Longitudinal spin structure of the nucleon:
COMPASS legacy

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Abstract. The COMPASS collaboration has dedicated a large fraction of its already twenty
year long existence to the study of the polarised structure of the nucleon. This presentation
addresses two of the investigated aspects: inclusive measurements and direct measurements of
the gluon polarised PDF, $\Delta g$. In the first case, a summary of what constitutes COMPASS
legacy is presented. This covers the actual measurements of structure function $g_1$, at both
high and low $Q^2$, and the information extracted from them in terms of polarised PDFs, axial
charge and Bjorken sum rule. In particular, the quark spin contribution to the nucleon spin is
determined to be $0.26 < \Delta \Sigma < 0.36$ and from COMPASS data alone the Bjorken sum rule is
verified to 9% accuracy. In the second case, the latest developments in the analysis of our data
on semi-inclusive single-hadron production at high-$p_T$ are reported.

1. Introduction
Polarised aspects of the nucleon structure are still less known than unpolarised ones. COMPASS
is one of the several experiments set up to gain further insight into this polarised structure
over the last twenty years. It fulfils its specifications by measuring double spin cross-section
asymmetries in lepton-nucleon scattering using muons of energy between 160 and 200 GeV.

2. Experimental apparatus and analysis methods
The experiment is a fixed target magnetic spectrometer. It is described in details in [1] and [2].
As far as polarised nucleon structure is concerned, it operates in conjunction with two other
equally important pieces of equipment: a polarised muon beam and a polarised target, which
fulfil the requisites of double spin measurements. The muon beam delivered leptons with a
natural, and hence stable, longitudinal polarisation of $\sim 80\%$ and an intensity of about $5 \times 10^7$
per second. The polarised target was designed with an impressive 1.2 m of length, compensating
the weakness of the beam intensity. It provided protons and deuterons polarised to $\sim 90\%$ and
$\sim 50\%$, respectively. It was subdivided into cells arranged along the beam axis and polarised
in opposite direction, allowing a simultaneous recording of the two nucleon spin states. The
polarisation configuration was regularly inverted.

The cross-section asymmetry $A_{\perp\perp}$ is extracted from the difference in number of interactions
for parallel and anti-parallel spins of muon and nucleon. For inclusive measurements, this
asymmetry when divided by the depolarisation factor is approximately equal to $A_1$. The latter,
together with the knowledge of the unpolarised structure function $F_2$ and the ratio $R$, give the
spin-dependent structure function $g_1$. The parametrisation of $F_2$ and $R$ were taken from Refs [3] and [4], respectively.

3. Results for $A_1^p$ and $g_1^p$ in the DIS region ($Q^2 > 1$ (GeV/c)$^2$)
Results presented here, Fig. 1, are based on data collected in 2007 [5] and 2011 [6]. In 2011 the beam energy was increased to 200 GeV to access higher values of $Q^2$ and lower values of $x$. Results on $A_1^p$ and $g_1^p$ for the two energies agree very well with each other and with the world data, Fig. 1, thus illustrating their weak dependence on $Q^2$. The strong point of COMPASS is that it reaches down to low values of Bjorken $x$, viz. $3.5 \times 10^{-3}$. A very interesting fact is that $g_1^p(x)$ stays finite and positive down to these low values.

![Figure 1](image1)

**Figure 1.** COMPASS results for $A_1^p$ and $g_1^p$ at 160 [5] and 200 GeV [6] in the DIS region. **Left:** mean value of $Q^2$ vs. $x$. **Middle:** $A_1^p$ vs. $x$ for both energies and at measured values of $Q^2$, compared to the other world data (EMC [7], CLAS [8], HERMES [9], E143 [4], E155 [10] and SMC [3]). **Right:** $g_1^p$ vs. $x$ for both energies and at measured values of $Q^2$, compared to the SMC measurements [3]. Bands at the bottom indicate the systematic uncertainties of the COMPASS data at 160 GeV (blue), 200 GeV (red) and SMC at 190 GeV (green).

4. Results for $A_1^d$ and $g_1^d$ in the DIS region ($Q^2 > 1$ (GeV/c)$^2$)
Results presented here, Fig. 2, are based on data collected in 2002-2004 [11] and 2006 [12]. Results on $A_1^d$ and $g_1^d$ from both samples agree very well with each other and with the world data, Fig. 2, thus illustrating their weak dependence on $Q^2$. Contrary to the behaviour of $g_1^p$ and to the hints from SMC [3], $g_1^d$ is compatible with zero at lowest measured $x$.

![Figure 2](image2)

**Figure 2.** COMPASS results for $A_1^d$ and $g_1^d$ for the data collected in 2002-2004 [11] and 2006 [12] in the DIS region. **Left:** mean values of $Q^2$ vs. $x$. **Middle:** $A_1^d$ vs. $x$ for both samples and at measured values of $Q^2$, compared to the other world data (CLAS [8], HERMES [9], SMC [3], E143 [4] and E155 [13]). **Right:** $g_1^d$ vs. $x$ for both samples combined and at measured values of $Q^2$ compared to the SMC measurements [3]. Bands at the bottom indicate the systematic uncertainties of the COMPASS data (blue) and SMC (green).
5. \textbf{A}_1 and \textbf{g}_1 in the non-perturbative \( Q^2 < 1 \text{ (GeV/c)}^2 \) region

The behaviour of \( g_1 \) in the region of lower \( Q^2 \), as opposed to that of DIS, is largely unknown. It is governed by soft physics processes and the transition to DIS is still not understood. For fixed-target experiments, low values of \( Q^2 \) are strongly correlated with low values of \( x \), where also physics of large parton densities sets up.

Measurements at low \( x \) and low \( Q^2 \) are scarce as they put high demands on event triggering and reconstruction. They were performed only by the Spin Muon Collaboration on proton and deuteron [3] and by COMPASS on the deuteron [14]. The latter, very precise results do not reveal any spin effects over the whole measured interval of \( x \). This very fact was understood as an early indication that \( \Delta G \), the total contribution of the gluon spin to the nucleon spin, is small [15].

Here we present new results obtained on the proton in the region \( 0.0062 \text{ (GeV/c)}^2 < Q^2 < 1 \text{ (GeV/c)}^2 \), \( 4 \times 10^2 < x < 4 \times 10^3 \), \( 0.1 < y < 0.9 \) and for \( W > 5 \text{ GeV} \). Four different 2-dimensional grids of kinematic variables are used: \((x, Q^2), (\nu, Q^2), (x, \nu)\) and \((Q^2, x)\); an example is shown in Fig. 3 (see Ref. [16] for the details). Resulting asymmetry \( A_1^p \) and structure function \( g_1^p \) are presented in Fig. 3. Very clear spin effects are seen at all \( x \).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{COMPASS results for \( A_1^p \) and \( g_1^p \) at low \( Q^2 \) for two muon energies. \textbf{Left}: kinematic range in \((Q^2, x)\) variables. \textbf{Middle}: \( A_1^p \) vs. \( x \) at measured values of \( Q^2 \), compared to data of HERMES [9] and SMC [3, 17]. \textbf{Right}: \( g_1^p \) vs. \( x \) at measured values of \( Q^2 \).}
\end{figure}

6. NLO QCD analysis of the \( g_1 \) world DIS data

The COMPASS results on the spin-dependent structure functions are used in an NLO QCD fit, together with the world data on \( g_1^p, g_1^d \) and \( g_1^\text{He} \) with \( Q^2 > 1 \text{ (GeV/c)}^2 \) and \( W^2 > 10 \text{ (GeV/c)}^2 \) [6, 12]. Renormalisation/factorisation scheme is \( \overline{\text{MS}} \). Fitted are the gluon helicity \( \Delta g \), the singlet \( \Delta g^S = \Delta(u + \bar{u}) + \Delta(d + \bar{d}) + \Delta(s + \bar{s}) \) and two non-singlet distributions \( \Delta q_3 = \Delta(u + \bar{u}) - \Delta(d + \bar{d}) \) and \( \Delta q_8 = \Delta(u + \bar{u}) + \Delta(d + \bar{d}) - 2\Delta(s + \bar{s}) \). The functional shape at input scale \( Q_0^2 \) is

\[
\Delta f_k(x) = \eta_k x^{\alpha_k} (1-x)^{\beta_k} (1+\gamma_k x)/(\int_0^1 x^{\alpha_k} (1-x)^{\beta_k} (1+\gamma_k x)dx)
\]

where \( k = S, 3, 8, g \) and \( \eta_k \) is the first moment of \( \Delta f_k(x) \) at \( Q_0^2 \). But several simplifications are applied, leaving a maximum of 11 free parameters. In particular, the non-singlet moments are fixed by the baryon decay constants: \( \eta_3 = F + D, \eta_8 = 3F - D \), assuming SU(2) and SU(3) symmetries. In addition, for the non-singlet and gluon distributions, \( \gamma = 0 \). For the gluons, \( \beta_g \) is fixed at the value of the unpolarised distribution in the MSTW PDF set [18]. The positivity constraint on \( q + \bar{q} \) and \( g \) is enforced by a \( \chi^2 \) penalty. Only statistical errors are considered in the fit; normalisations of data sets are allowed to vary, constrained by systematical uncertainties. Varying the input scale allows to generate sets of solutions. Doing this while fixing parameter \( \gamma_S \) to zero or not, generates two such sets, all elements of which describe the data equally well. Their envelope represents the systematical uncertainty.
Results of the QCD fit are shown in Fig. 4. The contribution of quark spin to the nucleon spin $\Delta \Sigma$ is found to be 0.31 with an uncertainty of $\pm 0.05$ that can be traced to the large uncertainty on $\Delta g$.

7. First moments of structure functions $g_1^d$ and $g_1^{NS}$

The first moment $\Gamma_1^d$ of $g_1^d$ allows for a determination of the flavour-singlet axial charge $a_0$, with the axial charge $a_8$ as an additional input. COMPASS data alone, when integrated over $x$ and extrapolated into the high $x$ region, give ([12]):

$$a_0(Q^2 = 3 \text{ (GeV/c)}^2) = 0.32 \pm 0.02_{\text{stat.}} \pm 0.04_{\text{syst.}} \pm 0.05_{\text{evol.}}$$

which is consistent with the value obtained from the COMPASS NLO QCD fit. The largest contribution to the uncertainty comes from the evolution to common $Q^2$ that data points need to undergo before integration, and hence, ultimately, from the uncertainty of $\Delta g$.

The non-singlet structure function, $g_1^{NS}$ is defined as:

$$g_1^{NS} = g_1^p - g_1^n = 2[g_1^p - g_1^d/(1 - 1.5\omega_D)]$$

where $\omega_D = 0.05 \pm 0.02$ is the contribution of the D-state in the deuteron. Its first moment is the right hand side of the fundamental Bjorken sum rule. The extraction of the $\Gamma_1^{NS}$ from COMPASS data alone, following a procedure similar to the above gives:

$$\Gamma_1^{NS} = 0.192 \pm 0.007_{\text{stat.}} \pm 0.0015_{\text{syst.}}$$

which leads to a validation of the sum rule at the level of 9%, see Ref. [12] for the details.
Figure 5. $\Delta g/g$ vs. $x$: LO analysis of COMPASS DIS data [19] in three ranges of $x_g$ compared to predictions from COMPASS NLO QCD fit [6] at $Q^2 = 3$ (GeV/c)$^2$. The three data points are not fully independent. Horizontal bars represent $1\sigma$ confidence intervals. Inner error bars represent statistical uncertainties and outer ones statistical and systematic uncertainties combined in quadrature.

8. Direct measurement of $\Delta g$

COMPASS has explored several avenues to better constrain the gluon polarised PDF, $\Delta g$, left largely undetermined by inclusive measurements. One of these is the semi-inclusive single-hadron production as a function of $p_T$, $p_T$ being the hadron transverse momentum with respect to the virtual photon.

A re-analysis [19] was performed on the data with $Q^2 > 1$ (GeV/c)$^2$ taken on polarised deuterons. It uses a novel method, putting the data to better use. It yields:

$$\langle \Delta g/g \rangle = 0.113 \pm 0.038_{\text{stat.}} \pm 0.036_{\text{syst.}}$$

for a weighted average of gluon momentum fraction $\langle x_g \rangle \approx 0.10$ and an average $Q^2$ of $3$ (GeV/c)$^2$. This result is compatible with and supersedes our previous result [20] obtained from the same $Q^2 > 1$ (GeV/c)$^2$ data. It favours a positive gluon polarisation in the measured $x_g$ range. The gluon polarisation is also extracted in three bins which correspond to three partially overlapping $x_g$ ranges. These are shown in Fig. 5. Within experimental uncertainties, the values do not show any significant dependence on $x_g$.

The above analysis relies on a Monte Carlo generator, viz. LEPTO [21], to go from the hadron to the parton level. And at parton level, it performs calculations at order $\mathcal{O}(\alpha_s)$, i.e LO for the gluon-induced subprocesses. For this reason, we consider our approach to be a LO analysis, although LEPTO is not strictly speaking a fixed-order pQCD program. This is a strong limitation, given that state-of-the-art QCD fits of polarised PDFs are now based on NLO pQCD calculations. Remains that the results demonstrate the potential of COMPASS single-hadron production data to constrain $\Delta g$. These are made available in Ref. [20], in the shape of raw double spin asymmetries in bins of $p_T$.

Single-hadron production at high $p_T$ was also measured by COMPASS in the low $Q^2$ quasi-real photoproduction region [22]. This time, a pQCD framework that was developed up to NLO is available to interpret the data [23]. Provided a technique known as ”threshold resummation” at next-to-leading logarithm (NLL) is applied, the calculations reproduce COMPASS unpolarised...
cross-section measurements [24] within theoretical uncertainties. This hence demonstrates the applicability of the framework to our kinematical domain. The resummation technique has recently been extended to the polarised case [25] and applied to the COMPASS case. Results show improved agreement with polarised PDFs from DSSV2014 [26], but the interpretation of the data is still affected by uncertainties arising from fragmentation functions.

9. Conclusions

Results presented in this paper constitute the COMPASS legacy on $g_1^p$ and $g_1^p$ both for the DIS ($Q^2 > 1$ (GeV/c)$^2$) and nonperturbative ($Q^2 < 1$ (GeV/c)$^2$) regions.

Direct measurements of the gluon distribution via semi-inclusive single-hadron production show promising results, although they are not yet in a shape that can be included in NLO QCD fits of polarised PDFs.

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