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Nuclear Rings, Nuclear Spirals, and Mass Accretion to Black Holes in Disk Galaxies

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Abstract. We use two-dimensional hydrodynamic simulations to study the formation of nuclear rings and nuclear spirals and the associated mass inflow rates at the centers of barred galaxies. We find nuclear rings form by the centrifugal barrier that the inflowing gas cannot overcome. The size of nuclear rings depend on various galaxy properties such as the bar strength, the bar pattern speed, and the bulge central density: they are smaller in galaxies with a stronger or slower bar, and with a more centrally concentrated bulge. Even a very weak bar potential can induce nuclear spirals that eventually develop into shocks. In galaxies with high shear, nuclear spirals are tightly wound and the shocks are inclined, forming a circumnuclear disk. On the other hand, galaxies with low shear produce loosely wound spirals and perpendicular shocks, without forming a circumnuclear disk. The mass inflow rates driven by the nuclear spiral shocks are enough to account for the observed level of AGN activities in Seyfert galaxies.

1. Introduction
One of the characteristic features of barred-spiral galaxies is the presence of nuclear rings and nuclear spirals at their centers (e.g., [1]). Nuclear rings are usually bright in Hα, indicating that they are actively forming new stars [2]. Observations suggest that about 20% of local spirals host star-forming nuclear rings [3]. The star formation rate (SFR) in nuclear rings appears to depend on the bar strength such that the SFR is quite small (∼0.1 M⊙ yr⁻¹) in strongly-barred galaxies, while it varies widely (∼0.1 – 10 M⊙ yr⁻¹) in weakly-barred galaxies [4], [5], [6]. The ring size also appears to depend on the bar strength, with a strongly-barred galaxy tending to have a smaller ring [5]. However, it is not well understood how the bar strength affects the ring formation and the associated SFR.

Nuclear spirals found at the very centers of barred galaxies [7], [8], [9] are thought to be a channel for gas inflows to feed supermassive black holes (e.g., [10], [11]). They even exist in galaxies with weak bar-like or oval potentials (e.g., [12], [13]). Statistically, weakly barred galaxies tend to harbor tightly wound nuclear spirals, while they are preferentially loosely wound in galaxies with a strong bar [9]. The linear theory on a curvature instability suggests that the shape of nuclear spirals should also depend on shear in the background rotation [14]. Since observed nuclear spirals are nonlinear, the linear prediction has to be checked by direct numerical simulations.

In order to study how nuclear rings and spirals form and evolve, we have been running a series of two-dimensional numerical simulations in which a bar is modeled by a fixed gravitational potential. These studies extend the previous works [15], [16], [17], [18] to explore the effects of the
sound speed [19], magnetic fields [20], bar strength [21], star formation [22], [23], bulge density and pattern speed [24], and background shear [25]. Here, we present the highlights of these studies, focusing on the properties of nuclear rings and spirals formed in our simulations. We refer the reader to the papers mentioned above for technical details as well as more quantitative results.

2. Nuclear Rings

To study how the ring size depends on various galaxy parameters such as the bar strength, pattern speed, and bulge density, we consider an initially-uniform gaseous disk subject to a fixed bar potential that rotates rigidly about the galaxy center at a pattern speed $\Omega_b = \Omega_b \hat{z}$. The gas is taken to be isothermal with a sound speed of $c_s = 10 \text{ km s}^{-1}$. All the simulations are run in a frame corotating with the bar. The basic hydrodynamic equations we solve read

$$\left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \Sigma = -\Sigma \nabla \cdot \mathbf{v},$$  \tag{1}

$$\left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{v} = -c_s^2 \frac{\nabla \Sigma}{\Sigma} - \nabla \Phi_{\text{ext}} + \Omega_b^2 \mathbf{R} - 2\Omega_b \times \mathbf{v},$$  \tag{2}

where $\Phi_{\text{ext}}$ is the external gravitational potential.

In our models, $\Phi_{\text{ext}}$ comes from four components: a stellar disk modeled by a Kuzmin-Toomre disk, a stellar bulge, a stellar bar, and a central black hole represented by the Plummer sphere with mass $M_{\text{BH}}$. For the bulge, we take a Hubble profile

$$\rho(r) = \rho_{\text{bul}} \left(1 + \frac{r^2}{r_b^2}\right)^{-3/2},$$  \tag{3}

where $\rho_{\text{bul}}$ and $r_b$ is the central density and scale length of the bar, respectively. In our models, the bar is modeled by Ferrers prolate spheroids with density distribution

$$\rho(r, \phi, z) = \begin{cases} \rho_{\text{bar}} (1 - g^2)^n, & \text{for } g < 1, \\ 0, & \text{elsewhere}, \end{cases}$$  \tag{4}

where $\rho_{\text{bar}}$ is the central density and $n = 1$ measures the degree of central density concentration. In Equation (4), $g$ is defined by

$$g^2(x, y, z) = \frac{y^2}{a^2} + \frac{x^2}{b^2} + \frac{z^2}{b^2},$$  \tag{5}

where $a = 5 \text{ kpc}$ and $b (\leq a)$ denote the semimajor and semiminor axes of the bar, respectively. The bar aspect ratio is $R = a/b$, which we control as a free parameter.

We run the following three series of models:

1. In the first series of models, we vary the aspect ratio $R$ in the range between 1.5 and 3.5 and the bar mass fraction $f_{\text{bar}} = M_{\text{bar}}/(M_{\text{bar}} + M_{\text{bul}})$ in the range between 0.08 and 0.6, where $M_{\text{bar}}$ and $M_{\text{bul}}$ denote the mass of the bar and bulge inside $r = 10 \text{ kpc}$, respectively. We fix the bar pattern speed to $\Omega_b = 33 \text{ km s}^{-1} \text{ kpc}^{-1}$ and the bulge central density to $\rho_{\text{bul}} = 10^{10} \text{ M}_\odot \text{ kpc}^{-3}$. The variations of the bar mass and the aspect ratio change the bar strength defined by

$$Q_b = \max_{r,\phi} \frac{F_T(r, \phi)}{F_R(r)},$$  \tag{6}
Figure 1. Azimuthally- and temporally-averaged ring radius $R_{\text{ring}}$ as a function of the bar strength $Q_b$. The symbols and errorbars give the mean values and the standard deviations.

where $F_T$ is the tangential force due to the non-axisymmetric bar potential and $F_R$ is the centrifugal force of the galaxy rotation (e.g., [26], [27], [28], [29]). Numerically, our galaxy models have

$$Q_b = \begin{cases} 0.58 f_{\text{bar}}^{0.89} (R - 1), & \text{for } n = 0, \\ 0.44 f_{\text{bar}}^{0.87} (R - 1), & \text{for } n = 1, \\ 0.38 f_{\text{bar}}^{0.70} (R - 1), & \text{for } n = 2, \end{cases}$$

[7]. These models allow to study how the bar strength affects the size of nuclear rings that form.

(2) In the second series of models, we vary $\Omega_b$ between 21 and 49 km s$^{-1}$ kpc$^{-1}$, while fixing $\rho_{\text{bul}} = 10^{10} \, M_\odot$ kpc$^{-3}$, and $R = 2.5$, $M_{\text{bar}} = 1.5 \times 10^{10} \, M_\odot$ corresponding to $f_{\text{bar}} = 0.3$. This series of models is useful to isolate the effect of the bar pattern speed on the ring formation.

(3) In the third series of models, we vary $\rho_{\text{bul}}$ from 1.2 to $4.0 \times 10^{10} \, M_\odot$ kpc$^{-3}$ by fixing $M_{\text{bul}} = 3.37 \times 10^{10} \, M_\odot$ and $\Omega_b = 33$ km s$^{-1}$ kpc$^{-1}$ to study the effect of the bulge central density (or compactness) on the ring size.

We use the CMHOG code to run the first series of models [21], while the simulations for the second and third series of models [24] utilize the Athena code. CMHOG is third-order accurate in space, has very little numerical diffusion, and solves the basic hydrodynamic equations in cylindrical geometry [18]. On the other hand, Athena is based on a higher-order Godunov scheme in Cartesian geometry that conserves mass and momentum within machine precisions [30], [31], [32]. In all models, we turn on the bar potential slowly over one bar revolution time $2\pi/\Omega_b$ in order to minimize transients in the flows caused by a sudden introduction of the bar potential.

2.1. Effects of the Bar Strength

The imposed non-axisymmetric bar potential provides perturbations for the gas that would otherwise follow circular orbits. The perturbations are strong enough to induce shocks, called
Figure 2. Azimuthally- and temporally-averaged ring radius $R_{\text{ring}}$ as functions of (a) the bar pattern speed $\Omega_b$ and (b) the bulge central density $\rho_{\text{bul}}$. The filled circles mark the mean values, with the errorbars corresponding to ring thickness.

dust lanes, located at the downstream side of galaxy rotation from the bar semimajor axis. Gas passing through the shocks loses angular momentum and thus experiences radial infalls. The inflowing gas tends to rotate faster as it moves radially inward, eventually forming a nuclear ring at the position where the inflowing gas achieves the initial velocity of galaxy rotation. The shape of the dust-lane shocks is well described by $x_1$ orbits, while nuclear rings closely follow $x_2$ orbits. This implies that the formation of $x_2$-type rings is caused by the centrifugal barrier that the inflowing gas cannot overcome [19], [20], [21].

Figure 1 plots the the azimuthally- and temporally-averaged ring radius $R_{\text{ring}}$ as a function of $Q_b$ for the first series of models [21]. It is apparent that $R_{\text{ring}}$ becomes smaller with increasing $Q_b$, entirely consistent with the observational results that a stronger bar hosts a smaller nuclear ring [5]. This is because a stronger bar induces stronger shocks and can thus take away a larger amount of angular momentum, making the gas move radially further in and form a ring closer to the center. In general, $R_{\text{ring}}$ in our models is smaller than the location of the inner Lindblad resonance (ILR), suggesting that the ring position is not determined by resonances with the bar potential but by the amount of angular momentum loss at dust-lane shocks.

2.2. Effects of the Pattern Speed

Figure 2(a) plots the azimuthally- and temporally-averaged ring radius $R_{\text{ring}}$ as a function of $\Omega_b$ for $x_2$-type rings obtained from the second series of models [24]. Models with $\Omega_b > 41$ km s$^{-1}$ kpc$^{-1}$ form $x_1$-type rings elongated along the bar semimajor axis. Apparently, the ring size decreases with increasing $\Omega_b$ almost linearly. This is presumably because the only the gas inside the corotation resonance loses angular momentum to move in, and corotation radius $R_{\text{CO}}$ is a decreasing function of $\Omega_b$. In addition, gas in models with smaller $\Omega_b$ takes longer time to be exposed to the full bar strength, so that it experiences a weaker bar torque at early time when a ring is beginning to form. The change of $R_{\text{ring}}$ over $\Omega_b = 21 - 41$ km s$^{-1}$ kpc$^{-1}$ is by less than a factor of 3.
2.3. Effects of the Bulge Central Density

Figure 2(b) plots the variation of $R_{\text{ring}}$ as a function of $\rho_{\text{bul}}$ for $x_2$-type rings obtained from the third series of models [24]. Models with $\rho_{\text{bul}} < 1.8 \times 10^{10} \text{M}_\odot \text{kpc}^{-3}$ produce $x_1$-type rings. Note that $R_{\text{ring}}$ is an increasing function of $\rho_{\text{bul}}$. Physically, this is because models with larger $\rho_{\text{bul}}$ rotates faster and can thus provide the required centrifugal barrier for the inflowing gas at larger radii. While $R_{\text{ring}}$ varies by a factor of about 2 over the variation of $\rho_{\text{bul}} = 1.8$ to $4.0 \times 10^{10} \text{M}_\odot \text{kpc}^{-3}$, $R_{\text{ring}}$ is insensitive to $\rho_{\text{bul}} \gtrsim 3 \times 10^{10} \text{M}_\odot \text{kpc}^{-3}$.

3. Nuclear Spirals and Gas Accretion

We now turn to nuclear spirals formed at the central parts of weakly barred galaxies. To study the effect of shear on the shape of nuclear spirals, we consider two galaxy models with rotational velocities

$$V = \begin{cases} 
65 + 95 \tanh \left( \frac{R - 0.07}{0.06} \right) - 50 \log R + 1.5(\log R + 3)^3, & \text{for MIL model}, \\
\left[ \frac{GM_{\text{BH}}}{R} + \left( \frac{V_0 R}{R_0 + R} \right)^2 \right]^{1/2}, & \text{for GAL model}, 
\end{cases}$$

where $V$ and $R$ for the MIL model are in units of km s$^{-1}$ and kpc, respectively, and $V_0 = 220$ km s$^{-1}$, $R_0 = 0.3$ kpc, and $M_{\text{BH}} = 3 \times 10^6 \text{M}_\odot$ for the GAL model. The MIL model is designed to simulate galaxies like the Milky Way with strong shear at the centers, while the GAL model is for galaxies with low shear, like NGC 3041 [33].

Figure 3 plots the radial distributions of $V$ as well as the angular frequencies $\Omega = V/R$ and $\Omega - \kappa/2$, where $\kappa^2 = R^{-2}d(\Omega^2 R^4)/dR$ is the epicycle frequency. The MIL model has strong shear near $R \sim 0.1$ kpc, while shear is overall weak in the GAL model. The gas is taken to be initially uniform with surface density $\Sigma_0$ and isothermal with a sound speed of $c_s = 10$ km s$^{-1}$. 

Figure 3. Radial profiles of (a) the rotational velocity $V$ and (b) the angular frequencies $\Omega$ (solid) and $\Omega - \kappa/2$ (dotted) of the MIL (red) and GAL (blue) models. The thin horizontal lines in (b) indicate the bar pattern speed $\Omega_b = 60$ and $30 \text{km s}^{-1} \text{kpc}^{-1}$ for the MIL and GAL models, respectively.
Figure 4. Snapshots of the gas surface density $\Sigma$ at $t = 0.3\,\text{Gyr}$ (upper panels) and $t = 1.3\,\text{Gyr}$ (lower panels) of the high-shear MIL model with $\Phi_0/c_s^2 = 0.1$. The circles in the left panels mark the ILR. The right panels zoom in the central 0.15 kpc regions. The dotted circles with radius 80 pc draw the regions influenced by shocks. Colorbars label $\log(\Sigma/\Sigma_0)$.

For the perturbing potential representing a weak bar or an oval distortion, we take a simple bi-symmetric form

$$\Phi_b(R, \phi, t) = \Phi_0 \cos(2\phi - 2\Omega_b t), \quad (9)$$

with amplitude $\Phi_0$ and pattern speed $\Omega_b$. To study the effect of the bar strength, we consider two models with $\Phi_0/c_s^2 = 0.1$ and 0.01. The bar pattern speed is taken to be $\Omega_b = 60$ and 30 km s$^{-1}$ kpc$^{-1}$ for the MIL and GAL models, respectively, which are indicated as thin horizontal lines in Figure 3(b). The corresponding ILR and corotation resonance are at $R_{\text{ILR}} = 1.00\,\text{kpc}$ and $R_{\text{CO}} = 3.32\,\text{kpc}$ in the MIL model, and at $R_{\text{ILR}} = 1.42\,\text{kpc}$ and $R_{\text{CO}} = 7.03\,\text{kpc}$ in the GAL model, respectively. We use Athena++, a newly developed grid-based code based on a higher-order Godunov scheme to evolve the gas in two-dimensional cylindrical geometry. The simulation domain extends from $R_{\text{in}} = 10\,\text{pc}$ to $R_{\text{out}} = R_{\text{CO}}$.

3.1. Density Structure

The imposed gravitational potential induces waves in the otherwise uniform disk that are gradually organized into a piecewise logarithmic spiral shape. The spiral waves grow as thermal
Figure 5. Same as Figure 4, but for the low-shear GAL model with $\Phi_0/c_s^2 = 0.1$. The radius of the dotted circles is 100 pc.

pressure align the apocenters of perturbed gas orbits inside the ILR [14]. They amplify further to become nonlinear as they propagate inward due to a geometric effect, eventually developing into shocks even for very weak potentials. Figures 4 and 5 plot the distributions of the gas surface density at $t = 0.3$ Gyr (upper panels) and 1.3 Gyr (lower panels) of the MIL and GAL models with $\Phi_0/c_s^2 = 0.1$, respectively. Clearly, the spirals exist only inside the ILR, denoted by the solid circles in both models, while the regions outside the ILR are almost featureless.

Comparison of Figures 4 and 5 reveals the following notable differences. First, the shape of the spirals depends sensitively on the background shear: spirals are tightly wound (with a pitch angle of $i_p \sim 10^\circ$) in the high-shear MIL model, while they are loosely wound (with $i_p \sim 35^\circ$) in the low-shear GAL model. Second, shocks are inclined in the MIL model, resulting in relatively small perpendicular Mach numbers of $M_{\perp} \sim 1.5$. On the other hand, the shocks in the GAL model are less inclined and unwind over time to be perpendicular, presumably due to an increase of the angular momentum flux [34]. The resulting perpendicular Mach numbers are $M_{\perp} \sim 2.5$. Third, the shocks take away angular momentum from the gas that encounters them, causing mass inflows. In the MIL model, the inflowing gas moves on more-or-less circular orbits, piles up near the center due to the geometric convergence effect, and forms a circumnuclear disk with radius of $\sim 20–30$ pc. In the GAL model, on the other hand, the inflowing gas after
Figure 6. Mass inflow rates $\dot{M}$ measured at the inner radial boundary $R_{\text{in}} = 10$ pc for the MIL (red) and GAL (blue) models. The solid and dotted lines correspond to the models with $\Phi_0/c_s^2 = 0.1$ and 0.01, respectively.

3.2. Mass Inflows

Figure 6 plots the mass inflow rates $\dot{M}$, normalized by $\Sigma_0$, measured at the inner radial boundary $R_{\text{in}} = 10$ pc of the simulation domain. The solid and dotted lines correspond to the models with $\Phi_0/c_s^2 = 0.1$ and 0.01, respectively. Overall, the low-shear GAL models have larger $\dot{M}$ than the high-shear MIL models since the shocks are stronger and perpendicular in the former. The mass inflow rate is approximately proportional to the strength of the imposed bar potential. The time-averaged mass inflow rates in our simulations can be written as

$$\langle \dot{M} \rangle = 10^{-5} f \left( \frac{\Sigma_0}{M_\odot \text{ yr}^{-1}} \right) \left( \frac{\Phi_0/c_s^2}{0.1} \right) M_\odot \text{ yr}^{-1},$$

(10)

where $f$ is a factor, of order unity, responsible for the effect of background shear [25]. Assuming that all the inflowing gas is accreted to a central black hole with mass $M_{\text{BH}}$, the corresponding Eddington ratio is

$$\lambda = 1.5 \times 10^{-2} f \left( \frac{\epsilon}{0.1} \right) \left( \frac{\Sigma_0}{10^2 M_\odot \text{ yr}^{-1}} \right) \left( \frac{\Phi_0/c_s^2}{0.1} \right) \left( \frac{M_{\text{BH}}}{3 \times 10^6 M_\odot} \right),$$

(11)

where $\epsilon \sim 0.1$ is the efficiency of an active galactic nucleus (AGN). Since Seyfert galaxies have $\lambda \sim 10^{-3} - 10^{-1}$ [35], [36], [37], this suggests that gas inflows driven by nonlinear nuclear spirals can account for observed levels of AGN activity in Seyfert galaxies.

4. Summary

Using two-dimensional hydrodynamic simulations, we have investigated the formation of nuclear rings and spirals and the associated mass inflow rates at galaxy centers. Our results can be summarized as follows.
(1) Nuclear rings form not by resonances but by the centrifugal barrier that the inflowing gas driven by the bar potential cannot overcome. This predicts that nuclear rings in more strongly barred galaxies are smaller in size, consistent with the results of our simulations and also with observations.

(2) The size of nuclear rings depend not only on the bar strength but also on the bar pattern speed and the bulge central density. Nuclear rings are larger in models with smaller pattern speed and/or larger bulge central density. This suggests that one should be careful in inferring the galaxy properties from the sizes of observed nuclear rings.

(3) Nuclear spirals exist even in very weekly barred galaxies. They amply due to the geometric convergence effect and eventually evolve into shocks, causing radial mass inflows. In high-shear models, nuclear spirals tend to be tightly wound and shocks are inclined. This results in relatively small rates of the mass inflows and the formation of a circumnuclear disk. In low-shear models, on the other hand, nuclear spirals are loosely wound and shocks are perpendicular, resulting in a comparatively large rate of the mass inflows and no formation of a circumnuclear disk.

(4) Even under a very-weak external potential, the mass inflow rates driven by nonlinear nuclear spirals are sufficient to power the observed level of AGN activity in diverse Seyfert galaxies.

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