Study on Composite Elastic Modulus of Elevated Temperature Diffusion Self-lubricating Material

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Abstract. Based on the pore structural characteristics of elevated temperature diffusion self-lubricating material, its two-phases composite model was established, and the composite elastic modulus of the materials was predicted by Mori-Tanaka method, as well as the results were compared with test data of Yaman in order to verify the validity of the proposed model. Moreover, the effects of lubricating oil characteristics on the composite elastic modulus of the materials were explored by taking the 4129 type of aerospace lubricating oil as an example. The research shows that the results calculated by the model agree well with the test data of Yaman, and the average error is less than 5%, which proves that the model can predict the composite elastic modulus of elevated temperature diffusion self-lubricating materials accurately. In addition, the composite elastic modulus of the materials decreases linearly with the increase of porosity, which indicates that the increase of lubricant content will cause a decline in the overall strength of the materials. Under the condition of oil lubrication, the composite elastic modulus of the materials decreases with the increase of temperature and velocity, and increases with the increase of the load.

1. Introduction
Porous materials have many special properties, e.g., low relative density, high specific strength, large specific surface area, light weight, sound and heat insulation capacity, energy absorption capacity as well as permeability [1-4], so it is often adopted to manufacture the ultra light structure, separation filter, silencer, heat insulation or heat storage material, vibration damping support, bioceramic material and so on, which is widely applied in the fields of aerospace, environmental protection, construction, petrochemical, metallurgy, machinery and medicine et al [5-7]. Recently, porous materials have been applied in the tribology as self-lubricating bearings at elevated temperature [8].
Elevated temperature diffusion self-lubricating material is a biphasic composite made up of a high strength cermet porous matrix and its stored lubricating fluid. The material is bio-inspired from the structure and function of the sweat gland of human [8]. Because the exudate amount of the lubricating fluid is influenced by the ratio of solid to liquid composition and the pore structure as well as the contact deformation of the matrix, the lubricating property of the material can be controlled probably, so it has a broad application space and development prospects.

However, in order to meet the requirements of high-speed and heavy load working conditions, it is urgent to solve the problem of mutual restraint between the lubrication effect and its strength of the material. The strength of elevated temperature diffusion self-lubricating material is closely related to its porous matrix and lubricating fluid characteristics. At present, the coupling relationship between the macroscopic mechanical properties of the solid-liquid composite and its components is still troubling the engineering field.

Some scholars have used the principle of micromechanics to determine the macro mechanical properties of composites considering the mechanical properties and micro geometric parameters of each component: e.g., based on the variational asymptotic homogenization theory, Zhong et al. adopted periodic representative cell element to establish a micromechanics model successfully, which can be applied to predict the effective properties of the hygrothermal elasticity of fiber reinforced composites and local field distribution in single cell element [9]. Lu et al. modified the Hirsch semi empirical model on the basis of the Voigt model and the Reuss model in order to study the micromechanical properties of particle reinforced composites [10]. Zhao et al. investigated the effect of crack inclusion on the elastic modulus of fiber reinforced composites based on Mori-Tanaka method [11]. However, the above studies are mainly aimed at multiple solid-solid composite, such as particle reinforced composites, fiber reinforced composites and so on, without considering the effect of liquid phase in porous matrix on the macro elastic modulus of solid-liquid composite.

The main aim of this study was to establish the solid-liquid biphasic microstructure model of the elevated temperature diffusion self-lubricating material based on its pore structure characteristics, and to predict the composite elastic modulus of the material by Mori-Tanaka method. Moreover, the influence of the characteristics of lubricating oil on the composite elastic modulus of the material was discussed on the basis of validation of the model.

2. Model and method

2.1. Analytical Model of the Material
The porosity of the high strength cermet porous matrix of elevated temperature diffusion self-lubricating material is low, generally not more than 30%, as shown in Fig. 1 [8].

![Figure 1. Micro structure of cermet porous matrix of elevated temperature diffusion self-lubricating material.](image-url)
The micropores distribute in the matrix uniformly, the difference in pore size is very small and the shape is fairly regular. When the micropores of the matrix are infused with lubricating fluid, it is advisable to establish the representative element [12] of the solid-liquid biphase complex for study the composite elastic properties of the material, as shown in Fig. 2.

![Figure 2. Representative element of solid-liquid biphase composite.](image)

For the representative element, it is assumed that the solid phase (cermet) of the material is one of homogeneous elastic materials, while the liquid (lubricating fluid) is considered as an elastic inclusion, which is considered to be full of pores. Therefore, the Mori-Tanaka method can be used to solve the related elastic parameters based on the idea of equivalent modulus.

### 2.2. Analytical Method to Solve Composite Elastic Modulus of the Material

**Assuming the pore in the representative element is not filled with any lubricating fluid initially. When the uniform stress of the far field \( \sigma \) is acted on the boundary of the element, the constitutive relation is as follows:

\[
\sigma = D_s \varepsilon
\]

\[
D_s = \frac{1}{3} (3K_s - 2G_s) \delta \delta + 2G_s I
\]

Where \( \sigma \) is the uniform stress of the far field acting on the boundary of the representative element, \( D_s \) is the elastic tensor of solid phase in representative element, \( \varepsilon \) is the strain for solid phase, \( K_s \) and \( G_s \) are the bulk modulus and shear modulus of the solid phase respectively, and \( I \) is 4 order unit tensor.

When the pore in the representative element is filled with the lubricating fluid, the average strain in the actual cermet solid changes into \( \varepsilon + \bar{\varepsilon} \), where \( \bar{\varepsilon} \) is the disturbance strain caused by the action of the lubricating fluid. Thus, the average stress of the cermet solid was obtained as

\[
\sigma_\varepsilon = \sigma + \bar{\sigma} = D_s (\varepsilon + \bar{\varepsilon})
\]

Where \( \sigma \) is the disturbance stress caused by the action of the lubricating fluid.

As the elastic properties of the filled lubricating fluid are different from the cermet solid, the average stress and the average strain of the liquid phase are different from the corresponding average in the solid phase under the external load. The differences were recorded as \( \sigma' \) and \( \varepsilon' \) respectively. The problem of stress disturbance of liquid phase can be solved by the principle of Eshelby equivalent inclusion.

\[
\sigma = \sigma + \bar{\sigma} + \sigma' = D_l (\varepsilon + \bar{\varepsilon} + \varepsilon') = D_l (\varepsilon + \bar{\varepsilon} + \varepsilon' - \varepsilon')
\]
Where $\sigma_l$ is the average stress of the liquid phase, $D_l$ is the elasticity tensor and $\varepsilon^*$ is the equivalent characteristic strain of the liquid phase.

$$D_l = \frac{1}{3} (3K_l - 2G_l) \delta \delta + 2G_l I \quad (5)$$

$$\varepsilon^* = S \varepsilon$$ \quad (6)

Where $K_l$ and $G_l$ are the bulk modulus and shear modulus of the liquid phase respectively, while $S$ is 4 orders Eshelby tensor, which is related to the elastic properties of solid phase and the shape of liquid phase. The average stress of the solid-liquid biphasic composite can be obtained by Mori-Tanaka method as

$$\sigma = (1 - \varphi) \sigma_s + \varphi \sigma_l \quad (7)$$

Where $\varphi$ is volume fraction of the liquid phase, i.e. lubricating fluid in the representative element of the solid-liquid biphasic composite, which is known as the porosity of the material. The disturbed strain caused by the liquid phase can be obtained by combining (1), (3), (4), (6) and (7).

$$\bar{\varepsilon} = - \varphi (S - I) \varepsilon^* \quad (8)$$

Combining (4), (6) and (8) resulted in

$$\begin{cases} 
\varepsilon^* = H \varepsilon \\
H = \frac{D_s - D_l}{D_l + (D_s - D_l) \left[ \varphi I + (1 - \varphi) S \right]} 
\end{cases} \quad (9)$$

The relationship between the mean strain $\bar{\varepsilon}$ and the stress was obtained.

$$\sigma = D_s \frac{1}{I + \varphi H} \bar{\varepsilon} \quad (10)$$

The equivalent elastic modulus of the solid-liquid biphasic composite can be obtained by (10).

$$D = D_s \frac{1}{I + \varphi H} \quad (11)$$

As the lubricating fluid in the micropores restricted the deformation of the cermet matrix into the pore, the rigidity of the micropores was increased, so the effect of the shape of the micropore can be ignored, and the shape of micropores was simulated as sphere [13]. For the spherical inclusion, its Eshelby tensor was gained as follows
The bulk modulus $K$ and shear modulus $G$ of the solid-liquid biphasic material were derived as (13) and (14) respectively

$$\frac{K}{K_s} = 1 + \frac{\varphi (K_i - K_s)}{K_i + (1 - \varphi) \frac{K_i - K_s}{K_i + \frac{4}{3} G_i}}$$ \hspace{1cm} (13)$$

$$\frac{G}{G_i} = 1 + \frac{\varphi (G_i - G_s)}{G_i + (1 - \varphi) \left[ \frac{G_i - G_s}{1 + \frac{9K_s + 8G_s}{6(K_i + 2G_i)} \right]}$$ \hspace{1cm} (14)$$

Therefore, the composite elastic modulus $E$ of the solid-liquid biphasic material, i.e. elevated temperature diffusion self-lubricating material can be obtained by elastic mechanics.

$$E = \frac{9GK}{3K + G}$$ \hspace{1cm} (15)$$

It can be seen that the composite elastic modulus can be predicted as long as the bulk modulus and shear modulus of the material and the volume fraction (i.e. porosity) of the lubricating fluid are known.

3. Validation of the analytical model
Yaman et al. [14] measured the bulk modulus and shear modulus of solid matrix of a high strength concrete, that were $K_s$=27.91GPa and $G_s$=18.45GPa respectively, and tested the elastic modulus of concrete with different porosity as shown in Table 1. Assuming that the elevated temperature diffusion self-lubricating material is water lubricated and its liquid phase is fresh water. When the temperature is 20℃, the volume modulus and the shear modulus of fresh water are $K_l$=2.25GPa and $G_l$=0GPa respectively [13]. Therefore, the composite elastic modulus of the solid-liquid biphasic material under the corresponding porosity were calculated by the above analytical model, and compared with the Yaman test data, as shown in Table 1.
Table 2. Composite elastic modulus of solid-liquid biphasic material.

| 1.1.1. $\phi$ (%) | 1.1.5. $K$ (GPa) | 1.1.6. $G$ (GPa) | 1.1.7. $E$ (GPa) | 1.1.8. $K$ (GPa) | 1.1.9. $G$ (GPa) | 1.1.10. $E$ (GPa) |
|-------------------|-----------------|-----------------|-----------------|----------------|----------------|-----------------|
| 1.1.8. 9.4        | 1.1.9. 23.5817  | 1.1.10. 15.3122 | 1.1.11. 37.7631 | 1.1.12. 38.43  |
| 1.1.13. 10.2      | 1.1.14. 23.246  | 1.1.15. 15.0693 | 1.1.16. 37.175  | 1.1.17. 35.94  |
| 1.1.18. 12.8      | 1.1.19. 22.1867 | 1.1.20. 14.3032 | 1.1.21. 35.3197 | 1.1.22. 35.11  |
| 1.1.23. 13.1      | 1.1.24. 22.0675 | 1.1.25. 14.217  | 1.1.26. 35.1109 | 1.1.27. 33.51  |
| 1.1.28. 15.5      | 1.1.29. 21.1352 | 1.1.30. 13.5433 | 1.1.31. 33.4789 | 1.1.32. 30.93  |
| 1.1.33. 15.9      | 1.1.34. 20.9835 | 1.1.35. 13.4337 | 1.1.36. 33.2133 | 1.1.37. 29.76  |
| 1.1.38. 16.1      | 1.1.39. 20.9079 | 1.1.40. 13.3791 | 1.1.41. 33.0811 | 1.1.42. 30.61  |
| 1.1.43. 17.1      | 1.1.44. 20.5341 | 1.1.45. 13.1092 | 1.1.46. 32.427  | 1.1.47. 31.57  |
| 1.1.48. 19.9      | 1.1.49. 19.5194 | 1.1.50. 12.3768 | 1.1.51. 30.6518 | 1.1.52. 30.93  |
| 1.1.53. 20        | 1.1.54. 19.484  | 1.1.55. 12.3513 | 1.1.56. 30.5899 | 1.1.57. 26.88  |
| 1.1.58. 20.8      | 1.1.59. 19.2028 | 1.1.60. 12.1484 | 1.1.61. 30.0982 | 1.1.62. 27.33  |
| 1.1.63. 21.1      | 1.1.64. 19.0983 | 1.1.65. 12.0731 | 1.1.66. 29.9155 | 1.1.67. 31.37  |

Fig. 3 shows the composite elastic modulus of the solid-liquid biphasic material compared between analytical results and Yaman test data. The results show that the analytical results are in good agreement with the Yaman test data, and the average error is less than 5%. Therefore, the analytical results have enough accuracy according to the requirements of the engineering calculation, which indicates that the analytical model can predict the composite elastic modulus of the elevated temperature diffusion self-lubricating material accurately.

In addition, as shown in Fig. 3, the composite elastic modulus of the material decreases linearly approximatively with the increase of porosity, which indicates that the increase of the proportion of lubricating fluid will lead to the decrease of the overall strength of the material.

![Analytical results vs Yaman test data](image)

**Figure 3.** Composite elastic modulus of the solid-liquid biphasic material compared between analytical results and Yaman test data.

4. Discussion
The type 4129 aero lubricating oil was taken as a liquid lubricant, and the influences of the lubricating oil characteristics on the composite elastic modulus of elevated temperature diffusion self-lubricating material were discussed by the analytical model. It was assumed that the material parameters of cermet matrix such as bulk modulus and shear modulus are invariable, i.e. $K_s=27.91\text{GPa}$ and $G_s=18.45\text{GPa}$ respectively.

It was found that the shear modulus of type 4129 aero lubricating oil is related to some factors such as temperature, velocity and load et al. [15]. The higher the temperature and velocity, the smaller the
viscosity of the lubricating oil, so the shear modulus of the lubricating oil decreases with temperature and velocity, and increases with the load. The specific parameters were shown in Table 2.

Table 2. Shear modulus of type 4129 Aero lubricating oil.

| temperature $t$ (°C) | the maximum Hertz stress $\sigma_{\text{Hmax}}$ (GPa) | velocity $v$ (m/s) | shear modulus $G_l$ (GPa) |
|----------------------|-----------------------------------------------|-------------------|--------------------------|
| 1.1.72. 15           | 1.1.73. 1.35                                  | 1.1.74. 3         | 1.1.75. 0.0650           |
|                      |                                               | 1.1.76. 5         | 1.1.77. 0.0581           |
|                      |                                               | 1.1.78. 7         | 1.1.79. 0.0575           |
|                      |                                               | 1.1.80. 9         | 1.1.81. 0.0570           |
|                      |                                               | 1.1.82. 11        | 1.1.83. 0.0566           |
|                      |                                               | 1.1.84. 15        | 1.1.85. 0.0530           |
|                      |                                               | 1.1.86. 20        | 1.1.87. 0.0525           |
|                      | 1.1.89. 1.00                                  |                   | 1.1.91. 0.0276           |
|                      | 1.1.92. 1.20                                  |                   | 1.1.93. 0.0482           |
|                      | 1.1.94. 1.35                                  | 1.1.90. 9         | 1.1.95. 0.0570           |
|                      | 1.1.96. 1.50                                  |                   | 1.1.97. 0.0595           |
| 1.1.98. 15           |                                               |                   | 1.1.101. 0.0573          |
| 1.1.102. 30          |                                               |                   | 1.1.103. 0.0080          |
| 1.1.104. 45          | 1.1.99. 1.2                                   | 1.1.100. 5        | 1.1.105. 0.0070          |
| 1.1.106. 60          |                                               |                   | 1.1.107. 0.0041          |

Assuming that there is no gas in the lubricating oil, its bulk modulus varies little with temperature and it is desirable to set a definite value for the bulk modulus, i.e. $K_l=1.38\text{GPa}$ [16]. Therefore, according to the analytical model, the curves of composite elastic modulus of the material versus velocity $v$, temperature $t$ and load (i.e. the maximum Hertz stress $\sigma_{\text{Hmax}}$) were obtained, as shown in Fig. 4, 5, and 6 respectively.

![Figure 4](image.png)

Figure 4. Curves of composite elastic modulus versus velocity ($t=15^\circ \text{C}$, $\sigma_{\text{Hmax}}=1.35\text{GPa}$, $\phi=20\%$).

Fig. 4 shows that under the condition of $t=15^\circ \text{C}$, $\sigma_{\text{Hmax}}=1.35\text{GPa}$ and $\phi=20\%$, the composite elastic modulus of the material decreases from 30.5277GPa to 30.5139GPa when the velocity increases from 3m/s to 20m/s. The rate of descent at a velocity less than 5m/s is much higher than that of faster than 5m/s, which indicates that the effect of velocity on the composite elastic modulus of the material decreases with the increase of velocity.

Fig. 5 demonstrates that under the condition of $\sigma_{\text{Hmax}}=1.2\text{GPa}$, $v=5\text{m/s}$, $\phi=20\%$, the composite elastic modulus of the material decreases rapidly from 30.5192GPa to 30.4647GPa when the temperature rises from 15$^\circ \text{C}$ to 30$^\circ \text{C}$, while the composite elastic modulus of the material decreases
slowly to 30.4604GPa when the temperature continues to rise to 60℃. It is indicated that the
temperature has a great influence on the composite elastic modulus of the material in the low
temperature zone, while the effect is negligible in the high temperature zone.

Figure 5. Curves of composite elastic modulus versus temperature (σHmax =1.2GPa, v=5m/s,
φ=20%).

Fig. 6 exhibits that under the condition of t=15℃, v=9m/s, φ=20%, the composite elastic modulus
of the material rises from 30.4864GPa to 30.5216GPa when the maximum Hertz stress increases from
1GPa to 1.5GPa. The larger the maximum Hertz stress is, the smaller the increase rate of the
composite elastic modulus.

Figure 6. Curves of composite elastic modulus versus the maximum Hertz stress (t=15℃, v=9m/s,
φ=20%).

5. Conclusion
(1) Taking water lubrication as an example, it is proved that the analytical model of solid-liquid biphase can accurately predict the composite elastic modulus of the elevated temperature diffusion self-lubricating material by comparing with the Yaman test data.

(2) The increase of lubricant content will result in the decrease of the overall strength of the elevated temperature diffusion self-lubricating material.

(3) The composite elastic modulus of the elevated temperature diffusion self-lubricating material decreases with temperature and velocity, and increases with the load.

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