Model-based motion simulation of delta parallel robot

Mahesh A Makwana1,* and Haresh P Patolia2

1College of Food Processing Technology & Bioenergy, Anand Agricultural University, Anand, Gujarat, India.
2Mechanical Engineering Department, Birla Vishvakarma Mahavidyalaya, V.V. Nagar, Gujarat, India.

* maheshmakwana@aau.in

Abstract. For a parallel configuration of a robot manipulator, the solution of Forward Kinematics (FK) is tough compared to Inverse Kinematics (IK). This paper presents the model-based motion planning of a delta parallel robot in Simulink’s SimScape environment. A model was developed and simulated for motion study. The developed model has been simulated to solve the FK of the parallel manipulator and to check its efficacy. First, a helix curve has been planned within the reachable workspace. Then IK was solved to extract angular positions of the biceps. Obtained angular positions then fed to SimScape model to run a simulation. The path taken by the end effector of the system calculated by simulation is in good approximation to the planned helix path. Further, visual simulation and motion analysis of delta parallel robot can be performed by Model-based simulation and solves mechanical design as well control system design problems. Experimental study also shows that the circular path designed for experiment is well followed in real time simulation.

1. Introduction

Parallel manipulator is one of the highly demanded robots in the manufacturing, dairy, food and agriculture industries. It possesses advantages like high acceleration, high precision, low inertia and high accuracy over serial manipulators. The only disadvantage of parallel configuration has is small work volume. But in micro-manufacturing, this limitation can be ignored [1]. For a better understanding of work volume, robot performance and manipulator control, the kinematic solution of robot manipulator including forward as well as inverse must be solved faster and accurately. The solution of FK of a parallel manipulator is highly complex and time-consuming when compared with the serial manipulator. Due to their utilization capacity in different industries, parallel manipulators were highly investigated in the past. Delta Parallel robot (DPR) is one of the types of parallel robot that is widely used in industries. Delta robot was invented in the early 1980s by Prof. Reymond Clevel of Laboratory of Robotics System, Swiss Federal Institute of Technology, Switzerland. Then after there are many modified delta robots have been developed for different applications.

Robot movement are often foreseen by incorporating FK and path/trajectory planning techniques [2]. Analytical solution of FK for DPR tends to the number of solutions with tedious computations. Its solution usually comprises of group of nonlinear equations which are extremely coupled and normally haven't any closed-form and distinctive answer. It is needed to solve several coupled nonlinear algebraic equations tends to several valid solutions [3]. Moreover, robot control can be affected by
singularities when executing different solutions to the FK problem. Hence, simulation can study the effect of singularities in controlling the robots [4]. Again, synthesizing of robots could be accomplished by any of the following models: the geometrical model, the kinematic model and the dynamic model. For that matter simulation programs are better choice consequent to advancement in compute technology [5]. In designing the future robots by keeping its safety, fast response, lower costs and better control design, simulation can provide a strong virtual platform to have safe operation and improved performance [6]. One such platform is MATLAB where two different approaches can be adopted for simulation as 1) Simulink which uses a block diagram approach and signal flows in unidirectional 2) SimScape uses physical modeling method in which the signal flow is bi-directional between blocks. SimScape models have less blocks and it is easy to comprehend and maintain. SimMechanics modelling differs from Denavit-Hartenberg representation as it represents mechanical parts by their mass centres, and place the origins of the coordinate frames of each body at its mass centre [7]. Hence using SimScape modeling motion study including a solution of IK and FK can be easily obtained. In order to have precise control of Robot, the kinematics study and mathematical modeling of the robot are to be done [8]. Using simulation manipulators work more reliably and correctly and better controller design may developed to fulfill agricultural and industrial related production intelligently [9]. After simulation in virtual environment motion error of actual can be analysed and an error compensation scheme may developed [10]. In this paper Delta parallel robot has been modeled using a physical modeling approach in the SimScape platform and simulated to analyze the motion of the system. Complex FK also has been solved with good efficacy. In the end conclusion and future scope has been discussed.

2. Model description of delta type parallel manipulator

The DPR, as shown in Figure 1, is made of three parallelograms attached to a moving platform (MP). Each chain is consisting of a bicep and forearm which in turn attached to the moving platform. The bicep is actuated by an actuator fixed to the manipulator base platform [11]. All actuators are placed 120º apart from each other. The moving platform can be manipulated by angular motion of the biceps and connected forearms. In the configuration, biceps are active limbs whereas forearms are passive limbs. Parallelogram design of manipulator ensures the parallel movement of moving platform. The robot is of Revolute-Universal-Universal (RUU) joint configuration. We consider $O$ as an absolute coordinate frame at the center of the base platform for further analysis. Vector $P$ which comprises the end-effector position is placed at the center of the MP with respect to $O$.

![Figure 1. Description of DPR.](image-url)
3. Kinematics of delta robot

Kinematics of the system is the study of the motion of the system without considering the cause behind it. In robotic studies Kinematics comprises of FK and IK. FK is a mathematical technique to calculate Cartesian space parameters of robotic manipulator by feeding joint space parameters, hence calculating \( x,y,z \) coordinates vector \( P=(p_x,p_y,p_z) \) of end-effector using given angular position vector expressed by \( \theta_0=(\theta_{i1},\theta_{i2},\theta_{i3}) \). Whereas IK is a mathematical technique to calculate joint space parameters of the robotic manipulator by feeding coordinate space parameters, hence calculating angular rotations of links using given coordinates. For given DPR configuration, IK calculates angular position vector for biceps expressed by \( \theta_i=(\theta_{i1},\theta_{i2},\theta_{i3}) \) using known \( x,y,z \) coordinates vector \( P=(p_x,p_y,p_z) \). Where \( \theta_{i1} \) is the angle between vector \( R \) and bicep \( L \) and \( \theta_{i2} \) is angle between bicep and forearm as shown in figure 2. A clockwise rotation from x-axis has been considered positive angular movement. \( \theta_{i3} \) is rotation of forearm in YZ plane.

![Figure 2. Geometrical parameters of DPR.](image)

![Figure 3. Absolute coordinate system.](image)
As shown in figure 3 \( \varphi_1 \) is an angle between \( X \)-axis to placement of actuator \( A1 \), \( \varphi_2 \) is angle between \( X \)-axis to \( A2 \) and \( \varphi_3 \) is angle between \( X \)-axis to \( A3 \). The end effector position \( P = (p_x, p_y, p_z) \) can be given from absolute coordinate system \( O \). The radius of base platform and moving platform, \( R \) and \( r \) respectively measured from the absolute coordinate system \( O \). The angular measurement taken about the \( Z \)-axis, both for a base and a moving platform [12].

### Table 1. Notations for the robot geometry.

| Description                   | Dimensions (in mm) |
|-------------------------------|--------------------|
| \( E_b \) base equilateral triangle side | 510                |
| \( E_p \) platform equilateral triangle side | 128                |
| \( L \) =Bicep length | 150                |
| \( l \) =Forearm parallelogram length | 380                |
| \( R \) = Base platform radius | 147.2              |
| \( R' \) = planar distance from \{0\} to a BPF vertex | 294.45             |
| \( r \) = Moving platform radius | 73.6               |
| \( r' \) = planar distance from \{P\} to a MPF vertex | 36.95              |

To model and solve the vector loop equation for FK of DPR using analytical method geometrical vertices, scalar points and vectors considered from base reference frame as shown in figure 4, and can be estimated as follows [13].

The revolute joint points of the base-platform equilateral triangle from base frame \( O \) where actuators are mounted can be expressed in vector notation as:

\[
^o A_1 = \begin{bmatrix} 0 \\ -R \\ 0 \end{bmatrix}, \quad ^o A_2 = \begin{bmatrix} \frac{R}{2} \\ \frac{R}{2} \\ 0 \end{bmatrix}, \quad ^o A_3 = \begin{bmatrix} -\frac{R}{2} \\ \frac{R}{2} \\ 0 \end{bmatrix}
\]

Similarly, the vertices of the moving-platform equilateral triangle can be expressed in vector notation as in equation (1):
To have the vertices of the base-platform equilateral triangle, one can express vector notation as in equation (2):

\[
\begin{align*}
O_{a1} &= \begin{bmatrix} -\frac{E_a}{2} \\ -R \\ 0 \end{bmatrix}; \\
O_{a2} &= \begin{bmatrix} 0 \\ R' \\ 0 \end{bmatrix}; \\
O_{a3} &= \begin{bmatrix} -\frac{E_a}{2} \\ -R \\ 0 \end{bmatrix}
\end{align*}
\]

where: \( R = \tan 90^\circ \times \frac{E_a}{2} \); \( R' = \frac{E_a}{2} \times \cos 30^\circ \); \( r = \frac{E_p}{2} \times \cos 30^\circ \).

Parallelogram design ensures the translational motion of end-effector and no orientations are allowed by the delta robot, three constraint states for the forearms for constant length can be written as in equation (4):

\[
\overline{BC}_i = \overline{OP}_i + \overline{PC}_i - \overline{OA}_i + \overline{AB}_i \\
\text{where: } i=1,2,3 \text{ and } \left[ \begin{array}{c} x \\ y \\ z \end{array} \right] = \overline{OP}_i
\]

Vector \( \overline{AB}_i \) which depends on the joint variables \( \theta_i = (\theta_{i1}, \theta_{i2}, \theta_{i3}) \) can be given as in equation (5):

\[
\overline{AB}_i = \begin{bmatrix} 0 \\ -L \cos \theta_i \\ -L \sin \theta_i \end{bmatrix}; \quad \overline{AB}_2 = \begin{bmatrix} \frac{E_p}{2} L \cos \theta_2 \\ \frac{E_p}{2} L \cos \theta_2 \\ -L \sin \theta_2 \end{bmatrix}; \quad \overline{AB}_3 = \begin{bmatrix} -\frac{E_p}{2} L \cos \theta_3 \\ \frac{E_p}{2} L \cos \theta_3 \\ -L \sin \theta_3 \end{bmatrix}
\]

Substituting all the above value in equation (4) for vector \( \overline{BC}_i \) for all three parallelogram yields in equation (6):

\[
\begin{align*}
\overline{BC}_1 &= \begin{bmatrix} x \\ y + L \cos \theta_1 + R - r \\ z + L \sin \theta_1 \end{bmatrix}; \\
\overline{BC}_2 &= \begin{bmatrix} x - \frac{E_p}{2} L \cos \theta_2 + \frac{E_p}{2} - \frac{E_p}{2} R \\ y - \frac{1}{2} L \cos \theta_2 + \frac{1}{2} R \\ z + L \sin \theta_2 \end{bmatrix}; \\
\overline{BC}_3 &= \begin{bmatrix} x + \frac{E_p}{2} L \cos \theta_3 - \frac{E_p}{2} + \frac{E_p}{2} R \\ y - \frac{1}{2} L \cos \theta_3 + \frac{1}{2} R \\ z + L \sin \theta_3 \end{bmatrix}
\end{align*}
\]

where: \( R - r = u; \quad r' - \frac{1}{2} R = w \).

So, three constraint equations yield the kinematic equation of delta parallel robot and given as equation (7):
\[
2L(y + u)\cos \theta_i + 2zL\sin \theta_i + x^2 + y^2 + z^2 + u^2 + L^2 + 2yu - l_i^2 = 0
\]
\[
-L\left(\sqrt{3}(x + v) + y + w\right)\cos \theta_i + 2zL\sin \theta_i + x^2 + y^2 + z^2 + v^2 + w^2 + L^2 + 2xv + 2yw - l_i^2 = 0
\]
\[
L\left(\sqrt{3}(x - v) - y - w\right)\cos \theta_i + 2zL\sin \theta_i + x^2 + y^2 + z^2 + v^2 + w^2 + L^2 - 2xv + 2yw - l_i^2 = 0
\]

(7)

Now, the absolute vector point Bi from absolute reference frame O can be given as: \(\overrightarrow{OB_i} = O\hat{A}_i + \overrightarrow{AB_i}\) for \(i = 1, 2, 3\) and expressed as equation (8):

\[
\overrightarrow{OB_i} = \begin{bmatrix}
0 \\
-R - L\cos \theta_i + r \\
-L\sin \theta_i
\end{bmatrix} \overrightarrow{OB_2} = \begin{bmatrix}
\frac{\sqrt{3}}{2}(R + L\cos \theta_2) \\
\frac{1}{2}(R + L\cos \theta_2) - R \sin \theta_2 \\
-\frac{1}{2}(R + L\cos \theta_2)
\end{bmatrix} \overrightarrow{OB_3} = \begin{bmatrix}
-\frac{\sqrt{3}}{2}(R + L\cos \theta_3) \\
\frac{1}{2}(R + L\cos \theta_3) - R \sin \theta_3 \\
-\frac{1}{2}(R + L\cos \theta_3)
\end{bmatrix}
\]

(8)

Now, since the moving platform orientation is always horizontal because of parallelogram structure, three virtual sphere centers can be defined as equation (9),

\[
\overrightarrow{O_{b,i}} = \overrightarrow{OB_i} - P_{p_i} \quad \text{for } i = 1, 2, 3
\]

\[
\begin{align*}
\overrightarrow{O_{b,1}} &= \begin{bmatrix} 0 \\ -R - L\cos \theta_1 + r \\ -L\sin \theta_1 \end{bmatrix} \\
\overrightarrow{O_{b,2}} &= \begin{bmatrix} \frac{\sqrt{3}}{2}(R + L\cos \theta_2) - \frac{R_p}{2} \\ \frac{1}{2}(R + L\cos \theta_2) - R \sin \theta_2 - R_p \\ -\frac{1}{2}(R + L\cos \theta_2) + \frac{R_p}{2} \end{bmatrix} \\
\overrightarrow{O_{b,3}} &= \begin{bmatrix} -\frac{\sqrt{3}}{2}(R + L\cos \theta_3) \\ \frac{1}{2}(R + L\cos \theta_3) - R \sin \theta_3 - R_p \\ -\frac{1}{2}(R + L\cos \theta_3) + \frac{R_p}{2} \end{bmatrix}
\end{align*}
\]

(9)

So, the FK unknown point \(\overrightarrow{OP} = O_{p} = [x \ y \ z]^T\) is the three spheres intersection which can be written in terms of vector center point and scalar radius as \((O_{b,1}, l_1)\), \((O_{b,2}, l_2)\) and \((O_{b,3}, l_3)\).

Now, by using above spheres definitions FK of delta parallel robot can be solve with three sphere interaction approach in which general equations (10, 11, 12) for three spheres are,

\[
\begin{align*}
(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 &= l_1^2 \\
(x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2 &= l_2^2 \\
(x - x_3)^2 + (y - y_3)^2 + (z - z_3)^2 &= l_3^2
\end{align*}
\]

(10)

(11)

(12)

To find unknown three sphere intersection point is \(\overrightarrow{OP} = O_p = [x \ y \ z]^T\) by expanding and subtracting (8) from (10) and (11) sequentially above equations yields in equation (13, 14);

\[
ax + by + cz = \lambda
\]

(13)

\[
\alpha_x x + \beta_y y + \gamma_z z = \lambda
\]

(14)

where: \(\alpha = 2(x_3 - x_1), \beta = 2(y_3 - y_1), \gamma = 2(z_3 - z_1), \alpha_x = 2(x_1 - x_2), \beta_y = 2(y_1 - y_2), \gamma_z = 2(z_2 - z_1), \lambda = l_1^2 - l_2^2 - x_1^2 - y_1^2 - z_1^2 + x_2^2 + y_2^2 + z_2^2\) and \(\lambda_x = l_2^2 - l_3^2 - x_1^2 - y_1^2 - z_1^2 + x_2^2 + y_2^2 + z_2^2\)

Further, by solving equations (13) and (14) to get an equations (15, 16) in terms of

\[
z = \frac{\lambda}{\gamma - \gamma_z}
\]

(15)

\[
z = \frac{\lambda_x}{\gamma_z - \gamma}
\]

(16)
By comparing both the above equations (15) and (16), equation (17) can be expressed:

\[ x = f(y) = \eta y + \kappa \]  

(17)

where, \( \eta = \frac{a_2 - a_1}{a} \) and \( \kappa = \frac{b_2 - b_1}{b} \).

Now, to obtain equation (18), \( z = f(y) \) by substituting equation (17) into equation (16):

\[ z = f(y) = \eta_i y + \kappa_i \]  

(18)

where:

\[ \eta_i = -\alpha_i \eta - \beta_i \]  
\[ \kappa_i = \frac{\lambda_i - \alpha_i \kappa}{\gamma_i} \]

Finally, substituting equations (17) and (18) into the (10), (11) and (12) gives:

\[ ay^2 + by + c = 0 \]  

(19)

Hence, the two solutions for \( y \) can be obtained by solving the quadratic equation (19):

\[ y_{\pm} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]  

(20)

Both \( y_{\pm} \) values are substitute into \( x \) to obtain intersection of three spheres as equation (21, 22):

\[ x_{\pm} = \eta y_{\pm} + \kappa \]  

(21)

\[ z = \eta_i y_{\pm} + \kappa_i \]  

(22)

So, two solutions with positive and negative intersections on either side of the base platform are \([x_+, y, z_+]^T\) and \([x_-, y, z_-]^T\). This analytical method thus has incorporated imaginary, singular and multiple solutions.

Model-based simulation for motion study can solve and analyses wide range of mechanics problems as FK, IK, velocity, acceleration and jerk. Also, it can calculate the dynamics of the system with ease. So, this paper presents solution of FK of DPR using simulation to end up with a single workable solution.

4. Model-based simulation of delta parallel robot

4.1. Physical Modeling of Three-dimensional Establishment of SimMechanics Model

For complex mechanism SimScape’s Sim Mechanics Toolbox provides faster and easier physical modeling using basic building blocks of mechanical, electrical and electronics components. Further, modeling, simulation and dynamics performance evaluation of different complex mechanisms can be studied. Researchers use SimMechanics to solve different engineering problems including rigid body and flexible mechanisms. For more complex mechanism this approach is very effective to construct a graphical simulation model using appropriate blocks.

SimMechanics can also be useful for robotic and mechatronics systems. In case of parallel configuration, it brings more suitability for modelling and research expansion for Delta parallel robot [14]. In this study, 3-D model of DPR is modeled using SimMechanics tool as shown in figure 5 to visualize the motion of a virtual model in an integrated “Mechanics Explorers tool”. The developed model is primarily consisting of rigid body blocks connected by joint blocks with added sensor [15].
5. Experimental analysis

5.1. Hardware used in delta parallel robot

The in-house developed system runs by ARDUINO UNO R3 micro controller as shown in figure 6 which sends the control signal to servo motors. All the three servo motors as shown in figure 7 have identical specifications having voltage rating 7.4V with 25 kg.-cm of torque capacity.
An experimental analysis has been carried out by making DPR to follow the circular path within the reachable workspace. Figure 8 shows an experimental setup which uses Arduino UNO as microcontroller for real time hardware in loop simulation.

![Experimental setup](image)

**Figure 8.** Experimental setup.

For real time control of DPR, all the three servo actuators have been actuated by sending control signals using Simulink model as shown in figure 9. Angular positions of servos in real time have been measured by Analog input block which gives output voltage. This outputs voltage finally converted to angles of servo motors.

![Simulink model](image)

**Figure 9.** Simulink model to control servo actuators of DPR.
6. Simulation results

After simulating the system for 20 seconds both in normal mode and external mode with controller, real time obtained data has been compared for 1) planned vs simulation and 2) experimental vs simulation. Comparison between end effector paths (xyz position data) got by simulation and planned path with each discrete time step has been plotted in figure 10.

Further, to check whether two groups of positional data differ from each other, both planned trajectory (group 1) and simulation positional data (group 2) is use for conducting t-test to compare the means of two groups. The results of t-test is shown in table 2. The value of $H = 0$ for all three coordinates shows that t-test does not reject the null hypothesis at the default 5% significance level. As p-values 0.9921, 0.9804 and 0.7900 respectively for $x$, $y$ and $z$ coordinates and are greater than 0.05, both the groups containing $xyz$ position data have no significant difference.

|   | $H$ | $P$  | $C_i$       | $tstat$ | $Df$  | $sd$  |
|---|-----|------|-------------|---------|-------|-------|
| x | 0   | 0.9921 | -6.6718 to 6.6048 | -0.0100 | 158   | 21.2569 |
| y | 0   | 0.9804 | -6.7732 to 6.6065 | -0.0246 |       | 21.4220 |
| z | 0   | 0.7900 | -8.3428 to 10.9479 | 0.2667  | 30.8858 |       |

Figure 10 shows planned vs Simulated Path comparison.

Figure 11 shows comparison between helix paths taken by end effector of DPR using simulation and actual planned. Blue helix is planned path within reachable workspace whereas orange helix is obtained by simulation of the developed model. Both the path is in close approximation with each other. Hence, this approach to solving FK is quick and efficient and converges in a single solution. The simulation results was also quantitatively analysed using t-test which shows that single FK solution has no significant difference compare to the originally planned path as $p$-values are greater than 0.05 for all the three coordinates $x$, $y$ and $z$ having 0.9921, 0.9804 and 0.7900 respectively.
6.1. Experimental vs simulation results
The DPR is run for 20 seconds in real time to follow the planned circular path. From the experiments it can be seen that the obtained angular positions data of servo motors by simulation as well by real time run is in close approximation as shown in figure 12.

Figure 11. Three-dimensional visualization of Planned vs Simulated Path.

Figure 12. Experimental vs simulation angular displacement of servo motors for circular path.

On x-axis time data has plotted and on y-axis angular positions for simulation as well as experimental runs are plotted. Overlapping of both the data ensures the good approximation during real time control of DPR.

7. Conclusion and Future Scope
Three dimensional physical model of three degree of freedom delta parallel robot has modeled in Simulink Simscape environment and simulated for helix path. The simulation result for FK is in good approximation with analytical solution of FK. Moreover, from experimental study, obtained results it has been concluded that the circular path taken by DPR in real time is in good approximation with
simulated path. Furthermore, numerous data which are tough to get using experiment can be obtain by such simulation, and hence the work efficiency improves. It helps to provide a natural reference for the selection of improved mechanism and design of a controller for the parallel robot.

Simulation has a very large scope of application in robotics and allied branches. In the future, different areas of the DPR like dynamics mapping, performance of manipulator, trajectory planning and hardware implementation will be explored. In future, the DPR will be experimentally analyzed for tracking three-dimensional complex curve path. Also, to solve FK, soft computing method will be employed and analyzed. For added efficiency in trajectory tracking different control designs will be carried out and performance of controller will be evaluated.

Acknowledgement

The authors are grateful to Anand Agricultural University for providing facilities to conduct the experiment. The authors would like to thank the editor and anonymous reviewers for their comments that help improve the quality of this work.

References

[1] Uzunovic T, Golubovic E, Baran E A and Sabanovic 2013 A Configuration Space Control of a Parallel Delta Robot with a Neural Network Based Inverse Kinematics 497–501
[2] Ang C K, Tang S H, Mashohor S and Arriffin M K A M 2014 Solving continuous trajectory and forward kinematics simultaneously based on ANN Int. J. Comput. Commun. Control 9 253–60
[3] Sadjadian H, Member H D T and Fatehi A 2005 Neural Networks Approaches for Computing the Forward Kinematics of a Redundant Parallel Manipulator Comput. Intell. 2 40–7
[4] Peidró A, Reinsono O, Gil A, Marin J M and Payá L 2015 A Virtual Laboratory to Simulate the Control of Parallel Robots IFAC-PapersOnLine 48 19–24
[5] Gouasmi M, Ouali M, Fernini B and Meghatria M 2012 Kinematic modelling and simulation of a 2-R robot using solidworks and verification by matlab/simulink Int. J. Adv. Robot. Syst. 9 245
[6] Choi H, Crump C, Duriez C, Elmquist A, Hager G, Han D, Hearl F, Hodgins J, Jain A, Leve F, Li C, Meier F, Negrut D, Righetti L, Rodriguez A, Tan J and Trinkle J 2021 On the use of simulation in robotics: Opportunities, challenges, and suggestions for moving forward Proc. Natl. Acad. Sci. 118
[7] Tlale N S and Zhang P 2005 Teaching the Design of Parallel Manipulators and Their Controllers Implementing MATLAB, Simulink, SimMechanics and CAD Int. J. Engng Ed. 21(5) pp. 838-845
[8] Yang X, wang S, Dong Y and Yang H 2017 D2 Delta Robot Structural Design and Kinematics Analysis 274(1) 12009
[9] Van Nam N, Ba Tan H and Quan N D 2021 An Image Processing Based Controller for a Three Degrees of Freedom Robotic Arm Applying New Technology in Green Buildings (ATiGB) pp 146–54
[10] Shang D, Li Y, Liu Y and Cui S 2019 Research on the Motion Error Analysis and Compensation Strategy of the Delta Robot Mathematics 7(5) 411
[11] Liu X-J 2006 Optimal Kinematic Design of a Three Translational DoFs Parallel Manipulator Robotica 24 239–250
[12] Brinker J, Corves B and Wahle M 2015 A Comparative Study of Inverse Dynamics based on Clavel’s Delta robot 14th IFToMM World Congr.
[13] Williams R L . 2016 The Delta Parallel Robot: Kinematics Solutions
[14] LIU C, CAO G and QU Y 2019 Motion simulation of Delta parallel robot based on Solidworks and Simulink IEEE 3rd Advanced Information Management, Communicates, Electronic and Automation Control Conference (IMCEC) pp 1683–1686
[15] Li M, Bi D and Xiao Z 2015 Mechanism Simulation and Experiment of 3-DOF Parallel Robot Based on MATLAB 483-488.