A QUANTITATIVE EXPLANATION OF THE OBSERVED POPULATION OF MILKY WAY SATELLITE GALAXIES.

SERGEY E. KOPOSOV, JAIYUL YOO, HANS-WALTER RIX, DAVID H. WEINBERG, ANDREA V. MACCIÒ, JORDI MIRALDA-ESCUDE

ABSTRACT

We revisit the well known discrepancy between the observed number of Milky Way (MW) dwarf satellite companions and the predicted population of cold dark matter (CDM) sub-halos, in light of the dozen new low luminosity satellites found in imaging data from the Sloan Digital Sky Survey (SDSS) and our recent calibration of the SDSS satellite detection efficiency, which implies a total satellite population far larger than these dozen discoveries. We combine a detailed dynamical model for the CDM sub-halo population with simple, physically motivated prescriptions for assigning a stellar content to each sub-halo, then apply observational selection effects and compare to the current observational census. Reconciling the observed satellite population with CDM predictions still requires strong mass-dependent suppression of star formation in low mass sub-halos: models in which the stellar mass is a constant fraction $F_\ast(\Omega_b/\Omega_m)$ of the sub-halo mass $M_{\text{sat}}$ at the time it becomes a satellite fail for any choice of $F_\ast$. However, previously advocated models that invoke suppression of gas accretion after reionization in halos with circular velocity $V_{\text{circ}} \leq V_{\text{crit}} \approx 35 \text{ km s}^{-1}$ can reproduce the observed satellite counts for $-15 \leq M_{\ast} \leq 0$. Successful models require $F_\ast \approx 10^{-3}$ in halos with $V_{\text{circ}} > V_{\text{crit}}$ and strong suppression of star formation before reionization in halos with $V_{\text{crit}} \approx 10 \text{ km s}^{-1}$; models without pre-reionization suppression predict far too many satellites with $-5 \leq M_{\ast} \leq 0$. In this successful model, the dominant fraction of stars formed after reionization at all luminosities. Models that match the satellite luminosity distribution also match the observed heliocentric radius distribution, and they reproduce the observed characteristic stellar velocity dispersion $\sigma_{\ast} \approx 5 - 10 \text{ km s}^{-1}$ of the SDSS dwarfs given the observed sizes ($\sim 50 - 200 \text{ pc}$) of their stellar distributions. The model satellites have $M(<300 \text{ pc}) \sim 10^7 M_\odot$ as observed even though their present day total halo masses span more than two orders of magnitude; the constancy of central masses mainly reflects the profiles of CDM halos. Our modeling shows that natural physical mechanisms acting within the CDM framework can quantitatively explain the properties of the MW satellite population as it is presently known, thus providing a convincing solution to the ‘missing satellite’ problem.

Subject headings: Galaxy: halo – Galaxy: structure – Galaxy: formation – Local Group

1. INTRODUCTION

The inflationary cold dark matter scenario predicts an initial fluctuation spectrum with power that continues down to small scales, and in consequence it predicts a mass function of dark matter halos that rises steeply towards low masses. A significant fraction of these halos survive as gravitationally self-bound units long after falling into more massive halos. As pointed out forcefully by [Klypin et al. 1999] and [Moore et al. 1999], the predicted number of sub-halos within a Milky Way-like galaxy halo greatly exceeded the then known numbers of Milky Way or Local Group dwarf satellites, when sub-halos and observed dwarfs were matched based on velocity dispersion or corresponding circular velocity (see also [Kaufmann et al. 1993]). This discrepancy between predicted and observed numbers has become known as the “missing satellite problem.”

Proposed solutions fall into three general categories. The first modifies the properties of dark matter or the primordial fluctuations from inflation in a way that eliminates the low mass dark matter sub-halos themselves (e.g. [Kamionkowski & Liddle 2000; Spergel & Steinhardt 2000; Bode et al. 2002; Zentner & Bullock 2003]). The second appeals to astrophysical mechanisms that suppress star formation in low mass halos so that they do not become observable dwarf satellites; photo-heating by the meta-galactic UV background is an attractive mechanism because it naturally introduces a cutoff at approximately the correct velocity scale (Bullock et al. 2000; Somerville 2002; Kravtsov et al. 2004). The third possibility, arguably a variant of the second, is that the numerous dwarf companions of the Milky Way actually exist but have been missed by observational searches.

In this paper we revisit the “missing satellite problem” with particular emphasis on the role of the new dwarf companions discovered in imaging data from the Sloan Digital Sky Survey (SDSS; York et al. 2000;
Adelman-McCarthy et al. 2008). There are now about a dozen of these (Wilman et al. 2005; Belokurov et al. 2006, 2007; Zucker et al. 2006; Irwin et al. 2007; Koposov et al. 2007; Walsh et al. 2007; a couple of systems still have ambiguous status), most of them at least an order of magnitude less luminous than the faintest of the previously known, “classical” satellites. Spectroscopic follow-up (e.g. Martin et al. 2007; Simon & Geha 2007; Geha et al. 2008) for many of them indicates that they are indeed dark matter dominated systems, even though most are fainter than typical globular clusters, as low as only ∼1000 L⊙ (e.g. Belokurov et al. 2007; Martin et al. 2008). Remarkably, almost all of the newly found faint satellite galaxies have stellar velocity dispersions in the range 3 – 10 km s−1, though their luminosities vary widely. Similarly, the total masses within the inner 300 pc span less than an order of magnitude (Strigari et al. 2008).

Since the SDSS imaging in which these satellites have been discovered covers only ∼ 20% of the sky, a naive accounting would increase the estimated number of Milky Way companions by 5 × 12 = 60, in addition to the ten classical satellites. However, Koposov et al. (2008) use a well-defined identification algorithm to show that the SDSS dwarfs are also subject to strong radial selection effects. Most of the newly discovered objects could only have been found within distances of 50-100 kpc, much smaller than the inferred virial radius of the Milky Way’s dark matter halo (∼ 280 kpc for ρvir/ρ = 340; Xie et al. 2008). The faintest SDSS dwarfs are detectable over only 1/1000 of the halo virial volume (including the factor of five for sky coverage). Walsh et al. (2008) have recently reached similar conclusions based on an independent identification algorithm and independent Monte Carlo tests.

Such analyses are the basis for ‘volume corrections’ for the faint Milky Way satellite population. With proper volume corrections applied, the luminosity function of faint Milky Way satellite galaxies turns out to be a rather shallow power law in the range −15 < M_V < −3 (Koposov et al. 2008). These results in turn imply that the number of satellites brighter than M_V = −3 is ∼80 or more, and the number above M_V = 0 could be a few hundred. Tollerud et al. (2008) reached a similar conclusion, adopting a radial satellite distribution based on the Via Lactea simulation of Diemand et al. (2007). Even this census counts only dwarfs that are above the effective surface brightness threshold for SDSS detection. With the Koposov et al. (2008) detection algorithm, this threshold is approximately 30 mag arcsec−2 (V-band), with a weak dependence on luminosity and distance. The dwarfs found in SDSS have surface brightnesses that range from 24 to 30 mag arcsec−2.

Studies of the high redshift Lyα forest indicate that the small scale power expected in the standard ΛCDM scenario (inflationary cold dark matter with a cosmological constant) is indeed present in the primordial fluctuation spectrum (Narayanan et al. 2008; Viel et al. 2005; Abazajian 2006; Seljak et al. 2006). Astrophysical suppression of star formation, and photo-ionization suppression in particular, has emerged as the most plausible and hence popular solution to the “missing satellite” conundrum. Within this category, there have been different proposals about what sub-halos host the observed dwarf satellites. Bullock et al. (2000) suggested that the observed dwarfs are those whose sub-halos assembled a substantial fraction of their mass before reionization, and thus before the onset of photo-ionization suppression.

Stoehr et al. (2002) suggested that the measured stellar velocity dispersions are well below the virial velocity dispersions of the dark matter sub-halos, and that the observed dwarfs occupy sub-halos that are still above the velocity threshold where star-formation suppression occurs. Kravtsov et al. (2004) used N-body simulations to show that roughly 10% of sub-halos lose a large fraction (∼ 90%) of their mass during dynamical evolution without being completely disrupted; they suggested that the observed dwarfs occupy sub-halos that were above the suppression threshold at the time they became satellites but have suffered extensive mass loss since then.

The new SDSS discoveries and their quantified detectability are the basis for the model-data comparison in this paper. We construct and test models of the Milky Way dwarf satellite population that incorporate Monte Carlo realizations of merger trees for 10^{12} M⊙ (main galaxy) halos, a detailed analytic model for the dynamical evolution and disruption of sub-halos, and a variety of recipes for assigning stellar masses to these sub-halos motivated by ideas in the existing literature. For most of our models, we assume that a sub-halo can only accrete gas to form stars (a) before the epoch of reionization or (b) after reionization if its virial velocity exceeds a critical threshold before it enters the Milky Way halo and becomes a satellite. The spirit of the exercise is similar to that of Bullock et al. (2000), but the dynamical modeling of sub-halos is more sophisticated, and we are now in a position to include directly the (strong) constraints imposed by the SDSS dwarfs accounting for the radial selection function found by Koposov et al. (2008).

In contrast to most previous studies, we treat the luminosity distribution as the primary test of models, rather than the stellar velocity dispersions or central masses (Strigari et al. 2007, 2008; Li et al. 2008;Marcaci et al. 2009), or the inferred but unobservable sub-halo circular velocities. This emphasis is motivated by the fact that the luminosity is the foremost quantity that matters for the observational selection. We consider stellar velocity dispersions and central masses as an additional test, but their interpretations are affected by the uncertainty in the dark matter profiles of the sub-halos associated with observed dwarfs.

2. THE POPULATION OF DARK MATTER SUB-HALOS IN THE MILKY WAY
Our model for the Milky Way satellites is based on the cold dark matter scenario, with each satellite forming initially in a separate dark matter halo that at some point falls into the Milky Way’s dark matter halo. We refer to the bound dark matter satellites orbiting in the Milky Way halo as sub-halos. A sub-halo may or may not correspond to a dwarf satellite galaxy, depending on whether it contains an observable number of stars. In this Section we describe our model for computing the dynamical evolution of sub-halos.

We use the dynamical dark-matter-only model of sub-halos developed by Yoo et al. (2007) to compute the sub-halo population and its orbital distribution. This model is described in detail in Yoo et al. (2007), where a much larger halo of $10^{15} M_\odot$ was considered as a model of a massive cluster of galaxies. Here we consider instead a final halo of $10^{12} M_\odot$ at the present time as a representation of the Milky Way galaxy. Despite the change in the final halo mass, the model remains basically the same as described in Yoo et al. (2007), so here we make only a brief summary of its description.

The model uses the extended Press-Schechter formalism to generate a Monte Carlo merger tree of the parent halo at the present time (Press & Schechter 1974; Bond et al. 1991). We follow the dynamical evolution of all the sub-halos with masses $M_{\text{sub}} > 10^6 M_\odot$ until they merge with the Milky Way and lose their mass below $M_{\text{sub}} = 10^5 M_\odot$. All halos start as isolated objects, and they grow in mass by accretion and mergers for as long as they remain isolated. At some redshift, $z_{\text{sat}}$, they merge into a larger halo (either the Milky Way or another object that will become a Milky Way sub-halo). After this merger, the object has become a satellite or sub-halo and it stops growing in mass. It can subsequently lose mass by tidal stripping when it passes near the center of its parent halo or undergoes encounters with other sub-halos. The sub-halo is subject to dynamical friction, which tends to shrink its orbit, and to random encounters with other sub-halos, which on average expand the orbit. The orbital eccentricity is also subject to random variations. The model allows for the presence of sub-halos within other sub-halos. When a sub-halo is disrupted, any sub-halos it contained are dispersed into the new, larger parent halo. This simple analytic model is able to reproduce the sub-halo mass function, in reasonably good agreement with that found in numerical N-body simulations (Zentner et al. 2005; Shaw et al. 2006; Yoo et al. 2007). For the present purpose, this approach has the advantage (over N-body) of easily affording the required mass resolution and multiple halo realizations.

We adopt a flat CDM cosmology with matter density $\Omega_m = 0.24$, baryon density $\Omega_b = 0.04$, power spectrum normalization $\sigma_8 = 0.8$, Hubble constant $h = 0.7$, and a primordial spectral index $n_s = 0.95$, consistent with recent measurements (Spergel et al. 2007; Tegmark et al. 2006). The matter power spectrum is computed by using the transfer function of Eisenstein & Hu (1999). We generate six Monte Carlo merger trees of a Milky-Way-sized halo. Each realization provides the sub-halo mass function, their orbital elements and density profiles at the present time. Our statistical results are the average of the six different realizations.

The dynamical model of Yoo et al. (2007) uses the Jaffe profile and its velocity dispersion to model sub-halos and their dynamical interactions, for reasons of numerical simplicity and because large galaxies that are tidally-limited satellites of a larger halo are reasonably well modelled by a Jaffe sphere for their baryon plus dark matter density profiles. However, the very low-mass dwarf satellites tend to be dominated by dark matter even in their inner parts. We therefore make an adjustment to better connect our Monte Carlo simulation results to the observed Milky Way dwarf galaxies: we use the sub-halo masses and orbital elements, which are the quantities most robustly computed in the Yoo et al. (2007) model, but we calculate the density profiles and velocity dispersions of sub-halos assuming that they have an NFW profile (Navarro et al. 1997). Using the standard spherical collapse model, the virial radius of an isolated halo is assigned as

$$R_{\text{vir}} = \left[ \frac{3 M_{\text{halo}}}{4 \pi \Delta_c \rho_{\text{crit}}(z)} \right]^{1/2},$$

(1)

where $\Delta_c = (18 \pi^2 + 82 x - 39 x^2)/(1 + x)$ (Bryan & Norman 1998), $x = -(1 - \Omega_m)/(\Omega_m(1 + z)^3 + 1 - \Omega_m)$, and the mean cosmic density is $\rho_{\text{crit}}(z) = \Omega_m \rho_c(1 + z)^3$. The halo concentration $c$ is computed using the relation from Bullock et al. (2001), scaled to $c_8 = 0.8$ according to Maccio et al. (2007), with $c = 0.8 \times 9 \times (M_\odot/10^9 h^{-1} M_\odot)^{-0.13}/(1 + z)$.

For the model in this paper, we use in particular the sub-halo masses at two different special epochs: $M_{\text{rei}} \equiv M_{\text{tot}}(z = z_{\text{rei}})$ when the universe ionizes and the photoionization background starts to suppress the star formation efficiency in low mass halos, and $M_{\text{sat}} \equiv M_{\text{tot}}(z_{\text{sat}})$ at the epoch when a halo merges into a larger halo and we presume that subsequent star formation and gas accretion is halted in the sub-halo. We shall also use below the halo circular velocity $V_{\text{circ}}$, which is the virial circular velocity, $V_{\text{circ}} \equiv [G M_{\text{tot}}/(R_{\text{vir}})]^{1/2}$. (Here $M_{\text{tot}}$ refers to the total mass including dark matter and a universal fraction of baryons.)

In Figure 1 we show the distribution of $M_{\text{sub-halo}}$ at redshifts $z = 0$ (left panel), $z = z_{\text{sat}}$ (middle panel) and $z = 8, 11, 14$ (right panel). As expected, the mass distribution is close to a power-law, except near the resolution limit of our simulations. Also, in Figure 2 we show the accretion history of the MW sub-halo population. By plotting the halo masses of the present day MW sub-halos at the time of accretion vs. the redshift at which they became satellites of larger halos. We see that most of the MW sub-halos became satellites at $z < 2$. Most of the accreted satellites have small circular velocities $V_{\text{circ}} < 20 \text{ km s}^{-1}$, so they lie in a range where gas accretion and star formation are likely to be suppressed after the epoch of reionization (Quinn et al. 1996; Thoul & Weinberg 1996; Bullock et al. 2000).

3. POPULATING THE DM HALOS WITH STARS

3.1. Recipes to assign stellar masses to sub-halos

To make direct observational predictions from these models, we populate each sub-halo in a given Monte-Carlo realization with stars according to a sequence of recipes, then test how many of these satellites could have been found within the SDSS. Some of these recipes are
stripping of the dark matter does not affect the stellar content of the satellite if it survives to the present day. Simulations suggest that these assumptions are reasonable but not perfect approximations (Simha et al. 2008; Peña-Rubia et al. 2008).

We begin with the simplest model (denoted Model 1A), that the stellar mass is a constant fraction of the sub-halo mass at the time of accretion into the main halo:

$$M_* = f_s \times M_{\text{sat}}.$$  (2)

The arguments of Klypin et al. (1999) and Moore et al. (1999) suggest that this model will fail badly, and we show that it does indeed fail despite the new satellite discoveries and the radial selection biases that affect them. There is ample evidence that the efficiency of star formation declines rapidly towards low masses even well above the dwarf satellite regime (e.g., van den Bosch et al. 2007). In Model 1B, we allow the stellar fraction to vary as a power law of $M_{\text{sat}}$ below a threshold $M_0$:

$$M_* = f_s \times \min \left( \left( \frac{M_{\text{sat}}}{M_0} \right) ^{\alpha} , 1 \right) \times M_{\text{sat}}.$$  (3)

Our second approach to modeling stellar masses includes the effects of a pervasive energetic radiation field after the epoch of reionization, which heats gas and hence keeps it from accumulating at the centers of low-mass halos. Calculations by Quinn et al. (1996) and Thoul & Weinberg (1996) showed that gas accretion in halos with the circular velocities below $V_{\text{circ}} \sim 30 - 40$ km s$^{-1}$ is strongly suppressed, while substantially larger halos are minimally affected (see also Weinberg, Hernquist, & Katz 1997; Gnedin 2000). In this spirit, we assume that halos below a critical circular velocity form no stars after reionization, and we thus assign stellar masses

$$M_* = \begin{cases} f_s \times M_{\text{sat}} & \text{if } V_{\text{circ}}(z_{\text{sat}}) > V_{\text{crit}} \\ f_s \times M_{\text{rei}} & \text{if } V_{\text{circ}}(z_{\text{sat}}) < V_{\text{crit}} \end{cases}.$$  (4)

This model (Model 2) has three adjustable parameters $- f_s , V_{\text{crit}}$, and $z_{\text{rei}}$ — with expectations that $V_{\text{crit}} \sim 20 - 40$ km s$^{-1}$ and $z_{\text{rei}} \sim 11$ (e.g. Weinmann et al. 2007).
expression," but this simply means that halos with $z_{\text{rei}}$ below a critical threshold form stars with very low efficiency (too low to produce observable satellites), most likely because of inefficient cooling rather than active feedback.

Our third class of models is similar to the second, but it replaces the sharp threshold of equation (4) with the continuous transition found in numerical simulations by Gnedin (2000), Hoeft et al. (2006), and Okamoto et al. (2008). The numerical results in these papers can be described fairly well by a formula similar to that in Gnedin (2000), with the fraction of baryons that cool in low mass halos suppressed by a factor $[1 + 0.26(V_{\text{crit}}/V_{\text{circ}}(z_{\text{sat}}))^3]^{-3}$, well after the reionization redshift, the critical velocity is found to be approximately independent of redshift.

Gnedin (2000) found $V_{\text{crit}} \sim 40$ km s$^{-1}$, but these results were artificially affected by numerical resolution (N. Gnedin, private communication). Hoeft et al. (2006) and Okamoto et al. (2008) find $V_{\text{crit}} \sim 25 - 30$ km s$^{-1}$. Including the pre-reionization contribution to $M_*$, this model (Model 3A) becomes

$$M_* = \frac{f_\star \times (M_{\text{sat}} - M_{\text{rei}})}{(1 + 0.26(V_{\text{crit}}/V_{\text{circ}}(z_{\text{sat}}))^3)} + f_\star \times M_{\text{rei}}.$$  \hfill (5)

The assumption that all halos can form stars before $z_{\text{rei}}$ may not be justified because in halos with virial temperature $T_{\text{vir}} \lesssim 10^4$ K ($V_{\text{circ}} \lesssim 10$ km s$^{-1}$) the gas does not get hot enough to cool by atomic processes, and simulations that include molecular cooling suggest that gas cooling and star formation is very inefficient in such halos (Haiman et al. 1997; Barkana & Loeb 1999; Machacek et al. 2001; Wise & Abel 2007; O’Shea & Norman 2008; Bovill & Ricotti 2008). We will therefore consider variant models (Model 3B) that eliminate stellar mass in pre-reionization halos below a critical threshold $V_{\text{crit, r}} \sim 10$ km s$^{-1}$. In Model 3B, halos with $V_{\text{circ}}(z_{\text{rei}}) < V_{\text{crit, r}}$ have stellar mass

$$M_* = \frac{f_\star \times M_{\text{sat}}}{(1 + 0.26(V_{\text{crit}}/V_{\text{circ}}(z_{\text{sat}}))^3)},$$  \hfill (6)

while halos with $V_{\text{circ}}(z_{\text{rei}}) > V_{\text{crit, r}}$ have mass given by equation (5).

To determine very roughly the plausible range of values for the stellar mass fraction $f_\star$, we refer to the results of Strigari et al. (2007), who derived $M(< r_{\text{tidal}})/L = 30 - 800 M_\odot/L_\odot$ for the classical dwarfs, and Simon & Geha (2007), who measured velocity dispersions for SDSS dwarfs and inferred total mass-to-light ratios of $140 - 1800 M_\odot/L_\odot$. For a stellar mass-to-light ratio $M_*/L_V = 1M_\odot/L_\odot$, we infer plausible values of $f_\star \sim 10^{-4} - 10^{-2}$, though these are very uncertain because all the dynamical mass-to-light ratio determinations suffer from the fact that the stars in luminous bodies of the dSphs probe only the inner parts of the dark matter potential wells. Another line of argument comes from matching the mean space density of dark matter halos to that of observed field dwarfs: Tinker & Conroy (2008) find $f_\star \approx 10^{-3.5}$ at absolute magnitude $M_r \approx -10$. In the rest of the paper, we will frequently refer to the stellar mass fraction normalized by the universal baryon fraction:

$$F_\star = \frac{f_\star}{\Omega_b/\Omega_m} = 6.25 f_\star.$$  \hfill (7)

Note that $f_\star$ and $F_\star$ refer to stellar fractions in halos where the efficiency is not suppressed, i.e., $V_{\text{circ}}(z_{\text{sat}}) > V_{\text{crit}}$. We will frequently refer to the quantity $(M_*/M_{\text{sat}}) \times (\Omega_m/\Omega_b)$ as the “star formation efficiency,” by which we mean the efficiency with which the halo converted the baryons available to it at $z_{\text{sat}}$ (for a universal baryon fraction) into stars observable at $z = 0$.

3.2. Detectability and observable properties of the simulated satellites

Color-magnitude diagrams for the faint dwarf spheroidal galaxies in the Milky Way halo show that the stellar populations are predominantly ‘old’ (older than several Gyrs) and metal poor ([Fe/H] $\lesssim -1$). To convert stellar masses to luminosities, we assume that all of our model dwarfs have a stellar mass-to-light ratio $M_*/L_V = 1M_\odot/L_\odot$ appropriate to an old, metal poor population (Bruzual & Charlot 2003; Martin et al. 2008). The light of the lowest luminosity dwarfs can be dominated by a handful of bright stars and thus subject to stochastic variations. We ignore this complication; our “luminosities” are simply scaled stellar masses: $L_V/L_\odot = M_*/M_\odot$. This seems appropriate, since the luminosities of the dwarfs galaxies are usually measured either by integrating over the luminosity function of old stellar population matched to the observed luminosity function of stars in dwarfs (Belokurov et al. 2008) or by averaging over possible stochastic variations of galaxy luminosity (Martin et al. 2008).

The detectability of a faint stellar MW satellite galaxy in an SDSS-like search depends on its luminosity and its distance from the Sun, as quantified by Koposov et al. (2008) (see also Walsh et al. 2008). On the basis of these results (Figure 12 of Koposov et al. 2008) we model the detectability of each simulated satellite as a binary deci-

---

**Table 1**  
List of models used

| Model Name | Present-Epoch Stellar Mass |
|-----------|-----------------------------|
| 1A        | $M_\star = f_\star \times M_{\text{sat}}$ |
| 1B        | $M_\star = f_\star \times \min((M_{\text{sat}}/M_\odot)^{\alpha}, 1) \times M_{\text{sat}}$ |
| 2         | $M_\star = \begin{cases} f_\star \times M_{\text{sat}} & \text{if } V_{\text{circ}}(z_{\text{sat}}) > V_{\text{crit}} \\ f_\star \times M_{\text{rei}} & \text{if } V_{\text{circ}}(z_{\text{sat}}) < V_{\text{crit}} \end{cases}$ |
| 3A        | $M_\star = \begin{cases} f_\star \times M_{\text{sat}} - M_{\text{rei}}/(1 + 0.26(V_{\text{crit}}/V_{\text{circ}}(z_{\text{sat}}))^3) & \text{if } V_{\text{circ}}(z_{\text{sat}}) > V_{\text{crit}},r \\ f_\star \times M_{\text{sat}} & \text{if } V_{\text{circ}}(z_{\text{sat}}) < V_{\text{crit}},r \end{cases}$ |
| 3B        | same as 3A for halos with $V_{\text{circ}}(z_{\text{sat}}) > V_{\text{crit}},r$, for halos with $V_{\text{circ}}(z_{\text{sat}}) < V_{\text{crit}},r$ |

Dunkley et al. 2008. The approach is similar to that of Bullock et al. 2000, except that we treat $V_{\text{crit}}$ as free parameter, and the stellar mass formed before the epoch of reionization is assigned using $M_\star = f_\star \times M_{\text{sat}}$, instead of simply dividing galaxies into “observable” or “unobservable” classes based on the fraction of the mass accreted by $z_{\text{rei}}$. 

---

8 We will refer to these models with “pre-reionization suppression,” but this simply means that halos with $V_{\text{circ}}(z_{\text{sat}})$ below a critical threshold form stars with very low efficiency (too low to produce observable satellites), most likely because of inefficient cooling rather than active feedback.
assign the heliocentric distance of the satellites.

Our simulations provide the current Galactocentric distance and orbital apocenter and pericenter for each sub-halo. This can be done straightforwardly if we assume that the stars are test particles — an assumption supported by the observed $(M/L)_\text{dyn}(<R_{\text{eff}}) \gg (M/L)_\text{st}(<R_{\text{eff}})$ — orbiting in an NFW potential with an isotropic velocity dispersion. Then we can use the Jeans equation (Jeans [1919]) to derive the velocity dispersion profile of stars:

$$\frac{d(\nu(r)\sigma^2(r))}{dr} + \nu(r)\frac{G M(r)}{r^2} = 0,$$

where $\nu$ is the density distribution of stars (see Strigari et al. [2007] for more detailed treatment). Here we assume that the density of stars follows a Plummer profile $\nu \propto [1 + (r/r_p)^2]^{-5/2}$ (Plummer [1911]), which seems to fit observed density profiles reasonably well (Wilkinson et al. [2002]; Belokurov et al. [2007]). The mass profile $M(r)$ used here is computed based on the virial radii and concentrations at the redshift $z_{\text{vir}}$ of sub-halo accretion. While the outer parts of the sub-halos are tidally stripped, Penarrubia et al. [2008] show that the stars and the inner part of the dark matter sub-halo are stripped only at a very late stage, when the sub-halo is close to complete disruption. They also show that the velocity dispersion in sub-halos is a function of the total dark matter mass remaining bound inside the luminous body and therefore remains nearly constant until this late stage.

After numerically solving the Jeans Equation, we compute the expected light-weighted velocity dispersion within the optical radius as

$$\sigma_* = \frac{\int \nu(r)\sigma^2(r) \, dx \, dy \, dz}{\int \nu(r) \, dx \, dy \, dz},$$

where the integration is done over a cylinder within a radius, $R = \sqrt{x^2 + y^2}$ equal to the Plummer radius of the galaxy; the integral extends over $\pm \infty$ in $z$. The stellar velocity dispersion depends on the radial extent of the stellar tracers, which cannot be predicted within our simple modeling context (see also Benson et al. [2002]). We therefore use the observed properties of the faint Milky Way satellites to choose stellar radii, based on Martin et al. [2008]. Specifically, we adopt Plummer radii $r_p = 150$ pc for $M_V < -5$, and for fainter dwarfs

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Predicted stellar mass functions of all satellites within the MW’s virial radius (280 kpc), for a variety of models. Left panel: The solid, dotted, and dashed lines represent, respectively, Model 1A with $F_s = 10^{-3}$ and Model 1B with $(F_s, M_0, \alpha) = (10^{-3}, 10^{10} M_\odot, 1)$ and $(10^{-3}, 10^{10} M_\odot, 2)$. Middle panel: The two curves show predictions of Model 2, with $F_s = 10^{-3}$, $z_{\text{vir}} = 11$, and $V_{\text{crit}} = 40$ km s$^{-1}$ (solid), and $V_{\text{crit}} = 20$ km s$^{-1}$ (dashed). Right panel: Thin solid, dashed, and dotted lines represent Model 3A with $(F_s, V_{\text{crit}}, z_{\text{vir}}) = (10^{-3}, 40$ km s$^{-1}, 11), (10^{-3}, 30$ km s$^{-1}, 11),$ and $(10^{-3}, 40$ km s$^{-1}, 8)$, respectively. The thick solid curve shows model 3B with $F_s = 10^{-3}$, $V_{\text{crit}} = 40$ km s$^{-1}$, $z_{\text{vir}} = 11$, and $V_{\text{crit}} = 10$ km s$^{-1}$. All curves reflect the average of six realizations of MW halos. These are the predicted complete satellite (stellar) mass functions, with no radial or sky coverage selection effects.
\end{figure}
we adopt a linear relation between log $r_p$ and $M_V$ with $r_p$ rising from 20 pc at $M_V = 0$ to 150 pc at $M_V = -5$.

The additional important component of the detectability is the tidal disruption of the satellite galaxies. Although our semi-analytic model of dark matter sub-halo evolution properly accounts for the tidal disruption of sub-halos, it does not allow for the possibility that stars have been dispersed in a tidal stream while a small core of the sub-halo survives. Here we simply classify a sub-halo as unobservable if its current tidal radius is less than the expected Plummer radius of the stellar body, which would imply substantial tidal disruption of the stellar component. We also presume that a satellite is unobservable if its host sub-halo has lost more than 99% of its original mass to tidal stripping.

4. RESULTS

4.1. Stellar mass function of the full satellite populations

Figure 3 shows the predicted distribution of the stellar masses of satellites within $R_{\text{virial}} = 280$ kpc, assuming $4\pi$ sky coverage and complete satellite detectability. In the left panel, the solid curve shows Model 1A with a constant $F_\star = 10^{-3}$, making the stellar mass function a scaled version of the dark matter sub-halo mass function. Introducing mass-dependent suppression, Model 1B with $\alpha = 1$ (dashed) and $\alpha = 2$ (dotted) lowers the low mass end of the stellar mass function as expected. Since this model also adopts $F_\star = 10^{-3} = \text{const.}$ above $M_{\text{sat}} = M_0 = 10^{10} M_\odot$, the high mass end of the mass function is unchanged.

The middle panel of Figure 3 shows Model 2, with post-reionization suppression of star formation in halos below a sharp circular velocity threshold, either $V_{\text{crit}} = 40 \text{ km s}^{-1}$ (solid) or $V_{\text{crit}} = 20 \text{ km s}^{-1}$ (dashed), where we have adopted $F_\star = 10^{-3}$ and a reionization redshift of $z_{\text{rei}} = 11$. The resulting stellar mass functions for the satellite galaxies are strongly bimodal, with the low mass portion corresponding to dwarfs in which all stars formed before reionization and the high mass portion corresponding to halos that exceed the critical velocity threshold before becoming satellites, $V_{\text{circ}}(z_{\text{sat}}) > V_{\text{crit}}$. The low mass portion is just a scaled version of the sub-halo mass function at $z = z_{\text{rei}}$. Above $M_\star \approx 10^{6.5} M_\odot$, the host halos are all massive enough to have star formation after $z_{\text{rei}}$, and the mass function is the same as that of Model 1. If the velocity threshold is lowered to $V_{\text{crit}} = 20 \text{ km s}^{-1}$, the high mass peak in the distribution of satellite stellar masses extends to lower values before photo-ionization suppression cuts it off.

The bimodal appearance of the middle panel of Figure 3 is a direct consequence of the sharp $V_{\text{circ}}$ threshold for photo-ionization suppression. The left hand panel shows predictions for several variants of Model 3A and 3B, with the Gnedin (2000) formula (Eq. 5) used to describe photo-ionization suppression. With this smooth suppression, the “pre-reionization” and “post-reionization” portions of the mass function join to form a smooth overall mass function. The low mass end of the mass function is now a mix of satellites that formed their stars before reionization and satellites with $V_{\text{circ}}(z_{\text{sat}}) < V_{\text{crit}}$ whose post-reionization star formation was strongly suppressed but not completely eliminated. Lowering the assumed reionization redshift from $z_{\text{rei}} = 11$ to $z_{\text{rei}} = 8$ boosts the stellar mass function below $M_\star = 10^4 M_\odot$. Conversely, if we eliminate pre-reionization SF in dwarfs with $V_{\text{circ}}(z_{\text{rei}}) < V_{\text{crit},r}$, for which the solid line, Model 3B, the number of satellites with $M_\star \leq 10^4 M_\odot$ drops by a large factor, while at higher masses the stellar mass function is unaffected. The difference between the thin and thick solid lines is the contribution of satellites that formed stars primarily before reionization in halos with $V_{\text{circ}}(z_{\text{rei}}) < 10 \text{ km s}^{-1}$, for $z_{\text{rei}} = 11$ and $V_{\text{crit}} = 40 \text{ km s}^{-1}$.

4.2. Distribution of observed dwarf satellite luminosities, $N(M_V)$

Figure 4 illustrates the impact of selection effects on the observable satellite population. For one realization of Model 3B (with parameters that yield a good match to observations), filled circles show satellites that would be detectable in an all-sky, SDSS-like survey (Koposov et al. 2008), and open circles show non-detectable satellites. The low end of the luminosity distribution, with $M_V \gtrsim -5$, is strongly affected by the radial selection bias. For direct comparison with observations, we therefore select only those model satellites whose combination of luminosity and distance would make them detectable. At the bright end, $M_V < -11$, we assume that existing photographic surveys are complete to $D_\odot = 280$ kpc, and we thus compare the total number of dwarfs across the whole sky to the total population of satellites within the virial radius in the simulation. For $M_V \geq -11$, we randomly select 1/5 of the model galaxies to mimic the 20% sky coverage of SDSS DR5, and we count only those satellites that would be detectable according to the criteria of...
SDSS and classical dwarfs are separated by the vertical line at $M_V = -11$; note that the y-axes for these two populations differ by a factor of five so that the model predictions (which incorporate a factor of 1/5 below $M_V = -11$ to account for SDSS sky coverage) are continuous across the boundary. Left Panel: Predictions of Model 1A, with $M_\star \propto M_{\rm sat}$, for three values of $F_\star$. For $F_\star = 10^{-4}$, the green band shows the bin-by-bin ±1σ range of the predictions from multiple realizations; the logarithmic width of this band is similar for other models. Model curves have been slightly smoothed with a polynomial filter. Right Panel: Comparison of Model 1A (red curve) to Model 1B, where the stellar mass fraction in halos with $M_{\rm sat} < 10^{10} M_\odot$ is $F_\star \propto M_{\rm sat}^{\alpha}$, with $\alpha = 1$ (green band) or $\alpha = 2$ (blue curve).

The luminosities, distances, and velocity dispersions of the observed Milky Way satellites that we use in all subsequent model - data comparisons were taken from various authors ([Mateo 1998; Metz & Kroupa 2007; Martin et al. 2008] and are compiled in Table 2). The sample of SDSS satellites used here consists of those systems above the 50\% completeness limits of Koposov et al. (2008). We do not include two systems, BooII and LeoV ([Walsh et al. 2007; Belokurov et al. 2008], which do not formally satisfy the very conservative selection limits from Koposov et al. (2008). These limits were chosen to avoid the issue of significant 'false positive' detections, at the expense of leaving out 2 objects that deeper follow-up found to be 'real'. For the analysis presented here it is most important that the same selection criteria are applied to the mock satellite observations and the SDSS data. As our analysis subsequently shows, such a small difference in sample size is smaller than the model halo to halo variation of number of galaxies. Therefore the inclusion of omission of these two objects does not affect our results significantly.

Anyway, as we will see later, the halo to halo variation of number of galaxies in our models is noticeable, so we believe that the fact that we do not include two galaxies should not affect our results significantly.

The left panel of Figure 5 compares our simplest model ($M_\star \propto M_{\rm sat}$, Model 1A) to the observed satellite counts, now including the satellite galaxy selection effects in the model. We randomly sample each of the six Monte Carlo halo simulations five times (choosing 1/5 of the faint satellites but always keeping the full set for $M_V < -11$), compute the mean model prediction as the mean of these 30 samplings, and compute the rms dispersion among these 30 in each absolute magnitude bin. Despite the selection bias against low luminosity satellites, this model fails drastically for any choice of $F_\star$, predicting a much steeper luminosity function than observed. For example, the model with $F_\star = 10^{-4}$ matches the observed counts near $M_V = -9$ but predicts far too many satellites fainter than $M_V = -6$. Selection effects and newly discovered satellites have not altered this basic discrepancy, first emphasized by Klypin et al. (1999) and Moore et al. (1999). The green band shows the 1\sigma dispersion in predicted counts, and it is clear that statistical fluctuations will not resolve the discrepancy either.

In the right panel we apply our purely empirical modification, $M_\star/M_{\rm sat} \propto M^\alpha$ below a halo mass $M_{\rm sat} = M_0 = 10^{10} M_\odot$ (Model 1B). With $F_\star = 10^{-3}$ and $\alpha = 2$, this model achieves reasonable agreement with the the observed $N_{\rm obs}(M_V)$ over the full range $0 \geq M_V \geq -15$. The agreement can be further improved by adjusting $F_\star$ and $M_0$, so it appears that this level of mass-dependent suppression is approximately what is needed to explain the observed shape of $N_{\rm obs}(M_V)$. Linear suppression ($\alpha = 1$, green band) is not sufficient, predicting an excess of faint dwarfs when normalized to the bright dwarfs. All of our models fail to match the brightest bin (comprised of the SMC and LMC); we defer discussion of this discrepancy to the end of this Section.
from pre-reionization star formation. The predicted $N_{\text{obs}}(V_V)$ distributions for Model 2, which has a sharp $V_{\text{crit}}$ threshold for the suppression of SF after reionization in small halos. As in Figure 3, the predicted $N_{\text{obs}}(V_V)$ is bimodal, with a bright peak corresponding to halos that exceeded $V_{\text{crit}}$ before $z_{\text{sat}}$ and a faint peak corresponding to stars formed before reionization. Raising the stellar fraction $F_*$ with other parameters fixed (red vs. blue) has no impact on the faint peak, but the bright peak extends to fainter magnitudes and grows in height because lower mass halos can now be populated with stars after reionization. Raising $z_{\text{rei}}$ (green vs. blue) with other parameters fixed has no impact on the bright peak, but it shifts the faint peak downwards in amplitude and slightly downwards in location because halos have accreted less mass by this higher redshift. While photo-ionization suppression reduces the discrepancy with the number of faint satellites seen in Model 1A, these sharp threshold models predict a gap between the faint and bright satellites that is clearly at odds with the data.

Figure 7 compares the Model 2 predictions with those of Model 3A, which uses the (Gnedin 2000) formula to incorporate a smoothly increasing suppression of the stellar mass fraction in halos with $V_{\text{crit}}(z_{\text{sat}}) \lesssim V_{\text{crit}}$. In both cases we use parameters $F_*=10^{-3}, V_{\text{crit}}=35$ km s$^{-1}$, $z_{\text{rei}}=11$. Model 3A is more physically realistic than Model 2, with a mass-dependent suppression that is calibrated on numerical simulations (and is approximately consistent with three independent numerical studies). Galaxies formed in halos with $V_{\text{crit}}(z_{\text{sat}}) \lesssim V_{\text{crit}}$ now fill the gap that was present in Model 2, producing a luminosity distribution that rises continuously from $M_V=-14$ down to $M_V=-2$, before radial selection effects finally cut it off. With these parameter choices, pre-reionization dwarfs dominate the counts (and exceed the observations) for $M_V \leq -4$, but suppressed post-reionization dwarfs dominate the counts at all brighter magnitudes.

Since Model 3 is both more physically realistic and more empirically successful than Models 1 and 2, we focus on it for the remainder of the paper, including Model 3B in which pre-reionization star formation is suppressed below a circular velocity threshold. Figure 8 systematically explores the impact of parameter variations in Models 3A and 3B. In the first three panels, the green band shows the Model 3A predictions for a fiducial set of parameter choices, $F_*=10^{-3}, V_{\text{crit}}=35$ km s$^{-1}$, and $z_{\text{rei}}=11$. Changing $F_*$ (top left) shifts the predicted distribution horizontally to higher or lower luminosities, with some change in shape at the faint end because of the luminosity dependence of radial selection effects. Changing $V_{\text{crit}}$ alters the predicted counts at intermediate luminosities, $-4 > M_V > -11$, while having little effect at the faint end (where pre-reionization dwarfs dominate) or at the bright end (where most galaxies exceed the highest threshold considered here). Changing $z_{\text{rei}}$ alters the height of the pre-reionization peak at faint luminosities but has minimal impact for $M_V < -7$.

With our fiducial parameter choices, Model 3A sub-
Fig. 8.— Predicted $N_{\text{obs}}(M_V)$ for Models 3A and 3B with a variety of parameter choices, in the same format as Figure 5. In the first three panels, green bands show Model 3A predictions for a reference parameter set $F_*=10^{-3}$, $V_{\text{crit}}=35\text{ km s}^{-1}$, $z_{\text{reion}}=11$. Red and blue curves show the impact of changing the stellar mass fraction to $F_*=10^{-2}$ or $10^{-4}$ (top left), the critical velocity threshold to $V_{\text{crit}}=45\text{ km s}^{-1}$ or $25\text{ km s}^{-1}$ (top right), or the reionization redshift to $z_{\text{reion}}=8$ or 14 (lower left). The lower right panel compares the prediction of this reference model (now shown by the red curve) to predictions of Model 3B with a pre-reionization critical threshold $V_{\text{crit},r}=6\text{ km s}^{-1}$ (green band) or $10\text{ km s}^{-1}$ (blue curve).
stantially overpredicts the number of satellites with $M_V \approx -3$. Raising the reionization redshift to $z_{\text{rei}} = 14$ erases this discrepancy, but this value of $z_{\text{rei}}$ seems implausible given the strong and rapidly evolving opacity of the intergalactic medium at $z \approx 6$ seen in quasar spectra (Fan et al. 2006), and it is only marginally consistent with the WMAP5 results. In the lower right panel, we return to $z_{\text{rei}} = 11$ but suppress pre-reionization star formation in halos with $V_{\text{circ}}(z_{\text{rei}}) < 6 \text{ km s}^{-1}$ (green) or $10 \text{ km s}^{-1}$ (blue), motivated by the inefficient gas cooling expected below the threshold for atomic line excitation (Model 3B). The $V_{\text{crit},r} = 10 \text{ km s}^{-1}$ model yields acceptable agreement with the observed number counts over the full range $0 \geq M_V \geq -15$. The $V_{\text{crit},r} = 6 \text{ km s}^{-1}$ model still yields an excess of faint satellites; results for $V_{\text{crit},r} = 8 \text{ km s}^{-1}$ (not shown) are nearly identical to those for $10 \text{ km s}^{-1}$, indicating that an $8 \text{ km s}^{-1}$ threshold is already sufficient to essentially eliminate the contribution of pre-reionization dwarfs. This pre-reionization suppression appears to be critical to explaining the number of dwarfs observed by the SDSS.

Within Model 3B, there is strong degeneracy between the values of $F_r$ and $V_{\text{crit}}$. Figure 9 shows that the parameter combinations $(F_r, V_{\text{crit}}) = (3 \times 10^{-3}, 25 \text{ km s}^{-1})$, $(10^{-3}, 35 \text{ km s}^{-1})$, and $(3 \times 10^{-4}, 45 \text{ km s}^{-1})$ all yield similar predictions and acceptable agreement with the observed number counts. The lower values of $V_{\text{crit}}$ are favored by the numerical studies of Hoefnagels et al. (2006) and Okamoto et al. (2008). For the remainder of the paper we will adopt $(F_r, V_{\text{crit}}, z_{\text{rei}}, V_{\text{crit},r}) = (10^{-3}, 35 \text{ km s}^{-1}, 11, 10 \text{ km s}^{-1})$ as the fiducial parameter values for Model 3B.

For this fiducial model, Figure 10 illustrates in more detail the relative importance of stars formed before and after reionization. For systems with $V_{\text{circ}}(z_{\text{rei}}) > V_{\text{crit},r}$, filled circles show the fraction of their stars that formed before reionization. For systems with $V_{\text{circ}}(z_{\text{rei}}) < V_{\text{crit},r}$, open circles show the fraction of stars that would have formed before reionization, but because of the $V_{\text{crit},r}$ threshold these galaxies have no pre-reionization stars in this model. At every satellite luminosity, the average fraction of pre-reionization stars is small, or even zero, but albeit for different reasons at high and low luminosities. The host halos for the brighter, “classical” dwarf satellites were typically massive enough at $z_{\text{rei}}$ to exceed $V_{\text{crit}}$, but that initial population of stars was subsequently swamped by the much larger post-reionization population. In contrast, the halos that now host the very faintest known satellites ($M_V > -4$) did not exceed $V_{\text{crit},r}$ at $z_{\text{rei}}$ and hence — in Model 3B — did not form any stars before $z_{\text{rei}}$. A small fraction of the satellites with $M_V \approx -5$ have large populations of pre-reionization stars; these are sub-halos that just exceeded $V_{\text{crit},r}$ at $z_{\text{rei}}$ but have low enough values of $V_{\text{circ}}(z_{\text{sat}})$ that their post-reionization star formation was strongly suppressed. If the pre-reionization threshold at $V_{\text{crit},r}$ were smooth rather than sharp, then some additional fainter systems might have significant fractions of pre-reionization stars. However, the general conclusion that pre-reionization star formation should be a small fractional contribution at all satellite luminosities seems fairly robust, provided this star formation is suppressed.
To reproduce the full satellite population, the efficiency of gas accretion and star formation must continue to rise with halo mass above $V_{\text{crit}}$, or at least it must be higher for the SMC and LMC hosts. Since the number of bright SMC and LMC-like objects in our model are determined mainly by one parameter $F_\ast$ (because these objects are not suppressed by the photo-ionization), that rise of star formation efficiency can not be accommodated with our simple model without introducing additional parameters.

4.3. Velocity dispersions, central masses, and radial distributions

As discussed in §4.2, predicting stellar velocity dispersions requires assumptions beyond those needed to compute $N_{\text{obs}}(M_V)$. In particular, we assume that the satellites’ host sub-halos have NFW profiles with concentration given by the theoretically expected mean $c(M)$ relation at $z_{\text{sat}}$, and that subsequent dynamical evolution (e.g., tidal stripping) does not alter the mass distribution of the inner parts of the sub-halo probed by the stars. We also take the observed stellar radii (20 – 150 pc, see §4.2 for details) as input rather than predicting them from a physical model. With these assumptions, the right panel of Figure 12 shows the predicted distribution of stellar velocity dispersions for Model 3B with our fiducial parameter choices. The characteristic value and narrow spread of velocity dispersions for the newly discovered SDSS dwarfs arises quite naturally from these models, despite the large range of stellar luminosities and host sub-halo masses. The predicted distribution is more sharply peaked than the observed one, probably because we did not include scatter in the halo concentration-mass relation and did not include observational uncertainties in the dispersion measurements. The mean value of $\sigma_\ast$ differs by < 20% between data and model, but we consider this small discrepancy is not worrisome, given the simplicity of our dynamical modeling.

The total masses of dwarf satellites are difficult to determine observationally because of the small extent of the stellar distributions relative to the expected extent of the dark matter sub-halo. However, Strigari et al. (2008) show that the total mass (principally dark matter) within a radius of 300 pc, $M_{300}$, can be inferred robustly from observations for nearly all of the known satellites. The top panel of Figure 13 compares the fiducial model predictions of $M_{300}$ to the Strigari et al. (2008) measurements. The model (red diamonds) naturally reproduces the key result of Strigari et al. (2008): over an enormous range of luminosities, the satellites have a narrow range of $M_{300}$, tightly concentrated around $10^7 M_\odot$. The theoretical prediction is artificially tight because we have not included scatter in halo concentrations, which would produce roughly 0.15 dex (rms) of scatter in $M_{300}$ (see Maccìo et al. 2009, figure 1). The model predicts a weak trend of $M_{300}$ with luminosity, which is not evident in the data (but is similar to that predicted by Maccìo et al. 2009).

While the $M_{300}$ range of the satellites is low, the range of total sub-halo masses (at $z = 0$) is more than three orders of magnitude, as shown in the middle panel of Figure 13. The trend of total mass with luminosity is much stronger than the trend for $M_{300}$, though there is a large scatter in mass at fixed luminosity because of tidal stripping. The near constancy of $M_{300}$ is a consequence of the density profiles of CDM halos: NFW ha-
los with the theoretically predicted $c(M)$ relation have only a weak dependence of $M_{300}$ on total mass over the range $\sim 10^7 - 10^{10} M_\odot$ that hosts observed Milky Way satellites (see Macciò et al. 2009 for further discussion). Thus our models and the models of Macciò et al. (2009) are able to reproduce the narrow observed range of $M_{300}$ without much difficulty (see also Li et al. 2008, who examine $M_{600}$ rather than $M_{300}$). We note, however, that if we also allow satellites to form stars with efficiency $F_\star = 10^{-3}$ before reionization (Model 3A), then the $M_{300}$ range for the lowest luminosity dwarfs, with $M_V > -3$, extends downwards to $M_{300} \sim 10^{6.5}M_\odot$ (blue circles in Figure 13). Thus, careful dynamical measurements for the faintest dwarfs could in principle distinguish whether they arise mainly from pre-reionization star formation or from highly suppressed post-reionization star formation in more massive halos. It is noticeable that our model as well as the models of Macciò et al. (2009) and Li et al. (2008) predicts that $M_{300}$ or $M_{600}$ should slightly increase with galaxy luminosity contradicting the observations, where there is no correlation at all of $M_{300}$ versus luminosity (Strigari et al. 2008). The reason of this disagreement is yet to be understood. It either can be caused by some problems with the data (selection effects or systematics in $M_{300}$ measurements) or by some astrophysical effects. For example Macciò et al. (2009) eliminates the correlation of $M_{300}$ versus luminosity by assuming that the inner profile of the halos with low concentration (i.e. massive halos) is modified during the process of tidal stripping (Kazantzidis et al. 2004).

Fig. 12.— Predictions of Model 3B with the fiducial parameters ($F_\star, V_\text{crit}, z_\text{sat}, V_{\text{crit}, \odot}) = (10^{-3}, 35$ km s$^{-1}, 11, 10$ km s$^{-1}$) compared to the observed distributions of absolute magnitude (left) and stellar velocity dispersions (right). The format of the left panel is the same as Figure 5. The right panel shows predicted and observed velocity dispersions only for the SDSS dwarfs — i.e., those with $M_V > -11$ — with data taken from Simon & Geha (2007).

Figure 14 compares the distribution of heliocentric distances of the MW satellites found in the SDSS to the predicted distribution for $M_V > -11$ satellites from our fiducial model. We show one distribution for each of the six Monte Carlo halo realizations. There are significant halo-to-halo variations in the predicted distributions, and the observed distribution follows the lower envelope of the predictions. The distance distribution is strongly influenced by the radial selection effects (the model predictions would be very different if we did not include them), but it also depends on the radial profile of sub-halos and the dependence of this profile on $M_{\text{sat}}$ and $z_{\text{sat}}$, so match-
Fig. 13.— Masses of the DM sub-halos within the central 300 pc (top), their total present-day masses (middle) and their masses at the time of accretion into larger halos (bottom). We only show halos hosting observable satellites within the MW virial radius, as a function of satellite luminosity. Red diamonds show all the observable galaxies from six realizations of the fiducial Model 3B with \((F_\star, V_{\text{crit}}, z_{\text{rei}}, V_{\text{crit}}, r) = (10^{-3}, 35 \text{ km s}^{-1}, 11, 10 \text{ km s}^{-1})\). Blue filled circles show the predictions of Model 3A, which includes pre-reionization dwarfs (or, equivalently, has \(V_{\text{crit}}, r = 0\)). Error bars show the estimates of \(M_{300}\) for observed MW satellites from (Strigari et al. 2008). Solid lines in the bottom panel show, from top to bottom, \(M_\star/M_{\text{sat}} = 10^{-5}, 10^{-4}, \) and \(10^{-3}\). Our models do not incorporate scatter in the concentration-mass relation; adding the theoretically expected scatter would add roughly 0.15 dex of rms scatter to the \(M_{300}\) predictions.

Fig. 14.— Comparison of the model predictions for the cumulative distance distribution of the satellite galaxies with those observed in the SDSS (black line). The predictions of the Model 3B with \((F_\star, V_{\text{crit}}, z_{\text{rei}}, V_{\text{crit}}, r) = (10^{-3}, 35 \text{ km s}^{-1}, 11, 10 \text{ km s}^{-1})\) are shown as red lines.

### TABLE 2

| Galaxy        | \(M_\star\) | \(\sigma_\star\) | \(D_\odot\) |
|---------------|-------------|-----------------|-------------|
| Bootes        | -6.3        | 6.6             | 60          |
| Canes Venatici II | -4.9        | 4.6             | 150         |
| Carina        | -9.4        | 6.8             | 100         |
| Coma          | -4.1        | 4.6             | 45          |
| Canes Venatici I | -8.6        | 7.6             | 220         |
| Draco         | -8.75       | 10.0            | 80          |
| Fornax        | -13.2       | 10.5            | 138         |
| Hercules      | -6.6        | 5.1             | 130         |
| Leo I         | -11.5       | 8.8             | 250         |
| Leo II        | -9.6        | 6.7             | 205         |
| Leo IV        | -5.0        | 3.3             | 60          |
| LMC           | -18.6       | 49              |             |
| Sagittarius   | -12.1       | 11.4            | 24          |
| Sculptor      | -11.1       | 6.6             | 80          |
| Sextans       | -9.5        | 6.6             | 86          |
| Segue I       | -1.5        | 4.3             | 23          |
| SMC           | -17.2       | 58              |             |
| Ursa Minor    | -9.0        | 9.3             | 66          |
| Ursa Major I  | -5.5        | 7.6             | 100         |
| Ursa Major II | -4.2        | 6.7             | 30          |
| Willman I     | -2.7        | 4.3             | 40          |

The satellite discoveries in the SDSS (Willman et al. 2005, Belokurov et al. 2006, 2007, Zucker et al. 2006, Irwin et al. 2007, Koposov et al. 2007, Walsh et al. 2007) have transformed our understanding of the MW’s dwarf satellite population, extending the luminosity range by two orders of magnitude and the implied number of sys-
tems by a factor of 20. Careful quantification of the SDSS satellite detection efficiency (Koposov et al. 2008; Walsh et al. 2008) allows models that specify the relation between dark matter sub-halos and their stellar content to be tested quantitatively against the observations. We have shown that CDM-based models incorporating previously advocated, physically plausible mechanisms for suppressing the stellar content of low mass halos can reproduce the observed properties of the known satellite population, including their numbers, luminosity distribution, stellar velocity dispersions, central masses, and heliocentric radius distribution. However, parameters of these models are tightly constrained, and alternative assumptions lead to conflict with the data. In summarizing our results, it is useful to review both what works and what doesn’t.

What works is a model in which the photo-ionizing background suppresses gas accretion onto halos with \( V_{\text{circ}}(z_{\text{sat}}) < V_{\text{crit}} \approx 35 \text{ km s}^{-1} \) (Quinn et al. 1996; Thoul & Weinberg 1996; Bullock et al. 2000), with the smooth mass-dependent suppression suggested by numerical simulations (eqn. 5; Gnedin 2000; Hoef et al. 2000; Okamoto et al. 2008), and inefficient molecular cooling (and/or stellar feedback) drastically reduces the efficiency of star formation in pre-reionization halos below the hydrogen atomic line cooling threshold \( V_{\text{crit}, r} \approx 10 \text{ km s}^{-1} \) (Haiman et al. 1997; Barkana & Loeb 1999; Machacek et al. 2001; Wise & Abel 2007; O’Shea & Norman 2008; Bovill & Ricotti 2008). There is some degeneracy between this model’s two main parameters, \( V_{\text{crit}} \) and \( F_{\ast} \), as shown in Figure 9, but with either parameter fixed the other is fairly well constrained (Figure 5). The other two parameters, \( z_{\text{rei}} \) and \( V_{\text{crit}, r} \), just need to be in a range that keeps pre-reionization star formation too low to affect the observable luminosity function. For the values \( V_{\text{crit}} = 25 - 35 \text{ km s}^{-1} \) favored by numerical simulations, \( F_{\ast} \) must be \( \lesssim 10^{-3} \), so even sub-halos above the \( V_{\text{crit}} \) threshold have star formation efficiency far lower than the values \( F_{\ast} \approx 0.1 - 0.4 \) found for bright galaxies (e.g., Pizagno et al. 2005; Maudelbaum et al. 2006; Dutton et al. 2007; Gnedin et al. 2007; Xue et al. 2008).

If we assign stellar extents based on observations, and make the reasonable dynamical assumptions discussed in §3.2, then our fiducial model naturally explains the characteristic value and narrow spread of stellar velocity dispersions found for SDSS dwarfs by Simon & Geha (2006; Dutton et al. 2007; Gnedin et al. 2007). The \( M_{300} \) values do not depend on the assumed stellar extent, and their narrow range arises from the theoretically predicted structure of CDM halos, which have a weak dependence of \( M_{300} \) on total halo mass over the range \( M_{\text{halo}} \approx 10^{8} - 10^{13} M_{\odot} \). Thus any CDM-based model that prevents formation of observable dwarfs in halos below \( \approx 10^{5} M_{\odot} \) should qualitatively reproduce the Strigari et al. (2007, 2008) results (e.g., Li et al. 2008; Macciò et al. 2009). Tempering this success, however, is the fact that the total halo mass in our model span three orders of magnitude; some of this range is a consequence of tidal stripping, but the span of \( M_{\text{sat}} \) values is only slightly narrower. The model, in combination with the radial selection biases found by Koposov et al. (2008), also explains the observed heliocentric radius distribution of the SDSS dwarfs, which tests the predicted Galactocentric radius distribution of subhalos and its dependence on mass and accretion redshift.

Many alternative models fail badly in reproducing the observed luminosity distribution. Models with constant \( M_{\ast}/M_{\text{sat}} \) predict far too many faint satellites relative to bright satellites. The SDSS discoveries and luminosity-dependent selection biases do not in themselves resolve the “missing satellite” discrepancy highlighted by Klypin et al. (1999) and Moore et al. (1999): strong mass-dependent suppression of star formation efficiency is still required to reconcile CDM predictions with observations. A simple model in which \( M_{\ast}/M_{\text{sat}} = 10^{-3}(\Omega_{b}/\Omega_{m})(M_{\text{sat}}/10^{10} M_{\odot})^{2} \) for \( M_{\text{sat}} < 10^{10} M_{\odot} \) is reasonably successful at matching the observations. This successful “empirical” model has a mass dependence of star formation efficiency roughly like that of the successful, physically motivated photo-ionization model (eqn. 5; note that \( M_{\text{sat}} \propto V_{\text{circ}}^{2} \) at fixed \( z_{\text{sat}} \)).

Models with sharp suppression of star formation below the photo-ionization threshold \( V_{\text{crit}} \) fail at intermediate luminosities, \( M_{V} \approx -8 \). Pre-reionization star formation can provide the population of faint dwarfs in such a model, but there is an unacceptable gap between the faint and bright populations (or, for parameter choices that fill the gap, there is an excess of dwarfs at other luminosities). It is striking, therefore, that the form of the mass-dependent photo-ionization suppression found in numerical simulations is just that required to match the shape of the observed luminosity distribution. However, the conversion of accreted baryons to stars must be very inefficient for our fiducial model to work, and it is not obvious why this conversion efficiency should be mass independent.

The most interesting of our “negative” conclusions is that star formation in halos before reionization must be extremely inefficient to avoid producing too many satellites in the range \( 0 \gtrsim M_{V} \gtrsim -6 \). Examination of Figure 5 suggests that the upper limit on the fraction of halo baryons converted to stars is a few \( \times 10^{-4} \) for \( z_{\text{rei}} = 11 \), or \( \times 10^{-3} \) if reionization is pushed back to \( z_{\text{rei}} = 14 \). Madau et al. (2008) have reached exactly the same conclusion, with a similar numerical value for the efficiency limit, using the Via Lactea II simulation instead of a semi-analytic method to predict the model sub-halo population. Suppression of star formation in halos below the hydrogen atomic line cooling threshold is physically plausible, as the metallicity is low and molecular cooling should be inefficient. For agreement with \( N_{\text{obs}}(M_{V}) \), we require pre-reionization suppression in halos with \( V_{\text{circ}}(z_{\text{rei}}) < V_{\text{crit}, r} \approx 10 \text{ km s}^{-1} \).

There are several caveats to these conclusions. First, as discussed in §4.2 reproducing the Magellanic Clouds requires that the most massive sub-halos have \( M_{\ast}/M_{\text{sat}} \sim 0.05 - 0.1 \), well above the \( F_{\ast} \sim 10^{-3} \) of our fiducial model. Thus, the photo-ionization suppression described by equation 5 must join onto a continuing increase of star formation efficiency with sub-halo mass above \( V_{\text{crit}} \), an increase that is presumably driven by other physical mechanisms. Indeed, there is nothing about our results that necessarily picks out photo-ionization as the sup-
pression mechanism in low mass sub-halos, but it is a
mechanism that comes in naturally (one might argue in-
evitably) at the desired scale (Bullock et al. 2000), and
the numerically calibrated form yields a good match to
the observed luminosity distribution.

In our fiducial model, even the faintest SDSS dwarfs
form most of their stars after reionization, but they have
\( V_{\text{circ}}(z_{\text{sat}}) \) far enough below \( V_{\text{crit}} \) that their star for-
mation is highly suppressed according to equation \( 6 \). The
SDSS dwarfs are physically a continuum with the clas-
sical dwarfs, and their much lower luminosities are a
consequence of the highly non-linear relation between star
formation efficiency and halo mass below \( V_{\text{crit}} \). Halos
with \( V_{\text{circ}}(z_{\text{crit}}) > V_{\text{crit}} \) form pre-reionization stars, but
in nearly all cases they grow large enough by \( z_{\text{sat}} \) that the
post-reionization population dominates by a large factor.
A small number of systems with \( M_{V} \approx -5 \) could have
large fractions of pre-reionization stars, but at any lu-
niminosity such systems are rare. These conclusions are
robust within our framework, but if we allowed for depar-
tures from our adopted prescriptions — in particular if
photo-ionization suppression for \( V_{\text{circ}} \ll V_{\text{crit}} \) were more
aggressive than equation \( 6 \) implies and pre-reionization
suppression weaker than we have assumed — then it
might be possible to construct models in which many
dwarfs with \( M_{V} \gtrsim -6 \) are pre-reionization “fossils.” The
efficiency of converting halo baryons to stars in these
systems must still be \( \sim 10^{-4} \) or less to avoid producing
too many faint satellites. [Bovill & Ricotti 2008] and
[Salvadori et al. 2009] have argued that halos cooling by
\( H_{2} \) before reionization naturally give rise to the physi-
cal and chemical properties of the SDSS dwarfs. How-
ever, even the low star formation efficiencies \( \sim 0.5\% -2\% \)
found by [Salvadori et al. 2009] appear far too high to be
consistent with the observed number counts. On the
other hand, [Busha et al. 2009] propose a model in which
post-reionization suppression of star formation is highly
efficient (a sharp threshold) but the star formation effi-
ciency in pre-reionization halos is strongly mass depen-
dent, effectively spreading the low luminosity peak evi-
dent in our Figure 6 up towards higher luminosities so
that it fills out the entire faint end of the luminosity
function.

A third caveat is that we do not explain the or-
igin of the observed stellar extents; we just show that
once the observed extents are adopted as inputs, then
the observed stellar velocity dispersions emerge natu-
urally. One possible explanation is that the baryons
in low mass halos condense until they reach a scale
at which the velocity dispersion is a few \( \text{km s}^{-1} \) and
that this minimum dispersion provides the conditions
necessary for star formation. We also have not at-
tempts to explain the chemical abundance distributions
or star formation histories of the satellites (see, e.g.,
[Orban et al. 2008, Salvadori, Ferrara, & Schneider 2008,
Salvadori et al. 2009].

A final caveat is that we have assumed that all dwarfs
luminous enough to be found in the SDSS also lie above
the surface brightness threshold for detection, which is
about 30 mag arcsec^{-2} [Koposov et al. 2008]. Since
some of the known satellites approach this threshold,
it is possible that others fall below it. A large popu-
lation of lower surface brightness dwarfs would change
the number counts that our model reproduces. Note
also that a large population of pre-reionization dwarfs
would be observationally allowed if they lie below the
surface brightness threshold; however, even in this sce-
nario the pre-reionization dwarfs do not account for the
presently known satellites. Deeper large area imaging
surveys, such as PanSTARRS, the Dark Energy Survey,
and LSST, will show whether the MW satellite popu-
lation includes a significant number of lower surface bright-
ness systems.

Our model makes several predictions that can be tested
by these upcoming surveys or by further follow-up studies
of known dwarfs. Deeper surveys should reveal many
more satellites, more than 200 with \( M_{V} < 0 \) and \( D_{\odot} <
400 \text{kpc} \) over the full sky, with the luminosity function
shown in Figure 11. Deep imaging of Andromeda and
other nearby galaxies can show whether they have similar
satellite systems, though these searches will not reach
the extremely low luminosities that can be probed in the
MW. Most satellites in our model have stellar extents
that are substantially smaller than the present-day tidal
radius of their host halo. Tidal tails and tidal disruption
should be rare, an implication that may be challenged
by photometric evidence on the profiles and shapes of
the ultra-faint galaxies, which have been interpreted as
signs of tidal distortion or disruption (e.g., Martin et
al. 2008). Measurements of the total sub-halo masses
of known dwarfs would provide a powerful test of the model
predictions in Figure 13, but the small stellar extents
may make such measurements impossible. Our models
predict that satellites continue to form stars down to
\( z_{\text{sat}} \) or below; and many observable systems should have
\( z_{\text{sat}} = 1 - 2 \) (see Figure 2). These predictions may be
testable with detailed stellar population modeling.

Our results greatly strengthen the argument
[Bullock et al. 2000, Benson et al. 2002, Somerville
2002, Kravtsov et al. 2004] that photo-ionization natu-
rally reconciles the CDM-predicted sub-halo population
with the observed dwarf spheroidal population, thus
solving the “missing satellite problem” highlighted by
[Klypin et al. 1999 and Moore et al. 1999].

The fiducial model presented here offers a detailed, quan-
titative resolution of this problem in light of new,
greatly improved observational constraints, while relying
on previously postulated and physically reasonable
mechanisms to suppress star formation in low mass
halos. The MW satellites provide a fabulous laboratory
for studying galaxy formation at the lowest mass scales,
and much remains to be understood about gas cooling,
star formation, feedback, and chemical enrichment in
these systems. These issues provide challenging targets
for numerical simulations and semi-analytic models,
whose predictions can be tested against detailed studies
of the dynamics and stellar populations of the known
dwarf satellites and of the many new satellites that will
be revealed by the next generation of sky surveys.

S. K. was supported by the DFG through SFB 439 and
by a EARA-EST Marie Curie Visiting fellowship. J. Y.
is supported by the Harvard College Observatory under
the Donald H. Menzel fund. D. W. acknowledges sup-
port from NSF grant AST-0707985 and the hospitality
of the Institut d’Astrophysique de Paris during part of
this work. S.K. acknowledges hospitality from the Kavli Institute for Theoretical Physics (KITP) Santa Barbara during the workshop “Building the Milky Way”. We thank James Bullock for his helpful comments on the paper and the anonymous referee for prompt review and constructive comments.

This paper relies heavily on data from the Sloan Digital Sky Survey. Funding for the SDSS and SDSS-II was provided by the Alfred P. Sloan Foundation, the Participating Institutions, the National Science Foundation, the U.S. Department of Energy, the National Aeronautics and Space Administration, the Japanese Monbukagakusho, the Max Planck Society, and the Higher Education Funding Council for England. The SDSS was managed by the Astrophysical Research Consortium for the Participating Institutions, which are listed at the SDSS Web Site, http://www.sdss.org/
Wilkinson, M. I., Kleyna, J., Evans, N. W., & Gilmore, G. 2002, MNRAS, 330, 778
Wise, J. H., & Abel, T. 2007, ApJ, 671, 1559
Xue, X. -, et al. 2008, ArXiv e-prints, 801, arXiv:0801.1232
York, D. G., et al. 2000, AJ, 120, 1579
Yoo, J., Miralda-Escudé, J., Weinberg, D. H., Zheng, Z., & Morgan, C. W. 2007, ApJ, 652, 26
Zentner, A. R., & Bullock, J. S. 2003, ApJ, 598, 49
Zentner, A. R., Berlind, A. A., Bullock, J. S., Kravtsov, A. V., & Wechsler, R. H. 2005, ApJ, 624, 505
Zucker, D. B., et al. 2006, ApJ, 643, L103