Optically induced topological superconductivity via Floquet interaction engineering

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We study the photo-induced superconductivity in a two-valley semiconductor with a massive Dirac type band structure. The superconducting phase results from the out-of-equilibrium excitation of carriers in the presence of Coulomb repulsion and is stabilized by coupling the driven semiconductor to a bosonic or fermionic thermal bath. We consider a circularly-polarized light pump and show that by controlling the detuning of the pump frequency relative to the band gap, different types of chiral superconductivity would be induced. The emergence of novel superconducting states, such as the chiral p-wave pairing, results from the Floquet engineering of the interaction. This is realized by modifying the form of the Coulomb interaction by projecting it into the states that are resonant with the pump frequency. We discuss a promising experimental platform to realize our proposal.

Introduction.— Possibility of generating superconductivity (SC) in driven systems has been long investigated in semiconductors 1, and it has been argued that under population inversion repulsive interactions can lead to a superconducting instability 2 5. Recent developments in Floquet band engineering 6 11 has revived interest in periodically driven interacting quantum phases of matter including periodically driven superconductors 12 18. In particular, recently such effects were studied in hexagonal semiconductors such as hexagonal Boron-Nitride or two dimensional transition metal dichalcogenides 19. It has been proposed that light-induced non-thermal population occupation can lead to interband superconducting correlations in the presence of repulsive interactions and fermionic or bosonic baths 20. Therefore, it is intriguing to question whether more exotic form of superconductivity could be achieved in such driven systems.

In this Letter, we show that the extension of these ideas could lead to creation and manipulation of topological superconducting phases. In particular, we show that optical pumping of electrons in such two-dimensional semiconductors can generate topologically non-trivial chiral SC 21 22 in the prethermal regime 23 24. The idea is illustrated in Fig.1(a), where we apply a circularly polarized laser field, in the presence of an external bath to create the population imbalance, required for the development of a non-equilibrium superconducting phase.

The key underlying mechanism for the development of unconventional superconductivity in our system is the following. By varying the frequency of the optical pump, we excite photocarriers of select momentum classes. Due to the optical valley polarization, this leads to an asymmetric occupation distribution around the resonance surfaces in the two valleys, as shown in Fig.1(b). This non-equilibrium occupation creates an effective interband population inversion around one of the valleys, as illustrated in Fig.1(c), which leads to an interband pairing of electrons for a repulsive density-density interaction, i.e., the population inversion effectively changes the in-
teraction sign. To study this pairing, the bare density-density interaction should be projected into the band basis, composed of Bloch wave functions. Due to the non-zero Berry curvature of Bloch wave functions around each valley, the effective interaction has a chiral nature, and can be decomposed into different angular momentum channels where each channel has a different dependence on the momentum of electrons. Combined with the fact that the momentum distribution of the excited phototcarriers are controlled by the frequency of the pump, our setup allows for engineering the dominant form of electron-electron interaction. Consequently, we find frequency regimes where a chiral p-wave pairing becomes more favorable than a s-wave pairing.

Our results indicate that the periodic drive could be a powerful tool to not only engineer a band, but also control the form and strength of the interaction. Previous examples include controlling the Haldane pseudo-electron interaction ($\epsilon_{a,k}$) and $|\psi_{a,k}\rangle$, where the valence and conduction bands are labeled by $\alpha = \{v,c\}$, respectively. To include the effect of the optical drive, we consider the minimal coupling $(k \rightarrow k + eA(t))$, where the laser field with a counter clock-wise polarization is described by the vector potential $A(t) = A_0(\cos \omega t, \sin \omega t, 0)$, where $A_0$ and $\omega$ label the amplitude and frequency of the pump, respectively and we have set $\hbar = 1$.

The Rabi frequency associated with the pump in the sublattice basis is

$$\Omega(t) = eA_0v
cos \omega t, \eta \sin \omega t, -2v\langle k_x \cos \omega t + \eta k_y \sin \omega t \rangle, \quad (4)$$

where we have only kept the linear terms in $A_0$.

For the electron-electron interaction, we consider a repulsive density-density potential $U(r - r')$, with the corresponding Hamiltonian

$$H_{e-e} = \int d^2rd^2r' \sum_{a,b} \psi_a^\dagger(r)\psi_b^\dagger(r')U(r - r')\psi_b(r')\psi_a(r), \quad (5)$$

where $\psi_a^\dagger(r)$ represents the electronic creation operator with the sublattice index $a$ at position $r$. To study the possibility of Cooper pairing between electrons, we assume that the dominant form of the interaction is a screened Coulomb interaction $\frac{g}{N}$. Therefore, in passing to the momentum space, such interactions can be treated as a constant coupling. Denoting the Fourier transform of interaction potential in Eq. (5) by $U_{kk'}$, this implies that $U_{kk'} = g/N$, where $g$ is the interaction strength and $N$ stands for the number of particles in the unit-cell. To further simplify our analysis, we restrict our interactions to intra-valley scattering events such that in $U_{kk'}$, $k$ and $k'$ belong to the same valley.

In order to create an effective population inversion, we need a thermal bath. Our bath can have a fermionic or bosonic nature, however, here, we only consider a bosonic bath composed of photons or phonons which is experimentally more feasible and leave the study of the fermionic bath to the supplemental. We assume that the bath has a continuous spectrum and can induce relaxation processes between the valence and conduction bands via absorption/emission of photons.

**Master equation.**—To examine the out-of-equilibrium nature of the superconducting phase in our system, we consider a thermal reservoir at a fixed temperature $T$ and governed by the Bose-Einstein distribution $n_B(\epsilon) = (1 + e^{-\beta \epsilon})^{-1}$, where $\beta = (k_B T)^{-1}$. Assuming that the

$K$ and $-K$ valleys. For simplicity, we ignore the effect of the physical spin and we expect when the spin-orbit coupling is not large compared to the semiconductor gap, the inclusion of the spin does not significantly affect our results. In the following, we denote the corresponding eigenenergies and eigenstates of the undriven Hamiltonian by $\epsilon_{a,k}$ and $|\psi_{a,k}\rangle$, respectively.

where $a_k = (nk_x, n\kappa k_y, m - \kappa k^2)$, with its components denoted by $d_i^\dagger$'s with $i = \{x,y,z\}$, and $K$ denoting the deviation from the $K$ or $-K$ points in the BZ, and $m$, $v$ and $\kappa$, corresponding to the band gap, Fermi velocity, and the band curvature, respectively. We have also used $k^2 = k_x^2 + k_y^2$ and $\eta = \pm 1$ to label the

Model. — The system considered in this Letter consists of a two-dimensional semi-conductor with honeycomb lattice structure, such as a single layer hexagonal Boron nitrate (h-BN) or transition metal dichalcogenides (TMD) \cite{30, 31}. The electronic band structure consists of two degenerate valleys and the broken inversion symmetry leads to a gap at two Dirac points $K$ and $K' = -K$ at the corners of Brillouin zone (BZ). The semiconductor is driven by a laser beam, whose frequency is slightly larger than the semiconductor gap. The Hamiltonian describing the system’s interaction is given by

$$H = H_K + H_{e-e}, \quad (1)$$

consisting of a driven kinetic term ($H_K$) and an electron-electron interaction ($H_{e-e}$).

The Hamiltonian for the driven semi-conductor has the form

$$H_K = \sum_{a,b} c_{a,k}^\dagger [(d_k + \Omega(t)) \tau_{ab} - \mu_k 1_{ab}] c_{b,k}, \quad (2)$$

where $c_{a,k}^\dagger$ is the electron creation operator of sublattice type $a$, $\tau_i$ with $i = \{x,y,z\}$ is the Pauli matrix acting on the sublattice space in the unit-cell, and $\mu_k$ is the chemical potential. The effective low-energy Hamiltonian for the two valleys corresponds to

$$d_k = (vk_x, \eta \kappa k_y, m - \kappa k^2), \quad (3)$$
We see that the Bloch wave functions which encompass the topological characteristics of the system, control the form of electron-electron interactions. The crucial effect of the Berry curvature of the band structure on the electron-electron interactions is embedded in the Bloch wave function overlaps in Eq. (10). The momentum dependence of Bloch overlaps which should be inserted in Eq. (10) can be decoupled in three channels according to their angular momenta,

$$\langle u_k^v|u_k^v\rangle = \sum_{l=0,1,2} f_k^{(l)} f_k^{(l)*} e^{-i(\phi_k - \phi_{k'})},$$  \hspace{1cm} (11)

where $f_k^{(0)} = (1 + d_z k/d_k)/2$, $f_k^{(1)} = v k/d_k$, $f_k^{(2)} = v^2 k^2/(2 d_k (d_k + d_z k))$ and $d_k = |d_k|$. For momenta close to the corners of BZ, the interaction Hamiltonian takes the form, $U_{kk'} = g (1 + \sqrt{2} |\mathbf{k}_x \cdot \mathbf{k}'_x - \mathbf{k}_y \cdot \mathbf{k}'_y - q^2|)/N$, where we have defined the Berry curvature at the Dirac point as $F = v^2/m^2$. Recently, such Berry phase induced modifications of the electron-electron interaction has been associated with the modification of excitonic spectrum \cite{35} \cite{39}.

**Mean-Field analysis.**— To study the possibility of the Cooper pair condensation, we use a mean-field (MF) approximation for the electron-electron interaction and express it as,

$$H_{e-e} = -\sum_{\eta = \pm 1; k k'} \Delta_k^{\eta \ast} U_{k k'}^{-1} \Delta_k^{\eta}$$

$$+ \sum_{\eta = \pm 1; k} (\Delta_k^{\eta \ast} c_{\eta K - k} c_{\eta K + k} + h.c.) \hspace{1cm} (12)$$

Notice that by explicitly using the valley momentum $\pm K$ and the valley index $\eta = (\pm)$, we have introduced two pairing order parameters $\Delta_k^{(\pm)}$, depending on whether the valence electrons around the $K$ valley and conduction electrons around $-K$ valley are bound to each other or vice versa. Employing the MF expression above, and truncating the hierarchy of many-body correlation functions, we can use Eq. (6) to write a closed set of equations for the occupation numbers $n_{\alpha, k}^\eta = \text{tr}(\rho_c c_{\alpha \eta}^\ast k c_{\alpha \eta} k)$, the polarization $\sigma_{k}^{\eta} = \text{tr}(\rho_c c_{\eta K + k} c_{\eta K - k})$, and the anomalous pairing $s_{k}^{\eta} = \text{tr}(\rho_c c_{\eta K - k} c_{\eta K + k})$. This approach leads to legitimate results at the onset of the SC phase transition, where the distinction between the Bogoliubov quasi-particles and electrons is negligible. The EOM for $n_{\alpha, k}^\eta$ and $\sigma_{k}^{\eta}$ in the absence of pairing are familiar and usually known as the optical Bloch equations in the literature \cite{30} \cite{11} . Thus, here we only present the EOM of the interband pairing and leave the detailed derivation of the EOMs to the supplemental,

$$\partial_t s_k^\eta = -i\epsilon_{\ell, k} s_k^\eta - i\Delta_k (1 + n_{\ell, -k}^\eta - n_{\ell, k}^\eta) - \frac{1}{2} \Gamma_s s_k^\eta \hspace{1cm} (13)$$
where the total decay rate is \( \Gamma_t = \Gamma_v + \Gamma_c \), and the energy gap between the conduction and valence band is \( \epsilon_{f,k} = \epsilon_{e,k} + \epsilon_{c,-k} \). Note that on the right side of this equation, the two occupation probabilities in the parenthesis belong to two different valleys. This is a manifestation of the Cooper pairing which couples quasiparticles at opposite momenta, and therefore different valleys. Hence, in general the EOMs at the two valleys should be solved together, details are discussed in the Supplementary Material.

We notice that in the steady state, which is reached at a time scale set by \( \Gamma \)'s, the interband pairing is controlled by the effective interband pairing populations \( n_{sc,k}^\eta \equiv 1 - n_{e,k}^\eta - n_{c,-k}^\eta \). We remark that since \( n_{e,k}^\eta \) and \( n_{c,-k}^\eta \) can be independently populated because of the optical valley selection rules, under non-equilibrium conditions the pairing population can acquire a finite value.

To explicitly demonstrate that a finite interband pairing can be obtained, we derive the self-consistency equation of the anomalous pairing. This equation in the presence of dissipation is retrieved from the saddle point of the total dissipative action [19] [42] [43]. At the onset of the phase transition the gap equation can be linearized with the total dissipative action [19, 42, 43]. At the onset of the Cooper pairing which couples quasiparticles at opposite momenta by the effective interband pairing populations \( U_{kk}^\eta \), the interband pairing is controlled to a time scale set by \( \Gamma \)'s, the interband pairing is controlled.

Varying \( \eta \) from Eq. (14) that only the states with negative momentum classes are resonantly excited. It is apparent that a finite interband pairing decouple and the critical value of the coupling strength is determined by,

\[
\frac{1}{g_{\text{crit}}^{(l)}} = \frac{1}{N} \sum_{k'} \frac{f_{k'}^{(l)}}{c_{f,k'}^2 + \Gamma_{f,k'}} \left( \frac{1}{2\zeta_{k'}^{(-)}} + 1 \right) - \frac{1}{2\zeta_{k'}^{(+)} + 1},
\]

where \( l = \{0, 1, 2\} \) can be ascribed to the angular momentum of the \( s, p \) and \( d \)-wave pairing modes and \( f_{k'}^{(l)} \)'s play the role of the SC form factors. Using this ansatz, the linearized gap equation for the three different types of pairing decouple and the critical value of the coupling strength is determined by,

\[
\frac{1}{g_{\text{crit}}^{(l)}} = \frac{1}{N} \sum_{k'} \frac{f_{k'}^{(l)}}{c_{f,k'}^2 + \Gamma_{f,k'}} \left( \frac{1}{2\zeta_{k'}^{(-)} + 1} - \frac{1}{2\zeta_{k'}^{(+)} + 1} \right),
\]

where on the right-hand side we have used the orthogonality of angular harmonics \( e^{i\phi_k} \), so that we only need to consider the contribution of \( U_{kk}^\eta \) in the \( l \)-th channel.

To perform the momentum integration in (17), since we have a continuum model, we assume an ultraviolet (UV) energy cutoff. As illustrated in Fig. (1c), the main contribution to \( n_{sc,k} \) comes from around the resonance energy surface where the detuning frequency vanishes \( \epsilon_{d,k} \sim 0 \). Hence, we consider a given UV cutoff \( E_A \) around this surface and show that the resulting phenomena are independent of the exact value of this parameter [45].

The frequency of the pump laser determines which momentum classes are resonantly excited. It is apparent from Eq. (14) that only the states with negative \( \eta_{sc,k} \) can form pairing. Moreover, since the projected Coulomb interaction has momentum dependence, the critical value of the coupling strength depends on the pump frequency.

After scaling units and making quantities dimensionless this behavior is depicted in Fig. (2) where we notice that the preferred form of pairing transforms from \( s \)-wave to
$p$-wave as the drive frequency increases. This transition can be ascribed to the momentum dependence of the SC form factors. In the inset of Fig. 2, these form factors are plotted as a function of the frequency associated with the momentum of the resonant energy surface. Here, we see a similar behavior as in the main plot of Fig. 2, where by increasing the frequency the initially dominant $s$-wave form factor becomes subdominant with respect to the $p$-wave form factor. We also note that the slight discrepancy between the frequency of the crossing points in the two plots is due to the integration over the form factors and the finite width of $n_{sc,k}$ in the gap equation which causes the frequency dependence of $g_{crit}$ lag behind that of $f^{1/3}$.

Other than the frequency of the pump, the critical value of the coupling depends on the amplitude of the pump. This is shown in Fig. 3, where the horizontal axis has been chosen to be the dimensionless parameter $evA_0/\Gamma$ which appears in the gap equation through $\xi_k$. We notice that unlike Fig. 2 the critical coupling strength of all the three SC modes always decreases as the pump amplitude increases. Specifically, in the low-intensity limit ($evA_0 \lesssim \Gamma$), the critical coupling is inversely proportional to the intensity $\propto (\frac{1}{evA_0})^2$ which could be associated with the fact that the peak value of excited population ($|n_{sc,k}^{(\pm)}| \approx \xi_k^{(\pm)}$) increases linearly with the intensity. At higher intensities, where $evA_0 \gtrsim \Gamma$, this behavior changes, since the width of $n_{sc,k}$, i.e., the number of momentum classes participating in pairing, keeps increasing. In other words, we do not see any saturating behavior by increasing the drive amplitude $evA_0/\Gamma$, because while the peak value of $|n_{sc,k}^{(\pm)}|$ in Fig. 1 is saturated with increasing $evA_0$, its width keeps increasing with the drive amplitude. In the inset of Fig. 3, the behavior of $g_{crit}$ in terms of the energy cut-off $E_{\Lambda}$ is shown. Once $E_{\Lambda}$ becomes comparable to the band gap, the cutoff dependence of $g_{crit}^{-1}$ becomes insignificant.

**Experimental feasibility.** While in general investigating the experimental feasibility of our proposal is material-dependent, here we provide an estimate for the required optical field amplitude based on the typical energy scales in the two-dimensional two-valley semiconductors. Specifically, to verify the feasibility of the realization $p$-wave SC in our model, we need to estimate the required critical coupling constant. The promising two-dimensional semiconductor to realize the phenomena outlined in this letter are h-BN with the band gap of the order of 6 eV [46] or TMDs with the band gap of the order of 2 eV [17]. The screened Coulomb interaction $g$ can be as large as $2 eV \mu$ [33], which is comparable to the band gap of these materials. Thus, based on Fig. 3, the ratio $evA_0/\Gamma$ should be of order $10^3$ or larger. Since, typically the inverse decay rate is of the order of picoseconds [48], this implies that the required Rabi frequency for our proposal should be at least 100 THz. Recently, such strong Rabi frequencies have been used to generate the light induced anomalous Hall effect in graphene [11]. In this work, a field strength $A_0$ of order $10^{-6}$ sV/m, was utilized. Given the Fermi velocity of the order $10^6$ m/s [49], this results in a Rabi frequency around 500 THz which is sufficient for our proposal.

The lack of screening, which leads to large Coulomb interaction is essential for development of SC phase. At the same time, the large Coulomb interaction also leads to large exciton binding energy in TMDs, and it might appear that exciton formation could compete with the formation of superconductivity. This issue can be avoided by strongly optically pumping the system well-above the band gap. More precisely, for the formation of excitons, the excited electrons should relax to the bottom of the conduction band by coupling to optical phonons. The effective energy associated with inverse relaxation rate of excited carriers to the bottom of the band through optical phonons is $2 \text{meV}$ [50] which is two orders of magnitude smaller than the Rabi frequency being more than $100 \text{meV}$. Correspondingly, in our derivation, we ignore the contributions of the density-density interaction leading to exciton formation.

In summary, we show an approach to create non-equilibrium superconducting chiral pairing in semiconductors, with optical drive, and discuss that the required tools to realize this proposal is within recent experimental capabilities. More broadly, such optical drives not only can make Floquet band engineering possible, but also can lead to engineering the form and strength of the effective interaction. Possible signature of unconven-
tional Floquet superconductivity in the form of optical signatures of transient edge states is an intriguing subject for future studies.

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Supplementary Material

DERIVING THE EQUATIONS OF MOTION

Bosonic Bath

In this section of the supplemental, we obtain the equations of motion (EOM) in the presence of a bosonic bath. In general, the bath used in our formalism and its coupling to our system can be described by the following Hamiltonian,

\[ H_b = \sum_k \nu_k b_k \]

\[ H_{s-b} = \sum_k \lambda_k \left( b_k c_{v,k}^\dagger c_{v,k} + h.c. \right). \]

Using such a bath, leads to a master equation as,

\[ \partial_t \rho_s(t) = -i[H_s, \rho_s] + \sum_{\alpha=v,c} \Gamma_\alpha \mathcal{L}[L_{\alpha,k}]\rho_s. \]

(3)

where the action of the Lindbladian superoperator \( \mathcal{L} \) with a quantum jump operator \( L \) on the density matrix \( \rho \), is defined as \( \mathcal{L}[L]\rho = L\rho L^\dagger - \frac{1}{2} \{ L^\dagger L, \rho \} \). While in general the decay rates depend on the coupling constant \( \lambda_k \) and the density of states of the bath, to simplify our formalism we consider constant decay rates, \( \Gamma_v = \Gamma_{n_B} \), and \( \Gamma_c = \Gamma(1+n_B) \), where \( n_B \) denotes the effective Bose-Einstein population of the bath which we assume is momentum-independent.

Here, we are interested in the EOM for the occupation probability, \( n_{\alpha,k}^v \), the polarization, \( \sigma_{k}^v \), and the anomalous pairing, \( s_k^v = \text{tr}(\rho, c_{-v,k}c_{v,k}^\dagger) \). For convenience of labeling we define the notation \( n_{\alpha_v,k}^v \equiv n_{\alpha_v,k}^v \), and \( n_{\alpha_c,k}^c \equiv \sigma_{k}^c \).

To derive the EOM for an arbitrary operator \( \mathcal{O} = \text{tr}(\rho, \hat{\mathcal{O}}) \) in the Schrödinger equation we use \( \partial_t \mathcal{O} = \text{tr}(\partial_t \rho, \hat{\mathcal{O}}) \). To write down the EOMs for these quantities we also need the following identities,

\[ \text{tr}(\hat{\mathcal{O}}[H, \rho]) = \text{tr}([\hat{\mathcal{O}}, H]\rho) \]

\[ \text{tr}(\hat{\mathcal{O}}\mathcal{L}[L]\rho) = \frac{1}{2}\text{tr}([\hat{\mathcal{O}}^\dagger, \mathcal{L}[L]\rho]) + \frac{1}{2}\text{tr}(\mathcal{L}[L]^{\dagger}\hat{\mathcal{O}}\rho). \]

We can study the contributions of the Hamiltonian and Lindbladian in the time evolution separately. For the kinetic Hamiltonian part with \( H_K = c_{\alpha,k}^\dagger b_{\alpha,k} c_{\beta,k} \) we can use the following identities,

\[ \partial_t n_{\mu c,k}^v \bigg|_{H_K} = \text{tr}(c_{\mu c,k}^\dagger b_{\alpha,k} c_{\alpha,v,k}) \partial_t \rho = -i \sum_{\alpha=v,c} (h_{\alpha c,k} n_{\mu c,k}^v - h_{\alpha c,k} n_{\alpha c,k}^v). \]

(6)

Similarly, for the interband pairing we get,

\[ \partial_t s_k^v \bigg|_{H_K} = \text{tr}(c_{-\alpha,k} b_{\alpha,k} c_{\beta,k}) \partial_t \rho = -i(\epsilon_{c,k} + \epsilon_{v,k}) s_k^v. \]

(7)

We can also compute the commutators of the anomalous pairing and the electron-electron interaction which is

\[ H_{\text{e-e}} = \sum_{n=\pm 1} \left( \Delta_k^v c_{-n,k} c_{v,k} + h.c. \right) + \text{const.} \]

(8)

The corresponding commutator becomes,

\[ \partial_t s_k^v \bigg|_{H_{\text{e-e}}} = -i\Delta_k (1 - n_{v,k}^v - n_{c,-k}^v). \]

(9)

For the Lindbladian contributions we employ Eq. (8) to obtain,

\[ \partial_t n_{\mu c,k}^v \bigg|_{\mathcal{L}} = -\Gamma n_{n_B n_{\mu c,k}}^v + \Gamma(1+n_B)n_{\mu c,k}^v, \]

\[ \partial_t n_{\alpha c,k}^c \bigg|_{\mathcal{L}} = \Gamma n_{n_B n_{\alpha c,k}}^c - \Gamma(1+n_B)n_{\alpha c,k}^c, \]

\[ \partial_t \sigma_k^v \bigg|_{\mathcal{L}} = -\Gamma \left( \frac{1}{2} + n_B \right) \sigma_k^v, \]

\[ \partial_t s_k^v \bigg|_{\mathcal{L}} = -\Gamma \left( \frac{1}{2} + n_B \right) s_k^v. \]

(10-13)
After combining the Hamiltonian and Lindbladian contributions, we get,

\begin{align}
\partial_t n^\eta_{v,k} & = -i(V^\eta_{x,k} - iV^\eta_{y,k})\sigma^\eta_{k} + i(V^\eta_{y,k} + iV^\eta_{x,k})\sigma^\eta_{k} - i\Delta^\eta s_{k} + \Gamma n_B n^\eta_{v,k} + \Gamma (1 + n_B)n^\eta_{v,k}, \\
\partial_t n^{-\eta}_{c,k} & = i(V^{-\eta}_{x,k} - iV^{-\eta}_{y,k})\sigma^{-\eta}_{k} - i(V^{-\eta}_{y,k} + iV^{-\eta}_{x,k})\sigma^{-\eta}_{k} - i\Delta^{-\eta} s^{-\eta}_{k} + \Gamma n_B n^{-\eta}_{v,k} - \Gamma (1 + n_B)n^{-\eta}_{v,k}, \\
\partial_t \sigma^\eta_{k} & = i(\epsilon_{c,k} - \epsilon_{v,k} - \omega)\sigma^\eta_{k} - i(V^\eta_{x,k} - iV^\eta_{y,k})(n^\eta_{v,k} - n^{-\eta}_{v,k}) - \Gamma \left( \frac{1}{2} + n_B \right) \sigma^\eta_{k}, \\
\partial_t s^\eta_{k} & = -i(\epsilon_{c,k} + \epsilon_{v,k})s^\eta_{k} - i\Delta^\eta_k (1 - n^\eta_{v,k} - n^{-\eta}_{c,-k}) - \Gamma \left( \frac{1}{2} + n_B \right) s^\eta_{k}. 
\end{align}

(S14) (S15) (S16) (S17)

Notice that should we want to take exciton formation into account, in the third equation above which is the EOM for the polarization, we need to consider the Hartree-Fock contribution of the electron-electron interaction in the particle-hole channel. This adds a term as \(-i \sum_{k'} \tilde{U}_{kk'} \sigma^\eta_{k'}\) on the right-hand side of the third equation above. We can show that in our system exciton formation and the Cooper instability do not compete with each other. Therefore, even in the presence of a finite density of excitons, we can still have a phase transition into a superconducting state. Thus, in our derivation we drop such terms to simplify our analysis. From the third and fourth equations above, we can obtain the anomalous pairing,

\[ s^\eta_{k} = \frac{\Delta^\eta (1 - n^\eta_{v,k} - n^{-\eta}_{c,-k})}{\epsilon_{c,k} - \epsilon_{v,k} - \omega}. \]

(S18)

and the polarization,

\[ \sigma^\eta_{k} = \frac{(V^\eta_{x,k} - iV^\eta_{y,k})(n^\eta_{v,k} - n^{-\eta}_{v,k})}{\epsilon_{c,k} + \epsilon_{v,k} + \Gamma \left( \frac{1}{2} + n_B \right)}, \]

(S19)

where \(\epsilon_{d,k} \equiv \epsilon_{c,k} - \epsilon_{v,k} - \omega\). These relations can be inserted in the first two EOMs in the steady state,

\begin{align}
0 & = \zeta^\eta_k \left( n^\eta_{c,k} - n^\eta_{v,k} \right) + \delta^\eta_k \left( 1 - n^\eta_{v,k} - n^{-\eta}_{c,-k} \right) - \gamma_v n^\eta_{v,k} + \gamma_c n^{-\eta}_{c,k}, \\
0 & = -\zeta^\eta_k \left( n^\eta_{c,k} - n^\eta_{v,k} \right) + \delta^\eta_k \left( 1 - n^{-\eta}_{v,k} - n^\eta_{c,-k} \right) + \gamma_v n^\eta_{v,k} + \gamma_c n^{-\eta}_{c,k}, 
\end{align}

(S20) (S21)

where we have defined

\[ \gamma_v \equiv \frac{n_B}{1 + n_B}, \quad \gamma_c \equiv \frac{1 + n_B}{1 + 2n_B}. \]

(S22)

The equations at the two valleys should be solved together. This gives,

\begin{align}
\left( \zeta^+_{k} + \delta^+_{k} + \gamma_v \right) \left( n^+_{v,k} - \frac{1}{2} \right) - \left( \zeta^+_{k} + \gamma_c \right) \left( n^+_{c,k} - \frac{1}{2} \right) = \frac{\gamma_c - \gamma_v}{2}, \\
\left( \zeta^-_{k} + \delta^-_{k} + \gamma_v \right) \left( n^-_{v,k} - \frac{1}{2} \right) - \left( \zeta^-_{k} + \gamma_c \right) \left( n^-_{c,k} - \frac{1}{2} \right) = \frac{\gamma_v - \gamma_c}{2}, \\
\left( \zeta^-_{k} + \delta^-_{k} + \gamma_c \right) \left( n^-_{c,k} - \frac{1}{2} \right) - \left( \zeta^-_{k} + \gamma_v \right) \left( n^-_{v,k} - \frac{1}{2} \right) = \frac{\gamma_c - \gamma_v}{2}, \\
\left( \zeta^+_{k} + \delta^+_{k} + \gamma_c \right) \left( n^+_{c,k} - \frac{1}{2} \right) - \left( \zeta^+_{k} + \gamma_v \right) \left( n^+_{v,k} - \frac{1}{2} \right) = \frac{\gamma_v - \gamma_c}{2},
\end{align}

(S23) (S24) (S25) (S26)

where the effective Rabi frequency and pairing amplitude are respectively given by,

\[ \zeta^\eta_{k} = \frac{(\tilde{\Omega}^\eta_{x,k} + \tilde{\Omega}^\eta_{y,k})}{\epsilon^2_{c,k} + (1 + n_B)^2 T^2}, \]

\[ \delta^\eta_{k} = \frac{|\Delta^\eta_k|^2}{\epsilon^2_{c,k} + (1 + n_B)^2 T^2}. \]

(S27) (S28)
The resulting equations can be rewritten in a matrix form,

\[
\begin{pmatrix}
\delta^{(+)}_{-k} + \delta^{(+)}_{k} + \gamma_v - \delta^{(-)}_{k} - \gamma_c \gamma_v - \delta^{(-)}_{k} + \gamma_c \\
\delta^{(-)}_{-k} - \gamma_v \delta^{(-)}_{k} + \gamma_v \\
0 \delta^{(+)}_{-k} \delta^{(-)}_{k} \gamma_v - \delta^{(+)}_{k} + \gamma_c
\end{pmatrix}
\begin{pmatrix}
\delta^{(+)}_{k} \\
\delta^{(-)}_{k} \\
\gamma_v
\end{pmatrix}
= \frac{1}{2} \begin{pmatrix}
\gamma_c - \gamma_v \\
\gamma_c + \gamma_v \\
\gamma_c
\end{pmatrix}.
\]

(S29)

Here, we are mainly interested in studying the onset of the SC phase transition which implies that we can ignore the pairing amplitude in the above equations. In the limit where the effective Rabi frequency around the $-K$ valley i.e. $\Omega^{(-)}_k$, is negligible, these probability populations become,

\[n_{v,k}^{(+)} = \frac{1}{2} + \frac{\gamma_v - \gamma_c}{2(\Delta^{(+)}_k + \gamma_v + \gamma_c)},\]

(S30)

\[n_{c,k}^{(+)} = \frac{1}{2} - \frac{\gamma_v - \gamma_c}{2(\Delta^{(+)}_k + \gamma_v + \gamma_c)},\]

(S31)

\[n_{c,-k}^{(-)} = 1,\]

(S32)

\[n_{c,-k}^{(-)} = 0.\]

(S33)

Let us further assume that $n_B = 0$ which results in $\gamma_v = 0$, and $\gamma_c = 1$. In this limit, it is evident that we can have an effective SC population inversion around one of the valleys because,

\[1 - n_{v,k}^{(+)} - n_{c,-k}^{(-)} \simeq \frac{1}{2} - \frac{1}{2(\Delta^{(+)}_k + 1)}\]

(S34)

\[1 - n_{v,-k}^{(-)} - n_{c,k}^{(+)} \simeq \frac{1}{2} - \frac{1}{2(\Delta^{(+)}_k + 1)}.\]

(S35)

We should hint that in the weak-drive limit, the right-hand side reduces to $+\delta^{(+)}_k$ and $-\delta^{(+)}_k$. More generally, we have,

\[1 - n_{v,k}^{(+)} - n_{c,-k}^{(-)} = \frac{-1}{2(1+2n_B)} \left( \frac{1}{2\Delta^{(+)}_k + 1} - \frac{1}{2\Delta^{(-)}_k + 1} \right),\]

(S36)

\[1 - n_{v,-k}^{(-)} - n_{c,k}^{(+)} = \frac{1}{2(1+2n_B)} \left( \frac{1}{2\Delta^{(+)}_k + 1} - \frac{1}{2\Delta^{(-)}_k + 1} \right).\]

(S37)

where we have used the fact that at every momentum we have $n_{v,k}^{(+)} + n_{c,k}^{(+)} = 1$. These equations lead to the linearized gap equation in the main text i.e. Eq. [15].

**FERMIONIC BATH**

Here, we show that we can obtain similar results with a fermionic bath at a fixed temperature $T$, described by the Hamiltonian [19]

\[H_b = \sum_{k,\alpha} \omega_{\alpha}(k) b_{\alpha,k}^\dagger b_{\alpha,k}.\]

(S38)

We consider a system-bath coupling which allows exchanging particles between the system and the reservoir,

\[H_{s-b} = \sum_{k,\alpha} t_{\alpha}(k) \left[ c_{\alpha,k}^\dagger b_{\alpha,k}^\dagger b_{\alpha,k}^\dagger + c_{\alpha,k}^\dagger b_{\alpha,k}^\dagger b_{\alpha,k} \right].\]

(S39)
Starting with the system-bath coupling term, we assume a thermal Fermi-Dirac distribution for the bath DOF at temperature \( T \), so that these DOF can be traced out. After applying the RWA and eliminating the oscillating terms, we arrive at the following master equation for the density matrix of the driven semiconductor:

\[
\partial_t \rho_s(t) = -i[H_s(t), \rho_s] + \sum_{k,\alpha=e,c} \Gamma_\alpha(k) \left( n_{\alpha,k}^F \mathcal{L}[^{\dagger}\rho_s]_{\alpha,k} + (1 - n_{\alpha,k}^F) \mathcal{L}[c_{\alpha,k}] \rho_s \right),
\]

where \( n_{\alpha,k}^F = n^F(\epsilon_{\alpha,k}) \) is the Fermi-Dirac distribution. The decay rates \( \Gamma_\alpha(k) = 2\pi \sum_\epsilon |\epsilon_{\alpha,k}^\nu(\epsilon_{\alpha,k})u_{\alpha,k}^\nu u_{\alpha,k}^\nu^* \) where \( \nu(\epsilon) \) represents the density of states of the bath's electrons at energy \( \epsilon \).

For the pairing amplitude between the valence \( \alpha = v \) and conduction band \( \alpha = c \), this yields

\[
\partial_t s_{k}\big|_H = \text{tr}(c_{c,k}^{\dagger} c_{c,k}^{\dagger} \partial_t \rho) = i(\epsilon_{c,k} + \epsilon_{v,k}) s_k.
\]

For the Lindbladian part we get,

\[
\partial_t \hat{\mathcal{O}} = \frac{1}{2} \Gamma_{\alpha\beta} n_{\alpha}^F \left( \text{tr}([c_{\alpha,k}, \hat{\mathcal{O}}]c_{\beta,k}^{\dagger} \rho) + \text{tr}(c_{\alpha,k}[\hat{\mathcal{O}}, c_{\beta,k}^{\dagger} \rho]) + \frac{1}{2} \Gamma_{\alpha\beta} (1 - n_{\alpha}^F) \left( \text{tr}([c_{\alpha,k}^{\dagger}, \hat{\mathcal{O}}]c_{\beta,k} \rho) + \text{tr}(c_{\alpha,k}^{\dagger}[\hat{\mathcal{O}}, c_{\beta,k} \rho]) \right) \right),
\]

where we have used the creation and annihilation operators in the rotating frame. Consequently, we can assume that oscillating terms in the rotating frame can be ignored. This way, we can time average over the Lindbladian which results in considering only the diagonal terms in the above with \( \alpha = \beta \).

\[
\partial_t \hat{\mathcal{O}} \big|_{\text{RW}} = \frac{1}{2} \Gamma_{\alpha\alpha} n_{\alpha}^F \left( \text{tr}([c_{\alpha,k}, \hat{\mathcal{O}}]c_{\alpha,k}^{\dagger} \rho) + \text{tr}(c_{\alpha,k}[\hat{\mathcal{O}}, c_{\alpha,k}^{\dagger} \rho]) + \frac{1}{2} \Gamma_{\alpha\alpha} (1 - n_{\alpha}^F) \left( \text{tr}([c_{\alpha,k}^{\dagger}, \hat{\mathcal{O}}]c_{\alpha,k} \rho) + \text{tr}(c_{\alpha,k}^{\dagger}[\hat{\mathcal{O}}, c_{\alpha,k} \rho]) \right) \right),
\]

Without loss of generality, in the rest of this section, we assume momentum independent dissipation rates and we label its diagonal components by \( \Gamma_{\alpha} \). The terms obtained from expanding the right-hand side are similar to the terms obtained in the Bosonic case. The final result of this expansion reads,

\[
\begin{align*}
\partial_t n_{\nu,v,k}^Q &= -i \left( V_{x,k}^\nu - i V_{y,k}^\nu \right) \sigma_{k}^{\nu*} + i \left( V_{x,k}^\nu + i V_{y,k}^\nu \right) \sigma_k^{\nu} + i \Delta_k^{\nu*} s_k - \Gamma_{\nu}(n_{\nu,k} - n_{\nu,k}^F), \\
\partial_t n_{\nu,v,k}^Q &= i \left( V_{x,k}^\nu - i V_{y,k}^\nu \right) \sigma_{k}^{\nu*} - i \left( V_{x,k}^\nu + i V_{y,k}^\nu \right) \sigma_k^{\nu} + i \Delta_k^{\nu*} s_k - \Gamma_{\nu}(n_{\nu,k} - n_{\nu,k}^F), \\
\partial_t \sigma_k^\nu &= i(\epsilon_{c,k} - \epsilon_{v,k} - \omega) \sigma_k^\nu - i \left( V_{x,k}^\nu - i V_{y,k}^\nu \right) \left( n_{\nu,k} - n_{\nu,k}^F \right) - \frac{1}{2} (\Gamma_{\nu} + \Gamma_{v}) \sigma_k^\nu, \\
\partial_t s_k^\nu &= -i (\epsilon_{c,k} + \epsilon_{v,k}) s_k^\nu - i \Delta_k^{\nu} \left( n_{\nu,k} + n_{\nu,-k} - 1 \right) - \frac{1}{2} (\Gamma_{\nu} + \Gamma_{v}) s_k^\nu,
\end{align*}
\]

where as before we have ignored the terms which are relevant in exciton formation. Next, we derive the steady state solution by assuming constant densities and pairing amplitudes in the rotating frame. We start by obtaining the equations for the anomalous pairing,

\[
s_k^\nu = -\frac{\Delta_k^{\nu} \left( 1 - n_{\nu,k} - n_{\nu,-k} - 1 \right)}{\epsilon_{c,k} - \epsilon_{v,k} - \frac{1}{2} \Gamma_{v}}
\]

where we have defined \( \epsilon_{c,k} = \epsilon_{c,k} + \epsilon_{v,k} \), and \( \Gamma_{v} = \Gamma_{c} + \Gamma_{v} \). In the next step, we consider the EOM for the polarization \( \sigma_k \),

\[
\sigma_k^\nu = \frac{\left( V_{x,k} - i V_{y,k} \right) \left( n_{\nu,k} - n_{\nu,k}^F \right)}{\epsilon_{c,k} + \frac{1}{2} \Gamma_{v}}.
\]

where we have defined \( \epsilon_{c,k} = \epsilon_{c,k} - \epsilon_{v,k} - \omega \). Inserting these two equations in the occupation probabilities we get,

\[
\begin{align*}
0 &= \zeta_k^\nu \left( n_{\nu,k} - n_{\nu,k}^F \right) + \delta_k^\nu \left( 1 - n_{\nu,k} - n_{\nu,-k} \right) - \gamma_{\nu} \left( n_{\nu,k} - n_{\nu,k}^F \right), \\
0 &= -\delta_k^\nu \left( n_{\nu,k} - n_{\nu,k}^F \right) + \delta_k^\nu \left( 1 - n_{\nu,-k} - n_{\nu,k} \right) - \gamma_{\nu} \left( n_{\nu,k} - n_{\nu,k}^F \right).
\end{align*}
\]
where the effective Rabi frequency and the effective pairing amplitudes are,
\[
\zeta_k = \frac{\Omega_{\eta, k}^2 + \Omega_{\eta, k}^2}{(c_{d, k}^2 + \frac{1}{4} \Gamma_l^2)}
\]
\[
\delta_k = \frac{|\Delta_k|^2}{(c_{d, k}^2 + \frac{1}{4} \Gamma_l^2)}.
\]

As in the bosonic case, we need to solve four equations simultaneously,
\[
\begin{align*}
    & (\zeta_k + \delta_k + \gamma_v) (n_{v, k}^{(+)} - \frac{1}{2}) - \zeta_k (n_{c, k}^{(+)} - \frac{1}{2}) + \delta_k (n_{c, k}^{(-)} - \frac{1}{2}) = \gamma_v (n_{v, k}^{(+)} - \frac{1}{2}), \\
    & (\zeta_k + \delta_k^{(+)} - \gamma_v) (n_{v, k}^{(+)} - \frac{1}{2}) - \zeta_k (n_{c, k}^{(+)} - \frac{1}{2}) + \delta_k (n_{c, k}^{(-)} - \frac{1}{2}) = \gamma_c (n_{v, k}^{(+)} - \frac{1}{2}), \\
    & (\zeta_k + \delta_k^{(-)} + \gamma_v) (n_{v, k}^{(-)} - \frac{1}{2}) - \zeta_k (n_{c, k}^{(-)} - \frac{1}{2}) + \delta_k (n_{c, k}^{(+)} - \frac{1}{2}) = \gamma_v (n_{v, k}^{(-)} - \frac{1}{2}), \\
    & (\zeta_k + \delta_k^{(+)} + \gamma_v) (n_{c, k}^{(+)} - \frac{1}{2}) - \zeta_k (n_{c, k}^{(+)} - \frac{1}{2}) + \delta_k (n_{c, k}^{(-)} - \frac{1}{2}) = \gamma_c (n_{c, k}^{(+)} - \frac{1}{2}).
\end{align*}
\]

where \( \Gamma_l = \Gamma_v + \Gamma_c, \gamma_{\alpha} = \Gamma_{\alpha}/\Gamma \). We can rewrite these equations in a matrix form,
\[
\begin{pmatrix}
    \zeta_k^{(+)} + \delta_k^{(+)} + \gamma_v & -\zeta_k^{(+)} & 0 & \delta_k^{(+)} \\
    -\zeta_k^{(+)} & \zeta_k^{(+)} + \delta_k^{(+)} + \gamma_v & 0 & -\zeta_k^{(-)} \\
    0 & \delta_k^{(-)} & \zeta_k^{(-)} + \delta_k^{(-)} + \gamma_v & 0 \\
    \delta_k^{(+) -} & 0 & -\zeta_k^{(-)} & \zeta_k^{(-)} + \delta_k^{(-)} + \gamma_v
\end{pmatrix}
\begin{pmatrix}
    n_{v, k}^{(+)} - \frac{1}{2} \\
    n_{v, k}^{(+)} - \frac{1}{2} \\
    n_{c, k}^{(-)} - \frac{1}{2} \\
    n_{c, k}^{(-)} - \frac{1}{2}
\end{pmatrix}
= \begin{pmatrix}
    \gamma_v (n_{v, k}^{(+)} - \frac{1}{2}) \\
    \gamma_c (n_{v, k}^{(+)} - \frac{1}{2}) \\
    \gamma_v (n_{v, k}^{(-)} - \frac{1}{2}) \\
    \gamma_c (n_{c, k}^{(+)} - \frac{1}{2})
\end{pmatrix}.
\]

In general one needs to invert the matrix on the left to find the solutions for the occupation probabilities. As the first step, we consider the linearized gap equation where we only consider the solutions of the above equation in the zeroth order of \( \Delta_k \). Furthermore, we consider the zero-temperature limit where \( n_{F, v/c} = 0 \) where \( \Delta_{v-k}^{(-)} = 0 \) for \( k \) around the \( K' \) Dirac cone. This yields,
\[
\begin{align*}
    & n_{v, k}^{(+) +} + n_{v, k}^{(-)} - 1 = \frac{\gamma_v \gamma_c (\zeta_k^{(+)} - \gamma_v \zeta_k^{(-)}) + (\gamma_c^2 - \gamma_v^2) \zeta_k^{(-)} + \gamma_v \gamma_c}{(\gamma_v \gamma_c + \gamma_v \gamma_c + \gamma_v \gamma_c)} + O(\Delta^2). \\
    & n_{v, k}^{(-) +} + n_{v, k}^{(-)} - 1 = \frac{\gamma_v \gamma_c (\zeta_k^{(-)} - \gamma_v \zeta_k^{(+)}) + (\gamma_c^2 - \gamma_v^2) \zeta_k^{(+)} + \gamma_v \gamma_c}{(\gamma_v \gamma_c + \gamma_v \gamma_c + \gamma_v \gamma_c)} + O(\Delta^2).
\end{align*}
\]

We can further simplify these relations in the limit that the Rabi frequency around the \( K' \) point is negligible,
\[
\begin{align*}
    & n_{v, k}^{(+) +} + n_{v, k}^{(-)} - 1 \approx \frac{-\gamma_v \zeta_k^{(+)} + \zeta_k^{(-)}}{(\gamma_v + \gamma_c) \zeta_k^{(+)} + \gamma_v \gamma_c} + \frac{\zeta_k^{(-)}}{\gamma_v}, \\
    & n_{v, k}^{(-) +} + n_{v, k}^{(-)} - 1 \approx \frac{-\gamma_v \zeta_k^{(-)} + \zeta_k^{(+)}}{(\gamma_v + \gamma_c) \zeta_k^{(-)} + \gamma_v \gamma_c} - \frac{\zeta_k^{(-)}}{\gamma_v}.
\end{align*}
\]

The above relations can be employed for the interband pairing which can be used to derive the gap equation. To perform this task we need to write the self-consistency definition of mean field order parameter. The result of this calculation yields,
\[
\Delta_k^\eta = -\sum_{k'} \tilde{U}_{kk'} \frac{\bar{c}_{d, k'} \bar{c}_{d, k'}}{\bar{c}_{d, k'}^2 + \frac{1}{4} \bar{\Gamma}_l^2} n_{sc, k}^\eta \Delta_k^\eta,
\]

where we have used the definition \( n_{sc, k}^\eta = 1 - n_{v, k}^\eta k - n_{c, k}^\eta k \). As in the bosonic bath case, we can see that this equation can be only satisfied around one of the valleys, which for our choice of the laser’s polarization will be the \( K \) valley.
Therefore, we can drop the valley index and rewrite this equation as,

$$\Delta_k = -\sum_{k'} \tilde{U}_{kk'} \frac{\epsilon_{t,k'}}{\epsilon_{t,k} + \Gamma_{t}^{2}} \left( \frac{\gamma_{c} \epsilon_{k}'^{(+)}}{(\gamma_{v} + \gamma_{c})\zeta_{k}^{(+)} + \gamma_{c} \gamma_{v}} - \frac{\zeta_{k}^{(-)}}{\gamma_{c}} \right) \Delta_{k'}.$$  (S57)

Since the only difference of this gap equation and the gap equation in the bosonic bath case is in the effective value of $n_{\nu,c,k}$, we can use the same ansatz for the pairing amplitude as before,

$$\Delta_{k}^{(l)*} = e^{-it\phi_{k}} f_{k}^{(l)} \Delta_{k}^{(l)}.$$  (S58)

Using this ansatz, we can evaluate the critical value of the coupling constant $g$ numerically. After employing the same integration method, we obtain a similar behavior for $g_{\text{crit}}$ as a function of the frequency of the pump, and we observe that a transition from a $s$-wave SC pairing to a $p$-wave pairing is possible.

### ROTATING WAVE TRANSFORMATION

The equations of motion (EOM) in our derivation are solved in the rotating frame. Here, we mention how we apply the required rotation. We first consider a generic traceless two-by-two Hamiltonian $h_{k} = d_{k} \cdot \tau$ where $\tau_i$ with $i = \{x, y, z\}$ are the Pauli matrices. The eigenstates of this Hamiltonian up to some phase factors are given by,

$$|u_{c,k}\rangle = \frac{1}{(2d_{k}(d_{k} + d_{z,k}))^{1/2}} \begin{pmatrix} d_{k} + d_{z,k} \\ d_{z,k} \end{pmatrix}, \quad |u_{v,k}\rangle = \frac{1}{(2d_{k}(d_{k} + d_{z,k}))^{1/2}} \begin{pmatrix} d_{-k} \\ -(d_{k} + d_{z,k}) \end{pmatrix},$$  (S59)

where $d_{\pm,k} = d_{x,k} \pm id_{y,k}$.

To apply the rotating wave approximation to a non-diagonal Hamiltonian, we need to first transform the Hamiltonian into the energy basis where it is diagonal and then apply a time-dependent rotation to the two energy levels so that the time dependence of the two transformed eigenstates becomes approximately the same. The first transformation is done through a similarity transformation by the unitary matrix $U_{k} = (|u_{v,k}\rangle \quad |u_{c,k}\rangle)$, where we have used the eigenstates of the undriven Hamiltonian, and the second transformation is realized by the diagonal time-dependent transformation $\text{diag}(e^{i\omega t/2}, e^{-i\omega t/2})$. The combination of these two transformation is $R_{k}(t) = (|u_{v,k}\rangle e^{i\omega t/2} \quad |u_{c,k}\rangle e^{-i\omega t/2})$. Therefore, denoting the electronic spinors in the lab frame and the rotating frame via $\Psi_{k}^{T} = (c_{v,k}, c_{c,k})$, and $\tilde{\Psi}_{k}^{T} = (\tilde{c}_{v,k}, \tilde{c}_{c,k})$, respectively, we have $\Psi_{k} = R_{k}(t) \tilde{\Psi}_{k}$. We apply this transformation to all of the terms in the Hamiltonian system. While the undriven Hamiltonian is trivially diagonalized, the pump Hamiltonian should be obtained after averaging over time. To evaluate the temporal average of the drive term it would be convenient to decompose the time dependent terms as,

$$\Omega(t) = \Omega_{c} \cos(\omega t) + \Omega_{s} \sin(\omega t).$$  (S60)

where $\Omega_{c} = eA_{0}v(1, 0, -2\frac{\pi}{v}k_{x})$ and $\Omega_{s} = eA_{0}v(0, \eta, -2\frac{\pi}{v}k_{y})$. Correspondingly, the expression that must be averaged over time is $R_{k}^{T}(t) \Omega(t) R_{k}(t)$.

Similarly, we need to transform electron-electron interaction term by rotating the electronic creation and annihilation operators and averaging over time. The final result of this calculation, in addition to the right-hand side of Eq. (10), has other contributions which include the overlap of the valence and conduction wave functions at close momenta which makes such terms negligible.

Here, we mention that in order to integrate the gap equation, we consider an energy cutoff with respect to the resonance surface. The resonance ring in the BZ is defined by $\omega = 2d(k_{\text{res}})$ demanding that,

$$k_{\text{res}} = \frac{1}{2} \sqrt{\frac{\omega^{2} - 4m^{2}}{v^{2} - 2mK}},$$  (S61)

where $k_{\text{res}}$ is the radius of the resonance ring. We can also use the above equation to define the integral bounds of the radial momentum through the UV energy cutoff $E_{\Lambda}$ as follows,

$$k_{\Lambda}^{(\pm)} = \frac{1}{2} \sqrt{\frac{(\omega \pm E_{\Lambda})^{2} - 4m^{2}}{v^{2} - 2mK}}.$$  (S62)
Furthermore, we note that we have used this relation in the main text to define the form factors as a function of the frequency $f^{(i)}(\omega)$. In particular, to determine the frequency where a transition from the $s$-wave SC to $p$-wave SC occurs, we should satisfy,

$$f^{(0)}(k_{\text{res}}(\omega)) = f^{(1)}(k_{\text{res}}(\omega)),$$

(S63)

where $f^{(0)}_k = (1 + d_{z,k}/d_k)/2$, $f^{(1)}_k = v k/d_k$. From here, we can observe that in order to satisfy this condition, it is desirable to have a positive band curvature $\kappa$ so that $d_{z,k}$ and correspondingly $f^{(0)}_k$ would decrease with the momentum. Therefore, since $f^{(1)}_k$ increases monotonically with the momentum, this condition can be satisfied with smaller values of the momentum deviation from the valley center.