Neutrinoless Double Beta Decay and $<\eta>$ Mechanism in the Left-Right Symmetric Model

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Abstract

The neutrinoless double beta decay is studied in the framework of left-right symmetric model. The coexistence of left and right handed currents induces rather complicated interactions between the lepton and hadron sectors, called $<\lambda>$ mechanism and $<\eta>$ mechanism in addition to the conventional effective neutrino mass $<m_\nu>$ mechanism. In this letter, we study the possible magnification of $<\eta>$ mechanism and the relatively vanishing of $<\lambda>$ mechanism. The importance to survey $0\nu\beta\beta$ decay of different nuclei for specifying new physics beyond the Standard Model is also discussed.
1 Introduction

Neutrinoless double beta ($0\nu\beta\beta$) decay is one of the key probes for the new physics beyond the Standard Model (BSM physics). In this letter, we consider this process in the framework of left-right ($L-R$) symmetric model \cite{1,2}, where the decay is concerned with the correlations between the $L$-handed light neutrinos and the $R$-handed heavy neutrinos. $L-R$ symmetric model in the SO(10) grand unified theory appears in the intermediate stage \cite{3,4} which includes

\[
SO(10) \supset SU(4)_{PS} \times SU(2)_L \times SU(2)_R \\
\supset SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}
\]

and is related with the wide varieties of BSM physics besides $0\nu\beta\beta$ decay, like baryo-genesis via lepto-genesis and dark matters etc. There are two conditions to realize $0\nu\beta\beta$ decay in the context of this framework \cite{5}.

1. $\nu_e$ should be the same as its anti-particle

   \[
   \nu_e = \overline{\nu_e}
   \]

2. the connecting neutrinos should have the same helicity. The latter condition is satisfied if neutrinos are massive or if the $R$-handed current coexists with the $L$-handed current. The first case of 2. is described as the well known effective neutrino mass,

\[
<m_\nu> = |\sum_j U_{ej}^2 m_j|.
\]

Here $U_{\alpha i}$ (Greek (Latin) indicates flavour (mass) eigenstate) is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix \cite{6,7} in $L$-handed current. Substituting the observed values,

\[
|U_{11}|^2/|U_{13}|^2 \approx 30.
\]

Then, the inverted hierarchy (IH) case enhances $<m_\nu>$ relative to the normal hierarchy (NH) case. Though the final answer to the hierarchy problem is given by observation, the theoretical predictions have been given by many models. One of the typical models is due to the predictive minimal SO(10) model \cite{4}. Based on this model, we fitted the low energy spectra of all quark and lepton masses and the CKM and the PMNS mixing angles and their phases. Our results prefer the NH manifestly to the IH: That is, inputting the observed lepton masses and the PMNS angles into the model, we compared the outputs of quark masses and CKM matrices \cite{8,9} in the model with the observations. We obtained $\chi^2 \leq 1$ for the NH case and $\chi^2 > 200$ for the IH case \cite{10}. In this model, the effective neutrino mass is also predicted including the Majorana phases as

\[
<m_\nu> \approx 1 \text{ meV}.
\]
On the other hand, the recent $0\nu\beta\beta$ experiment in the KamLAND-Zen [11] provides the most stringent upper limit on it. The half life $T_{1/2}$ of $0\nu\beta\beta$ decay in $^{126}\text{Xe}$ is

$$\frac{1}{T_{1/2}} = |G_{0\nu}|^2 \left( \frac{<m_\nu>}{m_e} \right)^2 < \frac{1}{2.3 \times 10^{26}\text{ yr}} @ 90\% \text{ C.L.} \quad (6)$$

Here $G_{0\nu}$ is the phase-space integral and $M_{0\nu}$ is the nuclear matrix element (NME), which leads to the $<m_\nu> = 36 - 156\text{ meV}$ [11, 12, 13], already in the inverted hierarchy regions. Efforts to reduce the ambiguities of NME in different nuclear models are in progress [14, 15, 16, 17].

We consider $0\nu\beta\beta$ decay in the $L-R$ symmetric model. The model generates contact, heavy neutrino and light neutrino exchange quark-quark interactions The quark-quark interactions then mapped onto contact, pion exchange and light neutrino exchange interactions between two nucleons classified in a general form in effective field theory [18, 19]. Here we focus on the light neutrino exchange mechanism in particular the $<\lambda>$ and the $<\eta>$ mechanisms in addition to the $<m_\nu>$ mechanism [5, 20]. The BSM physics appears in the leptonic currents, which restricts the structure of the hadronic current. Such interplay between leptonic and hadronic currents has been overlooked so far. Especially, $0\nu\beta\beta$ is very sensitive to the spatial momentum of neutrino propagator and interference between the vector and axial vector currents of nucleon, enhancing $<\eta>$ mechanism [5, 21, 22]. This term is also very crucial to the heavy right-handed neutrino mass. In this letter, surveying into these entanglements of interplay and solving them, we narrow down the general forms of $L-R$ symmetric models [18, 19], leading to the mechanism of $0\nu\beta\beta$ to $<m_\nu>$ and $<\eta>$ mechanisms if experiments reveal the non-null result around the present upper bound.

This letter is organized as follows. In Sec.2, we discuss the structure of the leptonic current, assuring the low energy seesaw mechanism. Its hadronic counterparts are studied in Sec.3. We show a simple understanding of the mechanism to enhance a sensitivity to the $<\eta>$ mechanism due to the V-A interference term. This mechanism has a potential to reveal $R$-handed current if the $0\nu\beta\beta$ decay is discovered above the NH region from the present and near future experiments. It is also discussed that the atom dependence (A-dependence) of $0\nu\beta\beta$ beta decay rate may clarify $<m_\nu>$ and/or $<\eta>$ mechanisms. Section 4 is devoted to discussions.

2 Right-handed weak current

We consider $L - R$ symmetric model [2] in this section. The weak Hamiltonian is given by

$$H_W = \frac{G_F \cos \theta_c}{\sqrt{2}} \left[ j_{L\mu}^\dagger \tilde{L}_\mu + j_{R\mu}^\dagger \tilde{R}_\mu \right] + H.c. \quad (7)$$
Figure 1: Diagrams of 0νββ decay. (a), (b), and (c) are \( < m_\nu >, < \lambda >, \) and \( < \eta > \)-mechanisms, respectively.

Here \( j_\mu \) \((J_\mu)\) indicates leptonic (hadronic) current, and the \( L \) and \( R \)-handed leptonic currents, \( j_{L\mu} \) and \( j_{R\mu} \), are given by

\[
j_{L\alpha} = \sum_l \bar{\ell}(x)\gamma_\alpha P_L\nu_{lL}(x),
\]

\[
j_{R\alpha} = \sum_l \bar{\ell}(x)\gamma_\alpha P_R\nu_{lR}(x),
\]

where

\[
P_L = \frac{1}{2}(1 - \gamma_5), \quad P_R = \frac{1}{2}(1 + \gamma_5).
\]

Also \( \nu_{lL}(N_{lR}) \) are \( L \)-handed \((R \)-handed\) weak eigenstates of the neutrinos, and

\[
\tilde{J}_L^\mu(x) = J_L^\mu(x) + \kappa J_R^\mu(x),
\]

\[
\tilde{J}_R^\mu(x) = \eta J_L^\mu(x) + \lambda J_R^\mu(x).
\]

Thus the system is mixed with rather simple leptonic world and composite hadronic world. There are many precedent works to have discussed 0νββ in the \( L - R \) symmetric models \cite{23} \cite{24} \cite{25} \cite{26} \cite{27} etc. We are not concerned with the detailed new calculations of hadronic models but to try to give a firm foundation for low energy seesaw mechanism and to make clear the connection of neutrino potential with hadronic NMEs. The main diagrams of 0νββ decay in the \( L - R \) symmetric model are depicted in Fig.1. In general, we have the other diagrams via
Here $A_R^0$ is the current experimental bound. $A_R^{(a),(b)}$ are the amplitude of (a) and (b) of Fig.2. Thus these diagrams give subdominant contributions. So we limit our arguments in Fig.1 hereafter. Its amplitude in closure approximation [20] is given as

$$R_{0\nu} = 4\sqrt{\frac{1}{2}} \left( \frac{G \cos \theta}{\sqrt{2}} \right)^2 \sum_{i=1}^{2} \sum_{\alpha, \beta} \int dx dy \int \frac{dk}{(2\pi)^3} e^{i k \cdot (y - x)} H_{\nu \mu} L_{\nu \mu},$$

where the lepton tensor $L_{\nu \mu}$ is

$$L_{\nu \mu} = \bar{e}_{p_{2, s'_2}(y)\gamma \nu} P_{\gamma} \frac{1}{2\omega} \left[ \frac{\omega \gamma^0 - \mathbf{k} \cdot \gamma + m_i}{\omega + A_1} - \frac{-\omega \gamma^0 - \mathbf{k} \cdot \gamma + m_i}{\omega + A_2} \right] P_{\alpha} e_{p_{1, s'_1}(x)}.$$  

(16)

Here $e_{p_{1, s'_1}(x)}$ are electron wave functions with the energy $e_i$, and the mixing matrices $U, V$ are omitted for simplicity. The energy denominator is given by $A_i = E_i < E_n > - E_i$ and $E_f + e_1 + e_2 = E_i$. Here $E_{i/f}$ and $< E_n >$ are energy of the initial/final nuclear state and the average energy of the intermediate nuclear state, respectively.

The nuclear tensor $H_{\nu \mu}$ is given by the matrix element of the nuclear weak current as

$$H_{\nu \mu} = \langle F| \tilde{J}_{\beta i}^\nu(y) \tilde{J}_{\alpha i}^{\mu\uparrow}(x)|I \rangle,$$

where $\tilde{J}_{L,R}^{\mu\uparrow}$ are given in (11) and (12).

The neutrino propagator becomes,

$$P_\alpha (\pm \omega \gamma^0 - \mathbf{k} \cdot \gamma + m_i) P_\beta = \begin{cases} m_i P_\alpha & (\alpha = \beta) \\ (\pm \omega \gamma^0 - \mathbf{k} \cdot \gamma) P_\beta & (\alpha \neq \beta). \end{cases}$$

(18)

In the presence of the R-handed current, we have $(\pm \omega \gamma^0 - \mathbf{k} \cdot \gamma) P_\beta$ in addition to (3). The spatial momentum exchanged between nucleon by neutrino is significantly larger than neutrino mass term, $|\mathbf{k}| \approx 100\text{MeV} \gg E_n - E_i, m_i$, which gives a significant effect to the decay rate.
This mechanism gives interesting interplay between particle physics and nuclear physics, whose explanation is the main theme of this paper.

The half life $T_{1/2}$ in this case \[5\] is given as

$$\frac{1}{T_{1/2}} = C_{mm}^{0} \left( \frac{<m_{\nu}>}{m_{e}} \right)^{2} + C_{m\lambda}^{0} \frac{<m_{\nu}>}{m_{e}} <\lambda> \cos \psi + C_{m\eta}^{0} \frac{<m_{\nu}>}{m_{e}} <\eta> \cos \psi$$

$$+ C_{\lambda\lambda}^{0} <\lambda>^{2} + C_{\eta\eta}^{0} <\eta>^{2} + C_{\lambda\eta}^{0} <\lambda><\eta>.$$ \(19\)

Here $C_{ab}^{0}$ includes NME and phase space integral. The other parts include BSM physics. The effective couplings $<\eta>$ and $<\lambda>$ are given as

$$<\lambda> = \lambda |\sum_{j} V_{ej}^{*} U_{ej}|,$$

$$<\eta> = \eta |\sum_{j} V_{ej}^{*} U_{ej}|.$$ \(20\)

$\psi$ is the relative phase between $<m_{\nu}>$ and $<\lambda>$ and $<\eta>$,

$$\psi = \text{arg} \left( \left( \sum_{j} m_{j} U_{ej}^{2} \right) \left( \sum_{j} U_{ej} V_{ej}^{*} \right) \right),$$ \(21\)

where $\sum'$ indicates the summation over only the light neutrinos. However, $U$ and $V$ are independent and we set $\psi = 0$ hereafter. The details of $\lambda$ and $\eta$ are given by \[30\] and \[31\].

We proceed to discuss the detailed structure of mixing matrices. Higgs sectors are composed of $(3,1,2)$, $(1,3,2)$ triplets ($\Delta_{L}, \Delta_{R}$, respectively) and bi-doublet $(2,2,0)$ ($\Phi$) under $SU(2)_{L} \otimes SU(2)_{R} \otimes U(1)_{B-L}$. They have the vacuum expectation values $v_{u}$, $v_{d}$, $v_{L}$, $v_{R}$ as

$$<\Phi>_{0} = \left( \begin{array}{c} \phi_{1}^{0} \\ \phi_{2}^{0} \end{array} \right), \quad v_{u} \quad 0 \quad 0$$ \(22\)

and

$$<\Delta_{L,R}>_{0} = \left( \begin{array}{c} \Delta_{L}^{++} \\ \Delta_{L}^{0} \end{array} \right), \quad 0 \quad 0 \quad v_{L,R}$$ \(23\)

The neutrino mass matrix is \[29\] \[30\] \[31\] \[32\]

$$M_{\nu} = \left( \begin{array}{cc} M_{L} & M_{D}^{T} \\ M_{D} & M_{R} \end{array} \right), \approx \left( \begin{array}{cc} 0 & M_{D}^{T} \\ M_{D} & M_{R} \end{array} \right).$$ \(24\)
Thus we have the extended Fermi couplings (7). In (8) and (9), \( \nu_L(N_R) \) are \( L \)-handed (\( R \)-handed) weak eigenstates of the neutrinos. Using 3 \( \times \) 3 blocks \( U, V, X, Y \), the mass eigenstates \( \nu', N' \) are given as

\[
\begin{pmatrix}
\nu \\
(N_R)\ell
\end{pmatrix}_L =
\begin{pmatrix}
U & X \\
V & Y
\end{pmatrix}
\begin{pmatrix}
\nu' \\
N'
\end{pmatrix}_L \equiv U \begin{pmatrix}
\nu' \\
N'
\end{pmatrix}_L.
\] (25)

That is,

\[
(\nu_L)_\alpha = U_{\alpha i} \nu'_{i} + X_{\alpha I} N'_{I}, \quad (N_R)^c_\alpha = V_{\alpha i} \nu'_{i} + Y_{\alpha I} N'_{I},
\] (26)

where \( \alpha \) (\( i \)) are the flavour (mass) eigenstates.

The constants \( \lambda \) and \( \eta \) in (7) are related to the mass eigenvalues of the weak bosons in the \( L \) and \( R \)–handed gauge sectors \( (W_L, W_R) \) as follows:

\[
W_L = W_1 \cos \zeta + W_2 \sin \zeta, \quad (27)
\]

\[
W_R = -W_1 \sin \zeta + W_2 \cos \zeta, \quad (28)
\]

\[
\frac{G_F}{\sqrt{2}} = \frac{g^2 \cos^2 \zeta}{8} \frac{M_{W_1}^2 \tan^2 \zeta + M_{W_2}^2}{M_{W_1} M_{W_2}}, \quad (29)
\]

\[
\lambda \equiv \frac{M_{W_1}^2 + M_{W_2}^2 \tan^2 \zeta}{M_{W_1} \tan^2 \zeta + M_{W_2}^2}, \quad (30)
\]

\[
\eta \equiv -\frac{(M_{W_2}^2 - M_{W_1}^2) \tan \zeta}{M_{W_1} \tan^2 \zeta + M_{W_2}^2}. \quad (31)
\]

Here \( M_{W_1} \) and \( M_{W_2} \) are the masses of the mass eigenstates \( W_1 \) and \( W_2 \), respectively, and \( \zeta \) is the mixing angle which relates the mass eigenstates and the gauge eigenstates. We are considering \( L - R \) symmetric model. The gauge boson mass is generated from (22) and (23) as

\[
M_W^2 = \left( \begin{array}{cc}
\frac{1}{2} g^2 (v_u^2 + v_d^2 + 2 v_L^2) & g^2 v_u v_d \\
g^2 v_u v_d & \frac{1}{2} g^2 (v_u^2 + v_d^2 + 2 v_R^2)
\end{array} \right)
\] (32)

and the mixing angle \( \zeta \) is

\[
\tan 2\zeta = \frac{2 v_u v_d}{v_R^2 - v_L^2} \approx \frac{2 v_u v_d}{v_R^2} = 2\xi \left( \frac{M_{WL}}{M_{WR}} \right)^2 \quad (33)
\]

with

\[
v_u^2 + v_d^2 = v_{ew}^2 \quad (34)
\]
\[ \xi = v_d/v_u = 1/\tan \beta \quad (35) \]

and

\[ M_{W2} = \sqrt{2} g_R v_R \geq 5 \text{TeV}. \quad (36) \]

In the \( L - R \) symmetric model, we set \( g_L = g_R \), which indicates further unification of at least rank five GUT, including \( SU(3) \) color. \( \tan \beta \) is constrained from the Yukawa coupling is renormalizable up to the GUT scale,

\[ 1 \leq \tan \beta \leq 60. \quad (37) \]

That is, the upper limit (lower limit) appears from the renormalizability of bottom (top) Yukawa coupling in GUT. Furthermore, in this case, large \( \tan \beta \) induces too rapid proton decay since the proton life-time is proportional to \( 1/\tan \beta^2 \) and \( \tan \beta \) is limited around 10 \[10\]. Reflecting these relatively low mass constraint, we will discuss on the low energy seesaw mechanism later.

Corresponding to Figure 1, we will consider \( 0\nu\beta\beta \) decay in this scheme:

• \( W_L - W_L \) diagram

\[ m_{\text{eff}}^{LL} = \sum_{i=1}^{3} U_{ei}^2 m_i + \sum_{i=1}^{3,6} k^2 X_{ei}^2 k^2 - M_I^2, \quad (38) \]

Here and in the subsequent discussions in this section, we write the subdominant terms in addition to (20), illustrating the seesaw structure. In the latter sum, 3 and 6 indicate type I (25) and Inverse seesaw mechanisms (47), respectively.

• \( W_R - W_R \) diagram

\[ m_{\text{eff}}^{RR} = \sum k^2 Y_{ei}^2 \frac{M_I^4}{k^2 - M_I^2} g_R^4 g_L^4 M_{WL}^4 M_{WR}^4, \quad (39) \]

with \( g_L = g_R \), which was suppressed compared with the others due to \( \left( \frac{M_{WL}}{M_{WR}} \right)^4 \).

• \( W_L - W_R \) diagram; the neutrino mixing (\( \lambda \)) and \( W_L - W_R \) mixing (\( \eta \))

\[ < \lambda > = \left( U_{ei} V_{ei}^s + X_{ei} Y_{ei}^s \frac{k^2}{k^2 - M_I^2} \right) \frac{M_{WL}^2}{M_{WR}^2}, \quad (40) \]

\[ < \eta > = \left( U_{ei} V_{ei}^s + X_{ei} Y_{ei}^s \frac{k^2}{k^2 - M_I^2} \right) (- \tan \zeta). \quad (41) \]

Here the first terms \( U_{ei} V_{ei}^s \) dominate, and let us estimate the magnitude of \( V_{ei} \). The naive type I seesaw (24) gives tiny value for the above quantities. We are interested in TeV scale
seesaw and consider the inverse seesaw mechanism \[36\] hereafter. Its $9 \times 9$ mass matrix is given by

$$M_\nu = \begin{pmatrix} 0 & M_T^T & 0 \\ M_D & 0 & M^T \\ 0 & M & \mu \end{pmatrix} \equiv \begin{pmatrix} 0_{3\times3} & M_{D3\times6}^T \\ M_{D6\times3} & M_{R6\times6} \end{pmatrix},$$

(42)

Their mass scales are $\mu \approx O(1)\text{keV}$, $M_D \approx O(100)\text{GeV}$, $M \approx O(1)\text{TeV}$. This matrix is approximately diagonalized by $9 \times 9$ mixing matrix $U [23,38,39]$ as

$$U \approx \begin{pmatrix} 1 - \frac{1}{2}M_D^T[M_R(M_R^T)]^{-1}M_D & M_D^T(M_R^T)^{-1} \\ -M_R^{-1}M_D & 1 - \frac{1}{2}M_R^{-1}M_D M_D^T(M_R^T)^{-1} \end{pmatrix}$$

as

$$U^T M_\nu U = \begin{pmatrix} m_{\text{light}} & 0_{3\times6} \\ 0_{6\times3} & M_{\text{heavy}}_{6\times6} \end{pmatrix}.$$ 

(44)

Here

$$m_{\text{light}} = M_D^T M^{-1} \mu (M^T)^{-1} M_D$$

(45)

and

$$M_{\text{heavy}} = M_R.$$ 

(46)

Thus $U$ of (25) in type I seesaw is modified in the inverse seesaw as

$$U = \begin{pmatrix} U & X \\ V & Y \\ W & Z \end{pmatrix},$$

(47)

where $\{U, V, W\}$ and $\{X, Y, Z\}$ are $3 \times 3$ and $3 \times 6$ matrices, respectively. All deviation from unitarity is determined by

$$\zeta = M_R^{-1} M_D.$$ 

(48)

It goes from (44) and (47),

$$\begin{pmatrix} V \\ W \end{pmatrix} = -M_R^{-1} M_D = - \begin{pmatrix} 0 & M \\ M & \mu \end{pmatrix}^{-1} \begin{pmatrix} M_D \\ 0 \end{pmatrix} = \frac{1}{M^2} \begin{pmatrix} -\mu M_D \\ M M_D \end{pmatrix}. \quad (49)$$

\[3\text{Such a low mass } M_R \text{ in SO(10) GUT is realized, for instance, by the radiatively generated } M_R \text{ model [37].}\]
Then
\[ V \approx O\left( \frac{\mu M_D}{M^2} \right) = O\left( \frac{m_{\text{light}}}{M_D} \right) \ll O(\zeta) \] (50)
and \( O\left( \frac{m_{\text{light}}}{M_D} \right) \) is also valid for type I seesaw. Thus, \( O(V) \) seems to be tiny. However, this is too naive estimation. This is because these estimations are due to the hierarchy assumption \( M \gg M_D \gg \mu \) and to the neglect of the generation. Alternative ideas may be free from the light neutrino mass constraint like (50). For type I seesaw, for example, if \( (M_D)_{i3} = 0 \) and \( (M_N^{-1})_{33} = 0 \), then, \( m_\nu = 0 \) and \( V \) is free from the light neutrino mass constraint. For the case of inverse seesaw, we may consider another mass hierarchy \( M \gg \mu \gg M_D \). Important is that there are windows of sizable \( V \). The order of the magnitude of \( V \) should be estimated from the observations.

As was shown in (37), \( < \eta > \) is not so suppressed and its contribution in (19) may be important when we take into account the large contribution from NME \( C_{\eta\eta}^{(0)} \).

3 Nuclear matrix elements and role of \( < \eta > \) mechanism

The sensitivity of the \( 0\nu \beta\beta \) decay to the R-handed current can be roughly summarized as follows. The decay rate calculated from the neutrino mass terms with \( < m_\nu > \sim 0.1 \text{ eV} \) corresponds to the R-handed current contribution of either \( < \eta > \sim 10^{-9} \) or \( < \lambda > \sim 10^{-7} \) [14]. The two order of magnitude difference of the sensitivity between the \( < \eta > \) mechanism and the \( < \lambda > \) mechanism comes from the interference between the nuclear vector and axial vector current. The combination of the \( V - A \) interference of nuclear current, which corresponds to NME \( \chi_R \) and \( \chi_P \) [5], and the phase space integral \( G_i \) enhances sensitivity to the \( < \eta > \) mechanism. In the previous section, it is argued that \( < \eta > \) may not be as tiny as usually assumed, which opens an interesting possibility to reveal R-handed current through the \( < \eta > \) mechanism. We revisit this mechanism, showing a simplified derivation of the relevant nuclear operator on the \( 0^+ - 0^+ \) \( 0\nu \beta\beta \) decay. We then estimate the allowed region of \( < \eta > \) from the current upper limit of the \( 0\nu \beta\beta \) decay rare.

The transition amplitude \( R_{0\nu} \) (15) can be written as

\[
R_{0\nu} = 4\sqrt{\frac{1}{2}} \left( \frac{G \cos \theta_c}{\sqrt{2}} \right)^2 \sum_i \sum_{\alpha,\beta} \int dx dy \int \frac{dk}{(2\pi)^3} e^{ik \cdot (y-x)} \frac{1}{2\omega} \left[ \frac{1}{\omega + A_1} + \frac{1}{\omega + A_2} \right] \times \tau_{p_2,s_2'}(y) \langle F | O(x,y) | I \rangle e^{\epsilon_{p_1,s_1}(x)} \]

where \( O(x,y) \) is the NME of the hadronic current. The interference terms of \( R \)-handed and \( L \)-handed current from the neutrino momentum \( (k) \) dependent term of the neutrino propagator

\[\text{We are grateful to Mimura on this indication.}\]
The Dirac matrices are for the electron spinor. Using the hadronic currents of (11) and (12), we keep the interference terms between the SM hadronic $L$-handed current and the BSM $L$-handed current $\eta J_L^\mu$ and the $R$-handed current $\lambda J_R^\mu$, while the $\kappa J_R^\mu$ term can be neglected. Using $J_L^\mu = V^\mu \pm A^\mu$, we keep only interference term between the vector and the axial vector current in $\mathcal{O}(x, y)$,

$$\mathcal{O}(x, y) = \{[V(y)k \cdot \gamma A(x) \mp A(y)k \cdot \gamma V(x)] \gamma_5 < \lambda > \}$$

Here $\mp$ is for $< \lambda >$ and $< \eta >$ term, respectively. We keep the contribution of the spatial component of the axial vector current, i.e. Gamow-Teller operator, as

$$\mathcal{O}(x, y) \sim [-i(V_0(y)A(x) \pm A(y)V_0(x)) \times k \cdot \gamma + i(V(y) \times A(x) \pm A(y) \times V(x)) \cdot k \gamma_0] \times \left( \begin{array}{c} < \lambda > \\ \gamma_5 < \eta > \end{array} \right)$$

Here $\pm$ is for $< \lambda >$ and $< \eta >$ term, respectively.

The $V - A$ interference terms remains only for the $< \eta >$ mechanism. The interference terms between the time component of the vector current($V_0$) and the space component of the axial vector current($A$) including s-wave and p-wave electron contribute to the $\chi_P$ term. The spatial components of the vector current($V$) and the axial current($A$) with the s-wave electron contribute to the $\chi_R$ term.

We examine further the $V - A$ term. Either emitted electrons or nuclear currents have to compensate the p-wave nature of neutrino propagator. The main contribution comes from the s-wave electron and momentum dependent nuclear magnetization current. $\mathcal{O}(x, y)$, which become scalar effective nuclear operator for the $0^+ \rightarrow 0^+$ transition, is given as

$$\mathcal{O}(x, y) = < \lambda > (k \times \mu(y) \cdot k \times A(x) - k \times A(y) \cdot k \times \mu(x))(-\gamma_0) \\
+ < \eta > (k \times \mu(y) \cdot A(x) + k \times A(y) \cdot k \times \mu(x))(\gamma_5 \gamma_0).$$

Here the magnetization current of the vector current is expressed as $V(x) = \nabla \times \mu(x)$. In the non-relativistic and impulse approximation of nuclear current, $\mu(x)$ and $A(x)$ are given by...
using the same spin-isospin flip operator \( \sim \tau^+ \sigma \) as \[40\],

\[
A(x) = \sum_i^A g_A(k^2) \tau_i^+ \sigma_i \delta(x - r_i),
\]

\[
\mu(x) = \sum_i^A \frac{g_V(k^2) + g_M(k^2)}{2M} \tau_i^+ \sigma_i \delta(x - r_i).
\]

Here \( g_A(0) = 1.27 \), \( g_V(0) + g_M(0) = 4.706 \) and \( M \) is mass of nucleon. Within this approximation, the \( < \lambda > \) term vanishes and only the \( < \eta > \) term remains. Therefore, the \( 0\nu\beta\beta \) decay for \( 0^+ - 0^+ \) transition can be sensitive probe for the \( < \eta > \) mechanism of \( R \)-handed current. Assuming the \( s \)-wave electron wave function can be approximated by constant, the amplitude \( R_{0\nu} \) is given by

\[
R_{0\nu} = 4 \sqrt{\frac{1}{2}} \left( \frac{G \cos \theta_c}{\sqrt{2}} \right)^2 \langle \gamma_\nu \gamma_0 \gamma_\nu \rangle \langle \sum_{i>j=1}^A M_{ij} |I \rangle,
\]

with

\[
M_{ij} = < \eta > \tau_i^+ \tau_j^+ \int \frac{dk}{(2\pi)^3} \frac{e^{ik\cdot(r_i-r_j)}}{\omega} \left[ \frac{1}{\omega + A_1} + \frac{1}{\omega + A_2} \right] g_A(k^2) \frac{g_V(k^2) + g_M(k^2)}{2M}
\]

\[
\times \left[ \frac{2}{3} k^2 \sigma_i \cdot \sigma_j - (k \cdot \sigma_i)(k \cdot \sigma_j) - \frac{1}{3} k^2 \sigma_i \cdot \sigma_j \right].
\]

The neutrino-exchange two-body operator \( M_{ij} \), whose spin and momentum structure is similar to the \( \rho \) meson exchange nucleon-nucleon potential. After \( k \) integration, effective two-body nuclear operator consists of spin-spin and tensor interaction. It is noticed that even though the light neutrino exchange mechanism, the operator becomes contact two-nucleon interaction by approximating \( \omega \sim k \) and \( A_i \sim 0 \). The short distance nature of the operator is well recognized and has been examined in detail. It is essential to include both the finite size of weak nucleon current for the effective nuclear operator and the short range correlation for the nuclear wave function. The form factors of axial vector and weak magnetism of nucleon are usually parameterized in a dipole form as \( g_A(k^2) = g_A/(1 - k^2/1.14GeV^2)^2 \) and \( g_V(k^2) + g_M(k^2) \sim (g_V + g_M)/(1 - k^2/0.71GeV^2)^2 \). Improved form factors and their uncertainties are analyzed from the analyses of electron and neutrino scattering data on proton and deuteron \[41, 42\]. The short range correlation, which is not taken into account in the model nuclear wave function, is taken into account by introducing short range correlation (SRC) function \[43, 44, 45, 46\]

\[
F(r) = 1 - ce^{-ar^2}(1 - br^2)
\]
$0 = 0.05$

**Figure 3**: Allowed region of $\langle \eta \rangle$ and $\langle m_\nu \rangle$ for $^{136}$Xe. a,b,c are evaluated using $C$’s of Refs. [48],[49] and [50] (model without p-n pairing), respectively.

with $r = |r_j - r_i|$. This correlation function vanishes at $\lim r = 0$ when $c = 1$. The suppression rates relative to those without the SRC are not affirmative, depending on nuclear models and various SRCs, from 5% to 30−40%. See the most recent result, Fig.10 of [16].

The role of BSM physics on the $0\nu\beta\beta$ decay has been studied extensively.(see references cited in [47].) Here we focus on $\langle \eta \rangle$ and mass mechanisms. Using the typical value of the ratio $C_{\eta\eta}^{(0)} / C_{mm}^{(0)} \sim 10^4$ to $10^5$, the decay rate for $\langle m_\nu \rangle \sim 100$ meV corresponds to $\langle \eta \rangle \sim 10^{-9}$. Actually, the decay rate is quadratic function of $\langle \eta \rangle$ and $\langle m_\nu \rangle$ for $\langle \lambda \rangle = 0$. Using current lower limit of $T_{1/2} > 2.3 \times 10^{26}$ years from KamLAND-Zen [11] and $C$’s given in Table 27 of Ref. [14], $\langle \eta \rangle$ and $\langle m_\nu \rangle$ constrained from the data are inside ellipses shown in Fig. 3. Here not only $V - A$ interference terms ($\chi_R, \chi_P$), $A - A$ and $V - V$ terms ($\chi_1-, \chi_2+$) are included.

The decay rate of a single nuclear species is not possible to reveal the mechanism of $0\nu\beta\beta$ decay including BSM physics. However, difference between the space and spin structure of the the effective nuclear operators for $\langle m_\nu \rangle$ and $\langle \eta \rangle$ mechanisms has a potential to generate $A$-dependence of the decay rate. $A$-dependence of the decay rate can be quantified by the normalized decay rate with respect to the reference $0\nu\beta\beta$ process for two extreme cases $\langle m_\nu \rangle$ alone ($\alpha = m_\nu$) and $\langle \eta \rangle$ alone ($\alpha = \eta$) as

$$R^\alpha_A = \frac{T^\alpha_{1/2}(A_{\text{reference}})}{T^\alpha_{1/2}(A)},$$

(61)

and deviation of the ratio $R_A = R^m_A / R^m_{\text{reference}}$ from one indicates an ability of the process to find BSM physics. We estimated $R_A$ using $C$’s from the Tables of [14]. In table [11] $R^\alpha_A$ and $R_A$ are calculated for [50] (model without p-n pairing), where $^{136}$Xe is chosen as $A_{\text{reference}}$. In this model,
Table 1: Ratio of decay rate $R_A^\nu$ evaluated using $C$’s of [50].

| Nucleus | $^{48}$Ca | $^{76}$Ge | $^{82}$Se | $^{96}$Zr | $^{100}$Mo | $^{116}$Cd | $^{128}$Te | $^{130}$Te |
|---------|---------|--------|--------|--------|--------|--------|--------|--------|
| $R_A^{\nu}$ | 0.75 | 0.51 | 1.2 | 3.0 | 0.47 | 0.39 | 0.095 | 2.1 |
| $R_A^\eta$ | 0.082 | 0.40 | 0.19 | 0.83 | 0.36 | 0.064 | 0.10 | 2.0 |
| $R_A = R_A^{\nu}/R_A^\eta$ | 0.11 | 0.77 | 0.15 | 0.28 | 0.76 | 0.16 | 1.1 | 0.94 |

$R_A$ is appreciably small for $^{48}$Ca, $^{82}$Se, $^{96}$Zr, $^{116}$Cd. The result suggests that A-dependence of the decay rate can be a key to disentangle the mechanism of $0\nu\beta\beta$ decay and to discover the BSM signal of $R$-handed current. A precise A-dependence of $R_A$ may not yet established theoretically and it is expected to narrow down model dependence of NME, especially for interaction range comparable with the nucleon size.

As for the heavy neutrino exchange mechanism, there may be some enhancements [19, 26, 51] due to $\pi\pi ee$ vertex from the effective operator, which generate pion range interaction,

$$C_{ab}^{i+} = (\gamma_L \tau^a \gamma_\mu q_L) (\gamma_R \tau^b \gamma_\mu q_R).$$  

We can obtain the NME due to this term using the master formula by [18, 19] 5

$$\mathcal{M}_\nu^{(9)} = -\frac{1}{2m_N^2} C_{\pi\pi L}^{(9)} \left( \frac{1}{2} M_{GT}^{AP} + M_{GT}^{PP} + \frac{1}{2} M_T^{AP} + M_T^{PP} \right).$$  

Here the notations and definitions are due to Appendix A.2 of [19]. Thus, this term enhances $\langle \lambda \rangle$ mechanism. However it may not change our order estimations because the original $C_{ij}$ is much larger than $C_{ij}$ with $i, j = m, \lambda$ by several orders.

### 4 Conclusion

We have studied $0\nu\beta\beta$ decay in the presence of $R$-handed current. We have tried to clarify the arguments of hadronic side and lepton’s BSM physical one. As is well known, if neutrino masses obey IH, $\langle m_\nu \rangle$ mechanism works around 50 meV already marginal to the present and near future experiments. From NME, $\langle \lambda \rangle$ mechanism is suppressed and $\langle \eta \rangle$ mechanism can dominate the decay rate even around the present or near future experimental limits. This is the case if the neutrino masses belong to NH which is much more probable than IH from theoretical reasons. Even if we get the non-null result in $0\nu\beta\beta$ decay in a single species, though it is the great achievement, we can not limit BSM physics. It is very important to survey this process in different nuclei to specify BSM physics.

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