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On generators and relations for higher level Zhu algebras and applications. (English)
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Summary: We give some general results about the generators and relations for the higher level Zhu algebras for a vertex operator algebra. In particular, for any element $u$ in a vertex operator algebra $V$, such that $u$ has weight greater than or equal to $-n$ for $n \in \mathbb{N}$, we prove a recursion relation in the $n$th level Zhu algebra $A_n(V)$ and give a closed formula for this relation. We use this and other properties of $A_n(V)$ to reduce the modes of $u$ that appear in the generators for $A_n(V)$ as long as $u \in V$ has certain properties (properties that apply, for instance, to the conformal vector for any vertex operator algebra or if $u$ generates a Heisenberg vertex subalgebra), and we then prove further relations in $A_n(V)$ involving such an element $u$. We present general techniques that can be applied once a set of reasonable generators is determined for $A_n(V)$ to aid in determining the relations of those generators, such as using the relations of those generators in the lower level Zhu algebras and the zero mode actions on $V$-modules induced from those lower level Zhu algebras. We prove that the condition that $(L(-1) + L(0))v$ acts as zero in $A_n(V)$ for $n \in \mathbb{Z}_+$ and for all $v$ in $V$ is a necessary added condition in the definition of the Zhu algebra at level higher than zero. We discuss how these results on generators and relations apply to the level $n$ Zhu algebras for the Heisenberg vertex operator algebra and the Virasoro vertex operator algebras at any level $n \in \mathbb{N}$.

MSC:
17B68 Virasoro and related algebras
17B69 Vertex operators; vertex operator algebras and related structures
81R10 Infinite-dimensional groups and algebras motivated by physics, including Virasoro, Kac-Moody, $W$-algebras and other current algebras and their representations
81T40 Two-dimensional field theories, conformal field theories, etc. in quantum mechanics

Keywords:
vertex operator algebras; conformal field theory; Virasoro algebra

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