Research Article

Optimal Control and Simulation for Enterprise Financial Risk in Industry Environment

Yanjun Liang,1 Wei-hua Zhang,2 Youjun Lu,1 and Zhong-Sheng Wang3

1School of Data Science and Information Engineering, Guizhou Minzu University, Guiyang 550025, China
2College of Economics and Management, Shanghai Ocean University, Shanghai 201306, China
3Department of Automation, Guangdong Polytechnic Normal University, Guangzhou 510000, China

Correspondence should be addressed to Wei-hua Zhang; whzhang@shou.edu.cn

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1. Introduction

In the period of postfinancial crisis, enterprises are facing constantly changing external conditions, many kinds of crises are emerging, and financial risks are more serious than before. Therefore, the construction of enterprise financial risk control or early warning system is imperative.

Enterprise financial risk control or early warning system has drawn much more attention recently, and there are about three kinds of work completed now. Firstly, the necessity of the construction of enterprise financial risk control or early warning system has been discussed [1–4]. For example, the relation of managing methods, financial risks, and financial systems is discussed in [3], and the authors think that sophisticated managing methods and financial systems may not do good to reduce financial risks, and in [4], the importance and standard process of financial risk engineering for electric power enterprises are emphasized, and the risks which are most likely to occur in business activities are identified. Secondly, the models of enterprise financial risk control or early warning system have been constructed and analyzed by many scholars [5–8]. In [5], a new financial early warning logit model is developed and improved the accuracy of prediction and stability. To forecast the bankruptcy risk of enterprises in Latin America and Central Europe in [6], the author has used statistical and soft computing methods to program the prediction models. To predict financial crises, based on a multinomial logit model, in [7], a new early warning system model is developed. In [8], an early warning model of China’s energy price is analyzed from the aspects of price fluctuation and price ratio structure through fitting the risk distributions of indices and applying the computable general equilibrium model. Furthermore, some methods are employed to control and to early warn financial risks [9–14]. Early warning system is treated as a pattern recognition problem in [9], and using a pattern classifier, based on distinctive features of economics, crisis critical and normal economical situations are distinguished. Systemic banking crises and early warning systems in low-income countries are studied, and a multinomial logit approach is proposed in [10]. To reduce abortions in dairy cattle in Denmark, a modified two-stage method for detecting an unusual increase in the abortion incidence is applied [11]. To reduce financial risks and early warn the
risks, an adaptive fuzzy measure by using the dynamic information in the single classifier pattern recognition results is proposed in [12]. For China’s burgeoning real estate enterprises, in [13], the z-score model is used to reduce the financial risk in early warning models. By using data mining, a financial early warning system is developed, and 15 risk indicators that affected financial distress are detected [14].

With the development of computer and internet technology, many scholars begin to study financial risk problem in the new environment. Based on data science and computer technology, financial risk is studied [15–20]. Recently, more and more scholars consider the dynamic property of financial risk control systems [21]. In this paper, the enterprise financial risk dynamical system model is established, and using the optimal control method, the optimal controller is designed to reduce the financial risk in the industry environment.

The remainder of this paper is structured as follows. In Section 2, enterprise financial risk dynamical model and optimal control problem in the industry environment are presented. In Section 3, optimal controller with an exponential decay rate and algorithm are designed for the financial risk control system. In Section 4, numerical experiments are presented. Finally, in Section 5, some conclusions are drawn.

2. Dynamical System Model and Optimal Control Problem

Enterprise financial activity is comprised by six subsystems, namely, purchase subsystem, production subsystem, sale subsystem, investment subsystem, financing subsystem, and profit subsystem, and they work together to complete the financial circulation of capital collection, investment, consumption, income, and distribution, and seven kinds of industry environment risks may emerge: industry resource risk, industry competition risk, industry life cycle risk, industry technological change risk, industry credit risk, industry tax rate risk, and industry interest rate risk. Enterprise finance system is a highly open system, and each subsystem is connected widely with the industry environment. In this paper, in order to establish an enterprise financial risk system, seven kinds of industry environment risks influencing enterprise financial activities are considered, and the system model is as follows:

$$\begin{align*}
\dot{z}_1 &= f_1 (z_1, z_2, \ldots, z_7), \\
\dot{z}_2 &= f_2 (z_1, z_2, \ldots, z_7), \\
\dot{z}_3 &= f_3 (z_1, z_2, \ldots, z_7), \\
\dot{z}_4 &= f_4 (z_1, z_2, \ldots, z_7), \\
\dot{z}_5 &= f_5 (z_1, z_2, \ldots, z_7), \\
\dot{z}_6 &= f_6 (z_1, z_2, \ldots, z_7), \\
\dot{z}_7 &= f_7 (z_1, z_2, \ldots, z_7),
\end{align*}$$

(1)

where $z_1$ is industry resource risk, $z_2$ is industry competition risk, $z_3$ is industry life cycle risk, $z_4$ is industry technological change risk, $z_5$ is industry credit risk, $z_6$ is industry tax rate risk, and $z_7$ is industry interest rate risk.

In (1), industry resource risk $z_1$ is determined by purchase price index of raw material, fuel, and power $z_{11}$ and supplier concentration ratio $z_{12}$; industry competition risk $z_2$ is indicated by concentration ratio $z_{21}$ and enterprise yearly increment rate $z_{22}$ of the industry; industry life cycle risk $z_3$ is represented by sales growth rate $z_{31}$ and investment in the fixed asset growth rate $z_{32}$ of the industry; industry technological change risk $z_4$ is influenced by industry technological investment rate and is controlled; industry credit risk $z_5$ is described by cash flow current ratio $z_{51}$ and bad debt rate $z_{52}$ of the industry; and industry tax rate risk $z_6$ and industry interest rate risk $z_7$ are determined, respectively, by tax rate level and interest rate level of the industry and are constants in general. According to economic principles, the enterprise financial risk system (1) is redescribed in the following form:

$$\begin{align*}
\dot{z}_{11} &= -k_{1111}z_{11} + k_{1112}z_{12} - k_{1121}z_{21} + k_{1122}z_{22} + k_{1131}z_{31} + k_{1132}z_{32} + k_{1151}z_{51}, \\
\dot{z}_{12} &= k_{1211}z_{11} - k_{1212}z_{12}, \\
\dot{z}_{21} &= -k_{2121}z_{21} + k_{2122}z_{22} - k_{2131}z_{31} + k_{2132}z_{32}, \\
\dot{z}_{22} &= -k_{2221}z_{21} - k_{2222}z_{22} + k_{2231}z_{31} - k_{2232}z_{32}, \\
\dot{z}_{31} &= -k_{3111}z_{31} - k_{3112}z_{12} - k_{3121}z_{21} + k_{3122}z_{22} - k_{3131}z_{31} + k_{3132}z_{32} + k_{3141}z_{4}, \\
\dot{z}_{32} &= k_{3221}z_{21} - k_{3222}z_{22} + k_{3231}z_{31} - k_{3232}z_{32} - k_{3241}z_{4} - k_{3251}z_{51} - k_{3252}z_{52}, \\
\dot{z}_{41} &= -k_{4121}z_{21} + k_{4222}z_{22} - k_{4311}z_{31} - k_{4322}z_{32} - k_{4411}z_{4} - k_{4511}z_{51} - k_{4522}z_{52} + bu, \\
\dot{z}_{51} &= -k_{5111}z_{11} - k_{5112}z_{12} + k_{5121}z_{21} - k_{5122}z_{22} + k_{5131}z_{31} + k_{5132}z_{32} - k_{5151}z_{51} - k_{5152}z_{52}, \\
\dot{z}_{52} &= k_{5221}z_{21} - k_{5222}z_{22} + k_{5231}z_{31} + k_{5232}z_{32} - k_{5251}z_{51} - k_{5252}z_{52},
\end{align*}$$

(2)

where $k_{ijmn} > 0$ is coefficient, $i = 1, 2, \ldots, 5$, $m = 1, 2$, $j = 1, 2$, and $n = 1, 2$, $u$ is control input, and $b$ is its coefficient.
\[
\dot{x}(t) = Ax(t) + Bu(t),
\]
\[
x(0) = x_0.
\] (3)

Optimal control theory is a subject to study and solve the problem of finding the optimal solution from all possible control schemes. In order to study the enterprise financial risk optimal control problem, choose an average performance index for system (3) as follows:

\[
J = \lim_{T \to \infty} \frac{1}{T} \int_0^T e^{2\alpha t} [x^T(t)Qx(t) + Ru^2(t)] dt,
\] (5)

where \(Q \in \mathbb{R}^{9 \times 9}\) are positive semidefinite matrices, \(R \in \mathbb{R}\) is a positive definite matrix, and \(\alpha \geq 0\) is the exponential decay rate. We can balance the control effect and control energy by changing the parameter of the performance index because the control effect is influenced by the weighting matrix \(Q\) and the control energy is altered by the weighting coefficient \(R\).

The objective of this paper is to find a control law \(u^*(t)\) for system (3) and make the value of performance index (5) a minimum.

3. Optimal Controller and Algorithm Design

Then, we design a controller for financial risk control system (3). The optimal control law with \(\alpha\) exponential decay rate can be presented in the following theorem.

**Theorem 1.** Consider the optimal control problem described by system (3) with performance index (5). The optimal control law \(u^*(t)\) exists and is unique. Its form is as follows:

\[
u^*(t) = -R^{-1}B^T P_1 x(t),
\] (6)

where \(P_1\) is the unique positive definite solution of the following Riccati matrix equation:

\[
(A + \alpha I)^T P_1 + P_1 (A + \alpha I) - P_1 SP_1 + Q = 0,
\] (7)

where \(S = R^{-1}B^T\).

**Proof.** Introduce model transform for system (3) with performance index (5):

\[
x = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9]^T = [z_{11}, z_{12}, z_{21}, z_{22}, z_{31}, z_{32}, z_4, z_{51}, z_{52}]^T,
\]

\[
B = [0, 0, 0, 0, 0, 0, b, 0, 0]^T,
\]

and

\[
\begin{bmatrix}
-k_{1111} & k_{1112} & -k_{1121} & -k_{1112} & k_{1131} & k_{1132} & 0 & 0 & k_{1151} \\
-k_{1121} & -k_{1212} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -k_{2121} & k_{2112} & -k_{2131} & k_{2132} & 0 & 0 & 0 \\
0 & 0 & -k_{2221} & -k_{2222} & k_{2231} & -k_{2232} & 0 & 0 & 0 \\
-k_{3111} & -k_{3112} & -k_{3121} & -k_{3122} & -k_{3131} & k_{3132} & k_{314} & 0 & 0 \\
0 & 0 & k_{3221} & -k_{3222} & -k_{3231} & -k_{3232} & -k_{324} & -k_{3251} & -k_{3252} \\
0 & 0 & -k_{421} & k_{422} & -k_{431} & -k_{432} & -k_{44} & -k_{451} & -k_{452} \\
-k_{5111} & -k_{5112} & k_{5121} & -k_{5122} & k_{5131} & k_{5132} & 0 & k_{5151} & -k_{5152} \\
0 & 0 & k_{5221} & -k_{5222} & k_{5231} & k_{5232} & 0 & 0 & -k_{5251} & -k_{5252}
\end{bmatrix}.
\] (4)

\[
\ddot{x}(t) = x(t)e^{\alpha t},
\]

\[
\ddot{u}(t) = u(t)e^{\alpha t},
\] (8)

\[
\dddot{\lambda}(t) = \dddot{\lambda}(t) + \frac{1}{\alpha} \dot{\lambda}(t),
\] (9)

then, we have

\[
\begin{bmatrix}
\dot{\lambda}(t) & \ddot{\lambda}(t) & \dddot{\lambda}(t)
\end{bmatrix} = \begin{bmatrix}
\hat{A} & \hat{B} & \hat{D}
\end{bmatrix} \begin{bmatrix}
\dot{\lambda}(t) & \ddot{\lambda}(t) & \dddot{\lambda}(t)
\end{bmatrix}
\] (10)

\[
\hat{A} = A + \alpha I,
\]

\[
\hat{B} = B,
\]

\[
\hat{D} = D,
\]

The maximum principle in optimal control is the necessary condition to obtain the optimal control in the problem of maximizing the objective functional, which is named after making the Hamiltonian function reach the maximum value. Applying the maximum principle to the optimal control problem in (10) and (11), the optimal control law can be written as follows:

\[
u^*(t) = -R^{-1}B^T \dddot{\lambda}(t),
\] (12)

where \(\dddot{\lambda}(t)\) is the solution to the following two-point boundary value problem:
\[-\lambda(t) = Q\dot{x}(t) + A^T\lambda(t),\]
\[\dot{x}(t) = A\bar{x}(t) - R^{-1}B^T\lambda(t),\]
\[\bar{x}(0) = x_0e^{\alpha t},\]
\[\lambda(\infty) = 0.\]

To solve (13), let
\[\lambda(t) = P_1\bar{x}(t).\]  \tag{14}

Substituting the equations of (10) and (12) into the first derivatives of (14), we get
\[\lambda(t) = P_1\dot{x}(t) = \left(P_1\bar{A} - P_1SP_1\right)\bar{x}(t).\]  \tag{15}

From equations 13 and (14), we obtain
\[\dot{\lambda}(t) = -\left(Q + A^TP_1\right)\bar{x}(t).\]  \tag{16}

Comparing the coefficients of (15) and (16), we obtain the matrix equation:
\[A^TP_1 + P_1\bar{A} - P_1SP_1 + Q = 0.\]  \tag{17}

Then, adding the model transform (9), we can obtain
\[(A + \alpha I)^TP_1 + P_1(A + \alpha I) - P_1SP_1 + Q = 0.\]  \tag{18}

In order to implement the control law described in Theorem 1, we design an algorithm as follows (Algorithm 1).

\begin{algorithm}
\caption{The algorithm to solve the enterprise financial risk optimal control law with exponential decay rate.}
\end{algorithm}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{$x_1$, $x_2$, and $x_3$ comparison curves. (a) Purchase price index of raw material, fuel, and power. (b) Supplier concentration ratio. (c) Concentration ratio.}
\end{figure}

4. Numerical Experiment

Considering the data of Turkey in 2007, the data of firms were obtained from the Turkish Central Bank with permission and are shown in Table 1 [14]. Financial data gained from balance sheets and income statements are used to calculate the matrices of the system.

Employing Matlab software, a numerical experiment is carried out for the proposed optimal vibration controller.

The main purpose of risk control is to reduce enterprise financial risks which indicate the limit of the enterprise financial activities. So, to evaluate the effectiveness of the proposed control strategy, the risks of the enterprise finance system are considered. Then, the corresponding curves of open loop and controlled by the proposed optimal controller are compared and shown in Figures 1–3.

The curves of the enterprise financial risks are shown in Figures 1–3, in which solid lines represent the open loop results of the enterprise financial risk system in industry environment and dotted lines describe the results of the enterprise financial risk system controlled by the proposed control strategy. It can be seen from these numerical results that the proposed optimal controller with $\alpha$ exponential decay rate is efficient, real-time, and robust in reducing the enterprise financial risks in industry environment. Enterprises, financial managers, and researchers can balance the control effect and control energy by adjusting the parameters of the performance index, namely, changing the value of the weighting matrix $Q$ and the weighting coefficient $R$, thereby improving further safety of the enterprise financial activities.
5. Conclusions

The industry environment is an external environment, the most basic of enterprises, and the influence of the macroenvironment for the enterprises generally spreads to specific enterprises through the industry environment; therefore, the adjustment of the enterprise internal environment is based on the requirements of the industry environment. In order to study the situation of enterprise finance, its industry environment and the relationship between them have to be

Table 1: Financial indicators that influenced financial risks.

| Financial indicators                                           | Value $(10^{-4})$ |
|-----------------------------------------------------------------|-------------------|
| Profit before tax to own funds                                  | 0.700             |
| Return on equity                                                | 0.300             |
| Cumulative profitability ratio                                  | 1.000             |
| Short-term liabilities to total loans                           | 1.000             |
| Total loans to total assets                                     | 230               |
| Interest expenses to net sales                                  | 11.0              |
| Fixed assets to long-term loans and own funds                   | 27.0              |
| Long-term liabilities to total liabilities                      | 0.800             |
| Gross profit to net sales                                       | 332               |
| Bank loans to total assets                                      | 12.0              |
| Inventory dependency ratio                                      | 1.00              |
| Own funds turnover                                              | 432               |
| Short-term receivables to total assets                          | 121               |
| Operating expenses to net sales                                 | 149               |
| Receivables turnover                                           | 0.400             |

Figure 2: $x_4$, $x_5$, and $x_6$ comparison curves. (a) Enterprise yearly increment rate. (b) Sales growth rate. (c) Investment in the fixed asset growth rate.

Figure 3: $x_7$, $x_8$, and $x_9$ comparison curves. (a) Industry technological change risk. (b) Cash flow current ratio. (c) Bad debt rate.
considered. In this paper, seven kinds of industry environment risks influencing enterprise financial activities are chosen as state variables and the enterprise financial risk dynamical system model is established. The optimal controller with exponential decay rate is designed for the enterprise financial risk system. Numerical simulation results demonstrated that the proposed strategy is efficient, real-time, and robust. In the next studies, we will focus on the nonlinear systems and time-delay systems of the enterprise financial risk control systems, and fuzzy control strategy [22–25] can also be applied to the subject.

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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