Dominant Andreev Reflection through Nonlinear Radio-Frequency Transport

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ABSTRACT

It is found that Andreev reflection provides a deterministic teleportation process at an ideal normal-superconductor interface, making it behave like an information mirror. However, it is challenging to control the Andreev reflection in a spatially-separated junction due to the mode mixing at the interface. We theoretically propose the laser-induced Andreev reflection between two-component Fermi superfluid and normal states without mode mixing via spatially-uniform Rabi couplings. By analyzing the tunneling current up to the fourth order, we find that the Andreev current exhibits unconventional non-Ohmic transport at zero temperature. The Andreev current gives the only contribution in the synthetic junction system at zero detunings regardless of the ratio of the chemical potential bias to the superfluid gap, which is in sharp contrast to that in conventional junctions. Our result may give a potential impact on theoretical and experimental study of quantum many-body phenomena, and also pave a way for understanding the black hole information paradox through the Andreev reflection as a quantum-information mirror.

INTRODUCTION

The study of transport phenomena in ultracold atomic systems can greatly improve our understanding of quantum many-body problems owing to controllability of microscopic parameters. By using Feshbach resonances, one can tune the interparticle scattering length, allowing to scan quantum many-body systems from the weakly-interacting to strongly-correlated regimes. This technique has successfully been applied to study ultracold Fermi gases in terms of crossover between the Bardeen-Cooper-Schrieffer (BCS) and Bose-Einstein Condensation (BEC) regimes. More recently, a variety of experiments with such Fermi gases have been done to observe various quantum transport phenomena including the direct current transport of bulk and mesoscopic systems.

One of these topics of current interest is the Andreev reflection, originally introduced by Andreev to explain the anomalous resistance of heat flow through a normal state-superconducting (N-S) interface. The Andreev process involves a conversion between a particle and a hole-like mode as well as creation or annihilation of a condensed pair in the BCS ground state, and exhibits unique characteristics different from conventional tunnelings.

In electron systems, the Andreev reflection has also attracted attention in terms of quantum tunneling phenomena such as the proximity effect and the Josephson effect. In addition, the presence of the Andreev reflection has been reported in charge neutral systems such as liquid Helium and ultracold Fermi gas.

Recently, it was pointed out that the Andreev reflection can be regarded as an analogue of Hawking radiation at a black hole event horizon. By assuming a momentum-conserved tunneling, the Andreev reflection can provide an information-mirroring process which is similar to the black hole evaporation as Hayden and Preskill’s proposal. In spite of the interesting connection, in reality it is challenging to control the Andreev reflection in the spatially-separated junction in which mode mixing at the interface occurs. Therefore, a specific system to experimentally simulate the information-mirror process is still lacking.

In this work, we propose a system provoking the momentum-conserved Andreev reflection without the mode mixing by applying multiple radio-frequency (rf) laser fields (see Fig. 1(a)). While the rf spectroscopy in ultracold atomic gases has been harnessed to extract the quasiparticle excitation, we consider double rf laser fields which transfer two hyperfine states and in the BCS superfluid phase to the third hyperfine state in the normal phase, to realize an effective N-S interface on the internal space. To illustrate this synthetic interface, beyond real dimensions we propose an extra synthetic dimension, where each site denotes an internal state of atoms (Fig. 1(b)). The synthetic interface separates the normal state from the superfluid state and the analogy between real spaces and internal spaces is valid regardless of nonlinear transport processes.

Following the idea above and using the Schwinger-Keldysh formalism, we study the laser-induced tunneling current between the superfluid- and normal-state reservoirs driven by the Rabi couplings and By
analyzing the current up to the fourth order in $\Omega_{1,3}$ and $\Omega_{4,3}$, we find that the Andreev reflection appearing at the nonlinear regime is the only transport process in the junction system at zero detunings. Contrary to the conventional wisdom in the N-S systems, the Andreev current is not suppressed in the supergap regime, where the chemical potential bias between the normal and superfluid reservoirs is greater than the superfluid gap. Moreover, the momentum-conserved Andreev current exhibits a non-Ohmic transport at zero temperature. Below, we take $k_B = \hbar = 1$ and the system volume is taken to be unity.

RESULTS

Model

The Hamiltonian of the normal-state reservoir $|3\rangle$ with the energy level $\omega_3$ is given by $H_3 = \sum_p (\epsilon_p + \omega_3) c_{p,3}^\dagger c_{p,3}$ with $\epsilon_p = p^2/(2m)$, and $c_{p,3}^\dagger$ ($c_{p,3}$) creates (annihilates) a fermion in state $|3\rangle$ with momentum $p$. The Hamiltonian of the superfluid-state reservoir is taken as

$$H_{SF} = \sum_{k,\sigma} (\epsilon_k + \omega_\sigma) d_{k,\sigma}^\dagger d_{k,\sigma}$$

$$- g \sum_{k,k',\sigma} d_{k+\frac{\omega_\sigma}{2},\sigma}^\dagger d_{-k+\frac{\omega_\sigma}{2},\sigma}^\dagger d_{-k'+\frac{\omega_\sigma}{2},\sigma}^\dagger d_{k'+\frac{\omega_\sigma}{2},\sigma}^\dagger,$$

where $d_{k,\sigma}^\dagger$ and $d_{k,\sigma}$ are respectively the creation and annihilation operators for fermions in states $|\sigma = \uparrow, \downarrow\rangle$ with momentum $k$ and energy level $\omega_\sigma$. Here, $g$ is the strength of attractive interaction in the superfluid reservoir. We then introduce Rabi couplings to induce a particle transfer between the reservoirs. Typically, the wavelengths of rf fields are large compared to the size of the atomic gas and the spatial dependence of Rabi couplings are ignorable. Thus, the corresponding Rabi coupling term can be expressed as

$$H_t = \sum_{k,\sigma} \left( e^{-i\omega_L,\sigma t} \Omega_{\sigma,3} d_{k,\sigma}^\dagger c_{k,3} + \text{H.c.} \right),$$

where $\omega_L,\sigma$ is the laser frequency. We note that $H_t$ retains the momentum conservation. The total Hamiltonian of the system is thus $H = H_3 + H_{SF} + H_t$. The particle current operator between two reservoirs is defined as

$$\hat{I} = -\hat{N}_3 = i \left[ N_3, H_t \right] = -i \sum_{k,\sigma} e^{-i\omega_L,\sigma t} \Omega_{\sigma,3} d_{k,\sigma}^\dagger c_{k,3} + \text{H.c.},$$

where $N_3 = \sum_p c_{p,3}^\dagger c_{p,3}$ is the particle number operator of the normal-state reservoir. Notice that the current expression above corresponds to the tunneling current expression between the spatially-separated reservoirs except for the presence or absence of the momentum conservation. In what follows, we consider the zero detunings as $\omega_\uparrow - \omega_\downarrow - \omega_{L,\uparrow} = \omega_\downarrow - \omega_\uparrow - \omega_{L,\downarrow} = 0$, where the usual quasiparticle current is suppressed.

Laser-induced tunneling current.

To study the tunneling current between the reservoirs, Schwinger-Keldysh Green’s function formalism is applied with the operators evolving with Hamiltonian $H_0 = H_3 + H_{SF}$. After performing the perturbative expansion of the tunneling current with respect to $H_t$, we evaluate the correlation functions in each reservoir with thermal equilibrium under the grand-canonical Hamiltonian $K_0 = H_0 - \mu_3 N_3 - \mu_3 N_3$ and $\mu_3$ are the chemical potentials of particles in normal state $|3\rangle$ and superfluid states, respectively, and $N_3 = N_3 + N_3$ is the particle number operator of the superfluid reservoir ($N_3$ is the spin-resolved one). In the following, $a(H_0^{(a)}) (t)$ and $a(K_0)^{(a)} (t)$ denote operator $a$ in the Heisenberg pictures of $H_0$ and $K_0$, respectively. Using the relations $d_{k,\sigma}^{(a)} (t) = \ldots$
Here, we introduced the expectation value of the current $I \equiv \langle \hat{I}(t,t') \rangle$, where

$$
\langle \hat{I}(t,t') \rangle = -i \sum_{k,\sigma} \int_C dt_1 \cdots \int_C dt_n \Omega_{\sigma,3} \langle T_C e^{-i(\mu t' - \mu t)} \hat{c}_{k,3}^{(\text{K})}(t) \hat{c}_{k,3}^{(\text{K})}(t') \rangle F(t_1) \cdots F(t_n) + \text{H.c.},
$$

(4)

and $\Delta \mu = \mu_3 - \mu_3$ denotes the chemical potential bias between two reservoirs. The integral in Eq. (4) is taken along the Keldysh contour $C$ and $T_C$ is the contour order-product operator. By using the Langreth rules, we can change the contour of integral into the real time axis, from $t = -\infty$ to $t = +\infty$, and write each perturbation in terms of Green’s functions. In addition, we introduce the Nambu representation in which Green’s functions adopt the form

$$
i \hat{G}_{d(c)}(k,t-t') = \langle T_C A_{d(c)}(t) A_{d(c)}^\dagger(t') \rangle,
$$

(6)

with vectors $A_d(t) = (d_{k,1}(t), d_{k,1}^\dagger(t))^T$ and $A_c(t) = (c_{k,3}(t), c_{k,3}^\dagger(t))^T$. We note that $\hat{G}_{d,c}(k,t-t')$ has no off-diagonal elements, while in the superfluid states the anomalous Green’s functions $G_{d12}$ and $G_{d21}$ are generated due to the nonzero value of gap parameter $\Delta$. After these manipulations, the leading-order contribution in frequency representation is obtained as

$$
I^{(1)} = -2 \sum_{k,\sigma} \int \frac{d\omega}{2\pi} \text{Re} \left[ G_{d11}^{\text{ret}}(k,\omega) G_{c11}^{\text{ret}}(k,\omega - \Delta \mu) + G_{d11}^{\text{ret}}(k,\omega) G_{c11}^{\text{ret}}(k,\omega + \Delta \mu) \right],
$$

(7)

with the lesser Green’s functions $G^<$ and retarded Green’s functions $G^{\text{ret}}$.

We are now in a position to evaluate the particle current up to the next-to-leading order ($n=3$). The two coupling constants $\Omega_{1,3}$ and $\Omega_{1,3}$ give rise to the contractions like $\langle d_{k,1}^\dagger d_{k,1}^\dagger \rangle$ and $\langle d_{k,1} d_{k,1}^\dagger \rangle$, which do not vanish in the presence of the superfluid. As a result, such contractions cause tunnelings with pair degrees of freedom including the Andreev reflection. The total current up to this order is obtained as $I = I^{(1)} + I^{(3)}_1 + I^{(3)}_2 + I_A$, where

$$
I^{(3)}_1 = 4 \sum_{k,\sigma,\sigma'} \Omega_{1,3}^2 \Omega_{3,3}^2 \int \frac{d\omega}{2\pi} \left[ \text{Im} G_{d11}(k,\omega) \right]^2 \left[ \text{Im} G_{c11}(k,\omega - \Delta \mu) \right]^2 \left[ f(\omega - \Delta \mu) - f(\omega) \right],
$$

(9)

$$
I^{(3)}_2 = 16 \Omega_{1,3}^2 \Omega_{3,3}^2 \sum_k \int \frac{d\omega}{2\pi} \left[ \text{Im} G_{d12}(k,\omega) \right]^2 \text{Im} G_{c11}(k,\omega - \Delta \mu) \text{Im} G_{c22}(k,\omega + \Delta \mu) \left[ f(\omega) - f(\omega - \Delta \mu) \right],
$$

(10)

$$
I_A = 8 \Omega_{1,3}^2 \Omega_{3,3}^2 \sum_k \int \frac{d\omega}{2\pi} \left[ \text{Im} G_{d12}(k,\omega) \right]^2 \text{Im} G_{c11}(k,\omega - \Delta \mu) \text{Im} G_{c22}(k,\omega + \Delta \mu) \left[ f(\omega - \Delta \mu) - f(\omega + \Delta \mu) \right].
$$

(11)

Andreev reflection in nonlinear rf current.

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$$

(10)

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$$

(11)
Here $I_1^{(3)}$ is the current corresponding to the nonlinear quasiparticle tunneling between the reservoirs shown in Fig. 2(b). On the other hand, $I_2^{(3)}$ and $I_A$ originate from the processes represented by Fig. 2(c), while the former corresponds to a transfer of a single particle (hole) in normal side with creation or annihilation of pairs in superfluid side as an intermediate state and the latter arises from the Andreev reflection. To see the detailed properties of each contribution, we take the standard forms of Green’s functions in the normal-state reservoir, in which case the imaginary parts are given by $\text{Im} \, G_{c,11}(k, \omega) = \text{Im} \, G_{c,22}(k, -\omega) = -\pi \delta(\omega - \xi_{k,3})$. For the superfluid reservoir, we take the mean-field form of Green’s functions, where the gap parameter $\Delta_S = \sum_{k'}(d_{-k'}, d_{k', \uparrow})$ arises and the imaginary parts read $\text{Im} \, G_{d,1,1}(k, \omega) = -\pi[\nu_k^2 \delta(\omega - E_k) + \nu_{k'}^2 \delta(\omega + E_{k'})]$, and $\text{Im} \, G_{d,1,2}(k, \omega) = \text{Im} \, G_{d,2,1}(k, \omega) = -u_k\nu_k \pi \delta(\omega + E_k) - u_{k'}\nu_{k'} \pi \delta(\omega - E_{k'})$. Note that chemical potentials $\mu_S$ and $\mu_S$ are included in $\xi_{k,3/S} = \epsilon_k - \mu_S/S$, $E_k = \sqrt{\xi_{k,S}^2 + \Delta_S^2}$, and $u_k, \nu_k = \sqrt{(1 \pm \xi_{k,S}/E_k)/2}$. Inserting these expressions into Eqs. (8)–(11), we find that $I_1^{(1)}$ and $I_1^{(3)}$ vanish, since there is no overlap between $\text{Im} \, G_{c,11}(k, \omega - \Delta \mu)$ and $\text{Im} \, G_{d,1,1}(k, \omega)$ as long as $\Delta_S \neq 0$. Similarly, Eq. (10) is also shown to vanish. After inserting $\text{Re} \, G_{d,1,2}(k, \omega) = -u_k\nu_k [(\omega - E_k)^{-1} (\omega + E_k)^{-1}]$ into Eq. (11) and performing the momentum integration, we obtain the Andreev current $I_A$ as

$$I_A = \Theta(\mu_S)\Omega_{\uparrow,3}^2 \Omega_{\downarrow,3}^2 \frac{m^2 \mu S/e^{\Delta \mu/T}}{m S/e^{\Delta \mu/T} - 1} = \frac{m \mu S/e^{\Delta \mu/T}}{m S/e^{\Delta \mu/T} - 1}.$$  (12)

The step function $\Theta(\mu_S)$ in Eq. (12) indicates that such tunneling occurs only when the chemical potential of superfluid states is positive. Notice that all the quantities in Eq. (12) can be determined in experiments and hence the result can be directly compared with the experimental result of the nonlinear rf current. This result is valid at weak tunneling coupling where tunneling term is taken as a perturbation, and thus will not be changed qualitatively by higher order corrections when $\Omega_{\sigma,3}$ is small.

If we take $\Delta \mu \to 0$ with finite temperature $T > 0$, $I_A$ will reduce to a linear form, $I_A = \kappa_0(T) \Delta \mu$, with the conductance $\kappa_0(T) = \Theta(\mu_S)\Omega_{\uparrow,3}^2 \Omega_{\downarrow,3}^2 m \sqrt{2 \mu S/(2 \pi \Delta_S^2 T)}$. Another interesting fact is that, at zero temperature with finite chemical potential bias, $I_A \propto \text{sgn}(\Delta \mu)$ does not depend on the magnitude of $\Delta \mu$. Such a non-Ohm’s transport characteristic is nontrivial, since in the conventional N-S interfaces the Andreev currents at a low bias basically obey the Ohm’s law even at zero temperature. Moreover, $I_A$ is not suppressed in the supergap regime ($\Delta \mu > \Delta_S$) in contrast to the conventional N-S case.

The tunneling current $I_A$ and the conductance $\kappa = I_A/\Delta \mu$ between the reservoirs are shown in Fig. 3 as functions of $\Delta \mu$. Here $I_A$ is normalized by a constant $\chi = \Omega_{\uparrow,3}^2 \Omega_{\downarrow,3}^2 m \sqrt{2 \mu S}/(2m)$ and $k_F = \sqrt{2 m E_F}$ are respectively the Fermi energy and Fermi momentum for the superfluid, while $\kappa$ is normalized by $\kappa_0$, the conductance at $\Delta \mu = 0$. In this figure, we take the values of $\mu_S$ and $\Delta_S$ as typical experimental values in the unitary limit, $\mu_S/E_F = 0.38$ and $\Delta_S/E_F = 0.47$, and the temperature is set as $T/T_F = 0.06$, where $T_F$ is the Fermi temperature. We note that the Andreev current in this figure exists not only in subgap, but also in supergap regions.

Moreover, in Fig. 4, we show the tunneling current at zero temperature as a function of dimensionless interaction strength $1/(ak_F)$, by using the result of $\mu_S$ and $\Delta_S$ obtained with the diagrammatic approach, where the scattering length $a$ in the superfluid reservoir is defined by $m/a = 1/a_0 + m A$, with the momentum cutoff $A$.

We can see that the current decreases monotonically from the BCS limit to the BEC limit as the attraction increases, and becomes zero at around $1/(ak_F) = 0.5$ where $\mu_S = 0$. This indicates that the Andreev reflection is abundant in the BCS regime, while disappear in the BEC system, even in the momentum-conserved tunneling processes. This behavior is consistent with that in the previous theoretical work on the spatial N-S junctions without the momentum conservation. Beyond our results that are accurate up to fourth order in $\Omega_{\sigma,3}$, the bosonic Andreev process, in which a pair of incident bosonic particles (holes) is transferred to a pair of bosonic holes (particles), may arise in the BEC limit. However, the leading order contribution of the bosonic Andreev process is pro-
native no Fermi surface exists in the normal phase for large negative \(\Delta \mu\) since the overlap between two spectra becomes small and \(\Delta \mu\) is positive so that the current is on the normal side. Still, \(\chi\) is the normalizing constant.

\(\Delta \mu\), the quasiparticle tunneling \(I^{(1)}\) and \(I^{(3)}\), and pair tunneling processes \(I^{(3)}_2\) will occur due to the overlap between spectral functions (Dirac functions) even at zero temperature. On the other hand, the nonzero detuning will only give a shift on \(\Delta \mu\) in the formula of Andreev current \(I_A\) Eq. (12): \(I_A(\mu_S, \Delta \mu) \rightarrow I_A(\mu_S, \Delta \mu + \delta)\) with \(\delta\) denoting the detuning. In this regard, the Andreev current, quasiparticle current, and pair tunneling current can coexist out of rf resonances. One way to distinguish the Andreev current is to tune the temperature above the superfluid critical temperature \(T_c\), where all components are in normal phase and the Andreev current disappears. By comparing the signal below \(T_c\) and the one above \(T_c\), the Andreev current can be extracted from the total signal. To do this, the broadened spectral functions at finite temperature should be taken into account accurately, which is left for future work.

Since two reservoirs are spatially overlapped in our synthetic N-S junction, there exist residual interactions between normal and superfluid components, apart from the strong interaction within the superfluid. This will cause self-energy shifts in each reservoir. A well-known correction in the weakly interacting limit is the Hartree shift \(\Sigma_3\), given by \(\Sigma_3 = \frac{4\pi}{m} N_3 \Sigma_3\) and \(\Sigma_3 = \sum_{\sigma} \frac{4\pi}{m} N_\sigma\), where \(a_{\sigma3}\) is the scattering length between components \(|\sigma\rangle\) and \(|3\rangle\). These give an effective shift of the chemical potentials as \(\mu_3^{\text{eff}} = \mu_3 - \Sigma_3\) and \(\mu_\sigma^{\text{eff}} = \mu_\sigma - \Sigma_\sigma\). Consequently, the chemical potential bias will be modified as \(\Delta \mu^{\text{eff}} = \mu_3^{\text{eff}} - \mu_\sigma^{\text{eff}} = \Delta \mu - \Sigma_3 + \frac{\Sigma_\sigma + \Sigma_\downarrow}{2}\), where

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**FIG. 4. Andreev current.** The solid line shows the Andreev current as a function of dimensionless coupling \(1/(ak_F)\), where \(k_F\) is the Fermi momentum and \(a\) is the scattering length. We take the limit that \(\Delta \mu/T \rightarrow \infty\) for simplicity and \(\Delta \mu\) is positive so that the current is on the normal side. Still, \(\chi\) is the normalizing constant.

**FIG. 5. Quasiparticle-tunneling current with broadened spectral functions.** \(X = 2mk_F(\Omega_{\sigma3}^2 + \Omega_{\downarrow3}^2)/\pi^2\) is the normalization constant for the current, where \(k_F\) is the Fermi momentum and \(\Omega_{\sigma3}\) represent the Rabi couplings. The black, blue, and purple solid lines depict the quasiparticle-tunneling current with the widths of spectra \(T/E_F = 0.02, 0.03, \) and 0.04, respectively. The values of the chemical potential \(\mu_S\) and gap energy \(\Delta_S\) are set to be \(\mu_S/E_F = 0.38\) and \(\Delta_S/E_F = 0.47\) as those in unitary limit, while \(T\) is set as \(T/T_F = 0.06(E_F\) and \(T\) are the Fermi energy and Fermi temperature of the superfluid, respectively).
\( \mu_{\text{eff}} = (\mu_{\text{i}} + \mu_{\text{f}})/2 \) is the averaged effective chemical potential in the superfluid. Therefore, by replacing \( \Delta \mu \) with \( \Delta \mu_{\text{eff}} \) in Eq. (12), the result is adapted to the case with weak residual interactions between reservoirs. On the other hand, an effective magnetic field \( h_{\text{eff}} = (\Sigma_{\text{i}} - \Sigma_{\text{f}})/2 \) arises in the superfluid due to the unbalanced residual interactions, even in the balanced mixture \( \mu_{\text{i}} = \mu_{\text{f}} \). However, it can be negligible if \( h_{\text{eff}} \) is sufficiently small compared to \( \Delta \).

In the case of the \(^6\text{Li}\) three-component mixture, it is known that the strong three-body loss shortens the system's lifetime\(^{49,50} \). The timescale of tunneling processes can be estimated by the uncertainty principle: \( \Omega \tau \geq \frac{1}{\pi \hbar} \) (where we take \( \Omega_{3,\uparrow} = \Omega_{3,\downarrow} \equiv \Omega \) for simplicity). Taking the typical magnitude of Fermi energy in \(^6\text{Li}\) Fermi gases as \( 10^3 \) Hz and \( \Omega/E_F \sim 0.1 \) to justify the perturbative treatment, we have \( \tau \sim 10^{-3} \)s, which is smaller than the timescale of atom losses. More precisely, the particle transport timescale can be estimated by using \( \tau' = \beta \kappa^{-1} \), where \( \beta = \frac{\partial \Delta \mu}{\partial \mu} \tau' \) is the compressibility of the reservoirs and \( \kappa \) is the conductance of the particle current. The dimension of \( \beta \) is given by \( \beta \sim \frac{\text{m}^2}{\text{MeV} \cdot \text{s}} \).

Therefore, the expression of the conductance \( \kappa \) is the compressibility \( \kappa_0 \) of the Andreev current, we have \( \tau' \sim \Delta_0^2 T/(\pi \Omega^3) \). In this sense, by adjusting the value of Rabi frequencies \( \Omega \), one can tune the transport timescale to be smaller than the timescale of atom losses. In either way, this indicates that one can measure the Andreev current before encountering the significant particle-number losses.

A promising way to detect the Andreev current with avoiding the three-body loss would be the preparation of the sole superfluid state with two-component fermions before applying the Rabi coupling. In this case, the normal phase is dilute and therefore largely negative \( \mu_3 \) (i.e., largely negative \( \Delta \mu \)) would be realized. As we showed in Fig. 3, still one may find the nonzero Andreev current in such a supergap regime. Moreover, in order to avoid the overlap of Feshbach resonances leading to the strong three-body losses, it is possible to use higher hyperfine states as the normal component\(^{32} \).

**DISCUSSION**

Our proposed system inducing the momentum-conserved tunneling may also be promising for understanding the black hole information paradox. Some similarities can be found between the momentum-conserved Andreev reflection and Hayden-Preskill model\(^{17,18} \), where certain final states in black hole allow the teleportation of information contained in matter falling into the black hole to the Hawking radiation going outwards\(^{53,54} \).

In our case, the BCS superfluid can be regarded as the black hole final states, which permit transferring of the quantum information encoded in an incident particle (hole) from the normal side to an outgoing hole (particle). Since the tunneling process is momentum-conserved, the hole is reflected with exactly the opposite momentum to the incident particle (hole), which ensures that the same information is teleported to the reflected one.

To further study this topic, we may prepare two hyperfine states in the normal side, for example, \(|3⟩\) and \(|4⟩\) with two Rabi couplings: \( \Omega_{3,3} \) and \( \Omega_{3,4} \). In this case, we can consider an incident particle at a superposition state of states \(|3⟩\) and \(|4⟩\), and investigate the reflected mode, which is expected to be in the same superposition state as the incident one. To be self-contained, we discuss how the information mirror process proposed in Ref.\(^{14} \) can be realized in this system. In the following, \(|1⟩_{k,3} \equiv c_{k,3}^\dagger |0⟩\) and \(|1⟩_{k,4} \equiv c_{k,4}^\dagger |0⟩\) are regarded as \(|1⟩_{k,3} \equiv c_{k,3}^\dagger |0⟩\) and \(|1⟩_{k,4} \equiv c_{k,4}^\dagger |0⟩\) in the normal side, respectively. We consider an incident mode from the normal phase \(|ψ⟩ = |a⟩ ⊗ (d_{q,\uparrow}^\dagger c_{q,\uparrow}^\dagger + d_{q,\downarrow}^\dagger c_{q,\downarrow}^\dagger)⟨G| \), where \(|G⟩ = |1⟩_{q,\uparrow} |1⟩_{q,\downarrow} \cdots \) denotes the fulfilled Fermi sea. The incident mode evolving with the tunneling Hamiltonian reads

\[
|φ(τ)⟩ = e^{-iτH_0'} |φ_c⟩ = \sin (\Omega τ) |φ_d⟩ + \cos (\Omega τ) |φ_c⟩,
\]

(13)

where \(|φ_d⟩ = (ad_{k,\uparrow}^\dagger + bd_{k,\downarrow}^\dagger) |0⟩\), and \( τ \) is the smallest time interval for the quantum system to make a change. While satisfying the uncertainty principle, \( τ \approx \pi/2Ω \) is adopted to permit a complete mode transfer, \( c_{k,σ}^\dagger → d_{k,σ}^\dagger \). The combined state then becomes \(|ψ⟩ → |φ_d⟩ ⊗ (d_{q,\uparrow}^\dagger c_{q,\uparrow}^\dagger + d_{q,\downarrow}^\dagger c_{q,\downarrow}^\dagger)⟨G| \). The BCS ground state \(|ψ_{\text{BCS}}⟩ = \prod_k (u_k + v_k d_{k,\uparrow}^\dagger d_{k,\downarrow}) |0⟩ \) treated as the final state is imposed on the combined state, which yields a reflected state,

\[
⟨ψ_{\text{BCS}} |ψ⟩ \propto |φ_h⟩.
\]

(14)

**Summary**

In this work, we investigate the particle tunneling through an effective N-S interface designed by two rf laser fields that hold the momentum conservation. By addressing the nonlinear response regime in terms of the Schwinger-Keldysh formalism, we find that the Andreev reflection is the only process passing through the synthetic interface up to the fourth-order perturbation in \( H_0 \). We succeed in obtaining the analytical solution of the current and show the dependence of Andreev current and conductance on the chemical bias between two reservoirs. We also demonstrate how the Andreev current at zero temperature varies with the interaction strength, from the BCS to BEC regime. Another interesting outcome is that, different from conventional cases, the present tunneling current totally violates Ohm's law at zero temperature.
We note the reflected mode $|\phi_h\rangle = (a h_q^+ |q\rangle + b h_q^+ |q\rangle) |0\rangle$, where $|0\rangle$ denotes the quasiparticle vacuum while $h_q^+ |q\rangle = |0_{q} \uparrow 0_{q} \downarrow \cdots \rangle$ and $h_q^+ |0\rangle = |1_{q} \uparrow 0_{q} \downarrow \cdots \rangle$ denote holes, is a hole-like mode in the same spin state as the incident mode. This indicates that the quantum information is transferred from the incident mode to the reflected one, which is known as the deterministic teleportation.

METHODS

We apply the Schwinger-Keldysh Green’s function formalism to calculate the tunneling current in a non-equilibrium steady state. We use the expanded Keldysh contour, which includes two parts along the real time axis: a forward contour (from $t = -\infty$ to $t = \infty$) and a backward contour (from $t = \infty$ to $t = -\infty$). The current can be expressed in terms of lesser Green’s functions

$$G_c^<(k, t, t') = i \langle c_k(t') c_k(t) \rangle,$$

$$G_d^<(k, t, t') = i \langle d^+_k(t') d_k(t) \rangle,$$

where $t$ and $t'$ respectively denote the time arguments on the forward and backward parts. The integral over the Keldysh contour can be changed into that over the real time axis according to the Langreth rules, which read

$$C(t, t') = \int_C dt_1 A(t, t_1) B(t_1, t'),$$

$$C^>(t, t') = \int_{-\infty}^{\infty} dt_1 [A^\text{ret.}(t, t_1) B^\text{ret.}(t_1, t') + A^\text{adv.}(t, t_1) B^\text{adv.}(t_1, t')],$$

$$C^\text{ret.}(t, t') = \int_{-\infty}^{\infty} dt_1 A^\text{ret.}(t, t_1) B^\text{ret.}(t_1, t').$$

Here $A$, $B$, and $C$ denote arbitrary time-ordering correlation functions and the superscripts “$>$” and “adv.” are respectively for greater and advanced correlation functions. Since the Green’s function $G^\text{ret.}(t, t')$ or $G^<(t, t')$ only depends on the time difference $t - t'$, its Fourier transform depends on a single frequency $\omega$. Therefore, we obtain Eq. (7) for the lowest order term of the momentum-conserved tunneling current. The expressions for higher order terms, $I_1^{(3)}$, $I_2^{(3)}$, and $I_4$ are obtained similarly.

DATA AVAILABILITY

Data supporting the findings of this study are available from the corresponding author upon reasonable request.

CODE AVAILABILITY

The code used for the numerical calculations in this study are available from the corresponding author upon reasonable request.

REFERENCES

1Chin, C., Grimm, R., Julienne, P. & Tiesinga, E. Feshbach resonances in ultracold gases. Rev. Mod. Phys. 82, 1295–1286 (2010). URL https://link.aps.org/doi/10.1103/RevModPhys.82.1225.

2Regal, C. A., Greiner, M. & Jin, D. S. Observation of resonance condensation of fermionic atom pairs. Phys. Rev. Lett. 92, 040403 (2004). URL https://link.aps.org/doi/10.1103/PhysRevLett.92.040403.

3Bartenstein, M. et al. Crossover from a molecular boson-einstein condensate to a degenerate fermi gas. Phys. Rev. Lett. 120, 120401 (2004). URL https://link.aps.org/doi/10.1103/PhysRevLett.120.120401.

4Zwerger, W. The BCS-BEC Crossover and the Unitary Fermi Gas 1 edn, Vol. 836 (Springer, Berlin, Heidelberg, 2012).

5Kramer, S., Esslinger, T. & Brantut, J.-P. Two-terminal transport measurements with cold atoms. Journal of Physics: Condensed Matter 29, 343003 (2017).

6Enss, T. & Thywissen, J. H. Universal spin transport and quantum bounds for unitary fermions. Annual Review of Condensed Matter Physics 10, 85–106 (2019).

7Andreev, A. F. Thermal conductivity of the intermediate state of superconductors. Zh. Eksperim. i Teor. Fiz. 6 (1964). URL https://www.osti.gov/biblio/4071888.

8Tinkham, M. Introduction to superconductivity (Corporer Courier, Corporation, 2004).

9Asano, Y. Andreev Reflection in Superconducting Junctions (Springer, 2021).

10Pannetier, B. & Courtous, H. Andreev reflection and proximity effect. Journal of Low Temperature Physics 118, 599–615 (2000). URL https://doi.org/10.1023/A:100465326825.

11Klapwijk, T. M. Proximity effect due to an andreev perspectve. Journal of Superconductivity 17, 593–611 (2004). URL https://doi.org/10.1007/s10948-004-0773-0.

12Enrico, M. P., Fisher, S. N., Guénau, A. M., Pickett, G. R. & Torizuka, K. Direct observation of the andreev reflection of a beam of excitations in superfluid B. Phys. Rev. Lett. 70, 1846–1849 (1993). URL https://link.aps.org/doi/10.1103/PhysRevLett.70.1846.

13Husmann, D. et al. Connecting strongly correlated superfluids by a quantum point contact. Science 350, 1498–1501 (2015).

14Manikandan, S. K. & Jordan, A. N. Andreev reflections and the quantum physics of black holes. Phys. Rev. D 96, 124011 (2017). URL https://link.aps.org/doi/10.1103/PhysRevD.96.124011.

15Manikandan, S. K. & Jordan, A. N. Bosons falling into a black hole: A superfluid analogue. Phys. Rev. D 98, 124043 (2018). URL https://link.aps.org/doi/10.1103/PhysRevD.98.124043.

16Manikandan, S. K. & Jordan, A. N. Black holes as andreev reflecting mirrors. Phys. Rev. D 102, 064026 (2020). URL https://link.aps.org/doi/10.1103/PhysRevD.102.064026.

17Hayden, P. & Preskill, J. Black holes as mirrors: quantum information in random subsystems. Journal of High Energy Physics 2007, 120–120 (2007). URL https://doi.org/10.1088/1126-6708/2007/09/020.

18Lloyd, S. & Preskill, J. Unitarity of black hole evaporation in final-state projection models. Journal of High Energy Physics 2014, 1–30 (2014). URL https://doi.org/10.1007/JHEP08(2014)126.
19. Mukherjee, B. et al. Spectral response and contact of the unitary fermi gas. Phys. Rev. Lett. 122, 203402 (2019). URL https://link.aps.org/doi/10.1103/PhysRevLett.122.203402.

20. Kinnunen, J., Rodríguez, M. & Tormá, P. Pairing gap and in-gap excitations in trapped fermionic superfluids. Science 305, 1128–1131 (2004). URL https://link.aps.org/doi/10.1126/science.1100792.

21. Chin, C. et al. Observation of the pairing gap in a strongly interacting fermi gas. Science 305, 1128–1130 (2004). URL https://link.aps.org/doi/10.1126/science.1100818.

22. Mancini, M. et al. Observation of chiral edge states with neutral fermions in synthetic hall ribbons. Science 349, 1510–1513 (2015). URL https://www.science.org/doi/10.1126/science.aad98736.

23. Devillard, P., Guyon, R., Martin, T., Safi, I. & Chakraverty, B. K. Andreev reflection off a fluctuating superconductor in the absence of equilibrium. Phys. Rev. B 66, 165413 (2002). URL https://link.aps.org/doi/10.1103/PhysRevB.66.165413.

24. Blonder, G. E., Tinkham, M. & Klappwijk, T. M. Transition from metallic to tunneling regimes in superconducting microcoaxial-nanotube devices current, charge imbalance, and supercurrent conversion. Phys. Rev. B 25, 4515–4532 (1982).

25. Cuevas, J. C., Martín-Rodero, A. & Yeyati, A. L. Hamiltonian approach to the transport properties of superconducting quantum point contacts. Phys. Rev. B 54, 7366–7379 (1996). URL https://link.aps.org/doi/10.1103/PhysRevB.54.7366.

26. Uchino, S. Role of nambu-goldstone modes in the fermionic-superfluid point contact. Phys. Rev. Research 2, 023340 (2020). URL https://link.aps.org/doi/10.1103/PhysRevResearch.2.023340.

27. Bardeen, J., Cooper, L. N. & Schrieffer, J. R. Theory of superconductivity. Phys. Rev. 108, 1175–1204 (1957).

28. He, Y., Chen, Q. & Levin, K. Radio-frequency spectroscopy and the pairing gap in trapped fermi gases. Phys. Rev. A 72, 011602 (2005). URL https://link.aps.org/doi/10.1103/PhysRevA.72.011602.

29. Tormá, P. & Zoller, P. Laser probing of atomic cooper pairs. Phys. Rev. Lett. 85, 487–490 (2000).

30. Ohashi, Y. & Griffin, A. Single-particle excitations in a trapped gas of fermi atoms in the bcs-bec crossover region. Phys. Rev. A 72, 013601 (2005). URL https://link.aps.org/doi/10.1103/PhysRevA.72.013601.

31. Tsuchiya, S., Watanabe, R. & Ohashi, Y. Photoemission spectrum and effect of inhomogeneous pairing fluctuations in the bcs-bec crossover regime of an ultracold fermi gas. Phys. Rev. A 82, 033629 (2010).

32. Bruun, G. M., Tormá, P., Rodríguez, M. & Zoller, P. Laser probing of cooper-paired trapped atoms. Phys. Rev. A 64, 033609 (2001). URL https://link.aps.org/doi/10.1103/PhysRevA.64.033609.

33. Schwenzer, I. Brownian motion of a quantum oscillator. Journal of Mathematical Physics 2, 407–432 (1961). URL https://doi.org/10.1063/1.1703727.

34. Keldysh, L. V. Diagram technique for nonequilibrium processes. Zh. Eksp. Teor. Fiz. 47, 1515–1527 (1964).

35. Stefanucci, G. & van Leeuwen, R. Nonequilibrium Many-Body Theory of Quantum Systems: A Modern Introduction (Cambridge University Press, 2013).

36. Fetter, A. L. & Walecka, J. D. Quantum Theory of Many-Particle Systems (McGraw-Hill, Boston, 1971). URL https://www.bibsonomy.org/bibtex/2a89d5cb6b22c7993781f1356e06ebc17f51/bosefree.

37. Horikoshi, M., Koashi, M., Tajima, H., Ohashi, Y. & Kuwata-Gonokami, M. Ground-state thermodynamic quantities of homogeneous spin-1/2 fermions from the bcs region to the unitarity limit. Phys. Rev. X 7, 041004 (2017). URL https://link.aps.org/doi/10.1103/PhysRevX.7.041004.

38. Tajima, H. et al. Strong-coupling corrections to ground-state properties of a superfluid fermi gas. Phys. Rev. A 95, 043625 (2017). URL https://link.aps.org/doi/10.1103/PhysRevA.95.043625.

39. Ku, M. J. H., Sommer, A. T., Cheuk, L. W. & Zwierlein, M. W. Revealing the superfluid lambda transition in the universal thermodynamics of a unitary fermi gas. Science 335, 563–567 (2012). URL https://www.science.org/doi/10.1126/science.1214987.

40. Hoina, S. et al. Goldstone mode and pair-breaking excitations in atomic fermi superfluids. Nature Physics 13, 943–946 (2017). URL https://doi.org/10.1038/s41567-017-0059.

41. Sekino, Y., Tajima, H. & Uchino, S. Mesoscopic spin transport between strongly interacting fermi gases. Phys. Rev. Research 2, 023152 (2020). URL https://link.aps.org/doi/10.1103/PhysRevResearch.2.023152.

42. Netiawan, F. & Hofmann, J. Analytic approach to transport in josephson junctions beyond the andreev approximation: General theory and applications to the bcs-bcs crossover. arXiv preprint arXiv:2105.01533 (2021).

43. Zapata, L. & Sols, F. Andreev reflection in bosonic condensates. Phys. Rev. Lett. 102, 180405 (2009).

44. Zapata, L., Albert, M., Parentani, R. & Sols, F. Resonant hawking radiation in bose–einstein condensates. New Journal of Physics 13, 063048 (2011).

45. Uchino, S. Asymmetry and nonlinearity of current-bias characteristics in superfluid–normal-state junctions of weakly interacting bose gases. Phys. Rev. A 106, L011303 (2022). URL https://link.aps.org/doi/10.1103/PhysRevA.106.L011303.

46. Schirotek, A., Shin, Y.-i., Schunck, C. H. & Ketterle, W. Determination of the superfluid gap in atomic fermi gases by quasi-particle spectroscopy. Phys. Rev. Lett. 101, 140403 (2008).

47. Kinnunen, J. J. Hartree shift in unitary fermi gases. Phys. Rev. A 85, 012701 (2012). URL https://link.aps.org/doi/10.1103/PhysRevA.85.012701.

48. Ottenstein, T. B., Lompe, T., Kohnen, M., Wenz, A. N. & Jochim, S. Collisional stability of a three-component degenerate fermi gas. Phys. Rev. Lett. 101, 203202 (2008).

49. Hucks, J. H., Williams, J. R., Hazlett, E. L., Stites, R. W. & O’Hara, K. M. Three-body recombination in a three-state fermi gas with widely tunable interactions. Phys. Rev. Lett. 102, 165302 (2009). URL https://link.aps.org/doi/10.1103/PhysRevLett.102.165302.

50. Brantut, J.-P. & Makhlin, Y. Thermoelectric heat engine with ultracold atoms. Science 342, 713–715 (2013).

51. Ketterle, W. & Zwierlein, M. W. Making, probing and understanding ultracold fermi gases. Riv. Nuovo Cim. 31, 247–422 (2008). URL https://doi.org/10.1393/nucr/i2008-10033-1.

52. Hawking, S. W. Particle creation by black holes. Communications in Mathematical Physics 43, 199–220 (1975). URL https://doi.org/10.1007/BF02345020.

53. Horowitz, G. T. & Maldacena, J. The black hole final state. Journal of High Energy Physics 2004, 008–008 (2004). URL https://doi.org/10.1088/1126-6708/2004/02/008.

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AUTHOR CONTRIBUTIONS.

T. Z. carried out the study. T. Z. wrote the first draft and H. T., Y. S., S. U., and H. L. edited the manuscript.

COMPETING INTERESTS.

All the authors discussed the results and reviewed the manuscript.

The authors declare no competing interests.