Untangling the spin of a dark boson in $Z$ decays

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ABSTRACT

We analyze the $Z$-boson decay $Z \to \gamma X$ into a photon ($\gamma$) plus a hypothetical light boson ($X$) belonging to a dark or secluded sector. Due to its feeble interactions with Standard Model fields, this dark boson is behaving as missing energy in the detector. We consider for $X$ the cases of spin-1 (massless dark-photon), spin-0 (axion-like), and spin-2 (graviton-like) particles and explore the way to untangle its spin origin. All these scenarios predict a universal signature for this decay, characterized by a single mono-chromatic photon in the $Z$ center of mass, with energy about half of the $Z$ mass, plus a neutrino-like missing energy associated to the $X$ boson. We show that if the $Z \to \gamma X$ signal should be discovered at $e^+e^-$ colliders, the angular distribution of the mono-chromatic photon in $e^+e^- \to Z \to \gamma X$ can provide a clean probe to discriminate between the $J = 1$ and alternative $J = 0/2$ spin nature of the $X$ dark boson.
1 Introduction

The lack of any experimental evidence at the LHC for a heavy New Physics (NP) above the TeV scale [1], as expected by the many NP scenarios beyond the Standard Model (SM) theory, is changing our perspective about the search for a NP. The accessible sector of NP could be instead made up of light new particles, feebly coupled to SM fields, as predicted by scenarios with dark or secluded sectors beyond the SM, where for instance the candidate(s) for dark matter might reside. The dark sector, consisting of new particles which are singlet under the SM gauge interactions, can indeed have its own long range interactions, characterized by massless or very light mediators, like the dark-photon, the quantum field associated to a $U(1)_D$ gauge invariance in the dark sector. These scenarios have motivated the search for weakly coupled light particles, as it can be seen by the many theoretical and experimental works on this subject [2].

In this framework, we focus on the effective couplings of a light and long-lived neutral $X$ boson with the neutral sector of electroweak gauge bosons of the SM. In particular, we explore, in a model independent way, the production of $X$ by means of the $Z$ boson decay into

$$Z \to \gamma X,$$

where $X$ is assumed to behave as missing energy in the detector.

The striking experimental signature of this decay, in the $Z$ rest frame, is then characterized by an isolated mono-chromatic photon, with energy (almost) half of the $Z$ mass, and missing energy with (almost) vanishing invariant mass for a massless (massive) $X$.

The best place to look for the process in Eq. (1) is at $e^+e^-$ colliders, where the main characteristic of the signature is maintained, although the mono-chromaticity of the photon is slightly spread by the initial Bremsstrahlung radiation. Moreover, rare $Z$ decays are expected to be investigated at the Future Circular Collider (FCC-ee), with its projected production of $10^{13}$ $Z$ bosons [3]. This process was already explored at the experimental level at the Large Electron-Positron Collider (LEP) via

$$e^+e^- \to Z \to \gamma + X,$$

where $X$ stands for no other detected neutral particles. Negative evidence for this signal, set a limit of $10^{-6}$ at the 95% CL on the corresponding branching ratio (BR), in the case of a massless final state $X$ [4]. On the other hand, at hadron colliders this signal would be rather difficult to detect, due to the challenging reconstruction of the $Z$ invariant mass and the large background of soft jets faking the missing energy.

This process has been recently analyzed in the case of $X$ as a massless dark-photon [5]. The dark photon scenario has been extensively analyzed in the literature, mainly for the massive case, and it is also the subject of many current experimental searches, see [6] and [7] for a more recent review. Most of the experimental searches focus on massive dark photons, where the $U(1)_D$ gauge field generates, through a potential kinetic mixing with the photon, a tree-level (milli-charged) interactions with ordinary charged SM particles. On the other hand, for a massless dark-photon the kinetic mixing can be fully rotated away leading to dark-photon interactions with ordinary matter mediated by effective higher-dimensional operators [8].
leading coupling of a massless dark photon to SM charged particles is provided by the magnetic-
and electric-dipole interactions \[8,9\], including the flavor-changing ones \[10\]. Phenomenological
implications of massless dark-photon scenarios have been recently explored in the framework
of Higgs boson \[11\] and rare Kaon decays \[12\].

Recently, in \[5\] it has been shown that the \(Z\) can decay at 1-loop into a photon and massless
dark photon without violating the Landau-Yang theorem \[13\], due to the fact that the dark
and the ordinary photon are distinguishable particles. An upper limit on the viable BR for
the decay \(Z \rightarrow \gamma \bar{\gamma}\) has been estimated to be of the order of \(O(10^{-9})\) \[5\], in the framework of a
simplified model of the dark sector. These results hold also for a massive dark photon, in the
limit of small mass, due to its own magnetic-dipole interactions with SM fields.

We will explore here the possibility that other \(X\) spin configurations can mimic the same
signature of a massless (or massive) dark photon in Eq. (1), and show how to disentangle
a genuine spin-1 dark-photon signal against possible \(X\) candidates of different integer spin.
We will assume an uncertainty of the order of a 1GeV in the invariant mass of the missing
energy, mainly due to the detector performance in the reconstruction of the missing mass.
Therefore, in alternative to the massless dark-photon, we consider at phenomenological level
the hypothetical scenarios of spin-0 and spin-2 particles with masses \(m_X\) below the 1GeV scale,
which are inspired by known theoretical frameworks.

In this respect, we consider first, as an alternative to the dark photon, \(X\) to be a light
axion-like particle (ALP), in both scalar and pseudoscalar scenarios. The ALPs have been
predicted in several SM extensions, mainly motivated by the solution to the strong-CP problem,
where the ALP is a QCD axion \[14\]), or being associated to pseudo-Nambu-Goldstones bosons
that correspond to spontaneously broken continuous symmetries (either in the visible or dark
sector), as well as to a moduli field in string models \[15\] [18]. The phenomenological aspects of
the ALPs have been extensively investigated in recent years, especially collider search of ALP’s
\[19\] [20]. The most severe constraints on the ALP couplings are in the range of masses below
the MeV scale, mainly due to low energy observables and constraints from astrophysics and
cosmology \[19\].

The process in Eq. (1), with \(X\) as ALP, has been considered in the literature and found
to have a viable BR as large as \(O(10^{-4})\) \[19\] [21], although these results hold for visible ALP
decays. We will show that, under the requirement to mimic the massless dark photon signature,
viable BRs as large as \(O(10^{-6})\) for \(Z\) decay in Eq. (1) with \(X\) an ALP, could be possible for
masses in the range of \(100\text{MeV} \lesssim m_{ALP} \lesssim 1\text{GeV}\).

Next, we consider a more exotic scenario for \(X\) as a ultralight massive spin-2 particle \(G\).
Fundamental massive spin-2 fields have been predicted by several extensions of gravity theories,
like the massive Kaluza-Klein (KK) excitations of the standard massless graviton in quantum
gravity (QG) theories in large extra-dimensions (ADD \[22\] and RS \[23\] scenarios), as well as
the massive graviton in the bi-metric theories \[24\] [26]. For the purposes of the present analysis,
we do not make any assumption about the origin of this field. Since we are only interested
in the phenomenological implications of \(Z \rightarrow \gamma G\) decay, we restrict the analysis to the effects
of the linear theory (with an on-shell \(G\) field as external source) in flat space-time, common
characteristic to many extended gravity scenarios. By consistency we assume the spin-2 field to
be universally coupled to the energy-momentum tensor of SM fields, as for the linear graviton-
like coupling to SM fields, with an effective scale \(\Lambda_G\). Then, the effective \(Z\gamma G\) vertex is predicted
as a function of $\Lambda_G$ to be finite, induced at 1-loop by SM fields running as virtual particles.

In order to avoid constraints from short-range gravity experiments (see [27] for a recent review) and mimic a neutrino-like signature, we restrict its mass to lie the range $eV \lesssim m_G \lesssim 1 GeV$, with an effective scale $\Lambda_G \geq TeV$, and require that it does not decay inside the detector. We will show, that for a spin-2 particle subject to these specific constraints, predictions for $\text{BR}(Z \to \gamma G)$ as large as $O(10^{-8})$ are possible, thus in the sensitivity range considered here for the $Z \to \gamma X$.

1 Now, assuming the process in Eq. (1) will be observed with a BR in the sensitivity range of $\text{BR}(Z \to \gamma X) \sim 10^{-12} - 10^{-6}$, given the possibility that $X$ might belong to one of these scenarios, one may wonder if its spin nature could be disentangled by analyzing the angular distributions of the outgoing photon. Clearly, the answer is not, if the $Z$ boson is unpolarized. Indeed, in the unpolarized $Z \to \gamma X$ decay the photon will be isotropically distributed, independently on the spin nature of the $X$ particle. However, a non-trivial angular distribution of the photon, that depends on the $X$ spin, can appear in the case of polarized $Z$ decays. Remarkably, one of the main features of the $e^+e^-$ colliders at the resonant $Z$ peak, is that the on-shell $Z$ boson is always produced polarized, thus transmitting the $Z$-spin correlations to the final state. In this regard, we will show that the angular distribution of the mono-chromatic photon in the $e^+e^- \to Z \to \gamma X$ process at the $Z$ peak can offer a clean probe to untangle the spin-1 nature of the $X$ boson against other possible spin-0/2 interpretations.

The paper is organized as follows. In section 2 we will give the expressions for the effective Lagrangians relevant to the decay $Z \to \gamma X$ for the three spin scenarios mentioned above, providing the corresponding amplitudes and total rates, as well as a discussion on the corresponding allowed range of branching ratios. In section 3 we analyze the angular distributions of polarized $Z$ decays in each spin $X$ scenario, while the corresponding results for a $Z$ produced in resonant $s$-channel at $e^+e^-$ colliders will be presented in section 4. Finally, our conclusions are reported in section 5.

2 Effective Lagrangians and Amplitudes

2.1 Spin-1: massless dark photon

We consider here the case of $X$ as a massless dark-photon $\bar{\gamma}$, which is effectively coupled to the photon $\gamma$ and $Z$ gauge boson. Generalization to the massive dark-photon in the limit of small mass are straightforward. We recall first the main results obtained in [5].

The decay in Eq. (1) with a spin-2 $X$ has been analyzed in [28] in the framework of the ADD scenario [22], predicting a possible BR of the order of $O(10^{-11})$ for $D=2$. However, that scenario is different from the present one, including its signature. There the observable production of $X$ is characterized by an almost continuum spectrum of massive KK excitations, behaving as an almost continuum spectrum of missing energy (each KK are coupled to the matter energy-momentum tensor with an effective $\Lambda_G$ equal to the Planck mass).
where \( e \) is the unit of electric charge, \( \Lambda \) is the scale of the new physics, the dimension-six operators \( \mathcal{O}_i \) are given by

\[
\mathcal{O}_1(x) = Z_{\mu\nu} \tilde{B}^{\mu\alpha} A^{\nu}_\alpha, \\
\mathcal{O}_2(x) = Z_{\mu\nu} B^{\mu\alpha} \tilde{A}^{\nu}_\alpha, \\
\mathcal{O}_3(x) = \tilde{Z}_{\mu\nu} B^{\mu\alpha} A^{\nu}_\alpha,
\]

the field strengths \( F_{\mu\nu} \equiv \partial_{\mu} F_{\nu} - \partial_{\nu} F_{\mu} \) for \( F_{\mu\nu} = (Z, B, A)_{\mu\nu} \), correspond to the \( Z \)-boson, \( B \)-photon, and photon fields, respectively, and \( \tilde{F}_{\mu\nu} \equiv \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \) is the dual field strength. The expression for the coefficients \( C_M \) in Eq. (7), derived in \cite{5}, can be found in Appendix.

As mentioned in the introduction, the Landau-Yang theorem \cite{13} can be avoided in the \( Z \to \gamma \bar{\gamma} \) due the fact that the photon and the massless dark-photon are distinguishable particles. Less obvious is how this effective vertex can be generated from a UV theory. In \cite{5} it has been demonstrated that the above Lagrangian in Eq. (3) arises at low energy as an effective 1-loop contribution, with SM fermions running in the loop, because the dark-photon does not have tree-level couplings with SM fields. Indeed, the leading coupling of a massless dark-photon to charged SM fermions is via magnetic- or electric-dipole operators, namely

\[
\mathcal{L}_{\text{dipole}} = \sum_f \frac{e_D}{2\Lambda} \bar{\psi}_f \sigma_{\mu\nu} \left( d^f_M + i\gamma_5 d^f_E \right) \psi_f B^{\mu\nu},
\]

where \( B^{\mu\nu} \) is the corresponding \( U(1)_D \) field strength of dark photon field, the sum runs over all the SM fields, \( e_D \) is the \( U_D(1) \) dark elementary charge (we assume universal couplings), \( \Lambda \) the effective scale of the dark sector, and \( \psi_f \) a generic SM fermion field. The scale \( \Lambda \) appearing in Eq. (3) is the same of Eq. (7). The magnetic- and electric-dipole coefficients \( d^f_M \) and \( d^f_E \) respectively, can be computed from a renormalizable UV completion theory for the dark sector \cite{5}.

If the dark-photon would have been coupled at tree-level with SM charged fermions (as for the ordinary photon or for the milli-charge couplings of massive dark-photon), the loop contribution would have been zero for each fermion running in the loop, in agreement with what is expected by the Landau-Yang theorem. Therefore, from the point of view of a renormalizable UV completion of the theory, the effective Lagrangian in Eq. (3) is the result of a 2-loop effect, including the effective dipole interactions that originate from 1-loop \cite{5}. The same conclusions hold for the massive dark-photon, since the effective \( Z\gamma\bar{\gamma} \) can be induced by its own dipole-type of interactions as in Eq. (7).

Analogously, the CP-odd Lagrangian induced by the electric-dipole moment is instead

\[
\mathcal{L}_{\text{eff}}^{(E)} = \frac{e}{\Lambda M_Z} C_E \mathcal{O}(x),
\]

where the dimension-six operator is

\[
\mathcal{O}(x) = Z_{\mu\nu} A^{\mu\alpha} B^{\nu}_\alpha.
\]

The expression for the coefficients \( C_E \) in Eq. (7) is reported in Appendix and in \cite{5}. The operators in Eq. (3) and Eq. (8) are \( CP \) even and odd respectively.
Concerning the decay $Z \rightarrow \gamma \bar{\gamma}$, the corresponding amplitudes in momentum space can be found in [5]. Finally, by taking into account the effective Lagrangians in Eq. (3) and Eq. (8) the total width for the unpolarized $Z$ decay is given by
\[ \Gamma(Z \rightarrow \gamma \bar{\gamma}) = \frac{\alpha M_Z^3}{6 \Lambda^2} \left( |C_M|^2 + |C_E|^2 \right), \] (10)
where $C_M = \sum_i C_i$. Same results hold for the massive dark photon in the massless limit, with the scale $\Lambda$ corresponding to its dipole-interactions in Eq. (7).

As discussed in [5], in the framework of a UV complete model for the dark sector, responsible to generate at 1-loop the dipole interactions in Eq. (7), it has been estimated that the largest allowed values for the BR could lie between $\text{BR}(Z \rightarrow \gamma \bar{\gamma}) \sim 10^{-11}$ and $\text{BR}(Z \rightarrow \gamma \bar{\gamma}) \sim 10^{-9}$, depending on the values of $\alpha_D$, the $U(1)_D$ coupling in the dark sector, and the $d_{M,E}^i$ couplings in the dipole-type of interactions in Eq. (7). However, these upper limits could be relaxed if a non-perturbative dynamics is responsible for these couplings potentially pushing up the BR close to the LEP upper bound of $\text{BR}(Z \rightarrow \gamma \bar{\gamma}) \simeq 10^{-6}$.

As mentioned in the introduction, the best place to study this kind of signature is at the $e^+e^-$ colliders. In particular, these BRs are in the ballpark of sensitivity of future $Z$ factories at $e^+e^-$ colliders, like for example the FCC-ee colliders [3].

2.2 Spin-0: ALP scalar and pseudoscalar

Here we consider a scenario for $X$ as an axion like particle (ALP), that can mimic the $Z \rightarrow \gamma X$ signature of a massless or ultralight dark photon. We consider both the scenarios for $X$ as massive scalar $\varphi_S$ and pseudoscalar $\varphi_P$ particles and require them to behave as missing energy in the detector.

Let assume that this process is induced by a UV physics well above the EW scale. In this case an effective low energy Lagrangian approach can be used. Then, we can parametrize the gauge-invariant contribution of the lowest dimensional operators of (dimension 5) to the corresponding effective Lagrangians as
\[ \mathcal{L}_{\text{eff}}^S = \frac{1}{\Lambda_S} \varphi_S Z_{\mu\nu} F^{\mu\nu}, \] (11)
\[ \mathcal{L}_{\text{eff}}^P = \frac{1}{\Lambda_P} \varphi_P Z_{\mu\nu} \tilde{F}^{\mu\nu}, \] (12)
where $\Lambda_{S,P}$ are the corresponding effective scales.

Using the Lagrangians in Eq. (12) the corresponding amplitudes $M_S$ ($M_P$) for the $Z$ decay into scalar (pseudoscalar) plus photon channel are
\[ Z(p) \rightarrow \gamma(k) \varphi_A(q) \] (13)
with $A = S, P$ are given by
\[ M_S = \frac{i}{\Lambda_S} \varepsilon_\mu(p) \varepsilon^{*\nu}(k) T_{\mu\nu}^S(p, k), \]
\[ M_P = \frac{i}{\Lambda_P} \varepsilon_\mu(p) \varepsilon^{*\nu}(k) T_{\mu\nu}^P(p, k), \] (14)
where $\hat{T}^{S}(p, k) = 2(\eta_{\mu\nu}(p \cdot k) - k_{\mu} p_{\nu})$ and $\hat{T}^{P}(p, k) = 4\epsilon_{\mu\nu\alpha\beta} p^{\alpha} k^{\beta}$, with $\eta_{\mu\nu}$ the Minkowski metric and $\epsilon_{\mu\nu\alpha\beta}$ the complete antisymmetric tensor. Then, the corresponding total decay widths in the $Z$ rest frame, neglecting the scalar/pseudoscalar mass contributions, are

$$\hat{\Gamma}_{A} \equiv \hat{\Gamma}(Z \rightarrow \gamma\varphi_{A}) = \frac{C_{A} m_{Z}^{3}}{24\pi \Lambda_{S}^{2}},$$ (15)

with $A = S, P$, where $C_{S} = 1$ and $C_{P} = 4$.

Now we consider some phenomenological implications of these results, in order to get a feeling with the expected BRs for the $Z \rightarrow \gamma\varphi_{A}$ decays. If we assume the interactions in Eq. (12), then the ALP is a stable particle and automatically satisfies the missing-energy signature. However, we conservatively consider a more realistic scenario, which is more theoretically justified. In particular, we assume the ALP to be effectively coupled, in addition to Eq. (12), two photons with the same strength as in Eq. (12), and require that it decays (in two photons) outside the detector.

Let us focus only on the scalar case, since the pseudoscalar scenario should give comparable bounds. At this aim, we consider in addition to Eq. (12), the existence of a new effective coupling to two photons in the Langrangian as

$$\mathcal{L}_{S}^{\text{eff}} \supset \frac{1}{\Lambda_{S}^{2}} \varphi F_{\mu\nu} F^{\mu\nu}.$$ (16)

The reason to consider also the two photon interaction is that, from the point of view of a UV completion of the theory, one cannot avoid the presence of this interaction, if the $Z\gamma\varphi_{S}$ coupling in Eq. (12) is present. Indeed, after the rotation into EW mass eigenstates, the two scales $\Lambda_{S}^{\gamma\gamma}$ and $\Lambda_{S}$ can be linearly related by coefficients proportional to the cosine and sine of the Weinberg angle $\theta_{W}$ [19]. Then, a part from special UV models where one of the two couplings is tuned to cancel or be suppressed, these two scales are expected to be of the same order. The same conclusion does not hold for the Yukawa-like coupling of the ALP to fermions, with respect to the effective interactions in Eqs. (12,16), where these two different kind of interactions could be really independent from each other [2]. In order to stick on the most simple but non-trivial scenario, we assume the ALP couplings to fermions vanishing or being strongly suppressed, thus not contributing to the total width. Then, since we are interested in the order of magnitude constraints on the effective scale $\Lambda_{S}$, we assume for simplicity $\Lambda_{S}^{\gamma\gamma} \sim \Lambda_{S}$, and set to zero all other ALP couplings to SM fields.

Under this setup, we can now analyze the constraints on the scalar or pseudoscalar mass against the corresponding effective scale $\Lambda$, that come from the requirement that the ALP does not decay inside the detector. Following the above considerations, total width of a scalar $X$ as ALP is given by

$$\hat{\Gamma}(S \rightarrow \gamma\gamma) = \frac{m_{S}^{3}}{16\pi \Lambda_{S}^{2}}.$$ (17)

As an example, notice that the effective scales in Eqs. (12,16) could be generated also in the absence of Yukawa couplings of the ALP to SM fermions, induced for instance by new heavy messenger scalar fields (EW charged and with trilinear couplings to ALP) running in the loop.
where \( m_S \) is the mass of the scalar ALP.

By requiring that the ALP does not decay inside the detector, that we conservatively take of length \( L = 10\text{m} \) for \( e^+e^- \) colliders, and assuming \( \Gamma(S \to \gamma\gamma) \) as the total width of ALP, we get

\[
\Lambda_S > 47 \left( \frac{m_S}{100\text{MeV}} \right)^2 \text{TeV}.
\]  

(18)

However, for masses below \( m_S < 100\text{MeV} \), stronger constraints from astrophysics and low energy experiments apply, that are of the order of \( \Lambda_S > 10^6 - 10^8\text{TeV} \) [19]. These can largely oversee the bounds in Eq. (18). For these lower bounds we predict \( \text{BR}(Z \to \varphi\gamma) < 10^{-13}(10^{-16}) \), corresponding to \( \Lambda_S > 10^5(10^6)\text{TeV} \) respectively. As we can see, these BRs are too small to be detected, even for the high statistics of \( Z \) that could be produced at the future FCC-ee collider.

Finally, we consider the next range of \( m_S \) masses, namely from 100 MeV up to the \( \mathcal{O}(1\text{GeV}) \), where the kinematic properties of a neutrino-like \( X \) signature might still hold, assuming the detector uncertainties does not allow to resolve \( X \) masses below 1GeV. In this range of mass, no strong constraints apply from astrophysics and collider experiments, being of the order of \( \mathcal{O}(1\text{TeV}) \) scale [19]. On the other hand, the bound in Eq. (18) gives a stronger constraint on the effective scale \( \Lambda_S \), which now reads \( \Lambda_S > 4.7 \times 10^9(10^{13})\text{TeV} \) for \( m_S \simeq 0.1(1)\text{GeV} \), corresponding to a BR of order \( \text{BR}(Z \to \gamma\gamma) \simeq 1.8 \times 10^{-6}(10^{-10}) \) respectively. As we can see, these BRs are even larger than the expected ones in \( Z \to \gamma\gamma \), and thus potentially candidates to the signature in Eq. (1). Analogous conclusions, with BRs of same order, can be obtained for the pseudoscalar case.

### 2.3 Massive spin-2 particle

As last example, we consider the case of a massive spin-2 particle \( X = G \), which is universally coupled to the total energy-momentum tensor \( T_{\mu\nu} \) of SM fields. As in the case of a massive graviton, this coupling reads

\[
L_G = -\frac{1}{\Lambda_G} T^{\mu\nu} G_{\mu\nu},
\]  

(19)

where \( G_{\mu\nu} \) is the field associated to the spin-2 particle \( G \). Since we assume \( G_{\mu\nu} \) not to be related to gravitational interactions, we take the effective scale \( \Lambda_G \) as a free parameter, uncorrelated from the Planck mass, and of the order of the TeV scale. This scale is reduced to the usual \( \Lambda_G^{-1} = \sqrt{8\pi G_N} \) relation in the ordinary case of massless graviton of General Relativity, with \( G_N \) the Newton constant. Since we do not make any hypothesis on the origin of the spin-2 field, we limit ourselves to the linear theory in flat space, avoing to enter into the issue of a consistent theory of massive spin-2 fields related to the non-linear massive graviton interactions. For the purposes of the present paper the coupling in Eq. (19) is sufficient to generate a finite (thus predictive) contribution at 1-loop for the effective \( ZG\gamma \) coupling. Indeed, due to the fact that \( G_{\mu\nu} \) is coupled to the conserved energy-momentum tensor \( T^{\mu\nu} \) of matter fields, the theory is renormalizable against radiative corrections of SM matter fields only, provided the \( G_{\mu\nu} \) is taken as an external field.
The free Lagrangian for the massive spin-2 is given by the usual term of the Fierz-Pauli Lagrangian \[29\] and we do not report its expression here. The corresponding Feynman rules for the \( G \) interaction in Eq. (19) can be derived from previous works on massive KK graviton productions in ADD scenarios \[30\], \[31\].

Now, we require that the mass \( m_G \) of the spin-2 particle is much smaller than the \( Z \) mass, but larger than the eV scale, in order to avoid the strong constraints from negative searches on the Newton law deviations at short distances \[27\].

The effective \( ZG\gamma \) coupling at low energy, is generated at 1-loop starting from the couplings in Eq. (19), with \( Z, G, \gamma \) external on-shell fields, in which only virtual SM fields run inside. As mentioned above, this contribution is finite due to the conservation of \( T^{\mu\nu} \) (at the zero order in \( 1/\Lambda_G \)). This vertex and the corresponding \( Z \rightarrow \gamma G \) decay has been computed in the context of quantum gravity in large extra dimension scenarios \[28\], with \( G \) the field of a generic massive spin-2 KK excitation of the standard graviton, and for the (massless) graviton in the Einstein theory \[32\].

Before entering in the discussion of the \( Z \rightarrow \gamma G \) decay, we analyze the bounds on \( m_G \) against the scale \( \Lambda_G \), obtained by requiring that \( G \) does not decay inside the detector, assumed as in section 2.2 of length \( L = 10m \). Since we are going to discuss a light \( G \) which decays into SM particles, as in the ALP case, we restrict the analysis to the range of masses

\[ \text{eV} \lesssim m_G \lesssim 1 \text{GeV}. \]

The tree-level total width of a spin-2 particle at rest, decaying into (massless) SM fermion pair \( f\bar{f} \), for the Lagrangian interaction in Eq. (19), is given by \[30\]

\[ \hat{\Gamma}(G \rightarrow f\bar{f}) = \frac{m_G^3 N_c}{80\pi\Lambda_G^2} \]  

where \( N_c = 1 \) and \( N_c = 3 \) for leptons and quarks respectively, while the corresponding one for the decay into two massless gauge bosons \( V \) is \[30\]

\[ \hat{\Gamma}(G \rightarrow VV) = \frac{N_V m_G^3}{40\pi\Lambda_G^2} \]

where \( N_V = 1 \) and \( N_V = 8 \) for \( V = \gamma \) (photons) and \( V = g \) (gluons) respectively.

Then, the total width of \( G \) in visible sector, corresponding to \( m_G = 1 \text{GeV} \) can be approximated to

\[ \Gamma(G \rightarrow \text{visible}) \sim 15\hat{\Gamma}(G \rightarrow \gamma\gamma), \]

where we neglected all fermion masses, and included channels in two photons, two gluons (assumed here to hadronize in two jets of light mesons), \( e^+e^-\mu^+\mu^- \), quark pairs \( q\bar{q} \) for \( q = u, d, s \).

In order to simplify the analysis, we divide the range of \( m_G \) in two regions, below and above the di-muon mass threshold \( 2m_\mu \). In the first region, only the two photon and electron pair channel contribute to the total width. For the second region, we assume the largest value for the total width \( \Gamma(G \rightarrow \text{visible}) \) corresponding to \( m_G = 1 \text{GeV} \), where all channels mentioned
above contribute, that is a quite good approximation for our estimate. Then, by requiring that
the spin-2 particle does not decay into visible states inside the detector – unlike the decay into
neutrino-pairs which is allowed – we get an upper bound on \( m_G \) versus \( \Lambda_G \) as in the ALP case, namely

\[
\Lambda_G \gtrsim 36 \left( \frac{m_G}{100\text{MeV}} \right)^2 \text{TeV}, \quad 1\text{eV} \lesssim m_G \lesssim 2m_\mu
\]

\[
\Lambda_G \gtrsim 113 \left( \frac{m_G}{100\text{MeV}} \right)^2 \text{TeV}, \quad 2m_\mu \lesssim m_G \lesssim 1\text{GeV}.
\]

Further theoretical constraints on this scenario should be imposed on the scale \( \Lambda \) that should
overseed the bounds in Eq. (24) for masses below 10 MeV. In particular, in order to suppress
potential large contributions from \textit{Bremsstrahlung} of \( G \) in high energy experiments, that would
break perturbative unitarity at the TeV energy colliders, we require that \( \Lambda_G > \mathcal{O}(1\text{TeV}) \). Then,
from these results we can see that for a mass range \( eV < m_G \sim 10\text{MeV} \) we have \( \Lambda_G \gtrsim 1\text{TeV} \),
while for \( m_G \sim 50(100)\text{MeV} \) we get \( \Lambda_G \gtrsim 28(113)\text{TeV} \).

Now, we compute the \( \text{BR}(Z \to \gamma G) \) as a function of the \( \Lambda_G \) scale. The corresponding
amplitude \( M_G \) for the process

\[
Z(p) \to \gamma(k) G(q)
\]

is induced at 1-loop and it is given by \cite{28}

\[
M_G = F_G \varepsilon^\mu_Z(p) \varepsilon^{\lambda\rho*}_G(q) \varepsilon^{\nu\sigma*}(k) V^{G\mu\nu\rho\sigma}(k,q)
\]

where \( \varepsilon^\lambda_G(q) \) is the polarization tensor of the massive spin-2 field. The \( F_G \) is a form factor
which is the result of a 1-loop computation. It depends only by SM parameters. Its expression
can be found in \cite{28,32} for massive and massless \( G \) respectively (with notation \( F_h \)). The
effective vertex \( V^{G\mu\nu\rho\sigma}(p,q) \) is \cite{28}

\[
V^{G\mu\nu\rho\sigma}(k,q) = (k_\lambda q_\nu - (k \cdot q) \eta_\nu\lambda)(k_\rho q_\mu - (k \cdot q) \eta_\mu\rho) + \{ \lambda \leftrightarrow \rho \}.
\]

The form factor \( F_G \) is \cite{28,32}

\[
F_G \simeq 0.41 \frac{\alpha}{\Lambda_G m_Z^2 \pi}.
\]

After computing the square of the amplitude and summed over all polarizations, mediating by
the initial ones, the unpolarized total width in the \( Z \) rest frame is

\[
\hat{\Gamma}_G = \frac{1}{576\pi m_Z^5} \left( m_Z^2 - m_G^2 \right)^5 \left( 7m_Z^2 + 3m_G^2 \right) |F_G|^2
\]

which, in the small \( m_G \) limit, reduces to \cite{30}

\[
\hat{\Gamma}_G = \frac{7m_Z^7}{576\pi} |F_G|^2 + \mathcal{O}(m_G^2/m_Z^2).
\]

\footnote{Notice that the massless limit of the width in Eq. (30) differs from the corresponding one for pure massless
graviton \cite{32}, by a overall factor 7/6, which is due to the sum over polarizations of massive graviton with respect
to the massless one. This is due to the known van Dam–Veltman discontinuity in the \( m_G \to 0 \) limit \cite{33}.}
The result in Eq. (29) is in agreement with the corresponding one in [28]. Numerically this gives

\[ \hat{\Gamma}_G \simeq 2.7 \times 10^{-9} \left( \frac{1\text{TeV}}{\Lambda_G} \right)^2 \text{GeV}, \quad (31) \]

corresponding to a branching ratio

\[ \text{BR}(Z \to \gamma G) = 1.1 \times 10^{-9} \left( \frac{1\text{TeV}}{\Lambda_G} \right)^2. \quad (32) \]

Finally, by using the results in Eqs. (24), (32), we find that a viable BR for the signal in Eq. (1) mediated by a long-lived spin-2 particle \( G \) in the range \( 10^{-12} \lesssim \text{BR}(Z \to \gamma G) \lesssim 10^{-9} \) is possible, for a mass range between \( 1 \text{eV} < m_G < 50 \text{MeV} \). For spin-2 masses above 50 MeV scale, the requirement of missing energy signature which is set in the upper bounds in Eq. (24), would exclude the BR above the \( 10^{-12} \) limit.

3 Polarized processes

Here we analyze the angular distributions for the decays \( Z \to X \gamma \), summed over all polarizations of final states, at fixed polarizations of the \( Z \) boson, for the three \( X \) scenarios discussed above. The reason to focus on the polarized processes is because the \( Z \) boson (on-shell) is always produced polarized at colliders, due to its couplings to SM fermions. We will show in more details this feature in the following, for the particular case of a \( Z \) boson production in a resonant \( s \)-channel at \( e^+e^- \) colliders.

In order analyze the polarized \( Z \) decays, we need to identify a special direction against which to consider its projections. In this respect, we choose a frame in which the \( Z \) is boosted, and identify this direction with the one parallel to the \( Z \) 3-momentum \( \vec{p}_Z \), that we choose along the \( z \)-axis, in particular

\[ p_Z = E_Z(1, 0, 0, \beta). \quad (33) \]

where \( \beta = \sqrt{1 - \frac{m_Z^2}{E_Z^2}} \) is the \( Z \) velocity. In this frame the differential \( Z \) decay width \( d\Gamma \) reads

\[ d\Gamma = \frac{|M|^2 m_Z^2}{32\pi E_Z^2 (1 - \beta z)^2} dz \quad (34) \]

where \( |M|^2 \) is the corresponding (Lorentz invariant) square modulus of the amplitude, \( z \equiv \cos \theta_\gamma \) with \( \theta_\gamma \) the angle between the \( Z \) and the photon 3-momenta. The distributions for the various spin cases \( S_X = 1, 0, 2 \) in this frame are discussed below.

**Massless dark photon** – We consider first the case of a \( X \) to be a massless dark photon. We anticipate here that the angular distributions of the photon for the polarized \( Z \) decay induced by magnetic and electric dipole moments interactions are the same.
We define the longitudinal ($L$) and transverse ($T$) $Z$ polarizations with respect to the $Z$ momentum in Eq. (33) respectively, corresponding to the eigenstates of spin projection along the $z$ axis $J_z = \pm 1$ and $J_z = 0$ respectively. Then, the final result for these distributions is

$$\frac{1}{\hat{\Gamma}} \frac{d\Gamma^{(T)}}{dz} = \frac{3}{4} \left( \frac{M_Z}{E_Z} \right)^5 \frac{1 - z^2}{(1 - \beta z)^4},$$  

(35)

$$\frac{1}{\hat{\Gamma}} \frac{d\Gamma^{(L)}}{dz} = \frac{3}{2} \left( \frac{M_Z}{E_Z} \right)^3 \frac{\beta - z}{(1 - \beta z)^4},$$  

(36)

where $\hat{\Gamma}$ is the total width in the $Z$ rest frame given in Eq. (10).

In Eq. (35), the distributions for the two transverse polarizations corresponding to $J_z = \pm 1$, include the average factor $1/2$ over initial polarizations. As a quick check, we can see that the angular distribution in the $Z$ rest frame ($\beta = 0$) for the unpolarized process, given by

$$\frac{d\Gamma}{dz}{|_{\beta = 0}} = \frac{2}{3} \left( \frac{d\Gamma^{(T)}}{dz} + \frac{d\Gamma^{(L)}}{dz} \right){|_{\beta = 0}} = \frac{\hat{\Gamma}}{2},$$  

(37)

is isotropic, in agreement with known theoretical expectations. Also, by integrating Eq. (37) at $\beta \neq 0$, the value of the total width in the moving frame $\int_{-1}^{1} dz \frac{d\Gamma}{dz} = \frac{m_Z}{E_Z} \hat{\Gamma}$ is recovered.

In the $Z$ rest frame, where any direction is equivalent, the angle $\theta_\gamma$ is identified here with the angle formed between the directions of photon-momentum and the $z$-axis, the latter being the axis where the $Z$ spin projections have determined values. We will see in the next section that, due to the $Z$ couplings to electrons, in the resonant production at $e^+e^-$ the $Z$ is mainly produced polarized at rest with transverse polarizations with respect to the beam axis.

**Scalar and pseudoscalar** – Now, we repeat the same analysis above, but in the case of $Z$ decays into photon plus a scalar $S$ or a pseudoscalar $P$, in the massless limit. Since the polarized angular distributions for the scalar and pseudoscalar cases are the same, we will show only one of them as a representative case. Then, the results for these distributions, normalized to the corresponding total width, are

$$\frac{1}{\hat{\Gamma}_I} \frac{d\Gamma^{(T)}}{dz} = \frac{3}{8} \left( \frac{M_Z}{E_Z} \right)^3 \frac{(1 + z^2)(1 + \beta^2) - 4\beta z}{(1 - \beta z)^4},$$  

(38)

$$\frac{1}{\hat{\Gamma}_I} \frac{d\Gamma^{(L)}}{dz} = \frac{3}{4} \left( \frac{M_Z}{E_Z} \right)^5 \frac{1 - z^2}{(1 - \beta z)^4},$$  

(39)

with $\hat{\Gamma}_I$ the total width for $I = S, P$ given in Eq. (15). As for the spin-1 case, one can check that in the unpolarized case, the $Z$ the distribution in the $Z$ rest frame is independent by the angle $\theta_\gamma$, and that by integrating in $\theta_\gamma$ the total width for the unpolarized distribution in Eq. (15) is recovered.

Remarkably, for the longitudinal and transverse polarizations, the corresponding distributions of the massless spin-1 and spin-0 case are different. These distributions are shown in Fig. 1 including the spin-2 cases $X = G$. 

12
Figure 1: Normalized distributions of $\cos \theta_\gamma$ in polarized $Z \rightarrow \gamma X$ decay, with $\theta_\gamma$ the angle between the directions of photon momentum and the $J_z$ spin axis of the $Z$ (see text for details), for the scenarios of $X = \bar{\gamma}, S, P, G$ final states. The distributions of transverse $T$ and longitudinal $L$ polarizations of the $Z$, corresponding to $J_z = \pm 1$ and $J_z = 0$ respectively, are shown in the left and right plots respectively. Normalized angular distribution for a $Z$ produced via $e^+e^- \rightarrow Z \rightarrow \gamma X$ are shown in the left plot, where the $\theta_\gamma$ is the angle of photon momentum with respect to the beam axis.

**Massive spin-2** – Following the same analysis above, we provide below the polarized angular distributions for the spin-2 case in the $Z \rightarrow \gamma G$ decay, in the $m_G$ massless limit, normalized to the corresponding total width in the $Z$ rest frame, in particular

$$\frac{1}{\Gamma_G} \frac{d\Gamma^{(T)}_G}{dz} = \frac{3}{8} \left( \frac{M_Z}{E_Z} \right)^2 \frac{(1 + z^2)(1 + \beta^2) - 4 \beta z}{(1 - \beta z)^4},$$

$$\frac{1}{\Gamma_G} \frac{d\Gamma^{(L)}_G}{dz} = \frac{3}{4} \left( \frac{M_Z}{E_Z} \right)^4 \frac{1 - z^2}{(1 - \beta z)^4},$$

where the total width $\Gamma_G$ is given in Eq. (30). As we can see from these results, the angular distributions of $Z$ for the spin-2 case have the same functional dependence by $\theta_\gamma$ of the corresponding scalar/pseudoscalar ones at fixed polarizations, see Eqs.(38),(39). They only differ in the boosted frame by different powers of $M_Z/E_Z$ in the overall coefficients. This equivalence holds only in the massless limit. Results of angular distributions are included in Fig.1.

### 4 Z decays at $e^+e^-$ colliders

In this section we will analyze the photon angular distributions coming from the $Z$-resonant process $e^+e^- \rightarrow Z \rightarrow \gamma X$ at the $Z$ peak. We will show that these distributions can be easily obtained from a particular linear combination of polarized $Z$ distributions analyzed above. This approach has the advantage to avoid the computation of the scattering cross section $e^+e^- \rightarrow Z \rightarrow \gamma X$. These results can also be applied to any final state.

In the center of mass frame of $e^+e^-$, the beam axis identifies a special direction, that we choose to be our third- or $z$-axis. In this frame, we choose the initial momenta along the beam direction, namely $p_{e^-} = (E, 0, 0, -E)$ and $p_{e^+} = (E, 0, 0, E)$, where $E = \sqrt{S}/2$ is the center of
mass energy (we neglect the electron mass). The transverse and longitudinal Z polarizations for a Z at rest can now be identified with respect to the beam axis.

In this frame, we define the two transverse Z polarizations vectors, for a Z at rest, as
\[ \varepsilon^\mu_Z(\pm) = \frac{1}{\sqrt{2}} (0, 1, \pm i, 0) , \]  
while for the longitudinal one, for a Z at rest, we have
\[ \varepsilon^\mu_Z(L) = (0, 0, 0, 1) . \]

In the case of a frame with boosted Z along the beam direction, in which the Z comes out with a velocity \( \beta = k_Z/E_Z \), with \( k_Z \) and \( E_Z \) its momentum and energy respectively, the corresponding results for the longitudinal polarization generalize to
\[ \varepsilon^\mu_Z(L) = 1/m_Z (k_Z, 0, 0, E_Z ) . \]

Then, concerning our final state, we identify the angle \( \theta_\gamma \) as the angle formed between the direction of the outgoing photon momentum and the initial electron momentum \( \vec{p}_{e^-} \), in particular for the photon 4-momentum we have
\[ k_\gamma = \frac{E_2}{2} (1, \sin \theta_\gamma \cos \phi_\gamma, \sin \theta_\gamma \sin \phi_\gamma, \cos \theta_\gamma ) , \]  
with \( \phi_\gamma \) the corresponding photon azimuthal angle.

Now, we can extract the \( \cos \theta_\gamma \) distributions of the final photon in \( e^+e^- \to Z \to \gamma X \), by using a linear combination of the same polarized Z angular distributions discussed in previous section, provided the \( \theta \) angle appearing in the \( z = \cos \theta \) distributions in Eqs. (35)–(41) is identified with the \( \theta \) angle defined in Eq. (45). In this linear combination, each contribution of the Z polarization \( \varepsilon^{(\lambda)}_Z \) to the width should be multiplied by a polarization-weight coefficient
\[ 0 \leq C^{(\lambda)}_Z \leq 1 \]  
where \( \sum_{\lambda=\pm,L} C^{(\lambda)}_Z = 1 \), corresponding to the Z production in resonance \( e^+e^- \) collision.

We find these coefficients \( C^{(\lambda)}_Z \) by performing the matching between the resonant \( e^+e^- \to Z \to X_f \) cross section (with \( X_f \) a generic final state) in the Breit-Wigner approximation, against the decay width of a polarized on-shell Z boson. These coefficients are universal, since they depend only by the initial states, which in this case are the \( e^+e^- \) from which the Z has been created. Therefore, these results could be applied to any final state.

In general, for \( e^+e^- \) collisions, a generic distribution of final states \( d\Gamma_f \) reads
\[ d\Gamma_f(e^+e^- \to Z \to X_f) = C^{+}_Z d\Gamma^+_f + C^{-}_Z d\Gamma^-_f + C^{L}_Z d\Gamma^L_f , \]  
where \( d\Gamma^+_f \) (\( d\Gamma^L_f \)) stand for the corresponding transverse (longitudinal) polarized distributions of the \( Z \to X_f \) decay and \( C^{\pm,L}_Z \) the corresponding polarization weights. For a Z boson at rest, we have
\[ C^{+}_Z = \frac{1}{2} \left( 1 \mp \frac{2g^{\mu}_V g^{\mu}_A}{(g^{\mu}_V)^2 + (g^{\mu}_A)^2} \right), \quad C^{L}_Z = O(m_e/M_Z) , \]
with $m_e$ the electron mass. As we can see from the above results Eq. (46), the contribution of the longitudinal polarization $\varepsilon_Z^L$ is strongly suppressed and vanishing in the limit of vanishing electron mass. This means that the $Z$ boson produced in resonance at $e^+e^-$ comes out mainly transverse polarized with respect to the beam direction. This is a well known result that can be easily understood in terms of chirality arguments and angular momentum conservation.

The relation in Eq. (46) can be applied to all kind of distribution of final states. In particular, it reproduces the well known result of angular distributions of fermion pair production $e^+e^- \rightarrow Z \rightarrow f\bar{f}$ in the $Z$ resonant region at the peak, including the contribution to the forward-backward (FB) asymmetry.

In general, for a boosted frame in which the resonant $Z$ is produced with speed $\beta$ along the beam direction, the polarization coefficients $C^\pm$ read

$$C^\pm = \frac{1}{2} \left( 1 \mp \frac{2g_V^e g_A^e}{(g_V^e)^2 + (g_A^e)^2} \frac{(1 - \beta)^2}{1 - \beta^2} \right).$$

These results could be also generalized to a resonant $Z$ production at hadron colliders via quark-antiquark annihilation, provided in Eq. (48) $g_V^e$ and $g_A^e$ are replaced with the corresponding $g_V^u$ and $g_A^u$ couplings to up and down quarks respectively.

The term proportional to $(\mp)$ coefficient in Eqs. (47)–(48) is responsible of parity violating contributions. We find that, in all spin cases analyzed here for the $Z \rightarrow \gamma X$ process, the two angular distributions $\frac{d\Gamma_f^T}{dz} = \frac{d\Gamma_f^\perp}{dz}$ are the same for all processes. This means that the $C^\pm$ polarization coefficients enter in the combination of $C^+ + C^- = 1$ for a $Z \rightarrow \gamma X$ decay produced in resonance at $e^+e^-$ colliders. This is due to the fact that, the $Z$ bosonic effective vertices discussed above do not introduce any parity violating contributions when the $Z$ is produced from an unpolarized $e^+e^-$ collider.

In conclusion, the photon angular ($\theta_\gamma$) distributions, coming from the resonant $Z$ boson produced in $e^+e^-$, are just given by the $d\Gamma_f^T/dz$ expressions reported in Eqs. (35)–(41), and are shown in the left plot of Fig.1 for the various $X$ scenarios.

From these results we could see that a massless dark-photon signature is indeed characterized by a central photon, produced at large angles $\theta$ with respect to the beam, while it is vanishing in the FB directions ($\theta = 0, \pi$). On the other hand, for the spin-0 and spin-2 cases the photon will be mainly emitted in the FB directions. This is also in agreement with results on photon angular distributions in the KK gravitons emission in the massless limit [28]. This behaviour can be easily understood by angular momentum conservation. Due to the conservation of chirality in the $Z$ couplings to initial $e^+e^-$ states, the total angular momentum $J_Z$ along the beam axis could be $J_Z = \pm 1$. On the other hand, at $\theta = 0, \pi$ where orbital angular momentum vanishes, the two final photon states can have either $J_Z = 2, 0$, but not $J_Z = 0$. This forces the angular distribution rate to vanish at $\theta = 0, \pi$ as shown in the left plot of Fig.1. This conclusion does not hold for the $Z$ decay into a spin-0 or spin-2 particles accompanied by a photon, for which the total $J_Z = 1$ is possible at $\theta = 0$, leaving to a non-vanishing distribution rate in the FB directions.

These results suggest that from the study of the photon-angular distributions of the $Z \rightarrow \gamma X$ decay at $e^+e^-$ it would be possible to disentangle the (massless) $J^P = 1^-$ nature of the $X$ particle from the other $J^P = 0^-, 2^-$ hypothesis. A more accurate analysis of the signal $Z \rightarrow \gamma\gamma$ process including the relevant backgrounds, as well as a test-statistic distributions to disentangle
the $J = 1^-$ against $J = 0^-/2^-$ hypothesis of the signal will be presented in a forthcoming paper \cite{34}.

5 Conclusions

We analyzed the decays of the $Z$ boson into $Z \to \gamma X$ with $X$ a long-lived light dark boson, assumed to behave as missing energy in the detector. We discussed three potential scenarios for $X$ based on their spin origin: a massless or ultralight dark photon for the spin-1, an ALP in both scalar and pseudoscalar cases for the spin-0, and a light spin-2 particle. For the spin-0 and spin-2 scenarios, the masses are assumed to be in the range of $[100\text{MeV} - 1\text{GeV}]$ for the ALP, and $[1\text{eV} - 1\text{GeV}]$ for the spin-2. Moreover, we required that the ALP and spin-2 particles do not decay inside the detector. We show that for these scenarios the largest BRs could be in the observable range of $10^{-12} \lesssim \text{BR}(Z \to \gamma X) \lesssim 10^{-6}$, depending on the spin and allowed values of the corresponding effective scales. All these BRs are in the ballpark of sensitivity range of future $Z$ factories at $e^+e^-$ colliders, like for instance the FCC-ee facility, with its projected production of $10^{13}$ $Z$ bosons \cite{3}.

These scenarios have in common the same signature, characterized by a mono-chromatic photon plus an almost neutrino-like missing energy. In case this signature should be discovered, a spin test to discriminate about the spin-1 dark photon origin against the spin-0/2 ones is proposed. Due to the fact that the $Z$ boson is always polarized when is resonantly produced at $e^+e^-$ colliders, we show that the spin-1 nature of $X$ could be disentangled from the spin-0 and spin-2, by analyzing the angular distribution of the mono-chromatic photon. The massless dark-photon signature is indeed characterized by a photon mainly produced central and at large angles with respect to the $e^+e^-$ beam axis. On the other hand, for the spin-0 and spin-2 cases (that have the same angular distributions) the mono-chromatic photon is mainly expected along the forward/backward directions.

In conclusion, due to the clean environment of the FCC-ee facility, the rare $Z \to \gamma X$ decay could be a golden place to search for a light $X$ dark boson, offering also the possibility of untangling its spin origin.

6 Appendix

We provide here the expression of the $C_{1-3}$ and $C_E$ coefficients appearing in Eqs.(3),(8) for the effective $Z\gamma\bar{\gamma}$ interactions, as a function of the $d_{M,E}^{M,E}$ coefficient in Eq. (7). By matching the on-shell amplitude for the $Z \to \gamma \bar{\gamma}$ process – as obtained by using the effective Lagrangian in Eqs.(3),(8) — with the corresponding one obtained by the one-loop computation with the
insertion of the dipole-operators in Eq. (7), we obtain [5]

\[
C_1 = - \sum_f \frac{d_M^f X_f}{4\pi^2} \left( 5 + 2B_f + 2C_f \left( m_f^2 + M_Z^2 \right) \right),
\]

\[
C_2 = -3 \sum_f \frac{d_M^f X_f}{4\pi^2} \left( 2 + B_f \right),
\]

\[
C_3 = 2 \sum_f \frac{d_M^f X_f}{4\pi^2} \left( 4 + 2B_f + C_f M_Z^2 \right),
\]

(49)

and

\[
C_E = \sum_f \frac{d_E X_f}{4\pi^2} \left( 3 + B_f + 2m_f^2 C_f \right),
\]

(50)

where \( X_f \equiv \frac{m_f}{M_Z^2} N_c^f g_A^f Q_f e_D \), with \( m_f \) the mass, \( g_A^f \) the Z axial coupling, \( Q_f \) the EM charge of SM fermions \( f \) in units of \( e \), and \( N_c = 1(3) \) for leptons (quarks). The sum over the index \( f \) runs over all EM charged SM fermions. The \( B_f \) and \( C_f \) terms are defined as

\[
B_f \equiv \text{Disc}[B_0(M_Z^2, m_f, m_f)],
\]

\[
C_f \equiv C_0(0, 0, M_Z^2, m_f, m_f),
\]

(51)

where \( B_0 \) and \( C_0 \) are the scalar two- and three-point Passarino-Veltman functions, respectively (see [35] for their explicit expressions), and \( \text{Disc}[B_0] \) stands for the discontinuity of the \( B_0 \) function. These terms are both finite functions which can be evaluated numerically, for example, by using the Package X [36].

7 Acknowledgments

We thank M. Fabbrichesi, L. Marzola, B. Mele, and A. Urbano, for useful discussions. EG is affiliated to the Institute for Fundamental Physics of the Universe (IFPU), Trieste, Italy. EG thanks the Department of Theoretical Physics of CERN for its kind hospitality during the preparation of this work.

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