Research evidence for mathematics education for students with visual impairment: A systematic review

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Research evidence for mathematics education for students with visual impairment: A systematic review

Oliv G. Klingenberg1, Anne H. Holkesvik2 and Liv Berit Augestad1,3*

Abstract: The aim of the study was to conduct a systematic review in order to synthesize the evidence-based literature on mathematics education among students with visual impairment (VI). Studies were identified through searches of electronic databases (SCOPUS, PubMed, ERIC, and Web of Science). The authors included articles published between 1 January 2000 and 31 October 2017. Eleven publications met the inclusion criteria, and seven studies had observational designs. The studies focused on teachers’ attitudes and experiences, the use of abacus, tactile graphics, and the development of mathematical concepts. The results showed that the studies had different aims and methods, and only four studies reported eye disorder diagnoses. The ability to choose suitable teaching strategies that involve individual instructions requires qualified and enthusiastic teachers who allow students to experience a sense of accomplishment and success. Additionally, it is important to gain information on eye disorder diagnoses and possible learning disabilities. The authors conclude that it is to be expected that students with VI but without cognitive disabilities will follow their grade level in mathematics. The findings highlight the need for more randomly
controlled, high quality trials in order to obtain more evidence and knowledge of mathematics education among students with VI.

Subjects: Teaching mathematics for students with visual impairment; Mathematics education; Mathematical skills

Keywords: visual impairment; Braille; mathematics education; abacus; tactile graphics; students; systematic review

1. Introduction

Vision enables important access to information that supports students’ development of mathematical concepts and procedures (Emerson & Anderson, 2018; Hershkowitz, Markovits, Rosenfeld, Ilani, & Eylon, 2018). Many basic mathematical concepts take the form of words that describe visual phenomena (Jones, 2018). Concepts such as big (or bigger), long (or longest), and between are comprehensible to sighted children because vision helps them to group objects according to their overall similarity. Sighted children can see relations between objects at a glance. For children with visual impairment (VI), such object relations are mental, because their hands can only explore (“look at”) one object at a time. Accordingly, their awareness or understanding of spatial concepts and directional concepts are more challenging to grasp compared with children without VI (Hatwell, 2003). Consequently, it is natural that many children with VI are offered fewer experiences of how mathematical concepts are used in daily-life communication than are sighted children (Bateman et al., 2018; Emerson & Anderson, 2018).

In the field of visual impairment, researchers have traditionally studied students with VI as one group of students, despite the fact that visual impairment can be caused by a variety of conditions (Warren, 1994, p. 2). Some students have congenital disorders, and some have lost their vision partially or completely during the course of their lives. Ophthalmological diagnoses, age of onset, comorbidity, and residual vision are significant variables that may influence the development of mathematical understanding. In addition, as among all students, cognitive function and personal interest are important factors in learning to think mathematically as well as to develop grade-appropriate skills.

Our experiences are similar to the results reported by Rapp and Rapp (1992), namely that students who use Braille are less likely to participate in advanced mathematics classes. In addition, as a population, students with VI tend to perform below their ability in mathematics compared with in other academic subjects (Beal & Shaw, 2008; Freeland, Emerson, Curtis, & Fogarty, 2010).

One factor that may limit the participation of blind and partially sighted learners in mathematics education may be what Emerson and Anderson (2018) call the trend toward more visually based mathematics materials. Many mathematics textbooks have visual images with important content information that is not transcribed or described in either Braille books or digital versions of the texts. This trend is making textbooks, which to some extent are already inaccessible to students with VI, even more challenging for blind and partially sighted learners.

Graphs, charts, diagrams, figures, and drawings are used as mathematical tools to communicate huge amount of data or relationships between variables in a simplified and concise way (Akar & Övez, 2018). An important mathematical skill is to gather information from such graphics and use it to solve problems (Rosenblum, Cheng, & Beal, 2018). However, curricular graphic materials are visual in nature and therefore many students with VI face considerable challenges in reading them (Rosenblum & Herzberg, 2015).

The provision of suitable learning materials is vital in mathematics education. The abacus is a tool for addition, subtraction, multiplication, and division by students with VI, in a similar way to
how pencil and paper are used for calculations by sighted students. Often, it is up to the teacher to decide whether the student should learn how to use an abacus. This may be a difficult decision because the effectiveness of the abacus has been questioned (Ferrell, 2011).

Technology can offer new opportunities and valuable access to information for students with VI by complementing the use of tactile devices, Braille, and large print with audio (Beal & Rosenblum, 2018; Pitchford, Kamchedzera, Hubber, & Chigeda, 2018; Supalo, Hill, & Larrick, 2014; Supalo, Isaacson, & Lombardi, 2013). Digitally enhanced learning tools may thus transform educational environments by making the learning process more encompassing, more engaging, and more collaborative (Bin Tuwaym & Berry, 2018; Metatla et al., 2018).

Self-esteem may play a profound role in all aspects of a child’s development (Breckler, Olson, & Wiggins, 2006; Brooks, 1992). A student’s self-esteem may be dependent upon his or her psychological adjustment, quality of life, adaptive behaviour, relationships with friends, motivation, school performance, and success in life (Augestad, 2017b; Brooks, 1992; Papadopoulos, Mtsiou, & Agaliotis, 2013; Saigal, Lambert, Russ, & Hoult, 2002). In STEM self-confidence is one factor that contributes to a student’s motivation to learn mathematics. Supalo et al. (2013, p. 196) state that “inadequate hands-on science experiences may inhibit development of self-confidence concerning one’s capacity to independently function in scientific endeavors, and that low self-confidence in the sciences may be associated with consideration of nonscientific college studies rather than studies in the sciences”. Another factor is inclusion in the student group and being able to learn together with peers, which may increase the student’s sense of well-being and his or her learning process. However, there is a lack of evidence about inclusion and collaborative learning in mathematics for the students with VI.

The purpose of our research was to examine the state of mathematics education research in the field of visual impairment. Following a literature search, we found a review published in 2006 in which the authors state:

Because no two interventions were alike, we were unable to establish distinct categories or apply meta-analytic techniques with any group of studies. Accordingly, our initial syntheses of the research in mathematics for students with visual impairments yield promising, but not best practices. Without replication, even promising practices are preliminary and perhaps misleading in the absence of further research (Ferrell, Buettel, Sebald, & Pearson, 2006, p. 11).

In a literature review published in 2013, the author focuses on STEM education (science, technology, engineering, and mathematics education) (Cryer, 2013), but we were unable to identify the results that focus only on mathematics education.

We found it relevant to conduct a systematic review and to evaluate the quality of the research and the effect of any intervention, with the aim of summarizing the evidence-based academic knowledge about mathematics education among students with VI.

2. Method

2.1. Design
We carried out a systematic review of pre-reviewed articles and described the study characteristics and major findings. In addition, we evaluated the quality of the studies and discussed possible interpretations of the results.

2.2. Search strategy
In order to identify relevant published articles, we searched the databases SCOPUS, PubMed, ERIC, and Web of Science using the following search terms: visual impairment, blind, low vision, mathematics, and education. We included articles published between 1 January 2000 and 31 October 2017 (Figure 1).
2.3. Criteria for inclusion and exclusion

We included studies of children and young adults with VI in the age range 5–25 years. The reason for the wide age range was that we wanted to include studies of students of school age. Students with VI may have major problems with accessing information, and it is possible they may need additional years to graduate from secondary or upper secondary school (Yurtay, Yurtay, & Adak, 2015). We included peer-reviewed articles written in English that report studies of humans. We identified 278 abstracts.
After expert reading of the 278 abstracts, we excluded publications on children and young adults who had VI as well as comorbidity or multiple disabilities. In addition, we excluded articles that describe analyses of the situation related to teaching tool and materials in a specific country, and articles that do not include separate data collection for individuals with VI. We ended up with 108 publications. After 55 duplications were excluded, the search process yielded 53 articles.

Two of us read the 53 publications. Some studies did not have original data collection. Due to the defined inclusion and the exclusion criteria, we decided that 25 of the publications fitted the aim of our study. We evaluated the 25 publications and found it natural to divide them into two separate groups according to length. The review of mathematics education for students with VI was thus based on 11 publications (Figure 1). The other publications will be presented in a separate paper related to digital learning among students with severe vision loss.

2.4. Data extraction
We used a standardized protocol and reporting form to abstract the following data from each publication: year of publication, first author's name, country in which the study was conducted, aim of the study, study design, age and number of persons in the study sample, number of persons with VI and number of sighted persons in each study, definition of VI, and main results of each study (Table 1). In Table 1, under the rubric study design, we include each study's dependent and independent variable. However, since the different studies did not have the same dependent and independent variables, we chose the main dependent variable and independent variables. The reason for this choice was number of words in the table and making the table optimal for reading. We used the date that the article was first published online as the publication year, not the date of the print edition.

2.5. Evaluation of the studies
We used the Quality Assessment Tool for Studies with Diverse Designs (QATSDD) to evaluate the selected studies (Sirriyeh, Lawton, Gardner, & Armitage, 2012). The tool, which was developed to assess the quality of studies with one topic but using different approaches or designs, has been found to have good reliability (Cohen's kappa, χ = 71.5) and good face validity (Sirriyeh et al., 2012). We used the version of QATSDD with 14 items related to quantitative studies. Each item was rated on a 4-point scale ranging from “not at all” (0), “very slightly” (1), “moderate” (2), to “completely” (3), with a maximum score of 42. The percentage score was calculated by dividing the actual score by the maximum score (i.e. 42). Studies scoring higher than 75% were considered “high quality”, between 50% and 75% “good quality”, between 50% and 25% “moderate quality”, and below 25% “poor quality”. The quality ratings are presented in Table 1.

3. Results
The 11 reviewed articles are summarized in Table 1.

After using the QATSDD, four studies were classified as “high quality”, six as “good quality”, and one study as “moderate quality”.

The studies had somewhat different aims and methods, but they all focused on improvements in mathematics education among students with VI. Two studies focused on the use of the abacus as a tool in the education (3, 4), and three studies focused on tactile graphics (1, 2, 7). Five focused on competence in mathematics (1, 6, 7, 8, 11), and three studies concerned environmental factors in the classroom and mathematics achievement (5, 9, 10).

The result showed that seven studies had an observational study design (2, 3, 4, 5, 6, 9, and 11 in Table 1), two used interviews (1, 8), and two had a mixed methods design, including an experimental study (7, 10). Study No. 10 used an intervention approach to examine the attitudes and instructional changes in teachers throughout a year of contact with itinerant teachers of the visually impaired, as well as reflective narratives on the use of supplemental adaptive resources. The results of the study
| Article number | Year | Authors | Country          | Aim                                                                 | Method                                                                 | Sample                                                                 | Main results (QATSSD)                                                                 |
|---------------|------|---------|------------------|----------------------------------------------------------------------|------------------------------------------------------------------------|------------------------------------------------------------------------|---------------------------------------------------------------------------------------|
| 1             | 2015 | Rosenblum & Herzberg | USA              | 1. According to students who are tactile readers, what qualities make a “good” tactile graphic? 2. Can students who are tactile readers accurately locate and identify requested information in tactile graphics? | Structured interviews about four different tactile graphics. Participants were asked to complete a series of tasks and share their impressions regarding the clarity of the graphics. D: Tactile readers InD: Access to tactile graphics, accurately locate and identify requested information in tactile graphics | 12 youths in grades 6–12 8 females, 4 males  Public school (n = 11) Specialized school for the blind (n = 1) Grade level in mathematics (n = 9) Below grade level (n = 1) Above grade level (n = 2) LCA (n = 4), ROP (n = 3), SOD (n = 2), CVI (n = 1), Peter’s syndrome (n = 1), and cancer of the optic nerve (n = 1) | Participants varied in their accuracy in locating specific information in the graphics. They all reported that what made a tactile graphic “good” was clarity of information. When inaccuracies of measurement occurred, all of the youths tended to report lengths longer than they were in reality. Youths reported that they could locate necessary information on the x-axis but reported difficulty in locating specific information on the y-axis, even though there were gridlines. (Good quality) |
| 2             | 2014 | Zebehazy & Wilton | Canada and USA  | To investigate the perceptions of teachers of students with VI with respect to the quality, importance, and instruction in the use of tactile and print graphics. | An online survey sent to networks via e-mail, e-discussion groups, and newsletter announcements. The survey link remained open for 6 months. D: Perceptions of teachers of students with VI InD: Quality of tactile and print graphics, to promote learning, direct instruction | 306 teachers responded, of which 221 (72.7%) completed the survey in full. Respondents (94%) worked with at least one student who used tactile graphics at some point in their career. It is not stated how many individuals received the survey. | Greater than half (n = 245) agreed or strongly agreed the importance of early exposure to tactile graphics. Only 22% agreed that they had adequate time with their students to teach the use of graphics. Close to one-third of the respondents indicated they did not teach their students how to make their own graphics in order to understand concepts. (Good quality) |
| Article number | Year | Authors                      | Country       | Aim                                                                 | Method                                                                 | Main results (QATSSD)                                                                                                                                 |
|----------------|------|------------------------------|---------------|----------------------------------------------------------------------|------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|
| 3              | 2013 | Rosenblum et al.             | USA and Canada| To gather data that could be used to determine how much emphasis should be placed on abacus instruction within university teacher training programmes | An online survey advertised on local and national e-bulletin boards in the field of VI<sup>1</sup>  
D: Teachers use and beliefs about computation with abacus  
InD: Demographic data, proficiency or exposure to methods and ratings of belief statements, skills learned | Statements with the highest level of agreement included one that indicated that when sighted individuals used pencil and paper for computation, an individual with VI<sup>1</sup> should be allowed to use an abacus. The data reported here support the concern that some teachers of the students with VI<sup>1</sup> were not taught specific computational skills that they may need to teach to students. Online webinars, an electronic bulletin board devoted to abacus skills and instructions, hands-on workshops, and interactive practice tools all have the potential to enrich teachers’ knowledge of the abacus.  
(Good quality)  

(Continued)
| Article number | Year | Authors | Country | Aim | Method | Main results (QATSSD^2) |
|----------------|------|---------|---------|-----|--------|-------------------------|
|                | 5    | 2012    | McDonnell et al. | USA | To determine whether PIS^7 and PIH^8 are associated with mathematics achievement for youths with VI in elementary and middle school | The SEELS^9 database was used. Data collection from several sources (i.e., parents, teachers, and school administrators) occurred up to three times for each student. Three waves, during 2000–2001, 2002, and 2004. Dependent: The Calculations subtest of the WJ-III ACH^10 measures mathematics computational skills, ranging in difficulty from simple to advanced. InD: PIS^7 a scale measuring the frequency of parent attendance at the school, PIH^8 a measuring the frequency of parental interaction with the child | Levels of PIS^7 were low for most students, with greatest levels when students were younger. PIS^7 was positively associated with mathematics achievement for students who began the study in elementary school. The effect of PIS^7 did not differ according to whether a cognitive disability was present. Levels of PIH^8 were high for most students, with successively lower levels reported at each wave except for students with a cognitive disability. PIH^8 significantly predicted mathematics achievement, and the authors conclude that students with VI^1 who are low achievers in mathematics may benefit most. (High quality) |
|                | 6    | 2012    | Klingenberg et al. | Norway | To estimate the number of Braille students who were educated according to their grade-level progression in mathematics, and to analyse the association between eye disorder diagnoses and progression in mathematics | A retrospective study design of clinical data, consisting of educational and medical information D: Students following grade level in mathematics: yes/no InD: Demographic data, eye-related variables, reading media | All students who had received Braille education in Norway between 1967 and 2007 (n = 248). 45 principal eye diagnoses | 141 students (57%) had been taught mathematics at grade level. In 19 of the 45 principal eye diagnoses, all of the students were educated according to the normative grade progress in mathematics. There were no statistically significant associations between gender and the abstracted variables or between mathematics education and reading media. (High quality) |

Table 1. (Continued)
| Article number | Year | Authors       | Country   | Aim                                                                                           | Method                                                                                           | Main results (QATSSD)                                                                 |
|---------------|------|---------------|-----------|-----------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------|
| 7             | 2012 | Klingenberg  | Norway    | To investigate the ways in which two students who read Braille completed geometry tasks and how they constructed mental representations of the shapes of objects | Educational experiment conducted as a four-day geometry course for Braille students               | The students complemented their multiple manual explorations with body movements and postures that corresponded to characteristic features of the shapes of objects. They used their whole body as an explorative instrument. They typically rotated small three-dimensional objects and experimented with the objects’ point of balance, and “played” with the physical characteristics of environments. They continually measured geometric properties, such as length, area, and volume, as part of their way of “seeing” the shapes of objects. (High quality) |
| 8             | 2012 | Jones et al. | USA       | To document the reported experiences of students with VI1 with respect to estimating measurements, as well as the students’ conceptualizations of linear distances and accurate estimations | Interviews                                                                                       | The results showed that the students lacked skills in making linear estimations and had difficulty in estimating metric lengths. The results indicated that the students with VI1 might not have connected informal estimation experiences with the skills required in formal scientific inquiry. Across the tasks, the students were able to make better estimates when measurements were close to the human scale, but the difficulty extended to measurements of centimetres. (Moderate quality) |
| Article number | Year | Authors          | Country | Aim                                                                 | Method                                                                 | Main results (QATSSD⁴)                                                                 |
|---------------|------|------------------|---------|---------------------------------------------------------------------|------------------------------------------------------------------------|------------------------------------------------------------------------------------------|
| 9             | 2012 | Giesen et al.    | USA     | To investigate whether an environment with more academic support in mathematics is helpful for students with VI¹ | Longitudinal Design: The SEELS⁵ database was used. D: Primary outcome measure of mathematical achievement, the applied problems subtest of the WJ-III ACH⁶. InD: Extent of academic supports, the SEELS⁵ Wave 2 School Characteristics Questionnaire, Cognitive disabilities. Sample: Data were limited to students with VI¹ (including blindness) and scores available on the mathematics achievement measure for at least one of three measurement time points (n = 292). Method: The data analyses do not show a simple overall relationship between extent of academic support in the school and mathematics achievement for either initial level or growth in achievement over time. Presence of a cognitive disability moderated a relationship between the extent of school support and mathematics achievement. The extent of academic support appeared to be positively related for VI¹ students without a cognitive disability. (High quality) |
| 10            | 2011 | Rule et al.      | USA     | To examine the attitudes of secondary teachers towards students with VI¹ in their science or mathematics classes before and after a year-long funded programme that provided curriculum materials and adaptive equipment | Three types of data collections: Survey of teachers' attitudes towards students with disabilities in STEM¹¹ classes. Pre-test and post-test questions regarding the teachers' anticipated and actual experiences during the project. A narrative about the challenges and successes that the teachers and their students experienced during the year. D: The attitude survey of science and mathematics teachers working with students with VI¹. I: Individual survey questions showing largest pre-post differences. | Sample: 15 science or mathematics teachers of 13 students with VI¹ (8 blind, 5 with low vision), in the age range 13–18 years, and who were performing at or above the level of their peers. Main results: Teachers expressed that teaching a student with VI¹ was less difficult than anticipated at the beginning of the course and they felt more prepared to meet the needs of such students. A total of 8 out of 15 teachers reported that they faced a struggle when they tried to implement the new materials and adaptive devices. Given appropriate tactile or auditory materials and/or enlarged visuals, 10 of 12 teachers reported that students with VI¹ were as successful as their classroom peers in science and mathematics. (Good quality) |
| Article number | Year | Authors          | Country       | Aim                                                                 | Method                                                                                      | Sample Definition of VI | Main results (QATSSD)
|---------------|------|------------------|---------------|----------------------------------------------------------------------|-------------------------------------------------------------------------------|--------------------------|---------------------------------------------------------------|
|               | 11   | Crollen et al.   | Belgium and Canada | 1. To determine whether blind children used their fingers to count and to show quantities  
2. To determine whether blind children who did not use their fingers or used them comparatively less achieved the same level of performance as sighted children  
3. To determine whether blind children used their fingers in the same way as their sighted peers when they were explicitly required to count and to show quantities with their fingers | Dependent (D)  
Independent variables (InD)  
3 test batteries:  
General cognitive abilities  
The spontaneous use of finger-counting  
Canonicity of the finger-based representations  
D: Blind children use their fingers to count  
InD: General cognitive abilities, characteristics of the children, working memory test | 14 children (7 girls and 7 boys)  
age range 7–13 years (mean age = 9.9)  
11 right-handed, 1 left-handed, and 2 ambidextrous  
All were born with VI and did not have any other disability.  
Principal ophthalmic diagnoses  
Sighted matched control group (mean age 9.3) | Compared with sighted controls, blind children used their fingers less spontaneously to count and in a less canonical way to count and show quantities. The absence of vision precluded the development of a typical finger numeral representation and suggested that the use of canonical finger-counting and finger-monitoring strategies relied on the visual recognition of particular hand shapes. (Good quality) |

1VI—visual impairment; 2QATSSD—Quality Assessment Tool for Studies with Diverse Designs; 3LCA—Leber congenital amaurosis; 4ROP—retinopathy of prematurity; 5SOD—septo-optic dysplasia; 6CVI—cortical vision impairment; 7PIS—parental involvement at school; 8PIH—parental involvement at home; 9SEELS—Special Education Elementary Longitudinal Study, a comprehensive study documenting the achievement, personal characteristics, and educational experiences of a sample of elementary special education students as they moved through the school system up to high school; 10WJ-III ACH—Woodcock-Johnson III Tests of Achievement; 11STEM—science, mathematics, engineering and technology
conducted by Rule et al. (10) support the claim that “given knowledgeable, supportive teachers, and with appropriate accommodations such as tactile or auditory materials, students with visual impairments can be as successful and engaged as other students in science and mathematics” (Rule, Stefanich, Boody, & Peiffer, 2011, p. 865).

The age range in the study samples varied from 6–18 years (1, 5, 6, 7, 8, 9, 10, 11). Three studies included only teachers (2, 3, 4). Studies 1, 6, and 7 included students in the age group 6–12 years, studies 8, 10, and 11 included students older than 12 years of age. The state-wide (USA) SEELS study included students in the age range 7–16 years (5, 9).

Four studies reported eye disorder diagnoses (1, 6, 7, 11). Four studies (5, 8, 9, 10) did not clearly define visual impairment, nor was visual function reported. In seven of the studies, the students were Braille readers (1, 2, 6, 7, 8, 10, 11), while print readers were included in two studies (2, 10). Five studies reported gender in the sample characteristics (1, 5, 7, 8, 11), but only 2 studies reported gender in the results (5, 7). Four studies reported teachers’ attitudes and experiences (2, 3, 4, 10).

Seven of the eleven articles are published in the Journal of Visual Impairment and Blindness, two in the Journal of Special Education, one in the Journal of Disability Policy Studies, one in the International Journal of Science Education, and one in the Journal of Experimental Psychology.

4. Discussion
The purposes of our systematic review was to evaluate the quality of the research and the effect of any interventions, with the aim of summarizing the evidence-based academic knowledge about mathematics education among students with VI. Our findings demonstrate the dearth of intervention studies of mathematics education and students with VI. This shortage is problematic in the field of visual impairment and mathematics education because it has an impact on the understanding of the best practices of students with VI. In addition, due to the different aims, methods, and samples reported in the 11 articles, it is to some extent difficult to summarize them with respect to evidence-based knowledge about mathematics education among students with VI. However, to facilitate a more comprehensive understanding of what can be learned from the research carried out to date, we have summarized the results under the following categories: Learning environment, Realistic expectations, Explicit and individual teaching, and Embodied mathematics.

4.1. Learning environment
Three articles report environmental factors that affect the mathematics learning process (Giesen, Cavenaugh, & McDonnall, 2012; McDonnall, Cavenaugh, & Giesen, 2012; Rule et al., 2011). McDonnall et al. (2012) recommend that both schools and parents should strive for continued high levels of parents’ involvement in schools. Nevertheless, learning independency is a main goal for students with VI, and the balance between self-determination and involvement can be a challenge.

In many cases, students with VI attend mainstream schools and are often the only student with vision impairment in their class (Augestad, 2017b; de Verdier, 2018). Rule et al. (2011) found that teachers’ experiences were that many mathematics lessons were difficult to teach when a student had vision loss. Nevertheless, many teachers experienced that an advantage of having a student with VI was that all students in the classroom benefited from using the tactile or electronic enlargement adaptations and receiving more modified and descriptive explanations.

The results of the study by Rule et al. (2011) support that given knowledgeable, supportive teachers and appropriate mathematics materials, students with VI can participate more fully in class activities. To choose the appropriate teaching strategies in mathematics requires highly qualified teachers, with enthusiasm and interest in achieving the curriculum goals set for their students. Rule et al. (2011, pp. 875–876) claim that teachers showed great motivation to modify their teaching strategies based on appropriateness for their students. The development of self-
evaluation skills is important for all students because it increases their self-esteem. Teachers who have both been made aware of and assisted in the purchase or creation of adaptive materials and activities have reported positive experiences for their students with VI, their classmates, and for themselves (Rule et al., 2011).

Giesen et al. (2012) similarly emphasize the consequences of well-educated teachers for academic support in schools regarding mathematics achievement for students with VI. However, the extent of school support and mathematics achievement is moderated when a student has a cognitive disability. One possible explanation for this surprising result is that students with VI and mental retardation, autism, traumatic brain injury, or multiple disabilities may have more varied needs for support (Giesen et al., 2012). This suggestion is supported by de Verdier (2018).

4.2. Realistic expectations

The findings from the review confirm that students with VI are a heterogeneous group with respect to achievement in mathematics (Klingenberg, Fosse, and Augestad (2012); Rule et al. (2011)). In a longitudinal sample studied over 40 years, Klingenberg et al. (2012) found that the majority (57%) of blind and partially sighted learners were taught mathematics at grade level (i.e. the same level as their sighted peers). In addition, there is evidence that blind and partially sighted learners performed well in STEM subjects and in some cases excelled in them. Rule et al. (2011) report that when students with VI were given appropriate curricular materials, 10 out of 12 were as successful in mathematics as their peers. These results show that it is important to expect that students with VI will perform as well as their sighted peers. However, we agree with Warren (1994, pp. ix–x) that teachers must provide appropriate learning environments and methods in order to ensure an optimal developmental course for their students.

The limited amount of explicit advice on either teaching methods or teaching materials is challenging for teachers of students with VI. The least ambiguous advice that emerged from our review is that the competence and attitudes of teachers improve significantly when they reflect on their own practice together with a supervisor.

If students with VI struggle with mathematics, teachers must consider whether they have cognitive difficulties. Loss of vision may not be the primary reason for learning difficulties in the subject. Erin and Koenig (1997) point out that visual disabilities and learning disabilities have many similar symptoms and that early identification of a visual disability often masks the presence of a learning disability. Some visual diagnoses have a neurological origin and may coexist with a learning disability.

4.3. Explicit and individual teaching

The use of graphics is one example of a curriculum theme for which students who are tactile readers may need additional time and direct instruction in order to learn specific skills and strategies (Rosenblum & Herzberg, 2015; Zebehazy & Wilton, 2014). Such students need accurate information and specific feedback about their skills to know whether they have completed a task accurately. Their efficiency may improve if new types of graphical materials are introduced systematically (Rosenblum & Herzberg, 2015), and it is important for them to have early exposure to new tactile materials (Zebehazy & Wilton, 2014).

The abacus can be a good learning aid for all students and its usage can be taught for a whole class of students. However, Rosenblum et al. (2013) support the findings by Ferrell (2011), as they found inconsistency in the teachers’ opinions on the use of the abacus as a valuable tool for computing in mathematics. However, most of the teachers considered that training in the use of the abacus should be included in their proposed list of teacher training competencies.

Abacus instruction is often introduced between preschool and second grade (5–7 years of age), but division with two-digit divisors is taught only to a relatively small number of students (Amato,
Amato et al.’s finding is in line with our personal experiences of the Norwegian school system, in which some teachers have not mastered the computation skill. They found that when teachers lacked knowledge about the use of abacuses, they hesitated to introduce them in their teaching (Amato et al., 2013; Rosenblum et al., 2013).

4.4. Embodied mathematics

Amato et al. (2013) recommend the use of an abacus to teach the concept of one-to-one correspondence. Their interesting recommendation relates to the study conducted by Crollen, Mahe, Collignon, and Seron (2011), who conclude that compared with sighted students, students with VI develop an understanding of numbers through different strategies. In their study, students who were blind did not spontaneously use their fingers to count and show quantities, probably because finger-counting and finger-counting strategies rely on visual recognition of particular hand shapes (Crollen et al., 2011).

Although fingers may not play a functional role in the development of a mature counting system in students with VI, Klingenberg (2012) and M. G. Jones, Forrester, Robertson, Gardner, and Taylor (2012) have observed how students with VI use their bodies as a measurement tool when making linear and spatial estimations. Their findings indicate there is an unconscious and intrinsic way of “seeing spatial object information”, but it is likely that the perceived distances are different from those measured in metric units. Hence, teachers must help a student with VI to connect informal experiences with the measuring skills required in formal mathematics. M. G. Jones et al. (2012) conclude that research is needed to document how students who use tactile learning media are able to form concepts of metric units and apply them in concrete and abstract contexts. This finding is in accordance with Klingenberg’s conclusion that students must be challenged to use measuring units and spatial concepts in many different activities and contexts (Klingenberg, 2012).

4.5. Gender

Lee and Kung (2018) found gender variations with respect to self-concept and achievement in mathematics. Boys had significantly higher score on self-concept compared with girls, whereas girls exhibited higher levels of achievement compared with boys. The finding is confirmed by the work of Ehrtmann and Wolter (2018), who report an interaction effect of gender and gender role orientation in mathematics. The presence of gender as a variable in a study design may therefore be important when students with VI are included in the study sample. However, the results in the reviewed articles did not show clear gender differences in attitude and skills in mathematics among the students with VI. Furthermore, the sample sizes were small in most of the studies.

4.6. Authors and journals

Most of the authors of the reviewed articles represent the academic fields of education and teaching. We consider it important that teachers with experience of practical teaching in schools should participate in study designs and report their knowledge. Their collaboration with members of other professionals may ultimately lead to mathematical education that is more beneficial for students with VI, since an academic background and competence may influence the focus of a study and the interpretation of published results.

Only 1 of the 11 articles is published in an open access journal. The results may therefore be more readily available for personnel in institutions with large journal subscriptions, but harder for educators and caregivers to access.

4.7. Eye disorder diagnosis

Students with VI are a heterogeneous group, not least because they have a wide range of visual functions. Therefore, it is important to account for visual diagnoses, visual condition, and visual history disorders (Augestad, Klingenberg, & Fosse, 2012; Rahi & Cable, 2003; Warren, 1994, pp. 196–203).
Four articles report eye disorder diagnoses (1, 6, 7, 11). The other articles do not define VI clearly. The diversity of eye pathologies implies that students differ in their preferences and possibilities concerning the choice of informatics tools (Carrière, 2012). In cases where information about sight functions were overlooked in the studies, the results reported in the articles have less validity. In addition, pupils with congenital VI often have multiple disabilities with coexisting neurodevelopmental disorders (de Verdier, Ek, Löfgren, & Fernell, 2018; Klingenberg et al., 2012), yet relevant information is lacking in most of the reviewed articles.

4.8. Quality of the studies
After using the Quality Assessment Tool for Studies with Diverse Designs (QATSDD) to evaluate the selected studies holistically, we classified only four studies as high quality. The included studies had a moderately explicit theoretical framework and did not include a representative sample of a target group of reasonable size. Additionally, for some of the studies, the rationale for the choice of data collection was scored as moderate. A further reason for the lack of high quality scores for the studies was the fit between stated research question and the format and content of the data collection tool. Most of the studies did not have good justification for the analytical method selected. We also registered that few authors discussed the strengths and limitations critically.

Ethical considerations are important in all scientific work. Seven of the 11 publications do not mention the word “ethics” and the authors do not inform about whether they had any approval from an ethical committee. This may be explained by different rules in different countries, and that the study design in some of the included studies did not require ethical approval.

4.9. Strengths and limitations
A strength of our search for literature was that the second author is an experienced professional librarian. A further strength is that we searched multiple databases, which resulted in articles from different academic fields. In addition, teaching mathematics for pupils with VI is the main field of interest for the first author, who has a PhD in the field and 40 years of work experience with special education for pupils with VI.

By contrast, a limitation of our literature search was that different definitions of VI were used in the studies. Most of the publications do not mention the diagnoses, progression of the disease, or the onset age of vision loss. In some of the studies, the age range of the participants was wide, which may have affected the results.

Confounding may have occurred in some of the studies. In addition, we strongly suspect there was bias in the studies that lacked randomization, especially those that included only a small convenience sample of children with VI. Unfortunately, most of the studies used this approach to obtain a more homogeneous sample. Thus, the study subjects might not have been representative of the target population. The conclusions drawn in some of the studies might not have been accurate in the cases where selection bias was not taken into account.

5. Conclusions
There is a lack of mathematics education research in the field of VI and much of the existing literature was published a long time ago (Ferrell, 2011; Holbrook, 2015). A lack of up-to-date research is problematic because it has an impact on the understanding of best practices of students with VI. To choose the right teaching strategies requires educated and enthusiastic teachers who enable students with VI to experience feelings of accomplishment and success.

The studies reported in the reviewed articles had different aims and methods, which made comparison and systematic analysis difficult. In general, it is challenging to conduct research on the education of students with VI. One factor is that visual impairment is a low-incidence disability and therefore it is problematic to identify an adequate and homogeneous group of study participants and to obtain enough
statistical power in a study (Ferrell, 2011). In addition, conducting research within the population is often expensive and logistically difficult (Ferrell, 2011; Holbrook, 2015). Convenience sampling was used in many of the studies reported in the reviewed articles, with a wide age range in the samples and small samples sizes. This finding is in line with findings of earlier reviews that included students with VI (Augestad, 2017a, 2017b). Our review highlights the need for increased numbers of randomized controlled trials of high quality to secure more evidence-based knowledge about mathematical education among students with VI.

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Author details
Oliv G. Klingenberg 1
E-mail: oliv.klingenber@statped.no
Anne H. Holkesvik 2
E-mail: anne.hildrum.holkesvik@statped.no
Liv Berit Augestad  3
E-mail: liv.berit.augestad@ntnu.no
ORCID ID: http://orcid.org/0000-0002-9466-5382
  1 Department of Visual Impairment, Statped midt, Heimdal, Norway.
  2 Department of Neurology and Movement Science, Norwegian University of Science and Technology (NTNU), Trondheim, Norway.

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The authors declare that there are no conflicts of interest regarding this article.

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References
Akar, N., & Övez, F. T. D. (2018). An anthropological analysis of the knowledge on graphics within middle school mathematics. Journal of Education and Future, 13, 95–119.

Amato, S., Hong, S., & Rosenblum, L. P. (2013). The obs: Instruction by teachers of students with visual impairments. Journal of Visual Impairment & Blindness, 107(4), 262–273. doi:10.1177/0145482X13130700403

Augestad, L. B. (2017a). Mental health among children and young adults with visual impairments: A systematic review. Journal of Visual Impairment & Blindness, 111(5), 411–425. doi:10.1177/0145482X1711100503

Augestad, L. B. (2017b). Self-concept and self-esteem among children and young adults with visual impairment: A systematic review. Cogent Psychology, 4(1), 1319652. doi:10.1080/23311908.2017.1319652

Bateman, A., Zhao, O. K., Bajcsy, A. V., Jennings, M. C., Toth, B. N., Cohen, A. J., ... Oliveira, M. A. (2018). A user-centered design and analysis of an electrostatic haptic touchscreen system for students with visual impairments. International Journal of Human-Computer Studies, 109, 102–111. doi:10.1016/j.ijhcs.2017.09.004

Beal, C. R., & Rosenblum, L. P. (2018). Evaluation of the effectiveness of a tablet computer application (App) in helping students with visual impairments solve mathematics problems. Journal of Visual Impairment & Blindness, 112(1), 5–19. doi:10.1177/0145482X1811200102

Beal, C. R., & Shaw, E. (2008). Working memory and math problem solving by blind middle and high school students: Implications for universal access. Paper presented at the Society for Information Technology & Teacher Education International Conference.Las Vegas, Nevada, USA.

Bin Tuwaym, S. T., & Berry, A. B. (2018). Assistive technology for students with visual impairments: A resource for teachers, parents, and students. Rural Special Education Quarterly, 37(4), 219–227. doi:10.1177/00000001355518773397

Breckler, S. J., Olson, J. M., & Wiggins, E. C. (2006). Social psychology alive. Belmont, CA: Thomson/Wadsworth.

Brooks, R. B. (1992). Self-esteem during the school years: Its normal development and hazardous decline. Pediatric Clinics of North America, 39(3), 537–550. doi:10.1016/S0031-3955(16)38343-2

Carrière, V. (2012). ICT and cooperation in learning/teaching in visually handicapped situation. Procedia – Social and Behavioral Sciences, 46, 701–705. doi:10.1016/j.sbspro.2012.05.184

Crolien, V., Mahe, R., Collignon, G., & Sera, X. (2011). The role of vision in the development of finger-number interactions: Finger-counting and finger-monting in blind children. Journal of Experimental Child Psychology, 109(4), 525–539. doi:10.1016/j.jecp.2011.03.011

Cryer, H. (2013). Teaching STEM subjects to blind and partially sighted learners RNIB website. Retrieved from https://www.rnib.org.uk/knowledge-and-research-hub/research-reports/education-research/stem

de Verdier, K. (2018). Children with blindness: Developmental aspects, comorbidity and implications for education and support (Doctoral dissertation). Department of Special Education, Stockholm University. Retrieved from https://www.diva-portal.org/smash/get/diva2:1206093/FULLTEXT01.pdf
Major causes, development and implementation of K-12 mathematics education in Israel: Issues and challenges.

Ehrtmann, L., & Wolter, I. (2018). The impact of students' gender-role orientation on competence development in mathematics and reading in secondary school. Learning and Individual Differences, 61, 256–264. doi:10.1016/j.lindif.2018.01.004

Emerson, R. W., & Anderson, D. (2018). What mathematical images are in a typical mathematics textbook? Implications for students with visual impairments. Journal of Visual Impairment & Blindness, 112(1), 20–32. doi:10.1177/0145482X18112001013

Erin, J. N., & Koenig, A. J. (1997). The student with a visual disability and a learning disability. Journal of Learning Disabilities, 30(3), 309–320. doi:10.1177/002221949703000306

Ferrell, K. A. (2011). What do we know, and how do we know it? The Educator, 57(2), 13–19. Retrieved from https://www.jumpa.com/en/document/full_screen/5814944/pdf-version-icevi

Ferrell, K. A., Buettel, M., Sebold, A. M., & Pearson, R. (2006). American Printing House for the blind mathematics research analysis. The University of Northern Colorado, National Center on Severe and Sensory Disabilities. Retrieved from http://www.pathstoliteracy.org/research/american-printing-house-blind-mathematics-research-analysis

Freeland, A. L., Emerson, R. W., Curtis, A. B., & Fogarty, K. (2018). Gender-role orientation on competence development and reciprocal relations among junior high school students in Taiwan. Eurasia Journal of Mathematics, Science and Technology Education, 14(4), 1239–1252.

McDonnell, M. C., Cavenaugh, B. S., & Giesen, J. M. (2012). The relationship between parental involvement and mathematics achievement for students with visual impairments. Journal of Special Education, 45(4), 204–215. doi:10.1177/0022466910365169

Metatla, O., Serrano, M., Joffraiss, C., Thieme, A., Kane, S., Branham, S., & Bennett, C. L. (2018). Inclusive education and innovations: Emerging opportunities for people with visual impairments. Paper presented in the Extended Abstracts of the 2018 CHI Conference on Human Factors in Computing Systems, Montreal, Canada.

Papadopoulos, K., Metsiou, K., & Agaliotis, I. (2011). Adaptive learning of children and adolescents with visual impairments. Research in Developmental Disabilities, 32(3), 1086–1096. doi:10.1016/j.ridd.2011.01.021

Pitchford, N. J., Kamchchedzera, E., Hubber, P. J., & Chigeda, A. L. (2018). Interactive apps promote learning of basic mathematics in children with special educational needs and disabilities. Frontiers in Psychology, 9, 262. doi:10.3389/fpsyg.2018.00262

Rahi, J. S., & Cable, N. (2003). Severe visual impairment and blindness in children in the UK. The Lancet, 362(9393), 1359–1365. doi:10.1016/S0140-6736(03)14631-4

Rapp, D., & Rapp, A. (1992). A survey of the current status of visually impaired students in secondary school Mathematics. Journal of Visual Impairment and Blindness, 86(2), 115–117.

Rosenblum, L. P., Cheng, L., & Beal, C. R. (2018). Teachers of students with visual impairments share experiences and advice for supporting students in understanding graphics. Journal of Visual Impairment & Blindness, 112(5), 475–487. doi:10.1177/0145482X1811200505

Rosenblum, L. P., & Herzberg, T. S. (2015). Braille and tactile graphics: Yths with visual impairments share their experiences. Journal of Visual Impairment & Blindness, 109(6), 173–184. doi:10.1177/0145482X1510900302

Rosenblum, L. P., Hong, S., & Amato, S. (2013). The abacus: Teachers' preparation and beliefs about their abacus preservice preparation. Journal of Visual Impairment & Blindness, 107(4), 274–286. doi:10.1177/0145482X1310700604

Rule, A. C., Stefanich, G. P., Boody, R. M., & Peiffer, B. (2011). Impact of adaptive materials on teachers and their students with visual impairments in secondary science and mathematics classes. International Journal of Science Education, 33(6), 665–687. doi:10.1080/09500693.2010.506200

Saigol, S. (2012). Educational Support for Blind Women and Girls in Egypt. Center for the Advancement of Disability Studies. University of South Carolina. Retrieved from http://www.pathstoliteracy.org/research/american-printing-house-blind-mathematics-research-analysis

Sensory Disabilities. Retrieved from stoliteracy.org/research/american-printing-house-blind-mathematics-research-analysis

Taylor, A. R. (2002). Conceptual understanding of the definite integral. Journal of Educational Studies in Mathematics, 47(3), 215–234. doi:10.1023/A:1018086801794-z

Klingenberg, O. G. (2012). Conceptual understanding of shape and space by Braille-reading Norwegian students in elementary school. Journal of Visual Impairment & Blindness, 106(8), 453–465. doi:10.1177/0145482X1210600802

Klingenberg, O. G., Rose, P., & Augestad, L. B. (2012). An examination of 40 years of mathematics education among Norwegian Braille-reading students. Journal of Visual Impairment & Blindness, 106(2), 93–105. doi:10.1177/0145482X1210600204

Lee, C.-Y., & Kung, H.-Y. (2016). Math self-concept and mathematics achievement: Examining gender variation and reciprocal relations among junior high school students in Taiwan. Eurasia Journal of Mathematics, Science and Technology Education, 14(4), 1239–1252.
Sirriyeh, R., Lawton, R., Gardner, P., & Armitage, G. (2012). Reviewing studies with diverse designs: The development and evaluation of a new tool. *Journal of Evaluation in Clinical Practice, 18*(4), 746–752. doi:10.1111/j.1365-2753.2011.01662.x

Supalo, C. A., Hill, A. A., & Larrick, C. G. (2014). Summer enrichment programs to foster interest in STEM education for students with blindness or low vision. *Journal of Chemical Education, 91*(8), 1257–1260. doi:10.1021/ed400585v

Supalo, C. A., Isaacson, M. D., & Lombardi, M. V. (2013). Making hands-on science learning accessible for students who are blind or have low vision. *Journal of Chemical Education, 91*(2), 195–199. doi:10.1021/ed3000765

Warren, D. H. (1994). Blindness and children: An individual differences approach. Cambridge, UK: Cambridge University Press.

Yurtay, N., Yurtay, Y., & Adak, M. F. (2015). An education portal for visually impaired. *Procedia – Social and Behavioral Sciences, 171*, 1097–1105. doi:10.1016/j.sbspro.2015.01.271

Zebehazy, K. T., & Wilton, A. P. (2014). Quality, importance, and instruction: The perspectives of teachers of students with visual impairments on graphics use by students. *Journal of Visual Impairment & Blindness, 108*(1), 5–16. doi:10.1177/0145482X1410800102