Flow and heat transfer characteristics of laminar mixed convection of water with sub-millimeter bubbles in a vertical channel

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Abstract. Laminar mixed-convection heat transfer is widely seen in compact heat exchangers. Injection of sub-millimeter bubbles is considered as one of the efficient techniques for enhancing laminar mixed-convection heat transfer for liquids. However, the effects of sub-millimeter-bubble injection on the laminar mixed-convection heat transfer are poorly understood. In this study, we experimentally investigate flow and heat transfer characteristics of the laminar mixed-convection of water with sub-millimeter bubbles in a vertical channel. The thermocouples and a PTV (Particle Tracking Velocimetry) technique are used for the temperature and velocity measurements, respectively. Tap water is used for working fluid and hydrogen bubbles generated by electrolysis of water are used as the sub-millimeter bubbles. The Reynolds number of the main flow ranges from 100 to 150. Our results show that the ratio of the heat transfer coefficient with sub-millimeter-bubble injection to that without injection decreases as the Reynolds number increases. It is found from the liquid velocity measurements that this decrease is mainly due to a decrease in the "bubble advection effect".

1. INTRODUCTION
The development of compact heat exchangers has been strongly desired with miniaturization of industrial equipment. In most cases of compact heat exchangers, the Reynolds number of the main flow is relatively low, so that the flow pattern and the classification of the convective heat transfer are respectively laminar flow and mixed convection. Therefore, detailed information on heat transfer characteristics and heat transfer enhancement techniques for laminar mixed-convection flows is required to develop high-performance compact heat exchangers.

So far, upward mixed-convection flows have been studied by a number of investigators (e.g., Oosthuizen and Hart, 1973; Gryzagoridis, 1975; Abdel-Wahed et al., 1976; Kitamura and Inagaki, 1987). In particular, Kitamura and Inagaki (1987) carried out experiments of the upward mixed convection for water along a vertical flat plate using thermocouples and a hot-film anemometer, and revealed transport mechanisms of heat and momentum for the upward mixed convection. In terms of the mixed-convection heat transfer enhancement for water, Celata et al. (1999) experimentally investigated effects of air injection on the mixed convection in a vertical pipe. In their study, a-few-millimeter bubbles were injected into the water flow. They showed that the heat transfer coefficient with air injection is up to 10 times higher than that without air injection under specific conditions. They also indicated that the heat transfer coefficient with air injection is higher than that using twisted-tape inserts at Re=1000–3000. These results clearly demonstrate that air injection is a highly efficient
technique for enhancing mixed-convection heat transfer. However, when air injection is applied to compact heat exchangers, it may happen that a-few-millimeter bubbles attach to the inside wall of a heated flow channel. This bubble attachment leads to a decrease in the heat transfer and an increase in the pressure drop, and consequently the performance of compact heat exchangers is reduced. One of the feasible solutions for this problem is to use bubbles whose diameters are smaller than a channel height (e.g., sub-millimeter bubbles), instead of a-few-millimeter bubbles. However, the flow and heat transfer characteristics of laminar mixed-convection flows with sub-millimeter bubbles are poorly understood.

The purpose of this study is to clarify flow and heat transfer characteristics of the laminar mixed convection of water with sub-millimeter bubbles in a vertical channel. The thermocouples and a PTV (Particle Tracking Velocimetry) technique are used for the temperature and velocity measurements, respectively.

2. EXPERIMENTAL SETUP

2.1 Apparatus

Figure 1 shows a schematic diagram of the apparatus. This apparatus consists of an upstream chamber, a vertical channel, a sub-millimeter-bubble generator, a vertical plate with uniform heat flux (vertical heated plate), a downstream chamber, a double-pipe heat exchanger, a low-temperature thermostatic bath (Asone: LTB-250) and a pump. The vertical channel made of transparent acrylic resin has dimensions of 2000 mm total length, 90 mm width and 8.6 mm height. Tap water of 20 ºC is used as the working fluid. The air temperature around the apparatus is kept at 20 ºC using air conditioning equipment. The x, y and z axes are, respectively, set along the streamwise, wall-normal and spanwise directions. Here, \( x=0 \), \( y=0 \) and \( z=0 \) are set at the starting point of the heating section, at the surface of the heated plate and at the center of the heated plate in the spanwise direction, respectively.

![Fig. 1 Schematic diagram of the apparatus](image)

2.2 Sub-Millimeter-Bubble Generator

In this study, we use hydrogen bubbles generated by electrolysis of water as the sub-millimeter bubbles. This bubble generation method using the electrolysis of water has two advantages. One is that the bubbles are generated in the vicinity of the vertical heated plate. The other is that the bubble volumetric flow rate is controlled simply but precisely. The bubble flow rate \( Q \) is defined as

\[
Q = \frac{1R_0T_v}{2F_p}, \quad (1)
\]
where $I$ is the electrical current through the electrodes, $R_0$ is the universal gas constant ($R_0=8.31\times10^9$ Pa·mm³/(K·mol)), $T_\infty$ is the temperature of the ambient liquid ($T_\infty=293$ K), $F$ is the Faraday constant ($F=9.65\times10^4$ A·s/mol) and $p$ is the pressure ($p=1.11\times10^5$ Pa).

A schematic diagram of the sub-millimeter-bubble generator is shown in Fig. 2. This bubble generator consists of 5-mm thick transparent acrylic resin plates, a 1-mm thick stainless-steel plate, a gold wire (0.1 mm in diameter and 55 mm long) and a carbon rod (15 mm in diameter and 60 mm long) and a charcoal absorber. The gold wire and the carbon rod are used as the cathode and the anode, respectively. The chlorine bubbles generated at the anode are completely removed using the charcoal absorber. The surface of the stainless-steel plate is sandpapered to enhance its hydrophilicity. As a result, the hydrogen bubbles generated at the cathode rise smoothly along this plate. The hydrogen bubbles are injected through a slot of this generator into the vertical channel. The slot is located at 996 mm downstream from the inlet of the vertical channel and the width of the slot is approximately 4 mm. The generation of the bubbles is performed from 390 to 610 s after the heating of the vertical plate starts.

**Fig. 2** Schematic diagram of the sub-millimeter-bubble generator

### 2.3 Vertical Heated Plate

Figure 3 shows a schematic diagram of the vertical heated plate. The stainless-steel foils (110 mm long, 25 mm wide, 0.02 mm thick) are attached to acrylic resin plate 1 (1000 mm high, 90 mm wide, 10 mm thick) at 0.5 mm intervals to electrically heat the plate under uniform heat flux conditions. The foils are connected in series with copper wires and are heated using the DC power supply. The connecting parts of the foils are on the back of the plate to minimize the disturbance to liquid flow. Acrylic resin plate 1 is entirely covered with 25-µm thick Kapton tape for electrical insulation. The air between acrylic resin plate 1 and 8-mm thick acrylic resin plate 3 reduces heat leakage from behind the heated plate. The heat loss through the back of the heated plate is estimated to be less than 2.5 % of the wall heat flux. To measure the surface temperature on the heated plate $T_W$, thermocouples are...

**Fig. 3** Schematic diagram of the vertical heated plate
inserted into 3.0 mm diameter holes on the back of the heated plate and the tips, which are covered with Kapton tape, are contact with the back surface of the foils. All the holes are completely filled with silicone sealant to prevent water from flowing into them. The leading edge of the heated plate is located 1000 mm downstream from the inlet of the vertical channel. The starting point of the heating section, namely, $x=0$ is located 50 mm downstream from the leading edge of the plate. The heating of the plate is performed in the region of $0 < x < 500$ mm. It is noted that the heated plate is flush with the vertical acrylic plate which is set upstream of the heated plate (see Fig. 2).

### 2.4 Double-Pipe Heat Exchanger

In our experiments, the liquid temperature in the upstream chamber is kept constant (20 ºC) using the double-pipe heat exchanger. A schematic diagram of the double-pipe heat exchanger is shown in Fig. 4. This heat exchanger consists of two pipes. The inner pipe made of aluminum is 40 mm i.d., 1500 mm total length and 1 mm thick, while the outer pipe made of transparent acrylic resin is 70 mm i.d., 1500 mm total length and 3 mm thick. Cool water, whose temperature is controlled by a sensor, is introduced into the outer pipe from the low-temperature thermostatic bath. The heat exchange is made between the warm water flowing in the inner pipe and the cool water flowing in the annulus.

![Fig. 4 Schematic diagram of the double-pipe heat exchanger](image)

### 2.5 Experimental Conditions

Table 1 shows the experimental conditions for the temperature and velocity measurements. In this table, $Re$ is the Reynolds number of the working fluid and is given by

$$Re = \frac{UH}{\nu}, \quad (2)$$

where $U$ is the streamwise mean velocity of the liquid in the cross-section at $z=0$ mm, $H$ is the height of the vertical channel and $\nu$ is the kinematic viscosity of the liquid. In this table, $q_w$ and $Q$ are the wall heat flux and the bubble flow rate, respectively.

| Temperature measurement | $Re$ = 100, 125, 150 |
|-------------------------|----------------------|
| Bubble flow rate        | $Q = 40$ mm$^3$/s    |
| Wall heat flux           | $q_w = 1480$ W/m$^2$ |
| Measurement position    | $x = 70, 170, 270$ mm, $z=0$ mm |

| Velocity measurement   | $Re$ = 100, 125, 150 |
|------------------------|----------------------|
| Bubble flow rate       | $Q = 40$ mm$^3$/s    |
| Wall heat flux          | $q_w = 0, 1480$ W/m$^2$ |
| Measurement position   | $x = 270$ mm, $z=0$ mm |

### 3. TEMPERATURE MEASUREMENT TECHNIQUE

#### 3.1 Temperature Measurement System

The temperature measurement system consists of K-type 100 µm thermocouples, a reference junction and a high-speed data acquisition unit (Yokogawa, MX100). The thermocouples, which are
accurate within ±0.12 °C, are used to simultaneously measure the surface temperature of the heated plate $T_w$ and the liquid temperature in the upstream chamber $T_{\infty}$. The thermocouples for measuring $T_w$ are set at $x=70, 170, 270$ mm and $z=0$ mm. The high-speed data acquisition unit is directly connected to a personal computer in order to record the thermoelectric force of the thermocouples in real time. A thermostatic bath, which is accurate within ±0.10 °C, is used for calibrating the thermocouple temperature readings. The sampling frequency for the temperature measurement is 5 Hz and the measurement period is 610 s.

### 3.2 Estimation of Local Heat Transfer Coefficient

The local heat transfer coefficient $h_x$ at each of the measurement positions is obtained from Eq. (3).

$$h_x = \frac{q_w}{T_w - T_{\infty}}, \quad (3)$$

where $q_w$ is estimated from Eq. (4).

$$q_w = \frac{\rho' I^2}{S A_{st}}. \quad (4)$$

In Eq. (4), $S$, $l$, $\rho'$ and $A_{st}$ are the cross-section area, total length, specific electrical resistance and overall surface area of the foil, respectively. The specific electrical resistance of each foil is $7.4 \times 10^{-7}$ Ω·m. The total uncertainty in the heat transfer coefficient with respect to both the thermocouple accuracy and the heat loss through the back of the heated plate is approximately 6%.

### 4. VELOCITY MEASUREMENT TECHNIQUE

#### 4.1 Velocity Measurement System

In this study, we use the PTV technique to conduct the liquid velocity measurements (e.g., Kitagawa et al., 2005; Murai et al., 2006). We also use a shadow image technique to obtain the bubble interface clearly (e.g., Tokuhiro et al., 1998).

Figure 5 shows a schematic diagram of the velocity measurement system. A digital color CCD camera (Imperx, VGA210-LC), which is set in front of the vertical channel, is used to capture the tracer particle and bubble images. Particles of 60 µm in mean diameter and 1.02 in specific gravity are used as the tracer particles and the particle concentration is approximately 100 ppm. A halogen-light source (Moritex, MHAB-150W) and blue LED arrays (Imac: $\lambda=470$ nm) are used for illuminating the tracer particle and bubble, respectively. The former is set at the side of the vertical channel, while the latter is set behind the vertical channel. A red light sheet with a thickness of 3 mm is produced with a line light guide, a colored glass filter, a red-pass filter and a slit. The measurement area is $264<\langle x \langle 276$ mm, $0<\langle y \langle 16$ mm and $-1.5<z<1.5$ mm. Images of 600×480 pixels, which are taken with the color CCD camera, are directly recorded by a personal computer. The spatial resolution is 0.027 mm/pixel. The

![Fig. 5 Schematic diagram of the velocity measurement system](image-url)
exposure time of the color CCD camera is 0.0005 s and the frame rate is 65 to 110 fps depending on
the different experimental conditions.

4.2 Estimation of Bubble Diameter
The diameter of bubbles is calculated from the equivalent area of each bubble image taken by
the CCD camera. Each bubble image is binarized by a binary labeling method (e.g., Yamamoto et al.,
1996). When the bubble image overlaps with other bubble images, its image is eliminated considering
the aspect ratio of the bubble. The uncertainty in the bubble equivalent diameter associated with
the binary labeling method is estimated to be approximately 0.4 % of the mean bubble diameter.

4.3 Estimation of Liquid Velocity Vector
The estimation procedure for the liquid velocity vector is described below.
(1) The location of the tracer particle centroid is estimated from the red image using a particle mask
correlation method, which was proposed by Takehara and Etoh (1999).
(2) Some of the particle locations obtained in step (1) are detected from light scattered at the bubble
interface and the surfaces of the particles that overlap with bubble images. The former depends
strongly on the bubble interface motion while the latter is affected by reflection or refraction at the
bubble interface. Therefore, both cause errors in the liquid velocity estimation. These particle
locations are completely eliminated using binarized bubble images.
(3) The liquid velocity vector is estimated using the velocity gradient tensor method (Ishikawa et al.,
2000) in which the velocity gradient tensor is considered in order to obtain the velocity vectors
with high accuracy.

When estimating the liquid velocity without bubbles (i.e., single-phase flow), steps (2) is omitted. The
uncertainty in the liquid velocity associated with the particle centroid detection is estimated to be 0.83
mm/s. Hence, the error in this value relative to the maximum liquid rise velocity (25.8 mm/s) is 3.2 %.

5. RESULTS AND DISCUSSION

5.1 Flow and Heat Transfer Characteristics without Sub-Millimeter-Bubble Injection

5.1.1 Liquid Mean Velocity
Figure 6 (a)–(c) shows the profiles of the liquid mean velocity at different Reynolds numbers. In
this figure, $u$ is the liquid mean velocity in the streamwise direction. The vertical axis is normalized by
$u_{0,max}$ which is the maximum value of the streamwise liquid mean velocity without bubble injection
and heating. For comparison, the profile of the Poiseuille flow is added to the figure. At each Reynolds
number, the experimental result without heating is in agreement with the profile of the Poiseuille flow.
The maximum difference between the two is approximately 4 %. Moreover, at each Reynolds number,
the liquid mean velocity in the vicinity of the heated wall is increased by heating. It is clear from this

![Profile of liquid mean velocity at different Reynolds numbers](image-url)
result that the flow with heating is the mixed-convection flow.

5.1.2 Local Heat Transfer Coefficient

Figure 7 shows the relationship between the Reynolds number and the local heat transfer coefficient at different measurement positions. At each measurement position, the local heat transfer coefficient increases with the Reynolds number. This originates from an increase in the streamwise liquid mean velocity near the heated wall, which is shown in Fig. 6. Moreover, at each Reynolds number, the local heat transfer coefficient decreases with an increase in the $x$. This is because the liquid flowing downstream is gradually heated by the heated plate. In Section 5.2.1, the local heat transfer coefficient obtained here is used as $h_{x0}$.

![Fig. 7 Relationship between Reynolds number and local heat transfer coefficient at different measurement positions](image)

5.2 Flow and Heat Transfer Characteristics with Sub-Millimeter-Bubble Injection

5.2.1 Heat Transfer Coefficient Ratio

Figure 8 shows the relationship between the Reynolds number and the heat transfer coefficient ratio $h_x/h_{x0}$ at different measurement positions. The heat transfer coefficient ratio ranges from 1.23 to 1.35. This clearly demonstrates that sub-millimeter-bubble injection enhances laminar mixed-convection heat transfer for water. In addition, the heat transfer coefficient ratio decreases as the Reynolds number increases. This means that sub-millimeter-bubble injection for the laminar mixed convection is effective under relatively lower Reynolds number conditions. Furthermore, the heat transfer coefficient ratio decreases with an increase in the $x$. This tendency was also seen in our previous paper regarding laminar natural convection flow with sub-millimeter bubbles (Kitagawa et al, 2008).

![Fig. 8 Relationship between Reynolds number and $h_x/h_{x0}$ at different measurement positions.](image)
5.2.2 Heat Transfer Gain

Figure 9 shows the relationship between the Reynolds number and the heat transfer gain $\eta$. The heat transfer gain is the ratio of the heat transfer rate obtained by the sub-millimeter-bubble injection to the power consumption of the sub-millimeter-bubble generation $P$ and is defined as

$$\eta = \frac{d_w A_{H} \left( \frac{h_x}{h_{x0}} - 1 \right)}{P}, \quad (5)$$

where $A_{H}$ is the surface area of the heated wall and is set to 500 mm×90 mm for convenience. $h_x/h_{x0}$ is obtained by averaging $h_x/h_{x0}$ at three measurement positions. At each Reynolds number, the heat transfer gain is much higher than 1, and the maximum value of the heat transfer gain is approximately 2.65. It is therefore considered that injection of sub-millimeter bubbles is a promising technique for enhancing laminar mixed-convection heat transfer for water.

![Fig. 9 Relationship between Reynolds number and heat transfer gain](image)

5.2.3 Bubble Diameter

Figure 10 shows the probability distribution of the bubble equivalent diameter $d$ at different Reynolds numbers. In this figure, $n$ is the number of bubbles for each equivalent diameter, and $N$ is the total of $n$. At each Reynolds number, the bubble diameter ranges from 0.3 to 0.8 mm. Additionally, at each Reynolds number, the probability distribution has a peak at $d=0.52$ mm. Therefore, the bubble diameter is independent of the Reynolds number.

![Fig. 10 Probability distribution of bubble equivalent diameter at different Reynolds numbers](image)

5.2.4 Bubble Projection Area Ratio

When many bubbles concentrate near the wall, overlaps of bubble images taken by a camera occur significantly. In the image processing, this overlap leads to a decrease in the number of bubbles detected near the wall. Therefore, in this study, the bubble projection area ratio is used to discuss the probability distribution of the bubble location and is defined as
where $A$ is the projection area and the subscription $L$ and $G$ are, respectively, the liquid and the bubble. Figure 11 shows the profiles of the bubble projection area ratio at different Reynolds numbers. The vertical axis is normalized by $A_{p,\text{sum}}$ (sum of all $A_p$). Each datum is averaged over a 0.2 mm interval. At each Reynolds number, bubbles are located in the region of $y/H=0-0.2$. That is, the dependency of the bubble location on the Reynolds number is weak.

5.2.5 Liquid Mean Velocity

Figure 12 shows the profiles of the liquid mean velocity in the streamwise direction at different Reynolds numbers. In this figure, the results for more than 1000 samples are presented. Here, a major reason for the lack of data in the region of $y/H=0-0.08$ is that in the liquid velocity measurements, the intensity of the red light sheet decreases remarkably because of bubble concentration. At all the Reynolds numbers, the liquid mean velocities with sub-millimeter-bubble injection are higher than those without injection. In particular, the increase in the liquid mean velocity is significant near the heated wall. This is deeply related to the high bubble concentration near the heated wall. The increase in the liquid mean velocity leads to advection in the warm liquid near the heated wall and consequently contributes heat transfer enhancement. In the following, this is called the "bubble advection effect".

Figure 13 shows the relationship between the Reynolds number and the rate of increase in the maximum liquid velocity. In this figure, $u_{\text{max,T}}$ and $u_{\text{max,S}}$ are, respectively, the maximum liquid mean velocity with and without bubble injection, which are obtained from Fig. 12. The rate of increase in the maximum liquid velocity becomes lower as the Reynolds number increases. This result means that...
the "bubble advection effect" decreases with an increase in the Reynolds number.

5.2.6 RMS Values of Liquid Fluctuation Velocities

Figure 14 shows the profiles of the RMS values of the liquid fluctuation velocities at different Reynolds numbers. In this figure, $u_{\text{rms}}$ and $v_{\text{rms}}$ are the RMS values of the liquid fluctuation velocities in the streamwise and wall-normal directions, respectively. In this figure, the results for more than 1000 samples are presented, similar to Fig. 12. At all Reynolds numbers, the RMS values of the liquid fluctuation velocities in both directions are increased by the bubble injection. Particularly, the increase in the RMS values near the heated wall is significant. This is the same tendency as the profile of the liquid mean velocity. This increase near the heated wall activates the mixing of warm and cool liquids and as a result, contributes heat transfer enhancement directly. Hereafter, this is called the "bubble mixing effect".

Figure 15 shows the relationship between the Reynolds number and the RMS values of the liquid fluctuation velocities near the heated wall (which are obtained from Fig. 14). In this figure, $u_{\text{rms,av}}$ and $v_{\text{rms,av}}$ are obtained by averaging $u_{\text{rms}}$ and $v_{\text{rms}}$ over the range of $y/H=0.08-0.2$, respectively. $u_{\text{rms,av}}$ and $v_{\text{rms,av}}$ tend to increase as the Reynolds number increases. The tendency of increase in the $v_{\text{rms,av}}$ with increasing Reynolds number is clear compared with that in the $u_{\text{rms,av}}$. These results mean that the "bubble mixing effect" increases with the Reynolds number.

From Figs. 13 and 15, it was clear that the "bubble mixing effect" increases but the "bubble advection effect" decreases as the Reynolds number increases. Therefore, we conclude that the decrease in the heat transfer coefficient ratio with increasing Reynolds number, shown in Fig. 8, is mainly due to a decrease in the "bubble advection effect".
6. CONCLUSIONS

We experimentally investigated flow and heat transfer characteristics of the laminar mixed convection of water with sub-millimeter bubbles in a vertical channel. The Reynolds number of the main flow ranges from 100 to 150. The conclusions are as follows.

1. The ratio of the heat transfer coefficient with sub-millimeter-bubble injection to that without injection (i.e., the heat transfer coefficient ratio) ranges from 1.23 to 1.35. Moreover, at each Reynolds number, the heat transfer gain is much higher than 1, and the maximum value of the heat transfer gain is approximately 2.65. It is therefore considered that injection of sub-millimeter bubbles is a promising technique for enhancing laminar mixed-convection heat transfer for water.

2. The heat transfer coefficient ratio decreases with as the Reynolds number increases. This means that sub-millimeter-bubble injection for the laminar mixed convection is effective under relatively lower Reynolds number conditions. According to the liquid velocity measurements, the "bubble mixing effect" increases but the "bubble advection effect" decreases as the Reynolds number increases. Therefore, we conclude that the decrease in the heat transfer coefficient ratio with increasing Reynolds number is mainly due to a decrease in the "bubble advection effect".

NOMENCLATURE

- \( A_{H} \) surface area of heated plate
- \( A_{st} \) overall surface area of stainless-steel foil
- \( d \) bubble equivalent diameter
- \( F \) Faraday constant
- \( h_{x} \) heat transfer coefficient with bubble injection
- \( h_{x0} \) heat transfer coefficient without bubble injection
- \( H \) channel height
- \( I \) electrical current
- \( l \) total length of stainless-steel foil
- \( n \) number of bubbles
- \( N \) sum of all \( n \)
- \( p \) pressure
- \( P \) power consumption of bubble generation
- \( Q \) bubble flow rate
- \( q_w \) wall heat flux
- \( R_{0} \) universal gas constant
- \( Re \) Reynolds number
- \( S \) cross-section area of stainless-steel foil

Fig. 15 Relationship between Reynolds number and RMS values of liquid fluctuation velocities near the heated wall
\( T \)  \text{ temperature} \\
\( u \)  \text{ liquid mean velocity in the streamwise direction} \\
\( u_{0,\text{max}} \)  \text{ maximum value of streamwise liquid mean velocity without bubble injection and heating} \\
\( U \)  \text{ streamwise mean velocity of liquid in the cross-section at } z=0 \text{ mm} \\
\( v \)  \text{ liquid mean velocity in the wall-normal direction} \\
\( x, y, z \)  \text{ streamwise, wall-normal and spanwise directions} \\

\textbf{Greek Letters} \\
\( \eta \)  \text{ heat transfer gain} \\
\( \nu \)  \text{ kinematic viscosity} \\
\( \rho' \)  \text{ specific electrical resistance of stainless-steel foil} \\

\textbf{Subscripts} \\
\( G \)  \text{ bubble} \\
\( L \)  \text{ liquid} \\
\( w \)  \text{ wall condition} \\
\( \infty \)  \text{ ambient condition} \\

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