Experimental Demonstration of High-Dimensional Quantum Steering With $N$ Measurement Settings

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High-dimensional (HD) quantum systems possess advantages of larger channel capacity, stronger noise resilience and higher security against attacks in quantum communication compared with qubit systems. Here, we experimentally demonstrate HD quantum steering criterion with $n$ measurement settings in high-noise environments. We verify the unbounded violation of steering equalities, revealing a high strength of steering without full-state tomography. Moreover, our results exhibit that the noise resilience can be enhanced with both extra dimension and measurement settings. We experimentally certify 11-dimensional steering with 60.5% isotropic noise, exceeding the upper bound of 2-setting steering criteria. Our work paves the way for the certification of HD quantum correlations with high noise in quantum information processing.

I. INTRODUCTION

One of the most fundamental characteristics of quantum systems is that distant objects may show stronger correlations than in the classical world. Quantum correlations can be strictly categorized into three hierarchies [1–3]: Entanglement, Einstein-Podolsky-Rosen (EPR) steering and Bell nonlocality, among which entanglement is the weakest and Bell nonlocality is the strongest. As a generalization of EPR paradox [4], steering was first introduced by Schrödinger in 1935 to describe Alice’s ability to remotely affect Bob’s state via local measurements [5]. The modern notion and operational framework of steering were formulated in the innovative work of Wiseman et al. [1] where they defined steering in terms of the impossibility to describe the conditional states at one party by the local hidden state (LHS) model. The certification of quantum steering is an asymmetric quantum information task where one party’s measurement devices are untrusted. This feature makes steering easier to perform than Bell nonlocality and have many applications in quantum information processing, such as subchannel discrimination [6, 7], one-sided device-independent quantum key distribution [8], secure quantum teleportation [9, 10] and one-sided device-independent randomness generation [11–14].

High-dimensional (HD) quantum system is expected to transcend limitations of qubits in some applications [15], due to its stronger nonlocal correlation [16, 17], larger channel capacity [18–20] and higher robustness against noise [21–23]. Recently, some works began to explore the HD steering effect [13, 14, 24–26]. In particular, Zeng et al. exhibited the strong noise robustness increasing with the extra dimension in HD steering [25]. However, these works are restricted to 2-setting steering criteria. The steering effect with $n$ measurement settings, where $n$ increases with the dimension $d$ of the system, has not been experimentally demonstrated. The extra measurement settings enable us to obtain more information about the underlying quantum state so that reveal more strength of steering. Moreover, the degree of nonlocality depends on the strength of the uncertainty relationship and quantum steering [27]. Recently, Marciniak et al. [28] exploited the intrinsic relationship between steering and the uncertainty principle, applied to mutually unbiased bases (MUBs) which offer strong uncertainty, and theoretically constructed the $n$-setting linear steering inequality which shows an unbounded violation. Interestingly, there is no similar simple form in the existing Bell inequalities with unbounded violation [29–32]. Compared with the 2-setting steering criteria, this criterion is theoretically more robust against white noise [33]. As the dimension goes to infinity, it can tolerate any amount of noise to detect steerability while 2-setting steering criteria only tolerate 50% at most.

In this work, we implement an experiment based on photons entangled in their orbital angular momentum (OAM) to explore HD steering effect with $n$ measurement settings. We first experimentally demonstrate the unbounded violation of steering equalities using measurements on MUBs, and obtain the degree of violation $V = 2.102 \pm 0.031$ in dimension of $d = 11$ while the violation in 2-setting criteria satisfies $V < 2$. Then, by introducing tunable isotropic noise, we further explore the noise resistance of HD steering and find that its performance is enhanced by increasing not only dimension but also measurement settings. In particular, we experimentally certify 11-dimensional steering with 60.5±1.4% isotropic noise, and in the dimension of $d = 5, 7$, the percentages of tolerated noise are also more than 50%, exceeding the upper bound of 2-setting criteria. The certification of HD quantum correlations is extremely demand-
The quantum bound, equals the number of measurement x a parties obtain result where MUBs and their standard construction [35]. Then we in-

d the scene of steering functional for the LHS model is

\[ V = \frac{S_Q}{S_{LHS}} = \frac{1 + d}{1 + \sqrt{d}}. \] 

We can see an unbounded quantum violation of O(\(\sqrt{d}\)) as dimension d goes to infinity, while the degree of violation in 2-setting criteria is \(V = 2/(1 + 1/\sqrt{d}) < 2\) [25]. It is worth to mention that this criterion directly uses probability distributions from few measurements, without the need of performing full-state tomography.

III. EXPERIMENTAL DEMONSTRATION OF UNBOUNDED VIOLATION OF STEERING INEQUALITIES

The experimental setup is shown in Fig. 1. A 405nm continuous wave laser beam is coupled to a single-mode fiber (SMF) to obtain pure fundamental Gaussian mode pump beam. The OAM entangled photons are generated through type-II spontaneous parametric down-conversion (SPDC) in a 5-mm-long periodically poled potassium titanyl phosphate (ppKTP) crystal. After the polarization beam splitter (PBS), the polarization orthogonal correlated photon pairs are separated. A half-wave plate (HWP) is used to rotate the polarization of the idler photon from vertical to horizontal, allowing it to be manipulated by the spatial light modulator (SLM). We make the idler photon reflect odd times to reverse its OAM process (SPDC) in a 5-mm-long periodically poled potassium titanyl phosphate (ppKTP) crystal.

\[ \text{FIG. 1. Experimental setup for verification of the } n\text{-setting steering criterion.} \]

SMF: single mode fiber; PPKTP: periodically poled potassium titanyl phosphate; DM: dichroic mirror; PBS: polarizing beam splitter; HWP: half wave plate; SLM: spatial light modulator; Filter: long-pass optical filter; SPAD: single-photon avalanche detector.

If Bob demonstrates that the correlation of their results violates this inequality, he will admit they share steerable states. The degree of violation, a natural way to quantify the strength of steering, can be defined as [28]

\[ S_{Q}\text{ is a prime power, we can always find } n = d + 1 \text{ MUBs and their standard construction [35]. Then we introduce the steering functional, which can be expressed as [28] } \]

\[ S_Q = \sum_{x=1}^{n} \sum_{a=1}^{d} \text{Tr} \left[ \left( A_{a|x} \otimes I_B^T \right) \rho_{AB} \right] \]

\[ = \sum_{x=1}^{n} \sum_{a=1}^{d} P(a, a|x, x), \] 

where \(P(a, a|x, x)\) denotes the joint probability that both parties obtain result a when measuring in the same basis x. The maximal quantum value of Eq.(2), which is called the quantum bound, equals the number of measurement settings \(S_Q = d + 1\).

II. UNBOUNDED VIOLATION OF STEERING INEQUALITIES

Consider a scenario featuring in two space-like separate observers, Alice and Bob, sharing a bipartite state \(\rho_{AB}\). Alice chooses one of her measurement settings \(\{A_{a|x}\}_{x,a}\), labeled by \(a = 1, \ldots, n\) and receives a result \(a \in \{1, \ldots, d\}\), where \(A_{a|x}\) denotes the positive operators satisfying \(\sum_{a} A_{a|x} = I\) for each x, thus remotely steers Bob’s subsystem to the conditional state

\[ \sigma_{a|x} = \text{Tr}_A \left[ \left( A_{a|x} \otimes I_B^T \right) \rho_{AB} \right]. \] 

Bob performs his measurements on the conditional state and confirms whether the correlation of their results violates the steering inequalities.

Here we consider the scenario where Alice and Bob share a HD maximally entanglement \(|\Phi_d\rangle = 1/\sqrt{d} \sum_{l=0}^{d-1} |l, l\rangle\) and perform ideal measurements on MUBs. Two orthonormal bases are called MUBs if the inner product of any vector from the first basis with any vector from the second basis is \(|\langle \varphi^a_w, \varphi^b_v \rangle|^2 = 1/\sqrt{d}\) [34]. In the scene d is a prime power, we can always find \(n = d + 1\) MUBs and their standard construction [35]. Then we introduce the steering functional, which can be expressed as [28]

\[ S_Q = \sum_{x=1}^{n} \sum_{a=1}^{d} \text{Tr} \left[ \left( A_{a|x} \otimes I_B^T \right) \rho_{AB} \right] \]

\[ = \sum_{x=1}^{n} \sum_{a=1}^{d} P(a, a|x, x), \] 

where \(P(a, a|x, x)\) denotes the joint probability that both parties obtain result a when measuring in the same basis x. The maximal quantum value of Eq.(2), which is called the quantum bound, equals the number of measurement settings \(S_Q = d + 1\).

If they share a separable state \(\rho = \sum_i p_i \rho_i^A \otimes \rho_i^B\), Alice can only steer Bob’s subsystem to the conditional state that satisfies the LHS model

\[ \sigma_{a|x} = \sum_i p_i \text{Tr}_A \left[ A_{a|x} \rho_i^A \right] \otimes \rho_i^B. \] 

The bound of steering functional for the LHS model is [33]

\[ S_{LHS} = \sum_{x=1}^{n} \sum_{a=1}^{d} \text{Tr}(A_{a|x} \sigma_{a|x}) \leq 1 + \sqrt{d}. \] 

If Bob demonstrates that the correlation of their results violates this inequality, he will admit they share steerable
FIG. 2. Experimental results. (a) Experimental 2-photons coincidence counts in computational basis \(|l_i⟩_A \langle l_j|_B\) with \(l_i, l_j = -5, ..., 5\). The coloured and transparent bars depict the coincidence counts with and without entanglement concentration. (b) Normalised 2-photons coincidence rates in two 11-dimensional MUBs (\(x = 1\) and 2). When Alice’s projective measurement is \(|ϕ_a^x⟩\), Bob measures on the complex conjugation bases \(|ϕ_b^x⟩^*\) (\(a, b = 0, ..., 10\)). (c) Experimental results of the steering functional \(S\) in Eq.(2) with dimension up to 11. The error bars are of the order of \(10^{-2}\), much smaller than the marker size.

in the OAM bases. Long-pass optical filters are used to reduce the detection of noise photons.

The original OAM entanglement produced by SPDC process is not a maximally entangled state owing to the limited spiral bandwidth [36]. To faithfully certify the unbounded violation of steering inequalities, we need to use the entanglement concentration technique called Procrustean method [37] to equalize different orders of OAM. This is generally done by choosing local operations matched to the spiral bandwidth for our SPDC source [38]. Fig. 2(a) shows our coincidence measurement results before and after entanglement concentration.

To test within \(d\)-dimensional subspace, for odd \(d\), we choose the modes \(l\) from \((-d-1)/2\) to \((d-1)/2\) as the computational basis. For even \(d\), we choose \(l\) from \(-d/2\) to \(d/2\), omitting the \(l = 0\) mode. And we construct the other \(d\) MUBs using these modes, respectively. The normalized coincidence rates in two 11-dimensional MUBs (\(x = 1\) and 2) are shown in Fig. 2(b).

Fig. 2(c) displays our experimental results of steering functional. The quantum bound and LHS model bound are also presented for comparison. We can see all the values of \(S_Q\) exceed the corresponding LHS bound in the dimensions of \(d = 2, 3, 4, 5, 7, 11\), thus successfully violate the steering inequalities and certify HD quantum steering with dimension up to \(d = 11\).

The experimental and theoretical values of violation of steering inequality in different dimensions are displayed in Fig. 3. It is worth noting that in dimension \(d = 11\) the experimental value of violation is \(V = 2.102 \pm 0.031\), which exceed the theoretical biggest violation in the 2-setting steering criterium. It means our results reveal a higher strength of steering.

IV. EXPERIMENTAL CERTIFICATION OF NOISE-RESILIENCE IN HD STEERING WITH \(N\) SETTINGS

We further explore the robustness of this method in the noisy environment. According to the theoretical prediction in Ref. [33], we consider Alice and Bob share a \(d\)-dimensional isotropic state \(ρ_{iso} = p|Φ^+⟩⟨Φ^+| + (1 - p)\hat{I}/d^2\), where \(p\) denotes the percentage of the pure maximal entanglement \(|Φ^+⟩ = (1/\sqrt{d})\sum_{l=-d/2}^{d/2}|l⟩_A|l⟩_B\) (\(l \neq 0\) for even \(d\)). The steering functional for the isotropic state can be calculated by

\[
S_{iso} = (d + 1)[p + (1 - p)/d].
\]

To violate the steering inequality \(S \leq 1 + \sqrt{d}\), the minimal percentage of the pure maximal entanglement

FIG. 3. Experimental results of violation \(V\) of the \(n\)-settings steering inequality with dimension up to 11. The error bars are of the order of \(10^{-2}\).
state is \( p_{\text{min}} = (d^4/2 - 1)/(d^2 - 1) \). For comparison, the minimal percentage in linear 2-setting steering criteria is \( p_{\text{min}}^{(2)} = 1/2[1 + (\sqrt{d} - 1)/(d - 1)] \) \cite{25}. As we can see, for growing system dimension, \( p_{\text{min}} \) would be lower, which means that the noise-resilience ability of the system becomes stronger. Importantly, as dimension goes to infinite, \( p_{\text{min}} \) gets close to 0, while \( p_{\text{min}}^{(2)} \) gets close to 50\%. So the n-setting criterion is theoretically more robust against white noise.

To experimentally verify this characteristic, we follow Ref. \cite{25} to construct a simplified tunable isotropic noise in a statistical sense:

\[
\frac{I/d^2}{d+1} = \frac{1}{d+1} \sum_{i,j=0}^{d-1} |l_i l_j \rangle_{AB} \langle l_i l_j | + \sum_{x=0}^{d-1} \sum_{a,b=0}^{d-1} |\varphi^a_x \varphi^b_x \rangle_{AB} \langle \varphi^a_x \varphi^b_x |,
\]

where \( \{|\varphi^a_x \rangle \}_x \) labeled by \( x = 0, \ldots, d - 1 \) denotes \( d \) MUBs except computational basis. Obviously, \( (d + 1)d^2 \) CGHs with same probability which correspond to the product states \( |l_i \rangle_A |l_j \rangle_B \) and \( \{|\varphi^a_x \rangle \}_x \) need to be performed in SLM1. When the incident photons are affected by these CGHs, their initial states transform into the corresponding states and covered all components in Eq. (7). Since the correspond measurement time is too long, we put regular diffraction grating which represents \( |\Phi^+ \rangle \) and these \( (d + 1)d^2 \) CGHs which represent isotropic noise into a random number pool to construct animations in SLM1. Then we calculate their emerging probability. The isotropic state contains two components, \( |\Phi^+ \rangle \langle \Phi^+ | \) and \( I/d^2 \) with probability \( p \) and \( 1 - p \). Going through all possibilities of the pure state \( |\Phi^+ \rangle = (1/\sqrt{d}) \sum_{i=1}^{(d+1)/2} |l_i \rangle_A |l_i \rangle_B \) takes at least \( dt \) time, while the ergodic time is \( d^2 t \) for the isotropic noise. Considering their relative probability, the ratio of pure entanglement and mixed isotropic noise is \( pd t : (1 - p)d^2 t \). So the emerging probability of regular diffraction grating is \( P = p/[p + (1 - p)d] \), the other CGHs share the probability of \( (1 - P)/(d + 1) d^2 \). Thus, we can obtain the tunable isotropic state \( \rho_{\text{iso}} \) by setting a certain \( p \).

The experimental results of \( S_{\text{iso}} \) as function of \( p \) in dimensions of \( d = 4, 5, 7, 11 \) are shown in Fig. 4(a), 4(b), 4(c) and 4(d), respectively. We set 5 levels of percentages \( p = 0.3 \pm 0.7 \) as points to fit the blue solid lines, while the blue dashed lines exhibit the range of error bars. The theoretical (red) and experimental lines are both linearly changed with \( p \), though there are discrepancies in the slopes. And the discrepancy becomes larger as \( d \) increases because of the nonideal entanglement source, increasing shot noise and crosstalk of different OAM modes. The black horizontal lines with the corresponding values denote the bound of LHS model.

The experimental results of \( p_{\text{min}} \) in Fig. 4(e) are obtained from the points where the blue lines intersect the black horizontal lines in Fig. 4(a), 4(b), 4(c) and 4(d). As we can see, the experimental minimal percentage of the pure state for \( d = 11 \) is \( p_{\text{min}} = 39.5 \pm 1.4\% \), which means that we can certify 11-dimensional steering with

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**FIG. 4.** Experimental results of noise-resilience. (a)-(d) Steering functional of isotropic state \( S_{\text{iso}} \) for different noise percentages in dimensions of \( d = 4, 5, 7, 11 \). (e) Experimental results of \( p_{\text{min}} \) as function of \( d \).
60.5 ± 1.4% isotropic noise. And for $d = 5, 7$, the percentages of tolerated noise also exceed 50%, which is the upper bound of 2-setting criteria. Although the discrepancy of $p_{\text{min}}$ between experiment and theory increases with the dimension increasing due to the other noises (shot noise and crosstalk), $p_{\text{min}}$ still decreases evidently. Thus, we can conclude that the noise resilience of HD steering effect is indeed enhanced by increasing both dimension and measurement settings. Our results also indicate that the $n$-setting steering criteria can reveal more strength of steering than 2-setting steering criteria. This is due to that extra measurement settings reveal more information about the underlying quantum system. This method have potential applications in the certification of HD nonlocal correlations with high noise in the quantum information processing. It is interesting to note that this could motivate the search for HD MUBs beyond prime power dimensions.

V. CONCLUSIONS

We have experimentally demonstrated $n$-setting HD steering with dimension up to $d = 11$ without full-state tomography in high-noise environments based on the OAM entanglement. The experimentally maximal violation of steering inequalities exceeds the upper bound of 2-setting criteria. More importantly, we discover that not only the extra dimension but also measurement settings enhance the noise resilience. In our experiment, we certify 11-dimensional steering with 60.5% isotropic noise. In the dimensions of $d = 5, 7$, the percentages of tolerated noise also exceed 50% which is the bound of 2-setting steering criteria. Obviously, our results indicate that $n$-setting criteria can reveal more strength of steering. Our work paves the way for the certification of HD quantum correlations with high noise in the quantum information precessing. In addition, noisy steerable states provide potential applications in the task of subchannel discrimination [6, 39]. Moreover, our method can be easily extended to other steering criteria with $n$ measurement settings, such as for generalized entropies [40, 41].

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