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Precise strain measurement in complex materials using Digital Volumetric Correlation and time lapse micro-CT data

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Abstract

Digital volumetric correlation (DVC) method can be employed for evaluation of full-volume displacement and strain fields in deformed material with clearly recognizable microstructure. This microstructure can be defined by complex inner geometrical topography, as trabecular bone and metal foam have for instance, or/and by significant variations of chemical composition, as aluminum grainy alloys have for instance. Volumetric image data were acquired using time lapse X-ray micro-CT during gradually specimen loading. Tetrahedral FE mesh was generated describing related volumetric data. The displacement fields were measured in set of control points by DVC utilizing vertices of this tetrahedral FE mesh. The advantage of the approach is that the resulting strain can be directly compared with FE simulation of the experiment. Problems of accuracy, especially in evaluation of the strain field from the noisy measured displacements and problems of computational efficiency of the method are discussed.

Keywords: Digital volumetric correlation, strain measurement, trabecular bone, micro-CT.

1. Introduction

Microstructural properties and spatial arrangement of the inner structure are the key factor for the overall mechanical properties of complex natural materials such as trabecular bone [1] or materials produced artificially such as metal foam [2]. In recent years 3-D imaging techniques were established which enables direct measurement of structural properties of these complex materials, namely microfocus Computed Tomography (micro-CT) [3-4]. Micro-CT can be used not only for non-destructive

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reconstruction of the inner microstructure and for measuring of some morphological properties of a material, but it can be successfully applied to record deforming microstructure under applied mechanical (or other) loading.

Digital Image Correlation (DIC) techniques are frequently used to measure the strains on the surface of a loaded sample for a wide range of both artificial and biological materials [5-6]. Precision of DIC has been shown to be well below 0.01 pixel and precision of the strains below 0.02%. However, precision and accuracy of the three-dimensional variant of DIC have not yet been fully established.

In this study a three-dimensional variant of digital image correlation algorithm - digital volumetric correlation (DVC) is used to quantify the displacements and strains in two materials with complex microstructure – trabecular bone and metal foam. The main focus of this article is in estimate the precision of the DVC method for displacement and strain measurements.

2. Materials and Methods

To evaluate the best parameters needed in the DVC method two sets of volumetric images were used. The first set is the data from time-lapse tomography of trabecular bone loaded gradually in compression and second set is a similar experiment performed on a sample of metal foam.

A cylindrical sample of trabecular bone (diameter=5mm, height=10mm) was gradually loaded up to 10% strain in ten 1% increments. After each load step the sample was allowed to relax for 20 minutes until the force value relaxed. The force was continuously recorded with a 100 N load cell (U9B, Hottinger Baldwin Messtechnik, Germany). Similar setup was used in case of a sample of metal foam. In this case, a larger sample was cut from a block of material (10x10x20mm). The sample was loaded incrementally in compression up to 10% total strain with applied force measured using a 2kN load cell.

Tomographic images of the deformed microstructures were acquired using microfocus X-ray source Hamamatsu L8601-01 with wolfram anode was used together with a large-area flat-panel detector Hamamatsu-mod. C7942CA, resolution 2240×2344 pixel resolution and 55 μm pitch, the sensitive area 120×120 mm².

![Fig. 1. Definition of the control points for DVC in trabecular bone sample: whole sample (left), cut-out of sample and its mesh (middle), full-volume displacement field in z-direction measured by DVC (right).](image)

2.1. Definition of the grid of control points for DVC

The reconstructed 3D image of the undeformed state was used to generate the tetrahedral mesh. To create this mesh the images were firstly segmented to contain only the bone or foam material (without empty spaces). Connected threshold, a region growing algorithm was used for this purpose in both cases. From the segmented images the surface mesh was developed using a fast Marching Cubes Algorithm [7].
After shape optimization of the surface mesh the volume of interest was filled with tetrahedral elements using Delaunay approach [8].

The all vertices of the resulting tetrahedral mesh were used directly as the control points at which the DVC method was applied, see Fig. 1. As the result, at each nodal point a displacement vector was estimated from which the strain tensor was calculated using coefficients of affine transformation between the reference and deformed coordinates.

2.2. Digital Volumetric Correlation

The digital volumetric correlation (DVC) method employed for evaluation of the full-volume displacement field is an extension of the well-known two-dimensional digital image correlation (DIC) [9] to all three spatial dimensions. Computational principles of these methods are very similar. The technique utilizes a sequence of consecutive 3D image data (at least two) that represents the process of the object displacement and deformation. In this sequence DVC observes a movement of individual sub-volume templates by employing the correlation technique, see Fig. 2. The sub-volume template is a small part of the full volume 3D data that has to contain a distinguishable part of the object inner structure.

The tracking algorithm works in two steps. First, an integer value of voxel displacement is evaluated using normalized cross-correlation (NCC). Let \((X,Y,Z)\) are coordinates of a central voxel of a reference sub-volume template in the reference image data. The NCC coefficient is computed in the vicinity of \((X,Y,Z)\) in the deformed image data until its maximum value is reached, see Figure 2 (right). The NCC coefficient is defined as:

\[
\bar{r} = \frac{\sum_{i} \sum_{j} \sum_{k} (T_{i,j,k} - \bar{T})(I_{i,j,k} - \bar{I})}{\sqrt{\sum_{i} \sum_{j} \sum_{k} (T_{i,j,k} - \bar{T})^2 \sum_{i} \sum_{j} \sum_{k} (I_{i,j,k} - \bar{I})^2}} \tag{1}
\]

where \(T\) is the reference sub-volume (template) and \(I\) is the deformed sub-volume, respectively. The maximum of the NCC coefficient is found using steepest-gradient method. The position of the maximum gives a new integer coordinates \(X',Y',Z'\) where the sub-volumes have the best correlation. Thus, the displacement vector (pixel accuracy) of the reference sub-volume template is given as:

\[
\bar{u} = [u, v, w] = [X' - X, Y' - Y, Z' - Z] \tag{2}
\]
Subsequently, these integer values of displacement are passed on as initial inputs into the second step which is 3D extension of Lucas-Kanade algorithm [10]. This step takes into account own deformation of the reference sub-volume (template). Lucas-Kanade algorithm is based on the minimizing of the sum of squared error between reference and deformed sub-volume:

\[ \sum x \left[ I(W(\bar{x}; \bar{p})) - T(\bar{x}) \right]^2 \]  

(3)

where \( W(\bar{x}; \bar{p}) \) describes a warp of the deformed sub-volume onto the reference sub-volume \( T \). In our case, the affine warp of the deformed sub-volume \( I \) is considered:

\[
W(\bar{x}; \bar{p}) = \begin{bmatrix}
1 + p_1 & p_4 & p_7 & p_{10} \\
p_2 & 1 + p_5 & p_8 & p_{11} \\
p_3 & p_6 & 1 + p_9 & p_{12}
\end{bmatrix}
\]

(4)

Algorithm searches the vector of parameters \( \bar{p} = [p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12}] \) minimizing the expression (3) by employing a non-linear optimization technique (the extension of the Inverse Compositional Algorithm to all three spatial dimensions). How the algorithm works is step by step described below.

**Pre-compute:**

1. Evaluate the gradient \( \nabla T \) of the sub-volume template \( T(\bar{x}) \)
2. Evaluate the Jacobian of the warp \( \frac{\partial W}{\partial \bar{p}} \) at \( (\bar{x}; \bar{o}) \)
3. Compute the steepest descent \( \nabla T \frac{\partial W}{\partial \bar{p}} \)
4. Compute the Hessian matrix \( H = \sum x \left[ \nabla T \frac{\partial W}{\partial \bar{p}} \right]^T \left[ \nabla T \frac{\partial W}{\partial \bar{p}} \right] \)

**Iterate:**

1. Warp \( I \) with \( W(\bar{x}; \bar{p}) \) to compute \( I(W(\bar{x}; \bar{p})) \)
2. Compute the error image \( E(\bar{x}) = I(W(\bar{x}; \bar{p})) - T(\bar{x}) \)
3. Compute the increment \( \Delta \bar{p} = H^{-1} \sum x \left[ \nabla T \frac{\partial W}{\partial \bar{p}} \right]^T E(\bar{x}) \)
4. Update the warp \( W(\bar{x}; \bar{p}) \leftarrow W(\bar{x}; \bar{p}) \circ W(\bar{x}; \Delta \bar{p})^{-1} \)

until \( \| \Delta \bar{p} \| \leq \varepsilon \)

Further details of the algorithm for 2D L-K is described in [11]. Note that the last three parameters \( p_{10}, p_{11}, p_{12} \) of \( \bar{p} \) describes the displacement of the central voxel of the reference sub-volume template, so the integer values of the displacement obtained in the first step serves as initial parameters \( p_{10}, p_{11}, p_{12} \). Thus, the initial vector of parameters passing into LK algorithm looks as \( p_0 = [0, 0, 0, 0, 0, 0, 0, u, v, w] \). After a few iteration, LK gives a new vector of re-fined parameters \( \bar{p} \) and by re-computation of the resulted \( W(\bar{x}; \bar{p}) \), a new sub-pixel destination of the central voxel of the sub-volume template is obtained.
3. Results

To evaluate the precision and accuracy of the DVC, numerical experiments with the micro-CT images of the undeformed (original, reference) state were performed. The image data of the reference state were even numerically distorted to represent different types of deformation (compression, rigid body motion, shear and torque). Influence of all parameters has been assessed in a parametric study in which a set of optimal values (smallest error versus computational time needed for correlation of one point to converge) was determined.

Size of the correlation window was found to be the most influential parameter in terms of the precision. As an optimal size of the correlation cube 35\times35\times35 pixels has been determined, which gives maximal error in displacements 0.00106 pixels and maximal error in strains 0.03923\% for the worst considered case (2\% deformation). Further increasing of the correlation cube don’t have significant influence for precision improvement. In terms of standard deviation, the results for the worst deformation state can be expressed as: 5.1\pm0.00051 pixels in displacement or 2\pm0.013\% in strains. Time needed for the DVC algorithm to converge at one point with this set of optimal parameters was 1.3 seconds in MATLAB on Intel Xeon processor 5500 series at 2.80GHz.

![Graph](image_url)

Fig. 3. Dependency of the precision of the DVC method on the size of correlation cube (from 13x13x13 up to 41x41x41): a) maximal error and standard deviation in displacements (left), maximal error and standard deviation in strains (right).

The loading experiments were evaluated with the set of above determined optimal parameters in a smaller volume (50x50x50 voxels in the middle part of the samples). Resulting tetrahedral meshes consisted of 35\times10^3 nodes and 154\times10^3 elements in case of trabecular bone and 47\times10^3 nodes and 193\times10^3 elements in case of metal foam sample. The DVC algorithm did not converge in 213 points only (average from all deformation states) in trabecular bone and in 407 points in metal foam. In these points, the displacement value was computed as the average displacement of nodal points of neighbouring elements. Example of resulting displacements in trabecular bone together with the FE model of the whole sample is presented in Fig. 1.

4. Conclusions

Presented DVC technique enables precise calculation of displacements and strains in a loaded microstructure. The underlying FE mesh serves not only for visualization but also verification of FE models against experiments. This is important e.g. in comparison of material models used for large-strain analysis, where material and geometrical nonlineairities are dominant.
The method has been tested using data of two experimentally and artificially loaded specimens. The precision was found to be sufficient for these types of experiments, where determination of the post-yield behaviour is needed and therefore applied strains are moderate or large. The typical feature of this method is that the precision of measured displacements are fine instead of computed strains that are affected by noisy displacements, see Fig. 3. Therefore, some sort of smoothing process of displacement field should to be taken into account in the future. For the experiments, it can be concluded, that both for the aluminium foam and the sample of trabecular bone the DVC method did not converge only in a negligible number of control points. It was possible for these points to calculate the displacements as average values from the neighbouring elements.

Described algorithms are computationally expensive and for large number of control points the evaluation of such an experiment is time consuming. It is not easy to reduce the number of nodal points because of the shape complexity of studies materials. However, since all the algorithms used in the study are in principle suitable for parallel computing, it is possible to use a multiprocessor system or to utilize modern GPUs. The method is suitable for parallelization using the CUDA architecture.

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