Temperature Dependence of Critical Velocity in a Bose Gas in a Moving Optical Lattice

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Abstract. We study the critical velocity of a Bose-condensed gas in a moving one-dimensional (1D) optical lattice potential at finite temperatures. Using a quasi-1D model of the Gross-Pitaevskii equation and the Bogoliubov equations, we explicitly include effects of thermal excitations in the radial direction. Within the Popov approximation, we calculate the temperature dependence of the Bogoliubov excitations with varying lattice velocity. From the condition of the negative excitation energy, we obtain the critical velocity as a function of the lattice depth and the temperature. We find that the critical velocity decreases rapidly with increasing the temperature; this result is consistent with the experimental observations. Moreover, the critical velocity shows a rapid decrease with increasing lattice depth. This tendency is much more significant than in the previous works ignoring the effect of thermal excitations in the radial direction.

1. Introduction
The occurrence of energetic and dynamical instabilities in a Bose-Einstein condensate (BEC) moving in a periodic (optical) potential is an interesting problem from the conceptual viewpoint, since it involves basic properties of superfluidity. Experimentally, energetic instabilities of BECs moving through optical lattices has been observed both in the weak and tight-binding regimes [1–3]. Applying Landau argument [4], the energetic instability of the superfluid state is attributed to the appearance of negative excitation energy, (which is known as Landau instability). Once a superfluid velocity is beyond its unique critical velocity, the current suffers friction leading to the decay of superfluidity. Since energetic instability requires the thermal component that receives the energy emitted by the condensate during the breakdown process, it is only observed at finite temperatures. However, most theoretical studies on the Landau instability of a Bose gas in an optical lattice used the zero-temperature GP equation. Moreover, they concentrated on the first Bloch band using the Bose-Hubbard model, and have ignored the effect of radial excitations. In order to provide a quantitative account for the critical velocity relevant to the experimental data, it is important to consider thermal excitation in the radial direction.

In this paper, we study the critical velocity of current carrying condensate in a 1D optical lattice at finite temperature. For this purpose, we calculate the Bogoliubov excitation energy in the framework of Hatrree-Fock-Bogoliubov-popov (HFB-Popov) approximation in a periodic potential, with explicitly including the effect of the radial excitations. In Sec. II, we derive a quasi-1D model of the Gross-Pitaevskii equation and the Bogoliubov equations for the Bose gas moving in a 1D optical lattice. Using the HFB-Popov approximation, we solve these equations to calculate the Bogoliubov excitation spectrum. In Sec. III, we calculate the critical velocity
of current carrying condensate in a lattice, which is determined from the condition of the negative excitation energy. We obtain the critical velocity as a function of the lattice depth with a fixed temperature, and compare it with the experimental data [1]. The magnitude of the critical velocity is found to be in reasonable agreement of the experimental data [1]. The critical velocity decreases with increasing lattice depth, which is consistent with the experimental result [1]. We also calculate the temperature dependence of the critical velocity with a fixed lattice depth. We find that the critical velocity drops very rapidly with increasing temperature.

2. Quasi 1D modeling of a current carrying condensate

We consider a Bose condensed gas in a combined potential of highly elongated harmonic trap and 1D optical lattice. In order to take into account this quasi-1D situation, we expand the field operator in terms of the radial wave function $\hat{\psi}(r) = \sum_\alpha \psi_\alpha(z) \phi_\alpha(x, y)$, where $\phi_\alpha(x, y)$ is the eigenfunction of the radial part of the single-particle Hamiltonian[5, 6]. Here $\alpha = (n_x, n_y)$ is the index of the single-particle state with the eigenvalue $\epsilon_{(n_x, n_y)} = h\omega_\perp(n_x + n_y + 1)$, where $\omega_\perp$ is the trap frequency in the radial direction. Within the HFB-Popov approximation, we obtain the generalized Gross-Pitaevskii (GP) equation[5, 6],

$$\mu \Phi_\alpha = \left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} + V_{op} + \epsilon_\alpha \right] \Phi_\alpha + \sum_{\alpha'\beta\gamma} g_{\alpha\alpha'\beta\gamma} \left( \Phi^*_{\beta} \Phi_{\alpha'} + 2 \Phi_{\beta} \left\langle \hat{\psi}_{\beta}^{*} \hat{\psi}_{\alpha'} \right\rangle \right),$$

(1)

where $V_{op}(z) = sE_R \cos^2(\frac{G}{2} z)$ is an optical lattice potential with the lattice constant $2\pi/G$. In this paper, we neglect the harmonic trap potential along the z-direction. The renormalized coupling constant is defined by $g_{\alpha\alpha'\beta\gamma} \equiv g \int dx dy \phi^{*}_\beta \phi^{*}_\alpha \phi_{\alpha'} \phi_{\gamma}$, where $g = \frac{4\hbar^2 \mu}{m}$ is the coupling constant determined by the s-wave scattering length $a$. As in Refs.[5, 6], we can approximate $\Phi_\alpha \approx \Phi \delta_{\alpha,0}$. Taking the usual Bogoliubov transformations for the noncondensate, $\hat{\psi}_\alpha(z, t) = \sum_{jk} \left[ \hat{\psi}_{jk}^{*} \hat{\alpha}_{jk} - \hat{\alpha}_{jk}^{*} \hat{\psi}_{jk} \right]$ we obtain the coupled Bogoliubov equations,

$$\hat{L}_\alpha \hat{u}_{ja} + \sum_{\alpha'} \left[ 2g^{\alpha\alpha'}_{\alpha} n_0 + g_{\alpha\alpha'\beta\gamma} \tilde{n}_{\beta\gamma} \right] \hat{u}_{ja'} - g^{\alpha\alpha'}_{\alpha} n_0 \hat{v}_{ja'} = E_j \hat{u}_{ja},$$

$$\hat{L}_\alpha \hat{v}_{ja} + \sum_{\alpha'} \left[ 2g^{\alpha\alpha'}_{\alpha} n_0 + g_{\alpha\alpha'\beta\gamma} \tilde{n}_{\beta\gamma} \right] \hat{v}_{ja'} - g^{\alpha\alpha'}_{\alpha} n_0 \hat{u}_{ja'} = -E_j \hat{u}_{ja},$$

(2)

where we have introduce the operator $\hat{L}_\alpha \equiv -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} + V_{op}(z) + \epsilon_\alpha - \mu$. We have also introduce the simplified notations $n_0(z) = |\Phi(z)|^2$, $g^{\alpha\alpha'}_{\alpha} = g_{\alpha\alpha'00}$, $\tilde{n}_{\beta\gamma} = \left\langle \hat{\psi}_{\beta}^{*} \hat{\psi}_{\gamma} \right\rangle$. As noted above, we only include the lowest mode ($\alpha = 0$) in the condensation wavefunction $\Phi$. Sums over the repeated indices $\beta, \beta'$ are implied in Eq. (2). These equations define the quasiparticle excitation energies $E_j$ and the quasi-particle amplitudes $\hat{u}_{ja}$ and $\hat{u}_{ja'}$. Using the solutions of Eq. (2), one can obtain the noncondensate density from $\tilde{n} = \sum_{\alpha} \tilde{n}_{\alpha\alpha}$, where $\tilde{n}_{\alpha\beta} = \sum_{jk} \left( u_{ja} u_{jb} + v_{ja} v_{jb} \right) N(E_j) + v_{ja} v_{jb}$ with $N(E_j) = 1/ \left| \exp(\beta E_j) \right| - 1$.

We expand the condensate wavefunction in terms of the reciprocal lattice vector $G$, $\Phi(z) = \sum_{n} e^{i(q+nG)z} C_n$, where $\hbar q$ is the velocity of the condensate. We also expand the Bogoliubov quasiparticle wavefunctions $\hat{u}_{ka}$ and $\hat{v}_{ka}$ in terms of the reciprocal lattice vector, $\hat{u}_{ka} = \sum_{n} u_{kn} e^{i(nG+k)z}$, $\hat{v}_{ka} = \sum_{n} v_{kn} e^{i(nG+k)z}$, where $k$ is the Bloch wavevector (or quasimomentum) of the excitations. The dispersion relation for the condensate moving through the optical lattice is obtained by solving the Bogoliubov equations in Eq. (2). Solving the coupled equations (1), and (2), we self-consistently determine the excitations spectrum $E_j$ and the condensate fraction at finite temperatures. Our calculation procedure is summarized in Refs. [5, 6]. Throughout
this paper we use the following parameters of the experiment of Ref. [1]: $m(^{87}\text{Rb}) = 1.44 \times 10^{-25}$ kg, $\omega_z/2\pi = 9.0$ Hz, $\omega_{\perp}/2\pi = 92$ Hz, scattering length $a = 5.82$ nm and the wavelength of the optical lattice $\lambda = 795$ nm. We fixed the number of atoms per lattice site as 200.

3. The critical velocity of current carrying condensate

![Figure 1](image)

**Figure 1.** The excitation spectrum of the current carrying condensate with a fixed lattice depth $s=1$ and temperature $T = 20\mu K$, for different values of the condensate momentum $q$ ($q/G = 0$, 0.01, 0.02). We plot the lowest radial branch (corresponding to axial excitations with no radial nodes).

In Fig. 1, we plot a typical excitation spectrum of the lowest phononic branch ($\alpha = 0$) for $s = 1$ at $T=20\mu K$ for different values of the condensate velocity. The $q = 0$ case corresponds to the condensate at rest. By increasing $q$, the dispersion law becomes asymmetric where the slope of the phononic branch decreases for the mode propagating backwards ($k < 0$). By further increasing $q$, the slope of the spectrum at $k \to -\infty$ approaches zero at a certain critical value $q_c$. Beyond $q_c$, low energy excitations become negative. Therefore, the superfluidity breaks down at $q_c$. For comparison, we also calculate the Bogoliubov sound velocity $c$ for $q = 0$ for different values of the temperatures $T$. The analytical expression for the Bogoliubov sound velocity $c$ is given by $c = (m^*\kappa)^{-1/2}$, where the effective mass $m^*$ and the compressibility $\kappa$ are defined as $m^* = \lim_{k \to 0} (\partial^2 E(k)/\partial k^2)^{-1}$ and $\kappa^{-1} = \bar{n} (\partial \mu/\partial \bar{n})$ [7].

In Fig. 2, we plot the temperature dependence of the critical velocity $q_c$ and the sound velocity $c$ with fixing the lattice depth as $s = 1$. It is clear that critical velocity is always lower than the sound velocity. We also find that critical velocity decreases rapidly with increasing temperature. The reduction of that critical velocity is more significant than that of the sound velocity. The reduction of both those velocities are due to the reduction of the condensate fraction with increasing temperature, which is larger for the condensate with a finite $q$ than the condensate at rest. This tendency is much more significant than in the previous works ignoring the effect of thermal excitations in the radial direction. It is thus important to include the effect of the thermal excitations in the radial direction for obtaining the comparable results with the experimental observations.

In Fig. 3, we plot the Bogoliubov sound velocity $c$ and the critical velocity $q_c$ as a function of lattice depth $s$ with fixing the temperature as $T = 20\mu K$. We find that the critical velocity rapidly decreases with increasing lattice depth $s$. Fig. 3 also shows that the critical velocity is always lower than the sound velocity $c$. The reductions of both the critical and sound velocities are due to the increase of the effective mass $m^*$ with increasing lattice depth. As shown in Ref. [8], $m^*$ depends on the condensate momentum, and it is larger for finite $q$. 

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4. Conclusion

In this paper, we studied the temperature dependence of the critical velocity of current carrying condensate in a 1D optical lattice, with explicitly including the effect of the radial excitations. While we treated the condensate wavefunction only with the lowest radial mode, we took into account the radial excitations for thermal cloud.

Within the Popov approximation, we calculated the temperature dependence of the Bogoliubov excitations with varying lattice velocity. From the condition of the negative excitation energy, we obtain the critical velocity as a function of the lattice depth and the temperature. For comparison, we also calculated the sound velocity of condensate at rest in a 1D optical lattice potentials. As shown in Fig.2, the critical velocity rapidly decrease with increasing temperature, much faster than the sound velocity. The difference between the sound velocity and critical velocity was observed in the experiment of Ref. [1]

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