A U-spin prediction for the CP-forbidden transition
\[ e^+e^- \rightarrow D^0\bar{D}^0 \rightarrow (K^+K^-)_D (\pi^+\pi^-)_D \]

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Abstract

The LHCb Collaboration has recently reported the discovery of direct CP violation in combined \( D^0 \rightarrow K^+K^- \) and \( D^0 \rightarrow \pi^+\pi^- \) decay modes at the 5.3\( \sigma \) level. Assuming U-spin symmetry (i.e., \( d \leftrightarrow s \) interchange symmetry) for the strong-interaction parts of these two channels, we find that their corresponding direct CP-violating asymmetries are \( A_{\text{CP}}(K^+K^-) \simeq (-7.7 \pm 1.5) \times 10^{-4} \) and \( A_{\text{CP}}(\pi^+\pi^-) \simeq (7.7 \pm 1.5) \times 10^{-4} \). The CP-forbidden transition \( e^+e^- \rightarrow D^0\bar{D}^0 \rightarrow (K^+K^-)_D (\pi^+\pi^-)_D \) on the \( \psi(3770) \) resonance is therefore expected to have a rate of \( O(10^{-10}) \) or smaller under U-spin symmetry, and it can be observed at a high-luminosity super-\( \tau \)-charm factory if at least \( 10^{10} \) pairs of coherent \( D^0 \) and \( \bar{D}^0 \) events are accumulated.

PACS number(s): 14.60.Pq, 11.30.Hv, 13.35.Hb.
Within the standard model charmed CP violation in \(D\)-meson decays is expected to be of \(\mathcal{O}(10^{-3})\) or smaller. The reason for this expectation is simply that the charmed unitarity triangle of the Cabibbo-Kobayashi-Maskawa (CKM) quark flavor mixing matrix \(V\), defined by the orthogonality relation \(V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} = 0\) in the complex plane as illustrated by Fig. 1 [1], is so sharp that the ratio of the CP-violating part to the CP-conserving part in many \(D\)-meson decays is essentially characterized by [2]

\[
\frac{|\text{Im}(V_{ub}^* V_{cb})|}{|V_{us}^* V_{cs}|} \sim \frac{|\text{Im}(V_{ub}^* V_{cb})|}{|V_{us}^* V_{cs}|} \sim A^2 \lambda^4 \eta \simeq 6.3 \times 10^{-4},
\]

where \(\lambda \simeq 0.224\), \(A \simeq 0.836\) and \(\eta \sim 0.355\) are the Wolfenstein parameters [3]. In other words, it is the smallest inner angle of all the six CKM unitarity triangles [4],

\[
\phi_{\text{charm}} \equiv \arg\left(-\frac{V_{ud}^* V_{cd}}{V_{us}^* V_{cs}}\right) \simeq A^2 \lambda^4 \eta \simeq 6.3 \times 10^{-4},
\]

that sets an upper bound on the weak-interaction parts of charmed CP violation. That is why the strength of CP violation in the charm sector is at most of \(\mathcal{O}(10^{-3})\) in the standard model even if there exist significant final-state interactions.

Figure 1: The charmed unitarity triangle of the CKM matrix defined by the orthogonality relation \(V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} = 0\) in the complex plane.

The above expectation is consistent with the first observation of direct CP violation in combined \(D^0 \to K^+K^-\) and \(D^0 \to \pi^+\pi^-\) decay modes, as recently reported by the LHCb Collaboration [5]. The explicit experimental result is

\[
\Delta A_{\text{CP}} \equiv A_{\text{CP}}(K^+K^-) - A_{\text{CP}}(\pi^+\pi^-) = (-15.4 \pm 2.9) \times 10^{-4},
\]

where \(A_{\text{CP}}(K^+K^-)\) and \(A_{\text{CP}}(\pi^+\pi^-)\) can simply be interpreted as the direct CP-violating asymmetries of \(D^0 \to K^+K^-\) and \(D^0 \to \pi^+\pi^-\) decays because both the \(D^0\)-\(\bar{D}^0\) mixing effect and the indirect CP-violating asymmetries are found to be negligibly small in this measurement. In this case we just make use of the definitions

\[
A_{\text{CP}}(K^+K^-) = \frac{\Gamma(D^0 \to K^+K^-) - \Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to K^+K^-) + \Gamma(D^0 \to K^+K^-)},
\]

\[
A_{\text{CP}}(\pi^+\pi^-) = \frac{\Gamma(D^0 \to \pi^+\pi^-) - \Gamma(\bar{D}^0 \to \pi^+\pi^-)}{\Gamma(D^0 \to \pi^+\pi^-) + \Gamma(\bar{D}^0 \to \pi^+\pi^-)},
\]

Figure 1
and neglect the effects of $D^0$-$\bar{D}^0$ mixing and indirect CP violation as a fairly reasonable approximation. Then the question is how to separately determine or estimate the values of $A_{\text{CP}}(K^+K^-)$ and $A_{\text{CP}}(\pi^+\pi^-)$ from their difference given in Eq. (3).

Assuming that direct CP violation arises from the interference between tree and one-loop (penguin) amplitudes of $D^0 \to K^+K^-$ or $D^0 \to \pi^+\pi^-$ decay as illustrated in Fig. 2, we find that $A_{\text{CP}}(K^+K^-) = -A_{\text{CP}}(\pi^+\pi^-)$ holds in the limit of U-spin symmetry (i.e., $d \leftrightarrow s$ interchange symmetry) for the strong-interaction parts of these two decays.\footnote{U-spin is an SU(2) subgroup of flavor SU(3) group, under which a pair of down ($d$) and strange ($s$) quarks forms a doublet, analogous to the isospin symmetry of up ($u$) and down ($d$) quarks. Under this symmetry $d$ and $s$ quarks are expected to couple equally to gluons at short distances in all the quark diagrams, and thus the breaking of $d \leftrightarrow s$ interchange symmetry mainly occurs at the hadron level and its effect is measured by the relevant decay constants and form factors \cite{6}.} To see this point clearly, let us write out their decay amplitudes in a universal way as follows:

$$A(D^0 \to K^+K^-) = T_s (V_{cs} V_{us}^*) \exp (i\delta_s) + P_s (V_{cb} V_{ub}^*) \exp (i\delta'_s) ,$$

$$A(D^0 \to \pi^+\pi^-) = T_d (V_{cd} V_{ud}^*) \exp (i\delta_d) + P_d (V_{cb} V_{ub}^*) \exp (i\delta'_d) , \quad (5)$$

where $T_q$ and $P_q$ (for $q = d, s$) are real and positive, $\delta_q$ and $\delta'_q$ (for $q = d, s$) stand respectively for the strong phases of tree and penguin amplitudes, and only the dominant bottom-quark contribution to the penguin loop is taken into account as a reasonable approximation. If the penguin-diagram contribution is neglected and the Wolfenstein phase convention \cite{7} for the CKM matrix is adopted, one will arrive at $A(D^0 \to K^+K^-) \simeq -A(D^0 \to \pi^+\pi^-)$ under U-spin symmetry \cite{8, 9, 10} because the latter assures $T_s = T_d, \delta_s = \delta_d$ and $\delta'_s = \delta'_d$ to hold. Since $K^+K^-$ and $\pi^+\pi^-$ are CP-even eigenstates, it is straightforward to have

$$A(\bar{D}^0 \to K^+K^-) = T_s (V_{cs} V_{us}^*) \exp (i\delta_s) + P_s (V_{cb} V_{ub}^*) \exp (i\delta'_s) ,$$

$$A(\bar{D}^0 \to \pi^+\pi^-) = T_d (V_{cd} V_{ud}^*) \exp (i\delta_d) + P_d (V_{cb} V_{ub}^*) \exp (i\delta'_d) . \quad (6)$$
Using Eqs. (5) and (6) to calculate the direct CP-violating asymmetries defined in Eq. (4), we immediately obtain

\[ A_{CP}(K^+K^-) \simeq +2A^2\lambda^4\eta_2\frac{P_s}{T_s} \sin(\delta_s - \delta'_s) , \]
\[ A_{CP}(\pi^+\pi^-) \simeq -2A^2\lambda^4\eta_2\frac{P_d}{T_d} \sin(\delta_d - \delta'_d) , \]

(7)

where \( P_q \) is expected to be comparable with \( T_q \) (for \( q = d, s \)) in magnitude. A combination of Eqs. (3) and (7) yields

\[ \frac{P_s}{T_s} \sin(\delta'_s - \delta_s) + \frac{P_d}{T_d} \sin(\delta'_d - \delta_d) \simeq 1.2 . \]

(8)

Given U-spin symmetry, we are left with \( P_s/T_s = P_d/T_d \), \( \delta_s = \delta_d \) and \( \delta'_s = \delta'_d \), and thus

\[ A_{CP}(K^+K^-) \simeq +\frac{1}{2}\Delta A_{CP} \simeq (-7.7 \pm 1.5) \times 10^{-4} , \]
\[ A_{CP}(\pi^+\pi^-) \simeq -\frac{1}{2}\Delta A_{CP} \simeq (+7.7 \pm 1.5) \times 10^{-4} , \]

(9)

together with \( (P_q/T_q) \sin(\delta'_q - \delta_q) \simeq 0.61 \) for \( q = d \) and \( s \).

In view of current experimental data on the branching fractions of \( D^0 \to \pi^+\pi^- \) and \( K^+K^- \) decay modes \[3\],

\[ \mathcal{B}(D^0 \to K^+K^-) = (3.97 \pm 0.07) \times 10^{-3} , \]
\[ \mathcal{B}(D^0 \to \pi^+\pi^-) = (1.407 \pm 0.025) \times 10^{-3} , \]

(10)

U-spin symmetry is apparently broken. But the ratio \( \sqrt{\mathcal{B}(D^0 \to K^+K^-)/\mathcal{B}(D^0 \to \pi^+\pi^-)} \simeq 1.68 \) can essentially be explained after the relevant phase-space factors, decay constants and form factors are taken into account. Therefore, we expect that the U-spin estimates of \( A_{CP}(K^+K^-) \) and \( A_{CP}(\pi^+\pi^-) \) made in Eq. (9) should be close to their true values. In fact, our U-spin results are essentially consistent with the predictions made by Cheng and Chiang in Ref. \[11\] and by Li, Lü and Yu in Ref. \[12\] with the consideration of SU(3) symmetry breaking effects and final-state interactions.

Instead of trying to estimate the magnitudes of \( T_q \) and \( P_q \) which involve quite a lot of hadronic (nonperturbative) uncertainties, we proceed to estimate the rate of the CP-forbidden transition \( e^+e^- \to D^0\bar{D}^0 \to (K^+K^-)_D(\pi^+\pi^-)_D \) on the \( \psi(3770) \) resonance with the help of Eqs. (9) and (10). On this resonance the \( D^0\bar{D}^0 \) pair with odd CP can be coherently produced, and thus its transition into the CP-even state \( (K^+K^-)_D(\pi^+\pi^-)_D \) is CP-forbidden. Here CP violation is measured by a nonzero rate rather than an asymmetry between a decay mode and its CP-conjugate progress, and hence it is of particular interest both theoretically and experimentally.
A generic formula for the branching fraction of such a CP-forbidden transition has been calculated in Ref. [13]. Given the fact that both CP violation in $D^0-\bar{D}^0$ mixing and indirect CP violation from the interplay of decay and $D^0-\bar{D}^0$ mixing are negligible in $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ channels [5], it is convenient for us to make use of the following simplified formula which only contains direct CP-violating effects:

$$\text{Rate} \approx 2\mathcal{B}(D^0 \to K^+K^-) \cdot \mathcal{B}(D^0 \to \pi^+\pi^-) \left| \frac{A(\bar{D}^0 \to K^+K^-)}{A(D^0 \to K^+K^-)} - \frac{A(D^0 \to \pi^+\pi^-)}{A(D^0 \to \pi^+\pi^-)} \right|^2$$

$$\approx 8A^4\lambda^8\eta^2\mathcal{B}(D^0 \to K^+K^-) \cdot \mathcal{B}(D^0 \to \pi^+\pi^-) \left| \frac{P_s}{T_s}e^{i(\delta_s' - \delta_s)} + \frac{P_d}{T_d}e^{i(\delta_d' - \delta_d)} \right|^2$$

$$\approx 32A^4\lambda^8\eta^2\mathcal{B}(D^0 \to K^+K^-) \cdot \mathcal{B}(D^0 \to \pi^+\pi^-) \left| \frac{P_q}{T_q} \right|^2 \approx 7.1 \times 10^{-11} \left| \frac{P_q}{T_q} \right|^2,$$  \quad (11)

where U-spin symmetry has finally been used. One can see that this transition rate is at most of $\mathcal{O}(10^{-10})$ if $P_q$ is comparable with $T_q$ (for $q = d, s$) in magnitude. Therefore, to see a single event of this kind of CP-forbidden transition requires at least $10^{10}$ $D^0\bar{D}^0$ pairs on the $\psi(3770)$ resonance for a perfect detection efficiency. A high-luminosity super-$\tau$-charm factory might be able to do this job in the future. However, to discover such a tiny CP-forbidden transition at the $5\sigma$ level, at least $10^{12}$ coherent $D^0\bar{D}^0$ pairs are needed.  

The author would like to thank H.Y. Cheng, H.B. Li and S. Zhou for very helpful discussions. This work was supported in part by the National Natural Science Foundation of China under Grant No. 11835013, and the Ministry of Science and Technology of China under Contract No. 2015CB856701.

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2The author would like to thank H.B. Li for pointing out this challenge.
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