Abstract

The Optical Transpose Interconnection System (OTIS) was proposed by Marsden et al. [Opt. Lett 18 (1993) 1083–1085] to implement very dense one-to-one interconnection between processors in a free space of optical interconnections. The system which allows one-to-one optical communications from \( p \) groups of \( q \) transmitters to \( q \) groups of \( p \) receivers, using electronic intragroup communications for each group of consecutive \( d \) processors, is denoted by \( \text{OTIS}(p, q, d) \). \( H(p, q, d) \) is the digraph which characterizes the underlying topology of the optical interconnection implemented by \( \text{OTIS}(p, q, d) \). A digraph has an \( \text{OTIS}(p, q, d) \) layout if it is isomorphic to \( H(p, q, d) \). Based on results of Coudert et al. [Networks 40 (2002) 155–164], we show that De Bruijn digraph \( B(d, n) \) has an \( \text{OTIS}(dp', dp'+1-d, d) \) layout if and only if \( \gcd(p', n+1) = 1 \). We also prove that \( H(p, q, d) \) is a line digraph if and only if \( d \mid \gcd(p, q) \).

Keywords—OTIS layout, line digraph, De Bruijn digraph.

1 INTRODUCTION

It is known that electronic interconnects perform better than optical interconnects when the distance is up to a few millimeters [13], while on a longer distance, the latter has many advantages over the former, like less crosstalk, less power consumption, higher speed, and high bandwidth channels at a single communication point [12, 25]. To take benefits from both optical and electronic technologies, Marsden et al. [20] proposed the Optical Transpose Interconnection System (OTIS), which has gained considerable attention [7, 8, 9, 10, 11, 12, 22, 25]. The idea of the OTIS system is to partition the processors into groups and use electronic interconnects for the intragroup communications (which are of short distance) while optical interconnects for the intergroup communications (which are of larger distance). For the purpose of designing a well-behaved OTIS system, we may hope to use some good topological structures as models for the electronic interconnects as well as the optical interconnects. Since arbitrary connections using optical links via lenses are harder to implement than using wires on a VLSI circuit, multi-chip module or printed circuit board, how to realize a given good topology as optical interconnects has been of special concern.

After the brief introduction of the background, we now turn to a mathematical abstraction of the OTIS layout problem. Let us follow the model of Coudert et al. [8]. For any two integers \( a \leq b \), write \([a, b] \) for the set \( \{ c \in Z \mid a \leq c \leq b \} \). Suppose \( p, q \) are two positive integers. Any \( a \in [0, pq - 1] \) can be uniquely expressed as \( a = iq + j \), where \( j \in [0, q - 1] \) and \( i \in [0, p - 1] \). We use the notation \((i, j)_{p, q}\) for such an \( a \) and say that \( a \) has first \((p, q)\)-coordinate...
i and second \((p, q)\)-coordinate \(j\). Let there be \(pq\) processors, each equipped with an optical transmitter/receiver pair. We label these processors with \([0, pq - 1]\). Set two planes of lenses arrays, one consisting of \(p\) lenses each corresponding to \(q\) transmitters for a group of processors with the same first \((p, q)\)-coordinate, the other consisting of \(q\) lenses each corresponding to \(p\) receivers for a group of processors with the same first \((q, p)\)-coordinate. These lenses establish optical links from transmitters of processor \((i, j)_{p,q}\) to receivers of processor \((q - 1 - j, p - 1 - i)_{q,p}\) for \(i \in [0, p - 1], j \in [0, q - 1]\), namely the directed optical links transpose both coordinates. For a divisor \(d > 1\) of \(pq\), we use electronic interconnects among processors with labels \([kd, kd + d - 1]\) for \(k \in [0, pq/d - 1]\). This way, we then have built an OTIS\((p, q, d)\) architecture. Viewing each group of processors \([kd, kd + d - 1]\) as a node and assigning as many arcs from node \([kd, kd + d - 1]\) to \([k'd, k'd + d - 1]\) as there are optical links from transmitters of the first group to receivers of the second, we obtain a digraph \(H(p, q, d)\), which reflects the optical intergroup communication pattern of the OTIS\((p, q, d)\) architecture. Note that we can also use two sets of processors in the above construction, one corresponding to the transmitters and the other the receivers. The basic connecting unit thus obtained may be cascaded to accommodate successive processing planes. It is not hard to see that the optical intergroup communication pattern of the resulting structure is a multistage interconnection network \([14]\) and \(H(p, q, d)\) characterizes its underlying topology. Observe that \(H(p, q, d)\) is a \(d\)-regular digraph on \(pq/d\) vertices. We now come to

**Definition 1.1** ([8] Definition 4.2) We say that a \(d\)-regular digraph \(G\) has an OTIS layout provided there are positive integers \(p, q\) such that \(G\) is isomorphic to \(H(p, q, d)\).

The OTIS layout problem is the problem to characterize all OTIS layouts for a given digraph and to find among all OTIS layouts the one which is optimal in some aspects, like using the fewest lenses, namely minimizing \(p + q\).

The technique of line digraph iterations proves to be useful in producing vast families of good network models \([3, 6, 15, 24]\). Particularly, for any positive integers \(d\) and \(n\), the \(n\)th line digraph of the complete digraph on \(d\) vertices with loops, \(L^n(K_d)\), called the \(n\)-dimensional \(d\)--ary De Bruijn digraph and denoted by \(B(d, n)\), has been the focus of much study as a very good interconnection structure \([4, 5, 8, 9, 11, 16, 19, 21]\). It thus seems natural to address the isomorphisms between \(H(p, q, d)\) and De Bruijn digraphs, or more generally, line digraphs.

Coudert et al. deduced the following characterization of OTIS\((d^{p'}, d^{q'})\) layouts for De Bruijn digraphs.

**Theorem 1.1** ([8] Lemma 4.4) Let \(p' + q' - 1 = n\). For any degree \(d\), \(B(d, n)\) and \(H(d^{p'}, d^{q'}, d)\) are isomorphic if and only if the permutation \(f\) of \(Z_n\) defined by

\[
 f(i) = \begin{cases} 
  i + p' & \text{if } i \in \{0, 1, \ldots, q' - 2\}; \\
  p' - 1 & \text{if } i = q' - 1; \\
  i + p' - 1 \pmod n & \text{otherwise}, 
\end{cases}
\]

is cyclic.

As a corollary, Coudert et al. ([8] Corollary 4.8) pointed out that whether or not \(B(d, n)\) has an OTIS\((d^{p'}, d^{q'})\) layout can be checked in \(O(n)\) time. Finally, they concluded [8] by indicating that their exhaustive search led them to

**Conjecture 1.1** If \(B(d, n)\) has an OTIS\((p, q)\) layout, then \(p, q\) must be powers of \(d\).
Our paper is an effort to characterize all OTIS layouts of De Bruijn digraphs. Making use of Theorem 1.1, we will show in Section 2 that De Bruijn digraph $B(d, n)$ has an OTIS($d^p$, $d^{n+1-p'}$) layout if and only if $\gcd(p', n+1) = 1$. Note that using Euclidean algorithm, we only need $O(\log n)$ time steps to evaluate $\gcd(p', n-1)$ ([23] Theorem 4.2.1) and thus it implies an improvement of the above-mentioned result of Coudert et al. from $O(n)$ to $O(\log n)$. As a step toward proving the conjecture of Coudert et al., we will prove in Section 3 that $H(p, q, d)$ is a line digraph if and only if $d \mid \gcd(p, q)$.

## 2 DE BRUIJN DIGRAPH

Let $p', q', n$ be three positive integers such that $p' + q' - 1 = n$. Define a permutation $g_{p', q'}$ on $[0, n - 1]$ by

$$g_{p', q'}(i) = \begin{cases} 
  i + p' & \text{if } i \in [0, q' - 2]; \\
  i + p' - q' & \text{if } i = q' - 1; \\
  i - q' & \text{if } i \in [q', n - 1].
\end{cases}$$

We will adopt the convenient notation $g$ for $g_{p', q'}$ hereafter. $g$ is just another representation of the $f$ as defined in Theorem 1.1. Thus we have

**Theorem 2.1** $B(d, n)$ and $H(d^p, d^{n+1-p'}, d)$ are isomorphic if and only if $g$ is a cyclic permutation on $[0, n - 1]$.

As we will see immediately, the form of $g$ is more suitable for an investigation of its cycle structure and the above trivial reformulation of Theorem 1.1 is indeed a key observation for us. Let $\lambda = \gcd(p', q')$. For each $i \in [0, n - 1]$, write $C_i$ for the set \{ $j \in [0, n - 1] \mid j \equiv i \pmod{\lambda}$\} and $O_i$ for the orbit of $i$ under the action of $g$. We use $| S |$ for the cardinality of any finite set $S$. The Kronecker Delta $\delta_{i,j}$ is defined as having value 1 when $i = j$ and 0 otherwise.

**Theorem 2.2** $g$ has exactly $\lambda$ orbits. Indeed, the partition of $[0, n - 1]$ into orbits of $g$ is the same with its partition into congruence classes modulo $\lambda$, namely $O_i = C_i$, $i \in [0, n - 1]$.

**Proof.** Clearly, it always holds $g(i) - i \equiv 0 \pmod{\lambda}$. This means $O_i \subseteq C_i$ for all $i \in [0, n - 1]$. But $[0, n - 1]$ is a disjoint union of $C_i$ for $i \in S = [0, \lambda - 2] \cup \{q' - 1\}$. Moreover, it is easy to see that $| C_i | = \frac{n + 1}{\lambda} - \delta_{i, q' - 1}$ for $i \in S$. Thus our goal is just to verify that $| O_i | \geq \frac{n + 1}{\lambda} - \delta_{i, q' - 1}$ for $i \in S$.

Take any $i \in S$. Let $\alpha_i = | O_i \cap [0, q' - 2] |$, $\beta_i = | O_i \cap \{q' - 1\} |$, and $\gamma_i = | O_i \cap [q', n - 1] |$. As $O_i$ is an orbit, we obtain $0 = \sum_{j \in O_i} j - \sum_{j \in O_i} j = \sum_{j \in O_i} (g(j) - j) = \alpha_i p' + \beta_i (p' - q') - \gamma_i q' = (\alpha_i + \beta_i)p' - (\beta_i + \gamma_i)q'$. Cancelling the common factor $\lambda$ of $p'$ and $q'$ yields $(\alpha_i + \beta_i)\frac{p'}{\lambda} = (\beta_i + \gamma_i)\frac{q'}{\lambda}$. Since $\gcd(p'/\lambda, q'/\lambda) = 1$, it follows that $\alpha_i + \beta_i$ is a multiple of $q'/\lambda$ and therefore $q'/\lambda \leq \alpha_i + \beta_i$. Similarly, we have $p'/\lambda \leq \beta_i + \gamma_i$. These two inequalities together implies that $\alpha_i + \beta_i + \gamma_i \geq \frac{p' + q'}{\lambda} - \beta_i = \frac{n + 1}{\lambda} - \beta_i$. But it holds $\alpha_i + \beta_i + \gamma_i = | O_i |$. Furthermore, we can derive from $O_i \subseteq C_i$ that $\beta_i = \delta_{i, q' - 1}$. So we have arrived at $| O_i | \geq \frac{n + 1}{\lambda} - \delta_{i, q' - 1}$, as desired. \hfill \blackslug

Notice that $\gcd(p', q') = \gcd(p', p' + q') = \gcd(p', n + 1)$. Consequently, by Theorem 2.1 and Theorem 2.2, we can establish the following characterization of OTIS layouts of De Bruijn digraphs.

**Theorem 2.3** For $p' \in [0, n + 1]$, $B(d, n)$ and $H(d^p, d^{n+1-p'}, d)$ are isomorphic if and only if $\gcd(p', n + 1) = 1$. 

3
3 LINE DIGRAPH

We remark that, assuming Conjecture 1.1, which holds trivially when \( d \) is a prime, Theorem 1 tells us that there are totally \( \phi(n + 1) \) different OTIS layouts for \( B(d, n) \), where \( \phi \) is the Euler’s totient function. But is Conjecture 1.1 really true in general cases? As a prominent characteristic of De Bruijn digraphs is their iterated line digraph structure [24], we are naturally led to the study of those parameters \( p \) and \( q \), such that for a fixed \( n \), \( H(p, q, d) \) is an \( n \)th iterated line digraph. This line of research requires some preliminary results on characterizing iterated line digraphs. A classic result is Heuchenne’s characterization of line digraphs [2, 17, 18], proved about 40 years ago. Indeed, our subsequent work on characterizing OTIS layouts of line digraphs is just based on it. For possible later use in tackling the problem for general iterated line digraphs, instead of merely presenting Heuchenne’s characterization, we include here a characterization of iterated line digraphs, which generalizes Heuchenne’s result and an earlier generalization of it due to Beineke and Zamfirescu [1].

A digraph is said to satisfy the \( n \)th Heuchenne condition [1, 18] if for any of its vertices \( u, v, w, x \) (not necessarily distinct) for which there exist \( n \)-walks from \( u \) to \( w \), from \( v \) to \( w \), and from \( v \) to \( x \), there must also exist an \( n \)-walk from \( u \) to \( x \). Restricting our attention to the case of \( n = 1 \), the following theorem is just Heuchenne’s characterization.

**Theorem 3.1** ([24] Theorem 7) Let \( G \) be a digraph without sinks or sources. Then \( G \) is an \( n \)th line digraph if and only if the following conditions are satisfied:

(I) There are no multiple \( n \)-walks between any pair of vertices;

(II) \( G \) satisfies both the \((n - 1)\)th and the \( n \)th Heuchenne conditions.

We are in a position to prove our main results for the OTIS layouts of line digraphs. For \( j \in [id, id + d - 1] \), we define \( \alpha(j) = i \), namely \( \alpha(j) = \lfloor \frac{j}{d} \rfloor \).

**Theorem 3.2** If \( d \mid \gcd(p, q) \), then \( H(p, q, d) \) is a line digraph.

**Proof.** For any \( i \in [pq/d] \), it can be uniquely expressed as \( i = (t, s)_p,q \). Let \( v_i \) be the vertex of \( H(p, q, d) \) corresponding to the interval \( M_i = [di, di + d - 1] \). It is straightforward to check that \( M_i = \{(t, ds)_{p, q}, \cdots, (t, ds + d - 1)_{p, q}\} \). Thus the out-neighbors of \( v_i \) can be enumerated as \( v_{id}, \cdots, v_{id-1} \) such that \((q - 1 - ds, p - 1 - t)_{q, p} \in M_{id}, \cdots, q - 1 - (ds + d - 1), p - 1 - t)_{q, p} \in M_{id-1} \). It follows that the out-neighbors of \( v_i \) are just \( v_{\alpha((q - 1 - ds, p - 1 - t), q, p)}, \cdots, v_{\alpha((q - 1 - (ds + d - 1), p - 1 - t), q, p)} \), which turns out to be \( v_{(q - 1 - ds, \alpha(p - 1 - t)), q, p}, \cdots, v_{(q - 1 - (ds + d - 1), \alpha(p - 1 - t)), q, p} \). Since these \( d \) vertices are obviously pairwise different, \( H(p, q, d) \) fulfills condition (I). Also, we see that for any two vertices \( v_i \) and \( v_{i'} \) with \( i = (t, s)_p,q \) and \( i' = (t', s')_p,q \), respectively, their out-neighbor set will be disjoint if \( s \neq s' \) or \( \alpha(p - 1 - t) = \alpha(p - 1 - t') \), and will be identical otherwise. This shows that condition (II) holds as well. By Heuchenne’s characterization, this then completes the proof. \( \blacksquare \)

Interestingly, the converse of Theorem 3.2 is also true, which provides partial support to Conjecture 1.1. Recall that for a digraph \( G \), its dual, written \( \overrightarrow{G} \), is the digraph obtained from \( G \) by reorienting each edge in the opposite direction as in \( G \).

**Theorem 3.3** If \( H(p, q, d) \) is a line digraph, then \( \gcd(p, q) \) is a multiple of \( d \).

**Proof.** Clearly, it holds \( H(p, q, d) = \overrightarrow{H(q, p, d)} \) and \( L(\overrightarrow{G}) = \overrightarrow{L(G)} \) for any digraph \( G \). Thus we only need to prove \( d \mid p \). Let us fix some notation before proceeding. Hereafter, the digraph...
$H(p, q, d)$ is simply called $H$. For any $i$, we denote by $I_i$ the interval $[\alpha((i, 0)_{p,q})d, \alpha((i, 0)_{p,q})d + d - 1]$ and by $v_i$ the vertex of $H$ which corresponds to $[id, id + d - 1]$. We first claim that $d \leq \min(p, q)$. Again, we only need to prove $d \leq p$ due to the fact $L(H) = L(G)$. Seeking a contradiction, suppose that $d > p$. Then, both $(0, 0)_{q,p}$ and $(1, 0)_{q,p}$ belong to $[0, d - 1]$, the interval corresponding to $v_0$. But $[pq - d, pq - 1]$ includes $(p - q, q - 1)_{p,q}$ and $(p - 1, q - 2)_{p,q}$, which send links to $(0, 0)_{q,p}$ and $(1, 0)_{q,p}$, respectively. Hence there are multiple arcs from $v_{pq - 1}$ to $v_0$, in violation of condition (I).

Next observe that condition (II) means that for the line digraph $H$ the relation $\sim$ of having a common out-neighbor is an equivalence relation on its vertex set. Because $H$ is $d$-regular, each equivalence class of $\sim$ has size $d$. Moreover, since the vertices of $H$ correspond to pairwise disjoint intervals of length $d$, the relation $\sim$ naturally extends to the equivalence relation $\sim$ on $[0, pq - 1]$ such that $i \approx j$ if and only if $v_{a(i)} \approx v_{a(j)}$. Each equivalence class under $\approx$ has a size $d$ times as large as that of an equivalence class of $\sim$ and thus contains $d^2$ elements.

To finish the proof, suppose, on the contrary, that $p$ is not a multiple of $d$. Note that $p \geq d$ and $d \mid pq$. It immediately follows that $p > d$ and $\alpha((q - 1, 0)_{q,p}) = \alpha((q - 2, p - 1)_{q,p})$. Because there are links from $(p - 1, 0)_{p,q}$ to $(q - 1, 0)_{q,p}$ and from $(0, 1)_{p,q}$ to $(q - 2, p - 1)_{q,p}$, the latter formula tells us that

$$\langle p - 1, 0 \rangle_{p,q} \approx \langle 0, 1 \rangle_{p,q}. \quad (1)$$

Further notice that for each $i \in [0, d - 1]$, $(i, 0)_{p,q}$ links to $(q - 1, p - 1 - i)_{q,p}$, which lies in the interval corresponding to the vertex $v_{pq - 1}$. Therefore, all the vertices $v_{a((i, 0)_{p,q})}$, $i \in [0, d - 1]$, are from a common $\sim$ equivalence class. Since the elements in the interval $I_i$ are evidently all $\approx$ equivalent to $(i, 0)_{p,q}$, we find that $\cup_{i = 0}^{d - 1} I_i$ belongs to one $\approx$ equivalence class, say $A$. But the fact that $d \leq q$ implies that $I_i \cap I_j = \emptyset$ as long as $i \neq j$. Consequently, we deduce from $|A| = d^2$ that $|A| \subseteq \cup_{i = 0}^{d - 1} I_i$, and henceforth $A = \cup_{i = 0}^{d - 1} I_i$. From $p > d$, we see that $I_{p - 1} \cap A = \emptyset$. In particular, this gives $(p - 1, 0)_{p,q} \notin A$, and hence $(0, 1)_{p,q} \notin A$, in virtue of Eq. (1). This is impossible as we surely have $(0, 1)_{p,q} \in I_0 \subseteq A$. This is the end of the proof. 

It is immediate from Theorem 3.3 that Conjecture 1.1 is true for $n = 1$. Hence, we know that there is a unique OTIS layout for $B(d, 1) = K_d^+$. 

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