More Meta-Stable Brane Configurations without D6-Branes

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Abstract

We describe the intersecting brane configurations, consisting of NS-branes, D4-branes(and anti-D4-branes), in type IIA string theory corresponding to the meta-stable nonsupersymmetric vacua of $\mathcal{N} = 1 \, SU(N_c) \times SU(N'_c) \times SU(N''_c)$ gauge theory with bifundamentals. By adding the orientifold 4-plane to these brane configurations, we also discuss the meta-stable brane configurations for other gauge theory with bifundamentals. Furthermore, we study the intersecting brane configurations corresponding to the nonsupersymmetric meta-stable vacua of other gauge theory with bifundamentals, by adding the orientifold 6-plane.
1 Introduction

In the standard type IIA brane configuration, the quark masses correspond to the relative displacement of the D6-branes(0123789) and D4-branes(01236) along the 45 directions geometrically. Then the eigenvalues of quark mass matrix correspond to the positions of D6-branes in 45 directions. See the review paper [1] for the gauge theory and the brane dynamics. The Seiberg duality in the classical brane picture can be accomplished by exchanging the locations of the NS5-brane(012345) and NS5’-brane(012389) along x6 direction each other.

The geometric misalignment of D4-branes connecting both NS5’-brane and D6-branes in the magnetic brane configuration can be interpreted as a nontrivial F-term condition in the gauge theory with massive flavors. Then the F-term equations can be partially cancelled by both recombination of flavor-D4-branes with the color-D4-branes and then movement of those D4-branes into the 45 directions. This phenomenon in magnetic brane configuration corresponds to the fact that some entries in the magnetic dual quarks acquire nonzero vacuum expectation values to minimize the F-term in the dual gauge theory side. Moreover, the remaining flavor-D4-branes that do not move to 45 directions, connecting to NS5’-brane, can move along 89 directions freely since D6-branes and NS5’-brane are parallel and this geometric freedom of meson field corresponds to the classical pseudomoduli space of nonsupersymmetric vacua of the gauge theory.

On the other hand, it is known that the NS-brane configuration in type IIA string theory, where there exist only two types of NS5-brane and NS5’-brane, preserves $\mathcal{N} = 2$ supersymmetry in four dimensions [1]. The geometry [2] of the coincident NS5-branes is characterized by the metric, the dilaton, and the field strength and is useful to construct the DBI action for D4-branes. In order to break the supersymmetry, one adds D4-branes and anti-D4-branes [3]. By adding D4-branes suspending between the NS5-brane and the NS5’-brane, and anti-D4-branes($\overline{D4}$-branes) suspending between the NS5-brane and the other NS5’-brane, the supersymmetry of this system is broken [3]. The low energy dynamics can be described by the gauge theory on the D4-branes. The brane configuration corresponding to the electric theory with vanishing mass for the bifundamentals consists of the left NS5’-brane, the middle NS5-brane and the right NS5’-brane and two sets of D4-branes suspended between two NS5’-branes. The gauge group is a product of two unitary gauge groups and there exist bifundamentals. For the nonvanishing mass for these bifundamentals, the relative displacement between the two NS5’-branes along the 45 directions occurs. By taking the Seiberg dual for one of two gauge group factors with nonvanishing masses for the bifundamentals, the magnetic dual theory has a cubic superpotential between the dual quarks and a meson which is nothing but
a quadratic term of bifundamentals in an electric theory. Also the linear term in a meson appears in this magnetic superpotential. Then the F-term equation for this meson field leads to the supersymmetry breaking. One finds that supersymmetry is broken classically but is restored quantum mechanically. It turns out the classical nonsupersymmetric vacuum becomes long-lived state [3].

As the distance between the two NS5'-branes along the 45 directions becomes zero, this brane configuration with D4- and $\overline{D}4$-branes can decay and the geometric misalignment between flavor-D4-branes arises, as before. Due to the presence of NS5-brane in this system, there exists an attractive force between the tilted D4-branes and NS5-brane. The explicit and careful computation of DBI action for these D4-branes in the background created by NS5-brane has been done by the work of [3] and this effect of the gravitational attraction leads to a curve for tilted D4-branes rather than a straight line. Then for small displacement of two NS5'-branes, the ground state is given by this “curved” brane configuration. As this displacement between two NS5'-branes is increased, the ground state brane configuration is given by “straight” brane configuration. The meta-stable vacua of [4] arise in some region of parameter space. In this description, the dual quarks are represented by the bifundamentals of product gauge group and the mass term is encoded by the relative displacement of two NS5’-branes in 45 directions, as we mentioned before. Note that there exist no D6-branes in this brane configuration [1]. When one of the NS5'-branes goes to infinity along the $x^6$ direction, then the corresponding gauge group becomes a global symmetry and the theory leads to a standard $\mathcal{N} = 1$ SQCD with fundamentals. In other regions, a generalization of [4] showing very similar qualitative phenomena in classical string theory occurs [3].

The focus on the new meta-stable brane configurations by adding an orientifold 4-plane and an orientifold 6-plane to the above brane configuration studied by [3], along the line of [5, 6, 7, 8], was given in [9]. When the former was added, no extra NS-branes or D-branes were needed. However, when the latter was added, the extra NS-branes or D-branes into the above brane configuration were needed in order to have a product gauge group.

In this paper, we continue to find out new meta-stable brane configurations which contain four NS-branes or six NS-branes, along the line of [3, 9], by starting from the known or new supersymmetric brane configurations in type IIA string theory. Compared with the previous approaches given by [5, 6, 7, 8], the superpotential in the magnetic theory has very simple form because there are no D6-branes in the brane configurations and this fact allows us to analyze the meta-stable vacua easily using the F-term equations and one loop effective potential. But

\[^1\] A replacement of D6-branes with NS5’-brane corresponds to the gauging of the flavor group(global symmetry) of the gauge theory realized on the D4-branes and this replacement might be useful to construct the phenomenological model building.
the number of NS-branes is increased for a given gauge theory with matters since the role of D6-branes is replaced by NS5'-brane. Some of the meta-stable brane configurations lead to the known meta-stable brane configuration corresponding to the gauge theory with less gauge group factors in the literature, by replacing the NS5'-brane with coincident D6-branes. Basically, the gauge group will be a triple product gauge group for four NS-branes with D4-branes. When we add an orientifold 4-plane into this brane configuration, the gauge group will be a triple product between symplectic gauge group and an orthogonal gauge group, alternatively, depending on the orientifold 4-plane charge. When we add an orientifold 6-plane into six NS-branes with D4-branes, one of the gauge group factor will be a symplectic or an orthogonal gauge group depending on the orientifold 6-plane charge and the other gauge group factor will be unitary.

In section 2, we describe the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N} = 1\ SU(N_c) \times SU(N'_c) \times SU(N''_c)$ gauge theory with the bifundamentals and deform this theory by adding the mass term for the bifundamental. We construct the three different dual magnetic theories by taking the Seiberg dual for each gauge group factor. Then we consider the nonsupersymmetric meta-stable minimum and present the corresponding intersecting brane configurations of type IIA string theory.

In section 3, we discuss the type IIA brane configuration, by adding the orientifold 4-plane to the brane configuration in section 2, corresponding to the electric theory based on the $\mathcal{N} = 1\ Sp(N_c) \times SO(2N'_c) \times Sp(N''_c)$ gauge theory with matters and deform this theory by adding the mass term for the bifundamental. Then we construct the corresponding dual magnetic theories by taking the Seiberg dual for each gauge group factor. We consider the nonsupersymmetric meta-stable minimum and present the corresponding intersecting brane configurations of type IIA string theory. We also comment on the case of $\mathcal{N} = 1\ SO(2N_c) \times Sp(N'_c) \times SO(2N''_c)$ gauge theory with matters.

In section 4, we discuss the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N} = 1\ Sp(N_c) \times SU(N'_c) \times SU(N''_c)$ gauge theory with matters and deform this theory by adding the mass term for the bifundamental. Then we construct the two different dual magnetic theories by taking the Seiberg dual for each unitary gauge group factor. We consider the nonsupersymmetric meta-stable minimum and present the corresponding intersecting brane configurations of type IIA string theory. Moreover, we also discuss the meta-stable brane configurations corresponding to the electric theory based on the $\mathcal{N} = 1\ SO(N_c) \times SU(N'_c) \times SU(N''_c)$ gauge theory by changing the orientifold 6-plane charge.

In section 5, we make some comments for the future directions.
2 Meta-stable brane configurations with four NS-branes

2.1 Electric theory

The type IIA brane configuration \[ \text{[10, 11]} \] corresponding to \( N = 1 \) supersymmetric gauge theory with gauge group

\[
SU(N_c) \times SU(N'_c) \times SU(N''_c)
\]

and with a field \( F \) charged under \( (N_c, N'_c) \), a field \( G \) charged under \( (N'_c, N''_c) \), and their conjugates \( \tilde{F} \) and \( \tilde{G} \) can be described by the left \( NS5'_L \)-brane(012389), the left \( NS5_L \)-brane(012345), the right \( NS5'_R \)-brane(012389), the right \( NS5_R \)-brane(012345), \( N_c \)-, \( N'_c \)- and \( N''_c \)-color D4-branes(01236). The fields \( F \) and \( \tilde{F} \) correspond to 4-4 strings connecting the \( N_c \)-color D4-branes with \( N'_c \)-color D4-branes while the fields \( G \) and \( \tilde{G} \) correspond to 4-4 strings connecting the \( N'_c \)-color D4-branes with \( N''_c \)-color D4-branes.

The left \( NS5'_L \)-brane is located at \( x^6 = 0 \) and let us denote the \( x^6 \) coordinates for the \( NS5'_L \)-brane, the \( NS5'_R \)-brane and the \( NS5_R \)-brane by \( y_1, y_2, y_2 + y_3 \) respectively.

The \( N'_c \) D4-branes are suspended between the \( NS5'_L \)-brane and the \( NS5'_R \)-brane, the \( N''_c \) D4-branes are suspended between the \( NS5'_R \)-brane and the \( NS5_R \)-brane. We draw this brane configuration in Figure 1A for the vanishing mass case \[ \text{[2]} \].

The gauge couplings of \( SU(N_c) \), \( SU(N'_c) \) and \( SU(N''_c) \) are given by a string coupling constant \( g_s \), a string scale \( \ell_s \) and the \( x^6 \) coordinates \( y_i \) for three NS-branes through

\[
g_1^2 = \frac{g_s \ell_s}{y_1}, \quad g_2^2 = \frac{g_s \ell_s}{y_2}, \quad g_3^2 = \frac{g_s \ell_s}{y_3}.
\]

For example, as \( y_3 \) goes to \( \infty \), the \( SU(N''_c) \) gauge group becomes a global symmetry and the theory leads to SQCD with the gauge group \( SU(N_c) \times SU(N'_c) \) and \( N''_c \) flavors in the fundamental representation.

There is no superpotential in Figure 1A since the \( NS5'_L \)-brane is perpendicular to two \( NS5'_ \)-branes and the \( NS5'_R \)-brane is perpendicular to two \( NS5 \)-branes. Let us deform this theory. Displacing the two \( NS5'_ \)-branes relative each other in the

\[ v \equiv x^4 + ix^5 \]

direction corresponds to turning on a quadratic mass-deformed superpotential for the fields \( F \) and \( \tilde{F} \) as follows:

\[
W = mF\tilde{F} \equiv m\Phi'
\]  

\[ \text{[2]} \text{There are similar brane configurations in the context of quiver gauge theory [12, 13].} \]
Figure 1: The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $SU(N_c) \times SU(N'_c) \times SU(N''_c)$ and bifundamentals $F, \tilde{F}, G$ and $\tilde{G}$ with vanishing(1A) and non-vanishing(1B) mass for the bifundamentals $F$ and $\tilde{F}$. The $N_c'$ D4-branes in 1A are decomposed into $(N_c' - N_c'')$ D4-branes which are moving to $+v$ direction in 1B and $N_c''$ D4-branes which are recombined with those D4-branes connecting between NS5'$_L$-brane and NS5$_R$-brane in 1B.

where the first gauge group indices in $F$ and $\tilde{F}$ are contracted, each second gauge group index in them is encoded in $\Phi'$ and the mass $m$ is given by

$$m = \frac{\Delta x}{2\pi \alpha'} = \frac{\Delta x}{\ell_s^2}. \tag{2.3}$$

The gauge-singlet $\Phi'$ for the first dual gauge group is in the adjoint representation for the second dual gauge group, i.e., $(1, (N_c' - N_c'')^2 - 1, 1) \oplus (1, 1, 1)$ under the dual gauge group (2.4). Then the $\Phi'$ is a $(N_c' - N_c'') \times (N_c' - N_c'')$ matrix. The NS5'$_R$-brane together with $(N_c' - N_c'')$-color D4-branes is moving to the $+v$ direction for fixed other branes during this mass deformation. In other words, the $N_c''$ D4-branes among $N_c'$ D4-branes are not participating in the mass deformation. Then the $x^5$ coordinate($\equiv x$) of NS5'$_L$-brane is equal to zero while the $x^5$ coordinate of NS5'$_R$-brane is given by $\Delta x$. Giving an expectation value to the meson field $\Phi'$ corresponds to recombination of $N_c$- and $N'_c$- color D4-branes, which will become $N_c$-color D4-branes in Figure 1A such that they are suspended between the NS5'$_L$-brane and the NS5'$_R$-brane and pushing them into the

$$w \equiv x^8 + ix^9$$

direction. We assume that the number of colors satisfies

$$N_c' \geq N_c \geq N_c''.$$

Now we draw this brane configuration in Figure 1B for nonvanishing mass for the fields $F$ and $\tilde{F}$.
2.2 Magnetic theory

By applying the Seiberg dual to the $SU(N_c)$ factor in (2.1), the two $NS5'_{L,R}$-branes can be located at the inside of the two NS5-branes, as in Figure 2. Starting from Figure 1A and moving the $NS5'_{R}$-brane with $(N'_c - N''_c)$ D4-branes to the $+v$ direction leading to Figure 1B and interchanging the $NS5'_{L}$-brane and the $NS5_L$-brane, one obtains the Figure 2A. Before arriving at the Figure 2A, there exists an intermediate step where the $(N'_c - N''_c)$ D4-branes are connecting between the $NS5_L$-brane and the $NS5'_{L}$-brane, $(N'_c - N''_c)$ D4-branes connecting between the $NS5'_{L}$-brane and $NS5'_{R}$-brane, and $N''_c$ D4-branes between the $NS5_L$-brane and the $NS5_R$-brane. By introducing $-N''_c$ D4-branes and $-N''_c$ anti-D4-branes between the $NS5_L$-brane and $NS5'_{L}$-brane, reconnecting the former with the $N'_c$ D4-branes connecting between $NS5_L$-brane and the $NS5'_{L}$-brane (therefore $N'_c - N''_c$ D4-branes) and moving those combined $(N'_c - N''_c)$ D4-branes to $+v$-direction, one gets the final Figure 2A where we are left with $(N_c - N''_c)$ anti-D4-branes between the $NS5_L$-brane and $NS5'_{L}$-brane.

When two NS5'-branes in Figure 2A are close to each other, then it leads to Figure 2B by realizing that the number of $(N'_c - N''_c)$ D4-branes connecting between $NS5_L$-brane and $NS5'_{R}$-brane can be rewritten as $(N_c - N''_c) + \tilde{N}_c$. If we ignore $N''_c$ D4-branes and $NS5_{R}$-brane from Figure 2B, then the brane configuration becomes the one in [3].

\[
SU(\tilde{N}_c = N'_c - N_c) \times SU(N'_c) \times SU(N''_c) \tag{2.4}
\]

Figure 2: The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group $SU(\tilde{N}_c = N'_c - N_c) \times SU(N'_c) \times SU(N''_c)$ corresponding to Figure 1B with D4- and $\overline{D}4$-branes(2A) and with a misalignment between D4-branes(2B) when the NS5'-branes are close to each other. The number of tilted D4-branes in 2B can be written as $N_c - N''_c = (N'_c - N''_c) - \tilde{N}_c$.

The dual gauge group is given by

\[
SU(\tilde{N}_c = N'_c - N_c) \times SU(N'_c) \times SU(N''_c). \tag{2.4}
\]
The matter contents are the field \( f \) charged under \( (\tilde{N}_c, \overline{N}_c, 1) \), a field \( g \) charged under \( (1, N'_c, \overline{N}_c) \), and their conjugates \( \tilde{f} \) and \( \tilde{g} \) under the dual gauge group \( (2.4) \) and the gauge-singlet \( \Phi' \) for the first dual gauge group in the adjoint representation for the second dual gauge group, i.e., \( (1, (N'_c - N''_c)^2 - 1, 1) \oplus (1, 1, 1) \) under the dual gauge group \( (2.4) \).

The cubic superpotential with the mass term \( (2.2) \) in the dual theory is given by

\[
W_{\text{dual}} = \Phi' f \tilde{f} + m \Phi'.
\] (2.5)

Here the magnetic fields \( f \) and \( \tilde{f} \) correspond to 4-4 strings connecting the \( \tilde{N}_c \)-color D4-branes (that are connecting between the NS5\(_L\)-brane and the NS5\(_R\)'-brane in Figure 2B) with \( N'_c \)-flavor D4-branes (that are a combination of three different D4-branes in Figure 2B). Among these \( N'_c \)-flavor D4-branes, only the strings ending on the upper \( (N'_c - N_c) \) D4-branes and on the tilted middle \( (N_c - N''_c) \) D4-branes in Figure 2B enter the cubic superpotential term. Although the \( (N'_c - N''_c) \) D4-branes in Figure 2A cannot move any directions, the tilted \( (N_c - N''_c) \)-flavor D4-branes can move \( w \) direction in Figure 2B. The remaining upper \( \tilde{N}_c \) D4-branes are fixed also and cannot move any direction. Note that there is a decomposition

\[
(N'_c - N''_c) = (N''_c - N'_c) + \tilde{N}_c.
\]

The brane configuration for zero mass for the bifundamental, which has only a cubic superpotential, can be obtained from Figure 2A by moving the upper NS5\(_R\)'-brane together with \( (N'_c - N''_c) \) color D4-branes into the origin \( v = 0 \). Then the number of dual colors for D4-branes becomes \( \tilde{N}_c \) between NS5\(_L\)-brane and NS5\(_L\)'-brane and \( N'_c \) between two NS5\(_L\)'-branes as well as \( N''_c \) D4-branes between NS5\(_R\)'-brane and NS5\(_R\)-brane. Or starting from Figure 1A and moving the NS5\(_L\)-brane to the left all the way past the NS5\(_L\)'-brane, one also obtains the corresponding magnetic brane configuration for massless case.

The brane configuration in Figure 2A is stable as long as the distance \( \Delta x \) between the upper NS5\(_L\)'-brane and the lower NS5\(_L\)'-brane is large, as in [3]. If they are close to each other, then this brane configuration is unstable to decay and leads to the brane configuration in Figure 2B. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group \( (2.4) \) and superpotential \( (2.5) \). The \( (N'_c - N''_c - \tilde{N}_c) \) flavor D4-branes of straight brane configuration of Figure 2B bend due to the fact that there exists an attractive gravitational interaction between those flavor D4-branes and NS5\(_L\)-brane from the DBI action, by following the procedure of [3], as long as the distance \( y_3 \) goes to \( \infty \) because the presence of an extra NS5\(_R\)-brane does not affect the DBI action. For the finite and small \( y_3 \), the careful analysis for DBI action is needed in order to obtain the bending curve connecting two NS5\(_L\)'-branes.
When the upper NS5'-brane (or NS5'\textsubscript{R}-brane) is replaced by coincident \((N'_c - N''_c)\) D6-branes and the NS5\textsubscript{R} is rotated by an angle \(\frac{\pi}{2}\) in the \((v, w)\) plane in Figure 2B, this brane configuration reduces to the one found in [14] where the gauge group was given by \(SU(n_f + n'_c - n_c) \times SU(n''_c)\) with \(n_f\) multiplets, \(n'_c\) multiplets, flavor singlets and gauge singlets. Then the present number \((N'_c - N''_c)\) corresponds to the \(n_f\), the number \(N_c\) corresponds to \(n'_c\) and the number \(N''_c\) corresponds to the \(n''_c\) of [14]. Note that the number of D4-branes touching NS5'\textsubscript{R}-brane in Figure 2B is equal to \((N'_c - N''_c)\).

The quantum corrections can be understood for small \(\Delta x\) by using the low energy field theory on the branes. The low energy dynamics of the magnetic brane configuration can be described by the \(\mathcal{N}=1\) supersymmetric gauge theory with gauge group (2.4) and the gauge couplings for the three gauge group factors are given by

\[
g^2_{1,\text{mag}} = \frac{g_s \ell_s}{y_1}, \quad g^2_{2,\text{mag}} = \frac{g_s \ell_s}{(y_2 - y_1)}, \quad g^2_{3,\text{mag}} = \frac{g_s \ell_s}{y_3}.
\]

The dual gauge theory has an adjoint \(\Phi'\) of \(SU(N'_c)\) and bifundamentals \(f, \tilde{f}, g\) and \(\tilde{g}\) under the dual gauge group (2.4) and the superpotential corresponding to Figures 2A and 2B is given by

\[
W_{\text{dual}} = h \Phi' f \tilde{f} - h \mu^2 \Phi', \quad h^2 = g^2_{2,\text{mag}}, \quad \mu^2 = -\frac{\Delta x}{2\pi g_s \ell_s^3}.
\]

Then \(f \tilde{f}\) is a \(\tilde{N}_c \times \tilde{N}_c\) matrix where the second gauge group indices for \(f\) and \(\tilde{f}\) are contracted with those of \(\Phi'\) while \(\mu^2\) is a \((N'_c - N''_c) \times (N'_c - N''_c)\) matrix. Although the field \(f\) itself is an antifundamental in the second gauge group which is a different representation for the usual standard quark coming from D6-branes, the product \(f \tilde{f}\) has the same representation for the product of quarks and moreover, the second gauge group indices for the field \(\Phi'\) play the role of the flavor indices, as in comparison with the brane configuration in the presence of D6-branes before.

Therefore, the F-term equation, the derivative \(W_{\text{dual}}\) with respect to the meson field \(\Phi'\) cannot be satisfied if the \((N'_c - N''_c)\) exceeds \(\tilde{N}_c\). So the supersymmetry is broken. That is, there exist three equations from F-term conditions:

\[
f \tilde{f} - \mu^2 = 0, \quad \text{and} \quad \Phi' f = 0 = \tilde{f} \Phi'.
\]

Then the solutions for these are given by

\[
<f> = \left( \mu 1_{\tilde{N}_c} \right), \quad <\tilde{f}> = \left( \mu 1_{\tilde{N}_c} \right), \quad <\Phi'> = \left( 0 \Phi_0' \right), (2.6)
\]
where the zero of \( f \) is a \((N'_c - N''_c - \tilde{N}_c) \times \tilde{N}_c\) matrix, the zero of \( \tilde{f} \) is a \(\tilde{N}_c \times (N'_c - N''_c - \tilde{N}_c)\) matrix and the zeros of \( \Phi' \) are \(\tilde{N}_c \times \tilde{N}_c \times (N''_c - N''_c - \tilde{N}_c)\), and \((N'_c - N''_c - \tilde{N}_c) \times \tilde{N}_c\) matrices. Then one can expand these fields around a point \((2.6)\), as in \([4, 15, 16, 17, 18]\) and one arrives at the relevant superpotential up to quadratic order in the fluctuation. At one loop, the effective potential \(V^{(1)}_{\text{eff}}\) for \(\Phi'_0\) leads to the positive value for \(m^2_{\Phi'_0}\) implying that these vacua are stable.

### 2.3 Other magnetic theory-I

Let us consider other magnetic theory for the same undeformed electric theory given in the subsection 2.1. Here we consider the different mass deformation. By applying the Seiberg dual to the \(SU(N'_c)\) factor in \((2.1)\), the two \(NS5'_{L,R}\)-branes can be located at the left hand side of the two \(NS5\)-branes, as in Figure 4.

![Figure 3: The \( \mathcal{N} = 1 \) supersymmetric electric brane configuration for the gauge group \( SU(N_c) \times SU(N'_c) \times SU(N''_c) \) and bifundamentals \( F, \tilde{F}, G \) and \( \tilde{G} \) with vanishing(3A) which is identical to Figure 1A and nonvanishing(3B) mass for the bifundamentals \( F \) and \( \tilde{F} \). This deformation is different from the one (2.2) given previously. In 3B, the \(NS5'_{L}\)-brane together with \(N_c D4\)-branes is moving to \(+v\) direction.](image)

By starting from Figure 3A which is the same as Figure 1A and moving the \(NS5'_{L}\)-brane with \(N_c D4\)-branes to the \(+v\) direction leading to Figure 3B and interchanging the \(NS5_L\)-brane and the \(NS5'_R\)-brane, one obtains the Figure 4A. Before arriving at the Figure 4A, there exists an intermediate step where the \(N_c\) D4-branes are connecting between the \(NS5'_L\)-brane and the \(NS5'_R\)-brane, \((N''_c - N'_c + N_c)\) D4-branes are connecting between the \(NS5'_{R}\)-brane and \(NS5_L\)-brane, and \(N''_c\) D4-branes are suspended between the \(NS5_L\)-brane and the \(NS5_R\)-brane. By moving the combined \(N_c\) D4-branes, obtained from reconnection of those D4-branes between \(NS5'_L\)-brane and the \(NS5'_R\)-brane and those D4-branes between
the NS5'_{R}-brane and NS5_{L}-brane (therefore between the NS5'_{L}-brane and the NS5_{L}-brane), to +v-direction, one gets the final Figure 4A where we are left with \((N'_e - N''_e)\) anti-D4-branes between the NS5'_{R}-brane and NS5_{L}-brane. We assume that the number of colors satisfies

\[N_c + N''_e \geq N'_e \geq N''_e.\]

When two NS5'-branes in Figure 4A are close to each other, it leads to Figure 4B by realizing that the number of \(N_c\) D4-branes connecting between NS5'_{L}-brane and NS5_{L}-brane can be rewritten as \((N'_e - N''_e)\) plus \(\tilde{N}'_e\).

Figure 4: The \(\mathcal{N} = 1\) magnetic brane configuration for the gauge group \(SU(N_c) \times SU(\tilde{N}'_e = N_c + N''_e - N'_e) \times SU(N''_e)\) corresponding to Figure 3B with D4- and \(\overline{D}\)4-branes(4A) and with a misalignment between D4-branes(4B) when the NS5'-branes are close to each other. The number of tilted D4-branes is equal to \(N'_e - N''_e = N_c - \tilde{N}'_e\) in 4B.

The dual gauge group is

\[SU(N_c) \times SU(\tilde{N}'_e = N_c + N''_e - N'_e) \times SU(N''_e).\]  \hspace{1cm} (2.7)

The matter contents are the field \(f\) charged under \((N_c, \tilde{N}'_e, 1)\), a field \(g\) charged under \((1, \tilde{N}'_e, N''_e)\), and their conjugates \(\tilde{f}\) and \(\tilde{g}\) under the dual gauge group \([2.7]\) and the gauge-singlet \(\Phi\) for the second dual gauge group in the adjoint representation for the first dual gauge group, i.e., \((N''_e - 1, 1, 1) \oplus (1, 1, 1)\) under the dual gauge group \([2.7]\). Then the \(\Phi\) is a \(N_c \times N_c\) matrix. All the \(N_c\) D4-branes participate in the mass deformation.

The cubic superpotential with the mass term in the dual theory \(^3\) is given by

\[W_{\text{dual}} = \Phi f \tilde{f} + m\Phi\]  \hspace{1cm} (2.8)

\(^3\)One can also construct the mass deformation by rotating NS5_{R}-brane and moving it to +v direction. Then the brane configuration will look like as the Figure 15.
where we define $\Phi$ as $\Phi \equiv F \tilde{F}$, the second gauge group indices in $F$ and $\tilde{F}$ are contracted, each first gauge group index in them is encoded in $\Phi$ and the mass $m$ is given by (2.3) where $\Delta x$ refers to the distance between two NS5'-branes along the $x^5$ direction in Figure 4A. Let us emphasize that although the $\Phi$ which has first gauge group indices looks similar to the previous $\Phi'$ which has second gauge group indices in (2.2), the group indices are different. Here the magnetic fields $f$ and $\tilde{f}$ correspond to 4-4 strings connecting the $\tilde{N}_c'$-color D4-branes (that are connecting between the $NS5'_L$-brane and the $NS5_L$-brane in Figure 4B) with $N_c$-flavor D4-branes (which are realized as corresponding D4-branes in Figure 4A). Although the $N_c$ D4-branes in Figure 4A cannot move any directions, the tilted $(N'_c - N'''_c)$-flavor D4-branes can move $w$ direction in Figure 4B. The remaining upper $\tilde{N}_c'$ D4-branes are fixed also and cannot move any direction. Note that there is a decomposition

$$N_c = (N_c' - N_c'') + \tilde{N}_c'.$$

The brane configuration for zero mass for the bifundamental, which has only a cubic superpotential, can be obtained from Figure 4A by moving the upper NS5'-brane (or $NS5'_L$-brane) together with $N_c$ color D4-branes into the origin $v = 0$. Then the number of dual colors for D4-branes becomes $N_c$ between two NS5'-branes, $\tilde{N}_c'$ between $NS5'_R$-brane and $NS5_L$-brane and $N_c''$ between $NS5_L$-brane and $NS5_R$-brane. Or starting from Figure 3A and moving the $NS5_L$-brane to the right all the way past the $NS5'_R$-brane, one also obtains the corresponding magnetic brane configuration for massless case.

The brane configuration in Figure 4A is stable as long as the distance $\Delta x$ between the upper NS5'-brane and the lower NS5'-brane is large. If they are close to each other, then this brane configuration is unstable to decay to the brane configuration in Figure 4B. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group (2.7) and superpotential (2.8). The $(N_c - \tilde{N}_c')$ flavor D4-branes of straight brane configuration of Figure 4B bend since there exists an attractive gravitational interaction between those flavor D4-branes and $NS5_L$-brane from the DBI action, as long as the distance $y_3$ is large because the presence of an extra $NS5_R$-brane does not affect the DBI action. For the finite and small $y_3$, the careful analysis for DBI action is needed in order to obtain the bending curve connecting two NS5'-branes. Or if $y_3$ goes to zero, then this extra $NS5_R$-brane plays the role of enhancing the strength for the NS5-branes and will affect both the energy of bending curve, $E_{\text{curved}}$, which is proportional to $\frac{1}{l}$ with $l \equiv \sqrt{k}\ell_s$ where $k$ is the number of NS5-branes and $\Delta x$ which depends on both $\frac{1}{l}$ and $l$ [3].

When the upper NS5'-brane (or $NS5'_L$-brane) is replaced by coincident $N_c$ (that is equal to the number of D4-branes touching the $NS5'_L$-brane) D6-branes, this brane configuration looks...
similar to the one found in [14] where the gauge group was given by $SU(n'_f+n_c-n'_c) \times SU(n_c)$ with $n_f$ multiplets, $n'_f$ multiplets, flavor singlets and gauge singlets. Then the present $N_c$ corresponds to the $n'_f$, the number $N'_c$ corresponds to $n'_c$, and $N''_c$ corresponds to the $n_c$ of [14].

The low energy dynamics of the magnetic brane configuration can be described by the $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group (2.7) and the gauge couplings for the three gauge group factors are given by

$$g^2_{1,\text{mag}} = \frac{g_s \ell_s}{(y_1 - y_2)}, \quad g^2_{2,\text{mag}} = \frac{g_s \ell_s}{y_2}, \quad g^2_{3,\text{mag}} = \frac{g_s \ell_s}{(y_2 + y_3)}.$$  

The dual gauge theory has an adjoint $\Phi$ of $SU(N_c)$ and bifundamentals $f, \tilde{f}, g$ and $\tilde{g}$ under the dual gauge group (2.7) and the superpotential corresponding to Figures 4A and 4B is given by

$$W_{\text{dual}} = h \Phi f \tilde{f} - h \mu^2 \Phi, \quad h^2 = g^2_{1,\text{mag}}, \quad \mu^2 = -\frac{\Delta x}{2\pi g_s \ell_s^3}. $$

Then $f \tilde{f}$ is a $\tilde{N}_c' \times \tilde{N}_c'$ matrix where the first gauge group indices for $f$ and $\tilde{f}$ are contracted with those of $\Phi$ while $\mu^2$ is a $N_c \times N_c$ matrix. The product $f \tilde{f}$ has the same representation for the product of quarks and moreover, the first gauge group indices for the field $\Phi$ play the role of the flavor indices when there are D6-branes before.

Therefore, the F-term equation, the derivative $W_{\text{dual}}$ with respect to the meson field $\Phi$ cannot be satisfied if the $N_c$ exceeds $\tilde{N}_c'$. So the supersymmetry is broken. That is, there exist three equations from F-term conditions: $f \tilde{f} - \mu^2 = 0$ and $\Phi f = 0 = f \tilde{f} \Phi$. Then the solutions for these are given by

$$< f > = \left( \begin{array}{c} \mu 1_{\tilde{N}_c'} \\ 0 \end{array} \right), \quad < \tilde{f} > = \left( \begin{array}{c} \mu 1_{\tilde{N}_c'} \\ 0 \end{array} \right), \quad < \Phi > = \left( \begin{array}{cc} 0 & 0 \\ 0 & \Phi_0 1_{(N_c - \tilde{N}_c')} \end{array} \right) \quad (2.9)$$

where the zero of $< f >$ is a $(N_c - \tilde{N}_c') \times \tilde{N}_c'$ matrix, the zero of $< \tilde{f} >$ is a $\tilde{N}_c' \times (N_c - \tilde{N}_c')$ matrix and the zeros of $< \Phi >$ are $\tilde{N}_c' \times \tilde{N}_c'$, $\tilde{N}_c' \times (N_c - \tilde{N}_c')$ and $(N_c - \tilde{N}_c') \times \tilde{N}_c'$ matrices. Then one can expand these fields around a point (2.9), as in [4, 15] and one arrives at the relevant superpotential up to quadratic order in the fluctuation. At one loop, the effective potential $V^{(1)}_{\text{eff}}$ for $\Phi_0$ leads to the positive value for $m^2_{\Phi_0}$ implying that these vacua are stable.

### 2.4 Other magnetic theory-II

One can think of the following dual gauge group

$$SU(N_c) \times SU(N'_c) \times SU(\tilde{N}'_c = N'_c - N_c') \quad (2.10)$$
by performing the magnetic dual for the last gauge group in (2.1). The electric brane configuration can be given in terms of Figure 1A or Figure 1A with an exchange between NS5-brane and NS5’-brane. Then for the latter, the resulting brane configuration is given by NS5_{L'}-brane, NS5_{L}-brane, NS5_{R'}-brane, and NS5_{R}-brane from the left to the right in the \( x^6 \) direction.

In order to obtain the above dual gauge group, we need to interchange between the NS5_{L'}-brane and the NS5_{R}-brane, as we did before. One can do this either by following the previous procedure or by looking at the Figure 2 from the negative \( w \) direction which is an opposite viewpoint, compared with Figure 2. In other words, we are looking at the Figure 2 from the other side of \( w \). Then the resulting brane configuration in this case can be obtained by taking a reflection for all the NS-branes, D4-branes and anti D4-branes with respect to the NS5_{L}-brane(rotating them to the left for fixed NS5_{L}-brane) in Figure 2A and Figure 2B. Then the \( \mathcal{N} = 1 \) magnetic brane configuration for the gauge group \( SU(N_c) \times SU(N'_c) \times SU(\tilde{N''}_c = N'_c - N''_c) \) corresponds to the Figure 5A' with D4- and \( \bar{D}4 \)-branes and the Figure 5B' with a misalignment between D4-branes when the NS5'-branes are close to each other. The number of tilted D4-branes in 5B' can be written as \( N''_c - N_c = (N'_c - N_c) - \tilde{N''}_c \). We do not present the Figures 5A' and 5B' here.

Let us consider other magnetic theory for the same electric theory given in the subsection 2.1 with Figure 1A. By applying the Seiberg dual to the \( SU(N'_c) \) factor in (2.1) and interchanging the NS5_{R'}-brane and the NS5_{R}-brane, one obtains the Figure 5A''. Before arriving at the Figure 5A'', there exists an intermediate step where \( N_c \) D4-branes between NS5_{L'}-brane and the NS5_{R}-brane, the \( N'_c \) D4-branes are connecting between the NS5_{L}-brane and the NS5_{R}-brane, and \( (N'_c - N''_c) \) D4-branes are connecting between the NS5_{R'}-brane and NS5_{R}-brane. By rotating NS5_L-brane by an angle \( \frac{\pi}{2} \) which will become NS5_{M'}-brane, moving it with the \( (N'_c - N_c) \) D4-branes to \(+v\) direction where we introduce \( (N'_c - N_c) \) D4-branes and \( (N'_c - N_c) \) anti D4-branes between the NS5_{R} and the NS5'_{R} -brane, one gets the final Figure 5A'' where we are left with \( (N''_c - N_c) \) anti-D4-branes between the NS5-brane and the NS5'_{R} -brane. When two NS5'-branes in Figure 5A'' are close to each other, then it leads to Figure 5B'' by realizing that the number of \( (N'_c - N_c) \) D4-branes connecting between NS5_{M'}-brane and NS5-brane can be rewritten as \( (N''_c - N_c) \) plus \( \tilde{N''}_c \).

The brane configuration in Figure 5A'' is stable as long as the distance \( \Delta x \) between the upper NS5'-brane and the lower NS5'-brane(or NS5'_{R} -brane) is large. If they are close to each other, then this brane configuration is unstable to decay to the brane configuration in Figure 5B''. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group and superpotential. The \( (N'_c - N_c - \tilde{N''}_c) \) flavor D4-branes of straight brane configuration of Figure 5B'' bend since there exists an attractive gravitational
interaction between those flavor D4-branes and NS5-brane from the DBI action. As mentioned in [9], the two NS5'-branes are located at different side of NS5-brane in Figure 5B" and the DBI action computation for this bending curve should be taken into account.

The matter contents are the field $f$ charged under $(N_c, N'_c, N''_c)$, a field $g$ charged under $(1, N'_c, N''_c)$ and their conjugates $\tilde{f}$ and $\tilde{g}$ under the dual gauge group \( (2.10) \) and the gauge-singlet $\Phi'$ which is in the adjoint representation for the second dual gauge group, in other words, $(1, (N'_c - N_c)^2 - 1, 1) \oplus (1, 1, 1)$ under the dual gauge group \( (2.10) \). Then the $\Phi'$ is a $(N'_c - N_c) \times (N'_c - N_c)$ matrix. Only $(N'_c - N_c)$ D4-branes can participate in the mass deformation.

The cubic superpotential with the mass term is given by

$$W_{\text{dual}} = \Phi' g \tilde{g} + m \Phi'$$

(2.11)

where we define $\Phi'$ as $\Phi' \equiv G \tilde{G}$ and the third gauge group indices in $G$ and $\tilde{G}$ are contracted, each second gauge group index in them is encoded in $\Phi'$. Here the magnetic fields $g$ and $\tilde{g}$ correspond to 4-4 strings connecting the $N''_c$-color D4-branes (that are connecting between the NS$5'_M$-brane and the NS5-brane in Figure 5B") with $N'_c$-flavor D4-branes. Among these $N'_c$-flavor D4-branes, only the strings ending on the upper $(N'_c - N''_c)$ D4-branes and on the tilted middle $(N'_c - N_c)$ D4-branes in Figure 5B" enter the cubic superpotential term. Although the $(N'_c - N_c)$ D4-branes in Figure 5A" cannot move any directions, the tilted $(N''_c - N_c)$-flavor D4-branes can move $w$ direction. The remaining upper $\tilde{N}_c''$ D4-branes are fixed also.
and cannot move any direction. Note that there is a decomposition
\[(N'_c - N_c) = (N''_c - N_c) + \tilde{N}''_c.\]

The brane configuration for zero mass for the bifundamental, which has only a cubic superpotential, can be obtained from Figure 5A” by moving the upper NS5’-brane together with \((N'_c - N_c)\) color D4-branes into the origin \(v = 0\). Then the number of dual colors for D4-branes becomes \(N_c\) between the \(NS5'_L\)-brane and the \(NS5'_M\)-brane, \(N'_c\) between the \(NS5'_M\)-brane and the NS5-brane and \(\tilde{N}''_c\) between NS5-brane and \(NS5'_R\)-brane. Or starting from Figure 1A and moving the \(NS5'_R\)-brane to the right all the way past the \(NS5'_R\)-brane, one also obtains the corresponding magnetic brane configuration for massless case.

The low energy dynamics of the magnetic brane configuration can be described by the \(\mathcal{N} = 1\) supersymmetric gauge theory with gauge group \((2.10)\) and the gauge couplings for the three gauge group factors are given by
\[g^2_{1, \text{mag}} = \frac{g_s \ell_s}{y_1}, \quad g^2_{2, \text{mag}} = \frac{g_s \ell_s}{(y_2 - y_3)}, \quad g^2_{3, \text{mag}} = \frac{g_s \ell_s}{y_3}.\]

The dual gauge theory has an adjoint \(\Phi'\) of \(SU(N'_c)\) and bifundamentals \(f, \tilde{f}, g\) and \(\tilde{g}\) under the dual gauge group \((2.10)\) and the superpotential corresponding to Figures 5A” and 5B” is given by
\[W_{\text{dual}} = h\Phi'g\tilde{g} - h\mu^2\Phi', \quad h^2 = g^2_{2, \text{mag}}, \quad \mu^2 = -\frac{\Delta x}{2\pi g_s \ell_s^3}.\]

Then \(g\tilde{g}\) is a \(\tilde{N}''_c \times \tilde{N}''_c\) matrix where the second gauge group indices for \(g\) and \(\tilde{g}\) are contracted with those of \(\Phi'\) while \(\mu^2\) is a \((N'_c - N_c) \times (N'_c - N_c)\) matrix. The product \(g\tilde{g}\) has the same representation for the product of quarks and moreover, the second gauge group indices for the field \(\Phi'\) play the role of the flavor indices.

Therefore, the F-term equation, the derivative \(W_{\text{dual}}\) with respect to the meson field \(\Phi'\) cannot be satisfied if the \((N'_c - N_c)\) exceeds \(\tilde{N}''_c\). So the supersymmetry is broken. That is, there exist three equations from F-term conditions: \(g\tilde{g} - \mu^2 = 0\) and \(\Phi'g = 0 = \tilde{g}\Phi'\). Then the solutions for these are given by
\[< g > = \left( \begin{array}{c} \mu 1 \\ 0 \end{array} \right), \quad < \tilde{g} > = \left( \begin{array}{c} \mu 1 \\ \tilde{N}''_c \end{array} \right), \quad < \Phi' > = \left( \begin{array}{c} 0 \\ 0 \end{array} \right) \Phi'1_{(N'_c - N_c) - \tilde{N}''_c}^{t} \]

where the zero of \(< g >\) is a \((N'_c - N_c - \tilde{N}''_c) \times \tilde{N}''_c\) matrix, the zero of \(< \tilde{g} >\) is a \(\tilde{N}''_c \times (N'_c - N_c - \tilde{N}''_c)\) matrix and the zeros of \(< \Phi' >\) are \(\tilde{N}''_c \times \tilde{N}''_c\), \(\tilde{N}''_c \times (N'_c - N_c - \tilde{N}''_c)\) and \((N'_c - N_c - \tilde{N}''_c) \times \tilde{N}''_c\) matrices. Then one can expand these fields around on a point, as in
and one arrives at the relevant superpotential up to quadratic order in the fluctuation. At one loop, the effective potential $V_{\text{eff}}^{(1)}$ for $\Phi_0'$ leads to the positive value for $m_{\Phi_0'}^2$ implying that these vacua are stable.

3 Meta-stable brane configurations with four NS-branes plus O4-plane

In this section, we add an orientifold 4-plane to the previous brane configurations and find out new meta-stable brane configurations. Or one can realize these brane configurations by inserting the extra NS-brane and O4-planes into the brane configuration [16].

3.1 Electric theory

The type IIA brane configuration [19] corresponding to $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group

$$Sp(N_c) \times SO(2N'_c) \times Sp(N''_c)$$

and with a field $F$ charged under $(2N_c, 2N'_c)$, a field $G$ charged under $(2N'_c, 2N''_c)$ can be described by the left $NS5'_L$-brane, the left $NS5_L$-brane, the right $NS5'_R$-brane, the right $NS5_R$-brane, $2N_c$, $2N'_c$ and $2N''_c$-color D4-branes as well as an $O4^{\pm}$-plane(01236) we should add. The $O4^{\pm}$-planes act as $(x^4, x^5, x^7, x^8, x^9) \rightarrow (-x^4, -x^5, -x^7, -x^8, -x^9)$ as usual and they have RR charge $\pm 1$ playing the role of $\pm 1$ D4-brane.

We draw this brane configuration in Figure 6A for the vanishing mass case.

There is no superpotential in Figure 6A. Let us deform this theory. Displacing the two NS5'-branes relative each other in the $+v$ direction corresponds to turning on a quadratic mass-deformed superpotential for the field $F$ as follows:

$$W = mFF \equiv m\Phi'$$

where the first gauge group indices in $F$ are contracted, each second gauge group index in $F$ is encoded in $\Phi'$ and the mass $m$ is given by (2.3). The gauge-singlet $\Phi'$ for the first dual gauge group is in the adjoint representation for the second dual gauge group, i.e., $(1, (N'_c - N''_c)(2N'_c - 2N''_c - 1), 1)$ under the dual gauge group (3.3). Then the $\Phi'$ is a $2(N'_c - N''_c) \times 2(N'_c - N''_c)$ matrix. The half $NS5'_R$-brane [20] together with $(N'_c - N''_c)$-color D4-branes is moving to the $+v$ direction(and their mirrors to $-v$ direction) for fixed other branes during this mass deformation. The $2N''_c$ D4-branes among $2N'_c$ D4-branes are not
The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $Sp(N_c) \times SO(2N'_c) \times Sp(N''_c)$ and bifundamentals $F$ and $G$ with vanishing (6A) and nonvanishing (6B) mass for the bifundamental $F$. The $2N'_c$ D4-branes in 6A are decomposed into $2(N'_c - N''_c)$ D4-branes which are moving to $\pm v$ direction in $\mathbb{Z}_2$ symmetric way in 6B and $2N''_c$ D4-branes which are recombined with those D4-branes connecting between $NS5'_R$-brane and $NS5_R$-brane in 6B.

participating in the mass deformation. Then the $x^5$ coordinate of $NS5'_L$-brane is equal to zero while the $x^5$ coordinates of half $NS5'_R$-brane are given by $\pm \Delta x$.

Giving an expectation value to the meson field $\Phi'$ corresponds to recombination of $2N'_c$- and $2N''_c$- color D4-branes, which will become $2N'_c$-color D4-branes in Figure 6A such that they are suspended between the $NS5'_L$-brane and the $NS5'_R$-brane and pushing them into the $w$ direction. We assume that the number of colors satisfies

$$N'_c \geq N_c + 2 \geq N''_c.$$

Now we draw this brane configuration in Figure 6B for nonvanishing mass for the field $F$.

### 3.2 Magnetic theory

By applying the Seiberg dual to the $Sp(N_c)$ factor in (3.1), the two $NS5'_{L,R}$-branes can be located at the inside of the two NS5-branes, as in Figure 7. Starting from Figure 6B and interchanging the $NS5'_L$-brane and the $NS5_L$-brane, one obtains the Figure 7A.

Before arriving at the Figure 7A, there exists an intermediate step where the $2(N'_c - N_c - 2)$ D4-branes are connecting between the $NS5_L$-brane and the $NS5'_L$-brane, $(N'_c - N''_c)$ D4-branes connecting between the $NS5'_L$-brane and $NS5'_R$-brane (and their mirrors), and $2N''_c$ D4-branes between the $NS5'_L$-brane and the $NS5_R$-brane. By introducing $-2N''_c$ D4-branes and $-2N''_c$ anti-D4-branes between the $NS5_L$-brane and $NS5'_L$-brane, reconnecting the former with the $N'_c$ D4-branes connecting between $NS5_L$-brane and the $NS5'_L$-brane (therefore $(N'_c - N''_c)$ D4-
Figure 7: The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group $Sp(\tilde{N}_c = N'_c - N_c - 2) \times SO(2N'_c) \times Sp(N''_c)$ corresponding to Figure 6B with D4- and 7A) and with a misalignment between D4-branes(7B) when the NS5'-branes are close to each other. The number of tilted D4-branes in 7B can be written as $N_c - N''_c + 2 = (N'_c - N''_c) - \tilde{N}_c$.

The dual gauge group is

$$Sp(\tilde{N}_c = N'_c - N_c - 2) \times SO(2N'_c) \times Sp(N''_c). \quad (3.3)$$

The matter contents are the field $f$ charged under $(2\tilde{N}_c, 2N'_c, 1)$, a field $g$ charged under $(1, 2N'_c, 2N''_c)$ under the dual gauge group (3.3) and the gauge-singlet $\Phi'$ that is in the adjoint representation for the second dual gauge group, i.e., $(1, (N'_c - N''_c)(2N'_c - 2N''_c - 1), 1)$ under the dual gauge group. That is, the $\Phi'$ is an $2(N'_c - N''_c) \times 2(N'_c - N''_c)$ antisymmetric matrix.

The cubic superpotential with the mass term (3.2) in the dual theory is given by

$$W_{\text{dual}} = \Phi'ff + m\Phi'. \quad (3.4)$$

Here the magnetic field $f$ corresponds to 4-4 strings connecting the $2\tilde{N}_c$-color D4-branes(that are connecting between the $NS5_L$-brane and the $NS5'R_L$-brane including the mirrors) with $2N'_c$-flavor D4-branes(that is a combination of three different D4-branes including the mirrors in Figure 7B). Among these $2N'_c$-flavor D4-branes, only the strings ending on the upper $2(N'_c - N_c - 2)$ D4-branes and on the tilted middle $2(N_c - N''_c)$ D4-branes including the mirrors in Figure 7B enter the cubic superpotential term. Although the $(N'_c - N''_c)$ D4-branes
in Figure 7A cannot move any directions, the tilted $2(N_c - N_c'' + 2)$-flavor D4-branes including the mirrors can move $w$ direction. The remaining upper $\tilde{N}_c$ D4-branes (and its mirrors) are fixed also and cannot move any direction. Note that there is a decomposition

$$(N_c' - N_c'') = (N_c - N_c'' + 2) + \tilde{N}_c.$$  

The brane configuration for zero mass for the bifundamental, which has only a cubic superpotential, can be obtained from Figure 7A by moving the upper and lower NS5'-branes together with $(N_c' - N_c'')$ color D4-branes into the origin $v = 0$. Then the number of dual colors for D4-branes becomes $2\tilde{N}_c$ between $NS5_L$-brane and $NS5'_L$-brane and $2N_c''$ between two NS5'-branes as well as $2N_c''$ between $NS5'_R$-brane and $NS5_R$-brane. Or starting from Figure 6A and moving the $NS5_L$-brane to the left all the way past the $NS5'_L$-brane, one also obtains the corresponding magnetic brane configuration for massless case.

The brane configuration in Figure 7A is stable as long as the distance $\Delta x$ between the upper NS5'-brane and the middle NS5'-brane is large. If they are close to each other, then this brane configuration is unstable to decay and leads to the brane configuration in Figure 7B. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group (3.3) and superpotential (3.4). The upper $(N_c' - N_c' - \tilde{N}_c)$ flavor D4-branes of straight brane configuration of Figure 7B bend due to the fact that there exists an attractive gravitational interaction between those flavor D4-branes and $NS5_L$-brane from the DBI action, as long as the distance $y_3$ goes to $\infty$ because the presence of an extra $NS5_R$-brane does not affect the DBI action. For the finite and small $y_3$, the careful analysis for DBI action is needed in order to obtain the bending curve connecting two NS5'-branes. Of course, their mirrors, the lower $(N_c' - N_c'' - \tilde{N}_c)$ flavor D4-branes of straight brane configuration of Figure 7B can bend and their trajectories connecting two NS5'-branes should be preserved under the O4-plane, i.e., $Z_2$ symmetric way.

When the upper and lower half $NS5'_R$-branes are replaced by coincident $(N_c' - N_c'')$ D6-branes and the $NS5_R$ is rotated by an angle $\frac{\pi}{2}$ in the $(v, w)$ plane in Figure 7B, this brane configuration reduces to the one found in [16] where the gauge group was given by $Sp(n_f + n'_c - n_c - 2) \times SO(2n'_c)$ with $2n_f$ multiplets, flavor singlet and gauge singlets. Then the present $(N_c' - N_c'')$ corresponds to the $n_f$, the number $N_c$ corresponds to $n_c$ and $N_c''$ corresponds to the $n'_c$ of [16]. However, the gauge group $Sp(N_c''')$ corresponds to the different gauge group $SO(2n'_c)$. When we discuss the subsection 3.5 and take the Seiberg dual for the middle gauge group, then it becomes $SO(2N_c) \times Sp(\tilde{N}_c = N_c + N_c'' - N_c' - 2) \times SO(2N_c'')$. Then the $N_c$ corresponds to the $n_f$, the number $N_c'$ corresponds to $n_c$ and $N_c''$ corresponds to the $n'_c$ of [16]. If we ignore $2N_c''$ D4-branes and $NS5_R$-brane from Figure 7B, then the brane configuration
becomes the one in [21, 6].

The dual gauge theory has an adjoint $\Phi'$ of $SO(2N'_c)$ and bifundamentals $f$ and $g$ under the dual gauge group (3.3) and the superpotential corresponding to Figures 7A and 7B is given by

$$W_{dual} = h\Phi'f f - h\mu^2\Phi', \quad h^2 = g_{2,\text{mag}}^2, \quad \mu^2 = -\frac{\Delta x}{2\pi g_s\ell_s^3}.$$ 

Then $ff$ is a $2\tilde{N}_c \times 2\tilde{N}_c$ matrix where the second gauge group indices for $f$ are contracted with those of $\Phi'$ while $\mu^2$ is a $2(N'_c - N''_c) \times 2(N'_c - N''_c)$ matrix. The product $ff$ has the same representation for the product of quarks and moreover, the first gauge group indices for the field $\Phi'$ play the role of the flavor indices, as we observed above for the comparison with the brane configuration in the presence of D6-branes.

Therefore, the F-term equation, the derivative $W_{dual}$ with respect to the meson field $\Phi'$ cannot be satisfied if the $2(N'_c - N''_c)$ exceeds $2\tilde{N}_c$. So the supersymmetry is broken. That is, there exist two equations from F-term conditions: $ff - \mu^2 = 0$ and $\Phi'f = 0$. Then the solutions for these are given by

$$< f > = \begin{pmatrix} \mu_{2,\tilde{N}_c}^1 & 0 \\ 0 & 0 \end{pmatrix}, \quad < \Phi' > = \begin{pmatrix} 0 & 0 \\ 0 & \Phi'_0 1_{(N'_c - N''_c - \tilde{N}_c)\otimes\sigma_2} \end{pmatrix}$$ (3.5)

where the zero of $< f >$ is a $2(N'_c - N''_c - \tilde{N}_c) \times 2\tilde{N}_c$ matrix and the zeros of $< \Phi' >$ are $2\tilde{N}_c \times 2\tilde{N}_c, 2\tilde{N}_c \times 2(N'_c - N''_c - \tilde{N}_c)$, and $2(N'_c - N''_c - \tilde{N}_c) \times 2\tilde{N}_c$ matrices. Then one can expand these fields around on a point (3.5), as in [4] and one arrives at the relevant superpotential up to quadratic order in the fluctuation. At one loop, the effective potential $V_{eff}^{(1)}$ for $\Phi'_0$ leads to the positive value for $m_{\Phi'_0}^2$ implying that these vacua are stable.

### 3.3 Other magnetic theory-I

Let us consider other magnetic theory for the same electric theory given in the subsection 3.1. By applying the Seiberg dual to the $SO(2N'_c)$ factor in (3.1), the two $NS5'_{L,R}$-branes can be located at the left hand side of the two NS5-branes, as in Figure 9.

The Figure 8A is the same as the one in Figure 6A and one moves half $NS5'_{L}$-brane together with $N_c$ D4-branes to $+v$ direction (and its mirrors to $-v$ direction) and is given by Figure 8B. Starting from Figure 8B and interchanging the $NS5_L$-brane and the $NS5'_R$-brane, one obtains the Figure 9A. Before arriving at the Figure 9A, there exists an intermediate step where the $N_c$ D4-branes are connecting between the $NS5'_L$-brane and the $NS5'_R$-brane (and their mirrors), $2(N''_c - N'_c + N_c + 2)$ D4-branes are connecting between the $NS5'_R$-brane and
Figure 8: The $N = 1$ supersymmetric electric brane configuration for the gauge group $Sp(N_c) \times SO(2N_c') \times Sp(N''_c)$ and bifundamentals $F$ and $G$ with vanishing(8A) which is the same as Figure 6A and nonvanishing(8B) mass for the bifundamental $F$. This deformation is different from the previous case (3.2). In 8B, the $2N_c$ D4-branes are moving to $\pm v$ directions in $Z_2$ symmetric way.

$NS5_L$-brane, and $2N_c''$ D4-branes are suspended between the $NS5_L$-brane and the $NS5_R$-brane. By moving the combined $N_c$ D4-branes, obtained from the reconnection of those D4-branes between the $NS5'_L$-brane and the $NS5'_R$-brane and those D4-branes between the $NS5'_R$-brane and the $NS5'_L$-brane(therefore between the $NS5'_L$-brane and the $NS5'_L$-brane), to $+v$-direction(and their mirrors to $-v$ direction), one gets the final Figure 9A where we are left with $2(N_c' - N_c'' - 2)$ anti-D4-branes between the $NS5'_R$-brane and $NS5_L$-brane. We assume that the number of colors satisfies

$$N_c + N_c'' \geq N_c' - 2 \geq N_c''.$$

When two NS5'-branes in Figure 9A are close to each other, then it leads to Figure 9B by realizing that the number of $N_c$ D4-branes connecting between $NS5_L'$-brane and $NS5_L$-brane in Figure 9A can be rewritten as $(N_c' - N_c'') + 2\tilde{N}_c'$. If we ignore $2N_c''$ D4-branes and $NS5_R$-brane and change the O4-plane charge(corresponding to change the symplectic gauge group into the orthogonal gauge group and vice versa) from Figure 9B, then the brane configuration becomes the one in [9].

The dual gauge group is

$$Sp(N_c) \times SO(2\tilde{N}_c = 2N_c + 2N''_c - 2N'_c + 4) \times Sp(N''_c).$$

(3.6)

The matter contents are the field $f$ charged under $(2N_c, 2\tilde{N}_c', 1)$, a field $g$ charged under $(1, 2\tilde{N}_c', 2N''_c)$ under the dual gauge group (3.6) and the gauge-singlet $\Phi$ for the second dual gauge group in the adjoint representation for the first dual gauge group, i.e.,

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Figure 9: The $N = 1$ magnetic brane configuration for the gauge group $Sp(N_c) \times SO(2\tilde{N}_c' = 2N_c + 2N_c'' - 2N_c' + 4) \times Sp(N_c'')$ corresponding to Figure 8B with D4- and $\overline{D}4$-branes(9A) and with a misalignment between D4-branes(9B) when the NS5'-branes are close to each other. The number of tilted D4-branes is equal to $N_c' - N_c'' - 2 = N_c - \tilde{N}_c'$ in 9B.

$(N_c(2N_c + 1), 1, 1)$ under the dual gauge group. Then the $\Phi$ is a $2N_c \times 2N_c$ matrix. All the $2N_c$ D4-branes are participating in the mass deformation.

The cubic superpotential with the mass term in the dual theory is given by

$$W_{\text{dual}} = \Phi f f + m\Phi$$

where we define $\Phi$ as $\Phi \equiv FF$ and the second gauge group indices in $F$ are contracted, each first gauge group index in them is encoded in $\Phi$. Although the $\Phi$ that has first gauge group indices looks similar to the previous $\Phi'$ that has second gauge group indices, the group indices are different. Here the magnetic field $f$ corresponds to 4-4 strings connecting the $2\tilde{N}_c'$-color D4-branes (that are connecting between the NS5$_L$-brane and the NS5$_L$-brane in Figure 9B) with $2N_c$-flavor D4-branes including the mirrors (which are realized as corresponding D4-branes in Figure 9A). Although the $N_c$ D4-branes (and its mirrors) in Figure 9A cannot move any directions, the tilted $(N_c' - N_c'' - 2)$-flavor D4-branes (and its mirrors) can move $w$ direction in Figure 9B. The remaining upper $\tilde{N}_c'$ D4-branes (and its mirrors) are fixed also and cannot move any direction. Note that there is a decomposition

$$N_c = (N_c' - N_c'' - 2) + \tilde{N}_c'.$$

The brane configuration for zero mass for the bifundamental, which has only a cubic superpotential, can be obtained from Figure 9A by moving the upper and lower NS5'-branes together with $N_c$ color D4-branes into the origin $\nu = 0$. Then the number of dual colors for

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4One can also construct the mass deformation by rotating $NS5_R$-brane and moving it to $\pm \nu$ direction, as in previous case in subsection 2.3. The brane configuration can be obtained easily.
D4-branes becomes $2N_c$ between two NS5'-branes, $2\tilde{N}_c'$ between $NS5'_R$-brane and $NS5_L$-brane and $2N''_c$ between $NS5_L$-brane and $NS5_R$-brane. Or starting from Figure 8A and moving the $NS5_L$-brane to the right all the way past the $NS5'_R$-brane, one also obtains the corresponding magnetic brane configuration for massless case.

The brane configuration in Figure 9A is stable as long as the distance $\Delta x$ between the upper NS5'-brane and the middle NS5'-brane is large. If they are close to each other, then this brane configuration is unstable to decay and leads to the brane configuration in Figure 9B. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group (3.6) and superpotential (3.7). The $(N_c - \tilde{N}_c')$ flavor D4-branes of straight brane configuration of Figure 9B bend due to the fact that there exists an attractive gravitational interaction between those flavor D4-branes and $NS5_L$-brane from the DBI action, as long as the distance $y_3$ goes to $\infty$ because the presence of an extra $NS5_R$-brane does not affect the DBI action. For the finite and small $y_3$, the careful analysis for DBI action is needed in order to obtain the bending curve connecting two NS5'-branes. Or if $y_3$ goes to zero, then this extra $NS5_R$-brane plays the role of enhancing the strength for the NS5-branes and will affect both the energy of bending curve, $E_{\text{curved}}$, and $\Delta x$ [3]. Of course, their mirrors, the lower $(N_c - \tilde{N}_c')$ flavor D4-branes of straight brane configuration of Figure 9B can bend and their trajectories connecting two NS5'-branes should be preserved under the O4-plane, i.e., $\mathbb{Z}_2$ symmetric way.

When the NS5'-brane(or $NS5'_L$-brane) is replaced by coincident $N_c$ D6-branes, this brane configuration looks similar to the one found in [16] where the gauge group was given by $SO(2n'_f + 2n_c - 2n'_c + 4) \times Sp(n_c)$ with $2n'_f$ multiplets, flavor singlet and gauge singlets. Then the present $N_c$ corresponds to the $n'_f$, the number $N'_c$ corresponds to $n'_c$ and $N''_c$ corresponds to the $n_c$ of [16].

The dual gauge theory has an adjoint $\Phi$ of $Sp(N_c)$ and bifundamentals $f$ and $g$ under the dual gauge group (3.6) and the superpotential corresponding to Figures 9A and 9B is given by

$$W_{\text{dual}} = h \Phi f f - h \mu^2 \Phi, \quad h^2 = g_{1,\text{mag}}^2, \quad \mu^2 = -\frac{\Delta x}{2\pi g_s \ell_s^3}.$$ 

Then $f f$ is a $2\tilde{N}_c' \times 2\tilde{N}_c'$ matrix where the first gauge group indices for $f$ are contracted with those of $\Phi$ while $\mu^2$ is a $2N_c \times 2N_c$ matrix. The product $f f$ has the same representation for the product of quarks and moreover, the first gauge group indices for the field $\Phi$ play the role of the flavor indices as we observed above.

Therefore, the F-term equation, the derivative $W_{\text{dual}}$ with respect to the meson field $\Phi$ cannot be satisfied if the $2N_c$ exceeds $2\tilde{N}_c'$. So the supersymmetry is broken. That is, there
exist two equations from F-term conditions: \( ff - \mu^2 = 0 \) and \( \Phi f = 0 \). Then the solutions for these are given by
\[
<f> = \left( \frac{\mu}{\Phi} \right), \quad <\Phi> = \left( \begin{array}{cc} 0 & 0 \\ 0 & \Phi_0 \end{array} \right)
\]
where the zero of \(<f>\) is a \(2(N_c - \tilde{N}_c') \times 2\tilde{N}_c'\) matrix and the zeros of \(<\Phi>\) are \(2\tilde{N}_c' \times 2\tilde{N}_c', 2\tilde{N}_c' \times 2(N_c - \tilde{N}_c')\) and \(2(N_c - \tilde{N}_c') \times 2\tilde{N}_c'\) matrices. Then one can expand these fields around on a point \(3.8\), as in [4] and one arrives at the relevant superpotential up to quadratic order in the fluctuation. At one loop, the effective potential \(V_{eff}^{(1)}\) for \(\Phi_0\) leads to the positive value for \(m_0^2\) implying that these vacua are stable.

### 3.4 Other magnetic theory-II

One can think of the following dual gauge group
\[
Sp(N_c) \times SO(2N_c') \times Sp(\tilde{N}_c'' = N_c' - N_c'' - 2)
\]
by performing the magnetic dual for the last gauge group in \(3.1\). The electric brane configuration can be given in terms of Figure 6A or Figure 6A with an exchange between NS5-brane and NS5'-brane. For the latter, the resulting brane configuration is given by \(NS5_L\)-brane, \(NS5'_L\)-brane, \(NS5_R\)-brane, and \(NS5'_R\)-brane from the left to the right in the \(x^6\) direction. One can take the magnetic dual either by following the previous procedure or by looking at the Figure 7 from the negative \(w\) direction which is an opposite viewpoint, compared with Figure 7. In other words, we are looking at the Figure 7 from the other side of \(w\).

Then the resulting brane configuration in this case can be obtained by taking a reflection for all the NS-branes, D4-branes and anti D4-branes with respect to the \(NS5_L\)-brane(rotating them to the left for fixed \(NS5_L\)-brane) in Figure 7A and Figure 7B. Then the \(N' = 1\) magnetic brane configuration for the gauge group \(Sp(N_c) \times SO(2N_c') \times Sp(\tilde{N}_c'' = N_c' - N_c'' - 2)\) corresponds to the Figure 10A' with D4- and \(D4\)-branes and the Figure 10B' with a misalignment between D4-branes when the \(NS5'\)-branes are close to each other. The number of tilted D4-branes in 10B' can be written as \(N_c'' - N_c + 2 = (N_c' - N_c) - \tilde{N}_c''\). We do not present the Figures 10A' and 10B' here.

We turn to the other case. Let us consider other magnetic theory for the same electric theory given in the subsection 3.1 with Figure 6A. By applying the Seiberg dual to the \(Sp(N_c'')\) factor in \(3.1\) from Figure 6A and interchanging the \(NS5'_R\)-brane and the \(NS5_R\)-brane, one obtains the Figure 10A''. Before arriving at the Figure 10A'', there exists an intermediate step where \(2N_c\) D4-branes between \(NS5'_L\)-brane and the \(NS5_L\)-brane, the \(2N_c'\) D4-branes
are connecting between the $NS_{5L}$-brane and the $NS_{5R}$-brane, $(N'_c - N''_c - 2)$ D4-branes are connecting between the $NS_{5R}$-brane and $NS_{5R}'$-brane (and their mirrors). By rotating $NS_{5L}$-brane by an angle $\frac{\pi}{2}$, moving it with the $(N'_c - N_c)$ D4-branes to $+v$ direction where we introduce $2(N'_c - N_c)$ D4-branes and $2(N'_c - N_c)$ anti D4-branes between the $NS_{5R}$-brane and the $NS_{5R}'$-brane, one gets the final Figure 10A” where we are left with $2(N''_c - N_c + 2)$ anti-D4-branes between the $NS_{5R}$-brane and the $NS_{5R}'$-brane. When two NS5’-branes in Figure 10A” are close to each other, then it leads to Figure 10B” by realizing that the number of $(N'_c - N_c)$ D4-branes connecting between $NS_{5M}$-brane and NS5-brane can be rewritten as $(N''_c - N_c + 2)$ plus $\tilde{N''}_c$.

The brane configuration in Figure 10A” is stable as long as the distance $\Delta x$ between the upper NS5’-brane and the middle NS5’-brane (or $NS_{5R}'$-brane) is large. If they are close to each other, then this brane configuration is unstable to decay to the brane configuration in Figure 10B”. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group and superpotential. The upper $(N'_c - N_c - \tilde{N''}_c)$ flavor D4-branes of straight brane configuration of Figure 10B” bend since there exists an attractive gravitational interaction between those flavor D4-branes and NS5-brane from the DBI action. As mentioned in [9], the two NS5’-branes are located at different side of NS5-brane in Figure 10B” and the DBI action computation for this bending curve should be taken into account. Of course, their mirrors, the lower $(N'_c - N_c - \tilde{N''}_c)$ flavor D4-branes of straight brane configuration of Figure 10B” can bend and their trajectories connecting two NS5’-branes should be preserved under the O4-plane, i.e., $Z_2$ symmetric way.

Figure 10: The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group $Sp(N_c) \times SO(2N'_c) \times Sp(\tilde{N''}_c = N'_c - N''_c - 2)$ with D4- and D4-branes (10A”) and with a misalignment between D4-branes (10B”) when the NS5’-branes are close to each other. The number of tilted D4-branes in 10B” can be written as $N''_c - N_c + 2 = (N'_c - N_c) - \tilde{N''}_c$. The deformation is related to the bifundamentals $G$. 

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The matter contents are the field \( f \) charged under \((2N_c, 2N_c', 1)\), a field \( g \) charged under \((1, 2N_c', 2\tilde{N}_c'')\) under the dual gauge group \((\ref{3.9})\) and the gauge-singlet \( \Phi' \) that is in the adjoint representation for the second dual gauge group, i.e., \((1, (N'_c - N_c)(2N_c' - 2N_c - 1), 1)\) under the dual gauge group. That is, the \( \Phi' \) is an \( 2(N'_c - N_c) \times 2(N'_c - N_c) \) antisymmetric matrix.

The cubic superpotential with the mass term in the dual theory is given by

\[
W_{\text{dual}} = \Phi'gg + m\Phi'
\]

where we define \( \Phi' \) as \( \Phi' \equiv GG \) and the third gauge group indices in \( G \) are contracted, each second gauge group index in \( G \) is encoded in \( \Phi' \). Although the \( \Phi' \) that has second gauge group indices looks similar to the previous \( \Phi \) that has first gauge group indices, the group indices are different. Here the magnetic field \( g \) correspond to 4-4 strings connecting the \( 2\tilde{N}_c'' \)-color D4-branes including the mirrors(that are connecting between the \( \text{NS}5'_M \)-brane and the \( \text{NS}5 \)-brane in Figure 10B”) with \( 2N'_c \)-flavor D4-branes. Among these \( 2N'_c \)-flavor D4-branes, only the strings ending on the upper \( 2(N'_c - N_c'' - 2) \) D4-branes and on the tilted middle \( 2(N'_c - N_c + 2) \) D4-branes in Figure 10B” enter the cubic superpotential term. Although the \( (N'_c - N_c) \) D4-branes(and its mirrors) in Figure 10A” cannot move any directions, the tilted \( (N'_c - N_c + 2) \)-flavor D4-branes(and its mirrors) can move \( w \) direction. The remaining upper and lower \( \tilde{N}_c'' \) D4-branes are fixed also and cannot move any direction. Note that there is a decomposition

\[
(N'_c - N_c) = (N'_c'' - N_c + 2) + \tilde{N}_c''.
\]

The brane configuration for zero mass for the bifundamental, which has only a cubic superpotential, can be obtained from Figure 10A” by moving the upper and lower \( \text{NS}5' \)-branes together with \( (N'_c - N_c) \) color D4-branes into the origin \( v = 0 \). Then the number of dual colors for D4-branes becomes \( 2N_c \) between two \( \text{NS}5' \)-branes and \( 2N'_c \) between the \( \text{NS}5'_M \)-brane and the \( \text{NS}5 \)-brane and \( 2\tilde{N}_c'' \) between the \( \text{NS}5 \)-brane and the \( \text{NS}5'_R \)-brane. Or starting from Figure 6A and moving the \( \text{NS}5'_R \)-brane to the left all the way past the \( \text{NS}5'_R \)-brane, one also obtains the corresponding magnetic brane configuration for massless case.

The dual gauge theory has an adjoint \( \Phi' \) of \( SO(2N'_c) \) and bifundamentals \( f \) and \( g \) under the dual gauge group \((\ref{3.9})\) and the superpotential corresponding to Figures 10A” and 10B” is given by

\[
W_{\text{dual}} = h\Phi'gg - h\mu^2\Phi', \quad h^2 = g_{2,\text{mag}}^2, \quad \mu^2 = -\frac{\Delta x}{2\pi g_s\ell_s^3}.
\]

Then \( gg \) is a \( 2\tilde{N}_c'' \times 2\tilde{N}_c'' \) matrix where the second gauge group indices for \( g \) are contracted with those of \( \Phi' \) while \( \mu^2 \) is a \( 2(N'_c - N_c) \times 2(N'_c - N_c) \) matrix. The product \( gg \) has the same
representation for the product of quarks and moreover, the first gauge group indices for the field $Φ'$ play the role of the flavor indices.

Therefore, the F-term equation, the derivative $W_{dual}$ with respect to the meson field $Φ'$ cannot be satisfied if the $2(N'_{c} − N_{c})$ exceeds $2\tilde{N}'_{c}$. So the supersymmetry is broken. That is, there exist two equations from F-term conditions: $gg − μ^2 = 0$ and $Φ'g = 0$. Then the solutions for these are given by

$$< g > = \begin{pmatrix} \mu_1 2\tilde{N}'_{c} \\ 0 \end{pmatrix}, \quad < Φ' > = \begin{pmatrix} 0 & 0 \\ 0 & Φ'_0 1_{(N'_{c} − N_{c} − \tilde{N}'_{c})σ_2} \end{pmatrix}$$

where the zero of $< g >$ is a $2(N'_{c} − N_{c} − \tilde{N}'_{c})$ matrix and the zeros of $< Φ' >$ are $2\tilde{N}'_{c} × 2\tilde{N}'_{c}$, $2\tilde{N}'_{c} × 2(N'_{c} − N_{c} − \tilde{N}'_{c})$, and $2(N'_{c} − N_{c} − \tilde{N}'_{c}) × 2\tilde{N}'_{c}$ matrices. Then one can expand these fields around on a point (3.11), as in [4] and one arrives at the relevant superpotential up to quadratic order in the fluctuation. At one loop, the effective potential $V_{eff}^{(1)}$ for $Φ'_0$ leads to the positive value for $m^2_{Φ'_0}$ implying that these vacua are stable.

### 3.5 Other magnetic theories-III

By changing the charges of O4-plane in previous brane configuration of Figure 6A, the type IIA brane configuration is realized by an $\mathcal{N} = 1$ supersymmetric gauge theory with

$$SO(2N_{c}) × Sp(N'_{c}) × SO(2\tilde{N}'_{c})$$

and corresponding matter contents. Then by deforming the theory by mass term and taking the magnetic dual on each gauge group factor, one gets meta-stable brane configurations. There exists an $\mathcal{N} = 1$ magnetic supersymmetric gauge theory with $SO(2\tilde{N}_{c} = 2N'_{c} − 2N_{c} + 4) × Sp(N'_{c}) × SO(2N'_{c})$ with matters which corresponds to Figure 7 with opposite O4-plane charges. Also there is an $\mathcal{N} = 1$ magnetic supersymmetric gauge theory with $SO(2N_{c}) × Sp(\tilde{N}'_{c} = N_{c} + N''_{c} − N'_{c}) × SO(2N''_{c})$ with matters which corresponds to Figure 9 with opposite O4-plane charges. Finally, there exists an $\mathcal{N} = 1$ magnetic supersymmetric gauge theory with $SO(2N_{c}) × Sp(N'_{c}) × SO(2\tilde{N}'_{c} = 2N'_{c} − 2N''_{c} + 4)$ with matters which corresponds to Figure 10 with opposite O4-plane charges. The remaining analysis can be done easily without any difficulty.

### 4 Meta-stable brane configurations with six NS-branes plus O6-plane

In this section, we add an orientifold 6-plane to the previous brane configuration for the product gauge group $[10]$ realized by three NS-branes, together with the extra mirrors for
them, and find out new meta-stable brane configurations. Or one can realize these brane
configurations by inserting the two outer NS-branes into the brane configuration \[22, 14\].

4.1 Electric theory

The type IIA brane configuration corresponding to \(\mathcal{N} = 1\) supersymmetric gauge theory with
gauge group

\[ Sp(N_c) \times SU(N'_c) \times SU(N''_c) \] (4.1)

and with a field \(F\) charged under \((2N_c, N'_c)\), a field \(G\) charged under \((N'_c, N''_c)\), and their
conjugates \(\tilde{F}\) and \(\tilde{G}\) can be described by the left \(NS5'_L\)-brane, the NS5-brane, the right \(NS5'_R\)-brane(and their mirrors), \(2N_c\), \(N'_c\) and \(N''_c\)-color D4-branes as well as O6-plane(0123789)\[22\].

The \(O6^-\)-plane acts as \((x^4, x^5, x^6) \rightarrow (-x^4, -x^5, -x^6)\) and has RR charge \(-4\).

Let us place an O6-plane at the origin \(x^6 = 0\) and let us denote the \(x^6\) coordinates for the
\(NS5'_L\)-brane, the NS5-brane and the \(NS5'_R\)-brane by \(x^6 = y_1, y_1 + y_2, y_1 + y_2 + y_3\) respectively.
Their mirrors can be understood similarly. The \(2N_c\) D4-branes are suspended between the
\(NS5'_L\)-brane and its mirror, the \(N'_c\) D4-branes are suspended between the \(NS5'_L\)-brane and the
NS5-brane(and their mirrors), and the \(N''_c\) D4-branes are suspended between the NS5-brane and the
\(NS5'_R\)-brane(and their mirrors). We draw this brane configuration in Figure 11A for the vanishing mass for the field \(G\).

The gauge couplings of \(Sp(N_c)\), \(SU(N'_c)\) and \(SU(N''_c)\) are given by a string coupling con-
stant \(g_s\), a string scale \(\ell_s\) and the \(x^6\) coordinates \(y_i\) for three NS-branes through

\[ g^2_1 = \frac{g_s \ell_s}{2y_1}, \quad g^2_2 = \frac{g_s \ell_s}{y_2}, \quad g^2_3 = \frac{g_s \ell_s}{y_3}. \]

As \(y_3\) goes to \(\infty\), the \(SU(N''_c)\) gauge group becomes a global symmetry and the theory
leads to SQCD with the gauge group \(Sp(N_c) \times SU(N'_c)\) and \(N''_c\) flavors in the fundamental
representation.

There is no superpotential in Figure 11A. Let us deform this theory. Displacing the two
NS5\(^'\)-branes relative each other in the \(+v\) direction corresponds to turning on a quadratic
mass-deformed superpotential for the field \(G\) as follows:

\[ W = mG\tilde{G} \equiv m\Phi'' \] (4.2)

\[From now on, when we say about NS-branes(NS5-brane or NS5\(^'\)-brane), they refer to those in positive
region of \(x^6\). Their mirrors in the negative region of \(x^6\) are understood with O6-plane while we are taking the
brane motion. In other words, there exist three NS-branes: \(NS5'_L\)-brane, NS5-brane and \(NS5'_R\)-brane from
Figure 11A.
where the second gauge group indices in $G$ and $\tilde{G}$ are contracted and the mass $m$ is given by (2.3). The gauge-singlet $\Phi''$ for the second dual gauge group is in the adjoint representation for the third dual gauge group, i.e., $(\mathbf{1}, \mathbf{1}, N''_c^2 - 1) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1})$ under the dual gauge group (1.3). The $\Phi''$ is a $N''_c \times N''_c$ matrix. The $NS5'_R$-brane together with $N''_c$-color D4-branes is moving to the $+v$ direction for fixed other branes during this mass deformation (and their mirrors to $-v$ direction). Then the $x^5$ coordinate of $NS5'_L$-brane is equal to zero while the $x^5$ coordinate of $NS5'_R$-branes is given by $\Delta x$. Giving an expectation value to the meson field $\Phi''$ corresponds to recombination of $N'_c$- and $N''_c$-color D4-branes, which will become $N''_c$ or $N'_c$-color D4-branes in Figure 11A such that they are suspended between the $NS5'_L$-brane and the $NS5'_R$-brane and pushing them into the $w$ direction. We assume that the number of colors satisfies

$$2N_c + N''_c \geq N'_c \geq 2N_c.$$ 

Now we draw this brane configuration in Figure 11B for nonvanishing mass for the fields $G$ and $\tilde{G}$. The geometry for three NS-branes in Figure 11B is the same as the one given by first three NS-branes in Figure 1B.

![Figure 11: The $N = 1$ supersymmetric electric brane configuration for the gauge group $Sp(N_c) \times SU(N'_c) \times SU(N''_c)$ and bifundamental $F, \tilde{F}, G$ and $\tilde{G}$ with vanishing(11A) and nonvanishing(11B) mass for the bifundamental $G$ and $\tilde{G}$. In 11B, the $NS5'_R$-brane together with $N''_c$ D4-branes is moving to $+v$ direction (and their mirrors to $-v$ direction).](image)

4.2 Magnetic theory

By applying the Seiberg dual to the $SU(N'_c)$ factor in (4.1), the $NS5'_L,R$-branes can be located at the outside of the two NS5-branes, as in Figure 12. Starting from Figure 11B and interchanging the $NS5'_L$-brane and the NS5-brane (and their mirrors), one obtains the Figure 12A.
Figure 12: The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group $Sp(N_c) \times SU(\tilde{N}_c' = 2N_c + N''_c - N'_c) \times SU(N''_c)$ corresponding to Figure 11B with D4- and $\overline{D}4$-branes(12A) and with a misalignment between D4-branes(12B) when the NS5’-branes are close to each other. The number of tilted D4-branes is equal to $2N_c$ D4-branes between the NS5-brane and its mirror. By reconnecting the $N'_c$ D4-branes connecting between the NS5-brane and the NS5’L-brane with the $N''_c$ D4-branes connecting between NS5’L-brane and the NS5’R-brane(12A) and moving those combined $N'_c$ D4-branes to $v$-direction(12B), one gets the final Figure 12A where we are left with $N'_c - 2N_c = N''_c - \tilde{N}_c'$ in 12B. The notation for the anti D4-branes is used for the bar on the number of those branes in 12A.

Before arriving at the Figure 12A, there exists an intermediate step where the $(N''_c - N'_c + 2N_c)$ D4-branes are connecting between the NS5-brane and the NS5’L-brane, $N''_c$ D4-branes are connecting between the NS5’L-brane and NS5’R-brane(12B) as well as $2N_c$ D4-branes between the NS5-brane and its mirror. By reconnecting the $N''_c$ D4-branes connecting between the NS5-brane and the NS5’L-brane with the $N'_c$ D4-branes connecting between NS5’L-brane and the NS5’R-brane and moving those combined $N'_c$ D4-branes to $v$-direction(12B), one gets the final Figure 12A where we are left with $(N'_c - 2N_c)$ anti-D4-branes between the NS5-brane and NS5’L-brane. When two NS5’-branes in Figure 12A are close to each other, it becomes Figure 12B by realizing that the number of $N'_c$ D4-branes connecting between NS5-brane and NS5’R-brane in Figure 12A can be rewritten as $(N'_c - 2N_c) + \tilde{N}_c'$. The brane configuration consisting of NS5-brane and two NS5’-branes in Figure 12B is exactly the same as those in Figure 2B.

The dual gauge group is given by

$$Sp(N_c) \times SU(\tilde{N}_c' = 2N_c + N''_c - N'_c) \times SU(N''_c).$$

(4.3)

The matter contents are the field $f$ charged under $(2N_c, \overline{N}_c', 1)$, a field $g$ charged under $(1, \overline{N}_c', \overline{N}_c')$, and their conjugates $\tilde{f}$ and $\tilde{g}$ under the dual gauge group (4.3) and the gauge-singlet $\Phi''$ for the second dual gauge group in the adjoint representation for the third dual gauge group, i.e., $(1, 1, N''_c^2 - 1) \oplus (1, 1, 1)$ under the dual gauge group. Then the $\Phi''$ is a $N''_c \times N''_c$ matrix.
The cubic superpotential with the mass term (4.2) is given by

$$W_{\text{dual}} = \Phi'' \tilde{g} g + m\Phi''.$$  (4.4)

Here the magnetic fields $g$ and $\tilde{g}$ correspond to 4-4 strings connecting the $\tilde{N}_c'$-color D4-branes (that are connecting between the NS5-brane and the $NS5'_R$-brane in Figure 12B) with $N''_c$-flavor D4-branes (which are realized as corresponding D4-branes in Figure 12A). Although the $N''_c$ D4-branes in Figure 12A cannot move any directions, the tilted $(N'_c - 2N_c)$-flavor D4-branes can move $w$ direction in Figure 12B (and its mirrors). The remaining upper $\tilde{N}_c'$ D4-branes are fixed also and cannot move any direction. Note that there is a decomposition

$$N''_c = (N'_c - 2N_c) + \tilde{N}_c'.$$

The brane configuration for zero mass for the bifundamental, which has only a cubic superpotential, can be obtained from Figure 12A by moving the upper NS5'-brane (or $NS5'_R$-brane) together with $N''_c$ color D4-branes into the origin $v = 0$ (and their mirrors). Then the number of dual colors for D4-branes becomes $2N_c$ between the NS5-brane and its mirror, $\tilde{N}_c'$ between NS5-brane and $NS5'_L$-brane and $N''_c$ between $NS5'_L$-brane and $NS5'_R$-brane. Or starting from Figure 11A and moving the NS5-brane to the left all the way past the $NS5'_L$-brane (and their mirrors), one also obtains the corresponding magnetic brane configuration for massless case.

The brane configuration in Figure 12A is stable as long as the distance $\Delta x$ between the upper NS5'-brane and the lower NS5'-brane is large. If they are close to each other, then this brane configuration is unstable to decay and leads to the brane configuration in Figure 12B. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group (4.3) and superpotential (4.4).

One can perform similar analysis in our brane configuration since one can take into account the behavior of parameters geometrically in the presence of O6-plane. Then the upper $(N''_c - \tilde{N}_c')$ flavor D4-branes of straight brane configuration of Figure 12B can bend due to the fact that there exists an attractive gravitational interaction between those flavor D4-branes and NS5-brane from the DBI action, by following the procedure of [3], as long as $y_1$ is very large. Then the mirror of NS5-brane does not affect the flavor D4-branes. On the other hand, if $y_1$ goes to zero, then the mirror of NS5-brane plays the role of enhancing the strength for the NS5-branes and will affect both the energy of bending curve, $E_{\text{curved}}$, and $\Delta x$. Of course, their mirrors, the lower $(N''_c - \tilde{N}_c')$ flavor D4-branes of straight brane configuration of Figure 12B can bend and their trajectories connecting two NS5'-branes should be preserved under the O6-plane, i.e., $Z_2$ symmetric way.
The low energy dynamics of the magnetic brane configuration can be described by the $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group (4.3) and the gauge couplings for the three gauge group factors are given by

$$g_{1,\text{mag}}^2 = \frac{g_s \ell_s}{2(y_1 + y_2)}, \quad g_{2,\text{mag}}^2 = \frac{g_s \ell_s}{y_2}, \quad g_{3,\text{mag}}^2 = \frac{g_s \ell_s}{(y_3 - y_2)}.$$  

The dual gauge theory has an adjoint $\Phi''$ of $SU(N''_c)$ and bifundamentals $f, \tilde{f}, g$ and $\tilde{g}$ under the dual gauge group (4.3) and the superpotential corresponding to Figures 12A and 12B is given by

$$W_{\text{dual}} = h\Phi'' g \tilde{g} - h \mu^2 \Phi'', \quad h^2 = g_{3,\text{mag}}, \quad \mu^2 = -\frac{\Delta x}{2\pi g_s \ell_s^3}.$$  

Then $g \tilde{g}$ is a $\vec{N}'_c \times \vec{N}'_c$ matrix where the third gauge group indices for $g$ and $\tilde{g}$ are contracted with those of $\Phi''$ while $\mu^2$ is a $N''_c \times N''_c$ matrix. The product $g \tilde{g}$ has the same representation for the product of quarks and moreover, the third gauge group indices for the field $\Phi''$ play the role of the flavor indices.

When the upper NS5'-brane (or $NS5'_R$-brane) is replaced by coincident $N''_c$ D6-branes in Figure 12B, this brane configuration looks similar to the one found in [14] where the gauge group was given by $SU(n_f + 2n'_e - n_c) \times Sp(n'_e)$ with $n_f$ multiplets and singlets. Then the present $2N'_c$ corresponds to the $2n'_e$, $N'_c$ corresponds to $n_c$, and $N''_c$ corresponds to the $n_f$ of [14].

Therefore, the F-term equation, the derivative $W_{\text{dual}}$ with respect to the meson field $\Phi''$ cannot be satisfied if the $N''_c$ exceeds $\vec{N}'_c$. So the supersymmetry is broken. That is, there exist three equations from F-term conditions: $g \tilde{g} - \mu^2 = 0$ and $\Phi'' g = 0 = \tilde{g} \Phi''$. Then the solutions for these are given by

$$< g > = \left( \begin{array}{c} \mu 1_{\vec{N}'_c} \\ 0 \end{array} \right), \quad < \tilde{g} > = \left( \begin{array}{c} \mu 1_{\vec{N}'_c} \\ 0 \end{array} \right), \quad < \Phi'' > = \left( \begin{array}{c} 0 \\ 0 \end{array} \right) \Phi''_0 1_{(N''_c - \vec{N}'_c)} \right) \right) (4.5)$$

where the zero of $< g >$ is a $(N''_c - \vec{N}'_c) \times \vec{N}'_c$ matrix, the zero of $< \tilde{g} >$ is a $\vec{N}'_c \times (N''_c - \vec{N}'_c)$ matrix and the zeros of $< \Phi'' >$ are $\vec{N}'_c \times \vec{N}'_c$, $\vec{N}'_c \times (N''_c - \vec{N}'_c)$ and $(N''_c - \vec{N}'_c) \times \vec{N}'_c$ matrices. Then one can expand these fields around on a point (4.3), as in [3] and one arrives at the relevant superpotential up to quadratic order in the fluctuation. At one loop, the effective potential $V_{\text{eff}}^{(1)}$ for $\Phi''_0$ leads to the positive value for $m^2_{\Phi''_0}$ implying that these vacua are stable.

### 4.3 Other magnetic theory

Let us consider other magnetic theory for the same electric theory given in the subsection 4.1. By applying the Seiberg dual to the $SU(N''_c)$ factor in (4.1), the $NS5'_{L,R}$-branes can be
located at the inside of the two NS5-branes, as in Figure 14. Starting from Figure 13B and interchanging the NS5-brane and the \( NS5'_{R} \)-brane(and their mirrors), one obtains the Figure 14A. The geometry for three NS-branes in Figure 13B is the same as the one given by first three NS-branes in Figure 3B.

Figure 13: The \( \mathcal{N} = 1 \) supersymmetric electric brane configuration for the gauge group \( Sp(N_c) \times SU(N'_c) \times SU(N''_c) \) and the bifundamentals with vanishing(13A) which is the same as the Figure 11A and nonvanishing(13B) mass for the bifundamental \( G \) and \( \tilde{G} \). This deformation is different from the one in (12). The \( N'_c \) D4-branes in 13A are decomposed into \( (N'_c - 2N_c) \) D4-branes which are moving to \(+v\) direction in 13B and \( 2N_c \) D4-branes which are recombined with those D4-branes connecting between \( NS5'_{L} \)-brane and its mirror in 13B.

Before arriving at the Figure 14A, there exists an intermediate step where the \( (N'_c - 2N_c) \) D4-branes are connecting between the \( NS5'_{L} \)-brane and the \( NS5'_{R} \)-brane, \( (N'_c - N''_c) \) D4-branes are connecting between the \( NS5'_{R} \)-brane and NS5-brane(and their mirrors) as well as \( 2N_c \) D4-branes between \( NS5'_{R} \)-brane and its mirror. By reconnecting the \( (N'_c - 2N_c) \) D4-branes connecting between the \( NS5'_{L} \)-brane and the \( NS5'_{R} \)-brane with the \( (N'_c - 2N_c) \) D4-branes connecting between \( NS5'_{R} \)-brane and the NS5-brane where we introduce \(-2N_c \) D4-branes and \(-2N_c \) anti D4-branes and moving those combined D4-branes to \(+v\)-direction(and their mirrors to \(-v\) direction), one gets the final Figure 14A where we are left with \( (N''_c - 2N_c) \) anti-D4-branes between the \( NS5'_{R} \)-brane and the NS5-brane. We assume that the number of colors satisfies

\[
N'_c \geq N''_c \geq 2N_c.
\]

When two NS5'-branes in Figure 14A are close to each other, then it leads to Figure 14B by realizing that the number of \( (N'_c - 2N_c) \) D4-branes connecting between \( NS5'_{L} \)-brane and NS5-brane in Figure 14A can be rewritten as \( (N''_c - 2N_c) \) plus \( \tilde{N''}_c \). The brane configuration consisting of NS5-brane and two NS5'-branes in Figure 14B is exactly the same as those in Figure 4B.
Figure 14: The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group $Sp(N_c) \times SU(N'_c) \times SU(\tilde{N}_c'' = N'_c - N''_c)$ corresponding to Figure 13B with D4- and $\bar{D}4$-branes (14A) and with a misalignment between D4-branes (14B) when the NS5'-branes are close to each other. The number of tilted D4-branes in 14B can be written as $N''_c - 2N_c = (N'_c - 2N_c) - \tilde{N}_c''$.

The dual gauge group is given by

$$Sp(N_c) \times SU(N'_c) \times SU(\tilde{N}_c'' = N'_c - N''_c)$$

(4.6)

The matter contents are the field $f$ charged under $(2N_c, N'_c, 1)$, a field $g$ charged under $(1, N'_c, \bar{N}_c')$ and their conjugates $\tilde{f}$ and $\tilde{g}$ under the dual gauge group (1.6) and the gauge-singlet $\Phi'$ which is in the adjoint representation for the second dual gauge group, in other words, $(1, (N'_c - 2N_c)^2 - 1, 1) \oplus (1, 1, 1)$ under the dual gauge group (4.6). Then the $\Phi'$ is a $(N'_c - 2N_c) \times (N'_c - 2N_c)$ matrix. Only $(N'_c - 2N_c)$ D4-branes can participate in the mass deformation.

The cubic superpotential with the mass term is given by

$$W_{\text{dual}} = \Phi' g \tilde{g} + m \Phi'$$

(4.7)

where we define $\Phi'$ as $\Phi' \equiv GG\tilde{G}$ and the third gauge group indices in $G$ and $\tilde{G}$ are contracted, each second gauge group index in them is encoded in $\Phi'$. Although the $\Phi'$ that has second gauge group indices looks similar to the previous $\Phi''$ that has third gauge group indices, the group indices are different. Here the magnetic fields $g$ and $\tilde{g}$ correspond to 4-4 strings connecting the $\tilde{N}_c''$-color D4-branes (that are connecting between the NS5'_L-brane and the NS5-brane in Figure 14B) with $N'_c$-flavor D4-branes. Among these $N'_c$-flavor D4-branes, only the strings ending on the upper $(N'_c - N''_c)$ D4-branes and on the tilted $(N''_c - 2N_c)$ D4-branes in Figure 14B enter the cubic superpotential term. Although the $(N'_c - 2N_c)$ D4-branes in Figure 14A cannot move any directions, the tilted $(N''_c - 2N_c)$-flavor D4-branes can move $w$
direction. The remaining upper $\tilde{N}_{c}''$ D4-branes are fixed also and cannot move any direction. Note that there is a decomposition

$$(N_c' - 2N_c) = (N_c'' - 2N_c) + \tilde{N}_{c}''.$$ 

The brane configuration for zero mass for the bifundamental, which has only a cubic superpotential, can be obtained from Figure 14A by moving the upper NS5'-brane together with $(N_c' - 2N_c)$ color D4-branes into the origin $v = 0$(and their mirrors). Then the number of dual colors for D4-branes becomes $2N_c$ between the NS5'$_L$-brane and its mirror, $N_c''$ between the NS5'$_L$-brane and the NS5'$_R$-brane and $\tilde{N}_c''$ between NS5'$_R$-brane and NS5-brane. Or starting from Figure 13A and moving the NS5-brane to the right all the way past the NS5'$_R$-brane(and their mirrors), one also obtains the corresponding magnetic brane configuration for massless case.

The brane configuration in Figure 14A is stable as long as the distance $\Delta x$ between the upper NS5'-brane and the lower NS5'-brane is large. If they are close to each other, then this brane configuration is unstable to decay and leads to the brane configuration in Figure 14B. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group (4.6) and superpotential (4.7). Then the upper $(N_c' - 2N_c - \tilde{N}_c'')$ flavor D4-branes of straight brane configuration of Figure 14B can bend due to the fact that there exists an attractive gravitational interaction between those flavor D4-branes and NS5-brane from the DBI action, as long as $y_1$ is very large. Of course, their mirrors, the lower $(N_c' - 2N_c - \tilde{N}_c'')$ flavor D4-branes of straight brane configuration of Figure 14B can bend and their trajectories connecting two NS5'-branes should be preserved under the O6-plane, i.e., $Z_2$ symmetric way.

When the upper NS5'-brane(or NS5'$_L$-brane) is replaced by coincident $(N_c'^{'} - 2N_c)$ D6-branes in Figure 14B, this brane configuration looks similar to the one found in [14] where the gauge group was given by $SU(n_f + n_c' - n_c) \times SO(n_c')$ with $n_f$ multiplets, bifundamentals, and singlets. Then the present $2N_c$ corresponds to the $n_c'$, $(N_c'^{'} - 2N_c)$ corresponds to $n_f$, and $N_c''$ corresponds to the $n_c$ of [14]. Note that $Sp(N_c)$ corresponds to $SO(n_c')$. Moreover, there is a meta-stable brane configuration for the gauge group given by $SU(n_c) \times SU(n_f' + n_c - n_c')$ with fundamentals, bifundamentals, an antisymmetric flavor, a conjugate symmetric flavor, and singlets where there are NS5'-brane, $O6^{\pm}$-planes, and eight semi infinite D6-branes at $x^6 = 0$. Then the our $2N_c$ corresponds to the $n_c$, the number $(N_c'^{'} - 2N_c)$ corresponds to $n_f'$, and our $N_c''$ corresponds to the $n_c'$ of [23].

The low energy dynamics of the magnetic brane configuration can be described by the $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group (4.6) and the gauge couplings for the
three gauge group factors are given by
\[
g_{1,mag}^2 = \frac{g_s \ell_s}{2y_1}, \quad g_{2,mag}^2 = \frac{g_s \ell_s}{(y_2 - y_3)}, \quad g_{3,mag}^2 = \frac{g_s \ell_s}{y_3}.
\]

The dual gauge theory has an adjoint \( \Phi' \) of \( SU(N'_c) \) and bifundamentals \( f, \tilde{f}, g \) and \( \tilde{g} \) under the dual gauge group (4.6) and the superpotential corresponding to Figures 14A and 14B is given by
\[
W_{dual} = h \Phi' g \tilde{g} - h \mu^2 \Phi', \quad h^2 = g_{2,mag}^2, \quad \mu^2 = -\frac{\Delta x}{2\pi g_s \ell_s^3}.
\]

Then \( g \tilde{g} \) is a \( \tilde{N}'_c \times \tilde{N}''_c \) matrix where the second gauge group indices for \( g \) and \( \tilde{g} \) are contracted with those of \( \Phi' \) while \( \mu^2 \) is a \( (N'_c - 2N_c) \times (N'_c - 2N_c) \) matrix. The product \( g \tilde{g} \) has the same representation for the product of quarks and moreover, the second gauge group indices for the field \( \Phi' \) play the role of the flavor indices, as above.

Therefore, the F-term equation, the derivative \( W_{dual} \) with respect to the meson field \( \Phi' \) cannot be satisfied if the \( (N'_c - 2N_c) \) exceeds \( \tilde{N}'_c \). So the supersymmetry is broken. That is, there exist three equations from F-term conditions: \( g \tilde{g} - \mu^2 = 0 \) and \( \Phi' g = 0 = \tilde{g} \Phi' \). Then the solutions for these are given by
\[
<g> = \begin{pmatrix} \mu^1 \tilde{N}'_c \\ 0 \end{pmatrix}, \quad <\tilde{g}> = \begin{pmatrix} \mu^1 \tilde{N}'_c \\ 0 \end{pmatrix}, \quad <\Phi'> = \begin{pmatrix} 0 \\ 0 \\ \Phi' \mu^1 (N'_c - 2N_c) - \tilde{N}'_c \end{pmatrix}
\]

where the zero of \( <g> \) is a \( (N'_c - 2N_c - \tilde{N}'_c) \times \tilde{N}'_c \) matrix, the zero of \( <\tilde{g}> \) is a \( \tilde{N}'_c \times (N'_c - 2N_c - \tilde{N}'_c) \) matrix and the zeros of \( <\Phi'> \) are \( \tilde{N}'_c \times \tilde{N}'_c, \tilde{N}'_c \times (N'_c - 2N_c - \tilde{N}'_c) \) and \( (N'_c - 2N_c - \tilde{N}'_c) \times \tilde{N}'_c \) matrices. Then one can expand these fields around a point (4.8), as in [4] and one arrives at the relevant superpotential up to quadratic order in the fluctuation. At one loop, the effective potential \( V_{eff}^{(1)} \) for \( \Phi'_0 \) leads to the positive value for \( m_{\Phi'_0}^2 \) implying that these vacua are stable.

4.4 Other magnetic theories

In this subsection, we add an orientifold 6-plane with positive charge to the previous brane configuration for the product gauge group [10] realized by three NS-branes, together with the extra mirrors for them, and find out new meta-stable brane configurations. Or one can realize these brane configurations by inserting the two outer NS-branes into the brane configuration [22] [14].
4.4.1 Electric theory

The type IIA brane configuration corresponding to $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group

$$SO(N_c) \times SU(N'_c) \times SU(N''_c)$$

and with a field $F$ charged under $(N_c, N'_c)$, a field $G$ charged under $(N'_c, N''_c)$, and their conjugates $\tilde{F}$ and $\tilde{G}$ can be described by the left NS5$^L$-brane, the NS5$'$-brane, the right NS5$^R$-brane(and their mirrors), $N_c$, $N'_c$- and $N''_c$-color D4-branes as well as $O6^+$-plane(0123789). The $O6^+$-plane acts as $(x^4, x^5, x^6) \rightarrow (-x^4, -x^5, -x^6)$ and has RR charge +4.

Let us place an $O6^+$-plane at the origin $x^6 = 0$ and let us denote the $x^6$ coordinates for the NS5$^L$-brane, the NS5$'$-brane and the NS5$^R$-brane by $x^6 = y_1, y_1 + y_2, y_1 + y_2 + y_3$ respectively. Their mirrors can be understood similarly. The $N_c$ D4-branes are suspended between the NS5$^L$-brane and its mirror, the $N'_c$ D4-branes are suspended between the NS5$^L$-brane and the NS5$'$-brane(and their mirrors), and the $N''_c$ D4-branes are suspended between the NS5$'$-brane and the NS5$^R$-brane(and their mirrors). We assume that the number of colors satisfies

$$N_c + N''_c \geq N'_c \geq N_c.$$

4.4.2 Magnetic theory

By applying the Seiberg dual to the $SU(N'_c)$ factor in (4.9) and interchanging the NS5$^L$-brane and the NS5$'$-brane(and their mirrors), one obtains the Figure 15A. Before arriving at the Figure 15A, there exists an intermediate step where the $(N''_c - N'_c + N_c)$ D4-branes are connecting between the NS5$'$-brane and the NS5$^L$-brane, $N''_c$ D4-branes are connecting between the NS5$^L$-brane and NS5$^R$-brane(and their mirrors) as well as $N_c$ D4-branes between NS5$'$-brane and its mirror. By rotating NS5$^R$-brane by an angle $\frac{\pi}{2}$, moving it with $N''_c$ D4-branes to $+v$-direction(and their mirrors to $-v$ direction), one gets the final Figure 15A where we are left with $(N'_c - N_c)$ anti-D4-branes between the NS5$^L$-brane and NS5-brane. When two NS5$'$-branes in Figure 15A are close to each other, it becomes Figure 15B by realizing that the number of $N''_c$ D4-branes connecting between NS5-brane and NS5$''_R$-brane can be rewritten as $(N'_c - N_c)$ plus $\tilde{N}'_c$.

The dual gauge group is given by

$$SO(N_c) \times SU(\tilde{N}'_c = N_c + N''_c - N'_c) \times SU(N''_c).$$

(4.10)
Figure 15: The $\mathcal{N} = 1$ magnetic brane configuration for the gauge group $SO(N_c) \times SU(\tilde{N}_c') = N_c + N_c'' - N_c') \times SU(N_c'')$ with D4- and $\overline{D4}$-branes (15A) and with a misalignment between D4-branes (15B) when the NS5'-branes are close to each other. Note that the number of D4-branes on the gauge group $SO(N_c)$ is equal to $N_c$ not $2N_c$. The number of tilted D4-branes is equal to $N_c' - N_c = N_c'' - \tilde{N}_c'$ in 15B. The deformation is related to the bifundamentals $G$ and $\tilde{G}$.

The matter contents are the field $f$ charged under $(N_c, \tilde{N}_c', 1)$, a field $g$ charged under $(1, \tilde{N}_c, N_c''')$, and their conjugates $\tilde{f}$ and $\tilde{g}$ under the dual gauge group (4.10) and the gauge-singlet $\Phi''$ for the second dual gauge group in the adjoint representation for the third dual gauge group, i.e., $(1, 1, N_c''^2 - 1) \oplus (1, 1, 1)$ under the dual gauge group. Then the $\Phi''$ is a $N_c'' \times N_c'''$ matrix.

The cubic superpotential with the mass term is given by (4.4) where we define $\Phi''$ as $\Phi'' \equiv G\tilde{G}$ and the second gauge group indices in $G$ and $\tilde{G}$ are contracted, each third gauge group index in them is encoded in $\Phi''$. Although the $\Phi''$ that has third gauge group indices looks similar to the previous $\Phi'$ that has second gauge group indices the group indices are different. Here the magnetic fields $g$ and $\tilde{g}$ correspond to 4-4 strings connecting the $\tilde{N}_c'$-color D4-branes (that are connecting between the NS5-brane and the NS5' $R$-brane in Figure 15B) with $N_c''$-flavor D4-branes (which are realized as corresponding D4-branes in Figure 15A). Although the $N_c''$ D4-branes in Figure 15A cannot move any directions, the tilted $(N_c' - N_c)$-flavor D4-branes can move $w$ direction in Figure 15B (and its mirrors). The remaining upper $\tilde{N}_c'$ D4-branes are fixed also and cannot move any direction. Note that there is a decomposition

$$N_c'' = (N_c' - N_c) + \tilde{N}_c'.$$

The brane configuration for zero mass for the bifundamental, which has only a cubic superpotential, can be obtained from Figure 15A by moving the upper NS5'-brane together with $N_c''$ color D4-branes into the origin $v = 0$ (and their mirrors). Then the number of dual colors for D4-branes becomes $N_c$ between the NS5$'_L$-brane and its mirror, $\tilde{N}_c'$ between
NS5′_L-brane and NS5-brane and \( N_c'' \) between NS5-brane and \( NS5'_R \)-brane.

The brane configuration in Figure 15A is stable as long as the distance \( \Delta x \) between the upper NS5'-brane and the lower NS5'-brane is large. If they are close to each other, then this brane configuration is unstable to decay to the brane configuration in Figure 15B. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group and superpotential. The upper \( (N_c'' - \tilde{N}_c') \) flavor D4-branes of straight brane configuration of Figure 15B bend since there exists an attractive gravitational interaction between those flavor D4-branes and NS5-brane from the DBI action. As mentioned in \[9\], the two NS5'-branes are located at different side of NS5-brane in Figure 15B and the DBI action computation for this bending curve should be taken into account.

The low energy dynamics of the magnetic brane configuration can be described by the \( \mathcal{N} = 1 \) supersymmetric gauge theory with gauge group \((4.10)\) and the gauge couplings for the three gauge group factors are given by the expressions in subsection 4.2.

The dual gauge theory has an adjoint \( \Phi'' \) of \( SU(N_c'') \) and bifundamentals \( f, \tilde{f}, g \) and \( \tilde{g} \) under the dual gauge group \((4.10)\) and the superpotential corresponding to Figures 15A and 15B is given by the expressions in subsection 4.2. Then \( g\tilde{g} \) is a \( \tilde{N}_c' \times \tilde{N}_c' \) matrix where the third gauge group indices for \( g \) and \( \tilde{g} \) are contracted with those of \( \Phi'' \) while \( \mu^2 \) is a \( N_c'' \times N_c'' \) matrix. The product \( g\tilde{g} \) has the same representation for the product of quarks and moreover, the third gauge group indices for the field \( \Phi'' \) play the role of the flavor indices.

When the upper NS5'-brane(or \( NS5'_R \)-brane) is replaced by coincident \( N_c'' \) D6-branes in Figure 15B, this brane configuration looks similar to the one found in \[14\] where the gauge group was given by \( SU(n_f + n_c' - n_c) \times Sp(n_c') \) with \( n_f \) multiplets and singlets. Then the present \( N_c \) corresponds to the \( n_c' \), \( N_c' \) corresponds to \( n_c \), and \( N_c'' \) corresponds to the \( n_f \) of \[14\].

Therefore, the F-term equation, the derivative \( W_{dual} \) with respect to the meson field \( \Phi'' \) cannot be satisfied if the \( N_c'' \) exceeds \( \tilde{N}_c' \). So the supersymmetry is broken. That is, there exist three equations from F-term conditions: \( g\tilde{g} - \mu^2 = 0 \) and \( \Phi'' g = 0 = \tilde{g} \Phi'' \). Then the solutions for these are given by the expressions in subsection 4.2. Then one can expand these fields around a point, as in \[11\] and one arrives at the relevant superpotential up to quadratic order in the fluctuation. At one loop, the effective potential \( V_{eff}^{(1)} \) for \( \Phi_0'' \) leads to the positive value for \( m_{\Phi_0''}^2 \) implying that these vacua are stable.

### 4.4.3 Other magnetic theory

Let us consider other magnetic theory for the same electric theory given in the subsection 4.4.1. By applying the Seiberg dual to the \( SU(N_c'') \) factor in \((4.9)\) and interchanging the NS5'-brane and the \( NS5'_R \)-brane(and their mirrors), one obtains the Figure 16A.
Before arriving at the Figure 16A, there exists an intermediate step where the \(N'_c\) D4-branes are connecting between the NS5\(_L\)-brane and the NS5\(_R\)-brane, \((N'_c - N'_c)\) D4-branes are connecting between the NS5\(_R\)-brane and NS5'-brane (and their mirrors) as well as \(N_c\) D4-branes between NS5\(_L\)-brane and its mirror. By rotating NS5\(_L\)-brane by an angle \(\frac{\pi}{2}\), moving it with \((N'_c - N_c)\) D4-branes to \(+v\) direction where we introduce \((N'_c - N_c)\) D4-branes and \((N'_c - N_c)\) anti D4-branes between the NS5\(_R\)-brane and the NS5'-brane (and their mirrors to \(-v\) direction), one gets the final Figure 16A where we are left with \((N''_c - N_c)\) anti-D4-branes between the NS5-brane and the NS5'\(_R\)-brane. We assume that the number of colors satisfies

\[ N'_c \geq N''_c \geq N_c. \]

When two NS5'-branes in Figure 16A are close to each other, then it leads to Figure 16B by realizing that the number of \((N'_c - N_c)\) D4-branes connecting between NS5'\(_L\)-brane and NS5-brane can be rewritten as \((N''_c - N_c)\) plus \(\tilde{N}''_c\). The brane configuration consisting of NS5-brane and two NS5'-branes in Figure 16B is exactly the same as those in Figure 5B''.

![Figure 16](image)

Figure 16: The \(\mathcal{N} = 1\) magnetic brane configuration for the gauge group \(SO(N_c) \times SU(N'_c) \times SU(N''_c) = N'_c - N''_c\) with D4- and \(\overline{D4}\)-branes(16A) and with a misalignment between D4-branes(16B) when the NS5'-branes are close to each other. The number of tilted D4-branes in 16B can be written as \(N''_c - N_c = (N'_c - N_c) - \tilde{N}''_c\). The deformation is different from the previous one.

The dual gauge group is given by

\[ SO(N_c) \times SU(N'_c) \times SU(N''_c) = N'_c - N''_c \]

(4.11)

The matter contents are the field \(f\) charged under \((N_c, N'_c, 1)\), a field \(g\) charged under \((1, N'_c, N''_c)\) and their conjugates \(\tilde{f}\) and \(\tilde{g}\) under the dual gauge group (4.11) and the gauge-singlet \(\Phi'\) which is in the adjoint representation for the second dual gauge group, in other words, \((1, (N'_c - N_c)^2 - 1, 1) \oplus (1, 1, 1)\) under the dual gauge group (4.11). Then the \(\Phi'\) is
a \((N'_c - N_c) \times (N'_c - N_c)\) matrix. Only \((N'_c - N_c)\) D4-branes are participating in the mass deformation.

The cubic superpotential with the mass term is given by \((4.7)\) where we define \(\Phi'\) as \(\Phi' \equiv G\tilde{G}\) and the third gauge group indices in \(G\) and \(\tilde{G}\) are contracted, each second gauge group index in them is encoded in \(\Phi'\). Although the \(\Phi'\) that has second gauge group indices looks similar to the previous \(\Phi''\) that has third gauge group indices, the group indices are different. Here the magnetic fields \(g\) and \(\tilde{g}\) correspond to 4-4 strings connecting the \(\tilde{N}_c''\)-color D4-branes (that are connecting between the \(NS5'_L\)-brane and the NS5-brane in Figure 16B) with \(N'_c\)-flavor D4-branes. Among these \(N'_c\)-flavor D4-branes, only the strings ending on the upper \((N'_c - N'_c)\) D4-branes and on the tilted \((N''_c - N_c)\) D4-branes in Figure 16B enter the cubic superpotential term. Although the \((N'_c - N_c)\) D4-branes in Figure 16A cannot move any directions, the tilted \((N''_c - N_c)\)-flavor D4-branes can move \(w\) direction. The remaining upper \(\tilde{N}_c''\) D4-branes are fixed also and cannot move any direction. Note that there is a decomposition

\[(N'_c - N_c) = (N''_c - N_c) + \tilde{N}_c''.\]

The brane configuration for zero mass for the bifundamental, which has only a cubic superpotential, can be obtained from Figure 16A by moving the upper NS5'-brane together with \((N'_c - N_c)\) color D4-branes into the origin \(v = 0\) (and their mirrors). Then the number of dual colors for D4-branes becomes \(N_c\) between the \(NS5'_L\)-brane and its mirror, \(N'_c\) between the \(NS5'_L\)-brane and the NS5-brane and \(\tilde{N}_c''\) between NS5-brane and \(NS5'_R\)-brane.

When the upper NS5'-brane (or \(NS5'_L\)-brane) is replaced by coincident \((N'_c - N_c)\) D6-branes in Figure 16B, this brane configuration looks similar to the one found in \([14]\) where the gauge group was given by \(SU(n_f + 2n'_c - n_c) \times Sp(n'_c)\) with \(n_f\) multiplets, bifundamentals, and singlets. Then the present \(N_c\) corresponds to the \(2n'_c\), \((N'_c - N_c)\) corresponds to \(n_f\), and \(N''_c\) corresponds to the \(n_c\) of \([14]\). Note that \(SO(N_c)\) corresponds to \(Sp(n'_c)\). Moreover, the meta-stable brane configuration corresponding to gauge group given by \(SU(n_c) \times SU(n'_f + n_c - n'_c)\) with fundamentals, bifundamentals, a symmetric flavor, a conjugate symmetric flavor, and singlets was given in \([23]\) where there exists NS5-brane on the O6-plane. Then our \(N_c\) corresponds to the \(n_c\), our \((N'_c - N_c)\) corresponds to \(n'_f\), and our \(N''_c\) corresponds to the \(n'_c\).

The brane configuration in Figure 16A is stable as long as the distance \(\Delta x\) between the upper NS5'-brane and the lower NS5'-brane is large. If they are close to each other, then this brane configuration is unstable to decay to the brane configuration in Figure 16B. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group and superpotential. The upper \((N'_c - N_c - \tilde{N}_c'')\) flavor D4-branes of straight brane
configuration of Figure 16B bend since there exists an attractive gravitational interaction between those flavor D4-branes and NS5-brane from the DBI action. As mentioned in [9], the two NS5'-branes are located at different side of NS5-brane in Figure 16B and the DBI action computation for this bending curve should be taken into account.

The low energy dynamics of the magnetic brane configuration can be described by the $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group (4.11) and the gauge couplings for the three gauge group factors are given by the expressions in subsection 4.3. The dual gauge theory has an adjoint $\Phi'$ of $SU(N'_c)$ and bifundamentals $f, \tilde{f}, g$ and $\tilde{g}$ under the dual gauge group (4.11) and the superpotential corresponding to Figures 16A and 16B is given by the one in subsection 4.3. Then $g\tilde{g}$ is a $\tilde{N}'_c \times \tilde{N}_c$ matrix where the second gauge group indices for $g$ and $\tilde{g}$ are contracted with those of $\Phi'$ while $\mu^2$ is a $(N'_c - N_c) \times (N'_c - N_c)$ matrix. The product $g\tilde{g}$ has the same representation for the product of quarks and moreover, the second gauge group indices for the field $\Phi'$ play the role of the flavor indices.

Therefore, the F-term equation, the derivative $W_{\text{dual}}$ with respect to the meson field $\Phi'$ cannot be satisfied if the $(N'_c - N_c)$ exceeds $\tilde{N}_c$. So the supersymmetry is broken. That is, there exist three equations from F-term conditions: $g\tilde{g} - \mu^2 = 0$ and $\Phi'g = 0 = \tilde{g}\Phi'$. Then the solutions for these are given by

$$
\begin{align*}
< g > &= \left( \begin{array}{c}
\mu 1 & \tilde{N}'_c \\
0 & 0
\end{array} \right), \\
< \tilde{g} > &= \left( \begin{array}{c}
\mu 1 & \tilde{N}'_c \\
0 & 0
\end{array} \right), \\
< \Phi' > &= \left( \begin{array}{c}
0 \\
0 \\
\Phi' 1 & 0 \\
\tilde{N}'_c & (N'_c - N_c - \tilde{N}_c)
\end{array} \right)
\end{align*}
$$

(4.12)

where the zero of $< g >$ is a $(N'_c - N_c - \tilde{N}'_c) \times \tilde{N}'_c$ matrix, the zero of $< \tilde{g} >$ is a $\tilde{N}'_c \times (N'_c - N_c - \tilde{N}_c)$ matrix and the zeros of $< \Phi' >$ are $\tilde{N}'_c \times \tilde{N}'_c$, $\tilde{N}'_c \times (N'_c - N_c - \tilde{N}_c)$ and $(N'_c - N_c - \tilde{N}_c) \times \tilde{N}_c$ matrices. Then one can expand these fields around on a point (4.12), as in [4] and one arrives at the relevant superpotential up to quadratic order in the fluctuation. At one loop, the effective potential $V^{(1)}_{\text{eff}}$ for $\Phi'_0$ leads to the positive value for $m_{\Phi'_0}^2$ implying that these vacua are stable.

5 Conclusions and outlook

The meta-stable brane configurations we have found are summarized by Figures 2, 4, 5, 7, 9, 10, 12, 14, 15 and 16. If we replace the NS5'-brane in Figures 2B, 7B with opposite O4-plane charge, 14B with opposite O6-plane charge, and 16B with opposite O6-plane charge, with the coincident D6-branes, those brane configurations become nonsupersymmetric minimal energy brane configurations found in [14], in [16], in [14], and in [14], respectively.

So far, we have considered the cases for even number of NS-branes, i.e., four and six. For odd cases, i.e., three and five NS-branes, the construction of meta-stable brane configuration...
has been done in [9]. So it is natural to ask what happens if there are seven NS-branes. When this extra seventh NS-brane is located at the O6-plane in section 4, then the gauge group will be the same as the one in section 2, i.e., $SU(N_c) \times SU(N'_c) \times SU(N''_c)$ with different matter contents. This can be obtained also from the brane configuration of [23] by adding two outer NS-branes. It would be interesting to find out how the meta-stable brane configurations appear.

Some different directions on the meta-stable vacua are present in recent relevant works [24]-[33] where some of them are described in the type IIB string theory. It would be very interesting to find out how the meta-stable brane configurations from type IIA string theory including the present work are related to those brane configurations from type IIB string theory.

Acknowledgments

I would like to thank D. Kutasov for discussions. I would like to thank Kyungho Oh, who passed away from cancer, for ongoing collaboration and discussions during the last 10 years and, in memory of him, I would like to dedicate this work to him. This work was supported by grant No. R01-2006-000-10965-0 from the Basic Research Program of the Korea Science & Engineering Foundation.

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