FACTORIZATION AND QCD ENHANCEMENTS IN THE COMPTON MECHANISM OF W AND Z BOSON HADROPRODUCTION

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It is argued that W/Z boson production in ultra-relativistic $pp$ collisions in the fragmentation region, subject to a kinematic cut on the boson transverse momentum $Q_\perp > Q_{\perp\text{min}}$, with $1 \text{ GeV} < Q_{\perp\text{min}} < M_{W/Z}$, must be dominated by the Compton mechanism $qg \to q'W/Z$. We propose applications for boson hadroproduction in this kinematics and formulate the factorization theorem. The analysis of QCD enhancements indicates that Regge behavior should manifest itself not only in the $s$ but also in $Q_\perp$-dependence of the boson production differential cross-section.

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1. INTRODUCTION

Resonant production of $W^\pm$ and $Z^0$ bosons at $hh$ colliders [1] has high probability and a clean experimental signature when the boson decays to a lepton pair, since both leptons have $p_T \sim M_V/2 \sim 40$ GeV ($V = W$ or $Z$), thus being well separated from the hadronic underlying event. That makes the electroweak (EW) boson production process a convenient quark-meter, well suited for determination of quark momentum distributions in hadrons, complementary to DIS [2], and also serve as a playground for new physics searches.

Due to the $W$ boson charge, measurement of its asymmetry is convenient for probing valence quark distributions [3,4]. As for $Z$-bosons, they have nearly the same differential distribution, but are easier to reconstruct from detection of 2 charged leptons, and serve as a benchmark. Albeit the formidable EW boson mass does not afford probing the smallest $x$ frontier for the given proton beam energy (at LHC in the central rapidity region $x_{1,2} \sim M_V/\sqrt{s} \sim 10^{-2}$), but in the fragmentation region one of the $x$ diminishes to $\sim 10^{-3}$, so in this kinematics the heavy boson hadroproduction may serve as a probe of small-$x$ physics, as well.

Historically, it was suggested by Drell and Yan [5] that the leading contribution to inclusive hadroproduction of a heavy gauge boson comes from $q\bar{q}$ annihilation, with $q$ and $\bar{q}$ carried by different hadrons (Fig. 1a). This approximation holds well at $pp$ collisions and for not very high energies ($\sqrt{s} \lesssim 200 \text{ GeV} \sim 0.5 \text{ TeV}$). However, in $pp$ collisions and at modern collider energies (particularly at LHC), antiquarks are by far less abundant than gluons [6], and since an EW boson can only be emitted from a quark line, the next mechanism to be considered is $gq \to qV$ (known as QCD Compton scattering, see Fig. 1b). Next, since at small $x$ gluons tend to be more abundant than even quarks, in the central rapidity region one also has to take into account boson production through $gq$ fusion. But as long as boson-gluon coupling is not direct, proceeding through an auxiliary (virtual or real) $q\bar{q}$ pair production (see Figs. 1c and 1d), $gq$ processes appear rather as a background for quark physics studies, and it is desirable either to arrange conditions in which their contribution is minor, or apply additional experimental selection criteria to exclude them. Most obviously, to suppress $gg$ fusion, it should suffice to work in the fragmentation region, where $q(x) \gg g(x)$. Besides that the process at Fig. 1d gives 2 jets [6], while that of Fig. 1c, as well as Fig. 1a − a minimum bias event with only small $Q_\perp \sim \Lambda \leq 1 \text{ GeV}$. To compare with, the contribution from the Compton mechanism is typically 1-jet and broadly distributed in the boson’s transverse momentum $Q_\perp$, out to $Q_\perp \sim M_V$ (which at inclusive treatment of $Q_\perp$ gives rise to $\ln M_V$). So, the criterion may be to select events with 1 jet balancing the boson transverse momentum, in favor of 2-jet and 0-jet events.

Strictly speaking, the physical distinction between the Drell-Yan (DY) and Compton mechanisms is not quite clear-cut, because one of the two Feynman diagrams of the Compton process (Fig. 2b) is topologically similar to that of $q\bar{q}$ pair production by

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1Gluons were not firmly established as partons at the time article [5] was written, whereas it had already been known from electron DIS at SLAC that charged partons are spin-1/2 fermions, and so existence of quarks and antiquarks was naturally hypothesized. Besides that, [5] actually discussed the case of finite $x$ for both fusing partons.
a gluon and subsequent annihilation of the $\bar{q}$ with the projectile quark. If in a proton all antiquarks stem from gluon splitting $g \to q\bar{q}$, then one may expect the DY mechanism to be contained in the Compton one. However, before the annihilation, the antiquark may interact with other constituents of the hadron and get non-trivially entangled with them, while in the Compton mechanism this possibility is not accounted for. At evaluation of the $Q_{\perp}$-integrated cross-section of boson production, that problem is circumvented by taking the approach similar to DIS — including the LO gluon pdf contribution into NLO antiquark pdf contribution at a higher factorization scale (actually $\sim M_s^2$) [7, 8].

At high $p_T$, nonetheless, one may be pretty sure that most of the (anti)quarks are due to direct gluon splitting. Although even an intrinsic, low-$p_T$ antiquark can scatter on another constituent (of the same hadron [9], or even of the opposite hadron [10]) and acquire high $p_T$, while staying non-trivially entangled with the hadron constituents, but the probability of hard scattering is $\sim \alpha_s^2(p_T^2)$, whereas that of direct production of $\bar{q}$ at high $p_T$ from gluon splitting is $\sim \alpha_s(p_T^2)$, i.e. of lower order in $\alpha_s$ and thus greater. A caveat is that there are several constituents in the proton to hard-scatter from, but from DIS experience, that should not make the proton obscure with respect to hard scattering, anyway. Besides that, the event shapes in these cases contain different number of minijets and can be rejected by additional experimental criteria. Therefore, at high $p_T$ the distinction between DY and Compton mechanisms is clearer.

![Diagram](image)

**Fig.1.** Contributions to $W$ and $Z$ boson production in a high-energy proton-proton collision. (a) $q\bar{q}$ annihilation; (b) $gg \to qV$ (Compton); (c) $gg$ fusion with virtual $q\bar{q}$ pair creation; (d) $gg$ fusion with real $q\bar{q}$ pair creation.

The above discussion suggests a possibility to experimentally isolate the Compton contribution by imposing a kinematic cut $Q_{\perp} > Q_{\perp,\text{min}} \gg \Lambda$, which suppresses the DY contribution, and working in the fragmentation region, which suppresses the contribution from $gg$-fusion. The contribution from the central rapidity region may otherwise be suppressed by constructing a $d\sigma(W^+) - d\sigma(W^-)$ difference. The stipulated high $Q_{\perp}$ will also alleviate the partonic subprocess description, getting rid of collinear gluon reabsorption by the non-annihilated active quark, and of the active quark irreducible rescatterings [10] (they are entirely absorbable into $g(x_g)$). Hence, it may be feasible to use this mechanism as well in global analysis of quark momentum distribution functions, provided the factorization procedure is appropriately formulated. In fact, the factorization procedure here must be similar to that in case of direct photon or jet production, with the proviso that we do not actually need to be involved in precise jet definition. That is rather natural since the final quark with high $p_T$ will emerge as a jet. Incidentally, let us note that tagging the flavor of the quark jet, e.g. its charm, in association with a $W$ boson will give access to the (sea) strangeness content of the nucleon (cf [3]).

Comparing the procedures of pdf determination from EW boson hadroproduction with a $Q_{\perp}$ cut and via inclusive $Q_{\perp}$ treatment, we should note the following. At LO, the DY mechanism is very convenient for pdf determination because from the reconstructed $V$-boson momentum one determines longitudinal momenta of both annihilating partons exactly. At NLO, though, due to an additional unregistered gluon in the final state it involves an $x$-convolution. For Compton mechanism, the convolution arises already at LO. However, since diagram 2b is similar to that of DY, and moreover, it appears to dominate, the convolution kernel for Compton is also rather singular, and perhaps even replaceable by a $\delta$-function. This is quite opposite, say, to the direct photon production case, where the emitted photon is always the lighter particle among the final products, and therefore tends to carry away only a small fraction of energy. For heavy boson production, the roles of the momentum-conserving radiator and the soft radiation are reversed: the emitted boson assumes most of the momentum while the left-over quark is wee. The dynamical reason is that diagram 2b dominates, wherein the final quark and the virtual space-like quark tend to be collinear with the parent gluon, and so the space-like quark manifests itself more like an antiquark.

![Diagram](image)

**Fig.2.** Feynman diagrams for the partonic subprocess of Fig. 1b (Compton scattering).

Ultimately, the dominance of diagram 2b may be utilized for probing the gluon distribution in a close analogy with probing quark distribution at DIS. Indeed, the virtual space-like ($\sim -M_s^2$) quark in Fig. 2b corresponds to the virtual photon in the DIS LO diagram, and this virtual quark knocks out a quasi-real gluon from the proton, converting it to a quark. However, at small $x_g$ one must beware of multiple gluon exchanges between the probing quark and the probed hadron, which can affect the universality of the prob-
ableistic gluon distribution. Those issues will be discussed in the next section.

2. FACTORIZATION FOR THE COMPTON PRODUCTION MECHANISM BEYOND GLUON DISTRIBUTIONS

In this section we outline the factorization procedure relating the fully differential cross-section of EW boson hadroproduction with the corresponding $qq \rightarrow q'V$ partonic cross-section. At small $x$ values, it may be important to formulate the factorization theorem non-perturbatively, beyond the notion of gluon distribution probability. The irreducible to single gluon exchange amounts to quark scattering off an intense and coherent gluonic field. In this respect the situation resembles that in QED, where there is a familiar factorization theorem beyond the perturbative treatment of interaction with the external field, first established for scattering in a Coulomb field [11], and then generalized to high-energy scattering in compact field of an arbitrary shape [12] (see also [13]). The theorem presents the process amplitude as a product of the electron spin-independent (eikonal) scattering amplitude and the electron spin-dependent perturbative amplitude of real photon emission at absorption of a virtual photon with the momentum equal to the total momentum transfer in scattering. In our case the initial gluon virtuality may be neglected compared with the emitted boson mass. Therefore we may regard the initial gluon as real, avoiding the issue of initial gluon distribution with the account of virtuality, and apply the Weizsäcker-Williams approximation. The latter, however, needs to be generalized, factoring out not the gluon flow but the full quark scattering amplitude.

Consider a non-diffractive high-energy $pp$ collision event containing a high-$p_T \bar{u}u$ pair. Suppose that by reconstructing the total momentum of the $\bar{u}u$ pair its mass is identified to be at the $W$ or $Z$ boson resonance[2] and the rapidity being $>1$, say, positive. The latter implies that this boson had most probably been emitted by one of the quarks of the hadron moving in the positive (forward) direction.

Owing to the Lorentz-contraction of ultra-relativistic hadrons, the interaction of the emitter quark with the opposite hadron proceeds very rapidly. Furthermore, owing to large value of $M_V$ compared to the typical hadronic energy scale 1 GeV, the boson emission from the quarks also passes very rapidly compared to the intra-hadron timescale. Hence, sufficiently reliable must be the impulse approximation, at which the emitting quark initial state is described by the (empirical) momentum distribution function, while the rest of the partons in that hadron are regarded as spectators taking no part in the boson production process. Thereby we reduce the problem to that of $V$ boson emission by a relativistic quark scattering on an hadron. Since due to

the boson heayness, the amplitudes of its emission from different quarks within one (forward moving) hadron do not interfere (the formfactor reduces to the number of quarks), the probability (differential cross-section) of boson production in the $pp$ collision comes as an integral of the correspondent quark-proton differential cross-section weighted with the quark pdf $f(x)$ in the first proton:

$$d\Sigma(P_1, P_2, Q) = \int_0^1 dx f(x) d\sigma(xP_1, P_2, Q) \quad \text{(fr. reg.),}$$

where $P_1^\mu, P_2^\mu$ are the initial hadron 4-momenta, $Q$ the final boson momentum, and $x$ the hadron momentum fraction carried by the emitter quark. The factorization scale for the quark pdf $f(x)$ will be determined later on.

Relation with the quark-hadron scattering differential cross-section

Applying the generalized Weizsäcker-Williams procedure to the differential cross-section of boson production in quark-hadron scattering, we obtain

$$\frac{d\sigma}{d\Gamma_Q} = 16\pi \frac{2E}{E' + p_z'} \frac{d\hat{\sigma}}{d\sigma_{\text{scat}}} k^2 \quad \text{(2)}$$

where

$$d\Gamma_Q = \frac{d^3Q}{(2\pi)^3 2Q_0},$$

and $d\hat{\sigma}$ may be related with QED of QCD virtual Compton cross-section:

$$\frac{d\hat{\sigma}}{dt} = \frac{1}{4\pi\alpha_{\text{em}}} \frac{d\sigma(\gamma^* \rightarrow eV)}{dt} = \frac{2N_c}{4\pi\alpha_s} \frac{d\sigma(qq \rightarrow qV)}{dt}.$$

The value of the gluon longitudinal momentum, or energy, $\omega$ is fixed by the 4-momentum conservation law and on-shellness of the final undetected quark.

The quark-hadron quasi-elastic scattering differential cross-section $d\sigma_{\text{scat}}$ encodes all non-perturbative aspects of fast quark-hadron interaction in a model-independent way. Earlier, the differential cross-section of quark-hadron scattering had already been introduced in the context of high-energy $pA$ [14] and $\gamma A, \gamma^* A$ collisions of nucleons and nuclei.

Representation (2) also expresses similarity with $kT$-factorization [15], but there $d\hat{\sigma}$ may go beyond the WW approximation, while in place of $d\sigma_{\text{scat}}$ one has the BFKL kernel describing the growth of the cross-section with the energy. In the next section we shall discuss the latter issue as well, along with other effects arising in QFT.

Relation with the integrated and unintegrated gluon distributions

If we could rely on an approximation that the high-energy small angle quark-hadron scattering proceeds only through a single $t$-channel gluon exchange (presumably with a running coupling constant), we might avoid detailed description of the hadron creating the color field. Then all we need to know is the

\[2\text{The resonance width } \Gamma \sim 2\text{GeV} \ll M_V \text{ will be neglected for a quasi-free particle.}\]
equivalent gluon flow. In the single gluon exchange approximation, the absorbed gluon may be treated on equal footing with the initial quark, and so the description of boson hadroproduction must become symmetric in terms of initial quark and gluon distributions.

Implementing the single gluon exchange approximation into the factorization procedure and comparing the final result with Eq. (2), we obtain a formula for the unintegrated gluon density (DGLAP type)

$$x_g(x_g, k^2_\perp, Q^2) = \frac{1}{\pi} \frac{2 N_c}{4 \pi \alpha_s(Q^2)} k^2 \frac{d \sigma_{scat}}{d^2 k_\perp}. \quad (3)$$

This function vanishes at $k^2_\perp \to 0$ due to factor $k^2_\perp$ as well as at $k^2_\perp \to \infty$ due to factor $d \sigma_{scat}/d^2 k^2_\perp$. Hence, somewhere in between it must have a maximum, but it is unobvious whether it belongs to the hard or soft region, and whether the decrease immediately beyond the maximum follows exponential or power law $\sim 1/k^2_\perp$. The existing parameterizations favor the hard scenario.

The corresponding conventional gluon density obtained by $k^2_\perp$-integration of Eq. (3) is

$$x_g(x_g, Q^2) = \frac{1}{\pi} \frac{2 N_c}{4 \pi \alpha_s(Q^2)} \int_0^{Q^2} d^2 k^2_\perp \frac{d \sigma_{scat}}{d^2 k^2_\perp}. \quad (4)$$

This may be compared with DIS in the dipole picture [23], and with the approach of [24].

**3. MODIFICATIONS ARISING IN QFT**

In ordinary quantum mechanics, the differential cross-section of $qh$ scattering appearing in Eqs. [2] and [3] would assume a finite value in the high-energy limit. But it is now well-known that QFT brings (fortunately, mild) modifications to the impulse approximation and the parton model, for a number of reasons. First of all, even a static field created by an ensemble of point-like partons has Coulomb singularities, resulting in a logarithmic dependence of the transport or radiative cross-section on some hard scale. Secondly, multiple emission of soft quanta and particle pairs in the central rapidity region generates various double logarithmic asymptotics in the cross-section, which upon resummation to all orders may turn into power-law modifications [10]. We shall discuss these effects by turn as applied to our specific problem.

**$Q_\perp$ as the natural factorization scale for Compton mechanism**

At practice, transverse momenta of final hadrons produced within the underlying event are usually not counted, and correspondingly, $d \sigma/dk_\perp$ must be integrated over the unconstrained momentum components of the initial gluon(s) — i.e., over $k_\perp$. In so doing, it seems reasonable to neglect the $k^2_\perp$-dependence of the Compton subprocess cross-section $d \sigma/dt$ provided $k^2_\perp \ll p \cdot k$. That leads to

$$\frac{d \sigma}{dt} \approx \frac{16 \pi^2}{E^2 + p^2} \frac{E}{E'} \frac{d \sigma_{scat}}{d^2 k^2_\perp} \int d^2 k_\perp k^2_\perp \frac{d \sigma_{scat}}{d^2 k_\perp}. \quad (5)$$

However, at large $k^2_\perp$ the scattering differential cross-section has Rutherford asymptotics (cf., e.g., [17]):

$$\frac{d \sigma_{scat}}{d^2 k^2_\perp} \sim \frac{2 \alpha_s^2(k^2_\perp)}{k^4_\perp} \left[ \left( 1 - \frac{1}{N^2_c} \right) \frac{N_g + N_g}{2} + N_g \right] \quad (6)$$

(with $N_c = 3$ the number of colors, and $N_q, N_g$ the mean numbers of quarks, antiquarks and gluons in the proton), and therewith the $k^2_\perp$-integral in [5] appears to be logarithmically divergent at the upper limit. That means that at sufficiently large $k^2_\perp$ one still needs to rely on the decrease of $d \sigma/dt$ with $k^2_\perp$, providing the additional convergence factor. The sensitivity of $d \sigma/dt$ to $k^2_\perp$ arises at

$$k_{\perp, \text{max}} \sim \min\{M_V, Q_\perp\}, \quad (7)$$

which should be used as the upper limit in $k^2_\perp$ integral in Eq. (5) and serve as a natural factorization scale. It is also to be used as a factorization scale for the quark pdf in Eq. (1), if we wish at determination of $Q_\perp$ to be able to neglect the initial quark transverse momentum. In what follows, we will be mostly considering the case $Q_\perp < M_V$, whereby $k_{\perp, \text{max}} \sim Q_\perp$.

That differs from the case of DY mechanism, where even at small $Q_\perp$ the natural factorization scale is $M_V$ [15,14], and is in the spirit of factorization in direct photon and jet production [20].

**Distribution of the color sources**

With asymptotics (6) and upper limit (7), the $k^2_\perp$-integral in (5) with constant $N_q, N_g$ would give $\frac{Q^2}{\Lambda^2}$. But in fact, $N_q, N_g \neq \text{const}$, because they express as integrals from pdfs, which diverge at low $x$. If the asymptotics of $f(x')$ is $\sim 1/x'$, as is motivated by the perturbation theory

$$N_q = \int_{k^2_\perp/xs}^1 dx' f(x') \sim \ln \frac{xs}{k^2_\perp}, \quad (8)$$

where $xs$ is the quark-hadron collision subenergy. Substituting Eqs. (6) to (5), we get

$$\int \frac{d \ln^2 \frac{k^2_\perp}{k_\perp}}{d^2 k^2_\perp} \sim \int \frac{Q^2}{\Lambda^2} \ln \frac{xs}{k^2_\perp} \ln \frac{xs}{k^2_\perp} \sim \frac{\Lambda^2}{2} \ln^2 \frac{xs}{k^2_\perp} \frac{Q^2}{\Lambda^2} \ln \frac{xs}{\Lambda Q_\perp}. \quad (9)$$

This equation is similar to the (Sudakov) double logarithms for the reggeized gluon, if $Q^2/\Lambda^2$ stands for $|t|$. But $\ln \frac{Q^2}{\Lambda^2}$ may be absorbed into pdf definition.

In a more empirical approach, however, the divergence proceeds as a power law [22]

$$f(x') \sim \left( \frac{1}{x'} \right)^{\alpha_p}, \quad \alpha_p > 1, \quad (10)$$

For $Q_\perp$-integrated distributions, the factorization scale is usually taken to be $\sim M_V^2$, as well — see [19].
and the factorization scale must be taken \( \ll Q_\perp \) (cf. CGC approach \[23\]). The \( x' \)-integration then gives

\[
N_q \sim \left( \frac{x_s}{k_\perp^2} \right) \Delta, \quad \Delta = \alpha_p - 1 > 0, \tag{11}
\]

Therewith, the \( k_\perp \)-integral in Eq. \[5\] converges on the upper limit

\[
\int d^2k_\perp k_\perp^2 \frac{d\text{rescat}}{dk_\perp^2} \sim (xs)^\Delta \int_{\Delta^2}^{\infty} \frac{dk_\perp^2}{(k_\perp^2)^{\gamma_0}} \sim (xs)^\Delta, \tag{12}
\]

and the result is independent of the factorization scale, provided \( \Delta \) is. That must correspond to the BFKL-regime \[22\].

**Small-\( x \) behavior of the gluon distribution**

Since at present we can not reliably calculate the gluon distribution function \textit{ab initio}, it is to be inferred on phenomenological basis. Although gluon is not directly observable outside of \( h h \) collisions, in DIS at small \( x \) its density must be proportional to that of sea quark, which, in turn, is \( \propto F_2(x, \mu^2) \). At typical scales of hadroproduction at LHC (\( Q_\perp^2 \approx 100 \text{ GeV}^2 \) and \( x_g \approx 10^{-3} \)), the DIS data for the nucleon structure function are not available, but the data come close, and seemingly admit safe extrapolation. One must also duly incorporate the dependence on the factorization scale \( \mu^2 \), since at so small \( x \) it is significant.

To extrapolate both \( x \) and \( Q^2 \) dependences, one may utilize the observation \[25\] that at \( x < 0.01 \) the DIS \( \gamma^* p \) cross-section

\[
\sigma_{\gamma^*p} = \frac{4\pi^2 \alpha_{em}}{\mu^2} F_2(x, \mu^2) = \sigma_{\gamma^*p}(\tau) \tag{13}
\]

obeys “geometrical scaling”, reducing to a function of a single variable

\[
\tau = \frac{\mu^2}{\mu_0^2} \left( \frac{x_g}{x_0} \right)^\lambda, \quad \mu_0 = 1 \text{ GeV}, \tag{14}
\]

with the best-fit parameters \[25\]

\[
\lambda \approx 0.3, \quad x_0 \approx 3 \cdot 10^{-3}. \tag{15}
\]

Furthermore, in the domain \( \tau \gg 1 \), to which our parameters belong, the dependence on \( \tau \) is a simple power law in itself:

\[
F_2(x, \mu^2) = \frac{\mu^2}{4\pi^2 \alpha_{em}} 40 \mu b \tau^{-\Delta/\lambda} \tag{15a}
\]

\[
= 0.35 \left( \frac{\mu^2}{\mu_0^2} \right)^{1-\Delta/\lambda} \left( \frac{x_0}{x} \right)^\Delta. \tag{15b}
\]

Next we note that phenomenologically the exponent in Eq. \[15a\] \( \Delta/\lambda \approx 0.75 \), and so in Eq. \[15b\] \( 1 - \Delta/\lambda \approx 0.25 \approx \Delta \), i.e. exponents for \( \mu^2 \) and \( 1/x \)-dependencies are equal. Theoretically, there might be some difference between them in connection that integral \( \int d^2k_\perp \) diverges and demands a cutoff at \( \sim \mu^2 \). But if for simplicity we assume the equality of the exponents, and utilize the relation

\[
\mu^2/x = W^2 + \mu^2 \approx W^2, \tag{16}
\]

\[
\alpha_s(\mu^2)x_g(x_g, \mu^2) \propto F_2(x, \mu^2) \approx 0.07 \left( \frac{W^2}{\mu_0^2} \right)^{0.25}. \tag{17}
\]

Recalling the relation with the differential cross-section \[9\], equation \[17\] is quite natural from the viewpoint of \( t \)-channel Reggeization. Then, we obtain the same result in any treatment – through pdfs or through \( qh \) scattering.

From the viewpoint of DGLAP equations, however, the factorization and Reggeization sooner or later must break down, and saturation to set in. In that case, the slope of \( Q_\perp \)-dependence should vary with the energy.

**Reggeization in the \( Q_\perp \)-dependence of the boson hadroproduction differential cross-section**

In general, the onset of an energy-dependence of the quark-hadron scattering amplitude, along with the strong difference between initial and final quark energies in the hard subprocess may affect the balance of Compton process Feynman diagrams, and in principle violate the gauge invariance. Fortunately, at \( Q_\perp \ll M_V \), only one of the two Feynman diagrams dominates, wherein the final quark interacts with the encountered proton. Then, the \( qh \) collision subenergy is counted by the energy of the final quark.

To estimate it, note that

\[
x_g \sim M_V^2/xs, \tag{18}
\]

\[
p' \cdot P_2 = \frac{p' \cdot k}{x_g} = \frac{p' \cdot k}{p \cdot k} x_s = \frac{p' \cdot x_s}{p \cdot p'} \sim M_V^2 x_s. \tag{19}
\]

The non-trivial thing about subenergy \[19\] is the \( Q_\perp^4/\mu_0^2 = \rho^2 \) factor. It means that as a result of Reggeization, the differential cross-section multiplies by \( \rho^4 \Delta \). That factor may also be considered as being due to the factorization scale \( Q_\perp \) dependence of the gluon structure function. The rest of the \( Q_\perp^2 \)-dependence comes from the partonic Compton differential cross-section, which in the perturbative description (the sum of diagrams 2a and 2b) goes as \( \sim Q_\perp^2 \) for \( \Lambda \ll Q_\perp \ll M_V \) and as \( \sim Q_\perp^4 \) for \( Q_\perp \geq M_V \). Hence, in the fragmentation region of rapidities, and intermediate region of boson transverse momenta \( \Lambda \ll Q_\perp \ll M_V \) the boson hadroproduction differential cross-section should behave as

\[
\frac{d\sigma}{dQ_\perp^2} \sim Q_\perp^{2-\Delta} \sim Q_\perp^{-1/2}. \tag{20}
\]

In the central rapidity region, \( Q_\perp \)-dependence of the quark pdf comes into play, and Eq. \[20\] modifies to

\[
\frac{d\sigma}{dQ_\perp^2} \sim Q_\perp^{4-\Delta} \sim Q_\perp^0 \tag{21}
\]

However, one must keep in mind that in the central region there are other contributing mechanisms besides the Compton one.

\[5\]
Sudakov form-factors in the Compton subprocess

The above discussed power-law increase of sea and gluon pdfs at small $x$ physically owes to the opening possibility of particle production in the central rapidity region, with the phase space indefinitely increasing with the collision energy. Theoretically it is connected with gluon Reggeization and double logarithmic asymptotics. But double log effects also arise in the hard subprocess, since $M_2^2$ is a large scale, while $Q_2^2$ is a smaller subscale. Physically, at color exchange the particle intensely emit soft and collinear radiation quants, but those do not essentially change the emitting particle energy, only alter the transverse momentum. For large-angle scattering, this is inessential, but for small-angle it is. Resummation of real and virtual contributions does not compensate completely, which leads to transverse Sudakov of real and virtual contributions does not compensate completely, which leads to transverse Sudakov form-factors.

The Sudakov resummation for the Drell-Yan production mechanism at $Q_⊥ \ll M_V$ received a great deal of attention (see, e.g., [29] for an overview). But for the Compton mechanism the implementation of the developed techniques is hampered by the presence of 3 eikonal lines instead of 2 in DY, though all belonging to one plane.

The Sudakov formfactors also lead to a hardening of the $Q_⊥$-dependence. But basically it is just a redistribution, whereas Regge effects are pure enhancements. So far, Tevatron [27] and LHC [28] agreed well with the predictions of existing Monte-Carlo generators. Thus, in order to discern Regge effects in the $Q_⊥$-dependence and disentangle them from Sudakov effects, more studies are required.

4. CONCLUSIONS

In the present note we have argued that the Compton mechanism of EW boson hadroproduction, when properly isolated from the other mechanisms, may serve for determination of quark distributions in the proton in the valence domain. We have established the recipe for the pdf factorization scale ($Q_⊥^2$ instead of $M_V^2$), and formulated qualitative predictions for modification of $Q_⊥$-dependence of the boson hadroproduction cross-section due to the physical manifestation of the factorization scale, which can also be viewed as manifestation of Reggeization in the quark-hadron scattering.

In conclusion, let us point out the experimental objectives ensuing from our study. In order to measure the pdfs in the proton it might suffice to deal with boson rapidity distributions, but to control the underlying partonic dynamics, and actually to test the non-trivial predictions of QCD in hard but non-perturbative regime, one also needs measurement of the $Q_⊥$-dependence. With the accumulation of statistics at LHC, it is desirable to measure at ATLAS and CMS the double-differential (in rapidity and $Q_⊥$) cross-section of $Z$-boson production. In the central rapidity region and in the fragmentation region the $Q_⊥$-dependencies may differ. The same procedure is to be repeated for (absolute) cross-sections of $W^+$ and $W^-$ boson production, since it is the difference of those cross-sections which determines the valence quark distribution function in the proton. Although the reconstruction of $W$ boson transverse momentum is more complicated because of unregistered neutrinos, at the Tevatron it was managed [29], and must be feasible at LHC.

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