Static balancing of flexural pivots with two symmetrically arranged pre-compressing springs

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Abstract. To overcome the spring back behavior of the flexural pivots, a novel statically balanced flexural pivot with two symmetrically arranged pre-compressing springs (SBFP-2PCS) is proposed. Five balancing design parameters and two design objectives of the SBFP-2PCS are specified. The design parameters are determined based on the stiffness analysis of the SBFP-2PCS. Samples of the SBFP-2PCS are built to verify the proposed balancing method. The test result of the average stiffness reduction is 92.5%. The proposed method is general for various flexural pivots with low axis drift.

1. Introduction

Flexural pivots transfer force or motion through elastic deformation and are widely used in high-precision instrumentations and flexible robots as compliant revolute joints [1-3]. However, flexural pivots have spring back behavior that may increase power requirements and diminish effective force feedback [4]. Static balancing is one strategy to overcome this disadvantage [5-10] and is already used in applications such as surgical instruments [11].

A flexural pivot is statically balanced if its elastic potential energy is constant in rotation [5]. This type of pivot is also called zero-stiffness flexural pivot (ZSFP). A statically balanced flexural pivot (SBFP) with zero stiffness can be designed by adding a balancing mechanism with negative stiffness into a flexural pivot with positive stiffness [6-10]. Several types of SBFP (i.e. ZSFP) have been proposed by adding different balancing mechanisms to various flexural pivots. Based on conventional zero stiffness pivots, Morsch et al. proposed a ZSFP by substituting a cross-axis flexural pivot (CAFP) with a conventional pivot, and a pair of compliant leaf springs with two springs [6]. This method is well-understood but requires optimization, and the proposed ZSFP has insufficient structural compactness and zero stiffness quality. Merriam et al. proposed a statically balanced flexural pivot with one pre-extending spring (SBFP-PES) [7]. The proposed SBFP-PES has a simple structure and satisfactory zero stiffness quality, but correction of the flexural pivot stiffness characteristics is required. Based on the building block approach, Bi et al. designed a ZSFP by using an inner-outer ring flexural pivot (IORFP) as a positive stiffness subsystem and a spring-crank mechanism as a negative stiffness subsystem [8]. Their method is convenient, the proposed ZSFP is satisfactory in zero stiffness quality, but the structure is rather complex. Liu et al. developed a ZSFP by combining an IORFP with a spring four-bar linkage (4BSL) [9]. The proposed ZSFP-4BSL has excellent zero stiffness quality but complex structure. Zhao et al. designed a near-zero stiffness rotational flexural pivot (NZSRFP) by using two pairs of leaf-springs to balance the generalized cross-spring pivot (GCSP) [10]. Their proposed NZSRFP can be applied to complex load conditions but is has insufficient compactness and
zero stiffness quality. Among the designs mentioned above, the SBFP-PES has the simplest structure. The ZSFP-4BSL has the most excellent zero stiffness quality (95%).

This paper designs a SBFP with simple structure and excellent zero stiffness quality by adding two symmetrically arranged pre-compressing springs to the GCSP. This design is novel because (1) it applies pre-compressing springs to the design of the SBFP, and (2) it proposes the new design of symmetrically arranged springs. Correspondingly, (1) a statically balanced flexural pivot with one pre-compressing spring (SBFP-PCS) is analyzed, and (2) a statically balanced flexural pivot with two symmetrically arranged pre-compressing springs (SBFP-2PCS) is proposed to achieve higher performance of the SBFP-PCS. Moreover, five balancing design parameters and two design objectives of the SBFP-2PCS are specified. The design process of the SBFP-2PCS and its stiffness characteristics are analyzed. The samples are built and tested. A practical design solution is proposed for application in precision machinery and flexible robots.

2. Static balancing design parameters and objectives
The static balancing approach of adding one pre-loaded spring is analyzed. Both the pre-extending spring and the pre-compressing spring are shown to qualify, and the balancing design parameters and objectives of the SBFP-PES and the SBFP-PCS are specified.

2.1. Static balancing design parameters
The SBFP with rotational stiffness \( k \) is combined of a flexural pivot with rotational stiffness \( k_\theta \) and a pre-loaded spring with stiffness \( k_l \). Figure 1 and Figure 2 illustrate the details.

![Figure 1. The SBFP-PES and its associated variables.](image1)

![Figure 2. The SBFP-PCS and its associated variables.](image2)

The SBFP rotates an angle \( \theta \) under a driving torque \( M = k\theta \). The rotational range of the SBFP is \( \theta \in [-\theta_{max}, \theta_{max}] \) and \( \theta_{max} \) is the maximum rotational angle to keep away from material failure. \( C \) is the rotational center of the SBFP. \( A_f \) and \( A_{m0} \) are the attachment points of the pre-loaded spring on the fixed rigid body and the motion rigid body. Vectors \( CA_f \), \( CA_{m}\), and \( AfAm_0 \) are obtained.

The distance \( d \) between the attachment point \( A_f \) and the rotational center \( C \) is

\[
d = \| CA_f \|
\]

(1)

The range of \( d \) is often determined by the size of the flexural pivot.

Figure 1(a) and Figure 2(a) show that when the rotational angle \( \theta \) of the SBFP is zero, \( CA_f \) and \( CA_{m0} \) are colinear. Define \( \xi \) as the attachment coefficient, there is

\[
CA_{m0} = \xi CA_f
\]

(2)

\( \xi \in [-2, 0) \cup (0, 1) \) is suggested in the application. When the SBFP rotates from \( 0^\circ \) to \( \theta_{max} \), the elastic potential energy of the flexural pivot is stored, and the elastic energy of the pre-loaded spring is released. Specifically, when \( \theta \) of the SBFP is zero, the deflection of the pre-loaded spring is maximum. Thus, the pre-loaded spring is a pre-extending spring when \( \xi < 0 \) (Figure 1), and a pre-compressing
spring when $\xi > 0$ (Figure 2). In reference [7] only $\xi = -1$ is considered. This paper verifies that adding one pre-compressing spring is viable in the design of the SBFP.

$x_f$ is the free length of the pre-loaded spring. When the SBFP rotates $\theta$, the length of the pre-loaded spring is $x_\theta$ and $x_\theta = |A_F A_a|$. When $\theta = 0^\circ$, the length of the pre-loaded spring is $x_0$ and $x_0 = |A_F A_a|$. The relative deformation $\delta$ of the pre-loaded spring is

$$\delta = \frac{x_f - x_0}{x_f}$$

When the SBFP rotates from $0^\circ$ to $\theta_{max}$, the pre-loaded spring is constantly deflected to provide a driving torque, i.e. $|x_f - x_0| > |x_{\theta_{max}} - x_0|$. Thus, $\delta > (|x_{\theta_{max}} - x_0| / x_{\theta_{max}})$.

In sum, five parameters, $k_\theta$, $k_l$, $\delta$, $\xi$, and $d$ should be determined in the balancing design of the SBFP-PES and the SBFP-PCS. $k_\theta$ indicates the stiffness characteristics of the flexural pivot, $k_l$ indicates the stiffness characteristics of the pre-loaded spring, and $\delta$, $\xi$, and $d$ indicate the location of the attachment points of the pre-loaded spring.

2.2. Static balancing design objectives

2.2.1. High stiffness reduction. The rotational stiffness of the $k$ SBFP is often close but not equal to zero due to friction, manufacture error and so on. Compared to the $k_\theta$ of the unbalanced flexural pivot, the rotational stiffness $k$ of the SBFP is significantly reduced. Define $\eta$ as the stiffness reduction coefficient of the SBFP, there is

$$\eta = (1 - |k/k_\theta|) \times 100\%$$

$\eta$ is the most important criterion of the SBFP. The closer $\eta$ is to 100%, the higher the zero-stiffness quality. The most important design objective of the SBFP is a high average stiffness reduction throughout the rotational range [6-10].

2.2.2. Low preload. Figures 1(c) and 2(c) show the force analysis of the motion rigid body of the flexural pivot. When the SBFP rotates $\theta$, the force applied to the motion rigid body of the flexural pivot by the pre-loaded spring is $P_\theta$. $P_\theta$ and the preload $P_0$ are directly proportional. The mechanics explanation of adding one pre-loaded spring to achieve static balancing is that $P_\theta$ produces an internal driving torque that reduces the external driving torque required by the SBFP. However, $P_\theta$ introduces three negative effects [12]. (1) $P_\theta$ may alter $k_\theta$, which requires a correction to $k_\theta$ before the design of the SBFP. (2) $P_\theta$ may cause internal stress, buckling, etc., and reduce the performance of the SBFP. (3) $P_\theta$ causes friction between the springs and their assemble bolts, which affects the zero-stiffness quality. Therefore, a low preload $P_0$ is also a design objective of the SBFP.

3. Static balancing of the SBFP-2PCS

To achieve static balancing, the pre-loaded spring needs only to apply an appropriate internal driving torque to the flexural pivot. A static balancing method of combining two symmetrically arranged pre-compressing springs with the GCSP is proposed, and the SBFP-2PCS is designed (Figure 3).

Figure 3. The SBFP-2PCS and its associated variables. (a) The SBFP-2PCS. (b) The GCSP. (c) Two symmetrically arranged pre-compressing springs.
The SBFP-2PCS avoids the negative effects such as stiffness correction, internal stress, etc. caused by \( P_\theta \). However, the friction caused by \( P_\theta \) is unresolved. Thus, the design objectives of the SBFP-2PCS are the same as those of the SBFP-PCS: high stiffness reduction \( \eta \) and low spring preload \( P_0 \).

Figure 3 demonstrates that this design requires the flexural pivot center to be the rotational center, i.e. the flexural pivot has negligible axis drift.

The static balancing of the SBFP-2PCS is achieved through the matching of the stiffness of the GCSP and the two pre-compressing springs. Three steps are taken as shown in sections 3.1 to 3.3.

### 3.1. Rotational stiffness of the flexural pivot

In Figure 2(b), the stiffness model of the GCSP under pure torque [2] is

\[
k_\theta = 8\left(3\lambda^2 - 3\lambda + 1\right) \frac{EI}{L}
\]

In Equation (5), \( \lambda \) is the geometrical parameter of the intersection of the flexure strips. \( E \) is Young’s modulus of the flexure material. \( I \) is the moment of inertia of the flexure strip and \( I = WT^3/12 \). \( W \) is the width of the flexure strip. \( T \) is the thickness of the flexure strip. \( L \) is the length of the flexure strip.

When the intersection of the flexure strips is set at 12.73%, i.e. \( \lambda = 0.1273 \), the axis drift of the flexural pivot under pure torque is approximately zero [3].

### 3.2. Stiffness characteristics of the pre-compressing spring

Figure 3(a) shows the length of the spring \( x_0 \) when the SBFP-2PCS rotates \( \theta \), \( x_0 \) when the SBFP-2PCS rotates 0°, and the free length \( x_f \). There are

\[
x_0 = d(1 - \xi), \quad x_0 = d\sqrt{\xi^2 + 1 - 2\xi \cos \theta}, \quad x_f = d\frac{1 - \xi}{1 - \delta}
\]

Figure 3(c) shows the stiffness model of the SBFP-2PCS. There is

\[
M = k\theta = k_\theta \theta - \frac{2\xi d^2 \sin \theta}{x_0} P_\theta = k_\theta \theta - \frac{2\xi d^2 \sin \theta}{x_0} k_1 (x_f - x_0)
\]

To achieve perfect static balancing, i.e. \( M = k\theta = 0 \) in Equation (7), the ideal stiffness of the pre-compressing spring \( k_{\text{ideal}} \) is

\[
k_{\text{ideal}} = k_\theta \frac{x_0}{2\xi d^2 x_f - x_0 \sin \theta} \frac{\theta}{1 - \delta} = k_\theta \frac{\sqrt{\xi^2 + 1 - 2\xi \cos \theta}}{1 - \delta} \frac{\theta}{\sqrt{\xi^2 + 1 - 2\xi \cos \theta} \sin \theta}
\]

\( k_{\text{ideal}} \) varies with \( \theta \). Figure 4 shows the relationship of \( k_{\text{ideal}} \) and \( \theta \) when \( d = 40.0 \) mm and \( k_\theta = 0.261 \) Nm/rad. Figure 5 shows the relationships of \( \eta \) to \( \theta \), and is used in section 3.3.1.

![Figure 4](image-url)  
**Figure 4.** The ideal stiffness \( k_{\text{ideal}} \) as functions of \( \theta \) for different \( \xi \) and \( \delta \) when \( d = 40.0 \) mm and \( k_\theta = 0.261 \) Nm/rad.

![Figure 5](image-url)  
**Figure 5.** The stiffness reduction \( \eta \) as functions of the rotational angle \( \theta \).
In practice, constant stiffness linear springs are often used as pre-compressing springs. $k_i$ is the constant stiffness of the pre-compressing spring. Two conclusions should be considered in choosing $k_i$.

1. The function of $k_{ideal}$ to $\theta$ monotonically increases.
2. To achieve a stable system, $k$ should be no less than zero. A negative stiffness system is unstable. When $\theta = 0^\circ$, if $k < 0$, the flexural pivot rotates under a minor perturbation. According to Equation (7), $k_i$ should be determined as the minimum $k_{ideal}$ in the rotational range.

Bring $\theta = 0^\circ$ into Equation (8). The constant stiffness $k_i$ of the pre-compressing spring is

$$k_i = \frac{x_0}{x_f - x_0} \frac{k_d}{2\xi d^2} = \frac{1-\xi}{\delta} \frac{k_d}{2\xi d^2}$$

(9)

3.3. Location of the attachment points of the pre-compressing spring

3.3.1. Analysis of the stiffness reduction. Bring Equations (7) and (9) into Equation (4). Because $k \geq 0$, the stiffness reduction $\eta$ of the SBFP is

$$\eta = \left(1 - \frac{k}{k_{ideal}}\right) \times 100\% = \frac{k_i}{k_{ideal}} \times 100\% = \left[1 - \frac{\sqrt{\xi^2 + 1 - 2\xi \cos \theta - (1 - \xi)}}{\delta \sqrt{\xi^2 + 1 - 2\xi \cos \theta}}\right] \sin \theta \times 100\%$$

(10)

Equation (10) indicates that $\eta$ equals to the ratio of $k_i$ to $k_{ideal}$. $\eta$ is only related to $\theta$, $\delta$, and $\xi$. Figures 5 to 7 show the relationships of $\eta$ to $\theta$, $\delta$, and $\xi$, respectively. Three conclusions are drawn.

1. The larger $\theta$ is, the lower $\eta$ is. Specifically, $\eta = 100\%$ when $\theta = 0^\circ$.
2. The larger $\delta$ is, the higher $\eta$ is. But a larger $\delta$ means larger internal stress, so $\delta < 0.5$ is suggested. Moreover, $\delta > (|x_{max} - x_0| / x_{max})$ is required (see section 2.1). For example, when $\theta_{max} = 21^\circ$ and $\xi = 0.2$, $\delta$ should be larger than 0.02. With the above considerations, $\delta = (1/3)$ is decided.
3. With the increase of $\xi$, $\eta$ decreases significantly. To achieve a higher $\eta$, $\xi < 0.3$ is suggested.

Figure 6. The stiffness reduction $\eta$ as functions of the relative deformation $\delta$ when $\theta$ is $21^\circ$.

Figure 7. The stiffness reduction $\eta$ as functions of the attachment coefficient $\xi$ when $\theta$ is $21^\circ$.

3.3.2. Analysis of the spring preload. When $\theta = 0^\circ$, the spring preload $P_0$ is

$$P_0 = k_i (x_f - x_0) = \left(\frac{1}{\xi} - 1\right) \frac{k_d}{2d}$$

(11)

$P_0$ is related to $k_{0k}$, $k_i$, $\xi$, and $d$. When $k_0$ and $k_i$ are determined (sections 3.1 and 3.2), two conclusions are drawn.

1. $P_0$ increases when $\xi$ decreases. To achieve a lower $P_0$, $\xi = 0.2$ is decided. Figure 5 shows that when $\xi = 0.2$ and $\delta = (1/3)$, the model result of the average stiffness reduction is 96.9% within the available stroke ($\pm 21^\circ$).
(2) $P_0$ is inversely proportional to $d$. To achieve a lower $P_0$, a relatively large $d$ should be chosen within the application space.

4. Test and verify of the samples

4.1. Parameter design of the SBFP-2PCS

4.1.1. Parameter design of the GCSP. The GCSP, whose $\lambda$ is 0.1273 and $\alpha$ is 60°, is chosen. $\alpha$ is half the intersection angle of the GCSP. The rotational range of the GCSP is designed as $\pm 21°$. Table 1 shows the parameters properties of the GCSP when the pivot is applied with a rotational torque.

| Material     | $\alpha$ (degree) | $\lambda$ | $E$ (GPa) | $L$ (mm) | $W$ (mm) | $T$ (mm) | $k_0$ (N·m/rad) |
|--------------|-------------------|-----------|-----------|---------|---------|---------|-----------------|
| AL7075-T6    | 60                | 0.1273    | 73        | 46.0    | 9.40    | 0.340   | 0.261           |

4.1.2. Parameter design of the GCSP. When parameters $k_0$, $d$, $\delta$, and $\xi$ are decided (see section 3.3), $k_i$ of the pre-compressing spring can be obtained from Equation (9). Table 2 shows the parameters properties of the two pre-compressing springs.

| $\delta$ | $\xi$ | $d$ (mm) | $k_i$ (N/m) | $P_0$ (N) |
|----------|-------|---------|-------------|-----------|
| 1/3      | 0.2   | 40.0    | 814.31      | 13.03     |

A diamond leaves string (DLS) that can be used as a linear stiffness compression spring with no guiding mechanism is designed in reference [8]. The DLS is used as the pre-compressing spring in this paper. Figure 8 provides the details about the DLS.

![Figure 8](image)

When $(T_d/L_d) << 1$ and $P_0\sin\alpha_d << 20$, the stiffness of the DLS has quasi-linear characteristics [8]

$$ k_i = \frac{P_0}{x_f - x_0} = \frac{12E I_d}{n L_d^3 \cos^2 \alpha_d} $$

(12)

In Equation (14), $E$ is Young’s modulus of the flexure material. $I_d$ is the moment of inertia of the leaves and $I_d = W_d T_d^3 / 12$. $W_d$ is the width of the leaves. $T_d$ is the thickness of the leaves. $L$ is the length of the leaves. $\alpha_d$ is the intersection angle between the undeformed leaves and the horizontal axis. $n$ is the string number of the diamond leaves.

When the parameters $n$, $L_d$, $W_d$, and $\alpha_d$ of the DLS are decided, $T_d$ can be obtained from Equation (12). Table 3 shows the parameter properties of the DLS.
Table 3. The parameters properties of the DLS.

| Material       | \(x_f\) (mm) | \(n\) | \(\alpha_d\) (degree) | \(L_d\) (mm) | \(W_d\) (mm) | \(T_d\) (mm) | \(E\) (GPa) |
|----------------|--------------|-------|------------------------|--------------|--------------|--------------|-------------|
| AL7075-T6      | 48           | 4     | 8                      | 20           | 9.40         | 0.334        | 73          |

4.2. Tests of the samples

Similar to the manufacturing method of the generalized triple-cross-spring flexure pivot proposed in [13], a laminated assemble scheme is used to the manufacture of the GCSP. Figure 9 shows the samples of the GCSP and the SBFP-2PCS. Figures 9(a) and 9(c) show the sample of the laminated sheet and two samples of the DLS manufactured by a high precision wire cutting machine (CHARMILLES ROBOFIL 380, Switzerland), respectively. Figure 9(b) shows the assembled sample of the GCSP. Figure 9(d) shows the sample of SBFP-2PCS, where two samples of the DLS are combined with the GCSP through four bolts.

Figure 9. The samples of the GCSP and the SBFP-2PCS. (a) The sample of the laminated sheet. (b) The sample of the GCSP. (c) Two samples of the DLS. (d) The sample of the SBFP-2PCS.

Figure 10 illustrates the stiffness test platform of the GCSP and the SBFP-2PCS. The platform is similar to that of the generalized triple-cross-spring flexure pivot [13]. The test platform is mainly composed of an angle measuring turntable, a torque sensor (Interface, USA), and a digital display instrument. The reading accuracy of the angle measuring turntable is 2′. The measuring range of the torque sensor is 0.2 Nm, whose maximum error is 0.1% of full scale.

Figure 10. The stiffness test platform of the GCSP and the SBFP-2PCS.

Figure 11. Comparison of the rotational stiffness between the test and the model.

Figure 12. Comparison of the stiffness reduction of the SBFP-2PCS between the test and the model.
Figure 11 shows the rotational stiffness test results of the GCSP and the SBFP-2PCS. The stiffness model result of the GCSP is 0.2606 Nm/rad, and its stiffness test result is 0.258 Nm/rad. The results of the stiffness model and the test are almost the same, with a relevant error of only 1.1%.

According to Figure 11 and Equation (4), the stiffness reduction of the SBFP-2PCS is obtained. Figure 12 provides the details. Within the available stroke (± 21°), the model result of the average stiffness reduction is 96.9%. The test results show that the stiffness of the SBFP-2PCS is reduced by 92.5% compared with the stiffness of the GCSP. The relevant error between the model result and the test result is 4.5%. The validity of the proposed static balancing method is verified.

Four reasons are identified to explain the error.

1) Friction. The spring preload is \( P_0 = 13 \) N in the designed SBFP-2PCS. Friction between the pre-compressing springs and their assembled bolts exists.

2) Inadequate manufacture precision. Due to limited manufacture precision, error exists in the size of flexures (especially the thickness), which causes error in stiffness. The stiffness test of the DLS is carried out by a force sensor and a mobile platform. The stiffness test results of the two samples of the DLS are 803.0N/m and 804.7N/m. The relevant errors from the model result 814.3N/m are 1.4% and 1.2%, respectively. A relevant error of 1.1% is also observed between the stiffness test result and the model result of the GCSP. These three errors are caused by inadequate manufacture precision.

3) Inadequate equipment accuracy. The rotational torque applied to the SBFP-2PCS is less than 0.015 Nm when \( \theta = 21^\circ \). The accuracy of the torque sensor is not enough when the angle is small.

4) Axis drift of the GCSP. Axis drift, though very low, exists when the flexural pivot rotates [3].

5. Conclusion

In this paper, the static balancing method of combining two symmetrically arranged pre-compressing springs with a flexural pivot is proposed and verified. Five design parameters and two design objectives of the SBFP-2PCS are specified. Design parameters are determined from the stiffness analysis of the SBFP-2PCS. The model result of the average stiffness reduction is 96.9% within the available stroke (± 21°). The proposed method is general for various flexural pivots with low axis drift.

To verify the proposed balancing method, samples of the SBFP-2PCS and the GCSP are built and tested. The test results show that the stiffness of the SBFP-2PCS is reduced by 92.5% compared with the stiffness of the GCSP. A relevant error of 4.5% between the test result and the model result verifies the accuracy of the proposed method. Compared to the works mentioned in the introduction, this paper is more specified in design parameters and objectives. The novel design of symmetrically arranged auxiliary springs is proposed, which successfully avoids some negative influences auxiliary springs apply to flexural pivots. The proposed SBFP-2PCS has simple structure and only one more spring than the SBFP-PES [7] (the simplest structure mentioned). The proposed SBFP-2PCS has excellent zero stiffness quality, which is only 2.5% lower than that of the ZSF-4BSL [9] (the most excellent zero stiffness quality mentioned). The SBFP-2PCS has an excellent reference value for application in low-frequency flexural joints, precision machinery, flexible robots, drive mechanism and so on.

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