On continued gravitational collapse

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Summary. – According to a widespread idée fixe, the spherically-symmetric collapse of a sufficiently massive celestial body of spherical shape should generate a black hole. I prove that this process generates simply an ordinary point mass. My argument is model-independent.

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1. – The necessary and sufficient condition that a Riemann-Einstein space-time admit the group of spatial rotations is that its $ds^2$ be reducible to the following form [1]:

$$ds^2 = A_1(r,t) \, dr^2 - A_2(r,t) \, dr^2 - A_3(r,t) \, d\omega^2, \quad (r \geq 0),$$

where

$$d\omega^2 = d\theta^2 + \sin^2\theta \, d\varphi^2.$$ 

By suitable substitutions: $r \rightarrow f_1(r,t), \ t \rightarrow f_2(r,t)$, eq. (1.1) becomes

$$ds^2 = B_1(r,t) \, dr^2 - B_2(r,t) \, dr^2 - r^2 d\omega^2, \quad (r \geq 0),$$

Let us put (cf. [2], [2bis])

$$R \equiv \left[ r^3 + (2M)^3 \right]^{1/3}, \quad (r \geq 0), \quad (c=G=1)$$

where $M$ is the mass of a given collapsing spherical body. Accordingly, we can write

$$ds^2 = C_1(R,t) \, dr^2 - C_2(R,t) \, dR^2 - R^2 d\omega^2.$$
Take ideally an instantaneous photograph of our contracting sphere at any time $t = \bar{t}$; its co-ordinate radius $r_a$ be equal to $\bar{r}_a$. Then, if $\bar{R}_a = [(\bar{r}_a)^3 + (2M)^3]^{1/3}$, by virtue of a well-known Birkhoff’s theorem, we have (see [2], [2bis]):

$$(1.4) \quad C_1(\bar{R}_a, \bar{t}) = \frac{\bar{R}_a - 2M}{\bar{R}_a},$$

$$(1.4') \quad C_2(\bar{R}_a, \bar{t}) = C_1^{-1}(\bar{R}_a, \bar{t}).$$

Now, $\bar{t}$ is just any time: this means that, since $r_a$ tends to zero, the star will reduce asymptotically to the origin of the space co-ordinates, i.e. it will become asymptotically a simple point mass, as described by the original Schwarzschild’s memoirs [2], [2bis].

Remark that the original form of Schwarzschild’s solution to the problem of a gravitating mass point at rest is diffeomorphic to the exterior part $(r > 2M)$ of the usual, standard form of solution, which is due to Hilbert [3], Droste [4], and Weyl [5]. (The invariance of the surface area $4\pi(2M)^2$ is only a geometrical curiousness, devoid of any physical significance).

2. – If in the mentioned standard form of solution ([3], [4], [5]) we substitute for $r$ the following function $f(r)$ (see [6]):

$$(2.1) \quad f(r) \equiv r + 2M,$$

for the spatial region external to the collapsing body we obtain

$$(2.2) \quad ds_{\text{ext}}^2 = \frac{r}{r + 2M} dr^2 - \frac{r + 2M}{r} dr^2 - (r + 2M)^2 d\omega^2.$$

(Obviously, eq. (2.2) can be obtained also from Schwarzschild’s $ds^2$ of paper [2] with the substitution $R \to r+2M$).
In lieu of eq. (1.1bis) we have:

$$\text{(2.3)} \quad ds^2 = D_1(r,t) \, dr^2 - D_2(r,t) \, dr^2 - (r+2M)^2 \, d\omega^2$$

At $t = \bar{r}$, if $r_a = \bar{r}_a$:

$$\text{(2.4)} \quad D_1(\bar{r}_a, \bar{r}) = \frac{\bar{r}_a}{\bar{r}_a + 2M}$$

$$\text{(2.4')} \quad D_2(\bar{r}_a, \bar{r}) = D_1^{-1}(\bar{r}_a, \bar{r})$$

But $r_a$ tends to zero, and our object will shrink asymptotically to a point mass. Again, no black hole has been engendered by the collapsing process.

3. – It is commonly believed that the final stage of a collapsing rotating star is a Kerr’s black hole. Now, I have proved (see [7]) that Kerr’s $ds^2$ is generated in reality by a simple spinning point mass, without event horizons, stationary-limit surface, ergo-sphere. The conclusion is obvious.

Vain is the chase of the black holes.

“Nicht Jeder wandelt nur gemeine Stege:
Du siehst, die Spinnen bauen luft’ge Wege.”

J. W. v. Goethe

HISTORICAL FINALE

In the Twenties of the 20th century the form of solution of the papers [3], [4], [5] was not the unique solution taken into consideration, Schwarzschild’s solution [2] had not yet fallen into oblivion. (Remark that, for very good reason, only the exterior part, $r>2M$, of the HDW-solution was regarded as valid by all the Fathers of Relativity. Magic and science fiction were extraneous to physics; no guru had brainwashed the community of physicists).
In 1922 some witty men \((lucus a non lucendo)\) proposed to make Schwarzschild’s form fully equivalent to whole HDW-form by the assumption that Schwarzschild’s \(r\) take also the negative values of the interval \(-2M \leq r < 0\). Obviously, the Fathers of Relativity rejected this physical and mathematical folly, which was refuted in a detailed way by Marcel Brillouin [8]. Recently, the above proposition was put forward anew by some uninformed authors, but a nonsense remains a nonsense even if it is dressed with a sauce \(à la mode\).

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