INTRODUCTION

The measurements of luminosity distances $d_L$ to the SN Ia stars as function of redshift have revealed the accelerated expansion of the Universe \cite{10, 11}. Until recently for the interpretation of collected data on large scale structure of the Universe models with positive cosmological constant $\Lambda$ were preferred by cosmologists. For such models one can calculate the evolution of matter density perturbations up to formation of gravitationally bound systems of galaxies and clusters of galaxies (see \cite{5} and references therein). Searching of plausible physical interpretation of $\Lambda$-constant has introduced into the astrophysics new terms: dark energy (DE) and quintessence for the notation of energy of unknown nature that repulses and involves the self-attracting matter into accelerated expansion. Classical $\Lambda$-constant is the simplest kind of such energy. Now the more general models of this component are under considerations (see for review \cite{9} and references therein). In some papers the assumption of absence of coupling between DE and matter is used. But in this case the matter density perturbations lead to perturbations of dark energy density \cite{7, 2, 3}. Other kind of DE is based on the assumption of homogeneous and isotropic distribution of this component. Such models predict energy flow from one component to another or in other words DE and matter are coupled in perturbed regions. In this paper we will analyze the evolution of matter density perturbations for both kinds of DE with constant equation of state and $\omega_{DE} = P_{DE}/\rho_{DE} \neq -1$, where $P(\rho)$ and $\epsilon(\rho)$ are pressure and energy density of DE respectively.

DARK ENERGY

The influence of DE upon dynamics of the Universe and evolution of matter density perturbations can be studied by analysis of Einstein equations and its presentation as ideal fluid with equation of state $P = \omega \rho$, where $\omega$ is negative. In the case of $\omega = -1$ we have $\Lambda$-constant or Lorentz-invariant dark energy which can be presented also by density of Lagrangian function $\mathcal{L} \equiv \mathcal{L} \left( \{ g_{ik} \}, \{ \frac{\partial \mathcal{L}}{\partial g_{ik}} \} \right)$ that satisfies the equation

$$\frac{1}{2} \sqrt{-g} \Lambda g_{ik} = \frac{\partial (\sqrt{-g} \mathcal{L})}{\partial g^{ik}} - \frac{\partial}{\partial x^l} \left( \frac{\partial (\sqrt{-g} \mathcal{L})}{\partial g^{lk}} \right),$$

where $\Lambda \equiv \Lambda \left( \{ g_{ik} \}, \{ \frac{\partial \mathcal{L}}{\partial g_{ik}} \} \right)$ is some arbitrary function. The Einstein $\Lambda$-constant has the physical interpretations of zero-point vacuum fluctuations, vacuum polarization or follows from some versions of supersymmetry theories. But all these interpretations converge to the fine tuning problem: at Planckian epoch the energy density of matter was by $\sim 120$ orders larger than dark energy one. This issue can be essentially relaxed when $\omega \neq -1$ and depends on time, this called dark energy of tracker field kind. For the such type of DE besides of the state equation and coupling of DE with matter we should define the vector of 4-velocity $(\vec{u}, (\vec{u})^2 = -1)$ which indicate the direction of energy flow. Using this vector we can define the 3d metrics tensor $h_{ik} = u_i u_k - g_{ik}$ in 4d space-time and thermodynamic parameters such as the energy density $\epsilon$, pressure density $P$ and anisotropic stress-tensor $\Pi_{ik}$:

$$\epsilon = T_{ik} u^i u^k, \quad P = \frac{1}{3} T_{ik} h^{ik}, \quad \Pi_{ik} = T_{jl} \left( h^i_j h^l_k - \frac{1}{3} h^{il} h_{ik} \right).$$
In cosmological applications the constant equation of state \( P = \omega \varepsilon \) with \( \omega = 0 \) for the case of dust-like pressureless matter, \( \omega = 1/3 \) for the case of electromagnetic field and \( \omega = -1 \) for the case of Einstein’s \( \Lambda \)-constant are used. For the case of scalar fields the general form of \( \omega = \omega(\tau) \) may exist. The scalar field has a density of Lagrangian: \( \mathcal{L} = \frac{1}{2} g^{ij} \partial_i \varphi \partial_j - V(\varphi) \), leading to \( \varepsilon(\varphi) = \frac{1}{2} \dot{\varphi}^2 + V(\varphi) \) and \( P(\varphi) = \frac{1}{2} \dot{\varphi}^2 - V(\varphi) \) for \( \varphi \) homogeneously and isotropically distributed on 3d hypersurface. The DE of such kind finds its interpretation in the framework of generalizations of gravitation theory, e.g. branes theory and theory of gravitation with more general geometry than Riemann’s one or some unified theories of fundamental physical interactions.

**MATTER AND DARK ENERGY**

For two-component model of Universe consisting of matter and DE the energy-stress tensor is represented by \( T^i_k = T^{(M)i}_k + T^{(DE)i}_k \). Conservation equations are written as \( \nabla_i (T^{(M)i}_k + T^{(DE)i}_k) = 0 \), or in other form \( \nabla_i T^{(M)i}_k = Q_k \) and \( \nabla_i T^{(DE)i}_k = -Q_k \), where \( \vec{Q} \equiv \{Q_k\} \) is vector of energy flow \( \vec{Q} \). For the general case one should define the vector of energy flow between two components.

We assume unperturbed DE and comoving matter and DE on the homogeneous and isotropic background. This simplification leads to \( \nabla_i T^{(DE)i}_k = 0 \), where \( \nabla_i \) is covariant derivative in isotropic and homogeneous space with metrics tensor \( \vec{g}_{ik} \). The vector of energy flow is \( Q_k = \nabla_i T^{(DE)i}_k - \nabla_i T^{(DE)i}_k \), that gives

\[
\frac{d\pi^i}{ds} = -\Gamma^i_{jk} \pi^j \pi^k + f^i,
\]

where \( \Gamma^i_{jk} \) and \( \Gamma^i_{jk} \) are Christoffel symbols defined for the metrics \( \vec{g}_{ik} \) and \( \vec{g}_{ik} \) respectively, \( f^i = (\Gamma^i_{jk} - \Gamma^i_{jk}) \pi^j \pi^k \) is vector of additional force needed for the homogeneous distribution of the DE in the regions of perturbations. The vector of energy flow is \( \dot{Q}_k = \frac{\delta}{\delta a} (\dot{P}(\vec{Q}) + \dot{\varepsilon}(\vec{Q})) \) while \( \dot{P}(\vec{Q}) + \dot{\varepsilon}(\vec{Q}) \) is the energy-reservoir in the system.

The real perturbed space-time presented by metrics \( \vec{g}_{ik} \). The motion equations for particles comoving to unperturbed background and DE (in the case of \( T^{(DE)}_{ik} = 0 \)) are following

\[
\frac{d\pi^i}{ds} = -\Gamma^i_{jk} \pi^j \pi^k + f^i,
\]

where \( T^{(DE)}_{ik} \) and \( T^{(DE)}_{ik} \) are Christoffel symbols defined for the metrics \( \vec{g}_{ik} \) and \( \vec{g}_{ik} \) respectively, \( \dot{f}^i = (\Gamma^i_{jk} - \Gamma^i_{jk}) \pi^j \pi^k \) is vector of additional force needed for the homogeneous distribution of the DE in the regions of perturbations. The vector of energy flow is \( \dot{Q}_k = \nabla_i T^{(DE)i}_k - \nabla_i T^{(DE)i}_k \), that gives

\[
\frac{d\pi^i}{ds} = -\Gamma^i_{jk} \pi^j \pi^k + f^i,
\]

where tensor \( W^i_{jk} = \Gamma^i_{jk} - \Gamma^i_{jk} \). Definition of perturbations is ambiguous and depends on a choice of gauge.

The metrics in the longitudinal gauge has a form \( ds^2 = a(\eta)^2 \left[ -(1 + 2\Psi(\eta)Y(x^\alpha))d\eta^2 + (1 + 2\Phi(\eta)Y(x^\alpha))\delta_{\beta\gamma}dx^\beta dx^\gamma \right] \) where \( \Phi(\eta) \) and \( \Psi(\eta) \) are Bardeen’s potentials \( \Phi \). Non-zero components of tensor \( W^i_{jk} \) according to this metrics are \( (\alpha, \beta = 1, 2, 3) \)

\[
\begin{align*}
W^i_{00} &= \dot{Y}, \\
W^i_{0a} &= W^i_{a0} = -k\dot{Y}Y_a, \\
W^i_{00} &= -k\dot{Y}Y^a, \\
W^i_{a0} &= \dot{\Phi}, \\
W^0_{a0} &= \dot{\Phi}Y_a, \\
W_{a0} &= \dot{\Phi}Y^a, \\
W^i_{\alpha\beta} &= \dot{\Phi}Y^a - k\dot{Y}Y^a, \\
W^i_{\alpha\beta} &= \dot{\Phi}Y^a \quad \text{and} \quad W^i_{\alpha\beta} = k\dot{Y}Y^a \quad \text{for} \quad \alpha \neq \beta \quad (\text{no sum on} \ \beta).
\end{align*}
\]

Thus the components of vector of energy flow and additional force are

\[
\begin{align*}
Q_0 &= 3\dot{\Phi}(\varepsilon^{(DE)} + P^{(DE)})Y, \\
Q_0 &= \dot{\Phi}(\varepsilon^{(DE)} + P^{(DE)})Y, \\
f^0 &= \dot{\Phi}Y, \\
f^\alpha &= \dot{Y}Y^\alpha
\end{align*}
\]

respectively.

**EVOLUTION OF MATTER DENSITY PERTURBATIONS**

Formation of the large scale structure of the Universe is described by linear theory of scalar perturbations. We use here the gauge-invariant approach presented in [1][4][5][8]. We have considered a case of two-components Universe with small perturbations in a component of the dust-like matter and no perturbations in a component of the DE. The non-zero components of energy-stress tensors for every component are

\[
\begin{align*}
T^{(M)}_{00} &= \varepsilon^{(M)}(1 + \delta^{(M)}Y), \\
T^{(M)}_{a0} &= (\varepsilon^{(M)} + T^{(M)}_{\alpha\alpha})Y_a, \\
T^{(M)}_{00} &= \varepsilon^{(M)} + T^{(M)}_{\alpha\alpha}, \\
T^{(M)}_{\alpha\beta} &= T^{(M)}_{\beta\alpha} = (1 + \varepsilon^{(M)}Y)(\delta_{\beta\alpha} + \Pi^{(M)}Y_{\beta\alpha}), \\
T^{(DE)}_{00} &= \varepsilon^{(DE)}, \\
T^{(DE)}_{\alpha\beta} &= T^{(DE)}_{\beta\alpha} = \varepsilon^{(DE)} \delta_{\alpha\beta},
\end{align*}
\]
where $\delta$ and $v$ are perturbations of energy density and velocity respectively, $\pi^{(M)}$ and $\Pi^{(M)}$ isotropic and anisotropic components of pressure perturbations (over-lines denote the background magnitudes). From Einstein’s equations $\delta G^{0}_{\alpha} = 4\pi G \delta T^{0}_{\alpha}$, $\delta c^{0}_{\alpha} = 4\pi G \delta T^{0}_{\alpha}$ and $\delta G^{\alpha}_{\alpha} = 4\pi G \delta T^{\alpha}_{\alpha}$ we obtain the following connection between perturbations of metrics and perturbations of matter density and velocity:

$$4\pi Ga^{2}(\pi^{(M)} + \Pi^{(M)})V^{(M)} = k \left( \frac{\dot{a}}{a} \Psi - \dot{\Phi} \right),$$

$$4\pi Ga^{2}\Pi^{(M)} = -k^{2}(\Phi + \Psi).$$

The conservation equations $\delta T^{(M)}_{\alpha \beta} = Q_{0}$ and $\delta T^{(M)}_{\alpha i} = Q_{\alpha}$ lead to the following equations for matter density and velocity perturbations:

$$\dot{D}^{(M)} + 3(\delta^{(M)} - \omega^{(M)})\frac{\dot{a}}{a}D^{(M)} + (1 + \omega^{(M)})kV^{(M)} + 3\omega^{(M)}\frac{\dot{a}}{a}\Gamma^{(M)} = -3\Phi \frac{\varepsilon^{(DE)}}{\varepsilon^{(M)}}(1 + \omega^{(DE)})$$

$$\dot{V}^{(M)} + \frac{\dot{a}}{a}(1 - 3s^{(M)}k) + k(\Psi - 3c^{(M)}_{s}2\Phi) - \frac{c^{(M)}_{s}k^{2}}{1 + \omega^{(M)}}D^{(M)} - \frac{\omega^{(M)}k}{1 + \omega^{(M)}}
\left[ \Gamma^{(M)} - \frac{3}{2} \left( 1 - \frac{3K}{k^{2}} \right) \Pi^{(M)} \right] = \frac{k\Psi}{\varepsilon^{(M)}} \frac{\varepsilon^{(DE)}}{1 + \omega^{(DE)}}.$$

where $V = v$, $D_{g} = \delta + 3(1 + \omega)\Phi$, $D = \delta + 3(1 + \omega)\frac{\dot{a}}{a}$, $\Gamma = \pi - \frac{c^{2}_{s}}{\omega} \delta$ are gauge-invariant amplitudes \[ ] [1]-[8] ($c^{2}_{s} = \dot{P}/\dot{\rho}$ is square of sound speed).

For DE component we have $\delta T^{(DE)}_{\alpha \beta} = -Q_{0}$ and $\delta T^{(DE)}_{\alpha i} = -Q_{\alpha}$ that gives the equations $\dot{D}^{(DE)} = 0$ and $\dot{V}^{(DE)} = 0$. If we suppose initial zero perturbations of dark energy ($D^{(DE)}_{in} = 0$, $\Pi^{(DE)}_{in} = 0$) then $\Pi^{(DE)} = V^{(DE)} = 0$.

For dust-like matter $\Pi^{(M)} = c^{(M)}_{s} = \omega^{(M)} = \Gamma^{(M)} = 0$ the conservation equations are simplified:

$$\dot{D}^{(M)} + kV^{(M)} = -3\Phi \frac{\varepsilon^{(DE)}}{\varepsilon^{(M)}}(1 + \omega^{(DE)}), \quad \dot{V}^{(M)} + \frac{\dot{a}}{a}V^{(M)} - k\Psi = k\Psi \frac{\varepsilon^{(DE)}}{\varepsilon^{(M)}}(1 + \omega^{(DE)}).$$

The set of equations [4]-[6] and [7] describes the evolution of scalar perturbations of matter in the Universe with DE which is homogeneously distributed over whole space. For the case of DE uncoupled with dust-like matter the right-hand sides of the equations [7] will be equal zero. In this case the energy density of DE will trace matter density perturbations by means of metrics perturbations, so, it will be perturbed [7] 2 3. The evolution of perturbation amplitudes of matter density $D^{(M)}_{g}$ for the two cases of DE with unperturbed energy density and perturbed one are shown in Fig.1. For calculation the following values of parameters have been used: the constant state equation parameter of DE is $\omega^{(DE)} = -0.8$, the current contents of DE and matter are respectively $\Omega^{(DE)} = 0.7$ and $\Omega^{(M)} = 0.3$, and dimensionless Hubble constant is $\tilde{h} = 0.65$. The amplitude $D^{(M)}_{g}$ is larger for case of unperturbed DE that is stipulated by flow of energy from DE component to matter one in perturbed region.

**CONCLUSIONS**

We have analyzed the evolution of matter density perturbations for two kinds of dark energy: unperturbed homogeneously distributed and uncoupled with dust-like matter. The expressions for energy flow between components [4] and additional force which keeps homogeneous distribution of DE [4] as well as the equations for evolution of matter density perturbations [4]-[6] and [7] are obtained. Their numerical solutions show that gauge-invariant amplitude of matter density perturbations grow slightly faster in the case of homogeneously distributed DE than in the case of dark energy uncoupled with matter. This difference can be explained by existence of flow of energy from DE to dust-like matter and additional force smoothing the DE.

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Figure 1. The dependence of $D_g^{(M)}$ on scale factor $a$ for two kinds of DE: uncoupled with dust-like matter (dashed line) and unperturbed DE (solid line). The state of DE is $\omega^{(DE)} = -0.8$ and other cosmological parameters are $\Omega^{(DE)} = 0.7$, $\Omega^{(M)} = 0.3$, $k = 10^{-1}$ Mpc$^{-1}$, $h = 0.65$.

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