On the inner dynamics between Fossil fuels and the carbon market: a combination of seasonal-trend decomposition and multifractal cross-correlation analysis

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Received: 27 May 2022 / Accepted: 27 October 2022 / Published online: 9 November 2022
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Abstract
This study examines the inner dynamics of multifractality between the carbon market (EU ETS) and four major fossil fuel energy markets: Brent Crude Oil (BRN), Richards Bay Coal (RBC), UK Natural Gas (NGH2), and FTSE350 electricity index (FTSE350) from January 04, 2016, to March 04, 2022. First, we decompose the daily price changes by applying seasonal and trend decomposition using loess (STL). Then, we examine the inner dynamics of multifractality and cross-correlation by employing multifractal detrended fluctuation analysis (MFDFA) and multifractal detrended cross-correlation analysis (MFDCCA) using the remaining components of the return series. Our findings reveal that all series and the cross-correlations of carbon and fossil fuels markets have multifractal characteristics. We find crude oil to be the most efficient market (lowest multifractal), while coal is the least efficient (highest multifractal). Only coal shows persistent, whereas the other markets exhibit antipersistent behavior. Interestingly, the coal and EU ETS pair demonstrates a higher degree of multifractal patterns. In contrast, the pair of natural gas and EU ETS exhibits the lowest multifractal characteristics among the energy markets. Only the crude oil market shows persistent cross-correlations in the multifractality. These findings have important academic and managerial implications for investors and policymakers.

Keywords Carbon market · Fossil fuel energy markets · Multifractality · Decomposition · Multifractal detrended fluctuation analysis (MFDFA) · Multifractal detrended cross-correlation analysis (MFDCCA)

Introduction
Environmental issues such as energy depletion, rising sea levels, and global warming pose a serious long-term threat to establishing a globally sustainable planet. To contain global warming under 2 °C, carbon emissions must be reduced by 25% by 2030 and to zero by 2070 (Tan et al. 2022). Since 1920, carbon emissions from fossil fuels (coal, oil, and gas) have become the primary source of anthropogenic emissions (Le Quéré et al. 2018). In addition, international conflicts such as the Russia-Ukraine war risks greater carbon pollution. It is already clear that military operations and frequent bombardments have a negative impact on air quality. Massive deforestation and even wildfires have been brought on by military operations. Such conflicts may directly hinder local and international efforts to tackle issues like climate

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change, sustainable development goals (SDGs), and pollution controls. As a result, a worldwide consensus has emerged to reduce reliance on fossil fuels while increasing nonfossil energy production and consumption (Davidson 2019; Shi et al. 2018; X. Shi et al. 2020a, b; Zhou et al. 2021). Over 130 countries, regions, and international organizations, including IPCC, UNFCC, and APPCDC, have set carbon neutrality objectives to promote the economic green transition (Tang et al. 2015). Numerous countries have adopted or are planning to establish policies aimed at energy saving, increasing energy efficiency, moving to greener fuels, and electrification (Tan et al. 2022). The carbon emission trading system (ETS) is one of the most cost-effective market-based financial instrument tools available to achieve the goal.

Since the signing of the Kyoto Protocol, numerous countries and regions have introduced ETS as a key policy tool, notably the United States Regional Greenhouse Gas Initiative (RGGI), the Chinese ETS pilots, and the European Union Emissions Trading System (EU ETS) (Guo et al. 2020). Over 96 countries recognize ETS as a critical tool for meeting their Paris Agreement carbon reduction targets (Liu and Zhang 2021). Unlike other mitigation techniques, ETS is based on the cap-and-trade principle, requiring governments to set a cap on a region’s carbon emissions and divide the cap into allowances and permits. Firms subject to carbon emissions may obtain their permits and allowances from the concerned government institutions or may trade emission permits with other businesses (Dai et al. 2018; Pollitt 2019).

The European Emissions Trading Scheme (EU ETS) is the world’s largest and first international carbon trading mandated scheme, functioning since 2005. It regulates around half of Europe’s total greenhouse gas (GHG) emissions and encompasses over 11,000 plants and industrial sites in 31 countries (Mandaroux et al. 2021). The mechanism governs the allocation of allowances on an annual basis. The system controls the yearly allotment of allowances. The European Union has pledged to cut GHG emissions in phases progressively, the fourth (2021–2030) of which is presently underway. The 1st phase (trial phase, 2005–2007) served as a “warm-up” phase before the Kyoto Protocol’s full adoption. The 2nd phase lasted from 2008 to 2012 and worked in tandem with the Kyoto Protocol, while the 3rd phase (2013–2020) was intended to cut carbon emissions by 20% by the end of 2020 (Dhamija et al. 2018). Interestingly, the nitrous oxide emissions from acid production, the PFC emissions from the aluminum sector, and the ammonium, ammonia, and petrochemical industries were all included in the 3rd phase. The emissions from the aviation industry (all European zone flights) were also included in 2012 (Ciesielska-Maciągowska et al. 2021).

The relationship between carbon emissions and fossil fuel usage has significant economic and environmental implications. Since fossil fuels primarily cause carbon emissions, greenhouse gas policies, regulations, and carbon prices are projected to raise the burning costs of fossil fuels. Businesses are encouraged to reduce their reliance on fossil fuels and increase their investment in renewable and low-carbon clean energy sources (Ji et al. 2018). For example, industrial sectors need an in-depth understanding of carbon and energy price to balance their fossil fuel consumption structures and decrease carbon emissions. Given that the EU is the world’s largest marketplace and energy user, changes in energy demand in non-European markets also cause price adjustments, complicating the pricing structure.

On the one hand, reduced fossil fuel prices result in greater consumption, which increases demand for carbon allowances or prices, encourages firms to develop clean energy technology, lowers the cost of renewable energy, and influences the cost of other energy sources. Furthermore, some research indicates that the connections are mostly the result of the electricity market’s association with the carbon and other fossil fuel markets (Boersen and Scholtens 2014; Keppler and Mansanet-Bataller 2010). The electricity industry, as the major member of the EU ETS, accounted for over 75% of all European Union allowances (Hintzmann 2010). Additionally, the growth in contemporary financial instruments and the trend of economic globalization are bringing diverse financial and non-financial assets together. On the other hand, information shocks may influence markets or potentially spread instability (Wang and Guo 2018).

A large body of empirical and theoretical research has shown the complicated linkages between fossil fuels and carbon prices (Xu et al. 2020). The factors contributing to these complications include energy demand and supply, weather conditions, institutional policies, and macroeconomic performance (Chen et al. 2022; Hammoudeh et al. 2014b), which impact greenhouse gas emissions and global warming conditions. Hence, a sound understanding of the complications in the cross-correlation of carbon and fossil fuels market structures is critical for the global environmental and economic growth reservations (Balcilar et al. 2016; Chen et al. 2022). The existing quantitative research on the relationship of fossil fuels with the carbon market uses conventional methodologies, i.e., the DCC-GARCH model (Zhang and Sun 2016), the quantile regression method (Hammoudeh et al. 2014a), the Markov-switching VAR model (Chevallier 2011), and the cointegrated vector autoregressive (CVAR) model (Fell 2010). However, such econometric models have flaws; for example, when assessing the link between fossil fuel and carbon price, such models use linear and stationary data assumptions. The market prices contain dynamic,
investigate sequence memory, also known as anti-persistence. The mono-fractal methods employed to detect complex fractal features in nonstationary time series. There are two types of fractal methods: mono-fractal and multifractal. The mono-fractal methods investigate sequence memory, also known as anti-persistence (negative autocorrelation) and persistence (positive autocorrelation) behavior. The rescaled range analysis (R/S) (Hurst 1951; Mandelbrot and Wallis 1969) and the detrended fluctuation analysis (DFA) (Peng et al. 1994) used to be the classic methods for studying mono-fractal features. These approaches are extensively used in financial and nonfinancial time series analysis (Gvozdanovic et al. 2012; Wang and Liu 2010; Wang et al. 2011). Scholars later discovered that financial time series’ complexity and multi-scale behavior challenge the single fractal models. According to Mandelbrot (1997), multifractal may better quantify the dynamical features of financial markets than single-fractal and has a broader use in empirical research. Hence, Kantelhardt et al. (2002) extended the DFA model to the multifractal detrended fluctuation analysis (MFDFA) to characterize multifractal features in the financial markets.

Furthermore, single-fractal and multifractal only explain the fractal properties of a single time series and cannot correlate with two-time series. Podobnik and Stanley (2008) addressed this issue by including the detrended fluctuation analysis in correlation analysis and presented a detrended cross-correlation analysis (DCCA) that quantifies the long memory characteristics of two nonlinear time series. By combining DCCA with multifractality, Zhou (2008) proposed the MFDCCA to examine the cross-correlation in the multifractality of two-time series with the same observations.

In light of the complexity inherent in carbon and fossil fuels energy markets, this study employs advanced econophysics-based techniques to explore the inner dynamics of multifractality in three distinct ways. We use the prices of EU ETS and four major fossil fuel energy markets: Brent Crude Oil (BRN), Richards Bay Coal (RBC), UK Natural Gas (NGH2), and FTSE350 electricity (FTSE350). First, we decompose the time series into trend, seasonal, and residual components, applying a versatile and robust filtering technique known as seasonal-trend decomposition using loess (STL). The STL approach allows us to uncover the core dynamics of asset returns while increasing data dependency. The patterns of multifractal behavior are then analyzed using the robust MF DFA, an extension of DFA. The MF DFA approach measures market efficiency using various scaling exponents in each financial time series. Finally, we use MF DCCA to reveal the complexity and multifractality of cross-correlations between the EU ETS and key fossil fuel energy markets. The inner dynamics of multifractality better explain the long memory, persistency, predictability, informational efficiency, and enigmatic behavior in the cross-correlation between EU ETS and fossil fuel energy markets. Furthermore, the findings provide implications for carbon and fossil fuel market regulators to make robust and strategic policy decisions and initiate necessary changes in energy consumption, market development, and greenhouse gas reduction.

Light review

Since its inception in 2005, the EU ETS has received a great deal of attention in the literature. The EU ETS contains time-series data that allows for capturing the relationship between the carbon market and other financial markets. Initial studies such as Christiansen et al. (2005) indicate that fossil fuel costs, fuel switching between gas and coal, weather, and policy and regulation influence carbon prices. For example, Mansanet-Bataller et al. (2007) show that carbon price is linked to the price of natural gas and oil, but not coal.

Zhang and Wei (2010) find strong dynamic cointegration between carbon and fossil fuels energy prices. However, the patterns of interdependence alter with time, albeit not significantly during periods of stability but noticeably during times of turmoil and crisis (Marimoutou & Soury 2015). Additionally, the impact of oil and coal prices on the carbon market is significantly positive during the low and high volatility regimes (Lutz et al. 2013). Chung et al. (2018) report that natural gas prices are causally affected by carbon pricing, but overall, fossil fuel energy markets are positively associated with the carbon market. The fossil fuel market drives the carbon price on long and medium timescales (Zhu et al. 2019).

The carbon market is intertwined with the crude oil market. Based on the MFDCCA approach, Zhuang et al. (2014) find cross-correlations in the multifractality between carbon and oil markets, where small fluctuations have more persistent behavior than large fluctuations. Moreover, these markets have also been proven inefficient, with carbon price volatility higher than crude oil price volatility. The oil market, in comparison to other energy markets, has the greatest impact on carbon prices (Ji et al. 2018; Zhang and Wei 2010). Furthermore, the interconnectedness in the volatility time series of carbon and oil markets is found to be higher than the return time series. Hammoudah et al. (2015) employ the nonlinear autoregressive distributed lag (NARDL) model and shows that oil prices have an asymmetric and negative long-run effect on allowance pricing. However, Cao and Xu (2016) show that the causality between crude oil and the carbon market is inadequate since crude oil prices are affected mainly by abnormal climates and major geopolitical events.
Concerning natural gas and coal, Creti et al. (2012) look into the factors that influence carbon prices throughout the first two phases (2005–2012) of the EU ETS. They discover that the switching price between coal and natural gas is significant. Furthermore, the volatility of the coal-to-carbon and carbon-to-natural gas markets has a considerable unidirectional spillover. Furthermore, in extreme cases, the volatility of the natural gas market exacerbates the overall volatility of the carbon market (Wang and Guo 2018). In contrast to other fossil fuel markets, Zhang and Sun (2016) found that coal prices are highly associated with the carbon market. Alberola et al. (2008) also report a negative impact of coal but a positive impact of natural gas prices on the carbon market.

The linkages between carbon and power markets have also piqued academic interest. Researchers are examining the relationship between electricity and carbon prices (Freitas and da Silva 2015; Hammoudeh et al. 2014b; Koop and Tole 2013). For example, Huisman and Kiliç (2015) and Castagneto-Gissey (2014) examined the carbon price pass-through rates in the power industry. Boersen and Scholtens (2014) studied the power markets of the UK, Spain, France, Northern Europe, and Germany and discovered a positive correlation between carbon markets and electricity prices. Nazifi and Milunovich (2010) examined the Nordic power market and discovered no Granger causation extending from carbon to electricity markets. However, Kepller and Mansanet-Bataller (2010) discovered that from 2005 to 2007, the carbon market drove the electricity market and vice versa in 2008. Aatola et al. (2013) studied the power markets of Northern Europe and Germany and discovered that carbon prices unidirectionally drove the electricity prices. Castagneto-Gissey (2014) revealed that carbon and power prices in Northern Europe and Germany have biddirectional causality. Cao and Xu (2016) also examined the UK market and reported mutual linear Granger causalities between the power and carbon markets.

The abovementioned literature on the carbon market and its interaction with energy markets lead to the conclusion that, as compared to other energy markets, the carbon and coal markets have a very strong and dynamic relationship. Natural gas prices have a positive impact on carbon prices, while the volatility of natural gas intensifies the overall volatility of the carbon market in extreme cases. However, unusual weather patterns and significant geopolitical tensions have a greater impact on crude oil prices than carbon prices. Furthermore, the results highlight the existence of bidirectional and mutual causalities between electricity and carbon markets. However, given the complexity of the carbon market, the literature on the carbon market and its relationship with other markets in the multifractal setting is scarce, which can give a wider and more robust perspective to these relationships. The conventional econometric methodologies that are frequently employed in literature on carbon prices mostly examine the connection between these markets in a linear perspective. Multifractality is central to the science of complexity and it finds applications in almost all essential areas of scientific activity, including physics (Muzy et al. 2008; Subramaniam et al. 2008), chemistry (Stanley and Meakin 1988, Udovichenko and Strizhak 2002), biology (Makowiec et al. 2009; Rosas et al. 2002), hydrology (Telesca et al. 2005), environment (Farjah 2019; Sipra et al. 2021), linguistics (Drożdż et al. 2016), Physiology (Nagy et al. 2017), psychology (Kelty-Stephen 2017; Stephen et al. 2012), behavioral sciences (Ihlen and Vereijken 2013), agriculture (Kim et al. 2011, McKenzie and Holt 2002), economics (Drożdż et al. 2010), finance (Delbianco et al. 2016, Zou and Zhang 2020), and even music (Jafari et al. 2007). Additionally, using conventional linear analysis techniques to study the financial markets is not reliable and robust. The presence of long memory in complex time series also prevents them from meeting requirements of the classical correlation coefficient-based techniques. On other hand, it is possible to understand the intrinsic complicated structure of the fractals by computing the multifractal spectrum. Furthermore, the combination of STL decomposition with MFDFA and MFDDCA to reveal the inner dynamics on the linkage between carbon market and fossil fuel energy markets makes this study different from the existing literature.

Data and methodology

Data

This study examines the daily prices of the European Union’s Emissions Trading System (EUETS) and four major fossil fuel markets: Brent Crude Oil (BRN), Richards Bay Coal (RBC), UK Natural Gas (NGH2), and FTSE350 electricity index (FTSE350). The EUETS is the world’s first and largest carbon emission trading market. According to Refinitiv’s worldwide carbon trading volume and price assessment report, the EUETS’s carbon trading volume has reached around 169 billion euros, accounting for roughly 87% of the worldwide carbon market. The daily closing prices are collected from DataStream, from January 04, 2016, to March 04, 2022.

Daily fluctuations in the carbon and fossil fuel markets were calculated for additional investigation. The daily logarithmic returns series were generated as follows:

\[ r_{t,j} = \ln \left( \frac{p_{t,j}}{p_{t-1,j}} \right) \]

where \( r_{t,j} \) is the daily return of each index \( j \) and \( p_{t,j} \) represents each index \( j \) at time \( \text{day}_t \).
The market names, symbols, and summary statistics are documented in Table 1.

We summarize the descriptive statistics in Table 1 and present the daily price fluctuation in Fig. 1 for all selected series. EUETS shows a growth pattern throughout our sample period, whereas a significant downside tendency was observed in March 2020, during the recent outbreak of the COVID-19 pandemic and the price war between Russia and Ukraine, all of the series, particularly the NGH2, hit new highs. The December 2021 high seems to have been driven by cold weather and a supply deficit in Russia, but the current high (March 2022) looks to have been caused by Russian-Ukrainian tensions and imposed sanctions on Russia. In terms of mean values, the average return of all series is positive. RBC exhibits the highest average return (0.14%), while FTSE350 shows the lowest average return (0.01%). Additionally, RBC shows the highest return (33%) and maximum loss (48%).

Except for NGH2, all sample series have negative skewness. The most volatile index is NGH2, followed by EUETS, RBC, and BRN, while the least volatile index is FTSE350 (see Fig. 2).

### Seasonal-trend decomposition using loess (STL)

Before using MFDFA and MFDCCA, we begin with a flexible and powerful filtering technique, the seasonal-trend decomposition with loess (STL). Cleveland et al. (1990) proposed STL, which is resistant to structural breaks and outliers and identifies nonlinear correlations to uncover the intrinsic dynamics of time series. Besides, because STL separates the remaining components by omitting intrinsically unpredictable trends and seasonal components due to trade cycles, it can show interactional dynamics (Laib et al. 2018a, b). Some recent studies (Aslam et al. 2020a, 2020b; Miloș et al. 2020) have used STL before MFDFA and MFDCCA. This method outperforms other classical methods such as TRAMO/SEATS or X11, due to its robustness with time-varying flexibility and outliers.

Let $Y_i$ as a time series variable decomposed into three components: trend $T_i$, seasonal $S_i$, and remainder $R_i$ from $v = 1$ to $N$. According to (Laib et al. 2018b; Miloș et al. 2020), these components can be mathematically denoted as:

$$Y_i = T_i + S_i + R_i \quad (2)$$

Cleveland et al. (1990) claim that STL uses locally weighted scatterplot loess smoother to decompose the time series. Assume that $x_i$ and $y_i$ measure dependent and independent variables from $i = 1$ to $n$. The loess curve denoted by $g^{(x)}(y)$ smooths the dependent variable $y_i$ given $x$ and can be calculated for any value along the scale of that independent variable $x$. We calculate $g^{(x)}$ in a following way. To begin with, we choose $q, (q \leq n)$ as a positive integer. The neighborhood weight $v_q(x)$ is given to values of $q$ of $x_i$ as per the distance between $x_i$ and $x$. The tricube function $W$ is represented as:

$$W(u) = \begin{cases} 
(1 - u^3)^3, & \text{for } 0 \leq u < 1 \\
0, & \text{for } u \geq 1 
\end{cases} \quad (3)$$

The $v_q(x)$ for any $x_i$ is:

$$v_q(x) = W\left(\frac{|x_i - x|}{\lambda_q(x)}\right) \quad (4)$$

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**Table 1** List of carbon and energy markets

| Symbol          | European Union allowance | Brent Crude Oil | Richards Bay Coal | UK Natural Gas | Electricity |
|-----------------|--------------------------|----------------|-------------------|----------------|-------------|
| Mean            | 0.0013                   | 0.0007         | 0.0014            | 0.0005         | 0.0001      |
| Standard deviation | 0.0304              | 0.0261         | 0.0264            | 0.0329         | 0.0157      |
| Kurtosis        | 3.6150                   | 20.9803        | 101.2568          | 3.9565         | 10.6176     |
| Skewness        | -0.4633                  | -1.2339        | -1.7706           | 0.1909         | -0.6153     |
| Range           | 0.3225                   | 0.4705         | 0.8055            | 0.3785         | 0.2424      |
| Minimum         | -0.1944                  | -0.2798        | -0.4803           | -0.1805        | -0.1222     |
| Maximum         | 0.1281                   | 0.1908         | 0.3251            | 0.1980         | 0.1202      |
| Count**         | 1590                     | 1591           | 1545              | 1590           | 1558        |

*Data ranges from January 4, 2016, to March 4, 2022

**For MFDCCA analysis, EU ETS number of observations depends on energy indices to make them equal

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2. [https://www.iea.org/reports/coal-2021/executive-summary](https://www.iea.org/reports/coal-2021/executive-summary)

3. The residuals (irregular or error) from the seasonal plus trend fit.
where the distance between $x_i$ and $x$ is denoted by $\lambda_q(x)$.

It is clear from the above equation that the values of $x_i$ away from $x$ contain small neighborhood weights $v_i(x)$ and, eventually, reach zero at $q_{th}$ farthest point. Then we compute the fitted value $g^{(k)}$ at $x_i$ with neighborhood weight $v_i(x) \text{ at}(x_i, y_i)$. $g^{(k)}$ appears smoother as the value of $q$ increases, while the weights approach to 1 as a result of $q$ tends to $\infty$.

The STL decomposition technique comprises two procedures: an inner and an outer loop. In the inner loop, the trend is fitted, and the seasonal component is calculated. The following are the sequential specifics of $k_{th}$ values:
The first step is called **detrending**, where we subtract the trending values of \( T^{(k)}_v \) from \( Y_v \) in order to construct a new detrended series as \( Y_v - T^{(k)}_v \). If there are missing values of \( Y_v \), the detrended series \( Y_v - T^{(k)}_v \) remains missing.

In the second step, named **Cycle-subseries Smoothing**, we regress each cycle-subseries by loess produced in the first step and temporarily record the resulting smoothed values as \( C^{(k+1)}_v \).

The third step, called **low-pass filtering of smoothed cycle subseries**, includes three sub-steps. The first step is a moving average of length \( n_p \), with \( n \) as the number of samples. The following step is also a moving average of length 3. Then, the loess is applied to the results of low-pass filtering, with the results as \( L^{(k+1)}_v \).

The fourth step is named as **detrending smoothed cycle subseries**, where the seasonal series are generated from \((K + 1)st\) loop \( S^{(k+1)}_v = C^{(k+1)}_v - L^{(k+1)}_v \) for \( v = 1toN \).

In the fifth step, we compute the deseasonalized series as \( Y_v - S^{(k+1)}_v \), while in the sixth step, the trend series \( T^{(k+1)}_v \) are generated when applied loess to \( Y_v - S^{(k+1)}_v \).

The following are the steps of the outer loop:

After the inner loop, we obtain the values of seasonal \( S_v \) and trend \( T_v \) components, while the remainder component \( R_v \) are computed as:

\[
R_v = Y_v - T_v - S_v
\]  

The \( R_v \) cannot be computed if there are missing values, while the neighborhood weight \( v_i(x) \) can only be defined in the case of \( Y_v \). The second loop, as per Brock et al. (1991), consists of interactive updates of \( S_v \) and \( T_v \) components without taking into account the current trend estimations. This technique ultimately divides series into cycle subseries, with 252 cycle subseries (average number of trading days per year).
Then we define the robustness of weight $\rho$ to examine the robustness and reliability of the residual component $R_v$ with $\rho_v$:

$$h = \text{median}(|R_v|)$$  \hspace{1cm} (6)

$$\rho_v = \frac{B(|R_v|)}{h}$$  \hspace{1cm} (7)

where $B$ denotes the bi-square weight and is estimated as:

$$B(u) = \begin{cases} (1 - u^2)^2, & \text{for } 0 \leq u < 1 \\ 0, & \text{for } u \geq 1 \end{cases}$$  \hspace{1cm} (8)

We kept the loess window, equivalent to the number of daily trading prices per month for the EU ETS and fossil fuel energy prices (21 days).$^4$

**Multifractal detrended analysis methods**

The majority of financial and energy markets literature has primarily concentrated on the assumption that the market prices follow Bachelier (1900)'s random walk hypothesis (RWH) and, hence, have a normal distribution. In this context, a variety of traditional, as well as fractal econometric methodologies, have been used to examine the efficiency of these markets. The usage of variance ratios, parametric autocorrelation, nonparametric tools, unit root, filter rules, and the GARCH models are among the most widely used models. However, according to the modeling of market microstructure, time is a crucial source of information and, hence, should be considered in the modeling. Engle and Russell (1998), in this background, created the autoregressive conditional duration (ACD) model, which expressly focuses on the modeling of times between events and relies on linear parameterization. The GARCH and ACD models have a lot of similarities, as shown by (Engle 2000; Engle and Russell 1998). On the other hand, nonlinear models claim that these linear models overlook the nonlinear dependence (Engle and Russell 1998). According to (Mandelbrot 1967), the market structures exhibits a number of universal characteristics, including fat tails (Gopikrishnan et al. 2001), volatility clustering (Kim and Eom 2008), long-term correlations (Alvarez-Ramirez et al. 2008), chaos (Adrangi et al. 2001), and fractals or multifractals (Ali et al. 2021; Aslam et al. 2021a, b). Multifractal-based methodologies offer a powerful tool for explaining these complexities, as well as the stylized facts, like long range dependence, scaling patterns, and self-similarity, which are standard theories underlying informational efficiency, are unable to reconcile.

The scaling exponents provide a clearer description of the multifractality in real-world time series data. Several nonlinearly interacting factors make up the complex systems of physical quantities, such as biological, ecological, technological, financial, as well as social, and they all have been shown to exhibit long-range correlations. Recent research aimed to shed light on the internal dynamics of these cross-correlations that occur in numerous simultaneously recorded time series (Aslam et al. 2022; Podobnik et al. 2009; Y. Shi et al. 2020a, b; Wątorek et al. 2019; Xiong et al. 2018; Zhao and Cui 2021).

Since the development of multifractal detrended cross-correlation analysis (MF-DCCA, or called MF-DXA) by Zhou (2008) reveal the multifractal features of two cross-correlated signals, DCCA and MF-DCCA have been widely discussed and used (Aslam et al. 2022; Jafari et al. 2007; Jiang and Zhou 2011; Kristoufek 2011; Zou and Zhang 2020).

**Multifractal detrended fluctuation analysis (MF DFA)**

After decomposition by STL, we employ MF DFA in a five-step procedure to look into multifractal characteristics. We assume $\{z_t, t = 1, \ldots, N\}$ as nonstationary time series, where $N$ denotes the number/length of observations.

To begin with, we construct the profile as $X_{(k)}$ by:

$$X_{(k)} = \sum_{t=1}^{k} (z_t - \bar{z})$$  \hspace{1cm} (9)

where, $k = 1, 2, 3, \ldots, N$, and the average of the overall time series is represented by $\bar{z}$.

The profile $X_{(k)}$ is then segmented into non-overlapping windows $N_s = \text{int}\frac{N}{s}$ of equal length $s$. Most of the time, $N$ is not the multiple of taken time scale $s$; hence, the short components at the end of profile could stay. To ensure that this end component is not overlooked, the same method is followed from the opposite end. Therefore, $2N_s$ components are estimated in total. This process $10 < s < \frac{N}{s}$ was introduced by Peng et al. (1994).

In the third step, we estimate the variance by calculating the local trend for every segment of $2N_s$ by a least-square fit of sampled time series:

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$^4$ The R package “stats” is used for STL decomposition. For further details, visit [https://www.rdocumentation.org/packages/stats/versions/3.6.2/topics/stl](https://www.rdocumentation.org/packages/stats/versions/3.6.2/topics/stl)
For each segment, $m = 1, \ldots, N_s$, and.

\begin{equation}
F^2(s, m) = \frac{1}{s} \sum_{j=1}^{s} \{ X[(m - 1)s + j] - x_m(j) \}^2
\end{equation}

(10) For $m = N_t + 1, \ldots, 2N_s$, where $x_m(j)$ is the polynomial fit in segments $m$. 

\begin{equation}
F^2(s, m) = \frac{1}{s} \sum_{j=1}^{s} \{ X[N - (m - N_s)s + j] - x_m(j) \}^2
\end{equation}

(11)
We obtain the $q^{th}$ order fluctuation functions by averaging all the segments from the second step.

$$F_{q(x)} = \left\{ \frac{1}{2N} \sum_{s=1}^{2N} |F^2(s,\rho)| \right\}^{1/q}$$ (12)

For any real value if $q \neq 0$ and, if $q = 0$:

$$F_{0(x)} = \exp \left\{ \frac{1}{4N} \sum_{s=1}^{2N} \ln[F^2(s,\rho)] \right\}$$ (13)

The $q$ parameter enables the differentiation of segments with small and large-scale fluctuations. Positive values of $q$ are responsible for large fluctuations, while negative values of $q$ indicate small fluctuations. However, $q = 2$ for detrended fluctuation analysis. It is important to note that $F_{q(x)}$ is the increasing function of $s$.

Lastly, we calculate the relationship of $F_{q(x)}$ with $s$, where the scaling exponent of the fluctuation function is fixed for any $q$. In the case of $F_{q(x)}$ as a power law, the time series are in the log–log scale for that particular $q$:

$$F_{q(x)} \sim S^{h(q)}$$ (14)

Here, we call $h(q)$ the generalized Hurst exponent, whereas $h(2)$ is called the Hurst exponent for stationary time series. Several researchers have employed the Hurst exponent (see, e.g., (Ali et al. 2021; Aslam et al. 2021a, b) in explaining the phenomenon of market efficiency (Fama 1970). The interpretation of $h(q)$ is concise and simple; for random walk behavior, the $h(q) = 0.5$, while for persistent $h(q) > 0.5$ and anti-persistent behavior $h(q) < 0.5$.

The Renyi exponent $\tau(q)$ is also one way to present the results of the Hurst exponent $h(q)$:

$$\tau(q) = qh(q) - 1$$ (15)

The multiple spectrum defined by $f(\alpha)$ shows if the time series is mono-fractal or multifractal:

$$f(\alpha) = q\alpha - \tau(q)$$ (16)

where $\alpha$ is the Holder exponent:

$$\alpha = h_q + q \frac{yh_q}{\gamma_q} - \tau_q$$ (17)

The higher variations in the values of the Hurst exponent $h(q)$ indicate the presence of stronger multifractal patterns. We measure the strength of multifractality through the width of the generalized Hurst exponent $\Delta h$ as:

![Fig. 4 The MFDSA findings of EUETS and four major fossil fuel energy indices. The fluctuation functions for $q = -10$, $q = 0$, and $q = 10$ are displayed at top. The middle shows the generalized Hurst exponent for each $q$. The bottom shows the multifractal spectrum.](image-url)
Table 2 Generalized Hurst exponents ranging from $Q=−10$ to $Q=10$

| Order | EU ETS | BRN | RBC | NGH2 | FTSE 350 |
|-------|--------|-----|-----|------|----------|
| −10   | 0.7563 | 0.7808 | 0.9487 | 0.7483 | 0.6516 |
| −9    | 0.7457 | 0.7741 | 0.9378 | 0.7395 | 0.6437 |
| −8    | 0.7332 | 0.7665 | 0.9244 | 0.729 | 0.6344 |
| −7    | 0.7183 | 0.7577 | 0.9077 | 0.7164 | 0.6234 |
| −6    | 0.7005 | 0.7476 | 0.8867 | 0.701 | 0.6104 |
| −5    | 0.6792 | 0.7361 | 0.8599 | 0.6819 | 0.5954 |
| −4    | 0.6541 | 0.7232 | 0.8259 | 0.658 | 0.5787 |
| −3    | 0.6249 | 0.7091 | 0.7844 | 0.6278 | 0.5619 |
| −2    | 0.5919 | 0.6944 | 0.7377 | 0.5898 | 0.5468 |
| −1    | 0.5558 | 0.6796 | 0.6897 | 0.5436 | 0.5332 |
| 0     | 0.5176 | 0.6627 | 0.6364 | 0.4915 | 0.5157 |
| 1     | 0.4783 | 0.6337 | 0.5635 | 0.4384 | 0.4854 |
| 2     | 0.4396 | 0.5788 | 0.4672 | 0.3887 | 0.4365 |
| 3     | 0.4034 | 0.5113 | 0.3747 | 0.3439 | 0.3766 |
| 4     | 0.3712 | 0.4555 | 0.3053 | 0.3049 | 0.3212 |
| 5     | 0.344 | 0.4156 | 0.2566 | 0.272 | 0.278 |
| 6     | 0.3216 | 0.3872 | 0.2219 | 0.2452 | 0.246 |
| 7     | 0.3034 | 0.3663 | 0.1964 | 0.2237 | 0.2223 |
| 8     | 0.2885 | 0.3502 | 0.177 | 0.2064 | 0.2042 |
| 9     | 0.2762 | 0.3376 | 0.1618 | 0.1923 | 0.1902 |
| 10    | 0.2661 | 0.3273 | 0.1495 | 0.1808 | 0.1789 |
| Δh   | 0.4902 | 0.4535 | 0.7992 | 0.5675 | 0.4727 |

$$\Delta h = h_{(q_{\text{min}})} - h_{(q_{\text{max}})}$$ \hfill (18)

### Multifractal detrended cross-correlation analysis (MF DCCA)

Zhou (2008) offered the multifractal detrended cross-correlation (MF DCCA) to cope with two nonlinear and non-stationary time series' multifractality by integrating the MF DFD A and DCCA. When we use the STL and the MF DCCA to examine the cross-correlation in the multifractality of carbon and fossil fuel markets makes, our study is more in-depth, detailed, and robust.

MF DCCA follows the same five steps as MF DFD A:

1. We take two STL remainder components \{\{X_i\}\} and \{\{Y_i\}\} time series of equal length, where $N$ denotes the number of observations and then construct the profile as follows:

$$X_i(t) = \sum_{t=1}^{i} (x_t - \bar{x}), \quad Y_i(t) = \sum_{t=1}^{i} (y_t - \bar{y}), \quad i = 1, 2, 3 \cdots, N$$ \hfill (19)

where:

2. The profile $\{\{X_i(t)\}\}$ and $\{\{Y_i(t)\}\}$ is then segmented into non-overlapping windows $N_s = \text{int}\frac{N}{s}$ of equal length $s$. However, the number of observations $N$ is not always a multiple of the time scale $s$, and hence the end part of the profile is omitted. This part of the series must not be overlooked; the same technique was performed but from the opposite side of $X_s(k)$ resulting in the segments 2 $N_s$.

3. This process is the same as done in MF DFA (Kantelhardt et al. 2002).

4. Then using ordinary least squares, we estimate the local trend $X_s(t)$ and $Y_s(t)$ of each segment, for each $v = 1, 2, \ldots, 2N_s$, from which the variance is calculated:

$$F^2(s, v) = \frac{1}{s} \sum_{i=1}^{s} |X[(v-1)s+i] - X_s(i)| \cdot |Y[(v-1)s+i] - Y_s(i)|$$ \hfill (20)

for each segment $v = 1, 2, \ldots, N_s$ and

$$F^2(s, ν) = \frac{1}{s} \sum_{i=1}^{s} |X[(v-s-N_s)s+i] - X_s(i)| \cdot |Y[(v-s-N_s)s+i] - Y_s(i)|$$ \hfill (21)

for $v = N_s, \ldots, 2N_s$.

5. We obtain the $q^{th}$ order fluctuation functions by averaging all the segments from the second step.

$$F_q(s, ν) = \left\{ \frac{1}{2N_s} \sum_{s=1}^{2N_s} F^2(s, ν) \right\}^{q/2}$$ \hfill (22)

for any $q \neq 0$, while for $q = 0$, it is given by

$$F_0(s) = \exp \left\{ \frac{1}{4N_s} \sum_{s=1}^{2N_s} \ln[F^2(s, ν)] \right\}$$ \hfill (23)

The $q$ parameter enables the differentiation of segments with small and large-scale fluctuations. The positive values of $q$ are responsible for large fluctuations, while negative values of $q$ indicate small fluctuations. However, $q = 2$ for the standard DCCA procedure. It is important to note that $F_q(s)$ is the increasing function of $s$.

Lastly, we calculate the relationship of $F_q(s)$ with $s$, where the scaling exponent of the fluctuation function is fixed for any $q$. In the case of $F_q(s)$ as a power law, the time series are in the log–log scale for that particular $q$.

$$F_q(s) \sim s^{\Delta H_q(q)}$$ \hfill (24)

The $\Delta H_q(q)$ scaling exponent in the above equation indicates the correlation in power-law between two taken time series, revealing how quickly the $F_q(s)$ of local fluctuations rise with an increase in scale $s$. In the case of two identical
time series \( y_1 \) and \( y_2 \), the MFDCCA relationship becomes a special case. The scaling exponents \( H_{xy}(q) \) and \( H_{xy}(2) \) have similar features as the MFDFA (Kristoufek 2011), where if there is no cross-correlation, \( h_{xy}(2) = 0.5 \), while for persistent cross-correlations, \( h_{xy}(2) > 0.5 \), and in the case of anti-persistent cross-correlations, \( h_{xy}(2) < 0.5 \).

We quantify the strength of multifractal characteristics by \( \Delta H \), which is suggested by Yuan et al. (2009) and can be mathematically expressed as:

\[
\Delta H = H_{\text{max}}(q) - H_{\text{min}}(q)
\]

(25)

We can determine the degree of multifractality of the various cross-correlations by looking at the values of \( H_{xy}(q) \). The Legendre transform yields the following:

\[
\alpha = H(q) + q H'_{xy}(q)
\]

(26)

Hence, \( f(\alpha) \) is the singularity spectrum and can be estimated as:

\[
f(\alpha) = q(\alpha - H_{xy}(q)) + 1
\]

(27)

Moreover, the \( \Delta \alpha \) is calculated as the width of the multifractal spectrum to examine the level of multifractality:

\[
\Delta \alpha = \alpha_{\text{max}} - \alpha_{\text{min}}
\]

(28)

For MFDFA and MFDCCA analysis, the R package, namely “MFDF DA” developed by (Laib et al. 2018a, b), was used.

**Empirical findings**

We decompose the data of EUETS and four major fossil fuel energy markets, namely BRN, RBC, NGH2, and FTSE350, using STL to separate their components from any occasional oscillations. The STL decomposition of carbon returns (EUETS) is shown in Fig. 2 in four graphs. The first graph presents the original returns data, while the second indicates the seasonal component, the third shows the trend

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5 The detailed documentation is available at https://www.rdocumentation.org/packages/MFDFA/versions/1.1/topics/MFDFA).
component, and the fourth is the remainder component. Figure 3 shows similar STL decomposition results for the four different energy markets. Except for the FTSE350, all of the series’ trend components were considerably reduced after the COVID-19 outbreak. On the other hand, the remaining components do not follow any particular pattern. The rest of the components of the sample series is then run through MFDFA and MFDCCA to look into how multifractality and cross-correlation work in the carbon and energy markets.

MFDFA results

This section presents the empirical findings of the MFDFA for the EUETS and four major fossil fuel energy markets. Each sample series is presented in Fig. 4 plots. The first/top row of plots indicates the log–log plots of the fluctuation function $F_q(s)$ vs. time scale $s$, while the second/middle row shows the slopes of the generalized Hurst exponent, and the third/bottom row shows the multifractal spectrum $f(a)$. In the log–log plots for the fluctuation functions and scales, the fitted lines are visible and picked up for $q = [-10, 0, 10]$. The fitted lines related to the generalized Hurst exponent (GHE) $h(q)$ in the second plots depend on $q$, which confirms the presence of multifractality in all series. Lastly, the plots of multifractal spectrum $f(a)$ also confirm the presence of multifractal structures, with a single-hump.

The results of GHE $h(q)$ for $q = [-10, 0, 10]$ are shown in Table 2, where negative and positive values of $q$ signify the small and large price fluctuations, respectively. All the values of $h(q)$ fluctuate with $q$ from $-10$ to $10$, indicating multifractality in all of the sample series. For instance, the results for EUETS show that $h(q)$ reaches a high of 0.76 at $q = -10$ and then drops to 0.52 at $q = 0$ and eventually to 0.27 at $q = 10$. The sampled energy markets also show similar results.

All of the series are multifractal, but the strength or the degree of multifractality varies. We measure the strength of multifractality through the range or width of $h(q)$, i.e., $\Delta h$ over the range $q \in [-10, 10]$. Higher values of $\Delta h$ indicate a stronger degree of multifractality and lower levels of market efficiency (see Table 2). The highest range of GHE is noted for RBC ($\Delta h = 0.80$), followed by NGH2 ($\Delta h = 0.57$), EUETS ($\Delta h = 0.49$), and FTSE350 ($\Delta h = 0.47$). BRN, on the other hand, has the lowest level of multifractality, with a
Δh of 0.45. Hence, the findings reveal that RBC is the least efficient, while BRN is the most efficient sample series.

Lastly, we calculate the classical Hurst exponent at q = 2, for the considered time series. The findings show that h(2) for BRN is 0.58, indicating that this series exhibits persistent and positive autocorrelation. This implies that a negative or positive return in a given period is most likely to be followed by another negative or positive return in the next period. On the contrary, the rest of the sample series; i.e., EUETS, RBC, NGH2, and FTSE350 show h(2) of 0.44, 0.47, 0.39, and 0.44, respectively, showing anti-persistent and negative autocorrelation, indicating that a negative or positive return in a given period is more likely to revert in the following period.

**MFDCCA results**

The section discusses the findings of MFDCCA for the remaining components of the pairs of carbon and fossil fuel energy markets. Similar to the previously discussed MF DFA, we calculate the fluctuation function \( F_{xy}(S) \) from the scaling order of \(-10t_0 + 10\). The log–log trend of \( F_{xy}(S) \) for \( q = -10(\text{black}), q = 0(\text{lightgrey}), \) and \( q = 10(\text{darkgrey}) \) is presented in Fig. 5 for the pairs of EUETS with the remaining components of energy returns, which vary with time length s. The log–log connection displays a well-shaped form within these four pairs, growing linearly with the scale s, implying a power-law correlation.

The generalized Hurst exponent \( H_{xy}(q) \) for EUETS and fossil fuel returns with \( q \) from \(-10 \) to \(+10 \) is shown in Fig. 6, which is nonlinear and exhibits a decline as the scale increases. The EUETS/BRN, for instance, has the highest value of \( H_{xy}(q) \) at \( q = -10 \), declined to 0.60 at \( q = 0 \), and finally dropped to 0.33 at \( q = 10 \) (see Table 3). This downward trend supports multifractal patterns between the variables studied. Other pairs of EUETS with sampled energy markets show similar patterns.

Table 3 shows the range of GHE (ΔH), which is significantly larger than zero, implying stronger multifractal behavior in the cross-correlations of EUETS and fossil fuel energy markets. However, multifractality’s strength in the cross-correlations varies across different energy markets. A higher degree of multifractal patterns is found for the pair EUETS/BRN (ΔH = 0.54), followed by EUETS/FTSE350 (ΔH = 0.47) and EUETS/BRN (ΔH = 0.38). In comparison, EUETS/NGH2 (ΔH = 0.36) exhibits the lowest multifractal characteristics among the energy markets. Figure 7 shows the width of multiple spectrums and corroborates our results.

These wider widths of multiple spectrums show greater fluctuations in the patterns of the various types of fluctuation distributions. These heterogeneous and varied patterns emerge as a result of the fluctuating nature of the examined fossil fuel energy markets. The wider widths of multifractal spectrums indicate that the random walk process does not generate them. The presence of multifractality in the cross-correlations of EUETS and energy markets shows that the behavior of variables is consistent with the adaptive market hypothesis (AMH), proposed by Kristoufek and Vosvrda (2013) and is consistent with prior empirical findings (Ferreira 2019; Hasan and Salim 2017; Ruan et al. 2016).

Lastly, we report \( H_{xy}(q = 2) \) in Table 3, which validates the persistent behavior of the cross-correlations in the multifractality of the examined pairs. The \( H_{xy}(q = 2) \) is higher than 0.5 only for EUETS/BRN, indicating that EUETS and BRN exhibit persistent cross-correlations, while \( H_{xy}(q = 2) \) for EUETS/RBC and EUETS/NGH2 is less than 0.5, indicating anti-persistent cross-correlation. On the contrary, in case of the EUETS/FTSE350, \( H_{xy}(q = 2) \) is around 0.5, revealing weak evidence of cross-correlation between EUETS and FTSE350. As indicated by (Kristoufek 2011), an \( H_{xy}(q = 2) \) greater than 0.5 suggests a cross-persistent series, where a positive or negative value of \( \Delta_{xy} \) is more likely to have another next positive or negative value of \( \Delta_{xy} \). Hence, a long-range cross-correlation implies that both time series have long memory characteristics in their own lags (Podobnik and Stanley 2008; Yuan et al. 2012). Furthermore, cross-correlation shows that a rise (or fall) in one variable is more

| Order | EUETS/BRN | EUETS/RBC | EUETS/NGH2 | EUETS/FTSE350 |
|------|----------|----------|-----------|---------------|
| -10  | 0.7096   | 0.7286   | 0.7022    | 0.6580        |
| -9   | 0.7027   | 0.7198   | 0.6918    | 0.6504        |
| -8   | 0.6949   | 0.7095   | 0.6796    | 0.6414        |
| -7   | 0.6862   | 0.6976   | 0.6651    | 0.6309        |
| -6   | 0.6766   | 0.6837   | 0.6480    | 0.6186        |
| -5   | 0.6663   | 0.6678   | 0.6282    | 0.6046        |
| -4   | 0.6554   | 0.6501   | 0.6055    | 0.5890        |
| -3   | 0.6441   | 0.6316   | 0.5806    | 0.5725        |
| -2   | 0.6324   | 0.6137   | 0.5540    | 0.5562        |
| -1   | 0.6195   | 0.5977   | 0.5266    | 0.5413        |
| 0    | 0.5982   | 0.5710   | 0.4987    | 0.5271        |
| 1    | 0.5795   | 0.5490   | 0.4732    | 0.5147        |
| 2    | 0.5440   | 0.4814   | 0.4491    | 0.4965        |
| 3    | 0.4991   | 0.3993   | 0.4276    | 0.4704        |
| 4    | 0.4548   | 0.3336   | 0.4087    | 0.4395        |
| 5    | 0.4183   | 0.2872   | 0.3922    | 0.4100        |
| 6    | 0.3904   | 0.2543   | 0.3780    | 0.3852        |
| 7    | 0.3691   | 0.2300   | 0.3658    | 0.3651        |
| 8    | 0.3528   | 0.2115   | 0.3552    | 0.3489        |
| 9    | 0.3399   | 0.1970   | 0.3462    | 0.3359        |
| 10   | 0.3295   | 0.1853   | 0.3383    | 0.3253        |
| Δh   | 0.3801   | 0.5433   | 0.3639    | 0.4719        |
likely to be followed by a rise (or fall) in another variable (Podobnik and Stanley 2008).

According to the abovementioned results, BRN appears to be cross-persistent with EUETS, implying that a rise (or fall) in returns of EUETS is more likely to be followed by a rise (or fall) in BRN’s returns. On the other hand, changes in returns of EUETS are likely to be followed by changes in returns of RBC and NGH2. The pair of EUETS/BRN is found to have a higher value of $H_{xy}(q = 2)$ in comparison to other pairs, suggesting that the returns of European union allowance (EUETS) have the strongest association with Brent crude oil returns, while the value of $H_{xy}(q = 2)$ of EUETS/FTSE350 exhibits a weak association between EUETS and the electricity market.

**Conclusion and discussion**

The European Union’s Emissions Trading System (EUETS) is a market mechanism that assigns a price to carbon and hence offers incentives to cut emissions as much as possible. Over the last 16 years, it has reduced the carbon emissions from energy-intensive and power generation businesses by 42.8 percent (Appunn 2021). Therefore, power generation from fossil fuels, notably coal burning, has become more expensive, making clean energy options more appealing. Likewise, businesses are encouraged to become more energy-efficient since they may sell their emissions permits. In this context, we examine the multifractal characteristics in the cross-correlation of EUETS and the four major fossil fuels energy markets: Brent Crude Oil (BRN), Richards Bay Coal (RBC), UK Natural Gas (NGH2), and FTSE350 electricity index (FTSE350) from January 04, 2016, to March 04, 2022. First, we decompose our data using seasonal and trend decomposition with loess (STL) to separate returns into seasonal, trend, and remainder. Then, we employ multifractal detrended fluctuation analysis (MFDFA) and multifractal detrended cross-correlation analysis (MFDCCA) to examine the inner dynamics of multifractality and cross-correlation.

Our MFDFA results suggest that all of the series under examination are multifractal to varying degrees. The RBC
has the most multifractal patterns, while BRN exhibits the least multifractal patterns among all the series, suggesting that BRN is the most efficient index, whereas RBC is the least efficient. Moreover, only BRN shows persistent (positive autocorrelation), while EUETS, RBC, NGH2, and FTSE350 exhibit anti-persistent behavior (negative autocorrelation). The results of MFDCCA indicate that all the pairs of EUETS with four major fossil fuels energy markets exhibit multifractality. Similar to MFDFA, the pair of EUETS/RBC demonstrates a higher degree of multifractal patterns, while EUETS/NGH2 exhibits the lowest multifractal characteristics among the energy markets. Furthermore, only EUETS/BRN exhibits persistent cross-correlations, while EUETS/RBC and EUETS/NGH2 show anti-persistent cross-correlations. EUETS/FTSE350, however, has relatively low cross-correlations in the multifractality. The following may account for these findings:

According to Agency (2011), around 80% of the emissions are caused by burning fossil fuels, and roughly 40% of these are caused by producing heat and electricity in the power sector, while coal combustion accounts for roughly three-quarters of all carbon emissions. For decades, coal has been the primary source of electricity in Europe and is still being used to produce about one-third of the electricity in Europe. Despite tighter climate policies and recent broad innovation in the clean and renewable energy mix, coal is likely to reappear as a substantial input and source of emissions (Aatola et al. 2013). As a result, the price of coal becomes a key predictor of the EUETS price, which is consistent with our findings. For instance, if EUETS prices increase, coal-fired electricity becomes costly, compelling firms to switch to gas, hence making gas-fired electricity more competitive. Therefore, gas prices generally favor the high EUETS prices over coal in the electricity generation sector (Singh 2021).

Interestingly, European gas prices have risen by over 140 percent, making it more cost-effective for certain producers to burn coal, which emits around twice as much carbon dioxide as gas plants (Twidale 2021a). Gas prices are rising due to supply shortages, making coal-fired power generation more profitable in the short term. Since coal emits more carbon than gas, power providers’ demand for EU and UK allowances is causing prices of carbon to rise. The cost of allowances could be influenced by gas and coal prices in the future. As a result, falling gas prices may stimulate a shift away from coal toward gas, hence lowering the cost of allowances. On the other hand, a hard winter that drives up gas demand could indicate a continued need for coal (Hodgson 2021). Furthermore, a factor is the recent escalation of tensions between Russia and the West over a force build-up on the Ukrainian border (Twidale 2021b).

Our findings on long-range or multifractality in cross-correlations of EUETS with fossil fuel markets have substantial modeling implications. Long-range cross-correlations, for example, could help better explain the linkages between carbon and fossil fuel energy markets. Hence, the nonlinear structure model outperforms the traditional econometric model. We can conclude from the multifractality degree that the coal market is less efficient than the other fossil fuel energy and carbon markets analyzed. The low efficiency of the coal market exposes it to external shocks, increasing the market’s vulnerabilities. Thus, investors should consider a wide range of market risks when developing a risk diversification strategy.

As discussed earlier, the carbon price is closely linked to various energy markets, particularly the coal market. However, countries fail to negotiate valid agreements, making the carbon price more volatile (Cao and Xu 2016). In addition, lack of adequate freedom in market transactions, too few market-maintaining bodies, excessive government intervention, etc., are some of the possible reasons contributing to the carbon market’s inefficiency.

We suggest that financial regulators should tighten financial regulation while improving the legal system and policy on information disclosure to reduce information asymmetry, allowing investors to make better investment decisions.

This study is limited to EUETS, which is a cornerstone of the EU’s policy to combat climate change and its key tool for reducing greenhouse gas emissions cost-effectively. There are other emerging carbon emission indices with diverse methodologies and relatively short histories. We have to admit that there may be different strengths of cross-correlations with fossil fuel indices due to a finite-size effect of the enlargement of multifractality for the limited size of the sample (Zhou 2012; Zhuang et al. 2014). Future studies can be extended by comparing and incorporating different carbon emission indices to reveal the finite-size effect.

Author contribution Faheem Aslam contributed to the study’s conception and design. Material preparation, data collection, and analysis were performed by Ijaz Ali and Inza Irfan. Hyder Ali and Fahd Amjad wrote the first draft of the manuscript. Faheem Aslam supervised the process. Finally, all authors reviewed, edited, and commented on previous versions of the manuscript. All authors read and approved the final manuscript.

Data availability The data set used in the study is available on request.

Declarations

Ethics approval Not applicable.

Consent to participate Not applicable.

Consent for publication Not applicable.

Competing interests The authors declare no competing interests.
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