On Performance Loss of DOA Measurement Using Massive MIMO Receiver with Mixed-ADCs

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Abstract—High hardware cost and high power consumption of massive multiple-input and multiple output (MIMO) are two challenges for the future wireless communications including beyond fifth generation (B5G) and sixth generation (6G). Adopting the low-resolution analog-to-digital converter (ADC) is viewed as a promising solution. Additionally, the direction of arrival (DOA) estimation is an indispensable technology for beam alignment and tracking in massive MIMO systems. Thus, in this paper, the performance of DOA estimation with mixed-ADC structure is firstly investigated. The Cramér-Rao lower bound (CRLB) for this architecture is derived based on the additive quantization noise model. Eventually, a performance loss factor and the associated energy efficiency factor is defined for analysis in detail. Simulation results show that the mixed-ADC architecture can strike a good balance among performance loss, circuit cost and energy efficiency. More importantly, just a few bits (up to 4 bits) of low-resolution ADCs can achieve a satisfactory performance for DOA measurement.

Index Terms—massive MIMO, DOA estimation, mixed-ADC, CRLB

I. INTRODUCTION

Direction of arrival (DOA) estimation has attracted lots of attention [1]. In the massive multiple-input multiple-output (MIMO) systems, DOA is a key technology for its integral role in many emerging applications, including unmanned aerial vehicle (UAV) communications, secure and precise wireless transmission systems [2], and millimeter wave massive MIMO for beyond fifth generation (B5G).

It is well-known that there are two main categories in DOA estimation methods: subspace based methods and parametric methods. Estimation of signal parameters via rotational invariance technique (ESPRIT) and multiple signal classification (MUSIC) are two most famous subspace based methods [3], [4]. Authors in [5] considered DOA estimation in hybrid massive MIMO systems. In order to tackle the phase ambiguity, a deep-learning-based method for a hybrid uniform circular array (UCA) was firstly proposed in [6]. Then, the performance of DOA estimation for low-resolution ADC structure was investigated in [7].

Replacing high-resolution ADCs with low-resolution ADCs is a promising solution to reduce the high hardware cost and high power consumption. However, it is hard to analyse the performance of nonlinear signal. [8] showed that the additive quantization noise model (AQNM) can be applied to eliminate that distortion caused by low-resolution ADCs. However, massive MIMO systems with pure low-resolution ADCs face some tricky challenges, such as time-frequency synchronization, channel estimation, and achievable rate [9]. Thus, a new structure, mixed-ADC structure, was proposed in [10] to make up the shortcomings in low-resolution structure. The performance of this system was investigated over the Rician fading channel in [9]. However, the DOA estimation for a massive MIMO system with mixed-ADC is still an open challenging problem.

To the best of our knowledge, DOA estimation in mixed-ADC massive MIMO systems has not been studied. It is crucial to investigate the performance and the energy efficiency (EE) of DOA estimation with mixed-ADCs, since the mixed-ADC is valuable in practical application due to its satisfactory performance and very low energy consumption. Our main contributions are summarized as follows:

1) The AQNM is adopted to establish the linear system model of the DOA estimation with mixed-ADCs in massive MIMO systems. Then, based on that model, we prove that subspace-based methods can be utilized without modification in this system.

2) In order to assess the performance, the closed-form expression of the CRLB for mixed-ADC structure is derived. In addition, by defining the performance loss factor, the specific performance loss can be calculated through theoretical computations.

3) Finally, we make an investigation on the EE of the mixed-ADC structure. The EE factor of DOA estimation with mixed-ADC is firstly proposed. And, by resorting to the energy consumption model, the optimal number of quantization bits for different proportions of high-resolution ADCs is given. Simulation results show that mixed-ADC architecture can achieve a better trade-off with a few bits (up to 4 bits) of ADCs in most applications.

II. SYSTEM MODEL

As shown in Fig. 11, we consider a uniform linear array (ULA) equipped with mixed-ADCs. The array has \( M_0 \) high-resolution ADCs and \( M_1 \) low-resolution ADCs. The ULA has \( M \) antenna elements, and we define \( \kappa \triangleq M_0/M, (0 \leq \kappa \leq 1) \).
will be quantized by high-resolution ADCs and low-resolution ADCs, respectively. Thus, the received signals experiencing the $M_0$ high-resolution ADCs can be written as $y_0(n) = x_0(t)|_{t=n} = a_0(\theta_0)s(n) + w_0(n), n = 1, 2, \cdots, N$, where $N$ is the number of snapshots. Furthermore, by leveraging on AQN[M][2], the received signals quantized by $b$-bit ADCs can be formulated as

$$y_1(n) = \mathbb{Q}(x_1(t)) \approx \alpha a_1(\theta_0)s(n) + \alpha w_1(n) + w_q(n), \quad (3)$$

where $w_q(n)$ denotes the quantization noise, $\mathbb{Q}(\cdot)$ is the quantization function, and $\alpha = 1 - \beta$ is the linear quantization gain, where $\beta = \frac{\mathbb{E}[\|x_1 - y_1\|^2]}{\mathbb{E}[\|x_1\|^2]}$ denotes the distortion factor of the low-resolution ADC. The accurate value of $\beta$ is listed in Table I when $b \leq 5$. For longer quantization bitlength (e.g., $b > 5$), the distortion factor $\beta$ can be approximated as $\beta \approx \frac{\sqrt{2\pi}}{2} \cdot 2^{-2b}, \ b \geq 6$. For a fixed channel realization, the covariance matrix of $w_q$ is given by

$$R_w = \alpha^2 \beta \text{diag}(\sigma_a^2 a_1(\theta_0)a_1^H(\theta_0) + I_{M_1})$$

Thus, $w_q$ can be modelled as $w_q \sim \mathcal{CN}(0, R_{w_f})$.

Thus, the overall received signal can be expressed as

$$y(n) \approx \begin{bmatrix} a_0(\theta_0)s(n) + w_0(n) \\ \alpha a_1(\theta_0)s(n) + \alpha w_1(n) + w_q(n) \end{bmatrix} \quad \text{(5)}$$

### III. Analysis for the Application of Root-MUSIC in Mixed-ADC Structure

The MUSIC proposed in [3] is a well-known subspace-based method for DOA estimation. In this section, we prove the fact that there is no change on MUSIC method.
When the array is equipped with pure high-resolution ADCs, the eigenvalue decomposition (EVD) of the covariance matrix $R'_y$ can be written as

$$R'_y = E[yy^H] = \sigma^2 a a^H + I_M$$

$$= E_S A S E_S^H + E_W A W E_W^H,$$  \(6\)

where the columns of $E_S$ and $E_W$ are the eigenvectors corresponding to the useful signal and channel noise, respectively. Those can span the signal subspace and noise subspace, which are orthogonal. Thus, the spectrum,

$$S(\theta) = \|E_W^H a(\theta)\|^2,$$  \(7\)

will be infinity when $\theta = \theta_0$. In practice, $R_y$ is estimated from sampled data by $R_y = \frac{1}{N} \sum_{n=1}^{N} y(n)y^H(n)$.

In mixed-ADC structure, the covariance matrix $R'_y$ can be casted as \(8\) at the bottom of next page, where $\gamma = E[ss^H] = \sigma^2 = \sigma_s^2 / \sigma_n^2$ is the input SNR of ADCs. Observing $8$, as the number of quantization bit increases, $R_c \to O_M$ and $R_c \ll \|R'_y\|$. $R_y \approx R'_y$ and $R_c$ can be considered as the error of $R'_y$. Thus, $R_y \approx R'_y$ and $R_c$ can be regarded as the error of $R'_y$. Now, let $\epsilon'$ and $\lambda'$ denote the eigenvalue and eigenvector of $R'_y$, respectively. Then, we have $R'_y \epsilon' = \lambda' \epsilon'$, which can be regarded as the linear equations $R'_y x = b$. And the condition number of $R'_y$ is given by

$$\text{cond}(R'_y) = \frac{\|\lambda'_{\text{max}}(R'_y)\|}{\|\lambda'_{\text{min}}(R'_y)\|} = \gamma + 1.$$  \(9\)

Obviously, $R'_y$ has a low condition number. And, we can conclude that $R'_y$ and $R_y$ will have the approximate eigenvalues and eigenvectors. Thus, MUSIC and other subspace-based methods can be applied without modification, like root-MUSIC and ESPRIT \(4\).

IV. PERFORMANCE LOSS AND ENERGY EFFICIENCY

In this section, to evaluate the performance of the massive MIMO for mixed-ADC architecture, we derive the CRLB for the mixed-ADC structure. Then, the performance loss factor and EE factor are firstly defined to seek a basic trade-off between the performance and power consumption in practical applications.

A. CRLB and Performance Loss

Now, let us define $d_m = (m-1)d$. And, according to Appendix, we have the CRLB for the mixed-ADC structure, \(10\), as shown at the bottom of this page, where $J_0 = \beta M_0 (M_0 - 1)(\gamma + 1)$ and $J = \alpha M (M - 1)$.

Furthermore, define the performance loss factor $\eta_{PL}$ by \(11\) at the bottom of this page, where $g = \beta (\gamma + 1)$. In massive MIMO systems, when the number of antenna elements increases without bound, $M \to \infty$, the $\eta_{PL}$ will converge to

$$\eta_{PL} \approx \frac{(g + \alpha)(g + k + \alpha)}{4(g + \alpha)(g^3 + \alpha) - 3(gk^2 + \alpha)^2}.$$  \(12\)

Due to $0 \leq \kappa \leq 1$, $\eta_{PL}$ is a linear monotonically increasing function of $\gamma$ for a fixed $\kappa$ and $b$. By contrary, $\eta_{PL}$ decreases as $\kappa$ and $b$ increase.

B. Energy Efficiency

To the best of our knowledge, no one has investigated the energy efficiency for DOA estimation with mixed-ADCs. Thus, with the help of the definition in \(9\), EE of DOA estimation is defined as

$$\eta_{EE} = \frac{CRLB^{-\frac{1}{2}}}{P_{total}} \text{ 1/degree/W},$$  \(13\)

where $P_{total}$ is the total power consumption in the massive MIMO system. $CRLB^{-1/2}$ represents the accuracy, which is the reciprocal of standard deviation lower bound and the unit of that is 1/degree. $P_{total}$ can be expressed as

$$P_{total} = P_{RF} + M_0 P_{ADC,H} + M_1 P_{ADC,L}$$  \(14\)

where $P_{RF} = P_{syc} + M(P_{LNA} + P_{mix} + P_{fz} + P_{FA}) + M_0 P_{AGC} + \rho M_1 P_{AGC}$, where $P_{syc}$, $P_{LNA}$, $P_{mix}$, $P_{fz}$, $P_{FA}$, $P_{AGC}$, $P_{ADC,H}$ and $P_{ADC,L}$ are the power consumption values for the frequency synthesizer, LNA, mixer, the active filter, the intermediate frequency amplifier, AGC, high-resolution ADCs, low-resolution ADCs, respectively. In addition, $\rho$ is the flag function determined by the low-resolution ADC’s bit, which is given by

$$\rho = \begin{cases} 0, & b = 1, \\ 1, & b > 1. \end{cases}$$  \(15\)

The power consumption of the ADC can be calculated by

$$P_{ADC} \approx 3V_{dd}^2 L_{min}(2B + f_{cor})$$

\(10^{-0.10205+4.8368}\),

where $B$ denotes the bandwidth of the signal, $V_{dd}$ is the supply voltage of converter, $L_{min}$ is the minimum channel length for the given CMOS technology, $f_{cor}$ is the corner frequency of the $1/f$ noise. \(16\) is established for the complete class of CMOS Nyquist-rate high speed ADCs \(11\).

V. SIMULATION RESULTS AND DISCUSSIONS

In this section, we provide the simulation results to analyse the impact of different $\kappa$ and $b$ on the performance loss $\eta_{PL}$. Furthermore, the simulation of MUSIC with $\kappa = 1/4$ and different $b$ is conducted, where all results are averaged over 8000 Monte Carlo realizations. In all simulations, it is assumed that two emitters are located in $0^\circ$ and $30^\circ$, the number of snapshots $K$ is 32 and the number of antenna elements $M$ is 128.

In Fig.\(2\) performance loss over $\kappa$ is illustrated. We consider two common SNR: $\gamma = 0$ dB and $\gamma = 10$ dB. Obviously, $\eta_{PL}$ decreases as $\kappa$ increases. If we set 1 dB as an acceptable minimum of performance loss, 1-bit ADCs could be considered when $\kappa > 0.9$. For a medium value of $\kappa$ ($0.2 < \kappa < 0.9$), ADCs with 2-bit or 3-bit are more suitable. In addition, when $\kappa$ decreases to 0.2 below, 3-bit ADCs even 4-bit ADCs can be adopted to achieve a satisfactory performance. Of course, in some special cases, more or less quantization bit of ADCs should be chosen to meet the practical requirement.

Fig.\(3\) plots curves of MUSIC spectrum for mixed-ADC arrays with different quantization bits. It is seen that MUSIC spectrums of arrays with 1-bit ADCs and 2-bit have lower peaks at the same directions. That proves that MUSIC method
was investigated, which showed that 1-4 bits’ ADCs are good choices in most applications. Finally, the mixed-ADC structure can achieve a satisfactory performance of DOA estimation with much less circuit cost and power consumption.

**Appendix**

**Derivation of CRLB for Full Digital Structure with Mixed-ADC**

In this section, we derive the CRLB for the massive MIMO with mixed-ADCs. In accordance with [1], the corresponding Fisher information matrix (FIM) $F$ can be written as

$$F = \text{Tr} \left\{ R_y^{-1} \frac{\partial R_y}{\partial \theta} \frac{\partial R_y}{\partial \theta} \right\}. \quad (17)$$

For convenient derivation, $s(n)$, $y(n)$, $a$ and $w(n)$ are abbreviated as $s$, $y$, $a$ and $w$ respectively in the following part. Now, to simplify the derivation, we reformulate $y$ as

$$y = Tas + Tw + q. \quad (18)$$

where

$$T = \begin{bmatrix} I_{M_0} & 0_{M_0 \times M_1} \\ 0_{M_1 \times M_0} & aI_{M_1} \end{bmatrix}. \quad (19)$$

and

$$q = \begin{bmatrix} 0_{M_0 \times 1} \\ w_F \end{bmatrix}. \quad (20)$$

Then, the $R_y$ is given by

$$R_y = E[yy^H] = \gamma Taa^HT^H + Q, \quad (21)$$

where

$$Q = \begin{bmatrix} I_{M_0} & 0_{M_0 \times M_1} \\ 0_{M_1 \times M_0} & [\sigma^2 + \alpha \beta (\sigma_s^2 + 1)]I_{M_1} \end{bmatrix}. \quad (22)$$

Thus,

$$\frac{\partial R_y}{\partial \theta_0} = \gamma T(\dot{a}a^H + a\dot{a}^H)T^H, \quad (23)$$

where $\dot{a}$ is the differential of $a$ to $\theta_0$, which is derived as

$$\dot{a} = \frac{d}{d\theta_0}a(\theta_0) = \frac{2\pi}{\lambda} \cos \theta_0 Da, \quad (24)$$

and

Figure 2. Performance comparison of the DOA estimation over $\kappa$.  

Figure 3. MUSIC spectrum for 32-element mixed-ADC ULA systems.  

VI. Conclusion  

In this paper, we built the DOA estimation for the ULA with mixed-ADC. Then, we proved that the subspace-based methods can be applied to this system without modification. Furthermore, we derived the CRLB and as the benchmark of the performance. Based on that, the performance loss and EE
where
\[
D = \begin{bmatrix}
  d_1 & 0 & \cdots & 0 \\
  0 & d_2 & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  0 & 0 & \cdots & d_M
\end{bmatrix}.
\]

Therefore, with the help of some properties of the trace in [13], FIM can be expressed as
\[
F = \sum_{m=1}^{M_0} d_m^2 + \frac{\alpha}{\beta \sigma_s^2 + 1} \sum_{m=M_0+1}^{M} d_m^2.
\]

Combining (30) and (31), \( F_b \) in (28) is represented by
\[
F_b = (\alpha^H T^H R_y^{-1} T) \left( \frac{Q^{-1}}{\gamma^{-1} + \xi} \right) Ta.
\]

Finally, substitute (32) into (27), which yields
\[
F = \gamma^2 \left( F_a + 2 F_b + F_c \right).
\]

And, the second part is expressed as
\[
\begin{align*}
\tilde{a}^H R_y^{-1} T a &= \left( Q^{-1} - \frac{Q^{-1} T a a^H T^H Q^{-1}}{\gamma^{-1} + \xi} \right) T a \\
&= \left( Q^{-1} - \frac{Q^{-1} T a a^H T^H Q^{-1}}{\gamma^{-1} + \xi} \right) T a \\
&= 4 \pi^2 \lambda^2 \cos^2 \theta_0 \left( \nu - \frac{\mu^2}{\gamma + \xi} \right),
\end{align*}
\]

where
\[
\nu = \sum_{m=1}^{M_0} d_m^2 + \frac{\alpha}{\beta \sigma_s^2 + 1} \sum_{m=M_0+1}^{M} d_m^2.
\]

The derivation of CRLB for the massive MIMO system with mixed-ADCs is completed.

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