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Abstract. The article presents a model of mechanical contact of a spherical tip and a product with small forces according to elastic deformations (further - deformations) taking into account the appearance of material deformations in the gap between them along the perimeter of the contact zone in the form of so-called piles-up. Dependences are obtained that link the gap with the deformation and the piles-up size. A graph is constructed for these parameters, the general form of which is close to the function of a single jump ~1(l-R_{pile}), where R_{pile} is the radius of the pile-up.

The developed design and operating principle of the scanning hybrid fiber-optical measuring head (FOMH) based on the use of a small-sized spatial light modulator in the form of a fiber-optic piezoscanner with the formation of scanning movements of the laser beam and the measurement of gaps between the tip and the product are described. The possibilities of providing high-precision measurements of laser beam deflection angles during scanning movements by using acousto-optic heterodyne interference measurement systems are considered.

1. Introduction

Contact measurements are still the main way to determine the size of products for most devices, in particular for measuring heads [1,2] in coordinate measuring machines, active control devices [3-5] and for thread control [6-11]. Mechanical contact of the tip with the product with the application of small forces, which is typical for contact measurements of product sizes, is physically similar to nanoindentation, which arose as a result of a long evolution from methods of hardness control, in particular, the Brinel method. One of the reserves for improving the accuracy of such measurements by using high-strength and optically transparent tips may be to compensate for the resulting elastic deformations (later - deformations) of the ld, for example, by on-line video measurements of the size of the contact zone l_{zc} [12,13]. But the accuracy of such measurement is limited by the error of the video recorder measurements and the appearance of specific reversible material deformations along the perimeter of the contact zone so-called piles (pile-up) that increase the l_{zc} with a corresponding increase in the calculated values of the ld.

In addition, ensuring high scanning accuracy requires the creation and use of a high-precision laser beam transverse displacement meter as a feedback sensor. The resolution of such meters based on matrix spatially sensitive recorders, allows to reach a level that is usually comparable to the pixel size, i.e. no higher than 3-4 μm. Resolution capabilities up to the subpixel level do not provide a drastic improvement, and the use of acousto-optic heterodyne interference measurement systems, whose resolution for lateral displacements of the laser beam can be \approx 12 nm [14], with an increase of more than 100 times, may be more promising.

2. Formulation of the problem

In connection with the above, the objectives of this work are to consider the following issues:
1) construction of a model of mechanical contact of the tip with the product under the condition of elastic piles,
2) features of building a scanning hybrid FOMH by using a small-sized spatial light modulator in the form of a fiber-optic piezoscanner [15-19],
3) increasing the accuracy of measuring the scanning movements of the laser beam by using acousto-optic heterodyne interference measurement systems.

3. Theory

The tasks of research involve consideration of issues of determining the dependency of the gap between the spherical tip and the workpiece in the presence of piles, the features of construction and principle of operation hybrid FOMH and scanning capabilities improve accuracy of measurement of angular deviations of the laser beam.

3.1. Geometric contact model

As shown above, in the process of mechanical contact in the gap between the spherical tip and the product along the perimeter of the contact zone, there are ideally completely reversible deformations of the material - piles-up. Video recording of such reversible piles-up is difficult, but images of plastic piles-up materials formed by the Berkovich pyramid in the process of measuring the hardness of materials using nanoindentation methods are shown in Figure 1.

![Figure 1](image)

Figure 1. Images of plastic piles-up formed by the Berkovich pyramid in the process of measuring the hardness of materials using nanoindentation methods [20].

To understand the nature of the change in the gap between the tip and the product, taking into account the formation of the bulk, a geometric scheme of mechanical contact between the tip and the product with a gap filled with elastic pile-up is constructed (Figure 2a, b). This scheme allows to calculate the gap consisting of the upper \( l_{gup} \) and lower \( l_{glo} \) gaps, defined as \( l_{gup} = l_{gup1} + l_{gup2} \) and \( l_{glo} = l_{glo1} + l_{glo2} \), respectively. In this case, the first components of each equation \( l_{gup1} \) and \( l_{glo1} \) are connected from above and below the rounded shape of the pile-up peak.

\[
\cos \alpha = \frac{|AB|}{|AF|} = \frac{|AD| - |CD| - |BC|}{|AE| + |EF|} = \frac{R_{tip} - l_{glo}}{R_{tip} + R_{pil}}. \tag{1}
\]

We can ignore the angular deviations of the laser beam when it is displaced at the level of units of micrometers corresponding to the size of the pile-up due to the significant excess of the tip radius \( R_{tip} \) over the pile-up radius \( R_{pil} \), \( \frac{R_{tip}}{R_{pil}} >> 500...1000 \). Therefore, we can use the Pythagorean theorem to calculate \( l_{gup1} \)

\[
l_{gup1} = R_{tip} - \sqrt{R_{tip}^2 - l_{dis}^2} \tag{2}
\]
and $l_{\text{gap}2}$

$$l_{\text{gap}2} = R_{\text{tip}} - \sqrt{R_{\text{tip}}^2 - l_{\text{dis}}^2}. \quad (3)$$

**Figure 2.** Model of pile-up formation in the gap between the tip and the product (a), with an enlarged scale (b)

However, taking into account the significant excess of $R_{\text{tip}}$ over $l_{\text{dis}}$: $R_{\text{tip}} > 1000 l_{\text{dis}}$, we know that the change of $l_{\text{dis, pil}}$ in expression (2) can be ignored and then obtained as a result

$$l_{\text{gap}} = R_{\text{pil}} - \sqrt{R_{\text{pil}}^2 - l_{\text{dis}}^2}. \quad (4)$$

The calculation of the lower gap $l_{\text{glo}}$ is made for the case when the pile-up has a rounded, protruding shape with the formation between the points $O$ and $L$ (Figure 2b) "depressions", i.e. the concavity of the product material that does not extend ("not bulging") beyond the OL line. The length of the lower gap $l_{\text{glo}}$ can be defined as the sum of the lengths of several segments:

$$l_{\text{glo}} = |ON| + |NM| + |ML| + \Delta l_{\text{glo}} = R_{\text{pil}} + l_{\text{dis}} + R_{\text{pil}} \cdot \tan \alpha + \Delta l_{\text{dis}} \cdot \tan \alpha = R_{\text{pil}} + l_{\text{dis}} + (R_{\text{pil}} + \Delta l_{\text{dis}}) \cdot \tan \alpha \quad (5)$$

Using the similarity of the triangles $ABF$ and $FHK$ (Figure 2a,b) we can write

$$\cos \alpha = \frac{|FH|}{|FK|} = \frac{|FH|}{|FK| + |JK|} = \frac{R_{\text{pil}}}{l_{\text{dis}} + l_{\text{dis}}},$$

where $l_{\text{dis, pil}}$ is the length of the segment $[NM]$, i.e. the initial, zero value of the lower gap. Then we can output an expression $l_{\text{dis, pil}} = R_{\text{pil}} \left( \frac{1}{\cos \alpha} - 1 \right)$ and set the formula (1) in it we get

$$l_{\text{glo}} = R_{\text{pil}} \left( \frac{R_{\text{pil}} + l_{\text{dis}} - R_{\text{pil}}}{R_{\text{pil}} - l_{\text{dis}} - R_{\text{pil}}} \right) = \frac{R_{\text{pil}} \cdot (2R_{\text{pil}} + l_{\text{dis}})}{R_{\text{pil}} - l_{\text{dis}} - R_{\text{pil}}} \approx \frac{R_{\text{pil}} \cdot (2R_{\text{pil}} + l_{\text{dis}})}{R_{\text{pil}}}. \quad (6)$$

Taking into account that $R_{\text{tip}} > R_{\text{pil}}$, we can assume that the lines (OL) and (EK) are parallel, and then as follows from the diagram in Figure 2b $R_{\text{pil}} = |FE| = |FH| = |FJ| = |FO| = |JN| = |ON| = |KM|$. We can write the equation $|ML| = |ML| = R_{\text{pil}} \cdot \tan \alpha$ due to the fact that the angle $\angle MKL = \alpha$, and $R_{\text{pil}} = |KM|$. As a result the equation for the lower gap $l_{\text{glo}}$ can be written

$$l_{\text{glo}} = R_{\text{pil}} + \frac{R_{\text{pil}} \cdot (2R_{\text{pil}} + l_{\text{dis}})}{R_{\text{pil}}} + R_{\text{pil}} \cdot \tan \alpha + \Delta l_{\text{dis}} \cdot \tan \alpha = R_{\text{pil}} + \frac{R_{\text{pil}} \cdot (2R_{\text{pil}} + l_{\text{dis}})}{R_{\text{pil}}} + (R_{\text{pil}} + \Delta l_{\text{dis}}) \cdot \tan \alpha \quad (7)$$
Using trigonometric equality $\tan \alpha = \sqrt{\frac{1}{\cos^2 \alpha} - 1}$, we can be converted the formula (1) to the form

$$tg \alpha = \sqrt{(R_{np} + R_{pil})^2 - (R_{np} - l_d - R_{pil})^2} \quad \text{and, by converting the numerator to account for the neglect of terms}$$

$$R_{pil}^2 \approx l_d^2 \approx (R_{pil} + l_d)^2 \approx 0,$$

we have

$$tg \alpha \approx \frac{\sqrt{R_{np}^2 + 2R_{np} \cdot l_d + R_{np}^2 + 2R_{np} \cdot l_d + 2R_{np} \cdot R_{pil}}}{R_{np} - l_d - R_{pil}} \approx \frac{2R_{np}(2R_{pil} + l_d)}{R_{np} - l_d - R_{pil}}.$$

And then change the expression (7) to the following form:

$$l_{glo} = R_{pil} + \frac{R_{pil} \cdot (2R_{pil} + l_d)}{R_{np}} + (R_{pil} + \Delta l_{dist}) \cdot \frac{\sqrt{2R_{np}(2R_{pil} + l_d)}}{R_{np} - l_d - R_{pil}} \quad (8)$$

If $l_{dist} = R_{pil}$ according to expression (4) we have $l_{gup} = R_{pil}$ and accordingly the formula for $l_g$ takes the form:

$$l_g = l_{gup} + l_{glo} = 2R_{pil} + \frac{R_{pil} \cdot (2R_{pil} + l_d)}{R_{np}} + (R_{pil} + \Delta l_{dist}) \cdot \frac{\sqrt{2R_{np}(2R_{pil} + l_d)}}{R_{np} - l_d - R_{pil}} \quad (9)$$

Then, taking into account all the above, the expression for the gap $l_g$ can be written as a system of equations for two ranges: the upper $l_{glo}$ range at $[0; R_{pil}]$ and the total $l_g$ range (upper and lower ranges: $l_{gup}+l_{glo}$) in the area exceeding the position $R_{pil}$: $[R_{pil} ; n \cdot R_{pil}]$:

$$l_g = \begin{cases} R_{pil} - \sqrt{R_{np}^2 - l_{dis}^2}, & \text{with } l_{dis} \in [0; R_{pil}] \\ 2R_{pil} + \frac{R_{pil} \cdot (2R_{pil} + l_d)}{R_{np}} + (R_{pil} + \Delta l_{dist}) \cdot \tan \alpha, & \text{with } l_{dis} \in [R_{pil}; n \cdot R_{pil}] \end{cases} \quad (10)$$

where $n$ is a natural number.

Based on the expression (10) in Figure 3 the graph for $l_g$ is constructed in the form close to the function of a single jump ~1 (l-R_{pil}). As you can see, this graph includes two sections [O; A] and [B;C] with smooth changes in the function $l_g$ (l_{dis}) corresponding to the two gaps tip-pile-up and tip-product. The section [A;B] is transitional from the first to the second of the above-mentioned gaps. The feature of this graph can be used to select $l$ values that exclude the influence of pile-up on the measurement process during scanning movements of the laser beam.

3.2. Design and operating principle of the scanning hybrid fiber-optic measuring head (FOMH).

In the development of a series of author's works on single-channel and multi-channel hybrid FOMH [1,2], a compromise version with scanning was developed. In order to improve the accuracy of contact measurements due to high-precision measurement of deformations in contact with the spherical tip of the product, a scanning hybrid FOMH was developed based on a new spatial light modulator in the form of a fiber-optic piezoscanner [15-19].
Figure 3. Dependence of the gap $l_g$ between the tip and the product from the displacement $l_{dis}$.

Figure 4a-e shows a scanning system (without an electromechanical unit that generates a trigger signal during mechanical contact) consisting of a low-coherence interferometer 1, a control unit 2, a fiber-optic piezoscanner 3, a first 4 and second 6 GRIN lens, a beam splitter 5, an optical system 7, a video recorder 8 with a matrix of $m \times n$ sensitive elements, an ultra-wide-angle optical system of the "fisheye" 9, and tip 10 made from transparent and high-strength material, such as corundum (leucosapphire, sapphire, ruby), aluminum oxynitride ($\text{AlON/Al}_{23}\text{O}_{27}\text{N}_{5}$), stishovite and similar.

During the contact measurements the piezoscanner 3 uses a signal from the control unit 2 to perform transverse oscillations (as an option, in a spiral with periods of expansion and contraction), respectively, deflecting the collimated microlens in the angular sector up to $100^\circ \times 100^\circ$ [22] (Figure 4b) an outgoing laser beam coming from a low-coherence interferometer 1. Then this laser beam, moving along a curved path through the first GRIN lens 4, is divided by the beam splitter 5 into two streams. The first stream after deflecting passes through the optical system 7 and illuminating the video recorder 8, which forms a reference.
channel and measures the current position of the center of the laser beam, allows forming a feedback signal for the control unit 2. The second stream after the beam splitter 5 passes sequentially through the second GRIN lens 6, a "fisheye" 9, tip 10, scanning by a collimated laser beam the surrounding space in the angular sector of the hemisphere up to \(180^\circ \times 180^\circ\) (azimuth and angle of location).

When the product 11 is illuminated and reflected from it, a portion of the laser radiation follows in the opposite direction through the tip 10, GRIN lenses 4 and 6, the beam splitter 5, and the piezoscanner 3 to the low-coherence interferometer 1. The algorithm for measuring it, as in the previously developed FOMH [1,2], is based on fixing the equality of the path difference \(l_{ci} = l_0\), set in the low-coherence interferometer 1, to the value of the gap \(l_g\) between the tip 10 and the product 11. By changing the \(l_{ci}\) and deflecting the laser beam, for example, along a spiral trajectory in the angular sector of the \(180^\circ \times 180^\circ\) hemisphere, it is possible to non-contact scan (feel) the surrounding space, and in particular, the product 11, and the occurrence of the \(l_{ci} = l_g\) event corresponds to the so-called "virtual touch" with the angular position of the illuminating beam normal to the surface of the product 11.

A series of non-contact coordinate measurements obtained in a spherical coordinate system, when probing for such a scanning FOMH, for each measured point of the product, will consist of a linear and two angular coordinates \((l_{ci}, \beta, \gamma)\). And its can be converted to the Cartesian coordinate system adopted in almost all coordinate measuring machines, given the coordinates of the FOMH (x0, y0, z0), the position of the piezoscanner \((X_{fomh}, Y_{fomh}, Z_{fomh})\), as well as taking into account the changes of video recorder 8: \(\beta = k_n l_{xmh}\) and \(\gamma = k_a l_{ymh}\):

\[
\begin{align*}
x_{ccm} &= x_{fomh} + x_0 + l_{ci} \cdot \sin \beta \cdot \cos \gamma, \\
y_{ccm} &= y_{fomh} + y_0 + l_{ci} \cdot \sin \beta \cdot \sin \gamma, \\
z_{ccm} &= z_{fomh} + z_0 + l_{ci} \cdot \cos \beta.
\end{align*}
\]  

(11)

where \(k_n, k_a, l_{xmh}, l_{ymh}\) - the conversion coefficients and the number of sensitive elements M×N of the video recorder 8.

Method for measuring the size of the contact zone by measuring a series of gaps \(l_{g1}, l_{g2}, \ldots, l_{gn}\) between the tip and the product near its border at various points with coordinates \((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\) with an error at the level of fractions of a micrometer

\[l_d = R_{op} - (R_{ip} - l_g) \cdot \cos (k \cdot l_{dis})\]  

(12)

So, for a series of measurements with movements of the laser scanning beam around the contact zone along a series of concentric circles, for example, equidistant rings with spiral transitions between them (Figure 2e) similar to Newton’s interference rings, a set of \(n\) values of deformations \(l_{d1}, l_{d2}, \ldots, l_{dn}\) is calculated by using the system of equations:

\[
\begin{align*}
l_{d1} &= R_{op} - (R_{ip} - l_{g1}) \cdot \cos (k \cdot l_{dis1}), \\
l_{d2} &= R_{op} - (R_{ip} - l_{g2}) \cdot \cos (k \cdot l_{dis2}), \\
& \quad \vdots , \\
l_{dn} &= R_{op} - (R_{ip} - l_{gn}) \cdot \cos (k \cdot l_{disn}).
\end{align*}
\]  

(13)

from which the desired average value \(l_d\) is determined:

\[l_d = \frac{1}{n} \sum_{i=1}^{n} l_{di},\]  

(14)

The obtained value of elastic deformations \(l_d\) is subtracted from the measurement result increasing the accuracy of measurements.
3. High-precision measurement of the lateral displacements of the laser beam during its scanning movements.

Using a video recorder based on matrix spatially sensitive elements as a feedback sensor for measuring the lateral displacements of the laser beam can be considered a standard solution. However, alternative measurement technologies have already been developed using single- and two-coordinate acousto-optic modulators (AOM) as spatially sensitive transducers (Figure 5). According to [14,23], when using a water-based AOM with a modulation frequency of \( f_{\text{mod}} \approx 8 \text{ MHz} \) and an ultrasonic wave length of \( \Lambda \approx 200 \mu\text{m} \), as well as a measurement scheme based on a phase-locked phase system with phase-digital conversion, the resolution can reach \( \approx 12 \text{ nm} \) [14] (Figure 5b). This is more than 100 times less than if you use a video recorder with dimensions of spatially sensitive elements \( \approx 3-4 \mu\text{m} \). As you can see, the use of measurement schemes based on AOM for accuracy indicators is more promising. The natural limitation of this measurement technology is the large mass-dimensional indicators and prospects for progress on which will be considered in further author’s research.

![Figure 5](image_url). Laser beam lateral displacements meters based the AOM: two single-coordinate (a), single-coordinate with phase-digital conversion in the phase-locked phase system (b) [14].

4. Experimental result

The experimental development and operation of individual component blocks are discussed in thematic articles: on low-coherence interferometers [24,25], fiber-optic piezoscanners [15-19], the formation of photopolymer lenses at the ends of optical fibers [21] and ultra-wide-angle optical systems of the “Fisheye” type [22]. Experimental studies conducted in [23] using two independent AO modulators confirmed the high resolution of measurements of the lateral displacements of the laser beam. The reliable operation of the two-coordinate AO modulator was also developed and experimentally confirmed (Figure 6).
5. Discussion of results

5.1. Mechanical contact of the tip with the product with the application of physical force is similar to nanoindentation, which arose as a result of a long evolution from methods of hardness control, similar to the Brinel method. The manifestation of elastic properties of product materials leads to the appearance of specific surface formations in the form of elastic piles-up along the perimeter of the contact zone, increasing its size. The use of methods for determining deformations with measuring the size of the contact zone between the tip and the product the appearance of such piles-up reduces the accuracy of the measurement and makes it necessary to take them into account in the calculations.

5.2. The gap function consists of two smooth sections and one non-linear section. The first smooth portion and formed the emerging non-linear elastic piles-up, while the third plot a smooth surface of the product. Measuring the gaps between the tip and the product outside the piles-up, i.e. after a nonlinear section with high-precision registration of two coordinates of the scanning laser beam, allows you to determine the size and spatial position of the contact zone without taking into account the piles-up and, accordingly, more accurately the deformation and size of the product.

5.3. For high-precision measurement of the lateral displacements of the laser beam in the scanning process, the most promising measurement systems are those using AOM as a coordinate-sensitive transducer and together with phase-locked phase or frequency systems, respectively, with phase-or frequency-digital conversion.

6. Conclusion

6.1. It is necessary to determine the actual values of the piles-up resulting from the contact of the spherical tip with the product for different materials with different clamping forces.

6.2. Additional research is required to improve high-precision systems for measuring lateral displacements based on AOM in the areas of expanding functional capabilities and reducing their mass and dimensions.

6.3. The issues discussed in the article allow us to make a significant contribution to the creation of a promising model of scanning hybrid FOMH with the highest tactical and technical characteristics in terms of functionality and measurement accuracy.

7. References

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