\[ \phi \rightarrow \pi^0\pi^0\gamma \] DECAY WITHIN A \( U(3) \times U(3) \) LINEAR SIGMA MODEL

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We show that the recently observed pion invariant mass distribution of the \( \phi \rightarrow \pi^0\pi^0\gamma \) decay can be satisfactorily described by the chiral \( U(3) \times U(3) \) Linear Sigma Model.

I. INTRODUCTION

Over the past years experimental evidence has accumulated for the existence of light scalar mesons \[5\], and different proposals exist for the \( \bar{q}q \) lowest lying scalar meson nonet. The Particle Data Group (PDG) \[6\] candidates for the ground state \( \bar{q}q \) scalar nonet are: the \( f_0(980) \), \( f_0(1370) \) and the \( f_0(400-1250) \) (or \( \sigma \) meson) for two states in the \( I = 0 \) sector; the \( a_0(980) \) and \( a_0(1450) \) for the isovector scalar meson, and the \( K^*_0(1430) \) for the isospinor scalar meson.

The small decay rate into two photons is among the most important drawbacks for the identification of the \( a_0(980) \) and \( f_0(980) \) as the \( \bar{q}q \) scalar isovector and isosinglet respectively. These decay rates have been calculated using a variety of approaches \[3-5\], in particular, in different versions of the quark model \[3\]. The generally accepted conclusion, is that the \( a_0(980), f_0(980) \rightarrow \gamma\gamma \) decay widths are not consistent with a \( \bar{q}q \) structure.

Moreover, the nearby mass degeneracy of these mesons suggest they are the scalar analogous of the \( \omega \) and \( \rho \) system, i.e. the \( a_0(980), f_0(980) \) are expected to be \( \bar{q}q \) ( \( q = u, d \) ) which however contradicts the strong coupling of the \( f_0(980) \) meson to the \( K\bar{K} \) system.

Other possibilities such as a molecule picture \[4\] and a \( \bar{q}q\bar{q}q \) structure \[5\] have been explored. Recently, it has been argued that the four-quark picture for these mesons is consistent not only with the two photon decays of these states but also with the \( \Phi \rightarrow f_0\gamma \rightarrow \pi\pi\gamma \) decay \[1\].

An alternative approach to the hadron physics in this energy region is provided by the \( U(3) \times U(3) \) chiral model which incorporates a nonet of scalar as well as a nonet of pseudoscalar particles \[7-12\]. In fact, the \( U_A(1) \) component of the \( U(3) \times U(3) \) symmetry exhibited by the light sector of QCD in the massless quark limit, is broken at the quantum level which in the model amounts to the possibility of incorporating otherwise forbidden terms in the interaction lagrangian. In this model, the \( f_0(980) \) turns out to be a mostly \( \bar{s}s \) meson whereas the \( a_0(980) \) meson is the chiral partner of the pion \[1,2\]; the reason for the nearby degeneracy of the \( a_0(980) \) and the \( f_0(980) \) being that the \( U_A(1) \) anomaly pushes up the \( a_0(980) \) mass while leaving untouched the \( f_0(980) \) meson.

The model has shown to be phenomenologically successful \[1,2,3\]. In particular the \( a_0(980) \rightarrow \gamma\gamma \) and \( f_0(980) \rightarrow \gamma\gamma \) decays are consistently accounted for in this framework \[3\], providing thus an explanation to the failure of the quark model calculations which do not take into account \( U_A(1) \)-breaking induced interactions.

This year, DA\( \phi \)NE, the high luminosity \( \phi \) factory, will perform precise measurements of \( \phi \) radiative decays. The Novosibirsk CMD and SND collaborations already reported, among others, measurements of \( \phi \rightarrow \pi^0\pi^0(\eta)\gamma \) and \( \pi^+\pi^-\gamma \) \[4\]. On the theoretical side the \( \phi \rightarrow \pi\pi\gamma \) has been considered by a number of authors \[3,5,13\]. In particular Bramon, Grau and Pancheri (B.G.P.) considered vector meson and chiral loop contributions. By itself the vector meson contribution turns out to be small whereas the chiral loops lead to a broad pion invariant mass spectrum which could easily be distinguished by experiments.
In this contribution we report calculations for the $\phi \to \pi^0 \pi^0 \gamma$ decay within the $U(3) \times U(3)$ model where intermediate scalar resonances naturally appear. From the theoretical point of view this is a clean process, since no final state radiation exists (as compared to the decay in charged pions) and the pseudoscalar mixing angle is not involved (as in the $\pi^0 \eta \gamma$ case).

II. SCALAR MESON CONTRIBUTIONS TO $\phi \to \pi^0 \pi^0 \gamma$

The process under consideration is generated at one loop level. The diagrams contributing to this process are depicted in Fig 1.

![Diagrams](image)

+ seagull diagrams

**FIG. 1. Contributions to $\phi \to \pi^0 \pi^0 \gamma$ in the LSM. Dashed lines denote pseudoscalar mesons (kaons in the loops and neutral pions in the final state) while dot-dashed lines denote (intermediate) isoscalar scalar mesons ($\sigma$ and $f_0(980)$).**

The amplitude arising from the scalar contributions is given by:

$$M(\phi \to \pi^0 \pi^0 \gamma) = e G(m_{\pi \pi}^2) T_{\mu\nu} \eta^{\mu\nu}$$

where

$$T_{\mu\nu} = Q_k g_{\mu\nu} - k_\mu Q_\nu$$

and

$$G(m_{\pi \pi}^2) = \frac{g_{\phi K^+ K^-}}{2\pi^2 M_\phi^2} FL(m_{\pi \pi}^2)$$

with the loop function

$$L(m_{\pi \pi}^2) = \frac{1}{2(a-b)} - \frac{2}{(a-b)^2} [f(\frac{1}{b}) - f(\frac{1}{a})] + \frac{a}{(a-b)^2} [g(\frac{1}{b}) - g(\frac{1}{a})].$$

where
\[ f(z) = \begin{cases} -\left( \arcsin \left( \frac{1}{2\sqrt{z}} \right) \right)^2 & z > \frac{1}{4} \\ + \left( \ln \frac{\eta_+}{\eta_-} - i\pi \right)^2 & z < \frac{1}{4} \end{cases} \]

\[ g(z) = \begin{cases} (4z - 1)^{\frac{3}{2}} \arcsin \left( \frac{1}{2\sqrt{z}} \right) & z > \frac{1}{4} \\ \frac{1}{2} (1 - 4z)^{\frac{3}{2}} \ln \frac{\eta_+}{\eta_-} - i\pi & z < \frac{1}{4} \end{cases} \]

with

\[ \eta_\pm = \frac{1}{2} [1 \pm (1 - 4z)^{\frac{1}{2}}], \quad a = \frac{M^2_\phi}{m_{K^+}^2}, \quad b = \frac{m_{\pi}^2}{m_{K^+}^2}. \] (5)

The F factor appearing in Eq.(3) contain the information on the coupling constants.

\[ F = 2(g_{KK\pi\pi} - \frac{g_{\sigma\pi\pi}g_{KK}}{m_{\pi}^2 - m_{\sigma}^2 + i\Gamma_\sigma m_{\sigma}} - \frac{g_{f\pi\pi}g_{fKK}}{m_{\pi}^2 - m_{f}^2 + i\Gamma_f m_{f}}) \] (6)

The three and four-meson couplings are given by the model \[8,10,12\] as

\[ g_{\sigma KK} = -\frac{m_{\sigma}^2 - m_{K}^2}{2f_{K}} (\cos\phi - \sqrt{2}\sin\phi); \quad g_{\sigma \pi \pi} = -\frac{m_{\sigma}^2 - m_{\pi}^2}{2f_{\pi}} \cos\phi \]

\[ g_{f KK} = -\frac{m_{f}^2 - m_{K}^2}{2f_{K}} (\sin\phi + \sqrt{2}\cos\phi); \quad g_{f \pi \pi} = -\frac{m_{f}^2 - m_{\pi}^2}{2f_{\pi}} \sin\phi \] (7)

The pion invariant mass spectrum is obtained

\[ \frac{d\Gamma}{dm_{\pi\pi}} = \frac{\alpha_{em}}{4\pi} \frac{m_{\pi}^2}{M_\phi} \left( \frac{g_{KK\pi\pi}}{4\pi} \right)^2 (\frac{M_\phi}{m_{K}^2})^4 |L(m_{\pi\pi}^2)|^2 |F|^2 (1 - \frac{m_{\pi}^2}{M_\phi^2})^3 \sqrt{1 - \frac{4m_{\pi}^2}{m_{\pi\pi}^2}} \] (8)

FIG. 2. \(dB(\phi \rightarrow \pi^0\pi^0\gamma)/dm_{\pi\pi} \times 10^{-8}\text{ MeV}^{-1}\) as a function of the dipion invariant mass. The experimental points are taken from the SND Coll. V.M. Aulchenko et.al. [14].
We observe from Eqs(7,8) that the energy spectrum depend on the scalar mixing angle (in the \{\|S\>,\|NS\>\) basis) \(\phi\) and the scalar meson masses. In particular is highly sensitive to the chosen value for the mixing angle. It is worth to remark that for a sigma meson mass above 600 MeV the theoretical predictions for the energy spectrum yield a disastrous result as compared with the experimental results. Our results reduces to those of B.G.P. [15] -upto an overall normalization factor- in the very heavy (and non-mixed) scalars limit (as compared to the typical 1GeV scale). We have been unable to trace back the difference in normalization of the two approaches (a factor \(\sqrt{3}\) in the amplitude).

The LSM results for the energy spectrum in Eq.(8) are shown in Fig. 2. We use \(\phi = -9^0\), \(m_f = 980\) MeV, \(\Gamma_f = 70\)MeV and \(m_{\sigma} = 560\) MeV in the numerical evaluations. The sigma width is dictated by the model as

\[
\Gamma_{\sigma} = \frac{3m_{\sigma}^3}{32\pi f_{\sigma}^2} \left((1 - \frac{m_{\sigma}^2}{m^2})\cos(\phi)\right)^2 \sqrt{1 - 4\frac{m_{\sigma}^2}{m^2}}.
\]  

(9)

In table 1 we also show the theoretical predictions arising from Eq(8) for the total and partial (i.e. integrated over a limited region of the energy spectrum) Branching Ratios. For comparison we also included the experimental results reported by the Novosibirsk groups. Within experimental errors, agreement is satisfactory.

| \(m_{\pi^0\pi^0}(MeV)\) | \(BR(CMD-2)(\times10^{-4})\) | \(BR(SND)(\times10^{-4})\) | \(BR_{TH}(\times10^{-4})\) |
|----------------------|-----------------|-----------------|-----------------|
| \(>550\)            | 1.06 \pm 0.09 \pm 0.06 | 1.00 \pm 0.07 \pm 0.12 | 0.99            |
| \(>700\)            | 0.92 \pm 0.08 \pm 0.06 | 0.50 \pm 0.06 \pm 0.06 | 0.90            |
| \(>900\)            | 0.57 \pm 0.06 \pm 0.04 | 1.14 \pm 0.10 \pm 0.12 | 0.53            |
| \(total(>2m_{\pi})\)| 1.08 \pm 0.17 \pm 0.09 | 1.14 \pm 0.10 \pm 0.12 | 1.08            |

So far we have used a Breit-Wigner to describe the sigma (and \(f_0(980)\)) propagator. It has been argued that the inclusion of the sigma width in this way strongly breaks chiral symmetry [20,21]. If we modify the sigma vertices in such a way that the Goldstone Boson nature of the pions is preserved as discussed in [21], the curve in Fig. 2 is modified. In this case agreement with experimental results is obtained for a lower sigma mass \(m_{\sigma} = 430\)MeV and the same mixing angle \(\phi = -9^0\).

Summarizing, the VEPP-2M SND and CMD2 experimental results for the \(\phi \rightarrow \pi^0\pi^0\gamma\) results are consistent with a mostly \(\bar{s}s\) \(f_0(980)\) meson provided we take into account the effects of the \(U_A(1)\) breaking in the scalar sector. This process gives also support to the existence of a scalar meson resonance (\(\sigma\)) in the 400-600 MeV. The process under consideration is highly sensitive to the scalar mixing angle and experimental results for this process require \(\phi \approx -9^0\) which is consistent with other estimates [14].

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IV. REFERENCES

[1] For a recent account on this we refer to the Proceedings of the Workshop on Hadron Spectroscopy, Frascati, Italy, March (1999): Frascati Physics Series Vol. XV, T. Bressani, A. Feliciello and A. Filippi Eds.

[2] Particle Data Group, Eur. Phys. Jour. C3 (1998).

[3] A. Bramon and M. Greco, Lett. Nuovo Cimento 2, 522 (1971); S.B. Berger and B.T. Feld, Phys. Rev. D8, 3875 (1973); S. Eliezer, J. Phys. G1, 701 (1975); J. Babcock and J. L. Rosner, Phys. Rev. D14, 1286 (1976); M. Budnev and A.E. Kaloshin, Phys. Lett. B86, 351 (1979); N.N. Achasov, S.A. Denayin and G. N. Shestakov, Z. Phys. C16, 55 (1982); E. P. Shabalin, JETP Lett. 42, 135 (1985); J. Ellis and J. Lanik, Phys. Lett. B175, 83 (1986); S. Narison, Phys. Lett B175, 88 (1986); C. A. Dominguez and N. Paver, Z. Phys. C39, 39 (1988); Z.P. Li, F.E. Close and T. Barnes, Phys Rev D43, 2161 (1991); A. S. Deakin et. al., Mod. Phys. Lett. A9, 2381 (1994).

[4] J. Wenstein and N. Isgur, Phys Rev. D27, 588 (1983); T. Barnes, Phys. Lett. B165, 434 (1985).

[5] N.N. Achasov, S.A. Denayin and G. N. Shestakov, Phys. Lett. B108, 134 (1982); Z. Phys. C16, 55 (1982); E. P. Shabalin, Sov. J. Nucl. Phys. 46, 485 (1987); N.N. Achasov and G. N. Shestakov, Z. Phys. C41, 309 (1988); N.N. Achasov and V.N. Ivanchenko, Nucl. Phys. B315, 465 (1989); N.N. Achasov, hep-ph/9803292.

[6] N. N. Achasov, hep-ph/9910540, hep-ph/9904223; N.N. Achasov, V.V. Gubin, hep-ph/9904431.

[7] M. Levy, Nov. Cim. LI A, 7247 (1967); S. Gasiorowicz and D.A. Geffen, Rev. Mod. Phys. 41, 531 (1969).

[8] J. Schechter and Y. Ueda, Phys. Rev. D3, 2874 (1981).

[9] M.D. Scadron, Phys. Rev. D26, 239 (1982); R. Delbourgo and M.D. Scadron, Int. J. Mod. Phys. A13, 657 (1998), hep-ph/9807504.

[10] M. Napsuciale, “Scalar meson masses and mixing angle in a U(3)xU(3) LSM”, hep-ph/9803396, unpublished.

[11] G. t’Hooft, Phys. Rep. 142 (1986) 357.

[12] N.A. Tornqvist, Eur. Phys. Jour. C11, 359 (1999).

[13] J.L. Lucio and M. Napsuciale, Phys. Lett. B454 (1999) 365.

[14] R.R Akhmetsin et.al, Phys. Lett. B462 (1999) 380; V.M. Aulchenko et.al, Phys. Lett. B440 (1998) 442, hep-ex/9807016.

[15] A. Bramon et.al, Phys. Lett B289 (1992) 97.

[16] A. Bramon et al. Phys. Lett. B283 (1992) 416.

[17] A. Bramon, G. Colangelo and M. Greco, Phys Lett. B287 263 (1992).

[18] J. L. Lucio and M. Napsuciale Phys. Lett. B331, 418 (1994).

[19] E. Oset "Radiative φ decays", contributions to DAFNE99, Frascati, Italy, Nov. (1999).

[20] N.N. Achasov and G.N. Shestakov, Phys. Rev. D49 5779 (1994).

[21] J.L. Lucio, M. Napsuciale and M. Ruiz-Altaba, hep-ph/9903420.