Abstract—This paper presents a novel approach to multicarrier Offset-QAM (OQAM) with conjugate root (CR) filters, which provides a more regular phase space. CR-OQAM is analyzed in the context of Generalized Frequency Division Multiplexing (GFDM), a waveform proposed for 5G applications and a simple method for combining GFDM/CR-OQAM with Alamouti’s space-time coding is presented. The symbol error rate (SER) performance in fading multipath channels is evaluated, showing that GFDM/CR-OQAM outperforms conventional GFDM.

Index Terms—Quadrature Amplitude Modulation, Multicarrier Modulation, Space-Time Coding

I. INTRODUCTION

With 5G on the horizon, new waveforms for the PHY layer are investigated that are suitable for the upcoming requirements [1]. In particular, good time-frequency localization (TFL) of the transmit signal is required to cope with asynchronous [2] and to provide a low out-of-band radiation, which is needed for spectral agility and aggregation of carriers. Filtered multicarrier systems [3] provide the means for good TFL by adaptation of the prototype filter and spectral agility is achieved by switching on and off certain subcarriers. High spectral efficiency is important to serve the increased demand for high speed data access [1]. Hence, a future waveform should transmit symbols at the Nyquist rate in order to not waste valuable time-frequency resources.

Even in a distortion-free channel, the Balian-Low theorem prohibits the distortion-free reconstruction of complex valued-symbols sent at Nyquist rate when using filters with good TFL [4]. Hence, in terms of SER, QAM multicarrier systems with good TFL perform worse than orthogonal systems without good TFL. One possibility to circumvent the BLT is to transmit real-valued symbols at twice the Nyquist rate, which was initially proposed in [1]. This gave rise to the well known OFDM/OQAM [5] and FBMC/OQAM [6] multicarrier systems where both orthogonality and good TFL is kept by using Offset-QAM (OQAM).

GFDM [7] is another candidate waveform for 5G. It uses standard QAM modulation, but its discrete and circular nature allows to avoid the BLT for certain configurations [8]. However, this significantly limits the set of possible GFDM configurations and furthermore, using well-localized filters in GFDM creates non-orthogonality which results in a performance degradation compared to orthogonal systems [8].

The contribution of this paper is two-fold: First, a modified version of multicarrier OQAM modulation, named conjugate-root (CR) Offset-QAM (CR-OQAM) is developed. So far, OQAM was only known to work with symmetric filters whereas CR-OQAM allows to use the class of non-symmetric CR filters which simplifies the phase space and provides means for an easier implementation. Second, the scheme is applied to GFDM in order to provide a well-localized orthogonal system and to improve the symbol error rate performance in fading multipath channels. Additionally, time-reversal spacetime-coding (TR-STC) is applied to GFDM/CR-OQAM to exploit multi-antenna diversity, showing that in GFDM STC can be combined with OQAM modulation which has been a known issue in FBMC/OQAM [9]. In [10] the related FBMC/OQAM is proposed, though neither CR-OQAM nor Alamouti-Coding is covered in depth.

The remainder of this paper is organized as follows: Section II describes the conventional orthogonal OQAM transceiver, whereas the proposed modification is presented in Section III. Section IV describes the application of CR-OQAM to GFDM and simulations of system performance are provided in section V. Finally, Section VI concludes this paper.

II. CONVENTIONAL OFDM/OQAM

In OFDM/OQAM, complex-valued data symbols $c_{k,m}$ are transmitted on $K$ subcarriers, where real and imaginary part are offset by $\frac{T}{2}$ where $T$ is the symbol duration. Each symbol is pulse-shaped with a symmetric, real-valued pulse shaping filter $g(t)$ [3]. The transmission equation is given by

$$x(t) = \sum_{k=0}^{K-1} \sum_{m \in \mathbb{Z}} (c_{k,m}^R g(t - mT) + j c_{k,m}^I g(t - mT - \frac{T}{2})) e^{j \omega_m t}$$

(1)

where $w = \exp(j2\pi F)$, $F = \frac{1}{T}$, and $c_{k,m}^R$ and $c_{k,m}^I$ are real and imaginary part of $c_{k,m}$, respectively. Note that, due to the factor $\frac{T}{2}$, adjacent subcarriers differ by a phase shift of $\frac{T}{2}$.

At the receiver, a matched filtering operation is carried out, i.e.,

$$\hat{c}_{k,m}^R = \Re \{x(t)e^{-j\omega_m t}g(-t)|_{t=mT}\}$$

(2)

$$\hat{c}_{k,m}^I = \Im \{x(t)e^{-j\omega_m t}g(-t)|_{t=(\frac{1}{2}+m)T}\}$$

(3)

where convolution $x(t)*y(t)$ is defined as

$$(x(t)*y(t))|_{t=\tau} = \int_{-\infty}^{\infty} x(t)y(\tau-t)dt.$$  

(4)

The corresponding block diagram of the OFDM/OQAM transceiver is presented in Fig. 1. For perfect reconstruction,
the orthogonality conditions between the \(k'\)th and \((k' + k)\)th subcarrier that are given by

\[
\mathbb{R} \left\{ (g(t)j^{-k}w^{-kt} \ast g(-t))_{t = mT} \right\} = \delta(k, m) \quad (5)
\]

\[
\mathbb{R} \left\{ (jg(t)j^{-k}w^{-kt} \ast g(-t))_{t = \left(\frac{1}{2} + m\right)T} \right\} = 0 \quad (6)
\]

\[
\mathbb{I} \left\{ (g(t)j^{-k}w^{-kt} \ast g(-t))_{t = mT} \right\} = 0 \quad (7)
\]

\[
\mathbb{I} \left\{ (jg(t)j^{-k}w^{-kt} \ast g(-t))_{t = mT} \right\} = \delta(k, m) \quad (8)
\]

must be fulfilled for all \(k, m\). With the relations \(\mathbb{R}\{ja\} = -\mathbb{I}\{a\}\) and \(\mathbb{I}\{ja\} = \mathbb{R}\{a\}\), \(5\) and \(6\) are equivalent to \(8\) and \(7\), respectively. These conditions are in particular fulfilled, when \(g(t)\) is a symmetric, half-Nyquist filter with band-limitation \(G(f) = 0, |f| \geq F\) [2].

The convolution product can be expressed with the Fourier transform \( \mathcal{F} \) by

\[
(g(t)w^{-kt} \ast g(-t))_{t = \tau} = \mathcal{F}^{-1}\{S_k(f)(\tau) = s_k(\tau), \quad (9)
\]

where \(S_k(f) = G(f - kF)G^*(f)\) is the spectrum of the intercarrier interference (ICI) from the \(k\)'th to the \((k' + k)\)th subcarrier, \(s_k(t)\) is the corresponding time domain function, \(G(f)\) is the Fourier transform of \(g(t)\) and \(\cdot^*\) denotes complex conjugation. Fig. 2(b) depicts \(s_1(t)\) for a root raised cosine (RRC) filter with roll-off \(\alpha = 0.75\).

### III. Conjugate-Root Multicarrier OQAM

In literature, so far only symmetric half-Nyquist filters were employed for multicarrier OQAM systems. In this work, we propose the application of non-symmetric conjugate root (CR) [11], [12] filters for multicarrier OQAM systems.

Assume an even \(H(f)\) with band limitation \(H(f) = 0\) for \(|f| \geq F\). Then, \(H(f)\) fulfills the 1st Nyquist criterion [11] iff

\[
\forall f \in [0, F] : H(f) + H(f - F) = 1. \quad (10)
\]

Its corresponding half-Nyquist filter is given by \(G(f) = \sqrt{H(f)}\) where the most prominent example is the symmetric raised cosine (RC) and root raised cosine (RRC) pair. The corresponding non-symmetric CR filter \(G^C(f)\) is constructed by

\[
G^C(f) = \begin{cases} H(f) + j\sqrt{(1 - H(f))H(f)} & f \geq 0 \\ H(f) - j\sqrt{(1 - H(f))H(f)} & f < 0 \end{cases} \quad (11)
\]

with impulse response \(g^C(t) = \mathcal{F}^{-1}\{G^C(f)\}\). Note that both \(G^C(f)\) and \(G^C(f)(G^C(f))^*\) are Nyquist filters (cf. [10]). Fig. 2(c) shows the time and frequency response of a standard RRC and its conjugate RRC (CRRC) version.

The ICI \(S^C_k(f)\) between two adjacent subcarriers when using CR filters is given by

\[
S_1(f) = \sqrt{H(f)H(f - F)} \quad (12)
\]

\[
S^C_1(f) = G^C(f - F)G^C(f)^* = jS_1(f), \quad (13)
\]

where the last equality follows from \([11]\) and \(H(f) = 1 - H(f - F)\) (cf. [10]). Therefore, since \(s_1(t) = js_1(t)\), (15)

the real and imaginary part of the ICI are exchanged when using a CR filter compared to its standard half-Nyquist version. Both \(S^C_k(f)\) and \(s^C_k(t)\) are presented in Fig. 2(c).

Accordingly, if a \(g(t)\) fulfills the orthogonality conditions \(5\)–\(8\), the corresponding \(g^C(t)\) fulfills

\[
\mathbb{R} \left\{ (g^C(t)w^{-kt} \ast g^C(-t))_{t = mT} \right\} = \delta(k, m) \quad (16)
\]

\[
\mathbb{R} \left\{ (jg^C(t)w^{-kt} \ast g^C(-t))_{t = \left(\frac{1}{2} + m\right)T} \right\} = 0 \quad (17)
\]

\[
\mathbb{I} \left\{ (g^C(t)w^{-kt} \ast g^C(-t))_{t = \left(\frac{1}{2} + m\right)T} \right\} = 0 \quad (18)
\]

\[
\mathbb{I} \left\{ (jg^C(t)w^{-kt} \ast g^C(-t))_{t = mT} \right\} = \delta(k, m). \quad (19)
\]

Note, that compared to \([5]–[8]\) the factor \(j^k\) has been removed. Consequently, an OFDM/CR-OQAM system that uses a non-symmetric CR filter \(g^C(t)\) can be described by the modulation equation

\[
x(t) = \sum_{k=0}^{K-1} (c^R_{k,m}g^C(t - mT) + jc^I_{k,m}g^C(t - mT - \frac{T}{2}))w^{kt}, \quad (20)
\]

the demodulation equations

\[
c^R_{k,m} = \mathbb{R}\{x(t)w^{-kt} \ast g^C(-t))\}_{t = mT} \quad (21)
\]

\[
c^I_{k,m} = \mathbb{I}\{x(t)w^{-kt} \ast g^C(-t))\}_{t = \left(\frac{1}{2} + m\right)T} \quad (22)
\]

and the block diagram which is depicted in Fig. 3.

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**Fig. 1.** Block diagram of the conventional OFDM/OQAM transceiver.

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**Table 1.** Comparison of OFDM/CR-OQAM with a standard RRC filter.

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**Fig. 2.** Time and frequency domain representation of the OFDM/CR-OQAM system.

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**Fig. 3.** Block diagram of the OFDM/CR-OQAM system with RRC and CRRC filters.
Due to the roll-off $\hat{F}$ and $0 \times$ the shift of $\pi$ can be understood as two parallel OFDM/PAM systems that are regular for the CR-OQAM case. Hence, OFDM/CR-OQAM is modeled in discrete base band with a sampling period $T_a$. The transmit signal exhibits a block structure and subsequent transmit signal with a timing difference of $\pi$ can be understood as two parallel OFDM/PAM systems that transmit with a timing difference of $T_a$. The receiver can be realized accordingly.

IV. APPLICATION OF CR-OQAM IN GFDM

OFDM/OQAM is a streaming based, filtered multicarrier system, where every symbol overlaps with its adjacent symbols in time. GFDM [7] is a filtered multicarrier system, where circular convolution is applied instead of linear. Hence, its transmit signal exhibits a block structure and subsequent blocks can be decoupled by a cyclic prefix (CP) to ease equalization.

GFDM is modeled in discrete base band with a sampling period $T_a$. The transmit signal $\tilde{x}$ can be described by the matrix equation

$$\tilde{x} = A \tilde{d},$$

where the columns of the matrix $A$ contain circular time-frequency shifted versions of a prototype transmit filter $g[n]$ with distance $K T_a$ in time and $1/(K T_a)$ in frequency, where $K$ is the number of subcarriers. $\tilde{d}$ contains the complex-valued data symbols to be transmitted with the block. By appending a CP, frequency domain channel equalization can be carried out at the receiver, yielding the signal $\hat{x}$ and zero-forcing (ZF) or matched filter detection (MF) is applied

$$\hat{d}_{ZF} = A^{-1} \tilde{x},$$

$$\hat{d}_{MF} = A^H \tilde{x},$$

where $(\cdot)^H$ denotes Hermitian conjugate. A main property of GFDM is its good TFL of the transmit filter, which allows to achieve a low out-of-band radiation and robustness against asynchronicity [7]. However, when using QAM modulation, the Balian-Low theorem prohibits orthogonality completely, which impacts MF performance while ZF detectors introduce noise-enhancement and only exist for certain parameter configurations [8]. Hence, with perfect synchronization, SER performance of GFDM is worse compared to an orthogonal system.

To circumvent this problem, QAM modulation can be applied, which provides orthogonality but still keeps the advantageous property of good time-frequency localization. Looking at the phase space of CR-OQAM (Fig. 4), the GFDM/CR-OQAM modulator is given by

$$\tilde{x} = A \Re\{\tilde{d}\} + j C K (A \Im\{\tilde{d}\}),$$

where $\Re\{}$ and $\Im\{}$ denote the real and imaginary part, respectively. Roll-off $\alpha = 0.75$.
where $C_u(\cdot)$ denotes a circular rotation of its argument by $u$ elements. At the receiver, the CR-OQAM detection with the matched filter is simply achieved by

$$\Re\{\tilde{d}\} = \Re\{A^H \tilde{x}\} \quad (26)$$

$$\Im\{\tilde{d}\} = \Im\{A^H C_{\frac{-K}{2}} \tilde{x}\}. \quad (27)$$

Time-reversal space-time coding (TR-STC) \cite{13} can be applied to GFDM/CR-OQAM to provide transmit diversity in a fading multipath environment. TR-STC was initially developed for single-carrier systems, where two subsequent time domain codewords are decoupled by a CP and jointly space-time encoded. For GFDM, the idea is to treat one block as a single-carrier codeword and apply the TR-STC onto two subsequent GFDM blocks \cite{12}. At the receiver, the time-domain signal of both GFDM blocks is recovered by the TR-STC and later on processed by the GFDM detector. The approach can be applied to both GFDM and GFDM/(CR-)OQAM, but GFDM/OQAM is expected to outperform conventional GFDM due to the kept orthogonality.

### V. PERFORMANCE EVALUATION

The symbol error rate performance of GFDM and GFDM/(CR-)OQAM with and without TR-STC has been evaluated in fading multipath channels. The power delay profile of the channel between the transmit and receive antennas is modelled with length 16 where the tap powers decay linearly in log scale from 0dB to -16dB. Each tap is multiplied by an i.i.d circular Gaussian random variable with variance $\sigma^2 = 1$. The GFDM parameters are given in Tab. \ref{table:1} and the simulation results are presented in Fig. \ref{fig:5} where perfect channel state information was available at the receiver. The theoretic equations for GFDM and orthogonal OFDM/OQAM are taken from \cite{7}. Note that GFDM/OQAM becomes orthogonal with the application of Offset-QAM and hence performs equal to OFDM/OQAM.

GFDM/OQAM outperforms conventional GFDM with and without TR-STC due to the orthogonality introduced by the OQAM modulation. The performance gap equals the noise-enhancement of GFDM, which is 0.8 dB in the presented configuration. Both GFDM and GFDM/(CR-)OQAM show a diversity gain of 2 when combined with TR-STC.

### VI. CONCLUSION

This paper has presented an alternative approach for the implementation of Offset-QAM when using non-symmetric conjugate root filters. With the new technique, no extra phase shift between subcarriers is required, which makes the CR-OQAM time-frequency phase space more regular and simplifies the implementation of CR-OQAM. Orthogonality conditions have been stated for the OFDM/CR-OQAM system and it has been shown that CR filters fulfill these conditions. CR-OQAM has been applied to GFDM to create an orthogonal system with good TFL. A space-time code aiming for transmit diversity was applied to GFDM/OQAM and it was shown to outperform the conventional space-time coded GFDM system.

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