Melting of nano-enhanced PCM inside finned radiator

N S Bondareva, N S Gibanov, M A Sheremet
Tomsk State University, Russia, Tomsk, Lenin str., 36, 634050

E-mail: bondarevans@mail.tsu.ru

Abstract. The two-dimensional melting process of paraffin enhanced by Al₂O₃-nanoparticles inside a copper radiator with various frequency of finning is studied numerically. The governing equations have been formulated in dimensionless stream function, vorticity and temperature. The obtained system of partial differential equations has been solved using the finite difference method. The influence of the frequency of the fins location on the melting process of paraffin with different nanoparticles volume fraction has been studied.

1. Introduction

In recent years phase change materials (PCM) have become more important in industries: paraffin increasingly is used for storage and transportation of energy in heat pipes, the use of paraffin in building blocks allows saving on air conditioning systems, in electronic devices and radio equipment a PCM is used for cooling [1–3].

The ways to increase efficiency of passive cooling system based on PCM are using of radiators with fins and metal foam. Also, it was shown that nanoparticles with increased thermal conductivity can be added to the material for the intensification of heat transfer [3–5]. It was experimentally obtained [4] that thermal conductivity increases by 48% and 67% by adding of 10% and 20% of nanomagnetite, respectively. However along with the intensification of heat conduction the viscosity increases and as a consequence an attenuation of convective heat transfer in melt occurs [6]. Therefore, the interaction of these two factors requires a more detailed numerical analysis. It should also be noted that the complex shape of the profile also has a significant impact on fluid dynamic in melt and heat transfer mode in the whole cavity [7, 8].

The aim of this work is numerical study of heat transfer inside the system containing the finning profile with various numbers of fins and filled with nano-enhanced paraffin.

2. Physical statement and mathematical model

The present paper is devoted to the numerical study of heat and mass transfer during the melting process inside heatsink of sizes (2H×H) based on copper profile filled with nano-enhanced phase change material (NePCM). NePCM is a mixture of the paraffin (n-octadecane) with Al₂O₃ nanoparticles. The source of constant volumetric heat generation is located in the lower part of the considered area. The fins height is equal to 0.6H and their number varied from 0 to 10. Initial temperature of system was equal to the outside temperature. The melt is considered to be a Newtonian fluid with Boussinesq approximation and the flow mode is laminar.
Equations of liquid flow and heat transfer in the variables "velocity – pressure – enthalpy" have the following form:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_m} \frac{\partial p}{\partial x} + \frac{\mu_m}{\rho_m} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right),
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_m} \frac{\partial p}{\partial y} + \frac{\mu_m}{\rho_m} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + (p\beta_m)_{nm} g(T - T_m).
\]

The energy equations for the liquid and solid paraffin have the following form:

\[
\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + v \frac{\partial h}{\partial y} = k_m \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]

\[
\frac{\partial h}{\partial t} = (k_i)_{nm} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right).
\]

Inside the profile and the heat source, the energy equation was solved taking into account the thermal properties of used materials (copper and silicon) and the magnitude of internal volumetric heat generation \(Q\).

The thermophysical properties of the nanomaterial were determined from the following relationships [5]:

\[
(p\beta)_{nm} = (1 - \Phi)(p\beta)_m + \Phi(p\beta)_sp, \quad (pc)_{nm} = (1 - \Phi)(pc)_m + \Phi(pc)_sp, \quad \rho_{nm} = (1 - \Phi)\rho_m + \Phi\rho_{sp},
\]

\[
c_{nm} = (pc)_{nm}/\rho_{nm}, \quad L_{nm} = (1 - \Phi)\rho_L/L_{nm}, \quad \mu_{nm} = 0.983e^{2.959\Phi}\mu_m.
\]

The thermal conductivity for nanomaterial can be determined as [9]:

\[
k_{nm} = \frac{k_{sp} + 2k_m - 2(k_m - k_{sp})\Phi}{k_{sp} + 2k_m + (k_m - k_{sp})\Phi} k_m + 5 \cdot 10^5 \beta_k \Phi \rho_m c_m \sqrt{\frac{k T_j}{\rho_p d_{sp} f(T, \Phi)}}
\]

where \(\beta_k = 8.4407(100\Phi)^{1.07304}\), \(\kappa = 1.381 \cdot 10^{-23} J/K\) is the Boltzmann constant, \(f(T, \Phi) = (2.817 \cdot 10^{-2}\Phi + 3.917 \cdot 10^{-3})T_j/T_0 + (-3.0669 \cdot 10^{-2}\Phi - 3.91123 \cdot 10^{-3})\) for the melt, \(T_0 = 273^\circ K, T_j = 320^\circ K\), for the solid paraffin \(f(T, \Phi) = 0\).
The governing equations of liquid flow were solved using the stream function \( \psi \) (\( u = \partial \psi / \partial y, \nu = -\partial \psi / \partial x \)), vorticity \( \omega \) (\( \omega = \partial v / \partial x - \partial u / \partial y \)), and the following dimensionless parameters:

\[ X = x/H, \quad Y = y/H, \quad U = u/V_0, \quad V = v/V_0, \quad V_0 = \sqrt{g\beta (T_m - T_f) H}, \quad \tau = H/V_0, \quad \Theta = (T - T_f) / (T_m - T_f), \quad \Psi = \psi / (V_0 H), \quad \Omega = \omega H / V_0. \]

The dimensionless form of equations (1)–(5) is:

\[
\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = -\Omega, \tag{6}
\]

\[
\frac{\partial \Omega}{\partial \tau} + U \frac{\partial \Omega}{\partial X} + V \frac{\partial \Omega}{\partial Y} = \frac{\mu_m / \mu_f}{\rho_m / \rho_f} \sqrt{\frac{\Pr}{\text{Ra}}} \nabla^2 \Omega + \frac{\left( \rho \beta \right)_{nm} / \left( \rho \beta \right)_f}{\rho_m / \rho_f} \frac{\partial \Theta}{\partial X}. \tag{7}
\]

The energy equations (4) and (5) for liquid and solid paraffin by using the smoothing function

\[
\varphi = \begin{cases} 
0, & \text{if } T < T_m - \eta \\
\frac{T - (T_m - \eta)}{2\eta}, & \text{if } T_m - \eta \leq T \leq T_m + \eta \\
1, & \text{if } T > T_m + \eta 
\end{cases}
\]

take the following form:

\[
\zeta(\varphi) \left[ \frac{\partial \Theta}{\partial \tau} + U \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y} \right] + \frac{\rho_m l_{nm}}{\rho_f L_f} \text{St} \cdot \left[ \frac{\partial \varphi}{\partial \tau} + U \frac{\partial \varphi}{\partial X} + V \frac{\partial \varphi}{\partial Y} \right] = \frac{\xi(\varphi)}{\sqrt{\text{Ra} \cdot \text{Pr}}} \nabla^2 \Theta. \tag{8}
\]

where functions \( \zeta \) and \( \xi \) can be defined as follows

\[
\zeta(\varphi) = \left( \frac{\rho c_s}{\rho c_l} \right)_{nm} + \varphi \left( \frac{\rho c_s}{\rho c_l} \right)_{nm} - \left( \frac{\rho c_s}{\rho c_l} \right)_{nm}, \quad \xi(\varphi) = \left( \frac{k_s}{k_f} \right)_{nm} + \varphi \left( \frac{k_s}{k_f} \right)_{nm} - \left( \frac{k_s}{k_f} \right)_{nm}. \]

Energy equations for the heat source and copper profile are

\[
\frac{\partial \Theta}{\partial \tau} = \frac{\alpha_s / \alpha_0}{\sqrt{\text{Ra} \cdot \text{Pr}}} \left( \nabla^2 \Theta \right) + \text{Os} \tag{9}
\]

\[
\frac{\partial \Theta}{\partial \tau} = \frac{\alpha_s / \alpha_0}{\sqrt{\text{Ra} \cdot \text{Pr}}} \nabla^2 \Theta. \tag{10}
\]

In the present formulation (1)–(10) the following parameters are used: \( x, y \) are Cartesian coordinates; \( X, Y \) are dimensionless coordinates; \( t \) is dimensional time; \( \tau \) is dimensionless time; \( g \) is gravitational acceleration; \( \mu \) is dynamic viscosity; \( \beta \) is the coefficient of thermal expansion in the melt; \( \rho \) is density; \( u, v \) are horizontal and vertical components of the velocity vector; \( U, V \) are dimensionless components of velocity; \( p \) is pressure; \( T \) is temperature; \( \Theta \) is dimensionless temperature; \( T_m \) is melting temperature; \( h \) is enthalpy, \( k \) is thermal conductivity, \( c \) is heat capacity, \( \Phi \) is volume fraction of nanoparticles; and the subscripts: \( s \) refers to solid paraffin, \( l \) – melt, \( m \) – paraffin, \( np \) – nanoparticles, \( nm \) – nano-enhanced paraffin.

Initial conditions in the whole system are \( \Theta = \Theta_0 \) and \( V = 0 \).

Boundary conditions are the following:

- on all rigid boundaries: \( \Psi = 0 \) and \( \Omega = -\nabla^2 \Psi \);
- on the side and bottom walls: \( \frac{\partial \Theta}{\partial n} = 0 \);
- on the top wall: \( \frac{\partial \Theta}{\partial Y} = -Bi (\Theta - \Theta_0) \).
In obtained dimensionless equations (6)–(10) and boundary conditions the following dimensionless parameters are used: the Prandtl number $Pr = \frac{\nu \rho c_p}{\lambda}$, the Rayleigh number $Ra = \frac{g\beta \Delta T H^3}{\nu \alpha}$, the Stefan number $Ste = \frac{L_i}{c_i \Delta T}$, the Ostrogradsky number $Os = \frac{q H^2}{\nu \lambda}$ and the Biot number $Bi = \frac{\alpha H}{\lambda}$, respectively. As the scales of temperature and length the following values were taken: $\Delta T = 60^\circ$ and $H = 0.01$ m.

Partial differential equations were solved using the uniform rectangular grid of $481 \times 201$ points by the finite difference method [7, 10]. The discrete elliptic equation for the stream function was solved by the successive over-relaxation method. The energy equation and the vorticity dispersion equation were solved using the locally one-dimensional Samarsky scheme.

As a result of the calculations, distributions of velocity vector and temperature at different levels of the melting process were obtained. Cases with different frequency of the fins location and the volume fraction of nanoparticles are considered. The study was focused on the influence of these parameters on the development of the convective regime in the melt and its interaction with an interphase boundary.

3. Results and discussions

The calculations were carried out for the following dimensionless parameters: $Pr = 48.36$, $Ra = 1.19 \cdot 10^6$, $Ste = 1.84$, $Os = 0.007$ and $Bi = 10$. The number of fins $n$ was varied from 0 to 10. The nanoparticles concentration $\Phi$ was varied from 0 to 0.08.
Addition of fins allows to increase the surface area of profile and to dissipate energy over the entire volume. Therefore, the finning profile has advantages over a flat plate. Efficiency of use phase change materials enhanced by nanoparticles depends on many factors. The figure 2 depicts the temperature fields for different numbers of fins. In the case without fins several upward flows form above the plate, in pure PCM it has curved form and different sizes. With the addition of a small amount of nanoparticles (Φ = 0.04), the temperature flares equalize, further increase in particles concentration leads to a change in the number of ascending flows. The addition of fins leads to more intense melting. At n = 3, Φ = 0 heated liquid ascends along the vertical faces and descends in central parts of spaces between the fins. When nanoparticles are added, the viscosity rises, and the velocities in the melt are reduced, which leads to the formation of additional ascending fluxes between the ribs and it accelerates of the melting process. Figure 3 shows that at n = 3 melting process occurs faster with adding of 4% nanoparticles.

The appearance of natural convection in the melt is accompanied by an acceleration of the melting process in the cases n ≤ 3. In the case of a flat plate a stronger circulation in the pure melt leads to intensive melting, however in nano-enhanced PCM this effect is not observed. At n = 5, in all three cases a similar flow structure is observed. When n is increased to 10, the space between the edges is heated faster due to thermal conduction and small circulation in a narrow space. Therefore with the development of weak convective heat transfer and expansion of the melt area the melting rate sharply decreases.

4. Conclusions

The effect of the frequency of the fins location on the melting modes of the material at different concentrations of nanoparticles is analyzed. It was shown that an increase in the number of ribs leads to an intensification of the melting process. Nano-enhanced phase change material melts faster at the initial stages, when the main mechanism of heat transfer is heat conduction. As the concentration of nanoparticles increases, the melting rate increases. However, with the development of a convective regime the situation changes to the opposite. A growth of the liquid viscosity shows that the circulation of liquid paraffin with the nanoparticles is weaker than the pure paraffin case. At the same time, it was shown that the flow structure changes with the addition of nanoparticles. In the case without fins the addition of nanoparticles can be effective only in the early stages of melting. A large
number \((n = 10)\) of ribs contributes to the development of conductive heat exchange, but the small space between the ribs prevents the development of circulation.

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