Anti-GZK effect in UHECR spectrum

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Abstract. In this paper we discuss the anti-GZK effect that arises in the framework of the diffusive propagation of Ultra High Energy (UHE) protons. This effect consists in a jump-like increase of the maximum distance from which UHE protons can reach the observer. The position of the jump is independent of the Intergalactic Magnetic Field (IMF) strength and depends only on the energy losses of protons, namely on the transition energy from adiabatic and pair-production energy losses. The Ultra High Energy Cosmic Rays (UHECR) spectrum presents a low-energy steepening approximately at this energy, which is very close to the position of the observed second knee. The dip, seen in the universal spectrum as a signature of the proton interaction with the Cosmic Microwave Background (CMB) radiation, is also present in the case of diffusive propagation in magnetic fields.

Keywords: cosmic rays, diffusion, intergalactic magnetic field

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INTRODUCTION

Recently a very interesting phenomenon, determined by the UHE proton propagation in IMF, has been found [1, 2]. It consists in a low energy steepening of the proton spectrum that occurs at energy below $10^{18}$ eV. This steepening is caused by an increase of the diffusive propagation time, that rapidly exceeds the age of the universe, and can be explained through the diffusive propagation of UHECR in IMF. The position of the steepening energy $E_s = 1 \times 10^{18}$ eV, is determined only by the proton energy losses on CMB and coincides with a good accuracy to the position of the 2nd knee observed in the CR spectrum [3]. In this paper we will discuss the main features of the diffusive propagation of UHECR in IMF, focusing our attention on the steepening energy scale $E_s$. Before entering the details of the diffusive propagation of UHECR protons let us review the main experimental evidences related to IMF.

The presence of an IMF is still an open question, the most reliable observations of this field are based on the measurement of the Faraday rotation (RM) of polarized radio emission [3]. The upper limit obtained with these measurements is $RM < 5$ rad/m², it implies an upper limit on the IMF that depends on the assumed scale of coherence length. For instance, according to [3], in the case of an inhomogeneous universe $B_{1c} < 4$ nG with a scale of coherence of about $l_c = 50$ Mpc. In general, as follows from the observations of Faraday rotation, the magnetic field is high, of the order of 1 µG with a coherence length $l_c = 1$ Mpc, in clusters of galaxies and radio lobes of radio galaxies [2]. Apart from observations, the IMF can be predicted, in principle, through Magneto-hydrodynamics (MHD) simulations. The main ambiguities in these simulations are related to the assumed seed magnetic field and to the capability of simulations to reconstruct the local Universe as we observe it (i.e. constrained [2] and unconstrained simulations [3]). Unfortunately, because of these uncertainties, MHD simulations are not completely conclusive, there are at least two opposite results in literature with predicted magnetic field in voids (filaments) that vary from $10^{-3}$ nG ($10^{-1}$ nG) [3] up to $10^{-1}$ nG (10 nG) [4].

While a direct evaluation of the IMF strength is still challenging, indirect informations about the UHECR propagation mode can be inferred from UHECR data. The analysis of the arrival directions of UHECR at energies $E > 10^{19}$ eV shows a small angle clustering within the angular resolution of the detectors. The AGASA detector has found 3 doublets and 1 triplet among 47 detected events [5]. This analysis is confirmed also by the combined data of different detectors [6] in which 8 doublets and 2 triplets are found in 92 collected events. This evidence can be well understood in terms of a rectilinear propagation of protons at the highest energies ($E > 10^{19}$ eV) with a random arrival of two (three) particles from the same source and a source number density of about $n_s \approx 10^{-5}$ Mpc$^{-3}$ [3]. However, the small angle clustering may survive in the case of UHE protons propagation in IMF [2, 10].

Another remarkable evidence of an almost rectilinear propagation of UHECR, in the energy range $2 - 8 \times 10^{19}$ eV, has been found by Tinyakov and Tkachev [11]. These authors have found a correlation between arrival directions of UHE particles in the AGASA and Yakutsk detectors and the directions of several BL-Lac objects, (i.e. AGNs with jet directed toward us). The combined evi-

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to the diffusive equation. Following [2], we will also use the Syrovatsky \cite{12} solution of protons at energies larger than 10^{19} eV.

**UHECR DIFFUSIVE PROPAGATION**

In order to describe the diffusive propagation in IMF of UHECR protons, we will use the Syrovatsky \cite{12} solution to the diffusion equation. Following \cite{2}, we will also assume a distribution of sources on a lattice; under this hypothesis the diffuse flux can be calculated as the sum over the fluxes from the discrete sources at distances $r_i$:

$$j_p^\text{diff}(E) = \frac{c}{4\pi} \frac{L_p K(\gamma_E)}{b(E) E_{\text{min}}^2} \int E dE_g \left( \frac{E_g}{E_{\text{min}}} \right)^{-\gamma_E} \times \exp \left[ - \frac{r_i^2}{4\lambda(E,E_g)} \right] \times \left( \frac{4\pi \lambda(E,E_g)}{2^3} \right)^{3/2},$$

where $b(E) = dE/dt$ is the proton energy losses (we have used $b(E)$ as computed in \cite{13}) and $Q_{\text{nj}} = \langle L_p K(\gamma_E) / E_{\text{min}}^2 \rangle (E_g / E_{\text{min}})^{-\gamma_E}$ is the particle generation rate per unit energy with $L_p$ the source luminosity (here we assume identical sources with the same luminosity) and $K(\gamma_E) = \gamma_E - 2$ (if $\gamma_E > 2$) a normalization constant.

The function $\lambda(E,E_g)$ is

$$\lambda(E,E_g) = \int E dE \frac{D(E)}{b(E)}$$

with $D(E)$ the diffusion coefficient. The quantity in \cite{2} describes the squared distance traversed by a proton in the direction of the observer, while its energy decreases from $E_g$ to $E$. The Syrovatsky parameter $\lambda(E,E_g)$ poses a natural cut to the flux contributing sources, it is clear from equation \cite{11} that a source at distance $r > 2\sqrt{\lambda(E,E_g)}$ gives a negligible (exponentially suppressed) contribution to the flux.

The Syrovatsky solution \cite{11} formally includes all propagation times up to $t \to \infty$ and the generation energy of particle is limited only by the energy that the source can provide $E_{\text{max}}$. Nevertheless, the propagation time should be smaller than the age of the universe $t_0$, this poses an additional limit to the generation energy represented by $E_g(t_0)$, which can be computed evolving backward in time the proton energy from $E$ at $t = 0$ up to $E_g$ at $t = t_0$. Therefore, to limit the propagation time, the upper limit of integration $E_g^\text{max}(E)$ in equation \cite{11} is fixed at the minimum between $E_{\text{max}}$ and $E_g(t_0)$. It is interesting to note that at energies $E < E_g = 1 \times 10^{18}$ eV, where only adiabatic energy losses are relevant, the upper limit of integration in \cite{11} is fixed by $E_g^\text{max}(E) = E_g(t_0)$ while at higher energies, where pair-production and photo-pion-production energy losses become relevant, the upper limit of integration is $E_g^\text{max}(E) = E_{\text{max}}$. This behavior of $E_g^\text{max}(E)$ is responsible for the low energy steepening of the UHECR diffusive spectrum. In Figure \cite{11} (right panel) we have reported the $E_g^\text{max}$ as function of the observed energy $E$, with $E_{\text{max}} = 1 \times 10^{22}$ eV.

Let us now concentrate on the diffusion coefficient that enters in $\lambda(E,E_g)$ function, see equation \cite{11}. Following \cite{2}, we assume diffusion in a random magnetic field with a strength $B_0$ on the coherence length $l_c$. This assumption determines the diffusion coefficient $D(E)$ at the highest energies when the Larmor radius of protons $r_L(E) = (B_0/\pi G)^{-1}(E/10^{18} \text{eV})$ Mpc becomes larger than $l_c$, namely $D(E) = D_0 (r_L/E)/l_c^2$ with $D_0 = c l_c/3$. 

![FIGURE 1. [Left Panel] Maximum generation energy $E_{\text{max}}^g$ defined as min$[E_g(E,t_0), E_{\text{acc}}^\text{max}]$, where $E_{\text{acc}}^\text{max}$ is the maximal acceleration energy and $t_0$ is the age of the universe (see text). [Right Panel] Maximal distance $r_{\text{max}}(E)$ to the contributing sources as function of the observed energy $E$. Three merging curves in the left-low corner give $r_{\text{max}}^\text{diff}(E)$, the Syrovatsky solution (1) formally includes all propagation times up to $t \to \infty$ and the dash-dotted curve gives $r_{\text{max}}^\text{max}(E)$, which numerically is very close to the energy-attenuation length $l_{\text{att}}(E) = [(1/cE) dE/dt]^{-1}$.](image)
At low energy (i.e. when $r_s(E) \lesssim l_c$) we have considered three different cases: (i) the Kolmogorov diffusion $D_K(E) = D_0(r_s(E)/l_c)^{1/3}$; (ii) the Bohm diffusion $D_B(E) = D_0(r_s(E)/l_c)$; (iii) an arbitrary case $D(E) \propto E^{\alpha}$ with $\alpha = 2$ as the extreme possibility, also in this case $D(E)$ is normalized by $D_0$ at $r_s = l_c$, that corresponds to an energy $E_c \approx 10^{18} (B_0/nG)(l_c/Mpc)$ eV. The diffusion length can be evaluated through the interpolation formula $l_{\text{diff}} = L_d + l^2/l_c$, with: $L_d = r_s$ and $L_d = l_c(r_s/l_c)^{1/3}$ in the Bohm and Kolmogorov cases respectively. At distances $r < l_{\text{diff}}(E)$ the proton propagation becomes rectilinear and the fluxes from individual sources on the lattice are computed in the rectilinear propagation limit. In this limit, taking into account the cosmological evolution of the Universe, the diffuse flux in our lattice model will be:

$$f_p^{\text{rec}}(E) = \frac{L_p K(\gamma_0)}{(4\pi E_{\text{min}})^2} \sum_i \left( \frac{E_g(E, \tilde{z}_i)}{E_{\text{min}}} \right)^{-\gamma_0} \times \frac{1}{r_s^2(1 + \tilde{z}_i)} \frac{dE_g(E, \tilde{z}_i)}{dE}$$

where $\tilde{z}_i$ is the red shift that, according to the standard cosmology, is associated to the source at distance $r_s$, the two quantities $E_g(E, \tilde{z}_i)$ and $dE_g(E, \tilde{z}_i)/dE$ can be computed according to [13].

Following [2] we fix a reasonable strength of the IMF: namely $B_0 = 1$ nG on the coherence scale $l_c = 1$ Mpc. This choice of $(B_0, l_c)$ is compatible with the limits that follows from the Faraday rotation measures and with a rectilinear propagation regime at the highest energies $E > 10^{19}$ eV. In order to reproduce, in our lattice model, the density of sources as follows from small angle clustering, we have chosen a separation between sources, i.e. a lattice spacing, of $d = 30$ Mpc that corresponds to a source space density of $n_s = 1/d^3 = 3.7 \times 10^{-5}$ Mpc$^{-3}$. In the computation of the fluxes we have assumed an injection spectrum with a single power law $\gamma_0 = 2.7$, starting from the minimum energy $E_{\text{min}} = 1$ GeV.

As follows from equation [4] the particles that, as a whole, contribute to the diffuse flux are those particles produced inside a sphere of radius $r_{\text{max}}(E) = 2 \sqrt{\lambda(E, E_{\text{max}})}$, the contribution to the flux is exponentially suppressed for particles produced at higher distances. In Figure [14] (left panel) we have reported, fixing $l_c = 1$ Mpc and $B_0 = 1$ nG the behavior of the maximum contributing distance $r_{\text{max}}(E)$ in the three cases Kolmogorov, Bohm, and $D(E) \propto E^{2}$. The dotted-dashed line represents the proton attenuation length $l_{\text{att}} = E(dE/dl)^{-1}$, that can be interpreted as the maximum contributing distance in the rectilinear propagation regime. Figure [14] (right panel) clearly illustrates the steepening of the diffuse spectrum at low energies. While the energy-attenuation length $l_{\text{att}}$ diminishes with energy and has the GZK steepening at energy $E \simeq 5 \times 10^{19}$ eV, the diffusive maximum distance $r_{\text{max}}(E)$ increases with energy and has a sharp jump at energy $E_{\text{eq}} \simeq 2 \times 10^{18}$ eV. The position of this jump is related only to the proton energy losses, it does not depend on the magnetic field configuration (i.e. on the diffusion parameters). The energy $E_{\text{eq}}$ corresponds to the proton energy at which pair-production energy losses become equal to adiabatic energy losses. At this energy the upper limit of integration in the Syrovatsky solution changes abruptly from $E_{g}(E, r_0)$ to the maximum energy $E_{\text{max}}$ that the source
can provide.

Let us consider now the UHECR spectra shown in Figure 2 (left panel). In this figure we report the diffusive and universal spectra, the latter corresponds to the rectilinear propagation (see equation (3)) in the case of a homogeneous distribution of sources (see [14] for a detailed discussion). The effect of the low energy steepening is clearly seen in the spectra, with the steepening energy \( E_s \) being independent of the diffusive regime chosen. According to the results presented in Figure 1 (right panel) for \( r_{\text{max}}(E) \) the flux below the steepening energy \( E_s \) is largest for the Kolmogorov diffusion and lowest in the case \( D(E) \propto E^2 \), with the Bohm diffusion between them. The source luminosity \( L_p \) needed to provide the observed spectrum is very high, for a distance between sources of \( d = 30 \) Mpc the required luminosity is \( L_p = 3.0 \times 10^{18} \) erg/s. To reduce the required emissivity one can assume that the acceleration mechanism works only starting from a somewhat higher minimum energy \( E_{\text{min}} \) (see [22] for a detailed discussion).

It is also worthwhile to stress that the feature of the dip, that signals in the experimental data for a proton dominated spectrum, is not washed out by the IMF in the diffusive approximation. The validity of this approximation is related to the validity of the Syrovatskii solution at energies \( E < 10^{18} \) eV. The diffusive equation, and hence its solution, are valid only in the case in which the energy losses and diffusion coefficient are time independent. At energies lower than \( 10^{19} \) eV, this is not the case, because the propagation time of protons approaches the age of the universe and the effect of the CMB temperature variation with time (red-shift) becomes important. However, our computations show a good agreement between the quasi-rectilinear regime of propagation and the exact rectilinear propagation, this agreement shows the approximate validity of the Syrovatsky solution at the discussed energies.

Following [1, 2, 15] we shall conclude by discussing shortly the transition from galactic to extragalactic cosmic rays. The remarkable feature of the diffusive spectra is the low-energy steepening at the fixed energy \( E_s \sim 1 \times 10^{18} \) eV, which provide the transition from extragalactic to galactic CR. This energy coincides approximately with the position of the 2nd knee \( E_{2K} \) and gives a non-trivial explanation of its value as \( E_{2K} \sim E_s \). Like in the above-mentioned works we shall assume that at \( E \geq 1 \times 10^{17} \) eV the galactic spectrum is dominated by iron nuclei and calculate their flux by subtracting the calculated flux of extragalactic protons from all-particle Akeno spectrum. For these calculations we shall fix the spectrum with magnetic configuration (1 nG, 1 Mpc), the Bohm diffusion and a separation between sources on the lattice \( d = 30 \) Mpc The calculated spectrum of galactic iron is shown in Figure 2 (right panel) by the dashed curve. This prediction should be taken with caution because obtained with a model-dependent calculations (assumption of the Bohm diffusion) and uncertainties involved in the Syrovatsky solution. However, it is interesting to note that the iron-nuclei spectrum in Figure 2 (right panel) is well described by the Hall diffusion [16] in the galactic magnetic field at energies above the knee.

**CONCLUSIONS**

We have analyzed the anti-GZK effect in the diffusive propagation of UHE protons. This effect consists in a sharp increase of the maximum distance \( r_{\text{max}}(E) \) from which UHE protons can arrive (Figure 1 (left panel)). The observational consequences of the anti-GZK effect is a low-energy steepening of the diffuse spectrum. While the shape of the steepening depends on the magnetic field configuration, the steepening energy \( E_s \) is practically independent of it. The steepening of the spectrum at \( E_s \sim 1 \times 10^{18} \) eV coincides with the 2nd knee observed in the CR spectra by most of the detectors and provides a natural transition from galactic iron nuclei to extragalactic protons.

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