Minority language and the stability of bilingual equilibria

Nagore Iriberri
Universitat Pompeu Fabra, Catalonia-Spain

José-Ramón Uriarte
University of the Basque Country, Basque Country-Spain

Abstract
We investigate a society with two official languages: A, shared by all individuals, and B, spoken by a bilingual minority. A model is developed in which the bilingual agents must make strategic decisions about the language to be used in a conversation. The decisions are taken under imperfect information about the linguistic type of the participants. We show that the bilingual population is optimally partitioned into two groups, one composed of agents who strategically hide their bilingual nature and the other composed of those who reveal it. As a consequence, in the interactions between members of the former group the language used is A, having therefore coordination failures on the minority language. We show that this mixed strategy Nash equilibrium has strong stability properties: it is evolutionarily stable and, dynamically, asymptotically stable for the one-population replicator dynamics. These properties might explain the difficulties encountered by language policies directed at promoting the use of minority languages.

JEL Classification Numbers: C72, D81.

Keywords
Imperfect information, language competition, majority/minority language, Nash equilibrium, replicator dynamics

Corresponding author:
José-Ramón Uriarte, University of the Basque Country, Avenida del Lehenakari Aguirre, 83 Bilbao 48015 Basque Country-Spain.
Email: jr.uriarte@ehu.es
**Introduction**

Abrams and Strogatz (2003) proposed a model for the dynamics of language death which has triggered a burst of research on language competition and diversity (e.g. Castelló et al., 2006; Mira and Paredes, 2005; Patriarca and Leppänen, 2004; Pinasco and Romanelli, 2006; Stauffer et al., 2007; Stauffer and Schulze, 2005; Wang and Minnet, 2005, 2008). The model describes two languages that compete with each other for speakers and predicts that there is no stable coexistence of the two languages; one will eventually drive the other to extinction. As noted by Wang and Minnet (2005), a significant weakness of this model is that only monolingual speakers of $A$ and $B$ are taken into account.

Bilingual societies, however, do exist and, in most of them, what we find is an asymmetric language competition. Typically, in these societies there are two languages: one majority language, denoted $A$, which is spoken by all individuals in the society, and another, denoted $B$, which is spoken by a bilingual minority (the $A$ and $B$ speakers). Often, $B$ is an endangered language which, to avoid extinction, must increase its speaker population both by promoting its transmission and use among the bilingual speakers, and by converting some of the $A$ monolingual speakers into bilingual speakers. This is the case of Welsh and Scottish Gaelic competing with English; the Basque language competing with French and Spanish; Breton, Catalan and the Occitan languages (Gascon, Provençal, Aranés) competing with French; Sami competing with Swedish, Norwegian and Russian; Frisian, spoken in the province of Friesland in The Netherlands, competing with Dutch; the Aboriginal languages of New Zealand and Australia competing with English; Native American languages (Quechua, Aimara, Guarani, among others) competing with English, Spanish, French, Portuguese and Dutch; languages from the republics of Russia competing with Russian – see Fishman (2001) for more examples. In all those cases, the minority language $B$ needs to increase its population share and expand its use to every social domain. Therefore, it is the language dynamics of $B$ deriving from the interactions that occur inside the bilingual population that are both empirically and theoretically relevant.

In the present paper, we assume a society whose constitutional law states that $A$ and $B$ are the official languages of the community and that they should have equal rights and be equally used and promoted. In this bilingual society every individual speaks $A$ and a minority speaks both $A$ and $B$. Thus, we have just two linguistic groups: the $A$ monoglots and the bilingual minority.

This paper does not study how and why an individual chooses to change from being a monoglot who can only use $A$ to a bilingual who is able to use both $A$ and $B$. Our view is that in bilingual societies with a majority and
minority language, the asymmetric competition between $A$ and $B$ cannot be properly described by means of models based on the assumption that language attractiveness increases monotonically with the proportion of its speakers and its status (Abrams and Strogatz, 2003; Minnet and Wang, 2008). The attractiveness of $A$ would always dominate that of $B$, and the status of $A$ in the community would also be much higher than that of $B$ (for asymmetric attractiveness see Pinasco and Romanelli, 2006). The study of this competitive situation would require models describing the language decisions concerning the use and learning of $B$. These decisions are guided by a mixture of loyalty to the language and its related culture, a sense of belonging to the community that supports that language, socially induced cultural habits, and a consideration of the practical advantages to be gained (Wickström, 2005).

This paper studies how bilingual individuals choose strategically between the majority language $A$ and the minority language $B$. The consensus among language planners and sociolinguists is that a minimal condition for the survival of a minority language is that it be spoken by its speech community (see, for example Crystal, 2001; Fishman, 2001; Krauss, 1992; Wurm, 2001). Hence, the model that we propose looks into bilingual speakers’ language use in relation to $B$. Furthermore, we study the kind of interactions that occur outside the historical stronghold communities of $B$, where individuals are aware of the monolingual or bilingual nature of other individuals. That is, we deal with interactions that might happen in ‘modern’ domains, such as justice administration, media, modern technologies, higher education and research and development, where competition with $A$ is harder and where, frequently, individuals are led to meet and interact with people whose bilingual or monolingual nature is unknown (ex-ante). This, we think, is a more plausible and interesting situation to study.

There is no language contact without social conflict (e.g. Grin, 2003; Nelde, 1997). Thus, in many real situations marking or labelling people to reveal their monolingual or bilingual type may be seen as a potential source of further political conflict. Therefore, in most one-time interactions with public and private administration, agents are not able to recognize others’ bilingual or monolingual nature. Further, analytically imperfect information is more relevant than perfect information because, under the assumptions of our model, bilingual individuals with perfect information will coordinate efficiently in $B$, as is frequently the case in real situations. In this setting, we assume that none of the two interacting individuals has or sends any signal that might provide information about his/her bilingual or monolingual nature.

We model the initial steps of the conversation that takes place in those types of interaction by means of a non-cooperative game of
imperfect information played by two individuals, called the Language Conversation Game (LCG). We first study under which strategic conditions language $B$ would be used in a conversation held in that kind of interaction. Later, the LCG is studied as a population game played by the bilingual population. The main issue is to know the role played by the strategic behaviour of bilingual agents in stabilizing Nash equilibria; that is, the language conventions built by the bilingual population. The paper seeks to answer the question posed by Wickström (2005), which, in the context of our model, would be: is there a dynamically stable bilingual Nash equilibrium?

As expected, imperfect information reduces considerably the use of the minority language, aggravating its minority status. Furthermore, this is independent of the length of the introduction to the conversation modelled by the game. A key element is the dissatisfaction, as mentioned by Fishman (1991), felt by bilinguals who frequently choose to use $B$ but are forced to change to the majority language $A$. We show that there is an evolutionary stable Bayesian mixed strategy Nash equilibrium. Dynamically, this equilibrium is asymptotically stable for the one population replicator dynamics. The equilibrium is a partition of the bilingual population in two groups: one composed of those who hide their bilingual nature; the other composed of those who reveal their bilingual type by choosing $B$, whatever the information they have. Interactions between members of the former group take place using the majority language $A$; only when they mingle with members of the latter group would the former group use $B$. Since the equilibrium has such strong stability properties, we can think of it as a language convention built by the bilingual population. The problem with this convention is that it is compatible with bilingual agents failing to coordinate in language $B$. As a consequence, we cannot conclude that the survival of $B$ outside the traditional activities – that is, in those where $A$ or stronger languages are dominant and interactions are frequently between anonymous agents – is guaranteed.

Mira and Paredes (2005) show that the key factor for the stability of bilingualism is the interlinguistic similarity of $A$ and $B$, and Patriarca and Leppänen (2004) show that $A$ and $B$ may coexist if they are concentrated in disjoint geographical areas. We instead prove the stability of a bilingual equilibrium assuming that $A$ and $B$ are linguistically distant, so that a conversation is only possible in one language; as mentioned above, we avoid one-shot interactions taking place in the stronghold areas of $B$. Castelló et al. (2006), using different tools, deal with the stabilizing role of the bilingual agents and the emergence of linguistic norms or conventions; they show that in the long run one language dominates and the other becomes
extinct, with the bilingual population splitting into communities in which only one language is used.

The paper is organized as follows. In section 2 we present a detailed description of the Language Conversation Game in its extensive form. In Section 3 we offer the equilibrium analysis. Finally, Section 4 concludes.

The Language Conversation Game

Assumptions

The Language Conversation Game (LCG) captures the language used in a simple social interaction between two agents: a shopkeeper and a potential buyer. This interaction takes place in a society with two official languages (so that the language choice is not trivial): language $A$, called majority language and spoken by every individual in the society, and language $B$, called minority language, spoken by a relatively small proportion of individuals and having a small range of social usage. Thus, even though by law both languages have the same legal status, the weaker $B$ must compete for speakers and to be used for the same social functions as $A$. Thus, there are two types of individuals in the society: the bilingual type, who speaks both $A$ and $B$ and therefore can choose between the two possible actions or languages ($a_{bi} = \{A, B\}$); and the monolingual type, who only speaks the majority language $A$ and therefore has no language choice ($a_{mo} = \{A\}$). Let $\alpha$ and $(1-\alpha)$ denote the given proportion of bilingual and monolingual individuals, respectively, at some point in time. We assume that $\alpha < (1-\alpha)$; it is also assumed that $\alpha$ is much smaller than $(1-\alpha)$.

Assumption 1 (Imperfect information). The players know their own type — whether they are bilingual or monolingual — but do not observe the other players’ type.

The players know that all individuals speak the majority language $A$ and that only a minority speaks $B$. In fact, the proportion of bilingual ($\alpha$) and monolingual ($1-\alpha$) individuals in the society is common knowledge. This setting represents the interaction among individuals who do not know each other and, therefore, each other’s bilingual or monolingual nature, as is the case in many real life situations. To our knowledge, the existence of imperfect information and its consequences with regard to the use of minority language has never been stressed in the literature. The case of perfect
Iriberri and Uriarte

information is trivial: two bilingual players would meet with probability $\alpha^2$, and would, very likely, coordinate on their preferred language (see Assumption 2, below).

The social interaction in which a shopkeeper (player I) and a potential buyer (player II) choose a language in which to communicate effectively is shown in Figure 1. Actions are taken sequentially, as in an ordinary conversation, so that one player starts the conversation and the other player replies. A bilingual shopkeeper starts the interaction using one of the two languages. On the one hand, if the shopkeeper chooses $B$, the potential buyer can recognize that the shopkeeper is bilingual and can reply using $A$ or $B$ if bilingual or $A$ if monolingual. On the other hand, if the shopkeeper chooses $A$, the potential buyer cannot distinguish whether the shopkeeper is bilingual or monolingual and can reply using $A$ or $B$ if bilingual or $A$ if monolingual. If the potential buyer replies with $B$ after hearing $A$, the shopkeeper has a second chance to decide whether or not to change her initial choice of language.

The presence of Nature represents each player’s unawareness of the other player’s type. Nature chooses bilingual players with probability $\alpha$ and monolingual players with $(1-\alpha)$. As it is described in the game tree, the second player can observe (hear) the language choice, but not the type. As a consequence, $B$ will be used if both players happen to be bilingual and happen to coordinate on $B$. In any other case, the majority language $A$ will be used. For example, when both players are bilingual but coordinate on $A$, or when a bilingual player is matched to a monolingual player independently of the choice of language made by the bilingual player, the interaction will necessarily take place in language $A$. Given that the monolingual type has no proper choice to make, we will only look at bilingual players’ information sets and actions.

In Figure 1, we can see that a bilingual player I has two information sets, called Bilingual Ia and Bilingual Ib. In bilingual Ia, player I knows she is bilingual; thus in this information set she must choose among the languages $A$ or $B$ in order to start a conversation with player II, whose type she cannot observe. Hence, player I is uninformed in this information set. The bilingual Ib information set starts a proper subgame: player I chooses after player II has revealed her bilingual nature by choosing $B$. Thus, player I is informed in this information set. The bilingual player II has also two information sets, which we will refer to as Minority Language (MiL) and Majority Language (MaL). The MiL information set starts a proper subgame. Here, player II may choose after player I has revealed her bilingual nature by deciding at Bilingual Ia to choose $B$. Hence, player II is informed in this information
Figure 1. The Language Conversation Game, where A denotes the majority language, B the minority language, c the ‘frustration’ cost, and α and 1−α the proportion of bilingual and monolingual individuals, respectively, in the society. The probability with which an action is chosen is given by {.}.
Iriberri and Uriarte

set. A bilingual player II makes choices in the MaL set after having heard player I start the conversation using the majority language A. In this set, player II cannot observe whether player I is bilingual or monolingual. Thus, this information set contains two nodes, x and y.

**Assumption 2 (Bilinguals’ language preference).** Bilingual players prefer to speak B rather than A. Formally, let $\succ_{bi}$ denote the preference relation of a bilingual agent, then $B \succ_{bi} A$.

Bilingual individuals are described as agents who are loyal to the minority language B. There are two reasons that could justify that preference relation. Pool (1986) introduced a game with perfect information and only bilingual players, where each player’s native language is the other’s second language, and assumed that both players prefer to speak their own native language. Along the same lines, we could interpret this assumption as considering all bilingual players’ native language to be B.1 Also, the culturally specific language of any society “is more than just a tool of communication for its culture. (…). Such a language is often viewed as a very specific gift, a marker of identity and a specific responsibility vis-à-vis future generations” (Fishman, 1991). This statement is valid for any language, but the minority condition of language B strengthens this conception. Bilingual players are aware that B is a minority language and therefore an endangered one. They consider that the only way to avoid its disappearance is by using it, such that, whenever possible, they have a preference for using B over A. Monolingual individuals have no choice and so no preference for one language over the other.

Before we present the third assumption, which is about payoff ordering, notice that there are three payoff relevant situations in the language game. First, bilingual players might coordinate on their preferred language B. In that case, we will assume both players get payoffs equal to $m$. Second, either bilingual or monolingual players might coordinate on the majority language A. In that case, we will assume both players get payoffs equal to $n$, because this was a voluntary coordination or choice. Finally, a bilingual player might try to coordinate on her preferred language B but fail to do so and end up using the majority language A. In this latter case, the interaction will take place in the majority language A and we will assume that the bilingual or monolingual player who chose A will get a payoff of $n$, while the bilingual player who tried to use B will get a payoff of $n-c$, where c represents the frustration or disenfranchisement cost due to the forced change in the use of language, from B to A.
Notice that since \( n \) is the payoff that a bilingual player could definitely obtain had he chosen to speak \( A \), the frustration cost \( c \) should be subtracted from \( n \). Thus, \((n-c)\) is the payoff to a bilingual player who, having chosen \( B \), is matched to someone monolingual (or bilingual) who uses language \( A \) and, therefore, ends up speaking \( A \). Given the three payoff relevant situations, we will now order the three payoffs in the following assumption.

**Assumption 3 (Payoff ordering).** For a given \( \alpha \), such that \( \alpha < (1 - \alpha) \), the payoff ordering is given by \( m > n > c > 0 \).

The first inequality, \( m > n \), is due to Assumption 2. It must be interpreted as bilingual players preferring \( B \) to \( A \), such that they will get a higher von Neumann and Morgenstern utility when they interact in their preferred language \( B \) than they would in \( A \). ‘Switching to a minority language is a very common means of expressing solidarity with a social group. The change signals to the listener that the speaker is from a certain background; if the listener responds with a similar switch, a degree of rapport is established’ (Crystal, 1987: ch. 60). The cost, \( c \), which we assume to be smaller than \( n \), is intended to capture the dissatisfaction – as mentioned by Fishman (1991) – felt by the bilingual player who must face the fact that, in many interactions, she is forced to use the majority language \( A \). Following Fishman (1991), the efforts made by the bilingual population to reverse language shift are an indication of dissatisfaction with the cultural life which is dominated by the majority language.

**Assumption 4 (Linguistic distance).** The linguistic distance (see Crystal, 1987) between \( A \) and \( B \) is sufficiently high that successful communication is only possible when the interaction takes place in one language.

This assumption is important to understand that the language choice is not a trivial one. In other words, it is not possible to have a conversation where one individual speaks \( A \) and the other \( B \), because a monolingual agent would not be able to understand what is being said when someone uses language \( B \). This also implies that when a monolingual individual interacts with a bilingual one, the interaction will necessarily take place in the majority language \( A \). Notice that this assumption might not be realistic in, say, some European regions, such as Galicia (northwest Spain), where it is possible to have a conversation in which one agent speaks \( A \) (Spanish) and the other replies using \( B \) (Galician), because the linguistic distance
between $A$ and $B$ is not too big – both are Romance languages (see Mira and Paredes, 2005 about the influence of language similarity in the long-run coexistence of $A$ and $B$). However, in other regions in which two official languages are in contact – such as, say, in the Basque Country – mixed language conversations are not common due to the size of the linguistic distance between $A$ (Spanish or French) and $B$ (Basque, which is a pre-Indo-European language). Some examples satisfying assumption 4 would be Welsh, Scottish and Irish relative to English, Breton relative to French, and most of the languages noted in the introduction to this article.

**Pure strategies and relevant information sets**

In the LCG of Figure 1, bilingual player I has four pure strategies: $S_I = \{BB, BA, AB, AA\}$. In each strategy, the left component is the action taken at the Bilingual Ia information set, and the right component is the action taken at her second information set, Bilingual Ib. Therefore, say, $AB$ describes the strategy where player I uses language $A$ first and then switches to language $B$. Bilingual player II has also four pure strategies: $S_{II} = \{BB, BA, AB, AA\}$. In each strategy, the left component is the decision taken at the MiL information set and the right component is the action taken at the MaL information set.

Notice that since, according to Assumption 2, bilingual players are B-loyal agents, their strategy sets could be further simplified. They will take care not to make the mistake of playing strategies that would clearly block the use of $B$. For player I, $A$ is dominated by $B$ in Bilingual Ib, while for player II, $A$ is dominated by $B$ in MiL. Therefore, the language loyalty behaviour of bilingual individuals coincides with the rational behavior, because $B$ is, in fact, the dominant choice in the two proper subgames of the LCG shown in Figure 1, one of which starts at player I’s Bilingual Ib information set and the other at player II’s MiL information set. This implies that any equilibria that does not include action $B$ in these two subgames cannot be Bayesian-perfect. Hence, a bilingual in the position of player I will avoid playing the strategies $BA$ and $AA$; similarly, in the position of player II, the strategies $AB$ and $AA$ will be avoided. Thus, in the model, the pure strategies that a bilingual individual in the position of player I will actually use are in the set $S'_{I} = \{BB, AB\}$, while those for the position of player II are in $S'_{II} = \{BB, BA\}$, making Bilingual Ia and MaL the relevant information sets.
Finally, it is important to analyse the language in which the social interaction takes place in each possible strategy combination. Four possible events or combinations of player types may occur: with probability $\alpha^2$ both players are bilingual; with probability $\alpha(1-\alpha)$ player I is bilingual and player II is monolingual; with probability $(1-\alpha)\alpha$ player I is monolingual and player II is bilingual; and, finally, with probability $(1-\alpha)^2$ both players are monolingual.

In Figure 2, the Language Matrix shows the language associated to each pair of pure strategies when played by bilingual players, which is the only non-trivial event on which we concentrate. In the rest of the three possible events, since at least one player is monolingual – and given Assumption 4 – the spoken language will be $A$. For instance, let us look at the strategy profile $(AB, BB)$. In this situation, the language that will be used in the interaction is determined as follows: player I starts, choosing $A$ in Bilingual Ia information set; player II, after hearing language $A$, does not know the type of player I, but takes the risk of suffering the cost $c$ by choosing $B$ at her MaL information set. Then, player I reaches her Bilingual Ib information set, where she switches to $B$. Thus, under this strategy profile, the minority language $B$ will actually be used. Note the language $B$ coordination failure resulting from the $(AB, BA)$ strategy profile.

**Equilibrium analysis**

In this section we carry out equilibrium analysis of the Language Matrix described in Figure 2. The exercise is on comparative statics. We shall first describe the Bayesian Perfect equilibria in pure strategies. The equilibrium analysis in mixed strategy equilibria is then presented.

**Pure equilibrium analysis: when is it optimal for bilingual individuals to use language $B$?**

We shall consider three regimes that derive from the assumption that could be made about the possible value of the frustration cost, $c$, felt by bilingual
players, vis-a-vis the benefit, \((m-n)\), they might obtain when they are able to use their preferred language.\(^3\)

**Proposition 1.** Let the proportion of bilingual individuals, \(\alpha\), be given with \(\alpha < 1 - \alpha\).

**Regime 1:** \(c = (m-n) \frac{\alpha}{(1-\alpha)}\); then the set of Bayesian-Perfect equilibria is \(BPN^1 = \{(AB, BA), (AB, BB), (BB, BA)\}\), where \((AB, BA)\) is the only equilibrium with dominant strategies for both players. The language spoken in this equilibrium is \(A\).

**Regime 2:** \(c > (m-n) \frac{\alpha}{(1-\alpha)}\); then there is only a unique Bayesian-Perfect equilibrium, \(BPN^2 = \{B, BA\}\). The language spoken in this equilibrium is \(A\).

**Regime 3:** \(c < (m-n) \frac{\alpha}{(1-\alpha)}\), then the set of Bayesian-Perfect equilibria is \(BPN^3 = \{(AB, BB), (BB, BA)\}\). The language spoken in both equilibria is \(B\).

The payoff combinations for each player in each of the equilibria and regimes can be seen in the Appendix.

For proof, see Appendix.

Even though bilingual agents are willing to use \(B\) and have payoff incentives to do so, Proposition 1 shows that in regimes 1 and 2 they fail to coordinate on the minority language \(B\). In all the equilibria they use \(A\) instead. As one might expect, the proposition shows that the use of minority language \(B\) is optimal only in regime 3. Probably less expected, however, is that the pure equilibria of regime 3, \((AB, BB), (BB, BA)\), are not good candidates for a language convention for bilingual individuals because they are neither dynamically stable nor evolutionarily stable. In fact, language coordination failures are also present in regime 3. We show this in the next section.

**Mixed strategy equilibrium: a bilingual language convention**

In both regime 1 and regime 2, the model shows that there is a unique Bayesian-perfect Nash equilibrium with undominated strategies, \((AB, BA)\), in which \(A\) is the spoken language. Thus, the social use of \(B\) that would help to guarantee its survival is lowered by the coordination failures that occur due to choices made under imperfect information. In other words, in regimes
1 and 2 the frustration cost $c$ is a curse for the bilingual agents. Given our interest in a regime with equilibria that allow the use of $B$, we shall investigate regime 3.

To capture real-life situations, the LCG described in Figure 1 should not be an interaction that occurs only once. This kind of game is played many times by many different people and the final outcome is not an individual one but a social construct. Therefore, a different analytical approach is needed. This section studies the issue of whether a language convention based on the use of $B$ could be built by the bilingual population. To this end, we are led to use an evolutionary approach to complete the equilibrium analysis.

We have seen above that, under Assumption 2, the set of pure strategies for a bilingual individual in the position of player I is $S'_I = \{BB, AB\}$ and the set for the position of player II is $S''_I = \{BB, BA\}$. We can interpret these pure strategies, irrespective of the bilingual agent’s playing position, as follows:

$s_1$: Always use $B$, whether you know for certain you are speaking to a bilingual individual or not. That is, play strategy $BB$.

$s_2$: Use $B$ only when you know for certain that you are speaking to a bilingual individual; otherwise, use $A$. This is strategy $AB$ if you start the conversation or strategy $BA$ if you follow the conversation.

Hence, we can say that the members of the bilingual population play the LCG having $S = \{s_1, s_2\}$ as their common strategy set. The LCG is henceforth viewed as a population game, and the members of the bilingual population are called agents.

In order to offer a reasonable and intuitive interpretation of the mixed strategy equilibria, let us assume that the bilingual population consists of a large but finite number of individuals who play a certain pure strategy $s_i$, $i = \{1, 2\}$ in a two-player game. The interactions are modelled as pairwise random matching between agents of the bilingual population; that is, no more than two (randomly chosen) agents interact at a time. The interaction takes place continuously over time. Let $N$ be the total population of bilingual agents in the society, $x$ the proportion of bilingual agents playing pure strategy $s_1$ at any point $t$ in time (time dependence is suppressed in the notation), and $(1-x)$ the proportion playing pure strategy $s_2$. Thus, in this setting, a mixed strategy, $(x, (1-x))$, is interpreted as a population state that indicates the bilingual population share of agents playing each pure strategy $s_i$, $(i = 1, 2)$. On the other hand, the payoffs of the game should not be interpreted as biological fitness, but as utility.
Iriberri and Uriarte

Proposition 2. In regime 3, there exists an interior symmetric mixed strategy Nash equilibrium in which the bilingual population play \( s_1 \) with probability \( x^* = 1 - \frac{c(1-\alpha)}{\alpha(m-n)} \). This equilibrium is evolutionarily stable – that is, a language convention – and asymptotically stable in the associated one-population replicator dynamics.\(^5\)

For proof, see the Appendix.

Features of the bilingual language convention

It is important to notice that in Regime 3, the LCG is a Hawk–Dove game (see, e.g., Binmore, 1992). In the payoff matrix, shown in the Appendix, \( s_1 \) is the Dove strategy while \( s_2 \) is the Hawk strategy. If both bilingual players behave like doves, they share payoffs and use language \( B \) in the interaction. If one behaves like a hawk and the other like a dove, the game shows that they will use \( B \), but in terms of payoffs, the dove gets smaller expected payoffs because of the risk taken in choosing \( B \) without knowing the bilingual or monolingual nature of the other party.

If both bilingual players behave like hawks, they hurt each other. That is, if they both play strategy \( s_2 \), the model shows that the language spoken in the interaction is \( A \). Hence, by playing strategy \( s_2 \), they do not realize that they are impeding each other’s willingness to use their preferred language.

The equilibria \((s_2, s_1) = (AB, BB)\) and \((s_1, s_2) = (BB, BA)\) are strong candidates to maintain language diversity because bilingual speakers use only \( B \) when they interact, and this is what is needed to keep the minority language alive. However, both equilibria are weak, in the sense that they are not dynamically stable. The mixed equilibrium, \( x^* \), on the other hand, could be said to be a bilingual equilibrium because both \( s_1 \) and \( s_2 \) are played by non-zero proportions of bilingual agents, \( Nx^* \) and \( N(1-x^*) \), respectively. Hence, both \( A \) and \( B \) will be spoken in interactions between bilingual agents.

The mixed strategy equilibrium is strong due to its stability properties, but it could be weak in terms of language diversity because the use of \( B \) in this equilibrium could be reduced to very low levels. This argument is developed in the next paragraphs.

1. **The Robustness of the A- B Mixture:** The mixed equilibrium \( x^* \in (0, 1) \) is robust in the sense that any alternative, pure or mixed, strategy will get smaller payoffs. That is, any small group of bilingual agents who experiment with an alternative strategy will do less well than the agents who stick to the incumbent mixed
strategy $x^*$. Therefore, there are no incentives to change the mixed strategy equilibrium, and for that reason, the equilibrium is said to be evolutionary stable; that is, $x^*$ is the language convention for the bilingual population based on the use of both $A$ and $B$ languages. This feature of $x^*$ could explain why in real situations it is so difficult to avoid the language $B$ coordination failures that occur between bilingual agents, leading them to use $A$ even though they are ready to use $B$ and have payoff incentives to do so.

2. **The Weakness of $B$ in the $A$-$B$ Mixture**: Proposition 1 shows that strategy $s_2$ is undominated for both players in regimes 1 and 2. In the evolutionary stable equilibrium $x^*$ of regime 3, there is also a proportion of bilingual agents, given by $1 - x^* = \frac{c(1-\alpha)}{\alpha(m-n)}$, who play strategy $s_2$. Thus, the model seems to predict that bilingual agents would recurrently use strategy $s_2$. In real situations one might expect that strategy $s_2$ would be widely chosen because, at first sight, it is non controversial and satisfies the principles of politeness (of Lakoff, 1973, and Brown and Levinson, 1987). Note that for the individual starting the conversation, $s_2 = AB$ suggests beginning the conversation with $A$, the language spoken by all the individuals. If a bilingual receiver answers using $B$ (that is, II takes the risk of playing $s_1$), $s_2 = AB$ suggests switching to $B$. On the other hand, for the individual responding, strategy $s_2 = BA$ suggests using only the language that player I started with. In other words, $s_2$ is perceived as non-imposing to monoglots and, by its embodied language switching, allows the choice and use of $B$. That is, it satisfies the rules of politeness of Lakoff, 1973. Furthermore, it is also perceived as ‘non face-threatening’ (see Brown and Levinson, 1987; Goffman, 1967) to both monolingual and bilingual individuals – it is respectful of the rights of both linguistic groups. The adoption of strategy $s_2$ conforms with politeness, which is thought to be basic for social order and cooperation (Brown and Levinson, 1987). Hence, by playing $s_2$, bilingual players just avoid a potential conflict.

However, the apparently polite, hawkish strategy $s_2$ is, as mentioned above, an obstruction for the social use of $B$. Strategy $s_1$, on the other hand, seems somewhat ‘face-threatening’ (against $s_1$ you must reveal you are monolingual if you do not speak $B$), but it is the only possibility for bilingual agents to induce the social use of their preferred language $B$. 
Conclusions

The LCG makes explicit the decision making of bilingual agents, who ought to make language choices continuously under imperfect knowledge of other individuals’ bilingual or monolingual nature. The language contact between $A$, spoken by all individuals, and $B$, spoken by a minority, produces a negative externality upon the use of $B$. This is illustrated by the situation in which two bilingual players meet and fail to coordinate on the minority language $B$, because they are not informed about each other’s bilingual nature. We show that this may happen even if bilingual agents are willing to speak $B$ and have payoff incentives to do so. Thus, language contact and imperfect information reduce considerably the use of the minority language, aggravating its minority status.

We have shown that there is an equilibrium where the bilingual agents shift between the use of $A$ and $B$. The equilibrium is shown to be robust, meaning evolutionary and asymptotically stable for the single population replicator dynamics. At the same time, however, this equilibrium, understood as a language convention, provides weak support for language diversity. The use that bilingual speakers make of $B$ in this equilibrium depends on the language contact situation (where $A$ is dominant in every social domain), imperfect information, politeness norms and other factors outside the model, such as the scarcity of formal and informal usages of the minority language and hardly developed oral and written discursive models, that may reduce the use of $B$ to very low levels.

A straightforward lesson we can learn from this paper is that eliminating the imperfect information structure of the game will improve considerably the use of the minority language among the bilingual players. It is fair to say that in some real life bilingual settings, even outside the stronghold areas of $B$, one might detect signals, such as a certain accent or specific physical attributes, that might help to deduce who is bilingual and who is not. This can ensure the use of $B$ and help to maintain the linguistic diversity of the society. But it should be taken into account that a language policy consisting of marking or labelling people in order to denote their bilingual nature could be an additional source of conflict.

Given the difficulty in measuring the use that bilingual players make of $B$ in anonymous situations, we believe experimental work is warranted. There are many questions of interest. In particular, it would be interesting to compare loaded and unloaded information treatment within a bilingual society; that is, comparing the individual behaviour in the LCG when $A$ and $B$ are just actions with no further meaning with the individual behaviour when $A$ and $B$ represent now the majority and minority languages, respectively. It would also be interesting to discover the speech conventions that appear in
an experimental setting and compare them with the prediction made in Proposition 2, and to study the influence of politeness in the strategies used and the gender differences of that influence. We leave those questions for further research.

**Funding**

Nagore Iriberri acknowledges financial support from Ministerio de Educación y Ciencia (SEJ2007-64340 and ECO2009-11213) and the support of Fundación Rafael del Pino and the Barcelona GSE Research Network and the Government of Catalonia. José Ramón Uriarte gratefully acknowledges financial support from the Spanish Government under the grants MCYT SEJ2006-05455 FEDER and MICINN ECO2009-11213 ERDF, the Basque Government under the grant GIC07/22-IT-223-07, and the University of the Basque Country, UPV 00043.321-15836/2004. The usual disclaimer applies.

**Acknowledgements**

We are grateful to Ehud Kalai, Ignacio Palacios-Huerta, Karl Schlag, Larry Samuelson, Reinhard Selten, Joel Sobel and Bengt-Arne Wickström for their comments and criticisms on different versions of this work. Special thanks to two referees of this journal for comments that helped to significantly improve the manuscript.

**Notes**

1. This assumption can be relaxed: where the bilingual types would be further divided into two different types, some might have the ordering assumed in Assumption 2 and others might have the opposite ordering. This is a possible extension but does not affect the main result shown in this paper.
2. In our game, Sequential Equilibrium and Perfect Bayesian Equilibrium are equivalent.
3. For the sake of completeness, we also include the cut-off regime where $c$ equals the weighted benefits.
4. Thus far, we have treated $c$ and $m$ as exogenously given parameters. But it is natural to think that steady increases of $\alpha$ would be accompanied by steady reductions of $c$ and $m$, since $B$ is changing its minority-language status. Thus, it could be thought that there exists a threshold level $\alpha_T$, such that for $\alpha > \alpha_T$ both $c$ and $m$ would be decreasing functions of $\alpha$, with $c = c(\alpha)$ approaching 0 and $m = m(\alpha)$ approaching $n$ as $\alpha (\geq \alpha_T)$ approaches 1. In that process, it could be assumed that $c = c(\alpha) < m = m(\alpha)$. This allows for the study of societies in which a transition has occurred from one regime to another.
5. It is known that the replicator dynamics could be derived from behaviours observed in social interactive learning settings: among others, the aspiration-based learning model of Binmore et al. (1995); decisions based on imitation and reinforcement of successful behaviour (Cabrales, 2000; Schlag, 1998; Weibull, 1995); and constant doubts based decision procedures, as in Cabrales...
and Uriarte (2010). The replicators are represented here by the pure strategies $s_1$ and $s_2$.

References

Abrams DM and Strogatz SH (2003) Modeling the dynamics of language death. *Nature* 424: 900 pp.

Binmore K (1992) *Fun and Games: A Text on Game Theory*. Lexington, KY: D.C. Heath.

Binmore K, Gale J and Samuelson L (1995) Learning to be imperfect: the ultimatum game. *Games and Economic Behavior* 8: 56–90.

Brown P and Levinson SC (1987) *Politeness: Some Universals in Language Usage (Studies in Interactional Sociolinguistics)*. Cambridge: Cambridge University Press.

Cabrales A (2000) Stochastic replicator dynamics. *International Economic Review* 41: 451–481.

Cabrales A and Uriarte JR (2012) Doubts and Equilibria. *Journal of Evolutionary Economics* . Published Online, DOI: 10.1007/s00191-012-0269-1 Mimeo.

Castelló X, Eguiluz VM and San Miguel M (2006) Ordering dynamics with two non-excluding options: bilingualism in language competition. *New Journal of Physics* 8: 308.

Crystal D (1987) *The Cambridge Encyclopedia of Language*. Cambridge: Cambridge University Press.

Crystal D (2001) *Language Death*. Cambridge: Cambridge University Press.

Fishman JA (1991) *Reversing Language Shift: Theoretical and Empirical Foundations of Assistance to Threatened Languages*. Clevedon: Multilingual Matters.

Fishman JA (2001) Why is it so hard to save a threatened language? In: Fishman JA (ed.) *Can Threatened Languages Be Saved?* Clevedon: Multilingual Matters: pp. 1–23.

Goffman E (1967) *Interaction Ritual*. Harmondsworth: Penguin, pp 1–23.

Grin F (2003) Language planning and economics. *Current Issues in Language Planning* 4(1): 1–66.

Krauss M (1992) The scope of the language endangerment crisis and recent response to it. In: Matsumura K (ed.) *Studies in Endangered Languages*. Papers from the international symposium on endangered languages. Tokyo, November 18-20, 1995 (pp. 101–112). Tokyo: Hituzi Syobo

Krauss, M. (1998). The scope of the language endangerment crisis and recent response to it. In K. Matsumara (Ed.), Studies in endangered languages. Papers from the international symposium on endangered languages. Tokyo, November 18-20, 1995 (pp. 101–112). Tokyo: Hituzi Syobo.

Lakoff, R. (1973) The logic of politeness; or mind your p’s and q’s, in Corum, C., Smith Stark, T.C. and Weiser, A. (eds), Papers from the 9th regional meeting of the Chicago Linguistic Society, pp. 292–305. Chicago: Chicago Linguistic Society.

Mira JB and Paredes A (2005) Interlinguistic similarity and language death dynamics. *Europhysics Letters* 69(6): 1031–1034.
Minnet JW and Wang W S-Y. (2008) Modelling endangered languages: The effects of bilingualism and social structure. Lingua, 118(1), 19–45.

Nelde P (1997) Language conflict. In: Coulmas F (ed.) The Handbook of Sociolinguistics. London: Blackwell, pp. 285–300.

Patriarca M and Leppänen T (2004) Modeling language competition. Physica A: Statistical Mechanics and its Applications 338 (1–2): 296–299.

Pinasco JP and Romanelli (2006) Coexistence of languages is possible. Physica A: Statistical Mechanics and its Applications 361: 355–360.

Pool J (1986) Optimal strategies in linguistic games. In: Ferguson CA and Fishman J (eds) The Fergusonian Impact: Sociolinguistics and the Sociology of Language, vol. 2. Berlin, New York and Amsterdam: Mouton de Gruyter. pp.157–171.

Schlag K (1998) Why imitate, and if so, how? A boundedly rational approach to multi-armed bandits. Journal of Economic Theory 78: 130–156.

Stauffer D and Schulze C (2005) Microscopic and macroscopic simulation of competition between languages. Physics of Life Reviews 2: 89–116.

Stauffer D, Castelló X, Eguiluz VM, et al. (2007) Microscopic Abrams-Strogatz model of language competition. Physica A: Statistical Mechanics and its Applications 374: 835–842.

Wang WS-Y and Minnet JW (2005) The invasion of language: emergence, change, and death. Trends in Ecology & Evolution 20(5): 263–269.

Wang WS-Y and Minnet JW (2008) Modelling endangered languages: the effects of bilingualism and social structures. Lingua 118: 19–45.

Weibull JW (1995) Evolutionary Game Theory. Cambridge, MA: The MIT Press.

Wickström B-A (2005) Can bilingualism be dynamically stable? Rationality and Society 17: 81–115.

Wurm SA (2001) Atlas of the World’s Languages in Danger of Disappearing. Paris and Canberra, ACT, Australia: UNESCO Publishing.

Appendix

Proof of Proposition 1

From Figure 1, player I’s and player II’s expected payoffs are given by the following expressions:

\[
EP_I = EP_I(BiI_a) + EP_I(BiI_b) = \alpha p[sm + (1-s)(n-c)] \\
+ \alpha (1-p)q[rm + (1-r)n] + (1-q)n \\
+ (1-\alpha)(1-p)n + (1-\alpha)p(n-c)
\]

\[
EP_{II} = EP_{II}(MiL) + EP_{II}(MaL) = \alpha p[sm + (1-s)n] \\
+ \alpha(1-p)q[rm + (1-r)(n-c)] + \alpha(1-p)(1-q)n
\]
Iriberri and Uriarte

\[+(1-\alpha)[q(n-c)+(1-q)n] \]

Bilinguals’ preference for B over A, presented in Assumption 2, means that \(r = s = 1\), such that both players’ expected payoffs simplify to:

\[ EP_I = \begin{cases} \alpha q(m-n)+n & \text{if } AB_p = 0 \\ \alpha(m-n)-(1-\alpha)c+n & \text{if } BB_p = 1 \end{cases} \]

\[ EP_{II} = \begin{cases} \alpha p(m-n)+n & \text{if } BA_q = 0 \\ \alpha(m-n)-(1-\alpha)c+n & \text{if } BB_q = 1 \end{cases} \]

Given (1) and (2), we can fill in the payoff profiles for each of the strategy combinations in Figure 2 and obtain the following matrix (where the common element \(n\) has been deleted):

|       | BB               | BA               |
|-------|------------------|------------------|
| BB    | \(\alpha(m-n)-c(1-\alpha),\alpha(m-n)-c(1-\alpha)\) | \(\alpha(m-n)-c(1-\alpha),\alpha(m-n)\) |
| AB    | \(\alpha(m-n),\alpha(m-n)-c(1-\alpha)\)          | \(0,0\)          |

Now, we can easily obtain the Bayesian perfect Nash equilibria for each of the three regimes.

**Proof of Proposition 2**

Recall the expected payoff matrix of Proposition 1. Also, remember the reinterpretation given to the pure strategies for both player roles and that in Regime 3, \(c < (m-n)\frac{\alpha}{(1-\alpha)}\). We can further normalize the payoff matrix, as follows:

\[ a_1 = a_{11} - a_{21} = \alpha(m-n)-c(1-\alpha)+n-[\alpha(m-n)+n] = -c(1-\alpha) < 0 \]

\[ a_2 = a_{22} - a_{12} = n-[\alpha(m-n)-c(1-\alpha)+n] = -\alpha(m-n)+c(1-\alpha) < 0 \]

\[ b_1 = b_{11} - b_{12} = \alpha(m-n)-c(1-\alpha)+n-[\alpha(m-n)+n] = -c(1-\alpha) < 0 \]

\[ b_2 = b_{22} - b_{21} = n-[\alpha(m-n)-c(1-\alpha)+n] = -\alpha(m-n)+c(1-\alpha) < 0 \]
Notice that, in Regime 3, this matrix is a symmetric two-player game (played by two bilingual agents) with the payoff structure of a *Hawk–Dove Game*. Hence, the game has two asymmetric equilibria: the two Bayesian-perfect Nash equilibria in pure strategies mentioned in Proposition 1, \((s_2, s_1) = (AB, BB)\)(s2, s1) = (AB, BB) and \((s_1, s_2) = (BB, BA)\), and a symmetric one in mixed strategies, \((x^*, 1-x^*) = (1 - \frac{c(1-\alpha)}{\alpha(m-n)} , \frac{c(1-\alpha)}{\alpha(m-n)})\), where \(x^* > 0\). The latter equilibrium is an evolutionary stable strategy — that is, a language convention — and, furthermore, is an asymptotically *stable population state* in the single population Replicator Dynamics (see Proposition 3.10 of Weibull, 1995):

\[
x = [\alpha(m-n)(1-x) - c(1-\alpha)]x(1-x)
\]

Notice that at the mixed strategy equilibrium, \(x^* = 1 - \frac{c(1-\alpha)}{\alpha(m-n)}\), \(\alpha(m-n)\)

\((1-x) - c(1-\alpha) = 0\) and so \(x = 0\). We can see that for any \(0 < x < 1 - \frac{c(1-\alpha)}{\alpha(m-n)}\), \(x\) increases toward \(x^*\), and for any \(1 > x > 1 - \frac{c(1-\alpha)}{\alpha(m-n)}\), \(x\) decreases toward \(x^*\).