Superalgebraic Truncations from $D=10$, $N=2$ Chiral Supergravity

Chang-Ho Kim
Department of Physics, Seonam University, Namwon, Chonbuk 590-170, Korea

Young-Jai Park*
Department of Physics and Basic Science Research Institute, Sogang University, C.P.O. Box 1142, Seoul 100-611, Korea

ABSTRACT

We study ten-dimensional $N=2$ maximal chiral supergravity in the context of Lie superalgebra SU(8/1). The possible successive superalgebraic truncations from ten dimensional $N=2$ chiral theory to the lower dimensional supergravity theories are systematically realized as sub-superalgebraic chains of SU(8/1) by using the Kac-Dynkin weight techniques.

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* E-mail address: yjpark@ccs.sogang.ac.kr
I. Introduction

There have been considerable interests in superalgebras which are relevant to many supersymmetric theories.\(^1\),\(^2\) Recently, M and F theories\(^3\) have been also tackled from the point of view of the general properties of the superalgebra.\(^4\) Supersymmetric extensions of Poincaré algebra in \(D\)-dimensional space-time were reviewed, and their representations (reps) for the supermultiplets of all known supergravity theories were extensively searched by Strathdee.\(^5\) This work has been an extremely useful guideline for studying supersymmetric theories. Cremmer\(^6\) developed the complicated method for consistent truncations by choosing a particular rep of real symplectic metric in order to derive \(N=6,4,2\) supergravities from \(N=8\) in five dimensions.

On the other hand, during last ten years, we have shown that superalgebras allow a more systematic analysis for finding the supermultiplets\(^7\),\(^8\) of several supergravity and type-IIB closed superstring theories by using the Kac-Dynkin weight techniques of \(SU(m/n)\) Lie superalgebra.\(^9\) In particular, we have shown that the massless reps of supermultiplets of the \(D=10, N=2\) chiral supergravity\(^10\) and the \(D=4, N=8\) supergravity\(^11\) belong to only one irreducible representation (irrep) of the \(SU(8/1)\) superalgebra using the Kac-Dynkin method.\(^12\) Recently, we have shown that all possible successive superalgebraic truncations from four-dimensional \(N=8\) theory to \(N=7,6,...,1\) supergravity theories are systematically realised as sub-superalgebra chains of \(SU(8/1)\) superalgebra.\(^13\)

In this letter, we show that the successive superalgebraic truncations from \(D=10, N=2\) chiral supergravity\(^9\) to possible lower dimensional nonmaximal theories can be easily realized as sub-superalgebra chains of \(SU(8/1)\) Lie superalgebra by using projection matrices.\(^14\) In Sec. II, we briefly recapitulate the mathematical structure of the \(SU(8/1)\) superalgebra related to \(D=10, N=2\) maximal chiral supergravity. In Sec. III, we explicitly show that supermultiplets of possible lower dimensional supergravity theories can be systematically obtained from \(SU(8/1)\) by successive superalgebraic dimensional reductions and truncations. The last section contains conclusions.

II. Kac-Dynkin Structure of \(SU(8/1)\) superalgebra

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In this section, let us briefly recapitulate the Kac-Dynkin Structure of SU(8/1) superalgebra. The Kac-Dynkin diagram of the SU(8/1) Lie superalgebra is

\[
\begin{array}{cccccccc}
w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 & w_8 \\
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \times
\end{array}
\]  

(1)

where the set \((w_1 w_2 \cdots w_8)\) characterizes the highest weight vector of an irrep.\(^{1,2}\) The components \(w_i\) \((i \neq 8)\) of this vector should be a nonnegative integer, while \(w_8\) can be any complex number. The last node denotes the simple odd root \(\beta_8\), while the seven white nodes in the Kac-Dynkin diagram denote the simple even roots \(\alpha_i\) \((i = 1, 2, \cdots, 7)\), which constitute SU(8) subalgebra.

The corresponding graded Cartan matrix is given by

\[
\begin{bmatrix}
2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0
\end{bmatrix}.
\]  

(2)

Note that each positive simple even root \(\alpha_i^+\) corresponds to the \(i\)-th column of the graded Cartan matrix, while the positive simple odd root \(\beta_8^+\) corresponds to the last column of the graded Cartan matrix. The negative simple roots \(\alpha_i^-\) and \(\beta_8^-\) are given by

\[
\alpha_i^- = -\alpha_i^+, \quad \beta_8^- = -\beta_8^+,
\]  

(3)

and other odd roots are easily obtained by

\[
\beta_i^\pm = [\alpha_i, \beta_{i+1}], \quad i = 1, 2, \cdots, 7.
\]  

(4)
Then, the action by an odd root $\beta_i \pm$ alternates a bosonic (fermionic) floor with a fermionic (bosonic) one.

The fundamental rep of SU(8/1) is $(1 0 0 0 0 0 0 0)$, and it has the substructure $[ (8, 1)_F \oplus (1, 1)_B ]$ in the SU(8)$\otimes$U(1) bosonic subalgebra basis, where the subscripts $F$ and $B$ stand for fermionic and bosonic degrees of freedom, respectively, as follows:

\[
(1 0 0 0 0 0 0 0) \\
\mid \text{ground} > (1 0 0 0 0 0 0) = (8, 1)_F \\
\downarrow \beta_i^- \\
\mid 1\text{st} > (0 0 0 0 0 0 1) = (1, 1)_B.
\]

On the other hand, the complex conjugate rep of the fundamental rep is given by $\overline{(0 0 0 0 0 0 - 1)} = [(1, 1)_B \oplus (\bar{8}, 1)_F]$. The even and odd roots consist of the adjoint rep $(1 0 0 0 0 0 0 - 1)$, which is obtained from the tensor product of the above two reps, as follows

\[
(1 0 0 0 0 0 0 - 1) \\
\mid \text{gnd} > (1 0 0 0 0 0 0 - 1) = \beta_i^+ \\
\mid 1\text{st} > (1 0 0 0 0 0 1 - 1) = \text{SU}(8) \\
(0 0 0 0 0 0 0) = \text{U}(1) \\
\mid 2\text{nd} > (0 0 0 0 0 0 1 0) = \beta_i^-.
\]

In general, there are two types of irreps of SU($m/n$), which are typical and atypical$^{1,9,15}$. All atypical reps of SU(8/1) are characterized by the last component of the highest weight. The atypicality condition$^9$ is given by

\[
w_8 = - \sum_{j=i}^7 w_j + i - 8, \quad 1 \leq i \leq 8.
\]

Note that since an odd root $\beta_i^-$ string is terminated in the full weight system for the case of the atypical rep such that $w_8$ satisfies Eq.(7) for a specific $i$, the atypical reps generally have not equal bosonic and fermionic degrees of freedom.

On the other hand, all the typical reps of SU(8/1) consist of nine floors and have equal bosonic and fermionic degrees of freedom. The typical, lowest dimensional rep is
(0 0 0 0 0 0 w_8) = [128_B ⊕ 128_F] = [1 ⊕ 8 ⊕ 28 ⊕ 56 ⊕ 70 ⊕ 56 ⊕ 28 ⊕ 8 ⊕ 1] for 

w_8 ≠ 0, −1, · · ·, −7. Particularly, this weight system with w_8 = −\frac{7}{2} satisfies both the typical and real properties. By using these properties, we have already shown that the typical rep (0 0 0 0 0 0 −\frac{7}{2}) is beautifully identified with the supermultiplets of the D=4, N=8 supergravity and D=10, N=2 chiral supergravity. \textsuperscript{12,13}

Now, let us consider the case of D=10, N=2 maximal chiral supergravity. Although the hidden symmetry of full theory on the shell is still not known, we have found that SO(8) ⊗ SO(2) ⊂ SU(8/1). Here, we have introduced a bigger symmetry SU(8) ⊃ SO(8) to preserve chirality, and the U(1) ≈ SO(2), which corresponds to the simple odd root, for N=2 supersymmetry.

Let h_i (i = 1, 2, . . . , 8) be Cartan sublagebras of SU(8/1) superalgebra. Then the U(1) sublagebra is composed of \sum_{i=1}^{8} i h^i to satisfy the supertraceless condition, and the U(1) supercharge generator should be Diag(1, 1, 1, 1, 1, 1, 1, 8). Then, the typical lowest dimensional rep (0 0 0 0 0 0 −\frac{7}{2}) = [128_B ⊕ 128_F] corresponds to the supermultiplets of D = 10, N = 2 chiral supergravity. The full contents of the representation and the field identifications are given by
It is interesting to note that this rep can be identified with a single scalar superfield $\Phi(x, \theta)$ treated by Green and Schwarz.\textsuperscript{10}

### III. Possible Superalgebraic Truncations

#### 3.1 $D=8$, $N=1$ Reduction

Now, let us consider the possible superalgebraic truncation to the eight dimensions. The supermultiplets of $D = 8$, $N = 2$ are in the rep space of SO(6) $\otimes$ Sp(2) symmetry. However, the irreps of SO(6) $\otimes$ Sp(2) are not fit in SU(8) $\otimes$ U(1) $\subset$ SU(8/1). Therefore, the case of the only possible superalgebraic truncation is to accommodate the supermultiplets of $D = 8$, $N = 1$ in the rep space of a maximal subalgebra SU(4) $\otimes$ SU(4/1) $\otimes$ U$^a$(1), which is simply obtained by removing the fourth node from the Kac-Dynkin diagram in Eq.(1). The supertraceless condition is satisfied by tak-
ing the $U^a(1)$ assignment as $\text{Diag.}(3,3,3,-4,-4,-4,-4)$. Note that the $U^a(1)$ subalgebra is composed of $[3h^1 + 6h^2 + 9h^3 + 12h^4 + 8h^5 + 4h^6 - 4h^8]$.

We find that this branching scheme describes the $D = 8, N = 1$ chiral theory. The light-like symmetry $SO(6) \approx SU(4)$ is realized through the subalgebraic chains as follows

\[
\begin{align*}
SU(8/1) & \rightarrow SU_V(4) \otimes SU_S(4/1) \otimes U^a(1) \\
& \rightarrow SU_V(4) \otimes SU_S(4) \otimes U^a(1) \otimes U^b(1) \\
& \rightarrow SU_{V+S}(4) \otimes U(1),
\end{align*}
\]

where the subscripts $V$ and $S$ mean vectorial and spinorial reps, respectively. A branching rule of the first step for the rep $(0 0 0 0 0 0 0 -7/2)$ is

\[
(0 0 0 0 0 0 0 -\frac{7}{2}) \rightarrow (0 0 0)(0 0 0 -\frac{7}{2})(2) \oplus (0 0 1)(0 0 0 -\frac{5}{2})(1) \\
\oplus (0 1 0)(0 0 0 -\frac{3}{2})(0) \oplus (1 0 0)(0 0 0 -\frac{1}{2})(-1) \\
\oplus (0 0 0)(0 0 0 \frac{1}{2})(-2),
\]

where the $U^a(1)$ supercharges are normalized by -7. The typical rep $(0 0 0 w_8)$ of $SU_S(4/1)$ has the content of $(8_B + 8_F) = (1 + \bar{4} + 6 + 4 + 1)$.

Then, the typical rep $(0 0 0)(0 0 0 -\frac{7}{2})(2)$ in Eq.(10) gives a Yang-Mills multiplet such as

\[
\begin{array}{cccc}
SU_V(4) \otimes SU_S(4) \otimes U^a(1) \otimes U^b(1) & SU_{V+S}(4) \otimes U(1) & \text{field} \\
(0 0 0)(0 0 0)(2)(-14) & (0 0 0)(-2) & \phi^1 \\
(0 0 0)(0 0 1)(2)(-11) & (0 0 1)(-1) & \chi^- \\
(0 0 0)(0 1 0)(2)(-8) & (0 1 0)(0) & A_\mu \\
(0 0 0)(1 0 0)(2)(-5) & (1 0 0)(+1) & \chi^+ \\
(0 0 0)(0 0 0)(2)(-2) & (0 0 0)(+2) & \phi^2.
\end{array}
\]
Here, the U(1) supercharge is given by $U(1) = \frac{1}{3}[4U^a(1) + U^b(1)]$. Note that since the rep $(0 \ 0 \ 0)(0 \ 0 \ 1\frac{1}{2})(-2)$ is the complex conjugation of the rep given by Eq.(11), one may also take it as a Yang-Mills multiplet.

On the other hand, the graviton multiplet is the rep $(0 \ 1 \ 0)(0 \ 0 \ 0 \ 0 \ -\frac{3}{2})(0)$ in Eq.(10) as follows

\[
\begin{array}{cccc}
SU_V(4) \otimes SU_S(4) \otimes U^a(1) \otimes U^b(1) & SU_{V+S}(4) \otimes U(1) & \text{field} \\
(0 \ 1 \ 0)(0 \ 0 \ 0)(0)(-6) & (0 \ 1 \ 0)(-2) & A^1_\mu \\
(0 \ 1 \ 0)(0 \ 0 \ 1)(0)(-3) & (0 \ 1 \ 1)(-1) & \Psi^- \\
 & (1 \ 0 \ 0)(-1) & \chi^- \\
(0 \ 1 \ 0)(0 \ 1 \ 0)(0)(0) & (0 \ 2 \ 0)(0) & e^a_\mu \\
 & (1 \ 0 \ 1)(0) & B_{\mu\nu} \\
 & (0 \ 0 \ 0)(0) & \phi \\
(0 \ 1 \ 0)(1 \ 0 \ 0)(0)(+3) & (1 \ 1 \ 0)(+1) & \Psi^+ \\
 & (0 \ 0 \ 1)(+1) & \chi^+ \\
(0 \ 1 \ 0)(0 \ 0 \ 0)(0)(+6) & (0 \ 1 \ 0)(+2) & A^2_\mu.
\end{array}
\]

Note that the other two reps $[(0 \ 0 \ 1)(0 \ 0 \ 0 \ -\frac{5}{2})(1) \oplus (1 \ 0 \ 0)(0 \ 0 \ 0 \ -\frac{1}{2})(-1)]$ in Eq.(10) make an extra gravitino multiplet at the $SU_{V+S}(4) \otimes U(1)$ stage, which should be removed for consistency in the $D = 8$, $N = 1$ theory.

3.2 $D=6$, $N=2$ Reduction

As you know, the underlying symmetry of $D = 6$, $(N_+, N_-) = (2, 0)$ chiral theory is $SO(4) \otimes Sp(2)$. But, let us try to accommodate this symmetry in the larger supersymmetry $SU(4/1)$, which contains $SU(2) \otimes SU(2/1)$ substructure given by the branching pattern
SU_{V}(4) \otimes SU_{S}(4/1) \rightarrow \text{Sp}_{V}(4) \otimes SU_{S}(2) \otimes SU_{S}(2/1) \\
\rightarrow [SU_{V}(2)]^2 \otimes [SU_{S}(2)]^2 \\
\rightarrow [SU_{V+S}(2)]^2 \approx SO_{V+S}(4). \quad (13)

Here, we use the branching pattern SU_{V}(4) \rightarrow \text{Sp}_{V}(4) \rightarrow SU_{V}(2) \otimes SU_{V}(2) for the vectorial space. Thus, the branching rule for the rep (0 1 0) of SU_{V}(4) should be (0 1 0) \rightarrow (0 1) \oplus (0 0) \rightarrow [(1)(1) \oplus (0)(0)] \oplus (0)(0). Note that we have lost the U(1) symmetry at the Sp_{V}(4) branching stage. On the other hand, we use the branching pattern SU_{S}(4/1) \rightarrow SU_{S}(2) \otimes SU_{S}(2/1) for the spinorial space. Then, the rep (0 0 0 -3/2) of SU_{S}(4/1) branches into the reps (0)(0 -3/2) \oplus (1)(1 -1/2) \oplus (0)(0 1/2). Finally the rep [(1)(1)][(1)(0 -1/2)] in [SU_{V}(2)]^2 \otimes [SU_{S}(2) \otimes SU_{S}(2/1)] basis reduces to the following reps of SO_{V+S}(4)

\[(2 1) \oplus (0 1)] \oplus [(2 2) \oplus (0 2) \oplus (2 1) \oplus (0 0)] \oplus [(2 1) \oplus (0 1)], \quad (14)\]

where the reps are denoted in Dynkin weights of SO(4) and the floors of SU(2/1) are distinguished by the square brackets. Note that they are equivalent to the following expression of Strathdee^{5}

\[(3, 2; 1) \otimes 2^2 \oplus (1, 2; 1) \otimes 2^2, \quad (15)\]

which are the reps of SO(4) \otimes \text{Sp}(2). Here, the rep 2^2 means \(1,2;1 \oplus 1,1;2\) in SO(4) \otimes \text{Sp}(2) basis. On the other hand, the rep (0)(0 -3/2) of SU(2) \otimes SU(2/1) is

\[(0)(0 - \frac{3}{2})\]
\[| \text{ground} > \quad (0)(0 - \frac{3}{2}) \]
\[| \text{1st} > \quad (0)(1 - \frac{3}{2}) \]
\[| \text{2nd} > \quad (0)(0 - \frac{1}{2}). \quad (16)\]
where the first floor gives \((1,2;1)\) and the ground and the second floors make \((1,1;2)\), that is, the Sp(2) indices are reproduced by the floors.

Similarly, the Yang-Mills and matter multiplets can be also easily identified with the reps \((0)_V (0)_V (1)_S (0 - 1/2)_S\) and \((0)_V (0)_V (0)_S (0 - 3/2)_S\), respectively. Note that the adjoint rep of SU\((2/1)\) is given by

\[
\begin{align*}
| \text{ground} > & \quad (1 - 1) = Q^+_{1/2} \\
| \text{1st} > & \quad (2 - 1) = \text{SU}(2) \\
| \text{2nd} > & \quad (1 0) = Q^-_{1/2}.
\end{align*}
\]

As a results, the Yang-Mills multiplet \((0 0 0)(0 0 0 - 7/2)(2)\) in \(D=8\), \(N=1\) reduces into \((0)(0 - 7/2) \oplus (1)(0 - 5/2) \oplus (0)(0 - 3/2)\) in \(D=6\), \(N=2\). The Yang-Mills multiplet of \(D=6\), \(N=2\) is \((1)(0 - 5/2) = [(2, 1) \oplus (2, 2) \oplus (2, 1)]\), while the other reps \((0)(0 w_8) = [(1, 1) \oplus (1, 2) \oplus (1, 1)]\) are matter multiplets.

It seems appropriate to comment on the truncations from \(D=6\) to \(D=4\) theories. Unfortunately, we cannot directly obtain the \(D=4\), \(N=4\), 3, 2, 1 from \(D=6\), \(N=2\) theory by the successive superalgebraic truncations because we already lost several U(1) informations at the \(D=6\) stage. However, the \(D=4\), \(N=8\) supergravity is effectively equivalent to the \(D=10\), \(N=2\) chiral supergravity, and these equivalence can be shown schematically in the SU\((8/1)\) Kac-Dynkin diagram

\[
\begin{array}{c}
\begin{array}{cccccccccc}
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
D=10 & N=8 & & & & & & & & N=2 \\
\end{array}
\end{array}
\]

In fact, this equivalence implies that as space-time dimensions are decreased by the consistent dimensional reduction, supersymmetry must be extended or vice versa.\(^{16}\)
According to this line, it seems enough to comment our previous result\textsuperscript{13} that the successive superalgebraic truncations from $D=4$, $N=8$ theory to $D=4$, $N=7,6,\ldots,1$ supergravity theories can be systematically realized as sub-superalgebraic chains of SU(8/1) superalgebra.

\section*{IV. Conclusion}

In conclusion, we have studied $D=10$, $N=2$ chiral supergravity in the context of SU(8/1) superalgebra. We have obtained possible regular maximal branching patterns in terms of Kac-Dynkin weight techniques. Then, we have shown that the possible superalgebraic truncations from the $D=10$, $N=2$ maximal chiral theory to the $D=8$, $N=1$, and $D=6$, $N=2$ theories can be systematically realized as sub-superalgebra chains of the SU(8/1) superalgebra. As results, we have explicitly identified the supermultiplets of the possible relevant lower dimensional theories, which have been classified in terms of super-Poincaré algebra by Strathdee, with irreps of SU($N/1$) superalgebra by using the systematic superalgebraic truncation method. Finally, through further investigations, we hope that our superalgebraic branching method will provide a deeper understanding of the structure of the supersymmetric systems including the M and F theories.

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