THE INTRINSIC DENSITY MATRICES OF NUCLEAR SHELL MODEL

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Abstract

A new method for the calculation of shell model intrinsic density matrices, defined as two-particle density matrices integrated over centrum of mass position vector of two last particles and accompanied with isospin variables, has been developed. Produced intrinsic density matrices are completely antisymmetric, translational invariant and do not employ a group theoretical classification of antisymmetric states. They are devoted for exact realistic density matrix expansion within the framework of Reduced Hamiltonian method. The procedures on a base of precise arithmetic for intrinsic density matrices calculation that involve any numerical diagonalization and orthogonalization were developed and implemented in computer code.

I. INTRODUCTION

Last time discovering of a exotic nuclei emphasize the deficiency of usual nuclei description methods based on central field approximation. The reason relay upon week exotic nuclei binding energy due to oversaturation of neutrons or protons. To describe such the systems large-space shell model expansions have to be considered, since the ordinary shell model is unable to manage deep nucleon–nucleon correlations. However shell model functions are implicitly redundant with many times encountering a numerous of the same intrinsic states. The reason is that excited state shell model wave function is representable as linear combination of the products of intrinsic wave function and centrum of mass (c.m.) wave function and only the first term in the series corresponding to the c.m. in ground state is needful. Every remaining term contains the one of excited intrinsic wave functions that was encountered in the first term of expansion of less excited state shell model wave function. The first attempt to solve this problem was the translationally invariant shell model [1–3], preliminary invoked to antisymmetrize the wave-function depending on intrinsic coordinates, leads only to sophisticated exercising in group theory which do not end in available results.

We propose a simple and effective procedures comprising simplicity of ordinary shell model and requirement of translational invariancy for wave-function calculation. The method relies on the number of statements simplifying the problem under consideration.

At first, despite the harmonic oscillator potential do not depend on spin and isospin coordinates, we refuse of orbital, spin and isospin spaces separation in model wave-functions.
Hence we loose an advantage of using the group theoretical methods for many-particle antisymmetrical states classification. Nevertheless, on the other hand, rejection of precise labelling of model wave-functions by quantum numbers of higher order groups, such as seniority, leads to simplification of the very antisymmetrization procedure. In this approach many-particle antisymmetrical states are characterized only by well defined set of quantum numbers: number of oscillator quanta $E$, angular momentum $J$, parity $\Pi$, isospin $T$ and only one additional integer quantum number $\Gamma' = 1, 2, \ldots r$ necessary for unambiguous enumeration of the states. Here $r$ is rank of the corresponding antisymmetrization operator matrix. Developed on this basis computational procedures allow unrestricted configuration space to be taken into account [4,5].

Second simplification refer to the 'spurious' states. Description of atomic nuclei can be carried out within the framework of ordinary shell model using intrinsic wave-function expansion in terms of shell model functions. The coefficients of this expansion can be obtained diagonalizing c.m. Hamiltonian in the shell model basis. Intrinsic motion of nucleons would be represented by subspace of c.m. Hamiltonian eigenvectors corresponding to minimal eigenvalue equal $\frac{3}{2}$.

At last, simplification of the A-particle system description is using two-particle density matrices instead of wave-functions. Since all operators of observables including intrinsic Hamiltonian and root-mean-square (r.m.s.) radius operators are symmetric ones, their expectation values calculations do not require the complete wave-function to be involved. Moreover two-particle density matrix is still redundant in its variables. We can use more simple quantity as density matrix integrated over c.m. position vector of two last particles, as so-called intracule [6]. In case of light nuclei intracule have to describe A-particle system with not excited c.m. and must be accompanied with isospin variables [6]. Such intracule we call intrinsic density matrix. Goal is that using intrinsic density matrices we have to deal only with $7 + 7$ orbital, spin and isospin variables, instead of wave-functions containing $5A - 3$ variables.

II. DEVELOPMENT OF THE METHOD

In the shell model it is assumed that free nucleons are moving in self–consistent central field. It can be approximated by an isotropic harmonic oscillator potential. The many–particle Hamiltonian used in shell model is

$$H = \sum_{i=1}^{A} \left\{ -\frac{\hbar^2}{2m} \Delta_i + \frac{1}{2} m \omega^2 r_i^2 \right\} = \sum_{i=1}^{A} h(\vec{r}_i) \quad (2.1)$$

Here $m$ denotes the nucleon mass, $\vec{r}_i$ the i-th nucleon radius vector, $\omega$ the angular frequency and $h(\vec{r}_i)$ the single-particle hamiltonian. Usually in the shell model only one eigenfunction of many–particle Hamiltonian (2.1) is taken into account. Hence the consideration is restricted by only one configuration (so–called ground configuration) characterized by minimal total oscillator quantum number. The consistent consideration could be achieved only on complete basis of eigenfunctions of Hamiltonian (2.1). To construct any antisymmetrical eigenfunction of many–particle $H$ (2.1) characterized with a good quantum numbers it is appropriate to define eigenfunctions of single-particle $h(\vec{r}_i)$ in a j-j coupled representation as

$$\psi_{eljtm,\alpha_1} (x) = R_{el} (r) \{ Y_i (\vec{r}) \otimes \alpha_{1/2} (\vec{\sigma}) \}_{jm_j} \alpha_{1/2 m_t} (\vec{\tau}) \quad (2.2)$$

Here $R_{el} (r)$ is a radial function, $Y_{im} (\vec{r})$ a spherical harmonics, $\alpha_{1/2m_s} (\vec{\sigma})$ a spin-1/2 function in spin space and $\alpha_{1/2 m_t} (\vec{\tau})$ a spin-1/2 function in isospin space. The single-particle variables
are $x_i \equiv \vec{r}_i \vec{\sigma}_i \vec{\tau}_i$ (a set of the corresponding radius-vector, spin and isospin variables). The $e, l$ and $j$ are the principal, orbital and total angular momentum quantum numbers, $m_l, m_s$ and $m_j$ are the magnetic projection quantum numbers of orbital, spin and total angular momentum respectively. The $m_t$ is projection of isospin defined such that $m_t = +1/2$ corresponds to a neutron state and $m_t = -1/2$ to a proton state. A vector coupling of the angle and spin functions to form a state of good total angular momentum is denoted by $\{ \ldots \otimes \ldots \} j m_t$.

Intrinsic properties of a nucleus can be described only by intrinsic wave function. It depends on $(A - 1)$ intrinsic (Jacobi) coordinates and spin-isospin variables. In general eigenfunctions of Hamiltonian (2.1) could be represented as linear combination of products of intrinsic wave functions and c.m. functions when all c.m. excitations are taken into account. Let us determine the system of orthonormalized Jacobi variables [7,8] according to the $2A - 1$ vertices Jacobi tree presented in Fig. 1. The upper vertices correspond to the single-particle variables $x_1 \ldots x_A$. The remaining $A - 1$ vertices (situated below the first ones) correspond to the Jacobi variables which could be of three types:

- firstly if vertex associated with Jacobi variable is not connected directly with any single-particle variable, the Jacobi variable is taken to be equal the corresponding Jacobi coordinate (as associated with $\xi_1 \equiv \vec{\xi}_1$)
- secondly that connected with only one of single-particle vertices is associated with Jacobi variable which is composed of Jacobi coordinate and directly reached single-particle spin-isospin variables (as $\xi_3 \equiv \vec{\xi}_3 \vec{\sigma}_{A-2} \vec{\tau}_{A-2}$)
- finally to that connected with two of single-particle vertices will be prescribed Jacobi variable which is composed of Jacobi coordinate and two sets of spin-isospin variables coming from directly connected single-particle vertices (as $\xi_2 \equiv \vec{\xi}_2 \vec{\sigma}_{A-1} \vec{\tau}_{A-1} \vec{\sigma}_{A} \vec{\tau}_{A}$).

Orthogonal transformation to Jacobi coordinates is

$$
\begin{align*}
\vec{\xi}_\alpha &= \sqrt{\frac{p_\alpha q_\alpha}{p_\alpha + q_\alpha}} \left[ \frac{1}{p_\alpha} \sum_{j \in \{p_\alpha\}} \vec{r}_j - \frac{1}{q_\alpha} \sum_{j \in \{q_\alpha\}} \vec{r}_j \right], \quad \alpha = 1 \ldots A - 1 \\
\vec{\xi}_0 &= \frac{1}{\sqrt{A}} \sum_{j=1}^{A} \vec{r}_j 
\end{align*}
$$

(2.3)

Here $p_\alpha$ is the number of single-particle vertices which could be reached while moving from the $\alpha$-th vertex upwards along the left edge; $\{p_\alpha\}$ is its numbers manifold and $q_\alpha, \{q_\alpha\}$—the same for the right edge. The $\vec{\xi}_0$ is proportional to the nucleus c.m. coordinate. For the vertex connected with two last single-particle vertices we obtain for instance $\vec{\xi}_2 = \frac{1}{\sqrt{2}}(\vec{r}_{A-1} - \vec{r}_{A})$. 

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In the Jacobi coordinates the Hamiltonian (2.1) takes the form

\[
H = \sum_{\alpha=0}^{A-1} \left\{ -\frac{\hbar^2}{2m} \Delta_{(\alpha)} + \frac{1}{2} m \omega^2 \xi_\alpha^2 \right\} \\
= \left\{ -\frac{\hbar^2}{2m} \Delta_{(0)} + \frac{1}{2} m \omega^2 \xi_0^2 \right\} + \sum_{\alpha=1}^{A-1} \left\{ -\frac{\hbar^2}{2m} \Delta_{(\alpha)} + \frac{1}{2} m \omega^2 \xi_\alpha^2 \right\} \\
= h(\vec{\xi}_0) + \sum_{\alpha=1}^{A-1} h(\vec{\xi}_\alpha) \equiv H_{\text{c.m.}} + H_{\text{intr}}.
\]

(2.4)

It implies that the Hamiltonian could be divided into two parts: c.m. Hamiltonian \(H_{\text{c.m.}}\) and Hamiltonian \(H_{\text{intr.}}\) representing the intrinsic motion of the nucleons in the system. The set of \(h(\vec{\xi}_0)\)’s eigenfunctions \(\Psi_{\text{elm}}(\vec{\xi}_0)\) consist of products of corresponding radial functions and spherical harmonics. Whereas \(h(\vec{\xi}_\alpha)\)’s single-particle eigenfunctions could be of three different types depending on number of single-particle variables directly connected with \(\alpha\)-th vertex (if any):

- firstly there are functions depending on Jacobi coordinates which have no direct connection with any single-particle coordinate (as \(\xi_1\)). Such the functions are products of radial function and spherical harmonic.
- secondly there are functions of the form (2.2) for Jacobi coordinates directly connected with one single-particle coordinate (e.g. \(\xi_3\)).
- thirdly there will be functions depending on a Jacobi coordinate and two sets of spin-isospin variables (e.g. \(\xi_2\)). They can be represented as product of the vector coupled orbital and the vector coupled two last particles in the list spin functions and vector coupled isospin functions of corresponding particles.
\[
\psi_{elj\pi tm_j m_t}(\xi_2) = \left\{ \psi_{el}(\vec{\xi}_2) \otimes \left\{ \alpha_1/2(\vec{\sigma}_A-1) \otimes \alpha_1/2(\vec{\sigma}_A) \right\}_s \right\}_{jm_j} \left\{ \alpha_1/2(\vec{\tau}_A-1) \otimes \alpha_1/2(\vec{\tau}_A) \right\}_{tm_t} (2.5)
\]

Here orbital function \( \psi_{el}(\vec{\xi}_2) \) is product of the radial function and spherical harmonic. The intrinsic wavefunctions \( \Psi_{E\Gamma J\Pi T M_j M_T}(\xi_1 \ldots \xi_{A-1}) \) are eigenfunctions of \( H_{\text{intr.}} \). Since the antisymmetrization procedure is rather cumbersome in Jacobi coordinates it is convenient to introduce the expansion of the product of c.m. ground state function and intrinsic wave-function in terms of shell model functions

\[
\Psi_{00}(\vec{\xi}_0) \Psi_{E\Gamma J\Pi T M_j M_T}(\xi_1 \ldots \xi_{A-1}) = \sum_{K \Delta} \Psi_{E K \Delta J\Pi T M_j M_T}(x_1 \ldots x_A) a_{K \Delta 00, \Gamma}^{E\Gamma J\Pi T} (2.6)
\]

The coefficients of this expansion \( a_{K \Delta 00, \Gamma}^{E\Gamma J\Pi T} \) can be obtained diagonalizing c.m. Hamiltonian \( H_{\text{c.m.}} \) in the shell model basis. The summation in this formula runs over all configurations \( K \) and additional quantum number \( \Delta \) (in spirit of the \( \Gamma \)). Here zeros indicate the c.m. ground state: principal and orbital angular momentum quantum numbers. The intrinsic wave-function depends on the \( 5A - 3 \) intrinsic Jacobi variables \( \xi_1 \ldots \xi_{A-1} \) where \( \xi_\alpha \) stands for the \( \vec{\xi}_\alpha \) together with one of the mentioned three types of the sets of spin and isospin variables.

Due to the identity of the nucleons it is convenient to restrict consideration only by two last particles in the list. General form of an expression of two-particle intrinsic density matrix is chosen to be

\[
Q^{E\Gamma J\Pi T, E'\Gamma' J\Pi T}_{\xi_2, \xi_2'}(\xi_2, \xi_2') = \frac{1}{[J, T]} \sum_{M_j, M_T} \int d\vec{\xi}_0 d\xi_1 d\xi_3 \ldots d\xi_{A-1} \Psi_{00}(\vec{\xi}_0) \Psi^*_0(\vec{\xi}_0) \times \Psi_{E\Gamma J\Pi T M_j M_T}(\xi_1, \xi_2 \ldots \xi_{A-1}) \Psi_{E'\Gamma' J\Pi T M_j M_T}(\xi_1, \xi_2' \ldots \xi_{A-1}) (2.7)
\]

where integration sign denotes integration over the continuous variables of the relative motion of \( A - 2 \) particles and summation over discrete variables of the corresponding nucleons according to the Jacobi tree in use. Here and in the following the notations as \( [J, T] \equiv (2J + 1)(2T + 1) \) is shortcut of number of states with corresponding angular momentum and isospin. The most convenient way to construct an antisymmetric wave function is the method of fractional parentage coefficients (CFP) developed in \( [4] \).

\[
\Psi_{E\Gamma J\Pi T M_j M_T}(\xi_1 \ldots \xi_{A-1}) = \sum_{(E\Gamma J\Pi T) \begin{array}{c} elj\pi t \end{array}(E\Gamma J\Pi T)} \left\{ \Psi_{E\Gamma J\Pi T}(\xi_1 \ldots \xi_{A-1}) \otimes \psi_{elj\pi t}(\xi_2) \right\}_{J\Pi T M_j M_T} (2.8)
\]

Here double bar indicates the grandparent state and \( elj\pi t \) characterize the separated subsystem. The summation in this formula spans subspace of grandparent \( (E\Gamma J\Pi T) \) and two particle \( elj\pi t \) states which satisfy the necessary selection rules and the energy, momentum and parity conservation conditions. The expansion coefficients have to be intrinsic CFP. By means of (2.8) it is possible to get the expression of the intrinsic density matrix

\[
Q^{E\Gamma J\Pi T, E'\Gamma' J\Pi T}_{\xi_2, \xi_2'}(\xi_2, \xi_2') = \sum_{elj'\pi t} \sum_{(E'\Gamma' J\Pi T) \begin{array}{c} elj'\pi t \end{array}(E'\Gamma' J\Pi T)} \left\{ \psi_{elj'\pi t}(\xi_2) \psi^*_{elj'\pi t}(\xi_2') \right\}_{j' t'} \frac{1}{[j, t]} \sum_{m_j, m_t} \psi_{elj\pi tm_j m_t}(\xi_2) \psi^*_{elj'\pi tm_j m_t}(\xi_2') (2.9)
\]
The intrinsic density matrix in harmonic oscillator representation is

\[
W_{eljnt,e'l'j'n't}(E'\Gamma'J'\Pi'T') = \sum_{\langle E\Gamma J\Pi T \rangle} \langle (E\Gamma J\Pi T); (eljnt) || (E\Gamma J\Pi T); (e'l'j'n't) || (E'\Gamma'J'\Pi'T') \rangle
\]  

(2.10)

This matrix could be defined avoiding the appearance of intrinsic CFP. The key is relation (2.9) between intrinsic and shell model wave functions. The usual shell model fractional parentage could be used to separate out two last particles in the list

\[
\Psi_{E\Gamma\Delta J\Pi M_j M_T}(x_1 \ldots x_A) = \sum_{\langle E\Gamma\Delta J\Pi M_j \rangle} \langle (E\Gamma\Delta J\Pi M_j); (elj_{A-1}) || (E\Gamma\Delta J\Pi M_j) \rangle \Phi_{(elj_{A-1}, (elj)_{A}) J''T''/x_{A-1}, x_A} (x_{A-1}, x_A)
\]  

(2.11)

Here inverted commas refer to the separated two-particle subsystem. For the sake of convenience of the notations we shall use comma to denote the nucleons variables with respect to which a wave-function is antisymmetric. The coefficients of introduced expansion (generalized CFP (GCFP) as defined by Levinson [9]) enable to express the antisymmetric shell model wave function of \( A \) nucleons in the form of linear combinations of products of antisymmetric functions of \( A - 2 \) nucleons and antisymmetric two-particle wave functions. The separation of the two nucleons from the initial configuration can be accomplished in all possible ways consistent with required triangular relations, thus giving rise to transformation matrix describing transformation between different momentum coupling schemes. In case when two nucleons are taken from different shells the GCFP can be expressed in terms of single shell one-particle CFP and corresponding transformation matrix

\[
\langle (E\Gamma\Delta J\Pi M_j M_T); (elj_{A-1}) || (E\Gamma\Delta J\Pi M_j) \rangle = (-1)^{\nu_r + n_p - 1} \left( \frac{2n_r n_p}{A(A - 1)} \right)^{1/2} \times \langle (elj)^{\nu_r}(\Delta JT)_r; (elj)_{r} || (elj)^{\nu_r}(\Delta JT)_{r} \rangle \langle (elj)^{n_p-1}(\Delta JT); (elj)_{p} || (elj)^{n_p}(\Delta JT)_{p} \rangle
\]  

(2.12)

Here single bar over the quantum numbers indicates the parent state, subscript \( r \) refers to the \( r \)-th shell in the configuration and superscript \( n_r \) is number of particles contained in the \( r \)-th shell. The integer number \( \nu_r = \sum_{i=r+1}^{k} n_i \), where sum runs over all shells standing to the right from the \( r \)-th shell. The \( \langle (elj)^{\nu_r-1}(\Delta JT)_r; (elj)_{r} || (elj)^{\nu_r}(\Delta JT)_{r} \rangle \) denotes the one-particle CFP of \( r \)-th shell. When two nucleons are separated from the same shell the GCFP definition contains two-particle CFP:

\[
\langle (E\Gamma\Delta J\Pi M_j M_T); (elj_{A-1}) || (E\Gamma\Delta J\Pi M_j) \rangle = \left( \frac{n_r(n_r - 1)}{A(A - 1)} \right)^{1/2} \langle (elj)^{\nu_r-2}(\Delta JT); (elj)^{\nu_r}(\Delta JT) \rangle \langle (elj)^{\nu_r}(\Delta JT)_r; (elj)^{\nu_r}(\Delta JT)_{r} || (elj)_{p} || (elj)^{n_p}(\Delta JT)_{p} \rangle
\]  

(2.13)
The simplest way of CFP matrix calculation is proposed in [4]. The method is based on the observation that the spectral decomposition of antisymmetrization operator matrix is not uniquely defined. The best choice is to set the upper triangle of CFP matrix equal to zero. This method was implemented in the program code and all s-d shells CFP were calculated. Corresponding two-particle CFP were obtained following the well-known Redmond formula. Transformation matrix describing momentum recoupling was calculated using its direct representation by sum of Clebsch-Gordan coefficients.

The intrinsic Jacobi variable \( \xi_2 \) could be introduced by expressing antisymmetric two-particle wave-functions \( \Phi_{((\ell j)_{A-1},(\ell j)_A)}^{\mu''T''}(x_{A-1}, x_A) \) in terms of single-particle eigenfunctions (2.2) and coupling them in proper order for well-known Talmi-Moshinsky transformation to apply. At first the antisymmetric wave functions \( \Phi_{((\ell j)_{A-1},(\ell j)_A)}^{\mu''T''}(x_{A-1}, x_A) \) should be expressed as a linear combination of not antisymmetrized coupled momentum wave-functions. Here and below we will indicate such functions with semicolon as separation mark for the variables in the list. In case when the nucleons are taken from the same shell we have

\[
\Phi_{((\ell j)_{A-1},(\ell j)_A)}^{\mu''T''}(x_{A-1}, x_A) = \frac{1}{2} \left[ 1 - (-1)^{J'' + T''} \right] \Phi_{((\ell j)_{A-1};(\ell j)_A)}^{\mu''T''}(x_{A-1}; x_A)
\]  

Here we suppress for brevity magnetic quantum numbers. When the two nucleons are taken from different shells linear combination is of the form

\[
\Phi_{((\ell j)_{A-1};(\ell j)_A)}^{\mu''T''}(x_{A-1}; x_A) = \frac{1}{\sqrt{2}} \left[ \Phi_{((\ell j)_{A-1};(\ell j)_A)}^{\mu''T''}(x_{A-1}; x_A) - (-1)^{J'' + T''} \Phi_{((\ell j)_A;(\ell j)_{A-1})}^{\mu''T''}(x_{A-1}; x_A) \right]
\]

We reveal the complete representation of the coupled momentum wave functions by the coupled orbital-spin and isospin functions

\[
\Phi_{((\ell j)_{A-1};(\ell j)_A)}^{\mu''T''M''_JM''_T}(x_{A-1}; x_A) = \left\{ \alpha_{1/2}(\bar{r}_{A-1}) \otimes \alpha_{1/2}(\bar{r}_A) \right\}_{T''M''_T} \times \left\{ \phi_{e_{A-1}l_{A-1}}(\bar{r}_{A-1}) \otimes \alpha_{1/2}(\bar{\sigma}_{A-1}) \right\}_{J_{A-1}} \otimes \left\{ \phi_{e_{A}l_{A}}(\bar{r}_A) \otimes \alpha_{1/2}(\bar{\sigma}_A) \right\}_{J_A} \right\}^{\mu''M''_J}
\]

Here orbital function is defined as \( \phi_{elm}(\bar{r}) = R_{el}(r)Y_{lm}(\hat{r}) \).

Second point concerns transition to the appropriate Jacobi coordinates. This transformation from single-particle variables \( x_{A-1} \) and \( x_A \) to Jacobi variable \( \xi_2 \) could be accomplished by assistance of Jacobi coordinates with nonpositive indices [8]. The laters are chosen to be proportional to the c.m. of corresponding subsystems.

\[
\bar{\xi}_{1-\alpha} = \frac{1}{\sqrt{p_\alpha + q_\alpha}} \sum_{j \in \{p_\alpha \cap q_\alpha\}} \bar{r}_j, \alpha = 1 \ldots A - 1
\]

For example \( \bar{\xi}_2 \) is accompanied with \( \bar{\xi}_1 = \frac{1}{\sqrt{2}}(\bar{r}_{A-1} + \bar{r}_A) \) which is simply proportional to the c.m. of the two last particles. When \( \alpha = 1 \) corresponding Jacobi coordinate with nonpositive index coincide with \( \bar{\xi}_0 \) defined in (2.3). This orthogonal transformation allowing to climb up and down the Jacobi tree is defined

\[
\begin{align*}
\xi_\alpha &= \sqrt{\frac{1}{1 + d_\alpha}} \xi_\mu - \sqrt{\frac{d_\alpha}{1 + d_\alpha}} \bar{\xi}_\nu \\
\bar{\xi}_{1-\alpha} &= \sqrt{\frac{d_\alpha}{1 + d_\alpha}} \bar{\xi}_\mu + \sqrt{\frac{1}{1 + d_\alpha}} \bar{\xi}_\nu
\end{align*}
\]  

(2.18)
Here $d_{\alpha} = p_{\alpha}/q_{\alpha}$, $\vec{\xi}_{-\mu}$ is Jacobi coordinate associated with vertex which could be reached while moving from the $\alpha$-th vertex upwards along the left edge; $\vec{\xi}_{-\nu}$ is the same for the right edge. In case when any of the upper vertices is bound to single-particle variables the corresponding single-particle coordinate is taken.

To apply this transformation it is necessary to interchange the order of coupling. It will be the so-called L-S coupling scheme.

\[
\left\{ \left\{ \phi_{e_{A-1}l_{A-1}}(\vec{r}_{A-1}) \otimes \alpha_{1/2}(\vec{\sigma}_{A-1}) \right\}_{l_{A-1}} \otimes \left\{ \phi_{e_{A}l_{A}}(\vec{r}_{A}) \otimes \alpha_{1/2}(\vec{\sigma}_{A}) \right\}_{l_{A}} \right\}_{J''M''J} = \\
= \sum_{Ls} \langle \langle (l_{A-1}, 1/2)J_{A-1}, (l_{A}, 1/2)J_{A} \rangle \rangle J'' \langle \langle (l_{A-1}, l_{A})L, (1/2, 1/2)s \rangle \rangle J'' \\
\times \left\{ \left\{ \phi_{e_{A-1}l_{A-1}}(\vec{r}_{A-1}) \otimes \phi_{e_{A}l_{A}}(\vec{r}_{A}) \right\}_{L} \otimes \left\{ \alpha_{1/2}(\vec{\sigma}_{A-1}) \otimes \alpha_{1/2}(\vec{\sigma}_{A}) \right\}_{s} \right\}_{J''M''J} \tag{2.19}
\]

Here $L$ is the orbital momentum of the relative motion of the two last nucleons and $s$ is the corresponding spin. Now can be used the Talmi - Moshinsky - Smirnov transformation

\[
\left\{ \left\{ \psi_{(el)A-1} (\vec{\xi}_{-1}) \otimes \psi_{el} (\vec{\xi}_{2}) \right\}_{LM} \right\}{el}_{A-1, el} = \\
= \sum_{J} \langle \langle (el)_{A-1}, (el)_{A} : L \rangle \rangle \langle \langle (el)_{-1}, el : L \rangle \rangle \psi_{(el)A-1} (\vec{\xi}_{-1}) \otimes \psi_{el} (\vec{\xi}_{2}) \right\}_{LM} \tag{2.20}
\]

here sum is restricted under energy $(e_{A-1} + e_{A} = e_{-1} + e)$ and parity conservation. The Talmi - Moshinsky - Smirnov coefficients $\langle \langle (el)_{A-1}, (el)_{A} : L \rangle \rangle \langle \langle (el)_{-1}, el : L \rangle \rangle \psi_{(el)A-1} (\vec{\xi}_{-1}) \otimes \psi_{el} (\vec{\xi}_{2})$ following $[10][11]$ are the elements of the matrix for the transition between the oscillator functions depending on coordinates which are related with orthogonal transformation $(2.18)$. Finally we must return to the momentum coupling defined in the $\psi_{elj\pi\tau m_{j}m_{l}}(\vec{\xi}_{2})$ functions

\[
\left\{ \left\{ \psi_{(el)A-1} (\vec{\xi}_{-1}) \otimes \psi_{el} (\vec{\xi}_{2}) \right\}_{LM} \right\}{el}_{A-1, el} = \\
= \sum_{J} \langle \langle (l_{-1}, l)J \rangle \rangle \langle \langle (l_{-1}, l, s)J' \rangle \rangle \psi_{(el)A-1} (\vec{\xi}_{-1}) \otimes \psi_{el} (\vec{\xi}_{2}) \right\}_{LM} \tag{2.21}
\]

Now we can express functions from the left side of the expression $(2.19)$ in terms of the functions from the right side of the expression $(2.21)$

\[
\left\{ \left\{ \phi_{e_{A-1}l_{A-1}}(\vec{r}_{A-1}) \otimes \alpha_{1/2}(\vec{\sigma}_{A-1}) \right\}_{l_{A-1}} \otimes \left\{ \phi_{e_{A}l_{A}}(\vec{r}_{A}) \otimes \alpha_{1/2}(\vec{\sigma}_{A}) \right\}_{l_{A}} \right\}_{J''M''J} = \\
= \sum_{(el)A-1, elsJ} \langle \langle (elj)_{A-1} \rangle \rangle \langle \langle (elj)_{A} \rangle \rangle \psi_{(el)A-1} (\vec{\xi}_{-1}) \otimes \psi_{el} (\vec{\xi}_{2}) \right\}_{LM} \tag{2.22}
\]

The coefficients of introduced expansion represents the collection of the transformation matrices from $(2.21)$ and $(2.19)$, and the Talmi - Moshinsky coefficients from the expansion $(2.20)$

\[
\langle \langle (elj)_{A-1} \rangle \rangle \langle \langle (el)_{-1}, elsJ \rangle \rangle = \sum_{(el)A-1, elsJ} \langle \langle (l_{-1}, l)L \rangle \rangle \langle \langle (l_{-1}, l, s)J'' \rangle \rangle \psi_{(el)A-1} (\vec{\xi}_{-1}) \otimes \psi_{el} (\vec{\xi}_{2}) \right\}_{LM} \tag{2.23}
\]

\[
\times \langle \langle (l_{A-1}, l_{A})J'' \rangle \rangle \langle \langle (l_{A-1}, l_{A})L, (1/2, 1/2)s \rangle \rangle \psi_{(el)A-1} (\vec{\xi}_{-1}) \otimes \psi_{el} (\vec{\xi}_{2}) \right\}_{LM} \tag{2.23}
\]

\[
\times \langle \langle (el)_{A-1}, (el)_{A} : L \rangle \rangle \langle \langle (el)_{-1}, el : L \rangle \rangle \psi_{(el)A-1} (\vec{\xi}_{-1}) \otimes \psi_{el} (\vec{\xi}_{2}) \right\}_{LM} \tag{2.23}
\]
Corresponding transformation matrices can be represented in terms of standard vector coefficients: 6-j and 9-j, thus we are led to the expression

\[
\langle((elj)_{A-1}; (elj)_A J'' |((el)_1, elsj) J'']\rangle = (-1)^{l_{A-1} + l_A + j'} \sqrt{[j_{A-1}, j_A, s, j]} \times \sum_L [L] \langle((elj)_{A-1}, (elj)_A ; L](elj)_1 \{l_{A-1} \frac{1}{s} \frac{l_A}{j_A} \frac{1}{j} \frac{j_{A-1}}{s} \frac{j_A}{j} \frac{j''}{l''} \} \ (2.24)
\]

It should be noted that presented coefficients weighted only not antisymmetrical functions as is stressed by semicolon. Taking into account the isospin part of the two-particle function we get the final form of the coefficients for the transition from the antisymmetrical two-particle shell model functions to the sought \(\psi_{elj\pi t,m_l} (\xi_2)\) vector coupled with the introduced functions \(\psi_{(el)-1} (\vec{\xi}_1)\)

\[
\langle((elj)_{A-1}, (elj)_A J''|T''; ((el)_{1}, elj\pi t) J''|T'')\rangle = \delta_{l,T'} \frac{1 - (-1)^{l+s+j}}{\sqrt{2} (1 + \delta_{elj_{N-1}, (elj)_N})} \times \langle((elj)_{A-1}, (elj)_A J''|((el)_{1}, elsj) J'')\rangle \ (2.25)
\]

This transformation allows us to re-express the two-particle shell model function expansion in the form

\[
\Phi_{((elj)_{A-1}, (elj)_A) J'' T'' M''_{J''} M''_{T''}} (x_{A-1}, x_A) = \sum_{elj,eljs} \{\alpha_{1/2}(\vec{\tau}_A-1) \otimes \alpha_{1/2}(\vec{\tau}_A)\} T''_{M''_T} \times \{\psi_{(el)-1} (\vec{\xi}_1) \otimes \psi_{el}(\xi_2) \otimes \alpha_{1/2}(\vec{\sigma}_A-1) \otimes \alpha_{1/2}(\vec{\sigma}_A)\} s_j j \times \langle((elj)_{A-1}, (elj)_A J''|T''; ((el)_{1}, elj\pi t) J''|T'')\rangle \ (2.26)
\]

Finally we obtain the eigenfunctions of \(A\)-particles Hamiltonian (2.1) with separated two groups of nucleons: first, containing two last particles which are described by antisymmetrical, coupled function, depending on Jacobi coordinates, and second, containing remaining set of particles, which are described by corresponding eigenfunction of \(A-2\) particles Hamiltonian depending on single-particle coordinates. That functions we call the \(A\)-particle oscillator functions with singled out dependence on intrinsic coordinates of two last particles in the list.

\[
\Psi_{E K \Delta J H T M^J M^T} (x_1 \ldots x_A) = \sum_{(E K \Delta J H T)} \sum_{elj\pi t} \sum_{J'' T''} \langle(E K \Delta J H T); ((elj)_1, elj\pi t) J'' T''|E K \Delta J H T \rangle \times \{\psi_{(elj)_1} (\vec{\xi}_1) \otimes \psi_{elj\pi t}(\xi_2)\} J'' T'' M^J M^T \ (2.27)
\]

The momentum coupling scheme between the functions can be arbitrary since density matrix is invariant with respect to orthogonal their transformation. Taking advantage of this will save us for one more recoupling looked for coupling scheme with \(\psi_{elj\pi t} (\xi_2)\) coupled at end.

The coefficients of the (2.27) functions expansion by introduced ones are composed of GCFP and coefficients (2.25).
\[ \langle (E \Delta J \Pi T) ; ((el)_{A-1}, el_j \pi t), J'' T'' || E \Delta J \Pi T \rangle = \sum_{(el)_{A-1}, (el)_A} \langle (E \Delta J \Pi T) ; ((el)_{A-1}, (el)_A) J'' T'' || E \Delta J \Pi T \rangle \times \langle ((el)_{A-1}, (el)_A) J'' T'' | ((el)-1, el_j \pi t) J'' T'' \rangle \quad (2.28) \]

This set of coefficients may satisfy the orthonormalization condition

\[ \sum_{(E \Delta J \Pi T) ; ((el)_{A-1}, el_j \pi t), J'' T''} \sum_{(E \Delta J \Pi T) ; ((el)_{A-1}, el_j \pi t), J'' T''} \langle (E \Delta J \Pi T) ; ((el)_{A-1}, el_j \pi t), J'' T'' || E \Delta J \Pi T \rangle \times \langle (E \Delta J \Pi T) ; ((el)_{A-1}, el_j \pi t), J'' T'' || E' \Delta' J \Pi T \rangle = \delta_{K \Delta, K' \Delta'} \quad (2.29) \]

Now we are able to get the final expression of intrinsic density matrix. Preliminarily it should be noted that due to orthogonality of transformations (2.3) and (2.18) it may hold equivalence for integration over two sets of variables

\[ \int d\vec{\xi}_0 d\xi_1 d\xi_3 \ldots d\xi_{A-1} \overset{\text{\(\ddots\)}}{=} \int dx_1 \ldots dx_{A-1} d\vec{\xi}_{-1} \quad (2.30) \]

Here the integration is taken in sense of the definition (2.7). Inserting in the density matrix (2.7) the linear combination of shell model functions with fixed c.m. state (2.6) and integrating after substitution of variables according (2.30) we get the sought intrinsic density matrix expression

\[ Q^{E \Gamma J \Pi T, E' \Gamma' J \Pi T} (\xi_2, \xi'_2) = \sum_{el'j' \pi t} \sum_{K \Delta, K' \Delta'} a^{E \Gamma J \Pi T}_{K \Delta, 00, \Gamma} a^{E' \Gamma' J \Pi T}_{K' \Delta', 00, \Gamma'} \]
\[ \times \sum_{(E \Delta J \Pi T) ; ((el)_{A-1}, el_j \pi t), J'' T''} \langle (E \Delta J \Pi T) ; ((el)_{A-1}, (el)_{-1}) J'' T'' || E \Delta J \Pi T \rangle \times \langle (E \Delta J \Pi T) ; ((el)_{A-1}, (el)_{-1}) J'' T'' || E' \Delta' J \Pi T \rangle \]
\[ \times \frac{1}{[j, t]} \sum_{m_j, m_t} \psi_{elj \pi t, m_j, m_t} (\xi_2) \psi^*_{elj' \pi t, m_j, m_t} (\xi'_2) \quad (2.31) \]

Comparing the expressions at the products of the functions \( \psi_{elj \pi t, m_j, m_t} (\xi_2) \psi^*_{elj' \pi t, m_j, m_t} (\xi'_2) \) in the intrinsic density matrix expressions (2.9) and (2.31) we are led to the final result

\[ W^{E \Gamma J \Pi T, E' \Gamma' J \Pi T}_{elj \pi , e'l' \pi t} = \sum_{(E \Delta J \Pi T) ; ((el)_{A-1}, el_j \pi t), J'' T''} \langle (E \Delta J \Pi T) ; ((el)_{A-1}, el_j \pi t), J'' T'' || E \Gamma J \Pi T \rangle \]
\[ \times \langle (E \Delta J \Pi T) ; ((el)_{A-1}, el_j \pi t), J'' T'' || E' \Gamma' J \Pi T \rangle \quad (2.32) \]

Presented coefficients are connected with outlined above coefficients (2.27) and (2.4) according to the relation

\[ \langle (E \Delta J \Pi T) ; ((el)_{A-1}, el_j \pi t), J'' T'' || E \Gamma J \Pi T \rangle = \sum_{K \Delta} \langle (E \Delta J \Pi T) ; ((el)_{A-1}, el_j \pi t), J'' T'' || E \Delta J \Pi T \rangle a^{E \Gamma J \Pi T}_{K \Delta, 00, \Gamma} \quad (2.33) \]

That coefficients project out the space of A-particle oscillator functions with singled out dependence on intrinsic coordinates of two last particles into intrinsic functions subspace with not excited c.m.\ldots.
The intrinsic density matrix in harmonic oscillator representation satisfy a usual normalization relation

\[ \sum_{elj\pi t} W_{elj\pi t,elj\pi t}^{E\Gamma J\Pi T,E'\Gamma' J'\Pi T} = \delta_{E\Gamma,E'\Gamma'} \]  

(2.34)

The presented expressions enable to obtain the full set of intrinsic density matrices describing \( A \)-nucleon system in isospin formalism after calculating the coefficients (2.33) weighting linear combination of that eigenfunctions of \( A \)-particles Hamiltonian (2.1) for which ones c.m. is in fixed state.

### III. COMPUTATIONAL RESULTS

Since momentum recoupling, antisymmetrization and transformation to Jacobi coordinates are orthogonal transformations a precise arithmetics could be applied. Instead of calculations with real numbers, which are connected with serious numerical instabilities, calculations were performed with numbers represented in the form \( n/(m\sqrt{k}) \), where \( n, m \) and \( k \) are integers.

On the basis of precise arithmetics were developed computational procedures of a number of well-known transformations coefficients and implemented in a computer code. As follows: one-particle single shell CFP [9], two-particle single shell CFP according to Redmond expression, Clebsh-Gordan coefficients, 6-j and 9-j vector coupling coefficients, momentum recoupling matrices, GCFP, Talmi coefficients.

Another set of computational procedures was developed for enumeration of the antisymmetric states by means of combinatorial calculations.

Outlined above computational procedures were used for general formalism, presented in the section Number 2, to implement in a computer code. For illustration let us propose the calculations of 6, 7 and 8 nucleon systems with minimal oscillator energy compatible with the Pauli exclusion principle were performed. The number of calculated intrinsic CFP and intrinsic density matrices for 6, 7 and 8 nucleon systems are: 255 and 41, 1345 and 66, 5021 and 138 respectively.

As an example we consider the case \( A = 6 \). In this case minimal total oscillator quantum number is \( E_{\text{min}} = 2 \). Hence 6 nucleon system can be in the three ground state configurations \( K_i \):

\[
K_1 \equiv \left(00{\frac{3}{2}}\right)^4 \left(11{\frac{1}{2}}\right)^2 : \quad JT = 01, 10,
\]

\[
K_2 \equiv \left(00{\frac{3}{2}}\right)^4 \left(11{\frac{3}{2}}\right) \left(11{\frac{1}{2}}\right) : \quad JT = 10, 11, 20, 21,
\]

\[
K_3 \equiv \left(00{\frac{1}{2}}\right)^4 \left(11{\frac{3}{2}}\right)^2 : \quad JT = 01, 10, 21, 30.
\]

Here \((elj)^n\) denotes the single shell. Displayed sequence of them taken in ascending quantum number order stands for configuration. According to the number of configurations with the same \( JT \) values, situated at the right, we have to obtain: one three-dimensional matrix with \( JT = 10 \), two two-dimensional matrices with \( JT = 01, 21 \) and three one-dimensional matrices with \( JT = 11, 20, 30 \). All shells and configurations are unambiguously characterized by their \( JT \) values, thus additional quantum numbers are not necessary. The same specification scheme
by $JT$ values will be valid and for coefficients of expansion of intrinsic wave-function by shell model functions. According to the Elliott and Skyrme theorem shell model states characterized by $E_{\text{min}}$ contains the nuclear c.m. in its ground state, hence $a_{K\Delta,00,\Gamma}^{E,JHT} = 1$. In this case the coefficients of expansion of intrinsic wave-function by shell model functions are the CFP of intrinsic function.

Let us consider the intrinsic CFP characterized by $JT = 30$. This is the simplest case since they can be generated only in daughter configuration $(00\frac{1}{2})^4 (11\frac{3}{2})^2$. Computational results are presented in Table 1.

Table 1
Intrinsic CFP of 6 nucleon system:
$E_{\text{min}} = 2$, $K = (00\frac{1}{2})^4 (11\frac{3}{2})^2$, $\Delta = 1$, $JT = 30$

| $K\Delta$ | $JT$ | $(el)_{-1}$ | $elsjt$ | state | $J''T''$ | CFP |
|------------|------|-------------|---------|--------|----------|-----|
| $(00\frac{1}{2})^2 10 \left(11\frac{3}{2}\right)^2 10 30$ | 31 | 00 | 00001 | $^1S_0$ | 01 | $\frac{1}{\sqrt{5}}$ |
| | 20 | 00110 | $^3S_1$ | 10 | $-\frac{1}{\sqrt{11}}$ |
| | 30 | | | | $-\frac{1}{\sqrt{15}}$ |
| | 40 | | | | $-\frac{1}{\sqrt{405}}$ |
| $(00\frac{1}{2})^2 11 \left(11\frac{3}{2}\right)^2 11 30$ | 10 | 11 | 00110 | $^3S_1$ | 20 | $-\frac{1}{2\sqrt{5}}$ |
| | 20 | 11121 | $^3P_2$ | 21 | $\frac{3}{2\sqrt{15}}$ |
| | 11 | 11010 | $^1P_1$ | 10 | $-\frac{1}{2\sqrt{15}}$ |
| | 21 | 00110 | $^3S_1$ | 20 | $-\frac{1}{2\sqrt{15}}$ |
| | | | | | $-\frac{1}{\sqrt{30}}$ |
| $(00\frac{1}{2})^4 00 00110 | 22 11111 | $^3P_1$ | 11 | $-\frac{1}{2\sqrt{5}}$ |
| | 11 | 00001 | $^1S_0$ | 10 | $\frac{1}{\sqrt{15}}$ |
| | 00 | 11121 | $^3P_2$ | 21 | $\frac{1}{\sqrt{30}}$ |
| | | | | | $-\frac{1}{\sqrt{30}}$ |

Composition of grandparent configuration $K\Delta$ is clearly seen from displayed notations where every single shell is specified by its total $JT$ values. Proposed in this paper set of quantum numbers $\Gamma\Pi\Delta M_T$ for enumeration of antisymmetrical states of $A$-particle system can be in natural way prolonged and for only two particles. The corresponding set will be $lj\pi\tau\mu_t$. Here orbital momentum $l$ plays the role of addition quantum number as $\Gamma$, since in the case of two particles $l$ is well defined quantum number. So we follow the more usual spectroscopic notation $2s+1\ell_j$ to denote the state of the subsystem containing two separated particles. To avoid overloading the Table 1 we show only the values of the indices which in turn were not repeated in the preceding row.

The obtained intrinsic CFP were used for intrinsic density matrices calculations according the definition (2.31). As an example we present intrinsic density matrix characterized by $E = E_{\text{min}} = 2$, $JT = 10$ when the subsystem of two separated particles is in state $^3S_1$. 

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\[ W^{2,10}(^3S_1) = \begin{pmatrix} 44 & 135 & -1 \\ 135 & 44 & 27 \sqrt{10} \\ -1 & 27 \sqrt{10} & 17 \sqrt{54} \end{pmatrix}. \]  

(3.1)

Here we simplify notations of (2.31) and display in superscripts only \( E \) and \( JT \). \( W^{2,10}(^3S_1) \) is symmetric matrix, thus only upper triangle is shown. It rows and the same columns are labelled by intrinsic CFP left hand bracket indices. Namely rows labels are:

- first - \( \left( \left( 00 \frac{1}{2} \right)^4 00, (11 \frac{1}{2})^2 10 \right) 10 \),
- second - \( \left( \left( 00 \frac{1}{2} \right)^4 00, \left( 11 \frac{1}{2} \right)^2 \frac{1}{2}, \left( 11 \frac{1}{2} \right)^2 \frac{3}{2} \right) 10 \),
- third - \( \left( \left( 00 \frac{1}{2} \right)^4 00, (11 \frac{1}{2})^2 10 \right) 10 \).

Here notations of Table 1 are used except that parentheses represents the momentum coupling order. Using the ordinary shell model notations for intrinsic CFP is justified by their clarity, origin of representation space construction procedures and preservation of space dimension under transformation to Jacobi coordinates. Last means that spaces labelled by \( \Gamma \) and \( K \Delta \) have equal dimensions.

Table 2

| Config. term | JT | 10 | 30 | 01 | 20 | 21 | 11 |
|--------------|----|----|----|----|----|----|----|
| \(^3S_1\)    | 4  | 15 | 17 | 2 | 1  | 3 | 3 |
| \(^3S'_1\)   | 1 | 135 | 135 | 15 | 1 | 3 | 3 |
| \(^1S_0\)    | 3 | 10 | 10 | 3 | 29 | 3 | 3 |
| \(^1S'_0\)   | 1 | 90 | 15 | 1 | 29 | 3 | 3 |
| \(^3P_0\)    | 13 | 270 | 270 | 19 | 1 | 30 | 30 |
| \(^3P_1\)    | 1 | 10 | 10 | 1 | 27 | 30 | 30 |
| \(^3P_2\)    | 1 | 5 | 5 | 1 | 27 | 30 | 30 |
| \(^1D_2\)    | 2 | 81 | 162 | 1 | 30 | 30 | 30 |
| \(^3D_1\)    | 2 | 81 | 162 | 1 | 30 | 30 | 30 |
| \(^3D_2\)    | 1 | 30 | 30 | 1 | 30 | 30 | 30 |

The empty places in the Table 2 means that there are any intrinsic density matrices of such kind. Here for the brevity in the configuration notations we don’t display the first closed shell \( \left( 00 \frac{1}{2} \right)^4 \) and suppressed \( e \) and \( l \) in the higher single shells notations.

Presented results imply that even in \( E_{\text{min}} \) approximation we can make same predictions concerning effects invoked by tensor character of nuclear forces. The display of variety effects arise from so-called mixing of states and have the same origin as in deuteron case. According to the reduced Hamiltonian method [3] presence of bound states of \( A \)-particle system is explained by existence of two-particle bound states of the so-called reduced Hamiltonian. Direct numerical integration of reduced Hamiltonian equations with some realistic nuclear interaction potential
could supply corresponding eigenvalues and percent of eigenstates mixing. For \( A = 6 \) that are only \(^1S_0\) and \(^3S_1 - ^3D_1\). The admixture of \(^3D_1\) term may consist about 30\%. This admixture must be exhausted by corresponding terms with higher orbital momentum in expansion of realistic density matrix by intrinsic ones. The less part of admixture is exhausted in lower energy approximations the more remain to the higher ones. Hence less pronounced contribution of \(^3D_1\) term in \(E_{\text{min}}\) approximation forces increasing influence of admixed terms arising from higher energy approximations. Increasing contribution of higher energy approximations in turn must increase the calculated nuclear interaction radius.

As an example we consider the nuclei \(^6\)Li and \(^6\)He, which ground states are characterized by \(JT = 10\) and \(JT = 01\) correspondingly. As can be seen from Table 2 the admixture of \(^3D_1\) term is absent for \(^6\)He whereas it is not the case for \(^6\)Li. Hence influence of higher energy approximation must be more pronounced in the \(^6\)He case and as consequence it must have the larger interaction radius, which is in full correspondence with experimental results.

### IV. CONCLUSIONS

The only quantities really needed for calculation of identical particle systems are two–particle density matrixes. The developed formalism is particulary pointed to description of energy spectrum formation mechanism and irregularities of root-mean-square radii (Halo effect) in light exotic nuclei. The method consistently lines the principles of antisymmetrization and translational invariancy and is implicitly based on Reduced Hamiltonian method. The proposed density matrices are very suitable for exact realistic density matrix expansion since it enable do not involve the realistic spectator functions, devoted for description of the remaining \(A - 2\) particle subsystem, into consideration. It is well-known that such oscillator many particle functions form slowly convergent series and thus the as large as possible set of states have to be used. This way the procedures of group theory for choosing the 'best state' loose their sense.

The calculation procedures described above including ones for computation of momentum recoupling matrices and Talmi-Moshinsky-Smirnov coefficients were implemented in computer code. The ordinary procedures for generating of complete many particle sets for large spaces usually suffer of buildup of numerical errors. Their presence could even lead to undefinite–ness of the obtained space dimension. Numerical instabilities do not concern the calculational procedures under preposition. All computations were performed with precise arithmetic using numbers represented in the form \(n/m/\sqrt{k}\), where \(n,m\) and \(k\) are integers. The application of the precise arithmetic is possible due to the orthogonal nature of the intrinsic density matrix formalism and the particulary character of developed calculational procedures that involve any numerical diagonalization and orthogonalization. The effectiveness of the proposed approach could be illustrated by the fact that intrinsic density matrices and related quantities of CFP nature for energy up to \(E_{\text{min}} + 2\) and nuclei up to \(A = 11\) could be calculated on 40 Mhz AT-386 within a few hours.
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