IDENTIFICATION OF HEAT EXCHANGE BOUNDARY CONDITIONS AT VARIOUS NATURAL AND TECHNOCENIC FACTORS

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Abstract. The development of permafrost areas requires reliable and long-term projected estimates in a changing climate and man-made impacts. Dangerous natural and man-made factors, such as snowfall, infiltration of precipitation, ice, thermokarst, etc., interact with the ground and determine the stability of the territories of the cryolithozone. In this regard, the work is devoted to the development of new algorithms and software for identifying the boundary conditions on the surface of frozen soil, taking into account the actual process of freezing and thawing of the pore solution of the soil.

Problem Formulation
The temperature field of frozen soils, taking into account the phase transition of pore water, is described by the following heat conduction problem (direct problem) [6]:

\[
c^\beta \rho^\beta \partial T^\beta \partial T^\beta = \frac{1}{r^n} \partial_r \left( \lambda^\beta r^n \partial T^\beta \partial r \right), \quad R_1 < r < R_N, \quad 0 < \tau <= \tau_m, \quad \beta = 1, N, \quad \lambda^\beta \frac{\partial T^\beta}{\partial r} = 0, \quad r = R_N, \quad 0 < \tau <= \tau_m, \quad T^\beta(r,0) = T^\beta_0(r), \quad R_1 <= r <= R_N, \quad \beta = 1, N, \tau = 0. \]

We see matching conditions at the junction of the two layers:

\[
T^\beta \bigg|_{R_\beta^-} = T^{\beta+1} \bigg|_{R_\beta^+},
\]

\[
\lambda^\beta \frac{\partial T^{\beta+1}}{\partial r} \bigg|_{R_\beta^-} = \lambda^{\beta+1} \frac{\partial T^{\beta+1}}{\partial r} \bigg|_{R_\beta^+}, \quad 0 < \tau <= \tau_m, \quad \beta = 1, n-1, \]

where

\[
c^\beta = c^\beta(T) = c^\beta_{sk} + c_i \omega^\beta_0 + (c_w - c_i) \omega^\beta_{uw}(T) + L \frac{\partial \omega^\beta_{uw}(T)}{\partial T},
\]
\[ \lambda^\beta = \lambda^\beta(T) = \lambda_f^\beta + (\lambda_f^\beta - \lambda_0^\beta) \frac{W_{uw}(T) - W_{tb}^\beta}{W_0^\beta - W_{tb}^\beta}, \]

It is required to restore the boundary condition on the ground surface \((s = 1)\) with \(r = R_1:\)

\[ -\lambda^1 \frac{\partial T^1}{\partial r} = q(\tau), \quad r = R_1, \quad 0 < \tau < \tau_m, \quad (4) \]

where the desired parameter is the function \(u(\tau) = q(\tau)\).

To restore the desired parameter, additional measurements of the temperature inside the sample are required:

\[ T(r_i, \tau) = T^a(\tau), \quad i = 1, n_t. \quad (5) \]

We formulate this problem as an optimal control problem: find the function \(u(\tau)\) from the minimum of the objective functional:

\[ J(u) = \sum_{i=1}^{n_t} \int_0^{\tau_m} p_i(\tau) \left( T(r_i, \tau) - T^a(r_i, \tau) \right)^2 d\tau \quad (6) \]

Here \(c\) is effective heat capacity of the soil, taking into account the phase transition in the temperature spectrum, \(J/(kg \, K)\); \(\rho\) is soil density, \(kg/m^3\); \(T\) is temperature, \(K\); \(\lambda\) is ground thermal conductivity, \(W/(m \, K)\), \(\tau\) is time, \(s\); \(r\) is spatial coordinate, \(m\); \(n = 0,1,2\) corresponds to Cartesian, cylindrical and spherical coordinates; \(\beta\) is layer index; \(R_1, R_2\) are area boundaries, \(m\); \(\tau_m\) is right boundary value of the time interval; \(L\) is specific heat of phase transition, \(J/kg\); \(W_{uw}, W_{tb}\) is unfrozen and tightly bound water, \%; \(q(\tau)\) is heat flow, \(W/m^2\); \(p_i(\tau)\) is weight factors with dimension \(K^{-2} \, c^{-1}\); \(T(r_i, \tau), T^a(r_i, \tau)\) is calculated and measured temperatures in the \(i\)-th point of the frozen array.

**Solution Algorithm**

Boundary inverse problems belong to the class of incorrect ones and are solved by special iterative methods. In this case, the conditions of existence and uniqueness of the solution of the boundary inverse problems of heat conduction [2]. Iterative methods are used with great success in solving ill-posed problems [2], [3], [7]. In this case, the number of iterations acts as a regularization parameter.

In the conjugate gradient method, the iteration sequence is constructed taking into account the previous direction of descent according to the following law:

\[ u^{s+1} = u^s - \beta_s S^s, \quad s = 0, 1, ... \]

\[ S^s = J'(u^s) - \gamma_s S^{s-1}, \quad S^0 = J'(u^0), \quad \gamma_0 = 0 \quad (7) \]

The calculation procedure for the conjugate gradient method is as follows. Starting from a certain point \(u^0\) the movement in the direction of the anti-gradient to the minimum is achieved, which is reached at the point \(u^1\). Then, the direction paired with the previous one, in which the transition to the next minimum point occurs, is determined. As a result, a new approximation of \(u^2\) is determined, and so on.

An important feature of conjugate gradients is that the number of iterations to achieve an exact minimum of a quadratic function is theoretically equal to \(m\) – the dimension of the desired vector. In practice, due to errors that appear in the course of the calculation, the convergence of
the method deteriorates, for this, it is recommended to “stir up” the iterative process, this allows to reduce the accumulation of computational errors.

\( \gamma_s \) parameter is defined in various ways depending on the properties of the target functional. So, for example, if the second Fréchet derivative function changes little, then the parameter is given using the formula:

\[
\gamma_s = - \frac{(J'(u^s), J'(u^{s-1}) - J'(u^s))}{||J'(u^s)||_R^m_0^2}. \tag{8}
\]

If the minimized functionality is close to quadratic, then it is determined by the expression:

\[
\gamma_s = - \frac{||J'(u^s)||^2_2}{||J'(u^{s-1})||^2_2}. \tag{9}
\]

Step size \( \beta_s \) of the conjugate gradient method is determined using the formula:

\[
\beta_s = \frac{\sum_{i=1}^{n_t} \int_0^{T_m} p_i(\tau) \nu_i \left( T_i - T_{\epsilon i} \right) d\tau}{\sum_{i=1}^{n_t} \int_0^{T_m} p_i(\tau) \nu_i^2 d\tau}, \tag{10}
\]

where \( \nu_i = \nu_i(r, \tau) \) is solution of the boundary value problem for increment.

The reliability of the proposed algorithm was tested on a model problem with an exact solution. As a result of a numerical experiment, it was established that the proposed algorithm has a good regularizing property [5].

**Numerical Research**

Example 1. According to the above proposed algorithm, the heat flux density was restored during the formation of icing in the Ulakan-Taryn Valley. The length of the icing was about 3 km, width ranged from 120-800 m [1].

The observation was made on four different stationary platforms: 1 – icing; 2 – spruce forest in the icing valley; 3 – pine forest on the watershed, as well as on the “Meadow” Site in Yakutsk on the terrace above the floodplain on the left bank of the Lena River.

Site 3 (pine forest) is 25 m higher from the icing valley at the watershed with total humidity up to 10\%, and the soil and vegetation layer is only 0.05 m. Numerical calculations show that the annual course of the heat flux density goes more than intensive compared to other sites (Fig. 1).

At the beginning of the winter period (November), the process of freezing in Icing Valley 1 and Site 2 is almost the same, but in January, ground water arrives, which increases the heat flow density in this massif. The height of the frost reaches up to 2 meters, but in each year its height varies depending on the external temperature and the volume of groundwater. The thawing process depends on the external temperature, the area of ice and sometimes ends at the end of July or does not completely thaw. The depth of thawing of water-saturated ground comes
Figure 1. Annual course of the density of the heat flux on the surface of the sites with some delay, but at the end of the summer period the depth of thawing of natural open ground is reached and due to the large total humidity, full freezing occurs at the most deadline.

The soil section of the “Meadow” Site consists of 0.0-0.3 m of sod-vegetable layer, which is a shielding low heat-conducting layer. Therefore, the heat flux of this area lies in the range from -15 to 18 W/m².

Example 2. Considered the restoration of heat flow in “Tuymaada” Station. The initial temperature distribution corresponds to the actual observations at the hospital. As \( T^\theta(r, \tau) \) temperature measurements at a depth of 0.2 m are given.

Figure 2. Annual variation of heat flows at the surface of herb meadow (1), bare plot (2) and the data of Pavlov (1979) (3)
From the graph of recovery of heat flux on the surface of the Site is shown (Fig. 2), that at the bare site, the freezing process (curve 1) is more intensive in the wintertime in comparison with the grass of the meadow (curve 2). In March – April, there is an intensive influx of heat flow in the bare area, and in the summertime, the flow intensity is almost the same. Curve 3, which reflects the density of the heat flux, was determined by a numerical method according to the heat balance observation data [4]. The figure shows that the absolute value of the heat flux according to Pavlov A.V. in winter is lower compared with the results of our observations. The intensity of the heat flux depends on the insulating properties of snow, which is determined by the height, density, and structure of the snow cover (density, temperature, water vapor, etc.). In the summer, the heat flux densities do not differ much, but a shading effect of vegetation appears.

Conclusion
With the help of the proposed algorithm, heat fluxes on the frozen soil surface were restored under various exogenous and man-made conditions (ice, snow removal, land cover, etc.).

The results of computational experiments show the high efficiency of the method of iterative regularization and the possibility of solving with it a wide range of nonlinear problems.

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