Nonstrange hybrid compact stars with color superconducting matter

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Realistic nonstrange hybrid compact stars with color superconducting quark matter in their interior are constructed. It is shown that a positively charged two-flavor color superconducting phase could naturally appear in the core of a hybrid star as one of the components of a globally neutral mixed phase. The negatively charged normal quark phase is the other component of the mixed phase. The quark core of the star is surrounded by another mixed phase made of hadronic and normal quark matter. The two mixed phases are separated by a sharp interface. Finally, the lowest density regions of the star are made of pure hadronic matter and nuclear crust.

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I. INTRODUCTION

At large densities, quantum chromodynamics (QCD) becomes a weakly interacting theory of quarks. The quarks tend to form a highly degenerate Fermi surface. Because of the existence of an attractive interaction in the color-antitriplet channel, the famous Cooper instability develops, and the ground state of the system is rearranged. The rearrangement is similar to the Bardeen-Cooper-Schrieffer (BCS) mechanism of low temperature superconductivity in metals and alloys. The ground state of dense quark matter is a color superconductor. It is characterised by spontaneous breaking (through the Anderson-Higgs mechanism) of the non-Abelian SU(3)_c color gauge group rather than the Abelian U(1)_em group of electromagnetism.

Weakly interacting quark matter at asymptotic densities was studied from first principles in Refs. However, the microscopic studies are not very reliable quantitatively at realistic densities that exist in nature (e.g., inside compact stars). The corresponding running coupling constant of QCD defined at the relevant scale of the quark chemical potential is large, and therefore the results cannot be trusted.

It might be appropriate to mention that the creation of cold dense quark matter with color superconducting properties in heavy ion collisions seems very unlikely to us (see, however, some speculations in Ref. ). In particular, it does not look plausible that one could avoid producing a large entropy per baryon in any type of a heavy ion collision. Therefore, a cold and dense environment, needed to support color superconductivity, could hardly be formed. A typical central region of a compact star, on the other hand, appears to be a very natural place where sufficiently cold and dense matter could exist. Indeed, the estimated central densities of such stars might be as large as 10^5 g/cm^3 (where \( \rho_0 \approx 0.15 \text{ fm}^{-3} \) is the saturation density), while their temperatures are in the range of 10 to 100 keV.

In order to study quark matter at realistic densities, currently there is no other real alternative but to rely on various phenomenological models (such, for example, as Nambu-Jona-Lasinio type models). The obvious shortcoming of this approach is the same as that of the microscopic approach: the results should always be interpreted with caution and treated as qualitative rather than quantitative. Nevertheless, if one accepts the possibility of color superconductivity in quark matter at realistic densities, existing in compact stars, it is not so important which specific model is used to estimate the effect of color superconductivity on properties of compact stars.

In general, QCD at high baryon density has a very rich phase structure. There are many possible color superconducting phases of quark matter made of one, two and three lightest quark flavors. Each of them is characterised by a unique symmetry breaking pattern and by a specific number of bosonic as well as fermionic gapless modes. In the rest of this paper, we are going to concentrate our attention almost exclusively on matter with two quark flavors. In order to better understand our motivation, let us also briefly mention other possibilities.

A one-flavor color superconductor is characterised by a condensate of Cooper pairs made of the same quark flavor. Thus, the corresponding wave function is symmetric in flavor. By taking into account that the condensation results from the attractive interaction between quarks in the color antitriplet antisymmetric channel, one finds that a spin zero configuration is forbidden by the Pauli principle. Therefore, it is a much weaker spin-1 condensate that gives rise to color superconductivity in one-flavor dense quark matter. The recent study in Ref. suggests that the presence of such a phase may still affect some properties of compact stars.
Perhaps, the most interesting color superconducting phase appears in quark matter with three light flavors. It is the so-called color-flavor-locked (CFL) phase \[\text{12}\]. In this phase the original gauge symmetry SU(3)_c and the global chiral symmetry SU(3)_L \times SU(3)_R break down to a global diagonal SU(3)_{c+L+R} subgroup. As a result, the quark quasiparticles get large gaps in their energy spectra and decouple from the low energy dynamics. Because of the Higgs mechanism all gluons also become massive and decouple. Then, it appears that the low energy spectrum of the CFL phase is dominated by one Nambu-Goldstone boson and nine massive pseudo-Nambu-Goldstone bosons \[\text{12}\]. This observation has interesting implications for the transport properties of the CFL phase \[\text{16}\].

In order to have the CFL phase in compact stars, the strange quark should be sufficiently light at the corresponding densities. However, the actual density dependence of the constituent, medium modified mass of the strange quark is not known in QCD. Taking this into account, one could imagine that the strange quark might be too heavy even to appear in baryonic matter at realistic conditions inside compact stars. Then, of course, the CFL quark phase could not be realized. In yet another scenario, the strange quarks could appear inside stars, but they would not participate in Cooper pairing. Then, a mixture of 2SC matter and normal matter of strange quarks could form. We denote such a phase 2SC+s.

Matter in the bulk of a compact star should be neutral (at least, on average) with respect to electric as well as color charges. Otherwise, the gravitational force would not be able to hold the star together. Also such matter should remain in \(\beta\)-equilibrium. These conditions turn out to play an important role in determining which color superconducting phases can and which cannot exist inside stars. Indeed, the strongest color superconducting phases appear when the Fermi momenta of different quark flavors are approximately equal. By enforcing the neutrality and \(\beta\)-equilibrium conditions, however, one does not necessarily get the Fermi surfaces of quarks that are best suitable for Cooper pairing. In some cases, these conditions may even appear to be inconsistent with color superconductivity.

Recently, it was argued in Ref. \[\text{18}\] that imposing the charge neutrality condition on quark matter inside compact stars would prevent the appearance of the 2SC+s phase, and would favor the appearance of the CFL phase instead. (Note that the strange quark mass was chosen too small to allow the appearance of the pure 2SC phase in Ref. \[\text{17}\].) This conclusion was essentially confirmed in a more rigorous study of Ref. \[\text{18}\] where it was found that the 2SC+s phase could exist only in a narrow window (about 10 or 15 MeV wide) of the baryon chemical potential around the midpoint \(\mu_B/3 \approx 450\) MeV. In this phase, a positive charge of the 2SC condensate is compensated by a negative charge of unpaired strange quarks. At smaller values of the chemical potential, no conventional 2SC was detected in Ref. \[\text{18}\]. It was shown in Ref. \[\text{19}\] that a new type of neutral 2SC phase, the so-called gapless 2SC phase develops at the corresponding densities. In this gapless 2SC phase, the conventional relations between the number densities of the pairing quarks are not valid.

While matter in the bulk of a star should satisfy the condition of charge neutrality, this condition should not necessarily be enforced locally. In fact, it is sufficient to make matter neutral on average, or \textit{globally}. This kind of matter appears naturally in mixed phases. Recently, studies of various mixed phases with color superconducting matter were presented in Ref. \[\text{21}\]. In this paper, we use this general idea to construct a realistic model of a compact star with two-flavor color superconducting matter at the core (for constructions using 2SC quark matter without mixed phases see Ref. \[\text{22, 23}\], and for constructions using CFL quark matter see Ref. \[\text{24, 25}\]).

This paper is organized as follows. In the next section, we present a quark model with a four-fermion interaction that we use to derive the quark matter equation of state. There, we also briefly discuss the phase diagram of the model, and the subtleties of enforcing the electrical neutrality condition in color superconducting matter. In Sec. \[\text{III}\] we present the Gibbs construction for the mixed phase of normal and color-superconducting quark matter. In Sec. \[\text{IV}\] we discuss a model for hadronic matter, and the corresponding equation of state with the charge neutrality condition imposed. In the same section, we apply the Gibbs construction to derive the equation of state of the hadron-quark mixed phase. The star properties are discussed in Sec. \[\text{V}\] and the summary and outlook are given in Sec. \[\text{VI}\].

\section{II. Quark Model}

We start our analysis from discussing the quark model that we use to derive the equation of state of baryonic matter at high density. In this study, we accept a rather conservative point of view that the strange quark is sufficiently heavy and does not appear at baryon densities that can be reached in compact stars without causing a gravitational instability. It is also appropriate to mention that we aim at a general qualitative rather than quantitative description of hybrid stars with color superconducting matter in their interior. These assumptions justify the use of the simplest SU(2) Nambu-Jona-Lasinio model of Ref. \[\text{24}\] in our study. The explicit form of the Lagrangian density reads:

\[\mathcal{L} = \bar{q}(i\gamma^\mu \partial_\mu - m_0)q + G_S \left[ (\bar{q}\tilde{q})^2 + (\bar{q}i\gamma_5\tau q)^2 \right] + G_D \left[ (i\bar{q}^C C \gamma^5 q)(i\bar{q}^C C \gamma^5 q)^C \right],\]

\[\text{(1)}\]

where \(q^C = C\bar{q}^T\) is the charge-conjugate spinor and \(C = i\gamma^2\gamma^5\) is the charge conjugation matrix. The quark field \(q \equiv q_0\) is a four-component Dirac spinor that carries flavor \((i = 1, 2)\) and color \((\alpha = 1, 2, 3)\) indices. \(\tau = (\tau^1, \tau^2, \tau^3)\) are Pauli matrices in the flavor space,
and \((\varepsilon)^{ik} \equiv \varepsilon^{ik}\), \((\varepsilon^b)^{\alpha\beta} \equiv \varepsilon^{\alpha\beta b}\) are antisymmetric tensors in flavor and color, respectively. We also introduced two independent coupling constants in the scalar quark-antiquark and scalar diquark channels, \(G_S\) and \(G_D\). The definition of this non-renormalizable NJL model is complete after introducing a momentum cut-off \(\Lambda\).

The values of the parameters in the NJL model are the same as in Refs. 14, 20: \(G_S = 5.0163\) GeV\(^{-2}\) and \(\Lambda = 0.6533\) GeV. Here we also consider only the chiral limit with \(m_0 = 0\). As for the strength of the diquark coupling \(G_D\), its value is taken to be proportional to the quark-antiquark coupling constant, i.e., \(G_D = \eta G_S\) with \(\eta = 0.75\). The choice \(\eta = 0.75\) corresponds to the regime of intermediate strength of the diquark coupling [14]. Regarding this point, we mention that our final results would remain qualitatively the same also for a wide range of weak and strong diquark couplings.

In \(\beta\)-equilibrium, the diagonal matrix of quark chemical potentials is given in terms of baryonic, electric and color chemical potentials. In particular,

\[
\mu_{ij,\alpha\beta} = (\mu_{ij} - \mu_e Q_{ij}) \delta_{\alpha\beta} + \frac{2}{\sqrt{3}} \mu_s \delta_{ij} (T_8)_{\alpha\beta},
\]

where \(Q\) and \(T_8\) are generators of \(U(1)_{em}\) of electromagnetism and \(U(1)_a\) subgroup of the color gauge group. The explicit expressions for the quark chemical potentials read

\[
\mu_{ur} = \mu_{ug} = \mu - \frac{2}{3} \mu_e + \frac{1}{3} \mu_s,
\]

\[
\mu_{dr} = \mu_{dg} = \mu + \frac{1}{3} \mu_e - \frac{1}{3} \mu_s,
\]

\[
\mu_{ub} = \mu - \frac{2}{3} \mu_e - \frac{2}{3} \mu_s,
\]

\[
\mu_{db} = \mu + \frac{1}{3} \mu_e - \frac{2}{3} \mu_s.
\]

One should notice that, in general, there are two generations of quarks. In particular, the same as in Refs. \[19, 20\]:

\[
\Omega = \Omega_0 - \frac{\mu_e^4}{12\pi^2} + \frac{m^2}{4G_S} = \frac{\Delta^2}{4G_D} - 2 \int_0^\Lambda \frac{p^2 dp}{\pi^2} \left(E(p) + \sqrt{E(p)^2 + \mu_e^2 + \Delta^2} + \sqrt{E(p)^2 + \mu_e^2 + \Delta^2} \right)
\]

where \(\mu_v^\pm \equiv \mu \pm \sqrt{(\delta \mu)^2 - \Delta^2}\). Note that the physical thermodynamic potential that determines the pressure, \(\Omega_{\text{phys}} = -P\), is obtained from \(\Omega\) in Eq. (14) after substituting the order parameter \(\Delta\) that solves the following gap equation:

\[
\frac{\partial \Omega}{\partial \Delta} = 0.
\]

In addition, homogeneous quark matter in the bulk of a compact star should be neutral with respect to color and electric charges. Therefore, for such matter, one should also impose extra two (local) conditions:

\[
n_s \equiv \frac{\partial \Omega}{\partial \mu_s} = 0, \quad n_e \equiv \frac{\partial \Omega}{\partial \mu_e} = 0.
\]
that the pressure difference than normal quark matter \cite{19}. Numerically, we find the potential, gapless 2SC matter has a slightly higher pressure than normal quark matter. For any given value of the baryon chemical potential, gapless 2SC matter has a slightly higher pressure than normal quark matter.\cite{10}. Numerically, we find that the pressure difference $\delta P$ lies somewhere between 1 and 2.5 MeV/fm$^3$. Therefore, the gapless 2SC phase is slightly more favorable than the neutral normal quark phase.

In addition to homogeneous (one-component) phases, one could also study various mixed phases of two-flavor quark matter. The most promising components for constructing mixed phases are (i) normal quark matter, (ii) gapless 2SC matter, and (iii) ordinary 2SC matter. The first two of them are included because they allow locally neutral phases by themselves. The last one, on the other hand, is included because it is the strongest color superconducting phase known. Clearly, this list of phases is not complete. One could also take into consideration other phases, e.g., various combinations of color superconductors with spin-1 condensates $\rho_{e}$. Our experience suggests that these latter would lead to less favorable one- and multi-component constructions. In order to prove this rigorously, a detailed study is required. This, however, is outside the scope of the present paper.

Inside mixed phases, the charge neutrality is satisfied “on average” rather than locally. This means that different components of mixed phases may have non-zero densities of conserved charges, but the total charge of all components still vanishes. In this case, one says that the local charge neutrality condition is replaced by a global one. Now, in absence of the local neutrality, the pressure of each phase could be considered as a function of baryon chemical potential, as well as a function of chemical potentials related to other conserved charges (e.g., $\mu_e$ and $\mu_\delta$ in the case of two-flavor quark matter at hand). In order to decide which mixed phase is the most favorable, one should utilize the so-called Gibbs construction. The details of this construction will be given in the next section.

While we intend to relax the local neutrality condition with respect to the electric charge, in this paper we always enforce the condition of local color neutrality. We cannot fully justify this constrain in all situations where mixed phases appear. However, one could speculate that color charged components are less likely to make energetically favorable mixed phases because of strong color Coulomb forces. Besides, in the two main constructions that we build in this paper, at least one of the components (normal quark or hadronic matter) of a mixed phase is expected to be locally color neutral. Therefore, our consideration here remains rather general.

The pressure of the main three phases of two-flavor quark matter as a function of the baryon and electric chemical potentials is shown in Fig. 1. In this figure, we also show the pressure of the neutral normal quark and gapless 2SC phases (two dark solid lines). In full agreement with the study of Ref. \cite{10}, the surface of the gapless 2SC phase extends only over a finite range of the values of $\mu_e$. It merges with the pressure surfaces of the normal quark phase (on the left) and with the ordinary 2SC phase (on the right).

It is interesting to notice that the three pressure surfaces in Fig. 1 form a characteristic swallowtail structure. As one could see, the appearance of this structure is directly related to the fact that the phase transition between color superconducting and normal quark matter, which is driven by changing parameter $\mu_e$, is of first order. In fact, one should expect the appearance of a similar swallowtail structure also in a self-consistent description of the hadron-quark phase transition. Such a description, however, is not available yet.

From Fig. 1 one could see that the surfaces of normal and 2SC quark matter intersect along a common line. This means that the two phases have the same pressure along this line, and therefore could potentially co-exist. Moreover, as is easy to check, normal quark matter is negatively charged, while 2SC quark matter is positively charged.
In this section we study the possibility of mixed phases in two-flavor quark matter. As we have already mentioned in the previous section, we restrict our consideration to only three most promising components: (i) normal quark matter, (ii) gapless 2SC matter, and (iii) ordinary 2SC matter. Before proceeding further, it would be appropriate to mention that some mixed phase with color superconducting components were also constructed in Ref. 21.

Let us start by giving a brief introduction into the general method of constructing mixed phases by imposing the Gibbs conditions of equilibrium 24–27. From the physical point of view, the Gibbs conditions enforce the mechanical as well as chemical equilibrium between different components of a mixed phase. This is achieved by requiring that the pressure of different components inside the mixed phase are equal, and that the chemical potentials (\(\mu\) and \(\mu_e\)) are the same across the whole mixed phase. For example, in relation to the mixed phase of normal and 2SC quark matter, these conditions read

\[
P^{(NQ)}(\mu, \mu_e) = P^{(2SC)}(\mu, \mu_e),
\]

\[
\mu = \mu^{(NQ)} = \mu^{(2SC)},
\]

\[
\mu_e = \mu_e^{(NQ)} = \mu_e^{(2SC)}.
\]

It is easy to visualize these conditions by plotting the pressure as a function of chemical potentials (\(\mu\) and \(\mu_e\)) for both components of the mixed phase. This is shown in Fig. 2. As should be clear, the above Gibbs conditions are automatically satisfied along the intersection line of two pressure surfaces (dark solid line in Fig. 2).

Different components of the mixed phase occupy different volumes of space. To describe this quantitatively, we introduce the volume fraction of normal quark matter as follows: \(\chi_{2SC}^{NQ} \equiv V_{NQ}/V\) (notation \(\chi_A^{B}\) means “volume fraction of phase A in a mixture with phase B”). Then, the volume fraction of the 2SC phase is given by \(\chi_{NQ}^{2SC} = (1 - \chi_{2SC}^{NQ})\). From the definition, it is clear that \(0 \leq \chi_{2SC}^{NQ} \leq 1\).

The average electric charge density of the mixed phase is determined by the charge densities of its components taken in the proportion of the corresponding volume fractions. Thus,

\[
n_c^{(MP)} = \chi_{2SC}^{NQ} n_c^{(NQ)}(\mu, \mu_e) + (1 - \chi_{2SC}^{NQ}) n_c^{(2SC)}(\mu, \mu_e).
\]

If the charge densities of the two components have opposite signs, one can impose the global charge neutrality condition, \(n_c^{(MP)} = 0\). Otherwise, a neutral mixed phase could not exist. In the case of quark matter, the charge density of the normal quark phase is negative, while the charge density of the 2SC phase is positive along the line of the Gibbs construction (dark solid line in Fig. 2). Therefore, a neutral mixed phase exists. The volume fractions of its components are

\[
\chi_{2SC}^{NQ} = \frac{n_c^{(2SC)}}{n_c^{(2SC)} - n_c^{(NQ)}},
\]

\[
\chi_{NQ}^{2SC} = 1 - \chi_{2SC}^{NQ} = \frac{n_c^{(NQ)}}{n_c^{(NQ)} - n_c^{(2SC)}}.
\]

After the volume fractions have been determined from the condition of the global charge neutrality, we could also calculate the energy density of the corresponding mixed phase,

\[
\varepsilon^{(MP)} = \chi_{2SC}^{NQ} \varepsilon^{(NQ)}(\mu, \mu_e) + (1 - \chi_{2SC}^{NQ}) \varepsilon^{(2SC)}(\mu, \mu_e).
\]

This is essentially all that we need in order to construct the equation of state of the mixed phase.

So far, we were neglecting the effects of the Coulomb forces and the surface tension between different components of the mixed phase. In a real system, however, these might be important. In particular, the balance between the Coulomb forces and the surface tension deter-
mines the geometries of different components inside the mixed phase.

In our case, nearly equal volume fractions of the two quark phases are likely to form alternating layers (slabs) of matter. The energy cost per unit volume to produce such layers scales as \( \sigma^{2/3}(n_{e}^{(2SC)} - n_{e}^{(NQ)})^{2/3} \) where \( \sigma \) is the surface tension \([28]\). Therefore, the quark mixed phase is a favorable phase of matter only if the surface tension is not too large. Our simple estimates show that \( \sigma_{\text{max}} \lesssim 20 \text{ MeV/fm}^2 \). However, even for slightly larger values, \( 20 \lesssim \sigma \lesssim 50 \text{ MeV/fm}^2 \), the mixed phase is still possible, but its first appearance would occur at larger densities, \( 3\rho_0 \lesssim \rho_B \lesssim 5\rho_0 \). The value of the maximum surface tension obtained here is comparable to the estimate in the case of the hadronic-CFL mixed phase obtained in Ref. \([28]\). The thickness of the layers scales as \( \sigma^{1/3}(n_{e}^{(2SC)} - n_{e}^{(NQ)})^{-2/3} \), and its typical value is of order 10 fm in the quark mixed phase. This is similar to the estimates in various hadron-quark and hadron-hadron mixed phases \([28, 29]\). While the actual value of the surface tension in quark matter is not known, in this study we assume that it is not very large. Otherwise, the homogeneous gapless 2SC phase should be the most favorable phase of nonstrange quark matter \([10]\).

Under the assumptions of this paper, the mixed phase of normal and 2SC quark matter is the most favorable neutral phase of matter in the model at hand. This should be clear from observing the pressure surfaces in Figs. 1 and 2. For a given value of the baryon chemical potential \( \mu = \mu_B/3 \), the mixed phase is more favorable than the gapless 2SC phase, while the gapless 2SC phase is more favorable than the neutral normal quark phase.

The validity of the quark model is limited when the baryon density decreases. From the physical point of view, at some point quark matter should become confined and the NJL model should fail to reproduce the correct equation of state of low density baryonic matter. Therefore, at low densities, it is appropriate to use a hadronic description of matter. This is discussed in the next section.

IV. LOW DENSITY REGION AND QUARK-HADRON PHASE TRANSITION

At densities around normal nuclear matter density \( \rho_0 \approx 0.15 \text{ fm}^{-3} \), the description of baryonic matter in terms of quarks could hardly be accurate. At such low densities, quarks are confined inside hadrons. Thus, it is more natural to use an effective hadronic model.

In the literature, there exist many hadronic models that have been studied in detail. In particular, the Walecka model \([30, 31]\) and its nonlinear extensions \([32, 33]\) are well known. These models have been quite successful and widely used for the description of finite nuclei, hadronic and neutron star matter. One could also use some hadronic versions of the NJL model \([34, 35, 36]\), as well as many others \([37, 38]\).

In the following we use a QCD-motivated hadronic chiral SU(3)\(_L\) × SU(3)\(_R\) model as an effective theory of strong interactions to describe the low density regime of the baryonic matter \([39, 40, 41]\). This model was found to describe reasonably well the hadronic masses of various SU(3) multiplets, finite nuclei, hypernuclei, excited nuclear matter and neutron star properties \([39, 40, 41]\). The basic features of the chiral model are: (i) Lagrangian of the model is constructed in accordance with the nonlinear realization of the chiral SU(3)\(_L\) × SU(3)\(_R\) symmetry; (ii) heavy baryons and mesons get their masses as a result of spontaneous symmetry breaking; (iii) the masses of pseudoscalar mesons (pseudo-Nambu-Goldstone bosons) result from an explicit symmetry breaking; (iv) a QCD-motivated field that describes the gluon condensate (dilaton field) is introduced.

It is expected that the phase transition between the hadronic phase and the normal quark phase is a first order phase transition at zero temperature and finite baryon chemical potential. Then, the hadronic and quark phases could co-exist in a mixed phase \([28, 29]\). This mixed phase should satisfy the Gibbs conditions of equilibrium which are similar to those discussed in the previous section, see Eqs. (18)–(20). The total energy density in the hadron-quark mixed phase is given by

\[
\epsilon^{(MP)} = \chi_H^{NQ} \epsilon^{(NQ)}(\mu, \mu_e) + \left(1 - \chi_H^{NQ}\right) \epsilon^{(H)}(\mu, \mu_e),
\]

where \( \chi_H^{NQ} \) denotes the volume fraction of normal quark matter inside a mixture with hadronic matter.

To visualize the Gibbs construction of the hadron-quark mixed phase, we plot the hadronic surface of the pressure along with the quark surfaces discussed in the previous section. Thus, Fig. 2 is replaced by Fig. 3. The new figure shows the surfaces of the pressure of the pure hadronic and quark phases as a function of their chemical potentials \( \mu \) and \( \mu_e \). The intersection lines of different surfaces indicate all potentially viable mixed phase constructions. Although the Gibbs conditions are satisfied along all these lines, not all of them could produce globally neutral phases (e.g., there are no neutral constructions along the light shaded solid lines in Fig. 3).

The dark solid line gives the complete, most favorable construction of globally neutral matter in general \( \beta \)-equilibrium. This line consists of three pieces. The lowest part of the curve (up to the point denoted by \( \square \) and \( P \lesssim 10 \text{ MeV/fm}^3 \)) corresponds to the pure confined hadronic phase. Within this region matter is mostly composed of neutrons with little fractions of protons and electrons to realize the charge neutrality and \( \beta \)-equilibrium. The hyperonic particles (\( \Lambda, \Sigma \), and \( \Xi \)) are not present in this lowest density region. Such particles would appear in the hadronic phase at considerably higher values of pressure and density.

The mixed phase of hadronic and normal quark matter starts at the baryonic density \( \rho_B \approx 1.49 \rho_0 \) which corresponds to the \( \square \)-point in Fig. 3. At this point the first bubbles of deconfined quark matter appear in the system. At the beginning of this hadron-quark mixed
phase, the deconfined bubbles are small but highly negatively charged, whereas hadronic matter, in which the bubbles are embedded, is slightly positively charged. The global charge neutrality condition reads

\[ n_e^{(MP)} \equiv \chi_{H}^{NQ} n_e^{(NQ)}(\mu, \mu_e) + (1 - \chi_{H}^{NQ}) n_e^{(H)}(\mu, \mu_e) = 0. \]

where \( n_e^{(H)} \) and \( n_e^{(NQ)} \) are the charge densities of hadronic and normal quark matter, respectively. This condition should be satisfied at each point along the middle part of the dark solid line (i.e., between the points denoted by \( \square \) and \( \triangle \)). With increasing density (from about 1.49\( \rho_0 \) up to 2.56\( \rho_0 \)), the volume fraction of hadronic matter decreases (down to about 0.59), while the fraction of normal quark matter increases (up to about 0.41).

It is quite remarkable that the hadron-quark mixed phase does not reach a point where the fraction of hadronic matter would vanish completely. This is in contrast to other examples of hadron-quark Gibbs constructions in the literature \[24,26,42,43\]. Instead, at baryon density about 2.56\( \rho_0 \) (a point denoted by \( \triangle \) in Fig. 3), the mixed phase is replaced by another mixed phase which is made of the normal and 2SC quark components. At this point, positively charged hadronic matter will be suddenly converted into positively charged color superconducting quark matter in the 2SC phase. As a result of this rearrangement, the energy density changes from 378 MeV/fm\(^3\) to 415 MeV/fm\(^3\). Right after the transition, the volume fractions of the 2SC and normal quark phases are 0.53 and 0.47, respectively.

The mixed phase of normal and 2SC quark matter above the point \( \triangle \) in Fig. 3 will remain the most favorable globally neutral phase of baryonic matter in our model. The volume fractions of the two components of this phase stay nearly constant with increasing density. This unusual property of the mixed phase originate from the simple fact that 2SC quark matter has a strong preference to remain positively charged. In principle, such a mixed phase could possibly be replaced by a neutral 2SC+ς or CFL phase (or, possibly, even by some new mixed phase) at sufficiently high densities when the strange quarks appear in the system. However, it may also happen that the stars with high enough densities become gravitationally unstable long before the strange quarks get a chance to appear.

The equations of state for quark and hybrid matter are shown in Fig. 4. The first equation of state corresponds to globally neutral quark matter which is a mixture of the normal quark and 2SC phases. This was discussed in Sec. 3 in detail, see Fig. 2. As for the equation of state of hybrid matter, it is constructed out of the equation of state of neutral hadronic matter and two Gibbs constructions in accordance with Fig. 3. As before, the points that indicate the beginning of two different mixed phases are denoted by \( \square \) and \( \triangle \) in Fig. 4.
V. STAR PROPERTIES

It is evident from Fig. 4 that no minimum appears in the equation of state of hybrid matter. Therefore, the corresponding compact stars can only be bound by the gravitational force. Because of a strong gravitational field inside and around such compact stars, we need to describe the gravitational field within the framework of general relativity as a curvature of spacetime. By knowing the behavior of the pressure and the energy density of matter inside the star, one can obtain the corresponding spacetime geometry by solving the Einstein equation,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi\kappa T_{\mu\nu}, \quad (27)$$

where $R_{\mu\nu}$ is the Ricci tensor, $g_{\mu\nu}$ is the spacetime metric and $T_{\mu\nu}$ is the energy-momentum tensor of matter inside the compact star.

The metric inside a non-rotating spherically symmetric star with the energy-momentum tensor $T_{\mu\nu}$ of an ideal fluid \[^44\] can be determined by solving the Tolman-Oppenheimer-Volkoff (TOV) equations \[^45\] that follow from the above Einstein equation. The metric is given by the following ansatz:

$$g_{\mu\nu} = \text{diag} \left( e^{2\nu(r)}, -\left( 1 - \frac{2m(r)}{r} \right)^{-1}, -r^2, -r^2\sin^2\theta \right), \quad (28)$$

and the explicit form of the TOV equations read

$$m(r) = 4\pi \int_0^r \epsilon(r_1)r_1^2 dr_1, \quad (29)$$

$$\frac{d\nu}{dr} = \frac{m(r) + 4\pi r^3 P(r)}{r[r - 2m(r)]}, \quad (30)$$

$$\frac{dP}{dr} = -[P(r) + \epsilon(r)]\frac{d\nu}{dr}. \quad (31)$$

(To simplify notation, we use units with $\kappa = c = 1$.) The functions $P(r)$ and $\epsilon(r)$ are the pressure and energy density at radius $r$ inside the star. By definition, $R$ is the radius of the star while $M = m(R)$ is the star mass. For a given equation of state $P(\epsilon)$ and a fixed central energy density $\epsilon_c \equiv \epsilon(r = 0)$ one can numerically integrate the above set of equations from the centre of the star up to its surface ($r = R$) where the pressure is zero, i.e., $P(R) = 0$, and

$$\nu(r) = \frac{1}{2} \ln \left( 1 - \frac{2M}{r} \right), \quad \text{for} \quad r \geq R. \quad (32)$$

The energy density profiles for hybrid stars with different central energy densities are displayed in Fig. 5. The star with the lowest central energy density in Fig. 5, $\epsilon_c = 210$ MeV/fm$^3$, is composed of pure confined hadronic matter, mainly neutrons, surrounded by a thin compact star crust consisting of leptons and nuclei. To get the equation of state for the crust, we use the results of Ref. \[^{10}\] for

\[\rho_B < 0.001 \text{ fm}^{-3}\] and the results of Ref. \[^{47}\] for $0.001 \text{ fm}^{-3} < \rho_B < 0.08 \text{ fm}^{-3}$.

The next energy density profile in Fig. 5 corresponds to the central energy density $\epsilon_c = 370$ MeV/fm$^3$. We see that the corresponding star already has a rather large core (the radius is about 8 km) consisting of a mixture of hadronic and normal quark matter. The core of the star is surrounded by a layer of hadronic matter and a crust.

As we saw in the previous section, there is no globally neutral baryonic matter that would produce the energy densities in the window between 378 MeV/fm$^3$ and 415 MeV/fm$^3$. Therefore, there are no stars that could have the central energy densities in this window either. At $\epsilon_c > 415$ MeV/fm$^3$, a quark core (made of the mixed phase of normal quark and 2SC matter) forms at the center of the star. Two examples of the corresponding energy density profiles are also shown in Fig. 5. The star with the central energy density $\epsilon_c = 500$ MeV/fm$^3$ contains a quark phase core with radius about 6 km. This core is separated from the layer of the hadron-quark mixed phase by a sharp interface (the corresponding point is denoted by $\triangle$ in Fig. 5). The most extreme stable star within this model (the star with the largest possible mass), has the central energy density $\epsilon_c = 1392$ MeV/fm$^3$. This star is mainly composed of the quark core surrounded by a relatively thin layer of the hadron-quark mixed phase, as well as the pure hadronic phase and the crust on the outside.

The dependence of the gravitational mass $M$ as a function of the central energy density $\epsilon_c$ for hybrid and quark...
stars are displayed in Fig. 6. The first appearance of the two mixed phases inside hybrid stars are marked with the symbols □ and △. The maximum mass hybrid star has the following properties: \( M_{\text{max}} = 1.81 M_\odot \), \( \epsilon_c = 1392 \text{ MeV/fm}^3 \), \( \rho_c = 7.58 \rho_0 \) and \( R = 10.86 \text{ km} \). Hybrid stars with higher masses or central densities would collapse to black holes because of gravitational instability.

The mass-radius relations for hybrid and quark stars are shown in Fig. 7. As one would expect, the pure quark stars composed of the mixed phase constructed in Sec. III have much smaller radii and the value of their maximum mass is slightly smaller, see Fig. 7. The difference between hybrid and pure quark stars is mostly due to the low density part of the equation of state. This is also evident from the qualitative difference in the dependence of the radius as a function of mass for the hybrid and quark stars with low masses. The corresponding study of compact stars is left for future work. It is worth to emphasize, however, that the properties of the hybrid stars lighter than the “○-star” in Figs. 6 and 7 would remain unchanged.

VI. SUMMARY AND OUTLOOK

In this paper, we constructed a realistic equation of state of nonstrange baryonic matter that is globally neutral and satisfies the condition of \( \beta \)-equilibrium. This equation of state might be valid up to densities of about \( 5 \rho_0 \), or even up to densities of about \( 8 \rho_0 \) in the most optimistic scenario. In our construction, matter at low density (\( \rho_B \lesssim 1.49 \rho_0 \)) is mostly made of neutrons with traces of protons and electrons. At intermediate densities (\( 1.49 \rho_0 \lesssim \rho_B \lesssim 2.56 \rho_0 \)), homogeneous hadronic matter is replaced by the mixed phase of positively charged hadronic matter and deconfined bubbles of negatively charged normal quark matter. The volume fraction of normal quark matter gradually grows with increasing
density. Before the volume fractions of two phases become equal, the mixed phase undergoes a rearrangement in which the hadronic component of matter turns into the two-flavor color superconductor. At higher densities ($\rho_B \gtrsim 2.75\rho_0$), only the quark mixed phase exists. This latter is composed of about equal fractions of normal and 2SC quark matter.

Previously, it was argued that 2SC quark matter could not appear in compact stars when the charge neutrality condition is imposed \[17\]. The main reason for this is a strong preference of 2SC quark matter to remain positively charged. Therefore, one may conclude that no color superconducting phases can appear inside compact stars if the baryon density is not large enough for the strange quarks to condense and participate in the formation of the neutral CFL phase. The present study reinstalls the status of 2SC quark matter as the most promising color superconducting phase in the central regions of compact stars.

Our study shows that positively charged 2SC quark matter appears naturally in a quark mixed phase at densities around $3\rho_0$. The other component of the globally neutral mixed phase is negatively charged normal quark matter. The corresponding construction turns out to be very stable. In particular, we observe that the volume fractions of the two quark components remain approximately the same with changing the baryon density. From the physical point of view, this rigidity of the mixed phase is connected with the nature of 2SC quark matter which tends to remain positively charged.

To the best of our knowledge, hybrid baryonic matter in this paper gives the first and the only example of a combination of two different Gibbs constructions that replace each other in the same system with changing the density. It should be clear that this special construction results from the existence of the triple point where hadronic matter coexists with two different quark phases. It is fare to mention that some speculations about the possibility of another type of a triple point were expressed in Ref. \[15\]. In our construction, at the triple point, the hadronic component of the first mixed phase is replaced by the 2SC quark component of the other. This could be interpreted as a first order phase transition that happens only in one of the components of the mixed phase. The volume fraction of the inert component, i.e., normal quark matter, changes by a small jump during this transition. This change accounts for the difference of the charge densities of hadronic and 2SC quark matter.

By making use of the equation of state of hybrid matter, we construct the corresponding non-rotating compact stars. We find that the largest mass hybrid star has the radius $10.86\,\text{km}$, the mass $1.81M_\odot$ and the central baryon density $7.58\rho_0$. This star has a large (8 km) quark core, and a relatively thin outer layers of hadronic matter and a crust. It also appears that the quark cores are quite large even for stars with relatively low central energy densities. Clearly, this is the reflection of the fact that the mixed quark phase starts to develop at rather low densities, $\rho_B \approx 2.75\rho_0$, in the NJL model used.

The quark core in a hybrid star is separated from the lower density hadron-quark layer by a sharp interface where the baryon density and the energy density change by a jump. In a real star, this interface should be naturally smoothed over the distances comparable to the physical sizes of different matter components in the mixed phase. Our estimates show that a typical thickness of the layers of different quark phases is about 10 fm. This should also be comparable to the physical sizes of components in the hadron-quark mixed phase around the interface \[28,29\]. This value sets the scale of the actual size of the interface. A more detailed study of the interface between the mixed phases is outside the scope of this paper.

In our analysis, we used a rather simple quark model and a specific hadronic model. This may suggest that most of our conclusions in this paper are not quite general. We would like to argue, however, that is not the case. In fact, the main results should remain qualitatively correct and model independent, because they are based on a simple and clear physical picture. For example, the fact that the most favorable 2SC quark phase appears to be positively charged is related to the nature of the phase itself rather than to any specific properties of the model. It is this fact that was used previously to justify that the CFL phase is more favorable than the homogeneous 2SC (or 2SC++ or $\phi$) phase in compact stars. Here we turn the same arguments around, and find that the mixed phase of normal and 2SC quark matter is the most favorable nonstrange globally neutral quark matter. Indeed, when converting all quark matter into a color superconductor is impossible, the next best possibility is to convert at least a fraction of it.

It is fare to mention that, in this paper, we made a rather conservative assumption that the strange quark is very heavy and it does not appear in quark matter even at rather high densities. In particular, the highest density reached in our model of the hybrid star without causing a gravitational instability is $7.58\rho_0$. If we accept this, the hybrid stars which are heavier than $\mu_B/3 \approx 450\,\text{MeV}$ if we accept this, the hybrid stars which are heavier than $\mu_B/3 \approx 450\,\text{MeV}$.

In the future, it would be interesting to study the physical properties of the hybrid stars that have not been addressed in this paper. For example, it would be interesting to study the neutrino emissivity and the mean free...
path of neutrinos inside quark cores of such stars. The corresponding results would be crucial for understanding the cooling mechanism of the hybrid stars. Also, it would be interesting to address the magnetic and various transport properties. The nature of the quark mixed phase suggests that a flux of the magnetic field should penetrate through the quark core. However, it might also be interesting to see how the flux is distributed inside the mixed phase, and how this may affect the star properties. Finally, it would be useful to generalize the current study to the case of rotating stars.

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