Electric Polarization Induced by Gravity in Fat Branes

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In the fat brane model, also known as the split fermion model, it is assumed that leptons and baryons live in different hypersurfaces of a thick brane in order to explain the proton stability without invoking any symmetry. It turns out that, in the presence of a gravity source \( M \), particles will see different four-dimensional (4D) geometries and hence, from the point of view of 4D-observers, the equivalence principle will be violated. As a consequence, we show that a hydrogen atom in the gravitational field of \( M \) will acquire a radial electric dipole. This effect is regulated by the Hamiltonian

\[
H_d = -\mu \mathbf{A} \cdot \delta \mathbf{r},
\]

where the electric field is replaced by the tidal acceleration \( \mathbf{A} \) due to the split of fermions in the brane and the atomic reduced mass \( \mu \) substitutes the electric charge.

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I. INTRODUCTION

In the braneworld models, our four-dimensional spacetime is viewed as a submanifold isometrically embedded in an ambient space with higher dimensions. The basic feature of this scenario is the confinement of matter and fields in the brane (when they have an energy lower than a certain level which is expected to be of the order of 1TeV, at least), while gravity has access to all dimensions \[1, 3\]. In this framework, the extra dimensions might be much larger than Planck length. As a matter of fact, in the RSII model it was shown that the extra dimension might even have an infinity length without any phenomenological conflict \[3\].

A modified version of the RSII model, known as fat brane, assumes that the brane has a thickness and that leptons and baryons live in different hypersurfaces of the thick brane \[4\]. The original motivation of this model is to explain the stability of protons without using any symmetry. The conservation of the baryonic number, that protects the proton from decaying, is just a consequence of the split of fermions in the thick brane, since this separation produces a strong suppression in couplings between quarks and leptons. On the other hand, gauge fields have access to all the brane. Thus, if the thickness of the brane is of TeV order then we might expect that traces of the extra dimensions could be detected in experiments at LHC \[4, 5\].

By virtue of the confinement, particles do not see the geometry of the whole bulk but they feel the induced metric on the hypersurface where they live. If there is no gravity source then all fermions see the same Minkowski spacetime. However, under the gravitational influence of a mass \( M \) in the brane, the induced metric will be different for distinct slices. This means that leptons and baryons will feel different geometries. From the point of view of 4D-observers (not aware of the extra dimensions), this will be seen as a violation of the equivalence principle, since particles in the same four-dimensional brane coordinates will feel different gravitational accelerations. This tidal acceleration \( \mathbf{A} \), due to the split of fermions in the extra dimension, produces an internal force in a hydrogen atom, inducing, in this way, an electric dipole in a tangential direction of the brane. As we shall see, the Hamiltonian associated with the interaction between the atom and the gravitational field of \( M \) contains a dipole term which has exactly the same form of the Stark Hamiltonian, \( H_d = -\mu \mathbf{A} \cdot \delta \mathbf{r} \) (where \( \delta \mathbf{r} \) denotes the internal relative coordinates), in the first order of \( GM \), where \( G \) is Newton’s gravitational constant.

II. GRAVITY IN THICK BRANES

In the context of thick brane models, the brane is usually described as a domain wall generated by a certain scalar field \( \phi \) \[1, 6\]. The presence of a mass \( M \) (a star or a black hole) localized in the brane will affect the domain wall solution. An exact solution for this system is not known so far. Solutions for lower dimension (2-brane) are known.
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surface of the scalar field anymore. For instance, the center of the domain wall (φ = 0), which originally coincides
with z = 0, is now, in the first order approximation, given by the equation z = −εk(r, 0). However, by a convenient
coordinate transformation, we can show that it is possible to restore a coordinate system adapted to the center of the
domain wall, i.e., coordinates in which the center is localized at the hypersurface z = 0. The great advantage of
working in these coordinates is the fact that “initial conditions”, i.e., the value that the correction functions assume
in the center of wall, can be easily established. For example, as the center corresponds to z = 0, then, we should have
k(r, 0) = 0. Another important condition can be immediately deduced based on the expectation that the metric
should be symmetric with respect to the center of the wall. As ∂z is the normal vector of the hypersurface z = 0, then,
it follows that in the center of the domain the first derivative with respect to z should be zero: f_z = m_z = h_z = 0. The
remaining set of the initial conditions, i.e., f(r, 0), m(r, 0), h(r, 0) and k_z(r, 0), can be determined by using the thin
brane solution as inspiration. In Ref. [12], Garriga and Tanaka found the metric produced by a matter distribution
with mass M localized in the thin brane in the first approximation order of GM. With the purpose to obtain a
connection with this thin brane solution, we are going to impose that in z = 0 our solution reproduces Garriga and
Tanaka’s result. This condition allows us to determine f(r, 0), m(r, 0), h(r, 0). With these choices, the last condition
k_z(r, 0) = 0 follows from the (zz)-component of the Einstein equations, which corresponds to a constraint equation.
Now by using the dynamical components of the Einstein equations (those that involve second derivatives with respect
to z) to propagate the initial conditions into the bulk, we find the metric around the center of the brane as a power

\[ ds^2 = -e^{2A(r,z)}dt^2 + e^{2B(r,z)}dr^2 + e^{2C(r,z)}d\Omega^2 + dz^2, \]

where z = 0 gives the localization of the center of the brane. Due to symmetry the scalar field will depend only on
the coordinates r and z, i.e., φ = φ(r, z).

Even when M = 0, the Einstein equations (5) \( G_{\mu\nu} = \kappa T^{(\phi)}_{\mu\nu} \) coupled to the scalar field equation \( \Box \phi - V'(\phi) = 0 \in five dimensions are not easy to solve. Here \( \kappa \) is the five-dimensional gravitational constant, \( T^{(\phi)}_{\mu\nu} \) is the usual
energy-momentum tensor of the scalar field subjected to the potential \( V(\phi) \) and \( c = 1 \). In some special situations,
when the potential is conveniently chosen, an exact solution of a self-gravitating domain wall can be obtained. For
example, taking \( V(\phi) = \lambda/4 \times (\phi^2 - \eta^2)^2 - \beta \lambda/3\eta^2 \times \phi^2 (\phi^2 - 3\eta^2)^2 \), the solution is [11]:

\[ ds^2 = e^{2a(z)} (-dt^2 + dr^2 + r^2 d\Omega^2) + dz^2, \]

\[ 2a(z) = -2\beta \ln \cosh^2 \frac{z}{\epsilon} - \beta \tanh^2 \frac{z}{\epsilon}, \]

\[ \phi = \eta \tanh \frac{z}{\epsilon}, \]

where \( \epsilon^2 = 2/\lambda \eta^2 \), \( \beta = \kappa \eta^2/9 \). This solution can be interpreted as a regularized version of the RSII brane model.
Indeed, taking the parameter \( \epsilon \) (the thickness of the wall) equal to zero, while keeping the condition \( \epsilon/2\beta = \text{const.} \equiv \ell \) (\( \ell \) defines the curvature radius of AdS5 space) the RSII solution is recovered [10].

Let us now consider a mass M describing a body or a black hole confined in the core of the domain wall. The
presence of this gravity source certainly will modify both the original metric and the scalar field. Considering the
amount of technical difficulty to solve this problem exactly, let us try to employ approximation methods [11]. At large
distances from M, where the weak field regime is valid, the modification can be treated as a small perturbation of
the original solution. In this case, we can write

\[ ds^2 = e^{2a} \left[ -(1 + f)dt^2 + (1 + m)dr^2 + r^2 (1 + h) d\Omega^2 \right] + dz^2, \]

\[ \phi = \eta \left( \tanh \frac{z}{\epsilon} + k \right), \]
series in $z$. In the first correction order, the line element for $z << \varepsilon$ and $r >> GM$ is given by
\[
ds^2 = -e^{2a} \left( 1 - \frac{2GM}{r} \left( 1 + \frac{2\ell^2}{3\varepsilon^2} - \frac{2\ell^2}{r^4} z^2 \right) \right) dt^2 \\
+ e^{2a} \left( 1 + \frac{2GM}{r} \left( 1 + \frac{\ell^2}{3\varepsilon^2} + \frac{\ell^2}{r^4} z^2 \right) \right) dr^2 \\
+ e^{2a} \left( r^2 - \frac{3GM\ell^2}{\varepsilon^2} z^2 \right) d\Omega^2 + dz^2,
\]
and the scalar field solution is simply $\phi = \eta \tanh z/\varepsilon$ in the first approximation order of $GM$.

### III. Electric Dipole Induced by Gravity

It is well known that the confinement of matter in a thick brane can be obtained by means of a Yukawa-type interaction between the Dirac field and the scalar field [1]. Under this interaction, the wave packet of a massless Dirac field has a peak at the center of the domain wall and decays exponentially in the extra dimension. When a non-null mass is taken into account, the peak is shifted by a certain amount that depends on the particle mass [14]. Therefore, electrons and quarks will be localized in different slices of the brane.

In this scenario, particles are in a bound state with respect to the transversal direction, but they might be free in the parallel direction. If we want to study the motion of a particle along the brane it is convenient to consider a classical approach for this problem. In order to achieve this, it is necessary first to provide a mechanism of confinement of test particles to the brane, which may simulate classically the confinement of the matter field. In Ref. [13], a particular Lagrangian based on the Yukawa interaction was proposed to describe the particle’s motion in this context. It was then shown that the new Lagrangian has the effect of increasing the effective mass of the particle due to the interaction with the scalar field, and this modification is sufficient to ensure the localization of the particle. This new Lagrangian was defined as $L^2 = -\left( m^2 + h^2 \varphi^2 \right) \tilde{g}_{AB} \dot{x}^A \dot{x}^B$, where $m$ is the rest mass of the particle in a free state and $h$ is the coupling constant of the interaction. The split of fermions in different slices can be added in our model by redefining the Lagrangian as
\[
L^2 = - \left( m^2 + h^2 (\varphi + \alpha m)^2 \right) \tilde{g}_{AB} \dot{x}^A \dot{x}^B, \tag{8}
\]
where $\alpha$ is a new parameter related to the interaction. Calculating the 5D-momentum $P_A = \partial L/\partial \dot{x}^A$ of the particle, we find $P_A P_A = -m^2 - h^2 (\varphi + \alpha m)^2 \equiv -m_{eff}^2$, for massive test particles ($\tilde{g}_{AB} \dot{x}^A \dot{x}^B = -1$). Then, we can verify directly that the effective mass $m_{eff}$ is now affected by the presence of the scalar field. Of course, the usual relation is recovered by turning off the interaction, i.e., by setting $h = 0$. It is worthy of mention that a similar kind of Lagrangian was also employed, in a different context, to describe the interaction between test particles and dilatonic fields [14].

Due to the interaction with the scalar field, particles will move with a proper acceleration $A^A = -\Pi^{AC} \tilde{g}_{CB} \ln \mathcal{M}$, which is the gradient of the mass potential $\mathcal{M} \equiv \tilde{g}_{AB} \dot{x}^A \dot{x}^B$ projected by the tensor $\Pi^{AC} \equiv \tilde{g}^{AC} + \tilde{x}^A \tilde{x}^C$ into the four-space orthogonal to the particles’ proper velocity $\dot{x}^A$.

When there is no additional gravity source ($\mathcal{M} = 0$) the metric in the bulk is given by (2) and the transversal motion decouples from the motion in the tangential direction. In this case, the first integral of the equation of motion in the $z$ direction can be obtained directly, also implying that the transversal motion is bounded by the mass potential $\mathcal{M}$ according to the equation $\mathcal{M} \ddot{z}^2 = \mathcal{E} - \mathcal{M}$, where $\mathcal{E}$ is a constant related to the initial condition of the motion. The function $\mathcal{M}$ plays the role of a confining potential and has a stable equilibrium point when the parameters $h$ and $\alpha$ satisfy appropriate conditions. It is important to stress here that the equilibrium position $z_0$ depends on the mass of the particle. If we admit that $z_0$ is small in comparison with the brane thickness $\varepsilon$, then we can show that the particle with mass $m$ is confined to a slice approximately specified by $z_0 = \sqrt{2/3} \alpha m \sqrt{\varepsilon/\ell c}$. As we can see, electrons are localized closer to the center than quarks. Of course we can manipulate the Lagrangian in order to get the inverse result, namely, quarks stuck in the center of the brane and electrons in an upper slice. For our purpose, what is important here is that we can formulate a simple classical model which has the essential characteristic of the fat brane model, namely, the split of leptons and baryons in different slices of the brane. On the other hand, in the tangential direction both fermions move freely in the same induced four-dimensional Minkowski spacetime.

This situation changes when we consider the presence of a mass $M$ in the brane. For the sake of simplicity, hereafter we are going to admit that quarks are confined in the center of the brane, while electrons will be stuck in some slice $z_0$. With this choice, we can admit that almost all the mass $M$ is localized in the center of the brane, since the baryons
are stuck in that hypersurface. It follows then that the metric will given by (1) and therefore leptons and baryons will see different four-dimensional geometries, since the induced metric depends on the value of $z$. As a consequence, the equivalence principle will be violated from the 4D perspective and this fact can produce interesting phenomena in the brane as, for example, the induction of an electric dipole in a hydrogen atom by gravitational effects. In order to investigate this, let us consider the motion of a particle in the spacetime with metric (7). Due to the symmetry of this spacetime, the energy $E$ and the axial angular momentum $L$ will be conserved and the particle’s motion should obey the following equations

$$-m_e \ddot{y}_t t = E$$  \hspace{1cm} (9)

$$m_e \ddot{y}_0 \phi \dot{\phi} = L$$  \hspace{1cm} (10)

We can also verify that $\theta = \frac{\pi}{2}$ is a solution of the equations. On the other hand, the motion in the $z$ direction is not decoupled from the radial motion. However, if the particle is in a circular motion or static ($r_0 = \text{const}$) then the effect of the mass $M$ is just to modify the equilibrium position of the particle by an amount of the order of $GM$. Analyzing the radial motion we can see that there are stable circular orbits for appropriate values of energy and angular momentum. Considering this, it follows from equations (9) and (10) that the angular frequency of a particle in a circular motion of radius $r_0$ is given by:

$$\omega^2 = \left(\frac{\dot{\phi}}{\ell}\right)^2 = \frac{GM}{r_0^3} \left(1 + \frac{2\ell^2}{r_0^2} - \frac{10\ell^2}{r_0} z_0\right).$$  \hspace{1cm} (11)

This clearly shows that the angular frequency depends on the mass of the particle through $z_0$. From the perspective of 4D observers, this mass dependence will be seen as a violation of the equivalence principle. In fact, in a circular orbit with the same radius $r_0$, a proton (stuck in the center of the brane) will move faster than an electron.

Now if we consider a hydrogen atom orbiting the mass $M$ with a certain angular frequency or static, then based on the previous reasoning we are led to expect that the radius of the electron’s orbit and the radius of the proton’s orbit should be different. As matter of fact, the center of the negative charge tends to circulate in an outer orbit in comparison with the proton’s orbit, as we shall see next.

In the weak field regime, we can consider that the gravitational potential of the mass $M$ is given by

$$\varphi = -\frac{GM}{r} \left(1 + \frac{2\ell^2}{3r^2} - \frac{2\ell^2}{r^4} z^2\right).$$  \hspace{1cm} (12)

Here it is important to note that, based on the equation (9), the warping factor can be incorporated in a redefinition of the mass of the particle and for this reason we might define the potential $\varphi$ without using it. Thus, the non-relativistic Hamiltonian of the hydrogen atom is

$$H = \frac{P_e^2}{2m_e} + \frac{P_p^2}{2m_p} + m_p \varphi_p + m_e \varphi_e + U,$$  \hspace{1cm} (13)

where $U$ is the potential energy of the electric and gravitational interaction between the proton and the electron. Introducing the coordinates of the center of mass $\mathbf{R}$ and the relative coordinates $\delta \mathbf{r} = \mathbf{r}_p - \mathbf{r}_e$, we can verify that $H$ contains an unusual dipole term due to the fact that electrons and protons live in different slices of the brane. This new term $H_d$ has the same form of the Stark Hamiltonian

$$H_d = -\mu \mathbf{A} \cdot \delta \mathbf{r},$$  \hspace{1cm} (14)

where the tidal acceleration $\mathbf{A}$ between the electron and the proton replaces the electric field. We must emphasize that $\mathbf{A}$ is the relative gravitational acceleration between the electron and the proton when they have the same four-dimensional brane coordinates. It is clear that the origin of $\mathbf{A}$ is the split of fermions in the extra dimensions. It can also be shown that

$$\mathbf{A} = -GM \left(\frac{10\ell^2}{R^7} z_0^2\right) \mathbf{R}.$$  \hspace{1cm} (15)

We should mention that, considering the procedure usually adopted to deal with atomic systems in gravitational field (which is based on the expansion of metric around the center of the mass of the system in Fermi coordinates), this tidal acceleration comes from the term $H_d$ is the proper velocity of the center of mass.
and $s^2 = (0, 0, 0, z_0)$ is the separation vector (or the relative coordinates). The semi-colon indicates the covariant derivative of the five-dimensional Riemann tensor $R_{\beta\gamma\lambda\mu}^\rho$.

The Hamiltonian $H_d$ will induce an electric dipole $\mathbf{p}$ in the hydrogen atom, whose direction tends to be aligned with the tidal field. In order to make a rough estimate of the dipole magnitude we are going to describe the atom following a semi-classical approach. First, we admit that the electron charge is uniformly distributed in a cloud around the proton. The tidal acceleration inside the hydrogen atom will produce a radial separation between the proton and the center of the negative charge, giving rise to an electric force $\mathbf{F}$ between them. To calculate $\mathbf{F}$, we are going to make some considerations. We begin by recalling that, according to the fat brane model, the particle state is described by a very narrow wave packet along the extra dimension and thus the wave function can be considered as a delta-distribution in $z$ direction. So we can assume that the electronic cloud is spread in a spherically symmetric 3-volume of the slice $z_0$. A sphere attached to a hypersurface constitutes a kind a 3D-disk from the bulk perspective. We are going to admit that the radius of this disk is equal to the Bohr radius $a$. Additionally, it is reasonable to expect that the radial separation $\delta r$ is not greater than $z_0$, and therefore, in our calculations, we have to take into account that in such domain the electric force has a 5D behavior. Considering all these assumptions, and also that $z_0 << a$, we can show that $\mathbf{F} = (2ke^2/e^4a^4)\delta r$, where $k$ is the known electrostatic constant in 4D spacetime.

Since at the equilibrium state the internal electric force and the tidal force are balanced, it follows that the electric dipole induced by gravity is proportional to the tidal acceleration $\mathbf{p} = (\pi^2a^4\mu/2ke^2)\mathbf{A}$. The proportionality factor defines the electric polarizability of the hydrogen atom induced by gravity and it can be written as $\alpha_G = (\pi^2\mu/2e)\alpha_E$, where $\alpha_E$ is the usual electric polarizability of the atom induced by electric fields. In order to make a comparison between the polarization induced by gravity and that induced by an electric field, let us consider the equivalent electric field $\mathbf{E}_{eq} \equiv \mu/e\mathbf{A}$, which is capable to produce the same acceleration $\mathbf{A}$ in a particle with charge $e$ and mass $\mu$. In a TeV-brane, the gravity-induced dipole will be approximately $10^9$ ($\approx a/\varepsilon$) greater than the dipole induced by the equivalent electric field. Despite this impressive magnification, as the tidal acceleration $\mathbf{A}$ produced by astronomical bodies will be very tiny, we should expect that gravity-induced dipole will be more significant in the presence of microscopic black holes.

### A. Final remarks

We would like to mention that in Ref. [16], it was shown that an effective Hamiltonian of a hydrogen atom, deduced from the covariant Dirac equation in a curved spacetime up to order $v^2/c^2$, contains terms that can mix opposite-parity state. However, these terms arise as a result of a relativistic effect in post-Newtonian approximation, while the electric polarization induced by gravity discussed here is a consequence of the split of fermions in the extra dimension.

Finally, we should also emphasize that in the fat brane, as electrons and protons are stuck in different slices of the brane, then every atom should have an electric dipole with a non-null component in the $z$ direction. However, we have shown here that the gravitational field of a mass $M$ will induce a tangential component in the atomic dipole and hence the atom will produce an electric field in the brane whose pattern can be recognized by 4D-observers.

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