Measurement of the Branching Ratios $\Gamma(D^{*+} \to D^+ \pi^0) / \Gamma(D^{*+} \to D^*_s \gamma)$ and $\Gamma(D^{*0} \to D^0 \pi^0) / \Gamma(D^{*0} \to D^0 \gamma)$

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(Dated: July 24, 2018)
Data samples corresponding to the isospin-violating decay $D_{s}^{+}\rightarrow D_{s}^{0}\pi^{0}$ and the decays $D_{s}^{+}\rightarrow D_{s}^{+}\gamma$, $D_{s}^{0}\rightarrow D_{s}^{0}\pi^{0}$ and $D_{s}^{0}\rightarrow D_{s}^{0}\gamma$ are reconstructed using 90.4 fb$^{-1}$ of data recorded by the BABAR detector at the PEP-II asymmetric-energy $e^{+}e^{-}$ collider. The following branching ratios are extracted: $\Gamma(D_{s}^{+}\rightarrow D_{s}^{0}\pi^{0})/\Gamma(D_{s}^{+}\rightarrow D_{s}^{+}\gamma) = 0.062 \pm 0.005$ (stat.) $\pm 0.006$ (syst.) and $\Gamma(D_{s}^{0}\rightarrow D_{s}^{0}\pi^{0})/\Gamma(D_{s}^{0}\rightarrow D_{s}^{0}\gamma) = 1.74 \pm 0.02$ (stat.) $\pm 0.13$ (syst.). Both measurements represent significant improvements over present world averages.

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The decay of any higher-mass $c\bar{s}$ meson into $D_{s}^{+}\pi^{0}$ violates isospin conservation, thus guaranteeing a small partial width. The amount of suppression is a matter of large theoretical uncertainty according to most models of charm-meson radiative decay. One such model suggests that the decay $D_{s}^{+}\rightarrow D_{s}^{+}\pi^{0}$ may proceed via $\pi^{0}$-$\eta$ mixing. Even including such considerations, the radiative decay $D_{s}^{+}\rightarrow D_{s}^{+}\gamma$ is still expected to dominate. The existence of isospin-violating decay modes such as $D_{s}^{+}\rightarrow D_{s}^{+}\pi^{0}$ is particularly relevant given the recent observations of two narrow new $D_{s}^{+}$ meson states $\Delta$. In particular, in contrast to the $D_{s}^{+}$ meson, there is no experimental evidence for the electromagnetic decay of the $D_{s}(2317)^{+}$; current measurements place the branching ratio to $D_{s}^{+}\gamma$ at less than 18% at 90% confidence level (CL).

Besides the $D_{s}^{+}\pi^{0}$ and $D_{s}^{+}\gamma$ final states, no other decay modes of the $D_{s}^{+}$ meson have been observed and none are expected to occur at a significant level. Only one previous observation of the decay $D_{s}^{+}\rightarrow D_{s}^{+}\pi^{0}$ is recorded in the literature, yielding a value of $0.062 \pm 0.018$ (stat.) $\pm 0.022$ (syst.) for the branching ratio $\Gamma(D_{s}^{+}\rightarrow D_{s}^{+}\pi^{0})/\Gamma(D_{s}^{+}\rightarrow D_{s}^{+}\gamma)$. The analysis presented here confirms this observation and provides a more precise measurement of this branching ratio.

The decay $D_{s}^{0}\rightarrow D_{s}^{0}\pi^{0}$, in contrast to $D_{s}^{+}\rightarrow D_{s}^{+}\pi^{0}$, does not violate isospin conservation and the world average for the branching ratio is $\Gamma(D_{s}^{0}\rightarrow D_{s}^{0}\pi^{0})/\Gamma(D_{s}^{0}\rightarrow D_{s}^{0}\gamma) = 1.625 \pm 0.20$. As for the $D_{s}^{+}$ meson, the $\pi^{0}$ and $\gamma$ decay modes are expected to saturate the decay width of the $D_{s}^{0}$ meson.

The results presented here are based on data recorded by the BABAR detector at the PEP-II asymmetric-energy $e^{+}e^{-}$ storage rings. The data sample, corresponding to an integrated luminosity of 90.4 fb$^{-1}$, was recorded at and approximately 40 MeV below the $\Upsilon(4S)$ resonance. Due to the unequal beam energies, the $e^{+}e^{-}$ center-of-mass system is boosted relative to the laboratory frame with $\beta\gamma \approx 0.55$. The BABAR detector and trigger are described in detail elsewhere. Charged particles are detected and their momenta measured by a silicon vertex tracker (SVT) consisting of five layers of double-sided silicon strip sensors and a cylindrical 40-layer drift chamber (DCH), both operating within a 1.5 T solenoidal magnetic field. Charged particle identification is provided by energy loss measurements in the SVT and DCH and by Cherenkov light detected in an internally reflecting ring imaging detector (DIRC). Photons are identified and their energies measured by an electromagnetic calorimeter (EMC) composed of 6580 CsI(Tl) crystals.

In the following paragraphs, the $D_{s}^{+}$ measurement is described first. The $D_{s}^{0}$ analysis, which uses similar procedures for signal extraction, is described afterward in less detail.

$D_{s}^{+}$ mesons are reconstructed via the decay sequence $D_{s}^{+}\rightarrow \phi\pi^{+}, \phi \rightarrow K^{+}K^{-}$. Kaons are identified by combining the energy deposited in the SVT and DCH with the information from the DIRC. Tracks not identified as kaons according to the particle identification criteria are required to fit successfully to a common vertex. Only combinations with a $K^{+}K^{-}\pi^{+}$ candidate are required to fit successfully to a common vertex. Only combinations with a $K^{+}K^{-}\pi^{+}$ invariant mass within 8 MeV/c$^{2}$ of the nominal $\phi$ mass are retained.

In $e^{+}e^{-}$ annihilation to charm quarks, the $c\bar{c}$ fragmentation process is characterized by the production of high-momentum (leading) charm hadrons. This property is exploited in order to reduce substantially the combinatorial background by retaining only those $\phi\pi^{+}$ candidates with scaled momentum $x_{p}$ greater than 0.6, where $x_{p}$ is defined as $x_{p}(D_{s}^{+}) = p^{\star}(D_{s}^{+})/p^{\star}_{\text{max}}(D_{s}^{+})$ and $p^{\star}(D_{s}^{+})$ is the momentum of the $D_{s}^{+}$ candidates in the $e^{+}e^{-}$ center-of-mass frame with $p^{\star}_{\text{max}}(D_{s}^{+}) = \sqrt{E_{\text{beam}}^{2} - m(D_{s}^{+})^{2}}$ as its maximum value.

The longitudinal polarization of the $\phi$ meson in the $D_{s}^{+}$ rest frame is used to reduce background by requiring that the absolute value of the cosine of the helicity angle, defined as the angle between the $\phi$ momentum direction in the $D_{s}^{+}$ rest frame and the momentum direction of either of the kaons in the $\phi$ rest frame, is 0.3 or greater.

The resulting $K^{+}K^{-}\pi^{+}$ invariant mass distribution is shown in Fig. 1. This distribution can be modeled by the sum of two Gaussian functions (to represent the signal) and a third-order polynomial (to represent the background). The resulting binned $\chi^{2}$ fit yields 73500 $\pm$ 300 events (statistical errors only). A $D_{s}^{+}$ candidate is retained if its invariant mass is within 12 MeV/c$^{2}$ of the nominal $D_{s}^{+}$ mass.

A $\pi^{0}$ candidate is reconstructed by combining two photon candidates that fulfill the following requirements. Each photon candidate is identified by a calorimeter cluster that is not associated with a charged track and has an energy in the laboratory frame of at least 45 MeV. Ad-
FIG. 1: The $K^+K^-\pi^+$ mass distribution. The dots represent data points with error bars corresponding to statistical uncertainties (these uncertainties are small enough that the error bars are difficult to distinguish). The solid curve shows the fitted function. The dashed curve indicates the background. $D_s^+$ candidates are defined by the region between the vertical dotted lines.

Additionally, to help remove the background from hadronic showers, the fractional lateral width [10], which describes the shape of the shower in the calorimeter, is required to be less than 0.55. The fiducial acceptance of photon candidates is restricted by the angular range of the EMC ($-0.92 \leq \cos \theta \leq 0.89$, where $\theta$ is the polar angle in the center-of-mass frame [8]).

A $\pi^0$ candidate is retained if it has a momentum $p^*$ in the $e^+e^-$ center-of-mass frame greater than 150 MeV/$c$. Furthermore, the absolute value of the cosine of the decay angle, $\theta^*$, which is defined as the angle between the direction of one of the photons in the $\pi^0$ rest frame and the direction of the $\pi^0$ candidate in the center-of-mass frame, is required to be less than 0.85. For $\pi^0 \rightarrow \gamma\gamma$ decay, the $\cos \theta^*$ distribution is uniform, while it peaks near $\pm 1$ for random $\gamma\gamma$ combinations.

Only $\gamma\gamma$ pairs within a specified mass interval are retained. This interval is defined by the values of mass at which the $\pi^0$ signal portion of a function fitted to the $\gamma\gamma$ mass distribution falls below 0.2 times its maximum value. This requirement accommodates the asymmetric shape of the $\gamma\gamma$ mass distribution and takes into account variations in detector calibration. A kinematic fit is applied to the surviving $\gamma\gamma$ pairs to constrain their mass to the nominal $\pi^0$ mass.

After combining the $D_s^+$ and $\pi^0$ candidates in a search for the decay $D_{s}^{*+} \rightarrow D_s^+\pi^0$, a fit is applied to the distribution of the mass difference $\Delta m(D_s^{*+}\pi^0) = m(K'^+K^--\pi^0) - m(K'^+K^++\pi^+)$. The fit function is the sum of a double Gaussian function to represent the signal and the function

$$f_1(\Delta m) = N \left( 1 - \exp \left( -\frac{\Delta m - m(\pi^0)}{\mu} \right) \right) \times \left( \Delta m^2 + a \Delta m + b \right),$$

where $m(\pi^0)$ is the $\pi^0$ mass, and $N$, $\mu$, $a$, and $b$ are free fit parameters to describe the background. The exponential term models the kinematic threshold; this threshold term has little influence on the background shape near the signal region. The result of this fit is shown in Fig. 2(a). A signal event yield of 560 ± 40 (statistical error only) is obtained.

For the reconstruction of the decay $D_{s}^{*+} \rightarrow D_s^+\gamma$, a calorimeter cluster that is not associated with a charged track is considered a photon candidate if it fulfills the following requirements: the energy must be 50 MeV or greater in the laboratory frame and 100 MeV or greater in the $e^+e^-$ center-of-mass frame and the fractional lateral width must be less than 0.8. To reduce the background due to photons from $\pi^0$ decay, a photon candidate is discarded if it forms a $\pi^0$ candidate with any other photon candidate in the same event. In this case, a $\gamma\gamma$ combination is considered a $\pi^0$ candidate if the invariant mass is in the range $115 < m(\gamma\gamma) < 155$ MeV/$c^2$ and if the total energy is at least 200 MeV in the $e^+e^-$ center-of-mass frame.

To obtain the $D_{s}^{*+} \rightarrow D_s^+\gamma$ signal event yield, a fit is applied to the distribution of the mass difference $\Delta m(D_s^{*+}\gamma) = m(K'^+K^-\gamma) - m(K'^+K^-\pi^+)$. The fit function is a sum of a third-order polynomial to model the background plus a function first introduced by the Crystal Ball collaboration [11] for the signal

$$f_2(\Delta m) = N \left\{ A \left( B - \frac{\Delta m - \mu}{\sigma} \right)^{-n} \exp \left( -\frac{(\Delta m - \mu)^2}{2\sigma^2} \right) \right\}$$

if $(\Delta m - \mu)/\sigma \leq \alpha$,  

$$\exp \left( -\frac{(\Delta m - \mu)^2}{2\sigma^2} \right)$$

if $(\Delta m - \mu)/\sigma \geq \alpha$,  

where $N$, $\mu$, $\sigma$, $n$, and $\alpha$ are free fit parameters and $A$ and $B$ are chosen such that the function and its first derivative are continuous at $(\Delta m - \mu)/\sigma = \alpha$. The fit result is shown in Fig. 2(b). A signal yield of 15 600 ± 200 events (statistical error only) is obtained.

The reconstruction efficiencies are determined using a Monte Carlo simulation based on 30 000 events for each $D_s^+$ decay mode. The simulated events are analyzed using the same procedure as for real data. By calculating the ratio of the number of reconstructed to generated events, efficiencies of $\epsilon(D_s^{*+}\pi^0) = 0.041 \pm 0.002$ and
TABLE I: A summary of the relative systematic uncertainties in the branching ratio measurements.

| Sources                | \(\Gamma(D_s^+ \to D_s^0 \pi^0)/\Gamma(D_s^+ \to D_s^+ \gamma)\) | \(\Gamma(D_s^0 \to D_s^0 \pi^0)/\Gamma(D_s^0 \to D_s^0 \gamma)\) |
|------------------------|---------------------------------------------------------------|---------------------------------------------------------------|
| Background shape       | 4.8                                                           | 0.1                                                           |
| Monte Carlo statistics | 5.0                                                           | 5.4                                                           |
| Signal model           | 3.6                                                           | 3.8                                                           |
| \(p^*\) dependence     | 6.8                                                           | 2.8                                                           |
| Quadrature Sum         | 10.2                                                          | 7.2                                                           |

\(\epsilon(D_s^+ \gamma) = 0.071 \pm 0.002\) are found for the two \(D_s^+\) decay modes. The efficiency ratio is \(\epsilon(D_s^+ \pi^0)/\epsilon(D_s^+ \gamma) = 0.58 \pm 0.03\) (statistical error only).

Various sources of systematic uncertainties are studied. To verify that the Monte Carlo events model the data correctly, \(\tau\) decays with one or two \(\pi^0\) mesons in the final state are studied to obtain energy-dependent Monte Carlo efficiency corrections for \(\pi^0\) mesons and photons. Although this procedure indicates that no correction is necessary, the errors on the correction functions represent uncertainties in the Monte Carlo model and contribute a systematic uncertainty of 3.6%.

To test for uncertainties in the background shape of the mass difference distributions, upper and lower sidebands in the \(K^+K^+\) and \(\gamma\gamma\) mass distributions are considered. Positive signal yields are expected in these sidebands from either mis-reconstructed or unassociated \(\pi^0\) candidates. To measure these yields, the same fit functions used to determine the signal yields are applied to the mass difference distributions of the sideband samples. Any discrepancy in yield so obtained from data and Monte Carlo simulation is considered a systematic uncertainty (4.8%). Most of this uncertainty is attributed to the relatively large background in the \(D_s^0 \pi^0\) decay mode.

The measurement of \(\Gamma(D_s^+ \to D_s^0 \pi^0)/\Gamma(D_s^+ \to D_s^+ \gamma)\) is repeated for the subsamples of candidates within various \(p^*\) intervals. By fitting either a constant function or a first-order polynomial to the branching ratio as a function of \(p^*\), it is possible to verify that the mass branching ratios are independent of \(p^*\) (see Fig. 3). Nevertheless, it is assumed conservatively that any \(p^*\) dependence arises from unknown momentum dependencies of the efficiencies that may not cancel in the branching ratios. The difference (6.8%) between the branching ratio represented by the constant function and the integral of the first-order polynomial is therefore reported as a systematic uncertainty.

The systematic uncertainties are summarized in Table I. Combining all contributions in quadrature, a total systematic uncertainty of 10.2% is derived for the measurement of \(\Gamma(D_s^+ \to D_s^0 \pi^0)/\Gamma(D_s^+ \to D_s^+ \gamma)\).

The ratio \(\Gamma(D_s^0 \to D_s^0 \pi^0)/\Gamma(D_s^0 \to D_s^0 \gamma)\), where \(D_s^0 \to K^- \pi^+\), is measured using the same selection criteria for the \(\pi^0\) and photon candidates as in the reconstruction of \(D_s^+ \to D_s^0 \pi^0\) and \(D_s^+ \to D_s^+ \gamma\). To be included in the \(D_s^0\) sample, a candidate \(K^-\) and \(\pi^+\) combination must yield an acceptable fit to a common vertex and the scaled momentum \(x_p\) of the resulting \(D_s^0\) candidate must be 0.6 or greater. Fitting the sum of a double Gaussian function and a third-order polynomial to the resulting \(K^-\pi^+\) invariant mass distribution (not shown) produces \((996.0 \pm 1.5) \times 10^3\) signal events (statistical error only). A \(K^-\pi^+\) combination is retained if its mass differs by less than 17 MeV/c\(^2\) from the nominal \(D_s^0\) mass.

The \(D_s^0\) candidates are combined with all \(\pi^0\) candidates; the resulting mass difference \(\Delta m(D_s^0 \pi^0) = m(K^-\pi^+\pi^0) - m(K^-\pi^+)\) is shown in Fig. 4(a). A fit using a double Gaussian for the signal and the function shown in Eq. 4 for the background yields \((69,000 \pm 450)\) signal events (statistical error only).

The \(D^0\) candidates are then combined with all photon candidates producing the distribution of the mass difference \(\Delta m(D^0 \gamma) = m(K^-\pi^-\gamma) - m(K^-\pi^-)\) shown in Fig. 4(b). In this case, the peak corresponding to \(D^0 \to D^0 \gamma\) signal is close to a large bump arising from...
the reflection of \( D^{*0} \rightarrow D^0 \pi^0 \) in which one photon is produced by \( \eta^0 \) decay (the same reflection appears in \( D_s^{*+} \) decay but with a lower rate and less distinctive shape). Most of this bump is avoided by limiting the analysis to \( \Delta m > 95 \text{ MeV}/c^2 \). The remainder of the background is modeled using the function

\[
f_3(\Delta m) = N \left( 1 + \exp \left( -\frac{\Delta m - m(\pi^0)}{\mu} \right) \right) \times (\Delta m^2 + a\Delta m + b). \tag{3}
\]

(Note that this function is similar to that of Eq. 1 but differs in the sign of the exponential term.) The signal is modeled by the Crystal Ball function (Eq. 2). The resulting fitted signal consists of 67 880±670 events (statistical errors only).

Efficiencies and systematic uncertainties are determined using the procedures described for \( \Gamma(D_s^{*+} \rightarrow D_s^{*+} \rightarrow D_s^{*+} \rightarrow D_s^{*+} \gamma) \). Efficiencies of \( \epsilon(D_s^{*+} \rightarrow D_s^{*+} \rightarrow D_s^{*+} \rightarrow D_s^{*+} \gamma) \) are 0.037±0.002 and \( \epsilon(D_s^{*0} \gamma) \) is 0.064±0.002 and an efficiency ratio of \( \epsilon(D_s^{*+} \rightarrow D_s^{*+} \rightarrow D_s^{*+} \gamma) / \epsilon(D_s^{*+} \rightarrow D_s^{*+} \gamma) \) is 0.86 ± 0.03 are found. The latter is consistent with the value of \( \epsilon(D_s^{*+} \rightarrow D_s^{*+} \gamma) \). The ratio \( \Gamma(D_s^{*0} \rightarrow D_s^{*0} \pi^0) / \Gamma(D_s^{*0} \rightarrow D_s^{*0} \gamma) \) is \( 1.74 \pm 0.02 \) (stat.) ± 0.13 (syst.) obtained.

The branching ratio measurements are summarized in Table II. By assuming that the \( D_s^{*+} \) meson decays only to \( D_s^{*+} \) and \( D_s^{*+} \), and that the \( D_s^{*0} \) meson decays only to \( D_s^{*0} \) and \( D_s^{*0} \gamma \), it is possible to calculate the branching fractions, which are also listed in Table II.

In summary, the branching ratio \( \Gamma(D_s^{*+} \rightarrow D_s^{*+} \pi^0) / \Gamma(D_s^{*+} \rightarrow D_s^{*+} \gamma) \) is 0.062 ± 0.005 (stat.) ± 0.006 (syst.) has been measured and is consistent with the previous measurement [3], but has higher precision. Also determined is the ratio \( \Gamma(D_s^{*0} \rightarrow D_s^{*0} \pi^0) / \Gamma(D_s^{*0} \rightarrow D_s^{*0} \gamma) \) = 1.74±0.02 (stat.)±0.13 (syst.). This result is in agreement with, but is more precise than, the world average [3].

It has been proposed that the decay \( D_s^{*+} \rightarrow D_s^{*+} \pi^0 \) proceeds via \( \eta^0 \rightarrow \pi^0 \) mixing and calculations based on Chiral perturbation theory [4] predict \( B(D_s^{*+} \rightarrow D_s^{*+} \pi^0) \approx 1–3\% \) based on current measurements of \( B(D_s^{*+} \rightarrow D_s^{*+} \gamma) = 1.6\% \) [5]. Newer theoretical estimates in a relativistic quark model [2] predict \( B(D_s^{*+} \rightarrow D_s^{*+} \pi^0) \approx 13\% \), somewhat larger than our measurement.

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\begin{table}[h]
\centering
\caption{Summary of the results. The first errors are statistical; the second represent systematic uncertainties.}
\begin{tabular}{lcc}
\hline
$\Gamma(D_s^{*+} \rightarrow D_s^{*+} \pi^0)/\Gamma(D_s^{*+} \rightarrow D_s^{*+} \gamma)$ & 0.062 ± 0.005 ± 0.006 & \\
$B(D_s^{*+} \rightarrow D_s^{*+} \pi^0)$ & 0.059 ± 0.004 ± 0.006 & \\
$B(D_s^{*+} \rightarrow D_s^{*+} \gamma)$ & 0.942 ± 0.004 ± 0.006 & \\
$\Gamma(D_s^{*0} \rightarrow D_s^{*0} \pi^0)/\Gamma(D_s^{*0} \rightarrow D_s^{*0} \gamma)$ & 1.74 ± 0.02 ± 0.13 & \\
$B(D_s^{*0} \rightarrow D_s^{*0} \pi^0)$ & 0.635 ± 0.003 ± 0.017 & \\
$B(D_s^{*0} \rightarrow D_s^{*0} \gamma)$ & 0.365 ± 0.003 ± 0.017 & \\
\hline
\end{tabular}
\end{table}

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