Majorana Neutrinos, CP-Violation, Neutrinoless Double-Beta and Tritium-Beta Decays

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Abstract

If the present or upcoming searches for neutrinoless double $\beta$––$((\beta\beta)_{0w})$ decay give a positive result, the Majorana nature of massive neutrinos will be established. From the determination of the value of the $(\beta\beta)_{0w}$-decay effective Majorana mass parameter $(\langle m \rangle)$, it would be possible to obtain information on the type of neutrino mass spectrum. Assuming 3-$\nu$ mixing and massive Majorana neutrinos, we discuss the information a measurement of, or an upper bound on, $\langle m \rangle$ can provide on the value of the lightest neutrino mass $m_1$. With additional data on the neutrino masses obtained in $^3\text{H}$ $\beta$–decay experiments, it might be possible to establish whether the CP-symmetry is violated in the lepton sector. This would require very high precision measurements. If CP-invariance holds, the allowed patterns of the relative CP-parities of the massive Majorana neutrinos would be determined.

1 Introduction

With the accumulation of more and stronger evidences for oscillations of the atmospheric and solar neutrinos, caused by neutrino mixing (see, e.g., [3]), the problem of the nature of massive neutrinos emerges as one of the fundamental problems in the studies of neutrino mixing. Massive neutrinos can be Dirac or Majorana particles. In the former case they possess a conserved lepton charge and distinctive antiparticles, while in the latter there is no conserved lepton charge and massive neutrinos are truly neutral particles identical with their antiparticles (see, e.g., [3]). Thus, the question of the nature of massive neutrinos is directly related to the question of the basic symmetries of the fundamental particle interactions.

The present and upcoming neutrino oscillation experiments will allow to make a big step forward in understanding the patterns of neutrino mass squared differences and of $\nu$-mixing, but will not be able to determine the absolute values of the neutrino masses and to answer the question regarding the nature of massive neutrinos. The $^3\text{H}$ $\beta$-decay experiments, studying the electron spectrum, are sensitive to the electron (anti-)neutrino mass $m_{\nu_e}$ and can give information on the absolute value of neutrino masses. The present bounds on $m_{\nu_e}$ from the Troitzk [1] and Mainz [2] experiments read (at 95% C.L.): $m_{\nu_e} < 2.5$ eV [4] and $m_{\nu_e} < 2.9$ eV [5]. There are prospects to increase the sensitivity of the $^3\text{H}$ $\beta$-decay experiments to $m_{\nu_e} \sim (0.3 - 1.0)$ eV [6] (the KATRIN project).

The problem of the nature of massive neutrinos can be addressed in experiments studying processes in which the total lepton charge $L$ is not conserved and changes by two units, $\Delta L = 2$. The process most sensitive to the existence of massive Majorana neutrinos (coupled to the electron) is the neutrinoless double $\beta$ ($((\beta\beta)_{0w})$–decay of certain even-even nuclei (see, e.g., [3]): $(A, Z) \rightarrow (A, Z + 2) + e^- + e^-$. If the $(\beta\beta)_{0w}$–decay is generated only by the left-handed charged current weak interaction through the exchange of virtual light massive Majorana neutrinos, the probability amplitude of this process is proportional to the “effective Majorana mass parameter”:

$$\langle m \rangle \equiv |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_1} m_2 + |U_{e3}|^2 e^{i\alpha_2} m_3,$$

where $m_j$ is the mass of the Majorana neutrino $\nu_j$, $U_{ej}$ is the element of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino (lepton) mixing matrix [4, 5], and $\alpha_j$, $j = 2, 3$, are two Majorana CP-violating phases [4]. If CP-parity is conserved we have $\alpha_2 = k\pi$ and $\alpha_3 = k\pi + \pi/2$. Many experiments are searching

\textsuperscript{a}To be published in the Proceedings of the Conference NANP’01, III International Conference on Non-Accelerator New Physics, Dubna (Russia), June 19-23 2001.
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for \((\beta\beta)_{0\nu}\)-decay. No indications that this process takes place were found so far. A most stringent constraint on the value of \(|<m>|\) was obtained in the \(^{76}\text{Ge}\) Heidelberg-Moscow experiment \([12]\) \(|<m>| < 0.35\text{ eV} (90\%\ C.L.)\). The IGEX collaboration has obtained \([13]\) \(|<m>| < (0.33 \pm 1.35)\text{ eV} (90\%\ C.L.)\). Higher sensitivity to the value of \(|<m>|\) is planned to be reached in several \((\beta\beta)_{0\nu}\)-decay experiments of a new generation \([14]\).

The present article represents a continuation of the studies of the physical implications of the possible future results on \(|<m>|\) and on \(m_{\nu_e}\) performed in Refs. \([13, 16, 17]\). The article is based on the work done in \([17]\). We generalize the results, derived in \([13]\) by using the best fit values of the input neutrino oscillation parameters, to the case when some of these parameters take values in their 90\% C.L. allowed regions. Earlier studies on the subject were performed, e.g., in Refs. \([18, 19, 20]\). Recent relevant studies include Ref. \([21]\).

2 Constraining or Determining the Lightest Neutrino Mass \(m_1\) and/or the Majorana CP-Violating Phases

In this Section, assuming \(3-\nu\) mixing, we discuss the information that future \((\beta\beta)_{0\nu}\)-decay and/or \(^3\text{H}\ \beta\)-decay experiments can provide on the lightest neutrino mass \(m_1\) and on the CP-violation generated by the Majorana CP-violating phases \(\alpha_{21}\) and \(\alpha_{31}\). We number the three neutrinos with definite mass \(\nu_i\) in such a way that their masses obey \(m_1 < m_2 < m_3\). The two cases of neutrino mass spectrum are analyzed: spectrum with normal hierarchy, \(\Delta m^2_\odot \equiv \Delta m^2_{21}\), and with inverted hierarchy, \(\Delta m^2_\odot \equiv \Delta m^2_{32}\), \(\Delta m^2_{21} > 0\) being the neutrino mass squared difference characterizing the solar neutrino oscillations (see, e.g., Refs. \([13, 17]\)). In both cases we use in the analysis which follows fixed values of the mixing parameters \(|U_{e3}|^2\) (or \(|U_{e1}|^2\)) and of the angle \(\theta_\odot\), which are constrained by the CHOOZ and the solar neutrino data \([1]\), respectively. We allow \(\Delta m^2_\odot\) and \(\Delta m^2_{\text{atm}}\), which characterizes the oscillations of atmospheric neutrinos, to vary within their 90\% C.L. allowed intervals found in the analyses of the solar and atmospheric neutrino data in Refs. \([23, 24]\). We denote the minimal and maximal values in these intervals by \((\Delta m^2_\odot)_{\text{MIN}}\), \((\Delta m^2_\odot)_{\text{MAX}}\), \((\Delta m^2_{\text{atm}})_{\text{MIN}}\) and \((\Delta m^2_{\text{atm}})_{\text{MAX}}\). The results thus obtained are summarized in Fig. 1 (normal neutrino mass hierarchy) and in Fig. 2 (inverted hierarchy).

2.1 Normal Mass Hierarchy: \(\Delta m^2_\odot \equiv \Delta m^2_{21}\)

If \(\Delta m^2_\odot = \Delta m^2_{21}\), for any given solution of the solar neutrino problem LMA MSW, LOW-QVO, SMA MSW, \(|<m>|\) can lie anywhere between 0 and the present upper limits, as Fig. 1 indicates. This conclusion does not change even under the most favorable conditions for the determination of \(|<m>|\), namely, even when \(\Delta m^2_{\text{atm}}\), \(\Delta m^2_\odot\), \(\theta_\odot\) and \(\theta\) are known with negligible uncertainty. The further conclusions in the case of the LMA MSW solution of the solar neutrino problem, which are illustrated in Fig. 1, are now summarized.

Case A. An experimental upper limit on \(|<m>|\), \(|<m>| < |<m>|_{\text{exp}}\), will determine a maximal value of \(m_1\), \(m_1 < (m_1)_{\text{max}}\). The value \((m_1)_{\text{max}}\) is fixed by one of the two equalities:

\[
|<m>|_{\text{exp}} = \left( m_1 \cos 2\theta_\odot - \sqrt{m_1^2 + (\Delta m^2_{\text{atm}})_{\text{MIN}} \sin^2 \theta_\odot} \right) \left( 1 - |U_{e3}|^2 \right) - \sqrt{m_1^2 + (\Delta m^2_{\text{atm}})_{\text{MAX}} |U_{e3}|^2} ;
\]

\[
|<m>|_{\text{exp}} = \left( m_1 \cos 2\theta_\odot - \sqrt{m_1^2 + (\Delta m^2_{\text{atm}})_{\text{MIN}} \sin^2 \theta_\odot} \right) \left( 1 - |U_{e3}|^2 \right) + \sqrt{m_1^2 + (\Delta m^2_{\text{atm}})_{\text{MAX}} |U_{e3}|^2} .
\]

Eq. \((2)\) is valid for \(\cos 2 \theta_\odot > (\Delta m^2_{\text{atm}} / m_3^2) \sin^2 \theta_\odot\), otherwise eq. \((3)\) should be used.

For the quasi-degenerate mass spectrum one has \(m_1 \gg \Delta m^2_\odot\), \(\Delta m^2_{\text{atm}}\), \(m_1 \simeq m_2 \equiv m_3 \equiv m_{\nu_e}\), and \([17, 21]\)

\[
(m_1)_{\text{max}} \simeq \frac{|<m>|_{\text{exp}}}{|\cos 2\theta_\odot (1 - |U_{e3}|^2) |} .
\]

\(^4\) In our further discussion we assume \(\cos 2 \theta_\odot > 0\), which is favored by the analyses of the solar neutrino data \([22]\). The modification of the relevant formulae and of the results in the case \(\cos 2 \theta_\odot < 0\) is rather straightforward.
If \( |\cos 2\theta_\odot (1 - |U_{ee}|^2) - |U_{ee}|^2| \) is sufficiently small, the upper limit on \( m_{\nu_e} \) obtained in \(^3\)H \( \beta \)-decay experiments could yield a more stringent upper bound on \( m_1 \) than the bound from the limit on \( |<m>| \).

**Case B.** A measurement of \( |<m>| = |\langle<m>| \rangle_{exp} \geq 0.02 \text{ eV} \) would imply that \( m_1 \geq 0.02 \text{ eV} \) and thus a mass spectrum with partial hierarchy or of quasi-degenerate type \([13]\). The lightest neutrino mass will be constrained to lie in the interval, \((m_1)_{min} \leq m_1 \leq (m_1)_{max}\), where \((m_1)_{max}\) and \((m_1)_{min}\) are determined respectively by eq. \((3)\) or \((3)\) and by the equation \((7)\).

\[
(m_1 \cos^2 \theta_\odot + \sqrt{m_1^2 + (\Delta m^2_\odot)_{MAX} \sin^2 \theta_\odot}(1 - |U_{e3}|^2)^2 + \sqrt{m_1^2 + (\Delta m^2_\text{atm})_{MAX} |U_{e3}|^2} = |<m>|_{exp}.
\]

The limiting values of \( m_1 \) correspond to the case of CP-conservation. If it holds that \( \Delta m^2_\odot \ll m_1^2 \) (i.e., for \( \Delta m^2_\odot \approx 10^{-4} \text{ eV}^2 \)), \((m_1)_{min}\) to a good approximation is independent of \( \theta_\odot \), and for \( \sqrt{\Delta m^2_\text{atm} |U_{e3}|^2} \ll m_1 \), which takes place in the case we consider as \( |U_{e3}|^2 \approx 0.05 \), we have \((m_1)_{min} = (|<m>|)_{exp} \). For \( |U_{e3}|^2 \ll \cos 2\theta_\odot \), which is realized in Fig. 1 for \( |U_{e3}|^2 \ll 0.01 \), practically all the region between \((m_1)_{min}\) and \((m_1)_{max}\), \((m_1)_{min} < m_1 < (m_1)_{max}\), corresponds to violation of the CP-symmetry. If \( |U_{e3}|^2 \) is non-negligible with respect to \( \cos 2\theta_\odot \), e.g., if \( |U_{e3}|^2 \approx (0.02 - 0.05) \) for the values of \( \cos 2\theta_\odot \) used to derive the right panels in Fig. 1, one can have \((m_1)_{min} < m_1 < (m_1)_{max}\) if the CP-symmetry is violated, as well as in two specific cases of CP-conservation \([17]\). One of these two CP-conserving values of \( m_1 \), corresponding to \( \eta_2 = -\eta_3 = -1 \), can differ considerably from the two limiting values (see Fig. 1). In general, the knowledge of the value of \( |<m>| \) alone will not allow to distinguish the case of CP-conservation from that of CP-violation.

**Case C.** It might be possible to determine whether CP-violation due to the Majorana phases takes place in the lepton sector if both \(|<m>|\) and \( m_{\nu_e} \) are measured. Since prospective measurements are limited to \((m_{\nu_e})_{exp} \geq 0.35 \text{ eV} \), the relevant neutrino mass spectrum is of quasi-degenerate type, \( m_1 \equiv m_2 \equiv m_3 \equiv m_{\nu_e} \) (see, e.g., \([15]\)) and one has \( m_1 > 0.35 \text{ eV} \). If we can neglect \(|U_{e3}|^2 \) (i.e., if \( \cos 2\theta_\odot \gg |U_{e3}|^2 \)), a value of \( m_{\nu_e} \equiv m_1 \), satisfying \((m_1)_{min} < m_{\nu_e} < (m_1)_{max}\), where \((m_1)_{min}\) and \((m_1)_{max}\) are determined by eqs. \((3)\) and \((3)\), would imply that the CP-symmetry does not hold in the lepton sector. In this case one would obtain correlated constraints on the CP-violating phases \( \alpha_1 \) and \( \alpha_3 \) \([15, 22]\). This appears to be the only possibility for demonstrating CP-violation due to Majorana CP-violating phases in the case of \( \Delta m^2_\odot \equiv \Delta m^2_21 \) under discussion \([17]\). In order to reach a definite conclusion concerning CP-violation due to the Majorana CP-violating phases, considerable accuracy in the measured values of \(|<m>|\) and \( m_{\nu_e} \) is required. For example, if the oscillation experiments give the result \( \cos 2\theta_\odot \leq 0.3 \) and \(|<m>| \approx 0.3 \text{ eV} \), a value of \( m_{\nu_e} \) between 0.3 eV and 1.0 eV would demonstrate CP-violation. This requires better than 30% accuracy on both measurements.

The accuracy requirements become less stringent if the upper limit on \( \cos 2\theta_\odot \) is smaller.

If \( \cos 2\theta_\odot > |U_{e3}|^2 \) but \(|U_{e3}|^2 \) cannot be neglected in \(|<m>| \) \), there exist two CP-conserving values of \( m_{\nu_e} \) in the interval \((m_1)_{min} < m_{\nu_e} < (m_1)_{max} \) \([17]\). The one that can significantly differ from the extreme values of the interval corresponds to the specific case of CP-conservation with \( \eta_2 = -\eta_3 = -1 \) (Fig. 1).

**Case D.** A measured value of \( m_{\nu_e}, \langle m_{\nu_e}\rangle_{exp} \geq 0.35 \text{ eV} \), satisfying \( \langle m_{\nu_e}\rangle_{exp} > (m_1)_{max} \), where \((m_1)_{max}\) is determined from the upper limit on \(|<m>| \), eq. \((4)\) or \((4)\), in the case the \((\beta\beta)_{0\nu}\)-decay is not observed, might imply that the massive neutrinos are Dirac particles. If \((\beta\beta)_{0\nu}\)-decay has been observed and \(|<m>| \, measured, the inequality \( \langle m_{\nu_e}\rangle_{exp} > (m_1)_{max} \), would lead to the conclusion that there exist contribution(s) to the \((\beta\beta)_{0\nu}\)-decay rate other than due to the light Majorana neutrino exchange (see, e.g., \([23]\) and the references quoted therein) that partially cancel the one from the Majorana neutrino exchange.

A measured value of \(|<m>| \), \(|\langle<m>| \rangle_{exp} \geq 0.01 \text{ eV} \), and a measured value of \( m_{\nu_e} \) or an upper bound on \( m_{\nu_e} \) such that \( m_{\nu_e} < (m_1)_{min} \), where \((m_1)_{min}\) is determined by eq. \((4)\), would imply that there are contributions to the \((\beta\beta)_{0\nu}\)-decay rate in addition to the ones due to the light Majorana neutrino exchange (see, e.g., \([23]\)), which enhance the \((\beta\beta)_{0\nu}\)-decay rate and signal the existence of new \( \Delta L = 2 \) processes beyond those induced by the light Majorana neutrino exchange in the case of left-handed charged current weak interaction.

**Case E.** An actual measurement of \(|<m>| \approx 10^{-2} \text{ eV} \) is unlikely, but it is illustrated in Fig. 1 to show the
interpretation of such a result. There always remains an upper limit on \( m_1, (m_1)_{\text{max}} \), determined by eq. (4) or by eq. (3). For \( |U_{e3}|^2 \geq |U_{\odot}|^2 \), where \( |U_{e3}|^2 \equiv \left( \frac{\Delta m^2_{\odot}}{\Delta m^2_{\text{atm}} \cdot \sin^2 \theta} \right) / \sqrt{\Delta m^2_{\text{atm}} \cdot \max} \), the cancellations between the different terms allow to have \( (m_1)_{\text{min}} = 0 \). Both the cases of CP-conservation and CP-violation are allowed. In principle, there exists the possibility to have regions of the parameter space where only the case of CP-violation is allowed. However, in order to establish that \( |<m>| \) and \( m_1 \) lie in those regions, they must be known with a precision which is far beyond that aimed at in the currently planned future experiments.

If \( |U_{e3}|^2 < |U_{\odot}|^2 \), and \( |<m>| \) lies in the interval \( |<m>|_{-} \leq |<m>| \leq |<m>|_{+} \), defined by

\[
|<m>|_{+} = \sqrt{(\Delta m^2_{\odot})_{\max} (\sin^2 \theta)_{\max} (1 - |U_{e3}|^2) + \Delta m^2_{\text{atm}} \cdot \max |U_{e3}|^2}, \tag{6}
\]

\[
|<m>|_{-} = \sqrt{(\Delta m^2_{\odot})_{\min} (\sin^2 \theta)_{\min} (1 - |U_{e3}|^2) - \Delta m^2_{\text{atm}} \cdot \max |U_{e3}|^2}, \tag{7}
\]

where \( (\sin^2 \theta)_{\min} \) and \( (\sin^2 \theta)_{\max} \) are respectively the minimal and maximal allowed values of \( \sin^2 \theta \) in the LMA solution region, the lower limit on \( m_1 \) goes to zero. All the CP-parity patterns in the case of CP-conservation as well as the violation of the CP-symmetry are possible. If \( |U_{e3}|^2 < |U_{\odot}|^2 \) and \( |<m>| < |<m>|_{-} \), \( (m_1)_{\min} \) is determined by the following equation:

\[
|<m>|_{\exp} = \left| m_1 \cos^2 \theta - \sqrt{m_1^2 + (\Delta m^2_{\odot})_{\min} (\sin^2 \theta)_{\min} (1 - |U_{e3}|^2)} + \sqrt{m_1^2 + (\Delta m^2_{\text{atm}} \cdot \max |U_{e3}|^2} \right|. \tag{8}
\]

Under the above conditions, the case of CP-conservation corresponding to \( \eta_{21} = \pm \eta_{31} = 1 \) will be excluded. At the same time both the case of CP-conservation with \( \eta_{21} = \pm \eta_{31} = -1 \) and that of CP-violation will be allowed.

It should be noted also that one can have \( |<m>| = 0 \) for \( m_1 = 0 \) in the case of CP-invariance if \( \eta_{21} = -\eta_{31} \) and the relation \( \sqrt{\Delta m^2_{\odot} \cdot \sin^2 \theta (1 - |U_{e3}|^2)} = \sqrt{\Delta m^2_{\text{atm}} \cdot |U_{e3}|^2} \) holds. Finally, there would seem to be no practical possibility to determine the Majorana CP-violating phases.

The analysis of the Cases A - E for the LOW-QVO solution of the solar neutrino problem leads to the same qualitative conclusions as those obtained above for the LMA MSW solution. The conclusions differ, however, in the case of the SMA MSW solution and we will discuss them next briefly [17]. An experimental upper limit on \( |<m>| \) (Case A) in the range \( |<m>|_{\exp} \geq 10^{-2} \) eV, would imply in the case of the SMA MSW solution, \( m_1 < |<m>|_{\exp} (1 - 2|U_{e3}|^2)^{-1} \). For values of \( |<m>| \approx 10^{-2} \) eV, the maximum and minimum values of \( m_1 \) are extremely close: \( (m_1)_{\min} \approx |<m>|_{\exp} \). As a result, a measurement of \( |<m>| \) (Case B) practically determines \( m_1, m_1 \approx |<m>| \). However, no information about CP-violation generated by the Majorana phases can be obtained by the measurement of \( |<m>| \) (or of \( |<m>| \) and \( m_{\nu_e} \) ) [15]. If both \( |<m>| \approx 0.02 \) eV and \( m_{\nu_e} \approx 0.35 \) eV would be measured (Case C), the relation \( m_1 \approx |<m>|_{\exp} \approx (m_{\nu_e})_{\exp} \) should hold. The conclusions in the Cases D and E are qualitatively the same as for the LMA MSW solution.

### 2.2 Inverted Mass Hierarchy: \( \Delta m^2_{\odot} \equiv \Delta m^2_{32} \)

Consider next the possibility of a neutrino mass spectrum with inverted hierarchy, which is illustrated in Fig. 2. A comparison of Fig. 1 and Fig. 2 reveals two major differences in the predictions for \( |<m>| \): if \( \Delta m^2_{\odot} \equiv \Delta m^2_{32} \), i) even in the case of \( m_1 \ll m_2 \ll m_3 \) (i.e., even if \( m_1 \ll 0.02 \) eV), \( |<m>| \) can exceed \( \sim 10^{-2} \) eV and can reach the value of \( \sim 0.08 \) eV [13], and ii) a more precise determination of \( \Delta m^2_{\text{atm}}, \Delta m^2_{\odot}, \theta \) and \( \sin^2 \theta = |U_{e1}|^2 \), can lead to a lower limit on the possible values of \( |<m>| \) [14]. For the LMA and the LOW-QVO solutions, \( \min(|<m>|) \) will depend, in particular, on whether CP-invariance holds or not in the lepton sector, and if it holds - on the relative CP-parities of the massive Majorana neutrinos. All these possibilities are parametrized by the values of the two CP-violating phases, \( \alpha_{21} \) and \( \alpha_{31} \), entering into the expression for \( |<m>| \). The existence of a significant lower limit on the possible values of \( |<m>| \) depends crucially in the cases of the LMA and LOW-QVO solutions on the minimal value of \( |\cos 2\theta|, |\cos 2\theta|_{\min} \), allowed by the data. Up to corrections \( \sim 5 \times 10^{-3} \) eV, the minimum value of \( |<m>| \) is for these two solutions
(see, e.g., [13]):

\[
(|<m>|)_{\text{MIN}} \simeq \sqrt{(\Delta m_{\text{atm}}^2)_{\text{MIN}} |(\cos 2\theta_\odot)|_{\text{MIN}}(1 - |U_{e1}|^2)}.
\]  

(9)

The \(\min(|<m>|)\) in eq. (9) is reached in the case of CP-invariance and \(\eta_{21} = -\eta_{31} = \pm 1\).

We shall discuss next briefly the implications of the results of future \((\beta\beta)_{0u}\)-decay and \(^3\text{H}\) \(\beta\)-decay experiments. We follow the same line of analysis we have used for neutrino mass spectrum with normal hierarchy. Consider the case of the LMA MSW solution of the solar neutrino problem.

**Case A.** An experimental upper limit on \(|<m>|, |<m>| < |<m>|_{\text{exp}}\), which is larger than the minimal value of \(|<m>|, |<m>|_{\text{MIN}}\), predicted by taking into account all uncertainties in the values of the relevant input parameters \((\Delta m_{\text{atm}}^2, \Delta m_\odot^2, \theta_\odot, \text{etc.)}, |<m>|_{\text{exp}} \geq |<m>|_{\text{MIN}},\) will imply an upper limit on \(m_1, m_1 < (m_1)_{\text{MAX}}\). The latter is determined by the equality:

\[
\left|\sqrt{m_1^2 + (\Delta m_{\text{atm}}^2)_{\text{MIN}} - (\Delta m_\odot^2)_{\text{MAX}} \cos^2 \theta_\odot} - \sqrt{m_1^2 + (\Delta m_{\text{atm}}^2)_{\text{MIN}} \sin^2 \theta_\odot}\right|(1 - |U_{e1}|^2) \pm m_1 |U_{e1}|^2 = |<m>|_{\text{exp}}.
\]  

(10)

where the + (−) corresponds to a negative (positive) value of the expression in the big round brackets [17].

For the quasi-degenerate neutrino mass spectrum \((m_1 \gg \Delta m_\odot^2, \Delta m_{\text{atm}}^2, m_1 \equiv m_2 \equiv m_3 \equiv m_\nu, \) \((m_1)_{\text{MAX}}\) is given by eq. (9) in which \(|U_{ee}|^2\) is replaced by \(|U_{ee}|^2\). Correspondingly, the conclusion that if \(|\cos 2\theta_\odot(1 - |U_{e1}|^2) - |U_{e1}|^2|\) is sufficiently small, the upper limit on \(m_1 \equiv m_\nu\), obtained in \(^3\text{H}\) \(\beta\)-decay, can be more stringent than the upper bound on \(m_1\), implied by the limit on \(|<m>|\), remains valid.

An experimental upper limit on \(|<m>|, \) which is smaller than the minimal possible value of \(|<m>|, |<m>|_{\text{MIN}}\), \(|<m>|_{\text{exp}} < |<m>|_{\text{MIN}}\), would imply that either i) the neutrino mass spectrum is not of the inverted hierarchy type, or ii) that there exist contributions to the \((\beta\beta)_{0u}\)-decay rate other than due to the light Majorana neutrino exchange (see, e.g., [23]) that partially cancel the contribution from the Majorana neutrino exchange. The indicated result might also suggest that the massive neutrinos are Dirac particles.

**Case B.** A measurement of \(|<m>| = |(\text{signal})|_{\text{exp}} > \sqrt{(\Delta m_{\text{atm}}^2)_{\text{MAX}}(1 - |U_{e1}|^2) \geq (0.04 - 0.08)\text{ eV},\) where we have used the 90\% C.L. allowed regions of \(\Delta m_{\text{atm}}^2\) and \(|U_{ee}|^2\) from [23], would imply the existence of a finite interval of possible values of \(m_1, (m_1)_{\text{MIN}} \leq m_1 \leq (m_1)_{\text{MAX}}\), with \((m_1)_{\text{MAX}}\) and \((m_1)_{\text{MIN}}\) given respectively by eq. (10) and by the equality:

\[
m_1 |U_{e1}|^2 + \left(\sqrt{m_1^2 + (\Delta m_{\text{atm}}^2)_{\text{MAX}} - (\Delta m_\odot^2)_{\text{MIN}} \cos^2 \theta_\odot} + \sqrt{m_1^2 + (\Delta m_{\text{atm}}^2)_{\text{MAX}} \sin^2 \theta_\odot}\right)(1 - |U_{e1}|^2) = |<m>|_{\text{exp}}.
\]  

(11)

In this case \(m_1 \geq 0.04\text{ eV}\) and the neutrino mass spectrum is with partial inverted hierarchy or of quasi-degenerate type [13]. The limiting values of \(m_1\) correspond to CP-conservation. For \(\Delta m_\odot^2 \ll m_1^2\), i.e., for \(\Delta m_\odot^2 \lesssim 10^{-4}\text{ eV}^2\), \((m_1)_{\text{MIN}}\) is to a good approximation independent of \(\theta_\odot\) and we have:

\[
\sqrt{(m_1)_{\text{MIN}}^2 + (\Delta m_{\text{atm}}^2)_{\text{MAX}}(1 - |U_{e1}|^2)} \equiv (|<m>|)_{\text{exp}}.
\]

For negligible \(|U_{ee}|^2\) (i.e., \(|U_{ee}|^2 \ll 0.01\) for the values of \(\cos 2\theta_\odot\) in Fig. 2), essentially all of the interval between \((m_1)_{\text{MIN}}\) and \((m_1)_{\text{MAX}}\), \((m_1)_{\text{MIN}} < m_1 < (m_1)_{\text{MAX}}\), corresponds to violation of the CP-symmetry. If the term \(\sim m_1 |U_{e1}|^2\) cannot be neglected in eqs. (10) and (11) (i.e., if \(|U_{ee}|^2 \geq (0.02 - 0.05)\) for the values of \(\cos 2\theta_\odot\) in Fig. 2), there exists for a fixed \(|<m>|_{\text{exp}}\) two CP-conserving values of \(m_1\) in the indicated interval [17], one of which differs noticeably from the limiting values \((m_1)_{\text{MIN}}\) and \((m_1)_{\text{MAX}}\) and corresponds to \(\eta_{21} = -\eta_{31} = 1\) (Fig. 2).

In general, measuring the value of \(|<m>|\) alone will not allow to distinguish the case of CP-conservation from that of CP-violation. In principle, a measurement of \(m_{\nu_e}\), or even an upper limit on \(m_{\nu_e}\), smaller than \((m_1)_{\text{MAX}}\), could be a signal of CP-violation, as Fig. 2 (upper panels) shows. However, unless \(\cos 2\theta_\odot\) is very small, the required values of \(m_{\nu_e}\) are less than prospective measurements. For example, as is seen in Fig. 2,
middle left panel, for $\cos 2\theta_\odot < 0.1$ and $|<m>| = 0.03$ eV, one needs to find $m_{\nu_e} < 0.35$ eV to demonstrate CP-violation.

If the measured value of $|<m>|$ lies in the interval $(|<m>|_\text{MIN}, |<m>|_\text{MAX})$, where

$$
|<m>|_\text{MAX} = \sqrt{(\Delta m^2_{\text{atm}})^\text{MIN} - (\Delta m^2_{\text{sol}})^\text{MIN}} \cos^2 \theta_\odot + \sqrt{(\Delta m^2_{\text{atm}})^\text{MIN}} \sin^2 \theta_\odot (1 - |U_{e1}|^2),
$$

$$
|<m>|_\text{MIN} = \sqrt{(\Delta m^2_{\text{atm}})^\text{MAX} - (\Delta m^2_{\text{sol}})^\text{MAX}} \cos^2 \theta_\odot - \sqrt{(\Delta m^2_{\text{atm}})^\text{MAX}} \sin^2 \theta_\odot (1 - |U_{e1}|^2),
$$

we would have $(m_1)_{\text{MIN}} = 0$. Furthermore, if one finds that $|<m>| < (|<m>|_\text{MAX} - |<m>|_\text{MIN})$, the case of CP-conservation corresponding to $\eta_{21} = \eta_{11} = \pm 1$ will be excluded. The observation of the $\beta\beta$0$\nu$-decay is not conserved. The conclusion would either indicate that the neutrino mass spectrum is not of the inverted hierarchy type, or that there exist contributions to the $\beta\beta$0$\nu$-decay rate other than to the light Majorana-$\nu$ exchange that partially cancel the contribution from the Majorana-$\nu$ exchange.

The above conclusions hold with minor modifications (essentially of the numerical values involved) for the LOW-QVO solution as well. In the case of the SMA MSW solution we have, as is well-known, $\sin^2 \theta_\odot < 1$ and $\Delta m^2_{\odot} \lesssim 10^{-5}$ eV$^2$ (see, e.g., [24]). Consequently, the analog of eq. (4) in Case A reads $(m_1)_{\text{MAX}} \equiv |<m>|_\text{exp}(1 - 2|U_{e1}|^2)^{-1}$. The conclusions in the Cases B - D are qualitatively the same as in the case of neutrino mass spectrum with normal hierarchy. In particular, a measured value of $|<m>| > |<m>|_\text{PH} \equiv \sqrt{(\Delta m^2_{\text{atm}})^\text{MIN} (1 - |U_{e1}|^2)}$, would essentially determine $m_1$, $m_1 \equiv (m_1)_{\text{MIN}} \equiv |<m>|_\text{exp}$. No information about CP-violation generated by the Majorana phases can be obtained by the measurement of $|<m>|$, or of $|<m>|$ and $m_{\nu_e}$. If both $|<m>|$ and $m_{\nu_e} > 0.35$ eV are measured, the relation $m_1 \equiv (m_1)_{\text{MAX}} \equiv |<m>|_\text{exp}$ should hold. If it is found that $|<m>| = \sqrt{\Delta m^2_{\text{atm}} (1 - |U_{e1}|^2)}$, one would have $0 \leq m_1 \leq (m_1)_{\text{MAX}}$, where $(m_1)_{\text{MAX}}$ is determined by eq. (1) in which effectively $\sin^2 \theta_\odot = 0$, $\cos^2 \theta_\odot = 1$, and $\Delta m^2_{\odot} = 0$. Finally, a measured value of $|<m>| < (|<m>|_\text{MIN})_{\text{MIN}} \equiv (|<m>|_\text{MIN})_{\text{MIN}} \equiv \sqrt{(\Delta m^2_{\text{atm}})^\text{MIN} (1 - |U_{e1}|^2)}$ would either indicate that there exist new contributions to the $\beta\beta$0$\nu$-decay rate, or that the SMA MSW solution is not the correct solution of the solar neutrino problem.

3 Conclusions

If $\beta\beta$0$\nu$-decay will be detected by present or upcoming experiments, we will conclude that neutrinos are massive Majorana particles and that the total lepton charge $L$ is not conserved. The observation of the $\beta\beta$0$\nu$-decay with a rate corresponding to $|<m>| \gtrsim 0.02$ eV, which is in the range of sensitivity of the future $\beta\beta$0$\nu$-decay experiments, can provide unique information on the type of neutrino mass spectrum and on the absolute values of neutrino masses. With additional information on the value of neutrino masses from $\beta$ decay experiments or the type of neutrino mass spectrum, one could obtain also information on the CP-violation in the lepton sector, and - if CP-invariance holds - on the relative CP-parities of the massive
Majorana neutrinos. The possibility of establishing CP-nonconservation requires high precision measurements. Given the precision of the future planned ($\beta\beta_{0ν}$-decay and $^3$H $\beta$-decay) experiments, it holds for a limited range of the values of the parameters involved.

4 Acknowledgements

S. P. and S. T. P. would like to thank L. Wolfenstein for stimulating discussions which are at the origin of this paper. S. P. is grateful to the organizers of this Workshop, for having been invited to such an interesting meeting which took place in a nice and stimulating atmosphere. The work of S.P. was partly supported by the Marie Curie Fellowship of the European Community program HUMAN POTENTIAL under contract number HPMT-CT-2000-00096.

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Figure 1: The dependence of $|<m>|$ on $m_1$ for $\Delta m^2_{21} = \Delta m^2_{31}$ in the case of 3-$\nu$ mixing and of the LMA MSW solution obtained at 90\% C.L. in ref.\cite{1}, for $\cos 2\theta_\odot = 0.1$ (upper panels), $\cos 2\theta_\odot = 0.3$ (middle panels), $\cos 2\theta_\odot = 0.54$ (lower panels), and for $|U_{e3}|^2 = 0.05$ (right panels) and $|U_{e3}|^2 = 0.01$ (left panels). The allowed values of $|<m>|$ are constrained to lie in the case of CP-conservation i) in the medium-grey region between the two thick solid lines if $\eta_{21} = \eta_{31} = 1$, ii) in the medium-grey region between the two long-dashed lines and the axes if $\eta_{21} = -\eta_{31} = 1$, iii) in the medium-grey region between the dash-dotted lines and the axes if $\eta_{21} = -\eta_{31} = -1$, iv) in the medium-grey region between the short-dashed lines if $\eta_{21} = \eta_{31} = -1$. In the case of CP-violation, the allowed region for $|<m>|$ covers all the grey region. Values of $|<m>|$ in the dark grey region signal CP-violation.
Figure 2: The dependence of $|<m>|$ on $m_1$ for $\Delta m_{2\odot}^2 = \Delta m_{3\odot}^2$ in the case of 3-$\nu$ mixing and of the LMA MSW solution obtained at 90\% C.L. in ref.\cite{[23]}, for $\cos 2\theta_{\odot} = 0$ (upper panels), $\cos 2\theta_{\odot} = 0.1$ (middle panels), $\cos 2\theta_{\odot} = 0.3$ (lower panels), and for $|U_{e1}|^2 = 0.05$ (right panels) and $|U_{e1}|^2 = 0.005$ (left panels). The allowed regions for $|<m>|$ correspond to: for $|U_{e1}|^2 = 0.005$ i) the medium-grey regions between the solid lines if $\eta_{21} = \eta_{31} = \pm 1$, ii) the medium-grey regions between the dashed lines (lowest left and middle left panels) or the dashed line (upper left panel) if $\eta_{21} = -\eta_{31} = \pm 1$, and all the grey regions if CP-invariance does not hold, and, for $|U_{e1}|^2 = 0.05$, iii) the solid lines if $\eta_{21} = \eta_{31} = 1$, iv) the long-dashed lines if $\eta_{21} = -\eta_{31} = 1$, v) the dashed-dotted lines if $\eta_{21} = -\eta_{31} = -1$, vi) the short-dashed lines if $\eta_{21} = \eta_{31} = -1$, and all the grey regions if CP-invariance does not hold. Values of $|<m>|$ in the dark grey region signal CP-violation.