Radiative energy shifts of accelerated atoms

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Abstract

We consider the influence of acceleration on the radiative energy shifts of atoms in Minkowski space. We study a two-level atom coupled to a scalar quantum field. Using a Heisenberg picture approach, we are able to separate the contributions of vacuum fluctuations and radiation reaction to the Lamb shift of the two-level atom. The resulting energy shifts for the special case of a uniformly accelerated atom are then compared with those of an atom at rest.

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1 Introduction

One of the most remarkable effects predicted by quantum field theory is the Lamb shift, the shift of the energy levels of an atom which is caused by the coupling to the quantum vacuum. It is known that this level shift can be modified by external influences like a cavity [1], for example. Its presence alters the mode structure of the vacuum and leads to a Lamb shift which is different from its free-space value.

In this paper, we study the effect of acceleration on radiative energy shifts. It may not seem obvious at first sight why acceleration should lead to a modification of the Lamb shift. To see this, one has to combine results from different subfields of physics. First we note that for a uniformly accelerated observer, the Minkowski vacuum appears as a thermal heat bath of so-called “Rindler particles”. This is usually interpreted as a consequence of the non-equivalence of the particle concept in inertial and accelerated frames [2, 3, 4]. The second ingredient we need is the fact that the presence of photons leads to additional energy shifts for atomic systems. This effect is called AC Stark shift or light shift [5] and is connected with the virtual absorption and emission of real photons. In particular, a thermal photon field causes such an effect [6, 7, 8]. Consequently, taking these results together, we can gain a heuristic insight why the Lamb shift of an atom is

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modified if the atom is uniformly accelerating. A corresponding effect is to be expected for other types of acceleration.

The first aim of this paper is the calculation of radiative energy shifts. It can be carried out in an elegant manner using the formalism of Dalibard, Dupont-Roc, and Cohen-Tannoudji (DDC) \[9, 10\]. This approach has the advantage that it allows also to separate the contributions of vacuum fluctuations and radiation reaction to the energy shifts. The independent treatment of these two effects has a tradition in Heisenberg picture quantum electrodynamics \[11, 12, 13, 14, 15\] and beyond \[16\]. In a previous paper \[17\], we have studied the influences of vacuum fluctuations and radiation reaction on the spontaneous transitions of a uniformly accelerated atom moving through the Minkowski vacuum It leads to a modified value of the Einstein A coefficient for spontaneous emission. In addition we have shown how the lack of a balance between vacuum fluctuations and radiation reaction causes a spontaneous excitation from the ground state. This gives an interpretation of the physics underlying the Unruh effect. For the radiative shift of atomic levels, the separate discussion of vacuum fluctuations and radiation reaction may also be interesting from a conceptual point of view, since in heuristic pictures the Lamb shift has been often associated with the notion of vacuum fluctuations alone \[18\]. This discussion is the second aim of this paper.

The fully realistic calculation of the influence of acceleration on the Lamb shift would require to deal with a multilevel atom coupled to the electromagnetic field. This will not be done in the present paper. To keep the discussion as clear and transparent as possible, we will restrict to the simplest nontrivial example, a two-level atom in interaction with a massless scalar quantum field. However, as we will see, most of the essential features of the full problem are also present in the simple model. The structure of the results can be seen clearly. Furthermore, we will discuss only the nonrelativistic contribution to the energy shift, i.e. we neglect the effects which are due to the quantum nature of the electron field. However, it is well known that the nonrelativistic part gives the dominant contribution to the Lamb shift. Since we are only interested in the structure of the results, we will concentrate on this part of the problem.

Our treatment applies the formalism of DDC to the case of a two-level atom coupled to a scalar quantum field and generalizes it to an arbitrary stationary trajectory of the atom. We will proceed as follows: We consider the time evolution of an arbitrary atomic observable \( G \) as given by the Heisenberg equations of motion. Since we are only interested in atom variables, we trace out the field degrees of freedom in the part of the Heisenberg equations that is due to the coupling with the field. It is then possible to identify in the resulting expressions unambiguously the part that acts as an effective Hamiltonian with respect to the time evolution of \( G \). The expectation value of this operator in an atom state \( |b\rangle \) gives the radiative energy shift of this level.

We must emphasize that the modification of radiative level shifts by acceleration which is considered here must carefully be distinguished from the direct influence of acceleration or curvature on the energy levels of an atom \[14, 20\]. These corrections are obtained in the simplest case by the inclusion of a term \( amx \) into the Dirac equation of the atom. They are assumed to be already included in the otherwise unperturbed energies \( \pm \frac{1}{2} \omega_0 \) of the atom. In contrast to this, the effects considered in this paper are true radiative corrections caused by the interaction of the atom with the quantum field.

The organization of the paper is the following: In Sec. 2, we define our model and set up
the Heisenberg equations of motion, which are solved formally. We apply the formalism of DDC to our model in Sec. 3 and generalize it to an atom moving on an arbitrary stationary trajectory through the Minkowski vacuum. In Sec. 4, we calculate the level shift for a two-level atom at rest. We find that for the symmetric operator ordering adopted here, the only contribution to relative energy shifts comes from vacuum fluctuations, while the effect of radiation reaction is the same for both levels. The result obtained for the scalar theory are then compared to the standard results for the electromagnetic field and a similar structure is found. Sec. 5 deals with a uniformly accelerated atom. Because the analysis becomes more involved in this case, we use some methods from quantum field theory in accelerated frames, which are discussed in the Appendix. As a result, we find that the contribution of vacuum fluctuations to the level shift is altered by the appearance of a thermal term with the Unruh temperature \( T = \hbar a/(2\pi c k) \). The contribution of radiation reaction is the same as for an atom at rest and does not contribute to relative shifts. Finally, we point out the similarity of the results to those obtained for the Lamb shift in a thermal heat bath [6, 7, 8].

2 Two-level atom interacting with a massless scalar quantum field

To investigate how the radiative energy shifts of atoms are modified by acceleration, we choose the simplest nontrivial example: a two-level atom in interaction with a real massless scalar field. We consider an atom on a timelike Killing trajectory \( x(\tau) = (t(\tau), \vec{x}(\tau)) \), which is parametrized by the proper time \( \tau \). It will be called stationary trajectory [21]. The important consequence of the stationarity is that the level spacing \( \omega_0 \) of the two states \( |-\rangle \) and \( |+\rangle \) of the atom does not depend on \( \tau \). The zero of energy is chosen so that the energies of the two stationary states are \( -\frac{1}{2}\omega_0 \) and \( +\frac{1}{2}\omega_0 \). As mentioned in the Introduction, a possible constant modification of the energy level spacing which is directly caused by the acceleration is assumed to be already included. The free Hamiltonian of the two-level atom which generates the atom’s time evolution with respect to the proper time \( \tau \) is given by

\[
H_A(\tau) = \omega_0 R_3(\tau)
\]

where we have written \( R_3 = \frac{1}{2}|+\rangle\langle+| - \frac{1}{2}|-\rangle\langle-| \), following Dicke [22].

The free Hamiltonian of the quantum field is

\[
\tilde{H}_F(t) = \int d^3k \omega_\vec{k} a_{\vec{k}}^\dagger a_{\vec{k}}.
\]

where \( a_{\vec{k}}^\dagger, a_{\vec{k}} \) are creation and annihilation operators ‘photons’ with momentum \( \vec{k} \). The Hamiltonian (3) governs the time evolution of the quantum field with respect to the time variable \( t \) of the inertial frame. However, to derive the Heisenberg equations of motion of the coupled system, we have to choose a common time variable. It turns out that it is most reasonable to refer generally to the atom’s proper time \( \tau \). The free Hamiltonian of the field with respect to \( \tau \) can be obtained by a simple change of the time variable in the Heisenberg equations:

\[
H_F(\tau) = \int d^3k \omega_\vec{k} a_{\vec{k}}^\dagger a_{\vec{k}} \frac{dt}{d\tau}.
\]
We couple the atom and the quantum field by the interaction Hamiltonian

\[ H_I = \mu R_2(\tau)\phi(x(\tau)). \]  

(4)

which is a scalar model of the electric dipole interaction. Here we have introduced \( R_2 = \frac{1}{2}i(R_- - R_+) \), where \( R_+ = |+\rangle\langle -| \) and \( R_- = |-\rangle\langle +| \) are raising and lowering operators for the atom. \( \mu \) is a small coupling constant. The field operator in (4) is evaluated along the world line \( x(\tau) \) of the atom.

In the solutions of the Heisenberg equations of the atom and field variables, two physically different contributions can be distinguished: (1) the free part, which is the part of the solution that goes back to the free Hamiltonian (1) and (3) and which is present even in the absence of the interaction, (2) the source part which is caused by the coupling between atom and field and represents their mutual influence:

\[ R_{\pm}(\tau) = R^f_{\pm}(\tau) + R^s_{\pm}(\tau), \]  

(5)

\[ R_3(\tau) = R^f_3(\tau) + R^s_3(\tau), \]  

(6)

\[ \phi(x(\tau)) = \phi^f(x(\tau)) + \phi^s(x(\tau)). \]  

(7)

Their explicit form has been given in [17].

3 Radiative energy shifts: The contributions of vacuum fluctuations and radiation reaction

To determine the Lamb shift of an accelerated atom, we use the physically appealing formalism of Dalibard, Dupont-Roc, and Cohen-Tannoudji (DDC) [9, 10]. We will generalize it for calculating level shifts to an atom on an arbitrary stationary world line \( x(\tau) \).

One attractive feature of the formalism is that it allows a separate discussion of the two physical mechanisms which both contribute to radiative energy shifts: the contributions of vacuum fluctuations and of radiation reaction. Let us first discuss how these two parts can be separated (for a more detailed discussion see [17]): Because of the coupling (4), the field operator \( \phi \) will appear in the Heisenberg equation of motion of an arbitrary atomic variable \( G \). According to (4), \( \phi \) can be split into its free and source part. The following physical mechanisms can be connected with these two contributions: (1) the part of the Heisenberg equation which contains \( \phi^f \) represents the change in \( G \) due to vacuum fluctuations. It is caused by the fluctuations of the field which are present even in the vacuum. (2) The term containing \( \phi^s \) represents the influence of the atom on the field. This part of the field in turn reacts back on the atom. This mechanism is called radiation reaction.

Explicitly, the contributions of vacuum fluctuations and radiation reaction to \( dG/d\tau \) are:

\[ \left( \frac{dG(\tau)}{d\tau} \right)_{vf} = \frac{1}{2}i\mu \left( \phi^f(x(\tau))[R_2(\tau), G(\tau)] + [R_2(\tau), G(\tau)]\phi^f(x(\tau)) \right), \]  

(8)

\[ \left( \frac{dG(\tau)}{d\tau} \right)_{rr} = \frac{1}{2}i\mu \left( \phi^s(x(\tau))[R_2(\tau), G(\tau)] + [R_2(\tau), G(\tau)]\phi^s(x(\tau)) \right), \]  

(9)

where we followed DDC [9] in choosing a symmetric ordering between atom and field operators. Because we are interested only in observables of the atom, we take the average
of (8) and (9) in the vacuum state of the quantum field. In a perturbative treatment, we take into account only terms up to second order in $\mu$. Proceeding in a similar manner as in Refs. [10, 17], it is then possible to identify in Eqs. (8) and (9) an effective Hamiltonian for atomic observables that governs the time evolution with respect to $\tau$ in addition to the free Hamiltonian (1). The averaged equations (8) and (9) can be written

$$\langle 0 | \left( \frac{dG(\tau)}{d\tau} \right)_{\nu f,rr} | 0 \rangle = i[H_{\nu f,rr}^{\text{eff}}(\tau), G(\tau)] + \text{non-Hamiltonian terms.} \tag{10}$$

where in order $\mu^2$

$$H_{\nu f}^{\text{eff}}(\tau) = \frac{1}{2} i \mu^2 \int_{\tau_0}^{\tau} d\tau' C^F(x(\tau), x(\tau')) [R_2^f(\tau'), R_2^f(\tau)] \tag{11}$$

$$H_{rr}^{\text{eff}}(\tau) = -\frac{1}{2} i \mu^2 \int_{\tau_0}^{\tau} d\tau' \chi^F(x(\tau), x(\tau')) \{ R_2^f(\tau'), R_2^f(\tau) \} \tag{12}$$

These effective Hamiltonians depend on the field variables only through the simple statistical functions

$$C^F(x(\tau), x(\tau')) = \frac{1}{2} \langle 0 | \{ \phi^f(x(\tau)), \phi^f(x(\tau')) \} | 0 \rangle, \tag{13}$$

$$\chi^F(x(\tau), x(\tau')) = \frac{1}{2} \langle 0 | [\phi^f(x(\tau)), \phi^f(x(\tau'))] | 0 \rangle \tag{14}$$

which are called the symmetric correlation function and the linear susceptibility of the field. The non-Hamiltonian terms in (10) describe the effects of relaxation.

The expectation values of the effective Hamiltonians (11) and (12) in an atomic state $| b \rangle$ represent the energy shift of the atomic level $| b \rangle$ caused by the coupling to the quantum field. The total shift contains the contribution of vacuum fluctuations as well as the contribution of radiation reaction. Taking the expectation values of (11) and (12), we obtain the radiative energy shifts of the level $| b \rangle$ due to vacuum fluctuations

$$\langle \delta E_b \rangle_{\nu f} = -i \mu^2 \int_{\tau_0}^{\tau} d\tau' C^F(x(\tau), x(\tau')) \chi^A_b(\tau, \tau'), \tag{15}$$

and due to radiation reaction

$$\langle \delta E_b \rangle_{rr} = -i \mu^2 \int_{\tau_0}^{\tau} d\tau' \chi^F(x(\tau), x(\tau')) C^A_b(\tau, \tau'), \tag{16}$$

where we have defined the symmetric correlation function

$$C^A_b(\tau, \tau') = \frac{1}{2} \langle b | \{ R_2^f(\tau), R_2^f(\tau') \} | b \rangle \tag{17}$$

and the linear susceptibility of the atom

$$\chi^A_b(\tau, \tau') = \frac{1}{2} \langle b | [R_2^f(\tau), R_2^f(\tau')] | b \rangle \tag{18}$$

As a result, we note that Eqs. (15) and (16) for the radiative energy shifts for atoms that move on an arbitrary stationary trajectory differs from the results for atoms at rest [8, 10].
only in that the statistical functions of the field (13) and (14) are evaluated along the world line of the atom which may now be an accelerated one. The statistical functions of the atom do not depend on the trajectory $x(\tau)$.

Below we will need the explicit forms of the different statistical functions. Those referring to the atom can be written

$$C^A_b(\tau, \tau') = \frac{1}{2} \sum_d \langle |b| R^f_2(0) |d\rangle |^2 \left( e^{i\omega_{bd}(\tau-\tau')} + e^{-i\omega_{bd}(\tau-\tau')} \right),$$  \hspace{1cm} (19)$$

$$\chi^A_b(\tau, \tau') = \frac{1}{2} \sum_d \langle |b| R^f_2(0) |d\rangle |^2 \left( e^{i\omega_{bd}(\tau-\tau')} - e^{-i\omega_{bd}(\tau-\tau')} \right),$$  \hspace{1cm} (20)$$

where $\omega_{bd} = \omega_b - \omega_d$ and the sum extends over a complete set of atomic states.

4 Radiative energy shift for an atom at rest

Let us first reproduce the standard result for the Lamb shift for an atom at rest in the scalar theory. It can be compared afterwards with the corresponding result for a uniformly accelerated atom. The statistical functions of the field for the trajectory

$$t(\tau) = \tau, \quad \vec{x}(\tau) = 0$$  \hspace{1cm} (21)$$

can be easily evaluated. We obtain

$$C^F(x(\tau), x(\tau')) = \frac{1}{8\pi^2} \int d\omega \omega \left( e^{-i\omega(\tau-\tau')} + e^{i\omega(\tau-\tau')} \right),$$  \hspace{1cm} (22)$$

$$\chi^F(x(\tau), x(\tau')) = \frac{1}{8\pi^2} \int d\omega \omega \left( e^{-i\omega(\tau-\tau')} - e^{i\omega(\tau-\tau')} \right).$$  \hspace{1cm} (23)$$

The contribution of vacuum fluctuations to the radiative shift of level $|b\rangle$ can be calculated from (15)

$$(\delta E_b)_{vf} = \mu^2 \frac{\sum_d \langle |b| R^f_2(0) |d\rangle |^2 \int_0^\infty d\omega \omega \left( \frac{P}{\omega + \omega_{bd}} - \frac{P}{\omega - \omega_{bd}} \right) ,}$$  \hspace{1cm} (24)$$

where $P$ denotes the principle value. The integral is logarithmically divergent, as expected for a nonrelativistic calculation of radiative shifts [24]. As is well known, the introduction of a cutoff is therefore necessary. The summation over $d$ displays the role of virtual transitions to other levels.

The relative energy shifts of the two levels due to vacuum fluctuations can be calculated by evaluating the $d$ summation for $|b\rangle = |\pm\rangle$. The resulting expression is

$$\Delta_{vf} = (\delta E_+)_{vf} - (\delta E_-)_{vf}$$

$$= \mu^2 \frac{\int_0^\infty d\omega \omega \left( \frac{P}{\omega + \omega_{bd}} - \frac{P}{\omega - \omega_{bd}} \right) ,}$$  \hspace{1cm} (25)$$

For the contribution of radiation reaction we obtain

$$(\delta E_b)_{rr} = -\mu^2 \frac{\sum_d \langle |b| R^f_2(0) |d\rangle |^2 \int_0^\infty d\omega \omega \left( \frac{P}{\omega + \omega_{bd}} + \frac{P}{\omega - \omega_{bd}} \right) ,}$$  \hspace{1cm} (27)$$
This expression diverges linearly. However, as can be seen by explicitly evaluating the sum, radiation reaction does not give a contribution to the relative shift of the two levels:

$$\Delta_{rr} = (\delta E_+ + \delta E_-) - (\delta E_+ - \delta E_-) = 0. \quad (28)$$

Thus the Lamb shift as the relative radiative energy shift of the two-level atom at rest is caused entirely by vacuum fluctuations:

$$\Delta_0 = \Delta_{\text{rad}} + \Delta_{\text{rr}} = \frac{\mu^2}{16\pi^2} \int_0^\infty d\omega \omega \left( \frac{\mathcal{P}}{\omega + \omega_{bd}} - \frac{\mathcal{P}}{\omega - \omega_{bd}} \right). \quad (29)$$

This expression agrees structurally with the standard result for the Lamb shift of a two-level atom [11, 12, 13, 14, 15]. The modifications are caused by the differences between the electromagnetic and the scalar theory.

We mention that Welton’s picture of radiative shifts [18], who tried to interpret the Lamb shift only in terms of vacuum fluctuations conforms with the fact that the energy shift (29) is caused solely by vacuum fluctuations. This feature is a peculiarity of the simple model of a two-level atom, however. For a real multilevel atom, the contribution of radiation reaction will be different for different levels, and a mass renormalization will become necessary.

5 Radiative energy shifts for a uniformly accelerated atom

Let us now consider the case of a uniformly accelerated two-level atom. It moves on the trajectory

$$t(\tau) = \frac{1}{a} \sinh a\tau, \quad z(\tau) = \frac{1}{a} \cosh a\tau, \quad x(\tau) = y(\tau) = 0, \quad (30)$$

where $a$ is the proper acceleration. The calculation of the statistical functions of the field turns out to be much more complicated for an accelerated atom than for an atom at rest. It is most convenient to employ methods from the quantum field theory in accelerated frames. In order to keep the discussion transparent, we have put the calculation into the Appendix and simply quote the results here:

$$C^F(x(\tau), x'(\tau')) = \frac{1}{8\pi^2} \int_0^\infty d\omega' \omega' \coth \left( \frac{\pi\omega'}{a} \right) \left( e^{-i\omega'(\tau-\tau')} + e^{i\omega'(\tau-\tau')} \right) \quad (31)$$

$$\chi^F(x(\tau), x'(\tau')) = \frac{1}{8\pi^2} \int_0^\infty d\omega' \omega' \left( e^{-i\omega'(\tau-\tau')} - e^{i\omega'(\tau-\tau')} \right) \quad (32)$$

We see that the expression (32) for the linear susceptibility of the field is formally identical for an accelerated atom and an atom at rest. However, with regard to the cutoff prescription to be given below, it is important to note that $\omega'$ in (31) and (32) denotes the energy in the accelerated frame, i.e. as measured by an observer on the trajectory (30).
The remaining calculation is straightforward. By substituting the statistical functions (19), (20), (31), and (32) into the general formulas (15) and (16) for the level shifts, we find

\[(\delta E_b)_{vf} = \frac{\mu^2}{8\pi^2} \sum_d |\langle b | R_2^f(0) | d \rangle|^2 \int_0^\infty d\omega' \omega' \coth \left( \frac{\pi \omega'}{a} \right) \text{Im} \left\{ \int_0^\infty du \left( e^{i(\omega'+\omega_d)u} - e^{i(\omega'-\omega_d)u} \right) \right\}, \tag{33} \]

\[(\delta E_b)_{rr} = -\frac{\mu^2}{8\pi^2} \sum_d |\langle b | R_2^f(0) | d \rangle|^2 \int_0^\infty d\omega' \omega' \text{Im} \left\{ \int_0^\infty du \left( e^{i(\omega'+\omega_d)u} + e^{i(\omega'-\omega_d)u} \right) \right\}. \tag{34} \]

The result for the radiative energy shift for a uniformly accelerated atom can now be obtained by evaluating the integrals. The contribution of vacuum fluctuations is

\[(\delta E_b)_{vf} = \frac{\mu^2}{8\pi^2} \sum_d |\langle b | R_2^f(0) | d \rangle|^2 \int_0^\infty d\omega' \omega' \left( 1 + \frac{2}{e^{2\pi\omega'/a} - 1} \right) \left( \frac{\mathcal{P}}{\omega' + \omega_0} - \frac{\mathcal{P}}{\omega' - \omega_0} \right). \tag{35} \]

Comparing this formula to Eq. (24) for an atom at rest, we note that the acceleration-caused correction is additive and contains a characteristic thermal term with the Unruh temperature \( T = \frac{\hbar a}{2\pi kc} \). On the other hand, the contribution of radiation reaction,

\[(\delta E_b)_{rr} = -\frac{\mu^2}{8\pi^2} \sum_d |\langle b | R_2^f(0) | d \rangle|^2 \int_0^\infty d\omega' \omega' \left( \frac{\mathcal{P}}{\omega' + \omega_0} + \frac{\mathcal{P}}{\omega' - \omega_0} \right), \tag{36} \]

is exactly the same as for an atom at rest. The uniform acceleration has no effect on the shift caused by radiation reaction and we find again no relative energy shift: \( \Delta_{rr} = 0 \).

Thus the Lamb shift \( \Delta \) can be obtained from the contribution of vacuum fluctuations (35) by evaluating the summation over \( d \) for each of the two levels separately:

\[
\Delta = \Delta_{vf} = (\delta E_+)_{vf} - (\delta E_-)_{vf} = \frac{\mu^2}{16\pi^2} \int_0^\infty d\omega' \omega' \left[ 1 + \frac{2}{e^{2\pi\omega'/a} - 1} \right] \left( \frac{\mathcal{P}}{\omega' + \omega_0} - \frac{\mathcal{P}}{\omega' - \omega_0} \right). \tag{37} \]

From the two terms in square brackets, we can distinguish two contributions in (37). The first one has the same functional form as \( \Delta_0 \) of (24) for an atom at rest. Because it is logarithmically divergent, the introduction of a cutoff is necessary. The only sensible way to do this is to impose the cutoff frequency in the rest frame of the atom. Since \( \omega' \) in (37) is the frequency in the accelerated system, this can be done quite naturally in the present formalism. The second contribution in (37) represents the modification of the Lamb shift caused by the acceleration and contains the thermal term. Inspection of the integral shows that this correction is finite. Equation (37) can thus be written

\[
\Delta = \Delta_0 + \frac{\omega_0 \mu^2}{16\pi^2} G \left( \frac{a}{2\pi\omega_0} \right), \tag{38} \]
where $\Delta_0$ is the Lamb shift (29) for $a = 0$ and $G(u)$ is defined by

$$G(u) = u \int_0^\infty dx \frac{x}{e^x - 1} \left( \frac{\mathcal{P}}{x + u^{-1}} - \frac{\mathcal{P}}{x - u^{-1}} \right).$$

The evaluation of the integral must be done numerically. The result is shown in Fig. 1. In the limit of small $u = a/(2\pi\omega_0)$, (38) can be approximated by

$$\Delta = \Delta_0 + \frac{a^2 \mu^2}{192 \pi^2 \omega_0}.$$  

The dependence on $a$ can be read off from Fig. 1. To estimate the order of magnitude of the acceleration needed for an appreciable effect, we demand $(\Delta - \Delta_0)/\omega_0 = \mu^2/(16\pi)$ and therefore $G(u) = 1$. This amounts to $a \approx c\omega_0$, which gives $a \approx 10^{24} \text{m/s}^2$ for an 1 eV transition.

Finally, we note the structural similarity of the equations (37) and (38) to the corresponding expressions obtained for the Lamb shift in a thermal heat bath [6, 7, 8]. They have in common the appearance of the factor in square brackets under the integral in (37) with $T = a/(2\pi)$. A heuristic connection between the two physical situations has already been given in the Introduction. The differences are due to the properties of the scalar theory and the fact that we considered a two-level atom instead of a real atom.

6 Conclusion

In this paper, we have discussed two main points. First, we studied the influence of acceleration on the radiative energy shift of atoms. The formalism can be applied to an atom on an arbitrary stationary trajectory. We considered here the special case of uniformly
accelerated motion. The resulting expression for the Lamb shift of a uniformly accelerated two-level atom is given in Eq. (37) and (38). Comparison with the corresponding formula (29) for an atom at rest shows that the correction consists in the appearance of the factor in square brackets, which contains the thermal term. This modification is structurally the same as for the Lamb shift in a thermal heat bath.

The second main goal of the paper has been the identification of the two physical mechanisms responsible for the radiative shifts. Using the formalism of DDC, we were able to discuss the contributions of vacuum fluctuations and radiation reaction separately. The effect of radiation reaction is the same for a uniformly accelerated atom as for an atom at rest. It does not affect relative energy shifts. It is interesting to note that also in the case of other radiative phenomena like the Unruh effect and spontaneous emission, radiation reaction is not influenced by acceleration [17]. The contribution of vacuum fluctuation, however, is modified by the thermal terms and gives the total effect.

Appendix: Statistical functions of the field for a uniformly accelerated atom

The statistical functions of the field for an accelerated atom can be most easily evaluated by using the well-known formalism of quantum field theory in accelerated frames [2]. For the details of the theory, we refer to the literature [4, 25, 26].

The two Rindler wedges \( R^+ \) (\( z > |t| \)) and \( R^- \) (\( -z > |t| \)) can be covered by Rindler coordinates \((\eta, x', y', \xi)\):

\[
t = \frac{\epsilon}{a} e^{\alpha \xi} \sinh a\eta, \quad z = \frac{\epsilon}{a} e^{\alpha \xi} \cosh a\eta, \quad x' = x, \quad y' = y,
\]

where \( \epsilon = \pm \) refers to the wedge \( R^\pm \). For \( \xi = x' = y' = 0 \), (41) reduces to (30), describing a uniformly accelerated observer with acceleration \( a \) on a trajectory with proper time \( \eta \).

The metric of Minkowski spacetime takes in terms of the coordinates (41) the following form

\[
d s^2 = e^{2\alpha \xi} (d\eta^2 - d\xi^2) - dx'^2 - dy'^2 \tag{42}
\]

and the wave equation becomes

\[
\left[ e^{-2\alpha \xi} \left( \partial_\eta^2 - \partial_\xi^2 \right) - \partial_{x'}^2 - \partial_{y'}^2 \right] \phi(x) = 0. \tag{43}
\]

It can be shown that the normalized mode solutions which have positive frequency with respect to the time variable \( \eta \) are

\[
\lambda^\epsilon \sim k_{\perp} = \frac{1}{2\pi^2} \sinh \frac{1}{2} \left( \frac{\pi \omega'}{a} \right) K_{i\omega'} \left( \frac{1}{a} k_{\perp} e^{\alpha \xi} \right) \exp \left( i\epsilon \vec{r} \right) \tag{44}
\]

in the wedge \( R^\epsilon \). They are identically zero in the opposite wedge. In (44), \( \omega' \) denotes the energy with respect to the time variable \( \eta \), \( \vec{r} = (x', y') \), and \( k_{\perp}^2 = k_{x'}^2 + k_{y'}^2 \). Quantization with respect to the mode functions (44) leads to the concept of Rindler particles [2]. However, Unruh [3] observed that the linear combinations

\[
v^\epsilon \sim \lambda^\epsilon = (1 - e^{-2\pi \omega'/a})^{-\frac{1}{2}} \left( \lambda^\epsilon + e^{-\pi \omega'/a} \lambda^\epsilon \right) \tag{45}
\]
have positive frequency with respect to the inertial time \( t \). Since they form a complete set in the wedges \( R^\pm \), it is possible to quantize the field using the Unruh modes (45).

The expansion of the field operators reads

\[
\phi(x) = \sum_\epsilon \int d\omega' \int d^2 k'_\perp \left( v^\epsilon_{\omega' k'_\perp}(x) b^\epsilon_{\omega' k'_\perp} + v^\epsilon_{\omega' k'_\perp}(x) b^\epsilon_{\omega' k'_\perp} \right)
\]

with creation and annihilation operators \( b^\epsilon_{\omega' k'_\perp}, b^\epsilon_{\omega' k'_\perp} \) which obey the usual commutation relations. The operators \( b^\epsilon_{\omega' k'_\perp} \) annihilate the Minkowski vacuum: \( b^\epsilon_{\omega' k'_\perp} |0\rangle = 0 \).

It is now possible to calculate the statistical functions of the field \( C^F(x(\tau), x(\tau')) \) and \( \chi^F(x(\tau), x(\tau')) \) as defined by (13) and (14). To do this, we use the field expansion (46). We stress, however, that no special physical status is admitted to the Unruh modes (45). They serve only as a calculational tool. Keeping this in mind, we consider the function

\[
\langle 0 | \phi(x(\tau)) \phi(x(\tau')) | 0 \rangle = \sum_\epsilon \int d\omega' \int d^2 k'_\perp v^\epsilon_{\omega' k'_\perp}(x(\tau)) v^\epsilon_{\omega' k'_\perp}(x(\tau'))
\]

and evaluate the expression for the trajectory \( \xi = x' = y' = 0, \tau = \eta' \). We find

\[
\langle 0 | \phi(x(\tau)) \phi(x(\tau')) | 0 \rangle = \frac{1}{8 \pi^4 a} \int d\omega' \int d^2 k'_\perp K^2_{i \omega'}
\]

\[
\times \left(\frac{1}{a} k'_{\perp}\right) \left( e^{i \omega'/a} e^{-i \omega'(\tau-\tau')} + e^{-i \omega'/a} e^{i \omega'(\tau-\tau')} \right).
\]

The statistical functions (13) and (14) can now be written

\[
C^F(x(\tau), x(\tau')) = \frac{1}{8 \pi^4 a} \int d^2 k'_\perp \int d\omega' K^2_{i \omega'}
\]

\[
\times \left(\frac{1}{a} k'_{\perp}\right) \cosh \left(\frac{\pi \omega'}{a}\right) \left( e^{-i \omega'(\tau-\tau')} + e^{i \omega'(\tau-\tau')} \right),
\]

\[
\chi^F(x(\tau), x(\tau')) = \frac{1}{8 \pi^4 a} \int d^2 k'_\perp \int d\omega' K^2_{i \omega'}
\]

\[
\times \left(\frac{1}{a} k'_{\perp}\right) \sinh \left(\frac{\pi \omega'}{a}\right) \left( e^{-i \omega'(\tau-\tau')} - e^{i \omega'(\tau-\tau')} \right).
\]

After the evaluation of the integral

\[
\int d^2 k'_\perp K^2_{i \omega'} \left(\frac{1}{a} k'_{\perp}\right) = \frac{a \pi^2 \omega'}{\sinh(\pi \omega'/a)},
\]

where we used Eq. (6.521.3) of Ref. [27], we obtain the expressions (31) and (32) for the statistical functions of the field for the uniformly accelerated atom.

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