Infinitely many global continua bifurcating from a single solution of an elliptic problem with a concave-convex nonlinearity

Rainer Mandel
Scuola Normale Superiore di Pisa, Italia
Rainer.Mandel@sns.it

Initiated by a paper of Ambrosetti-Brézis-Cerami [1] many existence and multiplicity results for boundary value problems on bounded domains like

\[
\begin{aligned}
-\Delta u &= \lambda|u|^{q-2}u + |u|^{p-2}u \quad \text{in } \Omega, \\
u &= 0 \quad \text{on } \partial\Omega
\end{aligned}
\]

for exponents \(1 < q < 2 < p < \infty\) were found using variational methods (see [1, 3, 5]).

A seminal result from [1] says that there is a positive number \(\Lambda_0\) such that (1) has two positive solutions for \(0 < \lambda < \Lambda_0\). Later the existence of infinitely many nontrivial solutions was proved in [3]. In a joint paper with T. Bartsch [2] we prove in the case of the annulus that these nontrivial solutions are located on mutually disjoint continua \(C_j\) of solutions having precisely \(j\) nodal domains which emanate from \((\lambda_0, u_0) = (0, 0)\) and satisfy \(\text{pr}(C_j) = (-\infty, \Lambda_j]\).

A study of these continua shows that they accumulate at \(R \geq 0 \times \{0\}\) and \(R \times \{\infty\}\) leading to bifurcation from \((\lambda, 0)\) when \(\lambda \geq 0\) and bifurcation from infinity at every \(\lambda \in \mathbb{R}\). The proofs are based on degree theory similar to [4]. Our method applies also to boundary value problems with more general convex-concave nonlinearities \(f_\lambda(|x|, u, |\nabla u|)\).

\[
\| \cdot \|_{C^1(\Omega)}
\]

\[
C_3 \quad C_2 \quad C_1 \quad C_0
\]

\[
\Lambda_0 \Lambda_1 \quad \Lambda_2 \quad \Lambda_3
\]

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