Breaking the Dark Degeneracy with the Drifting Coefficient of the Field Cluster Mass Function

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Abstract

We present a numerical analysis supporting the evidence that the redshift evolution of the drifting coefficient of the field cluster mass function is capable of breaking several cosmic degeneracies. This evidence is based on the data from the CoDECs and DUSTGRAIN-pathfinder simulations performed separately for various nonstandard cosmologies including coupled dark energy, $f(R)$ gravity, and combinations of $f(R)$ gravity with massive neutrinos as well as for the standard $\Lambda$ cold dark matter ($\Lambda$CDM) cosmology. We first numerically determine the field cluster mass functions at various redshifts in the range of $0 < z < 1$ for each cosmology. Then, we compare the analytic formula developed in previous works with the numerically obtained field cluster mass functions by adjusting its drifting coefficient, $\beta$, at each redshift. It is found that the analytic formula with the best-fit coefficient provides a good match to the numerical results at all redshifts for all of the cosmologies. The empirically determined redshift evolution of the drifting coefficient, $\beta(z)$, turns out to significantly differ among different cosmologies. It is also shown that even without using any prior information on the background cosmology the drifting coefficient, $\beta(z)$, can discriminate with high statistical significance the degenerate nonstandard cosmologies not only from the $\Lambda$CDM but also from one another. It is concluded that the evolution of the departure from the Einstein–de Sitter state and spherically symmetric collapse processes quantified by $\beta(z)$ is a powerful probe of gravity and dark sector physics.

Unified Astronomy Thesaurus concepts: Large-scale structure of the universe (902); Cosmological models (337)

1. Introduction

A cosmic degeneracy refers to the circumstance that a standard diagnostic fails to distinguish between different cosmologies with high statistical significance. For example, the cluster mass function, which is regarded as one of the most powerful probes of cosmology based on the large-scale structure, is unable to discriminate the effect of a low amplitude of the linear density power spectrum from that of massive neutrinos ($\nu$) (dubbed the $\sigma_8$—$\sum m_\nu$ degeneracy) in a massive neutrinos $\nu +$ cosmological constant $\Lambda +$ cold dark matter ($\nu$CDM) cosmology. Another example that the cluster mass function fails to discriminate from the $\Lambda$CDM cosmology is a coupled dark energy (cDE) model in which a scalar field DE coupled to DM particles follows a supergravity potential (Baldi et al. 2010). Since the cosmic degeneracy is caused by the limited sensitivity of a given standard diagnostic on which the degenerate models have almost the same effects, what is required to break it is to overcome the limitation by utilizing prior information from other independent diagnostics. In the aforementioned example, the $\sigma_8$—$\sum m_\nu$ degeneracy can be broken by prior information on the large-scale amplitude of the linear density power spectrum from the cosmic microwave background observations.

There are, however, a few cosmic degeneracies that have been found to be more difficult to break even by combining the priors from several independent diagnostics. A notorious example is the cosmic degeneracy between the $\Lambda$CDM + GR and the $\nu$CDM + MG cosmologies, where GR and MG stand for the general relativity and modified gravity, respectively (see, e.g., Baldi et al. 2014; Wright et al. 2019). All different versions of the MG theory adopt a common tenet that the apparent acceleration of the present universe is caused not by the dominance of the antigravitational $\Lambda$ at the present epoch but by the deviation of the gravitational law from the prediction of GR on cosmological scales. The consequence of this tenet is the existence of a long-range force, which in turn has an effect of enhancing the density power spectrum on the scales comparable to those affected by the suppression due to free streaming massive neutrinos (for a review, see Clifton et al. 2012).

In the theory of $f(R)$ gravity, the gravitational dynamics is defined by a modified Einstein–Hilbert action functional to which an arbitrary function of the Ricci scalar, $f(R)$, is introduced as a substitution for the Ricci scalar $R$ itself of the original action in GR (see, e.g., Buchdahl 1970; Starobinsky 1980; Hu & Sawicki 2007). Choosing as a viable MG the $f(R)$ gravity in which the Ricci scalar term, $R$, in the Einstein–Hilbert action functional is replaced by an arbitrary function, $f(R)$, Baldi et al. (2014) numerically investigated a possible cosmic degeneracy between the $\Lambda$CDM + GR and the $\nu$CDM + $f(R)$, and demonstrated that the two cosmologies cannot be discriminated from each other by several standard diagnostics such as the nonlinear density power spectra, halo bias, and cluster mass functions (see also García-Farieta et al. 2019; Hagstotz et al. 2019). The nonlinear growth rate functions, cluster velocity dispersions, and tomographic higher-order weak lensing statistics were proposed in subsequent works as candidate diagnostics that could be capable of breaking this cosmic degeneracy (Giocoli et al. 2018; Peel et al. 2018b; Hagstotz et al. 2019), also employing machine learning techniques (Peel et al. 2018a; Merten et al. 2019).
Very recently, Ryu & Lee (2020a) developed a new independent diagnostic based on the evolution of the drifting coefficient of the field cluster abundance and showed that this new diagnostic is capable of distinguishing between those dynamical DE cosmologies that are degenerate with the ΛCDM case in their linear density power spectra and cluster mass functions. In the follow-up work of Ryu & Lee (2020b), it was also found that the aforementioned σ_{bb}−ς_{M} degeneracy can be in principle broken by this new diagnostic. Our goal here is to explore whether or not this new diagnostic can break other cosmic degeneracies including that between the ΛCDM + GR and the ρCDM + MG models.

The key content of the upcoming sections are as follows. Section 2 will present a succinct review of the analytic model for the field cluster abundance on which the new diagnostic is based. Section 3.1 will present numerical evidence that the new diagnostic can distinguish between the standard ΛCDM and a DE cosmology that produce very similar cluster mass functions. Section 3.2 will present proof for the validity of the analytic formulae for the field mass function and drifting coefficient in an MG gravity cosmology. Section 3.3 will present numerical evidence that the new diagnostic can break the degeneracy between the standard ΛCDM+GR and the DE cosmologies. Section 4 will be devoted to summarizing the final results and discussing the follow-up work.

2. A Succinct Review of the Analytic Model

According to the generalized excursion set theory that incorporates the nonspherical collapse conditions and non-Markovian random-walk process (Corasaniti & Achitouv 2011a, 2011b), the differential mass function of the cluster halos can be written as

\[
\frac{dN(M, z)}{dM} = \frac{\bar{\rho}_b}{M} \left| \frac{d}{dM} \ln \sigma^{-1} \right| f[\sigma(M, z)],
\]

where \(\bar{\rho}_b\) is the mean mass density of the universe, \(\sigma(M)\) is the standard deviation of the linear density contrast field smoothed on the mass scale \(M\), and \(f(\sigma)\) is the multiplicity function characterized by two coefficients, \(\beta\), and \(D_B\):

\[
f(\sigma; D_B, \beta) \approx f^{(0)}(\sigma; D_B, \beta) + f^{(1)}_{\beta=0}(\sigma; D_B) + f^{(1)}_{\beta}(\sigma; D_B, \beta),
\]

\[
f^{(0)}(\sigma; D_B, \beta) = \frac{\delta_{cc}}{\sigma \sqrt{1 + D_B}} \frac{2}{\pi} \left\{ \frac{\delta_{cc}}{\sigma^2} - \frac{1}{2} \right\} \left[ 0, \frac{a \delta_{cc}^2}{2 \sigma^2} \right],
\]

\[
f^{(1)}_{\beta=0}(\sigma; D_B) = -\kappa \delta_{cc} \left\{ \frac{2a \delta_{cc}^2}{\sigma^2} - \frac{1}{2} \right\} \left[ 0, \frac{a \delta_{cc}^2}{2 \sigma^2} \right],
\]

\[
f^{(1)}_{\beta}(\sigma; D_B, \beta) = -\beta \delta_{cc} \left\{ f^{(1)}_{\beta=0}(\sigma; D_B) + \kappa \erfc\left( \frac{\delta_{cc}^2}{\sigma^2} \right) \right\},
\]

\[
f^{(1)}_{\beta}(\sigma; D_B, \beta) = \beta^2 a^2 \delta_{cc}^2 \kappa \left\{ \erfc\left( \frac{\delta_{cc}^2}{\sigma^2} \right) \right\}.
\]

with \(a \equiv 1/(1 + D_B)\), \(\kappa = \kappa_a\), \(\kappa = 0.475\), upper incomplete gamma function \(\Gamma(0, x)\), complementary error function \(\erfc(x)\), and critical density contrast for the spherical collapse \(\delta_{cc}\). The diffusion coefficient, \(D_B\), is a measure of the stochasticity of \(\delta\) caused by the influence of the perturbing neighbors as well as by the uncertainty in the identification of a cluster halo, while the drifting coefficient, \(\beta\), quantifies the deviation of the critical density contrast for the realistic nonspherical collapse, \(\delta_c\), from \(\delta_{cc}\) at a given \(\sigma(M)\).

Suggesting that for the isolated field clusters, the degree of the stochasticity of \(\delta_{cc}\) should be negligible (i.e., \(D_B = 0\), Lee 2012) modified the Corasaniti–Achitouv formalism to construct a single-parameter model for the field cluster mass function as

\[
dN_1(M, z) = \frac{\bar{\rho}_b}{M} \left| \frac{d}{dM} \ln \sigma^{-1} \right| f[\sigma(M, z); D_B = 0, \beta].
\]

In spite of having only a single parameter, this analytic model has been shown to be very successful in describing the field cluster mass functions in a wide redshift range not only for the standard ΛCDM cosmology but also for several nonstandard cosmologies including dynamical dark energy or massive neutrinos (Ryu & Lee 2020a, 2020b).

Although the exact value of \(\delta_{cc}\) has been known to weakly depend on the background cosmology as well as on the redshift (Eke et al. 1996; Pace et al. 2010), Ryu & Lee (2020a) regarded \(\delta_{cc}\) as a constant, setting it at the Einstein–de Sitter value of 1.686 (Gunn & Gott 1972), as done in the original formulation of the generalized excursion set mass function theory (Maggiore & Riotto 2010a, 2010b). In reality, the gravitational collapse proceeds in a nonspherical way, for which the actual critical density contrast, \(\delta_c\), departs from the idealistic spherical threshold, \(\delta_{cc}\). The cosmology dependence of \(\delta_{cc}\) is expected to overwhelm that of \(\delta_{cc}\), given that the degree of the nonsphericity of the collapse process is closely linked with the anisotropy of the cosmic web, which in turn possesses strong dependence on the background cosmology (e.g., Shim & Lee 2013; Naidoo et al. 2020). Unlike \(\delta_{cc}\), however, the value of \(\delta_c\) and its link to the initial conditions cannot be analytically derived from first principles due to the complexity associated with the nonspherical collapse process (Bond & Myers 1996).

Ryu & Lee (2020a) imparted any redshift and cosmology dependences of the collapse threshold at a given mass scale to the empirical parameter, \(\beta\), and showed that the redshift evolution of the empirically determined \(\beta(z)\) has a universal form of an inverse sine hyperbolic function of \(z\), regardless of the background cosmology:

\[
\beta(z) = \beta_A \sinh^{-1} \left\{ \frac{1}{q_z} (z - z_c) \right\},
\]

where \(\beta_A\), \(q_z\), and \(z_c\), represent three adjustable parameters, whose best-fit values depend on the background cosmology. In particular, the critical redshift, \(z_c\), defined as \(\beta(z_c) = 0\) (i.e., \(\delta_c(z_c) = \delta_{cc} = 1.686\)), has been found to depend most sensitively on the background cosmology (Ryu & Lee 2020a, 2020b), as it reflects not only how severely the real gravitational collapse...
deviates from the spherical symmetry but also how rapidly the universe evolves away from the Einstein–de Sitter state.

3. Evolution of the Drifting Coefficient in Nonstandard Cosmologies

3.1. Effect of cDE on $\beta(z)$

A cDE cosmology describes an alternative universe where the role of DE is played by a dynamical scalar field, $\phi$, coupled to DM particles through energy-momentum exchange. The DE–DM coupling that causes the time variation of DM particle mass (Wetterich 1995; Amendola 2000, 2004) generates a long-range fifth force via which the growth of structures can be enhanced (e.g., Mangano et al. 2003; Macciò et al. 2004; Mainini & Bonometto 2006; Pettorino & Baccigalupi 2008; Baldi et al. 2010; Wintergerst & Pettorino 2010, and references therein). Categorized by the shape of DE self-interaction potential, $V(\phi)$, as well as by the strength of the DE–DM coupling, $s(\phi) \equiv -d \ln m_{DM}/d\phi$,
a cDE cosmology has recently attained delving attentions since it has been found to provide a possible solution to the Hubble tension (Di Valentino et al. 2020).

To investigate the effect of cDE on the redshift evolution of $\beta(z)$, we utilize the data from the Large Coupled Dark Energy Cosmological Simulations (L-CoDECS) run by Baldi (2012a) with a modified version of the GADGET3 code, a nonpublic developers’ version of the widely used public code GADGET-2 (Springel 2005). The L-CoDECS is a series of $N$-body cosmological runs that simulate a standard $\Lambda$CDM and five different cDE cosmologies on a periodic box of linear size $1h^{-1}$ Gpc containing $1024^3$ collisionless DM particles of individual mass $m_{\text{DM}} = 5.84 \times 10^{10} h^{-1} M_\odot$, as well as an equal number of collisionless baryon particles of $m_{\text{baryon}} = 1.17 \times 10^{10} h^{-1} M_\odot$. The initial conditions of the standard $\Lambda$CDM cosmology were chosen to meet the constraints from the Seven-Year Wilkinson Microwave Anisotropy observations (Komatsu et al. 2011). The five different cDE cosmologies are divided into three categories: the constant DM–DE coupling and exponential potentials (EXP001, EXP002, and EXP003), the exponential DM–DE coupling and exponential potential (EXP008e3), and the constant coupling and supergravity potential (SUGRA). All cDE cosmologies simulated by the L-CoDECS were ensured to have a flat geometry, sharing the same values of the five key cosmological parameters, $h = 0.703$, $\Omega_{\text{CDM}} = 0.226$, $\Omega_{\text{DE}} = 0.729$, $\Omega_b = 0.0451$, $A_s = 2.42 \times 10^{-9}$, and $n_s = 0.966$. They differ from one another only in the potential shape and DM–DE coupling as well as in the linear density power spectrum amplitude, information on which are provided in the first four columns of Table 1. For a more detailed description of the cDE cosmologies and the L-CoDECS, we refer the readers to Baldi (2012a, 2012b).

The L-CoDECS simulations have been released with catalogs of gravitationally bound halos identified for each cosmology through a two-step process starting with a friends-of-friends (FoF) algorithm with a linking length parameter of $l_s = 0.2$ followed by a gravitational unbinding procedure of each individual FoF halo using the SUBFIND algorithm (Springel et al. 2001) that allows us to associate spherical

Figure 2. Same as Figure 1, but at $z = 1$.  

5 All data are available at the CoDECS website, http://marcobaldi.it/web/CoDECS.html.
overdensity masses and radii to each gravitationally bound main substructure. Selecting the cluster halos with virial masses \( M \gtrsim 3 \times 10^{13} h^{-1} M_\odot \) from the halo catalog and applying again the FoF algorithm with a linking length parameter of \( l_{sc} = 2 \times l_c \) to such a halo sample, we identify the clusters of cluster halos as marginally bound superclusters. Here, the choice of \( l_{sc} = 2 \times l_c \) for the FoF finding of the superclusters is made to guarantee \( D_B = 0 \) for the field cluster halos, as explained in Ryu & Lee (2020a).

Sorting out the field cluster halos as the superclusters that have only one member cluster halo, we create a sample of the field cluster halos at each redshift in the range of \( 0 \leq z \leq 1 \). The differential mass function of the field clusters halos is determined by computing the number density of the field cluster halos, \( dN_\ell/d \ln M \), whose masses fall in the differential bin of the logarithmic mass, \( [\ln M, \ln M + d \ln M] \), at each redshift. As done in Ryu & Lee (2020a), we employ the jackknife method to compute the errors in the determination of \( dN_\ell/d \ln M \). Splitting the sample of the field clusters into eight jackknife subsamples of equal size, we obtain \( dN_\ell/d \ln M \) separately from each subsample and then calculate the one standard deviation scatter of \( dN_\ell/d \ln M \) among the eight subsamples as errors at each logarithmic mass bin.

Using the linear density power spectrum of each cDE cosmology provided within the CoDECS public data release, we evaluate the linear density rms fluctuation, \( \sigma(M) \), and the analytic mass function of the field cluster, Equation (8), as well. The best-fit value of the drifting coefficient, \( \beta \), in Equation (8) is determined at each redshift by minimizing the following \( \chi^2 \) under the assumption that there is no correlation in \( dN_\ell/d \ln M \) at different mass bins:

\[
\chi^2(\beta) = \sum_{i=1}^{N_p} \frac{[n^a(\ln M_i) - n^b(\ln M_i; \beta)]^2}{\sigma_i^2},
\]

where \( n^a(\ln M_i) \) and \( n^a(\ln M_i; \beta) \) are the numerically obtained and analytically evaluated values of \( dN_\ell/d \ln M \) at the \( i \)th logarithmic mass bin, respectively, \( N_p \) is the number of the logarithmic mass bins, and \( \sigma_i \) is the jackknife error on \( n^a(\ln M_i) \). The error on \( \beta \) is then determined as the square root.

Figure 3. Empirically determined redshift evolution of the drifting coefficient of the field clusters (black filled circles) and fitting formula (red solid lines) for six different cosmologies.
of the inverse of the second derivative of $\chi^2$ with respect to $\beta$ as
$$\sigma_\beta = \left( \frac{d^2\chi^2}{d\beta^2} \right)^{-1/2}$$
(Ryu & Lee 2020a).

As done in Ryu & Lee (2020a), once the values of $\beta(z)$ are determined at various redshifts, we fit them to Equation (9) via a nonlinear least-squares regression procedure with the SciPy python code (Virtanen et al. 2020) to find the best-fit values of the three parameters, $\beta_A$, $q_z$, and $z_c$ and their associated errors $\sigma_{\beta_A}$, $\sigma_{q_z}$, and $\sigma_{z_c}$, respectively (see Table 1). Then, we calculate the statistical significance of the differences in the three parameters among the cosmologies as $\Delta\beta_A/\sigma_{\beta_A}$, $\Delta q_z/\sigma_{q_z}$, and $\Delta z_c/\sigma_{z_c}$, where $\Delta\beta_A$, $\Delta q_z$, and $\Delta z_c$ are the differences in the three parameters between two cosmologies, while $\sigma_{\beta_A}$, $\sigma_{q_z}$, and $\sigma_{z_c}$ correspond to the propagated errors in the determination of the differences.

Figure 1 (Figure 2) plots the numerically determined mass functions of the field cluster halos (filled black circles) as well as the analytic model (red solid line), Equation (8), with the best-fit value of $\beta$ for the six cosmologies at $z = 0$ ($z = 1$), respectively. In each panel, the analytic model for the $\Lambda$CDM case (black dashed line) is also plotted to clearly show the differences. Although the analytic model, Equation (8), succeeds in matching the numerical results at both of the
redshifts for all of the cDE cosmologies, the field cluster mass functions are found to be incapable of telling apart with high statistical significance the three cosmologies $\Lambda$CDM, EXP001, and SUGRA at both of the redshifts, $z=0$ and 1.

Figure 3 plots the redshift evolution of the empirically determined drifting coefficient, $\beta(z)$ (filled black circles), as well as the fitting formula (red solid lines) for the six cosmologies. In each panel, the fitting formula for the $\Lambda$CDM case (black dashed line) is also plotted to show the differences. As can be seen, the fitting formula expressed in terms of the inverse sine hyperbolic function, Equation (9), with the best-fit values of $q_z$, $\beta_A$, and $z_c$ indeed describes quite well the behaviors $\beta(z)$ for all of the six cosmologies. Note that the SUGRA can be distinguished by $\beta(z)$ from the $\Lambda$CDM with high statistical significance despite that the two cosmologies are mutually degenerate in the cluster mass functions. The statistical significance of the difference in the critical redshift parameter, $z_c$, between the $\Lambda$CDM and the SUGRA cosmologies is found to be as high as 7.48$\sigma$. Although $\beta(z)$ distinguishes with high statistical significance the other cDE cosmologies except for the EXP001 from the $\Lambda$CDM, it fails to break the degeneracy between the $\Lambda$CDM and the EXP001 cases, due to the extremely weak DM–DE coupling of the latter cosmology.

We have so far used prior information on the background cosmology for the determination of $dN_l/d\ln M$ and $\beta(z)$. In other words, to examine if $\beta(z)$ can break a cosmic degeneracy between two different cosmologies, we assume that information on the shape of the linear density power spectrum is available. In practice, however, this prior information is not available for the determination of $\beta(z)$. In particular, if a background cosmology is indistinguishable from the $\Lambda$CDM case by the standard diagnostics, then it may not be justified to make such a preemptive assumption about the shape of the linear density power spectrum. The EXP001 corresponds to this case where no prior information on the background cosmology should be assumed to be available in practice, since the standard diagnostics including the linear density power spectrum, mass function, etc., are unable to distinguish it from the $\Lambda$CDM case.

To deal with this degeneracy, we use the linear density power spectrum of the $\Lambda$CDM case, $P(k; \Lambda$CDM), for the

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**Figure 6.** Statistical significances of the differences among the $\Lambda$CDM, EXP001, and SUGRA that are mutually degenerate in the cluster mass functions.

**Table 2**

| Model  | $f_{\text{fit}}$ | $\beta_A$ | $\beta_L$ | $q_z$ | $z_c$ |
|--------|-----------------|-----------|-----------|-------|-------|
| $\Lambda$CDM | $...$ | 0.0 | 0.847 | $0.11 \pm 0.01$ | 0.22 \pm 0.06 | 1.24 \pm 0.03 |
| $fR4$ | $10^{-4}$ | 0.0 | 0.967 | $0.10 \pm 0.01$ | 0.16 \pm 0.04 | 1.39 \pm 0.03 |
| $fR5$ | $10^{-5}$ | 0.0 | 0.903 | $0.16 \pm 0.02$ | 0.50 \pm 0.11 | 1.40 \pm 0.04 |
| $fR6$ | $10^{-6}$ | 0.0 | 0.861 | $0.08 \pm 0.01$ | 0.09 \pm 0.04 | 1.24 \pm 0.04 |
| $fR4+0.3 \text{ eV}$ | $10^{-4}$ | 0.3 | 0.893 | $0.09 \pm 0.01$ | 0.29 \pm 0.09 | 1.52 \pm 0.06 |
| $fR5+0.15 \text{ eV}$ | $10^{-5}$ | 0.15 | 0.864 | $0.15 \pm 0.05$ | 0.85 \pm 0.45 | 1.73 \pm 0.14 |
| $fR6+0.06 \text{ eV}$ | $10^{-6}$ | 0.06 | 0.847 | $0.08 \pm 0.01$ | 0.11 \pm 0.04 | 1.27 \pm 0.04 |
computation of $\sigma(M)$ in $dN/d\ln M$ for the EXP001 case and compare the reevaluated analytic model with the numerical results to find the best-fit $\beta(z)$. That is, we redetermine $\beta(z)$ for the EXP001 case without using prior information on $P(k; \Lambda CDM)$ and compares it with the numerical results (black filled circles) for the EXP001 case at $z = 0$ (top panel) and $z = 1$ (bottom panel). As can be seen, in spite of no prior information on the background cosmology, the analytical mass function of the field clusters still describes quite well the numerical results at both of the redshifts for the EXP001 case.

Figure 5 plots the same as the top right panel of Figure 3 but without using prior information on $P(k; \text{EXP001})$. As can be seen, the EXP001 turns out to yield larger differences in $\beta(z)$ from the $\Lambda CDM$. The best-fit values of $\beta_A$, $q_z$, and $z_c$ for the EXP001 case listed in Table 1 correspond to the ones obtained without using prior information on $P(k; \text{EXP001})$. The statistical significance of the difference in $z_c$ between the $\Lambda CDM$ and the EXP001 is found to be as high as 2.53. Figure 6 summarizes the statistical significance of the difference in $z_c$ among the three cosmologies, $\Lambda CDM$, EXP001, and SUGRA, which are mutually degenerate in the field cluster mass functions. Although the degeneracy between the $\Lambda CDM$ and the EXP001 can be broken by $\beta(z)$ only with 2.53 significance, we speculate that a larger data set would improve the significance.

3.2. Effect of $f(R)$ Gravity on $\beta(z)$

In the theory of $f(R)$ gravity, the strength of a long-range fifth force is quantified by the absolute value of the derivative of $f(R)$ with respect to the Ricci scalar $R$ at the present epoch, $|f_{R_0}| \equiv |df/dR|_0$. A larger value of $|f_{R_0}|$ corresponds to a stronger fifth force, which would more severely enhance the small-scale density power (Hu & Sawicki 2007; Li & Barrow 2007). If neutrinos have a nonzero mass in an $f(R)$ gravity cosmology, however, the suppressing effect of the free streaming neutrinos on the small-scale density power spectrum could compensate the enhancing effect of the fifth force, resulting in a suppression of the deviations from the standard
ΛCDM + GR cosmology that each of these two scenarios would individually imprint on structure formation. In other words, the linear density power spectra may not be capable of distinguishing a certain combination of $f_R$ with $\sum m_\nu$ from the standard ΛCDM + GR cosmology, since they could have zero net effect on the amplitude of small-scale density perturbations (e.g., Baldi et al. 2014).

To investigate if $\beta(z)$ can also break the cosmic degeneracy between ΛCDM + GR and $\nu$CDM + $f(R)$, we use a subset of the data from the DUSTGRAIN-pathfinder $N$-body simulation suite that was conducted by Giocoli et al. (2018) on a box of volume $750^3 h^{-3}$ Mpc$^3$ for various $\nu$CDM + $f(R)$ cosmologies as well as the ΛCDM + GR cosmology. The DUSTGRAIN-pathfinder simulations were performed with the MG-GADGET code (Puchwein et al. 2013)—another modified version of GADGET3 implementing an adaptive mesh solver for the spatial fluctuations of the $f_R$ scalar degree of freedom—to trace the evolution of 768$^3$ DM particles of mass $8.1 \times 10^{10}$ $h^{-1} M_\odot$.

To simulate the $\nu$CDM + $f(R)$ cosmologies, the DUSTGRAIN-pathfinder adopted the widely used realization of $f(R)$ proposed by Hu & Sawicki (2007) and a particle-based implementation of massive neutrinos developed by Viel et al. (2010). Collapsed structures were identified through an FoF finder with a linking length parameter of $l_c = 0.16$ followed by the unbinding procedure implemented in the SUBFIND code to identify the halo center and its spherical overdensity mass and radius for all gravitationally bound objects in each cosmology, similarly to what is described above for the CoDECS simulations. For a detailed description of the technical details of the DUSTGRAIN-pathfinder simulations, see Giocoli et al. (2018).

Among the various cosmologies simulated by the DUSTGRAIN-pathfinder, we consider three different CDM+$f(R)$ (namely, $fR4$, $fR5$, and $fR6$ corresponding to $|f_R| = 10^{-4}$, $10^{-5}$, and $10^{-6}$, respectively) and three different $\nu$CDM+$f(R)$ (namely, $fR4+0.3$ eV, $fR5+0.15$ eV, and $fR6+0.06$ eV corresponding to $\sum m_\nu = 0.3$ eV, 0.15, and 0.06 eV, respectively) as well as the standard ΛCDM + GR (from here on, GR) with initial conditions set at the Planck values (Planck Collaboration et al. 2016). These seven different cosmologies were ensured to be flat, described by the common key cosmological parameter values of $h = 0.67$, $\Omega_m = 0.31$, $\Omega_{DE} = 0.67$, $\Omega_\nu = 0.0481$, $A_s = 2.2 \times 10^{-9}$, and $n_s = 0.97$. The first four columns of

\begin{figure}[ht]
\centering
\includegraphics[width=\textwidth]{figure8}
\caption{Same as Figure 7, but at $z = 1$.}
\end{figure}
Table 2 lists the values of $|f_{R0}|$, $\sum m$, and $\sigma_8$ for each of the seven cosmologies considered in the present work.

We first examine whether or not the analytic model for the field cluster mass function, Equation (8), is valid for the three CDM+$f(R)$ cosmologies. Analyzing the FoF halo catalogs in the redshift range $0 \leq z \leq 1$ and following the same procedure described in Section 3.1, we numerically determine $dN/d\ln M$ for the GR, $fR4$, $fR5$, and $fR6$ cases. To evaluate the analytic model, Equation (8), and compare it with the numerically determined $dN/d\ln M$ to derive $\beta(z)$ for each of the three $f(R)$ gravity cosmologies, we use the MGCAMB code (Lewis et al. 2000; Zhao et al. 2009; Hojjati et al. 2011; Zucca et al. 2019).

Figure 7 (Figure 8) depicts the same as Figure 1 (Figure 2) but for the $fR4$, $fR5$, and $fR6$ cases, revealing that the analytic model matches the numerical results quite well even for the $f(R)$ gravity models. As expected, the $fR4$ ($fR6$) case yields the most (least) abundant field clusters in the entire mass range. No statistically significant difference is found in $dN/d\ln M$ between the GR and the $fR6$ cases, indicating their mutual degeneracy in the field cluster mass functions. Figure 9 plots the same as Figure 3 but for the $fR4$, $fR5$, and $fR6$ cases. As can be seen, despite that the field cluster mass functions fail to distinguish between the GR and the $fR6$ cases, the field cluster drifting coefficient, $\beta(z)$, can break the degeneracy, showing a substantial difference between the two cosmologies.

It is worth noting the distinct redshift dependence of the difference in $\beta(z)$ between the GR and each $f(R)$ gravity cosmology. The $fR4$ case yields an almost redshift-independent shift of $\beta(z)$ from that of the GR case, while the other two cases show different redshift-dependent shifts between each other. That is, for the $fR5$ ($fR6$) case, the largest deviation of $\beta(z)$ from that of the GR case occurs at the low (high) redshift ends.

A qualitative explanation for this trend is provided in the following. For the $fR4$ case, the fifth force is basically always unscreened at low redshifts, which implies that the halos of all masses are equally affected by the fifth force, and that there is no sharp transition between screened and unscreened regions in the cosmic web.

For the $fR5$ and $fR6$ cases, the screening properties imply that the massive halos are screened, while less massive halos are...
not. This introduces a mass dependence on the deviation of the halo mass function from that of the GR case. In particular, for the $fR6$ case, the high-mass tail of the halo mass function should be almost unaffected and thus there should be an enhancement in the number of smaller-mass halos. This may have a different impact on the bias of halos at different masses, and consequently an impact on the definition of the field clusters (i.e., isolated massive objects) that could induce a different evolution of $\beta(z)$. A more quantitative analysis is required to fully understand the distinct differences in $\beta(z)$.
between the GR and each f(R) cosmology, which is, however, beyond the scope of this paper.

3.3. Combined Effect of f(R)+ν on β(z)

Now that the validity of the analytic model of the field cluster mass function for the f(R) gravity cosmology is confirmed, we repeat the whole process but for the fR4 +0.3 eV, fR5+0.15 eV, and fR6+0.06 eV cosmologies, which were shown to be degenerate with the GR in the standard statistics including the nonlinear density power spectrum, cluster mass functions, and halo bias (Baldi et al. 2014; Giocoli et al. 2018). Figure 10 (Figure 11) depicts the same as Figure 1 (Figure 2) but for the fR4+0.3 eV, fR5+0.15 eV, and fR6+0.06 eV cosmologies. As can be seen, the analytic model is still quite valid in matching the numerically obtained field cluster mass functions even for the f(R)+ν cosmologies. At z = 0, the three f(R)+ν cosmologies show no difference from the GR case in the field cluster mass functions. At z = 1, the differences in dN/ dln M between the f(R)+ν and the GR cases are slightly larger but still not statistically significant. Figure 12 plots the same as Figure 3 but for the fR4+0.3 eV, fR5+0.15 eV, and fR6+0.06 eV cosmologies. As can be seen, the fR4+0.3 eV cosmology yields a substantial difference in β(z) from the GR case, in spite of their mutual degeneracy in the standard statistics. Yet both of the fR5+0.15 eV and fR6+0.06 eV cosmologies show almost no difference in β(z) from the GR case.

As done in Section 3.1, we redetermine dN/ dln M for both of the fR5+0.15 eV and fR6+0.06 eV cases without using prior information on the shapes of their power spectra, which are plotted in Figure 13. As can be seen, the analytic model, Equation (8), still agrees quite well with the numerically obtained field cluster mass functions for both of the cases at both of the redshifts, despite that P(k; GR) is substituted for P(k; fR5 +0.15 eV) and P(k; fR6 +0.06 eV). The drifting coefficient, β(z), redetermined without using prior information is plotted in Figure 14, which reveals that the three cosmologies yield much larger differences in β(z).

For each cosmology, we determine the best-fit values of βh, qz, and zc by fitting Equation (9) to β(z) obtained without priors. Then, we calculate the statistical significance of the differences in the three parameters among the three cosmologies, which are shown in Figure 15. As can be seen, without using prior information on the linear density power spectra of the f(R)+ν cosmologies, the statistical significance of the differences in zc between the GR and the fR5+0.15 eV and between the fR6+0.06 eV and the fR5+0.15 eV are as high as 3.48 and 3.22, respectively.

Meanwhile, for the fR6+0.06 eV case, it turns out to be not zc but βh that is able to distinguish it from the GR case with Δβh/σβh = 2.03. The lower statistical significance of the differences in β(z) between the GR and the fR6+0.06 eV is likely to be at least partially due to the large errors caused by the relatively small box size of the DUSTGRAIN-pathfinder simulations. Given the distinct behaviors of β(z) between the GR and the fR5+0.06 eV shown in Figure 14, we suspect that if a halo sample from a larger simulations were used, the statistical significance would increase. The best-fit values of βh, qz, and zc for each of the seven cosmologies simulated by the DUSTGRAIN-pathfinder are shown in Table 2. For the fR6 +0.06 eV and fR5+0.15 eV cosmologies that are degenerate with the GR case in the standard statistics, what is listed in Table 2 are the best-fit values obtained without using priors in the shapes of the linear density power spectra.

4. Summary and Discussions

The new diagnostic developed by Ryu & Lee (2020a) traces the redshift evolution of the drifting coefficient, β(z), which is a
single parameter of the analytic model for the field cluster mass function derived by Lee (2012) in the frame of the generalized excursion set theory (Maggiore & Riotto 2010a, 2010b; Corasaniti & Achitouv 2011a, 2011b). Motivated by the recent finding that this diagnostic can in principle break the $\sigma_8 - \sigma_{\nu m}$ degeneracy (Ryu & Lee 2020b), we have studied whether or not the same diagnostic can break the degeneracy between the nonstandard and the standard $\Lambda$CDM+GR cosmologies by utilizing the data from the CoDECS and DUSTGRAIN-pathfinder simulations. For this study, we have considered 11 different nonstandard cosmologies that include 5 different cDE (EXP001, EXP002, EXP003, EXP008e3, and SUGRA), 3 different $f(R)$ gravity ($fR4$, $fR5$, and $fR6$), and 3 different $f(R)$ gravity+$\nu$ cosmologies ($fR4+0.3$ eV, $fR5+0.15$ eV, and $fR6+0.06$ eV).

Among the cDE and $f(R)$ gravity cosmologies, EXP001 and $fR6$ have been known to be very similar to the $\Lambda$CDM+GR in the linear density power spectra and cluster mass functions at $z = 0$, due to their extremely weak DM–DE coupling and fifth force, respectively. The three $f(R)$ gravity+$\nu$ cosmologies have been known to be degenerate not only with the $\Lambda$CDM+GR but also among one another in the standard diagnostics that include the cluster mass functions, halo bias, and nonlinear density power spectrum (Baldi et al. 2014; Giocoli et al. 2018). Analyzing the catalogs of the FoF bound objects for each cosmology, we have identified the field cluster halos and determined their mass functions at each redshift in the range of $0 \leq z \leq 1$. The best-fit value of the drifting coefficient $\beta$ has been found by adjusting the analytic model of Lee (2012) to the numerically determined field cluster mass functions. The fitting formula for $\beta(z)$ proposed by Ryu & Lee (2020a) has been used to assess the statistical significance of the differences in $\beta(z)$ among the cosmologies. This analysis has lead us to find the following.

1. The analytic model of Lee (2012) for the field cluster mass functions with the best-fit values of $\beta$ agrees excellently well with the numerical results at all redshifts for all of the nonstandard cosmologies.
2. The empirical formula of Ryu & Lee (2020a) for $\beta(z)$ works fairly well for all of the nonstandard cosmologies.
3. Despite that they produce very similar (field) cluster mass functions, the $\Lambda$CDM and the SUGRA cosmologies substantially differ in $\beta(z)$ from each other.
4. The degeneracy between the $\Lambda$CDM and the EXP001 in the field cluster mass functions can be broken by $\beta(z)$ with 2.53$\sigma$ significance without using any prior information on the linear density power spectrum, $P(k; \text{EXP001})$.

5. The degeneracy between the $\Lambda$CDM+GR and the $fR4 +0.3 \, \text{eV}$ in the linear density power spectra and (field) cluster mass functions can be broken by $\beta(z)$ with high statistical significance.

6. The degeneracy among the $\Lambda$CDM+GR, $fR5 +0.15 \, \text{eV}$, and $fR6 +0.05 \, \text{eV}$ cosmologies in the standard diagnostics can be broken by $\beta(z)$ with a minimum 2.01$\sigma$ significance, without using any prior information on the linear density power spectra.

To understand the advantage of using $\beta(z)$ as a cosmology discriminator, it may be worth comparing $\beta(z)$ with the standard diagnostics such as the linear density power spectrum, nonlinear density bispectrum, and cluster mass function. As for the linear density power spectrum, it deals with isotropically averaged densities and thus fails to capture independent information contained in the anisotropic nonlinear cosmic web about the background cosmology (Naidoo et al. 2020). As for the nonlinear density bispectrum that treats the nonlinear anisotropic density field, it is not readily observable, suffering from highly nonlinear halo bias and redshift space distortion effects. Regarding the cluster mass function, although it is free from the halo bias and redshift space distortion effect, it varies most sensitively with the value of $\sigma_8$. If two different cosmologies share an identical value of $\sigma_8$ (e.g., $\Lambda$CDM and SUGRA), the cluster mass function is apt to fail in telling them apart.
Meanwhile, the field cluster drifting coefficient, $\beta(z)$, deals with the nonspherical collapse occurring in the anisotropic cosmic web that contains additional information on the initial conditions. It is free from the halo bias and redshift space distortion effect, directly quantifying how the background cosmology deviates from the Einstein–de Sitter state that sensitively depends on the evolution of the energy contents of the universe. Notwithstanding, we have yet to find a direct link of $\beta(z)$ to the initial conditions, which weakens its power as a probe of gravity and dark sector physics. The very fact that the $fR5$ inverse sine hyperbolic function provides a fairly good probe of gravity and dark sector physics. The very fact that the inverse sine hyperbolic function provides a fairly good approximation to the empirically determined $\beta(z)$ for all of the cosmologies hints that it should be beyond a mere fitting formula. Our future work is in the direction of theoretically deriving $\beta(z)$ from first principles, providing a physical explanation for why $\beta(z)$ behaves as an inverse sine hyperbolic function of $z$ and establishing its direct link to the initial conditions.

Another advantage is high observational applicability of $\beta(z)$. As shown in Figure 14, the $fR5+0.15 \ eV$ and $fR6+0.06 \ eV$ cases considerably differ from the GR case in the low-$z$ values of $\beta(z) (z<0.5)$. In other words, it does not require a large sample of the high-$z$ clusters with $z>0.5$ to distinguish between the $f(R)+\nu$ and the GR cases with $\beta(z)$ in practice. Yet, to distinguish between the $fR5+0.15 \ eV$ and the $fR6+0.06 \ eV$ cases as well as between the $f(R)$ gravity and the cDE cosmologies with $\beta(z)$, it indeed requires a large cluster sample from a wide range of redshifts. The upcoming large-scale deep surveys such as the Large Synoptic Survey Telescope (LSST) or EUCLID that is expected to cover the redshift range up to $z \sim 2$ (Tyson 2002; Amendola et al. 2013) will improve prospects for $\beta(z)$ as a cosmological discriminator.

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**References**

Amendola, L. 2000, PhRvD, 62, 043511
Amendola, L. 2004, PhRvD, 69, 103524
Amendola, L., Appleby, S., Bacon, D., et al. 2013, LRR, 16, 6
Baldi, M. 2012a, MNRAS, 420, 430
Baldi, M. 2012b, MNRAS, 422, 1028
Baldi, M., Pettorino, V., Robbers, G., et al. 2010, MNRAS, 403, 1684
Baldi, M., Villaescusa-Navarro, F., Viel, M., et al. 2014, MNRAS, 440, 75
Bond, J. R., & Myers, S. T. 1996, ApJS, 103, 1
Buchdahl, H. A. 1970, MNRAS, 150, 1
Clifton, T., Ferreira, P. G., Padilla, A., et al. 2012, PhR, 513, 1
Corasaniti, P. S., & Achitouv, I. 2011a, PhRvL, 106, 241302
Corasaniti, P. S., & Achitouv, I. 2011b, PhRvD, 84, 023009
Di Valentino, E., Melchiorri, A., Mena, O., et al. 2020, PDU, 30, 100666
Eke, V. R., Cole, S., & Frenk, C. S. 1996, MNRAS, 282, 263
García-Farieta, J. E., Marulli, F., Veropalumbo, A., et al. 2019, MNRAS, 488, 187
Giocoli, C. M., Baldi, M., & Moscardini, L. 2018, MNRAS, 481, 2813
Gunn, J. E., & Gott, J. R. 1972, ApJ, 176, 1
Hagstotz, S., Costanzi, M., Baldi, M., et al. 2019, MNRAS, 486, 3927
Holjati, A., Pogosian, L., & Zhao, G. 2011, JCAP, 1108, 005
Hu, W., & Sawicki, I. 2007, PhRvD, 76, 064004
Komatsu, E., Smith, K. M., Dunkley, J., et al. 2011, ApJS, 192, 18
Lee, J. 2012, ApJ, 752, 40
Lewis, A., Challinor, A., & Lasenby, A. 2000, ApJ, 538, 473
Li, B., & Barrow, J. D. 2007, PhRvD, 75, 084010
Macciò, A. V., Quercellini, C., Mainini, R., et al. 2004, PhRvD, 69, 123516
Maggiore, M., & Riotto, A. 2010a, ApJ, 711, 907
Maggiore, M., & Riotto, A. 2010b, ApJ, 717, 515
Mainini, R., & Bonometto, S. 2006, PhRvD, 74, 043504
Mangano, G., Miele, G., & Pettorino, V. 2003, MPLA, 18, 831
Merten, J., Giocoli, C., Baldi, M., et al. 2019, MNRAS, 487, 104
Naidoo, K., Whiteway, L., Massara, E., et al. 2020, MNRAS, 491, 1709
Pace, F., Waizmann, J. C., & Bartelmann, M. 2010, MNRAS, 406, 1865
Peel, A., Lalande, F., Stark, J.-L., et al. 2018a, PhRvD, 100, 023508
Peel, A., Pettorino, V., Giocoli, C., et al. 2018b, A&A, 619, A38
Pettorino, V., & Baccigalupi, C. 2008, PhRvD, 77, 103003
Planck Collaboration, Ade, P. A. R., Aghanim, N., et al. 2016, A&A, 594, A24
Puchwein, E., Baldi, M., & Springel, V. 2013, MNRAS, 436, 348
Ryu, S., & Lee, J. 2020a, ApJ, 889, 62
Ryu, S., & Lee, J. 2020b, ApJ, 894, 65
Shim, J., & Lee, J. 2013, ApJ, 777, 74
Springel, V. 2005, MNRAS, 364, 1105
Springel, V., White, S. D. M., Tormen, G., et al. 2001, MNRAS, 328, 726
Starobinsky, A. A. 1980, PhLB, 91, 99
Tyson, J. A. 2002, Proc. SPIE, 4836, 10
Viel, M., Hachemt, M. G., & Springel, V. 2010, JCAP, 06, 015
Virtanen, P., Gommers, R., Uram, T. E., et al. 2020, NatMe, 17, 261
Wintergerst, N., & Pettorino, V. 2010, PhRvD, 82, 103516
Wintergerst, N., & Pettorino, V. 2019, PhRvD, 100, 023508
Wright, B. S., Koyama, K., Winther, H. A., et al. 2019, JCAP, 06, 040
Zhao, G., Pogosian, L., Silvestri, A., et al. 2009, PhRvD, 79, 083513
Zucca, A., Pogosian, L., Silvestri, A., et al. 2019, JCAP, 1905, 001

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