A Critical String Theory in (3+1)+4 Dimensions for Maximal Gauge Symmetry

\[ SU(3)_C \otimes SU(2)_L \otimes U(1)_{Y_W} \]

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Abstract

Redefining the vacuum state of a free twofold N=1 covariant supersymmetric string action as the one with all the world sheet fermionic excited states occupied, makes the theory free from anomalies in a (3+1)+4 dimensional space. One supersymmetry is associated with the dynamics of the oriented string in (3+1) dimensional Minkowski spacetime and the other with a 4-dimensional internal space adding trivial quantum numbers of \( SU(3)_C \) and non-trivial quantum numbers of \( SU(2)_L \) to the strings. Together, the longitudinal and time components of the world-sheet fermions add the weak hypercharge to a state to complete the Standard Model with three families of fermions. Even though the full spectrum contains both bosons and fermions, there is no space-time supersymmetry for the lack of triality. Consequences of the presence of both open and closed string tachyons are discussed.
We consider open strings moving in a (3+1)+4 dimensional space. There are two $N = 1$ local supersymmetries. One acts on four world-sheet scalar fields and four world-sheet Majorana spinors that are coordinates and components of vectors of the (3+1) dimensional Minkowski spacetime. The other supersymmetry acts on another set of world-sheet scalars and Majorana spinors that are coordinates and components of vectors of the four-dimensional Euclidean space. Therefore, the world-sheet action is

\[
S = -\frac{1}{2\pi} \int d^2 \sigma e \left\{ [h^{\alpha \beta} \partial_{\alpha} X^\mu \partial_{\beta} X_\mu - i \bar{\psi}^\rho \rho^\alpha \partial_{\alpha} \psi_\mu + 2 \bar{\chi}^1_{\alpha \rho} \rho^\alpha \partial_{\alpha} X_\mu + \frac{1}{2} \bar{\psi}^\rho \rho^\alpha \partial_{\alpha} \chi^1_{\beta}] \\
+ [h^{\alpha \beta} \partial_{\alpha} X^A \partial_{\beta} X^A - i \bar{\psi}^A \rho^\alpha \partial_{\alpha} \psi^A + 2 \bar{\chi}^2_{\alpha \rho} \rho^\alpha \partial_{\alpha} X^A + \frac{1}{2} \bar{\psi}^A \rho^\alpha \partial_{\alpha} \chi^2_{\beta}] \right\},
\]

where $\chi^1_{\alpha}$ and $\chi^2_{\alpha}$ are the two world-sheet gravitinos corresponding to the two local supersymmetries:

\begin{align*}
\delta X^{\mu,A} &= \bar{\epsilon}^{1,2} \psi^{\rho,A}, \quad \delta \psi^{\mu,A} = -i \rho^\alpha \epsilon^{1,2} (\partial_{\alpha} X^{\mu,A} - \bar{\psi}^{\rho,A} \chi_{1,2}^{\alpha}) \\
\delta e^a_{\alpha} &= -2 i \epsilon^{1,2} \rho^a \chi_{1,2}^{\alpha}, \quad \delta \chi^{1,2}_{\alpha} = \nabla_{\alpha} \epsilon^{1,2}.
\end{align*}

Here $\mu$ runs over the values $(0, 1, 2, 3)$ and $A$ runs over the values $(1, 2, 3, 4)$. The local Weyl transformation that leaves the action invariant is

\begin{align*}
\delta X^{\mu,A} &= 0, \quad \delta \psi^{\mu,A} = -\frac{1}{2} \Lambda \psi^{\mu,A} \\
\delta e^a_{\alpha} &= \Lambda e^a_{\alpha}, \quad \delta \chi^{1,2}_{\alpha} = \frac{1}{2} \Lambda \chi^{1,2}_{\alpha}.
\end{align*}

There is also another local fermionic symmetry given by

\begin{align*}
\delta \chi^{1,2}_{\alpha} &= i \rho^\alpha \eta^{1,2} \\
\delta \eta^a_{\alpha} &= \delta \psi^{\mu,A} = \delta X^{\mu,A} = 0.
\end{align*}

There are altogether four local bosonic symmetries: two world-sheet reparametrizations, one local Lorentz and one local Weyl scaling. Locally they can be used to gauge the four components of the zweibein into the standard form $e^a_{\alpha} = \delta e^a_{\alpha} (h_{++} = h_{--} = 0)$. Similarly, the four supersymmetries ($\epsilon^1, \epsilon^2$) and the four superconformal symmetries ($\eta^1, \eta^2$) can be used to set the eight components of $\chi^1$ and $\chi^2$ to zero. The Faddeev-Popov determinant for the residual reparametrization ($\sigma \to \sigma + \xi$) symmetry: $\delta h_{++} = \nabla_+ \xi_+ = 0$ and $\delta h_{--} = \nabla_- \xi_- = 0$ yields the pair of reparametrization ghosts and anti-ghosts ($c, b$). Similarly the residual
supersymmetries: $\delta \chi^{1,2}_\alpha = \nabla_\alpha \epsilon^{1,2} = 0$ yield two pairs of superconformal ghosts and anti-ghosts ($\gamma^1, \beta^1$) and ($\gamma^2, \beta^2$). Thus, the gauge-fixed world-sheet action becomes

$$S = \frac{1}{\pi} \int d^2 \sigma \left[ e^+ \partial^- b^+ + e^- \partial^+ b^- + \partial^\mu X^\mu \partial_\mu X \right. \tag{5}$$

$$\left. + \partial^- X^\mu \partial_\mu X^\mu + \psi^\mu \partial_- \psi^\mu + \psi^\mu \partial_\mu \psi^\mu - \beta^{1,3/2} \partial_- \gamma^{1/2}_1 + \beta^{1,3/2} \partial_+ \gamma^{1/2}_1, \right]$$

$$\left\{ \partial_+ X^A \partial_- X^A + \partial_- X^A \partial_+ X^A + \psi^A \partial_- \psi^A + \psi^A \partial_+ \psi^A \right\}$$

$$\left. + \beta^{2,3/2} \partial_- \gamma^{1/2}_2 + \beta^{2,3/2} \partial_+ \gamma^{1/2}_2 \right].$$

If we allow the time coordinate to vary from $-\infty$ to $+\infty$ and assume that the system is in thermal equilibrium at temperature $T$, then from the expression for the partition function $Z = Tr e^{-\frac{\mathcal{H}}{kT}}$, we can infer that $T = 0$, because the periodic time $\frac{1}{T}$ is infinite.

So fermions will fill all the energy levels up to the Fermi energy $E_F = \frac{1}{m} \left( \frac{N}{l} \right)^2$, where $N$ is the number of fermions and $l$ is the one dimensional spatial extension of the manifold. Hence the Fermi energy $E_F = \infty$ for massless fermions.

If the positive energy states of fermions were filled instead of the negative energy states, the Schwinger Terms [5,8] in the current algebras including the Virasoro algebra [9] would change sign [10–25]. Because, the anomaly term involves only the odd powers of the Fourier indices $(m)$ and replacing a vacant state with an occupied state, reverses the roles of the creation and annihilation operators of the fermions $(m \rightarrow -m)$. This corresponds to anti-normal ordering of the current-current commutators. Creation operators of the fermions are brought to the right of the annihilation operators in the anti-normal ordered expression for the current-current commutators.

The infinite Fermi Sea where all positive energy states are filled up with exactly one fermion per level according to Pauli principle is a unique state too like the canonical vacuum and all states built from this would only contain de-excitations (obtained by action of positive fourier modes) of the fermion. The scenario is analogous to Dirac’s original hole theory [26], where the canonical vacuum was defined to be the one with all negative energy states filled up.

From the previous discussions, it appears that the Virasoro anomalies would cancel in this case, because

$$c = 2[4(c_X + c_\psi) + c_c] + c_c \tag{7}$$

$$= 2[4(1 - \frac{1}{2}) + 11] - 26$$

$$= 0,$$

where $c$ is the central charge.
The solutions to the wave equations compatible with the proper boundary conditions are

\[ X^\mu = x^\mu + p^\mu \tau + \sum_{n \neq 0} \frac{1}{n} a_n e^{-in\tau} \cos n\sigma \]  

(8)

\[ X^A = x^A + q^A \tau + \sum_{n \neq 0} \frac{1}{n} a_n e^{-in\tau} \cos n\sigma \]  

(9)

\[ \psi^\mu_A = \sum_{-\infty}^{\infty} d_{nA} e^{-in(\tau \pm \sigma)} \]  

(10)

\[ c^\pm = \sum_{-\infty}^{\infty} c_n e^{-2in(\tau \pm \sigma)} \]  

(11)

\[ b_{\pm \pm} = \sum_{-\infty}^{\infty} b_n e^{-2in(\tau \pm \sigma)} \]  

(12)

\[ \gamma_{\pm 1} = \sum_{-\infty}^{\infty} \gamma_n e^{-2in(\tau \pm \sigma)} \]  

(13)

\[ \beta_{\mp 3} = \sum_{-\infty}^{\infty} \beta_n e^{-2in(\tau \pm \sigma)}. \]  

(14)

In the RNS model, the modes of the \( \psi^s \) are half-integral in the bosonic sector and integral in the fermionic sector of the string [27,28]. We assume that the modes of the superconformal ghosts are half-integral in both the bosonic and the fermionic sectors. Integral modes in the fermionic sector would lead to infinite degeneracy of the ground state due to the presence of the zero modes of the superconformal ghosts.

The modes of the vanishing components \( T_{++} \) and \( T_{--} \) of the energy momentum tensor give the Virasoro generators

\[ L^a_m = \frac{1}{2} \sum_{-\infty}^{\infty} \alpha_{m-n, \alpha_n} \]  

(15)

\[ L^d_m = \frac{1}{2} \sum_{-\infty}^{\infty} (n - \frac{m}{2})d_{m-n, d_n} \]

\[ L^c_m = \sum_{-\infty}^{\infty} (n - 2m)c_{m-n, b_n} \]

\[ L^\gamma_m = \sum_{-\infty}^{\infty} (n - \frac{3m}{2})\gamma_{m-n, \beta_n}. \]
We write the Virasoro algebra for each component as

\[
[L_{m}^{\alpha,\mu,A}, L_{-m}^{\alpha,\mu,A}] = 2m : L_{0}^{\alpha,\mu,A} : + \frac{m^3 - m}{12},
\]

\[
[L_{m}^{d,\mu,A}, L_{-m}^{d,\mu,A}] = 2m :: L_{0}^{d,\mu,A} :: + \frac{m^3 - m}{24} \quad \text{(in the bosonic sector)}
\]

\[
= 2m :: L_{0}^{d,\mu,A} :: - \frac{m^3 + 2m}{24} \quad \text{(in the fermionic sector)}
\]

\[
[L_{m}^{c}, L_{-m}^{c}] = 2m : L_{0}^{c} : - \frac{26m^3 - 2m}{12}
\]

\[
[L_{m}^{\gamma,1,2}, L_{-m}^{\gamma,1,2}] = 2m : L_{0}^{\gamma,1,2} : + \frac{11m^3 + m}{12}.
\]

Here :: :: is the symbol for anti-normal ordering.

One might wonder how the anomaly, which is basically a c-number central extension of an operator algebra, would have a different form with what looks like a partly upside down situation of the original Fock space (i.e. only as far as the fermions are concerned). The reason behind such a difference is the fact that the normalization coefficient of the Fermi Sea is ill-defined with respect to the canonical vacuum because of the infinite number of fermions that defines the former. One should note here that the two \( \mathcal{N} = 1 \) world-sheet supersymmetries that are manifest in the action are not quite reflected in the usual fashion in the states. The bosonic state created by the creation operator of a boson field on the vacuum will transform into a “hole” state of the fermions obtained by the action of the destruction operator of the fermion on the Fermi vacuum. In that sense it can be termed as anti-supersymmetry.

Combining all the contributions of the individual components, we get

\[
[L_{m}, L_{-m}] = 2m( : L_{0}^{\alpha} : + :: L_{0}^{d} :: + : L_{0}^{c} : + : L_{0}^{\gamma} :)
\]

\[+ \frac{2[4(1 - \frac{1}{2}) + 11] - 26}{12} m^3 \]

\[= \frac{2[4(1 - \frac{1}{2}) - 1] - 2}{12} m \]

\[= 2m( : L_{0}^{\alpha} : + :: L_{0}^{d} :: + : L_{0}^{c} : + : L_{0}^{\gamma} :)
\]

in the bosonic sector. Here,

\[
L_{0}^{\alpha} = \sum_{\mu, A} L_{0}^{\mu,\alpha, A}, L_{0}^{d} = \sum_{\mu, A} L_{0}^{d,\mu, A}, L_{0}^{c} = \sum_{i} L_{0}^{c, i}, L_{0}^{\gamma} = \sum_{i} L_{0}^{\gamma, i}.
\]

Now

\[
L_{0}^{d} = : L_{0}^{d} : - a_{d} = :: L_{0}^{d} :: + a_{d}.
\]

The normal ordering constant can be regularized with a \( \zeta \)-function [29] to yield

\[
:: L_{0}^{d} :: = : L_{0}^{d} : - 2.2.4 \frac{1}{48}
\]

\[= : L_{0}^{d} : - \frac{1}{3}.
\]
in the bosonic sector.

We assume that any physical state \( |\phi\rangle \) is annihilated by all the modes of the anti-ghost \( b \). So, it has ghost number

\[
U^c |\phi\rangle = \sum_{n=\pm} c_{-n} b_n |\phi\rangle = 0.
\]

Hence, from (15) we can write

\[
L^c_0 |\phi\rangle = 0.
\]

Though fermionic, we cannot demand that the reparametrization ghosts occupy all the positive energy states at zero temperature. Since \( N = 0 \) from (21) and \( m = 0 \) also, \( E_F \) is indeterminate from (6). This is why we do not anti-normal order the commutator \([L^c_0, L^c_{-m}]\) in (16).

The bosonization of the \((\beta, \gamma)\) system and the corresponding action are given by the expressions

\[
\gamma = e^{-i\varphi - i\kappa} \partial \varphi, \quad \beta = e^{i\varphi + i\kappa},
\]

and

\[
S_{\beta\gamma} = \int d^2 \sqrt{h} \left( \frac{1}{2\pi} \partial^0 \varphi \partial_{\alpha} \varphi - \frac{i}{4\pi} K_{\varphi} R \varphi + \frac{1}{2\pi} \partial^0 \kappa \partial_{\alpha} \kappa - \frac{i}{4\pi} K_{\kappa} R \kappa \right),
\]

where \( K_{\varphi} = 2 \) and \( K_{\kappa} = -1 \). In our case, there are two \( \varphi \)s \( \varphi_1 \) and \( \varphi_2 \) for two sets of superconformal ghosts.

Let \( \varphi = -i\varphi' \) and \( K_{\varphi} = iK_{\varphi'} \). Hence,

\[
S = \int d^2 \sqrt{h} \left( \frac{1}{2\pi} \partial^0 \varphi' \partial_{\alpha} \varphi' - \frac{i}{4\pi} K_{\varphi'} R \varphi' + \frac{1}{2\pi} \partial^0 \kappa \partial_{\alpha} \kappa - \frac{i}{4\pi} K_{\kappa} R \kappa \right).
\]

We fermionize \( \varphi' \) and \( \kappa \) to get two Dirac spinors

\[
\psi^{(1)} = e^{i\varphi'} = \sum_{n=\pm} d_n^{(1)} e^{-in\tau},
\]

and

\[
\psi^{(2)} = e^{i\kappa} = \sum_{n=\pm} d_n^{(2)} e^{-in\tau}.
\]

Thus,

\[
L^c_0 = \sum_{n=\pm} \text{nd}_n^{(1)} d_n^{(1)} + \sum_{n=\pm} \text{nd}_n^{(2)} d_n^{(2)}.
\]
The reparametrization ghosts and the superconformal ghost s are superpartners. So, it is natural to expect from (21) that the ghost number

\[ U^\gamma |\phi\rangle = \sum_{\gamma = -n}^{\gamma = n} \beta_{\gamma} |\phi\rangle \]  

\[ = \int d\sigma \partial^{\prime} |\phi\rangle \]  

\[ = \int d\sigma \bar{\psi}^{(1)}(1) \psi^{(1)} |\phi\rangle \]  

\[ = \sum_{-\infty}^{\infty} \bar{d}_{-n}^{(1)} d_{n}^{(1)} |\phi\rangle \]  

\[ = \int d\sigma \partial \kappa |\phi\rangle \]  

\[ = \int d\sigma \bar{\psi}^{(2)}(2) \psi^{(2)} |\phi\rangle \]  

\[ = \sum_{-\infty}^{\infty} \bar{d}_{-n}^{(2)} d_{n}^{(2)} |\phi\rangle \]  

\[ = 0. \]

We changed the definition of the ghost fields to write

\[ \gamma = e^{-i\phi + i\kappa} \]  

\[ \beta = e^{i\phi - i\kappa} \partial \kappa \]  

and

\[ U^\gamma = \int d\sigma \partial \kappa \]

in the fifth line of equation (29). So, from (29), physical states will be annihilated by both the positive and negative modes of \( \bar{\psi}^{(1,2)} \). It is the absolute vacuum and not the Dirac Sea, according to Dirac’s hole theory. Neither the positive nor the negative energy states of the particle are filled in this case. They can also be interpreted as a state completely filled with the resultant anti-particles, but containing no particles. Therefore, from (28) we can write

\[ L^\gamma |\phi\rangle = 0. \]  

Now,

\[ L^\gamma_{0} = : L^\gamma_{0} + : \frac{1}{12} \]  

and

\[ L^\gamma_{0} = : L^\gamma_{0} + : \frac{1}{24}. \]  

Substituting (20), (33) and (34) in (17) we get

\[ [L_m, L_{-m}] = 2m( : L^\alpha_{0} + : L^\delta_{0} + L^\gamma_{0} - \frac{1}{2}) \]

\[ = 2m(L_0 - \frac{1}{2}), \]
where
\[ L_0 = \omega_0^+ + \omega_0^d + 2 \omega_0^c + 2 \omega_0^\gamma. \] (36)

From (16) we can write
\[ [L_m, L_{-m}] = 2m(\omega_0^+ + 2 \omega_0^d + 2 \omega_0^c + \omega_0^\gamma - \frac{1}{2}) \] (37)
in the fermionic sector.

Now,
\[ :: \omega_0^d :: = 2 + \frac{2}{3}. \] (38)

Substituting (33), (34) and (38) in (37) we get
\[ [L_m, L_{-m}] = 2mL_0, \] (39)

Hence, from (35) and (39) we should write the string Hamilton ian for bosons as
\[ H = L_0 - a. \] (40)

The value of \( a \) depends on the state on which \( H \) acts, \( a = 0 \) if it is a fermion and \( a = \frac{1}{2} \) if it is a boson.

If the Fourier modes of the \( \psi_s \) are \( d_n \), the vacuum state under the interchange of the creation and the annihilation operators transforms as
\[ d_n |0\rangle = 0 \text{ for } n > 0 \] (41)
to
\[ d_n |0'\rangle = 0 \text{ for } n < 0. \] (42)
Hence, \( |0'\rangle \) is the Fermi vacuum.

It is obvious that to go from (41) to (42) we have to replace \( \sigma^\alpha \) with \( -\sigma^\alpha \) in (5). Under \( \sigma^\alpha \to -\sigma^\alpha \), the Virasoro anomaly for fermions changes from
\[ \langle 0 | [T_{++}(\sigma), T_{++}(\sigma')] |0\rangle = c\delta'(\sigma') \] (43)
to
\[ \langle 0' | [T_{++}(-\sigma), T_{++}(-\sigma')] |0'\rangle = -c\delta'(\sigma). \] (44)

Since there are no such states as \( |0'\rangle \) for bosons, signs of Virasoro anomalies cannot change for them. This is because, the replacement \( \sigma^\alpha \to -\sigma^\alpha \) does not change the sign of the kinetic energy for bosons in (5) and we can get back to its original form by replacing \( X(-\sigma) \) by a new \( X(\sigma) \).

The replacement \( \sigma^\alpha \to -\sigma^\alpha \) interchanges the positive frequency modes with the negative frequency modes. This is not allowed in the case of bosons but allowed
in the case of fermions, because if we interchange the boson creation and annihilation operators, the commutator $[a, a^\dagger] = 1$ will change sign, leading to negative norm states. This is not so for the anti-commutator $\{d, d^\dagger\} = 1$ of fermion creation and annihilation operators.

The descendant states for the definition of the Kac determinant [31] involving world-sheet fermions will be

$$|\psi\rangle = L_n L_m \cdots |h\rangle .$$

(45)

Thus, from (16) we can write

$$\langle h | L_{-m} L_m | h \rangle = -\langle h | [L_m, L_{-m}] | h \rangle$$

(46)

$$= -\langle h | (2m L_0 - \frac{m^3 - m}{24}) | h \rangle$$

$$= -\langle h | (2m h - \frac{m^3 - m}{24}) | h \rangle ,$$

where $L_0 |h\rangle = h |h\rangle$. Since $L_0$ is anti-normal ordered for world-sheet fermions, $h$ will be negative in this case from (19) and so the norm of (46) is positive [32].

Equations (35) and (39) indicate that our theory is very similar to the $D = 10, N = 1$ superstring theory in the light-cone gauge. But, if we replace $\sigma^\alpha$ with $-\sigma^\alpha$, the canonical vacuum transforms into the Fermi Sea. It has the occupation number $\sum_{n=1}^\infty 1 = \zeta(0) = -\frac{1}{2}$ with respect to the canonical vacuum. So bosons become fermions and vice versa. This is reflected in the equations (17) and (37).

From the above discussions, it is clear that bosonization is possible only if the theory is regular and canonical commutation relations are unambiguous. If we bosonize the $\psi_s$, we will get $c_\psi = \frac{1}{2}$ instead of $-\frac{1}{2}$ in (7). So anomalies will not cancel, and canonical commutators will be ill-defined. We can, however, bosonize them after the anomaly cancels. Various states can then be obtained from the ground state by the action of the ladder operators.

Since ghosts do not contribute to the zero-point energy, we cannot discard the longitudinal and time-like excitations without altering it. Physical states are admixtures of both, but their total contribution to the norms vanishes [33, 34].

World-sheet supersymmetry and residual reparametrization invariance are sufficient to gauge away the $+\,$ components of all nonzero mode oscillators of $X$ and allow us to set

$$\psi^+ = 0 ,$$

(47)

Therefore physical states will be annihilated by positive frequency modes of $\psi^+$ and thus, are equal mixtures of longitudinal and time like excitations of a particular frequency.\footnote{From (47)} We assume that $k^\mu = (k^0, 0, 0, k^3)$. Therefore,

$$|\phi\rangle = (d^0_n + d^3_n) |0\rangle$$

(51)

$$\langle d^0_n + d^3_n | \phi \rangle = 0$$

(48)
\[ \langle \phi | \phi \rangle = \langle 0 \rangle (d^0_n + d^3_n)(d^0_{-n} + d^3_{-n})|0 \rangle \]
\[ = \langle 0 \rangle \{(d^0_n, d^0_{-n}) + (d^3_n, d^3_{-n})\}|0 \rangle \]
\[ = \langle 0 \rangle (-1 + 1)|0 \rangle \]
\[ = 0. \quad (52) \]

Physical state conditions can be written as

\[ \langle L_{1,\text{matter}} + L_{2,\text{matter}} - a\delta_m \rangle |\phi \rangle = 0 \]

for \( m \geq 0 \), where \( a = 0 \) in the fermionic sector and \( \frac{1}{2} \) in the bosonic sector. We call \((\psi^\mu, X^\mu)\) collectively as \((1, \text{matter})\) and \((\psi^A, X^A)\) collectively as \((2, \text{matter})\).

We set

\[ \langle L_{1,\text{matter}} - a\delta_m \rangle |\phi \rangle = 0 \]

(54)

to get massless fermions (spin-\( \frac{1}{2} \) particles) and gauge bosons (spin-1 particles) in four dimensions. Hence,

\[ L_{2,\text{matter}} |\phi \rangle = 0. \quad (55) \]

If \( F_{1,2,\text{matter}} \) are the Fourier modes of the supercurrents \( G_{1,2,\text{matter}} = \psi^{\mu,A} \partial X^{\mu,A} \), we can write

\[ F_{1,2,\text{matter}} |\phi \rangle = 0, \quad (57) \]

From (54) and (55), we can write

\[ L_{1,\text{matter}}^1 = \alpha' p^2 + N_1 \]

\[ = a \]

(58)

and

\[ L_{2,\text{matter}}^2 = \alpha' q^2 + N_2 \]

\[ = 0. \]

(59)

Therefore,

\[ q = 0 \]

(60)

Hence,

\[ d^0_n |\phi \rangle = -d^3_n |\phi \rangle \]

or

\[ \langle \phi | d^0_{-n} d^0_n |\phi \rangle = \langle \phi | d^3_{-n} d^3_n |\phi \rangle \]

(49)

or

\[ \langle \phi | d^0_{-n} d^0_n |\phi \rangle = \langle \phi | d^3_{-n} d^3_n |\phi \rangle \]

(50)

\[ 10 \]
and

\[ N_2 = 0. \quad (61) \]

Though we should write

\[ L_0 = \alpha' p^2 + \alpha' q^2 + N \quad (62) \]

\[ = a, \]

in general, where \( N = N_1 + N_2 \).

From (58) \( p^2 = 1 \) for the ground state tachyon \((N_1 = 0)\) in the bosonic sector. Hence, the vertex operator for emission of the same state will be

\[ V_0(\tau) = e^{i\phi(\tau)} e^{i\mu X(\tau)}, \quad (63) \]

where \( \phi = \phi_1 \).

For \( N_1 = \frac{1}{2} \), the first excited state will be a massless vector boson. The vertex operator for emission of this state will be

\[ V_v(\tau) = e^{i\phi(\tau)} \psi_\mu(\tau) \zeta_\mu e^{i\mu X(\tau)}. \quad (64) \]

For the ground state in the fermionic sector, the condition \( F_{1,matter} |\phi\rangle = 0 \) yields the Dirac equation describing a massless spinor. The zero-mode condition \( F_{0, matter} |\phi\rangle = 0 \) is trivial in this case.

The vertex operator for emission of this state will be [35,37]

\[ V_s(\tau) = e^{i\phi(\tau)} \Theta_{0,1;\pm}(\tau) \Theta_{2,3;\pm}(\tau) e^{i\mu X(\tau)} \Theta_{1,2;\pm}(\tau) \Theta_{3,4;\pm}(\tau) e^{i\mu X(\tau)}, \quad (65) \]

where \( \Theta_{0,1;\pm} = e^{\pm i\frac{\mu}{2}} \) are the spin operators for the pair \((\psi_\mu=0, \psi_\mu=1)\), where \( \psi_\mu=0 \pm i\psi_\mu=1 = e^{\pm i\theta_1} \). Similarly spin operators for the pairs \((\psi_\mu=2, \psi_\mu=3), (\psi_\mu=4)\) can be written as \( \Theta_{1,2;\pm} = e^{\pm i\frac{\mu}{2}}, \Theta_{3,4;\pm} = e^{\pm i\frac{\mu}{2}} \). The products \( \Theta_{0,1;\pm} \Theta_{1,2;\pm} \Theta_{3,4;\pm} \) and \( \Theta = \Theta_{0,1;\pm} \Theta_{2,3;\pm} \Theta_{2,3;\pm} \Theta_{3,4;\pm} \) represent spinors of \( SO(3,1), SO(4) \) and \( SO(7,1) \) respectively.

Other vertex operators can be constructed in a similar way.

We can write the external states of a \( M \)-particle scattering amplitude as

\[ |\phi_M\rangle = \lim_{\tau \to \pm \infty} V(\tau) |0\rangle \quad (66) \]

and

\[ \langle \phi_1 | = \lim_{\tau \to -i\infty} V(\tau) |0\rangle. \quad (67) \]
Hence, from (63), (64) and (65), we can write

\[
|\phi_0^M\rangle = \lim_{\tau \to i\infty} V_0(\tau) |0\rangle \\
= \lim_{\tau \to i\infty} e^{i\varphi(\tau)} e^{ip.X(\tau)} |0\rangle \\
= e^{i\varphi_0} e^{ip.x} |0\rangle
\]  

(68)

and

\[
|\phi_v^M\rangle = \lim_{\tau \to i\infty} V_v(\tau) |0\rangle \\
= \lim_{\tau \to i\infty} e^{i\varphi_0} b_{-\frac{1}{2}} \zeta e^{ip.x} |0\rangle
\]  

(69)

\[
|\phi_s^M\rangle = \lim_{\tau \to i\infty} V_s(\tau) |0\rangle \\
= e^{i\varphi_0} b_{-\frac{1}{2}} \zeta e^{ip.x} |0\rangle
\]  

(70)

where

\[
\varphi = \varphi_0 + p_\varphi \tau + \text{oscillator terms.}
\]  

(71)

Since \(\varphi\) has negative kinetic energy in (24), we put \([\varphi_0, p_\varphi] = -[x, p]\) in the above equations.

We set \(\varphi_0 = 0\) in (68), (69) and (70) to write

\[
|\phi_0^M\rangle = e^{ip.x} |0\rangle
\]  

(72)

\[
|\phi_v^M\rangle = b_{-\frac{1}{2}} \zeta e^{ip.x} |0\rangle
\]

\[
|\phi_s^M\rangle = e^{\pm i\frac{\theta_1^0}{2}} e^{\pm i\frac{\theta_2^0}{2}} e^{\pm i\frac{\theta_3^0}{2}} e^{i\frac{\theta_4^0}{2}} e^{ip.x} |0\rangle
\]

It simply states that \(p_\varphi = 0\) for the external states.

For maximal gauge symmetry, we assume that \(x^4\) describes a \(CP^2\) to define the gauge symmetry \(SU(3)_C\) over it [38,39]. Equation (60) then states that physical states do not change under \(SU(3)_C\) transformations. We assume that the two coordinates \((\theta_0^0, \theta_2^0)\) we get after bosonization of the pairs \((\psi^{A=1}, \psi^{A=2})\) and \((\psi^{A=3}, \psi^{A=4})\) describe a \(S^2\) to define a gauge symmetry \(SU(2)_L\) on it. Right-chiral fermions are \(SU(2)_L\) singlets. Therefore, we set \(\theta_0^0 = \theta_2^0 = 0\) for them. It is impossible to rotate them in iso-space.

To complete the Standard Model [40,44] an extra coordinate compactified to a circle will be needed for the definition of the \(U(1)_{Y_W}\) gauge symmetry. This coordinate should merely add the weak hypercharge \(Y_W\), but no new dynamics to the string.
From (63), (64) and (65), we identify \( V \) with an operator \( W \) of conformal dimension \( \frac{1}{2} \) through the relation
\[
V = e^{i\phi} W, \tag{73}
\]
in the \( F_2 \) picture [1].

Hence,
\[
W_0 = e^{ip\cdot X} \tag{74}
\]
where \( p^2 = -1 \),
\[
W_\psi = \psi_\mu \zeta^\mu e^{i p\cdot X},
\]
where \( p^2 = 0 \) and
\[
W_s = \Theta e^{i p\cdot X},
\]
where \( p^2 = 0 \).

We define the new vertex operator in the \( F_1 \) picture [1, 30, 45] as
\[
V = [F_m, W], \tag{75}
\]
From (74) and (75) we write the vertex operator for tachyon emission as
\[
V_0 = [F_m, W_0] = \psi_\mu p_\mu e^{i p\cdot X}, \tag{76}
\]
where \( p^\mu = (\pm \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}) \) and \( (\psi_3 \pm i \psi_0) \frac{1}{\sqrt{2}} = e^{\pm i Y} \).

Assuming a mode expansion of \( Y \) similar to \( X \), we write
\[
Y = y + Y_W \tau + \text{oscillator terms}, \tag{77}
\]
to interpret \( y \) as the curled up coordinate to define the \( U(1)_{Y_W} \) gauge symmetry and the momentum \( Y_W = \pm 1 \) conjugate to it as the weak hypercharge of the state.

We absorb the longitudinal and time components of \( \psi^\mu \) into \( Y \) to write the world-sheet action as
\[
S = \int d^2 \sigma (\partial^\alpha X^\mu \partial_\alpha X_\mu + \partial^\alpha Y \partial_\alpha Y + \psi_\perp \partial_\perp \psi_\perp + \cdots), \tag{78}
\]
where \( \psi^\perp s \) are the transverse components of \( \psi^\mu \). We write \( Z^a = (X^\mu, Y) \), Thus,
\[
S = \int d^2 \sigma (\partial^\alpha Z^a \partial_\alpha Z_a + \psi_\perp \partial_\perp \psi_\perp + \cdots). \tag{79}
\]
The invariance of the action under rotations of coordinate axes in the $Z$ plane will allow us to set $p^a = (p, Y_W) = (p, 0)$, where $p^2 = 2$, as in the case of the tachyon of the bosonic string. Hence, $L_0 = 1$ in the $F_1$ picture of the bosonic sector.

From (74) and (75), we can write the vertex operator for the massless vector boson as

$$V_v = [F_m, W_v] = (\zeta_{\mu} \partial X^\mu + \psi_{\mu} p^\mu \psi_{\nu} \zeta_{\nu}) e^{ip \cdot X}$$

The second term in the second line of the above equation drops out from the transversality condition $\zeta_{\mu} p^\mu = 0$.

Similarly, the vertex operator for the ground state fermion can be written as

$$V_s = [F_m, W_s] = \psi_p \Theta e^{ip \cdot X}$$

We replace $\psi^0$ by $i \psi^0$ and set $p = (\pm \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}})$ in the above to get

$$V_s = \psi_p \Theta e^{ip \cdot X}$$

Thus,

$$L_0 = \alpha' p^2 + \alpha' q^2 + \alpha' Y_W^2$$

from (60).

The states $\Theta^2_{i, \pm} \Theta^2_{i, \pm}$ should correspond to

$$\begin{bmatrix} \nu_L \\ e_L \\ X_{UL} \\ X_{dL} \end{bmatrix}$$

for the first generation of fermions (there will be three such families as we shall see later). They should be conjugate isospinors. Hence, if the first has the hypercharge $Y_W = -1$, the second will have the hypercharge $Y_W = 1$. No particles, however, carry color quantum numbers according to (60).

We replace $\tau$ by $-i \tau$ in (66) and (67), to write the $M$-point primitive amplitudes at the tree level involving both bosons and fermions in the open string sector as

$$g^{M-2} \int_0^\infty d\tau_2 \cdots d\tau_{M-2} \langle \phi; 1 | V(2, \tau_2) \cdots e^{H_{\tau_{M-2}} V(M-2)} e^{-H_{\tau_{M-2}} V(M-1)} \phi; M \rangle.$$
We set $\tau_1 = \infty$, $\tau_M = -\infty$ and $\tau_{M-1} = 0$ with the three real parameters of $SL(2, R)$ in the above equation.

Since $L_0$ commutes with an integrated vertex operator, the factor $e^{H + M - 2}$ can be brought past successive vertices to the left until the Hamiltonian annihilates the external state. So the equation (85) reduces to

\[
\frac{1}{H} V(M - 2) e^{-H + M - 2} V(M - 1) |\phi; M\rangle
\]

Some of the vertices might be of $W$ type, the amplitude would vanish otherwise.

From (40) the string propagator $\frac{1}{H} = \frac{1}{L_0 - a} = \frac{1}{L_0 - a + L_0}$ can be expanded into powers of $\frac{L_0}{L_0 - a}$. The $L_0^a$ can then be brought past the subsequent vertices and propagators until it annihilates against the physical state at the right end of the tree. Here $L_0$ includes contributions of $(\psi^\mu, X^\mu)$, the superconformal ghosts $(\gamma^1, \beta^1)$ and the reparametrization ghosts $(c, b)$ to $L_0$ and $L_0^a$ includes contributions of $(\psi^A, X^A)$ and the superconformal ghosts $(\gamma^2, \beta^2)$ to the same.

Thus,

\[
\frac{1}{L_0 - a} = \frac{1}{L_0^a - a}.
\]

Thus, we can write the tree amplitude for $M$ bosons as

\[
g^{M-2} \langle \phi; 1 | V(2) \frac{1}{L_0} V(3) \frac{1}{L_0 - \frac{1}{2}} \cdots V(M - 1) |\phi; M\rangle
\]

where $|\phi\rangle$ is a physical state like (72) in the $F_2$ picture and $|\phi'\rangle = \frac{1}{L_0} \text{matter} |\phi\rangle$ is the corresponding state in the $F_1$ picture.

The expression for the amplitude is very similar to the $N = 1$, $D = 10$ superstring theory. After the arguments cited in [1], we can show that spurious states decouple from a physical tree.

From (86), we can write the amplitude of scattering of 2 fermions and $M - 2$ bosons as

\[
g^{M-2} \langle \text{Fermion}; 1 | W(2) \frac{1}{L_0} V(3) \cdots \frac{1}{L_0} V(M - 1) |\text{Fermion}; M\rangle
\]
From (75) we can write it as
\[ g^{M-2} \langle \text{Fermion}; 1 | W(2) \frac{F_0}{L_0} W(3) \]
\[ \cdots \frac{F_0}{L_0} W(M-1) | \text{Fermion}; M \rangle = g^{M-2} \langle \text{Fermion}; 1 | W(2) \frac{1}{F_0} W(3) \]
\[ \cdots \frac{1}{F_0} W(M-1) | \text{Fermion}; M \rangle , \]
where \( F_0 = F_0^1 \).

We note that \( CP^2 \) does not admit spinors. But the chirality content of the total particle spectrum of an internal manifold \( CP^2 \otimes M \) could well be different from that of the spinors on the \( CP^2 \) only [46]. From [54] it is evident that physical states are isospinors on \( S^2 \) also. So, we calculate the Betti-Hodge numbers of \( CP^2 \otimes S^2 \).

For \( S^2 \) or \( CP^1 \) they are given by the matrix
\[ b_{pq}^{S^2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]
and those for \( CP^2 \) are given by the matrix
\[ b_{pq}^{CP^2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \]

Therefore, for \( CP^2 \otimes S^2 \) they are given by the matrix
\[ b_{pq}^{CP^2 \otimes S^2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \]

This, after proper addition of cells becomes
\[ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} . \]

The matrix can be identified with the Hodge diamond of a six dimensional Calabi-Yau Manifold [47][50]. So, the number of families will be
\[ \frac{X}{2} = b_{11} - b_{21} \]
\[ = 3 . \]

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Only open strings with opposite charges at the same mass level can join ends to form closed strings. The charges of the massless fermions in the R-sector cannot match those of the massless gauge bosons in the NS-sector to cancel each other, because unlike in the $D = 10, N = 1$ theory, they are not gauginos. Therefore, NS-R states of the closed string like gravitinos will be absent. This is consistent with the fact that there is no supersymmetry in the open string spectrum. The presence of the ground state tachyon only in the NS sector, is also inconsistent with the presence of the NS-R states.

At lower energies, string theory is consistent with quantum field theory [51]. We can independently verify it from the $\alpha' \to 0$ limit of the primitive amplitudes [1]. Keeping in mind that three of the four real components of the Higgs field $(\phi_1, \phi_2, \phi_3, \phi_4)$ can be set at zero using the local $SU(2)$ invariance of the Standard Model, we should identify the ground state tachyon of the bosonic sector of the open string with the surviving field $\phi_1$ (say).

If we consider finite temperature field theory below a critical temperature, the universe stays in a false vacuum at the zero of $\phi_1$ in a supercooled state before rolling down to its true vacuum [52, 53]. To study the effect of this supercooling on the world-sheet action, we consider the nonlinear sigma model [54] that we regularize by dimensional regularization to yield

\begin{align}
S &= -\lim_{\epsilon \to 0} \frac{1}{4\pi\alpha'} \int d^{2(1+)\sigma} \sqrt{h} h^{\alpha\beta} \partial_\alpha X. \partial_\beta X (93) \\
 &= -\lim_{\epsilon \to 0} \frac{1}{4\pi\alpha'} \int d^{2(1+)\sigma} \sigma e^{\epsilon\phi} \partial X. \partial X (94) \\
 &= -\lim_{\epsilon \to 0} \frac{1}{4\pi\alpha'} \int d^{2(1+)\sigma} \sigma e^{\epsilon\phi} g_{\mu\nu} \partial X^\mu. \partial X^\nu (95) \\
 &= -\lim_{\epsilon \to 0} \frac{1}{4\pi\alpha'} \int d^{2(1+)\sigma} \sigma e^{\epsilon\phi} (\eta_{\mu\nu} - R^\rho_{\mu\rho\sigma} x^\rho x^\sigma) (96) \\
 &= -\lim_{\epsilon \to 0} \frac{1}{4\pi\alpha'} \int d^{2(1+)\sigma} \sigma \partial X. \partial X - \frac{1}{2\epsilon} \alpha' R_{\mu\nu} (97) \\
 &= -\lim_{\epsilon \to 0} \frac{1}{4\pi\alpha' \lambda} \int d^{2(1+)\sigma} \sigma \partial X. \partial X - \frac{1}{2\epsilon} \alpha' \lambda \eta_{\mu\nu} (98) \\
 &= -\lim_{\epsilon \to 0} \frac{1}{4\pi\alpha' \lambda} \int d^{2(1+)\sigma} \sigma \partial X. \partial X - \frac{1}{2\epsilon} \alpha' \lambda \eta_{\mu\nu} (99)
\end{align}

Reparametrization invariance of the world-sheet action allows us to set $h_{\alpha\beta} = \eta_{\alpha\beta} e^{\phi}$ in (94). We used Riemann normal coordinates to write $g_{\mu\nu} = g_{\mu\nu}(X) = \eta_{\mu\nu} - R^\rho_{\mu\rho\sigma}(X_0) x^\rho x^\sigma$ in (96) ($X = X_0 + x$ are locally inertial coordinates at $X_0$) and put the logarithmically divergent contraction $\lim_{\sigma \to 3\sigma'} < x^\rho(\sigma) x^\sigma(\sigma') >= \ldots$
We used the lambda-vacuum solution to the Einstein field equation \([55]\) to write \(R_{\mu\nu} = \lambda \eta_{\mu\nu}\) in \([58]\), where \(\lambda\) is proportional to the latent heat of the false vacuum per unit volume.

The \(\phi\) dependent term in \([29]\) breaks the conformal invariance of the worldsheet action. Hence, one can define only the subalgebra \(OSp(1|2)\) of the super-Virasoro algebra consistently. Therefore, there will be spin \(\leq 1\) states only in the matter sector. This together with the fact that the Regge slope \(\alpha'_{1-\alpha'^2}\) vanishes in the limit \(\epsilon \to 0\) for finite \(\lambda\) suggests that basic interactions will solely be described by particle physics, and not the string theory. From the last equation it is clear that the ground state energy of a string changes as a result of supercooling by an amount proportional to \(\lambda \phi_0\). From \([83]\), it is obvious that it can change the color charges \(q\) from the trivial to non-trivial values.

We can ignore the change of the ground state energy when \(\lambda\) becomes too small to write

\[
Y_W^2 + q^2 = g^2 (1 + 8) = 1
\]

from \([83]\) for the ground state fermion. Hence,

\[
g = Y_W = \frac{1}{3}
\]

the hypercharge of quarks. They, together with charged leptons, may derive Dirac mass through their interactions with the open string tachyons.

The quartic Higgs self-coupling \(\lambda_1\) we get from the low energy limit of the Veneziano amplitude \([56]\) is \(\lambda_1 \approx 1\). Therefore, the vacuum expectation value of the Higgs field

\[
<\phi_1> = \sqrt{-\mu^2 / \lambda_1} \approx 1
\]

where \(\mu\) is the mass of the Higgs boson. Hence, the Dirac mass of fermions derived through the Yukawa coupling \(\bar{\psi} \psi \phi_1\) is also of the order of unity.

The sterile right-handed neutrino with \(\theta^3_0 = \theta^4_0 = y = 0\), should exist \([57]\), because it is not projected out by any projection operator and can only interact via gravity. It derives a Majorana mass \([58]\) from its coupling \([\bar{\nu}_R \nu_R + \bar{\nu}_R (\nu_R)^c]\phi_2\) to the tachyon of the gravity sector. Since the four-tachyon amplitude \(A_4^{cl} \approx \sin \frac{\pi t}{8}\) in the closed string sector \([59]\) vanishes in the low energy limit \(t \to 0\), The Majorana mass of a right-handed neutrino should be very large.

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