Submesoscale contribution to subduction: Tracer and momentum fluxes

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Abstract

An important component of the carbon-cycle is subduction, for example of dissolved carbon, from the surface layers to depths of \((10^2 - 10^3)\,\text{m}\). Recently, attention has been focused on the contribution by small-scale, mesoscale, and submesoscale eddies. In the Southern Ocean, the M contribution to subduction was found to be negative and of an order of magnitude smaller than the positive one by vertical diffusion. Since there is now observational evidence that SM export organic carbon but they have not yet been included in subduction studies, the goal of this work is to derive the following results needed to carry out such studies: (a) OGCMs used in C-cycle studies solve the equations for the mean temperature, mean salinity, and mean concentration. We derive the forms of the 3-D arbitrary tracer fluxes (Reynolds Stresses) in terms of resolved fields. (b) The same OGCMs also solve the mean momentum equation. We derive the form of the SM momentum fluxes (Reynolds Stresses) also in terms of resolved fields. (c) It is shown that whether there is subduction or obduction depends on the ratio \(h/H\), where \(h\) is depth of the SM regime and \(H\) is the mixed-layer depth. We show that in the ACC the ratio depends on the specific location and that both subduction and obduction occur but with a topology different than that of mesoscales.

1. Introduction

An arbitrary tracer \(\tau\) can represent active tracers, e.g., mean \(T\), \(S\), and buoyancy and/or passive tracers, e.g., a mean concentration denoted by \(\tau\).

A central issue in carbon cycle studies, referred to as C-studies, is the export or subduction from the surface layers to depth of \(100 - 1000\,\text{m}\). In addition to sinking caused by large-scale water mass transformations, attention has been recently called to the contribution by small-scale, mesoscale \(M\), and submesoscale \(SM\) eddies. The rate of subduction or obduction depends on the ratio \(h/H\), where \(h\) is depth of the SM regime and \(H\) is the mixed-layer depth. We show that in the ACC the ratio depends on the specific location and that both subduction and obduction occur but with a topology different than that of mesoscales.

\[
S_d = - \left( \frac{\partial H}{\partial t} + \mathbf{u} \cdot \nabla H + \mathbf{w} \right) \tau + \left< \mathbf{w} \cdot \nabla \tau \right>
\]

where all the variables are computed at the bottom of the mixed layer, \(H\); \(\mathbf{u}, \mathbf{w}\) are the horizontal and vertical mean velocities; and \(\left< \mathbf{w} \cdot \nabla \tau \right>\) comprises small-scale turbulence (sst), mesoscale \(M\), and submesoscale \(SM\) eddies. The contributions of sst and M were analyzed by Mignone et al. [2006], Karleskind et al. [2011], and Gnanadesikan et al. [2015]. In particular, Bopp et al. [2015, Figure 4] showed that in the Southern Ocean, M contributes \(-0.08\,\text{PgC/yr}\) while vertical diffusion plays a leading role with a contribution of \(+0.69\,\text{PgC/yr}\).

Submesoscales generate a concentration vertical flux that has not yet been included in subduction studies. Since there is recent observational evidence that SM contribute to the export of organic carbon [Omand et al., 2013], in this work we parameterize the flux

\[
F_v(\tau) = \left< \mathbf{w} \cdot \nabla \tau \right>_{SM}
\]

However, even after (2) is known, to fully quantify the effect of SM, one needs more information. In fact, C-studies employ OGCMs that solve the equations for the mean \(T\), \(S\) which are also affected by SM fluxes.
which must be accounted for. These mean variables are solutions of the following equation
\( \text{Do} = \partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + \mathbf{w} \partial_z \mathbf{u} \):

\[
\text{Do} = \nabla \cdot \mathbf{F}_{\text{SM}}(\mathbf{u}) + \partial_z F_{\text{v}^\text{ext}}(\mathbf{u}) = Q,
\]

(3)

where \( \mathbf{F}_{\text{SM}}(\mathbf{u}) \) are the 3-D SM tracer fluxes, \( F_{\text{v}^\text{ext}}(\mathbf{u}) \) is the small-scale vertical flux, and \( Q \) represents external sources and sinks. Since the flux (2) cannot be used to describe the largest second term which we parameterized using the same procedure used to arrive at (6). The result is

\[
F_{\mathbf{v}}(\mathbf{u}) = \begin{pmatrix} 0 & \mathbf{u} \times \nabla \mathbf{u} \end{pmatrix}
\]

(4)

Finally, the mean momentum equation reads as follows:

\[
\text{Do} = \mathbf{u} + \mathbf{e}_3 \times \mathbf{u} + \nabla \cdot \mathbf{R} = - \rho_0^{-1} \nabla p + \mathbf{F}_{\text{ext}},
\]

(5)

where the tensor \( \mathbf{R} = \mathbf{u} \cdot \mathbf{u} + \mathbf{e}_3 \cdot \mathbf{u} \) represents the SM momentum fluxes (Reynolds stresses) whose parameterization is given in section 2; finally, \( f \) is the Coriolis parameter and the other symbols have the usual meaning.

In this work we derive the SM tracer fluxes (4) and the SM momentum fluxes needed in (5). The vertical flux is valid for arbitrary isotracer slopes, isopycnal slopes, and wind stress. In contrast to other parameterizations, the vertical flux depends on the SM kinetic energy. The diffusivity in the vertical flux turns out to be 3–4 orders of magnitude larger than the one by small scales. Though these large diffusivities are multiplied by the horizontal tracer gradients that are smaller than the vertical one (in the small-scale fluxes), the SM vertical tracer flux may be of the same order or even larger than one by the small scales, making SM a potentially important contribution to both the OGCM dynamic equations and the subduction process.

2. Submesoscale Tracer and Momentum Fluxes

SM operate in the diabatic D-regime in which water parcels no longer move along isopycnal surfaces as they do in the adiabatic A-regime. As Killworth [1997] pointed out, the proper representation of the D-regime must be in terms of vertical and horizontal fluxes rather than in terms of fluxes along and across surfaces of constant buoyancy. The SM fluxes (4) derived in Canuto and Dubovikov [2010, equation (6)] read as follows:

**Vertical flux:**

\[
F_{\mathbf{v}}(\mathbf{u}) = - \mathbf{\kappa}(z) \cdot \nabla_h \mathbf{u},
\]

(6)

where the diffusivity \( \mathbf{\kappa} \) has the following form (\( e_3 = 0, 0, 1 \)):

\[
\mathbf{\kappa}(z) = \frac{\text{Ro}^2}{1 + \text{Ro}^2} \left[ \mathbf{\kappa}(z) - \frac{1}{\text{Ro} / |f|} \mathbf{e}_2 \times \mathbf{\kappa}(z) \right],
\]

\[
\mathbf{\kappa}(z) \equiv \frac{1}{0} \begin{pmatrix} \mathbf{u}(z) & \mathbf{z} \end{pmatrix} dz - \frac{z}{h} \nabla_h \begin{pmatrix} \mathbf{u}(z) \\ z \end{pmatrix} dz,
\]

\[
\text{Ro} = \frac{(2\text{K}_{\text{SM}})^{1/2}}{r_s |f|}, \quad r_s = \frac{hN}{\pi |f|},
\]

(7)

**Horizontal flux:**

\[
F_{\mathbf{h}}(\mathbf{u}) = - \mathbf{\kappa}_{\text{SM}} \nabla_h \mathbf{u}, \quad \mathbf{\kappa}_{\text{SM}} = r_s K_{\text{SM}}^{1/2},
\]

(8)

where \( K_{\text{SM}} \) is the SM eddy kinetic energy and \( r_s \) is their horizontal extent.

The divergence of the momentum fluxes in equation (5) is given by \( \nabla \cdot \mathbf{R} = \mathbf{u} \cdot \nabla - \nabla \mathbf{u} + \mathbf{w} \partial_z \mathbf{u} \). We retain only the largest second term which we parameterized using the same procedure used to arrive at (6). The result is

\[
\nabla \cdot \mathbf{R} \cong \mathbf{w} \partial_z \mathbf{u} = -2A(\text{Ro}) \mathbf{e}_2 \times \mathbf{u}(z) + 2\text{Ro}^{-1} A(\text{Ro}) f \mathbf{u}(z), \quad A(\text{Ro}) = \frac{\text{Ro}^2}{1 + \text{Ro}^2},
\]

(9)

where \( \mathbf{u}(z) = \mathbf{u}(z) - h^{-1} \int_0^h \mathbf{u}(z) \). Several features of (6)–(9) are worth discussing. First, when the SM kinetic energy vanishes, \( \text{Ro} = 0 \) and the SM fluxes also vanish; in fact, if there are no sources to sustain the
SM eddy kinetic energy, there are no SM and no fluxes either. Second, one might expect a vertical flux to entail a vertical mean tracer gradient while in (6) there is the horizontal gradient. This is because (6) represents a baroclinic instability whereby horizontal gradients generate vertical fluxes. A vertical flux proportional to the vertical gradient is provided by sst which is already present in both relations (1) and (3). Third, since the presence of the 2-D mean velocity $u(z)$ in the vertical flux (2) may seem unusual, we must clarify its origin. One begins with the dynamic equations for traces and velocity which are separated into mean and fluctuating parts. Inserting them into the original equations and subtracting the equations for the mean values, yields the dynamic equations for $w_0, s_0$. Multiplying the first by $s_0$ the second by $w_0$, adding the results and averaging, yields the dynamic equation for the correlation $<w_0 s_0>$. The original dynamic equations contain the 2-D mean velocity $u(z)$ in the advective term which then becomes part of the vertical tracer flux (7). Fourth, the original dynamic equations contain nonlinear terms (in http://www.giss.nasa.gov/staff/vcanuto/canuto_nonlinearity_201507.pdf, we discussed the basic assumptions that allowed an analytic solution to be found for the fluxes of interest). If the latter are treated with a mixing length model, there appears a turbulent viscosity (diffusivity) which is the product of a velocity and a length which then appear in Ro that plays the role of a SM Rossby number. Fifth, a critical role is played by $h$, the depth of the SM regime that appears in the above fluxes and which is such that

$$F_v(0) = 0, \quad F_v(-h) = 0.$$  

However, the SM parameterizations (6)–(9) do not provide a model for $h$, specifically, its relation with the mixed-layer depth $H$. Analytic studies presented below indicate that the sign of tracer fluxes depend on the difference

$$h - H,$$

and whether in a given location there is subduction or obduction depends on whether (11) is positive or negative. Previous studies [McWilliams, 1985; Mensa et al., 2013; Veneziani et al., 2014] concluded that

$$h > H,$$

but since they were not specific to the ACC, it may not be legitimate to extrapolate (12) to that region. The model we present in section 5 reproduces relation (12) away from the ACC and shows that in the latter there are three distinct regions characterized by $h > H$, $h = H$ and $h < H$ (see Figure 1).

To better understand these features, we derive the analytic form of the tracer fluxes. To do so, we take the 2-D mean velocity $\mathbf{u}(z)$ as the sum of geostrophic and a-geostrophic (wind-driven) components

$$\mathbf{u} = u_g + u_{ag}, \quad u_g = \frac{1}{f} \mathbf{e}_2 \times \nabla_h D, \quad u_{ag} = -\frac{1}{f} \mathbf{e}_2 \times \partial_z \tau(z),$$

where $\tau(z)$ is the wind stress and in the second relation we used the fact that the horizontal buoyancy gradient is constant in the mixed layer [Mensa et al., 2013; Veneziani et al., 2014]. Using the second of (13) in (7) and then in (6), we obtain that at $z = -H$, Figure 1. ACC. Three year winter average of the depth $h$ of the SM regime in units of the mixed-layer depth $H$ computed from the potential density criterion $\Delta_r = 0.03$ kg m$^{-3}$. There are three distinct regions: $h > H$ (red regions), $h = H$ (white circle), and $h < H$ (blue regions). In the red regions, $H$ occurs within the D-regime and only a mesoscale parameterization of the D-regime is required; in the blue regions, $H$ occurs below the D-regime in the A-regime. Finally, the white circle where $h = H$ is less frequent than the red and blue regions.
Geostrophy:

\[ F_v(\tau)_g = \gamma_0 \nabla H \mathbf{B} \cdot \nabla H \tau + \gamma_0 \text{Ro}(\nabla H \mathbf{B} \times \nabla H \tau) \cdot \mathbf{e}_z \]  

(14)

which exhibits the dependence on (11). The two terms in the flux (14) represent the general combination of the \( \tau \) isoslopes (~\( -\nabla H \tau \)) with respect to the isopycnal slopes (~\( \nabla H \mathbf{B} \)). In the case of nitrates, Omand and Mahadevan [2013, Figure 3b] shows that the slopes \( s = -N^{-2} \nabla H \mathbf{B}, s_e = -(\partial \mathbf{e} / \partial z)^{-1} \nabla H \mathbf{C} \) are aligned in which case the first term in (14) dominates. Since \( \gamma_0 \) scales like \( H^3 \), it is large in winter when the mixed layers are deep and small in summer when mixed layers are shallow; however, since OGCMs need to describe mixed layers of arbitrary depths, one must include the wind contribution which, as we show next, is weakly dependent on \( H \) and becomes relevant in summer.

The derivation of the flux due to the a-geostrophic (wind) component of the 2-D mean velocity \( \mathbf{u}(z) \) is slightly more laborious. We combine the last of (13) with the “closure relation” \( \rho \nu \gamma \mathbf{a}_y = \tau(z) \), where \( \nu \) is the momentum diffusivity due to small-scale turbulent mixing. The result is a differential equation for \( \tau(z) \) that has the following solution, as it can be directly verified:

\[ \tau(z) = \alpha(z) w + f|f|^{-1} \beta(z) \mathbf{e}_z \times \mathbf{w}, \alpha(z), \beta(z) \equiv e^i (\cos \zeta, \sin \zeta), \zeta = z/\delta_e. \]  

(15)

Here \( \delta_e = (2\pi|f|^{-1})^{1/2} \) is the Ekman depth and \( x(0) = 1, \beta(0) = 0 \). A series of algebraic steps then lead to the following form of the flux:

A-geostrophic:

\[ F_v(\tau)_a = \gamma_2 (\mathbf{a}_w \times \mathbf{e}_z) \cdot \nabla H \tau + \gamma_3 \mathbf{a}_w \cdot \nabla H \tau, \]

\[ \gamma_2 = \frac{\text{Ro}^2}{1 + \text{Ro}^2} f^{-1} \rho^{-1} |H| \left( \frac{h}{H} - 1 \right), \quad \gamma_3 = \frac{\text{Ro}^2}{1 + \text{Ro}^2} f^{-1} \rho^{-1} |H| \left( \frac{h}{H} - 1 \right), \]  

(16)

which also exhibits the dependence on (11); in analogy with the Ekman buoyancy flux \( \text{EBF} = f^{-1} \rho^{-1} \mathbf{a}_w \times \mathbf{e}_z \cdot \nabla H \mathbf{B}, \) the first term in the flux (16) could also be expressed as an Ekman tracer flux (ETF). In analogy with (14), the two terms in the flux (16) represent the \( \tau \)-vertical flux for a general orientation of the \( \tau \) isoslopes with respect to the direction of the wind stress.

3. Comparison With a Previous Model

Omand et al. [2015] suggested the following form of the concentration vertical flux:

\[ F_v(\tau) = -K_{v(SM)} \frac{\partial \tau}{\partial z}, \quad K_{v(SM)} = N^2 F_v(\mathbf{B}) g. \]  

(17)

The fact that (17) was taken to be proportional to the vertical gradient of the mean concentration implies a physical process different from the baroclinic instabilities we have considered in deriving relations (14). Second, the diffusivity in (17) was taken to be the geostrophic component of (14) which, in the buoyancy case, becomes \( F_v(\mathbf{B}) = \gamma_0 |\nabla H \mathbf{B}|^2 \) and thus

\[ K_{v(SM)} = 4C_e |f|^{-1} s^2 N^2 H^2 \left( \frac{h}{H} - 1 \right), \]  

(18)

where we have adopted the symbol \( C_e \) used in Omand et al. [2015] where in this model \( C_e = \frac{\text{Ro}}{8 \pi^2 \text{Ro}^2} \). The value 0.07 taken in Omand et al. [2015] is recovered for \( \text{Ro} = 1 \) corresponding to a fixed value of the SM kinetic energy. However, section 6 shows that \( \text{Ro} \) is location dependent and \( \text{Ro} = 1 \) does not have a universal character. Using the characteristic values \( H = 100 \text{ m}, N^2 = 10^{-6} \text{s}^{-2}, s = 10^{-3} \), we obtain for \( \text{Ro} > 1 \)

\[ K_{v(SM)} \approx \frac{1}{2 \text{Ro}} \left( \frac{h}{H} - 1 \right) \text{cm}^2 \text{s}^{-1}. \]  

(19)
which is of the same magnitude as the one from small-scale mixing which, near the bottom of the MLD, was taken to be \cite{Bopp et al., 2015}

\[ K_v(c) \approx 1 \text{ cm}^2\text{s}^{-1}. \] (20)

Relations (19) and (20) suggest that (17) describes a mixing process different from (14)–(16), one that is more similar to the SST vertical mixing than to the baroclinic instabilities we have discussed.

4. Numerical Estimates

Using $N^2=10^{-6}s^{-2}$, $f=10^{-4}s^{-1}$, $H=10^2$ m, we obtain the following estimate for the “effective” SM geostrophic diffusivity in (14):

\[ K^0_{SM} \equiv c_N^2 N^2 s \approx 10^6 \text{ cm}^2\text{s}^{-1}. \] (21)

In the case of the a-geostrophic flux (16), we have

\[ K^a_{SM} \equiv \tau_w / f \approx 10^4 \text{ cm}^2\text{s}^{-1}, \quad \tau_w = 0.1 \text{ Nm}^{-2}, \] (22)

which means that

\[ K^0_{SM} \approx 10^4 K_v(SM), \quad K^a_{SM} \approx 10^4 K_v(SM). \] (23)

Though the difference of 3–4 orders of magnitude is somewhat offset by the fact that $|\nabla_h \tau| < |\tau|$, the SM vertical flux may be equal or even larger than the one by small scales that was recently shown to play a leading role in anthropogenic carbon subduction \cite{Bopp et al., 2015}.

5. Depth of the D-Regime

As already discussed, the determination of this variable is not a simple problem and since (12) was not arrived at using ACC data, it cannot be automatically extrapolated there. Since we were unable to find a model for $h$ of general validity, we suggest the following. Since only $M$ are present in both D and A regimes, to determine the extent of the D-regime, we recall that in the A-regime the mesoscales vertical buoyancy flux is given by

\textit{A-regime:}

\[ F_v(B) = \kappa_M N^2 |s|^2, \] (24)

where $\kappa_M$ is the 3-D mesoscale diffusivity and $s = -N^{-2} \nabla_h B$ is the slope of the isopycnals. In the D-regime the mesoscales buoyancy flux was derived in \textit{Canuto and Dubovikov} [2011, equations (35)] with the following result:

\textit{D-regime:}

\[ F_v(B) = -\kappa \cdot \nabla_h B = \kappa \cdot s N^2, \]

where

\[ \kappa = \kappa_M \omega \times e_z, \quad \tau_0 = \nabla \cdot \nabla \psi, \quad \kappa = \kappa_M \omega \times e_z, \quad \tau_0 = \nabla \cdot \nabla \psi, \quad \tau = \nabla \cdot \nabla \psi, \quad \tau_0 = \nabla \cdot \nabla \psi, \]

where $\kappa_M$ is the 3-D mesoscale diffusivity and $s = -N^{-2} \nabla_h B$ is the slope of the isopycnals. In the D-regime the mesoscales buoyancy flux was derived in \textit{Canuto and Dubovikov} [2011, equations (35)] with the following result:

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where $\kappa_M$ is the 3-D mesoscale diffusivity discussed in \textit{Chelton et al.} [2011] and whose form was derived in \textit{Canuto and Dubovikov} [2005, equation ((15)b)]. Since at the A-D interface, the fluxes of the A-D regimes must match, from equating (24) and (25), one obtains the following condition in which $\kappa_M$ is no longer present:

\textit{Depth of the D-regime, h:}

\[ s^2 = (\omega \times e_z) \cdot s, \] (26)

which can be rewritten in the alternative form
\[ h = \frac{\int \sqrt{s_h^2 + \mathbf{e_z}^2} \, ds}{\mathbf{u}_h \cdot (s_h \times \mathbf{e_z})}, \quad \mathbf{u}_h \equiv \mathbf{u}(h) - \int_0^h \mathbf{u}(z) \, dz - 2 \mathbf{u}_d, \]  

(27)

where \( s_h \) is the isopycnal slope at \( h \) and the drift velocity \( \mathbf{u}_d \) is given in Canuto and Dubovikov (2006, equations (4a)–(4f)). Since (27) is an implicit relation for \( h \), one begins with guesses of \( h, s_h, \) and \( \mathbf{u}_d \), evaluates the right-hand side of the first relation in (27) and repeats the procedure until the two sides match. We solved relation (27) and the results presented in Figure 1 show that, depending on the location in the ACC, the ratio \( h/H \) can be both smaller and larger than unity and that the conclusion result (12) cannot be simply transported to the ACC. In the red regions, the mixed-layer depth occurs within the D-regime and only a mesoscale parameterization of the D-regime is required; in the blue regions, the mixed-layer depth occurs below the D-regime in the A-regime. Along the white circle \( h = H \) which is far less frequent than the red and blue regions. Finally, we note that the D-regime fluxes (25) was assessed by Luneva et al. (2015) with a numerical simulation.

6. SM Kinetic Energy, Rossby Number \( \text{Ro} \)

\( \text{Ro} \) was obtained as solution of the energy balance production+dissipation where the production is contributed by two sources, the buoyancy flux obtained from (6) with \( \tau = 0 \) averaged over the depth \( h \) and the SM momentum flux-mean shear interaction \( -\mathbf{w} \cdot \mathbf{u} \mathbf{\partial}_h \mathbf{u} \); the SM momentum fluxes are parameterized in equation (9). The rate of dissipation of the SM eddy kinetic energy is computed using the Kolmogorov kinetic energy spectrum [e.g., Lesieur, 1987, equation ((4.1)). The SM forcing by baroclinic instabilities (represented by \( \nabla_h \mathbf{u} \mathbf{j} \)) and wind stress \( \tau_w \) appear in the following combinations (\( R_i \) is the geostrophic Richardson number):

\[ R_i = \frac{\rho E^2}{|\mathbf{u}|^2}, \quad \tau \equiv \frac{\tau_w / \rho}{h^2 N^2 |\mathbf{f}|}, \quad \text{EBF} = \frac{\rho^{-1} \mathbf{e_z} \cdot \nabla_h \mathbf{u}}{h^2 N^2 |\mathbf{f}|}, \quad \tau = \frac{(\tau_w / \rho)^2}{h^2 N^2 |\mathbf{f}|}. \]  

(28)

The resulting \( \text{Ro} \) is given by the algebraic relation

\[ (q_0 \pi^2)^{-1} \text{Ro}^2 = \Gamma_0 R_i^{-1} + \Gamma_1 \tau + \Gamma_2 \tau^2, \]  

(29)

where the dimensionless variables \( \Gamma_1s \) are defined as follows:

\[ 12 \Gamma_0 = A(1 + 2A) \text{Ro}^{-1}, \quad \Gamma_2 = 2A^2 \text{Ro}^{-1} \left( 1 - E^{-1/2} \right) \left( 1 - E^{-1/2} \right), \]  

\[ 2 \Gamma_1 = A \left[ 1 - E^{-1/2} \left( 1 - \text{Ro}^{-1} \right) \right] + \frac{1}{3} A^2 \text{Ro}^{-1}, \]  

(30)

where \( A = A(\text{Ro})/E = (\delta_e^2 / h^2) \), and \( q_0 = \left( \frac{3}{2} K \right)^{3/2} \) is given in terms of the Kolmogorov constant \( K \) which in 2-D turbulence is \( 4 < K < 8 \) [Danilov and Gurarie, 2000].

7. Conclusions

Following the conclusions about the contribution by \( M \) to subduction by previous authors [Mignone et al., 2006; Karleskind et al., 2011; Bopp et al., 2015; Gnanadesikan et al., 2015; Omand et al., 2015], we took the next step and considered the contribution of SM to the mean equations (3) for \( T, S, \varpi \), to the subduction rate (1) and to the mean momentum (5).

OGCMs used in C-cycle studies can now employ the parameterization (6)–(9), together with (29) and (30), so as to quantify the role of SM in subduction-obduction studies. The hope is that the results presented here can help to better quantify the “the strong sensitivity of the oceanic carbon cycle to changes in mixed-layer depth, ocean currents, and wind” found by previous studies.

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