Steady-State Analysis Based on Space Vector Model of Self-Excited Induction Generators

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Abstract. In order to attain more optimization for the procedure of transient and steady-state operation performance analysis and evaluation of self-excited induction generators (SEIGs), the paper converts the state-space transient mathematic model with real domain into the space vector transient model with complex domain based on the same two-phase stationary reference frame, described by which the physical systems can apply extensively the holistic analysis approach based on Lyapunov stability theory proposed in real domain. Through equivalent analysis, the extended form of the steady-state conditions and its mathematic description into the complex domain is obtained, that is, the extended quasi limit cycle and the steady-state operating points.

The analytic calculation for the extended steady-state condition provides successively the analytic solutions of exciting capacitance, stator frequency, as well as the extended steady state conditions under given rotor speed and magnetizing inductance. The practical calculation example verifies correctness and effectiveness of the approach and the analytic solution of the extended steady-state condition, with engineering reference values.

1. Introduction

Self-excited induction generators (SEIGs) have many advantages such as low cost, high reliability, easy maintenance, short-circuit self-protection. In construction of green small hydropower, small wind power and other energy-saving and environmentally friendly new energy micro-grid, the research on its theory and application technology have received increasingly more attention [1].

Considering many factors such as the cost-effectiveness of related hardware equipment, energy-saving efficiency, and control complexity, in-depth research on SEIG theory is extremely important. There are two main types of research methods. One is based on the steady-state equivalent circuit [2,3]. The analytic calculation is relatively simple, but the dynamic response cannot be evaluated. There are inevitably problems of calculation efficiency and convergence; the other is based on transients, etc. The effective circuit has a clearer physical meaning, and can calculate the dynamic response of the generator frequency and its various state parameters. The method also uses numerical calculation methods [4-6] at the beginning, so the calculation efficiency and versatility are not good.

The analysis based on the transient equivalent circuit is more comprehensive, universal and engineering reference value. Applying extended Hurwitz complex domain criterion to calculate the performance under resistive load conditions [7]; the approach based on the Lyapunov stability of the overall theory analysis can be applicable to more comprehensive and in-depth analysis [8], both the steady state [9] transient process of self-excited build-up and switching load is fully analysed [10]. Although these methods are relatively effective, the calculation process and effect are not good, and the results cannot even be calculated analytically. The extended Hurwitz criterion can be also applied to
analyse the steady-state performance [11], but the calculation needs to be further optimized, and the mechanism analysis needs to be integrated and in-depth to make the unified analysis of transient steady-state operation stability more applicable.

Based on the space vector transient mathematical model of SEIGs in the two-phase stationary $\alpha\beta$ reference frame, the paper proposes the extended steady-state conditions in the complex domain. Secondly, the steady-state conditions of the complex domain are analysed and the steady-state running stator frequency analytical formula and its possible variation range, the analytical formula and theoretical value range of the exciting capacitance under given load and rotor speed, and the analytical solution to the extended steady-state conditions is attained. Finally, the proposed approach has good agreement of the calculated results, simplicity of calculation and applicability of analytical formulas, which is applicable to more complex systems, through the verification of examples. The analytical formulas for the extended steady-state conditions in the complex domain are helpful for system configuration optimization and performance evaluation.

2. Space vector model of SEIGs

2.1. Space vector transient equivalent circuit

The voltage and flux-linkage equations of squirrel-cage induction generators can be equivalently transformed from the three-phase stationary reference frame to the two-phase stationary reference frame by the $abc/\alpha\beta$ transformation. Neglecting the winding slot effect, the length of the air gap can be considered even. The coefficient of the magnetizing inductance is thus transformed into a time-invariant coefficient in the arbitrary reference frame. The magnetizing inductance is consequently a function of only the state parameter, namely the magnetizing current. However, characteristics never change with state parameters’ coordinate transformation [12].

The space vector transient equivalent circuits in the two-phase stationary reference frame of SEIGs are shown in Figure 1, where the positive direction is indicated. The physical and state parameters represent real values transformed into the stator side. $p$ is the differential operator; $\vec{i}$ and $\vec{i}$ are the stator and rotor current vectors, respectively; $\vec{\psi}$ and $\vec{\psi}$ are the stator and rotor flux linkage vectors, respectively; $\vec{u}$ is the stator voltage vector; $R_s$ and $R_r$ are the stator and rotor resistances, respectively; $L_{ls}$ and $L_{lr}$ are the stator and rotor leakage inductances, respectively; $M$ is the magnetizing inductance between the stator and rotor windings; $C$ is the exciting capacitance; and $\omega$ is the rotor speed, which is constant. Because the magnetizing inductance is a function of the magnetizing current, which continuously varies with time, the transient equivalent circuits are nonlinear. Figure 1 gives $L_s=M+L_{ls}$, and $L_r=M+L_{lr}$, which are the stator and rotor self-inductances, respectively. The analytic formula of the exciting current at the transient operating states is

$$ I_m = |\vec{v}_m| = |\vec{i}_s + \vec{i}_r| $$

Figure 1. Space-vector equivalent circuit of SEIG in the stationary reference frame

To facilitate the analysis of the multivariable nonlinear transient mathematical model given by Figure 1, it is necessary to make the following assumptions: (i) Iron loss in induction machines is neglected. (ii) Only the state-space main wave component of the air-gap magneto-motive force and the time main
wave component of the electromotive force along with the current are considered. (iii) The effect of the
stator frequency and temperature on the winding resistance is neglected.

2.2. Space vector transient mathematical model
Since the subject content in this paper is continuation and further depth of that in the research [13], the
model adopted in the paper is the same space vector transient mathematical model, that is

\[
\dot{X} = AX
\]  

In (2), \(X = [\tilde{I}_s, \tilde{I}_r, \tilde{u}_s]^T\), \(\tilde{I}_s = i_{s \alpha} + j i_{s \beta}\), \(\tilde{I}_r = i_{r \alpha} + j i_{r \beta}\), \(\tilde{u}_s = u_{s \alpha} + j u_{s \beta}\), and

\[
A = \begin{bmatrix} L_s R_s + j \omega M^2 & MR_r - j \omega L_s M & 1 \\ MR_r + j \omega L_s M & \sigma L_s L_s & 0 \\ \frac{1}{C} & 0 & \frac{1}{CR_s} \end{bmatrix}
\]

where, \(\sigma\) denotes the leakage coefficient, \(\sigma = 1 - M^2/(L_s L_s)\).

As (1) shows, the magnetizing current \(I_m\) continuously changes during the transient process. On the
other hand, \(M\) has a fixed functional relation with the magnetizing current, which depends on the
magnetizing curve of the unloaded induction machine. When \(I_m\) is small, the generator operates in the
unsaturated region of the magnetic circuit; \(M\) can be considered a constant, and the transient system is
linear. Once \(I_m\) is larger than a certain value, the generator operates in the saturated region of the
magnetic circuit; \(M\) will decrease with increasing \(I_m\), and the transient system is nonlinear.

3. The extended form of the steady-state conditions in complex domain

3.1. The quasi limit cycle condition extended in complex domain
Since both transient mathematical model representations for SEIGs are the same in the basic properties,
the theoretical analysis approach proposed in [9] can be extended to the case in complex domain.

The mathematical description of the transient mathematical model represented by the complex matrix
for the build-up and the critical stability conditions of steady-state operation is: the initial transient state
of the system self-excitation build-up pressure or the linearization after local linearization at any other
transient operating point The characteristic equation (4) of the complex coefficient matrix \(A\) of the state
differential equation has a common point \(j \omega s\) with the imaginary axis.

\[
det(\lambda I - A) = 0
\]  

3.2. The quasi steady-state operating point condition extended in complex domain
The mathematical description of the extended steady-state operating point condition of the transient
mathematical model system represented by the complex coefficient matrix is: the transient operating
point \(Y\) in the complex number domain is the solution of the complex coefficient matrix equation (4).

\[
AY = j \omega Y
\]  

Then, similar to the SEIG transient mathematical model represented by the real coefficient matrix,
the extended steady-state conditions of the SEIG transient mathematical model system represented by
the complex coefficient matrix are composed of the quasi-limit cycle (3) and the extended steady-state
operating point (4) The steady-state condition for the expansion of the complex domain.

4. Analytic Calculation of the extended form of the steady-state conditions in complex domain

4.1. The quasi limit cycle
The real and imaginary parts expanded by equation (4) are zero respectively, we can get
\[
\begin{cases}
  h_1 \omega_s^2 + h_2 \omega_s + h_3 = 0 \\
  h_4 \omega_s^2 + h_5 \omega_s + h_6 = 0
\end{cases}
\] (5)

where,
\[
\begin{align*}
  h_1 &= \frac{1}{2} \left( \frac{L_s R_r}{L_r} + R_l \right) \\
  h_2 &= \frac{1}{2} \left( \frac{L_s R_r}{L_r} + R_l \right) \\
  h_3 &= \frac{1}{2} \left( \frac{L_s R_r}{L_r} + R_l \right) \\
  h_4 &= \frac{1}{2} \left( \frac{L_s R_r}{L_r} + R_l \right)
\end{align*}
\]

When the rotor speed \( \omega \) and load \( R_l \) are known, \( C \) can be given by the equation with the real part equal to 0, which is a function of stator frequency \( \omega_s \), rotor speed \( \omega \), and load \( R_l \).

The substitution of (6) for \( C \) in the second equation of (5) gives two positive real solutions for \( \omega_s \) as
\[
\begin{align*}
  \omega_{s,\text{min}} &= \frac{\omega}{2} + \frac{1}{2} \left( \sqrt{\Delta^2 - \sqrt{A}} \right) \\
  \omega_{s,\text{max}} &= \frac{\omega}{2} + \frac{1}{2} \left( \sqrt{\Delta^2 + \sqrt{A}} \right)
\end{align*}
\] (7)

where
\[
\begin{align*}
  \Delta &= -g_1 + 2g_2 + \frac{1}{4} g_3 \sqrt{\Delta} \\
  A &= g_1 + g_2 + g_3 = \frac{1}{2} \left( \sqrt{f_1 - \sqrt{-4 f_1^2 + f_2^2}} + \sqrt{f_1 + \sqrt{-4 f_1^2 + f_2^2}} \right)
\end{align*}
\]

The substitution of analytic formulas in (7) for \( \omega_s \) in (6) gives the analytic formulas for the exciting capacitance \( C \). In terms of \( \omega \), \( R_l \) and \( M \). Since the analytic function of the exciting capacitance \( C \) with respect to the stator frequency is a monotonically decreasing, \( \omega_s \) takes the maximum value of \( \omega_{s,\text{max}} \), \( C \) attains the minimum value; while \( \omega_s \) takes the minimum value \( \omega_{s,\text{min}} \), \( C \) takes the maximum value.

\[
\begin{align*}
  C_{\text{max}} &= \frac{L_s R_r \sigma (\eta^2 - \eta_1^2) - 4 R_l (R_r + R_l)}{R_l [L_s R_r (\omega^2 - \eta_1^2) - L_s (\omega + \eta_1)^2]} \\
  C_{\text{min}} &= \frac{L_s R_r \sigma (\eta^2 - \eta_2^2) - 4 R_l (R_r + R_l)}{R_l [L_s R_r (\omega^2 - \eta_2^2) - L_s (\omega + \eta_2)^2]}
\end{align*}
\] (8)

where, \( \eta_1 = \sqrt{\Delta} + \sqrt{A} \), \( \eta_2 = \sqrt{\Delta} - \sqrt{A} \), other parameters are the same as above. Under certain conditions of given speed and load, the value range of the exciting capacitance to ensure successful self-excitation and voltage build-up is: \( (C_{\text{min}}, C_{\text{max}}) \).

4.2. The quasi steady-state operating point

Assuming the voltage component of quasi steady-state operating point at certain time as \( \tilde{u}_o \), then
\[
A_i \tilde{i}_s \quad \tilde{u}_o \quad i \quad u_s^T\]
(9)

According to the computation method proposed by [9], (9) can be equivalently transformed into (10).

\[
GZ = U_s
\] (10)

where, \( Z = [i_s, \tilde{i}_s]^T \), and
\[
U_s = \left[ \frac{1}{\sigma L_s} - \frac{M}{\sigma L_s L_i} \sigma L_i \sigma L_i \right] u_s^T G = \left[ \begin{array}{c}
  -\frac{L_s R_r + j\omega M^2}{\sigma L_s L_i} - j\omega_i \quad \frac{M R_r + j\omega M}{\sigma L_i L_i} - j\omega_i \\
  \frac{M R_r + j\omega M}{\sigma L_i L_i} - j\omega_i \quad \frac{R_r - j\omega M}{\sigma L_i L_i} - j\omega_i \\
\end{array} \right] u_s^T
\]

Furthermore, the analytical formulas for the stator current \( \tilde{i}_s \) and \( \tilde{i}_s \), the rotor current \( \tilde{i}_r \), and the exciting current \( \tilde{i}_m \) calculated from the generator end and the stator winding respectively are:
\[
\begin{align*}
i_n &= \left(\frac{1}{R_s} + jC\omega_s\right)u_n, i_r &= -\frac{R_s + jL_s(\omega - \omega_s)}{d_c} u_s, \\
i_r &= -\frac{jM(\omega - \omega_s)}{d_c} u_r, i_m &= -\frac{R_s + j(L_s - M)(\omega - \omega_s)}{d_c} u_o,
\end{align*}
\]

where \( d_c = R_sR_c + L_sL_r\omega \omega_s + j[L_sR_o\omega_s - L_c(\omega - \omega_s)] \) is the common complex denominator of analytic formulas. The magnitude of the state parameter is the modulus of the corresponding complex vectors.

5. Case analysis and verification
In order to facilitate the verification of the correctness of the analytical calculation formula for the extended steady-state condition of the complex number domain, this paper adopts the conditions consistent with the literature [9].

The analysis is performed with a small induction machine with three phases and four poles, but the principles are applicable to large machines. Its rated values are \( P_N = 2.2 \text{ kW}, U_N = 380 \text{ V (Y-connected)}, I_N = 5 \text{ A}, f_s = 50 \text{ Hz}, \) and \( n_N = 1430 \text{ r/min}, \) and the parameters are \( n_P = 2, R_s = 3.383 \Omega, R_r = 2.973 \Omega, L_{ls} = L_{lr} = 8.479 \text{ mH}, \) and \( M_0 = 0.2875 \text{ H}. \) To facilitate analytical computation, this study uses an analytic approximation of the magnetizing curve obtained experimentally. The approximation of the magnetizing inductance curve was developed in [9] and verified experimentally. The inductance for different current regions is defined as follows:

\[
M = \begin{cases} 
0.2875, & I_m \leq 1.163 \\
3.500/(I_m + 11.01) & 1.163 < I_m \leq 2.162 \\
3.099/(I_m + 9.503) & 2.162 < I_m \leq 3.046 \\
2.519/(I_m + 7.152) & 3.046 < I_m \leq 4.553 \\
1.527/(I_m + 2.544) & I_m \geq 4.553
\end{cases}
\]

For the loaded build-up process, there must be remanence \( Y_0 \neq 0, \) and it can basically be regarded as a linear system. In the tiny neighborhood of the origin of the system equilibrium point, there is still \( M=0.2875 \text{ H}, \) so we might as well take \( R_l = 600 \Omega, \omega = 314 \text{ rad/s}, \) then it can be obtained from equations (9) and (10), the critical stability limit cycle conditions The stator frequency and excitation capacitance are: \( \omega_{s,\text{min}} = 172.41 \text{ rad/s}, C_{\text{max}} = 1857.3 \mu \text{F}; \omega_{s,\text{max}} = 311.98 \text{ rad/s}, C_{\text{min}} = 35.230 \mu \text{F}. \) The calculation results of the quasi steady-state conditions for each set of solutions are shown in Table 1.

| \( R_l \) = 600 \( \Omega \), \( M = 0.2875 \text{ H}, \omega = 314 \text{ rad/s} \) | \( \omega_{s,\text{min}} = 172.41 \text{ rad/s}, C_{\text{max}} = 1857.3 \mu \text{F} \) | \( \omega_{s,\text{max}} = 311.98 \text{ rad/s}, C_{\text{min}} = 35.230 \mu \text{F} \) |
|---|---|---|
| \( \omega_{s,\text{min}} \) | 15.238 | 15.238 |
| \( \omega_{s,\text{max}} \) | 1.1864 | 1.1864 |
| \( \omega_{c,\text{min}} \) | 14.765 | 0.2276 |
| \( \omega_{c,\text{max}} \) | 1.1630 | 1.1630 |
| \( \psi_0 \) | 1.1630 | 1.1630 |
| \( \psi_1 \) | 47.589 | 106.72 |

Table 1 shows that the Ist and Is calculation data results of the two analytical calculation formulas all satisfy \( I_{st} = I_s \). Obviously, due to the large difference between the stator frequency value and the rotor speed value of the first set of data, and the excitation capacitance value is also too large, it is difficult to achieve in practice. In addition, the stator current and the rotor current are both too large and the excitation current is too small, which is also practical. Physical conditions are not allowed, because this situation will make the generator magnetic circuit far too saturated and damage the equipment, so the first set of analytical solutions for the extended limit cycle conditions can be omitted. In summary, under the given conditions of rotor speed and load resistance value, limited by the actual operating conditions of the physical system, the analytical solution of the extended steady-state condition associated with the excitation capacitance value is unique, and the unique set of analytical solutions is
\[C = \frac{L_1 L_2 \sigma (\eta_2 - \omega^2) - 4 R_c (R_i + R_c)}{R_1 (L_2 L_3 (\omega^2 - \eta_2^2) - L_2 L_2 (\omega + \eta_2)^2), \omega = \frac{\omega}{2} \left( \sqrt{\Delta_2} + \sqrt{\Delta_1} \right), i_o = \frac{1}{R_i} + j C \omega_o u_o, i_s = -\frac{R_c + \frac{1}{d_c} (\omega - \omega_o)}{d_c} u_o, i_l = -\frac{-j M (\omega - \omega_o)}{d_c} u_o, i_u = \frac{-R_c + \frac{1}{d_c} (L_c - M) (\omega - \omega_o)}{d_c} u_o}\]

where, \( h_1, h_2, h_3, \eta_2, d_c, \Delta_1, \Delta_2, a, b, c, u_0 \) is the same as above.

6. Conclusion

Aiming at the problem that SEIGs is difficult to analyse and calculate under load, this paper proposes a steady-state operation based on the space vector model of the SEIG in the stationary two-phase reference frame Analytical calculation method of performance parameters. Different from the steady-state condition based on the mathematical model of the state space, this article can be regarded as an extended form in the complex domain of the steady-state condition, but it is simpler and more efficient than the calculation of the steady-state condition in the real number domain, mainly in that the order of the matrix equation is reduced half. The computational complexity and amount of computation are also greatly reduced. By expanding the analytical calculation of steady-state conditions, the results obtained are completely consistent with that by other methods. So it shows the correctness and effectiveness of the method in this paper, which can be applied to more complex systems.

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