Cooperative states and shift in resonant scattering of an atomic ensemble

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Keywords: dipole–dipole interaction, collective Lamb shift, superradiance, cooperativity, subradiance

Abstract

We investigate the spectral shift known as the collective Lamb shift in forward scattering for a cold dense atomic cloud. The shift results from resonant dipole–dipole interaction mediated by real and virtual photon exchange, forming many-body states displaying various super- and subradiant behaviour. However, the scattering spectrum reflects the overall contributions from these states but also averages out the radiative details associated with the underlying spin orders, causing ambiguity in determination and raising controversy on the scaling property of this shift. We employ a Monte–Carlo simulation to study how the collective states contribute to emission. We thus distinguish two kinds of collective shift that follow different scaling laws. One results from dominant occupation of the near-resonant collective states. This shift is usually small and insensitive to the density or the number of participating atoms. The other comes from large spatial correlation of dipoles, associated with the states of higher degree of emission. This corresponds to larger collective shift that is approximately linearly dependent on the optical depth. We further demonstrate that the spatial spin order plays an essential role in superradiant emission. Our analysis provides a novel perspective for understanding collective scattering and cooperative effects.

1. Introduction

Light-ensemble interaction has been an important topic drawing continuing attention for recent years thanks to its fundamental interest in quantum many-body physics and practical applications in various areas, such as atomic clocks and metrology \cite{1}, sensing and precision measurement \cite{2}, quantum simulation \cite{3}, quantum interface, memory, and network \cite{4–6}. In various proposals, schemes based on atomic ensembles are expected to have enhanced coupling strength for more efficient manipulation by increasing the number and/or density of atoms. As the system becomes sufficiently dense, cooperative effects such as super- and subradiance due to atoms’ dipole–dipole interaction start to emerge, leading to collective shift and even distortion of line shapes \cite{7,8}. These effects may, for instance, cause unwanted decoherence in quantum control and degrade the precision of optical atomic clocks \cite{9}. How to understand the cooperativity in atomic ensembles not only provides insightful perspectives for many-body physics in the presence of nontrivial competing interactions, but also helps develop quantum optical devices and applications more accurately.

One intriguing cooperative phenomenon is the emergence of the collective Lamb shift, the many-body version of the ordinary Lamb shift. The ordinary Lamb shift accounts for the vacuum fluctuations that perturb the electron’s orbital in an atom and introduce an energy shift \cite{10}. Understanding of this shift has opened a new subject now known as quantum electrodynamics. To correctly calculate the shift, contributions of all transition processes including virtual ones need to be properly dealt with. Similar consideration applies to many-body cases, where both the real and virtual processes of photon exchange mediate the dipole–dipole interaction, resulting in cooperative decay and energy shift of the collective states. Recently, such phenomena have attracted extensive attention, and have been discussed in various contexts including atomic clouds \cite{7,}.
one-dimensional atomic chains [18], ordered atomic arrays [19–21], nano-layer gases [22, 23], ensembles of nuclei [24], trapped ions [25], and artificial atoms [26, 27]. Another perspective views this shift as coupling between collective states, leading to Rabi-like excitation transfer among a few atoms [28].

A commonly-used technique to detect the collective shift is through scattering experiments, where one measures the emission spectrum while sweeping the probe frequency, and extracts the shift of the spectral peak. For weak probing, though it is valid to consider only the lowest excited manifold as in many experiments, there are \( N \) such singly-excited states with \( N \) atoms in an ensemble, and these states are shifted differently owing to competing dipole–dipole interaction. Unfortunately, the scattering spectrum only reflects the overall effect of superposed contributions from individual collective states. Some unrevealed spin orders relevant to the spectrum might be averaged out. Further, there have been controversies regarding the scaling nature of the collective shift in different geometries such as slabs and ellipsoids. Recent experiments have reported large collective shift in elongated atomic samples [7, 11]. In pancake-shaped clouds, however, the shift is negligible [17], yet can be restored by including inhomogeneous broadening [29, 30]. Changing the anisotropy also introduces unusual shift scaling [13]. For some specific geometries and densities, the lineshape becomes asymmetric and even displays two peaks in the profile, making the determination of the shift ambiguous [31].

To better understand the underlying mechanisms, this manuscript investigates the roles of many-body states by selecting an appropriate orthogonal basis. We utilise a Monte–Carlo simulation by randomly distributing atoms of controlled density within a specific geometry, and employ the standard non-Hermitian Schrödinger equation approach in the weak-field (single-excitation) limit. After ensemble averaging, the spectral profile exhibits a ‘three-peak’ structure with each peak characterised by distinct scaling properties due to the different roles played by the participating collective states. Our analysis is applicable to general cases but the significance of the three types of contributions may vary across different geometries and parameters, thus providing an explanation that helps resolve the controversy regarding the scaling nature of the collective shift. Furthermore, through this approach, we are able to identify the correspondence between the induced spin order and superradiance by examining the most emissive states.

This manuscript is organised as follows: section 2 summarises the formulation of the current problem and our approach for analysis. Section 3 presents the general results of the scattering spectrum and its correlation with the emissivity of cooperative states. Here, we discuss the atomic spatial correlations, i.e. spin orders, and their connection to the radiative behaviour. We further explore the effects of geometries, densities, and detection angles before concluding this work in section 4.

2. Method

We consider an ensemble of \( N \) two-level atoms randomly distributed in a localised region of certain geometry with \( r_i \) denoting the location of the \( i \)th atom assumed to be fixed in space. To probe the spectrum scattered by the singly-excited collective states and the associated Lamb shift, these atoms are driven weakly by a laser beam propagating along the \( +z \) direction, where the detuning \( \Delta \equiv \omega - \omega_0 \ll \omega_0 \), wavevector \( k \equiv \omega z/c \approx \omega_0 z/c \equiv 2\pi z/\lambda_0 \), Rabi frequency \( \Omega \ll \gamma_0 \), with \( \omega_0 (\omega) \) the atomic (laser) frequency, \( \lambda_0 \) the transition wavelength, and \( \gamma_0 \) the spontaneous emission rate, and \( c \) the speed of light. In the low-excitation limit, a quantum state can be of the form \( |\psi\rangle \approx |g_0 \cdots g_N\rangle + \sum_n b_n |g_0 \cdots e_n g_n\rangle \) with \( |b_n| \ll 1 \), which satisfies \( i\dot{\mathbf{B}}(t) = \mathbf{MB} + \Omega \mathbf{D} \) [32], where \( \mathbf{B} = [b_1, \ldots, b_1, \ldots, b_N]^T \), \( \mathbf{D} = [e^{ikr_1}, \ldots, e^{ikr_1}, \ldots, e^{ikr_N}]^T \), and

\[
\mathbf{M} = \begin{bmatrix}
-\Delta - \frac{\gamma_0}{2} & V_{12} & \cdots & V_{1N} \\
V_{21} & -\Delta - \frac{\gamma_0}{2} & \cdots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
V_{N1} & \cdots & \cdots & -\Delta - \frac{\gamma_0}{2}
\end{bmatrix}.
\]

The dipole–dipole interaction is given by

\[
V_{ij} = \frac{3\gamma_0}{4} \left[ (1 - \cos^2 \theta_{ij}) \frac{e^{ikr_{ij}}}{kr_{ij}} + (1 - 3 \cos^2 \theta_{ij}) \left( -i e^{ikr_{ij}} \frac{e^{ikr_{ij}}}{(kr_{ij})^2} + e^{ikr_{ij}} \frac{e^{ikr_{ij}}}{(kr_{ij})^2} \right) \right],
\]

depending on the separation \( r_{ij} = |r_i - r_j| \) between two atoms \( i \) and \( j \), wavenumber \( k = |k| \), and the angle \( \theta_{ij} \) between the distance vector \( r_i - r_j \) and the dipole orientation assumed to be the \( x \) direction for linear polarised driving field. Note that this is a special case where the mathematical structure is consistent with the classical dipole analysis [33]. The dipole–dipole interaction couples the \( N \) singly excited states, causing various degrees of shift depending on the spatial arrangement (called a configuration in this manuscript) of
the atoms. When $N$ is large, it is however unrealistic and meaningless to measure the shift of each collective state. Nevertheless, the scattering spectrum still catch the overall contributions, and can help identify the emergence and order of magnitude of the collective Lamb shift. But we still need to look into what are actually probed for further diagnosis.

By solving $B = -\Omega M^{-1} D$, we obtain the steady-state solution $b_i$, which determines the intensity of the forward scattering by $I \propto \sum_i b_i e^{-i k \cdot r_i}$.

The population distribution can then be obtained by expressing the steady state solution as

$$I_{ij} = \sum_i |b_i|^2 + \sum_{i\neq j} b_i^* b_j e^{-i (r_i - r_j)}.$$ 

We identify two parts: the incoherent scattering term $\sum_i |b_i|^2$ and the coherent one $\sum_{i\neq j} b_i^* b_j e^{-i (r_i - r_j)}$. The former exactly corresponds to the total excitation of individual atoms for a given probe detuning. The latter accounts for the spin–spin correlation modulated by the spatial phases. The resultant spectral profile of emission fluctuates drastically for a given spatial configuration of atoms, implying the sensitivity of the system to the actual parameters. The robustness of the results lies in the ensemble average as many configurations are considered, leading to a convergent coarse-grained profile.

To understand the spectrum, it is natural to relate the radiative properties to the many-body states that are relevant. Note that the coupling matrix $M$ is symmetric but complex. In some previous literature, it has been directly diagonalised to find the dynamics for the system [15, 34–36]. The collective basis states thus found are however not strictly orthogonal. Though the calculated dynamics is correct, it fails to associate the radiative behaviour with individual states because they are seriously overlapped [36]. Here, by noting that $M = -\Delta I + M_2 + i M_3$, where $I$ is an $N \times N$ identity matrix, $M_2$ and $M_3$ are the real and imaginary parts of $M + \Delta I$, respectively, we choose the orthogonal eigenbasis that diagonalises only the real part $M_2$ such that $M^D = R^T M_2 R$ with $[M^D]_{ij} = \delta_{ij}$. The eigenvalue $\varepsilon_j$ is associated with an eigenvector $R_j$ (the $j$th column of $R$), and can be understood as the shift of the $j$th collective state $|R_j\rangle = \sum_i R_j^{(i)} |g_i\rangle$, with $R_j^{(i)} = \langle R_j | i \rangle$. The population distribution can then be obtained by expressing the steady state solution as $B = \sum j p_j |R_j\rangle$, where the excitation over the state $R_j$ is given by $|p_j|_1$ with $p_j = \langle R^{-1} B \rangle_j$.

In order to quantity the intrinsic spectral contribution of a state, we define a probe-independent quantity called the emission capability associated with each state $|R_j\rangle$

$$\Pi_j = \sum_i |R_j^{(i)}|^2 e^{-i k r_i}.$$ 

Recall $I \propto \sum_{i,j} p_j |R_j^{(i)}|^2 e^{-i k r_i}$, and therefore $\Pi_j$ captures the degree to which the state $|R_j\rangle$ contributes to the emission behaviour. This aids in diagnosing the role of each collective state and offers a fresh perspective regarding super- and subradiance without explicitly taking into account the collective decay rates. It is important to note that the relevant quantities discussed in the subsequent sections have been ensemble-averaged, unless explicitly stated otherwise.

3. Results and discussions

3.1. Emission spectrum and collective shift

We first look at one of the most illustrative examples corresponding to a uniform dense cylindrical sample of radius comparable to a transition wavelength and length over a few wavelengths. With a probe laser incident along the cylindrical axis, figure 1(a) shows the corresponding forward scattering spectrum and excitation distribution to characterise the spectral contributions of these states. We can observe ‘three peaks’ in the spectrum by sweeping the probe detuning: the strongest left peak, the small tip in the centre, and the smooth hump on the right. Similar spectral profiles have also been discussed in [37, 38]. We find that the three peaks have different origins by taking into consideration the roles played by the participating collective states. Such diagnosis remain valid in general cases but the three types of effects may contribute differently depending on the geometry and parameters of a system. For instance, the smooth hump on the blue side only emerges in cases close to uniform distribution of atoms, and easily smears out in Gaussian samples. The large shift on the left is more pronounced in elongated systems, and gradually merges to the central peak as the anisotropy reduces. Note that the central peak is also shifted, as shown in the inset of figure 1(a). Most of the collective shift measurements refer to either the left or the central peak (the right hump is usually not very contrastive), yielding very different scaling nature. We also show the total excitation against the probe frequency for comparison, corresponding to the incoherent scattering part only. Apparently, the excitation curve cannot explain the three-peak structure, suggesting the dominant role played by the spin correlation in the emission spectrum.

In order to analyse the emissivity, we now turn to the orthogonal collective states defined in section 2. In figure 1(b), we plot the emission capability of such states against their energy shift $\varepsilon$ for a given single
Figure 1. (a) Forward scattering (blue) and total excitation (red) spectrum (ensemble averaged). Inset: zoomed plot near the central tip of the scattering spectrum. The three peaks are located at $\Delta = -7.3\gamma_0$, $-0.2\gamma_0$, and $+9.6\gamma_0$. (b) Emission capability versus the eigenenergy $\epsilon$ (compatible to the state shift) of the corresponding collective state. The scattered dots are from a single configuration and the blue curve is from ensemble average. The system used here is a cylindrical atomic ensemble of length $L = 3\lambda_0$ and radius $R = \lambda_0$. The laser incident direction is along the axis of the cylinder. The ensemble average is done by considering >5000 random configurations of 701 atoms uniformly distributed inside the cylinder with $\rho/k^3 = 0.3$, where $\rho$ is the number density. (The scattering spectrum and excitation curves are normalised by the values at $\Delta = 0$. The emission capability is in arbitrary units.)

Figure 2. (a) Density of states (normalised by the area underneath). Inset: zoomed plot around the tip, which is red shifted by about $0.2\gamma_0$ away from the single-atom resonance. (b) Histogram of the excitation distribution (arbitrary unit) of at various detuning. The system used here is a uniform cylindrical atomic ensemble of length $L = 3\lambda_0$ and radius $R = \lambda_0$. The ensemble contains 701 atoms, corresponding to a number density of $\rho/k^3 = 0.3$. The laser incident direction is along the axis of the cylinder. All curves are ensemble averaged.

Configuration (red dots) and after ensemble averaged (blue line). We find that the emission capability curve has two peaks around $\epsilon \approx -8.2\gamma_0$ and $\epsilon \approx +9.1\gamma_0$, implying that these states are highly capable of emission. The left peak does give rise to the most visible peak in the spectrum of figure 1(a). The right peak is also responsible for a small hump at the corresponding detuning of the spectral profile. This hump on the blue side is however less evident because the corresponding states are less populated. On the other hand, the states of small shift have low emission capability even though they are mostly populated. This ‘M-shaped’ feature of emission capability is generally observed in all atomic configurations, even in various geometries and densities. But the actual spectral curves are still determined by considering overall the emission capability, density of states, excitation, and cross-term interference of the collective states in detail, which vary from sample to sample of different parameters.

3.2. Density of states

By utilising the modes of $\mathbf{M}_R$, we plot the density of states as a function of a state’s shift $\epsilon$ in figure 2(a). The profile suggests that the majority of the collective states ($\sim 82.3\%$) have shift less than $5\gamma_0$. The density-of-state profile has a peak slightly shifted to the red side of the resonance ($\epsilon = 0$), which is responsible for the shift of the central spectral tip in figure 1(a). We can now discuss the excitation $|p_j|^2$ of the state $|R_i\rangle$ by applying a probe laser of frequency $\omega = \omega_0 + \Delta$. We plot the excitation histogram of different shift for various detuning $\Delta$ in figure 2(b), and find that the probe field excites only those states resonant to the laser frequency ($\epsilon \approx \Delta$) with a linewidth comparable to $\gamma_0$. The peak values of excitation also drops significantly for large $|\Delta|$ since the density of states is small for large $|\epsilon|$. 
As we sweep the detuning, we might have expected that there would be a significant peak in the centre of the spectrum, corresponding to the most probable excited states indicated by figure 2(a). In fact, this is not the case. Those most-populated states only contribute to a small tip in the spectrum. By contrast, the most significant peak appears on the red side ($\Delta \approx -7.3\gamma_0$ in figure 1(a)) with the corresponding peak excitation lower than that of $\Delta = 0$ by two orders of magnitude. This suggests that these states, though just a few of them, are much more radiative compared to the majority of the collective states.

We emphasise that the overall excitation should be independent of the choice of the basis as long as the probe beam is given. But, since different bases yield different eigenvalue distributions, the profile shown in figure 2(b) is only valid in terms of the modes of $\mathbf{M}_R$. Many previous studies also utilise the direct diagonalisation for the full, complex, coupling matrix $\mathbf{M}$ but such an approach does not afford an intuitive explanation for the frequency locations of spectral peaks in the frequency domain. This is because the complex eigenvalues do not straightforwardly correlate with frequency shifts and therefore it becomes challenging to directly link the peaks with the contributing states. Instead, our analysis, utilising the modes of the real part, establishes a self-consistent framework. Through this particular basis, it becomes apparent that explaining for the frequency locations of spectral peaks in the frequency domain. This is because the diagonalisation for the full, complex, coupling matrix $\mathbf{M}$.

3.3. Collective decay rates

This point of view is supported by examining the imaginary part $\gamma_0$ for the most-collective eigenstates, $\epsilon < \epsilon - \delta \epsilon$ with $\delta \epsilon \ll \gamma_0$ suggesting that higher emissivity is associated with greater collectivity. Thirdly, the converse may not hold true. For example, it can be observed that some of collective decay rates, despite the converse may not hold true. For example, it can be observed that some of collective decay rates, despite the high degree of collectivity, fall below $\gamma_0$ when approaching $\epsilon \approx 0$, indicating subradiant spin orders that lead to destructive interference in the resultant emission.

3.4. Spin correlations

Note that the probe detuning has nothing to do with the structure of the collective states $|\mathbf{R}_f\rangle$, which are determined solely by the interaction matrix, $\mathbf{M}_R$. But the probe sets a detuning window that selects which collective states are populated. Therefore, the scattered signal must reflect the spatial order of the picked states. The definition of emission capability equation (3) is a reminiscence of the Fourier transform of $\mathbf{R}_f$. We further define

$$\tilde{F}_f (k_f) = \sum_i R_{ij}^{(f)} e^{-ik_f a_i},$$

which bares the information of components with 'spatial frequencies' $k_f$ for a given state $|\mathbf{R}_f\rangle$. Here, we only focus on the spatial variation along the $+z$ propagation axis for forward scattering. To proceed, we take the ensemble average on the Fourier spectrum $F(\epsilon, k_f) = \langle \tilde{F}_f (k_f) \rangle$, that is, an average over $\mathbf{R}_f$'s whose eigenenergies are within $\epsilon$ and $\epsilon + \delta \epsilon$ with $\delta \epsilon \ll \gamma_0$ by considering many configurations, and show the results in figure 4(a). The dotted line refers to $k_f = 2\pi/\lambda_0$, taken as a reference of spatial order in periods of $\lambda_0$. 

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Figure 3. Spontaneous decay rates of the collective states $|R_j\rangle$ versus eigenenergies for a given configuration. The colour indicates the normalised participation ratio PR. The system used here is a uniform cylindrical ensemble of $L = 3\lambda_0$, $R = \lambda_0$ and $\rho/k^3 = 0.3$.

Figure 4. (a) Fourier spectrum of the spatial frequency versus eigenenergy with the colour darkness (arbitrary unit) representing $F(\epsilon, k_f)$. (b) Maximal spatial frequency extracted from (a). The system used here is a uniform cylindrical ensemble of $L = 3\lambda_0$, $R = \lambda_0$ and $\rho/k^3 = 0.3$. The direction of the spatial ordering is along the axis of the cylinder.

Further, the pattern of $F(\epsilon, k_f)$ on both the red and blue sides appears to be stripe-like. For a given $\epsilon$, we identify the spatial frequency $k_{f_{\max}}(\epsilon)$ that maximises $F(\epsilon, k_f)$, plotted in figure 4(b). We observe that the states who possess the most distinct spatial ordering (darkest spots in figure 4(a)) coincide with those of highest emission capability. This can be expected because the characteristic $k_{f_{\max}}(\epsilon)$ of these states is closest to $k$, i.e. matching the spatial modulation of the incident light. This spatial ordering implies the cooperative nature of superradiance, and thus contributes significantly to the emission. By contrast, the states of small $|\epsilon|$ do not display a clear spatial order. This is due to random phase cancellation from averaging out a huge amount of such collective states.

Note that although the ensemble is disordered in essence, the dipole–dipole interaction impose certain spatial ordering on the collective states and give rise to prominent emission. Thus, it is expected that the ordering-enhanced emission is more significant in atomic arrays. Indeed, it has been reported that the ordered arrays show a clearer doubled-peak profile in emission spectrum [40, 41]. Our results explain the origin of the optical response from an intrinsic point of view. The discreteness of $k_{f_{\max}}$ is a reminiscence of the standing wave conditions, for which we find the spatial frequency gap $\Delta k_{f_{\max}} \sim 2\pi/L$. But we do not have exactly $k_{f_{\max}} = k$. This might be due to the size effect of a finite cylinder.

3.5. Dependence on geometry and density

Now we discuss the collective shift in different geometries and densities. We focus on two exemplary cases: spherical and cigar-shaped samples. To better approximate the actual ensembles in experiments, we consider atomic clouds of Gaussian density distribution: $\rho(r) = \rho_0 \exp[-(x^2/\sigma_x^2 + y^2/\sigma_y^2 + z^2/\sigma_z^2)/2]$, where $\rho_0 = (2\pi)^{3/2}/(\sigma_x \sigma_y \sigma_z)^{-1}N$ is the peak density. For spherical samples, in figure 5(a) we present the scattering spectrum and total excitation profile of a typical case with a peak density $\rho_0/k^3 = 0.26$, $\sigma_{x,y,z} = \lambda_0$, and $N = 1000$. Here, we only observe the central peak, apparently due to significant population of the
near-resonant state. The one-peak feature remains for spherical Gaussian samples of $\sigma_{x,y,z}$ over a few wavelengths (not shown). In these cases, the occupation of the most radiative states is very low.

The left peak becomes more evident only when the radiative spatial order can be supported by the medium. We thus expect to observe a clear signal in elongated samples. We then consider a cigar-shaped sample with $\sigma_x = \sigma_y = 0.5\lambda_0$ and $\sigma_z = 4\lambda_0$ while keeping the same number and peak density. Figure 5(b) shows the spectrum presenting a significant left peak around $\Delta = -9.3\gamma_0$ together with the central one. By checking the excitation curve, we find strong asymmetry with respect to $\Delta = 0$. The near-resonant states are still highly populated but the excitation of red-shifted states is considerably larger than that in the spherical case. That of the blue-shifted states is now strongly suppressed. This explains the two-peak spectral profile.

We further examine the dependence of the collective shift on the density in terms of the optical depth $OD = 3N/(2k^2\sigma_x\sigma_y)$ in the cigar-shaped cases of $\sigma_x : \sigma_y : \sigma_z = 1 : 1 : 10$. We vary OD in two ways while keeping the aspect ratio the same: One is to fix the peak density and change the size. The other is to fix the size and change the density. The results are plotted in figure 5(c). Both of them present similar linear relations between shifts and OD but not exactly coincide. Their differences seem to reflect the finite-size effect, which needs further investigation. As a comparison, we also demonstrate the central-peak shift, usually smaller than $\gamma_0$ by an order of magnitude, showing very different dependence on OD. Note that this shift originates mainly from the central peak of the density of states, which can be obtained by looking at the histogram of the eigenenergy distribution. Since the matrix $M_{\xi}$ is traceless, these eigenenergies must add up to be zero, thus restricting the shift from varying sensitively against OD, and presenting no linear scaling.

3.6. Angular dependence of observation

Finally, figure 5(d) discusses an interesting angular dependence of the scattering spectrum in the spherical case of figure 5(a), which has also been reported in [32]. At $\Delta = 0$, the scattering intensity is mostly owing to near-resonant states, consisting of both incoherent and coherent contributions. It is usually the coherent part that dominates, causing the forward directional emission with enhanced intensity. However, since these states ($\varepsilon \approx 0$) does not possess distinct spatial order, the coherent scattering is only confined within a narrow angle about the forward direction. When the spectrum is detected at a finite but small angle, the central peak drops rapidly, revealing the two largely-shifted peaks. Since these two peaks correspond to states presenting distinct spatial ordering, and hence are more robust against the detection angle.
4. Conclusion

In summary, our investigation into the collective Lamb shift in forward scattering within a cold dense atomic cloud has unveiled nuanced insights into the phenomena of cooperative states and shift resonances. Through Monte-Carlo simulations, we have dissected the contributions of many-body states to the observed spectral shift, differentiating between two distinct types of collective shifts that adhere to separate scaling laws. Our analysis, underpinned by an ensemble-average approach, sheds light on the pivotal role of spatial spin orders in driving superradiant emission, offering a novel perspective on the complexities of cooperative effects in atomic ensembles.

Key to our findings is the delineation of collective shifts into two categories: one resulting from the dominant occupation of near-resonant collective states and another stemming from large spatial correlations of dipoles. This distinction not only clarifies the observed spectral phenomena but also illustrates the competitions between individual and collective atomic behaviours. The nuanced understanding of these shifts, especially their dependence on system parameters like density and geometry, provides critical insights into controlling and harnessing cooperative effects in quantum optical systems.

Looking ahead, our study opens several avenues for future exploration. The intricate roles that many-body states, characterised by their spatial orders, play in collective decay rates and linewidths merit further investigation. Additionally, our findings suggest a potential parallel between the relaxation dynamics of timed Dicke states [36] and the mathematical frameworks applied in our study, hinting at a broader applicability of our approach. Such inquiries not only promise to deepen our understanding of collective quantum phenomena but also pave the way for advancing the development of quantum optical devices and applications with enhanced precision and control.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Acknowledgments

We thank the support from MOST/NSTC of Taiwan under Grant Nos. 109-2112-M-002-022, 112-2112-M-002-001 and National Taiwan University under Grant No. NTU-CC-110L890106. G D L thanks Ying-Cheng Chen, Hsiang-Hua Jen, and Ming-Shien Chang for valuable discussion and feedback.

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