Abstract. A brief review is given on the study of the thermodynamic properties of spin models defined on different topologies like small-world, scale-free networks, random graphs and regular and random lattices. Ising, Potts and Blume-Capel models are considered. They are defined on complex lattices comprising Appolonian, Barabási-Albert, Voronoi-Delaunay and small-world networks. The main emphasis is given on the corresponding phase transitions, transition temperatures, critical exponents and universality, compared to those obtained by the same models on regular Bravais lattices.

1. Introduction

The study and characterization of magnetic systems on regular $d$-dimensional lattices, both experimentally and theoretically, has been well established during the last century (see, for instance, the book series edited by Domb and Green, and also edited by Domb and Lebowitz, on *Phase Transitions and Critical Phenomena* [1, 2]). Although the transition temperatures are model dependent and non-universal, the critical exponents are known to be universal. In general, the universality class depends on the spatial lattice dimension $d$, on the number of components $n$ (and symmetry) of the order parameter, and on the range of interactions. The agreement between theory and experimental results on real compounds has been reported to be excellent in this regard.

More recently, there has been a great deal of interest in studying networks, which are different from the regular crystalline Bravais lattices [3, 4, 5]. The study of such complex lattices, also called scale-free networks, has been mainly motivated by social organizations and computers connectivities [4], among others. It has been possible to recognize, in this way, networks ranging from networks in nature to networks of people as well.

Thus, it turns out quite interesting to understand the behavior of a magnetic system on such networks and also on random graphs. Despite being theoretical by now, this will certainly trigger the possibility of synthesizing experimental realizations of materials governed by random-lattices sites. Under this theoretical point of view, the magnetic model can be defined by usual spin interactions, where each spin is located on the sites of a complex lattice. The main questions that arise concern the possible existence of a first- or second-order (critical or multicritical) phase transition, and in the case of a second-order transition, the corresponding universality class of the model.
Table 1. Order of transition and critical exponents for the Ising and Potts models on regular lattices. In two dimensions we have the exact results and in three dimensions the values come from Monte Carlo simulations (see, for instance, reference [10]).

| model       | transition | $\beta/\nu$ | $\gamma/\nu$ | $1/\nu$ |
|-------------|------------|-------------|-------------|--------|
| 2d Ising-S  | 2nd        | 0.125       | 1.75        |        |
| 3d Ising-S  | 2nd        | 0.5185(15)  | 1.9630(30)  | 0.630(2) |
| 2d Potts-3  | 2nd        | 2/15 = 0.133| 26/15 = 1.733| 1.2    |
| 2d Potts-4  | 2nd        | 0.125       | 1.75        | 1.5    |
| 2d Potts > 4| 1st        | -           | -           | -      |

In this brief review we will consider complex lattices based on the Voronoi-Delauny (VD) tessellation [6], Barabási-Albert (BA) [3], Appolonian (AP) [7], and small-world (SW) [8] networks, and Erdős-Rényi (ER) [9] random graphs. The magnetic models applied to these networks are the Ising, Potts and Blume-Capel models, among others. In the next section, a short introduction of the Hamiltonian models and lattices will be presented. In the following sections we will give a brief summary of what has been done on each network and random graphs and, in the final section, we will present a discussion and a summary of the results.

2. Models, lattices, networks and graphs

2.1. Models

The Ising and Blume-Capel models can be defined by the following Hamiltonian

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j - H \sum_{i=1}^{N} \sigma_i + D \sum_{i=1}^{N} \sigma_i^2,$$

(1)

where the first sum is over nearest-neighbors on a d-dimensional lattice with $N$ sites, $\sigma_i$ is the state of a spin-$S$ with components $\sigma_i = -S, -S + 1, ... S - 1, S$, and $D$ is the crystal field. The general spin-$S$ simple Ising model is recovered when $D = 0$. There is no transition for the one-dimensional model and in the two and three dimensions the universality class is defined by the exponents given in Tab. 1.

On the other hand, the $q$-state Potts model can be written as

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} \delta_{\sigma_i,\sigma_j},$$

(2)

where $\delta_{\sigma_i,\sigma_j}$ is the Kronecker delta function and now $\sigma_i = 1, 2, ..., q$. Again, no transition is observed for the one-dimensional model. In two dimensions, which is the case treated in this work, the universality class is also given in Tab. 1 for $q \leq 4$ (for $q = 2$ it corresponds to the spin-1/2 Ising model). For $q > 4$ a first-order transition takes place.

The above models have also been treated in complex lattices. Some of these complex lattices will be briefly described below and more details can be found in references [3, 6, 7, 8, 9].

2.2. Directed and undirected Appolonian network

The Appolonian network is composed of $N = 3 + (3^n - 1)/2$ nodes, where $n$ is the generation number and $N$ the node number [7, 11]. On these AP structures we can introduce a disorder, in such a way that we redirect a fraction $p$ of the links. This redirecting results in a directed network, preserving the outgoing node of the redirected link but changing the incoming node. When $p = 0$ we have the standard AP networks, while for $p = 1$ we have something similar
to random networks [9]. In this procedure of the redirecting links, the number of outgoing links of each node is preserved even when $p = 1$ and the network still have hubs that are the most influential nodes. These networks display a scale-free degree distribution and a hierarchical structure. In the undirected case there exists the reciprocity of redirected link.

2.3. Directed and undirected small-world network
To generate the directed SW networks [8] we use a square grid and other irregular triangular. The disorder introduced here is identical to the procedure used on AP network cited above for both cases directed and undirected network.

2.4. Directed and undirected Erdős-Rényi random graphs
An ER random graph is formed by a set of $N$ vertices (sites) connected by $K$ links (bonds) [9]. With a probability $p$ a given pair of sites is connected by a bond type $p = 2K/N(N − 1)$. The connectivity of a site is defined as the total number of bonds connected to it, like $k_i = \sum_j l_{ij}$, where $l_{ij} = 1$ if there is a link between the sites $i$ and $j$ and $l_{ij} = 0$ otherwise. Random graphs are completely characterized by the mean number of bonds per site, or the average connectivity $z = p(N − 1)$. These links can be directed or undirected as well.

2.5. Directed and undirected Barabási-Albert Network
In the directed BA network, each new site added to the network selects (a preferential attachment proportional to the number of previous selections), with connectivity $z$, already existing sites or nodes as neighbors influencing it; the newly added site does not influence these neighbors. In the case of the undirected BA network, the newly added spin does influence these neighbors.

2.6. Directed and undirected Voronoi-Delaunay random lattices
In the undirected VD random lattice the construction of the lattice obey the following procedure: for each point in a given set of points in a plane, we determine the polygonal cell that contains the region of space nearest to that point than any other. Two cells are considered neighbors when they possess an extremity in common (Voronoi tessellation). From this Voronoi tessellation, we can obtain the dual lattice, or triangulation of Delaunay, by the following procedure. (i) When we have two neighbor cells, a link is placed between the two points located in the cells. (ii) From the links, we obtain the triangulation of space that is called the Delaunay lattice. (iii) The Delaunay lattice is dual to the Voronoi tessellation in the sense that its points correspond to cells, links to edges and triangles to the vertices of the Voronoi tessellation. The directed VD random lattices is constructed in the same way as the directed SW network.

3. Appolonian network
3.1. spin-1/2 Ising model
The Ising model has been studied on the triangular Appolonian networks with constant exchange interaction $J$ [11, 12] and with node-dependent interaction constants $J_{i,j}$ [11, 13]. The corresponding thermodynamic and magnetic properties have been obtained by formulating the problem in terms of transfer matrices. In both cases the results show no evidence of any phase transition either for the ferromagnetic or the antiferromagnetic model, as well as for short or long ranged interactions. Quenched random models, including ferromagnetic and antiferromagnetic bond dilution and Ising spin glasses have already been considered [14]. In this case the ordered phases persist up to infinite temperature over the entire range of disorder.
3.2. $q$-state Potts model

The $q$-state Potts model has been studied on Appolonian networks by using Monte Carlo simulations and transfer matrix techniques\[15\]. In this case, as for the spin-1/2 Ising model, no transition has been detected for any value of $q$.

4. Erdős-Rényi random graphs

4.1. Ising model

The spin-1/2 and spin-1 Ising model have been studied on directed and undirected Erdős-Rényi random graphs with different coordination numbers by using Monte Carlo simulations\[16\]. For spin-1/2, the model exhibits a critical temperature, below which there is a spontaneous magnetization, with mean-field exponents. Although the critical temperature depends on being directed or undirected graphs, the critical exponents are the same as the mean-field ones for both cases. The same qualitative results are also obtained for the spin-1 model. However, in this case, one has a first-order phase transition instead. These results are in agreement with those obtained for a non-equilibrium model defined on the same random Erdős-Rényi lattices \[17, 18\] (in this case, according to the Grinstein, Jayaprakash and He criterion \[19\], non-equilibrium and equilibrium models that present up-down symmetry may belong to the same universality class).

5. Barabási-Albert Network

5.1. Ising model

The spin-1/2 Ising model was first studied on undirected scale-free Barabási-Albert network (UBA) by Aleksiejuk et al. \[20\]. Through Monte Carlo simulations \[20\] they have shown that there exists a Curie temperature which logarithmically increases with the increasing of the system size $N$. After, Sumour et al. \[21, 22\] have treated the Ising model on a directed Barabási-Albert network (DBA) using standard Glauber kinetic Ising model to a fixed network. Unlike the previous results presented by Aleksiejuk et al. \[20\], they have shown that the spin-1/2 Ising model on DBA does not exhibit any phase transition for temperatures different from zero and confirmed that the corresponding Curie temperature presents an asymptotic Arrhenius law extrapolation for the relaxation time $\tau$ (defined as the first time when the sign of the magnetization flips) of the type $1/\ln \tau \propto T$. This means in fact that at all finite temperatures the magnetization eventually vanishes, i.e., no ferromagnetism is present. Lima et al. \[23\] have also studied the Ising model with spin $S = 1, 3/2$ and 2 on DBA and they have shown that no phase transition is present on these DBA networks.

5.2. $q$-state Potts model

The Potts model with $q = 3$ and 8 states have been studied on DBA through Monte Carlo simulations by Lima \[24\]. Surprisingly, different from the conventional results for the Potts model on regular two-dimensional lattices, the Potts model on DBA presented a phase transition of first order for both values of $q = 3$ and 8. Sumour and Lima \[25\] have studied the critical behavior of Ising and Potts models on semi-directed Barabási-Albert network (SDBA), recently studied by Sumour and Radwan \[26\], where now the number $N(k)$ of nodes with $k$ links each decays as $1/k^\gamma$ and the exponent $\gamma$ decreases from 3 to 2 for increasing $m$, where $m$ is the number of old nodes which a new node added to the network selects to be connected with. This behavior is totally different from $UBA$ and $DBA$ scale free networks where $\gamma = 3$ is universal, i.e., independent of $m$. For both Ising and Potts model the results showed no phase transition, in agreement with references\[21, 22\], and a Curie temperature to diverge at positive temperatures $T_c(N)$ by a Vogel-Fulcher law, with $T_c(N)$ increasing logarithmically with network size $N$. 
6. Small-World Network

6.1. Ising model

The one-dimensional spin-1/2 Ising model has been studied, via Monte Carlo simulations, on small-world network, where the small-world connections have a form proportional to \( r_{ij}^{-\alpha} \). It has been shown that any non-zero value of \( \alpha \) destroys the finite-temperature transition in the thermodynamic limit[27]. The results are different when the small-world interactions are independent of the length (\( \alpha = 0 \)), where a finite-temperature phase transition is observed [28, 29, 30, 31, 32, 33] with critical exponents smaller than the corresponding ones for the two-dimensional Ising model.

The two- [34, 35] and three-dimensional spin-1/2 model have also been treated by Monte Carlo simulations [36]. It has been shown that in the thermodynamic limit the phase transition has a mean-field character.

On the other hand, the two-dimensional model has been extended for greater values of spin, namely spin-1, 3/2 and 2 on directed small-world networks (DSW) [23]. In the DSW network [37], Lima et al. [23] showed that the two-dimensional Ising model exhibits a second and first-order phase transition for rewiring probability \( p = 0.2 \) and 0.8, respectively, for spin values \( S = 1, 3/2 \) and 2. For values \( p > p_c \sim 0.25 \) this model presents a first-order phase transition[38]. These results are, nevertheless, in accordance with the those from Sánchez et al. [37] for the corresponding non-equilibrium model.

6.2. \( q \)-state Potts model

Recently, Silva et al. [39], through Monte Carlo simulations, have studied the two-dimensional Potts models with \( q = 3 \) and 4 states on DSW network. They have found that this model exhibits a first-order and a second-order phase transition for \( q = 3 \), depending on the rewiring probability \( p \) values. However, for \( q = 4 \) the system presents only a first-order phase transition for any value of \( p \) different of zero. This critical behavior on DSW is different from the Potts model on a regular square lattice, where the second-order phase transition is present for \( q \leq 4 \) and a first-order phase transition takes place for \( q > 4 \).

7. Voronoi-Delaunay random lattices

7.1. Ising model

The spin-1/2 Ising model was first studied, via Monte Carlo simulations, on two-dimensional Voronoi-Delaunay random lattices by Espriu et al. [40]. Their results, obtained with Metropolis update algorithm, showed weak evidence that the critical exponents belong to the same universality class of the spin-1/2 Ising model on regular lattices in two-dimensions. Janke et al [41, 42], using single-cluster Monte Carlo update algorithm [43], reweighing techniques [44] and finite size scaling analysis, also studied the Ising model on the two-dimensional Voronoi-Delaunay random lattices and their results confirmed the initial evidence obtained by Espriu et al. [40], namely, that the Ising model on these random lattices belong in fact to the same universality class of the Ising model in regular two-dimensional lattices. Afterwards, Janke et al [45, 46], using the same algorithm earlier applied for the two-dimensional Ising model, studied the three-dimensional version on the VD random lattices. Again, their results showed that the system belongs to the same universality class of the Ising model in three-dimensional regular lattices. These new results, however, seem to be in disagreement with Harris criterion [47].

On the other hand, Lima et al. [48, 49] have also studied this model by assuming that the exchange coupling \( J \) varies with the distance \( r \) between the first neighbors as \( J(r) \propto e^{-\alpha r} \), with \( \alpha \geq 0 \). The model in two and three dimensions was simulated applying the single-cluster Monte Carlo update algorithm and the reweighing technique. The results also showed that this random system belongs to the same universality class as the pure two and three-dimensional ferromagnetic Ising model. In addition, within the context of persistence, Lima et al [50] have
studied the Ising model on VD random lattices using the zero-temperature Glauber dynamics. They showed that the model has exhibited blocking effect, which means that the persistence does not go to zero when time $t \to \infty$.

Models with spin greater than $1/2$ have also been treated by Monte Carlo simulations. While for the spin-1 Ising model one also gets the same universality class as for the model on regular lattices [51], the same is not true for the spin-3/2 case, where the critical exponents are different[52].

The spin-1 Blume-Capel was treated on directed Voronoi-Delauny lattices[51] and, as in the directed small-world case above, a continuous transition has been observed only for rewiring probabilities $p < p_c \sim 0.35$, while for $p > p_c$ a first-order transition takes place.

7.2. $q$-state Potts model

Janke et al. [53] have treated the Potts model with $q = 8$ states on two-dimensional VD random lattices using the same algorithm and technique from references [45, 46]. They have obtained the same first-order behavior as for the model on a regular square lattice. It is worth to stress that Chen et al. [54] have treated the quenched bond randomness of this model on regular square lattice and have obtained a phase transition which changed from first to second order. Afterwards, Lima et al. [55] have also studied the $q = 8$ Potts model on VD random lattices using the same coupling factor $J(r) \propto e^{-\alpha r}$ from references [48, 49]. Using Monte Carlo simulations, and the same procedure from references [45, 46], they have shown that for $\alpha = 0$ only first-order transition is present, in agreement with previous works [53]. However, for $\alpha > 0$ the $q = 8$ Potts model on VD random lattices displays a second-order transition with the critical exponents $\beta/\nu$ and $\gamma/\nu$ different from the Ising model on regular square lattices. For the $q = 3$ Potts model[56], the critical exponents are different from those of the model on a two-dimensional regular lattice, however, the exponents ratio $\beta/\nu$ and $\gamma/\nu$ seem to be identical to the corresponding ratio on regular lattices.

Recently, Lima [57] has studied the $q = 4, 6$ and $8$ Potts model. The $q = 4$ model exhibits a second-order transition for $\alpha = 0$, 0.5 and 1 with critical exponents that are $\alpha$ dependent, and different from those exponents on regular lattices. For $q = 6$ and $q = 8$, the system undergoes a first-order transition for $\alpha = 0$ and a second-order transition for $\alpha > 0$, with critical exponents again dependent on $\alpha$ and different from those on regular lattices.

7.3. Harris criterion and Voronoi-Delaunay random lattices

The Harris’s criterion[47] for regular lattices (for instance, square, triangular, cubic and hyper-cubic lattices) is based on the exponent of the specific heat of a pure system (free of impurities), to determine whether or not the system will change its critical behavior or universality class in the presence of impurities or some kind of topological disorder. Since then, it has been well established that when $\alpha < 0$ (not to be confused with the exchange decay exponent used in the previous sections) the critical behavior of the specific heat is unchanged when impurities are added to the magnetic system in a random way. On the other hand, for $\alpha > 0$ the system with randomness has a critical behavior different from the pure system case. However, for the marginal case, i.e, $\alpha = 0$ (two-dimensional Ising model) the Harris’s criterion predict that both earlier behavior can occur with a pure system in the presence of some type of disorder (for the case $\alpha = 0$ see, for instance, [58, 59] and references therein). However, as mentioned previously, Janke et al.[41, 42, 45, 46] and Lima et al [48, 49] showed that the Harris’s criterion fails on two and three-dimensional Ising model on the VD random lattices.

In order to better understand the Harris’s criterion on the VD random lattices Janke et al. [60] investigated the applicability of this criterion to the case of spin models on VD random lattices. They were interested in verifying whether the critical behavior depends on the degree of spatial correlations present in the random lattices, which was quantified by a wandering
exponent \( w \). They determined the numerical value of wandering exponent \( w = 0.5 \) for both Ising and Potts models. However, this value \( w = 0.5 \) corresponds to case \( \alpha = 0 \) of the Harris’s criterion and again, for these spin models on VD random lattices, nothing can be predicted about the corresponding critical behavior of magnetic models.

8. Discussion
Continuous symmetry spin models have recently been treated on complex topologies such as the XY model on dilute Lévi graphs [61]. We should stress that other non-magnetic systems have also been studied on such complex networks, for instance, the majority vote model on Archimedian lattices (see references [62, 63] and references therein) and the non-equilibrium contact-process on Voronoi-Delauny lattice [64], which are not being treated here. In Table 2 we try to summarize the main results of the magnetic models with discrete degree of freedom treated on random lattices. For the readers convenience, we have also in Table 1 the transition and critical exponents of the same models on regular lattices.

From Table 2 one can see that there is, in some cases, a change in the order of the transition and, in others, no transition at all. When the transition remains continuous, in general the exponents also change to another universality class, except for the spin-1/2 and spin-1 Ising model on the Voronoi-Delauny lattice and the spin-1 Ising model on the directed VD lattice, where the results for the exponents give an indication of belonging to the same universality class of the corresponding model on regular lattices. It is also clear that the Harris criterion seems not to be effective for these random lattices, because one cannot say a priori either what the order of the transition will be or the universality class of the possible second-order transition. One can argue, of course, that the simulations are still not in the true finite size scaling regime and a change on the results could be obtained by considering larger lattices. However, from the quality of the results shown in the simulations it seems not so plausible such an idea. Thus, apart from some lacking models still to be treated in order to complete Table 2, the question of the order of the transition and the corresponding universality class of the model defined on complex topologies seem to be an open question still to be answered in more general terms.

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Table 2. Order of transition and critical exponents (when not available it is indicated by dots) for different models on different random lattices. $S$ stands for the value of the Ising spin and $q$ for the number of states of the Potts model (or Ising-$S$ and Potts-$q$). $d$ stands for the dimension of the lattice, and $p$ for the rewiring probability of the corresponding directed random lattice. $\alpha$ gives the range of the interaction through $J(r) \propto e^{-\alpha r}$ (unless stated $\alpha = 0$). MF - mean-field.

| model          | transition | $\beta/\nu$ | $\gamma/\nu$ | $1/\nu$ | universality | Ref. |
|----------------|------------|-------------|--------------|--------|--------------|------|
| **Appolonian** |            |             |              |        |              |      |
| Ising-1/2      | no         | -           | -            | -      | -            | [11] |
| Potts          | no         | -           | -            | -      | -            | [13] |
| **Erdős-Rényi**|            |             |              |        |              |      |
| Ising-1/2      | 2nd        | 1           | 2            | 2      | MF           | [16, 17, 18] |
| Ising-1        | 1st        | -           | -            | -      | -            | [17, 18] |
| **Barabási-Albert** |      |             |              |        |              |      |
| Ising-$S$      | no         | -           | -            | -      | -            | [21, 22, 23] |
| Potts-$q$      | no         | -           | -            | -      | -            |      |
| **Small World**|            |             |              |        |              |      |
| 1d Ising-1/2   | 2nd        | ~ 0.0001    | 0.6(1)       | ...    |              | [28]-[33] |
| 2d Ising-1/2   | 2nd        | 1           | 2            | 2      | MF           | [36] |
| **Ising on directed SW** |           |             |              |        |              |      |
| S-1/2 any $p$  | 2nd        | 0.44(3)     | 1.129(6)     | 1.14(2)| ...          | [38] |
| S-1, $p < 0.25$| 2nd        | 0.40(3)     | 1.22(3)      | 1.16(5)| ...          | [38] |
| S-1, $p > 0.25$| 1st        | -           | -            | -      | -            | [38] |
| **Potts on directed SW** |           |             |              |        |              |      |
| $q=3$ $p=0.1$  | 2nd        | 0.24(5)     | 1.5(1)       | ...    | ...          | [39] |
| $q=3$ $p=0.9$  | 1st        | -           | -            | -      | -            | [39] |
| $q=4$ any $p$  | 1st        | -           | -            | -      | -            | [39] |
| **Voronoï-Delauny** |        |             |              |        |              |      |
| 2d Ising       |            |             |              |        |              |      |
| S-1/2          | 2nd        | 0.1208(92)  | 1.7503(59)   | 0.964(28)| Ising       | [42] |
| S-1            | 2nd        | 0.135(9)    | 1.751(4)     | 1.016(8)| Ising       | [51] |
| S-3/2          | 2nd        | 0.331(9)    | 1.467(9)     | 1.13(3)| ...          | [52] |
| 3d Ising       |            |             |              |        |              |      |
| S-1/2          | 2nd        | 0.51587(82)| 1.9576(13)   | 1.5875(12)| Ising       | [45] |
| **2d Potts**   |            |             |              |        |              |      |
| $q=3$ $\alpha=0$| 2nd        | 0.133(13)   | 1.764(18)    | 1.19(2)| ...          | [56] |
| $q=3$ $\alpha=0.5$| 2nd     | 0.118(12)   | 1.751(17)    | 1.07(1)| ...          | [56] |
| $q=3$ $\alpha=1$| 2nd        | 0.106(12)   | 1.754(18)    | 0.94(1)| ...          | [56] |
| $q=4$ $\alpha=0$| 2nd        | 0.143(9)    | 1.799(6)     | 1.377(9)| ...          | [57] |
| $q=4$ $\alpha=0.5$| 2nd    | 0.12(2)     | 1.70(2)      | 1.13(2)| ...          | [57] |
| $q=4$ $\alpha=1$| 2nd        | 0.10(2)     | 1.66(4)      | 0.93(3)| ...          | [57] |
| $q=6,8$ $\alpha=0$| 1st        | -           | -            | -      | -            | [53, 55, 57] |
| $q=6$ $\alpha=0.5$| 2nd        | 0.122(4)    | 1.53(5)      | 1.09(1)| ...          | [57] |
| $q=6$ $\alpha=1$| 2nd        | 0.14(1)     | 1.56(5)      | 0.91(1)| ...          | [57] |
| $q=8$ $\alpha=0.5$| 2nd        | 0.131(7)    | 1.20(8)      | 0.83(3)| ...          | [57] |
| $q=8$ $\alpha=1$| 2nd        | 0.126(8)    | 1.45(6)      | 0.79(3)| ...          | [57] |
| 2d Ising-1 on directed VD |           |             |              |        |              |      |
| $p < 0.35$     | 2nd        | 0.421(7)    | 1.101(4)     | 1.105(8)| Ising       | [51] |
| $p > 0.35$     | 1st        | -           | -            | -      | -            | [51] |
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