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Abstract. This work presents a discussion related to the teaching and learning of mathematical contents related to the study of exponential functions in a freshman students group enrolled in the first semester of the Science and Technology Bachelor’s (STB of the Federal University of Jequitinhonha and Mucuri Valleys (UFVJM). As a contextualization tool strongly mentioned in the literature, the modelling approach was used as an educational teaching tool to produce contextualization in the teaching-learning process of exponential functions to these students. In this sense, were used some simple models elaborated with the GeoGebra software and, to have a qualitative evaluation of the investigation and the results, was used Didactic Engineering as a methodology research. As a consequence of this detailed research, some interesting details about the teaching and learning process were observed, discussed and described.

1. Introduction

Mathematics gradually developed with the intellectual and practical needs of the society. The growth of this science had, as one of its precursors, the thought of Euclides, which aimed at the logical and axiomatic construction of Geometry, in addition to the useful inventions of Archimedes. With the emergence of new scientific challenges, new mathematical theories have been built to aid in problem-solving theories. Subsequently, these new theories were used to study and represent similar or entirely different problems. As an example, the logarithms that were commonly used only as an aid for procedural calculations, and today, are used to model several biological and physical phenomena, such as the study of the alcohol elimination from the organism, [1, 2, 3].

In [4], Burak and Klüber use psychology to discuss and emphasize the importance of student interest and their interaction along the teaching-learning process. This process, when using modeling approach as a didactical tool, is carried out in a shared knowledge way with students, unlike in the usual way, where the process is held only by the teacher. The modeling approach provides a greater interaction with the students, especially because it works with their interest, which generates an advance in the learning process.

There are several models in the literature that describe the process of alcohol elimination from the body. These models, according to their complexity, can address several specific parameters of the phenomenon, such as the concentration of alcohol in the stomach after ingestion, the rate of absorption of alcohol into the bloodstream, and the rate at which the liver metabolizes the alcohol [5]. For a better understanding of the ingested alcohol elimination model, there is a
rather interesting discussion on the subject in [6] and [7]. Also, [8] discusses the metrological control of alcohol meters, instruments for measuring the amount of alcohol in the blood.

To study these models it is important to know and understand the basic logarithmic and exponential functions concepts. In Brazil, the educational secondary institutions are the means by which the learning of these functions by the students traditionally occurs. In this way, this work started with some questions about how the teaching-learning process of these functions happens in the public schools. To analyze the teaching of these functions and to propose didactical interventions that can provide improvements in this process, we used Didactic Engineering (DE) as a research methodology. DE is more detailed in [9] and [10].

In this work, we made an in-depth discussion derived from [11], analysing teaching of exponential and logarithmic functions process. We also discussed the way these functions are addressed in Mathematics textbooks. Thus, we used modeling approach as a tool in the teaching-learning process, through the application of a model of alcohol elimination, which provides a relevant proposal for the freshman students of STB. The reader can find more details about STB in [12], [13] and [14].

An important factor to be highlighted is the fact that only 2% to 10% of the absorbed alcohol is eliminated, the body oxidizes the excess [7]. We adapted the initial hypotheses of the model, according to the proposal, to simplify the model and the students’ understanding.

2. The Modelling Approach and the Research Methodology

The research methodology used in this work will be presented in four stages. We will not describe the main characteristics of this methodology, but they are more detailed in [10].

In the first stage, which is called previous analysis, we investigated the exponential functions fundamental concepts. We analyzed these functions in the textbooks provided by the Brazilian education system. Then, we checked the deficiencies in the contextualization of this content. Finally, we detected that these gaps negatively impacted students’ understanding of the subject. More details about the procedures of this stage can be found detailed in [11].

The second stage is called the a priori analysis. At this stage, we choose the model 2 as a didactical approach with a computational graph produced with GeoGebra software (see Figure 1). We described the didactical approach as well as the actions we were expecting from the students on each stage of the research. In section 3 we will detail the two main goals that the teacher was trying to achieve.

In this second stage, we analyze the system (3) and observe that it represents the model (2). For the didactical modeling approach used in this research, we considered that the rate of elimination of alcohol ingested at a given time \( t \) is proportional to the amount of alcohol ingested at the same time. Considering \( C \) the alcohol concentration, measured in g/l (Grams per liter of alcohol) and \( \frac{dC}{dt} \) the rate of elimination of ingested alcohol measured in g/l/min (Grams per liter per minute), there is \( \alpha \) such as:

\[
\frac{dC}{dt} = \alpha \cdot C
\]  

(1)

Since the amount of alcohol decreases along the time, there is has a problem of decay in the phenomenon of the elimination of the alcohol. Then, considering \( \alpha = -k \):

\[
\frac{dC}{dt} = -k \cdot C
\]  

(2)

As the elimination of alcohol occurs exclusively by the lungs, we considered the rate of elimination of alcohol as \( k = 0.0075 \) g/l/min [7]. Considering the concentration as \( C = C(t) \) and the initial concentration in \( t_0 \) as \( C(t_0) = C_0 \), we have a Cauchy problem:
The solution of the system (3) is expressed as an exponential function:

$$C(t) = C_0 e^{-0.0075t}$$ (4)

Although the domain of the exponential functions is the real numbers set ($\mathbb{R}$), for the application of the modeling approach, we considered only the real positives numbers set ($\mathbb{R}^+$). GeoGebra was used to represent graphically the model (3), as appears in Figure 1.

**Figure 1.** Graphical representation of the model (3) produced by the teacher using GeoGebra.

In the third stage, called experimentation, the teacher executed his planning in the classroom and some diagnostic questionnaries were introduced to the students to evaluate their understanding about the subject and also about the didactical approach. Figure 2 shows the students producing their models. The reader can find all the questionnaires detailed on [11].

**Figure 2.** Students working in the experimentation stage.
In the last stage, called \textit{a posteriori analysis and validation}, the obtained results were compared with the goals that were related to the second stage, the \textit{a priori} analysis stage. Along this stage, we performed a stratification of the results obtained with the application of the didactic sequences. The collected information was separated and organized in groups. It is important that this analysis reaches the reality of the students’ production [15].

3. Discussions about the Research Methodology

A careful analysis was made with the data collected from the students answers after the application of some diagnostics questionnaires. Using direct observation and direct observation, two hypotheses were analyzed:

\textbf{Hypothesis A}: The use of computers, specially GeoGebra software as a didactic resource, encourages the possibilities of visualization, calculation, and interpretation.

\textbf{Hypothesis B}: The study of exponential and logarithmic functions, through growth and decay models, improves the understanding of these contents.

Regarding the construction of the slope field (Figure 3) related to the equation (2) in the analysis \textit{a priori} stage, it was expected that the students would connect the visualization of the slope field to the possible solutions of the system (3). We also expected that they could see and understand how the concentration decreasing occurs.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{slope_field.png}
\caption{Slope field produced by a student using GeoGebra.}
\end{figure}

In the model presentation and discussion process, the students were able to visualize the behavior of the concentration of the alcohol $C$ as a function that depends on the time, observing the slope field and already imagining which family of functions would be the solutions of the system. Particularly, one of the students described the phenomena behavior as a decreasing exponential function. So, with this report, it is possible to consider that the hypotheses A and B were validated.

It was expected from the students in the \textit{a priori} analysis, that after observing the models graphical solutions, they realized that as higher the value of $C$, the higher the alcohol concentration will be, and the faster this concentration will decay. It was detected in the \textit{a posteriori} analysis that the students perceived the relationship between the value of the alcohol concentration and the decay speed. It can be seen in Figure 4, a graphical construction produced by a student. In this construction, we can observe that he was varying the value of $k$. In this way the hypotheses A and B were validated.
Finally, in the *a priori* analysis, it was expected that the students were able to relate the decay of the body’s alcohol concentration to the exponential function with base $e$. It was also expected that the student would realize the advantage of writing the final solution as a function of the exponential type in the form $f(x) = be^{ax}$, because this expression displays not only the initial value $b = f(0)$, but also the coefficient $a$, which is closely connected to the growth/decay rate of the function.

In the *a posteriori* analysis was detected that the students realized that the natural-based exponential function appeared as a solution to represent the behavior of the alcohol elimination from the organism. The construction produced by the student is presented in Figure 5. In this way, we consider that the hypothesis B was validated.

*Figure 4.* Slope fields fixing $C_0 = 3.5$ and the variation of $k$
4. Final considerations

As is possible to observe in [11] research, this work opened some possibilities of proposals for didactic interventions that can actually add to the classroom some experiences that can contribute to the understanding of the mathematical specificities related to these functions by the students.

The modeling approach has proven to be an efficient and effective tool to teach exponential functions, especially when it awakens students’ interest in working with models that describe natural phenomena. Besides, it enables students to understand the behavior of functions dynamically; this is possible through the sliding commands that the GeoGebra presents, which makes it possible to comprehend the functions when their parameters vary.

This research can also serve as a suggestion of classroom applications, keeping the specificities of each teaching environment, as well as to start new investigations using modeling approach, GeoGebra software and DE. It is very important to emphasize the relevance of a proper and detailed planning of the activities as well as the organization of the information by the teacher to this research progress.

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Figure 5. Exponential function solution of the model produced by one of the students
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