Multi-quark states were predicted by Gell-Mann when the quark model was first formulated. Recently, numerous exotic states that are considered to be multi-quark states have been experimentally confirmed (four-quark mesons and five-quark baryons). Theoretical research indicates that the four-quark state might comprise molecular and/or tetraquark structures. We consider that the meson containing four different flavors $sbd$ should exist and decay via the $X(5568) \rightarrow B_s \pi$ channel. However, except for the D0 collaboration, all other experimental collaborations have reported negative observations for $X(5568)$ in this golden portal. This contradiction has stimulated the interest of both theorists and experimentalists. To address this discrepancy, we propose that the assumed $X(5568)$ is a mixture of a molecular state and tetraquark, which contributes destructively to $X(5568) \rightarrow B_s \pi$. The cancellation may be accidental and it should be incomplete. In this scenario, there should be two physical states with the same flavor ingredients, with spectra of $5344 \pm 307$ and $6318 \pm 315$. $X(5568)$ lies in the error range of the first state. We predict the width of the second state (designated as $S_2$) as $\Gamma(X_{S_2} \rightarrow B_s \pi) = 224 \pm 97$ MeV. We strongly suggest searching for it in future experiments.

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I. INTRODUCTION

A resonance named $X(5568)$ was reported by the D0 collaboration in the $B_s \pi^\pm$ invariant mass spectrum, where $B_s$ was reconstructed by the $J/\psi \phi$ final state. The mass and width were determined as $(5567.8 \pm 2.9^{+0.9}_{-1.9})$ MeV and $(21.9 \pm 6.4^{+2.5}_{-1.9})$ MeV, respectively. These new observations have stimulated great interest among both theorists and experimentalists because analyses indicate that it should be an exotic state with four different flavors ($sbd$ or its charge conjugates). If this observation is correct, then it is a non-ambiguous signal of the existence of the four-quark exotic state, although multi-quark states were predicted by Gell-Mann when the quark model was first formulated. Recently, several $X, Y, Z$ particles [2–11], have been discovered. However, in most of these states, the charm or bottom flavors are hidden, which makes the confirmation of a four-quark structure difficult. In principle, no law forbids an exotic state with open charm or bottom flavor. Therefore, we consider that such a meson (e.g., $X(5568)$) should exist in nature. In addition, we do not know whether the more favorable structure is a molecular state or a tetraquark, or even their mixture. Clearly, only experiments can give us the answer.

Except for the D0 collaboration, all other important experimental facilities throughout the world, including the LHCb collaboration [12], the CMS collaboration at Large Hadron Collider (LHC) [13], and the CDF collaboration at Fermilab [14], have claimed that no such state can be detected in the $X \rightarrow B_s \pi^\pm$ channel. At the end of 2017, the D0 collaboration again declared that they had confirmed the existence of $X(5568)$ from the decay $X(5568) \rightarrow B_s \pi^\pm$, where $B^0_s$ was reconstructed via a semileptonic decay $B^0_s \rightarrow \mu^\pm D^\mp_s$ [15], but its width shifted to a slightly smaller number $18.6^{+7.9}_{-6.0}(\mathrm{stat})^{+3.5}_{-3.3}(\mathrm{syst})$ MeV than the value measured previously. By contrast, the ATLAS collaboration [16] very recently announced a negative observation of $X(5568)$ in the $B_s \pi$ invariant mass spectrum, i.e., no significant signal was found.

The clear discrepancy between the results obtained by the D0 collaboration and others has led to a dispute because of the obvious significance of $X(5568)$ for understanding the quark model, and thus great efforts have made to resolve this issue. Burns and Swanson suggested [17] that an additional hadron should be undetectable in the $X(5568)$ production process. In addition, as generally argued, the possibility that $X(5568)$ represents a physical particle comprising four different flavors cannot be excluded.

In fact, various studies have provide different opinions about the mysterious $X(5568)$ [18–38]. If it does exist, this clearly raises the question about how it escapes detection.

It is well known that a four-quark state may be a hadronic molecule or a tetraquark [39]. According to theoretical computations by several groups, a pure molecular state or tetraquark makes a substantial contribution to $X(5568) \rightarrow B_s^\pm + \pi^\mp$ and should be “seen” by experimental scanning. Previous studies numerically computed the partial width of the mode in terms of various phenomenological models by assuming it is a molecule, whereas others performed computations by assuming that it is a tetraquark. It is interesting to note that regardless of whether $X(5568)$ was assumed to be a pure hadronic molecule or a tetraquark, the numerical estimates of the partial width of $X(5568) \rightarrow B_s^\pm + \pi^\mp$ were remarkably close (and close to the results obtained by the D0 collaboration). These findings suggest the following possible scenario. The $sbd$ exotic state is a mixture of a molecular state and tetraquark, and they contribute destructively to the golden channel $X(5568) \rightarrow B_s \pi$. The mixing parameter (mixing angle) determines their transition amplitudes, which almost cancel each other. We consider that this cancellation is accidental and incomplete, so a weak signal should exist. Moreover, the uncertainty is large for the recently measured width ($X(5568)$

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where $m$ and molecular state (Quantum Chromodynamics (QCD) sum rules and the center mass of 5757 MeV [22]) was obtained with a large uncertainty. In our calculation, we set $m_M = 5733 \pm 35$ MeV. The mass of the tetraquark has been estimated in several previous studies and different values were obtained. Stanciu [23] employed a simple quark model and determined the mass of the tetraquark as 5530 MeV. In the chiral quark model, Chen et al. [31] determined the tetraquark mass as about 6400 MeV. The scalar tetraquark given by [26] is 5708 MeV. According to [17], a simple sum of the total masses of the constituent quarks (subd) could be close to 6107 MeV (indeed, there is an arbitrariness when selecting the quark masses). If we set the binding energy as 131 MeV [20] or 225 MeV [23], the tetraquark mass $M_T$ should be $5929 \pm 47$ MeV.

After we know the off-diagonal matrix elements, we can diagonalize the matrix and obtain two eigenstates: $S_1$ and $S_2$, and one of them should be identified as $\chi(5568)$, although its mass is not exactly that determined by the D0 collaboration. We have

\[ |S_1 > = \cos \theta |T > + \sin \theta |M >, \]
\[ |S_2 > = - \sin \theta |T > + \cos \theta |M >, \]

where $|T >$ and $|M >$ are the pure tetraquark and molecule state, respectively.

Now, let us consider the hadronic matrix element of $S_1 \rightarrow B_s \pi$. According to [33, 34], $X(5568) \rightarrow B_s \pi$ was calculated in terms of the QCD sum rules while assuming that $X(5568)$ is a tetraquark. Similar results were obtained in these two studies. The coupling constant was obtained as $g_{X_s B_s \pi} = (10.6 \pm 2.1)$ GeV by [34]. We calculated this transition rate in the light front quark model while assuming that $X(5568)$ is a molecular state [35] and the corresponding coupling constant was determined as $g_{X_s B_s \pi} = (13.0 \pm 2.4)$ GeV.

In the mixture scenario, the hadronic transition amplitude is written as

\[ B_s \pi |H_{eff}| S_1 = \cos \theta < B_s \pi|H^{(1)}_{eff}|T > + \sin \theta < B_s \pi|H^{(2)}_{eff}|M >, \]

where $H_{eff} = H^{(1)}_{eff} + H^{(2)}_{eff}$. The effective interaction $H_{eff}$ can be divided into two parts, where $H^{(1)}_{eff}$ corresponds to a quark–antiquark exchange between diquark and antidiquark to make color singlet final mesons, whereas $H^{(2)}_{eff}$ is responsible for dissolved molecular state.

According to our strategy, i.e., letting the contributions of the tetraquark and molecular state fully cancel each other (almost), we can fix the mixing angle $\theta$ as $(\sim 50.8 \pm 7.8)^\circ$ when $S_1$ is regarded as the narrow $X(5568)$.

After substituting the values of $m_T$, $m_M$ and $\theta$ into Eq. (2) and diagonalizing the mass matrix, we have two eigenvalues comprising $m_{S_1}$ and $m_{S_2}$, which are $m_{S_1} = 5344 \pm 307$ MeV and $m_{S_2} = 6318 \pm 315$ MeV, respectively. It should be noted that $X(5568)$ lies in the error range of $m_{S_1}$. In this scheme, another physical state $S_2$ exists that should also decay into $B_s \pi$ because for $S_2$, the tetraquark and molecule components contribute constructively to the $B_s + \pi$ final state, and its partial width should be large. Using the values $g_{X_s B_s \pi}$, $g_{X_s B_s \pi}$ and the mass of $S_2$, we predict the width as $\Gamma(X_{S_2} \rightarrow B_s \pi) = 224 \pm 97$ MeV. Naturally, the experimental search for a wide resonance at $B_s \pi^* \pi$ channels is crucial for testing our ansatz.

We can also study the possible charmed partners of these states. Similar to the $BK$ case, the bare mass of the $DK$
molecular state is \(2311 \pm 35\) MeV, whereas the bare mass of a tetraquark of \(su \bar{c} \bar{d}\) is \(2589 \pm 47\) MeV, which is consistent with that reported by \[41\]. However, in our case, the tetraquark of \(su \bar{c} \bar{d}\) is not a physical state. Under heavy quark symmetry, we use the same mixing angle to obtain two physical states with masses of \(1759 \pm 414\) MeV and \(3141 \pm 417\) MeV. By contrast, \[25\] estimated the mass of the tetraquark state corresponding to \(X(5568)\) as 2262 MeV, which is slightly larger than our estimate for the first charm partner (\(1759 \pm 414\) MeV).

### III. CONCLUSION AND DISCUSSION

To reconcile the discrepancy between the results obtained for \(X(5568)\) by the D0 collaboration and most other important experimental collaborations, we propose that based on the quark model, an exotic state with four different flavors (\(s u \bar{c} \bar{d}\)) should exist but it is a mixture of a tetraquark and hadronic molecule. There should be two eigenstates comprising \(S_1\) and \(S_2\), which are the on-shell physical particles. For the lighter \(S_1\) with a mass similar to that assumed for \(X(5568)\), the tetraquark and molecule components contribute destructively to the \(B_s \pi^+\) final state. This cancellation allows it to escape detection. We consider that this cancellation is accidental and incomplete because no principle can ensure full cancellation. Therefore, it is possible that one collaboration has observed a small signal whereas others have not.

In our computations, we employed theoretical estimates of the masses of the tetraquark and molecular state as inputs. The hadronic transition matrix elements of \(\langle B_s \pi^+|\mathcal{H}_{eff}|S_1(S_2)\rangle\) were calculated in different models, so remarkable theoretical uncertainties might have been involved. Thus, we cannot guarantee accurate quantitative results, but the qualitative consequences are reasonable and acceptable.

Moreover, we predicted an extra exotic state \(|S_2\rangle\) as the partner of \(|S_1\rangle\) with the same quark contents, but different combinations of the tetraquark and molecular state. It is important to test our ansatz by searching for this new particle in the \(B_s \pi^+\) final state.

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