On the variance of the nodal volume of arithmetic random waves

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We discuss arithmetic random waves on the \(d\)-dimensional torus \(\mathbb{R}^d/\mathbb{Z}^d\). Their zero set and its volume are related to the study of linear correlations of lattice points on the sphere of radius \(\sqrt{n}\) in \(\mathbb{R}^d\) \((n \geq 2)\). In this talk we look for bounds on the variance of the nodal volume. The problem has been solved (in the stronger form of an asymptotic with power saving) in dimension \(d = 2, 3\) by using a combination of number theory and graph theory. In this seminar we will explain what is known in dimension \(d \geq 4\). As the dimension increases, analysis gives the best results. The main input is a result that follows from the proof of the \(L^2\)-decoupling conjecture by Bourgain and Demeter.