The properties of long-lived ions appearing in plasma diode in a course of nonlinear oscillations

L A Bakaleinikov, V I Kuznetsov and E Yu Flegontova

Ioffe Institute, 26 Politekhnicheskaya, St Petersburg, 194021, Russia

E-mail: bakal51@mail.ru

Abstract. Nonlinear oscillations may occur in a plasma diode when the ions are reflected from a potential barrier. It was found that long-lived ions are formed in the course of the oscillations of the potential. Based on the proposed analytical model of the oscillations of the potential the reason for the appearance of long-lived ions is elucidated and the main features of their distribution function are revealed. The analytical results are in good agreement with the results of simulation.

1. Introduction

The nonlinear oscillations of the electron virtual electrode (e-VE) develop in a vacuum diode with a monoenergetic electron flow when the current density exceeds the threshold value (the Bursian threshold). This phenomenon is widely used in the creation of powerful microwave generators (vircators). It was found that long-lived electrons appear in the interelectrode gap in the course of the oscillations of the electron virtual electrode – these are particles that approach the oscillating potential barrier at velocities close to zero, oscillate in the vicinity of e-VE for several periods of barrier oscillations, and then go to one of the electrodes [1]. This phenomenon was studied in detail in [2], where the reason for the existence of long-lived electrons was also clarified.

In [3], when studying a plasma diode with counter-streaming electron and ion flows, solutions exhibited nonlinear oscillations were found in the mode when there is an ion virtual electrode (i-VE). It was also found that the presence of the oscillating potential barrier for ions leads to a rather complex structure of the ion distribution function (DF) on the coordinate–velocity phase plane.

2. Statement of the problem and calculation results

The steady-state solutions in a plasma diode of plane geometry with counter-streaming electron and ion beams were studied in [4]. It was found that for given values of the interelectrode gap $L$ and the potential difference $U$ between the electrodes the problem of finding the potential distribution (PD) in a diode can have several stationary solutions. Among them, there was a class of solutions characterized by a PD with i-VE, which reflects part of the ions to the emitting electrode. The stability features of such solutions were studied numerically in [3]. The problem was set as follows.

It was assumed that a monoenergetic flow of electrons with velocity $v_{e,0}$ and density $n_{e,0}$ entered the interelectrode gap from the left electrode, and the ions were emitted from the right electrode with an average velocity $v_{i,0}$, density $n_{i,0}$ and DF in the form of a “gate”, i.e. DF is constant and not equal
to zero in narrow velocity interval in the vicinity of $v_{i,0}$. Particles move in the interelectrode gap collisionlessly in a self-consistent electric field. When a particle reaches any electrode it is absorbed.

When calculating the time-dependent process, it was taken into account that the electrons traveled a distance equal to the length of the interelectrode gap in less time than the ions moved a distance of the order of the Debye length. Therefore, the time step of the order of the time being required to ions to cover the Debye length was chosen. At each step, it was assumed that the distribution of electrons “instantly” adjusts to the PD that existed at this moment. As a result a time-dependent collisionless kinetic equation was solved for the ions by use of the ion time scale and the distribution of electrons at each moment was found from the stationary kinetic equation. When calculating ion DF a modified high-precision $E,K$-code [2], [5] was used.

Calculations were carried out in dimensionless units. The electron energy $W_{e,0} \equiv m_n v_{e,0}^2 / 2$ and Debye length at the left electrode $\lambda_D = (2\varepsilon_0 W_{e,0} / e^2 n_{e,0})^{1/2}$, where $e$ is the electron charge, $\varepsilon_0$ is the permittivity of free space, were chosen as the units of energy and length. The particle density was measured in units $n_{e,0}$, the electron velocity in units $v_{e,0}$ and the ion velocity in $v_{i,0} = v_{e,0} \sqrt{m/M}$ (here $m$ and $M$ are the masses of the electron and ion). In dimensionless variables the “gate” ion DF at the right electrode has the form

$$f_i(\delta, u) = \delta (\delta^2 - (u + 1)^2) / (2\Delta), \quad u < 0.$$  

(1)

Here $u$ and $\delta$ are the dimensionless ion velocity and distance between the electrodes, the gate width being $2\Delta \ll 1$.

During the simulations, it was found that nonlinear periodic oscillations develop in a diode with an interelectrode distance $\delta = L / \lambda_D = 3.52$ and a potential difference between the electrodes $V = eU / (2W_{e,0}) = 0.1$. In the course of the oscillation process, the maximum of the PD was either higher or lower than the value of the potential, at which the ions are reflected in the steady state corresponding to the considered values of the parameters $\delta$, $V$. In this case, the distribution of ions in the region between the oscillating i-VE and the right electrode is highly inhomogeneous. The density distribution had a sharp maximum, the position of which was in the vicinity of the maximum of the potential. During the oscillation process, the value and position of the ion density maximum oscillated with the frequency of field oscillations.

**Figure 1.** Dependence of the maximum potential value on its position during oscillation period calculated numerically (dashed curve) and found for the model potential (2) (solid curve).

**Figure 2.** Boundaries of the region of non-zero values of ion DF on the phase plane $(\zeta, u)$ at the moment $\tau = 15.8$ in the diode with $\delta = 3.52$, $V = 0.1$ and $\Delta = 0.01$. 

2
The dependence of the maximum potential value on its position during two periods of steady oscillations is represented in figure 1. It can be seen that the dependence forms a closed curve on the plane \((\zeta_M, \eta_M)\). It turned out that the presence of oscillating ion potential barrier leads to a rather complex structure of ion DF (figure 2). The analysis of this structure shows that along with the usual regions corresponding to particles that have overcome the potential barrier and reached the opposite electrode, or to particles that have reflected from the barrier and returned to the ion-emitting electrode, new regions appear in the process of oscillations. The number of these areas increases over time, and their width and distance between them decrease. This structure of the phase plane is very similar to the structure of the phase plane of long-lived electrons, which were studied in [2]. We assume that this structure is due to the existence of long-lived ions in the vicinity of the barrier. These ions approach the oscillating barrier at velocities close to zero, oscillate in the vicinity of the potential maximum for several periods of its oscillations, and then leave for one of the electrodes.

3. Long-lived ions in model potential

To investigate properties of the long-lived ions we use an analytical model of potential evolution as in [2] assuming that in the vicinity of the maximum the potential is a quadratic parabola, the value and position of which oscillate with a small amplitude \(\kappa\)

\[
\eta(\zeta, \tau) = \alpha^2 \left[ (\delta_a - \zeta_M^0)^2 - (\zeta - \zeta_M^0)^2 \right] / 2 + \alpha^2 \zeta_M^0 (\zeta - \delta_a) \kappa \cos(\Omega \tau) + V_a. \tag{2}
\]

Parameters \(\Omega\) and \(\kappa\) characterize the frequency and amplitude of i-VE oscillations. The parameters \(\delta_a, V_a\) and associated with them \(\alpha^2 = 2(\eta_M^0 - V_a)/(\delta_a - \zeta_M^0)^2\) are fitting ones, \(\eta_M^0\) and \(\zeta_M^0\) being the potential at the i-VE top and its position in the absence of oscillations (\(\kappa = 0\)). Note that this approximation is not valid for the entire gap, but it can be used to effectively study the ion behavior in the vicinity of the potential maximum. It follows from (2) that the time dependences of the position of PD maximum and its value are determined by the formulas

\[
\zeta_M(\tau) = \zeta_M^0 (1 + \kappa \cos(\Omega \tau)), \quad \eta_M(\tau) = \alpha^2 / 2 (\delta_a - \zeta_M^0)^2 + V_a = (\eta_M^0 - V_a)(\delta_a - \zeta_M^0)^2 + V_a. \tag{3}
\]

The dependence (3) represents a portion of a parabola lying within a rectangular area on the plane \((\zeta_M, \eta_M)\)

\[
\zeta_M^0 (1 - \kappa) \leq \zeta_M(\tau) \leq \zeta_M^0 (1 + \kappa), \quad (\eta_M^0 - V_a - \zeta_M^0)^2 + V_a \leq \eta_M \leq (\eta_M^0 - V_a - 1 + 2\zeta_M^0 \kappa \delta_a - \zeta_M^0)^2 + V_a.
\]

For the case studied numerically: \(\delta = 3.52, V = 0.1\), the following values of the fitting parameters were selected: \(\Omega = 2.04, \zeta_M^0 = 3.033, \eta_M^0 = 0.586, \kappa = 0.051, \delta_a = 3.484, V_a = 0.341, \alpha = 1.229\). The dependence (3) for these parameters is shown as a solid line in figure 1.

The trajectory of an ion leaving the collector at a time \(\tau_0\) with velocity \(u_0\) is determined by the equation

\[
d^2 \zeta / d\tau^2 = \alpha^2 (\zeta - \zeta_M^0) - \alpha^2 \zeta_M^0 \kappa \cos(\Omega \tau)
\]

with initial conditions

\[
\zeta(\tau_0) = \delta, \quad \zeta'(\tau_0) = u_0
\]

and has the form

\[
\zeta(\tau) = A_+ \exp(\alpha(\tau - \tau_0)) + A_- \exp(-\alpha(\tau - \tau_0)) + \zeta_M^0 (1 + B \kappa \cos(\Omega \tau)), \tag{4}
\]

\[
A_+ = |\delta - \zeta_M^0| u_0 / \alpha - B \zeta_M^0 \kappa \cos(\Omega \tau_0) \pm \arctan(\varphi) / 2, \quad B = 1 / (1 + \varphi^2), \quad \varphi = \Omega / \alpha. \tag{5}
\]

For the ion velocity \(u\) at a time \(\tau\) we get

\[
u(\tau) / \alpha = A_+ \exp(\alpha(\tau - \tau_0)) - A_- \exp(-\alpha(\tau - \tau_0)) - \zeta_M^0 B \Omega \kappa \sin(\Omega \tau). \tag{6}
\]
Let’s analyze the behavior of the ion at \( \alpha (\tau - \tau_0) \gg 1 \). At \( A_+ \leq 0 \) ion overcomes the potential barrier and reaches the left electrode. At \( A_+ > 0 \) it is reflected from the potential barrier and returns to the right electrode. In the case \( A_+ \approx 0 \) ion can oscillate for quite a long time near the potential maximum, until it joins either the first or the second group. This is just the long-lived ion. The departure parameters \( u_0 \) and \( \tau_0 \) of this ion lie in a narrow neighborhood of the curve

\[
u_0 = -\alpha \left\{ \delta - \phi_0 - \sqrt{B} \zeta_0 \kappa \cos[\Omega \tau_0 + \arctan(\phi)] \right\},
\]

which meets the case \( A_+ = 0 \). As we can see from (7), the interval of maximum change in the departure rates of long-lived ions \( \Delta u_0 \) is limited by the magnitude \( B \zeta_0 \kappa \). As the amplitude of the oscillation of the PD maximum \( \kappa \) increases, the value of the interval \( \Delta u_0 \) increases, but it cannot exceed \( 2 \Delta \). As we can see from (4), long-lived ions fluctuate for several periods in the vicinity of the potential maximum according to the law

\[
\zeta(\tau) = \phi_0 \left( 1 + B \kappa \cos(\Omega \tau) \right),
\]

that is, synchronously with the field oscillations. The amplitude of their oscillations is \( 1 + \varphi^2 \) time less than the amplitude of the i-VE ones. Since the ion velocity is less than the i-VE velocity and the ion and i-VE oscillation phases coincide, the oscillating potential barrier generates a "dynamic" potential trap for the long-lived ions, in which they oscillate.

Thus, the long-lived ions are caused by the fact that a portion of the particles emitted by the right electrode, when reaching the vicinity of i-VE, has a kinetic energy close to zero. Ions bounce off the barrier, but because they are moving slower than i-VE, the potential barrier is again in their path. The ions are again reflected from it, etc.

The presence of an exponentially growing over time term in formulae (4) leads to the fact that the ion trajectories are unstable relative to small perturbations of the initial conditions. Therefore, no matter how close to the curve (7) the point \((u_0, \tau_0)\) is, i.e. no matter how small the coefficient \( A_+ \) is, in a sufficiently long time the exponentially growing term will dominate over the others, and the ion will go to one of the electrodes. For this reason, any ion "lives" in the field (2) for a finite time.

Since long-lived ions oscillate in the vicinity of the PD maximum, groups of ions that have made different numbers of oscillations can be located at the same point of the gap. The model of potential (2) allows us to estimate the width of the velocity interval for each group at given point. Indeed, according to (8), long-lived ions with the same velocity can leave the collector at moments of time

\[
\tau_{0,s}^i = \left( \arccos(\left( u_0 + a \delta - a_0 \delta_0 \right) / \sqrt{B} \zeta_0 \kappa) - \arctan(\varphi + 2 \pi) / \Omega \right),
\]

\[
\tau_{0,s}^{-i} = \left( - \arccos(\left( u_0 + a \delta - a_0 \delta_0 \right) / \sqrt{B} \zeta_0 \kappa) - \arctan(\varphi + 2 \pi) / \Omega \right),
\]

\[
0 < \tau_{0,s}^i < \tau, \quad 0 < \tau_{0,s}^{-i} < \tau, \quad \alpha \left( \tau - \tau_{0,s}^i \right) > 1, \quad \alpha \left( \tau - \tau_{0,s}^{-i} \right) > 1.
\]

The interval of values \( \tau_{0,s}^i, \tau_{0,s}^{-i} \) for each group is determined by the interval \( 2 \Delta \) of the velocity \( u_0 \) values, and is small when inequalities \( \sqrt{B} \zeta_0 \kappa \gg \Delta, \Delta \ll 1 \) are true. It is easy to see from the relations (4), (6) that the arrival velocity at a fixed point \( \zeta \) at a time \( \tau \) is determined by the equality

\[
u(\tau) = \zeta - 2 A \exp\left( - \alpha \left( \tau - \tau_0 \right) \right) - \phi_0 B \Omega \kappa \sin(\Omega \tau) + \phi_0 (1 + B \kappa \cos(\Omega \tau)).
\]

Then for the velocity range \( \Delta u_i^s \) of the group of ions emitted at a time \( \tau_{0,s}^i \), we have

\[
\Delta u_i^s = 2 \alpha \Delta u \exp\left( - \alpha \left( \tau - \tau_{0,s}^i \right) \right).
\]

As a result of (9), (10), the spread \( \Delta A_i \) of coefficient \( A_0 \) values is determined only by the interval of the change in the velocity \( u_0 \) and does not depend on the departure time. Then for the ratio of interval widths we find

\[
\Delta u_i^s / \Delta u_i^{s+1} \approx \exp\left( \alpha \left( \tau_{0,s}^i - \tau_{0,s}^{i+1} \right) \right) = \exp\left( - 2 \pi \alpha / \Omega \right).
\]
With the chosen parameters of the approximation the ratio \( \frac{\Delta u_i^s}{\Delta u_i^{s+1}} \) is equal to 0.014. Thus, the analysis shows that the number of groups of long-lived ions increases, but remains finite with increasing time \( \tau \). In addition, when the travel time of the ion in the gap (i.e., the value \( \tau - \tau_0^{s+1} \)) increases, the width of the velocity intervals of these groups decreases.

Let us compare the value \( \frac{\Delta u_i^s}{\Delta u_i^{s+1}} \) with that for the ion DF simulation. To calculate the DF of ions arrived in the point \( \varsigma \) at the time moment \( \tau \) with velocity \( u \), the ion trajectories with initial conditions \( u(\tau) = u, \; \varsigma(\tau) = \varsigma \) were calculated back in time up to the intersection with the right electrode. When the departure velocity \( u_0(\varsigma_0) \) is in the interval \( [1 - \Delta, 1 + \Delta] \), the ion with velocity \( u \) contributes to the DF. As a result, there are a large number of narrow areas (zones) in the area located between i-VE and the right electrode on the phase plane. The position of the borders and the ratio of the widths of neighboring zones at \( \varsigma = 3.344 \) are shown in table 1. It can be seen that the ratios of velocity interval widths obtained in numerical calculations and calculated using the formula (11) corresponding to the potential (2) are in good agreement with each other.

Table 1. Parameters of arrival velocity intervals for the point \( \varsigma = 3.344 \) at the time moment \( \tau = 15.8 \).

| \( N_i \) zone number | Lower boundary \( u_0 \) | Upper boundary \( u_0 \) | The interval width \( \Delta \) | \( \frac{N_1}{N_2} \) | \( \Delta N_i / \Delta N_2 \) | \( \tau_0 \) |
|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| 1                      | -0.73658               | -0.70882               | 0.02776                | 4/2                    | 0.010                  | 15.614                  |
| 2                      | 0.47060                | 0.49186                | 0.02126                | 5/3                    | 0.010                  | 14.277                  |
| 3                      | 0.532353               | 0.536819               | 4.466 \cdot 10^{-4}   | 6/4                    | 0.014                  | 12.632                  |
| 4                      | 0.5400071              | 0.5402256              | 2.185 \cdot 10^{-4}   | 7/5                    | 0.014                  | 9.445                   |
| 5                      | 0.5410769              | 0.5411215              | 4.478 \cdot 10^{-5}   | 8/6                    | 0.013                  | 8.567                   |
| 6                      | 0.5411573              | 0.5411604              | 3.098 \cdot 10^{-6}   | 9/7                    | 0.018                  | 6.349                   |
| 7                      | 0.54117273             | 0.54117337             | 6.427 \cdot 10^{-7}   | 10/8                   | 0.013                  | 5.465                   |
| 8                      | 0.541173879            | 0.541173917            | 3.864 \cdot 10^{-8}   | 11/9                   | 0.013                  | 3.065                   |

4. Conclusion

The characteristics of the long-lived ions detected during calculations described in [3] are investigated. The reason for the appearance of such ions was found out. It is related to the fact that the ions actually fall into a dynamic potential trap. The finiteness of the lifetime of such ions is explained by the instability of their trajectories. It is shown that long-lived ions on the phase plane correspond to zones whose width, as well as the distance between them, decreases with the growth of the length of the ion trajectories. An analytical model is constructed that well describes the main characteristics of long-lived ions.

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