Dynamics of a Thin Shell in the Reissner–Nordström Metric

V. I. Dokuchaev and S. V. Chernov

Institute for Nuclear Research, Russian Academy of Sciences, pr. 60-letiya Oktyabrya 7a, Moscow, 117312 Russia
Lebedev Physical Institute, Russian Academy of Sciences, Leninskii pr. 53, Moscow, 119991 Russia

E-mail: dokuchaev@ms2.inr.ac.ru
E-mail: chernov@lpi.ru

Received June 15, 2009

Abstract — The dynamics of a thin spherically symmetric gravitating shell around an electrically charged Reissner–Nordström black hole is considered. The energy–momentum tensor of an electrically neutral shell is modeled by an ideal fluid with a polytropic equation of state. The dynamics of a shell with a dust equation of state can be traced completely analytically. The Carter–Penrose diagrams that describe the global geometry and all possible types of motions of a gravitating shell in the case of an eternal black hole have been constructed. The conditions have been found under which stable oscillatory motions of the shell take place. These transfer it successively from one universe to the next in an infinite series of identical universes. Such stable oscillatory shell motions are shown to be possible for an arbitrary polytropic equation of state of the shell.

DOI: 10.1134/S10637776110100024

1. INTRODUCTION

The model of thin gravitating shells that was first proposed by Israel [1] occupies an important place among the exactly solvable problems in general relativity. The formalism of the model of thin shells was subsequently developed in details and used for a wide class of cosmological and astrophysical problems [2–6]. In particular, when the phase transitions in the early universe are analyzed [7, 8], the model of thin shells is a very convenient formalism that allows the dynamics of the phase transitions themselves and the formation and evolution of baby universes to be traced in sufficient detail [9–25]. The phase transitions in the early universe begin with the formation of seed bubbles of a new vacuum [26–30]. This process is a quantum one, but the bubbles of a new vacuum pass into the classical stage of evolution due to their rapid expansion. The classical stage of dynamical evolution of vacuum bubbles was considered using the formalism of thin shells in many papers [5, 6, 31–40]. In astrophysics, the formalism of thin shells helps to analyze the relativistic properties of compact stellar systems [41]. In the field theory, models similar to the model of a thin shell were constructed when the decay dynamics of a metastable vacuum was studied [7, 8, 26–30, 42–45]. The model of thin shells is also convenient for the semiclassical description of quantum black holes [46–52]. The special case of a thin shell with a phantom equation of state falling to a black hole was considered in [53]. The dynamics of a thin rotating dust shell was studied in [54].

The possibility of a stable (oscillatory) motion of a shell in the metric of an electrically charged Reissner–Nordström black hole was recently discussed in [55]. However, the corresponding solution was not found. A spherically symmetric shell in the Minkowski [32], Schwarzschild [2, 6], Schwarzschild–de Sitter [5, 40] metrics, and the Friedmann–Schwarzschild universe [39] is known to be dynamically unstable. In the long run, the shell either collapses (falls to the central singularity) or expands infinitely. In this paper, we investigate in detail the dynamics of a spherically symmetric shell for the Reissner–Nordström metric and find the conditions under which the shell oscillations can be stable. In particular, this problem can be investigated completely analytically for a dust shell.

Kuchar [3] was among the first authors who considered the problem on the dynamical evolution of a shell in the Reissner–Nordström metric. In particular, he showed that the electric charge of the black hole could prevent the shell collapse, i.e., a bounce point could exist. Previously, Novikov [56] showed that the collapse of a charged sphere could stop and subsequently expand into another universe. The existence of a bounce point for a contracting shell can provide its oscillatory motion in the case of an eternal black hole whose global geometry contains an infinite number of identical universes. This possibility was discussed qualitatively in [55]. Below, we will find necessary conditions for the realization of such an oscillatory shell motion and the corresponding exact analytical solution. The same oscillatory shell motion can be assumed to be possible for the Kerr metric, because a rotating black hole has
a centrifugal barrier. There is no such possibility for a Schwarzschild black hole and the collapsing shell inevitably falls to the central singularity.

Everywhere below, we assume that the Greek indices $\alpha, \beta, \ldots$ correspond to four coordinates $t, r, \theta, \varphi$ in the four-dimensional spacetime, while the Latin indices $i, k, \ldots$ correspond to three coordinates $t, r, \theta$, and $\varphi$ on the shell.

2. THE EQUATIONS OF MOTION OF THE SHELL

Let there be a spherically symmetric hypersurface $\Sigma$ in the four-dimensional spacetime that divides the spacetime into two regions. We will denote the inner and outer parts by the subscripts “in” and “out.” We will describe each region inside and outside this hypersurface by the metric of an electrically charged Reissner–Nordström black hole. It has the well-known form

$$dS_{\Sigma}^2 = dt^2 - \rho^2 (\tau)d\Omega,$$

where $\tau$ is the proper time of an observer located on the shell and $\rho$ is the shell radius measured by the observer on the shell. The equations of motion of a thin shell were derived in many papers (see, e.g., [6]) and can be written as

$$[K_0^0] + [K_2^0] = 8\pi S_2^0,$$

$$\{K_0^0\} S_0^0 + 2\{K_2^0\} S_2^0 + [T_0^0] = 0,$$

$$[K_2^0] = 4\pi S_0^0 \frac{dS_0^0}{d\tau} + 2\frac{\partial}{\partial r}(S_0^0 - S_2^0) + [T_0^0] = 0,$$

where $K^0_2$ is the extrinsic curvature of the shell, $T_0^0$ is the energy–momentum tensor of matter inside and outside the shell, $S_0^0$ is the energy–momentum tensor of the shell itself, and the following notation is used: $[T] = T_{\text{out}} - T_{\text{in}}$ and $\{T\} = T_{\text{out}} + T_{\text{in}}$. The expressions for the extrinsic curvature in the Reissner–Nordström metric are [3, 6]

$$K_2^0 = -\frac{\rho}{\rho^2 + 1 - 2m/\rho + Q^2/\rho^2},$$

$$K_0^0 = -\sigma (\rho + m/\rho - Q^2/\rho^2),$$

where $\sigma = \pm 1$. It can be shown [57] that the signs of $\sigma$ coincide with those of the $R_+$ and $R_-$ spacetime regions. A charged black hole produces an electric field outside the black hole whose electromagnetic field tensor is [58]

$$F_{tr} = \frac{Q}{r^2} = -F_{rt}.$$

The corresponding energy–momentum tensor of the electromagnetic field is

$$4\pi T_{\frac{a}{b}} = -F^a_{\nu\lambda}F_{\rho\lambda} + \frac{1}{4}\epsilon_{\rho\lambda}F_{\gamma\delta}F^{\gamma\delta},$$

$$T^i_\tau = T^\theta_\varphi = -T^\varphi_\theta = -T^\rho_\rho = \frac{1}{8\pi r^4} Q^2.$$