Randall-Sundrum model with $\lambda < 0$ and bulk brane viscosity

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We study the effect of the inclusion of bulk brane viscosity on brane world (BW) cosmology in the framework of the Eckart’s theory, we focus in the Randall-Sundrum model with negative tension on the brane.

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I. INTRODUCTION

Extra dimensions theories have a long history, from the original work of Kaluza-Klein$^1$ to modern ideas of string theory$^2,3$. In particular the Randall-Sundrum scenario has got great attention in the last decade$^4,5$. From the cosmological point of view, brane world offers a novel approach to our understanding of the evolution of the universe. The most spectacular consequence of this scenario is the modification of the Friedmann equation. In these models, for instance in five dimensions, matter is confined to a four dimensional brane, while gravity can be propagated in the bulk, and can feel the extra dimension. From the perspective of string theory$^6$, brane world cosmology has been a big challenge for modern cosmology. For a review on BW cosmology see Ref.$^7$. For example, consequences of a chaotic inflationary universe scenario in a BW model was described$^8$, where it was found that the slow-roll approximation is enhanced by the modification of the Friedmann equation. Many works in these theories has been made considering a perfect fluid, and inclusion of imperfect fluid has been less considered in comparison with the former case, see for example Refs.$^9,10,11,12,13$. In this work we study the effect of bulk viscous brane in Randall-Sundrum model with negative tension ($\lambda < 0$) in a modified Friedmann-Lemaitre-Robertson-Walker (FLRW) model over the brane. Effects of the imperfect fluids in the brane world cosmology produce many differences with respect to standard cosmology scenarios.

The plan of the paper is as follows: In Sec. II we specify the effective four dimensional cosmological equations from a five dimensional Anti de Sitter brane world model. We write down the cosmological equations for an imperfect fluid. Also, we discussed the condition and consequences of thermodynamics equilibrium. Finally, we conclude in Sec. III.

II. BULK VISCOSITY AND THE ECKART’S THEORY IN RANDALL-SUNDRUM COSMOLOGICAL SCENARIO

One of the most spectacular consequences of the cosmological brane world scenario is the modification of the Friedmann equation, that has allowed to study some cosmological puzzles from a new perspective. The particular case of a five dimensional model and matter confined to a four dimensional Brane, has been acquiring great attention in the last time, for a review see Ref.$^14$ and references therein. We are going to consider an homogeneous and isotropic 4-brane described by the FLRW metric, in the case of flat universe where the gravitational sector of the field equations is described by a modified Friedmann equation given by

$$3H^2 = \rho \left( 1 \pm \frac{\rho}{2\lambda} \right),$$

where was considered that $8\pi G = 1$, the positive and negative signs are related to positive and negative brane tension.

The matter content sector satisfy

$$\dot{\rho} + 3H (\rho + p) = 0.$$  \hspace{1cm} (2)

These equations show that just the gravitational field can escape from the brane and propagate through the bulk, and that matter is confined on the brane. In order to study the effect of a viscous fluid on the brane, we include a viscous pressure on the continuity equation

$$\dot{\rho} + 3H (\rho + p + \Pi) = 0,$$  \hspace{1cm} (3)

where $\Pi$ represents viscous pressure over the brane. To show some virtues of this model, we first review the case with $\Pi = 0$, $p = \omega \rho$ and $\omega < -1$ that was studied by Srivastava in Ref.$^15$. From Eqs.$^1$ and $^2$, it is possible to obtain
an exact solution for the matter contents
\[ \rho(t) = \left\{ \mp \frac{1}{2\lambda} + \left[ \frac{1}{\rho_0} \pm \frac{1}{2\lambda} + \sqrt{\frac{3}{4} (1 + \omega) (t - t_0)} \right]^2 \right\}^{-1}. \]

In Ref. [15] the case of brane world model with \( \lambda > 0 \) was discussed, that describes an accelerated phantom universe that ends in a big-rip singularity in the time
\[ t_s = t_0 + \sqrt{\frac{4}{3(1 + \omega)}} \left[ \frac{1}{2\lambda} - \sqrt{\frac{1}{\rho_0} + \frac{1}{2\lambda}} \right]. \]

For a brane world model with \( \lambda < 0 \), a phantom universe that begins with an accelerated phase and then evolves towards a decelerated phase was found.

For our model, when \( \Pi \neq 0 \), it is useful to define the adimensional variable \( x = \rho/2\lambda \) that allow to rewrite Eq. (1) in the standard form
\[ 3H^2 = \rho_{\text{eff}}, \]

where the effective density is given by
\[ \rho_{\text{eff}} = 2\lambda x (1 \pm x). \]

Taking the derivative of Eq. (4) respect to cosmological time and using Eq. (3), we obtain the dynamical equation
\[ \dot{H} = -\frac{1}{2} (\rho_{\text{eff}} + p_{\text{eff}} + \Pi_{\text{eff}}), \]

where the effective pressures are given by
\[ p_{\text{eff}} = 2\lambda [\omega x (1 \pm 2x) \pm x^2], \]
\[ \Pi_{\text{eff}} = \Pi (1 \pm 2x), \]

and the effective state equation is given by
\[ \omega_{\text{eff}} = \frac{p_{\text{eff}} + \Pi_{\text{eff}}}{\rho_{\text{eff}}}, \]

where
\[ \omega_{\text{eff}}(x) = \frac{1}{1 \pm x} \left[ (1 \pm 2x) \left( \omega + \frac{\Pi}{2\lambda x} \right) \pm x \right]. \]

This equation has the Friedmann limit when \( x << 1 \) and standard cosmology is recovered. It is straightforward to see in this case that the effective barotropic index is given by \( \omega_{\text{eff}} \to \omega + \frac{\Pi}{2\lambda x} \), i.e., one recovers the standard four dimensional general relativity with a bulk viscosity. On the other hand, the strong limit \( x >> 1 \) is given by \( \omega_{\text{eff}} \to 2(\omega + \frac{\Pi}{2\lambda x}) + 1 \), here the quadratic term in the energy density dominates over other terms giving rise to a new kind of behavior for the Friedmann equation. Acceleration or deceleration phases without viscosity has been discussed in the literature [13, 14], and for the particular case of \( \lambda < 0 \) it is known as the bouncing brane [17, 18].

Assuming that our cosmological fluid is viscous and to characterize this novel behavior on the evolution in the universe, we are going to study our model in the context of non-causal thermodynamics that is known as Eckart Theory [19]. Although this model present some causality warning, it is the simplest alternative and has been widely considered in cosmology, see for example Refs. [20, 21, 22] and references therein. Because we assumed spherical symmetry, the shear viscosity does not play any role, and only the bulk viscosity has to be considered,
\[ \Pi(\rho) = -3\xi(\rho)H. \]

From a thermodynamical point of view, in conventional physics \( \xi \) needs to be positive; this is a consequence of the positive entropy change in irreversible processes, For \( \xi(\rho) \) we consider
\[ \xi(\rho) = \xi_0 \rho^n, \]
and the effective state equation (13) reads

$$\omega_{\text{eff}}(x, s) = \frac{1}{1 \pm x} \left[ (1 \pm 2x) \left( \omega - \sqrt{3\xi_0} (2\lambda)^{s-1/2} x^{s-1} \sqrt{x(1 \pm x)} \right) \pm x \right].$$  \hfill (13)

For $\lambda < 0$, it is well known that the universe exhibit bouncing and turnaround phases. These phases arise from the modifications to the Friedmann equation. A simple analysis of Eqs. (12)-(8) and Eq. (13) with a negative sign, allows to characterize the behavior of the brane universe filled with a viscous fluid. For example, if we take $s = 1/2$ it is straightforward to find a novel behavior of $\Pi_{\text{eff}}(x)$ from Eq. (13), note that in the case $s \neq 1/2$ the same behavior is obtained. In the interval $0 < x < 1/2$, $\Pi_{\text{eff}}(x) < 0$ admits a genuine interpretation in term of a bulk viscosity and therefore can be considerate as a source for entropy generation. On the other hand, when $1/2 < x < 1$ it is a quite problematic to give a conventional physics interpretation of $\Pi_{\text{eff}}(x) > 0$, this point will be discussed elsewhere. Moreover, the effective description shows the crossing of phantom divide $\omega_{\text{eff}} < -1$, this region correspond to a new phantom bounce, due to viscosity effects. In order to check the thermodynamic equilibrium, we study the if the limit $\Pi_{\text{eff}}/p_{\text{eff}} \ll 1$ is satisfied, here

$$\frac{\Pi_{\text{eff}}}{p_{\text{eff}}} = \sqrt{3\xi_0} \left| \frac{\sqrt{1-x(1-2x)}}{\omega(1-2x) - x} \right|.$$  \hfill (14)

in the interval $0 < x < 1/2$ where $\Pi_{\text{eff}}$ is a genuine viscosity. We note that Eq. (14) satisfies the limit $\Pi_{\text{eff}}/p_{\text{eff}} \ll 1$ if $-1 < \omega < -1/3$, therefore Eq. (14) satisfies the condition for thermodynamic equilibrium. When $\omega$ is outside this interval it is not possible to satisfy this condition.

For a model with $\lambda > 0$, it is possible to extend the previous analysis and to obtain an exact solution. First, we rewrite the continuity equation (2) in terms of the adimensional variable $x$

$$\dot{x} + (1 + \omega) x \sqrt{6\lambda x (1+x)} = 3\xi_0 (2\lambda)^s x^{s+1} (1 + x).$$  \hfill (15)

At early time in the evolution of the universe, where the viscosity ($\Pi$) could be relevant, we consider the quadratic correction as the dominant term in the Friedmann Eq. (1) [16]. Thus, Eq. (15) can be integrated as follow

$$t(x) = \frac{1}{B} \left( \frac{1}{x} + \int dx \frac{x^{s-2}}{x^s - B/A} \right).$$  \hfill (16)

where the constants $A$ and $B$ are given by

$$A = 3\xi_0 (2\lambda)^s \quad \text{and} \quad B = \sqrt{6\lambda(1 + \omega)}. \hfill (17)$$

An explicit solution of Eq. (16) is given by

$$t(H) = -\frac{1}{3(1 + \omega)} H^{-1} \frac{1}{s} \left( \frac{3^{(s+1)/2} \xi_0 (2\lambda)^{(s-1)/2}}{(1 + \omega)} H^s, 1, -\frac{1}{s}, \right),$$  \hfill (18)

where the Lerch transcendent $\Phi(z, s, a)$ function is defined by,

$$\Phi(z, s, a) = \sum_{n=0}^{\infty} \frac{z^n}{(n+a)^s}.$$  \hfill (19)

$\Pi_{\text{eff}} < 0$ in the interval allowed for $x$ and it is straightforward to verify that the condition for the thermodynamic equilibrium is not satisfied. A complete qualitative description for model with $\lambda > 0$ in the context of causal thermodynamics can be found in Ref. [13]. We want to finish this section with a comparison of our formalism with the structure that was obtained in the context of loop quantum cosmology with $k = 0$ [23] the modified Friedmann equation can be written as,

$$3H^2 = \rho \left( 1 - \frac{\rho}{\rho_c} \right),$$  \hfill (20)

where $\rho_c$ is the critical energy density set by quantum gravity. This scheme including viscosity admits the same analysis and shows some results quite similar to brane world cosmology in the presence of viscosity for $\lambda < 0$. 

III. DISCUSSION

In this work we have studied the effect of bulk viscous brane in Randall-Sundrum model. For non-causal thermodynamics, in the context the Randall-Sundrum model with $\lambda < 0$, we have considered the viscous pressure $\Pi(\rho) = -3\xi_0^s \rho s H$, and we have shown that in the interval $0 < x < 1/2$, $\Pi_{\text{eff}}(x) < 0$ admits a genuine interpretation in terms of bulk viscosity. Because $\Pi_{\text{eff}}(x) < 0$ we can interpret $\Pi_{\text{eff}}(x)$ as a source for entropy generation. Besides, the effective description shows the possibility to produced crossing of phantom divide $\omega_{\text{eff}} < -1$, this region corresponds to a new phantom bounce due to viscosity effects. Through of the study of the ratio $\Pi_{\text{eff}}/p_{\text{eff}}$, in order to check the thermodynamics equilibrium, we have shown that it is possible to satisfy $\Pi_{\text{eff}}/p_{\text{eff}} << 1$. On the other hand, when $1/2 < x < 1$, it is quite delicate to give a interpretation for $\Pi_{\text{eff}}(x) > 0$ in terms of conventional physics. We hope to clarify this point in the near future. For a Randall-Sundrum model with $\lambda > 0$, we had extended the previous analysis and found an implicit exact solution for the Hubble parameter. We also noted that $\Pi_{\text{eff}} < 0$ in all interval allowed for the variable $x$ and the condition of the thermodynamic equilibrium is not satisfied.

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