Modelling dielectric elastomer actuators using higher order material characteristics

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Abstract

Dielectric elastomers, a special class of electroactive polymers, have viscoelastic properties that strongly affect their dynamic performance. This paper contributes a high-order linear solid model together with an optimised parameter identification method to aid the selection of the model parameters. The paper also demonstrates that accurate modelling of the viscoelastic characteristics for commonly used dielectric elastomer (DE) material requires additional spring-damper combinations within a standard linear solid model. The effect of key parameters on the system dynamics in the frequency domain is elaborated and used to guide the parameter identification of the models. The increased effectiveness of higher order models that incorporate multiple spring-damper combinations is demonstrated using three experiments; (a) mechanical loading of a stacked sample over 0.01–5 Hz with strain variations up to 50%; (b) mechanical loading of a single-layer sample over 1–100 Hz with strain variations up to 10%; and (c) electrical actuation of a single-layer sample over 1–100 Hz using electric fields up to 20 MV m\(^{-1}\). Silicone and polyacrylate samples were tested to show the effect of viscoelastic properties in the frequency domain. The proposed method of parameter identification is optimised to capture the frequency response.

1. Introduction

Dielectric Elastomers (DEs) form a special class of Electro-Active Polymers (EAPs), which will have significant potential in the next generation of soft actuated systems. Their electro-mechanical properties allow conversion of energy between electrical and mechanical forms. DE-based actuators produce mechanical deformation in response to an applied voltage. The actuation includes low cost, high energy density, noise-free operation and does not require a rigid mechanical structure. In addition, the force density generated by DE Actuators (DEAs) is similar to that of human muscles [1–3]. Applications have been explored in robotics [4–6] and muscle-like actuation [7–9]. In particular, the relatively fast response of a DEA has potential for applications in pumps [10–13], valves [14–16], loudspeakers [17, 18] and optical positioning systems [19–21]. Moreover, DEs have also been shown to have capabilities in smart sensors [22, 23] and energy harvesters [24–28].

The deformation and dynamic force output of a DEA depends on the viscoelastic properties of the elastomer, which are often modelled as spring-damper rheological systems. In early work, Hackle et al [29] demonstrated that the standard linear solid model is a more suitable representation than the Kelvin-Voigt model that contains only one spring and one damper in parallel. Li et al [26] proposed an analytical model based on the standard linear solid representation to evaluate the effect of DE viscoelasticity on energy conversion. Hong [30] questioned the accuracy of the well-known material models [31–33] in simulating viscoelastic DE actuation, and developed a general field theory for the coupled electro-viscoelastic behaviour of dielectric elastomers. These models are yet to be validated experimentally. Choi et al [34] tested polyacrylate- and silicone-based biomimetic DEAs in the frequency domain up to 10 Hz and 100 Hz. A viscoelastic material model was derived in the early
part of this work to develop a control strategy. However, the presented frequency response ignored phase information, hence the accuracy of the modelling was not evaluated fully in the frequency domain.

Recent work has started to validate viscoelastic models of frequently used materials. Berselli et al [35] have validated experimentally the Zener model in simulating VHB 4910 in a constant-force DEA with stretch rates up to 10 mm s$^{-1}$. Hossain et al [36] proposed a modified Bergstrom-Boyce viscoelastic model with a finite strain linear evolution law to simulate VHB 4910 in various step–based standard testing. The accuracy of the model is excellent in simulating material deformation, which degraded for strain rates above 0.05 s$^{-1}$. Zhang et al [37] employed a constitutive model of the DEA to perform model-based control for creep elimination in VHB 4910 under constant stretch. The same research group also developed a combined Kelvin-Voigt-Maxwell model to simulate step responses of VHB 4910 for actuation strains up to 100% [38]. The models were accurate under limited loading conditions. The previously cited papers show research gaps:

(a) The dynamics of DEAs and the effect of viscoelasticity at different operating frequencies have not been characterised completely (theoretical and experimentally).

(b) The accuracy and flexibility of existing viscoelastic material models in simulating real materials is not satisfactory and can be improved.

Physically, an elastomer contains numerous molecular segments that have diverse dissipation properties due to different segment orientations. The relaxation of segments may not occur simultaneously, resulting in different characteristics at different frequencies. The Maxwell-Wiechert model [39], which contains multiple spring-damper combinations, may be a better option for DEA modelling. It has been used to analyse viscoelasticity and stress relaxation of fibres [40], electro-optic polymers [41] and composite beams [42, 43]. The main disadvantages of the Maxwell-Wiechert model in these applications are:

(c) It is inaccurate for short-term stress relaxation prediction.

(d) It requires a large number of parameters to compare well with non-linear viscoelastic models [44].

It remains unclear how well the Maxwell-Wiechert model would fit the viscoelastic behaviour of DE polymers in the frequency domain. Because the multiple spring-damper combinations can be tuned to represent the dynamics at different frequencies, the model may have the advantage in providing more realistic frequency responses for viscoelastic polymers. However, few works have used the Maxwell-Wiechert model to simulate DE devices. Zhang et al [45] employed the non-linear Maxwell-Wiechert model to investigate geometrical effects on the viscoelastic behaviour of DEs. The model shows the different dynamic capabilities of DEAs in various rectangular formats, but it is not validated. Hodgins et al [46] demonstrated good agreement of the Maxwell-Wiechert model incorporating an Ogden spring component up to 50 Hz. The model was not fully evaluated in frequency domain nor compared with existing material models.

To address the research gaps, this paper is focused on characterising DEs and DEAs in the frequency domain. The contribution of the paper is the provision of a parameter identification procedure for Maxwell-Wiechert representations of variable orders determined by the number of spring-damper combinations. The system dynamics of the model and its correlation to the system parameters are studied and the advantages of having higher orders in the material model are elucidated. Three experiments are considered to measure the dynamics of polyacrylate in mechanical loading and electric actuation. The validation of the proposed models is presented and compared with a conventional standard linear solid model. An elastic silicone was also tested to show how the viscoelastic polyacrylate characteristics differ in terms of dynamic response.

2. Theory

2.1. Higher order spring-damper rheological model under mechanical loading
Consider a Maxwell-Wiechert model for a viscoelastic DE as shown in figure 1. It generalises the spring-damper rheological model by having multiple sets of spring-damper combinations, whose number defines the model order. Hence the commonly used standard linear solid model that contains only one spring-damper combination is referred to as a first-order model ($n = 1$). When an external force $F$ is applied to the DE, the system deforms and the equations of motion for the $nth$-order model are
The generic model is simplified \((x_0' = x_0 = 0)\) to match an experimental setup where the DE is mechanically loaded on the top end and clamped onto a load cell for force measurement on the bottom end. Equation (1) therefore becomes

\[
\begin{align*}
& k_0(x_0 - x_0') + \sum_{i=1}^{n} k_i(x_0 - x_i) = F \\
& c_i(x_i - x_0') + k_i(x_0 - x_0) = 0 \\
& \quad \vdots \\
& c_n(x_n - x_0') + k_n(x_n - x_0) = 0
\end{align*}
\]

with associated parameters explained in Table 1.

![Figure 1. Nth-order viscoelastic solid model of DE that contains \(n\) sets of spring-damper combinations and a main spring of stiffness \(k_0\). The standard linear solid model occurs when \(n = 1\).](image)

Table 1. Parameters for the system as in figure 1 and equation (1).

| Parameters | Description |
|-----------|-------------|
| \(n\)     | Number of spring-damper subsystems \((n > 0)\) |
| \(x_{0,0}'\) | Displacements of both ends of the system |
| \(x_n\) | Displacement in the spring-damper combination |
| \(k_0\) | Stiffness of the main spring |
| \(c_k, c_n\) | Stiffness and damper rate of the spring-damper combination |
| \(F\) | Applied force |

Assuming an input displacement \(x_0\) and force \(F\) as the output, equation (2) can be expressed in state-space form as

\[
\begin{align*}
\mathbf{x}' &= A\mathbf{x} + B\mathbf{x}_0 \\
F &= C\mathbf{x} + D\mathbf{x}_0
\end{align*}
\]

2.2. Higher order spring-damper rheological model under electrical actuation

Actuation of a DE requires electrodes to cover an Active Region (AR) on the DE surface. This enables a voltage to be applied across the DE material causing it to expand under Maxwell pressure. In the system model, the actuation force generated is denoted by \(F_a\). The set of equation (2) are modified to
\[
\begin{align*}
&\begin{cases}
    c_1(\dot{x}_1 - \dot{x}_0) + k_1(x_1 - x_0) = 0 \\
    \vdots \\
    c_n(\dot{x}_n - \dot{x}_0) + k_n(x_n - x_0) = 0 \\
    \sum_{i=1}^{n} c_i(\dot{x}_i - \dot{x}_0) + k_0(x_0' - x_0) + F + F_A = 0
\end{cases} \\
\end{align*}
\]  

(4)

Taking the constraint in the experimental setup as \( \dot{x}_0^0 = x_0^0 = 0 \), equation (4) simplifies to

\[
\begin{align*}
&\begin{cases}
    c_1\dot{x}_1 + k_1(x_1 - x_0) = 0 \\
    \vdots \\
    c_n\dot{x}_n + k_n(x_n - x_0) = 0 \\
    \sum_{i=1}^{n} c_i\dot{x}_i + k_0x_0 + F_A = F
\end{cases} \\
\end{align*}
\]  

(5)

Equation (5) can be therefore presented in the state space form as

\[
\begin{align*}
\dot{x} &= Ax + BF_A \\
F &= Cx + DF_A
\end{align*}
\]  

(6)

where

\[
A = \begin{bmatrix}
1 & -\frac{k_1}{c_1} & 0 & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
& & & 1 & \cdots & 0 & -\frac{k_n}{c_n} 
\end{bmatrix}
\]

\[
B = [0 \ldots 0]^T
\]

\[
C = \left[ \sum_{i=1}^{n} k_i \ k_i \ldots -k_n \right]^T, \quad D = 1, \quad x = [x_0 \ x_1 \ \ldots \ x_n]^T
\]

Under uniaxial loading, when an external stress \( \sigma_1 \) and electric field \( E \) are applied to the DE, the equation of the state gives the stress in direction 1 as \([47]\)

\[
\sigma_1 + \varepsilon_0\varepsilon_0 E^2 = \lambda_1 \frac{\partial W(\lambda_0, \lambda_2, \lambda_3)}{\partial \lambda_1}
\]  

(7)

where \( W(\lambda_0, \lambda_2, \lambda_3) \) is the free-energy density function, \( \lambda_0, \lambda_1 \) and \( \lambda_3 \) are the pre-strains in the three orthogonal directions. Taking \( E = \frac{V}{z} \), the actuation force \( F_A \) is derived as

\[
F_A = \sigma_1\varepsilon_0\varepsilon_0 z = \frac{\varepsilon_0\varepsilon_0}{z} \varepsilon_0 z V^2
\]  

(8)

where \( V \) is the applied voltage, \( \varepsilon_0 \) is the dielectric permittivity in a vacuum, \( \varepsilon_r \) is the relative dielectric permittivity of the DE, \( w_{AR} \) is the width of the AR region and \( z \) is the thickness of the DE.

2.3. Dynamics of the first-order model

To understand fully the benefits of having a higher order model, it is important to know the influence of the springs and dampers on the system dynamics in the first order case. Given a sinusoidal displacement input as

\[
x_0(t) = X_0 \sin \omega t
\]  

(9)

the steady state dynamic force will have the form

\[
F(t) = F_0 \sin(\omega t + \phi)
\]  

(10)

where \( \omega \) is the operating frequency and \( \phi \) is the phase. The dynamic response of a typical first-order model (\( n = 1 \)) is shown in figure 2. At very low operating frequencies (\( \omega \to 0 \)), the damper \( c_1 \) isolates the spring \( k_1 \) and the system is approximated by the spring \( k_0 \). At relatively high operating frequencies (\( \omega \to 45 \text{ rad s}^{-1} \)), the damper acts as a strut. The system is therefore approximated by two springs in parallel, \( k_0 + k_1 \), defining the high frequency asymptotic characteristic. Over the frequency range between these two points, the dynamic stiffness of the system changes from \( k_0 \) to \( k_0 + k_1 \) and the frequency rate of transition depends on the damper \( c_1 \). Figure 2 also shows the phase of the first-order model. The curve rises rapidly and reaches the peak at the frequency where the dynamic stiffness curve has the highest rate of change. It then decreases gradually to zero.
2.4. Effect of having higher order

Now consider a second-order model that contains one main spring and two sets of spring-damper parallel combinations \( n = 2 \). Given the same sinusoidal displacement input as in equation (9), the dynamic stiffness and phase are shown in figure 3. The dynamic stiffness starts at \( k_0 \) and high frequency asymptote is \( k_0 + k_1 + k_2 \). The transition depends on \( c_1 \) and \( c_2 \). The phase curve has monotonic rise and a two-stage decrement. In comparison with the first-order model of figure 2, the second-order model allows more versatile definition of the dynamic stiffness and phase over the frequency range. Given equations (9) and (10), in the Laplace transform domain, the dynamic stiffness is given by

\[
k(s) = \frac{F_0}{X_0} = k_0 + \left( k_1 - \frac{1}{c_1 s + k_1} \right) + \left( k_2 - \frac{1}{c_2 s + k_2} \right)
\]

(11)

Setting \( s = j \omega \) gives the frequency dependence of the stiffness in complex form. It is therefore evident that

\[
\begin{cases} 
|k(j \omega)| = |A_1| + |A_2| + |A_3| \\
\arg(k(j \omega)) = \arg(A_1) + \arg(A_2) + \arg(A_3)
\end{cases}
\]

(12)

where

\[
A_1 = k_0, \quad A_2 = k_1 - \frac{1}{j \omega + k_1}, \quad A_3 = k_2 - \frac{1}{j \omega + k_2}
\]

(13)

The inequalities of equation (12) show that the dynamics of a second-order model are not simply a superposition of the individual parallel elements in equation (13), therefore having important implications when identifying parameters for the model.
2.5. Parameter identification
The process of parameter identification for a first-order model could be relatively straightforward. Given a measured dynamic stiffness characteristic, the lower and higher frequency values could be set to $k_0$ and $k_0 + k_1$, respectively. The damper rate, $c_1$, then be adjusted to give the best fit for the transition between the lower and higher frequency values.

In higher order models ($n \geq 2$), the parameter identification is more difficult because the spring-damper combinations are not simple superpositions (see equation (12)), therefore the fitting cannot be a sequential process for each parameter. Hence the scheme of figure 4 was formulated to include an iterative procedure to establish the higher order system parameters. The differences of the fitted model compared with measured characteristics are defined in terms of the mean absolute errors in magnitude and phase over the specified frequency range by

$$
\begin{align*}
\epsilon_{mag} &= \frac{\text{mean}}{\omega} |k_{sim} - k_{exp}| \\
\epsilon_{phase} &= \frac{\text{mean}}{\omega} |\phi_{sim} - \phi_{exp}|
\end{align*}
$$

The parameter identification is intended to be performed for DE samples under direct mechanical loading.

3. Experimental procedure
3.1. DE fabrication and configuration
To demonstrate the validity of the model, two types of DE having distinct viscoelastic properties were selected to be subjected to force measurements, polyacrylate- and silicone-based. The polyacrylate sample was VHB 4910.
from 3M\textsuperscript{TM} in a basic 1 mm thick film format. For the mechanical loading experiments, the following configurations were fabricated:

(a) A single-layer sample of area 50 mm $\times$ 50 mm for experiments having lower mechanical strains that would not induce buckling of the sample under cyclic loading (see figures 5(a) and (b)).

(b) A stacked format with an area of 10 mm $\times$ 10 mm and consisting of 10 layers with total thickness of 10 mm was used for high mechanical strain cyclic loading experiments (see figure 5(c)). Because the polyacrylate is highly viscous, rapidly changing compressive loads may cause buckling in a single-layer pre-strained sample. The thicker multi-layer sample has a higher buckling load threshold.

The silicone was PlatSil 7315, a clear Room Temperature Vulcanization (RTV) silicone rubber. It has a Pour Time of 20 min at room temperature, which gave sufficient time to mix the agents, degas and pour into the mould for curing. The degassing was performed under 90 Pa for 10 min in a vacuum oven. This silicone elastomer was also selected because of its low mixing viscosity at 2500 cP that eases degassing. The mould for curing the mixture into the film was made from flat glass plate. Insulating tape of thickness 150 $\mu$m was used to form the inner walls of the glass. Four layers of tape were used, hence producing a 600 $\mu$m thick silicone film for
the walls of the mould. Using the glass-based mould ensured that the film had a low surface roughness compared with a metallic mould that would have required precision machining. The drawback is that the tape is very flexible and may distort and have variable thickness when building up the walls. It may lead to slight errors in the thickness of the cured film. After the degassed mixture was poured into the mould, it took approximately 5 h to cure.

For electrical actuation experiments, the single-layer samples of polyacrylate and silicone included an AR of area 40 mm \( \times \) 40 mm. Cables were wired onto the clamps and the AR was extended to the clamp for the lead contact. The electrode material used was graphite powder, which was chosen to facilitate the electrode deposition using the screen printing technique. Figure 5 shows the sample configurations.

3.2. Measurement system setup

For mechanical loading at low frequencies, the stacked multi-layer sample of polyacrylate was clamped to a hydraulic loading machine, HClO from DARTECH\textsuperscript{TM} and to a load cell, LCBP-5 from OMEGA\textsuperscript{TM}. The load cell was capable of measuring forces up to 50 N. The stacked sample was clamped to give exposed length of 10 mm, then it was pre-strained by 100% to have the effective length of 20 mm. Cyclic loading was then applied under closed loop control to give peak-peak sinusoidal displacements as 2 mm (0 to 10% strain), 4 mm (0 to 20%), 6 mm (0 to 30%), 8 mm (0 to 40%), 10 mm (0 to 50%). At each level of cyclic strain, force measurements were recorded over 10 cycles at frequencies in the range of 0.01 Hz to 5 Hz and the samples were trained prior to taking measurements to remove the Mullins effect. The setup is shown in figure 5(d).

For mechanical loading at higher frequencies, the single layer samples (polyacrylate and silicone) were pre-strained, \( \lambda_{l,\text{pre}} = 1.2 \) for silicone, and \( \lambda_{l,\text{pre}} = 1.4 \) for polyacrylate, to achieve better force resolution. The samples were clamped to an electromagnetic shaker (Model 455 from LDS\textsuperscript{TM}). The displacements of cyclic testing were measured by a laser vibrometer (PSV-400 from Polytec\textsuperscript{TM}). A load cell connected to the sample at the bottom of the rig was used to take force measurements. The cell was purpose-made of aluminium and designed to measure forces up to 2 N. A command chirp signal that had a frequency range from 1 Hz to 100 Hz was used to induce the cyclic loading of the DE samples. The setup is shown in figure 6(a). For electrical actuation, the single-layer DE samples were pre-strained in similar manner as for higher frequency mechanical loading and clamped to the test rig and the load cell as shown in figure 6(b). The height of the top plate was adjustable for setting the samples with configured pre-strain, which was fixed during experiments. A high voltage (HV) generator, based on a HV DC-DC converter (module 15A24 from PPMT\textsuperscript{TM}), was used to amplify the input voltage (0–10 V) to the voltage output (0–15 kV) and actual voltage output was monitored using a built-in channel from the HV module. A resistor of 33 M\( \Omega \) was connected in parallel with the sample to reduce the lag from the HV generator due to the associated current discharge (figure 7(a)). The frequency responses of the samples were obtained using a Schroeder Phased Harmonic Sequence (SPHS) signal \cite{48} as the input voltage that covered a frequency range from 1 Hz to 100 Hz as shown in figure 7(b).

For all experiments, LabVIEW and the microprocessor, compactRIO, from NI\textsuperscript{TM} were used for data transmission.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{Figure6.png}
\caption{Test rig setups for the experiments: (a) mechanical loading at higher frequencies (b) electrical actuation.}
\end{figure}
3.3. Data correction to remove resonance effects

Depending on the mass of the clamp used to connect the DE samples to the load cell, and the stiffness of the load cell itself, resonance effects can occur. In our particular setting, resonance occurred between 40–60 Hz. Because the mass of the DE samples is negligible compared with that of the clamp, the resonance effects are not attributable to the characteristics of the DE samples. In order to obtain the true characteristics of the DE samples, a correction procedure was applied to remove the influence of the clamp-load cell resonance.

The dynamic stiffness characteristics of a DE sample under mechanical loading should be evaluated from

\[ k = \frac{F_{\text{DE}}}{Z_{\text{DE}}} \]  

where \( F_{\text{DE}} \) is the force exerted on the DE and \( Z_{\text{DE}} \) is the deformation of the DE sample. Due to resonance, the deflection of the load cell becomes significant when compared with the displacement input and DE deformation, hence the simplification of \( \dot{x}_0 = x_0 = 0 \) is no longer applicable. Given the force measured by the load cell, \( F \), and the stiffness of the load cell, \( k_L \), \( x_0 \) is given by

\[ x_0' = \frac{F}{k_L} \]  

If the displacement input due to mechanical loading is \( x_0 \), the deformation of the DE, \( Z_{\text{DE}} \), is given by

\[ Z_{\text{DE}} = x_0 - \frac{F}{k_L} \]  

The force exerted on the DE is obtained by removing the inertia force:

\[ F_{\text{DE}} = F - M\ddot{x}_0' \]  

where \( M \) is the mass of the clamp. Combining equations (16)–(18), the dynamics of the DE under mechanical loading are represented by the corrected expression

\[ \frac{F_{\text{DE}}}{Z_{\text{DE}}} = \frac{F - M\ddot{x}_0'}{x_0 - \frac{F}{k_L}} \]  

The correction formula of equation (19) may be implemented in the frequency domain. Given

\[ x_0'(t) = X_0' \sin(\omega t + \phi) \]
\[ \ddot{x}_0'(t) = -\omega^2 X_0' \sin(\omega t + \phi) \]  

where \( \phi \) is the same phase as in \( F \) (equation (10)), it follows in the complex form that

\[ \frac{F_{\text{DE}}}{Z_{\text{DE}}} = \frac{X_0 - \frac{F_0 e^{i\phi}}{k_L}}{X_0 - \frac{F_0 e^{i\phi}}{k_L}} \]

Similarly, in the case of electrical actuation, \( F_R = F_{A0} \sin \omega t \), the dynamics are represented by the complex form...
The values of $k_L$ and $M$ should be adjusted to suit the experimental setup. Under mechanical loading up to 5 Hz, a relatively stiff load cell was used to measure the force output of the stacked DE samples compared to that used for the single-layer samples up to 100 Hz. The clamps used were different in each case, but had similar masses. Resonance was experienced around 50 Hz for the single-layer tests, which is within the test range, hence the correction formulas were applied. However, for the stacked sample tests, the resonant frequency greatly exceeded the maximum forcing frequency of 5 Hz, hence the corrections were not applied.

4. Results

4.1. Mechanical loading over 0.01–5 Hz for the stacked polyacrylate sample

Figure 8 shows the mechanical loading results for the stacked polyacrylate sample under 100% pre-strain and peak-peak strain variations 0 to (10%, 20%, 30%, 40%, 50%). It is evident that the phase variations depend only weakly on the dynamic loading. However, the magnitudes vary between 2.3 N mm$^{-1}$ and 3 N mm$^{-1}$ at 5 Hz, higher strain variations giving reduced magnitudes.

Figure 9 shows the magnitude and phase variations averaged over the strain variations. These averaged curves were then used to fit first-, second- and third-order models following the iterative flow chart in figure 4. The third-order model provides a significantly better agreement with the experimental results in both magnitude and phase. The first- and second-order models have different dynamics, especially in the phase, compared with the measured characteristics. The reason is that these two lower order models lack the degrees of freedom to define the dynamics in phase and magnitude simultaneously. The third-order model is sufficient to

\[
\frac{F_{DE}}{F_A} = \frac{(F_0 + M\omega^2X'_0)e^{i\phi}}{F_{A0} - M\omega^2X'_0e^{i\phi}}
\]  

(22)

The values of $k_L$ and $M$ should be adjusted to suit the experimental setup. Under mechanical loading up to 5 Hz, a relatively stiff load cell was used to measure the force output of the stacked DE samples compared to that used for the single-layer samples up to 100 Hz. The clamps used were different in each case, but had similar masses. Resonance was experienced around 50 Hz for the single-layer tests, which is within the test range, hence the correction formulas were applied. However, for the stacked sample tests, the resonant frequency greatly exceeded the maximum forcing frequency of 5 Hz, hence the corrections were not applied.

Figure 8. The measured characteristics of the stacked polyacrylate samples under 100% pre-strain and peak-peak strain variations 0 to (10%, 20%, 30%, 40%, 50%).

Figure 9. Comparing the averaged dynamics with first-, second- and third-order models. The identified parameters are listed in table 2.
allow detailed definition of dynamics over a broader frequency range. However, the third-order model still deviates below the measured phase by up to 10 deg. for frequencies below 1 Hz (figure 9). This could be corrected by increasing the model order further. Table 2 shows the identified parameters for models and the evaluated errors in magnitude and phase. It shows that by having a second-order model, the errors are reduced by approximately 40%; by having a third-order model, the errors are reduced by approximately 80%.

Moreover, given the cross-sectional area of the sample as $A = 100 \text{ mm}^2$, the pre-strain $\lambda_{pre} = 2$ and the effective length of the sample as $l = 20 \text{ mm}$, the identified $k_0$ indicates the estimated elastic modulus of the polycrylate as 0.16 MPa, which is close to the expected value.

### 4.2. Mechanical loading over 1–100 Hz for single-layer silicone and polyacrylate samples

Figures 10 and 11 present the original and corrected results for the single-layer silicone and polyacrylate samples under mechanical loading up to 100 Hz. The pre-strains were $\lambda_{pre} = 1.2$ for silicone, and $\lambda_{pre} = 1.4$ for polycrylate, and the loading strains were less than 10% in peak-peak variation. The original results show the resonances are approximately at 50 Hz, with higher damping evidence in polycrylate. In the corrected results, the silicone has a near constant stiffness and zero phase. For polycrylate, more force is required to overcome the damping effects at higher frequencies. The stiffness and phase of the polycrylate therefore increase with frequency.

Figure 12 shows the corrected measured characteristics with the fitted first-order model for the silicone sample with the identified parameters listed in table 3. It shows that the first-order model fit is sufficient to predict the dynamics of the spring-like silicone. The model is accurate up to 70 Hz, however, measurement noise above 70 Hz is significant, which gives rise to the larger than expected (apparent) errors listed in table 4.

Figure 13 compares the corrected measured characteristics with models for first-, second- and third-order for the single-layer polycrylate and the identified parameters are listed in table 3. It shows the third-order model is clearly superior to the first-order model. As indicated in table 4, the errors of the third-order model are only 8% of those of the first-order model.

### 4.3. Electrical actuation over 1–100 Hz for single-layer silicone and polyacrylate samples

Figures 14 and 15 present the original and corrected measured characteristics for the single-layer silicone and the polycrylate samples under electrical actuation up to 100 Hz. The corrected results show dynamics of DE actuation between silicone and polycrylate due to their viscoelastic characteristics. The pre-strains were $\lambda_{pre} = 1.2$ for silicone, and $\lambda_{pre} = 1.4$ for polycrylate, and the driving voltage was up to 7 kV. According to
equation (8), the driving force $F_A$ is proportional to $V^2$, hence the results are presented in terms of $F_{DE}/V^2$. The original measured characteristics show resonance between 50–55 Hz. In the corrected measured characteristics, the silicone has a near constant dynamics and zero phase while the polyacrylate has decreasing magnitude and
negative phase throughout the frequency range. A higher driving voltage is required for the polyacrylate to
generate the same actuation force.

Figures 16 and 17 compare the corrected measured characteristics with the fitted models, parameters
identified from mechanical loading tests (table 3). It shows that the first-order model fits well with the dynamics
of the spring-like silicone. For polyacrylate, the samples undergo small deformation (<10%) during electrical
actuation, hence the benefit of having higher order models is less significant compared with that in mechanical

Figure 13. The corrected measured characteristics of the single-layer polyacrylate sample compared with the first-, second- and third-order models under mechanical loading, the identified parameters are listed in table 3.

Figure 14. The original and corrected measured characteristics of the single-layer silicone sample under electrical actuation up to 100 Hz.

Figure 15. The original and corrected measured characteristics of the single-layer polyacrylate sample under electrical actuation up to 100 Hz.
loading and is associated mainly with phase. In phase, the first-order model is accurate below 10 Hz, the second-order model remains accurate until 20 Hz and the third-order model fits well up to 50 Hz. It confirms that increasing model order allows more accurate modelling over a broader frequency range. However, the simulated dynamics deviate from the measured characteristics above 70 Hz because the actuation force is reduced and affected by measurement noise (e.g. low signal to noise ratio). Because the DEA model is derived based on the viscoelastic material model with added Maxwell pressure, good agreement between the models and the measured characteristics validate the accuracy of Maxwell approach for real materials, as Hong [30] had questioned.

5. Conclusions

This paper has considered the use of higher order Maxwell-Wiechert material models that contain multiple spring-damper combinations for more accurate representation of the viscoelastic characteristics of dielectric polymers. Compared with a conventional first-order model, the higher order representation allows the system dynamics to be shaped over frequency ranges. They offer more degrees of freedom in the parameter identification process, which leads to more accurate determination of the frequency response. An iterative procedure has been devised to guide the parameter identification such that magnitude and phase error bounds are within specified tolerances.

To demonstrate the model identification, two dielectric polymers, silicone and polyacrylate, were used. They were configured in a single-layer sheet format for higher frequency testing during which the mechanical strain variations were less than 10%, thus avoiding buckling of the sheet during the cyclic loading. Lower frequency testing with strain variations above 10% was performed using stacked layers of the polyacrylate, to eliminate the occurrence of buckling during the cyclic loading. The polymer samples were subjected cyclic loading and forces
were measured using load cells. The combination of low level forces and the low stiffness of the load cell used during the higher frequency testing led to resonance being experienced within the frequency range. A correction procedure was therefore implemented in which the inertia forces and load cell deflections were accounted for and eliminated from the dynamic frequency response measurements to reveal the characteristics of the polymer samples alone.

The experimental results and parameter identification for the silicone sample under mechanical loading indicated that a first-order model is appropriate, which is due to the spring-like behaviour of the polymer material. However, the polycarbonate samples required third-order models to achieve suitable accuracy to represent the viscoelastic characteristics. For the polycarbonate, third-order models improved the accuracy in modelling by 80% compared with the conventional first-order model.

The proposed higher order modelling and parameter identification of this paper should prove useful in the model-based control for polymer-based soft robotic systems.

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