The Argument against Quantum Computers

Gil Kalai

The Hebrew University of Jerusalem and IDC, Herzliya

Dedicated to the memory of Itamar Pitowsky

My purpose in this paper is to examine, in a general way, the relations between abstract ideas about computation and the performance of actual physical computers.

Itamar Pitowsky – The physical Church thesis and physical computational complexity 1990

Abstract

We give a computational complexity argument against the feasibility of quantum computers. We identify a very low complexity class of probability distributions described by noisy intermediate-scale quantum computers, and explain why it will allow neither good-quality quantum error-correction nor a demonstration of “quantum supremacy.” Some general principles governing the behavior of noisy quantum systems are derived. Our work supports the “physical Church thesis” studied by Pitowsky (1990) and follows his vision of using abstract ideas about computation to study the performance of actual physical computers.

1 Introduction

My purpose in this paper is to give an argument against the feasibility of quantum computers. In a nutshell, our argument is based on the following statement:

Noisy quantum systems will not allow building quantum error-correcting codes needed for quantum computation.

To support this statement we study the relations between abstract ideas about computation and the performance of actual physical computers, which are referred to as noisy intermediate-scale quantum computers or NISQ computers for short. We need to explain why it is the case that quantum error-correcting codes are needed to gain the computational advantage of quantum computing, and we also need to explain why quantum error-correcting codes are out of reach. The crux of the matter lies in a parameter referred to as the rate of noise. We will explain why reducing the rate of noise to the level needed for good quantum error-correction will already enable NISQ computers to demonstrate “quantum
supremacy.” We will also explain why NISQ computers represent a very low level computational complexity class which cannot support quantum supremacy. This provides a good, in-principle, argument for the infeasibility of both good quantum error-correcting codes and quantum supremacy.

We will show where the argument breaks down for classical computation. Rudimentary classical error-correcting codes are supported by the low-level computational complexity class that describes NISQ computers.

Pitowsky (1990) formulates and studies a strong form of the “physical” Church–Turing thesis (referred to later as the extended Church–Turing thesis (ECTT))\footnote{Pitowsky’s paper attributes the physical Church–Turing thesis to Wolfram (1985).}, namely,

“whether we can invoke physical laws to reduce a computational problem that is manifestly or presumably of exponential complexity, and actually complete it in polynomial time.”

and considers the then recently discovered model of quantum computers\footnote{Quantum computers were proposed by Feynman (1982), Deutsch (1985), and others. The idea can be traced to Feynman’s 1959 visionary lecture, “There is Plenty of Room at the Bottom.”} as a possible counterexample to this newly formulated physical Church–Turing thesis. Very strong support for “quantum supremacy” – the ability of quantum computers to perform certain tasks that classical computers cannot perform, – came four years after Pitowsky’s paper, with Peter Shor’s discovery (Shor 1994) that quantum computers can factor integers efficiently.

Our argument, which is in agreement with quantum mechanics, supports the validity of the extended Church–Turing thesis but does not rely on this thesis. Our argument is based on computational complexity considerations and thus manifests the vision of Pitowsky of using abstract ideas about computation to draw conclusions on the performance of actual physical computers, and of integrating computational complexity into theories of nature. Our computational complexity considerations refer to very low-level computational classes and rely on definite mathematical theorems; they do not depend on the unproven conjectures underlying our view of the computational complexity world, such as the $P \neq NP$ conjecture. The novelty as well as a certain weakness of my argument is that it uses asymptotic insights (with unspecified constants) to draw conclusions on the behavior of small- and intermediate-scale systems and, in particular, on the value of constants that naively appear to reflect just engineering capabilities.

The argument against quantum computers leads to various predictions on near-term experiments, and to general principles governing the behavior of noisy quantum systems. It may shed light on a variety of questions about quantum systems and computation. We emphasize that the argument predicts the failure of near-term experimental goals of many groups around the world to demonstrate on NISQ computers quantum supremacy and good-quality quantum error-correcting codes.

In Section 2 we will consider basic models of computation and basic insights about computational complexity. Our argument against quantum computers is presented and discussed in Section 3, along with concrete predictions on near-term experimental efforts. The argument crucially relies on the study by Kalai and Kindler (2014) of noise stability and sensitivity for systems of non-interacting bosons. This study is built on the theory of noise stability and noise sensitivity of Boolean functions developed by Benjamini, Kalai, and Schramm (1999). Section 4 discusses underlying principles behind the failure of quantum computers and some consequences. Section 5 concludes and Section 6 is about Itamar. This paper complements, updates, and presents in nontechnical language some of my earlier works on the topic of quantum computers; see Kalai (2016, 2016b, 2018), and earlier papers cited there.
2 Basic models of computation

Pitowsky (1990) described three types of computers where each type is more restrictive than the previous one: “the Platonist computer,” “the constructivist computer,” and “the finitist computer.” We will consider versions of the last two types. Our definitions rely on the notion of Boolean circuits.

2.1 Pitowsky’s “constructivist computer,” Boolean functions, and Boolean circuits

The “constructivist computer” is just a (deterministic) Turing machine, and associated with this notion is the class of all Turing machine computable, i.e., recursive functions. Can a physical machine compute a non-recursive function?

Itamar Pitowsky (1990)

Model 1 – Computers (circuits): A computer (or circuit) has \( n \) bits, and it can perform certain logical operations on them. The NOT gate, acting on a single bit, and the AND gate, acting on two bits, suffice for the full power of classical computing.

The outcome of the computation is the value of certain \( k \) output bits. When \( k = 1 \) the circuit computes a Boolean function, namely, a function \( f(x_1, x_2, \ldots, x_n) \) of \( n \) Boolean variables (\( x_i = 1 \) or \( x_i = 0 \)) so that the value of \( f \) is also 0 or 1.

2.2 Easy and hard problems and Pitowsky’s “finitist computer”

The “finitist computer,” is a deterministic, polynomial-time Turing machine (with a fixed finite number of tapes). Associated with this notion is the class of all functions which are computable in a number of steps bounded by a polynomial in the size of the input. As is well known, many important computational problems seem to lie outside the class \( P \), typically the so-called \( NP \)-hard problem.

Itamar Pitowsky (1990)

While general circuits can compute arbitrary Boolean functions, matters change when we require that the size of the circuit be at most a polynomial in the number of input variables. Here the size of the circuits is the total number of gates.

Model 2 – Polynomial size circuit: A circuit with \( n \) input bits of size at most \( An^c \). Here \( A \) and \( c \) are positive constants.

The complexity class \( P \) refers to problems that can be solved using a polynomial number of steps in the size of the input. The complexity class \( NP \) refers to problems whose solution can be verified in a polynomial number of steps. Our understanding of the computational complexity world depends on a whole array of conjectures: \( NP \neq P \) is the most famous one. Many natural computational tasks are \( NP \)-complete. (A task in \( NP \) is \( NP \)-complete if a computer equipped with a subroutine for solving this task can solve, in a polynomial number of steps, every problem in \( NP \).)

\[ ^3 \text{The precise relation between the Turing machine model (considered in Pitowsky 1990) and the circuit model is interesting and subtle but we will not discuss it in this paper.} \]
Let me mention two fundamental examples of computational “easiness” and “hardness.” The first example is: multiplication is easy, factoring is hard. Multiplying two \( n \)-digit numbers requires a simple algorithm with a number of steps that is quadratic in \( n \). On the other hand, the best-known algorithm for factoring an \( n \)-digit number to its prime factors is (we think) exponential in \( n^{1/3} \).

The second fundamental example of complexity theory is that “determinants are easy but permanents are hard.” Recall that the permanent of an \( n \)-by-\( n \) matrix \( M \) has a similar (but simpler) formula compared to the more famous determinant. The difference is that the signs are eliminated, and this difference accounts for a huge computational difference. Gaussian elimination gives a polynomial-time algorithm to compute determinants. By contrast, computing permanents is harder than solving \( \text{NP} \)-complete problems.

Classical circuits equipped with random bits lead to randomized algorithms, which are both practically useful and theoretically important.

**Model 3 – Randomized circuit:** A circuit with \( n \) bits of input and with one additional type of gate that provides a random bit. Again we assume that the circuit is of size at most \( An^c \) (here, again, \( A \) and \( c \) are positive constants). This time the output is a sample from a probability distribution on strings of length \( k \) of zeroes and ones.

---

4Factoring belongs to \( \text{NP} \cap \text{coNP} \) and therefore, according to the basic conjectural view of computational complexity, it is not an \( \text{NP} \)-complete problem. The hardness of factoring is a separate important computational complexity conjecture.

5The hardness of computing permanents defines an important computational class \( \#P \) (in words, number-P or sharp-P) that is larger than \( \text{PH} \) (the entire “polynomial hierarchy”).

6For simplicity, we use in this paper the term \( \text{P} \) (rather than \( \text{BPP} \)) to describe the computational power of polynomial-time
2.3 Quantum computers

We will now give a brief description (based on circuits) of quantum computers.

**Model 4 – Quantum computers:**

- A qubit is a piece of quantum memory. A qubit is a unit vector in $\mathbb{C}^2$, and the state of a qubit is a unit vector in a two-dimensional complex Hilbert space $H = \mathbb{C}^2$. The memory of a quantum computer (quantum circuit) consists of $n$ qubits and the state of the computer is a unit vector in the $2^n$-dimensional Hilbert space, i.e., $(\mathbb{C}^2)^{\otimes n}$.

- A Quantum gate is a unitary transformation. We can put one or two qubits through gates representing unitary transformations acting on the corresponding two- or four-dimensional Hilbert spaces. As for classical computers, there is a small list of gates that are sufficient for the full power of quantum computing.

- Measurement: Measuring the state of $k$ qubits leads to a probability distribution on $0 - 1$ vectors of length $k$.

Quantum computers allow sampling from probability distributions well beyond the capabilities of classical computers (with random bits). Shor’s famous algorithm shows that quantum computers can factor $n$-digit integers efficiently, in roughly $n^2$ steps! It is conjectured that quantum computers cannot solve NP-complete problems and this implies that they cannot compute permanents.

For the notions of noisy quantum computers and for fault-tolerant quantum computation that we discuss next, it is necessary to allow measurement throughout the computation process and to be able to feed the measurement results back into later gates.

2.4 Noisy quantum circuits

Next we consider noise. Quantum systems are inherently noisy; we cannot accurately control them, and we cannot accurately describe them. In fact, every interaction of a quantum system with the outside world amounts to noise.

**Model 5 – Noisy quantum computers:** A noisy quantum circuit has the property that every qubit is corrupted in every “computer cycle” with a small probability $t$, and every gate is $t$-imperfect. Here, $t$ is a small constant called the rate of noise.

Here, in a “computer cycle” we allow several non-overlapping gates to perform in parallel. We do not specify the precise technical meaning of “corrupt” and “$t$-imperfect,” but the following intuitive explanation (for a restricted form of noise called depolarizing noise) could be useful. When a qubit is corrupted then its state is replaced by a uniformly distributed random state on the same Hilbert state. A gate is $t$-imperfect if with probability $t$ the state of the qubits that are involved in the gate is replaced by a uniformly random state in the associated Hilbert space.

Computers (and polynomial-size circuits) with randomization. We also use the term $Q$ rather than $BQP$ (for decision problems) and Quantum Sampling (for sampling problems) to describe the computational power of polynomial-time quantum computers (and polynomial-size quantum circuits).
**Theorem 1:** “The threshold theorem.” If the error rate is small enough, noisy quantum circuits allow the full power of quantum computing.

The threshold theorem was proved around 1995 by Aharonov and Ben-Or (1997), Kitaev (1997), and Knill, Laflamme, and Zurek (1998). The proof relies on quantum error-correcting codes first introduced by Shor (1995) and Steane (1996).

A common interpretation of the threshold theorem is that it shows that large-scale quantum computers are possible in principle. A more careful interpretation is that if we can control noisy intermediate-scale quantum systems well enough, then we can build large-scale universal quantum computers. As we will see, there are good reasons for why we cannot control the quality of noisy intermediate-scale quantum systems well enough.

### 2.5 Quantum supremacy and NISQ devices

**Model 6 – Noisy intermediate-scale quantum (NISQ) computers:** These are simply noisy quantum circuits with at most 500 qubits.

The number 500 sounds arbitrary but there is a reason to regard NISQ circuits and other NISQ devices as an important separate computation model: NISQ circuits are too small to apply quantum error-correction for quantum fault-tolerance, since a single good-quality “logical” qubit requires a quantum error-correcting code described by a NISQ circuit with more than a hundred qubits.

A crucial theoretical and experimental challenge is to understand NISQ computers. Major near-future experimental efforts are aimed at demonstrating quantum supremacy using NISQ circuits and other NISQ devices, and are also aimed at using NISQ circuits to build high-quality quantum error-correcting codes.

For concreteness we mention three major near-term experimental goals.

**Goal 1:** Demonstrate quantum supremacy via systems of non-interacting bosons. (See Section 3.3.)

**Goal 2:** Demonstrate quantum supremacy on random circuits (namely, circuits that are based on randomly choosing the computation process, in advance), with 50–100 qubits.

**Goal 3:** Create distance-5 surface codes on NISQ circuits that require a little over 100 qubits.

In our discussion we will refer to the task of demonstrating with good accuracy boson sampling for 10–20 bosons as **baby goal 1**, to the task of building with good accuracy random circuits with 10–30 qubits, as **baby goal 2**, and to the task of creating distance-3 surface codes on NISQ circuits that require a little over 20 qubits, as **baby goal 3**. The argument presented in the next section asserts that attempts to reach Goals 1-3 will fail, and the difficulties will be evident already in the baby goals.

---

7John Preskill coined the terms “quantum supremacy” (in 2012) and “NISQ systems” (in 2018). Implicitly, these notions already play a role in Preskill (1998). The importance of multi-scale analysis of the quantum computer puzzle is emphasized in Kalai (2016).
Figure 2. NISQ circuits are computationally very weak and therefore unlikely to create quantum error-correcting codes needed for quantum computers.

3 The argument against quantum computers

3.1 The argument

In brief, the argument against quantum computers is based on a computational complexity argument for why NISQ computers cannot demonstrate quantum supremacy and cannot create the good-quality quantum error-correcting codes needed for quantum computation. We turn now to a detailed description of the argument, which is based on the following assertions on NISQ devices.

(A) Probability distributions described (robustly) by NISQ devices can be described by low-degree polynomials (LDP). LDP-distributions represent a very low-level computational complexity class well inside bounded-depth (classical) computation.

(B) Asymptotically low-level computational devices cannot lead to superior computation.

(C) Achieving quantum supremacy is easier than achieving quantum error-correction.

Concretely, we argue that Goals 1 and 2 both represent superior computation that cannot be reached by the low-level computational NISQ devices and that Goal 3 (creating distance-5 surface codes) is even harder than Goal 2.

We will discuss in Sections 3.3 and 3.4 our argument for assertion (A) based on the theory of noise sensitivity and noise stability, and, in particular, a detailed study of noisy systems of noninteracting bosons. It would be interesting to explore other theoretical arguments for (A).

Assertion (B) requires special attention and it can be regarded as both a novel and a weak link of our argument. There is no dispute that we can apply asymptotic computational insights to the behavior of computing devices in the small and intermediate scale when we know or can estimate the constants
involved. This is not the case here. The constants depend on (unknown) engineering abilities to control the noise. I claim that (even when the constants are unknown) the low-level asymptotic behavior implies or strongly suggests limitations on the computing power and hence on the engineering ability. Moreover, these limitations already apply for the intermediate scale, namely, for NISQ circuits, already when the number of qubits is rather small. In my view, this type of reasoning is behind many important successes regarding the interface between the theory of computing and the practice of computing.

Several researchers disagree with this part of my analysis and find it unconvincing.

There is substantial empirical evidence for (C) and experimentalists often refer to achieving quantum supremacy as a step toward achieving quantum error-correction. Quantum circuits are expected to exhibit probability distributions beyond the power of classical computers already for 70–100 qubits, while quantum error-correcting codes of the quality needed for quantum fault-tolerance will require at least 150–500 qubits. It will be interesting to conduct a further theoretical study of the hypothesis that the ability to build good-quality quantum error correction codes already leads to quantum systems that demonstrate quantum supremacy. There is good theoretical evidence for a weaker statement (C'). (Claims (A), (B), and (C') already support a weaker form of the argument against quantum computers.)

(C') High-quality quantum error-correcting codes are not supported by the very low-level computational power LDP of NISQ systems.

3.2 Predictions on NISQ computers

The quality of individual qubits and gates is the major factor for the quality of the quantum circuits built from them. One consequence of our argument is that there is an upper bound on the quality of qubits and gates, which is quite close, in fact, to what people can achieve today. This consequence seems counterintuitive to many researchers. As a matter of fact, the quantum-computing analogue of Moore’s law, known as “Schoelkopf’s law,” asserts, to the contrary, that roughly every three years, quantum decoherence can be delayed by a factor of ten. Our argument implies that Schoelkopf’s law will be broken before we reach the quality needed for quantum supremacy and quantum fault-tolerance. (Namely, now!) Our first prediction is thus that

(a) The quality of qubits and gates cannot be improved beyond a certain threshold that is quite close to the best currently existing qubits and gates. This applies also to topological qubits (Section 3.5).

8A nice example concerns the computation of the Ramsey number $R(k, r)$, which is the smallest integer $n$ such that in every party with $n$ participants you can find either a set of $k$ participants where every two shook hands, or a set of $r$ participants where no two shook hands. This is an example where asymptotic computational complexity insights into the large-scale behavior (namely, when $k$ and $r$ tend to infinity) explain the small-scale behavior (namely, when $k$ and $r$ are small integers). It is easy to verify that $R(3, 3) = 6$, and it was possible with huge computational efforts (and human ingenuity) to compute the Ramsey number $R(4, 5) = 25$ (McKay and Radziszowski 1995). It is commonly believed that computing $R(7, 7)$ and certainly $R(10, 10)$ is and will remain well beyond our computational ability. Building gates that are two orders of magnitude better than the best currently existing gates can be seen as analogous to computing $R(10, 10)$: we have good computational complexity arguments that both these tasks are beyond reach.

9Schoelkopf’s law was formulated by Rob Schoelkopf’s colleagues based on Schoelkopf’s experimental breakthroughs, and only later was adopted as a prediction for the future. An even bolder related prediction by Neven asserts that quantum computers are gaining computational power relative to classical ones at a “doubly exponential” rate. Our argument implies that Neven’s law does not reflect reality.
Our argument leads to several further predictions on near-term experiments, for example, on distributions of 0-1 strings based on a random quantum circuit (Goal 2), or on circuits aimed at creating good-quality quantum error-correcting codes such as distance-5 and even distance-3 surface codes (Goal 3).

(b) For a larger amount of noise, robust experimental outcomes are possible but they will represent probability distributions that can be expressed in terms of low-degree polynomials (LDP-distributions, for short) that are far away from the desired noiseless distributions.

(c) For a wide range of smaller amounts of noise, the experimental outcomes will not be robust. This means that the resulting probability distributions will strongly depend on fine properties of the noise and hence the outcomes will be chaotic.

In other words, in this range of noise rate the probability distributions are still far away from the desired noiseless distributions and, moreover, running the experiment twice will lead to very different probability distributions.

Following predictions (b) and (c) we further expect that

(d) The effort required to control \( k \) qubits to allow good approximations of the desired distribution will increase exponentially with \( k \) and will fail in a fairly small number of qubits. (My guess is that the number will be \( \leq 20 \).)

3.3 Non-interacting bosons

Caenorhabditis elegans (or C. elegans for short) is a species of a free-living roundworm whose biological study shed much light on more complicated organisms. Our next model, boson sampling can be seen as the C. elegans of quantum computing. It is both technically and conceptually simpler than the circuit model and yet it allows definite and clear-cut insights that extend to more general models and more complicated situations.

Model 7 – Boson Sampling (Troyansky and Tishby 1996; Aaronson and Arkhipov 2013): Given a complex \( n \)-by-\( m \) matrix \( X \) with orthonormal rows, sample subsets of columns (with repetitions) according to the absolute value-squared of permanents. This task is referred to as boson sampling.

Boson sampling represents a very limited form of quantum computers based on non-interacting bosons. (The number of bosons is \( n \), and each boson can be in \( m \) modes.) Quantum circuits can perform boson sampling efficiently on the nose! There is a good theoretical argument by Aaronson–Arkhipov (2013) that these tasks are beyond the reach of classical computers. If we consider non-interacting fermions rather than bosons, we obtain a similar sampling task, called fermion sampling, with determinants instead of permanents. This sampling task can be performed by a classical randomized computer (Model 3).

Model 8 – Noisy boson sampling (Kalai and Kindler 2014): Let \( G \) be a complex Gaussian \( n \)-by-\( m \) noise matrix (normalized so that the expected row norm is 1). Given an input matrix \( A \), we average the boson sampling distributions over \( \sqrt{(1-t)A + \sqrt{t}G} \). Here, \( t \) is the rate of noise.

To study noisy boson sampling we expand the outcome distribution in terms of Hermite polynomials in \( nm \) variables which correspond to the entries of the input matrix \( A \). The effect of the noise is exponential
Figure 3. The huge computational gap (left) between boson sampling (purple) and fermion sampling (green) vanishes in the noisy versions (right).

decay of high-degree terms and, as it turns out, the Hermite expansion for the boson sampling model is very simple and quite beautiful (much like the simplicity of the graph of synaptic connectivity of C. elegans). This analysis leads to the following theorems:

**Theorem 2 (Kalai and Kindler 2014):** When the noise level is constant, distributions given by noisy boson sampling are well approximated by their low-degree Fourier–Hermite expansion. (Consequently, these distributions can be approximated by bounded-depth polynomial-size circuits.)

**Theorem 3 (Kalai and Kindler 2014):** When the noise level is higher than $1/n$, noisy boson sampling is very sensitive to noise, with a vanishing correlation between the noisy distribution and the ideal distribution.

Theorem 2 asserts that noisy intermediate-scale systems of non-interacting bosons are, for every fixed level of noise, “classical computing machines”; namely, they can be simulated by classical computers. We do not expect classical computing devices to exhibit quantum supremacy. In this case, the argument is especially strong in view of the very low-level computational class described by Theorem 2, as well as in view of Theorem 3 that asserts that in a wide range of subconstant levels of noise the experimental outcomes will be unrelated to the noiseless outcomes. Furthermore, Theorem 3 strongly suggests that in this wide range of subconstant noise levels, experimental outcomes will be chaotic; namely, the correlation between the outcomes of two different runs of the experiment will also tend to zero.
3.4 From boson sampling to NISQ circuits

The following open-ended mathematical conjecture is crucial to part (A) of our analysis, as well as to predictions (b)–(d) in Section 3.2.

**Conjecture 4:** Theorems 2 and 3 both extend to all NISQ computers (in particular, to noisy quantum circuits) and to all realistic forms of noise.

(i) When the noise level is constant, distributions given by NISQ systems are well approximated by their low-degree Fourier expansion. In particular, these distributions are very low-level computationally.

(ii) For a wide range of lower noise levels, NISQ systems are very sensitive to noise, with a vanishing correlation between the noisy distribution and the ideal distribution. In this range, the noisy distributions depend on fine parameters of the noise.

Assertion (A) of our argument is supported by Conjecture 4(i). The conjecture asserts that, for every fixed level of noise, probability distributions described by NISQ circuits can be simulated by classical computers. As in the case of boson sampling, the argument is especially strong in view of the very low-level computational class described by Conjecture 4(i), and in view of Conjecture 4(ii) that asserts that in a wide range of subconstant levels of noise the experimental outcomes will be unrelated to the noiseless outcomes and, furthermore, the correlation between the outcomes of two different runs of the experiment will also tend to zero.

One delicate conceptual aspect of the argument is that it is a multi-scale argument. The threshold theorem asserts that noisy quantum circuits for a small enough level of noise support universal computation! However, in the NISQ regime the situation is different and Conjecture 4(i) asserts that NISQ circuits are “classical” and of a very low level of computational complexity even in the classical computational kingdom. It follows that in the NISQ regime quantum supremacy (e.g., Goal 2) is out of reach, and high-quality quantum error-correction (e.g. Goal 3) is therefore out of reach as well. This behavior of NISQ circuits implies lower bounds on the rate of noise that show that, in larger scales, the threshold theorem is not relevant.

Results by Gao and Duan (2018), and by Bremmer, Montanaro, and Shepherd (2017) give some support to Conjecture 4(i).

3.5 The scope of the argument

NISQ computers account for most of the effort to create stable qubits via quantum error-correction, to demonstrate quantum supremacy, and to ultimately build universal quantum computers. We expect (Section 4.1) that noise stability and the very low-level computational class LDP that we identified for noisy NISQ systems apply in greater generality.

Specifically, it is plausible that the failure of NISQ computers to achieve stable logical qubits extends to systems, outside the NISQ regime, that do not apply quantum error-correction. It is plausible that Google’s attempts to build “surface codes” with quantum circuits of 50–1000 qubits will meet the same fate as Microsoft’s attempts to build a single surface-code qubit based on non-Abelian anyons. The microscopic quantum process for Microsoft’s attempts is not strictly in the NISQ regime, but it is plausible that it represents the same primitive computational power of noisy NISQ circuits.
In brief, the extended argument against quantum computers is based on a paradox. On the one hand, you cannot achieve quantum supremacy without quantum error-correction. On the other hand, if you could build quantum error-correcting codes of the required quality then you could demonstrate quantum supremacy directly on smaller and simpler quantum systems that involve no quantum error-correction.

To sum

Noisy quantum systems that refrain from using error correction represent a low-level computational ability that will not allow building quantum error-correcting codes needed for quantum computation.

It is not easy to define quantum devices that “refrain from using error correction,” but this property holds for NISQ circuits that simply do not have enough qubits to use quantum error correction, and appears to hold for all known experimental processes aimed at building high quality physical qubits including toplogical qubits. What we see next is that the low level complexity class of NISQ circuits does allow building basic forms of classical error-correcting codes.

3.6 Noise stability, noise sensitivity, and classical computation

The study of noisy boson sampling relies on a study of Benjamini, Kalai, and Schramm (1999) of noise stability and noise sensitivity of Boolean functions. See Kalai (2018) where both these theories and their connections to statistical physics, voting rules, and computation are discussed. A crucial mathematical aspect of the theory is that the noise has a simple description in terms of Fourier expansion: it reduces “high frequency” coefficients exponentially and, therefore, noise stability reflects “low frequency” (low-degree) Fourier expansion.

The theory of noise stability and noise sensitivity posits a clear distinction between classical information and quantum information. The class \( \mathbf{LDP} \) of probability distributions that can be approximated by low-degree polynomials does not support quantum supremacy and quantum error-correction but it still supports robust classical information, and with it also classical communication and computation. The (“majority”) Boolean function, defined by \( f(x_1, x_2, \ldots, x_n) = 1 \) if more than half the variables have a value of 1, allows for very robust bits based on a large number of noisy bits or qubits and admits excellent low-degree approximations. It can be argued that every form of robust information, communication, and computation in nature is based on a rudimentary form of classical error-correction where information is encoded by repetition (or simple variants of repetition) and decoded by the majority function (or a simple variant of it). In addition to this rudimentary form of classical error-correction, we sometimes witness more sophisticated forms of classical error-correction.

3.7 The extended Church–Turing thesis revisited

The extended Church–Turing thesis asserts (informally) that

(ECTT) It is impossible to build computing devices that demonstrate computational supremacy.

Here, computational supremacy is the ability to perform certain computations far beyond the power of digital computers.

A classical computing device (or a \( \mathbf{P} \)-device) is a computing device for which we can show or have good reason to believe that it can be modeled by polynomial-size Boolean circuits.
We now formulate the weak extended Church–Turing thesis that reads

(WECTT) It is impossible to build a classical computing device that demonstrates computational supremacy.

WECTT seems almost tautological and it is in wide agreement. A popular example of an application of WECCT is to the human brain. As many argue, it is unlikely that quantum effects in the human brain lead to some sort of quantum computational supremacy. Another implication of WECCT is that the recent proposal by Johansson and Larsson (2017) to demonstrate Shor’s factoring algorithm on certain classical analog devices cannot be realistic.

The argument given in this section asserts that for constant level of noise, NISQ devices are $\mathbf{P}$-devices. Consequently, WECTT suffices to rule out quantum supremacy of NISQ devices. What makes this argument stronger is that we have good evidence that

1. Probability distributions described by NISQ devices for constant error rate are actually modeled by $\mathbf{LDP}$, a computational class well below bounded-depth computation, which is well below $\mathbf{P}$.

2. The outcomes of the computation process are chaotic for a wide range of subconstant levels of errors.

We argue that NISQ devices are, in fact, low-level classical computing devices, and I regard it as strong evidence that engineers will not be able to reach the level of noise required for quantum supremacy. Others argue that the existence of low-level simulation (in the NISQ regime) for every fixed level of noise does not have any bearing on the engineering question of reducing the level of noise. These sharply different views will be tested in the next few years.

In his comments on this paper, physicist and friend Aram Harrow wrote that my predictions are so sweeping that they say that hundreds of different research teams will all stop making progress. Indeed, my argument and predictions say that the goals of these groups (such as Goals 1–3 and other, more ambitious, goals) will not be achieved. Rather, if I am correct, these research teams will make important progress in our understanding of the inherent limitations of computation in nature, and the limitations of human ability to control quantum systems.

Aram Harrow also wrote: “The Extended Church–Turing Thesis (ECTT) is a major conjecture, implicating the work of far more than quantum computing. Long before Shor’s algorithm it was observed that quantum systems are hard in practice to simulate because we do not know algorithms that beat the exponential scaling and work in all cases, or even all practical cases. Simulating the 3 or so quarks (and gluons) of a proton is at the edge of what current supercomputers can achieve despite decades of study, and going to larger nuclei (say atomic weight 6 or more) is still out of reach. If the extended Church–Turing thesis is actually true then there is a secret poly-time simulation scheme here that the lattice QCD community has overlooked. Ditto for chemistry where hundreds of millions of dollars are spent each year simulating molecules. Thus the ECTT is much stronger than just saying we cannot build quantum computers.”

I agree with the spirit of Harrow’s comment, if not with the specifics, and the comment can serve as a nice transition to the next section that discusses a few underlying principles and consequences that follow from the failure of quantum computing.
4 The failure of quantum computers: Underlying principles and consequences

In this section we will propose some general underlying principles for noisy quantum processes that follow from the failure of quantum computers. (For more, see Kalai 2016b and Kalai 2018.)

4.1 Noise stability of low-entropy states

Principle 1: Probability distributions described by low-entropy states are noise stable and can be expressed by low-degree polynomials.

The situation for probability distributions described by NISQ systems appears to be as follows: noise causes the terms, in a certain Fourier-like expansion, to be reduced exponentially with the degree. Noise-stable states, namely, those states for which the effect of the noise is small, have probability distributions expressed by low-degree terms. Probability distributions realistically (and robustly) obtained by NISQ devices represent a very low-level computational complexity class, LDP, the class of probability distributions that can be approximated by low-degree polynomials. In the absence of quantum fault-tolerance this description extends to (low-entropy) quantum states that arbitrary quantum circuits and (other quantum devices) can reach.

Noise stability and descriptions by low-degree polynomials might be relevant to some general aspects of quantum systems in nature. In the previous section we considered one such aspect: the feasibility of classical information and computation. The reason that our argument against quantum computation does not apply for classical computation is that noise stability still allows basic forms of classical error-correction. In fact, every form of robust information, communication, and computation in nature is based on some rudimentary form of classical error-correction that is supported by low-degree polynomials. We will consider two additional aspects: the possibility of efficiently learning quantum systems, and the ability of physical systems to reach ground states.

Before we proceed I would like to make the following two remarks. The first remark is that the specific “Fourier-like expansion” required for the notions of noise stability and noise sensitivity is different for different situations. (The computational class LDP applies to all these situations.) An interesting direction for future research would be to extend the noise stability/noise sensitivity dichotomy and to find relevant Fourier-like expansions and definitions of noise for mathematical objects of high-energy physics. The second remark is that the study of noise stability is important in other scientific disciplines, and the assumption that systems are noise stable (aka “robust”) may provide useful information for studying them. Robustness is an important ingredient in the study of chemical and biological networks; see, e.g., Barkai and Leibler (1997).

Learnability

It is a simple and important insight of machine learning that the approximate value of a low-degree polynomial can efficiently be learned from examples. This offers an explanation for our ability to understand natural processes and the parameters defining them. Rudimentary physical processes with a higher entropy rate may represent simple extensions of the class of LDP for which efficient learnability is still possible, and this is an interesting topic for further research.
Figure 4. Low-entropy quantum states give probability distributions described by low degree polynomials, and very low-entropy quantum states give chaotic behavior. Higher entropy enables classical information.

Reaching ground states

Reaching ground states is computationally hard (NP-hard) for classical systems, and even harder for quantum systems. So how does nature reach ground states so often? The common answer relies on two ingredients: the first is that physical systems operate in positive temperature rather than zero temperature, and the second is that nature often reaches meta-stable states rather than ground states. However, these explanations are incomplete: we have good theoretical reasons to think that, for general processes in positive temperature, reaching meta-stable states is computationally intractable as well. First, for general quantum or classical systems, reaching a meta-stable state can be just as computationally hard as reaching a ground state. Second, one of the biggest breakthroughs in computational complexity, the “PCP theorem,” asserts (in physics language) that positive temperature offers no computational complexity relief for general (classical) systems.

Quantum evolutions and states approximated by low-degree polynomials represent severe computational restrictions, and the results on the hardness of reaching ground states no longer apply. This may give a theoretical support to the natural phenomenon of easily reached ground states.
4.2 Noise and time-dependent evolutions

Principle 2: Time-dependent (local) quantum evolutions are inherently noisy.

It is an interesting challenge to give a satisfactory mathematical description of what “time dependent” means. We proposed in Kalai (2016) to base a lower bound on the rate of noise on a measure of the non-commutativity of unitary operations. This measure of non-commutativity can also serve as an “internal” clock of the noisy quantum evolution. We also proposed in Kalai (2016) a restricted class of noisy evolutions called “smoothed Lindblad evolutions” defined via a certain smoothing operation that mathematically expresses the idea that in the absence of quantum error-correction, noise accumulates. An important consequence of Principle 2 is that there are inherent lower bounds on the quality of qubits, or, more concretely, a universal upper bound on the number of non-commuting Pauli operators you can apply to a qubit before it get destroyed. We note that lower bounds on the rate of noise in terms of a measure of non-commutativity arise also in Polterovich (2014) in the study of quantizations in symplectic geometry.

4.3 Noise and correlation

Principle 3: Entangled qubits are subject to positively correlated noise.

Entanglement, the quantum analog of correlations, is a notion of great importance in quantum physics and in quantum computing, and the famous cat state $\frac{1}{\sqrt{2}} |00\rangle + \frac{1}{\sqrt{2}} |11\rangle$ represents the simplest example of entanglement and the strongest form of entanglement between two qubits. Principle 3 asserts that the errors for the two qubits of a cat state necessarily have large positive correlation. This is an observed and accepted phenomenon for gated qubits and without quantum error-correction it is inherited to all pairs of entangled qubits. Error synchronization refers to a substantial probability that a large number of qubits, much beyond the average rate of noise, are corrupted. An important consequence of Principle 3 is that complicated states and evolutions lead to error-synchronization. Principle 3 leads to additional predictions about NISQ systems that can be tested for current experiments.

(e) Every pair of gated qubits will be subject to errors with large positive correlation.

(f) Every pair of qubits in a cat state will be subject to errors with large positive correlation.

(g) All experimental attempts to achieve Goals 2 and 3 will lead to a strong effect of error-synchronization.

We emphasize that our argument against quantum computers in Section 3 does not depend on Principle 3 and, moreover, the main trouble with quantum computers is that the noise rate of qubits and gates cannot be reduced toward the threshold for fault-tolerance. Correlation is part of this picture since, as is well known, the value of the threshold becomes smaller when there is a larger positive correlation for 2-qubit gated errors. We note (Kalai 2016) that Prediction (f) implies that the quality of logical qubits via quantum error-correction is limited by the quality of gates. See also Chubb and Flammia (2018) for an interesting analysis of the effect of correlated errors on surface codes.

A reader may question how it is possible that entangled states will experience correlated noise, even long after they have been created and even if the constituent particles are far apart and not directly
interacting. We emphasize that Principle 3 does not reflect any form of “non local” noise and that no “additional” correlated noise on far apart entangled qubits is ever assumed or conjectured. Principle 3 simply reflects two facts. The first fact is that errors for gated qubits are positively correlated from the time of creation. The second fact is that unless the error rate is small enough to allow quantum error-correction (and this may be precluded by the argument of the previous section), cat states created indirectly via a sequence of computational steps will inherit errors with large positive correlation.

**Remark:** Error-synchronization is related to an interesting possibility (Kalai 2016b) that may challenge conventional wisdom and can also be checked for near term NISQ (and other) experiments. A central property of Hamiltonian models of noise in the study of quantum fault-tolerance (see, e.g., Preskill 2013), is that error fluctuations are sub-Gaussian. Namely, when there are $N$ qubits, the standard deviation for the number of qubit errors behaves like $\sqrt{N}$ and the probability of more than $t\sqrt{N}$ errors decays as it does for Gaussian distribution. Many other models in mathematical physics have a similar property. However, it is possible that fluctuations for the total amount of errors for noisy quantum circuits and other quantum systems with interactions are actually super-Gaussian and perhaps even linear.

### 4.4 A taste of other consequences

The failure of quantum computers and the infeasibility of high-quality quantum error-correction have several others far-reaching consequences imposing severe restrictions of humans’ and nature’s ability to control quantum states and evolutions. Following the tradition of using cats for quantum thought experiments, consider an ordinary living cat.

If quantum computation is not possible, it will be impossible to teleport the cat; it will be impossible to reverse time in the life of the cat; it will be impossible to implement the cat on a very different geometry; it will be impossible to superpose the lives of two distinct cats; and, finally, even if we place the cat in an isolated and monitored environment, the life of this cat cannot be predicted.

These restrictions will already be in place when our “cat” represents realistic (but involved) quantum evolutions described by quantum circuits in the small scale, namely, with less than 50 qubits (baby goals 1–3 can serve as good candidates for such “cats”). See Kalai (2016b) for a detailed discussion.

### 5 Conclusion

The intrinsic study of computation transcends human-made artifacts, and its expanding connections and interactions with all sciences, integrating computational modeling, algorithms, and complexity into theories of nature and society, marks a new scientific revolution!

Avi Wigderson – *Mathematics and Computation* 2019.

Understanding noisy quantum systems and potentially even the failure of quantum computers is related to the fascinating mathematical theory of noise stability and noise sensitivity and its connections to the theory of computing. Our study shows how insights from the computational complexity of very low-level computational classes support the extended Church–Turing thesis. Exploring this avenue may have important implications for various areas of quantum physics. These connections between mathematics, computation, and physics are characterized by a rich array of conceptual and technical methods and points of view. In this study, we witness a delightfully thin gap between fundamental and philosophical issues on the one hand and practical and engineering aspects on the other.
6 Itamar

This paper is devoted to the memory of Itamar Pitowsky. Itamar was great. He was great in science and great in the humanities. He could think and work like a mathematician, and like a physicist, and like a philosopher, and like a philosopher of science, and probably in various additional ways. And he enjoyed the academic business greatly, and took it seriously, with humor. Itamar had an immense human wisdom and a modest, level-headed way of expressing it.

Itamar’s scientific path interlaced with mine in many ways. Itamar made important contributions to the foundations of the theory of computation, probability theory, and quantum mechanics, all related to some of my own scientific endeavors and to this paper. Itamar’s approach to the foundation of quantum mechanics was that quantum mechanics is a theory of non-commutative probability that (like classical probability) can be seen as a mathematical language for the other laws of physics. (My work, to a large extent, adopts this point of view. But, frankly, I do not quite understand the other points of view.)

In the late ’70s when Itamar and I were both graduate students I remember him enthusiastically sharing his thoughts on the Erdős–Turán problem with me. This is a mathematical conjecture that asserts that if we have a sequence of integers $0 < a_1 < a_2 < \cdots < a_n < \cdots$ such that the infinite series $\sum \frac{1}{a_n}$ diverges then we can find among the elements of the sequence an arithmetic progression of length $k$, for every $k$. Over the years both of us spent some time on this conjecture without much to show for it. This famous conjecture is still open even for $k = 3$. Some years later both Itamar and I became interested, for entirely different reasons, in remarkable geometric objects called cut polytopes. Cut polytopes are obtained by taking the convex hull of characteristic vectors of edges in all cuts of a graph. Cut polytopes arise naturally when you try to understand correlations in probability theory and Itamar wrote several fundamental papers studying them; see Pitowsky (1991). Cut polytopes came into play, in an unexpected
way, in the work of Jeff Kahn and myself where we disproved Borsuk’s conjecture (Kahn and Kalai 1993). Over the years, Itamar and I became interested in Arrow’s impossibility theorem (Arrow 1950), which Itamar regarded as a major 20th-century intellectual achievement. He gave a course centered around this theorem and years later so did I. The study of the noise stability and noise sensitivity of Boolean functions has close ties to Arrow’s theorem.

The role of skepticism in science and how (and if) skepticism should be practiced is a fascinating issue. It is, of course, related to this paper that describes skepticism over quantum supremacy and quantum fault-tolerance, widely believed to be possible. Itamar and I had long discussions about skepticism in science, and about the nature and practice of scientific debates, both in general and in various specific cases. This common interest was well suited to the Center for the Study of Rationality at the Hebrew University that we were both members of, where we and other friends enjoyed many discussions, and, at times, heated debates.

A week before Itamar passed away, Itamar, Oron Shagrir, and I sat at our little CS cafeteria and talked about probability. Where does probability come from? What does probability mean? Does it just represent human uncertainty? Is it just an emerging mathematical concept that is convenient for modeling? Does it change when we move from classical to quantum mechanics? When we move to quantum physics the notion of probability itself changes for sure, but is there a change in the interpretation of what probability is? A few people passed by and listened, and it felt like this was a direct continuation of conversations we had had while we (Itamar and I; Oron is much younger) were students in the early ‘70s. This was to be our last meeting.
Acknowledgement

Work supported by ERC advanced grants 320924, & 834735, NSF grant DMS-1300120, and BSF grant 2006066. The author thanks Yuri Gurevich, Aram Harrow, and Greg Kuperberg for helpful comments, and Netta Kasher for drawing Figures 1–5.

References

[1] S. Aaronson and A. Arkhipov, The computational complexity of linear optics, *Theory of Computing* 4 (2013), 143–252.

[2] D. Aharonov and M. Ben-Or, Fault-tolerant quantum computation with constant error, STOC ’97, ACM, New York, 1999, pp. 176–188.

[3] K. Arrow, A difficulty in the theory of social welfare, *Journal of Political Economy* 58 (1950), 328–346.

[4] N. Barkai, and S. Leibler. Robustness in simple biochemical networks, *Nature*, 387 (1997), 913.

[5] I. Benjamini, G. Kalai, and O. Schramm, Noise sensitivity of Boolean functions and applications to percolation, *Publications Mathématiques de l’Institut des Hautes Études Scientifiques* 90 (1999), 5–43.

[6] M. J. Bremner, A. Montanaro, and D. J. Shepherd, Achieving quantum supremacy with sparse and noisy commuting quantum computations, *Quantum* 1, 8 (2017).

[7] C. T. Chubb and S. T. Flammia, Statistical mechanical models for quantum codes with correlated noise, [arXiv:1809.10704](https://arxiv.org/abs/1809.10704).

[8] D. Deutsch, Quantum theory, the Church–Turing principle and the universal quantum computer, *Proceedings of the Royal Society of London A* 400 (1985), 96–117.

[9] R. P. Feynman, Simulating physics with computers, *International Journal of Theoretical Physics* 21 (1982), 467–488.

[10] X. Gao and L. Duan, Efficient classical simulation of noisy quantum computation, [arXiv:1810.03176](https://arxiv.org/abs/1810.03176).

[11] N. Johansson and J.-A. Larsson, Realization of Shor’s algorithm at room temperature, arXiv:1706.03215.

[12] J. Kahn and G. Kalai, A counterexample to Borsuk’s conjecture, *Bulletin of the American Mathematical Society* 29 (1993), 60–62.

[13] G. Kalai, The quantum computer puzzle, *Notices of the American Mathematical Society* 63 (2016), 508–516.

[14] G. Kalai, The quantum computer puzzle (expanded version), [arXiv:1605.00992](https://arxiv.org/abs/1605.00992).

[15] G. Kalai, Three puzzles on mathematics, computation and games, *Proceedings of the International Congress of Mathematicians* 2018, Rio de Janeiro, Vol. I, pp. 551–606.

[16] G. Kalai and G. Kindler, Gaussian noise sensitivity and BosonSampling, [arXiv:1409.3093](https://arxiv.org/abs/1409.3093).

[17] A. Y. Kitaev, Quantum error correction with imperfect gates, in *Quantum Communication, Computing, and Measurement*, Plenum Press, New York, 1997, pp. 181–188.
[18] E. Knill, R. Laflamme, and W. H. Zurek, Resilient quantum computation: Error models and thresholds, *Proceedings of the Royal Society of London A* 454 (1998), 365–384.

[19] B. D. McKay and S. P. Radziszowski, R(4,5)=25, *Journal of Graph Theory* 19 (1995), 309–322.

[20] I. Pitowsky, The physical Church thesis and physical computational complexity, *Iyun, A Jerusalem Philosophical Quarterly* 39 (1990), 81–99.

[21] I. Pitowsky, Correlation polytopes: Their geometry and complexity, *Mathematical Programming* A50 (1991), 395–414.

[22] L. Polterovich, Symplectic geometry of quantum noise, *Communications in Mathematical Physics* 327 (2014), 481–519.

[23] J. Preskill, Quantum computing: Pro and con, *Proceedings of the Royal Society of London A* 454 (1998), 469–486.

[24] J. Preskill, Sufficient condition on noise correlations for scalable quantum computing, *Quantum Information and Computing* 13 (2013), 181–194.

[25] P. W. Shor, Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer, *SIAM Rev.* 41 (1999), 303–332. (Earlier version, *Proceedings of the 35th Annual Symposium on Foundations of Computer Science*, 1994.)

[26] P. W. Shor, Scheme for reducing decoherence in quantum computer memory, *Physical Review A* 52 (1995), 2493–2496.

[27] A. M. Steane, Error-correcting codes in quantum theory, *Physical Review Letters* 77 (1996), 793–797.

[28] L. Troyansky and N. Tishby, Permanent uncertainty: On the quantum evaluation of the determinant and the permanent of a matrix, in *Proceedings of the 4th Workshop on Physics and Computation*, 1996.

[29] A. Wigderson, *Mathematics and Computation*, Princeton University Press, 2019.

[30] S. Wolfram, Undecidability and intractability in theoretical physics, *Physical Review Letters* 54 (1985), 735–738.