LETTER TO THE EDITOR

Effect of atomic transfer on the decay of a Bose-Einstein condensate

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Abstract. We present a model describing the decay of a Bose-Einstein condensate, which assumes the system to remain in thermal equilibrium during the decay. We show that under this assumption transfer of atoms occurs from the condensate to the thermal cloud enhancing the condensate decay rate.

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1. Introduction

Rapid advances in experimental techniques of laser cooling and trapping made it possible to achieve Bose-Einstein condensation (BEC) in weakly interacting systems. One of the many interesting aspects of BEC in dilute gases is the dynamics of growth and decay of the condensate [1, 2, 3]. Bose-Einstein condensates can have long lifetimes ranging from 2 s [1] up to more than 10 s [1, 5]. This finite lifetime is mainly caused by inelastic collisions between condensate atoms and by collisions with particles from the background gas, resulting in atom loss and heating of the system. Many experiments with condensates can be performed on a time scale short compared to the condensate lifetime, such that decay effects are not important. This is, however, not always the case and some experiments specifically focus on condensate decay [1]. Hence, it is important to model the decay of a condensate in detail, taking into account all processes which significantly contribute. In literature condensate decay due to inelastic collisions and collisions with background gas atoms as well as heating effects are discussed. To our knowledge, the transfer of atoms between the condensate and the thermal cloud occurring dynamically as a consequence of thermalization of the system has not yet been taken into account. We will show that this process plays an important role when the number of atoms in the condensate is of the same order or smaller than the number in the thermal cloud. Its effect could only be neglected in experiments with large condensate fractions which are often obtained by removal of most of the thermal part with an rf-knife, or in situations in which our assumption of fast thermalization is not fulfilled.

The present work concentrates on the decay of condensates in the presence of a considerably large thermal fraction. We assume thermal equilibrium during the decay of the condensate which is justified in most experiments with BEC in dilute atomic gases. Since elastic collision rates are typically large ($\gtrsim 10^3$ s$^{-1}$ [1, 4]), one expects thermalization to occur very rapidly [7] compared to the rate of change in thermodynamic variables during the decay. Using the simple condensate growth equation [2] we investigated numerically the effect of dynamical disturbances caused by atom loss and heating from inelastic collisions on a system originally in thermal equilibrium. We found that the system stays very close to thermal equilibrium under conditions typically encountered in the experiments with dilute atomic gases.

Although our model incorporates two- and three-body collisions it does not take into account subsequent secondary effects. For instance, the effects of induced local variations in the mean-field interparticle interaction [8] are neglected. Also secondary collisions of reaction products with atoms of the condensate or the thermal cloud are not accounted for. Avalanches, recently discussed by Schuster et al [9], are therefore beyond the scope of this work. They may be incorporated later.

This letter is organized as follows. In Sec. 2 we derive a simple analytical formula for the decay of a condensate of non-interacting bosons including only losses due to collisions with background gas particles. The expressions derived in this section show the existence of transfer of atoms between condensate and thermal cloud and the significant
reduction in condensate lifetime that may occur. In Sec. 3 we present a more complete dynamical model of condensate decay using the mean-field theory for weakly-interacting bosons and including also losses by inelastic two- and three-body collisions. In Sec. 4 we present, as an example, numerical simulations of condensate decay in metastable helium.

2. Atomic transfer

Apart from elastic collisions that keep the system in thermal equilibrium, in this section we assume the atoms to undergo only collisions with background gas particles, inducing atom losses. For simplicity we consider a system of non-interacting bosons. At a temperature $T$ below the critical temperature $T_c = \hbar \omega [N/g_3(1)]^{1/3}/k$ the number of atoms $N_T$ in the thermal cloud is given by [10]

$$N_T = g_3(1) \left(\frac{kT}{\hbar \omega}\right)^{3},$$

where $\omega = (\omega_x^2 \omega_y \omega_z)^{1/3}$ is the averaged frequency of the harmonic potential trapping the atoms, $N$ denotes the number of atoms and $g_n(u) = \sum_{k=1}^{\infty} u^k/k^n$, $g_3(1) \approx 1.202$. The energy $E_T$ contained in the thermal cloud is [10]

$$E_T = \hbar \omega \frac{\pi^4}{30} \left(\frac{kT}{\hbar \omega}\right)^4 = \alpha N_T^{4/3},$$

with $\alpha = \pi^4 \hbar \omega g_3^{-4/3}(1)/30$. To describe the equilibrium state we choose instead of the temperature $T$ and the total number of atoms $N$ as dynamical variables the number of atoms in the condensate $N_C$ and the number of atoms in the thermal cloud $N_T$.

For the background gas particles we assume room-temperature energies and a uniform distribution over the volume occupied by the trapped atoms. As the average kinetic energy of a background gas particle is much larger than the energy of an atom in the trap, the collision cross section does not depend on the latter and each collision removes one atom from the trap. After such a collision the total energy of the system $E$ is depleted on average by the mean energy per atom $E/N$. With these assumptions we obtain for the loss rate

$$\dot{N} = -\frac{1}{\tau} N = \dot{N}_C + \dot{N}_T = -\frac{1}{\tau} (N_C + N_T),$$

with $\tau$ denoting the trap lifetime. For the rate of change in the total energy $E$ of the system accompanying the atom loss we have

$$\dot{E} = -\frac{1}{\tau} E = \dot{E}_C + \dot{E}_T = -\frac{1}{\tau} (E_C + E_T),$$

where we used the energy contained in the condensate $E_C = \varepsilon_0 N_C$ with $\varepsilon_0 = \frac{1}{2} \hbar (\omega_x + \omega_y + \omega_z)$ being the ground state energy of the trap. Using Eq. (2) the time derivatives $\dot{E}_T = \frac{4}{3} \alpha N_T^{1/3} \dot{N}_T$ and $\dot{E}_C = \varepsilon_0 \dot{N}_C$ can be substituted in Eq. (3) and in combination with Eq. (3) we then find

$$\dot{N}_T = -\frac{1}{\tau} N_T \frac{1 - \varepsilon_0 N_T/E_T}{4/3 - \varepsilon_0 N_T/E_T} \approx -\frac{3}{4\tau} N_T,$$

(5)
where we used $\varepsilon_0 N_T/E_T \ll 1$, neglecting the energy of a condensate atom compared to the average energy per atom in the thermal cloud. Substituting in Eq. (3) yields

$$\dot{N}_C = -\frac{1}{\tau} \left( N_C + \frac{1}{4} N_T \right). \quad (6)$$

This simple analysis shows that for a condensate coexisting in thermal equilibrium with a thermal cloud in a trap neither $\dot{N}_C = -N_C/\tau$ nor $\dot{N}_T = -N_T/\tau$ holds for the decay induced by collisions with background particles. Conservation of energy and number of atoms combined with rapid thermalization inevitably results in transfer of atoms from condensate to thermal cloud thereby enhancing the decay rate of the condensate. Especially with a considerable fraction of thermal atoms in the system this affects the decay of the condensate. Only in the limit of a large condensate fraction Eq. (6) becomes $\dot{N}_C = -N_C/\tau$.

3. Interacting model

In order to parametrize the equilibrium state of the gas, we will use the temperature $T$ and the number of atoms in the condensate $N_C$ as independent variables fully describing the state of the system. We will start with the stationary “two-gas” model proposed by Dodd et al [11], in which atoms of the thermal cloud do not affect the condensate described by the stationary Gross-Pitaevskii (GP) equation. The thermal cloud atoms do not interact with each other (except for thermalization) but they are influenced by the condensate through the mean-field potential $2U_0 n_C(r)$. Here, the contact potential $U_0 = 4\pi \hbar^2 a/m$ expresses the binary interaction between atoms with scattering length $a$ and mass $m$, and $n_C(r)$ denotes the spatial density of the condensate which can be calculated from the GP equation. We additionally simplify the two-gas model by describing the thermal cloud semiclassically, replacing discrete states when evaluating statistical averages by a continuum of states. This way we obtain analytical formulas for the atomic density $n_T(r)$ and energy density $e_T(r)$ in the thermal cloud:

$$n_T(r) = \int \frac{d^3 p}{(2\pi \hbar)^3} \frac{1}{\exp \left( \left( \frac{p^2}{2m} + V_{\text{eff}}(r) - \mu \right)/kT \right) - 1}$$

$$= \lambda^{-3} g_{3/2} \left( e^{- (V_{\text{eff}}(r) - \mu)/kT} \right), \quad (7)$$

$$e_T(r) = \int \frac{d^3 p}{(2\pi \hbar)^3} \frac{\frac{p^2}{2m} + V_{\text{eff}}(r)}{\exp \left( \left( \frac{p^2}{2m} + V_{\text{eff}}(r) - \mu \right)/kT \right) - 1}$$

$$= \frac{3}{2} kT \lambda^{-3} g_{5/2} \left( e^{- (V_{\text{eff}}(r) - \mu)/kT} \right) + V_{\text{eff}}(r) \lambda^{-3} g_{3/2} \left( e^{- (V_{\text{eff}}(r) - \mu)/kT} \right), \quad (8)$$

where $\lambda = \sqrt{2\pi \hbar^2/mkT}$ is the thermal de Broglie wavelength, $\mu$ is the chemical potential and $V_{\text{eff}}(r) = V(r) + 2U_0 n_C(r)$. The explicit form of $n_C(r)$ and $\mu$ follows
from the GP equation. They depend on the number of atoms in the condensate $N_C$. By integrating the densities $n_C$ and $n_T$ over spatial degrees of freedom we obtain the number of atoms in the thermal cloud $N_T(N_C, T)$ and the energy of the thermal cloud $E_T(N_C, T)$.

We are interested in the time dependence of $N_C$ and $T$. As before, we will first consider the dynamics of the total number of trapped atoms $N$ and total energy $E$ as a function of $N_C$ and $T$. The advantage of this approach is that we do not need to state transfer terms explicitly; the transfer will follow from our analysis automatically.

The total loss rates $\dot{N}$ and $\dot{E}$ are related to $\dot{N}_C$ and $\dot{T}$ via

$$\dot{N} = \frac{\partial N_T}{\partial T} \dot{T} + \left( \frac{\partial N_T}{\partial N_C} + 1 \right) \dot{N}_C,$$  

$$\dot{E} = \frac{\partial E_T}{\partial T} \dot{T} + \left( \frac{\partial E_T}{\partial N_C} + \mu \right) \dot{N}_C,$$  

(9)

with $\mu = \partial E_C/\partial N_C$. The reparametrization is straightforward as we have already found explicit expressions of almost all terms appearing in Eq. (9).

In order to describe the dynamics of the system we must consider all relevant processes affecting its state. We include three main dynamical effects that may cause losses of atoms from the system: two-body inelastic collisions, three-body recombinations and collisions with background gas. In the following we will neglect all secondary collisions. Then the total atomic loss rate $\dot{N}$ is given by [12]:

$$\dot{N} = \frac{1}{\tau} N + 2\chi \int d^3r \left( \frac{1}{2!} n_C^2 + 2n_C n_T + n_T^2 \right) +$$

$$3\xi \int d^3r \left( \frac{1}{3!} n_C^3 + \frac{3}{2!} n_C^2 n_T + 3n_C n_T^2 + n_T^3 \right),$$  

(10)

where $\chi$ and $\xi$ are two- and three-body collision rate constants and $\tau$ is the lifetime of the trap.

Each term in Eq. (10) corresponds to a loss process that may occur in the system. For example the term $\xi \sum \frac{1}{2} n_C^2 n_T r(n_T(r))$ expresses the probability density of a three-body recombination between two condensate and one thermal cloud atom to take place at point $r$. In this event we lose all three atoms from the system, hence the factor 3 in front of the integral in Eq. (10). The energy lost in this particular process consists of two terms. First, an energy $2\mu$ carried by the two lost condensate atoms. Second, the energy carried by the thermal cloud atom just before the recombination. We assume that the change in internal energy per lost thermal particle is equal to the average energy per atom $e_T(r)/n_T(r)$ at point $r$. Then the energy loss rate by this process becomes

$$\xi \int d^3r \frac{3}{2!} n_C^2 n_T \left( 2\mu + \frac{e_T}{n_T} \right).$$  

(11)

Analogously, one can derive the proper rates corresponding to the other possible
collisions. Finally we end up with the expression for the total energy loss rate:

\[-\dot{E} = \frac{1}{\tau}(E_T + \mu N_C) + 2\chi \int d^3r \left[ \frac{e_T}{n_T} (n_T^2 + n_C n_T) + \mu \left( \frac{1}{2!} n_C^2 + n_C n_T \right) \right] +
\]

\[3\xi \int d^3r \left[ \frac{e_T}{n_T} \left( \frac{1}{2!} n_C^2 n_T + 2n_C n_T^2 + n_T^3 \right) + \mu \left( \frac{1}{3!} n_C^3 + \frac{2}{2!} n_C^2 n_T + n_C n_T^2 \right) \right]. \tag{12}\]

In equations (9), (10), and (12) $\dot{N}_C$, $\dot{E}$, $N_T$, $E_T$, and $\mu$ are functions of $N_C$ and $T$ only.

4. Results and conclusions

In this section we present some numerical solutions of the set of equations (9), (10), and (12). We have performed simulations of the condensate decay for the conditions of the Paris experiment on metastable helium [4]: an initial number of atoms in the condensate of $N_C(0) = 5 \times 10^5$, a trap lifetime $\tau = 35$ s, s-wave scattering length $a = 16$ nm, trap parameters $\omega_x = \omega_y = 1090$ Hz and $\omega_z = 115$ Hz, two-body inelastic collision rate $\chi = 1.5 \times 10^{-14}$ cm$^3$/s [13], and three-body recombination rate $\xi = 4 \times 10^{-27}$ cm$^6$/s [14]. The initial number in the thermal cloud is either $N_T(0) = 5 \times 10^5$ (Fig. 1) or $N_T(0) = 2 \times 10^6$ (Fig. 2), with initial temperatures of 1.5 $\mu$K and 2.5 $\mu$K, respectively. This gives densities still outside the regime where avalanches occur. The numerical solution $N_C(t)$ is shown together with results for a much simpler approach in which the transfer is absent. For the loss rates $\dot{N}_C$ and $\dot{N}_T$ again Eq. (10) is used, setting to zero the thermal or condensate density, respectively. Comparing both curves with the same initial conditions it follows that, when the existence of transfer is neglected, the lifetime of the trap is overestimated by 50% when the initial fraction of atoms in the thermal cloud is 0.5 and by more than a factor of 2 when this fraction is 0.8. The third (dotted) line in each figure shows the decay in the absence of a thermal cloud. The lifetime increases by a factor of 2.5 in Fig. 1 and a factor of 8 in Fig. 2, due to the absence of inelastic collisions between condensate and thermal cloud atoms. In both models the decay rate is larger with a thermal cloud, but in the full model the effect is enhanced by atomic transfer.

Another feature of condensate decay that can be studied on the basis of our model is the time dependence of the temperature during the decay. Usually it is assumed that the temperature is constant [1]. In both figures an inset shows the numerical solution for $T(t)$. Indeed, the temperature remains constant within 4%.

We have presented an equilibrium model of the decay of a Bose-Einstein condensate. Our analysis has shown that the assumption of sustained thermal equilibrium leads to transfer of atoms from the condensate to the thermal cloud which can significantly
Figure 1. Decay of a condensate of metastable helium atoms with $N_C(0) = 5 \times 10^5$ applying our model (solid line) in comparison with a simpler model neglecting thermalization (dashed line). The initial number of thermal atoms $N_T(0) = 5 \times 10^5$ [$T(0) \approx 1.5 \mu K$]. The dotted line represents the decay of a pure condensate: $N_T(0) = 0$ ($T = 0$ K). Inset: dynamics of the temperature in our model.

Figure 2. Decay of a condensate with a large thermal cloud: $N_T(0) = 2 \times 10^6$ [$T(0) \approx 2.5 \mu K$]. Further details as in the caption of Fig. 1.

enhance the condensate decay rate. This effect could be seen by an experimental examination of the decay rate as a function of the fraction of thermal atoms.

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