Reanalysis of QCD sum rules for nucleon with perturbative correction.

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Abstract
A new analysis of baryon sum rules is done with modern values of parameters and known perturbative corrections. The restriction for gluon and quark condensates and the new value of nucleon coupling constant are found.

1. Introduction

During last 20 years in the framework of QCD sum rules the large number of different hadron properties (both static and dynamic, as masses, various decay widths, structure functions and so on) was calculated in non-model way. It is well known that the only parameter in this approach are values of vacuum average of the operators appeared in OPE, like gluon condensate $b = (2\pi)^2 \langle (\alpha/\pi) G^a_{\mu\nu} G^a_{\mu\nu} \rangle$, quark condensate $a = -(2\pi)^2 \langle \bar{\psi}\psi \rangle$ etc. Except this parameters, from consideration of few simple correlator with two hadronic currents, (which usually are used to determine hadron masses), also the so-called coupling constants of hadronic current $g^h$ defined as $g^h \sim \langle 0|j^h|h \rangle$ are fixed. This coupling constants appear in a large number of other sum rules, where analogous hadron current is used. For example in the case of nucleon such "basic" role plays sum rule for on 2-point correlator

$$\Pi = \int e^{iqx} d^4x \langle 0 | T\eta(x)\overline{\eta}(0) | 0 \rangle$$

where $\eta$ is a current with proton quantum number. In what following we will choose it as

$$\eta = \varepsilon^{abc}(u^a C \gamma^\lambda u^b)\gamma^5 \gamma^\lambda d^c$$

This correlator (with various choices of the nucleon current) was first investigated in 1982 in the paper $\Pi$, where coupling constant $\lambda_N$, defined as $\langle 0 | \eta | p \rangle = \lambda_N v(q)$ was found. (Here $v(q)$ is the proton spinor $(\hat{q} - m)v(q) = 0$). Let us discuss this case more detail, because just the reanalysis of this sum rules with account of new theoretical and experimental results obtained since 1982 is our purpose.
2. Baryon sum rule

First the sum rules for baryon from correlator (1) was found by [1], a little later mistakes was corrected in [2]. One should note that authors of [1] used different choices of proton current, but later they come to conclusion that the choose we noted above (eq.(2)) is optimal (see [2]). Now it is widely used, so we also will discuss sum rules for this choice. The procedure of deriving sum rules is standard (and at nowadays well-known). From one side correlator (1) was calculated and OPE terms up to dimension \(d = 9\) are taken into account, from another it was saturated by physical resonances plus continuum, then equating this two representations the following sum rules were obtained [2]:

\[
M^6 E_2(s_0/M^2) L^{-4/9} + \frac{4}{3} a^2 L^{4/9} + \frac{1}{4} b M^2 E_0(s_0/M^2) L^{-4/9} - \frac{1}{3} a^2 m_0^2/M^2 = \bar{X}_N \exp(-m^2/M^2)
\]

(3)

\[
2a M^4 E_1(s_0/M^2) + \frac{272}{81} \frac{\alpha_s a^3}{\pi M^2} - \frac{1}{12} a b = m^2 \bar{X}_N \exp(-m^2/M^2)
\]

(4)

where

\[ E_2(x) = 1 - (1 + x + x^2/2)e^{-x}, \quad E_1 = 1 - (1 + x)e^{-x}, \quad E_0 = 1 - e^{-x}, \]

\[ L = \frac{\alpha_s(M^2)}{\alpha_s(M^2)}, \quad \bar{X} = 32\pi^4 \lambda_N^2, \quad m \text{ is proton mass and quark condensate } a \text{ and gluon condensate } b \text{ was defined above. Two sum rules (3),(4) correspond to the amplitudes at kinematical structures } \hat{p} \text{ and } I \text{ in correlator (1).} \]

Some examples of diagrams, corresponding to different operators contribution to this sum rules, are shown on fig1: Fig.1a correspond to unit operator contribution (first term in (3)), Fig.1b- to gluon condensate contribution (third term in (3)), Fig.1c- to four-quark operator contribution (second term in (3)), Fig.1d- to \(d = 8\) contribution (last term in (3)), Fig.1e- to quark condensate contribution (first term in (4)), Fig.1f- to quark-gluon condensate contribution, \(\langle 0 | \bar{\psi}(\lambda^a/2)G_{\mu\nu}\sigma^{\mu\nu}\psi | 0 \rangle = m_0^2 \langle \bar{\psi}\psi \rangle\), (it contribution found to be 0), Fig.1g- to \(d = 7\) contribution (last term in (4)), Fig.1h- to \(d = 9\) condensate contribution (second term in (4)).

Factorization hypothesis for all operator with \(d > 5\) was used, that’s why, a number of operators of dimension 6 (like \(\langle 0 | \bar{\psi}\Gamma\psi \bar{\psi}\Gamma\psi | 0 \rangle\), where \(\Gamma = I, \gamma^5, \gamma^\mu, \gamma^\mu\gamma^5, \sigma^{\mu\nu}\)) and operator with higher dimension were expressed in terms of \(a, b\) or \(m_0^2a\). One should note, that contribution of \(d = 6\) operators in sum rule (3) is rather large (and larger than operator \(d = 4\)) but the operators of \(d=8\) are small, so the standard condition that higher terms in OPE in sum rules should be less then 30% is fulfilled at \(d > 6\) (see [1, 2]).

Some years later perturbative correction to bare loop, quark condensate and four-quark condensate was calculated (in \(\overline{MS}\) scheme) (see [3] and [4]). It is necessary to note, that in calculation of perturbative corrections for \(d = 6\) four quark operator \(\langle 0 | \bar{\psi}\Gamma\psi \bar{\psi}\Gamma\psi | 0 \rangle\), one should not use factorization hypothesis from very beginning, but renormalize each operator separately, taking in account mixing and only in final answer use factorization hypothesis. This procedure was done by authors of see [3] very carefully, and for numerical estimations one can use the following relation, (based on result see [3])
\[
M^6 E_2(s_0/M^2) L^{-4/9} \left[ 1 + \left( \frac{53}{12} + \gamma_E \right) \frac{\alpha_s(M^2)}{\pi} \right] + \frac{M^2}{4} b E_0(s_0/M^2) + \\
+ \frac{4}{3} a^2 \left( 1 + \frac{\alpha_s(M^2)}{\pi} \left( -\frac{1}{9} + \frac{\gamma_E}{3} \right) \right) - \frac{1}{3} \frac{a^2 m_0^2}{M^2} = \frac{\pi^2}{\lambda N} e^{-m^2/M^2} \\
\]

(5)

\[
2aM^4 E_1(s_0/M^2) (1 + \frac{3}{2} \frac{\alpha_s}{\pi}) + \frac{272}{81} \frac{\alpha_s a^3}{\pi M^2} - \frac{1}{12} ab = m \frac{\pi^2}{\lambda N} e^{-m^2/M^2} \\
\]

(6)

(The results of paper \[5\] coincide with \[3\] for bare loop and quark condensate and for \(d = 6\) four quark condensate differ only in non-logarithmic term in \(\alpha_s\) correction. This difference can be connected with slightly difference factorization scheme).

In eq.(5,6) \(\alpha_s(M^2)\) should be accounted up to second term of perturbative expansion, but the better is to use well-known renormgroup relation for \(\ln Q^2/M^2 = -\frac{\alpha_s(Q^2)/\pi}{\alpha_s(\mu^2)/\pi} \frac{d \alpha_s}{d \ln \mu^2} = -\frac{\alpha_s(\mu^2)/\pi}{\beta(\alpha_s/\pi)}\)

where \(\beta(x) = \sum_0 \beta_n x^{n+2}\), and terms \(\beta_n\) up to fourth one can find in \[6,7\] (for review see, for example, \[8\]) and fix normalization point \(\mu\) at Z-bozon mass, where \(\alpha_s\) is well-known.

These results, on which eq. (5) is based, were obtained more then 15 years ago. From this time the situation with parameters changes drastically (and that’s why reanalysis of baryon sum rules seems to be necessary). First of all, \(\Lambda_{QCD}\) became about two time larger (the modern value is about 300 MeV). Also large uncertainity appear for the value of quark condensate. The well-known estimation of quark condensate, based on Gel-Mann-Oakes-Renner relation lead to value of quark condensate \(\langle \bar{\psi}\psi \rangle = -(243 MeV)^3\) (see for example review \[8\]). The normalization point for it supposed to be about 1GeV, (to be able use this relation in QCD calculation), which is doubtfull for relation obtained as low-energy theorem. In this case renorminvariant value \(\bar{a}^2 = \alpha_s a^2\) is equal

\[
\bar{a}^2 = 0.26 GeV^6 \\
\]

but accuracy is about 50% (see discussion in \[8\]). From other side, there are estimations of quark condensate from sum rules \(\tau\) lepton decays (for review see \[8\]) which lead to much larger value

\[
\bar{a}^2 = 0.47 \pm 0.14 GeV^6 \\
\]

In \[8\] it was supposed that real value should be close to \(\bar{a}^2 = 0.34 GeV^6\) which is close to the boundary of this two equations (upper for (6) and lower for(7)).

Finally, the value of gluon condensate \(b = (2\pi)^2 \langle (\alpha/\pi)G_{\mu\nu}G_{\mu\nu} \rangle\) which usually supposed to be 0.47 GeV\(^4\) was also changed last years as a result of changing \(\Lambda_{QCD}\) from one side and more precise analysis of the sum rules with higher terms of pertuirbative corrections with other. The modern estimation is (see review \[8\])

\[
b = 0.35 \pm 0.28 \\
\]

so one can see that uncertainty is very large, and the possibility that gluon condensate is equal to zero isn’t excluded.

In all this reasons we in our analysis of sum rule should vary
The parameter \( m_0^2 \) we choose in standard way equal to 0.8 GeV\(^2\) (we don’t vary it because terms proportional to this parameter are rather small). We choose continuum threshold \( s_0 = 2.3 \text{GeV}^2 \), according [2]. Numerical analysis show, that variation of \( s_0 \) in the reasonable region \( 2.1 \text{GeV}^6 < S_0 < \text{GeV}^6 \) give less than 10% variation in eq.(5,6)

Let us now discuss the sum rules (5,6). We see that we have two different equations for the \( \lambda N^2 \). That’s mean that the ratio \( R \) of this two equations should be close to unity and it deviation from unity should indicate the accuracy of the sum rule. But one should note that this sum rules has very high accuracy itself due the fact that large number of the OPE series are taken in account and also perturbative correction are accounted (if we for a moment forget about accuracy of condensates, i.e supposed that condensates are fixed). Estimations show, that accuracy of sum rules (5,6) are about 10%. An additional argument of such high accuracy is Borel mass behaviour, which is practically constant (see as an example fig.2, which we shall discuss little later). That’s why, from our point of view the deviation of the ratio \( R \) from 1 more than 15% indicates that choose of parameters from (10) is incorrect. So we can restrict the area of variation of \( b \) and \( \bar{a}^2 \). On Fig.3a,b the Borel mass dependence of ratio \( R \) is shown for different choices of \( b \) and \( \bar{a}^2 \). Thin, thick and dash lines correspond to \( b=0.48, \) 0.24 and \( 0 \text{ GeV}^4 \), fig.3a correspond to \( \bar{a}^2 = 0.34 \text{ GeV}^6 \), and fig. 3b - to \( \bar{a}^2 = 0.23 \text{ GeV}^6 \). One can see, that value \( b=0 \) should be excluded, \( b=0.24 \text{ GeV}^4 \) is allowed only at \( \bar{a}^2 = 0.23 \text{ GeV}^6 \), and \( b=0.48 \text{ GeV}^4 \) is good in whole region of \( \bar{a}^2 \), and for \( \bar{a}^2 = 0.23 \text{ GeV}^6 \) the agreement is the best.

Result for nucleon coupling \( \lambda N^2 \) for \( \bar{a}^2 = 0.23 \text{ GeV}^6 \) and two values of \( b = 0.24, 0.48 \text{ GeV}^4 \) are shown on Fig.2 (thick and thin line correspondingly). One can see that Borel mass dependence is almost constant, and estimated accuracy is less than 15%. Finally we come to following conclusions,

1. \( \lambda N^2 = 2.3 \text{ GeV}^6 \) with accuracy about 15%,
2. the reasonable lower value of gluon condensate is \( 0.24 \text{ GeV}^4 < b \)
3. \( \bar{a}^2 \) can vary in region from \( 0.23 \text{ GeV}^6 \), which correspond to standard value of quark condensate (see eq. (7) up to \( 0.34 \text{ GeV}^6 \), which is close to lower limit, obtained from sum rules for \( \tau \) lepton decay.
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Figure 1: Examples of diagram for different operator contribution, wavy lines correspond to gluon.
Figure 2: Dependence of proton coupling constant $\lambda_N^{-2}$ from Borel mass for gluon condensate $b=0.24$ and $0.48 GeV^4$
Figure 3: Dependence of ratio $R$ from borel mass for various set of gluon and quark condensates from (10)