Fermionic model of unitary transport of qubits from a black hole

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Inspired by a recent model of Osuga and Page, we propose an explicitly unitary fermionic toy model for transferring information from a black hole to the outgoing radiation. The model treats the unitary evolution as a composition of the Hawking pair creation outside the black hole and of pair annihilation inside the black hole.
I. INTRODUCTION

The black hole (BH) information (loss) paradox concerns difficulties around the issue of unitarity of BH evaporation (for recent reviews see [4, 8, 11, 17]). There are a lot of approaches proposed to date to analyse and resolve the paradox — some of them suggest to study simplified situations embodied in various qubit models (e.g., see [2, 5–7, 12–14, 16]). A successful model of BH evolution should include description of particle pair production according to the Hawking prescription, following gradual evaporation (“vanishing”) of the BH, and moreover it should be (or not) unitary. A model strictly motivated by actual physical phenomena would certainly be greatly appreciated, but in fact any model respecting at least general physical laws, even without any real physical mechanism built in, would be welcome as a “proof of concept”.

Recently Osuga and Page [16] (inspired by [1]) have proposed an explicitly unitary toy qubit transport model for BH evaporation (without firewalls). Another version of the model (with additional features) has been presented in [3]. In the present paper, building on the both models, we propose yet another toy qubit transport model for BH evaporation, which is explicitly unitary. Since the model, by assumption, operates on qubits and particle pair production scheme exactly follows the Hawking mechanism for fermions, we shall work in terms of fermion modes rather than bosonic ones. A new and important feature of our present proposal is explicit incorporation of the (fermion) Hawking pair creation mechanism into the chain of unitary processes. In other words, the global unitary evolution considered is given by the composition $U = U'' \cdot U'$, where $U'$ corresponds to creation of fermion pairs outside a BH according to the Hawking prescription, whereas $U''$ corresponds to annihilation of fermion pairs inside the BH (as described in [3]).

For reader’s convenience, we will follow conventions and notation of [16] (and of [3]) as closely as possible.

II. THE TOY MODEL

An initial total quantum state describing a newly formed (fermion) BH and “fermionic radiation” in the vacuum state is assumed in the following (partially product) form [3] (and
also cf. [16])

\[ |\Psi\rangle = \sum_{q_1, q_2, \ldots, q_N=0}^{1} A_{q_1 q_2 \ldots q_N} \bigotimes_{k=1}^{N} |q_k\rangle_{a_k} \otimes |O\rangle_{b_k c_k}. \]  

(1)

Here \(A_{q_1 q_2 \ldots q_N}\) are amplitudes for inner BH modes \(a_k\), which encode a quantum state of the BH, and the vacuum state for “fermionic radiation” is

\[ |O\rangle_{b_k c_k} \equiv |0\rangle_{b_k} \otimes |0\rangle_{c_k}, \]

(2)

where the Hawking (fermion) modes \(b_k\) and \(c_k\) are infalling and outgoing modes, respectively. The same range of indices \((k = 1, 2, \ldots, N)\) postulated for BH modes \(a_k\) and \(b_k, c_k\) pairs is not only a convenient computational simplification in our model but also it is a physically justified assumption, at least approximately (e.g., see [3]).

In the language of \(k\)-component blocks, the first step of (unitary) evolution denoted by \(U'_k\) yields the Hawking (fermion) pair for a single “\(k\)” mode, i.e.

\[ U'_k \left( |q_k\rangle_{a_k} \otimes |O\rangle_{b_k c_k} \right) = |q_k\rangle_{a_k} \otimes |H_1\rangle_{b_k c_k}, \]  

(3)

where the fermionic Hawking state can be chosen in the form (cf. Eq. (116) in [10])

\[ |H_1\rangle_{b_k c_k} \equiv \cos \omega_k |0\rangle_{b_k} \otimes |0\rangle_{c_k} + \sin \omega_k |1\rangle_{b_k} \otimes |1\rangle_{c_k}, \]  

(4)

with \(\omega_k\) determined by BH parameter(s).

The total (i.e. for all modes \(k\)) \(U'\)-evolution yields by virtue of (3) the total (intermediate) state

\[ |\Psi'\rangle = \sum_{q_1, q_2, \ldots, q_N=0}^{1} A_{q_1 q_2 \ldots q_N} \bigotimes_{i=1}^{N} |q_k\rangle_{a_k} \otimes |H_1\rangle_{b_k c_k}. \]  

(5)

We could possibly consider a slight generalization of the unitary evolution (3) allowing some unitary transformation \(U'_k|q_k\rangle_{a_k}\) of the internal BH mode \(a_k\) on the RHS of (3), but we ignore this option, because it would merely give rise to a redefinition of \(A\)-amplitudes \((A_{q_1 q_2 \ldots q_N} \mapsto A'_{q_1 q_2 \ldots q_N})\) in the final state.

The second step of (unitary) evolution denoted by \(U''_k\) yields particle pair annihilation inside the BH (see [3] and cf. Eq. (3.3) in [1116]), i.e.

\[ U''_k \left( |q_k\rangle_{a_k} \otimes |H_1\rangle_{b_k c_k} \right) = |O\rangle_{a_k b_k} \otimes |q_k\rangle_{c_k}, \]  

(6)

where the vacuum state \(|O\rangle_{a_k b_k}\) is defined analogously to (2) (with appropriate replacements of modes).
Consequently, for the entire evolution $U_k \equiv U_k'' \cdot U_k'$ we have

$$U_k (|q_k\rangle_{a_k} \otimes |O\rangle_{b_k c_k}) = |O\rangle_{a_k b_k} \otimes |q_k\rangle_{c_k},$$  

and the final total state assumes the form

$$|\Psi''\rangle = \sum_{q_1 q_2 \cdots q_N=0}^{1} A_{q_1 q_2 \cdots q_N} \bigotimes_{k=1}^{N} |O\rangle_{a_k b_k} \otimes |q_k\rangle_{c_k}. \quad (8)$$

Eq. (8) means that all information has been transferred from a BH to the outgoing radiation, and the BH is in the vacuum state.

Obviously, the total operators $U'$, $U''$, $U$ are the following tensor products of the above-defined $k$-mode operators

$$U' = \bigotimes_{k=1}^{N} U_k', \quad U'' = \bigotimes_{k=1}^{N} U_k'', \quad U = \bigotimes_{k=1}^{N} U_k' \cdot U_k' \equiv \bigotimes_{k=1}^{N} U_k,$$  

respectively, and

$$U' |\Psi\rangle = |\Psi'\rangle, \quad U'' |\Psi\rangle = |\Psi''\rangle, \quad U |\Psi\rangle \equiv U'' \cdot U' |\Psi\rangle = |\Psi''\rangle. \quad (10)$$

**III. UNITARY OPERATORS**

We shall now explicitly derive the implicitly defined unitary operators $U_k'$, $U_k''$ and $U_k$. To this end let us first observe the following elementary fact from linear algebra: namely, for each pair of orthonormal bases $\{|E_\Lambda\rangle\}, \{|E'_\Lambda\rangle\}$ ($\langle E_\Lambda|E_{\Lambda'}\rangle = \langle E'_\Lambda|E'_{\Lambda'}\rangle = \delta_{\Lambda\Lambda'}$) in a finite dimensional Hilbert space $\mathcal{H}$ we can construct an operator

$$U = \sum_\Lambda |E'_\Lambda\rangle \langle E_\Lambda|,$$  

which is explicitly unitary. Really, we can easily check that, e.g.

$$U^\dagger \cdot U = \sum_{\Lambda, \Lambda'} |E_\Lambda\rangle \langle E'_\Lambda|E'_{\Lambda'}\rangle \langle E_{\Lambda'}| = \sum_{\Lambda, \Lambda'} \delta_{\Lambda\Lambda'} |E_\Lambda\rangle \langle E_{\Lambda'}| = \mathbb{I}. \quad (12)$$

Since the total Hilbert space $\mathcal{H}$ is a tensor product of $N$ $k$-mode Hilbert spaces $\mathcal{H}_k$, i.e. $\mathcal{H} = \otimes_{k=1}^{N} \mathcal{H}_k$, we can confine our construction to the single $k$-mode space $\mathcal{H}_k = \mathcal{H}_{a_k} \otimes \mathcal{H}_{b_k} \otimes \mathcal{H}_{c_k}$, where $\dim \mathcal{H}_k = 2 \cdot 2 \cdot 2 = 8$. Then, our unitary operators will be defined by three 8-dimensional orthonormal ($k$-dependent) bases in $\mathcal{H}_k$. 
The first base, \([|E_\Lambda\rangle_k\]_\Lambda=0\), assumes the following standard form:

\[
|E_0\rangle_k = |0\rangle_{a_k} \otimes |0\rangle_{b_k} \otimes |0\rangle_{c_k} \\
|E_1\rangle_k = |0\rangle_{a_k} \otimes |0\rangle_{b_k} \otimes |1\rangle_{c_k} \\
\vdots \\
|E_7\rangle_k = |1\rangle_{a_k} \otimes |1\rangle_{b_k} \otimes |1\rangle_{c_k}
\]

(13)

(or \(|E_\Lambda\rangle_k = |\Lambda\rangle_{a_k b_k c_k}\), in short).

The second (\(\tau\)-dependent) base, \([|E_\Lambda' (\tau)\rangle_k\]_\Lambda=0\), is given by (for further convenience, we have also included expansions in terms of \(\tau\)):

\[
|E'_0(\tau)\rangle_k = |0\rangle_{a_k} \otimes \left[ \cos (\omega k \tau) |0\rangle_{b_k} \otimes |0\rangle_{c_k} + \sin (\omega k \tau) |1\rangle_{b_k} \otimes |1\rangle_{c_k} \right] \\
\equiv |0\rangle_{a_k} \otimes [H_{\tau}^+]_{b_k c_k} \\
\equiv \cos (\omega k \tau) |E_0\rangle_k + \sin (\omega k \tau) |E_3\rangle_k \\
= |E_0\rangle_k + \omega k \tau |E_3\rangle_k + \mathcal{O}(\tau^2) \\
|E'_1(\tau)\rangle_k = |0\rangle_{a_k} \otimes |0\rangle_{b_k} \otimes |1\rangle_{c_k} \equiv |E_1\rangle_k \\
|E'_2(\tau)\rangle_k = |0\rangle_{a_k} \otimes |1\rangle_{b_k} \otimes |0\rangle_{c_k} \equiv |E_2\rangle_k \\
|E'_3(\tau)\rangle_k = |0\rangle_{a_k} \otimes \left[ -\sin (\omega k \tau) |0\rangle_{b_k} \otimes |0\rangle_{c_k} + \cos (\omega k \tau) |1\rangle_{b_k} \otimes |1\rangle_{c_k} \right] \\
\equiv |0\rangle_{a_k} \otimes [H_{\tau}^+]_{b_k c_k} \\
\equiv \cos (\omega k \tau) |E_3\rangle_k - \sin (\omega k \tau) |E_0\rangle_k \\
= |E_3\rangle_k - \omega k \tau |E_0\rangle_k + \mathcal{O}(\tau^2) \\
|E'_4(\tau)\rangle_k = |1\rangle_{a_k} \otimes \left[ \cos (\omega k \tau) |0\rangle_{b_k} \otimes |0\rangle_{c_k} + \sin (\omega k \tau) |1\rangle_{b_k} \otimes |1\rangle_{c_k} \right] \\
\equiv |1\rangle_{a_k} \otimes [H_{\tau}^+]_{b_k c_k} \\
\equiv \cos (\omega k \tau) |E_4\rangle_k + \sin (\omega k \tau) |E_7\rangle_k \\
= |E_4\rangle_k + \omega k \tau |E_7\rangle_k + \mathcal{O}(\tau^2) \\
|E'_5\rangle_k = |0\rangle_{a_k} \otimes |0\rangle_{b_k} \otimes |1\rangle_{c_k} \equiv |E_5\rangle_k \\
|E'_6\rangle_k = |0\rangle_{a_k} \otimes |0\rangle_{b_k} \otimes |1\rangle_{c_k} \equiv |E_6\rangle_k \\
|E'_7(\tau)\rangle_k = |1\rangle_{a_k} \otimes \left[ -\sin (\omega k \tau) |0\rangle_{b_k} \otimes |0\rangle_{c_k} + \cos (\omega k \tau) |1\rangle_{b_k} \otimes |1\rangle_{c_k} \right] \\
\equiv |1\rangle_{a_k} \otimes [H_{\tau}^+]_{b_k c_k} \\
\equiv \cos (\omega k \tau) |E_7\rangle_k - \sin (\omega k \tau) |E_4\rangle_k \\
= |E_7\rangle_k - \omega k \tau |E_4\rangle_k + \mathcal{O}(\tau^2),
\]

where \([H_{\tau}^+]\) denotes a Hawking state orthogonal to the Hawking state \([H_{\tau}]\), and the auxiliary dimensionless “time” parameter \(\tau \in [0, 1]\) governs the unitary evolution (in particular, for \(\tau = 1\) the evolution is supposed to be complete).
The third base, \( \{|E''_{\Lambda} (\tau)\}_k\}_{\Lambda=0}^7 \), is given by:

\[
|E''_0\rangle_k = |0\rangle_{a_k} \otimes |0\rangle_{b_k} \otimes |0\rangle_{c_k} \equiv |E_0\rangle_k,
\]

\[
|E''_1(\tau)\rangle_k = \cos\left(\frac{\pi \tau}{2}\right) |0\rangle_{a_k} \otimes |0\rangle_{b_k} \otimes |1\rangle_{c_k} - \sin\left(\frac{\pi \tau}{2}\right) |1\rangle_{a_k} \otimes |0\rangle_{b_k} \otimes |0\rangle_{c_k}
\equiv \cos\left(\frac{\pi \tau}{2}\right) |E_1\rangle_k - \sin\left(\frac{\pi \tau}{2}\right) |E_0\rangle_k
= |E_1\rangle_k - \frac{\pi \tau}{2} |E_0\rangle_k + O(\tau^2).
\]

\[
|E''_2\rangle_k = |0\rangle_{a_k} \otimes |1\rangle_{b_k} \otimes |0\rangle_{c_k} \equiv |E_2\rangle_k,
\]

\[
|E''_3\rangle_k = |0\rangle_{a_k} \otimes |1\rangle_{b_k} \otimes |1\rangle_{c_k} \equiv |E_3\rangle_k,
\]

\[
|E''_4(\tau)\rangle_k = \sin\left(\frac{\pi \tau}{2}\right) |0\rangle_{a_k} \otimes |0\rangle_{b_k} \otimes |1\rangle_{c_k} + \cos\left(\frac{\pi \tau}{2}\right) |1\rangle_{a_k} \otimes |0\rangle_{b_k} \otimes |0\rangle_{c_k}
\equiv \cos\left(\frac{\pi \tau}{2}\right) |E_4\rangle_k + \sin\left(\frac{\pi \tau}{2}\right) |E_1\rangle_k
= |E_4\rangle_k + \frac{\pi \tau}{2} |E_1\rangle_k + O(\tau^2).
\]

\[
|E''_5\rangle_k = |1\rangle_{a_k} \otimes |0\rangle_{b_k} \otimes |1\rangle_{c_k} \equiv |E_5\rangle_k,
\]

\[
|E''_6\rangle_k = |1\rangle_{a_k} \otimes |1\rangle_{b_k} \otimes |0\rangle_{c_k} \equiv |E_6\rangle_k,
\]

\[
|E''_7\rangle_k = |1\rangle_{a_k} \otimes |1\rangle_{b_k} \otimes |1\rangle_{c_k} \equiv |E_7\rangle_k.
\]

Now we define (\( \tau \)-dependent generalizations of) the unitary operators according to the recipe (11) as follows:

\[
U'_k(\tau) = \sum_{\Lambda=0}^7 |E'_{\Lambda} (\tau)\rangle_k \langle E_{\Lambda}|_k,
\]

(16)

\[
U''_k(\tau) = \sum_{\Lambda=0}^7 |E''_{\Lambda} (\tau)\rangle_k \langle E'_{\Lambda} (\tau)|_k,
\]

(17)

and

\[
U_k(\tau) \equiv U''_k(\tau) \cdot U'_k(\tau) = \sum_{\Lambda,\Lambda'=0}^7 |E''_{\Lambda} (\tau)\rangle \langle E'_{\Lambda} (\tau) | E'_{\Lambda'} (\tau) \rangle \langle E_{\Lambda'}|
= \sum_{\Lambda=0}^7 |E''_{\Lambda} (\tau)\rangle \langle E_{\Lambda}|.
\]

(18)

We can easily confirm that for \( \tau = 1 \) the (explicitly) unitary operators (16), (17) and (18) act according to the rules (3), (6) and (7), respectively. For example, for \( U'_k (\equiv U'_k (1)) \) we
confirm that
\[
U'_k (\{q_k\}_{a_k} \otimes \{O\}_{b_k c_k}) = U'_k \left\{ (1 - q_k) |0\rangle_{a_k} + q_k |1\rangle_{a_k} \right\} \otimes \{O\}_{b_k c_k}
\]
\[
= \sum_{\Lambda=0} |E_\Lambda' (1)\rangle_k \langle E_\Lambda |_k [(1 - q_k) |E_0\rangle_k + q_k |E_4\rangle_k]
\]
\[
= (1 - q_k) |E_0' (1)\rangle_k + q_k |E_4' (1)\rangle_k = [(1 - q_k) |0\rangle_{a_k} + q_k |1\rangle_{a_k}] \otimes |H_1\rangle_{b_k c_k}
\]
\[
= |q_k\rangle_{a_k} \otimes |H_1\rangle_{b_k c_k},
\]
as expected (see (3)).

IV. PHYSICAL PICTURE

Now, let us determine a corresponding (infinitesimal) generator \(H'_k\) (“Hamiltonian”) for the unitary evolution operator \(U_k (\tau)\), i.e.
\[
U_k (\tau) = I_k - i\tau H_k + \mathcal{O}(\tau^2).
\]
To this end we will utilize expansions of the bases \(\{|E_\Lambda' (\tau)\rangle\}, \{|E_\Lambda'' (\tau)\rangle\}\) (in terms of the “time” parameter \(\tau\)) around the standard base \(\{|E_\Lambda\rangle\}\) given in (14) and (15).

To make a comparison to a known case (i.e. to the fermion squeezing operator [9, 18]), let us derive the form of the generator \(H'_k\) for the unitary transformation \(U'_k (\tau)\) (16). By virtue of (16), (14) and (20) (with unprimed quantities replaced by primed ones) we get
\[
U'_k (\tau) = I_k - i\tau \hat{H}'_k + \mathcal{O}(\tau^2).
\]
Introducing the identification:
\[
|0\rangle_{x_k} \langle 0|_{x_k} = \hat{x}_k \hat{x}_k^\dagger
\]
\[
|0\rangle_{x_k} \langle 1|_{x_k} = \hat{x}_k
\]
\[
|1\rangle_{x_k} \langle 0|_{x_k} = \hat{x}_k^\dagger
\]
\[
|1\rangle_{x_k} \langle 1|_{x_k} = \hat{x}_k^\dagger \hat{x}_k,
\]
for the modes \(x_k = a_k, b_k, c_k \ (k = 1, 2, \ldots, N)\), from (21) we obtain a representation of the operator \(\hat{H}'_k\) in the Fock space, i.e.
\[
\hat{H}'_k = \hat{b}'_k \hat{c}'_k + \text{H.c.},
\]
where “H.c.” means Hermitian conjugation. The operator (23) is known as a two-mode fermion squeezing operator, responsible for creation of fermion pairs in the framework of the Hawking effect (e.g., see Sect. 5.2 in [10]).
Let us now derive the generator $H_k$, and its Fock space counterpart $\hat{H}_k$, for the entire evolution $U_k(\tau)$. By virtue of (18), (15) and (20) we obtain

$$U_k(\tau) = I_k - i\tau \frac{i\pi}{2} (|E_1\rangle_k \langle E_4|_k - |E_4\rangle_k \langle E_1|_k) + O(\tau^2).$$

(24)

Implementing the identification (22) we get the corresponding Fock space generator ("Hamiltonian")

$$\hat{H}_k = i\frac{\pi}{2} \hat{a}^+_k \hat{b}_k \hat{b}^+_k \hat{c}^+_k + \text{H.c.}$$

(25)

One should note that the operator $\hat{H}_k$ is 4-linear, which should be contrasted with a bilinear structure of the squeezing operator (23) and a trilinear structure of the operator discussed in the context of the Hawking effect in [15]. According to (9) the total "Hamiltonian" is

$$\hat{H} = \sum_{k=1}^{N} \hat{H}_k$$

then.

Coming back to the global description of the BH unitary evolution, we would like to draw the reader’s attention to a possible interpretation, which is depicted in Fig. 1. Namely, Fig. 1 presents, in an intuitive way, the entire unitary process $U_k$ of qubit transfer from a BH to the outgoing radiation as a composition of the two processes, $U'_k$ and $U''_k$, i.e. the Hawking particle pair creation outside the BH and particle pair annihilation inside the BH, respectively. A Feynman-like diagram/line depicts the transfer of qubits as a kind of a tunneling phenomenon.
V. FINAL REMARKS

Primarily inspired by a recent paper of Osuga and Page [16], in particular by their Eq. (3.3), essentially the same as Eq. (3.3) in [1] (accidental coincidence of the numbers of the equations!), we have proposed a unitary toy model of BH evaporation, which is an extension of the model introduced in [3]. By virtue of the construction the model is explicitly unitary and it describes transport of qubits from a BH to the outgoing radiation. For interpretational simplicity and a more direct relation to particle language, we have decided to formulate our qubit model in terms of fermionic species. As a byproduct of our construction, for possible reference to other models of the Hawking effect and BH evaporation, besides the global evolution operator, we have determined its infinitesimal generator/version (“Hamiltonian”). In turn the global evolution, involving the Hawking creation as well as latter annihilation inside the BH, can intuitively be interpreted in terms of a tunneling phenomenon as depicted in Fig. [1].

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