Transient Effect of Negative Electric Current in Irradiated Semiconductors

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The peculiarities of electric current are studied occurring in semiconductors with strongly nonuniform distribution of charge carriers. The formation of such nonuniformities and the regulation of carrier mobilities can be realized by means of external irradiation, for instance, charge–particle beams and laser irradiation. The transient effect of negative electric current is shown to arise under some specific conditions.

I. INTRODUCTION

The study of electric transport in semiconductors is important for describing and modelling different semiconductor devices [1,2]. The action of external irradiation can result in the formation in a semiconductor sample of nonuniform distributions of charge carriers. Thus, ion–beam irradiation leads to the formation of a dense layer of ions located at the distance of their mean free path from the surface. Similar charged layers can be created by irradiating semiconductors with other beams of charged particles, say, electrons or positrons. If the injected charges can move, then the irradiated material behaves as an extrinsic semiconductor. The mobility of the charge carriers can be activated and regulated by involving additionally laser irradiation. Narrow laser beams can also be employed for creating nonuniform distributions of charge carriers.

Transport properties of semiconductors with essentially nonuniform distribution of carriers can be rather specific. For instance, in a sample, biased with an external constant voltage, the resulting electric current may turn against the latter [3,4]. Certainly, this can happen only as a short–time fluctuation after which the current turns back becoming positive [5,6]. The transient effect of negative electric current has been considered earlier [5,6] for simplified models. The aim of the present paper is to give a careful analysis of this effect under conditions typical of realistic semiconductor materials.

II. DRIFT–DIFFUSION EQUATIONS

Transport properties of semiconductors are usually described by the semiclassical drift–diffusion equations [1,2] consisting of the continuity equation

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot \vec{j}_i + \frac{\rho_i}{\tau_i} = \xi_i,$$  

(1)

with the drift–diffusion current

$$\vec{j}_i = \mu_i \rho_i \vec{E} - D_i \nabla \rho_i,$$  

(2)

and of the Poisson equation

$$\varepsilon \nabla \cdot \vec{E} = 4\pi (\rho_1 + \rho_2).$$  

(3)

Here $\rho_i = \rho_i(\vec{r},t)$ is a charge density, $\vec{E} = \vec{E}(\vec{r},t)$ is the electric field, and $\xi_i = \xi_i(\vec{r},t)$, generation–recombination noise [7]. The considered semiconductor is characterized by mobilities $\mu_i$, diffusion coefficients $D_i$, and a dielectric
permittivity $\varepsilon$. In what follows, two types of charge carriers are assumed, positive and negative, such that $\rho_1 > 0$, $\mu_1 > 0$ and $\rho_2 < 0$, $\mu_2 < 0$. The total density of electric current writes

$$j_{\text{tot}} = j_1 + j_2 + \varepsilon \frac{\partial \vec{E}}{\partial t},$$  \hspace{1cm} (4)$$

being the sum of the drift–diffusion current (2) and the displacement current.

Let us consider a plane device of area $A$ and width $L$, so that there is one space variable $x \in [0, L]$. Let the sample be biased with an external voltage $V_0$, which means that

$$\int_0^L E(x, t) \, dx = V_0. \hspace{1cm} (5)$$

Define the transit time

$$\tau_0 \equiv \frac{L^2}{\mu V_0}, \quad \mu \equiv \min\{\mu_1, |\mu_2|\}, \hspace{1cm} (6)$$

and also introduce the following characteristic quantities

$$\rho_0 \equiv \frac{Q_0}{AL}, \quad Q_0 \equiv \varepsilon AE_0, \quad E_0 \equiv \frac{V_0}{L}, \quad j_0 \equiv \frac{Q_0}{A\tau_0}, \quad D_0 \equiv \mu V_0, \quad \xi_0 \equiv \frac{\rho_0}{\tau_0}. \hspace{1cm} (7)$$

It is convenient to pass to dimensionless notation where the space variable $x$ is measured in units of $L$; time, in units of the transit time (6); and all other quantities in the corresponding units (7). Then the continuity equation (1) is written as

$$\frac{\partial \rho_i}{\partial t} + \mu_i \frac{\partial}{\partial x} (\rho_i \epsilon) - D_i \frac{\partial^2 \rho_i}{\partial x^2} + \frac{\rho_i}{\tau_i} = \xi_i$$ \hspace{1cm} (8)$$

and the Poisson equation (3) becomes

$$\frac{\partial E}{\partial x} = 4\pi (\rho_1 + \rho_2), \hspace{1cm} (9)$$

where the space and time variables are

$$0 < x < 1, \quad t > 0. \hspace{1cm} (10)$$

Equation (8) requires an initial and two boundary conditions. The first is given by the initial distribution of charge carriers

$$\rho_i(x, 0) = f_i(x) \quad (i = 1, 2). \hspace{1cm} (11)$$

As the boundary conditions, we may accept the absence of diffusion through the semiconductor surface, which reads

$$\frac{\partial}{\partial x} \rho_i(x, t) = 0 \quad (x = 0, \ x = 1). \hspace{1cm} (12)$$

The role of the boundary condition for Eq. (9) is played by the voltage integral (5) that in the dimensionless notation is

$$\int_0^1 E(x, t) \, dx = 1. \hspace{1cm} (13)$$

Note that by employing Eqs. (9) and (13), we may write the electric field in the form

$$E(x, t) = 1 + 4\pi \left[ Q(x, t) - \int_0^1 Q(x, t) \, dx \right], \hspace{1cm} (14)$$
where
\[ Q(x, t) = \int_0^x [\rho_1(x', t) + \rho_2(x', t)] \, dx'. \]

As the total current (4), we have
\[ j_{tot} = \left( \mu_1 E - D_1 \frac{\partial}{\partial x} \right) \rho_1 + \left( \mu_2 E - D_2 \frac{\partial}{\partial x} \right) \rho_2 + \frac{1}{4\pi} \frac{\partial E}{\partial t}. \] (15)

The quantities whose time behaviour will be of interest for us are the electric current through the semiconductor sample
\[ J(t) = \int_0^1 j_{tot}(x, t) \, dx \] (16)
and the electric current at the left and right surfaces of the device,
\[ J(0, t) = j_{tot}(0, t), \quad J(1, t) = j_{tot}(1, t). \] (17)

The electric current (16), using Eq. (15), can be written as
\[ J(t) = \int_0^1 [\mu_1\rho_1(x, t) + \mu_2\rho_2(x, t)] E(x, t) \, dx + D_1 [\rho_1(0, t) - \rho_1(1, t)] + D_2 [\rho_2(0, t) - \rho_2(1, t)], \] (18)
while for the currents (17) at the left and right surfaces, we get
\[ J(0, t) = J(t) + \int_0^1 [\gamma_1 Q_1(x, t) + \gamma_2 Q_2(x, t)] \, dx, \] (19)
where
\[ Q_i(x, t) = \int_0^x \rho_i(x', t) \, dx', \quad \gamma_1 \equiv \frac{1}{\tau_1} \quad (i = 1, 2), \]
and, respectively,
\[ J(1, t) = J(0, t) - \int_0^1 [\gamma_1 \rho_1(x, t) + \gamma_2 \rho_2(x, t)] \, dx. \] (20)

**III. NEGATIVE CURRENT**

In order to demonstrate that there exist conditions when the electric currents defined above can become negative, that is, directed against the applied voltage, let us take an illustrative example of narrow initial charge distributions which can be approximated by the form
\[ \rho_i(x, 0) = f_i(x) = Q_i \delta(x - a_i). \] (21)

Then from Eqs. (18)–(20), we have at the initial time
\[ J(0) = \mu_1 Q_1 E(a_1, 0) + \mu_2 Q_2 E(a_2, 0), \] (22)
\[ J(0, 0) = J(0) + \gamma_1 Q_1 (1 - a_1) + \gamma_2 Q_2 (1 - a_2), \] (23)
\[ J(1, 0) = J(0) - \gamma_1 Q_1 a_1 - \gamma_2 Q_2 a_2. \] (24)
For the electric field (14), we get

\[ E(x, 0) = 1 + 4\pi Q_1[a_1 - \Theta(a_1 - x)] + 4\pi Q_2[a_2 - \Theta(a_2 - x)] , \]

where \( \Theta(x) \) is the unit step function. Using this, for the total current (22), we find

\[ J(0) = \mu_1 Q_1 \left\{ 1 + 4\pi Q_1 \left( a_1 - \frac{1}{2} \right) + \Theta(a_2 - a_1) \right\} + \]

\[ + \mu_2 Q_2 \left\{ 1 + 4\pi Q_2 \left( a_2 - \frac{1}{2} \right) + \Theta(a_1 - a_2) \right\} . \] (25)

The relation between the currents (23) and (24) has the form

\[ J(0, 0) - J(1, 0) = \gamma_1 Q_1 + \gamma_2 Q_2 . \]

In what follows, we shall consider two particular cases, when the initial charge distributions are located at the same place and when they are separated.

A. Single–Layer Case

Assume that both charge distributions (21) are located at the same place

\[ a_1 = a_2 \equiv a . \] (26)

The initial electric field, then, is

\[ E(a, 0) = 1 + 4\pi Q \left( a - \frac{1}{2} \right) , \quad Q \equiv Q_1 + Q_2 . \] (27)

The total current (25) becomes

\[ J(0) = (\mu_1 Q_1 + \mu_2 Q_2) \left( 1 + 4\pi Q \left( a - \frac{1}{2} \right) \right) . \] (28)

Since \( \mu_i Q_i \geq 0 \), we always have \( \mu_1 Q_1 + \mu_2 Q_2 > 0 \). Hence, the current (28) can be negative if

\[ a < \frac{1}{2} - \frac{1}{4\pi Q} \quad (Q > 0) , \]

\[ a > \frac{1}{2} + \frac{1}{4\pi |Q|} \quad (Q < 0) . \] (29)

As far as \( 0 < a < 1 \), inequalities (29) are possible for

\[ |Q| > \frac{1}{2\pi} . \] (30)

Inverting Eq. (28), we may define the initial charge location

\[ a = \frac{1}{2} - \frac{1}{4\pi Q} \left[ 1 - \frac{J(0)}{\mu_1 Q_1 + \mu_2 Q_2} \right] \] (31)

as a function of the current \( J(0) \). Measuring the latter gives us the location (31).

The current (23) at the left surface is negative under the condition

\[ a \left( 4\pi Q - \frac{\gamma_1 Q_1 + \gamma_2 Q_2}{\mu_1 Q_1 + \mu_2 Q_2} \right) < 2\pi Q - 1 - \frac{\gamma_1 Q_1 + \gamma_2 Q_2}{\mu_1 Q_1 + \mu_2 Q_2} . \] (32)

Also, measuring \( J(0, 0) \), we may define the location
\[ a = \frac{(2\pi Q - 1)(\mu_1 Q_1 + \mu_2 Q_2) - (\gamma_1 Q_1 + \gamma_2 Q_2) + J(0, 0)}{4\pi Q(\mu_1 Q_1 + \mu_2 Q_2) - (\gamma_1 Q_1 + \gamma_2 Q_2)}. \]  

(33)

The current (24) at the right surface becomes negative when

\[ a \left( 4\pi Q - \frac{\gamma_1 Q_1 + \gamma_2 Q_2}{\mu_1 Q_1 + \mu_2 Q_2} \right) < 2\pi Q - 1. \]  

(34)

The location of the initial charge layer can be defined through \( J(1, 0) \) as

\[ a = \frac{(2\pi Q - 1)(\mu_1 Q_1 + \mu_2 Q_2) + J(1, 0)}{4\pi Q(\mu_1 Q_1 + \mu_2 Q_2) - (\gamma_1 Q_1 + \gamma_2 Q_2)}. \]  

(35)

B. Double–Layer Case

Now consider the case when, at the initial time, two charge layers, described by the distributions (21), are separated in space so that

\[ a_1 = a < a_2 = 1 - a. \]  

(36)

Then, substituting into the electric current (22) the electric fields

\[ E(a_1, 0) = 1 - 2\pi Q_1 + 4\pi a (Q_1 - Q_2), \]

\[ E(a_2, 0) = 1 + 2\pi Q_2 + 4\pi a (Q_1 - Q_2), \]

we have

\[ J(0) = \mu_1 Q_1 (1 - 2\pi Q_1) + \mu_2 Q_2 (1 + 2\pi Q_2) + 4\pi a (Q_1 - Q_2)(\mu_1 Q_1 + \mu_2 Q_2). \]  

(37)

This current is negative if

\[ 2a (Q_1 - Q_2) < \frac{\mu_1 Q_1^2 - \mu_2 Q_2^2}{\mu_1 Q_1 + \mu_2 Q_2} - \frac{1}{2\pi}. \]  

(38)

In particular, when \( Q_2 = -Q_1 \), Eq. (38) yields

\[ a < \frac{1}{4} - \frac{1}{8\pi Q_1}. \]

This inequality, since \( 0 < a < \frac{1}{2} \), gives \( Q_1 > \frac{1}{2\pi} \). The conditions for the currents (23) and (24) to be negative are

\[ \left[ 4\pi (Q_1 - Q_2) - \frac{\gamma_1 Q_1 - \gamma_2 Q_2}{\mu_1 Q_1 + \mu_2 Q_2} \right] a < \frac{2\pi (\mu_1 Q_1^2 - \mu_2 Q_2^2)}{\mu_1 Q_1 + \mu_2 Q_2} - \frac{\gamma_1 Q_1}{\mu_1 Q_1 + \mu_2 Q_2} - 1 \]  

(39)

and, respectively,

\[ \left[ 4\pi (Q_1 - Q_2) - \frac{\gamma_1 Q_1 - \gamma_2 Q_2}{\mu_1 Q_1 + \mu_2 Q_2} \right] a < \frac{2\pi (\mu_1 Q_1^2 - \mu_2 Q_2^2) + \gamma_2 Q_2}{\mu_1 Q_1 + \mu_2 Q_2} - 1. \]  

(40)

In this way, we see that each of the electric currents (22)–(24) can become negative at initial time, provided the corresponding conditions hold true.
IV. QUALITATIVE ANALYSIS

To understand the general physical picture, we need to solve Eqs. (8) and (9). The generation–recombination noise in Eq. (8) is usually modelled by the white Gaussian noise for which the stochastic averaging can be denoted by $\ll \ldots \gg$. This noise is defined by the mean

$$\ll \xi_i(x, t) \gg = 0$$

(41)

and by the correlation function

$$\ll \xi_i(x, t) \xi_j(x', t') \gg = \gamma_{ij} \delta(x - x') \delta(t - t') ,$$

(42)

where $\gamma_{ij}$ are the parameters characterizing the specific properties of the generation–recombination process.

An approximate solution of Eqs. (8) and (9) can be found by generalizing the method of scale separation [8-10] to the case of differential equations in partial derivatives. To this end, we have to find out which of the functions $E, \rho_1$ or $\rho_2$ could be considered as slow varying and, if so, with respect to what variables. In what follows we assume that

$$\left| \int_0^\infty \ll \rho_1(x, t) + \rho_2(x, t) \gg dt \right| < \infty ,$$

(43)

which will be confirmed a posteriori. Using Eqs. (9) and (43), we get

$$\lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau \ll \frac{\partial}{\partial x} E(x, t) \gg dt = 0 ,$$

(44)

hence we can tell that the function $E$ is, on average, slowly varying in space. Then, because of the voltage integral (13), we have

$$\int_0^1 \ll \frac{\partial}{\partial t} E(x, t) \gg dx = 0 ,$$

(45)

which suggests that $E$ is, on average, slowly varying in time. Thus, the function $E(x, t)$ can be treated as a quasi–invariant on average, with respect to both space and time. Keeping $E$ fixed in Eq. (8), we obtain a linear equation with respect to $\rho_i$, with constant coefficients. This equation, complimented by the initial conditions (11) and the boundary conditions (12), can be solved. The resulting solution looks a little too cumbersome and we shall not write it down here in full, since our aim in this section is to give only a qualitative analysis for understanding the general physical picture. Therefore, we shall simplify the solution by considering a thick sample for which, instead of the boundary conditions (12), we may formally take

$$\left| \lim_{x \to \pm \infty} \ll \rho_i(x, t) \gg \right| = 0 .$$

(46)

This simplification is equivalent to passing to an infinite sample, with the simultaneous continuation of $\rho_i$ outside the interval $[0, 1]$ by setting $\rho_i = 0$ for $x < 0$ and $x > 1$. Then an approximate solution of Eq. (8) reads

$$\rho_i = \rho_i^{reg} + \rho_i^{ran} ,$$

(47)

where the regular part

$$\rho_i^{reg}(x, t) = \int_{-\infty}^{+\infty} G_i(x - x', t) f_i(x') dx'$$

(48)

is caused by the initial distribution $f_i(x)$, the Green function being

$$G_i(x, t) = \frac{1}{2\sqrt{\pi D_i t}} \exp \left\{ - \frac{(x - \mu_i E t)^2}{4D_i t} - \gamma_i t \right\} ,$$

(49)

and the random part

$$\rho_i^{ran}(x, t) = \int_0^t \int_{-\infty}^{+\infty} G(x - x', t - t') \xi_i(x', t') dx' dt'$$

(50)
is generated by the noise $\xi_i(x,t)$.

The initial charge distribution can be modelled by the Gaussian

$$f_i(x) = \frac{Q_i}{Z_i} \exp \left\{ -\frac{(x-a_i)^2}{2b_i^2} \right\},$$

where $0 < a_i < 1$ and

$$Q_i = \int_0^1 f_i(x) \, dx, \quad Z_i = \int_0^1 \exp \left\{ -\frac{(x-a_i)^2}{2b_i^2} \right\} \, dx.$$

Strictly speaking, $f_i(x)$ is defined as zero for $x < 0$ and $x > 1$. But this restriction can be neglected in the thick–sample approximation which, by using the form (51) in the solution (48), yields

$$\rho_i^{reg}(x,t) = \frac{Q_i b_i}{Z_i \sqrt{b_i^2 + 2D_i t}} \exp \left\{ -\frac{(x-\mu_i E t - a_i)^2}{2b_i^2 + 4D_i t} - \gamma_i t \right\}.$$  \hspace{1cm} (52)

For the random solution (50), because of condition (41), we have

$$\ll \rho_i^{ran}(x,t) \gg = 0.$$  \hspace{1cm} (53)

And from the definition (42), it follows

$$\ll \rho_i^{ran}(x,t)\rho_j^{ran}(x',t) \gg =$$

$$\int_0^t \frac{\gamma_{ij}}{2\sqrt{\pi(D_i + D_j) t}} \exp \left\{ -\frac{|x - x' - (\mu_i - \mu_j) E t|^2}{4(D_i + D_j) t} - (\gamma_i + \gamma_j) t \right\} \, dt,$$  \hspace{1cm} (54)

where $\gamma_{ij}$ are defined in Eq. (42). From here

$$\lim_{t \to 0} \ll \rho_i^{ran}(x,t)\rho_j^{ran}(x',t) \gg = 0.$$  \hspace{1cm} (55)

Equations (52) and (53) show that $\ll \rho^{reg} \gg$ exponentially tends to zero as $t \to \infty$, thus, confirming inequality (43). The currents (18)–(20) are influenced by the noise through the correlators (54). The latter, according to Eq. (55), are small at short times. Therefore, at the beginning of the process, when $t \ll 1$, the role of noise is not important. This conclusion suggests that for considering transient effects, occurring at $t \ll 1$, the influence of the generation–recombination noise may be neglected.

**V. NUMERICAL SOLUTION**

To analyze more accurately the time behaviour of electric current, we have accomplished the numerical calculations of Eqs. (8) and (9) with the initial conditions (11) and (51) and the boundary conditions (12) and (13). In agreement with the previous section, the noise is neglected. All quantities are given in dimensionless units, as is explained in Sec. 2. Varying different parameters entering the problem, we fix $\mu_1 = 1$ and $Q_1 = 1$. We also keep in mind the relation $D_2 = 3D_1$ for the diffusion coefficients, typical of that for holes ($D_1$) and electrons ($D_2$). For short, we use the notation $\gamma_1 = \gamma_2 = \gamma$ and $b_1 = b_2 = b$. Figs. 1 and 2 present the results for the single–layer case, with $a_1 = a_2 = a$; and Figs. 3 and 4, for the double–layer case, when $a_1 = a$, $a_2 = 1 - a$. The values of the varying parameters are taken so that, when passing to dimensional units, they would correspond to the values characteristic for typical semiconductors [1,2]. The current $J(t)$, for $Q_1 \geq |Q_2|$, lies always between $J(0,t)$ and $J(1,t)$, so that $J(1,t) \leq J(t) \leq J(0,t)$, as is clear from Eqs. (18)–(20). Therefore, we concentrate our attention on the behaviour of the limiting quantities $J(1,t)$ and $J(0,t)$. The general behaviour of the latter is in agreement with the qualitative analysis of Secs. 3 and 4. The principal thing which was impossible to notice in the qualitative analysis is that the electric current can become negative not at $t = 0$, but at some finite time. Anyway, the occurrence of the negative current is a transient effect always happening at $t \ll 1$. The dependence of this effect on the physical parameters is thoroughly illustrated in the presented figures.

**Acknowledgement**

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We appreciate financial support from the University of Western Ontario, London, Canada, where this work was accomplished.

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**Figure Captions**

**Fig. 1.** Single–layer case.

(a) The electric current $J(0,t)$ at the left surface of a semiconductor sample for the parameters $a = 0.25$, $D_1 = D_2 = 0$, $\gamma = 1$, $\mu_2 = -3$, for different negative charges: $Q_2 = 0$ (solid line), $Q_2 = -0.25$ (long–dashed line), $Q_2 = -0.5$ (short–dashed line), $Q_2 = -0.75$ (dotted line), and $Q_2 = -1$ (dashed–dotted line).

(a') The electric current $J(1,t)$ at the right surface of semiconductor for the same parameters as in Fig. 1a.

(b) The electric current $J(0,t)$ at the left surface for the parameters $a = 0.25$, $\gamma = 1$, $\mu_2 = -3$, $Q_2 = -0.5$, and for different diffusion coefficients: $D_1 = 0$ (solid line), $D_1 = 10^{-3}$ (long–dashed line), $D_1 = 10^{-2}$ (short–dashed line), $D_1 = 10^{-1}$ (dotted line).

(b') The electric current $J(1,t)$ at the right surface for the same parameters as in Fig. 1b.

(c) The electric current $J(0,t)$ at the left surface for the parameters $a = 0.25$, $\gamma = 1$, $D_1 = 10^{-3}$, $Q_2 = -0.5$, and different mobilities of the negative charge carriers: $\mu_2 = -10$ (solid line), $\mu_2 = -5$ (long–dashed line), $\mu_2 = -3$ (short–dashed line).

(c') The electric current $J(1,t)$ at the right surface for the same parameters as in Fig. 1c.

**Fig. 2.** Single–layer case.

The electric currents at the left, $J(0,t)$ (solid line), and at the right, $J(1,t)$ (long–dashed line), surfaces for the diffusion coefficient $D_1 = 10^{-3}$ and different sets of other parameters:

(a) $a = 0.25$, $\gamma = 1$, $\mu_2 = -3$, $Q_2 = -0.5$.

(b) $a = 0.35$, $\gamma = 1$, $\mu_2 = -3$, $Q_2 = -0.5$.

(c) $a = 0.05$, $\gamma = 10$, $\mu_2 = -3$, $Q_2 = -0.5$.

(d) $a = 0.25$, $\gamma = 10$, $\mu_2 = -3$, $Q_2 = -0.5$.

(e) $a = 0.35$, $\gamma = 10$, $\mu_2 = -3$, $Q_2 = -0.5$.

(f) $a = 0.25$, $\gamma = 0.1$, $\mu_2 = -3$, $Q_2 = -0.5$.

(g) $a = 0.25$, $\gamma = 10$, $\mu_2 = -10$, $Q_2 = -0.5$.

(h) $a = 0.35$, $\gamma = 10$, $\mu_2 = -3$, $Q_2 = -1$.

**Fig. 3.** Double–layer case.
(a) The electric current $J(0, t)$ at the left surface of a semiconductor sample for the parameters $a = 0.1$, $\gamma = 1$, $\mu_2 = -3$, $Q_2 = -1$, and different diffusion coefficients: $D_1 = 0$ (solid line), $D_1 = 10^{-3}$ (long-dashed line), $D_1 = 10^{-2}$ (short-dashed line), $D_1 = 10^{-1}$ (dotted line).

(a') The electric current $J(1, t)$ at the right surface for the same parameters as in Fig. 3a.

(b) The left-surface current $J(0, t)$ for the parameters $a = 0.1$, $\gamma = 1$, $\mu_2 = -3$, $D_1 = 10^{-3}$, and different initial charges: $Q_2 = 0$ (solid line), $Q_2 = -0.25$ (long-dashed line), $Q_2 = -0.5$ (short-dashed line), $Q_2 = -0.75$ (dotted line), and $Q_2 = -1$ (dashed-dotted line).

(b') The right-surface current $J(1, t)$ for the same parameters as in Fig. 3b.

(c) The left-surface current $J(0, t)$ for the parameters $a = 0.1$, $\gamma = 1$, $D_1 = 10^{-3}$, $Q_2 = -1$, and different mobilities: $\mu_2 = -10$ (solid line), $\mu_2 = -5$ (long-dashed line), $\mu_2 = -3$ (short-dashed line).

(c') The right-surface current $J(1, t)$ for the same parameters as in Fig. 3c. For these parameters the left- and right-surface currents are indistinguishable, so the longer time interval is presented here.

(d) The left-surface current $J(0, t)$ for the parameters $a = 0.25$, $\mu_2 = -10$, $D_1 = 10^{-3}$, $Q_2 = -0.1$, and different relaxation widths: $\gamma = 25$ (solid line), $\gamma = 10$ (long-dashed line), $\gamma = 1$ (short-dashed line).

(d') The right-surface current $J(1, t)$ for the same parameters as in Fig. 3d.

Fig. 4. Double-layer case.

The electric currents at the left, $J(0, t)$ (solid line), and the right, $J(1, t)$ (long-dashed line), surfaces for the diffusion coefficient $D_1 = 10^{-3}$ and different sets of other parameters:

(a) $a = 0.1$, $\gamma = 1$, $\mu_2 = -3$, $Q_2 = -0.5$.
(b) $a = 0.1$, $\gamma = 10$, $\mu_2 = -3$, $Q_2 = -0.5$.
(c) $a = 0.1$, $\gamma = 10$, $\mu_2 = -3$, $Q_2 = -1$.
(d) $a = 0.25$, $\gamma = 10$, $\mu_2 = -3$, $Q_2 = -0.25$.
(e) $a = 0.25$, $\gamma = 10$, $\mu_2 = -10$, $Q_2 = -0.25$.
(f) $a = 0.25$, $\gamma = 10$, $\mu_2 = -10$, $Q_2 = -0.1$.
(g) $a = 0.25$, $\gamma = 100$, $\mu_2 = -10$, $Q_2 = -0.1$. 