Characterization and control of a microdisplay as a Spatial Light Modulator

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Abstract. This work introduces a new approximation of the Jones Matrix model, widely used for the prediction of the phase modulation of Twisted Nematic Liquid Crystal Displays (TN-LCDs). Furthermore, this work presents a new set of simple experiments that were carried out to reveal the optical characteristics of a Kopin's microdisplay and to verify the feasibility of a Spatial Light Modulator (SLM) prototype developed, to provide a phase modulation between 0 and 2π without modifying the amplitude of light passing through the device, which is called "Phase Only - SLM".

1. Introduction
Spatial Light Modulators (SLM) are optical systems that allow to modify in real time, the amplitude and phase of the light passing through them. Usually, the heart of these systems is a matrix of liquid crystal pixels called microdisplay.

In the last two decades there has been an interest in SLMs made from microdisplay obtained from the cannibalization of video projectors because they are easy to get and relatively cheap, but this implies the need to adapt the video projector's electronic circuit and raises an issue when several microdisplay with similar characteristics are needed.

Berreman[1] presented a theory of the behavior of light in liquid crystal. Saleh and Lu[2] proposed a model of the Twisted Nematic Liquid Cristal Displays (TN-LCDs) using Jones Matrix method and a characterization method that allows to find the model parameters. This model was extensively used because of its simplicity and first Coy et al.[3] and later Márquez et al.[4], proposed improvements to this model.

For this work it was developed a SLM prototype including the electronic control circuit with the intention of achieving a “Phase-Only Modulation” between 0 and 2π. A Kopin’s monochromatic transmissive microdisplay was used. There is no information available about the microdisplay’s optical properties, so it was necessary to perform a set of experiments to find the parameters of the model that best fit this microdisplay. Yamauchi[5] has shown the inherent ambiguity of the model and the typical characterization method[2]. Márquez et al.[4] proposed to use different wavelengths to eliminate the ambiguities.

In section 2, the Jones Matrix model is reviewed and the improvements made to this model are explained. Also, a new approximation to the model is proposed. The set of experiments that were
carried out to reveal the optical properties of the microdisplay is presented in section 3, where these experiments are also compared with previous techniques. In section 4, the parameters of the model are used to find a configuration suitable of Phase-Only SLM. Finally, in section 5, a breve detail of the electronic control board developed is given.

2. Matrix Model

Berreman’s theory predicts that the molecules inside the TN-LCDs follow the behavior described in figure 1(a). In the off state, no voltage is applied (solid curve $V = 0$) and the twist angle increases gradually with the distance $d$, between $0^\circ$ and $\alpha$ (figure 1(a1)); meanwhile the birefringence is maximum through the material (figure 1(a2)). When an electric field is applied (dotted and dashed curves) between two conductive plates sandwiching the liquid crystal, the molecules tilt, trying to align with the electric field. The birefringence through the material decreases following the behavior described in figure 1(a2). The twist angle increases through the material as described in figure 1(a1). It is important to note that the molecules near the borders tend to adopt the twist angle of the molecules of the alignment layers.

\[ D(\alpha, \beta, \Psi) = R^{-1}(\Psi) \cdot R^{-1}(\alpha) \cdot LC(\alpha, \beta) \cdot R(\Psi) \] (1)

Lu and Saleh presented a model of TN-LCDs based on Jones Matrix method, which describes in a simple way the behavior of the microdisplays. They assumed that the twist angle does not change with the applied voltage and the birefringence decreases uniformly through the material, as it is shown in figures 1(b1) and 1(b2) respectively. The resulting matrix is

\[ D(\alpha, \beta, \Psi) = R^{-1}(\Psi) \cdot R^{-1}(\alpha) \cdot \text{LC}(\alpha, \beta) \cdot R(\Psi) \] (1)

Figure 1.

TN-LCD Models
(a) Berreman
(b) Saleh and Lu
(c) Coy et al.
(d) Márquez et al.
(e) Proposed model
where \( R(\bullet) \) and \( \text{LC}(\alpha, \beta) \) are

\[
\text{LC}(\alpha, \beta) = \begin{bmatrix}
\cos(\Gamma) - j\beta \sin(\Gamma) & \frac{\alpha}{\Gamma} \sin(\Gamma) \\
-\frac{\alpha}{\Gamma} \sin(\Gamma) & \cos(\Gamma) + j\beta \frac{\sin(\Gamma)}{\Gamma}
\end{bmatrix}
\]

(2)

\[
R(\bullet) = \begin{bmatrix}
\cos(\bullet) & j\sin(\bullet) \\
-j\sin(\bullet) & \cos(\bullet)
\end{bmatrix}
\]

(3)

\( \alpha \) represents the total twist of the molecules inside the liquid crystal; \( \Psi \) represents the direction of the molecules in the frontal face of the microdisplay\(^1\); \( \Gamma = \sqrt{\alpha^2 + \beta^2} \); and \( \beta \) is proportional to the birefringence

\[
\beta = \frac{2\pi(n_e - n_o)d}{\lambda}
\]

(4)

where \( n_e \) and \( n_o \) are the extraordinary and ordinary refraction indices respectively, \( d \) is the microdisplay’s thickness and \( \lambda \) is the wavelength. In this model, only \( \beta \) depends on the applied voltage \( V \), and decreases monotonically with the increases of \( V \).

In an attempt to get a model closer to Berreman’s theory, Coy \textit{et al}. proposed the division of the model in three sections. There are two lateral sections where the twist angle of the molecules is the same as in the alignment layers. The width of these sections is \( m \) and remain fixed when a voltage is applied (figure 1(c1)). Also, the molecules in these sections can not tilt, and the birefringence is maximum (figure 1(c2)). The result is that these two lateral sections behave as retarder wave plates and the retard they introduce is \( \delta \) which is constant and independent of \( V \).

In the central section, the liquid crystal behaves in the same way as in Lu and Saleh’s model. The equation (1) becomes

\[
D(\alpha, \beta(V), \Psi, \delta) = R^{-1}(\Psi) \cdot R^{-1}(\alpha) \cdot S(\delta) \cdot \text{LC}(\alpha, \beta) \cdot S(\delta) \cdot R(\Psi)
\]

(5)

where \( S(\delta) \) is

\[
S(\delta) = \begin{bmatrix}
e^{-j \delta/2} & 0 \\
0 & e^{j \delta/2}
\end{bmatrix}
\]

(6)

Mármuez \textit{et al}. made another improvement to the model. They proposed that the width of the two lateral sections \( m \) increases with the applied voltage. In this approximation they assume, as it was proposed by Coy \textit{et al}., that the width of the section where the molecules have the same twist angle as in the alignment layer, is the same as the width of the section of maximum birefringence (figures 1(d1) and 1(d2)). This proposal leads to \( \delta(V) \), in equation (5), increasing monotonically with the applied voltage. Also the width of the central section, and consequently \( \beta \), is decreasing in the same way as in the model of Coy \textit{et al}.

The matter is that, as shown in Berreman’s theory, as long as the width of the section where the molecules tilt increases, the section where the twist angle is the same as in the alignment layers also increases. Wang and He\cite{6} proposed a model where they consider this effect. The disadvantage of their model is that, for any particular value of \( V \), the matrix of the display must be calculated as the product of several matrices, loosing the simplicity of the Saleh and Lu model. Based on the work of Wang and He\cite{6}, in this work it is proposed a modification to the Jones Matrix model where the width of the lateral and central sections of the model are considered to be independent. On one hand, \( m_1 \) denotes the width of the section where the twist angle is constant and equal to the alignment layers, increasing with \( V \), as it can be seen in

\(^1\) In figure 1 was taken \( \Psi = 0 \), for simplicity
figure 1(e1). On the other hand, \( m_2 \) represents the width of the section where the molecules do not tilt and the birefringence is maximum. As it is shown in figure 1(e2) \( m_2 \) decreases with \( V \).

As a result of this approximation, the following consideration can be made: the central section (where the twist angle increases gradually) reduces both its width and its birefringence with an increase of \( V \), leading to a monotonically decreasing \( \beta(V) \). The lateral section, where the twist angle remains constant increases its width and can be divided in two subsections: the portion where the birefringence is maximum \( (m_2) \) and the remaining portion \( (m_1 - m_2) \) where the birefringence also decreases. The result is that the retard introduced by the lateral section \( (\delta) \) will increase up to a certain point and eventually will decrease. Equation (5) is still valid, without the restriction that \( \delta \) must increase monotonically.

3. Characterization of the microdisplay

In order to obtain the parameters of the model and their dependence with \( V \), a set of experiments was carried out. The experimental setup is presented in figure 2. It includes a He-Ne 632.8nm wavelength laser, followed by a 1/4 wave plate and a polarizer. A Kopin’s monochromatic transmissive microdisplay was used, with a 320 × 240 resolution and a 11\( \mu \)m pitch. The total active area of the microdisplay is 3.3mm × 2.7mm. A second polarizer and a CCD camera complete the setup. The electronic control board developed, allows to command the microdisplay and apply different gray levels. In this display the gray level is proportional to the applied voltage.

The experiment consists in measuring the intensity transmitted through the microdisplay for different angles of the polarizers 1 and 2 (\( \theta_1 \) and \( \theta_2 \), respectively). The 1/4 wave plate is used to obtain right-handed circularly polarized light as shown in figure 3. This is accomplished when the wave plate’s slow axis[7] is aligned at \(-45^\circ\) with respect to the polarization angle of the (linearly polarized) laser.

The expression of intensity vs \( \theta_1 \) and \( \theta_2 \) with the dependence of the parameters of the microdisplay is presented in equation (7)

\[
I = \left[ \cos(\Gamma) \cos(\delta) - \frac{\beta}{\Gamma} \sin(\Gamma) \sin(\delta) \right] \cos(\alpha - \theta_2 + \theta_1) + \frac{\alpha}{\Gamma} \sin(\alpha - \theta_2 + \theta_1) \right]^2 + \\
\left[ -\frac{\beta}{\Gamma} \cos(\delta) - \cos(\Gamma) \sin(\delta) \right] \cos(\alpha + 2\Psi - \theta_1 - \theta_2) \right]^2
\]

(7)

The intensity was registered for the angles \( \theta_1 \in \{0^\circ, 5^\circ, \cdots, 175^\circ\} \) and \( \theta_2 \in \{0^\circ, 5^\circ, \cdots, 175^\circ\} \). The result of measuring at all those angles is shown in figure 4, in what it was called Intensity Matrix. This intensity matrix corresponds to the case when no voltage is applied. In this condition, \( \delta = 0 \) and \( \beta \) is maximum.
3.1. Determining $\alpha$ and $\beta_{\text{max}}$

Using a fitting algorithm, the parameters that best match with the measurements can be found. As it was mentioned, Yamauchi\cite{5} demonstrated that, in the experiment without an applied voltage, there is a different set of solutions that provides the same intensity matrix. Some of the set of parameters founded for this display are listed in table 1 as an example. Also, it can be seen that, for each solution, there are two possible values for $\Psi$.

To find the right solution, Márquez et al. have used different wavelengths but this requires having different lasers available, and a 1/4 wave plate for each one. It also requires to repeat the alignment process of the laser, being this a source of error in the results. In this work, to select the correct solution, the experiment is repeated applying the same voltage to all the pixels, using only the original wavelength. In figure 5, the evolution of the intensity matrix for the microdisplay under test is shown. The green circles indicate the displacement of the intensity maximum when the voltage increases.

![Figure 3. Schematic representation of the experimental setup](image1)

**Table 1.** Group of solutions that satisfies the model and the measurements

| N | $\alpha$ [$^\circ$] | $\beta_{\text{max}}$ [$^\circ$] | $\Psi_a$ - $\Psi_b$ [$^\circ$] |
|---|-----------------|-----------------|-----------------|
| 1 | -98.5           | 497.2           | -82.3           | 7.7  |
| 2 | -94.6           | 311.9           | -84.2           | 5.8  |
| 3 | -79.5           | 114.3           | -1.8            | 88.2 |
| 4 | 79.4            | 567.2           | -81.2           | 8.8  |
| 5 | 82.1            | 384.6           | -82.6           | 7.4  |
| 6 | 91.5            | 195.9           | -87.3           | 2.7  |

**Figure 5.** Intensity matrix with gray level: (a) 0; (b) 100; (c) 150
In figures 6(a), the simulation of the solution 5 is presented. It can be observed that the maximum of the matrix moves in the opposite direction to the measurements. The same behaviour was observed for all the other solutions where $\alpha > 0$, so all of them were discarded.

![Simulation of solution 5](image)

**Figure 6.** Simulated intensity matrices with different gray levels (a) Solution 5 (b) Solution 2

For all the solutions with $\alpha < 0$, with one exception, the simulated system pass through different states that do not appear in the measurements. The simulation of solution 2 is shown in figure 6(b) where these states are indicated.

Using a qualitative analysis of the measured intensity matrices for several applied voltages, the right solutions can be found which, in this case, is solution 3.

$$\alpha = -80^\circ \quad \beta_{\text{max}} = 114^\circ \quad \Psi = -1.8^\circ \text{ or } 88.2^\circ$$

### 3.2. Determining $\Psi$

It can be seen in equation (7) that for any combination of $\theta_1$ and $\theta_2$ and for any voltage applied (that implies different $\beta$ and $\delta$) the measured intensity has the ambiguity of two different values of $\Psi$: $\Psi_1$ and $\Psi_2 = \Psi_1 + 90^\circ$. This is because the intensity depends on the square value of $\cos(\alpha + 2\Psi - \theta_1 - \theta_2)$. As a result, $\Psi$ cannot be determined by using linearly polarized light as source and measuring only the transmitted intensity. The parameter $\Psi$ plays no role in the intensity but it does in the phase delay introduced by the SLM.

Several techniques[8, 9] have been proposed to eliminate this ambiguity. These techniques are simple but require a new experimental setup.

If the polarizer 1 is eliminated from the experimental setup presented in figure 2 the source of light is right-handed circularly polarized and it can be demonstrated that the intensity measured in dependence of $\theta_2$ is[10]

$$I = \frac{1}{2}[1 + r \cdot f(\theta)]$$
where \( r = \pm 1 \) and the sign of \( r \) depends exclusively on the different solutions of \( \Psi \) and \( f(\theta) \) has no dependency with \( \Psi \). As a result, the measured intensity will be different for the two possible solutions of \( \Psi \) and the only change it was made in the setup was the remotion of polarizer 1, with no additional alignment process needed.

\[
\text{Figure 7. Comparison between experimental measurements and model simulation: (a) Experimental measurements; (b) Intensity simulated with } \Psi = -1.8^\circ; \text{ (c) Intensity simulated with } \Psi = 88.2^\circ.
\]

In figure 7(a), the result of the experiment is presented. Figures 7(b) and 7(c) are the simulation of the model with the two possible solutions of \( \Psi \). It is easily distinguishable that the right solution is \( \Psi = -1.8^\circ \). Because the parameter \( \Psi \) is dependent on the reference frame adopted, its value can only be known with the precision of the alignment method.

3.3. Determining \( \beta(V) \) and \( \delta(V) \)

Finally, using the measurements partially presented in figure 5, the dependence of \( \beta(V) \) and \( \delta(V) \) can be recovered. Once again, the fitting algorithm is applied.

\[
\text{Figure 8. Result of parameter fitting for } \text{Gray Level} = 0, 5, \cdots, 255: \text{ (a) } \beta(V) \text{ (b) } \delta(V)
\]

The correspondence between the measurements and the proposed model can be observed in figures 8(a) and 8(b).
4. Phase modulation

Knowing the parameters of the model and their dependence with $V$, a configuration where the microdisplay acts as a Phase-Only SLM can be found. The classical configuration consists of using two polarizers, fixed at certain angles, with the microdisplay between them. The combination of angles $\theta_1$ and $\theta_2$ that gives the maximum phase modulation along with the minimum amplitude modulation, is searched. The optimum configuration founded for this microdisplay is presented in figure 9. The maximum phase modulation obtained is $115^\circ$ while the amplitude modulation is relatively low.

![Figure 9](image)

Figure 9. $\theta_1 = 72^\circ$ and $\theta_2 = 24^\circ$ (a) Amplitude modulation vs Gray Level; (b) Phase modulation[$^\circ$] vs Gray Level

5. Control of the Modulator

Usually, as it was already mentioned in the introduction, SLM are made from microdisplays obtained from the cannibalization of video projectors. Also there are commercial available SLMs but their cost is over U$S 5000 and exceeds the resources of South American research groups. In both cases, the modulators require a computer to operate because the control board included with them, performs a conversion of a video signal to the necessary signals to handle the microdisplay. This implies the need to have a computer constantly generating the video signal for the modulator.

Part of the work carried on, consisted in the development of an electronic control board for the microdisplay used in this work. The main goal was to develop an reasonably economical SLM that could function without requiring a computer.

The heart of the control device is a CPLD (Complex Programmable Logic Device). A CPLD consist of a large amount of logic gates and flip-flops that can be connected in any desired way. Because all these gates are inside the integrated circuit, the connection can only be made by means of a Hardware Descriptor Language (HDL) (in this case VHDL). To satisfy the timing specification of the display the logic control was implemented through a state machine, simplifying the design process.

In order to operate independently of a computer, the information must to be stored in the control board. In this, a 512KB static RAM that allows to store up to four different images was used. Because the chosen microdisplay has analog data inputs, a digital to analog converter (DAC) was also used. As the conversion needs to be synchronized with the control signals, the control of the DAC is also managed from the CPLD.

More details of the developed control board can be found in reference[10].
6. Conclusions
A revision of the different approximations of the Jones Matrix model was made and a new approximation of the model was proposed. This proposal was validated by experimental measurements. Also, the classical characterization method was discussed, and a simple procedure was proposed to eliminate the ambiguity in $\Psi$.

The Kopin’s microdisplay used in this work can only achieve 115° phase modulation at 632.8 nm. A greater phase modulation can be obtained if a 1/4 wave plate is used between the microdisplay and the second polarizer as it has been reported in reference[8], but this work makes emphasis on the modulation capability of the microdisplay and attempts to develop a SLM of a simple configuration.

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