Band ferromagnetism versus collective Kondo state
in lattice fermion models*

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ABSTRACT

It is becoming widely recognized that, contrary to earlier expectations, the usual one-band Hubbard model does not give an explanation for itinerant ferromagnetism. After reviewing the status of magnetic ordering in the one-band model, we discuss the possibility of ferromagnetism in some recently introduced two-band Hubbard models, and in generalized Anderson lattices. It is argued that these two classes of models are closely related and that it is their common feature that the ferromagnetic phase has to compete with a collective Kondo state.

§ 1. INTRODUCTION: MAGNETISM IN THE HUBBARD MODEL

Over three decades of intense research effort have been spent on mapping out the phase diagram of the single-orbital Hubbard model (Hubbard 1963)

\[ \mathcal{H} = -t \sum_{\langle i,j \rangle} \sum_{\sigma} (c_{i\sigma}^\dagger c_{j\sigma} + \text{H.c.}) + U n_{i\uparrow} n_{i\downarrow} \, . \]  

It has been usual to treat the case when the summation over \( i, j \) extends over the sites of a Bravais lattice (typically, the \( d \)-dimensional cubic lattice); then a single atomic orbital gives rise to a single tight binding band. In such cases, the expressions “single-orbital”, and “single-band” can be used synonymously. Recent work on carefully constructed non-Bravais lattices made us aware of the potential importance of single-orbital multi-band Hubbard models. Degenerate or multi-orbital Hubbard models and their extended versions are introduced by a completely different physical motivation, but they can be made formally equivalent to single-orbital models with a larger number of sites per unit cell.

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The extensive work on the simplest one-band case was done in the expectation that most of the major correlation phenomena are represented in its phase diagram, and are thus generic features of the Hubbard model, without the need for fine-tuning the parameters. Optimistically, one might have hoped to find Mott transition, ferro- and antiferromagnetism, incommensurate magnetic structures, charge and spin gaps, heavy fermions, and maybe even an electronic mechanism of (high-temperature) superconductivity. All seemed to go well when early Hartree–Fock calculations (Penn 1966) readily gave extensive domains of both ferro- and antiferromagnetism in the \( n-U/t \) plane (\( n = N/L \) is the band filling, \( L \) is the number of lattice sites, and \( N \) the number of electrons). It was taken for granted that allowing more complicated ordering patterns, richer Hartree–Fock phase diagrams can be found. But at least, there seemed to be no reason to doubt that a basic understanding of the most important kinds of magnetic ordering has been reached. One might have argued (all too naively, as it turns out) that for three-dimensional systems, fluctuation effects play a relatively minor role: they might shift the phase boundaries but would not alter the overall appearance of the phase diagram.

In recent years, this view has undergone substantial changes. Now we would rather say that while the single-band Hubbard model provides a solid basis for understanding antiferromagnetic Mott insulators, and probably also spin density wave states, the seemingly simplest kind of magnetic order: ferromagnetism, is not a generic Hubbard phenomenon. It is acknowledged that Nagaoka’s Theorem (Nagaoka, 1966) guarantees the existence of a fully polarized ground state in a singular limiting case: at \( U = \infty \), with one electron more (or less, depending on the sign of \( t \)) than what corresponds to exact half-filling. However, there seems to be no general argument which would permit the continuation of the Nagaoka state into an extended ferromagnetic region of the phase diagram (Takahashi 1982, Sütő 1991). The recently developed detailed understanding of the infinite-dimensional \((d = \infty)\) Hubbard model also speaks against the possibility of ferromagnetism. A perturbation expansion of the Landau Fermi-liquid parameters indicates that the tendency to a ferromagnetic instability is strongly suppressed (Müller–Hartmann 1989). A recent version of the ground state phase diagram shows an extended antiferromagnetic regime straddling the \( n = 1 \) line, flanked by SDW domains but no ferromagnetism (Freericks and Jarrell 1995). We have no proof which would exclude states with a finite polarization (apart from the Nagaoka limit) in the square lattice or simple cubic Hubbard models, but the region where high-spin states may still exist, keeps on shrinking (Wurth, Uhrig and Müller–Hartmann 1996), and it has become imaginable that these bipartite lattice models are never ferromagnetic at all.

The (possible) elimination of one of the major Hartree–Fock phases means that the Hubbard model is much more fluctuating than it was origi-
inally assumed. The essential reason is the purely on-site nature of the interaction: the Hubbard term “does not feel” the change of the space dimensionality \(d\). (In contrast, mean field theory would become exact at \(d \to \infty\) for any additional intersite interaction term.) Fluctuations tend to stabilize the paramagnetic phase. It is interesting to note that a variational theory which enforces local correlations by Gutzwiller-projecting the optimized Hartree–Fock states, still finds an extended regime of (unsaturated) ferromagnetism (Fazekas, Menge and Müller–Hartmann 1990). Since the evaluation is exact in \(d = \infty\), this finding points to the importance of correlations which are not described by the Gutzwiller Ansatz.

Regarding itinerant ferromagnetism as a major unsolved problem may seem surprising since it is well-known that the magnetic and structural properties of the iron-group elements, and their alloys, are described in impressive quantitative detail by density functional theories (Andersen, Madsen, Poulsen, Jepsen and Kollár 1977, Györfy, Kollár, Pindor, Stocks, Staunton and Winter 1984). The difficulty arises solely in the theory of lattice fermion models. In lattice fermion theories, one tries to understand ordering phenomena by studying models with a small local basis, and a restricted set of coupling constants (in the Hubbard model, we have just a single orbital, and one coupling constant). In contrast, for density functional theories the fundamental quantity is the electron density, and orbitals play a purely auxiliary role. One needs (in principle, infinitely) many local basis functions to describe the spatial variation of the electron density. The interaction is the true Coulomb interaction, with all sorts of inter-orbital matrix elements. If we wanted to translate a density functional calculation into a lattice fermion model reasonably faithfully, we would need a multi-band model with a large number of coupling constants. Even if such a model could be solved, and shown to be ferromagnetic, we might still complain about our lack of insight into the basic mechanism of ordering. Thus what we would really like is finding the “minimal” lattice model of ferromagnetism (presumably an extension of the Hubbard model) with as few parameters as possible. Ferromagnetism should be a robust phenomenon, appearing at intermediate coupling strengths, and extending over a substantial range of band filling. The question is which is the simplest, and at the same time physically relevant, modification of the usual single-band Hubbard model, which would give us such results. Ideally, we would like a model which explains ferromagnetism at about the same level of ease, clarity, and also confidence, as the simple Hubbard model does for antiferromagnetism. The exploration of several different routes is in progress, and apparently, there are a number of ways to get ferromagnetism. We do not know yet whether they are all realized in different systems, or we will eventually settle for a single relevant mechanism.

The major candidates are the extended Hubbard model (Strack and Vollhardt 1995, Kollar, Strack and Vollhardt 1996), carefully constructed
§ 2. FERROMAGNETISM IN TWO-BAND HUBBARD MODELS

One of the popular ways to get ferromagnetism in the Hubbard model is to consider lattices with triangular plaquettes. To understand why this idea works, let us begin with Tasaki’s (1996) toy model of ferromagnetism: a single triangle. Let the sites 1 and 2 at the base of the triangle be connected with the hopping matrix element $-t$, while site 3 at the top vertex of the triangle is connected with $-v$ to sites 1 and 2. Furthermore, we ascribe the site energy $\epsilon_3$ to the top site but not to sites 1 and 2. The Hubbard $U$ may also be chosen differently for site 3. The three-site Hubbard model is

$$
\mathcal{H}_3 = -t \sum_\sigma (c_{1\sigma}^\dagger c_{2\sigma} + c_{2\sigma}^\dagger c_{1\sigma}) + \epsilon_3 \sum_\sigma \hat{n}_{3\sigma} + U_3 \hat{n}_{3\uparrow} \hat{n}_{3\downarrow} \quad (2)
$$

$$
- v \sum_\sigma (c_{1\sigma}^\dagger c_{3\sigma} + c_{3\sigma}^\dagger c_{1\sigma} + c_{2\sigma}^\dagger c_{3\sigma} + c_{3\sigma}^\dagger c_{2\sigma}) + U \sum_{j=1}^2 \hat{n}_{j\uparrow} \hat{n}_{j\downarrow} .
$$

Following Penc et al. (1996), we consider the strong-coupling regime $U, U_3 \gg \epsilon_3 \gg |t|, |v|$, and take $N = 2$ electrons. Then the low-energy configurations have one electron at each of the base sites, and the effective Hamiltonian can be expressed by the spins $\mathbf{S}_1$ and $\mathbf{S}_2$. To get the exchange coupling, let us start from $| \uparrow \downarrow 0 \rangle = c_{1\uparrow}^\dagger c_{2\downarrow}^\dagger |0\rangle$. Then the sequence of three hopping events: $| \uparrow \downarrow 0 \rangle \to |0 \downarrow \uparrow \rangle \to | \downarrow \downarrow 0 \uparrow \rangle \to | \downarrow \uparrow 0 \rangle$ effects the exchange of the two spins. The matrix elements are $-v$, $-v$, and $-t$, and the two intermediate states have the excitation energy $\epsilon_3$. The effective Hamiltonian describes these third-order processes

$$
\mathcal{H}_{\text{eff}}^{(3)} = - \frac{4tv^2}{\epsilon_3^2} \mathbf{S}_1 \cdot \mathbf{S}_2 = J_3 \mathbf{S}_1 \cdot \mathbf{S}_2 . \quad (3)
$$

Obviously, the sign of $t$ controls the sign of $J_3$. In particular, the coupling is ferromagnetic if $t > 0$.

If $U$ and $U_3$ are not kept so artificially big, there are also contributions from the familiar second-order processes

$$
\mathcal{H}_{\text{eff}} = \mathcal{H}_{\text{eff}}^{(2)} + \mathcal{H}_{\text{eff}}^{(3)} = \left( \frac{4t^2}{U} - \frac{4tv^2}{\epsilon_3^2} \right) \mathbf{S}_1 \cdot \mathbf{S}_2 . \quad (4)
$$

The overall sign of the exchange coupling is decided by the competition between antiferromagnetic, and ferromagnetic, pieces of the kinetic exchange.
The toy model can be extended into a quasi-one-dimensional lattice model: the triangle-ladder (or railroad trestle) lattice which consists of the parallel chains of bottom sites, and top sites. The unit cell contains one bottom site, and one top site. Henceforth, let us denote bottom site operators by $c$, and top site operators by $d$. Our previous Hamiltonian $H_3$ can be generalized to

$$
H = -\sum_{j,\sigma} \left[ t(c_{j,\sigma}^\dagger c_{j+1,\sigma} + c_{j+1,\sigma}^\dagger c_{j,\sigma}) + t'(d_{j,\sigma}^\dagger d_{j+1,\sigma} + d_{j+1,\sigma}^\dagger d_{j,\sigma}) \\
+ v(c_{j,\sigma} d_{j,\sigma} + d_{j,\sigma}^\dagger c_{j,\sigma}) + v'(d_{j,\sigma}^\dagger c_{j+1,\sigma} + c_{j+1,\sigma} d_{j,\sigma}) \right] \\
+ \epsilon \sum_{j,\sigma} d_{j,\sigma}^\dagger d_{j,\sigma} + U_c \sum_j \hat{n}_{j,\uparrow} \hat{n}_{j,\downarrow} + U_d \sum_j \hat{n}_{j,\uparrow} \hat{n}_{j,\downarrow} ,
$$

where $\hat{n}_{j,\sigma} = d_{j,\sigma}^\dagger d_{j,\sigma}$, etc. $t'$ is the hopping connecting nearest-neighbour top sites, and we distinguish $v$ acting within a lattice cell from the inter-cell $v'$. Assuming again $U_c, U_d \gg \epsilon \gg$ all hopping amplitudes, and exact quarter filling so that the bottom chain can be filled with one electron per site, a systematic perturbation expansion (Penc et al. 1996) confirms that the effective spin-spin coupling is very similar to the toy model result (4)

$$
H_{\text{eff}} = \sum_j \left( \frac{4t^2}{U_c} - \frac{4tvv'}{\epsilon^2} \right) \mathbf{S}_j \cdot \mathbf{S}_{j+1}
$$

where the spins are sitting on the bottom chain. We get ferromagnetism if $U_c$ is large enough. Strictly at quarter-filling, the fully polarized system is an insulator because of a large single-particle gap, but it can be shown (Penc et al 1996) that the coupling remains ferromagnetic even if we move away from quarter-filling.

The plaquette mechanism gives a very transparent reason for finding ferromagnetism in insulating and metallic cases alike, but at a price: $U$ has to be so large that the leading (second-order) kinetic exchange is suppressed and the usually negligible higher-order processes become dominating. Besides, the presence of frustration has to be assumed in the sense that the plaquette product $\prod t_{ij}$ must have a definite sign. Finally, the possibility of the perturbation expansion relies on the seemingly ad hoc assumption of a large $\epsilon$. But at least, it is a hopeful feature that the ferromagnetic order is understood to be stable in a finite domain of the parameter space, not only in singular limiting cases. Our basic hope is that ferromagnetism extends beyond the range of validity of the perturbational argument, and then perhaps the entire phase can be understood by continuation from the strong-coupling regime where we have a well-founded argument. There are reasons to think that indeed such is the case. Takahashi’s (1982) numerical work on small clusters used $\epsilon = 0$; nevertheless, it indicated that three-site
exchange in triangular loops is instrumental in stabilizing strong ferromagnetism. Tasaki (1995) found that the ground state of the quarter-filled system is a fully saturated ferromagnet if the parameters of the Hamiltonian (5) satisfy a certain relationship. His argument requires that $U$ be large but it does not require a large $\epsilon$. Penc et al. (1996) study the model by a variety of techniques and find an extensive ferromagnetic phase which is continuously connected to the strong-coupling regime. Similar results were found for a related family of models by Sakamoto and Kubo (1996). Further evidence comes from our studies described below.

Starting an independent line of investigation, Müller–Hartmann (1995) studied a highly symmetrical version of (5), namely, the Hubbard chain with nearest- and next-nearest-neighbour hopping

$$\mathcal{H} = -\sum_{j,\sigma} \left[ t_1 (c_{j+1,\sigma}^\dagger c_{j,\sigma} + \text{H.c.}) + t_2 (c_{j+2,\sigma}^\dagger c_{j,\sigma} + \text{H.c.}) \right] + U \sum_j \hat{n}_{j\uparrow} \hat{n}_{j\downarrow}. \tag{7}$$

Gauge symmetry allows to fix the sign of $t_1 > 0$. For $t_2 > 0$, the model is not ferromagnetic, in the sense familiar from the usual one-dimensional (1-dim) Hubbard model. For $t_2 < 0$ there is a Nagaoka state (Mattis and Peña 1974) and the question arises whether this is just an isolated point, or it belongs to a finite domain of ferromagnetic states.

Noticing that the band structure develops two degenerate minima if $t_2/t_1 < -1/4$, Müller–Hartmann (1995) suggested that in this case, there may be a “low density route” to ferromagnetism. Let us start with the observation that for $N = 2$ electrons, the singlet and triplet states are degenerate if $U = 0$, and the ground state is certainly a triplet for $U > 0$. We may be wondering whether the high-spin state survives if we increase the number of electrons. Müller–Hartmann argued that this is the case, at least when $U \to \infty$, and the density is vanishingly small.

We have undertaken a detailed investigation of the extent of the ferromagnetically ordered state of the $t_1$--$t_2$ Hubbard chain in its parameter space (Pieri, Daul, Baeriswyl, Dzierzawa and Fazekas 1996; Daul, Pieri, Dzierzawa, Baeriswyl and Fazekas 1996). We started with exact diagonalization studies of small chains (with $L \leq 18$ sites, $L$ and $N$ even, periodic boundary conditions). We found that the ground state is either a singlet, or fully polarized. In particular, we found that the system has a high-spin ground state for large enough $U$ if two conditions are satisfied: 1) $t_2/t_1 < -1/4$, i.e., there are two degenerate band minima, and 2) $N$ is small enough ($N < N_{cr}(t_2/t_1)$) so that the fully polarized Fermi sea consists of two disjoint pieces. Our data show that the ferromagnetic phase is quite extended, ranging from low to intermediate densities, and appearing at intermediate coupling strengths (Pieri at al. 1996). This gives evidence that it is possible to get robust itinerant ferromagnetism in a single-orbital Hubbard model.

There remains, however, the unsettled question of the relationship of
the intermediate-density ferromagnetism to the Nagaoka state. Under the stated conditions, we did not find high-spin states for \( N \) between \( N_{cr} \) and the Nagaoka value \( N = L - 1 \). Though these (purely numerical) findings seem to be in nice accord with the suggestion of a hypothetical “valley-degeneracy-assisted” ferromagnetism, we feel somewhat uneasy about their interpretation. It should be emphasized that, taken in themselves, the numerical studies are as yet inconclusive. It is thus disturbing that we have no theoretical argument to show that the Nagaoka state is disconnected from the low-to-intermediate-density ferromagnetic region; rather on the contrary. Relying on the experience that the ferromagnetic state, whenever it exists, is fully saturated, we studied its stability by the single-spin-flip variational Ansatz introduced by Shastry, Krishnamurty and Anderson (1990). The resulting phase boundary shows a sharp cusp at the critical density \( n_{cr} \) where the two Fermi lakes merge into a single Fermi sea, but the critical \( U_{cr} \) keeps on rising continuously as \( n \) is increased from \( n_{cr} \) towards 1. Similar result is found with a more sophisticated trial state (Daul et al. 1996). Thus it would seem that though the critical density is likely to play a distinguished role in the stability criterion for ferromagnetism, it is not a boundary value, and the large-\( U \) behaviour is ferromagnetic for all \( 0 < n < 1 \). Further work is needed to dispel the remaining doubts.

Whatever its exact extent, the 1-dim \( t_1-t_2 \) Hubbard model does have a ferromagnetic phase which can be understood by continuation from the low-density limit. What about higher dimensions? It would have been nice if postulating the existence of degenerate band minima had opened the way to understanding the ferromagnetism of 3-dim metals by a similar scenario. Alas, we found that this possibility is ruled out (Pieri et al. 1996), reconfirming earlier results of a similar nature (Kanamori 1963, Caron and Kemeny 1971). 2-dim remains a borderline case where ferromagnetism can arise if the density of states at the bottom of the band is high enough but there is no clear-cut connection with the presence of degenerate minima.

§ 3. FERROMAGNETISM IN ANDERSON LATTICES

It is interesting to reinterpret the Hamiltonian (5) by postulating that \( c \) and \( d \), instead of referring to different sites in the unit cell, denote different orbitals at the same site. Let us assume that \( U_d \) is much bigger than \( U_c \); then (5) describes the hybridization of a strongly correlated band with a relatively weakly correlated band: it is an extended version of the well-known periodic Anderson model (PAM) which is often used to describe \( f \)-electron based heavy fermion systems (Fulde, Keller and Zwicknagl 1990). Compared to the standard PAM, the new features are: a finite inherent bandwidth for both bands; a non-zero Hubbard \( U \) for both orbitals; and the simultaneous presence of on-site \((-v)\) and inter-site \((-v')\) hybridization matrix elements.
We are thus led to posing the question about ferromagnetism in Anderson lattice models, and then also in the related Kondo lattice models (KLM). In any case, the perturbation theory argument leading to (6) is still in force, yielding the ferromagnetic exchange term $\sim -4tvv'/\epsilon^2$. But we have plenty of indications that the periodic Anderson model can be ferromagnetic even with purely on-site hybridization (i.e., with $v' = 0$). It has long been known that Gutzwiller-type treatments readily give a ferromagnetic instability (Fazekas and Brandow 1987; Reynolds, Edwards and Hewson 1992) but taken in themselves, these results are probably hardly less suspect than the Hartree–Fock instabilities. There is, however, supportive evidence from other approaches. A slave boson treatment (Möller and Wölfle 1993) finds a ferromagnetic phase above quarter-filling, in roughly the same regime where high-spin ground states are reported on the basis of a numerical study of the 1-dim PAM (Guerrero and Noack 1996). Turning to the KLM: a variety of techniques gave the rather astonishing result that the phase diagram of the 1-dim Kondo chain is largely covered by a ferromagnetic phase (Sigrist, Tsunetsugu, Ueda and Rice 1992; Moukouri and Caron 1995). We should note, however, that most of this is in the intermediate-to-strong-coupling regime where the KLM is not equivalent to an underlying PAM, so the relationship to the previously cited Anderson lattice results is not straightforward. The peculiarities of the 1-dim case make it difficult to analyse the proverbial competition between RKKY magnetism and Kondo states (Doniach 1977). In higher dimensions, the weak-coupling regime is presumably reserved for RKKY phases (including an RKKY ferromagnet at low band fillings), while at stronger couplings, an itinerant ferromagnet competes with a heavy electron liquid (Fazekas and Müller–Hartmann 1991). In any case, it is an irony of fate that the Hubbard model which was introduced to explain the properties of the iron group elements, is so reluctant to become ferromagnetic, while the Anderson and Kondo lattice models which were introduced to describe the often non-magnetic concentrated Kondo systems, have a hard time to avoid ferromagnetism.

We suggest that one should not hesitate to regard this observation as a clue, and think it over whether two-band models related to the Anderson and Kondo lattices are not the natural candidates to describe itinerant ferromagnetism. We have already indicated that there is a formal relationship between the PAM, and the Tasaki–Müller-Hartmann kind of two-band Hubbard models. However, the usual PAM discards $v'$ and with that, the plaquette exchange term in (6), which we identified as the essential reason for ferromagnetism in § 2. Still, the PAM seems to have a strong tendency to order ferromagnetically, for an as yet unclarified reason. One may suspect that this is reinforced by switching on $v'$, and this leads to the robust order of the two-band model (5).

It is well-established that the collective singlet (Kondo) state (which, at exact half-filling, becomes the Kondo insulator) is one of the possible ground
states of the PAM and the KLM. This state is characterized by a large mass enhancement, and a Luttinger Fermi surface (Shiba and Fazekas 1990). The extension of the PAM by the new terms present in (5) necessitates a re-investigation of the Kondo state. It is quite possible that a line of first-order phase transitions separates the fully developed ferromagnetic order from the heavy Fermi liquid. A good estimate of the ground state energy of the latter is a prerequisite for locating the phase boundary.

In a preliminary investigation (Itai and Fazekas 1996) we studied the effect of switching on a Hubbard $U$ for the conduction band on the Kondo energy. We used an extension of the well-known Gutzwiller method (Fazekas and Brandow 1987) to describe the non-magnetic ground state of the Hamiltonian (the notations now follow those customary for the PAM: the strongly correlated electrons are $f$-electrons, and they hybridize with a $d$-band)

$$
\mathcal{H} = \sum_{k,\sigma} \epsilon_d(k)d_{k\sigma}^\dagger d_{k\sigma} + \epsilon_f \sum_{j,\sigma} \hat{n}_j^f + U_f \sum_j \hat{n}_j^f \hat{n}_j^\dagger^f \\
+U_d \sum_j \hat{n}_j^d \hat{n}_j^d - v \sum_{j,\sigma} (f_{j\sigma}^\dagger f_{j\sigma} + d_{j\sigma}^\dagger d_{j\sigma})
$$

(8)

where the $k$ are wave vectors, and the $j$ are site indices. The $d$-bandwidth is $W$. We considered the strongly asymmetric Anderson model with $U_f \to \infty$ and the $f$-level $\epsilon_f < 0$ sufficiently deep-lying so that we are in the Kondo limit: $1 - n_f \ll 1$ where the $f$-valence is defined as $n_f = \langle \sum_\sigma \hat{n}_j^f \rangle$.

As a starting point, let us recall that for the ordinary PAM, the Kondo energy density is given by (Fazekas and Brandow 1987)

$$
E_K(0) = -W n_0^d \cdot \exp \left\{ -\frac{\mu_0(0) - \epsilon_f}{4(v^2/W)} \right\}
$$

(9)

where the subscripts 0 refer to the $v = 0$ values of the corresponding quantities.

Our essential new result is that the extended PAM (8) has the modified Kondo scale (Itai and Fazekas 1996)

$$
E_K(U_d) = -W n_0^d q_0^d \cdot \exp \left\{ -\frac{\mu_0(U_d) - \epsilon_f}{4(v^2/W)} \right\}
$$

(10)

Like (9), this holds in the weak-hybridization limit $v \ll W$. $q_0^d$ is the Gutzwiller–Brinkman–Rice (Gutzwiller 1965; Brinkman and Rice 1970) band narrowing factor, and $\mu_0(U_d)$ is the interaction-dependent chemical potential of the conduction band. We see that switching on $U_d$ changes the Kondo scale in two different ways: in the prefactor, and in the exponent. The prefactor describes the correlation-induced narrowing of the $d$-band. The exponent reflects that the promotion energy needed to raise an electron from the $f$-level to the chemical potential depends on $U_d$. 

9
The overall effect of switching on the conduction electron $U_d$ is the suppression of the lattice Kondo scale, i.e., of the binding energy of the singlet ground state. This can be qualitatively understood by an argument which is familiar from the Brinkman–Rice scenario (Brinkman and Rice 1970) of the Mott transition: $U_d$ suppresses polarity fluctuations in the $d$-orbitals, and thereby blocks the hybridization processes which, in second order, give rise to the effective Kondo coupling $J_K \sim 4\nu^2/(\mu_0(U_d) - \epsilon_f)$. Though in the usual Gutzwillerian framework this is the expected result, it should be mentioned that the conclusion about the monotonically decreasing nature of $|E_K(U_d)|$ is not undisputed. In a different interpretation, (8) is the lattice generalization of the Hamiltonian of an Anderson impurity in a Hubbard band; the study of the latter problem was initiated by Schork and Fulde (1994). While our result (10) is obtained from a variational trial state which can be regarded as the lattice version of the lowest-order Varma–Yafet state (Varma and Yafet 1976), the relative simplicity of the impurity problem allowed pushing forward to higher levels of the Varma–Yafet hierarchy (Schork 1996). It was found that the inclusion of higher-order processes can reverse the trend, and give an $|E_K(U_d)|$ which increases with $U_d$. However, relying on previous experience with the simple PAM, we were arguing (Fazekas and Itai 1996) that in the lattice case, postulating a Luttinger Fermi surface implies that electron–hole-type excitations have been largely taken into account, and the conclusion reached by using the simplest trial state is likely to remain essentially unaltered.

Having found that the Anderson–Hubbard (AH) lattice model (8) has interesting features, we are led to considering the similarly constructed Kondo–Hubbard (KH) lattice models. For a start, it is heartening to learn that they are ferromagnetic in a substantial range of the conduction band $U_d$ (Yanagisawa and Harigaya 1994). What about the Kondo scale? First, let us note that, in contrast to their impurity counterparts, AH and KH lattice models are not simply related; they represent physically different systems. In a KH model, the Kondo coupling between the localized ($f$) and conduction electron ($d$) spins is a fixed quantity, and switching on $U_d$ primarily leads to a more spin-polarizable electron gas, with which it is easier to bind into an overall singlet. We find it a plausible result that the lattice Kondo scale increases with $U_d$ (Shibata, Nishino, Ueda and Ishii 1996).

Obviously, there is still a lot to do to clarify the relative stability of the ferromagnetic and Kondo states. There is a good chance that in the Anderson–Hubbard lattice model, the Kondo scale is reduced by the conduction electron interaction, leading to an increased domain of ferromagnetism. This should hold even more clearly for an extended model which includes the inherent $f$-bandwidth, and maybe intersite hybridization terms, so that ferromagnetic plaquette exchange can appear. Investigating these effects may well contribute to the explanation of the apparently robust ferromagnetism observed in some two-band Hubbard models.
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