Abstract

We consider the Dirac equation with two different four-potentials of the plane electromagnetic waves. We derive the partial differential equation for the wave function, which is generalized form of the Volkov equation. We find the solutions of the Dirac equation for two orthogonal plane waves. We determine the modified Compton formula for the scattering of two photons on an electron.
1 Introduction

The application of a laser as a source of intense electromagnetic radiation enables to study the new class of physical processes which are running in the intense field of the electromagnetic wave. The probability of some processes are increased, or, decreased in comparison with the processes in vacuum. For instance the probability of the process \( \pi \rightarrow \mu + \nu \) is increased and the probability of the process \( \pi \rightarrow e + \nu \) is decreased.

Some processes which are forbidden in vacuum, are allowed in the intense field of the electromagnetic wave \((\nu \rightarrow \pi + \mu, \text{or, } e \rightarrow \pi + \nu)\). The polarization of the electromagnetic wave plays important role. The situation with the two electromagnetic wave is the next step and the future direction of the laser physics of elementary particles.

We consider here the electron described by the the Dirac equation for two different four-potentials of the plane electromagnetic waves. We derive the partial differential equation for the wave function, which is generalized form of the Volkov equation. We find the solutions of the Dirac equation for two orthogonal plane waves. We determine the modified Compton formula for the scattering of two photons on an electron.

The solution of the Dirac equation for the two waves with the perpendicular polarization was given for instance by Lyulka \[1,2,3,4\], who described the decay of particles in the two laser fields. The derivation of the two-wave solution is not explicitly involved in Lyulka articles. So, we investigate the situation and present our results.

The laser field is the detector of the new properties of elementary particles which cannot be revealed without laser. The situation with the two independent lasers can give evidently further information on the properties of elementary particles. At the same time the laser and the system of two lasers can be considered in chemistry as a specific catalyzer which was not known before the existence of the laser physics. So, laser methods in particle physics and chemistry can form in the near future the new scientific revolution, which is not describe in the standard monographs on the scientific revolutions.

To be pedagogically clear, we remind in the next section the derivation of the Volkov \[5\] solution of the Dirac equation in vacuum.

2 Volkov solution of the Dirac equation with massless photons

We follow the method of derivation and metric convention of \[6\]:

\[
(\gamma(p - eA) - m)\psi = 0. \tag{1}
\]

where

\[
A^\mu = A^\mu(\varphi); \quad \varphi = k\cdot x. \tag{2}
\]

We suppose that the four-potential satisfies the Lorentz gauge condition

\[
\partial_\mu A^\mu = k_\mu (A^\mu)' = (k_\mu A^\mu)' = 0, \tag{3}
\]

where the prime denotes derivative with regard to \( \varphi \). From the last equation follows
\[ kA = \text{const} = 0, \] (4)

because we can put the constant to zero. The tensor of electromagnetic field is

\[ F_{\mu\nu} = k_{\mu}A'_{\nu} - k_{\nu}A'_{\mu}. \] (5)

Instead of the linear Dirac equation (1), we consider the quadratic equation, which we get by multiplication of the linear equation by operator \((\gamma(p - eA) + m)\) [6]. We get:

\[ \left[(p - eA)^2 - m^2 - \frac{ie}{2} F_{\mu\nu} \sigma^{\mu\nu}\right] \psi = 0. \] (6)

Using \(\partial \mu(A^\mu \psi) = A^\mu \partial \psi,\) which follows from eq. (3), and \(\partial_\mu \partial^\mu = \partial^2 = -p^2,\) with \(p_\mu = i(\partial/\partial x^\mu) = i\partial_\mu,\) we get the quadratic Dirac equation for the four potential of the plane wave:

\[ [-\partial^2 - 2ie(A\partial) + e^2A^2 - m^2 - ie(\gamma k)(\gamma A')]\psi = 0. \] (7)

We are looking for the solution of the last equation in the form:

\[ \psi = e^{-ipx} F(\varphi). \] (8)

After insertion of eq. (8) into eq. (7), we get with \((k^2 = 0)\)

\[ \partial^\mu F = k^\mu F', \quad \partial_\mu \partial^\mu F = k^2 F'' = 0, \] (9)

the following equation for \(F(\varphi)\)

\[ 2i(kp)F' + [-2e(pA) + e^2A^2 - m^2 - ie(\gamma k)(\gamma A')]F = 0. \] (10)

The integral of the last equation is of the form [6]:

\[ F = \exp \left\{ -i \int_0^{kx} \left[ \frac{n^2}{(kp)} - \frac{e^2}{2(kp)} A^2 \right] d\varphi + \frac{e(\gamma k)(\gamma A)}{2(kp)} \right\} \frac{u}{\sqrt{2p_0}}. \] (11)

where \(u/\sqrt{2p_0}\) is the arbitrary constant bispinor.

All powers of \((\gamma k)(\gamma A)\) above the first are equal to zero, since

\[ (\gamma k)(\gamma A)(\gamma k)(\gamma A) = -(\gamma k)(\gamma k)(\gamma A)(\gamma A) + 2(kA)(\gamma k)(\gamma A) = -k^2A^2 = 0. \] (12)

where we have used eq. (4) and relation \(k^2 = 0\). Then we can write:

\[ \exp \left\{ e \frac{(\gamma k)(\gamma A)}{2(kp)} \right\} = 1 + e \frac{(\gamma k)(\gamma A)}{2(kp)}. \] (13)

So, the solution is of the form:

\[ \psi_0 = R \frac{u}{\sqrt{2p_0}} e^{is} = \left[ 1 + \frac{e}{2kp}(\gamma k)(\gamma A) \right] \frac{u}{\sqrt{2p_0}} e^{is}, \] (14)

where \(u\) is an electron bispinor of the corresponding Dirac equation.
\[(\gamma p - m)u = 0 \quad (15)\]
and we shall take it to be normalized by condition \(\bar{u}u = 2m\). The mathematical object \(S\) is the classical Hamilton-Jacobi function, which was determined in the form:

\[S = -px - \int_{0}^{kx} \frac{e}{(kp)} \left[ (pA) - \frac{e}{2} A^2 \right] d\varphi. \quad (16)\]

The current density is

\[j^{\mu} = \bar{\psi}_{\mu} \gamma^{\mu} \psi_{\mu}, \quad (17)\]

where \(\Psi\) is defined as the transposition of (14), or,

\[\bar{\psi}_{\mu} = \frac{\bar{u}}{\sqrt{2p_0}} \left[ 1 + \frac{e}{2kp} (\gamma A)(\gamma k) \right] e^{-iS}. \quad (18)\]

After insertion of \(\Psi_{\mu}\) and \(\bar{\psi}_{\mu}\) into the current density, we have:

\[j^{\mu} = \frac{1}{p_0} \left\{ p^{\mu} - eA^{\mu} + k^{\mu} \left( \frac{e(pA)}{(kp)} - \frac{e^2 A^2}{2(kp)} \right) \right\}. \quad (19)\]

### 3 The solution of the Dirac equation for two plane waves.

We suppose that the total vector potential is given as a superposition of the potential \(A\) and \(B\) as follows:

\[V_{\mu} = A_{\mu}(\varphi) + B_{\mu}(\chi), \quad (20)\]

where \(\varphi = kx\) and \(\chi = \kappa x\) and \(k \neq \kappa\).

We suppose that the Lorentz condition is valid. Or,

\[\partial_{\mu} V^{\mu} = 0 = k_{\mu} \frac{\partial A^{\mu}}{\partial \varphi} + \kappa_{\mu} \frac{\partial B^{\mu}}{\partial \chi} = k_{\mu} A_{\varphi}^{\mu} + \kappa_{\mu} B_{\chi}^{\mu}, \quad (21)\]

where the subscripts \(\varphi, \chi\) denote partial derivatives. The equation (21) can be written in the more simple form if we notice that partial differentiation with respect to \(\varphi\) concerns only \(A\) and partial differentiation with respect to \(\chi\) concerns only \(B\). So we write instead eq. (21).

\[\partial_{\mu} V^{\mu} = 0 = k_{\mu}(A_{\mu})' + \kappa_{\mu}(B_{\mu})' = k A' + \kappa B'. \quad (22)\]

Without loss of generality, we can write instead of equation (22) the following one

\[k_{\mu}(A_{\mu})' = 0; \quad \kappa_{\mu}(B_{\mu})' = 0; \quad \text{or,} \quad k A = const = 0; \quad \kappa B = const = 0, \quad (23)\]

putting integrating constant to zero.

The electromagnetic tensor \(F_{\mu\nu}\) is expressed in the new variables as in (5).
\[ F_{\mu\nu} = k_{\mu} A'_{\nu} - k_{\nu} A'_{\mu} + \kappa_{\mu} B'_{\nu} - \kappa_{\nu} B'_{\mu}. \]  

(24)

Now, we can write Dirac equation for the two potentials the form

\[ [-\partial^2 - 2ie(V\partial) + e^2 V^2 - m^2 - \frac{i}{2} e F_{\mu\nu} \sigma^{\mu\nu}] \psi = 0. \]  

(25)

where \( V = A + B \), \( F_{\mu\nu} \) is given by eq. (24) and the combination of it with \( \sigma \) is defined as follows:

\[ \frac{i}{2} e F_{\mu\nu} \sigma^{\mu\nu} = ie(\gamma k)(\gamma A') + ie(\gamma \kappa)(\gamma B') \]  

(26)

We will look for the solution in the standard Volkov form (8), or:

\[ \psi = e^{-ipx} F(\varphi, \chi). \]  

(27)

After performing all operations prescribed in eq. (25), we get the following partial differential equation for the unknown function \( F(\varphi, \chi) \):

\[ -2k\kappa F_{\varphi\chi} + (2ipk - 2ikB)F_{\varphi} + (2ip\kappa - 2ie\kappa A)F_{\chi} + (e^2(A + B)^2 - 2e(A + B)p - ie(\gamma k)(\gamma A_{\varphi}) - ie(\gamma \kappa)(\gamma B_{\chi}))F = 0. \]  

(28)

### 4 The solution of the Dirac equation for two orthogonal waves

The equation (28) is more simple if \( k\kappa = 0 \). Then, we write eq. (28) in the following form:

\[ aF_{\varphi} + bF_{\chi} + cF = 0, \]  

(29)

where

\[ a = 2ipk - 2iekB; \quad b = 2ip\kappa - 2ie\kappa A \]  

(30)

and

\[ c = e^2(A + B)^2 - 2e(A + B)p - ie(\gamma k)(\gamma A') - ie(\gamma \kappa)(\gamma B'). \]  

(31)

and the term of with the two partial derivations is not present because of \( k\kappa = 0 \).

For the field which we specify by the conditions

\[ kB = 0; \quad \kappa A = 0; \quad AB = 0, \]  

(32)

we have:

\[ 2ipk F_{\varphi} + 2ipk\kappa F_{\chi} + (e^2 A^2 + e^2 B^2 - 2epA - 2epB - ie(\gamma k)(\gamma A') - ie(\gamma \kappa)(\gamma B'))F = 0. \]  

(33)
Now, we are looking for the solution in the most simple form

\[ F(\varphi, \chi) = X(\varphi)Y(\chi). \] (34)

After insertion of (34) into (33) and division the new equation by \( XY \) we get the terms depending only on \( \varphi \), and on \( \chi \). Or, in other words we get:

\[
\left( 2ipk \frac{X'}{X} + e^2 A^2 - 2epA - ie(\gamma k)(\gamma A') \right) + \\
\left( 2ipk \frac{Y'}{Y} + e^2 B^2 - 2epB - ie(\gamma \kappa)(\gamma B') \right) = 0
\] (35)

So, there are terms dependent on \( \varphi \) and terms dependent on \( \chi \) only in eq. (35). The only possibility is that they are equal to some constant \( \lambda \) and \(-\lambda\). Then,

\[ 2ipkX' + (e^2 A^2 - 2epA - ie(\gamma k)(\gamma A'))X = \lambda X \] (36)
and

\[ 2ipkY' + (e^2 B^2 - 2epB - ie(\gamma \kappa)(\gamma B'))Y = -\lambda Y \] (37)

We put \( \lambda = 0 \) without lost of generality. Now, the solution of eq. (35) is reduced to the solution of two equations only. Because the form of the equations is similar to the form of eq. (14) we can write the solution of these equations as follows:

\[ X = \left[ 1 + \frac{e}{2(kp)}(\gamma k)(\gamma A) \right] \frac{u}{\sqrt{2p_0}} e^{iS_1}, \] (38)

with

\[ S_1 = \int_0^{kx} \frac{e}{(kp)} \left[ (pA) - \frac{e}{2} (A)^2 \right] d\varphi. \] (39)

and

\[ Y = \left[ 1 + \frac{e}{2(kp)}(\gamma \kappa)(\gamma B) \right] \frac{u}{\sqrt{2p_0}} e^{iS_2}, \] (40)

with

\[ S_2 = -\int_0^{\kappa x} \frac{e}{(kp)} \left[ (pB) - \frac{e}{2} (B)^2 \right] d\chi. \] (41)

The total solution is then of the form:

\[ \psi_p = \left[ 1 + \frac{e}{2(kp)}(\gamma k)(\gamma A) \right] \left[ 1 + \frac{e}{2(kp)}(\gamma \kappa)(\gamma B) \right] \frac{u}{\sqrt{2p_0}} e^{i(S_1(A) + S_2(B))}. \] (42)
5 The standard Compton process

In order to find the wave function of the electron in the two laser beams, let us first remind the well known Compton problem following from the Volkov solution. The pioneering articles of this problem was written by Goldman [7] and plenty of problems concerning the application of the Volkov solution can be seen in the Ritus article [8].

Let us consider electromagnetic monochromatic plane wave which is polarized in a circle. We write the four-potential in the form:

\[ A = a_1 \cos \varphi + a_2 \sin \varphi, \] (43)

where the amplitudes \( a_i \) are equal in magnitude and orthogonal, or,

\[ a_1^2 = a_2^2 = a^2, \quad a_1 a_2 = 0. \] (44)

Then, it possible to show that the Volkov solution for this situation is of the form [6]:

\[ \psi_p = \left\{ 1 + \left( \frac{e}{2(kp)} \right) [(\gamma k)(\gamma a_1) \cos \varphi + (\gamma k)(\gamma a_2) \sin \varphi] \right\} \frac{u(p)}{\sqrt{2q_0}} \times \]
\[ \exp \left\{ -ie \frac{(a_1 p)}{(kp)} \sin \varphi + ie \frac{(a_2 p)}{(kp)} \cos \varphi - iq \right\}, \] (45)

where

\[ q^\mu = p^\mu - e^2 \frac{a^2}{2(kp)} k^\mu \] (46)

follows from eq. (19) as a time-average value. In other words, \( q^\mu \) is the mean value of quantity \( p^\mu j^\mu \).

We know that the matrix element \( M \) corresponding to the emission of photon by an electron in the electromagnetic field is as follows [6]:

\[ S_{fi} = -ie^2 \int d^4 x \bar{\psi}_p (\gamma e^\ast) \psi_p \frac{e^{ik'x}}{\sqrt{2\omega'}} , \] (47)

where \( \psi_p \) is the wave function of an electron before interaction with the laser photons and \( \psi_p' \) is the wave function of electron after emission of photon with components \( k'^\mu = (\omega', k') \). The quantity \( e^\ast \) is the polarization four-vector of emitted photon.

The matrix element (47) involves the following linear combinations:

\[ e^{-i\alpha_1 \sin \varphi + i\alpha_2 \cos \varphi} \] (48)
\[ e^{-i\alpha_1 \sin \varphi + i\alpha_2 \cos \varphi} \cos \varphi \] (49)
\[ e^{-i\alpha_1 \sin \varphi + i\alpha_2 \cos \varphi} \sin \varphi, \] (50)

where
\[ \alpha_1 = e \left( \frac{a_1 p}{kp} - \frac{a_1 p'}{kp'} \right), \tag{51} \]

and

\[ \alpha_2 = e \left( \frac{a_2 p}{kp} - \frac{a_2 p'}{kp'} \right), \tag{52} \]

Now, we can expand exponential function in the Fourier series, where the coefficients of the expansion will be \( B_s, B_{1s}, B_{2s} \). So we write:

\[ e^{-i\alpha_1 \sin \varphi + i\alpha_2 \cos \varphi} = e^{-i\sqrt{\alpha_1^2 + \alpha_2^2} \sin(\varphi - \varphi_0)} = \sum_{s=-\infty}^{\infty} B_s e^{-is\varphi} \tag{53} \]

\[ e^{-i\alpha_1 \sin \varphi + i\alpha_2 \cos \varphi} \cos \varphi = e^{-i\sqrt{\alpha_1^2 + \alpha_2^2} \sin(\varphi - \varphi_0)} \cos \varphi = \sum_{s=-\infty}^{\infty} B_{1s} e^{-is\varphi} \tag{54} \]

\[ e^{-i\alpha_1 \sin \varphi + i\alpha_2 \cos \varphi} \sin \varphi = e^{-i\sqrt{\alpha_1^2 + \alpha_2^2} \sin(\varphi - \varphi_0)} \sin \varphi = \sum_{s=-\infty}^{\infty} B_s e^{-is\varphi} \tag{55} \]

The Coefficients \( B_s, B_{1s}, B_{2s} \) can be expressed by means of the Bessel function as follows \([6]\):

\[ B_s = J_s(z)e^{is\varphi_0} \tag{56} \]

\[ B_{1s} = \frac{1}{2} \left[ J_{s+1}(z)e^{i(s+1)\varphi_0} + J_{s-1}(z)e^{i(s-1)\varphi_0} \right] \tag{57} \]

\[ B_{2s} = \frac{1}{2i} \left[ J_{s+1}(z)e^{i(s+1)\varphi_0} - J_{s-1}(z)e^{i(s-1)\varphi_0} \right], \tag{58} \]

where the quantity \( z \) are now defined through the \( \alpha \)-components, or,

\[ z = \sqrt{\alpha_1^2 + \alpha_2^2} \tag{59} \]

and

\[ \cos \varphi_0 = \frac{\alpha_1}{z}; \quad \sin \varphi_0 = \frac{\alpha_2}{z}. \tag{60} \]

Functions \( B_s, B_{1s}, B_{2s} \) are related one to another as follows:

\[ \alpha_1 B_{1s} + \alpha_2 B_{2s} = sB_s, \tag{61} \]

which follows from the well known relation for Bessel functions:

\[ J_{s-1}(z) + J_{s+1}(z) = \frac{2s}{z} J_s(z). \tag{62} \]

The matrix element (47) can be written in the form \([6]\):

\[ S_{fi} = \frac{1}{\sqrt{2\omega'q_0q_0'}} \sum_s M^{(s)}_{fi} (2\pi)^4 i\delta^{(4)}(sk + q - q' - k'), \tag{63} \]
where the $\delta$-function involves the law of conservation:

$$sk + q = q' + k'; \quad s = 1, 2, 3, \ldots$$  \hspace{1cm} (64)

with the relation

$$q^2 = q'^2 = m^2(1 + \xi^2) \equiv m_*^2 = m^2 \left(1 - \frac{e^2 a^2}{m^2}\right),$$  \hspace{1cm} (65)

as it follows from eq. (46).

For $s = 1$, eq. (64) has the physical meaning of the conservation of energy-momentum of the one-photon Compton process, $s = 2$ has meaning of the two-photon Compton process and $s = n$ has meaning of the multiphoton interaction with $n$ photons.

It is possible to show, that the differential probability per unit volume and unit time of the emission of the $s$ harmonics is of the following form [6]:

$$dW_s = |M^{(s)}_{fi}|^2 \frac{d^3 k' d^3 q'}{(2\pi)^6 2\omega' 2q_0 2q_0} (2\pi)^4 \delta(4)(sk + q' - k').$$  \hspace{1cm} (66)

In order to obtain the probability of emission of photon, we must make some operation with the matrix element $M$. We will here not perform these operations. We concentrate our attention on the conservation law in the formula (66). It can be expressed by words as follows. The multiphoton object with the momentum $sk$ interacts with the electron of the momentum $q$, and the result is the electron with the momentum $q'$ and one photon with the momentum $k'$.

Now, let us consider the equation (64) in the form

$$sk + q - k' = q'.$$  \hspace{1cm} (67)

If we introduce the angle $\Theta$ between $k$ and $k'$, then, with $|k| = \omega$ and $|k'| = \omega'$, we get from the squared equation (67) in the rest system of electron, where $q = (m_*, 0)$, the following equation:

$$s \frac{1}{\omega'} - \frac{1}{\omega} = \frac{s}{m_*} (1 - \cos \Theta),$$  \hspace{1cm} (68)

which is modification of the original equation for the Compton process

$$\frac{1}{\omega'} - \frac{1}{\omega} = \frac{1}{m} (1 - \cos \Theta).$$  \hspace{1cm} (69)

So, we see that Compton effect described by the Volkov solution of the Dirac equation differs from the original Compton formula only by the existence of the renormalized mass and parameter $s$ of the multiphoton interaction.

We know that the last formula of the original Compton effect can be written in the form suitable for the experimental verification, namely:

$$\lambda' - \lambda = \Delta \lambda = 4\pi \frac{\hbar}{mc} \sin^2 \frac{\Theta}{2},$$  \hspace{1cm} (70)

which was used by Compton for the verification of the quantum nature of light.
6 The two-photon Compton process

In case of the two laser beams which are not collinear the experimental situation involves possibility that the two different photons can interact with one electron. The theory does not follow from the standard one-photon Volkov solution because in the standard approach the multiphoton interaction involve the collinear photons and not photons from the two different lasers. The problem was solved by Lyulka in 1974 for the case of the two linearly polarized waves [1]

\[ A = a_1 \cos \varphi; \quad B = a_2 \cos(\chi + \delta) \]  

with the standard conditions for \( \varphi, \chi, k, \kappa \). The quantity \( \delta \) is the phase shift.

The two-wave Volkov solution is given by eq. (42) and the matrix elements and appropriate ingredients of calculations are given by the standard approach as it was shown by Lyulka [1].

It was shown [1], that

\[ q^\mu = p^\mu - e^2 a_1^2 k^\mu - e^2 a_2^2 \kappa^\mu \]  

and

\[ m^2_* = m^2 \left( 1 - \frac{e^2 a_1^2}{m^2} - \frac{e^2 a_2^2}{m^2} \right). \]  

The matrix element involves the extended law of conservation. Namely:

\[ sk + t\kappa + q = q' + k' + \kappa', \]  

where \( s \) and \( t \) are natural numbers and the interpretation of the last equation is evident. The multiphoton objects with momenta \( sk \) and \( t\kappa \) interact with electron with momentum \( q \). After interaction the electron has a momentum \( q' \) and two photons are emitted with momenta \( k' \) and \( \kappa' \).

Instead of equation (74), we can write

\[ sk + q - k' = q' + \kappa' - t\kappa. \]  

From the squared form of the last equation and after some modification, we get the following generalized equation of the double Compton process for \( s = t = 1 \):

\[ \frac{1}{\omega'} - \frac{1}{\omega} = \frac{1}{m_*} (1 - \cos \Theta) + \frac{\Omega' - \Omega}{\omega \omega'} - \frac{\Omega \Omega'}{\omega \omega' m_*} (1 - \cos \Xi), \]  

where the angle \( \Xi \) is the angle between the 3-momentum of the \( \kappa \)-photon and the 3-momentum of the \( \kappa' \)-photon with frequency \( \Omega \) and \( \Omega' \) respectively.

Let us remark that if the frequencies of the photons of the first wave substantially differs from the frequency of photons of the second electromagnetic wave, then, the derived formula (76) can be experimentally verified by the same way as the original Compton formula. To our knowledge, formula (76) is not involved in the standard textbooks on quantum electrodynamics because the two laser physics is at present time not sufficiently developed.
7 Discussion

We have discussed the problem of the Dirac equation with the two-wave potentials of the electromagnetic fields. While the Volkov solution for one potential is well known for long time, the case with the two waves represents the new problem. To our knowledge, the Compton process with two beams was not investigated experimentally by any laboratory.

This article is in a some sense author’s continuation of the problems where the Compton, or Volkov solution plays substantial role [10, 11, 12, 13].

It is possible to consider the situation with sum of N waves, or,

\[ V = \sum_{i=1}^{N} A_i(\varphi_i) \quad \varphi_i = k_i x. \]  

(77)

The problem has obviously physical meaning because the problem of the laser compression of target by many beams is one of the prestigious problems of the today laser physics. The solution of that problem in the general form is not elementary and can be solved only by some laser institution such as the Lebedev institution of physics, the Livermore laser national laboratory and so on.

References

[1] V. A. Lyulka, ZhETF 67 Vol. 5(11) (1974) 1639.
[2] V. A. Lyulka, ZhETF 69 Vol. 3(9) (1975) 800.
[3] V. A. Lyulka, ZhETF 72 Vol. 3 (1977) 865.
[4] V. A. Lyulka, Journal of Nuclear Physics Vol. 5(11) (1985) 1211.
[5] D. M. Volkov, Zeitschrift für Physik 94 (1935) 250.
[6] V. B. Berestetzkii, E. M. Lifshitz and L. P. Pitaevskii, Quantum Electrodynamics, Moscow, Nauka, (1989). (in Russian).
[7] I. I. Goldman, ZhETF 46 (1964) 1412; ibid., Phys. Lett. 8 (1964) 103.
[8] V. I. Ritus, Trudy FIAN 111 5 (1979).
[9] N. B. Delone and V. P. Krainov, Multiphoton Processes in Atoms, 2nd ed., Springer-Verlag, Berlin, Heidelberg, New York, (2000).
[10] M. Pardy, Physics Letters A 243 (1998) 223.
[11] M. Pardy, Radiation Physics and Chemistry 61 (2001) 321.
[12] M. Pardy, International Journal of Theoretical Physics 42(1) (2003) 99.
[13] M. Pardy, International Journal of Theoretical Physics 43(1) (2004) 127.