Non–decoupling effects of the SM Higgs boson to one loop

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Abstract

We study the complete non-decoupling effects of the standard model Higgs boson to one loop. Using effective field theory methods, we integrate out the Higgs boson and represent its non-decoupling effects by a set of gauge invariant effective operators of the electroweak chiral Lagrangian. In a previous work, we analyzed the non-decoupling effects in the two and three-point Green’s functions of gauge fields. We complete here the calculation of the chiral effective operators by analyzing the four-point functions. We discuss in detail the relation between the renormalization of both the standard model and the effective theory, which is crucial for a correct understanding and use of the electroweak chiral Lagrangian. Some examples have been chosen to show the applicability of this effective Lagrangian approach in the calculation of low energy observables in electroweak theory.

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1 Introduction

Non-decoupling effects of heavy particles in the low energy observables have been for a long time an indirect but crucial test for the discovery of new particles. The most recent example is the top quark, which contributes significantly to the observables that measure the electroweak (EW) radiative corrections, as for instance $\Delta \rho$ \cite{1}. In particular, the latest global fit of the electroweak parameters from LEP data gives a quite strong constraint for the allowed top mass value \cite{2}. This indirect search supports the recently announced evidence of $t\bar{t}$ production at CDF \cite{3} and will certainly contribute to get a final confirmation of the existence of the quark top.

The non-decoupling effects of the Higgs boson are, however, weaker than in the case of the top quark due to the screening theorem \cite{4}. According to this theorem, the sensitivity in the low energy observables to the Higgs boson mass is at most logarithmic at one loop. This fact has made in the past a difficult task to disentangle the Higgs from the dominant fermionic effects and, therefore, no significant bound on the Higgs mass has been obtained so far from the analysis of LEP data. This situation, however, will likely improve in the near future due to the increasing precision of electroweak measurements and the forthcoming CDF data. A confirmation of the top quark is still needed, as well as a precise measurement of the top quark mass, but the important point is that it is now beginning to be plausible the search for evidence of the Higgs boson in electroweak radiative corrections.

On the other hand, regarding the theoretical aspects, the leading logarithmic Higgs mass effects in the low energy observables are known up to one-loop level since the pioneering works by Appelquist and Bernard \cite{5} and by Longhitano \cite{6}. Their strategy was to use the symmetry properties of the $SU(2)_L \times U(1)_Y$ gauged non-linear $\sigma$-model (GNL) \cite{7} along with a systematic power-counting analysis to provide a list of these logarithmic Higgs mass dependent terms. However, at present, it is known that these leading logarithmic terms are not sufficient to discriminate a heavy Higgs possibility from an alternative symmetry breaking scenario to which one requires to respect the same SM symmetries. These logarithmic contributions are a consequence of the general gauge and custodial symmetry requirements of the low energy structure of EW interactions and therefore they will be the same irrespective of the particular choice for the breaking dynamics, with the Higgs mass being replaced by some alternative physical mass. Thus, if one wants to reveal the nature of the symmetry breaking from low energy observables, one has to go beyond the leading logarithmic effects.

In this paper, we present the complete calculation of the leading and next-to-leading non-decoupling effects of a heavy Higgs boson in the SM to one loop level \cite{1}. These genuine Higgs boson effects cannot be obtained using the GNL as in \cite{5, 6}, but must be calculated directly from the evaluation of the one loop diagrams in the SM. Furthermore, in order to classify in a systematic way those effects, we will use here the electroweak chiral Lagrangian (EChL) \cite{8}. Our approach is based on effective field theory methods, in which the non-decoupling effects of a heavy Higgs boson are represented, at energies below the Higgs mass, by certain set of gauge invariant

\footnote{For simplicity, we will ignore the fermions in all the discussion but their contribution, which is assumed here to be the SM one, has to be added in any comparison with data.}
effective operators of the EChL.

The EChL is basically a non-linear sigma model coupled to the $SU(2)_L \times U(1)_Y$ gauge fields, where the Higgs field has been removed from the physical spectrum of the theory. The model respects the fundamental symmetries of the SM, namely $SU(2)_L \times U(1)_Y$ gauge invariance spontaneously broken to $U(1)_{em}$ and the custodial symmetry $SU(2)_C$ of the pure scalar sector, but does not include explicitly a particular dynamics for the symmetry breaking [8]-[14]. Although this Higgs-less parametrization of EW interactions is non-renormalizable, the theory can be rendered finite to one loop by adding gauge invariant operators up to dimension four. These effective operators parametrize the low energy effects of the underlying fundamental dynamics of the symmetry breaking. In particular, the EChL can be regarded as an effective theory of the SM in which the Higgs field has been integrated out, and its effects at energies well below the Higgs mass are parametrized by the chiral effective operators [8]. We believe that this kind of approach may be interesting for several reasons. First of all, it provides a gauge invariant way of separating the non-decoupling Higgs boson effects from the rest of the EW radiative corrections. On the other hand, the EChL is a general framework in which one can analyze the low energy effects not only of a heavy Higgs in the SM, but of more general breaking dynamics characterized by the absence of light modes [15]. It is then desirable to have the EChL that parametrizes a SM Higgs as a fundamental reference model.

In our previous work [8], we discussed a general procedure to obtain the EChL operators by matching the SM predictions in the limit of large Higgs mass with the predictions from the EChL to one loop order. The subset of EChL operators involved in the two and three-point Green’s functions of the gauge fields were also obtained there. In this work, we complete the calculation of the EChL operators by analyzing the four-point Green’s functions for gauge fields. We will also discuss here in more detail the relation between the renormalization of both the SM and the effective theory, which is crucial for understanding the true meaning of the effective operators. We will show with some explicit examples how to calculate observables in the EChL parametrization of EW interactions.

The paper is organized as follows. Section two is a survey of the electroweak chiral Lagrangian approach, in which we also include a brief discussion of the formal renormalization procedure of the effective theory and the matching conditions. Section three is devoted to describe the computation of the SM four-point Green’s functions in the large $M_H$ limit. The renormalization prescription chosen for the SM is fixed in this section to be the on-shell scheme. The matching equations will be solved in section four where we present the results for the complete set of EChL bare parameters. Section five is devoted to discuss the dependence of the bare EChL parameters on the renormalization prescription fixed for the underlying SM. We explain in section six the relation between the bare EChL parameters and the renormalized EChL parameters for different renormalizations of the effective theory. In section seven we explain how to compute observables to one loop with the EChL and demonstrate by choosing some simple observables that the result agrees with previous calculations in the literature. Finally, section eight is devoted to the conclusions.
The electroweak chiral Lagrangian

The EChL is the most simple effective theory of EW interactions that parametrizes the physics of the $SU(2)_L \times U(1)_Y$ breaking dynamics at low energies. The assumption made in this approach is that, whatever the $SU(2)_L \times U(1)_Y$ breaking interactions may be, the particles involved in the symmetry breaking are heavier than the W and Z bosons. The EChL is then a low energy formulation of EW interactions which contains just the "light" gauge and would-be Goldstone fields, satisfying the basic requirement of $SU(2)_L \times U(1)_Y$ gauge invariance spontaneously broken to $U(1)_{em}$:

$$\mathcal{L}_{\text{EChL}} = \mathcal{L}_{\text{NL}} + \sum_{i=0}^{13} \mathcal{L}_i.$$  \hspace{1cm} (1)

Its basic structure is a gauged non-linear sigma model $\mathcal{L}_{\text{NL}}$, where a non-linear parametrization of the would-be Goldstone bosons is coupled to the $SU(2)_L \times U(1)_Y$ gauge fields

$$\mathcal{L}_{\text{NL}} = \frac{v^2}{4} Tr \left[ D_\mu U^\dagger D^\mu U \right] + \frac{1}{2} Tr \left[ W_\mu W^{\mu\nu} + B_\mu B^{\mu\nu} \right] + \mathcal{L}_{\text{R}} + \mathcal{L}_{\text{FP}},$$ \hspace{1cm} (2)

where the bosonic fields have been parametrized as

$$U \equiv \exp \left( i \frac{\vec{\pi} \cdot \vec{\tau}}{v} \right), \quad v = 246 \text{ GeV}, \quad \vec{\pi} = (\pi^1, \pi^2, \pi^3),$$

$$W_\mu \equiv \frac{-i}{2} \vec{W} \cdot \vec{\tau},$$

$$B_\mu \equiv \frac{-i}{2} B \tau^3,$$ \hspace{1cm} (3)

and the covariant derivative and the field strength tensors are defined as

$$D_\mu U \equiv \partial_\mu U - g W_\mu U + g' U B_\mu,$$

$$W_{\mu\nu} \equiv \partial_\mu W_\nu - \partial_\nu W_\mu - g[W_\mu, W_\nu],$$

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu.$$ \hspace{1cm} (4)

The physical fields are given by

$$W^\pm_\mu = \frac{W^1_\mu \pm i W^2_\mu}{\sqrt{2}},$$

$$Z_\mu = c W^3_\mu - s B_\mu,$$

$$A_\mu = s W^3_\mu + c B_\mu,$$ \hspace{1cm} (5)

where $c = \cos \theta_w$, $s = \sin \theta_w$ and the weak angle is defined by $\tan \theta_w = g'/g$.

The second term in eq.(1) includes the complete set of $SU(2)_L \times U(1)_Y$ and CP invariant
operators up to dimension four that were classified by Longhitano in \[8\]:

\[\mathcal{L}_0 = a_0 g^2 v^2 \frac{1}{4} [Tr (TV_\nu)]^2\]
\[\mathcal{L}_1 = a_1 \frac{ig g'}{2} B_{\mu\nu} Tr (TW^{\mu\nu})\]
\[\mathcal{L}_2 = a_2 \frac{ig}{2} B_{\mu\nu} Tr (T[V^\mu, V^\nu])\]
\[\mathcal{L}_3 = a_3 g Tr (W_{\mu\nu}[V^\mu, V^\nu])\]
\[\mathcal{L}_4 = a_4 [Tr (V_\mu V_\nu)]^2\]
\[\mathcal{L}_5 = a_5 [Tr (V_\mu V^\mu)]^2\]
\[\mathcal{L}_6 = a_6 Tr (V_\mu V_\nu) Tr (TV^\mu) Tr (TV^\nu)\]
\[\mathcal{L}_7 = a_7 Tr (V_\mu V^\mu) [Tr (TV^\nu)]^2\]
\[\mathcal{L}_8 = a_8 g^2 \frac{1}{4} [Tr (TW_{\mu\nu})]^2\]
\[\mathcal{L}_9 = a_9 g \frac{1}{2} Tr (TW_{\mu\nu}) Tr (T[V^\mu, V^\nu])\]
\[\mathcal{L}_{10} = a_{10} [Tr (TV_\mu) Tr (TV_\nu)]^2\]
\[\mathcal{L}_{11} = a_{11} Tr ((D_\mu V^\mu)^2)\]
\[\mathcal{L}_{12} = a_{12} Tr (TD_\mu D_\nu V^{\nu\mu}) Tr (TV^\mu)\]
\[\mathcal{L}_{13} = a_{13} \frac{1}{2} [Tr (TD_\mu V_\nu)]^2\]  

where the basic building blocks are defined as

\[T \equiv U_T^3 U^\dagger, \quad V_\mu \equiv (D_\mu U) U^\dagger.\]  

The above particular base of invariants can be transformed into a new one with only 11 independent structures by making use of the classical equations of motion \[9\]. The new effective Lagrangian is then given in terms of a new set of chiral parameters \(\hat{a}_i\) given by: \(\hat{a}_1 = a_1 + a_{13}, \hat{a}_4 = a_4 - a_{13}, \hat{a}_5 = a_5 + a_{13}, \hat{a}_6 = a_6 - a_{13}, \hat{a}_7 = a_7 + a_{13}, \hat{a}_8 = a_8 + a_{13}, \hat{a}_{11} = \hat{a}_{12} = \hat{a}_{13} = 0; \hat{a}_i = a_i, i = 0, 2, 3, 9, 10.\) Both effective Lagrangians, however, will give rise to the same physical on-shell amplitudes. In this work, since we will not restrict ourselves to calculate on-shell matrix elements, we keep the complete basis given in eq.(8).

\(2\) There is an extra term \(\mathcal{L}_{14}\) proportional to \(\epsilon^{\mu\nu\alpha\beta}\) that is \(CP\) conserving but \(C\) and \(P\) violating. It is not relevant in case of absence of fermion contributions and will not be considered here.

\(3\) The relation with Longhitano’s notation is the following: \(a_0 = \frac{g^2}{\sqrt{2}} \beta_1; a_1 = \frac{g}{\sqrt{2}} \alpha_1; a_2 = \frac{g}{\sqrt{2}} \alpha_2; a_3 = -\alpha_3; a_4 = \alpha_4, i = 4, 5, 6, 7; a_8 = -\alpha_8; a_9 = -\alpha_9; a_{10} = a_{10}/2; a_{11} = \alpha_{11}; a_{12} = \alpha_{12}/2; a_{13} = \alpha_{13}.\) Notice that the definition of \(a_0\) is different here than in \[8\].
We will work in a generic $R_\xi$ gauge, the gauge fixing term $L_{R_\xi}$ and the Faddeev-Popov Lagrangian $L_{\text{FP}}^{\text{NL}}$ in eq. (1) were given in our previous work [8]. We refer the reader to this work for the detailed formulas and a discussion on these terms. It is worth just recalling here that $L_{\text{NL}}^{\text{FP}}$ does not coincide with the usual Faddeev-Popov Lagrangian of the SM due to the non-linearity of the would-be Goldstone bosons under infinitesimal $SU(2)_L \times U(1)_Y$ transformations. Furthermore, because of the non-linear realization of the gauge symmetry, some of the couplings in $L_{\text{NL}}^{\text{FP}}$ have different Feynman rules than in the SM. We collect in fig.(1) the subset of them that are relevant for the present calculation.

It is also important to mention that in a $R_\xi$ gauge, the complete electroweak chiral Lagrangian must be BRS invariant and include also effective operators involving the ghost fields. Longhitano[6] showed, however, that the subset of operators given above is sufficient to absorb the divergences of the gauged non-linear sigma model if the Landau gauge is chosen. As it will be shown later, we have demonstrated in this work that the set of operators given in eq.(6) is also enough, in a general $R_\xi$ gauge, to render finite the two, three and four-point Green’s functions with external gauge fields.

The non-linear sigma model in eq.(2) is not a renormalizable theory, as increasing the number of loops in a calculation implies the appearance of new divergent structures of higher and higher dimension. However, the EChL is an effective theory that can be renormalized order by order in the loop expansion. In particular, at one loop order, the new divergences generated by a one loop calculation with $L_{\text{NL}}$ can be absorbed into redefinitions of the effective operators given in eq.(6) [6]. Therefore, one can obtain finite renormalized Green’s functions if one makes a suitable redefinition of the fields and parameters of the EChL, among which the chiral parameters $a_i$ must be included [16]- [18]. Formally, one defines the renormalized quantities in the effective theory by the following relations

$$B^b_\mu = \hat{Z}_{B}^{1/2} B_\mu, \quad g^b = \hat{Z}_{g}^{-1/2} (g' - \hat{\delta}g'),$$
$$\hat{W}^b_\mu = \hat{Z}_{\hat{W}}^{1/2} \hat{W}_\mu, \quad g^b = \hat{Z}_{\hat{g}}^{-1/2} (g - \hat{\delta}g),$$
$$\hat{\pi}^b = \hat{Z}_{\hat{\pi}}^{1/2} \hat{\pi}, \quad v^b = \hat{Z}_{\hat{v}}^{1/2} (v - \hat{\delta}v),$$
$$\xi^b_B = \xi_B (1 + \hat{\delta}\xi_B), \quad \xi^b_W = \xi_W (1 + \hat{\delta}\xi_W),$$
$$a^b_i = a_i(\mu) + \delta a_i,$$

where the renormalization constants of the effective theory are $\hat{Z}_i \equiv 1 + \hat{\delta}Z_i$ and the superscript $b$ denotes bare quantities. We use the hatted notation to distinguish counterterms and Green’s functions in the effective theory from the corresponding quantities in the SM.

The 1PI renormalized Green’s functions of the effective theory to one loop will be generically denoted by

$$\hat{\Gamma}^R = \hat{\Gamma}^T + \hat{\Gamma}^C + \hat{\Gamma}^L,$$

where the superscript $R$ denote renormalized function and the superscripts $T$, $C$ and $L$ denote the tree level, counterterm and loop contributions respectively. We will discuss in section 6 the
on-shell renormalization of the effective theory, giving explicit expressions for the counterterms introduced in eq.(8). For the moment, in order to discuss the matching procedure, we will treat the counterterm contributions to the renormalized functions of eq.(9) just at a formal level.

We would like to focus now our attention on the chiral parameters. Once a particular renormalization scheme has been chosen to fix the counterterms of the effective theory, the renormalized $a_i(\mu)$ parameters remain as free parameters that can not be determined within the framework of the low energy effective theory. The values of the renormalized chiral parameters can be constrained from the experiment, as they are directly related to different observables in scattering processes \cite{11, 12, 21-23} and in precision electroweak measurements \cite{11-13, 24}, see also section 7); but to have any theoretical insight on their values, one has to relate the effective theory with a particular underlying fundamental dynamics of the symmetry breaking.

If the underlying fundamental theory is the standard model with a heavy Higgs, the chiral parameters can be determined by matching the predictions of the SM in the large Higgs mass limit and those of the EChL, at one loop level. By heavy Higgs we mean a Higgs mass much larger than any external momenta and light particle masses ($p^2, M_Z^2 \ll M_H^2$) so that one can make a low energy expansion, but smaller than say 1 TeV, so that a perturbative loop calculation is reliable.

We will impose here the strongest form of matching \cite{19} by requiring that all renormalized one-light-particle irreducible (1LPI) Green’s functions are the same in both theories at scales $\mu \leq M_H$. The 1LPI functions are, by definition, the Green’s functions with only light particles in the external legs and whose contributing graphs cannot be disconnected by cutting a single light particle line. This matching condition is equivalent to the equality of the light particle effective action in the two descriptions. Some other forms of matching have been discussed in the literature, by requiring the equality of the two theories at the level of the physical scattering amplitudes \cite{10} or connected Green’s functions \cite{25}. These requirements, however, complicate the calculation unnecessarily while give at the end the same results for the physical observables. There is also some discussion in the literature \cite{23} on the dependence of the Green’s functions on the parametrization chosen for the would-be Goldstone bosons. We will fix here the particular parametrizations of eq.(3) in the effective theory and eq.(12) in the SM. Of course, the physical observables will not depend on this particular choice.

In order to completely determine the chiral parameters in terms of the parameters of the SM, it is enough to impose matching conditions in the two, three, and four-point 1LPI renormalized Green’s functions with external gauge fields. We have worked in a general $R_\xi$-gauge to show that the chiral parameters $a_i$ are $\xi$-independent. We use dimensional regularization and the on-shell subtraction scheme.

The SM Green’s functions are non-local; in particular, they depend on $p/M_H$ through the virtual Higgs propagators. One has to make a large $M_H$ expansion to represent the virtual Higgs boson effects by the local effective operators $\mathcal{L}_i$. In this step, care must be taken since clearly the operations of making loop integrals and taking the large $M_H$ limit do not commute. Thus, one must first regulate the loop integrals by dimensional regularization, then perform the renormalization with some fixed prescription (on-shell in our case) and at the end take the
large $M_H$ limit, with $M_H$ being the renormalized Higgs mass. From the computational point of view, in the large $M_H$ limit we have neglected contributions that depend on $(p/M_H)$ and/or $(M_V/M_H, M_V = M_W, M_Z)$ and vanish when the formal $M_H \to \infty$ limit is taken. We show in appendix A one illustrative example of how to take the large $M_H$ expansion of the loop integrals.

The matching procedure can be summarized by the following relation among renormalized 1LPI Green’s functions

$$
\Gamma^{\text{SM}}_{\mu}(\mu) = \hat{\Gamma}^{\text{EChL}}_{\mu}(\mu), \quad \mu \leq M_H,
$$

(10)

where the large Higgs mass expansion of the SM Green’s functions has to be made as explained above. This matching condition imposes a relation between the renormalization of the SM and the renormalization of the effective theory. We have chosen to renormalize both theories in the on-shell scheme, so that the renormalized parameters are the physical masses and coupling constants. Therefore, the renormalized parameters are taken to be the same in both theories and the matching conditions will provide relations between the SM and the EChL counterterms.

The matching condition (10) represents symbolically a system of tensorial coupled equations (as many as 1LPI functions for external gauge fields) with several unknowns, namely the complete set of parameters $a^b_i$ that we are interested in determining. In our previous work, we solved the subset of coupled equations involving the two-point and three-point functions. From this subset, we were able to determine the chiral parameters $a^b_0, a^b_1, a^b_2, a^b_3, a^b_8, a^b_9, a^b_{11}, a^b_{12}$ and $a^b_{13}$. In section 4, we will solve the matching equations for the four-point Green’s functions, thus the set of EChL parameters for a heavy Higgs will be completed. But before that, we have to set a renormalization prescription for the standard model.

### 3 Renormalization of the standard model

We start by writing down the SM Lagrangian

$$
\mathcal{L}_{\text{SM}} = (D_{\mu} \Phi)^\dagger (D^{\mu} \Phi) + \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 + \frac{1}{2} \text{Tr} (W_{\mu \nu} W^{\mu \nu} + B_{\mu \nu} B^{\mu \nu}) + \mathcal{L}_{\text{R}_\xi} + \mathcal{L}_{\text{FP}},
$$

(11)

where

$$
\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 - i \phi_2 \\ \sigma + i \chi \end{pmatrix}, \quad (\pi_1, \pi_2, \pi_3) \equiv (-\phi_2, \phi_1, -\chi),
$$

$$
D_{\mu} \Phi = (\partial_{\mu} + \frac{1}{2} ig W_{\mu} \cdot \vec{r} + \frac{1}{2} ig' B_{\mu}) \Phi.
$$

(12)

$W_{\mu \nu}, B_{\mu \nu}$ are defined in eqs. (34), $\mathcal{L}_{\text{R}_\xi}$ and $\mathcal{L}_{\text{FP}}$ are the usual R$_\xi$ gauge fixing and Faddeev–Popov terms of the standard model.

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4 In some related literature on effective field theories, it has also been discussed. In that case, the matching procedure relates the running $\overline{\text{MS}}$-renormalized parameters, that are different in the fundamental and the effective theories.
We rescale the fields and parameters as follows

\[ \begin{align*}
B^b_\mu &= Z_B^{1/2} B_\mu, \\
\Phi^b &= Z_\Phi^{1/2} \Phi, \\
g^b &= Z_W^{1/2} (g - \delta g), \\
\mu^b &= Z_\mu^{1/2} (\mu - \delta \mu), \\
\xi_B^b &= \xi_B (1 + \delta \xi_B),
\end{align*} \]

(13)

where the renormalization constants of the SM are \( Z_i \equiv 1 + \delta Z_i \) and the superscript \( b \) denotes bare quantities.

We have chosen to renormalize the SM in the on-shell scheme. We choose the physical masses, \( M_H, M_W, M_Z \) and \( g \) as our renormalized parameters. The weak mixing angle is defined in terms of physical quantities, as it is usual in the on-shell scheme

\[ \cos^2 \theta_W \equiv \frac{M_W^2}{M_Z^2} \]

(14)

and from \( g \) and \( \theta_W \) one derives the coupling constant \( g' = g \tan \theta_W \).

The 1LPI renormalized Green’s functions in the standard model to one loop will be generically denoted by

\[ \Gamma^R = \Gamma^T + \Gamma^C + \Gamma^L, \]

(15)

where one has to consider the tree, counterterm and loop contributions of all the one light particle irreducible diagrams in the SM; that is, all the diagrams that cannot be disconnected by cutting a light (non-Higgs) particle line.

In principle, we should give now the whole set of renormalization conditions defining the SM on-shell scheme. However, to extract from the matching conditions the values of the chiral parameters \( a_i \), we only need for the moment to evaluate explicitly the SM counterterms that enter in the renormalization of the diagrams \( T_i \) in figs.(3), that is, the tree level diagrams with an intermediate Higgs boson. Furthermore, since we are doing a large \( M_H \) expansion, it will not be necessary to give the complete expressions for these SM counterterms, but just the leading terms that give non-negligible contributions (i.e. non vanishing in the large \( M_H \) limit) once they are plugged into the matching equations of the four-point functions. In summary, these considerations imply that we will need explicit expressions for the tadpole and Higgs mass counterterms to order \( M_H^4 \) and for the W and Z mass counterterms to order \( M_H^2 \). The other SM counterterms, \( \delta Z_W \) and \( \delta g \), have at most a logarithmic dependence on Higgs mass and give subleading contributions to the renormalization of the \( T_i \) diagrams. The \( \delta Z_W \) and \( \delta g \) counterterms, however, do contribute to the renormalization of the four-point Green’s functions through the renormalization of the tree level irreducible diagrams. We do not need to give explicit expressions for them because, as we will see, they appear in the matching through the differences \( \Delta Z_W = \delta Z_W - \delta \bar{Z}_W \) and \( \Delta g = \delta g - \delta \bar{g} \). The values of these differences will be extracted from the matching.

The renormalization of the scalar sector has been done following the work of Marciano and Willenbrock [26]. In order to fix the notation and renormalization for the tadpole we first write
down the SM Lagrangian for the scalar sector in terms of the would-be Goldstone boson fields $\phi^\pm \equiv (\phi_1 \mp i\phi_2)/\sqrt{2}$ and $\chi$ and the physical Higgs boson field $H$. In terms of the bare fields and parameters, it reads as follows

$$L_{\text{scalar}}^{\text{SM}} = \partial_\mu \phi^+ \partial^\mu \phi^- + \frac{1}{2} \partial_\mu \chi^b \partial^\mu \chi^b + \frac{1}{2} \partial_\mu H^b \partial^\mu H^b$$

$$-\lambda^b \left[ \phi^+ \phi^- (\phi^+ \phi^-) + (\chi^b)^2 + \left( (H^b)^2 + \frac{1}{4} (H^b)^2 \right)^2 \right]$$

$$-\lambda^b \left[ 2\phi^+ \phi^- H^b + (\chi^b)^2 H^b + (H^b)^3 \right]$$

$$-\lambda^b (v^b)^2 (H^b)^2 + \frac{\delta T}{v^b} (\phi^+ \phi^-) + \frac{1}{2} (\chi^b)^2 + \frac{1}{2} (H^b)^2 + \delta T H^b$$

$$+ (\xi - \text{dependent terms})$$

(16)

The tadpole counterterm is defined in terms of bare parameters as

$$\delta T \equiv v^b \left( (\mu^b)^2 - \lambda^b (v^b)^2 \right)$$

(17)

The renormalization condition for the tadpole is fixed such that the tadpole loop corrections $T$ are cancelled by the tadpole counterterm $\delta T$, or equivalently, such that the renormalized tadpole vanishes. From the computational point of view, one ignores all tadpole diagrams and tadpole counterterms, and includes the extra contributions to the renormalized scalar Green’s functions coming from the counterterm $\delta T/v$ in eq.(16) that is quadratic in the scalar fields.

The bare masses of the Higgs and gauge bosons are taken as

$$(M^b_H)^2 = 2 \lambda^b (v^b)^2$$

$$(M^b_W)^2 = (g^b)^2 (v^b)^2 / 4$$

$$(M^b_Z)^2 = \left( (g^b)^2 + (g'^b)^2 \right) (v^b)^2 / 4$$

so that the following relation among the basic bare parameters holds

$$\lambda^b = \frac{(g^b)^2 (M^b_H)^2}{8 (M^b_W)^2}$$

(19)

The renormalized masses are defined by

$$(M^b_H)^2 = M^2_H + \delta M^2_H$$

$$(M^b_W)^2 = M^2_W + \delta M^2_W$$

$$(M^b_Z)^2 = M^2_Z + \delta M^2_Z$$

so that the on-shell renormalization conditions

$$\text{Re} \left[ \Sigma^R_H (q^2 = M^2_H) \right] = 0, \quad T^R = T + \delta T = 0,$$

$$\text{Re} \left[ \Sigma^R_W (q^2 = M^2_W) \right] = 0, \quad \text{Re} \left[ \Sigma^R_Z (q^2 = M^2_Z) \right] = 0,$$

(21)

imply that eqs.(18,19) are also fulfilled by the renormalized quantities

$$M^2_H = 2\lambda v^2,$$

$$M^2_W = g^2 v^2 / 4,$$

$$M^2_Z = (g^2 + g'^2) v^2 / 4,$$

$$\lambda = \frac{g^2 M^2_H}{8 M^2_W}.$$

(22)

5 Notice that the sign chosen in the definition of $\delta T$ is opposite to the one in [26]
The renormalization conditions (21) fix the values of the SM counter terms to be

\[
\begin{align*}
\delta M^2_H &= -\text{Re} \left[ \Sigma^L_H (q^2 = M^2_H) \right] + \delta T / v, \\
\delta T &= -T, \\
\delta M^2_W &= \text{Re} \left[ \Sigma^L_W (q^2 = M^2_W) \right], \\
\delta M^2_Z &= \text{Re} \left[ \Sigma^L_Z (q^2 = M^2_Z) \right].
\end{align*}
\] (23)

If one wishes to keep just the non-vanishing contributions in the large \( M_H \) limit to the renormalization of the tree level diagrams \( T_i \), the computation of the unrenormalized self-energies of eq.(23) involve just the leading diagrams collected in fig.(2). These loop diagrams give the following values of the mass and tadpole counterterms in the on-shell scheme

\[
\begin{align*}
\frac{\delta M^2_H}{M^2_H} &= \frac{g^2 M^2_H}{M^2_W} \frac{1}{16 \pi^2} \left[ \frac{3}{2} \hat{\Delta} + 3 - \frac{3\sqrt{3}}{8\pi} \right] + \mathcal{O}(1), \\
\frac{\delta M^2_W}{M^2_W} &= \frac{g^2 M^2_H}{M^2_W} \frac{-1}{16 \pi^2} + \mathcal{O}(1), \\
\frac{\delta M^2_Z}{M^2_Z} &= \frac{g^2 M^2_H}{M^2_W} \frac{-1}{16 \pi^2} + \mathcal{O}(1), \\
\frac{\delta T / v}{M^2_H} &= -\frac{g^2 M^2_H}{M^2_W} \frac{1}{16 \pi^2} \frac{3}{8} \left[ \hat{\Delta} + 1 \right] + \mathcal{O}(1)
\end{align*}
\] (24)

where

\[
\hat{\Delta} = \Delta - \log \frac{M^2_H}{\mu^2}, \quad \Delta = \frac{2}{\epsilon} - \gamma_E + \log 4\pi, \quad \epsilon = 4 - D \quad (25)
\]

and \( \mu \) is the usual mass scale of dimensional regularization. We have checked the agreement of our expressions with the results of [26].

We have explicitly indicated in the formulas of the SM counterterms (24) that these expressions are truncated to a certain order in the \( 1/M_H \) expansion. This truncation is enough to keep the non-vanishing effects of a heavy Higgs in the evaluation of the following combination of SM counterterms

\[
\delta S = \frac{M^2_W}{g^2 M^2_H} \left( -\frac{\delta M^2_H}{M^2_H} + \frac{\delta T / v}{M^2_H} + \frac{\delta M^2_W}{M^2_W} \right)
\] (26)

that comes from the renormalization of the tree level diagrams \( T_i \) and appears explicitly in the matching equations for the four-point functions given in appendix B.

--

6 We denote by \(-i g_{\mu \nu} \Sigma_V\), (V=W,Z) and \(-i \Sigma_H\) the direct result from the Feynman diagrams. The tadpole loops are however denoted by \( iT \).

7 In the two-point functions however, it is necessary to go to the next order in the large \( M_H \) expansion of these terms [3].
4 Matching equations for the 4-point Green’s functions

In this section we present the results of our calculation of the 4-point Green’s functions, giving the set of matching equations that we have imposed and their solution. The master equation that summarizes the complete set of matching conditions for the renormalized four-point functions is the following:

\[ M_{abcd}^{T \mu \nu \rho \lambda} + M_{abcd}^{C \mu \nu \rho \lambda} + M_{abcd}^{L \mu \nu \rho \lambda} = \hat{M}_{abcd}^{T \mu \nu \rho \lambda} + \hat{M}_{abcd}^{C \mu \nu \rho \lambda} + \hat{M}_{abcd}^{L \mu \nu \rho \lambda}, \tag{27} \]

where \( abcd = \gamma \gamma WW, \gamma ZWW, ZZWW, WWWW, ZZZZ \).

The calculation of the one loop contributions \( M_L \) and \( \hat{M}_L \) is the most involved part. One must include all the 1PI diagrams in the EChL and all the 1LPI diagrams in the SM. 1LPI diagrams are those that cannot be disconnected by cutting a single light particle line, that is, a non-Higgs particle line. One must, in principle, account for all kind of diagrams with gauge, scalar and ghost fields flowing in the loops. However, some simplifications occur. Firstly, a subset of the diagrams that have only light particles in it is exactly the same in both models, and their contribution can be simply dropped out from both sides of the matching equation (27). This is the case, for instance, of the subset of diagrams with only gauge particles in them. Secondly, calculating explicitly every diagram in the four-point 1LPI SM functions and using the techniques given in app.A, we have checked that the diagrams involving both gauge and Higgs particles in the loops give vanishing contributions in the large \( M_H \) limit to the four-point functions. Only those with just scalars (Goldstone bosons or Higgs) particles in the loops do contribute with non-vanishing corrections in the large \( M_H \) limit to the matching equation (27). Finally, among the diagrams with pure scalar loops, there are some with only Goldstone boson particles. One would expect that these diagrams give the same contributions in the SM and the EChL, however they do not. The reason is the already mentioned differences in the Feynman rules of the vertices in fig.(1). These diagrams (denoted generically by \( D_i \) in figs.(3)) must therefore be included in both sides of the matching equation.

In fig.(3), we give the complete list of the tree level \( (T_i) \) and one loop \( (L_i, D_i) \) diagrams that give a contribution to the matching equations (27) of the \( \gamma \gamma WW, \gamma ZWW, ZZWW, WWWW, ZZZZ \) Green’s functions.

We give in appendix B the final result of the calculation of the tree, counterterms and loop contributions to the matching equations for the different Green’s functions. Each matching condition of the form given in eq.(B1) is a tensorial equation and therefore it provides several equations corresponding to the various independent tensorial structures. Furthermore, each of these equations can be written in the form of a polynomial in powers of \( c^2 \). In summary, one gets one equation "per" coefficient of the polynomial in each independent tensorial structure and in each Green’s function. The result including the equations from the two, three and four-point functions is a linear system with more equations than unknowns. The system turns out to be compatible, giving a strong consistency check of the calculation. By keeping just the independent set of equations from the four point functions, one gets:

\[ \text{This is also true for the three-point functions but it is false for the two-point functions where both pure scalar and mixed gauge-scalar loops contribute in the large } M_H \text{ limit.} \]
\[
\left( \Delta Z_W - \frac{\Delta g^2}{g^2} \right) \frac{1}{g^2} = \frac{1}{16\pi^2} \frac{-1}{12} \left[ \Delta \epsilon + \frac{5}{6} \right] \tag{28}
\]

\[
\left( \Delta Z_W - \frac{\Delta g^2}{g^2} \right) \frac{1}{g^2} + a_{11}^b = \frac{1}{16\pi^2} \frac{-1}{12} \left[ \hat{\Delta} \epsilon + \frac{4}{3} \right]
\]

\[
2a_3^b = \frac{1}{16\pi^2} \frac{-1}{12} \left[ \hat{\Delta} \epsilon + \frac{17}{6} \right]
\]

\[
a_3^b - a_{11}^b + a_{12}^b = \frac{1}{16\pi^2} \frac{-1}{24} \left[ \hat{\Delta} \epsilon + \frac{11}{6} \right]
\]

\[
2 \left( a_5^b + a_7^b \right) = \frac{1}{16\pi^2} \frac{1}{12} \left[ \frac{43}{2} \hat{\Delta} \epsilon + \frac{47}{3} \right] + \frac{M_W^2}{g^2 M_H^2} + \delta S
\]

\[
a_4^b + a_6^b - a_{11}^b + 2a_{12}^b = \frac{1}{16\pi^2} \frac{-1}{12} \left[ \hat{\Delta} \epsilon + \frac{7}{3} \right]
\]

\[
\left( \Delta Z_W - \frac{\Delta g^2}{g^2} \right) \frac{1}{g^2} + 2a_3^b - a_4^b - a_8^b + 2a_9^b - 2a_{13}^b = \frac{1}{16\pi^2} \frac{-1}{12} \left[ \hat{\Delta} \epsilon + \frac{5}{6} \right]
\]

\[
\left( \Delta Z_W - \frac{\Delta g^2}{g^2} \right) \frac{1}{g^2} + 2a_3^b + a_4^b + 2a_5^b - a_8^b + 2a_9^b - 2a_{13}^b = \frac{1}{16\pi^2} \frac{1}{24} \left[ \frac{37}{43} \hat{\Delta} \epsilon + \frac{55}{3} \right] + \frac{M_W^2}{g^2 M_H^2} + \delta S
\]

\[
2 \left( a_4^b + a_5^b + 2(a_6^b + a_7^b + 2a_{10}^b) \right) = \frac{1}{16\pi^2} \frac{1}{8} \left[ \frac{13}{3} \hat{\Delta} \epsilon + \frac{20}{3} \right] + \frac{M_W^2}{g^2 M_H^2} + \delta S
\]

where \( \hat{\Delta} \epsilon \) and \( \delta S \) have been defined in eqs. (25) and (26) respectively and we use the following notation for the differences of counterterms

\[
\Delta Q \equiv \delta Q - \hat{\delta} Q \quad \text{with} \quad Q = Z_B, Z_W, g^2, \text{etc}...
\]

The first four equations provide a check for the values of \( \Delta Z_W, \frac{\Delta g^2}{g^2}, a_{11}^b, a_3^b \) and \( a_{12}^b \) that we already obtained in our previous work from the calculation of the two and three-point functions. Finally, using these results and the values of \( a_8^b, a_9^b \) and \( a_{13}^b \) from [8] we can extract the genuine parameters of the four-point functions: \( a_4^b, a_5^b, a_6^b, a_7^b \) and \( a_{10}^b \). By solving the complete linear system of equations, one gets a unique solution for the bare electroweak chiral parameters given by:

\[
a_0^b = \frac{1}{16\pi^2} \frac{3}{8} \left( \Delta \epsilon - \log \frac{M_H^2}{\mu^2} + \frac{5}{6} \right),
\]

\[
a_1^b = \frac{1}{16\pi^2} \frac{1}{12} \left( \Delta \epsilon - \log \frac{M_H^2}{\mu^2} + \frac{5}{6} \right),
\]

\[
a_2^b = \frac{1}{16\pi^2} \frac{1}{24} \left( \Delta \epsilon - \log \frac{M_H^2}{\mu^2} + \frac{17}{6} \right).
\]
\[ a_3^b = \frac{-1}{16\pi^2} \frac{1}{24} \left( \Delta_\epsilon - \log \frac{M_H^2}{\mu^2} + \frac{17}{6} \right), \]

\[ a_4^b = \frac{-1}{16\pi^2} \frac{1}{12} \left( \Delta_\epsilon - \log \frac{M_H^2}{\mu^2} + \frac{17}{6} \right), \]

\[ a_5^b = \frac{M_H^2}{2g^2 M_W^2} - \frac{1}{16\pi^2} \frac{1}{24} \left( \Delta_\epsilon - \log \frac{M_H^2}{\mu^2} + \frac{79}{3} - \frac{27\pi}{2\sqrt{3}} \right), \]

\[ a_{11}^b = \frac{-1}{16\pi^2} \frac{1}{24}, \]

\[ a_6^b = a_7^b = a_8^b = a_9^b = a_{10}^b = a_{12}^b = a_{13}^b = 0. \] (30)

We would like to make some remarks on this result for the chiral parameters:

1. First of all, we agree with the \(1/\epsilon\) dependence of the \(a_i^b\) parameters that was first calculated by Longhitano [6] looking at the divergences of the non-linear sigma model. We see therefore that the divergences generated with the \(L_{NL}\) to one loop are exactly canceled by the \(1/\epsilon\) terms in the \(a_i^b\)’s.

2. The values of \(a_4^b\) and \(a_5^b\) agree with the results given in [10], where the equivalence theorem was used in comparing the scattering amplitudes for Goldstone bosons in the SM [27] and the EChL. These values, on the other hand, do not coincide with the values in [13] where just contributions from pure Higgs loops were considered.

3. It is important to realize that the matching procedure fixes completely the values of the bare parameters \(a_i^b\) in terms of the renormalized parameters of the SM.

4. Eqs.(30) give the complete non-decoupling effects of a heavy Higgs, that is, the leading logarithmic dependence on \(M_H\) and the next to leading constant contribution to the electroweak chiral parameters. The \(a_i\)’s are accurate up to corrections of the order \((p/M_H)\) where \(p \approx M_Z\) and higher order corrections in the perturbative expansion.

5. We demonstrate that the \(a_i\)’s are gauge independent, as expected.

6. There is only one effective operator, the one corresponding to \(a_5\), that gets a tree level contribution. Its expression in terms of renormalized SM parameters depends on the renormalization prescription that one has chosen in the standard model, on-shell in our case. We believe it is important at this point to clarify the relation among the \(a_i^b\)’s that correspond to an on-shell renormalization of the SM and their corresponding values if a different renormalization prescription for the SM is chosen. We will discuss this point in the following section.

By putting together the results of the two, three and four-point functions, one also obtains some relations among the counterterms of the two theories

\[ \Delta Z_W = \frac{-g^2}{16\pi^2} \frac{1}{12} \left( \Delta_\epsilon - \log \frac{M_H^2}{\mu^2} + \frac{5}{6} \right), \]
\[
\Delta Z_B = -\frac{g'^2}{16\pi^2} \frac{1}{12} \left( \Delta_\epsilon - \log \frac{M_H^2}{\mu^2} + \frac{5}{6} \right),
\]

\[
\Delta \xi_W = \Delta Z_W, \quad \Delta \xi_B = \Delta Z_B,
\]

\[
\frac{\Delta g^2}{g^2} = \frac{\Delta g'^2}{g'^2} = 0,
\]

\[
\Delta Z_\phi - 2\frac{\Delta w}{v} = \frac{g^2}{16\pi^2} \left[ -\frac{M_H^2}{8M_W^2} + \frac{3}{4} \left( \Delta_\epsilon - \log \frac{M_H^2}{\mu^2} + \frac{5}{6} \right) + \frac{1}{4} \xi Z \left( \Delta_\epsilon - \log \frac{M_Z^2}{\mu^2} + 1 \right) + \frac{1}{2} \xi W \left( \Delta_\epsilon - \log \frac{M_W^2}{\mu^2} + 1 \right) \right]
\]

(31)

These equations give the differences among the renormalization constants of the SM in the large \(M_H\) limit and those in the EChL, when the on-shell renormalization scheme is chosen in both theories. They are obtained here as a constraint imposed by the matching; one can also calculate them from the explicit expressions of the on-shell counterterms of the two theories and verify that these relations are indeed satisfied.

5 Dependence of the chiral parameters on the renormalization of the SM

In the previous section, we have given the values of the bare electroweak chiral parameters (30) when the SM is renormalized in the on-shell scheme. We would like now to discuss how these bare parameters are changed when a different renormalization scheme is chosen for the SM.

We have seen that the chiral parameter \(a_5\) is the only one that gets a tree level contribution when a heavy Higgs is integrated out, and therefore it is the only one whose expression in terms of the SM renormalized parameters will depend on the renormalization prescription of the SM. By using eqs. (22) and (30), one can rewrite \(a_5\) in terms of the on-shell renormalized scalar self-coupling \(\lambda\):

\[
a_5 = \frac{1}{16\lambda} - \frac{1}{16\pi^2} \frac{1}{24} \left( \Delta_\epsilon + \frac{79}{3} - \frac{27\pi}{2\sqrt{3}} \right).
\]

(32)

The easiest way to connect with a new renormalization scheme for the SM is to write down \(a_5\) in terms of the bare scalar self-coupling \(\lambda^b\). To this end, we first write down the relation \(\lambda\) between the renormalized self-coupling \(\lambda\) and the bare \(\lambda^b\):

\[
\lambda = \lambda^b \left( 1 + \frac{\delta M_H^2}{M_H^2} - \frac{\delta M_W^2}{M_W^2} \right)
\]

(33)

The contribution from the renormalization of \(g\) is of higher order in the large \(M_H\) expansion than the contributions from the renormalization of \(M_H\) and \(M_W\) and will be ignored here.
By substituting the values of the mass counterterms given in (24) in this equation, we get the relation among the bare and the renormalized self-coupling in the on-shell scheme

$$\lambda = \lambda^b \left[ 1 + \frac{1}{16\pi^2} \lambda^b \left( -12 \hat{\Delta}_c - 25 + \frac{9\pi}{\sqrt{3}} \right) \right]$$

(34)

Now, by plugging the last equation into eq.(32) one finds the value of $a^b_5$ in terms of $\lambda^b$

$$a^b_5 = \frac{1}{16\lambda^b} + \frac{1}{16\pi^2} \frac{1}{24} \left( 17 \hat{\Delta}_c + \frac{67}{6} \right).$$

(35)

In order to connect with a different renormalization prescription we simply substitute $\lambda^b$ in eq.(35) by its corresponding definition in terms of the new renormalized self-coupling. For instance, if one chooses the $\overline{\text{MS}}$ scheme, where the scalar self-coupling is defined by

$$\lambda^b = \lambda_{\overline{\text{MS}}} \left[ 1 + \frac{1}{16\pi^2} \lambda_{\overline{\text{MS}}} 12 \hat{\Delta}_c \right]$$

(36)

then $a^b_5$ in terms of the renormalized self-coupling $\lambda_{\overline{\text{MS}}}$ is given by

$$a^b_5 = \frac{1}{16\lambda_{\overline{\text{MS}}}} - \frac{1}{16\pi^2} \frac{1}{24} \left( \hat{\Delta}_c - \frac{67}{6} \right).$$

(37)

and for the rest of chiral parameters one gets exactly the same values as in eq.(30).

Another example is the renormalization prescription chosen by Gasser and Leutwyler in [17]. Their prescription for the renormalized coupling is given by [19]

$$\lambda^b = \lambda_{\text{GL}} \left[ 1 + \frac{1}{16\pi^2} \lambda_{\text{GL}} 12 \left( \hat{\Delta}_c + 1 \right) \right]$$

(38)

and the expression of $a^b_5$ in terms of the renormalized self-coupling $\lambda_{\text{GL}}$ is therefore

$$a^b_5 = \frac{1}{16\lambda_{\text{GL}}} - \frac{1}{16\pi^2} \frac{1}{24} \left( \hat{\Delta}_c + \frac{41}{6} \right).$$

(39)

and the rest of parameters remain again the same.

For comparison, it is interesting to translate the results of eqs.(30,39) to the usual notation in chiral perturbation theory

$$L^b_1 = \frac{l^b_1}{4} = a^b_5 = \frac{1}{16\lambda_{\text{GL}}} - \frac{1}{16\pi^2} \frac{1}{24} \left( \hat{\Delta}_c + \frac{41}{6} \right).$$

$$L^b_2 = \frac{l^b_2}{4} = a^b_4 = \frac{-1}{16\pi^2} \frac{1}{12} \left( \hat{\Delta}_c + \frac{17}{6} \right),$$

$$L^b_9 = \frac{l^b_6}{2} = a^b_3 - a^b_2 = \frac{-1}{16\pi^2} \frac{1}{12} \left( \hat{\Delta}_c + \frac{17}{6} \right),$$

$$L^b_{10} = \frac{l^b_5}{2} = a^b_1 = \frac{1}{16\pi^2} \frac{1}{12} \left( \hat{\Delta}_c + \frac{5}{6} \right).$$

(40)

\[\text{In ref.}[17] \ g_c \ \text{is what we call here} \ \lambda_{\text{GL}}. \ \text{The mass appearing in} \ \hat{\Delta}_c \ \text{is not the physical mass of the Higgs, but what they call} \ M_r. \ \text{The renormalized mass} \ M_r \ \text{was fixed in}[17] \ \text{such that} \ M_r^2 = 2\lambda_{\text{GL}}v^2.\]
These chiral parameters agree with the values found by Gasser and Leutwyler in [17] (see appendix B of this reference) and in [29]. We find it a quite remarkable agreement since they used functional methods to integrate out the Higgs particle in a linear sigma model where the gauge fields were considered as external sources and they used as well different techniques to study the large $M_H$ limit. This confirms the fact already mentioned that the contributions to the effective operators of dimension four that come from mixed gauge-scalar loops are subleading, in the large $M_H$ limit, compared to the pure scalar loops contributions. However, this is not the case for the dimension two custodial breaking operator $a_0$ that gets contributions from mixed gauge-scalar loops [8].

To conclude this section, we emphasize once more that the bare chiral parameters for a given underlying theory, the SM in our case, must be computed with a choice for the renormalization prescription of this theory. The explicit expression of the chiral parameters will vary from one prescription to another, but the numerical value remains the same, and the connection between different prescriptions can be clearly and easily established.

6 Renormalization of the effective theory

In this section we briefly describe the renormalization procedure in the effective theory. Given the effective Lagrangian of eq. (1), the first step is to redefine the fields and parameters of the Lagrangian according to eq. (8). It introduces, at a formal level, the set of counterterms of the effective theory $\delta Z_i, \delta g, \delta h$, etc, that need to be computed once a particular renormalization prescription scheme is chosen. We fix here the on-shell renormalization scheme as we did in the case of the SM. For practical reasons we prefer to choose the renormalization conditions as in reference [30], which are the most commonly used for LEP physics. In terms of the renormalized selfenergies these renormalization conditions read as follows

$$\hat{\Sigma}_{W}(M_W^2) = 0, \quad \hat{\Sigma}_{Z}(M_Z^2) = 0, \quad \hat{\Sigma}_{\gamma}(0) = 0, \quad \hat{\Sigma}_{\gamma Z}(0) = 0.$$

(41)

The renormalized self energies are computed in the effective theory as usual, namely, by adding all the contributions from the one loop diagrams and from the counterterms. We get the following expressions$^{11}$:

$$\hat{\Sigma}_{\gamma}(q^2) = \hat{\Sigma}_{\gamma}(q^2) + \left( s^2 \delta \hat{Z}_W + c^2 \delta \hat{Z}_B \right) q^2 + s^2 g^2 (a_8^b - 2a_1^b) q^2. $$

$$\hat{\Sigma}_{W}(q^2) = \hat{\Sigma}_{W}(q^2) + \delta \hat{Z}_W \left( q^2 - M_W^2 \right) - \delta \hat{M}_W. $$

$$\hat{\Sigma}_{Z}(q^2) = \hat{\Sigma}_{Z}(q^2) + \left( c^2 \delta \hat{Z}_W + s^2 \delta \hat{Z}_B \right) \left( q^2 - M_Z^2 \right) - \delta \hat{M}_Z + 2g^2 a_0^b M_Z^2 + \left( 2s^2 g^2 a_1^b + c^2 g^2 a_8^b + (g^2 + g'^2) a_{13}^b \right) q^2.$$

$^{11}$Notice that in our rotation defining the physical gauge fields, the terms in $s$ have different sign than in reference [30].
\[ \hat{\Sigma}_{\gamma Z}^R(q^2) = \hat{\Sigma}_{\gamma Z}^L(q^2) + sc \left( \delta \hat{Z}_W - \delta \hat{Z}_B \right) q^2 - sc \ M_Z^2 \left( \frac{\hat{\delta} g'}{g} - \frac{\hat{\delta} g}{g} \right) + \left( scg^2a_s^b - (c^2 - s^2)gg'a_1^b \right) q^2. \]  

(42)

where

\[ \delta \hat{M}_W^2 = M_W^2 \left( \delta \hat{Z}_W - 2 \frac{\hat{\delta} g}{g} - 2 \frac{\hat{\delta} v}{v} - \delta \hat{Z}_W \right), \]

\[ \delta \hat{M}_Z^2 = M_Z^2 \left( \delta \hat{Z}_W - 2c^2 \frac{\hat{\delta} g}{g} - 2s^2 \frac{\hat{\delta} g'}{g'} - 2 \frac{\hat{\delta} v}{v} - c^2 \delta \hat{Z}_W - s^2 \delta \hat{Z}_B \right), \]

\[ M_W^2 = g^2 v^2/4, \]

\[ M_Z^2 = (g^2 + g'^2) v^2/4, \]  

(43)

and the superscripts R and L denote renormalized and EChL loops respectively.

From eq.(43) the following relation among the W and Z mass counterterms is obtained

\[ \frac{\delta \hat{M}_Z^2}{M_Z^2} - \frac{\delta \hat{M}_W^2}{M_W^2} = 2s^2 \frac{\hat{\delta} g}{g} + 2c^2 \frac{\hat{\delta} g'}{g'} + s^2 \left( \delta \hat{Z}_W - \delta \hat{Z}_B \right) \]  

(44)

Finally, by requiring these renormalized self energies to fulfill eqs.(43) and taking into account that the \( U(1)_Y \) Ward identity implies \( \delta \hat{g'} = 0 \) one gets the following results for the values of the counterterms in terms of the unrenormalized selfenergies of the effective theory and the bare \( a_i \)’s:

\[ \delta \hat{M}_W^2 = \hat{\Sigma}_{\gamma W}(M_W^2), \]

\[ \delta \hat{M}_Z^2 = \hat{\Sigma}_{\gamma Z}(M_Z^2) + M_Z^2 \left( 2g_0^2 a_0^b + 2s^2 g^2 a_1^b + c^2 g^2 a_s^b + (g^2 + g'^2)a_{13}^b \right), \]

\[ \frac{\hat{\delta} g}{g} = - \frac{1}{sc} \frac{\hat{\Sigma}_{\gamma Z}^L(0)}{M_Z^2}, \]

\[ \frac{\hat{\delta} g'}{g'} = 0, \]

\[ \delta \hat{Z}_W = \frac{c^2}{s^2} \left( \frac{\hat{\Sigma}_{\gamma Z}(M_Z^2)}{M_Z^2} - \frac{\hat{\Sigma}_{\gamma W}(M_W^2)}{M_W^2} \right) + \frac{2}{s} \frac{\hat{\Sigma}_{\gamma Z}^L(0)}{M_Z^2} - \hat{\Sigma}_{\gamma Z}^L(0) + 2g_0^2 a_0^b + 2s^2 g^2 a_1^b + \frac{c^2 - s^2}{s^2} g^2 a_s^b + \frac{c^2}{s^2} (g^2 + g'^2)a_{13}^b, \]

\[ \delta \hat{Z}_B = \frac{\hat{\Sigma}_{\gamma W}(M_W^2)}{M_W^2} - \frac{\hat{\Sigma}_{\gamma Z}(M_Z^2)}{M_Z^2} - 2 \frac{s}{c} \frac{\hat{\Sigma}_{\gamma Z}^L(0)}{M_Z^2} - \hat{\Sigma}_{\gamma L}^L(0) \]

\[ - \left( 2g_0^2 a_0^b + g^2 a_s^b + (g^2 + g'^2)a_{13}^b \right). \]  

(45)

Now that we have at hand eqs.(43) the only parameters of the theory that still need to be renormalized are the electroweak chiral parameters \( a_i \). The following formal redefinition of the
The divergent part of the $a^b_i$ parameters, or equivalently the divergent part of the counterterms $\delta a_i$, are fixed by the symmetries of the effective theory and since the work of Longhitano [3] they are known to be

$$
\delta a_0|_{\text{div}} = \frac{1}{16\pi^2} \frac{3}{8} \Delta\epsilon, \quad \delta a_1|_{\text{div}} = \frac{1}{16\pi^2} \frac{1}{12} \Delta\epsilon, \\
\delta a_2|_{\text{div}} = \frac{1}{16\pi^2} \frac{1}{24} \Delta\epsilon, \quad \delta a_3|_{\text{div}} = \frac{1}{16\pi^2} \frac{1}{24} \Delta\epsilon, \\
\delta a_4|_{\text{div}} = -\frac{1}{16\pi^2} \frac{1}{12} \Delta\epsilon, \quad \delta a_5|_{\text{div}} = -\frac{1}{16\pi^2} \frac{1}{24} \Delta\epsilon, \\
\delta a_i|_{\text{div}} = 0, \quad i = 6, ..., 13.
$$

(47)

These universal divergent contributions to the chiral bare parameters imply in turn universal renormalization group equations for the renormalized parameters

$$
a_0(\mu) = a_0(\mu') + \frac{1}{16\pi^2} \frac{3}{8} \log \frac{\mu^2}{\mu'^2}, \quad a_1(\mu) = a_1(\mu') + \frac{1}{16\pi^2} \frac{1}{12} \log \frac{\mu^2}{\mu'^2}, \\
a_2(\mu) = a_2(\mu') + \frac{1}{16\pi^2} \frac{1}{24} \log \frac{\mu^2}{\mu'^2}, \quad a_3(\mu) = a_3(\mu') - \frac{1}{16\pi^2} \frac{1}{24} \log \frac{\mu^2}{\mu'^2}, \\
a_4(\mu) = a_4(\mu') - \frac{1}{16\pi^2} \frac{1}{12} \log \frac{\mu^2}{\mu'^2}, \quad a_5(\mu) = a_5(\mu') - \frac{1}{16\pi^2} \frac{1}{24} \log \frac{\mu^2}{\mu'^2}, \\
a_i(\mu) = a_i(\mu'); \quad i = 6, ..., 13.
$$

(48)

The value of the bare chiral parameters $a^b_i$, on the other hand, is completely determined by the matching procedure in terms of the renormalized parameters of the underlying physics that has been integrated out, as we have seen for the particular case of a heavy Higgs. However, for a given $a^b_i$, we still have to choose how to separate the finite part into the renormalized $a_i(\mu)$ and the counterterm $\delta a_i$ in eq.(46) such that their sum gives $a^b_i$. This second renormalization scheme concerns only to the effective theory. Therefore, in using a set of renormalized parameters $a_i(\mu)$ for a particular underlying theory, one must specify, in addition, how the finite parts of the counterterms in eq.(46) have been fixed.

In the case of the SM, where a heavy Higgs has been integrated out to one loop, the bare chiral parameters are given in eq.(30). They correspond to the on-shell renormalization of the underlying SM. Now, in order to present the corresponding renormalized parameters we have to fix the finite parts of the counterterms. For instance, if we fix the counterterms to include just the $\Delta\epsilon$ terms as in eq.(17), the renormalized chiral parameters for the SM with a heavy Higgs
Another example is the renormalization scheme chosen by Gasser and Leutwyler for the linear $O(N)$ sigma model in ref.\cite{17}. The values of the bare parameters for their choice of the renormalization scheme of the underlying sigma model were given in eqs.(40). Now, they fix instead the chiral counterterms to the following values:

$$
\delta L_1 = \frac{\delta l_1}{4} = \delta a_{5}^{GL} = \frac{1}{16\pi^2} \left( \frac{11}{6} - \log \frac{M_{GL}^2}{\mu^2} \right), \\
\delta L_2 = \frac{\delta l_2}{4} = \delta a_{4}^{GL} = \frac{1}{16\pi^2} \left( \frac{11}{6} - \log \frac{M_{GL}^2}{\mu^2} \right), \\
\delta L_9 = -\frac{\delta l_6}{2} = \delta a_{3}^{GL} - \delta a_{2}^{GL} = \frac{1}{16\pi^2} \left( \frac{11}{6} - \log \frac{M_{GL}^2}{\mu^2} \right), \\
\delta L_{10} = \delta l_5 = \delta a_{1}^{GL} = \frac{1}{16\pi^2} \left( \frac{11}{6} - \log \frac{M_{GL}^2}{\mu^2} \right).
$$

Consequently, they get the following renormalized chiral parameters for the sigma model\cite{17}:

$$
L_1(\mu) = \frac{l_1(\mu)}{4} = a_{5}^{GL}(\mu) = \frac{1}{16\lambda_{GL}} - \frac{1}{16\pi^2} \left( \frac{35}{6} - \log \frac{M_{GL}^2}{\mu^2} \right), \\
L_2(\mu) = \frac{l_2(\mu)}{4} = a_{4}^{GL}(\mu) = \frac{1}{16\pi^2} \left( \frac{11}{6} - \log \frac{M_{GL}^2}{\mu^2} \right), \\
L_9(\mu) = -\frac{l_6(\mu)}{2} = a_{3}^{GL}(\mu) - a_{2}^{GL}(\mu) = \frac{1}{16\pi^2} \left( \frac{11}{6} - \log \frac{M_{GL}^2}{\mu^2} \right), \\
L_{10}(\mu) = l_5(\mu) = a_{1}^{GL}(\mu) = \frac{1}{16\pi^2} \left( \frac{11}{6} - \log \frac{M_{GL}^2}{\mu^2} \right).
$$

\footnote{This particular renormalization of the chiral parameters was chosen in our previous work, where we called it $\overline{MS}$.} 

\footnote{Here we call $M_{GL}$ the renormalized mass $M_r$ of [17].}
7 Calculating observables with the EChL

In this section we will show, as an example, the explicit calculation of the radiative corrections to $\Delta \rho$ and $\Delta r$ within the electroweak chiral Lagrangian approach. These observables are defined in the effective theory in terms of the renormalized self-energies in the same way as in the fundamental SM, namely:

$$\Delta \rho \equiv \frac{\hat{\Sigma}^R_Z(0)}{M_Z^2} - \frac{\hat{\Sigma}^R_W(0)}{M_W^2},$$

$$\Delta r \equiv \frac{\hat{\Sigma}^R_W(0)}{M_W^2} + (\text{vertex + box}),$$

where

$$(\text{vertex + box}) \equiv \frac{g^2}{16\pi^2} \left(6 + \frac{7-4s^2}{2s^2} \log c^2\right),$$

and the renormalized self-energies can be computed as we have explained in section 6.

Once a renormalization scheme has been chosen, one can always express $\Delta \rho$ and $\Delta r$ in terms of unrenormalized self-energies and the $a_i^b$'s. For instance, in the on-shell scheme given by the conditions of eq.(41), one gets the particular values of the counterterms given in eqs.(45). Next, by plugging these counterterms into eqs.(42), one obtains the renormalized self-energies in terms of the unrenormalized ones and the $a_i^b$'s. Finally, by substituting these formulas into eqs.(52) the following expressions for $\Delta \rho$ and $\Delta r$ in the on-shell scheme are found

$$\Delta \rho = \frac{\hat{\Sigma}^L_Z(0)}{M_Z^2} - \frac{\hat{\Sigma}^L_W(0)}{M_W^2} + \frac{2s}{c} \frac{\hat{\Sigma}^L_{\gamma Z}(0)}{M_Z^2} + 2g^2a_0^b,$$

$$\Delta r = \frac{\hat{\Sigma}^L_W(0)}{M_W^2} - \frac{\hat{\Sigma}^L_W(M_W^2)}{M_W^2} + \frac{c^2}{s^2} \left[\frac{\hat{\Sigma}^L_W(M_W^2)}{M_W^2} - \frac{\hat{\Sigma}^L_Z(M_Z^2)}{M_Z^2} - \frac{2s}{c} \frac{\hat{\Sigma}^L_{\gamma Z}(0)}{M_Z^2}\right]$$

$$-2g^2 a_0^b + \frac{s^2 - c^2}{s^2} g^2(a_8^b + a_{13}^b) - 2g^2(a_1^b + a_{13}^b) + (\text{vertex + box}).$$

The explicit computation of the bosonic loop contributions in the effective theory, as well as the contributions from just $a_0^b$, and $a_1^b$ were found in [12,13]. We present here the complete result

$$\Delta \rho = \frac{g^2}{16\pi^2} \left[\frac{3s^2}{4c^2} \left(-\Delta_\epsilon + \log \frac{M_W^2}{\mu^2}\right) + h(M_W^2, M_Z^2)\right] + 2g^2a_0^b,$$

$$\Delta r = \frac{g^2}{16\pi^2} \left[\frac{11}{12} \left(\Delta_\epsilon - \log \frac{M_W^2}{\mu^2}\right) + f(M_W^2, M_Z^2)\right]$$

$$-2g^2 a_0^b + \frac{s^2 - c^2}{s^2} g^2(a_8^b + a_{13}^b) - 2g^2(a_1^b + a_{13}^b).$$
where
\[
\begin{align*}
  h(M_W^2, M_Z^2) &= \frac{1}{c^2} \log c^2 \left( \frac{17}{4s^2} - 7 + 2s^2 \right) + \frac{17}{4} - \frac{5s^2}{8c^2}, \\
  f(M_W^2, M_Z^2) &= \log c^2 \left( \frac{5}{c^2} - 1 + \frac{3c^2}{s^2} - \frac{17}{4s^2c^2} \right) - s^2(3 + 4c^2)F(M_Z^2, M_W, M_W) \\
  &+ I_2(c^2)(1 - \frac{c^2}{s^2}) + \frac{c^2}{s^2}I_1(c^2) + \frac{1}{8c^2}(43s^2 - 38) \\
  &+ \frac{1}{18}(154s^2 - 166c^2) + \frac{1}{4c^2} + \frac{1}{6} + \Delta \alpha + \left( 6 + \frac{7 - 4s^2}{2s^2} \log c^2 \right). \tag{55}
\end{align*}
\]

and \(F, I_1, I_2\) and \(\Delta \alpha\) can be found in [31]. In eqs.(54) there are apparently a divergent term and a \(\mu\)-scale dependent term. However, when one redefines the bare effective chiral parameters as usual, \(a_i^b = a_i(\mu) + \delta a_i\), it can be easily seen that the divergent terms are cancelled by the divergent parts of the \(\delta a_i\) and the \(\mu\)-scale dependence is cancelled by the scale dependence of the \(a_i(\mu)\). The observables \(\Delta \rho\) and \(\Delta r\) turn out to be finite and scale and renormalization prescription independent, as it must be. In particular, if we set the substraction scheme for the chiral counterterms to include just the \(\Delta \epsilon\) terms as in eq.(47), the following expressions for \(\Delta \rho\) and \(\Delta r\) in terms of renormalized chiral parameters are obtained
\[
\begin{align*}
  \Delta \rho &= \frac{g^2}{16\pi^2} \left[ \frac{3}{4} s^2 \log \frac{M_W^2}{\mu^2} + h(M_W^2, M_Z^2) \right] + 2g^2a_0(\mu), \\
  \Delta r &= \frac{g^2}{16\pi^2} \left[ -\frac{11}{12} \log \frac{M_W^2}{\mu^2} + f(M_W^2, M_Z^2) \right] \\
  &- 2g^2a_0(\mu) + \frac{s^2 - c^2}{s^2}g^2(a_8 + a_{13}) - 2g^2(a_1(\mu) + a_{13}). \tag{56}
\end{align*}
\]

Equations (56) are general and can be applied to any underlying physics for the symmetry breaking sector. If we want to recover the values of \(\Delta \rho\) and \(\Delta r\) in the particular case of the SM with a heavy Higgs, one just has to substitute the values of the chiral parameters in eqs.(49) into eqs.(56) to obtain
\[
\begin{align*}
  \Delta \rho &= \frac{g^2}{16\pi^2} \left[ -\frac{3}{4} s^2 \left( \log \frac{M_H^2}{M_W^2} - \frac{5}{6} \right) + h(M_W^2, M_Z^2) \right], \\
  \Delta r &= \frac{g^2}{16\pi^2} \left[ \frac{11}{12} \left( \log \frac{M_H^2}{M_W^2} - \frac{5}{6} \right) + f(M_W^2, M_Z^2) \right]. \tag{57}
\end{align*}
\]

which agrees with the result given in [31].

One can similarly obtain the heavy Higgs contributions to other relevant observables in electroweak phenomenology.
8 Conclusions

Given the present situation of remarkable improvement in electroweak precision measurements and the encouraging prospects for the future, we believe it is now imperative a good understanding of the SM Higgs boson radiative corrections. A heavy Higgs boson do not decouple from the low energy electroweak observables and therefore will leave some trace on them that could be observed with the future precision measurements.

In this paper, we have calculated the complete non-decoupling effects of the SM Higgs boson to one loop. They consist of the already known leading logarithmic Higgs mass dependent effects and the next to leading constant terms. Both effects are relevant and for not too large $M_H$ values can be of comparable magnitude.

We have classified these non-decoupling effects using the EChL approach, an effective field theory that respects the SM symmetries at low energies. Within this approach, the non-decoupling effects of a heavy Higgs boson are represented, at energies below the Higgs mass, by a subset of gauge invariant effective operators of the EChL. We have calculated in this work the values of the parameters of these chiral operators that represent the SM Higgs. It is our main result and is given in eq.(30). We get just seven non-vanishing relevant parameters summarizing the whole set of non-decoupling heavy Higgs effects to one loop. We have also discussed in detail in this work the relation between the renormalization of both the SM and the effective theory, with special emphasis in the on-shell scheme, that has been our particular choice here.

In conclusion, we believe that the approach followed in this work is interesting and useful because it provides a gauge invariant way of separating the non-decoupling Higgs boson effects from the rest of the EW radiative corrections and, on the other hand, it is a general framework in which one can analyze the low energy effects not only of an SM heavy Higgs, but of alternative strongly interacting symmetry breaking scenarios. The EChL that parametrizes the SM Higgs will then serve as a fundamental reference model.

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Appendix A

In this appendix, we give an explicit example of the large-$M_H$ techniques used in the calculation of the loop integrals.

We start by giving the m-theorem of G. Giavarini, C. P. Martin and F. Ruiz Ruiz \[32\] (reduced to the case of 1-loop integrals), which gives sufficient conditions for a loop integral to vanish in the large-m limit. Consider an integral of the form

\[
I(p, m) = m^\beta \int d^4k \frac{M(k)}{\prod(l_i^2 + m_i^2)^{m_i}}, \tag{A.1}
\]

where

\[
l_i = k + \sum_{j=1}^E b_{ij} p_j
\]

\[
m_i = 0 \text{ or } m \tag{A.2}
\]

and $M(k)$ is a monomial in the components of $k$. $\beta$ denote an arbitrary real number. The external momenta $p_1, ..., p_E$ lay in a bounded subdomain of $\mathbb{R}^4$. Let $d$ be the mass dimension of $I(p, m)$ and $\omega$ the minimum of zero and the infrared degree of $I(p, m)$ at zero external momenta. Then m-theorem: If the integral $I(p, m)$ is both UV and IR convergent by power counting at non-exceptional external momenta and $d - \omega < 0$, then $I(p, m)$ goes to zero when $m$ goes to infinity.

As an example, consider the $(H - \phi)$ loop correction to the $W$ self-energy given in fig.(2):

\[
e^2 4s^2 \mu^{4-D} \int \frac{d^Dk}{(2\pi)^D} \frac{(2k + q)_\mu (2k + q)_\nu}{[k^2 - \xi M_W^2][(k + q)^2 - M_H^2]} \tag{A.3}
\]

where $q$ is the $W$ external momentum and $D$ denotes the space-time dimension in dimensional regularization.

Let’s work out explicitly the large $M_H$ expansion of the most divergent part of this correction which comes from the integral

\[
I_{\mu\nu} = \mu^{4-D} \int \frac{d^Dk}{(2\pi)^D} \frac{k_\mu k_\nu}{[k^2 - \xi M_W^2][(k + q)^2 - M_H^2]} \tag{A.4}
\]

The superficial degree of UV divergence of $I_{\mu\nu}$ is 2 at $D = 4$. The first step is to rearrange algebraically the denominator that includes a light mass

\[
\frac{1}{k^2 - \xi M_W^2} = \frac{1}{k^2} + \frac{\xi M_W^2}{k^2(k^2 - \xi M_W^2)} \tag{A.5}
\]

so that one gets a term in the integral with the same degree of UV divergence but where the light mass is no more in the denominator, and a second term with still a light mass dependence but with the degree of UV divergence lowered by two. Applying this algebraic rearrangement as
many times as necessary until the last term gives a convergent integral (twice in the case of $I_{\mu \nu}$), one gets

$$I_{\mu \nu} = \mu^{4-D} \int \frac{d^D k}{(2\pi)^D} \left[ \frac{k_{\mu}k_{\nu}}{k^2[(k+q)^2-M_H^2]} + \frac{\xi M_W^2 k_{\mu}k_{\nu}}{k^4[(k+q)^2-M_H^2]} + \frac{\xi^2 M_W^4 k_{\mu}k_{\nu}}{k^4[k^2-\xi M_W^2][(k+q)^2-M_H^2]} \right]$$

$$= A_{\mu \nu} + B_{\mu \nu} + C_{\mu \nu}$$

Now the last term $C_{\mu \nu}$ is already UV and IR convergent. One can apply the Lebesgue dominated convergence theorem to see that the heavy Higgs mass limit can be safely taken inside the integral in this term, and therefore,

$$C_{\mu \nu} \to 0 \quad \text{when} \quad M_H \to \infty$$

Let’s develop now the term $A_{\mu \nu}$, (the other term $B_{\mu \nu}$ can be evaluated using the same procedure). We rewrite the denominators in $A_{\mu \nu}$, using again an algebraic identity

$$\frac{1}{(k+q)^2-M_H^2} = \frac{1}{k^2-M_H^2} - \frac{2kq+q^2}{k^2-\xi M_W^2}$$

One has to apply this identity as many times as necessary until one gets an integral that fulfills the conditions of the m-theorem. Using this identity three times in $A_{\mu \nu}$ one get’s

$$A_{\mu \nu} = \mu^{4-D} \int \frac{d^D k}{(2\pi)^D} \left[ \frac{k_{\mu}k_{\nu}}{k^2[k^2-M_H^2]} - \frac{k_{\mu}k_{\nu}(2kq+q^2)}{k^2[k^2-M_H^2]^2} + \frac{k_{\mu}k_{\nu}(2kq+q^2)^2}{k^2[k^2-M_H^2]^3} - \frac{k_{\mu}k_{\nu}(2kq+q^2)^3}{k^2[k^2-M_H^2]^3[(k+q)^2-M_H^2]} \right]$$

Now the last integral is finite at $D = 4$ and satisfies the requirements of the m-theorem. It then goes to zero as $D \to 4$ and $M_H \to \infty$. The other three terms in $A_{\mu \nu}$ can be evaluated using standard techniques, and one gets

$$A_{\mu \nu} = \frac{i}{16\pi^2} \left[ g_{\mu \nu} M_H^2 \left( \frac{3}{2} \right) - g_{\mu \nu} q^2 \left( \frac{1}{12} \Delta_\epsilon + \frac{5}{6} \right) + g_{\mu \nu} q^2 \left( \frac{1}{3} \Delta_\epsilon + \frac{1}{3} \right) \right]$$

Using the same techniques to evaluate the rest of terms one finally gets the large $M_H$ contribution of the $(H-\phi)$ correction to the $W$ self energy

$$\frac{e^2}{s^2} \frac{i}{16\pi^2} \left[ g_{\mu \nu} M_H^2 \left( \frac{3}{2} \right) + g_{\mu \nu} \xi M_W^2 \left( \frac{3}{2} \right) - g_{\mu \nu} q^2 \left( \frac{1}{12} \Delta_\epsilon + \frac{5}{6} \right) + g_{\mu \nu} q^2 \left( \frac{1}{12} \Delta_\epsilon + \frac{4}{3} \right) \right]$$

As a final comment, we would like to point a difference between these large-m techniques and the commonly used expansion in powers of the external momenta $q$. This large-m calculation gives us all the existing contributions up to an arbitrary power in the external momenta $q$, as far as they do not vanish in the large-$M_H$ limit. On the other hand, these techniques provide an extremely easy way to extract the non-vanishing large mass effects of any loop integral.
Appendix B

In this appendix, we present the results of the various terms contributing to the matching equations \( \ref{eq:matching-eq} \). Since we are interested mainly in the differences between the SM terms and the corresponding ones of the EChL, it is convenient to rewrite eq.\( \ref{eq:matching-eq} \) in the following form:

\[
\left( iM_{abcd}^{\mu\nu\rho\lambda} - \hat{iM}_{abcd}^{\mu\nu\rho\lambda} \right) + \left( iM_{abcd}^{C\mu\nu\rho\lambda} - \hat{iM}_{abcd}^{C\mu\nu\rho\lambda} \right) + \left( iM_{abcd}^{L\mu\nu\rho\lambda} - \hat{iM}_{abcd}^{L\mu\nu\rho\lambda} \right) = 0 \tag{B.1}
\]

The tree diagrams contributing to \( (iM^T - \hat{iM}^T) \) are displayed in fig.(3). The contributions from the counterterms are generated using eq.\( \ref{eq:counterterms-1PI} \) for the 1PI Green’s functions of the effective theory, \( \hat{iM}^C \), and eq.\( \ref{eq:counterterms-1LPI} \) for the 1LPI Green’s functions of the SM, \( iM^C \). For the difference of counterterms, we use the following notation:

\[
\Delta Q \equiv \delta Q - \hat{\delta Q} \quad \text{with} \quad Q = Z_B, Z_W, g^2, \text{etc}...
\]

The loop contributions to \( (iM^L - \hat{iM}^L) \) come from the explicit evaluation of all the one loop diagrams in fig.(3) in the large \( M_H \) limit. To perform this calculation we have used the techniques described in appendix A.

\[ \gamma\gamma WW \]

There are no differences in the tree level contributions

\[
\left( iM_{\gamma\gamma WW}^{T\mu\nu\rho\lambda} - \hat{iM}_{\gamma\gamma WW}^{T\mu\nu\rho\lambda} \right) = 0. \tag{B.2}
\]

The contributions from the counterterms are

\[
\left( iM_{\gamma\gamma WW}^{C\mu\nu\rho\lambda} - \hat{iM}_{\gamma\gamma WW}^{C\mu\nu\rho\lambda} \right) = -i g^4 s^2 \left[ 2 \left( \Delta Z_W - \frac{\Delta g^2}{g^2} \right) \frac{1}{g^2} \right] g^{\mu\nu} g^{\lambda\rho} \\
+ i g^4 s^2 \left( \Delta Z_W - \frac{\Delta g^2}{g^2} \right) \frac{1}{g^2} + a_{11}^\rho \left( g^{\mu\rho} g^{\lambda\nu} + g^{\lambda\mu} g^{\nu\rho} \right). \tag{B.3}
\]

The evaluation of diagrams in fig.(3.a) gives

\[
\left( iM_{\gamma\gamma WW}^{L\mu\nu\rho\lambda} - \hat{iM}_{\gamma\gamma WW}^{L\mu\nu\rho\lambda} \right) = \sum_{i=1}^{12} L_i + \sum_{j=1}^{3} (D_j - \hat{D}_j) = \\
-\frac{i g^4 s^2}{16 \pi^2} \left[ \frac{1}{6} \left( \hat{\Delta}_e + \frac{5}{6} \right) \right] g^{\mu\nu} g^{\lambda\rho} + \\
\frac{i g^4 s^2}{16 \pi^2} \left[ \frac{1}{12} \left( \hat{\Delta}_e + \frac{4}{3} \right) \right] \left( g^{\mu\rho} g^{\lambda\nu} + g^{\lambda\mu} g^{\nu\rho} \right). \tag{B.4}
\]

\[ ^{14} \text{We denote by } iM \text{ the direct result from Feynman diagrams} \]
There are no differences at tree level

\( \left( i M^{\mu \nu \rho \lambda}_{\gamma ZWW} - i \tilde{M}^{\mu \nu \rho \lambda}_{\gamma ZWW} \right) = 0. \)  

(B.5)

The contributions from the counterterms are

\[
\left( i M^{C, \mu \nu \rho \lambda}_{\gamma ZWW} - i \tilde{M}^{C, \mu \nu \rho \lambda}_{\gamma ZWW} \right) = -i g^4 s \left[ 2 \left( \Delta Z_W - \frac{\Delta g^2}{g^2} \right) \frac{c^2}{g^2} + 2a_3^b \right] g^{\mu \nu} g^{\lambda \rho} + i g^4 s \left[ \left( \Delta Z_W - \frac{\Delta g^2}{g^2} \right) \frac{1}{g^2} + a_{11}^b \right] c^2 
+ a_3^b - a_{11}^b + a_{12}^b \left( g^{\mu \rho} g^{\lambda \nu} + g^{\lambda \mu} g^{\nu \rho} \right). \]

(B.6)

The evaluation of diagrams in fig.(3.b) gives

\[
\left( i M^{L, \mu \nu \rho \lambda}_{\gamma ZWW} - i \tilde{M}^{L, \mu \nu \rho \lambda}_{\gamma ZWW} \right) = \sum_{i=1}^{18} L_i + \sum_{j=1}^{3} (D_j - \tilde{D}_j) =
-ig^4 s \frac{1}{16 \pi^2} \left[ \frac{1}{12} \Delta \epsilon (2c^2 + 1) + \frac{1}{72} (10c^2 + 17) \right] g^{\mu \nu} g^{\lambda \rho} 
+ ig^4 s \frac{1}{16 \pi^2} \left[ \frac{1}{12} \Delta \epsilon (c^2 + \frac{1}{2}) + \frac{1}{36} (4c^2 + \frac{11}{4}) \right] (g^{\mu \rho} g^{\lambda \nu} + g^{\lambda \mu} g^{\nu \rho}). \]

(B.7)

**ZZWW**

In this case, there are already differences at tree level since there is one diagram, \( T_1 \) in fig.(3c), with a tree level exchange of a Higgs boson. The large \( M_H \) limit of this diagram gives

\[
\left( i M^{T, \mu \nu \rho \lambda}_{ZZWW} - i \tilde{M}^{T, \mu \nu \rho \lambda}_{ZZWW} \right) = T_1 = \frac{i g^2 M_W^2}{c^2 M_H^2} g^{\mu \nu} g^{\lambda \rho} \]  

(B.8)

The contributions from the counterterms are

\[
\left( i M^{C, \mu \nu \rho \lambda}_{ZZWW} - i \tilde{M}^{C, \mu \nu \rho \lambda}_{ZZWW} \right) = +ig^4 c \left[ -2 \left( \Delta Z_W - \frac{\Delta g^2}{g^2} \right) \frac{c^4}{g^2} - 4c^2 a_3^b - 2(a_5^b + a_7^b) + \delta S \right] g^{\mu \nu} g^{\lambda \rho} 
+ ig^4 c \left[ \left( \Delta Z_W - \frac{\Delta g^2}{g^2} \right) \frac{1}{g^2} + a_{11}^b \right] c^4 - 2c^2 (-a_3^b + a_{11}^b - a_{12}^b) 
- (a_4^b + a_6^b - a_{11}^b + 2a_{12}^b) \left( g^{\mu \rho} g^{\lambda \nu} + g^{\lambda \mu} g^{\nu \rho} \right). \]

(B.9)

where \( \delta S \) is given by the following combination of SM counterterms

\[
\delta S = \frac{M_W^2}{g^2 M_H^2} \left( - \frac{\delta M_H^2}{M_H^2} + \frac{\delta T/v}{M_H^2} + \frac{\delta M_W^2}{M_W^2} \right). \]

26
and the SM counterterms $\delta M^2_H, \delta T$ and $\delta M^2_H$ are given in eq. (24). The evaluation of the diagrams in fig.(3.c) gives

$$
(i M^L_{ZZWW} - i \tilde{M}^L_{ZZWW}) = \sum_{i=1}^{47} L_i + \sum_{j=1}^{6} (D_j - \tilde{D}_j) = 
$$

$$
i g^4 \frac{1}{c^2} \frac{1}{16\pi^2} \left[ \frac{1}{6} \hat{\Delta}_\epsilon (-c^4 - c^2 + \frac{43}{4}) + \frac{1}{36}(-5c^4 - 17c^2 + 47) \right] g^{\mu\nu} g^{\lambda\rho} 
+ i g^4 \frac{1}{c^2} \frac{1}{16\pi^2} \left[ \frac{1}{12} \hat{\Delta}_\epsilon (c^4 + c^2 - 1) + \frac{1}{72}(8c^4 + 11c^2 - 14) \right] 
(g^{\mu\rho} g^{\lambda\nu} + g^{\lambda\mu} g^{\nu\rho}) . \tag{B.10}
$$

The differences at tree level are given by diagrams $T_1$ and $T_2$ of fig.(3.d). In the large $M_H$ limit we get

$$
(i M^T_{WWWW} - i \tilde{M}^T_{WWWW}) = T_1 + T_2 = ig^2 \frac{M^2_H}{M_H} (g^{\mu\nu} g^{\lambda\rho} + g^{\lambda\mu} g^{\nu\rho}) . \tag{B.11}
$$

The contributions from the counterterms are

$$
(i M^C_{WWWWW} - i \tilde{M}^C_{WWWWW}) = ig^4 \left( (\Delta Z_W - \Delta g^2/g^2) \frac{1}{g^2} + 2a_3^b - a_4^b - a_8^b + 2a_9^b - 2a_{13}^b \right) 2g^{\mu\nu} g^{\lambda\rho} 
+ ig^4 \left[ - (\Delta Z_W - \Delta g^2/g^2) \frac{1}{g^2} - 2a_3^b - a_4^b - 2a_5^b + a_8^b - 2a_9^b 
+ 2a_{13}^b + \delta S \right] (g^{\mu\rho} g^{\lambda\nu} + g^{\lambda\mu} g^{\nu\rho}) . \tag{B.12}
$$

The one loop diagrams are displayed in fig.(3.d) and their evaluation in the large $M_H$ limit gives

$$
(i M^L_{WWWWW} - i \tilde{M}^L_{WWWWW}) = \sum_{i=1}^{58} L_i + \sum_{j=1}^{12} (D_j - \tilde{D}_j) = 
$$

$$
i g^4 \frac{1}{16\pi^2} \left[ \frac{1}{12} \hat{\Delta}_\epsilon + \frac{5}{6} \right] 2g^{\mu\nu} g^{\lambda\rho} 
+ i g^4 \frac{1}{16\pi^2} \left[ \frac{37}{24} \hat{\Delta}_\epsilon + \frac{55}{72} \right] (g^{\mu\rho} g^{\lambda\nu} + g^{\lambda\mu} g^{\nu\rho}) . \tag{B.13}
$$

The differences at tree level are given by diagrams $T_1, T_2$ and $T_3$ of fig.(3.e). In the large $M_H$ limit we get

$$
(i M^T_{ZZZZ} - i \tilde{M}^T_{ZZZZ}) = T_1 + T_2 + T_3 = ig^2 \frac{M^2_H}{c^2 M_H} (g^{\mu\nu} g^{\rho\lambda} + g^{\mu\rho} g^{\nu\lambda} + g^{\lambda\mu} g^{\nu\rho}) . \tag{B.14}
$$
The contributions from the counterterms are

\[
(i M^{C \mu \nu \rho \lambda} - i \tilde{M}^{C \mu \nu \rho \lambda}) = \frac{g^4}{c^4} \left[ -2(a^b_4 + a^b_5) - 4(a^b_6 + a^b_7 + 2a^b_{10}) + \delta S \right] (g^{\mu \nu} g^{\rho \lambda} + g^{\mu \rho} g^{\lambda \nu} + g^{\lambda \mu} g^{\nu \rho}) .
\]  

(B.15)

The one loop diagrams are displayed in fig. (3.e) and their evaluation in the large \( M_H \) limit gives

\[
(i M^{L \mu \nu \rho \lambda} - i \tilde{M}^{L \mu \nu \rho \lambda}) = \sum_{i=1}^{75} L_i + \sum_{j=1}^{18} (D_j - \tilde{D}_j) = \frac{g^4}{c^4} \frac{1}{16\pi^2} \left[ 13\Delta_\epsilon + \frac{20}{3} \right] (g^{\mu \nu} g^{\rho \lambda} + g^{\mu \rho} g^{\lambda \nu} + g^{\lambda \mu} g^{\nu \rho}) .
\]  

(B.16)
Figure Captions

Fig.1 Feynman rules for the EChL couplings that differ from the SM. We have shown only those that are relevant for the present calculation.

Fig.2 One-loop diagrams that give a leading contribution to the SM combination of counterterms $\delta S$, as explained in the text.

Fig.3 1LPI standard model diagrams relevant for the matching of the four-point Green’s functions of gauge fields up to one loop. We have calculated all the existing diagrams including gauge, Goldstone boson and Higgs fields in the loops. We plot here only the diagrams that do not cancel at both sides of the matching equation, either because they do not exist in the EChL ($L_i, T_i$) or because they are different in the EChL and the SM ($D_i$). We have also restricted our plot to those diagrams that give a non-vanishing contribution to the matching in the large $M_H$ limit. Diagrams $D_i$ have to be calculated also in the effective theory, $\hat{D}_i$, using the Feynman rules given in fig.1.

All the momenta are taken incoming and arrows indicate negative charge flux.

3.a Diagrams for the $\gamma\gamma WW$ Green function. (P1) indicates that the diagram obtained exchanging the $W^+$ and $W^-$ external legs has also to be included. There are a total of 12 $L_i$ diagrams and 3 $D_i$ diagrams.

3.b $\gamma ZWW$ Green function. (P1) represents the diagram obtained exchanging the $W^+$ and $W^-$ external legs. There are 18 $L_i$ and 3 $D_i$ diagrams.

3.c $ZZWW$ Green function. (P1) represents the diagram obtained exchanging the $W^+$ and $W^-$ external legs. There are 47 $L_i$, 6 $D_i$ and 1 $T_i$ diagrams.

3.d $WWW$ Green function. (P3) indicates that the three diagrams obtained by exchanging the ($W^-_\mu \leftrightarrow W^-_\nu$), ($W^+\rho \leftrightarrow W^+\lambda$), and ($W^-_\mu \leftrightarrow W^-\rho$, $W^+\rho \leftrightarrow W^+\lambda$) external legs have to be also included. (P3)’ indicates the substitutions ($W^+\rho \leftrightarrow W^+\lambda$), ($W^+\rho \leftrightarrow W^-\nu$) and ($W^-_\mu \rightarrow W^+\lambda, W^+\rho \rightarrow W^+\nu, W^-_\nu \rightarrow W^-_\mu$). (P1) indicates the exchange ($W^+\rho \leftrightarrow W^+\lambda$). There are 58 $L_i$, 12 $D_i$ and 2 $T_i$ diagrams.

3.e $ZZZZ$ Green function. (P5) indicates the five diagrams obtained by the following permutations of the $(\mu\nu\rho\lambda)$ external Z’s: $(\mu\nu\lambda\rho)$, $(\mu\rho\nu\lambda)$, $(\mu\rho\lambda\nu)$ and $(\mu\lambda\rho\nu)$. (P2) indicates the exchange of $(\mu\nu\rho\lambda)$ by $(\mu\rho\lambda\nu)$ and $(\mu\lambda\nu\rho)$. There are 75 $L_i$, 18 $D_i$ and 3 $T_i$ diagrams.
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