The lightest scalar nonet

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First I review some previous work on the lightest scalars below 1.5 GeV, and how these scalars can be understood as unitarized $q\bar{q}$, $q\bar{q}q\bar{q}$ or meson-meson nonet states. The bare scalars are strongly distorted by hadronic mass shifts, and the lightest $I=0$ state becomes a very broad resonance of mass and width of about 500 MeV. This is the $\sigma$ meson required by models based on linear realization of chiral symmetry. Recently the light $\sigma$ has clearly been observed in $D \to \sigma\pi \to 3\pi$ by the E791 experiment at Fermilab and I discuss how this decay channel can be predicted in a Constituent Quark Meson Model (CQM), which incorporates heavy quark and chiral symmetries.

At the end I discuss the likely possibility that there are in fact two light scalar nonets, such as one mainly meson-meson (or 4-quark) nonet and one $q\bar{q}$ nonet. I point out that an interesting approximate description of this could be modelled by starting with two coupled linear sigma models. After gauging the overall symmetry one of these could be looked upon as the "Higgs sector of strong interactions", and the lightest scalar nonet becomes the corresponding Higgs nonet.

I. INTRODUCTION

This talk is partly based on published papers [1–5] on the lightest scalars including a few new comments, and partly on some work under progress. First I shall discuss the evidence for the light $\sigma$ and discuss shortly the linear sigma model, and explain how one can understand the controversial light scalar mesons with a unitarized quark model, which includes most well established theoretical constraints: (i) zeroes as required by chiral symmetry, (ii) all light two-pseudoscalar (PP) thresholds with flavor symmetric couplings in a coupled channel framework (iii) physically acceptable analyticity, and (iv) unitarity.

A unique feature of such a model is that it simultaneously describes a whole scalar nonet and one obtains a good representation of a large set of relevant data, with a few parameters, which all have a clear physical interpretation. Also consistency with unitarity can require that when the effective coupling becomes large enough one can have twice as many poles in the output spectrum, than the $q\bar{q}$-like poles one puts in as Born terms. The new poles can then be interpreted as being mainly meson-meson bound states, but strongly mixed with $q\bar{q}$ states.

Then I discuss the recently measured $D \to \sigma\pi \to 3\pi$ and $D_s \to f_0(980)\pi \to 3\pi$ decays, where the $\sigma$, respectively $f_0(980)$, is clearly seen as the dominant peak, and point out that these decays rates can be understood in a rather general model for the weak matrix elements. This indicates that the broad $\sigma(600)$ and the $f_0(980)$ belong to the same multiplet.

Finally the experimental fact that there seems to be too many light scalar mesons, presumably two nonets, one in the 1 GeV region (a meson-meson nonet) and another one near 1.5 GeV (a $q\bar{q}$ nonet), requires a new effective model for the light scalar spectrum.

I argue that two coupled linear sigma models may provide a first step for an understanding of such a proliferation of 18 light scalar states. After gauging the overall symmetry one could then look at the lightest scalars as Higgs-like bosons for the nonperturbative low energy strong interactions.

II. THE PROBLEMATIC SCALARS AND THE EXISTENCE OF THE $\sigma$

The interpretation of the nature of lightest scalar mesons has been controversial for long. There is no general agreement on where are the $q\bar{q}$ states, is there a glueball among the light scalars, are some of the too numerous scalars multiquark, $KK$ or other meson-meson bound states? These are fundamental questions of great importance in particle physics. The mesons with vacuum quantum numbers are known to be crucial for a full understanding of the symmetry breaking mechanisms in QCD, and presumably also for confinement.

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As for the light and broad $\sigma$ near 600 MeV, all authors do not even agree on its existence as a fundamental hadron, although the number of supporters is growing rapidly.

![Diagram of pole positions of the $\sigma$ resonance](image)

**Fig. 1** - The pole positions of the $\sigma$ resonance, as listed by the PDG (year 2000) under $f_0(400 - 1200)$ or $\sigma$ (filled circles), plotted in the complex energy plane (in units of MeV). The triangles represent the mass and width parameters (plotted as $m - i\Gamma/2$), which were reported at a Kyoto meeting in June 2000. I could not here distinguish between pole and Breit-Wigner parameters. The star is the $m - i\Gamma/2$ point obtained from the recent E791 experiment on $D \to \sigma\pi \to 3\pi$ ($m_\sigma = 478$ MeV, $\Gamma_\sigma = 324$ MeV) while the open circle is that obtained by CLEO for $\tau \to \sigma\pi\nu \to 3\pi\nu$.

A light scalar-isoscalar meson (the $\sigma$), with a mass of twice the constituent $u,d$ quark mass, or $\approx 600$ MeV, coupling strongly to $\pi\pi$ is of importance in all Nambu-Jona-Lasinio-like (NJL-like) models for dynamical breaking of chiral symmetry. In these models the $\sigma$ field obtains a vacuum expectation value, i.e., one has a $\sigma$-like condensate in the vacuum, which is crucial for the understanding of all hadron masses, as it explains in a simple way the difference between the light constituent and chiral quark mass. Then most of the nucleon mass is generated by its coupling to the $\sigma$, which acts like an effective Higgs-like boson for the hadron spectrum.

In Fig. 1 I have plotted with filled circles the results of 22 different analyses on the $\sigma$ pole position, which are included in the 2000 edition of the Review of Particle Physics [6] under the entry $f_0(400 - 1200)$ or $\sigma$. Most of these find a $\sigma$ pole position near 500-i250 MeV.

Also, at a recent meeting in Kyoto [7] devoted to the $\sigma$, many groups reported preliminary analyzes, which find the $\sigma$ resonance parameters in the same region. These are plotted as triangles in Fig. 1. Here it was not possible to distinguish between Breit-Wigner parameters and pole positions, which of course can differ by several 100 MeV for the same data. It must also be noted that many of the triangles in Fig. 1 rely on the same raw data and come from preliminary analyzes not yet published.

I also included in Fig. 1 (with a star) the $\sigma$ parameters obtained from the recent E791 Experiment at Fermilab [8], where 46% of the $D^{+} \to 3\pi$ Dalitz plot is $\sigma\pi$. The open circle in the same figure represents the $\sigma$ parameters extracted from the CLEO analysis of $\tau \to \sigma\pi\nu \to 3\pi\nu$ [9]. There is also a very clear, although still preliminary, signal for a light $\sigma$ (Breit-Wigner mass=$390^{+60}_{-30}$ and width=$282^{+77}_{-50}$) in a BES experiment [10] on $J/\psi \to \sigma\omega \to \pi\pi\omega$.

### III. THE NJL AND THE LINEAR SIGMA MODEL

The NJL model is an effective theory which is believed to be related to QCD at low energies, when one has integrated out the gluon fields. It involves a linear realization of chiral symmetry. After bosonization of the NJL model one finds essentially the linear sigma model (LeM) as an approximate effective theory for the scalar and pseudoscalar meson.
sector. About 30 years ago Schechter and Ueda [11] wrote down the $U3 \times U3$ $\sigma$M for the meson sector involving a scalar and a pseudoscalar nonet. This (renormalizable) theory has only 6 parameters, out of which 5 can be fixed by the pseudoscalar masses and decay constants ($m_{\pi}, m_{K}, m_{\eta}', f_{\pi}, f_{K}$). The sixth parameter for the OZI rule violating 4-point coupling must be small. One can then predict, with no free parameters, the tree level scalar masses [4], which turn out to be not far from the lightest experimental masses, although the two quantities (say Lagrangian mass vs. second sheet pole mass) are not the same thing, but can differ for the same model and data by well over 100 MeV.

The important thing is that the scalar masses are predicted to be near the lightest experimentally seen scalar masses, and not in the 1500 MeV region where many authors want to put the lightest $q\bar{q}$ scalars. The $\sigma$ is predicted [4] at 620 MeV with a very large width ($\approx 600$ MeV), which well agrees with Fig. 1. The $a_0(980)$ is predicted at 1128 MeV, the $f_0(980)$ at 1190 MeV, and the $\kappa$ or $K^*_0(1430)$ at 1120 MeV, which is still surprisingly good considering that loops or unitarity effects must be large as we discuss in the next section.

### IV. UNDERSTANDING THE S-WAVES WITHIN A UNITARIZED QUARK MODEL (UQM)

A few years ago I presented fits to the $K\pi, \pi\pi$ S-waves and to the $a_0(980)$ resonance peak in $\pi\eta$ [14]. In order to understand one of the main points of that work it is sufficient to look at the partial wave amplitude (PWA) in the case of one input resonance, such one $I=1/2$ ($K^*_0$) resonance, or one $I=1$ ($a_0(980)$) resonance. It can for $\pi\eta \to \pi\eta$ be written simply as:

$$A(s) = \frac{Im\Pi_{\pi\eta}(s)}{m_{\pi\eta}^2 + Re\Pi(s) - s + i Im\Pi(s)}, \tag{1}$$

where:

$$Im\Pi(s) = \sum_i Im\Pi_i(s) = -\sum_i \gamma_i^2(s - s_{A,i}) \frac{k_i}{\sqrt{s}} e^{-k_i^2/k^2} \theta(s - s_{th,i}),$$

$$Re\Pi(s) = \frac{1}{\pi} P.V. \int_{s_{th,i}}^{\infty} \frac{Im\Pi(s)}{s'} ds'.$$

Here the coupling constants $\gamma_i$ are related by flavour symmetry and OZI rule, such that there is only one over all parameter $\gamma$. The $s_{A,i}$ are the positions of the Adler zeroes, which are near $s = 0$. Eq. (1) can be looked upon as a more general Breit-Wigner form, where the mass parameter is replaced by an $s$-dependent function, “the running mass” $m_{\pi\eta}^2 + Re\Pi(s)$. This $s$-dependent mass coming from $Re\Pi(s)$ is crucial for understanding the scalars. Because of the S-wave thresholds, giving cusps, and the large effective coupling this cannot be neglected.

In the flavourless channels the situation is more complicated than in Eq. (1) since one has two $I = 0$ states, requiring a two dimensional mass matrix with $s$-dependent mass matrix and mass mixing. Note that the sum runs over all light PP thresholds, which means three for the $a_0(980)$: $\pi\eta, K\bar{K}, \pi\eta'$ and three for the $K^*_0(1430)$: $K\pi, K\eta, K\eta'$, while for the $f_0$'s there are five channels: $\pi\pi, K\bar{K}, \eta\eta, \eta'\eta'$.

In Figs. 2 and 3 I show the running mass, $m_{\pi\eta}^2 + Re\Pi(s)$, and the width-like function, $-Im\Pi(s)$, for the $I=1/2$ and $I=1$ channels. The crossing point of the running mass with $s$ gives the 90° mass of the $a_0(980)$. The magnitude of the $K\bar{K}$ component in the $a_0(980)$ is determined by $-\frac{d}{ds} Re\Pi(s)$, which is large in the resonance region just below the $K\bar{K}$ threshold. These functions fix the PWA of Eq. (1).

In Ref. [2] the $\sigma$ was missed because only poles nearest to the physical region were looked for, and the possibility of the resonance doubling phenomenon, discussed below, was overlooked. Only a little later we realized with Roos [3] that two resonances ($f_0(980)$ and $f_0(1370)$) can emerge although only one $s\bar{s}$ bare state is put in. Then we had to look deeper into the second sheet and found the broad $\sigma$ as the dominant singularity at low mass.

In fact, it was pointed out by Morgan and Pennington [12] that for each $q\bar{q}$ state there are, in general, apart from the nearest pole, also image poles, usually located far from the physical region. As explained in more detail in Ref. [1], some of these can (for a large enough coupling and sufficiently heavy threshold) come so close to the physical region that they make new resonances. And, in fact, there are more than four physical poles with different isospin, in the output spectrum of the UQM model, although only four bare states, of the same nonet, are put in!. The $f_0(980)$ and the $f_0(1370)$ of the model thus turn out to be two manifestations of the same $s\bar{s}$ state. Similarly the $a_0(980)$ and the $a_0(1450)$ could be two manifestations of the ud state.
Fig. 2 - The running mass $m_0 + \text{Re}\Pi(s)$ and $\text{Im}\Pi(s)$ of the $a_0(980)$. The strongly dropping running mass at the $a_0(980)$ position, below the $K\bar{K}$ threshold contributes to the narrow shape of the peak.

Fig. 3 - The running mass and width-like function $-\text{Im}\Pi(s)$ for the $K_{10}^0(1430)$. The crossing of $s$ with the running mass gives the $90^\circ$ phase shift mass, which roughly corresponds to a naive Breit-Wigner mass, where the running mass is put constant.
This phenomenon can manifest itself by two crossings with the running mass \( m_0^2 + \text{Re}\Pi(s) \), one near the threshold and another at higher mass. Then each crossing is related to a different pole. This happens if the coupling is strong enough and one would generate an extra bound state pole, below the threshold. (For \( s \rightarrow K\bar{K} \) one is close to that situation, and in the case of \( u\bar{s} \rightarrow K\pi \) a 50% larger coupling physical coupling could bind \( K\pi \). That bound state pole would then be a true "2-meson bound state" of the two pseudoscalars making up the threshold in question. The "binding mechanism" is then generated by the sum of \( s \)-channel loops required by unitarity. Such binding is of course, in general, related to crossed channel exchanges, which in the UQM are simulated by the form factor.

Although the details of any modelling can be criticised, the conclusion remains: \textit{Strong enough couplings can generate new bound states or resonances not present in the input or in the Born terms represented by an effective Lagrangian.}

Only after realizing that this resonance doubling is important the light and broad \( \sigma \) was found in the model. (See Ref. \[4\] for details). Another important effect that the model can explain is the large mass difference between the \( a_0 \) and \( K_0^* \). Because of this large mass splitting many authors argue that the \( a_0 \) is \( q\bar{q} \) states, since in addition to being very close to the \( KK \) threshold, they are much lighter than the first strange scalar, the \( K_0^*(1430) \). Naively one expects a mass difference between the strange and nonstrange meson to be of the order of the strange-nonstrange quark mass difference, or a little over 100 MeV.

Figs. 2 and 3 explain why one can easily understand this large \( K_0^*(1430) - a_0 \) mass splitting as a secondary effect of the large pseudoscalar mass splittings, and because of the large mass shifts coming from the loop diagrams involving the PP thresholds. If one puts Figs. 2 and 53 on top of each other one sees that the 3 thresholds involving the PP thresholds. If one puts Figs. 2 and 53 on top of each other one sees that the 3 thresholds all lie relatively close to the \( a_0 \) and \( K_0^* \), and all 3 contribute to a large mass shift. On the other hand, for the \( K_0^*(1430) \), the \( SU3_f \) related thresholds \((K\pi, K\eta')\) lie far apart from the \( K_0^* \), while the \( K\eta \) nearly decouples because of the physical value of the pseudoscalar mixing angle.

This large mass of the \( K_0^*(1430) \) is also one of the reasons why several authors want to have a lighter strange meson, the \( \kappa \), near 800 MeV. Cherry and Pennington \[13\] have strongly argued against its existence. But the E791 experiment see some evidence for such a light \( \kappa \) in \( D^+ \rightarrow K^- \pi^+ \pi^+ \). Here the signal is much less evident than the \( \sigma \) in \( D \rightarrow 3\pi \), but the \( \kappa \) improves the \( \chi^2 \) in the region dominated by the \( \kappa^0 \) (890). One should try a more sophisticated Breit-Wigner amplitude for the S-wave, as that in Eq. (1), before one makes more definite statements about the \( \kappa \) It could be something like a virtual bound \( K\pi \) state.

\[ V. \ D \rightarrow \sigma\pi \rightarrow 3\pi \ \text{AND} \ D_s \rightarrow F_0(980)\pi \rightarrow 3\pi \]

The recent experiments studying charm decay to light hadrons are opening up a new experimental window for understanding light meson spectroscopy and especially the controversial scalar mesons, which are copiously produced in these decays.

In particular we refer to the E791 study of the \( D \rightarrow 3\pi \) decay \[8\] where it is shown how adding an intermediate scalar resonance with floating mass and width in the Monte Carlo program simulating the Dalitz plot densities, allows for an excellent fit to data provided the mass and the width of this scalar resonance are \( m_\sigma \simeq 478 \) MeV and \( \Gamma_\sigma \simeq 324 \) MeV. This resonance is a very good candidate for the \( \sigma \). To check this hypothesis we adopt the E791 experimental values for its mass and width and using a Constituent Quark Meson Model (CQM) for heavy-light meson decays \[15\] we compute the \( D \rightarrow \sigma\pi \) non-leptonic process via \textit{factorization} \[16\], taking the coupling of the \( \sigma \) to the light quarks from the linear sigma model \[17\]. In such a way one is directly assuming that the scalar state needed in the E791 analysis could be the quantum of the \( \sigma \) field of the linear sigma model. According to the CQM model and to factorization, the amplitude describing the \( D \rightarrow \sigma\pi \) decay can be written as a product of the semileptonic amplitude \( \langle \sigma|A_{(dc)}^\mu(q)|D^+ \rangle \), where \( A^\mu \) is the axial quark current, and \( \langle \pi|A_{(\bar{u}\bar{s})}^\mu(q)|\text{VAC} \rangle \). The former is parameterized by two form factors, \( F_1(q^2) \) and \( F_0(q^2) \), connected by the condition \( F_1(0) = F_0(0) \), while the latter is governed by the pion decay constant \( f_\pi \). As far as the product of the two above mentioned amplitudes is concerned, only the form factor \( F_0(q^2) \) comes into the expression of the \( D \rightarrow \sigma\pi \) amplitude. Moreover we need to estimate it at \( q^2 \simeq m_\pi^2 \), that is the physically realized kinematical situation. The CQM offers the possibility to compute this form factor through two quark-meson 1-loop diagrams that we call the \textit{direct} and the \textit{polar} contributions to \( F_0(q^2) \). These quark-meson loops are possible since in the CQM one has effective vertices (heavy quark)-(heavy meson)-(light quark) that allow us to \textit{compute} spectator-like diagrams in which the external lines represent incoming or outgoing heavy mesons while the internal lines are the constituent light quark and heavy quark propagators.

In Figs. 2 and 3 of Ref. \[3\] we show respectively the \textit{direct} and the \textit{polar} diagrams for the semileptonic amplitude \( D \rightarrow \sigma \), the former being characterized by the axial current directly attached to the constituent quark loop, the latter involving an intermediate \( D(1^+) \) or \( D(0^-) \) state. These two diagrams are computed with an analogous technique and one finally obtains a determination of the direct and polar form factors \( F_0^{\text{dir,pol}}(q^2) \). The extrapolation to \( q^2 \simeq m_\pi^2 \simeq 0 \)
is safe for the direct form factor while is not perfectly under control for the polar form factor since the latter is more reliable at the pole $q^2 \approx m_p^2$, $m_P$ being the mass of the intermediate state. We take into account the uncertainty introduced by this extrapolation procedure and signaled by the fact that we find $F_0^{\text{pol}}(0) \neq F_1^{\text{pol}}(0)$ (computing $F_0$ from the polar diagram with $0^-$ intermediate polar state and $F_1$ from that with intermediate $1^+$ state). Our estimate for $F_0(0) = F_0^{\text{pol}}(0) + F_0^{\text{dir}}(0) = 0.59 \pm 0.09$ is in reasonable agreement with an estimate of $F_0(m_H^2) = 0.79 \pm 0.15$ carried out in $\chi$ using the E791 data analysis and a Breit-Wigner like approximation for the meson-quark loops are computed substituting the meson vertices with the heavy meson field expressions found by Heavy Quark Effective Theory (HQET) (since CQM incorporates heavy quark and chiral symmetries) and the quark lines with the propagators of the heavy and light constituent quarks. The light constituent mass $m$ is fixed by a NJL-type gap equation that depends on $m$, and on two cutoffs $\Lambda$ and $\mu$ in a proper time regularization scheme for the diverging integrals. The ultraviolet cutoff $\Lambda$ is fixed by the scale of chiral symmetry breaking, $\Lambda_\chi \approx 4\pi f_\pi$, and we consider $\Lambda = 1.25$ GeV. The remaining dependence of $m$ on the choice of the infrared cutoff $\mu$ has an expression similar to that of a ferromagnetic order parameter, $m(\mu)$ being different from zero for $\mu$ values smaller than a particular $\mu_c$, and zero for higher values. When $\mu$ ranges from 0 to 300 MeV, the value of $m$ is almost constant, $m = 300$ MeV, dropping for higher $\mu$ values. A reasonable light constituent quark mass is certainly 300 MeV and this clearly leaves a 300 MeV open window for choosing the infrared cutoff. Enforcing the kinematical condition for the meson to decay to its free constituent quarks, which must be possible since the CQM model does not incorporate confinement, requires $\mu \approx m$. Therefore we pick up the $\mu = 300$ MeV value. The results are quite stable against $10-15\%$ variations of the UV and IR cutoffs.

The CQM semileptonic $D \to \sigma$ transition amplitude is represented by the loop integrals associated to the direct and to the polar contributions. The result of the integral computations must then be compared with the expression for the hadronic transition element $\langle \sigma|A|D \rangle$ and this allows to extract the desired $F_{0,1}$ form factors. An estimate of the weight of $1/m_c$ corrections can also be taken into account.

This computation indicates that the scalar resonance described in the E791 paper can be consistently understood as the $\sigma$ of the linear sigma model. Of course a calculation such as the one here described calls for alternative calculations and/or explanations of the E791 data for a valuable and useful comparison of point of views on the $\sigma$ nature.

A similar computation was performed for the related process $D_s \to f_0(980)\pi \to 3\pi$ with the same model. The agreement with the data indicates that the two resonances the $\sigma$ and the $f_0(980)$ belong to a similar flavour multiplet.

VI. TWO COUPLED LINEAR SIGMA MODELS FOR THE TWO SCALAR NONETS

As we have seen in the previous discussion there seems to be a proliferation of light scalar mesons. Let us assume that we have two two light nonets of scalar mesons below about, say 1.7 GeV, one of which is the $q\bar{q}$ nonet expected from QCD or the quark model, while the other is e.g. meson-meson bound states, but also in a nonet. In order to have a realistic effective model at low energies for the scalars we need an effective chiral quark model, which includes all scalars and pseudoscalars, and where the chiral symmetry is broken by the vacuum expectation values of the scalar fields. The simplest such chiral quark model is the $U(N_f) \times U(N_f)$ linear sigma model (LoSM). Let us model each scalar multiplet by a LoSM. If one has only one scalar multiplet one writes as usual for a gauged LoSM,

$$\mathcal{L}(\Sigma) = \frac{1}{2} \text{Tr}[D_\mu \Sigma D_\mu \Sigma^\dagger] + \frac{1}{2} \mu^2 \text{Tr}[\Sigma \Sigma^\dagger] - \lambda \text{Tr}[\Sigma \Sigma^\dagger \Sigma \Sigma^\dagger] + \ldots$$

(2)

Here $\Sigma$ contains 9 scalar and 9 pseudoscalar fields. (As usual, $\Sigma$ is a $3 \times 3$ complex matrix, $\Sigma = S + iP = \sum_{a=0}^{8}(s_a + ip_a)\lambda_a/\sqrt{2}$, in which $\lambda_a$ are the Gell-Mann matrices, normalized as $\text{Tr}[\lambda_a \lambda_b] = 2 \delta_{ab}$, and where for the singlet $\lambda_0 = (2/N_f)^{1/2} 1$). If one adds a relatively small term $\lambda' \text{Tr}[\Sigma \Sigma^\dagger]^2$ the $a_0(980)$ is split from the $\sigma(600)$. One can add further terms of higher dimension, and like in Sec. III an anomaly term $\det \Sigma + \det \Sigma^\dagger$ and terms which break the symmetries explicitly, but these are here not important for our qualitative discussion.

Thus each meson in Eq. (2) belongs to a flavour nonet. In the quark model this means it can be a $q\bar{q}$ meson. But, it need not be $q\bar{q}$. E.g., meson-meson interactions are well known to be repulsive in exotic channels, but attractive in octet or singlet channels. Therefore, one expects bound meson-meson states only for octets or singlets. So we can have another set of states $\hat{\Sigma}$, which let us assume, are also described approximately by a similar Lagrangian as above, $\tilde{\mathcal{L}}(\hat{\Sigma})$, but with another set of parameters ($\hat{\mu}^2, \hat{\lambda}$). We have thus doubled the spectrum and initially we have two scalar, and two pseudoscalar multiplets, altogether 36 states for three flavours.

Then it is natural to introduce a coupling between the two sets of multiplets, which can break the relative symmetry. The full Lagrangian for both $\Sigma$ and $\hat{\Sigma}$ thus becomes,

$$\mathcal{L}_{\text{tot}}(\Sigma, \hat{\Sigma}) = \mathcal{L}(\Sigma) + \tilde{\mathcal{L}}(\hat{\Sigma}) + \frac{\epsilon^2}{4} \text{Tr}[\Sigma \Sigma^\dagger + h.c.] .$$

(3)
Without the $\epsilon^2$ term one has for 3 flavours two independent $U(3) \times U(3)$ symmetries ($(\mathbb{C}^1 \times \mathbb{C}^1)^2$), but the $\epsilon^2$ term breaks this down to one exact over-all $U(3) \times U(3)$ and one broken relative $U(3) \times U(3)$ symmetry. A similar scheme was discussed recently by Black et al. [20].

Now let (differently from [20]), both $\Sigma$ and $\hat{\Sigma}$ have vacuum expectation values $\langle v \rangle$ and $\hat{\langle v \rangle}$ even if $\epsilon = 0$ ($v = \mu^2/4\lambda + \mathcal{O}(\epsilon^2)$, $\hat{v} = \hat{\mu}^2/4\lambda + \mathcal{O}(\epsilon^2)$).

Then the $2 \times 2$ submatrix between two pseudoscalars with same flavour becomes to order $\epsilon^2$:

$$m^2(0^{-+}) = \left( \begin{array}{c} 4\lambda v(\epsilon)^2 - \mu^2 - \epsilon^2 \\ 2\lambda \langle v \rangle (\epsilon)^2 - \hat{\mu}^2 \end{array} \right) = +\epsilon^2 \left( \begin{array}{cc} \hat{v}/v - 1 \\ 0 \\ -1 \\ v/\hat{v} \end{array} \right), \tag{4}$$

which is diagonalized by a rotation $\theta$ ($\tan (\theta) = v/\hat{v}$), such that the eigenvalues are 0 and $\epsilon^2 v\hat{v}/(v^2 + \hat{v}^2)$:

$$\begin{pmatrix} c & -s \\ s & c \end{pmatrix} m^2(0^{-+}) \begin{pmatrix} c & -s \\ s & c \end{pmatrix} = \epsilon^2 v\hat{v} + \hat{v}^2 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{5}$$

Here $s = v/\sqrt{v^2 + \hat{v}^2}$ and $c = \hat{v}/\sqrt{v^2 + \hat{v}^2}$. The approximation is of course valid only if neither $v$ nor $\hat{v}$ vanishes. In fact, one would expect $v \approx \hat{v}$ phenomenologically. Thus one has one massive $|\pi >$ and one massless $|\hat{\pi} >$ would-be pseudoscalar multiplet. Denoting the the original pseudoscalars $|p >$ and $|\hat{p} >$, we have $|\pi >= c|p > - s|\hat{p} >$, and $|\hat{\pi} >= s|p > + c|\hat{p} >$. The mixing angle is determined entirely by the two vacuum expectation values, and is large if $v$ and $\hat{v}$ are of similar magnitudes, independently of how small $\epsilon^2$ is. On the other hand the scalar masses and mixings are only very little affected if $\epsilon^2/\mu^2 - \hat{\mu}^2$ is small. They are still close to $\sqrt{2\mu}$ and $\sqrt{2\hat{\mu}}$ as in the uncoupled case.

Furthermore, denoting (when $\epsilon = 0$) the axial vector current obtained from $L(\Sigma)$ by $j_{A\mu}$ and the one from $L(\hat{\Sigma})$ by $\hat{j}_{A\mu}$, then their sum is exactly conserved ($\partial_{\mu}[j_{A\mu} + \hat{j}_{A\mu}] = 0$) because of the masslessness of $|\pi >$ and, because of the still exact overall axial symmetry. But $j_{A\mu}$ or $\hat{j}_{A\mu}$ is only "partially conserved", $\partial_{\mu} j_{A\mu} = \sqrt{N_f c s} \sqrt{v^2 + \hat{v}^2})m_\pi^2$, because of the $\epsilon^2$ term, which explicitely breaks the relative symmetry. If one identifies this with PCAC, and $m_\pi$ with the pion mass then we have

$$f_\pi = \sqrt{N_f c s} \sqrt{(v^2 + \hat{v}^2)}, \tag{6}$$

$$m_\pi^2 = \epsilon^2 (v^2 + \hat{v}^2)/v\hat{v}. \tag{7}$$

In order that this should have anything to do with reality, one must of course get rid of the massless Goldstones $\pi$. By gauging the overall axial symmetry $i (D_\mu \Sigma - \partial_\mu \Sigma - i\frac{1}{2}g[\lambda_i A_i \Sigma + \Sigma \lambda_i A_i])$. I argue that the conventional Higgs mechanism absorbs the massless modes $|\pi >$ from the model, but these degrees of freedom enter instead as longitudinal axial vector mesons and give these mesons mass $m_A^2 = 2g^2 (v^2 + \hat{v}^2)$. This is similar to the work of Bando et al. [21] on hidden local symmetries.

The main prediction of this scheme is that one must have doubled the light scalar meson spectrum, as seems to be experimentally the case. Of course in order to make any detailed comparison with experiment one must also break the flavour symmetry, and unitarize the model, which is not a simple matter.

The dichotomic role of the pions in conventional models, as being at the same time both the Goldstone bosons and the $q\bar{q}$ pseudoscalars, is here resolved in a particularly simple way: One has originally two Goldstone-like pions, out of which only one remains in the spectrum, and which is a particular linear combination of the two original pseudoscalar fields.

The two scalar multiplets remain as physical states and one of these (formed by the $\sigma(600)$ and the $a_0(980)$ in the case of two flavours, or the $\sigma$, $a_0(980)$, $f_0(980)$ and the $\kappa$ in the case of three flavours) can then be looked upon as the Higgs multiplet of strong nonperturbative interactions when a hidden local symmetry is spontaneously broken.

VII. CONCLUSIONS

I have in this talk first pointed out that there is strong evidence (Sec. II) for the existence of the light $\sigma(600)$, and that the linear sigma model (Sec III) is not an unreasonable approximation for the lightest nonet. A more detailed
understanding requires a unitarized model (Sec IV) whereby one can understand how the masses are shifted, the widths and mixings are distorted, and even why new scalar meson-meson resonances can be created in nonexotic channels. These effects are important because of the large effective coupling, and because of the nonlinearities due to the S-wave cusps.

The recent observation of $\sigma(600)$ and $f_0(980)$ by the E791 experiment on $D$ and $D_s$ decays to three pions was discussed in Sec. V, and it was pointed out that these rates can be understood within a rather general constituent quark model, with a standard effective Lagrangian for the weak matrix elements.

At the end, in Sec VI, I discussed some preliminary work [19] concerning how one could model the fact that there seems to be growing experimental evidence for two light nonets of scalars, one in the 500-1000 GeV region ($\sigma(600)$, $a_0(980)$, $f_0(980)$, $\kappa(?)$), and another one near 1.3-1.7 GeV ($f_0(1370)$, $f_0(1500)/f_0(1710)$, $a_0(1450)$, $K^*_0(1430)$). I suggested a linear sigma model as a "toy model" for each nonet, each one with its separate vacuum expectation value. Then after coupling these two models through a mixing term and gauging the overall symmetry, one can argue that one of the nonets (the lighter one) is a true Higgs nonet for strong interactions.

VIII. ACKNOWLEDGEMENTS

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