The Drude-Smith Model for Conductivity: de novo Derivation and Interpretation

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(Dated: August 19, 2020)

The Drude-Smith model successfully describes the frequency and phase-resolved electrical conductivity data for a surprisingly broad range of systems, especially in the terahertz region. Still, its interpretation is unclear since its original derivation is flawed. We use an intuitive physical framework to derive the Drude-Smith formula for systems where microscopically free charges are accumulated on a mesoscopic scale by localized scatterers. Within this framework, the model allows us to quantify the microscopic momentum relaxation time of the charges and the fraction of mesoscopically localized charges in addition to the direct current limit of the conductivity. We show that the Drude-Smith model is unique among different Drude-Lorentz models because the relaxation time of the free carriers also determines the frequency and damping of the resonance of the bound charges.

Characterising charge carrier dynamics by conductivity spectra:

Charge carriers in conductors and semiconductors form the basis of several important technologies, including computers, semiconductor lasers, and light-emitting devices. These technologies consist of increasingly small structures. There is, therefore, both a technological and fundamental interest in characterizing and understanding the properties of charge carriers in both bulk and nanostructured materials. A very suitable way of characterizing charge carriers dynamics in different materials and material configurations is through their frequency-dependent conductivity, or, equivalently, dielectric response. The response of mobile carriers and polaron charge is dictated by carrier-phonon interactions leading to randomization of the carrier momentum typically occurring on (sub-)picosecond time scales, giving rise to dispersion in the dielectric response on meV energy scales. Scattering from defects also typically occurs on that time- and energy scale. The dielectric response in the same energy range is modified for carriers that undergo different types of transport, such as hopping transport in non-crystalline semiconductors. The ability to probe charge carriers in the (sub-)meV energy or, equivalently, megahertz to terahertz frequency range, therefore, allows their detailed characterization through the distinct spectral signatures in this frequency range. Empirically, the Drude-Smith (DS) model describes such frequency-resolved electrical conductivity data measured in a wide variety of systems, ranging from liquid metal, for which the model was first formulated,[1] over percolated metals[4–7] and other nanostructures[8–15], amorphous metals[16], graphene[17], semiconductors[18–22], to organic conductors[23–26]. The feature that all these systems have in common is a restriction of charge carrier motion that is translated into the observation of a reduced conductivity at low frequencies. The original derivation of the model by Smith was based on microscopic arguments[1], but is flawed to the point that it contradicts itself. This has left the meaning of the model and its parameters unclear. Observed Drude-Smith conductivities have, therefore, sometimes been interpreted in terms of Smith’s proposal preferential one-off backscattering, or alternatively as a more mesoscopic charge confinement induced by grain boundaries or as nanoscale disorder[27–30]. On occasion, the Drude-Smith model has just been considered an empirical approximation, e.g. of hopping transport, without substantial physical interpretation[18]. Hence a clear understanding of the Drude-Smith type conduction, and an unambiguous physical assignment of the parameters employed describing it, are desirable.

Here, we derive the Drude-Smith equation based on mesoscopic arguments. This may explain why the formula can describe microscopically very different systems. We start by explaining the problems and self-contradiction in Smith’s original derivation.

Smith’s problematic physical picture and contradicting assumptions:

Smith[1] motivated his model by considering the impulse response of n identical, non-interacting particles per unit volume carrying a charge e whose scattering process is governed by Poisson statistics, that means each scattering event is independent from all preceding events. The current density $j$ of such a system has the following response in the time domain[1]:

$$j(t) = \exp \left( -\frac{t}{\tau_c} \right) + \sum_{s=1}^{\infty} \beta_s \frac{t^s}{s!} \exp \left( -\frac{t}{\tau_c} \right)$$  \hspace{1cm} (1)

where $\tau_c$ is the expected time between collisions, $s$ is the number of collisions that a charge carrier has undergone up to time $t$. At time $t = 0$, an electric field had accelerated the charge carrier to a certain velocity, and $\beta_s$ reflects the expected fraction of this original velocity the charge carrier has retained after $s$ col-
lisions. In case of classical trajectories, "$\beta_s$ is the expectation of $\cos(\theta)$ after $s$ collisions." [1] $\theta$ is the angle between the original and final velocity. Smith correctly states that for independent collisions, $\beta_s = (\beta_1)^s$ and eq. (1) simplifies to $j(t)/j(0) = \exp(-t/\tau)$, where $\tau = \tau_r/(1 - \beta_1)$ is the velocity relaxation time. This response is the well-known Drude result [1] equivalent to a complex frequency-domain conductivity $\tilde{\sigma}$ of

$$\tilde{\sigma}(\omega) = \frac{ne^2\tau}{m} \frac{1}{1 - i\omega \tau} = \frac{\sigma_0}{1 - i\omega \tau}$$ \hspace{1cm} (2)

where $\omega$ is the angular frequency and $m$ the mass of each particle. We abbreviate the zero-frequency limit of the conductivity (i.e., DC conductivity) for this case of entirely free charges as $\sigma_0$.

Smith then made the assumption that $\beta_1 \neq 0$ and $\beta_s = 0$ for all $s \geq 2$. This implies for the conductivity that [1]:

$$\tilde{\sigma}_{DS}(\omega) = \sigma_0 \frac{1}{1 - i\omega \tau_C} \left( 1 + \frac{\beta_1}{1 - i\omega \tau_C} \right)$$ \hspace{1cm} (3)

Physically, this means each charge carrier scatters into a preferential direction, defined by $\beta_1$, as a consequence of its first scattering process after time 0, and then completely randomly forever after. The special spectral shape that Smith wanted to capture with his model appears when $\beta_1$ is negative, implying the moving electron charge is preferentially scattered in the backward direction, but only on the first scattering event. Smiths model thus implies the very strange physical picture of some kind of counter attached to each charge carrier. All counters of all carriers would somehow be initialized at time 0. Each counter would then count every scattering event the carrier undergoes, directing it to start scattering randomly after the first event. To compound the oddity of this physical picture, we remember that $t = 0$ is an arbitrary choice. The odd physical picture is problematic for Smiths model; a self-contradiction makes Smiths derivation of his formula mathematically wrong. Smith starts by assuming a Poisson distributed scattering process, allowing the use of eq. (1). But by definition, events of a Poisson process must be statistically independent. Choosing any $\beta_s \neq (\beta_1)^s$ assumes a correlation between scattering events, which therefore precludes the use of eq. (1). A Poisson process must always lead to the Drude shape of eq. (2). Hence the model is not internally consistent.

\textit{Deriving Smiths formula for a localized and a continuous scattering process}

Here, we offer a self-consistent derivation of the Drude-Smith formula that is general enough to explain the observation of Drude-Smith type conduction in a wide range of microscopically different materials. We start similarly to Smith, and consider a material with free carriers which relax their velocity within the time $\tau$ due to scattering. A constant applied field $E$ would lead to a current $j$. After the field is switched off at time $t = 0$, the current decays exponentially (fig. 1).

$$E(t) = E_0 \Theta(-t) \hspace{1cm} j(t \geq 0) = E_0 \sigma_0 \exp(-t/\tau)$$ \hspace{1cm} (4)

We obtain the frequency domain current by Fourier transformation:

$$\tilde{j}(\omega) = \tilde{E}(\omega) \frac{\sigma_0}{1 - i\omega \tau}$$ \hspace{1cm} (5)

This is the Drude response. Now we insert an obstacle in our material (fig. 1b)). When we induce a current $j_I$ in the material, the obstacle will reflect a fraction $C$ of the applied current. After the initial current $j_I$ is switched off at $t = 0$, the reflected current $j_B$ will relax exponentially with the same characteristic relaxation time $\tau$.

$$j_I(t) = j_0 \Theta(-t) \hspace{1cm} j_B(t \geq 0) = j_0 \cdot (-C) \exp(-t/\tau)$$ \hspace{1cm} (6)

This implies a frequency domain relationship:

$$\tilde{j}_B(\omega) = \tilde{j}_I(\omega) \frac{(-C)}{1 - i\omega \tau}$$ \hspace{1cm} (7)

Taking the initial current spectrum $\tilde{j}_I$ as the result of an applied electric field $\tilde{E}$, we can easily derive $\tilde{j}_B$ in the frequency domain (fig. 1c)).

$$\tilde{j}_B(\omega) = \tilde{E}(\omega) \frac{\sigma_0}{1 - i\omega \tau} \frac{(-C)}{1 - i\omega \tau}$$ \hspace{1cm} (8)

The net current spectrum $\tilde{j}$ as a result of the electrical field at the obstacle is

$$\tilde{j}(\omega) = \tilde{j}_I + \tilde{j}_B = \tilde{E}(\omega) \frac{\sigma_0}{1 - i\omega \tau} \left( 1 - \frac{C}{1 - i\omega \tau} \right)$$ \hspace{1cm} (9)

This is Smiths conductivity formula. For now, it only describes the current at the obstacle, not far away from it.

The reflection of the current caused by the mesoscopic obstacle induces a charge accumulation between the obstacle itself and any other position $x_1$ in the material (Fig. 1d)). We can call on the continuity equation to ensure that the formula is correct for all positions along the direction $x$ of current and field. In the frequency domain, the continuity equation reads:
FIG. 1.  

**a)** Conductive material  
- $E(t) = E_0 \Theta(-t)$  
- Steady-state conductivity $\sigma_0$  
- Relaxation time $\tau$  

$$j(t) = E_0 \sigma_0 e^{\frac{t}{\tau}}$$  

$$j(\omega) = \tilde{E}(\omega) \frac{\sigma_0}{1 - i\omega\tau}$$

**b)** Barrier  
- $j_I(t) = j_0 \Theta(-t)$  
- Reflected current $j_B(t) = -j_0 C e^{\frac{t}{\tau}}$  
- Back-reflected current $j_B(\omega) = j_I(\omega) \frac{-C}{1 - i\omega\tau}$

**c)**  
$$\tilde{E}(\omega)$$  
$$+$$  
$$=$$  

$$j(\omega) = \tilde{E}(\omega) \frac{\sigma_0}{1 - i\omega\tau} \left(1 - \frac{C}{1 - i\omega\tau}\right)$$

**d)**  
- $j_I + j_B$  
- $\tilde{j}_I + \tilde{j}_D + \tilde{j}_B$

The total current, including the displacement current, is therefore constant. This picture of spatial variations of electrical currents and fields due to localized scattering centers was introduced for the direct current by Landauer more than ten years before Smith. Landauer explains in detail how to derive the fraction $C$ of the microscopically free current dammed up by the obstacles based on their microscopic scattering cross-section and arrangement. For the macroscopic conductivity model, we do not need to know the microscopic details of the velocity distribution of the charge carriers or any details about the obstacles apart from the fact that they are localized. Localized means that the mesoscopic obstacles are far enough away from each other so that only a tiny fraction of back current is reflected again by another obstacle. Hence, the obstacles must be separated by distances ex-
ceeding the mean free path of the continuous scattering mechanism. In the limiting case of many obstacles and little background scattering, the build-up of back reflections upon back reflections leads to a similar situation as that described by eq. (1), which will result again in a Drude model with the obstacle scattering dominating the resulting Drude relaxation time. At this point, we should mention that the reflection from the obstacles does not need to be specular or particularly in the backward direction. Any interaction with the obstacle that leads to a change in current \( \partial j/\partial x \) will cause a charge accumulation and, therefore, localization. The model parameter \( C \) does not directly relate to microscopic scattering (backwards or otherwise). Rather, it is the fraction of the flow of microscopically free charges that is damped up by the localized obstacles, causing charge accumulation.

**Distinguishing Drude-Smith from other conductivity models:**

Now that we have derived the Drude-Smith formula, we investigate what makes it empirically successful and unique. We rewrite Smith’s conductivity in the following notation, to show its place in the Drude-Lorentz formalism:

\[
\tilde{\sigma}_{DS}(\omega) = \frac{\sigma_0}{1 - i\omega\tau} \left( 1 - \frac{C}{1 - i\omega\tau} \right) \quad (12)
\]

\[
= \frac{\sigma_0(1 - C)}{1 - i\omega\tau} - i\omega\tau C \sigma_0 \left( \frac{1}{2i\omega\tau} - (\omega\tau)^2 \right) \quad (13)
\]

We remind that the relaxation time \( \tau \) has a positive value, conductivity \( \sigma_0 \) is also a positive parameter and \( C \) is a parameter between 0 and 1. Note that we take \( C \) as positive, which is opposite to the current convention, as the convention is still based on the misidentification of \( C \) with Smith’s \( \beta_1 \). The product version in eq. (12) is close to the original and conventional formulation. The sum version eq. (13) allows more insightful comparisons with other conduction models, since it shows that the Drude-Smith formula is a special case of a Drude-Lorentz conductivity \( \tilde{\sigma}_{DL} \). The general Drude-Lorentz formula for a system with one species of free charge carriers and a single resonance of bound charges (e.g., a phonon mode or an exciton transition) is:

\[
\tilde{\sigma}_{DL}(\omega) = \frac{\sigma_{DC}}{1 - i\omega\tau_D} - i\omega\gamma / \omega^2_0 - (\omega / \omega_0)^2 \quad (14)
\]

We can identify the DC-limit of the conductivity \( \sigma_{DC} \) with \( \sigma_{DC} = \sigma_0(1 - C) \) in the Drude-Smith case. The steady-state limit of the polarization resonance \( P_0 \) is given by the product \( \tau C \sigma_0 \) of all three parameters of the Drude-Smith model. \( C \) is the parameter describing the importance of the bound resonance relative to the free carrier response, distinguishing Drude-Smith from plain Drude. When mapping the Drude-Lorentz perspective onto the Drude-Smith formula, the truly unique feature is the importance and universality of the relaxation time \( \tau \). It is not only equivalent to the Drude relaxation time \( \tau_D \) of the free carrier response, but it also determines both the damping rate \( \gamma = 2/\tau \) and the resonance frequency \( \omega_0 = 1/\tau \) of the Lorentz oscillator. Moreover, as the direct current conductivity \( \sigma_{DC} \) is also proportional to the relaxation time, we conclude that each of the five parameters of the general Drude-Lorentz formalism depends on \( \tau \) in the Drude-Smith case. The universality of the relaxation time sets the Drude-Smith model apart from other Drude-Lorentz type conduction models. The fact that the same relaxation time guides the behavior of both free and bound charges indicates that the charges are the same kind of microscopically free charge carriers, undergoing the same microscopic dissipation (scattering) processes. In conclusion, the parameter \( C \) denotes the fraction of these microscopically free charges that is confined on a larger, mesoscopic scale; \( \tau \) is the microscopic momentum relaxation time that determines free and confined carrier relaxation as well as confined carrier resonance.

Many other models, despite starting from different microscopic hypotheses, arrive at versions of the macroscopic Drude-Lorentz conductivity similar to Drude-Smith. Their adoption is, however, restricted to very specific cases, compared to the apparent universal applicability of the Drude-Smith model. Here, we present some of those models in more detail, illustrating their similarities and points of contrast with the macroscopic Drude-Smith conduction and the challenge to resolve those distinctions.

As first, we consider the case of a localized surface plasmon. This is an example of a resonance of microscopically free charge carriers which are mesoscopically confined in a conductor, surrounded by a dielectric. Nienhuys and Sundström [32] describe the complex conductivity of a surface plasmon localized in a small conductive particle \( \sigma_p(\omega) \) by

\[
\tilde{\sigma}_p(\omega) = \frac{\epsilon_0 \omega_p^2 \tau_D}{1 - i\omega\tau_D \left( \frac{\omega}{\omega_0} \right)^2} \quad (15)
\]

\( \epsilon_0 \omega_p^2 \tau_D \) is the formula for the DC conductivity \( \sigma_{DC} \) of the Drude material of the plasmonic particle. The permittivity of vacuum \( \epsilon_0 \) is a constant, not a parameter. \( \tau_D \) is the Drude relaxation time, \( \omega_p \) the plasma frequency of the conductive material. The particle plasmon resonance frequency \( \omega_P \) is proportional to the plasma frequency by factors depending on the geometry of the particle and the surrounding dielectric, resulting in a resonance frequency that is lower, but of the same order of magnitude as the plasma frequency \( \omega_P \) of the bulk material. The plasma frequency hence influences both the magnitude
and resonance frequency. This is the key conceptual difference to the Drude-Smith model, where the relaxation time plays the most prominent role. The DC-limit of the plasmon conductivity is 0, since all carriers are mesoscopically localized in the particle. Therefore, only if the surface plasmon frequency coincides with half the relaxation rate, the plasmon model becomes equivalent to a limiting case of a Drude-Smith model with $C = 1$. One might consider that a combination of isolated particles and percolation paths leads to an effective medium that is a combination of free Drude conductivity and a plasmonic part. When interrogating the conductivity at frequencies low compared to the relaxation rate and plasma resonance, this combination of plasmons and free carriers will be hard to distinguish from the Drude-Smith model.

We continue with the idea of an effective medium. We can construct a very crude effective circuit for a percolated medium (Fig 2a): a percolation path represented by resistor $R_1$ and a discontinuous path where the conductive parts are summed up by resistor $R_2$ and the gaps by capacitance $C_2$. The total complex conductance $\tilde{\Sigma}_{eff}$ will be

$$\tilde{\Sigma}_{eff}(\omega) = \frac{\Sigma}{1 - i\omega \tau_D} \left(1 - \frac{b}{1 - i\omega \tau_{RC}}\right). \quad (16)$$

$\tau_D$ is the Drude relaxation time of the material in the resistor, $\Sigma$ is the DC conductance of the resistor elements $R_1$ and $R_2$ in parallel, $b$ is $R_1/(R_1 + R_2)$ and the RC-response time of the capacitive branch $\tau_{RC}$ is $R_2/C_2$. The similarities between this and Smiths model are apparent. For frequencies sufficiently lower than $1/(\tau_{RC}\tau_D)$ the two models will again be hard to distinguish. The key difference is the appearance of $\tau_{RC}$, a second time constant unrelated to $\tau_D$.

Another model with two distinct time constants was derived by Cocker et al. [34] for the case of microscopic, Maxwell-Boltzmann distributed charges confined by reflecting walls (Fig 2b)). For totally reflecting walls, Cocker et al. derive a conductivity

$$\tilde{\sigma}_{Co}(\omega) = \frac{n e^2 \tau_1 / m}{1 - i\omega \tau_1} \left(1 - \frac{1}{1 - i\omega \tau_2}\right) \quad (17)$$

with $\tau_1 = \left(\frac{1}{\tau_D} + \frac{2v}{L}\right)^{-1}$ and $\tau_2 = \left(\frac{12v}{L} \sqrt{\frac{\tau_D}{L + 2v\tau_D}}\right)^{-1}$. $L$ is the distance between the reflecting walls, $v$ the average speed of the electrons (the thermal velocity). The formula was derived for a fixed reflectance set to 1; therefore, a distinct $C$ parameter is lacking. Cocker et al.'s more general formula for variable reflectance does contain terms which can be interpreted as the $C$ parameter in the Drude-Smith formula [34]. The macroscopic conductivity is essentially the same as the effective circuit eq. (16), both conductivities contain two different relaxation times $\tau_1$ and $\tau_2$ instead of the one relaxation time of the Drude-Smith model. Obviously, these two-relaxation times models will be hard to distinguish from Drude-Smith when their two relaxation times are similar to each other. Even for a larger difference between the two relaxation times, the difference to the Drude-Smith result will only show up as a slightly wider peak of the two-relaxation time model and can only be re-

![Fig. 2.](image-url)

Fig. 2 a): Equivalent circuit that may serve as a representation of an effective medium, i.e. for a conductor perforated by holes/scratches. The resulting conductivity model appears similar to Smiths model, with the ratio between resistances $R_1$ and $R_2$ replacing Smiths $C$ parameter. However, the decay time of the confined charges is given by the RC-time $\tau_{RC}$ of the capacitive branch, not by the momentum relaxation time $\tau_D$ that determines the inductance of $R_1$. Therefore, two time-constants matter, contrary to the single relaxation time in the Drude-Smith conductivity.

Fig. 2 b): Reproduced following Cocker et al. [34], who consider (Maxwell-Boltzmann distributed) charge carriers in a box with reflecting walls. This microscopic model is also a possible scenario for the localization of carriers in very specific scenarios. However, just as the equivalent circuit, the relaxation time of the confined portion of the carriers is different from that of the free ones; here, it depends on the distance $L$ between reflecting walls. Again, we have two time-constants instead of the single constant seen in the Drude-Smith model.
solved close to the resonance frequency \( \omega_0 = \sqrt{\frac{1}{\tau_1 \tau_2}} \). Similarly, a Drude-Smith response will look very similar to that predicted by the plasmon model above the resonance frequency, even if a sizeable fraction of the flow of charges is not damped. These two cases are displayed in Fig. 3 a). Fig. 3 b) displays a Drude-Lorentz conductivity, which approximates an electron-hole plasma that has partially condensed into excitons. The relation between the free carrier relaxation time \( \tau_D \), the resonance frequency \( \omega_0 \) and the damping rate \( \gamma \) was chosen to approximate photo-excited ZnO at 30 K \( [35] \). For ZnO \( [35] \), up to ca. 1.5 THz \( (\omega \tau_D \approx 0.1) \), the conductivity of excitons and plasma will be hard to distinguish from a Drude-Smith case. Figures 3 a) and b) illustrate that the macroscopic conductivity needs to be measured very precisely in a suitable frequency range to empirically distinguish between Drude-Smith and other models describing resonances of confined charges.

### Conclusion:

The Drude Smith model arises when mesoscopically localized obstacles dam the flow of a current of microscopically free carriers, which undergo continuous dissipation by microscopic scattering events, in analogy to what has been formulated in the Drude model. This means one can draw the following conclusions when identifying Drude-Smith conductivity behavior:

1. Microscopically free carriers exist in the system.
2. At least two different classes of scattering processes exist.
   
   (a) A microscopic class, that can be treated as a spatially homogeneous background.
   
   (b) A mesoscopic class of obstacles localized on the scale of the mean free path due to background scattering.
3. The fraction of charge carriers banked up by the localized scatterers in steady-state is represented by the confinement parameter \( C \).
4. \( \tau \) is the velocity relaxation time due to the microscopic scattering process.
5. \( \sigma_0 \) is the DC-conductivity the system would have if the mesoscopic obstacles were removed.
6. \( \sigma_{DC} = \sigma_0(1 - C) \) is the DC-limit of the conductivity.

Several other models for a combination of bound and free carriers lead to similar conductivities over most of the frequency range. In order to identify the Drude-Smith behavior from experimental conductivity data and justify the full mesoscopic interpretation, a comparative hypotheses test with those models should be performed. Despite being successfully adopted, thanks to its limited amount of parameters and their apparent simple meanings, the Drude-Smith model had to be considered a mere
phenomenological expression up to this point. This work, on the other hand, provides an alternative to the original derivation whose basic premise of preferential backscatter- ing must lead to a simple Drude, not Drude-Smith conductivity. Our results show that the Drude-Smith model should be considered as the time-dependent ver-
sion of Landauer’s idea of localized scatters damming the
flow of microscopically free charge carriers.

Acknowledgements We thank Hai Wang for fruitful discussions.

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[1] N. Smith, Drude theory and the optical properties of li-
quid mercury, Physics Letters A 26, 126 (1968)

[2] N. V. Smith, Classical generalization of the Drude for-
mula for the optical conductivity, Physical Review B - Con-
densed Matter and Materials Physics 64, 155106
(2001)

[3] J. Clerouin, P. Noiret, V. N. Korobenko, and A. D. 
Rakhel, Direct measurements and ab initio simulations for ex-
panded fluid aluminum in the metal-nonmetal transition range. 
Physical Review B - Condensed Matter and Materials Physics 78, 1 (2008)

[4] M. Walther, D. G. Cooke, C. Sherstan, M. Hajar, M. R.
Freeman, and F. A. Hegmann, Terahertz conductivity of thin gold films at the metal-insulator percolation transition. 
Physical Review B 76, 125408 (2007)

[5] A. Thoman, A. Kern, H. Helm, and M. Walther. Nano-
structured gold films as broadband terahertz antireflection coatings, 
Physical Review B 77, 1 (2008).

[6] T. Brandt, M. Hövel, B. Gompf, and M. Dressel, 
Temperature- and frequency-dependent optical proper-
ties of ultrathin Au films, Physical Review B - Con-
densed Matter and Materials Physics 78, 1 (2008).
arXiv:0808.2759

[7] K. Yoshioka, Y. Minami, K. I. Shudo, T. D. Dao, 
T. Nagao, M. Kitajima, J. Takeda, and I. Katayama, 
Terahertz-field-induced nonlinear electron delocalization in Au nanostructures. [Nano Letters 15, 1036 (2015)]

[8] M. C. Beard, G. M. Turner, J. E. Murphy, O. I. Micic, 
M. C. Hanna, A. J. Nozik, and C. A. Schmuttenmaer, 
Electronic Coupling in InP Nanoparticle Arrays, Nano 
Letters 3, 1695 (2003)

[9] D. G. Cooke, A. N. MacDonald, A. Hrycwig, J. Wang, 
Q. Li, A. Meldrum, and F. A. Hegmann, Transient ter-
ahertz conductivity in photoexcited silicon nanocrystal films, Physical Review B - Con-
densed Matter and Materials Physics 73, 101103(PhysRevB.73.193311) (2006).

[10] G. W. Guglietta, B. T. Diroll, E. A. Goulding, J. L. 
Fordham, S. Li, C. B. Murray, and J. B. Baxter, Lifetime, 
mobility, and diffusion of photoexcited carriers in ligand-
exchange lead selenide nanocrystal films measured by time-resolved terahertz spectroscopy, ACS Nano 9, 1820 (2015)

[11] J. B. Baxter and C. A. Schmuttenmaer, Conductivity of 
ZnO nanowires, nanoparticles, and thin films using time-resolved terahertz spectroscopy, Journal of Physical 
Chemistry B 110, 25229 (2006)

[12] S. A. Jensen, R. Ulbricht, A. Narita, X. Feng, K. Müller, 
T. Hertel, D. Turchinovich, and M. Bonn, Ultrafast photono-
t conductivity of graphene nanoribbons and carbon nan-
otubes, Nano Letters 13, 5925 (2013)

[13] D. G. Cooke, A. Meldrum, and P. Jepsen, Ultrabroad-
band terahertz conductivity of Si nanocrystal films, Applied Physics Letters 101, 1 (2012)

[14] L. V. Titova, T. L. Cooker, D. G. Cooke, X. Wang, 
A. Meldrum, and F. A. Hegmann, Ultrafast percolative 
transport dynamics in silicon nanocrystal films, Physical 
Review B - Condensed Matter and Materials Physics 83, 1 (2011)

[15] W. H. Evers, J. M. Schins, M. Aerts, A. Kulkarni, P. Cas-
pio, M. Berthe, B. Granddidier, C. Delerue, H. S. Van 
Der Zant, C. Van Overbeek, J. L. Peters, D. Vannackelbergh, and L. D. Siebbeles, High charge mobility in two-
dimensional percolative networks of PbSe quantum dots connected by atomic bonds, Nature Communications 6, 10.1038/ncomms9195 (2015).

[16] S. Bonetti, M. Hoffmann, M.-J. Sher, Z. Chen, S.-H. Yang, M. Samant, S. Parkin, and H. Dürr, THz-Driven 
Ultrafast Spin-Lattice Scattering in Amorphous Metallic 
Ferromagnets, Physical Review Letters 117, 087205 (2016).

[17] J. D. Buron, F. Pizzocchero, B. S. Jessen, T. J. Booth, 
P. F. Nielsen, O. Hanssen, M. Hille, E. Whitheway, P. U. 
Jepsen, P. Boggild, and D. H. Petersen, Electrically con-
tinuous graphene from single crystal copper verified by terahertz conductance spectroscopy and micro four-point 
probe, Nano Letters 14, 6348 (2014)

[18] H. Némeč, P. Žel, and V. Sundström, Charge transport in nanostructured materials for solar energy conversion studied by time-resolved terahertz spectroscopy, Journal of Photochemistry and Photobiology A: Chemistry 215, 123 (2010)

[19] T.-T. Kang, M. Yamamoto, M. Tanaka, A. Hashimoto, 
A. Yamamoto, R. Sudo, A. Noda, D. W. Liu, and K. Ya-
namoto, Terahertz characterization of semiconductor alloy AlInN: negative imaginary conductivity and its mean-
ing, Optics Letters 34, 2507 (2009).

[20] M. C. Beard, J. L. Blackburn, and M. J. Heben, Photo-
generated Free Carrier Dynamics in Metal and Semicon-
ductor Single-Walled Carbon Nanotube Films, NANO LETTERS 8, 10.1021/nl801913y (2008).

[21] K. Iwamoto, T. Mori, S. Kajitani, H. Matsumoto, 
N. Toyota, K. Suekuni, M. A. Avila, Y. Saiga, and 
T. Takabatake, Optical conductivity spectra of rattling phonons and charge carriers in the type-VIII clathrate 
Ba8Ga16Sn30, Physical Review B - Condensed Matter and Materials Physics 83, 1 (2011)

[22] D. Turchinovich, J. M. Hvam, and M. C. Hoffmann, Self-
phase modulation of a single-cycle terahertz pulse by non-
linear free-carrier response in a semiconductor, Physical 
Review B - Condensed Matter and Materials Physics 85, 1 (2012) arXiv:1202.5153

[23] P. Krauspe, D. Tsokkou, M. Causa, E. Buchaca-
domingo, Z. Fei, M. Heiney, N. Stengelin, and N. Banerji, 
Terahertz short-range mobilities in neat and intermixed 
regions of polymer:fullerene blends with controlled phase morphology, Journal of Materials Chemistry B 6, 22301 (2018).

[24] B. Pattengale, J. Neu, S. Ostresh, G. Hu, J. A. Spies, 
R. Okabe, G. W. Brudev, and C. A. Schmuttenmaer, Metal-Organic Framework Photoconductivity via Time-
Resolved Terahertz Spectroscopy, *Journal of the American Chemical Society* **141**, 9793 (2019)

25] D. G. Cooke, F. C. Krebs, and P. U. Jepsen, Direct observation of sub-100 fs mobile charge generation in a polymer-fullerene film, *Physical Review Letters* **108**, 1 (2012)

[26] Z. Jin, D. Gehrig, C. Dyer-Smith, E. J. Heilweil, F. Laquai, M. Bonn, and D. Turchinovich, Ultrafast terahertz photoconductivity of photovoltaic polymer-fullerene blends: A comparative study correlated with photovoltaic device performance, *Journal of Physical Chemistry Letters* **5**, 3662 (2014)

[27] P. D. Cunningham and L. Michael Hayden, Carrier dynamics resulting from above and below gap excitation of P3HT and P3HT/PCBM investigated by optical-pump terahertz-probe spectroscopy, *Journal of Physical Chemistry C* **112**, 7928 (2008)

[28] X. Ai, M. C. Beard, K. P. Knutsen, S. E. Sheheen, G. Rumbles, and R. J. Ellingson, Photoinduced charge carrier generation in a poly(3-hexylthiophene) and methanofullerene bulk heterojunction investigated by time-resolved terahertz spectroscopy, *Journal of Physical Chemistry B* **110**, 25462 (2006)

[29] F. W. Han, W. Xu, L. L. Li, and C. Zhang, A generalization of the Drude-Smith formula for magneto-optical conductivities in Faraday geometry, *Journal of Applied Physics* **119**, 245706 (2016)

[30] D. Tsokkou, A. Othonos, and M. Zervos, Carrier dynamics and conductivity of SnO 2 nanowires investigated by time-resolved terahertz spectroscopy, *Applied Physics Letters* **100**, 133101 (2012)

[31] R. Landauer, Spatial Variation of Currents and Fields Due to Localized Scatterers in Metallic Conduction, *IBM Journal of Research and Development* **1**, 223 (1957)

[32] H. K. Nienhuys and V. Sundström, Influence of plasmons on terahertz conductivity measurements, *Applied Physics Letters* **87**, 10.1063/1.1977213 (2005)

[33] X. Zou, J. Luo, D. Lee, C. Cheng, D. Springer, S. K. Nair, S. A. Cheong, H. J. Fan, and E. E. Chia, Temperature-dependent terahertz conductivity of tin oxide nanowire films, *Journal of Physics D: Applied Physics* **45**, 465101 (2012)

[34] T. L. Cocker, D. Baillie, M. Buruma, L. V. Titova, R. D. Sydora, F. Marsiglio, and F. A. Hegmann, Microscopic origin of the Drude-Smith model, *PHYSICAL REVIEW B* **96**, 205439 (2017)

[35] E. Hendry, M. Koeberg, and M. Bonn, Exciton and electron-hole plasma formation dynamics in ZnO, *Physical Review B - Condensed Matter and Materials Physics* **76**, 1 (2007)