Polarization Alignment in JVAS/CLASS flat spectrum radio surveys

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We present a detailed statistical analysis of the alignment of polarizations of radio sources at high redshift. This study is motivated by the puzzling signal of alignment of polarizations from distant quasars at optical frequencies. We use a coordinate invariant measure of dispersion to test for alignment of polarizations of widely separated sources. The data from JVAS/CLASS 8.4-GHz surveys are used for our study. We find that the data set with polarization flux greater than 1 mJy shows a significant signal of alignment at distance scales of order 150 Mpc. The significance of alignment decreases as we go to larger distances. In contrast the data set with polarization flux less than 1 mJy does not show significant alignment at short distances. However this set, as well as the full data sample, shows a very significant signal at large distances of order 500-850 Mpc. Due to the presence of relatively large error in the low polarization sample, this signal needs to be tested further with more refined data. Surprisingly, we also find that, at large distances, high polarization sample shows anomalously large scatter in comparison to random data sets, generated by shuffling data polarizations. We suggest that this might arise since the distribution of polarizations is not exactly uniform. We also study the signal by imposing a cut on the error in polarization. We find that the significance of alignment increases with decrease in fractional error. We are unable to attribute our results to known sources of bias. We discuss a possible physical explanation of our results.

Keywords: polarization, galaxies: high-redshift, galaxies: active

1. Introduction

The Big Bang model assumes that the Universe is homogeneous and isotropic on large distance scales. However there currently exist several observations which appear to violate this basic assumption. In particular the radio polarizations from radio galaxies appear to show a large scale dipole pattern across the sky. The possibility that the signal observed might arise due to bias was raised by. The signal was dismissed by who found that it is not present in a larger data set. However argued that the signal is present if we consider all the radio sources for which the relevant information, i.e. the polarization position angle and the galaxy orientation angle, is available in the literature. The relevant observable in this study was the difference of these two angles, which is independent of the coordinate system. It is also interesting that the dipole axis found aligns closely with the Cosmic Microwave Background Radiation (CMBR) dipole axis. Furthermore a recent study finds a dipole anisotropy in the brightness of radio sources which is much larger than what is predicted by the Doppler effect due to local motion. This also indicates a dipole axis which may be of cosmological origin and well aligned with the CMBR dipole.
The optical polarizations from quasars also show alignment over cosmologically large distances. The distance scale of these correlations is of the order of Gpc. Furthermore the Cosmic Microwave Background Radiation (CMBR) data shows several features which are not consistent with Big Bang cosmology. We point out that the WMAP science team has argued that some of the claimed anomalies in CMBR data set may arise due to a posteriori choice of statistics to test for a particular effect. In other words one notices a particular odd feature in the data and then devises a statistic to test its significance. Such a procedure is likely to overestimate the significance of the detected anomaly. In view of this it is extremely interesting that the alignment axis of CMBR quadrupole and octopole radio dipole axis, and the two point correlations in the optical polarizations all align very closely with the CMBR dipole axis and point roughly in the direction of the Virgo cluster. The maximum angular separation between any of these two axes is found to be 20°. If we remove the CMB dipole, the remaining axes align even more closely with one another. Furthermore there have been claims of violation of isotropy in cluster peculiar velocities and galaxy surveys. Remarkably the cluster peculiar velocities also indicate a direction close to the CMB dipole if we include the highest redshift data. Finally we mention that spiral galaxies reveal an interesting signal of parity violation.

There exist many attempts to theoretically explain these observations, most of which assume violation of the cosmological principle. An interesting possibility, which is completely consistent with Inflationary Big Bang model, is that the Universe was inhomogeneous and anisotropic at very early stage, before the epoch of inflation. It evolves into a homogeneous and isotropic de Sitter space-time during inflation. It has been shown that, for a wide class of models, there exists a parameter range such that the modes generated during this early phase can re-enter the horizon much before the current era and hence can affect present observations. These can, in principle, generate the observed anisotropies.

In, the authors compiled a catalogue of radio polarizations from distant radio galaxies. Motivated by the observed linear polarization alignment in quasar data at visible wavelengths, studied the possibility of a similar effect at radio frequencies using this catalogue. No significant effect was detected. The alignment in optical polarizations is seen over cosmologically large distances of order Gpc and the phenomena is very puzzling. There are several possible models which aim to provide an interpretation of this alignment. One possible explanation is the large scale correlations in the intergalactic magnetic field. The intergalactic magnetic field may be seeded in the early Universe and presence of large scale correlations in this field cannot be ruled out within the Big Bang cosmological model. We still require a mechanism for how such a magnetic field generates large scale correlations in the optical polarizations. One possibility is that this is caused by mixing of photons with hypothetical pseudoscalars. This effect is frequency dependent and is much smaller at radio frequencies. Hence it may be consistent with absence or reduced alignment effect in radio polarizations. There have also
been other proposals to explain this effect.\textsuperscript{28, 29}

In the present paper we analyze the radio data in order to study possible alignment of radio polarizations. Here we extend the work of \textsuperscript{24} by considering the fact that polarization angles depend on the coordinate system used. In order to properly analyze the presence or absence of large scale alignment one needs to define a coordinate invariant statistics.\textsuperscript{9} The basic point is that, in order to compare two polarization angles on the surface of the celestial sphere, one needs to parallel transport one of them to the position of the second along the great circle joining the two points. The contribution due to parallel transport may be negligible in most cases, if the two points are separated by a small distance. However it becomes very important if we are testing alignment over large distances. In \textsuperscript{24} the authors restricted their analysis to data which has polarization flux density greater than 1 mJy in order to select data which only contains sources with significant polarization detection. Here we study the signal both in the high polarization and the low polarization data sample. Furthermore we study data after imposing a cut on the fractional error in polarization since sources with small error are likely to be most reliable.

A potential source of bias is the error in the removal of residual instrumental polarization.\textsuperscript{23, 24} This can lead to large scale correlations in polarizations even when none are present. It is clear that this effect will dominate for sources which have low polarization flux. Hence one can evaluate its contribution by focussing on data with low polarizations. Another source of error is the positive bias that arises in the degree of polarization. This arises since the degree of polarization depends on the squares of Q and U and always acquires a positive value. However this cannot affect the alignment statistics which are based only on the linear polarization angle. We also point out that errors in polarization position angle (PA) calibration can reduce the true significance of alignment that might be present in data. This will lead to a systematic error in the observed PA which may be different in different runs.\textsuperscript{23} Hence, this can mask an alignment effect present in the data, but should not introduce spurious alignments.

2. Data Selection

We use data available in the catalogue produced by \textsuperscript{23} It contains a total of 12743 core dominated flat spectrum radio sources and lists their angular positions and the Stokes $I, Q$ and $U$ parameters. Since the redshift of most of these sources is unknown, we assume that these sources are roughly at the same redshift, equal to unity. The input observable for the alignment study is the polarization angle. The calibration methods and the catalogue production has been discussed in detail in \textsuperscript{23}.

In \textsuperscript{24} the authors imposed a cut on polarization flux density to include only sources with polarization flux greater than 1 mJy. We also impose this cut on the data. The resulting set contains a total of 4400 sources. We shall refer to this as set 1. We also consider the data sample with polarization flux greater than 2 mJy (set 2), less than 1 mJy (set 3) and less than 0.5 mJy (set 4). These contain 2468,
8342 and 5283 sources respectively. We note that the sources lie dominantly in the Northern hemisphere. Furthermore there are very few sources along the Galactic plane.

The distribution of polarization angles for the data sets with different cuts is shown in Fig. 1. We notice that the distribution in all cases shows a small bump for \( PA < 90^\circ \) and a corresponding dip for \( PA > 90^\circ \). This trend is more prominent for data with small polarizations. We may quantify this non-uniformity by making a fit with the von-Mises distribution,

\[
f(\theta, \kappa) = \frac{1}{2\pi I_0(\kappa)} \exp \left[ \kappa(\theta - \theta_0) \right]
\]

where \( I_0 \) is the modified Bessel function of order 0. In making our fit we set \( \theta = 2PA \). The null hypothesis is that the distribution is uniform, i.e. \( \kappa = 0 \). The best fit values of \( (\kappa, \theta_0/2) \) are found to be \((0.064, 40^\circ)\), \((0.070, 46^\circ)\) and \((0.063, 37^\circ)\) for the full data, \( Pol > 1 \) and \( Pol < 1 \) respectively. Using likelihood analysis, we find that the fit is significant at 4.7 \( \sigma \), 3.6 \( \sigma \) and 2.8 \( \sigma \) respectively for these cases. Hence the distribution shows a significant deviation from uniformity. We do not understand the reason for this non-uniformity. In any case, as we discuss in the next section, the non-uniformity in distribution does not affect our results.

3. Statistical Procedure

All astronomical observations are made on the hypothetical celestial sphere and any directional measurement on this sphere corresponds to a particular coordinate system. For example, the polarization angles are measured in a local frame, formed by two unit vectors \( \hat{\phi} \) and \( \hat{\theta} \). These unit vectors depends on the coordinate system used, i.e. they depend on which direction we choose as our North Pole, and hence one cannot directly compare vectors at different positions on the celestial sphere. The proper procedure to compare such vectors is to transport one of them to the position of the second along the geodesic joining the two positions. A detailed procedure has been discussed in [9] and we follow this procedure for comparing polarization angles of two different sources.

We next define statistics in order to quantify the alignment. This requires a measure of the difference of polarization angles at different sites. Let the polarization angles at site \( i \) and \( j \) be \( \psi_i \) and \( \psi_j \) respectively. We associate the unit vectors \( \hat{v}_i = [\cos(2\psi_i), \sin(2\psi_i)] \) and \( \hat{v}_j = [\cos(2\psi_j), \sin(2\psi_j)] \) with these polarization angles. The factor of two is required since a polarization angle \( \psi = 0 \) is identified with \( \psi = 180^\circ \). The variable \( 2\psi \) takes values over the entire range 0 to 360\(^\circ\). This is discussed in more detail by [36]. A convenient measure of alignment between vectors \( \hat{v}_i \) and \( \hat{v}_j \) is given by their dot product, \( \hat{v}_i \cdot \hat{v}_j = \cos(2\psi_i - 2\psi_j) \). This cosine measure takes into account the angular nature of polarizations and can also be motivated by the von Mises distribution, relevant for data on a circle. Taking into account the contribution due to parallel transport, the dot product becomes \( \cos[2(\psi_i + \Delta_{i \to j}) - 2\psi_j] \), where \( \Delta_{i \to j} \) is the contribution due to parallel transport from site \( i \) to site \( j \) along a great circle.
An alternate measure of alignment is $|\pi/2 - |\psi_i - \psi_j||$ where the angles are in radians.

Using the measures discussed above, we define three statistics, which can be used to test for alignment. We first define the statistic, $S_D$, which has been used in earlier studies. Consider the source $k$ along with its $n_v$ nearest neighbours. A useful measure of alignment of polarizations in the neighbourhood of site $k$ is given by,

$$d_k = \frac{1}{n_v} \sum_{i=1}^{n_v} \cos[2(\psi_i + \Delta_{i\rightarrow k}) - 2\bar{\psi}_k].$$

(2)

Here $\psi_i$ are the polarization angles of the nearest neighbours of the $k^{th}$ site and the factor $\Delta_{i\rightarrow k}$ arises due to parallel transport from $i \rightarrow k$. The sum in Eq. 2 includes the site $k$ also. Essentially the polarizations of the nearest neighbours are first parallel transported to the site $k$ and then tested for alignment. We next maximize $d_k$ as a function of $\bar{\psi}_k$. The resulting value of $\bar{\psi}_k$ is interpreted as the mean polarization angle at the site $k$ and the corresponding maximum of $d_k$ takes.
on higher values for data with lower dispersions and vice versa. The statistic may now be defined as:

$$S_D = \frac{1}{n_s} \sum_{k=1}^{n_s} d_k \bigg|_{\text{max}}$$

(3)

where $n_s$ is the total number of samples in the data. For example, for set 1, $n_s = 4400$. A large value of $S_D$ indicates a strong alignment between polarization vectors.

An alternate statistic that we also use is defined as,

$$S_D' = \frac{1}{n_s} \sum_{k=1}^{n_s} d'_k$$

(4)

where

$$d'_k = \frac{1}{n_v} \sum_{i=1}^{n_v} \cos \left[ 2(\psi_i + \Delta_{i\rightarrow k}) - 2\psi_k \right].$$

(5)

Note that here $\psi_k$ is the polarization angle at the site $k$. Hence, if we ignore the correction due to parallel transport, $d'_k$ is simply the sum of cosines of the differences of twice the polarization angle at site $k$ with its nearest neighbours. A third statistic we use is defined as,

$$Z_D = \frac{1}{n_s} \sum_{k=1}^{n_s} d''_k$$

(6)

with

$$d''_k = \frac{1}{n_v} \sum_{i=1}^{n_v} \left| \frac{\pi}{2} - \left| (\psi_i + \Delta_{i\rightarrow k}) - \psi_k \right| \right|.$$

(7)

In our analysis we mostly use statistic $S_D$ since this was also used earlier in the analysis of optical data. We use the statistics $S_D'$ and $Z_D$ to confirm the results obtained by $S_D$.

The significance of alignment in the data set is computed as follows. We first compute the statistic for a given number of nearest neighbours, $n_v$, of any source. This statistic is compared with the result for a large number of random samples, which are generated by two different procedures. In Procedure I, the random samples are generated by shuffling all the PAs among different sources. In Procedure II, these are generated from a uniform distribution of polarization angles. Procedure I tests for local alignment of polarizations given the polarization distribution of data. Procedure II, in contrast, tests for alignment relative to a uniform distribution. In most of our simulations we use a total of 1000 random samples for a given number of nearest neighbours. The probability or P-value that the alignment seen in data might arise as a random fluctuation is equal to the number of random samples which show a larger value of statistic in comparison to the real data divided by the total number of random samples. Almost all the P-values quoted in results are computed directly by this procedure. In some cases, a reliable estimate of P-value
was obtained only after enhancing the number of random samples since we did not find any random sample exceeding the statistic of the real data. In some cases, the P-value was found to be too small and a direct estimate was impractical. In this case it was estimated by computing the sigma value. Here we assume that the distribution of random samples is approximately Gaussian and determine its mean and standard deviation. This provides an estimate of the sigma values, which can be used to compute the P-values. We find that this slightly underestimates the significance in comparison to a direct computation. However the difference is found to be negligible for our purpose.

The distribution of $P_A$s is slightly non-uniform, as shown in Fig. 1. This non-uniformity does not affect our results as long as we use Procedure I for generating the random samples. This is because, in this case, the random samples are generated from the same distribution as the real data. However we shall get somewhat larger significance if we use Procedure II, which generates random samples from the uniform distribution. Procedure I really tests the alignment of sources, within a local neighbourhood, independent of their distribution. Hence, for example, it would yield a null result for alignment if all the sources have the same $P_A$s. This is discussed further in section 5.

The number of nearest neighbours, $n_v$, of any source are computed by assuming that all the sources are located at the same redshift of 1.0. The redshift information of these sources is not currently available and hence we make this assumption. Essentially here we are ignoring the third dimension and determining the nearest neighbours only on the basis of the angular separations. It is likely that in many cases this will lead to a wrong assignment of the set of nearest neighbours of a source. However this cannot generate alignment in a data set if none is present. If a data set shows alignment then it will affect the detailed numerical results. For example, let us assume that the radio polarizations are aligned over a small distance scale of a few Mpc, but show no alignment over larger distances. Our nearest neighbour assignment may include some sources which are in fact much further away. It is clear that these sources which are mis-identified as nearest neighbours will only add noise to the signal and reduce the significance of alignment. In Fig. 2 we show the relationship between the number of nearest neighbours and the mean comoving distance from a source within which these nearest neighbours reside. Here the mean is taken over the entire sample. We have assumed the standard Lambda Cold Dark Matter model for computing the comoving distance.

4. Results

The distribution of statistic, $S_D$, for random samples is shown in Fig. 3 for $n_v = 10$. Here we use 1000 random samples generated by shuffling data for radio sources in set 1. The value of statistic for the real data set is also shown. We find that a Gaussian provides a good fit to the random sample distribution. The fit values for 1000 random samples are found to be, mean = 0.2836 and standard deviation=0.00495.
Fig. 2. The mean distance (Mpc) among sources as a function of the number of nearest neighbours \( n_v \) for data with polarization flux greater than 1, i.e. set 1.

If the number of samples are reduced to 500 we again find a good fit with mean \( = 0.2834 \) and standard deviation\( = 0.00492 \). We have explicitly verified that a direct evaluation of the significance agrees well with that determined by the Gaussian fit. Hence the significance we quote is reliable, despite the small number of nearest neighbours.

In Fig. 4 significance of alignment, i.e. P-values, for data set 1 a function of the number of nearest neighbours. In Fig. 5 we show the significance more clearly for small number of nearest neighbours. We do not find a significant alignment for
$n_v > 11$. This is in agreement with the results obtained in\cite{24} However for lower values of $n_v$ we find significant alignment, with significance greater than 3 sigmas, for several values of $n_v$. In\cite{24} this region was never explored. The strongest signal is observed in the neighbourhood of $n_v = 10$. The mean comoving distance among sources for 10 nearest neighbours is about 150 Mpc. Hence our results show that for polarization flux greater than 1 mJy, the radio sources show alignment over distance scale of order 150 Mpc. This set is expected to be least contaminated with bias and hence the alignment effect we observe is most likely of physical origin. We point out that we do not find a significant signal of alignment for very small values of $n_v$, as shown in Fig. 5. A similar phenomenon was seen even in the optical alignment. In this case also alignment was not seen for very small $n_v$. This can be explained by the presence of large fluctuations at small $n_v$. We discuss this further in section 6.

In Fig. 4 we notice that, for large $n_v$, the significance of alignment continues to decrease with increase in $n_v$. If, beyond a certain value of $n_v$, the polarizations were randomly aligned, we would have expected that the sigma value would fluctuate about 0. In order to study this in more detail we have computed the significance for larger values of $n_v$. The results for the P-values are shown in Fig. 6 as filled squares. We find that the P-value continues to increase and rises above 0.95 beyond $n_v = 500$. Equivalently, the sigma value falls below $-2$ beyond $n_v = 500$. The P-value continues to increase till about $n_v = 1000$ and then starts decreasing. This implies that for $n_v$ between 500 and 1000, the data set shows larger scatter in comparison to a random sample. We have verified that this happens only for the
actual data used. If instead we use a randomly generated sample, then the sigma values indeed fluctuate about zero, as expected. This confirms the correctness of our procedure and simulations. However surprisingly the data sample shows unusual scatter for large \( n_v \).

In order to understand the behaviour at large \( n_v \) better we generated the random samples directly without shuffling the real data. The distribution of random samples is assumed to be uniform, i.e. constant as a function of the polarization position angle. The resulting P-values are also shown in Fig. 5 as plusses. We find that, in general, this analysis yields higher significance in comparison to what is obtained by shuffling the polarizations among different sources. At low \( n_v \) we obtain a trend similar to that seen in Figs. 4 and 5. However at large \( n_v \) we see a very different behaviour. The P-value rises to about 0.4 at \( n_v = 500 \). Beyond this it first falls and then again starts increasing. This analysis shows that the data set does not show unusual scatter in comparison to a truly random sample. It is only with respect to the shuffled sample that we obtain the anomalous result. This trend might be attributed to the non-uniformity of the distribution of polarizations in the real sample, as discussed in section 2.

In Fig. 7 we show the \( S_D \) values for data set with polarization flux greater than 2 mJy (set 2). The results for set 1 are shown for comparison. We find that the values of \( S_D \) do not change significantly between these two sets. The resulting P-
Fig. 6. The P-value as a function of the number of nearest neighbours, \( n_v \), for data set 1 with statistic \( S_D \) using Procedure I (filled circles) and Procedure II (plusses) for generating the random samples. The upper panel shows the expanded y-axis in the range 0.9 to 1 to show the points for \( n_v \geq 500 \) using Procedure I with better clarity.

values for set 2 are shown in Fig. 8. We observe that the significance of alignment reduces. This reduced significance can be explained by the fact that the number of samples in this set is smaller in comparison to set 1 by a factor of about 0.6. We should compare the results with these two cuts at fixed distance. Hence we should compare the results for \( n_v = 10 \) for set 1 with \( n_v = 6 \) for set 2. The mean value of the statistic of the random samples for a given \( n_v \) is same for both the samples. We find that the mean is equal to 0.3668 and 0.3669 for \( n_v = 6 \) for set 1 and 2 respectively. The fluctuations for set 2 are larger than those for set 1 by a factor of \( \sqrt{1/0.6} \) due to the smaller size of the sample. We find that for \( n_v = 10 \), set 1, the standard deviation = 0.00495, whereas for \( n_v = 6 \) and set 2, the standard deviation = 0.00698, consistent with our claim. This change in the standard deviation of the random samples explains the difference in significance between the two samples.

We next study the alignment further by imposing a cut directly on the error...
Fig. 7. The statistic $S_D$ as a function of the number of nearest neighbours, $n_v$, for data set with polarization flux greater than 2 mJy (set 2). The results for set 1 are shown for comparison.

Fig. 8. The P-value as a function of the number of nearest neighbours, $n_v$, for data set 2.
in the polarization flux. In Fig. 9 we show the results after imposing the cuts, error \( \leq 5\% \), \( \leq 10\% \) and \( \leq 20\% \) on data using statistic \( S_D \). The number of sources remaining after these cuts are 1058, 1728 and 2889 respectively. The error in the polarization angle, in radians, is also approximately equal to the fractional error in the polarization flux. Hence these results can also be interpreted in terms of
a cut on the error in PAs. A 5% error on the polarization flux leads to a cut of approximately 3° on the PA. We clearly see that at small values of $n_v$, the data set with smallest error shows the highest significance. For example, data with error $\leq 5\%$ shows better than 3 sigma significance for $n_v < 10$. This set is clearly the most reliable and provides further evidence that the effect we observe is not caused by bias. In comparing with the results shown in Fig. 5 we note that the sample size for error $\leq 5\%$ is much smaller, roughly one-fourth of the sample used in Fig. 5. For a homogeneous sample, the significance of alignment is expected to increase with the sample size. Furthermore the physical distance for any value of $n_v$ in this sample would be roughly double the distance scale for same $n_v$ for data set 1, which has 4400 sources. This shows that the sample with error $\leq 5\%$ shows results with similar or better significance in comparison to set 1.

In Figs. 10 and 11 we show the results obtained using statistics $S_D'$ and $Z_D$ respectively for different cuts using Procedure I. Here the results are obtained by ignoring the contribution due to the parallel transport, which can be neglected for small distances. We have explicitly verified that the results remain practically unchanged irrespective of whether we include or exclude it’s contribution. We find that the results obtained with statistics $S_D'$ and $Z_D$ are consistent with those obtained with $S_D$. In general these statistics show higher significance in comparison to $S_D$. We obtain similar results with these statistics if we use Procedure II.
In Fig. 11 we show the polarizations for set 1 averaged over 150 Mpc, the distance scale of alignment, in Galactic coordinates. Here we divide the celestial sphere by choosing a set of fixed latitudes in equatorial coordinates, which are further subdivided to create equal area segments. This leads to a total of 273 segments. We show the average polarization angle in each region. The main purpose of this plot is to visually detect any systematic pattern in polarization on large angular scales. We observe that at low latitude, several polarizations appear to be locally aligned with the equator, i.e. have PA equal to $\pi/2$ radians. This applies particularly to sources lying at Galactic longitudes, $l \leq \pi$ radians. We are unable to detect any other striking pattern in Fig. 12.

We next determine if the alignment of polarizations with Galactic latitude is statistically significant. Let $\chi_i$ represent the polarization angle of source $i$ in Galactic coordinates. We define the statistic

$$S_G = \sum_{i=1}^{n_s} \cos(2\chi_i - 2\chi_0)\bigg|_{\text{max}}$$

The sum is maximized with respect to $\chi_0$. The resulting value of $\chi_0$ is interpreted as the mean polarization angle. The corresponding value of $S_G$ gives an estimate of how well the polarizations are aligned with the local longitude. A large value implies very good alignment. We use this statistic to test for alignment of set 1 with the Galactic coordinates. We find that $S_G = 0.00721$. In this case also $n_s = 4400$. 

Fig. 11. The P-values (lower graph) for polarization flux $> 1$, error $\leq 5\%$, error $\leq 10\%$ and error $\leq 20\%$. Here the random samples are generated using Procedure I.
Comparing with 1000 random samples, the P-value in this case is 0.78. Hence we do not find significant alignment. We have repeated this analysis by including data only at very low Galactic latitudes and also by keeping sources with $l \leq \pi$ radians. The significance does rise in this case but remains much lower than 2 sigmas. We next apply this statistic by fixing the value of $\chi_0 = \pi/2$. In this case again the entire data set does not show a significant signal. The significance for data at low Galactic latitudes in this case is higher but still not equal to 2 sigmas. We do find a significant result at 2 sigmas if we also impose the cut $l \leq \pi$ radians. For $|b| \leq 0.1$ radians and $l \leq \pi$ radians, we find $P = 0.03$ with the number of sources, $n_s = 77$. For $|b| \leq 0.07$ radians and $l \leq \pi$ radians, we obtain $P = 0.011$ with $n_s = 36$. With further reduction in $|b|$ the P-value starts to rise. Hence we find a weak signal of alignment with Galactic equator at low latitudes. We do not understand the cause of this alignment. However since the signal is relatively weak and obtained only in a small subset of data, it might also be attributed to a statistical fluctuation. We have also determined how our results vary if we remove the region $|b| < 10^\circ$. We do not find a significant change in results for any of the cuts discussed in this paper.
4.1. Low Polarizations

We have so far only considered results for data which is expected to be most reliable. For comparison, we next show results for data with low polarizations. In Fig. 13, we show the significance of alignment for the data set with polarization flux less than 1.0 mJy (set 3) and less than 0.5 mJy (set 4). For set 3 we find that P-value is large for small $n_v$ and then rapidly decreases with increasing $n_v$. This trend is exactly opposite to that seen in Fig. 5 for data set 1. The fact that data with low polarization flux shows no signal of alignment at small $n_v$ provides further evidence that the alignment seen in set 1 cannot be attributed to bias. This is because a bias will dominantly affect data with low polarization flux and any signal which arises due to bias should be more prominent in this set. This point is further reinforced by data corresponding to polarization flux less than 0.5 mJy. In this case we do not find significant alignment over the entire range of $n_v$ shown in Fig. 13.

In Fig. 13 we observe a high significance of alignment for $n_v > 15$ for data set 3. This data set, however, has higher error in comparison to set 1. For this reason we have confined ourselves mostly to set 1. One may speculate that the high significance we see in this set at larger distances arises due to bias. However this interpretation is not consistent since set 4 shows no effect. The bias would have affected this sample the most. We add that we have studied set 4 at larger values of $n_v$ and found no signal of alignment with Procedure I. In contrast set 3 continues to yield a significant signal for larger $n_v$, as shown in Fig. 14. Here the P-values are found to be very small for some $n_v$. Hence we compute them by using the sigma values.
Fig. 14. The P-values as a function of the number of nearest neighbours, \( n_v \), for set 3.

Fig. 15. The P-value for set 3 using Procedure I (plusses) and II (crosses) as a function of \( n_v \). Here we have used statistic \( S_D' \).

Here we see that for set 3, the significance continues to increase up to \( n_v = 200 \). This corresponds to a distance scale of about 500 Mpc.

In Fig. 15 we show the P-value over a much larger range for set 3. The corre-
The P-value for set 4 using Procedure I (plusses) and II (crosses) as a function of $n_v$. Here we have used statistic $S'_D$.

The corresponding result for set 4 is shown in Fig. 16. Here we use the statistic $S'_D$ instead of $S_D$. This is because $S_D$ takes very long computation time for large values of $n_v$. We have verified that the two statistics give similar results in all cases where the results for both have been obtained. This can also be seen by comparing the results of Fig. 10 with those of Figs. 9a and 5. The results of Fig. 15 are also consistent with those obtained with statistic $S_D$, Fig. 14, where we find the most significant result at $n_v \approx 200$. However at larger $n_v$ we find an additional dip in the P-value at $n_v \approx 600$. This corresponds to a distance scale of about 850 Mpc. Beyond $n_v = 600$, the significance reduces monotonically. A very interesting feature of Fig. 15 is the difference in results seen with Procedure I and II. We find that Procedure II in general yields higher significance. The difference continues to increase until $n_v \approx 2500$ and then starts decreasing. We observe a very high significance at $n_v \approx 2500$ with Procedure II. At this $n_v$, Procedure I yields a null result. This indicates that the polarizations have a tendency to point in the same direction over very large distances. Procedure I does not yield significant result at $n_v = 2500$ since in this case the random samples will also have this correlation. This trend is very similar to that seen in Fig. 6 at large $n_v$. It is possible that this very large scale correlation arises due to instrumental bias since it affects almost the entire data. This is further indicated by the results for set 4, seen in Fig. 16. Here we find that Procedure I does not yield a significant result for any value of $n_v$. However Procedure II yields a very strong signal at large $n_v$. The strongest signal is seen at $n_v \approx 1100$ and is similar to the trend seen in set 3 at $n_v \approx 2500$. We discuss this issue in further detail in Section 5.
We find a similar trend with cuts on degree of polarization \((dop)\). For \(n_v > 40\), the data with \(dop > 1\%\) shows alignment whereas the remaining set does not. The complete data set of 12743 sources yields a 5.4 sigma signal of alignment at \(n_v = 250\), which also corresponds to a distance scale of order 500 Mpc. The signal decays below 3 sigma only for \(n_v > 750\).

The results seen for set 3 and 4 using Procedure I can be explained by realizing that a physical effect of alignment is likely to be masked in a sample which has large error. For larger values of \(n_v\) the fluctuations get reduced since each source is compared with a larger number of nearest neighbours. Hence the significance is expected to get enhanced. This is precisely the effect seen in data. We point out that sets 1 and 2 also show a similar trend with low significance for very small \(n_v\). In the case of set 4, the error is very large. This is most likely the reason for the lack of signal seen in this set.

Due to the presence of relatively large error in low polarization data set, we suggest that these results be tested further with more refined observations. If these results are confirmed with more refined data, then it would suggest the existence of correlations on larger distance scale for data with low polarization flux.

5. Bias

The JVAS/CLASS data may contain bias due to error in the removal of residual instrumental polarization\(^{23,24}\). In different observation sessions, the observations were made on different regions of the sky. Faulty removal of instrumental bias could lead to large scale correlations in these regions. We expect that a bias might affect the low polarization sample the most. It is possible that the bias is so large that all the data set is significantly affected. In that case we cannot draw any conclusions from the data. Here we assume that the bias is relatively small. The fact that set 4 does not lead to a significant signal of alignment gives us considerable confidence that bias is not responsible for the correlations we observe using Procedure I. It is possible that the non-uniformity in distribution, i.e. an excess of sources with PAs lying between 0° and 90°, seen in Fig. 1 might be caused by instrumental bias. This non-uniformity indicates the presence of some systematic effect which affects the data globally. Furthermore it is roughly correlated with the equatorial coordinate system with the shift occurring close to 90°, which also suggests that it might be caused by bias. Alternatively it might arise due to some local effect, caused, for example, by the ionosphere. As we discuss below, any such bias, which might affect the entire data, cannot generate the correlations we report. Furthermore there is evidence in the data which suggests that the non-uniformity in the distribution, irrespective of its origin, affects our results primarily at very large \(n_v\), where the significance of the signal observed with Procedure I is much reduced.

In our analysis we have mostly used Procedure I which generates random samples by shuffling real PAs among different sources. In this case each random sample has the same PA distribution as the real data. This procedure naturally eliminates
some contributions which might arise due to bias. In particular, a bias which might affect the entire sample in the same manner would not yield a significant signal of alignment with Procedure I. Let us consider the extreme case where the polarization angles in the entire data set are biased in the same manner, i.e. they point in the same direction, up to some contribution due to random error. In this case the distribution of PAs will be maximally skewed. This bias cannot yield the alignment signal we observe using Procedure I since the bias will affect the random samples as much as it affects real data. The random samples will also have the same PA values at all sites as the real sample. The difference between the two will arise primarily due to random error. The statistic of random samples will be exactly equal to that of real sample up to fluctuations due to random error in PAs. Hence we do not expect to get a significant signal in this case.

In the present sample it is more likely that different regions of the sky might be biased differently. Let us assume that this bias acts over very large distance scales. In that case it is reasonable to assume that the non-uniformity in the distribution, seen in Fig. 1, is caused by this bias. It is, therefore, interesting to determine the influence of this non-uniformity on our results. As we have already explained in section 3, this would affect the results of Procedure II but not Procedure I, which preserves the distribution of the real data while generating random samples. The difference in results between Procedures I and II, therefore, provides us with an estimate of the effect caused by the non-uniformity in distribution. This difference can be seen in Figs. 6, 15 and 16 for sets 1, 3 and 4 respectively. It is relatively small for small $n_v$ with Procedure I giving a smaller significance. As discussed in section 4, the main difference comes at large $n_v (\sim 2000)$ for set 1. Here Procedure I yields an anomalously low significance (high P-value) and, in contrast, Procedure II yields a mild (2 sigma) evidence of correlation. The strongest difference between the two procedures is seen at $n_v \approx 2000$, almost half of the entire sample. This indicates the presence of a systematic effect which affects nearly half the sample in the same manner. This trend is seen even more prominently in sets 3 and 4. Here Procedure II yields very significant result ($\sim 6\sigma$) at $n_v = 2500$ and 1100 respectively, whereas Procedure I yields a null result at these $n_v$ values. For set 1, Procedure II shows a dip in P-value at $n_v = 2000$ since the random samples in this case are free from the systematic effect present in data. In contrast, the random samples corresponding to Procedure I will also have such a systematic component. Hence the possibility that these might yield a larger statistic than the real data is not unlikely. This argument also explains the results seen in set 3 and 4. We conclude that the non-uniformity in distribution only affects our results at very large $n_v$ and does not generate the correlations that we report with Procedure I.

Finally, we address the issue of bias raised in Battye et al. The authors found a significant bias present in the polarization angles in the NVSS survey. It was found that the polarization angles have a tendency to be close to multiples of 45°. As can be seen from Fig. 11, this bias is absent for all the data sets we study in this paper. We do find a peak close to 45° but no peaks at 90°, 135° and 180°. Hence, although
the distribution shows deviation from uniformity, the trend is different from that discussed in Battye et al. Here we have argued that it is unlikely that the correlations we observe with Procedure I arise due to bias in the data. Specifically we have examined the effect of a bias which might affect the data globally. We have shown that it does not affect our results corresponding to Procedure I. Further evidence for absence of bias comes from the fact that data with low polarizations (set 4) shows no effect. However we cannot rule out the possibility of bias entirely and the issue may be best resolved by more refined data.

6. Physical Explanation

In this section we provide a possible physical explanation for the results obtained in this paper. We first notice that the alignment observed in set 1 (Pol. flux $> 1$ mJy) is over a distance scale of order 150 Mpc. At such distances the Universe does not show homogeneity and isotropy. For example, we observe superclusters of galaxies with length scales of order 100 Mpc. Hence we cannot rule out correlations among galaxies at such distances within the framework of the Big Bang cosmology. The distance scale observed also agrees well with the observed peak in the galaxy correlations using Sloan Digital Sky Survey, as predicted by the Big Bang cosmological model. The peak indicates an enhancement in the galaxy-galaxy correlations at a distance scale of 150 Mpc. We still require a physical mechanism which would cause such an alignment. One possibility is the large scale correlation in the intergalactic magnetic field if the intergalactic magnetic field has a primordial origin, then it could show correlations over large distance scales. Such correlations could generate the observed alignment by either affecting the electromagnetic radiation during propagation through intergalactic medium or by intrinsically aligning the sources at high redshift. We further speculate that correlations in the intergalactic magnetic field might also show enhancement at the scale of 150 Mpc, as observed in the galaxy correlations. This might explain the strong alignment seen at this distance scale, corresponding to $n_v = 10$, as shown in Fig. 5.

The correlation seen in the low polarization sample (set 3) is explained by posulating correlations in the magnetic field at larger distances of the order of 500 Mpc. The low polarization sample does not show alignment at short distances since the error in this sample is much larger. Hence the effect becomes apparent only when we compare a particular source with a sufficiently large number of nearest neighbours. In this case the fluctuations in the estimate of $d_k$ at any point get reduced. We demonstrate this phenomenon with an explicit simulation. We divide the whole sphere in 12 equal area parts using HEALPix. We generate a total of 3072 sources distributed uniformly over the sphere. For each segment the source PAs are generated from a Gaussian distribution with fixed mean and large stan-

\[^{a}http://healpix.jpl.nasa.gov/\]
The P-value for a simulated low polarization sample which has large error. The sample has correlations over large distances, as described in text. As expected, we see that the significance of alignment increases with the number of nearest neighbours.

standard deviation, equal to 60°. The standard deviation is taken to be large in order to simulate the large error in real PA values. The mean value in different segments is chosen randomly. We next determine the significance of alignment in this data set, considering only the sources in the northern hemisphere. The resulting P-values are shown in Fig. 17. We observe that at very low \( n_v \), the correlation is not significant. However we see a strong signal for large \( n_v \), in agreement with what is observed in set 3. This also explains the trend seen in set 1 and 2 and in optical data, where a significant signal is not seen at very low \( n_v \). The significance gets enhanced for larger values of \( n_v \).

The high polarization sample (set 1) does not show alignment signal at distance scale of order 500 Mpc since these sources may have evolved due to galaxy clustering. Hence these show correlations only upto the clustering distance scale. The magnetic field of these sources would be influenced by the cluster magnetic field and hence is likely to loose coherence with sources at larger distances. We speculate that sources which have low polarizations may not be significantly affected by clustering of galaxies. This may be another reason why these sources do not show an effect at short distances.

The above explanation for low polarizations requires that magnetic field shows correlations on large distances of the order of 500 Mpc. Such correlations may be present in the primordial model of intergalactic magnetic field. Hence one may be able to explain these also within the framework of Big Bang cosmology. However a
more detailed analysis is required to verify this possibility, which we postpone to further research.

It is natural to expect that both the optical and radio alignment might have a common origin, which may be the intergalactic magnetic field. However, in detail, these two may show different behaviour since there are many physical effects, such as extinction, mixing with hypothetical pseudoscalar particles, which affect optical but not radio frequencies. A possible scenario is that the radio alignment arises due to intrinsic alignment of galaxies, generated by the large distance correlations in the intergalactic magnetic field. It is generally expected that primordial magnetic field provides the seed for the magnetic field of these sources. The linear polarization of radio waves from these sources would be directly correlated with the direction of their magnetic field and may lead to the observed alignment in radio polarizations. The optical alignment, however, might arise dominantly due to propagation. For example, it might acquire correlations over larger distance due to mixing with pseudoscalar particles in background magnetic field. This phenomenon does not significantly affect radio polarizations. Hence despite the observed difference in the scale of alignment, it is possible that intergalactic magnetic field might explain both observations.

Our proposal that polarization alignment arises due to intrinsic alignment of sources may be tested by study of the corresponding jet angles. Information about jet angles is available for a small subsample of our data set. The histogram of the difference between the jet PA and the polarization PA shows a peak at 90°. Due to this correlation between the two PAs, the jet angles may also show alignment seen in polarization PAs. For set 1, we are able to unambiguously find only 520 sources for which information about the jet angles is also available. We have tested this sample for alignment at small $n_v$. It does not show a significant signal either for polarization PA or jet angles. This null result might arise since this sample is too small and hence the signal of alignment would be weak. Furthermore even for $n_v = 2$, the mean distance between galaxies is larger than 150 Mpc. Beyond this distance the correlations in set 1 decay rapidly. Hence the results of this study of jet angles is inconclusive. We suggest that this proposed explanation should be tested further by a more detailed survey.

If our proposed explanation in terms of intergalactic magnetic field is applicable, then it might imply that sources which are dominated by local effects might not show the alignment effect. These may be sources which show relatively strong magnetic field, which are generated during their evolution. In order to study this possibility we eliminated the sources with degree of polarization ($dop$) greater than 10 % in set 1. We find that this leads to an enhancement of the signal at $n_v = 2$. The P-value now reduces to 0.04 instead of 0.11 seen in the entire sample. For larger values of $n_v$ we do not see a clear trend. On average we find a larger P-value which may arise due to reduced number of sources. It is likely that local effects would be most prominent for smaller values of $n_v$. Hence our results give a suggestive but rather weak evidence that local effects may destroy coherence in sources. These
7. Conclusions

The possibility that sources at high redshift might show correlations at very large distance scales was first indicated by Here it was found that optical polarizations from quasars show alignment over very large distances. A similar test at radio frequencies was first investigated in who found a null result. Here we test for alignment in radio polarizations using a coordinate invariant statistics. We confirm the results of obtained in data which includes only sources with polarization flux greater than 1 mJy. However they did not explore the possibility of alignment at small distances. We find a significant signal for small number of nearest neighbours. The distance scale of alignment here is found to be of order 150 Mpc. At such distances, galaxies are expected to show clustering and the Universe is not expected to be homogeneous and isotropic. Hence the results we find are not inconsistent with Big Bang cosmology.

At larger distances the data set with polarization flux greater than 1 mJy does not yield a significant signal of alignment. However, surprisingly, we find that this set shows unusually large scatter. The significance of alignment tends to fall below $-2\sigma$ at large distance. This anomalous behaviour is seen only when comparing with respect to randomly shuffled data. It is not observed if the random data set is generated from a uniform distribution. We argue that this anomalous behaviour most likely arises since the distribution of observed polarizations is mildly non-uniform. It is possible that this non-uniformity may be caused by bias. However we have shown that it has not effect on our results, as long as we use Procedure I. In any case, due to the presence of such anomalies, it is necessary to test our results with more refined data.

We also find that data with polarization flux less than 1 mJy does not show a significant signal for short distances or, equivalently, small number of nearest neighbours. If the signal was caused by bias, then it would have been dominant in this set. This provides us considerable confidence that the signal of alignment we observe is likely of physical origin. We find a similar trend if we impose cuts on data based on the error in polarization flux. The significance of alignment in data with low error is found to be large at short distances and the effect disappears at large distances. We also find that the significance at small distances becomes better as the error becomes smaller.

The low polarization data, with polarization flux less than 1 mJy, as well as the complete data set, does, however, show a very significant signal alignment at larger distances. The strongest signal is seen at distances of order 500-850 Mpc with significance better than $5\sigma$. Although this set has larger error, it is unlikely that this signal is caused by bias. This is because the sample with polarization flux less than 0.5 mJy does not show significant correlations for any value of $n_v$ with Procedure I.
A bias would have affected this set the most. This signal might indicate the presence of correlations in intergalactic magnetic field at distance scales larger than the scale of galaxy clusters. Due to the presence of large error in the low polarization sample, more refined data is required in order to properly study this effect.

The signal of alignment we find is similar but not identical to that found by\cite{Hutsemékers1998} for optical polarizations. There the signal of alignment was found over cosmologically large distances whereas here the alignment is seen only over somewhat smaller distances of order 150 Mpc for the sample with polarization flux greater than 1 mJy. The sample with polarization flux less than 1 mJy, however, shows alignment for distances of order 500 Mpc.

We have also suggested a physical model which may explain our results. The large scale alignment of radio polarizations is explained by assuming that the background intergalactic magnetic field, which may be of primordial origin, has large scale correlations in real space\cite{Hutsemékers2001,Hutsemékers2004}. This magnetic field provides the seed for the magnetic field of the radio sources. Hence some large distance correlations may be present in magnetic field of these sources also. The radio polarizations would be directly aligned with the source magnetic field and hence show large distance alignment.

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