A step toward the calibration of the RGB Tip as a Standard Candle

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ABSTRACT

The absolute I magnitude of the Tip of the Red Giant Branch ($M_{I}^{TRGB}$) is one of the most promising Standard Candles actually used in astrophysics as a fundamental pillar for the Cosmological Distance Scale. With the aim of improving the observational basis of its calibration, we have obtained an accurate estimate of the ($M_{I}^{TRGB}$) for the globular cluster ω Centauri, based (a) on the largest photometric database ever assembled for a globular, by Pancino et al. (2000), and (b) on a direct distance estimate for ω Centauri, recently obtained by Thompson et al. (2001) from a detached eclipsing binary. The derived value $M_{I}^{TRGB} = -4.04 \pm 0.12$ provides, at present, the most accurate empirical zero-point for the calibration of the $M_{I}^{TRGB} - [\text{Fe/H}]$ relation, at $[\text{Fe/H}] \sim -1.7$. We also derived a new empirical $M_{I}^{TRGB} - [\text{Fe/H}]$ relation, based on the large IR dataset of red giants in Galactic Globular Clusters recently presented by Ferraro et al. (2000). This database (extending up to $[\text{Fe/H}] = -0.2$) covers a more appropriate metallicity range, for extragalactic applications, than previous empirical calibrations (limited to $[\text{Fe/H}] \leq -0.7$). The proposed relation is in excellent agreement with the newly determined zero-point.¹

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1. Introduction

While the use of the luminosity of the Tip of the Red Giant Branch (TRGB) as a standard candle dates back to 1930 (see Madore & Freedman (1998) and references therein), the development of the method as a safe and viable technique is relatively recent (Lee, Freedman & Madore (1993), hereafter L93). In a few years it has become a widely adopted technique, finding fruitful application also within the HST Key project on the Extragalactic Distance Scale (see Ferrarese et al. (2000a,b), and references therein). The underlying physical processes are clearly understood and well rooted in the theory of stellar evolution (Madore & Freedman 1998). The method is particularly useful to estimate distances to those stellar systems that do not contain Cepheids, such as early type galaxies, and it can be applied to galaxies as far as $\sim 12$ Mpc with the current instrumentation. The key observable quantity is the magnitude of the bright end (the tip) of the Red Giant Branch (RGB), that corresponds to a sharp cut-off in the RGB Luminosity Function (LF), measured in the Cousin’s $I$ passband$^3$. In this passband the magnitude of the tip shows a very weak (if any) dependence on metallicity (Da Costa & Armandroff (1990), hereafter DA90). The feature can be identified by applying the Sobel’s filter, an edge-detection algorithm, to the LF of the upper RGB (see Sakai, Madore & Freedman (1996), hereafter S96, for a standard application$^4$). The possible biases have been well characterized and quantified by means of numerical simulations by Madore & Freedman (1995) (hereafter MF95).

In this context, we are performing an extensive study of the Red Giant population in Galactic Globular Clusters (GGC). In particular, we have derived an accurate calibration of the RGB photometric properties as a function of metallicity, both in the optical and in the IR (Ferraro et al. 1999, 2000). The final aim of this program is to use stellar populations in GGCs as calibrators for the TRGB method, taking advantage of the large samples that can be easily assembled with the new generation of array detectors and cameras. As a part of this general project, in this paper we present (i) a very robust empirical calibration of $M^\text{TRGB}_I$ at

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$^1$Based on data taken at the European Southern Observatory, Chile, as part of the ESO observing programmes 62.L-0354, 63.L-0349 and 65.L-0463.

$^3$Hereafter, when we will refer to the $I$ passband, we will always mean the Cousin’s $I$ passband, unless we specify otherwise.

$^4$From now on, we will refer to their method for the detection of the TRGB as to the standard analysis.
[Fe/H] \sim -1.7, derived by applying the standard analysis to a very large sample of RGB stars in the globular cluster \omega Centauri (NGC 5139), and based on a direct distance estimate recently obtained by Thompson et al. (2001) (hereafter T01) for a detached eclipsing binary in this cluster, and (ii) a new empirical calibration of the $M_I^{TRGB} - [Fe/H]$ relation based on a large, homogeneous IR database of RGB stars in Galactic Globular Clusters, recently published by Ferraro et al. (2000) (hereafter F00).

The aim of this paper is to provide a first, robust pillar for a well rooted empirical calibration of $M_I^{TRGB}$ as a standard candle, extending over a wide range in metallicity. This will constitute the basis for a secure application of the TRGB method as an extragalactic distance indicator.

2. A new $M_I^{TRGB} - [Fe/H]$ relation

The most widely adopted calibration of the absolute I magnitude of the TRGB ($M_I^{TRGB} = M_{bol}^{TRGB} - BC_I$) was derived by L93 from the following relations, which give the bolometric magnitude of the tip ($M_{bol}^{TRGB}$) as a function of metallicity:

$$M_{bol}^{TRGB} = -0.19[Fe/H] - 3.81$$

and the I bolometric correction ($BC_I$) as a function of the intrinsic color $(V - I)_0$:

$$BC_I = 0.881 - 0.243(V - I)_0$$

both obtained by DA90. Using accurate V and I photometry of a small number of RGB stars in eight selected globular clusters in the metallicity range $-2.2 \leq [Fe/H] \leq -0.7$, DA90 estimated the apparent I magnitude of the TRGB ($I^{TRGB}$) as the magnitude of the brightest and reddest giant in their samples. They converted $I^{TRGB}$ to absolute magnitudes by adopting the RR Lyrae distance scale $M_V(RRLy) = 0.17[Fe/H] + 0.82$, from Lee, De-Marque & Zinn (1990), and finally to $M_{bol}$ using Eq. 2. Note that, in this context, the most relevant observational fact presented by DA90 is that $M_I^{TRGB} = -4.0 \pm 0.1$, in the sampled range of metallicity.

The above calibrations suffer from two major sources of uncertainty:

\footnote{From 20 to 110 stars per cluster were observed by DA90 in the upper \sim 4 mag of each RGB.}
• The evolution of stars along the RGB becomes faster as their luminosity increases along the path toward the TRGB. Thus, most of the cluster light has to be observed to correctly sample the fastest evolutionary phases (Renzini & Fusi Pecci 1988). MF95 stated that acceptable detections of the TRGB can be obtained if more than 50 stars are sampled within 1 mag from the tip and that fine estimates can be obtained only sampling more than 100 stars in that range. The DA90 samples are much poorer than this, therefore their $I^{TRGB}$ estimates may be affected by the systematics associated with small number statistics.

• The RR Lyrae distance scale is quite uncertain. While there is now some agreement on the slope of the $M_V(RRLy) - [Fe/H]$ relation, the actual zero points is still hardly debated (see Cacciari (1999), for a recent review).

The most recent calibration of the $M_{bol}^{TRGB} - [Fe/H]$ relation has been obtained by F00, from homogeneous near-infrared photometry of a large sample of RGB stars in nine Galactic Globular Clusters. Several hundred giants were observed in the upper $\sim 4$ mag of each RGB. Moreover, the data-set presented by F00 extends to a much higher metallicity with respect to DA90, covering the range $-2.2 \leq [Fe/H] \leq -0.2$. The TRGB was identified by using the brightest non-variable RGB star in K band. F00 adopted the new distance scale presented by Ferraro et al. (1999) which, although still based on the Horizontal Branch, was obtained by using a new semi-empirical approach (see Ferraro et al. (1999) for details). F00 provided $M_{bol}^{TRGB}$ as a function of [Fe/H] in two metallicity scales: (i) the Carretta & Gratton (1997) scale and (ii) the global metallicity ([M/H]) scale, which takes into account also the $\alpha$–elements abundances. To obtain a more direct comparison with the calibration by L93, we used the data from F00 to derive $M_{bol}^{TRGB}$ as a function of [Fe/H] in the Zinn & West (1984) metallicity scale, and obtained:

$$M_{bol}^{TRGB} = -0.12[Fe/H] - 3.76. \quad (3)$$

The larger samples used by F00, the wider metallicity range covered and the minor impact of the extinction and bolometric corrections on infrared luminosities, suggest that eq. 3 has to be preferred to eq. 1. Note, however, that the final $M_{bol}^{TRGB} - [Fe/H]$ relations derived by the two independent calibrations of $M_{bol}^{TRGB}$ are in agreement within the uncertainties (see Sect. 4 and Fig. 4).

In order to derive the $M_{I}^{TRGB}$ from the new empirical calibration, we need to combine eq. 3 with eq. 2, following the relation $M_{I}^{TRGB} = M_{bol}^{TRGB} - BC_I$. The actual $BC_I$ depends implicitly on metallicity, since $BC_I$ depends on the $(V - I)$ color of the TRGB (eq. 2) which, in turn, depends on metallicity. To make such a dependence explicit, we plot in Fig.
\( (V - I)^{TRGB}_0 \) as a function of \([\text{Fe/H}]\) for the six clusters studied by DA90. The data are best fitted by a second order polynomial whose equation is also reported in Fig. 1. We adopt this relation to eliminate the dependency of \( M^{TRGB}_I \) on \((V - I)\) color and we finally obtain the following relation:

\[
M^{TRGB}_I = 0.14[Fe/H]^2 + 0.48[Fe/H] - 3.66
\]  

which provides the calibration of \( M^{TRGB}_I \) as a function of the metallicity only (in the Zinn & West (1984) scale).

Eq. (3) and (4) represent a substantial improvement with respect to previous work, since they are based on the largest IR RGB samples in GGCs available in the literature. Note that the F00 survey sampled a significative fraction of the cluster light (up to \( \sim 80\% \)). However even in this large data-set the number of RGB stars in the upper 1 mag bin from the TRGB is < 50 in all the cases, and the brightest star detection is possibly prone to low number statistics effects. This is due, in most cases, to intrinsic poorness of the cluster population, since the majority of GGCs simply do not contain a sufficient number of RGB stars to adequately sample the upper RGB. Thus extensive observations of the most massive GGCs are urged in order to properly calibrate the above relations. In the following sections we report our results on the most massive Galactic globular: \( \omega \) Centauri.

3. The determination of the TRGB luminosity in \( \omega \) Centauri

3.1. Why \( \omega \) Centauri?

\( \omega \) Centauri is a nearby and well studied cluster. It is the most luminous globular cluster in the Milky Way system, therefore even the fastest evolutionary phases are well populated and it is possible to observe a quite large sample of RGB stars, fulfilling the prescriptions of MF95. Here we adopt the huge photometric \((B,I)\) database presented by P00, consisting of more than 220,000 stars observed with the WFI camera at the 2.2 ESO-MPI telescope and extending over a \( \sim 34' \times 33' \) field roughly centered on the cluster. The absolute photometric calibration is accurate to within \( \pm 0.02 \) mag. The observed field extends over \( \sim 24 \) core radii (van Leeuwen et al. 2000), i.e. an area enclosing 90\% of the cluster light. Thus, virtually all the bright stars of \( \omega \) Centauri are included in the adopted database. Considering that \( \omega \) Centauri is the most luminous globular cluster of the whole Galaxy, this photometric database is the largest sample that can be obtained in the Galactic globular cluster system.

Moreover, \( \omega \) Centauri was the first globular in which a detached eclipsing binary sys-
Fig. 1.— \((V-I)_0^{TRGB}\) as a function of \([\text{Fe/H}]\) for the six clusters studied by DA90. All data are taken from DA90. The curve represents the best fits to the data, the corresponding equation is also reported. The \(rms\) scatter is 0.04 mag.

\[
(V-I)_0^{TRGB} = 0.581[\text{Fe/H}]^2 + 2.472[\text{Fe/H}] + 4.013
\]
tem, \textit{OGLEGC17}, was discovered (Kaluzny et al. 1996). This enabled T01 to obtain a
direct distance estimate, independent of any other distance scale. The distance of detached,
eclipsing double line spectroscopic binaries can in fact be obtained by coupling the physical
parameters of the system with the relations between color and surface brightness by Barnes
& Evans (1976). This method is basically \textit{geometrical}, since the distance is obtained by
comparison between the linear and angular size of the binary members (see Lacy (1977);
Kruszewski and Semeniuk (1999)). The final distance obtained by T01 is $d = 5385 \pm 300$
pc, which corresponds to $(m - M)_0 = 13.65 \pm 0.11$, if we assume their adopted extinction
value, $E(B - V) = 0.13 \pm 0.02$. Their result is in good agreement with previous estimates.
According to T01, the error bar on the distance modulus can be significantly reduced as
soon as better light and velocity curves will be obtained for \textit{OGLEGC17}. Thus, a significant
improvement of the quoted measure of the distance modulus has to be expected in the near
future. A further independent distance estimate based on a geometrical method could soon
be provided also by the comparison of the linear and the angular velocity dispersion from the
large database of proper motions by van Leeuwen et al. (2000), once supported by detailed
dynamical modeling.

In deriving $M_I^{TRGB}$ another fundamental ingredient can still be greatly refined and im-
proved in the near future: the \textit{reddening}. For instance, Mateo et al. (1995) report that the
$(V - I)_0$ color at minimum light of RRab variables is universal and independent of metallicity.
Thus, reliable $E(V - I)$ estimates can be obtained from the observed $(V - I)$ at minimum
light and large samples would greatly improve the accuracy. For \omega Centauri, in particular, a
$(V,I)$ survey of RR Lyrae can potentially provide a reddening estimate with an uncertainty
of $\pm 0.01$ mag or lower, since this populous cluster contains more than 70 RRab (Rey et al.
2000).

In conclusion, \omega Centauri seems to fit all the requirements to obtain an excellent cali-
bration of $M_I^{TRGB}$ and still leaves considerable room for future improvements.

A possible caveat may be associated with the wide metallicity distribution observed in
this cluster (see P00 and references therein). However, it has been shown that the largely
dominant population is metal poor [Norris, Freeman & Mighell (1996); Suntzeff & Kraft
(1996), P00]. The peak of such population occurs at $[Fe/H] \sim -1.7$ [Suntzeff & Kraft
(1996), P00] and we will adopt this as the characteristic metallicity of the cluster.
3.2. $M_I^{\text{T RGB}}$ in ω Centauri

Fig. 2 shows the $(I, B - I)$ Color Magnitude Diagram (CMD) of the brightest 3 mag in the RGB of ω Centauri: 1777 stars are reported in this diagram, most of them being RGB stars. The contribution of AGB stars is negligible for $I < 11$. The photometric errors are lower than 0.02 mag in both passbands, over the whole magnitude range plotted in the diagram. The contamination by foreground sources is negligible in this region of the CMD and the crowding is not a serious concern for such bright stars in this nearby and relatively loose cluster.

The P00 sample contains 185 stars in the upper magnitude bin, which is almost twice of what recommended by MF95 for an optimal detection of the TRGB.

The edge-detector filter is applied on a smoothed version of the LF of the RGB, following the standard analysis as described by SMF96 (see their appendix). In Fig. 2 panel (a) the RGB LF for $I < 11$ is reported. A sharp cut-off is clearly evident at $I \sim 9.8$. The same LF is shown in Fig. 2 panel (b) as a smoothed histogram, while the edge-detector filter response to the smoothed LF is reported in panel (c). The main peak in the filter response indicates the cut-off point: the TRGB is clearly and unambiguously detected at $I = 9.84 \pm 0.1$, the associated uncertainty is the Half Width at Half Maximum (HWHM) of the peak.

Moreover, the large sample of giants observed in this cluster allows us to adopt a more refined approach to the quantification of the uncertainty, by applying a bootstrap technique (see Babu & Feigelson (1996), and references therein). To do this, we randomly extracted a subsample containing 80% of the stars from the global sample of all RGB stars with $I < 11$, we repeated the measure of $I^{\text{T RGB}}$ on the extracted subsample with the same technique as above and recorded the final result. Panel (d) in Fig. 2 shows the distribution of 10,428 such estimates on randomly extracted subsamples: the mean of the distribution is $< I > = 9.835$ and the standard deviation is $\sigma = 0.04$ mag. The estimate of $I^{\text{T RGB}}$ turns out to be remarkably robust to statistical fluctuations, with 83% of the estimates falling within $\pm 1\sigma$ from the mean. We adopt the bootstrapped $\sigma$ as the uncertainty on our estimate of the apparent $I$ magnitude of the tip. The final result is: $I^{\text{T RGB}} = 9.84 \pm 0.04$.

It is now straightforward to provide $M_I^{\text{T RGB}}$ of ω Centauri as a function of distance modulus and reddening:

$$M_I^{\text{T RGB}} = 9.84(\pm 0.04) - K \cdot E(B - V) - (m - M)_0$$  \hspace{1cm} (5)

where $K = A_I/E(B - V)$, $A_I$ is the amount of extinction in the $I$ passband, $E(B - V)$ is the reddening and $(m - M)_0$ is the true distance modulus. Since the P00 data-set sampled
Fig. 2.— \((I,B-I)\) CMD of the brightest 3 mag of the RGB of \(\omega\) Centauri, from P00
Fig. 3.— The LF for the RGB stars with $I < 11$ is shown as a histogram in panel (a) and as a smoothed histogram in panel (b). The arrow in panel (a) indicates the cut off of the LF corresponding to the TRGB. The smoothed histogram is multiplied by an arbitrary constant to facilitate comparison with panel (a). The response of the edge-detection algorithm is shown in panel (c), while the distribution of the bootstrapped estimates is shown in panel (d). The location of $I_{TRGB}$ is unambiguously identified by the peaks in panels (c) and (d) at $I = 9.84 \pm 0.1$. The associated uncertainty is the Half Width at Half Maximum (HWHM) of the peak.
virtually the whole cluster population in this range of magnitudes, it is unlikely that the $M_{TRGB}^I$ estimate will be significantly improved by new observations. On the other hand, all other terms of Eq. 5 can be subject to substantial improvements in their estimates, as discussed in Sect. 3.1. Here we adopt the distance and reddening by T01 and the reddening laws by Dean, Warren & Cousins (1978)$^6$, with the same assumptions of DA90, i.e., $A_I \simeq 1.76E(B-V)$. We obtain:

$$M_{TRGB}^I = -4.04 \pm 0.12$$

where all the sources of uncertainty have been taken into account. The main contributor to the error budget remains the estimate of the distance modulus.

4. Comparison with other calibrations

It is worth checking how the existing empirical and theoretical calibrations compare with the above estimate of $M_{TRGB}^I$, at the relevant metallicity $[Fe/H] \sim -1.7$.

In Figure 4 our estimate of $M_{TRGB}^I$ in $\omega$ Centauri is reported as a big black dot in the $M_{TRGB}^I$ vs. $[Fe/H]$ plane. The horizontal error bar represents the range in metallicity covered by the dominant population of the cluster (see Fig.11 in Suntzeff & Kraft (1996)).

The empirical calibrations discussed in Section 2 are also plotted: the heavy solid curve is the new calibration based on the large IR database by F00 (eq. 4), while the L93 calibration (eq. 1 plus eq. 2 from DA90) has been converted to a function of metallicity, following the procedure described in Sect. 2, and it is plotted as a dotted-dashed curve. The horizontal long dashed line represents the recent result by Ferrarese et al. (2000a) (hereafter Fe00a) who calibrated the TRGB as a secondary indicator by using Cepheid distances in a small set of nearby galaxies where both Cepheids and the TRGB have been detected. They found $M_{TRGB}^I = -4.06 \pm 0.07$ (random) $\pm 0.13$ (systematic), in very good agreement with our estimate, despite their larger uncertainty. In Fig. 4, we plotted also the region delimited by the two empirical calibrations discussed above as a shaded area, in order to point out the region of the plane where most of the empirical estimates lie.

Salaris & Cassisi (1998) (hereafter SC98) provided a theoretical relation of $M_{TRGB}^I$ as

$^6$As far as we know Dean, Warren & Cousins (1978) are the only ones providing a reddening law for the Cousin’s $I$ passband. All other sources we were able to retrieve refer to Johnson’s $I$, a quite different filter with different $A_I/E(B-V)$ values.
Fig. 4.— Comparison between the calibration of $M_1^{TRGB}$ obtained for ω Centauri and different $M_1^{TRGB}$ – [Fe/H] relations. The big black dot is our estimate of $M_1^{TRGB}$ in ω Centauri, where the horizontal error bar represents the range in metallicity comprising the dominant population of the cluster. Different curves represent different calibrations: the short dashed curve shows the calibration by SC98 (their eq. 5); the dotted curve represents the same calibration with different assumptions for the bolometric correction (eq. 6 in SC98); the dashed-dotted curve is the empirical calibration by L93 (eq. 1 plus eq. 2, reported in the $M_1^{TRGB}$ – [Fe/H] plane as described in Sect. 2); the heavy continuous curve represents the empirical calibration we have derived from the F00 database (our eq. 4); the long dashed line reports the calibration of $M_1^{TRGB}$ as a secondary, indicator provided by Fe00a. The shaded area delimits the region of the plane where most of the empirical relations lie. Note that both the ω Centauri measure and the Fe00a calibration fall within this region.
a function of the global metallicity \([M/H]\), instead of \([\text{Fe/H}]\). We convert their \([M/H]\) to \([\text{Fe/H}]\) according to the prescriptions of Salaris, Chieffi & Straniero (1993) and assuming \([\alpha/\text{Fe}] = +0.28\) over the considered metallicity range (see Ferraro et al. (1999)). Using two different bolometric corrections, SC98 derived two slightly different relations (their eq. 5 and 6), which are reported in Fig. 4 as a short dashed curve and as a dotted curve, respectively.

As can be seen from Fig. 4, the theoretical calibrations are systematically (\(\sim 0.2\) mag) brighter than empirical ones, as already noted by SC98 and Fe00a. The strong constraint provided by the TRGB luminosity in \(\omega\) Centauri seems to favor the empirical calibrations. However, as stated by Castellani, Degl’Innocenti & Luridiana (1993) and F00, theoretical relationships should be considered as upper limits for the TRGB luminosity, since they refer to the nominal red giant luminosity at the He flash.

5. Summary and Conclusions

We have used some of the most recent results from the study of Galactic Globular Clusters to improve the observational basis of the calibration of the TRGB method as a powerful distance indicator.

In particular, we have derived a robust estimate of \(M_T^{TRGB}\) in the globular cluster \(\omega\) Centauri, based \((a)\) on the application of the standard technique on the very large sample of RGB stars by P00, and \((b)\) on a direct distance estimate from a detached eclipsing binary by T01. Our result turns out to be in excellent agreement with previous empirical calibrations. Expected improvements in the distance and reddening estimates may likely reduce the uncertainties of our calibrating point to \(\sim \pm 0.06\) mag. Thus, eq. 5 shall be considered as a very firm point of the TRGB calibration (at \([\text{Fe/H}] \sim -1.7\)) for the future, as well as an important observational test for stellar evolutionary models. However, even at the present level of accuracy, the \(M_T^{TRGB}\) measure derived here is the less uncertain calibrating point for the \(M_T^{TRGB}-[\text{Fe/H}]\) relation.

A new empirical \(M_T^{TRGB}-[\text{Fe/H}]\) relation (eq. 4) was also derived. It turns out to be in excellent agreement with the derived \(M_T^{TRGB}\) of \(\omega\) Centauri as well as with other empirical calibrations. The main advantages of the new relation derived here with respect to the previous ones (L93, DA90) are: \((i)\) it is based on more solid observational basis, since it has been derived from the largest IR database of RGB stars in Galactic globulars available in the literature (F00), \((ii)\) it has been calibrated over a larger range of metal abundances, extending the metallicity limit from \([\text{Fe/H}] \leq -0.7\) (L93, DA90) to \([\text{Fe/H}] \leq -0.2\); for this reason, eq. 4 has to be considered a much more appropriate tool for applications to external
galaxies, which, in general, are significantly more metal rich than $[Fe/H] > -0.7$.

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