Constraints on inert dark matter from metastability of electroweak vacuum

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Abstract

Inert scalar doublet model of dark matter can be valid up to the Planck scale. We briefly review the bounds on the model in such a scenario and identify parameter spaces that lead to absolute stability and metastability of the electroweak vacuum.
I. INTRODUCTION

Although LHC has unearthed the standard model (SM) Higgs boson [1–3], so far, it has failed to identify any sign of new physics beyond SM. New physics at the TeV scale is invoked in the form of supersymmetry, extra dimensions etc. to address the issue of the hierarchy problem. Absence of such new physics can imply that the solution of hierarchy problem lies somewhere else. Hence, it is likely that SM could be valid upto the Planck scale ($M_{Pl}$), above which, gravity is expected to dominate over other forces.

However, we know that the presence of dark matter (DM) and non-zero neutrino masses indicate physics beyond the SM. Hence, it is worthwhile to explore a scenario where SM, augmented by a low energy DM model, is valid upto $M_{Pl}$. Earlier we had explored such a scenario where a gauge singlet scalar is added to the SM to pose as a weakly interacting massive particle (WIMP) DM [4]. In the present work, we extend the SM by a scalar doublet protected by a $Z_2$ symmetry. The model is popularly known in the literature as the inert doublet (ID) model, first proposed by Deshpande and Ma [5].

Given the experimental measured values for $M_t$, $M_h$ and $\alpha_s$, the electroweak (EW) vacuum is reported to reside in a metastable state within the framework of SM. This has been verified in the literature [6–10] only very recently using state of the art next-to-next-to leading order (NNLO) loop corrections contributing to the Higgs effective potential. In Ref. [4], this analysis was extended to the case of a singlet scalar DM model. The stability of the EW vacuum was shown to depend on new physics parameters. In this paper, we extend such analysis to the ID model. We assume that ID DM is the only DM particle which saturates the entire DM relic density. In this context, we review the constraints on the parameters of the ID model.

A detailed study on the ID parameter space was recently performed in Ref. [11] indicating bounds from EW stability, perturbativity, collider study, electroweak precision tests (EWPT) etc., considering DM annihilation to two-body final states only. In Ref. [12] DM relic density was calculated including three-body final states as well. An updated detailed analysis on the ID parameter space was performed in Refs. [13, 14] reflecting the impact of the $M_h \approx 126$ GeV Higgs boson discovery at the LHC. Ref. [13] looked at renormalisation group evolution (RGE) of model parameters to check the validity of ID model at higher energies. The constraints from the measurements of diphoton decay channel of Higgs boson on the ID parameter space was also discussed. High scale validity of this model in presence of a right-handed neutrino has been looked at in Ref. [15]. The influence of ID in the Higgs effective potential was studied in Ref. [16] to explore the possibility of electroweak symmetry breaking à la Coleman-Weinberg [17]. High scale validity of two Higgs doublet model has been explored in Ref. [18, 19] with broken $Z_2$ symmetry considering tree level potential only.

In this paper we improve earlier studies on ID model parameter space by including radiative
corrections to the scalar potential to explore stability of the electroweak vacuum. In particular, we permit the Higgs quartic coupling to assume small negative values to render a metastable EW vacuum. As our analysis improves the scalar potential considering radiative corrections, the tree level stability constraints used in earlier analyses need to be reviewed. In addition, as we demand the theory to be valid up to $M_{Pl}$, the model parameters at the EW scale are constrained by the requirement that they satisfy all the bounds up to the $M_{Pl}$.

The paper is organised as follows. Section II starts with an introduction of ID model followed by a discussion of effective scalar potential and RG running of parameters of the model. Constraints on the model are discussed in Section III. Tunnelling probability of the metastable vacuum and the restrictions on the parameters to avoid a potential unbounded from below, are mentioned in Section IV. A detailed study of the parameter space identifying regions of EW vacuum stability and metastability is performed in Section V with the help of various phase diagrams. Section VI contains a short discussion on Veltman’s conditions in the context of ID model. We finally conclude in Section VII.

II. INERT DOUBLET MODEL

In this model, the standard model is extended by adding an extra $SU(2)$ doublet scalar, odd under an additional discrete $Z_2$ symmetry. Under this symmetry, all standard model fields are even. The $Z_2$ symmetry prohibits the inert doublet to acquire a vacuum expectation value.

The scalar potential at the tree level is given by,

$$V(\Phi_1, \Phi_2) = \mu_1^2|\Phi_1|^2 + \lambda_1|\Phi_1|^4 + \mu_2^2|\Phi_2|^2 + \lambda_2|\Phi_2|^4$$

$$+ \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1|^2 + \lambda_5^2 \left[ (\Phi_1^\dagger \Phi_1)^2 + h.c. \right],$$

where, the SM Higgs doublet $\Phi_1$ and the inert doublet $\Phi_2$ are given by,

$$\Phi_1 = \left( \frac{1}{\sqrt{2}} (G^+ (v+h+iG^0)) \right), \quad \Phi_2 = \left( \frac{1}{\sqrt{2}} (H^+ (H+iA)) \right).$$

$G^\pm$ and $G^0$ are Goldstone bosons and $h$ is the SM Higgs.

$\Phi_2$ contains a CP even neutral scalar $H$, a CP odd neutral scalar $A$, and a pair of charged scalar fields $H^\pm$. The $Z_2$ symmetry prohibits these particles to decay entirely to SM particles. The lightest of $H$ and $A$ can then serve as a DM candidate.

After electroweak symmetry breaking, the scalar potential is given by

$$V(h, H, A, H^\pm) = \frac{1}{4} \left[ 2\mu_1^2(h+v)^2 + \lambda_1(h+v)^4 + 2\mu_2^2(A^2 + H^2 + 2H^+H^-) + \lambda_2(A^2 + H^2 + 2H^+H^-)^2 \right]$$

$$+ \frac{1}{2} (h+v)^2 \left[ \lambda_3 H^+H^- + \lambda_5 A^2 + \lambda_5 H^2 \right].$$

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where,
\[
\lambda_{L,S} = \frac{1}{2} (\lambda_3 + \lambda_4 \pm \lambda_5) .
\] (2.2)

Masses of these scalars are given by,
\[
\begin{align*}
M_h^2 &= \mu_1^2 + 3\lambda_1 v^2, \\
M_H^2 &= \mu_2^2 + \lambda_L v^2, \\
M_A^2 &= \mu_2^2 + \lambda_S v^2, \\
M_{H^\pm}^2 &= \mu_2^2 + \frac{1}{2}\lambda_3 v^2.
\end{align*}
\]

For \(\lambda_4 - \lambda_5 < 0\) and \(\lambda_5 > 0\) (\(\lambda_4 + \lambda_5 < 0\) and \(\lambda_5 < 0\)) implies that \(A\) (\(H\)) is the lightest \(Z_2\) odd particle (LOP). In this work, we take \(A\) as the LOP and hence, as a viable DM candidate. Choice of \(H\) as LOP will lead to similar results.

As we will explain later, for large DM mass, \(M_A \gg M_Z\), appropriate relic density of DM is obtained if \(M_A, M_H, M_{H^\pm}\) are nearly degenerate. Hence, in anticipation, we define
\[
\begin{align*}
\Delta M_H &= M_H - M_A, \\
\Delta M_{H^\pm} &= M_{H^\pm} - M_A.
\end{align*}
\]

so that the new independent parameters for the ID model becomes \(\{M_A, \Delta M_H, \Delta M_{H^\pm}, \lambda_2, \lambda_5\}\).

Here we have chosen \(\lambda_S\) as we treat \(A\) as the DM particle.

One-loop effective potential for \(h\) in \(\overline{\text{MS}}\) scheme and the Landau gauge is given by
\[
V_{1}^{\text{SM+ID}}(h) = V_{1}^{\text{SM}}(h) + V_{1}^{\text{ID}}(h)
\] (2.3)

where,
\[
V_{1}^{\text{SM}}(h) = \sum_{i=1}^{5} \frac{n_i}{64\pi^2} M_i^4(h) \left[ \ln \frac{M_i^2(h)}{\mu^2(t)} - c_i \right].
\] (2.4)

\(n_i\) is the number of degrees of freedom. For scalars and gauge bosons, \(n_i\) is positive, whereas for fermions it is negative. Here \(c_{h,G,f} = 3/2\), \(c_{W,Z} = 5/6\) and \(\mu(t) = M_Z \exp(t)\). \(M_i\) is given as
\[
M_i^2(h) = \kappa_i(t) h^2(t) - \kappa_i'(t).
\]

\(n_i, \kappa_i\) and \(\kappa_i'\) can be found in Eqn. (4) in Ref. \[20\] (see also Refs. \[21-24\]).

The additional contribution to the one-loop effective potential due to the inert doublet is given by \[16\],
\[
V_{1}^{\text{ID}}(h) = \sum_{j=H,A,H^+,H^-} \frac{1}{64\pi^2} M_j^4(h) \left[ \ln \left( \frac{M_j^2(h)}{\mu^2(t)} \right) - \frac{3}{2} \right]
\] (2.5)
where
\[ M_j^2(h) = \frac{1}{2} \lambda_j(t) h^2(t) + \mu_j^2(t) \]  (2.6)
with \( \lambda_A(t) = 2\lambda_S(t) \), \( \lambda_H(t) = 2\lambda_L(t) \) and \( \lambda_{H,\pm}(t) = \lambda_3(t) \).

In the present work, in the Higgs effective potential, SM contributions are taken at two-loop level \([6, 7, 25, 26]\), whereas the ID scalar contributions are considered at one-loop only.

For \( h \gg v \), the Higgs effective potential can be approximated as
\[ V_{\text{SM+ID}}^\text{eff}(h) \simeq \lambda_{1,\text{eff}}(h) \frac{h^4}{4}, \]  (2.7)
with
\[ \lambda_{1,\text{eff}}(h) = \lambda_{1,\text{SM}}(h) + \lambda_{1,\text{ID}}(h), \]  (2.8)
where \([6]\),
\[
\lambda_{1,\text{SM}}(h) = \sum_{j=L,S} e^{4\Gamma(h)} \left[ \lambda_1(\mu = h) + \lambda_1^{(1)}(\mu = h) + \lambda_1^{(2)}(\mu = h) \right] \\
\lambda_{1,\text{ID}}(h) = \sum_{j=L,S,3} e^{4\Gamma(h)} \left[ \delta_j \frac{\lambda_j^2}{64\pi^2} \left( \ln(\delta_j \lambda_j) - \frac{3}{2} \right) \right].
\]
Here \( \delta_j = 1 \) when \( j = L, S \), and \( \delta_j = \frac{1}{2} \) for \( j = 3 \) and
\[ \Gamma(h) = \int_{M_t}^{h} \gamma(\mu) \, d\ln \mu. \]

Anomalous dimension \( \gamma(\mu) \) of the Higgs field takes care of its wave function renormalization. As quartic scalar interactions do not contribute to wave function renormalization at one-loop level, ID does not alter \( \gamma(\mu) \) of SM. The expressions for the one- and two-loop quantum corrections \( \lambda_{1,\text{eff}}^{(1,2)} \) in SM can be found in Ref. \([6]\). All running coupling constants are evaluated at \( \mu = h \).

To compute the renormalisation group (RG) evolution of all the couplings, we first calculate all the SM couplings at \( M_t \) taking care of the threshold corrections as in Ref. \([4, 7, 8]\). Then we evolve them up to \( M_{\text{Pl}} \) using RG equations \([27–30]\). The corresponding \( \beta \)-functions include three-loop SM effects and one-loop ID contributions as presented in Appendix A. If DM mass \( M_A \) is larger than \( M_t \), then the ID starts to contribute after the energy scale \( M_A \). For \( M_A < M_t \), the contributions of ID to the \( \beta \)-functions are rather negligible for the running from \( M_A \) to \( M_t \) as evident from the expressions.

To help the reader in reproducing our results, we provide in Table I a specific set of values of \( \lambda_i \) at \( M_t = 173.1 \text{ GeV} \) and at \( M_{\text{Pl}} = 1.2 \times 10^{19} \text{ GeV} \) for \( M_h = 125.7 \text{ GeV} \) and \( \alpha_s(M_Z) = 0.1184 \). In Fig. 1, we explicitly show running of the scalar couplings \( (\lambda_i) \) for this set of parameters. We see that for this specific choice of parameters, \( \lambda_1 \) assumes a small negative value leading to a metastable EW vacuum as discussed in the following sections. This set is chosen to reproduce the DM relic density at the right ballpark.
### TABLE I

| $M_t$ | $\lambda_S$ | $\lambda_L$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_5$ | $\lambda_1$ |
|-------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
|       | 0.001       | 0.039       | 0.10        | 0.0399      | 0.00003     | 0.038       | 0.127       |
| $M_{Pl}$ | 0.045       | 0.081       | 0.125       | 0.087       | 0.038       | 0.036       | -0.009      |

A set of values of all ID model coupling constants at $M_t$ and $M_{Pl}$ for $M_A = 573$ GeV, $\Delta M_{H^\pm} = 1$ GeV, $\Delta M_H = 2$ GeV, $\lambda_S(M_Z) = 0.001$.

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**FIG. 1.** RG evolution of the couplings $\lambda_i$ ($i = 1, \ldots, 5$), $\lambda_L, \lambda_S$ for the set of parameters in Table I.

### III. CONSTRAINTS ON ID MODEL: A BRIEF REVIEW

ID model parameter space is constrained from theoretical considerations like absolute vacuum stability, perturbativity and unitarity of the scattering matrix. Electroweak precision measurements and direct search limits at LEP put severe restrictions on the model. The recent measurements of Higgs decay width at LHC puts additional constraints. The requirement that the ID DM saturates the DM relic density all alone restricts the allowed parameter space considerably. Although these bounds are already discussed in the literature, here we apply these bounds with the requirement that the model needs to be valid till $M_{Pl}$. 
A. Vacuum stability bounds

The tree level scalar potential potential \( V(\Phi_1, \Phi_2) \) is stable and bounded from below if

\[
\lambda_{1,2}(\Lambda) \geq 0, \quad \lambda_3(\Lambda) \geq -2\sqrt{\lambda_1(\Lambda)\lambda_2(\Lambda)}, \quad \lambda_{L,S}(\Lambda) \geq -\sqrt{\lambda_1(\Lambda)\lambda_2(\Lambda)} \tag{3.1}
\]

where the coupling constants are evaluated at a scale \( \Lambda \) using RG equations. However, these conditions become non-functional if \( \lambda_1 \) becomes negative at some energy scale to render the EW vacuum metastable. Under such circumstances we need to handle metastability constraints on the potential differently, which we pursue in the next section.

B. Perturbativity bounds

The radiatively improved scalar potential remain perturbative by requiring that all quartic couplings of \( V(\Phi_1, \Phi_2) \) satisfy

\[
|\lambda_{1,2,3,4,5}| \leq 4\pi \tag{3.2}
\]

C. Unitarity bounds

Unitarity bounds on \( \lambda_i \) are obtained considering scalar-scalar, gauge boson-gauge boson and scalar-gauge boson scatterings \[31\]. The constraints come from the eigenvalues of the corresponding S-matrix \[32\]:

\[
|\lambda_3 \pm \lambda_4| \leq 8\pi, \quad |\lambda_3 \pm \lambda_5| \leq 8\pi \\
|\lambda_3 + 2\lambda_4 \pm 3\lambda_5| \leq 8\pi \\
|-\lambda_1 - \lambda_2 \pm \sqrt{(\lambda_1 - \lambda_2)^2 + \lambda_4^2}| \leq 8\pi \\
| -3\lambda_1 - 3\lambda_2 \pm \sqrt{9(\lambda_1 - \lambda_2)^2 + (2\lambda_3 + \lambda_4)^2}| \leq 8\pi \\
| -\lambda_1 - \lambda_2 \pm \sqrt{(\lambda_1 - \lambda_2)^2 + \lambda_5^2}| \leq 8\pi \tag{3.3}
\]

D. Bounds from electroweak precision experiments

Bounds ensuing from electroweak precision experiments are imposed on new physics models via Peskin-Takeuchi \[33\] \( S, T, U \) parameters. The additional contributions from ID are given by \[32\] \[34\]

\[
\Delta S = \frac{1}{2\pi} \left[ \frac{1}{6} \ln \left( \frac{M_H^2}{M_{H^\pm}^2} \right) - \frac{5}{36} + \frac{M_H^2 M_A^2}{3(M_A^2 - M_H^2)^2} + \frac{M_A^4(M_A^2 - 3M_H^2)}{6(M_A^2 - M_H^2)^3} \ln \left( \frac{M_A^2}{M_H^2} \right) \right] \tag{3.4}
\]
and,

$$\Delta T = \frac{1}{32\pi^2\alpha v^2} \left[ F(M_{H^\pm}^2, M_A^2) + F(M_{H^0}^2, M_H^2) - F(M_A^2, M_H^2) \right]$$

(3.5)

where the function $F$ is given by

$$F(x, y) = \begin{cases} 
\frac{x+y}{2} - \frac{xy}{x-y} \ln \left( \frac{x}{y} \right), & x \neq y \\
0, & x = y 
\end{cases}$$

(3.6)

We use the NNLO global electroweak fit results obtained by the Gfitter group [35],

$$\Delta S = 0.06 \pm 0.09, \ \Delta T = 0.1 \pm 0.07$$

(3.7)

with a correlation coefficient of +0.91, fixing $\Delta U$ to zero. More stringent constraints on $\Delta S$ and $\Delta T$ are obtained this way as, instead, if $U$ is fitted as well, the bounds become rather relaxed:

$$\Delta S = 0.05 \pm 0.11, \ \Delta T = 0.09 \pm 0.13, \ \Delta U = 0.01 \pm 0.11$$

(3.8)

with correlation coefficients of +0.90 between $\Delta S$ and $\Delta T$, −0.59 between $\Delta S$ and $\Delta U$, −0.83 between $\Delta T$ and $\Delta U$.

**FIG. 2.** Allowed parameter space in $\Delta M_{H^\pm} - \Delta M_H$ plane for $M_A = 70$ GeV and $\lambda_S = 0.007$. Constraints from $S$ and $T$ parameters are shown by solid black, green, red and dashed purple lines. The blue band corresponds the 3σ variation in $\Omega h^2 = 0.1198 \pm 0.0026$ [35]. On the brown region the unitarity bound is violated on or before $M_{\text{Pl}}$. The cross-hatched region is excluded from LEP II data.
To assess the implications of $S$ and $T$ constraints on ID model, in Fig. 2, in $\Delta M_{H^\pm} - \Delta M_H$ plane, we display various constraints for $M_A = 70$ GeV and $\lambda_S = 0.007$. This is the maximum value of $\lambda_S$ for the given DM mass, allowed by LUX [37] direct detection data at 1$\sigma$. The blue region allowed by relic density constraints, shift upwards for smaller $\lambda_S$. Constraints on $\Delta S$ and $\Delta T$, as mentioned in Eqn. (3.7), are marked as black, green and red solid lines. We see that the 1$\sigma$ bound on $\Delta T$ is the most stringent one. The black line corresponding to the lower limit on $\Delta S$ at 1$\sigma$ can also be interesting. But the LEP II bounds, represented by the cross-hatched bar, takes away a considerable part. The line representing $\Delta S$ upper limit is beyond the region considered and lies towards the bottom-right corner of the plot. In this plot, increasing $M_H$ enhances $S$, but $T$ gets reduced. In addition, if we demand unitarity constraints to be respected up to $M_{Pl}$, assuming no other new physics shows up in between, the parameter space gets severely restricted. In this plot, a small window is allowed by $\Delta T$ only at 2$\sigma$, which satisfies DM relic density constraints. As mentioned earlier, this window gets further reduced with smaller $\lambda_S$ (the relic density band takes an ‘L’ shape as shown in Fig. 5).

However, such severe electroweak precision constraints are artefacts of using Eqn. (3.7). If we use Eqn. (3.8) instead, some parameter space is allowed by $\Delta T$ even at 1$\sigma$, as indicated by the dashed purple lines.

E. Direct search limits from LEP

The decays $Z \to AH$, $Z \to H^+H^-$, $W^\pm \to AH^\pm$ and $W^\pm \to HH^\pm$ are restricted from $Z$ and $W^\pm$ decay widths at LEP. It imply $M_A + M_H \geq M_Z$, $2M_{H^\pm} \geq M_Z$, $M_{H^\pm} + M_{H,A} \geq M_W$. More constraints on ID model can be extracted from chargino [38] and neutralino [39] search results at LEP II: The charged Higgs mass $M_{H^\pm} \geq 70$ GeV. The bound on $M_A$ is rather involved: If $M_A < 80$ GeV then $\Delta M_H$ should be less than $\sim 8$ GeV, or else, $M_H$ should be greater than $\sim 110$ GeV (see Fig. 4).

F. Bounds from LHC

In ID model, Higgs to diphoton signal strength $\mu_{\gamma\gamma}$ is defined as

$$\mu_{\gamma\gamma} = \frac{\sigma(gg \to h \to \gamma\gamma)}{\sigma(gg \to h \to \gamma\gamma)_{SM}} \approx \frac{Br(h \to \gamma\gamma)_{ID}}{Br(h \to \gamma\gamma)_{SM}}$$

using the narrow width approximation for production cross-section of $\sigma(gg \to h \to \gamma\gamma)$ and the fact that $\sigma(gg \to h)$ in both the SM and ID are the same.

Now if the ID particles have masses less than $M_h/2$, $h \to \text{ID}$, ID decays are allowed. In that
In case, 

$$\mu_{\gamma\gamma} = \frac{\Gamma(h \rightarrow \gamma\gamma)_{\text{ID}}}{\Gamma(h \rightarrow \gamma\gamma)_{\text{SM}}} \frac{\Gamma_{\text{tot}}(h \rightarrow \text{SM, SM})}{\Gamma_{\text{tot}}(h \rightarrow \text{SM, SM}) + \Gamma_{\text{tot}}(h \rightarrow \text{ID, ID})},$$  

(3.10)

where [40],

$$\Gamma(h \rightarrow \text{ID, ID}) = \frac{v^2}{16\pi M_h} \lambda^2_{\text{ID}} \left(1 - \frac{4M^2_{\text{ID}}}{M^2_h}\right)^{1/2},$$  

(3.11)

where for ID = A, H, H\(^\pm\), \(\lambda_{\text{ID}} = \lambda_S, \lambda_L, \sqrt{2}\lambda_3\).

In case, the ID particles are heavier than \(M_h/2\),

$$\mu_{\gamma\gamma} = \frac{\Gamma(h \rightarrow \gamma\gamma)_{\text{ID}}}{\Gamma(h \rightarrow \gamma\gamma)_{\text{SM}}}.$$

(3.12)

In ID model, the \(H^\pm\) gives additional contributions at one-loop. The analytical expression is given by [41]

$$\Gamma(h \rightarrow \gamma\gamma)_{\text{ID}} = \frac{\alpha^2 m^3_h}{256\pi^3 v^2} \left| \sum_f N^c_f Q^2 f y_f F_{1/2}(\tau_f) + y_W F_1(\tau_W) + Q^2_{H^\pm} \frac{v_{\mu_{hH^+H^-}}}{2m^2_{H^\pm}} F_0(\tau_{H^\pm}) \right|^2$$

(3.13)

where \(\tau_i = m^2_i/4m^2_f\). \(Q_f, Q_{H^\pm}\) denote electric charges of corresponding particles. \(N^c_f\) is the color factor. \(y_f\) and \(y_W\) denote Higgs couplings to \(f \bar{f}\) and \(WW\). \(\mu_{hH^+H^-} = \lambda_3 v\) stands for the coupling constant of \(hH^+H^-\) vertex. The loop functions \(F_{(0,1/2,1)}\) are defined as

$$F_0(\tau) = -[\tau - f(\tau)]\tau^{-2},$$

$$F_{1/2}(\tau) = 2[\tau + (\tau - 1)f(\tau)]\tau^{-2},$$

$$F_1(\tau) = -[2\tau^2 + 3\tau + 3(2\tau - 1)f(\tau)]\tau^{-2},$$

where,

$$f(\tau) = \begin{cases} 
\left(\sin^{-1} \left(\frac{\sqrt{\tau}}{\sqrt{\tau + 1}}\right)\right)^2, & \tau \leq 1 \\
\frac{1}{4} \ln \left[\frac{1 + \sqrt{1 - \tau}}{1 - \sqrt{1 - \tau}}\right] - i\pi, & \tau > 1
\end{cases},$$

(3.14)

From the diphoton decay channel of Higgs at LHC, the measured values are \(\mu_{\gamma\gamma} = 1.17 \pm 0.27\) from ATLAS [42] and \(\mu_{\gamma\gamma} = 1.14^{+0.26}_{-0.23}\) from CMS [43].

One can see that a positive \(\lambda_3\) leads to a destructive interference between SM and ID contributions in Eqn. (3.13) and vice versa. Hence, for ID particles heavier than \(M_h/2\), \(\mu_{\gamma\gamma} < 1\) (\(\mu_{\gamma\gamma} > 1\)) when \(\lambda_3\) is positive (negative). However, if these ID particles happen to be lighter than \(M_h/2\), they might contribute to the invisible decay of Higgs boson. Using the global fit result [44] that such an invisible branching ratio is less than \(\sim 20\%\), in Eqn. (3.10), the second ratio provides a suppression of \(\sim 0.8 - 1\).

Now can we work with a negative \(\lambda_3\) in ID model? We will discuss this at the end of this section. For the benchmark points used in this paper, we have worked only with positive values of \(\lambda_3\), allowed at 1\(\sigma\) by both CMS and ATLAS experiments.
G. Constraints from CMBR and Dark matter direct search limits

The ID dark matter candidate \( A \) can self-annihilate into SM fermions. Once the DM mass is greater than \( W \)-mass, so that the DM can annihilate into a pair of \( W \) bosons, the cross section increases significantly, thereby reducing DM relic density. Hence, it becomes difficult to saturate \( \Omega h^2 \) after \( \sim 75 \) GeV with positive \( \lambda_S \), although for \( M_A < 75 \) GeV, both signs of \( \lambda_S \) can be allowed to arrive at the right DM relic density \( \Omega h^2 \).

The role of the sign of \( \lambda_S \) can be understood from the contributing diagrams to the \( AA \to W^+W^- \) annihilation processes. Four diagrams contribute: the \( AAW^+W^- \) vertex driven point interaction diagram (henceforth referred to as the \( p \)-channel diagram), \( H^+ \) mediated \( t \) - and \( u \)-channel diagrams and \( h \) mediated \( s \)-channel diagram. For \( AA \to ZZ \) annihilation, the \( t \)- and \( u \)-channel diagrams are mediated by \( H \). A negative \( \lambda_S \) induces a destructive interference between the \( s \)-channel diagram with the rest, thereby suppressing \( AA \to W^+W^-, ZZ \) processes. For DM mass of 75–100 GeV, this can be used to get the appropriate \( \Omega h^2 \) \cite{15}. To avoid large contributions from \( t \)- and \( u \)-channel diagrams and coannihilation diagrams, the \( M_H \) and \( M_{H^\pm} \) can be pushed to be rather large \( \gtrsim 500 \) GeV. However, to partially compensate the remaining \( p \)-channel diagram by the \( s \)-channel one, \( \lambda_S \) assumes a large negative value \( \sim -0.1 \), which is ruled out by the DM direct detection experiments. That is why in the ID model, DM can be realised below 75 GeV, a regime we designate as the ‘low’ DM mass region.

At ‘high’ DM mass \( M_A \gtrsim 500 \) GeV, one can get the right \( \Omega h^2 \) due to a partial cancellation between different diagrams contributing to \( AA \to W^+W^- \) and \( AA \to ZZ \) annihilation processes. For example, in \( AA \to W^+W^- \), the \( p \)-channel diagram tends to cancel with the \( H^+ \) mediated \( t \)- and \( u \)-channel diagrams \cite{13} in the limit \( M_A \gg M_W \), and the sum of amplitudes of these diagrams in this limit is proportional to \( M_{H^\pm}^2 - M_A^2 \). Hence, at high \( M_A \), a partial cancellation between these diagrams is expected for nearly degenerate \( M_{H^\pm} \) and \( M_A \). Similarly, for \( AA \to ZZ \), a cancellation is possible when the masses \( M_H \) and \( M_A \) are close by. For \( M_A \gtrsim 500 \) GeV, keeping the mass differences of \( M_{H^\pm} \) and \( M_H \) with \( M_A \) within 8 GeV, such cancellations help reproduce the correct \( \Omega h^2 \). It is nevertheless worth mentioning that such nearly degenerate masses will lead to coannihilation of these \( Z_2 \) odd ID scalars \cite{40} to SM particles. Despite such near degeneracy, both \( H \) and \( H^+ \), being charged under the same \( Z_2 \) as \( A \), decay promptly to the LOP \( A \), so that they do not become relics. We use \texttt{FeynRules} \cite{17} along with \texttt{micrOMEGAs} \cite{48, 49} to compute relic density of \( A \).

DM direct detection experiments involve \( h \)-mediated \( t \)-channel process \( AN \to AN \) with a cross section proportional to \( \lambda_S^2/M_A^2 \) in the limit \( M_A \gg M_N \):

\[
\sigma_{A,N} = \frac{m_n^2}{\pi} f^2 m_N^2 \left( \frac{\lambda_S}{M_A M_H^2} \right)^2
\]

(3.15)

where \( f \approx 0.3 \) is the form factor of the nucleus. \( m_r \) represents the reduced mass of the nucleus.
and the scattered dark matter particle.

Thus, \( \lambda_S \) is constrained from non-observation of DM signals at XENON100 \([50, 51]\) and LUX \([37]\). For \( M_A = 70 \) GeV, the ensuing bound from LUX \([37, 52]\) data at 1\( \sigma \) is \(|\lambda_S| < 0.007\).

The constraint on \( \lambda_S \) from DM direct detection experiments gets diluted with \( M_A \) (see Eqn. (3.15)). Hence, for ‘low’ DM mass, direct detection bounds are more effective. At high mass, the relic density constraints are likely to supersede these bounds. For example, for \( M_{DM} = 573 \) GeV, the upper limit on \(|\lambda_S|\) is 0.138 from LUX. However, to satisfy the relic density constraints from combined data of WMAP and Planck within 3\( \sigma \), \( \lambda_S \) can be as large as 0.07 only.

Within the framework of ID model it is possible to explain the observations in various indirect DM detection experiments \([14, 53]\) for some regions of the parameter space. In this paper, however, we do not delve into such details as such estimations involve proper understanding of the astrophysical backgrounds and an assumption of the DM halo profile which contain some arbitrariness.

**Sign of \( \lambda_3 \)**

Whether \( \lambda_3 \) can be taken as positive or negative, depends on the following:

- If ID model is not the answer to the DM puzzle, so that both relic density and direct detection constraints can be evaded, no restriction exists on the possible sign of \( \lambda_3(M_Z) \). Otherwise, the following two cases need be considered.

- A negative \( \lambda_3(M_Z) \) implies

  \[
  \lambda_S(M_Z) < -\frac{1}{v^2}(M_{H^\pm}^2 - M_A^2) .
  \]

  As we are considering \( A \) as the DM candidate, so that \( M_A < M_{H^\pm} \), \( \lambda_S(M_Z) \) is always negative when \( \lambda_3(M_Z) < 0 \). For low DM mass, the splitting \((M_{H^\pm} - M_A) \gtrsim 10 \) GeV as otherwise DM coannihilation processes cause an inappreciable depletion in \( \Omega h^2 \). For \( M_A = 70 \) GeV, this implies a lower bound \( \lambda_S(M_Z) \lesssim -0.025 \), which violates the DM direct detection bound \(|\lambda_S| < 0.007\). Hence, for low DM mass, a negative \( \lambda_3(M_Z) \) is not feasible.

- For high mass DM, the right relic density can be obtained when the splitting \((M_{H^\pm} - M_A) \sim \) a few GeV or less. The above logic then implies that a negative \( \lambda_3 \) does not put any severe restriction on \( \lambda_S \) to contradict DM direct detection bounds as earlier. Hence, for high DM mass, \( \lambda_3(M_Z) \) can assume both the signs. Moreover, due to propagator suppression for large \( M_{H^\pm} \) in the \( h\gamma\gamma \) vertex, the ID contribution to \( \mu_{\gamma\gamma} \) is negligibly small and hence, the sign of \( \lambda_3(M_Z) \) is not constrained by measurements on \( \mu_{\gamma\gamma} \) as well.
• If at any scale, $\lambda_3$ is negative while $\lambda_1 > 0$, then the bound (3.1) must be respected.

• If at some scale, $\lambda_1 < 0$, then a negative $\lambda_3$ makes the potential unbounded from below, as mentioned in the following section. This means one can start with a negative $\lambda_3(M_Z)$, but with RG evolution when $\lambda_1$ turns negative, $\lambda_3$ evaluated at that scale must be positive. Such parameter space does exist. Here, we note a significant deviation of our analysis from earlier analyses which did not allow a negative $\lambda_1$. For example, in Ref. [13] a negative $\lambda_3(M_Z)$ was not allowed from stability of the Higgs potential if the theory has to be valid up to $10^{16}$ GeV.

IV. TUNNELING PROBABILITY AND METASTABILITY

![Graph](image)

FIG. 3. (a) Tunneling probability $P_0$ dependence on $M_t$. The left band (between dashed lines) corresponds to SM. The right one (between dotted lines) is for ID model for DM mass $M_A = 573$ GeV. Constraints from WMAP and Planck measured relic density, as well as XENON 100 and LUX DM direct detection null results are respected for these specific choice of parameters. Light-green band stands for $M_t$ at $\pm 1\sigma$. (b) $P_0$ is plotted against Higgs dark matter coupling $\lambda_S(M_Z)$ for different values of $\lambda_2(M_Z)$.

In the standard model, the present measurements on $M_h$ and $M_t$ indicate that the electroweak vacuum, in which the Universe is at present residing, may be a false one. From this metastable vacuum, it might tunnel into a deeper true vacuum, residing close to $M_{Pl}$.  

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The decay probability of the EW vacuum to the true vacuum at the present epoch can be expressed as \[ 6, 54, 55 \]
\[ P_0 = 0.15 \frac{\Lambda_B^4}{H^4} e^{-S(\Lambda_B)} \]
where the action is given by
\[ S(\Lambda_B) = \frac{8\pi^2}{3|\lambda_1(\Lambda_B)|}. \]
\( S(\Lambda_B) \) is called the action of bounce of size \( R = \Lambda_B^{-1} \). The value of \( R \) for which \( S(\Lambda_B) \) is minimum, gives the dominant contribution to the tunnelling probability \( P_0 \). It occurs when \( \lambda_1(\Lambda_B) \) is minimum, i.e., \( \beta \lambda_1(\Lambda_B) = 0 \). Here we neglect loop \[ 54 \] and gravitational corrections \[ 56, 57 \] to the action as in Ref. \[ 4 \].

The presence of inert doublet induces additional contributions to \( \beta \lambda_1 \) (see Eqn. (A2)). As a result, which is generic for all scalars, \( \lambda_1 \) receives a positive contribution compared to the SM, which pushes a metastable vacuum towards stability, implying a lower \( P_0 \).

Electroweak metastability in ID model has been explored earlier in the literature, albeit, in a different context \[ 58–60 \]. If \( H^+ \) gets a VEV, there could exist another charge violating minimum. If instead, \( A \) receives a VEV, another CP violating minimum could pop up. But these vacua always lie higher than the usual EW vacuum. If \( Z_2 \) is broken introducing additional soft terms in the Lagrangian, then the new \( Z_2 \)-violating minimum can be lower than the usual \( Z_2 \)-preserving EW minimum. As in our present work, \( Z_2 \) is an exact symmetry of the scalar potential, such cases need not be considered. However, as mentioned earlier, if at some scale before \( M_{\text{Pl}} \), \( \lambda_1 \) becomes negative, there might exist a deeper minimum which is charge-, CP- and \( Z_2 \)-preserving and lying in the SM Higgs \( h \) direction.

Whether the EW vacuum is metastable or unstable, depends on the minimum value of \( \lambda_1 \) before \( M_{\text{Pl}} \), which can be understood as follows. For EW vacuum metastability, the decay lifetime should be greater than the lifetime of the Universe, implying \( P_0 < 1 \). This implies \[ 4, 54 \]
\[ \lambda_1(\Lambda_B) > \lambda_{1,\text{min}}(\Lambda_B) = \frac{-0.06488}{1 - 0.00986 \ln \left( \frac{v}{\Lambda_B} \right)} . \]
Hence we can now re-frame the vacuum stability constraints on ID model, when \( \lambda_1 \) runs into negative values, implying metastability of EW vacuum. We remind the reader that in ID model instability of the EW vacuum cannot be realised as addition of the scalars only improve the stability of the vacuum.

- If \( 0 > \lambda_1(\Lambda_B) > \lambda_{1,\text{min}}(\Lambda_B) \), then the vacuum is metastable.
- If \( \lambda_1(\Lambda_B) < \lambda_{1,\text{min}}(\Lambda_B) \), then the vacuum is unstable.
- If \( \lambda_2 < 0 \), the potential is unbounded from below along the \( H, A \) and \( H^\pm \)-direction.
• If $\lambda_3(\Lambda_I) < 0$, the potential is unbounded from below along a direction in between $H^\pm$ and $h$.

• If $\lambda_L(\Lambda_I) < 0$, the potential is unbounded from below along a direction in between $H$ and $h$.

• If $\lambda_S(\Lambda_I) < 0$, the potential is unbounded from below along a direction in between $A$ and $h$.

In the above, $\Lambda_I$ represents any energy scale for which $\lambda_1$ is negative.

The tunnelling probability $P_0$ is computed by putting the minimum value of $\lambda_{1,\text{eff}}$ in Eqn. (4.2) to maximise $S(\Lambda_B)$. In Fig. 3(a), we have plotted $P_0$ as a function of $M_t$. The right band corresponds to the tunnelling probability for our benchmark point as in Table I. For comparison, we plot $P_0$ for SM as the left band in Fig. 3(a). 1σ error bands in $\alpha_s$ and $M_h$ are also shown. The error due to $\alpha_s$ is more significant than the same due to $M_h$. As expected, for a given $M_t$, the presence of ID lower tunnelling probability. This is also reflected in Fig. 3(b), where we plot $P_0$ as a function of $\lambda_S(M_Z)$ for different choices of $\lambda_2(M_Z)$, assuming $M_h = 125.7$ GeV, $M_t = 173.1$ GeV and $\alpha_s = 0.1184$. Here DM mass $M_A$ is also varied with $\lambda_S$ to get $\Omega h^2 = 0.1198$. For a given $\lambda_S(M_Z)$, higher the value of $\lambda_2(M_Z)$, $P_0$ gets smaller, leading to a more stable EW vacuum.

V. PHASE DIAGRAMS

The stability of EW vacuum depends on the value of parameters at low scale, chosen to be $M_Z$. In order to show the explicit dependence of EW stability on various parameters, it is customary to present phase diagrams in various parameter spaces.

In Fig. 4, we show the LEP constraints in $M_H - M_A$ plane as in Ref. 39. We update this plot identifying regions of EW stability and metastability. As we are considering a scenario where ID model is valid till $M_{\text{Pl}}$, there are further limits from unitarity. The relic density constraint imposed by WMAP and Planck combined data is represented by the thin blue band. The choice of $\lambda_2(M_Z)$ does not have any impact on relic density calculations, but affects EW stability as expected. In this plot, for higher values of $\lambda_2(M_Z)$, the region corresponding to EW metastability will be smaller. The chosen parameters satisfy LUX direct detection bound.

As small splitting between $M_A$, $M_H$ and $M_{H^\pm}$ leads to some cancellations between diagrams contributing to DM annihilation, $\Delta M_{H^\pm}$ and $\Delta M_H$ are often used as free parameters in ID model. In Fig. 5, we present constraints on this parameter space for $M_A = 70$ GeV. As before, the brown region corresponds to unitarity violation before $M_{\text{Pl}}$. For small $\Delta M_{H^\pm}$ and $\Delta M_H$, $\lambda_{3,4,5}$ are required to be small, which leads to little deviation from SM metastability. The metastable region is shown by the yellow patch, which shrinks for larger $\lambda_2$. The blue band reflects relic density constraint for $\lambda_S(M_Z) = 0.001$. For such small $\lambda_S(M_Z)$, the $h$-mediated $s$-channel diagram in $AA \rightarrow WW$ or $AA \rightarrow ZZ$ contributes very little. $H^+$ or $H$ mediated $t$- and $u$-channel diagrams are also less
FIG. 4. Constraints in $M_H - M_A$ plane. The cross-hatched region is excluded from LEP [39]. Choosing $M_{H^\pm} = 120$ GeV and $\lambda_S(M_Z) = 0.001$, relic density constraint is satisfied at $3\sigma$ on the blue band. The green (yellow) region corresponds to EW vacuum stability (metastability). The solid brown line correspond to $M_H = M_A$. The grey area on the left to it is of no interest to us as we have chosen $M_H > M_A$. The dashed brown line shows the LEP I limit. On the brown region, unitarity constraints are violated before $M_{Pl}$.

important than the quartic vertex driven diagram due to propagator suppression. This explains the ‘L’ shape of the blue band. For higher values of $\lambda_S(M_Z)$, the shape of the band changes and ultimately leads to a closed contour. It appears that due to LEP constraints, EW vacuum metastability is almost ruled out. Although the LEP constraint permits $\Delta M_H < 8$ GeV, allowing a narrow strip towards left, the relic density constraints cannot be satisfied on this strip as it leads to increased rate of DM coannihilation processes, leading to a dip in $\Omega h^2$. But as we will see later, if $M_t$ and $\alpha_s$ are allowed to deviate from their respective central values, for some region in this parameter space, it is possible to realise a metastable EW vacuum.

To delineate the role of $M_H$ in EW vacuum stability, in Fig. 4 and 5, $\lambda_2(M_Z)$ was chosen to be small. Now to demonstrate the effect of $\lambda_S$, we will now present in Fig. 6 phase diagrams in $\lambda_S(M_Z) - M_A$ plane. Panel (a) deals with low DM masses. For $\Delta M_{H^\pm} = 40$ GeV and $\Delta M_H = 40$ GeV, part of the allowed relic density band (blue) is allowed from LEP constraints (cross-hatched band). The entire parameter space correspond to EW vacuum stability. Choosing small $\Delta M_{H^\pm}$ and $\Delta M_H$, which imply small values of $\lambda_{3,4,5}$, can lead to metastability. But those regions are excluded by LEP. Again, metastability can creep in if $M_t$ and $\alpha_s$ are allowed to deviate from their central values.
FIG. 5. Phase diagram in $\Delta M_H - \Delta M_{H\pm}$ plane for $M_A = 70$ GeV. The green and yellow regions correspond to EW vacuum stability and metastability respectively. The cross-hatched band is excluded from LEP. The brown region suffers from unitarity violation before $M_{P1}$. The blue band reflects relic density constraint at $3\sigma$.

In Fig. 6(b), we study the same parameter space for ‘high’ DM masses. As mentioned before, to obtain the correct relic density, smaller mass splitting between various ID scalars need to be chosen. For $\Delta M_{H\pm} = 1$ GeV and $\Delta M_H = 2$ GeV, the $3\sigma$ relic density constraint is shown as the blue band. The blue dashed line demarcates the boundary between stable (green) and metastable (yellow) phases of EW vacuum. Choice of small values of $\Delta M_{H\pm}$ and $\Delta M_H$, in turn, leads to a large region pertaining to EW metastability.

To illustrate the sensitivity to the mass splitting, in Fig. 6(b), we present another relic density band (red) when $\Delta M_{H\pm} = 5$ GeV and $\Delta M_H = 2$ GeV. The corresponding boundary between the phases is denoted by the red dashed line. The region on the right imply EW stability (green and yellow regions does not apply to this case). As for high DM masses, EW metastability can be attained for a sizeable amount of the parameter space, $\lambda_2(M_Z)$ need not be chosen to be very small to maximise the metastable region for the sake of demonstration.

The fact, that for SM the EW vacuum stability is ruled out at $\sim 3\sigma$, is demonstrated by a phase space diagram in the $M_t - M_h$ plane [6,7]. In Ref. [4], similar diagrams were presented for a singlet scalar extended SM. To demonstrate the impact of ID scalars to uplift the EW vacuum metastability, we present phase diagram in $M_t - M_h$ plane for two sets of benchmark points in Fig. 7. Panel (a) is drawn for $M_A = 70$ GeV, $\Delta M_{H\pm} = 11.8$ GeV, $\Delta M_H = 45$ GeV, $\lambda_S(M_Z) = 0.001$ and $\lambda_2(M_Z) = 0.1$. For panel (b) the set of parameters in Table I is being used. Both sets of parameters
are chosen so that they respect the WMAP and Planck combined results on DM relic density and the direct detection bounds from XENON100 and LUX. As in Ref. [4], the line demarcating the boundary between stable and metastable phases of EW vacuum is obtained demanding the two vacua are at the same depth, implying \( \lambda_1(\Lambda_B) = \beta_\lambda(\Lambda_B) = 0 \). The line separating metastable phase from the unstable one is drawn using the conditions \( \beta_\lambda(\Lambda_B) = 0 \) and \( \lambda_1(\Lambda_B) = \lambda_{1,\text{min}}(\Lambda_B) \), as in Eqn. (4.3). The variations due to uncertainty in the measurement of \( \alpha_s \) are marked as dotted red lines. In each panel, the dot representing central values for \( M_h \) and \( M_t \) is encircled by 1\( \sigma \), 2\( \sigma \) and 3\( \sigma \) ellipses representing errors in their measurements. According to Fig. 7(a), EW vacuum stability is allowed at 1.4\( \sigma \), whereas, in Fig. 7(b), it is excluded at 2.2\( \sigma \), indicated by blue-dashed ellipses.

As in the literature SM EW phase diagrams are also presented in \( \alpha_s(M_Z) - M_t \) plane [8, 61], we do the same in ID model as well. In Fig. 8 we use the same sets of benchmark parameters, as in Fig. 7. As a consistency check, one can note that the EW vacuum is allowed or ruled out at the

FIG. 6. Phase diagram in \( \lambda_S(M_Z) - M_A \) plane for \( \lambda_2(M_Z) = 0.1 \). Panel (a) stands for ‘low’ DM mass. The blue band corresponds the 3\( \sigma \) variation in \( \Omega h^2 \) when \( \Delta M_{H^\pm} = 40 \) GeV and \( \Delta M_H = 40 \) GeV. LEP direct search constraints are represented by the cross-hatched band at the bottom. Entire green region imply EW vacuum stability. Panel (b) stands for ‘high’ DM masses. The relic density band (blue) now correspond to \( \Delta M_{H^\pm} = 1 \) GeV and \( \Delta M_H = 2 \) GeV. The corresponding stable and metastable phases for EW vacuum are represented by green and yellow patches respectively. The relic density band (red) corresponds to \( \Delta M_{H^\pm} = 5 \) GeV and \( \Delta M_H = 2 \) GeV. For this, the boundary separating the EW phases is denoted by the red dashed line.
FIG. 7. Phase diagrams in $M_h - M_t$ plane. Panels (a) and (b) stand for ‘low’ and ‘high’ DM masses respectively. Regions of absolute stability (green), metastability (yellow), instability (red) of the EW vacuum are also marked. The grey zones represent error ellipses at 1, 2 and 3σ. The three boundary lines (dotted, solid and dotted red) correspond to $\alpha_s(M_Z) = 0.1184 \pm 0.0007$. Details of benchmark points are available in the text.

same confidence levels.

To study the impact of non-zero ID couplings, however, it is instructive to study the change in the confidence level (σ) at which EW stability is modified with respect to these couplings. As in Ref. [4], we plot in Fig. 9 σ against $\lambda_S(M_Z)$ for different values of $\lambda_2(M_Z)$. We vary $M_A$ along with $\lambda_S(M_Z)$ to keep DM relic density fixed at $\Omega h^2 = 0.1198$ throughout the plot. Note that changing $\lambda_2(M_Z)$ does not alter $\Omega h^2$. The masses of other ID particles are determined using $\Delta M_{H^\pm} = 1$ GeV and $\Delta M_H = 2$ GeV. The parameter space considered does not yield too large DM-nucleon cross section, inconsistent with XENON 100 and LUX DM direct detection null results. For a specific value of $\lambda_2(M_Z) = 0.1$, with increase of $\lambda_S(M_Z)$, the confidence level at which EW is metastable (yellow region) gets reduced and becomes zero at $\lambda_S(M_Z) \simeq 0.042$. After this, EW vacuum enters in the stable phase (green). With further increase in $\lambda_S(M_Z)$, the confidence level at which EW is stable keeps on increasing. To illustrate the role of $\lambda_2(M_Z)$, we use two other values in the same plot. The value of $\lambda_S(M_Z)$ at which the EW vacuum enters in the stable phase, increases with decrease in $\lambda_2(M_Z)$, as expected. The yellow and green marked regions are not applicable when $\lambda_2(M_Z) = 0.05, 0.15$. 
FIG. 8. Phase diagrams in $M_t - \alpha_s(M_Z)$ plane for the same sets of benchmark points as in Fig. 7. Notations used are also the same as in Fig. 7.

FIG. 9. Dependence of confidence level at which EW vacuum stability is excluded (one-sided) or allowed on $\lambda_5(M_Z)$ and $\lambda_2(M_Z)$. Regions of absolute stability (green) and metastability (yellow) of EW vacuum are shown for $\lambda_2(M_Z) = 0.1$. The positive slope of the line corresponds to the stable electroweak vacuum and negative slope corresponds to the metastability.
VI. VELTMAN’S CONDITIONS

Veltman’s condition implies that the quadratic divergences in the radiative corrections to the Higgs mass can be handled if the coefficient multiplying the divergence somehow vanishes \([62]\). In SM, it suggests, the combination

\[
6\lambda_1 + \frac{9}{4}g_2^2 + \frac{3}{4}g_1^2 - 12y_t^2 = 0.
\]

In ID model, both \(\mu_1\) and \(\mu_2\) receive quadratically divergent radiative corrections. Veltman’s conditions imply \([34, 63]\).

\[
6\lambda_2 + 2\lambda_3 + \lambda_4 + \frac{9}{4}g_2^2 + \frac{3}{4}g_1^2 - 12y_t^2 = 0
\]

To satisfy both, \(\lambda_2 = \lambda_1 - 2y_t^2\). The present data indicate that \(\lambda_1 - 2y_t^2\) is always negative up to \(M_{\text{Pl}}\), assuming no other new physics creeps in between. But a negative \(\lambda_2\) renders the potential unbounded from below. Hence, it is not possible to satisfy Veltman’s condition in a scenario where only the ID model reigns the entire energy regime up to the \(M_{\text{Pl}}\). It all happens as \(\mathbb{Z}_2\) forbids fermionic interactions of the inert doublet \(\Phi_2\).

VII. SUMMARY AND CONCLUSION

If the standard model is valid up to the Planck scale, the present measurements on the masses of top quark and Higgs indicate the presence of a deeper minimum of the scalar potential at a very high energy scale, threatening the stability of the present electroweak vacuum. State of the art NNLO calculations performed to evaluate the probability that the present EW vacuum will tunnel into the deeper vacuum lying close to \(M_{\text{Pl}}\) suggest that the present EW vacuum is metastable at \(\sim 3\sigma\). The lack of stability might be the artefact of incompleteness of the SM.

Although LHC is yet to find any signal suggesting existence of any new physics beyond the standard model of particle physics, various other experimental evidences point towards the existence of dark matter, which so far could have escaped detection in colliders and DM direct detection experiments. Hence, it is important to look into the problem of EW stability in a scenario which addresses the issue of DM as well. In particular, we extend SM adding an inert scalar doublet, offering a viable DM candidate and assume that this model is valid up to \(M_{\text{Pl}}\).

In this paper, our intention is twofold. First, in such a scenario we have consolidated the bounds imposed on ID model. As we are demanding validity of the model up to \(M_{\text{Pl}}\), the RG evolution of the couplings can disturb the unitarity of the S-matrix governing various scattering processes,
which in turn imposes stringent limits on the parameter space at the EW scale. In this light, we present a consolidated discussion on the existing bounds on ID model.

The other goal of this paper is to check the stability of EW vacuum in ID model. If the ID DM happens to be the only DM particle, which saturates the observed DM relic density, can the ID model modify the stability of the EW stability? Note that rather than considering new physics effects close to the Planck scale, here new physics is added at EW scale only. It is well known that addition of a scalar can improve stability of the EW vacuum. But if we are to solve both the DM and EW vacuum stability problems in the context of the ID model, it is important to study the parameter space which allows us to do so. As ID introduces a few new parameters and fields, the study of the parameter space is quite involved when we consider radiatively improved scalar potential, containing SM NNLO corrections and one-loop ID contributions. Inclusion of these NNLO corrections are mandatory to reproduce the correct confidence level at which EW vacuum is metastable in the SM. Requiring the potential bounded from below at all scales below $M_{Pl}$ puts severe constraints on the model. We show the sensitivity of the parameters towards EW vacuum stability. We see that for DM masses of 70 GeV, the allowed parameter space corresponds to absolute stability unless we allow some deviation of $M_t$ and $\alpha_s$ from their measured central values. For higher DM masses more than 500 GeV, it is possible to realise a metastable EW vacuum for a large parameter space. This vacuum will have a longer lifespan than the SM one as addition of scalars improves stability of the EW vacuum.

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Appendix A: One-loop beta functions

The beta functions of the quartic coupling parameters for the ID model are defined as

$$\beta_{\lambda_i} = 16\pi^2 \frac{\partial \lambda_i}{\partial \ln \mu}.$$  \hspace{1cm} (A1)
The expressions at one-loop are given by [13, 64]

\[
\begin{align*}
\beta_{\lambda_1} &= 24\lambda_1^2 + 2\lambda_2^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + \lambda_5^2 \\
&+ \frac{3}{8} (3g_4^2 + g_4^1 + 2g_2^2g_1^2) - 3\lambda_1 (3g_2^2 + g_1^2) \\
&+ 4\lambda_1 \left( y_r^2 + 3y_b^2 + 3y_t^2 \right) - 2 \left( y_4^2 + 3y_4^1 + 3y_4^0 \right), \\
\beta_{\lambda_2} &= 24\lambda_2^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + \lambda_5^2 \\
&+ \frac{3}{8} (3g_4^2 + g_4^1 + 2g_2^2g_1^2) - 3\lambda_2 (3g_2^2 + g_1^2), \\
\beta_{\lambda_3} &= 4 (\lambda_1 + \lambda_2) (3\lambda_3 + \lambda_4) + 4\lambda_3^2 + 2\lambda_3^2 + 2\lambda_5^2 \\
&+ \frac{3}{4} (3g_4^2 + g_4^1 - 2g_2^2g_1^2) - 3\lambda_3 (3g_2^2 + g_1^2) \\
&+ 2\lambda_3 \left( y_r^2 + 3y_r^1 + 3y_b^2 \right), \\
\beta_{\lambda_4} &= 4\lambda_4 (\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_4) + 8\lambda_5^2 \\
&+ 3g_2^1 g_1^2 - 3\lambda_4 (3g_2^1 + g_1^2) \\
&+ 2\lambda_4 \left( y_r^2 + 3y_r^1 + 3y_b^2 \right), \\
\beta_{\lambda_5} &= 4\lambda_5 (\lambda_1 + \lambda_2 + 2\lambda_3 + 3\lambda_4) \\
&- 3\lambda_5 (3g_2^2 + g_1^2) \\
&+ 2\lambda_5 \left( y_r^2 + 3y_r^1 + 3y_b^2 \right).
\end{align*}
\]

Let us note the \(y_t\) dependence of these expressions. While \(\beta_{\lambda_1}\) is dominated by \(y_4^1\) term, \(\beta_{\lambda_2}\) does not depend on \(y_t\). The \(y_t\) dependence of other \(\beta_{\lambda_i}\)s are softened by the corresponding \(\lambda_i\) multiplying the \(y_2^t\) terms.

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