The development of scanning near-field optical microscopy (SNOM) has already led to enormous progress in studies of different nanoscale phenomena. Among other things, this technique is widely used for image formation of various nanoobjects, single molecule fluorescence detection, and for laser-induced ablation of a sample close to the tip apex (see [1, 2]). In the recent years SNOM methods have been employed in studies of quantum dots [3] and single-walled carbon nanotubes [4]. The near-field optical microscopy has opened a possibility for direct observation of fine features in the self-focusing effect [5], for imaging the light propagation in photonic crystal waveguides [6] and the electromagnetic local density of states of optical corrals [7].

To improve efficiency of the aperture probes for the SNOM technique one needs simultaneously to increase their transmittance and spatial resolution capability. In the present letter we study theoretically one possible way to enhance the optical transmittance through a metallized near-field probe with a subwavelength aperture. We suggest to work in the visible region using a short probe with a large taper angle with a core consisting of a semiconducting matter with a large taper angle of in the visible and near-infrared wavelength range. It is shown that in this spectral range the use of a short silicon probe instead of a glass one allows to achieve a strong (up to $10^2 - 10^3$) enhancement in the transmission efficiency.

The three-dimensional theory developed in the present letter is valid for all taper angles including large ones which are especially suitable for large transmittance [12, 13]. The theory is based on the exact analytic description of the conical waveguide eigenmodes inside a probe with a dissipative matter in its core and perfectly conducting metallic walls. For a loss-free dielectric core, similar approach has been developed in our recent work [10]. Here we consider the case when the semiconducting core has a complex dielectric function which is a necessary feature of the visible region. This generalization requires more than a trivial analytic continuation of our previous results due to the necessity of describing the effects associated with frequency dispersion and light absorption inside the core matter. Such effects lead to a number of new features in light transmission through semiconducting probes, which discuss below.

We consider here time-harmonic fields inside a cone whose core consists of a dissipative medium and whose walls are perfectly conducting. In spherical coordinates the Helmholtz equation for the Hertz function $U$ of the electromagnetic field inside a cone is

$$
\nabla^2 U + k_0^2 \mu_0 \varepsilon_0 U = 0
$$

where $k_0 = \frac{2\pi}{\lambda}$ is the wave number, $\mu_0$ and $\varepsilon_0$ are the permeability and permittivity of free space, respectively, and $\lambda$ is the wavelength of light. The solution of this equation can be written in the form

$$
U(r, \theta, \phi) = \frac{1}{r} \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} \frac{J_n(k_0 \rho_n \sin \theta)}{\rho_n} \cdot W_n^m(\rho_n \sin \theta) \cdot e^{im\phi}
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where $J_n$ and $Y_n$ are the Bessel and Neumann functions, respectively, and $W_n^m$ are the spherical vector wave functions. The coefficients $C_{mn}$ can be determined from the boundary conditions at the tip apex and the transmittance through the cone.

The purpose of this letter is to develop an analytic theory for transmission of visible light through a probe with a core made of a dissipative semiconducting matter. Here we are encouraged by the transmission SNOM experiments with a silicon probe at $\lambda = 1.06 \, \mu m$ [8, 9], which indicate that such probes are very promising in the near-infrared (IR) region. The theoretical support for this work comes from a comparative numerical analysis of the transmission efficiencies of glass and silicon probes at $\lambda = 1.3 \, \mu m$ [11]. We have to note however, that the above theory is restricted by the use of a two-dimensional model with a loss-free dielectric core and small taper angle of $15^\circ$. The further experimental success was achieved in [11], where the authors have employed a pyramidal Si probe that was entirely coated with a thin metal film to increase the transmission efficiency in the near-IR region ($\lambda = 830 \, \text{nm}$). An extremely high throughput (2.3 %) was obtained in this experiment with a resolution capability about 85 nm. In the IR region the light absorption in Si is sufficiently small, but in the visible region the imaginary part of its dielectric function rapidly increases with a decrease of $\lambda$. So, dissipation of the electromagnetic energy inside the silicon core and frequency dispersion of its dielectric function become important and should be taken into account to get an adequate physical pattern of light transmission through the probe.

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\[ \frac{\partial^2 U}{\partial r^2} + \frac{1}{r^2} \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial U}{\partial \theta} \right) + \frac{1}{r^2} \frac{\partial^2 U}{\partial \varphi^2} + k^2 U = 0. \]  
\( (1) \)

Here \( r \) is the distance from the cone vertex, \( \theta \) and \( \varphi \) are the polar and azimuthal angles. For a dissipative matter, the wave number \( k \) is complex: \( k = \omega \sqrt{\varepsilon' \mu}/c, \) \( \sqrt{\mu} = n + i \kappa, \) where \( c \) is the speed of light, \( n \) and \( \kappa \) are the refractive index and the attenuation coefficient. Assuming the permeability \( \mu = 1, \) the real \( \varepsilon' = \Re \{ \varepsilon \} \) and imaginary \( \varepsilon'' = 3n \{ \varepsilon \} \) parts of the frequency-dependent dielectric function \( \varepsilon(\omega) = \varepsilon'(\omega) + i \varepsilon''(\omega) \) are given by \( \varepsilon' = n^2 - \kappa^2 \) and \( \varepsilon'' = 2n \kappa \) (see Ref. \[14\]).

The relevant solution of Eq. (1) corresponding to the standing wave with vanishing amplitude at the cone vertex \( (r = 0) \), has the form

\[ U = R(r) P^m_n(\cos \theta) e^{i m \varphi}, \quad R = C r j_n(kr), \]  
\( (2) \)

at which the radial dependence \( R(r) \) of the Hertz function \( (2) \) is expressed through the spherical Bessel function of the first kind \( j_n(z) \) of a complex argument with the index \( n \) not equal to an integer. Here \( C \) is a constant. The dependence on the polar angle \( \theta \) is determined by the associated Legendre function of the first kind \( P^m_n(\cos \theta) \) with power \( n \) and order \( m \) (\( m \) is an integer).

For the transverse magnetic (TM) field modes the boundary condition at an interface between a core medium and a perfectly conducting metallic coating of strong thickness depends upon the value of \( \varphi \), associated Legendre function of the first kind \( P^m_n(\cos \theta) \) with power \( n \) and order \( m \) (\( m \) is an integer). Each choice of numbers \( n \) and \( m \) in this equation \( n \) denotes the number of its root) determines a possible TM\(_{mn}\) mode. The eigenvalues \( \nu_{mn} \) strongly depend upon the value of \( \theta_0 \) such that \( \nu_{mn} \) decreases with an increase of \( \theta_0 \). In the most interesting case of the lowest-order TM\(_01\) mode, the projections of electric \( E \) and magnetic \( H \) fields onto the corresponding axes of spherical coordinates \((r, \theta, \varphi)\), take the form

\[ E_r = \frac{\nu(n+1)}{r^2} R(r) P_n(\cos \theta), \quad E_{\theta} = \frac{\partial R(r)}{\partial r} \frac{\partial P_n(\cos \theta)}{\partial \theta}, \]  
\( (3) \)

\[ H_\varphi = \frac{\iota \omega}{c} \left[ \varepsilon' + \varepsilon'' \right] \frac{1}{r} R(r) \frac{\partial P_n(\cos \theta)}{\partial \theta}. \]  
\( (4) \)

while the \( H_r, H_\theta, \) and \( E_\varphi \) components are equal to zero.

In a dissipative medium with frequency-dependent dielectric function \( \varepsilon = \varepsilon' + \varepsilon'' \) and permeability \( \mu = \mu' + i \mu'' \), the general expressions (see \[14\]) for the time-averaged densities of the electric, \( w_E = w_r + w_\theta, \) and magnetic, \( w_m = w_\varphi, \) fields are given by

\[ w_d = \frac{1}{16\pi} \left( \frac{d(\omega \varepsilon')}{d\omega} \right) \left( |E_r|^2 + |E_\theta|^2 \right), \]  
\( (5) \)

\[ w_m = \frac{1}{16\pi} \frac{d(\omega \mu')}{d\omega} |H_\varphi|^2, \]  
\( (6) \)

To determine the near-field transmission coefficient of a truncated conical waveguide we introduce the quantities

\[ W_\beta(r) = 2\pi r^2 \int_0^{\theta_0} w_\beta(r, \theta) \sin \theta d\theta, \]  
\( (7) \)

which represent the integrals of \( w_r, w_\theta, \) or \( w_\varphi \), taken over a part of spherical surface lying inside the cone \((0 \leq \theta \leq \theta_0, 0 \leq \varphi \leq 2\pi)\) at a given distance \( r \) from the cone vertex. With the help of Eqs. (2), and (3)-(7), these integrals can be evaluated explicitly. The resulting expressions for \( W_r, W_\theta, \) and \( W_\varphi, \) take the form

\[ W_r = \left( \frac{C}{8} \right) \frac{d(\omega \varepsilon')}{d\omega} \frac{\nu(n+1)}{\nu(n+1)} I_\nu j_{\nu+1} \left( \frac{n(\nu+1)}{\nu(n+1)} \frac{\omega r}{c} \right)^2, \]  
\( (8) \)

\[ W_\theta = \left( \frac{C}{8} \right) \frac{d(\omega \varepsilon')}{d\omega} I_{\nu} j_{\nu+1} \left( \frac{(n+1)\nu}{(n+1)\nu} \frac{\omega r}{c} \right)^2, \]  
\( (9) \)

\[ W_\varphi = \left( \frac{C}{8} \right) |\varepsilon| \left( \frac{\omega r}{c} \right)^2 I_{\nu} j_{\nu+1} \left( \frac{(n+1)\nu}{(n+1)\nu} \frac{\omega r}{c} \right)^2. \]  
\( (10) \)

Here the angular integral \( I_\nu \) is given by

\[ I_\nu = \int_0^{\theta_0} \left( \frac{\partial P_n(\cos \theta)}{\partial \theta} \right)^2 \sin \theta d\theta. \]  
\( (11) \)

The integral energy density \( W_{int} = W_r + W_\theta + W_\varphi \) can now be evaluated as the sum of Eqs. (8)-(10). At small distances from the cone vertex \((r \ll \lambda_c)\) it exhibits a rapid power fall, \( W_{int} \propto \left( |k| r \right)^{2\nu}, \) with a decrease of \( r \). At distances \( r \) from the cone vertex much greater than the wavelength in the core medium \((r \gg \lambda_c)\), the asymptotic expression for \( W_{int} \) takes the form

\[ W_{int} = \frac{\left( \frac{C}{16} \right)}{\nu} I_{\nu} \left\{ \left[ \frac{d(\omega \varepsilon')}{d\omega} + |\varepsilon| \right] \cosh \left( \frac{r}{\lambda_c} \right) \right. \]  
\( (12) \)

\[ + \left. \left[ \frac{d(\omega \varepsilon')}{d\omega} - |\varepsilon| \right] \cos \left( \frac{2\pi r \varepsilon}{\lambda_c} - \nu \pi \right) \right\}. \]

Here \( \lambda_c = c/2k \omega \) is the attenuation length. It is evident that the main feature in the radial dependence is determined by a factor \( \cosh (r/\lambda_c) \), which reflects the influence of light absorption inside a dissipative core of a near-field probe. Another important point
is that the integral energy density $W_{\text{tot}}(r)$ exhibits an oscillatory behavior at distances far from the cone vertex $r \gg c/\omega |n + i\kappa|$. These additional oscillations are the result of the frequency-dependent dielectric function; they are absent if the core is made of a loss-free medium (this is the case of a probe with a glass core, $\varepsilon = \text{const}$ and $\kappa = 0$, see [13]). However, for a lossy matter the amplitudes of the electric ($W_{\text{el}} \propto d(\omega \varepsilon')/d\omega$, Eqs. (8), (9)) and magnetic ($W_{\text{m}} \propto |\varepsilon|$, Eq. (10)) components of the integral energy densities are not equal and can be significantly differed from each other. Therefore, the oscillations of the electric, $W_{\text{el}}$, and magnetic, $W_{\text{m}}$, energies do not compensate each other far from the cone vertex. Note also that in the presence of light absorption there is a significant difference in magnitudes of the energy fluxes associated with the incident and the reflected waves. Their ratio $|S_{\text{i}}/S_{\text{m}}|$ is equal to $\exp(-2r/r_{\kappa})$. So, this difference becomes particularly important in the range of $r \gg r_{\kappa}$.

Further we evaluate the optical transmittance of a conical waveguide with a dissipative matter in its core. For a probe tapered to a subwavelength diameter it is necessary to distinguish a near-field transmission coefficient of a waveguide itself and the resulting transmission coefficient to the far-field zone (see [1, 13]). The near-field transmission coefficient, $T$, can be expressed in terms of the time-averaged energy densities associated with the output and the input fields of the waveguide. For spherical waves inside a cone, this coefficient can be defined as the ratio $T = W_{\text{out}}^{\text{tot}}/W_{\text{tot}}^{\text{in}}$ of the time-averaged energy density $W_{\text{out}}^{\text{tot}} \equiv W_{\text{tot}}(z_0)$ at the exit plane $z = z_0$ of a probe integrated over the aperture cross section $2\pi rdp$ with radius $a$ ($a = z_0 \tan \theta_0$) to the corresponding integral energy density $W_{\text{in}}^{\text{tot}}$ at the waveguide entrance with radial coordinate $r_{\text{in}}$. In a dissipative medium, the latter is given by $W_{\text{in}}^{\text{tot}} = \alpha W_{\text{tot}}(r_{\text{in}})$ (see [1]). The factor $\alpha = [1 + \exp(-2r_{\text{in}}/r_{\kappa})]^{-1}$ shows a fraction of the integral energy density at $r = r_{\text{in}}$, associated with the incident wave alone. So, the contribution of the reflected wave turns out to be completely removed.

For the subwavelength aperture $2a \ll \lambda_{\text{c}}$, the basic expressions (8, 10) for $W_{\text{e}}, W_{\text{g}}$, and $W_{\varphi}$ at the exit of a conical waveguide, can be expanded in power series of $|k| r_{\text{out}}$, where $r_{\text{out}} = a/\sin \theta_0$ is the corresponding radial coordinate. For the value of $W_{\text{tot}}(r_{\text{in}})$ at the waveguide entrance ($r_{\text{in}} \gg \lambda_{\text{c}}$) we use the expression $W_{\text{tot}}(r) \propto \cosh(r/r_{\kappa})$, averaged over the fast oscillations of Eq. (12). Then, the resulting expression for the near-field transmission coefficient takes the form

$$T \propto \left(\frac{\omega |n + i\kappa| a}{c \sin \theta_0}\right)^{2\nu(\theta_0)} \cosh^{-1}\left[\frac{r_{\text{in}}}{r_{\text{in}}(\omega)}\right].$$

(13)

The eigenvalues $\nu \equiv \nu_01$ of the TM$_{01}$ mode in Eq. (13) exhibit rapid fall with an increase of the taper angle ($\nu_01 = 4.083, 2.548, 1.777$, and $1$ at $\theta_0 = \pi/6, \pi/4, \pi/3, \text{and } \pi/2$, respectively). Thus, Eq. (13) describes well all major features of light transmission through the subwavelength aperture in a conical waveguide with a dissipative matter in its core. It is seen that the values of $T$ are strongly dependent on the ratio $a/\lambda$, the taper angle $2\theta_0$, and the refractive index $n$. Moreover, according to (13) the transmission coefficient $T$ is proportional to $\cosh^{-1} \xi$, where $\xi = l/r_{\kappa}$ is the ratio of the length of the probe edge $l$ to the attenuation length $r_{\kappa} = c/2\kappa\omega$ (at small $a$ we have $r_{\text{in}} \approx l$). It is clear that the high transmission efficiency of a semiconductor probe can be achieved in the wavelength region far from the peak in its absorption band ($\kappa \ll n$). If additionally $r_{\kappa} \gg l$, than one can put $r_{\kappa} \to \infty$. Then, Eq. (13) is reduced to especially simple form $T \propto (\omega a/c \sin \theta_0)^{2\nu(\theta_0)}$. This is the case of a loss-free dielectric core. In the opposite case of large losses ($r_{\kappa} \ll l$), the transmission coefficient behaves like $T \propto (\omega a/c \sin \theta_0)^{2\nu(\theta_0)} \exp(-\xi)$. This reflects the strong influence of light absorption on the value of $T$.

Now we apply our theory for studies of the transmittance of the visible and near-IR radiation through the aperture-type metallized silicon probe. In the wavelength region from 580 nm ($\hbar \omega = 1.5$ ev) down to 400 nm ($\hbar \omega = 3.1$ ev), the refractive index $n$ of Si increases monotonically from 3.67 to 5.57 and the attenuation coefficient $\kappa$ grows from 0.005 to 0.387. This results in quite different influence of light absorption in Si in the near-IR and the short-wavelength part of the visible spectrum.

In Fig. 1 we present the wavelength dependences of the near-field transmission coefficient of the metallized silicon probe for the most interesting case of large taper angle $2\theta_0 = 90^\circ$ and for various values of the aperture diameter $2a$. To demonstrate a dependence of light absorption inside the Si core on the length of the probe edge, we calculated the transmission coefficient $T$ for various values of $l$. As expected, $T$ is strongly dependent on the aperture diameter in full agreement with simple formula (13) derived in this work. However, the wavelength dependence, obtained in the present work for the Si probe, differs dramatically from the case of a loss-free dielectric core (a glass fiber), for which $T \propto (a/\lambda)^{2\nu}$. As is evident from Fig. 1, the transmittance of the silicon probe strongly varies over the spectrum. If the length of the probe edge is not too large ($l \lesssim 10$ $\mu$m), the transmission coefficient increases first as the wavelength decreases from the IR region, reaches its maximum at a definite wavelength $\lambda_{\text{max}}$, and then strongly falls at $\lambda \ll \lambda_{\text{max}}$ in the short-wavelength part of the visible spectrum. The position of the maximum $\lambda_{\text{max}}$ and the maximal value of $T_{\text{max}}$ depends on the specific geometrical parameters of the probe.

It is important to stress that this maximum in the transmission efficiency of a silicon probe lies at $\lambda \sim 550–800$ nm (see Fig. 1). This occurs despite the fact that the attenuation length $r_{\kappa}$, associated with the imaginary part
of the dielectric function of Si, considerably decreases in the visible region compared to the near-infrared one. For example, at $\lambda = 633$ nm, 532 nm, and 488 nm, the respective values of the attenuation length $r_\kappa$ turn out to be equal to 2.66 $\mu$m, 0.84 $\mu$m and 0.45 $\mu$m, in contrast with 13.53 $\mu$m at $\lambda = 830$ nm.

In summary, it follows from our calculations that at large taper angles high values of the near-field transmission coefficient can be achieved for a passage of visible light through Si core of an optical probe. To illustrate the enhancement in the transmittance of Si probes in comparison with conventional fiber ones we compare the present estimates for Si with those obtained for a core with small $n$ (glass or SiO$_2$). Although the taper angles of fiber probes do not usually exceed 40° which additionally restricts their efficiency, we use the value $2\theta_0 = 90°$ and $n = 1.55$ to make comparison with our recent results [13]. For the probe with the length $l = 2 \mu$m and the aperture diameter $2a = 50$ nm we get $T_{Si}/T_{glass} = 2.2$, 14.45, and 71 for $\lambda = 488$, 532, 633, and 830 nm, respectively. For the same parameters, but $2\theta_0 = 60°$ we have $T_{Si}/T_{glass} = 43$, 240, 800, and 960. According to our theory, the enhancement occurs as a result of competition between two factors: the rise of $n$ and the decrease of the attenuation length $r_\kappa$ [13]. As follows from our results, the former factor is more important in the most part of the visible spectrum, provided the near-field probe length is sufficiently short (no more than several $\mu$m), such that effects associated with the light absorption are not too strong. We also would like to point out that in case of an entirely coated probe the surface plasmon-polariton propagation along the metallic cladding is likely to further increase the resulting transmittance in accordance with the mechanism discussed in Ref. [12].

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FIG. 1: The near-field transmission coefficient, $T$, of the conical waveguide ($2\theta_0 = 90°$) with the Si core vs $\lambda = 2\pi c/\omega$. Curves 1, 2, 3, and 4 correspond to the aperture diameter $2a = 100$ nm, 70 nm, 50 nm, and 25 nm, respectively. The length of the probe edge is $l = 2 \mu$m (a), 4 $\mu$m (b), 8 $\mu$m (c).