The Use of Integral Transform in Calculating Structured Interaction of Layered Soil Caused by Dynamic Loads

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Abstract. This study applies the Integral Transform Method (ITM) to solve the problems caused by dynamic loads that work on structures and its interactions with the soil. The structure and part of the soil from the half space layer are modeled with Finite Elements (FEM) where at the boundary must be coupled with the dynamic stiffness matrix from the half space calculating used Integral Transform. To obtain the dynamic stiffness matrix of the half space, the wave equation in the elastic half space which is a coupled partial differential equations must be uncoupled first with Helmholtz’s potential then it is used integral transform to find the wave equation in the transform domain. To get a response to the original domain, reverse transformation is performed using Inverse Integral Transform. This method is known as ITM - FEM Coupling.

1. Introduction
The study using integral transform has been examined by previous researchers which it was found difficult without integral transform solution from the original problem. In soil-structure interaction, part of the soil is considered as part of the upper structure and calculated using the Finite Element Method (FEM) approach. This method has been developed by Luco (1972), Dasgupta (1976) and Gaul (1976). For applications in the time domain, several approaches have been developed by Wolf (1988) and Schapertans (1996). Zirwas (1996) uses the ITM-FEM coupling method in the interaction of soil structure for 2 D problem, where a part of the soil can be considered as part of the upper structure. Rastandi, examined further study from Zirwas for the 3 D issue, where the structure and part of soil of the half space layer are modeled with finite elements and dynamic matrices calculated by finite element analysis [1]-[17]. The scope of the study is explained as follows:

(1) The dynamic load that works is limited to the harmonic load \( p_0 \, e^{i\omega t} \) that consists of real and imaginary.

(2) Property from the soil in each of the same layers is considered unchanged.

(3) The problem that will be analysed in this study is the response of the half space layer along with the structure above. The response that will be reviewed is the vertical and horizontal direction of movement at all nodal points of the half space layer and the structure above.

(4) The element that will be used for the structure is the bar element while for the half space layer is a 3D solid element so that at each nodal point there are only 3 DOF which are the displacement in the x, y and z directions.

(5) The property of the soil from the half space layer can be taken variously but in this study there are only used 2 different properties of soil; for the half space layer and for half space
This study used several computer programs that made from the Math lab program to calculate matrices and represent graphical obtained from the results of the study.

The equation of motion of a discrete system such as the FEM model for harmonic excitation as follows:

\[
[M] \{ \ddot{u} \} + [C] \{ \dot{u} \} + [K] \{ u \} = \{ P_0 \} e^{i\omega t}
\]

which

\[M = \text{Mass matrix in global coordinate}
\]
\[C = \text{Damping Matrix in global coordinate}
\]
\[K = \text{Stiffness matrix in global coordinate}
\]
\[\ddot{u} = \text{Displacement in global coordinate}
\]
\[\dot{u} = \text{Velocity in global coordinate}
\]
\[\ddot{u} = \text{Acceleration in global coordinate}
\]

Before calculating the value of each matrix in global coordinates, it is necessary to calculate first the value of each matrix in local coordinates. The amount of the mass matrix \( (m^e) \), stiffness matrix \( (k^e) \) and damping matrix \( (c^e) \) of each structure elements in local coordinate are:

\[
[m^e] = \iiint \rho [N]^T [N] \, dV
\]
\[
[k^e] = \iiint [B]^T [D] [B] \, dV
\]
\[
[c^e] = \iiint \xi [N]^T [N] \, dV
\]

In which

\[N = \text{shape function of the element}
\]
\[B = \text{strain and displacement of the element}
\]
\[D = \text{constitutive matrix for isotropic material}
\]
\[\xi = \text{damping ratio}
\]

Then, Mass matrix \([M^e]\), stiffness matrix \([K^e]\) and damping matrix \([C^e]\) of each element in global coordinate are calculated. Furthermore, mass matrix \([M]\), stiffness matrix \([K]\) and damping matrix \([C]\) of structure are additions from each elements of matrix.

\[
[M] = \sum_{e=1}^{n} [M^e]
\]
\[
[K] = \sum_{e=1}^{n} [K^e]
\]
\[
[C] = \sum_{e=1}^{n} [C^e]
\]

In this study it only uses solid elements of 8 solid nodal (hexahedron) for model of soil from the half space layer and space truss element for model of the steel tower, where each nodal point has 3 DOF, which is the translation direction of x, y and z.
In the equation (1) \( \{ \bar{u} \} \) consists of 2 components which are real and imaginary or it can be written as follows:

\[
\{ \bar{u} \} = \{ u_R \} + \{ u_I \} = \{ \bar{U} \} e^{i\omega t}
\] (8)

In which \( \{ \bar{U} \} \) is complex amplitude

By adding value of \( \{ u \} \), \( \{ \hat{u} \} \) and \( \{ \bar{u} \} \) in equation (1) it obtains:

\[
\begin{bmatrix}
-\text{[M]} \\ \text{[C]} i \omega + \text{[K]}
\end{bmatrix} \text{U} = \{ \text{F}_0 \}
\] (9)

If the coefficient of damping \( c \) in matrix \( \text{[C]} \) is substituted with damping ratio \( \xi \), it obtains:

\[
[ \text{D} ] = [ \text{K} ] + 2 [ \text{K} ] \xi i - \omega^2 [ \text{M} ]
\] (10)

The equation (10) can be simplified as:

\[
[ \text{D}^{FE} ] = [ \text{K} ] ( 1 + 2 \xi i ) - \omega^2 [ \text{M} ]
\] (11)

Matrix \( [ \text{D}^{FE} ] \) is known as dynamic stiffness matrix.

Damping ratio \( \xi \) do not depend on the frequency and it can be consider as constant in all the structure. \( [ \text{D}^{FE} ] \) can also be decomposed into sub matrices \( [ \text{D}^{FE}_{ss} ] \), \( [ \text{D}^{FE}_{sh} ] \), \( [ \text{D}^{FE}_{hs} ] \) dan \( [ \text{D}^{FE}_{hh} ] \)

The equation of motion for Finite element structure can be formulated as follow:

\[
\begin{bmatrix}
\text{p}^{FE} \\ \text{u}^{FE}
\end{bmatrix} =
\begin{bmatrix}
[ \text{D}^{FE}_{ss} ] & [ \text{D}^{FE}_{sh} ] \\ [ \text{D}^{FE}_{hs} ] & [ \text{D}^{FE}_{hh} ]
\end{bmatrix}
\begin{bmatrix}
\text{u}^{FE} \\ \text{u}^{\infty}
\end{bmatrix}
\] (12)

Considering that on the interface area (Γ) there is no external force, the structure as shown in figure (2) can be divided into 2 sub structure which are \( \Omega_{FE} \) and \( \Omega_{\infty} \)

Equilibrium on the interface area as shown in figure (2) is:

\[
\sigma_{h}^{FE} - \sigma_{h}^{\infty} = 0
\] (13)

Or on the discrete system

\[
\{ \text{p}^{FE}_h \} - \{ \text{p}^{\infty}_h \} = 0
\] (14)

and

\[
\{ \text{u}^{FE}_h \} - \{ \text{u}^{\infty}_h \} = 0
\] (15)

Relation between \( \{ \text{C} \} \) and \( \{ \text{u}^{\infty}_h \} \) can be written as follow:

\[
\{ \text{u}^{\infty}_h \} = [ \text{TR} ] \{ \text{C} \}
\] (16)

In which \( \text{TR} \) is transform matrix from the base of \( \{ \text{C} \} \) to displacement \( \{ \text{u}^{\infty}_h \} \) and the amount of \( \text{TR} \) is equivalent as displacement of nodal ‘h’ that can be calculated using ITM method.

From the previous study, it has been calculated dynamic matrix from half space, there is:

\[
[D^{\infty}] = \int [U_{lmm}]^T [T_{lnm}] d\Gamma_s
\] (17)

If there is no external force on the interface area (Γ), both of the sub structure can be combined and the result is:

\[
\begin{bmatrix}
\{ \text{p}^{FE}_{h} \} \\ \{ \text{0} \}
\end{bmatrix} =
\begin{bmatrix}
[D^{FE}_{ss}] \\ [TR]^T [D^{FE}_{sh}] \\
[TR]^T [D^{FE}_{hs}] \end{bmatrix}
\begin{bmatrix}
[TR]^T [D^{FE}_{hh}] [TR] + [D^{\infty}] \\ \{ \text{C} \}
\end{bmatrix}
\] (18)
Figure 2. two sub structure system in equilibrium

Figure 3 shows the P force = 20000 kg which works on the soil surface and displacement of $U_z$ is calculated using ITM method (figure 3a) and ITM-FEM method (figure 3b). The result as seen in figure (4) for displacement of real and imaginary $U_z$ in solid caused by P loads = 20000 kg in layer $z = 0$ m and from the picture, it can be concluded that the ITM-FEM method is accurate to be used in calculations for centralized loads.

![Figure 3. P force = 20000 kg which works on the soil surface.](image)

If the loads which work is block load with the size of 4.00 x 4.00 m$^2$ with $P_{\text{total}} = 20000$ kg so that the displacement of real and imaginary $U_z$ in solid that caused by block load in layer $z = 0$ m as showed in the figure (5) and based on this picture, it can be concluded that the ITM-FEM method is accurate to be used in calculating block load.
Figure 4. Real and imag displacement $U_z$ in solid caused by $P = 20000$ kg, layer $z = 0$ m.

Figure 5. Real and imag $U_z$ for block load $P_{tot} = 20000$ kg, $bx=by = 2$ m, layer $z= 0$ m.

Soil:
- Layer half space: $E = 1. \times 10^6$ kg/m$^2$
  $\mu = 0.35$
  $\rho = 1700$ kg/m$^3$
  $\xi = 0.02$

- Half space: $E = 1.\times 10^7$ kg/m$^2$
  $\mu = 0.20$
  $\rho = 2000$ km/m$^3$
  $\xi = 0.02$

Concrete Raft: $E = 2.0 \times 10^9$ kg/m$^2$
- $\mu = 0.15$
- $\rho = 2400$ kg/m$^3$
- $\xi = 0.02$

Steel $E = 2.0 \times 10^10$ kg/m$^2$
- $\mu = 0.20$
- $\rho = 7850$ kg/m$^3$
- $\xi = 0.02$

The steel tower and concrete piles are modeled with space truss elements while the soil and concrete raft are modeled with 3D hexahedron solid elements. Figure (6) showed Finite element mesh, steel tower and concrete pile. Dynamic Stiffness Matrix from Finite Element [$D^{FE}$] that has been calculated before:
$$[D^{FE}] = [K] (1 + 2 \xi i) - \omega^2 [M]$$

(19)

$\xi$ is damping ratio taken for $= 2\%$ and $\omega$ is frequency from external force taken for $= 10$ rad/second.

**Figure 6.** Finite element mesh, concrete raft, steel tower and pile.

The result is obtained as follows:

**Figure 7.** Real displacement of $U_x$, $U_y$ and $U_z$ for concrete raft + pile of $20 \times 20$ cm –6.00m, $\omega = 10$ rad/sec, $\xi = 0.02$, layer $z = 0$ m.
Figure 8. Displacement of real and imag Ux, Uy dan Uz for concrete raft + pile of 20 cm - 6.00m, $\omega = 10$ rad/sec, $\zeta = 0.02$, for layer $z = 0, 2$ and 4 m.

2. Conclusions

The ITM - FEM method can be used to calculate the response of the structure and part of the soil that are modeled with Finite Elements caused by dynamic loads. The advantage of this method is it can calculate the response of the surface soil around the half space layer further as we want, just by recalculating the dynamic matrix of the half space layer. This ITM-FEM method can be used for various types of soil properties from the half space layer and the properties of each layer are considered constant. The more soil properties taken, the more complex the calculation will be.

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