Selection of glazing material for Trombe wall applications in Johannesburg using the TOPSIS methodology

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Abstract-
Glazing are functional components of Trombe walls, they determine to a great extent the performance of the system. selection of optimal glazing for trombe wall application was considered in this article and the TOPSIS methodology was employed. TOPSIS is a multi-criteria decision-making algorithm which means a technique for order preference by similarity to ideal solution. considered in this article were four different glazing types and four criteria. the single glazing was the closest to the ideal solution followed by the double pane glazing with low emissivity coating, double pane glazing with high emissivity coating and the triple pane glazing respectively

Key words: Trombe wall, TOPSIS.

1. Introduction
Material selection in engineering applications determines to a great extent the performance of the system [1]. Trombe walls are passive solar structure uses in space conditioning in buildings, its consists of a glazing and a massive wall separated by an air gap with vents on the top and bottom of the wall [2]. The glazing material used in trombe walls has been identified as one of the major components that determine the overall performance on the system [3]. Yilmaz et al, in their research work reported that a single pane glazing improved the performance of a trombe wall installed in turkey during daytime hours due to high transmission of visible radiation while starzi et al, in Italy reported that a double pane glazing performed better due to reduced heat loss from the building during the winter [4,5]. Coating of glazings with various kinds of photosensitive materials has also been considered by researchers to improve their performance. Richman et al used a low emissivity coating on the glass to reduce radiative losses to the ambient during heating operation [6]. Zalewski et al, carried out simulation studies on the effect of glazing materials on the performance of trombe wall, this research was conducted for two different cities; Trappes (2.01°E, 48.46°N) and Carpentras (5.03°E, 44.08°N) respectively, the result of the study showed that a double pane glazing with low emissivity coating performed better than the standard double glazing. From the foregoing discussion, it is evident that the type of glazing used on a trombe wall influences its performance, this, therefore, justifies the need for this study. The cite of interest for this study is Johannesburg. The technique for order preference by similarity to ideal solution otherwise known as TOPSIS is a multi-criteria decision-making approach. This method is based on the concept that the best alternative should have the shortest Euclidean distance from the ideal solution [7]. Notable
researchers have employed the TOPSIS technique in selecting materials to meet specific objectives. Kumar et al applied the TOPSIS technique in selecting the most suitable material to serve their design purpose. Their normalized matrix table consists of seven alternatives and six different criteria, the result presented indicated that the nitrated steel was the most suited for use [8]. Shanian et al also implemented the TOPSIS technique in material selection for a metallic bipolar plate for polymer electrolyte fuel cell. A few other multi-criteria decision techniques have been explored by researchers in literature such as the AHP, FUZZY AHP, PSI, Bayesian algorithm etc. This study seeks to evaluate four types of glazing based on four different criteria using the TOPSIS technique and proffers the best to be used in trombe wall applications in Johannesburg.

2. Methodology
The decision for material selection will be done in three stages as depicted in figure 1 below: the first stage is the goal, which is the selection of an optimum glazing material from the various types of glazing considered. The second stage is the list of criteria to be considered while selecting the glazing types and the third stage presents the glazing alternatives. Furthermore, we make a succinct explanation of the glazing criteria being considered.

SHGC
The solar heat gain coefficient is the fraction of solar radiation transmitted or absorbed through a window glazing and subsequently released into the building. It has values within the limits of \( \{0,1\} \) [9]. It can be expressed mathematically below.

\[
SHG = \tau G_i + N_i \alpha G_i = G_i (\tau + N_i \alpha) = G_i * SHGC
\]

\( SHG \) = solar heat gain, \( G_i \) = incident solar radiation, \( \tau \) = transmissivity of the glazing, \( \alpha \) = absorptivity of the glazing, \( N_i \) = fraction released into the building.

U-value
This is a measure of the quantity of either the heat gains through glazing due to the thermal conductivity of the glazing material and the temperature difference across the glazing [10].

\[
Q = A \Delta T U
\]

\[
U = \frac{Q}{A \Delta T}
\]

\( Q = \) heat flow rate through the glazing, \( A = \) area of glazing, \( \Delta T = \) temperature gradient across the glazing

Visible transmittance
This is a fraction of the visible spectrum of sunlight transmitted through glazing, it is expressed as numbers between 0 and 1. Glazing with a high visible transmittance allows more visible light [9].

\[
T_{vis} = \frac{I}{I_o}
\]

\( I = \) transmitted sunlight through the glazing
\( I_o = \) incident sunlight on the glazing
2.1 The TOPSIS METHOD

The conceptual framework for the Topsis methodology was developed by Hwang and Yoon in 1981 [11]. TOPSIS is a numerical method for solving a multi criteria decision making problems and has found great applications in engineering material selection problems as highlighted in the preceding section of this study.

When given $m$ alternatives which are dependent on $n$ criteria with values expresses as positive real number $X_{ij}$, with the task of selecting the best material. $X_{ij}$ is known as the performance value of an $i$ alternative and $j$ criteria. The procedure would be described in steps as follows

**Step 1**
First, the performance value matrix table is set up, the order of the matrix depends on the number of criteria and alternatives being considered.

$$X_{ij} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \quad (1)$$

**Step 2**
The performance values are normalized using the mathematical operation presented below

$$\bar{X}_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}} \quad (2)$$

Where the $i =$ number of alternative and $j$ number of criteria
Step 3
The normalized performance values are weighted using equation (3).

\[ V_{ij} = w_j \times \bar{X}_{ij} \]  
(3)

\( w_j \) is the weight assigned to each criteria and expressed mathematically as \( w_j = 1/n \)  
(3α)

Step 4
For each alternative, we select values which are closest to the ideal best and ideal worst of the system under consideration.

\( V_{ij}^+ \) denotes the ideal best value while \( V_{ij}^- \) denotes the ideal worst values.

Step 5
The Euclidean distance from the ideal best and Ideal worst values are calculated using equations 4.0 and 5.0 respectively.

\[ S_i^+ = \left[ \sum_{j=1}^{m} (V_{ij}^+ - V_j^+) \right]^{0.5} \]  
(4)

\[ S_i^- = \left[ \sum_{j=1}^{m} (V_{ij}^- - V_j-) \right]^{0.5} \]  
(5)

Step 6
The performance score \( P_i \) is calculated using equation 6.0.

\[ P_i = \frac{S_i^-}{S_i^+ + S_i^-} \]  
(6)

The performance score is an indication of a closeness to the ideal solution, it is within an interval of \([0, 1]\), the value closest to 1 represents the ideal best solution while the value closest 0 represents the ideal worst solution.

The performance value matrix table for the four types of glazing and four criteria is presented below. The types of glazing considered are single pane, a double pane with low emissivity coating, a double pane with high emissivity coating, and triple pane glazing. These four glazing types are considered under the following criteria, namely; the solar heat gain coefficient, the U-value, the visible transmittance and cost.

| Types of Glazing | SHGC | U-Value | T-vis | Cost(R/SQM) |
|------------------|------|---------|-------|-------------|
| Single Pane      | 0.855| 5.68    | 0.901 | 300         |
| Double Pane L    | 0.755| 2.83    | 0.817 | 500         |
| Double Pane H    | 0.589| 1.4     | 0.708 | 500         |
| Triple Pane      | 0.407| 0.68    | 0.625 | 800         |
3. Result and discussions

The normalization of the performance values are executed as prescribed in equation 2.0. Presented below is the table of the normalized performance value. The normalized values are in the interval of \([0,1]\).

Table 3.1: Normalised decision matrix table

| Types of Glazing | SHGC       | U-Value   | T-vis     | Cost (R/SQM) |
|------------------|------------|-----------|-----------|--------------|
| Single Pane      | 0.629593   | 0.869294  | 0.585117  | 0.270501     |
| Double Pane L    | 0.570684   | 0.433116  | 0.530567  | 0.450835     |
| Double Pane H    | 0.433719   | 0.214262  | 0.459781  | 0.450835     |
| Triple Pane      | 0.299701   | 0.10407   | 0.40588   | 0.721336     |

Assigning of weights to various criteria is a measure of the significance of that criteria to the performance of the system being considered, for general cases where no particular choice is in mind, equation 3a is used in assigning of weights to the various criteria for consideration. The weighted normalised matrix table is generated by proper implementation of equations 3.0 and 3a respectively. The result of the operation is presented below.

Table 3.2: Weighted normalized decision matrix table

| Types of Glazing | SHGC       | U-Value   | T-vis     | Cost (R/SQM) |
|------------------|------------|-----------|-----------|--------------|
| Single Pane      | 0.157398   | 0.217323  | 0.146279  | 0.067625     |
| Double Pane L    | 0.142671   | 0.108279  | 0.132642  | 0.112709     |
| Double Pane H    | 0.10843    | 0.053566  | 0.114945  | 0.112709     |
| Triple Pane      | 0.074925   | 0.026018  | 0.10147   | 0.180334     |

The ideal best and ideal worst values are evaluated depending on the interest of the investigator, for a Trombe wall to function optimally we desire a high U-value of the glazing materials to be used, as well as high SHGC and T-vis values but a low cost of materials. The maximum values of the thermal properties of glazing and low-cost price reflect our ideal best solution while low thermal properties and high-cost price represent the ideal worst values.

Table 3.3: Computation of the Ideal best and worst values

| Types of Glazing | SHGC       | U-Value   | T-vis     | Cost (R/SQM) |
|------------------|------------|-----------|-----------|--------------|
| Single Pane      | 0.157398   | 0.217323  | 0.146279  | 0.067625     |
| Double Pane L    | 0.142671   | 0.108279  | 0.132642  | 0.112709     |
| Double Pane H    | 0.10843    | 0.053566  | 0.114945  | 0.112709     |
| Triple Pane      | 0.074925   | 0.026018  | 0.10147   | 0.180334     |

The Euclidean distance from the ideal best and ideal worst solutions are evaluated using equations 5 and 6 respectively, the values are summarized in table 5.0 below.
Table 3.4: Euclidean distance from the ideal best and ideal worst values

| Types of Glazing | SHGC     | U-Value  | T-vis   | Cost(R/SQM) | $S^+_i$ | $S^-_i$ |
|------------------|----------|----------|---------|-------------|---------|---------|
| Single Pane      | 0.157398 | 0.217323 | 0.146279| 0.067625    | 5.3E-07 | 0.241   |
| Double Pane L    | 0.142671 | 0.108279 | 0.132642| 0.112709    | 0.120   | 0.130   |
| Double Pane H    | 0.10843  | 0.053566 | 0.114945| 0.112709    | 0.180   | 0.081   |
| Triple Pane      | 0.074925 | 0.026018 | 0.10147 | 0.180334    | 0.241   | 4.8E-07 |
| $V^+_i$          | 0.157398 | 0.217323 | 0.146279| 0.067625    |         |         |
| $V^-_i$          | 0.074925 | 0.026018 | 0.10147 | 0.180334    |         |         |

The performance score of the various glazing alternatives was computed, this reflects how close or how far an alternative is to the ideal glazing material. The performance score is sometimes called closeness coefficient, and it falls within the range of positive real numbers $0 \leq P_i \leq 1$. The single glazing had the highest performance score of 0.99 followed by the double glazing with low emissivity coating with a performance value of 0.52, the double glazing with high emissivity coating had a performance score of 0.31 and the triple pane glazing with the least performance score of 1.98E-06.

Table 3.5: Performance score for glazing alternatives

| Types of Glazing | SHGC     | U-Value  | T-vis   | Cost(R/SQM) | $S^+_i$ | $S^-_i$ | $P_i$   |
|------------------|----------|----------|---------|-------------|---------|---------|---------|
| Single Pane      | 0.157398 | 0.217323 | 0.146279| 0.067625    | 5.3E-07 | 0.241   | 0.999   |
| Double Pane L    | 0.142671 | 0.108279 | 0.132642| 0.112709    | 0.120   | 0.130   | 0.521   |
| Double Pane H    | 0.10843  | 0.053566 | 0.114945| 0.112709    | 0.180   | 0.081   | 0.312   |
| Triple Pane      | 0.074925 | 0.026018 | 0.10147 | 0.180334    | 0.241   | 4.8E-07 | 1.98E-06|
| $V^+_i$          | 0.157398 | 0.217323 | 0.146279| 0.067625    |         |         |         |
| $V^-_i$          | 0.074925 | 0.026018 | 0.10147 | 0.180334    |         |         |         |

4. Conclusion
The TOPSIS methodology has been used extensively by researchers in material selection and performance evaluation. We have implemented this process in the selection of glazing material for Trombe wall systems. Glazing performances vary with location and applications which informed the need for this research. Having considered four types of glazing, the single had the closest performance score which makes it ideal for installation in the city of Johannesburg.

5. Recommendation
It is recommended that other multi-criteria decision making methodologies such as AHP, Fuzzy AHP, grey’s rational analysis and Bayesian algorithm be applied in selection of glazing materials and the result compared with that of the TOPSIS method.
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