Analysis of the planetary mass uncertainties on the atmospheric retrieval accuracy

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ABSTRACT

Context. The properties of the atmospheres of the exoplanets depend on several interconnected parameters, making it difficult to determine them. The mass of the planets plays a role in determining the scale height of atmospheres, similarly to that covered by the average molecular weight of the gas. Analogously, the clouds, which mask the real atmospheric scale height, make difficult to correctly derive atmospheric properties.

Aims. We investigated the relevance of planetary mass knowledge in spectral retrievals, identifying in which cases a mass measurement is needed for clear or cloudy, primary or secondary atmospheres, and at which precision, in the context of the ESA M4 Ariel Mission.

Methods. We used TauREx to simulate the Ariel transmission spectra of representative targets of the Ariel mission reference sample assuming different scenarios: a primordial cloudy atmosphere of a hot-Jupiter and hot-Neptune and a secondary atmosphere of a super-Earth, also in presence of clouds. We extract information about various properties of the atmospheres for the cases of unknown mass, or mass with different uncertainty. We also test how the signal-to-noise impacts the atmospheric retrieval for different wavelength ranges.

Results. We accurately retrieved the primordial atmospheric composition independently from mass uncertainties for clear atmospheres, while the uncertainties increased for high altitude clouds. We highlighted the importance of signal-to-noise ratio in the Rayleigh scattering region of the spectrum, which is crucial to retrieve the clouds pressure and to accurately retrieve all the other parameters. For the secondary atmosphere cases a mass uncertainty no larger than 50% is sufficient to retrieve the atmospheric parameters, even in presence of clouds.

Conclusions. Our analysis suggests that even in worst case scenarios a 50% mass precision level is enough for producing reliable retrievals, while an atmospheric retrieval without any knowledge of a planetary mass could lead to biases in cloudy primary atmosphere and in secondary atmosphere.

Key words. atmospheres

1. Introduction

In the last decade our knowledge about the exoplanet atmospheres has been revolutionised. The majority of planets for which we have obtained detailed atmospheric information transit their parent star. Atmospheres of about sixty exoplanets have been observed using transmission spectroscopy. Through modelling of the transmission spectra of exoplanets we are able to extract information about various properties and processes in the atmosphere (Charbonneau et al. 2002; Tinetti et al. 2007; Swain et al. 2008; Kreidberg et al. 2014; Schwarz et al. 2015; Sing et al. 2016; Hoeijmakers et al. 2018; de Wit et al. 2018; Tsiaras et al. 2019; Brogi & Line 2019; Welbanks et al. 2019; Edwards et al. 2020; Changeat & Edwards 2021; Changeat et al. 2022; Roudier et al. 2021; Yip et al. 2021a). This is commonly done through a forward model, which generates a spectrum from atmospheric parameters and a parameter estimation scheme, which samples the parameter space to calculate the probability distribution of the set of parameters. This method, called atmospheric retrieval, has become a fundamental tool to explain individual observations from transit, eclipse and phase curves spectroscopy at both low and high-resolution.

With NASA’s Kepler (Borucki et al. 2010) and Transiting Exoplanet Survey Satellite (TESS, Ricker et al. 2015) we have already identified a large number of targets suitable for atmospheric characterisation with the Hubble Space Telescope (HST) as well as the James Webb Space Telescope (JWST, Greene et al. 2016). A new generation of observatories from space and the ground and dedicated missions will come online, offering a broader spectral coverage, higher signal-to-noise ratio (S/N), allowing us to study a significantly larger number of targets. The ESA-Ariel mission alone was designed for this purpose: it will provide transit, eclipse and phase-curve spectra of hundreds of planets. It is expected to revolutionise our understanding of the physical and chemical properties of a large and diverse sample of extrasolar worlds. To maximise the science return of Ariel, the observations will be performed in four tiers (Tinetti et al. 2021), each one with different binning of the spectra in order to reach the required signal-to-noise ratio, and for a decreasing number of targets aiming to obtain both an unprecedented statistics of planetary atmospheres and their full characterisation for a number of benchmark cases.

Most of the planets with mass measurements, mainly coming from radial velocity follow-up confirmations, have typical error
bars of the order of 10%, in particular for planet with $M > 0.1\ M_J$. Planets smaller than Neptune have larger mass errors, often larger than 40-50%. This uncertainty may contribute to the degeneracy in retrieving the mean molecular weight of the atmosphere, especially when clouds are present (Batalha et al. 2019).

In a previous work, Changeat et al. (2020) using a set of simulations have performed the atmospheric retrieval studying the influence of the knowledge of the planetary mass on the retrieved parameters. In particular they found that for clear-sky gaseous atmospheres the results obtained when the mass is known or retrieved as a free parameters are the same. In the case of a secondary atmospheres the retrievals are more challenging due to the higher degree of freedom for the atmospheric main components. In particular when clouds are added the mass uncertainties may impact substantially the retrieval due to the degeneracy with the mean molecular weight.

In this context, we aim to understand how precisely we have to estimate the planetary mass in order to robustly characterise the atmosphere.

According to Edwards & Tinetti (2022), the mission reference sample (MRS) of Ariel will contain a selection of planets that could be observed in the prime mission lifetime. About 2000 will be included in the MRS and half of them will be actually investigated by Ariel. Today about 570 of them are confirmed planets and for ~ 500 of them we have an estimates of the mass. In Fig. 1 we reported the mass-radius relation for the planets that have a mass estimates and we highlighted with red dots the targets analysed in this work.

![Mass-Radius distribution of the planets of the mission reference sample (MRS) for which we have an estimation of the mass. In red we highlighted the targets analysed in this work (data courtesy by Edwards).](image)

The results of this work could provide an input for the radial velocity campaigns that should therefore prioritise the most impacted planets. This paper is organised as follows. We describe the methodology used for the retrieval analysis in Sect. 2.1. In Sect. 2.2 we present the analysis performed for the primordial atmosphere cases. The impact of the signal-to-noise ratio on the atmospheric retrieval is discussed in Sect. 2.3. Section 2.4 and Section 2.5 present the retrieval analysis of the clear and cloudy secondary atmosphere cases. Our conclusions follow in Sect. 3.

### 2. Retrieval Analysis

#### 2.1. Methodology

In order to analyse the atmospheric retrieval accuracy and how this depends on the planetary mass uncertainties we used the open-source TauREX 3.1, the new version of TauREX (Waldmann et al. 2015a,b), a fully Bayesian inverse atmospheric retrieval framework (Al-Refaie et al. 2021), to simulate different atmospheric configuration with different star-planet systems and perform retrievals. It uses the highly accurate line lists from the ExoMol (Tennyson et al. 2016), HITRAN (Rothman & Gordon 2014) and HITRAN (Gordon et al. 2016) database to build forward and retrieval models. The molecular cross sections were taken from ExoMol (H$_2$O, Polyansky et al. (2018); CO, Li et al. (2015); CH$_4$, Yurchenko et al. (2017)).

For each tested case we used TauREX in forward mode to generate high-resolution theoretical spectrum. We focused only on transit spectra. We specified the main properties of the star and the planet and the main constituents of the atmosphere using their relative abundances. Then, by convolving the high-resolution spectrum through the instrument model (ArielRad v. 2.4.6, Mugnai et al. (2020), Ariel Payload v. 0.0.5, ExoRad v. 2.1.94) we simulated a spectrum as observed by ARIEL and we used it as the input of the retrieval. The instrument model was obtained for each target and to simulate the Ariel Tier-2 performance we took into account the number of transit required for the Tier-2 to obtain the adequate S/N.

We investigated the parameter space with the nested sampling algorithm MultiNest (Feroz et al. 2009) with 500 live points and an evidence tolerance of 0.5.

In Section 2.2, we tested the case of a hypothetical hot-Jupiter, with parameters based on HD 209458b (see Table 1). In order to investigate the benefits on an increased accuracy in the planetary mass estimation we performed the retrieval when the mass is totally unknown, and so is retrieved as a free parameter, and when we know the mass with an uncertainty of 40% and 10%. Also, in order to test the atmospheric retrieval for a smaller planet, we performed a retrieval for a hot-Neptune around a G star, with parameters based on HD 219666b (see Table 1) considering even in this case a mass uncertainty of 40% and 10%.

In Sec. 2.3, we also discussed the importance to guaranteeing an adequate signal-to-noise ratio by performing the retrieval for the same cases but considering 10.5th-magnitude stars. Furthermore, we compared the retrieval performed on the same object considering different uncertainties at different wavelength ranges to investigate if the retrieval is more sensitive to a specific range of the spectrum.

In Section 2.4, we investigated the case of a hypothetical super-Earth, with parameters based on HD 97658b (see Table 2), one of the target of the ARIEL Target List (Edwards et al. 2019).

We tested three different atmospheric configurations by considering a heavy atmosphere containing a significant fraction of H$_2$O, CO and N$_2$, respectively. Also, in order to test the impact of the mass uncertainties onto the retrieval of the atmospheric properties we considered in our analysis three different mass uncertainties (10%, 30%, 50%). In Section 2.5, we investigated the case of cloudy N$_2$-dominated secondary atmospheres. In order to test the difference in the retrieval of a atmosphere dominated by active gases, that are characterised by traceable molecular features directly observable in the spectrum, we analysed two other different scenarios in which we considered a H$_2$O- and a CO-dominated atmosphere.
Table 1: Planetary and stellar parameters used to produce the forward models and the boundary used in our retrieval analyses for the primordial atmosphere of the Hot-Jupiter and the Neptunian planet.

| Stellar Parameters | HD 209458 | HD 219666 |
|--------------------|-----------|-----------|
| Sp. type           | G0 V      | G5 V      |
| \( R_p (R_\odot) \) | 1.19\(^{(a)}\) | 1.03\(^{(c)}\) |
| \( M_p (M_\odot) \) | 1.23\(^{(a)}\) | 0.92\(^{(c)}\) |
| \( T_p (K) \)      | 6091\(^{(a)}\) | 5527\(^{(c)}\) |
| \( d (pc) \)       | 48\(^{(a)}\) | 94\(^{(c)}\) |
| \( m_c \)          | 7.65\(^{(b)}\) | 9.81\(^{(d)}\) |

| Hot Jupiter - HD 209458b | Input | Boundary |
|--------------------------|-------|----------|
| \( R_p (R_\odot) \)  | 1.39  | (0.9,1.5) |
| \( M_p (M_\odot) \)  | 0.73  | (0.5,1) |
| \( T_p (K) \)        | 1450  | (100,400) |

| Neptunian planet - HD 219666b | Input | Boundary |
|-------------------------------|-------|----------|
| \( R_p (R_\odot) \)  | 0.42  | (0.4,0.44) |
| \( M_p (M_\odot) \)  | 0.05  | (0.02,0.07) |
| \( T_p (K) \)        | 1041  | (100,400) |

Notes. \(^{(a)}\) Stassun et al. (2017) \(^{(b)}\) del Burgo & Allende Prieto (2016) \(^{(c)}\) Esposito et al. (2019) \(^{(d)}\) Hogg et al. (2000).

Table 2: Planetary and stellar parameters used to produce the forward models for the secondary atmosphere of a Super-Earth planet.

| Parameters                  | HD 97658 |
|-----------------------------|----------|
| Sp. type                    | K1 V     |
| \( R_p (R_\odot) \)         | 0.73\(^{(a)}\) |
| \( M_p (M_\odot) \)         | 0.85\(^{(a)}\) |
| \( T_p (K) \)               | 5212\(^{(b)}\) |
| \( d (pc) \)                | 21.546\(^{(b)}\) |
| \( m_c \)                   | 7.78\(^{(b)}\) |
| \( R_p (R_\odot) \)         | 0.189    |
| \( M_p (M_\odot) \)         | 0.02611  |
| \( T_p (K) \)               | 720.33   |

Notes. \(^{(a)}\) Howard et al. (2011) \(^{(b)}\) Ellis et al. (2021).

2.2. Primordial Atmosphere

To investigate the contribution of the planetary mass uncertainties onto the retrieval of a primary atmosphere we simulated a spectrum of a Hot-Jupiter based on HD 209458b and its parent star.

In a previous work Changeat et al. (2020) have already performed a retrieval on this object, comparing the case in which the planetary mass is assumed to be known and in which it is retrieved as a free parameters. In particular they found that for a clear sky atmosphere the knowledge of the mass does not impact the results. However, if clouds are modelled some discrepancies appear only in the retrieval of the radius when the cloud pressure gets closer to \(10^{-3}\) bar and also the retrieved mass becomes less accurate.

Here, we want to investigate the benefits of an increased accuracy in the planetary mass estimation on the atmospheric retrieval. We used the same parameters used by Changeat et al. (2020). In particular, for trace gases, we have included H\(_2\)O, CH\(_4\), and CO, with mixing ratios \(10^{-2}\), \(5 \times 10^{-6}\) and \(10^{-4}\), respectively. We first simulated a clear sky atmosphere case and then we tested the behaviour of the retrievals when clouds are present with four different configurations (P\(_{\text{clouds}}\) = \(10^{-3}\), \(10^{-2}\), \(5 \times 10^{-2}\) and \(10^{-3}\) bar for our worst-case scenario). For each scenario we performed the retrieval for three cases: in the first case, the planetary mass is retrieved as a free parameters (we used a large boundary range, by supposing a mass uncertainty of about 100%, so we can assume the mass as totally unknown); in the second, we supposed to know the mass with an uncertainty of 40% and in the third case with an uncertainty of 10%.

We also performed the retrieval for a Neptunian planet around a G star to investigate how the retrieval depends from the planet characteristic.

The parameters used to generate the forward model and the prior bounds employed for each fitted parameter are reported in Table 1.

In Fig. 2 we compare the results obtained for a hot-Jupiter orbiting around a G star, as a function of clouds pressure, in the case when we know the mass with an uncertainty of 40% (in green), of 10% (in magenta), and in which the mass is totally unknown (in orange). The discrepancies in the retrieval of the radius, which appear when the clouds pressure gets closer to \(10^{-3}\) bar, and which are the same as obtained by Changeat et al. (2020), disappear when we performed the retrieval considering a mass uncertainty of about 40% or lesser. In these cases the retrieved radius for high altitude clouds is within \(1\sigma\) of the true value. Also, for all the parameters we obtained a more accurate and precise retrieval when we know the mass with an uncertainty of 40% also in case with high altitude clouds.

Focusing on the retrieval of the mass, in Fig. 3 we compare the results of the normalised retrieved mass of each tested case obtained for the mass as totally unknown and for the mass with an uncertainty of 40% and 10%. The mass is well retrieved for all cases with clouds at low altitudes even when we totally unknown the mass. The retrieved mass becomes less accurate when the clouds pressure is lower than \(10^{-2}\) bar. With a mass uncertainty of 40% we significantly increase the accuracy and precision in the normalised retrieved mass. Indeed, in this case the mass is well retrieved even for high altitude clouds and the retrieved values are within \(1\sigma\) with the true values. Additionally, we note that while a better estimation of the mass (mass uncertainty of 10%), could allow us to retrieve the mass and the radius with more precision also in the cases with high altitude clouds, we do not observe significant difference between the results ob-
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Fig. 2: Comparison between the results obtained from the retrieval performed for the case of a hot-Jupiter around a G star when the mass is known with an uncertainty of 40% (in green), of 10% (in magenta) and when is totally unknown (in orange) as a function of clouds pressure. In the gray area we reported the results obtained for a $P_{\text{clouds}} = 10^{-3}$ bar assuming noise decreased by a factor of two. The size of the box and the error bar represent the points within 1$\sigma$ and 2$\sigma$ of the median of the distribution (highlighted with solid-lines), respectively. The blue line is the real value.

The results obtained from the retrieval of the analysed cases are summarised in Table C.1 in Appendix C.

To test the atmospheric retrieval in the case of smaller planet we simulated a spectrum of a Neptunian planet, based on HD 219666b around its host star (Table 1). We used the same parameters of the hot-Jupiter case for the atmospheric composition. In Tab. C.2 in Appendix C we summarised the results obtained from this test. In this case some discrepancies appear in the atmospheric retrieval when the clouds pressure gets closer to $10^{-3}$ bar. All the other parameters are well retrieved even for high altitude clouds. Focusing on the retrieval of the mass, we note that the mass is well retrieved for all the cases at lower altitudes, while the retrieved mass becomes less accurate when the clouds pressure decreases. In all tested cases for neptunian planets we can refine the mass to within 20% provided the initial mass uncertainty is $\leq 40\%$.

We note that the discrepancies obtained in the atmospheric retrieval at high altitude clouds does not disappear when the mass is known with an uncertainty of 10%. In this case while the retrieval of the mass increase in precision, as expected, the mass seems to not impact on the retrieval of the atmospheric composition.

2.3. Signal-to-noise ratio impact on the atmospheric retrieval at different wavelength ranges

Changeat et al. (2020) have highlighted the importance of guaranteeing the adequate signal-to-noise ratio when we observe heavy secondary atmosphere, by suggesting that an adequate
it is not compatible at 1 uncertainties obtained for the 8th-magnitude star case and also uncertainties of all the fitted parameters increase with respect to the implies a lower S with solid-lines), respectively. The blue line is the real value.

As expected, in the case of 10.5th-magnitude star the uncertainties obtained in the previous section, in which we considered 8th-magnitude star regardless of the height of the clouds. In Fig. 9 we show the S/N is necessary to estimate correctly the mass through transit spectroscopy.

Here, we test the importance of the S/N for the primary atmosphere. To this purpose, in Fig. 4, we compare the results obtained in the previous section, in which we considered 8th-magnitude G star, with the results obtained for the same planet supposed in the previous section, orbiting around a 10.5th-magnitude star. Of course, a higher magnitude for the star implies a lower S/N. The results of this test are summarised in Table C.3 in Appendix C.

As expected, in the case of 10.5th-magnitude star the uncertainties of all the fitted parameters increase with respect to the uncertainties obtained for the 8th-magnitude star case and also increase when the clouds pressure decreases. The retrieved values are within 1σ of the true values excepted for the CO mixing ratio for which the accuracy decrease at high altitude clouds and it is not compatible at 1σ with the true value.

Focusing on the retrieval of the mass, see Fig. 5, we note that the mass is retrieved for all cases but with less accuracy and precision, and still within 1σ of the true value, in the case of 10.5th-magnitude star regardless of the height of the clouds.

In addition, we investigated the impact of the S/N of specific range of the spectrum onto the atmospheric retrieval. We performed a retrieval analysis of a jovian planet around a G star considering a clouds pressure of $10^{-1}$ bar. We decided to split the spectrum in six different ranges (see Fig. 6), each of which dominated by a different atmospheric features, in order to understand which range of the spectrum provides the main contribution to the retrieval.

For each retrieval we changed the error bars of the points within one of the six ranges of the spectrum from $σ = 3 \times 10^{-5}$ (the yellow band in Fig. 7) to $σ = 5 \times 10^{-5}$ (the green boxes) and $σ = 10^{-5}$ (the orange boxes).

Also, to better understand the contribution of the spectrum at low wavelengths we decide to performed other two cases: in the first case we totally excluded the first point (the magenta box in Fig. 7), in the second case we excluded all the points of the first section (the cyan box).

Fig. 8 suggests that we are not able to correctly retrieve the clouds pressure when we totally exclude the points of the first section. This result confirms that the wavelength range between 0.5 and 2 μ contains information of the features of the clouds, as also suggested by Yip et al. (2021a,b). This result highlights the importance of the continuous wavelengths coverage of the blue end of the spectrum that allow us to fit for more complicated cloud models and probe the presence of species like H$_2$O and CH$_4$.

For all the other parameters, excepted for the temperature, we can see an increase of the uncertainties with a decrease in the S/N in the first section. Since in the first section the main contribution to the spectrum is due to the clouds component, this result suggests that all the parameters of the retrieval, excepted the temperature, are impacted by the clouds pressure knowledge. However, whilst the cloud is harder to constrain, in case at low altitude clouds we are still able to constrain the atmospheric parameters, which is encouraging if there were cases where we cannot use the shortwave region.

2.4. Secondary Atmosphere

The atmospheric retrieval of Earths and super-Earths are challenging because the mean molecular weight, $μ$, is completely unconstrained (assumption of $μ \sim 2.3$ is no longer valid). Furthermore, diatomic background gases such as H$_2$ and N$_2$ referred to as spectrally inactive gases, do not exhibit strong vibrational absorptions bands, so they have not directly observable features in the spectrum. Additionally lower-mass planets tend to not have precise mass measurements.

To investigate how the mass uncertainties could impact in the retrieval of low-mass planets we considered a secondary atmosphere consisting of element heavier than H/He. The super-Earth simulated here is based on HD 97658b. The parameters used in our model are reported in Table 2.

We considered a N$_2$-dominated atmosphere and we used the inactive gas N$_2$ to increase the mean molecular weight of the atmosphere and simulate a heavy atmospheres around a rocky planet. We also included H$_2$O and CH$_4$ as trace gases fixing their absolute abundances at $10^{-4}$ and $6 \times 10^{-4}$ respectively. The rest of the atmosphere is filled with a combination of H$_2$ and He. We considered four different scenarios with different values for the mean molecular weight ($μ = 2.3$, $N_2$/He = $10^{-10}$; $μ = 5.2$, $N_2$/He = $1$; $μ = 7.6$, $N_2$/He = $2$; $μ = 11.1$, $N_2$/He = $4$) to explore different composition of the atmosphere. The highest considered mean molecular weight was selected to have atmospheric features detectable by an instrument such as Ariel, but other worst-case scenario for this planet could exist. For instance, a pure Venus-like CO$_2$ atmosphere would not be detectable without impacting the science objectives of the Ariel mission. Also, in order to test the impact of the mass uncertainties we performed the retrieval considering a mass uncertainty of about 10%, 30% and 50% and the case in which the mass is totally unknown (by using very large boundary for the mass parameter). In Fig. 9 we show the impact of the mass uncertainties on the atmospheric retrievals of different scenarios in which we considered heavy secondary atmospheres represented by increasing values of $μ$. In Appendix C we summarised the results and in Appendix A we reported some examples of corner plots obtained from these analyses.
Fig. 4: Comparison between the results obtained from the retrieval performed for the case of a hot-Jupiter around a 8th-magnitude G star (in green) and around a 10.5th-magnitude G star (in orange) as a function of clouds pressure, assuming mass uncertainty of about 40%. The size of the box and the error bar represent the points within 1σ and 2σ of the median of the distribution (highlighted with a red and an orange solid line), respectively. The blue line is the real value.

Fig. 5: Comparison between the retrieved mass for the case of a hot-Jupiter around a 8th-magnitude G star (in green) and around a 10.5th-magnitude G star (in orange) as a function of clouds pressure (mass uncertainty of about 40%). Scale colours is the same adopted for Fig. 4.

Fig. 6: Example of spectrum obtained for a primordial atmosphere case with a cloud pressure of $10^{-3}$ bar. We highlighted the range of the spectrum in which we expected the main contribution of H$_2$O, CH$_4$ and CO. We selected six different ranges of the spectrum and we increased the S/N in each of them to investigate the contribution of each range on the retrieval (the points at the edge of the ranges belong to both of the adjacent selected sections).
Fig. 7: Test of the impact of the S/N in each of the selected range of the spectrum performed on the primordial atmosphere of the hot-Jupiter around a 8th-magnitude G star. In green the retrieval performed considering an error of about $5 \times 10^{-5}$. In orange the retrieval obtained with an error of about $1 \times 10^{-4}$. The yellow band highlights the values retrieved in the original case ($\delta \simeq 3 \times 10^{-5}$). The magenta and cyan boxes represent the distributions of the values obtained by performing the retrieval without the first point or without the entire section 1 of Fig. 6, respectively. The blue line highlights the true value.

Fig. 8: Impact of the S/N in each of the selected range of the spectrum on the retrieved clouds pressure. The scale colour and the description of the figure are the same adopted in Fig. 7.

We note that a mass estimation with an uncertainty equal or lesser than 50% could help us to better constrain the mean molecular weight for all the tested cases with different mean molecular weight.

From this plot we do not see significant differences in the retrieved atmospheric parameters obtained performing the retrieval when we know the mass with different uncertainties; this is not surprising, because for the high mean molecular weight atmosphere the scale height is relatively small, so changes in the gravity will not produce such large differences in the spectrum. However, as expected the retrieved mass shows a correlation with the mass uncertainty.

In Fig. 9 we highlight the MAP (maximum-a-posteriori) parameters with the circle points. In the cases with higher $\mu$ some discrepancies appear in the temperature and $\text{H}_2\text{O}$ retrieved values with respect to the true values, and in some cases we obtained retrieved values that are not within $1\sigma$ of the true values. Although, we note that in these cases, even when we have a larger distribution, the MAP values obtained from the retrieval are totally consistent with the true values.

However, we note some discrepancies in the retrieved MAP of $\text{N}_2$/He when $\mu = 2.3$. This result suggests that we are not able to constrain this ratio. In these cases we only define a possible range of values and some performed tests proved us that this result does not depends from the choice of the prior limits.
Furthermore, from Fig. 10 we can see a slight trend between the mean molecular weight and the mass uncertainties. In particular, we note that for a mass uncertainty lower than 50% we are able to retrieve the mean molecular weight with higher accuracy (mostly if we consider the MAP values) with respect to the unknown mass cases, in particular for the heavier atmospheres and with a slight increase precision when we performed the retrieval with a mass uncertainty of 10%. This could be probably due to the higher accuracy/precision in the retrieval of the N$_2$/He when we consider a mass uncertainty of 10%.

All these results suggest that we could be able to correctly retrieve the atmospheric parameters of a secondary atmosphere with a clear sky even when we know the mass with an uncertainty of 50%, despite we considered our worst-case scenario to assess the degeneracy between the mass and the mean molecular weight. Our analysis, also, suggests that this degeneracy is intrinsic to secondary atmospheres and not directly connected with the mass uncertainty. Despite that, precisely by virtue of this degeneracy, a more accurate estimate of the mass obtained from an independent determination could help to break the degeneracy, increasing the accuracy in the determination of the abundances of the fill gases.

We also tested the atmospheric retrieval of an analogue scenario but considering a 10.5th-magnitude stars. The results obtained from this test are reported in Table C.5. In this case, we obtained similar results with respect to the previous case in which we considered a 8th-magnitude star. However, due to the lower S/N, we obtained a larger uncertainties for all the parameters, also for the cases with lower mean molecular weight.

2.5. Cloudy Secondary Atmosphere

Finally, we investigate the case of cloudy secondary atmospheres.

Small planets might not have a H$_2$-dominated atmosphere and the dominant gas is often unknown. We decide to investigate three different scenarios: in the first scenario we considered a nitrogen-dominated atmosphere representative of a rocky planet to investigate the retrieval results and compare it with the case without clouds (see Sec. 2.4). Also, in order to evidence the difference in the atmospheric retrieval when an active gas dominates in the transmission spectrum we analysed the second and third scenarios in which we considered a H$_2$O-dominated and a CO-dominated atmosphere, respectively. Atmospheres dominated by species such as H$_2$O or CO would have traceable molecular features directly observable in the spectrum, as we can see from Fig. 11 in which we compare the observed spectrum and
the fitted model obtained for a N$_2$-, H$_2$O- and CO-dominated atmosphere in the case of $\mu = 5.2$ and with a $P_{\text{clouds}} = 5 \times 10^{-2}$ bar. These cases represent a more favourable scenario for the inverse models with respect to the N$_2$-dominated.

In Appendix C we report the results obtained from the analysis of the cloudy secondary atmosphere in the three different scenarios and in all the configuration of mean molecular weight for the clouds pressure $10^{-1}$ and $10^{-3}$ bar.

**N$_2$-dominated Atmosphere** Fig. 12 shows the case $\mu = 5.2$ in which we compared the results obtained for different clouds pressure. In this scenario the atmosphere is lighter and presents a better signal. From Fig. 12h we can see that with a mass uncertainty equal or lesser than 30% we significantly increase the accuracy and the precision on the retrieval of N$_2$/He, in particular in the cases with higher clouds pressure while, if we consider the MAP values, we increase the accuracy also in the worst scenario with lower clouds pressure. These results are reflected in the determination of the mean molecular weight. Indeed, from Fig. 12e we note that with a mass uncertainty equal or lesser than 30% we are able to retrieve the mean molecular weight and, as we expect, the width of the values distributions increase (and consequently also the uncertainties associated to the median values) with decreasing the clouds pressure. It seems that the mass uncertainty does not impact the retrievals of the CH$_4$ mixing ratio. The H$_2$O mixing ratio, see Fig. 12f, shows some discrepancies between the retrieved values and the true values although the MAP values are compatible with the true values. However, these results do not show a correlation with the mass uncertainty while they could be connected with the discrepancies showed in the clouds pressure retrieval (see Fig. 12d).

In Fig. 13 we considered the heaviest scenario ($\mu = 11.1$). In this case we are not able to constrain the mean molecular weight. From Fig. 13e we can see that the retrieved $\mu$ tends to be larger than the true value but these results are not correlated with the mass uncertainties. Additionally, as we expected, the retrieved $\mu$ present larger uncertainties when the clouds pressure decreases.

Fig. 13d suggests that the mass uncertainties do not impact the retrieved clouds pressure. Indeed, we do not see significant discrepancies in the retrieved distribution with respect to the mass uncertainty. However, we note a better compatibility between the true values and the MAP values when we considered a mass uncertainties of 10%.

About the atmospheric parameters, the CH$_4$ mixing ratio is well retrieved also when the clouds pressure get closer to $10^{-3}$ bar, while we are not able to accurately retrieve the H$_2$O mixing ratio, in particular for clouds pressure lower than $10^{-2}$ bar. Here, additional observations are needed to increase the S/N and to constrain the mean molecular weight.

**H$_2$O and CO-dominated Atmosphere** In Fig. 14 we show the results obtained from the retrieval of H$_2$O-dominated secondary atmosphere ($\mu = 5.2$). In this case the mass uncertainties do not significantly impact the retrieval. Here, we are able to constrain the H$_2$O/He with a slightly increased accuracy for lower mass uncertainties. Also, the clouds pressure and the mean molecular weight are well retrieved, also in the cases with lower clouds pressure. In the worst considered scenario, when the clouds pressure get closer to $10^{-3}$ bar, the retrieved $\mu$ is within 2$\sigma$ of the true value, while the MAP value its closer to the true value.

Also, we performed the retrieval for our worst-case scenario, $\mu = 11.1$ (see Fig. B.1 in Appendix B). From this test we confirmed that for this target we need more observations in order to achieve an adequate S/N. The lower S/N prevent us from correctly retrieve the mean molecular weight which is for all cases larger than the true value. The accuracy in the retrieved $\mu$, that is within 2$\sigma$ of the true value, does not depends on the the mass uncertainties. The uncertainties of the CH$_4$ increase by several orders of magnitude with respect to the case $\mu = 5.2$. This is because, in the case of $\mu = 11.1$ the water features tend to dominate the methane features, present in the redder region of the spectrum, leading to greater uncertainty in the retrieval of CH$_4$.

An analogue behaviour is obtained for the CO-dominated atmosphere (see Fig. 15 and Fig. B.2 in Appendix B). In particular, in this scenario we note a slight trend with the mass uncertainties.
Fig. 12: Impact of the mass uncertainties on the retrieval for different scenarios of cloudy secondary $N_2$-dominated atmospheres in the case of $\mu = 5.2$. The different coloured boxes represent the different mass uncertainties. The blue line highlights the true value. The points alongside the boxes highlight the MAP (maximum-a-posteriori) parameters obtained for each analysed case. The size of the box and the error bar represent the points within $1\sigma$ and $2\sigma$ of the median of the distribution (highlighted with solid-lines), respectively.

in the retrieved CH$_4$ mixing ratio (see Fig. 15g). This increased accuracy in the retrieved CH$_4$ mixing ratio could be linked to the presence of a prominent CO feature in the redder part of the spectrum that allows to better describe and constrain the CH$_4$ component.

All these results confirm that with a more favourable scenario, represented by an atmosphere with a main gas producing spectral signature, we could be able to better constrain the atmospheric parameters and the mean molecular weight. However, these results seem not to be strongly correlated with the mass uncertainty although in some cases a better estimate of the mass can help to obtain more accurate retrievals.
Di Maio et al.: Analysis of planetary mass uncertainties on atmospherical retrieval accuracy

(a) Radius

(b) Temperature

(c) Mass

(d) Clouds

(e) $\mu$

(f) H$_2$O mixing ratio

(g) CH$_4$ mixing ratio

(h) N$_2$/He

Fig. 13: Results obtained from the retrieval of N$_2$-dominated atmosphere in the case of $\mu=11.1$. The different coloured boxes represent the different mass uncertainties. The blue line highlight the true value. The points alongside the boxes highlights the MAP (maximum-a-posteriori) parameters obtained for each analysed case. The size of the box and the error bar represent the points within 1$\sigma$ and 2$\sigma$ of the median of the distribution (highlighted with solid-lines), respectively.

3. Conclusions

In this paper we performed several tests in order to investigate the impact of the planetary mass uncertainties in atmospheric retrieval and to identify in which cases a mass measurements and which precision is needed in presence of clear or cloudy, primary or secondary atmosphere, in the context of the ESA Ariel Mission.

We have considered different scenarios to determine the level of planet mass precision required for robust atmospheric characterisation. We selected three representative targets from the Ariel MRS. For the primordial atmosphere we considered a hot Jupiter and a hot Neptunian and we tested also the importance of the S/N on the retrievals and the role of the spectral bands. We also investigated the retrieval of a secondary atmosphere of a super-Earth, also in presence of clouds. For each planet, we conducted the re-
Fig. 14: Retrieval results obtained from different scenarios of cloudy secondary H$_2$O-dominated atmosphere in the case of $\mu=5.2$. The different coloured boxes represent the different mass uncertainties. The blue line highlights the true value. The points alongside the boxes highlight the MAP (maximum-a-posteriori) parameters obtained for each analysed case. The size of the box and the error bar represent the points within 1$\sigma$ and 2$\sigma$ of the median of the distribution (highlighted with solid-lines), respectively.

Our conclusions are as follows:

1. In the hot-Jupiter case we are able to accurately retrieve the atmospheric composition of the atmosphere with an accuracy that does not depend on the mass uncertainty. In the worst analysed scenario, when the clouds pressure get closer to $10^{-3}$ bar, there is a small discrepancy in the retrieval of the radius that disappears when we performed the retrieval considering a mass uncertainty of about 40% or lower. For all the other parameters the uncertainties increase for high altitude clouds that can be partially mitigated by increasing the S/N.

2. We could use the atmospheric analysis to estimate the mass of hot-Jupiters with more precision than that we already know. For example, we can increase the precision level
Fig. 15: Retrieval results obtained from different scenarios of cloudy secondary CO-dominated atmosphere in the case of $\mu=5.2$. The different coloured boxes represent the different mass uncertainties. The blue line highlights the true value. The points alongside the boxes highlight the MAP (maximum-a-posteriori) parameters obtained for each analysed case. The size of the box and the error bar represent the points within 1$\sigma$ and 2$\sigma$ of the median of the distribution (highlighted with solid-lines), respectively.

1. Of the mass estimation from 40% to 10-25% depending on clouds presence.
2. For faint stars the uncertainties of all the fitted parameters increase, confirming the relevance of S/N, independently from our knowledge on the mass.
3. Analogue considerations can be made about the hot-Neptunian case. We note increased uncertainties in presence of high altitude clouds and in particular a worst estimation of the CO mixing ratio and of the temperature when the clouds pressure get closer to $10^{-3}$ bar. However, these results are independent from the planetary mass uncertainties.
4. The study about how the S/N at different wavelength ranges impacts on the retrieval highlighted the importance of blue end of the Ariel spectrum without which we could not be able to retrieve the cloud pressure, bringing us to a less accurate determination of the other parameters.
5. In the N$_2$-dominated secondary atmosphere case, when we do not consider the presence of clouds, a minimum knowl-
edge of the mass of about 50% allows us to significantly increase the accuracy and the precision of the retrieval, that only slightly improves further if we consider a better estimation of the mass.

7. For a cloudy $N_2$-dominated secondary atmosphere we need an estimation of the mass with an uncertainty of about 50% to correctly retrieve the mean molecular weight. The uncertainties of all the parameters increase for a clouds pressure lower than $10^{-7}$ bar. A better estimation of the mass could moderately help in the determination of the atmospheric parameters in particular increasing the accuracy of the maximum probability values.

8. The test performed for a $H_2O$- and CO-dominated atmosphere highlighted that in presence of a main gas producing spectral signatures we should be able to better constrain the atmospheric parameters and the mean molecular weight. Also in this case a minimum uncertainty of 50% on the mass is sufficient to measure the atmospheric parameters.

Our analysis indicates that, even in the worst cases investigated in this work, in order to obtain an accurate atmospheric characterisation a 50% mass precision level is sufficient. Going into an atmospheric characterisation without any knowledge of a planetary mass could compromise our ability to retrieve the atmospheric composition in cloudy primary atmospheres and in secondary atmospheres.

These results can be used in the preparation and target prioritisation of RV surveys supporting atmospheric characterisation studies. In the preparation of the Ariel mission, this work can help in defining the strategy of a RV monitoring for those targets included in the MRS still lacking of a measurement of their mass.

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Appendix A: \( \text{N}_2 \)-dominated clear sky secondary atmosphere \( \delta M=50\% \)

**Fig. A.1:** Retrieval results obtained for \( \text{N}_2 \)-dominated clear sky secondary atmosphere in the case of \( \mu=2.3 \) and \( \delta M = 50\% \). The blue, green and red vertical solid lines highlight the true, MAP and median values, respectively, while the vertical dashed-lines represent the values at 1\( \sigma \) from the median.
Fig. A.2: Retrieval results obtained for N$_2$-dominated clear sky secondary atmosphere in the case of $\mu=5.2$ and $\delta M = 50\%$. The blue, green and red vertical solid lines highlight the true, MAP and median values, respectively, while the vertical dashed-lines represent the values at 1σ from the median.
Fig. A.3: Retrieval results obtained for N$_2$-dominated clear sky secondary atmosphere in the case of $\mu=7.6$ and $\delta M = 50\%$. The blue, green and red vertical solid lines highlight the true, MAP and median values, respectively, while the vertical dashed-lines represent the values at 1$\sigma$ from the median.
Fig. A.4: Retrieval results obtained for N₂-dominated clear sky secondary atmosphere in the case of μ=11.1 and δM = 50%. The blue, green and red vertical solid lines highlight the true, MAP and median values, respectively, while the vertical dashed-lines represent the values at 1σ from the median.
Appendix B: $\text{H}_2\text{O}$- and CO-dominated atmospheres for $\mu = 11.1$

Fig. B.1: Retrieval results obtained from different scenarios of cloudy secondary $\text{H}_2\text{O}$-dominated atmosphere in the case of $\mu = 11.1$. The different coloured boxes represent the different mass uncertainties. The blue line highlights the true value. The points alongside the boxes highlight the MAP (maximum-a-posteriori) parameters obtained for each analysed case. The size of the box and the error bar represent the points within $1\sigma$ and $2\sigma$ of the median of the distribution (highlighted with solid-lines), respectively.
Fig. B.2: Retrieval results obtained from different scenarios of cloudy secondary CO-dominated atmosphere in the case of $\mu=11.1$. The different coloured boxes represent the different mass uncertainties. The blue line highlights the true value. The points alongside the boxes highlight the MAP (maximum-a-posteriori) parameters obtained for each analysed case. The size of the box and the error bar represent the points within 1$\sigma$ and 2$\sigma$ of the median of the distribution (highlighted with solid-lines), respectively.
### Appendix C: Tables with the results of the analysis of Sect. 2

Table C.1: Results from the retrieval performed for a hot-Jupiter around a G star when we the mass is totally unknown and when we known it with an uncertainty of about 40% and 10%.

| Parameters | Clear Sky | Clouds | Clear Sky | Clouds | Clear Sky | Clouds |
|------------|-----------|--------|-----------|--------|-----------|--------|
| Mass       | <5%       | <10%   | >50%      |        | <2%       | <10%   |
| Radius     |           |        |           |        | <2%       | <10%   |
| Temperature|           |        |           |        | <2%       | <10%   |
| H₂O        | ✔         | ✔      | ✔         | ✔      | ✔         | ✔      |
| CH₄        | ✔         | ✔      | ✔         | ✔      | ✔         | ✔      |
| CO         | ✔         | ✔      | ✔         | ✔      | ✔         | ✔      |
| Clouds     | ✔         | ✔      | ✔         | ✔      | ✔         | ✔      |

**Notes.** Green: well retrieved values; Red: values within 2σ or more of the true value.

Table C.2: Results from the retrieval performed for a neptunian planet around a G star when we know the mass with an uncertainty of about 40% and 10%.

| Parameters | Clear Sky | Clouds | Clear Sky | Clouds |
|------------|-----------|--------|-----------|--------|
| Mass       | <2%       | <20%   | <2%       | <10%   |
| Radius     |           |        |           |        |
| Temperature|           |        |           |        |
| H₂O        | ✔         | ✔      | ✔         | ✔      |
| CH₄        | ✔         | ✔      | ✔         | ✔      |
| CO         | ✔         | ✔      | ✔         | ✔      |
| Clouds     | ✔         | ✔      | ✔         | ✔      |

**Notes.** Green: well retrieved values; Red: values within 2σ or more of the true value.

Table C.3: Results from the retrieval performed for a hot-Jupiter around a 8th-magnitude G star and a 10.5th-magnitude G star.

| Parameters | Clear Sky | Clouds | Clear Sky | Clouds |
|------------|-----------|--------|-----------|--------|
| Mass       | <2%       | <10%   | <10%      | ≃ 20%  |
| Radius     |           |        |           |        |
| Temperature|           |        |           |        |
| H₂O        | ✔         | ✔      | ✔         | ✔      |
| CH₄        | ✔         | ✔      | ✔         | ✔      |
| CO         | ✔         | ✔      | ✔         | ✔      |
| Clouds     | ✔         | ✔      | ✔         | ✔      |

**Notes.** Green: well retrieved values; Red: values within 2σ or more of the true value.
Table C.4: Retrieved values obtained from the analyses performed on a N$_2$-dominated secondary atmosphere of a super-earth around a 8th-magnitude M star in the different considered scenarios.

| Parameter               | True value | 10% | 30% | 50% | Unknown          |
|-------------------------|------------|-----|-----|-----|-----------------|
| $\delta M$              | 0.026      | 0.026$^{+0.001}_{-0.001}$ | 0.026$^{+0.002}_{-0.002}$ | 0.026$^{+0.002}_{-0.002}$ | 0.026$^{+0.002}_{-0.002}$ |
| $\mu$                   | 7.23       | 7.23$^{+0.001}_{-0.001}$ | 7.23$^{+0.002}_{-0.002}$ | 7.23$^{+0.002}_{-0.002}$ | 7.23$^{+0.002}_{-0.002}$ |
| $\log(N/He)$            | -3.2       | -3.2$^{+0.001}_{-0.001}$ | -3.2$^{+0.002}_{-0.002}$ | -3.2$^{+0.002}_{-0.002}$ | -3.2$^{+0.002}_{-0.002}$ |
| Temperature             | 720.33     | 720.33$^{+0.001}_{-0.001}$ | 720.33$^{+0.002}_{-0.002}$ | 720.33$^{+0.002}_{-0.002}$ | 720.33$^{+0.002}_{-0.002}$ |
| $\log(H_2)$             | -4         | -4$^{+0.001}_{-0.001}$ | -4$^{+0.002}_{-0.002}$ | -4$^{+0.002}_{-0.002}$ | -4$^{+0.002}_{-0.002}$ |
| $\log(CH_4)$            | -3.2       | -3.2$^{+0.001}_{-0.001}$ | -3.2$^{+0.002}_{-0.002}$ | -3.2$^{+0.002}_{-0.002}$ | -3.2$^{+0.002}_{-0.002}$ |
| $\log(N_2/He)$          | -10        | -7$^{+0.001}_{-0.001}$ | -7$^{+0.002}_{-0.002}$ | -7$^{+0.002}_{-0.002}$ | -7$^{+0.002}_{-0.002}$ |
| $\log(Mass)$            | 0.026      | 0.026$^{+0.001}_{-0.001}$ | 0.026$^{+0.002}_{-0.002}$ | 0.026$^{+0.002}_{-0.002}$ | 0.026$^{+0.002}_{-0.002}$ |
| $\log(Radius)$          | 1.89       | 1.89$^{+0.001}_{-0.001}$ | 1.89$^{+0.002}_{-0.002}$ | 1.89$^{+0.002}_{-0.002}$ | 1.89$^{+0.002}_{-0.002}$ |
| $\log(O/O)$             | 4          | 4$^{+0.001}_{-0.001}$ | 4$^{+0.002}_{-0.002}$ | 4$^{+0.002}_{-0.002}$ | 4$^{+0.002}_{-0.002}$ |

Notes. We reported the median values with its 1σ error and the MAP (maximum-a-posteriori) values in square brackets.
Table C.5: Retrieved values obtained from the analyses performed on a N\textsubscript{2}-dominated secondary atmosphere of a super-earth around a 10.5th-magnitude M star in the different considered scenarios.

| Parameter | True value | \( \mu = 2.3 \) | \( \mu = 5.2 \) | \( \mu = 7.6 \) | \( \mu = 11.1 \) |
|-----------|------------|-----------------|-----------------|-----------------|-----------------|
| Mass      | 0.026      | 0.026\textsuperscript{[0.029]} \textsuperscript{[0.025]} | 0.028\textsuperscript{[0.027]} \textsuperscript{[0.025]} | 0.030\textsuperscript{[0.029]} \textsuperscript{[0.026]} | 0.032\textsuperscript{[0.030]} \textsuperscript{[0.029]} |
| Radius    | 0.189      | 0.190\textsuperscript{[0.188]} \textsuperscript{[0.189]} | 0.190\textsuperscript{[0.190]} \textsuperscript{[0.190]} | 0.190\textsuperscript{[0.190]} \textsuperscript{[0.190]} | 0.190\textsuperscript{[0.190]} \textsuperscript{[0.190]} |
| Temperature | 720.33 | 772.99 \textsuperscript{[737]} | 803.19 \textsuperscript{[693]} | 624.30 \textsuperscript{[505]} | 624.30 \textsuperscript{[505]} |
| log(H\textsubscript{2}O) | -4 | -5.0\textsuperscript{[4.3]} \textsuperscript{[4.7]} | -5.0\textsuperscript{[4.3]} \textsuperscript{[4.7]} | -5.0\textsuperscript{[4.3]} \textsuperscript{[4.7]} | -5.0\textsuperscript{[4.3]} \textsuperscript{[4.7]} |
| log(CH\textsubscript{4}) | -3.22 | -3.2\textsuperscript{[3.4]} \textsuperscript{[3.6]} | -3.2\textsuperscript{[3.4]} \textsuperscript{[3.6]} | -3.2\textsuperscript{[3.4]} \textsuperscript{[3.6]} | -3.2\textsuperscript{[3.4]} \textsuperscript{[3.6]} |
| H\textsubscript{2}/He | 6.67 | 6.5\textsuperscript{[6.5]} \textsuperscript{[6.10]} | 6.5\textsuperscript{[6.5]} \textsuperscript{[6.10]} | 6.5\textsuperscript{[6.5]} \textsuperscript{[6.10]} | 6.5\textsuperscript{[6.5]} \textsuperscript{[6.10]} |
| log(N\textsubscript{2}/He) | -10 | -7\textsuperscript{[8]} \textsuperscript{[11.7]} | -7\textsuperscript{[8]} \textsuperscript{[11.7]} | -7\textsuperscript{[8]} \textsuperscript{[11.7]} | -7\textsuperscript{[8]} \textsuperscript{[11.7]} |
| \( \mu \) | 2.3 | 2.3\textsuperscript{[2.29]} \textsuperscript{[2.29]} | 2.3\textsuperscript{[2.29]} \textsuperscript{[2.29]} | 2.3\textsuperscript{[2.29]} \textsuperscript{[2.29]} | 2.3\textsuperscript{[2.29]} \textsuperscript{[2.29]} |

Notes. We reported the median values with its 1\( \sigma \) error and the MAP (maximum-a-posteriori) values in square brackets.
Table C.6: Retrieved values obtained from the analyses performed on a N$_2$-dominated cloudy secondary atmosphere (P$_{\text{clouds}} = 10^{-3}$ bar) of a super-earth around a 8th-magnitude M star in the different considered scenarios.

| Parameter | True value | $\mu = 2.3$ | $\mu = 5.2$ | $\mu = 7.6$ | $\mu = 11.1$ |
|-----------|------------|-------------|-------------|-------------|-------------|
| Mass      | $0.266 \pm 0.005$ | $0.026 \pm 0.006$ | $0.028 \pm 0.006$ | $0.028 \pm 0.006$ | $0.026 \pm 0.006$ |
| Radius    | $0.189 \pm 0.009$ | $0.191 \pm 0.009$ | $0.193 \pm 0.009$ | $0.193 \pm 0.009$ | $0.191 \pm 0.009$ |
| Temperature | $720.333$ | $727 \pm 7$ | $747 \pm 7$ | $768 \pm 7$ | $798 \pm 7$ |
| log(H$_2$O) | $-4$ | $-6 \pm 1$ | $-5.8 \pm 0.9$ | $-5.7 \pm 0.8$ | $-5.7 \pm 0.8$ |
| log(CH$_4$) | $-3.22$ | $-3.6 \pm 0.9$ | $-3.7 \pm 0.9$ | $-3.6 \pm 0.9$ | $-3.6 \pm 0.9$ |
| log(N$_2$/He) | $-10$ | $-7.4 \pm 10$ | $-7.4 \pm 10$ | $-7.4 \pm 10$ | $-7.4 \pm 10$ |
| log(P$_{\text{clouds}}$) | $-3$ | $-2.6 \pm 0.3$ | $-2.5 \pm 0.3$ | $-2.5 \pm 0.3$ | $-2.5 \pm 0.3$ |
| $\mu$ | $2.3$ | $2.281 \pm 0.006$ | $2.280 \pm 0.006$ | $2.281 \pm 0.006$ | $2.281 \pm 0.006$ |
|          | $5.2$ | $9 \pm 4$ | $2.6 \pm 1$ | $2.5 \pm 2$ | $2.8 \pm 2$ |
|          | $7.6$ | $-2.4 \pm 4$ | $-1.7 \pm 4$ | $-1.7 \pm 4$ | $-1.7 \pm 4$ |
|          | $11.1$ | $10 \pm 4$ | $10 \pm 4$ | $10 \pm 4$ | $10 \pm 4$ |

Notes. We reported the median values with its 1$\sigma$ error and the MAP (maximum-a-posteriori) values in square brackets.
Table C.7: Retrieved values obtained from the analyses performed on a N$_2$-dominated cloudy secondary atmosphere (P$_{\text{clouds}} = 10^{-1}$ bar) of a super-earth around a 8th-magnitude M star in the different considered scenarios.

| Parameter | True value | 10% | 30% | 50% | Unknown |
|-----------|------------|-----|-----|-----|---------|
| Mass | 0.026 | 0.026 | 0.026 | 0.026 | 0.026 | 0.026 |
| Radius | 0.189 | 0.189 | 0.189 | 0.189 | 0.189 | 0.189 |
| Temperature | 720.33 | 727.32 | 729.35 | 728.30 | 728.35 | 729.31 |
| log(H$_2$O) | -4 | -3.9 | -4.2 | -4.0 | -4.3 | -4.0 |
| log(CH$_4$) | -3.22 | -3.11 | -3.4 | -3.1 | -3.1 | -3.2 |
| log(N$_2$/He) | -10 | -7.4 | -7.4 | -7.6 | -7.6 | -9.0 |
| log(P$_{\text{clouds}}$) | -1 | -1.1 | -0.9 | -1.1 | -0.9 | -1.0 |
| $\mu$ | 2.3 | 2.289 | 2.289 | 2.288 | 2.289 | 2.289 |

| Parameter | True value | 10% | 30% | 50% | Unknown |
|-----------|------------|-----|-----|-----|---------|
| Mass | 0.026 | 0.026 | 0.027 | 0.037 | 0.075 | 0.064 |
| Radius | 0.189 | 0.189 | 0.190 | 0.190 | 0.190 | 0.190 |
| Temperature | 720 | 789 | 757 | 685 | 788 | 777 |
| log(H$_2$O) | -4 | -6.4 | -3.9 | -6.4 | -6.4 | -6.4 |
| log(CH$_4$) | -3.22 | -3.3 | -2.6 | -3.4 | -3.3 | -3.3 |
| log(N$_2$/He) | -1 | 0.1 | -0.1 | -0.1 | -0.1 | -0.1 |
| log(P$_{\text{clouds}}$) | -1 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 |
| $\mu$ | 5.2 | 6.6 | 6.6 | 6.6 | 6.6 | 6.6 |

| Parameter | True value | 10% | 30% | 50% | Unknown |
|-----------|------------|-----|-----|-----|---------|
| Mass | 0.026 | 0.026 | 0.027 | 0.027 | 0.085 | 0.084 |
| Radius | 0.189 | 0.189 | 0.190 | 0.190 | 0.190 | 0.190 |
| Temperature | 720.33 | 896 | 849 | 860 | 681 | 778 |
| log(H$_2$O) | -4 | -7.1 | -4.3 | -7.1 | -7.1 | -7.1 |
| log(CH$_4$) | -3.22 | -3.3 | -2.6 | -3.3 | -3.3 | -3.3 |
| log(N$_2$/He) | 0.3 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 |
| log(P$_{\text{clouds}}$) | 0.5 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 |
| $\mu$ | 7.6 | 12.5 | 11.8 | 11.8 | 11.8 | 11.8 |

| Parameter | True value | 10% | 30% | 50% | Unknown |
|-----------|------------|-----|-----|-----|---------|
| Mass | 0.026 | 0.026 | 0.026 | 0.030 | 0.080 | 0.092 |
| Radius | 0.189 | 0.189 | 0.190 | 0.190 | 0.190 | 0.190 |
| Temperature | 720.33 | 984 | 873 | 929 | 615 | 624 |
| log(H$_2$O) | -4 | -8.1 | -5.5 | -8.1 | -8.1 | -8.1 |
| log(CH$_4$) | -3.22 | -3.4 | -3.6 | -3.4 | -3.4 | -3.4 |
| log(N$_2$/He) | 0.6 | 1.2 | 1.1 | 1.2 | 1.2 | 1.2 |
| log(P$_{\text{clouds}}$) | -1 | 0.3 | 0.0 | 0.4 | 0.4 | 0.4 |
| $\mu$ | 11.1 | 20.5 | 18.9 | 18.9 | 18.9 | 18.9 |

Notes. We reported the median values with its 1σ error and the MAP (maximum-a-posteriori) values in square brackets.
Table C.8: Retrieved values obtained from the analyses performed on a H$_2$O-dominated cloudy secondary atmosphere ($P_{\text{clouds}} = 10^{-3}$ bar) of a super-earth around a 8th-magnitude M star in the different considered scenarios.

| Parameter | True value | $\mu = 2.3$ | $\mu = 5.2$ | $\mu = 7.6$ | $\mu = 11.1$ |
|-----------|------------|--------------|--------------|--------------|--------------|
| $dM$      |            | 10%          | 30%          | 50%          | Unknown      |
| Mass      | 0.026      | 0.026$^{+0.002}_{-0.002}$ [0.024] | 0.029$^{+0.005}_{-0.005}$ [0.024] | 0.032$^{+0.006}_{-0.006}$ [0.024] | 0.034$^{+0.007}_{-0.007}$ [0.029] |
| Radius    | 0.189      | 0.191$^{+0.004}_{-0.004}$ [0.188] | 0.192$^{+0.005}_{-0.005}$ [0.187] | 0.194$^{+0.006}_{-0.006}$ [0.187] | 0.194$^{+0.006}_{-0.006}$ [0.191] |
| Temperature | 720.33   | 711$^{+46.8}_{-46.8}$ [715] | 736$^{+9.7}_{-9.7}$ [715] | 758$^{+10.6}_{-10.6}$ [721] | 774$^{+10.8}_{-10.8}$ [737] |
| log(CH$_4$) | -3.22    | -3.7$^{+0.5}_{-0.5}$ [-3.0] | -3.7$^{+0.3}_{-0.3}$ [-3.0] | -3.8$^{+0.3}_{-0.3}$ [-3.0] | -3.7$^{+0.6}_{-0.6}$ [-3.4] |
| log(H$_2$O/He) | -1.22 | -1.7$^{+0.2}_{-0.2}$ [-0.9] | -1.8$^{+0.3}_{-0.3}$ [-1.4] | -1.9$^{+0.4}_{-0.4}$ [-1.0] | -1.8$^{+0.6}_{-0.6}$ [-1.4] |
| log($P_{\text{clouds}}$) | -3 | -2.5$^{+0.5}_{-0.5}$ [-3.2] | -2.4$^{+0.5}_{-0.5}$ [-3.0] | -2.3$^{+0.5}_{-0.5}$ [-3.3] | -2.3$^{+0.5}_{-0.5}$ [-2.8] |
| $\mu$      | 2.3        | 2.32$^{+0.03}_{-0.02}$ [2.50] | 2.31$^{+0.03}_{-0.02}$ [2.35] | 2.30$^{+0.03}_{-0.02}$ [2.45] | 2.31$^{+0.03}_{-0.02}$ [2.34] |

| Parameter | True value | $\mu = 5.2$ | $\mu = 7.6$ | $\mu = 11.1$ |
|-----------|------------|--------------|--------------|--------------|
| $dM$      |            | 10%          | 30%          | 50%          |
| Mass      | 0.026      | 0.026$^{+0.002}_{-0.002}$ [0.023] | 0.028$^{+0.006}_{-0.006}$ [0.032] | 0.029$^{+0.007}_{-0.007}$ [0.034] | 0.077$^{+0.014}_{-0.014}$ [0.079] |
| Radius    | 0.189      | 0.194$^{+0.003}_{-0.003}$ [0.188] | 0.194$^{+0.006}_{-0.006}$ [0.191] | 0.194$^{+0.007}_{-0.007}$ [0.190] | 0.195$^{+0.007}_{-0.007}$ [0.193] |
| Temperature | 720       | 744$^{+15.1}_{-15.1}$ [820] | 733$^{+24.7}_{-24.7}$ [790] | 707$^{+34.8}_{-34.8}$ [816] | 720$^{+34.8}_{-34.8}$ [820] |
| log(CH$_4$) | -3.22    | -3.5$^{+0.2}_{-0.2}$ [-3.0] | -3.5$^{+0.2}_{-0.2}$ [-3.1] | -3.5$^{+0.2}_{-0.2}$ [-3.2] | -4.4$^{+0.2}_{-0.2}$ [-4.2] |
| log(H$_2$O/He) | 0.24 | 0.9$^{+0.1}_{+0.0}$ [0.4] | 0.9$^{+0.1}_{+0.0}$ [0.4] | 0.8$^{+0.1}_{+0.0}$ [0.3] | -0.8$^{+0.1}_{+0.0}$ [-0.9] |
| log($P_{\text{clouds}}$) | -3 | -1.0$^{+0.2}_{-0.2}$ [-3.4] | -0.7$^{+0.2}_{-0.2}$ [-2.6] | -0.6$^{+0.2}_{-0.2}$ [-2.8] | -0.4$^{+0.2}_{-0.2}$ [-1.9] |
| $\mu$      | 5.2        | 10$^{+2.8}_{-2.8}$ [6] | 10$^{+2.8}_{-2.8}$ [6] | 10$^{+2.8}_{-2.8}$ [5] | 2.6$^{+0.3}_{-0.3}$ [2.5] |

| Parameter | True value | $\mu = 7.6$ | $\mu = 11.1$ |
|-----------|------------|--------------|--------------|
| $dM$      |            | 10%          | 30%          |
| Mass      | 0.026      | 0.026$^{+0.002}_{-0.002}$ [0.026] | 0.028$^{+0.006}_{-0.006}$ [0.027] | 0.034$^{+0.009}_{-0.009}$ [0.034] | 0.065$^{+0.021}_{-0.021}$ [0.044] |
| Radius    | 0.189      | 0.191$^{+0.001}_{-0.001}$ [0.189] | 0.191$^{+0.001}_{-0.001}$ [0.191] | 0.191$^{+0.001}_{-0.001}$ [0.190] | 0.191$^{+0.001}_{-0.001}$ [0.190] |
| Temperature | 720.33   | 584$^{+28.3}_{-28.3}$ [742] | 530$^{+23.3}_{-23.3}$ [722] | 581$^{+24.6}_{-24.6}$ [699] | 729$^{+24.6}_{-24.6}$ [815] |
| log(CH$_4$) | -3.22    | -7$^{+1.2}_{-1.2}$ [-4] | -7$^{+1.2}_{-1.2}$ [-4] | -6$^{+1.2}_{-1.2}$ [-3] | -7$^{+1.2}_{-1.2}$ [-3] |
| log(H$_2$O/He) | 0.966 | 1.3$^{+0.2}_{+0.1}$ [1.2] | 1.3$^{+0.2}_{+0.1}$ [1.7] | 1.2$^{+0.2}_{+0.1}$ [0.6] | 0.9$^{+0.2}_{+0.1}$ [0.7] |
| log($P_{\text{clouds}}$) | -3 | -1.6$^{+0.2}_{-0.2}$ [-2.9] | -0.7$^{+0.2}_{-0.2}$ [-2.3] | -0.6$^{+0.2}_{-0.2}$ [-2.3] | -1.1$^{+0.2}_{-0.2}$ [-2.8] |
| $\mu$      | 11.1       | 14$^{+2.8}_{-2.8}$ [13] | 13$^{+2.8}_{-2.8}$ [16] | 13$^{+2.8}_{-2.8}$ [8] | 11$^{+2.8}_{-2.8}$ [8] |

Notes. We reported the median values with its 1$\sigma$ error and the MAP (maximum-a-posteriori) values in square brackets.
Table C.9: Retrieved values obtained from the analyses performed on a H₂O-dominated cloudy secondary atmosphere (P_{\text{clouds}} = 10^{-1} \text{ bar}) of a super-earth around a 8th-magnitude M star in the different considered scenarios.

| Parameter | True value | 10% | 30% | 50% | Unknown |
|-----------|------------|-----|-----|-----|---------|
| δM        | 0.026      | 0.027 ±0.002 [0.026] | 0.027 ±0.005 [0.027] | 0.027 ±0.003 [0.027] | 0.027 ±0.000 [0.027] |
| Radius    | 0.189      | 0.191 ±0.000 [0.190] | 0.191 ±0.001 [0.189] | 0.191 ±0.002 [0.190] | 0.191 ±0.000 [0.19] |
| Temperature| 720.33     | 704 ±0.05 [703] | 715 ±0.05 [724] | 717 ±0.06 [725] | 717.815 ±0.056 [713.822] |
| log(CH₄)  | -3.22      | -3.5 ±0.3 [-3.3] | -3.5 ±0.3 [-3.2] | -3.5 ±0.3 [-3.4] | -3.503 ±0.027 [-3.411] |
| log(H₂O/He) | -1.22     | -1.6 ±0.5 [-1.3] | -1.6 ±0.5 [-1.2] | -1.6 ±0.5 [-1.5] | -1.594 ±0.046 [-1.436] |
| log(P_{clouds}) | -1   | 0.6 ±0.1 [-0.9] | 0.5 ±0.1 [-0.9] | 0.5 ±0.1 [-0.8] | 0.534 ±0.040 [-0.693] |
| μ         | 2.3        | 2.34 ±0.15 [2.35] | 2.34 ±0.20 [2.38] | 2.33 ±0.09 [2.33] | 2.331 ±0.006 [2.336] |

μ = 5.2

| Parameter | True value | 10% | 30% | 50% | Unknown |
|-----------|------------|-----|-----|-----|---------|
| δM        | 0.026      | 0.026 ±0.002 [0.028] | 0.028 ±0.006 [0.042] | 0.031 ±0.011 [0.024] | 0.062 ±0.022 [0.032] |
| Radius    | 0.189      | 0.189 ±0.000 [0.189] | 0.189 ±0.001 [0.190] | 0.189 ±0.002 [0.189] | 0.189 ±0.000 [0.190] |
| Temperature| 720        | 733 ±0.05 [716] | 735 ±0.05 [725] | 720 ±0.05 [719] | 826 ±0.044 [696] |
| log(CH₄)  | -3.22      | -3.2 ±0.3 [-3.3] | -3.2 ±0.3 [-3.2] | -3.2 ±0.3 [-3.1] | -3.2 ±0.3 [-3.2] |
| log(H₂O/He) | 0.24      | 0.29 ±0.10 [0.21] | 0.29 ±0.10 [0.21] | 0.2 ±0.10 [-0.2] | 0.14 ±0.09 [0.34] |
| log(P_{clouds}) | -1   | 0.4 ±0.1 [-1.8] | 0.4 ±0.1 [-1.8] | 0.4 ±0.1 [-1.8] | 0.4 ±0.1 [-1.8] |
| μ         | 5.2        | 5.5 ±0.15 [5.0] | 5.2 ±0.17 [5.4] | 4.7 ±1.13 [5.8] | 2.6 ±0.15 [4.1] |

μ = 7.6

| Parameter | True value | 10% | 30% | 50% | Unknown |
|-----------|------------|-----|-----|-----|---------|
| δM        | 0.026      | 0.026 ±0.002 [0.027] | 0.026 ±0.007 [0.027] | 0.026 ±0.011 [0.023] | 0.070 ±0.021 [0.063] |
| Radius    | 0.189      | 0.189 ±0.000 [0.189] | 0.189 ±0.001 [0.188] | 0.189 ±0.002 [0.189] | 0.190 ±0.001 [0.190] |
| Temperature| 720.33     | 842 ±0.05 [723] | 844 ±0.05 [736] | 820 ±0.05 [669] | 787 ±0.197 [759] |
| log(CH₄)  | -3.22      | -3.3 ±0.2 [-3.2] | -3.3 ±0.2 [-3.2] | -3.2 ±0.2 [-3.2] | -3.2 ±0.2 [-3.2] |
| log(H₂O/He) | 0.59      | 0.9 ±0.1 [0.6] | 0.9 ±0.1 [0.6] | 0.9 ±0.1 [0.7] | 0.9 ±0.1 [0.7] |
| log(P_{clouds}) | -1   | 0.2 ±0.1 [-2.4] | 0.13 ±0.1 [-2.4] | 0.13 ±0.1 [-2.4] | 0.13 ±0.1 [-2.4] |
| μ         | 7.6        | 10.5 ±0.8 [8] | 10.4 ±1.1 [11] | 10.4 ±1.1 [8] | 3.2 ±0.6 [3.4] |

μ = 11.1

| Parameter | True value | 10% | 30% | 50% | Unknown |
|-----------|------------|-----|-----|-----|---------|
| δM        | 0.026      | 0.026 ±0.002 [0.028] | 0.029 ±0.006 [0.024] | 0.031 ±0.009 [0.020] | 0.066 ±0.025 [0.019] |
| Radius    | 0.189      | 0.189 ±0.000 [0.189] | 0.189 ±0.001 [0.188] | 0.189 ±0.002 [0.189] | 0.190 ±0.001 [0.189] |
| Temperature| 720.33     | 883 ±0.25 [712] | 899 ±0.26 [854] | 886 ±0.31 [715] | 970 ±0.417 [713] |
| log(CH₄)  | -3.22      | -5 ±0.1 [-3] | -5 ±0.1 [-3] | -5 ±0.1 [-3] | -5 ±0.1 [-3] |
| log(H₂O/He) | 0.996     | 1.5 ±0.3 [0.9] | 1.4 ±0.3 [1.1] | 1.4 ±0.3 [1.6] | 1.4 ±0.3 [1.6] |
| log(P_{clouds}) | -1   | -0.5 ±0.2 [2.1] | -0.10 ±0.3 [-2.02] | -0.01 ±0.3 [0.66] | 0.2 ±0.2 [-1.5] |
| μ         | 11.1       | 15 ±0.15 [10] | 14 ±0.15 [12] | 14 ±0.15 [11] | 9 ±0.15 [15] |

Notes: We reported the median values with its 1σ error and the MAP (maximum-a-posteriori) values in square brackets.
Table C.10: Retrieved values obtained from the analyses performed on a CO-dominated cloudy secondary atmosphere ($P_{\text{clouds}} = 10^{-3}$ bar) of a super-earth around a 8th-magnitude M star in the different considered scenarios.

| Parameter                  | True value | 10%        | 30%        | 50%        | Unknown   |
|----------------------------|------------|------------|------------|------------|-----------|
| $\delta M$                 | 0.026      | [0.026, 0.026] | [0.029, 0.029] | [0.031, 0.031] | [0.037, 0.037] |
| Radius                     | 0.189      | [0.191, 0.191] | [0.192, 0.192] | [0.193, 0.193] | [0.196, 0.196] |
| Temperature                | 720.33     | [727, 727] | [751, 751] | [762, 762] | [813, 813] |
| log(H$_2$O)                | -4         | [-5, -5]   | [-6, -6]   | [-7, -7]   | [-8, -8]   |
| log(CH$_4$)                | -3.22      | [-3.5, -3.5] | [-3.5, -3.5] | [-3.5, -3.5] | [-3.8, -3.8] |
| log(CO/He)                 | -10        | [-10, -10]  | [-10, -10]  | [-10, -10]  | [-10, -10]  |
| log($P_{\text{clouds}}$)  | -3         | [-3, -3]    | [-3, -3]    | [-3, -3]    | [-3, -3]    |
| $\mu$                      | 2.3        | [2.28, 2.28]  | [2.28, 2.28]  | [2.28, 2.28]  | [2.28, 2.28]  |

| $\mu = 2.3$ |          |            |            |            |           |
|-------------|------------|------------|------------|------------|-----------|
| $\delta M$ | 0.026      | [0.026, 0.026] | [0.029, 0.029] | [0.031, 0.031] | [0.037, 0.037] |
| Radius      | 0.189      | [0.191, 0.191] | [0.192, 0.192] | [0.193, 0.193] | [0.196, 0.196] |
| Temperature | 720.33     | [727, 727] | [751, 751] | [762, 762] | [813, 813] |
| log(H$_2$O) | -4         | [-5, -5]   | [-6, -6]   | [-7, -7]   | [-8, -8]   |
| log(CH$_4$) | -3.22      | [-3.5, -3.5] | [-3.5, -3.5] | [-3.5, -3.5] | [-3.8, -3.8] |
| log(CO/He)  | -10        | [-10, -10]  | [-10, -10]  | [-10, -10]  | [-10, -10]  |
| log($P_{\text{clouds}}$) | -3        | [-3, -3]    | [-3, -3]    | [-3, -3]    | [-3, -3]    |
| $\mu$       | 5.2        | [2.29, 2.29]  | [2.29, 2.29]  | [2.29, 2.29]  | [2.29, 2.29]  |

| $\mu = 5.2$ |          |            |            |            |           |
|-------------|------------|------------|------------|------------|-----------|
| $\delta M$ | 0.026      | [0.026, 0.026] | [0.029, 0.029] | [0.031, 0.031] | [0.037, 0.037] |
| Radius      | 0.189      | [0.191, 0.191] | [0.192, 0.192] | [0.193, 0.193] | [0.196, 0.196] |
| Temperature | 720.33     | [727, 727] | [751, 751] | [762, 762] | [813, 813] |
| log(H$_2$O) | -4         | [-5, -5]   | [-6, -6]   | [-7, -7]   | [-8, -8]   |
| log(CH$_4$) | -3.22      | [-3.5, -3.5] | [-3.5, -3.5] | [-3.5, -3.5] | [-3.8, -3.8] |
| log(CO/He)  | -10        | [-10, -10]  | [-10, -10]  | [-10, -10]  | [-10, -10]  |
| log($P_{\text{clouds}}$) | -3        | [-3, -3]    | [-3, -3]    | [-3, -3]    | [-3, -3]    |
| $\mu$       | 7.6        | [2.29, 2.29]  | [2.29, 2.29]  | [2.29, 2.29]  | [2.29, 2.29]  |

| $\mu = 7.6$ |          |            |            |            |           |
|-------------|------------|------------|------------|------------|-----------|
| $\delta M$ | 0.026      | [0.026, 0.026] | [0.029, 0.029] | [0.031, 0.031] | [0.037, 0.037] |
| Radius      | 0.189      | [0.191, 0.191] | [0.192, 0.192] | [0.193, 0.193] | [0.196, 0.196] |
| Temperature | 720.33     | [727, 727] | [751, 751] | [762, 762] | [813, 813] |
| log(H$_2$O) | -4         | [-5, -5]   | [-6, -6]   | [-7, -7]   | [-8, -8]   |
| log(CH$_4$) | -3.22      | [-3.5, -3.5] | [-3.5, -3.5] | [-3.5, -3.5] | [-3.8, -3.8] |
| log(CO/He)  | -10        | [-10, -10]  | [-10, -10]  | [-10, -10]  | [-10, -10]  |
| log($P_{\text{clouds}}$) | -3        | [-3, -3]    | [-3, -3]    | [-3, -3]    | [-3, -3]    |
| $\mu$       | 11.1       | [2.29, 2.29]  | [2.29, 2.29]  | [2.29, 2.29]  | [2.29, 2.29]  |

| $\mu = 11.1$ |          |            |            |            |           |

Notes. We reported the median values with its 1σ error and the MAP (maximum-a-posteriori) values in square brackets.
Table C.11: Retrieved values obtained from the analyses performed on a CO-dominated cloudy secondary atmosphere ($P_{clouds} = 10^{-1}$ bar) of a super-earth around a 8th-magnitude M star in the different considered scenarios.

| Parameter | True value | 10% | 30% | 50% | Unknown |
|-----------|------------|-----|-----|-----|---------|
| δM        | 0.026      | 0.026 ±0.002 [0.026] | 0.026 ±0.002 [0.026] | 0.026 ±0.002 [0.026] | 0.026 ±0.002 [0.027] |
| Radius    | 0.189      | 0.188 ±0.003 [0.189] | 0.189 ±0.003 [0.189] | 0.188 ±0.003 [0.189] | 0.189 ±0.003 [0.190] |
| Temperature | 720.33  | 728 ±3.4 [714] | 731 ±3.7 [718] | 732 ±3.9 [727] | 729 ±3.9 [735] |
| log(H2O)  | -4        | -3.8 ±0.6 [-4.0] | -3.9 ±0.6 [-4.0] | -3.9 ±0.6 [-4.0] | -3.9 ±0.6 [-4.3] |
| log(CH4)  | -3.22     | -3.1 ±0.4 [-3.2] | -3.1 ±0.4 [-3.3] | -3.1 ±0.4 [-3.2] | -3.1 ±0.4 [-3.4] |
| log(CO/He) | -10      | -8 ±4 [-7] | -8 ±4 [-6] | -8 ±4 [-12] | -8 ±4 [-11] |
| log(Pclouds) | -1      | -1.2 ±0.6 [-1.0] | -1.1 ±0.5 [-1.0] | -1.2 ±0.6 [-1.0] | -1.1 ±0.6 [-0.9] |
| μ         | 2.3       | 2.29 ±0.002 [2.261] | 2.289 ±0.009 [2.261] | 2.290 ±0.004 [2.261] | 2.288 ±0.008 [2.261] |

| Parameter | μ = 2.3 |
|-----------|---------|
| δM        | 0.026   |
| Radius    | 0.190   |
| Temperature | 720.33  | 796 ±2.0 [761] | 781 ±2.3 [717] | 785 ±2.5 [698] | 809 ±2.4 [749] |
| log(H2O)  | -4      |
| log(CH4)  | -3.22   |
| log(CO/He) | 0       |
| log(Pclouds) | -1      |
| μ         | 5.2     |

| Parameter | μ = 5.2 |
|-----------|---------|
| δM        | 0.026   |
| Radius    | 0.190   |
| Temperature | 720.33  | 888 ±2.9 [668] | 913 ±3.0 [604] | 864 ±1.9 [725] | 751 ±1.4 [705] |
| log(H2O)  | -4      |
| log(CH4)  | -3.22   |
| log(CO/He) | 0.3     |
| log(Pclouds) | -1      |
| μ         | 7.6     |

| Parameter | μ = 7.6 |
|-----------|---------|
| δM        | 0.026   |
| Radius    | 0.190   |
| Temperature | 720.33  | 988 ±3.0 [671] | 905 ±3.3 [734] | 941 ±1.7 [659] | 839 ±1.0 [803] |
| log(H2O)  | -4      |
| log(CH4)  | -3.22   |
| log(CO/He) | 0.6     |
| log(Pclouds) | -1      |
| μ         | 11.1    |

| Parameter | μ = 11.1 |
|-----------|---------|
| δM        | 0.026   |
| Radius    | 0.190   |
| Temperature | 720.33  | 988 ±3.0 [671] | 905 ±3.3 [734] | 941 ±1.7 [659] | 839 ±1.0 [803] |
| log(H2O)  | -4      |
| log(CH4)  | -3.22   |
| log(CO/He) | 0.6     |
| log(Pclouds) | -1      |
| μ         | 11.1    |

Notes. We reported the median values with its 1σ error and the MAP (maximum-a-posteriori) values in square brackets.