Why do pre-service teachers use the two-variable linear equation system concept to solve the proportion problem?

M Irfan¹, T Nusantara², Subanji², Sisworo², Z Wijayanto¹, S A Widodo¹

¹Universitas Sarjanawiyata Tamansiswa, ²Universitas Negeri Malang

E-mail: muhammad.irfan@ustjogja.ac.id

Abstract. Pre-service teachers need a good understanding of proportion concept so that later they can become teachers who can understand the concepts of mathematics to pre-service teachers. In this article discusses the mistakes of pre-service teachers when choosing the two-variable linear equation system concept to solve the proportion problem. This research is a qualitative research. Subjects in this study were two pre-service teachers of the fourth-semester mathematics education study program at one of the private universities in Yogyakarta. The research subjects were selected purposively with the following considerations: pre-service teachers who used the concept of a two-variable linear equation system to solve proportion and good communication skills so that researchers could explore the thinking process in determining problem-solving strategies. The findings of this study are that pre-service teachers experience interference in determining the knowledge that must be chosen to solve the problem of proportion.

1. Introduction

Mistakes that usually occur when the problem is resolved, overcome the right problem. Many researchers have discussed problems in the context of the problem of proportion, namely: students use non-proportion methods to solve problems [1,2]; students experience when creating a pseudo-sample word problems associated with the proportion [3][4]; students do not understand the problem thoroughly, so they only do procedurally [5]; students experience difficulties when solving speed problems [6]; students do not understand the variables used, do not understand the meaning of the proportion relationship that has been made, and do not understand the keywords of the problem given [7].

Of these various problems, many researchers have tried to provide solutions so that errors can be minimized. To improve proportional reasoning, there needs to be a good connection between what is thought and what will be written [5]. In addition, students need concrete objects that are close to students' lives to improve students' proportional reasoning [6–9]. Of the various ways that have been implemented to school students, there are also students who experience errors when solving proportions.

Usually, the problem presented in the proportion material is only the ratio of one quantity (single proportion). For example, "Heru can travel 1km in 10 minutes, how many minutes does Heru need to take 5km?" Even though in everyday life, we often encounter problems in proportion with more than one quantity (multiple proportions) [10]. For example, when driving, not only the distance and travel time are taken into account, but speed, fuel, density, and road conditions. In this study, researchers tried to present a proportion problem with three quantities, namely: meatballs, noodles, and servings.
The question is based on real life and is very close to students. This problem was designed as a proportion matter, but the researcher saw that there was an error that occurred when resolving the problem. The reason is that there are two quantities of objects that are compared, there is a possibility that students are wrong in choosing a problem-solving strategy.

Researchers observed two private universities in two cities, namely Yogyakarta and Malang. Based on these observations, researchers found that there were still many pre-service teachers who had problems when solving proportions. The researcher gave two questions. First, Rafi always makes the same amount of coffee every day. For 30 days, Rafi spent 600 gr of coffee and 1 kg of sugar. If today he buys 500 grams of coffee and \( \frac{3}{4} \) kg of sugar, then how many days can Rafi make coffee with available supplies? Secondly, Mrs. Tumini needed 90 meatballs and 3 kg of noodles to make 30 servings of meatballs. How many meatballs and noodles are needed to make 100 servings of meatballs? This is shown from the percentage of pre-service teachers who can answer correctly the questions given by the researcher. The results show that as many as 86% of pre-service teachers answered correctly in the first question and 28% in the second question. From these observations, researchers found that preservice teachers experienced difficulties when faced with multiple proportion problems. The number of textbooks that contain formal formulas and algorithms makes pre-service teachers less understanding of everyday problems [11]. When pre-service teachers solve proportions, they only carry out procedures without understanding what they are doing [12].

Researchers believe that there are still many pre-service teachers who experience errors in solving the problem of multiple proportions. When pre-service teachers experience errors when solving proportion problems, there is a possibility that they do not have a deep conceptual understanding. If they do not have a strong understanding, it is certainly a problem when they have started teaching as math teachers, both in elementary, middle and high school.

It is important for researchers to conduct research related to pre-service teachers errors in solving problems [2,10,12–14]. The purpose of this study was to determine the reasons why they used elimination strategies and substitutions which are commonly used to solve the problem of two-variable linear equations when solving proportion problems. The hope is that this research can be used as a reflection for higher education providers, to prepare their pre-service teachers for their deep understanding of school mathematics, especially proportion. These pre-service teachers who later become mathematics teachers at school, of course, must have an understanding of in-depth proportions so that their pre-service teachers do not experience many interruptions in constructing mathematical concepts, especially proportions. Therefore, this research question is “Why do pre-service teachers use the concept of linear equation two variables to solve the problem of proportion?”

2. Method
This study involved 120 students from the fourth semester Mathematics Education program, 2016/2017 academic year at Private Universities in the city of Yogyakarta. The student has taken education from elementary to high school, including vocational schools (with concentration: accounting, health, marketing, computer, and network engineering, etc.). Thus, student input in the school year is very diverse and can be said to be in the second level. All students were given a matter of proportions, “Mrs. Tumini needs 90 meatballs and 3 kg noodles to make 30 portions of meatballs. How many meatballs and noodles are needed to make 100 servings of meatballs?”[12]”. Then, they worked on the question on the paper that had been provided for 20 minutes. After that, researchers examined their answers. The researcher separates the right answer and the wrong answer. From the student's work, the researcher chose the wrong answer because the researcher wanted to know the mistakes made by students when solving the problem. The researcher chooses the subject by considering several things, including communication skills, unique answers, and suggestions from several lecturers. The uniqueness of the answers chosen by the researcher is the answers that use nonproportional methods to solve the problem of proportions, the use of unusual proportional relationships, and the incompatibility between the questions and the answers are written. Based on these criteria, researchers obtained two research subjects. Of the two students, the researchers conducted interviews one by one to explore in-depth
information related to their work. On average, researchers need about 20-30 minutes to interview a student. The goal is to find out why they are wrong in solving problems and why they use the strategies they choose. In the end, researchers can find out students' mistakes in solving the problem of proportion. The aim is to find out why they use the two-variable linear equation system concept when solving proportions.

3. Result and Discussion

When the researcher gave the question: Mrs. Tumini needed 90 meatballs and 3 kg of noodles to make 30 servings of meatballs. How many meatballs and noodles are needed to make 100 servings of meatballs?, 55% of the student's answers are wrong. Among these errors, there is a unique answer, namely students use the system concept of linear two-variable equations to solve the problem of comparison. We know that elimination and substitution methods are often used to solve the problem of a linear equation (non-proportion) system. In fact, the question given by the researcher is a matter of proportion. This finding is like the findings of previous studies which showed that students used non-standard methods (additives, affine, multiplication) to solve proportion problems [1,2,11,12,15]. Students use the elimination and substitution method because they assume that the two quantities in the problem are two variables. Because there are two variables, the problem is considered a matter of the two-variable linear equation system. So they use elimination and substitution methods.

Figure 1 and 2, the subject does not use proportion methods to solve problems. He instead used the method of elimination and substitution to solve the problem of proportion. This is a unique phenomenon, that students use non-proportion methods to solve problems compared to [1,2,10,15]. From the subject's work, he obtained the answers x = -3.1 and y = 103 (see Figure 1). At the beginning of the work, the subject performs an example, namely by using the variable x for meatballs and the variable y for noodles. So, 90 meatballs and 3 kg noodles to make 30 servings of meatballs are changed to 90x + 3y = 30, and the meatballs and noodles needed to make 100 portions of meatballs are changed to x + y = 100. Then, from the existing equation, the system is formed two variable linear equation. After using the elimination and substitution method, x = -3.1 and y = 103 were found. According to him, the interpretation of x = -3.1 and y = 103 was to make 100 servings of meatballs requiring -3.1 meatballs and 103 noodles. From the interpretation of the answers obtained, it can be seen that the subject is merely answering the question by performing an imitation procedure [16,17]. He does not reflect on what he has done. Could meatballs amount to -3.1?
To dig further information, researchers conducted interviews with the subject about the answers to the questions given. Interview fragments as shown in Dialogue 1.

**Dialogue 1**

| Researcher | : This question is included in the subject matter? |
| Subject    | : The system of linear equations of two variables, sir. |
| Researcher | : Why? |
| Subject    | : Because there are two variables, meatballs, and noodles. |
| Researcher | : Because there are two variables, then use SPLDV? |
| Subject    | : Yes, as far as I know, sir. |
| Researcher | : the portion is not a variable? |
| Subject    | : Not sir. It's like a two variable linear equation, the problem that is often encountered, sir. Suppose the price of 2 books and 3 pencils is Rp. 7,000.00. The price of 3 books and 1 pencil is Rp. 7,000.00, what is the price of 4 books and 3 pencils? |
| Researcher | : Oh that? So, the problem that I gave is the same as the example of your problem. |
| Subject    | : yes sir. |
| Researcher | : Can it be done in other ways, the problem I gave earlier? |
| Subject    | : Confused sir. |

From the interview in Dialogue 1, the subject understands the problem of proportion as a problem of a two-variable linear equation system. He considers that meatballs and noodles are two variables, so in planning, problem-solving using elimination and substitution methods to find the values of $x$ and $y$. After obtaining the values $x = -3.1$ and $y = 103$, the subject does not reflect his work. Are the values $x = -3.1$ and $y = 103$ fulfilling the existing equation? Could it be that $x$ is negative? When the researcher asks why $x$ is negative, it cannot answer correctly.

The subject 2 worked out like subject 1 by elimination and substitution method. He uses a linear system of two variables to answer questions. What distinguishes the answer of Subject 1 and subject 2 is in the value of $x$. If subject 1 is obtained the value $x = -3.1$ in subject 2 the value of $x$ is obtained $3.1 \approx 3$. Though, as in Figure 4, the subject writes $3x + 3y = 300 - (90x + 3y = 30)$. Supposedly, it gets $87x = 270$, and $x = -3.1$. This means, in addition to understanding the proportion that has not been good, subject 2 also has an understanding of the operation of algebraic forms that are not good. Whereas to understand proportions, it is necessary to understand the fractions and algebraic reasoning well [4,14,18,19].

The researcher conducted an interview with Subject 2 regarding the work on the second question. The interview quote is as shown in Dialogue 2.

**Dialogue 2**

| Researcher | : Why do you use SPLDV? |
| Subject    | : Because there are two pack objects, meatballs and noodles. |
| Researcher | : $87x = 270$ where did you get it? |
| Subject    | : from $3x + 3y = 300$ minus $90x + 3y = 30$. |
| Researcher | : Is that correct? |
| Subject    | : mmm, it should be $-87x$ sir. |
| Researcher | : Why did you write $87x$? |
| Subject    | : so that the $x$ is not negative, sir. |
Subject 2 realized that he made a mistake when writing $87x = 270$. He wrote $87x = 270$ to avoid negative $x$ values. He is aware that the value of $x$ cannot be negative. However, he is like subject 1, using SPLDV to solve the proportion problem with two objects.

Proportion problems are very closely related to problems in everyday life. To understand the problem of proportion is not easy. Proportion problems are complex problems and the basis for understanding algebra. The complexity of the problem is in proportion because in proportion it can be related to other material. The findings of this study support previous research which found that many students were wrong in solving problems because they did not understand the problem and inaccuracy in choosing problem-solving strategies [10, 19]. Prospective teacher students are the next generation of teachers who will teach students at the school. If the teacher has an understanding of the concept of superficial proportion, of course, the student also experiences the same thing. Therefore, prospective teacher students need to review the lessons in elementary, middle and high school.

4. Conclusion
The pre-service teacher has difficulty in solving unusual problems. This is the result of routine procedures and formal rules that are always used to solve mathematical problems. A pre-service teacher using the concept of linear equations two variables when solving comparison problems are clearly a big problem. They use the concept of a linear equation system of two variables because they assume the problem faced is a problem of linear equations, even though the problem is proportions. This happens because they are not able to properly coordinate the scheme of knowledge that is owned by the problem at hand. As a result, pre-service teacher mistakenly recalls memories in the brain / long-term memory. As a result, the encapsulation process becomes inappropriate, so the resulting object is wrong. The object, in this case, is when they write the formula for a two-variable linear equation system and solve it.

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