Dimensionless critical shear stress for sediment particles under varying slope conditions

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Abstract
One of the most fundamental and practical issues in sedimentology is making predictions on initial sediment motion. Many widely used bedload sediment-transport models are based on the concept that sediment transport either begins at or can be scaled by, a constant value of the dimensionless critical shear stress. Flume experiments were conducted for three different diameters of spherical particles under varying channel slope. A model is presented to determine the dimensionless critical shear stress for the incipient motion of spherical particles on nonhorizontal slopes. The findings of the study have been illustrated with mathematical relationships and graphical representations for various combinations of input variables.

Keywords: Spherical particles, sediment transport, particle size, critical shear stress

Introduction
One of the most fundamental and practical issues in sedimentology is making predictions on initial sediment motion. For researchers and engineers who deal with sediment movement in the water, the idea of incipient motion has been seeking attention. To route sediment through river networks, restore river functionality and habitat, and mitigate debris flows initiated from channel beds, sediment transport predictions are required. Many researchers have investigated sedimentary particle’s incipient movement through experimental and theoretical analysis (Ippen and Verma (1955); Meland and Norrman (1966); Grass (1970); Mantz (1977); Wilberg and Smith (1987); Dey (1999); Dey et al. (1999); Dey and Debnath (2000); and Dey (2001)) [7, 9, 6, 8, 15, 1, 3, 4]. The first experimental study on the incipient motion was carried out by Shields (1936). He developed the concept of sediment flux and created a term relating the dimensionless critical shear stress \( \theta \) (also called Shields parameter, Shields stress, or the Shields entrainment function) to the critical particle Reynolds number(Rep) at the onset of incipient motion.

\[
\theta = \frac{\tau_{oc}}{(\rho_s-\rho)gd} \quad (1)
\]

\[
\text{Rep} = \frac{U_*d}{v} \quad (2)
\]

Where \( \tau_{oc} \) is the critical shear stress; \( \rho \) the water density, \( \rho_s \) is the sediment density, \( g \) is the gravitational acceleration, \( d \) is the sediment size, \( \text{Rep} \) is the particle Reynolds number, \( U_* = \sqrt{\tau_{oc}/\rho} \) is the shear velocity, \( v \) is the kinematic viscosity of water.

Novak and Nalluri (1984) conducted experiment on incipient motion of single and grouped particles over fixed beds in circular and rectangular section flume. They found relationship for critical values of velocity or shear stress is a function of particle size, shape, and density. But they did not find the effect of slope on incipient motion of solitary particle. Shvidchenko and Pender (2000) found that the critical Shield stress or dimensionless critical shear stress is dependent on the bed slope and revealed that critical conditions not only dependent on grain size but also on relative depth. They found the effect of slope on dimensionless critical shear stress for bulk of sediment transport. After reviewing available literature it was found that very less number of work done to know the effect of slope on dimensionless critical shear stress.
So considering these research as motivation for this study the effect of slope on individual particle Critical Shield stress had been analyzed. The objective of this research is to investigate the effect of the size of coarse solitary spherical particles on the incipient motion and the effect of relative depth and slope on the dimensionless critical shear stress. A mathematical model is developed for dimensionless critical shear stress in terms of relative depth and slope.

Materials and Methods

Experimental Setup

Laboratory experiments were conducted using a 7 m long metallic rectangular hydraulic tilting flume which has 6m long, 0.30 m wide, and 0.60 m deep testing section for the experiment. To allow a visual view of the flow patterns along the flume, the sidewall was built with thick transparent Perspex sheets. The water supply to the flume is regulated through regulatory valves mounted on the water supply line of the flume. The flow measurement is done through a water flow meter fitted in the supply line near the upstream end of the flume. The flume has two adjustable gates with rack and pinion arrangement and is provided with a pipe railing in total length between the gates for movement of pointer gauge trolley over the entire length of the flume. The uniform flow condition was maintained by monitoring the depth with a mobile point gage attached to the channel.

Particles

Particles of different sizes were cast in cement to model sediment grains. Three different sizes of spherical particle having diameter d = 2.5 cm, 3.2 cm, and 3.6 cm were cast in cement. The specific weight of these particles was 2.00 g/cm³ and are taken to maintain the same specific weight of these particles constructed by using cement.

Experimental Procedure

In this experiment, a spherical sediment particle of given size was placed in the desired position on the bed at 3.5 m distance from the upstream end of the flume to get a fully developed flow. The bed slope of the flume was adjusted at 0.025%, 0.050%, and 1%. The discharge was increased incrementally with the help of regulating valve until incipient motion was reached. Once the incipient condition was reached the discharge Q and the corresponding flow depth h were registered with the help of electromagnetic flow meter and point gage, respectively. For each three particles, three different discharges corresponding to the initiation of particle motion, were recorded for three different slopes on smooth bed.

Results and Discussions

Shields (1936) is pioneer to define the critical shear stress at which the individual particles of spherical shape are on the verge of motion by a unidirectional stream flow at horizontal bed. Shields (1936) gave a empirical thresholds curve known as Shields type curves which plotted as dimensionless critical shear stress as a function of particle Reynolds number. The particles used in this study is casted in cement material to simulate the shape of a spherical particle having different sizes and Shields type curve dependency on slope is also examined. The data used in this analysis is listed in Table 1 and presented graphically Fig. 1(a-c). It was clearly indicated that at 0.25% slope the dimensionless critical shear stress value is less than 0.50% and 1% slope. The rate of increase in dimensionless critical shear stress for particular size of particles is dependent on slope. Particle Reynolds number was increases with size of the particle and also increases with slope. It was observed that with increase in slope relative depth of the particle decreases for a particular size of sediment particle.

Table 1: Dimensionless Parameters at incipient motion of spherical particles on different bed slope

| Slope (%) | Diameter (cm) | Dimensionless critical shear stress (θ) | Particle Reynolds Number (Rep) | Relative depth (R/d) |
|-----------|---------------|----------------------------------------|-------------------------------|---------------------|
| 0.25      | 2.5           | 0.00420                                | 803.677                      | 1.686               |
|           | 3.2           | 0.00374                                | 1110.987                     | 1.536               |
|           | 3.6           | 0.00367                                | 1292.972                     | 1.461               |
| 0.50      | 2.5           | 0.00766                                | 1085.669                     | 1.538               |
|           | 3.2           | 0.00688                                | 1505.971                     | 1.411               |
|           | 3.6           | 0.00670                                | 1746.897                     | 1.333               |
| 1         | 2.5           | 0.00952                                | 1209.990                     | 0.955               |
|           | 3.2           | 0.00865                                | 1688.411                     | 0.887               |
|           | 3.6           | 0.00830                                | 1944.611                     | 0.826               |

Fig. 1 (a). Shows that dimensionless critical shear stress increases with increase in slope. It means that particles of given size require higher shear stress to erode at steeper slope. Because lower relative depth caused greater resistance to the particle of given size (Fig. 1(b)). The variation of the dimensionless critical shear stress with the particle Reynolds number at the incipient condition for different sizes of spherical particle is shown in Fig. 1(c). Here it is clearly indicated that dimensionless critical shear stress decreases with increase in particle Reynolds number.

Regression Analysis

A linear regression analysis in which more than one independent variable involved is called MLR. The advantage of MLR is that it is simple, which shows how dependent variables related with independent variables (Snedecor and Cochran 1981).

Regression analysis is used to know the effect of two or more variables independent variables on dependent variable. In this study Multiple linear regression was done using Microsoft Excel 2019 multiple linear regression toolbox. Mathematical relationships for dimensionless critical shear stress (θ) of all three sediment particle sizes developed in terms of diameter (d, m) and Slope (S, in fraction) using multiple linear regressions were of the following form.

\[ \theta = 0.006 + 0.615S - 0.084d \]  
\[ R^2 = 0.8768 \]

Mathematical relationships for dimensionless critical shear stress (θ) of all three sediment particle sizes developed in terms of relative depth (R/d, m) and Slope (S, in fraction) using multiple linear regressions were of the following form.

\[ \theta = -0.010 + 1.284S + 0.007 R/d \]  
\[ R^2 = 0.9390 \]

These mathematical models indicated a significant dependence of dimensionless critical shear stress on diameter (d, m) and Slope (S, in fraction) with more than 0.80 coefficient of multiple determination. The model also describe...
that dimensionless critical shear stress better correlated with relative depth than particle size only.

![Graph](image)

**Fig 1:** Dimensionless critical shear stress ($\theta$) variations with (a) slope (%), (b) relative depth (R/d) and (c) particle Reynolds number ($R_{ep}$)

**Conclusion**

The concept of entrainment threshold is the main issue of sediment transport. On the basis of flume study a mathematical relationship between dimensionless critical shear stress, slope, and relative depth has been established. The model developed on the basis of flume experiment was observed that dimensionless critical shear stress better correlated with relative depth than particle size only. The dimensionless critical shear stress increases with increase in slope. Lower the value of relative depth for steeper slope the higher is the value of dimensionless critical shear stress for a given grain size.

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