Measurements of $K_{e4}$ and $K^{\pm} \to \pi^{0}\pi^{0}\pi^{\pm}$ decays

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The NA48/2 experiment at the CERN SPS collected in 2003 and 2004 large samples of the decays $K^{\pm} \to \pi^{\pm}e^{\pm}\nu_{e}$ ($K_{e4}^{\pm}$), $K^{\pm} \to \pi^{0}\pi^{0}e^{\pm}\nu_{e}$ ($K_{e0}^{00}$) and $K^{\pm} \to \pi^{0}\pi^{0}\pi^{\pm}$. From the $K_{e4}^{\pm}$ form factors and from the cusp in the $M_{200}^{2}$ distribution of the $K^{\pm} \to \pi^{0}\pi^{0}\pi^{\pm}$ events, the $\pi\pi$ scattering lengths $a_{0}^{0}$ and $a_{2}^{0}$ could be extracted. This measurement is a fundamental test of Chiral Perturbation Theory ($\chi PT$).

1 Introduction

The single-flavour quark condensate $\langle 0 | \bar{q}q | 0 \rangle$ is a fundamental parameter of $\chi PT$, determining the relative size of mass and momentum terms in the expansion. Since it can not be predicted theoretically, its value must be determined experimentally, e.g. by measuring the $\pi\pi$ scattering lengths, whose values are predicted very precisely within the framework of $\chi PT$, assuming a big quark condensate [1], or of generalised $\chi PT$, where the quark condensate is a free parameter [2].

The $K_{e4}^{\pm}$ decay is a very clean environment for the measurement of $\pi\pi$ scattering lengths, since the two pions are the only hadrons and they are produced close to threshold. The only theoretical uncertainty enters through the constraint [3] between the scattering lengths $a_{2}^{0}$ and $a_{0}^{0}$. In the $K^{\pm} \to \pi^{0}\pi^{0}\pi^{\pm}$ decay a cusp-like structure can be observed at $M_{200}^{2} = 4m_{\pi}^{2}$, due to re-scattering from $K^{\pm} \to \pi^{+}\pi^{-}\pi^{\pm}$. The scattering lengths can be extracted from a fit of the $M_{200}^{2}$ distribution around the discontinuity.

2 Experimental setup

Simultaneous $K^{+}$ and $K^{-}$ beams were produced by 400 GeV energy protons from the CERN SPS, impinging on a beryllium target. The kaons were deflected in a front-end achromat in order to select the momentum band of $(60 \pm 3)$ GeV/c and focused at the beginning of the detector, about 200 m downstream. For the measurements presented here, the most important detector components are the magnet spectrometer, consisting of two drift chambers before and

*On behalf of the NA48/2 collaboration
two after a dipole magnet and the quasi-homogeneous liquid krypton electromagnetic calorimeter. The momentum of the charged particles and the energy of the photons are measured with a relative uncertainty of 1\% at 20 GeV. A detailed description of the NA48/2 detector can be found in Ref. [4].

3 $K^\pm \rightarrow \pi^+\pi^- e^\pm \nu_e$

Analysing part of the 2003 data, $3.7 \times 10^5 K^\pm e^4$ events were selected with a background contamination of 0.5\%. The background level was estimated from data, using the so-called “wrong sign” events, i.e. with the signature $\pi^+\pi^- e^\mp \nu_e$, that, at the present statistical level, can only be background, since the corresponding kaon decay violates the $\Delta S = \Delta Q$ rule and is therefore strongly suppressed [5]. The main background contributions are due to $K^\pm \rightarrow \pi^+\pi^- \pi^\pm$ events with $\pi \rightarrow e\nu$ or a pion mis-identified as an electron. The background estimate from data was cross-checked using Monte Carlo simulation (MC).

3.1 Form factors

The form factors of the $K^\pm e^4$ decay are parametrised as a function of five kinematic variables [6] (see Fig. 1): the invariant masses $M_{\pi\pi}$ and $M_{e\nu}$ and the angles $\theta_\pi$, $\theta_e$ and $\phi$. The matrix element

$$T = \frac{G_F}{\sqrt{2}} V_{us}^* (p_\nu) \gamma_\mu(1 - \gamma_5) v(p_e) (V^\mu - A^\mu)$$

contains a hadronic part, that can be described using two axial ($F$ and $G$) and one vector ($H$) form factors [7]. After expanding them into partial waves and into a Taylor series in $q^2 = M_{\pi\pi}^2/4m_{\pi^\pm}^2 - 1$, the following parametrisation was used to determine the form factors from the experimental data [8, 9]:

$$F = (f_s + f'_s q^2 + f''_s q^4) e^{1i\delta_0(q^2)} + f_p \cos \theta_\pi e^{1i\delta_1(q^2)}$$

$$G = (g_p + g'_p q^2) e^{1i\delta_1(q^2)}$$

$$H = h_p e^{1i\delta_1(q^2)}.$$  

In a first step, ten independent five-parameter fits were performed for each bin in $M_{\pi\pi}$, comparing data and MC in four-dimensional histograms in $M_{e\nu}$, $\cos \theta_\pi$, $\cos \theta_e$ and $\phi$, with 1500 equal population bins each. The second step consisted in a fit of the distributions in $M_{\pi\pi}$ (see Figs. 3, 2), to extract the (constant) form factor parameters. The $\delta = \delta_0^0 - \delta_1^1$ distribution was

Figure 1: Topology of the $K^\pm e^4$ decay.
Figure 2: \( \delta = \delta_0^0 - \delta_1^1 \) distribution as a function of \( M_{\pi\pi} \). The points represent the results of the first-step fits, the line is fitted in the second step.

Figure 3: \( F, G \) and \( H \) dependence on \( M_{\pi\pi} \). The points represent the results of the first-step fits, the lines are fitted in the second step.
Figure 4: Invariant mass distribution in logarithmic scale of the $K_{e4}^{00}$ events selected from the 2003 data (crosses) compared to the signal MC (red) plus physical (yellow) and accidental (blue) background.

fitted with a one-parameter function given by the numerical solution of the Roy equations [3], in order to determine $a_0^0$, while $a_0^2$ was constrained to lie on the centre of the universal band. The following preliminary result was obtained:

$$f'/f_s = 0.169 \pm 0.009_{stat} \pm 0.034_{syst}$$

$$f''/f_s = -0.091 \pm 0.009_{stat} \pm 0.031_{syst}$$

$$f_p/f_s = -0.047 \pm 0.006_{stat} \pm 0.008_{syst}$$

$$g_p/f_s = 0.891 \pm 0.019_{stat} \pm 0.020_{syst}$$

$$g'_p/f_s = 0.111 \pm 0.031_{stat} \pm 0.032_{syst}$$

$$h_p/f_s = -0.411 \pm 0.027_{stat} \pm 0.038_{syst}$$

$$a_0^0 = 0.256 \pm 0.008_{stat} \pm 0.007_{syst} \pm 0.018_{theor},$$

where the systematic uncertainty was determined by comparing two independent analyses and taking into account the effect of reconstruction method, acceptance, fit method, uncertainty on background estimate, electron-ID efficiency, radiative corrections and bias due to the neglected $M_{ee}$ dependence. The form factors are measured relative to $f_s$, which is related to the decay rate. The obtained value for $a_0^0$ is compatible with the $\chi PT$ prediction $a_0^0 = 0.220 \pm 0.005$ [10] and with previous measurements [11, 12].

4 $K^{\pm} \rightarrow \pi^0\pi^0e^{\pm}\nu_e$

About 10,000 $K_{e4}^{00}$ events were selected from the 2003 data and about 30,000 from the 2004 data with a background contamination of 3% and 2%, respectively. The background level was estimated from data by reversing some of the selection criteria and was found to be mainly due to $K^{\pm} \rightarrow \pi^0\pi^0\pi^{\pm}$ events with a pion mis-identified as an electron (see Fig. 4). The branching fraction was measured, as a preliminary result from the 2003 data only, normalised to $K^{\pm} \rightarrow \pi^0\pi^0\pi^{\pm}$:

$$BR(K_{e4}^{00}) = (2.587 \pm 0.026_{stat} \pm 0.019_{syst} \pm 0.029_{ext}) \times 10^{-5},$$
Figure 5: Left: $M_{00}^2$ of the selection $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$ data events. The arrow indicates the position of the cusp. Right: angle between the $\pi^\pm$ and the $\pi^0$ in the $\pi^0\pi^0$ centre of mass system. The points represent the data, the three curves, the MC distribution for different values of $k'$. 

where the systematic uncertainty takes into account the effect of acceptance, trigger efficiency and energy measurement of the calorimeter, while the external uncertainty is due to the uncertainty on the $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$ branching fraction. This result is about eight times more precise than the best previous measurement [13].

For the form factors the same formalism is used as in $K_{e4}^{\pm}$, but, due to the symmetry of the $\pi^0\pi^0$ system, the $P$-wave is missing and only two parameters are left: $f'_s/f_s$ and $f''_s/f_s$. Using the full data sample, the following preliminary result was obtained:

$$f'_s/f_s = 0.129 \pm 0.036_{\text{stat}} \pm 0.020_{\text{syst}}$$
$$f''_s/f_s = -0.040 \pm 0.034_{\text{stat}} \pm 0.020_{\text{syst}},$$

which is compatible with the $K_{e4}^{\pm}$ result.

5 $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$

From 2003 data, about 23 million $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$ events were selected, with negligible background. The squared invariant mass of the $\pi^0\pi^0$ system ($M_{00}^2$) was computed imposing the mean vertex of the $\pi^0$s, in order to improve its resolution close to threshold. At $M_{00}^2 = 4m_{\pi^\pm}^2$, the distribution shows evidence for a cusp-like structure (see Fig. 5) due to $\pi\pi$ re-scattering. Fitting the distribution with the theoretical model presented in Ref. [14] and using the un-perturbed matrix element

$$M_0 = A_0(1 + \frac{1}{2}g_0u + \frac{1}{2}h'u^2 + \frac{1}{2}k'^2v^2),$$

the following result was obtained [15], assuming $k' = 0$ [16]:

$$g_0 = 0.645 \pm 0.004_{\text{stat}} \pm 0.009_{\text{syst}}$$
\[
\begin{align*}
  k' &= -0.047 \pm 0.012_{\text{stat}} \pm 0.011_{\text{syst}} \\
  a_2 &= -0.041 \pm 0.022_{\text{stat}} \pm 0.014_{\text{syst}} \\
  a_0 - a_2 &= 0.268 \pm 0.010_{\text{stat}} \pm 0.004_{\text{syst}} \pm 0.013_{\text{theor}},
\end{align*}
\]

where the \( a_0 - a_2 \) measurement is dominated by the uncertainty on the theoretical model.

In a further analysis, evidence was found for a non-zero value of \( k' \) (see Fig. 5):

\[
k' = 0.0097 \pm 0.0003_{\text{stat}} \pm 0.0008_{\text{syst}},
\]

where the systematic uncertainty takes into account the effect of acceptance and trigger efficiency.

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