Electron capture on $^8B$ nuclei and Superkamiokande results

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Abstract

The energy spectrum of recoil electrons from solar neutrino scattering, as observed by Superkamiokande, is deformed with respect to that expected from SSM calculations. We considered $\nu - e$ scattering from neutrinos produced by the electron–capture on $^8B$ nuclei, $e^- + ^8B \rightarrow ^8Be^* + \nu_e$, as a possible explanation of the spectral deformation. A flux $\Phi_{eB} \simeq 10^4$ cm$^{-2}$ s$^{-1}$ could account for Superkamiokande solar neutrino data. However this explanation is untenable, since the theoretical prediction, $\Phi_{eB} = (1.3 \pm 0.2)$ cm$^{-2}$ s$^{-1}$, is smaller by four orders of magnitude.
The energy spectrum of recoil electrons from solar neutrino scattering, as reported by Superkamiokande (SK), deviates from Standard Solar Models (SSM) predictions at energies near and above $E_0 = 13$ MeV \(1\). This feature can be interpreted in several ways:

i) as a distortion of the $^8B$ neutrino spectrum, due to neutrino oscillations between sun and earth \(2, 3\);

ii) as an excess of hep neutrinos\(4, 5\), by about an order of magnitude with respect to SSM estimates;

iii) as a combination of the two solutions above.

The problem of neutrino oscillations is so important that any alternative explanation of the data, although unlikely, has to be investigated carefully. In this spirit we consider the case of neutrinos from electron-capture on $^8B$ nuclei:

$$e^- + ^8B \rightarrow ^8Be^* + \nu_e \rightarrow 2\alpha + \nu_e,$$

as a possible source of the spectral distortion observed by SK. The energy spectrum of these neutrinos, which we refer to as $eB$ neutrinos, is peaked near $E_{eB} = 15.5$ MeV \(7\) with a full width half maximum $\Delta = 1.4$ MeV, see fig.1. The energy spectrum of recoil electrons from $eB$ neutrinos is practically flat up to about $E_{eB} - m_e/2$, contrary to that from $^8B$ neutrino scattering, which is a decreasing function of electron energy and vanishes near 14 MeV. A substantial flux of $eB$ neutrinos could then mimic the shape of the electron spectrum reported by SK.

In section 1, we look quantitatively at this idea, determining how many $eB$ neutrinos are required to account for SK data. In section 2, we compare the result obtained with the theoretical predictions for the $eB$ neutrino flux.

## 1 How many $eB$ neutrinos are needed?

SK has recently presented a measurement of the energy spectrum of recoil electrons from solar neutrino scattering, corresponding to 504 days of data taking \(1\). By assuming the SSM estimate of the hep neutrino flux \(3\), $\Phi_{hep}^{SSM} \simeq 2 \times 10^3$ cm$^{-2}$ s$^{-1}$ and an undeformed $^8B$ neutrino spectrum, with an arbitrary normalization, they obtained a $\chi^2/D.O.F. = 25.3/15$, corresponding to a 4.6 % confidence level \(1\). The poor fit is due mainly to the behaviour of the energy-bins above 13 MeV.

Escribano et al. \(4\) suggested that a hep flux significantly larger than the SSM estimate could reproduce the observed spectrum. Bahcall et al. \(5\) have shown that a flux $\Phi_{hep} \geq 20 \times \Phi_{hep}^{SSM}$ could actually mimic the SK spectrum.

Alternatively, one can keep the SSM prediction for hep neutrinos and look for other high energy neutrino sources. Since the average energy of $eB$ neutrinos is roughly twice
than that of hep neutrinos and since the $\nu - e$ scattering cross section increase linearly with energy, one expects that a flux $\Phi_{eB} \simeq 10 \times \Phi_{hep}^{SSM} \simeq 2 \times 10^4 \text{ cm}^{-2} \text{ s}^{-1}$ could be sufficient to account for the high energy behaviour of SK data.

In order to provide a quantitative estimate, let us analyse the data by using as free parameters $\alpha = \Phi_{eB}/\Phi_{SSM}^{8B}$ and $\delta = \Phi_{eB}/\Phi_{SSM}^{8B}$, where $\Phi_{SSM}^{8B} = 5.15 \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$ is the SSM prediction for the $^8B$ neutrino flux [1]. We define, in analogy with [1], the following $\chi^2$:

$$\chi^2 = \sum_{i=1}^{16} \left\{ \frac{R_i^{SSM} - \alpha \times B_i^{8B} / \Phi_{SSM}^{8B}}{(1 + \delta \times \beta / \Phi_{SSM}^{8B}) (1 + \gamma \times \gamma)} \right\}^2 + \gamma^2 + \beta^2 . \tag{2}$$

In the previous relation $R_i$ is the number of solar neutrino events observed in the $i$-th energy-bin; $SSM_i$ is the number of events in the same energy bin due to $^8B$ neutrinos, for a total flux $\Phi_{SSM}^{8B}$; $B_i$ is the same number due to $eB$ neutrinos, again for a total flux $\Phi_{SSM}^{8B}$; the quantities $\delta_{i,exp}$, $\delta_{i,cal}$, $\sigma_i$, defined as in [1], take into account correlated and uncorrelated theoretical and experimental errors; the free parameters $\beta$ and $\gamma$ are used for constraining the variation of correlated systematic errors. For each value of $\delta$ we determined the parameters $\alpha$, $\beta$ and $\gamma$ so as to determine the minimum of eq. (2), $\chi^2_{min}$, see fig. 2.

As expected, a large $eB$ neutrino flux produces a steep increase in the high energy tail of the Superkamiokande normalized spectrum, see fig. 3. The best-fit is obtained when $\Phi_{eB} = 1.1 \times 10^4 \text{ cm}^{-2} \text{ s}^{-1}$, corresponding to $\chi^2_{min}/D.O.F. = 15.7/14$. Acceptable fits are anyhow obtained for $\Phi_{eB}$ in the range $(0.3 - 2) \times 10^4 \text{ cm}^{-2} \text{ s}^{-1}$, see fig. 2.

2 Theoretical evaluation of the $eB$ neutrino flux

Boron ($^8B$) is produced in the sun, according to the following reaction

$$^7Be + p \rightarrow ^8B + \gamma$$

and it undergoes $\beta^+$ decay

$$^8B \rightarrow ^8Be^* + e^+ + \nu_e \rightarrow 2\alpha + e^+ + \nu_e \tag{3}$$

or electron capture reaction

$$^8B + e^- \rightarrow ^8Be^* + \nu_e \rightarrow 2\alpha + \nu_e . \tag{4}$$

The process under consideration is an allowed transition: in fact (see ref. [10]) $J^P(^8B) = J^P(^8Be^*) = 2^+$. In this case, the ratio $R$ between electron capture probability ($\Gamma_{eB}$) and $\beta^+$ decay probability ($\Gamma_\beta^+$) does not depend on the matrix elements of the transition operator between the nuclear states. A simple phase–space calculation, assuming that the electron

*The quantities $SSM_i$ and $B_i$ have been calculated taking into account the energy resolution of SK [3], as described e.g. in [3]
number density at nuclear site \( n_e(0) \) can be approximated by the average electron number density \( n_e \), gives immediately

\[
R = \frac{1}{8\pi} \left( \frac{\hbar c}{m_e c^2} \right)^3 \times \left( \frac{E_{eB}}{m_e c^2} \right)^2 \times f^{-1} \times n_e ,
\]

(5)

where, for later convenience, we show explicitly the dimensionless phase–space factor associated to \( \beta^+ \) decay, \( f \approx [(E_{eB} - m_e c^2)/m_e c^2]^5/30 \approx 7.1 \times 10^5 \). For \( n_e \approx 5.4 \times 10^{25} \text{cm}^{-3} \) as suggested by SSM, one has \( R \approx 4 \times 10^{-8} \) and consequently

\[
\Phi_{eB} = R \times \Phi_{SSM} = 2 \times 10^{-1} \text{cm}^{-2} \text{s}^{-1} ,
\]

(6)
i.e. five orders of magnitude lower than that required to account for SK data.

It is anyhow useful to estimate \( \Phi_{eB} \) with a better accuracy. With respect to the naive estimate given previously, one should consider the effects of interactions with the solar plasma. The distortion of the positron wave function in the \( \beta^+ \) decay rate can be described as a modification of the dimensionless phase–space factor \( f \), which is now given by \( f = 5.70 \times 10^5 \) \cite{13,14}. Moreover the electron density at nucleus \( n_e(0) \) is larger than \( n_e \) and, consequently, the ratio \( R \) has to be enhanced, with respect to eq. (5), by a factor

\[
\omega = \frac{n_e(0)}{n_e}.
\]

(7)

For a precise estimate of \( \omega \) one has to take into account: \( i) \) distortion of electron wave functions in the Coulomb field of nucleus \cite{15} \( ii) \) electron capture from bound states \cite{16} \( iii) \) screening effects \cite{16,17}. Let us discuss the problem in some detail, following the lines of Gruzinov and Bahcall who recently produced a clear and comprehensive analysis of the \( ^7\text{Be} \) electron capture in the sun \cite{18}:

\( i) \) Because of the Coulomb field of the nucleus, the wave functions of continuum electron states differ from plane waves. The rate of electron capture from continuum has then to be corrected by an enhancement factor \( \omega_c \) \cite{15}:

\[
\omega_c = \left[ \frac{\left\langle \psi_{\text{coul}}(0) \right\rangle}{\left\langle \psi_{\text{free}}(0) \right\rangle} \right]^2 = \left( \frac{m_e c^2}{kT} \right)^{\frac{3}{2}} \times (Z\alpha) \times 2 \times (2\pi)^{\frac{1}{2}} \times I(\beta) ,
\]

(8)

where the average is taken over electron thermal distribution. In the previous relation \( T \) is the Sun temperature, while \( I(\beta) \) is a correction factor of order unity, defined e.g. in \cite{14}. For \( R/R_{\odot} \approx 0.05 \), which corresponds to the solar region where the production of \( ^8\text{Be} \) neutrinos is maximal, the density enhancement at nucleus due to electron in continuum states is \( \omega_c = 3.82 \).

\( ii) \) As pointed out by Iben, Kalata e Schwartz \cite{16}, under solar conditions bound electrons give a substantial contribution to the electron density at the nucleus. The bound state enhancement factor is given by \cite{18}:

\[
\omega_b = \pi^{\frac{1}{2}} \times \hbar^{3} \times \left( \frac{m_e kT}{2} \right)^{-\frac{3}{2}} \sum_n \left( \frac{Z}{a_0 n^3} \right) \exp \left( \frac{Z^2 e^2}{2n^2 a_0 kT} \right) ,
\]

(9)
where $a_0$ is the Bohr radius. For $R/R_\odot \simeq 0.05$, the bound state enhancement factor is $\omega_b = 2.94$. The total density enhancement factor is then $\omega_c + \omega_b = 6.76$

iii) Screening effects reduce the electron density at nucleus for both bound [16] and continuum electron states [17]. If the temperature is sufficiently high and if the screened potential can by described by

$$V(r) = -\frac{Ze^2}{r} \exp(-r/R_D), \quad (10)$$

where $R_D$ is the Debye radius, by using a thermodynamical argument one finds [18]:

$$\omega = \exp\left(-\frac{Ze^2}{kT R_D}\right) \times (\omega_b + \omega_c). \quad (11)$$

For $R/R_\odot \simeq 0.05$ the total density enhancement factor, due to screening effects, is reduced to $\omega = 5.34$. The small difference between this value of $\omega$ and that given by [18] is due to the fact that they have been calculated for slightly different solar regions. Relation (11) is not so straightforward, especially because of the possible inadequacies of the Debye screening theory [19] and because of the relatively large thermal fluctuations which could results from the small number of ions in a Debye sphere [20]. For the similar case of $^7\text{Be}$ electron capture, Gruzinov & Bahcall have performed a detailed analysis of the problem, concluding that relation (7) is accurate at the 2% level.

By using the previous relations we can determine the ratio between electron capture and $\beta^+$ decay rates. We obtain:

$$R = 2.6 \times 10^{-7} \times (1 \pm 0.02) \quad (12)$$

This value is about 30% larger than previous estimates [13] which took into account only continuum electron states contribution. By using the SSM estimate of the $^8\text{B}$ neutrino flux, which is uncertain by about 17% [3], one concludes

$$\Phi_{\epsilon B} = R \times \Phi_{\epsilon B}^{\text{SSM}} = 1.3 \times (1 \pm 0.17) \text{ cm}^{-2}\text{s}^{-1}. \quad (13)$$

The predicted neutrino flux is lower by a factor $10^4$ than required to account for SK data and the calculation method is robust. We conclude that $eB$ neutrinos cannot explain the spectral distributions of solar neutrino events reported by SK.

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Figure 1: Normalized energy spectra of $^8B$, hep and eB neutrinos.
Figure 2: The minimum of eq.(2) for a given value of $\Phi_{\epsilon B}$.
Figure 3: Observed electron energy spectrum normalized to SSM expectations (dots). The solid line is the prediction for $\Phi_{eB} = 1.1 \times 10^4 \text{ cm}^{-2} \text{s}^{-1}$. 