Doppler effect and frequency-shift in optics

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Abstract

A critical review of frequency-shift phenomena a la Doppler effect is presented. The importance of Fermi’s theory of 1932 is pointed out, and it is argued that there exist gaps in our understanding of this phenomena at a fundamental level. Alternative mechanism in terms of photon number oscillations is suggested for polarization changing experiments. The physical reality of single photon is revisited with a new interpretation of zero point energy. Total energy of the photon comprises of translational (wave-like) and rotational (extended particle-like) energy.

1 Introduction

Doppler effect is a well established phenomenon, and has found many applications. Physical mechanism of Doppler effect for sound waves is very transparent as explained in elementary textbooks, however for light waves after the advent of the theory of relativity and rejection of aether hypothesis there arise subtle problems in its interpretation. The most cogent argument for Doppler effect in the case of electromagnetic (EM) waves is based on the invariance of the phase of the wave (treating frequency and wave vector as a four-vector) under Lorentz transformation. Considering the fact that all the known mechanisms to change the frequency of a monochromatic EM wave
depend on a dynamic interaction process or an active circuit (or optical) element, purely kinematic origin of frequency shift is intriguing. If the claim is made for a single photon, the physical explanation becomes obscure; in fact the word photon is used rather casually in the literature, see a critique in [1]. In the present paper, we analyze fundamental issues involved in the phenomena of frequency shifts for light waves a la Doppler effect and the generalizations.

In the next section, standard Doppler effect is defined, and frequency-shift phenomena are introduced. A critique on the relativistic treatment and the quantum theory of Doppler effect is presented in Sec. 3. Basic issues on phase and amplitude are discussed in Sec. 4, and an alternative interpretation of frequency shift is suggested. In Sec. 5 a picture of single photon is envisaged in which translatory periodic motion and internal rotation in the transverse plane represent the photon. It is argued that total energy of photon is equally divided between linear motion and spin. An outline of experimental test for the alternative interpretation at single photon level is presented. Concluding remarks constitute the last section.

2 Doppler effect and the generalizations

Relative motion between the source generating a wave and the observer receiving the wave leads to a change in the frequency of the wave—such a kinematic effect is known as Doppler effect. Obviously to define relative motion one has to introduce inertial frame of reference (IFR). The weakness of gravitational interaction allows one to assume that IFR could be reasonably defined. Since sound wave is a mechanical disturbance that propagates in a material medium, IFRs for source, observer and wave are easily defined, and a moving source (or an observer) counting different frequency makes physical sense. In the case of light, the postulated EM disturbance propagates in vacuum with constant wave velocity, c i.e. the velocity c is independent of the IFRs. Light emitted from moving (stationary) source is shifted in frequency as observed by a stationary (moving) observer given by the relation

\[ \nu_0 = \gamma \nu_c (1 - \beta \cos \theta) \]  

(1)

\[ \tan \theta' = \frac{\sin \theta}{\gamma (\cos \theta - \beta)} \]  

(2)
Here $\nu_0$ and $\nu_e$ are the observed and emitted frequencies of light respectively, $\beta = v/c, \gamma = \sqrt{1 - \beta^2}$, $v$ is the relative velocity between the source and the observer, and the wave vectors of light (in different IFRs) make angles $\theta$ and $\theta'$ with respect to $v$. To first order in $\beta$ with $\theta = 0$, the standard Doppler shift is obtained from Eq. (1)

$$\nu_0 = \nu_e (1 - v/c) \quad (3)$$

There is a frequency shift in the direction perpendicular to the motion of the source, termed as transverse Doppler effect. For $\theta = \pi/2$, Eq. (1) gives

$$\nu_0 = \nu_e \gamma \quad (4)$$

Note that both linear and transverse Doppler effects arise due to the translational motion of the bodies, let us designate them by TDE (translational Doppler effect). Garetz [2] introduced the term angular Doppler effect (ADE) to denote frequency shift caused by the rotation of the light-emitting body such that the axis of rotation and the direction of propagation of light are parallel. The treatment given by Garetz is applicable to polarized plane waves, while Allen et al [3] consider finite-sized orbital angular momentum (OAM) carrying beams to obtain azimuthal Doppler shift analogous to ADE. The notion of rotational frequency shift (RFS) was discussed in 1997 [4] based on non-relativistic quantum mechanical treatment for a rotating radiating two-level system interacting with quantized EM field. Authors of [4] note that, “The RFS should not be confused with the ordinary linear Doppler shift observed for rotating objects (for example, stars or galaxies) that is due to the instantaneous linear motion of the emitter. This linear Doppler shift is maximal in the plane of the rotation while the RFS is maximal along the angular velocity, that is in the direction perpendicular to the instantaneous velocity. Thus the RFS competes with the quadratic Doppler shift rather than with the linear Doppler shift”. Note that the frequency shifts caused by moving reflecting mirrors or rotating wave plates have the origin in the Doppler effect (TDE or ADE). Garetz [2] gives a nice discussion based on Beth experiment for interpreting frequency shift caused by rotating half-wave plates [5].

In his letter, Garetz [2] also interprets rotational Raman spectra and fluorescence doublets in terms of ADE. There is another class of frequency-shift phenomenon that occurs in the non-inertial frames. It is well known that the gravitational force in general theory of relativity is postulated to be an
effect of curved pseudo-Riemannian space-time, therefore one can anticipate gravitational red shifts. In a simple example [6], one can construct a space-time metric for a rotating coordinate system with a uniform angular velocity of rotation. Assuming synchronization of clocks at each instant of time the shift in frequency of light can be calculated. In general, the difference in the gravitational potential at two different points determines the gravitational frequency shift. In 1960, Pound and Rebka verified gravitational redshift for gamma rays using Mossbauer effect, and Hay et al measured the frequency shift of 14 KeV gamma rays in a rotating system [7].

3 Frequency shifts-physical mechanisms

3.1 Simple explanations

The standard approach to Doppler effect is based on the invariance of the phase factor \((k \cdot r - \omega t)\) under Lorentz transformation from one IFR to another. Jackson observes [8] that, ‘the phase of a wave is an invariant quantity because the phase can be identified with the mere counting of wave crests in a wave train, an operation that must be the same in all inertial frames’. In [8] the second postulate of Einstein’s special relativity is simply stated as ‘speed of light is independent of its source’. The question arises: what is the meaning of observing ‘a wave train of light’ in a specific IFR? Dingle in a short monograph [9] touches upon this problem to some extent. He says that the coordinates appearing in the phase of the wave train are in a frame in which the source of light is at rest, and notes that, "It is not necessary, of course, that we should think of this equation as representing a wave motion in the space between the source of light and the observer, although it is frequently convenient to do so. The facts of observation are all equally well expressed if we regard the various quantities as representing something characterizing the source of light itself ". His discussion on the question whether the frequency change is an intrinsic property of source or not is interesting, however the physical mechanism for the Doppler effect is not clarified. Note that the speed \(c\) represents the motion of the constant phase surfaces independent of IFRs. The literature does not address the problem: to which physical system, i.e. the source or the plane EM wave, the coordinates \((r, t)\) and the quantities \((k, \omega)\) belong. Merely counting of wave crests is not enough.
Einstein in his 1905 paper \cite{10} claims that aether is superfluous since an 'absolutely stationary space' provided with special properties is not required, and it is not necessary to introduce a velocity-vector to a point in the empty space in which the electromagnetic processes occur. Note that he does not give physical meaning to frames of reference in relative uniform translatory motion, and the definition of the velocity of light, namely, light path divided by time interval is too simplistic. The second postulate of relativity says, 'Any ray of light moves in a stationary system of coordinates with the determined velocity c, whether the ray be emitted by a stationary or a moving body'. In Sec.7 of his paper a theory of Doppler effect is given. Following Einstein let us consider a frame K in which a source of light emits waves, and an observer is moving with velocity v relative to infinitely distant source of light. Next the connecting line source-observer is considered where an observer is at rest in the moving system K. The space-time coordinates in the phase factor are assumed to be that of the source frame, and while the wave reaches the observer traversing in vacuum with constant velocity the space-time coordinates of the phase factor are now changed to that of the observer frame. Invariance of phase immediately leads to the Doppler shift and aberration. The most puzzling aspect is that intervening inert empty space between source and observer somehow is capable to affect the internal constitution of light i.e. apparently no physical mechanism is needed to change the frequency of the light wave. Relativistic world view merely asserts that space and time are not absolute and the phase invariance from one frame to another changes the space-time coordinates and correspondingly changes in frequency and wave vector.

Alternative to wave theory, particle picture of light is used in \cite{2}. Though Garetz cites Sommerfeld’s book \cite{11}, Doppler effect from ‘photon point of view’ seems to have been given by Schrodinger \cite{12}. Fermi’s 1932 article \cite{13} presents this approach nicely, and gives a quantum theory of Doppler effect using p.A form of atom-radiation interaction.

Following Fermi, let us consider a two-level quantum system, e.g. an atom with energy levels $E_1$ and $E_2$. Let

$$h
\nu = E_2 - E_1$$

Assuming the atom to be at rest, the emitted frequency is $\nu_e$. Supposing the atom to be in the excited state, $E_2$, and moving with velocity $v$, the frequency of the emitted radiation is calculated using the energy and momentum conservation. A little algebra finally leads to the Doppler shift, Eq.
It appears that there is an inconsistency. Why should there be no recoil for the atom in the rest frame? The conservation of momentum gives

\[ mu = -h\nu'/c \]  

where \( u \) is the velocity acquired by the atom initially at rest. The initial energy of the atom is \( E_2 \) (being in the excited state), and after emission of radiation it is \( E_1 + (1/2)mu^2 \). Thus the emitted energy is

\[ h\nu' = E_2 - E_1 - \frac{1}{2}mu^2 \]  

Evidently \( \nu' \) is not equal to \( \nu_e \), and there is a shift in frequency. It is only when recoilless emission (i.e. Mossbauer like effect) is enforced that the emitted frequency is exactly \( \nu_e \).

### 3.2 Quantum theory

Dirac’s quantum theory of radiation finds beautiful application to explain classical phenomena like light propagation in vacuum and the Lippman fringes in Fermi’s paper [13]. Here we present the main ingredients of this theory as applied to the Doppler effect. The atom, the radiation field and the atom-radiation interaction are treated as a single system. The radiation field enclosed in a finite volume of space, \( V \) is represented by quantized oscillators in the plane wave approximation. The interaction term is obtained by the transformation \( p \rightarrow p - eA \), and assumed to act as a small perturbation. In the nonrelativistic formulation, the standard perturbation theory is used to calculate the time evolution of the probability amplitudes of the Schrödinger wave function. In the Doppler shift phenomenon, a simple hydrogen-like atom is considered. The Hamiltonian for this system is given by

\[ H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + U(q_1 - q_2) + \sum_s \frac{1}{2}(p_s^2 + \omega_s^2q_s^2) - \frac{e}{m_1}A(q_1).p_1 + \frac{e}{m_2}A(q_2).p_2 \]  

The interaction term proportional to \( A^2 \) is neglected. Here \( q_1(q_2) \), \( m_1(m_2) \) and \( p_1(p_2) \) are respectively the coordinate, mass and momentum of proton (electron); \( U \) is the Coulomb binding energy. The fourth term in \( H \) represents quantized radiation, \( H_r \), and the last two terms represent the interaction Hamiltonian, \( H' \). The vector potential is given by

\[ A = c\sqrt{\frac{8\pi}{V}} \sum_s (\hat{e}_s q_s \sin \Gamma_s) \]  

where \( \Gamma_s \) is the phase shift.
Here $e_s$ is a unit vector along $A$, $k_s$ is the propagation vector, and $\beta_s$ is a constant phase.

Fermi makes two points: in order to account for the impulse of the radiation, the center of mass motion has to be incorporated in the theory, and the usual assumption of the constancy of the phases. $\Gamma_s$ does not hold in this case. Defining new variables, namely the center of mass coordinate $R$ and relative coordinate $q$, we have

$$ R = \frac{m_1 q_1 + m_2 q_2}{M} $$

$$ q = q_1 - q_2 $$

Here the total mass is $M = m_1 + m_2$. The corresponding momenta are given by

$$ P = p_1 + p_2 $$

$$ p = \frac{m_2 p_1 - m_1 p_2}{M} $$

The Hamiltonian of the system in new variables is transformed to the unperturbed part

$$ H_0 = \frac{P^2}{2M} + \frac{p^2}{2m} + U(q) + H_r $$

and the interaction part

$$ H' = -\frac{e}{m} \sqrt{\frac{8\pi}{V}} \sum_s \hat{e}_s \cdot p q_s \sin \Gamma_s $$

Note that $m = m_1 m_2 / M$, and the assumption is made that the phases $\Gamma_s$ depend only on $R$, i.e.

$$ \Gamma_s = k_s \cdot R + \beta_s $$

In Eq. (15), the first term represents the gross motion of the atom, and the next two terms define internal states. Assuming that the wave function of the system can be written as a product of the eigenfunctions of gross motion, internal state and the radiation we have

$$ \Psi = \Psi_{cm} \Psi_{int} \Psi_r $$

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and the energy eigenvalue is given by the sum of the corresponding eigenvalues. The probability amplitudes will satisfy a first order (time) differential equation, and involve the matrix elements for the interaction $H'$. The matrix elements nicely separate into a product of integrals with respect to $R$, $q$ and $q_s$. The integral involving $R$ looks like

$$I = \int e^{-iK_n \cdot R} \sin(k_n \cdot R + \beta_n) e^{iK_m \cdot R} dR$$

Here $\hbar \mathbf{K}$ is the momentum of the atom. Exponential representation of sine function immediately leads to the momentum conservation law

$$K_m - K_n \pm k_s = 0$$

Together with the energy conservation, this equation leads to the Doppler shift as explained in the preceding sub-section.

### 3.3 Critique

During past more than a decade, there has been a great deal of activity on the gross motion of atoms subjected to laser light beams. The theory of a moving two-level atom interacting with the light beam adequately brings out the physics of the phenomena such as atomic beam deflection and laser cooling. In the cold atom optics, at very low temperatures where the atomic de Broglie wavelength is of the same order as the wavelength of the light, the gross atomic motion needs to be quantized. We refer to [14] for basic theory and original references cited therein. Allen et al [3] generalize the canonical theory for the radiation-pressure effects on the gross motion of the atom [15] replacing the plane wave representation by the Laguerre-Gaussian modes to obtain the azimuthal Doppler shift. Let us consider the Hamiltonian given in [3] for a two-level atom of resonant frequency $\omega_o$, interacting with light mode of frequency $\omega$

$$H = \hbar \omega \pi^+ \pi + \frac{P^2}{2M} + U(R) + \hbar \omega a^+ a - i\hbar [\pi^+ a f(R) - f^*(R) a^+ \pi]$$

Here $\pi$ and $\pi^+$ are ladder operators for internal two-levels, and $a$ and $a^+$ are the annihilation and creation operators respectively of the radiation field. The operator $f(R)$ arises from the dipole interaction term $-E \cdot r$. Comparing Eq. (21) with Fermi’s theory, Eqs. (15) and (16) it is straightforward to
recognize that for a single mode radiation field both have nice correspondence except that Fermi uses $p.A$ interaction term. Note that the operators $q_s$ and $p_s$ in the radiation field Hamiltonian can be expressed in terms of non-Hermitian operators $a$ and $a^\dagger$.

$$q_s = \sqrt{\frac{\hbar}{2\omega}}[a_s + a^\dagger_s]$$  \hspace{1cm} (22)$$

$$p_s = i\sqrt{\frac{\hbar\omega}{2}}[a^\dagger_s - a_s]$$  \hspace{1cm} (23)$$

Thus it would seem that the quantum theory of Doppler effect has been rediscovered. Regarding the choice of dipole interaction term, see [16] for a critical analysis of the problem.

Quantum theory of rotational effects given in [3] seems quite logical. On the other hand, the theory given in [4] is based on an electron bound by a time dependent potential $V(r(t))$ where the coordinate $r$ does not bring out the gross motion clearly. At a fundamental level, it is true that the separation into internal and external parts of a system is unsatisfactory, but the claim that a uniform translation does not affect internal energy level difference while a uniform rotation does, remains rather obscure in [4]. In fact, it is not clear if RFS is equivalent to the azimuthal Doppler shift. In a recent report [17] it has been claimed that the RFS has been observed for the first time in a molecular system.

Let us contrast the relativistic explanation and quantum theory of the Doppler effect. The Lorentz transformations and the invariance of phase $(k.r - \omega t)$ under them do not throw any light on the mechanism of energy-momentum transfer that clearly arises in the quantum theory. If we consider the phase factor $\Gamma_s$ in quantum theory then it would appear that the coordinates in $(k.r - \omega t)$ should correspond to the source. It is intriguing that in the relativistic explanation, one merely asserts that let there be an IFR moving with a velocity $v$ relative to another IFR, but does not point out the physical process by which a stationary object acquires uniform motion. Similar to this, in quantum theory, one asserts the collapse of the wave function to a certain eigenstate. In this sense, both relativistic and quantum theories are unsatisfactory. It is interesting to note that Einstein in his 1917 paper on quantum theory of radiation [18] justifies the necessity of classical EM theory saying: 'Whatever the form of the theory of electromagnetic processes, surely in any case the Doppler principle and the aberration will remain valid'. In
the background of the developments preceding relativity paper and the formative years of quantum theory such an obscure and paradoxical approach to Doppler effect seems understandable, however with our present knowledge of light, photon and light-matter interaction we must address the fundamental issues afresh.

A careful examination of quantum theory shows that the transition of the atomic internal state is not instantaneous, there is a finite lifetime. In relativity, the time dilation is often used to explain the longer lifetime of a moving unstable particle as compared to the static one. In our previous work [19] we argued that kinematic explanation for an irreversible process posed a paradox. Critical analysis of the concept of time in relativity led us to suggest that the different IFRs are characterized by differing constant potentials. Similar to the Aharonov-Bohm effect or geometric phase, under suitable conditions the observable effects of constant potentials become manifest: change in lifetime and Doppler effect could be such phenomena [20]. We refer to [21] for detailed exposition of the ideas on time.

Preceding discussion shows that the quantum theory of Doppler shift takes into account the effect of radiation pressure on the gross quantized motion of the atom. It is also known that the lifetime of a two-level system can be calculated using Fermi’s golden rule [13]. We suggest that a self-consistent theory treating both the Doppler effect and lifetime dilation simultaneously would be a great advancement. A possible approach being investigated is based on the incorporation of Berry’s connection and non-equivalence of IFRs for a two-level atom plus radiation system. Let us recall that the standard theory for a quantum system is often simplified in terms of two sets of variables. First, one set of variables is kept constant, and the problem is solved for the other set of dynamical variables. Next the first set of variables is varied to obtain the effective dynamics [22]. The induced vector potential or Berry’s connection is shown to arise in the effective dynamics. Wilkens in an interesting paper [23] considers spontaneous emission of a moving atom including the Roentgen interaction term in the Hamiltonian. Here the magnetic dipole-like term arises due to the motion of the radiating atom. Cresser and Barnett [24] elucidate the basic problem considering a moving electric dipole that generates a magnetic dipole moment and the Doppler shift, and argue that these physical processes ‘conspire’ to give time dilation effect for the lifetime of the decaying atom as demanded by special relativity. Is it not strange that time dilation kinematic effect is unreservedly accepted for a decaying irreversible process? Our suggestion that
time dilation effect is an analogue of geometric phase effect becomes plausible in the light of Wilken’s work [25]. Similar to the Aharonov-Bohm effect one expects topological phase for a moving dipole [25], see also [26]. Another related interesting result in quantum optics is that of the enhancement or inhibition of spontaneous emission if the surrounding quantum vacuum is modified [27]. In a somewhat different context it has been argued [28] that Doppler shift and broadening of spectral lines both could be explained based on Wolf effect.

4 Phase and amplitude of light wave

In the review on angular momentum of light [29], a brief discussion is given on the RFS. A rotating half-wave plate shifts the frequency of a circularly polarized light wave, while for a circularly polarized Laguerre-Gaussian modes a rotating Dove prism and half-wave plate give rise to the frequency shift. A recent review [30] dwells upon the energy exchange mechanism for the frequency shifts, while the frequency shift in the context of geometric phase was suggested by Simon et al [31]. In fact, the significance of energy conservation for Doppler shift was clear in Fermi’s analysis [13], and for the angular Doppler effect, Garetz [2] considers the torque for a rotating wave plate, and the work done on or by the wave plate for the changes in the spin angular momentum of light. Note that the experiments discussed in [2] are carried out for the classical light beams though the author uses photon point of view. In the preceding section we have analyzed the physical mechanisms responsible for the Doppler frequency shifts, and pointed out that atom-radiation interaction is crucial for frequency changes of radiation though some questions remain unsettled. Could the frequency of a monochromatic light wave be changed by rotating wave plates? If the frequency of a photon is its intrinsic attribute, could it be changed by a rotating optical element? We confine our attention to the polarization changing effects in the following, and seek an alternative interpretation of the experimental results.

The experiments show that the intensity of light transmitted through a rotating wave plate behaves sinusoidally with time. For an ideal case of lossless transmission and perfect fringe visibility, we have

$$I_t = I_i [1 + \cos(\theta_0 + 2\Omega t)]$$  \hspace{1cm} (24)

The wave plate is assumed to rotate at an angular frequency of $\Omega$, and
the time-dependence of output intensity, \(I_t\) is interpreted as a frequency shift of \(\pm 2\Omega\). We argue that a time-dependent phase does not necessarily imply the frequency change. For a plane monochromatic EM wave, the electric field vector can be written as

\[
E = E_0 e^{i(k \cdot r - \omega t)}
\]

The Poynting vector, \(S\), is in the direction of \(k\), normal to both electric and magnetic field vectors. The magnitude of \(S\) averaged over time period gives the intensity of the plane wave.

\[
I = \frac{c}{8\pi} E_0 E_0^* \tag{26}
\]

It is clear that the intensity does not depend on the frequency of the wave, however starting with the vector potential \(A\) and applying the transversality condition (i.e. radiation gauge)

\[
A = A_0 e^{i(k \cdot r - \omega t)} \tag{27}
\]

The expression for the intensity in terms of \(A_0\) is easily calculated to be

\[
I = \frac{\omega^2}{2\pi c} A_0 A_0^* \tag{28}
\]

Eq. (28) shows the frequency dependence of the intensity. Recall that the total energy i.e. the sum of kinetic and potential energy of a classical simple harmonic oscillator is given by

\[
E_{tot}^c = \frac{1}{2} m \omega^2 x_m^2 \tag{29}
\]

where \(x_m\) is the amplitude, and \(\omega\) is the angular frequency of the oscillator. Note the similarity of the expression of the intensity I vide Eq. (28) with \(E_{tot}^c\).

Returning to the problem of rotating wave plate, light wave propagating along, let us say \(z\)-axis, can be represented by a row vector \([E_x\ E_y]\) and its Hermitian adjoint by a column vector. The effect of an optical element using Jones calculus can be described in terms of a 2x2 transmission matrix. A rotating half-wave plate introduces a time dependent phase shift depending on \(\Omega\). Bretanaker and LeFloch \[32\] rightly note that Jones calculus leads to the observed phase shift, and seek angular momentum exchange mechanism to interpret the frequency shift. Authors invoke energy exchange using the
energy of N photons equal to \(N\hbar\omega\), and change in \(\omega\) due to rotating wave plate. In laser physics, one uses the notion of energy flux in terms of the number of photons, \(N\)

\[
U_f = \frac{cN\hbar\omega}{V}
\]  

Assuming that the total number of photons in the light wave consists of spin \(+\hbar\), \(N_+\) and spin \(-\hbar\), \(N_-\), we suggest that the time-dependent phase arises due to the modulation of photon numbers in contrast to the frequency change suggested in [32].

Since the experiments measure the intensity oscillations, it is not obvious how to distinguish the photon number oscillations from the frequency shifts. Novel experimental scheme would have to be devised that markedly depends on the number of photons in the light wave. The experiments based on photoelectric effect is one of the possibilities however quantum theory of radiation shows that the photon number operator is not directly observable [18] p.629, therefore it is not clear whether it would lead to conclusive results.

5 Single photon

Instead of classical light wave, the frequency-shift phenomena at a single photon level appears to be an attractive idea. The first fundamental issue in this case is, of course, whether photon has physical reality or it is merely a mathematical construct in the form of vacuum excitation. Lamb has severely criticized the concept of photon used by ‘the laser community’ [33], however for a balanced critique we refer to [1]. In quantum optics, it is asserted that the radiation enclosed in a cavity has discrete electric field with an amplitude of

\[
E_{0r} = \sqrt{\frac{\hbar\omega}{2V}}
\]  

and for \(n\) photons the quantum of electric field is \(E_{0r} \sqrt{n}\). In [34] Knight draws attention to the experiments that seem to test the consequences of the quantized electric field. In spite of the recent advances in quantum optics, some fundamental questions have remained unresolved; besides the problems reviewed in [1] we refer to a passionate critique on the Copenhagen interpretation of quantum mechanics by Post [35]. Of particular significance is the anticipation of a zero-point energy by Planck in 1912 discussed in detail in [35]. Randomness in the mutual phases of an ensemble of classical harmonic
oscillators was considered by Planck, and for thermodynamic equilibrium an average zero-point energy of $\hbar \omega / 2$ per oscillator was necessary. To summarize: the meaning of electric field amplitude and phase for a single photon deserve a careful attention whether one rejects or accepts the physical reality of photon [36].

To gain insight, we revisit elementary considerations. Note that the electric field amplitude $E_{or}$ depends on the frequency of the radiation. Eqs. (26) and (30) do indicate such a relationship, but there maybe a deeper reason at the quantization level. The canonical variables $(q_s, p_s)$ are expressed in terms of creation and annihilation operators vide Eqs. (22) and (23), and the frequency dependent factors are absorbed in the definitions. Essentially similar step appears in quantum oscillator that has total energy

$$E_{tot}^q = (n + 1/2)\hbar \omega$$

(32)

In the textbooks, it is usually proved that for any value of $n$, the expectation values of kinetic and potential energy are equal and each one is half of the total energy just as in the case of classical oscillator. However, the crucial difference between the classical and quantum oscillators is not mentioned, namely the absence of an amplitude factor in Eq. (32). The expression (31) for $E_{or}$ is based on the calculation of the expectation value of the square of the field operator since

$$< n | \hat{E} | n > = 0$$

(33)

In the case of a classical oscillator the amplitude of $x$ is $x_m$, while the momentum amplitude is $m \omega x_m$. Assuming that the product of the two is constant, and setting it to be $\hbar$ we get

$$m \omega x_m^2 = \hbar$$

(34)

The energy of the oscillator becomes

$$E_{tot}^c = \hbar \omega / 2$$

(35)

In this form, the zero-point energy corresponds to a single oscillator unlike the randomized phases for the ensemble of oscillators in Planck’s work or vacuum quantum fluctuations in quantum optics. Though as yet no physically sound and concrete model of photon has been developed, it is generally
believed that photon has energy $\tilde{h}\omega$, momentum $h\nu/c$, and spin angular momentum $\pm \tilde{h}$. I have never been able to understand why a simple natural question is not asked: What is the energy of the photon associated with its spin angular momentum? It is quite possible that it may have something to do with Einstein’s reluctance to attach much significance to photon spin [36] and for historical reason that quantum optics formalism takes care of spin as polarization index for all practical purposes. In the light of expression (35), let us visualize photon as an object possessing internal rotational motion of an extended structure, and translational motion as a periodic propagating disturbance. Recall that the kinetic energy for a classical particle having momentum $p$ is $(1/2)pv$, and the rotational energy is equal to $(1/2)L\omega$, where $L$ is the angular momentum. Now we split the energy $\tilde{h}\omega$ of photon as follows:

$$\tilde{h}\omega = \frac{1}{2}\tilde{h}\omega + \frac{1}{2}\left(\frac{h\nu}{c}\right)c$$

(36)

Note that the expression (36) looks like $(1/2)L\omega + (1/2)pv$. Further the translational periodic motion as a harmonic oscillator would have the energy $(1/2)h\omega\nu$, Eq. (35) that would be consistent with the second term on the right hand side of Eq. (36). In [36] it has been speculated that internal and external periodicities are in synchronization for the extended structure of photon moving in vacuum.

An important significance of the present photon model is on the wave-particle duality. It is known that the first enunciation of the complementarity principle by Bohr was based on the simple argument: energy and momentum represent particle attributes while frequency and wavelength represent extended wave, therefore, the Planck-Einstein relationship between them imply duality. In the present model the extended structure transverse to the direction of the propagation has particle-like attribute, and the periodic translatory motion would give rise to wave-like effects. The radical revision of the classical picture envisaged here is that of discarding the description based on point particle and instantaneous dynamical variables. Just as zero point energy is not some mysterious quantum vacuum effect, the Heisenberg’s uncertainty relation merely represents finite discrete spatial and temporal units such that the product of the size of the photon and its momentum is equal to the Planck constant.

The intriguing questions regarding the frequency dependent amplitude, photon number oscillations, and frequency shifts will assume distinct significance for single photon experiments i.e. the transmission of a photon
through a rotating wave plate. The time scales in the interference experiments would be most crucial. In the context of our photon model the polarization-dependent experiments would show particle behavior, while the momentum exchange experiments would show wave-like aspects. We expect important implications on the black body radiation physics which will be discussed elsewhere.

6 Conclusion

In this paper, we have critically reviewed the physical mechanism for the Doppler shift, and drawn attention to Fermi’s quantum theory that considered quantized gross motion of the atom more than seven decades ago. In spite of the well known relativistic explanation and quantum theory of Doppler effect, we point out that there exist gaps in our understanding of this phenomenon. We suggest a self consistent theory taking into account the atom-radiation interaction for life-time dilation of a moving atom and the Doppler shift will provide new insights.

In the case of frequency-shift phenomena observed using passive optical elements (e.g. rotating wave plates), we offer an alternative mechanism in terms of the photon number oscillations for the polarization changing experiments. Some intriguing aspects of amplitude and phase of the light wave are discussed, and implications on the physical model of photon are indicated. A conclusive proof whether the intensity oscillations observed experimentally are due to frequency-shift or time-dependent amplitude due to photon number oscillations appears difficult, however at a single photon level it should be possible to distinguish the two.

Different strands on the frequency shift phenomena for electromagnetic radiation indicate the need for replacing the obscure and counter-intuitive physics of the Doppler effect by an underlying simple unifying principle. I believe this principle will emanate from the understanding of the physics of aether or quantum vacuum or what I prefer to call manifest dynamical space [37].
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