The running of coupling constants and unitarity in a finite electroweak model

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We investigate the properties of a Finite Electroweak (FEW) Theory first proposed in 1991 [1–5]. The theory predicts a running of the electroweak coupling constants and a suppression of tree level amplitudes, ensuring unitarity without a Higgs particle. We demonstrate this explicitly by calculating $W^+_L W^-_L \rightarrow W^+_L W^-_L$ and $e^+ e^- \rightarrow W^+_L W^-_L$ in both the electroweak Standard Model and the FEW model.

1. Introduction

In the Standard Model (SM) of particle physics, the Higgs field, a spin zero field with a nonzero vacuum expectation value is proposed, leading to spontaneous symmetry breaking and a generation of fermion and vector boson masses. Additionally, the Higgs field also resolves the issue of unitarity, by precisely canceling out badly behaved terms in the tree-level amplitude of processes involving longitudinally polarized vector bosons, for instance $W^+_L W^-_L \rightarrow W^+_L W^-_L$ or $e^+ e^- \rightarrow W^+_L W^-_L$. The challenge to any theory that aims to compete with the SM without introducing a Higgs particle is to generate the correct fermion and boson masses on the one hand, and ensure unitary behavior for these types of scattering processes on the other.

First proposed in 1991 [1], our Finite Electroweak (FEW) Theory employs three principal ideas [5]:

- Fermions acquire dynamically generated masses through self-energy [4];
- A non-local regularization scheme [6,7] ensures that the theory is finite to all orders and does not violate unitarity;
- A non-trivial choice of the path integral measure factor, consistent with the regularization scheme, leads to spontaneous symmetry breaking and the generation of vector boson masses.

The theory is described in detail in a companion paper [5]. In the present work, we focus on a key prediction of the theory: the running of the off-shell $W$ and $Z$ boson masses. This prediction leads to a running of the electroweak coupling constants $g$, $g'$ and $e$, as we demonstrate below. This running of the coupling constants is sufficient to suppress the scattering amplitudes at high energies, ensuring that the amplitudes remain finite and that, as anticipated, unitarity is indeed not violated.

In the first part of this paper, we show how the massive vector boson propagator of the FEW theory leads to a running of the off-shell mass and the consequent
running of the electroweak coupling constant. In the second part, we calculate the on-shell rest masses of the $W$ and $Z$ bosons along with the $\rho$ parameter. Lastly, in the third part we demonstrate via explicit calculation how the running of $g$ is sufficient to suppress the scattering amplitude in $W^+_L W^-_L \to W^+_L W^-_L$ and $e^+e^- \to W^+_L W^-_L$ processes.

In this paper, we use the metric signature $(+,−,−,−)$, and set $\hbar = c = 1$. The Feynmann rules of the FEW theory are listed in the Appendix.

2. Vector boson masses and the running of the electroweak coupling constant

In the FEW theory, vector boson propagators take the form [5]:

$$iD_{\mu\nu}^f = -i \left( \frac{\eta_{\mu\nu} - p_{\mu} p_{\nu}}{p^2 - \Pi_{\mu\nu}^T} + \frac{\xi p_{\mu} p_{\nu}}{p^2 - \xi \Pi_{\mu\nu}^L} \right),$$

(1)

where $\Pi_{\mu\nu}^T$ and $\Pi_{\mu\nu}^L$ are the transversal and longitudinal components, respectively, of the vacuum polarization tensor $\Pi_{\mu\nu}$ due to fermion loops, and are given by

$$\Pi_{\mu\nu} = \Pi_{\mu\nu}^T (\eta_{\mu\nu} - p_{\mu} p_{\nu}) + \Pi_{\mu\nu}^L p_{\mu} p_{\nu}.$$  

(2)

It is easy to see that the propagator (1) reduces to the SM vector boson propagator if we set $\Pi_{\mu\nu}^T = \Pi_{\mu\nu}^L = m^2_V$, where $m_V$ is the vector boson rest mass. For longitudinally polarized vector bosons, the gauge-dependent term in the propagator vanishes, as can be verified by explicit calculation. In what remains, $\Pi_{\mu\nu}^T$ replaces the usual mass squared term in the denominator.

The value of $\Pi_{\mu\nu}^T$ has been calculated explicitly in the FEW theory [5] as a function of the four-momentum $q$. For $W^\pm$ bosons:

$$\Pi_{W^\pm}^T(q^2) = \frac{g^2 \Lambda_W^2}{(4\pi)^2} \sum_{q^L} (K_{m_1 m_2}(q^2) - L_{m_1 m_2}(q^2) + 2 P_{m_1 m_2}(q^2)),$$

(3)

where $q^L$ indicates summation over left-handed fermion doublets with masses $m_1$ and $m_2$, $\Lambda_W$ is the non-local energy scale, and

$$K_{m_1 m_2}(q^2) = \int_0^{\frac{q^2}{\Lambda_W^2}} d\tau (1 - \tau) \left[ \exp \left( -\tau \frac{q^2}{\Lambda_W^2} - f_{m_1 m_2} \right) + \exp \left( -\tau \frac{q^2}{\Lambda_W^2} - f_{m_2 m_1} \right) \right],$$

(4)

$$P_{m_1 m_2}(q^2) = -\frac{q^2}{\Lambda_W^2} \int_0^{\frac{q^2}{\Lambda_W^2}} d\tau (1 - \tau) \left[ E_1 \left( \frac{q^2}{\Lambda_W^2} + f_{m_1 m_2} \right) + E_1 \left( \frac{q^2}{\Lambda_W^2} + f_{m_2 m_1} \right) \right],$$

(5)

$$L_{m_1 m_2}(q^2) = \int_0^{\frac{q^2}{\Lambda_W^2}} d\tau (1 - \tau) \left[ f_{m_1 m_2} E_1 \left( \frac{q^2}{\Lambda_W^2} + f_{m_1 m_2} \right) + f_{m_2 m_1} E_1 \left( \frac{q^2}{\Lambda_W^2} + f_{m_2 m_1} \right) \right],$$

(6)
The running of the vector boson masses as a function of momentum is illustrated in Fig. 1.

When we rewrite the theory's Lagrangian in terms of massive vector bosons \[4,5\], the Lagrangian picks up a finite mass contribution from the total sum of polarization graphs:

\[
L_m = \frac{1}{8} v^2 g^2 [(W_1^\pm)^2 + (W_2^\pm)^2] + \frac{1}{8} v^2 [g^2 (W_3^\pm)^2 - 2g g' W_3^\pm B + g'^2 B^2]
\]

\[
= \frac{1}{4} g^2 v^2 W_+^\mu W_-^\mu + \frac{1}{8} v^2 (W_3^\pm, B_\mu) \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix},
\]

where \( W_\pm^\mu = (W_1^\mu \pm i W_2^\mu)/\sqrt{2} \) and \( v \) is the electroweak symmetry breaking scale (which, in the SM, is the vacuum expectation value of the Higgs scalar). We see that we have the usual symmetry breaking mass matrix in which one of the eigenvalues
of the $2 \times 2$ matrix in (10) is zero, which leads to:

$$m_W = \frac{1}{2} v g, \quad m_Z = \frac{1}{2} v (g^2 + g'^2)^{1/2}, \quad m_A = 0.$$  \hfill (11)

Consistency between (9) and (11) requires the running of the constants $g$ and $g'$ in (11). Starting with the $W$ mass, we obtain

$$\frac{g^2(q^2)}{g^2(m_Z^2)} = \frac{\Pi_W^T(q^2)}{\Pi_W^T(m_Z^2)}. \hfill (12)$$

This relationship defines a running of the electroweak coupling constant $g$, which is illustrated in Fig. 2 (left). Furthermore, from (11) and (12), we compute

$$v^2 = \frac{4\Pi_W^T(m_Z^2)}{g^2(m_Z^2)} \simeq (245 \text{ GeV})^2. \hfill (13)$$

In a similar fashion, using the $Z$ mass we obtain

$$\frac{g^2(q^2) + g'^2(q^2)}{g^2(m_Z^2) + g'^2(m_Z^2)} = \frac{\Pi_Z^T(q^2)}{\Pi_Z^T(m_Z^2)}, \hfill (14)$$

which establishes the running of $g'(q^2)$. These relationships also allow us to calculate the running of the Weinberg angle $\theta_w$, which is defined through the ratio of the coupling constants $g$ and $g'$ as

$$\cos \theta_w = \sqrt{\frac{g^2 + g'^2}{g}}. \hfill (15)$$

The running of $\sin^2 \theta_w$ is shown in Fig. 2 (right).

We emphasize that the running of $g$ and $\theta_w$ described in this section arises as a result of the running of the vector boson masses, and does not incorporate radiative corrections, which are the usual origin of the (much more moderate) running of these quantities in the SM.

3. Vector boson masses and the $\rho$ parameter

In the previous section, we demonstrated how the rest masses of off-shell vector bosons run with energy. The same formalism can be used to compute the on-shell
rest mass of vector bosons by demanding that the following consistency equation be satisfied:

\[ m_V^2 = \Pi_T^f(m_V^2), \quad (V = W, Z). \] (16)

In particular, we can postulate this equation for the Z-boson \((V = Z)\). The left-hand side becomes the Z-boson mass, which is known to high accuracy. The expression on the right-hand side involves fermion masses, the electroweak coupling constant, the Weinberg angle, and the non-local energy scale \(\Lambda_W\). Of these, all but \(\Lambda_W\) are known from experiment; therefore, the equation

\[ m_Z^2 = \Pi_T^{Z_f}(m_Z^2, \Lambda_W), \] (17)

where we explicitly indicated the dependence of the right-hand side on \(\Lambda_W\), can be solved for \(\Lambda_W\). Solving numerically, we obtain

\[ \Lambda_W \simeq 541.9 \text{ GeV}, \] (18)

where the precision of \(\Lambda_W\) is determined by the accuracy to which \(m_Z\) is known, and is not significantly affected by the lack of precision of the known fermion masses, including that of the top quark. With this result at hand, we can move on to the equation for the W-boson mass,

\[ m_W^2 = \Pi_T^{W_f}(m_W^2), \] (19)

which we can solve for \(m_W\), to obtain

\[ m_W \simeq 80.05 \text{ GeV}. \] (20)

The comparable prediction from the SM without radiative corrections is \(m_W \simeq 79.95 \text{ GeV}\). The fact that our predicted value is slightly closer to the observed value of \(m_W = 80.398 \pm 0.025 \text{ GeV}\) than the SM prediction is consistent with our expectation that radiative corrections, which we have not yet calculated, will be somewhat smaller in magnitude due to suppression by the non-local operator than in the SM.

The relationship between the vector boson masses and the Weinberg angle is customarily captured in the form of the parameter

\[ \rho = \frac{m_W^2}{m_Z^2 \cos^2 \theta_w}. \] (21)

Using the observed value of \(m_Z = 91.1876 \pm 0.0021 \text{ GeV}\), the computed value of \(m_W\), and the Weinberg angle at the Z-pole \((\sin^2 \theta_w = 0.2312 \text{ GeV})\), we get the non-trivial result

\[ \rho \simeq 1.0023. \] (22)

\textsuperscript{a}The calculation presented in this section is, in effect, the reverse of that performed in \(\textsuperscript{[2]}\); instead of using the known values of \(m_W\) and \(m_Z\) to obtain \(\Lambda_W\) and \(m_t\), we use \(m_Z\) and \(m_t\) to obtain \(\Lambda_W\) and \(m_W\), and contrast the latter with observation.
We note, however, that before the computed value of $\rho$ can be compared to observation, radiative corrections must be taken into account, as their magnitude is similar to the difference between our computed value of $\rho$ and the trivial SM tree-level value of $\rho = 1$.

4. $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$ scattering

Without the Higgs particle, the SM would violate unitarity in scattering processes that involve longitudinally polarized vector bosons. The scattering of two vector bosons results in a divergent term proportional to $s$. A less rapid divergence, proportional to $\sqrt{s}$, occurs when fermions annihilate into a pair of vector bosons.

In the SM, these divergences are canceled by terms due to tree-level processes that involve the Higgs boson. A theory that does not incorporate a Higgs scalar must offer an alternate mechanism to either cancel or suppress the badly behaved terms, in order to maintain unitarity.

The scattering of two longitudinally polarized $W$ vector bosons can take place through one of the following processes:

\[ W^- \gamma/Z^0 W^- + W^+ \gamma/Z^0 W^+, \]

in addition to the 4$W$ vertex shown in (A-12).

In order to facilitate an efficient calculation, we use the center-of-mass frame of reference and align the momenta of the incoming particles with the $x_3$-axis. Furthermore, we align the frame such that the scattering takes place in the $x_2x_3$ plane. In this frame of reference, the momenta of the incoming and outgoing particles can be written as

\[ \begin{aligned}
    p_1 &= \{E, 0, 0, |p|\}, \\
    p_2 &= \{E, 0, 0, -|p|\}, \\
    p_3 &= \{E, 0, |p|\sin\theta, |p|\cos\theta\}, \\
    p_4 &= \{E, 0, -|p|\sin\theta, -|p|\cos\theta\},
\end{aligned} \]

where $E$ is the energy of the incoming $W^-$ particle, $p$ is its 3-momentum, and $\theta$ is the scattering angle. In terms of the center-of-mass energy $\sqrt{s}$, these quantities can be expressed as

\[ \begin{aligned}
    E &= \frac{1}{2}\sqrt{s}, \\
    |p| &= \sqrt{\frac{s}{4} - m_W^2}.
\end{aligned} \]

The spatial components of polarization vectors of longitudinally polarized vector bosons are aligned with the respective 3-momenta; the inner product of the polarization vector and the 4-momentum is zero, i.e., $\eta^{\mu\nu} p_\mu \epsilon_\nu = 0$. From this, one can
compute the unit longitudinal polarization vectors as
\begin{align}
\epsilon_1 &= \frac{1}{m_W} \{ |p|, 0, 0, E \}, \\
\epsilon_2 &= \frac{1}{m_W} \{ |p|, 0, 0, -E \}, \\
\epsilon_3 &= \frac{1}{m_W} \{ |p|, 0, E \sin \theta, E \cos \theta \}, \\
\epsilon_4 &= \frac{1}{m_W} \{ |p|, 0, -E \sin \theta, -E \cos \theta \}.
\end{align}

To facilitate efficient calculation of the vertices in (23), we define the following function:
\begin{equation}
V_3(\epsilon_1, \epsilon_2, p_1, p_2) = -ig\left( (\epsilon_1 \cdot \epsilon_2)(p_1 - p_2) + \epsilon_2 [\epsilon_1 \cdot (p_1 + 2p_2)] - \epsilon_1 [\epsilon_2 \cdot (2p_1 + p_2)] \right).
\end{equation}

Utilizing this function and denoting the photon and \( Z \)-boson propagators as \( D_A^{\mu\nu} \) and \( D_Z^{\mu\nu} \), respectively, we can write the matrix element that corresponds to the s-channel processes in (23) as
\begin{equation}
i\mathcal{M}_s = -iV_3(\epsilon_1, \epsilon_2, -p_1, -p_2)\nu \nu V_3(\epsilon_4, \epsilon_3, p_4, p_3)\nu (\sin^2 \theta_w D_A^{\mu\nu} + \cos^2 \theta_w D_Z^{\mu\nu})
\end{equation}
\[\begin{aligned}
&= -ig^2 \cos \theta \frac{(2m_W^2 + s^2)(4m_W^2 - s)}{4m_W^4} \left[ \sin^2 \theta_w \frac{\cos^2 \theta_w}{(p_1 + p_2)^2} + \frac{\cos^2 \theta_w}{(p_1 + p_2)^2 - \Pi_{Zf}^T} \right],
\end{aligned}\]
where we used the vector boson propagators of the FEW theory.

The t-channel processes in (23) can be calculated similarly:
\begin{equation}
i\mathcal{M}_t = -iV_3(\epsilon_1, \epsilon_3, -p_1, p_3)\nu \nu V_3(\epsilon_4, \epsilon_2, p_2, -p_1)\nu (\sin^2 \theta_w D_A^{\mu\nu} + \cos^2 \theta_w D_Z^{\mu\nu})
\end{equation}
\[\begin{aligned}
&= ig^2 \left[ 2c_1 m_W^2 - (2c_1 - 3)(5c_1 - 6) \frac{s^2}{2} + (c_2 + 8c_1 - 12)(c_1 - 2) \frac{s^2}{8m_W^2} \\
&\quad - (c_1 - 2)^2 (c_1 + 2) \frac{s^2}{32m_W^2} \right] \times \left[ \sin^2 \theta_w \frac{\cos^2 \theta_w}{(p_1 - p_3)^2} + \frac{\cos^2 \theta_w}{(p_1 - p_3)^2 - \Pi_{Zf}^T} \right],
\end{aligned}\]
where we used the shorthand \( c_1 = \cos \theta + 1 \).

Finally, we need to evaluate the 4\( W \) graph (A-12):
\begin{equation}
i\mathcal{M}_4 = ig^2 \left[ (c_3^2 - 12) \frac{s^2}{16m_W^2} - (3c_3 - 10) \frac{s}{2m_W^2} \right],
\end{equation}
where we used \( c_3 = \cos \theta + 3 \).

Summing the matrix elements that we obtained so far, we get
\begin{equation}
i\mathcal{M}_M = i\mathcal{M}_s + i\mathcal{M}_t + i\mathcal{M}_4 = ig^2 \left[ (\cos \theta + 1)(4m_W^2 - 3\Pi_{Zf}^T \cos^2 \theta_w) \frac{s}{8m_W^4} + O(1) \right].
\end{equation}

In the SM, \( \Pi_{Zf}^T \cos^2 \theta_w = m_W^2 \) and we get
\begin{equation}
i\mathcal{M}_M = ig^2 \left[ \frac{\cos \theta + 1}{8m_W^2} s + O(1) \right],
\end{equation}
Fig. 3. The tree-level scattering amplitude of longitudinal $W^\pm$ bosons at a scattering angle $\theta = \pi/2$. The SM (blue dotted line) predicts an asymptotically constant amplitude at high energy. Without the Higgs particle (red dashed line) the amplitude is divergent. In the FEW theory (black solid line) this divergent amplitude is suppressed by the running of the electroweak coupling constant.

which clearly violates unitarity for large $s$. However, this behavior is corrected by the addition of the $s$-channel Higgs process:

\[
\begin{split}
W^- \quad H \quad W^- \\
W^+ \quad W^+ 
\end{split}
\]  

The associated matrix element becomes

\[
i\mathcal{M}_H = ig^2 \left\{ \sin^2 \theta s^3 + [m_H^2 (\cos^2 \theta - 2 \cos \theta + 5) + 8m_W^2 (\cos \theta - 1)]s^2 \\
+ 8[m_W^4 (3 - 5 \cos \theta) + m_W^2 m_H^2 (\cos \theta - 3)]s + 32[m_W^6 (\cos \theta - 1) + m_W^4 m_H^2] \right\}
\]

\[
\left/ 8m_W^2 (s - m_H^2) [\cos (\theta - 1)s + 4(\cos \theta - 1)m_W^2 + 2m_H^4] \right.
\]  

In the high energy limit, we get

\[
i\mathcal{M}_H = -ig^2 \left[ \frac{\cos \theta + 1}{8m_W^2} s + \mathcal{O}(1) \right],
\]

canceling out the bad behavior in (39). The resulting matrix element becomes

\[
i\mathcal{M}_{SM}(W^+_L W^-_L \rightarrow W^+_L W^-_L) = ig^2 \left[ \frac{\cos^2 \theta + 3}{4 \cos^2 \theta (1 - \cos \theta)} - \frac{m_H^2}{2m_W^2} + \mathcal{O}(s^{-1}) \right].
\]  

In the case of the FEW theory, no such additive cancelation takes place. However, the running of the electroweak coupling constant is such that at high $s$, $g(s)s \simeq \text{const.}$, which is sufficient to ensure that unitarity is not violated (Fig. 3).

Though this calculation is instructive, before the computed amplitudes can be used to make experimentally testable predictions, radiative corrections must be
taken into account. Computing the amplitude of $W^+W^- \rightarrow W^+W^-$ scattering with radiative corrections will be a subject of our future research.

5. $e^+e^- \rightarrow W^+_L W^-_L$ scattering

The production of $W^+_L W^-_L$ pairs from electron-positron collisions can take place via one of the following processes \[9\]:

$$e^- \gamma/Z^0 W^+ \rightarrow e^- W^-.$$ (44)

To set up the problem, we once again work in the center-of-mass system, with the momenta of the incoming electron and positron coinciding with the $x^3$-axis, and the scattering taking place in the $x^2x^3$ plane. Denoting the 4-momenta of the incoming $e^-$ and $e^+$ with $p_1$ and $p_2$, and those of the outgoing $W^-$ and $W^+$ with $p_3$ and $p_4$, we get

$$p_1 = \{E, 0, 0, |p_e|\},$$ (45)
$$p_2 = \{E, 0, 0, -|p_e|\},$$ (46)
$$p_3 = \{E, 0, |p| \sin \theta, |p| \cos \theta\},$$ (47)
$$p_4 = \{E, 0, -|p| \sin \theta, -|p| \cos \theta\},$$ (48)

where $E$ and $p$, the center-of-mass energies and $W$ 3-momenta are defined by \[28\] and \[29\], and the electron 3-momentum is

$$|p_e| = \sqrt{\frac{s}{4} - m_e^2}.$$ (49)

Using the Dirac basis for the $\gamma$-matrices, the spinor $u$ corresponding to a given fermion 3-momentum $p$ and spin $\sigma$ ($\sigma = \pm 1/2$) can be constructed as

$$u(p, \sigma) = \frac{1}{\sqrt{2}} \begin{pmatrix} A_+ - A_- \sigma \cdot p & 0 \\ 0 & A_+ + A_- \sigma \cdot p \end{pmatrix} \begin{pmatrix} \chi(\sigma) \\ \chi(\sigma) \end{pmatrix},$$ (50)

where

$$A_{\pm} = \sqrt{\sqrt{p^2 + m_e^2} \pm m_e},$$ (51)
$$\chi(\sigma) = \begin{pmatrix} 1/2 + \sigma \\ 1/2 - \sigma \end{pmatrix},$$ (52)

and the matrix-valued vector $\sigma$ comprises the three Pauli-matrices. As we are using a metric with $(+, -, -, -)$ signature, the Dirac conjugate of a spinor is given by

$$\bar{u} = u^\dagger \gamma^0.$$ (53)
Antiparticle spinors are written as

\[ v(p, \sigma) = 2\sigma\gamma^5 u(p, -\sigma). \] (54)

Denoting the spinors of the \( e^- \) and \( e^+ \) particles with \( u \) and \( v \), respectively, the \( s \)-channel annihilation through \( \gamma \) and \( Z^0 \) can be calculated as

\[ iM_s = -gV_3(\epsilon_3, \epsilon_4, p_3, p_4)_\mu \left[ \bar{v} \left( \frac{\gamma^\mu \sin^2 \theta_w}{s} + \frac{(1 - \gamma^5 - 4\sin^2 \theta_w)\gamma^\mu}{s + \Pi^s_{Zf}} \right) u \right]. \] (55)

In the limit of large \( s \), keeping only terms with powers of \( s \) higher than 0, this expression simplifies to

\[ iM_s = g^2 \left[ \sin \theta \frac{m_e^2}{4m_W^2} s + i \frac{m_e \cos \theta}{2m_W^2} \sqrt{s} + O(1) \right]. \] (56)

The term proportional to \( s \) in this divergent expression is canceled by the \( t \)-channel neutrino exchange. This can be written as

\[ iM_t = -\frac{g^2}{2} \bar{v} \gamma^\mu \left\{ \frac{1}{2} \gamma^5 \frac{(p_1 - p_3)^2 - m_e^2}{2} \gamma^\nu \frac{1 - \gamma^5}{2} u \epsilon_{3\mu}^* \epsilon_{4\nu}^* \right\}. \] (57)

In the high-energy limit, we get

\[ iM_t = g^2 \left[ -\frac{\sin \theta}{4m_W^2} s - i \frac{m_e(1 + \cos \theta)}{2m_W^2} \sqrt{s} + O(1) \right], \] (58)

leading to

\[ i(M_s + M_t) = -ig^2 \left[ \frac{m_e}{2m_W^2} \sqrt{s} + O(1) \right]. \] (59)

In the sum \( M_s + M_t \), the terms proportional to \( s \) cancel completely. These terms arise from the interaction of the ‘proper’ helicity components in the incoming electrons: \( e_L e^+_R \) or \( e_R e^+_L \). This is consistent with the fact that in the interaction Lagrangian, there are interaction terms combining fermion doublets with fermion doublets, and fermion singlets with fermion singlets, but no interaction terms combine fermion doublets and fermion singlets together.

However, as the electron is massive, its spinor exhibits schizophrenic ‘wrong’ helicity behavior [10], which leads to the terms proportional to \( \sqrt{s} \). These terms do not completely cancel, and unitarity is violated until one introduces the Higgs boson. A fermion-antifermion pair (and, in particular, a lepton-antilepton pair) can annihilate into a Higgs particle, which in turn can decay into a \( WW \) pair:

\[ \begin{array}{c}
  e^- \\
  H \\
  e^+ \\
\end{array} \quad \begin{array}{c}
  W^- \\
  W^+ \\
\end{array}. \] (60)

The matrix element associated with this process can be written as

\[ iM_H = ig^2 \bar{v} \frac{m_e}{2(s - m_H^2)} \gamma^\mu \epsilon_{3\mu}^* \epsilon_{4\nu}^*. \] (61)
Fig. 4. The scattering amplitude, at a scattering angle of $\theta = \pi/2$, of electrons and positrons annihilating into longitudinally polarized $W^\pm$ bosons as a function of the center-of-mass energy $\sqrt{s}$, measured in GeV. The SM result (blue dotted line) is indistinguishable from the SM result that was calculated without the Higgs particle (red dashed line), as due to the smallness of $m_e$, the divergent term that is proportional to $m_e \sqrt{s}$ does not begin to dominate until much higher energies. Our Higgless theory, however, predicts a significant suppression of the amplitude even at moderate energies.

This matrix element is identically zero if the electron and positron have opposite helicities. However, if they have the same helicity, we obtain

$$iM_H = ig^2 \frac{m_e}{2m_W^2} \frac{s - 2m_W^2}{s - m_H^2} \sqrt{s - 4m_e^2}. \quad (62)$$

At high $s$, we obtain

$$iM_H = ig^2 \left[ \frac{m_e}{2m_W^2} \sqrt{s} + O(1) \right], \quad (63)$$

which is exactly the term needed to cancel out the bad $\sqrt{s}$ terms we encountered earlier.

These results are shown in Fig. 4. Although the cancelation due to the Higgs boson is an important feature of the SM, this is not really visible at the energy levels that can be achieved by accelerator, due to the smallness of the electron mass. The FEW theory, however, does predict a significant suppression of the scattering amplitude even at moderate energies, and this may lead to experimentally testable predictions. However, before our predictions can be tested by experiment, it is important to account for radiative corrections, as at specific energies, the magnitude of these corrections may be comparable to the difference between our predictions and those of the SM.

6. Conclusions

In this paper, we studied some of the specific consequences of a Higgless quantum field theory first proposed by Moffat [1]. The non-local regularization scheme employed by the theory guarantees unitarity by design; we have seen the explicit manifestation of this effect through the computed running of the electroweak coupling constants and the resulting suppression of divergent amplitudes in processes.
that involve longitudinally polarized vector bosons. At energy levels that are reachable by either present day accelerators, the LHC, or accelerators that will become operational in the not too distant future, our theory makes predictions that noticeably differ from the predictions of the SM. However, before these predictions can be tested with precision, much work remains. Most importantly, the running of $g$, $\sin^2 \theta_w$, and the amplitudes of scattering processes must be calculated with radiative corrections taken into account, in order to remove the inevitable infrared divergences that arise in tree level results at small values of the scattering angle, and to achieve the desired precision in the results.

Appendix A. Feynman rules of the Standard Model and the FEW theory

To investigate $W_L^+W_L^- \rightarrow W_L^+W_L^-$ and $e^+e^- \rightarrow W_L^+W_L^-$ events, we require the following vertex rules and propagators (note that we use the $(+,−,−,−)$ metric signature):

\[ H \quad \text{---} \quad f \quad \frac{i}{p^2 - m_H^2}, \quad (A-1) \]

\[ \mu \quad \gamma \quad \nu \quad \frac{i}{p^2 - m_f^2}, \quad (A-2) \]

\[ \mu \quad Z^0 \quad \nu \quad -i \frac{1}{p^2} \left[ \eta^{\mu\nu} - (1 - \xi) \frac{p^\mu p^\nu}{p^2} \right], \quad (A-3) \]

\[ \mu \quad Z^0 \quad \nu \quad -i \frac{1}{p^2} \left[ \eta^{\mu\nu} p^2 - p^\mu p^\nu \right] + \frac{\xi p^\mu p^\nu}{p^2 - \xi p^2}, \quad (A-4) \]

\[ W_\alpha^+ \quad H \quad ig m_W \eta_{\alpha\beta}, \quad (A-5) \]

\[ W^-_\beta \quad (p_1) \quad A_{\gamma}(q) \quad -ig \sin \theta_w [(p_2 - p_1)_{\gamma} \eta_{\alpha\beta} + (q - p_2)_{\alpha} \eta_{\beta\gamma} + (p_1 - q)_{\beta} \eta_{\gamma\alpha}], \quad (A-6) \]

\[ W^-_\beta \quad (p_2) \quad W^-_\alpha \quad (p_1) \quad Z^0_{\gamma}(q) \quad -ig \cos \theta_w [(p_2 - p_1)_{\gamma} \eta_{\alpha\beta} + (q - p_2)_{\alpha} \eta_{\beta\gamma} + (p_1 - q)_{\beta} \eta_{\gamma\alpha}], \quad (A-7) \]
For the vertices, all momenta are assumed to be pointing inward towards the vertex.

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