Hydroelastic Oscillations of a Three-layer Plate Interacting with Vibrating Stamp

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Abstract. Longitudinal and bending oscillations of a three-layer plate interacting with vibrating stamp through a thin layer of a viscous incompressible fluid are investigated. We study the plane problem. The plate is considered as a three-layer package consisting of two bearing layers with an incompressible filler between them. The broken normal hypothesis is used to cinematically describe a three-layer package. A mathematical model including Navier-Stokes equations, continuity equation and three-layer plate dynamics equations with the corresponding boundary conditions is developed. No-slip conditions, pressure conditions at the channel edges and simply supported plate edges conditions are selected as boundary ones. The hydrodynamic parameters distribution laws of the liquid layer and the elastic displacements of the three-layer plate are found by mines of the perturbation method.

1. Introduction
At present, the multilayer materials and structural elements are widely used in various industries: industrial and civil construction, energy, aerospace engineering, electronics, etc. [1]. Therefore the mathematical modeling questions of layer structures behavior at its static and dynamic loadings by force factors of various natures are of theoretical and practical interest. On the other hand, hydroelasticity problems studies usually consider homogeneous structural elements [2]. For example, in [3] a hydroelastic response model of IC engine wet cylinder liner as a beam interacting with an ideal liquid is considered. In references [4-7], the hydroelastic oscillations of plates surrounded by an ideal liquid or interacting with an ideal fluid flow are studied.

The chaotic oscillations of a plate interacting with an ideal fluid flow are investigated in [8]. Reference [9] considers the stability and dynamic problem of an elastic plate being a part of the border dividing the areas filled with viscous incompressible fluid. Oscillations of an infinite beam interacting with a viscous liquid layer are studied in [10]. Reference [11] is devoted to forced oscillations of an elastic fixed rigid body interacting with viscous liquid thin layer. Bending oscillations of a cantilever beam surrounded by a viscous incompressible liquid or located in its flow are considered in [12, 13]. References [14-16] deal with hydroelastic models of narrow and tapering channels filled with a
viscous liquid. The hydroelastic oscillations of plates resting on Winkler or Pasternak foundation are studied in [17-21]. Free bending oscillations of a composite cantilever plate immersed in an ideal liquid are investigated in [22]. References [23, 24] consider three-layer walls bending vibrations of the channels with viscous fluid without taking into account the shear stresses effect of a liquid and inertia forces of three-layer walls in the longitudinal direction. In this paper is devoted to the study of these factors’ influence.

2. Statement of the Problem
Let us consider a narrow plane channel through which a viscous liquid moves due to an assigned pressure difference at the channel edges (Fig. 1). The channel walls are formed by a three-layer plate and a vibrating stamp. The stamp oscillates in a vertical plane according to the given harmonic law: $z = z_m f(\omega t)$, $f(\omega t) = \sin(\omega t)$. Here $z_m$ is the stamp oscillations amplitude, $\omega$ is the oscillations frequency, $t$ is the time. The plate is a three-layer package consisting of two bearing layers and an incompressible filler between them. The bearing layers thickness are $h_1$ and $h_2$, the filler thickness is $2\varepsilon$. We use the broken normal hypothesis for the kinematics description of a three-layer package [1]. The three-layer plate is simply supported at its edges.

Let us consider Cartesian coordinate system $Oxyz$. Its center is connected with the center of the filler median surface in the unperturbed state. The top bearing layer of the plate contacts with the viscous incompressible fluid thin layer in the channel. The thickness of the fluid layer is $\delta_0$. We suppose the plate size $\ell$ is significantly greater than $\delta_0$, i.e. $\ell >> \delta_0$. The fluid completely fills the narrow channel between the vibrating stamp and the three-layer plate. Considering the liquid dynamics, we take into account its tangential and normal stresses acting on the boundary line. The static pressure difference $\Delta p = p^- - p^+$ is set at the channel edges. Here $p^-$ is the pressure at the left edge, $p^+$ is the pressure at the right edge. We study the regime of steady harmonic oscillations, as due to fluid viscosity the transient processes will quickly damp [25]. We suppose the oscillations amplitudes of the plate and the stamp to be significantly less than the fluid layer thickness.

3. The Theory and Solution
We obtained the dynamic equations of three-layered plate by using Lagrange principal of virtual displacements and taking into account the inertia forces work [26]. These equations are obtained in the form of:

$$
\begin{align*}
\alpha_1 \frac{\partial^2 u}{\partial x^2} + 6 \frac{\partial^2 \phi}{\partial x^2} - 4 \frac{\partial^3 w}{\partial x^3} - m_1 \frac{\partial^2 u}{\partial t^2} - m_6 \frac{\partial^2 \phi}{\partial x \partial t^2} + m_7 \frac{\partial^2 w}{\partial x^2 \partial t} = -q_{xx}, \\
\alpha_6 \frac{\partial^2 u}{\partial x^2} + 2 \frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^3 w}{\partial x^3} - a_5 \frac{\partial^2 \phi}{\partial t^2} - m_6 \frac{\partial^2 u}{\partial x \partial t^2} - m_2 \frac{\partial^2 \phi}{\partial x^2} - m_3 \frac{\partial^2 w}{\partial x^2 \partial t} = 0, \\
\alpha_3 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^3 w}{\partial x^3} - a_4 \frac{\partial^2 \phi}{\partial t^2} - m_1 \frac{\partial^2 u}{\partial x \partial t^2} - m_7 \frac{\partial^2 w}{\partial x^2 \partial t} - m_3 \frac{\partial^2 w}{\partial x \partial t^2} = -q_{zz},
\end{align*}
$$

$$
m_1 = \rho_1 h_1 + \rho_2 h_2 + \rho_3 2\varepsilon, \quad m_2 = c^2 (\rho_1 h_1 + \rho_2 h_2) + 2m_5, \quad m_3 = c (\rho_1 h_1 (c + h_1/2) + \rho_2 h_2 (c + h_2/2)) + m_5, \quad m_4 = \rho_1 h_1 (c^2 + h_1 + (1/3)h_1^2) + \rho_2 h_2 (c^2 + h_2 + (1/3)h_2^2) + m_5, \quad m_5 = (2/3)\rho_3 c^3, \quad m_6 = (\rho_1 h_1 - \rho_2 h_2) c,
$$

Figure 1. The narrow plane channel.
\[ m_j = \rho_j h_j (c + 1/2 h_j) - \rho_2 h_2 (c + 1/2 h_2), \quad a_1 = K_1^+ h_1 + K_2^+ h_2 + 2 K_3^+ c, \quad a_2 = c^2 (K_1^+ h_1 + K_2^+ h_2 + (2/3) K_3^+ c), \]
\[ a_3 = c (K_1^+ h_1 (c + (1/2) h_1) + K_2^+ h_2 (c + (1/2) h_2) + (2/3) K_3^+ c^2), \]
\[ a_4 = K_1^+ h_1 (c^2 + c h_1 + (1/3) h_1^2) + K_2^+ h_2 (c^2 + c h_2 + (1/3) h_2^2) + (2/3) K_3^+ c^3, \quad a_5 = 2 G_3 c, \]
\[ a_6 = c (K_1^+ h_1 - K_2^+ h_2), \quad a_7 = K_1^+ h_1 (c + (1/2) h_1) - K_2^+ h_2 (c + (1/2) h_2), \]
\[ K_3^+ = K_f + 4/3 G_f. \]

Where \( j = 1, 2, 3 \) is the layer number, \( G_j \) is the \( j \)-th layer shear modulus, \( K_j \) is the \( j \)-th layer bulk modulus, \( \rho_j \) is the \( j \)-th layer material density, \( u \) is the longitudinal plate displacement, \( w \) is the plate deflection, \( \varphi \) is the rotation angle of the deformed normal in the plate filler, \( q_s \) is the normal stress in the viscous fluid layer, \( q_{sh} \) is the shear stress in the viscous fluid layer.

We consider the equations (1) with boundary conditions:
\[ w = \partial^2 w / \partial x^2 = \partial u / \partial x = \partial \varphi / \partial x = 0 \text{ at } x = \pm \ell. \] (2)

The motion of viscous incompressible fluid in a narrow channel formed by a three-layered plate and vibrating stamp can be considered as a creeping one [27]. The creeping fluid dynamic equations are presented as:
\[
\frac{1}{\rho} \frac{\partial \rho}{\partial t} = \nabla \cdot \left( \frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_z}{\partial z^2} \right), \quad \frac{1}{\rho} \frac{\partial \rho}{\partial z} = \nabla \cdot \left( \frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_x}{\partial z^2} \right), \quad \frac{\partial u_x}{\partial t} + \frac{\partial u_x}{\partial x} = 0, \]
(3)

where \( u_x, u_z \) are fluid velocity projections on the coordinate axis, \( \rho \) is the fluid density, \( v \) is the coefficient of kinematic viscosity, \( p \) is the pressure.

The boundary conditions of equations (3) are the no-slip conditions and the conditions for the pressure at the edges:
\[ u_x = 0, \quad u_z = dz/dt \text{ at } z = \delta_0 + c + h_1, \quad u_x = \partial u / \partial t, \quad u_z = \partial w / \partial t \text{ at } z = w + c + h_1, \] (4)
\[ p = p^- \text{ at } x = -\ell, \quad p = p^+ \text{ at } x = \ell. \] (5)

Let us introduce dimensionless variables and small parameters:
\[
\lambda = z_m / \delta_0 << 1, \quad \psi = \delta_0 / \ell << 1, \quad \zeta = (z - c - h_1) / \delta_0, \quad \xi = c/\ell, \quad \tau = \alpha t, \quad w = w_m W, \quad u = u_m U, \]
(6)
\[
\varphi = \varphi_m \Phi, \quad p = \rho \nu z_m \alpha \delta_0 \nu^2 / \partial \psi^2 - 1, \quad u_z = z_m \omega U \psi, \quad u_z = (z_m \omega / \psi) U \psi. \]

Substituting (6) into equations (3)-(5), we obtain the dimensionless hydrodynamics problem of viscous liquid layer in a zeroth-term approximation for small parameters \( \psi \) and \( \lambda \):
\[
\partial P / \partial \xi = \partial^2 U_\zeta / \partial \zeta^2, \quad \partial P / \partial \zeta = 0, \quad \partial U_\zeta / \partial \xi + \partial U_\xi / \partial \zeta = 0, \]
(7)
\[ U_\zeta = 0, \quad U_\xi = df / d\tau \text{ at } \zeta = 1, \quad U_\zeta = 0, \quad U_\xi = (w_m / z_m) \partial W / \partial \tau \text{ at } \zeta = 0, \]
(8)
\[ P = p^- \text{ at } \xi = -1, \quad P = p^+ \text{ at } \xi = 1. \]

Solving this problem we find:
\[ U_\zeta = (\partial P / \partial \xi) (\xi^2 - \zeta^2) / 2, \quad U_\xi = (w_m / z_m) \partial W / \partial \tau + (\partial^2 P / \partial \xi^2) (3 \xi^2 - 2 \zeta^2) / 12, \]
(9)
\[ P = 6 (\xi^2 - 1) df / d\tau - 12 w_m z_m - \int^{\xi}_{-1} \frac{\partial W}{\partial \tau} d \tilde{\xi} d \tilde{x} + 6 (\xi + 1) w_m z_m - \frac{1}{2} \int^{\xi}_{-1} \frac{\partial W}{\partial \tau} d \tilde{\xi} d \tilde{x} + \frac{1}{2} [P^+ + P^- - \xi (P^+ - P^-)], \]
\[
\frac{\partial U}{\partial \xi} / \frac{\partial \zeta}{\partial \xi} \bigg|_{\xi=0} = -6\xi \frac{df}{d\tau} + 6(w_m / z_m) \int \frac{1}{-1} \partial W / \partial \tau d\tau \xi - 3(w_m / z_m) \int \partial W / \partial \tau d\tau \xi + (P^- - P^+) / 4.
\]

The normal and shear stresses of fluid acting on the boundary line take the form of:

\[
q_{zz} = -\rho v z_m \omega(\delta_0 \psi^2)^{-1} P \text{ at } \zeta = 0, \quad q_{xx} = \rho v z_m \omega(\delta_0 \psi^{-1})^{-1} \partial U / \partial \zeta \text{ at } \zeta = 0.
\]

Considering the boundary conditions (2), the equations solution (1) is presented in the form of:

\[
U = -\sum_{k=1}^{\infty} ((2k-1)\pi/2)(Q_k^0 + Q_k(\tau)) \sin(((2k-1)\pi/2)\xi) + \bar{Q}_k(\tau) k\pi \cos k\pi \xi,
\]

\[
\Phi = -\sum_{k=1}^{\infty} ((2k-1)\pi/2)(S_k^0 + S_k(\tau)) \sin(((2k-1)\pi/2)\xi) + \bar{S}_k(\tau) k\pi \cos k\pi \xi,
\]

\[
W = \sum_{k=1}^{\infty} (R_k^0 + R_k(\tau)) \cos(((2k-1)\pi/2)\xi) + \bar{R}_k(\tau) \sin k\pi \xi.
\]

Here the upper index 0 means the solution, corresponding to the static pressure.

By substituting (6), (9)-(11) into (2) and expanding \(p^-, p^+, \phi_p, \phi_p^+\) in the series of trigonometric functions of the longitudinal coordinate \(\xi\), we obtained the system of linear algebraic equations for the definition \(R_k^0, Q_k^0, S_k^0, \bar{Q}_k, \bar{S}_k\) and the system of the linear differential equations for the definition \(R_k(\tau), Q_k(\tau), S_k(\tau)\). However, for the stationary harmonic oscillations case, the differential equations system transforms into an algebraic one. The algebraic equations can be easily solved in relation to the desired values. As a result we obtain the following expressions:

\[
S_k^0 = \frac{w_m R_k^0}{\varphi_m} a_3 - \frac{u_m Q_k^0}{\varphi_m} a_6, \quad R_k^0 = \frac{\ell}{w_m} \left( b_{2e}(a_7 - a_6 a_3 / a_2 e) - b_{2e}(a_4 - a_6^2 / a_2 e) \right),
\]

\[
Q_k^0 = \frac{1}{w_m} \left( b_{2e}(a_7 - a_6 a_3 / a_2 e)^2 - b_{2e}(a_4 - a_6^2 / a_2 e) \right), \quad \bar{Q}_k = \frac{w_m}{\varphi_m} a_3 - a_6 a_6,
\]

\[
\bar{S}_k = \frac{w_m}{\varphi_m} a_3 - a_6 a_6, \quad \bar{R}_k = (p^- - p^+)^k (1 - (1)^k) \frac{2}{k\pi} \left( \frac{a_1 a_2 - a_6^2}{a_2 a_7 + a_4 a_6 - 2 a_3 a_6 a_7 - a_2 a_2 a_4} \right),
\]

\[
Q_k(\tau) = (z_m / w_m) B_2 \exp(i(\tau + \phi_\theta)), \quad R_k(\tau) = (z_m / w_m) B_2 \exp(i(\tau + \phi_\theta)) / \sqrt{A^2 + B^2},
\]

\[
S_k(\tau) = (a_2 w_m R_k(\tau) - a_1 u_m Q_k(\tau)) / (\varphi_m a_2 w_m), \quad \tau \phi_\theta = (B_1 \tilde{A} - B_1 \tilde{B}) / (\tilde{A}_1 \tilde{A} - \tilde{B}_1 \tilde{B}), \quad \tau \phi_\theta = \tilde{B}_2 B_1 \tilde{A} - \tilde{B}_2 B_1 \tilde{B},
\]

\[
\tilde{A}_1 = 2b_{33}(1-k) \left( \frac{(2k-1)\pi}{2\ell} \right)^2 K_1(\omega - 2b_{31} - (2k-1)\pi / (2\ell))^3 K_2(\omega), \quad \tilde{B}_1 = A_3 A_1 - A_1 A_3, \quad \tilde{B} = A_3 b_{13} - A_1 b_{33},
\]

\[
\tilde{B}_2 = 2A_{12}(-1)^k ((2k-1)\pi / (2\ell))^3 K_2(\omega - 2A_{32} - (2k-1)\pi / (2\ell))^2 K_1(\omega),
\]

\[
A_1 = (a_1 a_{22} - a_2 a_{21}) / a_{22}, \quad A_2 = (a_1 a_{22} - a_2 a_{23}) / a_{22}, \quad A_3 = (a_3 a_{22} - a_3 a_{22}) / a_{22},
\]
\[
A_{32} = \frac{a_{33}a_{22} - a_{12}a_{23}}{a_{22}}, \quad \alpha_{11} = \frac{(2k-1)\pi}{2} \left[ a_{11} \left( \frac{(2k-1)\pi}{2} \right)^2 - \omega^2 m_1 \right], \quad \alpha_{12} = \frac{(2k-1)\pi}{2} \left[ a_{12} \left( \frac{(2k-1)\pi}{2} \right)^2 - \omega^2 m_2 \right], \quad \alpha_{22} = \frac{(2k-1)\pi}{2} \left[ a_{22} \left( \frac{(2k-1)\pi}{2} \right)^2 + a_5 - \omega^2 m_2 \right], \quad \alpha_{23} = \frac{(2k-1)\pi}{2} \left[ a_{23} \left( \frac{(2k-1)\pi}{2} \right)^3 - \omega^2 m_3 \left( \frac{(2k-1)\pi}{2} \right)^2 \right], \quad \alpha_{31} = \frac{(2k-1)\pi}{2} \left[ a_{31} \left( \frac{(2k-1)\pi}{2} \right)^3 - \omega^2 m_3 \left( \frac{(2k-1)\pi}{2} \right)^2 \right], \quad \alpha_{32} = \frac{(2k-1)\pi}{2} \left[ a_{32} \left( \frac{(2k-1)\pi}{2} \right)^3 - \omega^2 m_3 \left( \frac{(2k-1)\pi}{2} \right)^2 \right], \quad \alpha_{33} = \frac{(2k-1)\pi}{2} \left[ a_{33} \left( \frac{(2k-1)\pi}{2} \right)^4 - \omega^2 m_1 - \omega^2 m_4 \left( \frac{(2k-1)\pi}{2} \right)^2 \right], \quad b_{13} = \frac{2}{(2k-1)\pi} K_\omega, \quad a_{21} = \frac{(2k-1)\pi}{2} \left[ a_{21} \left( \frac{(2k-1)\pi}{2} \right)^2 - \omega^2 m_6 \right], \quad b_{31} = \frac{2}{(2k-1)\pi} K_2 \omega, \quad K_1 = 6 \rho^2 v/\gamma h_0^2, \quad K_2 = 6 \rho^2 v/(\gamma h_0)^2, \quad b_{ic} = (p^+ - p^-) \left( \frac{2}{(2k-1)\pi} \right)^2, \quad b_{ic} = (p^+ + p^-) \left( \frac{2}{(2k-1)\pi} \right)^2, \quad b_{ic} = (p^+ - p^-) \left( \frac{2}{(2k-1)\pi} \right)^2, \quad b_{ic} = (p^+ + p^-) \left( \frac{2}{(2k-1)\pi} \right)^2. \]

4. Conclusion
The obtained mathematical model can be used to investigate the three-layered plate resonance oscillations, where the plate is a wall of the channel with viscous incompressible fluid inside. We can investigate the three-layered plate tense-deformed state, as well as, to define its resonance longitudinal and bending oscillations. The expression for fluid layer pressure provides a possibility to determine the liquid pressure in the channel relying on the given frequency rate vibrating stamp. Thus, the obtained result gives the opportunity to forecast cavitation emergence in the fluid and define the vibrating stamp frequency causing cavitation in the liquid.

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