Beauty hadron lifetime ratios and the problem of $\tau(\Lambda_b)$

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We describe a theoretical approach to compute inclusive widths of heavy hadrons, based on a systematic expansion in the inverse powers of the heavy quark mass. The method reproduces quite well the experimental ratios of $B$ mesons lifetimes. As for $\tau(\Lambda_b)/\tau(B_d)$, we present a QCD sum rule calculation of the $O(m_b^{-3})$ contribution to $\tau(\Lambda_b)$; we conclude that such correction is not responsible of the small experimental value of $\tau(\Lambda_b)/\tau(B_d)$.

1. INCLUSIVE DECAYS OF HEAVY HADRONS

A simple theoretical way to consider inclusive decays of hadrons containing one heavy quark is the Spectator Model, which states that only the heavy quark participates in the transition, while the light degrees of freedom are unaffected by the process. As a result, the model predicts that all the hadrons with the same heavy quark should have the same lifetime.

On the other hand, the experimental ratios of beauty hadron lifetimes are:

$$\frac{\tau(B^-)}{\tau(B_d)} = 1.06 \pm 0.04 \frac{\tau(B_s)}{\tau(B_d)} = 0.99 \pm 0.05 \quad (1)$$

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.79 \pm 0.06 \quad . \quad (2)$$

Eq. (2) shows that the ratio $\tau(\Lambda_b)/\tau(B_d)$ significantly differs from the Spectator Model prediction.

One can employ a more refined theoretical approach, based on QCD, to compute inclusive decay widths of heavy hadrons; the method consists in an expansion in powers of $m_Q^{-1}$. By using the optical theorem, the inclusive width of a hadron containing the heavy quark $Q$ into a final state with assigned quantum numbers $f$ can be written as:

$$\Gamma(H_Q \to X_f) = \frac{2 Im <H_Q|\hat{T}|H_Q>}{2M_{H_Q}}$$

where $\hat{T}(Q \to X_f \to Q)$ is the transition operator describing $Q$ with the same momentum in the initial and final state:

$$\hat{T} = i \int d^4x \, T[\mathcal{L}_w(x)\mathcal{L}_w^0(0)] \quad . \quad (4)$$

$\mathcal{L}_w$ is the effective weak lagrangian for the decay $Q \to X_f$. The time ordered product in (4) is computed by an operator product expansion (OPE):

$$\hat{T} = \sum_i C_i \mathcal{O}_i$$

where $\mathcal{O}_i$ are local operators ordered by increasing dimension, and $C_i$ are coefficients ordered by increasing powers of $1/m_Q$. As a result, the inclusive width of a heavy hadron $H_b$ (from now on we will refer to the beauty sector) is represented by a sum of expectation values over $H_b$ of operators with increasing dimension:

$$\Gamma(H_b \to X_f) = \frac{G^2_F m_b^5}{192\pi^3} |KM|^2 \left[ c_3^f <H_b|\bar{b}b|H_b> \frac{1}{2M_{H_b}} \right. \right.$$

$$+ \left. \frac{c_5^f}{m_b} <H_b|\bar{b}ig\sigma \cdot Gb|H_b> \frac{1}{2M_{H_b}} \right. \right.$$

$$+ \left. \sum_i c_{6i}^f \frac{m_b^3}{m_b} |H_b|\mathcal{O}_i^6|H_b> \frac{1}{2M_{H_b}} + O\left(\frac{1}{m_b^4}\right) \right].$$

$KM$ is the CKM matrix element relevant in the considered decay.
The first operator in (6) is $\bar{b}b$, which has dimension $D = 3$; next, the chromomagnetic operator $O_G = \bar{b}^a_2 \sigma_{\mu \nu} G^{\mu \nu} b$, responsible of the mass splitting between hadrons which differ only for the heavy quark spin orientation, has $D = 5$; finally, $O^b_3$ have $D = 6$ and contribute to $O(m_b^{-3})$ in the expansion. $<\bar{b}b | H_b > = < H_b | \bar{b}b | H_b > / 2 M_{H_b}$ can be further reduced by using the heavy quark equation of motion in the limit $m_b \to \infty$:

$$<\bar{b}b | H_b > = 1 + \frac{< O_G | H_b >}{2 m_b^2} - \frac{< O_T | H_b >}{2 m_b^2} + O\left( \frac{1}{m_b^3} \right).$$

(7)

$O_\pi = \bar{b}(i \tilde{D})^2 b$ is the heavy quark kinetic energy operator due to its residual motion inside the hadron.

Some important features should be noticed in (6). The first term of the expansion reproduces the Spectator Model result; $O(m_b^{-1})$ terms are absent since $D = 4$ operators cannot contribute, being reducible, by equation of motion, to the $D = 3$ operator $\bar{b}b$; finally, $O_G$ and $O_\pi$ are spectator blind, i.e. they are not sensitive to the light flavour.

$O(m_b^{-3})$ terms come from four quark operators, which can be classified as follows:

$$O^q_{V-A} = \bar{b} \gamma_\mu q_L \bar{q} L \gamma_\mu b_L$$

$$O^q_{S-P} = \bar{b} R \gamma_\mu q_L \bar{q} R$$

$$T^q_{V-A} = \bar{b} \gamma_\mu t^a q_L \bar{q} L \gamma_\mu t^a b_L$$

$$T^q_{S-P} = \bar{b} R t^a q_L \bar{q} L t^a b_R$$

(8)

where $t^a = \lambda^a/2$, $\lambda^a$ being the Gell-Mann matrices. Their contribution accounts for the presence of the spectator quark, which participates in the decay through the mechanisms known as weak annihilation and Pauli interference (the latter depends on the presence of two identical quarks in the final state).

In order to use the result (6) one has to know the matrix elements of all the previously mentioned operators. Let us define:

$$\mu^2_G(H_b) = \frac{< H_b | O_G | H_b >}{2 M_{H_b}}.$$  

(9)

$\mu_2^G(H_b)$ depends on the spin $J$ of $H_b$: $\mu_2^G = -2[J(J + 1) - \frac{3}{2}] \lambda_2$. It enters with $\mu_2^G$ in the mass formula for a heavy hadron in the $m_b \to \infty$ limit:

$$M_{H_b} = m_b + \bar{\Lambda} + \frac{\mu_2^G}{2 m_b} + O\left( \frac{1}{m_b^2} \right).$$

(11)

The parameters $\bar{\Lambda}$, $\mu_2^G$, $\lambda_2$ are independent of the heavy quark mass. For $B$ mesons, $\mu_2^G$ can be derived experimentally, since it is related to the mass splitting: $\mu_2^G(B) = 3(M_B^2 - M_{B_s}^2)/4 \simeq 0.36 \ GeV^2$. On the other hand: $\mu_2^G(\Lambda_b) = 0$, since the light degrees of freedom have zero total angular momentum relative to the heavy quark.

Various theoretical determinations exist for $\mu_2^G(B_d)$. From the mass formula (11) and using experimental data on the mass of the $\Lambda_b$ baryon, one obtains:

$$\mu_2^G(B_d) = 0.002 \pm 0.024 \ GeV^2$$

(12)

and hence it can be assumed: $\mu_2^G(B_d) \simeq \mu_2^G(\Lambda_b)$.

In order to evaluate the matrix elements of $D = 6$ operators, different approaches must be used for mesons and baryons. For $B$ mesons, factorization approximation gives:

$$< B_q | O^q_{V-A} \bar{b} q > = \frac{2 M_{B_q}}{f_{B_q}}$$

$$< B_q | O^q_{S-P} \bar{b} q > = \frac{(m_b + m_q)}{2 M_{B_q}}$$

$$< B_q | T^q_{V-A} \bar{b} q > = < B_q | T^q_{S-P} \bar{b} q > = 0$$

(13)

$f_{B_q}$ being the $B_q$ decay constant. The same technique cannot be applied to baryons, so that one must use other approaches. In the case of $\Lambda_b$, heavy quark symmetries reduce the relevant operators to two:

$$\tilde{O}^q_{V-A} = \bar{b} L \gamma^\mu q_L \bar{q} L \gamma_\mu \gamma_5 q_L \bar{q} L$$

$$O^q_{V-A} = \bar{b} L \gamma^\mu q_L \bar{q} L \gamma_\mu \gamma_5 q_L \bar{q} L.$$  

(14)

\footnote{This is confirmed by QCD sum rule estimates.}
One can parameterize the matrix elements as follows:

\[
\begin{align*}
<\hat{O}_{V-A}^\mu >_{\Lambda_b} &= \frac{f_B^2 M_B}{48} r, \\
<\hat{O}_{V-A}^\mu >_{\Lambda_b} &= -\tilde{B} \frac{<\hat{O}_{V-A}^\mu >_{\Lambda_b}}{2 M_{\Lambda_b}}; \tag{15}
\end{align*}
\]

in the valence quark approximation: \( \tilde{B} = 1 \).

It should be stressed that only large values of the parameter \( r \) in \([3]\) \((r = 3 - 4)\) could explain the observed difference between \( \tau(\Lambda_b) \) and \( \tau(B_d) \).

Calculations based on quark model \([8]\) or exploiting the experimental splitting: \( M_{\Sigma_b} - M_{\Xi_b} \) give values of \( r \) smaller than required. In order to have a different theoretical estimate, we computed by QCD sum rules the matrix elements of the relevant \( D = 6 \) operators over \( \Lambda_b \). This calculation will be described in the next section.

2. QCD SUM RULE CALCULATION OF \( <\hat{O}_{V-A}^\mu >_{\Lambda_b} \)

The starting point to evaluate \( <\hat{O}_{V-A}^\mu >_{\Lambda_b} \) by QCD sum rules within the Heavy Quark Effective Theory (HQET) is the correlation function:

\[
\Pi_{CD}(\omega, \omega') = (1 + \gamma_5) C_D \Pi(\omega, \omega') = i^2 \int dx \, dy \, e^{i(\omega x - \omega' y)}
\]

\[
<0|T[J_C(x)\tilde{O}^\mu_{V-A}(0) J_D(y)]|0 > \tag{16}
\]

where \( C, D \) are Dirac indices; the variable \( \omega' \) is related to the residual momentum of the incoming (outgoing) baryonic current \( p^\mu = m_b v^\mu + k^\mu \) with \( k^\mu = \omega' v^\mu \). A suitable interpolating field for \( \Lambda_b \), in the limit \( m_b \to \infty \), is given by \([10]\):

\[
J_C(x) = \epsilon^{ijk} (q^T(x) \Gamma q^j(x)) (h^k_v X C(x) \tag{17}
\]

where \( T \) means the transpose, \( i, j, k \) are colour indices, \( h_v \) is the effective heavy quark field, \( \Gamma = C \gamma_5 (1 + b \gamma^\tau) \) and \( C \) is the charge conjugation operator. \( \tau \) is the \( \Lambda_b \) light flavour matrix:

\[
\tau = \frac{1}{\sqrt 2} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{18}
\]

The parameter \( b \) is fixed to \( b = 1 \) from the QCD sum rules analysis of \( f_{\Lambda_b} \), defined by:

\[
<0|J_C|\Lambda_b(v) >= f_{\Lambda_b}(v) j_C \quad (\psi_v \text{ is the spinor for a } \Lambda_b \text{ of four-velocity } v).
\]

The hadronic representation of the correlator \([11]\) can be obtained saturating it by baryonic states, and considering the double pole contribution in the variables \( \omega \) and \( \omega' \):

\[
\Pi^{had}(\omega, \omega') = \langle \hat{O}^\mu_{V-A} \rangle_{\Lambda_b} \frac{f_{\Lambda_b}^2}{2} x
\]

\[
\frac{1}{(\Delta_{\Lambda_b} - \omega)(\Delta_{\Lambda_b} - \omega')} + \ldots \tag{19}
\]

at the value \( \omega = \omega' = \Delta_{\Lambda_b} \). \( \Delta_{\Lambda_b} \) represents the binding energy of the light degrees of freedom in \( \Lambda_b \): \( M_{\Lambda_b} = m_b + \Delta_{\Lambda_b} \) and must be derived within the same QCD sum rule framework. On the other hand, in the Euclidean region, for negative values of \( \omega, \omega' \), the correlator \([11]\) can be computed in QCD, in terms of a perturbative contribution and of vacuum condensates. The result can be written in a dispersive form:

\[
\Pi^{OP E}(\omega, \omega') = \int d \sigma d \sigma' \frac{\rho_{\Pi}(\sigma, \sigma')}{(\sigma - \omega)(\sigma' - \omega')} \tag{20}
\]

where possible subtraction terms have been omitted. The explicit expression of the computed spectral function \( \rho_{\Pi} \) can be found in \([11]\).

A sum rule for \( <\hat{O}_{V-A}^\mu > \) can be derived by equating \( \Pi^{had} \) and \( \Pi^{OP E} \). Moreover, invoking a global duality ansatz, the contribution of the higher resonances and of the continuum in \( \Pi^{had} \) in \([11]\) can be modeled as the QCD term outside the region \( 0 \leq \omega \leq \omega_c, 0 \leq \omega' \leq \omega_c \), with \( \omega_c \) an effective threshold. Finally, the application of a double Borel transform to both \( \Pi^{OP E} \) and \( \Pi^{had} \) in the momenta \( \omega, \omega' \):

\[
B(E_1) \frac{1}{\sigma - \omega} = \frac{1}{E_1} e^{-\sigma/E_1},
\]

\[
B(E_1) \frac{1}{\Delta - \omega} = \frac{1}{E_1} e^{-\Delta/E_1}, \tag{21}
\]

(and similar for \( \omega' \) with the Borel parameter \( E_2 \)) allows us to remove the subtraction terms in \([21]\).
that are polynomials in $\omega$ or $\omega'$ (the Borel transform of a polynomial vanishes). Moreover, the convergence of the OPE is factorially improved by the transform, and the contribution of the low-lying resonances in $\Pi^{\text{had}}$ is enhanced for low values of the Borel variables. The symmetry of the spectral function in $\sigma, \sigma'$ suggests the choice $E_1 = E_2 = 2E$. The final sum rule reads:

$$\frac{f^2}{2}(1 + b)^2 \exp(-\frac{\Delta_{\Lambda_b}}{E}) \langle \hat{O}^q_{-A} \rangle_{\Lambda_b}$$

$$= \int_0^{\omega_c} \int_0^{\omega_c} d\sigma d\sigma' \exp(-\frac{\sigma + \sigma'}{2E}) \rho_{11}(\sigma, \sigma') \quad (22)$$

The result of the numerical analysis of (22) is depicted in Fig. 1. A stability window is observed, starting at $E \simeq 0.2$ GeV and continuing towards large values of $E$; in this range the result for $\langle \hat{O}^q_{-A} \rangle_{\Lambda_b}$ is independent of the external parameter $E$. The variation of the result with $E$ and with $\omega_c$ provides us with an estimate of the accuracy of the numerical outcome. We find (22):

$$\langle \hat{O}^q_{-A} \rangle_{\Lambda_b} \simeq (0.4 - 1.20) \times 10^{-3} \text{ GeV}^3$$

a result corresponding to $\tau \simeq 0.1 - 0.3$.

As for $\hat{B}$ in (13), since our computational scheme considers only valence quark processes, a sum rule for the matrix element in (13) would give $\hat{B} = 1$. The explicit calculation confirms this conclusion. Using the computed $\tau$ and $\hat{B}$ in the formulae in ref. [5], we get: $\tau(\Lambda_b)/\tau(B_d) \geq 0.94$.

3. CONCLUSIONS

We presented a method based on the $1/m_Q$ expansion to compute inclusive widths of heavy hadrons. We also described the QCD sum rule calculation of the matrix element $\langle \hat{O}^q_{-A} \rangle_{\Lambda_b}$ contributing to $\mathcal{O}(m^{-3})$ to the $\Lambda_b$ lifetime. The result: $\tau(\Lambda_b)/\tau(B_d) \geq 0.94$ implies that such correction cannot explain the observed difference between $\tau(\Lambda_b)$ and $\tau(B_d)$. If the experimental data will be confirmed, a theoretical reanalysis of the problem would be required, as suggested for example in [12,13].

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