Sensitivity study and structural optimization of an aircraft wing in the maximum-stress and flutter constraints

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Abstract. Optimization process is very important to be done since the preliminary design step, not only for the static load aspect but also dynamic load as well. In the past, aeroelastic behavior aspect is often neglected in this stage and consequently becomes a problem at the later stage of design. This paper contains multi-constraints design sensitivity and optimization which include both maximum stress constraint and damping flutter constraint. Sensitivity study was performed to assist optimization process and makes the process more efficient. Wing structure was modeled as skins, ribs, spar-webs, and spar-caps. Optimization process was done by changing thickness of each component and beam dimension. The methodology used in this paper is a gradient-based optimization developed by MSC Nastran. The final cycle of this optimization has successfully reduced structural weight up to 33% without violating static and flutter constraint.

Keywords: sensitivity study, structural optimization, flutter constraint

1. Introduction

In the design of aerospace structures, it is important to make a structure to be save and as light as possible. A lightweight structure is not only beneficial for a structural aspect but also for other aspects such aerodynamic, cost and aircraft operations. Optimization is needed since the preliminary design stage in order to make an efficient configuration of structure that is as light as possible while satisfy safety requirement.

In order to conduct optimization process for an aircraft structures, a designer should not think for only one aspect, for example, only to satisfy aerelastic requirement, or static requirement; since each aspect may produce different optimization result. In the past, aeroelastic aspect often neglected in the preliminary design stage. An aircraft designer just concentrates on how to make the aircraft safe from static loading. As a result, for several cases, aeroelastic behavior becomes a problem later. In order to correct this problem, engineers had to make significant additional effort to find feasible solutions. Those solutions are usually non-optional, expensive repair solutions, which must be introduced fairly late in the design process.

An optimization problem consists of maximizing or minimizing a real function by systematically choosing input values from within an allowed set and computing the value of the function. More
generally, optimization includes finding “best available” values of some objective function given a defined domain (or a set of constraints), including a variety of different types of objectives functions and different types domains. A basic goal of design optimization is to automate the design process by using a rational, mathematical approach to yield improved designs. It is useful to produce more efficient designs while having maximum margins of safety, to perform trade-off or feasibility studies, to assist in design sensitivities studies, or to correlate test data and analysis results (model matching).

Optimization studies have been around since a long time. A lot of academic publications have been conducted to get better optimization method. Multidisciplinary objective for design and optimization has been considered to get a comprehensive and accurate result considering many aspects as constraints [1].

It has been presented elsewhere about the development of the structural optimization that begins with the first analytical work in structural optimization [1,2]. Also, other provides theoretical lower bounds on the weight of trusses and although highly idealized, offer considerable insight into the structural optimization problem and the design process [2]. The 1940s and early 1950s is the era of development of component optimization, such as the works of Shanley [3]. During this period, the availability of the digital computer led to the application of linear programming techniques for plastic design of frames. Digital computers were becoming commonly available, and the finite element method was offering a designer a tool for analysis of increasingly complex structures.

Nowadays, MSC Nastran [4] has developed optimization program using gradient-based method to answer structural designer expectation. Nastran SOL 200 has provided optimization code for many applications. This research will study optimization of wing structures for static strength and flutter constraints. This research focus on optimizing a wing structure to make it as-light-as-possible while satisfying static strength and flutter requirements using gradient-based optimization method.

2. Wing-box model

Figure 1 shows wing-box model studied in this paper. It contains skins, spar (spar-web and spar-cap), and ribs. The engine is modeled as a concentrated mass and is positioned in its center of gravity. Fuel is modeled as distributed mass that attached in 24 nodes connected to spar.

In a finite element model, skins, spar-webs and ribs are modeled as shell properties, while spar-caps are modeled as beam properties. The material for all the components is Al 2024-T3. Table 1 shows material properties of Al 2024-T3. For convenient, skins are divided into 5 areas as shown in figure 2; ribs into 4 areas as shown in figure 3 and spar-web is just divided into front and rear spar-web as shown in figure 4. Upper spar-caps are modeled as ‘T’ beam as shown in figure 5 (left) and lower spar-caps are modeled as inverted ‘T’ beam.

![Figure 1. Finite Element Model](image1.png) ![Figure 2. Skin Properties](image2.png)

| Table 1. Material Properties |
|-----------------------------|
| **Al 2024-T3**              |
| Density 2.78 g/cc           |
| Modulus of elasticity 73.1 GPa |
| Poisson’s ratio 0.33        |
| Ultimate tensile strength 469 Mpa |
3. Design Variables and Constraints
The objective of this research is to conduct optimization process in order to minimize the weight of the wing structure while satisfying safety requirement. Optimization process is done by changing thickness of skins, spar-webs, and rib. For the case of spar-caps, optimization process is done by changing the beam properties as shown in figure 5; except the length of the beam. Table 2 consists of initial value, minimum values and maximum values of optimization process for each variable.

There are two constraints in this research. The first one is the stress constraints. Al-2024 has an ultimate tensile strength of 469 MPa; this value become stress constraint to limit the stress value. The second constraint is flutter. In order to avoid flutter occurrence in certain speed, the damping of the structures should be 0.3 at 500 m/s.

| Table 2. Design variable |
|---------------------------|

| Variabel                  | Initial value (mm) | Min value (mm) | Max value (mm) |
|---------------------------|--------------------|----------------|----------------|
| Skin 1/2/3/4/5            | 4.5                | 0.8            | 10             |
| Ribs 1                    | 2.2                | 1              | 10             |
| Ribs 2                    | 2                  | 1              | 10             |
| Ribs 3                    | 1.5                | 1              | 10             |
| Ribs Bulk                 | 3                  | 1              | 10             |
| Spar-web                  |                    |                |                |
| Front                     | 8.5                | 1              | 10             |
| Rear                      | 5.6                | 1              | 10             |
| Upper Spar-cap (Front/Rear)|                  |                |                |
| H                         | 20                 | 10             | 20             |
| W                         | 20                 | 10             | 20             |
| T1                        | 5                  | 5              | 10             |
| T2                        | 5                  | 5              | 10             |
| Lower Spar-cap (Front/Rear)|                  |                |                |
| H                         | 20                 | 10             | 20             |
| W                         | 20                 | 10             | 20             |
| T1                        | 5                  | 5              | 10             |
| T2                        | 5                  | 5              | 10             |

4. Optimization Process
4.1 Basic Optimization Problem
Nastran uses basic optimization problem statement to optimize the structure. A basic optimization problem statement is defined as follows:
Find X to minimize (or maximize) \( F(X) \) objective, subjected to Eq. 1 – 4.

\[
\begin{align*}
  g_j(X) & \leq 0 & j = 1, \ldots, n_g & \text{inequality constraints} \\
  h_k(0) & = 0 & k = 1, \ldots, n_h & \text{equality constraints} \\
  x_i^l & \leq x_i \leq x_i^u & i = 1, \ldots, n & \text{side constraints} \\
  X & = \{x_1, x_2, \ldots, x_n\} & \text{design variables}
\end{align*}
\]

The objective function is a scalar quantity to be minimized. Side constraints are placed on design variables to limit the region of search. The inequality constraints are expressed in a less than or equal to zero form by convention. Equality constraints if present must be satisfied exactly at optimal design.

### 4.2 Methodology

In general, Nastran’s methodology can be summarized in figure 6.

**Figure 6. Nastran’s Optimization Methodology**

It begins with initial design that was used in the first analysis. In the structural analysis step, it can perform several analyses. In this paper, structural analysis consists of not only static load analysis but also flutter analysis. This paper uses two constraints; damping constraint for flutter and stress constraint for static load. Constraint screening is used to identify which constraint that is likely to drive the redesign process. If the result of analysis has not satisfied convergent criteria, a sensitivity analysis must be performed together with constraint screen result to make approximate model. Optimization is done with approximation model to improve efficiency. Based on the result from the optimizer, the design is updated. The cycle keeps going on until convergent criteria was satisfied.

### 4.3 Nastran Optimization

The optimization algorithms in MSC Nastran belong to the family of methods generally referred to as “gradient-based”. This is because, in addition to function values, they use function gradients to assist in the numerical search for an optimum. To begin the optimization process, the direction of the search must be known. In general, the gradient of our objective function should be known as well as some of the constraint functions. First, forward finite difference approximation of a derivative is used to determine the gradient. For a single independent variable, the first-forward difference is given by Eq. 5.
For most practical design tasks, a vector of design variables, the resultant vector of partial derivatives, or gradient of the function $\nabla F(X)$ should be the main concerns. Note that this requires a number of steps in this given direction, which is equivalent to a number of function evaluations in numerical optimization for a search direction $S$ and a vector of design variables $X$, the new design at the conclusion of our search in this direction can be written as in Eq. 6.

$$X^1 = X^0 + \alpha^* S^1$$

This relation allows updating a potentially huge number of design variables by varying the single parameter $\alpha$. The new objective and constraints can now be expressed as in Eq. 7 and Eq. 8.

$$F^1 = F(X^0 + \alpha^* S^1)$$

$$g^1_j = g_j(X^0 + \alpha^* S^1) \quad j = 1, ..., n_g$$

From this new point in the design space, the new gradients can be computed and establish another search direction based on this information. The process is repeated until the objectives are met.

### 4.4 Sensitivities Study

Design sensitivity coefficients are defined as the rate of change of a particular response quantity $r$ with respect to a change in a design variable $x$ or $\partial r / \partial x$. These coefficients are evaluated at a design characterized by the vector of design variables, giving in Eq. 9:

$$\lambda_{ij} = \left. \frac{\partial r_j}{\partial x_i} \right|_{x^0}$$

Static response stress or strain are determined from the displacement solution. For an arbitrary response, $r$, the functional dependency on the displacement is written as in Eq. 10:

$$r = r(U, X)$$

The sensitivities of the responses with respect to changes in the design variables are approximated using first forward finite differences as below to obtain the required sensitivities (Eq. 11)

$$\frac{\partial r_j}{\partial x_i} \approx \frac{\Delta r_j}{\Delta x_i} = \frac{r_j(X^0 + \Delta x_i U + \Delta U) - r_j(X^0, U^0)}{\Delta x_i}$$

Weight sensitivities are calculated using matrix $[\Delta M]$ and matrix $[DR_g]$. The latter matrix provides the displacement at all the degrees of freedom in the model based on unit displacement at the six degrees-of-freedom, Eq. 12:

$$[\Delta W] = [DR_g]^T [\Delta M] [DR_g]$$

Flutter sensitivities analysis begin with flutter equation given by Eq. 13:
Flutter sensitivity computes the rates of change of the transient decay rate coefficient \( \gamma \) with respect to changes in the design variables. \( \gamma \) is defined in connection with the complex eigenvalue \( p \), by Eq. 14:

\[
p = \omega (\gamma + i) = p_R + p_I
\]

The sensitivity calculation for \( \Delta \gamma \) is expressed in terms of the sensitivities of the eigenvalue as in equation 15:

\[
\frac{\partial \gamma}{\partial x} = \frac{1}{p_1} \left( \frac{\partial p_R}{\partial x} - \frac{\gamma \partial p_I}{\partial x} \right)
\]

5 Result and Discussion

5.1 Sensitivities Study

In order to make optimization process more efficient, Nastran does sensitivities study first. Equation 9 gives gradient equation for each variable and response. In this paper, sensitivities study was done in eight elements in the root area that give critical results. Each element gives different sensitivity coefficient for every response and design variable. For every unit dimension of design variable, the response will change according to sensitivity coefficients.

Figure 7 shows sensitivity coefficients with respect to weight response. It shows that skin 1 mostly gives highest sensitivity coefficient. Figure 8 also shows that skin 1 gives highest sensitivity coefficients for almost every point according to maximum stress response. It makes sense, because skin 1 has the widest area, therefore, the weight will change significantly with the change of skin thickness. Skin 1 also placed in root area where the stresses are maximum. Therefore, skin 1 affects maximum stress significantly. Sensitivity coefficient for different response has different value. Nastran picks the most significant value to determine the direction of the search. Later it can be seen that this sensitivity will strongly decide the design optimization.

Figure 7. Weight Sensitivities

Figure 8. Maximum Stress Sensitivities
Figure 9 shows sensitivity coefficients according to flutter response. According to weight and maximum stress response, the most significant coefficient is skin1, followed by front spar-web. According to flutter sensitivities study, the most significant variables are skins not only for skin 1 but also skin 4 and skin 5. Although skin 4 and skin 5 sensitivities coefficients are large enough, but if it is compared on the value of flutter response coefficients and weight or maximum stress response coefficients, flutter sensitivities has far smaller value than maximum stress or weight sensitivities so that maximum stress constraint affects the design variable more significantly.

![Flutter Sensitivities](image)

**Figure 9. Flutter Sensitivities**

### 5.2 Design Variable Histories

From design sensitivities it was found that skin 1 has largest sensitivity coefficient. From Figure 10, skin 1 is the thickest compared with other skins.

![Skin Thickness History](image)

**Figure 10. Skin Thickness History**

![Spar-web Thickness History](image)

**Figure 11. Spar-web Thickness History**

Figure 8 also shows that front spar web has highest sensitivity coefficient after skin1. Therefore, in figure 11 for the first design cycle, front spar-web gradient is positive.
Sensitivity coefficients for ribs are small compared to other design variable. Therefore, all rib’s thickness is kept to minimum except for ribs bulk. Although the direction kept decreasing, the thickness of ribs bulk still the largest between other ribs. It is rational, since ribs bulk is the place to mount the engine.

Figure 12 and 13 shows ribs thickness and spar-caps inertia history. From figure 13 we know that rear spar-cap gives higher inertia than front spar-cap.

5.3 Weight Objective
Figure 14 shows weight history from the initial design until final design. It shows that sensitivities study is very important. It makes optimization process becomes far more efficient. Since the initial design, design sensitivities had determined the direction of the search, then optimization process continued until reach convergence criteria. For only one cycle, reduction of weight had reached 191 kg or 25% of the structural weight. For the final design, optimization process had reduced 255 kg or 33% of the structural weight.

| Table 3. Initial-Final Constraints Comparison |
|---------------------------------------------|
| Initial | Final | Constraint |
| Flutter speed (m/s) | 850   | 500   | 500   |
| Max stress (Pa)   | 1.78E+05 | 2.93E+08 | 3.13+08 |

**Figure 12.** Ribs Thickness History  
**Figure 13.** Spar-cap Inertia History  
**Figure 14.** Weight History
Table 3 shows comparison between initial design and final design constraint. The initial design has very large flutter speed (850 m/s), optimization process had limited the flutter speed to 500 m/s when exceeding structural damping (0.3) so the final flutter speed becomes 500 m/s. The initial design had maximum stress 178 MPa whereas the material can sustain stress until 313 MPa, after optimization process structure become more efficient, the maximum stress becomes 293 MPa.

6. Conclusions
Design sensitivities and optimization process had been applied to the wing box model using MSC Nastran software. The sensitivities study for each design variable of weight, maximum stress, and flutter response. Also, it shows that the design-variable change from initial to final cycle. Structural weight of wing had been reduced successfully until 33% without violating static and flutter constraints. Further sensitivity study, the static-load-subcase gives more critical structure condition than flutter-subcase shown by sensitivity coefficients of flutter response far smaller than maximum-stress-response sensitivity-coefficients.

References
[1] Vanderplaats G N 1981 Structural Optimization – Past, Present, and Future Vol. 20 No.7 (AIAA Journal)
[2] Maxwell C 1869 Scientific Papers (Dover Publications, 2, 175-177).
[3] Shanley 1952 Weight-Strength Analysis of Aircraft Structures (Dover; 2nd edition)
[4] MSC Software Corporation 2010 MD Nastran 2010: Design Sensitivity and Optimization User’s Guide (Santa Ana: MSC Software Corporation)
[5] MSC Software Corporation 2010 MSC Software Corporation MD Nastran 2010: Getting Started with MD Nastran User’s Guide (Santa Ana: MSC Software Corporation).
[6] Powell M J 1981 Optimization Algorithms in 1979 Committee on Algorithms Newsletter 5 (Mathematical Programming Society 2-16)