Research on Differential Pricing Method under a Special Nonlinear Supply Constraint

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Abstract—Differential pricing strategy is one of the typical pricing strategies applied in practice, which is named for its potential price difference. Among them, second-degree price discrimination is widely used in real life as the pricing strategy most in line with the actual situation of the market. Under the condition of a typical non-linear supply function, this paper used the cost thought to realize the application of differential pricing method to the problem of second-degree price discrimination. The results show that the more segments of supply interval, the greater the cost increment of monopoly firms in the case of equal interval. At the same time, the relationship between the number of segments and the multiple of the maximum cost increase made by the market supervision department is obtained; in the state of complete competition, the static game analysis is made for the simultaneous decision-making situation of two oligarchic enterprises. When the market supervision department stipulates that the total cost of the two competitors in the market increases by at least times the maximum cost increment, the two firms can implement n segment pricing discrimination separately. And when the market share of the two firms is stable, the segment interval of differential pricing can be obtained.

Keywords—cost thinking; supply constraints; differential pricing; game theory

I. INTRODUCTION

In recent years, the theory of price discrimination has received common concern, of which the second-degree price discrimination is the most widely used. At present, there has been a more comprehensive study on the second-degree price discrimination. A large number of literatures at home and abroad have carried out in-depth analysis of it. Tang Xiaowo et al.[1] studied the necessary and sufficient conditions for maximizing profit of monopoly firms under the term of linear demand function, pointing out that the necessary and sufficient condition for monopoly firms to obtain the maximum surplus of consumers is to divide the demand interval equally. Chen Shaogang et al.[3] proposed that the more segments of demand, the greater the consumer surplus obtained by monopoly firms, but the increase of acquisition will decrease gradually (When pricing in two segments, 50 % can be obtained; while 66.7 % in three stage pricing; 75 % in four stage pricing and 80 % in five stage pricing). Therefore, two to three stage pricing is appropriate in practice. Tang Xiaowo[2] further discussed that under the condition of non-linear demand function, the maximization condition of monopoly firms revenue is that the length of each segment increases monotonously.

II. THE RELATION BETWEEN COST INCREASE AND SEGMENTATION NUMBER UNDER EQUAL INTERVAL

In theory, the more segments of supply interval, the greater the cost increment of monopoly firms. The extreme case is the first-degree price discrimination. But in the real market, this is impossible. For monopoly firms, because the cost increases substantially, which is also a complete disadvantage, so it is particularly important to determine the number of segments in the supply interval.

Let the supply function be a special kind of non-linear function. \[ P = aQ^2 + bQ + c, \] (\(a > 0, b, c\) is constant). When the market price is determined to be \( P_0 \), the supply is also determined to be \( Q_0 \). In the above assumption, the total cost of the manufacturer is \( TC = cQ_0 \), at this time, the cost is the...
minimum production cost. Now divide the supply interval \([0, Q_n]\) into \(n\) segments, so that \(Q_i = Q_0\) and 
\[
Q_i = \frac{Q_n}{n} Q_i = \frac{2Q_n}{n}, \ldots, Q_i = \frac{iQ_n}{n}, \ldots, Q_e
\]
At this time, the total cost of the manufacturer is: 
\[
TC' = cQ_i + P_1(Q_2 - Q_i) + \cdots + P_{n-1}(Q_n - Q_{i-1})
\]  
(1)  
Among them, 
\[
P_i = aQ_i^2 + bQ_i + c
\]  
(2)  
\[
Q_{i+1} - Q_i = \frac{Q_n}{n} \quad (i = 1, \ldots, n - 1)
\]  
(3)  
Substituting equations (2) and (3) into equation (1) yields: 
\[
TC' = cQ_i + \frac{Q_n}{n}(P_1 + P_2 + \cdots + P_{n-1})
\]
\[
eQ_i + \frac{Q_n}{n}\left[aQ_i + Q_2 + \cdots + Q_{i-1}\right] + b(Q_i + Q_{i+1} + \cdots + Q_n) + (n-1)c
\]
\[
= aQ_i(1 - \frac{i}{n})^2 + bQ_i(\frac{n-1}{2}) + cQ_i
\]
Obviously, it can be seen from the above equation that as the number of segmentation intervals increases, the cost of the enterprise also increases, which is consistent with the linear supply situation. According to the above formula, the maximum cost when \(n \to \infty\) can be calculated: 
\[
\max TC' = \frac{aQ_n^3}{4} + \frac{bQ_n^2}{2} + cQ_n
\]
This is consistent with the actual situation. Therefore, when performing second-degree price discrimination, the maximum cost increase for monopolists is 
\[
\max(\Delta TC') = \frac{aQ_n^3}{4} + \frac{bQ_n^2}{2}
\]
However, in the actual situation, if the monopolist obtains this maximum cost increase, it is completely inferior to the monopolist. Therefore, this maximum cost increase is not realistic. First consider from the perspective of marketing, the relevant national market supervision department stipulates that the increase in the cost of the manufacturer in the market should be at least \(\theta (0 < \theta < 1)\) times the maximum cost increase \(\max TC'\), in order to meet the minimum cost requirement. which is 
\[
TC' - TC = \theta \max(\Delta TC')
\]
Solving the above equation, we can find the relationship between the number of interval segments \(n\) and the multiple of the maximum cost increase specified by the market supervision department. 
\[
\theta = \frac{n - 1}{n}
\]
When \(\theta = 0.8\), \(n=5\), when \(\theta = 0.9\), \(n=10\). It can be seen that in the second price discrimination of the equal supply interval, it is unnecessary to increase the number of segments, and the manufacturers are not willing to bear the result. When the market supervision department stipulates that the cost is at least 0.8 times the maximum cost increment, it is only necessary to divide the supply interval into 5 segments to meet the requirements.

III. DIFFERENTIAL PRICING METHOD IN COMPETITIVE MARKET BASED ON A CLASS OF SPECIAL NONLINEAR SUPPLY CONSTRAINTS

\(n\)-segment price discrimination between two firms under complete competition

The previous analysis in this paper is based on the fact that there is only one monopoly manufacturer in the market to provide products, but the actual situation is that there are more than two manufacturers in the market to participate in competition, and each manufacturer is in a state of complete competition. At this time, as far as the price of products is concerned, each manufacturer can not be a price maker, but can only passively become a price acceptor under the competitive state. In such a competitive market, it will still reach a static equilibrium state, so the market share is introduced according to the actual situation, assuming that the market share of manufacturer 1 is \(\beta\), the market share of manufacturer 2 is \(1 - \beta\).

Assuming that suppliers 1 and 2 supply \(Q_1\) and \(Q_2\) respectively, and satisfy \(Q_1^2 + Q_2^2 = Q_n\), they divide their supply intervals into \(n\) segments, as follows: 
\[
\left[0, Q_1^2\right] = \left[0, Q_0^2\right] \cup \left[0, Q_1^2\right] \cup \cdots \cup \left[0, Q_n^2\right]
\]
\[
\left[0, Q_2^2\right] = \left[0, Q_2^2\right] \cup \left[0, Q_2^2\right] \cup \cdots \cup \left[0, Q_n^2\right]
\]
Since both manufacturers are subject to the market supply curve, there is 
\[
P_i = aQ_i^2 + bQ_i + c \quad (i = 1, 2, 3, \ldots, n)
\]
\(P_i\) is the price determined by the sum of the two suppliers at the \(i\)-th subsection point. So there are two firms whose cost-increasing function is 
\[
\Delta TC^{(1)} = \left[Q_1 - Q_i^2\right]P_1 - b + \left[Q_2 - Q_i^2\right]P_2 - b + \cdots + \left[Q_n - Q_i^2\right]P_{n-1} - b
\]
\[
= \sum_{i=1}^{n-1} \left[Q_i - Q_i^2\right]aQ_i^2 + Q_i^2 + bQ_i^2 + c(Q_i - Q_i^2)
\]
\[
\Delta TC^{(2)} = \left[Q_1 - Q_i^2\right]P_1 - b + \left[Q_2 - Q_i^2\right]P_2 - b + \cdots + \left[Q_n - Q_i^2\right]P_{n-1} - b
\]
\[
= \sum_{i=1}^{n-1} \left[Q_i - Q_i^2\right]aQ_i^2 + Q_i^2 + bQ_i^2 + c(Q_i - Q_i^2)
\]
Manufacturer 1 needs to decide \( Q_1^i, Q_2^1, \ldots, Q_n^i \) to achieve the minimum cost increment and manufacturer 2 to achieve the maximum cost increment, then there are

\[
\frac{\partial \Delta TC^{(1)}}{\partial Q_i^1} = 0 \quad (i = 1, 2, 3, \ldots, n-1) \quad (4)
\]

From the above formula, the reaction function of manufacturer 1 can be obtained.

Similarly,

\[
\frac{\partial \Delta TC^{(2)}}{\partial Q_i^2} = 0 \quad (i = 1, 2, 3, \ldots, n-1) \quad (5)
\]

The reaction function of manufacturer 2 can be obtained.

So the decision space of manufacturer 1 is

\[
Q_1 = \left\{ (Q_1^1, Q_2^1, \ldots, Q_n^1) \mid Q_i^1 \in [0, Q_n^i] \text{ 并且满足 } \frac{\partial \Delta TC^{(2)}}{\partial Q_i^1} = 0 \right\}
\]

The decision space of manufacturer 2 is

\[
Q_2 = \left\{ (Q_1^2, Q_2^2, \ldots, Q_n^2) \mid Q_i^2 \in [0, Q_n^i] \text{ 并且满足 } \frac{\partial \Delta TC^{(1)}}{\partial Q_i^2} = 0 \right\}
\]

Since both \( \Delta TC^{(1)} \) and \( \Delta TC^{(2)} \) are continuous functions, and the simultaneous equations (4) and (5), the solution can be obtained in the form of

\[
Q_1 = \left\{ (Q_1^1, Q_2^1, \ldots, Q_n^1) \mid \bar{f}_1(Q_n^i) = f_1(\beta Q_n) \right\}
\]

\[
Q_2 = \left\{ (Q_1^2, Q_2^2, \ldots, Q_n^2) \mid \bar{f}_2(Q_n^i) = f_2((1 - \beta) Q_n) \right\}
\]

Assume that in the market, the market share of the two manufacturers is fixed, that is,

\[
Q_1^n = \beta Q_n
\]

\[
Q_2^n = (1 - \beta) Q_n
\]

Substitute into the above formula, there is

\[
Q_1 = f_1(\beta Q_n)
\]

\[
Q_2 = f_2((1 - \beta) Q_n)
\]

At this point, the cost of the two manufacturers increases by \( \theta \) times the maximum cost increase. For the manufacturer, in order to achieve this equilibrium, the respective market share cannot be lower than \( \frac{2^{n-1} - 1}{2^n - 1} \), that is,

\[
\frac{2^{n-1} - 1}{2^n - 1} < \beta \leq \frac{2^{n-1}}{2^n - 1}
\]

Conclusion: When the market supervisory department stipulates that the total cost of the two competitors in the market increases by at least \( \theta \) times the maximum cost increment, the two manufacturers can separately implement n-segment pricing price discrimination. When the market share of the two manufacturers is stable, the segmentation interval for performing differential pricing is

\[
Q_1 = f_1(\beta Q_n) = f_1(\beta Q_n)
\]

\[
Q_2 = f_2((1 - \beta) Q_n)
\]

At this time, the market share of both manufacturer 1 and manufacturer 2 must be greater than \( \frac{2^{n-1} - 1}{2^n - 1} \).

IV. CONCLUSION AND OUTLOOK

The application of differential pricing method in demand analysis has been studied in detail. In view of the supply and under the condition of a typical non-linear demand function, this paper proves that the more segments in the supply interval, the greater the cost increment of the monopoly firms. In the meanwhile, this paper obtains the relationship on the manufacturers cost stipulated by the market supervision department between the multiples of the maximum cost increment and the number of segments. In the state of complete competition, the static game analysis is made for the simultaneous decision-making situation of two oligarchic enterprises. When market-related supervisory departments stipulate that the total cost of two competing firms in the market increases by at least times the maximum cost increment, the two firms can implement n segment pricing discrimination separately. And when the market share of the two firms is stable, the segment interval of differential pricing can be obtained. In this paper, the conditions to be satisfied for market share of the manufacturer 1 and the manufacturer 2 are given simultaneously. The results not only have theoretical significance, but also have certain practical value. However, in the n-segment pricing mode by stages, because of the difficulty in solving the corresponding response function, this paper points out the existence of Nash equilibrium and the dependency relation of segment pricing mode on market share. Moreover, this paper will further study the specific solution results of n-segment pricing mode. The solution of these problems will make the pricing method of second-degree price discrimination more theoretical completeness and practical operability.

REFERENCES

[1] Tang Xiaowo. Conditions for maximizing the profits of monopoly firms under second-degree price discrimination [J]. Journal of University of Electronic Science and Technology, 1998, (02). (In Chinese)

[2] Tang Xiaowo. Further research on second-degree price discrimination [J]. Journal of Management Science, 2001, (1). (In Chinese)

[3] Chen Shaogang, Tang Xiaowo, Zhao Shurong. Nash Equilibrium of Second-degree Price Discrimination in the Case of Two Manufacturers [J]. Application of System Engineering Theory and Method, 2003, 12 (4): 303-305.(In Chinese)

[4] Berry S T Estimating discrete-choice models of product differentiation [J].1994, (25).

[5] Tang Xiaowo, Zeng Yong, Li Shiming, et al. Management Economic Analysis-Theory and Application [M]. Chengdu: University of Electronic Science and Technology Press, 2000. (In Chinese)
[6] Zhang Weiying. Game Theory and Information Economics [M]. Shanghai: Shanghai Sanlian Bookstore, Shanghai People's Publishing House, 1996. (In Chinese)

[7] Xie Shiyu. Economic Game Theory (2nd Edition) [M]. Shanghai: Shanghai Sanlian Bookstore, Fudan University Press, 2002. (In Chinese)

[8] Wang Yankai, Chen Shaogang, Zhu Ruidong. Research on differential pricing method under linear supply constraint [J]. Modern commercial industry, 2015, 36 (16): 65-68.