Students’ difficulties in probabilistic problem-solving

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Abstract. There are many errors can be identified when students solving mathematics problems, particularly in solving the probabilistic problem. This present study aims to investigate students’ difficulties in solving the probabilistic problem. It focuses on analyzing and describing students errors during solving the problem. This research used the qualitative method with case study strategy. The subjects in this research involve ten students of 9th grade that were selected by purposive sampling. Data in this research involve students’ probabilistic problem-solving result and recorded interview regarding students’ difficulties in solving the problem. Those data were analyzed descriptively using Miles and Huberman steps. The results show that students have difficulties in solving the probabilistic problem and can be divided into three categories. First difficulties relate to students’ difficulties in understanding the probabilistic problem. Second, students’ difficulties in choosing and using appropriate strategies for solving the problem. Third, students’ difficulties with the computational process in solving the problem. Based on the result seems that students still have difficulties in solving the probabilistic problem. It means that students have not able to use their knowledge and ability for responding probabilistic problem yet. Therefore, it is important for mathematics teachers to plan probabilistic learning which could optimize students probabilistic thinking ability.

1. Introduction

Probability plays an important role in everyday human life. Knowledge of probability can help people in reading a newspaper, evaluating the information, analyzing the accuracy and probability of the event, and making the prediction or decision based on the information [1]. In fact, Shaughnessy stated that no other subdivision of mathematical knowledge is more important for students as important as probability theory and statistics [2]. Not only important in everyday life, but probability theory is also required in other disciplines as basic science and plays an important role in improving critical reasoning [3]. Therefore, it is important for all students to develop an awareness of probability concept and use it appropriately in solving problems [4]. In Indonesian Curriculum, probability theory was first introduced at junior high school on probabilistic material [5]. It is important for a student for learning probability concept to be able to solve and make a decision on problems.

Problem-solving becomes essential in mathematics learning process for connecting students with real situations [6]. By solving mathematics problems, students learn how to think, have persistence and curiosity, and confident when confronted with a foreign situation outside the class [4]. It appears that learning mathematics and problem-solving have interrelationship and dependence on each other. On the one hand, mathematics learning can improve students’ problem-solving skills, while on the other hand, problem-solving activities can be an appropriate tool for teaching mathematics concepts
and procedures. It means that solving the probabilistic problem can be an essential process for students in learning probability concept and improving problem-solving skill.

The importance of the probability concept and probabilistic problem-solving in mathematics learning is contrary to Indonesian student learning outcomes in the probabilistic material. Based on the report of Trend in International Mathematics and Science Study (TIMSS) 2011 [7], the average score of Indonesian students in the data and chance domain is 376. The score is below the mathematics average score of 386. On another hand low achievement in probabilistic material is also indicated from the national examination results of last three years in junior high school in Indonesia, particularly Central Java, as shown in Table 1 [8-10]. It shows that students’ score in statistic and chance always decreased.

| Year | SMP Al-Irsyad (%) | Central Java (%) | Indonesia (%) |
|------|------------------|------------------|--------------|
| 2014 | 60.00            | 58.85            | 60.44        |
| 2015 | 49.17            | 52.64            | 60.78        |
| 2016 | 38.81            | 40.82            | 46.73        |

Source: Badan Standar Nasional Pendidikan (BSNP)

The low achievement of Indonesian students on the probabilistic material indicates students’ inability and difficulties in solving probabilistic problem-solving. As some experts point out that probabilistic problem solving can be very difficult for students [11]. The natural misconceptions about the probabilistic concepts and the problem of using the word or probability term in mathematics also become difficult for many students. This is possible because to solve the problem students as problem solvers must think abstractly about the situation presented and then represent the situation using mathematical concepts. Also, probabilistic problem solving requires a combination of procedural, conceptual and real-world knowledge [12].

The difficulty may lead the student to make errors in solving the probabilistic problem [13, 14]. In this case, students’ errors in solving the problems may occur either written or orally [15]. By analyzing students’ errors, it can be identified students difficulties in solving the probabilistic problem [12]. Error analysis is an attempt to observe, investigate, and clarify students’ errors with certain rules to find an explanation of the error [16, 17]. One of the methods used to analyze students’ errors is to classify them into certain categories based on students’ work results.

There are three types of errors when students are solving problems proposed by Kiat. The errors include (a) conceptual error due to failure to grasp the concepts involved in the problem or errors that arise from failure to appreciate the relationships involved in the problem; (b) procedural error arising from failure to carry out manipulations or algorithms despite having understood the concepts behind the problem; and (c) technical errors due to a lack of mathematical content knowledge in other topics or errors due to carelessness [18]. While Veloo et al. stated that students made conceptual errors, careless errors, problem-solving errors, and value errors during solving mathematics problem [19].

O’Connell investigated students’ errors in solving the probabilistic problem. It classified students’ errors in categories of misunderstanding of text, conceptual errors, procedural errors, and computational error. The error of understanding the text is an error that arises from the lack of understanding of the problem. A conceptual error is an error involving a basic concept or probability theory. Procedural errors are errors arising from incorrect application or formula. An arithmetic error is an error involving calculations [12].

It is important to know what types of students’ error in solving the probabilistic problem. Student error is analyzed to know the difficulties experienced by the student causing the error. It is important for the teachers as their consideration to design and plan better learning strategy. Furthermore, it leads to improving students’ achievement and probabilistic problem-solving ability. Meese said that teachers
need to understand students’ potential, problems and learning difficulties to implement effective teaching strategy and to produce meaningful learning among students [20]. Thus, this study aims to analyze and describe the errors and difficulties of students during the probabilistic problem-solving process.

2. Methods
This research is a type of descriptive analysis research using the qualitative method and case study strategy. Qualitative research is a study to understand the phenomenon experienced by the subject obtained through descriptions in the form of words [21]. Sturman stated that case study was a general term for the exploration of an individual, group or phenomenon [22]. Thus, qualitative research with case study strategy is a study that intends to explore and understand the subject and phenomenon in depth through verbal description. In this research, the phenomenon referred to students’ errors and difficulties in probabilistic problem-solving.

This research was conducted at SMP Al-Irsyad in Surakarta, Indonesia in August 2017. It is based on Table 1 that the national examination results on the statistic and chance material always decreased and below the national and provincial averages score. Subjects in this research involve ten students of 9th grade selected with purposive sampling technique. The instruments of this research included a set of probabilistic problem-solving test and an interview guideline. Subjects were required to complete probabilistic problems before interview. The probabilistic problem-solving test results and interviews were analyzed qualitatively using Miles and Huberman steps to describe students’ errors and difficulties in probabilistic problem solving [23]. Activities in the data analysis include data reduction, data display, and verification. The first thing the researchers do is to analyze the probabilistic problem-solving test results to find out the error made by the students. After that, the students were interviewed to investigate the difficulties relates to the error in probabilistic problem-solving.

3. Result and Discussion
In this section, the results and discussion of research are presented. It includes analysis of probabilistic problem-solving result and interviews of the subject. It proposed three categories of difficulties based on analysis of subject errors in solving the probabilistic problem.

3.1. The difficulty of understanding the problem
The difficulty in understanding the problem refers to students’ inability in understanding the meaning of the probabilistic problem. It is indicated by students’ errors in identifying and representing the information in the problem. It is also indicated that the students fail to understand the meaning of the unknown of the problem. An example of students’ errors in solving a probabilistic problem which relates to this category of difficulty is shown in Figure 1.

*Question 1.1a:* There is a box contained five red marbles, seven blue marbles, and six white marbles. If you are asked to draw a marble from the box with closed eyes, what kind of marble is possible to draw?

**S3:**

\[\text{5 merah, 7 biru, 6 putih} \]

**S5:**

\[\text{5 merah, 7 biru, 6 putih, berubut} \]

*Figure 1. Example of Students’ Answer for the Difficulty of Understanding the Problem*

Based on the result of Figure 1, it seems that both subject S3 and S5 can write the information on the problem correctly, but fails to understand the meaning of the unknown. Both of them cannot identify correctly the meaning of the question. Subject S3 seems to think the question means the possible marbles to draw. Subject S5 seems to think the question means the probability of drawing a
marble. It is supported by the results of interview of the subject. Subject S3 stated that the answer to the question due to the most marbles contained in the box. On the other hand, subject S5 said that 1/18 based on the argument choosing one marble of 18 marbles contained in the box.

Based on the result of problem-solving and interview, it seems that students are difficult to understand the problem. It is indicated by students’ inability and fails to identify the unknown of the problem correctly. In this case, students do not know that the question relates to sample space. It is supported by the research of Abdullah et al. [13] that find students’ errors in understanding the meaning of the question. It is also related to students' text comprehension errors find by O'Connell [12]. This kind of errors relates to student’s difficulty in understanding the problem.

3.2. The difficulty of choosing and using strategy
The difficulty of choosing and using strategy refers to students’ difficulty in identifying an appropriate strategy and using it for solving the problem. It can be identified by students’ errors in arising and carrying out the formula or algorithm for solving the problem. An example of student’s errors in solving a probabilistic problem which is related to this category of difficulty is shown in Figure 2.

*Question 4:*
Dice and coin are thrown together
a. Determine all possible events!
b. Determine the probability of prime and image events!

*S4:*

![Figure 2. Example of Students’ Answer for the Difficulty of Choosing and Using Strategy](image)

Based on the Figure 2, it seems that subject S4 is not able to identify the appropriate strategy for solving the probabilistic problem. Subject’s answer to the question 4.a indicates the size of sample space rather than to describe it. While for the question 4.b, the subject seems to answer the probability of prime event and image events separately. It indicates that subject cannot arise and carry out the strategy or formula or algorithm correctly to answer the question. It is also supported by the result of an interview with the subject. Subject S4 stated that all possible events for the experiment are 12 because 2 (possible events of a coin) multiplied by 6 (possible events of dice). Also, subject S4 also said that the probability of prime and image event is 1/4 and 1/12. It is based on students’ argument that the probability of the prime event is 1/4, while the probability of image event is 1/12.

Based on the result of students problem-solving and interview, it seems that students are difficult to identify the appropriate strategy for solving the probabilistic problem. Students are not able to choose and use the formula and algorithm relates to the question of the problem. It is possibly caused by students misunderstanding about probability concept and the inability for remembering strategies for solving the problem. Most of the students still make errors in regards their conceptual and procedural understanding [12, 18, 19] which is related to this category of difficulty. Students are difficult in choosing and using the strategy for solving the probabilistic problem due to the lack of conceptual and procedural understanding of probability.
3.3. The difficulty of computational process
The difficulty of the computational process relates to students inability and fail in the calculation. It is indicated by students’ errors regarding the arithmetic operation. An example of students’ errors in solving a probabilistic problem which is related to this category of difficulty is shown in Figure 3.

**Question 1.b:**
There is a box contained five red marbles, seven blue marbles, and six white marbles. If you are asked to draw a marble from the box with closed eyes, determine the probability of drawing each kind of marbles!

S1:

![Figure 3. Example of Students’ Answer for the Difficulty of Computational Process](image)

Based on Figure 3, it seems that subject can apply a formula to determine the probability of an event but is failing to make the calculation. Subject S1 make errors regarding the arithmetic process. In this case, subject S1 can apply a formula for determining the probability of drawing each kind of marbles. Furthermore, subject S1 aims to simplify the result, but there is an error identified regarding fraction operation. The result of the interview also indicates that subject aims to simplify 5/18, 7/18, 6/18 by dividing the denominator with the numerator. It is not the right arithmetic process of simplifying the fraction.

Based on the problem-solving result and interview seems that subject still make errors in arithmetic operation [12], even the subject can determine the formula of probability. It is possibly caused by students careless and inaccuracy while computational process [18, 19, 24]. Also, students may not evaluate their problem-solving process which leads them to the errors. It means that subjects still have difficulties regarding computational process in solving the probabilistic problem.

4. Conclusion
Based on the results and discussion, it can be concluded that students still had difficulties in solving the probabilistic problem. It can be identified from the errors had been made by students when solved the probabilistic problem. The difficulties experienced by students in probabilistic problem solving in this research can be divided into three categories. The first difficulty relates to students' difficulties in understanding probabilistic problems. This was indicated by the student's errors while identifying a question or unknown problems. Second, students' difficulties were choosing and using appropriate strategies to solve probabilistic problems. This was indicated by the inability of students to identify and use the appropriate strategies. This was due to students' misconceptions about the concept of probability and the inability to recall strategies for solving the probabilistic problem. The third difficulty related to students' difficulties when they were completing the computation or computation process. This was indicated by the student's error about the arithmetic operation performed. This could be due to student inaccuracies, while they were computing process. In other words, students did not evaluate or re-check the problem-solving process.

Based on the result of the research, it seemed that students had a difficulty while they were solving the probabilistic problem. That was indicated that students had not been able to use their knowledge and ability to respond optimally the probabilistic problems. Therefore, it is important for mathematics teachers to plan probabilistic material learning that can optimize students' probabilistic thinking skills. Learning plan can be the selection of models, methods, strategies, techniques and learn media probabilistic material so can minimize the student difficulties and errors in probabilistic problem-solving process.
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