Evaluation of the dynamical characteristics of fluid flow caused by collapse of a non-spherical near-surface bubble

T C Le¹, V I Melikhov¹, O I Melikhov¹ and S E Yakush²

¹ National Research University Moscow Power Engineering Institute, Krasnokazarmennaya 14, Moscow 111250, Russia
² Institute for Problems in Mechanics of the Russian Academy of Sciences, Ave. Vernadskogo 101 Bldg 1, Moscow, 119526, Russia

E-mail: oleg.melikhov@erec.ru

Abstract. The paper is devoted to numerical study of the influence of the initial shape of a vapor bubble on the surface impact of the water jet emerging due to the bubble collapse. This problem is relevant to spreading of high-temperature melt under a layer of subcooled water. The vapor bubbles formed on the melt-water interface, upon complete penetration in water, are condensing rapidly to produce high-velocity water micro-jets directed towards the melt. Impact of these jets causes upward melt splashing. Superposition of these processes forms a dynamic layer where melt is mixed with water; the presence of this premixed layer can be a pre-requisite for steam explosion. In this work, numerical modelling by the boundary element method is performed for the collapse of a bubble of an oblate spheroid. It is shown that the water jets generated in this process possess the impulse comparable to that generated by a collapsing spherical vapour bubble of the same volume. By the numerical simulations and subsequent estimates it is obtained that collapse of non-spherical bubbles near the melt-water interface produces melt splashes of the height of few centimetres, which is sufficient for the occurrence of steam explosions.

1. Introduction

In certain severe accident scenarios at nuclear power plants featured by reactor core meltdown, direct contact of the core melt with water is possible, which can lead to energetic interaction (steam explosion) potentially capable of damaging the reactor containment [1]. Due to high relevance to nuclear power safety, this phenomenon has been studied quite intensively over the past few decades; nevertheless, many questions still remain open due to the complexity of multiple coupled factors involved [2]. In particular, it was believed for a long time that melt-water interaction in stratified geometry (with heavy melt spreading under lighter water) is of secondary importance and low conversion ratio because in this configuration there cannot develop a zone where the melt is mixed sufficiently with water to create conditions for powerful steam explosion [1]. However, recent experiments performed at the Royal Institute of Technology (KTH) in Sweden [3–5] overturned this point of view. In the tests carried out on PULiMS and SES facilities, few dozen kilograms of molten binary oxidic materials were poured at atmospheric pressure in a pool filled to the level of 20 cm by subcooled water. Water subcooling was varied in the range 5–30°C, the melt temperature ranged between 940°C and 1420°C (melt superheat with respect to the freezing temperature was 70–206°C). Eutectic Bi₂O₃-WO₃ and ZrO₂-WO₃ compositions were used as corium simulant materials.
In the video recordings [3–5], the melt jet passage of the 200-mm air gap between the nozzle orifice and water level, its penetration to water, and subsequent spreading under water were registered. As the authors of the experiments [3–5] note, melt spreading over the pool bottom was featured by formation, growth, and collapse of vapor bubbles on the water-melt interface; the size of some bubbles was as large as few centimeters. The time of spontaneous steam explosions after the beginning of melt pouring was between 1.1 and 15 s. In the majority of experiments, the primary explosion was followed by a secondary one with the delay of 1 s or even less. The conversion ratio, expressing the ratio of the mechanical work of the explosion and the initial thermal energy of the melt, was estimated to be about 1%. In the PULiMS-E6 experiment, only a single steam explosion was registered, which renders this experiment attractive for the analysis, due to the absence of the secondary explosions occurring in uncontrolled conditions.

This experiment was studied numerically in [6] by MC3D code based on the multiphase fluid mechanics models. As the initial condition, a zone where melt was premixed with water was set, and explosion was then initiated in this zone; the simulation predictions were compared with the experimental data. From the simulation results, the parameters of the premixed zone were estimated. The same parameters were estimated in [7] by a completely different approach, based on the Hugoniot analysis for thermal detonation in melt-water mixtures. Both approaches gave close mass of melt (about 2 kg) and water (about 1.5 kg) in the premixed zone.

As the mechanism for the formation of premixed zone, the following hypothesis was put forward in [5]. Vapor bubbles are formed on the melt-water interface; after their complete development they are surrounded completely by subcooled water, therefore, they are shrinking and collapsing due to vapor condensation. Since their collapse occurs near the melt-water interface, high-speed water jets directed towards the melt are generated (similar to the cumulative jets known in cavitation). By impacting the surface, these jet transfer their momentum to the melt; this momentum is then reflected from the pool bottom causing upward melt splashing. Superposition of these processes forms a dynamic layer where melt is mixed with water.

The experimental study on the effect of water subcooling performed in [5] revealed that at low subcooling (5°C) no steam explosion occurred. A rather thick (30–60 mm) vapor blanket was formed between the melt and water. It is stated in [3] that the melt splashes, even if occurred at this water subcooling, were not visible behind the vapor. Thus, in this case no significant premixing developed, and no explosion occurred. At 14°C water subcooling, an explosion with the lowest conversion ratio of the whole experimental series was registered. In the remaining experiments, water subcooling was 25–30°C, and the conversion ratio reached about 1%. The authors note that at such subcooling levels a continuous vapor layer between the melt and water was absent (or was very thin); at the same time, melt splashes were as high as 10 cm.

Consider in more detail the formation of vapor film in the experiments [3–5]; this question is essential because, as it was just noted, this film affects significantly the possibility of vapor explosion occurrence. Upon the contact of a high-temperature melt with water, evaporation proceeds on the interface surface, and the released vapor forms a film separating the melt and water. The vapor film receives a heat flux from the melt, transferring it to the water-vapor interface where some fraction of the heat goes to evaporation, and the remaining part is transferred to water. In most of the experiments [3–5], formation of a continuous vapor film was not observed; however, systematic growth and collapse of vapor bubbles was recorded. At the atmospheric pressure, water cannot (in the sense of thermodynamical equilibrium) come into direct contact with the melt having the temperature of about 1000°C; it can be expected that a thin vapor film must always be present, however, it was not recorded in the experiments due to its small thickness. In this configuration, with the lighter vapor located under heavier water, small perturbations of the vapor-water interface can grow exponentially (in the linear theory) by the Rayleigh-Taylor instability, resulting in the formation of bubbles.

Stability of a vapor film separating water from a hot surface underneath it was considered theoretically in [8]. By the linear analysis taking into account the phase change on the water-vapor interface it was shown that solutions exist in the form of waves on the interface having the nature other
than capillary or gravitational. It was shown that, unlike the isothermal problem where Rayleigh-Taylor instability develops inevitably, in boiling dynamic stabilization can occur in some parameter range, leading to the existence of an oscillating vapor film under a heavy water layer. Applicability of the theory [8] to the problem of liquid melt spreading under the water has yet to be studies, although it can qualitatively explain the existence of visible vapor film fragments at high subcoolings.

It was underlined in [5] that the hypothesis on the mechanism of premixed layer formation in the case of stratified melt and water configuration needs its confirmation. Partially, such confirmation was performed in [9] where, on the basis of a model of compressive and conductive vapor, as well as incompressible and conductive water, estimates were obtained for the kinetic energy of liquid gained due to hot vapor bubble collapse. It was shown that at high vapor superheats the dynamics of bubble collapse is close to the Rayleigh solution [10] for a cavitation bubble, i.e., a bubble with the internal pressure equal to the saturation pressure at the water temperature. On the basis of this conclusion, the impulse gained by water due to collapse of a bubble near the melt-water interface was obtained. The bubble parameters were taken like in cavitation, while the flow of water and deformation of bubble boundary was calculated by the boundary element method (BEM) [11]. The melt was approximated by a rigid plane because its density is about eight times higher than that of water; the presence of thin vapor film was neglected. The water impulse calculated in [9] allowed the impact of water jet on the melt, and the respective reaction of melt, to be evaluated. It was shown that the water momentum transferred to the melt can cause splashes of melt to the heights of few centimeters, which agrees well with the experimental observations [3–5].

The current paper continues the study [9], extending the analysis to collapse of non-spherical vapor bubbles and its effect on the underlying melt layer. The importance of considering non-spherical bubbles which can appear, for example, due to pressure gradient in liquid, was first emphasized in [12], where collapse of a bubble having the shape of an oblate spheroid was considered. It was shown that the aspect ratio of the spheroid significantly affects the parameters of the generated cumulative jet: as the axial size of the bubble is decreasing, the jet is becoming thinner, while its velocity increases sharply. In [13], detailed analysis of the cavitation bubble dynamics depending on its distance to wall and deviation of its initial shape from spherical was performed. For ellipsoidal initial perturbations of spherical bubble shape, the aspect ratio interval was determined where a jet directed normally towards the surface is developing; this interval was shown to be quite narrow. Thus, since the initial non-sphericity of a bubble affects its collapse significantly, the current paper is focused on the bubble shape influence on the jet impact on the wall, in application to melt-water premixing in stratified steam explosions.

2. Problem formulation and numerical method
We consider a bubble initially located near a rigid boundary; the geometry of the problem is presented schematically in figure 1. The bubble is assumed to have the shape of an oblate spheroid, with the polar radius $a$ and equatorial radius $b$. Initially, the bubble is touching the bottom boundary. The liquid is considered as incompressible and inviscid, its velocity field is potential, i.e., $v = \nabla \Phi$. The continuity equation $\nabla v = 0$, as well as the momentum equation (expressed as the Bernoulli equation), are described in terms of the potential $\Phi$ in the following form:
Figure 1. Bubble geometry and main parameters.

\[ \Delta \Phi = 0 \]  \hspace{1cm} (1)

\[ \frac{\partial \Phi}{\partial t} + \frac{1}{2} (\nabla \Phi)^2 + \frac{P - P_\infty}{\rho_f} = 0 \]  \hspace{1cm} (2)

where \( t \) is the time, \( p \) is the pressure in fluid, \( P_\infty \) is the pressure at infinity, \( \rho_f \) is the fluid density.

On the bubble surface \( \Gamma \) the pressure is constant and equal to the saturated vapor pressure at the temperature of liquid \( P_c \):

\[ P|_\Gamma = P_c \]  \hspace{1cm} (3)

The coordinates of the bubble surface points (\( x \) is the position vector of a point on \( \Gamma \)) are related to the velocity field by

\[ \frac{dx}{dt} = v \]  \hspace{1cm} (4)

On the bottom boundary \( z = 0 \), zero velocity of fluid is posed (impermeability condition)

\[ v|_{z=0} = \frac{\partial \Phi}{\partial z}|_{z=0} = 0 \]  \hspace{1cm} (5)

At infinity \( (r^2 + z^2 \rightarrow \infty) \) the velocity potential \( \Phi \) tends to zero.

Equations (1)–(5) are solved numerically. Advancement in time of the bubble boundary coordinates according to (4) and of the velocity potential at those points from (2) is performed by the first-order Euler scheme. The velocity potential \( \Phi \) is updated by the boundary element method (BEM) [11], which enables the fluid velocity on the bubble boundary to be updated on each time step.

3. Numerical modelling results and analysis

Numerical simulations were carried out for different aspect ratios of the oblate spheroid \( e = b/a = 1 - 1.15 \). As a reference case, a spherical bubble of the radius \( a_0 \) was considered; the semi-axes of the spheroid (polar radius \( a \) and equatorial radius \( b \)) were calculated from the condition that the volume of the spheroidal bubble be equal to that of the reference spherical bubble.

In figure 2, the evolution of bubble shape with time is presented for different aspect ratios \( e \). The results obtained agree, both qualitatively and quantitatively, with previous works on the collapse of a near-wall bubble [9, 12, 13]. It can be seen that, with the increase in the aspect ratio \( e \) (i.e., with more pronounced deviation from sphericity), the cumulative jet is becoming thinner, and it reaches the lower boundary faster.
Figure 2. The shape of collapsing bubble at different aspect ratio $e$ (the wall is shown by the dashed line). On each curve, the non-dimensional time $t^* = t \left( a_0 \sqrt{\rho_f / (p_a - p_v)} \right)$ is indicated.

The quantity which characterizes the directional fluid flow generated by the collapsing boundary, is the Kelvin impulse, namely, its vertical component

$$I_z = -\rho_f e_z \int_{\Gamma} \phi n \ d\Gamma$$

(6)

(the other two components of the Kelvin impulse are zero in the axially symmetric problem). In (6), $e_z$ is the unit vector in the $z$ coordinate direction, $n$ is the unit surface normal vector pointing into the bubble. The Kelvin impulse is well-known in fluid dynamics, providing a meaningful way to quantify unsteady flows near deformable bodies immersed in liquid, including collapsing cavities and
cavitating bubbles; a review of this concept with necessary references can be found in [14]. For the problem in question, the Kelvin impulse (6) characterizes the flow momentum directed towards the wall, and can be used to calculate the wall impact [15]. Note that the directional flow arises only for near-wall bubble collapse; in the infinite liquid it would be zero due to problem symmetry.

In figure 3, the non-dimensional Kelvin impulse $I_z' = \frac{I_z}{\rho_f (p_a - p_c)} \sqrt{\frac{p_a - p_c}{\rho_f}}$ during the bubble collapse is plotted as a function of time for different aspect ratios $e$. It can be seen that in the course of bubble evolution the Kelvin impulse grows monotonically, reaching its maximum at the instant of bubble collapse.

![Figure 3](image3.png)

**Figure 3.** Non-dimensional Kelvin impulse for different aspect ratio $e$.

In figure 4, the maximum non-dimensional velocity of water jet $v_{jx}^* = \frac{v_{jx}}{\sqrt{(p_a - p_c) / \rho_f}}$ and Kelvin impulse $I_z'$ at the instant of bubble collapse are shown as functions of the aspect ratio $e$. It can be seen that, for larger deviation from sphericity (increase in $e$), the velocity of cumulative jet is growing significantly, however, the Kelvin impulse is decreasing to a certain extent. This occurs because for more non-spherical bubbles the jet diameter (and the mass of water in the jet) is decreasing.

![Figure 4](image4.png)

**Figure 4.** Maximum non-dimensional jet velocity and Kelvin impulse as functions of the bubble aspect ratio $e$. 

6
In order to evaluate the impact of water jet from a collapsing non-spherical bubble on the underlying melt layer, we take the approach presented in [9]. Namely, consider the impingement of a water jet on the melt surface and assume that a melt droplet is thrown up due to the momentum gained by the melt and reflected from the pool bottom. From the momentum conservation, we can assume that the Kelvin impulse of the water jet is fully transferred to the melt droplet:

\[ I_{z,\text{max}}(e) = m_m w_m \]  

(7)

where \( m_m \) and \( w_m \) are the mass and initial vertical velocity of the melt droplet. For conservative estimates, neglect the drag acting on the moving droplet, as well the Archimedian force, and apply the mechanical energy conservation for a drop rising in the gravity field. Its maximum rise height \( \Delta h \) is obtained from

\[ \frac{m_m w_m^2}{2} = m_m g \Delta h \]  

(8)

Take \( w_m = I_{z,\text{max}}(e)/m_m \) from (7) and substitute it into (8), assuming that the melt droplet is spherical with a radius \( a_m \) (the melt drop mass is \( m_m = 4\pi a_m^3 \rho_m \), \( \rho_m \) is the melt density) to obtain

\[ \frac{a_m}{a_0} = \left( \frac{0.0285 p_0 - p_r}{\rho_f g} \right)^{1/6} \left( \frac{\rho_f}{\rho_m} \right)^{1/3} \left( \frac{I_{z,\text{max}}(e)}{a_0^2 [\rho_f (p_0 - p_r)]^{1/2}} \right)^{1/3} \frac{1}{\Delta h^{1/6}} \]  

(9)

Note that the expression in the third parentheses on the right-hand side is, in fact, the non-dimensional maximum Kelvin impulse \( I_{z,\text{max}}^* \) presented in figure 4.

For the estimates, take the parameters of the experiments [5]: at water temperature \( T_f = 348 \text{ K} \) and pressure \( p_0 = 10^5 \text{ Pa} \), we have \( \rho_f = 974.86 \text{ kg/m}^3 \), \( p_r = 0.38595\cdot10^5 \text{ Pa} \), the melt density is \( \rho_m = 7811 \text{ kg/m}^3 \). Using \( I_{z,\text{max}}^*(e) \) from figure 4, we obtain the melt-to-bubble radius ratio \( a_m/a_0 \) can be obtained as a function of the initial aspect ratio \( e = b/a \). In figure 5, these dependencies are plotted for three values of the melt rise height \( \Delta h = 3, 5, \text{ and } 7 \text{ cm} \).

Figure 5. Relative size of melt droplets as a function of bubble aspect ratio obtained for three melt rise heights \( \Delta h \).
It can be seen that as non-sphericity of the bubble becomes more pronounced, the Kelvin impulse is decreasing; nevertheless, even such non-spherical bubbles can affect the melt significantly. In the range of aspect ratios $e = 1 - 1.15$ the bubble can produce a water micro-jet capable of knocking out a melt drop of radius $a_n = (0.4 - 0.5)a_0$ which can rise to the heights $\Delta h = 3 - 7$ cm above the melt surface. It is noted in [5] that bubbles of few centimeters in size were observed in the experiments. Thus, if we take the initial bubble volume, which corresponds to the equivalent radius of the sphere 1 cm, the characteristic radii of the melt droplets penetrating the mixing layer of 5 cm depth can be about 4–5 mm.

At higher initial aspect ratios, the scheme of bubble collapse is changing, as is shown in figure 6. A shrinking bubble is divided in two parts. Where before a jet was formed, a small bubble is detaching which remains, for some time, connected by a thin neck with the main bubble. An annular jet is formed which collapses, connecting the neck. The BEM simulation was performed until this connection instant. It can be concluded from the physical considerations that after the closure of the neck connecting the two bubbles, two opposite micro-jets will be formed, [12]. One of the jets is directed towards the surface, the second one is directed upwards, into the smaller bubble. Therefore, in this case a cumulative jet hitting the melt surface will also appear, although it will be formed somewhat differently (after contraction of the neck between the two bubbles). This process is more difficult for calculation with the boundary element method because the initial simply connected vapor domain splits into two unconnected ones; its calculation requires modifications to the existing BEM code and will be performed in the future studies.

![Bubble shape evolution](image)

**Figure 6.** Bubble shape evolution for high aspect ratio $e = 1.3$, (wall is shown by the dashed line). The non-dimensional time is $t^* = \frac{t}{\rho_0 \sqrt{\frac{\rho_f}{(p_o - p_e)}}}$.

4. Conclusions
Influence of the initial shape of a collapsing bubble on the wall impact of the cumulative jet is studies. The bubble was taken initially as an oblate spheroid, with its polar axis normal to the surface. It is shown that with the increase in the aspect ratio the velocity of cumulative jet grows significantly, while, its diameter is decreasing. Nevertheless, the Kelvin impulse, characterizing the overall water impact on the surface, is reduced only moderately with the growth in the aspect ratio. It can be concluded that such non-spherical bubble can produce a substantial impact on the surface. In application to the problem of stratified steam explosion, it was obtained that impact of such jets can
result in the splashes where melt droplets of the 4–5 mm radius rise to the height of 3–8 cm. These droplets can mix with the water, so that a coarsely premixed layer develops above the propagating melt. In this premixed zone, conditions for the initiation and propagation of steam explosion waves observed experimentally can exist.

Further studies will be devoted to detailed modeling of melt-water interaction in the three-layer configuration (melt, vapor film, water) in order to verify the estimated based on the momentum conservation principles. In particular, it will be necessary to study the secondary flows following the primary bubble collapse, where instability of the vapor film can play an important role, leading to periodic splashing of the spreading melt.

Acknowledgments
This research was funded by Russian Science Foundation (RSF) under Grant 18-19-00289.

References
[1] Berthoud G 2000 Vapor explosions Annu. Rev. Fluid Mech. 32 573–611
[2] Meignen R, Raverdy B, Buck M, Pohlner G, Kudinov P, Ma W, Brayer C, Piluso P, Hong S-W, Leskovar M, Uršič M, Albrecht G, Lindholm I and Ivanov I 2014 Status of steam explosion understanding and modelling Ann. Nucl. Energy 74 125–33
[3] Konovalenko A, Karbojian A and Kudinov P 2012 Experimental results on pouring and underwater liquid melt spreading and energetic melt-coolant interaction. In: Proceedings of The 9th International Topical Meeting on Nuclear Thermal-Hydraulics, Operation and Safety (NUTHOS-9), Kaohsiung, Taiwan, September 9–13, N9P0303
[4] Grishchenko D, Konovalenko A, Karbojian A, Kudinova V, Bechta S and Kudinov P Insight into steam explosion in stratified melt-coolant configuration 2013 In: 15-th International Topical Meeting on Nuclear Reactor Thermal Hydraulics, NURETH 15, May 12-17, Pisa, Italy, Paper 599
[5] Kudinov P, Grishchenko D, Konovalenko A and Karbojian A 2017 Premixing and steam explosion phenomena in the tests with stratified melt-coolant configuration and binary oxidic melt simulant materials Nucl. Eng. Des. 314 182–97
[6] Leskovar M, Centrih V and Uršič M 2016 Simulation of steam explosion in stratified melt-coolant configuration Nucl. Eng. Des. 296 19–29
[7] Iskhakov A S, Melikhov V I, Melikhov O I, Yakush S E, Chung L T 2019 Hugoniot analysis of experimental data on steam explosion in stratified melt-coolant configuration Nucl. Eng. Des. 347 151–79
[8] Sinkevich O A 2015 Waves on the surface of a boiling liquid at various medium stratifications J. Exp. and Theor. Physics 121 321–35
[9] Melikhov V I, Melikhov O I, Yakush S E and Chung L T 2020 Evaluation of energy and impulse generated by superheated steam bubble collapse in subcooled water. Nucl. Eng. Des. [In press]
[10] Rayleigh, Lord 1917 VIII. On the pressure developed in a liquid during the collapse of a spherical cavity. Phil. Mag. 34(200) 94–8
[11] Brebbia C A, Telles J C F and Wrobel L C 1984 Boundary Element Techniques (Springler-Verlag)
[12] Voinov O V and Voinov V V 1976 On the scheme of cavitation bubble collapse near a wall and formation of a cumulative jet Doklady Akademii Nauk 227 63-6 [In Russian]
[13] Aganin A A, Ilgamov M A, Kosolapova L A and Malakhov V G 2013 Collapse of a cavitation bubble in liquid near a solid wall Vestnik Bashkirskogo Universiteta (Bulletin of Bashkir University) 18 15–21 [In Russian]
[14] Blake J R, Leppinen D M and Wang Q 2015 Cavitation and bubble dynamics: The Kelvin impulse and its applications Interface Focus 5 20150017 1–15
[15] Pearson A, Blake J R and Otto S R 2004 Jets in bubbles J. Eng. Math. 48 391–412