The structure of the $T = 1$ iso-triplet hypernuclei, $^7\text{He}$, $^7\text{Li}$ and $^7\text{Be}$ within the framework of an $\alpha + \Lambda + N + N$ four-body cluster model is studied. Interactions between the constituent subunits are determined so as to reproduce reasonably well the observed low-energy properties of the $\alpha N$, $\alpha \Lambda$, $\alpha NN$ and $\alpha \Lambda N$ subsystems. Furthermore, the two-body $\Lambda N$ interaction is adjusted so as to reproduce the $0^+ - 1^+$ splitting of $^7\text{H}$. Also a phenomenological $\Lambda N$ charge symmetry breaking (CSB) interaction is introduced. The $\Lambda$ binding energy of the ground state in $^7\text{He}$ is predicted to be 5.16(5.36) MeV with (without) the CSB interaction. The calculated energy splittings of the $3/2^+-5/2^+$ states in $^7\text{He}$ and $^7\text{Li}$ are around 0.1 MeV. We point out that there is a three-layer structure of the matter distribution, $\alpha$ particle, $\Lambda$ skin, proton or neutron halo, in the $^7\text{He}(J = 5/2^+)$, $^7\text{Li}(J = 5/2^+)$ and $^7\text{Be}(J = 1/2^+)$ states.

I. INTRODUCTION

A new stage in hypernuclear physics has been opened by the $\gamma$-ray spectroscopy for $\Lambda$ hypernuclei, where level structures of the order of keV are revealed systematically. In order to extract valuable information on hypernuclear structure and underlying $\Lambda N$ interactions from these extremely precise data, it is therefore indispensable to utilize accurate models for the many-body wave functions.

Our special concern in this work is the structure of a multiplet of $\Lambda$ hypernuclei specified by an isospin $T$, which have provided us with many interesting subjects so far. For example, in the case of the $T = 1$ multiplet with mass number $A = 7$, $^7\text{He}$, $^7\text{Li}$ and $^7\text{Be}$, their core nuclei are neutron or proton halo nuclei. When a $\Lambda$ particle is added to the core nuclei, $^6\text{He}$, $^6\text{Li}(T = 1)$ and $^6\text{Be}$, the resultant hypernuclear systems become more stable against neutron or proton emission. Hereafter, $T = 1$ excited states of $^6\text{Li}$ and $^7\text{Li}$ are denoted as $^6\text{Li}^*$ and $^7\text{Li}^*$. This stabilization is caused by the so-called ”glue-like” role of the $\Lambda$ [3]. Thanks to the role of $\Lambda$ particle, we can expect an interesting possibility that neutron (proton) drip lines in $\Lambda$ hypernuclei are extended far away from those in ordinary nuclear systems.

In the past, the level structures in $^7\text{He}$, $^7\text{Li}(T = 1)$ and $^7\text{Be}$ were studied with the three-body $^5\text{He} + N + N$ model [2], where only the even-state $\Lambda N$ interaction was used. In Ref. [2], we pointed out that there appear halo or skin structures in the ground state or some excited states of these hypernuclei. Recently, the experimental energy of the $T = 1 \ J = 1/2^+$ state of $^7\text{Li}$ has been observed through the high-resolution $\gamma$-ray experiment [3]. Furthermore, it is proposed to produce $^7\text{He}$ by ($e,eK^+$) at JLAB. One aim in the present work is to discuss halo or skin structure in the extended framework of an $\alpha + \Lambda + N + N$ four-body model.

Another interesting subject to discuss the spin-doublet state, $5/2^+ - 3/2^+$ in $^7\text{He}$ and $^7\text{Li}(T = 1)$. It is considered that these excited $5/2^+ - 3/2^+$ doublets are related intimately to the spin-dependent potentials of the $\Lambda N$ interaction. Therefore, it is important to discuss these splitting energies to determine the spin-dependent parts of the $\Lambda N$ interaction.

In our previous work [4], the spin-doublet structures of $^7\text{Li}$ in $T = 0$ states and the underlying spin-dependent interactions were investigated successfully in the $\text{op} \Lambda$ four-body cluster model. Here, the $\text{op}$ and $\text{on}$ interactions were chosen so as to reproduce the corresponding phase shifts, and the $\Lambda \sigma$ interaction was done so as to reproduce the experimental value of $B_{\Lambda}(^7\text{He})$, and the $\Lambda N$ spin-spin (spin-orbit) interaction was fitted so as to be consistent with the $0^+ - 1^+$ ($5/2^+ - 3/2^+$ spin-doublet energy separation in $^7\text{He}$. In the present work, our four-body analyses for $^7\text{Li}(T = 0)$ is extended straightforwardly to the $T = 1$ multiplet ($^7\text{He}$, $^7\text{Li}^*$, $^7\text{Be}$), where an asterisk stands for the $T = 1$ excited states.

An important subject related to the isospin multiplet of $\Lambda$ hypernuclei is the charge symmetry breaking (CSB) components in $\Lambda N$ interactions. The most reliable evidence for the CSB interaction appears in the $\Lambda$ binding energies $B_{\Lambda}$ of the $A = 4$ members with $T = 1/2$ ($^4\text{He}$ and $^4\text{H}$). Then, the CSB effects are attributed to the differences $\Delta_{CSB} = B_{\Lambda}(^4\text{He}) - B_{\Lambda}(^4\text{H})$, the experimental values of which are $0.35 \pm 0.06$ MeV and $0.24 \pm 0.06$ MeV for the ground ($0^+$) and excited ($1^+$) states, respectively.
There exist mirror hypernuclei in the $p$-shell region such as the $T = 1$ multiplet with $A = 7$ ($^7\text{He}$, $^7\text{Li}^*$, $^7\text{Be}$), $T = 1/2$ multiplet with $A = 8$ ($^8\text{Li}$, $^8\text{Be}$), $T = 1/2$ multiplet with $A = 10$ ($^{10}\text{Be}$, $^{10}\text{B}$), and so on. Historically, some authors mentioned CSB effects in these $p$-shell $\Lambda$ hypernuclei \cite{5,6}. However, there is no microscopic calculation of these hypernuclei taking account of the CSB interaction.

It is well known that the experimental values $\Delta_{CSB}$ can be fitted phenomenologically by an effective spin-independent CSB interaction. On the other hand, in the case of a meson-theoretical model an OPE-type CSB potential is derived through a $\Lambda$ - $\Sigma^0$ mixing effect, where the triplet CSB interaction is much stronger than the singlet one due to the tensor-force contribution. This feature is in strong disagreement with that in the phenomenological force which is almost spin-independent. This difference between triplet and singlet CSB interactions appears in the elaborate four-body calculations for $^7\text{He}$ and $^7\text{Li}$ with use of the Nijmegen soft core model (NSC97e model) \cite{7}, in which the CSB components are generated by the mass difference within the $\Sigma$-multiplet mixed in $\Lambda$ states and the $\Lambda$ - $\Sigma^0$ mixing effect.

Because the origin of the CSB interaction is not yet settled, we treat it phenomenologically in the present study: Similarly to Ref. \cite{8}, the CSB interaction is determined so as to reproduce the values of $\Delta_{CSB}$ obtained from the $\Lambda$ binding energies of $^3\text{He}$ and $^4\text{He}$. Then, the $T = 1$ triplet hypernuclei with $A = 7$ ($^7\text{He}$, $^7\text{Li}^*$, $^7\text{Be}$) are studied with use of this CSB interaction in the four-body cluster model. Additionally, the CSB effects in the $T = 1/2$ doublet hypernuclei with $A = 8$ are investigated within the $\alpha\Lambda$ and $\alpha^3\text{He}\Lambda$ cluster models for $^8\text{Be}$ and $^8\text{Li}$, respectively.

In this work, we study $A = 7$ hypernuclei within the framework of an $\alpha + \Lambda + N + N$ four-body model so as to take account of the full correlations among all the constituent baryons. Two-body interactions among constituent particles are chosen so as to reproduce all the existing binding energies of the sub-systems ($\alpha N$, $\alpha\Lambda N$, $\alpha\Lambda$, and so on). This feature is important in the analysis of the energy levels of these hypernuclei. Our analysis is performed systematically for ground and excited states of $\alpha\Lambda NN$ systems with no more adjustable parameters in this stage, so that these predictions offer important guidance for the interpretation of the upcoming hypernucleus experiments such as the $^7\text{Li}(e^-,e'K^+)\ ^7\text{He}$ reaction at Thomas Jefferson National Accelerator Facility (JLAB).

In Sec. II, the microscopic $\alpha\Lambda NN$ and $NN\Lambda$ four-body calculation method is described. In Sec.III, the interactions are explained. The calculated results and the discussion are presented in Sec.IV. Sec. 5 is devoted to the discussion on the charge symmetry breaking effects obtained for the $A = 7$ and $8$ systems. The summary is given in Sec. VI.

\section{II. Four-Body Cluster Model and Method}

The models employed in this paper are the same as those in our previous work \cite{4}. Namely, we employ the $\alpha + \Lambda + N + N$ model for the $A = 7$ hypernuclei (Fig.1) and the $\alpha + N + N$ model for the $A = 6$ nuclei (Fig.3 in Ref.\cite{4}), where all the rearrangement channels are taken into account. The Schrödinger equation is given by

$$ (H - E) \Psi_{JM,TT_i}(\Lambda Z) = 0, \quad (2.1) $$

$$ H = T + V_{N_1N_2} + \sum_{i=1}^{2} (V_{\alpha N_i} + V_{\alpha N_i}) + V_{\alpha\Lambda} + V_{\text{Pauli}}, \quad (2.2) $$

where $V_{\alpha N_i}$ is the interaction between the $\alpha$ particle and $i$-th nucleon and $V_{\alpha\Lambda}$ is the $\alpha\Lambda$ interaction, which are explained in the next section. The Pauli principle between the $\alpha$ particle and the two nucleons is taken into account by the Pauli projection operator $V_{\text{Pauli}}$, which is the same as in Ref.\cite{4}. The total wave function is described as a sum of amplitudes of all the rearrangement channels shown in Fig.1 in the $LS$ coupling scheme:

$$ \Psi_{JM,TT_i}(\Lambda Z) = \sum_{c=1}^{9} \sum_{nl,NL,\nu,\lambda} \sum_{IK} \sum_{ss} C_{nl,NL,\nu,\lambda,IK,ss}^{(c)} \Phi^{(c)}(\alpha) \times A \left[ \left[ \left[ \left[ \right] \right] \right] \right]_{JM} \times \left[ \eta_2^{(N_1)} \eta_2^{(N_2)} \right]_{TT_i}, \quad (2.3) $$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Jacobi coordinates for all the rearrangement channels \cite{c = 1 \sim 9} of the $\alpha + \Lambda + N_1 + N_2$ four-body system. Two nucleons are to be antisymmetrized.}
\end{figure}
where the notations are the same as in Ref. 4. Also, the definitions of the Gaussian basis functions and the Gaussian ranges are the same as those in the case of the \( A = 4 \) hypernuclei.

The eigenenergy \( E \) in Eq. (2.2) and the \( C \) coefficients in Eq. (2.3) are determined by the Rayleigh-Ritz variational method. The angular momentum space of \( l, L, \lambda \leq 2 \) was found to be sufficient to obtain good convergence of the calculated results as described below.

### III. INTERACTIONS

#### A. Charge symmetry parts

We recapitulate here the charge symmetric parts of the \( V_{\alpha N}, V_{NN}, V_{\alpha A}, \) and \( V_{AN} \) interactions employed in our \( \alpha N\Lambda \) systems.

For \( V_{\alpha N} \), we employ the effective potential proposed in Ref. 9, which is designed so as to reproduce well the low-lying states and low-energy scattering phase shifts of the \( \alpha n \) system. The Pauli principle between nucleons belonging to the \( \alpha \) and the valence nucleon is taken into account by the orthogonality condition model (OCM) 10. As for the \( NN \) interaction \( V_{NN} \), we use the AV8 11 potential, derived from the AV18 12 by neglecting the \((L\cdot S)\) term.

The interaction \( V_{\alpha A} \) is obtained by folding the \( \Lambda N \) G-matrix interaction derived from the Nijmegen model F(NF) 13 into the density of the \( \alpha \) cluster 14, its strength being adjusted so as to reproduce the experimental value of \( B_{\alpha}(^{3}He) \).

For \( V_{AN} \), we employ effective single-channel interactions simulating the basic features of the Nijmegen model NSC97f 15, where the \( \Lambda N-\Sigma N \) coupling effects are renormalized into \( \Lambda N-\Lambda N \) parts: We use three-range Gaussian potentials so as to reproduce the \( \Lambda N \) scattering phase shifts calculated from the NSC97f, and then their second-range strengths in \( ^{3}E \) and \( ^{1}E \) states are adjusted so that calculated energies of \( 0^{+}, 3^{+} \) states in the \( NNN \) four-body calculation reproduce the observed splittings of \( ^{4}_{\Lambda}H \). Furthermore, the spin-spin parts in the odd states are tuned to get the experimental values of the splitting energies of \( ^{4}_{\Lambda}Li \). The symmetric LS (SLS) and anti-symmetric LS (ALS) parts in \( V_{AN} \) are chosen so as to be consistent with the \( ^{9}_{\Lambda}Be \) data as follows: The SLS and ALS parts derived from NSC97f with the G-matrix procedure are represented in the two-range form, and then the ALS part is strengthened so as to reproduce the measured \( 5/2^{+}, 3/2^{+} \) splitting energy with the \( 2\alpha + \Lambda \) cluster model 16. The parameters of the \( \Lambda N \) interactions are given in

\[
V_{\Lambda N}(r) = \sum_{i=1}^{3} \frac{1 + P_{r}}{2} \left( v_{0}^{even} + \sigma_{\Lambda} \cdot \sigma_{N} v_{\alpha N}^{even} \right) e^{-\beta_{\Lambda N} r^{2}} + \frac{P_{r}}{2} \left( v_{0}^{odd} + \sigma_{\Lambda} \cdot \sigma_{N} v_{\alpha N}^{odd} \right) e^{-\beta_{\Lambda N} r^{2}},
\]

and listed in Table I(a).

The calculated energies of the \( 0^{+} \) states in \( ^{6}_{\Lambda}He \) and \( ^{6}_{\Lambda}Li^{*} \) are \( -0.59 \) MeV and unbound with the respect to the \( \alpha + N + N \) three-body breakup threshold, which are less bound than the observed values, \( -0.98 \) MeV in \( ^{6}_{\Lambda}He \) and \( -0.14 \) MeV in \( ^{6}_{\Lambda}Li \). Considering that it is of vital importance in our cluster model to reproduce accurately the binging energy of all subcluster systems, we introduce an effective three-body \( \alpha N\Lambda \) interaction phenomenologically, the form of which is assumed as

\[
V_{\alpha NN}(r_{1}, r_{2}) = \sum_{i=1}^{2} v_{i} e^{-\beta_{i} r_{1}^{2} - \beta_{2} r_{2}^{2}}, \quad (3.2)
\]

where \( r_{1} \) and \( r_{2} \) are Jacobian coordinates for \( C = 1 \) and 2 in Fig. 3 of Ref. 1.

This interaction includes four parameters \((\beta_{i}, v_{i})\), which cannot be determined completely by the two binding energies of \( ^{6}_{\Lambda}He \) and \( ^{6}_{\Lambda}Li^{*} \) only. Then, the condition to reproduce the experimental value of \( ^{4}_{\Lambda}Li^{*} \) is found to give a strong constraint for the parameters. The determined values of parameters are \((\beta_{1}, v_{1}) = (0.444 \text{ fm}^{-2}, 244.8 \text{ MeV})\), \((\beta_{2}, v_{2}) = (0.128 \text{ fm}^{-2}, -20.4 \text{ MeV})\).

#### B. Charge symmetry breaking interaction

It is out of our scope in this work to explore the origin of the CSB interaction. We assume here the CSB interaction with an one-range Gaussian form

\[
V_{NCB}(r) = \frac{\tau_{z}}{2} \left[ 1 + P_{r} \left( v_{0}^{even, CSB} + \sigma_{\Lambda} \cdot \sigma_{N} v_{\alpha N}^{even, CSB} \right) e^{-\beta_{even} r^{2}} + \frac{1 - P_{r}}{2} \left( v_{0}^{odd, CSB} + \sigma_{\Lambda} \cdot \sigma_{N} v_{\alpha N}^{odd, CSB} \right) e^{-\beta_{odd} r^{2}} \right],
\]

which includes spin-independent and spin-spin parts. In the cases of the four-body calculations of \( ^{4}_{\Lambda}H \) (\( nnp\Lambda \)) and \( ^{4}_{\Lambda}He \) (\( npp\Lambda \)), the contributions of the odd-state interactions are negligibly small and their strengths cannot be determined: We take \( v_{0}^{odd, CSB} = 0 \), and \( v_{\sigma N}^{odd, CSB} = 0 \). The range parameter, \( \beta_{even} \), is taken to be \( 1.0 \text{ fm}^{-2} \). The parameters \( v_{0}^{even, CSB} \) and \( v_{\sigma N}^{even, CSB} \) are determined phenomenologically so as to reproduce the values of \( \Delta CSB \) derived from the \( \Lambda \) binding energies of \( 0^{+} \) and \( 1^{+} \) states in the four-body calculation of \( ^{4}_{\Lambda}He \). Then, we obtain \( v_{0}^{even, CSB} = 8.0 \text{ MeV} \) and \( v_{\sigma N}^{even, CSB} = 0.7 \text{ MeV} \). The calculated \( B_{\Lambda} \) of \( 0^{+} \) and \( 1^{+} \) states in \( ^{4}_{\Lambda}Li \) are 1.99 MeV and 0.98 MeV, respectively. Those in \( ^{4}_{\Lambda}He \) are 2.35 MeV and 1.17 MeV, respectively. In these calculations including the CSB interactions, the parameters in the CS parts are slightly modified from those in Table Ia for fine fitting of the experimental \( B_{\Lambda} \) values. In Table I(a), the modified values of parameters are given in the parentheses.

In order to extract the information about the odd-state part of CSB, it is necessary to study iso-multiplet hypernuclei in the \( p \)-shell region. A suitable system for such
a study is $^7\Lambda$He, in which the core nucleus $^6\text{He}$ is in a bound state. (On the contrary, valence protons in $^8\text{Be}$ are unbound.) Our four-body calculation for this system has to be powerful to extract the accurate information. Though there is no data about $^7\Lambda$He at present, the coming experiments at JLAB will give us valuable data for our analyses.

Another example in the $p$-shell region is iso-doublet hypernuclei $^8\text{Li}$ and $^8\Lambda\text{Be}$, whose experimental values of $B\Lambda$ are obtained in emulsion. Then, it is interesting to see the contribution of the CSB interaction to the $B\Lambda$ values of these hypernuclei. For applications to these nuclei, we used $\Lambda t$ and $\Lambda^3\text{He}$ potential for CP states defined by

\[
V_{\Lambda t}(\mathbf{r}, \mathbf{r'}) = \sum_{i=1}^{3} \frac{1}{2} \left[ V^{i, \text{CSB}}_{\Lambda t} + s_\Lambda \cdot s_i V^{i, \text{CSB}}_{\Lambda^3\text{He}} \right] e^{-\mu_i r^2} \delta(\mathbf{r} - \mathbf{r'}) + \left( U^{i, \text{CSB}}_{\Lambda t} + s_\Lambda \cdot s_i U^{i, \text{CSB}}_{\Lambda^3\text{He}} \right) e^{-\delta_i (\mathbf{r} - \mathbf{r'})^2} \delta(\mathbf{r} - \mathbf{r'})
\]

where $x$ denotes $\Lambda$ or $^3\text{He}$. The parameters are listed in Table I(b). The CSB part for $\Lambda t$ and $\Lambda^3\text{He}$ is given by

\[
V_{\Lambda t}^{\text{CSB}}(\mathbf{r}, \mathbf{r'}) = \frac{1}{2} \left[ V^{0, \text{CSB}}_{\Lambda t} + s_\Lambda \cdot s_i V^{i, \text{CSB}}_{\Lambda^3\text{He}} \right] e^{-\mu_{\text{CSB}} r^2} \delta(\mathbf{r} - \mathbf{r'}) + \left( U^{0, \text{CSB}}_{\Lambda t} + s_\Lambda \cdot s_i U^{i, \text{CSB}}_{\Lambda^3\text{He}} \right) e^{-\delta_{\text{CSB}} (\mathbf{r} - \mathbf{r'})^2} \delta(\mathbf{r} - \mathbf{r'})
\]

The parameters for even-state are adjusted so as to reproduce the data within the $\Lambda t$ and $\Lambda^3\text{He}$ cluster models for $^4\text{He}$ and $^4\Lambda\text{He}$, respectively. The parameters are $V^{0, \text{CSB}}_{\Lambda t} = 0.38$ MeV, $U^{0, \text{CSB}}_{\Lambda t} = -0.03$ MeV, $\mu_{\text{CSB}} = 0.06$ fm$^{-2}$, $\delta_{\text{CSB}} = 0.679$ fm$^{-2}$ for $^4\text{He}$, and the same value with the opposite sign for $^4\Lambda\text{He}$. As explained later, also the odd-state CSB interaction is introduced phenomenologically so as to reproduce the $B\Lambda$ values of $^4\text{Li}$ and $^4\Lambda\text{Be}$. The determined parameters are $V^{0, \text{CSB}}_{\Lambda^3\text{He}} = -0.93$ MeV, $U^{0, \text{CSB}}_{\Lambda^3\text{He}} = -12.0$ MeV, $\mu_{\text{CSB}} = 0.223$ fm$^{-2}$, $\delta_{\text{CSB}} = -0.14$ MeV, $U^{0, \text{CSB}}_{\Lambda^3\text{He}} = -0.95$ MeV, $\gamma_{\text{CSB}} = 0.203$ fm$^{-2}$ and $\delta_{\text{CSB}} = 0.254$ fm$^{-2}$ for $^4\text{Li}$ and the same value with the opposite sign for $^4\Lambda\text{Be}$. It is notable here that the odd-state CSB is of far longer range than the even-state one.

IV. RESULTS

First, let us show the level structures of the $T = 1$ states calculated with the $\alpha + \Lambda + N + N$ four-body model using the same $\Lambda N$ interaction in Ref.[4]. We calculated the bound states in those $\Lambda$ hypernuclei.

In Fig.2 to Fig.4 and Table II, we show the level structures of $A=7$ hypernuclei calculated without the CSB interaction. In each figure, hypernuclear levels are shown in four columns in order to show separately the effects of even- and odd-state $\Lambda - N$ interactions and also the SLS and ALS interactions. Even if the CSB interactions are switched on, their small contributions do not alter the features of these figures. At first glance, the obtained $\Lambda$ states become less bound by 1 MeV in the order of $^7\Lambda\text{He}$, $^7\Lambda\text{Li}$, and $^7\Lambda\text{Be}$, because the repulsive Coulomb-force contributions increase in this order. In these figures, the calculated energy spectra of low-lying states of core nuclei, $^6\text{He}$, $^6\text{Li}$, and $^6\Lambda\text{Be}$ are also drawn in order to demonstrate the $\Lambda$-binding effects. Here, $^6\text{He}$ and $^6\text{Li}$ are nucleon-bound states, and the $^6\text{He}$ and $^6\Lambda\text{Be}$ interactions are adjusted so as to reproduce the observed energy spectra. On the other hand, $^6\text{Be}$ is an nuclear-unbound system. In order to extract the $B\Lambda$ value of $^7\Lambda\text{Be}$, it is needed to subtract the total energy of the lowest $^6\Lambda$Be resonant state from the calculated ground-state energy of $^7\Lambda\text{Be}$. The energy positions of resonant states are determined by the real scaling method [15]. The obtained lowest state in

| $i$ | (a) $\Lambda N$ interaction | (b) $^4\Lambda\text{He}$ interaction |
|-----|-------------------------------|---------------------------------|
| $\beta_{\Lambda N}^i$ | 0.391 | 1.5625 | 8.163 |
| $v_{\Lambda N}^{0,\text{even}}$ | -3.94 | -126.1(-126.4) | 1943 |
| $v_{\Lambda N}^{0,\text{odd}}$ | -0.003 | 17.5(18.0) | -374.1 |
| $v_{\Lambda N}^{0,\text{odd}}$ | -1.43 | 72.8 | 3247 |
| $v_{\Lambda N}^{0,\text{odd}}$ | -0.26 | -61.35 | -270.9 |

| $\gamma_i$ | 0.2033 | 0.2033 | 0.2033 |
| $\delta_i$ | 0.3383 | 0.8234 | 2.521 |
| $U^{0,\text{even}}_{\Lambda N}$ | -1.995(-1.998) | -36.389(-36.99) | 156.9(156.9) |
| $U^{0,\text{odd}}_{\Lambda N}$ | 0.029(0.028) | 4.246(4.242) | -18.75(-18.73) |
| $U^{0,\text{odd}}_{\Lambda N}$ | -1.455(-1.457) | -17.791(-17.814) | 620.2(618.9) |
| $U^{0,\text{odd}}_{\Lambda N}$ | 0.552(0.553) | 1.329(1.326) | -212.8(-213.3) |
$^6\text{Be}$ is a $0^+$ broad resonance, whose energy is 0.79 MeV. Thus, the experimental resonant energy 1.37 MeV cannot be reproduced, when the $\alpha N$, $NN$ and $\alpha NN$ interactions are adopted so as to reproduce the bound-state energies of $^6\text{He}$ and $^6\text{Li}^*$. It is particularly interesting to see the glue-like role of the $\Lambda$ particle in $A=7$ hypernuclear systems. Though the ground state of $^6\text{Be}$ is unbound, the $\Lambda$ participation leads to a bound state below the lowest $^5\Lambda\text{He}+p+p$ threshold, the binding energy of which is about 1.3 MeV. The ground states of the core nuclei $^6\text{He}$ and $^6\text{Li}^*$ are weakly bound by 1.02 and 0.12 MeV below the $\alpha + N + N$ threshold. Owing to an additional $\Lambda$ particle, those of $^7\Lambda\text{He}$ and $^7\Lambda\text{Li}^*$ become rather deeply bound by about 2 $\sim$ 3 MeV below the respective lowest thresholds. It should be noted, here, that the calculated values of $B_\Lambda$ of $^7\Lambda\text{Li}^*$ and $^7\Lambda\text{Be}$ are in good agreement with the experimental values, as shown in Table I. The 5/2$^+$ and 3/2$^+$ excited states in $^7\Lambda\text{Li}^*$ are predicted to be in weakly bound states with the respect to the $^5\Lambda\text{He}+p$ threshold. On the other hand, the corresponding states in $^7\Lambda\text{He}$ are in deeper bound states by about 1.3 MeV with respect to the $^5\Lambda\text{He}+n$ threshold. This difference is because the $\alpha p$ Coulomb repulsion in the former is not active in the latter.

In the past calculation [2], the uppermost bound states in $^7\Lambda\text{He}$, $^7\Lambda\text{Li}^*$ and $^7\Lambda\text{Be}$ were 5/2$^+$, 3/2$^+$ and 1/2$^+$ states, respectively. These states are very weakly bound structures, and exhibit halo or skin structures having long tails in density distributions of valence nucleons. In comparison with these calculations, performed in the limited three-body model space ($^5\Lambda\text{He} + N + N$), all states in $A=7$ systems become deeper bound in the present four-body model. This tendency is reasonable because in the present calculations the excitation effects of a $\Lambda$ particle are fully taken into account in the treatment with use of the $\Lambda N$ effective interactions chosen consistently with the four-body model space. It is instructive to compare the tail behavior of the density distributions of valence nucleons in the four-body model with those in the three-body model. We derive here the nucleon density distributions.
FIG. 3: (color online). Calculated energy levels of $^6\text{Li}^*$ and $^7\Lambda\text{Li}^*$. The charge symmetry breaking potential is not included in $^7\Lambda\text{Li}^*$. The level energies are measured from the particle breakup threshold.

of $5/2^+$ states in $^7\Lambda\text{He}$ and $^7\Lambda\text{Li}$ and that of $1/2^+$ state in $^7\Lambda\text{Be}$ using the two models.

In Table III we list the calculated values of the r.m.s radii between $\alpha$ and $N$ ($\bar{r}_{\alpha-N}$) and those between $\alpha$ and $\Lambda$ ($\bar{r}_{\alpha-\Lambda}$) in our four-body models of $^7\Lambda\text{He}$, $^7\Lambda\text{Li}^*$ and $^7\Lambda\text{Be}$. As shown here, the values of $\bar{r}_{\alpha-\Lambda}$ in these systems are larger than those of $\bar{r}_{\alpha-N}$, indicating that the distributions of valence nucleons are of longer-ranged tail than those of $\Lambda$'s in the respective systems. However, all r.m.s radii in the four-body models are shorter than those in the three-body models [2], that is the four-body binding energies in the present model are larger than the three-body ones in the previous model. This means that the distributions of nucleons and $\Lambda$ around $\alpha$ obtained in the four-body models are more compact than those in the three-body models.

In order to see the structures of these systems visually, in Fig.5 we draw the density distributions of $\Lambda$ (dashed curve) and valence neutrons (solid curve) of the $5/2^+$ states in $^7\Lambda\text{He}$ and $^7\Lambda\text{Li}^*$ and of the $1/2^+$ state in $^7\Lambda\text{Be}$. For comparison, here, also a single-nucleon density in the $\alpha$ core is shown by the dotted curve. In each case, the density distribution of the $\Lambda$ has a shorter-ranged tail than that of the two valence nucleons, but is extended significantly far away from the $\alpha$ core. This structure can be nicely imaged as three layers of matter distribution composed of an $\alpha$ core, a $\Lambda$ skin and a neutron (proton) halo. Here, the proton-density distribution in the $5/2^+$ state of $^7\Lambda\text{Li}^*$ has a particularly longer tail than those in the others due to the very weak binding of the halo proton from the lowest $^6\Lambda\text{He}+p$ threshold.

It is considered that the $3/2^+-5/2^+$ spin-doublet states in $^7\Lambda\text{He}$ and $^7\Lambda\text{Li}^*$ give valuable information about the underlying spin-dependence of the $\Lambda N$ interaction. Let us investigate these states straightforwardly with use of the $\Lambda N$ interaction determined in the analysis for the $T = 0$
spin-doublet states in $^7\Lambda$Li. The results for $^7\Lambda$He and $^7\Lambda$Li* are displayed in Fig.2 and Fig.3, respectively. Because their features are not different from each other, here we pick up the former case.

Then, let us remark how the energies of the $3/2^+ - 5/2^+$ spin-doublet states are changed by adding the components of AN interaction successively. We see that the resultant energy splitting of $5/2^+ - 3/2^+$ states in $^7\Lambda$He is given as about 0.1 MeV, being the combined contributions from the spin-spin, SLS and ALS interactions as explained below. We can see the same tendency in $^7\Lambda$Li* in Fig.3.
FIG. 6: The calculated energy levels of $^6$He, $\Lambda^7$He, $^6$Li, $\Lambda^7$Li, $^6$Be and $\Lambda^7$Be with spin-spin and spin-orbit $\Lambda N$ interactions. The charge symmetry breaking potential is not included in the calculated energies of $A=7$ hypernuclei. The energies are measured from the particle breakup threshold.

It should be noted here that the splitting energies of the $T=1$ $3/2^+-5/2^+$ states are much smaller than those of the $T=0$ $1/2^+-3/2^+$ and $5/2^+-7/2^+$ doublet states in $\Lambda^7$Li given in Ref. [2]. To understand the reason for the difference between the $T=1$ and $T=0$ doublet splittings, first we remark that the spin-isospin structure of $NN\Lambda$ system on the $\alpha$ core is $[(NN)_{sT}\Lambda]_S$ (cf. Eq. (2.3)). In the case of $T=1$ states, the corresponding $nn$ pair is in spin-singlet states ($s=0$, spin antiparallel), while in $\Lambda^7$Li ($T=0$) the $np$ pair outside the $\alpha$ core is in a spin-triplet state ($s=1$, spin-parallel). In general the numbers of $\Lambda N$ triplet and singlet bonds are different between the $J_>$ and $J_<$ partner states. Thus difference in spin-value of $(NN)_{s=1}\text{ or }0$ leads to the different contributions of the $\Lambda N$ spin-spin interactions to the doublet splittings. Let us see in more detail how the $\Lambda N$ spin-spin interactions contribute to the $3/2^+-5/2^+$ splitting in $\Lambda^7$He ($T=1$). Both doublet states are composed of the $L=2$ $(nn)_{s=0,T=1}$ pair in the spin-singlet state coupled to the $s$-state $\Lambda$. As mentioned above, the situation is notably different from that of the $5/2^+-7/2^+$ doublet in $\Lambda^7$Li ($T=0$) which is based on the $[L=2\text{ (}pn)_{s=1,T=0}]_{J=3^+}$ core state and therefore the $J_>_3^+$ partner is characterized by the spin-stretched configuration. In contrast to the $T=0$ case, both of the $J_<_3^+$ state and the $J_>_5^+$ state in $\Lambda^7$He ($T=1$) include $\Lambda N$ spin-singlet and spin-triplet states. However, we find that the contribution of the $\Lambda N$ spin-singlet state is negligibly small in the $J_>_5^+$ state. As a result the even-state spin-spin part of the $\Lambda N$ interaction gives rise to the splitting energy of about 0.31 MeV (See “even” column.). In addition, when the odd-state interaction is switched on, the
TABLE II: Calculated energies of the low-lying states of (a) $^7\text{He}$, (b) $^7\text{Li}^*$, and (c) $^7\text{Be}$ without the charge symmetry breaking potential, together with those of the corresponding states of $^6\text{He}$, $^6\text{Li}$, and $^6\text{Be}$, respectively. $E$ stands for the total interaction energy among constituent particles. The energies in the parentheses are measured from the corresponding lowest particle-decay thresholds $^5\text{He} + N$ for $^7\text{He}$ and $^7\text{Li}^*$ and $^5\text{He} + p + p$ for $^7\text{Be}$. The calculated r.m.s. distances, $\bar{r}_{\alpha-N}$, $\bar{r}_{\alpha-\Lambda}$ are also listed for the bound state.

(a) $^6\text{He}(\alpha nn)$ \hspace{1cm} $\tilde{\Lambda}$He($\alpha nn\Lambda$)

| $J^\pi$ | 0$^+$ | 2$^+$ | 1/2$^+$ | 3/2$^+$ | 5/2$^+$ |
|--------|--------|--------|--------|--------|--------|
| $E$ (MeV) | $-1.02$ | $0.82$ | $-6.39$ | $-4.73$ | $-4.65$ |
| $E^{\exp}$ (MeV) | $-0.98$ | $0.83$ | | | |

(b) $^6\text{Li}(\alpha np)$ \hspace{1cm} $\tilde{\Lambda}$Li($\alpha np\Lambda$)

| $J^\pi$ | 0$^+$ | 2$^+$ | 1/2$^+$ | 3/2$^+$ | 5/2$^+$ |
|--------|--------|--------|--------|--------|--------|
| $E$ (MeV) | $-0.12$ | $1.77$ | $-5.40$ | $-3.75$ | $-3.66$ |
| $E^{\exp}$ (MeV) | $-0.14$ | $1.67$ | | | |

(c) $^6\text{Be}(\alpha pp)$ \hspace{1cm} $\tilde{\Lambda}$Be($\alpha pp\Lambda$)

| $J^\pi$ | 0$^+$ | 2$^+$ | 1/2$^+$ | 3/2$^+$ | 5/2$^+$ |
|--------|--------|--------|--------|--------|--------|
| $E$ (MeV) | $0.79$ | $2.93$ | | | |

ALS does not efficiently work in the 5/2$^+$ state, because the spin-singlet component is small in this state. As a result, the energy splitting of 5/2$^+$-3/2$^+$ states including both spin-spin and spin-orbit terms in $\tilde{\Lambda}$He leads to 0.08 MeV. We can see the same tendency in $\tilde{\Lambda}$Li$^*$ and the resultant splitting is 0.09 MeV as shown in Fig.3. If the experimental energy resolution becomes good enough to discuss the present splitting energy, we would have a chance of getting information about the spin-dependent parts of the $\Lambda N$ interaction.

There still remain certain effects of the $\Lambda N$ tensor interaction on the doublet splittings. In this paper for the $T = 1$ isoscalar states ($A = 7$), however, we apply the prescription adopted in the analysis of the $T = 0$ $\tilde{\Lambda}$Li states [4] and therefore we do not include the tensor component. Here we note that the $\Lambda N - \Lambda N$ tensor contribution is small compared to the spin-spin interaction, however another tensor effect comes from the $\Lambda N - \Sigma N$ coupling. In fact, accounting for $\Sigma - \Lambda$ coupling by modifying the $\Lambda N$ interaction alters its effect on doublet splitting, and hence introduces an uncertainty in the calculation. According to the $\Sigma$-mixing studied within the shell model [14], the energy shifts amount to several tens of keV in some of the $T = 0$ states of $\tilde{\Lambda}$Li. The cluster model estimates for such effect will be discussed in the next stage.

V. CHARGE SYMMETRY BREAKING EFFECTS

A. CSB effects in $A = 7$ four-body models

Let’s focus on the ground states in $\tilde{\Lambda}$He and $\tilde{\Lambda}$Be and the $T = 1$ 1/2$^+$ state in $\tilde{\Lambda}$Li, which are the members of the iso-triplet. The CSB effect has to be reflected also in their binding energies in the same way as in the $T = 1/2$ iso-doublet members $^4\text{H}$ and $^5\text{He}$.

As explained in sec. IVB, we introduce the phenomenological CSB potential with the central-force component only. The CSB part of the two-body $\Lambda N$ interaction is fixed to reproduce the averaged energy spectra of $^4\text{H}$ and $^5\text{He}$, and then the CSB part is adjusted so as to reproduce simultaneously the energy levels of these hypernuclei. The spin-spin part of the CSB can be determined by performing this adjusting procedures both for the 0$^+$ and 1$^+$ states.

First, in Fig. 6, we show the energy spectra of $A = 7$ hypernuclei without the CSB interaction. The ground-state energy of $\tilde{\Lambda}$He is $-6.39$ MeV with the respect to the $\alpha + n + n + \Lambda$ four-body breakup threshold. With increase of the proton numbers, the Coulomb repulsion becomes more and more effective as going from $\tilde{\Lambda}$Li$^*$ to $\tilde{\Lambda}$Be. Recently in KEK-E419 experiment [3], they produced the $T = 1$ 1/2$^+$ state of $\tilde{\Lambda}$Li. The observed value of $B_A = 5.26$ MeV is in good agreement with our calculated value 5.28 MeV. In the case of $\tilde{\Lambda}$Be, there are the old emulsion data giving $B_A = 5.16$ MeV. This value should be compared
with our obtained value 5.21 MeV. Then, the $B$ value in the ground $1/2^+$ state of $^7\Lambda$He is predicted to be 5.36 MeV without taking the CSB effect into account.

Next, let’s consider the CSB effects in $A = 7$ iso-triplet hypernuclei. In Fig. 7, we show the energy spectra of those hypernuclei calculated with the CSB interaction switched on. In the $^7\Lambda$Li case, the CSB interaction brings about almost no contribution to the $\Lambda$ binding energies, because there is one proton and one neutron outside the $\alpha$ core and the $\Lambda\alpha$ and $\Lambda p$ CSB interactions cancel with each other. On the other hand, the CSB interaction works repulsively (+0.20 MeV) and attractively (−0.20 MeV) in the $^7\Lambda$He and $^7\Lambda$Be cases, respectively. Therefore, our result indicates that if the experimental energy resolution is as good enough as less than 0.2 MeV, the CSB effect could be observed in these cases. It should be noted here that only the even-state part of our CSB interaction is taken into account in consistent with the observed binding energies of $^4\Lambda$H and $^4\Lambda$He.

In the $^7\Lambda$Be case, the $\Lambda$ energy becomes more bound by 0.2 MeV due to the attractive CBS interaction between the $\Lambda$ and two protons, that is $B = 5.44$ MeV. The experimental $B$ value is found to be reproduced without the CSB effect and the inclusion of the CSB contribution goes unfavorably. In order to reproduce the binding energy of $^7\Lambda$Be, the CSB interaction seems to be vanishing or even of opposite sign from that in the $A = 4$ system. There still remains a problem in our treatment for the $^7\Lambda$Be system: The calculated value 0.79 MeV of the lowest resonance energy of the $^8\Lambda$Be is not in agreement with
repurpose reproduce the experimental value of $\Delta B_A^{(8)}$, here, let us have a try to introduce an odd-state CSB interaction phenomenologically, whose contributions in the $A = 4$ systems are negligible: We found that the experimental values of $B_A$ for $^8$Li and $^8$Be can be reproduced by adding a rather long-ranged odd-state interaction with the opposite sign of the even state CSB interaction described in Eq.(3.5). The $B_A$ values of $^8$Li and $^8$Be calculated with both even-state and odd-state CSB interactions are 6.81 MeV and 6.83 MeV, respectively, which are in good agreement with the data.

The present framework for the $A = 8$ iso-multiplet systems has a sort of limitation in the sense that the $t(3^+)$ cluster is assumed to have 3 nucleons of the same size of those in $\alpha$. However, the results for both systems of $A = 7$ and 8 are qualitatively consistent with each other, and the odd state of the CSB interaction are found to have an opposite sign of the even state CSB interaction determined at $A = 4$ hypernuclei.

In the near future, we expect to have the observed $B_A$ of $^7$He from the $(e,e'K^+)$ reaction experiment done at JLAB. On the basis of the coming data, it might be possible to gain information on the odd-state CSB interactions. Another example to clarify the even- and odd-state CSB interactions is to study $^{10}$Be with an $\alpha N N$ four-body model. This four-body calculation is in progress. Also, we hope to observe the $B_3$ of this hypernucleus by $^{10}$Be$(e,e'K^+)^{10}$Be at JLAB in the future.

VI. SUMMARY

We have studied the structures of the $T = 1$ triplet hypernuclei ($^6$He, $^7$Li and $^8$Be) within the framework of $\alpha + \Lambda + N + N$ four-body model. In the previous paper this four-body model proved to work successfully in the detailed analysis of the $T = 0$ energy levels of $^7$Li which are best known through the high-resolution $\gamma$-ray measurements. The present framework is also a natural extension of the previous calculations performed with the $^6$He$+N + N$ three-body model in which the $\Lambda$ particle motion was confined to form the $^5$He ground state.

The major conclusions are summarized as follows:

(1) On the basis of reasonable $\alpha(p,n)$, $\alpha p$, $\alpha N$ and $N A$ interactions, which well describe the binding energies of all sub-cluster units ($\alpha p$, $\alpha N$ and $N A$), we have made extensive and successful structure analyses for the $T = 1$ states of $A = 7$ iso-triplet hypernuclei. One of the non-trivial and important outcomes is that the observed $B_A$ value of the $T = 1 1/2^+$ state in $^7$Li is reproduced nicely with the use of the $\alpha N$ and $\Lambda N$ interactions determined in $T = 0$ states of $^5$Li. Also the $B_3$($^8$Be) observed in emulsion is reproduced well, though there still remains a problem that the unbound $^8$Be $0^+$ state is calculated at a bit lower position in comparison with the observed resonance energy. The $\Lambda$ binding energy for $^7$He ($J = 1/2^+$), which has not been observed so far, is calculated to be around 5.16–5.36 MeV (with or without the CSB
interaction). This result will be tested when the result of the $^7\text{Li}(e,e'K^+)^5\text{He}$ experiment comes from JLAB.

(2) As one of the purposes of the extended calculations, we have carefully tested whether the $3/2^+$ and $5/2^+$ spin-doublet excited states ($s_{1/2} \Lambda$ coupled to the $2^+$ excited core) are bound or not, since they were calculated previously to be just above the nucleon breakup threshold (weakly unbound) as a result of the limited three-body model of $\Lambda\text{He}+N+N$. It is interesting to see the gluelike role of the $\Lambda$ particle carefully when it is added to the core nuclei having a nucleon halo structure as concerned here. In this paper the four-body calculation, which allows free motion of $\Lambda$, gives a clear prediction that the excited spin-doublet states in $\Lambda\text{He}$ ($^7\!\!\Lambda\text{Li}$) become bound, respectively, at $1.3 \text{ MeV}$ ($0.3 \text{ MeV}$) below the lowest nucleon-breakup threshold $^9\text{He} + n$ ($^9\text{He} + p$). The energy splitting between these $T=1$ doublet states comes from the spin-spin and spin-orbit interactions, which is calculated to be around $0.1 \text{ MeV}$. If any coincidence experiment is available and the energy resolution is good enough to resolve the $0.1 \text{ MeV}$ splitting, one would have a chance of extracting information on the spin-dependent interactions. In $^7\Lambda\text{Be}$, however, we do not expect to get the corresponding bound excited states.

(3) It is interesting to get the three-layer structure of the matter distributions in the $T=1$ iso-triplet hypernuclei, which consist of the $\Lambda$ particle coupled to the core nuclear having neutron or proton halo. The typical numbers of the r.m.s. radii for the $^7\text{He}(J=5/2^+)$, $^7\!\!\text{Li}^*(J=5/2^+)$ and $^7\Lambda\text{Be}(J=1/2^+)$ states are calculated to be $\bar{r}_\alpha = 1.4 \text{ fm}$ for innermost $\alpha$, $\bar{r}_{\alpha-\Lambda} = 2.8 \text{ fm}$ for the $\Lambda$ distribution, and $\bar{r}_{\alpha-n} = 3.8 \text{ fm}$ for the outermost valence nucleon distribution.

(4) The charge symmetry breaking effects in light $p$-shell hypernuclei have been investigated quantitatively for the first time on the basis of the phenomenological CSB interaction which describe the experimental energy difference between $B_{\Lambda}(^4\text{H})$ and $B_{\Lambda}(^7\Lambda\text{Be})$. Here we found that the inclusion of this CSB interaction gives rise to push up the $^7\text{He}$ energy by $0.20 \text{ MeV}$, but it pushes down the $^7\Lambda\text{Be}$ energy by $0.20 \text{ MeV}$. In $^7\!\!\Lambda\text{Li}^*$, the level energies remain unchanged by adding the CSB interaction because of cancellation between contribution of valence proton and neutron on $\alpha$. Comparing the calculated value of $B_{\Lambda}(^7\Lambda\text{Be})$ with the emulsion data, it seems that the CSB interaction makes the agreement worse. In the case of $^7\Lambda\text{Be}$, however, there remains a problem of treating the unbound $^6\text{He}$ core within our framework. The CSB effect is expected to appear more clearly in the coming data of $^7\Lambda\text{He}$, whose core nucleus $^6\text{He}$ is a bound system. Next, we have tried to explain the binding energy difference of $T=1/2$ iso-doublet $A=8$ hypernuclei ($^7\!\!\Lambda\text{Li}$, $^7\Lambda\text{Be}$), adopting the phenomenological three-body models of $\alpha + t + \Lambda$ and $\alpha + ^3\text{He} + \Lambda$, respectively. The energy difference between $^7\!\!\Lambda\text{Li}$ and $^7\Lambda\text{Be}$, obtained in emulsion, cannot be reproduced accurately with use of our CSB interaction. Thus, our analyses for $p$-shell hypernuclei demonstrate that the CSB interaction determined in the $^7\text{H}$ and $^7\Lambda\text{He}$ doublet is not necessarily consistent with the experimental $B_{\Lambda}$ values of $^7\Lambda\text{Be}$, $^7\!\!\Lambda\text{Li}$ and $^7\Lambda\text{Be}$ in emulsion.

(5) As a trial, we have introduced the odd-state component of the CSB interaction, which is of a longer range than the even-state one. In order to reproduce the experimental data of $^7\!\!\Lambda\text{Li}$ and $^7\Lambda\text{Be}$, it is found to be necessary that the sign of the odd-state part is opposite with respect to that of the even part. It is likely that such an odd-state CSB interaction plays some role in the above $A=7$ four-body systems.

It is known that the CSB are generated essentially by the mass difference within the $\Sigma$-multiplet mixed, and the $\Lambda - \Sigma^0$ mixing in the meson-theoretical model. Thus, in order to get a firm conclusion on this matter, it is necessary to perform four-body calculation of $A=4 \Lambda$ hypernuclei and $A=7 \Lambda$ hypernuclei taking $NNNN\Lambda$ and $NNN\Sigma$ and $\alpha NNN$ and $\alpha \Sigma NN$, respectively. These types of calculation are in progress.

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