Calculation of the efficiency of the planetary gear K-V-V without a carrier

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Abstract. The work is dedicated to the calculation and improvement of the efficiency of the novel planetary gears without a carrier. The feature of the K-V-V gears is the misalignment of the axles of the driving wheel, the driven wheel and the supporting wheel. Three types of the gearing are considered: 1) involute; 2) involute with the track rollers; 3) articulated. Calculated values of efficiency at characteristic gear ratios are higher than, for example, worm gears, especially for variants with support rollers and gear-hinge gears. The articulated gear (variant 3) is much simpler than the involute gear with the track rollers (variant 2).

1. Introduction
Planetary gears are widely used in mechanical engineering, instruments and tools industries. The well-known planetary gear groups are 2K-H, K-H-V and 3K according to the classification of V.N. Kudryavtsev [1]. The main disadvantage of such transmissions is the presence of a complex and low-tech part - a carrier designed for rigid relative fixation of several satellites. There are non-carrier transmissions - 3K [2], but most of them have very low efficiency.

2. K-V-V transmission design and kinematic capabilities
We have proposed [2, 3] a non-carrier planetary gear, which should be designated K-V-V or 2K-V according to V.N. Kudryavtsev (figure 1).

![Figure 1. Non-carrier planetary gear K-V-V.](image-url)
The transmission contains a fixed wheel (3) with internal teeth and satellites of the outer (2, 4, 5) and central (1) groups located inside it. If the wheel (1) is not related as a satellite, but as the central gear of the planetary mechanism, then the transmission should not be assigned to the K-V-V group, but to the 2K-V group. The largest satellite (2) of the outer group is a driven wheel. Any other satellite can be the driving wheel. But the maximum gear ratio is achieved when the driving wheel belongs to the central group of satellites. Other satellites (4, 5) are intermediate wheels; they are not loaded with torque. The rotation is fed to the shaft of the wheel (1) and is taken from the shaft of the wheel (2). In this case, the direction of rotation changes to the opposite. A feature of this transmission is that all the axles of the driving (1), driven (2) and supporting (3) wheels do not coincide with each other. The gear ratio of the mechanism is calculated by the following equation:

$$i_{1-203} = \frac{1 + \frac{z_3}{z_1}}{1 - \frac{z_3}{z_2}} \tag{1}$$

where $z_3$ is the number of teeth of the central wheel (3); $z_1$, $z_2$ - the number of teeth of the driving (1) and driven (2) floating satellites.

Figure 1 presents a mechanism having the following parameters: $z_1 = 35$, $z_2 = 62$, $z_3 = 100$, $z_4 = 21$. After the substituting of the numbers of gear teeth in the equation (1), we obtain:

$$i_{1-203} = \frac{1 + \frac{100}{35}}{1 - \frac{100}{62}} = -6.29$$

The calculated gear ratios that can be obtained in gearboxes of this type are listed in table 1.

| $z_3$=100 | $z_1$ = 10 | $z_1$ = 15 | $z_1$ = 20 | $z_1$ = 25 | $z_1$ = 30 | $z_1$ = 35 | $z_1$ = 40 | $z_1$ = 45 | $z_1$ = 50 |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| $z_2$     | 87        | 82        | 77        | 72        | 67        | 62        | 57        | 52        | 47        |
| $i_{1-203}$ | -73.61    | -34.92    | -20.08    | -12.85    | -8.79     | -6.29     | -4.64     | -3.49     | -2.66     |

Kinematic pairs formed by links of the transmission mechanism can be performed in different ways. The simplest way from the technological point of view is the use of gears with an involute profile (figure 2, a). In another transmission variant (figure 2, b) the pitch circles of the involute gears are duplicated by track rollers. In the third case (figure 2, c) the gears have circular tooth profiles. The gears formed by such teeth will be related as the articulated gears.

This article is dedicated to a comparative calculation of the K-V-V transmission efficiency depending on the design of the gears and the parametric ratios that affect the gear ratio.

![Figure 2. Options for gearing mechanism.](image-url)
3. The analysis and calculation of the efficiency of a mechanism with involute teeth without track rollers

The scheme of forces acting on the transmission elements (figure 1) is shown in figure 3. For given gear sizes the resulting reaction force $R_{ij}$ is the same in all kinematic pairs. Accept the following: $R_{12} = R_{23} = R_{14} = R_{34} = 1$.

The pressure angles $\lambda_{ij}$ (i.e. the angles between the direction of the force $R_{ij}$ and the common tangent of the interacting pitch circles $i$ and $j$) are found from the figure 3: $\lambda_{34} = \lambda_{14} = 65^\circ54'$, $\lambda_{23} = \lambda_{12} = 20^\circ51'$. Note that this example was considered earlier [3]. In this article it is used as the initial version of the design of the gear mechanism to be improved.

![Figure 3. Force scheme.](image)

The angles of engagement $\alpha_{w14}$, $\alpha_{w34}$, $\alpha_{w12}$, $\alpha_{w23}$ are determined as a result of geometric calculation of the corresponding gears. Using the KOMPAS library "Shafts and mechanical transmissions 2D", we obtain: $\alpha_{w14} = 30^\circ$, $\alpha_{w34} = 30^\circ$, $\alpha_{w12} = 20^\circ$, $\alpha_{w23} = 20^\circ$.

In a mechanism without track rollers (figure 2, a) gears have two points of engagement. So the forces $R_{pij}$ and $R_{oij}$ act on two sides of the tooth: the "working" (index - p) and the "reverse" (index - o) as can be seen in figure 4. The values of these forces are calculated by the equations (2) and (3):

![Figure 4. Decomposition of forces for involute gears without track rollers.](image)
\[
R_{pij} = R_{ij} \frac{\sin(\lambda_{ij} + \alpha_{wlij})}{\sin 2\alpha_{wlij}} \tag{2}
\]

\[
R_{oij} = R_{ij} \frac{\sin(\lambda_{ij} - \alpha_{wlij})}{\sin 2\alpha_{wlij}} \tag{3}
\]

When \( R_{14} = R_{34} = R_{23} = R_{12} = 1 \), for the dimensions of the mechanism corresponding to the figure 1 and figure 3, we obtain.

\[
R_{p14} = 1.416; \quad R_{p34} = 1.416; \quad R_{p23} = 1.018; \quad R_{p12} = 1.018;
\]
\[
R_{o14} = 0.974; \quad R_{o34} = 0.974; \quad R_{o23} = 0.023; \quad R_{o12} = 0.023.
\]

The efficiency \( \eta \) of a mechanism is the ratio of the net power \( P_{pol} \) to the consumed power \( P_{zat} \) (which includes power losses \( P_{pot} \)).

\[
\eta = \frac{P_{pol}}{P_{zat}} = \frac{P_{pol}}{(P_{pol} + P_{pot})}. \tag{4}
\]

The net power:

\[
P_{pol} = P_2 = M_2 \cdot \omega_2 = R \cdot h_2 \cdot \omega_2 \tag{5}
\]

where: \( M_2 \) is the moment on the driven wheel; \( R \) – is the force \( R = 1 \); \( h_2 \) – shoulder on the driven wheel (determined according to the drawing); \( \omega_2 \) – angular speed of the driven wheel.

Transmission losses \( P_{pot} \) are the sum of losses in all gears \( P_{poti} \). So equation (4) can be transformed to:

\[
\eta = \frac{R \cdot h_2 \cdot \omega_2}{R \cdot h_2 \cdot \omega_2 + \sum_{i=1}^{n} P_{poti}} \tag{6}
\]

Losses in two-point engagements of the mechanism under consideration occur on the “working” \( P_{pi} \) and on the “reverse” \( P_{oi} \) sides of the tooth. The power losses for each gear is approximately determined by the equation [1]:

\[
P_{poti} = V_{cp} \cdot f_i \cdot (R_{pi} + R_{oi}), \tag{7}
\]

where: \( f_i \) is the calculated friction coefficient; \( V_{cp} \) – average slip velocity in engagements. We will take approximately [1]:

\[
V_{cp} = 2 \cdot V_p \cdot (1/ z_{sch} + 1/ z_k), \tag{8}
\]

where: \( V_p \) is the calculated peripheral speed at the poles of the engagements; \( z_{sch}, z_k \) – numbers of teeth for the gear and the wheel.

The peripheral speed \( V_p \) is calculated using the driven wheel when the carrier is stopped:

\[
V_p = \omega_2 \cdot r_2 \cdot (1 - i_{H-2}), \tag{9}
\]

where: \( r_2 \) – is the radius of the driven wheel; \( i_{H-2} \) – gear ratio from carrier to the driven wheel calculated by means of the following equation

\[
i_{H-2} = 1 / (1 - z_3 / z_2). \tag{10}
\]

After the substituting of the average velocity \( V_{cp} \) into the equation (7) we obtain the following expression:
Substituting the power losses of all elements into the equation (6) and performing the required transformations, we obtain the final equation for calculating the transmission efficiency:

\[
\eta = \frac{h_2}{h_2 + 2 \cdot r_2 \cdot (1 - i_{H-2}) \cdot \sum_{i=1}^{z} \left( R'_{ni} \left( \frac{1}{z_{schi}} \pm \frac{1}{z_{ki}} \right) \right)},
\]

(12)

where: \( R'_{ni} = R_{pi} + R_{oi} \) (provided that R = 1).

For the mechanism shown in Fig. 1 and Fig. 3 the efficiency is calculated by the equation (12). Let’s substitute the following values: \( h_2 = 57.97 \text{ mm}, \ r_2 = 31 \text{ mm}, \ i_{H-2} = 1.632, \ f = 0.1, \ R_{p14} = 1.416, \ R_{p21} = 1.018, \ R_{p34} = 1.416, \ R_{p32} = 1.018, \ R_{o14} = 0.974, \ R_{o21} = 0.023, \ R_{o34} = 0.974, \ R_{o32} = 0.023, \ z_1 = 35, \ z_2 = 62, \ z_3 = 100, \ z_4 = 21 \) in equation (12) and obtain:

\[
\eta = \frac{57.94}{57.94 + 2 \cdot 31.0 \cdot (1 + 1.632) \cdot 0.1 \sum_{i=1}^{z} (2.39 \cdot 0.076 + 1.04 \cdot 0.045 + 2.39 \cdot 0.038 + 1.04 \cdot 0.006)} = 0.916.
\]

4. The calculation of the efficiency of the mechanism with involute teeth and track rollers

The presence of track rollers does not affect the acting reaction forces \( R_{ij} \) (figure 3). However, these rollers perceive a radial load \( R_k \) instead of the back of the tooth \( R_o \) (figure 5). As a result, the reaction on the working side of the tooth is much lower:

\[
R_{prij} = R \frac{\cos \lambda}{\cos \alpha_{w}};
\]

(13)

**Figure 5.** Decomposition of forces for involute gears with track rollers.

Substituting the values of the angles \( \alpha_{w} \) and \( \lambda \) into equation (13), we obtain:

- \( R_{p14} = 0.471 \);
- \( R_{p34} = 0.471 \);
- \( R_{p12} = 0.984 \);
- \( R_{p23} = 0.984 \).
There is no slip in the contact of the rollers, where the reactions $R_k$ act, so the losses here can be neglected at this stage of the analysis. $R_{pij}$ values are substituted into the equation (12):

$$\eta = \frac{57.94}{57.94 + 2 \cdot 31.0 \cdot (1+1.632) \cdot 0.1 \sum_{i=1}^{x} \left( 0.471 \cdot 0.076 + 0.984 \cdot 0.045 + 0.471 \cdot 0.038 + 0.984 \cdot 0.006 \right)}$$

$$\eta = 0.972$$

In the considered example an efficiency of 0.972 was obtained. Note that this value is 6% higher than for a mechanism without track rollers.

5. Efficiency of the mechanism with articulated gears

When the teeth with circular profiles contact with each other, the resulting force $R_{ij}$ is not replaced by side reactions $R_p$ and $R_o$, but is perceived by the circular surface of the tooth (figure 6).

![Figure 6. Forces in articulated gear.](image)

The arising friction force $F_i = f \cdot R$ in this case will be the same for all the four gears. And the sliding velocities $V_{ci}$ in the engagements formed by different wheels depend on the number of teeth of these wheels.

$$V_{ci} = \frac{\pi}{2} \cdot V_p \cdot \left( \frac{1}{z_{sch}} \pm \frac{1}{z_k} \right)$$

(14)

As a result, the equation for the efficiency of a mechanism with articulated gears will be the following:

$$\eta = \frac{h_2}{h_2 + \frac{\pi}{2} \cdot r_2 \cdot (1 - i_{H-2}) \cdot f \sum_{i=1}^{x} \left( R \left( \frac{1}{z_{sch}} \pm \frac{1}{z_{ki}} \right) \right)}.$$ 

(15)

For the mechanism shown in Fig. 1, we get:

$$\eta = \frac{57.94}{57.94 + 1.57 \cdot 31.0 \cdot (1+1.632) \cdot 0.1 \sum_{i=1}^{x} \left( (1/62 - 1/100) + (1/35 + 1/62) + (1/21 + 1/62) + (1/21 - 1/100) \right)}$$

$$\eta = 0.965$$

This value is slightly lower than for a mechanism with track rollers, but significantly higher than for a mechanism without track rollers.
6. Comparison of the efficiency of non-carrier K-V-V planetary gears with different types of engagements

The schemes of the mechanism (figure 1) with different gear ratios used to calculate reaction forces were taken using the restrictions on the pressure angles in gears and the absence of contact of the surfaces of wheels (1) and (3). The involute gear angles $\alpha_{in}$ are obtained for the characteristic tooth numbers and gear displacement coefficients. The calculation results are presented in figure 7.

![Figure 7](image)

**Figure 7.** The dependence of the efficiency of non-carrier K-V-V planetary gear with different engagements on the gear ratio: 1 - involute without track rollers; 2 - involute with track rollers; 3 - articulated gears.

Thus, the efficiency of all considered gears decreases with increasing gear ratio. The efficiency of gears with track rollers and with articulated gears is significantly higher than with involute gears without track rollers in the studied range of gear ratios. Losses in articulated gears are only 15…20% higher than in gears with track rollers, but 2 times lower than in involute gears without track rollers.

7. Conclusions

K-V-V gears allow to get gear ratios in a wide range, approximately from 1 to 70. This is the main advantage of the mechanism. A feature of the K-V-V gears is that all three axles (the leading, the driven and the supporting wheel) do not coincide with each other. This feature of the mechanism should be considered a disadvantage. In practice, the use of such gears will cause additional difficulties with the layout, with shaft seals, etc.

The efficiency of the mechanism, especially for versions with track rollers and with articulated gears, is quite high. Higher than, for example, in worm gears. The variant with articulated gears is simpler than the variant with track rollers in design and manufacturing (in the case of modern 2D technologies).

The intended application of K-V-V gears: control mechanisms, winches, hoists, wrenches and other relatively slow-moving devices.

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