Fractality of pomeron-exchange processes in diffractive DIS

Zhang Yang
Institut für theoretische Physik, FU Berlin
Arnimallee 14, 14195 Berlin, Germany
E-mail: zhang@physik.fu-berlin.de

By using Monte Carlo simulation of pomeron exchange model, the dependence of the fractal behavior of the pomeron induced system in deep inelastic lepton-nucleon scattering upon the diffractive kinematic variables is found rather robust and not sensitive to the distinct parameterization of the pomeron flux factor and structure function. Based on this characteristic fractal plot, a feasible experimental test of the phenomenological pomeron-exchange model in DESY ep collider HERA is proposed.

High energy elastic and diffractive processes have long been described by using the phenomenology of Regge theory by the t-channel exchange of mesons and, at high energy, by the leading vacuum singularity, i.e. the pomeron. Because of the ignorance of the nature of the pomeron and its reaction mechanisms, there exists different kinds of approaches and parameterizations of pomeron dynamics in current Regge theory. And all the calculation results concerning the collective aspects of the diffractive processes, such as cross section of hard diffraction, the distribution of large rapidity gap, jet production in hard diffractive processes etc. were found very sensitive to the distinct parameterization of the pomeron. In this respect, it is natural to ask the following questions: Is there a way to test and justify the pomeron exchange model by using current diffractive experimental equipment (e.g. DESY ep collider HERA) while the criterion to the experimental measurements does not depend upon concrete parameterization of pomeron? If yes, what is the characteristic behavior of the pomeron exchange model in the expected experimental measurements?

Having in mind that the fractal and fluctuation pattern of the multiparticle production reveals the nature of the correlations of the spatial-temporal evolutions in both levels of parton and hadron and is, therefore, sensitive to the interact dynamics of the high-energy processes, it has been proposed to investigate the fractal behavior of the diffractively produced system by calculating the scaled factorial moments of the multihadronic final state. In this talk, we find the dependence of the fractal behavior of the pomeron induced system upon the diffractive kinematic variables is rather robust and not sensitive to the different parameterization of the phenomenological pomeron model in the deep inelastic lepton-nucleon scattering (DIS). So the characteristic plot
about fractal behavior of the diffractively produced system can be considered as a clear experimental test of the pomeron exchange model within DESY $ep$ collider HERA.

The fractal (or intermittency) behavior of the diffractively produced system in DIS (and also hadron-hadron collider) can be extracted by measuring the $q$-order scaled factorial moments (FMs) of the final-state hadrons excluding the intact proton from the incident beam, which are defined by

$$F_q(\delta x) = \frac{1}{M} \sum_{m=1}^{M} \frac{\langle n_m(n_m - 1)\ldots(n_m - q + 1) \rangle}{(n_m)^q},$$

where, $x$ is some phase space variable of the multihadronic final-state, e.g. (pseudo-)rapidity, the scale of phase space $\delta x = \Delta x/M$ is the bin width for a $M$-partition of the region $\Delta x$ in consideration, $n_m$ is the multiplicity of diffractively produced hadrons in the $m$th bin, $\langle \cdots \rangle$ denotes the vertical averaging with the different events for a fixed scale $\delta x$.

The manifestation of the fractality and intermittency in high energy multiparticle production refers to the anomalous scaling behavior of FM $F_q(\delta x) \sim (\delta x)^{-\phi_q} \sim M^{\phi_q}$, as $M \to \infty, \delta x \to 0$.

The $q$-order intermittency index $\phi_q$ can be connected with the anomalous fractal dimension $d_q$ of rank $q$ of spatial-temporal evolution of high energy collisions as

$$d_q = \phi_q/(q - 1)$$

Pomeron exchange in Regge theory has been used to describe successfully the main features of the high energy elastic and diffractive process. While waiting for the experimental measurements of DESY $ep$ collider HERA (and also hadron-hadron collider) for the above-mentioned fractal behavior of the diffractively produced system, let us now take a closer look at what we can learn from the current pomeron exchange theory.

Pomeron factorization allows the diffractive hard scattering cross-section to be written as

$$\frac{d^4\sigma(ep \to e + p + X)}{dx_t dt d\beta dQ^2} = f_{pP}(t, x_P) \frac{d^2\sigma(eP \to e + X)}{d\beta dQ^2},$$

where $x_B$ and $Q^2$ are the usual deep-inelastic variables; $x_P$ is the longitudinal momentum fraction of pomeron emitted by proton; and $\beta = x_B/x_P$. The first factor of the right-hand-side of Eq. (4) is the pomeron flux, i.e. probability of emitting a pomeron from the proton; the second factor, i.e. lepton-pomeron
hard cross section which is assumed to be independent of negative mass-squared \( t \), can be calculated in the way that
\[
\frac{d^2\sigma(eP \rightarrow e + X)}{d\beta dQ^2} = \int d\beta' G(\beta') \frac{d^2\hat{\sigma}_{\text{hard}}(e + \text{parton} \rightarrow e + X)}{d\beta dQ^2},
\]
(5)
if we could figure out the density \( G(\beta') \) of the quarks and gluons with fraction \( \beta' \) of the pomeron momentum. Needless to say, hard scattering cross section \( d\hat{\sigma}_{\text{hard}}(e + \text{parton} \rightarrow e + X) \) is to be computed in perturbation theory and the Born-level cross section \( d\hat{\sigma}_{\text{hard}} \) is proportional to \( \delta(\beta - \beta') \). But both pomeron flux and structure function are still main uncertainties in pomeron model, and it is even unknown whether the pomeron consist mainly of gluons or of quark (see last Donnachie and Landshoff’s papers in Ref. 3), although measurements of hard diffractive scattering have been performed in both lepton-hadron and hadron-hadron collider.

In addition to these theoretical uncertainties there is also a uncertainty in the \( Q^2 \) evolution of the parton densities of the pomeron. Numerical calculations using ordinary QCD evolution equation (Altarelli-Parisi or DGLAP), and GLR-MQ equation in which the inverse recombination processes of partons has been taken into account, turned out that the \( Q^2 \) evolution of the pomeron structure function can be very much different depending upon whether the non-linear recombination term of the QCD evolution equation is included or not. Furthermore, depending upon the initial parton distribution at a given momentum scale which is unknown, the size of nonlinear term may become too large for the QCD evolution equation to be reliable without further, but also unknown, correction. By assuming both leading and subleading Regge trajectory, a fit according to the NLO DGLAP evolution equations to HERA data of \( F_2^{(3)}(x_P, \beta, Q^2) \) has favored a rather peculiary “one-hard-gluon” distribution for the pomeron. Since what we try to pursue in this note is to find out whether and in which range the fractal behavior of pomeron induced system depends on varieties of different parameterization of pomeron, we leave the possible anomalous scaling behavior in the QCD evolution processes to the further discussion.

In typical kinematic region of hard diffractive processes of DESY \( ep \) collider HERA (say, i.e. \( M_X > 1.1 \text{GeV} \), and \( x_P < 0.1 \)), major properties of the diffractive events can be well reproduced by RAPGAP generator (see, e.g. 4, 6, 15). In the following intermittency analysis of the lepton-nucleon diffractive process, we use RAPGAP generator 4 to simulate the pomeron exchange processes, in which the virtual photon \((\gamma^*)\) will interact directly with a parton constituent of the pomeron for a chosen pomeron flux and structure function. In addition to the \( O(\alpha_{em}) \) quark-parton model diagram \((\gamma^*q \rightarrow q)\), the photon-gluon
fusion ($\gamma^* g \rightarrow q\bar{q}$) and QCD-Compton ($\gamma^* q \rightarrow qg$) processes are generated according to the $O(\alpha_{em} \alpha_s)$ matrix elements. Higher order QCD corrections are provided by the colour dipole model as implemented in (ARIADNE)\textsuperscript{17}, and the hadronization is performed using the JETSET\textsuperscript{18}. The QED radiative processes are included via an interface to the program HERACLES\textsuperscript{19}.

![Figure 1: The second-order scaled factorial moments $F_2$ getting from MC simulation of RAPGAP generator versus the number $M$ of subintervals of 3-dimensional ($\eta, p_\perp, \phi$) phase space in log-log plot, and the intermittency index $\phi_2$ correspondingly. The different kinds of points denote different parameterization of the pomeron flux factor $f_{P}(t, x_{P})$ and structure function $G(\beta)$, see text for detail.](image)

By choosing the concrete pomeron flux factor $f_{P}(t, x_{P})$ and the pomeron structure function $G(\beta)$, we generate 100,000 MC events, and calculate the second-order factorial moments in 3-dimensional ($\eta, p_\perp, \phi$) phase space, where the pseudorapidity $\eta$, transverse momentum $p_\perp$ and the azimuthal angle $\phi$ are defined with respect to the sphericity axis of the event. The cumulative
variables $X$ translated from $x = (\eta, p_\perp, \phi)$, i.e.

$$X(x) = \int_{x_{\text{min}}}^{x} \rho(x)dx / \int_{x_{\text{min}}}^{x_{\text{max}}} \rho(x)dx,$$

were used to rule out the enhancement of FMs from a non-uniform inclusive spectrum $\rho(x)$ of the final produced particles. The obtained result of second-order FM versus the decreasing scale of the phase space is shown in Fig. 1 in double logarithm. There exists obviously anomalous scaling behavior in the pomeron induced interaction, so we fit the points in Fig. 1 to Eq. (2) with least square method and extract the intermittency index $\phi_2$. In Fig. 1, we have also shown the Monte Carlo result of the second-order factorial moments for different kinds of the parameterization of the pomeron flux factor and structure function. In Fig. 1(a), (b), and (c), we keep the pomeron structure function $G(\beta)$ fixed as $\beta G(\beta) = 6\beta(1 - \beta)$ but vary the pomeron flux factor $f_{pP}(t, x_P)$ as $f_{pP}(t, x_P) = \beta^2_{x_P(t)} x_{x_P}^{-2}\alpha_{x_P(t)}$, $f_{pP}(t, x_P) = \frac{1}{2\pi x_P} (6.38e^{-8|t|} + 0.424e^{-3|t|})$ and $f_{pP}(t, x_P) = \frac{9.32}{x^3} (F_1(t)^2 x_{x_P}^{1-2\alpha_{x_P(t)}})$ respectively. The fractal behaviors of the pomeron induced system keep almost unchanged for different flux factor. On the contrary we keep the pomeron flux factor fixed as $f_{pP}(t, x_P) = \frac{\beta^2_{x_P(t)} x_{x_P}^{-1-2\alpha_{x_P(t)}}}{16\pi x_P^2}$ in Fig. 1(a), (d), (e), and (f), but vary the pomeron structure function as $\beta G(\beta) = 6\beta(1 - \beta)$, $\beta G(\beta) = 6(1 - \beta)^5$, $\beta G(\beta) = (0.18 + 5.46\beta)(1 - \beta)$ and $\beta G(\beta) = 0.077\pi\beta(1 - \beta)$ respectively (see Ref. 3 for all these parameterisations of pomeron flux and structure function which are extensively used). For a given pomeron flux, the fractal behaviors become weaker when the pomeron become softer. In Fig. 1(d) the parton distribution is as soft as that in proton, the intermittency index is smallest, which is understandable since if the hard parton in pomeron is involved it is more possible to evoke jets and then the anomalous short-range correlation in the final-state so that the intermittency index increases, and vice versa.

It is of the special interest to investigate the dependence of the fractal behavior of the pomeron induced system upon the diffractive kinematic variables. We generate 500,000 events by RAPGAP Monte Carlo generator, and divided the whole sample into 10 subsamples according to the diffractive kinematic variables, e.g. $x_B$. For each subsample, we calculate the second order scaled FM and the intermittency index $\phi_2$, and to see how the fractal behavior of the pomeron induced multihadronic final states depends upon the considered kinematic variable. In Fig. 2 is shown the dependence of the second order intermittency index $\phi_2$ on the different diffractive kinematic variables. Since it is well known that the gluon density increase sharply as $x_B$ decreases in small-$x_B$ region, the MC result from pomeron model in Fig. 2(a) means that
the anomalous fractal dimension \( d_2 \) of the diffractively produced system decreases with increasing gluon density, which is not inconceivable if one takes into account the fact (see, e.g. [9, 11]) that the effect of superposition of fractal systems can remarkably weaken the intermittency of whole system.

\[
\begin{align*}
\phi_2 & \quad \log_{10} x_B \\
\phi_2 & \quad \log_{10} x_p \\
\phi_2 & \quad \beta \\
\phi_2 & \quad Q^2 \\
\phi_2 & \quad -t
\end{align*}
\]

Figure 2: The dependence of second-order intermittency index \( \phi_2 \) in RAPGAP Monte Carlo implementation upon different kinematic variables. The different shapes of points denote different parameterization of pomeron flux factors and the structure functions in the same way as that in Fig. 1.

Obvious dependence of \( \phi_2 \) on pomeron momentum fraction \( x_P \) of a hadron and parton momentum fraction \( \beta \) of a pomeron as shown in Fig. 2(b) and (c) implies that, the intermittency calculated here can not be only referred to the hadronization processes and there should be substantial correlations between the fractal behaviours and pomeron dynamics. In Fig. 2(a) and (b), the intermittency index \( \phi_2 \) is less than 0 for the lower \( x_B \) and \( x_P \), which can be imputed to the constraint of the momentum conservation in the high energy process. Since the Leading Proton Spectrometer (LPS) has been used in ZEUS detector to detect protons scattered at very small angles (say, \( \leq 1 \) mrad), which make it possible to measure precisely the square of the four-momentum transfer \( t \) at the proton vertex, we also showed in Fig. 2(c)
the $t$-dependence of second order intermittency index in the $t$-region of LPS detector, i.e. $0.07 < -t < 0.4 \text{ GeV}^2$. To be different from the results of other kinematic variables, the fractal index for the $\gamma^* \mathbb{P}$ system doesn’t depend upon the $t$.

Especially, we calculate the intermittency index shown in Fig. 2 using different kinds of pomeron parameterization. We denote the different shapes of points in Fig. 2 for the different kinds of the pomeron parameterization, just in the same way as that in Fig. 1. Just as mentioned above, in conventional investigation, the pomeron theory has been used to compare with the data about cross section of hard diffraction, the rapidity distribution of large rapidity gaps and jet rapidity distribution and jet shape etc., where the results of the pomeron model concerning these collective nature of diffractive process were found very sensitive to the parameterization of the pomeron, and the experimental data in different aspects preferred different kinds of parameterization. It is remarkable that the dependence of the intermittency index, which concerned with the inherent scaling behaviors of diffractive processes, upon the diffractive kinematic variables in this implementation of the pomeron exchange model are rather robust and almost the same for the different parameterization of the pomeron flux and structure function!

Having in mind that the multiplicity measurement of multihadronic production is available in DESY $ep$ collider HERA and especially the multiplicity moments and KNO scaling behaviours of final hadrons in deep-inelastic processes have been already studied in recent year, it is feasible and urgent to check this characteristic fractal plot in DESY $ep$ collider HERA. Substantial revision would be necessary in the manner in which we have treated diffraction if it should turn out that experimental measurements differ drastically from this characteristic plot presented here.

**Acknowledgments**

I would like to thank T. Meng, R. Rittel and K. Tabelow for the helpful discussion, H. Jung for correspondence, and the Alexander von Humboldt Stiftung for financial support. Thanks are also due to W. Kittel for the invitation and arrangement of my talk, and his organizing such an enjoyable session. Because of the efforts of N.G. Antoniou, L. Kontraros and the other members of the organizing committee, the symposium was extremely well organized.

**References**

1. T. Regge, Nuov. Cim. **14**, 951 (1959), *ibid.* **18**, 947(1960).
2. G. Chew and S. Frautschi, Phys. Rev. Lett. 7, 294 (1961); G. Chew, S. Frautschi and S. Mandelstam, Phys. Rev. 126, 1202 (1962).
3. G. Ingelman and P. Schlein, Phys. Lett. B 152, 256 (1985). H. Fritzsch and K. Streng, Phys. Lett. B 164, 391 (1985). E. Berger, J. Collins, D. Soper and G. Sterman, Nucl. Phys. B 286, 704 (1987). A. Donnachie and P. V. Landshoff, Nucl. Phys. B 244, 322 (1984); Phys. Lett. B 191, 309 (1987).
4. H. Jung, Comput. Phys. Commun. 86, 147 (1995).
5. UA8 Collaboration, R. Bonino, et al., Phys. Lett. B 211, 239 (1988), A. Brandt, et al., ibid. B 297, 417 (1992).
6. H1 Collaboration, P. Marage, in: Proceeding of DIS 97, Chicago, edited by J. Repond and D. Krakauer, (AIP 1997), p. 570; ZEUS Collaboration, J. Terron, ibid., p. 574; J. Hernandez, ibid., p. 592.
7. M. Derrick et al., ZEUS Coll., Z. Phys. C 68, 569 (1995).
8. A. Bialas and K. Peschanski, Nucl. Phys. B 273, 703 (1986); B 308, 857 (1988).
9. For a recent review of fractal (intermittency) in high energy physics, see e.g., E.A. De Wolf, I.M. Dremin and W. Kittel, Phys. Rep. 270, 1 (1996).
10. Zhang Yang, Phys. Rev. D 57, R1327 (1998);
11. P. Lipa and B. Buschbeck, Phys. Lett. B 223, 465 (1989); R. C. Hwa, Phys. Rev. D 41, 1456 (1990); Zhang Yang, Liu Lianshou and Wu Yuanfang, Z. Phys. C 71, 499 (1996).
12. G. Ingelman and K. Prytz, Z. Phys. C 58, 285 (1993).
13. Yu. L. Dokshitzer, JETP 46, 641 (1977); V. N. Gribov and L. N. Lipatov, Sov. J. Nucl. Phys. 15, 438 (1972); G. Altarelli and G. Parisi, Nucl. Phys. B 126, 298 (1977).
14. L. V. Gribov, E. M. Levin and M. G. Ryskin, Phys. Rep. 100, 1 (1983); A. H. Mueller and J. Qiu, Nucl. Phys. B 268, 427 (1986).
15. C. Adloff et al., H1 Coll., Z. Phys. C 76, 613 (1997);
16. Zhang Yang, (in preparation).
17. L. L"onnblad, Comput. Phys. Commun. 71, 15 (1992).
18. T. Sjostrand, Comput. Phys. Commun. 39, 347 (1986).
19. A. Kwiatkowski, H. Spiesberger and H. M"ohring, Comput. Phys. Commun. 69, 155 (1992);
20. A. Bialas and M. Gazdzicki, Phys. Lett. B 252, 483 (1990); W. Ochs, Z. Phys. C 50, 339 (1991).
21. Liu Lianshou, Zhang Yang and Deng Yue, Z. Phys. C 73, 535 (1997).
22. See, e.g. M. Derrick et al., ZEUS Coll., Z. Phys. C 67, 93 (1995); S. Aid et al., H1 Coll., ibid. C 72, 573 (1996).