On a New Type of Information Processing for Efficient Management of Complex Systems

Victor Korotkikh and Galina Korotkikh
School of Computing Sciences
Central Queensland University
Mackay, Queensland, 4740
Australia

It is a challenge to manage complex systems efficiently without confronting NP-hard problems. To address the situation we suggest to use self-organization processes of prime integer relations for information processing. Self-organization processes of prime integer relations define correlation structures of a complex system and can be equivalently represented by transformations of two-dimensional geometrical patterns determining the dynamics of the system and revealing its structural complexity. Computational experiments raise the possibility of an optimality condition of complex systems presenting the structural complexity of a system as a key to its optimization. From this perspective the optimization of a system could be all about the control of the structural complexity of the system to make it consistent with the structural complexity of the problem. The experiments also indicate that the performance of a complex system may behave as a concave function of the structural complexity. Therefore, once the structural complexity could be controlled as a single entity, the optimization of a complex system would be potentially reduced to a one-dimensional concave optimization irrespective of the number of variables involved its description. This might open a way to a new type of information processing for efficient management of complex systems.

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I. INTRODUCTION

It is a challenge to manage complex systems efficiently without confronting NP-hard problems. To address the situation we consider the description of complex systems in terms of self-organization processes of prime integer relations and suggest to use the processes for information processing.

II. THE HIERARCHICAL NETWORK OF PRIME INTEGER RELATIONS

The description is realized through the unity of two equivalent forms, i.e., arithmetical and geometrical. We briefly present the forms in order to introduce the hierarchical network of prime integer relations. More details may be found in and suggest to use the processes for information processing.

A. THE ARITHMETICAL FORM

In the arithmetical form a complex system is characterized by hierarchical correlation structures built in accordance with self-organization processes of prime integer relations. As each of the correlation structures is ready to exercise its own scenario and there is no mechanism specifying which of them is going to take place, an intrinsic uncertainty about the complex system exists. At the same time, the information about the correlation structures can be used to evaluate the probability of an observable to take each of the measurement outcomes. Therefore, the arithmetical form of the description provides the statistical information about a complex system.

The form reveals nonlocal correlations without reference to signals as well as the distances and local times of the parts. Thus, the arithmetical form suggests that parts of a complex systems may be far apart in space and time and yet remain interconnected with instantaneous effect on each other, but no signalling. Namely, if a correlation structure of a system is selected and some parts are specified, then through the prime integer relations in control of the system cause the other parts to change accordingly, an event takes place. Once the changes have been realized, the event is fixed in space and time with respect to the reference frames of the parts. For the parts the effect of the event has not necessarily be the same, but for each part it is ap-
appropriately determined by the prime integer relations. However, the prime integer relations at work for a system have no causal power to effect systems controlled by separate prime integer relations. As a result, information about the systems is blocked for the observers of the system.

B. THE GEOMETRICAL FORM

Specified by two parameters $\varepsilon > 0$ and $\delta > 0$ the geometrical form arises as the self-organization processes of prime integer relations find isomorphic realization in terms of transformations of two-dimensional geometrical patterns [1]. As a result, hierarchical structures of prime integer relations defining the correlation structures of a complex system become equivalently represented by hierarchical structures of geometrical patterns determining the dynamics of the system and revealing its complexity.

The quantitative description of the system turns out to be given by the description of the geometrical patterns [1, 2].

Figure 2 shows a hierarchical structure of geometrical patterns, which for given $\varepsilon$ and $\delta$ is isomorphic to the hierarchical structure of prime integer relations depicted in Figure 1. A scale invariant property and a renormalization group transformation come to our attention, as we consider the connection between a geometrical pattern and the geometrical patterns it is made of.

Although the geometrical patterns are not triangles at level $l$, $l = 2, 3, 4$, yet the boundary curve, as the graph of the function $\Psi_1^{[l]}$, is such that the area $S_l$ of a geometrical pattern can be simply given, as if it were a triangle, by

$$S_l = \frac{W_l H_l}{2},$$

where $W_l$ and $H_l$ are its width and height.

The renormalization group transformation at level 4 defines

$$\varepsilon' = 2^4 \varepsilon, \quad \delta' = 2^3 \delta$$
and uses a coarse-grained procedure replacing the geometrical pattern made up of 8 geometrical patterns at level 1 by their enlarged version with the boundary curve as the graph of the function $\Psi_2^{[1]}$. Each of the geometrical patterns at level 1 is elementary in the sense that it is fully specified by a prime integer relation made up of two integers without further internal structure. The width $2\varepsilon$ and the height $\delta_2$ of the renormalized geometrical pattern at level 4 are given in terms of $\varepsilon'$ and $\delta'$ in the same way as the width $2\varepsilon$ and the height $\delta_{[1]}$ of a geometrical pattern at level 1 are given in terms of $\varepsilon$ and $\delta$. The two geometrical patterns at level 4 have the same width, height and area 

$$\int_{t_0}^{t_{16}} \Psi_1^{[4]}(t)dt = \int_{t_0}^{t_{16}} \Psi_2^{[1]}(t)dt,$$

but the lengths of their boundary curves are different (Figure 2).

The arithmetical and geometrical forms unite the dynamics and the structure of a complex system as two sides of the same entity in the preservation of the system as a whole. In particular, at one side the dynamics of the parts is determined to produce spacetime patterns of the parts to fit precisely into the geometrical patterns of the complex system. Under this condition at the side the corresponding prime integer relations can provide the relationships between the parts for the complex system to exist. If the spacetime patterns do not fit even slightly, then one or more of the relationships are not in place and the complex system collapses.

To measure the complexity of a system in terms of self-organization processes of prime integer relations a concept of structural complexity is introduced [1]. Starting with the integers at the zero level, the self-organization processes of prime integer relations progress to different levels and thus produce a hierarchical complexity order. In particular, the higher the level the self-organization processes progress to, the greater is the structural complexity of a corresponding system. In our description systems can be compared in terms of complexity by using two equivalent forms: in structure - by the hierarchical structures of prime integer relations and in dynamics - by the hierarchical structures of geometrical patterns.

Remarkably, based on integers and controlled by arithmetical only self-organization processes of prime integer relations can describe complex systems by information not requiring further simplification.

### III. TESTING A NEW MEDIUM FOR INFORMATION PROCESSING

The correlation structures of a complex system contain information about the parts. By changing some parts the information can be processed as the other parts change in accordance with the correlation structures. This shows the importance of self-organization processes of prime integer relations for information processing. Namely, for a given problem they could be used to build the correlation structures of a system in processing information demonstrating the optimal performance for the problem.

As a result, we suggest the hierarchical network of prime integer relations as a new medium for information processing and are interested in its navigating properties. It would be important if for any problem the performance of a system could behave as a concave function of its structural complexity. Guided by this property the performance global maximum for a problem could be efficiently found. It would be also beneficial if at the global maximum the structural complexities of the system and the problem could be related through an optimality condition.

The optimality condition might be interpreted as follows: if through arithmetical interdependencies emerging in the hierarchical network between a computing system and a problem a new building block is formed at the highest possible level, then the optimal performance takes place. Or, in other terms, a computing system finds the solution to a problem, once arithmetical interdependencies emerging between the system and the problem provide a channel to obtain the desired information.

It is worth to note that since the correlation structures of a system are completely determined by prime integer relations, which are equivalent to two-dimensional geometrical patterns, the entropy of the system, measuring its information content, can be connected with the areas of the two-dimensional patterns. Thus, in our approach there is a general connection between entropy and area.

Computational experiments have been conducted to test the navigating properties. In particular, an optimization algorithm $\mathcal{A}$, as a complex system, of $N$ computational agents minimizing the average distance in the travelling salesman problem (TSP) is developed. The agents work in parallel and start in the same city by choosing the next city at random. Then an agent at each step visits the next city by using one of the two strategies: a random strategy or the greedy strategy.

In the solution of a problem with $n$ cities the state of the agents at step $j, j = 1, ..., n-1$ can be described by a binary sequence $s_j = s_{j1}...s_{jN}$, where $s_{ij} = +1$, if agent $i, i = 1, ..., N$ uses the random strategy and $s_{ij} = -1$, if the agent $i$ uses the greedy strategy. The dynamics of the complex system is realized as the agents step by step choose their strategies and can be encoded by an $N \times (n-1)$ binary strategy matrix

$$S = \{s_{ij}, i = 1, ..., N, j = 1, ..., n-1\}.$$  

We try to change the structural complexity of the algorithm $\mathcal{A}$ monotonically by forcing the system to make the transition from regular behaviour to chaos by period-doubling. To control the system in this transition a parameter $v, 0 \leq v \leq 1$ is introduced. It specifies a threshold point dividing the interval of current distances passed by the agents into two parts, i.e., successful and unsuccessful. This information is provided for an optimal if-then rule that each agent uses to choose the next strat-
strategy. The rule relies on the Prouhet-Thue-Morse (PTM) sequence
\[ +1 - 1 - 1 + 1 - 1 + 1 + 1 - 1 \ldots \]
and has the following description:
1. if the last strategy is successful, continue with the same strategy.
2. if the last strategy is unsuccessful, consult PTM generator which strategy to use next.

Remarkably, for each problem \( p \) tested from a class \( \mathcal{P} \) it has been found that the performance of the algorithm \( \mathcal{A} \) indeed behaves as a concave function of the control parameter with the only global maximum at a value \( v^*(p) \). The global maximums \( \{v^*(p), p \in \mathcal{P}\} \) are of interest to probe whether the structural complexities of the algorithm \( \mathcal{A} \) and the problem are related through an optimality condition. For this purpose strategy matrices
\[ \{S(v^*(p)), p \in \mathcal{P}\} \]
corresponding to the global maximums \( \{v^*(p), p \in \mathcal{P}\} \) are used and the structural complexities of the algorithm \( \mathcal{A} \) and a problem \( p \) are approximated as follows. The structural complexity \( C(\mathcal{A}(p)) \) of the algorithm \( \mathcal{A} \) is approximated by the quadratic trace
\[ C(\mathcal{A}(p)) = \frac{1}{N^2} tr(V^2(v^*(p))) = \frac{1}{N^2} \sum_{i=1}^{N} \lambda_i^2 \]
of the variance-covariance matrix \( V(v^*(p)) \) obtained from the strategy matrix \( S(v^*(p)) \), where \( \lambda_i, i = 1, \ldots, N \) are the eigenvalues of \( V(v^*(p)) \).

The structural complexity \( C(p) \) of the problem \( p \) is approximated by the quadratic trace
\[ C(p) = \frac{1}{n^2} tr(M^2(p)) = \frac{1}{n^2} \sum_{i=1}^{n} (\lambda_i')^2 \]
of the normalized distance matrix
\[ M(p) = \{d_{ij}/d_{\text{max}}, i, j = 1, \ldots, n\}, \]
where \( \lambda_i', i = 1, \ldots, n \) are the eigenvalues of \( M(p) \), \( d_{ij} \) is the distance between cities \( i \) and \( j \) and \( d_{\text{max}} \) is the maximum of the distances.

To reveal the optimality condition the points with the coordinates
\[ \{x = C(p), y = C(\mathcal{A}(p)), p \in \mathcal{P}\} \]
are considered. The result has been indicative of a linear relationship between the structural complexities and thus suggests an optimality condition of the algorithm \( \mathcal{A} \):

If the algorithm \( \mathcal{A} \) demonstrates the optimal performance for a problem \( p \), then the structural complexity \( C(\mathcal{A}(p)) \) of the algorithm \( \mathcal{A} \) is in the linear relationship
\[ C(\mathcal{A}(p)) = 0.67C(p) + 0.33 \]
with the structural complexity \( C(p) \) of the problem \( p \).

According to the optimality condition if the optimal performance takes place, then in terms of the structural complexity the dynamics of the algorithm \( \mathcal{A} \) is in a certain relation with the structure of the problem \( p \), i.e., the distance network with the vertices as the cities and the edges specifying the pairwise distances.

The optimality condition is a practical tool. For a given problem \( p \) by using the distance matrix we can calculate the structural complexity \( C(p) \) of the problem \( p \) and from the optimality condition find the structural complexity
\[ C(\mathcal{A}(p)) = 0.67C(p) + 0.33. \]

Then to obtain the optimal performance of the algorithm \( \mathcal{A} \) for the problem \( p \) we need to tune the control parameter for the algorithm \( \mathcal{A} \) to function with the structural complexity \( C(\mathcal{A}(p)) \).

IV. THE ORDER OF THE PROCESSES AND EFFICIENT QUANTUM ALGORITHMS

The computational results point that by using self-organization processes of prime integer relations it may be possible to design classical algorithms comparable to efficient quantum algorithms.

Quantum algorithms rely on the practical use of entanglement, whose sensitivity challenges the development of relevant technologies. In a TSP quantum algorithm the wave function would be evolved to maximize through the amplitudes the probability of the shortest routes to be measured. However, there is no general direction known for the evolution to take in order to make the quantum algorithm efficient. In this regard the majorization principle seems to play an important role. It provides a local navigation, but without information about the whole landscape.

While the nature of quantum entanglement is yet to be understood in our approach the nonlocal correlations are known from their origin in the self-organization processes of prime integer relations. The question is whether this knowledge could be used to provide computational resources comparable to quantum computation.

In this paper we focus on whether such a resource could be obtained from the nonlocal correlations although used in classical computation, but with the order of the processes preserved. If that could be possible, then the order of the self-organization processes would establish a general direction for efficient computation. Remarkably, the experiments raise the possibility that if the evolution goes in this direction, then the performance landscape becomes concave.

In the experiments we use the parameter to control the correlation structures of the computing system with their consequences observed in the routes taken by the agents. To help in associations with the quantum case
the average distance for a value $v$ of the parameter can be written as
\[
\bar{D} = \frac{1}{N} (\gamma_{i_1, 1, ..., i_{n-1}}(v) d(1, ..., n-1) + \ldots + \gamma_{n-1, 1, ..., 1}(v) d(1, ..., n-1))
\]
where $\gamma_{i_1, 1, ..., i_{n-1}}(v)$ is the number of the agents followed the route $[i_1, 1, ..., i_{n-1}]$, $d([i_1, 1, ..., i_{n-1}])$ is its distance and the $n$ cities are labelled by $0, 1, ..., n-1$ with the initial city by $0$. The interpretation of the coefficient
\[
\frac{\gamma_{i_1, 1, ..., i_{n-1}}(v)}{N}
\]
as the probability of the route $[i_1, 1, ..., i_{n-1}]$ suggests that the minimization of the average distance considered in the algorithm $\mathcal{A}$ can be connected with the maximization of the probability of the shortest route considered in TSP quantum algorithms.

Significantly, the experiments have shown that in the case of the algorithm $\mathcal{A}$ the maximization of the probability turns out to be a one-dimensional concave optimization. In its course the computing system adapting to the problem got more complex or simpler until the global maximum is reached and the structural complexities of the computing system and the problem become related through the optimality condition. More connections might arise if the wave function could be involved in the description of the correlation structures.

We note that the algorithm $\mathcal{A}$ shares a common feature with Shor’s algorithm, which also relies on the PTM sequence $[6]$.  

V. CONCLUSIONS

We have suggested that self-organization processes of prime integer relations could be used for information processing.

In particular, for a given problem self-organization processes of prime integer relations could be used to build the correlation structures of a system in processing information demonstrating the optimal performance for the problem. Remarkably, the processes can be equivalently represented by transformations of two-dimensional geometrical patterns determining the dynamics of the system and revealing in the information processing its structural complexity.

The information processing would be distinctive, because self-organization processes of prime integer relations can define the correlation structures of a system without reference to the distances, local times and signals between the parts.

Computational experiments testing competitive advantages of the information processing have been presented. They raise the possibility of an optimality condition of complex systems: if the structural complexity of a system is in a certain relationship with the structural complexity of a problem, then the system demonstrates the optimal performance for the problem.

The optimality condition presents the structural complexity of a system as a key to its optimization. From its perspective the optimization of a system could be all about the control of the structural complexity of the system to make it consistent with the structural complexity of the problem.

Importantly, the experiments also indicate that the performance of a complex system may behave as a concave function of the structural complexity. Therefore, once the structural complexity could be controlled as a single entity, the optimization of a complex system would be potentially reduced to a one-dimensional concave optimization irrespective of the number of variables involved its description. This might open a way to a new type of information processing for efficient management of complex systems.

[1] V. Korotkikh, A Mathematical Structure for Emergent Computation, Kluwer Academic Publishers, Dordrecht/Boston/London, 1999.
[2] V. Korotkikh and G. Korotkikh, Description of Complex Systems in terms of Self-Organization Processes of Prime Integer Relations, in Complexus Mundi: Emergent Patterns in Nature, M. M. Novak (ed.), World Scientific, New Jersey/London, 2006, pp. 63-72, arXiv:nlin.AO/0509008.
[3] V. Korotkikh and G. Korotkikh, On an Irreducible Theory of Complex Systems, InterJournal of Complex Systems, 1751, 2006, arXiv:nlin.AO/0606023.
[4] R. Orus, J. Latorre and M. A. Martin-Delgado, Systematic Analysis of Majorization in Quantum Algorithms, arXiv:quant-ph/0212004.
[5] N. Gisin, Can Relativity be Considered Complete? From Newtonian Nonlocality to Quantum Nonlocality and Beyond, arXiv:quant-ph/0512168.
[6] K. Maity and A. Lakshminarayan, Quantum Chaos in the Spectrum of Operators Used in Shor’s Algorithm, arXiv:quant-ph/0604111.