FIELD THEORY NEAR THE CRITICAL TEMPERATURE

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ABSTRACT
Field theory at nonvanishing temperature beyond perturbation theory is discussed for the $N$-component $O(N)$-symmetric scalar theory. We compute the effective potential directly in three dimensions using an exact evolution equation for an effective action with an infrared cutoff. A suitable truncation is solved numerically. We obtain a detailed quantitative picture of the scaling form of the equation of state in the vicinity of the critical temperature of a second order phase transition.

1. Introduction
The most prominent application of field theory at nonzero temperature in particle physics concerns the symmetry restoration in spontaneously broken gauge theories at high temperature. In the standard model of electroweak interactions gauge bosons and fermions acquire a mass due to a nonvanishing vacuum expectation value of the Higgs doublet. According to arguments first given by Kirzhnitz and Linde\fnote{Talk given at the International School of Subnuclear Physics, 34th Course: Effective Theories and Fundamental Interactions, Erice-Sicily, 3-12 July 1996.} this expectation value vanishes at sufficiently high temperature and the electroweak symmetry is restored. This phase transition\fnote{For large Higgs boson masses around $m_H \gtrsim 80$ GeV there might be no phase transition but rather an analytical crossover.\footnote{Talk given at the International School of Subnuclear Physics, 34th Course: Effective Theories and Fundamental Interactions, Erice-Sicily, 3-12 July 1996.}} is expected to have occurred in the early universe which may have important consequences like the possible creation of the excess of matter compared to antimatter (baryon asymmetry).\fnote{For large Higgs boson masses around $m_H \gtrsim 80$ GeV there might be no phase transition but rather an analytical crossover.\footnote{Talk given at the International School of Subnuclear Physics, 34th Course: Effective Theories and Fundamental Interactions, Erice-Sicily, 3-12 July 1996.}} Another challenge is the study of QCD at nonvanishing temperature which is also relevant for heavy ion collisions in the laboratory. At temperatures around a few hundred MeV one expects dramatic changes in the structure and symmetries of hadronic matter. Of course, there are other fields of application of nonzero temperature field theory as for instance condensed matter physics.

The prototype for investigations concerning the symmetry restoration at high temperature is the $N$-component scalar field theory with $O(N)$-symmetry. For $N = 4$ it describes the scalar sector of the electroweak standard model in the limit of vanishing gauge and Yukawa couplings. It is also used as an effective model for the chiral phase transition in QCD in the limit of two quark flavors. In condensed matter physics $N = 3$ corresponds to the well known Heisenberg model used to describe the ferromagnetic phase transition. There are other applications like the helium superfluid transition ($N = 2$), liquid-vapor transition ($N = 1$) or statistical properties of long polymer chains ($N = 0$).\fnote{For large Higgs boson masses around $m_H \gtrsim 80$ GeV there might be no phase transition but rather an analytical crossover.\footnote{Talk given at the International School of Subnuclear Physics, 34th Course: Effective Theories and Fundamental Interactions, Erice-Sicily, 3-12 July 1996.}
The equation of state (EOS) for a magnetic system is specified by the free energy density per volume (here denoted by $U$) as a function of arbitrary magnetization $\phi$ and temperature $T$. All thermodynamic quantities can be derived from the function $U(\phi, T)$. For example, the response of the system to a homogeneous magnetic field $H$ follows from $\partial U/\partial \phi = H$. This permits the computation of $\phi$ for arbitrary $H$ and $T$. There is a close analogy to quantum field theory at nonvanishing temperature. Here $U$ corresponds to the temperature dependent effective potential as a function of a scalar field $\phi$. For instance, in the $O(4)$-symmetric model for the chiral phase transition in two flavor QCD the meson field $\phi$ has four components. In this picture, quark masses are associated with the source $H \sim m_q$ and one is interested in the behavior during the phase transition (or crossover) for $H \neq 0$. The temperature and source dependent meson masses and zero momentum interactions are given by derivatives of $U$.

The applicability of the $O(N)$-symmetric scalar model to a wide class of very different physical systems in the vicinity of the critical temperature $T_{cr}$ at which the phase transition occurs is a manifestation of universality of critical phenomena. There exists a universal scaling form of the EOS in the vicinity of a second order phase transition which is not accessible to standard perturbation theory. The quantitative description of the scaling form of the EOS will be the main topic of this talk. To compute the effective potential $U$ which encodes the EOS we employ a nonperturbative method. It is based on an exact evolution equation for an effective action $\Gamma_k$ with an infrared cutoff $\sim k$. From $\Gamma_k$ the standard effective action $\Gamma$, i.e. the generating functional of $1PI$ Green functions, is recovered when the additional infrared cutoff $\sim k$ is removed. Though I will refrain from going into technical details, an introduction to the method and some results for the EOS will be presented.

2. Effective three dimensional behavior of high temperature field theory

Quantum field theory at nonzero temperature $T$ can be formulated in terms of an Euclidean functional integral where the ‘time’ dimension is compactified on a torus with radius $T^{-1}$. If the characteristic length scale of the considered physical problem is much larger than the inverse temperature the compactified dimension cannot be resolved (‘dimensional reduction’). One therefore observes an effective three dimensional behavior of the high temperature quantum field theory. If the phase transition is second order the correlation length becomes infinite and the effective three dimensional quantum field theory is dominated by classical statistics. In particular, the critical exponents which describe the singular behavior of various quantities near the phase transition are those of the corresponding classical statistical system. Analogous considerations are valid for sufficiently weak first order phase transitions.

The calculation of the effective potential $U(\phi, T)$ in the vicinity of the critical temperature of a second order phase transition is an old problem. One can prove through a general renormalization group analysis the Widom scaling form of the EOS

$$H = \phi^\delta f \left( (T - T_{cr})/\phi^{1/\beta} \right).$$
Only the limiting cases $\phi \to 0$ and $\phi \to \infty$ are quantitatively well described by critical exponents and amplitudes. The critical exponents $\beta$ and $\delta$ have been computed with high accuracy\textsuperscript{8,15} but the scaling function $f$ is more difficult to access. A particular difficulty for a direct computation in three dimensions arises from the existence of massless Goldstone modes in the phase with spontaneous symmetry breaking for models with continuous symmetry ($N > 1$). They introduce severe infrared divergencies\textsuperscript{16} within perturbative expansions. A related example for the breakdown of perturbation theory is the symmetric phase of the electroweak theory where the gauge bosons are massless. In addition, within a perturbative description one is restricted to small expansion parameters. For instance, in the three dimensional $O(N)$-symmetric model no small coupling characterizes the interactions at the phase transition. One therefore has to employ nonperturbative methods which resolve the infrared problem.

A standard method to cope with infrared singularities is the ‘$\epsilon$-expansion’ which is a double series expansion in powers of the coupling constant $g$ and $\epsilon = 4 - d$. It has been used to compute the scaling function $f$ in second order in $\epsilon$ (third order for $N = 1$)\textsuperscript{8}. To study the three dimensional theory one sets the expansion parameter $\epsilon = 1$. The validity of this expansion is not clear a priori since the expansion parameter is not small. A nonperturbative computational method is provided by lattice Monte-Carlo simulations\textsuperscript{19}. Nevertheless second order or weak first order phase transitions are very demanding due to a finite lattice size and an infinite or large correlation length. In particular, it is notoriously difficult to distinguish by these methods between a weak first order and a second order nature of the transition. The nonperturbative method which allows here an unambiguous answer and which we will consider in the following relies on the use of an ‘exact renormalization group equation’ (ERGE). ERGEs have been formulated in many different but formally equivalent ways\textsuperscript{13,17}. Though these equations are exact there is practically no chance to solve them without approximations. The main challenge is to find a suitable approximation scheme that describes all the relevant physics of the considered problem. It is this point where the particular formulation of an ERGE becomes important. Here we will use the effective average action method\textsuperscript{18} which allows nonperturbative calculations in a feasible way. Before turning to the method in section 4 we consider some of its results for the scaling EOS for $O(N)$-symmetric models.\textsuperscript{9} They have been obtained from an approximate numerical solution of the effective potential $U$ directly in three dimensions (cf. sect. 4).

3. Scaling equation of state

Eq. (I) establishes the scaling properties of the EOS. They have been explicitly verified by our numerical solution\textsuperscript{9}. In particular, the function $f$ depends on only one scaling variable $x$. In our conventions $x = -\delta \kappa_A/\phi^{1/\beta}$ where $\delta \kappa_A$ is proportional to the deviation from the critical temperature $\delta \kappa_A = A(T)(T_{cr} - T)$ with $A(T_{cr}) > 0$ and $\phi = \sqrt{\phi_a \phi^a}$ ($a = 1, \ldots, N$). If one plots the logarithm of $f$ as a function of the logarithm of its argument $x$ one can easily consider some limits and compare with known values of critical exponents and amplitudes. Figs. 1 and 2 show our results for $\log(f)$ and $\log(df/dx)$ as a function of $\log|\phi|$ for $N = 1$ and $N = 3$. Here $x > 0$
corresponds to temperatures above the critical temperature (symmetric phase) and $x < 0$ accordingly to $T < T_{cr}$ (phase with spontaneous symmetry breaking).

As an example we consider the limit $x \to \infty$. One observes that $\log(f)$ becomes a linear function of $\log(x)$ with constant slope $\gamma$. In this limit the universal function takes the form

$$\lim_{x \to \infty} f(x) = (C^+)^{-1}x^\gamma.$$  

The amplitude $C^+$ and the critical exponent $\gamma$ characterize the behavior of the 'unrenormalized' squared mass or inverse susceptibility

$$\bar{m}^2 = \chi^{-1} = \lim_{\phi \to 0} \left( \frac{\partial^2 U}{\partial \phi^2} = (C^+)^{-1}|\delta \kappa_\Lambda|^\gamma \phi^{\delta-1-\gamma/\beta} \right).$$

We have verified the scaling relation $\gamma/\beta = \delta - 1$ that connects $\gamma$ with the exponents $\beta$ and $\delta$ appearing in the Widom scaling form (1). For the singular behavior of the susceptibility for $\delta \kappa_\Lambda \to 0$ in the symmetric phase one finds $\chi = C^+|\delta \kappa_\Lambda|^{-\gamma}$ with $\gamma = 1.258(1.465)$ for $N = 1(3)$. In another limit, $x \to 0$, the scaling function becomes a constant, $f(0) = D$, and according to eq. (1) one finds $H = D\phi^\delta$ ($\delta = 4.75(4.78)$ for $N = 1(3)$).

The spontaneously broken phase is characterized by a nonzero value $\phi_0$ of the minimum of the effective potential $U$ with $H = (\partial U/\partial \phi)(\phi_0) = 0$. The appearance of spontaneous symmetry breaking below $T_{cr}$ implies that $f(x)$ has a zero $x = -B^{-1/\beta}$ and one observes a singularity of the logarithmic plot in fig. 2. In particular, according to eq. (1) the minimum behaves as $\phi_0 = B(\delta \kappa_\Lambda)^\beta$ ($\beta = 0.336(0.388)$ for $N = 1(3)$). The critical exponents like $\gamma$, $\beta$ and $\delta$ are universal whereas the scaling function $f$ is universal up to arbitrary normalizations of $x$ and $f(x)$. There are only two independent scales in the vicinity of the transition point which can be related to the deviation from the critical temperature and to the external source. All models in the

Figure 1: Logarithmic plot of the scaling function $f$ in the symmetric phase.
same universality class can be related by a multiplicative rescaling of $\phi$ and $\delta \kappa_\Lambda$ or $(T_{cr} - T)$. Accordingly there are only two independent amplitudes and exponents respectively.

Apart from the given examples we have extracted other quantities which characterize the asymptotic behavior for the sake of comparison. The critical exponents and amplitude ratios we have calculated typically deviate by a few per cent from the values obtained by other methods\textsuperscript{8,15}. We expect the error to be related to the size of the anomalous dimension $\eta \simeq 4\%$. For recent lattice results for the EOS and a comparison see ref. 19. A more detailed discussion of the EOS, including semi-analytical expressions for the scaling function $f(x)$ for $N = 1$ and $N = 3$ and an alternative parametrization of the EOS in terms of renormalized quantities, can be found in ref. 9.

4. Effective average action and exact evolution equation

In the following we turn to the method we have used to derive the above results. It relies on the effective average action\textsuperscript{18} $\Gamma_k$. For a theory described by a classical action $S$, the effective average action results from the integration of fluctuations with characteristic momenta larger than a given infrared cutoff $\sim k$. To be explicit we consider a $k$-dependent generating functional with $N$ real scalar fields $\chi_a$,

$$W_k[J] = \ln \int D\chi \exp \left( -S[\chi] - \Delta_k S[\chi] + \int d^d x J_a(x) \chi^a(x) \right)$$

where the k-dependence arises from the additional term

$$\Delta_k S[\chi] = \frac{1}{2} \int \frac{d^d q}{(2\pi)^d} R_k(q) \chi_a(-q) \chi^a(q).$$

Without this term $W_k$ becomes the usual generating functional for the connected Green functions. Here the infrared cutoff function $R_k$ is required to vanish for $k \to 0$.
and to diverge for \( k \to \infty \) and fixed \( q^2 \). This can be achieved, for example, by the choice \( R_k(q) = Z_k q^2 e^{-q^2/k^2} (1 - e^{-q^2/k^2})^{-1} \) where \( Z_k \) denotes an appropriate wave function renormalization constant which will be defined below. For fluctuations with small momenta \( q^2 \ll k^2 \) the cutoff \( R_k \simeq Z_k k^2 \) acts like an additional mass term and prevents their propagation. For \( q^2 \gg k^2 \) the infrared cutoff vanishes such that the functional integration of the high momentum modes is not disturbed. The expectation value of \( \chi \) in the presence of \( \Delta_k S[\chi] \) and \( J \) reads \( \phi^a \equiv \chi^a = \delta W_k[J]/\delta J_a \). We define the effective average action as

\[
\Gamma_k[\phi] = -W_k[J] + \int d^d x J_a(x) \phi^a(x) - \Delta_k S[\phi].
\]  

As a consequence, as the scale \( k \) is lowered \( \Gamma_k \) interpolates from the classical action \( S = \lim_{k \to \infty} \Gamma_k \) to the standard effective action \( \Gamma = \lim_{k \to 0} \Gamma_k \), i.e. the generating functional of 1PI Green functions.\(^{18}\) Lowering \( k \) results in a successive inclusion of fluctuations with momenta \( q^2 \gg k^2 \) and therefore permits to explore the theory on larger and larger length scales. One can view \( \Gamma_k \) as the effective action for averages of fields over a volume of size \( \sim k^{-d} \) and the approach is similar in spirit to the block spin action\(^{20,13}\) on the lattice. The interpolation property of \( \Gamma_k \) can be used to 'start' at some high momentum scale \( \Lambda \) where \( \Gamma_\Lambda \) can be taken as the classical or short distance action and to solve the theory by following \( \Gamma_k \) to \( k \to 0 \). The scale dependence of \( \Gamma_k \) can be described by an exact evolution equation\(^{18}\)

\[
\frac{\partial}{\partial t} \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \left\{ \left( \Gamma^{(2)}_k[\phi] + R_k \right)^{-1} \frac{\partial R_k}{\partial t} \right\}
\]  

where \( t = \ln(k/\Lambda) \). The evolution is described in terms of the exact inverse propagator \( \Gamma^{(2)}_k(q, q') = \delta^2 \Gamma_k/\delta \phi_a(-q) \delta \phi^b(q') \). The trace involves a momentum integration as well as a summation over internal indices. The additional cutoff function \( R_k \) with a form like the one given above renders the momentum integration both infrared (IR) and ultraviolet (UV) finite. In particular, the direct implementation of the additional mass term \( R_k \simeq Z_k k^2 \) for \( q^2 \ll k^2 \) into the inverse average propagator makes the formulation suitable for dealing with theories which are plagued by infrared problems in perturbation theory.

Though the evolution equation for the effective average action is exact it remains a complicated functional differential equation. In practice one has to find a truncation for \( \Gamma_k \) and obtains approximate solutions. Our ansatz represents the lowest order in a systematic derivative expansion of \( \Gamma_k \),

\[
\Gamma_k = \int d^d x \left\{ U_k(\rho) + Z_k \partial_\mu \phi^a \partial^\mu \phi^a \right\}, \quad a = 1, \ldots, N.
\]  

Here \( \phi^a \) denotes the \( N \)-component real scalar field and \( \rho = \frac{1}{2} \phi^a \phi_a \). We keep for the potential term the most general \( O(N) \)-symmetric form \( U_k(\rho) \) since \( U(\rho) = \lim_{k \to 0} U_k(\rho) \) encodes the equation of state. The wavefunction renormalization is approximated by one \( k \)-dependent parameter \( Z_k \). The first correction to this ansatz would include
field dependent wave function renormalizations $Z_k(\rho)$ plus functions not specified in
eq. (8) which account for a different index structure of invariants with two derivatives
for $N > 1$. The next level involves invariants with four derivatives and so on. Concerning
the equation of state the truncation of the higher derivative terms typically generates
an uncertainty of the order of the anomalous dimension $\eta$. For $N = 1$ the weak $\rho$-dependence of $Z_k$ has been established explicitly at the critical
temperature. We define $Z_k$ at the minimum $\rho_0$ of $U_k$ and at vanishing momenta $q^2$,
i.e. $Z_k = Z_k(\rho = \rho_0; q^2 = 0)$. The $k$-dependence of this function is determined by
the anomalous dimension $\eta(k) = -d(\ln Z_k)/dt$.

If the ansatz (8) is inserted into the evolution equation for the effective average
action (7) one obtains flow equations for the effective average potential $U_k(\rho)$ and
for the wave function renormalization constant $Z_k$ (or equivalently the anomalous
dimension $\eta$). These have to be integrated starting from some short distance scale $\Lambda$
and one has to specify $U_\Lambda$ and $Z_\Lambda$ as initial conditions. The short distance potential
is taken to be a quartic potential

$$U_\Lambda(\rho) = -\bar{\mu}_\Lambda^2 \rho + \frac{1}{2} \bar{\lambda}_\Lambda \rho^2$$  \hspace{1cm} (9)

and $Z_\Lambda = 1$. For a study of the behavior in the vicinity of the phase transition, it is
convenient to work with dimensionless renormalized fields $\bar{\rho} = Z_k k^{2-d} \rho$ and we also
switch to a dimensionless potential $u_k = k^{-d} U_k$. The evolution equation for $u_k(\bar{\rho})$
reads\textsuperscript{12,18,21} (primes denote derivatives with respect to $\bar{\rho}$ at fixed $t$)

$$\frac{\partial u_k}{\partial t} = -du_k + (d - 2 + \eta)\bar{\rho}u'_k + 2v_d(N - 1)l_0^d(u'_k; \eta) + 2v_d l_0^d(u'_k + 2\bar{\rho}u''_k; \eta)$$  \hspace{1cm} (10)

where $v_d^{-1} = 2^{d+1} \pi^d/2 \Gamma(d/2)$ with $v_3 = 1/8 \pi^2$. The anomalous dimension $\eta$ is given
in our truncation by\textsuperscript{12,18}

$$\eta(k) = \frac{16v_d}{d} \kappa(u''_k(\kappa))^2 m_{2,2}^d(2\kappa u''_k(\kappa); \eta)$$  \hspace{1cm} (11)

with $\kappa$ the location of the minimum of the potential, $u'_k(\kappa) = 0$. The functions $l_0^d$
and $m_{2,2}^d$ result from the momentum integration in the flow equation (10) for $\Gamma_k$. They
equal constants of order one for vanishing arguments and decay fast for arguments
much larger than one.\textsuperscript{12,22} As a consequence these functions account for the decoupling
of modes with masses much larger than the scale $k$. For the minimum of the dimensionless potential at $\kappa \neq 0$ one easily recognizes in eq. (10) the contribution due
to the $(N - 1)$ massless Goldstone bosons with dimensionless mass term $u'_k(\kappa) = 0$
and the radial mode with dimensionless mass term $2\kappa u''_k(\kappa)$.

At a second order phase transition the correlation length diverges and accordingly
there is no mass scale present in the theory. In particular, one expects a scaling
behavior of the effective average potential. Exactly at a second order phase transition $u_k$
should be given by a $k$-independent (scaling) solution $\partial u_k/\partial t = 0$. The EOS
involves the potential away from the critical temperature. Its computation therefore
Figure 3: The evolution of $u_k'(\tilde{\rho})$ as $k$ is lowered from $\Lambda$ to zero.

requires the solution of the full partial differential equation (10) for the dependence of $u_k$ on the two variables $\tilde{\rho}$ and $k$. Suitable numerical methods for its solution are discussed in ref. 22.

In the remaining part of this section we consider the theory in three dimensions ($d = 3$) with the classical potential (9) as initial condition. For given $\bar{\lambda}_\Lambda/\Lambda$ there is a critical value for the minimum $\kappa_\Lambda = \bar{\mu}_\Lambda^2/(\bar{\lambda}_\Lambda \Lambda) = \kappa_{cr}$ of the classical potential for which the scaling solution is approached in the limit $k \to 0$. In particular, the minimum $\kappa$ and the anomalous dimension $\eta$ take on constant (fixed point) values $\kappa(k) = \kappa_\star$ and $\eta(k) = \eta_\star$. The order of the phase transition becomes apparent from the observation that the order parameter $\rho_0 \sim \lim_{k \to 0} k^{1+\eta_\star} \kappa_\star = 0$ vanishes at the phase transition.

To demonstrate the behavior of the scale dependent effective potential near the phase transition we consider the shape of $u_k'(\tilde{\rho})$ for different values of the scale. In fig. 3 the numerical solution of the function $u_k'(\tilde{\rho})$ is plotted for $d = 3$ and $N = 1$ and various values of $t = \ln(k/\Lambda)$. The evolution starts at $k = \Lambda$ ($t = 0$) where no integration of fluctuations has been performed. We arbitrarily choose $\bar{\lambda}_\Lambda/\Lambda = 0.1$ and fine tune $\kappa_\Lambda \approx \kappa_{cr}$ so that the scaling solution is approached at later stages of the evolution. If $\kappa_\Lambda$ is interpreted as a function of temperature the small deviation $\delta\kappa_\Lambda = \kappa_\Lambda - \kappa_{cr}$ is proportional to the deviation from the critical temperature. For the example in fig. 3 a value slightly above the critical temperature is used ($\delta\kappa_\Lambda < 0$). As $k$ is lowered $u_k'(\tilde{\rho})$ deviates from the initial linear shape of $u_\Lambda'(\tilde{\rho})$. By staying near the scaling solution for several orders of magnitude in $k$, the system looses memory of the initial conditions at the short distance scale $\Lambda$. As a result, after $u_k'(\tilde{\rho})$ has evolved away from the scaling solution, its shape is independent of the choice of $\bar{\lambda}_\Lambda/\Lambda$ for the classical theory. This property gives rise to the universal behavior near
second order phase transitions discussed in section 3. Eventually, for the final part of the evolution as $k \to 0$, the theory settles down either in the symmetric phase ($\delta \kappa_\Lambda < 0$) as is demonstrated in fig. 3 with $\kappa = 0$ and positive constant mass term $m^2 = \lim_{k \to 0} k^2 u_k'(0)$, or $\kappa$ grows in such a way that $\rho_0 = \lim_{k \to 0} Z_k^{-1} k \kappa$ approaches a constant value indicating spontaneous symmetry breaking ($\delta \kappa_\Lambda > 0$). The scaling EOS obtains from $U'(\rho) = \lim_{k \to 0} Z_k k^2 u_k'(\tilde{\rho})$ with $H/\phi_\delta = U'/\phi_\delta^{-1} = f$ as a function of $x = -\delta \kappa_\Lambda/\phi^{1/\beta} (\phi = \sqrt{2}\tilde{\rho})$.

5. Conclusions and outlook

We have arrived at a detailed quantitative picture of the equation of state in the vicinity of the critical temperature of a second order phase transition. The method we have used is free of IR or UV divergencies and provides a practical tool for calculations not accessible to perturbation theory. It relies on the effective average action $\Gamma_k$ which interpolates between the classical action $S (k = \Lambda)$ and the effective action $\Gamma (k = 0)$. Its scale dependence is described by an exact nonperturbative evolution equation.

Though not discussed in the present talk, the method can be applied as well to a description of first order phase transitions along the lines presented above. In particular, the scaling equation of state for weak (fluctuation induced) first order phase transitions in scalar matrix models has been obtained. Work in progress focuses on the equation of state for an effective quark-meson model at nonzero temperature, relevant for the study of the chiral phase transition in QCD.

The average action approach allows a well-defined description of a rich spectrum of critical phenomena like the observation of critical exponents, tricritical points and crossover behavior. Further applications at nonzero temperature in various levels of truncations include studies of the crossover behavior in two-scalar theories or the description of the Kosterlitz-Thouless phase transition in two space dimensions. Extensions to gauge theories have been e.g. applied to the abelian Higgs model and to the electroweak phase transition at nonzero temperature. As should be pointed out the method is not restricted to a discussion of the universal behavior of the theory which can be observed near or at second order or sufficiently weak first order phase transitions. It also permits the description of nonuniversal physics such as the behavior of the theory away from the critical temperature for the four dimensional theory at nonvanishing temperature.

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