Wealth distribution and Pareto’s law in the Hungarian medieval society

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Abstract

The distribution of wealth in the Hungarian medieval aristocratic society is reported and studied. The number of serf families belonging to a noble is taken as a measure of the corresponding wealth. Our results reveal the power-law nature of this distribution function, confirming the validity of the Pareto law for such a society. The obtained Pareto index $\alpha = 0.92$ is however smaller than the values currently reported in the literature. We argue that the value close to 1, of the Pareto index is a consequence of the absence of a relevant economic life in the targeted society, in agreement with the prediction of existing wealth distribution models for the idealized case of independently acting agents. Models developed to explain city populations may also be adapted to justify our results.

Key words: Wealth distribution, Econophysics, Pareto’s Law
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1 Introduction

At the end of the XIX century the economist Vilfredo Pareto [1] discovered a universal law regarding the wealth distribution in societies. His measurement results on several European countries, kingdoms and cities for the XV-XIX centuries revealed that the cumulative distribution of wealth (the probability

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that the wealth of an individual is greater than a given value) exhibits a universal functional form. Pareto found that in the region containing the richest part of the population, generally less than the 5% of the individuals, this distribution is well described by a power-law (see for example [2] for a review). The exponent of this power-law is denoted by \( \alpha \) and named Pareto index. In the limit of low and medium wealth, the shape of the cumulative distribution is fitted by either an exponential or a log-normal function.

The power-law revealed by Pareto has been confirmed by many recent studies on the economy of several corners of the world. The presently available data is coming from so apart as Australia [3], Japan [4,5], the US [6], continental Europe [7,8], India [9] or the UK [10]. The data is also spanning so long in time as ancient Egypt [11], Renaissance Europe [12] or the 20th century Japan[13]. Since it is difficult to measure wealth, most of the available data comes from tax declarations of individual income (which is assumed to be proportional to wealth). There are however some other databases obtained from different sources, like the area of the houses in ancient Egypt [11] or the inheritance taxation or the capital transfer taxes [14]. The results mostly back Pareto’s conjecture on the shape of the wealth distribution.

In the present paper, we present and discuss some recent results of wealth distribution in a medieval society – the Hungarian aristocratic society around the year 1550. The wealth of the nobles is estimated and expressed in the number of owned serf (villein) families, a measure generally used by historians. The case under study offers a somehow idealized example of a society without a relevant wealth-exchange mechanism and our results give further evidence for the universal nature of Pareto’s law.

2 Statistical Physics approach to Pareto’s law

Typically, the presence of power-law distributions is a hint for the complexity underlying a system, and a challenge for statistical physicists to model and study the problem. This is why Pareto law is one of the main problems studied in Econophysics. Since the value found by Pareto for the scaling exponent was around 1.5, Pareto law is sometimes related to a generalized form of Zipf’s law [15] and referred to as Pareto-Zipf law. According to Zipf’s law, many natural and social phenomena (distribution of words frequency in a text, population of cities, debit of rivers, users of web sites, strength of earthquakes, income of companies, etc) are characterized by a cumulative distribution function with a power-law tail with a scaling exponent close to 1.

It is however important to notice that, in contrast to what happens with most exponents in Statistical Physics, the Pareto index \( \alpha \), may change from
one society to another, and for the same society can also change in time depending on the economical circumstances [5,13]. The measured values of $\alpha$ for the individuals wealth/income distribution span a quite broad interval, typically in the $1.5 - 2.5$ range; a recent study of the Indian society [9] reports however, much lower values ($0.81 - 0.92$). This large variation of $\alpha$ indicates the absence of universal scaling in this problem – a feature which models designed to describe the wealth distribution in societies should be able to reproduce.

Models for wealth distribution are essentially defined by a group of agents, placed on a lattice, that interchange money following pre-established rules. The system will eventually reach a stationary state where some quantities, as for instance the cumulative distribution of wealth $P_\geq(w)$, may be measured. Following these ideas, Bouchaud and Mézard [16] and Solomon and Richmond [18,17] separately proposed a very general model for wealth distribution. This model is based on a mean field type scenario with interactions of strength $J$ among all the agents and on the existence of multiplicative fluctuations acting on each agent’s wealth. The obtained wealth distributions adjust well to the phenomenological $P_\geq(w)$ curve. Their mean-field results predict that Pareto index $\alpha$ should increase linearly with the strength of interaction between the agents and that $\alpha = 1$ for the case of independent agents ($J = 0$).

Similar conclusions were obtained by Scaffeta et al [19], who considered a nonlinear version of the model and from other regular lattice based models as those in Refs. [20] and [21].

Given the known complex nature of social networks (see [22] for a recent review), models were also considered in which the economic transactions take place on a complex small-world or scale-free network [13,23,24]. More recently, a model was proposed [25] where the network structure is not predefined but rather dynamically coupled to the wealth-exchange rule. This model successfully reproduces both the characteristics of the modern family relation networks and the measured shape of the wealth distribution curves, accounting for variations in $\alpha$ in the $1.7 - 2.0$ interval.

From the studies and modeling efforts made by the statistical physics community, one can conclude that the emergence of Pareto law can be understood from the multiplicative nature of the fluctuations of the agents’ wealth and from the dynamics of wealth on the underlying complex social network. Both the experimental results and the models suggest that societies in which there is strong wealth-exchange among the agents are characterized by higher values of the Pareto index, whereas societies of isolated agents, where each agent increases or decreases his wealth in a multiplicative and uncorrelated manner, are usually characterized by $\alpha$ close to 1. Our measurement results on the Hungarian medieval society confirm this rule.
3 Wealth distribution measurements in the Hungarian medieval society

To our knowledge, there is no available data concerning the wealth distribution and the Pareto law for the Central-Eastern European aristocratic medieval societies. A flourishing economic life, barter and wealth exchange developed very slowly in this part of Europe. A centralized and documented taxation system was also introduced relatively late. Of course, this undeveloped economic life makes it hard to collect uniform, relevant and large-scale data on this society. However, once such data is obtained, it could be of decisive importance, since such a society represents an idealized case, where the agents are acting roughly independently. The underlying social network and the wealth exchange on it, are expected to have no major influence, since the aristocratic families were more or less self-supporting and no relevant barter existed. This simplified economic system provides thus an excellent framework to test, in a trivial case, the prediction of wealth distribution models.

The first centralized data for 47 districts of the medieval Hungary (10 of them under Turkish occupation) is dated from 1550. The data, in a rough format, is available in a recent book [26] dealing with property relations of the XVI century Hungary. For each district, the nobles are arranged alphabetically and their wealth is grouped in six categories: number of owned serf families and their lands, unused lands, poor people living on their land, new lands, servants and others. Using a method accepted by historians [27], the number of owned serf families is taken as a common measure of the total wealth of a noble family.

After a careful analysis of all districts and summing up the wealth for those families that owned land in several districts, we obtained a dataset that is usable for wealth distribution studies. We also imposed a lower cut-off value, chosen to be 10 serf families, and disregard thus the low and medium wealth aristocrats. This cut-off is necessary since historians suggest that the database is not reliable in this ranges. With the above constraints, our final database [28] had data for 1283 noble families and 116 religious or city institutions. Considering an average of five persons per family (a generally accepted value by the historians and sociologists specialized in the targeted medieval period) we obtain that our sample contains around 6400 people. This represents the top 8% of the estimated 80000 aristocrats living in Hungary at that period, and 0.2 – 0.3% of the estimated total population (2.7 millions) of Hungary in 1550.

Ranking the considered families after their decreasing wealth and plotting the rank as a function of wealth for each family, gives the tail of the cumulative distribution function for wealth up to a proportionality constant (equal to
Fig. 1. The rank of the top 8% aristocrat families and institutions as a function of their estimated total wealth on a log-log scale. Measurement results for the Hungarian noble society in the year 1550. The total wealth of a family is estimated as the number of owned serf families. The power-law fit suggests a Pareto index $\alpha = 0.92$. The results obtained from our dataset are plotted in Figure 1.

According to Pareto law, this curve should have a power-law nature, thus plotting it on a log-log scale should give a straight line with a negative slope, which yields the Pareto index. The power-law scaling is nicely visible on two decades in the figure, thus confirming Pareto law with an exponent $\alpha = 0.92$. This value is comparable with the ones obtained for the Indian society [9] and much smaller than all the other values reported in the literature.

As explained above, a value of $\alpha$ close to 1 is in agreement with our expectations for the studied society. The validity of the Pareto law and the value of the Pareto index will not change considerably (we get $\alpha = 0.95$) if we study only the wealth distribution of the 1283 aristocratic families and neglect from the database the 116 institutions. Studying however only the wealth distributions of the institutions will not give a Pareto-like tail at all, and on the log-log scale we will get a constantly decreasing slope (empty circles in Figure 1.) It is also observable from Figure 1, that the Pareto law breaks down in the limit of the very rich families, where the wealth is bigger than 1000 serf families.
families. This is presumably a finite-size effect and such results are observable in other databases too ([5,9]).

In order to have some information on the time evolution of the Pareto index, the wealth distribution of the Hungarian aristocratic families in the 1767-1773 period was also studied. For this period we had rough data [29] available only for 11 districts. The sample was much smaller than for the year 1550: we had only 531 families and 65 institutions with total wealth greater than one serf family. However, as a compensation for the smaller database, in this case the wealth of each family is given with three markers: the owned number of serf families, the exact number of owned serfs and the total size of the owned land. To our great surprise, we obtained for each marker that the cumulative distribution function of wealth does not give the expected power-law behavior. Instead of a straight line with a negative slope, we found a constantly decreasing slope in the log-log plot of the rank as a function of wealth (Figure 2). The fact that we used only data for 11 districts cannot be the reason for the breakdown of the Pareto scaling – we checked that, on the same 11 districts, the 1550 data still gives the Pareto power-law tail. We believe the reason why the Pareto law is not valid for this database, is that a large wealthy part of the society is missing. Indeed, in the mid XVIII century in Hungary there were already many wealthy non-noble families of merchants, bankers, rich peasants, whose wealth exceeded the wealth of small or middle class aristocrats. This large category of relatively wealthy families were not landowners and had no serfs, so they are simply not present in the considered database. After our estimates, our wealth distribution data gives a reliable picture of the society only for the wealthier aristocrats, with wealth greater than 100 serf families. It is believed that the wealth of non-noble families that owned no land and serfs could not exceed this threshold. As observable however in the data from 1550, for wealth values larger than 1000 serf families finite-size effects are dominant, and the scaling breaks down. There is thus a very short wealth interval (one decade) where the data is trustful. Fitting a power-law on this interval leads to a Pareto-index $\alpha = 0.99$, which is a reasonable value. This value is also bigger than the one measured in year 1550, suggesting thus a more active economic life. From this Pareto index value it is possible to estimate the wealthy part of the society missing from our data (see Appendix A). The results obtained in such a manner are reasonable ones.

4 Discussion and conclusions

Our study shows that the cumulative wealth distribution of the Hungarian aristocratic families in the year 1550 exhibits a power-law shape with a Pareto index $\alpha = 0.92$. This result is a surprise in some sense, since it is generally believed that the Pareto index should be bigger than 1. The fact that the
Fig. 2. The rank of noble families and institutions as a function of their estimated total wealth on a log-log scale. Data for the Hungarian noble society between the years 1767-1773. The total wealth of a family is estimated in the number of owned serf families. The power-law fit suggests a Pareto index $\alpha = 0.99$.

measured value of $\alpha$ is close to 1 is however predicted by existing wealth-exchange models in the no trade limit, which we believe is applicable to the type of society under study.

In the model introduced by Bouchaud and Mézard [16], if we consider the $J = 0$ case, the wealth $w_i$ of each family varies in an independent, stochastic multiplicative manner, according to

$$\frac{dw_i}{dt} = \eta_i(t)w_i + \xi_i(t)$$

(1)

where $\eta_i(t)$ is presumably a Gaussian distributed random variable with zero mean ($<\eta_i>_t = 0$), and $\xi_i$ is an uncorrelated random noise with zero mean ($<\xi_i(t)>_t = 0$ and $<\xi_i(t)\xi_i(t')>=C\delta(t-t')$). This additive noise is necessary to prevent wealth extinction and stabilize the power-law type solution in a finite system. Following the solution given in [16] we immediately get $p(w) \propto w^{-2}$ for the tail of the wealth probability density function $p(w)$, which yields a Pareto index $\alpha = 1$ for the cumulative distribution function. The fact that the measured exponent is smaller than 1 cannot be explained within this model.
The wealth distribution obtained by us has a shape and scaling exponent very similar to the population size distribution of large cities [15,30]. Zanette and Manrubia [30] successfully justified this behavior using a simple reaction-like model (which is a generalization of the Zeldovich model for intermittency [31]).

Inspired by Zanette et al’s model, one can consider thus that the wealth of noble $i$ fluctuates in time due to several processes, either exogenous (wars, meteorological conditions affecting harvest sizes,...) or endogenous (good or bad administration, gambling,...). So one can assume that $w_i(t+1) = \lambda_j w_i(t)$ due to process $j$, which occurs with probability $p_j$. If there are $n$ such processes ($\sum_{j=1}^{n} p_j = 1$) it is sufficient to require that $\langle \lambda \rangle = \sum_{j=1}^{n} \lambda_j p_j = 1$ and to add some noise term to prevent collapse to 0 to obtain (see [30,32]) that $p(w) \propto w^{-2}$ is a stationary solution and thus $\alpha = 1$. This simple reaction-like stochastic model is also appropriate thus to understand a Pareto index close to 1 for a system composed by independently evolving agents.

We have thus argued that the obtained Pareto index, close to 1, can be justified in the context of several simple models, compatible with the socio-economical characteristics of the real system under study. Recalling that a similar value of $\alpha$ was recently presented for contemporary Indian society, one may speculate that this is not a mere coincidence, but rather evidence for the minor role of exchanges in the dynamics of wealthier Indian people.

The value smaller than 1 obtained for the Pareto index in the Hungarian medieval society is however still a puzzle, and probably further modeling effort is still necessary to account for it.

5 Appendix A

Let us focus on wealth values greater than one serf family. If we assume that for wealthy families the cumulative distribution is scaling as $P_{\geq}(w) \propto w^{-1}$, the $p(w)$ wealth distribution density function has the form:

$$p(w) = \frac{C}{w^2}$$

(2)

The $C$ constant can be determined by taking into account that we have 83 families and institutions with wealth between 100 and 1000 serfs

$$\int_{100}^{1000} \frac{C}{w^2} dw \approx \frac{C}{100} \approx 83,$$

(3)

leading to $C = 8300$. From here it is immediate to estimate the $N_1$ total
number of families or institutions in the targeted 11 districts of Hungary that have wealth greater than 1 serf family:

\[ N_1 = \int_{W_0}^{\infty} \frac{C}{w^2} dw \approx C = 8300 \]  

(W_0 stands for the biggest reported wealth value, W_0 = 6600). This estimate can be confronted with the results of the census made between 1784-1787. For the studied 11 districts the total population was estimated to be around 1.6 million, with 45 thousand aristocrats and 33 thousand rich burgers. Taking again 5 members per family, our database shows that only around 531 \times 5 = 2655 aristocrats had wealth bigger than the considered 1 serf family limit. This value is only 6% of the total estimated aristocrats in the society. Most of the aristocrats at that time had thus no considerable fortune except their noble title. The missing 8300 − 600 = 7700 families corresponding to roughly 7700 \times 5 = 38500 persons, should be mostly rich burgthers or some richer peasants. This estimate is in reasonable agreement with the census from 1784-1787.

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