Local ROI Reconstruction via Generalized FBP and BPF Algorithms along More Flexible Curves

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We study the local region-of-interest (ROI) reconstruction problem, also referred to as the local CT problem. Our scheme includes two steps: (a) the local truncated normal-dose projections are extended to global dataset by combining a few global low-dose projections; (b) the ROI are reconstructed by either the generalized filtered backprojection (FBP) or backprojection-filtration (BPF) algorithms. The simulation results show that both the FBP and BPF algorithms can reconstruct satisfactory results with image quality in the ROI comparable to that of the corresponding global CT reconstruction.

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1. INTRODUCTION

It is well known that the primary disadvantage of X-ray CT is ionizing radiation which may induce cancers and cause genetic damages with a probability related to the radiation dose. Thus, reducing the dose as low as possible is a general rule for practical medical applications. One effective way is to reduce the region or volume to be imaged. For example, reconstruction from truncated projections, which is also referred to as local region reconstruction, has been extensively studied. The existent algorithms can be categorized into three categories: (1) iterative methods [1, 2] which reconstruct the local ROI by minimizing I-divergence or maximizing statistical likelihood; (2) analytic methods such as those based on wavelet-based multi-resolution analyses [3, 4]; and (3) lambda tomography methods [5–7] which recover a gradient-like function of an object function.

To reconstruct a long object such as a patient, Katsevich proposed an exact and efficient FBP reconstruction algorithm [8, 9] for a standard cone-beam helical scanning trajectory. Zou and Pan derived a BPF algorithm [10]. Later, several groups independent generalized both the FBP [11, 12] and BPF [13–18] algorithms to more general scanning loci. These results offer a new opportunity to reconstruct the local ROI from truncated data. Based on the idea of combining global and local data for improved ROI reconstruction [3, 19–21], here we propose a local region reconstruction scheme which is novel in terms of the reconstruction methods, that is, the utilization of the recently developed generalized FBP and BPF algorithms. The major characteristic is to deliver a normal radiation dose to a local ROI that may contain the cancerous tissue while applying a very low radiation dose to the structures outside the ROI.

In the following, the generalized FBP and BPF algorithms are briefly reviewed in Section 2. The ROI reconstruction scheme is described in Section 3. Simulated results and analysis are presented in Section 4. Finally, in Section 5 we conclude this paper.

2. REVIEW OF GENERALIZED FBP AND BPF ALGORITHMS

Based on Katsevich’s FBP formula [9] for standard helical cone-beam CT, Ye and Wang derived a generalized FBP formula for exact image reconstruction from cone-beam data collected along a flexible three-dimensional (3D) curve [12]. The key step is to choose a filtering direction based on the general condition (see [12, equation 3.25]). A natural choice is the direction of the generalized PI-segment, also referred to as a chord. As a result, the generalized FBP method does not require the uniqueness of the chord. In fact, it can be used to reconstruct images on any chord as long as a scanning curve runs from one endpoint of the chord to the other endpoint. We implemented this generalized formula and applied it to reconstruct images from data collected along a nonstandard saddle curve [22].

By interchanging the order of the Hilbert filtering and backprojection operation in Katsevich’s FBP formula [9], Zou and Pan obtained a BPF formula for a standard helical
cone-beam CT [10], which reconstructs an object only from the minimum data in the Tam-Danielsson window. Our group contributed the first proof of the general validity of the BPF formula in the case of nonstandard spirals as well as other more general curves [14, 15, 22, 23]. Independent work on exact cone-beam reconstruction in the case of a general scanning curve was also independently reported by several groups [11, 13, 17, 18]. Although there are several schemes for constructing cone-beam inversion algorithms in a general case [24–27] before these recent publications, the generalized exact cone-beam reconstruction algorithms are explicit and straightforward. Note that earlier than our SPIE paper [14], Palamodov once formulated a general inverse formula [28]. Unfortunately, his formula is not theoretically exact [29]. Since a 2D locus can be regarded as a special class of 3D curves, we can readily obtain the corresponding generalized fan-beam reconstruction formulas [30]. Therefore, in this paper, we will discuss the reconstruction of a local ROI from truncated fan-beam and cone-beam data collected along flexible 2D/3D loci.

3. RECONSTRUCTION SCHEME

It is well known that a reconstructed 2D ROI from local truncated projection data suffers from image cupping and intensity shifting artifacts since the local CT problem is not uniquely solvable [31]. Based on the fact that the artifacts mainly distribute in low frequencies, the local ROI reconstruction can be improved by combining the truncated local projections and very few noisy global projections. Therefore, we can deliver a normal radiation dose to the local ROI that may contain the cancerous tissue while applying a very low radiation dose to the structures outside the ROI. As shown in Figure 1, there are two types of detectors: one is a local detector (solid thick line) which collects truncated local projections at a normal radiation dose rate, and the other is a global detector (dotted thin line) which collects global projections with very low radiation dose. For practical applications, the two types of detectors can be combined into one. As illustrated in Figure 1(a), this can be physically implemented by adding some lead filters in the current pre-patient collimator. Composed to the local normal-dose projections, the global low-dose projections can be obtained in two ways: (a) reducing the number of photons for each detector aperture; (b) shortening the scanning time by reducing the number of projections. The global data and local data can be acquired in two scans [21] or the same scan with an ROI beam filtering technique [20].

If the global and local data are acquired in two scans, an interpolation procedure is required to combine the two datasets. Assume that both the local and global detectors have the same scanning geometry. The local normal-dose projections \( P_{\text{local}} \) are finely sampled, while the global low-dose projections \( P_{\text{global}} \) are coarsely sampled. The two datasets can be combined as

\[
P_{\text{global}} = \begin{cases} 
P_{\text{local}} & \text{if } P_{\text{local}} \text{ is defined}, \\
 \text{linearly interpolated } P_{\text{global}} & \text{otherwise},
\end{cases}
\]  

(1)

where \( P_{\text{global}} \) denotes the linear interpolation of \( P_{\text{global}} \) on the fine grid of \( P_{\text{local}} \). The whole interpolation procedure is illustrated in Figure 2 for fan-beam geometry. Once the combined global dataset \( P_{\text{global}}^{c} \) is obtained, the local ROI can be reconstructed by either the generalized FBP or BPF method. For the implementation details of the reconstruction algorithms, please refer to our previous papers [22, 23, 30]. It is pointed out that the global dataset \( P_{\text{global}}^{c} \) can be directly acquired using an ROI beam filtering technique [20].

4. SIMULATION RESULTS

According to the analysis of our CMCT system [19], the organ dose, \( \text{DOSE} \), is proportional to the total photons flux \( \Phi \). Assume that there are \( N \) photons emitted from the X-ray source to each detector aperture, the number of detectors is \( S \), and the number of projections is \( P \), \( \Phi \) will be proportional to \( N \times S \times P \), that is,

\[
\text{DOSE} \propto \Phi \propto N \times S \times P.
\]  

(2)

Hence, the ratio between the dose from global projections and that from local truncated data roughly is

\[
\text{DOSE}_{\text{ratio}} = \frac{\text{DOSE}_{\text{global}}}{\text{DOSE}_{\text{local}}} = \frac{N_{\text{global}}}{N_{\text{local}}} \times \frac{S_{\text{global}}}{S_{\text{local}}} \times \frac{P_{\text{global}}}{P_{\text{local}}},
\]  

(3)
where the subscribes “global” and “local” indicate the global and local datasets, respectively.

Assume that among the $N$ photons emitted by the X-ray source only $\tilde{N}$ photons arrive at the detector aperture due to attenuation in the object, and that the number of photons $\tilde{N}$ obeys a Poisson distribution. Since the dose level as described by noise variance is inversely proportional to the number of the detected photons $\tilde{N}$, we can simulate projections at different dose levels in terms of Poisson random variables as a function of the number of the detected photons (see the appendix). To demonstrate the performance of the proposed algorithms, we implemented an X-ray projection simulator, and the generalized FBP and BPF algorithms in Matlab on a PC with all the computational intensive part coded in C++ language.

As shown in Figure 1(a), in our first set of numerical tests, a 2D differentiable Shepp-Logan phantom (DSLP) [32] was confined in a disk of radius $r = 10$ cm, and centered at the origin of the natural coordinate system. The X-ray source was rotated along a circular locus of radius $R = 50$ cm. Collinear local and global detectors were combined with the local detector arranged in the middle of the global detector. The distance between the detector and the X-ray source was $D = 100$ cm. Along a 40.82 cm length, 500 detector elements were distributed for global data, while 250 detector elements were spanned over 20.41 cm for local data. The local detectors covered all the projections of an ROI of radius $r' = 5.08$ cm. In our simulation, 720 local truncated normal dose and 36 global low dose projections were equiangularly acquired in two scans. We set $N = 1.0 \times 10^8$ per detector element for collection of local data at the normal dose. For different low-dose levels, global data were acquired, and the local ROI reconstructed by both the FBP and BPF methods, as shown in Figure 3. Furthermore, Table 1 includes the dose ratios and signal-to-noise ratio (SNR) in the ROI.

As the baseline, the last column of Table 1 gives the case when the global dataset was obtained using the same dose level and projection number as for the local dataset. From Table 1 and Figure 3, it can be observed that (a) there were some low-frequency artifacts at the edges of the ROI reconstructed by the FBP method; (b) there were some strip artifacts along the PI-segments all over the ROI reconstructed by the BPF method; (c) the FBP method offered better image quality in terms of SNR than the BPF method; (d) both the FBP and BPF methods reconstructed satisfactory results by incorporating some low-dose global projections, at an additional cost of less than 1% purely local CT dose.

5. DISCUSSION AND CONCLUSION

Since the exact FBP and BPF algorithms are utilized, the proposed scheme is different from the existent ROI beam filtering technique [20] in which the approximate Feld-kamp algorithm is used. Although our interpolation method (1) appears similar to the multi-resolution analysis method (MRAM) [3, 4, 21], we emphasize that the former is different from the latter in terms of the radiation dose. Both the global and local data are typically at the same dose level (noise level) in the MRAM, while here the dose level of the global data is far lower than that of the local data. Therefore, the contribution of this paper is to combine a normal-dose local ROI scan [3, 19–21] with a low-dose global scan, and apply the exact FBP and BPF reconstruction algorithms such as those described in [12, 15].

In conclusion, we have evaluated a new local ROI reconstruction scheme from data collected along scanning curves. By combining normal-dose local projections with some low-dose global projections, we have enhanced a local normal-dose dataset to a global dataset. Both the generalized FBP and BPF algorithms have been tested to reconstruct a local ROI. The simulation results have shown that both the FBP and BPF algorithms can produce excellent results with a minimal increment to the dose needed for purely local CT.

Figure 2: Illustration of combining local normal-dose data and global low-dose data in fan-beam geometry.
Figure 3: Reconstructed local ROI images of the DSLP with different low-dose levels for global projections. The left column was reconstructed by the FBP method while the right column by the BPF method. Each image includes a reconstructed image with a display window [1.0, 1.04] and a difference image against the real image with a display window [−0.02, 0.02]. The global projections for the first row were collected with the same $N$ and $P$ as that for the local projections. The white circle indicates the ROI. $N$ for global data with the second and third rows were $10^5$, $10^3$, respectively.

Table 1: Dose ratios and SNRs for different low-dose levels.

| $N$     | $10^7$ | $10^6$ | $10^5$ | $10^4$ | $10^3$ | $10^2$ |
|---------|--------|--------|--------|--------|--------|--------|
| DOSE$_{ratio}$ | $10^{-2}$ | $10^{-3}$ | $10^{-4}$ | $10^{-5}$ | $10^{-6}$ | $10^{-7}$ | 2.0   |
| SNR with FBP (dB) | 61.29   | 61.12   | 60.95   | 58.06   | 49.75   | 31.00   | 61.70 |
| SNR with BPF (dB)  | 61.03   | 59.78   | 55.65   | 46.89   | 37.76   | 25.86   | 61.72 |

APPENDIX

Simulation of Poisson noise

When a material is bombarded by high-speed electrons, the X-ray photons are produced. After their interactions (photoelectric, Compton, and coherent scattering) with the material, some photons are absorbed or scattered. In other words, they are attenuated when they go through the material. The attenuation process can be described by the Lambert-Beer law in the case of single energy photons. As a result, the number of the transmitted photons is a random variable obeying the Poisson distribution [33].

Assume that $N$ photons are emitted from the X-ray source towards each detector aperture, we can simulate the Poisson noise inherent with projection data from the mathematical phantom as follows.
Step 1. Compute the projection data $p$ by linear integration.

Step 2. Compute the expected photon number arriving at the detector, according to the Lambert-Beer law $N = N \exp(-\mu_w p)$, where $\mu_w$ is the X-ray linear attenuation coefficient for water.

Step 3. Generate a Poisson random variable $\tilde{N}$, with the mean and variance being equal to $N$.

Step 4. Compute noisy projection data $\tilde{p} = (1/\mu_w) \ln(N/\tilde{N})$.

Note that X-ray projection data $p$ computed in Step 1 are based on the relative linear attenuation coefficients that have
been normalized with respect to the linear attenuation coefficient for water $\mu_w$ [32]. Hence, in Step 2 the normalized projection data $p$ must be converted to have realistic magnitudes by multiplying the normalized data $p$ with the water coefficient $\mu_w$. Hence, with the formula $\mathcal{N} = N \exp(-\mu_w p)$, we can mimic the real X-ray attenuation process through the human head.

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