Lifshitz scaling effects on the holographic paramagnetic-ferromagnetic phase transition

B. Binaei Ghotbabadi · A. Sheykhi · G. H. Bordbar

Received: 6 May 2021 / Accepted: 7 September 2021 / Published online: 25 September 2021
© The Author(s), under exclusive licence to Springer Science+Business Media, LLC, part of Springer Nature 2021

Abstract
We disclose the effects of Lifshitz dynamical exponent $z$ on the properties of holographic paramagnetic-ferromagnetic phase transition in the background of Lifshitz spacetime. To preserve the conformal invariance in higher dimensions, we consider the Power-Maxwell (PM) electrodynamics as our gauge field. We introduce a massive 2-form coupled to the PM field and perform the numerical shooting method in the probe limit by assuming the PM and the 2-form fields do not back-react on the background geometry. The obtained results indicate that the critical temperature decreases with increasing the strength of the power parameter $q$ and dynamical exponent $z$. Besides, the formation of the magnetic moment in the black hole background is harder in the absence of an external magnetic field. At low temperatures, and in the absence of an external magnetic field, our result show the spontaneous magnetization and the ferromagnetic phase transition. We find that the critical exponent takes the universal value $\beta = 1/2$ regardless of the parameters $q, z, d$, which is in agreement with the mean field theory. In the presence of an external magnetic field, the magnetic susceptibility satisfies the Curie-Weiss law.

Keywords Paramagnetic-ferromagnetic · Phase Transition · Lifshitz

Contents
1 Introduction ............................................. 2
2 The holographic set-up ........................................ 4
3 Numerical calculation for spontaneous magnetization and susceptibility ..................... 7
1 Introduction

The Bardeen, Cooper and Schrieffer (BCS) theory of superconductivity is a well known microscopic theory for studying the weakly coupled low temperature superconductors [1,2]. However, this microscopic theory suffers to explain the pairing mechanism of the materials with strongly coupled interaction at high temperature. Therefore, it is important to find an alternative approach for describing the high temperature superconductors. The correspondence between a strongly coupled conformal field theory (CFT) in \( d \)-dimensions and a weakly coupled gravity theory in \( (d+1) \)-dimensional anti-de Sitter (AdS) spacetime provides a powerful tool to shed the light on the mechanism of high temperature superconductors [3–5]. Investigation of the electronic properties of materials and magnetism by employing AdS/CFT duality is one of the applications of this approach [6–15]. The first holographic superconductor considers a four-dimensional Schwarzschild-\( AdS \) black hole coupled to a Maxwell and a scalar fields in the simple model [16]. A variety of the holographic dual models have been explored in the literatures (see e.g. [17–28] and reference therein).

The holographic paramagnetic-ferromagnetic phase transition in a dyonic Reissner-Nordstrom-\( AdS \) black brane is another example which gives a starting point for exploration of complicated magnetic phenomena and quantum phase transition [29]. This model was extended by introducing two antisymmetric tensor fields which correspond with two magnetic sublattices in the materials [30]. In the framework of usual Maxwell electrodynamics, the physical properties of holographic paramagnetism-ferromagnetism phase transition have been investigated by some authors [30–36]. Although in AdS/CFT the action should be considered as an effective approach of string theory, in general, for applications of gauge/gravity duality such as the holographic superconductor model, the gravitational model could not be studied as well as those which satisfy the behavior of boundary theory and the condition of string theory. The employing different electromagnetic actions can modify the dynamics of dual theory. Since in this viewpoint, the nonlinear electrodynamic theories correspond to the higher derivative corrections to the Abelian gauge fields; therefore it can be useful for these kinds of investigations. These nonlinear electrodynamics carry more information than that of the Maxwell field, and they have been interesting subjects of research in the recent years. As an example, the effect of BI-like electrodynamics parameter on the paramagnetism-ferromagnetism phase transition has studied in Refs. [37,38]. Considering Born-Infeld-likes electrodynamics, it has been observed that the higher nonlinear corrections make the formation of magnetic moment being harder, decreasing the critical temperature and changes the condensation gap in the absence of external magnetic field. The properties of holographic superconductor with conformally invariant PM electrodynamics have been studied in Refs. [39–45]. Now it is interesting for us to investigate how the power-law Maxwell field as another type of nonlinear electrodynamics, affects the paramagnetism-ferromagnetism phase
transition. Here, one of the reasons is that the Power-Maxwell field takes the special asymptotical solution near boundary which is different from all known cases.

On the other hand, in the framework of condensed matter physics, a dynamical scaling appears near the critical point with the scale transformation turns to be [46]

\[ t \rightarrow \lambda^z t, \quad x_i \rightarrow \lambda x_i, \quad z \neq 0, \quad (1) \]

where \( z = 1 \) corresponds to usual AdS spacetime. It was pointed out that at the quantum critical point there is a Lifshitz scaling similar to Eq. (1) [47]. A lot of attempts have been carried out in Lifshitz scaling by using the holographic approach (see e.g. [48–53]). However, the most of the previous works regarding the effects of Lifshitz scaling on holographic superconductors were done by considering a SU(2) Yang-Mills gauge field in the bulk in the presence of Maxwell electrodynamics [48,49]. The holographic paramagnetism-ferromagnetism phase transition in the four and five-dimensional Lifshitz spacetime has been explored in [54] by introducing a massive 2-form field coupled to the Maxwell field. It has been confirmed that the Lifshitz dynamical exponent \( z \) contributes evidently to magnetic moment. Holographic model for ferromagnetic phase transition in the Lifshitz black hole in the presence of Born-Infeld-like nonlinear electrodynamics has been investigated in Ref. [55]. It was observed that, in case of larger dynamical exponent \( z \), the exponential form of nonlinear electrodynamics correction leads to the smaller value for critical temperature and the magnetic moment comparing with the logarithmic and Born-Infeld types of nonlinear electrodynamics [55].

In our previous work [56], we explored the effects of PM nonlinear electrodynamics on the properties of holographic paramagnetic-ferromagnetic phase transition in the background of Schwarzchild-AdS black hole. We wonder how other backgrounds affect the paramagnetism-ferromagnetism phase transition, especially the Lifshitz spacetime as an example of the non-relativistic spacetimes. Therefore, following the studies of the holographic ferromagnetic-paramagnetic phase transition in the presence of nonlinear electrodynamics [55], here we would like to extend the investigation on this system in the background of Lifshitz spacetime by taking into account the nonlinear PM electrodynamics. The motivation for taking into account the PM electrodynamics instead of Maxwell one comes from the fact that the Maxwell field is conformally invariant, and hence the corresponding energy momentum tensor is traceless, only in four dimensions. A natural question then arises: Is there an extension of Maxwell action in arbitrary dimensions that is traceless and hence possesses the conformal invariance? The answer is positive and the conformally invariant Maxwell action in \( d \)-dimensional spacetime is given by [57],

\[ I_{PM} = \int d^d x \sqrt{-g} (-F)^q, \quad (2) \]

where \( F = F_{\mu \nu} F^{\mu \nu} \) is the Maxwell invariant and \( q \) is the power parameter. One can easily check that action (2) is invariant under conformal transformation \( g_{\mu \nu} \rightarrow \Omega^2 g_{\mu \nu} \) and \( A_\mu \rightarrow A_\mu \). The associated energy-momentum tensor of the above action is given by
\[ T_{\mu\nu} = 2 \left( q F_{\mu\rho} F_{\nu}^{\rho} F_{\nu}^{\sigma q^{-1}} \right) - \frac{1}{4} g_{\mu\nu} F_{\nu}^{\sigma q}, \] \hspace{1cm} (3)

It is easy to check that the above energy-momentum tensor is traceless for \( q = d/4 \). The theory of conformally invariant Maxwell field is considerably richer than that of the linear standard Maxwell field and in the special case (\( q = 1 \)) it recovers the Maxwell action [58–60]. It is worthwhile to investigate the effects of exponent \( q \) on the paramagnetism-ferromagnetism phase transition in the Lifshitz background. To be more general, in this work, we consider not only the conformal case where \( d = 4q \), but also the arbitrary value of \( q \). This allows us to consider more solutions from different perspective [61] and brings rich physics in studying holographic paramagnetism-ferromagnetism phase transition. This holographic model can provide a powerful tool to analyze phenomena involving magnetization. In particular, we shall investigate how the PM electrodynamics influences the critical temperature and magnetic moment. Interestingly, we find that the effect of sub-linear PM field can lead to the easier formation of the magnetic moment at higher critical temperature with respect to other kinds of nonlinear electrodynamics. In other words, the critical temperature increases with decreasing the value of the power parameter. For higher values of the power parameter, the gap in the magnetic moment in the absence of magnetic field, is smaller which in turn exhibits that the condensation is formed harder.

We shall focus on four- and five-dimensional holographic paramagnetic-ferromagnetic phase transition in probe limit by neglecting the back reaction of both gauge and the 2–form fields on the background geometry. We employ the numerical shooting method to investigate the features of our holographic model. All theses holographic ferromagnetic models are constructed only in the relativistic spacetimes. We wonder whether this model still hold in non-relativistic spacetimes for example the Lifshitz spacetime, which is our motivation in this paper.

This paper is organized as follows; In sect. 2, we introduce the action and basic field equations of the holographic model for paramagnetic-ferromagnetic phase transition in the Lifshitz black hole with PM electrodynamics. In sect. 3, we employ the shooting method for our numerical calculations and obtain the critical temperature and magnetic moment. We also study the magnetic susceptibility density. In the last section, we summarize our results and concluding remarks.

2 The holographic set-up

In many condensed matter systems, it can be found that the phase transitions governed by fixed points which exhibit the anisotropic scaling of spacetime \( t \rightarrow b^z t, \quad x \rightarrow bx \), where \( z \) is the Lifshitz dynamical exponent. Let us consider the gravity description dual for this scaling in the \( d-\)dimensional spacetime of the form

\[ ds^2 = -r^{2z} f(r) dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 \sum_{i=1}^{d-2} dx_i^2, \] \hspace{1cm} (4)
with
\[ f(r) = 1 - \left( \frac{r_+}{r} \right)^{z+d-2}, \tag{5} \]
where \( r_+ \) is the event horizon radius of the black hole. The Hawking temperature of black hole, on the horizon, which can be interpreted as the temperature of CFT, is given by \[ 62\]
\[ T = \frac{f'(r_+)}{4\pi} = \frac{(z+d-2)r_+^z}{4\pi}. \tag{6} \]

The critical points with \( z > 1 \) are often called to be non-relativistic. The Lifshitz spacetime in a \( d \)-dimensional background can be realized by the following action
\[ S = \frac{1}{2\kappa^2} \int d^d x \sqrt{-g} \left( R - 2\Lambda + L_1(F) + \lambda^2 L_2 \right), \tag{7} \]
where \( \kappa^2 = 8\pi G \) with \( G \) is Newtonian gravitational constant, \( g \) is the determinant of metric, \( R \) is Ricci scalar and \( \Lambda = -(z+d-3)(z+d-2)/2l^2 \) is the cosmological constant of \( d \)-dimensional AdS spacetime with radius \( l \). The action of Power-Maxwell filed is taken as power-law function of the form \( L_1(F) = -\beta F^q \), where \( \beta \) is a constant, \( q \) is the power parameter of the Power-Maxwell field \[ 39,57\]. Here \( F = F_{\mu\nu}F^{\mu\nu} \) is the Maxwell invariant in which \( F_{\mu\nu} = \nabla_{[\mu} A_{\nu]} \) and \( A_{\mu} \) is the gauge potential of U(1) gauge field. Clearly \( q = 1 \) corresponds to the Maxwell electrodynamics, \( L_1 = -F/4 \), and the Einstein-Maxwell theory is recovered. In action (7) the Lagrangian density \( L_2 \) consists a \( U(1) \) field \( A_{\mu} \) and a massive 2-form field \( M_{\mu\nu} \) in \( (d) \)-dimensional spacetime and is given by \[ 33\]
\[ L_2 = -\frac{1}{12}(dM)^2 - \frac{m^2}{4} M_{\mu\nu}M^{\mu\nu} - \frac{1}{2} M^{\mu\nu}F_{\mu\nu} - \frac{J}{8} V(M), \]
where \( dM \) is the exterior differential of 2-form field \( M_{\mu\nu} \), \( \lambda \) and \( J \) are two real parameters with \( J < 0 \) for producing the spontaneous magnetization and \( \lambda^2 \) characterizes the back reaction of two polarization field \( M_{\mu\nu} \), and the Maxwell field strength on the background geometry. In addition, \( m \) is the mass of 2-form field \( M_{\mu\nu} \) being greater than zero \[ 33\] and \( dM \) is the exterior differential 2-form field \( M_{\mu\nu} \). \( V(M_{\mu\nu}) \) is a nonlinear potential of 2-form field \( M_{\mu\nu} \) describing the self-interaction of polarization tensor which should be expanded as the even power of \( M_{\mu\nu} \). In this model, for simplicity, we take the following form for the potential
\[ V(M) = (\ast M_{\mu\nu}M^{\mu\nu})^2 = [\ast (M \wedge M)]^2, \tag{8} \]
where \( \ast \) is the Hodge star operator \[ 33\].
Varying action (7), we can get the equations of motion for the matter fields as
\[ 0 = \nabla^\tau (dM)_{\tau \mu \nu} - m^2 M_{\mu \nu} - J (\ast M_{\tau \sigma} M^{\tau \sigma}) (\ast M_{\mu \nu}) - F_{\mu \nu} \]  
(9)

\[ 0 = \nabla^\mu \left( q F_{\mu \nu} (\mathcal{F})^{q-1} + \frac{\lambda^2}{4} M_{\mu \nu} \right). \]  
(10)

In the probe limit, we can neglect the back reaction of the 2-form and PM fields on the background Lifshitz geometry (4). In order to explore the effects of Lifshitz dynamical exponent \( z \) and power parameter \( q \) on the holographic ferromagnetic phase transition, we take the self-consistent ansatz with matter fields based on the Lifshitz spacetime as follows,

\[ M_{\mu \nu} = - p(r) dt \wedge dr + \rho(r) dx \wedge dy, \]  
(11)

\[ A_\mu = \phi(r) dt + B x dy, \]  
(12)

where \( B \) is a uniform magnetic field which is considered as an external magnetic field of dual boundary field theory. \( \rho(r), p(r) \) and \( \phi(r) \) which are two components of the polarization field and the electrical potential, respectively, are function of \( r \). Inserting this ansatz into Eqs. (9) and (10), nontrivial equations of motion in \( d \)-dimensional Lifshitz spacetime can be obtained as follows,

\[ 0 = \rho'' + \rho' \left[ \frac{f'}{f} + \frac{(z + 1) - (6 - d)}{r} \right] - \frac{\rho}{r^2 f} \left[ m^2 + \frac{4 J p^2}{r^{2 z - 2}} \right] + \frac{B}{r^2 f}, \]

\[ 0 = \left( m^2 - \frac{4 J \rho^2}{r^4} \right) p - \phi', \]

\[ 0 = \phi'' + \frac{\phi'}{r (2q - 1) \phi'^2 - B^2 r^{2 z - 6}} \left[ -B^2 r^{2 z - 6} (z - (d + 3) + 4q) + \phi'^2 (2q (z - 1) - \left[ z + (6 - d) \right]) \right] + \frac{\lambda^2}{2q + 1 r^{2 z + 4}} \left( p' + \frac{d - z - 1}{r} p \right) \times \left[ \frac{(\phi'^2 - B^2 r^{2 z - 6})^{2-q}}{(2q - 1) \phi'^2 - B^2 r^{2 z - 6}} \right], \]  
(13)

where the prime stands for the derivative with respect to \( r \). In the limiting case where \( d = 4, q = 1 \) and \( z = 1 \), the above equations reduce to the standard holographic paramagnetism-ferromagnetism phase transition models discussed in Ref. [33]. In order to solve Eq. (13) numerically, we need to seek the boundary conditions for \( \rho, \phi \) and \( p \) near the black hole horizon at the spatial infinity. Therefore, in additional to \( f(r_+) = 0 \), and due to the fact that the norm of the gauge field namely \( g_{\mu \nu} A^{\mu} A^{\nu} \) should be finite at the horizon, it is required \( \phi(r_+) = 0 \) by considering the regularity.
condition for $\rho(r_+)$ at the horizon. The asymptotic solutions for matter fields governed by the field equations (13) near the boundary ($r \to \infty$) are given by

$$
\phi(r) \sim \mu - \frac{\sigma^{2q-1}}{r^{ \frac{(d-3)+2q + z(1-2q)}{2q-1} - 1}},
$$
$$
p(r) \sim \frac{\sigma^{2q-1}}{m^2 r^{ \frac{(d-3)+2q + z(1-2q)}{2q-1} - 1}},
$$
$$
\rho(r) \sim \frac{\rho_-}{r^{\Delta_-}} + \frac{\rho_+}{r^{\Delta_+}} + \frac{B}{m^2}, \quad (14)
$$

where $\mu$ and $\sigma$ are respectively interpreted as the chemical potential and charge density in the dual field theory, and

$$
\Delta_\pm = \frac{(6-d) - z}{2} \pm \frac{1}{2} \sqrt{4m^2 - [z - (6-d)]^2}. \quad (15)
$$

The coefficients $\rho_+$ and $\rho_-$ are two constants correspond to the source and vacuum expectation value of dual operator in the boundary field theory when $B = 0$. Therefore, condensation happens spontaneously below a critical temperature when we set $\rho_+ = 0$. Considering $B \neq 0$, the asymptotic behavior is governed by external magnetic field $B$. It is important to note that, unlike other nonlinear electrodynamics such as Born-Infeld-like electrodynamics, the boundary condition for the gauge field $\phi$ depends on the power parameter $q$ of the PM field and the dynamical exponent $z$, [63,64]. Using the asymptotic behaviour given by Eq. (14) and the fact that $\phi$ should be finite as $r \to \infty$, it leads to

$$
\frac{(d - 3) + 2q + z(1 - 2q)}{2q - 1} - 1 > 0, \implies \frac{1}{2} < q < \frac{d - 2 + z}{2z}. \quad (16)
$$

Thus, the power parameter $q$ cannot take an arbitrary value and is bounded by the Lifshitz exponent $z$ and spacetime dimensions $d$. Besides, the asymptotic behavior of $p(r)$ confined by these parameters. Basically $q$ should satisfy the above condition, e.g., for $z = 3/2$ in four dimensions ($d = 4$) we have $q < 7/6$, while in five dimensions ($d = 5$), the upper bound for power parameter becomes $q < 9/6$.

In the remaining part of this paper, we will study the holographic ferromagnetic-paramagnetic phase transition numerically.

## 3 Numerical calculation for spontaneous magnetization and susceptibility

In the previous section, we have investigated the holographic paramagnetic-ferromagnetic phase transition in arbitrary dimensions and derived all expressions in terms of spacetime dimension $d$. However, in the numerical calculations one needs to specify the
spacetime dimensions $d$. Thus, in this section we focus on 4D and 5D in our numerical calculations which are more realistic from physical point of view. We set our calculations in the grand canonical ensemble where the chemical potential $\mu$ is fixed. The expression of magnetic moment in the background of Lifshitz black hole will be changed as

$$N = -\lambda^2 \int \frac{\rho}{2r^{d-z-1}} dr.$$  \hspace{1cm} (17)

We can obtain the basic features of the model by choosing $m^2 = -J = 1/8$ and $\lambda = 1/2$ as a typical example in the numerical computation. This is due to the fact that the choice of parameters will not qualitatively modify our results. Using the shooting method [6], we can solve Eq.(13) numerically to investigate the holographic phase transition, and discuss the effects of PM electrodynamics and the dynamical exponent $z$ on the magnetic moment in different dimensions. Hereafter, we define the dimensionless coordinate $Z = r_+/r$ instead of $r$, since it is easier to work with it. The numerical calculation may be justified by virtue of the field equation symmetry, $r \rightarrow ar, \quad t \rightarrow a^{-z}t, \quad f \rightarrow a^2 f, \quad \phi \rightarrow a\phi, \quad \mu \rightarrow a^z \mu$.

We can use the above scaling symmetry, and obtain the solution of Eqs. (13) with the same chemical potential as it is discussed in Ref. [56] in details. We consider the cases of different power parameter $q$ in 4D and 5D spacetime with different allowed values of Lifshitz parameter. We present our results in Figs.1 and 2. In these figures, we plot the magnetic moment for $d = 4$ and $d = 5$, for allowed values of $z$, with different values of power parameters. When the temperature is lower than $T_C$, we find that in the absence of an external magnetic field, the spontaneous condensate of $\rho$ (magnetic moment) in the bulk appears. In the vicinity of critical temperature, the numerical results show that the second order phase transition happens which its behavior can be obtained by fitting this curve ($N \propto \sqrt{T - T_C}$). The results have been shown in Tables 2 and 4. We find that there is a square root behavior for the magnetic moment versus temperature, and the critical exponent is the same as that of mean field theory for these two dimensions. In other words, the holographic paramagnetism-ferromagnetism phase transition still exists in Lifshitz black hole spacetime in the presence of PM electrodynamics, similar to the cases of Born-Infeld-like nonlinear electrodynamics [55].

From Figs.1 and 2, we observe that with increasing the value of power parameter $q$, the magnetic moment decreases for the fixed values of $z$ in different dimensions. Similar behavior is seen for the cases of Born-Infeld-like nonlinear electrodynamics. This implies that the magnetic moment is harder to form in the presence of nonlinear electrodynamics. This is consistent with the results given in Ref. [37,38]. This behavior has been also seen for the holographic superconductor in the Schwarzschild-AdS black hole, where the three types of nonlinear electrodynamics make scalar condensation harder to be formed [63].

In Tables 1 and 3 our numerical results for critical temperature with different values of $q$ and $z$ are presented. For $d = 4$, when $z \rightarrow 1$, in the Maxwell limit ($q \rightarrow 1$), our numerical results reproduce the results of Ref. [37]. We see from Tables 1 and 3 that
the critical temperature $T_c$ increases by decreasing the power parameter for fixed $z$. As the power parameter $q$ becomes larger, the critical temperature decreases. It means that the magnetic moment is harder to be formed. This behavior has been reported previously in Ref. [37], too. Figs. 1 and 2 confirms the above results. Similarly, for a constant value of $q$ and comparing the situation of $z = 3/2$ with $z = 7/4$ for $d = 4$ and $z = 3/2$ with $z = 17/10$ for $d = 5$, we see that the critical temperature $T_c$ will decrease with the increasing $z$, and the magnetic moment will become smaller for PM nonlinear electrodynamics. This fact can be found from Figs. 1 and 2. It is found that the spontaneous condensate in the absence of external magnetic field appears. The results are consistent with those of mean field theory as can be seen in Tables 2 and 4. From these Figs. 1 and 2, we find that the increasing value of PM parameter $q$ makes the magnetic moment smaller for fixed values of $z$, which is similar to the cases of nonlinear electrodynamics. It is easy to find that as $z$ increases, the magnetic moment decreases for different values of $q$ in $d = 4$, which is understood by comparing Fig. 1(a) and (b). One can find the same results for $d = 5$ from Figs. 2(a) and (b). However, there is different behavior for magnetic moment in case of $q = 3/4$ regardless of the

---

Fig. 1 The behavior of magnetic moment $N$ and the critical temperature with different values of power parameter $q$, in different dynamical exponents $z$ for $m^2 = 1/8$ and $J = -1/8$ for $d = 4$.

Fig. 2 The behavior of magnetic moment $N$ and the critical temperature with different values of power parameter $q$, in different dynamical exponents $z$ for $m^2 = 1/8$ and $J = -1/8$ for $d = 5$.

---

Since our chosen masses should satisfy the Breitenlohner-Freedman (BF) bound, $4m^2 - [z - (6 - d)]^2 > 0$, therefore by considering $m^2 = 1/8$, the dynamical exponent should be confined with $(6 - d) - 2m < z < 2m + (6 - d)$. So we have chosen the values for dynamical exponent as $z = 3/2, 7/4$ for $d = 4$. For $d = 5$, we have considered $z = 3/2, 17/10$ for dynamical exponent. Thus, for these choice of the parameter $m$ and $d$, the values of Lifshitz dynamical exponent is restricted $z < 2$. 

---

1 Since our chosen masses should satisfy the Breitenlohner-Freedman (BF) bound, $4m^2 - [z - (6 - d)]^2 > 0$, therefore by considering $m^2 = 1/8$, the dynamical exponent should be confined with $(6 - d) - 2m < z < 2m + (6 - d)$. So we have chosen the values for dynamical exponent as $z = 3/2, 7/4$ for $d = 4$. For $d = 5$, we have considered $z = 3/2, 17/10$ for dynamical exponent. Thus, for these choice of the parameter $m$ and $d$, the values of Lifshitz dynamical exponent is restricted $z < 2$. 

---

Springer
Table 1: Numerical results of $T_c/\mu$ for different values of $q$ in different dynamical exponents $z$ for $d = 4$

| $d = 4$ | $z = 3/2$ | $z = 7/4$ |
|---------|-----------|-----------|
| $q = 3/4$ | 2.7245 | 2.2219 |
| $q = 4/4$ | 0.8584 | 0.4490 |

Table 2: The magnetic moment $N$ with different values of $q$ and $z$ for $d = 4$

| $z$ | $3/2$ | $7/4$ |
|-----|-------|-------|
| $q = 3/4$ | 8.5730$(1 - T/T_c)^{1/2}$ | 8.3168$(1 - T/T_c)^{1/2}$ |
| $q = 1$ | 3.6318$(1 - T/T_c)^{1/2}$ | 2.3252$(1 - T/T_c)^{1/2}$ |

Table 3: Numerical results of $T_c/\mu$ for different values of $q$ in different dynamical exponents $z$ for $d = 5$

| $d = 5$ | $z = 3/2$ | $z = 17/10$ |
|---------|-----------|-------------|
| $q = 3/4$ | 2.8317 | 2.5128 |
| $q = 4/4$ | 1.5054 | 1.2261 |
| $q = 5/4$ | 1.1286 | 0.8736 |

Table 4: The magnetic moment $N$ with different values of $q$ and $z$ for $d = 5$

| $z$ | $3/2$ | $17/10$ |
|-----|-------|--------|
| $q = 3/4$ | 3.4779$(1 - T/T_c)^{1/2}$ | 3.5974$(1 - T/T_c)^{1/2}$ |
| $q = 1$ | 2.3363$(1 - T/T_c)^{1/2}$ | 2.2563$(1 - T/T_c)^{1/2}$ |
| $q = 5/4$ | 1.8779$(1 - T/T_c)^{1/2}$ | 1.7178$(1 - T/T_c)^{1/2}$ |

values of $z$. It can be understood from these figures that by increasing $z$, the magnetic moment increases. Therefore, by increasing $z$, both the magnetic moment and the critical temperature decrease for $q = 1, 5/4$. However, for $q = 3/4$, we observe a different behavior in which by increasing $z$, although the critical temperature decreases, the magnetic moment increases as well.

The magnetic susceptibility density is one of the characteristic properties of ferromagnetic material. Let us turn on the external magnetic field to investigate the response of magnetic moment $N$, which is described the static susceptibility density as

$$\chi = \lim_{B \to 0} \frac{\partial N}{\partial B}.$$  \hspace{1cm} (18)

This feature can be obtained based on the previous analysis which has been discussed in Ref. [33] where it has been found that the susceptibility obeys Curie-Weiss law,

$$\chi = \frac{C}{T + \theta}, \quad T > T_C,$$  \hspace{1cm} (19)
Fig. 3 The behavior of inverse susceptibility density as a function of temperature with different values of $q$, at different dynamical exponent $z$ for $d = 4$

where $C$ and $\theta$ are two constants. For PM nonlinear electrodynamics, Figure 3 shows the behavior of susceptibility density near the critical temperature in $4D$ for $q = 3/4, 1$ and $z = 3/2$ and $7/4$. The same calculation for $d = 5$ is presented in Fig. 4 for $q = 3/4, 1, 5/4$ and $z = 3/2$ and $17/10$. In the paramagnetic phase for all considered cases, we observe that when the temperature is reduced, the magnetic susceptibility increases for the fixed nonlinear parameter $q$ and also dynamical exponent $z$. Moreover, the magnetic susceptibility satisfies the Curie-Weiss law of the ferromagnetism near the critical temperature regardless of the value of $q$ in different dimensions. In the high temperature regime, one can see that when the temperature decreases, $\chi$ increases, and the susceptibility obeys the Curie-Weiss law. These results have been presented in Tables 5 and 6. Obviously we observe that, for fixed allowed value of dynamical exponent $z$, the coefficient in front of $T/T_c$ increases, when the power parameters $q$ increases too, while for fixed value of $q$, this coefficient decreases with increasing $z$. Meanwhile, we see that the absolute value of $\theta/\mu$ will decrease for $q = 1, 5/4$ by increasing the dynamical exponent $z$. We find different behavior for $q = 3/4$, as it can be found from these tables, the absolute value of $\theta/\mu$ increases by increasing the dynamical exponent $z$ (see Tables 5 and 6).

4 Conclusions

To summarize, we studied the physical properties of holographic paramagnetic-ferromagnetic phase transition in the background of $d$-dimensional Lifshitz black
Fig. 4 The behavior of inverse susceptibility density as a function of temperature with different values of $q$, at different dynamical exponent $z$ for $d = 5$.

holes. We considered the PM electrodynamics, and obtained the effects of nonlinear power parameter $q$ and dynamical Lifshitz exponent $z$ on the phase behavior of the system. We performed numerical shooting method for studying our holographic model. We observed that for this kind of nonlinear electrodynamics, both the power parameter and dynamical exponent can make the condensation harder to form, and the critical temperature and magnetic moment decrease as well for any dimension. It was confirmed that the enhancement in power parameter of electrodynamics and dynamical exponent, cause the paramagnetic phase more difficult to be appeared. Our data confirms these results. We observed that for fixed values of dynamical exponent, the effect of increasing power parameter causes the lower values for the critical temperature in our model. Besides, for smaller values of the power parameter, the gap in the magnetic moment in the absence of magnetic field is larger which in turn exhibits that the condensation is formed easier. In a fixed value of power parameter, by increasing
the dynamical exponent, the magnetic moment decreases which causes the condensation to be formed harder. Although for \( q = 3/4 \) in \( d = 5 \) this behavior is opposite. The behavior of the magnetic moment is always as \((1 - T/T_c)^{1/2}\). This is in agreement with the result of mean field theory. One can conclude that the explicit form of nonlinear electrodynamics and the anisotropy of spacetime with different dimensions do not have any effect on the relationship. Moreover, in the presence of external magnetic field, the inverse magnetic susceptibility near the critical point behaves as \((C T + \theta)\) for different values of power parameters with different allowed values of dynamical exponent in different dimensions, and it satisfies the Curie-Weiss law. The absolute value of \( \theta / \mu \) decreases by increasing the dynamical exponent for \( q = 1, 5/4 \), while for \( q = 3/4 \), the absolute value of \( \theta / \mu \) increases by increasing the dynamical exponent \( z \).

### Acknowledgements

We are grateful to the referee for constructive comments which helped us improve our paper. We thank the Research Council of Shiraz University. The work of A.S. has been supported financially by the Research Institute for Astronomy & Astrophysics of Maragha (RIAAM), Iran.

### References

1. Bardeen, J., Cooper, L.N., Schrieffer, J.R.: Microscopic theory of superconductivity. Phys. Rev. 106, 162 (1957)
2. Bardeen, J., Cooper, L.N., Schrieffer, J.R.: Theory of superconductivity. Phys. Rev. 108, 1175 (1957)
3. J. M. Maldacena, The large-N limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys. 2, 231 (1998) [arXiv:9711200]
4. S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Gauge theory correlators from non-critical string theory, Phys. Lett. B 428, 105 (1998) [arXiv:9802109]
5. E. Witten, Anti-de Sitter space and holography, Adv. Theor. Math. Phys. 2, 253 (1998) [arXiv:9802150]
6. Hartnoll, S.A.: Lectures on holographic methods for condensed matter physics. Class. Quant. Grav. 26, 224002 (2009). [arXiv:0903.3246]
7. Herzog, C.P.: Lectures on holographic superfluidity and superconductivity. J. Phys. A 42, 343001 (2009). [arXiv:0904.1975]
8. McGreevy, J.: Holographic duality with a view toward many-body physics. Adv. High Energy Phys. 2010, 723105 (2010). [arXiv:0909.0518]
9. Herzog, C.P.: Analytic holographic superconductor. Phys. Rev. D 81, 126009 (2010). [arXiv:1003.3278]
10. Gubser, S.S.: Breaking an Abelian gauge symmetry near a black hole horizon. Phys. Rev. D 78, 065034 (2008). [arXiv:0801.2977]
11. Montull, M., Pomarol, A., Silva, P.J.: The holographic superconductor vortex. Phys. Rev. Lett. 103, 091601 (2009). [arXiv:0906.2396]
12. Donos, A., Gauntlett, J.P., Sonner, J., Withers, B.: Competing orders in M-theory: superfluids, stripes and metamagnetism. JHEP 1303, 108 (2013). [arXiv:1212.0871]
13. Albash, T., Johnson, C.V.: A holographic superconductor in an external magnetic field. JHEP 0809, 121 (2008). [arXiv:0804.3466]
14. Montull, M., Pujolas, O., Salvio, A., Silva, P.J.: Magnetic response in the holographic insulator/superconductor transition. JHEP 1204, 135 (2012). [arXiv:1202.0006]
15. Iqbal, N., Liu, H., Mezei, M., Si, Q.: Quantum phase transitions in holographic models of magnetism and superconductors. Phys. Rev. D 82, 045002 (2010). [arXiv:1003.0101]
16. Hartnoll, S.A., Herzog, C.P., Horowitz, G.T.: Building a holographic superconductor. Phys. Rev. Lett. 101, 031601 (2008). [arXiv:0803.3295]
17. C. Lai, Q. Pan, J. Jing, Y. Wang, Analytical study on holographic superfluid in AdS soliton background, Phys. Lett. B 757, 65 (2016), [arXiv:1601.00134]
18. M. Rogatko, K.I. Wysokinski, Condensate flow in holographic models in the presence of dark matter, J. High Energy Phys. 1610, 152 (2016), [arXiv:1608.00343]
19. X.M. Kuang, E. Papantonopoulos, Building a holographic superconductor with a scalar field coupled kinematically to Einstein tensor, J. High Energy Phys. 1608, 161 (2016), [arXiv:1607.04928] [hep-th]
20. S.A.H. Mansoori, B. Mirza, A. Mokhtari, F.L. Dezaki, Z. Sherkatghanad, Weyl holographic superconductor in the Lifshitz black hole background, JHEP 1607, 111 (2016), [arXiv:1602.07245]
21. Y. Ling, P. Liu, C. Niu, J.P. Wu, Z.Y. Xian, Holographic entanglement entropy close to quantum phase transitions, JHEP 1604, 114 (2016), [arXiv:1502.03661]
22. R.G. Cai, L. Li, L.F. Li, R.Q. Yang, Introduction to holographic superconductor models, Sci. China, Phys. Mech. Astron. 58 (6), 060401 (2015), [arXiv:1502.00437]
23. Wu, Y.B., Lu, J.W., Zhang, C.Y., Zhang, N., Zhang, X., Yang, Z.Q., Wu, S.Y.: Lifshitz effects on holographic p-wave superfluid. Phys. Lett. B 741, 138 (2015), [arXiv:1412.3689]
24. B. Binaei Ghotbabadi, M. Kord Zangeneh, A. Sheykhi, One-dimensional backreacting holographic superconductors with exponential nonlinear electrodynamics, Eur. Phys. J. C 78, 381 (2018), [arXiv:1804.05442]
25. M. Mohammadi, A. Sheykhi and M. Kord Zangeneh, Analytical and numerical study of backreacting one-dimensional holographic superconductors in the presence of Born-Infeld electrodynamics, Eur. Phys. J. C 78, 654 (2018), [arXiv:1805.07377]
26. M. Mohammadi, A. Sheykhi and M. Kord Zangeneh, One-dimensional backreacting holographic p-wave superconductors, Eur. Phys. J. C 78, 984 (2018), [arXiv:1901.10540]
27. M. Mohammadi, A. Sheykhi, Conductivity of the holographic p-wave superconductors with higher order corrections, Eur. Phys. J. C 79, 473 (2019), [arXiv:1908.07992]
28. Mohammadi, M., A.: Sheykhi, textitConductivity of the one-dimensional holographic p-wave superconductors in the presence of nonlinear electrodynamics. Phys. Rev. D 100, 086012 (2019)
29. Cai, R.G., Yang, R.Q.: Paramagnetism-ferromagnetism phase transition in a dyonic black hole. Phys. Rev. D 90, 081901 (2014), [arXiv:1404.2856]
30. Cai, R.G., Yang, R.Q.: Holographic model for the paramagnetism/antiferromagnetism phase transition. Phys. Rev. D 91, 086001 (2015). [arXiv:1404.7737]
31. Cai, R.G., Yang, R.Q.: Coexistence and competition of ferromagnetism and p-wave superconductivity in holographic model. Phys. Rev. D 91, 026001 (2015). [arXiv:1410.5080]
32. N. Yokoi, M. Ishihara, K. Sato and E. Saitoh,Holographic realization of ferromagnets, Phys. Rev. D 93, 026002 (2016) [arXiv:1508.01626]
33. R. G. Cai and R. Q. Yang, Antisymmetric tensor field and spontaneous magnetization in holographic duality, Phys. Rev. D 92, 046001 (2015) [arXiv:1504.00855]

34. R. G. Cai, R. Q. Yang, Y. B. Wu and C. Y. Zhang, Massive 2-form field and holographic ferromagnetic phase transition, JHEP 021, 1511 (2015) [arXiv:1507.00546]

35. R. G. Cai and R. Q. Yang, Understanding strongly coupling magnetism from holographic duality [arXiv:1601.02936]

36. R. G. Cai, R. Q. Yang, Y. B. Wu and C. Y. Zhang, Holographic paramagnetism-ferromagnetism phase transition with the nonlinear electrodynamics, Can. J. Phys. 95, 450 (2017), [arXiv:1609.05040]

37. C. Y. Zhang, Y. B. Wu, Y. N. Zhang, H. Y. Wang and M. M. Wu, Holographic paramagnetism-ferromagnetism phase transition in the Born-Infeld electrodynamics, Nucl. Phys. B 914, 446 (2017), [arXiv:1609.03918v1]

38. Wu, Y.B., Zhang, C.Y., Lu, J.W., Fan, B., Shu, S., Liu, Y.C.: Holographic paramagnetism-ferromagnetism phase transition in the Born-Infeld electrodynamics. Phys. Lett. B 760, 469 (2016)

39. Jing, J., Pan, Q., Chen, S.: Holographic superconductors with Power-Maxwell field. JHEP 11, 045 (2011), [arXiv:1106.5181]

40. Jing, J., Jiang, L., Pan, Q.: Holographic superconductors for the Power-Maxwell field with backreactions. Class. Quantum Grav. 33, 025001 (2016)

41. A. Sheykhi, H. R. Salahi, A. Montakhab, Analytical and Numerical Study of Gauss-Bonnet Holographic Superconductors with Power-Maxwell Field, JHEP 04, 058 (2016) [arXiv:1603.00075]

42. Salahi, H.R., Sheykhi, A., Montakhab, A.: Effects of backreaction on power-maxwell holographic superconductors in gauss-bonnet gravity. Eur. Phys. J. C 76, 575 (2016). [arXiv:1608.05025]

43. A. Sheykhi, F. Shamsi, S. Davatolhaghi The upper critical magnetic field of holographic superconductor with conformally invariant Power Maxwell electrodynamics, Nucl. Phys. B 921, 112 (2020). [arXiv:1609.05040]

44. D. Hashemi Asl, A. Sheykhi, Meissner-like effect and conductivity of power-Maxwell holographic superconductors, Phys. Rev. D 101, 026012 (2020), [arXiv:1905.11810]

45. A. Sheykhi, D. Hashemi Asl, A. Dehyadegari, Conductivity of higher dimensional holographic superconductors with nonlinear electrodynamics, Phys. Lett. B 781, 139 (2018) [arXiv:1803.05724]

46. Kachru, S., Liu, X., Mulligan, M.: Gravity duals of lifshitz-like fixed points. Phys. Rev. D 78, 106005 (2008). [arXiv:0808.1725]

47. A. Sheykhi, F. Shamsi, S. Davatolhaghi The upper critical magnetic field of holographic superconductor with conformally invariant Power Maxwell electrodynamics, Can. J. Phys. 95, 450 (2017), [arXiv:1609.05040]

48. D. Hashemi Asl, A. Sheykhi, Meissner-like effect and conductivity of power-Maxwell holographic superconductors, Phys. Rev. D 101, 026012 (2020), [arXiv:1905.11810]

49. A. Sheykhi, D. Hashemi Asl, A. Dehyadegari, Conductivity of higher dimensional holographic superconductors with nonlinear electrodynamics, Phys. Lett. B 781, 139 (2018) [arXiv:1803.05724]

50. A. Sheykhi, F. Shamsi, S. Davatolhaghi The upper critical magnetic field of holographic superconductor with conformally invariant Power Maxwell electrodynamics, Can. J. Phys. 95, 450 (2017), [arXiv:1609.05040]

51. Z. Sherkatghanad, B. Mirza, F. Lalehgani Dezaki, Exponential nonlinear electrodynamics and backreaction effects on Holographic superconductor in the Lifshitz black hole background, Int. J. Mod. Phys. D 26, 1750175 (2017), [arXiv:1708.04289]

52. Zhao, Z., Pan, Q., Jing, J.: Notes on analytical study of holographic superconductors with Lifshitz scaling in external magnetic field. Phys. Lett. B 735, 438 (2014). [arXiv:1311.6260]

53. M. Natsuume and T. Okamura, Holographic Lifshitz superconductors: Analytic solution, Phys. Rev. D 97, 066016 (2018), [arXiv:1801.03154]

54. Z. Sherkatghanad, B. Mirza, F. Lalehgani Dezaki, Exponential nonlinear electrodynamics and backreaction effects on Holographic superconductor in the Lifshitz black hole background, Int. J. Mod. Phys. D 26, 1750175 (2017), [arXiv:1708.04289]

55. Z. Sherkatghanad, B. Mirza, F. Lalehgani Dezaki, Exponential nonlinear electrodynamics and backreaction effects on Holographic superconductor in the Lifshitz black hole background, Int. J. Mod. Phys. D 26, 1750175 (2017), [arXiv:1708.04289]

56. Z. Sherkatghanad, B. Mirza, F. Lalehgani Dezaki, Exponential nonlinear electrodynamics and backreaction effects on Holographic superconductor in the Lifshitz black hole background, Int. J. Mod. Phys. D 26, 1750175 (2017), [arXiv:1708.04289]

57. Zhao, Z., Pan, Q., Jing, J.: Notes on analytical study of holographic superconductors with Lifshitz scaling in external magnetic field. Phys. Lett. B 735, 438 (2014). [arXiv:1311.6260]

58. M. Natsuume and T. Okamura, Holographic Lifshitz superconductors: Analytic solution, Phys. Rev. D 97, 066016 (2018), [arXiv:1801.03154]

59. Z. Sherkatghanad, B. Mirza, F. Lalehgani Dezaki, Exponential nonlinear electrodynamics and backreaction effects on Holographic superconductor in the Lifshitz black hole background, Int. J. Mod. Phys. D 26, 1750175 (2017), [arXiv:1708.04289]

60. Zhao, Z., Pan, Q., Jing, J.: Notes on analytical study of holographic superconductors with Lifshitz scaling in external magnetic field. Phys. Lett. B 735, 438 (2014). [arXiv:1311.6260]
59. Sheykhi, A., Hendi, S.H.: Power-law entropic corrections to Newton law and Friedmann equations. Phys. Rev. D 84, 044023 (2011)
60. Sheykhi, A., Hendi, S.H.: Rotating black branes in $f(R)$ gravity coupled to nonlinear Maxwell field. Phys. Rev. D 87, 084015 (2013)
61. A. Dehyadegari, M. Kord Zangeneh and A. Sheykhi, *Holographic conductivity in the massive gravity with power-law Maxwell field*, Phys. Lett. B 773, 344 (2017) [arXiv:1703.00975]
62. Pan, Q., Wang, B., Papantonopoulos, E., Oliveira, J., Pavan, A.B.: Holographic superconductors with various condensates in Einstein-Gauss-Bonnet gravity. Phys. Rev. D 81, 106007 (2010). [arXiv:0912.2475]
63. Zhao, Z., Pan, Q., Chen, S., Jing, J.: Notes on holographic superconductor models with the nonlinear electrodynamics. Nucl. Phys. B 871, 98 (2013). [arXiv:1212.6693]
64. Jing, J., Pan, Q., Chen, S.: Holographic superconductor/insulator transition with logarithmic electromagnetic field in Gauss-Bonnet gravity. Phys. Lett. B 716, 385 (2012). [arXiv:1209.0893]

**Publisher's Note** Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.