Risk Analysis of Put Options in Urban Gas Supply Chain

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Abstract: Effected by the economic situation and the oil market, the mismatch between supply and demand has occurred in urban gas market. Moreover, the development pattern of urban gas in China also aggravates the imbalance between supply and demand. In such situations, the phenomenon of overstock always takes place for the urban gas company. It has been proved that put options can provide flexibility of downward adjustment to cope with supply-demand imbalance and improve the performance of the supply chain. There are few researches on whether put options bring risks to the urban gas company or not. This paper focuses on the risk analysis of introducing put options to the supply chain of urban gas industry.

Key Words: Urban gas, Put options, Supply chain management, Risk analysis

1. Introduction

Since 2014, the increase of gas consumption in China began to be slowdown because of slow economic growth and low prices of international oil, which has brought serious problems in the gas supply chain, especially on the mismatch between supply and demand. Meanwhile, the development of urban gas in China mainly focuses on the upstream of urban gas supply chain, such as resources exploitation and the import of overseas resources, rather than the downstream clients. This situation aggravates the imbalance between supply and demand. In face of such situation, it’s a major challenge for the urban gas company to deal with the balance between supply and demand.

Currently, there are two different modes existing in China urban gas supply. One is that urban gas company purchases gas from its supplier and sell it to downstream customers such as businesses, residents, small and medium enterprises. The other is that the supplier supplies gas to end-users directly\(^6\). This paper focuses on the former one because it is the main supply mode in China urban gas market. Generally, the urban gas companies are required to sign long-term supply contracts with the suppliers, while the contracts between the urban gas companies and the end-users are short-term. As a result, if the actual demand of the downstream customer changes, it will cause huge losses to urban gas company because the order quantity may be hard to change.

In the traditional supply chain system, to get more demand information to increase the forecast accuracy, urban gas company generally wants to delay the order as late as possible. At the same time, urban gas company also wants the supplier to increase production to deal with the demand uncertainty. However, the supplier wants to get the order as soon as possible in order to obtain steady demand. This contradiction always leads to the bad performance of the supply chain.

Put options has been proved to improve the performance of supply chain effectively, especially when facing the supply-demand imbalance market condition. Put options give the buyer the right to decrease orders, one for each option. In this paper, we consider a gas supply chain including a supplier, an urban gas company and an industrial end-user. And put option contract model has two stages, the first one is that the urban gas company places an initial order at the beginning of the planning horizon \((t_0)\), and the supplier accepts the order. The second stage is that urban gas company adjusts the initial order according to updated demand forecast information during the selling season, and the supplier starts to deliver the gas according to the urban gas company’s final order. Compared with the traditional inflexible supply mode, the flexible put option contract can contribute to a win-win relationship for both the supplier and the urban gas company. Although it has been proved that put options can increase the performance of the urban gas supply chain, there are few researches on whether put options bring risks to the urban gas company or not. If the expected profit at the beginning of the planning horizon increases, one considers that option contract can improve the performance of the supply chain. However, the long lead time between the first stage and the second stage may increase the uncertainty of the supply chain. As a result, introducing option contract may also bring about risks. To our best knowledge, there is no research on weather introducing put options will bring about risks or not. This paper focuses on the risks of put options in urban gas supply chain.

The rest of the paper are organized as follows. In section 2, we present a literature review. We introduce a flexible
supply contract with put options into urban gas supply chain in Section 3. Section 4 analyzes the risks of put options in urban gas supply chain. Finally, the summary and suggestion are given in Section 5.

2. Literature Review

In recent years, many scholars study the supply contract in gas supply chains. Xiao et al. (2011) [9] analyze the status and problems of gas supply and give suggestions from the perspective of the whole gas industrial chain coordination. Zhang (2013) [11] evaluates and summarizes the theoretical research of natural gas industrial chains. Based on the characteristics of the gas industry, he concludes the suggestions to comprehensive upgrading of gas industry chain. Zhang et al. (2011) [10] prove that suitable revenue sharing contract can coordinate oil and gas supply chain. However, they do not consider the revenue sharing contract is suitable for the urban gas supply chain.

There are few researches on applying option contracts to the urban gas supply chain. However, a lot of researchers have proved that option contracts can provide flexibility to cope with supply-demand imbalance and improve the performance of the supply chain. Chen and Parlar (2007) [2] consider an extension of the single-period stochastic inventory model where put option can be purchased to reduce losses resulting from low demand. Xue et al. (2015) [8] employ put option contract to develop the optimal ordering and hedging policies for a risk-averse newsvendor. They derive structural results on the optimal ordering and put option decisions with the CVaR downside risk measure. Li et al. (2012) [3] analyze the retailer’s stock, the optimal option strategies and the optimal production strategies of manufacturer in the background of price and market demand uncertainty. A conclusion is drawn that both sides can achieve win-win results with option contract price policy.

Some scholars research supply chain with risk preference and introduce options contract to avoid risk and coordinate supply chain. Mu et al. (2009) [4] analyze the capacity reservation contract by the Conditional Value-at-Risk and propose the optimization models under risk-neutral and risk-aversion assumptions. Those models illustrate that the retailer’s risk attitude has important influence on the optimal quantity of capacity reservation. The more risk-averse the retailer is, the smaller the optimal capacity reservation quantity is. Buzacott et al. (2011) [1] study commitment-option contracts with forecast updates and take mean-variance as the standard of the risk of options. Wang et al. (2012) [7] obtain the retailer’s optimal initial order and quantity of options purchased, and analyze the degree of risk-aversion and the parameters’ effect on optimal decision. Tian et al. (2014) [5] construct two-stage supply chain consisting two different loss-averse components supplier and an assembler and introduce the mechanism of option contracts to coordinate the assembly supply chain. Most of the scholars study risk attitude of supply chain members. However, few scholars consider the risks of introducing flexible options contract to the urban gas supply chain.

3. Model

We use the following notation throughout the paper:

- \( Q_o \): quantity of an initial order of urban gas company
- \( Q_{nv} \): order quantity of urban gas company
- \( q_o \): number of options purchased
- \( q_e \): number of options exercised
- \( w \): unit wholesale price of urban gas
- \( w_o \): unit option price of urban gas
- \( w_e \): unit exercise price of urban gas
- \( r \): unit retail price of urban gas
- \( p \): unit shortage cost of urban gas
- \( v_b \): unit salvage value of urban gas for the urban gas company
- \( \xi \): actual demand during the selling season
- \( f_L(\cdot) \): probability density function (pdf) of \( L \)
- \( F_L(\cdot) \): cumulative density function (cdf) of \( L \)
- \( f_{D,L}(\cdot) \): conditional pdf of demand for \( L = l \)
- \( F_{D,L}(\cdot) \): conditional cdf of demand for \( L = l \)

3.1 Newsvendor model

We regard the traditional urban gas supply chain as a newsvendor model (Fig.1). In this model, the supplier and urban gas company are independent of each another. In Fig.1, the urban gas company places the initial order quantity \( Q_{nv} \) based on the demand forecasting at \( t_0 \) (the beginning of the planning horizon) and pays the unit wholesale price \( w \) each unit to the supplier. Then, the supplier produces gas during \( t_0 \) to \( t_1 \). At \( t_1 \) (the beginning of the selling season), the supplier delivers gas to the urban gas company. During the selling season, urban gas company earns revenues from each satisfied demand and incurs shortage cost \( p \) for any unsatisfied demand. At \( t_2 \) (the end of the selling season), urban gas company gains the income through the salvage of unsold gas.

3.2 Options-based model: supply contract with put options

Put options give the urban gas company the right, but not the obligation, to adjust the initial order quantity. At \( t_0 \), the urban gas company places an initial order \( Q_o \) with a wholesale price \( w \) and purchases \( q_o \) options at a unit cost \( w_o \). The supplier commits to produce at the quantity of \( Q_o \). At \( t_1 \), the urban gas company adjusts the initial order quantity by exercising put options according to the updated demand forecast if necessary. At \( t_2 \), the urban gas company salvages the unsold gas at value \( v_b \) (see Fig.2).

3.3 Analysis

We make the following assumptions to rationalize above model:

1. \( r > w > v_b \)
2. \( w \geq w_e \geq 0 \)
3. \( w \geq w_o \geq 0 \)
4. \( w + w_o + (w - w_e) \leq r + p \)
We assume that the wholesale price and retail price \( r \) is exogenously determined. Let \( D(D \geq 0) \) represent the random demand in the selling season. At \( t_0 \), the urban gas company expects a uniformly distributed demand \( D \) over the interval \( [L - m, L + m] \), where the average demand \( L \) is unknown, but it is again expected to be uniformly distributed over \( [\gamma - n, \gamma + n] \). At \( t_1 \), urban gas company specifies the \( L \) value, \( L = l \), based on the latest forecast, which is updated during the supplier’s long lead-time from \( t_0 \) to \( t_1 \). We further assume that \( \gamma \geq m + n \) to ensure \( D \geq 0 \). The forecasting error can be measured by \( m \) and \( n \). Since \( L \) and \( D \) are continuous uniform distribution over \( [\gamma - n, \gamma + n] \) and \( [L - m, L + m] \), respectively, the pdf and cdf of \( L \) as well as those of \( D \) are as follows:

\[
f_l(l) = \frac{1}{2n}, \quad l \in [\gamma - n, \gamma + n],
\]

\[
F_L(l) = \frac{1}{2n} (l - \gamma + n), \quad l \in [\gamma - n, \gamma + n],
\]

\[
f_D(l) = \frac{1}{2m}, \quad l \in [l - m, l + m],
\]

\[
F_D(l) = \frac{1}{2m} (l - l + m), \quad l \in [l - m, l + m].
\]

### 3.3.1 Newsvendor model

Based on the above description, the urban gas company’s expected profit at \( t_0 \) is:

\[
\Pi_{nv}(Q_{nv}) = -w Q_{nv} + \int_{Q_{nv} - m}^{Q_{nv} + m} [r \xi + v_b(Q_{nv} - \xi)]f(\xi)d\xi f_L(l)dl + \int_{Q_{nv} - m}^{Q_{nv} + m} [r \xi + v_b(Q_{nv} - \xi)]f(\xi)d\xi f_L(l)dl + \int_{Q_{nv} - m}^{Q_{nv} + m} [rQ_{nv} - p(\xi - Q_{nv})]f(\xi)d\xi f_L(l)dl + \int_{Q_{nv} - m}^{Q_{nv} + m} [rQ_{nv} - p(\xi - Q_{nv})]f(\xi)d\xi f_L(l)dl.
\]

The first term represents the purchase cost. The second, third and fourth terms are the expected profits during the
selling season, when \( l \) is in the intervals \([\gamma - n, Q_{nv} - m] \), \([Q_{nv} - m, Q_{nv} + m] \) and \([Q_{nv} + m, \gamma + n] \), respectively. When \( L \) is in the interval \([\gamma - n, Q_{nv} - m], Q_{nv} > l + m \) holds, i.e., the urban gas company’s order quantity is more than the actual demand. Therefore, overstock occurs. Part three the urban gas company’s order quantity is more than the

\[
\max \Pi_t(Q_0, q_0) = -wQ_0 - w_0q_0 + E(\Pi_t(q_c, Q_0, q_0|l))
\]

where

\[
E(\Pi_t(q_c, Q_0, q_0|l)) = \int_{\gamma - n}^{Q_{nv} - m} \Pi_t(q_c, Q_0, q_0|l)f_L(l)dl + \int_{Q_{nv} - m}^{Q_{nv} - m} \Pi_t(q_c, Q_0, q_0|l)f_L(l)dl + \int_{Q_{nv} - z(w_n)}^{Q_{nv} - z(w_n)} \Pi_t(Q_0 - z(w_n) - q_c, Q_0, q_0|l)f_L(l)dl + \int_{Q_{nv} + m}^{\gamma + n} \Pi_t(0, Q_0, q_0|l)f_L(l)dl + \int_{Q_{nv} - z(w_n)}^{Q_{nv} + m} \Pi_t(0, Q_0, q_0|l)f_L(l)dl
\]

\[
\Pi_t(q_c, Q_0, q_0|l) = -w_c q_c + \int_{-m}^{l+m} [r\xi + v_b(Q_0 - q_c - \xi)]f(\xi)d\xi
\]

\[
\Pi_t^2(q_c, Q_0, q_0|l) = w_c q_c + \int_{-m}^{l+m} [r\xi - p(\xi - Q_0)]f(\xi)d\xi
\]

\[
\Pi_t^3(q_c, Q_0, q_0|l) = w_c q_c + \int_{Q_{nv} - q_c}^{Q_{nv} - q_c} [r(Q_0 - q_c) - p(\xi - Q_0 + q_c)]f(\xi)d\xi
\]

Taking the derivative of \( \Pi_t(q_c, Q_0, q_0) \) with respect to \( Q_0 \) and \( q_0 \) respectively, and equating them to zero, we get the urban gas company’s optimal policies at \( t_0 \):

\[
Q_0 = \gamma + n - \frac{2n(w + w_0 - w_p)}{r + p - w_c} - \frac{m(w_c - v_b)}{r + p - v_b}
\]

\[
q_0 = \frac{2n(r + p - w)}{r + p - w_c} - \frac{2nw_0(r + p - v_b)}{(r + p - w_c)(w_c - v_b)}
\]

4. Risk Analysis

Since the demand uncertainty is high and anything is possible to happen during the long lead-time, it is difficult to say that supply contract with put options model can also improve the buyer’s performance at the beginning of the selling season. That is, performance is improved at the beginning of the planning horizon, but worse performance may occur at the second stage. As a result, the introduced options contract may cause the possibility of risks for urban gas company. Therefore, this paper focuses on the risks of introducing put options in urban gas supply chain.

We formulate the profit difference \( \Delta(l) \) (urban gas company’s expected profit of supply contract with put options model minus that of newsvendor model at \( t_1 \)). It takes value of \( \Delta(l) \) as the criterion to estimate the risks. If \( \Delta(l) < 0 \), the urban gas company has risks, i.e., supply contract with put options model provides the urban gas company with lower profit than newsvendor model. If \( \Delta(l) \geq 0 \), there is no risk.

This paper uses numerical study to analyze the variation of the profit difference \( \Delta(l) \) under different options price \( w_0 \). In order to study whether the supply contract with put options model brings risks to urban gas company or not, we set the basic parameters as follows: \( \gamma = 1000, n = 500, m = 50, r = 10, w = 7, w_0 = 1, w_c = 6, p = 10, v_b = 1 \).

In Fig.3, the effect of urban gas company risk for various values of average demand \( l \) and options price \( w_0 \). Horizontal coordinate represents average demand \( l \) and the vertical coordinate represents profit difference \( \Delta(l) \) at \( t_1 \). In Fig.3, the profit of introduced put options may be lower than that of the newsvendor model at \( t_1 \). As the average demand \( l \) increases, \( \Delta(l) \) first decreases, then increases and finally remains unchanged. When \( w_0 = 0.1 \), the interval where \( \Delta(l) < 0 \) exists, i.e., introducing put options may cause risks, and \( \Delta(l_{\text{min}}) = -309 \). When \( w_0 = 2 \), \( \Delta(l_{\text{min}}) = -761 \). That means the risk interval becomes larger and the probability of risks increases. However, if the cost of obtaining flexibility is too high, purchasing option is meaningless to the urban gas company. For example, when \( w_0 \) increases to 5, the urban gas company prefers to choose the newsvendor model. Therefore, decision of the supply contract with put options model is the same as that of the newsvendor model, i.e., \( \Delta(l) = 0 \).

5. Conclusion

In this paper, we consider a single-period two-stage supply contract between a supplier and an urban gas company. The urban gas company purchases gas from the supplier and sells it to end customers. This paper analyzes the risks of introducing put options for the urban gas company, and analyzes the risks of the urban gas company under different options price by numerical study.

We introduce variable \( \Delta(l) \) to estimate the risk. If the introduced put options bring about risks for the urban gas company, \( \Delta(l) < 0 \). Otherwise, \( \Delta(l) \geq 0 \). We find that although the flexibility of put options can improve the performance of the supply contract, while the cost for such flexibility will increase the risks for the urban gas
At the same time, when the cost of purchasing options is too high, the urban gas company prefers to choose the newsvendor model but not purchasing options.

**Acknowledgements**

This work is supported by the National Social Science Foundation of China (No. 13CGL056), Development Research Center of Oil and Gas, Sichuan (SKB16-01).

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