A Parametric Study of Springback For Compensation Strategies

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Abstract. In this article, we perform a parametric study of the springback phenomenon. The effect of material parameters related to the work hardening and the thickness is studied by using computer experiments and statistical methods. First, a sensitivity analysis is performed using a fractional factorial design and a linear regression model. After determining the important factors, a Taguchi analysis is performed to estimate the optimum value of the parameters for robustness against springback. Next, we create a Gaussian process meta-model trained with the data generated via Latin hypercube sampling. This meta-model is used to better understand the nonlinearity of the response and the effect of parameter interactions. Finally, by using a Monte Carlo simulation on the meta-model we determine how the variability of the input parameters propagate to the response (springback). The pipeline explained in this work can help with establishing an effective strategy for the springback compensation.

1. Introduction

Minimizing or alleviating springback is a hard task due to its complexity [1, 2]; it usually requires corrections through trial and error. Finite element simulations along with compensation methods such as displacement adjustment (DA) and force descriptor (FD) have shown to be effective in providing insight on how to modify the tools to minimize the springback deviation [3, 4, 5]. The quantitative results, however, can not be considered as final; in practice, several rounds of tryouts follow. A Guideline is proposed in [6] for the springback prediction and compensation considering the effect of mesh density and integration scheme. Various improvements and modifications are proposed [7, 8, 9] to increase the efficacy of the compensation strategies (DA, FD) for the advanced/ultra high strength steels (A/UHSS).

Although considerable efforts have been expended to improve the prediction and compensation of springback, the problem still eludes an accurate solution due to its complexity and sensitivity. The variability inherent in the inputs such as material properties can render a prediction useless. One may carefully tune a simulation to get acceptable results for a fixed set of parameters only before finding out that it fails for the next batch of material. Therefore, it is vital to consider how the variability in the input parameters can affect the simulation results before investing in a costly detailed analysis.

The influence of material scatter on the sheet metal formability and springback was investigated in a number of studies [10, 11, 12, 13, 14]. In these researches, either experimentally gathered data [10] or a proxy multivariate distribution for data [12] was used. Stochastic analyses were performed via a direct or Monte Carlo simulation. It was shown that variability in the strain hardening parameters, elastic modulus and anisotropy have impact the springback prediction.
The importance of a proper parameter window selection for the robustness and reliability of the design was established [11, 15]. Performing a probabilistic analysis by direct simulation is very time consuming and uneconomical; an alternative is to employ an effective meta-model that accurately mimics the behaviour of the system in the parameter range of interest. Various meta-models are utilized to study forming and springback of sheet metals. Artificial neural networks (ANN) [16], Kriging which is a Gaussian process model [17] and the response surface method (RSM) [18] are good candidates depending on the level of complexity required. To provide the training data set for these models, design of experiment (DOE) techniques such as factorial, central composite and the Latin hypercube are often used [19].

In this study, we systematically investigate the effect of material parameters and the thickness on the springback of a U channel (similar to Numisheet 93 benchmark). Three types of independent but complementary statistical analyses are performed, namely, variable screening, optimum value estimation and probabilistic analysis. The aim is to introduce tools and a pipeline that can assist with a more informed design for springback.

2. Workflow and Methodology

The parametric study of this research comprises the following three parts: (1) variable screening using fractional factorial DOE, (2) optimum parameters estimation using Taguchi method and (3) Probabilistic analysis using Gaussian process (GP) metamodel and Monte Carlo simulation. Inputs to these analyses include material parameters and the thickness; the output (response) is the springback maximum displacement. The data generation and analysis pipeline used for this study is shown in Figure 1. First, an appropriate design of experiment (DOE) is selected and the associated parameter table is generated. Then, LS-Dyna incremental solver is invoked for the forming and springback simulations; results are dumped into data files and the maximum displacement is extracted. The parameter table and the response form a data frame that is used for the analyses mentioned above. The selection of DOE is based on the statistical analysis to be performed. This will be described in the subsequent sections. The simulation of forming and
springback is performed by using the LS-Dyna dynamic explicit and nonlinear implicit methods, respectively [20]. A reasonably fine initial mesh with Quad elements was created and the mesh adaptivity was activated. A Python script was developed to automatically generate the DOE data, run the simulations and extract the response (i.e., maximum displacement).

In the next phase, statistical analyses were performed using the Python packages Pandas, Scikit-learn, Scipy and Numpy [21]. Throughout this study, it is assumed that the formed part is acceptable (i.e., without tear, wrinkling or other defects) and the focus is put on the springback response.

3. Parameters

3.1. Fixed parameters
A schematic diagram of the sheet metal forming setup is shown in Figure 2. The shape deviation of the part due to springback is depicted in Figure 3. Stamping of this U channel produces relatively large springback deformations; this allows a better analysis as the influence of the parameters will be more pronounced and discernible from the numerical errors (round off and discretization). The dimensions of the tools, the friction coefficient $f$ and Poisson’s ratio $\nu$ are held fixed, see Table 1.

| Blank width (mm) | L (mm) | H (mm) | $R_1$ (mm) | $R_2$ (mm) | $f$    | $\nu$ |
|------------------|-------|-------|------------|------------|-------|-------|
| 35               | 360   | 70    | 10         | 10         | 0.125 | 0.3   |

3.2. Control Parameters
In this article, the sheet metal thickness and some important material properties are considered as control parameters; that is we can vary their values in the computer experiment. In sheet metal forming by drawing, plasticity characteristic of the material is of prime importance. In theory, the material behaviour beyond yield is described by a yield criterion and a hardening rule which are usually calibrated with the experimental data. The yield surface is generally of the form $g(\sigma, \alpha) - \sigma_0(\beta) = 0$, where $g$ is a scalar function of the stress tensor $\sigma$ and some parameters $\alpha$. The work hardening is described by $\sigma_0$ which depends on hardening parameters $\beta$. For this study, hardening is assumed to be isotropic, given by $\sigma_0 = \sigma_y + K \epsilon^n$ in which $\sigma_y$ is the initial yield stress, $\epsilon$ is the true plastic strain, $K$ is the coefficient of strength and $n$ is the hardening exponent. Regarding the yield surface, Hill’s planar model [22] is used (Dyna MAT37).

The control parameters selected for this study are listed in Table 2. The range of parameters considered are typical values for steel and aluminum. In the following sections, the parameters range are specified for each analysis.

4. Variable Screening
Before every detailed analysis, it is worth to screen the variables to find out the most important parameters. To determine the influence of each parameter on the springback, we perform a sensitivity analysis. The computer experiments are designed based on a two-level fraction factorial design of resolution four (i.e., $2^{6-2}$) which creates 16 tests.

With this DOE, we can study the sensitivity of the solution to the primary parameters $K, n, E, \sigma_y, r, t$ and the interactions between them, namely, $K : n, n : t, K : E$. The sensitivities
Table 2. Control parameters.

|   | Description                              | Value |
|---|------------------------------------------|-------|
| 1 | Coefficient of strength [MPa]            | K     |
| 2 | Hardening exponent [-]                   | n     |
| 3 | Young’s modulus [MPa]                    | E     |
| 4 | Yield stress [MPa]                       | σ_y   |
| 5 | Transverse anisotropy [-]                | r     |
| 6 | Thickness [mm]                           | t     |

are determined using a linear regression; the coefficients are indicative of the influence of the parameters on the response. Here, the absolute values of the coefficient are taken as the effect.

The regression model may be written as

\[ y = \alpha + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} \beta_{ij} x_i x_j + \varepsilon, \]

where \( y \) is the response, \( \beta_i \) are the coefficient of primary variables (parameters), \( \beta_{ij} \) are those of the interaction terms, \( k \) is the number of primary factors and \( p \leq k \) is used to for interaction factors. In reality, there is always noise (\( \varepsilon \)) but computer experiments are almost deterministic (modulo different round off errors in parallel execution) and noise of this form can be neglected.

In the resolution four (IV) design, the effect of primary variables is only aliased with three-terms interactions (e.g., \( x_1 x_2 x_6 \) or \( K : n : t \)) whose effects are often very small.

The levels chosen for this analysis are listed in Table 3. Pareto charts shown in Figure 4 depict the effect of primary parameters and their interactions for both steel and aluminum for the ranges selected here. It is observed that the primary parameters are the most significant, as expected; among the interaction terms, \( K : E, n : t, n : E \) and \( K : t \) appear to have larger effects than other interactions.

It should be noted that, this analysis is to only determine the important parameters. To determine the optimum values more refined analysis with higher levels is required.

Table 3. Parameter levels for steel and aluminum in screening analysis.

| Material | Level | K [MPa] | n  | E [MPa] | σ_y [MPa] | t [mm] | r     |
|----------|-------|---------|----|---------|-----------|--------|-------|
| Steel    | -1    | 5.0e2   | 0.2| 1.95e5  | 2.0e2     | 0.6    | 1.0   |
|          | 1     | 7.0e2   | 0.4| 2.10e5  | 4.0e2     | 1      | 2.0   |
| Aluminum | -1    | 3.0e2   | 0.2| 7.0e4   | 0.97e2    | 0.6    | 0.6   |
|          | 1     | 5.0e2   | 0.4| 8.0e4   | 1.90e2    | 1      | 1.0   |

5. Optimum parameter value estimation

An estimation to the optimum values of the parameters for a robust design can be achieved via Taguchi method [23]. In this method, a regular fractional factorial DOE for the control factors is extended with an outer array that considers the effect of variability and noise in the environment. In the computer experiments, however, the design factors are the primary parameters and the noise is, in fact, the variability of these factors.

The aim of Taguchi is to determine the best level of parameters at which the target (response) is less sensitive (more robust) to the variability in the source (here, the variation around the
nominal value of the respective parameter). This goal is achieved by minimizing a loss function or maximizing a signal-to-noise (S/N) ratio. It is important to note that, the effect of interaction terms are not considered as they are shown to be relatively small (see Figure 4).

Since the springback deformation is undesired, the best performance will be achieved when its value is at minimum or ideally zero; that is “smaller the better”. For this performance criterion, the S/N is defined as $SN = -10 \log(\sum_j y_j^2/n)$ in which $y_j$ are the maximum deformation magnitudes due to springback for every experiment. Here, the five primary factors $t, \sigma_y, n, K, r$ are considered as control and $E$ is held constant. The DOE matrix is created based on Taguchi’s L18 orthogonal design which comprises 18 tests. The parameters range and the selected levels are listed in Table 4.

The results of the Taguchi analysis is represented in the form of S/N and mean value curves for steel and aluminum in Figure 5. The influence of the primary parameters (without interactions)

| Table 4. Parameter levels for steel and aluminum in Taguchi analysis. |
|-------------------------|-------|--------|--------|-----|-----|
| Material                | Level | $t$ [mm] | $\sigma_y$ [MPa] | $K$ [MPa] | $n$ | $r$ |
| Steel, $E = 2.06E5$     | 1     | 0.6     | 2.07E2           | 5.80E2    | 0.16 | 1   |
|                         | 2     | 0.8     | 2.96E2           | 6.98E2    | 0.2  | 1.1 |
|                         | 3     | 1       | 4.10E2           | 8.92E2    | 0.22 | 1.2 |
| Aluminum, $E = 7.56E4$  | 1     | 0.6     | 9.78E1           | 3.27E2    | 0.13 | 0.66|
|                         | 2     | 0.8     | 1.47E2           | 4.24E2    | 0.21 | 0.73|
|                         | 3     | 1       | 1.90E2           | 5.20E2    | 0.23 | 1   |

Figure 4. Pareto chart of the parameters effect for (a) steel and (b) aluminum.

Figure 5. Taguchi analysis (top) for steel and (bottom) for aluminum; (left) signal to noise ratio and (right) mean mean displacement versus parameters levels.
on the springback of the U channel for the range of parameters considered here is discussed in
the following: as thickness \( t \) increases the springback decreases; the trend might not be as
monotonic if a more refined design was considered or the interaction terms were included. For
Yield stress \( \sigma_y \), it is known that parts formed from metals with higher yield and higher tensile
strength have larger springback. The effect of Hardening exponent \( n \) is not very clear as its
influence depends on the range of effective plastic strain. For Coefficient of strength \( K \), it is
observed that springback increases with \( K \). Transverse anisotropy for steel with \( 1.0 \leq r \leq 2.0 \)
does not have much effect. However for aluminum with \( 0.6 \leq r \leq 1.0 \), it has a positive effect.

The optimum levels of the factors are determined in two steps: (i) find levels which correspond
to maximum S/N for each parameters and (ii) find levels for which the mean becomes less and
also does not affect the S/N much. According to the trends shown in Figure 5, the best levels
are obtained and listed in Table 5. It is seen that for both materials, the best levels for the

| Factor     | \( t \) | \( \sigma_y \) | \( n \) | \( K \) | \( r \) |
|------------|-------|-------------|-------|-------|------|
| Steel      | 3     | 1           | 3     | 1     | 1    |
| Aluminum   | 3     | 1           | 2     | 1     | 1    |

S/N and mean happen to be identical. Utilizing these parameter levels will lead to a response
(springback) that is at a minimum and has the least sensitivity to the variation of the inputs.
These optimums are obtained in a statistical sense for the parameter range specified; the more
accurate optimum values may be determined by a global optimization technique such as the
Genetic Algorithm (GA) or Simulated Annealing (SA) using direct simulations or an accurate
meta-model.

6. Probabilistic analysis
In the screening and Taguchi study, we considered only two and three levels for the parameters
range; as the purpose of these analyses were to identify important factors and an estimation for
the optimum values. This coarseness, however, can overlook nonlinearities and subtle trends in
the response.

In this section, first we develop an accurate meta-model for the springback response using finer
levels. Then a Monte Carlo probabilistic analysis is performed to study how the variability of the
inputs propagate to the output. This knowledge enables us to predict the product rejection rate
and thereby to optimize the process by selecting the right window of parameters. A schematic
diagram of the workflow is shown in Figure 6.

![Figure 6. A Schematic diagram for the workflow of the probabilistic analysis.](image)

The parameter range of steel reported in Table 3 is used for the analysis of this section. For
refinement, 36 levels are considered for each parameter. In the calculations, the value for each
parameter is standardized to lay in the range \([0, 1]\). For each parameter, this is achieved by
subtracting the min from and dividing by the range of the parameter (i.e. max -min).
6.1. Latin hypercube sampling (LHS)
When the number of levels increase, the number of experiments from a factorial design becomes unmanageable, soon. For example, for the 6 parameters of this study a 5 level full factorial DOE results in 15625 tests. To mitigate this issue, we use the Latin hypercube sampling DOE [24]. LHS technique allows for having a higher number of levels with less experiments by filling the parameter space in a way that every experiment happens at a unique set of levels. Here, the LHS is performed by considering a uniform distribution for each parameter.

6.2. Meta-model
Running direct simulation for each and every case is time and resource consuming. Therefore, it is advantageous to use an accurate surrogate model that predicts the response.

To this end, we used a Gaussian process (GP) learning method [25] to create our meta-model. In models based on GP (e.g., Kriging), the value of the response for an unseen point is determined by a measure of similarity between the point and the training data. This measure also called covariance function should be selected so that it gives the best performance for the application at hand. This can be done in a cross-validation test. In comparison to the artificial neural networks (ANN), the GP models resemble a three-layer ANN for which the hidden layer has GP as the activation function.

In this research, first we split the data into the training and test sets (3:1). The GP model is fit with the three covariance functions (i) linear, (ii) cubic and (iii) generalized exponential; then the performance of each case on the test set is quantified by using the mean square error (MSE). Results indicated that to achieve the best bias-variance trade-off, the cubic function should be used. Subsequently, the model is fit/trained using the whole data set.

Figure 7 shows the response surface created with GP model for \( w_{\text{max}} \) as a function over \( K : E, n : t, n : E \) and \( K : r \); the first three showed strong interactions based on the screening analysis for steel (see Figure 4). The contour lines indicate that trends are not monotonic and the nonlinearities exist. These results are in agreement with the observation made from Taguchi analysis.

6.3. Monte Carlo analysis
To figure out the effect of input variability on the output, a Monte Carlo simulation is performed. We assume that the input parameters have normal distributions with the mean \( \mu \) and a standard deviation \( \sigma \). Analysis is performed by generating 10,000 random sample points from the parameter/s distribution and feeding them into the meta-model.

In the first investigation, we set the mean for all the parameters at 0.5 (i.e., mean of their range) and the standard deviation to 0.1 (\( \sigma = 0.1 \)). In the second investigation, \( K \) is set to be of the forgoing normal distribution while all other parameters are fixed at 0.5; results for these cases are shown in Figure 8. For the sake of analysis completeness, an arbitrary cut-off value of 30.0 mm is considered for the springback. The rejection rate (probability) for each case is obtained using the cumulative distribution function (CDF); for case 1 (all params.) the rejection rate is 20% while it is 1% for the second case. Using the method presented, the appropriate window of all parameters can be identified either by using a global optimization technique or visually for specific cases. The statistical information for the two cases studied here are summarized in Tables 6 and 7.

7. Conclusion
Given the importance of springback prediction and compensation, we embarked on a parametric study of this phenomenon using a variety of statistical tools. The parameters considered are related to material and the thickness. Specifically, the following analyses were performed: (1) variable screening by which important parameters are identified, (2) Optimum value estimation
Figure 7. Response surface and contours of maximum displacement versus parameters in range [0, 1]; darker colors represent higher values; the parameter values are standardized to lie in the range [0, 1]; the value of other parameters are set at 0.5 (i.e. the average).

Table 6. Statistics for all the parameters and the response.

| Stats. | All Param. | Response [mm] |
|--------|------------|---------------|
| $\mu$  | 0.5        | 27.88         |
| $\sigma$ | 0.1        | 2.46          |
| min    | 0.14       | 18.89         |
| max    | 0.86       | 39.15         |

20% rejection for $w_{\text{cut-off}} = 30.0$

Table 7. Statistics for $K$ and the response; other parameters at 0.5.

| Stats. | $K$ | Response [mm] |
|--------|-----|---------------|
| $\mu$  | 0.5 | 27.82         |
| $\sigma$ | 0.1 | 1.04          |
| min    | 0.14 | 25.85         |
| max    | 0.86 | 31.52         |

1% rejection for $w_{\text{cut-off}} = 30.0$
using Taguchi method and (3) probabilistic analyses using a GP meta model and Monte Carlo simulations. Although the finite element simulation of sheet metal forming has improved greatly, the uncertainties in the input data should be taken into account for a more reliable design.

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