Nature’s nonlocality must be boundlessly multipartite: an experimental demonstration under strict locality condition

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Nonlocality captures one of the counter-intuitive features of Nature that defies classical intuition. Recent investigation reveals that our physical world is boundlessly multipartite nonlocal, i.e., there are multipartite correlations in nature that cannot be reproduced by sharing fewer-partite nonlocal resources even with globally shared randomness. Here we report an experimental demonstration of genuine tripartite nonlocality in network under strict locality constraints, which are ensured by space-like separating all relevant events and employing fast quantum random number generators and high-speed polarization measurements. In particular, for a photonic quantum triangular network we observe a locality-loophole free violation of the Bell-type inequality by 7.57 standard deviations for a tripartite GHZ state of fidelity (93.05 ± 0.25)%.

Quantum nonlocality manifests as strong correlations observed by spatially separated observers performing local measurements on a shared quantum system, which cannot be explained by any classical causal theory. Nonlocality in the simplest bipartite scenarios known as Bell nonlocality \[1\,2\] has been conclusively confirmed by numerous experiments \[3\,4\] via loophole-free violations of Bell inequalities \[1\,2\]. Besides its fundamental interest, bipartite nonlocality has found novel applications in many quantum information tasks such as device-independent quantum cryptography \[5\,6\] and randomness certification \[10\,11\]. Moving to scenarios with three or more parties, multipartite nonlocality exhibits a much richer and more complex structure \[2\,12\]. Of particular interest is the strongest nonlocality form – genuine multipartite nonlocality (GMN).

The notion of GMN was first introduced and studied by Svetlichny in 1987 \[13\], and he derived a three-particle inequality for detecting genuine tripartite nonlocality, which has been experimentally verified \[14\,15\]. Much later, his work was generalized to scenarios featuring an arbitrary number of particles \[17\,18\] as well as arbitrary dimensions \[19\,20\]. However, Svetlichny’s original definition of GMN is based on local operations with classical communications (LOCC), which is inconsistent with an operational framework of GMN that requires non-signaling conditions \[21\,22\]. Interestingly, with non-signaling distributions, e.g., Popescu-Rohrlich boxes \[23\], two-party correlations predicted by quantum theory can always be achieved. Moreover, it has been shown that some tripartite nonlocal correlations can be simulated by bipartite resources in alternative physical theories such as Box-world \[24\]. However, the box-world theory cannot reproduce all quantum correlations even we allow to share a global classical randomness \[25\,26\], which is a realistic classical resource. Notably, restricted by nonsignaling conditions, Svetlichny’s original GMN can be observed in any network built by sharing only bipartite nonlocal resources, e.g., bipartite entanglement \[27\].

A natural question now arises (Fig. 1): Are there correlations in Nature irreproducible by sharing only fewer-partite nonlocal resources together with global randomness? Recently, Coiteux-Roy et al. answered

![Diagram of a triangular network](image-url)

**Fig. 1.** A triangular network features three observers (gray squares) A, B, and C for Alice, Bob, and Charlie, respectively, with \((x, a), (y, b),\) and \((z, c)\) being their inputs and outputs. The question of interest is whether or not the correlations \(P_Q(abc|xyz)\) observed on the network on the left-hand side, in which each observer receives a particle from the tripartite-entangled quantum source (green starburst) and performs local measurements, can be simulated by the correlations obtained on the network on the right-hand side, in which the three observers are connected by nonclassical bipartite resources \((\delta_{ij} with i, j = A, B, C)\) and global shared randomness \(\lambda_{ABC}\).
this question in positive from a theory-agnostic perspective [28, 29], taking into account all causal theories compatible with device replication (i.e., refer to generalized probabilistic theories (GPTs)), including classical theory, quantum theory, non-signaling boxes, and any hypothetical causal theory. In the framework of local operations and shared randomness (LOSR), they refined Svetlichny’s notion of GMN to genuine LOSR multipartite nonlocality or GMN in network. By exploiting the inflation technique, which is widely used in theory-independent correlations [28, 29], they derived a device-independent Bell-type inequality that is satisfied by all multipartite correlations arising from sharing fewer-partite nonlocal resources and global randomness. From the violations to the Bell-type inequality by N-partite GHZ states for all finite N they thus proved that Nature’s nonlocality must be boundlessly multipartite in any causal GPTs.

In this Letter, we implement the proposed Bell-type test in a state-of-art photonic quantum network under strict locality constraints, i.e., all the parties involved be space-like separated. This requirement is crucial in analyzing Bell-type inequality violation as potential locality loopholes might be exploited by adversaries and also enforces the non-signaling conditions with classical communication between the parties being forbidden. In details, we prepare a triggered three-photon GHZ state from two independent entangled pair sources, and distributed the state to three space-like separated observers Alice, Bob, and Charlie. The locality loophole is closed by space-like separating the relevant events and using fast quantum random number generators (QRNGs) and high-speed polarization analyzers. We experimentally show that the produced tripartite correlations cannot be simulated by any bipartite nonlocal resources with LOSR, i.e., they are genuinely LOSR tripartite nonlocal. We expect our work will stimulate further experimental investigation of genuinely multipartite nonlocal correlations to better understand our Nature.

The genuine LOSR tripartite nonlocality proposed by Coiteux-Roy et al. is guaranteed by violations to the device-independent inequality arising from combining two intertwined games [28, 29], respectively detecting (1) some nonclassical resources albeit possibly bipartite, and (2) some tripartite resource albeit possibly classical. For (1), the Bell game, they exploit the standard CHSH-Bell test between Alice and Bob, conditioned on Charlie’s output result C_1 = 1, which reads

\[ I_{\text{Bell}}^{C_1=1} := \langle A_0 B_0 | C_1=1 \rangle + \langle A_0 B_1 | C_1=1 \rangle + \langle A_1 B_0 | C_1=1 \rangle - \langle A_1 B_1 | C_1=1 \rangle, \]

where subscript represents for the observer’s setting choices and all observables take either ±1. In standard Bell game, \( I_{\text{Bell}}^{C_1=1} \) can reach the value of \( 2\sqrt{2} \), which necessitates nonclassical resources. For (2), all observers are required to give the same outputs, which can take either the two values ±1. In this tripartite consistency game (i.e., Same game), the correlation is defined as [30]

\[ I_{\text{Same}} := \langle A_0 B_2 \rangle + \langle B_2 C_0 \rangle, \]

and the perfect score is \( I_{\text{Same}} = 2 \).

We notice that \( A_0 := A_{x=0} \) appears in both games, thus Alice cannot distinguish which of the two games she is participating in. This prevents her from playing the two games separately and she has to optimize Eq. 1 and 2 simultaneously with her input \( x = 0 \). Actually, it is impossible for Alice to decouple the two games, which indicates that performing well at both games (1) and (2) would require dependence on a genuinely LOSR tripartite nonlocal resources [30].

With the inflation techniques [28, 29], Coiteux-Roy et al. then combine the two aforementioned games in one scenario. If each two parties from three space-like separated observers Alice, Bob, and Charlie share a bipartite nonlocal resource and each party performs some local measurements, e.g., \( A_x, B_y, \) and \( C_z \) with random inputs \( x \in \{0, 1\}, y \in \{0, 1, 2\}, \) and \( z \in \{0, 1\} \) (Fig. 2 (a)), with outcomes \( a, b, c = \pm 1 \), then the resulting joint outcome probabilities \( p(abc|xyz) \) satisfy the following device-independent Bell-type inequality (in slightly different but equivalent form)

\[ F := I_{\text{Bell}}^{C_1=1} + 4I_{\text{Same}} - \frac{8}{1 + \langle C_1 \rangle} \leq 2, \]

where \( F \) is the three-party correlation function and its calculations from \( p(abc|xyz) \) are in Supplementary. Therefore a violation to the above inequality signatures the genuine LOSR tripartite nonlocality.

There are quantum correlations that violate the Bell-type inequality above. In fact, we distribute tripartite GHZ state \( |\text{GHZ}_3\rangle = (|000\rangle + |111\rangle)/\sqrt{2} \) in a triangular network and set Alice’s, Bob’s, and Charlie’s measurements as \( A_x \in \{ Z, X \}, B_y \in \{ \frac{X+Z}{\sqrt{2}}, \frac{2X}{\sqrt{2}} \}, \) and \( C_z \in \{ Z, X \} \), respectively. Here \( Z \) and \( X \) are the standard Pauli operators. In this case, the tripartite quantum strategy yields a maximum violation of \( F = 2\sqrt{2} \). For a mixture of the \( |\text{GHZ}_3\rangle \) state with white noise, violation of the Eq. 3 requires a fidelity of ≥ 93%.

Our experimental configuration is illustrated in Fig. 2. To violate the inequality in Eq. 3 we first prepare a triggered \( |\text{GHZ}_3\rangle \) state that can be efficiently created by combining two EPR sources \( (S_1 \text{ and } S_2) \) at a polarization beam splitter (PBS), as shown in Fig. 2 (b). We use a pulse pattern generator (PPG) to send out 250 MHz trigger signals, and the PPG situated in source \( S_2 \) is used as the master clock to synchronize all operations. In each source, a distributed feedback (DFB) laser is triggered to emit a 2 ns 1558 nm laser pulse, which is carved into 80 ps with an intensity modulator (IM). The laser pulses are frequency-doubled in a PPMgLN crystal.
after passing through an erbium-doped fiber amplifier (EDFA). We then use the produced 779 nm pump laser to drive a Type-0 spontaneous parametric down-conversion (SPDC) process in the second PPMgLN crystal to create a polarization-based Sagnac loop. Each source produces pairs of photons in the Bell state $|\Phi^+\rangle$. The beeline distances are 104 m, 106 m, 89 m, and 110 m between Alice-S1, S1-Charlie, Charlie-S2, and S2-Bob, respectively, with an upper bounded uncertainty of 1 m. Their optical fiber links are 112.63 m, 124.9 m, 109.6 m, and 125.48 m respectively, with uncertainty within 0.10 m. (b) The $|\Phi^+\rangle$ state is created by projecting one photon over the diagonal basis $|\pm\rangle$ (trigger) from a four-photon GHZ state after post-selection. (c) In each source, a pair of polarization-entangled photons in state $|\Phi^+\rangle$ is prepared by pumping a PPMgLN crystal in a Sagnac loop (details see text and Ref. [32]). (d) and (e), each observer performs the local measurements on their received photon. The measurement choices are decided by a quantum random number generator (QRNG) situated there. In each node, a high-speed single-photon polarization modulation (SPPM) is implemented in order to vary the direction of the produced photon. The measurement choices are decided in real time by a fast quantum random number generator (QRNG) situated there. Note that the local measurements performed by Charlie and Alice, we space-like separate observers Charlie and Alice, we space-like separate to close locality loopholes, such that for example any setting the three-party correlation function $F'$. The timing and layout of the experiment are critical for calculating the three-party correlation function $F'$.

The observers then perform local measurements on their photon from the produced $|\Phi^+\rangle$ state. Alice and Charlie perform one of two measurements $A_x$ and $C_x$, respectively, while Bob measures one of three bases $B_y$. Their setting choices for are decided in real time by a fast quantum random number generator (QRNG) situated there. Note that the QRNG at Bob’s station outputs four random bits, thus we discard the redundant random bit. All random bits from QRNG sources pass the NIST randomness tests [33] (please refer to [34] for more information about the QRNGs). To realize the fast measurement setting choice, we implemented a high-speed high-fidelity single-photon polarization modulation (SPPM), which consists of two fixed quarter-wave plates (QWP), two Faraday rotator (FR) and electro-optical phase modulator (PM), shown in Fig. 2 (see Supporting Information in Ref. [34] for details). The SPPM varied photons’ polarization at a rate of 250 MHz with a fidelity of ~ 99% with random inputs. For Charlie, his single-photon detection events were analyzed in time and recorded by a field-programmable gate array (FPGA). Additionally, all setting results from QRNGs and detection events for Alice and Bob were recorded by a time-to-digital converter (TDC). All locally stored data are collected by a separate computer, which are used for calculating the three-party correlation function $F'$.

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number generation for setting choices (QRNG\textsubscript{C}) from the events of finishing single photon detection by Alice (M\textsubscript{A}), and vise versa. In each experimental trial, the time elapses of a QRNG to generate a random bit that determine the setting choices for the received photon are both 53±2 ns for QRNG\textsubscript{C} and QRNG\textsubscript{A}. The time elapse of measurement events defined as the interval between a photon enters the loop interferometer in the SPPM (Fig. 2) and the time of SNSPD outputs a signal for M\textsubscript{C} and M\textsubscript{A} are 44.9±0.5 ns and 44.6±0.5 ns, respectively. Their analysis are described in the left panel of Fig. \ref{fig:3} (a), where M\textsubscript{A} is 156.3±4 ns outside the light cone of QRNG\textsubscript{C} and M\textsubscript{C} is 73.5±4 ns outside the light cone of M\textsubscript{A}, satisfying the locality condition here. We summarize all relevant results for the other two slices in Fig. \ref{fig:3} (a) (middle and right panels), with the labels defined using the same conversion. All the time-space relations are drawn to scale. The analysis is summarized and detailed in the Supplementary.

To check the quality of the prepared \(|\text{GHZ}_3\rangle\), we reconstruct its density matrix using a maximum-likelihood quantum state tomography. The average triggered three-photon rate is 0.3 Hz and we collect around 6700 s for each setting. The fidelity is calculated to be \((93.05±0.25)\%\), satisfyingly the requisite fidelity \(\geq 93\%\). Detailed tomography results see Supplementary. We then evaluate the experimental violation of the inequality given by Eq. \ref{eq:3} and record 33770 four-fold coincidence detection events over 171725 seconds. As shown in Fig. \ref{fig:3} (b), we obtain the correlation of \(F = 2.338 ± 0.044\), which is beyond the bipartite GPT bound by 7.57 standard deviations. That means the observed correlations via three-photon GHZ state cannot be reproduced by any two-way GPT resources with local operations and unlimited shared randomness, i.e., it is genuinely LOSR tripartite nonlocal.

Base on a optical quantum network under strict locality constraints, we have experimentally demonstrated that Nature’s tripartite nonlocality cannot be simulated from any bipartite nonlocal causal theories. In our experiment, the locality loophole is closed by space-like separating relevant events and employing fast QRNGs and high-quality high-speed single-photon measurements. The three-photon GHZ state produced in our setup depends on a post-selection of four-fold coincidences. To create state without post-selecting, one could prepare them in heralded event-ready way such as using cascaded SPDC sources \cite{16} in the future. Furthermore, we expect to close detection loopholes \cite{35, 37} in the next step by using high-efficiency photon sources \cite{38} and detectors.

Note added – After finishing our experiment, we became aware of two similar experimental works without closing locality loopholes \cite{39, 40}.

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