \( f \) is greater than unity.
This is the overlap region between our classical simulations and kinetic theory and it is gratifying to see that they agree. This agreement further suggests that LPM interference effects are automatically included in the classical simulation.

What we have discussed above are the initial energy and number distributions of gluons produced in nuclear collisions. The subsequent evolution of the system (when the occupation number $f$ falls below unity) is beyond the scope of the classical approach—it does however provide the initial conditions for the evolution. The approach to equilibrium in the McLerran–Venugopalan model was first investigated by Mueller [25]. He initially considered only elastic scattering and derived a Landau–type equation for the single particle distributions. This equation has since been solved numerically and the approach to equilibrium studied quantitatively [27]. One finds that elastic processes are very inefficient in driving the system to equilibrium (see also Ref. [30]). Baier et al. have argued that $2 \to 3$ processes are not suppressed relative to $2 \to 2$, and that they may actually be more efficient. For a more detailed discussion of thermalization, we refer the reader to their paper and to Son’s talk at this conference [28].

4. The CGC confronts RHIC data

Data from RHIC were presented at this conference and some of it has already been published. How does the CGC picture fare when confronted with the data? The predictions of various models for the RHIC multiplicity have been summarized nicely by Eskola in his QM2001 talk [31] (see also Ref. [32]). Our prediction in Ref. [24] does quite well. This however was fortuitous since from Eq. 5 it is clear that the multiplicity depends very sensitively on the saturation scale—the latter clearly has at least a 10% uncertainty.

A more sensitive test is the centrality dependence of the multiplicity. Kharzeev and Nardi plotted the charged particle multiplicity per participant pair at pseudorapidity $\eta = 0$ versus the number of participants. They find that both the CGC and Glauber models give nearly identical predictions [33]. These in turn agree well with the data [34].

A consequence of the CGC picture is that $< p_t^2 > \propto dN/d\eta/\pi R^2$, as is suggested by Eq. 4. This dependence has been verified recently for the E735 $p\bar{p}$ data at the Tevatron as well as the UA(1) data [35]. It also works for the NA49 Heavy Ion CERN SPS data. First comparisons with the RHIC data are also promising [36].

In Ref. [24], we found that the momentum distributions of the produced gluons were nearly universal functions that scaled as $f \equiv f(p_t/\Lambda_s)$. Whether the RHIC data reflects this can be easily checked because $\Lambda_s$ should be different for different centralities. Juergen Schaffner–Bielich has checked that the RHIC data for charged particles satisfies this property by rescaling the $p_t$ scale by $p_t \to \lambda p_t$, where $\lambda$ was an apparently arbitrary scaling constant. What is truly remarkable however is that the scaling constant is not arbitrary—it corresponds closely to the ratio of the saturation scales at the two centralities. If this scaling constant is used instead to determine $\Lambda_s$ at the different centralities and if then the number of charged particles per participant pairs at $\eta = 0$ is plotted versus the number of participants (a la Kharzeev–Nardi), it agrees well with the data [36].

So far so good. However, the ratio of $R_i = E_i/N_{\text{gluon}} \sim 1.5\Lambda_s$ computed by us [23,24] is too large relative to the data [37]. This is not necessarily a problem because $R_i$ is not a conserved quantity and depends on the complex dynamics of the space–time evolution.
Our simulation only computes the initial value of this ratio. Both number changing processes a la Baier et al. and the expansion of the system should decrease this ratio. If the former is the dominant process, one might be throwing the baby out with the bath water because the $\alpha_S$ dependence of the multiplicity might change. A more mundane possibility is that the function $f_E$ which was computed in SU(2) might be significantly lower for SU(3). That’s one reason to press ahead with SU(3) simulations of the problem. As pointed out by Dumitru in his QM2001 talk, $R_i$ is a sensitive discriminant of the space–time dynamics in various models [30]. Detailed space–time simulations are in progress [38].

Tests of the CGC scenario in the coming years include the energy dependence of $\Lambda_s$, event by event fluctuations, and heavy quark production among others. We have recently computed the likelihood of the formation of CP–odd domains in the classical approach [39].

5. Outlook

The theoretical foundations of the classical approach to nuclear scattering clearly need to be on a firmer foundation. This is especially pressing in light of recent theoretical advances in small $x$ physics [8]. There are several practical improvements that need to be done independently of the above. The extension to SU(3) and relaxing the strict boost invariance assumption are on the top of the agenda. A related interesting problem is: does the hot gluonic matter thermalize? At least asymptotically, this problem should have a solution. Recent attempts to address this problem are encouraging but much work (hopefully aided by new ideas) remains. Finally, as more and more RHIC data pour in over the years, we expect that they will provide a sensitive test of the ideas discussed here. We live in interesting times!

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