Influence of Geometrical Parameters of the Cross Section, Strength and Deformability of the Materials Used on Stress-strain State of Three-layered Reinforced Concrete

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Abstract. New concrete materials and their structures are increasingly diverse, so it requires extensive methods of calculating reinforced concrete structures, including multi-layered reinforced concrete structures. For usage in the practice of modern construction of multilayer structures made of concrete with different physic-mechanical characteristics, it is compulsory to conduct numerical studies of the stress-strain state of these structures under different types of loading. This paper analyzes the effect of cross section and concrete materials for the stress-strain state of three-layer reinforced concrete with light weight concrete in the internal layer and heavy concrete in the external layers. The analysis was carried out on three-layer beam samples with thickness 250mm, external layers of heavy concrete B12.5, B15, B20, B25 and the thickness from 40mm to 120mm; and internal layer from lightweight concrete - polystyrene concrete B0.5, B0.75, B1 and B1.5. The results of the study indicate that when the height and material of the extern layer changes, it greatly affects the stress and deformation state of the three-layer beam under the different loads. The obtained scientific results enable to determine rational parameters about geometric cross-section and materials used for various structural solutions of multilayer reinforced concrete structures. This would limit the number of actual test samples, increasing the efficiency of the experiments and practical production of three-layer structures.

1. Introduction

One of the main advantages of multilayer enclosing structures is to provide high thermal protection properties with the middle layer from light-weight concrete, low thermal conductivity. They occupy an intermediate position between conventional reinforced concrete structures and combined composite section with extern layers of reinforced concrete and the middle layer of different types of thermal insulation materials, characterized by low strength and high deformation. However, the choice of their rational parameters in the design is associated with the development of calculation methods.

There have been studies on the stress-deformation state of three-layer reinforced concrete beams with the middle layer from lightweight concrete materials and insulation under loading. The results of studies [1, 2, 3, 4] show that cracks do not appear in multilayer reinforced concrete structures until the structure is elastic and the tension in the stretched zone reaches the ultimate tensile resistance, and the ratio of stresses and strains is subject to Hooke's law.
In publications [6, 7, 8, 9, 12], the author focused on effect of the elastic modulus of the materials using in the middle and extern layers for the stress-deformation state of the three-layer reinforced concrete sandwich under different loads. However, the thickness of the external and internal concrete layers of multi-layer beams, as well as the properties of materials used in these layers of multi-layer reinforced concrete have not been carefully considered.

In this study, the authors analyze the effect of geometrical parameters of the cross section, the strength and deformability of the materials used on the stress-strain state of three-layer reinforced concrete.

2. Materials and methods of research

In three-layered reinforced concrete slabs with a light self-weight and high insulation efficiency, commonly concrete class B12.5 – B30 would be used for the external layers. In the middle layer, lightweight concrete with low thermal conductivity properties is used, with a compression resistance of the order of 1.5 MPa [10].

This article considers three-layered beams with a width of 160 mm, height of 250 mm and length of 3000 mm. For external layers, concrete of class B12.5, B15, B20 and B25 (table 1) is commonly used, with the thickness from 40mm to 120mm. For the middle layer, lightweight concrete with low thermal conductivity is used, with polystyrene concrete class B0.5, B0.75, B1 and B1.5.

The armature is obtained with 2 diameters of 8 mm (A-500), $\sigma_y = 475.2$ MPa, $\sigma_u = 660.5$ MPa, modulus of elasticity of the armature $E_s = 206,000$ MPa.

![Diagram of concrete compression](image)

**Table 1.** Parameters and characteristics of concrete in structural layers [10, 11]

|                | External layer | Internal layer |
|----------------|----------------|----------------|
|                | B12.5 | B15 | B20 | B25 | B0.5 | B0.75 | B1 | B1.5 |
| $R_b$ (MPa)    | 9.5   | 11  | 15  | 18.5| 0.57 | 0.84  | 1.1| 1.61 |
| $R_{bt}$ (MPa) | 1     | 1.15| 1.4 | 1.6 | 0.34 | 0.44  | 0.51| 0.61 |
| $E_b$ (MPa)    | 21000 | 25000| 27000| 30000| 500 | 650  | 850| 1100 |
| $\nu$          | 0.2   | 0.2 | 0.2 | 0.2 | 0.2 | 0.2   | 0.2| 0.2 |
Figure 2. Parameters of the experimental model

Figure 3. Calculating schemes of distribution of forces and deformations in three-layer sections at the time of crack formation

In the publication of the authors [6, 8, 9, 11], the design scheme for the distribution of forces and deformations in a three-layer reinforced concrete section with a monolithic bond of layers is shown in figure 3.

In this study, it is assumed that before cracking, the greatest relative lengthening of the extreme stretched fiber of the concrete is $2\frac{R_{b_{\text{ext}}}}{E_{\text{ext}}}$. In the ensuing discussion, the relative elongations and the corresponding stresses in the concrete in the marked sections in figure 3a and 3b.

The height of the compressed zone before the formation of cracks is determined from the equilibrium condition of external forces and internal forces in the reinforcement and concrete:

$$N_{b_{1}}+N_{b_{2}}+N_{s}=N_{b_{1}}+N_{b_{2}}+N_{s}$$

Forces of compressed shelf; compressed internal layer and compressed reinforcement: $N_{b_{1}}=\sigma_{\text{ext}} h_{b} b+0.5(\sigma_{\text{ext}} - \sigma_{\text{ext}}) h_{b} b$; $N_{b_{2}}=0.5\sigma_{\text{ext}} (x-h) b$; $N_{s}=(\varepsilon_{s} E_{s}) A_{s}$.

Forces of stretched external layer: $N_{b_{1}}=R_{b_{\text{ext}}} h_{b} b$ with calculating schemes figure 4a and $N_{b_{1}}=\sigma_{\text{ext}}^{44} h_{b} b+0.5(\sigma_{\text{ext}}^{44} - \sigma_{\text{ext}}^{44}) h_{b} b$ with calculating schemes figure 4b.

Forces of stretched internal layer and stretched reinforcement: $N_{b_{2}}=0.5\sigma_{\text{int}}^{44} (h-x-h) b$; $N_{s}=(\varepsilon_{s} E_{s}) A_{s}$.
In the section (4-4) of the cross section (the point of contact between the middle layer and the outer layer in the stretching zone), if the deformation \( \varepsilon_{b}^{4-4} = \varepsilon_{b0} \frac{h-x-h_{1}}{h-x} \geq \frac{R_{m}}{E_{ext}} \), the strength of the concrete of the external layer reaches the tensile strength of concrete \( R_{bt1} \).

From equation (3) the height of the compressible concrete zone is determined by the expression:

\[
x = \frac{b_{h_{1}}^{2} + b_{h_{1}}h - \frac{E_{h_{1}}}{E_{best}} b[h_{1}^{2} - (h-h_{1})^{2}] + 2a[A_{x}^{'} + (h-a)A_{x}]}{3h_{1}b + 2\frac{E_{silent}}{E_{best}} b(h-2h_{1}) + 2a(A_{x} + A_{y})}
\]  

(4)

From expression (4) change \( x \) to expression \( \varepsilon_{b}^{4-4} = \varepsilon_{b0} \frac{h-x-h_{1}}{h-x} \geq \frac{R_{m}}{E_{ext}} \) we get the result:

\[
\frac{E_{ext}}{E_{best}} \left( \frac{7bh_{1}^{2} - 2bhh_{1} + 2a[A_{x}^{'} + (h-a)A_{x} - (A_{x} + A_{y})(h-2h_{1})]}{b(h-2h_{1})(h-4h_{1})} \right) \leq 1
\]  

(5)

If the deformation \( \varepsilon_{b0}^{4-4} < \frac{R_{b,ext}}{2} \), the strength of the concrete of the external layer is less than the determination of the tensile strength of the concrete \( R_{bt1} \) (mean \( \sigma_{b,ext}^{4-4} = \varepsilon_{b0} \frac{h-x-h_{1}}{h-x} \frac{E_{b,ext}}{E_{ext}} < R_{b,ext} \)).

From equation (3) the height of the compressible concrete zone is determined by the expression:

\[
x = \frac{b_{h_{1}}^{2} + b_{h_{1}}h - \frac{E_{h_{1}}}{E_{best}} b[h_{1}^{2} - (h-h_{1})^{2}] + 2a[A_{x}^{'} + (h-a)A_{x}]}{3h_{1}b + 2\frac{E_{silent}}{E_{best}} b(h-2h_{1}) + 2a(A_{x} + A_{y})}
\]  

(6)

From expression (6) change \( x \) to expression \( \varepsilon_{b}^{4-4} = \varepsilon_{b0} \frac{h-x-h_{1}}{h-x} < \frac{R_{m}}{E_{ext}} \) we get the result:

\[
\frac{E_{ext}}{E_{best}} \left( \frac{7bh_{1}^{2} - 2bhh_{1} + 2a[A_{x}^{'} + (h-a)A_{x} - (A_{x} + A_{y})(h-2h_{1})]}{b(h-2h_{1})(h-4h_{1})} \right) \geq 1
\]  

(7)

For elements without prestressing, the magnitude of the moment perceived by the normal section, when cracks are formed, taking into account the effect of concrete shrinkage, is determined by the expression:

\[
M_{cr} = R_{b}W_{pl} - M_{p}
\]  

(8)

where \( W_{pl} \) - the elastic-plastic moment of resistance of the reduced section for the extreme stretched fiber under the assumption that there is no longitudinal force;

\( M_{p} \) - the moment of force \( P \) relative to the axis passing through the sound point furthest from the stretched zone. Within the scope of this study, ignore the effect of concrete shrinkage (\( M_{b} = 0 \)).

The deflection from bending in mid-span of beam specimens to cracking is calculated according to the formula:

\[
f_{M} = \frac{M}{\varphi_{b}E_{best}J_{red}} P_{m}^{2}
\]  

(9)
The deflection from the transverse forces in any section of the beam element with the coordinate \( x \) is determined by the formula:

\[
f_Q = \int \frac{kQ(x)}{GF} dx + C_Q
\]

where: \( C_Q \) - integration constant equal to zero at the hinge support of the beam;\n\( G \) - shear modulus of material; \( G = \frac{E_0}{(2(1 + \nu))} \)\n\( k \) - the coefficient that takes into account the shape and dimensions of the cross section, the values of which are determined by the formula:

\[
k = \frac{F}{J_{\text{red}}} \int \frac{S(z)}{b(z)} dz
\]

where: \( F \) and \( J_{\text{red}} \) - the area and moment of inertia of the reduced section, respectively; \( S \) and \( b \) - the static moment and the width of the cut-off part of the reduced section, respectively.

3. Results and discussions

In the first series-1 of three-layer reinforce concrete beams, the authors analyze the three-layers concrete beams have the same thickness of 250mm. The external layer thickness changes from 40mm to 120mm, and the material is concrete B12,5, B15, B20 and B25 (table. 2). The internal layer material is polystyrene concrete B0,75. The results of the analysis are presented in table. 3, in figure 4, 5.

Table 2. Parameters of materials used in three-layer reinforced concrete beams

| Layer of beams          | Series 1 | Series 2 |
|-------------------------|----------|----------|
| External layer          | B12,5    | B15      |
| Interior layer          | B0,75    | B0,75    |

Table 3. Comparison of the moment and deflection of the three-layer elements at the time of crack formation

| \( h_1 \), mm | \( h_2 \), mm | The moment of a multi-layer reinforced concrete beam (\( M_{\text{cr}} \)), kN.m | B1 | B2 | B3 | B4 |
|---------------|---------------|---------------------------------|----|----|----|----|
| 40            | 170           | 2.540 /100.0%                   | 2.835 /100.0% | 3.200 /100.0% | 3.297 /100.0% |
| 50            | 150           | 2.854 /112.4%                   | 3.197 /112.8% | 3.631 /113.5% | 3.755 /113.9% |
| 60            | 130           | 3.136 /123.5%                   | 3.523 /124.3% | 4.018 /125.6% | 4.168 /126.4% |
| 70            | 110           | 3.381 /133.1%                   | 3.807 /134.3% | 4.359 /136.2% | 4.532 /137.5% |
| 80            | 90            | 3.559 /140.1%                   | 4.013 /141.5% | 4.605 /143.9% | 4.794 /145.4% |
| 90            | 70            | 3.722 /146.5%                   | 4.201 /148.2% | 4.830 /150.9% | 5.033 /152.7% |
| 100           | 50            | 3.876 /152.6%                   | 4.379 /154.5% | 5.041 /157.6% | 5.258 /159.5% |
| 110           | 30            | 4.027 /158.6%                   | 4.553 /160.6% | 5.247 /164.0% | 5.477 /166.1% |
| 120           | 10            | 4.177 /164.5%                   | 4.725 /166.7% | 5.452 /170.4% | 5.693 /172.7% |

Note: the upper part of the sign "/" represents the value, the under part of the sign "/" represents the percent %
Figure 4. Change of crack resistance moment of reinforced concrete three-layered beams of Series-1

(a) The value of $M_{cr}$ of three-layered beam samples - Series 1

(b) The percent of $M_{cr}$ of three-layered beam samples - Series 1

Figure 5. The moment of crack resistance $M_{cr}$ of three-layered beams - Series 1
The results of the analyses in table 3, in figures 4 and 5 indicate that type of concrete materials used and the thickness of the external layer have a great influence on the crack resistance ($M_{crc}$) of three-layered reinforced concrete beams. At the external layer thickness $h_1 = 40$ mm, when the concrete of the external layer changes from B12.5 ($R_b = 9.5$ MPa) to B25 ($R_b = 18.5$ MPa), the moment $M_{crc}$ of the reinforced concrete beam three-layered increased by about 30%; when the external layer thickness $h_1$ from 60 to 120 mm, the corresponding $M_{crc}$ change is from 32 to 36%.

In the beam samples B1; B2; B3 and B4 of series-1, the material used by the outer and inner layers of the three-layered beams remains constant, when the outer layer thickness varies from 40 mm to 120 mm, making the crack resistance of the beam greatly changed. $M_{crc}$ of B1 beam sample changes to 164.5%, The change of $M_{crc}$ corresponds to samples of B2 beams; B3 and B4 respectively 166.7%; 170.4% and 172.7%.

In the beams of series-2, the authors surveyed three-layer reinforced concrete beams (B5; B6; B7 and B8) with the thickness 250 mm, the external layer of concrete material B15 and the internal layer of polystyrene concrete materials B0.5, B0.75, B1 and B1.5 (table 2). Results of analyses of changes $M_{crc}$ of the beams are shown in table 4 and figure 6.

**Table 4.** Comparison of the moment of the three-layer elements at the time of crack formation

| No | External layer thickness ($h_1$, mm) | Internal layer thickness ($h_2$, mm) | B5     | B6     | B7     | B8     |
|----|-------------------------------------|-------------------------------------|--------|--------|--------|--------|
| 1  | 40                                  | 170                                 | 2.821  | 2.835  | 2.854  | 2.877  |
| 2  | 50                                  | 150                                 | 3.187  | 3.197  | 3.211  | 3.229  |
| 3  | 60                                  | 130                                 | 3.516  | 3.523  | 3.533  | 3.545  |
| 4  | 70                                  | 110                                 | 3.802  | 3.807  | 3.813  | 3.821  |
| 5  | 80                                  | 90                                  | 4.010  | 4.013  | 4.017  | 4.022  |
| 6  | 90                                  | 70                                  | 4.200  | 4.201  | 4.203  | 4.206  |
| 7  | 100                                 | 50                                  | 4.378  | 4.379  | 4.380  | 4.381  |
| 8  | 110                                 | 30                                  | 4.552  | 4.553  | 4.553  | 4.553  |
| 9  | 120                                 | 10                                  | 4.725  | 4.725  | 4.725  | 4.725  |

*Note: the upper part of the sign "/" represents the value, the under part of the sign "/" represents the percent %*

**Figure 6.** The moment of crack resistance $M_{crc}$ when changing the thickness of the external layer
The analysis results in table 4 indicate that, when changing the polystyrene concrete of the middle layer from B0.5 ($R_b = 0.57$ MPa) to B1.5 ($R_b = 1.61$ MPa), provided that moment $M_{crc}$ of beams three-layered reinforced concrete does not change significantly when $h_1$ remains unchanged. However, when $h_1$ changes from 40mm to 120mm, $M_{crc}$ changes from 100% to 164.2%.

In this analysis, the authors consider changing the deflection of the three-layer reinforced concrete beams in series-1 and series-2, by changing the type of concrete in the external and internal layers, and changing thickness of the external layers of them. The analytical results are shown in table 5 and in figure 7.

### Table 5. The deflection of the three-layered elements at the time of the start of crack formation

| N° | External layer thickness ($h_1$, mm) | Internal layer thickness ($h_2$, mm) | The deflection of three-layer beams ($f_{crc}$) at the time of the start of crack formation, cm |
|----|------------------------------------|-------------------------------------|------------------------------------------------------------------------------------------------|
|    |                                    |                                     | B1       | B2       | B3       | B4       | B5       | B6       | B7       | B8       |
| 1  | 40                                 | 170                                 | 0.183    | 0.188    | 0.194    | 0.198    | 0.208    | 0.194    | 0.183    | 0.176    |
| 2  | 50                                 | 150                                 | 0.190    | 0.196    | 0.202    | 0.207    | 0.218    | 0.202    | 0.190    | 0.181    |
| 3  | 60                                 | 130                                 | 0.196    | 0.202    | 0.209    | 0.214    | 0.226    | 0.209    | 0.196    | 0.187    |
| 4  | 70                                 | 110                                 | 0.201    | 0.208    | 0.214    | 0.218    | 0.231    | 0.214    | 0.201    | 0.192    |
| 5  | 80                                 | 90                                  | 0.200    | 0.208    | 0.214    | 0.218    | 0.229    | 0.214    | 0.201    | 0.192    |
| 6  | 90                                 | 70                                  | 0.199    | 0.207    | 0.211    | 0.215    | 0.225    | 0.211    | 0.201    | 0.193    |
| 7  | 100                                | 50                                  | 0.195    | 0.204    | 0.207    | 0.210    | 0.218    | 0.207    | 0.199    | 0.193    |
| 8  | 110                                | 30                                  | 0.191    | 0.198    | 0.201    | 0.203    | 0.208    | 0.201    | 0.196    | 0.191    |
| 9  | 120                                | 10                                  | 0.184    | 0.191    | 0.193    | 0.194    | 0.196    | 0.193    | 0.191    | 0.190    |

### Figure 7. The deflection of three-layered reinforced concrete beams with changes the external layer thickness

Results of deflection analysis of three-layered reinforced concrete beams in series -1 (B1, B2, B3 & B4) show that with the same height of beams ($h = 250$mm), polystyrene concrete materials in the internal layer is constant, when changing the external layer materials from concrete B12.5 to B25, the beam deflection at the time of the crack began to form changes insignificantly (from 0.183 to 0.198 cm). The deflection at the beginning of cracking reaches the maximum value with $h_1 = 70$mm. This is possible due to the deflection induced by $f_M$ and deflection caused by cutting $f_Q$ are large, so the total ($f_M + f_Q$)
reaches the maximum value. And the higher the grade of concrete used in the external layer, the higher the deflection at the time of the start of crack formation.

For three-layered reinforced concrete beams in series-2, the deflection at the time of crack formation varies considerably more than the beam deflections in series-1. The higher the grade of concrete used in the internal layer, the smaller the deflection at the time of the start of crack formation.

4. Conclusions

- Geometrical characteristics of concrete sections greatly affect stress and deformation of three-layered reinforced concrete beams. When thickness of the external layer changes, it can cause crack resistance \( M_{\text{crc}} \) to vary from 164\% to 173\% and the deflection \( f \) changes about 10\%.
- The materials used in the external and internal layers affect the crack resistance of three-layered reinforced concrete beams. However, the cracking resistance \( M_{\text{crc}} \) in the case of the changing external layer concrete materials is significantly increased more than the case of the changing internal layer materials.
- The deflection at the time of the start of crack formation reaches the maximum value when the external layer thickness is an intermediate value \( h_1 = 70\text{mm} \) and reaches the minimum when \( h_1 \) is value \( h_1 = 40\text{mm} \) or the value \( h_1 = 120\text{mm} \).

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