Partial decay widths of baryons in the spin-momentum operator expansion method

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Abstract. The covariant operator expansion method used by the Bonn-Gatchina group for the analysis of the meson photoproduction data is extended on the case of meson electro-production reactions. The angular dependence of the partial waves is deduced and the obtained amplitudes are compared with those used in other analyses of the electro-production reactions.

1 Introduction

Reactions with pseudoscalar mesons in the final state provide the main part of the information about spectrum and properties of hadron resonances. The final states with pseudoscalar mesons only are easy to measure and the data are relatively easy to analyze: for example, in the case of pion-pion scattering only measurement of the differential cross section provides the full information about partial wave amplitudes. In many cases such analysis can be performed in two steps. At the first step the measured angular distribution is analyzed at fixed energy and partial wave amplitudes are extracted with some precision. At second step the energy dependence of the extracted amplitudes is analyzed and their analytical structure is determined.

However, in the case of photoproduction of the mesons off nucleon the analysis of the data is a more complicated issue. To perform the full analysis of a single pseudoscalar meson production reaction at least eight independent polarization observables should be measured. In many cases such information is not available and the data are analyzed in so called energy dependent approach. Here the partial waves are extracted from the simultaneous analysis of the energy and angular distributions. Thus an observation of a resonance in a particular partial wave notably reduces the number of parameters and the resonance properties can be defined from the restricted number of measured observables. Moreover the great advantage of such approach is the possibility for a combined analysis of reactions with different final (or initial) states. In this case the polarization observables measured in one reaction can provide a key information for the analysis of the data measured in another reaction.

Another important source for the information about baryon resonances and their properties is the meson electro-production reactions. Such data allows us to study the dependence of the resonance production couplings on the mass of the virtual photon and therefore about size and internal structure of the resonances. It is a vital information which can help to understand the nature of the baryon states and properties of the strong interactions.

Originally the fully covariant Bonn-Gatchina formalism was developed for the analysis of the meson photoproduction reactions: it is described in details in the paper [1]. This approach was successfully used for the analysis of the data measured by the CB-ELSA, CLAS and MAMI collaborations. It also was applied by the HADES collaboration for the analysis of the pion induced meson production data. In this paper the Bonn-Gatchina approach is extended for the analysis of the electro-production data. Our formalism is compared with the covariant approach [2] suggested earlier.

2 Decay of the resonance into two spinless particles

The orbital angular momentum operators for \(L \leq 3\) are:

\[
X^{(0)} = 1, \quad X^{(1)}_{\mu_1 \mu_2} = k^\perp_{\mu_1}, \quad X^{(2)}_{\mu_1 \mu_2 \mu_3} = \frac{3}{2} \left( k^\perp_{\mu_1} k^\perp_{\mu_2} - \frac{1}{3} k^2_{\mu_1} g^\perp_{\mu_1 \mu_2} \right), \quad X^{(3)}_{\mu_1 \mu_2 \mu_3 \mu_4} = \frac{5}{2} \left[ k^\perp_{\mu_1} k^\perp_{\mu_2} k^\perp_{\mu_3} - \frac{k^2}{3} \left( g^\perp_{\mu_1 \mu_2} k^\perp_{\mu_3} + g^\perp_{\mu_1 \mu_3} k^\perp_{\mu_2} + g^\perp_{\mu_2 \mu_3} k^\perp_{\mu_1} \right) \right].
\]

The operators \(X^{(L)}_{\mu_1 \ldots \mu_L}\) for \(L \geq 1\) can be written in the form of the recurrence expression:

\[
X^{(L)}_{\mu_1 \ldots \mu_L} = k^\perp_\alpha Z^\alpha_{\mu_1 \ldots \mu_L}.
\]
For higher states, the operator can be calculated using the matrices. In the standard representation the structure of the fermion propagator

\[ Z_{\mu_1 \ldots \mu_L}^{\alpha} = \frac{2L - 1}{L^2} \left( \sum_{i=1}^{L} X_{\mu_1 \ldots \mu_{i-1} \mu_{i+1} \ldots \mu_L}^{(L-1)} g_{\mu_i \alpha} - \right. \]
\[ \left. \frac{2}{2L - 1} \sum_{i,j=1 \atop i < j}^{L} g_{\mu_i \mu_j} X_{\mu_1 \ldots \mu_{i-1} \mu_{i+1} \ldots \mu_{j-1} \mu_{j+1} \ldots \mu_L}^{(L-1)} \right) \]  

(2)

Other useful properties of the orbital momentum operators are listed in Appendix.

The projection operator \( O_{\mu_1 \ldots \mu_L}^{\mu'_1 \ldots \mu'_L} \) is constructed from the metric tensors \( g_{\mu \nu} \) and has the following properties:

\[ X_{\mu_1 \ldots \mu_L}^{(L)} O_{\mu_1 \ldots \mu_L}^{\mu'_1 \ldots \mu'_L} = X_{\mu'_1 \ldots \mu'_L}^{(L)} , \]
\[ O_{\alpha_1 \ldots \alpha_L}^{\mu_1 \ldots \mu_L} = O_{\mu_1 \ldots \mu_L}^{\alpha_1 \ldots \alpha_L} . \]  

(3)

The projection operator projects any tensor with \( n \) indices onto tensors which satisfy the properties \( (1) \). For the lowest states, \( O = 1 \), \( O_0^{\mu_1} = g_{\mu_1} \)

\[ O_{\mu_1 \mu_2}^{\mu_1 \mu_2} = \frac{1}{2} \left( g_{\mu_1 \nu_1} g_{\mu_2 \nu_2} + g_{\mu_2 \nu_1} g_{\mu_1 \nu_2} - \frac{2}{3} g_{\mu_1 \mu_2} g_{\nu_1 \nu_2} \right) . \]  

(4)

For higher states, the operator can be calculated using the recurrent expression:

\[ O_{\mu_1 \ldots \mu_L}^{\mu_1 \ldots \mu_L} = \frac{1}{L^2} \left( \sum_{i,j=1 \atop i < j}^{L} g_{\mu_i \nu_i} g_{\mu_j \nu_j} + g_{\mu_i \nu_j} g_{\mu_j \nu_i} - \frac{2}{3} g_{\mu_i \mu_j} g_{\nu_i \nu_j} \right) - \]
\[ \frac{4}{(2L - 1)(2L - 3)} \times \]
\[ \sum_{i,j=1 \atop i < j}^{L} g_{\mu_i \nu_i} g_{\mu_j \nu_j} \cdots \sum_{i,j=1 \atop i < j}^{L} g_{\mu_i \nu_i} g_{\mu_j \nu_j} \]  

(5)

The tensor part of the boson propagator is defined by the projection operator. Let us write it as

\[ F_{\mu_1 \ldots \mu_L}^{\mu'_1 \ldots \mu'_L} = (-1)^L O_{\mu_1 \ldots \mu_L}^{\mu'_1 \ldots \mu'_L} . \]  

(6)

3 The structure of the fermion propagator

The wave function of a fermion is described as Dirac bispinor, as object in Dirac space represented by \( \gamma \) matrices. In the standard representation the \( \gamma \) matrices have the following form:

\[ \gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} , \quad \gamma = \begin{pmatrix} 0 & \sigma \\ -\sigma & 0 \end{pmatrix} , \quad \gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \]  

(7)

where \( \sigma \) are \( 2 \times 2 \) Pauli matrices. In this representation the spinors for fermion particles with momentum \( p \) are:

\[ u(p) = \frac{1}{\sqrt{p^0 + m}} \begin{pmatrix} (p_0 + m) \omega \\ (p \sigma) \omega \end{pmatrix} , \]
\[ \bar{u}(p) = \frac{(\omega^*(p_0 + m), -\omega^*(p \sigma))}{\sqrt{p^0 + m}} . \]  

Here \( \omega \) represents a 2-dimensional spinor and \( \omega^* \) the conjugated and transposed spinor. The normalization condition can be written as:

\[ \bar{u}(p) u(p) = 2m \sum_{\text{polarizations}} u(p) \bar{u}(p) = m + \hat{p} \]  

(9)

We define \( \hat{p} = p^0 \gamma_0 \).

The structure of the fermion propagator \( P_{\mu_1 \ldots \nu_L}^{\mu'_1 \ldots \nu_L} \) was considered in details in [11]. The propagator is defined as

\[ P_{\nu_1 \ldots \nu_L}^{\mu_1 \ldots \mu_L} = \frac{F_{\mu_1 \ldots \mu_L}^{\nu_1 \ldots \nu_L}}{M^2 - s - iMT} , \]  

(10)

where

\[ F_{\mu_1 \ldots \nu_L}^{\mu'_1 \ldots \nu_L} = (-1)^n \sqrt{s + \hat{P}} \gamma^{\xi_1 \ldots \xi_n} \gamma^{\beta_1 \ldots \beta_n} \]  

(11)

Here, \( (\sqrt{s + \hat{P}}) \) corresponds to the numerator of fermion propagator describing the particle with \( J = 1/2 \) and \( n = J - 1/2 \) (\( \sqrt{s} = M \) for the stable particle). We define

\[ P_{\xi_1 \ldots \xi_n}^{\beta_1 \ldots \beta_n} = \frac{n + 1}{2n + 1} \left( g_{\xi_1 \beta_1} - \frac{n}{n + 1} \sigma_{\xi_1 \beta_1} \right) \prod_{i=2}^{n} g_{\xi_i \beta_i} \]
\[ \sigma_{\alpha_i \sigma_j} = \frac{1}{2} (\gamma_\alpha_i \gamma_\alpha_j - \gamma_\alpha_j \gamma_\alpha_i) . \]  

(12)

As in [1], we introduced the factor \((1/2\sqrt{s})\) in the propagator which removes the divergency of this function at large energies. For the stable particle it means that bispinors are normalized as follows:

\[ \bar{u}(kN) u(kN) = 1 , \quad \sum_{\text{polarizations}} u(kN) \bar{u}(kN) = \frac{m + \hat{k}_N}{2m} . \]  

(13)

Here and below, \( \hat{k} \equiv k^0 k_\perp \).

It is useful to list the properties of the fermion propagator:

\[ P_{\mu_1 \ldots \nu_L}^{\mu'_1 \ldots \nu_L} P_{\nu_1 \ldots \nu_L}^{\mu_1 \ldots \nu_L} = 0 , \]
\[ \gamma_\mu P_{\nu_1 \ldots \nu_L}^{\mu'_1 \ldots \nu_L} = 0 , \]
\[ P_{\alpha_1 \ldots \alpha_L}^{\mu_1 \ldots \mu_L} P_{\nu_1 \ldots \nu_L}^{\alpha_1 \ldots \alpha_L} = (-1)^n \gamma_{\mu} P_{\nu_1 \ldots \nu_L}^{\alpha_1 \ldots \alpha_L} , \]
\[ \hat{P} F_{\mu_1 \ldots \nu_L}^{\mu'_1 \ldots \nu_L} = \sqrt{s}  F_{\mu_1 \ldots \nu_L}^{\mu'_1 \ldots \nu_L} . \]  

(14)

3.1 \( \pi N \) vertices

The states with \( J = L + 1/2 \), where \( L \) is the orbital momentum of the \( \pi N \) system, are called `+` states \((1/2^+, 3/2^+, 5/2^+, \ldots)\). The states with \( J = L - 1/2 \) are called `−` states \((1/2^−, 3/2^−, 5/2^−, \ldots)\). The correspondent vertices are \( (n = J - 1/2) \):

\[ N^+_{\mu_1 \ldots \mu_n} (k^0) u(kN) = (n_{\mu_1 \ldots \mu_n} (k^0) u(kN)) \]
\[ N^-_{\mu_1 \ldots \mu_n} (k^0) u(kN) = i \gamma_\mu N^+(n_{\mu_1 \ldots \mu_n} (k^0) u(kN)) . \]  

(15)

Here, \( u(kN) \) is the bispinor of the final–state nucleon.
In the c.m.s. of the reaction this amplitude can be rewritten as
\[ A_{\gamma N} = \omega' [G(s, t) + H(s, t)i(\sigma n)] \omega' , \]
\[ G(s, t) = \sum_L [(L+1)F_L^+(s) - LF_L^-(s)]P_L(z) , \]
\[ H(s, t) = \sum_L [F_L^+(s) + F_L^-(s)]P'_L(z) , \]
where \( \omega \) and \( \omega' \) are nonrelativistic spinors and \( \alpha \) is a unit vector normal to the decay plane. The \( F \)-functions are defined as follows:
\[ F_L^+(s) = (|k|q)^L \chi_L^* \frac{\alpha_L}{2L+1} BW_L^+(s) , \]
\[ F_L^-(s) = (|k|q)^L \chi_L^* \frac{\alpha_L}{L} BW_L^-(s) , \]
\[ \chi_L = \sqrt{\frac{m_N + qN0}{2m_N}} , \quad \chi'_L = \sqrt{\frac{m_N + qN0}{2m_N}} , \]
where \( L = n \) stands for ‘+’ states and \( L = n + 1 \) for ‘-’ states.

### 4 The electro production amplitudes

#### 4.1 The ‘+’ states

For the states with \( n \geq 1 \), three vertices can be constructed of the spin and orbital momentum operators. For the ‘+’ states the vertices are:
\[ V_{\alpha_1 \cdots \alpha_n}^{(1)}(k^-) = \gamma^\mu_\alpha \gamma^\nu_\beta \chi^{(0)}_{\alpha_1 \cdots \alpha_n} (k^-) , \]
\[ V_{\alpha_1 \cdots \alpha_n}^{(2)}(k^-) = \gamma^\mu_\alpha \gamma^\nu_\beta \chi^{(n+2)}_{\alpha_1 \cdots \alpha_n} (k^-) , \]
\[ V_{\alpha_1 \cdots \alpha_n}^{(3)}(k^-) = \gamma^\mu_\alpha \gamma^\nu_\beta \chi^{(n)}_{\alpha_1 \cdots \alpha_n} (k^-) g_{\mu \nu} , \]
\[ (18) \]

The first vertex is constructed using the spin 1/2 operator and \( L = n \) orbital momentum operator, the second one has \( S = 3/2, L = n + 2 \) and the third one \( S = 3/2 \) and \( L = n \). In case of photoproduction, the second vertex is reduced to the third one and only two amplitudes (one for \( J = 1/2 \)) are independent.

#### 4.2 The ‘-’ states

For the decay of a ‘-’ state with total spin \( J \) into \( \gamma N \), the vertex functions have the form:
\[ V_{\alpha_1 \cdots \alpha_n}^{(1)}(k^-) = \gamma^\mu_\alpha \gamma^\nu_\beta \chi^{(n+1)}_{\alpha_1 \cdots \alpha_n} (k^-) , \]
\[ V_{\alpha_1 \cdots \alpha_n}^{(2)}(k^-) = \gamma^\mu_\alpha \gamma^\nu_\beta \chi^{(n)}_{\alpha_1 \cdots \alpha_n} (k^-) , \]
\[ V_{\alpha_1 \cdots \alpha_n}^{(3)}(k^-) = \gamma^\mu_\alpha \gamma^\nu_\beta \chi^{(n-1)}_{\alpha_1 \cdots \alpha_n} (k^-) g_{\mu \nu} . \]
\[ (19) \]

These vertices are constructed of the spin and orbital momentum operators with \( (S = 1/2, L = n + 1), (S = 3/2, L = n + 1) \) and \( (S = 3/2, L = n - 1) \). As in case of “+” states, the second vertex provides us the same angular distribution as the third vertex. For the first and third vertices, the width factors \( W_{ij} \) are equal to

### 4.3 Single meson electro-production

General structure of the single-meson electro-production amplitude in c.m.s. of the reaction is given by
\[ J_{\mu} = if_{L1} \bar{\sigma}_\mu + f_{L2}(\sigma q) \frac{\epsilon_{\mu ij} \sigma^i \xi_j}{|k|q} + iF_3 \frac{(\sigma k)}{|k|q} \tilde{q}_\mu + iF_4 \frac{(\sigma q)}{|q|^2} \tilde{q}_\mu , \]
\[ + iF_5 (\sigma k) \tilde{k}_\mu + iF_6 (\sigma q) k^\mu \]
where \( q \) is the momentum of the nucleon in the \( \pi N \) channel and \( k \) the momentum of the nucleon in the \( \gamma N \) channel calculated in the c.m.s. of the reaction. The \( \sigma_i \) are Pauli matrices.
\[ \tilde{q}_\mu = q_\mu - \frac{qk}{|k|^2} k^\mu = q_\mu - z k^\mu \frac{|q|}{|k|} \]
\[ (21) \]

The functions \( F_i \) have the following angular dependence:
\[ F_1(z) = \sum_{L=0}^{\infty} [(LM_L^+ + E_L^+)P_L^+(z) + [(L+1)M_L^- + E_L^-]P_{L-1}^+(z) , \]
\[ F_2(z) = \sum_{L=1}^{\infty} [(L+1)M_L^+ + LM_L^-]P_L^+(z) , \]
\[ F_3(z) = \sum_{L=1}^{\infty} [E_L^+ - M_L^+]P_{L+1}^+(z) + [E_L^- + M_L^-]P_{L-1}^+(z) , \]
\[ F_4(z) = \sum_{L=2}^{\infty} [M_L^- - E_L^- - M_L^-]P_L^+(z) , \]
\[ F_5(z) = \sum_{L=0}^{\infty} [(L+1)M_L^- P_{L+1}^+(z) - LM_L^- P_{L-1}^+(z) , \]
\[ F_6(z) = \sum_{L=1}^{\infty} [LM_L^- - (L+1)M_L^- P_L^+(z) \]
\[ (22) \]

Here \( L \) corresponds to the orbital angular momentum in the \( \pi N \) system, \( P_L(z), P_L^+(z), P_L^-(z) \) are Legendre polynomials and tier derivatives, \( z = (kq)/(|k|q) \), and \( E_L^\pm \) and \( M_L^\pm \) are electric and magnetic multipoles describing transitions to states with \( J = L \pm 1/2 \).

The single-meson production amplitude via the intermediate resonance with \( J = n + 1/2 \) (we take pion photoproduction as an example) has the general form:
\[ A^{\pm} = g_{\pi N}(s) \bar{u}(q_N) N^{\pm}_{\alpha_1 \cdots \alpha_n} (q^-) \times \]
\[ \frac{F_{\beta_1 \cdots \beta_n}^{\alpha_1 \cdots \alpha_n}}{M^2 - s - iM_{tot}L^2} V_{\xi_\mu}^{(i)}(k^-) u(k_N) g_i(s) \epsilon_\mu . \]
\[ (23) \]
4.4 Positive sector

For the positive amplitudes $L = n$. The spin $\frac{1}{2}$ amplitude has the structure:

$$A^{1+}_\mu = \bar{u}(qN) X^{(n)}_{\alpha_1 \ldots \alpha_n} (q^+)$$
$$F^{\alpha_1 \ldots \alpha_n}_{\beta_1 \ldots \beta_n} \gamma_\mu i\gamma_5 \chi_{\beta_1 \ldots \beta_n} (k^+) u(kN)$$

(24)

where

$$F_1^{1+} = \lambda_n P'_{n+1}$$
$$F_2^{1+} = \lambda_n P'_n$$
$$F_3^{1+} = 0$$
$$F_4^{1+} = 0$$
$$F_5^{1+} = +\lambda_n P'_{n+1}$$
$$F_6^{1+} = -\lambda_n P'_n$$

(25)

Therefore

$$E^{1+}_n = M^{1+}_n = L^{1+}_n = \frac{\lambda_n}{n+1}$$

(28)

The second ($S = \frac{3}{2}$) amplitude has the structure:

$$A^{3+}_\mu = \bar{u}(qN) X^{(n)}_{\alpha_1 \ldots \alpha_n} (q^+)$$
$$F^{\alpha_1 \ldots \alpha_n}_{\mu \beta_1 \ldots \beta_n} \gamma_{\mu \nu} \gamma_5 \chi_{\beta_1 \ldots \beta_n} (k^+) u(kN)$$

(29)

where

$$\lambda_n = \frac{\alpha^{(n)}}{2n + 1} (|k||q|)^n \chi_i \chi_f$$

(26)

$$\alpha^{(n)} = \prod_{j=1}^{n} \frac{2j - 1}{j} \quad \alpha^{(0)} = 1$$

(27)

Therefore

$$E^{3+}_n = M^{3+}_n = L^{3+}_n = \frac{\lambda_n}{n+1}$$

(31)

The third amplitude has the structure:

$$A^{2+}_\mu = \bar{u}(qN) X^{(n+1)}_{\alpha_1 \ldots \alpha_n} (q^+)$$
$$F^{\alpha_1 \ldots \alpha_n}_{\beta_1 \ldots \beta_n} \gamma_{\mu \nu} \gamma_5 \chi_{\beta_1 \ldots \beta_n} (k^+) u(kN)$$

(32)

$$F_1^{2+} = \xi_n P'_{n+1}$$
$$F_2^{2+} = 0$$
$$F_3^{2+} = \xi_n P'_{n+1}$$
$$F_4^{2+} = -\xi_n P'_n$$
$$F_5^{2+} = -\xi_n (n+2) P'_{n+1}$$
$$F_6^{2+} = \xi_n (n+2) P'_n$$

(33)

where

$$\xi_n = \frac{|k|^2 (2n + 1)}{(n+2) (n+1)} \lambda_n = \alpha^{(n)} |k|^{n+2} |q|^n$$

(34)

Therefore

$$E^{2+}_n = \xi_n \quad M^{2+}_n = 0 \quad L^{2+}_n = -\xi_n \frac{n+2}{n+1}$$

(35)

4.5 Negative sector

For the negative amplitudes $L = n + 1$. The spin $\frac{1}{2}$ amplitude has the structure:

$$A^{-1}_\mu = \bar{u}(qN) X^{(n+1)}_{\alpha_1 \ldots \alpha_n} (q^+)$$
$$F^{\alpha_1 \ldots \alpha_n}_{\beta_1 \ldots \beta_n} \gamma_{\mu \nu} \gamma_5 \chi_{\beta_1 \ldots \beta_n} (k^+) u(kN)$$

(36)

where

$$\zeta_{n+1} = \frac{\alpha^{(n+1)}}{n+1} (|k||q|)^{n+1} \chi_i \chi_f$$

(38)

Therefore

$$-M^{-1}_{n+1} = E^{-1}_{n+1} = L^{-1}_{n+1} = \frac{\zeta_{n+1}}{n+1}$$

(39)

For the third negative amplitude (spin $\frac{3}{2}$):

$$A^{-3}_\mu = \bar{u}(qN) X^{(n+1)}_{\alpha_1 \ldots \alpha_n} (q^+)$$
$$F^{\alpha_1 \ldots \alpha_n}_{\mu \beta_1 \ldots \beta_n} \gamma_{\mu \nu} \gamma_5 \chi_{\beta_1 \ldots \beta_n} (k^+) u(kN)$$

(40)

where

$$\zeta_{n+1} = \frac{\alpha^{(n+1)}}{n+1} (|k||q|)^{n+1} \chi_i \chi_f$$

(38)

Therefore

$$F_1^{-3} = \theta_{n-1} P'_{n}$$
$$F_2^{-3} = 0$$
$$F_3^{-3} = \theta_{n-1} P'_n$$
$$F_4^{-3} = -\theta_{n-1} P'_n$$
$$F_5^{-3} = n \theta_{n-1} P'_n$$
$$F_6^{-3} = n \theta_{n-1} P'_n$$

(41)
where
\[ \varrho_{n+1} = \frac{\alpha^{(n-1)}}{n(n+1)} |k|^n |q|^{n+1} \chi_{i} \chi_{j} \]
\[ (42) \]
Therefore
\[ M_{n+1}^2 = 0 \quad E_{n+1}^2 = \varrho_{n+1} \quad L_{n+1}^2 = \frac{n}{n+1} \]
\[ (43) \]
For the second amplitude from negative sector:
\[ A_{\mu}^2 = \bar{u}(qN)X_{\alpha_1 \ldots \alpha_n}(q^+) \gamma\mu \gamma_5 F_{\bar{\beta}_1 \ldots \beta_n} X_{\beta_1 \ldots \beta_n}(k^-)u(kN) \]
\[ (44) \]
\[ \Delta_n = \frac{\alpha^{(n)}}{(n+1)^2} (|k||q|)^{n+1} \chi_{i} \chi_{j} \]
\[ (46) \]
Therefore
\[ M_{n+1}^2 = 0 \quad E_{n+1}^2 = -\Delta_n \quad L_{n+1}^2 = -\Delta_n \]
\[ (47) \]
Remember that \( \chi_{i} = mN + qN \) and \( \chi_{j} = mN + kN \). For the \( '-' \) states, where \( L = n+1 \), the corresponding equations are
\[ E_{L}^{(\pm 2)} = -\frac{\alpha^{(L)}}{\sqrt{2M^2}} \sqrt{g_{\mu} g_{\nu}} [|k||q|] L^2 g_1(s) - iMT_{tot} \]
\[ M_{L}^{(\pm 2)} = -E_{L}^{(\pm 2)} \]
\[ L_{L}^{(\pm 2)} = -\frac{\alpha^{(L-2)}}{(L-1)\sqrt{2M^2}} \sqrt{g_{\mu} g_{\nu}} [|k||q|] L^2 g_3(s) - iMT_{tot} \]
\[ M_{L}^{(\pm 2)} = 0 \]
\[ (48) \]
These formulae are different from the correspondent expressions given in [11] by the factor \( (-1)^n \) which enters now in the resonance propagator. All other formulae given in [11] for the single meson photoproduction are not changed due to this redefinition.

The second \( (S = \frac{3}{2}) \) amplitude has the structure:
\[ A_{\mu}^2 = \bar{u}(qN)X_{\alpha_1 \ldots \alpha_n}(q^+) F_{\mu_1 \ldots \mu_n} \gamma\mu \gamma_5 \chi_{\beta_1 \ldots \beta_n} X_{\beta_1 \ldots \beta_n}(k^-)u(kN) \]
\[ (49) \]

5 The gauge invariant vertices

5.1 The \('+' \) states

Here we have three vertices,
\[ V_{\alpha_1 \ldots \alpha_n}^{(1-\mu)}(k^+) = \gamma_5 \gamma_{\mu} X_{\alpha_1 \ldots \alpha_n}(k^+) \]
\[ V_{\alpha_1 \ldots \alpha_n}^{(2-\mu)}(k^+) = \gamma_5 \gamma_{\mu} X_{\alpha_1 \ldots \alpha_n}(k^+) \]
\[ V_{\alpha_1 \ldots \alpha_n}^{(3-\mu)}(k^+) = \gamma_5 \gamma_{\mu} X_{\alpha_1 \ldots \alpha_n}(k^+) \]
\[ (50) \]

The vertices (1) and (3) are used to fit the photo-production reactions. Let us consider the vertex 2 with a propagator of the baryon state:
\[ F_{\delta_1 \ldots \delta_n}^{(\alpha_1 \ldots \alpha_n)} V_{\alpha_1 \ldots \alpha_n}^{(2+\mu)}(k^+) = F_{\delta_1 \ldots \delta_n}^{(\alpha_1 \ldots \alpha_n)} \gamma_5 \gamma_{\mu} X_{\alpha_1 \ldots \alpha_n}(k^+) = \]
\[ \frac{\alpha^{(n+2)}}{\alpha^{(n)}} F_{\delta_1 \ldots \delta_n}^{(\alpha_1 \ldots \alpha_n)} \gamma_5 \gamma_{\mu} X_{\alpha_1 \ldots \alpha_n}(k^+) \]
\[ (51) \]

Taking into account that
\[ F_{\delta_1 \ldots \delta_n}^{(\alpha_1 \ldots \alpha_n)} g_{\alpha \beta} = 0 \quad F_{\delta_1 \ldots \delta_n}^{(\alpha_1 \ldots \alpha_n)} g_{\mu \nu} = 0 \]
we obtain that this vertex can be written as:
\[ \frac{\alpha^{(n+2)}}{\alpha^{(n)}} F_{\delta_1 \ldots \delta_n}^{(\alpha_1 \ldots \alpha_n)} \times \]
\[ \left( k^+ \gamma_5 k^ nob \right) X_{\alpha_1 \ldots \alpha_n}(k^+) \]
\[ - \frac{k^2}{2n+3} \left( V_{\alpha_1 \ldots \alpha_n}^{(1+\mu)}(k^+) + n \right) V_{\alpha_1 \ldots \alpha_n}^{(3+\mu)}(k^+) \]
\[ (53) \]

It means that instead of \( V_{\alpha_1 \ldots \alpha_n}^{(2+\mu)}(k^+) \) one can use the vertex:
\[ \tilde{V}_{\alpha_1 \ldots \alpha_n}^{(\beta+\mu)}(k^+) = \frac{1}{2} k^+ i \gamma_5 k^\beta N \gamma_\mu X_{\alpha_1 \ldots \alpha_n}(k^+) \]
\[ (54) \]

Let us calculate the convolution of the vertices with photon momentum. Remember:
\[ k^\mu = \frac{1}{2} (k^N + k^\gamma) \]
\[ (55) \]
Thus:
\[ k^\gamma g_{\mu \nu} = -k^\mu \gamma^\rho k^\nu = -k^2 \]
\[ (56) \]

Therefore:
\[ V_{\alpha_1 \ldots \alpha_n}^{(1+\mu)}(k^+) k^\gamma = \frac{1}{2} k^+ i \gamma_5 X_{\alpha_1 \ldots \alpha_n}(k^+) \]
\[ \tilde{V}_{\alpha_1 \ldots \alpha_n}^{(2+\mu)}(k^+) k^\gamma = \frac{1}{2} k^+ i \gamma_5 X_{\alpha_1 \ldots \alpha_n}(k^+) k^\gamma \]
\[ (57) \]

It means that the gauge invariant operator can be made as:
\[ V_{\alpha_1 \ldots \alpha_n}^{(1+\mu)}(k^+) = V_{\alpha_1 \ldots \alpha_n}^{(2+\mu)}(k^+) - \frac{1}{k^2} \tilde{V}_{\alpha_1 \ldots \alpha_n}^{(2+\mu)}(k^+) \]
\[ (58) \]
Which leads to a very simple expression:

\[ V_{\alpha_1...\alpha_n}^{G(1+)}(k^\perp) = \gamma_\mu i \gamma_5 X_{\alpha_1...\alpha_n}(k^\perp) \]  

(59)

The third vertex should be considered with propagator of the resonance. Thus the convolution with photon momentum:

\[ F_{\beta_1...\beta_n}^{\alpha_1...\alpha_n} V_{\alpha_1...\alpha_n}^{G(3+)}(k^\perp) k_\mu = -F_{\beta_1...\beta_n}^{\alpha_1...\alpha_n} \gamma_\mu i \gamma_5 X_{\alpha_1...\alpha_n}(k^\perp) = -F_{\beta_1...\beta_n}^{\alpha_1...\alpha_n} k^\perp \gamma_5 X_{\alpha_1...\alpha_n} \]  

(60)

Which coincides with vertex (1). It means that gauge invariant combination could be

\[ V_{\alpha_1...\alpha_n}^{G(3+)}(k^\perp) = V_{\alpha_1...\alpha_n}^{G(1+)}(k^\perp) \]  

(61)

which is used in some of articles. However for us it is easier to use another combination:

\[ V_{\alpha_1...\alpha_n}^{G(3+)}(k^\perp) = V_{\alpha_1...\alpha_n}^{G(1+)}(k^\perp) - \frac{1}{k^\perp} V_{\alpha_1...\alpha_n}^{G(2+)}(k^\perp) \]  

(62)

In the presence of the resonance propagator it can be rewritten as:

\[ V_{\alpha_1...\alpha_n}^{G(3+)}(k^\perp) = \gamma_\mu i \gamma_5 X_{\alpha_1...\alpha_n}(k^\perp) g_{\mu\alpha_n} \]  

(63)

The second vertex can be written in the gauge invariant form using the property:

\[ k^\perp (g_{\mu\nu} - \frac{k^\gamma P_\nu}{(P k^\perp)}) = k^\perp + \frac{(k^\gamma P_\mu)}{(P k^\perp)} P_\mu \]  

(64)

Taking into account that:

\[ k^\perp = (k^\gamma)^2 - \frac{(k^\gamma P)^2}{P^2} \quad k^\perp = -k^\perp \]  

(65)

We obtain:

\[ k^\perp (g_{\mu\nu} - \frac{k^\gamma P_\mu}{(P k^\perp)}) = -k^\perp + \frac{(k^\gamma P)}{(P k^\perp)} P_\mu \]  

(66)

Using eq. (63) we obtain:

\[ F_{1}^{2+} = F_{1}^{2+} \frac{\alpha^{(n)}}{\alpha^{(n+2)}} + \frac{k^2}{2n+3} (F_{1}^{1+} + n F_{1}^{3+}) \]  

(70)

Taking into account:

\[ \xi_0 = \frac{\alpha^{(n)}}{\alpha^{(n+2)}} = -\frac{k^2}{2n+3} \]  

(71)

\[ \tilde{F}_{1}^{2+} = 0 \quad \tilde{F}_{2}^{2+} = 0 \quad \tilde{F}_{3}^{2+} = 0 \quad \tilde{F}_{4}^{2+} = 0 \quad \tilde{F}_{5}^{2+} = k^2 \lambda_n \tilde{P}_{n+1} \quad \tilde{F}_{6}^{2+} = -k^2 \lambda_n \tilde{P}_{n} \]  

(72)

we obtain:

\[ E_{1}^{2+} = M_{1}^{2+} = 0 \quad L_{1}^{2+} = \frac{k^2}{n+1} \lambda_n \]  

(73)

Then the first vertex will be:

\[ A_{\mu}^{(1+)} = \bar{u}(q N) X_{\alpha_1...\alpha_n}^{(n)} (q^+) P_{\mu \beta_2...\beta_n}^{\alpha_1...\alpha_n} V_{\alpha_1...\alpha_n}^{G(3+)}(k^\perp) u(k_N) \]  

(74)

Then we obtain:

\[ \tilde{F}_{1}^{G(1+)} = \lambda_n \tilde{P}_{n+1} \quad \tilde{F}_{2}^{G(1+)} = \lambda_n \tilde{P}_{n} \quad \tilde{F}_{3}^{G(1+)} = \tilde{F}_{4}^{G(1+)} = 0 \quad \tilde{F}_{5}^{G(1+)} = \tilde{F}_{6}^{G(1+)} = 0 \]  

(75)

Therefore

\[ L_{1}^{G} = \frac{\lambda_n}{n+1} \quad L_{1}^{G} = 0 \]  

(76)

The second (S = 3/2) amplitude has the structure:

\[ A_{\mu}^{G(3+)} = \bar{u}(q N) X_{\alpha_1...\alpha_n}^{(n)} (q^+) P_{\mu \beta_2...\beta_n}^{\alpha_1...\alpha_n} V_{\alpha_1...\alpha_n}^{G(3+)}(k^\perp) u(k_N) \]  

(77)

and

\[ \tilde{F}_{1}^{G(3+)} = 0 \quad \tilde{F}_{2}^{G(3+)} = -\frac{\lambda_n}{n} \tilde{P}_{n} \quad \tilde{F}_{3}^{G(3+)} = -\frac{\lambda_n}{n} \tilde{P}_{n+1} \quad \tilde{F}_{4}^{G(3+)} = -\frac{\lambda_n}{n} \tilde{P}_{n} \quad \tilde{F}_{5}^{G(3+)} = \tilde{F}_{6}^{G(3+)} = 0 \]  

(78)

Therefore

\[ E_{1}^{3+} = -n M_{1}^{3+} = \frac{\lambda_n}{n+1} \quad L_{1}^{3+} = 0 \]  

(79)

Thus we obtain a behavior of the CGLN functions without any kinematical problems.
5.2 The ‘-’ states

For the decay of a ‘-’ state with total spin \( J \) into \( \gamma N \), the vertex functions have the form:

\[
V_{\alpha_1\ldots\alpha_n}^{(1-)}(k^+) = \gamma_{\gamma\mu} X_{\alpha_1\ldots\alpha_n}^{(n+1)}(k^+) ,
\]
\[
V_{\alpha_1\ldots\alpha_n}^{(2-)}(k^+) = X_{\alpha_1\ldots\alpha_n}^{(n+1)}(k^+) ,
\]
\[
V_{\alpha_1\ldots\alpha_n}^{(3-)}(k^+) = X_{\alpha_2\ldots\alpha_n}^{(n-1)}(k^+) g_{\alpha_1\mu} .
\]

(80)

If one follows this idea we obtain the following vertices:

\[
V_{\alpha_1\ldots\alpha_n}^{G(1-)}(k^+) = \gamma_{\gamma\mu} X_{\alpha_1\ldots\alpha_n}^{(n+1)}(k^+) ,
\]
\[
V_{\alpha_1\ldots\alpha_n}^{G(2-)}(k^+) = X_{\alpha_1\ldots\alpha_n}^{(n+1)}(k^+) \left( P_{\mu} (k^\gamma)^2 - k_{\gamma} \right) ,
\]
\[
V_{\alpha_1\ldots\alpha_n}^{G(3-)}(k^+) = X_{\alpha_2\ldots\alpha_n}^{(n-1)}(k^+) g_{\alpha_1\mu} .
\]

(81)

If one follows this idea we obtain the following expressions. For the first vertex:

\[
\mathcal{F}_{1}^{(1-)} = - \zeta_{n+1} P_n'
\]
\[
\mathcal{F}_{2}^{(1-)} = 0
\]
\[
\mathcal{F}_{3}^{(1-)} = 0
\]
\[
\mathcal{F}_{4}^{(1-)} = 0
\]
\[
\mathcal{F}_{5}^{(1-)} = 0
\]
\[
\mathcal{F}_{6}^{(1-)} = 0
\]

(82)

where

\[
\zeta_{n+1} = \frac{\alpha_{(n+1)}}{n+1} \langle |k||q\rangle^{n+1} \chi_i \chi_f
\]

(83)

And therefore for this vertex: Therefore

\[
-M_{n+1}^{1-} = E_{n+1}^{1-} = \frac{\zeta_{n+1}}{n+1} \quad \mathcal{L}_{n+1}^{1-} = 0
\]

(84)

The third vertex:

\[
\mathcal{F}_{1}^{(3-)} = g_{n-1} P_n'
\]
\[
\mathcal{F}_{2}^{(3-)} = 0
\]
\[
\mathcal{F}_{3}^{(3-)} = g_{n-1} P_n''
\]
\[
\mathcal{F}_{4}^{(3-)} = 0
\]
\[
\mathcal{F}_{5}^{(3-)} = 0
\]
\[
\mathcal{F}_{6}^{(3-)} = 0
\]

(85)

where

\[
g_{n-1} = \frac{\alpha_{(n-1)}}{n(n+1)} \langle |k||q\rangle^{n+1} \chi_i \chi_f
\]

(86)

\[
M_{n+1}^{3-} = 0 \quad E_{n+1}^{3-} = g_{n-1} \quad \mathcal{L}_{n+1}^{3-} = 0
\]

(87)

For the second vertex we obtain:

\[
\mathcal{F}_{1}^{G(2-)} = 0
\]
\[
\mathcal{F}_{2}^{G(2-)} = 0
\]
\[
\mathcal{F}_{3}^{G(2-)} = 0
\]
\[
\mathcal{F}_{4}^{G(2-)} = 0
\]
\[
\mathcal{F}_{5}^{G(2-)} = -(n+1) P_n' \quad \mathcal{F}_{6}^{G(2-)} = (n+1) P_{n+1}'
\]

(88)

where

\[
\Delta_n = \frac{\alpha_{(n)}}{n+1} \langle |k||q\rangle^{n+1} \chi_i \chi_f
\]

(89)

Therefore

\[
M_{n+1}^{2-} = E_{n+1}^{2-} = 0 \quad \mathcal{L}_{n+1}^{2-} = \Delta_n
\]

(90)

6 The connection with other vertex definitions

In the article [2] the electro-production amplitudes were introduced as

\[
A_k^{(\pm)} = \bar{u}_{\beta_1\ldots\beta_n}(P) v_{\alpha_1\ldots\beta_n} u(k_1)
\]

(91)

where for the ‘+’ sector:

\[
I_k^{(\pm)1}_{\beta_1\ldots\beta_n} = \sqrt{s} (q_{\beta_1\gamma_\mu} - \bar{q} g_{\beta_1\mu}) q_{\beta_2} \cdots q_{\beta_n} i \gamma_5
\]
\[
I_k^{(\pm)2}_{\beta_1\ldots\beta_n} = (q_{\beta_1} P_\mu - (q \bar{P}) g_{\beta_1\mu}) q_{\beta_2} \cdots q_{\beta_n} i \gamma_5
\]
\[
I_k^{(\pm)3}_{\beta_1\ldots\beta_n} = (q_{\beta_1} q_\mu - \bar{q} \bar{g}_{\beta_1\mu}) q_{\beta_2} \cdots q_{\beta_n} i \gamma_5
\]

(92)

Here \( q \) was used in [2] to define the vector of the virtual photon and therefore \( q \equiv k^3 \). The vector \( P_\mu = \frac{1}{2} (P + k_1) \) is \( P_\mu = -q_\mu / 2 \). We also use our definition of \( \gamma_5 \) matrix \( \gamma_5 = \gamma_0 \gamma_1 \gamma_2 \gamma_3 \) which is differ from \([2]\) by the factor \( i \). Another difference is that in the article [2] the normalization of the resonance polarization vectors is defined as:

\[
\bar{u}(k_1) u(k_1) = 2m
\]
\[
\bar{u}_{\beta_1\ldots\beta_n}(P) u_{\beta_1\ldots\beta_n}(P) = (-1)^n 2 \sqrt{s}
\]

(93)

while our definition is:

\[
\bar{u}(k_1) u(k_1) = 1
\]
\[
\bar{u}_{\beta_1\ldots\beta_n}(P) u_{\beta_1\ldots\beta_n}(P) = (-1)^n
\]

(94)

It means that our amplitudes are different by the coefficient

\[
N = \frac{1}{2 \sqrt{n m} \sqrt{s}}
\]

(95)

However we introduced in our propagator an additional factor \((-1)^n\). The orbital momentum operator in our definition depends on the relative momentum between nucleon and photon. It can be rewritten through the relative momentum of the photon and nucleon as:

\[
\mathcal{X}_{\alpha_1\ldots\alpha_n}^{(n)} (k^+) = (-1)^n \mathcal{X}_{\alpha_1\ldots\alpha_n}^{(n)} (q^+)
\]

(96)
Therefore we have two factors \((-1)^n\) which compensate each another. If one use definitions of the polarization vectors from [2] our vertices can be written as:

\[
\begin{align*}
\hat{V}^{G(1+)}_{\beta_1...\beta_n}(k^\perp) &= N_{\gamma}^{\alpha_1+} i \gamma_{\alpha_1...\alpha_n} (\gamma^-) \\
\hat{V}^{G(2+)}_{\beta_1...\beta_n}(k^\perp) &= -N q^\perp i \gamma_{\alpha_1...\alpha_n} (\gamma^-) \left( P_\mu \frac{q^2}{(Pq)} - q_\mu \right) \\
\hat{V}^{G(3+)}_{\beta_1...\beta_n}(k^\perp) &= N_{\gamma}^{\alpha_1} i \gamma_{\alpha_1...\alpha_n} (\gamma^-) (q_\mu g_{\mu \beta_1}) \\
\end{align*}
\]

(97)

If one use the definition of momenta and polarization vectors like in [2].

The corresponding amplitudes can be expressed as:

\[
A^{\pm}_\mu = \bar{u}_{\beta_1...\beta_n}(P) V^{G(1+)}_{\beta_1...\beta_n} u(k_1)
\]

(98)

where bispinors are taken as defined in [2]. The relation between these amplitudes (currents) and amplitudes given in (II.1) of article [2] are:

\[
A^{1+}_\mu = N_{\gamma}^{\alpha_1+} \sqrt{s} A^{(+)}_{K\mu} + \frac{q_\perp}{\sqrt{s}} \left( A^{(+)}_{K\mu} (\sqrt{s} - m) - \frac{1}{2} (\sqrt{s} + m) A^{(+)}_{K\mu} \right)
\]

\[
A^{2+}_\mu = N_{\gamma}^{\alpha_1+} \left[ \frac{q_\perp}{\sqrt{s}} A^{(+)}_{K\mu} + \frac{s - m^2}{2} A^{(+)}_{K\mu} \right]
\]

\[
A^{3+}_\mu = N_{\gamma}^{\alpha_1+} \left[ \frac{q_\perp}{\sqrt{s}} A^{(+)}_{K\mu} + \frac{1}{2} (\sqrt{s} - m) A^{(+)}_{K\mu} \right]
\]

Here

\[
\chi = m + \sqrt{s} - \frac{(P_\mu)}{\sqrt{s}} = m + \frac{(P k_1)}{\sqrt{s}}
\]

(100)

and

\[
q_\perp^2 = k_\perp^2 = q^2 - \frac{(P q)^2}{s}
\]

(101)

For the 1\,+\, sector \(J^P = 1/2^+, 3/2^+, \ldots\) the vertices in [2] are defined as:

\[
\begin{align*}
\Gamma^{(-)}_{31...\beta_n} &= -\sqrt{s} (q_\mu + q_\mu \beta_n) q_{\beta_2} \ldots q_{\beta_n} \\
\Gamma^{(-)}_{31...\beta_n} &= -q_\mu \beta_n (q_\mu \beta_n) q_{\beta_2} \ldots q_{\beta_n} \\
\Gamma^{(-)}_{31...\beta_n} &= -q_{\beta_n} q_\mu + q_{\beta_n} q_\mu \beta_n q_{\beta_2} \ldots q_{\beta_n}
\end{align*}
\]

(102)

As before, our amplitudes can be rewritten as:

\[
\begin{align*}
V^{G(1-)}_{\alpha_1...\alpha_n}(k^\perp) &= -N_{\gamma}^{\alpha_1+} \frac{1}{2} X^{(n)}_{\alpha_1...\alpha_n} (\gamma^-) \\
V^{G(2-)}_{\alpha_1...\alpha_n}(k^\perp) &= N X^{(n)}_{\alpha_1...\alpha_n} (\gamma^-) \left( P_\mu \frac{q^2}{(Pq)} - q_\mu \right), \\
V^{G(3-)}_{\alpha_1...\alpha_n}(k^\perp) &= -N X^{(n-1)}_{\alpha_1...\alpha_n} (k^-) g_{\alpha_1\mu}.
\end{align*}
\]

(103)

Then we obtain the following relation:

\[
\begin{align*}
A^{2-}_\mu &= N_{\gamma}^{\alpha_1+} \left[ \chi A^{(-)}_{K\mu} (\sqrt{s} + m) A^{(-)}_{K\mu} - \frac{\sqrt{s} - m}{2} A^{(-)}_{K\mu} \right] \\
A^{3-}_\mu &= N_{\gamma}^{\alpha_1+} \left[ \frac{q_\perp}{\sqrt{s}} A^{(-)}_{K\mu} + \frac{s - m^2}{2} A^{(-)}_{K\mu} \right]
\end{align*}
\]

(104)

For the states with spin 1/2 the situation is more complicated. The vertices in [2] are defined as:

\[
\begin{align*}
\Gamma^{(+)}_{\mu} &= \left( q^2 q_\mu - \frac{q_\mu}{\sqrt{s}} \right) \gamma_5 \\
\Gamma^{(+)2}_\mu &= \left( (\bar{P} q) \gamma_\mu - \bar{q} \bar{P} q \gamma_5 \right)
\end{align*}
\]

(105)

Thus we obtain the following relation:

\[
\begin{align*}
A^{1+}_\mu &= N_{\gamma}^{\alpha_1+} \frac{1}{\sqrt{s}} \left[ \frac{1}{2} A^{(+)}_{K\mu} - \frac{\sqrt{s} - m}{\sqrt{s} + m} A^{(+)}_{K\mu} \right] \\
A^{2+}_\mu &= N_{\gamma}^{\alpha_1+} \left[ q^2 A^{(+)}_{K\mu} + \frac{m^2}{2} A^{(+)}_{K\mu} \right]
\end{align*}
\]

(106)

and for the 1/2\,+\, state the vertices in [2] are

\[
\begin{align*}
\Gamma^{(-)}_{\mu} &= \left( q_\mu q_\mu^2 - \frac{q_\mu}{\sqrt{s}} \right) \gamma_5 \\
\Gamma^{(-)}_{2\mu} &= \left( \bar{q} \bar{P} q \gamma_\mu - (\bar{P} q) \gamma_5 \gamma_\mu \right)
\end{align*}
\]

(107)

And we have the following relation:

\[
\begin{align*}
A^{1-}_\mu &= N_{\gamma}^{\alpha_1+} \left[ -\frac{1}{2} A^{(-)}_{K\mu} + \frac{\sqrt{s} + m}{\sqrt{s} - m} A^{(-)}_{K\mu} \right] \\
A^{2-}_\mu &= N \left[ q^2 A^{(-)}_{K\mu} + \frac{m^2}{2} A^{(-)}_{K\mu} \right]
\end{align*}
\]

(108)

7 Summary

We develop the formalism for the analysis of the meson electro-production reaction. The method is fully based on the covariant approach used by the Bonn-Gatchina group for the analysis of the meson photoproduction data and can be naturally used for a combined analysis of the meson electro and photoproduction reactions.

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