Prediction for \( \alpha_3(M_Z) \) in a string-inspired model

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Abstract

We apply the Renormalisation Group Evolution (RGE) to analyse the phenomenological implications of an extended supersymmetric model, for the value of the unification scale and the strong coupling at the electroweak scale. The model we consider is predicted to exist in Calabi-Yau string compactifications with Wilson line mechanism for \( E_6 \) symmetry breaking, contains additional matter beyond the MSSM spectrum and avoids the “doublet-triplet” splitting problem in the Higgs sector. The calculation is analytical in two-loop order and includes the effects of the heavy thresholds due to the additional matter considered. The value of \( \alpha_3(M_Z) \) can be brought within the experimental limits without a significant change of the unification scale from the MSSM prediction.

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1 Introduction

The Minimal Supersymmetric Standard Model (MSSM) currently provides the “standard” framework for the study of the physics beyond the Standard Model. The MSSM model takes into account the constraints from the negative searches so far for an experimental signature for Supersymmetry and gives circumstantial evidence for supersymmetric unified theories, such as the unification of the gauge couplings or the weak mixing angle prediction. The aforementioned constraints and predictions may also be fulfilled by other models, of string origin, and this would suggest that the MSSM is only a minimal candidate model for the physics beyond the Standard Model. Such string inspired models are regarded as “low-energy” limits of string theory, and they predict below the compactification scale a much richer spectrum (than that of the MSSM) including additional states at the high scale corresponding to predicted vector-like states under standard model gauge group. The presence of such additional states in conjunction with the low-energy input can lead to a phenomenology different from that of the MSSM. As a specific example which we address in this letter, consider the measurement of the strong coupling at the electroweak scale. The world average value \( \alpha_3(M_Z) = 0.119 \pm 0.002 \), situated rather close to the one loop value predicted in the MSSM is below the two-loop MSSM “bottom-up” prediction (of \( \approx 0.125 \)) obtained from the RGE equations for the gauge couplings. While at one-loop level this prediction depends on the symmetry of the model and the multiplet content, at two-loop level a dependence of \( \alpha_3(M_Z) \) on the high scale thresholds is induced, and this may indicate that the mismatch between the MSSM two-loop prediction and the experiment is due to our lack of understanding of the physics at the high scale. For this reason at least, exploring the phenomenological implications of string theory predicted models with additional intermediate scales finds enough justification and it may also help in selecting and restricting the number of viable string inspired models.

The exact structure of the spectrum that string-inspired models predict to exist (in addition to the MSSM spectrum) depends in general on the particular class of models one considers to investigate. In reference [3] the authors presented a class of extended supersymmetric models, as the low-energy limit of a string model with Calabi-Yau compactification [4] and Wilson line breaking [5] mechanism for the \( E_6 \) symmetry.

In a generic example of this class of models, the “low-energy” spectrum below the compactification scale contained the MSSM spectrum plus (pairs of) complete five and ten-dimensional \( SU(5) \) multiplets, “vector-like” under the Standard Model gauge group. The phenomenological implications of this case were discussed in [3, 7] for the case when perturbation theory applies up to the unification scale. It was found that, due to a “mixing” between the heavy thresholds and the two-loop contributions of the vector-like states to the running of the gauge couplings, there is only a small (two-loop) increase in the unification scale from the MSSM prediction. The increase factor was \( \approx 3 \), too small to make an agreement with the weakly coupled heterotic string prediction [8] which gives an unification scale \( \approx 20 \) times larger than that of the MSSM. The aforementioned factor of increase was accompanied by a small (two-loop) increase from the MSSM prediction for the strong coupling at electroweak scale [3].

Reference [3] also predicted another interesting possibility for the “low-energy” spectrum predicted by the same class of string-inspired models and this will be further analysed in this paper. This more specific model predicts not only complete five dimensional representations of \( SU(5) \) in addition to the

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1 Vector-like states are not protected by the chiral symmetry of the SM and are therefore heavy.
2 In the following by MSSM spectrum we understand the three generations of quarks and leptons with their superpartners, an appropriate gauge sector containing the gauge bosons and gauginos and the two Higgs doublets and superpartners, without any \( SU(3) \) Higgs triplets.
3 For the case when the unification of the gauge couplings takes place in the non-perturbative regime, when the unified coupling is assumed to be large, one way to make predictions was presented in [3], although the errors which affect them might be large as discussed in [3] (Section 5).
MSSM spectrum, but also a “split multiplet” structure, triplet under the $SU(3)$ group of the Standard Model. The model has therefore the nice feature of avoiding the “doublet-triplet” splitting problem which appears in the context of (Supersymmetric) Grand Unified Group Theories (GUT). In these theories, the Higgs multiplet content is a $5 + \overline{5}$ pair and consequently, the bare masses of the Higgs doublet and Higgs colour triplet have to be equal. To avoid proton decay mediated by the latter at a rate forbidden by the experimental constraints, the Higgs (colour) triplet has to be heavy enough to suppress such processes. In the meantime, the $SU(2)$ Higgs doublet must be light enough to explain the mass origin at the electroweak scale. This leads to the so-called “doublet-triplet” splitting problem, specific to supersymmetric GUT theories as well as to other theories which have such Higgs spectrum assignment.

In addition to avoiding the “doublet-triplet” splitting problem, this last specific model provides the unification of the gauge couplings, even in the absence of a grand unified group such as $SU(5)$ or larger. The spectrum predicted by this string inspired model contains below the compactification scale a pair $3 + \overline{3}$ and an arbitrary number (say $n + 1$) of extra pairs $5 + \overline{5}$ of $SU(5)$ in addition to the matter fields (three families) and the gauge sector of the Minimal Supersymmetric Standard Model, but without its Higgs content.

A coupling of the triplet of the “incomplete” $SU(5)$ representation, $3 + \overline{3}$, to the triplet component of a five dimensional representation of the form $\lambda \phi 35$ can naturally lead to a large mass (due to their vector-like character under SM group) term for the triplets, via a symmetry breaking mechanism when the (Standard Model singlet) Higgs field $\phi$ acquires a v.e.v., through a mechanism detailed in [6]. The same mechanism applies to the extra $n$ (vector-like) pairs of $5 + \overline{5}$ which also acquire a large mass (through couplings $\lambda \phi 55$ via the same mechanism [3] which provides a natural explanation for the origin of such mass terms). The bare value of this mass, assumed to be the same for all $5$’s is denoted by $\mu_g$ for further reference. The (remaining) doublet components of the initial pair $5 + \overline{5}$ are left uncoupled and thus light and can therefore account for the Higgs content of the MSSM.

It is the purpose of this letter to examine in some detail the phenomenological implications of this model, in a two-loop analytical approach. The predictions we make refer to the value of $\alpha_3(M_z)$, the unification scale itself and the scale of the intermediate matter $\mu_g$.

The reason for performing a two-loop analytical investigation of this model is three-fold; as mentioned, the discrepancy between the MSSM prediction for $\alpha_3(M_z)$ and the experiment arises mainly due to the two-loop corrections, and thus threshold dependence at the high scale plays a significant role. Moreover, any better candidate model than the MSSM should eliminate such discrepancy at this level of accuracy. Finally, the spectrum predicted by our model contains in addition to the $SU(3)$ triplet components, complete $SU(5)$ representations which are known to change the low energy prediction for $\alpha_3(M_z)$ at two-loop level only [6]. The present approach also provides an analytical method to examine, in two loop order the RGE prediction for the strong coupling and may be applied to other (string-inspired) models with spectrum different from that of the MSSM.

We show that $\alpha_3(M_z)$ can be reduced from the MSSM value (of $\approx 0.125$ or larger) and be brought within the experimental limits [2] of $0.119 \pm 0.002$, while keeping the unification scale $M_g$ close to that of the MSSM for an intermediate scale $\mu_g$ within one order of magnitude below $M_g$.

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4We refer here to the model which in [3] was called the “unconventional” case.
5Equivalently, we can say that the spectrum just below the compactification scale (and before the decoupling of any heavy state as we lower the scale) contains $n$ pairs $5 + \overline{5}$ and $2$ pairs $3 + \overline{3}$ in addition to the MSSM spectrum.
6We take as an input parameter the unified gauge coupling, to predict the intermediate scale and not vice-versa for the reason that the intermediate scale tends to have a flat behaviour for large range of values for $\alpha_g$, which can induce numerical instabilities of the solution, see Figures 3 and 4 of ref. [3] for a similar case.
7A similar situation exists for the unification scale prediction as well.
8The exact value of the strong coupling prediction in the MSSM depends on the assumptions made for the low energy (TeV scale) supersymmetric spectrum.
2 Predictions from the Renormalisation Group Evolution

The standard tool to exploring the phenomenological consequences of our model is the Renormalisation Group Evolution (RGE) for the gauge couplings, for which we take as (low-energy) boundary conditions the well known values of $\alpha_1(M_Z)$ and $\alpha_2(M_Z)$, obtained from measured electromagnetic coupling and weak mixing angle at the electroweak scale. To apply this tool we need to know the multiplet content, which was detailed in the Introduction, and the symmetry group, which is just the Standard Model gauge group, with $SU(5)$ normalisation for the $U(1)_Y$ coupling. Thus, the one-loop beta function before the decoupling of any extra (complete or incomplete) multiplet is given by

$$b_i^* = b_i + \Delta b_i + n$$

with $b_i$ the MSSM one-loop beta function, $b_i = (33/5, 1, -3)$, and with $\Delta b_i = 4 \times \{1/5, 0, 1/2\}$ to account for two pairs of triplets or equivalently four triplet states, hence the factor 4 in the definition of $\Delta b_i$. After the decoupling of all additional states, the one-loop beta function is just that of the MSSM, namely $b_i$. With some loss of generality we restrict ourselves to the case when the extra states (triplets and 5-plets) have the same bare mass which we called $\mu_g$ and this assumption does not reintroduce the “doublet-triplet” problem. The general case of considering different masses for $3 + \bar{3}$ and $5 + \bar{5}$ pairs can be done following the present approach, although introducing one further mass parameter would make the analysis less tractable.

To evaluate the full two-loop “running” of the gauge couplings, including the effects of the heavy thresholds, we use the integral form of the “NSVZ beta function”. This has been computed in $[10]$ and $[11]$ (see also $[12]$) and is given by

$$\beta(\alpha)^{SVZ} \equiv \frac{d\alpha}{d(ln \mu)} = -\frac{\alpha^2}{2\pi} \left[ 3 T(G) - \sum_\sigma T(R_\sigma) (1 - \gamma_\sigma^{SVZ}) \right] \left( 1 - T(G) \frac{\alpha}{2\pi} \right)^{-1}$$

with the definition ($\mu$ is the running scale)

$$\gamma_\sigma^{SVZ} = -\frac{d ln Z_\sigma}{d ln \mu}$$

and where $T(G)$ and $T(R_\sigma)$ represent the Dynkin index for the adjoint representation and for $R_\sigma$ representation respectively (not necessarily the fundamental one). The above sum runs over all matter fields $\sigma$ in representation $R_\sigma$ and this includes the extra heavy states in addition to the low energy spectrum of the MSSM.

Following the details given in $[1]$ to integrate the beta function given above, we find that, to all orders in perturbation theory, the gauge couplings run, in the presence of the extra matter, as follows

$$\alpha^{-1}_i(M_Z) = -\delta_i + \alpha^{-1}_g + \frac{b_i}{2\pi} \ln \frac{M_g}{M_z} + \frac{n + \Delta b_i}{2\pi} \ln \frac{M_g}{\mu_g} - \frac{\beta_{i,H_1}}{2\pi} \ln Z_{H_1}(M_Z) - \frac{\beta_{i,H_2}}{2\pi} \ln Z_{H_2}(M_Z)$$

$$-\frac{\beta_{i,g}}{2\pi} \ln \left[ \frac{\alpha_g}{\alpha_i(M_Z)} \right]^{1/3} - \sum_{j=1}^3 \sum_{\phi_j} \frac{\beta_{i,\phi_j}}{2\pi} \ln Z_{\phi_j}(M_Z)$$

where $b_1 = 33/5$, $b_2 = 1$, $b_3 = -3$ and where $\beta_{i,\phi_j} \equiv T(R_{\phi_j})$, $i = \{1, 2, 3\}$, are the contributions to one-loop beta function$[1]$ of the matter fields $\phi_j$ ($j$=generation index), while $\beta_{i,g} \equiv T^a(G)$ is the one-loop beta function for the pure gauge (+gaugino) sector; the Higgs (+higgsino) sector contribution is

$^a$We also used that the one-loop beta function is $b = -3T(G) + \sum T(R_\phi)$, where the sum runs over all chiral supermultiplets in representation $R_\phi$. 

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stands for the unification scale of our model. We have
\[ \beta_i = \begin{pmatrix}
\frac{3}{10} & \frac{1}{10} & \frac{4}{5} & \frac{1}{5}
\frac{1}{2} & \frac{3}{2} & 0 & 0
0 & 1 & 0 & \frac{1}{2} \ \frac{1}{2}
\end{pmatrix}_{i,\phi_j} \]
\[ \beta_{i,g} = \begin{pmatrix}
0 \\
-6 \\
-9 \\
0
\end{pmatrix}; \quad \beta_{i,H,2} = \begin{pmatrix}
\frac{3}{10}
\end{pmatrix}
\]
with \( \beta_{i,\phi_j} \) independent of the values of \( j \). The field \( \phi_j \) runs over the set \( \phi_j = \{l_L, q_L, e_R, u_R, d_R \} \), in this order, with \( j \) as generation index. The coefficients \( \delta_i \) in eq.(3) represent the low energy supersymmetric thresholds and they would be equal to zero if supersymmetry were valid at the electroweak scale. Their value of the strong coupling at electroweak scale.

The uncertainty in the low-energy supersymmetric spectrum (i.e. the value of \( \delta_i \)) can be taken into account in our present approach by allowing in our final results, a range of values for the MSSM variables which are present in our final expressions.

To compute the two-loop running for the gauge couplings, only a one-loop expression for the wavefunction renormalisation coefficients is required. Note that in the two-loop approximation there is no regularisation ambiguity which arises only in three-loop order. At \( M_z \) scale the one-loop expressions for \( Z \)'s have the following structure

\[ Z_F(M_z) = \prod_{k=1}^{3} \left[ \frac{\alpha_g}{\alpha_k(M_g)} \right]^{b_k} \left[ \frac{\alpha_k(M_g)}{\alpha_k(M_z)} \right]^{b_k} \]
\[ = \prod_{k=1}^{3} \left[ \frac{\alpha_g}{\alpha_k(M_g)} \right]^{2C_k(F)} \left[ \frac{\alpha_k(M_g)}{\alpha_k(M_z)} \right]^{b_k} \]
\[ = \prod_{k=1}^{3} \left[ \frac{\alpha_g}{\alpha_k(M_g)} \right]^{2C_k(F)} \left[ \frac{\alpha_k(M_g)}{\alpha_k(M_z)} \right]^{b_k}
\]

where \( F \) stands for any Higgs or MSSM chiral field. Strictly speaking in the expressions of \( Z \) factors we should have used the mean mass of the extra states \( \tilde{\mu} \) instead of \( \mu_g \); however this difference is an additional radiative effect and thus is of two-loop order for \( Z \)'s or of three loop order for the gauge couplings, and can be neglected in our two loop calculation. From equations (3) and (5) we find the following RGE equations

\[ \alpha_i^{-1}(M_z) = -\delta_i + \frac{\alpha_g^{-1}}{2\pi} \ln \left( \frac{M_g}{M_z} \right) + \frac{n + \Delta b_i}{2\pi} \ln \left( \frac{M_g}{\mu_g} \right) - \frac{1}{2\pi} \sum_{j=1}^{3} \tilde{Y}_{ij} \ln \left[ \frac{\alpha_g}{\alpha_j(\mu_g)} \right] + \frac{1}{4\pi} \sum_{j=1}^{3} \frac{b_{ij}}{b_j} \ln \left[ \frac{\alpha_g}{\alpha_j(M_z)} \right]
\]
\[ \tilde{Y}_{ij} = \frac{n + \Delta b_j}{b_j} \left[ \frac{1}{2} b_{ij} - \delta_{ij} \lambda_j \right]
\]

with \( \lambda_1 = 0, \lambda_2 = 2, \lambda_3 = 3 \) while \( \delta_i, i = \{1, 2, 3\} \), stand for the low-energy (TeV scale) supersymmetric thresholds.

\[ ^{10} \text{The MSSM quantities used as an input in our calculation will be the unified coupling, the unification scale and the value of the strong coupling at electroweak scale.} \]
From eq.(3) we can again see that the presence of $\mu_g$ in the two-loop term $\ln(\alpha_g/\alpha(\mu_g))$ instead of the mean physical mass $\tilde{\mu}$ of the additional multiplets we consider would account for an additional (three-loop) radiative effect and we neglect it, as it is beyond our two-loop approximation for the running of the gauge couplings. Thus, we can say that in two-loop order the extra states contribute to the gauge couplings running through their common bare mass only.

At this point we would like to emphasize that the result of equation (3) can also be obtained using the “standard” RGE equations, integrated in two-loop order, with appropriate taking into account of the heavy thresholds that the additional states we consider bring in. Using a radiative dressing of the masses of the additional states and following the approach of ref. 3 one will obtain the same result. As it was the case there, there is a cancellation of the heavy thresholds against the two-loop contributions of the additional states we considered and as a consequence in eq.(3) only the two-loop MSSM beta function appears, and not that in the presence of additional states$^{11} \Box 5 + \Box 3$. Eq.(3) is actually more general; if one considers vector-like matter in addition to the MSSM sector, with some arbitrary $\delta b_j$ contribution to one-loop beta function, the two-loop RGE equations have a form similar to that of eqs.(3) and (6) with the replacements $n + \Delta b_i \rightarrow \delta b_i$ and $b_j^* \rightarrow b_j + \delta b_j$.

To compute the unification scale $M_g$, the strong coupling $\alpha_3(M_z)$ and the value of the mass scale $\mu_g$ we must make some assumptions about the low energy supersymmetric spectrum which affects the running of the gauge couplings through the terms $\delta_i$, as seen from eq.(3). Since the effects of the low energy supersymmetric thresholds on the predictions of the MSSM are relatively known$^{14}$, we prefer to express our predictions as a change to the MSSM predictions which all have this dependence included (and assume that $\delta_i$’s have equal values to those of the MSSM). We therefore consider the two-loop running of the gauge couplings in the MSSM, which is of the form (the MSSM variables are labelled with an “o” index to distinguish them from those of our extended model)

$$\alpha_i^{o-1}(M_z) = -\delta_i + \alpha_i^{o-1} + \frac{b_i}{2\pi} \ln \left[ \frac{M_g^o}{M_Z} \right] + \frac{1}{4\pi} \sum_{j=1}^{3} \frac{b_{ij}}{b_j} \ln \left[ \frac{\alpha_j^o}{\alpha_j^o(M_Z)} \right]$$

We can then substitute$^{12}$ the values of $\delta_i$ from the above equation into eq.(3) and impose that in both the MSSM and our model, the values of $\alpha_1(M_z)$ and $\alpha_2(M_z)$ are taken equal with the corresponding experimental value, so $\alpha_1(M_z) = \alpha_1(M_g)$ and $\alpha_2(M_z) = \alpha_2(M_z)$. We then compute the analytical expressions for the factor of increase of the unification scale $M_g/M_g^o$, the strong coupling $\alpha_3(M_z)$ (in terms of $\alpha_3(M_g)$), and the bare mass of the extra states, $\mu_g$. This can be done following the approach of 3 and using that, under two-loop terms we can substitute the arguments of the “log” terms by their one loop values, as the difference would be of higher order. This means that $\ln(\alpha_g/\alpha_j(\mu_g)) = \ln(1 + b_j^* \alpha_g/(2\pi) \ln(M_g/\mu_g))$ and we further replace $\ln(M_g/\mu_g)$ by its one loop analytical expression, which is correct for a two loop running for the gauge couplings.

After some tedious algebra we find the following two-loop analytical results

$$\frac{M_g}{M_g^o} = \exp \left\{ \frac{2\pi}{7n-1} \left( \alpha_g^{-1} - \alpha_g^{o-1} \right) \right\} \left[ \frac{\alpha_g}{\alpha_g^o} \right]^{\mathcal{E}_1} \left[ 1 + \frac{18 \alpha_3^o(M_z)}{1 - 7n} \left( \alpha_g^{-1} - \alpha_g^{o-1} \right) \right]^{\mathcal{E}_2} \times \prod_{j=1}^{3} \left\{ 1 + \frac{7b_j^*}{1 - 7n} \left[ 1 - \frac{\alpha_j}{\alpha_j^o} \right] \right\}^{D_j}$$

$^{11}$Also note the strong similarity of eq.(3) with that of eq.(7) of ref.3.

$^{12}$In the MSSM we have $\alpha_3^o \approx 0.0433$, $M_g^o \approx 3 \times 10^{16}$ GeV and $\alpha_3^o(M_z) = 0.125$ or larger. These results are obtained in two loop approximation, using $\alpha_1(M_z)$ and $\alpha_2(M_z)$ as an input from experimental values for electromagnetic coupling and weak mixing angle.
\[
\alpha_3^{-1}(M_z) = \alpha_3^0(M_z) + \frac{18(\alpha_g^{-1} - \alpha_g^0)}{1 - 7n} + \frac{73 + 17n}{2(7n - 1)\pi} \ln \left\{ 1 + \frac{18\alpha_3^0(M_z)}{1 - 7n} \left( \alpha_g^{-1} - \alpha_g^0 \right) \right\} \\
+ \frac{2(467 + 235n)}{11\pi(1 - 7n)} \ln \left[ \frac{\alpha_g}{\alpha_g^0} \right] + \sum_{j=1}^{3} \ln \left\{ 1 + \frac{7b_j^*}{1 - 7n} \left[ 1 - \frac{\alpha_g}{\alpha_g^0} \right] \right\}^{A_j}
\]

(10)

\[
\frac{\mu_g}{M_g^o} = \exp \left\{ \frac{16\pi}{7n - 1} \left( \alpha_g^{-1} - \alpha_g^0 \right) \right\} \left[ \frac{\alpha_g}{\alpha_g^0} \right]^{E_3} \left\{ 1 + \frac{18\alpha_3^0(M_z)}{1 - 7n} \left( \alpha_g^{-1} - \alpha_g^0 \right) \right\}^{E_4} \\
\times \prod_{j=1}^{3} \left\{ 1 + \frac{7b_j^*}{1 - 7n} \left[ 1 - \frac{\alpha_g}{\alpha_g^0} \right] \right\}^{H_j}
\]

(11)

where

\[
E_1 = \frac{285 + 341n}{33(7n - 1)}, \quad E_2 = \frac{4(3 + n)}{3(1 - 7n)}, \quad E_3 = \frac{2621 + 341n}{33(7n - 1)}, \quad E_4 = \frac{100 + 4n}{3(1 - 7n)}
\]

(12)

\[
D_j = \left\{ \frac{18 - 77n}{660(7n - 1)b_1^3}, -3n(4 + 5n) \right\} \frac{4(2 + n)(3 + n)}{4(7n - 1)b_2^3} \frac{3(7n - 1)b_3^3}{(7n - 1)b_3^3}
\]

(13)

\[
A_j = \left\{ \frac{-29 + 57n}{220(7n - 1)\pi b_1^3}, \frac{81n(5 + 2n)}{4(7n - 1)\pi b_2^3}, \frac{2(2 + n)(-19 + n)}{(7n - 1)\pi b_3^3} \right\}
\]

(14)

\[
H_j = \left\{ \frac{-268 - 27n + 385n^2}{660(7n - 1)b_1^3}, -3n(125 + 13n) \right\} \frac{4(50 + 27n + n^2)}{4(7n - 1)b_2^3} \frac{3(7n - 1)b_3^3}{(7n - 1)b_3^3}
\]

(15)

The above analytical solution to eq. (11) agrees well with the numerical one. To find a numerical solution we just solved numerically the system of three equations obtained from subtracting eq. (11) from (14) to eliminate the δi’s and also replaced \( \ln(\alpha_g/\alpha_j(\mu_g)) \) by \( \ln(\alpha_g/\alpha_j(\mu_g)) = \ln(1 + b_i^*\alpha_g/(2\pi)\ln(M_g/\mu_g)) \). The agreement between the two approaches is good, within less than 1% relative error for \( \alpha_3(M_z) \), 5% relative error for the factor \( M_g/M_g^o \), and 10% relative error for \( \mu_g/M_g^o \). The larger error exists when the coupling \( \alpha_g \) is larger and is also due to the presence of the logarithmic dependence the terms involving \( M_g \) and \( \mu_g \) come with in the RGE equations.

3 Numerical Results

In this section we analyse the results and the phenomenological implications of eqs. (11), (14) and (15).

Figure 1 shows the ratio of the unification scales \( M_g/M_g^o \) for different values of \( n \) in function of the ratio \( \alpha_g/\alpha_g^0 \). We observe that this ratio is less than unity for most of the parameter space and the effect of extra states we added does not bring the unification scale closer to the weakly coupled heterotic string scale which is a factor of \( \approx 20 \) above the MSSM value. However, for large \( n \) the ratio \( M_g/M_g^o \) is very close to unity, and therefore the change induced in this case from the MSSM scale, is very small. (Note that the perturbative calculation is valid for large \( n \), as long as \( n\alpha \approx \kappa\mathcal{O}(4\pi) \), with \( \kappa < 1 \). For \( n \geq 20 \) we find \( M_g \) above 0.8\( M_g^o \) for most values of \( \alpha_g \), and therefore the change induced by the extra matter to the MSSM unification scale is insignificant. This is a result of the presence of two competing effects, the reducing of the scale (at one-loop level) due to the \( SU(3) \) triplet states and the opposite effect of increasing the scale due to the complete five dimensional multiplets.

Such opposite effects are also manifest in the predicted value of \( \alpha_3(M_z) \). In Figure 2 we presented this value for different \( n \) in function of the ratio \( \alpha_g/\alpha_g^0 \). We observe that we can accommodate values of \( \alpha_3(M_z) \) smaller than in the MSSM and in better agreement with the experimental value.
\( \alpha_3(M_z)_{\text{exp}} = 0.119 \pm 0.002 \), provided that the value of the unified coupling is marginally increased from the MSSM value, for the case of small \( n \). For \( n \geq 20 \) \( \alpha_3(M_z) \) is within the experimental limits for a larger range of values of \( \alpha_g \). The effect of reducing the strong coupling is essentially due to the presence of the colour triplets we considered. The result is somewhat expected as, unlike models which include complete \( SU(5) \) representations to the MSSM spectrum and where complete representations introduced the same term \( n \ln(M_g/\mu_g) \) in the running of the gauge couplings \([6]\), the situation here is different because the similar contribution is now \((n + \Delta b_i) \ln(M_g/\mu_g)\), with \( \Delta b_i \) standing for the triplets’ contribution (see eqs. \([3]\), \([6]\)). This means that the relative behaviour of the gauge couplings running is already changed at one-loop order from the MSSM prediction due to the presence of the \( SU(3) \) triplets while the complete five dimensional multiplets bring a two-loop additional increasing effect \([6]\). We would like to note that the input MSSM value for \( \alpha_3(M_z) \) considered here was 0.125; this represents the lower limit prediction of a “bottom-up” approach in the MSSM case, and therefore the predictions we made for \( \alpha_3(M_z) \) could increase slightly if the input for \( \alpha_3(M_z) \) is above this value.

Figure 3 shows the ratio \( \mu_g/M_g^0 \) in terms of \( \alpha_g \) and this determines the value of one of these when the other is fixed. For the parameter space with good predictions for \( \alpha_3(M_z) \) we find that the (bare) intermediate scale is in the region of \( 3 \times 10^{15} \) GeV, only a factor of \( \approx 10 \) below the standard MSSM unification scale. The large value for the intermediate scale avoids an enhancement of the proton decay rate by the colour triplet states.

4 Conclusions

In this work we have considered the phenomenological implications of a string-motivated model, which predicts below the compactification scale the existence of \( n \) extra pairs \( 5+\overline{5} \) of \( SU(5) \) states and 2 pairs of \( SU(3) \) triplets in addition to the MSSM spectrum. The motivation for studying this model originates in the suggestion that this might also solve the “doublet-triplet” splitting problem, commonly faced by Grand Unified Group-based theories. The strong coupling at the electroweak scale can be reduced
Figure 2: The values of $\alpha_3(M_z)$ plotted in function of the ratio $\alpha_g/\alpha_g^0$ for different values of $n$.

Figure 3: The values of $\log_{10}[\mu_g/M_g^0]$ plotted in function of the ratio $\alpha_g/\alpha_g^0$ for different values of $n$. 
below the value of the two-loop MSSM prediction and be brought into better agreement with the experiment while the value of the unification scale, in two-loop order, remains close to the MSSM prediction.

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