Quadragulation

\begin{center}
\begin{tikzpicture}
  \node (1) at (0,0) {1};
  \node (2) at (1,1) {2};
  \node (3) at (1,-1) {3};
  \node (4) at (2,0) {4};
  \node (5) at (2,-1) {5};
  \node (6) at (3,0) {6};
  \node (s) at (0,-3) {s};
  \node (t) at (3,3) {t};
  \node (e) at (1,2) {e};

  \draw (1) -- (2) -- (4);
  \draw (1) -- (3) -- (5);
  \draw (3) -- (5) -- (6);
  \draw (4) -- (5) -- (6);
  \draw (2) -- (4); \draw (2) -- (6);
  \draw (3) -- (5); \draw (3) -- (6);
  \draw (1) -- (s);
  \draw (6) -- (t);
  \draw (s) -- (t);
  \draw (s) -- (e);
  \draw (t) -- (e);
\end{tikzpicture}
\end{center}
Quadragulation

Separating Decomposition

[Felsner]
Quadragulation

Separating Decomposition

1 2 3 4 5 6

cw first

in color is outgoing

[Felsner]

cw last
Quadragulation

Separating Decomposition

connect bicolored corners

cw first

cw last

in color is outgoing

[Felsner]
Separating Decomposition

2-page book embedding

Alternating layout

[Felsner]
2-page book embedding

[Felsner]
Contact Representation

[Felsner]
We want to construct this squaring

We are given these two bipolar orientations
We are given these two bipolar orientations.
## Spanning Trees (Laplacian Matrix)

|     | s  | a  | b  | c  | d  | t  |
|-----|----|----|----|----|----|----|
| s   | *  | A  | B  | C  | 0  | 0  |
| a   | A  | *  | 0  | D  | 0  | 0  |
| b   | B  | 0  | *  | E  | F  | 0  |
| c   | C  | D  | E  | *  | G  | H  |
| d   | 0  | 0  | F  | G  | *  | J  |
| t   | 0  | 0  | 0  | H  | J  | *  |

* = minus row sum
Spanning Trees

ABCFH ABCFJ ABCGH ABCGJ
ABCHJ ABDFH ABDFJ ABDGH
ABDGJ ABDHJ ABEFH ABEFJ
ABEGH ABEGJ ABEHJ ABFGH
ABFGJ ABFHJ ACEFH ACEFJ
ACEGH ACEGJ ACEHJ ACFGH
ACFGJ ACFHJ ADEFH ADEFJ
ADEGH ADEGJ ADEHJ ADFGH
ADFGJ ADFHJ BCFHJ BCDFJ
BCDGH BCDGJ BCDHJ BDEFH
BDEFJ BDEGH BDEGJ CDEGH
CDEGJ CDEHJ CDFGH CDFGJ
CDFHJ

⇐ Terms in the determinant of the Laplacian
Perfect Squaring [Brooks, Smith, Stone, Tutte ’40]

Unique smallest perfect squaring [Duijvestijn ’78]
Visibility graph and its planar dual
Visibility graph and its planar dual
Visibility graph and its planar dual
Visibility graph and its planar dual
Red spanning tree
Red spanning tree and Blue dual spanning tree

\((T, T^*)\)
Red spanning tree and Blue dual spanning tree

\((T, T^*)\)
\((S, S^*)\)
Tiling by Squares
\[ \gamma_t = \{A, B, C\} \]
Blocking Polyhedra

\[ \hat{B} = A = \begin{bmatrix} 3 & 1/2 \\ 2 & 1 \\ 1 & 2 \end{bmatrix} \quad \hat{A} = B = \begin{bmatrix} 0 & 2 \\ 1/4 & 1/2 \\ 1/3 & 1/3 \\ 1 & 0 \end{bmatrix} \]

\[ B_1 = (0, 2) \]
\[ B_2 = (1/4, 1/2) \]
\[ B_3 = (1/3, 1/3) \]
\[ B_4 = (1, 0) \]

\[ A_1 = (3, 1/2) \]
\[ A_2 = (2, 1) \]
\[ A_3 = (1, 2) \]