MEASURING CLUSTER PECULIAR VELOCITIES AND TEMPERATURES AT CENTIMETER AND MILLIMETER WAVELENGTHS

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ABSTRACT
I present a detailed investigation of issues related to the measurement of peculiar velocities and temperatures using Sunyaev-Zel’dovich (SZ) effects and estimate the accuracy to which peculiar velocities and gas temperatures of distant galaxy clusters could be measured. With μK sensitivity on arcminute scales at several frequencies it will be possible to measure peculiar velocities to an accuracy of ~130 km s⁻¹ and gas temperatures to better than 1 keV. The limiting factor for the accuracy of tpec is the presence of bulk motions within the galaxy cluster, even for apparently relaxed clusters. The accuracy of the temperature is mainly limited by noise. These results are independent of redshift, provided the clusters of interest are distant (z ≥ 0.15). Using only three frequencies, the optimal strategy is to place one observing frequency in the Rayleigh-Jeans region (ν < 40 GHz), one near 150 GHz, and the third at 300 GHz or higher. Measurements at the null of the thermal SZ effect are of marginal utility, other than as a foreground/background monitor.

Subject headings: cosmic microwave background — cosmology: theory — galaxies: clusters: general — large-scale structure of universe

On-line material: color figures

1. INTRODUCTION
Observations of the Sunyaev-Zel’dovich (SZ) effect (Sunyaev & Zel’dovich 1972) are currently at a mature stage. Highly significant detections are now routine and the next generation of instruments is about to exploit the SZ effect to provide deep surveys of galaxy clusters (for a recent review see Carlstrom, Holder, & Reese 2002). All current and near-future instruments are devoted to studies of the thermal SZ effect. There are a host of yet more subtle distortions of the cosmic microwave background (CMB) spectrum that contain a wealth of information related to the cluster’s peculiar velocity (tpec), the gas temperature (T) of the intracluster medium, and the optical depth to Thomson scattering (τ).

In this paper we investigate the most significant of the subtler distortions, namely those due to the line-of-sight motion (the kinetic SZ effect) and the distortion of the spectrum due to relativistic effects (the relativistic thermal SZ effect). These effects are discussed in several reviews (Rephaeli 1995; Birkinshaw 1999) and are presented in great detail by several authors (Sunyaev & Zel’dovich 1980; Rephaeli 1995; Challinor & Lasenby 1998; Sazonov & Sunyaev 1998; Itoh, Kohyama, & Nozawa 1998; Nozawa, Itoh, & Kohyama 1998; Molnar & Birkinshaw 1999; Dolgov et al. 2001).

At sensitivities below ~1 μK some higher order effects could be important such as multiple scatterings and transverse velocities. At such low levels, CMB anisotropies will also be a very difficult contaminant. For simplicity, we restrict ourselves to the most significant effects.

The importance of using the SZ effect to its fullest potential comes from its redshift independence. As a spectral distortion of the CMB, it redshifts along with the CMB and the amplitude of the distortion does not suffer from cosmological dimming. For a cluster of fixed physical properties the SZ decrement along a given line of sight will be the same. However, it will project into a different angular size at different redshifts, so a given experiment with a fixed angular resolution will sample different parts of the cluster at different redshifts.

If there are clusters at redshift z ~ 2 (as there should be in standard models of structure formation), they will be very faint in X-rays, and perhaps undetectable if the gas has been preheated by an early burst of star formation. An independent probe of the gas temperature would be invaluable, and preliminary steps along these lines are being made (Hansen, Pastor, & Semikoz 2002). Measurements of peculiar velocities would allow reconstruction of the large-scale density field on scales comparable to the horizon (Dore, Knox, & Peeb 2002). A firm understanding of the large-scale density inhomogeneities would allow new tests of galaxy formation and provide a view of the evolution of structure.

The best frequencies for observations of these more subtle SZ effects have not been systematically addressed. It is usually assumed that the null of the thermal SZ effect is optimal, but it will be shown below that this is not optimal in the sense of signal-to-noise ratio for measuring peculiar velocities.

In the next section I lay out the physical effects that allow such powerful tests. In ß 3.1 investigate the observing frequencies that are best suited for such an investigation. A discussion of the particular challenge offered by primary CMB anisotropies is presented in § 4. The details of the simulations and map-making methods are outlined in § 5 followed by a summary and discussion of practical issues.

In all that follows, CMB and SZ temperature differences (∆T) will refer to thermodynamic temperatures (not, for example, brightness temperature). In the Rayleigh-Jeans region this distinction is not very important, but at higher frequencies the differences become large.

2. SZ EFFECTS
CMB photons have roughly a 1% probability of interacting with free electrons in the deep potential wells of galaxy clusters. Compton scattering leads to an exchange of energy between the cool photons and hot electrons, causing a distortion of the CMB spectrum. This is known as the thermal SZ effect. Relativistic effects in Compton scattering and the
electron energy distribution both depend on the electron gas temperature, providing a means of probing the gas temperature. The general form of the thermal SZ effect is a temperature decrement at low frequencies and an increment at high frequencies. The main effects of the gas temperature are to slightly reduce the amplitude of the distortion (in a frequency dependent way) and to shift the null of the distortion to a slightly higher frequency.

The kinetic SZ effect arises from a bulk velocity of the galaxy cluster along the line of sight. The scattered CMB photons essentially pick up a slight redshift or blueshift from the peculiar velocity of the cluster. The imprint on the CMB is a purely thermal distortion, i.e., the spectrum is exactly that of a blackbody at a slightly higher or lower temperature. The imprint on the CMB is photons essentially pick up a slight redshift or blueshift from slightly higher frequency.

A short-dashed curve shows the thermal SZ effect (i.e., \( kT/mc^2 \)) and only 1% of photons, on average, undergo scattering. Therefore, the cumulative effect relative to the 2.73 K background is on the order of one part in \( 10^4 \). The kinetic effect is even smaller, since \( \tau_{\text{spec}}/c \) is typically on the order of \( 10^{-3} \), making the typical fractional energy exchange an order of magnitude below that expected for the thermal effect. The distortions of the thermal SZ effect due to the gas temperature are comparable in magnitude to the kinetic SZ effect. Importantly, all of these effects have different spectral behavior, manifesting themselves differently as a function of frequency. Figure 1 shows some SZ effects as a function of frequency, both for temperature and intensity. The dominant effect is clearly the nonrelativistic thermal SZ effect, showing the well-known decrement at low frequencies and increment at higher frequencies. Note that the optical depth for the lower temperature cluster has been scaled to provide the same net Comptonization. Both peculiar velocity and the gas temperature shift the spectrum, but the effect of the velocity on the temperature decrement (or increment) is constant with frequency, while the gas temperature provides a nontrivial spectral signature. This could allow a well-designed experiment with several observing frequencies to separate these effects and measure the gas temperature and peculiar velocity.

In Figure 1 it is apparent that the relativistic temperature shifts are comparable to the velocity effects. Compared with the input spectrum the differences are typically on the order of 10% or smaller, so a cluster with a 500 \( \mu K \) central decrement (Rayleigh-Jeans) would have velocity and temperature effects at below the 50 \( \mu K \) level. Compared with noise levels that are possible for upcoming experiments this is a large signal, but obtaining maps at such high signal-to-noise ratio at several frequencies will be challenging.

3. OBSERVING FREQUENCIES

Separation of the various SZ components requires at least three observing frequencies (there are three unknowns: \( \tau, T, v \)). To address the best positioning of these frequencies, a small numerical experiment was performed.

We take the range of observable frequencies to be 10–350 GHz, with lower frequencies likely contaminated by radio point sources and higher frequencies possibly contaminated by Galactic dust and extragalactic dusty galaxies. In steps of 10 GHz, we assumed the first frequency was at a frequency in the range 10–300 GHz. For each of these values, we then stepped \( \nu_2 \) in 10 GHz intervals over all frequencies from \( \nu_1 + 10 \) to 330 GHz and then allowed the third frequency \( \nu_3 \) to be in the range \( \nu_2 + 10 \) to 350 GHz.

At each position we use the fitting functions of Nozawa et al. (1998) to calculate the expected SZ temperature decrement or increment. We use the Fisher matrix formalism to estimate errors in the three parameters for each \((\nu_1, \nu_2, \nu_3)\) point. This method essentially approximates the likelihood function \( L \) as a Gaussian function of parameters \( p \) near its peak and assumes that the curvature (second derivative) at the peak gives a good estimate of uncertainties.

We define the Fisher matrix as

$$ F_{ij} = \left< \frac{\partial^2 \ln L}{\partial p_i \partial p_j} \right> $$

where the angled brackets indicate an ensemble average over all realizations of data. If the data products are taken to be \( \Delta T_{\text{obs}} \), the thermodynamic temperature decrements or increments at frequencies \( \nu = \nu_\alpha \) and with experimental measurement uncertainties \( \sigma_\alpha \), then the likelihood of the parameters for a given set of data is simply

$$ -2 \ln L = \chi^2 = \sum_{\alpha=1}^3 \left[ \frac{\Delta T_{\text{obs}} - \Delta \hat{T}(\nu_\alpha, p)}{\sigma_\alpha} \right]^2, $$

where \( \Delta \hat{T} \) is simply the model prediction as a function of frequency and fiducial model.

In this case, the Fisher matrix reduces to (after ensemble averaging)

$$ F_{ij} = \sum_{\alpha=1}^3 \frac{1}{\sigma_\alpha^2} \frac{\partial \Delta \hat{T}(\nu_\alpha, p)}{\partial p_i} \frac{\partial \Delta \hat{T}(\nu_\alpha, p)}{\partial p_j}, $$

where all derivatives are evaluated at the point where the \( p \) take their fiducial values.
Derivatives are calculated numerically by stepping from the fiducial model by ±0.1 keV in temperature, ±10 km s⁻¹ in velocity, and ±1% of the fiducial τ. The minimum 1 σ single-parameter uncertainties, marginalized over other parameters are then simply (F⁻¹)ᵢᵢ. Note that a Gaussian prior in a parameter is easily applied by simply adding 1/σᵢ² to the diagonal element Fᵢᵢ, where σᵢ² is the variance of the prior on parameter i.

In Figure 2 we show the square root of the determinant of the inverse Fisher matrix as a function of frequencies, assuming equal variances at each point of σᵢ = 1 μK. The square root of the determinant of the inverse Fisher matrix is a measure of the volume of the ellipsoid in parameter space that would give the single-parameter 68% confidence regions. For uncorrelated parameters, the determinant would simply be ∏ᵢ=₁ σᵢ², where σᵢ² is the variance in parameter i.

The best results are obtained if the first frequency is as low as possible and the third frequency is as high as possible. For practical purposes, all frequencies below about 90 GHz work well, as do all frequencies above 300 GHz. What is interesting is that the second frequency is best placed at 150 GHz, not at the null of the thermal effect. Frequencies near the null of the thermal effect (∼220 GHz) are not particularly good places to try to get constraints on parameters. Furthermore, higher order terms to the observed spectrum can become significant near the null, complicating the analysis.

As a concrete example of why the thermal null is not necessarily a good place, take frequencies of 150, 220, and 300 GHz. In this case, the main effect of the temperature correction is to reduce the thermal SZ spectrum at the two end frequencies and to shift the thermal SZ null to a slightly higher frequency. Without an external temperature measurement, this can be degenerate with a slightly lower Comptonization (τ[kT/mₑc²]), which reduces the overall amplitude of the curve, and a peculiar velocity away from the observer, which slightly shifts the curve toward lower temperature decrements/increments. Whether or not this degeneracy can be broken will depend somewhat on the fiducial model.

Note that we have assumed uniform errors at each frequency in thermodynamic temperature difference measurements (not intensity). This is highly idealized, in that foregrounds and noise are highly frequency dependent. As motivation for this choice, we assume that this sort of experiment would be a natural outgrowth of a small-angle CMB experiment, where it might be expected that the experimental goal would be something close to uniform temperature sensitivity as a function of frequency.

We have ignored finite bandwidth issues, which could be especially problematic at the null of the thermal effect but should not be a large problem at the frequencies that turn out to be optimal. All quantities are relatively smoothly varying over a typical bandwidth of 20 GHz, as shown in Figure 1.

Pointing offsets and beam uncertainties in the map must be carefully controlled. For sub-μK errors, the beam and pointing must be known to a scale small compared with cluster core radii (roughly 30″), thus requiring arcsecond pointing and arcsecond beam modeling. The beam modeling is particularly demanding. The effect of beam uncertainties is required to introduce less than 1% error. Put another way, the resulting image must have a dynamic range of roughly 1000:1 or better.

If one has gas temperatures from X-ray spectroscopy, the constraints on parameters might be expected to improve significantly. We added a term to the Fisher matrix corresponding to an independent gas temperature measurement of ±1 keV. This does not significantly change the preferred frequency placement, with the lowest variances coinciding either with or without external gas temperature information. At a sensitivity of 1 μK the gas temperature measurement from the SZ data alone will be sufficiently good that a ±1 keV measurement is not of much additional use for frequencies that are nearly optimal and μK sensitivity. For nonoptimal frequency coverage or less sensitive measurements, additional information on the gas temperature provides significant leverage for velocity information. However, from hydrodynamical simulations, it would not be expected that the X-ray emission weighted temperature will agree with the mean electron temperature (which is relevant here) to better than 1 keV (Mathiesen & Evrard 2001). This would suggest that the SZ data itself may be better suited for temperature measurements than X-ray spectroscopy. SZ measurements provide a γ-weighted temperature, rather than the nₑ² weighting of X-ray spectroscopy. The pressure profile should be more extended than nₑ², leading to a more representative gas temperature. However, this is still not the mean electron temperature that is required to estimate the optical depth relevant for peculiar velocities.

From the Fisher matrix estimates, single-parameter uncertainties on gas temperature, peculiar velocity, and optical depth, assuming a noise level of 1 μK per frequency, are δT ∼ 0.5 keV, δv ∼ 30 km s⁻¹ and δτ ∼ 6 × 10⁻⁴ for a fiducial model of T = 6 keV, vₑc = 200 km s⁻¹, τ = 0.01. These results will scale inversely with the assumed value of τ and will scale linearly with the assumed noise level.

For a fiducial model with a positive peculiar velocity (away from the observer) the component separation is much more effective, providing an estimate of the velocity that is more precise by roughly a factor of 2. This behavior was recently noted by Aghanim et al. (2003). The best constraints on velocity shown in Figure 3 are comparable to what would be expected from the thermal noise level, while the higher velocity errors

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Fig. 2.—Uncertainty volumes as a function of observing frequencies. Dashed contours show size of uncertainty volume for v₁ on x-axis and v₂ on y-axis, while solid contours instead are for v₁ on the x-axis. Contours are in steps of 0.2 dex relative to smallest volume. Fiducial model in this case has Tₑ = 12 keV, vₑ = 200 km s⁻¹, τ = 0.01. [See the electronic edition of the Journal for a color version of this figure.]
The two largest sources of secondary anisotropies are the thermal SZ effect and the Ostriker-Vishniac (OV) effect. For the OV contribution we simply assume a flat band power contribution of 1 μK (Hu & Dodelson 2002). This is a contribution that is a real signal on the sky (unlike thermal noise in the detectors, for example) that will be completely correlated at different frequencies and is therefore only a source of confusion in assigning a physical meaning to the separated component that has a purely thermal component. Thus, this is a source of noise, just like the primary CMB contamination, that can be considered a source of noise in estimating peculiar velocities, but not a source of noise for the component separation or the estimates of gas temperature and optical depth.

The thermal SZ effect anisotropy power is strongly non-Gaussian, and since we are subtracting out the contribution of the cluster in question it is not clear what level of thermal SZ contamination from unresolved clusters will remain. Furthermore, there is significant uncertainty in the small-scale power expected, due to the unknown effects of nongravitational processes (such as gas cooling and feedback from star formation) on the intracluster medium. Using Press-Schechter arguments, it is estimated that the rms power on arcminute scales from unresolved SZ clusters (i.e., after bright sources have been removed) would be ~3 μK (Holder & Carlstrom 2001), assuming $\sigma_0 = 1$, or ~2 μK for $\sigma_0 = 0.9$. These estimates are uncertain at the level of factors of 2. This will only contribute confusion to the separation of temperature and optical depth, since this component will obviously have an SZ spectrum, with a different effective temperature than the main cluster. The bias in the derived temperature should be roughly comparable to its fractional contribution to the total spectrum. For a 500 μK decrement (in the Rayleigh-Jeans limit) the bias in the derived temperature and optical depth will be at the percent level or lower. Therefore, the bias due to residual SZ contamination in the peculiar velocity estimate will also be very small.

To investigate the effects of CMB contamination of peculiar velocity estimates, we used CMBFast\(^1\) (Seljak & Zaldarriaga 1996) to generate a power spectrum (including lensing) of a $\Lambda$CDM model and included 1 μK (flat band power) additional noise due to secondary anisotropies. As a cluster model, for simplicity we assumed the electron number density profile is given by $n_e(r) \propto (1 + r^2 / r_c^2)^{-1}$, $r$ is the three-dimensional radius and $r_c$ is a characteristic scale radius. This profile formally diverges at large radii, but since the CMB anisotropy dominates at large radii this is not a problem. The Fourier transform of the projected surface density for this profile is analytic. Denoting the core radius in angular units as $\theta_c$, the central temperature decrement or increment $\Delta T_0$, and working in the flat sky approximation, the profile as a function of multipole $\ell$ is given by

$$\Delta T(\ell) = \Delta T_0 2\pi \theta_c \exp \left(-\frac{\theta_c \ell}{\ell}\right). \quad (4)$$

More centrally concentrated profiles will have relatively more power at higher $\ell$, reducing the importance of the CMB; our adopted profile is relatively conservative, in that most clusters have been observed to have more concentrated profiles. We allow a range of angular core radii.

The contribution per ln interval to the central decrement/increment in temperature for this profile is $\ell^2 \Delta T(\ell) / 2\pi$. This can be compared with the noise power per ln interval given by

\(^1\) See http://www.cmbfast.org.
Fig. 4.—Comparison of kinetic SZ signal with confusion from primary CMB and detector noise. Top curve at low multipoles shows the primary CMB, with the effect of detector noise with a $\sigma = 30''$ beam shown as the exponential rise at high $\ell$. From left to right, the bottom curves show the contribution to the signal per ln interval in $\ell$ of a cluster with a core radius of 120'', 60'', 30'', and 15''. An optical depth of 0.01 and a peculiar velocity of 300 km s$^{-1}$ has been assumed. Note that the noise term adds in quadrature per ln interval. [See the electronic edition of the Journal for a color version of this figure.]

$\ell(\ell + 1) \delta T / 2\pi$. In Figure 4 we show the rms noise per ln interval compared with the temperature contribution per ln interval for cluster models with several cluster angular core radii. A central temperature difference of 27.3 $\mu$K was assumed, corresponding to a peculiar velocity of 300 km s$^{-1}$ for a central optical depth of 0.01. Of course, the optical depth and core radius are not independent, and a smaller core radius would require a higher optical depth for the same optical depth. However, it is instructive to assume a fixed central signal to understand the effects of the spatial filtering. For angular core radii below an arcminute the kinetic SZ is plainly well above the noise contribution. The highest curve at low $\ell$ shows the contribution of the primary CMB anisotropy, with the effects of detector noise and a $\sigma = 30''$ Gaussian beam shown by the rapid rise at high $\ell$.

Following Haehnelt & Tegmark (1996) we can construct a matched filter to estimate the signal-to-noise ratio that can be expected with optimal filtering of the CMB. The optimal filter is shown in Figure 5, where it can be seen that all structure on scales larger than about an arcminute are filtered out.

In the absence of thermal noise in the detector the best possible signal-to-noise ratio for a 300 km s$^{-1}$ cluster with $\theta_c = 30''$ is approximately 6.5, which can be translated into roughly 50 km s$^{-1}$ of noise. When thermal noise is included at the level of 1 $\mu$K per arcminute pixel (with a Gaussian beam of $\sigma_{\text{beam}} = 30''$) the signal-to-noise ratio drops to 3.5, indicating that the combination of thermal detector noise confusing the small scales and CMB contamination confusing the large scales adds an equivalent peculiar velocity uncertainty of 85 km s$^{-1}$. All velocity errors assume an optical depth of 0.01 and will scale inversely with $\tau$.

For core radii above approximately 2'' or beam sizes larger than about 3'' peculiar velocity estimates will be hopelessly confused by CMB contamination for all but the largest velocities, as pointed out by, for example, Haehnelt & Tegmark (1996). Measurements will still provide good sensitivity to large-scale bulk flows when several cluster measurements are combined. For distant clusters and sensitive experiments with small beams the CMB contamination in individual clusters can be well below 100 km s$^{-1}$. For less massive clusters, with smaller optical depths, the CMB contamination will be more important (as measured in units of velocity), with the noise in units of velocity scaling inversely with $\tau$.

It is important to keep in mind that the primary CMB contamination only affects errors in the determination of the peculiar velocity and does not affect either the errors in $\tau$ and $T_c$ or the optimal frequencies, assuming that high-dynamic range maps are available.

5. SIMULATIONS

For an isothermal galaxy cluster with a unique peculiar velocity, the results of the previous section are sufficient to estimate uncertainties. In reality, however, galaxy clusters are not isothermal and bulk flows on the order of the sound speed can persist for up to 10% of the Hubble time. In order to investigate the effects of such phenomena we turn to numerical simulations of galaxy clusters.

The simulations that were used have been described elsewhere (Holder et al. 2000; Mohr, Mathiesen, & Evrard 1999; Mohr & Evrard 1997) and were graciously provided by Professor A. Evrard of the University of Michigan. The cosmological model was a flat low-density universe with matter density (in units of the critical density) of 0.3 and a Hubble constant of 80 km s$^{-1}$ Mpc$^{-1}$. The simulation method is P$^3$M-SPH (Evrard 1988), where the hydrodynamics are done using the SPH method and the gravity is done with a particle-particle method for nearby particles and long-range forces are calculated from a grid. A dark matter only simulation was performed in a large volume to identify the positions of galaxy clusters, and all particles that ended up within the cluster virial radius were traced back to the initial conditions and replaced with more particles, some dark matter and some gas, each of...
smaller mass. Large particles outside the virial radius acted to include the effects of large-scale tidal fields. The net momentum of the simulation volume for each galaxy cluster was zero.

Cooling, heating, and feedback from star formation were not included in these simulations. This is important, since this leads to cluster profiles that are typically more concentrated than observed clusters. This will affect the signal-to-noise estimates moderately but should not drastically affect measures of velocity substructure.

Each gas particle in an SPH scheme has a smoothing radius that roughly corresponds to the mean distance to its Nth nearest neighbor, where N is about 24. We made projected maps of the relevant properties of the gas distribution (optical depth, average velocity, gas temperature, mean Comptonization, SZ effects) along the three principal axes of the simulation by spreading the electrons associated with each gas particle uniformly within a disk of radius equal to roughly one-third the SPH smoothing length. While this had the desirable effect of making the maps smoother (visually) by reducing shot noise, the quantitative effects of the smoothing kernel were negligible.

Three clusters were selected from the z = 0.5 outputs of the simulations, as the three most massive clusters in the simulation set at that redshift. For each particle in the simulation, the gas temperature was used to estimate its SZ effect contribution to the projected map, again using the fitting functions of Nozawa et al. (1998). By adding up the contribution of each particle we made synthetic SZ maps of each galaxy cluster, including the effects of relativistic corrections and peculiar velocities. The resulting maps had a resolution of 25''. The results are not sensitive to the resolution of the maps in that higher resolution did not have any discernible effects. This angular resolution also approximately corresponded to the spatial resolution of the simulation.

At each point in the map we used the SZ maps and the Fisher matrix methods of the previous section to calculate the expected measurement accuracy that would be possible in a high-resolution, sensitive experiment. An angular diameter distance of 1000 Mpc was assumed and pixel noise of 1 μK. CMB anisotropy will make this contribution larger, but this will only affect estimates of peculiar velocities at the level of roughly 50 km s⁻¹, as outlined above.

Subarcminute resolution maps with μK sensitivity are well beyond current instrumentation, but currently planned instruments such as the South Pole Telescope or Atacama Cosmology Telescope are expecting μK sensitivity with beams of roughly 1.5', and these requirements are well within the reported target sensitivities for ALMA. However, the small instantaneous field of view for ALMA will necessitate extensive mosaicking to synthesize a sufficiently large field to do SZ work. Adding smaller telescopes and lower frequencies than currently planned will improve these prospects. The limiting systematic for these instruments should be foreground contamination and not primary CMB anisotropies or detector noise.

As might be expected, the most important determinant of success in parameter measurement is the optical depth, as shown in the simulation maps in Figure 6. All of the uncertainties trace the optical depth map fairly well. In all clusters, within the central few arcminutes the gas temperature can be measured to better than 1 keV and the velocity to better than 100 km s⁻¹. Introducing external gas temperature information improves the size of the region over which reliable velocity measurements are possible to roughly 7', in the absence of CMB contamination.

The velocity uncertainty maps are much more ragged than the gas temperature uncertainty maps. This is mainly due to "lumpiness" in the projected velocity and gas temperature maps. In many ways, the velocity is the most difficult quantity to measure, because it relies on a good measure of the optical depth, which in turn rests on a reliable gas temperature measurement. Being at the "end of the line" leads to increased sensitivity to small-scale inhomogeneities. In addition, the sensitivity of the velocity uncertainties to the underlying model (see Fig. 3) enhances inhomogeneities.

The resulting temperature measurement is effectively ρ-weighted, since the leading contribution to the relativistic corrections scales as ρT_e. This should be intermediate between the mean electron temperature and the X-ray emission–weighted temperature.

Such accurate measures of the bulk velocity over scales larger than several arcminutes will be virtually impossible. While the gas temperature has a unique spectral signature, the effects of the bulk velocity have a spectral signature that is identical to the intrinsic primary and secondary anisotropies in the CMB. Furthermore, accurate gas temperature measurements at large radii will be severely affected by confusion from other distant clusters of galaxies that are nearby in projection.

Figure 5 showed that peculiar velocity measurements of galaxy clusters can only be performed in the inner 2' to 4' because of severe contamination by CMB anisotropy. Bulk velocities within the central few arcminutes of the cluster, due to somewhat recent mergers can therefore be a large source of confusion. Bulk flows moving at roughly the sound speed (roughly 1000 km s⁻¹) containing 10% of the mass would be expected to mimic a signal of the entire cluster moving at 100 km s⁻¹. This is not a particularly unlikely occurrence, as a 10:1 mass ratio merger should happen quite regularly. These bulk flows should remain intact for a few crossing times. For a 1000 km s⁻¹ flow crossing 1 Mpc, the crossing time is roughly 1 Gyr.

In Table 1 we show two measures of the effects of bulk flows on peculiar velocity measurements using the simulated galaxy clusters. We calculated the average velocity within the central 2' of three projections of the three simulated clusters. This was done either as an average of the individual pixels or as an optical depth weighted average of the pixels. A top-hat window was assumed, rather than the optimal filter discussed above. For the simulation that is clearly a merging system, the velocities were separately calculated for the central regions of each clump and then averaged. The individual clumps for the merger event had individual velocities of ~500 km s⁻¹ in opposite directions. It would be readily apparent from the thermal SZ map that this cluster is a double in the process of merging and it would be straightforward to take this into account in the modeling.

The bulk velocities in the central regions are contributing a random uncertainty of more than 100 km s⁻¹ to the peculiar velocity. This provides an estimate of the typical errors that would be incurred due to bulk flows within the intracluster medium. The dark matter in the simulation has no net momentum, so a perfect reflection of the bulk velocity would mean no net velocities. The rms velocities from these three simulations are 135 km s⁻¹ for the pixel-averaged velocity and 141 km s⁻¹ for the optical depth weighted average velocity. While it may be possible to reduce this confusion somewhat, and more simulations are required to quantify this more precisely, it is clear that measurements of peculiar velocities to an accuracy better than 100 km s⁻¹ will be very difficult. This is in agreement with Haehnelt & Tegmark (1996), who found that bulk motions within a simulated galaxy cluster led to a ~10% misestimate of the input 1000 km s⁻¹ peculiar velocity,
suggesting that the bulk flows are contributing $\sim 100$ km s$^{-1}$ uncertainties. This has also been found independently by Nagai, Kravtsov, & Kosowsky (2002).

Because individual measurements will not be noise-limited, it may be possible to use the distribution of velocities rather than velocities weighted by optical depth, in the same way optical studies use galaxy velocity distributions to estimate the mean redshift. The coherence length of the bulk flows are unfortunately not much smaller than a few arcminutes, so it is unlikely that this will lead to a large improvement.

**Fig. 6.**—Surface maps of simulated galaxy clusters. From top to bottom are three different simulated clusters. From left to right columns show maps of optical depth, possible measurement accuracies on gas temperature and peculiar velocity without and with an independent measure of the gas temperature of $\pm 1$ keV. The maps are $14'$ on a side, with each tick mark showing 1'. Contour levels in left panels are at multiples of $2 \times 10^{-3}$, in second panels from left correspond to 0.5, 1, and 2 keV, and in two right panels show 20, 50, and 100 km s$^{-1}$.

| TABLE 1 | MEAN BULK VELOCITIES |
|----------|-----------------------|
| WEIGHTING SCHEME | Simulation 1 | Simulation 2 | Simulation 3 |
| Pixel | $v_x$ | $v_y$ | $v_z$ | $v_x$ | $v_y$ | $v_z$ | $v_x$ | $v_y$ | $v_z$ |
| 150 | 135 | 80 | 170 | $-75$ | 20 | $-75$ | $-281$ | $-10$ |
| $\tau$ | 115 | 140 | 65 | 240 | $-100$ | 65 | $-80$ | $-250$ | $-5$ |

*Note.*—Mean bulk velocities (km s$^{-1}$) in central 2' of three projections of three simulated clusters, for two pixel weighting schemes (optical depth weighting or simple pixel averaging.)
6. DISCUSSION AND CONCLUSIONS

Measurements of peculiar velocities and temperature of galaxy clusters at microwave frequencies will soon be possible. We have shown that multifrequency, sensitive observations could measure peculiar velocities to an accuracy of roughly 100 km s\(^{-1}\). Measurements of gas temperatures will be useful, with uncertainties possibly smaller than could be achieved with X-ray spectroscopy. The redshift independence of the SZ effect will make this an extremely powerful tool for studies of distant clusters. Exploring differences between X-ray emission weighted and SZ emission weighted temperature maps will no doubt be interesting.

Foregrounds and backgrounds will be important barriers to such precise studies of peculiar velocities (Blain 1998; Fischer & Lange 1993). The primary anisotropies of the CMB will become problematic on scales larger than a few arcminutes, and point source removal will be very difficult. It is not expected that Galactic dust will be a problem, but very little is known about Galactic dust on arcminute scales. It has not yet been demonstrated that the atmosphere will not be a problem for ground-based experiments, but there is no a priori evidence that there will be a problem. Interfrequency calibration to the requisite precision will be a significant technical challenge.

Point sources come in (at least) two varieties. At frequencies below ~90 GHz radio point sources (primarily extra-galactic AGN but also star-forming galaxies) have historically been a problem for SZ measurements and they are unlikely to go away. The best solution seems to be simultaneous monitoring at extremely high (a few arcseconds) resolution. At higher frequencies, dusty star-forming galaxies are ubiquitous. If no source subtraction is done, confusion could easily be at the level of 10 \(\mu\)K, comparable to the kinetic SZ signal (Blain 1998). Currently, very little is known about these sources, making spectral subtraction (measuring at a higher frequency where the SZ signal is negligible) difficult; the ultimate solution may require something like ALMA to remove the point sources at mm wavelengths. There is no evidence for variability in these sources, so it will not be necessary to do the subtraction simultaneously, as is required for the occasionally variable radio point sources. However, at current levels of understanding, dusty galaxies could be the single largest source of uncertainty in determining peculiar velocities.

The best frequencies for observation turn out to not include the null of the thermal SZ effect. The best strategy is to have a Rayleigh-Jeans band (below 90 GHz), a high-frequency band (above 300 GHz), and a band near 150 GHz. Much has been made of the null of the thermal SZ effect, and it will be important as a check for systematic errors, but it is not a particularly good frequency for cluster studies. In reality, at least one more frequency would be essential as a consistency check and foreground (or background) monitor; a channel near the null could therefore still be very useful.

Bulk velocities within the cluster, combined with contamination from the anisotropies of the CMB, lead to a limit of roughly 100 km s\(^{-1}\) on the possible accuracy of kinetic SZ velocity measurements. This is much higher than what would be expected from considerations of the background noise levels and the few tens of km s\(^{-1}\) that arise from the difficulty in choosing the appropriate definition of velocity. More work on simulations could shed significant light on optimal strategies for estimating the true peculiar velocity as well as provide a much better estimate of the distribution of errors that could be expected from an ensemble of galaxy cluster peculiar velocity measurements. However, at current sensitivities and resolution we are not approaching the limits set by bulk velocities.

Measurements at centimeter and millimeter wavelengths are opening a new window on cosmology. It will soon be possible to measure gas temperatures and peculiar velocities to good accuracy out to \(z \sim 2\), allowing unprecedented tests of structure formation as well as an excellent understanding of the topography of much of the observable universe.

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