Using capillarimetry to assess the tortuosity of reservoir rocks

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Abstract. The use of a dumbbell void model is adequate for quantifying hydraulic tortuosity from reservoir parameters. The dumbbell model of the void space involves the alternation of filtering channels of the rock with pores (macrocapillaries) and interporeal constrictions (microcapillaries). Hydraulic tortuosity is physically explained by the expansion of the flow lines of the filtration flow in the pores and contraction in the interporeal tubules. Residual water is confined mainly to clay particles that line the pore channels. Since the residual water is immobile, it leads to a narrowing of the open area of the pores and, consequently, to a specific decrease in hydraulic tortuosity.

1. Introduction

The efficiency of liquid hydrocarbon reserves development is largely determined by the reservoir properties of reservoir rocks [1–10]. At the same time, the reservoir properties depend on the structure of the void space. The void space structure is understood as the features of the rock's microstructure, which are determined by the components' size, shape, and relative position.

The structural elements of the void space include concepts such as porosity, specific surface area, and pore geometry [11]. These factors ensure the interconnection of almost all the most important physical properties of the rock: absolute permeability, relative phase permeabilities, porosity parameter, piezo conductivity coefficient.

These structural elements can be determined in laboratory conditions with an accuracy sufficient for practical purposes concerning porosity and specific surface area.

When solving the problem of the relationship between various parameters of the rock, it becomes necessary to quantitatively characterize the third parameter, namely the geometry of the void space.

The geometry of the void space of rocks is so ambiguous that its quantitative characteristics are highly complex.

Therefore, the only way to solve this problem is to create mathematical models of the void space, which determine simplified versions of the pore geometry and make it possible to establish quantitative relationships between the porosity and reservoir properties of reservoir rocks: porosity coefficients, specific surface area, absolute and relative permeability coefficients, residual water saturation.

Many works by domestic and foreign scientists are devoted to studying these relationships [12–15]. Structural models used by researchers are divided into two main classes: granular and capillary.
Capillary models of void space have historically appeared later and have proven to be more effective. Of these, the simplest is the model of parallel capillaries of constant cross-section. Even such a simple model gives satisfactory results in many cases [16].

A perfect model is the model of parallel capillaries of different cross-sections per a particular distribution of the sizes of pore channels.

This model is closer to the real rock but does not consider the change in the cross-sections of the pore channels along the filtration line.

The relationships between the reservoir properties obtained using this model are close, but not in all cases.

For example, the dependence of the residual oil saturation on other parameters of the productive formation, which characterize the reservoir properties, cannot be specifically substantiated [16].

It is known that alternating pores and interpore narrowings represent each pore channel of real rock. At the same time, it is evident that the pores determine the capacity, and the interpore narrowings determine the filtration properties of the reservoir. Unfortunately, this indisputable fact is not considered in any way in the existing capillary models.

The model of a bundle of capillaries of different cross-sections is used as the basis for interpreting data from the study of the capillary pressure of core samples from reservoirs.

Under the conditions of Western Siberia, capillarimetric studies are carried out for each productive formation in laboratory conditions on a collection of core samples. The values of porosity and absolute permeability are measured.

Capillary curves represent the dependence of the capillary pressure on the water saturation of the void space of core samples with different filtration and capacity properties. This circumstance and the reliance of capillary pressure on water saturation lead to the following consequence. Capillary curves contain information about the void space structure and the nature of the relationship between the filtration and capacity properties of a particular reservoir [17, 18].

2. Materials and methods

As noted above, the void space structure has a significant impact on the quantitative relationships between the filtration and capacity properties of reservoir rocks. In this regard, let us consider the quantitative assessment of the absolute permeability coefficient from capillary pressure curves.

To calculate the absolute permeability $K_{pr}$, many researchers propose the following formula:

$$K_{pr} = \frac{K_p}{\beta_0} \int_0^\rho r^2 g(r) dr,$$

where $K_p$ is the coefficient of porosity;

$g(r)$ is the density of pore channel size distribution;

$r$ is radius of pore channels.

Now let us transform this formula. To do this, consider that:

$$g(r) = \frac{dK_v}{dr},$$

where $K_v$ is the current water saturation of the sample;

$$p_k = \frac{\beta_0}{r},$$

where $p_k$ is the capillary pressure;

$$\beta_0 = 2\sigma \cos \theta,$$

where $\sigma$ is the surface tension; $\theta$ is the contact angle.

After substituting the expression for $g(r)$ and the radius of the pore channel through the capillary pressure into formula (1), we obtain:

$$K_{pr} = \frac{K_p \beta_0^2}{8} \int_0^1 \frac{dK_v}{p_k^{1.0}}.$$

(2)
The permeability values calculated by formula (2) differ significantly from the permeability of the samples obtained experimentally in laboratory conditions.

When determining the permeability from the capillary pressure curve, Purcell introduced an additional factor $\lambda$ into equation (2) and called it the lithological factor, which considers the difference between a simple capillary model and the structures of the void space of real rocks.

In the study of absolute permeability, Purcell processed a collection of 27 sandstone samples from the Wilhawks and Paluxy fields (USA).

For each of the samples in laboratory conditions, the porosity, permeability, and capillary pressure curve were determined. Then, the values of $\lambda$ were calculated from these data, which varied within a fairly wide range from 0.085 to 0.363 [14].

Nevertheless, an average value of $\lambda_{av} = 0.216$ was found, with the help of which the theoretical values of permeability were calculated. Comparison of theoretical values with experimental data showed their quite satisfactory convergence.

N. Burdine, I. Fett, and H. Dykstra considered the lithological coefficient in Purcell's formula to be the reciprocal of the tortuosity of the pore channels.

Following Kozeny-Karman, we finally rewrite formula (2) in the following form

$$K_{pr} = \frac{K_p}{8\pi^2} \int_{K_v}^{1.0} \frac{dK_v}{r^2(K_v)}$$

where $T_g$ is the hydraulic tortuosity.

To calculate the absolute permeability coefficient using formula (3), we use the generalized mathematical model of the capillary pressure curve for reservoirs of Western Siberia, proposed in [18]:

$$\ln(p_r r_0) = a + b \ln(K_v^*) + c \ln^2(K_v^*)$$

where $p_r r_0$ is the dimensionless capillary pressure; $r_0 = \left(\frac{K_{pr}}{K_p}\right)^{1/2}$ – a parameter having the dimension of a radius; $K_v^* = \frac{K_v - K_{vo}}{1 - K_{vo}}$ – normalized water saturation; $K_v$ – current water saturation of the rock; $K_{vo}$ – residual water saturation; $a, b, c$ – fixed parameters determined by statistical processing of capillary curves.

Our studies show that parameter $a$ characterizes the initial section, parameter $b$ the plateau-like part, and parameter $c$ the section of the curve near the vertical asymptote, corresponding to a sharp increase in capillary pressure. This area is the area of minimum values of the pore channel sizes ($r \leq 1 \mu m$).

Analyze shows that the integral in formula (1) physically expresses the average value of the squared radius of the pore channels.

The contribution of the capillaries of the minimum size to the value of the integral is very insignificant.

Analysis shows that the integral in formula (1) is determined mainly by pore channels, the dimensions of which are close to the maximum radius.

In connection with the above, the third term on the right-hand side of formula (4) can be neglected without loss of accuracy.

Then we get:

$$\ln(p_r r_0) = a + bln(K_v^*)$$

Suppose that the normalized water saturation $K_v^*=1$. In this case, the capillary pressure is equal to $p_0$ – the initial pressure.

Then the following formula arises:
\[ \ln(p_0r_0) = a. \] (6)

From this, an expression for the initial capillary pressure can be obtained:
\[ p_0 = \frac{\exp(a)}{r_0} = \frac{\exp(a)}{\sqrt{K_{prr}K_p}}. \] (7)

Now, in formula (5), instead of \( a \), we substitute its expression (6) through the initial pressure. We get:
\[ \ln(p_pr_0) = \ln(p_0r_0) + b \ln(K'_p). \]

Further transformation gives:
\[ \ln \left( \frac{p_s}{p_0} \right) = \ln(K'_p). \]
\[ \frac{p_s}{p_0} = (K'_p)^b. \]

The final formula is the following:
\[ K^*_{pr} = p \left( K'_p \right)^b. \]

Now we substitute the expression for the capillary pressure in the formula (3):
\[ K_{mp} = \frac{K_p\beta_0^2}{8T_g^2} \int 0.0 1.0 \frac{dK'_p}{(K'_p)^{2b}} = \frac{K_p\beta_0^2}{8T_g^2} \int 0.0 1.0 \frac{(1-K_{vo}) (K'_p)^{-2b+1}}{p_0^2 - 2b+1} = \frac{K_p\beta_0^2}{8T_g^2} (1-K_{vo}) 1 - 2b. \] (8)

Finally, we get the following formula for the hydraulic radius:
\[ T_g = e^{-\frac{a}{1-K_{vo}}} \frac{1}{2\sqrt{1-2b}}. \] (9)

3. Results and Discussion
Consider the procedure for calculating hydraulic tortuosity using formula (9).

For the calculation, we will use the results of laboratory capillarimetric studies.

The figure shows the graphs of the dependence of the dimensionless capillary pressure on the logarithmic coordinate system for many core samples from the AV1 formation of the Urievskoe field in Western Siberia.

In this case, individual points correspond to fixed values of capillary pressures: 0.014; 0.026; 0.056; 0.105 MPa.

As follows from the figure, the capillary pressure curves in the logarithmic coordinate system are converted with high accuracy into straight lines. In this case, the free term of the straight line equation corresponds to parameter \( a \), the slope to parameter \( b \).
Figure 1. Graphs of dependence of dimensionless pressure $y = \ln(p_r/p_0)$ from the normalized water saturation $x = \ln(K_p^*)$

Similar constructions were performed for all investigated core samples.

Table 1 presents data on reservoir properties and the results of calculating the parameters $a$ and $b$ of 17 core samples from the $AIV^3$ formation of the Urievskoye field.

| №  | Open porosity, $K_{p0}$ | Permeability coefficient, $K_{pr}$ | Residual water saturation, $K_{v0}$ | $a$     | $b$     | $T_g$   | $T_{gl}$ |
|----|------------------------|-------------------------------|-----------------------------------|---------|---------|---------|---------|
| 1  | 0.254                  | 0.0285                        | 0.479                             | -2.6    | -1.13   | 2.7     | 3.4     |
| 2  | 0.225                  | 0.0138                        | 0.594                             | -3.176  | -1.3495 | 3.9     | 3.2     |
| 3  | 0.241                  | 0.0556                        | 0.427                             | -2.8807 | -1.2595 | 3.6     | 3.7     |
| 4  | 0.227                  | 0.0059                        | 0.657                             | -3.0954 | -1.5603 | 3.1     | 2.8     |
| 5  | 0.235                  | 0.2516                        | 0.338                             | -2.8225 | -1.2914 | 3.6     | 4.0     |
| 6  | 0.221                  | 0.0114                        | 0.575                             | -3.1138 | -1.4282 | 3.6     | 3.2     |
| 7  | 0.247                  | 0.214                         | 0.325                             | -2.9537 | -1.4368 | 3.9     | 3.9     |
| 8  | 0.25                   | 0.2228                        | 0.331                             | -3.0075 | -1.5332 | 4.0     | 3.9     |
| 9  | 0.238                  | 0.1865                        | 0.327                             | -3.1254 | -1.6524 | 3.6     | 3.7     |
| 10 | 0.25                   | 0.5486                        | 0.261                             | -2.8875 | -1.4898 | 3.8     | 4.0     |
| 11 | 0.259                  | 0.7204                        | 0.221                             | -2.9145 | -1.4552 | 4.1     | 3.8     |
| 12 | 0.235                  | 0.1326                        | 0.349                             | -3.1979 | -1.7156 | 4.6     | 4.0     |
| 13 | 0.253                  | 1.51113                       | 0.176                             | -3.0246 | -1.4108 | 4.7     | 4.3     |
| 14 | 0.249                  | 1.0283                        | 0.222                             | -2.7573 | -1.3086 | 3.7     | 4.2     |
| 15 | 0.243                  | 0.46                          | 0.242                             | -2.9662 | -1.4837 | 4.2     | 4.2     |
| 16 | 0.234                  | 0.289                         | 0.324                             | -3.0204 | -1.4743 | 4.2     | 4.1     |
| 17 | 0.257                  | 0.942                         | 0.233                             | -2.8487 | -1.4176 | 3.8     | 4.1     |

The analysis shows that the hydraulic tortuosity $T_g$ of the samples from the Urievskoye field varies
within small limits; the average is 3.8. However, there is a tendency to decrease tortuosity with an increase in residual water saturation in low-permeability reservoirs.

Consider the relationship between hydraulic tortuosity and the geometry of the void space. The integral value in formula (1) corresponds to the mean value of the square of the radius of the filtration channels.

By the value of capillary pressure per the Laplace formula, we determine the minimum cross-section of pore channels.

If each pore channel is represented as an alternation of pores and interpore contractions (dumbbell model), the considered integral estimates the cross-sections of interpore contractions.

Formula (1) also contains the porosity coefficient, which is obviously determined by the average pore section.

Thus, part of the formula (1) is determined by the pore size, and the size of interpore constrictions determines the other part of the formula (1). These values explain the main incorrectness of formula (1).

In addition, the right side of the formula includes only effective (filtering) pore channels, and the left side (open porosity) – all pore channels: both filtering and non-filtering.

Obviously, the permeability coefficient, calculated from the capillary pressure curves, should be determined only by the minimum size of the pore channels, the size of the interpore constrictions.

Suppose this explanation is the case in formula (1). In that case, it is necessary to pass to effective porosity (instead of open) and then divide it by the ratio of pore cross-sections and interpore narrowings.

In [4], an expression is given for the ratio of the electrical cross-sections of the pores $S_p$ and interpore channels $S_k$.

$$\frac{S_p}{S_k} = 1 + \frac{P_pK_p^{-1}}{p(1-p)} \approx \frac{P_pK_p}{p(1-p)}$$

where $P_p$ is the porosity parameter; $p$ is the linear proportion of microcapillaries (interpore constrictions); $1-p$ is linear proportion of pores.

It was shown in [4] that the product $p(1-p) = 0.25$.

According to V.N. Dakhnov, the electrical parameter of porosity is expressed by the following formula:

$$P_p = \frac{a}{K_p^m},$$

where $a, m$ are parameters that are constant for a given reservoir.

It is natural to assume that the hydraulic porosity parameter is expressed by the same formula, with the only difference that the parameters $a$ and $m$ may differ slightly.

Thus, for the square of hydraulic tortuosity, we finally obtain the following formula:

$$T_H^2 = 4P_pK_p(1 - K_{vo}) = \frac{4a}{K_p^{m-1}}(1 - K_{vo}).$$

Finally, the hydraulic tortuosity is

$$T_H = 2\sqrt{\frac{a(1-K_{vo})}{K_p^{m-1}}}. \quad (11)$$

For example, the Table 1 shows the values of the hydraulic tortuosity $T_{H1}$, calculated from the known values of the parameters of the core samples according to the formula (11).

Comparison of the tortuosity values obtained from the data of capillary studies $T_s$ and calculated from the reservoir properties of $T_{H1}$ using formula (11) shows their satisfactory convergence.
4. Conclusion

For a quantitative assessment of hydraulic tortuosity by reservoir parameters of the formation, it is proposed to use a dumbbell model of the void space. The filtration channels of the rock are represented by alternating pores (macrocapillaries) and interpore narrowings (microcapillaries).

Hydraulic tortuosity is physically explained by the expansion of the flow lines of the filtration flow in the pores and contraction in the interporeal tubules.

Residual water is confined mainly to clay particles that line the pore channels. Since the residual water is immobile, it leads to a narrowing of the open area of the pores and, consequently, to a specific decrease in hydraulic tortuosity.

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