Manipulating Acoustic Wavefront by Inhomogeneous Impedance and Steerable Extraordinary Reflection

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We unveil the connection between the acoustic impedance along a flat surface and the reflected acoustic wavefront, in order to empower a wide variety of novel applications in acoustic community. Our designed flat surface can generate double reflections: the ordinary reflection and the extraordinary one whose wavefront is manipulated by the proposed impedance-governed generalized Snell's law of reflection (IGSL). IGSL is based on Green's function and integral equation, instead of Fermat's principle for optical wavefront manipulation. Remarkably, via the adjustment of the designed specific acoustic impedance, extraordinary reflection can be steered for unprecedented acoustic wavefront while that ordinary reflection can be surprisingly switched on or off. The realization of the complex discontinuity of the impedance surface has been proposed using Helmholtz resonators.

Results

Theory: steerable extraordinary reflection and switchable ordinary reflection. The inhomogeneous SAI $Z_n$ of the flat surface can be expressed as a complex, whose real and imaginary parts may change spatially. In order to reduce the complexity of modeling as the beginning attempt, we set the real part as a spatial constant. Later we prove that the spatial varying of the real part cannot support our results, which is derived in detail in Supplementary Information. We consider

$$Z_n(y,\alpha) = A \left[ 1 - i \tan \frac{\psi(y)}{2} \right],$$

(1)
where $A$ is an arbitrary constant irrelevant to any spatial change and $\psi(y)$ is the variable for the imaginary part. Note that $\omega$-dependency on the right hand side of Eq. (1) has already been included in $\psi(y)$. The total acoustic pressure $p$ in the upper space satisfies the integral equation:

$$p(y,z,\omega) = p_i + p_\rho - \frac{k_0}{2\pi \sqrt{y^2 + z^2}} e^{ik_0 \sqrt{y^2 + z^2}}.$$  \hspace{1cm} (2)

$$\rho_0 c_0 \cos \theta^o \int_{-\infty}^{\infty} e^{i\psi(y_\rho)} p(y_\rho, 0, \omega) e^{-ik_0 y \sin \theta_\rho} dy_\rho.$$  

\[ \text{Reflected Pressure Field (Pa)} \]

**Figure 1**  
(a) For a flat interface with an inhomogeneous SAI, the angle of $p_{ro}$, i.e., $\theta_{ro}$, is not influenced, while $p_{re}$ occurs simultaneously and $\theta_{re}$ is controlled by IGSL. (b) If SAI is properly controlled, $p_{ro}$ is null. (c) Ultrasound with unit amplitude and $v = 300$ Krad/s impinges upon SAI surfaces in water. The SAI along the flat surface generates both $p_{ro}$ and $p_{re}$ when an arbitrary $A$ is chosen in Eq.(1). (d) A particular SAI is chosen according to Eq.(7). $\psi(y) = -100\sqrt{3}y$ is selected throughout. (e),(f) Simulation results based on impedance discontinuity with relations between $l$ and $y$ enclosed, corresponding to the cases (c) and (d) respectively. (g) Realization schematics by hard-sidewall tubes of designed lengths.
where \( p_i \) denotes the incidence; \( \rho_0 \) and \( c_0 \) are the density and the speed of sound in the upper space in Fig. 1(a); \( k_0 = \omega/c_0 \) is the wave number; \( \theta^* \) is constant; \( \theta_{re} \) is the angle of \( p_{re} \). According to Supplementary Information, both \( p_{ro} \) and \( p_{re} \) exist for a general A, implying the unusual double reflections:

\[
p_{ro} \propto \frac{2A \cos \theta_i - \rho_0 c_0}{2A \cos \theta_i + \rho_0 c_0} \exp[i k_0 (y \sin \theta_i + z \cos \theta_i)];
\]

\[
p_{re} \propto \int_{-\infty}^{\infty} e^{i\psi(y)} e^{i k_0 y (\sin \theta_i - \sin \theta_{re})} dy.
\]

After applying the first-order approximation and the stationary phase approximation to Eq. (4), the relation between \( \theta_{re} \) and the incident angle \( \theta_i \) is unveiled:

\[
\begin{align*}
    k_0 (\sin \theta_i - \sin \theta_{re}) &= d \frac{\psi(y)}{dy} \\
    Z_0(y,\omega) &= A \left[ 1 + i \tan \left( \frac{\psi(y)}{2} \right) \right].
\end{align*}
\]

Note that based on our derivation, only when the inhomogeneous SAI along the flat surface is expressed in form of Eq. (1) can our IGSL survive. Although IGSL’s appearance is similar to GSL, its physical meaning of \( \psi(y) \) are dramatically different. Fundamentally, the variable of our IGSL Eq. (5) is about the value of surface acoustic impedance instead of the abrupt propagating phase change. Moreover, IGSL only serves to steer \( p_{re} \), at will, with no influence on the direction of \( p_{ro} \), as illustrated in Fig. 1(a). In Supplementary Information we highlight the irrelevance between GSL and our proposed IGSL. In addition, GSL mentions the extra accumulated phases along wave-propagation paths, but it is still relying on graphical methods to find out the relation between the configuration of the passive antenna array and the needed phase in optics. However, we do not have the passive antenna in acoustics. Here, IGSL Eq. (5) and Eq. (1), serving as an explicit design rule, provide us the feasible way based on a different mechanism in acoustics.

Eq. (5) also sheds light on an extreme angle (similar to critical angle):

\[
\theta_{re} = \begin{cases} 
    \arcsin \left(1 - \frac{1}{k_0} \frac{\psi(y)}{dy}\right), & \text{if } \frac{\psi(y)}{dy} < 0 \\
    \arcsin \left(1 + \frac{1}{k_0} \frac{\psi(y)}{dy}\right), & \text{if } \frac{\psi(y)}{dy} > 0
\end{cases}
\]

above which \( p_{re} \) becomes evanescent in the upper space. Eq. (6) holds only when \(-1 \leq 1 - \frac{1}{k_0} \frac{\psi(y)}{dy} \leq 1\). Otherwise, \( p_{re} \) becomes evanescent.

Usually, both \( p_{ro} \) and \( p_{re} \) will coexist as shown in Fig. 1(a), suggesting double reflections, while IGSL only controls \( \theta_{re} \). Hence, it is interesting to eliminate \( p_{ro} \) as shown in Fig. 1(b), by means of a particularly selected value of \( A \). Eq. (3) suggests that \( A = (\rho_0 c_0)/(2 \cos \theta_{re}) \) can make \( p_{ro} \) vanish, i.e., \( p_{ro} \) is switched off, as shown in Fig. 1(b). The corresponding SAI of the flat surface then becomes

\[
Z_0(y,\omega) = \frac{\rho_0 c_0}{2 \cos \theta_{re}} \left[1 - i \tan \left( \frac{\psi(y)}{2} \right) \right].
\]

**Verification: continuous impedance and discontinuous impedan-ce.**

Supposing the gradient of \( \psi(y) \) along the flat interface is constant, we notice Eq. (4) turns out to be a Dirac Delta without approximation. From Eq. (5), we predict the wavefront of \( p_{ro} \) will propagate in the form of a plane acoustic wave, independent of \( y \). We select water (\( \rho_0 = 1 \text{ kg/m}^3; c_0 = 1500 \text{ m/s}^{2/3} \)) as the background medium, \( \omega = 300 \text{ K rad/s} \) as the circular frequency, \( e^{-ik_0 z} \) as the normal incident plane ultrasound, and a linear form \( \psi(y) = -100\sqrt{3}y \) in Eq. (7).

\( \theta_{re} \) is theoretically found to be \(-60^\circ\) by IGSL, validated by our simulation in Fig. 1(d), \( p_{ro} \) is thoroughly suppressed thanks to the specific \( A \) chosen according to Eq. (1). In contrast, in Fig. 1(c), the same parameters are kept except for another \( A \), whose value is arbitrarily taken to be \( \rho_0 c_0 \). It clearly shows that \( p_{ro} \) occurs and interferes with \( p_{re} \), but \( p_{re} \) still keeps the same, verifying our theoretical formulation. In terms of phenomena, the designed inhomogeneous SAI Eq. (1) essentially implies the changes of both the propagating phases and amplitudes, only by which the effect of double reflections may occur. In terms of physics, the extra momentum supplied by the metasurface is employed to compensate the momentum mismatch between the incident acoustic beams and the diffracted beams. Therefore, for the double backward propagating beams, \( p_{ro} \) is the most pervasive specular reflection, while \( p_{re} \) is attributed to the diffraction of higher order.

Fig. 1(d) suggests the possibility of negative reflection for \( p_{re} \) which is further verified for oblique incidence in Fig. 2. In Fig. 2(a), because of the inhomogeneous SAI and the arbitrary \( A \) in Eq. (1), both \( p_{ro} \) and \( p_{re} \) occur. Fig. 2(b) depicts the same situation except for \( p_{ro} \) being switched off as a result of the specifically chosen \( A \) according to Eq. (7), while the red line \( p_{re} \) stays the same as that in Fig. 2(a). The blue braces represent the region of negative \( p_{re} \). It is noteworthy that \( p_{ro} \) does not exist if \( \theta_{re} \) is beyond the extreme angle \( \theta_{re} = -30^\circ \) in Eq. (6), corresponding to the purple dots. One field simulation is provided in Supplementary Information.

As depicted in Fig. 1(g), we propose one plausible realization schematic for the general SAI of Eq. (1), where all hard-sidewall tubes with one pressure-release termination are gathered and juxtaposed perpendicularly to the flat interface. Observed at the top view, each tube has a square cross section whose width is \( d \), with four enclosed hard sidewalls (black). Then observed at the side view, the upside open termination of each tube constitutes a perfect SAI of the interface, while the other end sealed by a thin film (orange) serves as the pressure-release termination\(^{11} \). The upper space and the interior of each tube are filled with water, without separation. The light blue indicates air downside, which is isolated from water by the thin film.

The SAI of each tube at the opening facing the upper space is\(^{12} \):

\[
Z_0(y,\omega) = \frac{\rho_0 c_0 k_0^2 d^2}{2\pi} - i\rho_0 c_0 \tan \left[ k_0 l(y) + k_0 \Delta l \right],
\]

where \( l(y) \) is the length of each tube and \( \Delta l = 0.6133d/\sqrt{\pi} \) is the effective end correction. By comparison of Eq. (1) and Eq. (8), it is required that \( A = \rho_0 c_0 k_0^2 d^2/(2\pi) \) and \( A \tan \left[ \psi(y)/2 \right] = \rho_0 c_0 \tan \left[ k_0 l(y) + k_0 \Delta l \right] \), leading to the value of the spacing \( d \) for impedance discretization and the dependence between \( l(y) \) and \( \psi(y) \):

\[
\begin{align*}
    d &= \sqrt{(2\pi A)/(\rho_0 c_0 k_0^2)} \\
    l(y) &= \frac{1}{k_0} \arctan \frac{k_0^2 d^2}{2\pi} \tan \left( \frac{\psi(y)}{2} \right) + \frac{2k_0^2}{\omega} - \Delta l
\end{align*}
\]

where the arbitrary integer \( n \) is required to be set suitably to make \( l \) a positive value. Thus, the change of \( \psi \) along \( y \), representing the control of \( p_{re} \), is interpreted as the change of \( l \), implying one straightforward realization based on discontinuous impedance. Thus, the inhomogeneity of the acoustic impedance is strictly paraphrased into the inhomogeneity of the tube-array structure, resulting in our acoustic metasurface. At the side view in Fig. 1(g), the solid red contour indicates one arbitrary function of \( l(y) \) calculated from Eq. (9). Based on the discretization \( d \) calculated from Eq. (9) as well, we are able to find \( d \) and the corresponding height \( l(y) \), marked with the yellow dots. Note that the top of the tube array is aligned into a flat surface (red dashed line), above which acoustic waves impinge. Thus, the change of tube lengths will not affect the flatness of the surface. In addition, thanks to the property of the arc-tangent in Eq. (9), the tube-array metasurface is within a thin layer without the space-coiling-up technique\(^{13} \). It is also noteworthy that because of the intrinsic differences between optics and acoustics, so far we cannot obtain the mechanism-analog of the optical metasurface, which is based on resonances and independent with the thickness or
effective propagating lengths, but we can achieve the phenomenon-analog in acoustics using the tube array. In principle, because tubes can be regarded as Helmholtz resonators, complex SAI at each pixel can be realized by a suitable arrangement of resonators, as the analog of the complex electric impedance realized by the combination of resistance, capacitance and inductance. In addition, we know that only the real part, the electric resistance, consumes energy while the imaginary part does not. In the same manner in acoustics, the energy loss is theoretically only attributed to the real part of the surface impedance in Eq. (8), i.e., the loss in our case is caused by the imaginary part, re. In the same manner in acoustics using the tube array. In principle, because tubes can be regarded as Helmholtz resonators, complex SAI at each pixel can be realized by a suitable arrangement of resonators, as the analog of the complex electric impedance realized by the combination of resistance, capacitance and inductance. In addition, we know that only the real part, the electric resistance, consumes energy while the imaginary part does not. In the same manner in acoustics, the energy loss is theoretically only attributed to the real part of the surface impedance in Eq. (8), i.e., the loss in our case is caused by the imaginary part, re.

Using this method, we reproduce Fig. 1(c)(d) by realistic impedance discontinuity, so as to verify our proposed realization. In Fig. 1(e)(f), d = 0.0125 and 0.00886 are selected respectively according to Eq. (9), and the corresponding contours of the tube length l in terms of the location y are illustrated as the red lines, respectively. Fig. 1(e) shows strong interference between pre and pro, while Fig. 1(f) shows the nearly undisturbed pro, coinciding with Fig. 1(c) and (d) respectively.

**Application: acoustic illusion and ipsilateral focusing.** To demonstrate IGSL’s capability of designing novel acoustic devices, we metamorphose acoustic pressure fields everywhere through SAI manipulation as simulated in Fig. 3. This deceptive effect is obtained by manipulating plane wavefronts generated by a virtual reflector or focusing illumination, governed by the control of pro, i.e., IGSL. Under these scenarios, we need to consider nonlinear forms of $\psi(y)$. New phenomena are thus expected when $\theta_{re}$ becomes spatially varying.

It is found that the acoustic deception can be created via IGSL, e.g., $\psi(y) = 0.7 y^2$ in Eq. (7), resulting in $p_{re} = 0$. Correspondingly, $\theta_{re}$ in Fig. 3(a) is a position-dependent function $\sin \theta_{re} = 0.14 y$, in which case $p_{re}$ fans out as demonstrated in Fig. 3(a), verifying our theory. Here the spacing d for impedance discretization is 0.1772 and the relations between f and y derived from Eq.(9) are enclosed in Fig. 3. Therefore, IGSL can be employed to camouflage a flat surface as if there were a curvilinear object at the origin instead of the physical planar interface. The dual effect allowing a curved reflector to mimic a flat mirror, by manipulating the convex wavefronts into planar wavefronts, was reported in plasmonic regime19.

Furthermore, the SAI can be designed to make acoustic waves reflected by a planar interface focused as well. In optics, a flat lens with metallic nanoantennas of varying sizes and shapes can consequently converge the transmitted light to a focal point44. Note that the optical focusing controlled by optical GSL is on the other side of incoming lights, i.e., on two sides of the flat surface in the

**Figure 2 | sin$\theta_{re}$, re versus sin $\theta$ when $k_0 = 10$ rad/m and $\psi(y) = -5 y$.** $p_{re}$ and $p_{ro}$ emerge simultaneously in (a). In (b), only $p_{ro}$ occurs for the same parameters of (a) except A. The purple dot denotes $\sin \theta_i$ in Eq. (6).

**Figure 3 | Wavefront metamorphosis via SAI interface, with impedance discontinuity d = 0.1772.** A plane acoustic wave of $\theta = 15$ Krad/s is normally incident in water. Only reflected acoustic pressure is plotted. (a) The SAI of Eq.(7) with $\psi(y) = 0.7 y^2$ is set along the flat surface. $p_{re}$ diverges into a curved wavefront. (b) The SAI of Eq.(7) with $\psi(y) = -10(\sqrt{y^2 + 4^2} - 4)$ is set. $p_{re}$ converges to a focal point in the two-dimensional case.
transmission mode. In acoustics, we employ an inhomogeneous SAI flat surface to focus $p_{re}$ in the reflection mode by IGSL without $p_{ro}$.

This ipsilateral focusing in Fig. 3(b), is thus found in the planar geometry in acoustics for the first time. In Eq. (7), a hyperbolic form is set: $\psi(y) = -k_0(\sqrt{y^2 + f^2} - f)$ ($f$ being the given focal length) for the SAI of the flat interface. $p_{re}$ from different angles constructively interferes at the ipsilateral focal point, as if the waves emerge from a parabolic surface. The parameters in Fig. 3(b) are the same as those in Fig. 3(a) except for the specific hyperbolic SAI form $\psi(y) = -10(\sqrt{y^2 + 4^2} - 4)$, with the designed focal point at $(y = 0, z = 4)$ and $p_{ro}$ suppressed. In addition, the simulated acoustic pressure by impedance discretization at the focal point is well confined at $(y = 0, z = 4)$.

Interestingly, the imaging at the same side was previously presented for electromagnetic waves. In acoustics, our ipsilateral imaging is achieved by translating all the stringent requirements of the half-space chiral materials into an inhomogeneous impedance surface. In electromagnetism, ipsilateral imaging can be achieved as well by surface gratings or antenna arrays. However, the polarization of incident electromagnetic waves is always closely related to the effect of focusing. Therefore, the ipsilateral imaging in acoustics by IGSL has no polarization constraints thanks to the acoustic wave nature, i.e., longitudinal vibration.

**Application: conversion from propagating acoustic waves to surface acoustic waves.** Beyond the acoustic-field metamorphosis, we further establish a kind of acoustic cognitive deception about a SAI surface converting propagating acoustic waves (PAW) to surface acoustic waves (SAW) in Fig. 4, by means of IGSL. The extreme angle $0^\circ$ in Eq. (6) demands $\psi(y) = \pm 10\ y$. Therefore, we set the SAI of Eq. (7) slightly over that extreme, e.g., $\psi(y) = -11\ y$ for $y < 0$ and $\psi(y) = 11\ y$ for $y > 0$ are set along the flat interface symmetrically with respect to the z. In Fig. 4(a), the bidirectional surface acoustic waves are attributed to the coupling effect governed by the diffracted evanescent $p_{re}$ which propagates along the metasurface. Owing to the inhomogeneous SAI interface, the ideally perfect conversion comes true in acoustics except for a little diffraction. Physically, the SAI along the flat surface provides an extra momentum to compensate the momentum mismatch between propagating waves and surface waves in acoustics, resulting in the high efficiency conversion. In contrast, if one uses a constant SAI Eq. (7) with $\psi(y) = 11$ along the flat surface (the homogeneous SAI does not generate $p_{re}$; only $p_{ro}$ occurs), the reflected sound pressure level in Fig. 4(c) is almost uniformly spread over the space.

Fig. 4(b) clearly demonstrates that the acoustic field is well confined in the region close to the interface and attenuated quickly to around 0 Pa away from the interface, revealing the nearly perfect conversion. Interestingly, it shows in that the electromagnetic-varying metasurface is able to prevent the propagating electromagnetic waves from being reflected back to the upper space. Hence, our PAW-SAW conversion in acoustics, originating from a distinguished mechanism, is differentiated from.

In Fig. 4, we notice such technology is functional as an alternative invisible acoustic cloak by trapping the acoustic field in the vicinity of the coating, resulting in much lower signal of reflection. It may pave the avenue to the large size acoustic invisibility since it is only dependent on the surface technique instead of wave-interaction based metamaterial acoustic cloaking. It will also be promising to consider the time-varying surface technique in acoustics with non-reciprocal diffraction in the future.

**Discussion**

Here, IGSL is established for novel manipulation of acoustic wavefronts. Due to the lack of abrupt-phase-changing surface structures in acoustics, we resort to specific acoustic impedance as the variable to tweak the reflection. IGSL, which can simultaneously generate the switchable $p_{ro}$ and the steerable $p_{re}$, provides us the explicit connection between our designed SAI and the reflected field, serving as the

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**Figure 4 | Conversion from PAWs to SAWs via SAI interface.** The PAW with unit amplitude and $\omega = 15\ Krad/s$ is normally incident in water. Only reflected acoustic pressure is plotted. (a) The SAI of Eq. (7) is set to be $\psi(y) = -11\ y$ for $y < 0$ and $\psi(y) = 11\ y$ for $y > 0$. SAWs are bifurcated at the normal incidence (1Pa) and confined near the surface. (b) The reflected sound pressure level of (a). (c) The reflected sound pressure level when a homogeneous SAI is adopted instead.
design rule in acoustics. We not only demonstrate intriguing acoustic
manipulations but also provide insightful realization schemes. As a
few examples, we demonstrate acoustic disguise, acoustic planar lens,
acoustic ipsilateral imaging and acoustic PAW-SAW conversion.
These novel effects will inspire new technologies on acoustic wave
engineering, leading to unprecedented applications.

Methods
For theoretical derivations, we used Green’s function, the integral equation Eq. (2)
and Born approximation. The detailed theoretical development is elaborated in
Supplementary Information. For the numerical calculations, we used the Finite
Element Method by means of COMSOL Multiphysics. The left, right and top sides of
the meshed domain are set as plane wave radiation conditions, while the bottom side
is set as the impedance boundary with a certain value.

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Author contributions
J.Z. and C.W.Q. developed the theory, performed the numerical experiments, and prepared
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C.W.Q. conceived the idea.

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