Improving the Performance of the LSTM and HMM Models via Hybridization

Larkin Liu ∗
Department of Mechanical and Industrial Engineering
University of Toronto
larkin.liu@alum.utoronto.ca

Yu-Chung Lin
Department of Statistical Sciences
University of Toronto
kohjerry.lin@utoronto.ca

Joshua S. Reid
Department of Software Engineering
University of Waterloo
js2reid@uwaterloo.ca

Abstract: Language models based on deep neural neural networks and traditional stochastic modelling has become both highly functional and effective in recent times. In this work a general survey into the two types of language modelling is conducted. We investigate the effectiveness of a combination of the Hidden Markov Model (HMM) with the Long Short-Term Memory (LSTM) model via a process known as hybridization, which we introduce in this paper. This process involves combining the substitution of hidden state probabilities of the HMM into those of the LSTM. We conduct Monte Carlo sampling to produce training and validation of the data in order to produce robust results. The experimental results of this work displayed an increase in the predictive accuracy of LSTM model when hybridized with the HMM.

Keywords: Natural Language Processing, Hidden Markov Model, Long Short-Term Memory, Hybridization

1 Introduction

Language modelling has been an integral part of providing an understanding of the nature of language to capture its meaning. In order to improve the machine understanding of language using sequential models, we seek to explore two prominent areas of statistical language models, the Hidden Markov Model (HMM), and a Recurrent Neural Network (RNN) architecture, known commonly as Long Short-Term Memory (LSTM). Under a discrete stochastic modelling framework, HMM’s were first introduced in Rabiner [1] to classify speech signals. First used to automate AT&T’s voice activated call center, the revolutionary technology allowed computers to robustly characterise speech, and form a basic understanding of spoken words. HMM’s have since become a definitive benchmark for the state-of-the-art for speech recognition, and text recognition. Around the same period, RNN’s were introduced by Rumelhart et al. [2], however, the training complexity of the model was far too high and not commensurate with the hardware capabilities at the time. In the 21st century, with the introduction of more advanced hardware for deep learning model training, came a wave of applications for the RNN for both voice, text recognition, [3], [4], [5] and machine translation [6]. In parallel, an early form of neural language model was developed in Bengio et al. [7], displaying promising results in statistical language modelling.

LSTM’s were the first introduced in Hochreiter and Schmidhuber [8], specifically to combat the vanishing gradient problem, which will be further addressed in Section 1.2. Research has been performed to validate the effectiveness of the LSTM, not only in its ability to ameliorate the vanishing/exploding gradient problem, but also to capture long term dependencies in text, allowing the

∗ Alumni of the University of Toronto at time of authorship.

Working journal paper developed using the CoRL 2019 template.
model to keep in memory long term explanatory observations to make classifications and predictions. We reference the work of Krakovna and Doshi-Velez [9] and the procedure for hybridizing the two models. [9] and [10] showed that both the LSTM and the HMM is capable of identifying special occurrences in text, such as punctuation marks, vowels etc. In our work, the HMM model parameters are combined with the LSTM model parameters, in what is referred to as a sequential training method [9], in our work we refer to this as hybrid stochastic gradient descent (HSGD). We compare the performance of the hybrid model with that of a standalone LSTM on the ability to forecast the next character in the sliding window accurately.

1.1 The Hidden Markov Model (HMM)

Discrete time Hidden Markov Models (HMM) are stochastic models which have a wide range of applications for modelling stochastic processes for various applications. Discrete time HMMs are ideal for modelling discrete auto-correlated processes, where the observed variables depend on an unobservable hidden state, \( S_t \) which obey the Markov property, indicating that the conditional probability of the immediate next state depends only on the present state. The observable symbols at the current time epoch, \( t \) are conditionally dependent only on the hidden state at the time \( t - 1 \). HMMs have been applied in industry to many practical use-cases, most notably in the field of speech recognition [1]. However, HMM do extend to a variety of use cases, such as text classification [9], fraud detection [11], and autonomous driving [12].

We denote the hidden state at time \( t \), \( S_t \) as a condition representing some underlying state of the sequential text language. The relationship between hidden states, \( S_t \), follow a Markov Property, as stated in Eq. (1).

\[
P(S_{t+1}|S_1, \ldots, S_t) = P(S_{t+1}|S_t) \tag{1}
\]

For \( M \) observation symbols, and \( N \) hidden states, the parameters of the HMM include the transition probability between hidden states denoted as matrix \( A = \{a_{ij}\} \). Also the emission matrix \( B = \{b_n(m)\} \) for each hidden state, and the initial probability distribution \( \pi = \pi(n_0) \) at time \( t = 0 \). In order to estimate the parameters of the model we use the Forward-Backward Algorithm outlined in [1]. The forward probability \( \alpha_t(j) \) is defined as the probability of being at state \( S_t = j \) while observing sequences \( O_{0:t} \), as referenced in Eq. (2). And the backward probability, \( \beta_t(i) \) is defined as the probability of observing the sequence into the future \( O_{t+1:T} \) while currently at state \( S_t = i \), as demonstrated in Eq. (3). The canonical representation of the forward and backward probabilities are expressed in Equations (2) and (3).

\[
\alpha_t(j) = b_j(O_t) \sum_{i=1}^{N} \alpha_{t-1}(i) a_{ij} \tag{2}
\]

\[
\beta_t(i) = \sum_{j=1}^{N} \beta_{t+1}(j) b_j(O_{t+1}) a_{ij} \tag{3}
\]

The parameters for multiple sequence HMM’s can be estimated using the Baum-Welch Algorithm [1]. It is possible to estimate the parameters \( A, B \) and \( \pi \), using the Baum-Welch Algorithm for
multiple sequences. We define $\gamma_t(i)$ as observing the specific full sequence $O(0 : T)$ provided the knowledge of the system being in state $S_t = i$, as defined in Eq. (4) for $n_h$ hidden states.

$$\gamma_t(i) = \frac{\alpha_t(i) \beta_t(i)}{\sum_{i=0}^{n_h} \alpha_t(i) \beta_t(i)} \quad \text{(4)}$$

We define $\zeta_t(i, j)$ as the probability of $S_t = i$ transitioning to $S_{t+1} = j$ given the sequence of observations $O_{0:T}$, as defined in Eq. (5).

$$\zeta_t(i, j) = \frac{\alpha_t(i) a_{ij} b_{j}(S_{t+1}) \beta_{t+1}(j)}{P(O_{0:T})} \quad \text{(5)}$$

Using the Baum-Welch algorithm, iterating through 1 to $c$ iterations, we iterate until convergence for the HMM parameters $\theta^{(c)}_{hmm} = (\pi^{(c)}, A^{(c)}, B^{(c)})$.

$$\pi^{(c)}_i = \gamma_0(i) \quad \text{(6)}$$

$$A^{(c)}_{ij} = \frac{\sum_{t=0}^{T-1} \zeta_t(i, j)}{\sum_{t=0}^{T-1} \gamma_t(i)} \quad \text{(7)}$$

$$B_{i}(k)^{(c)} = \frac{\sum_{t=0}^{T} I(O_t = k) \gamma_t(i)}{\sum_{t=0}^{T} \gamma_t(i)} \quad \text{(8)}$$

### 1.2 Recurrent Neural Networks and Long Short-Term Memory Models (LSTM)

Recurrent Neural Networks (RNN) are a class of neural networks capable of processing sequential data using internal memory cells to represent temporal information. Theoretically, RNNs keep track of artificial long-term dependencies, but back-propagated gradients often vanish or explode as the lengths of sequences grow. Long Short-Term Memory (LSTM), a special architecture of RNN, proposed by Hochreiter and Schmidhuber [8] was created to alleviate the vanishing gradient problem. A standalone LSTM cell, shown in Figure 2, consists of three gates: input gate, output gate, and forget gate. The gates serve as regulating functions that control flow of information within the cell. The core of LSTM follows the equation, expressed in Eq. (9).

$$c_t = f_t * c_{t-1} + i_t * g_t \quad \text{(9)}$$

The cell state, $c_t$, serves as the memory of LSTM. It runs through the entire unit and is updated through linear interactions before feeding to the next LSTM unit. The LSTM cell chooses to retain a portion of its previous cell state through the forget gate $f_t$ adding new information based on current input through the input gate $i_t$. The forget gate, $f_t$, illustrated in Eq. (10) determines how much of the previous information is retained through the cell. Another crucial component, the input gate, $i_t$, dictates how much new information is incorporated into the updated cell state from the previous cell output, as illustrated in Eq. (11). We denote $g_t$ as the gate that contains the information used to update $c_t$ illustrated in Eq. (11). Finally, we denote $h_t$ serves as the output of the LSTM cell at time $t$, as stipulated in Eq. (13).

$$f_t = \sigma(W_{if} x_t + b_{if} + W_{hf} h_{t-1} + b_{hf}) \quad \text{(10)}$$

$$i_t = \sigma(W_{ix} x_t + b_{i} + W_{hi} h_{t-1} + b_{hi}) \quad \text{(11)}$$

$$g_t = \tanh(W_{ig} x_t + b_{ig} + W_{hg} h_{t-1} + b_{hg}) \quad \text{(12)}$$

$$h_t = \sigma(W_{io} x_t + W_{ho} h_{t-1} + b_o) * \tanh(c_{t-1}) \quad \text{(13)}$$

Contrary to the standard RNN architecture, a LSTM cell chooses how much information to retain from previous cells and what to add to the current cell state, $c_t$. As the gradient computations are connected to the forget gate activations, the vanishing gradient problem is greatly alleviated by creating a path for necessary information to flow through $f_t$. The ability to learn additively enables LSTM
cells to capture long-term dependencies much more efficiently than standard RNNs do. Since its inception, different flavours of LSTM have been invented to improve upon its existing architectures. Peephole LSTMs\[13\], which let gate layers borrow information from the cell state, was invented in 2000 to help RNNs learn precise timings. With the rise of computational power in the 2000s, LSTMs achieved record performances in natural language processing, handwriting recognition, and achieved 17.7% error rate on the classic TIMIT phoneme recognition benchmark\[14\]. Major breakthroughs in model performances encouraged a new wave of research targeting improved architectures of LSTM’s. The proven effectiveness of the LSTM model in multiple applications revolving around discrete time series forecasting provide justification into the use of the LSTM model as a benchmark for character classification accuracy of this project.

![Architecture of a standalone LSTM cell](image)

### 2 Methodology

The text data used for this experiment was the Tiny Shakespeare corpus. Each character undergoes one hot encoding of 64 possible alphanumeric characters, with a distinction made between upper and lower case characters. The corpus was structured in the form of a sliding window $\mathbf{z}$, of $\mathbf{w} = n_h + 1$ sequential characters, where $n_h$ is also the number of hidden states. To be specific, for each window of text, $\mathbf{z}, \mathbf{z}_{0:w-1}$ were selected as the input data, and $\mathbf{z}_{1:w}$ as the target variable, therefore providing a one-to-one mapping of a character to the character immediately proceeding it within the sliding window. The recurrent neural networks used in this work were implemented using PyTorch, and the HMM model was implemented using the \textit{hmmlearn} package in Python. The tests were run on a computer with an AMD Ryzen 2600 CPU and an Nvidia GTX 1070 graphics card.

#### 2.1 Hybridization of HMM and LSTM

In this experiment, the HMM model was trained using the Baum-Welch Algorithm (BW) as stipulated in Section 1.1.

We reference the work of Krakovna and Doshi-Velez \[9\] on the hybridization of HMM’s and LSTM’s. Our process of creating a hybrid model between HMM and LSTM involves first training the HMM parameters separately, generating a set of independent parameters, $\theta_{hmm}$. Subsequently, the same text sequence, we run a modified version of Stochastic Gradient Descent, which we refer to as \textit{Hybrid Stochastic Gradient Descent} (HSGD), where some of the learned parameters from the HMM, are fixed in place for the LSTM training, indicating that the parameters are static. We provide an implementation of this in PyTorch. In the sequentially trained hybrid model, the hidden cell state, $c_t$ is the equivalent of the emission probability matrix, $\mathbf{B}$, of the HMM for $n_h$ hidden states. Therefore, the LSTM training process Equations (10) to (13) are unchanged, however, we keep $c_t$ fixed at each iteration, and learn the rest of the LSTM parameters.

\[ c_t = \mathbf{B} \] (14)
Equation (14) serves as the fundamental link HMM and the training of the LSTM. This effectively allows the LSTM to fill in the gap where the HMM cannot learn. We seek to replicate and extend on the methodology of [9] and [10], and expand understanding of the work. Similarly, in these experiments, the model was trained with 10 hidden states and evaluate comparative model performance. Furthermore, these hyperparameters were expanded to 15, 25, 35, 50 and 100 hidden states to examine the effectiveness of hidden states on $\mathcal{L}(\theta)$. The HMM hidden state emission probabilities are inserted in place into the cell state of the LSTM. Therefore, $c_t$ derives from the HMM model parameters, $\theta_{hmm}$. This step as hybridization in this work, producing hybrid model parameters, $\theta_H$.

![Figure 3: Process of model training and model validation.](image)

### 3 Experimental Results

The generation of the training and validation data is completed via Monte Carlo Sampling (MC) producing $m$ observations, we set $m = 1000$ for both training and validation. We do this to ensure that there is sufficient randomness in the data to make the model evaluation as robust as possible. We measure the performance of each model based on its ability to accurately predict the subsequent character. We define the $\psi(\theta, x)$ as the accuracy of classification for one sliding window of text $x$ of size $w-1$, as illustrated in Eq. (15). Subsequently, compute model accuracy as the average accuracy of each window of data in the Monte Carlo validation set, of size $m$, denoted by Eq. (16), which represents the likelihood of the model. The log-likelihood of a model $\mathcal{L}(\theta)$ is simply the natural logarithm of $\Psi(\theta, x)$ referenced in Eq. (15).

$$
\psi(\theta, x) = \frac{\sum_{i=0}^{w-1} 1(x_i = \text{argmax}_{\hat{x}_i} P(\hat{x}_i|\theta, x))}{w-1}
$$

$$
\Psi(\theta, x) = \frac{\sum_{i=0}^{m} \psi(\theta, x_m)}{m}
$$

[9] displayed an improvement to the traditional HMM model and LSTM model in terms of predictive accuracy as measured by predictive log-likelihood of the next character, $\mathcal{L}(\theta)$. Likewise in this experiment, comparing the model on its results we see that the log-likelihood of the hybrid model $\mathcal{L}(\theta_H)$ is almost always greater than its LSTM counterpart $\mathcal{L}(\theta_L)$. This provides strong evidence for the value of Hybridization on improving the robustness of character sequence predictions using both HMM’s and LSTM’s. In reference to Table 1, it was observed that an increase in the number of hidden states does not guarantee an increase the predictive accuracy of the model. We hypothesise that the reason for this degradation of performance after a certain number of hidden states is due to the fact that there exists a limit on the number hidden states which accurately capture the nature of the character sequence. Therefore, although we have demonstrated the effectiveness of hybridization, selecting the optimal number of hidden states remains an open area of research.

Due to the small number of neurons, it was found that training with a GPU was slower than a CPU. This is due to GPU’s having a large number of weaker processing units that are well suited for the calculation of high amounts of many small operations, such as linear algebra, that can be highly parallelized Oh and Jung [16]. Having few neurons in each layer means that there is little parallelization that can be performed, so the more powerful computing units in a CPU perform faster.

In 2014, a dramatic variation of LSTM, the Gated Recurrent Unit (GRU), was introduced by Cho et al. [17] to simplify the LSTM architecture. GRU removes the cell state and requires fewer tensor operations to train, which speeds up the training process and provides a viable alternative to LSTM’s.
An application of this modelling technique could fundamentally speed up our model training period, and could be left to future exploration. Also in recent years, attention-based LSTM’s [18], which allows for dependencies regardless of the differences in input and output sequences, have achieved impressive model performances in image classification and speaker verification tasks. In particular, the Transformer[19], a specific type of the attention mechanism, can dramatically speed up the training process and has achieved record-breaking performances in machine translation tasks. Recent developments of LSTM’s continue to shed new light on future advancements of RNNs. Again these advances in RNN architecture could enhance both the modelling accuracy and model training period of our hybridization approach.

### 4 Conclusion

In this work, we provide a brief survey of the HMM, and LSTM, illustrating the key aspects of stochastic modelling and deep learning for the purpose of text prediction. We further demonstrate the effectiveness of combining the parameters of a trained HMM, into the training process of an LSTM, resulting on an improvement to the predictive capabilities of the model, in terms of log-likelihood.

In our experiment, we opted to apply Monte Carlo Sampling to construct both the training and validation set, in order to provide more robustness in the modelling and validation steps. We observe that an increase in the number of hidden-states does not guarantee an increase in the predictive accuracy of the model, suggesting that there may exist a limit on the number of latent states which can be used to accurately model the text sequences. Furthermore, an area of improvement is to modify our GPU based code in order to improve the training times of the model, and not rely on CPU based computation. In summary, this work has advanced the understanding of stochastic and deep learning based models aimed at classifying text patterns.
References

[1] L. R. Rabiner. A tutorial on hidden markov models and selected applications in speech recognition. In PROCEEDINGS OF THE IEEE, pages 257–286, 1989.

[2] D. E. Rumelhart, G. E. Hinton, and R. J. Williams. Neurocomputing: Foundations of research. chapter Learning Representations by Back-propagating Errors, pages 696–699. MIT Press, Cambridge, MA, USA, 1988. ISBN 0-262-01097-6. URL http://dl.acm.org/citation.cfm?id=65669.104451.

[3] R. Jozeowicz, W. Zaremba, and I. Sutskever. An empirical exploration of recurrent network architectures. In Proceedings of the 32Nd International Conference on International Conference on Machine Learning - Volume 37, ICML’15, pages 2342–2350. JMLR.org, 2015. URL http://dl.acm.org/citation.cfm?id=3045118.3045367.

[4] Z. C. Lipton. A critical review of recurrent neural networks for sequence learning. CoRR, abs/1506.00019, 2015. URL http://arxiv.org/abs/1506.00019.

[5] T. Hughes and K. Mierle. Recurrent neural networks for voice activity detection. In ICASSP, pages 7378–7382, 2013.

[6] M. X. Chen, O. Firat, A. Bapna, M. Johnson, W. Macherey, G. Foster, L. Jones, N. Parmar, M. Schuster, Z. Chen, Y. Wu, and M. Hughes. The best of both worlds: Combining recent advances in neural machine translation. CoRR, abs/1804.09849, 2018. URL http://arxiv.org/abs/1804.09849.

[7] Y. Bengio, R. Ducharme, P. Vincent, and C. Janvin. A neural probabilistic language model. J. Mach. Learn. Res., 3:1137–1155, Mar. 2003. ISSN 1532-4435. URL http://dl.acm.org/citation.cfm?id=944919.944966.

[8] S. Hochreiter and J. Schmidhuber. Long short-term memory. Neural Comput., 9(8):1735–1780, Nov. 1997. ISSN 0899-7667. doi:10.1162/neco.1997.9.8.1735. URL http://dx.doi.org/10.1162/neco.1997.9.8.1735.

[9] V. Krakovna and F. Doshi-Velez. Increasing the interpretability of recurrent neural networks using hidden markov models. arXiv:1606.05320, 2016.

[10] A. Karpathy, J. Johnson, and F. Li. Visualizing and understanding recurrent networks. CoRR, abs/1506.02078, 2015. URL http://arxiv.org/abs/1506.02078.

[11] L. Liu. Comparative study between statistical fraud detection methods on ecommerce networks, 2017.

[12] S. Liu, K. Zheng, L. Zhao, and P. Fan. A driving intention prediction method based on hidden markov model for autonomous driving. CoRR, abs/1902.09068, 2019. URL http://arxiv.org/abs/1902.09068.

[13] G. F. A. and S. J. Recurrent nets that time and count. In Proceedings of the IEEE-INNS-ENNS International Joint Conference on Neural Networks. IJCNN 2000. Neural Computing: New Challenges and Perspectives for the New Millennium, volume 3, pages 189–194 vol.3. IEEE, 2000. ISBN 0-7695-0619-4. doi:10.1109/ijcnn.2000.861302. URL http://dx.doi.org/10.1109/ijcnn.2000.861302.

[14] A. Graves, A. rahman Mohamed, and G. E. Hinton. Speech recognition with deep recurrent neural networks. CoRR, abs/1303.5778, 2013. URL http://dblp.uni-trier.de/db/journals/corr/corr1303.html#abs-1303-5778.

[15] G. Chevalier. The lstm cell. Wikipedia, 2018. URL https://commons.wikimedia.org/wiki/File:The_LSTM_cell.png.

[16] K.-S. Oh and K. Jung. Gpu implementation of neural networks. Pattern Recognition, 37(6):1311 – 1314, 2004. ISSN 0031-3203. doi:https://doi.org/10.1016/j.patcog.2004.01.013. URL http://www.sciencedirect.com/science/article/pii/S0031320304000524.
[17] K. Cho, B. van Merriënboer, C. Gulcehre, D. Bahdanau, F. Bougares, H. Schwenk, and Y. Bengio. Learning phrase representations using RNN encoder–decoder for statistical machine translation. In Proceedings of the 2014 Conference on Empirical Methods in Natural Language Processing (EMNLP), pages 1724–1734, Doha, Qatar, Oct. 2014. Association for Computational Linguistics. doi:10.3115/v1/D14-1179. URL https://www.aclweb.org/anthology/D14-1179.

[18] D. Bahdanau, K. Cho, and Y. Bengio. Neural machine translation by jointly learning to align and translate, 2014. URL http://arxiv.org/abs/1409.0473. cite arxiv:1409.0473Comment: Accepted at ICLR 2015 as oral presentation.

[19] A. Vaswani, N. Shazeer, N. Parmar, J. Uszkoreit, L. Jones, A. N. Gomez, Ł. Kaiser, and I. Polosukhin. Attention is all you need. In Advances in Neural Information Processing Systems, pages 5998–6008, 2017.