Estimation of residual stress of metal material with yield plateau by continuous spherical indentation method

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Abstract

In this paper, a novel method was proposed to measure the residual stress and plasticity parameters of metal materials with yield plateau through continuous spherical indentation test. The indentation energy was selected as the indentation parameter to establish a dimensionless equation between the residual stress and material parameters through dimensional analysis method. The effects of residual stress and plastic parameters on the indentation response tests are studied, and the dimensionless function expressions are determined respectively when the specimen with or without residual stress based on the finite element analysis data of different residual stress levels. Based on the self-established inverse analysis, the yield strength ($\sigma_y$), strain hardening exponent ($n$), ratio coefficient ($a$) and residual stress ($\sigma_r$) can be obtained through indentation test. At last, the validation of the method presented in this paper was verified by simulating the material (SS400 and SM490) with the known material parameters.

1. Introduction

In the mechanical manufacturing process, residual stresses often exist in a variety of metal parts and components, because of welding or uneven plastic deformation. The residual stress in component has a great influence on fatigue strength, corrosion cracking, dimensional stability and machining accuracy [1, 2]. It is great important to accurately measure the residual stress in structures for evaluating the mechanical properties of materials. In recent years, the indentation method has the characteristics of wide application, convenient operation, low cost and non-destructive measurement, which has attracted close attention from major research institutions [3]. Indentation technology is widely used to measure residual stress and material mechanical parameters, such as strength, modulus of elasticity and yield strength [4, 5].

Tsui [6] and Boshakov [7] had studied residual stress by experimental and finite element simulation method respectively. It was found that there was a bilinear relationship between residual stress and indentation hardness. Suresh and Giannakopoulos [8] had proposed a new model for calculating the residual stress based on the nanoindentation technique. The method uses the load displacement curve to determine the residual stress by establishing the relationship among the contact area ratio with and without stress, the hardness and residual stress of the material under the condition of a fixed indentation depth or load. The indentation contact area ratio is replaced by the indentation load ratio, which solves the measurement problem of the indentation area and avoid large measurement error [4]. Chen [9] carried out finite element simulation on the equivalent biaxial residual stress of ideal materials. It was found that the residual stress has high sensitivity to contact hardness, contact stiffness and indentation work, and was established according to the dimensional analysis method. According to the dimensional analysis method, the dimensionless function relationship between the residual stress and the material parameters is established, but the method needs to know the contact radius of the indentation. However, the above method is only for the ideal elastoplastic material model or the stress-strain relationship of the material is presented as a power-exponential function.
There is a part of materials that continue to be subjected to external load and deformation after the material reaches yield strength, but the internal stress of the material does not increase immediately. This material is called material containing yield plateau, and the coefficient $\alpha$ is used to reflect the material’s yield plateau. In 2017 Pham and Kim [10] performed indentation simulation experiments on metal materials with yielding plateau, and selected contact stiffness $S$, indentation depth $h$ and indentation work $W$ as indentation parameters, and gave four dimensionless equations. Due to the large number of coefficients in the equation, the solution is more complicated. Based on the analysis of the finite element simulation results, Wang [11, 12] proposed a fitting equation for the indentation response parameters and material properties. The relationship between residual stress, material properties and indentation parameters were established by dimensional analysis. Then based on the inverse analysis algorithm, three unknown parameters are estimated from the dimensionless functions. By comparing the relative errors of the dimensionless equations at different depths, the optimal combination of three different indentation depths is determined. In this paper, finite element simulation analysis is carried out on 350 kinds of metal materials with different residual stress parameters. According to the dimensional analysis method, the specific dimensionless function expressions with and without residual stress are established respectively, and a new inverse analysis method is proposed [10]. The analytical method was used to calculate the yield strength $\sigma_y$, strain hardening exponent $n$, the coefficient $\alpha$ and residual stress $\sigma_r$ of the material. The effectiveness of the proposed method is verified by numerical simulation of typical metal materials in engineering.

2. Material model and dimension analysis

2.1. Metal material constitutive relationship model
There is a part of the material in the engineering, when it reaches the yield point, it does not enter the strengthening stage immediately. Instead, it undergoes a transition process. As shown in figure 1, the constitutive equation of the corresponding material can be represented with the model proposed by Pham [13]. As in equation (1), where the transition part is assumed to be a straight line and is considered the ideal plastic plateau.

$$
\sigma = \begin{cases} 
E\varepsilon & (\varepsilon < \varepsilon_y) \\
\sigma_y & (\varepsilon_y < \varepsilon < \varepsilon_{st}) \\
\sigma_y \left[ 1 + E(\varepsilon - \varepsilon_{st}) / (\alpha \sigma_y) \right]^n & (\varepsilon > \varepsilon_{st}) 
\end{cases}
$$

where $\sigma$ is the stress, $E$ is the elastic modulus, $\varepsilon$ is the strain, $n$ is the strain hardening exponent, and $\alpha$ is the coefficient (the ratio of the initial plastic strain $\varepsilon_{st}$ to the initial yield strain $\varepsilon_y$). It is found that the range of $\alpha$ varies from 7 to 23 [14].

2.2. Dimensional analysis
Indentation test is carried out on the metal material containing residual stress using a spherical indenter, and the corresponding indentation curve of the material can be obtained. Figure 2 is a typical continuous indentation load-displacement curve. When the maximum indentation depth is $h_{max}$, the corresponding indentation load is $P_{max}$. When the indentation load $P$ is completely removed, the corresponding depth is the residual indentation
The shaded part $W_p$ in the figure 2 is the plastic deformation work, which is represented by the loading curve, the unloading curve and the area enclosed by the coordinate axis. The shaded part $W_e$ in the figure is the elastic recovery work, which is represented by the unloading curve and the axis enclosed area, the total loading work $W_t$ is the area enclosed by the loading curve and the coordinate axis.

For a specimen with residual stress $\sigma_i$, during the loading and unloading process, the indentation load $P$ is mainly affected by the following parameters, which can be represented by an equation.

$$p = f(E, v_i, E_i, v_i, \sigma_y, n, \sigma_i, h, R),$$

where $E$ and $E_i$ are the elastic modulus of the test specimen and the indenter respectively; $v$ and $v_i$ are the Poisson’s ratio of the test specimen and the indenter respectively; $h$ is the indentation depth of the indenter; $R$ is the radius of spherical indenter.

Using the simplified elastic modulus $E^* = [(1 - v_i^2)/E_i + (1 - v^2)/E]^{-1}$, equation (2) can be rewritten as:

$$p = f(E^*, \sigma_y, n, \sigma_i, \alpha, h, R),$$

(3)

According to the dimensional analysis and the $\Pi$ theory proposed by Brand [16], the yield strength and the indentation depth are selected as dimensionally-independent variables, while other parameters are dimensionally-dependent variables. Equation (3) can be expressed as:

$$p = \sigma_y h^2 \prod_{\alpha} \left( \frac{E^*}{\sigma_y}, \frac{\sigma_i}{\sigma_y}, n, \alpha, \frac{h}{R} \right),$$

(4)

It can be found that the number of independent variables of the function is reduced from 6 to 4 after dimensionless processing. When the indentation depth is $h_m$, by integrating the load-displacement curve, the loading work or the total pressing work $W_t$ can be obtained, as shown in the following equation:

$$W_t = \int_{h_m}^{h_m} p dh = \sigma_y h_m^2 \prod_{\beta} \left( \frac{E^*}{\sigma_y}, \frac{\sigma_i}{\sigma_y}, n, \alpha, \frac{h_m}{R} \right),$$

(5)

Similar to equation (5), the load-displacement curve is integrated from the residual indentation depth to the total indentation depth to obtain the unloading work or elastic recovery work $W_e$, as shown in the following equation:

$$W_e = \int_{h_m}^{h_m} p dh = \sigma_y h_m^2 \prod_{\beta} \left( \frac{E^*}{\sigma_y}, \frac{\sigma_i}{\sigma_y}, n, \alpha, \frac{h_m}{R} \right),$$

(6)

In the process of indentation, the radius $R$ of the indenter is known, and at the same time, the indentation depth $h_m$ is also the initial value set before the experiment, so the indentation can be obtained by combining the equations (5) and (6). Dimensional equation of work ratio can be expressed as follows:
Where $W_i - W_f$ represents the plastic indentation work $W_p$, as shown in figure 2. Indentation work ratio $(W_i - W_f) / W_i$ is selected to establish the dimensionless equation between the residual stress and mechanical parameters of materials. Indentation work is selected as the analytic parameter, which can uniquely represent the indentation response of a material containing residual stress.

3. Finite element analysis of indentation simulation experiment

Using ABAQUS finite element analysis software the indentation process of metal materials with residual stress is simulated in this paper. Considering the problem of symmetrical boundary conditions, the two-dimensional axial symmetric model is chosen here, as shown in figure 3. The indenter is a 0.5 mm rigid spherical indenter. It is considered that the shape of the indenter is constant during the indentation process. In order to reduce the number of elements in the model, the pressed specimen is a cylinder with a radius and height of 5 mm. The mesh of the contact between the indenter and the specimen is refined. The mesh size is from coarse to fine, and the mesh element type is CAX4R. There are 2280 elements in the model. As show in figure 3, the contact between the spherical indenter and the sample is face-to-face contact, and the depth of each pressing step can reach 5 μm.

Cao [5] believes that the contact friction between the indenter and the specimen is considered to be frictionless when the maximum indentation depth $h_m / R$ is not more than 0.08. The maximum indentation depth in this paper is 0.02 mm, which is considered to be no friction between the contact surfaces.

In this paper, by applying prestress in the radial direction of the whole model, the double equiaxed residual stress is simulated. Uniform tensile stress and uniform compressive stress simulates residual stress. The specimen and the indenter are set by axial symmetric constraints, the model can be moved along the axis of symmetry, and the bottom end of the sample is fixed.

In this paper 350 kinds of different material parameters were selected for finite element analysis to determine the dimensionless equation $\prod_{i=1}^{n} \left( \frac{E}{\sigma_y} \right) \left( \frac{\sigma_r}{\sigma_y} \right)^{1/a}$. The modulus of elasticity and yield strength ratio $E / \sigma_y$ is 250 to 1000, strain hardening exponent $n$ is 0.1 to 0.5, the ratio coefficients $a$ are 7, 14 and 23 respectively, the ratio of residual stress $\sigma_r$ and yield strength $\sigma_y$ are taken as $-0.8$ to $0.8$. Poisson’s ratio $\nu$ is fixed at 0.3. Specific material parameters are shown in table 1.

4. Numerical analysis of spherical indentation

4.1. Influence of element mesh size of the model

In order to study the effect of finite element mesh size on simulation analysis in the model with residual stress, the mesh at the contact part of the indenter was re-divided. The range of minimum mesh size in the model was from 0.0128 mm to 4.267 μm, and the total number of elements increased from 2280 to 19 584. Figure 4. shows a comparison of the effects of the original mesh and the re-divided mesh on the load-displacement curve in the continuous indentation experiment, taking $E / \sigma_y = 650$, $\nu = 0.28$, $n = 0.3$, $\sigma_r / \sigma_y = 0.6$. The result shows that the
change in the number of meshes and the minimum mesh size has little effect on the simulation results. Therefore, in order to save computational time and improve working efficiency, the minimum mesh size is selected as 0.0128 mm in this paper.

4.2. Effect of residual stress on plastic indentation zone
During the indentation experiment, the plastic contact between the indenter and the specimen is plastically deformed. In order to study the influence of residual stress on the indentation plastic zone and the influence of the model size on the simulation results, it was discussed by numerical simulation. Here, the material parameters elastic modulus 180 GPa, yield strength 240 MPa, strain hardening index 0.3 and ratio coefficient 14 were used to carry out three sets of spherical indentation simulation experiments, respectively, and the residual stress $\sigma_r / \sigma_y$ was introduced to $-0.8, 0$ and $0.8$, as shown in figure 5. When the indentation depth $h_{im}$ is 0.02 mm and the indentation contact radius is about 0.125 mm, the plastic zone in the figure contains the yield transition phase. As can be seen from the figure, with the increase of residual stress, the plastic deformation area is increasing and the elastic deformation area is decreasing. Material is difficult to yield and resist the deformation of the material because of the material contains compressive residual stress. When the materials contain residual tensile stress, the material under the indenter will cause larger area plastic deformation, because the material is easier to yield.

In this paper, the simulated spherical indenter radius is 0.5 mm. From the simulation results, it is found that the surface radius of the plastic deformation zone is about 3.10 ~ 4.03 times of the contact radius of the indentation, and the diameter of full plastic zone at position of the maximum indentation depth is between 0.776 mm to 1.008 mm, and the simulated specimen is a cylinder with a diameter of 10 mm, which is about 12.89 ~ 9.92 times the radius of the plastic deformation zone. It can be considered that the sample size of the finite element model is reasonable.

| $E$ [GPa] | $\sigma_y$ [MPa] | $E / \sigma_y$ | $n$ | $\alpha$ | $\sigma_r / \sigma_y$ |
|----------|-----------------|--------------|----|--------|-----------------|
| 100      | 400             | 250          | 0  | 7      | -0.8            |
| 140      | 350             | 400          | 0.1| 14     | -0.4            |
| 150      | 300             | 500          | 0.3| 23     | 0               |
| 168      | 280             | 600          | 0.5| 3      | 0.4             |
| 180      | 240             | 750          |    |        | 0.8             |
| 198      | 220             | 900          |    |        |                 |
| 200      | 200             | 1000         |    |        |                 |

Table 1. Material properties of FE analysis.

![Figure 4. The contrast of load-depth curves with different mesh sizes.](image-url)
4.3. The influence of $\alpha$ on indentation load-displacement curve

For the measurement of residual stress of metal materials containing yield plateau, a ratio coefficient $\alpha$ needs to be introduced. Therefore, different $\alpha$ is used to analyze and compare the impact of load-displacement curve. Here, the material parameters elastic modulus 180 GPa, yield strength 240 MPa, strain hardening index 0.3 were used to carry out three sets of spherical indentation simulation experiments, the ratio coefficients $\alpha$ were 7, 14 and 23, respectively, and the indentation depth $h_m$ was 0.02 mm. As shown in figure 6, the ratio coefficient has a significant influence on the indentation response. Under the same indentation depth, the indentation load decreases with the ratio coefficient increases. It means that the smaller the coefficient of ratio, the greater the stress required when the same strain occurs, that is, the stress required to occur at the same strain is greater. In addition, it can be seen from the figure that the slopes of the unloading curves under different coefficients are similar, so the influence of the magnitude of the ratio coefficient on the depth of the residual indentation is relatively small.

4.4. Determination of dimensionless function

4.4.1. Determination of dimensionless function without residual stress

The dimensionless relationship between the indentation work and the material parameters and residual stress has been established by dimension analysis. When the metal material does not contain residual stress, equation (7) will be expressed by the following equation.

$$\frac{W_i - W_e}{W_i} = \prod_{k=1}^{7} \left( \frac{E^k}{\sigma_y}, n, \alpha \right),$$  

In order to determine the specific function form of the dimensionless equation $\prod_{k=1}^{7}$, the finite element simulation of 84 kinds of metal materials without residual stress in table 1 was carried out, and the simulation results were analyzed. Figure 7. shows the relationship between work ratio $(W_i - W_e)/W_i$ and $\ln(E^k/\sigma_y)$, for different ratio coefficient $\alpha$ and strain hardening exponent $n$, indentation work ratio $(W_i - W_e)/W_i$ and $\ln(E^k/\sigma_y)$ may be expressed by a cubic polynomial. Indentation work ratio $(W_i - W_e)/W_i$ increases when

Figure 5. Simulation of indentation under different residual stresses.

(a) $\sigma_r/\sigma_y = -0.8$  
(b) $\sigma_r/\sigma_y = 0$  
(c) $\sigma_r/\sigma_y = 0.8$
ln\(\frac{E^*}{\sigma_y}\) become large. When \(\frac{E^*}{\sigma_y}\) is relatively large, the stress required for straining of the material is relatively small, that is, the material is more prone to yield during the indentation process. When the spherical indenter is pressed to the maximum depth during the loading process, the plastic deformation area inside the material is relatively large. Since the load \(P\) is smaller, the total work \(W_t\) is relatively small. During the unloading
process, the material is mainly plastically deformed, so the elastic recovery is small. It is resulted that the elastic recovery work $W_e$ is smaller, the indentation work ratio \( \frac{W_e}{W_i} \) is relatively large. Similarly, it can be found that when the ratio coefficient $\alpha$ is constant, all curves shift upward with the decrease of strain hardening index $n$, because the material is more prone to yield when $n$ is smaller. When $n=0$ and the stress of the material reaches yield strength, there will be no elastic deformation and it will completely enter the plastic stage. The indentation work ratio \( \frac{W_e}{W_i} \) will approach 1 when $\frac{E^*}{\sigma_f} = 1000$ and $n = 0$.

In addition, it can be found from the above figure that when the ratio coefficient $\alpha$ is small, the influence of the strain hardening index $n$ on the indentation work ratio is relatively large.

Take the simulation results of $\frac{E^*}{\sigma_f} = 500$ as an example, the relationship between the ratio coefficient $\alpha$ and $n$ is further studied. As shown in figure 8, each parameter point is on the fitting surface, and the relationship can be represented by a two-dimensional quadratic polynomial, whose $R^2$ value is 0.9978.

Through the above analysis, the fitting of the numerical simulation results of 84 kinds of metal materials without residual stress can obtain the specific function expression of the dimensionless equation $I_d$, as shown in the following formula:

$$\frac{W_i - W_e}{W_i} = b_1 + b_2^*\alpha + b_3^*n + b_4^*\alpha^*n + b_5^*\alpha^2n + b_6^*\alpha^*n^2 + b_7^*\alpha^2$$

$$+ b_8^*n^2 + c_1^* + c_2^* \ln \left( \frac{E^*}{\sigma_f} \right) + c_3^* \left( \ln \left( \frac{E^*}{\sigma_f} \right) \right)^2 + c_4^* \left( \ln \left( \frac{E^*}{\sigma_f} \right) \right)^3,$$

(9)

The specific values of coefficients $b_i (i = 1 \sim 8)$ and $c_i (i = 1 \sim 4)$ in the above equation are shown in table 2:

### Table 2. Coefficient of fitting function without residual stress.

| $b_i$    | $c_i$    |
|----------|----------|
| 1        | 5.5710E-01 | -5.2557E+00 |
| 2        | -1.7658E-04 | 2.8021E+00  |
| 3        | -9.8790E-02 | -3.8222E-01 |
| 4        | 7.4272E-03  | 1.7830E-02  |
| 5        | -1.7407E-04 |                      |
| 6        | 5.2485E-04  |                      |
| 7        | 5.9905E-06  |                      |
| 8        | -1.8797E-02 |                      |

4.4.2. Determination of dimensionless function with residual stress

For the metal materials with a total of 350 kinds of different parameter combinations in table 2 with residual stress $\sigma_r / \sigma_f = -0.8 \sim 0.8$, the dimensionless equation $I_d$ in equation (7) can be determined by the finite element analysis results. The indentation work ratio $\left( \frac{W_i - W_e}{W_i} \right)$ can be expressed by the quadratic polynomial of $E^*/\sigma_f$ and $n$ when $\sigma_r / \sigma_f$ and $\alpha$ were fixed. The representative fitting surface was shown in Figure 8.
When the value of $/s_\text{y} = 0.4$ and $\alpha = 14$ were fixed. The value of $R^2$ is 0.999. It can be found that the data points are almost distributed over the surface by finite element analysis.

Similarly, when fixed $E^*/\sigma_y$ and $n$, the indentation work ratio $(W_i - W_e)/W_i$ can be expressed by the quadratic polynomial of $/s_\text{y}$ and $\alpha$. The representative fitting surface was shown in figure 10, when the value of $E^*/\sigma_y = 750$ and $n = 0.3$ are fixed. The value of $R^2$ is 0.9985. When the residual stress is in the tensile stress state, the larger the residual stress, the smaller the ratio coefficient $\alpha$ and the smaller the indentation work ratio $(W_i - W_e)/W_i$. In the same time, when the material $\alpha$ is smaller, the effect of residual stress on the material indentation response is greater.

Through the above analysis, the dimensionless equation can be represented by the product form of two binary quadratic polynomials. By fitting the data of the finite element analysis of 350 kinds of different parameter combinations, the specific function expression can be obtained, as shown in equation (10).

$$
\frac{W_i - W_e}{W_e} = d_1 + d_2 \ln \frac{E^*}{\sigma_y} + d_3 n + d_4 \ln \frac{E^*}{\sigma_y} n + d_5 \left( \ln \frac{E^*}{\sigma_y} \right)^2 n + d_6 \ln \frac{E^*}{\sigma_y} n^2
$$

$$
+ d_7 \left( \ln \frac{E^*}{\sigma_y} \right)^2 + d_8 n^2 f_1 + f_2 \frac{\sigma_y}{\sigma_y} + f_3 \alpha + f_4 \frac{\alpha}{\sigma_y} + f_5 \frac{\sigma_y}{\sigma_y} \alpha^2 + f_6 \frac{\sigma_y}{\sigma_y} \alpha + f_7 \left( \frac{\sigma_y}{\sigma_y} \right)^2 + f_8 \alpha^2,
$$

(10)

The fitting coefficients $d_i(i = 1–8)$ and $f_i(i = 1–8)$ in the equation are listed in the following table 3:
5. Inversion analysis and experimental verification

5.1. The theory of inverse analysis method

Two specific dimensionless function expressions with and without residual stress have been given by analyzing the simulation results. In this section, a new set of inverse calculation methods based on the indentation energy method is given, and the parameters and residual stress of the unknown metal materials are determined by the indentation test. Figure 11 is a flow chart of the inverse analysis method. The experiment assumes that the elastic modulus $E$ is known, and the method is mainly divided into three steps.

| $d_i$   | $f_i$     |
|---------|-----------|
| 1       | 3.5751E-01| -1.7173E + 00 |
| 2       | -2.4719E-01| -1.5515E-02 |
| 3       | -3.2902E-01| -4.6277E-03 |
| 4       | 1.1675E-01| 1.1110E-04 |
| 5       | -9.3376E-03| 9.4439E-05 |
| 6       | 8.6504E-03| 9.9745E-07 |
| 7       | 1.6803E-02| -8.3403E-03 |
| 8       | -4.6882E-02| 9.7360E-05 |

Figure 11. A flow chart of the reverse analysis method.
Step 1. Indentation response parameters are extracted. The spherical indenter was used for the with and without residual stress test of the measured metal materials. The real indentation work ratio \((W_1 - W_2)/W_1\) when the material without residual stress and the real indentation work ratio \((W_2 - W_2)/W_2\) with residual stress were extracted from the load-displacement curve.

Step 2. Calculation of the parameters \(\sigma_y, n\) and \(\alpha\). The range of \(E/\sigma_y, n, \alpha\) is set, and different combinations of each parameter are established and substituted into equation (9). The relative error \(e\) of the inverse indentation work ratio and the real indentation work ratio is calculated. The control error \(e\) is no more than 5%.

Step 3. Calculate the residual stress. In the first step, the real indentation work ratio of the materials containing residual stress is obtained. The parameter combinations determined in the second step are substituted into equation (10), and the residual stress \(\sigma_r\) of the materials is calculated.

5.2. Inverse analysis method for verification of typical metal materials
In order to verify the effectiveness of this method, this section will carry out indentation simulation experiments on two common metal materials with yield plateau (SS400 and SM490) to verify the inverse analysis method proposed. In the literature [11], the mechanical properties of the two materials are given by tensile experiments, as shown in table 4. Three sets of indentation experiments were carried out for each material. Two groups of materials to be tested were artificially introduced with residual stress \(\sigma_r\) of \(\pm 0.6 \sigma_y\). The other set of indentation tests were conducted without residual stress. The spherical indenter was used for indentation test. Indentation load-displacement curve could be obtained at a depth \(h_m = 0.02\) mm. As shown in figure 12, when the material contains compressive residual stress, the required indentation load \(P\) is higher than the material without residual stress at the same indentation depth \(h\), and the same indentation is used when the material contains tensile residual stress. The required indentation load \(P\) at depth \(h\) is lower than that without residual stress. When the compressive residual stress is contained in the material, the yield resistance of the material will be improved, making the material difficult to yield. The effect of tensile residual stress on the indentation load is more obvious. As the indentation depth increases, the load difference becomes larger and larger. In addition, it can be seen from the figure that after complete unloading, the residual indentation depth of the material under the three sets of simulation experiments is similar, which indicates that the residual stress has little effect on the unloading process of the metal material indentation experiment.

| Materials | \(E\) [GPa] | \(\sigma_y\) [MPa] | \(n\) | \(\alpha\) |
|-----------|-------------|-----------------|------|--------|
| SS400     | 198.6       | 313.6           | 0.243| 9.8    |
| SM490     | 208.9       | 395.2           | 0.230| 12.1   |

Table 4. Properties of metal materials obtained by tensile test.
By processing the load-displacement curve in figure 11, the indentation response parameters and the total work \( W_t \) and the true elastic recovery work \( W_e \) can be obtained. The indentation parameters under different residual stress states are listed in table 5.

The indentation parameters in table 5 are substituted into equation (9). According to the inverse analysis method, \( E^*/\sigma_y, n \) and \( \alpha \) of the two metal materials SS400 and SM490 can be calculated. And then brought into equation (10), determine the magnitude of the residual stress, and finally to determine the positive and negative of the residual stress according to the load-displacement curve. If the curve is lower than the load-displacement curve without residual stress, the tensile residual stress in the material is positive. If the curve is higher than the load-displacement curve without residual stress, the compressive residual stress in the material is negative.

The inverse results are listed in tables 6 and 7. As can be seen from the table, the calculated yield strength \( \sigma_y \), strain hardening index \( n \) and ratio coefficient \( \alpha \) are all within a very small relative error to the actual value of the material. The maximum error is no more than 5%. When the residual stress is introduced into the two metal materials, the inverse calculation error is 8.71% and 12.30%. And the inverse calculation error is 10.90% and 8.79% when the tensile residual stress is introduced. It should be noted that the error of the inverse calculation residual stress \( \sigma_r \) can be reduced by subdividing the input parameter \( E^*/\sigma_y, n \) and \( \alpha \) intervals.

5.3. Experimental verification of spherical indentation

5.3.1. Experimental material parameters

In this paper, Q345R plate is selected as the test material for indentation test. In the process of indentation, Saint Venard’s principle can be considered to be satisfied. Indentation tester was shown in figure 13 and sample was shown in figure 14. We took two samples with number M and N respectively, and the size of samples is 20 * 20 * 8 mm. The material parameters were obtained by conventional tensile test. The modulus of elasticity \( E = 189.808 \) GPa, \( \sigma_y = 328.3 \) MPa, strain hardening index \( n = 0.133 \). X-ray diffraction (XRD) technology is used to measure the residual stress on the surface of the sample. Three points are removed from each sample for measurement. The specific values are shown in table 8. In addition, we have done the finite element indentation simulation and indentation test of the material at the same time. The load-displacement curves obtained by finite element simulation and indentation test of the material have good consistency, as shown in figure 15.

| Table 5. Indentation parameters under different residual stresses. |
|--------------------------|--------------------------|--------------------------|
| \( \sigma_r/\sigma_y \)   | SS400                   | SM490                   |
| \( W_t [N \cdot \text{mm}] \) | \( W_r [N \cdot \text{mm}] \) | \( W_t [N \cdot \text{mm}] \) | \( W_r [N \cdot \text{mm}] \) |
| 0.6                      | 0.85989                 | 0.06546                 | 0.6                      | 1.01459                 | 0.08484 |
| 0                        | 0.79031                 | 0.05698                 | 0                        | 0.92370                 | 0.07292 |
| 0.6                      | 0.68396                 | 0.04592                 | 0.6                      | 0.78725                 | 0.05731 |

| Table 6. Reverse analysis results of material parameters. |
|--------------------------|--------------------------|--------------------------|
| Materials                | \( \sigma_y \) [MPa]    | \( n \)                  | \( \alpha \)              |
|                          | Calculated value         | Error                    | Calculated value          | Error                    | Calculated value          | Error                    |
| SS400                    | 308.1                    | 1.75%                    | 0.247                    | 1.65%                    | 9.74                     | 0.61%                    |
| SM490                    | 391.9                    | 0.84%                    | 0.237                    | 3.04%                    | 11.59                    | 4.21%                    |

| Table 7. Reverse analysis results of residual stress. |
|--------------------------|--------------------------|--------------------------|
| Materials                | SS400                   | SM490                   |
| \( \Delta \sigma_r \)     | \( \sigma_r \) [MPa]    | \( \sigma_r \) [MPa]    |
| Input                     | \( \sigma_y \) [MPa]    | \( \sigma_y \) [MPa]    |
| Output                    | \( \sigma_y \) [MPa]    | \( \sigma_y \) [MPa]    |
| Error                     | 8.71%                   | 10.90%                  | 12.30%                  | 8.79%                   |

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Figure 13. Indentation tester.

Figure 14. The diagram of indentation specimen.

Table 8. The residual stress on specimen surfaces at different welding parts.

| Q345R | Point 1 (MPa) | Point 2 (MPa) | Point 3 (MPa) | average value (MPa) |
|-------|--------------|--------------|--------------|--------------------|
| M     | −310.69      | −288.21      | −292.10      | −297.00            |
| N     | −141.58      | −98.61       | −105.44      | −115.21            |

Figure 15. Comparison between FE analysis and experimental p-h curves.
5.3.2 Analysis of inversion analysis results

Through the continuous spherical indentation test, the indentation work data with the indentation depth \( h/R \) of 0.01, 0.06 and 0.07 can be obtained, which are listed in table 9. Taking these data as input data, the mechanical parameters of materials can be calculated by inversion analysis method. As shown in table 10, it can be found that the error of strain hardening index is within 7%, and the error of yield strength and residual stress is about 10%.

6. Conclusions

(1) In this paper, the stress-strain relationship model of the yielding plateau metal material is given. Based on the dimensional analysis, the indentation work ratio \( (W_t - W_r)/W_r \) is selected as the indentation parameter to establish the dimensionless equation between the residual stress and the material parameters. The metal materials of 350 kinds of different parameters were simulated to determine the specific dimensionless function expression between the indentation work ratio and the material parameters with and without residual stress. At the same time, a new set of inverse analysis methods is given. The yield strength \( \sigma_y \), strain hardening index \( n \), ratio coefficient \( \alpha \) and residual stress \( \sigma_r \) can be obtained by performing two spherical indentation tests on the residual stress specimen. By indentation simulation experiments on two known mechanical parameters SS400 and SM490, it is found that the error of \( \sigma_y \), \( n \) and \( \alpha \) is less than 5%, and the error of \( \sigma_r \) is about 10%.

(2) By discussing the numerical simulation results, it is found that the internal plastic deformation region of the material increases with the increase of residual stress. When the maximum indentation depth is 0.02 mm, the surface radius of the plastic deformation zone is about 3.10–4.03 times of the contact radius of the indentation, and the radius of the simulated specimen is about 12.89–9.92 times of the radius of the plastic deformation zone. The establishment of the specimen size is reasonable.

(3) The influence of the ratio coefficient \( \alpha \) on the spherical indentation load-displacement curve is analyzed. It is found that the ratio coefficient has a significant influence on the indentation response during the loading process. Under the same indentation depth, the indentation load decreases as the ratio coefficient increases. In addition, the effect of the ratio coefficient on the residual indentation depth is relatively small during the unloading process.

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| Table 9. Indentation parameters at different indentation depths. |
|-----------------|-----------------|-----------------|-----------------|
| Sample M       | Sample N        |
| h/R            | \( W_t (N \cdot \text{mm}) \) | \( W_r (N \cdot \text{mm}) \) | \( W_t (N \cdot \text{mm}) \) | \( W_r (N \cdot \text{mm}) \) |
| 0.1            | 0.225489        | 0.025798        | 0.211604        | 0.024618        |
| 0.6            | 8.886383        | 0.373186        | 8.637925        | 0.365131        |
| 0.7            | 12.045652       | 0.46474         | 11.736968       | 0.453402        |

| Table 10. Material properties determined using proposed reverse algorithm. |
|-----------------|-----------------|-----------------|-----------------|
| Q345R           | \( \sigma_y \)  | \( n \)         | \( \sigma_r \)  |
| M               | Calculated value | Error (%)       | Calculated value | Error (%)       | Calculated value | Error (%)       |
| N               | 302.7            | 11.08%          | 0.0812          | 3.21%           | −329.49         | 10.34%          |
|                 | 295.4            | 13.22%          | 0.0779          | 7.04%           | −103.67         | 10.02%          |
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