An optical transition-edge sensor with high energy resolution

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Abstract
Optical transition-edge sensors (TESs) have shown an energy resolution for resolving the number of incident photons at the telecommunication wavelength. However, a higher energy resolution is required for biological imaging and microscopic spectroscopy. In this study, we tested an Au/Ti (10/20 nm) bilayer TES that showed a high energy resolution. The high energy resolution was achieved by lowering the critical temperature $T_c$ to 115 mK, and the resultant energy resolution was 67 meV full width at half maximum (FWHM) at 0.8 eV. When $T_c$ was reduced to 115 mK, the theoretical resolution would scale up to 30 meV FWHM, considering that the typical energy resolution of optical TESs was 150 meV and $T_c$ was 300 mK. To investigate the difference between the theoretical expectation (30 meV) and the measured value (67 meV), we measured the complex impedance and current noise of the TES. We found excess Johnson noise in the TES; the excess Johnson noise term $M$ was 1.5 at a bias point where the resistance was 10% of the normal resistance. For reference, the above mentioned TES was compared with a TES showing typical energy resolution (156 meV FWHM). We also discussed factors that improved or inhibited the energy resolution.

Keywords: transition-edge sensor, thermodynamic noise, complex impedance, single-photon detector, photon-number resolving detector

(Some figures may appear in colour only in the online journal)

1. Introduction

Transition-edge sensors (TESs) are superconducting single-photon detectors [1] and can resolve the energy of a single photon by detecting a slight change in temperature due to the absorption of the photon [2]. The detector design is optimized to detect photons of desired energy, such as gamma-ray, x-ray, ultraviolet, visible, and near-infrared [3, 4]. We have developed optical TESs to detect visible and near-infrared photons. An optical TES can resolve the number of photons absorbed in the detector. This photon-number resolution is essential in fields such as quantum computing [5], quantum information [6–8], and quantum metrology [9, 10]. In these fields, a fast detector response is required. The sizes of optical TESs are set small to obtain sub-\mu s responses. A TES has achieved a time constant of 150 ns [11]. The small size also allows for efficient coupling to an optical fibre. A 105 meV full width at half maximum (FWHM) was reported as the highest energy resolution [12], which is sufficient to identify the number of photons in telecommunication wavelength bands.

Recently, new applications of optical TESs have been proposed, e.g. the combination of an optical TES and a scanning microscope [13, 14]. In this combined system,
two-dimensional colour images were obtained by resolving the energy of each photon. When multicolour imaging is performed using photon detectors without an energy resolution, filters are generally required. To obtain colour images with such detectors, monochromatic images captured with filters transmitting light at wavelengths of interest are overlaid. A TES does not necessitate this procedure and provides multicolour images without filters.

A TES has a low dark count rate and can be used for imaging under weak illumination. It is sensitive to photons at wavelengths ranging from near-infrared to visible light and provides visible and near-infrared images in a wide bandwidth. This feature can open a new window for observation of biological samples. The combined system allows the use of multiple fluorescent dyes for imaging [12]. The number of usable dyes is determined by the energy resolution of a TES. The spacing between adjacent emission peak wavelengths of dyes must be wider than the wavelength resolution. This system can also be used for microscopic spectroscopy, where the microscope focuses on a fixed point and collects photons over a long period to obtain a spectrum. As the energy resolution of a detector becomes higher, measured spectra can provide more information.

In photon-number counting, the requirement on the energy resolution is less stringent. A detector must exhibit an energy resolution sufficiently high to avoid an overlap between peaks of adjacent photon-number states. To reduce an error in the photon-number down to 1%, the peaks should be separated by more than 5.2σ, where σ denotes the standard deviation of the peaks. For this purpose, the typical energy resolution of optical TESs (150 meV) is sufficient for use at 1550 nm telecommunication band. Moreover, for biological imaging and microscopic spectroscopy, higher resolution is desired.

The energy resolution ∆EFWHM is expressed in terms of the wavelength resolution as ∆EFWHM = λ²∆λFWHM/hc, where λ denotes the wavelength of incident photons, h denotes Planck constant, and c denotes the speed of light. When the energy resolution is 0.1 eV, the wavelength resolution at 550 nm (green) is 24 nm, which may be comparable to the spacing between adjacent emission peak wavelengths of dyes. To separate these peaks, the energy resolution should be improved.

When the bath temperature is set to be the theoretical energy resolution of a TES is given by [2]

\[
\Delta E_{\text{FWHM}} = 2\sqrt{2\ln 2} \sqrt{\frac{4kT_c^2 C}{\alpha_f}} \sqrt{n(1+2/\beta_f)(1+M^2)/2},
\]

where \( k \) denotes the Boltzmann constant, \( T_c \) denotes the critical temperature, \( C \) denotes the heat capacity, \( \alpha_f = \frac{T - T_c}{R} \) denotes the temperature sensitivity, \( \beta_f = \frac{1}{\alpha_f T_c} \) is the current sensitivity, \( R \) denotes the resistance of the TES, \( I \) denotes the current flowing through the TES, \( n = 5 \) in the electron-phonon limited conductance exponent, and \( M \) expresses the excess Johnson noise term. We assume here that ratio between the bath temperature \( T_{\text{bath}} \) and \( T_c \) is negligible, i.e., \( T_{\text{bath}}/T_c \ll 1 \). The energy resolution can be improved by lowering \( T_c \), the heat capacity \( C \), and \( \beta_f \) and increasing \( \alpha_f \). At present, there is no established method to control \( \alpha_f \) and \( \beta_f \). To reduce the heat capacity, the size of a TES should be reduced. The size should be comparable to or larger than the mode field diameter (MFD) of an optical fibre to obtain the high detection efficiency of photons. The MFD of single-mode fibres is typically several \( \mu \)m, e.g., 3.2 \( \mu \)m at 1550 nm for UHNA-7. When a TES is smaller than MFD, it enhances the coupling loss and deteriorates the detection efficiency [15]. The size of a TES is typically 8 \( \mu \)m × 8 \( \mu \)m, which is sufficient with MFD. To obtain a high energy resolution while maintaining the detection efficiency, we chose to lower \( T_c \). A low-\( T_c \) TES has a reduced heat capacity, which is proportional to \( T_c \) with a given volume. From equation (1), the energy resolution is proportional to \( T_c^{-4/5} \). The typical \( T_c \) and energy resolution of our TESs are 300 mK and 150 meV, respectively. When this is lowered to 100 mK, the resolution is expected to be 30 meV.

In this study, we show that the energy resolution reached 67 meV FWHM at 0.8 eV by lowering \( T_c \) to 115 mK. The energy resolution was significantly enhanced but did not reach the expected resolution of approximately 30 meV. To investigate the difference between the theoretical expectation and the measured value, the current noise of the detector was measured and compared with the theoretical noise. For TESs designed to detect x-ray photons, excess Johnson noise is one of the limiting factors [16–18]. In this study, we show that there was the excess Johnson noise in the optical TES, assess if it significantly contributed to the energy resolution deterioration, and discuss the major factors inhibiting the energy resolution from reaching below 50 meV.

We also tested an optical TES that showed a typical energy resolution (156 meV FWHM at 1.46 eV) and compared the two TESs to discuss their differences and similarities.

2. Measurements and results

2.1. Optical TESs

In this study, we tested two TESs: one that achieved the high energy resolution (TES2 in table 1) and another that showed a typical resolution (TES1 in table 1). They have a Ti/Au thickness of 20/10 nm. Their size was 8 \( \mu \)m × 8 \( \mu \)m. The device fabrication was described in [15]. The critical temperature (\( T_c \)) of TES2 was 115 mK, whereas that of TES1 was 143 mK. The difference in \( T_c \) could be because they were fabricated in different batches. The small area and thin thickness of the TESs may also contribute to the variation of \( T_c \). The TESs were embedded in optical cavities optimized to maximize the detection efficiency at wavelengths of interest (950 and 1550 nm for TES1 and TES2, respectively). The TESs were cooled in a dilution refrigerator. The bath temperature was set to 7 mK.

The energy resolution was measured using pulsed lasers. Figure 1 shows the response of TES2 to a pulsed laser at 1550 nm (0.8 eV). The energy resolution was 67 meV. Because TES1 was designed to detect photons at around 950 nm, an 850 nm (1.46 eV) pulsed laser was used. The measured energy resolution was 156 meV. The system detection efficiency and the fall time of signals are shown in table 1. To obtain the time constant, the TESs were biased at 10% of...
Table 1. Parameters of TESs.

|                  | TES1                          | TES2                          |
|------------------|-------------------------------|-------------------------------|
| Material         | Ti (20 nm)/Au (10 nm)         | Ti (20 nm)/Au (10 nm)         |
| Size (µm²)       | 8 x 8                         | 8 x 8                         |
| Critical temperature (mK) | 143                            | 115                           |
| Normal resistance (Ω) | 2.7                            | 2.8                           |
| Energy resolution (FWHM) (meV) | 156 (at 1.46 eV)               | 67 (at 0.8 eV)                |
| Thermal model    | Single-block                  | Two-block                     |
| C/α_I (1 K⁻¹) at 0.1Rn | 4.1 x 10⁻¹⁸                    | 4.8 x 10⁻¹⁹                   |
| Excess Johnson noise term M at 0.1Rn | 0.95                           | 1.5                           |
| System detection efficiency (%) | 58 (at 1.46 eV)               | 60 (at 0.8 eV)                |
| Time constant (µs) | 10.3                           | 4.6                           |
| Critical current (µA) | 1.7                            | 7.3                           |

Figure 1. TES2 response on a pulsed laser at 1550 nm (0.8 eV). (a) The energy resolution of an Au/Ti bilayer TES (TES2 in table 1) reached 67 meV FWHM at 0.8 eV (the single-photon peak) by lowering Tc (115 mK). The solid line is fit to a Gaussian function. (b) Electrical signals.

the normal resistance. Table 1 also shows the critical current at 7 mK.

2.2. Complex impedance

We measured the complex impedance to extract parameters characterizing the TESs and calculate the theoretical values for the current noise and energy resolution. The complex impedance is determined by a thermal model of a TES. The simplest model shown in figure 2(a) is a single-block model comprising a TES and the thermal bath. This model described the behaviour of optical TESs well [19, 20]. The complex impedance of a single-block TES at an angular frequency ω can be expressed as follows: [2, 21]

\[ Z_{TES}(\omega) = R(1 + \beta_I) + \frac{R\mathcal{L}}{1 - \omega^2\tau_s}, \]

where \( \mathcal{L} \) denotes the constant-current loop gain given by \( \mathcal{L} = R^2\alpha_I/G_{tes,b}T \), \( G_{tes,b} \) denotes the thermal conductance between the TES and the thermal bath, and \( \tau_s \) denotes a time constant given by \( \tau_s = \tau_0/(1 - \mathcal{L}) \), where \( \tau_0 \) is an intrinsic time constant, \( C/G_{tes,b} \).

The complex impedance was measured by injecting small signals from a network analyzer into a voltage-biased TES on a cold stage at 7 mK. The current flowing through the TES was measured using a SQUID. Twisted pair cables for signal transmission from the room temperature electronics to the cold readout and vice versa. Detailed descriptions of the measurement method was found in [22]. As shown in figure 3(a), the complex impedance of TES1 can be well-described by equation (2). Figure 4(a) shows the extracted \( \mathcal{L} \) and \( \beta_I \).

For consistency check, the measured time constant of TES1 was compared with that calculated from the parameters extracted from the complex impedance. When the shunt resistance in the bias circuit is negligible, the fall time \( \tau_{eff} \) can be expressed as follows [2]:

\[ \tau_{eff} = \frac{C}{G} \frac{1 + \beta_I}{1 + \beta_I + \mathcal{L}}. \]

The calculated time constant using equation (3) was 9.7 µs and was comparable with the measured time constant of 10.3 µs.

On the other hand, the responses of TES2 deviated from that expected from the single-block model (figure 3(b), solid lines). Instead of the single-block model, a two-block model (figure 2(b)) was adopted for TES2. The two-block model includes an additional thermal block. When the thermal conductance between the additional body and the thermal bath is zero, the model represents a hanging model. This model
describes a TES with an absorber [16, 23]. When the thermal conductance between the TES and the thermal bath is zero, it represents an intermediate model [24]. We show that the behaviour of TES2 can be explained by the two-block model in section 2.3. The complex impedance given by the model is expressed as [24]

\[ Z_{TES}(\omega) = R(1 + \beta_I) + \frac{R L_{\text{eff}}(2 + \beta_I)}{1 - L_{\text{eff}}} \left[ 1 + i \omega \tau_I - g_{\text{two-body}} \frac{1 - L_{\text{eff}}}{1 - i \omega \tau_I} \right], \]  

(4)

where \( \tau_I \) denotes the time constant of the additional body, \( L_{\text{eff}} \) denotes the effective loop gain and \( g_{\text{two-body}} \) is a function of thermal conductance in the system. These parameters can be expressed as follows:

\[ L_{\text{eff}} = \frac{R I^2 \alpha_I}{(G_{\text{tes},1}(T_{\text{tes}}) + G_{\text{tes},b})}, \]  

(5)

\[ \tau_I = \frac{C_{\text{tes}}}{(G_{\text{tes},1}(T_{\text{tes}}) + G_{\text{tes},b})(1 - L_{\text{eff}})}, \]  

(6)

\[ \tau_1 = \frac{C_1}{G_{\text{tes},1}(T_1) + G_{1,b}}, \]  

(7)

where \( C_1 \) and \( T_1 \) denote the heat capacity and temperature of an additional body, respectively, \( G_{\text{tes},1} \) denotes the thermal conductance between the TES and additional body, and \( G_{1,b} \) denotes the thermal conductance between the additional body and thermal bath. As shown in figure 3(b) (dashed lines), the measured complex impedance of TES2 was fitted well with equation (4). Assuming that \( g_{\text{two-body}} \) and \( \tau_1 \) were independent of a bias point, they were 0.326 and 11.4 \( \mu s \), respectively, as extracted from the measured complex impedance shown in figure 3(b). \( \tau_1 \) was similar to \( \tau_0 \) (13.9 \( \mu s \) at 0.5 \( R_n \)). Note that the measured fall time (4.6 \( \mu s \)) differs from the value calculated using equation (3) (2.3 \( \mu s \)), which is derived from the single-block model. To obtain the time constant for the two-block model, three coupled linear differential equations are needed to be solved.

In TES2, although most essential parameters can be extracted from the complex impedance, some parameters, such as the ratio between \( G_{\text{tes},b} \) and \( G_{\text{tes},1} \), remain unknown. Therefore, \( G_{\text{tes},b} \) is unknown, and \( \alpha_I \) cannot be derived. However,
Figure 4. Parameters extracted from the measured complex impedance: (a) current sensitivity, and (b) loop gain.

Figure 5. Current noise of TES1. (a) Noise at 0.2 \( R_n \). TFN and Johnson noise were calculated using parameters extracted from the complex impedance shown in figure 3(a). The temperature at the shunt resistor is assumed to be the same as the bath temperature. (b) Noise at several bias points. The black lines show the sum of the TFN, Johnson noise, shunt resistor Johnson noise, and readout noise. The excess Johnson noise term is considered as a function of the bias point.

C/\( \alpha I \), which is an essential parameter for the energy resolution, can be directly extracted from the complex impedance. In the two-block model, \( C/\alpha I \) can be calculated using equations (5) and (6), \( L_{eff} \), and \( \tau_I \). In the single-block model, from equation (1), the energy resolution is proportional to \( \sqrt{C/\alpha I} \), assuming that the readout noise is zero. This also applies to the two-block model, as shown in appendix. The values of \( C/\alpha I \) extracted from the complex impedance measurements are listed in table 1. TES2 showed smaller \( C/\alpha I \) than TES1. This agrees with the fact that TES2 exhibited a better energy resolution. Using equation (4), the current sensitivity and effective loop gain of TES2 were extracted from the complex impedance (figure 4(b)).

2.3. Thermodynamic noise

In this subsection, we compare the measured noise with the theoretical noise level to evaluate the excess Johnson noise. The results of noise measurements for TES1 and TES2 are shown in figures 5 and 6, respectively.

The theoretical noise requires the thermal conductance between a TES and the heat bath as an input parameter. The parameter is used for calculating the thermal fluctuation noise (TFN) [2]. The thermal conductance is associated with the Joule power dissipation of a TES. The power dissipation is equal to the power flow to the heat bath and can be expressed as follows:

\[
P_J = K(T^n - T_{bath}^n),
\]

where \( n \) denotes the exponent determined by the nature of the thermal link to the heat bath. The parameter \( K \) is associated with the thermal conductance by \( G = nKT^{n-1} \). The cool-down mechanism of the TESs can be explored by measuring the Joule power dissipation as a function of the bath temperature. The measured \( P_J \) for TES1 and TES2 at a bias point of 0.5 \( R_n \) were fitted with equation (9), and the resultant exponents were \( n = 5.07 \) and 4.95, respectively. This implies that the TESs were cooled down by electron-phonon coupling.

The measured thermal conductance of TES1 \( (G_{tes,b}) \) was \( 9.9 \times 10^{-12} \) W K\(^{-1}\). For TES2, which agreed well with the two-block model, the thermal conductance was considered as a function of thermal conductance in the system \( (G_{tes,b}, G_{1,b} \) and \( G_{tes,1}) \), and the measured value was \( 6.3 \times 10^{-12} \) W K\(^{-1}\).

In the following, we show that the current noise in TES2 was well-fitted with the two-block model. In this model, we can define the effective thermal conductance \( G_{eff} \), which is equal to the thermal conductance extracted from the Joule power dissipation. We also show that we can calculate the
theoretical current noise without knowing $G_{\text{tes,b}}$ or the exact form of $G_{\text{eff}}$.

Using $G_{\text{eff}}$ and equation (6), the effective heat capacity can be defined as $C_{\text{eff}} = \gamma G_{\text{eff}}(1 - \mathcal{L}_{\text{eff}})$. $C_{\text{eff}}$ of TES2 was $1.3 \times 10^{-16}$ J K$^{-1}$ at 0.1$R_b$, while the heat capacity of TES1 was $1.9 \times 10^{-16}$ J K$^{-1}$ at 0.1$R_b$ and was comparable with TES2. According to $C/\alpha_T$ (table 1) and $C_{\text{eff}}$, $\alpha_T$ of TES2 is estimated to be an order of magnitude higher than TES1.

To calculate the theoretical current noise, we need the responsibility of the TES to small signals. The responsivity $s_0(\omega)$ can be associated with the complex impedance $Z_{\text{TES}}$ by [24]

$$s_0(\omega) = \frac{1}{(Z_{\text{tes}} + R_b + i\omega L)0} \frac{Z_{\text{tes}} - R_0(1 + \beta j)}{R_0(2 + \beta)}, \quad (10)$$

where $R_{sh}$ denotes the shunt resistance in the bias circuit, 18 mΩ in our setup, $L$ denotes the SQUID input impedance and is 18 nH. The Johnson noise is expressed as follows:

$$|I_0|^2 = 4kT_{\text{TES}} \frac{(1 + 2\beta)(1 + M^2)}{R_0(2 + \beta)} \left[ \frac{Z_{\text{tes}} + R_0}{Z_{\text{tes}} + R_b + i\omega L} \right]^2. \quad (11)$$

The Johnson noise due to the shunt resistor in the bias circuit is $|I_0|^2 = 4kT_{sh}R_b/|Z_{\text{tes}} + R_b + i\omega L|^2$, where $T_{sh}$ denotes the temperature of the shunt resistor. The contribution of $|I_0|^2$ to the total current noise was negligible (figures 5(a) and 6(a)). Notably, the temperature at the shunt resistor is assumed to be the same as the bath temperature.

Next, we consider the TFN current noise of the single-block model. It is expressed as follows:

$$|I_0|^2_{\text{TFN}} = 4kT_{\text{TES}}G_{\text{tes,b}}F(T_{\text{tes}}, T_{\text{bath}}) \left| s_0(\omega) \right|^2 \equiv P_{\text{tes,b}} \left| s_0(\omega) \right|^2, \quad (12)$$

where $P_{\text{tes,b}}$ denotes the power spectral density of the TFN. $F(T_{\text{tes}}, T_{\text{bath}})$ depends on the thermal link. In the radiative limit, it becomes $F(T_{\text{tes}}, T_{\text{bath}}) = \left[ (T_{\text{bath}}/T_{\text{tes}})^\tau_1 + 1 \right] / 2$ [25].

The measured current noise in TES1 shown in figure 5 agrees well with the sum of the theoretical and readout noises. The theoretical noise was calculated using the measured thermal conductivity and parameters extracted from the complex impedance. The theoretical noise is a sum of the TFN noise, the Johnson noise including the excess term $M$, and the shunt resistor Johnson noise. $M$ was calculated at each bias point (figure 7).

The TFN of the two-block model can be expressed as follows: [24]

$$|I_0|^2_{\text{TFN}} = \left| s_0(\omega) \right|^2 \left[ \frac{P_{\text{tes,b}}^2 + P_{\text{tes,1}}^2}{1 + \left| \frac{P_{\text{tes,1}}}{P_{\text{tes,b}}} \right|^2 \left[ \frac{G_{\text{tes,b}}(T_{\text{tes}})}{G_{\text{tes,1}}(T_{\text{tes}})} \right]^2 + G_{1,b}^2 - 1 \right] \right], \quad (13)$$

where $P_{\text{tes,b}}$ denotes the TFN between the TES and additional thermal block, and $P_{1,b}$ is that between the additional block and the thermal bath.

As shown in figure 6(b), the measured noise fitted well to equation (13). The parameters that can be extracted from the fit are $P_1 \equiv P_{\text{tes,b}}^2 + P_{\text{tes,1}}^2$ and $P_2 \equiv P_{\text{tes,1}}^2 \left( \left( \frac{G_{\text{tes,b}}(T_{\text{tes}})}{G_{\text{tes,1}}(T_{\text{tes}})} \right)^2 + G_{1,b}^2 - 1 \right)$. We found $P_1/P_{\text{TFN,b}} = 4.12$ and $P_2/P_{\text{TFN,b}} = -1.15$, where $P_{\text{TFN,b}} = \sqrt{2kT_{\text{eff}}G_{\text{eff}}(T_{\text{bath}}/T^{\mu+1} + 1)}$ is the TFN assuming that TES2 follows the single-block model and its thermal conductance to the heat bath is $G_{\text{eff}}$. Notably, $\tau_1$ extracted from the complex impedance was used for the fits. To determine $P_{\text{tes,b}}$, $P_{\text{tes,1}}$, $G_{1,b}$, and $T_{\text{bath}}$ individually, additional experiments may be necessary. Without knowing these values, the measured noise was able to be fitted to equation (13), and the excess Johnson noise term was obtained (figure 7).

### 2.4. Thermal models

In this section, we discuss models that could explain the behaviour of the TESs based on detector physics. In complex impedance measurements, the response of TES1 was effectively explained by the single-block model (figure 2(a)). A
with resistance $R_1$, the electrical differential equation is as follows:

$$L \frac{d}{dt} (I + I_1) = V - \left( R_L + \left( \frac{1}{R} + \frac{1}{R_1} \right)^{-1} \right) (I + I_1), \quad (14)$$

where $I_1$ denotes the current flowing through $R_1$ and $V$ denotes the voltage across the bias circuit. We assume that $R_1$ is constant.

The linearized differential equation is given by

$$L \frac{d}{dt} (\Delta I + \Delta I_1) = \Delta V - (\Delta I + \Delta I_1) R_L - \frac{(\alpha R I)}{T_{\text{tes}}} \Delta T_{\text{tes}} + (1 + \beta) R \Delta I_1. \quad (15)$$

When $R_1 \gg R$, in the frequency domain, $\Delta I_{1,\omega} = \frac{Z_{\text{tes}}(\omega)}{R_1} \Delta I_1$, and thus $\Delta I_1$ is much smaller than $\Delta I$. When $|\Delta I_1| \ll |\Delta I|$, equation (15) can be reduced to the equation without the resistance $R_1$. Therefore, the contribution of $I_1$ to ETF is negligible.

Next, we will show that $\Delta I_1$ does not affect the thermal response of the additional body. The linearized equation regarding the additional body is

$$C_1 \frac{d \Delta T_{1}}{dt} = G_{\text{tes},1}(T_{\text{tes}}) \Delta T_{\text{tes}} - (G_{1,b}(T_1) + G_{\text{tes},1}(T_1)) \Delta T_{1} + 2 V_{\text{tes}} \Delta I_1, \quad (16)$$

where $V_{\text{tes}} = R_1 I_1$ is the voltage across the TES. To show that $V_{\text{tes}} \Delta I_1$ is negligible in equation (16), the linearized equation regarding the time derivative of the change in temperature of the TES should be considered,

$$C_{\text{tes}} \frac{d \Delta T_{\text{tes}}}{dt} = V_{\text{tes}} (2 + \beta) \Delta I + \frac{R_1}{T_{\text{tes}}} G_{\text{tes},1}(T_{\text{tes}}) - G_{\text{tes},1}(T_{\text{tes}}) \Delta T_{\text{tes}} + G_{\text{tes},1}(T_1) \Delta T_{1}. \quad (17)$$

Comparing equations (16) and (17), the terms $G_{\text{tes},1}(T_{\text{tes}}) \Delta T_{\text{tes}} - G_{\text{tes},1}(T_1) \Delta T_{1}$ should be in the same order as $V_{\text{tes}} (2 + \beta) \Delta I$, which is much greater than $V \Delta I_1$. Thus, the term regarding $\Delta I_1$ in equation (16) is negligible. Therefore, the behaviour of a TES in parallel with large resistance can be described by the two-block model which assumes that an additional body is not heated by the current flowing through it. The Johnson noise of $R_1$ is also negligible, which is obvious from equation (11).

2.5. Energy resolution

We examined the measured current noise of the TESs and found that it matched the sum of the TFN and Johnson noise term implemented. Now, we can readily calculate the expected energy resolution based on the

![Figure 7](https://example.com/figure7.png) Excess noise parameter $M^2$ as a function of a bias point.

more detailed model of TES1 is presented by a thin metal film directly deposited on an insulating substrate (figure 8(a) [26]). The relaxation time of phonons in the TES was thought to be much faster than probe signals for measuring the complex impedance. Therefore, the phonon system can be effectively treated as the heat bath, and the detector response was well-described by the single-block model.

In the case of TES2, its thermal model became more complicated. From the measured complex impedance and current noise, the thermodynamic model was determined to be the two-block model. However, the components of the additional block (electrons or photons, etc) remained unknown. In this study, we present a model that could explain the behaviour of TES2.

Considering that the time constant of the additional block ($\tau_1$) was similar to that of the TES ($\tau$) when the loop gain was zero, it is straightforward to assume that the block could be an electron system. When the system is cooled by electron-phonon weak coupling, the time constant is determined by the temperature. Therefore, the temperature should be similar to $T_{\text{TES}}$.

To maintain the temperature above the bath temperature, the electron system must be heated. In our setup, the only mechanism that could heat the electron system was Joule dissipation. Thus, the additional block should have the resistance $R_1$. To maintain consistency with the two-block model, the current flowing through the block must be negligible and does not affect the electrothermal feedback (ETF) of the TES, implying that $R_1$ must be much larger than $R_{\text{TES}}$ and was connected in parallel with the TES. The model could have the form shown in figure 8(b). The large resistance could be created by a partial volume in the TES. The volume should be normal regardless of the voltage applied to the TES. The large resistance may be because the cross section of the volume is small.

We show that large resistance in parallel with a TES does not affect ETF and the set of equations is reduced to that derived by the two-block model. When a TES is in parallel
Figure 8. Schematic of models for (a) TES1 and (b) TES2. In both TESs, the relaxation time of phonons is much faster than probe signals for measuring the complex impedance. Therefore, the phonon system can be effectively treated as the thermal bath.

Table 2. Energy resolution at 0.1\(R_n\).

|                      | TES1 | TES2 |
|----------------------|------|------|
| Measured energy resolution \((\Delta E_m)\) (meV) | 156  | 67   |
| Energy resolution calculated from the measured current noise \((\Delta E_c)\) (meV) | 89   | 40   |
| Energy resolution without excess Johnson noise (meV) | 81   | 34   |
| Energy collection efficiency \(\eta\) (%) | 63   | 88   |
| \(\Delta E_c/\eta\) (meV) | 141  | 46   |
| \(\Delta E_{other}\) (meV) | 67   | 50   |

If the current noise is only the source for fluctuations in the measured energy of a photon, the measured energy resolution \(\Delta E_m\) should agree with the calculated energy resolution obtained by \(\Delta E_c/\eta\). However, as in table 2, \(\Delta E_c/\eta\) is significantly smaller than \(\Delta E_m\). The difference can be written as \(\sqrt{\Delta E_m^2 - \Delta E_c^2/\eta^2} \equiv \Delta E_{other}\). There must be some unexplained degradation in the energy resolution. In TES2, whose energy resolution is less than 100 meV, \(\Delta E_{other}\) significantly influenced the energy resolution. Therefore, it is necessary to prioritize the reduction of \(\Delta E_{other}\) to obtain higher energy resolution. As such, we should understand the unexplained sources.

2.6. Towards higher energy resolution

From the noise measurements, the optical TESs have the excess Johnson noise. The effect of the excess Johnson noise on the energy resolution of TES2 was relatively small. It was strongly affected by fluctuations due to unexplained sources.

It has been considered that the reduction of the current noise is crucial to improve the energy resolution of an optical TES. Therefore, a TES should have low-\(T_c\) and \(\beta\), a small heat capacity, high \(\alpha_1\), and low excess Johnson noise. We have successfully fabricated such a TES.

The study of the excess Johnson noise may be crucial for improving the energy resolution, though its contribution is small, as shown in table 2. For the TESs designed to detect x-ray photons, the excess Johnson noise is known to be one of the limiting factors. Optical TESs have a different detector design than x-ray TES; the optical TESs have no membrane or absorber and are an order of magnitude smaller in size. Thus, the cause of the excess Johnson noise in optical TES could also differ. Many groups have studied the excess Johnson noise in x-ray TES. However, the research on the excess Johnson noise in optical TESs is inadequate. This is an issue that needs to be addressed in the future.

To obtain higher energy resolution, the source of \(\Delta E_{other}\) should be investigated and reduced. The factors of \(\Delta E_{other}\) could be (1) nonlinear response, (2) error in the pulse height.
estimate of a signal by the optimal filtering method, and (3) downconversion phonon noise [27].

We discussed the energy resolution in the small-signal limit, assuming that detector response is linear. If there is nonlinearity in a TES, the energy resolution calculated based on the small-signal theory may deviate from the measured resolution.

In a linear detector, the pulse height should be proportional to the total energy of photons. When a linear TES is irradiated with a pulsed laser, the pulse height is proportional to the number of photons simultaneously absorbed there. In reality, as shown in figure 9, the measured pulse height was fitted to a quadratic function rather than a linear function. The quadratic function was used to convert the pulse height to the energy and then the energy resolution was obtained. TES2 showed more obvious nonlinear responses than TES1 and is in large-signal limit. Notably, the pulse height spectra including figure 1 were calibrated using a quadratic function, from which the energy resolution was obtained.

It is questionable whether it is optimal to fit with a quadratic function over the entire energy range. In figure 9, the region where TES2 responded linearly was unclear. There was a possibility that the TES responded linearly to small-energy deposition, such as a region from 0 eV to the single-photon peak. The single-photon peak in the pulse height spectrum was fitted with the Gaussian function, was converted to the energy spectrum using a linear function and the resultant energy resolution was 62 meV. The change in the resolution was 10% from the energy calibration using the quadratic function. To find an optimal function for the energy calibration, large-signal-limit models [3] should be studied. In the case of TES1, which exhibited better linearity, applying the same procedure to the measured pulse height spectrum resulted in an energy resolution of 150 meV. The change in the resolution was 4%, which was smaller.

The nonlinear responses are also linked to the optimal analysis of the pulse height. In the analysis, the static noise is assumed. However, a large change in the resistance (or a bias point) invalidates this assumption, possibly causing an error in the pulse height and deteriorating the energy resolution. In future work, to understand the response of a TES in more detail, we will study the nonlinear detector response between the small-signal limit and large signal region where a TES is saturated.

To improve the energy resolution, lowering $T_e$ further could still be an option in fields where slow detector response is acceptable. In applications that require high energy resolution, such as biological imaging and microscopic spectroscopy, the counting rate of incident photons is occasionally low, and a slow detector response may be acceptable. For a low-$T_e$ TESs its heat capacity becomes smaller. Therefore, with a given energy of a photon, a change in the temperature and resistance will be larger, and the responses will be nonlinear. Therefore, it is imperative to have a better understanding of nonlinearity.

In this study, we focused on the current noise. Another paramount source of fluctuations in the measured energy is a downconversion phonon noise. The noise is independent of the current noise and is originated from fluctuations in the number of phonons escaping into a substrate. Degradation due to downconversion phonon noise is greater at higher photon energy [27]. In TES2, the energy resolution at 1.46 eV was 77 meV, which was lower than that at 0.8 eV. To determine what the primary source of the degradation is, detailed studies of downconversion phonon noise and the nonlinear response of the TES are necessary. It should be noted the energy resolution of the two-photon peak of the pulsed laser at 0.8 eV (the total energy is comparable to 1.46 eV) was 78 meV.

The downconversion phonon noise could be reduced by forming a TES on the membrane to limit the escape paths of phonons. This membrane structure was implemented in another study [28] to reduce energy loss and enhance the energy collection efficiency $\eta$. We have achieved a fairly high $\eta$ without a membrane. However, nonuniformity in $\eta$ was quite high, i.e. typically $\eta$ was from 50% to 90%. By adopting the membrane structure, we could improve $\eta$ and the nonuniformity.

3. Conclusion

In conclusion, the energy resolution of an optical TES reached 67 meV at 0.8 eV by lowering the critical temperature $T_c$ and achieving a high loop gain. The measurements of complex impedance and current noise show that the behaviour of the TES was consistently described by a two-block model. We calculated the theoretical values of the current noise, compared them with the measured data, and found that there was excess Johnson noise. The excess Johnson noise term $M$ was 1.5 at a bias point where the resistance was 10% of the normal resistance. Another TES with a typical energy resolution (156 meV) was also tested, and it showed a similar $M$. Though the excess Johnson noise contributed to the deterioration of the energy resolution, it was not a dominant source of the current noise. We found unexplained fluctuations in the measured energy. The contribution of the unexplained noise to the energy resolution was comparable to that of the current noise.
In future work, further lowering $T_c$ could achieve an energy resolution below 50 meV. In parallel, we should understand and mitigate the unexplained noise. One of the possible sources of this noise could be the nonlinear response of the TESs. In this study, we assumed that the small-signal limit and TESs responded linearly. The discrepancy between the measured values and theoretical expectations could be explained by nonlinearity. The nonlinearity in the detector response will become more obvious as $T_c$ is lower and hence the heat capacity decreases. Understanding nonlinearity will become more essential in future research.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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Appendix. Energy resolution in case of two-block model

We will show that the energy resolution of a TES described by the two-block model is proportional to $C/\alpha_l$.

Using equations (9), (13) and (18), the energy resolution is

$$\Delta E_{\text{FWHM}} = 2\sqrt{(2\ln 2)} \times \left[ \int_0^\infty \frac{4}{P_1 + \frac{\omega c}{L_{\text{eff}}}(1 + \omega^2 \tau^2)} + \frac{P_2}{1 + \omega^2 \tau^2} \frac{df}{\sqrt{a + bx^2 - \frac{1}{b}}} \right]^{-1/2}$$

(A.1)

$$= 2\sqrt{(2\ln 2)} \left[ \frac{1}{2\pi \tau_1 |P_2|} \int_0^\infty \frac{4}{\sqrt{a + bx^2 - \frac{1}{b}}} \frac{dx}{\sqrt{a + bx^2 - \frac{1}{b}}} \right]^{-1/2}$$

(A.2)

$$a = \frac{P_1}{|P_2|} + \frac{V_{\omega,\text{ets}}^2 I_0^2}{|P_2| L_{\text{eff}}^2}, \quad (A.3)$$

$$b = \frac{V_{\omega,\text{ets}}^2 I_0^2}{|P_2| L_{\text{eff}}^2} \tau^2, \quad (A.4)$$

where $V_{\omega,\text{ets}} = \sqrt{4kT \text{TES} R_0 (1 + 2 \beta)}$, $I_0$ is the current flowing through the TES, $M = 0$, $s = \omega \tau$ and the strong ETF is assumed. With the large loop gain, the complex impedance $Z_{\text{ets}}$ gives the same from the single-block model.

In the strong ETF regime ($b \ll 1$), equation (A.4) simplifies to

$$\Delta E_{\text{FWHM}} = 2\sqrt{(2\ln 2)} \left[ \frac{1}{\tau_1 |P_2|} \left\{ \frac{b}{a - b + \frac{a - b}{x^2}} \right\}^{1/2} \left( 2\pi \tau_1 \right)^{-1/2} \left( \frac{b}{a - b + \frac{2b}{a - b}} \right)^{1/2} \right]^{-1/2}$$

(A.5)

$$\simeq 2\sqrt{(2\ln 2)} \tau_1 |P_2| \sqrt{ab}$$

(A.6)

$$= \sqrt{\frac{4kT^2C}{\alpha_l} \frac{P_1}{P_{\text{TN0}}} \frac{n(1 + 2\beta)(1 + \frac{1}{(T/T_n)\alpha_l + 1})}{1 - (T/T_n)^n}}.$$  \quad (A.7)

Therefore, the energy resolution of the two-block model is proportional to $C/\alpha_l$.

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