Capillary Wave-Detection Algorithm Based on Cylindrical Solitary Waves

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Abstract. In multiphase systems the thermodynamic and rheological properties of the interfacial layer have influences on the overall system behavior. The geometrical properties of capillary waves are extremely useful in the characterization of surface parameters. In Shmyrov et al 2019, the modified capillary wave technique was successfully used to perform experiments, to register surface profiles and to realize data processing, yet the algorithm for semi-manual data processing proposed by the authors is labor-intensive and time-consuming. It appears that the analysis of the data is complex because of specific geometry problems, presence of noise at different scales, and data gaps. In this study, the algorithm for analysis of a surface instantaneous profiles formed due to capillary waves propagating is proposed. It was found that the noise and the useful signal have different scales. Moreover, the structure of the useful signal is defined, and therefore it becomes possible to study the noise part of the signal and the patterns of the useful signals. On the basis of the preliminary knowledge about the signal structure, the algorithm has been developed to overcome the above mentioned problems. The proposed method has been tested on the set of semi-synthetic data and provides a reliable result.

1. Introduction
The study of the mechanical properties of liquid interfaces has a large relevance for many technological and natural processes involving multiphase systems characterized by the presence of several specific phases such as liquid films, emulsions and foams. In liquid/liquid (liquid/gas) systems the thermodynamic and rheological properties of the interfacial layer have an increasingly effective influence on the overall system behavior [1]. Although there are many experimental tools for assessing interphase boundary parameters (Wilhelmy plate technique, Langmuir balance, drop shape analyses, capillary waves), the development of fast, robust and easy to implement methods still remains one of the key research problems.

If the viscosity ($\eta$) of coexisting fluids is not extremely high ($\eta<0.1$ P) and the surface tension is not too small (>10 dyn/cm), then capillary waves exist [2]. The basic theory for surface waves on fluids has been known since Kelvin’s [3] time, but extensive experimental studies started much later [4, 5]. A solution to the problem of capillary-gravitational waves was derived from the linearized Navier-Stokes equation subject to boundary conditions at the fluid surface and infinity depth $H$. In its simplest form the equation contains surface tension $\sigma$ as a restoring force and bulk shear viscosity as a damping mechanism. The well-known Kelvin equation relates $\sigma$ of a fluid to the frequency $\nu$ and wavelength $\lambda$ of the capillary waves

$$\frac{\sigma}{\rho_1 + \rho_2} = \frac{\nu^2 \lambda^3}{2\pi}$$  

(1)

where $\rho_1$ are the densities of the fluid under study [6]. This formula holds true for $H \gg \lambda$; viscosity $\eta$ is neglected. Using more accurate approaches [7, 8] makes it possible to take both factors ($H$ and $\eta$) into account. All these formulas include $\lambda$ in the 3rd power, and therefore it is extremely important to reduce errors in defining $\lambda$.

The capillary wave method has been intensively developed over the past decade [7] with the purpose to...
characterize surface tension and surface viscosity. Modified versions of this method differ in the way of wave excitation [9] and registration. In the majority of works, the single point registration procedure includes scattering microwave radiation [10], interferometry of He-Neon laser radiation [11], and laser light scattering [12].

To the best of our knowledge, [13] was the first work in which analysis of moire surface images was performed. In recent study [14], measurements of water surface topography were made using a background oriented schlieren technique that is based on the apparent distortion of a background pattern image reflected by the deformed surface. In [15], we have introduced a modified version of the capillary wave method in which surface waves were excited by the acoustic wave generated by a speaker and transmitted to the liquid surface by a wave guide. The 3D instantaneous surface profile of the capillary wave was measured by the digital interferometry technique [17]. Real time surface profile measurements are performed automatically. The output of the registration system is that it provides a field height needed to approximate the surface under study. In our previous papers, most of the data processing has been done manually, which significantly increases the time until the result will be available. This study is focused on the development of a robust algorithm able to accurately extract capillary wave geometrical characteristics from the 3D surface profile.

Key factors that hamper the data analysis are as follows. 1) The physical setup is not vibroisolated and, as a result, the whole surface vibrates. The entire meniscus can also be distorted, this leads to the presence of large scale noise. 2) The experimental setup is highly centralized, and the capillary wave is excited in the center of the cuvette. Although the cuvette center coincides with the optical axis of the camera, nevertheless the small uncertainty in defining the wave center arises. 3) Image fragment overlapping by the parts of the experimental setup. 4) Inevitable presence of small scale noise. It is essential that the noise and the desired signal of the capillary wave profile exhibit different spatial frequencies and structures which makes it possible to develop a special function for extracting the useful signals.

In this paper an efficient and robust automatic algorithm to extract capillary wave geometrical characteristics from the 3D surface profile is developed. The idea underlying the algorithm is to convolute the experimental data with a set of solitary cylindrical wave profiles of different scales.

2. Algorithm description
An example of the water surface with excited capillary wave at frequency 2020 Hz, reconstructed from the interferogram by means of the spatial phase shifting method using the IntelliWave (ESDI, USA) software, is presented in Fig. 1. The experimental setup has been described in detail in [15, 19]. All factors that complicate the data analysis (large scale noise, overlapped parts of the image, small scale noise) are listed in the Introduction, presence in this image. It is necessary to extract only a cylindrical capillary wave from this data and to define its characteristics with high accuracy.

![Image](image_url)

**Figure 1.** Experimental results for the water surface at excitation frequency of 2020 Hz. The frame size is 311 × 311 pixels (1.53 × 1.53 cm²).

An instantaneous cylindrical solitary wave (independent of the angular coordinate ϕ) profile is defined as:

$$\psi(r, a, b) = \frac{1}{rb} \left(1 - \left(\frac{r-b}{a}\right)^2\right) e^{-\frac{1}{2} \left(\frac{r-b}{a}\right)^2},$$  \hspace{1cm} (2)
where \( r \) is the radial coordinate, \( b \) is the position of the wave maximum, and \( a \) is the wave scale. The wave is determined for positive \( r \) and \( b \). At \( b\to0 \) and \( r\to\infty \), the mean value of \( \psi(r,a,b) \) tends to infinity. We consider the wave located a distance from the center, which is reasonable because the central part of the frame is always overlapped by the parts of the experimental setup, and therefore the central area is always dropped from the consideration.

To reduce the calculation time, the total quantity of data in analyzing matrix should be downsized, which can be reached by chopping off values with close to zero meaning. For different scales, the error of deviation of the average value of \( \psi \) from zero is different. Therefore, the value of the ratio of the average value of \( \psi(r,a,b) \) to the modulus of its average value \( \psi(r,a,b) \) is chosen as a criterion for accuracy evaluation. Within the range of variation \( r \) (as \( b-3a \), \( b+3a \)), it is no more than 0.01\%, which is sufficient for the calculations.

Fig. 2 shows the typical instantaneous profile of the wave represented by Eq.2. Owing to the factor \( 1/b \), the introduced wave conserves energy. For the defined \( a \), the total value of \( \psi(r,a,b)^2 \) does not depend on \( b \). The vertical cross-sections of \( \psi(r,a,b) \) by the plane \( C[r,0,z] \) for \( a = 2.2 \) pixels and different \( b \) are equal to 20, 30 and 40 pixels (Fig. 3).

The block diagram, shown in Fig. 4, outlines the following steps involved in the algorithm. Experimental data \((Data(x,y))\) size is \(311 \times 311 \) pixels, the scale coefficient is \((4.93 \pm 0.04) \times 10^{-3} \) cm/pixel, the size of the whole frame is \((1.53 \times 1.53) \) cm\(^2\), and hereafter the matrix with these pixels is used. Then, we introduce the interval and resolution of scales, \([a_{\min}, a_{\max}] \) and \( da \), respectively. The range of wave displacement \([b_{\min}], [b_{\max}] \) and its step \( db \) are also downloaded at the beginning of the program execution. The accuracy evaluation time is proportional to \((a_{\max}-a_{\min})/da \times (b_{\max}-b_{\min})/db \), and therefore the parameter range should be chosen carefully in the case of a time limit.

After that, going through all values of \( a \) and \( b \) in the range of \( a_{\min}-a_{\max} \) and \( b_{\min}-b_{\max} \), with steps \( da \) and \( db \) correspondingly, we form the kernel \( k_{a,b,x,y} \) by using formula (2) and transforming cylindrical coordinates into Cartesian ones. The wave symmetry center of the test wave is put in the center of the image. At this stage, it is essential to export the kernel \( k_{a,b,x,y} \) so that the data processing time can be reduced.

In order to exclude from the calculation the overlapped image components, the user should introduce an array \( Mask(x,y) \) - a binary matrix which indicates “gaps” in data - regions in \( Data(x,y) \) which should not be taken into account. When the test wave approaches the gap (gray rectangles in both panels of Fig. 5), it should be modified to keep the mean value zero and the energy constant. Getting closer to the gap, the mean value of the analyzed wave becomes nonzero. There are several possibilities to overcome this difficulty, namely, the wave should be modified in some way to keep the mean value zero. In this study, we do not change the wave amplitude or frequency but we simply cut the part of the wave which is influenced by the data gap keeping the mean value of the analyzing function zero. In the left panel of Fig. 5, two limiting wave positions from the left and right sides of the gap are presented. Due to this, the wave approaching the gap causes a slope of the zero value for \( \psi \). The effect of the data gap depends on the analyzed wave scale, i.e., the wave of smaller scale has the part which experiences less effect compared to that of larger scale (Fig. 5, right panel).
Figure 4. Block diagram of the algorithm implementation. \( \text{Data} (x, y) \) - experimental data, \( \alpha_{\text{min}} \), \( \alpha_{\text{max}} \) - maximum, minimum values of the test wave scale, and \( da \) - variation step. The variables \( b_{\text{min}} \), \( b_{\text{max}} \), \( db \) characterize the wave displacement. A four dimensional array \( k_{a,b,x,y} \) is the kernel calculated by Eq. 2. \( \text{Mask} (x, y) \) is the binary array introduced together with the experimental data. It indicates those \( \text{Data} (x, y) \) elements, which are neglected in the calculations. Mask array \( m(a, b, x, y) \) is the 4D array which characterizes the dependence of the data gaps on \( a \) and \( b \). The variables \( dx \) and \( dy \) identify the symmetry center of the capillary wave in the limit of \( dc \). \( S(a, b) \) is the resulting array of energy dependence on \( a \) and \( b \).

Figure 5. Cross section of the \( \psi(r, a, b) \) wave for the data gap shown by gray rectangles. In the left panel, \( a = 2.2 \) pixels, and waves with maximum \( b_1 = 19 \) pixels, \( b_2 = 41 \) pixels. In the right panel, wave parameters are as follows: \( a_1 = 2.6, b_1 = 17 \) (dot-dashed curve), \( a_2 = 2.1 \) pixels, \( b_2 = 20 \) pixels (dashed curve), \( a_3 = 1.6 \) pixels, \( b_3 = 22 \) pixels (solid line).

Taking into account the problem geometry, we develop a wave adaptation technique. For the wave \( \psi(r, a, b) \) where \( b > 3a \) and \( b - 3a < r < b + 3a \) at defined \( a \) and \( b \) have common points with the \( \text{Mask} (x, y) \), we cut the sector of the analyzed function so that the input of the remaining wave increases. In Fig. 6, the rectangular gap is shown in grey, and several instant solitary wave profiles are given. The green contours indicate the parts of \( k(a, b) \) which are neglected. The way how the instant solitary wave profiles are adopted is seen in the regions shown by green contours \( \psi(r, a, b) = 0 \). For every \( a \) and \( b \), the calculation of these contours yields the functions of \( (x, y) \) and the array \( m(a, b, x, y) \).
In the majority of the experiments, the setup components integrated in the frame are the same, so \( m(a, b, x, y) \) is assumed to be constant in the full experimental data set. To reduce the calculation time, it is essential to save \( m(a, b, x, y) \) together with \( k(a, b, x, y) \). The product of these two kernels gives an adapted kernel which can be used in a set of experimental data. The convolution of \( Data(x, y) \) with the kernel is defined as follows:

\[
S(a, b) = \sum_{x=1}^{N} \sum_{y=1}^{N} D(x, y) m(a, b, x, y) k(a, b, x, y),
\]

summing over the \( b \)

\[
h(a) = \sum_{b=\text{min}}^{b=\text{max}} S(a, b)
\]

we obtain \( h(a) \) which is similar to the power of the oscillations with the scale \( a \). The scale \( a' \) at which \( h(a) \) has maximum value corresponds to the closest scale in the vector \( a \) to the wavelength.

Further we output \( S(a', b) \) and interpolate it with the function of the damping cylindrical wave as

\[
W(l) = \frac{A_0}{\sqrt{l}} e^{\beta l} Re[e^{i(kl + \psi_0)}],
\]

where \( l \) is the radial coordinate, \( A_0 \) is the wave amplitude, \( \beta \) is the damping coefficient, and \( k \) is the wave number equal to \( 2\pi/a \), in which \( a \) is the wavelength, and \( \psi_0 \) is the initial phase. To determine the properties of the wave from the analysis of the experimental data, it is necessary to evaluate \( k \) and \( \beta \).

In a real experiment, it is difficult to identify the position of the wave source; the error of several pixels commonly occurs. The closest coordinates of the center \( c_x, c_y \) and the possible values of \( dx \) and \( dy \) are usually given. The specification of the center position is realized as follows. We first move the initial data in the range of \([-dx, dx; -dy, dy]\) pixels and then calculate \( h(a) \) for every center position. Finally, we summarize \( h(a) \) over \( a \) and define the center position at which this sum reaches its maximum.

3. Algorithm testing

Let us now consider the artificial image of the capillary wave which is similar to the one obtained on the basis of the experimental results. It is constructed by analyzing the large scale field \( LS \), the high frequency white noise \( N \), and the capillary wave itself (Fig.7). The source of the wave is moved from the geometrical center.

When the capillary wave and testing wave \( psi(r, a, b) \) centers coincide, the total value of \( h(a) \) will be maximal, whereas the deviation of the wave centers leads to a decrease in \( h(a) \) total value. Thus we define \( C_1 = \sum_{a=\text{min}}^{a=\text{max}} h(a) \).

\( a. \) Quasi-synthetic data

The data on real large-scale surface distortion were taken from the experiment with pure water without
wave excitation. The experiment aimed to study the effect of large scale noise. Together with the meniscus distortion this data contains small scale noise typical for the experiment. The artificial damped cylindrical wave

\[ x(k_0, \beta_0) = \frac{A_0}{\sqrt{\text{Re}[e^{i(\omega t - k l + \psi a)}]}}, \]

where \( k_0 = 0.300 \) 1/pixel, \( \beta_0 = 0.010 \) 1/pixel, \( t = 0 \) is added. Its center is shifted from the geometrical center of the image. The 3D profile obtained as described above is given in the left panel of Fig. 8, and the radial component of the profile (black solid line) - in its right panel. This figure also presents the recovered signal \( S(a, b) \) (green points), and the interpolation of \( S(a, b) \) by equation (5) (red dashed line). The error in \( k \) definition is 0.002 1/pixel (0.6%), and the error in \( \beta \) is 0.0002 1/pixel (2%).

**Figure 7.** An example of the large scale field (left panel), circular wave profile with small scale noise (middle panel) and the final image obtained by combining two fields (right panel).

The following numerical experiment was performed to estimate the error in the whole range of \( k \) and \( \beta \) variations. As a measure of precision, we used the relative errors \( \Delta k = (k_0 - k_m)/k_0 \) 100% and \( \Delta \beta = (\beta_0 - \beta_m)/\beta_0 \) 100%, where the index 0 indicates the parameters of the artificial wave, and the \( m \) shows the calculated parameters analysis of the results has revealed that the higher \( k_0 \), the lower the error in \( k_m \) and \( \beta_m \) definition. In the case under consideration, the low \( k \) (0.2 1/pixel) means that there are seven waves within the image; excluding edge effects, the number of waves reduces to approximately five. Even in the case of such long waves and real values of \( \beta_0 \), the error in \( \beta_m \) definition is appropriate. It should be pointed out here that the approach developed in [cite shmyrov 2019 capillary] cannot be applied to such images. In the \( k_0 \) range close to the majority of experiments and under reasonable \( \beta_0 \), the error is lower than 0.5%. In the range of \( k_0 \) from 0.3 to 0.7 1/pixel, \( \beta \) can be obtained with the accuracy of 5%.

**Figure 8.** Implementation of the algorithm. Black solid line corresponds to the input data of the synthetic capillary wave (\( k_0 = 0.300 \) 1/pixel, \( \beta = 0.010 \) 1/pixel) added to the real large-scale field. Green points correspond to the recovered capillary wave, and red dashed line indicates the result of data interpolation. Parameters of the recovered wave are as follows (\( k_m = 0.2981/\text{pixel} \) and \( \beta_m = 0.0098 \) 1/pixel).
The overlapping of the frame areas by the parts of the experimental setup leads to the highest error in the calculations. This error is associated with the position of the gaps in the data. The gaps on the image boundaries related to the wave amplitude, which is usually rather small especially at large $\beta$, does not influence the results. The large gap in the center of the image leads to higher error. We provide here a test with a rectangular mask, which is placed to cover the center of the image closing the center. The area of the mask is varied from 5 to 15\% of the image. For $k_0 = 0.3$ and $f_0 = 0.01$, the error in $k$ definition does not exceed 1\% and $\beta$ - 40\%.

b. Analysis of real experimental data

In this section, the algorithm implementation is demonstrated on the data obtained in the experiment with pure water; the excitation frequency is 2020 Hz (Fig. 1). The input of both large and small scales noise is clearly seen. There are gaps close to the boundaries of the cuvette and in the regions overlapped by parts of the experimental setup. In the left panel of Fig. 9, we demonstrate the element of the adaptive kernel - an instantaneous wave profile ($a = 4.4$ pixels, $b = 55$ pixels) with gaps.

![Image](https://example.com/image.png)

**Figure 9.** Fragment of the adopted kernel ($a = 4.4$ pixels, $b = 55$ pixels) with gaps (left panel) and the result of calculations (right panel). Blue points indicate $S(a', b)$, and green line - the result of $S(a, b)$ fitting for the experimental data with pure water.

The results of convolution of the experimental data (Fig.1) with the kernel and, consequently, identification of an optimal filtration mode are shown in Fig. 9 by dots. The green solid line demonstrates the fitting of dots in the range between the red dashed vertical lines. The wavelength is $\lambda = 9.67$ pixels, as the scale coefficient is $(4.93 \pm 0.04) \times 10^{-3}$ cm/pixel. The wavelength is 0.0476 cm. Substitution of all values in Eq. 1 gives surface tension obtained as $\sigma = (69.9 \pm 1.5)$ dyn/cm. Note that the measurement is highly sensitive to the scale coefficient. The deviation of the obtained data from the table data (72 dyn/cm) can be attributed to some disadvantages of the experimental design.

4. Discussion and conclusions

In this study the algorithm for analysis of a 3D surface instantaneous profiles formed due to the propagation of capillary waves is proposed. The details of the experimental setup are described in Ref.[15, 16]. The surface distortion is very small which makes possible the study of the 3D surface profile by means interferometry. There are three main factors that complicate the analysis of the obtained data: large and small scale noises, uncertainty of the wave excitation source, and gaps in the data. The noise and the useful signal have different scales. Moreover, the structure of the useful signal is defined, and therefore it becomes possible to study the noise part of the signal and the patterns of the useful signals. On the basis of the preliminary knowledge about the signal structure, the algorithm has been developed to overcome the above mentioned problems.

The algorithm was tested on artificial, quasi-synthetic and experimental data. It allows one to reveal the wave-number and damping coefficients with sufficient accuracy. It is known that the surface tension $\lambda^3$, as well as $\lambda$, should be defined with high accuracy. It has been demonstrated that the proposed method can be used to identify the circular wave pattern in the image even in the presence of both large and small scale noises. The adaptive techniques developed for the gap in data. This paper is
a continuation of the previous studies devoted to the development of capillary wave techniques [15, 19]. However, most operations on data were performed in these works in manual mode. The version of the algorithm proposed in this study is able to operate in automatic mode. This reduces the processing errors and saves the researcher time. The developed method can be utilized in the laboratory software employed in the research on phenomena encountered in interphase hydrodynamics. Due to its ability to noninvasively detect capillary waves with a small amount of liquid in the frequency range which was inaccessible before, it holds great promise for chemistry, biology and hydrodynamics.

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