Modelling of Two-sided Assembly Line Balancing Problem with Resource Constraints

Muhammad Razif Abdullah Make¹, Mohd Fadzil Faisae Rashid¹*, Muhamad Magffierah Razali¹
Manufacturing Focus Group, Faculty of Mechanical Engineering, Universiti Malaysia Pahang, 26600, Pekan, Pahang, Malaysia.

Email: ffaisae@ump.edu.my

Abstract. Two-sided assembly line balancing (2S-ALB) problems is practically useful in improving the production of large-sized high-volume products. Many published papers have proposed various approaches to balance this well-known ALB problem. However, little attention is given in formulating the 2S-ALB problems. In this paper, 2S-ALB is modelled with four different objective functions comprising minimization of workstations, mated-workstation, idle time and resource constraints. In different with existing model, this paper also considers resource constraint with a mathematical modelling formulation in solving the 2S-ALB problems. The modelling procedures are present for each objective functions with a simple 2S-ALB example problem. Then, the anticipated performance solution is obtained from the test problem.

1. Introduction
The assembly line was firstly introduced in manufacturing system by Henry Ford in 1913. The aim of his idea is to develop the automobile plants for mass production used, with some sort of lines customization. Basically assembly line will consist of a set of a workstation that connected each other in linear fashion. Considering the much numbered of task and workstation presented in assemblies, the problem balancing approach are made. Assembly Line Balancing (ALB) problem formally presented with optimization objective and capable of enhancing the production rate or even the assembly lines. In 1993 Bartholdi has successfully become a pioneer initiating a new type of ALB problem consists with two sides of assemblies, known as Two-sided Assembly Line Balancing (2S-ALB) problem [1].

The 2S-ALB is classified under the General Assembly Line Balancing (GALB) problem, besides the Simple Assembly Line Balancing (SALB) problem. Further research on 2S-ALB is continued in [2-4] studies after Bartholdi in 1993, since the importance of 2S-ALB is highly recognized in the manufacturing industries. Importantly as 2S-ALB for being highlighted through its strength and potency in the manufacturing of a large-sized product. It is definitely managed to improve the assembly productivity throughout the installation. Compared to SALB, with single line operation the 2S-ALB remarkably advantageous since able to shortens the assembly line, save spaces, reduces the cost of tools and fixtures, reduces the throughput time and fit to cut the material handling [1]. These clearly give a good reason for utilizing the 2S-ALB into the assembly line.

In practice, 2S-ALB has firstly proposed in 1993 by Bartholdi in using an interactive computer program with the first fit heuristic method [1]. The successful study focusing on the large-sized product has continued by Kim, Kim et al. (2000) by adopting a genetic algorithm (GA) method [2].
Then, in 2001 a study on maximizing the work relatedness and slackness using a group assignment procedure has been developed by Lee, Kim et al. in solving the 2S-ALB problem [3]. Meanwhile, in 2008 another meta-heuristic approach for 2S-ALB has been proposed by Baykasoglu and Dereli [4]. They have applied an ant colony optimization (ACO) method in the 2S-ALB problem for a domestic product. The highly potential 2S-ALB problem has successfully influenced the other researcher to start a new stage in ALB problem studied. A complex combination with the 2S-ALB problem has been reported through several presented types of research. Such in Simaria and Vilarinho (2009), Özcan and Toklu (2009), Chutima and Chimklai (2012) and Yuan, Zhang et al. (2015) who previously proposed the combination of mixed-model and two-sided ALB problem adopting a different meta-heuristic method as their solving approach [5-8]. Meanwhile, in Kucukkoc and Zhang (2014) a mixed-model parallel two-sided assembly line balancing problem with an agent-based ant colony optimization approach was firstly introduced [9]. The idea of combining 2S-ALB problem with the other ALB model seem totally change the assembly configuration, however it effectively gives more favourable advantages.

This paper is focused on presenting the mathematical modelling for minimizing a multi-objective function implementing the 2S-ALB optimization problem. Our 2S-ALB is a general model emphasize four objective functions, comprising the minimization number of the workstation, mated-workstation, idle time and resource constraint. Besides, the particular feature of 2S-ALB also will be highlighted with a certain model as the numerical examples.

2. 2S-ALB Problem modelling
The 2S-ALB problem has been effectively studied using a specified method since last two decades. The recognition of 2S-ALB problem crucially influences numerous researcher to look forward adopting this ALB problem in their studies [10, 11]. The workstation of the one-sided assembly line is prepared in the linear line of the production system. Figure 1 illustrates the example of the one-sided assembly line with several numbers of the workstation. Each workstation will have certain assigned task and must be completed before moving to another workstation for another task or job. The product will be prepared after all tasks on every workstation are executed.

![Figure 1. One-sided assembly line](image1)

However, a different arrangement of workstation has been offered by two-sided assembly line. The illustration of two-sided assembly line has been depicted in figure 2. For two-sided assembly line the length of workstation logically is shorter than a one-sided assembly line [12, 13], since the tasks and the workstation is divided into two parallel lines. Therefore, it rationally shows some space efficient of the two-sided assembly line. A shorter line is able to reduce the material handling cost, besides the tools and fixture might be placed in the middle between the two parallel workstations (mated-workstation) [1, 12-14]. Furthermore, the two-sided assembly line allowed the task to be executed at the same time for both parallel workstations (left and right) along the assembly lines. However, the precedence relation for each assigned tasks should seriously be assessed before being executed on a specific workstation. The precedence relation among tasks will be discussed in the following subsection.

![Figure 2. Two-sided assembly line](image2)
On the other hand, Bartholdi also expressed the difficulty in permitting the two parallel mated-workstations to work for the same time [1]. The idle workstation will turn into a problem in order to attempt a balanced assembly line. Besides, for each assigned tasks it also possesses with a different particular duration that seems hard to be aligned and balance the assembly lines. Therefore, a comprehensive study is needed in making the two-sided assembly line be balanced and completely able to optimize the ALB problems.

2.1 Precedence relation

A precedence relation graph is constructed to shows an overview of the tasks to be performed. Figure 3 presents the precedence relation graph with some details. As shown below (figure 3) the circle indicates as the assigned task and each task is associated with a processing time and operational direction label \((t, d)\). While the arrows linked represent the relation between each task.

In two-sided assembly lines, the operational direction is allowed to be carried out on the same parallel workstation of both sides (left and right) on the same product. Due to the use of both sides of the lines, the additional operating direction has been classified into three different groups and must be obeyed. Three classified group of the operational direction: the left side (L), the right side (R), and either side (E). For left and right side the task execution is absolute and need to be implemented in the following position. Meanwhile, for either side, the execution of tasks is practicable to be performed on any side of the workstation. All the information concerning the assembly processes is disclosed on top of each precedence task.

2.2 Computational data presentation

From the precedence diagram in figure 3, the relation between each task is important to be transformed into a digital format. The aim is to establish the precedence matrix that could be understood by the computer. Table 1 indicates the information of the precedence graph that being changed into a language for the computational purpose. This precedence matrix consists of ones and zeros values. The value of one indicates the precedence between task \(i\) and the next task \(j\). While, zero means no precedence between both task \(i\) and \(j\).
Besides, the information of time and operational direction \((t, d)\) on the precedence relation graph also needs to be modified for the computational purpose. The data matrix on Table 2 presents the example of detail data of assembly information, in which each value brings a particular meaning. The time column on Table 2 represents the processing period of each task, while on side column three different values indicate different sides of the operational direction \((1=\text{left side}, \ 2=\text{either side}, \ 3=\text{right side})\). The other columns denoted as resource constraint data details.

### Table 1. Precedence matrix

| i\(j\) | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1     | 0   | 0   | 0   | 1   | 0   | 0   | 0   | 0   | 0   |
| 2     | 0   | 0   | 0   | 0   | 1   | 1   | 0   | 0   | 0   |
| 3     | 0   | 0   | 0   | 0   | 0   | 1   | 0   | 0   | 0   |
| 4     | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
| 5     | 0   | 0   | 0   | 0   | 0   | 1   | 1   | 0   | 0   |
| 6     | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 1   |
| 7     | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
| 8     | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
| 9     | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |

### Table 2. Data matrix

| task | time | side | resource constraint |
|------|------|------|---------------------|
| 1    | 2    | 1    | 1 2 0 0 0           |
| 2    | 3    | 3    | 3 0 0 0 0           |
| 3    | 2    | 2    | 2 3 0 0 0           |
| 4    | 3    | 1    | 1 0 0 0 0           |
| 5    | 1    | 3    | 3 0 0 0 0           |
| 6    | 1    | 2    | 2 3 0 0 0           |
| 7    | 2    | 2    | 1 2 3 0 0           |
| 8    | 2    | 1    | 2 0 0 0 0           |
| 9    | 1    | 2    | 1 3 0 0 0           |

### 2.3 Multi-objective modelling and execution

In this paper, the 2S-ALB is evaluated with four objectives; to minimize the number of the workstation, mated-workstation, idle time and resource constraint. The general assumptions of the problem are as follows: -

- The workstation numbers are not fixed and no absence of any workstation either left or right along the assembly line.
- The maximum operational cycle time is fixed and could not be exceeded.
- Three different operational directions of the left side, either side and the right side are presented in numerical order.
- The assembly operation /task can be started at the same time on both sides for every mated-workstation.
- The low level of operator skills is ignored, for the assembly tasks will be assembled at the same rate and pace.
- Each workstation can only be assigned by a single operator.
Considering the above assumptions, the parameters that are involved in permitting all the objective functions are as follow.

\[
\begin{align*}
J & : \text{number of mated-workstation } j = 1, 2, ..., J \\
I & : \text{number of one-sided workstation } i = 1, 2, ..., I \\
F & : 1, \text{if there is any space availability on the operating time, otherwise, } 0 \\
N & : \text{number of resource utilization } n = 1, 2, ..., N \\
X_{ms} & : 1, \text{if mated-workstation } j \text{ is utilized for both side of the line, otherwise, } 0 \\
Y_s & : 1, \text{if mated-workstation } j \text{ is utilized for only one side of the line, otherwise, } 0 \\
m_t & : \text{maximum processing time } t = 1, 2, ..., T \\
r_t & : \text{operational time of the task on the workstation } j \\
p_v & : \text{maximum gap value in space availability} \\
q_v & : \text{minimum gap value in space availability} \\
R_s & : 1, \text{if resource is utilized in workstation } j, \text{otherwise, } 0
\end{align*}
\]

The mathematical model of the problem can be formulated as follows.

\[
\begin{align*}
f1 &= \min \sum_{j=1}^{J} X_{ms} \\
f2 &= \min \sum_{j=1}^{J} 2FX_{ms} + \sum_{i=1}^{I} Y_s \\
f3 &= \min \sum_{t=1}^{T} (m_t - r_t) + \sum_{t=1}^{T} F(p_v - q_v) \\
f4 &= \min \sum_{n=1}^{N} R_s
\end{align*}
\]

The objective function in equation (1) aims to minimize the number of mated-workstation. Where \(X_{ms}\) shows the sum of utilized number of mated-workstations. Objective function (2) indicates the other aims of minimizing the number of utilized workstations. Meanwhile, objective function (3) represent the minimization of idle time value, considering the operational processing time \(m_t, r_t\) and the space time availability \(p_v, q_v\) on each workstation. Then, in objective function (4) the minimization of resource constrain is described with the summation of resource utilization \(R_s\).

3. Numerical example
This section presents a numerical example to explain the objective functions in section 2. For this purpose, the example in figure 3 is considered. In this example, the cycle time is fixed with 6 units of times. Therefore, each workstation completely allowed to have the unlimited assign task but it should not exceed 6 unit of cycle time. Figure 4 illustrates the task distribution layout of 2S-ALB problems. In distributing the tasks to both sides of the lines formerly the assembly sequence associated with tasks will firstly be generated. As the example, one assign sequence is proposed comprising of 9 assembly tasks: 2,1,3,6,5,4,9,8,7. Then, according to each time and side data \((t, d)\), the task will be assigned to the appropriate workstation. However, in filling the job for workstation sometimes the precedence will turn as a constraint, thus causing an idle time. The shaded area represents as idle time with no assigned...
task /job. The task distribution and layout will be different if the generated sequence is changed. Means that, the total idle time will also change, either turned into a longer or shorter period of time.

![Figure 4. 2S-ALB task distribution](image)

The analysis and evaluation of this 2S-ALB problem modelling also depicted from figure 4. Along the assembly line, four different aim will be calculated associate to the 9 assigned task (problem example) from figure 3.

3.1 Minimize the number of mated-workstation
The particular equation of this aims is subjected to equation (1), while the assembly line illustration is depicted in figure 4. The calculation of the first objective function ($f_1$) of minimizing the number of mated-workstation is presented below. The two-sided workstation consists of left (L) and right (R) is recognized as mated-workstation. Thus the computation of $f_1$ is calculated by the summation of utilized mated-workstation ($j$). It will be summed before multiplied by $X_{ms}$ condition, either 1 or 0. As the sum of mated-workstation ($1 + 1$), then it is multiplied by 1 as both side utilization of the workstation.

$$f_1 = \min \sum_{j=1}^{J} X_{ms}$$  \hspace{1cm} (1)

$$f_1 = (1 + 1) \times 1$$

$$f_1 = 2$$

2.4 Minimize the number of workstation
While the second objective function is subjected to equation (2). The assembly line illustration also depicted from figure 4. The totalled of mated-workstation will be multiplied by 2 and $X_{ms}$ condition. Then it should be added to the total of one-sided utilized workstation as its evaluation. As $f$ is equal to $f_1$, the number of mated-workstation is set to be two. The value of $f$ will be multiplied by 2 and 1 as the condition of both sided workstation utilization, ($2 \times 1$). After that, the total of one /single sided utilized workstation will be summed with the previous value for $f_2$ values. $Y_{s}$ is the condition for single sided utilized workstation, either 1 or 0.
2.5 Minimize idle time

Idle time is a wasted duration with no assigned task. Associated as a void space its graphically shown in figure 4 with shaded area. The best balanced line approximately has equal processing time. Therefore, minimizing the idle time is able to enhance the assembly operation line. The idle time formulation is presented in equation (3). The total subtraction of maximum processing time (cycle time) and the task time of each workstation, will be added together with the space availability on every workstation if any. The value of maximum processing time will be fixed on 6 unit of time before subtracting with the operation task time for each workstation (4, 6, 5, 3). As the example on the first workstation with 4 unit of processing time, \( (6 - 4) \). Then, the total needs to add with another total gap /space along the assembly line for each workstation if any, such on the second workstation \( (4 - 3) \). The deduction of maximum and minimum gap space will be summed for every workstation and multiplied with the \( F \) condition. The value will be totaled and come out as \( f^3 \) value

\[
f_3 = \min \sum_{j=1}^{J} 2JX_{ms} + \sum_{i=1}^{I} Y_s
\]

\[
f_2 = [2(2) \times 1] + [0(0)]
\]

\[
f_2 = 4
\]

\[
f_3 = \min \sum_{j=1}^{J} (m_j - n_j) + \sum_{i=1}^{I} (p_i - q_i)
\]

\[
f_3 = [(6 - 4) + (6 - 6) + (6 - 5) + (6 - 3)] + [(4 - 3) \times 1]
\]

\[
f_3 = 7
\]

2.6 Minimize resource constraint

As shown in Table 3, the tasks and resource have divided into four number of workstation. Since that, the resource utilization of machine and tools seem is repeated even for the same workstation. The minimization of the resource used significantly help in reducing the cost of tools and fixture. For this example, it has 3 types of fixtures that used in completing all of the tasks. The resource row indicated the fixtures used with one type of machine (M1) and two types of tools (T1 and T2). However, the same machine and tools which located in the same workstation practically turned into a waste. The calculation on minimizing the resource constraint is presented below. The total resource used for every workstation is calculated emphasizing machine and tools. Such in workstation 1 possessing with 1 type of machine and 2 type of tools. Therefore, as the total 3 resource is considered from workstation 1. The resource used for another workstation also calculated to be summed before its multiplied with \( R_s \) condition which either 1 if the resource is utilized in the workstation \( j \) or 0, otherwise.
4. Discussion
A slight different have shown in 2S-ALB compared to one-sided assembly. The deterministic performance time with a specific order of precedence tasks leads to the idle time formation. Previously in one-sided assembly, the ordered task and workstation are presented on a single straight line (figure 1). The preferred task will be assign to the workstation one to another until the product is complete and ready to use. But compared to two-sided assembly a short duration of idle time might be available on a certain workstation. This happens because of the precedence preferred assigned side along the assembly line. Although the idle time is issued in the 2S-ALB problem, it is believed more effective for improving the productivity. Such as able to avoid the excessive workload, minimize wasted or over processing leads to excess inventory, and have less processing time with higher production rate compared to one-sided assemblies.

Another different initiated from this 2S-ALB modelling paper is the resource constraint utilization. Table 3 demonstrates the resource utilization on every workstation with assigned task. The repeated used of the resource (machine and tools) in executing the task along the assembly line could be reduced by implementing the 2S-ALB operation. By installing 2S-ALB configuration the resource utilization of machine and tools used definitely will be minimized when that fixture are placed in the middle (left and right) between two mated-workstation. The minimum number of machine tools and fixture surely reduced the maintenance cost apart of adding the same machine and tools. Over numerous benefit and advantages, the 2S-ALB is known from the literature to successfully provide the high efficiency in operational lines. Practically the accomplishment of 2S-ALB has abundantly proven in the assembly of a large-sized product, such bus and trucks [2] and automobile [3]. Even so, the success has been continued for a domestic product and is presented in Baykasoglu and Dereli in 2008 studies [4]. From all of the success, it shown the 2S-ALB is potentially could be used in another field of studies, such printing and food processing industry. This could potentially enhance the 2S-ALB implementation in other various fields.

5. Conclusion
In this paper, the model of two-sided assembly line balancing (2S-ALB) problem was discussed. This study was undertaken to design a multi-objective problem with four different objective functions. The minimization number of workstation, mated-workstation, idle time and resource constraint was introduced based on the real world problem. Further on the 2S-ALB modelling evaluation of four aims is presented in details with a particular equation and calculation. The obtained results have described
the 2S-ALB features. In addition, the comparison of the performance of 2S-ALB modelling is also stated in this study to give some clear explanation with its advantages. From the results of this modelling, it shows the 2S-ALB could have a better performance in optimization.

For future work, we might extend the proposed model of the 2S-ALB problem for another optimization phase. Its involve with the implementation of computational optimization approach such GA and ACO method. This modelling also fits for other optimization problem in the area dealing with other objectives. Besides, a larger 2S-ALB problem, with more number of assign task associate to the assemblies is necessary to discuss in the future.

Acknowledgments
The authors would like to acknowledge the Ministry of Higher Education, Malaysia and Universiti Malaysia Pahang for supporting this research under the FRGS grant RDU140103.

References
[1] Bartholdi, J.J., Balancing two-sided assembly lines: a case study. *International Journal of Production Research*, 1993. **31**(10): p. 2447-2461.
[2] Kim, Y.K., Y. Kim, and Y.J. Kim, Two-sided assembly line balancing: A genetic algorithm approach. *Production Planning & Control*, 2000. **11**(1): p. 44-53.
[3] Lee, T.O., Y. Kim, and Y.K. Kim, Two-sided assembly line balancing to maximize work relatedness and slackness. *Computers & Industrial Engineering*, 2001. **40**(3): p. 273-292.
[4] Baykasoglu, A. and T. Dereli, Two-sided assembly line balancing using an ant-colony-based heuristic. *The International Journal of Advanced Manufacturing Technology*, 2008. **36**(5-6): p. 582-588.
[5] Simaria, A.S. and P.M. Vilarinho, 2-ANTBAL: An ant colony optimisation algorithm for balancing two-sided assembly lines. *Computers & Industrial Engineering*, 2009. **56**(2): p. 489-506.
[6] Özcan, U. and B. Toklu, Balancing of mixed-model two-sided assembly lines. *Computers & Industrial Engineering*, 2009. **57**(1): p. 217-227.
[7] Chutima, P. and P. Chimklai, Multi-objective two-sided mixed-model assembly line balancing using particle swarm optimisation with negative knowledge. *Computers & Industrial Engineering*, 2012. **62**(1): p. 39-55.
[8] Yuan, B., et al., An effective hybrid honey bee mating optimization algorithm for balancing mixed-model two-sided assembly lines. *Computers & Operations Research*, 2015. **53**: p. 32-41.
[9] Kucukkoc, I. and D.Z. Zhang, Simultaneous balancing and sequencing of mixed-model parallel two-sided assembly lines. *International Journal of Production Research*, 2014. **52**(12): p. 3665-3687.
[10] Abdullah Make, M.R., M.F.F. Ab. Rashid, and M.M. Razali, A review of two-sided assembly line balancing problem. *The International Journal of Advanced Manufacturing Technology*, 2016: p. 1-21.
[11] Rashid, M.F.F., W. Hutabarat, and A. Tiwari, A review on assembly sequence planning and assembly line balancing optimisation using soft computing approaches. *The International Journal of Advanced Manufacturing Technology*, 2011. **59**(1): p. 335-349.
[12] Khorasanian, D., S.R. Hejazi, and G. Moslehi, Two-sided assembly line balancing considering the relationships between tasks. *Computers & Industrial Engineering*, 2013. **66**(4): p. 1096-1105.
[13] Tuncel, G. and D. Aydin, Two-sided assembly line balancing using teaching–learning based optimization algorithm. *Computers & Industrial Engineering*, 2014. **74**: p. 291-299.
[14] Purnomo, H.D. and H.-M. Wee, Maximizing production rate and workload balancing in a two-sided assembly line using Harmony Search. *Computers & Industrial Engineering*, 2014. **76**: p. 222-230.