A POSSIBLE PHYSICAL CONNECTION BETWEEN HELIUM-RICH STELLAR POPULATIONS OF MASSIVE GLOBULAR CLUSTERS AND THE UV UPTURN OF GALACTIC SPHEROIDS

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ABSTRACT

We discuss a possible physical connection between helium-rich ($Y \geq 0.35$) stellar populations of massive globular clusters (GCs) and the ultraviolet (UV) upturn of galactic spheroids by using analytical and numerical models. In our model, all stars are initially formed as bound or unbound star clusters (SCs) formed from giant molecular clouds (GMCs) and the SCs can finally become GCs, open clusters, and field stars depending on the physical properties of their host GMCs. An essential ingredient of the model is that helium-rich stars are formed almost purely from gas ejected from massive asymptotic giant branch stars. The helium-rich star formation is assumed to occur within massive SCs if the masses of the progenitor GMCs are larger than a threshold mass ($M_{\text{thres}}$). These massive SCs can finally become either massive GCs or helium-rich field stars depending on whether they are disintegrated or not. Using this model, we show that if the initial mass functions (IMFs) in galactic spheroids are mildly top-heavy, then the mass fractions of helium-rich main-sequence stars ($f_{\text{He}}$) can be as large as $\sim 0.1$ for $M_{\text{thres}} = 10^7 M_{\odot}$. $f_{\text{He}}$ is found to depend on IMFs and $M_{\text{thres}}$ such that it can be larger for shallower IMFs and smaller $M_{\text{thres}}$. The inner regions of galactic spheroids show larger $f_{\text{He}}$ in almost all models. Based on these results, we suggest that if the UV upturn of elliptical galaxies is due to the larger fractions of helium-rich stars, then its origin can be closely associated with top-heavy IMFs in the galaxies.

Key words: galaxies: elliptical and lenticular, cD – globular clusters: general – stars: formation – ultraviolet: galaxies

Online-only material: color figures

1. INTRODUCTION

Recent observational and theoretical studies of the Galactic globular clusters (GCs) have suggested that some of the massive GCs ($\omega$ Cen and NGC 2808) have significant fractions of helium-rich stars (e.g., Bedin et al. 2004; Norris 2004; Lee et al. 2005a; Piotto et al. 2005, 2007; Renzini 2008). The observed unusual level of helium enhancement ($\Delta Y/\Delta Z \approx 70$; Piotto et al. 2005) and sizable fractions of helium-rich stars in these GCs have driven many theoretical studies to make great efforts in understanding where and how helium-rich stars can be formed (e.g., Bekki & Norris 2006; D’Antona & Ventura 2007; D’Ercole et al. 2008). As discussed by Renzini (2008), helium-rich stars can be formed from gaseous ejecta from massive asymptotic giant branch (AGB) stars with initial masses ($m_1$) ranging from $3 M_\odot$ to $8 M_\odot$, though the ejecta should not be diluted by interstellar gas (ISM) to keep the original high helium abundances of the ejecta.

Code & Welch (1979) investigated the integrated light of seven elliptical and S0 galaxies for a wavelength range of 1550–4250 Å and found that some of them show an increase in energy at the shortest wavelengths. Since this discovery of the UV upturn, the origin of the UV upturn has been extensively discussed both in observational and theoretical studies (e.g., Bertola et al. 1980; Burstein et al. 1988; Greggio & Renzini 1990; Hirsch et al. 1992; Dorman et al. 1995; Brown et al. 1997; O’Connell 1999; Yi et al. 1997, 1998, 2005). Although there can be a number of stellar candidates responsible for the UV upturn in galaxies, one of the most promising ones is old horizontal-branch stars (e.g., see Yi 2008 for a recent review). It is suggested that enhanced helium abundances can play an important role in the formation of hot stars responsible for the UV upturn (Yi 2008).

Brown et al. (2003) investigated the UV emission in eight early-type galaxies at $z = 0.33$ (a look-back time of 3.9 Gyr) and found that the UV emission in these galaxies is significantly weaker than it is in the current epoch. Observational studies by Galaxy Evolution Explorer (GALEX) investigated the UV properties of bright cluster galaxies (BCGs) in clusters at $z < 0.2$ and compared them with those of nearby giant elliptical galaxies (e.g., Lee et al. 2005b; Lee et al. 2007). Lee et al. (2007) concluded that the observed evolution of $F_{\text{UV}} - V$ color is consistent with a model in which the dominant FUV source is hot horizontal-branch stars. Yi et al. (2011) investigated correlations between the strength of the UV upturn and global galactic properties (e.g., luminosity) in BCGs and did not find any remarkable correlations (see also Loubser & Sánchez-Blázquez 2011 for similar results). Yi et al. (2011) therefore concluded that the helium sedimentation scenario proposed by Peng & Nagai (2009) cannot be supported by their observational results.

These observational and theoretical studies appear to suggest that there can be helium-rich stellar populations in diverse objects with dramatically different masses and sizes ranging from GCs to BCGs. Although it would be likely that different astrophysical objects with helium-rich stars have different origins, it would be possible that they can have a common origin. Previous theoretical studies, however, have not yet provided a unified picture for the origins of helium-rich stars in GCs, galactic bulges, and elliptical galaxies with the UV upturn. It is therefore worthwhile to construct a model to discuss the origins of helium-rich stars observed in GCs and galactic spheroids in a self-consistent manner.

The purpose of this paper is to present a new model which can provide a possible explanation for the origins of helium-rich stars with $Y \geq 0.35$ in GCs and galactic spheroids in a
self-consistent manner. Based on the model, we mainly discuss what types of initial mass functions (IMFs) are required to explain the observed possible fraction of helium-rich stars in galactic spheroids with the UV upturn (e.g., Chung et al. 2011). The most important ingredient of the model is that helium-rich stars are formed almost purely from gas ejected from massive AGB stars with \( M_{\odot} \lesssim m_1 \lesssim 8 M_{\odot} \) with no/little dilution of the gas with ISM. This formation of new stars from AGB ejecta has also been included in recent theoretical models of GC formation (e.g., D’Ercole et al. 2008).

In the present study, we assume that such formation of helium-rich stars almost directly from AGB ejecta can be possible only within star clusters (SCs) formed from massive giant molecular clouds (GMCs). By considering that the vast majority of stars are observed to be initially formed as bound or unbound SCs (e.g., Lada & Lada 2003), we assume that all stars form as SCs from GMCs in the model. Some massive SCs can retain gaseous ejecta from AGB stars and consequently have new helium-rich stars formed from the ejecta, and the helium-rich stars can become field stars of their host galaxies when the SCs become disintegrated. Such massive SCs can finally become GCs or nuclear SCs (or stellar galactic nuclei) depending on their birth places, if they are not destroyed by their host galaxies. The IMFs and GMCMFs are key parameter that can determine mass fractions of helium-rich stars in galaxies. By using this new model, we discuss the origin of physical properties of the Galactic GCs and the UV upturn in galactic bulges and elliptical galaxies.

Although it remains less clear what fractions of stars should be helium-rich stars in galactic spheroids with the UV upturn (Yi 2008), Chung et al. (2011) have recently suggested that if about 11% of stellar populations in galactic spheroids are helium-rich stars, then they can show the UV upturn for a given IMF. In their models, they considered that the major source of far-UV flux originates from metal-poor and helium-enhanced hot horizontal-branch stars. We therefore consider that if the fractions of helium-rich stars in galactic spheroids are less than \( \sim 0.1 \), the spheroids are unlikely to show the UV upturn in the present models. We investigate in what physical conditions (e.g., IMFs) the fractions of helium-rich stars can be above 0.1 for galactic spheroids to show clearly the UV upturn.

A possible physical link between helium-rich stellar populations and the UV upturn of galactic spheroids has already been pointed out by a number of authors (e.g., Yi 2008). Thus, the main point of the present study is not to propose the importance of helium-rich stellar populations in the origin of the UV upturn but to discuss how galactic spheroids can have significant fractions of helium-rich stellar populations. It should also be stressed that the present model is idealized and less realistic in some points (e.g., no chemical evolution of galaxies): the model should be regarded as a first step toward better understanding the origin of helium-rich stars in GCs and galaxies. More sophisticated numerical simulations including SC formation processes in galaxies will need to be done in our future studies to address the UV upturn problem in a much more quantitative way.

The plan of the paper is as follows: in the next section, we describe a model which enables us to estimate (1) the mass fraction of helium-rich stars in a single GC and (2) the mass fraction of helium-rich main-sequence (MS) stars in a galaxy for a given IMF and GMCMF. In Section 3, we present the results on the number/mass fraction of massive GCs with helium-rich stars in the Galaxy and the mass fractions of helium-rich MS stars in galaxies with different IMFs and GMCMFs. In Section 4, we discuss important implications of the present results in terms of the origin of helium-rich stars in GCs, the Galactic bulge, and elliptical galaxies. We also discuss the origin of the radial gradients of helium-rich stars in galactic spheroids in this section. We summarize our conclusions in Section 5.

2. THE MODEL

2.1. SCs as Fundamental Building Blocks of Galaxy Formation

2.1.1. SCs as Fundamental Building Blocks of Galaxy Formation

We adopt a scenario in which (1) all stars in galaxies are formed as bound or unbound SCs from GMCs and (2) helium-rich stars with \( Y \gtrsim 0.35 \) can be formed from gas ejected from AGB stars only in massive SCs. Some SCs can become field stars in their host galaxies after disintegration and others can become massive GCs or nuclear SCs (or stellar nuclei) in galaxies. Furthermore, if massive SCs with helium-rich stars are destroyed by their host galaxies, then the stars become helium-rich field populations in the galaxies. Therefore, formation efficiencies of massive SCs and IMFs in GCs and galaxies are key parameters which can determine their mass fractions of helium-rich stars. This scenario is referred to as the “cluster disintegration scenario” (CDS) just for convenience in the present study. Since a number of possibly unfamiliar acronyms are used in the present study, their physical meanings are briefly summarized in Table 1 in order for readers to understand more clearly the present paper.

| Acronyms | Meaning |
|----------|---------|
| CDS | Cluster disintegration scenario |
| SC | Star cluster |
| GC | Globular cluster |
| HNS | Helium-normal stars |
| HRS | Helium-rich stars |
| MS | Main-sequence (stars) |
| MGC | Massive GC (i.e., SCs with HRS) |
| GMCMF | GMC mass function |

In the CDS, stars formed directly from GMCs are assumed to have “normal” \( Y \) that is observed in \( \text{H} \) regions of galaxies (e.g., Peimbert et al. 2007) and predicted from canonical chemical evolution (i.e., \( \Delta Y/\Delta Z \approx 2 \)), and they are referred to as “HNS” (helium-normal stars). After these first generations of stars form, then the second generation of stars can form from gaseous ejecta of AGB stars among HNS without dilution of the ejecta with ISM. These second generations of stars can show large degrees of helium enhancement (i.e., \( \Delta Y/\Delta Z \gtrsim 4 \)) which cannot be expected in canonical chemical evolution models in which ISM and gaseous ejecta from stars are assumed to well mix and then form new stars. These second generations of stars are referred to as “HRS” (helium-rich stars) just for convenience. As demonstrated by many recent theoretical studies (e.g., D’Antona et al. 2010), massive AGB stars can eject gas with enhanced helium abundances (\( \Delta Y \approx 0.07 \)). Therefore it is quite reasonable to assume that HRS can have high helium abundances. Figure 1 illustrates a whole picture for the formation histories of two different populations (HNS and HRS) from GMCs in the CDS.
GMCs in the CDS. The evolution of newly formed HNS (blue stars) depends on the masses of their host GMCs ($m_{\text{gmc}}$). HNS formed in lower mass GMCs with $m_{\text{gmc}} < M_{\text{thres}}$ cannot form HRS (red stars) from gaseous ejecta of their AGB stars owing to the shallow gravitational potentials of the SCs. The HNS on the other hand, if $m_{\text{gmc}} > M_{\text{thres}}$, then gaseous ejecta of AGB stars among HNS can be converted into new HRS (red stars) owing to the deeper gravitational potentials of the SCs. The stars in the SCs can become either field HNS/HRS or main components of massive SCs with HRS depending on the physical properties of the SCs (i.e., on whether the SCs become disintegrated). The field HRS in galactic spheroids are responsible for the UV upturn in the CDS.

(A color version of this figure is available in the online journal.)

Our recent numerical simulations on secondary star formation from AGB ejecta in SCs have shown that the ejecta can be converted efficiently into new stars only in massive SCs with masses ($m_{\text{sc}}$) larger than $\sim 10^6 M_\odot$ (Bekki 2010, 2011). This is because AGB ejecta can be accumulated in the deep potential wells of massive SCs to form high-density gaseous regions with the densities exceeding the threshold gas density ($\rho_{\text{thres}}$) for star formation ($\rho_{\text{thres}} \gtrsim 10^{3} - 10^{5}$ atoms $\text{cm}^{-3}$). We accordingly consider that AGB ejecta can be converted into new stars without the dilution with ISM if some physical conditions are met for SCs. We assume that (1) $m_{\text{sc}}/m_{\text{gmc}}$ is constant for all SCs and (2) the HRS formation is possible within SCs only if the mass of the gaseous GMCs of their host GMCs can exceed a threshold mass ($M_{\text{thres}}$). The introduction of $M_{\text{thres}}$ is therefore consistent with recent numerical simulations by D’Ercole et al. (2008) and Bekki (2011).

2.1.3. Y in HRS

It depends on the chemical yields of AGB stars whether HRS can have $Y \gtrsim 0.37$ expected for blue MS stars in $\omega$ Cen and NGC 2808 (e.g., Renzini 2008) and for the UV upturn in elliptical galaxies (e.g., Yi 2008). Although D’Antona et al. (2005) suggested that more massive AGB stars with $m_{\text{s}} \geq 6 M_\odot$ can have $Y = 0.40$, AGB stars with lower $m_{\text{s}}$ ($\leq 5 M_\odot$) could have smaller $Y$ (see Renzini 2008 for a more detailed discussion on this). It should be noted here that Portinari et al. (2010) suggested helium-rich populations in GCs to have $Y \approx 0.3$ rather than $Y \approx 0.4$. Considering these recent results, we adopt a model in which HRS formed from gaseous ejecta of AGB stars with $3 M_\odot \leq m_{\text{s}} \leq 8 M_\odot$ can become helium-rich populations of GCs and galactic spheroids. As discussed in our previous study (Bekki & Norris 2006), HRS with higher $Y$ can be formed from ejecta of type II supernovae if the ejecta do not mix with ISM. However, we do not discuss whether and how HRS can be formed from the ejecta of type II supernovae in the present study. Some observational results suggest that the helium-enrichment parameter $\Delta Y/\Delta Z$ can be larger than 4 (e.g., $5.3 \pm 1.4$ for local K-dwarf stars of the Galaxy; Gennaro et al. 2010). Therefore, it would be possible that even in canonical chemical evolution models, metal-rich stars with $Z \sim 0.03$ can have large $Y (\geq 0.35)$ for the primordial helium content of $Y = 0.24$. Since we do not include chemical evolution of GCs and galaxies in the present study, we do not discuss the above possibility either.

2.2. IMF

We consider that HNS and HRS are formed in SCs with different IMFs. The adopted IMF in number is defined as follows:

$$\psi(m) = m_{\text{sc},0} m_1^{-\alpha}, \quad (1)$$

where $m_1$ is the initial mass of each individual star and the slope $\alpha = 2.35$ corresponds to the Salpeter IMF. The normalization factor $m_{\text{sc},0}$ is a function of $m_{\text{sc}}, m_1$ (lower mass cutoff), and $m_u$ (upper mass cutoff):

$$m_{\text{sc},0} = m_{\text{sc}} \left( \frac{2 - \alpha}{m_u - m_1} \right)^{\alpha}, \quad (2)$$

where $m_1$ and $m_u$ are set to be free parameters in the present study. The IMF slopes for HNS and HRS are denoted as $\alpha_1$ and $\alpha_2$, respectively, and $m_{\text{l},1}$ ($m_{\text{l},2}$) and $m_{\text{u},1}$ ($m_{\text{u},2}$) are for HNS (HRS).

Gaseous ejecta from AGB stars with masses ranging from $m_{\text{agb}}$ to $m_{\text{agb}}$ can contribute to the formation of HRS and HRS are fixed at $3 M_\odot$ and $8 M_\odot$, respectively. The total mass of AGB ejecta within an SC ($m_{\text{agb}}$) is accordingly described as

$$M_{\text{AGB}} = \int_{m_{\text{agb}}}^{m_0} m_{\text{ej}} \psi(m) dm, \quad (3)$$

where $m_{\text{ej}}$ describes the total gas mass ejected from an AGB star with initial mass $m_1$ and final mass $m_{\text{agb}}$. We derive an analytic form of $m_{\text{ej}} (= m_1 - m_{\text{f}})$ from the observational data of Weidemann (2000) by using the least-squares fitting method and find

$$m_{\text{agb}} = 0.91 m_1 - 0.444. \quad (4)$$

Using these equations, we estimate the mass fraction of AGB ejecta ($f_{\text{agb}}$) in each individual SC for a given set of IMF parameters. $f_{\text{agb}}$ is defined as follows:

$$f_{\text{agb}} = \frac{M_{\text{AGB}}}{m_{\text{sc}}}. \quad (5)$$

In order to discuss an increase of helium mass in an SC ($\Delta M_{\text{He}}$) due to gas ejection of massive AGB stars, we adopt a formula used by Renzini (2005):

$$\Delta M_{\text{He}} = 0.15 (m_1 - 3) M_\odot. \quad (6)$$

We estimate the mass fraction of the fresh helium gas ($f_{\text{He}}$) in each individual SC as follows:

$$f_{\text{He}} = \frac{f_{\text{agb}} \Delta M_{\text{He}} \psi(m) dm}{m_{\text{sc}}}. \quad (7)$$

It is suggested that $f_{\text{He}}$ can be 0.7% for a Salpeter IMF and $m_1 = 0.5 M_\odot$ (Renzini 2008). If super AGB stars with...
8 \leq m_{1} \leq 10 M_{\odot} can also contribute to the production of helium-rich gas, then \( f_{\text{m,He}} \) can be 0.009 (Renzini 2008). These possibly small \( f_{\text{m,He}} \) led several authors to suggest that the original GCs are much more massive than the present ones (e.g., Bekki & Norris 2006).

2.3. Mass Fractions of HRS in GCs

We investigate the mass fractions of HRS on the MS (simply referred to as “MS HRS”) in the present GCs (\( f_{\text{He}} \)). In order to calculate the MS turn-off mass (\( m_{\text{TO}} \), we use the following formula (Renzini & Buzzoni 1986):

\[ m_{\text{TO}}(t_{e}) = 0.0558 \log t_{e}^{2} - 1.338 \log t_{e} + 7.764, \tag{8} \]

where \( m_{\text{TO}} \) is in solar units and time \( t_{e} \) in years. We assume that the ages of GCs and galactic spheroids are 12 Gyr and thus \( m_{\text{TO}} = 0.885 M_{\odot} \). Although the present results can hardly depend on age differences of \( \sim300 \) Myr (corresponding to the turnoff (TO) epoch of stars with \( m_{1} = 3 M_{\odot} \)) between HNS and HRS, we consider that \( m_{\text{TO}} = 0.885 M_{\odot} \) for HNS and \( m_{\text{TO}} = 0.889 M_{\odot} \) for HRS.

The initial total mass of HRS in an SC (\( m_{\text{sc,2}} \)) is given as

\[ m_{\text{sc,2}} = \epsilon_{\text{sf,2}} M_{\text{AGB}}. \tag{9} \]

where \( \epsilon_{\text{sf,2}} \) is the star formation efficiency for stars formed from AGB ejecta. For a strongly bound SC to be formed, the SC should not lose a significant fraction of gas left behind from star formation: \( \epsilon_{\text{sf,2}} \) needs to be larger than 0.5 (e.g., Hills 1980). We adopt \( \epsilon_{\text{sf}} = 1.0 \) in the present study. The total mass of MS HNS (\( m_{\text{sc,msns}} \)) in an SC is given as

\[ m_{\text{sc,msns}} = \int_{m_{1,1}}^{m_{\text{TO}}} m_{\psi}(m) dm. \tag{10} \]

The total mass of MS HRS (\( m_{\text{sc,msrs}} \)) in an SC is given as follows:

\[ m_{\text{sc,msrs}} = \epsilon_{\text{sf,2}} \int_{m_{1,2}}^{m_{\text{TO}}} m_{\psi}(m) dm, \tag{11} \]

where the normalization factor of the IMF (\( \psi \)) is determined by \( m_{\text{sc,2}} \). Therefore, the mass fraction of MS HRS in an SC is given as

\[ f_{\text{He}} = \frac{m_{\text{sc,msrs}}}{m_{\text{sc,msns}} + m_{\text{sc,msrs}}}. \tag{12} \]

Thus \( f_{\text{He}} \) depends on \( \alpha_{1}, \alpha_{2}, m_{1,1}, m_{u,1}, m_{1,2}, \) and \( m_{u,2} \).

2.4. GMC MF

In order to discuss the number/mass fraction of MGCs in the GC system (GCS) of the Galaxy and masses fractions of HRs in galactic spheroids, we introduce GMCMFs. A galactic spheroid is initially composed of numerous GCMCs from which unbound and bound SCs can be formed. We adopt the following GMCMF for GCMCs:

\[ \Psi(m_{\text{gmc}}) = M_{\text{g,0}} m_{\text{gmc}}^{-\beta}, \tag{13} \]

where \( \beta \) describes the slope of a GMCMF and is observed to be 1.6–1.7 for galaxies in the Local Group (Rosolowski 2005). The normalization factor \( M_{\text{g,0}} \) is a function of the total gas mass in a galactic spheroid (\( M_{g} \)), \( m_{\text{gmc,1}} \) (lower mass cutoff), and \( m_{\text{gmc,u}} \) (upper mass cutoff):

\[ M_{\text{g,0}} = \frac{M_{g} \times (2 - \beta)}{m_{\text{gmc,u}}^{2-\beta} - m_{\text{gmc,1}}^{2-\beta}}, \tag{14} \]

where \( m_{\text{gmc,1}} \) and \( m_{\text{gmc,u}} \) are set to be \( 10^{3} M_{\odot} \) and \( 10^{5} M_{\odot} \), respectively, in the present study.

We consider that \( \beta \) could be different in different regions of a galaxy and in different galaxies and thus investigate models with different \( \beta (1.1 \leq \beta \leq 2.0) \). We assume that star formation efficiencies (\( \epsilon_{\text{sf,1}} \)) within host GCMCs are constant, though they can be different between different GCMCs. This is because we focus on mass fractions of HRS in GCs and galactic spheroids and need to more clearly show their dependences on IMFs and GMCMFs. The present results do not depend so strongly on \( m_{\text{gmc,1}} \) and \( m_{\text{gmc,u}} \) for reasonable ranges of these two.

2.5. Formation Efficiency of Bound SCs in GCMCs

Some SCs get disintegrated and consequently become field stars in their host galaxies and others survive disintegration and finally become GCs. SCs with HRS can finally become massive GCs (like \( \omega \) Cen and NGC 2808), which are referred to as “MGCs”: other GCs with no HRS are simply referred to as GCs. The total mass of SCs with HRS formed in a galaxy is a key parameter which determines the total helium mass of the galaxy. Therefore, the formation efficiency of bound SCs (\( \epsilon_{\text{bsc}} \)) in GCMCs is one of the most important parameters in the present study. Here \( \epsilon_{\text{bsc}} \) is defined such that if \( \epsilon_{\text{bsc}} = 1 \) (0.1), then each (one in ten) GCMC can form an SC that is strongly bound so as to form stars from AGB ejecta.

Although it remains observationally unclear what a reasonable value is for \( \epsilon_{\text{bsc}} \), our previous simulations showed that formation efficiencies of GCs in starbursting galaxy mergers can be much higher in comparison with isolated disk galaxies owing to the rather high pressure of ISM (Bekki et al. 2002). The simulations suggested that gas can be converted into strongly bound GCs rather than field stars (or weakly bound SCs) in major galaxy mergers. These results imply that if galactic spheroids are formed from major mergers, they can have rather high \( \epsilon_{\text{bsc}} \). We discuss \( \epsilon_{\text{bsc}} \) later in Section 4.

2.6. Number and Mass Fractions of MGCs in the Galaxy

We discuss the number and mass fractions of MGCs (\( F_{n,\text{gmc}} \) and \( F_{m,\text{gmc}} \), respectively) that can form “genuine GCs” that are observed to show O–Na anticorrelations and thus evidence for the presence of multiple stellar populations (e.g., Carretta et al. 2010) in the Galactic GCS. We consider that MGCs with \( m_{\text{gmc}} \geq 10^{6} M_{\odot} \) (or \( m_{\text{sc}} \geq 10^{5} M_{\odot} \) for \( \epsilon = 0.1 \)) form “genuine GCs.” The threshold GMC mass for the genuine GCs is denoted as \( m_{\text{gmc,gc}} \) for convenience. We estimate \( F_{n,\text{gmc}} \) as follows:

\[ F_{n,\text{gmc}} = \frac{\int_{m_{\text{gmc}}}^{m_{\text{gmc,\text{thres}}}} \epsilon_{\text{bsc}} \Psi(m) dm}{N_{\text{gc}}}, \tag{15} \]

where \( N_{\text{gc}} \) is the total number of genuine GCs and is given as

\[ N_{\text{gc}} = \int_{m_{\text{gmc}}}^{m_{\text{gmc,\text{thres}}}} \epsilon_{\text{bsc}} \Psi(m) dm. \tag{16} \]

Likewise, \( F_{m,\text{gmc}} \) is estimated as follows:

\[ F_{m,\text{gmc}} = \frac{\int_{m_{\text{gmc}}}^{m_{\text{gmc,\text{thres}}}} \epsilon_{\text{bsc}} \epsilon_{\text{sf,1}} m_{\psi}(m) dm}{M_{\text{g}}} \tag{17} \]

where \( M_{\text{g}} \) is the total stellar mass of the genuine GCs and is given as

\[ M_{\text{g}} = \int_{m_{\text{gmc}}}^{m_{\text{gmc,\text{thres}}}} \epsilon_{\text{bsc}} \epsilon_{\text{sf,1}} m_{\psi}(m) dm. \tag{18} \]
2.7. Mass Fractions of HRS in Galactic Spheroids

The total stellar mass of a galactic spheroid \( (M_s) \) is estimated as follows:
\[
M_s = \int_{m_{gmc,1}}^{m_{gmc,u}} \epsilon_{sf,1} m \Psi(m) dm. \tag{19}
\]
The total mass of HRS in the galactic spheroid \( (M_{s,hrs}) \) is estimated as follows:
\[
M_{s,hrs} = \int_{M_{besc}}^{m_{gmc,u}} \epsilon_{sf,1} \epsilon_{sf,2} \epsilon_{besc} m \Psi(m) dm. \tag{20}
\]
In these equations, the term \( \epsilon_{besc} \) is included only in Equation (20), because whether SCs are bound or unbound does not matter in estimating the total stellar mass \( M_s \). The mass fraction of HRS in a galactic spheroid \( (F_{He,1}) \) is therefore given as follows:
\[
F_{He,1} = \frac{M_{s,hrs}}{M_s}. \tag{21}
\]
Although this \( F_{He,1} \) enables us to understand what fraction of stars can be HRS in a galactic spheroid, it is different from \( F_{He} \) which is more useful when the origin of galactic spheroids with HRS is discussed.

The total mass of MS stars in a galactic spheroid \( (M_{s,ms}) \) is estimated as follows:
\[
M_{s,ms} = \int_{m_{gmc,1}}^{m_{gmc,u}} \epsilon_{sf,1} f_{ms,1} m \Psi(m) dm, \tag{22}
\]
where \( f_{ms,1} \) is the mass fraction of MS stars among all stars and depends on the IMF parameters of HNS \( (e.g., \alpha_1) \). The total mass of MS HRS \( (M_{s,mshrs}) \) is estimated as follows:
\[
M_{s,mshrs} = \int_{M_{besc}}^{m_{gmc,u}} \epsilon_{sf,1} \epsilon_{sf,2} \epsilon_{besc} f_{ms,2} m \Psi(m) dm, \tag{23}
\]
where \( f_{ms,2} \) is the mass fraction of MS HRS among all stars formed as HRS and determined by IMF parameters of HRS. Therefore \( F_{He} \) is given as follows:
\[
F_{He} = \frac{M_{s,mshrs}}{M_{s,ms}}. \tag{24}
\]

In the present study, \( \epsilon_{sf,1} \) is assumed to be independent of \( m_{gmc} \) and accordingly the present results on \( F_{He} \) do not depend on \( \epsilon_{sf,1} \). Since we focus on \( F_{He} \) in galactic spheroids, it is unimportant whether low-mass SCs are bound (or bound) to become field stars (open/GCs) after SC disintegration. The parameter \( \epsilon_{besc} \) is therefore unimportant for GMCs with \( m_{gmc} \leq M_{besc} \). We assume that \( \epsilon_{besc} = 1 \) for GMCs that form MGCs in all models.

### Table 2

| Parameters | Physical Meanings |
|------------|------------------|
| \( \alpha_1 \) | The IMF slope for HNS |
| \( \alpha_2 \) | The IMF slope for HRS |
| \( m_{l,1} \) | The lower mass cutoff of the IMF for HNS |
| \( m_{u,1} \) | The upper mass cutoff of the IMF for HNS |
| \( m_{l,2} \) | The lower mass cutoff of the IMF for HRS |
| \( m_{u,2} \) | The upper mass cutoff of the IMF for HRS |
| \( \beta \) | The slope of the GMC mass function (GMCMF) |
| \( M_{besc} \) | The threshold mass for MGC formation |

It may be possible that \( F_{He} \) is significantly overestimated owing to the adopted assumption of \( \epsilon_{besc} = 1 \). We discuss this point later in Section 4. Thus the free parameters are \( \alpha_1, \alpha_2, m_{l,1}, m_{l,2}, m_{u,1}, m_{u,2}, \beta, \) and \( M_{besc} \) in the present study. Tables 2 and 3 briefly summarize the physical meanings of these parameters and the definition of physical quantities investigated in the present study \( (e.g., \ F_{He} ) \). respectively, so that readers can understand more clearly the present results.

### 3. RESULTS

#### 3.1. GCs

Figure 2 shows how \( f_{ms,He} \) and \( f_{ms,agb} \) depend on the IMF parameters \( \alpha_1, m_{l,1}, \) and \( m_{u,1} \). Clearly, \( f_{ms,He} \) is rather small \((\sim 0.005)\) for a canonical IMF with \( \alpha_1 = 2.35, m_{l,1} = 0.1 \, M_{\odot}, \) and \( m_{u,1} = 100 \, M_{\odot} \). This result means that only a small fraction of original stellar mass in an SC can be fresh helium gas that can be used for the formation of HRS for a canonical IMF. However,
would be highly unlikely for most GCs, these peculiar IMFs (top-heavy IMF). Although top-heavy IMFs with \( f \) for different IMF slopes for HRS (\( f \sim 0.2 \)) can show different parameter dependences of \( \alpha \). Given that these SCs cannot be disintegrated due to mass loss through type II supernova explosions. These SCs are likely to become disintegrated before secondary star formation from AGB ejecta can proceed. Thus it is unlikely that SCs with large \( f \) can now be observed as bound GCs.

SCs with moderately top-heavy IMFs (\( \alpha < 2 \)) can have significant fractions of HRS (\( f \sim 0.2 \)) if these SCs cannot be disintegrated due to mass loss through type II supernova explosions. It should however be stressed that \( \epsilon_{sf} \) can be higher than those estimated in the present study. It should also be stressed that if rather small \( m_{u,1} (< 30 M_{\odot}) \) is adopted, then \( f \) can be higher than the values shown in Figure 3.

### 3.2. The Fraction of MGCs in the Galactic Halo

By assuming that all of the Galactic GCs were formed from GMCs, we here briefly discuss what fraction of GCs were originally formed as MGCs with HRS (like \( \omega \) Cen and NGC 2808). In the present study, SCs formed from GMCs with \( m_{gmc} \geq M_{\text{thres}} \) can efficiently convert their AGB ejecta into new stars without dilution by ISM so that they can have HRS. Other less massive GCs that evolve from SCs formed from GMCs with \( m_{gmc,gc} \leq m_{gmc} < M_{\text{thres}} \) can finally become genuine GCs with no/few HRS. SCs formed from GMCs with \( m_{gmc} \leq m_{gmc,gc} \) cannot become genuine GCs in the present study. Given that the Galactic GCs with significant fractions of helium-rich populations (e.g., \( \omega \) Cen) have total masses larger than \( 10^6 M_{\odot} \), we consider that \( M_{\text{thres}} = 10^7 M_{\odot} \) (for \( \epsilon_{sf} = 0.1 \)) is reasonable.

Figure 4 clearly shows that although the number fraction of MGCs is small (\( F_{\text{m,gmc}} \sim 0.16 \)) for a reasonable \( \beta = (1.7) \) and \( M_{\text{thres}} = 10^7 M_{\odot} \), the mass fraction (\( F_{\text{m,gmc}} \)) is quite significant (>0.6). Both \( F_{\text{n,gmc}} \) and \( F_{\text{m,gmc}} \) are larger for smaller \( \beta \) and smaller \( M_{\text{thres}} \), owing to the larger fractions of GMCs with \( m_{gmc} \geq M_{\text{thres}} \). The essential reason for large \( F_{\text{m,gmc}} > (0.5) \) in some models is that GMCMFs have slopes larger than \( -2 \) (i.e., \( \beta < 2 \)). It should be stressed that \( \epsilon_{bsc} \) is assumed to be constant for all SCs in deriving these results. If host GMCs for MGCs have higher \( \epsilon_{bsc} \) than those for other GCs, then \( F_{\text{n,gmc}} \) and \( F_{\text{m,gmc}} \) can be even larger than those shown in Figure 4.

### 3.3. Galactic Spheroids

Figure 5 shows that the mass fraction of HRS (\( F_{\text{HRS}} \)) in the model with \( \beta = 1.7 \) and \( M_{\text{thres}} = 10^7 M_{\odot} \) is rather small (\( <0.05 \)) for canonical IMFs (\( \alpha_1 = 2.35 \)). Since \( \epsilon_{bsc} \) is assumed to be 1 for all GMCs with \( m_{gmc} \geq M_{\text{thres}} \) in the present study,
galactic spheroids are unlikely to have significant fractions of MS HRS even if the mass fractions of MS HRS can be larger for smaller \( m_{\text{thres}} \) and smaller \( \beta \), because there are larger numbers of low-mass GMCs with \( m_{\text{gmc}} \geq m_{\text{thres}} \), IMFs of HNS in galactic spheroids should be top-heavy (\( \alpha_1 \leq 2 \)) for the spheroids to have significant fractions of MS HRS (\( F_{\text{He}} \geq 0.1 \)). It should be stressed, however, that the observationally suggested \( F_{\text{He}} \) (~0.1) for galactic spheroids with the UV upturn (Chung et al. 2011) can be explained by the models with mildly top-heavy IMFs of \( \alpha_1 \sim 2 \) that are not so exotic.

It is clear that \( F_{\text{He}} \) is larger for larger \( \alpha_2 \) for a given set of \( \alpha_1, \beta, \) and \( M_{\text{thres}} \). This is because larger numbers of low-mass HRS \( (m_1 < 3 M_\odot) \) can be formed for larger \( \alpha_2 \). Also, \( F_{\text{He}} \) can be larger for smaller \( \beta \) and smaller \( M_{\text{thres}} \), because larger numbers of GMCs with \( m_{\text{gmc}} \geq m_{\text{thres}} \) can be formed. These results mean that whether galactic spheroids can have higher \( F_{\text{He}} \) \( (\geq 0.1) \) depends on the physical properties of GMCs and the physical processes of secondary star formation in MGCs. It should be stressed here that both \( \epsilon_{\text{eff},2} = 1 \) and \( \epsilon_{\text{psc}} = 1 \) are assumed in these estimations: top-heavy IMFs are required for galactic spheroids to have significant fractions of MS HRS even in these maximum formation efficiencies of HRS and MGCs.

As expected from Figure 5, \( F_{\text{He}} \) is rather small (~0.05) for canonical IMFs with \( \alpha_1 = 2.35 \) and \( \alpha_2 = 2.35 \). Irrespective of \( \alpha_2, \beta, \) and \( M_{\text{thres}} \), IMFs of HNS in galactic spheroids should be top-heavy \( (\alpha_1 \leq 2) \) for the spheroids to have significant fractions of MS HRS \( (F_{\text{He}} \geq 0.1) \). It should be stressed, however, that the observationally suggested \( F_{\text{He}} \) (~0.1) for galactic spheroids with the UV upturn (Chung et al. 2011) can be explained by the models with mildly top-heavy IMFs of \( \alpha_1 \sim 2 \) that are not so exotic.

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4. DISCUSSION

4.1. The Lost GCs with Helium-rich Stars

The Galactic GCs can sink into the inner region of the bulge owing to dynamical friction against the halo within ~13 Gyr, if their initial masses ($m_{gc}$) are enough large (Binney & Tremaine 1987). By assuming that the Galaxy has a singular isothermal sphere, the timescale of dynamical friction for a GC ($t_{df}$) can be estimated as follows:

$$ t_{df} = 2.3 \left( \frac{\ln \Lambda}{10} \right)^{-1} \left( \frac{r_1}{2 \text{kpc}} \right)^2 \left( \frac{v_c}{220 \text{pc}} \right) \left( \frac{m_{gc}}{10^5 \text{M}_\odot} \right)^{-1} \text{Gyr}, $$

(25)

where $\ln \Lambda$, $r_1$, and $v_c$ are the Coulomb logarithm, the initial distance of the GC from the Galactic center, and the circular velocity of the Galaxy. In the above estimation, the reference value of $m_{gc}$ was set to be higher, because GCs can have significantly higher masses at their birth. The above equation means that initially massive GCs with HRS have already sunk into the central region of the Galactic bulge owing to dynamical friction, if their $r_1$ are less than 2 kpc. These GCs cannot be observed as Galactic halo GCs and contribute to the formation of helium-rich stars in the bulge.

In order to estimate what fraction of massive GCs in the GCS of the Galaxy has been already lost in the bulge, we consider that the GCS has a spherical distribution with a density profile of $\rho(r) \propto r^{-3.5}$. This is consistent with that observed for the Galactic GCs (Djorgovski & Meylan 1994). The GCS is distributed within 35 kpc of the Galaxy and has a half-number radius of 5 kpc. We here investigate a threshold radius $R_{\text{thres}}$ for which $t_{df}$ can be smaller than 13 Gyr for a given $m_{gc}$. GCs with $r_1 < R_{\text{thres}}$ can spiral into the center of bulge within 13 Gyr so that they cannot be observed as Galactic halo GCs. It is found that ~49% of GCs with $m_{gc} = 10^7 \text{M}_\odot$ have already been lost for the GCS radial profile adopted above (i.e., $R_{\text{thres}} = 4.7$ kpc). The fraction of these lost GCs ($F_{\text{lost}}$) is smaller for smaller $m_{gc}$: $F_{\text{lost}} = 0.37$ for $m_{gc} = 5 \times 10^5 \text{M}_\odot$ and 0.06 for $m_{gc} = 10^6 \text{M}_\odot$.

These results imply that what we can now observe as MGCs (e.g., $\omega$ Cen and NGC 2808) could be (51–63)% of the original MGCs ($m_{gc} > 5 \times 10^5 \text{M}_\odot$) formed in the Galactic halo. The lost MGCs would have been destroyed by the Galactic bulge and consequently their HRS would have been dispersed into the inner bulge region: the stars could be now observed as HRS in the bulge. This selective loss of MGCs due to dynamical friction might well occur in galactic bulges and giant elliptical galaxies. Sohn et al. (2006) revealed a large number of GCs with strong stellar populations. These GCs contain large fractions of HRS, then M87 could have already swallowed a fraction of these possible MGCs to add their HRS to its main stellar spheroid.

4.2. Top-heavy IMFs for the Origin of the UV Upturn in Elliptical Galaxies and BCGs

The present study has demonstrated that if IMFs are top-heavy (i.e., $\alpha_1 < 1.5$), then the SCs could become disintegrated before AGB ejecta can be recycled and converted into new stars. About 45% in mass of stars can become stars with masses larger than $8 \text{M}_\odot$ (thus supernova) in SCs for $\alpha_1 = 1.95$, $m_{1,1} = 0.1 \text{M}_\odot$, and $m_{u,11} = 100 \text{M}_\odot$. These SCs cannot become disintegrated and thus can continue secondary star formation, because more than 50% of their masses can still remain in SCs (e.g., Hills 1980). However, if $\alpha_1 = 1.5$, $m_{1,1} = 0.1 \text{M}_\odot$, and $m_{u,11} = 100 \text{M}_\odot$, then the mass fractions of supernovae in SCs are 0.74 so that SCs can lose most of their original masses. These SCs are highly likely to get disintegrated shortly after supernova explosions owing to their mass loss. Therefore, too top-heavy IMFs ($\alpha_1 < 1.5$) are not ideal for galactic spheroids to have helium-rich stellar populations.

Peng & Nagai (2009) proposed that sedimentation of helium in clusters of galaxies can be responsible for the formation of HRS in BCGs. They also predicted that the UV flux strength is stronger in more massive, low-redshift, and dynamically relaxed BCGs. Their predictions, however, have been recently challenged by an observational study by Yi et al. (2011) which found no correlation between the UV strength and rank/luminosities of BCGs and showed no clear difference in UV upturn fraction or strength in BCGs. The present study provides an alternative explanation for the origin of the BCGs with the UV upturn: BCGs show the UV upturn because IMFs at their formation were top-heavy ($\alpha_1 < 2$) and therefore they have larger $F_{\text{thres}}$. In the CDS with top-heavy IMFs, some BCGs cannot show the UV upturn, because most stars are formed as observational properties of galactic spheroids such as their luminosity evolution and chemical abundances?
SCs with canonical IMFs (whereas some can owing to top-heavy IMFs).

However, the origin of the UV upturn in BCGs would not be so simple as the CDS with top-heavy IMFs envisages. Recent theoretical models have shown that hierarchical merging of smaller galaxies can play a vital role in the formation of BCGs (e.g., De Lucia & Blaizot 2007). The IMFs are observationally suggested to be different between faint and luminous galaxies (e.g., Hovesten & Glazebrook 2008). Therefore, if BCGs were formed by numerous mergers between galaxies with different luminosities in clusters, then not just the IMFs but also the merging histories (e.g., fractions of faint/luminous galaxies) can be key determinants for whether BCGs can show the UV upturn.

4.3. Halo–spheroid Connection

Recently Martell et al. (2011) have revealed that about 3% of the Galactic halo stars can have depleted carbon and enhanced nitrogen abundances that are very similar to the chemical abundances observed for the “second generation” of stars in GCs. Since these stars with characteristic chemical abundances can be formed from gaseous AGB ejecta of the “first generation of stars” (e.g., Bekki et al. 2007; D’Ercole et al. 2008), they suggested that (1) these stars originate from GCs and (2) a minimum 17% of the present-day mass of the Galactic stellar halo was originally formed in GCs. Although it is difficult for observational studies to directly estimate helium abundances of stars in GCs, Braggiglia et al. (2010) have recently estimated the average enhancement in the helium mass fraction \( \alpha \) between the first and second generations (corresponding to HNS and HRS, respectively, in the present study) for 19 GCs with the Na–O anticorrelation. The estimated average enhancement is about 0.05–0.11, which means that a significant fraction of stars in these GCs can have HRS. If \( \sim 10\% \) of all GC stars are HRS, then about 1.7% of the halo stars can be regarded as HRS.

This smaller number of 1.7% implies that the Galactic halo cannot be identified as a galactic component with the UV upturn. Then what mechanism is responsible for the possible large difference in the mass fraction of HRS (\( F_{\text{He}} \)) between the Galactic stellar halo and galactic spheroids? The present CDS suggests that most stars in the Galactic halo can originate from disintegration of low-mass SCs with canonical IMFs whereas those of galactic spheroids originate from stars initially in more massive SCs with top-heavy IMFs. Furthermore, as discussed in Section 4.1, the more massive SCs (or GCs) with HRS can rapidly sink into the central regions of their hosts so that they cannot contribute to the formation of HRS in their halo regions: these more massive GCs can preferentially become the building blocks of the central spheroidal components in galaxies. Thus it is highly likely that \( F_{\text{He}} \) can be significantly different between halos and spheroids in galaxies.

4.4. High Formation Efficiencies of Bound Massive SCs at the Epoch of Spheroidal Formation

It should be stressed here that \( \epsilon_{\text{binc}} \sim 1 \) has so far been assumed: GMCs with \( m_{\text{gmc}} \geq M_{\text{thres}} \) need to almost always form bound SCs that can finally form helium-rich stars. If \( \epsilon_{\text{binc}} \sim 0.1 \), then it is very hard for galactic spheroids to have \( F_{\text{He}} \sim 0.1 \) even for very top-heavy IMFs (e.g., \( \alpha_1 = 1 \)). Only spheroids with \( \alpha_1 \sim 1 \) and \( m_{\text{gmc}} \sim 8 M_\odot \) (i.e., peculiar top-heavy IMFs) can have \( F_{\text{He}} \) as large as 0.1 for \( \epsilon_{\text{binc}} \sim 0.1 \). Photometric studies of super star clusters (SSCs) in the starbursting luminous infrared galaxy Arp 220 showed that the nuclear SSCs in Arp 220 contribute to \( \sim 20\% \) of the total bolometric luminosity of Arp 220 (Shioya et al. 2001). Larsen & Richtler (2000) found that the formation efficiency of SCs can be higher in local regions with high star formation rate per unit area for disk galaxies. These observations imply that (1) a significant fraction of new stars are formed as SCs in a starburst or in high-density star-forming regions and thus (2) galactic spheroids can have high cluster formation efficiencies if they were formed from massive starbursts in high-density environments.

However, owing to the lack of extensive observational studies on formation efficiencies of massive SCs in galaxies, it is currently very hard to discuss what a reasonable value of \( \epsilon_{\text{binc}} \) is for massive GMCs that can be progenitors of bound SCs with HRS. If \( \epsilon_{\text{binc}} \) can be significantly larger than 0.1, then the present CDS is promising as the origin of the UV upturn in galactic spheroids, though top-heavy IMFs are still required. Thus, ultimately speaking, the origin of the UV upturn is closely related to the formation processes of MGCs from massive GMCs at the epoch of galaxy formation in the present CDS.

4.5. Helium-rich Populations in the Galactic Bulge?

Nataf et al. (2011) revealed that the metal-rich stellar populations of the Galactic bulge can have \( Y \sim 0.35 \) based on observational results on the physical properties of the red giant branch bump (RGBB). They provided some important implications for the metal-rich HRS in the bulge and suggested that galactic bulges in general can have HRS like the bulge. The present results imply that if the Galactic bulge had a top-heavy IMF at its formation, then the present stellar populations can have a significant fraction (\( \sim 0.1 \)) of HRS. Given that previous chemical evolution models of the Galactic bulge (e.g., Ballero et al. 2007; Tsujimoto et al. 2010) suggested a moderately top-heavy IMF \( (\alpha \sim 2.05) \) for the bulge, it is possible that the Galactic bulge can have a significant fraction of HRS owing to the top-heavy IMF.

As suggested by Nataf et al. (2011), the dominant populations of the RGBB (with [Fe/H] > 0) in the bulge can have a rather high \( Y (\sim 0.35) \): not just an \( \sim 10\% \) of the bulge population needs to have \( Y \sim 0.35 \) to explain their observational results. If more than 30% of the metal-rich stellar populations have \( Y \sim 0.35 \), then the IMF slope \( (\alpha) \) for the populations should be well less than 1.5 for \( \beta = 1.7 \) and \( M_{\text{thres}} = 10^7 M_\odot \). The required IMF is significantly more top-heavy than those suggested by chemical evolution studies (e.g., \( \alpha \sim 2.05 \); Tsujimoto et al. 2010) and thus would not explain the observed global properties of the bulge such as the MDF of the bulge stars. However, if metal-rich stellar populations of the bulge have a very top-heavy IMF and if other populations have moderately top-heavy IMFs, then it would be possible that both the observed high fraction of metal-rich HRS and global chemical properties in the bulge can be self-consistently explained. This possibility needs to be explored in our future studies based on detailed chemical evolution models.

4.6. The Origin of the Correlation between the Strength of UV Upturn and Mg\(_2\) Index

Recent observational studies of 48 nearby early-type galaxies by the SAURON project (Bureau et al. 2011) have confirmed the presence of a negative correlation between FUV–V color and Mg line strength originally proposed by Burstein et al. (1988). The present study provides the following possible explanation for the origin of this “Burstein relation” (i.e., a correlation of
Mg$_2$ with ($1550 - V$) color). More massive elliptical galaxies can retain more efficiently the ejecta of supernovae so that they can finally have higher metallicities of their stars (e.g., Arimoto & Yoshii 1987). If more massive elliptical galaxies have shallower (i.e., more top-heavy) IMFs, then they can have large $F_{\text{He}}$ and thus show the stronger UV upturn. Therefore more massive (or luminous) elliptical galaxies can have higher metallicities and thus higher Mg$_2$ as well as a stronger UV upturn. Given the observed positive correlations between luminosities, Mg indices, and velocity dispersions in elliptical galaxies (e.g., Faber & Jackson 1976; Bender et al. 1992), more massive (or luminous) elliptical galaxies can have higher Mg$_2$, higher stellar velocity dispersion, and a stronger UV upturn. Thus, the dependence of IMF slopes on galactic masses/luminosities can be responsible for the origin of the Burstein relation.

Although Hoversten & Glazebrook (2008) revealed that galaxies significantly fainter than the Galaxy show steeper IMFs, it remain observationally unclear how IMF slopes depend on global galactic properties and formation environments of galaxies. Therefore it is too premature to conclude whether the CDS with IMF variation is promising or not. As reviewed by Yi (2008), only a small fraction of elliptical galaxies show the UV upturn, which needs an explanation. If elliptical galaxies are formed from merging of smaller galaxies with different IMFs, then the origin of the UV upturn would not be so simple as the CDS explains.

The observed strong correlation between Mg$_2$ and ($1550 - V$) color (Burstein et al. 1988) suggests that metallicities play a role in the formation of the correlation. Figure 1 in Burstein et al. (1988) also showed a large difference in ($1550 - V$) colors ($\sim 2.5$ mag) among elliptical galaxies with different Mg$_2$ ranging from $\sim 0.20$ to $0.36$. It is unclear whether this large difference can be quantitatively explained by IMF variation alone, because the present study cannot predict ($1550 - V$) colors as a function of $F_{\text{He}}$. It would be possible that the combination of high metallicities and large $F_{\text{He}}$ can make ($1550 - V$) colors significantly bluer in elliptical galaxies. Accordingly our future study will include chemical evolution and thereby discuss the dependences of $F_{\text{He}}$ on metallicities in elliptical galaxies.

4.7. Radial Gradients of Helium-rich Stars in Galactic Spheroids

Carter et al. (2011) have recently investigated radial gradients of the FUV excess in 52 galaxies observed by GALEX and found that some of them show a positive gradient in the (FUV–NUV) color. They therefore suggested that the observed gradient can be due to a helium abundance gradient and furthermore that the presence of the gradient can place a strong limit on the importance of dry mergers in elliptical galaxy formation. In order to discuss the origin of the observed radial gradients of (FUV–NUV) color, we have constructed a toy $N$-body model in which a galactic spheroid is formed from merging of numerous SCs with the mass fraction of HRS being a free parameter. The details of the model are given in the Appendix.

Figure 7 shows the time evolution of spatial distributions of HNS and HRS in galactic spheroids composed initially of numerous SCs for the standard model. As SCs closely interact with one another in the early-evolution phase of the galactic spheroid, smaller and less massive SCs are destroyed by the tidal field of the host spheroid and by larger and more massive SCs. The stars (HNS) initially in less massive SCs are dispersed during destruction of their host SCs and finally become field HNS. On the other hand, massive SCs with $m_{\text{gmc}} \geq M_{\text{thres}}$ and thus with HRS cannot be destroyed efficiently during early multiple SC interaction. These massive SCs can sink into the inner region of the spheroid owing to dynamical friction against field stars (HNS) and then interact dynamically with other massive SCs there. Consequently, the massive SCs can get disintegrated there and their HRS can be dispersed to become field HRS there. Since the inner region of the spheroid can be finally dominated by stars originally from HRS of massive SCs, the inner region can have a larger fraction of HRS within $\sim 1$ Gyr dynamical evolution.

Figure 8 shows how radial $F_{\text{He},t}$ gradients depend on the model parameters, $M_{\text{thres}}$ and $Y$, for a given $\beta (=1.7)$. Clearly, all models show negative gradients in the sense that inner regions of galactic spheroids have higher fractions of HRS. For example, $F_{\text{He},t}$ can be $\sim 0.08$ within the central 200 pc in the standard model, though it is rather small ($\sim 0.01$) in the outer region ($R \sim 1.8$ kpc). The parameter $M_{\text{thres}}$ can control the central value of $F_{\text{He},t}$ such that $F_{\text{He},t}$ can be larger for smaller $M_{\text{thres}}$. The $F_{\text{He},t}$ gradients can be smaller in models with $Y = 0$ in which there is no mass–size relation for SCs. The rather flat radial distribution derived is due to higher degrees of dynamical mixing of SCs with different mass in these models. Since $F_{\text{He}}$ is proportional to $F_{\text{He},t}$ for a given set of IMFs, these results on $F_{\text{He},t}$ gradients can be true for $F_{\text{He}}$. The present study accordingly demonstrates that galactic spheroids can show negative radial gradients of $F_{\text{He},t}$ and $F_{\text{He}}$ and therefore negative gradients of the strength of the UV upturn (i.e., stronger in their inner regions). The present study thus can clearly explain the above observational result by Carter et al. (2011) if the observed color gradients are due largely to helium abundance gradients.

5. CONCLUSIONS

We have adopted a model in which (1) all stars are formed as bound or unbound SCs from GMCs and (2) HRS with $Y \geq 0.35$ can be formed from gas ejected from AGB stars only in massive SCs with $m_{\text{gmc}} \geq M_{\text{thres}}$. Some SCs can become field stars in their host galaxies after disintegration and others can become massive GCs or nuclear SCs (or stellar nuclei) in galaxies. If massive SCs with HRS are destroyed by their host galaxies, then HRS become helium-rich field stars in the galaxies. We have investigated mass fractions of MS HRS ($f_{\text{He}}$ and $F_{\text{He}}$) in GCs and galactic spheroids and their dependences on IMFs, $M_{\text{thres}}$, and GMCMFs based on the CDS ("cluster disintegration scenario"). We have also investigated the radial gradients of $F_{\text{He}}$ in galaxies with mass–size relations of GMCs by using $N$-body simulations on dynamical evolution of multiple cluster systems. We summarize our principal results as follows.

1. The mass ratios ($f_{\text{He,agb}}$) of gaseous ejecta from massive AGB stars with $3 M_\odot \leq m_1 \leq 8 M_\odot$ to initial total masses of SCs are typically $\sim 0.08$ for a canonical IMF ($\alpha_1 = 2.35$) with reasonable lower and upper cutoff masses. If this gas can be efficiently converted into new stars with enhanced helium abundances (i.e., HRS) in SCs, then the mass fractions of MS HRS among all MS stars ($f_{\text{He}}$) are $\sim 0.08$ for canonical IMFs and an SC age of 12 Gyr. Only if the original SCs have top-heavy IMFs (e.g., $\alpha_1 \sim 1.5$) can the $f_{\text{He}}$ be larger than 0.3 for $\alpha_2 \geq 1.85$.

2. The initial number and mass fractions of massive GCs (MGCs) with HRS ($F_{\text{He,mgc}}$ and $F_{\text{in,mgc}}$, respectively) among the Galactic GCs formed from GMCs with $m_{\text{gmc}} \geq 10^6 M_\odot$ are $\sim 0.16$ and 0.66, respectively, for $M_{\text{thres}} = 10^5 M_\odot$. We have $\beta = 1.7$. Both $F_{\text{He,mgc}}$ and $F_{\text{in,mgc}}$ are larger for
Figure 7. Time evolution of the distributions of HNS (blue) and HRS (red) projected onto the $x$-$y$ plane for the standard model. The time ($T$ in units of Gyr) that has elapsed since the simulation starts is shown in the upper left corner of each panel.

(A color version of this figure is available in the online journal.)

Figure 8. Radial gradients of mass fractions of HRS ($F_{\text{He},t}$) in models with different parameters: $\gamma = 0.5$ and $M_{\text{thres}} = 10^7 M_\odot$ (blue), $\gamma = 0.5$ and $M_{\text{thres}} = 3 \times 10^7 M_\odot$ (red), $\gamma = 0.5$ and $M_{\text{thres}} = 5 \times 10^7 M_\odot$ (green), and $\gamma = 0.0$ and $M_{\text{thres}} = 10^7 M_\odot$ (magenta).

(A color version of this figure is available in the online journal.)

3. The mass fractions of HRS among all stars ($F_{\text{He},t}$) in galactic spheroids are $\sim 0.05$ for $M_{\text{thres}} = 10^7 M_\odot$, $\beta = 1.7$, $f_{\text{m,agb}} = 0.1$, and $\epsilon_{\text{bsc}} = 1$. $F_{\text{He},t}$ of galactic spheroids are larger for smaller $M_{\text{thres}}$ and smaller $\beta$ for a given IMF. The mass fractions of MS HRSs among all MS stars ($F_{\text{He}}$) in galactic spheroids are 0.06 for canonical IMFS ($\alpha_1 = 1.35$, $M_{\text{thres}} = 10^7 M_\odot$, $\beta = 1.7$, and a galaxy age of $\sim 12$ Gyr). $F_{\text{He}}$ in galactic spheroids can be larger for smaller $M_{\text{thres}}$ and smaller $\beta$ for a given IMF, and it can be significant ($> 0.1$) if the original SCs (i.e., building blocks of the spheroids) have top-heavy IMF (e.g., $\alpha_1 < 2$). These results suggest that if the observed UV upturn in bright elliptical galaxies is due to the larger fractions ($\sim 10\%$) of HRS within them, the IMFs need to be top-heavy. If $\epsilon_{\text{bsc}}$ is as small as $\sim 0.1$, then bright elliptical galaxies are unlikely to show the UV upturn owing to the small fractions of HRS.

4. The Galactic bulge can have a larger $F_{\text{He}}$ if it had a top-heavy IMF at its formation in the CDS. Given a number of suggestions on the top-heavy IMF of the bulge by chemical evolution models of the bulge, the observed possible presence of helium-rich stars with $Y \sim 0.35$ in the bulge (e.g., Nataf et al. 2011) can be understood in the context of the CDS. HRS in MGCs that were initially located in the Galactic halo and had sunk into the bulge can be currently observed as field HRS in the bulge, though the contribution of such stars to the entire helium-rich.
population is rather minor. It is doubtless worthwhile for observational studies to investigate whether or not the bulge has a larger fraction of HRS in the inner regions.

5. If \( M_{\text{hres}} \) and \( \beta \) are constant in elliptical galaxies with different physical properties, then elliptical galaxies with shallower (or more top-heavy) IMFs can show larger fractions of HRS. Therefore, if more massive/luminous elliptical galaxies have shallower IMFs, then they can have larger fractions of HRS. More massive elliptical galaxies can retain more efficiently the ejecta of supernovae so that they can finally have higher metallicities of their stars (e.g., Arimoto & Yoshii 1987). Therefore, if the Burstein relation (a correlation between UV flux and Mg index; Burstein et al. 1988) in bright elliptical galaxies is due to the correlation between \( F_{\text{He}} \) and Mg indices, then the relation can be understood in terms of shallower IMF slopes (\( \alpha_f \)) in more massive/luminous elliptical galaxies.

6. Galactic spheroids can show negative radial gradients of \( F_{\text{He},t} \) and \( F_{\text{He}} \) (i.e., higher in inner parts) for a reasonable set of model parameters. This is mainly because massive SCs with HRS can sink rapidly into the inner regions of galaxies owing to dynamical friction and disintegrate there. The HRS initially in the SCs can be dispersed to become field HRS there after disintegration of the host SCs. Therefore, they can show a stronger UV upturn in their inner regions, if \( F_{\text{He}} \) can determine the strength of the UV upturn. The observed spatial distribution of the FUV excess in elliptical galaxies can be understood in the context of formation and evolution of MGCs with HRS. The central regions of galactic spheroids can be composed of two different stellar populations with normal helium abundances and enhanced properties (e.g., Mg2 index) can be discussed in the context of IMF slopes of galaxies in a self-consistent manner. A disadvantage of the CDS is that it is unclear whether and in what physical conditions \( \epsilon_{\text{gmc}} \) can be \(< 1\) for massive GMCs in galaxies. The formation of HRS is possible if helium-rich gas from AGB stars is converted into new stars without dilution of the gas by ISM with normal helium abundances. In the CDS, the inner regions of massive SCs are assumed to be the production sites of HRS. It would be possible that AGB ejecta can be converted into new cold gas clouds outside SCs and consequently into new stars in deep potential wells of galactic spheroids, if there is no/little cold gas with normal helium abundance there. We need to investigate where and how AGB ejecta from field stars and SCs can be converted into new stars in galaxies using a more sophisticated chemodynamical simulation in our future studies.

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**APPENDIX**

**N-BODY MODELS FOR MULTIPLE SC EVOLUTION**

We investigate dynamical evolution of a galactic spheroid composed of numerous SCs using collisionless N-body simulations based on an idealized model for galactic spheroids. The main aim of this numerical investigation is to illustrate the radial gradients of the mass fractions of HRS (\( F_{\text{He},t}(R) \)) in the spheroids. This idealized model is used only in the present preliminary study for the origin of helium-rich stars in galactic spheroids, and more sophisticated models including chemical evolution due to gaseous ejection from supernovae and AGB stars will be used in our future studies. However, we consider that this “toy” model enables us to grasp some essential ingredients of the formation and evolution of radial \( F_{\text{He},t} \) gradients in galactic spheroids.

A galactic spheroid has an initial total mass \( M_{\text{gal}} \) and a size \( R_{\text{gal}} \) and is composed of SCs of total number \( N_{\text{sc},t} \). The radial distribution of SCs is described by a Plummer profile (e.g., Binney & Tremaine 1987) with a scale length of 0.2\( R_{\text{gal}} \). Each SC with a mass \( m_{\text{sc}} \) and a size \( r_{\text{sc}} \) is composed of many stars and has a Plummer density profile with a scale length of 0.2\( r_{\text{sc}} \). A galactic spheroid is assumed to be initially in dynamical equilibrium, so the velocities of each SC are given according to the velocity dispersion profile of the Plummer model adopted.

An SC is composed of HNS and HRS and these two populations have different initial distributions within the SC. Recent numerical simulations have shown that secondary star formation from AGB ejecta can proceed mostly in the central regions of MGCs (e.g., D’Ercole et al. 2008; Bekki 2010, 2011). Guided by these results, HNS and HRS are located at 0.1\( r_{\text{sc}} \) \(< R \leq r_{\text{sc}} \) and \( R < 0.1r_{\text{sc}} \) respectively. The mass fraction of HRS is calculated for a given \( \beta \) and \( M_{\text{hres}} \). The total particle number in an SC is proportional to \( m_{\text{sc}} \) so that masses of stellar particles can all be the same.

A progenitor GMC for a bound/unbound SC has a mass \( m_{\text{gmc}} \) and a size \( r_{\text{gmc}} \). If we use the observed relation between sizes of GMCs discovered by Larson (1981) and the observed typical mass and size of GMCs in the Galaxy (e.g., Solomon et al. 1979), then the \( m_{\text{gmc}} - r_{\text{gmc}} \) relation can be described as follows:

\[
 r_{\text{gmc}} = 40 \times \left( \frac{m_{\text{gmc}}}{5 \times 10^5 \text{M}_\odot} \right)^{\gamma} \text{pc}, \quad (A1)
\]

where \( \gamma \approx 0.5 \). We investigate models with different \( \gamma \), because initial SCs can have mass–size relations different from those of GMCs. We consider that when SCs are formed, SCs have a mass–size relation similar to the one above and thus assume that \( r_{\text{sc}} \) can be the same as \( r_{\text{gmc}} \). We present mainly the results of the “standard” dynamical model in which \( M_{\text{gal}} = 10^9 \text{M}_\odot \), \( R_{\text{gal}} = 2 \text{kpc} \), \( \beta = 1.7 \), \( M_{\text{hres}} = 10^7 \text{M}_\odot \), \( m_{\text{gmc},t} = 10^5 \text{M}_\odot \), \( m_{\text{gmc},u} = 10^6 \text{M}_\odot \), and \( \gamma = 0.5 \), though we investigate models with different parameters. In order to perform numerical simulations, we use the latest version of GRAPE (GRAVity PipE, GRAPE-DR), which is the special-purpose computer for gravitational dynamics (Sugimoto et al. 1990). The total particle number used in a simulation is \( \approx 200,000 \), which we consider to be reasonable for this preliminary investigation. The gravitational softening length is set to 20 pc for all models. We focus only on the final radial gradients of \( F_{\text{He},t} \) in galactic spheroids formed from dynamical evolution of numerous SCs.

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