Complexity analysis of charged dynamical dissipative cylindrical structure in modified gravity

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Abstract This article focuses on the formulation of some scalar factors which are uniquely expressed in terms of matter variables for dynamical charged dissipative cylindrical geometry in a standard gravity model $\mathcal{R}+\Phi Q$ ($\Phi$ is the coupling parameter, $Q = \mathcal{R}_{\varphi \theta} T^{\varphi \theta}$) and calculates four scalars by orthogonally decomposing the Riemann tensor. We find that only $\gamma_{TF}$ involves inhomogeneous energy density, heat flux, charge and pressure anisotropy coupled with modified corrections, and thus call it as complexity factor for the considered distribution. Two evolutionary modes are discussed to study the dynamics of cylinder. We then take the homologous condition with $\gamma_{TF} = 0$ to calculate unknown metric potentials in the absence as well as presence of heat dissipation. The stability criterion of the later condition is also checked throughout the evolution by applying some constraints. We conclude that the effects of charge and modified theory yield more complex system.

1 Introduction

The accelerating expansion of our universe has recently been viewed from several remarkable cosmic observations such as redshift and distance-luminosity relationship of type IA Supernovae [1, 2]. The study of current nature of the universe within the context of general relativity (GR) suffers from some shortcomings like fine-tuning and cosmic coincidence. In view of this, researchers modified GR to find a suitable solution to such issues and include the effects of rapid expansion. The immediate generalization of GR was proposed to study cosmological consequences at large scale, named as $f(R)$ theory [3]. The Einstein–Hilbert action was modified by replacing the Ricci scalar with its generic function $f(R)$ to get the effects of this extended theory. Numerous astrophysicists [4–7] studied different models in this framework and obtained physically feasible compact structures by employing multiple approaches.

Bertolami et al. [8] presented the notion of coupling between matter and geometry in $f(R)$ gravity for the very first time by engaging the effects of geometry in matter Lagrangian through insertion of the Ricci scalar. A couple of years ago, multiple extensions of GR encompassing such interaction were proposed that prompted astronomers to explore the physical feasibility of modified gravitational models. This idea has recently been generalized at action level by Harko et al. [9] by introducing $f(R, T)$ theory, in which $T$ is trace of the energy-momentum tensor (EMT). The contribution of the EMT in analytic functional of any theory results in its nonzero divergence contrary to GR and $f(R)$ framework. A large body of literature [10–14] exists to analyze the effects of coupling on self-gravitating structures and found several remarkable results in this theory. Haghani et al. [15] generalized this gravity by adding a factor $Q$ which is the contraction of the Ricci tensor and EMT. This theory explains the inflationary era of our cosmos properly. They also studied some cosmological applications corresponding to three different $f(R, T, Q)$ models like $\mathcal{R} + \lambda Q$, $\mathcal{R}(1 + \lambda Q)$ and $\mathcal{R} + \zeta \sqrt{|\mathcal{T}|} + \lambda Q$.

By adopting first two of the above models along with matter Lagrangian as $L_m = \rho - P$, Sharif and Zubair [16] discussed thermodynamical laws of black hole and computed some acceptable values of the coupling parameter $\lambda$. They also generalized energy bounds for this scenario and obtained some constraints for which those bounds show viable behavior [17]. They found that energy conditions do not hold for negative values of $\lambda$. Odintsov and Sáez-Gómez [18] discussed several cosmological solutions in $f(R, T, Q)$ gravity by reconstructing their corresponding gravitational action. Sharif and Waseem [19] analyzed three different compact stars in this context along with matter Lagrangian as $L_m = -P_\rho - P_\perp$. They concluded that these systems show stable behavior near the center for $L_m = -P_\rho$ only. Yousaf et al. [20–23] computed modified structure scalars for effective EMT with and without charge in spherical system and discussed the evolution of non-static self-gravitating structures. The complexity of self-gravitating systems has also been measured through a scalar $\gamma_{TF}$ [24–26]. We have obtained some stable anisotropic solutions by employing multiple approaches in this context [27–31].
The self-gravitating structures whose interior is cylindrically symmetric have been supported by the existence of cylindrical gravitational waves. The study of such geometrical objects produces significant consequences, and thus motivated many astrophysicists to investigate their fundamental features. The pioneering study of these massive systems have been done by Bronnikov and Kovalechuk [32]. Wang [33] determined analytic solutions to the field equations corresponding to four-dimensional cylindrical geometry along with a massless scalar field. They further observed the formation of a black hole as a result of collapse of such body. The influence of electromagnetic field on massive objects plays a considerable role in studying their evolution and stability. The attractive nature of gravity can be overcome through the magnetic as well as Coulomb forces. A huge amount of charge is needed to hamper the gravitational attraction and sustain the stable behavior of self-gravitating systems. Bekenstein [34] investigated collapse of charged spherical structure and found that charge reduces the collapse rate. The same result has been produced by Esculpi and Aloma [35] while studying collapsing phenomenon for anisotropic charged distribution. Sharif and Azam [36] examined the impact of charge on the evolution of cylindrically symmetric system. Takisa and Maharaj [37] discussed anisotropic gravitating body by considering polytropic equation of state and found the profiles of matter variables which are consistent with earlier treatments.

Numerous massive and highly dense structures (stars and galaxies) are the main constituents that made the visible portion of our universe. These self-gravitating systems incorporate different physical quantities such as energy density, pressure, heat flow in their interiors that may cause to make them complex. A mathematical definition of structural complexity is required in terms of physical factors. In this regard, López-Ruiz et al. [38] defined complexity for the very first time in terms of information and entropy. This definition was initially employed on two simplest physical systems (perfect crystal and ideal gas). The molecules in former structure are symmetrically arranged throughout and thus has zero entropy whereas it is maximum in ideal gas as particles are randomly distributed. Moreover, ideal gas and perfect crystal contain maximum and less data (or information) in accordance with their structural composition, respectively. However, both patterns have no complexity.

Later, this concept was proposed in terms of disequilibrium but failed because the complexity of both the structures has been found to be zero under this definition [39, 40]. Another definition was suggested through energy density that replaced the probability distribution [41, 42], nonetheless this was insufficient as the interior of compact geometry may involve some other variables (heat flux, pressure and temperature, etc.). Herrera [43] recently redefined this concept and stated that complexity can be measured in terms of energy density inhomogeneity and pressure anisotropy inside a static sphere. He named a particular scalar as the complexity factor (encompassing all aforesaid parameters) that comes from orthogonal decomposition of the Riemann tensor. Sharif and Butt [44, 45] analyzed the effects of charge on this factor and also studied for uncharged cylindrical fluid source. Herrera et al. [46] then studied a dynamical dissipative system and discussed some evolutionary patterns along with kinematical/dynamical quantities. This work has also been generalized to the axially symmetric space-time [47]. Sharif and Majid [48–50] found several solutions for self-gravitating systems by extending this definition in Brans–Dicke scenario. The complexity for anisotropic configurations has also been analyzed in the context of $f(R, T)$ and $f(G, T)$ theories [51–54].

This article addresses evolution and complex composition of the charged dynamical cylinder involving the effects of heat dissipation in $f(R, T, R_{\psi\theta} T^{\psi\theta})$ gravity. The paper is outlined as follows. We introduce basic formalism of this modified theory and calculate the corresponding field equations as well as Bianchi identities for the model $R + \zeta R_{\psi\theta} T^{\psi\theta}$ in Sect. 2. Section 3 discusses four structure scalars that come from orthogonal splitting of the Riemann tensor. We further study the evolution of considered matter source through some evolutionary modes in Sect. 4. The unknown metric potentials of cylindrical geometry are determined in the absence/presence of heat dissipation in Sect. 5. Section 6 explores some conditions which may deviate the system from complexity-free scenario. Section 7 summarizes all our findings.

2 The $f(R, T, R_{\psi\theta} T^{\psi\theta})$ Gravity

The generic function of $R$, $T$ and $Q$ in place of the Ricci scalar in the Einstein–Hilbert action (with $\kappa = 8\pi$) provides the following form [18]

$$S_{f(R, T, R_{\psi\theta} T^{\psi\theta})} = \int \sqrt{-g} \left\{ \frac{f(R, T, R_{\psi\theta} T^{\psi\theta})}{16\pi} + L_{\mathcal{M}} + L_{\mathcal{E}\mathcal{M}} \right\} d^4x,$$

(1)

where $L_{\mathcal{M}}$ and $L_{\mathcal{E}\mathcal{M}}$ are the Lagrangian densities corresponding to matter distribution and electromagnetic field, respectively. The execution of the variational principle on the action (1) yields the field equations as

$$G_{\psi\theta} = T^{(\text{EFF})}_{\psi\theta} = \frac{8\pi}{f_{\mathcal{R}} - L_{\mathcal{M}} f_{\mathcal{Q}}} (T_{\psi\theta} + \mathcal{E}_{\psi\theta}) + T^{(C)}_{\psi\theta}.$$

(2)

Here, $G_{\psi\theta}$ is the Einstein tensor, $T^{(\text{EFF})}_{\psi\theta}$ is termed as the EMT in modified framework, $T_{\psi\theta}$ is the anisotropic matter EMT and $\mathcal{E}_{\psi\theta}$ is the electromagnetic tensor. The last factor in the above equation has the form

$$T^{(C)}_{\psi\theta} = - \frac{1}{(L_{\mathcal{M}} f_{\mathcal{Q}} - f_{\mathcal{R}})} \left\{ \left( f_{T} + \frac{1}{2} R_{T\theta} \right) T_{\psi\theta} + \left\{ \frac{R}{2} \left( \frac{f}{R} - f_{R} \right) - L_{\mathcal{M}} f_{T} \right\} \right\}.$$
to which test particles in the gravitational field start their motion in non-geodesic path. Consequently, we have

\[
- \frac{1}{2} \nabla_\theta \nabla_\omega (f_Q T^\theta_\omega) + g_{\theta_\omega} - \nabla \frac{1}{2} \left( f_Q T^\theta_\omega \right) - (g_{\theta_\omega} \Box - \nabla \theta \nabla_\omega ) f_R
- 2 f_Q R^\theta_\omega + \nabla_\omega \nabla_\theta [ T^\theta_\omega ] + 2 ( f_Q R^\theta_\omega + f_T g^{\theta_\omega} ) \frac{\partial^2 L_M}{\partial g^{\theta_\omega} \partial g^{\omega_\theta} } \right].
\]

The partial differentiation of \( f \) is \( f_R = \frac{\partial f(R, T, Q)}{\partial R} \), \( f_T = \frac{\partial f(R, T, Q)}{\partial T} \) and \( f_Q = \frac{\partial f(R, T, Q)}{\partial Q} \). Also, \( \nabla \) is the covariant derivative and \( \Box \) is the D’Alambert operator whose mathematical expression is \( \Box = \frac{1}{\sqrt{-g}} \partial_{\theta} (\sqrt{-g} g^{\theta_\omega} \partial_\omega ) \). The most suitable choice of the matter Lagrangian in this case is \( L_M = -\frac{1}{4} N_{\theta_\omega} N^{\theta_\omega} \) which results in \( \frac{\partial^2 L_M}{\partial g^{\theta_\omega} \partial g^{\omega_\theta} } = -\frac{1}{2} N_{\theta_\omega} N_{\omega_\theta} \). Here, \( N_{\theta_\omega} = O_{\theta\omega} - O_{\omega\theta} \) is known as the Maxwell field tensor and \( O_{\theta\omega} \) serves as the four potential.

The \( \mathbb{EMT} \) describing anisotropic configuration with heat dissipation is

\[
T_{\theta_\omega} = \mu K_{\theta} K_{\omega} + P h_{\theta_\omega} + \Pi_{\theta_\omega} + \xi ( K_{\theta} W_{\omega} + W_{\theta} K_{\omega} ),
\]

where \( W_{\theta}, K_{\theta}, h_{\theta_\omega}, \) and \( \xi \) are the four-vector, four-velocity, projection tensor and heat flux, respectively, which satisfy the relations \( K_{\theta} K^{\theta} = -1 \), \( K_{\theta} W^{\theta} = 0 \), \( S_{\theta_\omega} S^{\theta_\omega} = 0 \), \( W_{\theta} W^{\theta} = 1 \). The remaining terms are described as

\[
P = \frac{P_r + 2 P_L}{3}, \quad h_{\theta_\omega} = g_{\theta_\omega} + K_{\theta} K_{\omega},
\]

\[
\Pi_{\theta_\omega} = \Pi \left( W_{\theta} W_{\omega} - \frac{h_{\theta_\omega}}{3} \right), \quad \Pi = P_r - P_L.
\]

It is important to mention here that pressure generally exists in three different directions for anisotropic cylindrically symmetric structure [55, 56], but the matter content (4) is not the most general form of the fluid distribution, rather it is a restricted case.

As this theory involves components of fluid configuration coupled with geometry, thus the equivalence principle does not hold and divergence of the corresponding \( \mathbb{EMT} \) is no more conserved, i.e., \( \nabla \theta T^{\theta_\omega} \neq 0 \). This causes the exertion of an extra force due to which test particles in the gravitational field start their motion in non-geodesic path. Consequently, we have

\[
\nabla T_{\theta_\omega} + \varepsilon_{\theta_\omega} = \frac{2}{2 f_T + R f_Q + 16 \pi} \left[ \nabla_{\theta} ( f_Q R^{\theta_\omega} T_{\theta_\omega} ) + \nabla_{\theta} ( L_M f_T )
- g_{\theta_\omega} \nabla_{\theta} ( f_Q L_M ) - \frac{1}{2} \nabla_{\theta} T^{\theta_\omega} ( f_T g_{\theta_\omega} + f_Q R_{\theta_\omega} )
- \frac{1}{2} \left( \nabla_{\theta} ( R f_Q ) + 2 \nabla_{\theta} f_T \right) T_{\theta_\omega} \right].
\]

The trace of \( f(R, T, Q) \) field equations yields

\[
3 \nabla_{\omega} f_R - R \left( \frac{T}{2} f_Q - f_R \right) - T ( 8 \pi + f_T ) + \frac{1}{2} \nabla_{\omega} \nabla_{\theta} ( f_Q T^{\theta_\omega} )
+ \nabla_{\omega} V_{\theta} ( f_Q T^{\theta_\omega} ) - 2 f ( R f_Q + 4 f_T ) L_M + 2 R_{\theta_\omega} T^{\theta_\omega} f_Q
- 2 g^{\theta_\omega} \frac{\partial^2 L_M}{\partial g^{\theta_\omega} \partial g^{\omega_\theta} } ( f_T g^{\theta_\omega} + f_Q R^{\theta_\omega} ) = 0.
\]

The gravitational effects of \( f(R, T) \) theory can be obtained by considering \( f_Q = 0 \), whereas \( f_T = 0 \) with previous limit gives \( f(R) \) gravity. The electromagnetic tensor is

\[
\varepsilon_{\theta_\omega} = \frac{1}{4 \pi} \left[ \frac{1}{4} g_{\theta_\omega} N^{\theta_\omega} N_{\mu_\omega} - N_{\omega_\omega} N^{\theta_\omega} \right],
\]

and Maxwell equations have the form

\[
N_{\omega_\theta} = 4 \pi J^{\theta_\omega}, \quad N[\theta, \omega] = 0,
\]

where \( J^{\theta_\omega} = \sigma K^{\theta_\omega} \). Here, \( J^{\theta_\omega} \) and \( \sigma \) indicate the current and charge densities, respectively.

We adopt a standard model of the form (proposed by Haghani et al. [15])

\[
f(R, T, R_{\theta_\omega} T^{\theta_\omega}) = f_1(R) + f_2 ( R_{\theta_\omega} T^{\theta_\omega} ) = R + \Phi R_{\theta_\omega} T^{\theta_\omega}.
\]

This model offers several solutions showing an oscillatory behavior for positive values of \( \Phi \), while \( \Phi < 0 \) produces the cosmic scale factor which has a hyperbolic cosine-type dependence. It is worth noting that the value of the coupling parameter within its observed range guarantees physical feasibility of the corresponding gravity model. Some acceptable values of \( \Phi \) have been explored under which the obtained solutions with respect to the model (8) for isotropic configuration show stable behavior [16, 17].

We take restricted form of cylindrically symmetric dynamical line element to examine the interior as

\[
ds^2 = -A^2 dt^2 + B^2 dr^2 + C^2 (d\theta^2 + a^2 d\zeta^2),
\]

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where \( A = A(t, r) \) and \( B = B(t, r) \) are dimensionless, while \( C = C(t, r) \) has the dimension as of \( r \). Also, \( \alpha \) is supposed to be a constant with dimension of inverse length. The charge inside the cylinder is defined as

\[
s(r) = 4\pi \int_0^r \sigma B C^2 dr,
\]

and the matter Lagrangian becomes \( \mathcal{L}_M = \frac{i^2}{2c^2} \). The four-velocity, heat flux and four-vector in comoving scenario are characterized as

\[
\mathcal{K}^\mu = \delta^\mu_\rho A^{-1}, \quad \zeta^\mu = \delta^\mu_\nu B^{-1}, \quad \mathcal{W}^\mu = \delta^\mu_\nu B^{-1}.
\]

The quantity \( Q \) of the model (8) becomes

\[
Q = -\frac{1}{A^3B^3C^3} \left[ \mu(2\dot{\mathcal{C}}AB^3 + \ddot{\mathcal{B}}AB^2C - A''A^3BC - 2\dot{\mathcal{A}}\dot{\mathcal{C}}B^3 - 2\dot{\mathcal{A}}'C'\dot{A}^2B \\
+ A'B'A''C' - \dot{\mathcal{A}}\dot{\mathcal{B}}B^2C + 4\xi AB(\dot{\mathcal{C}}'AB - A'\dot{\mathcal{C}}B - \dot{\mathcal{B}}C'\dot{A}) \\
+ P_1(2C''A^3B - \ddot{\mathcal{B}}AB^2C + A''A^3BC - 2\dot{\mathcal{B}}\dot{\mathcal{C}}AB^2 - 2B'C^3A^3 \\
- A'B'A^2C + \ddot{\mathcal{A}}\dot{\mathcal{B}}B^2C) + \frac{2P_1}{C}(C''A^3B - \dot{\mathcal{C}}AB^3C - \dot{\mathcal{C}}^2AB^3 \\
- \dot{\mathcal{B}}\dot{\mathcal{C}}AB^2C + \ddot{\mathcal{A}}\dot{\mathcal{C}}CB^3 + C^2A^3B - B'C^3C + A'C'A^2BC) \right].
\]

where \( = \frac{\partial}{\partial t} \) and \( \frac{\partial}{\partial t} \). The non-vanishing components of the field equations (2) are

\[
8\pi \left( \frac{\mu}{A^3B^3} + \frac{\bar{\sigma}}{A^3B^3C^3} + \frac{\bar{T}_0}{A^3B^3C^3} + \frac{\bar{\epsilon}_0}{A^3B^3C^3} \right) = -\frac{1}{B^2} \left( \frac{C''}{C} + \frac{2\dot{C}}{C} - \frac{2\dot{C}'}{C} \right) \frac{\dot{B}}{C} + \frac{\dot{C}}{A^2} \left( \frac{\dot{\mathcal{C}}}{C} + \frac{2\dot{\mathcal{B}}}{C} \right),
\]

\[
8\pi \left( \frac{\bar{\sigma}}{A^3B^3C^3} + \frac{\bar{T}_1}{A^3B^3C^3} + \frac{\bar{\epsilon}_1}{A^3B^3C^3} \right) = -\frac{1}{B^2} \left( \frac{2\dot{C}}{C} - \frac{2\dot{C}}{C} \right),
\]

\[
8\pi \left( \frac{\bar{P}_0}{A^3B^3C^3} + \frac{\bar{T}_2}{A^3B^3C^3} + \frac{\bar{\epsilon}_2}{A^3B^3C^3} \right) = -\frac{1}{B^2} \left( \frac{2\dot{C}}{C} - \frac{2\dot{C}}{C} \right),
\]

where \( \bar{\mu} = \frac{\mu}{1 - \frac{\alpha^2}{2\rho^2}} \), \( \bar{\sigma} = \frac{\sigma}{1 - \frac{\alpha^2}{2\rho^2}} \), \( \bar{P}_0 = \frac{P_0}{1 - \frac{\alpha^2}{2\rho^2}} \), \( \bar{P}_\perp = \frac{P_\perp}{1 - \frac{\alpha^2}{2\rho^2}} \) and \( \bar{\rho} = \frac{\rho}{1 - \frac{\alpha^2}{2\rho^2}} \). The terms \( \bar{T}_0, \bar{T}_1, \bar{T}_2 \) and \( \overline{\bar{\epsilon}_0}, \overline{\bar{\epsilon}_1}, \overline{\bar{\epsilon}_2} \) as well as their corresponding charge components \( \overline{\bar{\epsilon}_0}, \overline{\bar{\epsilon}_1}, \overline{\bar{\epsilon}_2} \) are modified corrections to the field equations whose values are given in Appendix A.

The non-null components of Bianchi identity through Eq. (7) are given as

\[
\bar{T}^\mu_\nu \mathcal{K}_\rho = -\frac{1}{A} \left( \bar{\mu} + (\mu + \bar{P}_\perp) \bar{B} + 2(\mu + \bar{P}_\perp) \frac{\dot{\mathcal{C}}}{C} \right) - \frac{1}{B} \left( \frac{\dot{\mathcal{C}}'}{\mathcal{C}} + \frac{2\dot{\mathcal{C}}'}{\mathcal{C}} \right) \frac{\dot{\mathcal{C}}}{C},
\]

\[
\bar{T}^\mu_\nu \mathcal{W}_\rho = \frac{1}{A} \left( 2\zeta \left( \frac{B'}{B} + \frac{C'}{C} \right) + \frac{\dot{\zeta}}{C} \right) + \frac{1}{B} \left( P_\perp + 2(\mu + \bar{P}_\perp) \frac{C'}{C} + (\mu + \bar{P}_\perp) \frac{\dot{\mathcal{C}}'}{C} \right) \frac{\dot{\mathcal{C}}}{C}.
\]

(16)

and

\[
\bar{T}^\mu_\nu \mathcal{W}_\rho = \frac{1}{A} \left( 2\zeta \left( \frac{B'}{B} + \frac{C'}{C} \right) + \frac{\dot{\zeta}}{C} \right) + \frac{1}{B} \left( P_\perp + 2(\mu + \bar{P}_\perp) \frac{C'}{C} + (\mu + \bar{P}_\perp) \frac{\dot{\mathcal{C}}'}{C} \right) \frac{\dot{\mathcal{C}}}{C}.
\]

(17)
where the terms on the right hand side of the above equations confirm the non-conservation of this gravity. The values of $Z_1$ and $Z_2$ are included in Appendix A. Some dynamical terms such as expansion scalar, nonzero components of shear tensor as well as four-acceleration are defined as

$$\Theta = \frac{1}{A} \left( 2 \frac{\dot{C}}{C} + \frac{\dot{B}}{B} \right), \quad \text{(18)}$$

$$\sigma_{11} = \frac{2}{3} B^2 \sigma, \quad \sigma_{22} = \frac{\sigma_{33}}{\alpha^2} = -\frac{1}{3} C^2 \sigma, \quad \text{(19)}$$

$$\sigma^{\psi \phi} \sigma_{\psi \phi} = \frac{2}{3} \sigma^2, \quad \sigma = \frac{1}{A} \left( \frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right), \quad \text{(20)}$$

$$a_1 = \frac{A'}{A}, \quad a = \sqrt{\sigma} a_\psi = \frac{A'}{AB}. \quad \text{(21)}$$

It should be noted that two scalars are needed to determine the shear tensor in the case of general cylindrically symmetric fluid [56]; however, we define a single scalar function (20) due to restricted class of the cylindrical space-time.

The effect of expansion scalar and shear on the fluid distribution can be analyzed by the alternative form of Eq. (13) as

$$4\pi (\tilde{z} - \tau_0^{1(\psi)} - \xi_0^{1(\psi)}) = \frac{1}{3} (\Theta - \sigma)' - \sigma C' \left( \frac{1}{3} D_R (\Theta - \sigma) - \sigma \right), \quad \text{(22)}$$

where the proper radial derivative is symbolized as $D_R = \frac{1}{C} \frac{d}{dr}$. The formula of C-energy can be employed to calculate mass of the cylindrical geometry [57]. Thus the following equation provides relation between C-energy and the mass function as

$$m(t, r) = \frac{\dot{E}}{8} (1 - \Theta - \frac{\sigma}{\alpha^2} \nabla_\psi \nabla_\psi \dot{\tau}) \quad \text{(23)}$$

where $\dot{\tau} = \theta \xi$, $\xi$ is the circumference radius and $\xi$ symbolizes specific length. Mathematically, we have $\dot{\Omega}^2 = \eta_{1(\psi)} \eta_{1(\psi)}$ and $\Omega^2 \theta^2 = \eta_{1(\psi)} \eta_{2(\psi)}$ in which $\eta_{1(\psi)} = \frac{\dot{\theta}}{\alpha^2}, \eta_{2(\psi)} = \frac{\dot{\phi}}{\alpha^2}$. Here, $\dot{E}$ is defined as the gravitational energy per specific length. Equation (23) yields the mass in terms of metric potentials as

$$m = C \left[ \frac{1}{4} - \left( \frac{C'}{B} \right)^2 + \left( \frac{\dot{C}}{A} \right)^2 + \frac{s^2}{C^2} \right]. \quad \text{(24)}$$

We study the evolution of charged dynamical cylinder by utilizing the definition of proper time derivative, i.e., $D_T \equiv \frac{1}{C} \frac{d}{dt}$. During the collapse of an astronomical object, the continuous reduction of its radius occurs, as a result of which the velocity of fluid in the interior turns to be negative, i.e.,

$$U = D_T C < 0. \quad \text{(25)}$$

Equations (24) and (25) provide the relationship between the C-energy and velocity as

$$\frac{\dot{E}}{B} = \frac{C'}{B} = \sqrt{U^2 + \frac{1}{4} + \frac{s^2}{C^2} - \frac{2m}{C}}. \quad \text{(26)}$$

We use the definition of $D_T$ to express the energy variation inside cylindrical object as

$$D_T \dot{m} = -4\pi \left[ \left( \tilde{F}_r + \tau_1^{1(\psi)} - \frac{s^2}{8\pi C^4} + E_1^{1(\psi)} \right) U + \left( \tilde{z} - \tau_0^{1(\psi)} - \xi_0^{1(\psi)} \right) \frac{\dot{\tau}}{\dot{E}} \right] \quad \text{(27)}$$

while it comes out to be in terms of $D_R$ as

$$D_R \dot{m} = 4\pi \left[ \left( \tilde{\mu} + \tau_0^{0(\psi)} + \frac{s^2}{8\pi C^4} + E_0^{0(\psi)} \right) + \frac{1}{32\pi C^2} \right] + \left( \tilde{z} - \tau_0^{1(\psi)} - \xi_0^{1(\psi)} \right) \frac{\dot{U}}{\dot{E}} \quad \text{(28)}$$

which further yields

$$\frac{3\dot{m}}{C^3} = 4\pi \left( \tilde{\mu} + \tau_0^{0(\psi)} + \frac{s^2}{8\pi C^4} + E_0^{0(\psi)} \right) - \frac{4\pi}{C^3} \int_0^1 \left[ D_R \left( \tilde{\mu} + \tau_0^{0(\psi)} + \frac{s^2}{8\pi C^4} + E_0^{0(\psi)} \right) \right] C^3 dr + \frac{3U}{CE} \left( \tilde{z} - \tau_0^{1(\psi)} - \xi_0^{1(\psi)} \right) C^3 dr + \frac{3}{8C^2} + \frac{3s^2}{2C^2}. \quad \text{(29)}$$
The term $\frac{1}{8\pi}$ in the above equation is obvious from C-energy (23). The Weyl tensor ($C_{\nu\theta\rho\sigma}^\gamma$) gives the amount of stretch by which a massive body educes nearby celestial structures due to fluctuations in its gravitational field. There are two independent components which completely define this tensor, namely magnetic and electric parts which are expressed, respectively, as

$$H_{\nu\theta} = \frac{1}{2} \eta_{\omega\nu\theta} C_{\nu\theta}^{\gamma\rho} K^{\rho\sigma} K^{\sigma\mu},$$

$$E_{\nu\theta} = C_{\nu\theta\varphi} K^{\mu\nu} K^{\sigma}.$$  

For the general case of cylindrical fluid, the magnetic part is non-vanishing and depends on a scalar, while it disappears in the current (restricted) setup. Moreover, the electric part can also be expressed in an alternative way as

$$E_{\nu\theta} = \varepsilon \left( W_{\nu} W_{\theta} - \frac{h_{\nu\theta}}{3} \right),$$  (30)

where the value of electric scalar is

$$\varepsilon = \frac{1}{2A^2} \left[ \frac{C}{C'} - \frac{B}{B'} - \left( \frac{C}{A} + \frac{\dot{A}}{A} \right) \left( \frac{C}{B} - \frac{\dot{B}}{B} \right) \right] + \frac{1}{2B^2} \left[ \left( \frac{C'}{C} - \frac{A'}{A} \right) \right] \times \left( \frac{C'}{C} + \frac{B'}{B} \right) + \frac{A''}{A} - \frac{C''}{C} - \frac{1}{2C^2}. \quad (31)$$

We would like to point out here that we describe the electric Weyl tensor in terms of a single scalar function due to the constrained character of the cylinder, whereas for the general cylindrically symmetric fluid, it is defined in the form of two scalars [56].

The effects of tidal force can also be described by the Weyl tensor and can be studied by the following equation due to the scalar (31) as

$$\frac{3\bar{m}}{C^3} = -\varepsilon + 4\pi \left\{ \left( \bar{\mu} + T_0^{(0)}(C) + \frac{z^2}{8\pi C^4} + \varepsilon_0^{(0)}(C) \right) - \Pi_1^{(E)} - \varepsilon(C) \right\} + \frac{3}{8C^2} + \frac{3s^2}{2C^4},$$ \quad (32)

where $\varepsilon(C) = \varepsilon_1^{(C)} - \varepsilon_2^{(C)}$, $\Pi_1^{(E)} = \Pi_1^{(C)}$, $\Pi_1 = \frac{\bar{m}}{1 - \frac{2m}{C^2}}$ and $\Pi_1^{(C)} = T_1^{(C)}$.  

3 Structure Scalars

Herrera et al. [58] proposed an extensive approach to split the Riemann tensor orthogonally. We use this approach in $f(R, T, R_{\nu\theta} T^{\nu\theta})$ gravity which yields some tensors and can further be split in their trace and trace-free parts. These scalars must associate with certain physical variables of the configuration. The Riemann tensor, the Weyl tensor as well as modified EMT and its trace can be interlinked through the following equation as

$$R_{\nu\varphi\rho\gamma}^{\omega\nu\theta} = C_{\nu\theta}^{\gamma\rho} + 16\pi \Omega_{[\nu}^{(E)} e^{[\omega}_{[\varphi} \delta_{\rho]}^{\sigma]} + 8\pi \Omega_{(E)} \left( \frac{1}{3} \delta_{[\nu}^{\sigma] \delta_{\varphi]}^{\rho} - \delta_{[\nu}^{[\varphi} \delta_{\rho]}^{\sigma]} \right),$$ \quad (33)

where antisymmetric property of the indices ($\omega, \varphi, \gamma, \theta$) is used. Here, $S_{[\nu}^{(E)} e^{[\omega}_{[\varphi} \delta_{\rho]}^{\sigma]} = T_{[\nu}^{(E)} e^{[\omega}_{[\varphi} \delta_{\rho]}^{\sigma]} + e^{[\omega}_{[\varphi} \delta_{\rho]}^{\sigma]}$, Equation (33) produces certain tensors, i.e., $Y_{\nu\theta}$ and $X_{\nu\theta}$ (after some lengthy calculations) as

$$Y_{\nu\theta} = R_{\nu\omega\theta} K^{\mu\nu} K^{\rho\gamma},$$ \quad (34)

$$X_{\nu\theta} = \ast R_{\nu\omega\theta} K^{\mu\nu} K^{\rho\gamma} = \frac{1}{2} \eta_{\nu\omega} R_{\nu\theta} K^{\mu\nu} K^{\rho\gamma},$$ \quad (35)

where $\eta_{\nu\omega}$ and $R_{\nu\omega\theta}^{\ast}$ are the Levi–Civita symbol and the dual Riemann tensor, respectively, defined as $R_{\nu\omega\theta}^{\ast} = \frac{1}{2} \eta_{\omega\nu\theta} R_{\nu\sigma\gamma}^{\sigma\nu\gamma}$. The alternate expressions of tensors (34) and (35) are

$$Y_{\nu\theta} = \frac{h_{\nu\theta}}{3} Y_T + \left( W_{\nu} W_{\theta} - \frac{h_{\nu\theta}}{3} \right) X_T,$$ \quad (36)

$$X_{\nu\theta} = \frac{h_{\nu\theta}}{3} X_T + \left( W_{\nu} W_{\theta} - \frac{h_{\nu\theta}}{3} \right) X_T,$$ \quad (37)

where $Y_T$, $X_T$, $Y_{TF}$ and $X_{TF}$ are trace and trace-free parts, respectively. In this scenario, these scalar functions come out to be

$$X_T = \frac{1}{1 - \frac{2m}{C^2}} \left( 8\pi \mu - \frac{z^2}{C^4} \right) \left( \frac{1}{2} \Phi R + 1 \right) + X_1^{(C)},$$ \quad (38)
\[ \chi_{TF} = -\varepsilon - \frac{1}{1 - \frac{\Phi T}{2\pi^2}} \left( 4\pi \Pi - \frac{s^2}{C^4} \right) \left( \frac{1}{2} \Phi R + 1 \right), \]  
\[ (39) \]

\[ \gamma_T = \frac{1}{1 - \frac{\Phi T}{2\pi^2}} \left\{ 4\pi (\mu + 3P_r - 2\Pi) + \frac{s^2}{C^4} \left( \frac{1}{2} \Phi R + 1 \right) + \chi^{(C)}_2 \right\}, \]  
\[ (40) \]

\[ \gamma_{TF} = \varepsilon - \frac{1}{1 - \frac{\Phi T}{2\pi^2}} \left( 4\pi \Pi - \frac{s^2}{C^4} \right) \left( \frac{1}{2} \Phi R + 1 \right) + \chi^{(C)}_3, \]  
\[ (41) \]

where the values of modified corrections \( \chi^{(C)}_1, \chi^{(C)}_2 \) and \( \chi^{(C)}_3 \) are given in Appendix B. It has been mentioned earlier that we have imposed restrictions on geometrical structure and the fluid distribution. As a result, we obtain only one structure scalar corresponding to the trace-free part of the electric component of the Riemann tensor \( \gamma_{\varphi \varphi} \). The scalar \( \chi_T \) incorporates only homogeneous energy density while factor \( \gamma_T \) encompasses local anisotropic pressure as well along with modified corrections.

The nature of any geometrical configuration (in the absence as well as presence of charge) can be understood through the study of several state variables such as energy density and radial/tangential pressure. It is observed from Eq. (42) that scalar \( \gamma_{TF} \) entails all physical parameters such as effective inhomogeneous energy density, dissipation flux, charge and pressure anisotropy. Another factor appears in the orthogonal splitting is \( \chi_{TF} \) (39) that helps to analyze the inhomogeneity of the energy density of fluid configuration as

\[ \chi_{TF} = -8\pi \tilde{\Pi} - 4\pi (\Pi^{(C)} + \varepsilon^{(C)}) + \chi^{(C)}_3 - \Phi R \left( 4\pi \tilde{\Pi} - \frac{s^2}{C^4} \right) \]
\[ + \frac{s^2}{C^4} + \frac{4\pi}{C^3} \int_0^\tau C^3 \left[ \frac{D_R}{C_E} \left( \tilde{\mu} + \tau_0^{(C)} + \frac{s^2}{8\pi C^4} + \varepsilon_0^{(C)} \right) \right] d\tau \]
\[ - 3 \left( \tilde{\xi} - \tau_1^{(C)} - \varepsilon_0^{(C)} \right) \frac{U}{C_E} ] C' d\tau. \]  
\[ (42) \]

It is observed from Eq. (42) that modified scalar \( \gamma_{TF} \) comprises all physical parameters such as effective inhomogeneous energy density, dissipation flux, charge and pressure anisotropy. Another factor appears in the orthogonal splitting is \( \chi_{TF} \) (39) that helps to analyze the inhomogeneity of the energy density of fluid configuration as

\[ \chi_{TF} = -4\pi \int_0^\tau C^3 \left[ \frac{D_R}{C_E} \left( \tilde{\mu} + \tau_0^{(C)} + \frac{s^2}{8\pi C^4} + \varepsilon_0^{(C)} \right) \right] d\tau \]
\[ + \frac{4\pi}{C_E} \left( \tilde{\Pi} - \frac{s^2}{C^4} \right) \left( \frac{1}{2} \Phi R + 1 \right) + \chi^{(C)}_3, \]  
\[ (43) \]

4 Different modes of evolution

The nature of any geometrical configuration (in the absence as well as presence of charge) can be understood through the study of several state variables such as energy density and radial/tangential pressure. It is observed from Eq. (42) that scalar \( \gamma_{TF} \) entails the combination of all these quantities together with dissipation flux and charge in association with modified corrections. Thereby, we choose it as the complexity factor for non-static cylindrical distribution influenced from electromagnetic field. Subsequently, the condition \( \gamma_{TF} = 0 \) leads to the complexity-free system. Two evolutionary patterns (homologous evolution and homogeneous expansion) are considered in the following subsections to examine the dynamical changes in the interior of self-gravitating object. We will construct some limitations which ultimately leads to the less complex system throughout the evolution.

4.1 Homologous evolution

The term homologous refers to the system which has same pattern throughout. The core of a compact object becomes so heavy after the inward fall of all the material into it, due to which that body collapses. Nonetheless, the radial distance and velocity of the fluid are directly related to each other in homologous collapse. Thus, we can say that the core attracts all the matter at the same rate during the collapse which consequently emits much more gravitational radiations dissimilar to the body whose core collapses initially. Equation (22) in terms of velocity of the fluid has the form

\[ D_R \left( \frac{U}{C} \right) = \frac{1}{C} \left\{ \sigma + \frac{4\pi}{C_E} \left( \tilde{\xi} - \tau_1^{(C)} - \varepsilon_0^{(C)} \right) \right\}. \]  
\[ (44) \]

After integrating Eq. (44), we have

\[ U = x(t) C + C \int_0^\tau C' \left\{ \sigma + \frac{4\pi}{C_E} \left( \tilde{\xi} - \tau_1^{(C)} - \varepsilon_0^{(C)} \right) \right\} d\tau. \]  
\[ (45) \]
where \( x(t) \) serves as an integration function. The final form of the velocity of collapsing cylindrical distribution at the boundary can be determined as

\[
U = C \left[ \frac{\Psi}{C^2} - \int_{r_0}^{r_\infty} \frac{C'}{C} \left( \sigma + \frac{4\pi C}{E} \left( \bar{\xi} - T_{0}^{1(C)} - E_{0}^{1(C)} \right) \right) dr \right].
\]  

(46)

The deviation of cylindrical structure from homologous mode can be studied through some significant factors, i.e., heat dissipation and shear scalar. We can observe homologous evolution inside the system \([59, 60]\), if effects of the above integrand disappears. Consequently, Eq. (46) is left with \( U \sim C \) leads to \( U = x(t)C \) and \( x(t) = \frac{\Psi}{C^2} \). The homologous condition for the fluid influenced from electromagnetic field has the form

\[
\frac{4\pi BC}{C^2} (\bar{\xi} - T_{0}^{1(C)} - E_{0}^{1(C)}) + \sigma = 0.
\]  

(47)

4.2 Homogeneous expansion

The constraint \( \Theta' = 0 \) is required to discuss another phenomenon, called as homogeneous expansion. This phase takes place when the rate at which cosmic bodies collapse or expand is not dependent on \( r \), unlike preceding mode. This constraint becomes together with Eq. (22) as

\[
4\pi (\bar{\xi} - T_{0}^{1(C)} - E_{0}^{1(C)}) = -\frac{C'}{3B} \left[ \frac{3\sigma}{C} + \mathcal{D}_R(\sigma) \right].
\]  

(48)

The simultaneous use of the homologous condition (47) and Eq. (48) yields \( \mathcal{D}_R(\sigma) = 0 \). Due to the regularity condition at the core, we have \( \sigma = 0 \) which makes Eq. (48) as

\[
\bar{\xi} = T_{0}^{1(C)} + E_{0}^{1(C)},
\]  

(49)

which discloses the incorporation of dissipative effects due to \( f(R, T, \mathcal{R}_\varphi \mathcal{R}^{\varphi}) \) corrections and thus opposing \( GR \), where the homogeneous evolution results in non-dissipative and shear-free matter source \([46]\).

5 Some kinematical and dynamical considerations

In this section, some physical entities are analyzed to choose the simplest possible evolutionary mode. Equation (22) along with the homologous condition (47) give

\[
(\Theta - \sigma)' = \left( \frac{3\mathcal{C}}{AC} \right)' = 0.
\]  

(50)

We take the metric potential \( C(t, r) \) as a separable function of both coordinates. Thus, we obtain \( A' = 0 \) [i.e., \( a = 0 \) from Eq. (21)] leads to the geodesic fluid. Further, we put \( A = 1 \) without any loss of generality. On the contrary, Eqs. (18) and (20) for \( A = 1 \) yields

\[
\Theta - \sigma = \frac{3\mathcal{C}}{C},
\]  

(51)

which provides \( (\Theta - \sigma)' = 0 \), and hence recovering the homologous condition (50). Thus, the necessary and sufficient condition for the dynamical cylinder to evolve in homologous mode is that the fluid must follow geodesic path. In \( GR \), the absence of heat dissipation \( (\bar{\xi} = 0 \Rightarrow \bar{\xi} = 0) \) implies the disappearance of the shear \( (\sigma = 0) \) in the matter source as opposed to \( f(R, T, \mathcal{Q}) \) framework where we have

\[
\sigma = \frac{4\pi BC}{C^2} (T_{0}^{1(C)} + E_{0}^{1(C)}).
\]  

(52)

Equation (48) produces the shear scalar corresponding to homogeneous pattern as

\[
\sigma = \frac{y(t)}{C^3} + \frac{12\pi}{C^3} \int_{r_0}^{r} \frac{BC^3}{C^2} (T_{0}^{1(C)} + E_{0}^{1(C)}) dr,
\]  

(53)

where \( y(t) \) is an arbitrary integration function. Furthermore, it is deduced that homogenous pattern implies homologous condition \( (\sigma = 0 \Rightarrow U \sim C) \) when heat dissipation and modified corrections are neglected. For the current cylindrical setup, the C-energy can be linked with collapse rate as

\[
\mathcal{D}_c U = -\frac{\dot{m}}{C^2} - 4\pi C \left( \dot{\mathcal{P}}_r + T_{1}^{1(C)} - \frac{s^2}{8\pi C^4} + E_{1}^{1(C)} \right) + \frac{1}{8C} + \frac{s^2}{2C^3}.
\]  

(54)
We combine Eq. (54) with the scalar $\gamma_{TF}$ (41) to get

$$\frac{3D_{TF}^{\parallel}}{C} = \gamma_{TF} - \frac{\Phi R}{2} \left( 4\pi \Pi - \frac{\bar{s}^2}{C^4} \right) - 4\pi \{ \bar{\mu} + \bar{\tau}_0^{(C)} + \epsilon_0^{(C)} \} + 3(\bar{\rho} + \bar{T}_1^{(IC)} + \epsilon_1^{(IC)}) + 2 \bar{\Pi} - \Pi^{(C)} - \epsilon^{(C)} \} - \chi_3^{(C)}.$$  

(55)

Using Eqs. (12), (14) and (15), we have

$$4\pi \{ \bar{\mu} + \bar{T}_0^{(C)} + \epsilon_0^{(C)} \} - 2(\bar{\Pi} - \Pi^{(C)} - \epsilon^{(C)}) + 3(\bar{\rho} + \bar{T}_1^{(IC)} + \epsilon_1^{(IC)}) \}

= - \frac{\bar{\rho}^2}{C^2} - \frac{\bar{\rho}^2}{C^4},$$  

(56)

and

$$\frac{3D_{TF}^{\parallel}}{C} = 3\frac{\bar{\rho}}{C}.$$  

(57)

Equations (55)--(57) simultaneously lead to

$$\frac{\bar{\rho}}{C} = \gamma_{TF} - \frac{\Phi R}{2} \left( 4\pi \Pi - \frac{\bar{s}^2}{C^4} \right).$$  

(58)

It should be mentioned here that the complexity-free structure can be obtained by taking

$$\frac{\bar{\rho}}{C} = \gamma_{TF} - \frac{\Phi R}{2} \left( 4\pi \Pi - \frac{\bar{s}^2}{C^4} \right).$$

(59)

Firstly, for the non-dissipative case, we take $\gamma_{TF} = 0 = \Pi^{(EFF)} = \zeta = \sigma$ at some initial time (say, $t = 0$) for which Eq. (59) yields

$$\frac{12\pi \bar{C}^2}{BC} \left( \bar{T}_0^{(IC)} + \bar{\epsilon}_0^{(IC)} \right) + \frac{\Phi}{16\pi + \Phi R} \left( \bar{\rho} \frac{\bar{s}^2}{C^4} \right) = 0.$$  

(60)

Together use of this equation and the time derivative of Eq. (42) (at $t = 0$), we have

$$\frac{\partial}{\partial t} \left[ \frac{1}{C^3} \int_0^t C^3 \left\{ (\bar{\mu} + \bar{T}_0^{(IC)} + \bar{s}^2 + \bar{\epsilon}_0^{(IC)}) \right\} \right] dr = 4\pi \bar{T}_0^{(IC)} + 2\pi \Phi (\bar{\Pi}^R) - 4\pi \bar{\epsilon}_0^{(IC)} + 8\pi \bar{\epsilon}^{(C)} + \frac{4\pi \bar{C}^2}{BC} = 0.$$

(61)
The stability of $\mathcal{Y}_{TF} = 0$ depends on state determinants (density and pressure). We see from Eq. (61) that the system could depart from stability if the interior configuration involves inhomogeneous energy density, pressure anisotropy and charge. Thus, in this scenario of charged configuration, the electromagnetic field also disturbs stability of the considered setup. Moreover, by substituting $\sigma = \mathcal{Y}_{TF} = 0$ in Eq. (59), we have the most general case involving dissipation flux as

$$-rac{4\pi}{B} \left\{ \bar{s}' - \frac{C'}{C} \left( \bar{s} + 3\bar{s}^{(C)} + 3s^{(C)} \right) \right\} + \frac{\Phi}{16\pi + \Phi R} \left( \mu \tilde{\mathcal{R}} + \bar{\rho} \tilde{\mathcal{R}}' - B \right) - \bar{\mathcal{Y}}_{TF}$$

$$+ \frac{\Phi}{2} \left( \bar{\mathcal{R}}^{\tilde{\mathcal{R}}^2} - \frac{3\bar{\mathcal{C}}}{C} \right) + \frac{\Phi R \bar{s}^2}{C^4} + \chi^{(C)}_{3} + \bar{\chi}^{(C)}_{3} + 4\pi \bar{\varepsilon}^{(C)} - 4\pi \bar{\varepsilon}^{(C)} - 8\pi \bar{\Pi}$$

$$- 4\pi \bar{\Pi}^{(C)} - 4\pi \bar{Z}_{1} + 4\pi \bar{T}_{0}^{(C)} + \frac{12\pi}{C} \left( \bar{T}_{0}^{(C)} + \bar{T}_{2}^{(C)} + \bar{\varepsilon}^{(C)} + 3\varepsilon^{2(C)} \right) = 0. \tag{62}$$

This shows that stability is now affected also by the heat flux.

7 Final remarks

Our universe contains plenty of astronomical bodies whose astonishing nature prompted many researchers to study their complex structures. This paper is devoted to studying different physical variables representing interior of the cylindrical fluid distribution that make the celestial object more complex in $f(R, T, \mathcal{R}_{\sigma}, T^{n\sigma})$ framework. A standard model $R + \Phi \mathcal{R}_{\sigma} T^{n\sigma}$ has been considered in this regard to study the matter-geometry coupling effects on non-static space-time influenced from electromagnetic field. We have considered anisotropic matter distribution coupled with heat flux. The Riemann tensor has been split orthogonally through Herrera’s technique which resulted in four scalars, each of them is uniquely defined in terms of particular physical parameters. Following reasons justify the adoption of $\mathcal{Y}_{TF}$ from four resulting candidates as the complexity factor.

1. This factor has been recognized as the best candidate for the complexity factor in $\mathcal{G}_{R}$ [43] as well as $f(R, T, \mathcal{R}_{\sigma}, T^{n\sigma})$ gravity [24–26] for static uncharged/charged space-times. Thus, $\mathcal{Y}_{TF}$ can be recovered from Eq. (41) for static scenario.

2. All physical quantities such as energy density inhomogeneity, heat dissipation, pressure anisotropy and charge together with correction terms should be included in complexity factor which is ensured only by this factor.

We have considered two simplest evolutionary patterns for self-gravitating dynamical structure, i.e., homogeneous expansion and homologous evolution, and studied them in modified gravity. We have constructed the possible solutions for metric potentials in the case of dissipation as well as non-dissipation with the help of homologous condition (47) and $\mathcal{Y}_{TF} = 0$. We have also discussed several factors under which the system deviates from complexity-free scenario.

It has been noticed that the strong non-minimally coupled model (8) makes dynamical cylinder more complex due to incorporation of product terms of the matter variables and metric potentials. We have considered the homologous fluid to be of geodesic nature (i.e., $\Lambda = 1$), and thus this mode was suggested as the simplest pattern of evolution. The fulfilment of the requirement $\mu = \bar{s} = \Pi = 0$ provides complexity-free structure ($\mathcal{Y}_{TF} = 0$) in $\mathcal{G}_{R}$, while an additional condition $(8\pi / B + 4\pi \bar{\Pi}^{(C)} + 2\pi \bar{\rho} \bar{\Pi} - \bar{\chi}^{(C)}_{3} = 0)$ is needed to disappear the complexity factor in this gravity. It is found that this modified theory does not provide shear-free ($\sigma = 0$) structure even in non-dissipative case ($\bar{\sigma} = 0$), in contrast to $\mathcal{G}_{R}$, thus the dynamical cylinder becomes more complex in the presence of charge and modified corrections. This phenomenon has been investigated for minimal/non-minimal $f(R, T)$ models. The compatibility of the simplest evolutionary modes with each other has been observed for the case of minimal coupling, and otherwise, not [52]. The results we obtained for the considered $f(R, T, Q)$ model are compatible with those of $f(R, T)$ gravity. We have analyzed the stability of vanishing complexity factor and figured out some factors that enforced the system to deviate from its stability. It is worth mentioning here that all our results can be recovered in $\mathcal{G}_{R}$ [46] by vanishing the coupling parameter $\Phi$ and charge.

Data Availability This manuscript has no associated data.

Appendix A

The $f(R, T, Q)$ corrections in the field equations (12)–(15) are

$$T_{00}^{(C)} = \frac{\Phi}{8\pi (1 - \frac{\Phi R}{2C})} \left\{ \mu \left( \frac{3B}{2B^2} + \frac{3\bar{A}B}{2AB} + \frac{3\bar{C}}{C} - \frac{\bar{A}C}{C^2} + \frac{3AA''}{2B^2} + \frac{2A^2}{B^2} \right) \right\}$$

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\[ T_{01}^{(C)} = \frac{\Phi}{8\pi (1 - \frac{\Phi}{2C})} \left\{ \mu \left( -\frac{\dot{A}A'}{A^2} + \frac{\dot{B}'B}{2A^2} - \frac{2\ddot{A}'C}{AC} - \frac{3\dot{A}'C'}{2A} + \frac{2\ddot{B}'C}{AC} - \frac{2\dot{A}'C}{AC} - \frac{3\dot{A}'C}{2A} \right) + \frac{2\muA'}{AC} \right\}, \]

\[ T_{11}^{(C)} = \frac{\Phi}{8\pi (1 - \frac{\Phi}{2C})} \left\{ \mu \left( -\frac{\dot{A}A'}{A^2} + \frac{\dot{B}'B}{2A^2} - \frac{2\ddot{A}'C}{AC} - \frac{3\dot{A}'C'}{2A} + \frac{2\ddot{B}'C}{AC} - \frac{2\dot{A}'C}{AC} - \frac{3\dot{A}'C}{2A} \right) + \frac{2\muA'}{AC} \right\}, \]

\[ T_{22}^{(C)} = \frac{\Phi}{8\pi (1 - \frac{\Phi}{2C})} \left\{ \mu \left( -\frac{\dot{A}A'}{A^2} + \frac{\dot{B}'B}{2A^2} - \frac{2\ddot{A}'C}{AC} - \frac{3\dot{A}'C'}{2A} + \frac{2\ddot{B}'C}{AC} - \frac{2\dot{A}'C}{AC} - \frac{3\dot{A}'C}{2A} \right) + \frac{2\muA'}{AC} \right\}.
\[ - \frac{C A' \dot{C}}{A^2 B} - \frac{2 C \dot{B} C'}{A B^3} - \dot{\zeta} \left( \frac{C^2 A'}{A^2 B} + \frac{C C'}{A B} \right) - \dot{\zeta'} \left( \frac{C^2 \dot{B}}{A B^2} + \frac{C \dot{C}}{A B} \right) - \frac{\dot{\zeta} C^2}{A B} + \frac{C^2 Q}{2} \right], \quad (A4)
\]
\[
\varepsilon_{\theta 0}^{(C)} = \frac{\Phi s^2}{8 \pi A B^3 C^5 (1 - \frac{\Phi s^2}{2 \Sigma})} \left\{ 2A^3 B C'' + A^2 B C A'' - 2A^2 B \dot{C} - 2A^3 B' C' \right. \\
- \left. A^2 C A' B' - A B^2 C \dot{B} + B^2 C \dot{A} B + \frac{A^3 B^3 C R}{2} \right\}, \quad (A5)
\]
\[
\varepsilon_{\theta 1}^{(C)} = \frac{\Phi s^2}{8 \pi A B C^5 (1 - \frac{\Phi s^2}{2 \Sigma})} \left\{ (A B^2 C' - B A' \dot{C} - A \dot{B} C') \right. \\
+ \left. 2B^3 \dot{A} C + A^2 C A' B' - B^2 C \dot{A} B - \frac{A^3 B^3 C R}{2} \right\}, \quad (A6)
\]
\[
\varepsilon_{\varphi 2}^{(C)} = \frac{\Phi s^2 R}{16 \pi C^2 (1 - \frac{\Phi s^2}{2 \Sigma})}. \quad (A8)
\]

The terms \( Z_1 \) and \( Z_2 \) in Eqs. (16) and (17) are

\[
Z_1 = \frac{2 \Phi}{16 \pi + \Phi \mathcal{R}} \left[ \left( \frac{\xi B \mathcal{R}}{A} \right)^{10} - \left( \frac{\xi B \mathcal{R}}{A} \right)^{11} - (\mu \mathcal{R})^0 - (\mu \mathcal{R})^0 + \frac{2 \xi^2 \dot{C} g^{00}}{C^5} \right] \\
- \mathcal{G}^{01} \left[ \frac{2 \xi^2 C'}{C^5} \right] + \frac{1}{A^2} \left[ R_{00} \left( \frac{\mu}{A^2} - \frac{2 \mu A}{A^3} \right) + 2 \left( \frac{\xi}{A B} - \frac{\xi A}{A} - \frac{\xi B}{A} \right) \right] \\
\times \mathcal{R}_{01} + \mathcal{R}_{11} \left( \frac{\dot{P}_x}{B^2} - \frac{2 \dot{P}_y}{B^3} \right) + 2 \mathcal{R}_{22} \left( \frac{\dot{P}_x^2}{C^2} - \frac{2 \dot{P}_y C}{C^3} \right) \right], \quad (A9)
\]
\[
Z_2 = \frac{2 \Phi}{16 \pi + \Phi \mathcal{R}} \left[ (P, \mathcal{R})^{10} + (P, \mathcal{R})^{11} - \frac{2 \xi^2 \dot{C} g^{00}}{C^5} \right] \\
- \mathcal{G}^{11} \left[ \frac{2 \xi^2 C'}{C^5} \right] - \frac{1}{A^2} \left[ R_{00} \left( \frac{\mu^2}{A^2} - \frac{2 \mu A}{A^3} \right) + 2 \left( \frac{\xi}{A B} - \frac{\xi A}{A} - \frac{\xi B}{A} \right) \right] \\
\times \mathcal{R}_{01} + \mathcal{R}_{11} \left( \frac{\dot{P}_x}{B^2} - \frac{2 \dot{P}_y B}{B^3} \right) + 2 \mathcal{R}_{22} \left( \frac{\dot{P}_x^2}{C^2} - \frac{2 \dot{P}_y C}{C^3} \right) \right]. \quad (A10)
\]

Appendix B

The scalars (38)–(41) encompass modified corrections which are

\[
\chi_1^{(C)} = - \frac{8 \pi \Phi}{1 - \frac{\Phi s^2}{2 \Sigma}} \left[ \left\{ \mathcal{R}_{\psi \varphi} h^{\psi \psi} \right\} \left( P - \frac{\pi}{3} + \frac{s^2}{8 \pi C^4} \right) - \frac{Q}{2} + \frac{1}{2} \nabla_\varphi \nabla_\psi \Omega^{\psi \varphi} - \frac{1}{2} \nabla_\psi \Omega^{\psi \psi} \right] \\
+ \mathcal{R}_{\psi \varphi} h^{\psi \psi} \left( P - \frac{\pi}{3} + \frac{s^2}{8 \pi C^4} \right) - \frac{Q}{2} + \frac{1}{2} \nabla_\varphi \nabla_\psi \Omega^{\psi \varphi} - \frac{1}{2} \nabla_\psi \Omega^{\psi \psi} \right] - 2 g^{\psi \psi} \mathcal{R}_{\psi \varphi \psi \varphi} \frac{a^2 L_{\mathcal{M}}}{\partial g^{\psi \psi} \partial g^{\psi \varphi}} \right], \quad (B1)
\]
\[
\chi_2^{(C)} = \frac{8 \pi \Phi}{1 - \frac{\Phi s^2}{2 \Sigma}} \left[ \left\{ \frac{1}{2} \nabla_\varphi \nabla_\psi \Omega^{\psi \psi} \right\} - \left\{ (\mu - 3 P) - \mathcal{K}^{\psi} \mathcal{K}^{\psi} \varphi \Omega_{\psi \varphi} + \mathcal{K}^{\psi} \mathcal{K}^{\psi} \varphi \Omega_{\psi \psi} \right\} + \frac{3 Q}{2} + 4 \mathcal{R}_{\psi} \left( P + \frac{\pi}{3} + \frac{s^2}{8 \pi C^4} \right) \right] \\
\left. \left\{ \frac{1}{2} \nabla_\varphi \nabla_\psi \Omega^{\psi \psi} \right\} - \left\{ (\mu - 3 P) - \mathcal{K}^{\psi} \mathcal{K}^{\psi} \varphi \Omega_{\psi \varphi} + \mathcal{K}^{\psi} \mathcal{K}^{\psi} \varphi \Omega_{\psi \psi} \right\} + 6 \mathcal{R}_{\psi} \left( \mu - \frac{s^2}{8 \pi C^4} \right) \right] + \frac{3 Q}{2} + 4 \mathcal{R}_{\psi} \left( P + \frac{\pi}{3} + \frac{s^2}{8 \pi C^4} \right) \right] \\
+ \frac{\xi^2}{24 \pi C^4} \right] \left\{ \frac{1}{2} \nabla_\varphi \nabla_\psi \Omega^{\psi \psi} \right\} - \left\{ (\mu - 3 P) - \mathcal{K}^{\psi} \mathcal{K}^{\psi} \varphi \Omega_{\psi \varphi} + \mathcal{K}^{\psi} \mathcal{K}^{\psi} \varphi \Omega_{\psi \psi} \right\} + 6 \mathcal{R}_{\psi} \left( \mu - \frac{s^2}{8 \pi C^4} \right) \right] \\
+ \frac{\xi^2}{24 \pi C^4} \right] \left\{ \frac{1}{2} \nabla_\varphi \nabla_\psi \Omega^{\psi \psi} \right\} - \left\{ (\mu - 3 P) - \mathcal{K}^{\psi} \mathcal{K}^{\psi} \varphi \Omega_{\psi \varphi} + \mathcal{K}^{\psi} \mathcal{K}^{\psi} \varphi \Omega_{\psi \psi} \right\} + 6 \mathcal{R}_{\psi} \left( \mu - \frac{s^2}{8 \pi C^4} \right) \right] \\
+ \frac{3 Q}{2} + 4 \mathcal{R}_{\psi} \left( P + \frac{\pi}{3} + \frac{s^2}{8 \pi C^4} \right) \right] \left\{ \frac{1}{2} \nabla_\varphi \nabla_\psi \Omega^{\psi \psi} \right\} - \left\{ (\mu - 3 P) - \mathcal{K}^{\psi} \mathcal{K}^{\psi} \varphi \Omega_{\psi \varphi} + \mathcal{K}^{\psi} \mathcal{K}^{\psi} \varphi \Omega_{\psi \psi} \right\} + 6 \mathcal{R}_{\psi} \left( \mu - \frac{s^2}{8 \pi C^4} \right) \right] \\
+ \frac{3 Q}{2} + 4 \mathcal{R}_{\psi} \left( P + \frac{\pi}{3} + \frac{s^2}{8 \pi C^4} \right) \right] \left\{ \frac{1}{2} \nabla_\varphi \nabla_\psi \Omega^{\psi \psi} \right\} - \left\{ (\mu - 3 P) - \mathcal{K}^{\psi} \mathcal{K}^{\psi} \varphi \Omega_{\psi \varphi} + \mathcal{K}^{\psi} \mathcal{K}^{\psi} \varphi \Omega_{\psi \psi} \right\} + 6 \mathcal{R}_{\psi} \left( \mu - \frac{s^2}{8 \pi C^4} \right) \right]. \quad (B2)
\[ \chi^{(C)}_{\psi\phi} = -\frac{4\pi\Phi}{1 - \Phi\Phi} \left[ \begin{array}{c} \left( h_{\psi} h_{\phi} \omega_{\phi} \omega_{\psi} - \omega_{\psi} \omega_{\phi} / R_{\phi \psi} - \omega_{\psi} \omega_{\phi} - \omega_{\psi} \omega_{\phi} - \omega_{\phi} \omega_{\psi} \right) - R_{\psi \phi} \end{array} \right] \]

\[ \begin{array}{c} + \frac{1}{2} \omega_{\psi} \omega_{\phi} \omega_{\psi} \omega_{\phi} \omega_{\phi} \omega_{\psi} + 2g^{\psi\phi} R^{\alpha\beta} \frac{\partial^2 L_M}{\partial \phi^{\alpha\beta}} \frac{\partial^2 L_M}{\partial \phi^{\alpha\beta}} \end{array} \] \]

(B2)

\[ \begin{array}{c} \varepsilon = \frac{2\pi}{C} \left( \Phi R \frac{2}{1 - \Phi\Phi} \right) \left[ \chi^{-1}(\chi'2C^2 - \chi'2\Phi) - \chi^{-1}(\chi'2C^2 - \chi'2\Phi) + 2C^2s^2 
\end{array} \]

(B3)

The complexity-free and homologous conditions in the absence of dissipation flux are

\[ \begin{array}{c} \varepsilon = \frac{2\pi}{C} \left( \Phi R \frac{2}{1 - \Phi\Phi} \right) \left[ \chi^{-1}(\chi'2C^2 - \chi'2\Phi) - \chi^{-1}(\chi'2C^2 - \chi'2\Phi) + 2C^2s^2 
\end{array} \]
\[ + x_3 x_8 x_{16} - 8 \pi x_9 x_{16} - 512 \pi^3 + ( - 8 \pi x_{14} + x_{10} (x_{14} - 8 \pi) + x_9 x_{16} + 64 \pi^2 )
\times x_2 (x_3 (8 \pi - x_{10}) (8 \pi - x_{21}) + x_5 x_9 (x_{21} - 8 \pi) + x_4 (8 \pi - x_{10} x_{20} - x_9
\times x_2) (x_1 (x_1 (s^2 \Phi - 2 C^4) + 2 C^4 s^2 x_6) + (x_7 (s^2 \Phi - 2 C^4) - 2 C^4 s^2 x_{11})
\times x_3) - x_4 (x_7 (8 \pi - x_{14}) (2 C^4 - s^2 \Phi) + x_9 (x_{12} (s^2 \Phi - 2 C^4) - 2 C^4 s^2 x_{17})
\times 2 C^4 s^2 x_{11} (8 \pi - x_{14})) + (( - x_5 x_9 - x_3 (x_{10} - 8 \pi)) x_{15} + x_4 ( - 8 \pi x_{14}
\times x_10 (x_{14} - 64 \pi^2)) (x_1 x_9 (8 \pi - x_{21}) (2 C^4 - s^2 \Phi) + x_3 (8 \pi
\times x_2) + x_5 x_9 (8 \pi - x_{21}) - 2 C^4 s^2 x_{12} x_9 x_{21} + 2 C^4 x_4 x_9 x_{18} + 2 C^4 x_4 x_7 x_{20} - s^2 \Phi x_{4} x_9 x_{18}
\times x_2 - s^2 \Phi x_{4} x_7 x_{20}) \right) = 0. \] (B4)

\[(x_4 (8 \pi - x_{14}) + x_3 x_{15}) ( - x_3 (2 C^4 - s^2 \Phi) - 2 C^4 s^2 x_{11}) + x_9 (x_{15} (s^2 \Phi
\times x_2) + 2 C^4 s^2 x_6) + x_4 (x_{12} (2 C^4 - s^2 \Phi) + 2 C^4 s^2 x_{17})) + (( - x_5 x_9 - x_3
\times x_10 (x_4 - 8 \pi) x_{15} + x_4 (x_{10} (x_{14} - 8 \pi) + x_9 x_{16} + 64 \pi^2)) ( - x_4
\times x_10 (x_{14} - 8 \pi) + x_9 ( - x_4 x_{19} - 8 \pi x_{21} + x_2 (x_{21} - 8 \pi) + 64 \pi^2))
\times ((x_5 x_8 + x_2 ( - 8 \pi x_{19}) x_{15} - x_4 (x_9 x_{13} + x_8 (x_{14} - 8 \pi))) x_3 (8 \pi - x_{10})
\times x_10 (8 \pi - x_{21}) + x_5 x_9 (x_{21} - 8 \pi) + x_4 ((8 \pi - x_{10} x_{20} - x_9 x_{22}))^{-1} (x_4 (x_9
\times x_10 + x_9 x_{14} - 8 \pi)) - (x_3 x_8 + (x_2 - 8 \pi) x_9) x_{15}) (((8 \pi - x_{10}) (8 \pi - x_{21})
\times x_3 + x_5 x_9 (x_{21} - 8 \pi) + x_4 ((8 \pi - x_{10} x_{20} - x_9 x_{22})) (x_15 ( - x_9 (x_1 (2 C^4
\times x_2) + 2 C^4 s^2 x_{16}) + x_3 ((x_7 (2 C^4 - s^2 \Phi) + 2 C^4 s^2 x_{11}) - x_4 ((8 \pi - x_{14})
\times (2 C^4 - s^2 \Phi) + x_9 (x_{12} (s^2 \Phi - 2 C^4) - 2 C^4 s^2 x_{17}) + 2 C^4 s^2 x_{11} (8 \pi - x_{14}))
\times (( - x_5 x_9 + x_3 (x_{10} - 8 \pi)) x_{15} + x_4 ((8 \pi - x_{14} + x_10 (x_{14} - 8 \pi) + x_9 x_{16}
\times x_2) + 64 \pi^2)) (2 C^4 - s^2 \Phi) + x_3 (8 \pi - x_{21}) (x_7 (2 C^4 - s^2 \Phi) + 2
\times C^4 s^2 x_{11}) - 16 \pi C^4 s^2 x_6 x_9 + 2 C^4 s^2 x_{4} x_{11} x_{20} + 2 C^4 s^2 x_6 x_9 x_{21} - 2 C^4 s^2 x_4 x_9
\times x_2 + 2 C^4 x_4 x_9 x_{18} + 2 C^4 x_4 x_7 x_{20} - s^2 \Phi x_{4} x_9 x_{18} - s^2 \Phi x_{4} x_7 x_{20}) \right) = 0. \] (B5)

The homologous condition in the presence of heat dissipation is

\[
\tilde{\xi} = \frac{8 \pi}{8 \pi - x_{10}} \left[ \frac{x_7}{8 \pi} \left( 1 - \frac{s^2 \Phi}{2 C^4} \right) + \frac{s^2 x_{11}}{8 \pi} + (16 \pi C^4 \left( ( - x_5 x_9 - x_3 (x_{10} - 8 \pi) \right)
\times x_15 + x_4 ((- 8 \pi x_{14} + x_{10} (x_{14} - 8 \pi) + x_9 x_{16} + 64 \pi^2)) ( - x_4 x_8 x_{20} + x_3 x_8
\times (x_{21} - 8 \pi) + x_9 ( - x_4 x_{19} - 8 \pi x_{21} + x_2 (x_{21} - 8 \pi) + 64 \pi^2)) + ((x_3 x_8
\times (x_2 - 8 \pi) x_9) x_{15} - x_4 (x_9 x_{13} + x_8 (x_{14} - 8 \pi))) x_3 (8 \pi - x_{10}) (8 \pi - x_{21})
\times x_5 x_9 (x_{21} - 8 \pi) + x_4 ((8 \pi - x_{10} x_{20} - x_9 x_{22}))^{-1} (x_8 (x_3 (8 \pi - x_{10})
\times (8 \pi - x_{21}) + x_5 x_9 (x_{21} - 8 \pi) + x_4 (8 \pi - x_{10} x_{20} - x_9 x_{22})) (x_15 (9 x_2^2
\times C^4 x_6 - x_{11} (2 C^4 - s^2 \Phi)) - x_3 (x_7 (2 C^4 - s^2 \Phi) + 2 C^4 s^2 x_{11}) - x_4 (x_7 (8 \pi
\times x_{14}) (2 C^4 - s^2 \Phi) + x_9 (x_{12} (s^2 \Phi - 2 C^4) - 2 C^4 s^2 x_{17}) + 2 C^4 (8 \pi - x_{14})
\times s^2 x_{11}) + (( - x_5 x_9 - x_3 (x_{10} - 8 \pi)) x_{15} + x_4 ((- 8 \pi x_{14} + x_{10} (x_{14} - 8 \pi)
\times x_9 x_{16} + 64 \pi^2)) (x_1 x_9 (8 \pi - x_{21}) (2 C^4 - s^2 \Phi) + x_3 ((2 C^4 - s^2 \Phi) x_{17} + 2 C^4
\times s^2 x_{11} (8 \pi - x_{21}) - 16 \pi C^4 s^2 x_6 x_9 + 2 C^4 s^2 x_{4} x_{11} x_{20} + 2 C^4 s^2 x_6 x_9 x_{21} - 2 s^2
\times C^4 x_{4} x_{9} x_{23} + 2 C^4 x_{4} x_{9} x_{18} + 2 C^4 x_{4} x_7 x_{20} - s^2 \Phi x_{4} x_9 x_{18} - s^2 \Phi x_{4} x_7 x_{20}) \right) \right]
\times \frac{s C'}{4 \pi B C}.
\] (B6)

The values of \( x_i \), \( i = 1, 2, 3, ..., 23 \) used in Eqs. (B4)–(B6) are

\[ x_1 = - \frac{1}{B^2} \left( \frac{2 C''}{C} - \frac{2 B' C'}{BC} - \frac{B^2}{C^2} + \frac{C'^2}{C} \right) + \frac{\dot{C}}{C} \left( \frac{2 \dot{B}}{B} + \frac{\dot{C}}{C} \right). \] (B7)
\[
\chi_2 = \Phi \left( \frac{2 \dot{B} \dot{C}}{BC} - \frac{3 \ddot{B}}{2 B^2} + \frac{\dot{C}^2}{C^2} - \frac{3 \ddot{C}}{C} + \frac{\mathcal{R}}{2} \right), \\
\chi_3 = \Phi \left( \frac{2 \dot{B} \dot{C}}{BC} - \frac{2 \dot{B}' \dot{C}'}{B'^2 C} - \frac{\dot{B} \ddot{B}'}{B} + \frac{\dot{B}' \ddot{C}'}{B'^2 C} + \frac{\mathcal{C}''}{B'^2 C} \right), \\
\chi_4 = \Phi \left( \frac{B' \dot{C}' + \dot{B} \dot{C}}{B^3 C} - \frac{C'^2}{B^2 C^2} - \frac{C''}{B^3 C} + \frac{\dot{C}}{C} \right), \\
\chi_5 = \Phi \left( \frac{5 \dot{B} \dot{C}'}{B'^2 C} + \frac{2 \dot{C}' \dot{C}}{B'^2 C} - \frac{\dot{C}'}{B C} \right), \\
\chi_6 = \frac{1}{C^4} + \frac{\Phi}{B'^3 C^5} \left( 2 \dot{B} B'^2 \dot{C} + 2 B' \dot{C}' + \ddot{B} B'^2 C - 2 B C'' - \frac{\mathcal{R} B^3 C}{2} \right), \\
\chi_7 = -\frac{1}{B} \left( \frac{2 \dot{B} \dot{C}}{BC} - \frac{2 \dot{C}'}{C} \right), \\
\chi_8 = \chi_9 = \Phi \left( \frac{2 \dot{B} \dot{C}'}{BC} - \frac{2 \ddot{B} \dot{C}'}{B'^2 C} \right), \\
\chi_{10} = \Phi \left( \frac{2 \dot{B} \dot{C}'}{B^3 C} - \frac{2 B' \dot{C}'}{B^3 C} - \frac{\dot{B} \ddot{B}'}{B} - \frac{2 \ddot{C}'}{C} + \frac{\mathcal{R}}{2} \right), \\
\chi_{11} = \frac{\Phi}{B^3} \left( \dot{B} C' - B \dot{C}' \right), \\
\chi_{12} = \frac{C'^2}{B^2 C^2} - \frac{2 \dot{C}'}{C} - \frac{\dot{C}''}{C^2} - \frac{1}{C^4}, \\
\chi_{13} = \Phi \left( \frac{B}{2 B} - \frac{\dot{C}}{C} - \frac{\dot{C}'}{C^2} \right), \\
\chi_{14} = \Phi \left( \frac{3 \ddot{C}''}{B^2 C} - \frac{2 \dot{B} \dot{C}'}{B^3 C} - \frac{2 B' \dot{C}'}{B^3 C} - \frac{3 \dot{B} \dot{C}}{B^2 C} - \frac{3 \ddot{B}}{2 B} - \frac{C'^2}{B^2 C^2} + \frac{\mathcal{R}}{2} \right), \\
\chi_{15} = \Phi \left( \frac{C'^2}{B^2 C^2} - \frac{B' \dot{C}'}{B^3 C} - \frac{\dot{B} \ddot{B}'}{B'^2 C} + \frac{\dot{C}''}{C^2} - \frac{\ddot{C}}{C} \right), \\
\chi_{16} = \Phi \left( \frac{2 \dot{B} \dot{C}'}{B'C} - \frac{4 \dot{B} \dot{C}}{B'^2 C} - \frac{2 \dot{C}' \dot{C}}{B'^2 C} \right), \\
\chi_{17} = \frac{1}{C^4} - \frac{\Phi}{B'^3 C^5} \left( \ddot{B} B'^2 C + 2 B^3 \dot{C}' - \frac{\mathcal{R} B^3 C}{2} \right), \\
\chi_{18} = \frac{1}{B^3} \left( \frac{\ddot{C}''}{C} - \frac{2 B' \dot{C}'}{B'C} - \frac{\dot{B} \ddot{B}'}{B'^2 C} - \frac{\dot{B} \ddot{C}}{B} - \frac{\ddot{C}}{C} \right), \\
\chi_{19} = -\Phi \left( \frac{\dot{B} \dot{C}}{BC} + \frac{\ddot{B}}{2 B} \right), \\
\chi_{20} = \Phi \left( \frac{B' \dot{C}'}{B^3 C} - \frac{2 B' \dot{C}'}{B^3 C} - \frac{\ddot{B} \dot{C}}{B'C} - \frac{\ddot{B}}{2 B} \right), \\
\chi_{21} = \Phi \left( \frac{2 \ddot{C}''}{B^2 C^2} - \frac{2 \dot{B} \dot{C}'}{B^3 C} - \frac{2 \dot{B} \dot{C}}{B^2 C} - \frac{2 \ddot{C}'}{C} - \frac{2}{C^2} + \frac{\mathcal{R}}{2} \right), \\
\chi_{22} = \Phi \left( \frac{\dot{B} \dot{C}'}{B^3 C} - \frac{2 \dot{B} \dot{C}'}{B^3 C} - \frac{\ddot{B} \dot{C}'}{B'^2 C} \right), \\
\chi_{23} = \frac{1}{C^4} \left( 1 + \frac{\Phi \mathcal{R}}{2} \right).
\]

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