Entropic issues in contemporary cosmology

D.H. Coule

Institute of Cosmology and Gravitation, University of Portsmouth, Mercantile House, Portsmouth PO1 2EG.

Abstract

Penrose [1] has emphasized how the initial big bang singularity requires a special low entropy state. We address how recent brane cosmological schemes address this problem and whether they offer any apparent resolution. Pushing the start time back to $t = -\infty$, or utilizing maximally symmetric AdS spaces, simply exacerbates or transfers the problem. Because the entropy of de Sitter space is $S \leq 1/\Lambda$, using the present acceleration of the universe as a low energy ($\Lambda \sim 10^{-120}$) inflationary stage, as in cyclic ekpyrotic models, produces a gravitational heat death after one cycle. Only higher energy driven inflation, together with a suitable, quantum gravity holography style, restriction on ab initio degrees of freedom, gives a suitable low entropy initial state. We question the suggestion that a high energy inflationary stage could be naturally reentered by Poincare recurrence within a finite causal region of an accelerating universe.

We further give a heuristic argument that so-called eternal inflation is not consistent with the 2nd law of thermodynamics within a causal patch.

PACS numbers: 04.20, 98.80.Hw

Key Words: entropy, inflation, branes, cyclic ekpyrotic cosmology
1. Introduction

When cosmological models are extrapolated back in time they generally reach an epoch when quantum gravity is necessary. This fortuitously allows us to sweep away many outstanding problems of the big bang theory and await their resolution when the correct theory is available. There is also the hope that within such a theory the “entrance” to the universe will become apparent. Of course we are too impatient to await an ultimate theory, and believing cosmology can provide feedback to this search, we set goals or define problems to be resolved.

The other side of the coin is that almost anything can be allowed during this unknown quantum gravity region and this is now causing a crisis of possibilities. For example, the pre-big bang [3] and ekpyrotic models [4] push the start time back to minus infinity. They therefore almost to incredible accuracy satisfy the Perfect Cosmological Principle (see eg.[5]) over huge epochs: the universe behaves mostly through its lifetime like the steady-state universe (see [6] for review) which was earlier believed discounted. Other approaches allow the fundamental constants to vary [7] and so confuse what puzzles of cosmology are actually paramount [8]. For example the holography principle [9] suggests only one degree of freedom per Planck area $\hbar G/c^3$. But note how this is altered, for example, in models where $c$ initially tends to infinity the initial number of degrees of freedom becomes potentially unbounded as the Planck area vanishes.

In this paper we wish to consider how these new, and apparently radical, cosmological models address a fundamental question. Why is entropy within the universe able to increase and so allow the generalized principle of the 2nd law of thermodynamics [10] to arise? Such arguments can help decide whether the models are at least plausible, or if the problems are simply transferred into understanding the initial state.

Penrose has suggested that the Weyl curvature tensor could be related to the gravitational entropy [1]. Although counter examples to this idea are known since some anisotropic models evolve to gravitational waves with zero Weyl curvature [11], it does give some idea of how some singularities can be more ordered than others. The usual big bang singularity presumably having small entropy or negligible Weyl curvature while a big crunch could have large entropy or a dominant Weyl component. The actual definition of gravitational entropy will not be so important for our discussion since it is believed known how to define it when cosmological horizons are present, in
for example, de Sitter space.

2.0 Standard Inflation

There is an interesting property of de Sitter space: the higher its temperature the lower is the entropy. This allows a high energy cosmological constant \( \Lambda \), or equivalent scalar potential \( V(\phi) \), to still have low entropy. Since \([12]\)

\[
S \leq \frac{1}{\Lambda} \approx \frac{1}{V(\phi)}
\]

(1)

where the temperature is given by

\[
T = \Lambda^{1/2} \approx V(\phi)^{1/2}
\]

(2)

Strictly speaking this is the entropy within the causal event horizon. This only agrees with the total entropy of the space at the minimum size of a closed de Sitter universe. The total entropy can be infinite in the non-closed case, unless one suitably compactifies the metric - see eg \([13]\). This de Sitter case can be contrasted with the case of gravity being simply attractive. For example, larger Black holes which maximize the entropy have lower temperatures \([14]\).

This example of holography, that bounds the entropy of de Sitter, prevents a singularity being present or else would allow violations in the generalized 2nd law cf.\([15]\). Therefore, if such principle, is valid it has a drastic consequence for inflation: it means that degrees of freedom are being restricted \textit{ab initio} with an increasing Planck sized \( V(\phi) \) potential. The “no hair” property of inflation (see eg. \([16]\)) is implemented automatically. This is helpful since in a classical analysis it is known that any upper bound in the potential can lead to ambiguous predictions from inflation: the flatness would still be arbitrary \([17-18]\) using a classical canonical measure \([19]\): essentially because the kinetic energy \( \dot{\phi}^2 \) can diverge. To state this differently: without holography there is potentially an infinite number of extra degrees of freedom that could be added within a Planck volume cf. \([20]\). These cannot be smoothed by a finite amount of inflation. This resolves a problem outlined by Penrose \([2]\), that if the quantum gravity region was “fractal” inflation would just transfer this fuzziness to large scales. In contrast if a large cosmological constant already saturates the holography bound it naturally keeps the space-time smooth and of low entropy. If one conversely tried to maximize the entropy then we would obtain the opposite i.e. \( \Lambda \to 0 \), and indeed people have argued that this could be a reason why the cosmological
constant is so small today [21]. This argument is however deficient since maximizing the entropy would not allow further increase for the 2nd law to hold.

Can a large $V(\phi)$ be justified? Somewhat counterintuitive the minimum size of closed de Sitter space is $\sim \Lambda^{-1/2}$, so that a smaller size corresponds to larger $\Lambda$, or scalar potential $V(\phi)$. In the limit $V(\phi) \to 1$ the minimum size approaches the Planck length. Using quantum cosmology one envisions that tunneling to this minimum from zero size can occur[22]. However, this is rather an extravagant extrapolation of usual quantum mechanical reasoning since space-time itself is coming into existence unlike the way electrons are treated in for example Alpha decay of nuclei. The closed de Sitter model itself also suffers from fragility[24]: if the curvature is removed the minimum size limit is removed. Also if a matter component is added the universe becomes instead the Lemaitre model (see eg.[5]) which also starts from a singularity at zero scale factor. One can still attempt to quantize such models but the results will depend upon arbitrary constants that depend upon the relative matter components present. Although in some simple models the quantum measure for inflation is still found to be more likely than purely classical reasoning would suggest [25].

If instead of quantum conception we wish to use inflation to amplify a “small bang” universe other problems are apparent. Inflationary conditions necessarily have a singularity in their past [26,27]. To be consistent the earlier singularity must also have a very low entropy, so this would still need an adequate explanation. If the singularity was a more generic high entropy one it is difficult to envision how a low entropy domain that subsequently inflates could develop. The gravitational entropy $S_W$ from the Weyl component must be strongly constrained if not to dominate the Sitter entropy $S$. In summary, inflation still leaves unanswered either the quantum mechanism that produces the initial large smooth field $V(\phi)$ or for the purely classical case what explains the preceding smooth singularity. Although the holography principle can help restrict degrees of freedom that require smoothing, the notion of a field itself is not consistent with such a principle. This is at present a serious limitation of inflationary theory, particularly when consid-

\[^1\text{Note that while the total entropy of the space will subsequently increase during the expansion the total entropy in the collapsing de Sitter phase is decreasing which would apparently violate the 2nd Law of thermodynamics. This helps explains the concern of Price that deflation should also be included [23].}\]
ered close to the Planck scale.

3.0 Low energy inflation

If inflation is driven by a small value of $\Lambda$ the maximum possible entropy increases. For example using the present apparent value of the cosmological constant as an inflationary phase the corresponding entropy is $S \sim 10^{120}$. This is the maximum possible entropy allowed within the horizon and agrees with the value given by Penrose [1] for a Black hole encompassing the total mass within the present horizon size. This value will only occur after a further time $t > H^{-1} \sim 10^{60} t_{pl}$, as particle production saturates this bound.

In any case inflation does not force local gravitating system apart. The so-called “no hair” theorems of de Sitter space exclude positive curvature [28]. If one requires that super massive Black holes should first evaporate it takes approx $\sim 10^{140} t_{pl}$ [14]. After such time scales the universe will have around the maximum entropy: being composed of Hawking radiation with photons at a temperature $\sim 10^{-28} K$ [12]. Note that although the present 3K background radiation will become redshifted below this value, and contribute negligible entropy, the quantum effects will dominate. Once the maximum entropy is reached further evolution or life becomes impossible. Although by using a weaker power-law expansion $a \sim t^p$ with $p > 1$, the Hubble parameter $H \rightarrow 0$ as $t \rightarrow \infty$. This would allow the maximum allowed entropy to continue growing and somewhat alleviate a gravitational heat death [29].

As mentioned, ambiguities occur when using a canonical measure for the typical classical solutions. If the inflationary potential is bounded above and the energy density can be bigger than this value ambiguities in the flatness occur [17,18]. Reducing the maximum height of this inflationary potential exacerbates the problem further [18]. This is also symptomatic of arbitrary values of anisotropy or inhomogeneity that would result if they were introduced into the model [18]. The upshot of this work is that any finite amount of inflation does not itself restrain one to a isotropic and flat FRW universe. Other principles have to be introduced to prevent such unwanted values, in for example, the curvature. Making the model cyclic does not help in this regard since the question simply becomes why this cyclic model and not that one with different properties of curvature, anisotropy? Also bear in mind that inflation does not destroy curvature but only dilutes it during the expansionary phase. If the universe subsequently collapses it will simply reappear again, with the anisotropy and finally rotation dominating as the scale factor tends to zero.
4.0 Eternal inflation

Once inflation occurs it is suggested that quantum fluctuations can always keep a region of the universe inflationary [30]. If one tries to extend this argument backwards in time to allow an always expanding inflationary universe it apparently fails [31]. This is not surprising since only the closed de Sitter model is geodesically complete: even violating the weak energy condition as in the steady state model does not remove this geodesic incompleteness [32].

There is a simpler way to see this concern. The greater the amount of inflation the smaller is the initial region that expands to form our universe. If this initially falls within the Planck region there is no reason to believe that space-time is continuous on this scale but rather notions in quantum gravity suggest it is discrete. If space has a cut-off at the Planck scale then indeed only a finite amount of inflation can have happened. Indeed one can argue that space must be fairly smooth even at sub-Planckian scales for inflation to be usable. There has been some works that modify the dispersion relations at high energy but this is only a small departure to the assumption of smoothness [35].

We can also tentatively question the eternal inflationary mechanism to the future. Firstly, once quantum gravity is present the scale factor becomes dimensional together with the velocity $\dot{a}$. As inflation continues this velocity increases exponentially $\dot{a} \sim \exp(\lambda t) \to \infty$. This could induce quantum effects as the space expands increasingly rapidly.

It further assumes that quantum fluctuations are present with wavelength $\sim \hbar^{-1}$. These cause the potential to grow $V(\phi) \to V(\phi) + \delta V(\phi)$ in certain domains. But from expression (1) this means the entropy has decreased in violation of the 2nd law of thermodynamics. Should this ever be allowed even if many domains are coming into existence? In general the typical wavelength of Hawking radiation is $\sim \Lambda^{1/2}$ where $\Lambda$ is the area of the event horizon. As $\Lambda$ becomes increasingly large is becomes difficult to justify quantum coherence being maintained so that a superposition of modes can be justified. This becomes particularly suspect with low energy inflation where the fluctuations being produced are of order the present size of the universe $\sim 10^{60}\ell_p$. One would expect that they should decohere by interacting with the environment.

\footnote{There is an attempt [33], which closely resembles the Hoyle-Narlikar model [34] to patch together two flat de Sitter universes to overcome this restriction. But time runs in opposite direction away from an infinite null boundary of low entropy connecting the two universes.}

6
and that the field would behave classically, with no violation of the weak energy condition, on large scales. The universe would always roll towards the minimum of the potential and inflation would have a finite future duration. Although this seems clearer in the low-energy inflationary case it is also possible that this also occurs for high energy inflation where the event horizon is now only of typical area $\sim 10^6 l_{pl}^2$. If quantum gravity “measures” the system cf.[1] the necessary quantum coherence for the eternal mechanism will be lost. There is also a contradiction with the holography principle as $A \rightarrow l_{pl}^2$. Degrees of freedom are being restricted and so “no hair” is being allowed. But Hawking radiation, which is simply quantum mechanics, should excite all possible particles that are present in the underlying theory. All matter modes’ fluctuations should be generated and these could provide an environment for destroying coherence. However, too many species make the theory unstable to collapse by black hole generation [36]. A related point was made in ref.[37] that expected “zero-point” fluctuations in radiation affected the implementation of the Hartle-Hawking [38] scheme in quantum cosmology.

This argument against eternal inflation can be contrasted with a recent argument of Turok[69], who also suggests that the typical evolution does not display the eternal mechanism. His argument does not seem complete since he first ignores the quantum effects on the classical evolution, and then points out the quantum component is anyway subdominant. The argument presented above would first decohere the quantum effects into an averaged classical evolution, which obeying the 2nd law of thermodynamics within any causal domain, will always cause rolling down the potential.

5.0 Universe proliferation

There is another scheme for providing an infinite number of inflationary universes that I find rather suspect. This is the idea that when Black holes are created in de Sitter space and after evaporating they leave behind disjoint universes [39].

This is because the Schwarzschild-de Sitter metric has an infinite number of repetitions in its Penrose diagram [32]. But if a black hole forms by gravitational collapse in our present universe we don’t claim that the left hand universe in the Penrose diagram of Schwarzschild is suddenly created. We rather claim that the physical Cauchy development of a collapsing star excludes this region. Likewise in our present universe that appears to be accelerating the formation of black holes does not imply that the other universes
in maximally extended Schwarzschild-de Sitter suddenly come into existence
and suddenly become relevant for us physically. Indeed by collapsing some
matter we apparently would violate causality by suddenly having created an
infinity of universes beyond our horizon. The whole present universe would
have to change throughout in response to universes now appearing at the
edges. During this time all classical mechanics, including general relativity,
causes to be valid and is simply being violated throughout the universe; an
extreme form of horizon problem. Now, does quantum mechanics make this
argument at all reasonable? The reasoning is that metric A can become B
by simply calculating the relevant actions of the two metrics and this gives
some probability for going from one to another. But this only makes sense
if classical behaviour of the universe can be entirely suspended. This
might be possible at the Planck scale but for a closed de Sitter universe the total
volume of the universe is rapidly becoming larger than the Planck volume for
times $t > t_{pl}$. Studies in loop quantum geometry suggest that space rapidly
becomes classical at scales above Planck size, see ref. [40] for review. This
would prevent the sort of proliferation of space-time processes outlined which
although having a small action are not confined to Planck length scales.

6.0 Semi-eternal cosmology

We have in mind cosmologies that start at time $t = -\infty$ before simulating
a big bang at time $t = 0$. Since the singularity at time $t = 0$ has to be smooth
(low Weyl curvature) we have to have an even lower entropy at the initial
start $t = -\infty$. One would expect the entropy to grow significantly during
a semi-infinite interval of time. For example in the pre-big bang model any
slight initial classical perturbations will grow during the collapsing phase.
Likewise in colliding brane schemes like the original ekpyrotic one [4], the
initial branes have to have small entropy that cannot dominate over the
entropy produced by the collisions.

Another example is an eternal brane produced by achieving a bounce
that prevents a singularity forming. For a Reissner-Nordstrom AdS bulk the
Friedmann equation becomes modified, such that [41,42]

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \left( \rho + \frac{\rho^2}{2\lambda} \right) + \frac{M}{a^4} - \frac{Q^2}{a^6}$$

where $M$ and $Q$ represent the mass and charge of the bulk space. In
the Friedmann equation the $M$ term behaves like radiation while the charge
$Q$ violates the weak-energy condition. For a perfect fluid equation of state $p = (\gamma - 1)\rho$, a bounce can occur for matter softer than that of dust, i.e. $\gamma < 1$. Again the problem still to be overcome is that classical perturbations will tend to grow rapidly during the collapsing phases. This has already been investigated in ref. [43] where the scalar field perturbation are prone to diverge as the singularity is approached. Even if the bounce can proceed the initial state’s order is difficult to justify and would require other principles for its explanation.

There are some related attempts to use quantum cosmology to give an initially large universe that could subsequently collapse cf.[44]. For reasons already discussed in the previous section this is difficult to envision. There was an earlier argument that tried to give a reason for the present value of $\Lambda$ but implicitly assumed the universe had just suddenly “quantum created” itself into its present size [45]. This could be discounted by arguing that any possible Euclidean space-time structure should be constrained to small scales only [46].

7.0 Cyclic ekpyrotic cosmology

This model [47] tries to employ a low energy driven inflationary stage as a means of explaining the smooth state required before a collapse to a bounce can proceed. Although this model is driven by the behaviour of branes in higher dimensions we will only consider the usual approximation of describing it by a scalar field model in 4 dimensions. The model uses a scalar potential with a negative region, but somewhat arbitrarily the weak energy condition is not allowed to be violated, and enforced by keeping only large and positive kinetic energy. Apart from the problems inherent in low energy inflation it also suffers from problems found earlier with the pre-big bang model [48]. Because the kinetic energy dominates as the scale factor goes to zero the Planck problem of the usual big bang model occurs [49]. This is true for any model that doesn’t violate the strong-energy condition and means the scale factor must be extremely large at the Planck time. The scale factor gains units of length whenever quantum gravity is introduced. One can see this by extrapolating back the presently observable universe adiabatically to the Planck time. Another way to see this is to consider the holography principle or its weaker requirement that only one degree of freedom per Planck volume is allowed. If we are to explain the required entropy today in radiation $\sim 10^{90}$ we require $a > 10^{30}l_{pl}$ back at the Planck time. The bounce must occur long before the quantum scale when string
theory might be expected to allow further unknown phenomena. If we insist on allowing the universe to approach the actual Planck size the values of the various quantities e.g. energy density, will rapidly go beyond values that we have any confidence in describing: the Planck problem [49].

The question of quantum fluctuations has produced much debate and argument [50]. Fluctuations are produced in expanding models that have an event horizon. But for a collapsing universe the time reversal of a particle horizon is an event horizon [5]: so perturbations are produced in contracting models with scale factor \( a \sim (-t)^p \) with \( 0 < p < 1 \). A serious concern is readily apparent. The usual horizon problem (presence of a particle horizon) only concerns times below Planck times where the expansion becomes faster than light. Simply altering the behaviour of the scale factor within the Planck time of the singularity can remove entirely the presence of the particle horizon [51]. Likewise in the collapsing universe the event horizon required for quantum fluctuations depends on the behaviour of the scale factor at below Planck times. If the scale factor changes its behaviour before this time strictly speaking an event horizon is never produced. One should instead be able to define a vacuum state until a Planck time before the impending singularity that does not produce particles. We will ignore further this concern and assume such perturbations are actually produced.

It has been known for some time that a kinetic energy or stiff equation of state \( p = 1/3 \) produces a blue spectrum \( n = 3 \) of fluctuations [51]. If one wishes to produce a scale invariant \( n = 0 \) spectrum during a collapsing phase one requires a dust equation of state or \( p = 2/3 \) [52]. This is in contrast to the expansionary case where only a cosmological constant gives such a spectrum. Although the kinetic energy will undoubtedly dominate as the bounce approaches there is a period when the negative cosmological constant also contributes to the equation of state and gives slow contraction \( p \to 0 \). In this limit it still produces a blue spectrum with \( n = 2 \) as does a collapsing radiation \( p = 1/2 \) case [54]. But because of the enormous time scales involved (\( 10^{140}t_{pl} \) for super massive black holes to evaporate) without fine tuning, only the fluctuations during the previous kinetic dominated phase should be relevant for our present early universe. It is presently no further

\[^3\]One might worry that the apparent speed of sound is unphysical for \( 0 < p < 1/3 \), since it corresponds to a super stiff equation of state \( \gamma > 2 \). If the kinetic energy was constrained \( |V(\phi)| > \dot{\phi}^2 \) pole-law collapse with an effective \( \gamma < 0 \) could result [53].
than a fraction $10^{-80}$ of its total lifetime between bounces.

Returning to the entropy question. It is suggested that inflation dilutes the entropy of matter before the collapse occurs. But this ignores the quantum particle creation that will saturate the de Sitter entropy bound. In the language of ref.[55] a gravitational heat death occurs. It is true that classically matter is swept across the horizon but the event horizon radiates an entropy that represents possible information lost beyond the event horizon. If the entropy could go to zero as required to reset the universe, it would mean that entropy can be destroyed by de Sitter space. This is like a classical black hole with $S = 0$ that likewise appears to have destroyed entropy. But when quantum mechanics is introduced the entropy of Black holes and of cosmological event horizons is given by the corresponding $\sim (\text{Horizon Area})/4l_{pl}^2$, so that the generalized 2nd law still holds.

Once this entropy is produced it is difficult to envision how it might be removed, or to allow an entropy gap between gravitational entropy and matter entropy to develop [55]. The photons will become increasingly blue shifted as the singularity is approached. The total entropy within causal contact will be at least $\sim 10^{120}$. Before the next universe cycle occurs this needs to be somehow dissipated or else the photons never be allowed to gain a high temperature. The authors of ref.[47] seem to suggest that these photons, become, in the words of ref.[56], frozen and so no longer contribute dynamically to the entropy. Having to dissipate such a large entropy, a factor $\sim 10^{30}$ times larger than the present entropy of the observable universe, would seem at best to be extravagant, if not contrary to the generalized 2nd law of thermodynamics.

It might be argued that the actual brane scale factors are always expanding [47] and the problem is only due to the effective theory. Although this might help alleviate the bounce the entropy is due to the cosmological constant on the branes themselves. By the way if the scale factors always grow more than they collapse all physical scales come from increasingly smaller scales as more cycles of the model occur. Eventually all scales would come from below the 11 dimensional Planck scale or other quantum gravity scale where the theory was no longer valid. The model would still have a beginning that requires further explanation.

Unless there was some sort of fractal spacetime structure that could constantly be magnified the model breaks down. But this contradicts our present notions of having a fundamental Planck scale -see eg.[40]. If the collapse time...
before the next “big crunch” was shortened before the quantum particle creation saturated the bound, then we still have problems with large black holes going into the bounce.

Also if the collapsing epochs are only a small disturbance from mostly expanding behaviour (as claimed when working with the brane scale factors) a geodesic incompleteness theorem could also be readily obtained cf. [31]. In ref.[47] it is claimed that such a theorem is not too serious because all particles are created afresh at each new bounce. But if there is always Hawking radiation present, as we argue, this reasoning is invalid and particles cannot be entirely diluted away prior to each bounce. This continuance of particles is necessary for a related geodesic incompleteness argument to be made and would mean that somewhere a beginning to the model’s evolution would still be required.

8.0 Cyclic universe by Poincare recurrence?

Are there other ways in which a cyclic universe might be possible which do not suffer a continual increase in entropy? In this section we just wish to restate how difficult such a scenario appears with our current knowledge of physics.

For example, it has been suggested [57] that, in the apparently de Sitter phase the universe is entering, the universe can naturally reenter a low entropic state, or high energy inflationary phase, given sufficient time to provide a suitable fluctuation. This is because the number of states within the cosmological horizon is finite \( \sim 10^{120} \) and Poincare recurrence should enable the space to return to any previous state. This is a old idea in cosmology—see ref.[11] for a discussion, but is generally discounted because cosmological models are all actually unbounded [58]. We know from studies in “universe creation in the lab” that the created universe expands not into the original universe but into an entirely new space [59]. A singularity is left behind in the original universe and the presence of singularities is one way of evading the possibility of Poincare recurrence [58]. One could avoid the creation of a singularity if the new inflationary phase extended into an antitraped region, so being larger that the horizon size of the low energy background [26,27]. This is a vast size and for similar reasons as were discussed in section 5) it is difficult to believe quantum tunneling could occur over such scales vastly larger than the Planck scale. One such model is described in ref.[60] where quantum bubbles of false vacuum are created from a low energy inflationary phase. Such bubbles are dependent on the background spacetime
in order not to collapse, they do not simply supplant the original spacetime as a finite cosmological model allowing eternal return would require. Again it is an example of an unbounded system which escapes possible Poincare recurrence. Note that these actual examples of possible inflationary universe generation are apparently not consistent with horizon complementarity or other claimed sacrosanct principles which led the authors of ref.[57] to the opposite conclusion. This is not too surprising since as emphasised in ref.[61] our present notions of entropy/horizons are not well established for highly dynamic spacetimes as in the example above with new universes being produced.

9.0 Conclusions

Explaining the low initial entropy of the universe still seems far from being resolved. In inflationary universes only at the minimum of the closed de sitter $a \sim \cosh t$, is the total gravitational entropy of the space minimized. The entropy tends to infinity as $t \to \pm\infty$. So only by expunging the collapsing region (or starting at $t = 0$) can we be consistent with the 2nd law that entropy should only increase. But doing this means the model needs a further constraint to cause it to start at the minimum. Within this explanation lies the answer to why the entropy is initially low. Quantum cosmology tries to address this question but it is a vast extrapolated from the usual domain of quantum mechanics. Where space-time itself is already in existence. The Hartle-Hawking proposal [38] also seems to favour maximizing the entropy $\Lambda \to 0$, to also produce a low energy inflation -see however [62] for alleviating this prediction.

Inflation is believed to smooth space, but because of a holography or Planck cut-off principle, the number of degrees of freedom initially present is severely restricted. This helps make inflation more durable, unlike a purely classical case that leads to indefinite values in various quantities. But if inflation is premised on these degrees of freedom being absent it does not explain the whole story.

If inflation is driven by a small cosmological constant then even within the event horizon itself, the entropy, taking its de Sitter value, becomes too large. Although classically one expects the entropy to be approaching zero, Hawking radiation from the event horizon fills the space with particles. Once gravitational and matter entropies are in equilibrium a heat death results. Even if the cosmological constant decays away the photons will blue shift during collapse producing vastly more entropy at any time in the next cycle.
than at any corresponding time in our present universe. A similar entropy
problem to that in the original Tolman cyclic model that also limits the
number of allowed cycles -see eg.[11]. Claiming that this entropy is no longer
“dynamic” for the next cycle goes against the belief in black hole physics that
information should always be recoverable. Neither, is there an expansion
phase immediately after inflation that would allow an entropy gap, between
gravity and radiation, to again develop cf. [55].

This is therefore a serious problem with schemes like the cyclic ekpyrotic
one, that utilize low energy inflation in the hope of producing low entropy
conditions before a subsequent bounce can occur. Some recent work [63] also
suggests that a high energy inflationary stage should be added to the scheme.

Other brane schemes that push the start back to time $t = -\infty$ only give
more time for the entropy to grow and so require even more orderly initial
states. Sometimes they use a bulk space which satisfies the Perfect Cosmolog-
cal Principle so no evolution is allowed. It then becomes difficult to explain
why dynamic branes in low entropy states are then produced [42]. Or else
the problem simply transfers to understanding the low entropic, or orderly,
bulk space. Some schemes try to use a prior inflationary phase within the
bulk before bubbles of Anti-de sitter (AdS) space are later produced [64,65].
Depending on various arbitrary scales entering the scalar potential such an
inflationary phase is not unstable to bubble formation even if a region of neg-
ative potential is present [65-67]. Provided the AdS “well” is not too deep, in
comparison to the false vacuum value of the potential, bubbles cannot form
without energy conservation violation. If such bubbles do actually form they
generally collapse and do not expand to fill all the previously inflationary
region [65]. Most of this difficulty stems from now having a dynamical bulk
so that the full maximal symmetry of DeSitter or AdS is no longer present.
Instead the symmetry is at most that of Roberston-Walker type: homogene-
ity and isotropy, where a negative cosmological constant simply produces a
rapid collapse to a ‘big crunch’ singularity [42].

The original remarks of Penrose [1,2] that the present time-symmetric
laws appear insufficient to explain the initial state of the universe still seems
pertinent. Using string cosmology to give some explanation for the low en-
tropy state seems a daunting task, but until then our cosmological models
are missing some vital component.
Acknowledgement
I should like to thank David Wands and Alexei Nesteruk for helpful advice.
References

1. R. Penrose, “The Emperor’s new mind” (Oxford University Press, Oxford) 1989

2. R. Penrose, “Difficulties with inflationary cosmology” 14th Texas symposium, ed. E.J. Fenyves (New York Academy of Science, New York) 1989.

3. M. Gasperini and G. Veneziano, Astro. Phys. 1 (1993) p.317.

4. J. Khoury, B.A. Ovrut, P.J. Steinhardt and N. Turok, Phys. Rev. D 64 (2001) p.123522.

5. W. Rindler, “Essential Relativity 2nd edn.” (Springer-Verlag: New York) 1977.

6. F. Hoyle, G. Burbidge and J.V. Narlikar, “A different approach to cosmology” (Cambridge University Press: Cambridge) 1999.

7. J.W. Moffat, Int. J. Mod. Phys. A 2 (1993) p.351.
   A. Albrecht and J. Magueijo, Phys. Rev D 59 (1999) p.043516.
   J.D. Barrow and J. Magueijo, Class. Quant. Grav. 16 (1999) p.1435.

8. D.H. Coule, Mod. Phys. Lett. A 14 (1999) p.2437.

9. G. ’t Hooft, preprint gr-qc/9310026.
   L. Susskind, J. Math. Phys. 36 (1995) p.6377.

10. J.D. Bekenstein, Nuovo Cim. Lett. 4 (1972) p.737.
    Phys. Rev. D 7 (1973) p.2333.

11. J.D. Barrow and F.J. Tipler, “The anthropic cosmological principle” (Oxford University Press: Oxford) 1986.

12. G.W. Gibbons and S.W. Hawking, Phys. Rev. D 15 (1977) p. 2738.

13. J.P. Luminet, Phys. Rep. 254 (1995) p.135.

14. S.W. Hawking, Com. Math. Phys. 43 (1975) p.199.
15. J.D. Bekenstein, Acta. Phys. Polon. B 32 (2001) P.3555.
16. D.S. Goldwirth and T. Piran, Phys. Rep. 214 (1992) p.223.
17. D.N. Page, Phys. Rev. D 36 (1987) p.1607.
18. D.H. Coule, Class. Quant. Grav. 12 (1995) p.455.
19. G.W. Gibbons, S.W. Hawking and J.M. Stewart, Nucl. Phys. B 281 (1987) p.736.
   S.W. Hawking and D.N. Page, Nucl. Phys. B 298 (1988) p.736.
20. F. Englert, preprint hep-th/9911185
21. P. Horava and D. Minic, Phys. Rev. Lett. 85 (2000) p.1610.
22. A. Vilenkin, Phys. Rev. D 30 (1984) p.509.
   A.D. Linde, Sov. Phys. JEPT 60 (1984) p.211.
   V.A. Rubakov, Phys. Lett. B 148 (1984) p.280.
   Y.B. Zeldovich and A.A. Starobinsky, Sov. Astron. Lett. 10 (1984) p.135.
23. H. Price, “Time’s arrow and Archimedes’ point” (Oxford University Press: Oxford) 1996.
24. R.K. Tavakol and G.F.R. Ellis, Phys. Lett. A 130 (1988) p.217.
   A.A. Coley and R.K. Tavakol, Gen. Rel. Grav. 24 (1992) p.835.
25. D.H. Coule and J. Martin, Phys. Rev. D 61 (2000) p.063501.
26. T. Vachaspati and M. Trodden, Phys. Rev. D 61 (2000) p.023502.
27. D.H. Coule, Phys. Rev. D 62 (2000) p.124010.
28. R.M. Wald, Phys. Rev. D 28 (1983) p.2118.
   I. Moss and V. Sahni, Phys. Lett. B 178 (1986) p.159.
29. E. Witten, preprint hep-th/0106109
30. A.D. Linde, Phys. Lett. B 175 (1986) p.395.
31. A. Borde, A.H. Guth and A. Vilenkin, preprint gr-qc/0110012
32. S.W. Hawking and G.F.R. Ellis, “The large scale structure of space-time”, (Cambridge University press: Cambridge) 1973.
33. A. Aguirre and S. Gratton, Phys. Rev. D 65 (2002) p.083507.
34. F. Hoyle and J.V. Narlikar, Proc. Roy. Soc. A 277 (1964) p.1.
35. R.H. Brandenberger and J. Martin, Mod. Phys. Lett. A 16 (2001) p.999.
   A.A. Starobinsky, JETP Lett. 73 (2001) p.371.
36. R. Brustein, D. Eichler and S. Foffa, Phys. Rev. D 65 (2002) p.105013.
37. J.R. Gott and Li-Xin Li, Phys. Rev. D 58 (1998) p.023501.
38. J.B. Hartle and S.W. Hawking, Phys. Rev D 28 (1983) p.2960.
39. R. Bousso, Phys. Rev. D 60 (1999) p.063503.
   Phys. Rev. D 58 (1998) p.083503.
40. A. Ashtekar, preprint math-ph/0202008 and references therein.
41. C. Barcelo and M. Visser, Phys. Lett. B 482 (2000) p.183.
   preprint hep-th/0004050.
42. D.H. Coule, Class. Quant. Grav. 18 (2001) p.4265.
43. A.B. Batista, J.C. Fabris and S.V.B. Goncalves, Class. Quant. Grav. 18 (2001) p.1389.
44. D. Green and W.G. Unruh, preprint gr-qc/0206068.
45. A. Strominger, Nucl. Phys. B 319 (1989) p.722.
46. D.H. Coule, Mod. Phys. Lett. A 10 (1995) p.1989.
47. P.J. Steinhardt and N. Turok, Phys. Rev. D 65 (2002) p.126003.
48. D.H. Coule, Class. Quant. Grav. 15 (1998) p.2803.
49. Y.B. Zeldovich, “My universe: selected reviews” (Harwood Academic Press) 1992 p.95
50. D.H. Lyth, Phys. Lett. B 524 (2002) p.1.
   R. Brandenberger and F. Finelli, JHEP 0111 (2001) p.056.
   J. Hwang, Phys. Rev. D 65 (2002) p.063514.
   J. Martin, P. Peter, N. Pinto Neto and D.J. Schwarz, Phys. Rev. D 65
   (2002) p.123513.

51. T. Padmanabhan, “structure formation in the universe”, (Cambridge
    University Press: Cambridge) 1993 p.359.

52. Y.B. Zeldovich and I.D. Novikov, “The structure and evolution of the
    universe: relativistic astrophysics vol.2” (Chicago University Press:
    Chicago)1983 p.666.

53. D.H. Coule, Phys. Lett. B 450 (1999) p.48.

54. D. Wands, Phys. Rev. D 60 (1999) p.023507.

55. J.D. Barrow, New. Astron. 4 (1999) p.333.
    see also P.C.W. Davies, in “Physical origins of time asymmetry”, eds.
    J.J. Halliwell, J. Perez-Mercader and W.H. Zurek, (Cambridge Univer-
    sity Press; Cambridge ) 1994.

56. R. Brustein, Phys. Rev. Lett. 84 (2000) p.2072.

57. L. Dyson, M. Kleban and L. Susskind, preprint hep-th/0208013.
    see also D. Bak, preprint hep-th/0208046.

58. F.J. Tipler, Nature 280 (1979) p.203.

59. S. Blau, E. Guendelman and A.H. Guth, Phys. Rev D 35 (1987) p.1747.
    E. Fahri and A.H. Guth, Phys. Lett. B 183 (1987) p.149.
    E. Fahri, A.H. Guth and J. Guven, Nucl. Phys. B 339 (1990) p.417.

60. J. Garriga and A. Vilenkin, Phys. Rev. D 57 (1998) p.2230.

61. A. Corichi and D. Sudarsky, Mod. Phys. Lett. A 17 (2002) p.1431.

62. D.N. Page, Phys. Rev. D 56 (1997) p.2065.

63. G. Felder, A. Frolov, L. Kofman and A. Linde, Phys. Rev. D 66 (2002)
    p.023507.
64. M. Bucher, preprint [hep-th/0107148].

65. U. Gen, A. Ishibashi and T. Tanaka, Phys. Rev. D 66 (2002) p.023519.

66. S. Coleman and F. De Luccia, Phys. Rev. D 21 (1980) p.3305.

67. S. Weinberg, Phys. Rev. Lett. 48 (1982) p.1776.

68. V.A. Berezin, V.A. Kuzmin and I.I. Tkachev, Phys. Rev. D 36 (1987) p.2919.

69. N. Turok, Class. Quant. Grav. 19 (2002) p.3449.