Detecting large extra dimensions with optomechanical levitated sensors

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Numbers of tabletop experiments have made efforts to detect large extra dimensions for the range from solar system to submillimeter system, but the direct evidence is still lacking. Here we present a scheme to test the gravitational law in 4+2 dimensions at microns by using cavity optomechanical method. We have investigated the probe spectrum for coupled quantum levitated oscillators in optical cavities. The results show that the spectral splitting can be obtained once the large extra dimensions present. Compare to the previous experiment, the sensitivity can be improved by the using of a specific geometry and a shield mirror to control and suppress the effect of the Casimir background. The weak frequency splitting can be optically read by the pump-probe scheme. Thus we can detect the gravitational deviation in the bulk based ADD model via spectroscopy without the isoelectronic technique.

I. INTRODUCTION

Why gravity is so weak compared to the other known forces in nature? This can be recast by the hierarchy problem, the seeming disparity (16 orders of magnitude) between the Planck mass and standard model electroweak scale. This problem can be overcome by adding new dimensions in the large extra dimension (bulk) model, namely the ADD model, which was first developed by Arkani-Hamed, Dimopoulos, and Dvali[1-3]. They proposed that for \( n = 2 \), the extra dimensions could be as large as a millimeter and the measurements of gravity may observe the transition from \( 1/r^2 \) to \( 1/r^4 \) Newtonian gravitation. In addition to this possibility, they found that their model could describe the hierarchy between the Planck mass and the electroweak symmetry breaking scale in terms of the large size of the extra dimensions. Therefore testing general relativity and its Newtonian limit at short distances has become particularly important in light of recent theoretical developments[4-13].

On the other hand, significant advances have been witnessed in studying the characteristic and application of the cavity optomechanical system with high-Q mechanical nanoresonators (NRs)[14]. Nano- and micromechanical devices can be coupled to optical cavities directly via radiation pressure leading to a variety of important properties, such as optical self-focusing[15], optomechanical entanglement[16] and optomechanically induced transparency (OMIT)[17,18]. Recently much effort has been directed toward optically levitating nanoscale mechanical oscillators in ultrahigh vacuum such as nanospheres[19,20], nanodiamonds[21,22], microdisk[23,24], and even the living organisms[25]. The laser trapped objects has no physical contact to the environment, leads to ultralow mechanical damping. In the absence of other noise sources, the quality factors of optical levitated nano-optomechanics can exceed \( 10^{12} \) with pressures of \( 10^{-10} \) Torr[19]. Owing to its exceptional mechanical properties, optically levitation appears to be an excellent candidate for the NRs in nanoelectromechanical systems (NEMS), which are of great interest for applications in fundamental science and precision measurement[26-28].

In the present work, by combining the cavity optomechanics and the ADD model, we investigate the optomechanical system consisted of coupled quantum levitated oscillators and cavities. Then we propose a design for non-Newtonian gravity detection at short range with the levitated sensors. The results show that the sharp enhanced transparency peaks with ultra-narrow linewidth can be induced through the coupling between the optical cavity and the levitated resonator. The contribution of extra dimensions will be able to show itself clearly on the probe spectrum by spectral splitting, performing a test of Newton’s law at microns. We also consider the Casimir effect as the main background force noise in the micro-scale optomechanical system. The constraints on the compactification distances are deduced as last, the super-resolution can be achieved by using the oscillators with high Q factor in vacuum. We expect that the proposed scheme could be applied to probe the large extra dimensions or set a new upper limit on the hypothetical long-range interactions which naturally arise in many extensions to the standard model[29,30].

II. THEORY FRAMEWORK

A schematic of our setup is sketched in Fig.1(a), where a dielectric microdisk and a nanosphere are optically levitated in two optical cavities. The micro- or nano-scale objects is attracted to the anti-node of the field. The
resulting gradient in the optical field provides a sufficiently deep optical potential well which allows the object to be confined in a number of possible trapping sites, with precise localization due to the optical standing wave [25,31,32]. We use a trapping laser to levitate the microdisk in the left cavity and use the other beam to trap the nanosphere in the right cavity. Then the left cavity is driven by a strong pump laser and probed by a weak probe laser. The mechanism can be explained as a quantum coupling induced normal mode splitting (NMS) in the four-wave mixing (FWM) process. The Fig. 1(b) is the energy level description of this process. In the present paper, the trapped microdisk and nanosphere are treated as quantum-mechanical harmonic oscillators and their masses are \(m_1\) and \(m_2\), mechanical frequencies \(\omega_1\) and \(\omega_2\), and damping rates \(\gamma_1\) and \(\gamma_2\), respectively. The Hamiltonian can be regarded as

\[
H_\omega = \sum_{j=1,2} \hbar \omega_j a_j^+ a_j,
\]

where \(a_j^+\) and \(a_j\) are the bosonic creation and annihilation operators for the two vibrational resonators. We use \(H_c = \hbar \omega_c c^+ c\) to describe the Hamiltonian of the left cavity mode, here \(\omega_c\) and \(c(c^+)\) denote the oscillation frequency and the annihilation (creation) operator of the cavity. The radiation pressure of the cavity gives rise to the optomechanical coupling \(H_g = -\hbar g_1 c^+ c (a_1^+ + a_1)\), where \(g_1\) is the single-photon coupling rate between the microdisk and the left cavity, it has the typical value of \(\sim 2\pi \times 1.2Hz\) [31].

Then we consider the gravity interaction in the system. The ADD theory establishes an effective Planck scale to coincide with the electroweak scale by allowing gravity to travel in extra dimensions. It is possible that all of the particles and fields of the standard model are trapped on this brane, providing an explanation for why we have never observed more than our three spatial dimensions. On the other hand, gravity would be able to travel in \((4+n)\) dimensions, where \(n\) is the number of extra dimensions. According to this model, the Planck scale is not fundamental but determined by the volume of the extra dimensions \(M_{Pl}^4 \sim M_{Pl}^{2(n+2)+n} V_n\). By this process in ADD, the hierarchy problem is nullified and a modification of Newtonian gravity is proposed for the ranges smaller than the compactification length. When the separation between the masses decreases to the point where \(r \ll R_s\) (\(R_s\) denotes the size of the extra dimensions), the usual inverse square law of gravity changes to a new power-law [3]

\[
V(r) = -\frac{m_1 m_2}{M_{Pl}^{n+2} r^{n+1}} \frac{1}{r_0^{n+1}}.
\]

The separation can be regarded as \(r = r_0 + x_1 - x_2\), where \(x_{1,2}\) denote the displacements of mechanical oscillators from their equilibrium positions, \(r_0\) denotes the distance between equilibrium positions of the disk and sphere. For equal-size extra dimensions and toroidal compactification, \(V_n = (2\pi R_s)^n\) and the size of the extra dimensions,

\[
R_s = 10^{30/n-17} mm \times \left(\frac{1TeV}{m_{EW}}\right)^{1 + \frac{n}{2}}.
\]

Here the electroweak scale \(m_{EW}\), also known as the Fermi scale, is the energy scale around \(246 GeV\). For \(n = 1, R_s \sim 10^{19} cm\) implying deviations from Newtonian gravity over solar system distances. In the case of \(n = 2, R_s \sim 1 mm\), it is particularly exciting, since it is the scale currently being probed by a number of tabletop gravity experiments. In our scheme the levitated resonators held in trapping potentials are separated by a distance of \(r_0 = 8\mu m\), thus the condition of \(r \ll R_s\) can be physically achieved. Expanding Eq. (1) in the condition of \([x_1], [x_2] \ll r_0\) and working to the lowest order, we can obtain the term of gravitational potential for \(4+n\) dimensions as

\[
V_{int} \approx -G_{4+n} m_1 m_2 \frac{1}{r_0^{n+1}} - \frac{c_{n,1}}{r_0^{n+2}} \frac{(x_1 - x_2)^2}{r_0^{n+1}},
\]

where \(G_{4+n} \sim M_{Pl}^{-(2+n)}\) is the fundamental gravitational constant in the full \(4+n\) dimensional spacetime, \(C_{n,k} = (-n-1)!...(-n-k)!/k!\) is the binomial coefficient. The first term is constant. The second term represents a steady force does not affect the interactional dynamics. The term proportional to \(x_1 x_2\) represents the lowest-order coupling between the resonators’ motions. In the regime of \(\omega_c, \omega_j \gg g_1\), the Hamiltonian of gravitational interaction can be obtained by quantizing mechanical oscillators within rotating wave approximation [33]

\[
H_{int} = C_{n,2} \frac{G_{4+n} m_1 m_2}{r_0^{n+3}} |x_1 x_2| \cong \hbar \beta (a_1^+ a_2 + a_1 a_2^+),
\]

and

\[
\beta (n) = C_{n,2} \frac{G_{4+n} \sqrt{m_1 m_2}}{r_0^{n+3}} \sqrt{\omega_1 \omega_2},
\]

The coefficient \(\beta\) can be defined as the bulk induced coupling rate, which reveals the gravitational strength between two oscillators in \(4+n\) dimensions.

For smaller separations (\(\ll 10^{-4} m\)), Casimir forces provide the dominant background force, and we expect the scheme can set the best limits in this regime. In the original point of view, the Casimir effect is derived from the change of the total energy of vacuum due to the presence of two plane perfect reflectors. In this global approach, the Casimir energy is the part of two plane perfect reflectors. In this global approach, the Casimir energy is the part of two plane perfect reflectors. In this global approach, the Casimir energy is the part of two plane perfect reflectors. The term proportional to \(x_1 x_2\) represents the lowest-order coupling between the resonators’ motions. In the regime of \(\omega_c, \omega_j \gg g_1\), the Hamiltonian of gravitational interaction can be obtained by quantizing mechanical oscillators within rotating wave approximation [33]

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where $\alpha_V = 3\epsilon_0 V(\epsilon_2 - 1)/(\epsilon_2 + 2)$ is the electric polarizability, and $V$ is the volume of the nanosphere, $\epsilon_2$ is the dielectric constant of the nanosphere. Thereby the total interaction Hamiltonian, including Casimir interaction, can be written as $H_{\text{int}} = \hbar(\beta + \beta_{\text{casimir}}) \cdot (a_1^+ a_2 + a_1 a_2^+)$. Based on the discussion above, the whole system can be considered as an optomechanical system where the microdisk's vibrational mode is coupled to a single cavity mode with the coupling rates $g_1$, meanwhile interacts with the other levitated oscillator(nanosphere) through the gravitational strength $\beta$ and the Casimir coupling rate $\beta_{\text{casimir}}$. By applying the pump-probe field, we can obtain the Hamiltonian of the whole system as

$$H = H_c + H_\omega + H_\beta + H_{\text{int}} - i\hbar \Omega_{pu}(c - c^+),$$

$$- i\hbar \Omega_{pr}(ce^{i\delta t} - c^+e^{-i\delta t}).$$

(7)

The pump laser owns the driving amplitude $\Omega_{pu} = \sqrt{2P_{pu}/\kappa/\hbar \omega_{pu}}$ and the frequency $\omega_{pu}$. The probe beam has the driving amplitude $\Omega_{pr} = \sqrt{2P_{pr}/\kappa/\hbar \omega_{pr}}$ with the frequency $\omega_{pr}$, where $P_{pu}(P_{pr})$ is the input power of the pump (probe) field and $\kappa$ is the total decay rate of the left cavity.

### III. FORECASTS

We use a silicon nitride microdisk and a silica nanosphere with density $\rho_1 = 2.7g/cm^3$, $\rho_2 = 2.3g/cm^3$ and the dielectric constant $\epsilon_1 = 4$, $\epsilon_2 = 2$, respectively. The mechanical frequency $\omega_j$ depends on the intracavity intensity, it can be modulated by the power of trapping beam[32]. The cavities are driven with two trapping lasers of wavelength $\lambda = 1.5\mu m$ and power $P_1 = 5.06W$ and $P_2 = 2.87W$ for a disk and sphere, respectively, corresponding to an axial trap frequency of $\omega_1/2\pi = \omega_2/2\pi = 10kHz$.

The quality factor one would get for levitated resonators are solely determined by the air molecule impacts. Random collisions with residual air molecules provide the damping $\gamma_j = \omega_j Q_j^{-1}$ and thus, the quality factor due to the gas dissipation can be defined as $Q_1 = \pi \omega_2 \rho_1 \nu d/32\rho$ for the microdisk and $Q_2 = \pi \omega_2 \rho_2 r_2^2/16\rho$ for the nanosphere[24]. Here $\nu = \sqrt{k_B T/m_{\text{gas}}}$ is the thermal velocity of the gas molecules, $p$ the gas pressure. Let us consider the levitated microdisk with the radius $r_1 = 75\mu m$, thickness $d = 1\mu m$ and the levitated nanosphere with the radius $r_2 = 120nm$. We get $Q_1 = 4.3 \times 10^{11}$ and $Q_2 = 8.8 \times 10^{10}$ for the ultralow pressure $p = 10^{-11} \text{mbar}$, at room temperature ($T = 300K$).

The coupling strength $\beta$ can be obtained from Eq.(5) by setting different $n$. In what follows, we take the pump- cavity detuning $\Delta_{pu} = 0$. Here we use the Heisenberg equation of motion to solve the Hamiltonian of levitation-cavity system. By solving the Heisenberg equation, we can obtain the transmission of the probe beam, $|t|^2$, defined as the ratio of the output and input field amplitudes at the probe frequency[35] (see the supplementary).

Then we depict the transmission $|t|^2$ of the probe beam as a function of the probe-pump detuning $\delta$ in Fig.2. At first we assume $\beta = \beta_{\text{casimir}} = 0$, then we get an enhanced peak which is located at $\delta = \omega_1 = 10kHz$, just corresponds to the fundamental frequency of the levitated microdisk as shown by the black curve. Without the presence of extra dimensions($n = 0$), we can only consider the Casimir-Polder coupling $\beta_{\text{casimir}} = 2.9 \times 10^{-7}Hz$. Then we can find the resonance peak suffers a splitting in the spectrum characterized by the blue curve. By applying the extra dimensions based ADD theory, for $n = 2$, $\beta = 1.4 \times 10^{-6}Hz$. The resonance frequency splitting can be amplified significantly in the spectrum as shown by the red curve. Here the transmitted spectrum of the probe laser can be effectively modulated by the number of extra dimensions. Without any interaction, one can obtain significant transmission of the probe laser at the resonant region. When gravity deviates from $1/r^2$ in 4+2 extra dimensions, considering the Casimir background force, the enhanced peak splits and separates. The linear increase of $L$ with $\beta$ reminds us of the possibility to detect the gravity strength between resonators by measuring the separation in the transmission spectrum. Their relationship can be expressed by $L = 2(\beta + \beta_{\text{casimir}})$, which strongly reveals the deviation from gravitational inverse-square law, namely, a sign indicative of large extra dimensions. The resolution depends on the full width of half maximum(FWHM) of the peak, thus the minimal detectable coupling strength $\beta_{\min} = L_{\min}/2 = \text{FWHM}/2$. Considering the peak

![FIG. 1.](image-url)
FWHM in Fig. 2 approximates $2 \times 10^{-7}$ Hz, one can obtain $\beta_{\text{min}} \approx 0.1 \mu H z$.

The levitated resonators with shorter separation can set better constraints since the gravitational strength rapid rise with decreasing the displacement $r_0$. However the system with smaller $r_0$ will suffer lager Casimir background force noise which goes up in proportion to the 6th power of $r_0$. We depict Fig. 3 to show the coupling-displacement function of the bulk and Casimir interaction. Then we depict the horizontal dash black line to show the minimal detectable coupling rate $\beta_{\text{min}}$ which corresponds to $0.1 \mu Hz$. The solid lines represent the detectable space, the dot lines represent the undetectable region. We can find that a separation of range of $8 \mu m$ is a good compromise between the two limit factors (separation and noise). In these conditions, the Casimir coupling is smaller than the gravitational coupling, $\beta_{\text{Casimir}} \approx \beta/5$, thus we expect the contribution of the bulk will be able to show itself clearly on the probe spectrum, performing a test with low Casimir background noise. Moreover, as shown in Fig. 1(a), the middle mirror between two cavities can be used to minimize the electrostatic and Casimir background forces by preventing direct coupling between the masses. Thus the shield mirror can attenuate this Casimir interaction even further, rendering the effect negligible. This means that we can first fix the trapping frequency of the microdisk at $\omega_1 = 10 kHz$, and adjust the frequency of the nanosphere approach to $\omega_1$ via modulating the trapping power in the right cavity. If the bulk exist, then we can obtain the vibrational mode splitting of the microdisk resonator clearly on the probe spectrum, on the contrary, the resonance peak will fix on the original frequency without the bulk. In the past few years, the new forces measurement in the Casimir regime relies on the isoelectronic technique[5,10,13]. This technique will induce vibration noises between the bimorph and the single-crystal silicon cantilever, the sensitivity is not able to probe forces below the level of aN[5]. Compare to the previous experiment, the sensitivity can be improved by the using of a specific geometry (sphere-plane interaction) to generate the Casimir-Polder potential and the shield mirror to suppress the effects of the Casimir background. Then the pump-probe technology can be used to read the weak frequency splitting. We list all the main optomechanical parameters in the Table 1.

The mechanism underlying these effects can be explained as four-wave mixing (FWM) in a three energy levels system. The simultaneous presence of a pump field and a probe field generates a radiation pressure force at the beat frequency, which drives the motion of the oscillator near its resonance frequency. In Fig. 1(b), we let $|N\rangle$, $|n_1\rangle$ and $|n_2\rangle$ denote the number states of the cavity photon, microdisk phonons, and nanosphere phonons respectively. $|N, n_1, n_2\rangle \leftrightarrow |N+1, n_1, n_2\rangle$ transition changes the cavity field, $|N+1, n_1, n_2\rangle \leftrightarrow |N, n_1+1, n_2\rangle$ transition is caused by the radiation pressure coupling. In the system, the coupling between two resonators adds a fourth level which can be mainly modified by the gravitational interaction. The coupling breaks down the symmetry of the OMIT interference, the single OMIT transparency window is split into two transparency windows, which yields the quantum coupling induced NMS as shown in Fig. 2.

### IV. CONSTRAINTS AND LIMITS

The spectral resolution depends on the full width at half maximum (FWHM) of the oscillation peak. The result shows that smaller linewidth can be achieved by increasing the Q factor of the levitated resonators. Fig. 4 presents the constraints on $R_*$ for the number of extra
dimensions, \( n = 1, 2, 3 \). The sloping lines represent the calculated coupling rate \( \beta \) for different \( n \) and \( R_* \). Then we depict the horizontal dash black line to indicate the limits of detectable coupling strength \( \beta_{\text{min}} \). \( R_* \) and \( \beta \) are constrained to be larger than the values of the intersections in the picture. The solid lines in the figure represent the detectable parameter space, the dot lines represent the undetectable region. The arrow indicates that the sensitivity can be improved through optomechanical oscillators with higher Q factors. Considering the minimum measurable gravitational coupling rate \( \beta_{\text{min}} = 0.1 \mu Hz \), we get the precision for the forces measurement as \( F_{\text{min}} = 2.2 \times 10^{-4} aN \). The Q factor of the optically trapped particles is limited only by collisions with residual air molecules, thus the sensitivity can be improved by decreasing the air pressure. The lower pressure limit of sputter-ion pumps is in the range of \( 10^{-11} \) mbar. Lower pressures in the range of \( 10^{-12} \) mbar can only be achieved when the sputter-ion pump works in a combination with other pumping methods[36,37]. In our considerations, a conservative value is taken \( p = 10^{-11} \) mbar, which is usually required for achieving ultrahigh-Q mechanical oscillators and the ultrasensitive measurements in the levitated optomechanical system[12,19,23,24,38]. If we choose a lower air pressure \( (p = 10^{-12} \) mbar) , this scheme will gain a sensitivity of \( F_{\text{min}} = 4.0 \times 10^{-5} aN \). Our scheme yields a 4-5 orders of magnitude improvement for the force sensing in the micro-scale[5].

The fundamental fluctuation processes impose the ultimate limits upon the sensitivity of the detection. Frequency stability \( \Delta \omega_{\text{min}} \) is key to performance of microresonators and their applications in the thermomechanical noise system. For the case of the high quality factor \( Q \gg 1 \), the minimum detectable frequency shift limited by thermomechanical fluctuations of the levitated microdisks can be estimated via[39],

\[
\Delta \omega_{\text{min}} \approx \sqrt{\frac{k_B T \omega_1 \Delta f}{E_c Q_1}}. \tag{8}
\]

where, \( \Delta f = 1/2 \pi f_{\text{min}} \) the measurement bandwidth which is dependent upon the measurement averaging time \( \tau \). \( E_c = m_1 \omega_1^2 \langle \xi_1^2 \rangle \) represents the maximum drive energy, \( \langle \xi_1^2 \rangle \) is the maximum root mean square(rms) level produced a predominantly linear response. For a Gaussian field distribution, the nonlinear coefficients are given by \( \xi = -2/W^2[40] \), here \( W \approx 75 \mu m \) is the trapping beam waist. For the displacements \( |x_c| \ll \xi^{-1/2} = 5 \times 10^{-5} m \) the nonlinearity is negligible. In our considerations, \( x_c \) is taken to be 2 orders of magnitude smaller, we choose \( x_c \approx 10^{-5} m \). Considering the quality factor of the microdisk \( Q_1 = 4.3 \times 10^{11} \), we can obtain \( \Delta \omega_{\text{min}} = 4.5 \times 10^{-8} Hz < \beta_{\text{min}} \) for the measurement bandwidth \( \Delta f = 10^{-3} Hz \) at the room temperature. It is corresponding to the measurement time \( \tau = 160 s \). The result shows that the thermomechanical noise can be controlled at a level lower than the claimed sensitivity.

V. CONCLUSION

We have analyzed the large extra dimensions induced coupling between two levitated resonators with quantum optomechanics. Our study reports a design for probing gravitational deviation in the range of 8um via the pump-probe optical technology in cavity. The transparency peak will suffer a distinct splitting and get apart with
the non-Newtonian gravity being taken into consideration. The gravitational strength, characterized by the coupling strength $\beta$, can also be determined by the splitting distance $L$. The Casimir-Polder coupling rate is about 5-fold smaller than the gravitational strength, and can be attenuated even further by the shield mirror. It allows the precision measurement in the Casimir regime without the isoelectronic technique. We hope that the precision can be significantly enhanced by experiments in ultrahigh vacuum.

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I. THE HEISENBERG EQUATIONS

The Hamiltonian of the system in a rotating frame at the pump frequency \( \omega_{pu} \) and the probe pulse frequency \( \omega_{pr} \) reads as follow:

\[
H = \hbar \omega_c c^+ c + \hbar \omega_1 a_1^+ a_1 + \hbar \omega_2 a_2^+ a_2 - \hbar g c^+ c (a_1^+ + a_1) + h(\beta + \beta_{\text{casimir}}) \cdot (a_1^+ a_2 + a_1 a_2^+) \\
- i \hbar \Omega_{pu} (c - c^+) - i \hbar \Omega_{pr} (e^{i \delta t} - e^{-i \delta t}).
\]

(1)

We define the operator \( S_j = a_j^+ + a_j \), the detuning of the probe and pump field \( \delta = \omega_{pu} - \omega_{pr} \) and the pump-cavity detuning \( \Delta_{pu} = \omega_c - \omega_{pu} \). In what follows, we deal with the mean response of the system to the probe field in the presence of the coupling, let \( \langle c \rangle, \langle c^+ \rangle \) and \( \langle S_{1,2} \rangle \) be the expectation values of operators \( c, c^+ \) and \( S_{1,2} \), respectively. According to the Heisenberg equation of motion and the commutation relations \( [c, c^+] = 1, [a, a^+] = 1 \). In a frame rotating with the frequency of the pump field, the temporal evolutions of \( c \) and \( S_{1,2} \) can be obtained and the corresponding equations are given by adding the damping terms

\[
\frac{d}{dt} \langle c \rangle = -(i \Delta_{pu} + \kappa) \langle c \rangle + i g_1 \langle S_1 \rangle \langle c \rangle + \Omega_{pu} + \Omega_{pr} e^{-i \delta t},
\]

(2)

\[
\frac{d^2}{dt^2} \langle S_1 \rangle + \gamma_1 \frac{d}{dt} \langle S_1 \rangle + \left( \omega_1^2 + \beta^2 \right) \langle S_1 \rangle - \beta (\omega_1 + \omega_2) \langle S_2 \rangle = 2 g_1 \omega_1 \langle c^+ \rangle \langle c \rangle, 
\]

(3)

\[
\frac{d^2}{dt^2} \langle S_2 \rangle + \gamma_2 \frac{d}{dt} \langle S_2 \rangle + \left( \omega_2^2 + \beta^2 \right) \langle S_2 \rangle - \beta (\omega_1 + \omega_2) \langle S_1 \rangle = 2 g_1 \beta \langle c^+ \rangle \langle c \rangle.
\]

(4)

To solve these equations, we make the ansatz as follows:

\[
\langle c(t) \rangle = c_0 + c_+ e^{-i \delta t} + c_- e^{i \delta t},
\]

(5)

\[
\langle S_1(t) \rangle = S_{10} + S_{1+} e^{-i \delta t} + S_{1-} e^{i \delta t},
\]

(6)

\[
\langle S_2(t) \rangle = S_{20} + S_{2+} e^{-i \delta t} + S_{2-} e^{i \delta t}.
\]

(7)

Substituting Eqs.(5),(7) into Eqs.(2)-(3), respectively, equating terms with the same time dependence, and working to the lowest order in \( \Omega_{pr} \) but to all orders in \( \Omega_{pu} \), we can obtain

\[
c_0 = \frac{\Omega_{pu}}{i \Delta_{pu} + \kappa - i \delta - ig_1 S_{10}},
\]

(8)

\[
c_+ = \frac{\Omega_{pr} + i c_0 g_1 S_{1+}}{i \Delta_{pu} + \kappa + i \delta - ig_1 S_{10}},
\]

(9)

\[
c_- = \frac{i c_0 g_1 S_{1-}}{i \Delta_{pu} + \kappa + i \delta - ig_1 S_{10}},
\]

(10)

and

\[
(\omega_1^2 + \beta^2) S_{10} - (\beta (\omega_1 + \omega_2)) S_{20} = 2 g_1 \omega_1 c_0^2
\]

(11)

\[
S_{1+} = \frac{\beta (\omega_1 + \omega_2) S_{2+} + 2 g_1 \omega_1 (c_0^* c_+ + c_0 c_+^*)}{-\delta^2 - i \gamma_1 \delta + \omega_1^2 + \beta^2},
\]

(12)

\[
S_{1-} = \frac{\beta (\omega_1 + \omega_2) S_{2-} + 2 g_1 \omega_1 (c_0^* c_- + c_0 c_-^*)}{-\delta^2 - i \gamma_1 \delta + \omega_1^2 + \beta^2},
\]

(13)

\[
(\omega_2^2 + \beta^2) S_{20} - (\beta (\omega_1 + \omega_2)) S_{10} = 2 g_1 \beta c_0^2
\]

(14)

\[
S_{2+} = \frac{\beta (\omega_1 + \omega_2) S_{1+} + 2 g_1 \beta (c_0^* c_+ + c_0 c_+^*)}{-\delta^2 - i \gamma_2 \delta + \omega_2^2 + \beta^2},
\]

(15)

\[
S_{2-} = \frac{\beta (\omega_1 + \omega_2) S_{1-} + 2 g_1 \beta (c_0^* c_- + c_0 c_-^*)}{-\delta^2 - i \gamma_2 \delta + \omega_2^2 + \beta^2}.
\]

(16)

Solving Eqs. (8)-(16), we obtain in the steady state,

\[
c_+ = \frac{\Omega_{pr} [(A_1 A_2 - W^2)(U - V) + Y \omega_0]}{(A_1 A_2 - W^2)(U^2 - V^2) + 2 V Y \omega_0},
\]

(17)

and

\[
\Omega_{pu}^2 = [\kappa^2 + (\Delta_{pu} - g_1 S_{10})^2] \omega_0,
\]

(18)
with $\omega_0 = |c_0|^2$, $U = \kappa - i\delta$, $V = i\Delta_p - ig_1 S_{10}$, $A_j = -\delta^2 - i\gamma_j \delta + \omega_j^2 + \beta^2 (j = 1, 2)$, $W = \beta(\omega_1 + \omega_2)$, $R_1 = 2g_1 \omega_1$, $R_2 = -2g_1 \beta$, $Y = ig_1 (A_2 R_1 + W R_2)$. Then $S_{10}$, $S_{20}$ can be resolved by

$$((\omega_1^2 + \beta^2)S_{10} - WS_{20}) = R_1 \omega_0, \quad (19)$$

$$((\omega_2^2 + \beta^2)S_{20} - WS_{10}) = R_2 \omega_0. \quad (20)$$

The transmission of the probe beam, defined as the ratio of the output and input field amplitudes at the probe frequency is then given by using the input-output relation[1],

$$t(\omega_{pr}) = \frac{\Omega_{pr}/\sqrt{2\kappa}}{\Omega_{pr}/\sqrt{2\kappa}} = 1 - \frac{2\kappa c_+}{\Omega_{pr}}. \quad (21)$$

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