Topology Optimization to Reduce Electromagnetic Force Induced Vibration for the Specific Frequency of PMSM Motor Using Electromagnetic-Structural Coupled Analysis

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Abstract: Vibration and noise reduction are very important in electric vehicle driving motors. In this study, topology optimization of housing was performed to reduce vibration in a specific frequency caused by electromagnetic force generated by a permanent magnet synchronous motor (PMSM). The vibration induced by the electromagnetic force of the motor was calculated using electromagnetic-structural coupled analysis. Then, the magnitude of the acceleration for a specific frequency at which peak occurs in the rectangular and circular shape housing concept design model was reduced by using the topology optimization method. As a result, the rectangular and circular shape housing design reduced 92.9% and 96.0%, respectively. Finally, the vibration was effectively reduced while maintaining the electromagnetic characteristics of the motor, for which topology optimization was conducted while not changing the rotor or stator shape design (electromagnetic design factor) but by changing the motor housing shape design (mechanical and structural design factor).

Keywords: PMSM; electromagnetic force; vibration; topology optimization; vibration reduction; electromagnetic-structural coupled analysis; specific frequency

1. Introduction

Recently, as the applications of electric vehicles are becoming increasingly widespread, the demand for high-output electric motors used for driving such vehicles is also increasing. In this regard, the permanent magnet synchronous motor (PMSM) is widely used for driving electric vehicles due to its high torque output and high efficiency [1]. However, the vibrations and noise of such motors are important factors to increase the vehicular ride comfort [2].

The vibrational source in PMSMs is stated as a bigger problem in the stator structure vibration characteristics by electromagnetic forces than electromagnetic vibration characteristics such as cogging torque and torque ripple [3]. The structural vibrations of the PMSM is induced by the electromagnetic force acting in the radial direction, which is excited by the stator [4]. Since this force exhibits electromagnetic characteristics and induces structural vibrations of the motor, an electromagnetic-structure coupling analysis method that simultaneously considers electromagnetic and structural aspects should be used to understand the vibration characteristics of the motor.

To understand these characteristics of electric motors, arising due to the electromagnetic force, several studies using the electromagnetic-structure coupling analysis have been conducted [5–13]. F. Ishibashi et al. [5] applied the electromagnetic force obtained through the Maxwell stress tensor method of a small induction motor to the finite element method, compared the vibration mode and amplitude with the experimental results, and confirmed...
Islam et al. [6] studied the vibrations induced by the electromagnetic field according to the pole/slots and winding configuration of the motor by co-simulating the structure with the electromagnetic field based on a finite element method (FEM) and quantified the radial force and displacement. Lin et al. [7] analyzed the relationship between different combinations of pole/slots of the PMSM and vibration/noise peaks using a multi-physics model. Nam et al. [8] compared the one-way method and two-way coupled analysis method on the finite element model of a motor by combining magnetic and structural vibrations. Lin et al. [9] analyzed electromagnetic noise and sound quality in variable speed range using a multiphysics model of a permanent magnet synchronous motor. Deng et al. [10] presented a model that can analyze the electromagnetic vibration and noise of an external rotor axial flux in wheel motor over a wide speed range, and the results were verified through simulations and experiments. Cho et al. [11] investigated the necessity of a 3D full finite element model using multi-physics to understand the vibration characteristics of BLDC induced by electromagnetic forces. Gao et al. [12] experimentally verified vibration and noise simulations, analyzed the vibration effects of specific frequencies through the stiffness evaluation of the stator, and filtered the main excitation frequencies.

Vibrations induced by the electromagnetic force of the motor degrades the ride comfort and eventual performance of electric vehicles [13]. Therefore, from the aspect of motor electromagnetic design, various studies have been conducted to reduce such structural vibrations. Hur et al. [14] developed a notch to reduce the vibration for removing the harmonics of the IPM-type BLDC motor. Gan et al. [15] applied the skewing effect to the stator to reduce vibrations in switched reluctance motors (SRM). Hong et al. [16] proposed a motor shape with reduced vibrations by conducting a parametric study to change the shape of the pole and yoke of the stator of an SRM motor. Takiguchi et al. [17] modified the driving current to reduce the third harmonic component in the sum of radial forces generated in the stator teeth, thereby reducing the acoustic noise and vibration of the motor. Zhang et al. [18] proposed, based on a special dual-branch three-phase PMSM, a method that uses a known modified space vector PWM technique.

More recently, variable studies have been conducted [19–26] to reduce vibrations by determining the shape of the rotor and stator of the motor using optimization methods for complex design of various shapes. In particular, Jung et al. [19] performed a design optimization of the rotor using the response surface methodology optimization method to reduce the vibration induced by electromagnetic force in the IPMSM. Cho et al. [20] used a notch in the stator teeth to reduce the vibration induced by the electromagnetic force in the PMSM, and a design optimization was performed for the position and width. Kim et al. [21] performed a design optimization of the rotor and stator of the IPM motor using the PQRSM optimization method to reduce the electromagnetic force-induced vibration. Lin et al. [22] reduced electromagnetic vibration and noise by adjusting the slot opening width and magnet shape and deriving the optimal skewing angle of the magnet. Xie et al. [23] were optimized to change the turn ratio and number of parallel branches to reduce the vibration induced by the electromagnetic force. Wang et al. [24] optimized rotor slotting to improve the PMSM’s NVH (Noise, Vibration, Harshness) performance to reduce the amplitude of harmonics.

In these studies, the vibration was reduced by decreasing the force harmonic of the electromagnetic force and changing the shape of the rotor and stator, which are considered electromagnetic design factors. However, since this factor determines the torque output performance designed by the motor designer, varying them may cause other electromagnetic problems such as cogging torque and torque ripple. However, the motor housing is considered a mechanical and structural design factor, which determines the vibration performance without affecting the output torque performance and electromagnetic characteristics. Therefore, research is needed to change the shape of the motor housing (mechanical and structural design factors) rather than the shape of the rotor and stator (electromagnetic design factors) to reduce the vibration induced by the electromagnetic forces.
In this paper, topology optimization of the motor housing was conducted to reduce the electromagnetic force induced vibration of a specific frequency in the operation state of a PMSM. The electromagnetic-structural coupled analysis method was used to calculate the structural vibration induced by the electromagnetic force of the motor. Firstly, the electromagnetic force in the radial direction generated from the stator tooth surface of the motor was calculated by using 2D electromagnetic field analysis. Secondly, the electromagnetic force induced vibration was derived from 3D FEM based frequency response analysis of the initial housing shapes of two types, which are rectangular and circular. Finally, the topology optimization of the motor housing was performed to reduce the magnitude of the acceleration for the specific frequency at which the peak occurs. Then, each of motor housings was redesigned based on the result of the topology design optimization and the effect of vibration reduction was confirmed.

As a result, the research proposes a process of designing housing of motor with reduced vibration, which can improve the ride comfort of an electric vehicle.

2. Electromagnetic Force Induced Vibration Analysis Using Electromagnetic-Structural Coupled Analysis

2.1. Electromagnetic-Structural Coupled Analysis

In this study, the electromagnetic field design of a 4 pole 24 slots permanent magnet synchronous motor was used in shown in Figure 1a. And the Figure 1b shows the electromagnetic field design 2D cross section of the PMSM. Table 1 shows the detailed specifications of the PMSM motor with a rated speed of 2000 rpm and a rated output of 1500 W. The 3D shape model of the stator is shown in Figure 1c,d.

Table 1. PMSM Specification.

| Specification       | Quantity      |
|---------------------|---------------|
| Rated Power         | 1500 W        |
| Pole                | 4 poles       |
| Slots               | 24 slots      |
| Rated Current       | 6.2 A         |
| Rated Speed         | 2000 rpm      |

(a) (b)

Figure 1. Cont.
Firstly, a 2D electromagnetic field analysis based on a finite element was performed on a cross section of the motor. Magnetic flux density in an airgap was generated from the interaction of the rotating magnetic field in the stator and poles in the rotor. The magnetic flux density that was calculated by the electromagnetic field analysis excited an electromagnetic force on the surface of the stator teeth. The exited electromagnetic force was calculated by using Maxwell stress tensor, as shown Equation (1)

\[
P_r(\theta, t) = \frac{B_r^2(\theta, t) - B_t^2(\theta, t)}{2\mu_0}, \quad P_t(\theta, t) = \frac{B_r(\theta, t)B_t(\theta, t)}{2\mu_0}
\]

where, \(P_r\) is radial electromagnetic force, \(P_t\) is tangential electromagnetic force, \(B_r\) is radial magnetic flux density, \(B_t\) is tangential magnetic flux density, \(\mu_0\) is magnetic permeability in the airgap.

Secondly, the electromagnetic force calculated through electromagnetic field analysis was applied to each node of the stator tooth surface of the housing initial model. Frequency response analysis was performed with a source, with electromagnetic force applied to the motor structure model, which included the stator and the housing. Then, the acceleration responses were calculated at the evaluation point. The frequency response analysis using modal superposition is represented as Equation (2).

\[
[M] \sum_{i=1}^{N} \{\Phi_i\} \ddot{x}_i + [C] \sum_{i=1}^{N} \{\Phi_i\} \dot{x}_i + [K] \sum_{i=1}^{N} \{\Phi_i\} x_i = \{F\}
\]

where, \(x_i\) is nodal displacement, \(\{\Phi_i\}\) is \(i^{th}\) mode shape, \(\{F\}\) is electromagnetic nodal force, \([M], [C], [K]\) are mass matrix, damping matrix, and stiffness matrix, respectively.

2.2. Electromagnetic Force Calculation

The motor to be studied is a 4-pole 24-slot permanent magnet synchronous motor, and a 2D electromagnetic transient analysis based on finite element analysis was performed by applying a 3-phase rated current under the condition of a 2000 rpm rated operation state. The magnetic flux density of PMSM is calculated as shown in Figure 2. The electromagnetic force at the surface of the stator tooth is calculated by Maxwell stress tensor using the computed magnetic flux density. The force acts as the main vibration source of the electric machine [27]. The calculated force is divided into radial and tangential forces on the surface of the stator teeth. In general, the tangential force is neglected because it is very small compared to the radial force [28]. The distribution of electromagnetic force calculated on the stator tooth surface by Equation (1) is shown in Figure 3.
Figure 2. Magnetic Flux Density distribution.

Figure 3. Radial Force distribution.

As the rotor rotates at a constant speed, the electromagnetic force is periodically generated on the surface of the stator teeth due to the effect of the pole. The main frequency of the exciting force is defined as shown in Equation.

\[ f = \frac{n}{60} \times 2p \]  

where \( n \) is the number of revolutions per minute of the motor and \( p \) is the pole pair.

The motor used in this study is a 4 pole 24 slot permanent magnet synchronous motor that rotates at a constant speed of 2000 rpm. Figure 4a shows the electromagnetic force in the radial direction excited on the stator tooth surface according to time and spatial harmonic order. For the electromagnetic force in the radial direction according to time and space, the electromagnetic force components due to time and spatial harmonics are shown in Figure 4b through FFT (Fast Fourier Transform). The force harmonic of the electromagnetic force due to the pole effect generates the 4th spatial harmonic order at the 4th time frequency 133.3 Hz, and the 24th spatial order is generated by the slot effect.
2.3. Frequency Response Analysis

The radial electromagnetic force at the stator tooth surface calculated by 2D electromagnetic field analysis is applied to the stator tooth surface of the 3D finite element model. The periodic electromagnetic forces which have harmonic orders excite at each node on the tooth surface. The stator and housing of the motor have mode characteristics according to their structural shape. The peak in the vibration response occurs due to resonance between the structural natural frequencies and the electromagnetic force harmonic orders [23].

In this study, the electromagnetic force harmonic orders that induce the vibration of the entire structure of 4 poles 24 slots PMSM in 2000 rpm operation state is generated in $f = 133.3$ Hz order by Equation (3). Frequency response analysis was performed on the entire 3D PMSM model which includes housing, and the vibration response of the entire PMSM structure caused by the electromagnetic force harmonics orders was calculated.

2.4. Topology Optimization Using Electromagnetic-Structural Coupled Analysis

The topology optimization was conducted to minimize the vibration response of the entire PMSM model that was calculated using electromagnetic-structural coupled analysis. Topology optimization is the technique that, based on element density as a design factor, derives an optimal structure according to the designer’s purpose [29].

In this paper, the homogenization method, one of the topology optimization methods, was used to derive the optimal structure. The method is an optimization technique that uses an infinite number of microscale holes to derive an optimal structure instead of removing the entire finite element [30].

Figure 5 shows the procedure of topology optimization using electromagnetic-structural coupled analysis. In each iteration, the structural vibration response at a specific frequency is calculated using electromagnetic-structure coupling analysis. Then, it is a method of
optimizing the design with a structure that minimizes the acceleration response of a specific frequency at the evaluation point.

![Diagram](image_url)

**Figure 5.** Procedure of topology optimization using electromagnetic-structural coupled analysis.

### 3. Topology Optimization for Motor Housing Design

#### 3.1. Topology Optimal Design Formulation

Motor design is largely divided into two types. It is an electromagnetic design that determines the output performance depend on the design of the stator and rotor, and a structural design that determines the structural performance depend on the design of housing. The vibration response peaks occur due to resonance between the excited electromagnetic force and the modal characteristics of the motor structure. However, the structural mode can avoid resonance with the excited force harmonic order frequencies by changing the housing design of the motor. Therefore, in this study, the structural design of the motor is changed to optimize the housing design to reduce the vibration transmitted from the stator, which is the electromagnetic force excitation point, to the evaluation point.

The problem formulation of topology optimization is defined as Equation (4).

\[
\text{Find } \rho_j \in [0,1], (j = 1, \cdots, N) \\
\text{Minimize } \max_\Omega \ddot{u}(f) \\
\text{Subject to } \int_{\Omega} \rho \; d\Omega > 0.3 \\
[\Phi]^T [M][\Phi] \cdot \{\ddot{u}(f)\} + [\Phi]^T [K][\Phi] \cdot \{u(f)\} = [\Phi]^T \{F(f)\}
\]

where, \( \rho_j \) is the element density of \( j^{th} \) element, \( \ddot{u}(f) \) is the acceleration magnitude at the evaluation node for the specific frequency \( f \), \( \Omega \) is design domain, and \( N \) is the number of elements which belong to the design area.

In this study, two concept housing designs were constructed, rectangular and circular shape, in order to apply the topology optimization method. The peak of the acceleration response occurs at a specific frequency for each concept housing design due to interaction between the electromagnetic force harmonic orders and modal characteristics of the structure. Finally, in order to reduce the acceleration response peak, the objective function was defined at the specific frequency, and the topology optimization was performed.
3.2. Rectangular Shape Housing Initial Design

The initial housing of the rectangular shape of 140 mm × 140 mm × 138 mm was designed as shown in Figure 6. The size of the initial design was determined in consideration of the housing size of the existing motor. The vibration response evaluation point was selected as the node in the center of the housing which was located closest to the stator excited by the electromagnetic force, and frequency response analysis was performed.

![Figure 6. Rectangular shape housing initial design.](image)

Based on the results of the frequency response analysis, topology optimization was performed to reduce the acceleration response of a target frequency with the largest peak acceleration response in the frequency range above 1000 Hz where vibration and noise of the motor are issues. The topology optimization converged to the optimal solution through 12 iterations, and the result is shown in Figure 7a. The topology optimization result can give the optimal design direction for the designer’s purposes. Therefore, the topology optimization result was redesigned considering the actual design possibility and environment as shown in Figure 7b.

![Figure 7. Result of Topology Optimization: Rectangular shape housing design (a) Topology optimization result of element density (b) Redesign model.](image)

Figure 8a shows the compared results of the acceleration response at the evaluation point of the initial housing and the optimized redesign housing by using electromagnetic-structural coupled analysis. The acceleration response of the initial design model occurs at the harmonic orders of 133.3 × k Hz, which is the fundamental electromagnetic excitation frequency harmonic orders. The largest acceleration peak occurred at 3200 Hz due to resonance with the stator of the motor and the structural mode of the housing. Figure 8b which shows the narrow frequency range and the topology optimization result shows that the vibration response of the target frequency is significantly reduced, and it can confirm the effective vibration reduction design.
3.3. Circular Shape Housing Initial Design

The initial housing design of the circular shape was constructed in the shape of a hollow cylinder as shown in Figure 9. The model is 25 mm thick and 138 mm high. The frequency response analysis was performed by selecting the vibration response evaluation point as the node in the center of the housing which is located closest to the stator excited by the electromagnetic force.

Based on the frequency response analysis results, the topology optimization was performed to reduce the acceleration response of target frequency with the largest acceleration peak response. The topology optimization converged to the optimal solution through 37 iterations, and the result is shown in Figure 10a. In the same way as in rectangular shape housing, the topology optimization result needs to be redesigned to consider the actual design possibility and environment. Therefore, Figure 10b shows redesigned results of the topology optimization of the circular housing model.
Figure 10. Result of Topology optimization: Circular shape housing design (a) Topology optimization result of element density (b) Redesign model.

Figure 11a,b show the result of comparing the acceleration response at the measuring point of the redesigned housing and the existing housing using electromagnetic-structure coupling analysis. The acceleration response of the initial design model occurs in a harmonic order component of 133.3 Hz, which is the fundamental EM excitation frequency component. A large response occurred at 1866 Hz due to resonance with the stator of the motor and the structural mode of the circular type housing. The phase optimization result confirmed that the vibration response of the corresponding frequency was significantly reduced, and the vibration reduction effect was more effective than the initial design.

3.4. Result and Discussion of Topology Optimization for Each Initial Design

The comparison result of the acceleration response, reduced through the topology optimization design for a specific frequency, where the vibration response of the rectangular and circular shape housing design was a problem, is shown in Table 2. For each housing model, the acceleration response was reduced from 28.09 m/s² to 1.99 m/s², and from 12.03 m/s² to 0.48 m/s² in the rectangular and circular shape housing design at a specific frequency, respectively. Finally, the acceleration response at a specific frequency was significantly reduced in redesigned shapes using the result of topology optimization.
Table 2. Comparison of acceleration response of Rectangular and Circular shape housing design.

|                      | Specific Frequency | Initial Model | Optimized Model |
|----------------------|-------------------|---------------|-----------------|
| Rectangular shape    | 3200 Hz           | 28.09 m/s²    | 1.99 m/s²       |
| housing design       |                   |               |                 |
| Circular shape       | 1866 Hz           | 12.03 m/s²    | 0.48 m/s²       |
| housing design       |                   |               |                 |

The peak of the acceleration response of the motor occurred when the frequency component of the housing mode due to structural characteristics matched with $133.3 \times k$ Hz, which is the harmonic force of the stator due to the electromagnetic characteristics of the motor.

In the case of the initial housing design, a peak occurred due to resonance at a specific frequency. It can avoid resonance with the excitation source due to the shift of the structural mode. Therefore, in this study, in order to avoid resonance at a specific frequency, the structural mode of the housing was shifted through the topology design optimization to minimize the resonance effect by the electromagnetic force.

4. Conclusions

In this study, the topology optimization process using electromagnetic-structural coupled analysis was established in order to reduce vibration at a specific frequency caused by electromagnetic force of PMSM.

First, the radial electromagnetic force acting at each stator tooth surface was calculated using the electromagnetic-structural coupled analysis under the operating conditions of the PMSM. The calculated electromagnetic force showed characteristics of time and space, which depend on the speed of motor and poles/slots. Then, two types of 3D housing conceptual design models, rectangular and circular, were constructed, and the calculated electromagnetic force was applied to the stator tooth surface.

Finally, topology optimization was performed using a frequency response analysis, having the objective function of minimizing the acceleration response of a specific frequency where the peak vibration response of each housing design occurred. Then, the housing was redesigned based on topology optimization results, which showed effective vibration reductions of 92.9 and 96.0 % for the rectangular-shaped and circular-shape housing designs, respectively.

In the future, this study will include a study to derive a motor housing design that minimizes the user’s discomfort due to high frequency noise through optimization using acoustic noise.

Author Contributions: Conceptualization, G.U.J.; Investigation, G.U.J., K.J. and J.M.; Methodology, S.C.; Project administration, C.-W.K.; Supervision, C.-W.K.; Writing—original draft, G.U.J. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) [2019R1A2C1090228], by the Human Resources Program in Energy Technology of the Korea Institute of Energy Technology Evaluation and Planning(KETEP) grant funded by the Ministry of Trade, Industry and Energy(MOTIE) of the Republic of Korea (Grant no. 20194010101790), and by the Technology Innovation Program (20004627) funded By the Ministry of Trade, Industry & Energy(MOTIE, Korea).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.
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