Numerical analysis of fractional coronavirus model with Atangana-Baleanu derivative in Liouville-Caputo sense

M Goyal*, A K Saraswat1 and A Prakash2
1Department of Mathematics, Institute of Applied Sciences and Humanities, GLA University, Mathura 281406, India
2Department of Mathematics, National Institute of Technology, Kurukshetra 136119, India

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Abstract: The novel coronavirus which emerged at the end of the year 2019 has made a huge impact on the population in all parts of the world. The causes of the outbreak of this deadliest virus in human beings are not yet known to the full extent. In this paper, an investigation is carried out for a new convergent solution of the time-fractional coronavirus model and a reliable homotopy perturbation transform method (HPTM) is used to explore the possible solution. In the presented model, the Atangana-Baleanu derivative in the Liouville-Caputo sense is used. The variations of the susceptible, the exposed, the infected, the quarantined susceptible (isolated and exposed), the hospitalized and the recovered population with time are presented through figures and are further discussed. The effects of selected parameters on the population with the time are also shown through figures. The convergence of solution by the HPTM is shown through tables. The results reveal that the HPTM is efficient, systematic, very effective, and easy to use in getting a solution to this new time-fractional mathematical model of coronavirus disease.

Keywords: Fractional calculus; Fractional coronavirus model; Atangana-Baleanu derivative in Liouville-Caputo sense; Homotopy perturbation transform method (HPTM); He’s polynomials

Mathematics Subject Classification: 26A33; 92D30; 37M05

1. Introduction

In December 2019, the Republic of China updated the World Health Organization (WHO) on some special cases of pneumonia in its Wuhan city by some unknown causes. Later, on January 7, 2020, it was revealed by some independent research laboratories in China through deep sequencing and etiological investigations that the causative agent for these cases of pneumonia may be a novel coronavirus. On January 10, 2020, the WHO labeled novel coronavirus as 2019-nCoV. The WHO later declared this epidemic, a global pandemic, on March 11, 2020.

The genome of coronavirus is a positive-sense single-stranded RNA with 3′–poly-A tail and 5′–cap structure. The corona virus genome, the largest identified RNA virus, is 30 kb in length. They can cause infection in the upper and lower respiratory system, and the gastrointestinal system of humans. They can attack the nerve endings of human beings, livestock, bats, birds, and animals [1]. The occurrence of respiratory syndrome proved the prospect of human-to-human and animal-to-human transmission of coronavirus [2].

The coronavirus distresses different population in a dissimilar manner. Mostly, the infected population gets minor to moderate illness and recovers without getting hospitalized. The common symptoms of infection are tiredness, body pain, high fever, running nose, and dry cough. However, some less shown symptoms are headache, sore throat, pain, diarrhoea, etc. It spreads primarily through the breathing droplets of the infected person. To slow down the spread of this virus and to protect all, everyone should wear a face mask covering the mouth, and nose and secure it under chin, often wash both hands with the water and soap for a minimum of 20 s, observe own health daily, and get vaccinated with the authorized vaccines.
At present, more than 220 countries are affected by this virus. Globally, there are 276,436,619 confirmed coronavirus cases that include 5,374,744 deaths conveyed to the WHO till December 23, 2021. As of December 22, 2021, about 8,649,057,088 doses of vaccines have been taken by people all over the world [3]. Globally, India is second on the list of confirmed coronavirus cases after the USA.

To control the transmission risk of this disease, a considerable amount of research has been carried out. Tang et al. [4] proposed a susceptible-exposed-infectious-recovered (SEIR) deterministic model based on its medical progress. Samui et al. [5] developed a predictive model to control its transmission. Chatterjee et al. [6] used Monte–Carlo simulation to study the stochastic model for the healthcare impact of this epidemic. Kahn and McIntosh [7] presented the history and advancement of the coronavirus. Owolabi and Atangana [8] modeled HIV/AIDS transmission with a fractional derivative. Christopher et al. [9] applied the differential transformation technique to analyze the coronavirus model. Pathak et al. [10] analyzed a susceptible-infectious-recovered (SIR) model with asymptotic function. Khan and Atangana [11] presented the dynamics of coronavirus and developed a model for its transmission from bats to an unknown receiver to the human beings. Yousefpour et al. [12] designed a model for the spread of coronavirus based on the real-time data from Wuhan.

The fractional-order derivatives can give a more accurate interpretation of natural happenings than integer-order models due to their ability to describe heretical and memory-related qualities. The arbitrariness in the order of the derivative gives more degrees of freedom in design and analysis which results in precise modeling, improved robustness in control, and better flexibility in signal processing. The diffusion process or the double-layer charge distribution can be better described with a system of fractional order. The characterization of ceramic bodies, viscoelastic materials, fractal structures, the decay rate of fruits and meat, the study of corrosion in a metal surface, etc. are some interesting areas of its applications. A Fractional-order system is used to study real-time events, for example, volcanic phenomena, earthquake propagation, design of pharmacokinetics, modeling of the human skin, lungs, and many more. When space–time is discontinuous, the fractal theory is used to describe many phenomena [13].

The differential equations which manage the systems with memory features are fractional differential equations (FDE). They are used to solve a variety of problems in biology [14], fluid mechanics [15], visco-elasticity [16], signal processing [17], vibration [18], neuro-physiology [19], and other fields. Mathematical investigations are carried out to state various phenomena of biological [20, 21], medical [22–28], physical [29–31], and other fields of importance. In general, a fractional differential equation does not have an exact solution therefore a numerical or an approximate solution is targeted [32]. The reliability of the solution is an essential feature [33].

The homotopy perturbation scheme (HPM) was introduced by He [34, 35]. It is a united form of perturbation method and the homotopy in topology. He [36] proposed the creation of a homotopy equation by including an auxiliary term that does not affect the initial solution ($p = 0$) and the real solution ($p = 1$). When $p = 0$, the system of equations is frequently reduced to a simplified form that admits a relatively simpler solution. As the value of $p$ approaches 1, the system goes through a series of deformations, each of which has a solution, similar to the preceding stage of deformation. At $p = 1$, the system eventually returns to its original form of the equation, and the final stage of deformation yields the desired result. He developed the HPM with the two expanding parameters which are effective for an equation having two nonlinear terms. One of the most impressive aspects of the HPM is that it only requires a few perturbation terms to achieve a relatively accurate solution [37]. A wide range of linear and nonlinear problems have been solved using the HPM [38–41]. It was found that the HPM can overcome the challenges associated with calculating Adomian polynomials [42].

The perturbation method has limitations such as, the solution may have a series of smaller parameters that have difficulties but in general, a nonlinear problem possesses no such type of parameters. The HPM was modified [43–45] by incorporating an acceleration parameter that led to rapid convergence. Filobello-Nino et al. [46] proposed the modified HPM as an enhanced perturbation method. Li and He [47] connected the enhanced perturbation approach with the parameter expansion technology. More details of Li–He’s modification of the HPM are in [48–50]. But, such methods have limitations such as massive computation with more time consumption. So, they require linkage with a transform operator. The hybrid methods using integral transforms [51–55] are useful to get a solution to a nonlinear differential equation.

The homotopy perturbation Laplace transform (HPTM) method is also a collective form of the HPM and Laplace’s transformation. The HPTM shows how Laplace’s transformation may be used to get a solution to an FDE by tackling the HPM. In literature, the HPTM is also called as He-Laplace method [56–59]. This method is capable of mixing two strong computational schemes for examining an FDE. It also reduces time and the computation work as compared to existing schemes simultaneously preserving the efficiency of results.

He’s fractional derivative is preferred for a function that is continuous but not necessarily differentiable. We know
that the kernels of Caputo and Riemann–Liouville fractional operators are singular [60, 61] although they are non-local. Riemann–Liouville derivative is Markovian only and obeys power-law while Caputo–Fabrizio derivative is just non-Markovian and has only exponential decay. Ain and He [62] proposed a two-scale fractal theory [63] to overcome the problem of scaling, as different scales produce different results. He and El-Dib [64] used a fractal Zhiber–Shabat oscillator to introduce the basic properties of a two-scale fractal differential equation.

Atangana and Baleanu [65] suggested Atangana-Baleanu (AB) fractional derivative with a non-local, non-singular kernel. It is based on the property of Mittag–Leffler kernel and has all the benefits of other fractional operators. Atangana and Baleanu derived the fractional integrals by using Laplace transform. AB derivative has both Markovian and non-Markovian properties with stretched exponential waiting time following power law and Brownian motion. It is deterministic as well as stochastic and is widely used to model the real-life problems. It was further developed by Atangana and Koca [66] by presenting some of its new properties [67–70]. AB derivative is used in the modeling of infectious diseases [25], computational biology [26], non-equilibrium processes in physics, fluctuations, and random processes.

The purpose of this article is to provide a convergent solution to the time-dependent fractional mathematical coronavirus model. This article is presented as follows. In Sect. 1, there is an introduction. Some fractional operators are defined with their properties in Sect. 2. In Sect. 3, the presented fractional coronavirus model is examined and discussed. The existence and uniqueness analysis of the solution is also shown with the Picard-Lindelof approach. In Sect. 4, the basic idea of the HPTM with convergence analysis is given. The solution of the model is also provided. In Sect. 5, the results are discussed. In Sect. 6, the conclusion is given.

2. Preliminaries

We now give properties with definitions and properties of fractional calculus operators [71].

**Definition 2.1** Riemann–Liouville fractional derivative is defined as.

\[ D^\alpha_x (f(x)) = \frac{1}{\Gamma(n - \alpha)} \frac{d^n}{dx^n} \int_0^x (x - t)^{n-\alpha-1} f(t) \, dt \]

**Definition 2.2** Jumarie’s modification [72] of the Riemann–Liouville fractional derivative is.

\[ D^\alpha_x (f(x)) = \frac{1}{\Gamma(n - \alpha)} \frac{d^n}{dx^n} \int_0^x (x - t)^{n-\alpha-1} f(t) \, dt \]

\[ D^\alpha_{x_0} f(x) = \frac{1}{\Gamma(n - \alpha)} \frac{d^n}{dx^n} \int_{x_0}^x (x - t)^{n-\alpha-1} f(t) \, dt + \frac{x^\alpha}{\Gamma(\alpha + 1)} \int_{x_0}^x \phi(t) E_\alpha \left[ \frac{\alpha}{\alpha - 1} (\rho - s)^{\alpha - 1} \right] \, ds \]

where \( N \) is a normalization of the function such that, \( N(0) = N(1) = 1 \).

**Definition 2.4** Let \( \phi \in C^1(a, b), \ b > a, \ a \in [0, 1] \) be a differentiable function, then.

\[ ABC_{a}^{\alpha} \{ \phi(t) \} = \frac{N(\alpha)}{1 - \alpha} \int_a^\rho \phi'(s) E_\alpha \left[ -\frac{\alpha}{1 - \alpha} (\rho - s)^{\alpha} \right] \, ds \]

**Definition 2.5** Liouville-Caputo derivative of fractional order \( \alpha > 0 \) on space \( \mathbb{R} = (-\infty, \infty) \) is

\[ \frac{d^\alpha}{dt^\alpha} \phi(t) = \frac{1}{\Gamma(n - \alpha)} \int_0^t \left( \frac{d^n}{dt^n} \phi(t) \right) (t - s)^{n-\alpha-1} \, ds \]

\[ \frac{d^\alpha}{dt^\alpha} \phi(t) = \frac{1}{\Gamma(n - \alpha)} \int_0^t \left( \frac{d^n}{dt^n} \phi(t) \right) (t - s)^{n-\alpha-1} \, ds \]

**Definition 2.6** Mittag–Leffler function is presented by the series,

\[ E_\alpha(z) = \sum_{m=0}^{\infty} \frac{z^m}{\Gamma(1 + \alpha m)}, \ z > 0, \ z \in C. \]

**Definition 2.7** Let \( \phi \in C^1(a, b), \ b > a, \ a \in [0, 1] \) be a differentiable function, then.

\[ ABC_{a}^{\alpha} \{ \phi(t) \} = \frac{N(\alpha)}{1 - \alpha} \int_a^\rho \phi'(s) E_\alpha \left[ -\frac{\alpha}{1 - \alpha} (\rho - s)^{\alpha} \right] \, ds \]

where \( N \) is a normalization of the function such that, \( N(0) = N(1) = 1 \).

\[ ABC_{a}^{\alpha} \{ \phi(t) \} = \frac{N(\alpha)}{1 - \alpha} \int_a^\rho \phi'(s) E_\alpha \left[ -\frac{\alpha}{1 - \alpha} (\rho - s)^{\alpha} \right] \, ds \]

**Definition 2.8** For \( 0 < \alpha < 1 \), the fractional integral operator in AB sense of order \( \alpha \) is

\[ ABC_{\alpha}^{\alpha} \{ \phi(t) \} = \frac{1}{\Gamma(\alpha)} \int_a^\rho \phi(t) E_\alpha \left[ \frac{\alpha}{\alpha - 1} (\rho - s)^{\alpha - 1} \right] \, ds \]
Definition 2.9 Laplace transform of AB derivative of function $\varphi(t)$ in Caputo sense is given as,

$$L\left[\frac{\partial^\alpha}{\partial t^\alpha} \{\varphi(t)\}\right](s) = \frac{N(x)}{1-x} \left(\left[\frac{(s)^x L\{\varphi(t)\}(s) - (s)^{x-1} \varphi(0)}{(s)^x + \frac{a}{1-x}}\right]\right),$$

where $x = \frac{\alpha}{\beta}$.

Definition 2.10 For $\varphi \in C[a, b]$, the following result holds:

$$\|\frac{\partial^\alpha}{\partial t^\alpha} \{\varphi(t)\}\| \leq \frac{N(x)}{1-x} \|\varphi(t)\|,$$

where $\|\varphi(t)\| = \max_{a \leq t \leq b} |\varphi(t)|$.

Further, the AB derivative in the Liouville-Caputo sense fulfills the Lipschitz condition,

$$\frac{\partial^\alpha}{\partial t^\alpha} \{\varphi_1(t)\} - \frac{\partial^\alpha}{\partial t^\alpha} \{\varphi_2(t)\} \leq \sigma \{\varphi_1(t)\} - \{\varphi_2(t)\},$$

where $\sigma = \frac{\alpha}{\beta}$.

3. Model description

The presented model is based on the model developed by Tang et al. [4]. They proposed a model with a system of eight simultaneous differential equations and graded the population as susceptible ($S$), exposed ($E$), infected with symptoms ($I$), infected without symptoms ($A$), susceptible quarantined ($S_q$), exposed ($E_q$), hospitalized ($H$), and the recovered population ($R$). The dynamics of novel coronavirus transmission is governed by a system of coupled differential equations as follows:

$$\begin{align*}
\frac{dS}{dt} &= -c\{\beta + q(1 - \beta)\}S(I + \theta A) + \lambda S_q, \\
\frac{dE}{dt} &= c\beta(1 - q)S(I + \theta A) - \sigma E, \\
\frac{dI}{dt} &= \varrho \sigma E - (\delta_I + \alpha + \gamma_I)I, \\
\frac{dA}{dt} &= \sigma(1 - q)E - \gamma_A A, \\
\frac{dS_q}{dt} &= cq(1 - \beta)S(I + \theta A) - \lambda S_q, \\
\frac{dE_q}{dt} &= \beta c_q S(I + \theta A) - \delta_q S_q, \\
\frac{dH}{dt} &= \delta_I I + \delta_q E_q - (b + \gamma_H)H, \\
\frac{dR}{dt} &= \gamma_I I + \gamma_A A + \gamma_H H, 
\end{align*}$$

where $\beta$ stands for the probability of coronavirus transmission per contact, $\theta$ is transmissibility multiple, $c$ denotes the contact rate, $q$ is the quarantined rate of exposed, $\sigma$ represents the transition rate of exposure to the infected, $\lambda$ is the rate at which quarantined uninfected were released, $q$ indicates the probability of having symptoms among the infected, $\varrho$ is the disease induced death rate, $\delta_I$ means the rate of transition of symptomatic infected to the quarantined infected, $\delta_q$ represents the transition rate of exposed quarantined to quarantined infected. $\gamma_r$, $\gamma_A$ and $\gamma_H$ specify the rates of recovery of the infected, the asymptomatic infected, and the quarantined infected people respectively.

The integer-order derivative is local. The integer-order model does not include memory effects. The non-integer derivative holds a non-locality property. The forthcoming system state is dependent also on earlier states. The arbitrary order represents the index of memory [73]. The fractional model administers the working stage with the memory process. Therefore, the models with fractional-derivatives [18] adhere to reality. Padmavathi et al. [74] developed this model as a fractional model. The memory in time is significant for the coronavirus model which makes its fractional modeling appropriate. So, extending the dynamics of coronavirus with the features of the memory effect is an inspiring factor in investigating a time-dependent coronavirus model.

The power law decay [18] in solution is found with the fractional order $\alpha$ of the derivative in contrast to the exponential decay in arbitrary Brownian motion. Therefore, to examine a more realistic model, the integer model is changed to a non-integer order model using the ABC derivative [65, 66, 69] as follows:
\( \text{ABC}_0 D^\alpha_t S = -w_1 S(t + \alpha A) + \lambda S_a \)
\( \text{ABC}_0 D^\alpha_t E = w_2 S(t + \alpha A) - \sigma E \)
\( \text{ABC}_0 D^\alpha_t I = \sigma \tau E - w_3 I \)
\( \text{ABC}_0 D^\alpha_t A = w_4 E - \gamma A \)
\( \text{ABC}_0 D^\alpha_t S_a = w_5 S(t + \alpha A) - \lambda S_a \)
\( \text{ABC}_0 D^\alpha_t E_a = w_6 S(t + \alpha A) - \delta S_a \)
\( \text{ABC}_0 D^\alpha_t H = \delta I + \delta_a E_a - w_7 H \)
\( \text{ABC}_0 D^\alpha_t R = \gamma I + \gamma A + \gamma H \)

with the initial conditions,

\[
S(0) = S_0, \quad E(0) = E_0, \quad I(0) = I_0, \quad A(0) = A_0, \quad S_a(0) = S_{a0}, \quad E_{a0}, \quad H_0, \quad R_0 \quad (t = 0) \]

Here,

\[
w_1 = c(\beta + a(1 - \beta)), \quad w_2 = c\beta(1 - a), \quad w_3 = \delta_I + b + \gamma, \quad w_4 = \sigma(1 - \tau), \quad w_5 = ca(1 - \beta), \quad w_6 = \beta ca, \quad w_7 = b + \gamma H.
\]

Here, \( a \) is the quarantine rate of the exposed population, \( b \) is the disease-induced rate of death, \( S_a \) is susceptible quarantined, \( E_0 \) is quarantined susceptible exposed population, \( \sigma \) is the probability of symptomatic infected. \( S_0, E_0, I_0, A_0, S_{a0}, E_{a0}, H_0, R_0 \) are constants. \( S, \ E, \ I, \ A, \ S_a, \ E_a, \ H, \ R \) are the differentiable functions of time \( t \). \( 0 < x \leq 1 \) is the order of \( AB \) derivative. The full model information is available in [74]. Amouch and Karim [75] computed the basic reproduction number of the COVID-19 model and studied the local stability of the disease-free equilibrium in terms of basic reproduction number.

The system given by Eq. (2) is equivalent to the following:

\[
S(t) - S_0(t) = \frac{1}{B(x)} \left\{ \begin{array}{c}
-w_1 S(t) \left( \frac{I(t)}{\theta A(t)} \right) + \lambda S_a(t) \\
+w_2 S(t) \left( \frac{I(t)}{\theta A(t)} \right) - \sigma E(t) \\
\end{array} \right\} d\alpha, \\
E(t) - E_0(t) = \frac{1}{B(x)} \left\{ \begin{array}{c}
w_2 S(t) \left( \frac{I(t)}{\theta A(t)} \right) - \sigma E(t) \\
\end{array} \right\} d\alpha, \\
I(t) - I_0(t) = \frac{1}{B(x)} \left\{ \begin{array}{c}
\sigma \tau E(t) \\
\tau \end{array} \right\} d\alpha, \\
A(t) - A_0(t) = \frac{1}{B(x)} \left\{ \begin{array}{c}
w_4 E(t) \left( -\gamma A(t) \right) \\
\gamma \end{array} \right\} d\alpha, \\
H(t) - H_0(t) = \frac{1}{B(x)} \left\{ \begin{array}{c}
\delta I(t) \\
\gamma \end{array} \right\} d\alpha, \\
R(t) - R_0(t) = \frac{1}{B(x)} \left\{ \begin{array}{c}
\gamma I(t) \\
\gamma \end{array} \right\} d\alpha,
\]

where \( B \) is normalization of function and, \( B(0) = B(1) = 1 \).

The above is iteratively written as:

\[
S_0(t) = S_0, \quad E_0(t) = E_0, \quad I_0(t) = I_0, \quad A_0(t) = A_0, \\
S_{a0}(t) = S_{a0}, \quad E_{a0}(t) = E_{a0}, \quad H_0(t) = H_0, \quad R_0(t) = R_0
\]
3.1. Existence and uniqueness analysis of solution

(Picard- Lindelof approach)

Let us define the operators below to show the proof of the existence of the solution as:

\[ f_1(t, S) = -w_1S(t + 0A) + \lambda S_a, \]
\[ f_2(t, E) = w_2S(t + 0A) - \sigma E, \]
\[ f_3(t, I) = \sigma E - w_3I, \]
\[ f_4(t, A) = w_4E - \gamma_A A, \]
\[ f_5(t, S_a) = w_5S(t + 0A) - \lambda S_a, A, \]
\[ f_6(t, E_a) = w_6S(t + 0A) - \delta_a S_a, \]
\[ f_7(t, H) = \delta_h I + \gamma_A A - w_h H, \]
\[ f_8(t, R) = \gamma_I + \gamma_A A + \gamma_H H. \]

It is obvious that \( f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8 \) are respectively the contraction for \( S, E, I, A, S_a, E_a, H, R. \)

Let,

\[ P_1 = \sup \| f_1(t, S) \|, \quad P_2 = \sup \| f_2(t, E) \|, \quad P_3 = \sup \| f_3(t, I) \|, \quad P_4 = \sup \| f_4(t, A) \|, \]
\[ P_5 = \sup \| f_5(t, S_a) \|, \quad P_6 = \sup \| f_6(t, E_a) \|, \quad P_7 = \sup \| f_7(t, H) \|, \quad P_8 = \sup \| f_8(t, R) \|, \]

where,

\[ W_{u,v_1} = [t - u, t + u] \times [S - v_1, S + v_1] = U \times V_1 \]
\[ W_{u,v_2} = [t - u, t + u] \times [E - v_2, E + v_2] = U \times V_2 \]
\[ W_{u,v_3} = [t - u, t + u] \times [I - v_3, I + v_3] = U \times V_3 \]
\[ W_{u,v_4} = [t - u, t + u] \times [A - v_4, A + v_4] = U \times V_4 \]
\[ W_{u,v_5} = [t - u, t + u] \times [S_a - v_5, S_a + v_5] = U \times V_5 \]
\[ W_{u,v_6} = [t - u, t + u] \times [E_a - v_6, E_a + v_6] = U \times V_6 \]
\[ W_{u,v_7} = [t - u, t + u] \times [H - v_7, H + v_7] = U \times V_7 \]
\[ W_{u,v_8} = [t - u, t + u] \times [R - v_8, R + v_8] = U \times V_8 \]

Banach fixed point theorem is applied to the above set using metric, implied by uniform norm,

\[ \| f(t) \|_\infty = \sup_{t \in [t, t+a]} \| f(t) \|_1 \] (10)

Picard’s operator amid both functional spaces of the continuous functions is:

\[ G : W(U, V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8) \rightarrow W(U, V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8) \]

It is defined as below:
\[ GX(t) = X_0(t) + F(t, X(t)) \frac{1 - \alpha}{B(\alpha)} + \frac{\alpha}{B(\alpha) \Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha - 1} F(\tau, X(\tau)) d\tau \]  

where \( X \) is the matrix of  

\[
X(t) = \begin{bmatrix}
S(t) \\ E(t) \\ I(t) \\ A(t) \\ S_0(t) \\ E_0(t) \\ H(t) \\ R(t)
\end{bmatrix}, 
X_0(t) = \begin{bmatrix}
S_0(t) \\ E_0(t) \\ I_0(t) \\ A_0(t) \\ S_{00}(t) \\ E_{00}(t) \\ H_0(t) \\ R_0(t)
\end{bmatrix}, 
F(t, X(t)) = \begin{bmatrix}
f_1(t, S) \\ f_2(t, E) \\ f_3(t, I) \\ f_4(t, A) \\ f_5(t, S_0) \\ f_6(t, E_0) \\ f_7(t, H) \\ f_8(t, R)
\end{bmatrix}
\]  

To get a good result, we suppose that the presented physical problem fulfills,  
\[
||X(t)||_\infty \leq \max \left \{ v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8 \right \}.
\]  

Now, \( ||GX(t) - X_0(t)|| \)  
\[
= ||F(t, X(t)) \frac{1 - \alpha}{B(\alpha)} + \frac{\alpha}{B(\alpha) \Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha - 1} F(\tau, X(\tau)) d\tau ||
\]  
\[
\leq \max \left \{ v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8 \right \}.
\]  

where \( P = \max \left \{ P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8 \right \} \).  

Also, we request that,  
\[
u < \frac{v}{P}.
\]  

In addition, we now evaluate the following:  
\[
||GX_1(t) - GX_2(t)||_\infty = \sup_{t \in U} |X_1(t) - X_2(t)|
\]  

Using Eq. (12), we have,  
\[
||GX_1(t) - GX_2(t)||
= ||\{ F(t, X_1(t)) - F(t, X_2(t)) \} \frac{1 - \alpha}{B(\alpha)} + \frac{\alpha}{B(\alpha) \Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha - 1} \{ F(\tau, X_1(\tau)) - F(\tau, X_2(\tau)) \} d\tau||
\]  
\[
\leq ||F(t, X_1(t)) - F(t, X_2(t))|| \frac{1 - \alpha}{B(\alpha)} + \frac{\alpha}{B(\alpha) \Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha - 1} ||F(\tau, X_1(\tau)) - F(\tau, X_2(\tau))|| d\tau
\]  
\[
\leq q \frac{1 - \alpha}{B(\alpha)} + \frac{\alpha u^2}{B(\alpha) \Gamma(\alpha)} q ||X_1(t) - X_2(t)||
\]  
\[
\leq q ||X_1(t) - X_2(t)||
\]  

As \( F \) is a contraction and with \( q < 1 \), we write,  
\[
u q < 1.
\]  

4. HPTM scheme for the time-dependent coronavirus model

Consider a time-dependent coronavirus model given by Eq. (2). To the analysis of the HPTM scheme, take a coupled system of FDE as:  
\[
aABC D^\alpha_a u(t) + R_1(u, v) + N_1(u, v) = f_1(t),
\]  
\[
aABC D^\alpha_a v(t) + R_2(u, v) + N_2(u, v) = f_2(t),
\]  

with initial values,  
\[
u(0) = u_0, v(0) = v_0.
\]  

Here, \( aABC D^\alpha_a u(t) \) and \( aABC D^\alpha_a v(t) \) are AB derivatives of the function \( u(t) \) and \( v(t) \) respectively in Liouville-Caputo sense. \( R_1 \) and \( R_2 \) are linear differential operators. \( N_1 \) and \( N_2 \) are nonlinear differential operators. \( f_1(t) \) and \( f_2(t) \) are source terms. Also, \( 0 < \alpha \leq 1 \).  

By using Laplace's transform on Eq. (18) and simplifying by definition 2.9, we find,
\[ u(t) = u_0 - L^{-1} \left[ \left( 1 - \alpha + \frac{s^\alpha}{\Gamma(\alpha)} \right) L[R_1(u,v) + N_1(u,v) - f_1(t)] \right], \]
\[ v(t) = v_0 - L^{-1} \left[ \left( 1 - \alpha + \frac{s^\alpha}{\Gamma(\alpha)} \right) L[R_2(u,v) + N_2(u,v) - f_2(t)] \right]. \]

(20)

By HPM, we write the results as,
\[ u(t) = \sum_{i=0}^{\infty} \rho^i u_i(t), \]
\[ v(t) = \sum_{i=0}^{\infty} \rho^i v_i(t), \]

(21)

Here, the homotopy parameter, \( \rho \in [0, 1] \).

Nonlinear terms are,
\[ N[u(t)] = \sum_{i=0}^{\infty} \rho^i H_i(u), \]
\[ N[v(t)] = \sum_{i=0}^{\infty} \rho^i H_i(v). \]

(22)

Here, \( H_i(u) \) and \( H_i(v) \) are He’s polynomial of \( u(t) \) and \( v(t) \). They are given by,
\[ H_i(u) = \frac{\partial^i}{\partial \rho^i} \left[ N \left( \sum_{k=0}^{\infty} \rho^k u_k \right) \right]_{\rho=0}, \quad i = 0, 1, 2, 3, \ldots, \]
\[ H_i(v) = \frac{\partial^i}{\partial \rho^i} \left[ N \left( \sum_{k=0}^{\infty} \rho^k v_k \right) \right]_{\rho=0}, \quad i = 0, 1, 2, 3, \ldots, \]

(23)

Using Eq. (20) and (21) and applying HPTM, Laplace transform and He’s polynomials, we get,
\[ \sum_{k=0}^{\infty} \rho^k u_k(t) = f_1(t) - pL^{-1} \left[ \left( 1 - \alpha + \frac{s^\alpha}{\Gamma(\alpha)} \right) L[R_1(u,v) + N_1(u,v)] \right], \]
\[ \sum_{k=0}^{\infty} \rho^k v_k(t) = f_2(t) - pL^{-1} \left[ \left( 1 - \alpha + \frac{s^\alpha}{\Gamma(\alpha)} \right) L[R_2(u,v) + N_2(u,v)] \right], \]

(24)

Here, \( f_1(t) \) and \( f_2(t) \) come from initial values and the source term.

Equating coefficients of powers of \( \rho \) in Eq. (24), we get,
\[ p^0 : u_0(t) = f_1(t), \]
\[ v_0(t) = f_2(t), \]
\[ p^k : u_k(t) = -L^{-1} \left[ \left( 1 - \alpha + \frac{s^\alpha}{\Gamma(\alpha)} \right) L[R_1(u_{k-1},v_{k-1}) + H_{k-1}(u,v)] \right], \]
\[ k > 0, k \in \mathbb{N} \]
\[ v_k(t) = -L^{-1} \left[ \left( 1 - \alpha + \frac{s^\alpha}{\Gamma(\alpha)} \right) L[R_2(u_{k-1},v_{k-1}) + H_{k-1}(u,v)] \right], \]
\[ k > 0, k \in \mathbb{N} \]

(25)

Therefore, the result can be obtained as a series solution,
\[ u(t) = \lim_{\rho \to 1} \sum_{i=0}^{\infty} \rho^i u_i(t) \]
\[ v(t) = \lim_{\rho \to 1} \sum_{i=0}^{\infty} \rho^i v_i(t). \]

(26)

4.1. Algorithm of HPTM

Step I. Apply the Laplace transform on each side of the given fractional-order Eq. (18).

Step II. Apply inverse Laplace transform on Eq. (19) to find \( u(t) \), \( v(t) \) using initial values.

Step III. Express the nonlinear terms, denoted by \( N \), in terms of He’s polynomials.

Step IV. Equate the coefficients of powers of \( \rho \) in Eq. (24).

Step V. Expressions for \( u_k(t) \) and \( v_k(t); k \geq 0 \) are found using Eq. (25).

Step VI. The solution of Eq. (18) by HPTM is obtained by using Eq. (26).

4.2. Convergence Analysis

**Theorem 1** [57] Suppose \( Q_i(t) \) and \( Q(t) \) are defined in Banach space \( \{C[0, 1], ||.||\} \). If there exists \( 0 < \delta < 1 \), such that,
\[ ||Q_{i+1}(t)|| \leq \delta ||Q_i(t)||, \forall i \in \mathbb{N}, \]

then the HPTM solution \( \sum_{i=0}^{\infty} Q_i(t) \) converges to \( Q(t) \) of the time-dependent model with ABC derivative.

**Theorem 2** [57] If there exists \( 0 < \delta < 1 \), in such a way that \( ||Q_{i+1}(t)|| \leq \delta ||Q_i(t)||, \forall i \in \mathbb{N}, \)

then the maximum absolute truncated error of HPTM solution (26) of the time-dependent model with ABC derivative is approximated as:
\[ \left| Q(t) - \sum_{i=0}^{j} Q_i(t) \right| \leq \frac{\delta^{j+1}}{1-\delta} ||Q_0(t)||. \]

4.3. Execution of HPTM

Now, we shall implement the HPTM to our model in Eq. (2) to illustrate its relevance and effectiveness.

Using Laplace’s transform on both flanks of Eq. (2) using definition 2.9, we get,
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\[ L(S) = \frac{S_0}{s} + \left(1 - \alpha + \frac{s}{s^2}\right)L\left(-w_1S(I + \theta A) + \lambda S_a\right), \]
\[ L(E) = \frac{E_0}{s} + \left(1 - \alpha + \frac{s}{s^2}\right)L\left(w_2S(I + \theta A) - \sigma E\right), \]
\[ L(I) = \frac{I_0}{s} + \left(1 - \alpha + \frac{s}{s^2}\right)L\left(\sigma \tau E - w_3I\right), \]
\[ L(A) = \frac{A_0}{s} + \left(1 - \alpha + \frac{s}{s^2}\right)L\left(w_4E - \gamma_A A\right), \]
\[ L(S_a) = \frac{S_{a0}}{s} + \left(1 - \alpha + \frac{s}{s^2}\right)L\left(w_5S(I + \theta A) - \lambda S_a\right), \]
\[ L(E_a) = \frac{E_{a0}}{s} + \left(1 - \alpha + \frac{s}{s^2}\right)L\left(w_6S(I + \theta A) - \alpha S_a\right), \]
\[ L(H) = \frac{H_0}{s} + \left(1 - \alpha + \frac{s}{s^2}\right)L\left(\delta I + \delta_a E_a - w_7H\right), \]
\[ L(R) = \frac{R_0}{s} + \left(1 - \alpha + \frac{s}{s^2}\right)L\left(\gamma I + \gamma_A A + \gamma_H H\right), \]

Using inverse transform of Laplace, we find,

\[
\begin{align*}
\sum_{0}^{\infty} p^n S_n(t) &= S_0 + pL^{-1}\left(1 - \alpha + \frac{s}{s^2}\right)L\left(-w_1\sum_{0}^{\infty} p^n H_n + \lambda \sum_{0}^{\infty} p^n S_a\right), \\
\sum_{0}^{\infty} p^n E_n(t) &= E_0 + pL^{-1}\left(1 - \alpha + \frac{s}{s^2}\right)L\left(w_2\sum_{0}^{\infty} p^n H_n - \sigma \sum_{0}^{\infty} p^n E_n\right), \\
\sum_{0}^{\infty} p^n I_n(t) &= I_0 + pL^{-1}\left(1 - \alpha + \frac{s}{s^2}\right)L\left(\sum_{0}^{\infty} p^n \left(\sigma \tau E_n - w_3I_n\right)\right), \\
\sum_{0}^{\infty} p^n A_n(t) &= A_0 + pL^{-1}\left(1 - \alpha + \frac{s}{s^2}\right)L\left(\sum_{0}^{\infty} p^n \left(w_4 E_n - \gamma_A A_n\right)\right), \\
\sum_{0}^{\infty} p^n S_{an}(t) &= S_{a0} + pL^{-1}\left(1 - \alpha + \frac{s}{s^2}\right)L\left(w_5\sum_{0}^{\infty} p^n H_n - \lambda \sum_{0}^{\infty} p^n S_{an}\right), \\
\sum_{0}^{\infty} p^n E_{an}(t) &= E_{a0} + pL^{-1}\left(1 - \alpha + \frac{s}{s^2}\right)L\left(w_6\sum_{0}^{\infty} p^n H_n - \alpha \sum_{0}^{\infty} p^n E_{an}\right), \\
\sum_{0}^{\infty} p^n H_n(t) &= H_0 + pL^{-1}\left(1 - \alpha + \frac{s}{s^2}\right)L\left(\sum_{0}^{\infty} p^n \left(\delta I_n + \delta_a E_{an} - w_7H_n\right)\right), \\
\sum_{0}^{\infty} p^n R_n(t) &= R_0 + pL^{-1}\left(1 - \alpha + \frac{s}{s^2}\right)L\left(\sum_{0}^{\infty} p^n \left(\gamma I_n + \gamma_A A_n + \gamma_H H_n\right)\right).
\end{align*}
\]
where $\sum_{0}^{\infty} p^n H_n = S_0(I_n + \theta A_n)$.

Using $H_0 = S_0(I_0 + \theta A_0)$, and equating coefficients of powers of $p$ in Eq. (29), we get,

\[ p^0 : S_0(t) = S_0, E_0(t) = E_0, I_0(t) = I_0, A_0(t) = A_0 \]

\[ p^1 : S_1(t) = \{-S_0(ac(1-\beta) + c\beta)(I_0 + A_0\theta) + S_{a0}\lambda\} \times \left(1 - \alpha + \frac{r^2\alpha}{\Gamma[1+2\alpha]}\right) \]

\[ E_1(t) = \{(1-a)cS_0\beta(I_0 + A_0\theta) - E_0\sigma\}\left(1 - \alpha + \frac{r^2\alpha}{\Gamma[1+2\alpha]}\right) \]

\[ I_1(t) = -I_0(b + \gamma_I + \delta_I) + E_0\sigma \left(1 - \alpha + \frac{r^2\alpha}{\Gamma[1+2\alpha]}\right) \]

\[ A_1(t) = \{-A_0\gamma_A + E_0\sigma(1-\tau)\} \left(1 - \alpha + \frac{r^2\alpha}{\Gamma[1+2\alpha]}\right) \]

\[ S_{a1}(t) = (acS_0(1-\beta)(I_0 + A_0\theta) - S_{a0}\lambda) \left(1 - \alpha + \frac{r^2\alpha}{\Gamma[1+2\alpha]}\right) \]

\[ E_{a1}(t) = \{-E_0\delta_a + acS_0(1-\beta)(I_0 + A_0\theta)\} \left(1 - \alpha + \frac{r^2\alpha}{\Gamma[1+2\alpha]}\right) \]

\[ H_1(t) = \{-H_0(b + \gamma_H) + E_0\delta_a + I_0\delta_I\} \left(1 - \alpha + \frac{r^2\alpha}{\Gamma[1+2\alpha]}\right) \]

\[ R_1(t) = \{A_0\gamma_A + H_0\gamma_H + I_0\gamma_I\} \left(1 - \alpha + \frac{r^2\alpha}{\Gamma[1+2\alpha]}\right) \]

\[ p^2 : S_2(t) = \lambda(acS_0(1-\beta)(I_0 + A_0\theta) - S_{a0}\lambda) \times \left\{\left(1 - \alpha\right)^2 + \frac{2r^2(1-\alpha)x\alpha}{\Gamma[1+2\alpha]} + \frac{r^2x^2\alpha}{\Gamma[1+2\alpha]}\right\} \]

\[ E_2(t) = -\sigma\{(1-a)cS_0\beta(I_0 + A_0\theta) - E_0\sigma\} \times \left\{\left(1 - \alpha\right)^2 + \frac{2r^2(1-\alpha)x\alpha}{\Gamma[1+2\alpha]} + \frac{r^2x^2\alpha}{\Gamma[1+2\alpha]}\right\} \]

\[ I_2(t) = \left[\begin{array}{c}
(b^2I_0 - \sigma\{(a-1)A_0\gamma_A + E_0\gamma_I + \delta_I + \sigma\})r + \frac{2I_0(\gamma_I + \delta_I)}{-E_0\sigma}\gamma_I^2 + 2I_0\delta_I + \delta_I^2
\end{array}\right] \times \left\{\left(1 - \alpha\right)^2 + \frac{2r^2(1-\alpha)x\alpha}{\Gamma[1+2\alpha]} + \frac{r^2x^2\alpha}{\Gamma[1+2\alpha]}\right\} \]

\[ A_2(t) = \left[\begin{array}{c}
A_0\gamma_A^2 + (a-1)cS_0\beta\sigma(-1 + \tau)
\end{array}\right] \times \left\{\left(1 - \alpha\right)^2 + \frac{2r^2(1-\alpha)x\alpha}{\Gamma[1+2\alpha]} + \frac{r^2x^2\alpha}{\Gamma[1+2\alpha]}\right\} \]

\[ S_{a2}(t) = \{-S_0\beta\gamma_a + E_0\sigma\gamma_a\} \times \left\{\left(1 - \alpha\right)^2 + \frac{2r^2(1-\alpha)x\alpha}{\Gamma[1+2\alpha]} + \frac{r^2x^2\alpha}{\Gamma[1+2\alpha]}\right\} \]

\[ E_{a2}(t) = \{-E_0\delta_a + acS_0(1-\beta)(I_0 + A_0\theta)\} \times \left\{\left(1 - \alpha\right)^2 + \frac{2r^2(1-\alpha)x\alpha}{\Gamma[1+2\alpha]} + \frac{r^2x^2\alpha}{\Gamma[1+2\alpha]}\right\} \]

\[ H_2(t) = \{b^2H_0 + H_0\gamma_H + acS_0\beta\gamma_a + E_0\gamma_a\gamma_H\} \times \left\{\left(1 - \alpha\right)^2 + \frac{2r^2(1-\alpha)x\alpha}{\Gamma[1+2\alpha]} + \frac{r^2x^2\alpha}{\Gamma[1+2\alpha]}\right\} \]

\[ R_2(t) = -\left\{\left(1 + \alpha\right)^2 + \frac{2r^2(1-\alpha)x\alpha}{\Gamma[1+2\alpha]} + \frac{r^2x^2\alpha}{\Gamma[1+2\alpha]}\right\} \]

and so on. Like this, the rest iterates of the HPTM solution can be found using Mathematica software package. Hence, the solution is,

\[ S(t) = \lim_{p \to 0} \frac{\sum_{n=0}^{\infty} p^n S_n(t)}{p^n S_n(t)}, \quad S_a(t) = \lim_{p \to 0} \frac{\sum_{n=0}^{\infty} p^n S_a(t)}{p^n S_a(t)} \]

\[ E(t) = \lim_{p \to 0} \frac{\sum_{n=0}^{\infty} p^n E_n(t)}{p^n E_n(t)}, \quad E_a(t) = \lim_{p \to 0} \frac{\sum_{n=0}^{\infty} p^n E_a(t)}{p^n E_a(t)} \]

\[ I(t) = \lim_{p \to 0} \frac{\sum_{n=0}^{\infty} p^n I_n(t)}{p^n I_n(t)}, \quad H(t) = \lim_{p \to 0} \frac{\sum_{n=0}^{\infty} p^n H_n(t)}{p^n H_n(t)} \]

\[ A(t) = \lim_{p \to 0} \frac{\sum_{n=0}^{\infty} p^n A_n(t)}{p^n A_n(t)}, \quad R(t) = \lim_{p \to 0} \frac{\sum_{n=0}^{\infty} p^n R_n(t)}{p^n R_n(t)} \]

5. Numerical results and discussion

Numerical results are obtained for $S(t), E(t), I(t), A(t), S_a(t), E_a(t), H(t)$ and $R(t)$ at distinct values of fractional order $\alpha$ in AB derivative in the Liouville–Caputo sense. A solution of Eq. (2) is found by using the HPTM. Figure 1 shows the behavior of the susceptible population with time for $0.7 \leq \alpha \leq 1$. In Fig. 1, susceptibles increase with time which is an obvious result. Figures 2 depict the behavior of the susceptible population for
different probabilities of transmission per contact at $\alpha = 0.5, 1$ respectively. Figures 2 show that as the probability of transmission per contact increases, the susceptibles increase. The susceptible population increases rapidly at $\alpha = 1$ as compared to $\alpha = 0.5$. Figure 3 displays the behavior of the exposed population with time for $0.7 \leq \alpha \leq 1$. In Fig. 3, the exposed population increases with time. Figures 4 show the behavior of the exposed population at different rates of contact. The exposed population rises with the rise in contact rate. Increment in the exposed population is less at $\alpha = 0.5$ as compared to $\alpha = 1$. Figure 5 explains the behavior of the symptomatic infected population with the time for $0.7 \leq \alpha \leq 1$. In Fig. 5, the symptomatic infected population increases as time increase with the increase in the value of $\alpha$. Figures 6 depict the behavior of the symptomatic infected population to the recovery rate of the infected at $\alpha = 0.5, 1$. In Fig. 6, the symptomatic infected population reduces as the recovery rate of infected individuals increases. These figures show
that the number of the infected population is less for $\alpha = 0.5$ in comparison to $\alpha = 1$. Figure 7 displays the behavior of the asymptomatic infected population with time for $0.7 \leq \alpha \leq 1$. In Fig. 7, the asymptomatic infected population rise with time. Figure 8 explains the behavior of asymptomatic infected population to the recovery rate of the asymptomatic infected population. In Fig. 8, the asymptomatic infected population decreases with a rise in the recovery rate of the asymptomatic infected population. Figure 9 shows the behavior of the quarantined susceptible isolated population. In Fig. 9, this population increases with time for $0.7 \leq \alpha \leq 1$. Figs. 10 depict the behavior of quarantined susceptible isolated population with the change in the quarantined rate of the exposed people. In Fig. 10, the quarantined susceptible isolated population increases as the quarantined rate of exposed people increases. Figure 11 shows the behavior of the quarantined susceptible exposed population with the time for
0.7 \leq \alpha \leq 1. In Fig. 11, this population increases with time. Figure 12 explains the behavior of the quarantined susceptible exposed population to the quarantined rate of exposed individuals. In Fig. 12, the quarantined susceptible exposed population increases as the quarantined rate of exposed individuals increases. Figure 13 displays the effect of the hospitalized population with time for 0.7 \leq \alpha \leq 1. In Fig. 13, the number of hospitalized ones increases with time. Figures 14 show the behavior of hospitalized population to the transition rate of symptomatic infected to the quarantined infected people. In Fig. 14, the hospitalized people reduce as the transmission rate of symptomatic infected to the quarantined infected people increases. Figure 15 shows the behavior of the recovered population with the time for 0.7 \leq \alpha \leq 1. In Fig. 15, this population increases with time. Figures 16 shows the behavior of the recovered population to the recovery rate of quarantined
infected people. In Fig. 16, the recovered population increases as the recovery rate of quarantined infected increases.

The estimates of parameters [8] are displayed in Table 1a and initial values [8] are given in Table 1b. The absolute errors between subsequent iterations for the distinct population at order \( \alpha \) for different values of time \( t \) are shown in Tables 2, 3, 4, 5, 6, 7, 8, 9, 10. We observe that the error is negligible, which confirms that the solution is convergent, positive, and bounded.

Novel Coronavirus is a grave threat to the health of mankind. The coronavirus communicated from animals to humans. The diseases carried from animals to humans are a substantial threat to human health as humans do not have existing antibodies to defend themselves against the disease. So, the research work is making firm progress to combat such threats and the analysis of the presented model (2) is apt. Scientists are keen on predicting future
Table 1 Estimates of parameters for COVID-19 in Wuhan, China [4]

| S. no | Parameter | Estimated mean value | S. no | Parameter | Estimated mean value |
|-------|-----------|----------------------|-------|-----------|----------------------|
| 1     | $\beta$   | $2.1011 \times 10^{-8}$ | 7     | $b$       | $1.7826 \times 10^{-5}$ |
| 2     | $c$       | 14.781               | 8     | $\delta_t$ | 0.13266               |
| 3     | $a$       | $1.8887 \times 10^{-7}$ | 9     | $\delta_s$ | 0.1259                |
| 4     | $\sigma$  | 0.142857             | 10    | $\gamma_t$ | 0.33029               |
| 5     | $\lambda$ | 0.7142857            | 11    | $\gamma_A$ | 0.13978               |
| 6     | $\tau$    | 0.86834              | 12    | $\gamma_H$ | 0.11624               |

Table 2 Assessment of Initial values for COVID-19 in Wuhan, China [4]

| S. No | Initial value of Parameters | Estimated Mean Value |
|-------|-----------------------------|----------------------|
| 1     | $S_0$                       | 10893000             |
| 2     | $E_0$                       | 16000                |
| 3     | $I_0$                       | 2000                 |
| 4     | $A_0$                       | 1000                 |
| 5     | $S_{00}$                    | 167000               |
| 6     | $E_{00}$                    | 0                    |
| 7     | $H_0$                       | 1000                 |
| 8     | $R_0$                       | 2000                 |

trends and solutions to such disease models using efficient and reliable schemes. The presented model reveals an innovative feature of order $\alpha$ of AB derivative by HPTM, which was missing in [4].

6. Conclusions

In this analysis, a new time-dependent model of coronavirus disease with eight variables is established and then analyzed for its possible solution using a reliable approach. The homotopy perturbation technique via the Laplace transform is applied to get the solution of this arbitrary order model. The effects of order $0.7 \leq \alpha \leq 1$ of AB

Table 3 A comparison study of error between sequential iterations for $S(t)$ at order $\alpha = 0.5, 1$ for different values of $t$

| $|S_2 - S_1|$ | $|S_3 - S_2|$ | $|S_4 - S_3|$ |
|-----------|----------------|----------------|
| $x = 0.5$ | $x = 0.5$       | $x = 0.5$      |
| $x = 1$  | $x = 1$         | $x = 1$        |
| 0        | 1.0921166E+7    | 2.905038E+4    |
| 0.20     | 1.0935380E+7    | 4.43328E+4     |
| 0.40     | 1.0941267E+7    | 5.07665E+4     |
| 0.60     | 1.0945785E+7    | 5.57444E+4     |
| 0.80     | 1.0949594E+7    | 5.996879E+4    |
| 1.0      | 1.0952949E+7    | 6.371153E+4    |

Table 4 A comparison study of error between sequential iterations for $E(t)$ at $\alpha = 0.5, 1$ for different $t$

| $|E_1 - E_0|$ | $|E_2 - E_1|$ | $|E_3 - E_2|$ |
|-------------|---------------|---------------|
| $x = 0.5$   | $x = 0.5$     | $x = 0.5$     |
| $x = 1$    | $x = 1$       | $x = 1$       |
| 0          | 1.372606E+5   | 2.436365E+4   |
| 0.20       | 1.257857E+5   | 3.780269E+4   |
| 0.40       | 1.210326E+5   | 4.355961E+4   |
| 0.60       | 1.173855E+5   | 4.805263E+4   |
| 0.80       | 1.143108E+5   | 5.189141E+4   |
| 1.0        | 1.116019E+4   | 5.531212E+3   |
Table 5 A comparison study of error between sequential iterations for $I(t)$ at $x = 0.5, 1$ for different $t$

| $t$  | $|I_1 - I_0|$ | $|I_2 - I_1|$ | $|I_3 - I_2|$ |
|------|---------------|---------------|---------------|
|      | $x = 0.5$     | $x = 0.5$     | $x = 0.5$     |
|      | $x = 1$       | $x = 1$       | $x = 1$       |
| 0    | 1.470579E+4   | 2.000E+3      | 5.109336E+3   |
| 0.20 | 1.20342E+4    | 0.178232E+3   | 7.557376E+3   |
| 0.40 | 1.092758E+4   | 0.1576463E+3  | 8.549729E+3   |
| 0.60 | 1.007845E+4   | 0.1364695E+3  | 9.302585E+3   |
| 0.80 | 9.362598E+3   | 0.1152927E+3  | 9.231468E+3   |
| 1.0  | 8.731919E+2   | 9.411585E+3   | 8.701126E+2   |

Table 6 A comparison study of error between succeeding iterations for $A(t)$ at $x = 0.5, 1$ for different $t$

| $t$  | $|A_1 - A_0|$ | $|A_2 - A_1|$ | $|A_3 - A_2|$ |
|------|---------------|---------------|---------------|
|      | $x = 0.5$     | $x = 0.5$     | $x = 0.5$     |
|      | $x = 1$       | $x = 1$       | $x = 1$       |
| 0    | 9.194214E+3   | 1.000E+3      | 6.482542E+2   |
| 0.20 | 8.787593E+3   | 9.677686E+3   | 8.643795E+2   |
| 0.40 | 8.619166E+3   | 9.355371E+2   | 9.354456E+2   |
| 0.60 | 8.489926E+3   | 9.033057E+2   | 8.926485E+2   |
| 0.80 | 8.380973E+3   | 8.710743E+2   | 1.017379E+3   |
| 1.0  | 8.284892E+2   | 8.388429E+2   | 1.044444E+2   |

Table 7 A comparison study of error between consecutive iterations for $S_n(t)$ at $x = 0.5, 1$ for different $t$

| $t$  | $|S_{n1} - S_{n0}|$ | $|S_{n2} - S_{n1}|$ | $|S_{n3} - S_{n2}|$ |
|------|---------------------|---------------------|---------------------|
|      | $x = 0.5$           | $x = 0.5$           | $x = 0.5$           |
|      | $x = 1$             | $x = 1$             | $x = 1$             |
| 0    | 1.422503E+5        | 1.67000E+5          | 2.563358E+4        |
| 0.20 | 1.29761E+5         | 1.571001E+5        | 3.91918E+4         |
| 0.40 | 1.245877E+5        | 1.472003E+5        | 4.491335E+4        |
| 0.60 | 1.206182E+5        | 1.373004E+5        | 4.934125E+4        |
| 0.80 | 1.172717E+5        | 1.274005E+5        | 5.310358E+4        |
| 1.0  | 1.143233E+5        | 1.175007E+5        | 5.643929E+4        |

Table 8 A comparison study of error between succeeding iterations for $E_n(t)$ at $x = 0.5, 1$ for different $t$

| $t$  | $|E_{n1} - E_{n0}|$ | $|E_{n2} - E_{n1}|$ | $|E_{n3} - E_{n2}|$ |
|------|---------------------|---------------------|---------------------|
|      | $x = 0.5$           | $x = 0.5$           | $x = 0.5$           |
|      | $x = 1$             | $x = 1$             | $x = 1$             |
| 0    | 6.86E - 4           | 6.45E - 4           | 4.32E - 5           |
| 0.20 | 1.061E - 3          | 0.261E - 4          | 9.71E - 4           |
| 0.40 | 1.221E - 3          | 0.529E - 4          | 1.106E - 3          |
| 0.60 | 1.345E - 3          | 0.804E - 4          | 1.209E - 3          |
| 0.80 | 1.452E - 3          | 0.1085E - 3         | 1.297E - 3          |
| 1.0  | 1.546E - 3          | 0.1372E - 3         | 1.374E - 3          |
derivative in Liouville-Caputo sense on the solution of this model are displayed through figures and tables. The effect of chance $\beta$ of transmission per contact ($1.1011 \times 10^{-8} \leq \beta \leq 3.1011 \times 10^{-8}$) to the susceptible population, rate of contact ($13.781 \leq c \leq 15.781$) to the exposed population, a recovery rate of infected individuals ($0.32029 \leq \gamma_t \leq 0.34029$) to symptomatic infected population, a recovery rate of asymptomatic infected ($0.12978 \leq \gamma_a \leq 0.14978$) to the symptomatic infected, the quarantined rate of exposed ($0.8887 \times 10^{-7} \leq a \leq 2.8887 \times 10^{-7}$) to the quarantined susceptible isolated and exposed population, transition rate of symptomatic infected to the quarantined infected ($-0.88376 \leq \delta_t \leq 0.11624$) to the hospitalized and recovery rate of quarantined infected ($\gamma_H = 0.01624, 0.11624, 0.21624$) to the recovered population are also discussed in detail through figures. The results unveil that the fractional-order $\alpha$ is vital against the transmission of this disease. They also specify that the HPTM needs fewer iterations to get reliable results. It is recognized that the HPTM is capable of lessening the calculation size. From the tables, we find that absolute error between the consecutive iterations reduces admirably as the iterations increase for diverse $\alpha$. It represents that the solution to this model is positive, bounded, and convergent. However, it is also interesting to note that this scheme works efficiently even when the exact solution for the corresponding non-fractional model is unknown. It is predicted that a mathematical model focused on the outcomes of its vaccination may be a future topic of research. To curb the menace of waves of coronavirus in India, early identification, effective medication and vaccination are mandatory for all human beings along with applying all safety measures and mandatory precautions. However, memory effects are inducted in the presented model and we got an efficient and reliable solution by the HPTM.

**Table 9** A comparison study of error between consecutive iterations for $H(t)$ at $\alpha = 0.8, 1$ for different $t$

| $t$  | $|H_1 - H_0|$ | $H_2 - H_1|$ | $H_3 - H_2|$ |
|------|---------------|---------------|---------------|
|      | $\alpha = 0.8$ | $\alpha = 0.8$ | $\alpha = 0.8$ |
| 0.10 | 9.701876E+2   | 1.000E+3      | 2.488699E+1   |
| 0.20 | 9.498954E+2   | 9.850938E+2   | 3.708936E+1   |
| 0.30 | 9.348568E+2   | 7.01876E+2    | 4.434588E+1   |
| 0.40 | 9.213194E+2   | 5.528136E+2   | 4.957722E+1   |
| 0.50 | 9.254689E+2   | 9.254689E+2   | 5.594074E+1   |

**Table 10** A comparison study of error between succeeding iterations for $R(t)$ at $\alpha = 0.8, 1$ for different $t$

| $t$  | $|R_1 - R_0|$ | $R_2 - R_1|$ | $R_3 - R_2|$ |
|------|---------------|---------------|---------------|
|      | $\alpha = 0.8$ | $\alpha = 0.8$ | $\alpha = 0.8$ |
| 0    | 1.5417E+3     | 2.000E+3      | 3.69054E+2    |
| 0.20 | 1.310438E+3   | 1.816689E+3   | 4.744011E+2   |
| 0.40 | 1.214634E+3   | 1.63336E+3    | 5.100022E+2   |
| 0.60 | 1.141128E+3   | 0.145004E+3   | 5.327874E+2   |
| 0.80 | 1.079159E+3   | 0.126672E+3   | 5.489389E+2   |
| 1.0  | 1.024564E+3   | 0.10834E+3    | 5.60851E+2    |

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**Declarations**

**Conflict of interest** The authors have no relevant financial or non-financial interests to disclose.

**Availability of data and material** Data sharing does not apply to this article as no datasets were generated or analyzed during the current study.

**Code Availability** Not applicable.
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