Spin-states in multiphoton pair production for circularly polarized light

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Scalar and fermionic particle pair production in rotating electric fields is investigated in the nonperturbative multiphoton regime. Angular momentum distribution functions in above-threshold pair production processes are calculated numerically within quantum kinetic theory and discussed on the basis of a photon absorption model. The particle spectra can be understood if the spin states of the particle-antiparticle pair are taken into account.

Introduction.– Photoelectron angular distributions (PAD) are well known in chemistry and atom physics, where the focus is on studying ionization spectra and understanding the inner structure of molecules and atoms [1]. Viewing multiphoton pair production as the highly relativistic analogue of the ionization of hydrogen, we can expand the concept of angular momentum transfer via photon absorption to the strong-field QED regime.

In theory, particle production is similar to atomic ionization [2], e.g., above-threshold effects in the multiphoton regime [3]. Specifically, the particle under consideration absorbs more photons than necessary in order to transit to a continuous state, which manifests in a series of peaks in the momentum spectrum. In both scenarios, the peak positions are determined by the number of absorbed photons and the field-dependent threshold [4–7]. A partial wave analysis, however, has been performed only in atomic physics, where a model for angular momentum transfer was introduced, see Refs. [1] [8] for reviews and further information on this topic as well as Ref. [9] for a recent experimental verification.

With regards to strong-field matter creation, the focus has been on understanding the different mechanism to create particles in the first place, see Refs. [10] [11] [12] and [13]. Furthermore, in contrast to atomic ionization there has been only one successful experiment carried out so far [14]. However, as laser development progresses the strong-field regime becomes more accessible, thus enabling studies on particle creation processes on a regular basis, see Refs. [15] [16] for detailed information on planned projects.

In order to study multiphoton pair production in electric fields we compute distribution functions using a phase-space formalism [17] [18]. To be more specific, we work with quantum kinetic theory (QKT) [19]. One big advantage of QKT is, that it is easy to obtain results for fermionic as well as bosonic pair production, since both are described by a set of time-dependent ordinary differential equations. Additionally, the framework allows to freely choose the polarization of the laser beams [20–22]. On top of that, we obtain direct access to the particle’s momentum spectrum [19]. Such spectra are then discussed on the basis of a photon absorption model taking into account energy and angular momentum conservation.

In this article we only consider simple, time-dependent, rotating electric fields [21] [22]. Such configurations provide the perfect setting for testing our absorption model. In particular, the orientation of the photon spin is fixed making selection rules much easier to apply.

Throughout this paper we use natural units $\hbar = c = 1$ and measure all dimensionful quantities in terms of the mass of the particles $m$.

Electric field pulse.– In order to study pair production for circularly polarized light, we have chosen the following model for the vector potential

$$\mathbf{A}(t) = -\frac{\varepsilon E_{cr}}{2\omega} \exp \left(-\frac{t^2}{\tau^2}\right) \begin{pmatrix} \sin(\omega t) \\ \cos(\omega t) \end{pmatrix},$$

with the critical field strength $E_{cr} = m^2/e$, the peak field strength $\varepsilon$, the pulse length $\tau$ and the photon energy $\omega$. The electric field is derived from this expression.

We have chosen the model to mimic a standing wave formed by two counter-propagating laser beams with propagation direction $\pm \mathbf{z}$. To form a standing wave pattern as given by eq. the pulse has to be left-handed and the other must be right-handed. As photons are spin-1 bosons we assume without loss of generality the helicity to be quantum. Hence, every photon not only transfers an energy quant of $\omega$ to the particle-antiparticle pair, but also increases its total angular momentum by one.

Quantum Kinetic Theory.– All numerical results are based upon a phase-space approach belonging to a particular class of quantum kinetic theories. As we perform calculations for spatially homogeneous fields the governing equations of motion take on a numerically favorable form. As a result, we are able to obtain very accurate numerical solutions allowing us to compare the computed data with predictions from the model.
In the Weyl gauge, the quantum kinetic equations for spin-1/2 particles can be written as \[ \partial_t \mathbf{s}^v - 2 \mathbf{p} \cdot \mathbf{t}_1^v = -2 \frac{Q(t)}{\omega(t)}, \] (2)
\[ \partial_t \mathbf{v}^v + 2 \mathbf{p} \times \mathbf{a}^v + 2 \mathbf{t}_1^v = 2 \frac{e \mathbf{E}(t) - Q(t) \mathbf{p}(t)}{\omega(t)}, \] (3)
\[ \partial_t \mathbf{a}^v + 2 \mathbf{p} \times \mathbf{v}^v = 0, \] (4)
\[ \partial_t \mathbf{t}_1^v + 2 \mathbf{p} \cdot \mathbf{s}^v - 2 \mathbf{v}^v = 0 \] (5)
with initial conditions
\[ \mathbf{s}^i_0 = 0, \quad \mathbf{v}^i_0 = \mathbf{a}^i_0 = \mathbf{t}_1^i_0 = 0. \] (6)

Following Refs. \[17\] \[18\] we can associate the individual terms in the differential equation as mass density \( \mathbf{s}^v \), current density \( \mathbf{v}^v \), spin density \( \mathbf{a}^v \) and magnetic moment density \( \mathbf{t}_1^v \). We have used abbreviations for the one-particle energy \( \omega(t) = \sqrt{1 + \mathbf{p}(t)^2} \), the kinetic momentum \( \mathbf{p}(t) = q - e \mathbf{A}(t) \) and the auxiliary variable \( Q(t) = \frac{e \mathbf{E}(t) \cdot \mathbf{p}(t)}{\omega(t)^2} \).

The equations of motion for scalar particles, on the other hand, are given by \[18\]
\[ \partial_t f^v - \mathbf{p}(t)^2 (g^v + h^v) - 2g^v = 2 \frac{\mathbf{p}(t)^2}{\omega(t)}, \] (7)
\[ \partial_t g^v + \mathbf{p}(t)^2 f^v + 2f^v = -\frac{1}{2} \frac{(1 + \mathbf{t}_1^v)^2}{\omega(t)} Q(t), \] (8)
\[ \partial_t h^v - \mathbf{p}(t)^2 f^v = -\frac{1}{2} \frac{\mathbf{p}(t)^2 Q(t)}{\omega(t)}. \] (9)

Again, the initial conditions have been incorporated into the equations of motion, thus
\[ f^v_0 = g^v_0 = h^v_0 = 0. \] (10)
From Refs. \[18\] we know to identify \( \mathbf{f}^v = \mathbf{p}(t) (g^v + h^v) \) as current density and \( h^v \) as mass density. The term \( f^v \) does not seem to have a direct physical interpretation.

The time integration was performed using a Dormand-Prince Runge-Kutta integrator of order 8(5,3) \[23\].

Throughout this article we will discuss the results on the basis of the spin-dependent particle number density \( f_\sq \), which is evaluated at asymptotic times. Hereby, the electron-positron distribution function is given by
\[ f_{1/2}(p_x, p_z) = \frac{\mathbf{s}^v + \mathbf{p} \cdot \mathbf{v}^v}{\omega} \bigg|_{t \to \infty} \] (11)
and the particle number density for spinless particles is given by
\[ f_0(p_x, p_z) = \frac{2h^v + \mathbf{p} \cdot \mathbf{v}^v}{\omega} \bigg|_{t \to \infty}. \] (12)

In the parameter region of interest the particle momentum spectrum at asymptotic times (vanishing vector potential) is axially symmetric in \((p_x, p_y)\). For the sake of simplicity, we therefore have \( p_y = 0 \) and refer to \( p_x \) as in-plane or parallel momentum. In this regard, \( p_z \) defines the perpendicular momentum.

**Photoparticle angular distribution.** In order to enable multiphoton particle production the total photon energy \( \hbar \omega \) has to exceed not only the particles’ rest mass but also their oscillatory energy in the background field \( 2m_\ast \). Here, \( m_\ast \) describes the particle’s effective mass in a homogeneous, oscillating background field
\[ m_\ast = m \sqrt{1 + \xi^2} \approx m \sqrt{1 + \frac{\xi^2 m^2}{4 \omega^2}}, \] (13)
where \( \xi = \frac{\hbar}{m} \sqrt{-\langle A_\mu A_\mu \rangle} \). Moreover, energy conservation laws dictate the particle’s kinetic energy and as a consequence its momentum reads \[21\]
\[ \mathbf{p}^2 = \left( \frac{n_\omega}{2} \right)^2 - m_\ast^2. \] (14)

From eq. \[14\] we can already deduce that particle distributions form shells in phase space, which can be classified by the number of photons absorbed \[23\]. The effective mass model, however, cannot explain why each shell falls off differently.

For the case under consideration it is useful to perform the partial wave analysis for the particle distribution and not the quantum amplitudes. Within the absorption model, given a specific shell \( N \) (n-photon absorption), the particle’s angular distribution is given by \[24\]
\[ I(\theta, \varphi) \propto \sum_{L=0}^{2N} \sum_{M=-L}^{L} B_{LM} Y_{LM}(\theta, \varphi), \] (15)
with the coefficients \( B_{LM} \), the spherical harmonics \( Y_{LM} \) and the angular momentum quantum numbers \( L \) and \( M \). As our focus in this manuscript is on circularly polarized light only, the probability distribution simplifies drastically
\[ I_S(\theta) = M_S \sin(2(N-S) \theta), \] (16)
where \( S \) describes the total particle spin and \( M_S \) states the spin-dependent creation rate. The angle \( \theta \) is defined as the angle between the field’s propagation direction and the particle’s ejection direction.

**Scalar pair production.** The shell structure as well as the lack of pronounced interference patterns can be
observed in our results. In Fig. 1 where we employed a background field with peak strength \( \varepsilon = 0.075 \), frequency \( \omega = 0.55m \) and pulse length \( \tau = 250m^{-1} \), we recognize a perfectly regular pattern of three peaks at momenta \( p_4 = 0.4556m \), \( p_5 = 0.9419m \) and \( p_6 = 1.311m \). Here, the indices enumerate the shells with above-threshold peaks caused by the absorption of 4, 5 and 6 photons. 

The predictions obtained by the effective mass model yield \( p_{*,4} = 0.4532m \), \( p_{*,5} = 0.9413m \) and \( p_{*,6} = 1.3107m \), which support our interpretation. 

As stated above, the model for the background field \( \Delta \) describes photons with helicity +1 only. Hence, in an \( n \)-photon process the total angular momentum before particle creation takes place is equal to the number of photons \( n \) in the process. As angular momentum must be conserved, the total angular momentum of the produced particle pair has to be \( n \), too. Hence, every photon absorbed increases the total angular momentum of the particles by one. As these particles do not possess an intrinsic spin the total angular momentum is equal to the pairs’ orbital angular momentum \( L \).

As we also aim to obtain a quantitative understanding of the processes, we test for non-trivial scaling of the \( n \)-photon peaks. Hence, we have solved the governing equations of scalar QKT (7)-(9) for various values of the peak field strength \( \varepsilon \) at the effective momenta \( p_4 = (p_*,4,0) \) for 4-, 5- and 6-photon processes, c.f. Fig. 2. Monitoring each peak individually has the advantage that we can ensure we are probing a fundamental observable; in Ref. [4] the total yield was discussed, which suffers from having multiple sources of contributions. We fit a simple model of the form \( \varepsilon^{2n} \) to the data, because the intensity of a laser beam coincides with the number of photons in the beam \( I \sim \varepsilon^2 \propto n \). The exponents were calculated to be \( x_4 = 7.94 \), \( x_5 = 9.96 \) and \( x_6 = 11.96 \) with confidence intervals of \((7.92,7.95)\), \((9.96,9.97)\) and \((11.96,11.97)\) at 95% confidence. For comparison, our model predicts an increase of \( \varepsilon^{2n} \), thus de facto excluding any non-trivial effects at low field strengths.

Electron-positron pair production. – Electron-positron momentum spectra generally look similar compared to their scalar counterparts. The differences stem from the fact that there are two options for spin alignment in a pair of spin-1/2 particles; parallel and anti-parallel. In the former, both spins point in the same direction thus their total spin is given by \( S = 1/2 + 1/2 = 1 \); while in the latter the total spin adds up to \( S = 0 \). Hence, in an \( n \)-photon absorption process there are now two options for the particles orbital angular momentum \( L \). As a result, the model for fermionic pair production can be put in the form

\[
I(\theta) = \varepsilon^{2n} \left| M_0(\omega) \sin^2(\theta) + M_1(\omega) \sin^{2(n-1)}(\theta) \right|,
\]

where \( M_i \) can take on negative values. This is slightly different compared to the simpler case of scalar pair production, where no \( S = 1 \) state exists.

One consequence of the existence of an excited spin state becomes apparent when investigating particles with vanishing momentum \( p \). Every photon absorbed by the particle pair adds +1 to its total angular momentum. In case of scalar pair production, or if spins are aligned anti-parallel \((S = 0)\), all angular momentum is carried by the quantum number \( L \). As a non-zero orbital momentum requires a non-zero linear momentum, pair production cannot happen in this case. The only option for particle
creation at vanishing linear momentum $p$ is a one-photon process, where the particle spins align parallel, thus absorbing the photon angular momentum and resulting in $L = 0$.

Interestingly, one-photon absorption is also the only way to obtain particle ejection in the propagation direction of the laser beams. If the energy of one and only one photon exceeds the rest energy of a fermionic particle pair, we obtain a closed shell in momentum space. This is because only for the one-photon process an electron-positron pair in an $S = 1$ spin state can absorb all the photon angular momentum and have a vanishing orbital angular momentum.

Generally, there are contributions from both spin-states to the electron-positron momentum spectrum. In this regard, it is interesting to discuss the particle spectrum as a function of the angle $\theta$, where $\theta = 90^\circ$ corresponds to $(p_x, 0)$, see Fig. 3.

Solving scalar QKT for a 4-photon event ($\varepsilon = 0.075$ and $\omega = 0.55m$) we obtain a particle distribution $(p_x = 0.456m)$ which is perfectly described by our model: here $M_0 = 0.9987 (0.998, 0.999)$ with 95% confidence interval and $R^2 = 0.999997$ using normalized quantities. In the case of fermionic particles, we obtain a distribution function $f_{1/2}(\theta)$ with contributions from an $S = 0$ as well as from an $S = 1$ state. Assuming that for a given field configuration the particle creation rates for the attainable spin states do not change (besides a trivial factor of 2 due to combinatorics in $M_0$), we can subtract the zero-spin production probability from the total probability yielding the contribution from the $S = 1$ state alone.

Applying our model to fit the data we find the numerical results being well described by a $\sin^6$ function, as we obtain $M_1 = 1.01$ with confidence interval $(1.00, 1.02)$ at 95% certainty and $R^2 = 0.9994$. It is remarkable to see the contribution from the $S = 1$ state being much higher than the contribution from the singlet state. In a way, this situation resembles the status regarding orthohelium and parahelium with orthohelium ($S = 1$) having a lower rest energy.

Discussion.— Particle pair production is a complex process and observables are very sensitive to changes in the background fields. Nevertheless, it seems as if we have found a way to understand certain features of multiphoton pair production intuitively.

The specific procedure in which we obtained the particle creation rates $M_i$ is, in its current form, only applicable to purely circularly polarized fields. Although the generic model [15] can be applied for arbitrary field configurations, the composition of the particle spectra in terms of angular momentum contributions takes on the particularly simple form [16] only in case of rotating fields. For linearly polarized light, for example, the particles final angular momentum becomes a sum over many terms, because the initial photons could have been left- or right-handed.

Our model is also limited to the nonperturbative multiphoton regime in pair production for Keldysh parameter $\gamma_\omega = \omega/(m\varepsilon) > 1$. Moreover, the studied momentum signatures only emerge for multi-cycle pulses $\omega\tau \gg 1$. In a few-cycle pulse, the individual photon energies are varying too much, thus the clear characteristic peaks cannot form in the final particle spectrum.

Another issue arises in experimental studies of pair production, e.g., the collision of two laser pulses at an arbitrary angle. In such a case the photons carry momentum, thus linear momentum conservation has to be taken into account effectively excluding some processes. Nevertheless, the way to interpret the momentum shells does not change.

Finally, charge-conjugation and parity invariance pose additional constraints on multiphoton pair production. For an even (odd) number of photons $n$, the (scalar) momentum distribution has to vanish for vanishing momentum $p$ independent of the polarization of the incoming beams. These constraints become especially important for linearly polarized fields, where, in principle, the angular momentum transfer-picture allows for pair production at $p = 0$ for higher photon numbers. Within the context of this work, the model coincides with all selection rules.

Summary.— We have demonstrated that, in multiphoton pair production, the distribution functions for scalar and spin one-half particles in rotating electric fields follow a specific, intuitive pattern. On the basis of accurate numerical solutions of quantum kinetic theory it is straightforward to obtain the spin-dependent particle creation amplitudes. Although the procedure presented in this article is based on circularly polarized waves, the underlaying model can be readily extended to arbitrary field configurations.
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