IMAGE COMPRESSION BY USING WALSH AND FRAMELETT TRANSFORM

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ABSTRACT

In this paper, Framelet and Walsh transform are proposed for transformation, and then using arithmetic coding for compression an image. The goal is to achieve higher compression ratio by applying two levels Framelet transform (FLT), and then apply 2D Walsh-Hadamard transform (WHT) on each 8x8 block of the low frequency sub-band, while all other sub-bands are ignored. Experimental results show that the proposed algorithm gets best possible solution for tradeoff between compression ratio (size of image) and quality of compressed image, Peak Signal to Noise Ratio (PSNR). The simulation was carried using MATLAB software package version 2014. In this work, experiments were carried out on the gray scale and colored images

KEYWORDS: Walsh Hadamard Transform (WHT); Framelet Transform (FLT); Peak Signal to Noise Ratio (PSNR); Compression Ratio (CR); Arithmetic coding.
INTRODUCTION

Image compression is an inevitable solution for image transmission since the channel bandwidth is limited and the demand is for faster transmission (Alsayyh and Dzulkifli, 2012). The two major types of compression algorithms: lossless compression and lossy compression. Lossless compression is used for applications that require an exact reconstruction of the original data, while lossy compression is used when the user can tolerate some differences between the original and reconstructed representations of the data. In return for accepting this distortion in the reconstruction, we can generally obtain much higher compression ratios than is possible with lossless compression (Khalid Sayood, 2006). Transform based image coding is one of the popular image compression method. Transform when applied on images; change the image pixels to frequency domain coefficients. Desirable property of transforms is that most of the image energy is concentrated only in few significant transform coefficients. Retaining these significant coefficients and eliminating remaining coefficients results in image compression. Discrete Cosine Transform (DCT) and wavelet transform are commonly used transform methods for image compression. They are used in JPEG and JPEG 2000 respectively (Kekre et al., 2014). Recent advancements in this area show that transform based coding combined with other compression method results in better performance. In this paper, Framelet transform is used with Walsh transform for image compression. Framelet is very similar to wavelets, but has some important differences. Framelet has two or more high frequency filter banks, which produces more sub bands in decomposition. This can achieve better time-frequency localization ability in signal processing. Moreover, Framelet is more robust (Jiao and Lin, 2010). And the Walsh-Hadamard transform (WHT) is an invertible linear transform and is widely used in many practical image compression systems because of its compression performance and computational efficiency (Thida and Aye, 2015). The elements of the basis vectors of the Hadamard Transform take only the binary values ±1 and are, therefore, well suited for digital hardware implementations of image processing algorithms. Hadamard transform offers a significant advantage in terms of a shorter processing time as the processing involves simpler integer manipulation (compared to floating point processing with DCT) and the ease of hardware implementation than many common transform techniques. So it is computationally less expensive than many other orthogonal transforms (Veeraswamy and Srinivaskumar, 2017). In this work, will be applying 2D Walsh-Hadamard transform on each 8x8 block of the low frequency sub-band of Framelet transform, while all other sub-bands are ignored to get higher compression ratio. Such method is lossy compression method, then entropy coding of the quantized coefficients by Arithmetic coding (Arithmetic coding gives
greater compression than Huffman method, is faster for adaptive models, and clearly separates
the model from the channel encoding (Moffat, et al, 1998). Quantization is the process of
reducing the number of possible values of a quantity, thereby reducing the number of bits
needed to represent it. Quantization is a lossy process and implies in a reduction of the color
information associated with each pixel in the image (Thida and Aye, 2015).

2. DISCRETE WAVELET TRANSFORM (DWT)
In DWT, image is divided into four sub bands as shown in Fig. 1 a. These sub bands are formed
by separable applications of horizontal and vertical filters. Coefficients that are represented as
sub bands LH1, HL1 and HH1 are detail images while coefficients are represented as sub band
LL1 is approximation image. The LL1 sub band is further decomposed to obtain the next level
of wavelet coefficients as shown in Fig. 1b (Kumar and Agarwal, 2015).

Fig. 1. Decomposition DWT.

Fig. 2 (a and b) show that the filter bank structure for (2D analysis DWT and 2D synthesis
DWT), respectively.

Fig. 2. Filter bank for 2D- DWT Analysis and Synthesis (Barni, 2004).
3. FRAMELET TRANSFORM (FLT)

The three-channel filter bank, which is used to develop the FLT corresponding to a wavelet frame based on a single scaling function $\phi(t)$ and two distinct wavelets $\Psi_1(t)$ and $\Psi_2(t)$ the extra wavelet here makes this system an over complete one. It follows that $\phi(t)$, $\Psi_1(t)$ and $\Psi_2(t)$ satisfies the dilation and wavelet equations (Choi, 2007).

$$\phi(t) = \sqrt{2} \sum h_0(n) \phi(2t - n)$$

$$\Psi_i = \sqrt{2} \sum h_i(n) \phi(2t - n), \quad i = 1, 2$$

The scaling function $\phi(t)$ and the wavelets $\Psi_1(t), \Psi_2(t)$ are defined through these equations by the low-pass (scaling) filter $h_0(n)$ and the two high-pass (wavelet) filters $h_1(n)$ and $h_2(n)$, where the two distinct wavelets $\Psi_1(t)$ and $\Psi_2(t)$ are specifically designed to be offset from one another by one half as follows:

$$\Psi_1(t) \equiv \Psi_2(t - 0.5)$$

Where the filters $h_1(n)$ and $h_2(n)$ should satisfy the Perfect Reconstruction (PR) condition. This means that the input and output of the two filters are expected to be the same (Choi, 2007).

3.1. Filter bank structure for 2D framelet

To perform the FLT on 2D matrix, the transform first should be alternatively applied to the rows, then to the columns of the resulting matrices. This gives rise to nine 2-D sub-bands, one of which is the 2-D low pass scaling filter, and the other eight of which make up the eight 2-D wavelet filters, as shown in Fig. 3 (Abdulkareem, 2012).

![Diagram](image)

Fig. 3. Analysis and synthesis Stages of a 2-D Single Level FLT.
3.2. **computational of FLT for 2-D signal using separable method**

A separable 2-D FLT can be obtained by alternating between rows and columns i.e., it processes each row in order and then processes each column of the result. Non-separable methods work in both matrix dimensions at the same time. A 2-D separable transform is equivalent to two 1-D transforms in series. It is implemented as 1-D row transform followed by a 1-D column transform on the data obtained from the row transform. To compute a single level discrete framelet transform for 2-D signal using separable method, the next steps should be followed (Abdulkareem, 2012):

- **Checking dimensions**: Input matrix should be of size $N \times N$, where $N$ must be even and $N \geq \text{length (analysis filters)}$.

- **Construct a transformation matrix**: For an $N \times N$ matrix input 2-D signal $X$, construct a $\frac{3N}{2} \times N$ transformation matrix, $W$, using length-7 coefficients filter (for example) given as (Abdulkareem, 2012):

\[
W = \begin{bmatrix}
    h_0(0) & h_0(1) & h_0(2) & h_0(3) & h_0(4) & h_0(5) & h_0(6) & 0 & 0 & \ldots & 0 & 0 \\
    0 & 0 & h_0(3) & h_0(1) & h_0(2) & h_0(3) & h_0(4) & h_0(5) & 0 & h_0(5) & \ldots & 0 & 0 \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
    h_0(2) & h_0(3) & h_0(4) & h_0(5) & h_0(6) & 0 & 0 & \ldots & 0 & 0 & h_0(0) & h_0(1) \\
    h_1(0) & h_1(1) & h_1(2) & h_1(3) & h_1(4) & h_1(5) & h_1(6) & 0 & 0 & \ldots & 0 & 0 \\
    0 & 0 & h_1(1) & h_1(2) & h_1(3) & h_1(4) & h_1(5) & h_1(6) & 0 & h_1(5) & \ldots & 0 & 0 \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
    h_1(2) & h_1(3) & h_1(4) & h_1(5) & h_1(6) & 0 & 0 & \ldots & 0 & 0 & h_1(0) & h_1(1) \\
    h_2(0) & h_2(1) & h_2(2) & h_2(3) & h_2(4) & 0 & 0 & \ldots & 0 & 0 \\
    0 & 0 & h_2(2) & h_2(3) & h_2(4) & h_2(5) & h_2(6) & 0 & 0 & \ldots & 0 & 0 \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
    h_2(2) & h_2(3) & h_2(4) & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & h_2(0) & h_2(1) \\
\end{bmatrix}
\]

- **Transformation of input rows by applying matrix multiplication to the $\frac{3N}{2} \times N$ constructed transformation matrix by the $N \times N$ input matrix.**

\[
Y = [W]^{\frac{3N}{2}}_{N} \times [X]_{N \times N}
\]

- **Transformation of input columns**: can be done as follows (Abdulkareem, 2012):
  - Transpose the row transformed $\frac{3N}{2} \times N$ matrix resulting from step (3).
  - Apply matrix multiplication to the $\frac{3N}{2} \times N$ constructed transformation matrix by the $N \times N$ column matrix.
\[ YY = [W]_{\frac{3N}{2} \times N}^{3N} \times [Y]_{\frac{N}{2} \times N}^{T} \]  

The final framelet transformed matrix is equal to:

\[ Y_{0} = [YY]_{\frac{3N}{2} \times \frac{3N}{2}}^{T} \]

### 3.3. Computation of IFLT for 2-D signal using separable and non-separable methods

To reconstruct the original signal from the discrete framelet transformed signal, Inverse Framelet Transform (IFLT) should be used. The inverse transformation matrix is the transpose of the transformation matrix as the transform is orthogonal, so to compute a single level 2-D inverse discrete framelet transform using separable method, the next steps should be followed (Abdulkareem, 2012; Qasim, 2010):

- Let \( Y_{0} \) be \( \frac{3N}{2} \times \frac{3N}{2} \) framelet transformed matrix.
- Construct \( N \times \frac{3N}{2} \) reconstruction matrix, using transpose of transformation matrix, \([W]^{T}\).
- Reconstruction of columns: by applying matrix multiplication to the \( N \times \frac{3N}{2} \) reconstruction matrix by the \( \frac{3N}{2} \times \frac{3N}{2} \) framelet transformed matrix.

\[ YYY = [W]_{\frac{3N}{2} \times N}^{T} \times [Y_{0}]_{\frac{3N}{2} \times \frac{3N}{2}} \]  

- Reconstruction of rows: can be done as follows:
  - Transpose the column reconstructed matrix resulting from step (3).

\[ YYX = [YYX]^{T}_{\frac{3N}{2} \times N} \]  

- b) Apply matrix multiplication by multiplying the reconstruction matrix with the resultant transpose matrix.

\[ XX = [W]_{\frac{3N}{2} \times \frac{3N}{2}}^{T} \times [YYX]_{\frac{3N}{2} \times N} \]  

\[ X = XX^{T} \]

To compute a single level inverse framelet transform for 2-D signal using non-separable method, the next steps should be followed:

- Let \( Y_{0} \) be \( \frac{3N}{2} \times \frac{3N}{2} \) framelet transformed matrix.
- Construct \( N \times \frac{3N}{2} \) reconstruction matrix \( W^{T} \) using transformation matrix.
Reconstruction of the input matrix by multiplying the reconstruction matrix by the input matrix and by the transpose of the reconstruction matrix.

\[ X = W^T \times Y_0 \times W \]

A single-stage framelet transformation consists of nine frequency bands. These subbands are shown in Fig. 4. Since \( L \) is a low-pass filter \( h_0(n) \) while both \( H_1 \) and \( H_2 \) are high-pass filters \( (h_1(n) \) and \( h_2(n)) \), the \( H_1 \ H_1 \ H_1 \ H_2 \) subbands each has a frequency-domain support comparable to that of the \( HH \) subband in a DWT. A similar scheme creates the \( H_1 \ L, H_2 \ L, LH_1 \) and \( LH_2 \) subbands with the same frequency-domain support as the corresponding \( HL(LH) \) subbands of the DWT, but with twice as many coefficients. Finally, note that there is only one subbands \( LL \) with the same frequency-domain as the \( LL \) subbands in a DWT (Abdulkareem, 2012).

![Fig. 4. Matrix subbands after a single-level decomposition FLT.](image)

4. WALSH–HADAMARD TRANSFORM (WHT)
Discrete Fourier transform (DFT), discrete cosine transform (DCT), and Walsh–Hadamard transform (WHT) are widely used in the image processing applications. These linear image transforms are chosen in the image processing application because of their flexibility, energy compaction, and robustness. These transforms effectively extract the edges and also provide energy compaction in the state-of-the-art methods. Among all these transforms, WHT is very gorgeous one because of its simplicity and its computational efficiency. The major properties of WHT are same as that of other image transforms. The basis vector components of WHT are orthogonal, and it is having binary values \((\pm 1)\) only (Verma and Kadian, 2014).

The discrete Walsh-Hadamard transform matrices are rearrangements of discrete Hadamard matrices, which are of particular importance in coding theory. A Hadamard matrix of order \( N \) is defined as an \( N \times N \) matrix \( H \), with the property that \( HH^T = NI \), where \( I \) is the \( N \times N \) identity
matrix. Hadamard matrices whose dimensions are a power of two can be constructed in the following manner (Khalid Sayood, 2006):

\[ H_{2N} = \begin{bmatrix} H_N & H_N \\ H_N & -H_N \end{bmatrix} \]

with \( H_1 = [1] \). Therefore,

\[ H_2 = \begin{bmatrix} H_1 & H_1 \\ H_1 & -H_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \]

\[ H_4 = \begin{bmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \]

\[ H_8 = \begin{bmatrix} H_4 & H_4 \\ H_4 & -H_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 \end{bmatrix} \]

The DWHT transform matrix \( H \) can be obtained from the Hadamard matrix by multiplying it by a normalizing factor so that \( HH^T = I \) instead of \( NI \), and by reordering the rows in increasing sequency order. The sequency of a row is half the number of sign changes in that row. In \( H_8 \) the first row has sequency 0, the second row has sequency \( 7/2 \), the third row has sequency \( 3/2 \), and so on. Normalization involves multiplying the matrix by \( 1/\sqrt{N} \). Reordering the \( H_8 \) matrix in increasing sequency order, we get:

\[ H = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 \end{bmatrix} \]

Because the matrix without the scaling factor consists of \( \pm 1 \), the transform operation consists simply of addition and subtraction. For this reason, this transform is useful in situations where minimizing the amount of computations is very important.
5. PROPOSED ALGORITHM

Fig. 5 shows the flow chart of proposed algorithm:

![Flow chart of proposed algorithm](image)

In this work, 2-Level 2-Dframelet decomposition is applied on the original image, then apply Walsh transformation on low frequency domain and ignore all high frequency domains. Then apply arithmetic code. The decompressed image is reconstructed by applying inverse Walsh transform and inverse framelet transform.

6. COMPRESS AND DECOMPRESS COLOR IMAGES

This algorithm is proposed for compressing the color images. First the RGB colors images are converted into Y C_b C_r form, then applying proposed algorithm on each layer independently, this means each layer from Y C_b C_r are compressed as a gray scale image. Fig. 6 shows that proposed algorithm is applied on each Y C_b C_r layer.
Fig. 6: RGB Layers are Converted to YCrCb Layer, and then Compressed by proposed Algorithm.

For decompression color images, apply decompression on each layer then collect all layers in one matrix YCrCb and convert YCrCb format to RGB color image.

7. RESULTS AND DISCUSSION

Fig. 7 shown some gray scale images with proposed algorithm and the results in Tables 1, 2, 3 and 4 shown the performance of PSNR and CR compared with other algorithms

Table 1. Performance comparison between proposed algorithm and other algorithms for lena image.

| Algorithm                                      | CR     | PSNR    | Time (s) |
|------------------------------------------------|--------|---------|----------|
| Proposed algorithm using Walsh and discrete wavelet transform (db1) | 26.1542 | 26.0871 | 2.3585   |
| Proposed algorithm using Walsh and discrete wavelet transform (db3) | 22.826  | 29.1551 | 2.6486   |
| Proposed algorithm using Walsh and discrete wavelet transform (db5) | 23.4371 | 29.4685 | 2.657    |
| Proposed algorithm using Walsh and Framelet transform          | 26.0762 | 27.6329 | 2.6234   |
| Efficient image compression using artificial neural network (Verma and Kadian, 2014) | 1.75    | 26.87   | 81.42    |
Table 1 illustrates that the proposed algorithm using Walsh and framelet transform is the best choice for get higher compression ratio with good quality for image by compared with the other algorithms.
Table 2. PSNR and Time values for 26:1 compression for lena image in different algorithms.

| Algorithm                                           | PSNR     | Time (s) |
|-----------------------------------------------------|----------|----------|
| Hybrid DCT-VQ (Rehna and Kumar, 2011)               | 25.3996  | 43.35    |
| Modified Levenberg-Marquardt Method (Prena, 2011)   | 22.3675  | 2169.579 |
| Proposed algorithm using Walsh and Framelet transform | 27.6329  | 2.6234   |

From the results tabulated in Table 2, the time taken in proposed algorithm is lower than in other algorithms and gives higher PSNR.

Table 3. Performance comparison between proposed algorithms at barbara image.

| Algorithm                                           | CR       | PSNR     | Time (s) |
|-----------------------------------------------------|----------|----------|----------|
| Proposed algorithm using Walsh and discrete wavelet transform (db1) | 25.1216  | 21.5519  | 2.5467   |
| Proposed algorithm using Walsh and discrete wavelet transform (db3) | 21.8472  | 22.4649  | 2.8162   |
| Proposed algorithm using Walsh and discrete wavelet transform (db5) | 22.6553  | 22.7160  | 2.6952   |
| Proposed algorithm using Walsh and Framelet transform | 25.5850  | 21.9662  | 2.6177   |

Table 4. Performance comparison between proposed algorithms at eye image.

| Algorithm                                           | CR       | PSNR     | Time (s) |
|-----------------------------------------------------|----------|----------|----------|
| Proposed algorithm using Walsh and discrete wavelet transform (db1) | 31.1705  | 30.4274  | 2.2451   |
| Proposed algorithm using Walsh and discrete wavelet transform (db3) | 27.4181  | 32.0927  | 2.4667   |
| Proposed algorithm using Walsh and discrete wavelet transform (db5) | 28.4167  | 32.2037  | 2.4154   |
| Proposed algorithm using Walsh and Framelet transform | 31.8716  | 31.3748  | 2.3854   |

Table 5. Performance comparison between proposed algorithms at mandrill image.

| Algorithm                                           | CR       | PSNR     | Time (s) |
|-----------------------------------------------------|----------|----------|----------|
| Proposed algorithm using Walsh and discrete wavelet transform (db1) | 23.7859  | 19.1179  | 2.6967   |
| Proposed algorithm using Walsh and discrete wavelet transform (db3) | 21.0541  | 20.5527  | 2.9726   |
| Proposed algorithm using Walsh and discrete wavelet transform (db5) | 20.8348  | 19.9292  | 2.9293   |
| Proposed algorithm using Walsh and Framelet transform | 24.5937  | 19.2109  | 2.8851   |
The above comparison of Barbara & eye & mandrill images in Tables 3, 4 and 5, respectively, show that the proposed algorithm using Walsh and framelet transform gives the best results for compression ratio with maintaining good quality for images.

Finally testing the proposed algorithm on Lena colour image. The comparison between original and decompressed image is shown in Fig. 8:

![Original image Lena.bmp](image1.png) ![Decompressed Lena](image2.png)

*Fig. 8. Comparison between Original Colour Test Image and its Decompressed Image.*

Good image quality and good compression ratio for the color images are illustrate in Table 6 after applying proposed algorithm.

| Algorithm                                                                 | CR   | PSNR   | Time (s) |
|---------------------------------------------------------------------------|------|--------|----------|
| Proposed algorithm using Walsh and discrete wavelet transform (db1)       | 12.1737 | 27.0997 | 5.0602   |
| Proposed algorithm using Walsh and discrete wavelet transform (db3)       | 12.6186 | 29.4078 | 5.3341   |
| Proposed algorithm using Walsh and discrete wavelet transform (db5)       | 11.1592 | 29.7541 | 5.5396   |
| Proposed algorithm using Walsh and Framelet transform                     | 12.5288 | 28.4541 | 5.2383   |

8. CONCLUSION

The evaluation of performance using some objective criteria such as CR and PSNR show that the proposed algorithm based on WHT with FLT provides an important compression ratio while keeping a good quality of reconstructed image. A comparative study between the proposed algorithm and the algorithm with DWT. The properties of Framelet transform helped the algorithm to get good image quality, where the main reason for using Framelet transform is to reduce the image dimensions, while maintaining image quality. So, the high-frequencies coefficients are ignored.
9. REFERENCES

Moh’dAli Moustafa Alsayyh and Mohamad Dzulkifli, “Image Compression Using Hybrid Technique” Faculty of Computer Science and Information System, University Technology Malaysia, Skudai, Johor Bahru, Malaysia, Information and Knowledge Management, ISSN 2224-5758, Vol. 2, No.7, 2012.

Khalid Sayood, “Introduction to data Compression” University of Nebraska, 3rd Edition, 2006.

H. B. Kekre, Tanuja Sarode and Prachi Natu, “Color Image Compression using Hybrid Wavelet Transform with Haar as Base Transform”, Computer Engineering Department, MPSTME, NMIMS University, International Journal of Scientific and Research Publications, Vol. 4, Issue 6, 2014.

Runhai Jiao and Biying Lin, "A Digital Image Watermarking Method Based on Tight Framelet", International Conference on Web Information Systems and Mining, China, IEEE, 2010.

Khin Thida and Nang Aye, “Image Compression Based on Modified Walsh Hadamard Transform (MWHT)”, Department of Information Technology, Mandalay Technological University, The Republic of the Union of Myanmar, Proceedings of 3rd ISERD International Conference, Singapore, 2015.

K. Veeraswamy and S. Srinivaskumar, "Designing Quantization Table for Hadamard Transform based on Human Visual System for Image Compression" ECE Dept., JNTUCE, Kakinada, A.P, India. ICGST-GVIP Journal, Volume 7, Issue 3, 2017.

A. Moffat, et al, "Arithmetic Coding Revisited", ACM Transactions on Information Systems, Vol. 16, No. 3, pp. 256–294, 1998.

Prateek Kumar and Sandeep Kumar Agarwal, “A Color Image Denoising By Hybrid Filter for Mixed Noise” Department of Electronics Communication Engineering, Rustamji Institute of Technology, BSF Academy, Tekanpur, Gwalior (M. P.), India, Journal of Current Engineering and Technology, Vol. 5, No. 3, 2015.

Mauro Barni, "Document and Image Compression", Dekker Inc., 2004.

M. Choi, "Introduction of a Symmetric Tight Wavelet Frame to Image Fusion Methods Based on Substitutive Wavelet Decomposition", Korea Advanced Institute of Science and Technology, Korea, 2007.
Ziad Q. Abdulkareem, "Framelet-based OFDM System", M.Sc. Thesis, University of Technology, Baghdad, Iraq, 2012.

Wisam Qasim, "Optical Flow Estimation using Framelet Transform", M.Sc. Thesis University of Technology, Iraq, 2010.

Priyali Verma and Swapna Kadian, "Efficient Image Compression using Artificial Neural Network", department of electronics and communication, Jaypee University of Engineering & Technology, India, IJIACS, ISSN 2347-8616, Volume 3, Issue 1, April 2014.

Rehna. V. J. and Jeya Kumar.M.K. "Hybrid Approaches to Image Coding: A Review", Department of Electronics & Communication Engineering, Noorul Islam University, India, IJACSA, Vol. 2, No. 7, 2011.

Prena K. "A Study on Image Compression with Neural Networks using Modified Levenberg Maruardt Method", Global Journal of Computer Science and Technology, vol. 11, issue 3, 2011.