Self-interacting dark matter with a vector mediator: kinetic mixing with $U(1)_{(B-L)_3}$ gauge boson

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Abstract. A spontaneously broken hidden $U(1)_h$ gauge symmetry can explain both the dark matter stability and the observed relic abundance. In this framework, the light gauge boson can mediate the strong dark matter self-interaction, which addresses astrophysical observations that are hard to explain in collisionless cold dark matter. Motivated by flavoured grand unified theories, we introduce right-handed neutrinos and a flavoured $B-L$ gauge symmetry for the third family $U(1)_{(B-L)_3}$. The unwanted relic of the $U(1)_h$ gauge boson decays into neutrinos via the kinetic mixing with the $U(1)_{(B-L)_3}$ gauge boson. Indirect detection bounds on dark matter are systematically weakened, since dark matter annihilation results in neutrinos. The kinetic mixing between $U(1)_{(B-L)_3}$ and $U(1)_Y$ gauge bosons are induced by quantum corrections and leads to an observable signal in direct detection experiments of dark matter. This model can also explain the baryon asymmetry of the Universe via the thermal leptogenesis. In addition, we discuss the possibility of explaining lepton flavour universality violation in semi-leptonic $B$ meson decays that is recently found in the LHCb experiment.
1 Introduction

The nature of dark matter (DM) is an longstanding mystery in cosmology and particle physics. If DM consists of some new particle, it needs to be long-lived and its relic density should explain the observed amount. A simple framework to explain these aspects is to introduce a gauge symmetry $U(1)_h$, which is broken to some discrete group. The DM particle is stable due to the unbroken discrete group and the correct relic density is obtained through the DM annihilation into light gauge bosons $Z_h$.

The light gauge boson mediates a DM self-interaction, which can address small-scale issues in structure formation of collisionless cold dark matter (see, e.g., Refs. [1, 2]). The self-interaction is velocity-dependent as astrophysical observations prefer [3]. The cross section per mass is $\sigma/m \gtrsim 1 \text{cm}^2/\text{g}$ in (dwarf) galaxies to explain, e.g., diversity in galaxy rotation curves [4–6] (see also Ref. [7]). Meanwhile it diminishes to $\sigma/m \lesssim 0.1 \text{cm}^2/\text{g}$ in galaxy clusters to be compatible, e.g., with the inferred core of relaxed galaxy clusters [3] (see also Ref. [8]). The circular velocity in galaxy clusters is of order $v \sim 1000 \text{km}/\text{s}$, while that in dwarf galaxies is of order $v \sim 30\text{km}/\text{s}$. It implies that a velocity-dependent self-interaction is preferred.

$Z_h$ tends to be stable, while one can make the $U(1)_h$-breaking scalar decay into two $Z_h$'s. Thermally produced $Z_h$ may overclose the Universe if it is stable. One may introduce a kinetic mixing between $Z_h$ and the $U(1)_Y$ gauge boson $B$ so that $Z_h$ can decay into an
electron-positron pair or photons. However, late-time DM annihilation followed by the $Z_h$ decay is largely disfavored by indirect detection constraints, e.g. from cosmic microwave background (CMB) anisotropies (see, e.g., Ref. [9]). One way to avoid the overclosure of $Z_h$ and these constraints is to make it decay only into standard model (SM) neutrinos.

A similar line of constructing a viable self-interacting DM model was pursued in Ref. [10], where $U(1)_h$ is identified as a flavoured lepton gauge symmetry $U(1)_{L_\mu - L_\tau}$. The MeV-scale $L_\mu - L_\tau$ gauge boson decays predominantly into neutrinos since charged lepton channels are kinematically forbidden. On other hand, the gauge coupling needs to be rather small to satisfy constraints from muon anomalous magnetic moment and thus the mediated self-interaction is also small. This is why the MeV-scale $U(1)_{L_\mu - L_\tau}$-breaking scalar was considered as a scalar mediator of the DM self-interaction. In this paper, we propose a self-interacting DM model with a vector-mediator [11–18].

We consider flavoured $U(1)_{B - L}$ gauge symmetries; we introduce a $B - L$ gauge symmetry $U(1)_{(B - L)_i}$ for each family $(i = 1, 2, 3)$ in the SM sector. The anomaly cancellation implies that there is a right-handed neutrino $N_{R_i}$ in each family. This model could be extended to grand unified theories such as $[SO(10)]^3$ [19] (see also Ref. [20]). We assume that $U(1)_{(B - L)_3}$ is spontaneously broken around the electroweak scale. MeV-scale $Z_h$ decays predominantly into neutrinos through kinetic mixing between $U(1)_h$ and $U(1)_{(B - L)_3}$ since channels into quarks and charged leptons are not kinematically allowed. In our model, we make the mass of the $U(1)_h$-breaking scalar larger than $2 \times m_{Z_h}$ so that the scalar field can decay into two $Z_h$'s. Quantum corrections give a kinetic mixing between $U(1)_{(B - L)_3}$ and $U(1)_Y$ gauge bosons because there are bicharged particles in the SM sector. Because of these kinetic mixings, our DM can be detected by direct detection experiments in the near future.

Interestingly, we can realize the seesaw mechanism [21–24] and the thermal leptogenesis [25] (see, e.g., Refs. [26–29] for recent reviews) by assuming $U(1)_{(B - L)_1}$ and $U(1)_{(B - L)_2}$ to be spontaneously broken at the scale above $10^9$ GeV. We assume that electroweak-scale $N_{R_3}$ is stable because of a $Z_2$ symmetry so as not to washout the $B - L$ asymmetry via its decay [30]. Our model can also explain the recent measurement of lepton flavour universality violation in semi-leptonic $B$ meson decays [31, 32] because the $U(1)_{(B - L)_3}$ gauge boson mediates flavour universality violating interactions for mass eigenstates of quarks and leptons.

This paper is organized as follows. First, we specify our model of DM and flavoured $U(1)_{(B - L)_i}$. We introduce a spontaneously broken $U(1)_h$ gauge symmetry in the dark sector. The $U(1)_{(B - L)_3}$ is spontaneously broken around the electroweak scale so that the kinetic mixing with $Z_{(B - L)_3}$ leads to the decay of $Z_h$ into SM neutrinos. In Sec. 3, we discuss the cosmology of this model. In particular, we discuss that there are two candidates of DM in this model: the vector-like fermion in the hidden sector and $N_{R_3}$. The former one has the self-interaction through the massive gauge boson exchange and is assumed to be the dominant component of DM. Then, we discuss the compatibility with the present collider experiments and future detectability in Sec. 4. Sec. 5 is devoted to the conclusion.

2 Model

We introduce three right-handed neutrinos $N_{R_i}$ and flavoured $U(1)_{(B - L)_i}$ gauge symmetries $(i = 1, 2, 3)$ that are spontaneously broken by vacuum expectation values (VEVs) of $\Phi_i$ at the energy scale of $v_{\Phi_i}$. We make the third family of right-handed neutrino stable by introducing a $Z_2$ symmetry. We also introduce another complex scalar field $\Psi$ and a vector-like fermion
pair $\chi$ and $\bar{\chi}$ that are charged under a hidden gauge symmetry $U(1)_h$. The field $\Psi$ is assumed to obtain a nonzero VEV to break $U(1)_h$ spontaneously at the energy scale of $v_\psi$. The charge of $\Psi$ is taken to be three in units of that of $\chi$ to forbid Yukawa interactions with $\chi$ or $\bar{\chi}$. The charge assignment of the newly introduced particles are summarized in Table 1, where we omit the first and second families for simplicity. We denote the gauge bosons of $U(1)_{(B-L)i}$ and $U(1)_h$ by $Z_{(B-L)i}$ and $Z_h$, respectively.

The Lagrangian is given by

$$L = L_{\text{SM}} + L_{\text{kin}} + L_{1,2} + L_3 + L_h,$$

$$L_{1,2} = -\frac{1}{2} \sum_{i=1}^{2} y_R^i \Phi_i N_R_i N_R_i - \sum_{i,j=1}^{2} y_R^{ij} H N_R_i L_j + \text{h.c.} - \sum_{i=1}^{2} V_{\Phi_i}(\Phi_i),$$

$$L_3 = -\frac{1}{2} y_R^3 \Phi_3 N_R_3 N_R_3 + \text{h.c.} - V_{\Phi_3}(\Phi_3),$$

$$L_h = -m_\chi \chi \bar{\chi} + \text{h.c.} - V_{\Psi}(\Psi),$$

where $L_{\text{kin}}$ represents the canonical kinetic terms including the gauge interactions and $H$ is the SM Higgs field.

The scalar fields are assumed to be unstable at the origins of the potentials and obtain nonzero VEVs $v_{\phi_i}$ and $v_{\psi}$ at the stable minima. We denote the perturbations around the minima as $\phi_i$ and $\psi$ as

$$\Phi_i = \frac{1}{\sqrt{2}} (v_{\phi_i} + \phi_i),$$

$$\Psi = \frac{1}{\sqrt{2}} (v_{\psi} + \psi).$$

After the spontaneous symmetry breaking (SSB), the gauge bosons $Z_{(B-L)i}$ and $Z_h$ obtain masses such as $m_{Z_{(B-L)i}} = 2g_{(B-L)i} v_{\phi_i}$ and $m_{Z_h} = 3g_h v_\psi$, respectively. The right-handed neutrinos obtain masses of $m_{N_R_i} = y_R^i v_{\phi_i}/\sqrt{2}$ via the Yukawa interaction. The SM neutrinos obtain small masses via the seesaw mechanism.

Because of the flavoured symmetry, the proper structure of Yukawa interactions cannot be generated in the simplest setup. To generate the proper Yukawa matrices, one may introduce (I) $U(1)_{(B-L)3}$-charged scalars in addition to $U(1)_{(B-L)3}$-neutral vector-like fermions that mix with the SM quarks and leptons [19]; or (II) an additional Higgs doublet that is charged under $U(1)_{(B-L)3}$ [33, 34]. We do not go into further detail about these possibilities in this paper and just assume that the following flavour structure arises appropriately.

The SM Yukawa matrices can be diagonalized by a unitary rotation for each field: $f = U_f f'$. Although each unitary matrix is not observable in the SM, except for $U_{uL}^T U_{dL} = V_{\text{CKM}}$

| Table 1. Charge assignment. |
|-----------------------------|
| $N_{R3}$ | $\Phi_3$ | $\chi$ | $\bar{\chi}$ | $\Psi$ |
| $U(1)_{(B-L)3}$ | $-1$ | $2$ | $0$ | $0$ | $0$ |
| $U(1)_h$ | $0$ | $0$ | $1$ | $-1$ | $3$ |
| $Z_2$ | $-1$ | $+1$ | $+1$ | $+1$ | $+1$ |
and $U_{eL}^\dagger U_{\nu L} = U_{PMNS}$, it affects the interactions with the $Z_{(B-L)_3}$ boson. The interactions with the $Z_{(B-L)_3}$ boson are given by

$$\mathcal{L} \supset - \sum_f g Q_f Z_{(B-L)_3}^\mu J_{f,\mu},$$

(2.7)

$$J_{f,\mu} = \bar{f} (U_f)^i_3 (U_f)_{3j} \gamma_\mu f^j.$$  

(2.8)

$Z_{(B-L)_3}$ can mediate interactions between different families in the mass eigenstate even if only the third family fermions are charged under $U(1)_{(B-L)_3}$ in the interaction basis.

In this paper, we simply assume that the rotations of the right-handed fermions are suppressed and the 2-3 family rotations of the left-handed fermions exist in addition to $V_{CKM}$ and $U_{PMNS}$ such as

$$U_{eL} = R^{23}(\theta_l), \quad U_{\nu L} = R^{23}(\theta_l) U_{PMNS},$$

(2.9)

$$U_{dL} = R^{23}(\theta_q), \quad U_{uL} = R^{23}(\theta_q) V_{CKM}^\dagger,$$  

(2.10)

where $R^{23}(\theta)$ is a rotation in the 23 sector by an angle $\theta$. In particular we assume that $R^{13}(\theta)$ does not arise so that $Z_h$ does not decay into electrons via the kinetic mixing with $Z_{(B-L)_3}$.

3 Cosmology of the model

3.1 Thermal leptogenesis

We can generate the lepton asymmetry by the thermal leptogenesis via the decay of the first and second family right-handed neutrinos. We assume that the reheating temperature after inflation is as high as the mass of the lighter one among these neutrinos so that they can be produced from the thermal plasma. The lepton asymmetry can be generated by their decay. Since the $B + L$ symmetry is broken by the non-perturbative effect, we can generate the baryon asymmetry from the lepton asymmetry. The observed baryon asymmetry can be explained when the lighter one is heavier than about $10^9$ GeV [25].

If the third family right-handed neutrino has a Yukawa interaction with the SM particles, the $B - L$ symmetry violating interaction may be in equilibrium after the thermal leptogenesis and the lepton asymmetry may be washed out. To avoid this washout effect, we impose a $Z_2$ symmetry on $N_{R_3}$. As a result, it is stable and can be a DM candidate.

If we do not introduce the $Z_2$ symmetry on $N_{R_3}$, the Yukawa coupling with the SM fields should be small enough to suppress the washout effect. The decay rate of $N_{R_3}$ is given by

$$\Gamma_{N_{R_3}} \simeq \sum_i \frac{|y^R_{3i}|^2}{8\pi} m_{N_{R_3}},$$

(3.1)

where $y^R_{ij}$ ($i, j = 1, 2, 3$) is a Yukawa matrix for the interaction between $N_{R_i}$ and the SM lepton doublet $L_j$. The washout effect should not be efficient, $\Gamma_{N_{R_3}} \lesssim H$, until the temperature of the Universe decreases to the mass of $N_{R_3}$. Thus we require

$$\sqrt{\sum_i |y^R_{ij}|^2} \lesssim 2 \times 10^{-7} \left( \frac{m_{N_{R_3}}}{1 \text{ TeV}} \right)^{1/2},$$

(3.2)

to avoid the washout effect.
3.2 Dark matter

There are two DM candidates in our model. We identify $\chi$ and $\bar{\chi}$ as the dominant component of DM, while $N_{R3}$ is the subdominant component. Their thermal relic densities are determined as

$$\Omega_i h^2 \approx 0.12 \left( \frac{3 \times 10^{-26} \text{ cm}^3 \text{s}^{-1}}{\langle \sigma v \rangle} \right),$$

with the $s$-wave annihilation cross section times relative velocity $(\sigma_i v)$.

3.2.1 Weakly-interacting DM: $N_{R3}$

The annihilation of $N_{R3}$ proceeds through the $U(1)_{(B-L)}$ gauge interaction and the Yukawa interaction with $\Phi_3$. We found in Ref. [30] that the dominant process is a $s$-wave annihilation channel $N_{R3}N_{R3} \rightarrow Z_{(B-L)}\phi$ if it is kinematically allowed and $m_{N_{R3}} \gg m_{Z_{(B-L)}} \phi$. The cross section is given by

$$\langle \sigma_{N_{R3}} v \rangle (N_{R3}N_{R3} \rightarrow Z_{(B-L)}\phi) \sim \frac{\pi\alpha^2_{(B-L)_3}}{4m^4_{N_{R3}} m^4_{Z_{(B-L)_3}}} \left[ m^4_\phi - 2m^2_\phi (4m^2_{N_{R3}} + m^2_{Z_{(B-L)_3}}) + (4m^2_{N_{R3}} - m^2_{Z_{(B-L)_3}})^2 \right]^{3/2},$$

where $\alpha_{(B-L)_3} \equiv g^2_{(B-L)_3}/(4\pi)$. The resulting amount of $N_{R3}$ can be then estimated as

$$\Omega_{N_{R3}} h^2 \approx 1.4 \times 10^{-2} \left( \frac{m_{N_{R3}}}{1 \text{ TeV}} \right)^{-2} \left( \frac{m_{Z_{(B-L)_3}}}{70 \text{ GeV}} \right)^4 \left( \frac{\alpha_{(B-L)_3}}{10^{-4}} \right)^{-2},$$

for $m_{N_{R3}} \gtrsim m_{Z_{(B-L)_3}} \phi$. We assume that $N_{R3}$ is the subdominant component of DM: $\Omega_{N_{R3}} h^2 \ll (\Omega_{DM} h^2)_{\text{obs}} \approx 0.12$. Then we obtain

$$y_{R3}^{-1} v_{\phi_3} \ll 2 \text{ TeV}.$$  \hspace{1cm} (3.6)

The Yukawa coupling $y_{R3}$ cannot be arbitrary large because of the Unitarity bound. One may also require that the Landau pole does not appear below the Planck scale, which leads to $y_{R3} \lesssim 1.2$ [30]. Then we obtain $v_{\phi_3} \approx 2.4 \text{ TeV}$ from Eq. (3.6).

Although the dominant annihilation channel is $s$-wave and its cross section is not suppressed at the galactic center, $N_{R3}$ is not the dominant component of the DM and hence its indirect detection signals can be neglected.

3.2.2 Self-interacting DM: $\chi$ and $\bar{\chi}$

For $\chi$ and $\bar{\chi}$, the annihilation cross section is given by [35]

$$\langle \sigma_{\chi} v \rangle (\chi \bar{\chi} \rightarrow Z_h Z_h) \sim \frac{\pi\alpha^2_h}{m^4_\chi},$$

$$\langle \sigma_{\chi} v \rangle (\chi \bar{\chi} \rightarrow Z_h \psi) \sim \frac{9\pi\alpha^2_h}{4m^2_\chi},$$

for $m_\chi \gtrsim m_{Z_h}$. The Yukawa coupling $y_{\chi}$ cannot be arbitrary large because of the Unitarity bound.
where \( \alpha_h \equiv g_h^2/(4\pi) \). The total abundance of \( \chi \) and \( \bar{\chi} \) is twice larger than Eq. (3.3) because there are \( \chi \) and \( \bar{\chi} \), each abundance of which is determined by the thermal freeze out. The resulting amount of \( \chi \) and \( \bar{\chi} \) can be then estimated as

\[
\Omega_{\chi} h^2 \approx 0.13 \left( \frac{m_\chi}{40 \text{ GeV}} \right)^2 \left( \frac{\alpha_h}{10^{-3}} \right)^{-2}.
\]  

(3.9)

The massive gauge boson \( Z_h \) mediates the self-interaction of \( \chi \) and \( \bar{\chi} \). It is convenient to use the transfer cross section defined by [37]\(^2\)

\[
\sigma_T \equiv \frac{1}{2} \left( \sigma_T^{(\chi\chi)} + \sigma_T^{(\bar{\chi}\bar{\chi})} \right),
\]  

(3.10)

\[
\sigma_T^{(\chi\chi)} = \int d\Omega (1 - \cos \theta) \left( \frac{d\sigma_{(\chi\chi)}}{d\Omega} \right),
\]  

(3.11)

When one computes \( \sigma_T \), one encounters three regimes [37]: Born regime \((\alpha_h m_\chi/m_{Z_h} \ll 1)\), classical regime \((\alpha_h m_\chi/m_{Z_h} \gtrsim 1 \& m_\chi v_{\text{rel}}/m_{Z_h} \gg 1)\), and resonance regime \((\alpha_h m_\chi/m_{Z_h} \gtrsim 1 \& m_\chi v_{\text{rel}}/m_{Z_h} \ll 1)\). In the Born regime, one can rely on the perturbative calculation and find an analytic expression in Refs. [37, 39, 40]. In the classical and resonance regimes, one needs to solve the Schrödinger equation to take into account non-perturbative effects related to multiple exchange of \( Z_h \). Meanwhile, fitting formulas can be found in the classical regimes [37, 40, 41, 42]. In the resonance regime, an approximate formula can be obtained in the Hulthén potential [37].

Kinematics of dwarf and low-surface brightness galaxies indicate that \( \sigma_T / m_\chi \approx 1 - 10 \text{ cm}^2/\text{g} \) for the DM velocity of order 30 km/s [3]. On the other hand, observations of galaxy clusters prefer \( \sigma / m_\chi \lesssim 0.1 \text{ cm}^2/\text{g} \) for the velocity of order 1000 km/s [3]. If the cross section saturates this upper bound, we can also explain the inferred density cores in the galaxy clusters [8]. The desirable parameter region is mostly in the resonance regime (see, e.g., Ref. [10]), where the parameter dependence of the self-scattering cross section is non-trivial. In this paper, we do not pin down the precise values of \( m_\chi \) and \( m_{Z_h} \) because they are not sensitive to other observables. Instead, we simply use an approximate formulas found in Ref. [37] with the replacement of \( \cos \theta \rightarrow |\cos \theta| \) in Eq. (3.11) (see footnote 2) to check if the self-interaction cross section is within a desirable range. We find that the above constraints can be satisfied when \( m_{Z_h} \approx 10-100 \text{ MeV} \) and \( m_\chi \approx 10-100 \text{ GeV} \).

### 3.3 Dark radiation

We assume that the mass of the dark Higgs boson \( \psi \) is larger than twice that of \( Z_h \), so that it can decay into two \( Z_h \)’s. We make \( Z_h \) unstable by introducing a kinetic mixing between \( U(1)_{(B-L)_3} \) and \( U(1)_h \):

\[
-\frac{1}{2} \epsilon_2 F_{(B-L)_3}^{\mu \nu} F_{h \mu \nu},
\]  

(3.12)

where \( F_{(B-L)_3} \) and \( F_{h} \) denote the field strengths of \( U(1)_{(B-L)_3} \) and \( U(1)_h \), respectively. Then \( Z_h \) can decay into tau-neutrinos \( \nu_\tau \) via the mixing with \( Z_{(B-L)_3} \). We remark that the decay

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1 Depending on \( m_{Z_h} \), the Sommerfeld enhancement can be significant [36]. When we focus on the regime where a large self-scattering cross section alleviates small-scale issues, \( m_\chi \lesssim 100 \text{ GeV} \) is free from this subtlety.

2 Replacing \( 1 - \cos \theta \) by \( 1 - |\cos \theta| \) is suggested since backward scattering has nothing to do with phase space redistribution as forward scattering [38, 39].
of $Z_h$ into muons $\mu$ or taus $\tau$ is kinematically forbidden for $m_{Z_h} \lesssim 200$ MeV. The other decay of $Z_h$ into electrons $e$ is suppressed since $Z_{(B-L)_3}$ does not directly couple to $e$ under our assumption of the Yukawa structure [see Eq. (2.9)]. Thus the late time DM annihilation into $Z_h$ results in $\nu_\tau$ and thus is harmless.

The decay rate can be estimated as

$$\Gamma_{Z_h} \sim \alpha_{(B-L)_3} \epsilon_2 \left( \frac{m_{Z_h}}{m_{Z_{(B-L)_3}}} \right)^4 m_{Z_h}. \quad (3.13)$$

We require that $Z_h$ decays into $\nu_\tau$ long before the neutrino decoupling; otherwise only the temperature of $\nu_\tau$ is enhanced by the decay of $Z_h$ and the energy density of $\nu_\tau$ may exceed the upper bound on that of dark radiation. This can be satisfied when $\Gamma_{Z_h} \gtrsim H|_{T=1}$ MeV, where $H$ is the Hubble expansion rate at temperature $T$. It gives the lower bound on the mixing parameter as

$$\epsilon_2 \gtrsim 4 \times 10^{-2} \left( \frac{m_{Z_{(B-L)_3}}}{70 \text{ GeV}} \right)^2 \left( \frac{m_{Z_h}}{10 \text{ MeV}} \right)^{-5/2} \left( \frac{\alpha_{(B-L)_3}}{10^{-4}} \right)^{-1/2}. \quad (3.14)$$

Even if $Z_h$ decays into $\nu_\tau$ long before the neutrino decoupling, the thermalized $Z_h$ can still enhance only the temperature of $\nu_\tau$ after the neutrino decoupling. This constraint is evaded for $m_{Z_h} \gtrsim 10$ MeV [10, 43].

Furthermore, one needs to take account of $Z_h$ possibly dominating the energy density of the Universe. It takes place if the decay rate of the dark Higgs boson is much smaller than the Hubble expansion rate when the temperature is comparable to the mass of dark Higgs boson, $\Gamma_{Z_h} \lesssim H|_{T=m_{Z_h}}$. If $m_{Z_h} > 1$ MeV and $\Gamma_{Z_h} \lesssim H|_{T=1}$ MeV, the Hubble expansion rate during the big bang nucleosynthesis is dominated by non-relativistic $Z_h$ and affects the big bang nucleosynthesis critically.\(^3\) If $m_{Z_h} > 1$ MeV and $\Gamma_{Z_h} \gtrsim H|_{T=1}$ MeV, the $Z_h$ domination does not impact the big bang nucleosynthesis, but still dilutes the baryon asymmetry. To evade such a wash out, the lower bound of the mixing should satisfy

$$\epsilon_2 \gtrsim 4 \times 10^{-1} \left( \frac{m_{Z_{(B-L)_3}}}{70 \text{ GeV}} \right)^2 \left( \frac{m_{Z_h}}{10 \text{ MeV}} \right)^{-3/2} \left( \frac{\alpha_{(B-L)_3}}{10^{-4}} \right)^{-1/2}. \quad (3.15)$$

If this condition is not satisfied, the amount of the entropy production due to the $Z_h$ decay can be estimated as

$$\Delta \equiv \frac{s_i a_i^3}{s_f a_f^3} \simeq \frac{m_{Z_h}}{T_d} \quad (3.16)$$

$$\simeq 10 \left( \frac{\epsilon_2}{4 \times 10^{-2}} \right)^{-1} \left( \frac{m_{Z_{(B-L)_3}}}{70 \text{ GeV}} \right)^2 \left( \frac{m_{Z_h}}{10 \text{ MeV}} \right)^{-3/2} \left( \frac{\alpha_{(B-L)_3}}{10^{-4}} \right)^{-1/2}, \quad (3.17)$$

where $a_i$ ($a_f$) is the scale factor, $s_i$ ($s_f$) is the entropy density before (after) the $Z_h$ domination, and $T_d$ is the decay temperature of $Z_h$. The constraint (3.15) can be avoided if the generated baryon asymmetry is larger than the observed value by this factor. This can be

\(^3\) This point seems missing in Ref. [35], where the lightest particle in a hidden sector ($Z_h$ in our case) is the dark Higgs boson. They take about 1.5 MeV as a reference value of the dark Higgs boson mass and require its lifetime to be shorter than $10^5$ sec not to affect the CMB spectral distortion (see, e.g., Ref. [44]). However, the lifetime of the dark Higgs boson should be shorter than $\mathcal{O}(1)$ sec not to dominate the energy density of the Universe.
realized when the first and second right-handed neutrinos are heavier than $10^9$ GeV at least by the same factor.

Here we comment on another possible mechanism of the entropy production, which could be relevant in models with spontaneous symmetry breaking. As for a dynamics of $U(1)_h$ breaking in the hidden sector, thermal inflation may occur at the time of the phase transition if the mass of the gauge boson $m_{Z_h}$ is many orders of magnitude larger than that of the symmetry-breaking field $m_\psi$. This effect washes out the baryon asymmetry, so that we should avoid such a thermal inflation. We discuss the condition to avoid thermal inflation in Appendix A and check that it does not occur in our model. However, we note that it is non-trivial in other models with hierarchical mass scales.

### 3.4 DM direct and indirect detection constraints

There may be couplings between scalar fields like $\lambda H |\Phi|^2 |\phi|^2$ and $\lambda H |\psi|^2$. Since they are irrelevant in the above discussion, we take the loop induced values as natural choices. For example, the former interaction arises at the two loop level as

$$\lambda \sim y^2 t \alpha^2 \frac{(B-L)^3}{(4\pi)^2}.$$  

It results in the mixing between the SM Higgs and $\phi$, which leads to spin-independent $N_{R3}$-nucleon scatterings. However, $N_{R3}$ is the subdominant component of DM and hence easily evades the constraint from the direct detection experiments for DM. For the same reason, the indirect detection constraint on $N_{R3}$ is also weakened.

The kinetic mixing between the $U(1)_Y$ and $U(1)_{(B-L)_3}$ gauge bosons arises at the one-loop level:

$$\mathcal{L}_{\text{kin}} \supset -\frac{1}{2} \epsilon_1 F_{\mu\nu} F^{\mu\nu}_{(B-L)_3},$$  

$$\epsilon_1 \approx \frac{2 g_Y g_{(B-L)_3}}{9\pi^2} \ln \left( \frac{\Lambda}{\mu} \right) \approx 10^{-2} \left( \frac{\alpha_{(B-L)_3}}{10^{-4}} \right)^{1/2} \ln \left( \frac{\Lambda}{10^{16} \text{ GeV}} \frac{10^2 \text{ GeV}}{\mu} \right),$$  

where $g_Y$ is the $U(1)_Y$ gauge coupling. Here $\mu$ is the energy scale considered and $\Lambda$ is a cutoff scale at which the kinetic mixing vanishes. When $U(1)_Y$ is unified into a non-abelian gauge symmetry, we should take $\Lambda$ to be the grand unification scale of order $10^{16}$ GeV.\(^4\) Through the kinetic mixings parametrized by $\epsilon_1$ and $\epsilon_2$, we obtain the following effective interaction at a low energy scale:

$$\mathcal{L} \supset b_p \bar{\chi} \not{\gamma}^\mu \chi \not{p}_\mu p,$$  

where $p$ represents the proton. The coefficient $b_p$ is given by

$$b_p \approx -\frac{e g_h \cos \theta_W \epsilon_1 \epsilon_2}{m_{Z_{(B-L)_3}}^2},$$  

where $e$ is the electromagnetic charge and $\theta_W$ is the Weinberg angle.

\(^4\) We implicitly consider $SU(5) \times U(1)_{(B-L)_3} \times U(1)_h$ gauge theory as an effective theory, where $SU(5)$ breaks down to the SM gauge groups at the GUT scale. In this case, the kinetic mixing between the $U(1)_Y$ and $U(1)_h$ gauge bosons are forbidden above the GUT scale while the one between the $U(1)_{(B-L)_3}$ and $U(1)_h$ gauge bosons is allowed by the symmetry $SU(5) \times U(1)_{(B-L)_3} \times U(1)_h$. Although the former one can be induced after the GUT symmetry breaking, it depends on the detail of the model. In this paper, we assume that the mixing parameter is suppressed enough so that we can avoid the constraints coming from DM direct detection experiments.
For a given nucleus $A Z N$, the coefficient of the coupling is given by $b_N = Z b_p$, where we neglect the contribution comes from the neutron. Then the spin-independent $\chi$-nucleon scattering cross section is given by

$$\sigma_N = \frac{1}{\pi} \frac{\mu_N^2}{A^2} b_N^2 F_{\text{FL,T}}^2(E_R)$$

$$\simeq 7 \times 10^{-48} \text{ cm}^2 \left( \frac{\alpha_h}{10^{-3}} \right) \left( \frac{\epsilon_1}{10^{-2}} \right)^2 \left( \frac{\epsilon_2}{10^{-2}} \right)^2 \left( \frac{m_Z(B-L)_3}{70 \text{ GeV}} \right)^{-4},$$

(3.22) (3.23)

(see, e.g., Ref. [45]), where $\mu_N$ is the $\chi$-nucleon reduced mass and $F_{\text{FL,T}}^2(E_R)$ is the form factor defined such that $F_{\text{FL,T}}(0) = 1$. This is just below the present upper bound reported by XENON1T for $m_\chi = 20 - 100 \text{ GeV}$ [46]. XENONnT [47], DarkSide-20k [48], and LUX-ZEPLIN [49] can search DM with a cross section smaller by a factor of order 10. DARWIN can detect DM if the cross section is above the neutrino coherent scattering cross section [50], which is around $10^{-49} \text{ cm}^2$ for $m_\chi \simeq 20 - 100 \text{ GeV}$.

As we stressed, the late-time annihilation of the dominant component of DM, $\chi$ and $\bar{\chi}$, predominantly results in $\nu_\tau$. Since the detection of neutrino signals is quite challenging and the constraint is very weak [51], this does not lead to observable effects on astrophysical experiments.

4 Collider constraints

As discussed in Sec. 2, we assume that the appropriate flavour structure of the SM Yukawa matrices comes from some UV physics (see, e.g., Ref [19]). The CKM and PMNS matrices are attributed to the up-type left-handed quarks and leptons, respectively. We allow for additional 2-3 family rotations of left-handed quarks and leptons.

4.1 No additional physical phase

First we discuss the constraints from collider experiments, when there are no additional physical phases, $\theta_\ell = \theta_q = 0$, following Ref. [30].

A relevant constraint on the $Z(B-L)_3$ mass comes from the lepton flavor universality violation in $Y$ decays. The lepton flavor universality ratio is modified in the presence of $Z(B-L)_3$ as

$$R_{\tau\mu}(Y(1S)) \equiv \frac{\Gamma_{Y(1S) \to \tau^+ \tau^-}}{\Gamma_{Y(1S) \to \mu^+ \mu^-}}$$

$$\simeq \left( 1 + \frac{\alpha(B-L)_3}{\alpha} \frac{m_Y^2}{m_{Z(B-L)_3}^2 - m_Y^2} \right)^2 ,$$

(4.1) (4.2)

where $m_Y \simeq 9.46 \text{ GeV}$ is the Upsilon mass and $\alpha = e^2/(4\pi)$. The BaBar experiment places the constraint on this ratio as $R_{\tau\mu} = 1.005 \pm 0.013\text{(stat.)} \pm 0.022\text{(syst.)}$ [52]. In the limit of $m_{Z(B-L)_3}^2 \gg m_Y^2$, we obtain

$$\left( \frac{m_{Z(B-L)_3}}{70 \text{ GeV}} \right)^2 \left( \frac{\alpha(B-L)_3}{10^{-4}} \right)^{-1} \gtrsim 1.7 \times 10^{-2} ,$$

(4.3)

corresponding to $v_{\phi_3} \gtrsim 130 \text{ GeV}$. (4.4)
This is consistent with the upper bound on $v_{\phi_3}$ from the $N_{R_3}$ abundance [see Eq. (3.6)].

Since the $U(1)_{(B-L)_3}$ gauge boson is coupled with the third-family quarks, it can be produced in hadron colliders. The dominant production process is a Drell-Yan process from the bottom quark pair: $b\bar{b} \rightarrow Z_{(B-L)_3}$. Resonance searches in the $\tau\bar{\tau}$ final state place the constraint for $200 \text{ GeV} \lesssim m_{Z_{(B-L)_3}} \lesssim 4 \text{ TeV}$ [53]. This constraint, $\alpha_{(B-L)_3} \lesssim 10^{-3}$ for $m_{Z_{(B-L)_3}} \sim 200 \text{ GeV}$, would not be quite stringent when compared to others.\(^5\)

The kinetic mixing between the $Z_{(B-L)_3}$ and $U(1)_Y$ gauge bosons changes the mass eigenstates and interactions of the vector mesons. In particular it leads to a shift in the SM $Z$ boson mass. In the mass basis, the physical mass of the SM $Z$ boson is given by

\[
m^2_Z = \frac{m^2_{Z_0} - m^2_{Z_{(B-L)_3}} \sin^2 \xi}{\cos^2 \xi},
\]

\[
\tan 2\xi = \frac{2\Delta (m^2_{Z_{(B-L)_3}} - m^2_{Z_0})}{(m^2_{Z_{(B-L)_3}} - m^2_{Z_0})^2 - \Delta^2},
\]

\[
\Delta = -m^2_{Z_0} \sin \theta_W \tan \epsilon_1,
\]

where $m_{Z_0} = m_{W^\pm}/\cos \theta_W$ (see, e.g., Ref. [56]). The mass of the SM $Z$ boson is tightly constrained by electroweak precision measurements and is consistent with the SM prediction. In this paper we require that the mass difference is smaller than the current experimental uncertainty of 0.0021 GeV [57]. The mass of the SM $Z$ boson is tightly constrained by electroweak precision measurements and is consistent with the SM prediction. Then we can plot a constraint in the $\alpha_{(B-L)_3} - m_{Z_{(B-L)_3}}$ plane as shown by the blue region in Fig. 1. The orange region in the upper left corner is excluded by the $\Upsilon$ decay measurement. We also plot a green region which is excluded by the flavour physics which we will discuss in Sec. 4.2. The dashed lines are a couple of reference values of the parameters we will use in Fig. 2.

The Higgs-portal interaction $\lambda_{H\Phi_3} |H|^2 |\Phi_3|^2$ may provide indirect signals of Higgs invisible decays if the SM Higgs can decay into $N_{R_1} N_{R_3}$, $Z_{(B-L)_3} Z_{(B-L)_3}$, or $\phi_3 \phi_3$. The constraint, however, can be easily evaded unless the Higgs-portal coupling is as large as $O(1)$.

### 4.2 Non-zero new physical phases

Next we allow the additional physical phases to be non-zero. The following discussion is based on Ref. [19].

#### 4.2.1 Semi-leptonic $B$ decays

To study semi-leptonic $B$ decays, it is convenient to use the following effective Hamiltonian at the low energy:

\[
\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left(C^i_9 \mathcal{O}_9^i + C^i_{10} \mathcal{O}_{10}^i + (C^i_{L})_j \mathcal{O}^{ij}_L \right),
\]

\[
\mathcal{O}_9^i = \frac{\alpha}{4\pi} (\bar{s}_L \gamma^\mu b_L) (\bar{l}_L \gamma^\mu l_L),
\]

\[
\mathcal{O}_{10}^i = \frac{\alpha}{4\pi} (\bar{s}_L \gamma^\mu b_L) (\bar{l}_L \gamma^\mu \gamma^5 l_L),
\]

\[
\mathcal{O}^{ij}_L = \frac{\alpha}{2\pi} (\bar{s}_L \gamma^\mu b_L) (\bar{l}_L \gamma^\mu \gamma^5 l_L).
\]

\(^5\)Our reference value of $m_{Z_{(B-L)_3}}$ is 70 GeV, which is slightly out of this range. We expect that the constraint does not change drastically (see also Refs. [54, 55]).
Figure 1. Parameter region in the $\alpha_{(B-L)_3^3-m_{Z_{(B-L)_3}}}$ plane. The orange and blue shaded regions are constrained by the $\Upsilon$ decay and kinetic mixing, respectively, when $\theta_t = \theta_q = 0$. In the green shaded regions, one cannot find a region in the $\sin \theta_\ell$-$\sin \theta_q$ plane, where the flavor constraints are satisfied. The dashed lines are the reference values of the parameters we use in Fig. 2.

After integrating out $Z_{(B-L)_3}$, we obtain the deviation from the SM contributions for $\mu$ such as

$$\delta C^\mu_9 = -\delta C^\mu_{10} = -\frac{\pi}{\alpha \sqrt{2} G_F} \frac{g_1^2}{3 m_{Z_{(B-L)_3}}^2} \frac{s_{\theta_q} \sin \theta_q}{s_{\theta_t}^2} \left( \frac{\alpha_{(B-L)_3}}{10^{-4}} \right) \left( \frac{m_{Z_{(B-L)_3}}}{70 \text{ GeV}} \right)^2 \ll 1.5 \times 10^{-2},$$

(4.12)

where $V_{tb}$ denotes the $tb$ component of $V_{CKM}$ and so on. Hereafter in this subsection, we also use $s_{\theta_q} = \sin \theta_q$ and so on. The LHCb experiment reported a lepton flavour universality violation in semi-leptonic $B$ decays \([31, 32]\), which is represented by the ratio of

$$\mathcal{R}_{K}^{(*)} = \frac{\Gamma(B \to K^{(*)}\mu^+\mu^-)}{\Gamma(B \to K^{(*)}e^+e^-)}.$$  

(4.13)

The tension with the SM prediction is around the 4$\sigma$ level \([58–63]\). This can be explained by the $Z_{(B-L)_3}$ contribution when $\delta C^\mu_9 \in [-0.81, -0.48]$ (1$\sigma$ interval) \([64]\). Using $|V_{tb}| \approx 1.0$ and $|V_{ts}| \approx 3.9 \times 10^{-2}$ \([57]\), we then require

$$8.7 \times 10^{-3} \lesssim s_{\theta_q} \sin \theta_q \left( \frac{\alpha_{(B-L)_3}}{10^{-4}} \right) \left( \frac{m_{Z_{(B-L)_3}}}{70 \text{ GeV}} \right)^2 \lesssim 1.5 \times 10^{-2}.$$  

(4.14)

The $B$ meson can decay also into neutrinos: $B \to K^{(*)}\nu\bar{\nu}$. The deviation from the SM contribution is given by

$$\delta C^ij = \delta C^i \left( U_{\nu_L}^{*} \right)_{3i} \left( U_{\nu_L} \right)_{3j},$$  

(4.15)

$$\delta C^i = -\frac{\pi}{\alpha \sqrt{2} G_F} \frac{g_1^2}{3 m_{Z_{(B-L)_3}}^2} \frac{s_{\theta_q} \sin \theta_q}{s_{\theta_t}^2} \left( \frac{\alpha_{(B-L)_3}}{10^{-4}} \right) \left( \frac{m_{Z_{(B-L)_3}}}{70 \text{ GeV}} \right)^2 \ll 1.5 \times 10^{-2}.$$  

(4.16)
The ratio to the SM prediction is given by

\[
R_{\nu \bar{\nu}} \equiv \frac{\Gamma}{\Gamma_{SM}} = 1 + \frac{2}{3} \left( \frac{\delta C_{\nu}}{C_{\nu}^{SM}} \right) + \frac{1}{3} \left( \frac{\delta C_{\nu}}{C_{\nu}^{SM}} \right)^2 ,
\]

(4.17)

where \( C_{\nu}^{SM} \approx -6.35 \) [65]. The experimental upper bound is \( R_{\nu \bar{\nu}} < 4.3 \) at the 90% CL [66, 67], which gives

\[
s_\theta c_\theta \left( \frac{\alpha_{B-L}}{10^{-4}} \right) \left( \frac{m_{Z/3}}{70 \text{ GeV}} \right)^2 \lesssim 2.6 \times 10^{-1} .
\]

(4.18)

Combining this with Eq. (4.14), we obtain

\[
|s_\theta| \gtrsim 0.18 .
\]

(4.19)

### 4.2.2 \( B_s - \bar{B}_s \) and \( D^0 - \bar{D}^0 \) mixings

\( Z_{(B-L)3} \) also contributes to the \( B_s - \bar{B}_s \) mixing via the following effective Lagrangian:

\[
\mathcal{L} \supset - \frac{g^2_{(B-L)3} s_\theta c_\theta}{18m^2_{Z_{(B-L)3}}} (\bar{s}\gamma^\mu b_L)^2 .
\]

(4.20)

This gives a deviation of the \( B \) meson mass difference from the SM prediction as

\[
C_{B_s} = \frac{\Delta m_{B_s}}{\Delta m_{B_s}^{SM}} = 1 + \frac{4\pi^2 c(m_{Z_{(B-L)3}})}{G^2_F m^2_W V_{ub} V_{us}} \frac{g^2_{(B-L)3} s_\theta^2 c^2_\theta}{18m^2_{Z_{(B-L)3}}} ,
\]

(4.21)

where \( c(m_{Z_{(B-L)3}}) \approx 0.8 \) [68, 69], \( S(m^2_W/m^2_{Z}) \approx 2.30 \) [70], and \( \eta_B \approx 0.84 \) [71, 72]. The experimental constraint is \( 0.899 < C_{B_s} < 1.252 \) (95% CL interval) [73]. Then we obtain

\[
|s_\theta c_\theta| \left( \frac{\alpha_{B-L}}{10^{-4}} \right)^{1/2} \left( \frac{m_{Z/3}}{70 \text{ GeV}} \right)^{-1} \lesssim 2.0 \times 10^{-1} .
\]

(4.22)

This upper bound is comparable to that in Eq. (4.18), in the parameter region we are interested in.

The \( D^0 - \bar{D}^0 \) mixing is induced by the \( Z_{(B-L)3} \) exchange effective interaction of

\[
\mathcal{L} \supset - \frac{g^2_{(B-L)3} c_D^2}{18m^2_{Z_{(B-L)3}}} (\bar{u}\gamma^\mu c_L)^2 ,
\]

(4.23)

where

\[
c_D \equiv (V_{ub}c_\theta - V_{us} s_\theta) (V^*_{cb}c_\theta - V^*_{cs} s_\theta) .
\]

(4.24)

This results in

\[
\Delta m_{NP}^{D_D} = \frac{2}{3} f_D^2 B_D m_D c(m_{Z_{(B-L)3}}) \frac{g^2_{(B-L)3} c_D^2}{18m^2_{Z_{(B-L)3}}} ,
\]

(4.25)
where \( f_D \approx 207.4 \text{ MeV} \) [74] and \( B_D \approx 0.757 \) [75] are calculated by the lattice quantum chromodynamics. The \( D^0 \) meson mass is \( m_D \approx 1.86 \text{ GeV} \). The mass difference calculated in the SM has large uncertainties [76], so that we cannot evaluate the total (SM+NP) mass difference robustly. In this paper we simply require that the new contribution does not exceed the experimental data, which is \( 4 \times 10^{-4} < \Delta m_{D}^{\text{NP}}/\Gamma < 6.2 \times 10^{-3} \), where \( \Gamma \approx 2.44 \times 10^{12} \text{/sec} \) is the average decay width of \( D^0 \) and \( \bar{D}^0 \) [57, 77]. Since \( |V_{ub}| \approx 4.1 \times 10^{-3} \), \( |V_{us}| \approx 0.22 \), \( |V_{cb}| \approx 4.1 \times 10^{-2} \), and \( |V_{cs}| \approx 1.0 \), this constraint is stringent particularly for \( c_{\theta_4} \ll 1 \). Then we obtain

\[
|s_{\theta_4}| \left( \frac{\alpha_{(B-L)_3}}{10^{-4}} \right)^{1/4} \left( \frac{m_{Z(B-L)_3}}{70 \text{ GeV}} \right)^{-1/2} \lesssim 1.5 \times 10^{-1}.
\]

(4.26)

for \( s_{\theta_4} \gg 0.04 c_{\theta_4} \).

### 4.2.3 Lepton flavour violation

Lepton flavour violating processes are also induced by \( Z_{(B-L)_3} \) interactions. The most important effective interaction is

\[
\mathcal{L} \supset \frac{g_{(B-L)_3}^2}{m_{Z(B-L)_3}^2} s_{\theta_4} c_{\theta_4} \bar{\tau} \gamma^\rho \mu L \bar{\gamma}_\rho L \mu L, \tag{4.27}
\]

which gives the \( \tau \) decay into \( 3 \mu \). The resulting branching ratio is given by

\[
\text{Br}(\tau \to 3\mu) = \frac{m_\tau^5}{48\pi \Gamma_\tau} \frac{\alpha_{(B-L)_3}^2}{m_{Z(B-L)_3}^4} s_{\theta_4} c_{\theta_4}^2, \tag{4.28}
\]

where \( m_\tau \approx 1.78 \text{ GeV} \) and \( \Gamma_\tau \approx (2.9 \times 10^{-13} \text{sec})^{-1} \approx 2.3 \times 10^{-12} \text{ GeV} \) are the mass and decay width of the tau lepton, respectively. The experimental upper bound is \( \text{Br}(\tau \to 3\mu) < 2.1 \times 10^{-8} \) at the 90\% CL [78]. Thus we obtain

\[
|s_{\theta_4} c_{\theta_4}| \left( \frac{\alpha_{(B-L)_3}}{10^{-4}} \right) \left( \frac{m_{Z(B-L)_3}}{70 \text{ GeV}} \right)^{-2} \lesssim 3.1 \times 10^{-2}.
\]

(4.29)

For the reference values of the parameters, \( \alpha_{(B-L)_3} = 10^{-4} \) and \( m_{Z(B-L)_3} = 70 \text{ GeV} \), this constraint implies that \( |s_{\theta_4}| \lesssim 0.32 \), which is compatible with Eq. (4.19).

### 4.2.4 \( Z_{(B-L)_3} \) production and decay in colliders

The mixing in the lepton sector leads to new decay channels of \( Z_{(B-L)_3} \): \( Z_{(B-L)_3} \to \mu \bar{\mu} \) and \( Z_{(B-L)_3} \to \mu \tau \). We can place a constraint by using the di-muon search by the ATLAS and CMS collaborations at the LHC experiment for 200 \text{ GeV} < \( m_{Z_{(B-L)_3}} < 4 \text{ TeV} \) [79, 80]. The constraint is similar to \( Z_{(B-L)_3} \to \tau \bar{\tau} \) discussed in Sec. 4.1 and would not be quite stringent.

The SM \( Z \) boson can decay into \( \mu \bar{\mu} Z_{(B-L)_3} \) followed by \( Z_{(B-L)_3} \to \mu \bar{\mu} \). The ATLAS collaboration reported an upper bound on the branching ratio of \( Z \to 4\mu \) as \( \text{Br}(Z \to 4\mu) < [3.2 \pm 0.25\text{(stat.)} \pm 0.13\text{(syst.)}] \times 10^{-6} \) [81]. This can be interpreted as an upper bound on \( g_{(B-L)_3}^2 s_{\theta_4}^4 \) for a given \( m_{Z_{(B-L)_3}} \), where \( g_{(B-L)_3}^2 s_{\theta_4}^4 \) comes from a coupling for the \( Z_{(B-L)_3} \) production process and another \( s_{\theta_4}^2 \) comes from a branching ratio of \( Z_{(B-L)_3} \) decay into \( \mu \bar{\mu} \).\(^{6}\)

\(^{6}\) The \( Z_{(B-L)_3} \) boson can decay into \( \mu \bar{\mu} \) via the kinetic mixing with U(1)_Y. Since this process does not dominate for \( s_{\theta_4} \) satisfying (4.19), we neglect this contribution.
The relevant process is Electron Positron Collider (CEPC) \cite{84}, International Linear Collider (ILC) \cite{85,86}, and 14 TeV data. In this case, the high-luminosity LHC would observe a dilepton signal for discussed that the constraint will be improved by a factor about 5 by using 3000 fb⁻¹ of magnitude weaker for \( m_{Z(B-L)_3} \gtrsim 70 \text{ GeV} \). We find that this constraint is negligible in most of the parameter region we are interested in.

Because of the kinetic mixing, \( Z(B-L)_3 \) can be produced by hadron colliders via the Drell-Yan process, which leads to a clear dilepton signal with an invariant mass about the \( Z(B-L)_3 \) boson mass. In Ref. \cite{83}, they considered the case where a hidden gauge boson is coupled with the SM sector only via the kinetic mixing and estimated that the 8 TeV LHC with 20 fb⁻¹ luminosity put a constraint on the kinetic mixing parameter as \( \epsilon_1 \lesssim 0.005-0.01 \) for the gauge boson mass range of 10-70 GeV. In our model, the \( Z(B-L)_3 \) boson can be produced via the kinetic mixing\(^7\) and decay into \( \mu \bar{\mu} \) via the flavour mixing. The constraint can be interpreted as a bound on \( \epsilon_1 s_\theta^2 \), where \( s_\theta^2 \) comes from the branching ratio into \( \mu \bar{\mu} \). However, this does not give a strong constraint on \( s_\theta \) when \( \alpha(B-L)_3 \lesssim 10^{-4} \). It was also discussed that the constraint will be improved by a factor about 5 by using 3000 fb⁻¹ of 14 TeV data. In this case, the high-luminosity LHC would observe a dilepton signal for \( \alpha(B-L)_3 \sim 10^{-4} \).

The \( Z(B-L)_3 \) gauge boson can also be produced by lepton colliders through the kinetic mixing \( \epsilon_1 \) and its decay signal can be searched by the future \( e^+ e^- \) colliders, such as Circular Electron Positron Collider (CEPC) \cite{84}, International Linear Collider (ILC) \cite{85,86}, and Future Circular Collider (FCC-ee) \cite{87}. The relevant process is \( e^+ e^- \rightarrow \gamma Z(B-L)_3 \) followed by \( Z(B-L)_3 \rightarrow \mu^+ \mu^- \). Projected constraints were discussed in Ref. \cite{88} in the case where the dark photon couples to the SM particles only via the kinetic mixing with the U(1)\(_Y\) gauge boson. The upper bound on the kinetic mixing parameter was found to be about 0.003. Again, we could interpret their result in the same way as discussed above. The future lepton colliders would observe a signal of \( Z(B-L)_3 \) gauge boson for \( \alpha(B-L)_3 \sim 10^{-4} \).

### 4.3 Summary of the collider constraints

Now we shall put together all the constraints discussed in this section. The result is shown in Fig. 1, where the shaded regions are excluded by the constraints. Note that all the shown constrains depend on \( \alpha(B-L)_3 \) and \( m_{Z(B-L)_3} \) only via a combination of \( m_{Z(B-L)_3}^2 / \alpha(B-L)_3 \). In the figure, we take \( m_{Z(B-L)_3}^2 / \alpha(B-L)_3 = (7 \text{ TeV})^2 \) (left panel) or \( (30 \text{ TeV})^2 \) (right panel).

We change the value of \( m_{Z(B-L)_3}^2 / \alpha(B-L)_3 \) and find that there is an allowed region when \( (3 \text{ TeV})^2 \lesssim m_{Z(B-L)_3}^2 / \alpha(B-L)_3 \lesssim (49 \text{ TeV})^2 \) corresponding to 0.4 TeV \( \lesssim v_{\phi_3} \lesssim 6.9 \text{ TeV} \). This is shown in Fig. 1 as the green-shaded region. From Fig. 1, we can see that there is a certain parameter region where we can explain the lepton flavour universality violation in semi-leptonic \( B \) decays consistently with the constraints coming from the kinetic mixing.

### 5 Conclusion

We have proposed a model of DM whose stability is guaranteed by a discrete symmetry that is a subgroup of a spontaneously broken hidden U(1)\(_h\) gauge symmetry. The massive

\(^7\) The \( Z(B-L)_3 \) boson can also be produced from \( b \bar{b} \), in which case the constraint can be interpreted as a bound on \( g g_{Z(B-L)_3} / e s_\theta^2 \), where \( f \sim \mathcal{O}(\alpha) \), with the quantum chromodynamics fine-structure constant \( \alpha_s \), represents a factor coming from parton distribution functions of \( b \) and \( \bar{b} \).
gauge boson $Z_h$ is assumed to be much lighter than the DM and mediates the velocity-dependent DM self-interaction that are suggested by small-scale issues in structure formation of collisionless cold dark matter. The observed abundance of DM is explained by the thermal relic via the freeze-out mechanism. Motivated by flavoured grand unified theories, we have also introduced right-handed neutrinos and flavoured $B-L$ gauge symmetries. The unwanted relic of $Z_h$ can then decay into neutrinos via the kinetic mixing with the electroweak scale $U(1)_{(B-L)_3}$ gauge boson $Z_{(B-L)_3}$. This model can also explain the baryon asymmetry of the Universe via the thermal leptogenesis.

Although the hidden sector couples to the SM sector only via a kinetic mixing with the $U(1)_{(B-L)_3}$ gauge boson, it predicts detectable DM signals in direct detection experiments. Our model also predicts a relatively light $U(1)_{(B-L)_3}$ gauge boson, which leads to interesting signals in collider phenomenology. In particular, we have found that we can explain the lepton flavour universality violation in semi-leptonic $B$ meson decays recently found in LHCb experiment. The $U(1)_{(B-L)_3}$ gauge boson can also be searched by the future high-luminosity LHC experiment and $e^+e^-$ colliders such as CEPC, ILC, and FCC-ee. These experiments would observe signals when the fine-structure constant for $U(1)_{(B-L)_3}$ is of order $10^{-4}$.

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A Phase transition and thermal inflation

In this Appendix, we comment on the effects of the phase transition from the SSB of $U(1)_h$, in which the dark Higgs boson $\Psi$ develops the VEV $v_\Psi$.\footnote{We do not discuss the phase transitions of the $U(1)_{(B-L)}$ breaking fields $\Phi$, because their potentials may be complicated by additional scalars and heavy fermions that are required to reproduce the proper Yukawa structure (see, e.g., Ref. [19]).}

Since it breaks the local Abelian gauge symmetry, cosmic strings form through the phase transition. However, their effects are negligible in our model because the energy density of cosmic strings is suppressed by a factor of the VEV squared in the Planck units when compared to the total energy density of the Universe.

If the VEV of a scalar field is much larger than its mass, the potential energy before the phase transition may be much larger than the energy of the thermal plasma. In this case, the energy density of the Universe becomes dominated by the former vacuum energy and a mini inflation called a thermal inflation occurs through the phase transition [89, 90]. The duration of the thermal inflation depends on the ratio between the VEV and (zero-temperature+thermal) mass of the SSB field. After the thermal inflation, the vacuum energy will be released into the radiation and the entropy production proceeds. As a result, the baryon asymmetry is diluted due to the entropy production at the time of this reheating.

Here, we give a quantitative estimate of the dilution of the thermal relic through the entropy production. The (zero temperature+thermal) potential of $\Psi$ is given by

$$V(|\Psi|) = \lambda \left(|\Psi|^2 - \frac{1}{2} v_\Psi^2\right)^2 + V_T(\Psi), \quad (A.1)$$

where $\lambda$ is a quartic coupling constant. The mass of $\psi$ at the vacuum is given by $\sqrt{2\lambda} v_\psi$. The thermal potential $V_T$ from $\psi$ and the gauge boson $Z_h$ is approximately given by

$$V_T(|\Psi|) = \left(\frac{\lambda}{3} + q_\psi^2 g_h^2 / 4\right) T^2 |\Psi|^2, \quad (A.2)$$

where $q_\psi (= 3)$ is the charge of $\Psi$. At a high temperature, the thermal potential dominates and $\Psi$ is stabilized at $\Psi = 0$. When the temperature becomes lower than the critical temperature $T_c$, which is given by

$$T_c = \sqrt{\frac{\lambda}{\lambda/3 + q_\psi^2 g_h^2 / 4}} v_\psi, \quad (A.3)$$

the potential becomes unstable at $\Psi = 0$ and the scalar field starts to oscillate around the true vacuum at $|\Psi| = v_\psi / \sqrt{2}$. We define a dilution factor as the ratio of the initial to the final comoving entropy density as

$$\Delta \equiv \frac{s_f a_f^3}{s_i a_i^3} = 1 + \frac{4}{3} g_s(T_{RH,\psi}) T_{RH,\psi} (2\pi^2/45) g_{ss}(T_c) T_c^3, \quad (A.4)$$

where (I) $a_i$ ($a_f$) is the scale factor, $s_i$ ($s_f$) is the entropy density before (after) the thermal inflation; (II) $V(0) = \lambda v_\psi^4 / 4$; and (III) $g_{ss}$ ($g_s$) is the effective number of relativistic degrees of freedom for entropy (energy) density as a function of $T$. The reheating temperature of
oscillating $\psi$ satisfies $T_{RH,\psi} < \left[30V(0)/\{g_*(T_{RH,\psi}) \pi^2\}\right]^{1/4}$. To avoid the washout effect due to the thermal inflation, we require $\Delta \approx 1$ corresponding to

$$\left(\frac{m_Z^2}{m_h^2} + \frac{2}{3}\right) \lesssim \lambda^{-1/2} \left(\frac{4\sqrt{2} \pi^2 g_*(T_c) g_*(T_{RH,\psi})}{15 g_*(T_{RH,\psi})}\right)^{2/3} \left(\frac{15}{2\pi^2 g_*(T_{RH,\psi})}\right)^{1/6}.$$  \hspace{1cm} (A.5)

It follows that the mass of gauge boson cannot be arbitrary larger than the mass of the SSB field. This condition is easily satisfied in our model though it is non-trivial in other models with hierarchical mass scales.

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