Gluonia in QCD with massless quarks

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Abstract

I briefly review the estimate of the gluonia masses, decay and mixings in QCD with massless quarks from QCD spectral sum rules and some low-energy theorems. The data in the $0^{++}$ channel can be explained with some maximal gluonium-quarkonium mixing schemes, which then suggest a large violation of the OZI rule, similar to the case of the $\eta'$ in the $U(1)_A$ sector.

1For more details and complete references, see S.N., hep-ph/9612457 (1996) [Nucl. Phys. B (in press)] and QCD 97 International Euroconference, Montpellier, hep-ph/9710281 (1997) [Nucl.Phys. B (Proc. Suppl.) (in press)].
1 Introduction

The possible existence of the gluon bound states (gluonia or glueballs) or/and of a gluon continuum is one of the main consequences of the non-perturbative aspects of QCD. In this talk, I summarize our recent results.

2 Gluonia masses from QCD spectral sum rules

We shall consider the lowest-dimension gauge-invariant currents $J_G$ built from two gluon fields with the quantum numbers of the $J^{PC} = 0^{++}, 2^{++}$ and $0^{++}$ gluonia. The former two enter the QCD energy-momentum tensor $\theta_{\mu \nu}$, while the third one is the $U(1)_A$ axial anomaly. We shall also consider the three-gluon current associated to the $0^{++}$ gluonia. We shall work with the generic two-point correlator:

$$\psi_G(q^2) \equiv i \int d^4x \ e^{i q \cdot x} \langle 0 \mid T J_G(x) (J_G(0)) \dagger \mid 0 \rangle,$$

where its QCD expression can be parametrized by the usual perturbative terms plus the non-perturbative ones due to the vacuum condensates in the Wilson expansion. In the massless quark limit $m_q = 0$, the dominant non-perturbative contribution is due to the dimension-four gluonic condensate $\langle \alpha_s G^2 \rangle \simeq (0.07 \pm 0.01)$ GeV$^4$, recently estimated from the $e^+e^-$ data and heavy-quark mass splittings. We shall study the SVZ Laplace sum rules:

$$\mathcal{L}_G(t) = \int_{t_{\Sigma}}^{\infty} dt \exp(-t \tau) \frac{1}{\pi} \text{Im} \psi_G(t), \quad \text{and} \quad \mathcal{R}_G \equiv -\frac{d}{dt} \log \mathcal{L}_G, \quad (1)$$

where $t_{\Sigma}$ is the hadronic threshold. The latter, or its slight modification, is very useful, as it is equal to the resonance mass squared, in the simple duality ansatz parametrization of the spectral function:

$$\frac{1}{\pi} \text{Im} \psi_G(t) \simeq 2 f^2_G M^4_G \delta(t - M^2_G) + \text{"QCD continuum"} \Theta(t - t_c),$$

where the decay constant $f_G$ is analogous to $f_\pi = 93.3$ MeV; $t_c$ is the continuum threshold which is, like the sum rule variable $\tau$, an a priori arbitrary parameter. Our results in Table 1 satisfy the $t_c$ and $\tau$ stability criteria, whilst the upper bounds have been obtained from the minimum of $\mathcal{R}_G$ combined with its positivity. Our results show the mass hierarchy $M_S < M_P \approx M_T$, which suggests that the scalar is the lightest gluonium state. However, the consistency of the different subtracted and unsubtracted sum rules in the scalar sector requires the existence of an additional lower mass and broad $\sigma_B$-meson coupled strongly both to gluons and to pairs of Goldstone bosons (a case similar to the $\eta'$). The effect of the $\sigma_B$-meson can be missed in a one-resonance parametrization of the spectral function, and in the present lattice quenched approximation. The values of $\sqrt{t_{\Sigma}}$, which are about equal to the mass of the next radial excitations, indicate that the mass splitting between the ground state and the radial excitations is much smaller (30%) than for the $\rho$ meson (about 70%), so that one can expect rich gluonia spectra in the vicinity of 2.0–2.2 GeV. We also conclude that:

- The $\zeta(2,2)$ is a good $2^{++}$ gluonium candidate because of its mass (see Table 1) and small width in $\pi\tau$ ($\leq 100$ MeV). However, the associated value of $t_c$ can suggest that the radial excitation state is also in the 2 GeV region, which should stimulate further experimental searches.
- The $E/\pi$ (1.44) or other particles in this region is too low for being the lowest pseudoscalar gluonium. One of these states is likely to be the first radial excitation of the $\eta'$ because its coupling to the gluonic current is weaker than the one of the $\eta'$ and of the gluonium (see Table 1).

3 Widths of the scalar glueon

- The hadronic widths can be estimated from the vertex: $V(q^2) = \langle H_1 \mid \theta^\mu_1 \mid H_2 \rangle$, where: $q = p_1 - p_2$, $H \equiv \pi, \eta_1, \sigma_B$, and $B$ refers to the unmixed bare state. It satisfies the constraints $V(0) = M^2_B$ and, in the chiral limit, $V'(0) = 1$. Saturating it with the three resonances $H \equiv \sigma_B$, $\sigma'_B$ and $G$, one obtains the first and second NV sum rules (Goldberger–Treiman-like relation):

$$\frac{1}{4} \sum_{S \equiv \sigma_B, \sigma'_B, G} g_{SSH} \sqrt{3} f_S \simeq 2 M^2_H,$$

$$\frac{1}{4} \sum_{S \equiv \sigma_B, \sigma'_B, G} g_{S\pi\pi} \sqrt{3} f_S / M^2_S = 1. \quad (2)$$

Higher dimension condensates, including the ones due instanton–anti-instanton ($D = 11$) operators, can be neglected at the sum rule optimization scale. UV renormalon and some eventual other effects induced by the resummation of the QCD series, and not included in the OPE, are estimated from the last known term of the truncated series, as is done in the extraction of $\alpha_s$ from $\tau$ decays.

Throughout this paper, we shall use, for three active flavours, the value of the QCD scale $\Lambda = (375 \pm 125)$ MeV. An analogous large violation of the OZI rule is also necessary for explaining the proton spin crisis. These features are consequences of the $U(1)$ symmetry, which is not the case of the vector meson $\phi$ of the $SU(3)_F$ symmetry. The small quarkonium-gluonium (mass) mixing angle allows us to expect that the observed meson mass is about the same as the one in Table 1.
We identify the $G$ with the $G(1.5 \sim 1.6)$ at GAMS (an almost pure gluonium candidate), and the $\sigma_B$ and $\sigma_B'$ (its radial excitation) with the broad resonance below 1 GeV and the $f_0(1.37)$. In this way, we obtain the predictions in Table 2, showing the presence of gluons inside the wave functions of the broad $\sigma_B$ and $\sigma_B'$, which decay copiously into $\pi \pi$, signalling a large violation of the OZI rule in this channel. For the $G$ meson, we deduce $g_{G\eta\eta} \approx \sin \theta_P g_{G\eta\eta}'$ ($\theta_P = -18^\circ$ being the pseudoscalar mixing angle), implying a ratio of widths of about 0.22 in agreement with the GAMS data $r \approx 0.34 \pm 0.13$, but suggest that the Crystal Barrel particle having $r \approx 1$ is a mixing between this gluonium and other states. We also expect that the $4\pi$ decay of the $G(1.6)$ are mainly $S$-waves initiated from the decay of pairs of $\sigma_B$.

- The $\gamma\gamma$ widths of the $\sigma_B$, $\sigma_B'$ and $G$ can be obtained (Table 2) by identifying the Euler-Heisenberg box two gluons–two photons effective Lagrangian, with the scalar-$\gamma\gamma$ Lagrangian, while the $J/\psi \rightarrow \gamma S$ radiative decays can be estimated, using dispersion relation techniques, to this effective interaction.

### 4 “Mixing-ology” for the decay widths of scalar mesons

- We consider that the observed $f_0$ and $\sigma$ result from the two-component mixing of the $\sigma_B$ and $S_2 \equiv \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$ unmixed bare states. Using the prediction on $\Gamma(\sigma_B \rightarrow \gamma\gamma)$, and the experimental width $\Gamma(f_0 \rightarrow \gamma\gamma) \approx 0.3$ keV, one obtains a maximal mixing angle: $\theta_S \approx (40 \sim 45)^\circ$, which indicates that the $f_0$ and $\sigma$ have a large amount of gluons in their wave functions. Then, one can deduce (in units of GeV):

\[
g_{f_0\pi^+\pi^-} \simeq (0.1 \sim 2.6), \quad g_{f_0K^+K^-} \simeq -(1.3 \sim 4.1),
\]

which can give a simple explanation of the exceptional property of the $f_0$ (strong coupling to $\bar{K}K$: $\pi\pi$ and $\bar{K}K$ data, and $\phi \rightarrow \gamma f_0$ decay).

- The $f_0(1.37), f_0(1.50)$ can be well described by a $3 \times 3$ mixing scheme involving the $\sigma'_B(1.37), S_3(\bar{s}s)(1.47)$ and $G(1.5)$ gluonium, and contains a large amount of glue and $\bar{s}s$ components. The different widths are given in Table 3. The $f_J(1.71)$, if confirmed to be a $0^{++}$, can be essentially composed by the radial excitation $S_3'(\bar{s}s)$ due to its main decay into $\bar{K}K$.

### 5 Conclusions and acknowledgements

I have shown that QCD spectral sum rules plus some QCD low-energy theorems can provide a plausible explanation of the complex gluonia spectra.

I thank the CERN theory division for its hospitality.

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6 This situation is quite similar to the case of the $\eta'$ in the pseudoscalar channel (mass given by its gluon component, but strong coupling to quarkonia).
### Table 1: Unmixed gluonia masses and couplings from QSSR

| $J^P C$ | Name | Mass [GeV] | $f_G$ [MeV] | $\sqrt{s_c}$ [GeV] |
|---------|------|------------|-------------|-------------------|
| 0$^{++}$ | $G$  | 1.5 ± 0.2  | 390 ± 145  | 2.0 ~ 2.1        |
|         | $\sigma_B$ | 1.00 (input) | 1000      |                   |
|         | $\sigma'_B$ | 1.37 (input) | 600       |                   |
|         | $3G$   | 3.1        | 62         |                   |
| 2$^{++}$ | $T$   | 2.0 ± 0.1  | 80 ± 14    | 2.2               |
| 0$^{-+}$ | $P$   | 2.05 ± 0.19| 8 ~ 17     | 2.2               |
|         | $E/\iota$ | 1.44 (input) | $7 : J/\psi \rightarrow \gamma \iota$ | |

### Table 2: Unmixed scalar gluonia and quarkonia decays

| Name | Mass [GeV] | $\pi^+ \pi^-$ [MeV] | $K^+ K^-$ [MeV] | $\eta \eta'$ [MeV] | $(4\pi)^S_S$ [MeV] | $\gamma \gamma$ [keV] |
|------|------------|------------------|----------------|----------------|-------------------|----------------|
| Glue |            |                  |                |                |                   |                |
| $\sigma_B$ | 0.75 ~ 1.0 | 0.2 ~ 0.5 | $SU(3)$ | $SU(3)$ | 0.2 ~ 0.3 | |
|         | (input)   |                  |                |                |                   |                |
| $\sigma'_B$ | 1.37 | 0.5 ~ 1.3 | $SU(3)$ | $SU(3)$ | 43 ~ 316 | 0.7 ~ 1.0 |
|         | (input) |                  |                |                |                   |                |
| $G$   | 1.5 | $\approx 0$ | $\approx 0$ | $1 ~ 2$ | $5 ~ 10$ | $60 ~ 138$ | 0.2 ~ 1.8 |
| Quark |            |                  |                |                |                   |                |
| $S_2$ | 1. | 0.12 | $SU(3)$ | $SU(3)$ | 0.67 | |
| $S'_2$ | $1.3 \approx \pi'$ | 0.30 ± 0.15 | $SU(3)$ | $SU(3)$ | 4 ± 2 | |
| $S_3$ | 1.47 ± 0.04 | 73 ± 27 | $15 \pm 6$ | 0.4 ± 0.04 | |
| $S'_3$ | $\approx 1.7$ | 112 ± 50 | $SU(3)$ | 1.1 ± 0.5 | |

### Table 3: Predicted decays of the observed scalar mesons

| Name | $\pi^+ \pi^-$ [MeV] | $K^+ K^-$ [MeV] | $\eta \eta'$ [MeV] | $(4\pi)^S_S$ [MeV] | $\gamma \gamma$ [keV] |
|------|------------------|----------------|----------------|----------------|----------------|
| $f_0(0.98)$ | 0.2 ~ 134 | Eq. [3] |                   | $\approx 0.3$ | (exp) |
| $\sigma(0.75 ~ 1)$ | 300 ~ 700 | $SU(3)$ | $SU(3)$ | 0.2 ~ 0.5 | |
| $f_0(1.37)$ | 22 ~ 48 | $\approx 0$ | $\leq 1$ | $\leq 2.5$ | 150 | $\leq 2.2$ |
| $f_0(1.5)$ | 25 | 3 ~ 12 | $1 \sim 2$ | $\leq 1$ | 68 ~ 105 | $\leq 1.6$ |
| $f_J(1.71)$ | $\approx 0$ | 112 ± 50 | $SU(3)$ | $\approx 0$ | 1.1 ± 0.5 |