Joint aggregation of cardinal and ordinal evaluations with an 
application to a student paper competition

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Abstract

An important problem in decision theory concerns the aggregation of individual rankings/ratings 
into a collective evaluation. We illustrate a new aggregation method in the context of the 2007 MSOM’s 
student paper competition. The aggregation problem in this competition poses two challenges. Firstly, 
each paper was reviewed only by a very small fraction of the judges; thus the aggregate evaluation is 
highly sensitive to the subjective scales chosen by the judges. Secondly, the judges provided both car-
dinal and ordinal evaluations (ratings and rankings) of the papers they reviewed. The contribution here 
is a new robust methodology that jointly aggregates ordinal and cardinal evaluations into a collective 
evaluation. This methodology is particularly suitable in cases of incomplete evaluations—i.e., when the 
individuals evaluate only a strict subset of the objects. This approach is potentially useful in manage-
rial decision making problems by a committee selecting projects from a large set or capital budgeting 
involving multiple priorities.

Keywords: Consensus ranking, group ranking, student paper competition, decision making, incom-
plete ranking aggregation, incomplete rating aggregation.

1 Introduction

We present here a new framework for group decision making in which a group of individuals, or judges, 
collectively ranks all of the objects in a universal set. This framework takes into consideration the pairwise 
comparisons implied by the individuals’ evaluations, and furthermore, it is first to combine ordinal rankings 
with cardinal ratings so as to achieve an aggregate ranking that represents as well as possible the individuals’ 
assessments, as measured by pre-set penalty functions.

Group-ranking problems are differentiated by whether the evaluations are given in ordinal or cardinal 
scales. An ordinal evaluation, or ranking, is one where the objects are ordered from “most preferred” to 
“least preferred” in the form of an ordered list (allowing ties). On the other hand, a cardinal evaluation, or 
rating, is an assignment of scalars, which are cardinal scores/grades, to the objects evaluated. In a rating, 
the difference between the scores of two objects indicates the magnitude of separation between such objects. 
Depending on the type of evaluations to be aggregated, group-ranking problems are referred to as ranking 
aggregation problem or rating aggregation problems.
Previous work addressed either the rankings alone aggregation problem (e.g. Kemeny and Snell 1962; Arrow 1963; Bartholdi et al. 1989; Hochbaum and Levin 2006), or the ratings alone aggregation problem (e.g. Keeney 1976; Saaty 1977), but not both. One of the primary contributions here is the technique that permits to jointly aggregate rankings and ratings into a collective evaluation. The individual evaluations input to a group-ranking problem can be complete or incomplete. In cases when each individual in the group ranks (rates) all of the objects in the universal set, then the ranking (rating) is said to be complete, or full list; otherwise, it is said to be incomplete, or partial list. The framework developed here is applicable when the judges’ ratings and rankings are incomplete.

The power of the framework developed here is illustrated in ranking the participants of the 2007 MSOM’s student paper competition (SPC). This SPC aggregate ranking problem poses challenges that are unique to that scenario:

1. The judges provided both ratings and rankings of the papers they reviewed. This requires to reconcile the possibly conflicting two types of evaluations.
2. The incompleteness of the evaluations was extreme: Each judge evaluated fewer than a tenth of the papers, and each paper was reviewed by fewer than a tenth of the judges. This caused the aggregation to be subject to the “incomplete evaluation” phenomenon bias, in which the individual scales used by the judges affect the average scores, even if the preference ordering of all the judges agree with each other. Also, outlier scores that are too low or too high tend to dominate the aggregate score of the papers.

The issue of subjective scales is well recognized within the aggregate ranking literature. French (1988) argues that, the value difference functions (the rating scales) of two individuals involve an arbitrary choice of scale and origin and thus the same numeric score from two different judges generally do not have the same meaning. Similarly, in the context of international surveys, a large number of studies (see, for example, Baumgartner and Steenkamp 2001; Smith 2004; Harzing 2006) show that the responses across different countries do not have the same meaning. In particular, these studies showed that even when asking respondents to rate each object using a simple 5-point rating scale, there are significant differences in the response styles between countries. One example of a difference in response style, that arises even when using a simple 5-point rating scale, is that in some countries there is a tendency to use only the extreme categories while in others there is a tendency to use only the middle categories. Another example of a difference in response style, is that in some countries there is a tendency to use only the top categories while in others there is a tendency to use only the bottom categories.

In a decision making set up when the judges provide scores, one can generate implied pairwise comparisons that reflect the intensity of the preference. This is done by letting this intensity be the difference in the scores for the two respective objects (these are called additive comparisons further discussed later). Hochbaum and Levin (2006) demonstrated that an aggregate rating that minimizes the penalties for differing from the individual judges’ implied pairwise comparisons overcomes the issue of using different parts of the scale and is less sensitive to subjective scales than the use of cardinal scores alone. These type of penalties are called separation penalties, and the optimization problem that seeks to assign scores that minimize the total separation penalties is called the separation problem. The separation-deviation (SD) model, proposed in (Hochbaum 2004, 2006; Hochbaum and Levin 2006), considers an aggregate rating scenario where the input to the rating process is given as separation gaps and point-wise scores. A separation gap is a quantity that expresses the intensity of the preference of one object i over another j by one particular judge. A point-wise score is a cardinal score of an object. The SD optimization problem combines the objective of minimizing the penalties of the deviation of the assigned scores to the point-wise scores assigned by judges to each object and the minimization of the separation penalties. For any choice of penalty functions the aggregate rating obtained by solving the SD model is a complete-rating that minimizes the sum of penalties on deviating from the given point-wise scores and separation gaps. The SD model is solved in polynomial time if the penalty functions are convex. It is NP-hard otherwise.

2
In our problem setting the judges provided only point-wise scores. Therefore there are no pairwise comparisons provided directly. Instead we use here the pairwise comparisons implied by the scores. The mechanism we propose here uses the SD model for both the rankings and ratings provided by the judges. For the rankings the penalty functions proposed are not convex. We “convexify” those functions and attain an optimization model that combines the separation and deviation penalties for deviating from the rankings and from the ratings of all judges. This is the first aggregate decision model that combines both ordinal and cardinal inputs.

The advantages of the mechanism proposed are obvious in comparison to standard approaches. It is easy to recognize a discrepancy in scores given to the same object by different judges. However, it is possible that the scores given are very close, yet each one is assigned from a different subjective scale. For one judge the score of 7 out of 10 can indicate the top evaluation, whereas for another it may mean the very bottom. Such scale differences cannot be identified by considering the variance of the scores alone. Our optimal solution to the SD problem, with the given penalty functions, allow to identify immediately the largest penalty pairs which, if large enough, indicate that different judges disagreed significantly on the comparison between such pairs of objects. This permits to identify inconsistencies and outliers that could be judges who are too lenient or too strict, or for other reasons had intensity of preference substantially different from the others. As such the methodology proposed not only provides an aggregate ranking, but also clarifies the disagreements and inconsistencies that allow to go back and possibly investigate the reasons for those outliers.

The paper is organized as follows: Section 2 provides a literature review on some relevant aggregate group-decision making techniques for rankings and ratings aggregation. Section 3 describes the evaluation methodology used in the 2007 MSOM’s SPC, and gives examples where the differences in scale used by the judges are evident. Section 4 reviews the models and distance metrics used to construct the penalty functions and defines the notions of consensus ranking and consensus rating used here. Section 5 describes the methodology for the combined use of the given ratings and rankings in order to obtain the aggregate ranking-ranking pair. Section 6 uses the methodology presented in Section 5 to rank the contestants in the 2007 MSOM’s SPC and analyzes the obtained results. Finally, Section 7 provides comments on our group-decision making framework and its usefulness for different applications and decision-making scenarios.

2 Literature Review

The ranking aggregation problem has been studied extensively, especially in the social choice literature. In this context, one of the most celebrated results is Arrows’s impossibility theorem (Arrow, 1963), which states that there is no “satisfactory” method to aggregate a set of rankings. Kenneth Arrow defined a satisfactory method as one that satisfies the following properties: universal domain, no imposition, monotonicity, independence of irrelevant alternatives, and non-dictatorship.

Kemeny and Snell (1962) proposed a set of axioms that a distance metric between two complete rankings should satisfy. They proved that these axioms were jointly satisfied by a unique metric distance. This distance between two rankings is measured by the number of rank reversals between them. A rank reversal is incurred whenever two objects have a different relative order in the given rankings. Similarly, half a rank reversal is incurred whenever two objects are tied in one ranking but not in the other. Kemeny and Snell defined the consensus ranking as the ranking that minimizes the sum of the distances to each of the input rankings. Bartholdi et al. (1989) showed that the optimization problem that needs to be solved to find the Kemeny-Snell consensus ranking is NP-hard.

Following the work of Kemeny-Snell, several axiomatic approaches have been developed to determine consensus. For instance, Bogart (1973) developed an axiomatic distance between partial orders. One of the applications of Bogart’s distance is to determine a consensus partial order from a set of partial orders. Moreno-Centeno (2010) developed an axiomatic distance between incomplete rankings that is used here.
The difficulties presented by Arrow’s impossibility theorem and the NP-hardness of finding the Kemeny-Snell’s consensus ranking can be overcome by replacing ordinal rankings by (cardinal) ratings. Following this direction, Keeney (1976) proved that the averaging method satisfied all of Arrow’s desirable properties. In the averaging method, the consensus rating of each object is the average of the scores it received. The most immediate drawback of this approach is that the averaging method implicitly requires that all judges use the same rating scale; that is, that all individuals are equally strict or equally lenient in their score assignments. This work also ignores the aspect of pairwise comparisons, which is essential to the Kemeny-Snell model.

Pairwise comparisons intensities are the input to Saaty’s Analytic Hierarchy Process technique (Saaty 1977). There, the optimal scores are found by the principal eigenvector technique. The readers are referred to (Hochbaum 2010) for an analysis of the principal eigenvector method in the context of aggregate decision making.

The separation-deviation model of (Hochbaum 2004, 2006; Hochbaum and Levin 2006) addresses the computational shortcomings of the Kemeny-Snell model, and the decision quality inadequacies of the principal eigenvector method. This model takes point-wise scores and potentially also pairwise comparison as inputs. It is the building block of the mechanism proposed here. As pointed out above, the respective separation-deviation optimization problem is solvable in polynomial time if all the penalty functions are convex (Hochbaum and Levin 2006).

The rating aggregation problem has also been studied in the context of multi-criteria decision making literature. Hochbaum and Levin (2006) showed the equivalence between the rating aggregation problem and the multi-criteria decision making problem. In this context, the non-axiomatic ELECTRE (Brans et al. 1975) and PROMETHEE (Brans and Vincke 1985) methods (and their extensions) solve the rating aggregation problem that arises from a multi-criteria decision problem by transforming it in some sense to a ranking aggregation problem. This transformation is claimed to be needed because each criterion is evaluated on a different scale.

3 The Data

The data used here is the evaluations for the 2007 MSOM’s SPC. There were 58 papers submitted to the competition and 63 judges participated in the evaluation process. Each of the 63 judges evaluated only three to five out of the 58 papers; and each of the 58 papers was evaluated by only three to five out of the 63 judges. Each judge reviewed and evaluated the assigned papers on the attributes (scale):

A) Problem importance/interest (1–10),
B) Problem modeling (0–10),
C) Analytical results (0–10),
D) Computational results (0–10),
E) Paper writing (1–10), and
F) Overall contribution to the field (Field contribution, for short) (1–10).

On each attribute, the judges assigned scores according to the score guidelines provided (see Table I). In addition, each judge also provided an ordinal ranking of the papers he/she reviewed (1 = best, 2 = second best, etc.).
Table 1: Interpretation of each numerical score. The journals considered are: MSOM, Operations Research (OR) and Management Science (MS).

| Score | Definition / Interpretation |
|-------|----------------------------|
| 10    | Attribute considered is comparable to that of the best papers published in the journals. |
| 8,9   | Attribute considered is comparable to that of the average papers published in the journals. |
| 7     | Attribute considered is at the minimum level for publication in the journals. |
| 5,6   | Attribute considered independently would require a minor revision before publication in the journals. |
| 3,4   | Attribute considered independently would require a major revision before publication in the journals. |
| 1,2   | Attribute considered would warrant by itself a rejection if the paper were submitted to the journals. |
| 0     | Attribute considered is not relevant or applicable to the paper being evaluated. |

Although precise score interpretations were provided to the judges (Table 1), they nevertheless appeared to have differed significantly in their evaluation and must have interpreted the scale differently. Examples of this phenomenon are illustrated for paper 43, in Table 2, and paper 26, in Table 3. To maintain the anonymity of judges and papers the judge and paper identification numbers were assigned randomly.

Table 2: Evaluations received on paper 43.

| Judge | Problem Importance | Problem Modeling | Analytical Results | Computational Results | Paper Writing | Field Contribution | Paper Ranking |
|-------|--------------------|------------------|-------------------|----------------------|--------------|-------------------|--------------|
| 47    | 9                  | 8                | 8                 | 8                    | 9            | 9                 | 1            |
| 6     | 6                  | 4                | 2                 | 4                    | 6            | 4.5               | 1            |
| 55    | 9                  | 6                | 0                 | 9                    | 8            | 6                 | 2            |
| 2     | 7                  | 7                | 2                 | 6                    | 7.5          | 4                 | 3            |

A detailed examination of Table 2 illustrates that paper 43 received in the Problem Modeling category a score of 8 by one judge (meaning that the Problem Modeling in the paper is comparable to that in an average paper published in MSOM, OR and MS), and a score of 4 by other judge (meaning that the problem modeling in the paper requires a major revision before publication in MSOM, OR and MS). These score differences are not insignificant. Another example of the differences between the judges’ evaluations is found on the Analytical Results category. In this category, a judge gave a score of 8 (meaning that the analytical results in the paper are comparable to those in an average paper published in MSOM, OR and MS), two judges gave a score of 2 (meaning that the analytical results in the paper are so bad that the paper should be rejected by MSOM, OR and MS), and the remaining judge considered that the category was not applicable to the paper (thus assigned the value of zero).

Table 3: Evaluations received on paper 26.

| Judge | Problem Importance | Problem Modeling | Analytical Results | Computational Results | Paper Writing | Field Contribution | Paper Ranking |
|-------|--------------------|------------------|-------------------|----------------------|--------------|-------------------|--------------|
| 21    | 8                  | 10               | 8                 | 8                    | 5            | 8                 | 3            |
| 24    | 8                  | 9                | 8                 | 10                   | 7            | 8                 | 1            |
| 14    | 7                  | 2                | 3                 | 2                    | 2            | 2                 | 5            |
| 26    | 8                  | 8                | 7                 | 8                    | 8            | 7                 | 3            |
| 49    | 10                 | 7                | 6                 | 9                    | 9            | 8                 | 1            |

In Table 3 the data shows that judge 14’s evaluations were not on the same scale as the evaluations of the other judges. In particular, in all attributes (with the exception of Problem Importance) judge 14 gave a score indicating that the paper would be rejected by MSOM, OR and MS; on the other hand in every attribute all of the other judges considered the paper is worth of publishing (some of their evaluations even indicate that the paper would be among the best papers published in MSOM, OR and MS!). Such discrepancies in the judges’ evaluations are quite common throughout the data.

Henceforth, we use as the input point-wise ratings the (cardinal) scores only on the attribute “Overall Contribution to the Field” (“Field Contribution”, for short). This is because the authors and the head judge of the 2007 MSOM’s SPC, believe that, among all the attributes that were scored according to the cardinal scale in Table 1 (i.e., excluding the ordinal paper ranking), this attribute is the single most important attribute evaluated.
4 Preliminaries

This section gives the notation used throughout the rest of the paper, and reviews the concepts of: the separation-deviation (SD) model, distance between incomplete ratings, and distance between incomplete rankings.

4.1 Notation

Let $V$ be the ground set of $n$ objects to be rated; without loss of generality, we assign a unique identifier to each element so that $V = \{1, 2, \ldots, n\}$. The judges are $K$ individuals. Each judge $k$, $k \in \{1, 2, \ldots, K\}$, provides a set of scores, or ratings vector, $a^{(k)}$ of the objects in a subset $A^{(k)}$ of $V$. Thus $a^{(k)}_j$ is the score of object $j$ by the $k^{th}$ individual, and $a^{(k)}_j$ is undefined if the $k^{th}$ individual did not rate object $j$. Without loss of generality, we assume that the scores are integers contained in a pre-specified interval $[\ell, u]$. The range of the ratings is defined as $R = u - \ell$.

We say that judge’s $k$ implied pairwise comparison, or separation gap of $i$ to $j$ is $p^{(k)}_{ij}$ where

$$p^{(k)}_{ij} = \begin{cases} a^{(k)}_i - a^{(k)}_j, & \text{if } i \in A^{(k)} \text{ and } j \in A^{(k)} \\ \text{undefined}, & \text{otherwise.} \end{cases}$$

Analogously, in the ordinal setting of the incomplete-ranking aggregation problem, each judge $k$ provides an incomplete ranking $b^{(k)}$ of the objects in $B^{(k)}$, a subset of $V$. Here $b^{(k)}_i$ is the rank (an ordinal number) of object $i$ in the ranking provided by the $k^{th}$ individual, and $b^{(k)}_i$ is undefined if individual $k$ did not rank object $i$.

The implied separation gaps for ordinal rankings are $\text{sign}(b^{(k)}_i - b^{(k)}_j)$ for $i, j \in B^{(k)}$, where the sign function is defined as:

$$\text{sign}(x) = \begin{cases} -1, & \text{if } x < 0 \\ 0, & \text{if } x = 0 \\ 1, & \text{if } x > 0. \end{cases}$$

For a vector of scores, or ratings, $a$ of a set of objects, we denote by $\text{rank}(a)$ the complete ranking of those objects obtained by sorting the objects according to their scores in $a$. For example, the vector of scores $(4, 5, 3, 2, 7, 3)$ corresponds to the ranking $(2, 1, 3, 5, 3)$.

4.2 Review of the Separation-Deviation Model

The SD model can be applied to group-decision making problems where the input is given as pairwise comparisons and/or point-wise scores. In the model formulation, the variable $x_i$ is the aggregate score of the $i^{th}$ object, and the variable $z_{ij}$ is the aggregate separation gap of the $i^{th}$ over the $j^{th}$ object. The separation gaps must be consistent. A set of separation gaps, $p_{ij}$, is said to be consistent if and only if for all triplets $i, j, k$, $p_{ij} + p_{jk} = p_{ik}$.

In (Hochbaum 2010; Hochbaum and Levin 2006) it was proved that the consistency of a set of separation gaps is equivalent to the existence of a set of scores $\omega_i$ for $i = 1, \ldots, n$ so that $p_{ij} = \omega_i - \omega_j$.

The mathematical programming formulation of the SD model is:

$$\begin{align*}
\text{(SD)} \quad & \min_{x, z} \sum_{k=1}^{K} \sum_{i=1}^{n} \sum_{j=1}^{n} f^{(k)}_{ij} (z_{ij} - p^{(k)}_{ij}) + \sum_{k=1}^{K} \sum_{i=1}^{n} g^{(k)}_{i} (x_{i} - a^{(k)}_{i}) \\
\text{subject to} \quad & z_{ij} = x_{i} - x_{j} \quad i = 1, \ldots, n; \quad j = 1, \ldots, n \\
& \ell \leq x_{i} \leq u \quad i = 1, \ldots, n \\
& x_{i} \in \mathbb{Z} \quad i = 1, \ldots, n. \end{align*}$$

(1.1)
The function \( f_{ij}^{(k)}(\cdot) \) penalizes the difference between the aggregate separation gap of the object pair \((i, j)\) and the \(k\)th reviewer’s separation gap of the object pair \((i, j)\). The function \( g_i^{(k)}(\cdot) \) penalizes the difference between the aggregate score of object \(i\) and the \(k\)th reviewer’s score of object \(i\). In order to ensure polynomial-time solvability, the functions \( f_{ij}^{(k)}(\cdot) \) and \( g_i^{(k)}(\cdot) \) must be convex. In the context of rating aggregation, the penalty functions assume the value 0 for the argument 0; meaning that if the output separation gap for \(i\) an \(j\), \(z_{ij}\) agree with \(p_{ij}^{(k)}\) then \(f_{ij}^{(k)}(z_{ij} - p_{ij}^{(k)}) = f_{ij}^{(k)}(0) = 0\). If \(i \notin B(k)\), then \(g_i^{(k)}(\cdot)\) is set to the constant function 0; similarly, if at least one of \(i\) or \(j \notin B(k)\), then \(f_{ij}^{(k)}(\cdot)\) is set to the constant function 0. Constraints (1b) enforce the consistency of the aggregate separation gaps conforming to the aggregate rating.

It was proved in (Hochbaum, 2004, 2006; Hochbaum and Levin, 2006) that problem (SD) is a special case of the convex dual of the minimum cost network flow (CDMCNF) problem. The most efficient algorithm known for the CDMCNF has a running time of \(O(mn \log n^2 \log (u - \ell))\) (Ahuja et al., 2003), where \(m\) is the total number of given separation gaps, and \(n\) is the number of objects. Ahuja et al. (2004) presented an alternative algorithm that uses a minimum-cut algorithm as a subroutine.

### 4.3 Distance between Incomplete-Ratings

Defining a penalty function on separation gaps is equivalent to quantifying the distance between them. Cook and Kress (1985) proposed a distance between complete ratings. This distance function was adapted to incomplete ratings in (Moreno-Centeno, 2010). It was shown that for a set of desirable properties this adaption, called normalized projected Cook-Kress distance (NPCK), is the only one that satisfies all those properties.

Given two incomplete ratings \(a^{(1)}\) and \(a^{(2)}\), the NPCK distance between the implied separation gaps is

\[
d_{\text{NPCK}}(a^{(1)}, a^{(2)}) = C \sum_{i \in \mathcal{A}^{(1)} \cap \mathcal{A}^{(2)}} \sum_{j \in \mathcal{A}^{(1)} \cap \mathcal{A}^{(2)}} |p_{ij}^{(1)} - p_{ij}^{(2)}|,
\]

where

\[
C = \left(4 \cdot R \cdot \left| \frac{\mathcal{A}^{(1)} \cap \mathcal{A}^{(2)}}{2} \right| \cdot \left| \frac{\mathcal{A}^{(1)} \cap \mathcal{A}^{(2)}}{2} \right| \right)^{-1}.
\]

\(C\) is a normalization constant that guarantees that \(0 \leq d_{\text{NPCK}}(\cdot, \cdot) \leq 1\) and \(R\) is the range of the ratings, \(R \equiv u - \ell\). We note that \(d_{\text{NPCK}}(a^{(1)}, a^{(2)}) = 0\) indicates a total agreement between the ratings \(a^{(1)}\) and \(a^{(2)}\), and \(d_{\text{NPCK}}(a^{(1)}, a^{(2)}) = 1\) indicates a total disagreement between the ratings \(a^{(1)}\) and \(a^{(2)}\). The normalization is important so that the distances in problem (4) are comparable to each other even when the individuals rate a different number of objects. The normalization constant \(C\) was chosen to address the following difficulties: (a) Each of the distances in problem (4) are between a complete rating \(x^{(c)}\) and an incomplete rating. (b) The number of objects rated by each incomplete rating are different; therefore the distances in problem (4) are over different dimensional spaces (the distance only considers the objects rated by the incomplete rating). (c) Distances in higher dimensional spaces tend to be bigger than distances in lower dimensional spaces; specifically, observe that the number of summands in equation (2) is the square of the number of objects rated by the incomplete rating.

In (Moreno-Centeno, 2010) the consensus rating, \(x^{(c)}\), is the optimal solution to the following optimization problem:

\[
\min_x \sum_{k=1}^{K} d_{\text{NPCK}}(a^{(k)}, x).
\]

The problem of finding the consensus rating is as a special case of the SD model and therefore solvable in polynomial time.
4.4 Distance between Incomplete-Rankings

Given a set of incomplete rankings, \( \{b^{(k)}\}_{k=1}^{K} \), the consensus ranking is defined as the complete ranking closest to the given incomplete rankings. Kemeny and Snell [1962] proposed a distance between complete rankings. This distance function was adapted to incomplete rankings in [Moreno-Centeno, 2010]. It was shown that for a set of desirable properties this adaption, called normalized projected Kemeny-Snell distance (NPKS), is the only one that satisfies all those properties.

Given two incomplete rankings \( b^{(1)} \) and \( b^{(2)} \), the NPKS distance between them is calculated as follows:

\[
d_{\text{NPKS}}(b^{(1)}, b^{(2)}) = D \sum_{i \in B^{(1)}} \sum_{j \in B^{(2)}} \frac{1}{2} |\text{sign}(b_i^{(1)} - b_j^{(1)}) - \text{sign}(b_i^{(2)} - b_j^{(2)})|, \tag{5}\]

where \( D = \left( |B^{(1)} \cap B^{(2)}|^2 - |B^{(1)} \cap B^{(2)}| \right)^{-1} \). \( D \) is a normalization constant that guarantees that \( 0 \leq d_{\text{NPKS}}(\cdot, \cdot) \leq 1 \). When \( d_{\text{NPKS}}(b^{(1)}, b^{(2)}) = 0 \) there is a total agreement between \( b^{(1)} \) and \( b^{(2)} \), and when \( d_{\text{NPKS}}(b^{(1)}, b^{(2)}) = 1 \) there is a total disagreement between \( b^{(1)} \) and \( b^{(2)} \). The normalization is important so that the distances in problem (6) are comparable to each other even when the individuals rank a different number of objects. The normalization constant \( D \) was chosen to address the following difficulties: (a) Each of the distances in problem (6) are between a complete ranking \( x^{(o)} \) and an incomplete ranking. (b) The number of objects ranked by each incomplete ranking are different; therefore the distances in problem (6) are over different dimensional spaces (the distance only considers the objects ranked by the incomplete ranking). (c) Distances in higher dimensional spaces tend to be bigger than distances in lower dimensional spaces; specifically, observe that the number of summands in equation (5) is the square of the number of objects ranked by the incomplete ranking.

The distance \( d_{\text{NPKS}}(b^{(1)}, b^{(2)}) \) has the following natural interpretation: The distance between two incomplete rankings is proportional to the number of rank reversals between them. Where a rank reversal is incurred whenever two objects have a different relative order in the rankings \( b^{(1)} \) and \( b^{(2)} \). Similarly, half a rank reversal is incurred whenever two objects are tied in one ranking but not in the other ranking.

In [Moreno-Centeno, 2010] the consensus ranking, \( x^{(o)} \), is the optimal solution to

\[
\min x \sum_{k=1}^{K} d_{\text{NPKS}}(b^{(k)}, x). \tag{6}\]

4.5 Convexifying the Rankings Penalty Function

In contrast to problem (4), problem (6) is NP-hard. We propose here to convexify the nonlinear sign functions in \( d_{\text{NPKS}}(\cdot, \cdot) \) as suggested in [Moreno-Centeno, 2010]:

\[
h_{ij}^{(k)}(z_{ij}) = \begin{cases} 
0 \frac{z_{ij}+1}{2} & \text{if sign}(b_i^{(k)} - b_j^{(k)}) = -1 \\
\max \left\{ \frac{-z_{ij}}{2}, \frac{z_{ij}}{2} \right\} & \text{if sign}(b_i^{(k)} - b_j^{(k)}) = 0 \\
\max \left\{ \frac{1-z_{ij}}{2}, 0 \right\} & \text{if sign}(b_i^{(k)} - b_j^{(k)}) = 1
\end{cases} \tag{7}\]

The following formulation is then a convex version of problem (6):

\[
\min x, z \sum_{k=1}^{K} D_k \sum_{i \in B^{(k)}} \sum_{j \in B^{(k)}} h_{ij}^{(k)}(z_{ij}) \tag{8a}
\]

subject to \( z_{ij} = x_i - x_j \) \( i = 1, \ldots, n; \) \( j = 1, \ldots, n \). \tag{8b}
We conclude this section by observing that, for the rankings given by the judges in the 2007 MSOM’s SPC, the optimal solution to convexified problem (8) is a good approximation to the optimal solution of problem (6). That is, the distance $d_{\text{NPKS}}(\cdot, \cdot)$ between the optimal solution to problem (6), (obtained using the implicit hitting set approach problem of Karp and Moreno-Centeno [2013]), and the optimal solution to the convex approximation, problem (8), is only $0.1606$. This is further discussed in Section 6.

5 Joint Aggregation of Ratings and Rankings

This section describes the model to jointly aggregate the ratings and the rankings. The goal of this model is not only to fairly represent the judges’ rating and the judges’ rankings, but also to balance the cardinal and ordinal evaluations. We refer to this optimization model as the Combined Aggregate raTing problem, or (CAT).

The input to (CAT) is a set of ratings $\{a^{(k)}\}_{k=1}^K$ and a set of rankings $\{b^{(k)}\}_{k=1}^K$. (CAT) is a combination of the rating aggregation problem (4) and the ranking aggregation problem (6). In order to guarantee that ratings rankings weigh equally in the optimization problem (CAT), both distance functions, $d_{\text{NPCK}}$ and $d_{\text{NPKS}}$, are normalized. Note that one can weigh these distances differently if justified by the circumstances of the decision context. Also, the choice of $d_{\text{NPCK}}$ and $d_{\text{NPKS}}$ as penalty functions, or distances, can be replaced by other distances between incomplete ratings and between incomplete rankings, respectively.

\[
(\text{CAT}) \quad \min_{x} \sum_{k=1}^K d_{\text{NPCK}}(a^{(k)}, x) + \sum_{k=1}^K d_{\text{NPKS}}(b^{(k)}, \text{rank}(x))
\]

We next establish that (CAT) is NP-hard by reducing problem (6) (which is NP-hard) to it.

**Lemma 1.** (CAT) is NP-hard.

**Proof.** Given an instance of problem (6), a set of incomplete rankings $\{b^{(k)}\}_{k=1}^K$, one can transform it (in polynomial time) to an instance of (CAT) as follows. Keep unchanged $\{b^{(k)}\}_{k=1}^K$, and create a set of ratings $\{a^{(k)}\}_{k=1}^K$ such that each rating evaluates exactly one object (the choice of object is irrelevant; moreover all of the ratings can evaluate the same object). From the definition of $d_{\text{NPCK}}$ (equation (2)), it follows that, for every $x$, the first summand in (CAT) will be equal to 0. Therefore, with this choice of ratings, $\text{rank}(x^*)$, where $x^*$ is the optimal solution to (CAT), will be the optimal solution to problem (6). $\square$

The (nonlinear, nonconvex) mathematical programming formulation of (CAT) is:

\[
\min_{x, z} \sum_{k=1}^K c_k \sum_{i \in A^{(k)}} \sum_{j \in A^{(k)}} |z_{ij} - p_{ij}^{(k)}| + \sum_{k=1}^K d_k \sum_{i \in B^{(k)}} \sum_{j \in B^{(k)}} \frac{1}{2} |\text{sign}(z_{ij}) - \text{sign}(b_j^{(k)} - b_j^{(k)})|
\]

subject to

\[
\begin{align*}
    z_{ij} &= x_i - x_j & i &= 1, \ldots, n; & j &= 1, \ldots, n \\
    \ell &\leq x_i & i &= 1, \ldots, n \\
    x_i &\in \mathbb{Z} & i &= 1, \ldots, n.
\end{align*}
\]

The convexification of the objective of problem (10), as described in Section 4.4, results in the convex formulation:

\[
\min_{x, z} \sum_{k=1}^K c_k \sum_{i \in A^{(k)}} \sum_{j \in A^{(k)}} |z_{ij} - p_{ij}^{(k)}| + \sum_{k=1}^K d_k \sum_{i \in B^{(k)}} \sum_{j \in B^{(k)}} \frac{1}{2} |\text{sign}(z_{ij}) - \text{sign}(b_j^{(k)} - b_j^{(k)})|
\]

subject to

\[
\begin{align*}
    z_{ij} &= x_i - x_j & i &= 1, \ldots, n; & j &= 1, \ldots, n \\
    \ell &\leq x_i & i &= 1, \ldots, n \\
    x_i &\in \mathbb{Z} & i &= 1, \ldots, n.
\end{align*}
\]

The convexification of the objective of problem (10), as described in Section 4.4, results in the convex formulation:
$$\min_{x,z} \sum_{k=1}^{K} C_k \sum_{i \in A^{(k)}} \sum_{j \in A^{(k)}} \left| z_{ij} - p^{(k)}_{ij} \right| + \sum_{k=1}^{K} D_k \sum_{i \in B^{(k)}} \sum_{j \in B^{(k)}} h^{(k)}_{ij}(z_{ij})$$  \tag{11a}

subject to

$$z_{ij} = x_i - x_j \quad i = 1, \ldots, n; \quad j = 1, \ldots, n$$  \tag{11b}

$$\ell \leq x_i \leq u \quad i = 1, \ldots, n$$  \tag{11c}

$$x_i \in \mathbb{Z} \quad i = 1, \ldots, n$$  \tag{11d}

where,

$$h^{(k)}_{ij}(z_{ij}) = \begin{cases} 
0, & \text{if sign}(b_j^{(k)} - b_i^{(k)}) = -1 \\
\frac{z_{ij} + 1}{2}, & \text{if sign}(b_j^{(k)} - b_i^{(k)}) = 0 \\
\frac{z_{ij} - 1}{2}, & \text{if sign}(b_j^{(k)} - b_i^{(k)}) = 1.
\end{cases}$$  \tag{11e}

Problem (11) is a special case of the convex SD model and thus solvable in polynomial time.

**Remark:** Note that in equations (10a) and (11e), the argument of the sign function is $b_j^{(k)} - b_i^{(k)}$ and not $b_i^{(k)} - b_j^{(k)}$ as in equations (5) and (7). This is because of the classical convention that in the given ratings high cardinal numbers are assigned to the most preferred objects; while in the given rankings high ordinal numbers are assigned to the least preferred objects.

The optimal solution to (CAT) is a combined aggregate rating-ranking pair which is denoted by $x^{(car)}$, and its implied ranking is denoted by rank($x^{(car)}$).

Next, we propose two mechanisms to identify inconsistencies in the given evaluations (e.g. outliers, judges that are too lenient or too strict, etc.). This information is helpful so that (say) the lead decision maker initiates an investigation of the nature of the discrepancies and acts appropriately (for example, by discussing these inconsistencies with the judges and promote a discussion with the objective of alleviating them).

The first mechanism is to use the solution $x^{(car)}$ to identify (a) judges whose evaluations differ the most with the rest of the evaluations and (b) objects such that the judges evaluating them had particularly divergent evaluations. These judges (objects) are those that assigned (received) scores that disagree the most with $x^{(car)}$. Specifically, we use the separation penalty to identify the judges whose evaluations are at the farthest distance from $x^{(car)}$ (i.e., have the highest separation penalty). Specifically, the contribution of judge $k$ to the separation penalty is

$$\sum_{i \in A^{(k)}} \sum_{j \in A^{(k)}} C_k \left| (x_i^{(car)} - x_j^{(car)}) - (a_i^{(k)} - a_j^{(k)}) \right|. \tag{12}$$

Similarly, we use the separation penalty to identify the objects such that the judges evaluating them had particularly divergent evaluations. These objects are those with the highest contribution to the separation penalty. The contribution of object $i$ to the separation penalty is

$$\sum_{k \in A^{(k)}} \sum_{j \in A^{(k)}} C_k \left| (x_i^{(car)} - x_j^{(car)}) - (a_i^{(k)} - a_j^{(k)}) \right|. \tag{13}$$

The second mechanism to identify inconsistencies in the given evaluations is based on Brans and Vincke’s PROMETHEE method (Brans and Vincke 1985). The mechanism is to aggregate the consensus rating $x^{(c)}$ (solution to problem (4)) and the consensus rating $x^{(o)}$ (solution to problem (6)) into a partial
order \((P, T, I)\) as follows:

- \(a\) is preferred to \(b\) \((a \stackrel{P}{\preceq} b)\) if
  \[\begin{align*}
  &\begin{cases}
    x^{(c)}(a) > x^{(c)}(b) & \text{and } x^{(o)}(a) \geq x^{(o)}(b) \\
    x^{(c)}(a) \geq x^{(c)}(b) & \text{and } x^{(o)}(a) > x^{(o)}(b)
  \end{cases}
  \end{align*}\]  
  (14a)

- \(a\) and \(b\) are tied \((a \stackrel{T}{\approx} b)\) if
  \[
x^{(c)}(a) = x^{(c)}(b) \quad \text{and} \quad x^{(o)}(a) = x^{(o)}(b)\]
  (14b)

- \(a\) and \(b\) are incomparable \((a \stackrel{I}{\not\approx} b)\) otherwise. (14c)

Thus, by construction, the partial order \((P, T, I)\) summarizes the agreement (or lack thereof) between the consensus rating \(x^{(c)}\) and the consensus ranking \(x^{(o)}\).

Section 6 illustrates these mechanisms and their usefulness for identifying objects whose evaluations deserve special attention/further discussion.

### 6 Results

We illustrate here how to use the proposed mechanism in the ranking of the contestants of the 2007 MSOM’s SPC. These results are compared to those obtained by aggregating only the cardinal evaluations, and those obtained by aggregating only the ordinal evaluations.

Table 4 gives the consensus rating \((\text{optimal solution to problem (4)}) \times \text{x}\) (optimal solution to problem (4)); the (approximate) consensus ranking \((\text{optimal solution to problem (8)}) \times \text{x}\); and, the combined aggregate rating \((\text{optimal solutions to problem (11)}) \times \text{x}\) and ranking \((\text{optimal solutions to problem (11)}) \times \text{x}\). (optimal solutions to problem (11)).

| Paper | \(x^{(c)}\) | \(x^{(o)}\) | rank\((x^{(o)})\) | rank\((x^{(c)})\) | Paper | \(x^{(c)}\) | \(x^{(o)}\) | rank\((x^{(o)})\) | rank\((x^{(c)})\) |
|-------|--------|--------|-------------|-------------|-------|--------|--------|-------------|-------------|
| 1     | 3      | 4      | 2           | 3           | 20    | 5      | 24     | 5           | 5           |
| 2     | 5.5    | 24     | 5           | 23          | 21    | 4.5    | 41     | 4           | 41          |
| 3     | 5      | 41     | 4           | 41          | 22    | 4      | 41     | 4           | 41          |
| 4     | 5.5    | 24     | 5           | 23          | 23    | 5      | 24     | 5           | 23          |
| 5     | 4.5    | 41     | 4           | 41          | 24    | 4.5    | 41     | 4           | 41          |
| 6     | 6.5    | 9      | 6           | 8           | 25    | 5.5    | 9      | 6           | 8           |
| 7     | 6.5    | 24     | 5           | 23          | 26    | 6.5    | 24     | 5           | 23          |
| 8     | 6      | 9      | 6           | 8           | 27    | 6      | 9      | 6           | 8           |
| 9     | 5.5    | 9      | 6           | 8           | 28    | 5      | 24     | 5           | 23          |
| 10    | 6      | 3      | 7           | 2           | 29    | 5      | 41     | 4           | 41          |
| 11    | 6      | 3      | 7           | 2           | 30    | 4      | 41     | 4           | 41          |
| 12    | 5      | 24     | 5           | 23          | 31    | 5      | 41     | 4           | 41          |
| 13    | 6.5    | 9      | 6           | 8           | 32    | 6      | 9      | 6           | 8           |
| 14    | 7.5    | 9      | 6           | 8           | 33    | 5.5    | 9      | 6           | 8           |
| 15    | 5      | 41     | 4           | 41          | 34    | 4.5    | 9      | 5           | 23          |
| 16    | 4      | 53     | 3           | 53          | 35    | 4.5    | 53     | 3           | 53          |
| 17    | 6.5    | 9      | 6           | 8           | 36    | 4.5    | 53     | 3           | 53          |
| 18    | 3.5    | 53     | 3           | 53          | 37    | 4.5    | 53     | 3           | 53          |
| 19    | 5.5    | 24     | 5           | 23          | 38    | 4.5    | 53     | 3           | 53          |
| 20    | 2.5    | 55     | 2           | 58          | 39    | 4.5    | 53     | 3           | 53          |
| 21    | 4.5    | 41     | 4           | 41          | 40    | 4.5    | 53     | 3           | 53          |
| 22    | 4      | 41     | 4           | 41          | 41    | 4.5    | 53     | 3           | 53          |
| 23    | 4.5    | 41     | 4           | 41          | 42    | 4.5    | 53     | 3           | 53          |
| 24    | 5.5    | 24     | 5           | 23          | 43    | 4.5    | 53     | 3           | 53          |
| 25    | 5      | 24     | 5           | 23          | 44    | 4.5    | 53     | 3           | 53          |
| 26    | 6.5    | 24     | 5           | 23          | 45    | 4.5    | 53     | 3           | 53          |
| 27    | 7.5    | 9      | 6           | 8           | 46    | 4.5    | 53     | 3           | 53          |
| 28    | 4.5    | 53     | 3           | 53          | 47    | 4.5    | 53     | 3           | 53          |
| 29    | 6      | 9      | 6           | 8           | 48    | 4.5    | 53     | 3           | 53          |

In Table 4, the consensus rating \(x^{(c)}\) is non-integral because some of the judges assigned fractional scores (in particular they assigned grades that are multiple of 1/2). To appropriately handle the judges’ fractional grades, we decided to set the ‘grading unit’ to 1/2. From an optimization point of view, this represents no problem, since the separation-deviation problem can be solved in any pre-specified precision.

Next we give a specific example of objects/papers whose ratings and ranking are in conflict with several other objects/papers. In particular, paper 14 has the highest consensus score, however this conflicts with several papers (e.g., paper 54) that have a lower consensus score but a higher consensus rank. The evaluations received by papers 14 and 54 are given in Table 5. The number of papers reviewed by each judge and
the average Field Contribution (FC) they gave are given in Table 6. The adjusted FC, obtained by dividing the paper’s FC by the judge’s average FC, is given in Table 7. From these tables we observe the following:

1. The ordinal evaluations of paper 54 seem better than those of paper 14. This explains in part why paper 54 has a better consensus rank than paper 14.
2. The average FC of paper 14 (5.6) is only slightly bigger than that of paper 54 (5.5). This explains in part why paper 14 has a better consensus score than paper 54.
3. It seems that judge 44, who evaluated paper 14, was remarkably lenient, while judge 30, who evaluated paper 14, was remarkably strict. This suggests that the FC of ’5’ given by these two judges is not comparable. Note that the adjusted FC of paper 14-judge 44 is of 0.71; while the adjusted FC of paper 54-judge 30 is of 1.39. Moreover, the average adjusted FC of paper 14 and 54 are 1.10 and 1.28, respectively.
4. All of this suggests that paper 54 deserves a collective evaluation better than that of paper 14.

In the combined aggregate rating-ranking pair, $x^{(cat)}$, which is the optimal solution to (CAT), and in its implied ranking $\text{rank}(x^{(cat)})$, paper 54 is rated and ranked higher than paper 14; this, as discussed previously, seems appropriate. In contrast, the consensus rating $x^{(c)}$ ranks paper 14 higher than 54. This provides some evidence that indeed the combined aggregate rating-ranking pair better represents the judges’ evaluations/opinions than the consensus rating $x^{(c)}$, which takes into consideration only the ratings provided by the judges.

### Table 5: Evaluations of papers 14 and 54.

| Paper | Judge | Field Contribution Score | Paper Ranking |
|-------|-------|--------------------------|---------------|
| 14    | 35    | 6                        | 1             |
| 14    | 23    | 6                        | 1             |
| 14    | 48    | 7                        | 1             |
| 14    | 57    | 4                        | 4             |
| 14    | 44    | 5                        | 4             |
| 54    | 30    | 5                        | 1             |
| 54    | 32    | 4                        | 4             |
| 54    | 25    | 6                        | 1             |
| 54    | 22    | 7                        | 1             |

### Table 6: Evaluation statistics of the judges that evaluated papers 14 and 54.

| Judge | Number of Papers Evaluated | Average Field Contribution |
|-------|---------------------------|-----------------------------|
| 35    | 4                         | 4.50                        |
| 23    | 4                         | 4.25                        |
| 48    | 4                         | 5.25                        |
| 57    | 4                         | 5.75                        |
| 44    | 5                         | 7.00                        |
| 30    | 5                         | 3.60                        |
| 32    | 4                         | 5.25                        |
| 25    | 5                         | 4.00                        |
| 22    | 4                         | 4.75                        |

### Table 7: Adjusted Field Contribution received by papers 14 and 54.

| Paper | Judge | Adjusted Field Contribution |
|-------|-------|----------------------------|
| 14    | 35    | 1.33                       |
| 14    | 23    | 1.41                       |
| 14    | 48    | 1.33                       |
| 14    | 57    | 0.70                       |
| 14    | 44    | 0.71                       |
| 54    | 30    | 1.39                       |
| 54    | 32    | 0.76                       |
| 54    | 25    | 1.50                       |
| 54    | 22    | 1.47                       |

Next, we use the partial order $(P, T, I)$ (created as described in Section 5) to highlight the discrepancies...
between the consensus rating $x^{(c)}$ and the consensus ranking $x^{(o)}$. Figure 1 gives a graphical representation of the partial order that highlights the pairs of objects where $x^{(c)}$ and $x^{(o)}$ disagree on their relative order (that is, those object pairs that are members of the set $I$ in the partial order $(P,T,I)$).

From Figure 1 we observe the following: (a) Paper 14 has the highest consensus score, however this conflicts with several papers (e.g., paper 54) that have a lower consensus score but a higher consensus rank (this agrees with the analysis given above). (b) Paper 20 (lower left corner of Figure 1) should receive the lowest consensus evaluation. (c) Although the agreement between $x^{(c)}$ and $x^{(o)}$ is not perfect, there are subsets of papers should receive a lower (or higher) collective evaluation than others. For example, the papers $\{1, 38, 18, 16, 22, 28, 50, 55\}$ should receive a collective evaluations higher than that of paper 20; lower than or equal to that of papers $\{5, 21, 23, 52\}$; and lower than the rest of the papers.
Figure 1: The papers (circled) are ordered (top to bottom) in decreasing consensus score. There is an arc between two papers whenever the lower rated paper has a better ranking than a higher rated paper.
In the 2007 MSOM’s SPC, papers 38, 14, 10, 1 and 42 had the highest contributions to the separation penalty. As noted previously, this indicates that these papers are those whose evaluations are not consistent/deserve further discussion. For example, paper 38—a very low rated paper in the consensus rating—received scores from 2 to 5 and was ranked by all but one of the judges as their least preferred paper (see Tables 8 and 9). In particular, paper 38 was the second most preferred paper of judge 9; perhaps because this judge received other papers with less quality than paper 38? We believe this is not the case since, as shown in Table 10, the paper ranked last by judge 9 was paper 10. As noted above, paper 10 is also among the highest contributors to the separation penalty. Paper 10 received three high evaluations and 2 very low evaluations (see Table 11). Therefore, we believe that, in order to get a better consensus, the scores/ranks of paper 38 and paper 10 should be discussed by the judges assigned to these two papers.

Table 8: Evaluations of paper 38.

| Judge | Contribution | Paper Score | Ranking |
|-------|--------------|-------------|---------|
| 30    | 3            | 5           |
| 41    | 2            | 5           |
| 44    | 3            | 5           |
| 9     | 5            | 2           |
| 20    | 5            | 4           |

Table 9: Evaluation statistics of the judges that evaluated paper 38.

| Judge | Number of Papers Evaluated | Average Field Contribution |
|-------|-----------------------------|----------------------------|
| 30    | 5                           | 3.60                       |
| 41    | 5                           | 5.00                       |
| 44    | 5                           | 7.00                       |
| 9     | 5                           | 4.60                       |
| 20    | 4                           | 7.25                       |

Table 10: Evaluations of judge 9.

| Paper | Contribution | Paper Score | Ranking |
|-------|--------------|-------------|---------|
| 10    | 3            | 5           |
| 19    | 4            | 3           |
| 38    | 5            | 2           |
| 50    | 4            | 3           |
| 58    | 7            | 1           |

Table 11: Evaluations of paper 10.

| Judge | Contribution | Paper Score | Paper Ranking |
|-------|--------------|-------------|---------------|
| 33    | 7            | 1           |
| 41    | 7            | 1           |
| 19    | 2            | 3           |
| 15    | 6            | 1           |
| 9     | 3            | 5           |

Next we analyze the combined aggregate rating $\mathbf{x}^{(cat)}$ and ranking $\text{rank}(\mathbf{x}^{(cat)})$ (solution to problem (11)). We make the following observations:

1. The consensus rating, $\mathbf{x}^{(c)}$, has a total rating distance (equation (4)) of 7.3611.
2. The consensus ranking, $\mathbf{x}^{(o)}$, has a total ranking distance (equation (6)) of 13.8500.
3. (a) The combined aggregate rating, $\mathbf{x}^{(cat)}$, has a total rating distance (equation (4)) of 8.1667.
   (b) The combined aggregate ranking, $\text{rank}(\mathbf{x}^{(cat)})$, has a total ranking distance (equation (6)) of 13.9333.

This shows that, in this case, the combined aggregate rating $\mathbf{x}^{(cat)}$ and ranking $\text{rank}(\mathbf{x}^{(cat)})$ achieve a very good compromise. In particular, $\mathbf{x}^{(cat)}$ remains almost as close as the consensus rating $\mathbf{x}^{(c)}$ to the judges’ ratings, and $\text{rank}(\mathbf{x}^{(cat)})$ remains almost as close as the consensus ranking $\text{rank}(\mathbf{x}^{(c)})$ to the judges’ rankings.

7 Concluding Remarks

We propose here a new framework for group decision making that aggregates both cardinal and ordinal input evaluations (referred to as ratings and rankings, respectively). Our framework consists on finding the rating-ranking pair that minimizes the sum of the rating-distances from the rating to the given ratings plus the sum of the ranking-distances from the ranking to the given rankings.
The effectiveness of the new framework is illustrated by ranking the contestants of the 2007 MSOM’s student paper competition. We provide evidence that obtaining a combined aggregate cardinal and ordinal evaluations better represents the judges’ opinions as compared to a rating that aggregates only the judges’ cardinal evaluations or only the judges’ ordinal evaluations.

Aggregating incomplete evaluations is challenging because the aggregate evaluation is prone to be biased by the judges’ subjective scales; for example, objects assigned to a particularly strict (lenient) judge have an advantage (disadvantage) compared to those objects not assigned to this specific judge. Our framework identifies these inconsistencies in the given evaluations. This information is helpful so that the lead decision maker can initiate an investigation of the nature of the conflicts and act accordingly (for example, by having the specific judges discuss, and possibly resolve, these inconsistencies).

The problem of aggregating complete evaluations (in which all judges evaluate all objects) is a special case of the problem of aggregating incomplete evaluations (in which the judges are allowed to evaluate only some of the objects). Therefore our framework is also applicable to aggregating complete evaluations.

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