Fine structure of alpha decay of even-even trans-lead nuclei – an intriguing nuclear structure paradigm

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Abstract.

The systematics of the experimental data of alpha-decay fine structure in the even-even nuclei above lead are examined. The representation of relative branching ratios (b.r.) and hindrance factors (HF) for the $2^+_1$, $4^+_1$, and $6^+_1$ states within the valence correlation schemes (as a function of $N_pN_n$ or $P = N_pN_n/(N_p + N_n)$) proves useful, as it provides smooth trajectories that can be discussed in parallel with the development of collectivity. It is shown that practically all recent theoretical calculations are not able to account for some of the most conspicuous features presented by the evolution of the branching ratios and hindrance factors. These are: a practically exponential increase of the b.r.’s and HF’s for the $2^+_1$ state of collective nuclei (above $P \approx 4.0$); a pronounced maximum around $P \approx 7.5$ of the same quantities for the $4^+_1$ state, a region where these nuclei have usually been considered as well deformed rotors; and the decrease of these quantities for the $6^+_1$ state in the interval of $P$ from 4.0 to about 8.0.

The observed evolutions of these fine structure quantities do not appear to be correlated with different structure indicators deduced from the low-lying level schemes, with the exception of the HF of the $2^+_1$ state which is reasonably well correlated with a parameter that describes the nuclear rigidity, indicating the necessity of going beyond the rigid rotor description of the nuclei considered in this study. It is also necessary that a simultaneous description of both electromagnetic transitions and alpha decay fine structure data is achieved by the theoretical calculations.

1. Introduction

Although it is one of the oldest observed nuclear structure phenomena, alpha decay remains a very important experimental tool for the investigation of unstable nuclei, especially the superheavy ones. The fine structure of the alpha decay was experimentally observed by Salomon Rosenblum in 1929 [1], and soon after that explained by Gamow [2] as due to the population of the ground and excited states in the daughter nucleus. The population of the excited states in the residual nucleus is usually much weaker than that of the ground state, and a detailed theoretical understanding of the experimentally observed alpha decay branching ratios has not been achieved yet.

In this work we review the experimental data on the fine structure of the alpha decay of even-even trans-lead nuclei. This subject was partially covered in a previous paper where, however, only hindrance factors were considered [3]. Here, we consider the systematics of both relative branching ratios and of hindrance factors. Conspicuous features of these systematics are discussed, that are not fully understood theoretically at present. The usefulness of the
presentation of these quantities within the valence correlation schemes, as proposed in [3], is also reinforced.

2. Systematics of branching ratios and hindrance factors

The considered alpha-decay data are those of the even-even daughter nuclei with \( Z \geq 82 \) for which at least the branch for the \( 2^+_1 \) state was measured besides that of the ground state. In addition to the \( 2^+_1 \) state, other states considered in our analysis were the \( 4^+_1 \) and \( 6^+_1 \) states that were measured in most of the rotational nuclei. For other excited states the measured data are less numerous and are not included in this study. The measured branching ratios and the corresponding hindrance factors for these states of the ground state band were collected and examined. Systematics of these quantities were published before by Ellis and Schmorak [4] (for \( A \geq 229 \)), and by Akovali [5], but we have also checked these experimental data with the most recent ENSDF evaluations [6].

For self-consistency we review the experimental observables and different quantities based on these and used in the comparison with theoretical calculations. The quantities experimentally determined in alpha decay are the \( Q \)-values (which determine the excitation energies of the states populated in the daughter nucleus), the branching ratios \( B_{r,i} \) (for the state \( i \)), and the half-life \( T_{1/2} \) of the alpha-decaying state in the parent nucleus (in our case, the \( 0^+ \) ground state). For each state \( i \) one then defines a partial half-life \( T_{1/2,i} \) and a partial width \( \Gamma_i \):

\[
T_{1/2,i} = \frac{T_{1/2}}{B_{r,i}}, \quad \Gamma_i = \frac{\hbar \ln 2}{T_{1/2,i}}
\]

(1)

For comparison with theory one usually factorizes the widths as

\[
\Gamma = \delta^2 P
\]

(2)

where \( \delta^2 \) is called reduced width, which mainly contains the nuclear structure information, and \( P \) is the penetrability of the alpha particle through the Coulomb barrier. The hindrance factors are defined as:

\[
HF_i = \frac{\delta_{gs}^2}{\delta_i^2} = \frac{B_{r,gs} P_i}{B_{r,i} P_{gs}} = \frac{\Gamma_{gs}}{\Gamma_i} \frac{P_i}{P_{gs}}
\]

(3)

One sees that for the even-even nuclei, the hindrance factor defined for the ground state to ground state transition is, by definition, \( HF = 1 \). The hindrance factors are model dependent quantities, because they contain the penetrabilities that can be calculated in different ways. On the other hand, the reduced widths are essentially determined by an alpha formation amplitude, therefore they depend on the structure of the two states implied in the decay process. Definition (3) also shows that in calculating the hindrance factors one corrects for the barrier penetrabilities, leaving, in principle, mainly the dependence on the structure of the initial and final states. This makes the \( \alpha \)-decay a strong spectroscopic tool for investigating the properties of the nuclear states. Actually, the structure information obtained from the alpha decay (the reduced widths) can be compared to the similar spectroscopic information obtained from the direct \( \alpha \)-pickup (d,\( ^6\text{Li} \)) reaction. However, for the set of nuclei that we study, there are only a small number of such studies [7, 8].

In ENSDF, the listed HF-values are based on the recipe of Preston (spherical case, rectangular potential well) [9]. Another definition often encountered is that of Rasmussen [10] (also spherical case, but using a more realistic alpha-nucleus potential). Deformation dependent penetrabilities, best suited to most of the nuclei in this set, were also recently considered (see, for example, Refs. [11, 12]).

In total, we have found experimental branching ratios and corresponding \( HF \) values for a number of 62 \( 2^+_1 \) states, 29 \( 4^+_1 \) states, and 17 \( 6^+_1 \) states, respectively, in daughter nuclei with
Z ≥ 82. As already stated, preliminary results based on the examination of the hindrance factors were published in [3].

The evolution of the experimental branching ratios and hindrance factors can be investigated by representing them as functions of different variables, like mass number A, neutron number N, etc. Thus, in Ref. [4] their evolution with A was presented. We prefer, as in Ref. [3], to use the so-called valence correlation scheme representation, because although the A-representation shows a relatively smooth evolution above mass 220, where nuclei become collective, it does not distinguish between isobars. The VCS are based on the use of the $N_pN_n$ [13, 14], or $P = N_pN_n/(N_p + N_n)$ [15] quantities, where $N_p$ ($N_n$) represent the number of active protons (neutrons) counted with respect to the nearest magic number. It is known [13, 14, 15] that these type of representations give similar, rather compact trajectories for the evolution of different structure indicators (such as, e.g., $R(4/2) = E(4^+_1)/E(2^+_1)$, or $B(E2; 2^+_1 \rightarrow 0^+_0)$); also, they have the advantage that the observed evolutions can be correlated with the evolution of the collectivity (for example, it was shown that in all nuclear regions the transitional nuclei pass into deformed ones in the region $P \approx 4.0 - 5.0$ [15]). In our case, we have counted the active numbers of nucleons (particles or holes) in the daughter nucleus, with respect to the shell gap numbers 82 and 126 for protons, and 126 and 184 for the neutrons, respectively.

![Figure 1](image_url)

**Figure 1.** Experimental relative branching ratios for the first three excited states of the ground state (quasi)band, as a function of the $P$ factor, where $P = N_pN_n/(N_p + N_n)$.

Fig. 1 shows the evolution of the relative branching ratios for the three considered excited states, as a function of the $P$-factor. Like in the case of the hindrance factors [3], rather compact trajectories are obtained, especially for the nuclei that start developing collective features ($P$ larger than about 4.0). Conspicuous features (also remarked for the HF values)
are: the exponential increase for the $2^+$ state above $P \approx 6.0$, the pronounced maximum at $P \approx 7.5$ for the $4^+$ state, and the decrease for the $6^+$ state, almost out of phase with the $4^+$ state variation.

Figures 2 and 3 show a comparison of these experimental data with the results of some recent theoretical calculations. In Fig. 2 are shown the results of calculations performed with a unified model for $\alpha$-decay and $\alpha$-capture (UMADAC) [16], and those of a Coulomb and proximity potential model for deformed nuclei (CPPMDN) [17]. Very similar results are provided by other calculations, like those based on the generalized liquid drop model [18], or those based on the Gamow theory with a square well potential barrier penetration [19]. One can see that a reasonable description is obtained with CPPMDN for the vibrational and transitional nuclei ($P$ below $\sim 4.0$, while for the rest of the nuclei, especially the well deformed ones, none of these models is able to explain the main experimental features emphasized above.

**Figure 2.** Comparison of experimental relative branching ratios with theoretical model calculations. CPPMDN – Coulomb and proximity potential model for deformed nuclei [17]; UMADAC – Unified model for alpha decay and alpha capture [16].

Figure 3 shows the same type of comparison with results of stationary coupled channels calculations [20, 21]. Again, while for the vibrational and transitional nuclei [20] a very good agreement is achieved, for the deformed nuclei [21] the exponential increase for the $2^+$ state and the maximum observed for the $4^+$ state are not accounted for. One should emphasize that a rigid rotor model was employed for the deformed nuclei.

Hindrance factors as adopted by ENSDF [6] were presented and discussed in [3]. We mention
that hindrance factors calculated by using different calculations for the penetrabilities, such as those from Refs. [6, 10, 21] generally differ from each other just in absolute value, but present similar evolutions.

**Figure 4.** Comparison of experimental and calculated hindrance factors, data from Ref. [21].

As an example, Fig. 4 shows the experimental and theoretical hindrance factors calculated with the Coupled Channel method [20, 21]. As in the case of the branching ratios (Fig. 3), again a good agreement is obtained for the vibrational and transitional nuclei, while for the deformed ones there is a systematic deviation with respect to the experimental data.

**Figure 5.** Relative branching ratios and hindrance factors (as calculated in Ref. [21]) for the $2^+$ and $4^+$ states, represented as a function of the VCS $P$-factor.

Fig. 5 shows, however, the clear advantage of the $P$-representation for the hindrance factors. The branching ratios show a larger scattering of the data points (for the $2^+$ state one can even
distinguish the isotopic chains). By correcting for the barrier penetrability and thus obtaining the HF mainly containing structure information, the scattering of the data is much reduced. For both $2^+$ and $4^+$ states one obtains rather compact trajectories for the collective nuclei ($P$ above 4.0), therefore this kind of plot lends itself to the prediction of values for nuclei that are not measured yet.

3. Discussion
In Fig. 6 we present different structure indicators, extracted from the data of the low-lying excited states, and try to correlate their evolution with those observed for the $\alpha$-decay fine structure quantities discussed above. These quantities comprise the following. (i) The moment of inertia, as evaluated from the energy of the $2^+$ state with the rigid rotor formula; in graph (a) it is also shown the rigid body of inertia calculated for an ellipsoid of quadrupole deformation $\beta_2$ (either experimental, or calculated [22]). (ii) the $J_0$ and $J_1$ Harris parameters of the fit with

![Graphs showing different nuclear structure indicators.](image)
the variable moment of inertia (VMI) [23] formula

\[ E = \frac{1}{2} \omega^2 (J_0 + \frac{3}{2} \omega^2 J_1). \]  

(iii) the ratio \( R(4/2) = E(4^+_1)/E(2^+_1) \); (iv) the experimental quadrupole deformation parameter \( \beta_2 \), and the hexadecapole deformation parameter as predicted by Ref. [22].

An examination of these parameters shows that our set of nuclei undergoes a rather "normal" transition from vibrational to rotational nuclei. Thus, at \( P \approx 5.0 \), \( R(4/2) \) reaches values that are close to the rotational limit 3.33, as shown in Ref. [14]. In the region of large \( P \), the experimental moment of inertia stabilizes at about half the rigid body value, similar to the rare earths deformed region. The quadrupole deformation \( \beta_2 \) smoothly increases with \( P \), with values above 0.25 for \( P \geq 6.0 \). The predicted [22] hexadecapole deformation \( \beta_4 \) has a sudden change of slope in the region of well deformed nuclei (at about \( R(4/2) = 3.25 \)). An interesting feature is revealed by the examination of the energies of the yrast line with the VMI formula (4). The \( J_0 \) parameter is roughly identical with the moment of inertia extracted from the \( 2^+_1 \) state energy. The \( J_1 \) parameter decreases with the increase of \( P \). Within the VMI [23], the inverse of \( J_1 \) is twice the value of a stiffness parameter \( C \) that measures the rigidity of the nucleus. Thus, relatively small \( J_1 \) values associated with large \( J_0 \) indicate an increased rigidity – a behavior that is closer to that of a rigid rotor (not in the sense of an increase of the moment of inertia, but in that the intrinsic structure of the nucleus changes little with increasing excitation energy, or rotation). We see that the rigidity of the nuclei considered here increases with increasing \( P \).

![Figure 7](image.png)

**Figure 7.** Correlation between the hindrance factors of the \( 2^+_1 \) state (as calculated in Ref. [21]) and the inverse of the \( J_1 \) Harris parameter of eq. (4), which is proportional (see text) with the rigidity parameter. The linear fit to the data gives a correlation coefficient \( r = 0.89 \).

We have tried to find correlations between the structure indicators discussed above and the \( \alpha \)-decay fine structure quantities. The only useful correlation observed has been that between the HF of the \( 2^+ \) state and the stiffness parameter (the rigidity of the nucleus) – Fig. 7. This fact may indicate why by using the rigid rotor model the calculations (Figs. 3, 4) cannot
reproduce the exponential increase experimentally observed for both branchings and HF’s. On the other hand, the origin of the conspicuous maximum observed for the $4^+$ state still remains very intriguing, especially as it is in a region of large $P$ (about 7.5), where these nuclei are usually regarded as good rotors and no sudden changes are expected. Also, the decrease observed for the $6^+$ state, which takes place in the same time with the strong increase for the $4^+$ state (Figs. 1,2), has no simple explanation.

Recent coupled channel calculations performed by Ni and Ren [24] report an improvement in the description of the fine structure data. They consider coupling between five channels (up the the $8^+$ state), and an improved consideration of the dynamics of the core. The results, essentially contained in their Fig. 1, show the following. There is still a certain underestimation of the branchings for the $2^+$ state, that increases with the mass number (or $P$, in our case). There is also an improvement in the description of the $4^+$ state values, however the data show a large scattering and do not seem to present the neat maximum around mass 240 (corresponding to the Pu isotopes with mass 236 to 244, with $P$ around 7.5). The description of the $6^+$ states is similar to that of the $4^+$ state. While the general description of the fine structure data is improved, and clearly indicates that much of the observed effects may come from the couplings between channels, we lack a “simple” explanation of conspicuous effects such as the well defined maximum in the $4^+$ state data. As observed in the present study, the consideration of a variable nuclear rigidity of the deformed nuclei in this region may be very important. It is also important that calculations that give a good description of the alpha decay fine structure data give also a good description of the electromagnetic transition probabilities of the excited states [25].

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