Partial teleportation of entangled atomic states

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In this paper we propose a scheme for partially teleporting entangled atomic states. Our scheme can be implemented using only four two-level atoms interacting either resonantly or off-resonantly with a single cavity-QED. The estimation of losses occurring during this partial teleportation process is accomplished through the phenomenological operator approach technique.

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INTRODUCTION

Quantum teleportation, first suggested by Bennett et al. [1], is one of the cornerstones of quantum information and computation [2, 3, 4]. The crucial ingredient characterizing this phenomenon is the transfer of information between noninteracting systems at the expense of a quantum channel. This issue has received great attention since its pioneer proposal, mainly after its experimental realizations from 1997 onwards [5–8]. In the meantime, various proposals have been suggested for implementing teleportation, for instance, teleportation of trapped wave fields [16, 17, 18], teleportation of running wave fields [10–15], teleportation of running wave fields in the state that we want to teleport; ii) an entangled state of particles in the state we want to teleport; iii) a joint measurement on particles previously entangled with atom 2 when the atom-field interaction is resonant, and (12)

\[
|\phi\rangle_{12} = C_0 |g\rangle_1 |e\rangle_2 + C_1 |e\rangle_1 |g\rangle_2 ,
\]

where \(C_0\) and \(C_1\) are unknown coefficients obeying \(|C_0|^2 + |C_1|^2 = 1\), and \(|g\rangle\) (|e\rangle) is the atomic ground (excited) state. The state (12) can be prepared, for instance, by the method presented in Ref. [28], where two two-level atoms interact simultaneously with a single mode of a cavity-field.

The Hamiltonian describing the atom-field interaction, in the interaction picture, is

\[
H_I = \hbar \lambda a^\dagger \sigma^- + a \sigma^+ ,
\]

when the atom-field interaction is resonant, and

\[
H_I = \frac{\hbar \lambda^2}{\delta} a^\dagger a \sigma^+ \sigma^- ,
\]

when the atom-field interaction is off-resonant. This condition is valid provided that \(\pi \lambda^2 \ll \delta^2 + \gamma^2\), where \(\pi\) is the mean photon number and \(\gamma\) is the damping rate for the cavity-field. Here \(a^\dagger\) and \(a\) are the creation and annihilation operators for the cavity field mode, \(\sigma^+\) and \(\sigma^-\) are the raising and lowering operators for the atom, \(\lambda\) is the atom-field coupling constant, and \(\delta = \omega - \omega_0\) is the detuning between the cavity field frequency \(\omega\) and the atomic frequency \(\omega_0\).

To compose the nonlocal channel, a third atom (initially prepared in the excited state \(|e\rangle_{3}\)) interacts resonantly with the cavity field mode \(A\) in vacuum state \(|0\rangle_A\), according to Eq. (2) (see Fig. 1a). Adjusting the atom-field interaction time to \(t = \pi/4\lambda\), the nonlocal channel will be given by

\[
|\psi\rangle = \frac{1}{\sqrt{2}} (|e\rangle_3 |0\rangle_A - i |g\rangle_3 |1\rangle_A ) .
\]
At this point, Alice has the atom 1 and the cavity, while atoms 2 and 3 are sent to Bob. The state of the whole system composed by the two-level atoms and the cavity mode field is

$$|\psi\rangle_{\text{total}} = \frac{1}{2} \left[ |\psi^+\rangle_{1A} (C_0 |e\rangle_3 |e\rangle_2 + C_1 |e\rangle_3 |g\rangle_2) + |\psi^-\rangle_{1A} (C_0 |e\rangle_3 |e\rangle_2 - C_1 |e\rangle_3 |g\rangle_2) + |\Phi^+\rangle_{1A} (C_0 |e\rangle_3 |e\rangle_2 - C_1 |g\rangle_3 |g\rangle_2) + |\Phi^-\rangle_{1A} (C_0 |e\rangle_3 |e\rangle_2 + C_1 |g\rangle_3 |g\rangle_2) \right],$$

(5)

where, for convenience, we have defined the Bell states $|\psi\rangle_{1A}$ and $|\Phi\rangle_{1A}$ as

$$|\psi\rangle_{1A} = \frac{1}{\sqrt{2}} (|g\rangle_1 |1\rangle_A + |e\rangle_1 |0\rangle_A),$$

(6)

$$|\Phi\rangle_{1A} = \frac{1}{\sqrt{2}} (|g\rangle_1 |0\rangle_A + |e\rangle_1 |1\rangle_A).$$

(7)

As in the OP, the teleportation is completed after Alice measuring on particle 1 and cavity $A$ and sending her result to Bob, whom will know which unitary operation to accomplish on its particles in order to recover the entangled state that Alice wanted to teleport. Note, however, that different from the OP, when comparing the teleported state resulting from Eq. (5) with the state to be teleported, Eq. (4), we see that partner 1 was replaced by particle 3. This explains the “partial teleportation” term used. The experimental setup is shown, step by step, in Fig. 2f. Here, we show how Alice must proceed to perform the Bell state measurements.

**Bell State Measurements**

First, atom 1 crosses a Ramsey zone $R$, adjusted to produce the following evolutions

$$|e\rangle \rightarrow \frac{1}{\sqrt{2}} (|g\rangle + |e\rangle)$$

(8)

and

$$|g\rangle \rightarrow \frac{1}{\sqrt{2}} (|g\rangle - |e\rangle).$$

(9)

Then, atom 1 crosses the cavity interacting off-resonantly with mode $A$, with the interaction time adjusted to $\chi t = \pi$ (with $\chi = \chi^2/\delta$), resulting in the evolutions $|g\rangle_1 |0\rangle_A \rightarrow |g\rangle_1 |0\rangle_A$, $|g\rangle_1 |1\rangle_A \rightarrow |g\rangle_1 |1\rangle_A$, $|e\rangle_1 |0\rangle_A \rightarrow |e\rangle_1 |0\rangle_A$, and $|e\rangle_1 |1\rangle_A \rightarrow |e\rangle_1 |1\rangle_A$. Next, atom 1 crosses another Ramsey zone $R'$ adjusted like the Ramsey zone $R$ (see Eqs. (8,9)). As a consequence, the states of the Bell basis evolve as

$$|\psi\rangle_{1A} \rightarrow \frac{1}{\sqrt{2}} |g\rangle_1 |1\rangle_A \pm |0\rangle_A),$$

(10)

$$|\Phi\rangle_{1A} \rightarrow \frac{1}{\sqrt{2}} |e\rangle_1 |0\rangle_A \pm i |1\rangle_A).$$

(11)

Fig. 1b shows the passage of the atoms 1 and 2 in the schematic setup. By selective atomic state detection on atom 1 it is possible to know if the joint state is $|\psi\rangle_{1A}$ or $|\Phi\rangle_{1A}$. Next, we have to discern the phases ($\pm$) of the Bell states. With the Ramsey zone $R$ turned off, another two-level atom (atom 4) in the ground state $|g\rangle_4$ is sent through the cavity to interact resonantly with mode $A$ (see Fig. 1c) as indicated by Eq. (2), with the interaction time $t = \pi/2\lambda$, thus resulting in the following evolutions: $|g\rangle_4 |0\rangle_A \rightarrow |g\rangle_4 |0\rangle_A$ and $|g\rangle_4 |1\rangle_A \rightarrow -i |e\rangle_4 |0\rangle_A$. Next, atom 4 crosses the Ramsey zone $R'$ (according to Eqs. (8,9)) such that the Bell-states are written as

$$|\psi\rangle_{1A} |g\rangle_4 \rightarrow \left\{ \begin{array}{ll} |g\rangle_1 |e\rangle_4 |0\rangle_A & \text{if (+)} \\ |g\rangle_1 |e\rangle_4 |0\rangle_A & \text{if (–)} \end{array} \right.,$$

(12)

$$|\Phi\rangle_{1A} |g\rangle_4 \rightarrow \left\{ \begin{array}{ll} |e\rangle_1 |g\rangle_4 |0\rangle_A & \text{if (+)} \\ |e\rangle_1 |g\rangle_4 |0\rangle_A & \text{if (–)} \end{array} \right..$$

(13)

Thus, by measuring atom 4 we will be able to distinguish between the phase (–) or (+). The perfect discrimination be-
TABLE I: Results of the teleportation scheme. BSM denotes the resulting measurement on atom 1 and field mode A. Unitary operation denotes the required operation by Bob after Alice communicating her results. The $\sigma_j$ is the Pauli operator $\sigma_j$ acting on atom 3.

| BSM | $|\psi\rangle_{AB}$ | Unitary operation |
|-----|----------------|------------------|
| $|\Psi^+\rangle_{1A}$ | $C_0|g\rangle_3|e\rangle_2 + C_1|e\rangle_3|g\rangle_2$ | $I_3 \otimes I_2$ |
| $|\Psi^+\rangle_{1A}$ | $C_0|g\rangle_3|e\rangle_2 - C_1|e\rangle_3|g\rangle_2$ | $\sigma_3x \otimes I_2$ |
| $|\Phi^+\rangle_{1A}$ | $C_0|e\rangle_3|g\rangle_2 - C_1|g\rangle_3|e\rangle_2$ | $\sigma_3y \otimes I_2$ |
| $|\Phi^-\rangle_{1A}$ | $C_0|e\rangle_3|e\rangle_2 + C_1|g\rangle_3|g\rangle_2$ | $\sigma_3x \otimes I_2$ |

DECAY OF THE FREE ATOMIC EXCITED STATE

Phenomenological operator approach (POA)

Here we observe that the coupling of the atomic states to a surrounding environment $\mathcal{E}$ can be described by the relations

$$|g\rangle|\mathcal{E}\rangle \xrightarrow{U_t} |g\rangle\hat{T}_0|\mathcal{E}\rangle, \quad (14)$$

$$|e\rangle|\mathcal{E}\rangle \xrightarrow{U_t} |e\rangle\hat{T}_e|\mathcal{E}\rangle + |g\rangle\hat{T}_g|\mathcal{E}\rangle, \quad (15)$$

where $|\mathcal{E}\rangle$ denotes the initial state of the environment, the operators $\hat{T}$, acting on this state, account for the atom-environment coupling, and $U_t$ denotes an unitary operation mixing the atom to its environment. We will assume the environment $|\mathcal{E}\rangle$ in the vacuum state, which is an excellent approximation for high-Q cavities in the microwave domain. Accordingly, we assume that $\hat{T}_0 = 1$, $\hat{T}_e = \hat{f}(t) = e^{-\kappa t}1$, $\hat{T}_g = \sum_j g_j(t)\hat{b}_j$, with $\sum_j |g_j(t)|^2 = 1 - e^{-2\kappa t}$, $\kappa$ denoting the spontaneous decay rate of the atomic excited state, $I$ is the identity operator, $\hat{b}_j$ is the creation operator, having a corresponding annihilation operator $\hat{b}_j$, of the $j$th oscillator mode of the environment, and $t$ is the time elapsing after the atom suffering a given excitation. With these assumptions, it is straightforward to verify that the superposition $|\langle g\rangle + |e\rangle\rangle / \sqrt{2}$ leads to the reduced density operator

$$\rho = \frac{1}{2}\{\exp(-\kappa t)|e\rangle\langle e| + [2 - \exp(-\kappa t)]|g\rangle\langle g| + \exp(-\kappa t)|e\rangle\langle g| + |g\rangle\langle e|\}. \quad (16)$$

Note that the evolution (14) and (15) are consistent with the well-known result that an unstable atomic state decays exponentially. In this case, the phenomenological-operator evolution leads to the same atomic density operator as the one we obtain using an ab-initio master equation approach. Moreover, due to recent advances in high-Q cavities, we will neglect the damping time of the mode $A$.

Decay of the teleported state

To estimate the losses, we assume the whole state starting to decay after the preparation of the quantum channel. In the first step, the phenomenological operators used to introduce damping effects, Eqs. (14) and (15), are applied to the whole system until the time $t$. Then, for each excitation suffered by the atoms during the teleportation process, a new phenomenological operator is included, which modifies the decay probability of the atomic states, and as a consequence, the fidelity of the whole teleportation process. Summarizing the applications of the phenomenological operators from the beginning, i.e., since the preparation of the quantum channel until the end of our teleportation protocol, which occurs at the instant that the fourth atom is detected, we have to apply them soon after i) the atom 1 crossing the first Ramsey zone ($t_1$); ii) the atom 1 crossing the second Ramsey zone ($t_2$); iii) the atom 4 interacting resonantly with the mode field cavity ($t_3$); iv) the atom 4 crossing the Ramsey zone ($t_4$). After the inclusion of the decay via POA, the state of the whole system by the time the teleportation is concluded becomes a mixture, being represented by a reduced operator density when the reservoir is traced out. In our estimation, we take the case of the teleported state in Bob hands when Alice measures the Bell state $|\Psi^+\rangle_{1A} (|g\rangle_1|e\rangle_2)$. The corresponding fidelity is shown in Fig. ??.

In fact, taking $t_1 = 2\mu s$, as reported in [31], we will have $t_2 \approx 5 \times 10^{-4}s + 2\mu s + t_1$, which is the necessary time for the atom 1 to interact dispersively with the cavity field and to cross the Ramsey zone, $t_3 = 10^{-4}s + t_2$, which is the necessary time for the atom-field resonant interaction, and $t_4 = 2\mu s + t_3 \approx 6, 06 \times 10^{-4}s$, which is much shorter than the atomic decay $\kappa^{-1} \approx 10^{-2}s$, being the fidelity at this time 0.99 as can be seen from Fig. ??.

Moreover, as the time goes on, the decay becomes faster and the fidelity is reduced to $2/3$ at the instant $t_f = 5, 78 \times 10^{-3}s$. Therefore, the effective time during which the teleported state is at our disposal for further operations is $t_f - t_4 = 5, 17 \times 10^{-3}s$.
state of a particle
ice shares with Bob a nonlocal channel composed by the joint
Alice performing a Bell measurement on the states of particles
cavity) and a particle
considering up to
fidelity does not suffer a significant modification when con-
several other proposals have appeared, modifying slightly or
[18]. In our scheme, Alice has an atomic state to be teleported,
plored a kind of teleportation named by
The plot of the fidelity is shown in Fig.
C
(b) The behavior of the fidelity for the fixed values of the coefficients
state and the value of the coefficients of the state to be teleported.
the fidelity and its dependence with both the life-time of the atomic
FIG. 2: Decay effects of the teleported state. In (a) The behavior of
the cavity. We assume that \(f_j(t_j; \tilde{t}_j)\) is a Gaussian distribution
centered around \(\tilde{t}_j\), e.g.,

\[
f_j(t_j; \tilde{t}_j) = \frac{1}{\Delta_j \sqrt{2\pi}} \exp \left(-\frac{(t_j - \tilde{t}_j)^2}{2\Delta_j^2}\right),
\]

where \(\Delta_j = x\tilde{t}_j\) and \(x\) is a parameter related to the uncertainty in the atomic velocity (around 0.5% according to recent experiments [33]), and therefore in the requested interaction times \(\tilde{t}_j\). Thus, the density operator of the system including the fluctuation effect is written as

\[
\rho = \int \int \int_{-\infty}^{\infty} \left[ \prod_{j=1}^{N} f_j(t_j; \tilde{t}_j) dt_j \right] |\psi\rangle_{total} \langle \psi|.
\]

Here, for simplicity, we consider \(N = 1, 2, 3\) to describe the three interactions between atoms 1 and 4 and the cavity. Following the steps in Ref. [32], we obtain the fidelity given by

\[
F = N^2 \left[ \frac{1}{2} C_0^4 \left( e^{3/2 x^2 \pi^2} + e^{1/2 x^2 \pi^2} + 2 e x^2 \pi^2 \right) e^{-3/2 x^2 \pi^2} + (1 - C_0^2) (2 - 2 C_0^2) 
- 2 C_0^2 \left( -e^{1/2 x^2 \pi^2} - 1 + C_0^2 e^{1/2 x^2 \pi^2} + C_0^2 \right) e^{-3/4 x^2 \pi^2} \right],
\]

with

\[
N = \left( \sqrt{2 C_0^2 e^{-1/2 x^2 \pi^2} + 3 - 2 C_0^2 - e^{-1/2 x^2 \pi^2}} \right)^{-1}.
\]

The plot of the fidelity is shown in Fig. ?? . Note that the fidelity does not suffer a significant modification when considering up to 3% of uncertainty in the interaction time.

**COMMENTS AND CONCLUSION**

Since the teleportation protocol by Bennett et al. [1], several other proposals have appeared, modifying slightly or substantially the original protocol. In this paper we have explored a kind of teleportation named by partial teleportation [18]. In our scheme, Alice has an atomic state to be teleported, given by an entanglement of particles 1 and 2. Besides, Alice shares with Bob a nonlocal channel composed by the joint state of a particle \(A\) (represented by a single mode of a high Q cavity) and a particle 3 (represented by an atomic state). After Alice performing a Bell measurement on the states of particles \(A\) and 1, and informing Bob her result, the following interesting result emerges, after the usual rotation by Bob: particle 3 takes exactly the role of particle 1 in the entanglement addressed to Alice, but in Bob location. As the entanglement between the particles 1 and 2 is broken and a new entanglement between the particles 3 and 2 is created in a different place, this characterizes a partial teleportation. Note that different from Ref. [26], in our scheme the teleportation occurs in only one particle of the entangled pair. To estimate losses occurring during and after the teleportation process, we have used the phenomenological operator approach (POA), as introduced in Ref. [27]. The fluctuation effect in the atom-field interaction time due to the uncertainty in atomic velocities was also considered, showing a small variation in the fidelity. Taking experimental parameters from recent experiments in QED-cavity [30], we believe that this scheme can experimentally be accomplished using nowadays technology.
FIG. 3: Fidelity of the teleported state considering the fluctuation effects in the atom-field interaction time. $C_0$ is the coefficient to be teleported and $x$ is a parameter of uncertainty in the interaction time due to the uncertainty in atomic velocities.

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