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FORMULATION OF BLOCK SCHEMES WITH LINEAR MULTISTEP METHOD FOR THE APPROXIMATION OF FIRST-ORDER IVPS

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ABSTRACT
In this paper, formulation of an efficient numerical schemes for the approximation first-order initial value problems (IVPs) of ordinary differential equations (ODE) is presented. The method is a block scheme for some \( k \)-step linear multi-step methods \((k = 1, 2 \text{ and } 3)\) using the Hermite Polynomials as a basis function. The continuous and discrete linear multi-step methods (LMM) are formulated through the technique of collocation and interpolation. Numerical examples of ODE have been examined and results obtained show that the proposed scheme can be efficient in solving initial value problems of first order ODE.

**Keywords:** linear multi-step method, ordinary differential equations, initial value problems, Hermite polynomials.

INTRODUCTION
Numerous problems in Sciences and Engineering are modelled using ordinary differential equations (ODEs). Most of these differential equations do not have analytical solutions which makes numerical methods an option for solving these problems. There are two major discrete variable methods for approximating the solutions of ODEs, namely, one step and linear multi-step methods.

In this paper, we proposed an efficient numerical scheme to solve numerically first order IVPs. The proposed method is a block scheme for some \( k \)-step linear multistep methods \((k = 1, 2 \text{ and } 3)\) using Hermite polynomial as the basis functions. Also, we give the discrete methods used in block and implement it for solving some existing IVPs in the literature. In this paper, we consider the general form of the first order initial value problems.

\[ y'(x) = f(x, y(x)), \quad y(x_0) = y_0 \]  \hspace{1cm} (1)

Many researchers had developed interest on improving the numerical solution of initial value problems (IVPs) of ordinary differential equation. Consequently, the development of a class of methods called block methods is one of the outcomes. Okunuga and Ehigie (2009) derived two-step continuous and discrete linear multistep methods using power series as a basis function. Akinfenwa et al. (2011) developed a four step continuous block hybrid method with four non-step points for the direct solution of first order initial value problem. Odekunle et al. (2012) developed a continuous linear method using interpolation and collocation for the solution of first-order ODE with constant step size. James et al. (2013), proposed a continuous block method for the solution of second order IVPs with constant step size, the method was developed by interpolation and collocation of power series approximate solution to generate a continuous linear multistep method. A block procedure with linear multistep methods using Legendre polynomials was done by Abualnaja (2015). However, he did not include the block schemes. Okedayo et al. (2018) developed on modified Legendre collocation block method for solving initial value problems of first order ODEs. Also, Okedayo et al. (2018), developed a continuous Laguerre collocation block method for solving initial value problems of first order ordinary differential equations. However, in this paper, Hermite polynomial is used as a basis function to derive some block methods for the solution of first order initial value problem (1).
FORMULATION OF THE METHODS

We consider the approximate solution of the perturbed form of (1) in the power series below

\[ y_k(x) = \sum_{i=0}^{k} c_i \psi_i(x), \quad x_n \leq x \leq x_{n+k} \]  \hspace{1cm} (2)

where

\[ \psi_i(x) = x^i, \quad i = 0, 1, 2, \ldots, k \]  \hspace{1cm} (3)

Substituting (2) into (1) and add \[ \lambda H_k(x) \] where \( \lambda \) is the perturbed term and \( H_k(x) \) is the Hermite polynomial of degree \( k \) valid in \( x_n \leq x \leq x_{n+k} \) we have

\[ \sum_{i=0}^{k} c_i \psi_i'(x) = f(x, y) + \lambda H_k(x) \]  \hspace{1cm} (4)

We shall consider cases where \( k = 1, 2 \) and \( 3 \) in (2) and (3)

The Hermite polynomial is given by \( H_i(x) \), \( i = 0, 1, 2, 3 \)

\[ H_0(x) = 1, H_1(x) = 2x, H_2(x) = 4x^2 - 2, H_3(x) = 8x^3 - 12x \]  \hspace{1cm} (5)

These polynomials are gotten from the Hermite Rodrigue’s formula

\[ H_n(x) = e^{x^2} \left( -1 \right)^n \frac{d^n}{dx^n} (e^{-x^2}) \]  \hspace{1cm} (6)

In this paper, we are going to use the set of polynomials in (5) to formulate the block schemes. Using these polynomials in (5) in the interval \( [x_n, x_{n+k}] \), we introduce the change of variable to define the Hermite polynomial as

\[ x = \frac{2\bar{x} - (x_{n+k} + x_n)}{(x_{n+k} - x_n)}, \quad k = 1, 2, 3 \]  \hspace{1cm} (7)

Abualnaja (2015)

For \( k = 1 \),

In this case, take \( H_1(x) = 2x \), since \( k = 1 \) and use equation (7). Collocate equation (7) at \( x_n \) and \( x_{n+1} \) and solve to obtain

\[ x = \frac{2\bar{x} - (x_{n+k} + x_n)}{x_{n+k} - x_n} = \frac{2x_n - (x_{n+1} + x_n)}{x_{n+1} - x_n} = \frac{-x_{n+1} + x_n}{x_{n+1} - x_n} = -1 \]  \hspace{1cm} (8)

Substitute the value of \( x \) into \( H_1(x) = 2x \) and obtain \( H_1(x) = -2 \)

Also, following the same procedure for \( x_{n+1} \) we have

\[ x = \frac{2\bar{x} - (x_{n+k} + x_n)}{x_{n+k} - x_n} = \frac{2x_{n+1} - (x_{n+2} + x_n)}{x_{n+2} - x_n} = \frac{x_{n+1} + x_n}{x_{n+2} - x_n} = 1 \]  \hspace{1cm} (9)

Hence \( H_1(x) = 2 \)

From equation (2.2), we deduce that \( \omega_0'(x) = 0 \) and \( \omega_1'(x) = 1 \). Then substituting into (2.3), we have

\[ f(x, y) = c_1 - \lambda H_1(x) \]  \hspace{1cm} (10)

Thus collocate (10) at \( x_{n+1}, i = 0, 1 \) and interpolate (2) at \( x = x_n \), we obtain a system of three equations with \( c_i, (i = 0, 1) \)
and parameter $\hat{\lambda}$. The system of the equations are given as:

$$
\begin{align*}
    y_n &= c_0 + c_1 x \\
    f_n &= c_1 + 2\hat{\lambda} \\
    f_{n+1} &= c_1 - 2\hat{\lambda}
\end{align*}
$$

Solve (11) to obtain

$$
\hat{\lambda} = \frac{1}{4} (f_n - f_{n+1}), \quad c_1 = \frac{1}{2} (f_n + f_{n+1}), \quad c_0 = y_n - \frac{x_n}{2} (f_n + f_{n+1})
$$

From (2) we obtain

$$
\bar{y} = c_0 + c_1 x
$$

Now, the required numerical scheme of the proposed method for $y_{n+1}$ will be obtain if we collocate (12) at $x = x_{n+1}$ and substitute the value of $c_0, c_1$ and $\hat{\lambda}$. Hence, we obtain

$$
y_{n+1} = y_n + \frac{h}{2} (f_n + f_{n+1})
$$

Now for $k = 2$

In this case, $H_2(x) = 4x^2 - 2$, since $k = 2$ and use equation (7), then collocate the equation at $x_n, x_{n+1}$ and $x_{n+2}$ and solve to obtain

$$
x = \frac{2x - (x_{n+1} + x_n)}{x_{n+1} - x_n} = \frac{2x_{n+1} - (x_{n+2} + x_n)}{x_{n+2} - x_n} = \frac{-x_{n+2} + x_n}{x_{n+2} - x_n} = -1
$$

Substitute the value of $x$ into $H_2(x) = 4x^2 - 2$ and obtain Hence $H_2(x) = 2$

Using the same procedure for $x_{n+1}$ to have

$$
x = \frac{2x - (x_{n+1} + x_n)}{x_{n+1} - x_n} = \frac{2x_{n+1} - (x_{n+2} + x_n)}{x_{n+2} - x_n} = \frac{-x_{n+2} + x_n}{x_{n+2} - x_n} = 1
$$

Now, put $x_{n+1}$ to be $x_n + h$ and $x_{n+2}$ to be equal to $x_n + 2h$ in equation (15) to obtain $x = 0$. Thus, substitute $x$ into $H_2(x) = 4x^2 - 2$ to get $H_2(x_{n+1}) = -2$

Also for $x_{n+2}$, we have

$$
x = \frac{2x - (x_{n+1} + x_n)}{x_{n+1} - x_n} = \frac{2x_{n+1} - (x_{n+2} + x_n)}{x_{n+2} - x_n} = \frac{-x_{n+2} + x_n}{x_{n+2} - x_n} = 1
$$

Hence, we substitute the value of $x$ into $H_2(x) = 4x^2 - 2$. Therefore $H_2(x_{n+2}) = 2$

From (3), we deduce that $\psi'_0 = 0, \psi'_1 = 1$ and $\psi'_2 = 2x$. Then equation (4) reduces to the form

$$
f(x, y) = c_1 + 2xc_2 - \hat{\lambda}H_2(x)
$$

We now collocate (17) at $x_{n+i}, i = 0, 1, 2$ and interpolate (2) at $x = x_n$ to obtain a system of four equations with $c_i (i = 0, 1, 2)$ and parameter $\hat{\lambda}$
\[ y_n = c_0 + c_1x_n + c_2x_n^2 \]
\[ f_n = c_1 + 2x_nc_2 - 2\lambda \]
\[ f_{n+1} = c_1 + c_2x_{n+1} + 2\lambda \]
\[ f_{n+2} = c_1 + c_2x_{n+2} - 2\lambda \]  
(18)

Solving the system (18) results to
\[ \lambda = \frac{1}{8}(2f_{n+1} - f_{n+2} - f_n) \]
\[ c_2 = \frac{1}{4h}(f_{n+2} - f_n) \]
\[ c_1 = \frac{1}{4h}(2hf_{n+1} - 2x_nf_{n+2} + 2x_nf_n - hf_{n+2} + 3hf_n) \]
\[ c_o = \frac{1}{4h}(4hy_n - 2hx_nf_{n+1} + hx_nf_{n+2} - 3hx_nf_n + x^2f_{n+2} - x^2f_n) \]  
(19)

From (2), we obtain
\[ \bar{y} = c_0 + c_1x + c_2x^2 \]  
(20)

The required numerical scheme is obtained if we collocate equation (19) at \( x = x_{n+1} \) and substitute \( c_0, c_1, c_2 \) and \( \lambda \) as
\[ y_{n+1} = y_n + \frac{h}{4}(f_{n+2} + 2f_{n+1} + f_n) \]  
(21)

**Now consider \( k = 3 \)**

In this case, take \( H_2(x) = 8x^3 - 12x \) since \( k = 3 \) and use equation (7). Collocate this equation at \( x_n, x_{n+1}, x_{n+2} \) and \( x_{n+3} \) and solve to have
\[ x = \frac{2y - (x_{n+1} + x_n)}{(x_{n+1} - x_n)} = \frac{2x_n - (x_{n+3} + x_n)}{x_{n+3} - x_n} = \frac{-x_{n+3} + x_n}{x_{n+3} - x_n} = -1 \]  
(22)

Thus substitute the value of \( x \) into \( H_3(x) = 8x^3 - 12x \) and obtain \( H_3(x) = 4 \)

Following the same process for \( x_{n+1} \) we have
\[ x = \frac{2x_{n+1} - (x_{n+3} + x_n)}{x_{n+3} - x_n} \]

Put \( x_{n+1} = x_n + h, x_{n+3} = x_n + 3h \) and obtain
\[ \frac{2(x_n + h) - (x_n + 3h + x_n)}{x_n + 3h - x_n} = -\frac{1}{3} \]

By substituting the value of \( x \) into \( H_3(x) = 8x^3 - 12x \), then \( H_3(x) = \frac{100}{27} \)

Follow the same procedure for \( x_{n+2} \) and have
\[ x = \frac{2x_{n+2} - (x_{n+3} + x_n)}{x_{n+3} - x_n} \]. Putting \( x_{n+2} = x_n + 2h, x_{n+3} = x_n + 3h \) and substituting into the above equation to obtain
\[ \frac{2(x_n + 2h) - (x_n + 3h + x_n)}{x_n + 3h - x_n} = \frac{1}{3}. \]

Substituting the value of \( x_n + 3h + x_n \) into \( H_3(x) = 8x^3 - 12x \), then \( H_3(x) = -\frac{100}{27} \).

Following the same procedure for \( x_{n+3} \), we have

\[ x = \frac{2x_n - (x_{n+k} + x_n)}{(x_{n+k} - x_n)} = \frac{2x_{n+3} - (x_{n+3} + x_n)}{x_{n+3} - x_n} = 1 \]

Thus, substitute the value of \( x_n + 3h + x_n \) into \( H_3(x) = 8x^3 - 12x \), and have \( H_3(x) = -4 \).

Thus, from equation (3), we deduce that \( \psi'_3(x) = 0, \psi'_2(x) = 1, \psi'_1(x) = 2x, \psi'_0(x) = 3x^2 \).

Equation (3) reduces to the form

\[ f(x, y) = c_1 + 2xc_2 + 3x^2c_3 - \lambda H_3(x) \]  

We now collocate (23) at \( x_{n+i} \) \((i = 0, 1, 2)\) and interpolate (2) at \( x = x_n \) to obtain a system of five equations with \( c_i \)\((i = 0, 1, 2, 3)\) and parameter \( \lambda \) as

\[ y_n = c_0 + c_1x_n + c_2x_n^2 + c_3x_n^3 \]
\[ f_n = c_1 + 2c_2x_n + 3c_3x_n^2 - 4\lambda \]
\[ f_{n+1} = c_1 + 2c_2x_{n+1} + 3c_3x_{n+1}^2 - \frac{100}{27} \lambda \]
\[ f_{n+2} = c_1 + 2c_2x_{n+2} + 3c_3x_{n+2}^2 + \frac{100}{27} \lambda \]
\[ f_{n+3} = c_1 + 2c_2x_{n+3} + 3c_3x_{n+3}^2 + 4\lambda \]

Solving the system (24) resulted to

\[ \lambda = \frac{9}{128} (f_n - 3f_{n+1} + f_{n+2} - f_{n+3}) \]
\[ c_3 = \frac{1}{12h^2} (f_n - f_{n+1} - f_{n+2} + f_{n+3}) \]
\[ c_2 = -\frac{1}{96h^2} (123h^2 f_n - 81h^2 f_{n+1} + 81h^2 f_{n+2} - 27h^2 f_{n+3} + 122hf_n x_n - 126hf_n x_n - 18hf_{n+2} x_n + 22f_{n+3} x_n + 24 f_n x_n^2 - 24 f_{n+1} x_n^2 - 24 f_{n+2} x_n^2 + 24 f_n x_n + 3x_n^2) \]
\[ c_1 = \frac{1}{96h^2} (123h^2 f_n x_n - 81h^2 f_{n+1} x_n + 81h^2 f_{n+2} x_n - 27h^2 f_{n+3} x_n + 161hf_n x_n^2 - 63hf_{n+2} x_n^2 + 11hf_{n+3} x_n^2 + 8 f_n x_n^3 - 8 f_{n+1} x_n^3 - 8 f_{n+2} x_n^3 + 8 f_{n+3} x_n^3 - 96h^2 y_n) \]

Again, from (2) we have

\[ \hat{y} = c_0 + c_1 x + c_2 x^2 + c_3 x^3 \]  

The required numerical scheme is then obtained if we collocate (25) at \( x_{n+1} \) and substituting for \( c_0, c_1, c_2 \) and \( c_3 \) as
FORMULATION OF BLOCK...  

\[ y_{n+1} = y_n + \frac{h}{48} (35f_n - 13f_{n+1} + 41f_{n+2} - 15f_{n+3}) \]  

(26)

Now we formulate the block schemes for the polynomials of cases \( k = 1, 2 \) and 3

For \( k = 1 \), collocate equation (12) at \( x = x_{n+1}, x_{n+2}, x_{n+3} \) to obtain

\[
\begin{align*}
y_{n+1} &= y_n + \frac{h}{2} (f_n + f_{n+1}) \\
y_{n+2} &= y_n + h(f_n + f_{n+1}) \\
y_{n+3} &= y_n + \frac{3h}{2} (f_n + f_{n+1})
\end{align*}
\]  

(27)

For \( k = 2 \), collocate equation (20) at \( x = x_{n+1}, x_{n+2}, x_{n+3} \) to obtain

\[
\begin{align*}
y_{n+1} &= y_n + \frac{h}{4} (f_n + 2f_{n+1} + f_{n+2}) \\
y_{n+2} &= y_n + \frac{h}{2} (f_n + 2f_{n+1} + f_{n+2}) \\
y_{n+3} &= y_n + \frac{3h}{4} (6f_n + 2f_{n+1} + f_{n+2})
\end{align*}
\]  

(28)

For \( k = 3 \), collocate equation (25) at \( x = x_{n+1}, x_{n+2}, x_{n+3} \) to obtain

\[
\begin{align*}
y_{n+1} &= y_n + \frac{h}{48} (35f_n - 13f_{n+1} + 41f_{n+2} - 15f_{n+3}) \\
y_{n+2} &= y_n + \frac{h}{48} (33f_n + 13f_{n+1} + 67f_{n+2} - 17f_{n+3}) \\
y_{n+3} &= y_n + \frac{3h}{8} (f_n + 3f_{n+1} + 3f_{n+2} + f_{n+3})
\end{align*}
\]  

(29)

ERROR ANALYSIS OF THE METHOD

In this section, we discuss the order, the error constant and convergence of the proposed block schemes. The proposed schemes in this paper belong to the class of linear multi-step method (LMM) which is of the form

\[
\sum_{j=0}^{k} \alpha_j(x) y(x_{n+j}) = h \sum_{j=0}^{k} \beta_j(x) f(x_{n+j})
\]  

(30)

According to Sastry (2008) and Lambert (1981), define the order and error constant associated with (30) to be the linear difference operator

\[
\ell[y(x); h] = \sum_{j=0}^{k} \left\{ \alpha_j y(x + jh) - h \beta_j y'(x + jh) \right\}
\]  

(31)

Assuming that \( y(x) \) is continuously differentiable on the interval \([a, b]\), we can expand the terms in (31) as a Taylor series about the point \( x \) to obtain the expansion

\[
\ell[y(x); h] = c_0 y(x) + c_1 h y'(x) + c_2 h^2 y''(x) + \ldots + c_q h^q y^{(q)}(x)
\]  

(32)

where the constant coefficients \( c_q, q = 0, 1, \ldots \) are given as follows:

\[
c_0 = \sum_{j=0}^{k} \alpha_j, \quad c_1 = \sum_{j=0}^{k} (j \alpha_j - \beta_j), \quad \ldots, \quad c_q = \sum_{j=0}^{k} \left[ \frac{1}{q!} j^q \alpha_j - \frac{1}{(q-1)!} j^{q-1} \beta_j \right], \quad q = 2, 3, \ldots
\]  

(33)
We say that the methods are said to be order p if 
\[ c_0 = c_1 = c_2 = \ldots = c_p = 0, c_{p+1} \neq 0 \]
Therefore the \( c_{p+1} \) is the error constant.
Hence, we establish that (27), (28) and (29) are of the following orders and error constants respectively.

when \( k = 1 \), \( p = 1 \) and \( c_{p+1} = \frac{1}{4} \)
when \( k = 2 \), \( p = 2 \) and \( c_{p+1} = -\frac{1}{6} \)
when \( k = 3 \), \( p = 4 \) and \( c_{p+1} = -\frac{864}{23040} \)

**NUMERICAL EXAMPLES**

In this section, two initial value problems were solved using the proposed method for \( k = 2 \) and \( k = 3 \) and compare the numerical results obtained with the exact solution and those obtained from using the method of Okedayo et al (2018) in order to test the efficiency of the schemes.

**Example 1**
Consider the following IVP
\[ y'(x) = -y, \; y(0) = 1 \]
with the exact solution \( y(x) = e^{-x} \)

**Example 2**
Consider the IVP
\[ y'(x) = x - xy, \; y(0) = 0 \]

The numerical results of these examples are depicted in Tables 1, 2, 3 and 4 with \( k = 2 \) and \( k = 3 \) with constant step size of \( h = 0.1 \) respectively. In tables 1 and 3 we presented a comparison of the obtained numerical results using the proposed scheme with the exact solution and Table 2 and 4 presents the comparison of the results obtained from proposed scheme, the exact solution and those numerical results obtained from Okedayo et al. (2018).

**Table 1: A comparison of numerical results of proposed Scheme at \( k = 2 \) with exact solution for Example 1**

| x-value | BLS \( k = 2 \) | Exact | \(|\text{Exact-BLS}|\) |
|---------|----------------|-------|-----------------|
| 0.0     | 1.000000       | 1.000000 | 0.000000       |
| 0.1     | 0.905090       | 0.904837 | 4.253×10^{-4} |
| 0.2     | 0.818181       | 0.818730 | 5.49×10^{-4}  |
| 0.3     | 0.749092       | 0.740818 | 8.274×10^{-3} |
| 0.4     | 0.675227       | 0.670320 | 4.907×10^{-3} |
| 0.5     | 0.608761       | 0.606531 | 2.23×10^{-3}  |
| 0.6     | 0.548760       | 0.548811 | 5.1×10^{-3}   |
| 0.7     | 0.504028       | 0.496585 | 7.443×10^{-3} |
| 0.8     | 0.448139       | 0.449328 | 1.189×10^{-3} |
| 0.9     | 0.416232       | 0.406569 | 9.663×10^{-3} |
| 1.0     | 0.373893       | 0.367879 | 6.014×10^{-3} |
Table 2: A comparison of numerical results of proposed scheme at $k = 3$ with exact solution and LM for Example 1

| x-value | BLS | LM | Exact | |Exact-BLS| |Exact-LM|
|---------|-----|----|-------|--------------|--------------|
| 0.0     | 1.000000 | 1.000000 | 1.000000 | 0.000000 | 0.000000 |
| 0.1     | 0.904808 | 0.905953 | 0.904837 | 2.9×10^{-5} | 1.116×10^{-3} |
| 0.2     | 0.818705 | 0.820856 | 0.818730 | 5.0×10^{-5} | 2.126×10^{-3} |
| 0.3     | 0.740823 | 0.743857 | 0.740818 | 5.0×10^{-5} | 3.039×10^{-3} |
| 0.4     | 0.670304 | 0.674185 | 0.670320 | 1.6×10^{-5} | 3.865×10^{-3} |
| 0.5     | 0.606516 | 0.611143 | 0.606531 | 1.5×10^{-5} | 4.612×10^{-3} |
| 0.6     | 0.548820 | 0.554100 | 0.548811 | 9.0×10^{-6} | 5.289×10^{-3} |
| 0.7     | 0.496576 | 0.502486 | 0.496585 | 9.0×10^{-6} | 5.901×10^{-3} |
| 0.8     | 0.449321 | 0.455784 | 0.449328 | 7.0×10^{-6} | 6.456×10^{-3} |
| 0.9     | 0.406578 | 0.413527 | 0.406569 | 9.0×10^{-6} | 6.958×10^{-3} |
| 1.0     | 0.367876 | 0.375290 | 0.367879 | 3.0×10^{-6} | 7.411×10^{-3} |

Table 3: A comparison of numerical results of proposed scheme at $k = 2$ with exact solution for Example 2

| x-value | BLS | Exact | |Exact-BLS| |
|---------|-----|-------|--------------|
| 0.0     | 0.000000 | 0.000000 | 0.000000 |
| 0.1     | 0.005452 | 0.004785 | 6.67×10^{-4} |
| 0.2     | 0.018904 | 0.018901 | 1.97×10^{-4} |
| 0.3     | 0.042261 | 0.033013 | 9.218×10^{-3} |
| 0.4     | 0.058942 | 0.056862 | 2.078×10^{-3} |
| 0.5     | 0.117427 | 0.115602 | 1.825×10^{-3} |
| 0.6     | 0.121342 | 0.166729 | 4.5387×10^{-2} |
| 0.7     | 0.178557 | 0.216929 | 1.1828×10^{-2} |
| 0.8     | 0.235772 | 0.255531 | 1.9758×10^{-2} |
| 0.9     | 0.274301 | 0.292022 | 1.37121×10^{-2} |
| 1.0     | 0.327572 | 0.353368 | 2.5706×10^{-2} |
Table 4: A comparison of numerical results of proposed scheme at $k = 3$ with exact solution and LM for Example 2

| x-value | BLS $k = 3$ | LM | Exact | |Exact-BLS| |Exact-LM|
|---------|-------------|-----|-----|---------|----------|---------|
| 0.0     | 0.0000000   | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 0.1     | 0.005073    | 0.005892 | 0.004785 | 2.88×$10^{-5}$ | 1.027×$10^{-4}$ |
| 0.2     | 0.018214    | 0.020465 | 0.018901 | 6.87×$10^{-4}$ | 1.564×$10^{-3}$ |
| 0.3     | 0.039568    | 0.037475 | 0.033013 | 6.555×$10^{-3}$ | 4.402×$10^{-3}$ |
| 0.4     | 0.064584    | 0.078865 | 0.056862 | 7.222×$10^{-3}$ | 2.2003×$10^{-3}$ |
| 0.5     | 0.107592    | 0.120033 | 0.115602 | 8.01×$10^{-3}$ | 4.4431×$10^{-3}$ |
| 0.6     | 0.161899    | 0.159353 | 0.166729 | 4.83×$10^{-3}$ | 7.376×$10^{-3}$ |
| 0.7     | 0.212809    | 0.221055 | 0.216929 | 4.1214×$10^{-3}$ | 2.362×$10^{-2}$ |
| 0.8     | 0.250814    | 0.279245 | 0.255531 | 2.4716×$10^{-2}$ | 2.3715×$10^{-2}$ |
| 0.9     | 0.308725    | 0.328348 | 0.292022 | 1.6703×$10^{-2}$ | 3.6326×$10^{-2}$ |
| 1.0     | 0.384329    | 0.398478 | 0.353368 | 3.0961×$10^{-2}$ | 1.35202×$10^{-1}$ |

Tables 1, 2, 3 and 4 shows that the proposed schemes approximate the solutions of initial value problems given in Examples 1 and 2 as the absolute errors are convergent. Also, the absolute errors presented in Tables 2 and 4 show that the proposed schemes compared favourably with the method of Okedayo et al. (2015) applied to the given numerical examples.

CONCLUSION
In this research work, a class of three new block schemes for the approximation of initial value problems of first order ordinary differential equations using Hermite polynomial as a basis function has been obtained. The proposed method was used to solve numerically some initial value problems and the results compared with the exact solutions and the method of Okedayo et al. (2015). From the numerical results, it is observed that the new schemes were capable for solving first order IVPs as generated results compared favorably with the existing method and the exact solutions. The method is very simple to implement.

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