Probing the matter term at long baseline experiments

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Abstract

We consider $\nu_{\mu} \rightarrow \nu_e$ oscillations in long baseline experiments within a three flavor oscillation framework. A non-zero measurement of this oscillation probability implies that the $(\frac{1}{3})$ mixing angle $\phi$ is non-zero. We consider the effect of neutrino propagation through the matter of earth’s crust and show that, given the constraints from solar neutrino and CHOOZ data, matter effects enhance the mixing for neutrinos rather than for anti-neutrinos. We need data from two different experiments with different baseline lengths (such as K2K and MINOS) to distinguish matter effects unambiguously.

Recent results from the Super-Kamiokande have sparked tremendous interest in neutrino physics \cite{1,2}. The deficits seen in the solar and atmospheric neutrino fluxes can be very naturally explained in terms of neutrino oscillations. Since the energy and the distance scales of the solar neutrino problem are widely different from those of the atmospheric neutrino problem, the neutrino oscillation solution to these problems requires widely different values of mass-squared differences \cite{3,4}. A minimum of three mass eigenstates, and hence three neutrino flavors, are needed for a simultaneous solution of solar and atmospheric neutrino problems. Since LEP has shown that three light, active neutrino species exist \cite{5}, it is natural to consider all neutrino data in the framework of oscillations between all the three active neutrino flavors.

The flavor eigenstates $\nu_\alpha (\alpha = e, \mu, \tau)$ are related to the mass eigenstates $\nu_i (i = 1, 2, 3)$
by a unitary matrix $U$

$$|\nu_\alpha\rangle = U|\nu_i\rangle. \quad (1)$$

As in the quark sector, $U$ can be parametrized in terms of three mixing angles and one phase. A widely used parametrization, convenient for analyzing neutrino oscillations with matter effects, is

$$U = U^{23}(\psi) \times U^{\text{phase}} \times U^{13}(\phi) \times U^{12}(\omega), \quad (2)$$

where $U^{ij}(\theta_{ij})$ is the two flavor mixing matrix between the $i$th and $j$th mass eigenstates with the mixing angle $\theta_{ij}$. We assume that the vacuum mass eigenvalues have the pattern $\mu_3^2 \gg \mu_2^2 \gg \mu_1^2$. Hence $\delta_{31} \simeq \delta_{32} \gg \delta_{21}$, where $\delta_{ij} = \mu_i^2 - \mu_j^2$. The larger $\delta$ sets the scale for the atmospheric neutrino oscillations and the smaller one for solar neutrino oscillations.

The angles $\omega, \phi$ and $\psi$ vary in the range $[0, \pi/2]$. For this range of the mixing angles, there is no loss of generality due to the assumption that $\delta_{31}, \delta_{21} \geq 0$.

It has been shown that in the above approximation of one dominant mass, the oscillation probability for solar neutrinos is a function of only three parameters ($\delta_{21}, \omega$ and $\phi$) and that of the atmospheric neutrinos and long baseline neutrinos is also function of only three parameters ($\delta_{31}, \phi$ and $\psi$). In each case, the three flavor nature of the problem is illustrated by the fact that the oscillation probability is a function of two mixing angles. The phase is unobservable in the one dominant mass approximation because, in both solar and atmospheric neutrino oscillations, one of the three mixing angles can be set to zero.

The value of $\delta_{31}$ preferred by the Super-K atmospheric neutrino data is about $2 \times 10^{-3}$ eV$^2$. For this value of $\delta_{31}$, CHOOZ data sets a very strong constraint

$$\sin^2 2\phi \leq 0.2. \quad (3)$$

If $\phi = 0$, then the solar and atmospheric neutrino oscillations get decoupled and become two flavor oscillations with relevant parameters being ($\delta_{21}, \omega$) and ($\delta_{31}, \psi$) respectively.

It is interesting to look for the consequences of non-zero $\phi$. A non-zero $\phi$ leads to $\nu_\mu \to \nu_e$ oscillations in atmospheric neutrinos and long baseline experiments. Due to theoretical
uncertainty in the calculation of atmospheric neutrino fluxes, it will be very difficult to
discern the effect of a small $\phi$ from the atmospheric neutrino data. Recently a proposal was
made to look for matter enhanced $\nu_\mu \rightarrow \nu_e$ oscillations in atmospheric neutrino data, which
can be significant even if $\phi$ is small $^{11}$. Here we consider the effect of matter on $\nu_\mu \rightarrow \nu_e$
oscillations in a three flavor framework at long baseline experiments.

The relation between the flavor states and mass eigenstates can be written in the simple
form,
\[
\begin{pmatrix}
\nu_e \\
c_\phi \nu_\mu - s_\phi \nu_\tau \\
s_\phi \nu_\mu + c_\phi \nu_\tau
\end{pmatrix} =
\begin{pmatrix}
c_\phi & 0 & s_\phi \\
0 & 1 & 0 \\
-s_\phi & 0 & c_\phi
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix},
\]
(4)
where $c$ stands for cosine and $s$ stands for sine. This equation is obtained by first setting
$U_{\text{phase}} = I$ and $\omega = 0$ in equation (2). Then substitute the resulting form $U$ in equation (3)
and multiply it from left by $U_{23}^\dagger (-\psi)$. From (3) we see that $\nu_2$ has no $\nu_e$ component and
hence is decoupled from any oscillation involving $\nu_e$. Thus the calculation of oscillation
probability is essentially a two flavor problem. The three flavor nature is present in the fact
that the oscillations occur between $(\nu_e \rightarrow (s_\phi \nu_\mu + c_\phi \nu_\tau))$. The vacuum oscillation probability
is calculated to be
\[
P_{\mu e} = \sin^2 \psi \sin^2 2\phi \sin^2 \left(\frac{1.27 \delta_{31} L}{E}\right),
\]
(5)
where $\delta_{31}$ is in eV$^2$, the baseline length $L$ is in km and the neutrino energy $E$ is in GeV.
For low energies the phase $1.27\delta_{31} L/E$ oscillates rapidly and $P_{\mu e}$ becomes insensitive to it.
However, for the range of energies for which the phase is close to $\pi/2$, $P_{\mu e}$ varies slowly as
shown in figures (1)-(3). If the spectrum of the neutrinos in long baseline experiments peaks
in this range, then $\delta_{31}$ and the mixing angles can be determined quite accurately.

Given the CHOOZ constraint on $\phi$, Super-Kamiokande atmospheric neutrino data fix
$\sin^2 2\psi \simeq 1$ or $\sin^2 \psi \simeq 0.5$. Substituting this value in equation (3), we find that $P_{\mu e} \leq 0.1$.
The neutrino beams at K2K and MINOS have a 1% $\nu_e$ contamination. This background
and the systematic uncertainties set a limit to the smallest value of $P_{\mu e}$ (and $\phi$) that can
be measured. With the current estimates of the systematic uncertainties, the sensitivity of K2K is similar to that of CHOOZ \((\sin^2 2\phi \sim 0.2)\) but MINOS is capable of measuring values of \(\phi\) as small as \(3^\circ (\sin^2 2\phi \geq 0.01)\) \cite{2}. Once K2K starts running, its systematics will be better understood and its sensitivity is likely to improve.

In both K2K and MINOS experiments, the neutrino beam travels through earth’s crust, where it encounters matter of roughly constant density \(3 \text{ gm/cc}\). This traversal leads to the addition of the Wolfenstein term to the \(e - e\) element of the \((\text{mass})^2\) matrix, when it is written in the flavor basis \cite{13}. The Wolfenstein term is given by

\[
A = 0.76 \times \rho(\text{gm/cc}) \times E(\text{GeV}) \times 10^{-4} \text{ eV}^2. \tag{6}
\]

For \(\rho\) of a few gm/cc and \(E\) of a few GeV, \(A\) is comparable to the value of \(\delta_{31}\) set by atmospheric neutrino data. This can lead to interesting and observable matter effects in long baseline experiments. The interactions of \(\nu_\mu\) and \(\nu_\tau\) with ordinary matter are identical. Hence equation (6) can be used to include matter effects in a simple manner because the problem, once again, is essentially a two flavor one. It is easy to see that \(\psi\) is unaffected by matter and the matter dependent mixing angle \(\phi_m\) and the mass eigenvalues \(m_1^2\) and \(m_3^2\) are given by \cite{8}

\[
\tan 2\phi_m = \frac{\delta_{31} \sin 2\phi}{\delta_{31} \cos 2\phi - A}, \tag{7}
\]

\[
m_1^2 = \frac{1}{2} \left[ (\delta_{31} + A) - \sqrt{(\delta_{31} \cos 2\phi - A)^2 + (\delta_{31} \sin 2\phi)^2} \right], \tag{8}
\]

\[
m_3^2 = \frac{1}{2} \left[ (\delta_{31} + A) + \sqrt{(\delta_{31} \cos 2\phi - A)^2 + (\delta_{31} \sin 2\phi)^2} \right]. \tag{9}
\]

The above equations hold for the propagation of neutrinos. For vacuum propagation, the mass eigenvalues and mixing angles for neutrinos and anti-neutrinos are the same. However, to include matter effects for anti-neutrinos, one should replace \(A\) by \(-A\) in equations (7), (8) and (9). Note that, since \(A\) is always positive, matter effects enhance the neutrino mixing angle and suppress the anti-neutrino mixing angle if \((\delta_{31} \cos 2\phi)\) is positive and vice-verse if \((\delta_{31} \cos 2\phi)\) is negative. Since we have taken \(\delta_{31}\) to be positive, matter effects enhance neutrino mixing if \(\cos 2\phi\) is positive or \(\phi < \pi/4\). The CHOOZ constraint from equation (3)
on $\sin^2 2\phi$ sets the limit $\phi \leq 13^\circ$ or $\phi \geq 77^\circ$. For the latter possibility, $\cos 2\phi$ is negative. However, this large a value of $\phi$ is forbidden by the three flavor analysis of solar neutrino data, which yields the independent constraint $\phi \leq 50^\circ$ \cite{1,2}. Hence the enhancement of the mixing angle will be for neutrinos rather than for anti-neutrinos. This is good news because the beams in long baseline experiments consist of neutrinos overwhelmingly.

The matter dependent oscillation probability can be calculated in a straight forward manner from equation (4) in terms of $\psi, \phi_m$ and $\delta_{31}^m = m_3^2 - m_1^2$.

$$P_{\mu e}^m = \frac{1}{2} \sin^2 2\phi_m \sin^2 \left( \frac{1.27\delta_{31}^m L}{E} \right),$$  \hspace{1cm} (10)

where we have substituted the Super-Kamiokande best fit value $\sin^2 \psi = 1/2$. As discussed above, the matter effects enhance the mixing angle for neutrinos and the matter modified oscillation probability $P_{\mu e}^m$ will be greater than the vacuum oscillation probability $P_{\mu e}$, for a range of energies. For some neutrino energy, the Mikheyev-Smirnov resonance condition \cite{14}

$$\delta_{31} \cos 2\phi = A$$  \hspace{1cm} (11)

can be satisfied and near this energy $\phi_m \sim \pi/4$. Unfortunately, this does not lead to any dramatic increase in $P_{\mu e}^m$ because, near the resonance, when $\sin^2 2\phi_m$ is maximized, the matter dependent mass-squared difference $\delta_{31}^m$ is minimized \cite{15}. In fact, near the resonance, $P_{\mu e}^m \geq P_{\mu e}$. By differentiating equation (11) with respect to $E$, we can calculate the energy at which $P_{\mu e}^m$ is maximized. The highest energy at which this occurs is just below the highest energy at which $P_{\mu e}$ is maximised, that is the energy for which the phase $1.27\delta_{31} L/E = \pi/2$. At this energy, the $\phi_m$ is significantly higher than $\phi$ and hence $P_{\mu e}^m$ will be measurably higher than $P_{\mu e}$.

Since the baseline of K2K is 3 times smaller than that of MINOS, the energy where the its phase becomes $\pi/2$ is smaller by a factor of 3 compared to similar energy for MINOS. Because the highest energy maximum (at which the phase is $\pi/2$) occurs at different energies, the increase in $\nu_\mu \rightarrow \nu_e$ oscillation probability, due to the energy dependent enhancement of the
mixing angle, is different for the two experiments. We find that, at their respective highest energy maxima, $P_{\mu e}^m \simeq 1.1 P_{\mu e}$ for K2K and $P_{\mu e}^m = 1.25 P_{\mu e}$ for MINOS. This is illustrated in figure (1) for $\phi \simeq 12.5^o$ (which is just below the CHOOZ limit) and $\delta_{31} = 2 \times 10^{-3}$ eV$^2$ (which is the best fit value for Super-Kamiokande atmospheric neutrino data). This conclusion is independent of the value of $\delta_{31}$ and is illustrated in figures (1)-(3), for different values of $\delta_{31}$ (and $\phi = 12.5^o$). A similar conclusion was obtained in a recent paper, where it was demonstrated that the relation between $P_{\mu e}^m$ and $P_{\mu e}$ is almost independent of $\phi$.[16]

We wish to emphasize that data from at least two different experiments with different baselines is needed to state unambiguously, whether matter effects are playing a role in neutrino oscillations. Suppose we have data from only one experiment. We can obtain allowed values of $\delta_{31}$ and $\phi$ by fitting either $P_{\mu e}$ or $P_{\mu e}^m$ to this data. The two analyses will give different values of mixing angle. Since we don’t apriori know the value of vacuum mixing angle, we can’t say which result is correct. However, as mentioned above, matter effects lead to different enhancement of oscillation probability for K2K and MINOS as shown in figure (1). This difference can be exploited to distinguish matter effects. The data from each experiment, K2K and MINOS, should be analyzed twice, once using $P_{\mu e}$ as the input and the second time with $P_{\mu e}^m$ as the input. In the first analysis, the allowed value of the mixing angle from MINOS will be significantly higher than from K2K, if the matter effects are important. Matter effects can be taken to be established, if the second analysis gives the same values of $\phi$ for both K2K and MINOS.

In conclusion, we find that matter effects enhance $\nu_\mu \rightarrow \nu_e$ oscillations at long baseline experiments K2K and MINOS. This enhancement can be large enough to be observable. However, one must combine the data from both experiments before making a definite statement about the effect of the matter term.

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FIG. 1. Plots of $P_{\mu e}^m$ for MINOS (solid line), $P_{\mu e}$ for MINOS (long-dashed line), $P_{\mu e}^n$ for K2K (short-dashed line), $P_{\mu e}$ for K2K (dotted line) vs $E$ for $\delta_{31} = 2 \times 10^{-3}$ eV$^2$ and $\phi = 12.5^\circ$.

FIG. 2. Plots of $P_{\mu e}^m$ (solid line), $P_{\mu e}$ (long-dashed line) vs $E$ for MINOS $\delta_{31} = 6 \times 10^{-3}$ eV$^2$, and $\phi = 12.5^\circ$. 
FIG. 3. Plots of $P_{\mu e}^m$ (solid line), $P_{\mu e}$ (long-dashed line) vs $E$ for MINOS $\delta_{31} = 10^{-2} \text{ eV}^2$, and $\phi = 12.5^o$. 