Adjusting Beliefs via Transformed Fuzzy Priors

T Rattanadamrongaksorn¹, D Sirikanchanarak¹,³, J Sirisrisakulchai² and S Sriboonchitta²

¹Faculty of Economics, Chiang Mai University, Chiang Mai 50200, Thailand
²Puey Ungphakorn Center of Excellence in Econometrics, Faculty of Economics, Chiang Mai University, Chiang Mai 50200, Thailand
³Bank of Thailand, Bangkok 10200, Thailand

tanarat_ra@cmu.ac.th, http://orcid.org/0000-0002-4401-0011

Abstract. Instead of leaving a decision to a pure data-driven system, intervention and collaboration by human would be preferred to fill the gap that machine cannot perform well. In financial applications, for instance, the inference and prediction during structural changes by critical factors; such as market conditions, administrative styles, political policies, etc.; have significant influences to investment strategies. With the conditions differing from the past, we believe that the decision should not be made by only the historical data but also with human estimation. In this study, the updating process by data fusion between expert opinions and statistical observations is thus proposed. The expert’s linguistic terms can be translated into mathematical expressions by the predefined fuzzy numbers and utilized as the initial knowledge for Bayesian statistical framework via the possibility-to-probability transformation. The artificial samples on five scenarios were tested in the univariate problem to demonstrate the methodology. The results showed the shifts and variations appeared on the parameters of the distributions and, as a consequence, adjust the degrees of belief accordingly.

1. Introduction

There exist the situations that changes in important factors such as technological constraints, individual preferences, market conditions, etc. occur profoundly and permanently which are called structural changes and cause the shifts of the properties of data. Thus, the inference and prediction by the existing parameters are unreliable. Fortunately, the estimation based solely on human intuition could be made by the so-called expert opinion. For example in, but not limited to, the financial application, as soon as the new tax law is enacted, the exporter raise their price even before they really pay for the new cost. As a matter of fact, the calculation in backgrounds such as labor wages, material prices and other related costs are complex and time-consuming but the expert can neglect those details and pursue with their approximation coming out of the thin air. Most of the time, surprisingly, their expert opinions are reasonably correct. We therefore propose the incorporation of the existing historical observations with this subjective data which are handled well by fuzzy set theory [1].

The fuzzy set theory is the generalization of set theory to represent the non-random uncertainty. Fuzzy set is characterized by a set of pairs—the elements and their possibilities, \([\theta, \pi(\theta)]\). The element, \(\theta\), is a considered parameter while its pair is the possibility of occurring in the specific value. The concept is efficient in dealing with the linguistic expressions by fuzzy number which is the representation of quantity that cannot be described precisely due to the incomplete knowledge.
2. Representing linguistic terms by fuzzy priors

Rather than specifying the knowledge about data by probability distributions, it is much easier and more practical for the person in the field to convey the messages about something verbally such as “at least”, “most positive”, “exactly”, etc. In this study, we therefore translate the linguistic terms into mathematical expressions by fuzzy numbers. Fuzzy number is the mathematical expression of the quantity or parameter that takes the form of \( N = (a, b, c, d) \) whose possibilities \((0,1,1,0)\) are usually omitted from writing. The translation of the linguistic terms into fuzzy representations can be achieved by defining the fuzzy numbers that has been adapted from and introduced in the previous works [2,3]. Currently, our collection of fuzzy representations grows from the original work to the list in Figure 1. Because later we integrate these representations into Bayesian framework as priors, we may call them as fuzzy prior.

The examples of fuzzy priors for our experiment are 1) No distribution, 2) Uniform, 3) Mean-Variance, 4) Interval, and 5) Left-skewed and the parameters for generating statistical data in normal model are shown in Table 1.

| Name            | Largest possible | Most possible | \( \theta \) centroid\(^a\) | \( \pi \) centroid\(^a\) | Mean | SD | Size |
|-----------------|------------------|---------------|-----------------------------|-----------------------------|------|----|------|
| No distribution | 50               | 50            | 50                          | 1.00                        | 0    | 5  | 100  |
| Uniform         | [20,80]          | [20,80]       | 50                          | 1.00                        | 0    | 5  | 100  |
| Mean-Variance   | [20,80]          | 50            | 50                          | 0.50                        | 0    | 5  | 100  |
| Interval        | [20,80]          | [40,60]       | 50                          | 0.33                        | 0    | 5  | 100  |
| Left-skewed     | [20,80]          | 60            | 53                          | 0.50                        | 0    | 5  | 100  |

\(^a\) \( \theta \) centroid, \( \bar{\theta} = \int \theta \, \pi(\theta) \, d\theta / \int \pi(\theta) \, d\pi \), and \( \pi \) centroid, \( \bar{\pi} = \int \theta \, \pi(\theta) \, d\pi / \int \pi(\theta) \, d\pi \), are the expectations of fuzzy numbers on the corresponding axes but the latter is not applicable in this work.

There are, however, the incompatibilities between the usages of expert opinions and historical observations because they are based on the different theories and, on the other hand, utilizing only either of them is loss of opportunity. Taking a closer look into the real-life event, we often find that even the data are structural changed but their behavior may remain almost unchanged. For instance, the expected price of a stock has been elevated due to the new appointment of its administrative team but the variation is not altered because of unchanged market conditions. This implies that while there are needs for the new evaluation of data, the existing knowledge should not be neglected. The better alternative is to combine both data via, we propose, the possibility-to-probability transformation.
a) No Distribution

Only possible at $\theta_1$

b) Uniform

Equally possible between $\theta_2$ and $\theta_3$

c) Mean-Variance

At least at $\theta_4$, At most at $\theta_6$, and Most possible at $\theta_5$

d) Intervals

Most possible between $\theta_8$ and $\theta_9$, Largest possible between $\theta_7$ and $\theta_{10}$

e) Left-skewed

Increasingly possible from $\theta_{11}$ to $\theta_{12}$

f) Right-skewed

Decreasingly possible from $\theta_{13}$ to $\theta_{14}$

g) Negatively half-distributed

Not possible less than $\theta_{15}$, Most possible more than $\theta_{16}$, But less than 0

h) Positively half-distributed

Not possible more than $\theta_{18}$, Most possible less than $\theta_{17}$, But more than 0

Figure 1. Fuzzy representations
3. Transforming expert opinions to probability distributions
In this study, we consider the transformation exclusively from the possibility to probability due to the loss of information from the inverse operation. In order to obtain the probability distribution from fuzzy number, the transformation is activated for M cycles by: 1) randomly select the possibility from the uniform distribution which is called alpha-cut fuzzy number at the selected possibility to obtain a new fuzzy number, $A_{\alpha}$, 2) randomly choose the possibility, $\pi_{\alpha}$, from the first step, 3) sort the results in descending order, 4) repeat the above steps for $M$ times, and 5) calculate the transformed probabilities by [4]:

$$P_i(x) = \sum_{j=i}^{M-1} \frac{\pi_j(x) - \pi_{j+1}(x)}{j},$$

where $\max(\pi_{\alpha}) > \pi_1 > \pi_2 > ... \pi_j > ... > \pi_{M-1} > \pi_M = \min(\pi_{\alpha})$. The possibilities are transformed by accumulating the differences between the consecutive possibilities and divided by its decreasing order of transformation, $j$, from the current (order $i$) to the one before the last order, $M - 1$. As a result, the distribution shapes of the transformed probabilities will gradually increase from the start on both sides with acceleration and reach its peak at the highest probability.

The results of transformations are depicted in the left column of Figure 2. The transformed probabilities in the case of no distribution and uniform are unchanged due to the fact that the estimations are definitely possible which are normally the rare cases, however. The remaining cases show the changes on the side of the distributions which are lower in transformed probabilities than those in possibilities but the shapes still follow their fuzzy priors. On the sides of fuzzy numbers, the concave appeared and noticeably more curved in the triangular fuzzy numbers (Mean-Variance and Left-skewed) than those in the trapezoid (Interval) because more possibilities can be identified thus induced more probabilities.

4. Updating beliefs by Bayes’ theorem
Bayesian approach is a statistical method that assumes randomness in parameter. It is attractive in its updating mechanism through the Bayes’ theorem:

$$P(\theta|X) = \frac{P(\theta)P(X|\theta)}{P(X)},$$

where $\theta$ and $X$ represent parameters and data respectively. The equation indicates that the conditional probability of parameter given data or Posterior, $P(\theta|X)$ is proportional to the product of the initial belief in the probability of interested parameters or Prior, $P(\theta)$ and the joint probability of data or Likelihood, $P(X|\theta)$ and normalized by the Marginal Likelihood, $P(X)$—which is usually ignored in computation. The formula can be interpreted that as the evidence is available, the prior is updated by or combined with the probability of data that yields in the probability of posterior.

The results of our experiments shown in Figure 2 and Table 2 were obtained by the Markov-Chain Monte-Carlo (MCMC) simulation. The posteriors were developed from the mixtures of likelihoods and priors. The means of the distributions (50.04-59.75) stand approximately on the centroids (50-53) of the fuzzy priors. The shapes of distributions were developed from the original shapes of the priors but deteriorated on the sides close to the data. The ranges of distributions are close to the ranges of fuzzy priors that the minimums (20.21-25.25) and maximums (76.10-79.89) are around the lower and upper bounds of fuzzy priors (20 and 80 respectively). The exception is on the case of no-distribution that has a little of variation causing merely by the variance of randomness, $u \sim Unif(0,1)$, in the sampling algorithm. Both illustrations and numerical results confirm the significant impacts from the expert opinions by the differences from those by the MLE obviously seen in the figure and table, even though the numerical results are not included.
Figure 2. Left – Priors’ Fuzzy (circle or solid) numbers and Transformed (left, circle or dotted) probability distribution. Right – Results from Maximum Likelihood Estimation (dashed) vs Transformed (circle or dotted) and Posterior (solid) probability distributions by the proposed method.
Table 2. Statistics of the estimates by the MLE and the posteriors by the proposed method.

| Method               | Mean   | SD    | Min.  | Max.  | Median | Mode  | C.I. / C.R. | Size |
|----------------------|--------|-------|-------|-------|--------|-------|-------------|------|
| MLE                  | -0.57  | 4.96  | -14.46| 13.34 | 0.08   | 0.59  | [-10.03,9.10] | 100  |
| Proposed method      |        |       |       |       |        |       |             |      |
| No distribution      | 50.04  | 1.00  | 45.64 | 53.62 | 50.04  | 50.03 | [48.08,51.96] | 10000|
| Uniform              | 59.75  | 14.64 | 20.21 | 79.89 | 62.58  | 75.02 | [26.82,79.07] | 10000|
| Mean-Variance        | 53.98  | 10.20 | 23.30 | 76.10 | 53.58  | 50.63 | [33.05,73.11] | 10000|
| Interval             | 54.94  | 11.40 | 22.74 | 78.03 | 56.08  | 58.76 | [31.66,74.11] | 10000|
| Right-skewed         | 58.25  | 9.45  | 25.25 | 77.30 | 59.41  | 60.06 | [36.30,74.14] | 10000|

* C.I. and C.R. are abbreviations for Confidence Interval (for MLE) and Credible Interval (for the proposed method) respectively.

5. Conclusions

In the situation that the statistical analysis is insufficient, we propose the fusion of the expert opinions and historical data through the Bayes’ updating mechanism with the possibility-to-probability transformation. The proposed methodology includes 1) Representing linguistic terms by fuzzy priors, 2) Representing linguistic terms by fuzzy priors, and 3) Updating beliefs by Bayes’ theorem.

With the artificial samples, we were able to initiate the various cases of the initial knowledge. The benefit was that the results from the different formulations could easily be traced and understood. The posterior probabilities from the experiments appeared to be compromised between the prior knowledge and the past observations. The outcomes from our experiments were biased by the priors but varied by the likelihoods. In simpler terms, the positions of the results were indicated by the expert opinions but their shapes were influenced by the distributions of the existing data.

This research contributes to the acquisition and utilization of subjective data within Bayesian framework. The method is more natural and easier than the traditional implementations by the expert opinions that can be translated from linguistic terms into fuzzy numbers which are intuitive and flexible to users. However, it also pays the price in computation time and its accuracy depends very much on the quality of expert opinions. The method is so general that can be adapted to the applications in various disciplines that require the intervention of and adjustment by human. The concurrent [5] and future research include the more practical real-life situations like multivariate inference and optimization which involve much complex and require more sophisticated techniques.

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