Superposing spacetimes: lessons from analogue gravity.

Carlos Barceló

Instituto de Astrofísica de Andalucía (IAA-CSIC),
Glorieta de la Astronomía, 18008 Granada, Spain

Luis J. Garay

Departamento de Física Teórica and IPARCOS,
Universidad Complutense de Madrid, 28040 Madrid, Spain and
Instituto de Estructura de la Materia (IEM-CSIC), Serrano 121, 28006 Madrid, Spain

Gerardo García-Moreno

Departamento de Física Teórica and IPARCOS,
Universidad Complutense de Madrid, 28040 Madrid, Spain and
Instituto de Astrofísica de Andalucía (IAA-CSIC),
Glorieta de la Astronomía, 18008 Granada, Spain
Abstract

We analyze Penrose’s ideas concerning the role of gravity in the quantum-mechanical state reduction from the perspective of analogue-gravity models. We first review a line of work which argues that it is not enough to find a satisfactory quantization scheme for gravity in the description of a matter-gravity system. Instead of that; the standard rules of quantum mechanics as they are actually applied to matter systems need to be modified when the gravitational interaction is relevant. Among these ideas we focus on Penrose’s argument for the absence of macroscopic quantum superpositions. He argues that such phenomenon has its roots in the idiosyncratic aspects of gravity, which become relevant just when the system is macroscopic and hence able to generate a sufficiently strong gravitational field, and not in the environmental decoherence. We then present an analogue model consisting of a Bose-Einstein condensate in a double well potential, where it is possible to build states that would correspond to putative superpositions of spacetimes. Such configurations are unstable and the source of instability from a microscopic point of view can be related to the absence of a well-defined causal structure in the effective geometric description. We also discuss some additional lessons that can be taken from these instabilities.

* carlos@iaa.es
† luisj.garay@ucm.es
‡ gerargar@ucm.es
CONTENTS

I. Introduction 3

II. The quantum puzzle 7

III. Gravity’s role in quantum state reduction 11
   A. Karolyhazy’s and Diósi’s ideas 11
   B. Penrose’s idea 14
   C. Quantum mechanics with stochastic time 17

IV. Analogue gravity model for state reduction due to gravity 20
   A. Condensates in a double well 21
   B. Effective spacetimes from BEC’s 27
   C. Stability of the ground states 28
   D. Attempting to generate a superposition of two effective spacetimes 30
   E. Same localization, different causality 32

V. Summary and Discussion 34

Acknowledgments 38

References 38

I. INTRODUCTION

Twentieth century physics has left us two separate paradigms to understand Nature. On the one hand, we have the quantum paradigm. Quantum Field Theory (QFT), which is the result of putting together the laws of quantum mechanics and special relativity, is the most successful framework we have nowadays for explaining three of the four fundamental interactions currently known: the electromagnetic force, the weak force and the strong force. On the other hand, we have the gravitational interaction. The paradigm of General Relativity constitutes our current understanding of gravity as a theory of spacetime, which is, however, purely classical. For about one hundred years, people have tried to extend General Relativity to the quantum realm, in such a way that it follows an underlying scheme similar
to the other fundamental interactions. However, gravity has shown itself to be resilient to quantization within the QFT paradigm, mainly because of not being perturbatively renormalizable \cite{1,2} (see also \cite{3} for a pedagogical explanation). In addition, different alternative attempts to quantize gravity have been tried, among them string theory \cite{5-8} and canonical quantum gravity \cite{9} in its modern formulation in terms of loop quantization \cite{10,11} are the leading approaches. Although their development is an important test bench for the interplay between the quantum and the gravitational realms, it is fair to say they are still far from providing a compelling understanding of the interface between both paradigms.

Given this situation, there has been a relatively small but steady trend pointing out that maybe one should not quantize gravity but *gravitizate* quantum mechanics, in Penrose’s words \cite{12}. At least, these approaches suggest that both paradigms should be “deformed” to succeed in being joined together. In this respect, this kind of ideas also suggest that maybe classical and quantum dynamics are just two approximate regimes of an all embracing universal dynamics \cite{13-15}. Part of the motivation for this trend comes from some arguably unsatisfactory characteristics of the quantum paradigm, that we will briefly recall in next section.

One particular key cornerstone of these alternative points of view relates to one of the crucially distinctive properties of quantum mechanics with respect to classical mechanics: it allows superpositions \cite{16}. These superpositions, although at the core of the formulation, do not seem to permeate to the macroscopic world: we do not perceive in any way superpositions of macroscopic configurations. There has to be a mechanism that dynamically accounts for this process of losing quantum coherence, which is often called quantum state reduction. A plausible mechanism for this reduction of state can be found within the standard formalism of quantum mechanics through a process known as environmental decoherence \cite{17-19} (see next section). However, there is an obvious question concerning this decoherence: are there other sources of fundamental decoherence which appear somewhere along the way between the macroscopic world and the microscopic worlds? It is important to highlight that experimentally speaking we do not know for sure whether the generic decoherence that we observe in Nature is always caused by a partial tracing over actual degrees of freedom of environmental type or there are other sources of decoherence mixed in. These hypothetical new sources of fundamental decoherence should affect more strongly macroscopic systems than elementary systems, as in the latter superpositions are clearly observed. In the case in
which some fundamental decoherence emerges on the way towards the macroscopic world, we would have a sound alternative to the otherwise seemingly unavoidable manyworld view of reality [20, 21].

In seeking for the existence of a fundamental source of macroscopic decoherence a natural question arises. Being qualitatively different from the rest of the interactions, does gravity play a role in this reduction of state? In the literature, there are several arguments or mechanisms appealing to gravity as the cause of a fundamental decoherence. In section III we will provide a quick overview of those ideas. Within this set of ideas we will follow more closely the language put forward by Penrose [12, 22–24]. Essentially, he proposed that quantum configurations which differ sufficiently in their associated gravitational fields (and hence, which would represent superpositions of different gravitational fields) should decohere as a fundamental principle. Strictly speaking, this would mean a modification of the laws of quantum mechanics once they are put together with General Relativity, realizing his idea of the need for a gravitization of quantum mechanics. Using his language, in this paper we work out an analogue-gravity model in terms of Bose-Einstein condensates (BECs) where it is possible to analyze the putative quantum state resulting from the superposition of different gravitational fields. Let us explain what we mean by that. Many analogue models of gravity have as underlying substratum a non-relativistic quantum many-body system. Within this framework, it is not difficult to find situations and regimes in which one has the emergence of collective quantum excitations, i.e. a rearranging of the structure of the system such that the low-energy excitations are well-described by weakly interacting local fields propagating on top of a relativistic background structure. This captures most of the kinematical aspects of a general-relativistic gravitational theory which, to some extent, we can summarize as having a fixed arbitrary geometry in which the light-cones change from one spacetime point to another. Within this setting one can ask whether one can engineer situations such that the degrees of freedom encoded in the emergent local fields perceive a superposition of two different causalities. This is the main inquire that we address in this work.

For concreteness, we base our discussion in BECs. BECs are systems which behave classically (owing to their strong coherence) although they live in an ultraquantum regime (its constituents are almost in their one-particle vacuum state). One can perfectly describe a BEC as a classical fluid with just a peculiar pressure law (the quantum pressure) that
recalls its quantum origin \cite{25, 26}. For instance, BECs are very much used as analogue
gravity models for simulating black holes and other gravitational configurations \cite{27}. A
flowing BEC which exceeds the (long wavelength) velocity of acoustic waves can produce
a configuration analogous to a general relativistic black hole \cite{28}. The flowing condensate
plays the role of a background black hole spacetime, and on top of it one can analyze the
behaviour of quantum (phononic) fluctuations, or in some other regime, classical acoustic
waves. In this way Steinhauer and collaborators have found strong evidence for the creation
of Hawking phonon pairs owing to the formation of a horizon in these systems \cite{29}.

In this paper we want to investigate what would happen if a BEC was engineered to be in a
state which is a quantum superposition of two otherwise classical configurations that from the
analogue gravity perspective could represent two distinct background spacetimes. We will
show that such configurations are highly unstable and indeed the system will tend to rapidly
decay to a more stable configuration. From this perspective, our analogue model captures
the main features of Penrose mechanism. By looking at the analogue model, one would
pinpoint some particularities of classical macroscopic entities exhibiting causal behaviour.

Here is an outline of the rest of the article. In section II we briefly summarize two
related problems concerning the standard formulation of quantum mechanics, the absence
of macroscopic superpositions in nature which are allowed in principle and the intrinsic
statistical nature of its predictions. Furthermore, we catalogue the main ways of solving
these problems and briefly discuss them. In section III we focus on the possible role played
by gravity in explaining why we do not observe macroscopic quantum superpositions. We
review several approaches that attempt to solve the first of the problems mentioned, with
special focus on Penrose’s ideas which are formulated in a language that is closer to our
analogue model. Section IV constitutes the core of the article. We first introduce a model
of Bose-Einstein condensates in a double well and perform an analysis of the stability of
the ground states of the model according to the possible values of the parameters entering
the Hamiltonian. We use this model to establish an analogy between these condensates and
effective spacetimes for the propagation of sound waves. We find that the states that would
correspond to a putative superposition of spacetimes develop an instability under small
perturbations that makes them collapse, preventing their formation. This impossibility for
performing effective superpositions of spacetimes in an analogue model whose substratum is
fully quantum, supports Penrose’s idea that macroscopic superpositions would necessarily
decay due to the instability effective superposition of spacetimes carry with them. We finish the article summarizing the analogue model that we have introduced and putting forward the conclusions that can be drawn up from our analysis.

II. THE QUANTUM PUZZLE

Quantum mechanics, as described by just a dynamical wavefunction, faces two related problems. The first one is how to accommodate the apparent absence of superpositions of macroscopic systems. As the formalism \textit{per se} does not suggest a division between microscopic and macroscopic systems, the same rules for superposition should apply to both realms. The second problem, is how to account for the specific although probabilistic results of actual acts of measurement in the laboratories, i.e., how to account for the apparent intrinsic statistical result of individual acts of measurement. The formalism of quantum mechanics just provides us with a statistics for the outcome of a huge number of experiments repeated under identical conditions. Although a pragmatic attitude towards interpreting the origin of these probabilities can be taken, it is quite undesirable and problematic from a more realistic point of view. The reason for this is that such a pragmatic approach directly assumes that no reality or meaning can be given to the observations. Nowadays, such a “statistical” interpretation is not problematic from an epistemological point of view but our opinion is that it would hardly be a complete description of the physical world. This interpretation essentially removes the previous problem by simply removing the question from the list of questions we can ask, i.e., it is a dissolution of the problem. However, other interpretations of quantum mechanics pretend to answer this question in order to, first, be able to assign an ontological nature to the systems underlying observations and, second, open the door to modifications of the theory with a potential observational impact which might be obscure or counterintuitive within this pragmatic view. We can catalogue these alternative best known ways of solving the connundrum as (see for instance the review [30]):

- **Copenhagen interpretation**: It postulates a (non-precise) division between macroscopic and microscopic entities following classical and quantum evolution laws respectively [31]. It incorporates two distinct evolution laws for quantum systems [32]: the unitary evolution, when no external measurement is taken, and the non-unitary evolution (state reduction or collapse), when one measures a quantum property of the
system using a classical apparatus. This law has a probabilistic nature. In practice these assumptions have proved to work extremely well. However, many researchers have tried to provide a deeper understanding of this separation. Furthermore, from a physical point of view the division itself seems rather unnatural and somehow an *ad hoc* imposition: for the theory to be predictive we would additionally need an exhaustive classification of the systems according to their classical or quantum behaviour. This suggests that a more conceptually satisfying underlying explanation might exist which effectively agrees with the Copenhagen interpretation.

**Decoherence and many worlds:** Many scientist believe that the disappearance of correlations in the way towards the macroscopic world is caused by a phenomenon often called environmental decoherence [17][19]. This phenomenon can be described within standard linear quantum mechanics and it is a consequence of the fact that we never observe the entire world but restrict our observations to partial open subsystems, with the rest of the world acting as an environment. Interactions of the system with the environment, which are always present, tend to suppress the correlations of the system with itself in favour of correlations of the system with the environment. Then, the effect of partial tracing over the environment degrees of freedom, typically difficult to monitor, leads to a reduction of the state of the open system much alike the standard collapse of the wave function in the Copenhagen interpretation. In fact, from the viewpoint of environmental decoherence the non-unitary collapse of the wave function when measuring a quantum property with a classical (macroscopic) apparatus is just an effective description of a very efficient (almost instantaneous) process of decoherence by the environment. From this viewpoint, it is also reasonable to expect that the lifetime of quantum superpositions would diminish progressively as a system becomes more and more macroscopic, since their interactions with the environment are usually enhanced by the complexity of such an object. Here the words *macroscopic* and *complexity* are used in an intuitive non-technical manner. From a mathematical point of view, this is an explanation of how we can pass from a quantum probability distribution (a general density matrix) to a classical probability distribution (a density matrix diagonal in some basis, the so-called pointer basis) which has no trace of quantum correlations. The existence of macroscopic superpositions is not fundamentally
forbidden, only highly suppressed. However, this mechanism by itself, does not solve the second quantum puzzle we have mentioned. The final description of a measurement process is a probabilistic mixture of all the possible outcomes. In fact there is nothing fundamental forbidding that the lost correlations are recovered in the far future. For finite systems, such a recovery is inevitable: recurrences always occur, although the timescales at which they occur (the Poincaré recurrence time) might be too huge for typical systems [33]. Again, this is something that we do not see but we do not know whether the reason is fundamental or just practical. Accepting the absence of any fundamental evolution law other than the unitary evolution leads naturally to a manyworld interpretation of quantum mechanics, if one insists on taking a realist approach and neglects the possibility of having hidden variables.

The manyworlds interpretation put forward by Everett [20, 21] assumes that only the unitary evolution really exists. Results of a measurement different from the one we actually obtain, live in different coexistent worlds we do not have access to. This interpretation displays several problems, for instance, being the evolution deterministic it is not clear how Born’s probability rule is recovered for repeated experiments under identical conditions [30]. For more potentially unpleasant philosophical consequences see [34].

The combination of the decoherence mechanism and the manyworlds interpretation applied to the already reduced quantum states leads to a conceptual framework which can be currently considered as the “establishment view”, paraphrasing [30]. However, it is important to remark that this viewpoint is far from being sufficiently verified and free of conceptual puzzles [35].

- **Bohmian mechanics**: In this theoretical framework, originally conceived by de Broglie and later further pursued and formalized by Bohm [36, 37], the wavefunction is an abstract physical entity that guides the trajectories of real classical particles or fields, which are treated as in classical statistical mechanics. Measurement is a purely classical process, and the collapse of the wave function after measurement only encodes a reselection of the statistical properties of the particles in the ensemble. Bohmian mechanics is experimentally equivalent to the Copenhagen interpretation (except perhaps in situations of non-equilibrium [38, 39]). Its interest resides in the fact that
it is a proof of principle that an ontological interpretation of quantum mechanics is possible.

• **Modified quantum-mechanical theories**: Even acknowledging the success of the standard quantum mechanical formalism, since the very birth of quantum mechanics there has been a minoritarian but steady stream of works trying to find modified quantum mechanical frameworks free from some of the puzzles of the standard formulation. On the one hand, there are many proposals prescribing non-linear generalizations of quantum mechanics which could combine in a single dynamical law both the unitary and the non-unitary evolution \[13, 15, 40, 42\]. On the other hand, there are more or less vague attempts to add to the standard formalism a fundamental source of decoherence. These approaches attempt to solve the first of the two problems we have presented by introducing a modified dynamics which forbids superposition of macroscopic objects. Some of them try to introduce non-linearities in the Schrödinger equation in order to have a single equation describing the classical and quantum regimes, for a recent review of these ideas see \[43\]. Other proposals try to introduce some kind of stochastic dynamics in order to model the processes of localization of the wave function, being the proposals of Ghirardi-Rimini-Weber \[44\] one of the most famous examples. There are other proposals which associate the fundamental loss of coherence to the omnipresence of gravity \[22, 24, 43, 54\]. Here we are interested on these proposals, and specially on the one put forward by Penrose, whose language we will follow in our presentation of an analogue model. These approaches tend to suggest that both the classical and quantum dynamics are only effective approximations of an universal dynamics. In other words, both kinds of dynamics seem to correspond to certain limits of a more general theory.

A strong criticism to these kind of fundamental modifications of quantum mechanics was presented in \[55\], suggesting that dynamical laws which map pure state to mixed state are not viable since they either violate causality or energy conservation. In \[56\], the authors argue that evolution laws transforming pure states into mixed states can be chosen in such a way that causality and energy-momentum conservation hold at all scales, exemplifying it with some non-Markovian models. Furthermore, they point out that it is possible to choose a subset of the Markovian models analyzed in \[55\]
avoiding causality problems and such that all the possible deviations from the ordinary dynamics, including the energy conservation violations, are confined to a given sector of the Hilbert space, avoiding conflicts with experimental results. In [57], the authors argue that the belief that energy conservation violation is inevitable with non-unitary evolution laws comes from the misbelief that any environmental decoherence requires energy transfer among the system and the environment. They emphasize that it is possible to build quantum mechanical systems, like the ones considered in [58, 59], which interacts with an environment in a way such that the evolution erases the quantum coherences of the system and exact energy conservation holds.

III. GRAVITY’S ROLE IN QUANTUM STATE REDUCTION

As we have already mentioned there has been a steady trend trying to see whether the standard formalism of quantum mechanic fails in some regime [30, 44, 60–67]. Furthermore, there have also been many attempts to relate this violations of quantum mechanics with a gravitational origin [22–24, 45–54], and [12, 68–76]. Here we are going to briefly review these ideas. We will put a special emphasis on the proposal put forward by Penrose, which will be the language used in the rest of the paper.

A. Karolyhazy’s and Diósi’s ideas

In 1966 Karolyhazy suggested that the absence of a clear quantum behaviour in macroscopic bodies could be (at least partially) caused by the fluctuations in spacetime itself [45]. Our interpretation of what he was saying is the following (at the risk of going further than he intended).

Starting from the viewpoint that spacetime is not a fundamental structure but a more phenomenological structure that serves to describe macroscopic behaviours of rulers and clocks, it is reasonable to think that the actual quantum nature of microscopic ruler and clock would transfer some of its fuzziness into the very definition of spacetime. But then, when a quantum system moves under the influence of a fuzzy spacetime, its effect is to prevent the wave function of macroscopic systems to behave too much quantum mechanically, i.e. for instance, it would prevent the wave function of a macroscopic body to widely spread. In
turn, the close-to-classical behaviour imprinted in the macroscopic matter would make the spacetime, controlled by this matter, to behave also close to classically. Years later, Diósi proposed similar ideas \[50\]. He built on top of another proposal for reduction of macroscopic fluctuations due to Ghirardi, Rimini and Weber (GRW model).

The GRW model \[44\] is just a modification of standard quantum mechanics in order to account for the reduction of state. In fact, the only difference with standard quantum mechanics appears in the dynamical law; whereas in the quantum case evolution is governed by a uniparametric family of unitary operators \(U_t\), in this case we introduce sudden jumps of the wavefunction that tend to spatially localize the system. In the GRW model, these sudden jumps that we include to introduce spontaneous localization processes at scales \(L\), the first free parameter in the theory, are modelled by a Poissonian distribution with a certain frequency \(\lambda\) which is the second free parameter in the theory.

For concreteness, let us consider the case of just one nonrelativistic particle. The master equation for its density matrix is given by

\[
\frac{d}{dt}\rho(t) = -i[H,\rho(t)] - T[\rho(t)],
\]

(1)

\(\rho\) representing as usual the density matrix of the system, \(H\) its Hamiltonian (the first part accounts for the unitary evolution, that is, standard quantum mechanics), and the last term models the spontaneous localization process. The operator \(T\), in the position basis, can be written as

\[
\langle x|T[\rho(t)]|y\rangle = \lambda \left(1 - \exp \left[-(x - y)^2/4L^2\right]\right) \langle x|\rho(t)|y\rangle.
\]

(2)

Thus, we see that the effect of the new term is to suppress non-diagonal (delocalized states) elements of the density matrix in the position basis. An operator \(T_i\) would appear for each particle in the case we are considering many-particle systems. As showed in \[44\], for typical Hamiltonians the center-of-mass degrees of freedom decouple from the internal ones. The result of such a decoupling in this limit is that the center-of-mass follows an equation of motion of the type (2) while the internal degrees of freedom follow an ordinary Schrödinger equation of motion. Furthermore, the parameter \(\lambda\) in (2) is replaced with \(N\lambda\), being \(N\) the number of particles of the system. Thus, the model is able to account qualitatively for the absence of macroscopic superpositions while keeping the microscopic dynamics approximately untouched \[30\]. Actually, it illustrates what we already advanced in the introduction: the more complex and macroscopic a system is, the faster its quantum
correlations tend to decay. In this context, the measure of how complex and macroscopic a system is would be associated with the number of particles that compose it. At this level, the GRW model has two free parameters and experiments have put bounds on their possible values. To our knowledge, this model has not been fully discarded yet [66, 67].

The GRW proposal constitutes a representative of some models that want to provide a unified framework for the observed microscopic quantum dynamics and the absence of macroscopic superpositions. They typically introduce stochastic differential equations to account for both phenomena, like the ones introduced by Diósi [50] which tried to assign a gravitational origin to the localization processes. He introduced the QMUDL (Quantum Mechanics with Universal Density Localization) model, where instead of the wave-function (as in the GRW model) it is the probability density (and thus the matter density) that undergoes localization, following Karolyhazy’s proposal [45]. Furthermore, the parameters characterizing the length scale and frequency of these localization processes are dimensionally fixed in terms of the Newton constant (realizing, again, the ideas of Karolyhazy) leaving just one pure number as a free parameter in the theory.

Diósi’s model was able to phenomenologically account for the possible role that gravity might play in the quantum state reduction. His main achievement was to take Karolyhazy’s vague ideas, concerning gravity and quantum mechanics, and capture them in a simple model which differs from standard Quantum Mechanics. Within this line of thought, later on Penrose gave different arguments explaining why and how gravity might be a source of decoherence, but with similar phenomenological consequences. In some sense, Diósi’s arguments are much more phenomenological than Penrose’s, which are more theoretical in nature. We will review them in the next subsection.

A step further in the formulation of these ideas was taken in the past years, by introducing models in which gravity did not only act as a source of decoherence for the quantum system but also the backreaction of the quantum system on the gravitational field in the Newtonian regime was included. These models rely on the result of [77], where it was shown that it is possible to give a phenomenological description for the interaction of a classical variable with a quantum degree of freedom modelling it as a continuous measurement, at the expense of necessarily introducing fluctuations on the classical variables. Two proposals which are built on top of these principles are the Kafri-Taylor-Milburn [78] model and the Tilloy-Diósi model [79]. The former model applies only to two particles moving in one di-
dimension where the Newtonian potential can be linearized, while the latter is general within the regime of applicability of Newtonian gravity. A recent analysis [80], where some natural extensions of the Kafri-Taylor-Milburn model are introduced to include an arbitrary number of particles, shows that all these generalizations are straightforwardly ruled out experimentally or internally inconsistent. They conclude that just the Tilloy-Diósí model is a viable description of a quantum mechanical system interacting with a classical gravitational field through this continuous measurement process and, actually, it reduces to the original proposal of Diósí and the one of Penrose which we will explore in detail in the next section for the gravitationally induced quantum reduction of state [79].

B. Penrose’s idea

Here we will review Penrose’s idea of how gravity may be responsible for the reduction of state in quantum mechanics ([12, 22–24]). In order to do that, let us assume initially that both gravity and matter have a quantum character. A mild assumption is that there should exist states which represent an almost classical stationary spacetime sourced by an equivalently almost classical stationary distribution of matter which we represent as $|\psi, G_{\psi}\rangle$, where $\psi$ is associated with a particular matterial configuration and $G_{\psi}$ represents the gravitational field generated by the matter content $\psi$. Following the superposition principle of quantum mechanics, within the set of possible states one has to include some of the form

$$\alpha |\psi, G_{\psi}\rangle + \beta |\phi, G_{\phi}\rangle,$$

(3)

with the two terms independently representing almost classical configurations; the complex coefficients $\alpha, \beta$ satisfy $|\alpha|^2 + |\beta|^2 = 1$, in order to keep the state normalized. The key observation is that, even if each of the states in this combination is stationary, it is not clear whether (3) is a stationary state or not. In stationary globally hyperbolic spacetimes, we have a well defined notion of stationary state since we have a global timelike Killing vector $K$ that generates time translations. It is natural to use this classical notion in order to define stationarity in the quantum scenario we are dealing with, since gravity should be well described (at least approximately) by a classical spacetime due to the assumptions we are making. Thus, stationary states $|S_E\rangle$ are those eigenstates of the Killing vector operator $\hat{K}$,
which turn out to be also states of well defined energy,

$$-i\hat{K}|S_E\rangle = E|S_E\rangle,$$

where $|S_E\rangle$ represents a generic stationary state of the form $|\psi, G_\psi\rangle$ with energy $E$. The key observation is that the state (3) is a superposition of states of the form $\alpha|S_E\rangle + \beta|S'_E\rangle$, both stationary states with respect to their own Killing vector field $K, K'$ respectively. The problem is that we have, in some sense, a superposition of spacetimes and the concept of time translation in this superposed spacetime is an ill posed concept. To be able to talk about stationary states in this superposed spacetime, we should be able to identify the corresponding time translation generator on it but, in fact, Penrose argues that there is a fundamental difficulty with this concept and it can be quantified in some cases. This uncertainty brings with it another uncertainty in the concept of energy for that superposed state, an uncertainty that can be explicitly estimated in a clear way in certain situations. This provides us with an energy uncertainty $\Delta$ that, in accordance with the usual application of the Heisenberg uncertainty principle to estimate the lifetime of unstable particles, is related to the lifetime of the superposed state via the expression $\tau \sim \hbar/\Delta$. Thus, the superposition of almost classical spacetimes is an ill posed concept since it is unstable. In fact, even in Newton-Cartan spacetimes where time coordinates can be identified between both spacetimes and, thus, there is a canonical correspondence between the various space sections of one space-time and the other, there is no canonical way of identifying points of the spatial sections of one spacetime and the other. It is this lack of definite pointwise identification between spatial sections that leads to problems when trying to define the notion of time-translation for the superposition of gravitational fields.

In fact, for the case of superpositions of two slightly different spacetimes (for instance those generated by the same lump of mass in two slightly different places), Penrose proposed a way to quantify this ill-possedness. We expect to be able to make an approximate local spatial identification between the two spacetimes by applying the equivalence principle to those states and identifying regions of both spacetimes where free fall agrees. However, for the superposed spacetimes, there is no way to make the spatial identification in such a way that free fall agrees everywhere; so the natural thing to do is to minimize the difference between free fall motions. In the Newtonian limit, Penrose argued that the measure of incompatibility in the identification between spacetimes is $\Delta$, basically the gravitational
self-energy of the difference between the mass distributions of each of the two locations of the lump of mass:

\[
\Delta = -4\pi G \int d^3x d^3y (\rho(x) - \rho'(x)) (\rho(y) - \rho'(y)) / |x - y|, \tag{5}
\]

where \(\rho, \rho'\) are the mass densities of the two lumps.

Coming back to the discussion of the introduction, we notice that Penrose’s model measures how complex and how macroscopic a system is in terms of the gravitational field it can create. It is similar to Díosí’s model, where the complexity is associated also with the mass of the system (and, thus, with the gravitational field that it can produce). Presented in this way, assuming \(\tau = \hbar / \Delta\) the model lacks free parameters and it has been recently discarded experimentally: the relaxation times this model implies are much bigger than the ones observed in \([76]\). We can always introduce a dimensionless parameter since Penrose arguments just suggest that the decay times scale like \(\tau \sim \hbar \Delta^{-1}\), but they do not fix the proportionality constant. For a careful analysis on the impact of this mechanism for gravitationally induced decoherence of physical systems and comparison with the expected environmental decoherence, see \([81]\).

At this point, it is better to pause and discuss some recent ideas that try to give a physical meaning to superposing spacetimes. The way in which these ideas try to make sense of it is by giving to such superpositions an operational meaning, without entering the discussion of how such superpositions can be embedded in a full theory of quantum gravity. For this purpose, one can consider variations of an Unruh-deWitt detector, which constitutes the simplest local probe one can think of, i.e. a two-level system following a timelike trajectory in spacetime and coupled to quantum fields propagating on top of a fixed geometry \([82]\). An extended framework was examined in \([83, 84]\), where it was discussed how to consider also superpositions of trajectories for such detectors, instead of fixed ones. This idea was used in \([85]\) in order to analyze concrete superpositions of de Sitter spacetimes translated one with respect to the other but with the same curvature, proving that such situation is diffeomorphic to a superposition of the trajectories of a single detector for a single de Sitter manifold, applying the results of \([86]\). Furthermore, in \([85]\) they also try to address the problem of considering superpositions of de Sitter spacetimes with different curvatures assuming they just have one manifold in a superposition of some parameters describing such geometry (in this case, the characteristic radius of curvature of de Sitter).
Another, line of work in this direction is the one which takes advantage of the so-called Quantum Reference Frame [87]. Considering again that a superposition of semiclassical spacetimes is a meaningful concept, in [88] they introduce a generalization of the Einstein Equivalence Principle into the quantum realm to consider the Quantum Reference Frames. Given a local probe propagating on top of a superposition of both spacetimes, it exists a transformation between Quantum Reference Frames that leaves the system in a state where the local probe propagates in a locally Minkowskian spacetime effectively decoupling from the gravitational field, even if the gravitational field remains in a superposition in a neighbourhood of the local probe. We notice that this framework might provide a way out to Penrose concerns about the ill-posedness of the concept of stationarity for superposed spacetimes, at the prize of introducing a local probe and defining the concept of stationarity subjected to this operational meaning. We could argue that a sensible definition of stationarity is the following: A superposition of semiclassical spacetimes is a stationary state if for a family of local probes, the state for the gravitational field after disentangling it from the local probes propagating in the superposed spacetimes, is stationary with respect to the proper time of such local probes. Although this provides a way out to the dilemma posed by Penrose that does not point towards a break of the unitarity of the quantum theory, it is not completely satisfactory due to the necessity of introducing additional ingredients in the construction. Furthermore, it is far from clear whether a superposition of semiclassical geometries like the ones considered by Penrose would be stationary according to this definition.

C. Quantum mechanics with stochastic time

We see that Penrose arguments for the gravity induced reduction of state in quantum mechanics rely on the ill-posedness of the concept of time translation on this superposition of spacetimes. This is the ultimate reason for the instability of the superposition of macroscopic distributions of matter and the existence of a characteristic decay time for them. Thus, we can assume that, forgetting about the gravitational fields (tracing over them in some sense) will lead us to the necessity of describing quantum mechanics in terms of some kind of stochastic time that fluctuates. That is, no stable superpositions of the form (3) exist due to the inability to define a time variable with respect to the one in which they are stationary.
states. This leads us to the conclusion that a way to model this phenomenon is to assume that, whatever time variable we choose to describe that system, it will be a stochastic variable. We will follow the discussions presented in [58, 89].

If we make superpositions of slightly different distributions of matter, we expect the time variable to have small fluctuations and thus the laws of quantum mechanics will be slightly affected. In other words, we expect that some of the fuzziness of matter is transferred to the spacetime itself and, from a quantum mechanical perspective, to our choice of time to perform evolution. In order to model these features, we will define the ideal time $s$ in terms of which the Schrödinger equation is written, as $s = t + \delta(t)$, being $t$ a generic time (it could be, for example, the proper time of some observer or the coordinate time of some of the spacetimes we are superposing) and $\delta(t)$ a stochastic variable that represents small fluctuations due to the uncertainty in our choice of time. This uncertainty models our inability to choose one privileged time or, in other words, the impossibility to identify pointwise both spacetimes.

With this, the Schrödinger equation for our generic density matrix can be written as follows

$$i\partial_s \rho(s) = [H, \rho(s)],$$

which can be rewritten in terms of the variable $t$ as

$$i\partial_t \rho(t + \delta(t)) = [H, \rho(t + \delta(t))](1 + \alpha(t)),$$

being $\alpha(t) = \frac{d\delta(t)}{dt}$. This quantity can be interpreted as the error due to our choice of time and we assume that there exists some probability distribution functional $P[\alpha(t)]$ for it. Moreover, in order to describe the phenomenon of the instability of superpositions in terms of this formalism, we will assume it to be stationary since we will be concerned with superpositions of stationary spacetimes. In addition, we will assume that there is no systematic rate of increase (or decrease) of the fluctuations for the situations we are considering and thus $\langle \alpha(t) \rangle = 0$.

Moving to the interaction picture ($\rho \rightarrow \rho' = e^{iHt}\rho e^{-iHt}$) and integrating in $t$, we can reach the following integro-differential equation for the density matrix

$$i\partial_t \rho'(t + \delta(t)) = \alpha(t)[H, \rho(t)] - i \int_0^t dt' \alpha(t')\alpha(t)[H, [H, \rho'(t + \delta(t))]].$$

\[1\text{ Notice that this error could be straightforwardly corrected.}\]
Assuming that time fluctuations are small enough, we can perform a power expansion of $\rho$ and neglect terms of order greater than $\alpha^3$. Moreover, the density matrix we have access to is the averaged density matrix $\langle \rho(t + \delta(t)) \rangle = \rho_{\text{eff}}(t)$ over the space of possible fluctuations. Averaging the expression above, using the fact that $\mathcal{P}[\alpha(t)]$ is stationary, and the considerations above, we obtain the following expression in the interaction picture

$$i\partial_t \rho_{\text{eff}}(t) = -i \int_0^t d\tau c(\tau)[H, [H, \rho_{\text{eff}}'(t - \tau)]] + \mathcal{O}((\langle \alpha(t)^3 \rangle), \quad (9)$$

where we have introduced the time correlation function for the fluctuations $c(t - t') = \langle \alpha(t)\alpha(t') \rangle$. The assumption of small correlations in the fluctuations allows us to neglect the $\mathcal{O}((\langle \alpha(t)^3 \rangle)$ terms, since huge ones would lead to highly non-local effects that could be observed in principle.

We will assume that the characteristic time of correlations of the fluctuations are much smaller than the characteristic time of evolution of the system. Then, we expect the density matrix to not evolve significantly within a correlation time and thus we can replace $\rho'_{\text{eff}}(t - \tau)$ with $\rho'_{\text{eff}}(t)$ and take the limit of integration to infinity. This approximation, often called the Markov approximation, allows to obtain a coarse grained density matrix, i.e. a density matrix which does not capture fluctuations at time-scales shorter than the characteristic correlation time of the stochastic variable. This approximation can be done in two steps which, to some extent, are independent. The first one is replacing $\rho'_{\text{eff}}(t - \tau)$ with $\rho'_{\text{eff}}(t)$, which means assuming the evolution of the density matrix is much slower than the typical correlation time of the bath, something ensured by choosing a weak enough coupling. The resulting equation of this first step is often called the Redfield equation. The second step in this approximation is to replace the upper limit of integration by $\infty$, something that can be done as long as the time correlation function $c(\tau)$ decays fast enough at large $\tau$.

Since we are concerned with stationary superpositions of times, it seems plausible to assume that memory effects in our system will have little impact. Moving back to the Schrödinger picture and putting altogether these approximations, the density matrix dynamics is encoded in the following master equation

$$i\partial_t \rho_{\text{eff}} = [H, \rho_{\text{eff}}] - i\xi[H, [H, \rho_{\text{eff}}]]. \quad (10)$$

$\xi$ is a parameter with units of time resulting from the integration of the correlation function $c(t)$ and it controls the characteristic timescale of decay for the correlations. Clearly, the first
term is the usual unitary part while the second term, which is diffusive but dissipationless, is responsible for the reduction of state, as an effective residue of the stochastic time we need to use to describe our system. This means that the pointer basis, i.e. the basis in which the correlations of the density matrix decay exponentially is the energy basis. Explicitly, assuming that the spectrum of $H$ is discrete, we can write the evolution of the density matrix in the basis of eigenstates of $H$. We label those states as $|n, g_n\rangle$, with $g_n$ accounting for the possible degeneracies of the spectrum and being $H |n, g_n\rangle = E_n |n, g_n\rangle$ their definitory property.

$$\langle m, g_m | \rho_{\text{eff}}(t) | n, g_n \rangle = \langle m, g_m | \rho_{\text{eff}}(0) | n, g_n \rangle e^{-i(E_n - E_m)t} e^{-(E_n - E_m)^2 \xi t}.$$  \hspace{1cm} (11)

IV. ANALOGUE GRAVITY MODEL FOR STATE REDUCTION DUE TO GRAVITY

The ideas advocated by Penrose and others are still at the heuristic level, trying to push in an alternative direction in the search for a unification of gravity and the quantum. At the same time, they provide a way-out to some of the problems associated with the standard formulation of quantum mechanics. In that sense, they do not constitute completely defined theories of the interplay between gravity and the quantum. However, analogue models of gravity have been proved to be powerful tools in providing precisely defined theories that exemplify how we dissolve a classical spacetime structure into a quantum regime. What plays the role of spacetime in an analogue model is nothing fundamental (as required by Karolyhazy, for instance), but only a useful descriptive tool to understand low-energy excitations in the system. In this work we want to construct a translation dictionary from Penrose set-up to their counterparts in an analogue gravity system. This might help to give additional concreteness to his proposal.

The concrete analogue model we are going to discuss in what follows is based on a Bose-Einstein condensate (BEC) in dilute gases. In more general terms it is a quantum many-particle system constructed on top of a Galilean world (i.e. non-relativistic). We use these ingredients not because we believe the underlying world should be like this but because they are simple enough to start providing insights. Furthermore, they have proven to be experimentally feasible in order to simulate analogue spacetimes of interest.

We know that a system of weakly interacting bosonic particles can develop condensation
at low temperatures: a macroscopic number of particles populates the same state. Once the system condenses, in the case of BECs with repulsive interactions there is a long-wavelength regime in which the system behaves as a (quantum) fluid with acoustic excitations (classical and quantum) living on top of an overall fluid flow or effective spacetime. Now, our inquiry is essentially the following: what does it mean to superpose two analogue gravity configurations? Are such superpositions stable. For example, we can try to superpose two black hole configurations with wildly different parameters (e.g. surface gravity, etc.) occupying the same location; or two equal or different configurations but now residing at different locations. To start digging into these questions we will first analyze a simple model of BEC with two potential wells in which to localize the composing particles. This system is complex enough to sustain the emergence of an effective classical spacetime, while keeping the underlying full quantum structure of the model. Therefore, we will use it to address the previous questions of superposing different effective spacetimes and analyze whether such a superposition breaks down.

The remain of the section is structured as follows. In subsection IV A we will review the main properties of the model that we will consider for bosons in a double well, the Bose-Hubbard model with two sites. In subsection IV B we will explain which are the main properties that a Bose-Einstein condensate needs to fullfill for its sound wave excitations to admit an effective description in terms of a Klein-Gordon equation propagating on top of a curved geometry. In subsection IV C we will explore the stability of the ground states of the Bose-Hubbard model depending on the possible values of its parameters. In subsection IV D we will explore the implications of this stability analysis and discuss the constraints that we have to impose to the model in order to describe an effective spacetime. Furthermore, we will explore the possibility of building an effective superposition of spacetimes in this model, in order to establish an analogy with Penrose ideas. Finally, in subsection IV E we will conclude by extending our discussion to the possibility of creating superposition of similar geometries in the same location.

A. Condensates in a double well

Let us assume that we have scalar bosons in a symmetric double well potential where we will label the wells by $i = 1, 2$; we will make the simplifying assumption for the moment
that there is only one relevant state in each well and particles within a well have a contact
interaction controlled by the parameter $U$ which can be either repulsive ($U > 0$) or attractive
($U < 0$). Also, we will assume that we have a term describing tunneling between the wells
controlled by the parameter $t$ which is the amplitude of probability for tunneling between
both wells. The bigger the parameter $t$, the higher the potential barrier will be. The
Hamiltonian describing these particular interactions is the Bose-Hubbard Hamiltonian [93]
with just “two sites” in the lattice

$$H = -t(a_1^\dagger a_2 + a_2^\dagger a_1) + \frac{U}{2} [n_1(n_1 - 1) + n_2(n_2 - 1)],$$

where $a_i^\dagger, a_i$ represent the usual creation and annihilation operators that annihilate
and create particles in the well $i$, with $n_i = a_i^\dagger a_i$ being the number of particles in the well
$i$. Furthermore, we will work under the assumption of having a fixed number of particles
$N = n_1 + n_2$. For the non-interacting Hamiltonian, i.e. Eq. (12) with only the tunneling
term, the solutions are explicit and can be found via an ordinary Bogolyubov transformation.
For the regime in which the tunneling is highly suppressed, just the interacting term survives,
which is diagonal in the number basis.

We will now describe the main properties of the states which will be interesting for our
purposes: the ground state of the free Hamiltonian and the ground state for the Hamiltonian
with $t = 0$, i.e., the ones with no hopping between the two wells, either with attractive
$U < 0$ and repulsive interactions $U > 0$. They will serve as approximate ground states
for the regimes of hopping domination $t \gg |U|N$ and interaction domination $t \ll |U|N$,
respectively. The intermediate ground states between both regimes are not so useful for our
purposes and can be found numerically. However, understanding the evolution of the system
from one regime to the other as we adiabatically increase the parameters sheds some light
onto the behaviour of the model and it can be well captured by the ansatz provided in [94].
We will review those ansätze and discuss the intermediate regime at the end of the section,
although such details can be safely skipped by the reader just interested in the realization
of the kind of configurations Penrose has in mind in BECs.

Prior to discussing the ground states on the extreme regimes above, we need to introduce
the concept of the single particle reduced density matrix $\rho_{SP}$. Such density matrix is defined
as

$$(\rho_{SP})_{ij} = \text{tr} \left( \rho a_i^\dagger a_j \right),$$

(13)
where $\rho$ represents the density matrix of the system. According to Leggett’s nomenclature [95], a condensate will be fragmented if its single particle density matrix has two macroscopic eigenvalues, i.e., two eigenstates with eigenvalue of order $O(N)$. Intuitively, this would correspond to a situation in which condensation occurs simultaneously in two different single-particle states. Armed with this tool, let us now analyze the three ground states of interest: the free case, the repulsive interacting case and the attractive interacting case.

**Free condensate:** The free case can be explicitly solved via an ordinary Bogolyubov transformation, described by the following $SO(2)$ rotation within the space of operators $\{a_1^\dagger, a_2^\dagger\}$:

$$b_{1,2}^\dagger = \frac{a_1^\dagger \pm a_2^\dagger}{\sqrt{2}}.$$

Thus, we find that for the single-particle subspace the eigenstates of the Hamiltonian are the symmetric and antisymmetric states with energies $-t$ and $t$, respectively. Thus, for the $N$ bosons, the ground state will be given by the fully symmetric state

$$|C\rangle = \frac{1}{\sqrt{2^N N!}} \left(a_1^\dagger + a_2^\dagger\right)^N \vert \text{vac} \rangle,$$

because they do not interact among themselves. The $C$ stands for the coherent state, since it corresponds to $N$ bosons coherently delocalized among the two wells with equal probability of finding them in one of the wells. The single particle density matrix for this state reads

$$\rho_{\text{SP}}(C) = \frac{N}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

This state has a single macroscopic eigenvalue $N$, that is, the ground state is a single condensate representing a condensation on the state which is a superposition of particles in one well and the other. According to Leggett’s classification [95], it corresponds to a non-fragmented condensate since we just have one macroscopic eigenvalue of the single-particle density matrix. Physically, it corresponds to a condensation on the single-particle state which is approximately a superposition of the two Gaussians peaked around the center of each well in position space, describing the ground state of each well. Since the ground state $|C\rangle$ is a linear combination of number states $|n_1, n_2\rangle = a_1^{\dagger n_1} a_2^{\dagger n_2} |\text{vac} \rangle / \sqrt{n_1! n_2!}$, the number of particles in each well has enormous fluctuations. We can compute it by writing the coherent
state \(|C\rangle\) in the number basis. For even \(N\), we have

\[
|C\rangle = \sum_{\ell=-N/2}^{N/2} \Psi^{(0)}_\ell |\ell\rangle,
\]

(17)

where \(|\ell\rangle \equiv |\frac{N}{2} + \ell, \frac{N}{2} - \ell\rangle\) and

\[
\Psi^{(0)}_\ell = \left( \frac{N!}{2^N (\frac{N}{2} + \ell)! (\frac{N}{2} - \ell)!} \right)^{1/2} \approx e^{-\ell^2/N} / (\pi N/2)^{1/4}.
\]

(18)

The fluctuations in the number of particles on each well are given by

\[
\langle \Delta n_i^2 \rangle_C = \langle (n_i - \langle n_i \rangle)^2 \rangle_C = N/4.
\]

(19)

with \(i = 1, 2\). Let us move on to discuss the interacting case \(t = 0, U \neq 0\). For this purpose, taking into account that the total number of particles is conserved \(n_1 + n_2 = N\), we can rewrite the interacting part of the Hamiltonian as

\[
\frac{U}{2} [n_1(n_1 - 1) + n_2(n_2 - 1)] = \frac{U}{4} \left[ (n_1 - n_2)^2 + N^2 - 2N \right],
\]

(20)

which parametrizes the interaction by the difference in the number of particles on each well.

**Strong repulsive interactions:** We will begin with the repulsive interactions \((U > 0)\). Such case clearly favors the minimum difference in the number of particles between the two wells. Thus, for even \(N\) which we consider for simplicity, the ground state for the repulsive interactions is clearly

\[
|F\rangle = \frac{a_1^{\dagger N/2} a_2^{\dagger N/2}}{(N/2)!} |\text{vac}\rangle.
\]

(21)

Its single particle density matrix is

\[
\rho_{SP}(F) = \frac{N}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},
\]

(22)

which corresponds to a fragmented condensate since it has a macroscopic eigenvalue \(N/2\) with multiplicity 2. It corresponds to two uncorrelated condensates having half the particles, each one located at one of the two wells. This result for the ground state in this limit is quite intuitive since having one more particle than the half of them on one of the two wells would imply paying a penalty in energy. The fluctuations in the number of particles on each well vanish identically

\[
\langle (\Delta n_i)^2 \rangle_F = 0.
\]

(23)
Thus, in this case, we have a fragmented condensate with a well defined number of particles on each well.

**Strong attractive interactions:** For attractive interactions $U < 0$, the situation is the opposite; the favoured states are those containing a huge difference in the number of particles on each well. This means, the ground state is the subspace spanned by the vectors $\{|N,0\rangle,|0,N\rangle\}$. However, if we begin with the coherent state $|C\rangle$, the ground state of the free theory, and we turn on adiabatically the attractive interactions, the state that we will reach once $t$ becomes negligible will be a concrete superposition of both vectors (further details of how this state is reached will be provided at the end of the section). Actually, it corresponds to a Schrödinger-cat like state

$$|\text{cat}\rangle = \frac{1}{\sqrt{2}}(|N,0\rangle + |0,N\rangle).$$

(24)

It is also fragmented since its single-particle density matrix has also two eigenvalues

$$\rho_{SP}(\text{cat}) = \frac{N}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

(25)

being equal to $\rho_{SP}(F)$, the one that we had for the Fock state. Particle number fluctuations on each of the wells allow us to distinguish between both states though: in this case they do not vanish but

$$\langle (\Delta n_i)^2 \rangle_{\text{cat}} = N^2/4.$$

(26)

Thus, we have seen that although the Fock and cat states are fragmented according to the standard definition of Leggett (their one-particle density matrices have two macroscopic values), they physically correspond to very different states. While the Fock state has a definite number of particles on each well, the cat state has an indefinite number of particles on each well; the latter corresponds to having the bosons delocalized.

Until now, we have presented the three states that will be relevant for our purposes. They are the coherent state $|C\rangle$, which is the ground state of the free theory $t \neq 0, U = 0$; the Fock state $|F\rangle$, which is the ground state of the repulsive interacting theory $t = 0, U > 0$; and the cat state $|\text{cat}\rangle$, which is the ground state of the attractive interacting theory $t = 0, U < 0$. The coherent state corresponds to a non-fragmented condensation while the Fock and cat state correspond to fragmented condensates. With respect to fluctuations in the number of particles on each well, the Fock state has no fluctuations on the number of particles on each
well while the coherent and cat state have enormous fluctuations on the number of particles on each well. These are the relevant features of these three states which will play a role in the building of our analogue models. We now provide additional details of how the model behaves for intermediate regimes, which can be safely skipped by the readers who want to go directly through the details of the analogy.

**Intermediate interactions:** We will analyze what happens as we tune on the interactions adiabatically, i.e., we begin with the free Hamiltonian and assume we slowly turn on the interactions in such a way that the ground state of the free theory accommodates to the ground state of the interacting theory. Although numerically it is possible to obtain explicitly the wave function of the ground state for the interacting theory, as we advanced, we will take advantage of the ansatz introduced in [94] and use their families of states as analytic states capturing the main properties of the ground states and having scalar product with them very close to one.

These ansätze for the interacting theory will be based on the ground state wave function for the non-interacting theory, the \( U = 0 \) case of the Hamiltonian (12). The starting point will be to write the ground state in terms of the number basis

\[
|\Psi\rangle = \sum_{\ell=-N/2}^{N/2} \Psi_\ell |\ell\rangle,
\]  

(27)

and write the Schrödinger equation for this system \( H |\Psi\rangle = E |\Psi\rangle \), which gives the following expression

\[
E \Psi_\ell = -t_{\ell+1} \Psi_{\ell+1} - t_\ell \Psi_{\ell-1} + U \ell^2 \Psi_\ell,
\]  

(28)

with

\[
t_\ell = t \sqrt{(N/2 + \ell)(N/2 - \ell + 1)}.
\]  

(29)

The problem is equivalent then to a one-dimensional tight-binding model in a harmonic potential with non-uniform tunneling matrix elements [94]. The non-uniformity is such that wave functions \( \Psi_\ell \) with large amplitudes near \( \ell \sim 0 \) always have less energy than the ones that spread around different values of \( \ell \). Actually, in the free case the wave function was a narrow Gaussian centered at \( \ell = 0 \).

With this expression for the eigenvalue problem, let us start by considering the repulsive case \( (U > 0) \). These interactions make the coherent state [18] squeeze in an even narrower distribution. The family of states introduced in [94] for capturing this evolution from the
coherent to the Fock state is

$$\Psi_\ell(\sigma) = \frac{e^{-\ell^2/\sigma^2}}{(\pi \sigma^2/2)^{1/4}}.$$  \hspace{1cm} (30)

As $\sigma^2$ varies from $\sqrt{N}$ to small values, the initial coherent state $|C\rangle$ starts looking much more like the Fock state. Actually we can obtain a relation between $\sigma$ and the value of $U$ by taking a continuum limit on (28). In this limit, the equation reduces to that of a harmonic oscillator potential and we can obtain the value of $\sigma(U)$ since that problem is exactly solvable: $\sigma^{-2} = (2/N)(1 + UN/t)^{1/2}$. The single particle density matrix for these states reads

$$\rho_{SP} = \frac{N}{2} \begin{pmatrix} 1 & e^{-1/(2\sigma^2)} \\ e^{-1/(2\sigma^2)} & 1 \end{pmatrix}.$$  \hspace{1cm} (31)

It has eigenvalues $\frac{N}{2}(1 \pm e^{-1/\sigma^2})$ and the fluctuation in the number of particles on each well is $\langle (\Delta n_i)^2 \rangle = \sigma^2/2$. As the gaussian width increases, the eigenvalues vary from $(N,0)$ to $(N/2,N/2)$ and $\sqrt{\langle (\Delta n_i)^2 \rangle}$ varies from $\sqrt{N}$ to 0.

Let us discuss now the case of attractive interactions $U < 0$. When the interaction is attractive, states having a huge difference in the number of particles on each well, or, in other words, a huge amount of particles on a single well, are favored. Thus, the effect of slowly turning on an attractive interaction is to split the Gaussian of the noninteracting state (18) into a symmetric distribution with two peaks, a process which is captured by the family of states

$$\Psi_\ell(a) = K \left( e^{-(\ell-a)^2/2\sigma'^2} + e^{-(\ell+a)^2/2\sigma'^2} \right),$$  \hspace{1cm} (32)

being $2a$ is the separation between the peaks, $\sigma'$ their width and $K$ is a normalization factor. As $a$ varies from 0 to $N/2$ and $\sigma'$ reduces at the same time from $1/\sqrt{N}$ to 0, the coherent state of the free theory evolves to the cat state (which is reached in the $U \to -\infty$ limit).

\textbf{B. Effective spacetimes from BEC’s}

In order to build an analogue gravity model, a crucial necessary condition is that the system can exhibit in some regime \textit{causality properties}. For the Bose-Einstein condensate, this means that it should be able to accommodate the propagation of sound-like excitations within the system, which correspond to some of the collective excitations of the system. A fundamental ingredient for the propagation of this collective excitations is the existence of
an effective repulsion among the constituents of the BEC. This does not mean that there can not be any attractive interactions among the elementary constituents of the BEC, it just means that at distances short enough the dominating interaction needs to be repulsive. Actually, this requirement is needed in order to prevent a destabilization of the system that produces a “bosenova” \cite{96}.

Within the catalogue of available analogue models, Bose-Einstein condensates are one of the best spacetimes analogues we have \cite{27}, mainly because they are very clean and relatively insensitive to external noise. For this system, acoustic perturbations propagating on top of the condensed fluid are well described by the Klein-Gordon equation on top of an effective spacetime whose properties are related to the properties of the condensed fluid. This holds for acoustic perturbations whose wavelength is greater than the so-called healing length of the condensate (which roughly speaking is the length at which inhomogeneities of the atomic density composing the condensate are smoothen out).

The coherent state resulting from taking $U = 0$ or the cat state that results from taking $U < 0$ need to be understood as effective, in the sense that for a realistic system they would be slightly modified due to the presence of repulsive interactions. However, it is safe to assume that there exists a regime in which these repulsive interactions are mild enough to take the coherent and cat state as good approximations to the state of the system, but they still admit a propagation of sound-like excitations. Furthermore, in principle, one could build a Schrödinger-cat-type state \cite{24} even in the case of repulsive interactions. For example, one could adiabatically switch on the attractive interactions with $U < 0$ and once the system settles in the appropriate cat state turn the interaction towards the repulsive regime.

\textbf{C. Stability of the ground states}

We saw that even though one-particle density matrices of the Fock and cat states were the same, they described completely different states: while the Schrödinger-cat state had enormous number fluctuations, the Fock state had zero number fluctuations; that is, higher order correlation functions are needed to characterize these different fragmented states.

Moreover, this huge number fluctuations in the Schrödinger-cat state is telling us something about the stability of the system: it is obvious that such a delocalized system must tend to be unstable under local perturbations which act independently on each of the wells.
In fact, let us consider a generic interaction modelled by the Hamiltonian

$$H' = \epsilon \left( c_1^\dagger a_1 + c_1 a_1^\dagger \right), \quad (33)$$

where $\epsilon$ is a small parameter controlling the perturbation, $a_1$ is the usual annihilation operator for bosons in the first well and $c_1^\dagger$ is the creation operator for a generic excited state of the first well and, consequently, representing a local perturbation of the first well. Under this small perturbation, the Fock state $|F\rangle$ is robust since it simply changes to another Fock state $|F'\rangle$ under the action of this Hamiltonian operator

$$|F'\rangle = H'|F\rangle \sim c_1^\dagger a_1^\dagger N/2 a_2^\dagger N/2 |\text{vac}\rangle, \quad (34)$$

which shows the robustness of Fock states under standard local perturbations. However, the cat state does not display such robustness. Actually, the cat state tends to collapse to a localized condensate once we take them into account

$$H'|\text{cat}\rangle \sim c_1^\dagger a_1^\dagger (N-1) |\text{vac}\rangle, \quad (35)$$

since this state corresponds to a localized state on the first well. It corresponds to a number state with $N-1$ particles on the ground state and one particle in the excited state described by the $c$ operator.

The conclusion we extract from this analysis is that the system tends to avoid being in delocalized macroscopic superpositions since they are unstable under local perturbations that do not coherently hit the system on the two places. We have just considered interactions of the type $|F\rangle$ which will be contained in a generic condensed matter Hamiltonian. This can be seen by noting that, in general, we can work in the second quantized formalism in which the fundamental object we consider is the field $\psi(x)$,

$$H_{\text{int}} = \int d^3x h(\psi^\dagger(x), \psi(x)), \quad (36)$$

whose expansion in a concrete basis of one-particle states reads $\psi = \sum_i \left( a_i f_i + a_i^\dagger f_i^* \right)$. The previous simplification was equivalent to saying that we just focused on the mode $f_1$ which was an excited state of the first well. In practice we will have a tower of excited states above them that can be regarded as perturbations of the form $|F\rangle$. 

29
D. Attempting to generate a superposition of two effective spacetimes

Now that we have described the three main states that can appear in a Bose-Hubbart model, and their stability properties, it is time to discuss them at the light of their ability to provide superpositions of effective spacetimes.

Our aim here is to consider a model whose substratum is an intrinsic quantum system, admitting an effective geometrical description in some regime, and analyze the implications and viability of producing a state which would be understood within the effective geometrical description as a putative superposition or mixture of spacetimes. For that purpose, we first need to identify the states that would correspond to superpositions of spacetimes. We have two natural candidates for this purpose: the coherent and the cat states. From an effective spacetime perspective, the Fock state would correspond to an incoherent mixture of two spacetimes. This means that the propagation of sound on each of the two wells would occur independently of what is happening on the other well.

The coherent state can be taken as representing a system in a macroscopically delocalized state. However, we have seen that this state appears at the cost of eliminating the local interactions between the composing bosons. Having negligible local interactions, this state lack the possibility of producing a rich causal physics. For instance, the state develops no acoustic excitations, it is does not really serves as an analogue model of gravity.

We are left with the cat state as the only one that we can identify, at least momentaneusly, with a superposition of two different spacetimes, since it is a superposition of two states representing an effectively curved geometry on each of the wells.

The following question now appears: if we construct a condensate like the cat state, i.e., what we would claim to be a would-be “superposition of spacetimes” in the analogue system, what happens with the potential acoustic perturbations propagating on top of such a spacetime: is there an effective spacetime resulting from this superposition? The idea is that there should be a limitation in describing the acoustic perturbations propagating on top of such a superposition as acoustic perturbations on a new spacetime. Otherwise, this would mean that the superposition of analogue spacetimes would be also a spacetime, contrary to what one would expect to happen with real gravitational systems. Indeed, such limitation appears in the form of a huge instability of the system under consideration. As we discussed in subsection LVC, the cat state tends to decay to a Fock state under
small perturbations. The propagation of sound-like excitations itself would be enough to destabilize such superposition. This translates into a decay of the putative superposition of spacetimes into a mixture of two spacetimes, i.e., two effective spacetimes which behave independently from each other on each of the wells. This is somehow what Penrose argues should happen when trying to superpose two different spacetimes, as we discussed in detail in Section III B.

To make more explicit the analogy between both mechanisms, we can think about the source of such instability. On the one hand, for the condensates the instability under local perturbations that the cat state suffers comes from the huge fluctuations in the particle number density that the system displays. Recalling that the speed of sound at a point in the condensate $c$ is proportional to the local number density of particles $n_0$, huge fluctuations in the number of particles would induce huge fluctuations of the underlying causal structure for the propagation of sound waves in the effective geometry. Such fluctuations arise because the cat state is highly delocalized. On the other hand, Penrose argues that the source of instability for the superpositions of spacetimes, from a purely geometric point of view, is the inability to make sense of the notion of stationarity for such states. The absence of a well defined notion of time, i.e. the absence of a well-defined parameter with respect to which we can talk about evolution when considering these kind of superpositions makes impossible to regard the system as having some kind of causal structure. Although some fuzziness of the causal structure when entering the quantum regime is to be expected, attempting to superpose two radically different causalities drives us completely into this regime and thus, the causal structure itself dilutes and becomes a meaningless concept. Thus, the reason for the decay of this delocalized superpositions seems to be the huge instability these states develop due to the absence of a well-defined notion of causality.

Apart from the instability under perturbations of this state which we have already discussed, an additional impediment appears. This impediment is the following: if we consider the evolution of such a system according to the Gross-Pitaevskii equation which describes generic fluid condensates, the interference is such that the healing length of this condensate is much bigger than the healing lenght of either of the condensates. Consequently, the effective spacetime picture breaks at much bigger wavelengths and we conclude that the superposition of two spacetimes rapidly breaks down. This manifests the impossibility of finding an analogue of a superposition of spacetimes, since such a concept appears ill-defined.
in the gravitational sector and, as such, we would expect a fundamental limitation to find it in an analogue version.

Although the dynamical mechanism responsible for forbidding these configurations is different in the analogue and for instance, Penrose’s gravitational proposal (which is to be expected, since the analogy holds just at a kinematical level), we see that the inability to produce stable superpositions of semiclassical spacetimes is a robust idea. Something very similar happens when one tries to use analogue models for constructing effective spacetimes in which causality properties are compromised: although in the gravitational description energy conditions need to be violated in non-trivial ways by including exotic matter content [27], other kinds of limitations are found for the analogue system which make the difficulty of violating causality principles also a robust idea. We postpone a detailed analysis of how for these superpositions the effective acoustic description breaks for future works.

E. Same localization, different causality

The double well Bose-Hubbard model does not allow us to consider the next situation we want to consider concerning the superposition of semiclassical spacetimes. What we want now is to have a BEC sitting in one single localization but representing the superposition of two analogue gravity models with different causalities. We will try to analyze whether this is at all possible or not.

As the simplest situation, one could think of a homogeneous BEC at rest but with a superposition of sound velocities. The sound velocity in a condensate, \( c^2 = gn_0/m \), is controlled by the coupling constant \( g \), the effective mass of the bosons \( m \) and their number density \( n_0 \). The numbers \( g \) and \( m \) are parameters of the system which are in principle not subject to quantum rules. For example, the coupling constant can be controlled by an external magnetic field by using Feschbach resonances [97]. Then, we can ask, whether we can engineer this magnetic field to be in a quantum superposition. The magnetic field we speak about is a macroscopic entity so what we are doing with this inquiry is just translating the problem of generating a macroscopic superposition from one place to another. Instead, it is typically assumed that the values of the parameter of a system are phenomenological values to which one could in principle associate infinitely precise values. As the causality of the system depends directly on these values one could think that they imprint a classical
flavor into the notion of causality.

The other quantity entering the definition of the sound velocity is $n_0$. It is true that this quantity can have fluctuations, but again, is it sensible to think of a superposition of two macroscopically distinguishable values? In a condensed matter system the number of particles is completely fixed; the creation of new particles is forbidden by a huge energy gap. It is true that if one was hypothetically able to generate a BEC in an ultrarelativistic regime, the possibility of having a superposition of states with different number of particles would exist. But again, would it be possible to have a stable superposition of two mutually non-interacting gases each having a different number of particles? The possibility does not sound realistic, the very notion of macroscopic (thermodynamic) behaviour would try to produce states with a peaked distribution for the number of particles. Actually, the standard ensemble in terms of which condensed matter systems are described is the Grand Canonical Ensemble, which allows not only for energy fluxes but also particle density fluxes between the system and the environment. In such scenarios, thermal equilibrium is needed to reach thermodynamical equilibrium although it is not sufficient. Chemical equilibrium is also required, which means having equal chemical potentials for the bath and the system. If the number of particles is not highly peaked, it is impossible to reach a stationary state.

The only element of the causality in a BEC remaining is the flow velocity of the fluid. The flow velocity that appears in the effective analogue gravity metric in a BEC corresponds to a macroscopic occupation of a particular phase structure for the mono-particle wave functions. Once again, the natural question is whether it is possible to have a stable superposition of condensates with different phases or not. The situation would be parallel to our discussion of the stability properties of the cat and Fock states, respectively. Depending on whether we perform a superposition of the two states with all the particles on each phase state or a more democratic Fock-like state in which half of the particles are on each of the phase states, we would produce an unstable state or a stable state, respectively. The additional difference is the external potential which, for the formation of standard condensates, always favors one of the two phase states. Thus, although under the absence of external potentials it would be possible to generate a stable Fock-like state, in a real systems the particles would always tend to be projected onto one of the two fixed phases states. Thus, this qualitative discussion suggests also the inability to perform superpositions of different causalities, i.e. to have a BEC sitting on a single location but giving rise to different causal structures for
the propagation of sound-like waves.

V. SUMMARY AND DISCUSSION

The problem of explaining why macroscopic superpositions are not observed in experiments even though they are allowed by the principles of quantum mechanics is one of the main fundamental problems in modern physics, concerning the foundations of quantum mechanics. Another fundamental, long-standing problem in physics is the reconciliation between the basic principles of quantum mechanics and those of General Relativity. These problems are typically presented as different conceptual problems. However, being gravity the only interaction that remains elusive to an extension to the quantum realm, it is natural to ask whether it may play a role in the solution (or disolution) of the open problems in the foundations of quantum mechanics. After presenting the main approaches towards solving the first of these problems without any mention to gravity, we have reviewed a minoritarian although continuous line of research that has been explored steadily throughout the years that has argued for the existence of a relation among this problem and gravity.

Our aim in the second part of this work has been to present an analogue model of gravity where it is possible to analyze the properties of the states corresponding to would-be superposition of spacetimes. These kinds of states, as we have reviewed in the first part of the work, seem to be relevant for understanding the potential role that gravity might play in quantum state reduction. The quantum state reduction is essentially the loss of quantum coherences, which means the diagonalization of the density matrix on a pointer-basis. The idea of gravity playing a possible role in this mechanism radicates in the properties that make gravity different from the rest of the fundamental interactions. Although several ideas concerning this plausible possibility have been presented throughout the years in the literature, we have focused on Penrose’s proposal since its language appears as more direct when trying to reproduce its main idea in an analogue gravity model. In Penrose proposal, the underlying mechanism responsible for the reduction of state is the very ill-defined nature of the notion of causal structure in a superposition of two spacetimes, understanding spacetimes as the closest notion of a smooth geometry that we would have in a full quantum theory, i.e., semiclassical states.

In an attempt to clarify and characterize the main features of this quantum state reduc-
tion induced by gravity models, we have studied a toy model of a Bose-Einstein condensate in a double well which captures their main features. This toy model shows three non-trivial phases: a coherent phase, when interactions are negligible; the fragmented uncorrelated phase, whose representative in the regime in which the hopping dominates is the Fock state; and the fragmented correlated phase, whose representative when the interactions dominate is the so-called cat state. We have seen that the coherent state cannot lead to analogue physics exhibiting causal behaviour. The Fock state would be, from the gravitational perspective, a statistical mixture of spacetimes, void of quantum coherences. Such a state offers no conceptual problem from the quantum gravity perspective and, as such, its condensed matter representative in this analogue model is stable under perturbations and is a robust state. Finally, the cat state, which represents the superposition of two spacetimes, results to be highly unstable under perturbations. As we have seen, perturbations tend to collapse it into a Fock-like state. This is exactly the same feature advanced by Penrose: the superposition of spacetimes is geometrically ill-posed and results in a decay of such states. In that sense, the superposition of spacetimes (even stationary ones) would be unstable in quantum gravity. The rate at which this destabilization occurs is proportional to the complexity and macroscopicity of the system. Penrose characterizes this complexity of the superposition with the difference in gravitational fields that these spacetimes can produce. For our toy model, the complexity and macroscopicity would be given by the number of particles that condense, and the relative relevance or strength of the local causality, represented by the ratio between the interaction $U$ and the hopping parameter $t$. Penrose also goes one step further and argues that such decay corresponds to a quantum reduction of the state, leaving a statistical mixture. In our toy analogue model, the evolution from a cat state to a Fock state is unitary in the underlying quantum mechanical many-body description. However, from the point of view of obtaining an intrinsic effective quantum mechanical description that applies only to the effective degrees of freedom propagating on the effective geometries, i.e. the phononic excitations on each of the two wells, one would trace over the excited states of the BEC. As a result, one would clearly obtain an effective non-unitary evolution. However, the reader should notice that understanding this effective non-unitary evolution as caused by some form of tracing over unobservable degrees of freedom would be at this stage an epistemological choice. For observers that do not have access to the whole tower of excited states, such choice is irrelevant since both of them produce the same phenomenology.
Furthermore, the quantity that controls the instability of the cat state for the BEC under local perturbations is the number fluctuation. On the other hand, Penrose ideas concerning the reduction of state due to gravity allude to the inability to define stationarity for superposed spacetimes and, thus, to the dissolution of the underlying causal structure for such kind of states. The identification of both is straightforward once we recall that the sound speed is proportional to the local number density of bosons, and, thus, huge fluctuations in the number density induce huge fluctuations of the underlying causal structure.

It is interesting to comment on the interplay between the ideas presented in this article and the more explored route of applying different quantization schemes to the gravitational degrees of freedom. Although all the observables for quantum gravity in the Dirac sense need to be non-local (and in that sense will also have an emergent flavour as Carlip has already pinpointed [98]), for the purposes of low-energy physics we typically need to consider semiclassical observables [99]. This means that we typically compute and consider observables with respect to a privileged background structure. For those kinds of observables, it might happen that the matrix elements among different semiclassical geometries are almost zero. This would mean that, from a low-energy perspective, the superposition of two (or more) such states describing an effective semiclassical geometry would be indistinguishable from a statistical mixture; in other words, the coherences among both states would be irrelevant and hence lost. This means that from a low energy perspective, the relevant quantum states would be those giving rise to effective geometries and they would effectively break into superselection sectors. Actually, such situation would be parallel to the discussion of the BEC’s in the double-well potential if one could only probe one of the wells. This situation being realized in nature would mean that when we attempt to generate a macroscopic superposition of an object able to generate a macroscopic gravitational field, we would find a fundamental limit. This situation would result in a dramatic modification of the laws of quantum mechanics when we involve macroscopic superpositions and, in that sense, very similar to the ones explored here since gravity would be “the responsible in deciding” which configurations are macroscopic. The main difference would be that, unlike in Penrose’s proposal for instance, local experiments would never detect loss of quantum coherence due to this effective superselection. Thus, for an internal observer, the underlying fundamental quantum-gravitational description or the effectively modified quantum laws would be merely epistemological choices without further phenomenological consequences. This would be a
line of reasoning similar to the one Coleman introduced concerning the interpretation of the \( \alpha \)-parameters arising from integrating out Euclidean wormholes \[49\]. Coleman argued that no local observer would experience loss of coherence due to the effect of Euclidean wormholes, since their effect was to introduce the so-called \( \alpha \)-parameters which broke the theory into superselection sectors \[49\], contrary to some previous claims in the literature \[100, 101\].

The toy model that we have used in this work could make the reader believe that we suggest that Penrose’s proposal could be just another form of environmental decoherence. This means one in which one traces out some high-energy, unobservable degrees of freedom associated with a suitable ultraviolet completion of quantum gravity. Indeed, one could imagine tracing out a huge band of high-energy states from quantum gravity, playing the role of the perturbations corresponding to excited states of the BEC in a double-well analyzed (those created by the \( c_1^\dagger \) operator in our simple toy model), and the net effect would be similar to that of a suitable environment. For instance, it is commonly believed that such huge amount of states exist even for single spacetime configurations, as the black-hole microstate counting suggests \[102, 103\]. If such degrees of freedom were finite and bounded (i.e. like oscillators), they would inevitably lead to (quasi) recurrences. Although this points out a potential difference from the experimental point of view, in standard situations the characteristic time scales for those recurrences can be ridiculously huge, leading in practice to decoherence. At this point it might appear that we inevitably reach the conclusion that, if a local observer did not have access to those states, it would be impossible to distinguish between the phenomenology of Penrose’s model and standard decoherence explanations. They would correspond to just epistemological choices encoding the same phenomenology.

However, this possible logic is not the one we want to emphasize with this work. Let us speculate just a bit. First of all, assuming that we are low-energy entities taking part on states with no macroscopic superpositions. Any attempt to produce a macroscopic superposition from the low-energy sector might simply find a strong resistance, so strong that one might find it appropriate to consider it a fundamental phenomenological limit (similar in spirit but maybe not in the details to the speed of light limit in these systems). Unfortunately, at this stage we do not have a specific proposal to implement such a limit, nor how to work out its derived consequences. In any case, from this perspective one might wonder whether looking for signals of decoherence is the best experimental strategy.

Secondly, if the final effective theory that happens to work phenomenologically deviates
from standard quantum mechanics, then, what would be the reason for trying to maintain a standard quantum-mechanical framework for the hypothetical underlying theory? Specially considering that the strongest support for the standard quantum-mechanical framework is precisely its phenomenological power. At the end of the day, the philosophy of our work has been to just try to identify fundamental reasons explaining why a Penrose-like proposal might take place in nature. Reasons that might transcend the toy model we have considered here to describe the underlying physics.

The lesson we seem to be learning is that, although macroscopic superpositions might be relevant for very high energies (this situation is of course logically possible), they are qualitatively irrelevant at low energies. We have exemplified the apparent universality of the impossibility of generating a superposition of two such effective geometries, even if its microscopic constituents whose collective excitation give rise to a spacetime have a quantum nature, with a toy model consisting of BEC’s in a double-well potential. We think this reconsideration of the problems concerning the foundations of quantum physics from the point of view of an emergent nature of geometrical notions serves to point towards a direction for the reconciliation of quantum mechanics and general relativity which most of the approaches towards quantum gravity miss.

ACKNOWLEDGMENTS

GGM would like to thank Alfredo Luis for very useful discussions and Flaminia Giacomini for useful correspondence. Financial support was provided by the Spanish Government through the projects FIS2017-86497-C2-1-P, FIS2017-86497-C2-2-P (with FEDER contribution), FIS2016-78859-P (AEI/FEDER,UE), and by the Junta de Andalucía through the project FQM219. CB and GGM acknowledge financial support from the State Agency for Research of the Spanish MCIU through the “Center of Excellence Severo Ochoa” award to the Instituto de Astrofísica de Andalucía (SEV-2017-0709). GGM acknowledges financial support from IPARCOS through “Ayudas para cursar estudios de Doctorado del Instituto de Física de Partículas y del Cosmos”.

[1] G. ’t Hooft and M. J. G. Veltman, Ann. Inst. H. Poincare Phys. Theor. A 20, 69 (1974).
[2] M. H. Goroff, A. Sagnotti, and A. Sagnotti, *Physics Letters B* **160**, 81 (1985).

[3] A. E. M. van de Ven, *Nucl. Phys. B* **378**, 309 (1992).

[4] A. Shomer, (2007), arXiv:0709.3555 [hep-th]

[5] M. B. Green, J. H. Schwarz, and E. Witten, *Superstring Theory: 25th Anniversary Edition* Cambridge Monographs on Mathematical Physics, Vol. 1 (Cambridge University Press, 2012).

[6] M. B. Green, J. H. Schwarz, and E. Witten, *Superstring Theory: 25th Anniversary Edition* Cambridge Monographs on Mathematical Physics, Vol. 2 (Cambridge University Press, 2012).

[7] J. Polchinski, *String theory. Vol. 1: An introduction to the bosonic string* Cambridge Monographs on Mathematical Physics (Cambridge University Press, 2007).

[8] J. Polchinski, *String theory. Vol. 2: Superstring theory and beyond* Cambridge Monographs on Mathematical Physics (Cambridge University Press, 2007).

[9] J. A. Wheeler, Adv. Ser. Astrophys. Cosmol. **3**, 27 (1987).

[10] T. Thiemann, (2001), arXiv:gr-qc/0110034.

[11] C. Rovelli, *Quantum Gravity* Cambridge Monographs on Mathematical Physics (Cambridge University Press, 2004).

[12] R. Penrose, *Found. Phys.* **44**, 557 (2014).

[13] C. Barceló, R. Carballo-Rubio, L. J. Garay, and R. Gómez-Escalante, *Phys. Rev. A* **86**, 042120 (2012).

[14] G. García, L. Ares, and A. Luis, *Annals of Physics* **411**, 167961 (2019).

[15] P. Morgan, *Annals of Physics* **414**, 168090 (2020).

[16] A. Zeilinger, *Rev. Mod. Phys.* **71**, S288 (1999).

[17] H. D. Zeh, *Foundations of Physics* **1**, 69 (1970).

[18] W. H. Zurek, *Phys. Rev. D* **24**, 1516 (1981).

[19] W. H. Zurek, *Phys. Rev. D* **26**, 1862 (1982).

[20] H. Everett, *Rev. Mod. Phys.* **29**, 454 (1957).

[21] J. A. Wheeler, *Rev. Mod. Phys.* **29**, 463 (1957).

[22] R. Penrose, *The Emperor’s New Mind: Concerning Computers, Minds, and the Laws of Physics* Oxford landmark science (Oxford University Press, 2016).

[23] R. Penrose, in *13th Conference on General Relativity and Gravitation (GR-13)* (1992).

[24] R. Penrose, *Gen. Rel. Grav.* **28**, 581 (1996).
[25] W. Ketterle, D. S. Durfee, and D. M. Stamper-Kurn, *Making, probing and understanding Bose-Einstein condensates*, Tech. Rep. (1999).

[26] L. Pitaevskii and S. Stringari, *Bose-Einstein Condensation and Superfluidity*, International Series of Monographs on Physics (OUP Oxford, 2016).

[27] C. Barcelo, S. Liberati, and M. Visser, *Living Rev. Rel. 8, 12 (2005)* arXiv:gr-qc/0505065

[28] L. J. Garay, J. R. Anglin, J. I. Cirac, and P. Zoller, *Phys. Rev. A 63, 023611 (2001)* arXiv:gr-qc/0005131

[29] J. Steinhauer, *Nature Physics 12*, 959 (2016).

[30] A. Bassi, K. Lochan, S. Satin, T. P. Singh, and H. Ulbricht, *Rev. Mod. Phys. 85*, 471 (2013).

[31] N. Bohr, *Nature 121*, 580 (1928).

[32] J. von Neumann and R. Beyer, *Mathematical Foundations of Quantum Mechanics*, Goldstine Printed Materials (Princeton University Press, 1955).

[33] P. Bocchieri and A. Loinger, *Phys. Rev. 107*, 337 (1957)

[34] G. Bacciagaluppi, http://philsci-archive.pitt.edu/504/2/cracow.pdf (), accessed: 26-04-2021.

[35] G. Bacciagaluppi, http://stanford.library.usyd.edu.au/entries/qm-decoherence/ (), accessed: 26-04-2021.

[36] D. Bohm, *Phys. Rev. 85*, 166 (1952)

[37] D. Bohm and B. Hiley, *The Undivided Universe: An Ontological Interpretation of Quantum Theory*, Physics, philosophy (Routledge, 1993).

[38] A. Valentini, *Physics Letters A 156*, 5 (1991)

[39] A. Valentini, *Physics Letters A 158*, 1 (1991)

[40] R. Galazo, I. Bartolomé, L. Ares, and A. Luis, *Physics Letters A 384*, 126849 (2020).

[41] J. Oppenheim, (2018), arXiv:1811.03116 [hep-th].

[42] J. Oppenheim and Z. Weller-Davies, (2020), arXiv:2011.15112 [hep-th].

[43] A. Paredes, D. N. Olivieri, and H. Michinel, *Physica D 403*, 132301 (2020).

[44] G. C. Ghirardi, A. Rimini, and T. Weber, *Phys. Rev. D 34*, 470 (1986).

[45] F. Karolyhazy, *Il Nuovo Cimento A* (1965-1970) *42*, 390 (1966).

[46] A. Komar, *International Journal of Theoretical Physics 2*, 157 (1969).

[47] F. Karolyhazy, *Magyar Fizikai Folyoirat 22*, 23 (1974).

[48] L. Diosi, *Physics Letters A 120*, 377 (1987).

[49] S. R. Coleman, *Nucl. Phys. B 307*, 867 (1988).
[50] L. Diósi, \textit{Phys. Rev. A} \textbf{40}, 1165 (1989).

[51] N. Gisin, \textit{Helv. Phys. Acta} \textbf{62}, 363 (1989).

[52] G. Ghirardi, R. Grassi, and A. Rimini, \textit{Phys. Rev. A} \textbf{42}, 1057 (1990).

[53] I. C. Percival, \textit{Proc. Roy. Soc. Lond. A} \textbf{451}, 503 (1995) \texttt{arXiv:quant-ph/9508021}.

[54] P. Pearle and E. Squires, \textit{Foundations of Physics} \textbf{26}, 291 (1996).

[55] T. Banks, L. Susskind, and M. E. Peskin, \textit{Nuclear Physics B} \textbf{244}, 125 (1984).

[56] W. G. Unruh and R. M. Wald, \textit{Phys. Rev. D} \textbf{52}, 2176 (1995).

[57] W. G. Unruh and R. M. Wald, \textit{Rept. Prog. Phys.} \textbf{80}, 092002 (2017) \texttt{arXiv:1703.02140 [hep-th]}.

[58] I. L. Egusquiza, L. J. Garay, and J. M. Raya, \textit{Phys. Rev. A} \textbf{59}, 3236 (1999).

[59] W. G. Unruh, \textit{Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences} \texttt{https://royalsocietypublishing.org/doi/pdf/10.1098/rsta.2012.0163}.

[60] E. P. Wigner, “Remarks on the mind-body question,” in \textit{Philosophical Reflections and Syntheses}, edited by J. Mehra (Springer Berlin Heidelberg, Berlin, Heidelberg, 1995) p. 247.

[61] I. Bialynicki-Birula and J. Mycielski, \textit{Annals of Physics} \textbf{100}, 62 (1976).

[62] P. Pearle, \textit{Phys. Rev. D} \textbf{13}, 857 (1976).

[63] J. Ellis, J. S. Hagelin, D. Nanopoulos, and M. Srednicki, \textit{Nuclear Physics B} \textbf{241}, 381 (1984).

[64] P. Pearle, \textit{Phys. Rev. A} \textbf{39}, 2277 (1989).

[65] G. Ghirardi, P. Pearle, and A. Rimini, \textit{Phys. Rev. A} \textbf{42}, 78 (1990).

[66] W. Feldmann and R. Tumulka, \textit{Journal of Physics A: Mathematical and Theoretical} \textbf{45}, 065304 (2012).

[67] M. Bilardello, S. Donadi, A. Vinante, and A. Bassi, \textit{Physica A: Statistical Mechanics and its Applications} \textbf{462}, 764 (2016).

[68] A. Frenkel, \textit{Foundations of Physics} \textbf{32}, 751 (2002).

[69] B. L. Hu and E. Verdaguer, \textit{Living Rev. Rel.} \textbf{11}, 3 (2008) \texttt{arXiv:0802.0658 [gr-qc]}.

[70] D. Giulini and A. Großardt, \textit{Classical and Quantum Gravity} \textbf{28}, 195026 (2011).

[71] S. L. Adler, (2014), \texttt{arXiv:1401.0353 [gr-qc]}.

[72] B. L. Hu, \textit{J. Phys. Conf. Ser.} \textbf{504}, 012021 (2014) \texttt{arXiv:1402.6584 [gr-qc]}.

[73] A. Sharma and T. P. Singh, \textit{International Journal of Modern Physics D} \textbf{23}, 1442007 (2014) \texttt{https://doi.org/10.1142/S0218271814420073}.

[74] S. Bera, S. Donadi, K. Lochan, and T. P. Singh, \textit{Foundations of Physics} \textbf{45}, 1537 (2015).
[75] T. P. Singh, J. Phys. Conf. Ser. 626, 012009 (2015) arXiv:1503.01040 [quant-ph].

[76] S. Donadi, K. Piscicchia, C. Curceanu, L. Diósi, M. Laubenstein, and A. Bassi, Nature Physics 17, 74 (2021).

[77] L. Diósi and J. J. Halliwell, Phys. Rev. Lett. 81, 2846 (1998).

[78] D. Kafri, J. M. Taylor, and G. J. Milburn, New J. Phys. 16, 065020 (2014) arXiv:1401.0946 [quant-ph].

[79] A. Tilloy and L. Diósi, Phys. Rev. D 96, 104045 (2017).

[80] J. L. Gaona-Reyes, M. Carlesso, and A. Bassi, Phys. Rev. D 103, 056011 (2021).

[81] R. Howl, R. Penrose, and I. Fuentes, New J. Phys. 21, 043047 (2019) arXiv:1812.04630 [quant-ph].

[82] W. G. Unruh, Phys. Rev. D 14, 870 (1976).

[83] L. C. Barbado, E. Castro-Ruiz, L. Apadula, and C. Brukner, Phys. Rev. D 102, 045002 (2020) arXiv:2003.12603 [quant-ph].

[84] J. Foo, S. Onoe, and M. Zych, Phys. Rev. D 102, 085013 (2020).

[85] J. Foo, R. B. Mann, and M. Zych, “Schrödinger’s cat for de sitter spacetime,” (2021), arXiv:2012.10025 [gr-qc].

[86] M. Zych, F. Costa, and T. C. Ralph, (2018), arXiv:1809.04999 [quant-ph].

[87] F. Giacomini, E. Castro-Ruiz, and C. Brukner, Nature Commun. 10, 494 (2019) arXiv:1712.07207 [quant-ph].

[88] F. Giacomini and C. Brukner, (2020), arXiv:2012.13754 [quant-ph].

[89] L. J. Garay, International Journal of Modern Physics A 14, 4079 (1999) https://doi.org/10.1142/S0217751X99001913.

[90] H. Breuer, P. Breuer, F. Petruccione, and S. Petruccione, The Theory of Open Quantum Systems (Oxford University Press, 2002).

[91] Á. Rivas and S. Huelga, Open Quantum Systems: An Introduction SpringerBriefs in Physics (Springer Berlin Heidelberg, 2011).

[92] A. G. Redfield, IBM J. Res. Dev. 1, 19 (1957).

[93] M. Lewenstein, A. Sanpera, and V. Ahufinger, Ultracold Atoms in Optical Lattices: Simulating quantum many-body systems (OUP Oxford, 2012).

[94] E. J. Mueller, T. Ho, M. Ueda, and G. Baym, Phys. Rev. A 74, 033612 (2006).
[95] A. Leggett, *Quantum Liquids: Bose Condensation and Cooper Pairing in Condensed-matter Systems*, Oxford Graduate Texts (OUP Oxford, 2006).

[96] E. A. Donley, N. R. Claussen, S. L. Cornish, J. L. Roberts, E. A. Cornell, and C. E. Wieman, *Nature* **412**, 295 (2001).

[97] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, *Rev. Mod. Phys.* **82**, 1225 (2010).

[98] S. Carlip, *Stud. Hist. Phil. Sci. B* **46**, 200 (2014), arXiv:1207.2504 [gr-qc].

[99] S. B. Giddings, D. Marolf, and J. B. Hartle, *Phys. Rev. D* **74**, 064018 (2006).

[100] S. W. Hawking, *Phys. Lett. B* **195**, 337 (1987).

[101] S. B. Giddings and A. Strominger, *Nuclear Physics B* **306**, 890 (1988).

[102] I. Agullo, J. Fernando Barbero, E. F. Borja, J. Diaz-Polo, and E. J. S. Villasenor, *Phys. Rev. D* **82**, 084029 (2010), arXiv:1101.3660 [gr-qc].

[103] A. Strominger and C. Vafa, *Phys. Lett. B* **379**, 99 (1996), arXiv:hep-th/9601029.