Direct observation of vortices in an electron fluid

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Vortices are the hallmarks of hydrodynamic flow. Recent studies indicate that strongly-interacting electrons in ultrapure conductors can display signatures of hydrodynamic behavior including negative nonlocal resistance [1–4], higher-than-ballistic conduction [5–7], Poiseuille flow in narrow channels [8–11], and a violation of the Wiedemann-Franz law [12]. Here we provide the first visualization of whirlpools in an electron fluid. By utilizing a nanoscale scanning superconducting quantum interference device on a tip (SQUID-on-tip) [13] we image the current distribution in a circular chamber connected through a small aperture to an adjacent narrow current-carrying strip in the high-purity type-II Weyl semimetal WTe$_2$. In this geometry, the Gurzhi momentum diffusion length and the size of the aperture determine the vortex stability phase diagram. We find that vortices are present only for small apertures, whereas the flow is laminar (non-vortical) for larger apertures. Moreover, near the vortical-to-laminar transition, we observe a single vortex in the chamber splitting into two vortices, a behavior that can occur only in the hydrodynamic regime but is not expected for ballistic transport. These findings suggest a novel mechanism of hydrodynamic flow in thin pure crystals: the spatial diffusion of electrons’ momenta is enabled by small-angle scattering at the planar surfaces, instead of the routinely invoked electron-electron scattering, which becomes extremely weak at low temperatures. This surface-induced para-hydrodynamics, which mimics many aspects of the conventional hydrodynamics, including vortices, opens new avenues for exploring and utilizing electron fluidics in high-mobility electron systems.
Recent years have seen a quest for systems in which strong electron-electron interactions and scattering lead to electron flows governed by hydrodynamics [14], as in viscous fluids, rather than by Ohmic transport. Common fluids display two distinct hydrodynamic regimes [15]: laminar flows in which neighboring sheets move at gradually varying velocities, and turbulent flows characterized by eddies and vortices with counterflow that develop into chaotic behavior at large scales. The transition from laminar to turbulent flow is usually associated with nonlinear fluid dynamics, described by the Navier–Stokes equations. Yet, hydrodynamic vortices in Newtonian fluids occur already in linear Stokes flow [16].

In contrast to common fluids, which routinely display hydrodynamic phenomena, evidence for hydrodynamics in electron fluids has remained scarce [17,18] until recently. The advent of high purity single crystals, clean van der Waals heterostructures, and high mobility 2D systems has accelerated the observation of fluid-like behavior in semiconductors and semimetals [1–12,19–26], triggering many theoretical works [19,27–46]. Recently, laminar Poiseuille flow in narrow strips has been demonstrated in graphene [8–10] and in WTe$_2$ [11] by employing Hall potential imaging and diamond nitrogen-vacancy magnetometry, supporting the hydrodynamic nature of electron fluids in these systems. Yet, the most striking behavior ubiquitous in common fluids -- the formation of vortices and turbulence -- has not yet been observed in electron fluids despite numerous theoretical predictions based on linear [47–49] and nonlinear [29,34,50] hydrodynamics. Transport measurements showing negative nonlocal resistance in the vicinity of the current injection point are suggestive of electron whirlpools in graphene and GaAs heterostructures [1–4]. Recent studies, however, indicate that the observed negative potentials may arise in ballistic and hydrodynamic regimes even without an actual electron backflow [3,32,35]. Hence, direct observation of vortices and the study of their properties remains an outstanding challenge in electron fluids.

The conventional picture of electron fluids [14] involves two length scales -- the momentum-relaxation length $l_{mr}$ describing momentum transfer from electrons to the lattice and the length $l_{ee}$ describing momentum transfer between the carriers due to electron-electron collisions. A fluid-like regime that can harbor vortices and other hydrodynamic effects is expected when $l_{ee} \ll l_{mr}$, at the lengthscales $l_{ee} < W < l_{mr}$, where $W$ is system size. In common metals, the shortest length scale is instead $l_{mr}$. In this case, transport is described by Ohm’s law $J = -\sigma \nabla \phi = \sigma E$ and the continuity equation $\nabla \cdot J = 0$, where $\phi$ is the electrostatic potential and $E$ is the electric field. In this regime, no vortices can exist since $\nabla \times J = \sigma \nabla \times E = 0$ in the steady state. Here, $J$ and $\sigma$ denote the 2D current density and conductivity. In ultraclean systems at low temperatures, however, very large values of $l_{mr} \gg W$ can be achieved. If both $l_{ee}$ and $l_{mr}$ exceed $W$, the transport is ballistic, wherein electrons propagate essentially unimpeded, with scattering occurring mainly at system edges. Ballistic transport can also lead to vortices [3,22,35,45].

In this work, we provide a direct visualization of vortices in an electron fluid, occurring at low temperatures in what can be best described as a para-hydrodynamic regime. In our system, the conventional notion of electron viscosity is not relevant because of the large $l_{ee}$ values. Instead, the role of $l_{ee}$ is taken on by energy-conserving small-angle scattering, presumably originating from long-wavelength disorder such as surface roughness. This is a one-particle process that remains operational at arbitrarily low temperatures. The rate associated with this process governs an effective viscosity, yielding Stokes equation with an ohmic term. Para-hydrodynamics occurs in the absence of electron-electron scattering, but mimics many aspects of hydrodynamics, including vortices.

Since vortical flow requires large $l_{mr}$ both in the hydrodynamic and ballistic regimes, we have set out to synthesize ultraclean single crystals of the Weyl semimetal WTe$_2$ as described in Methods, achieving residual resistance ratio ($RRR$) of over 3,000 (Extended Data Fig. 1). The crystals were exfoliated into flakes of thickness $d$ of 23 to 48 nm and patterned into various geometries using e-beam lithography and plasma etching (Methods). The primary geometry consists of a central strip of width $W = 550$ nm with two truncated circular chambers of radius $R = 900$ nm connected to its sides through apertures defined by the opening angle $\theta \leq 180^\circ$ (see AFM images in Fig. 1c and Extended Data Fig. 9a-c). Analogous geometries
were patterned in Au films of similar thicknesses for comparison. An ac current with rms amplitude $I_0$ of 1 to 400 µA was applied to the samples at $T = 4.5$ K. The corresponding out-of-plane component of the Oersted field $B_z(x, y)$ was imaged with a superconducting quantum interference device fabricated on the apex of a sharp pipette (SQUID-on-tip, SOT) [13]. Scanning was performed at a height $h = 50$ nm above the sample surface (see Figs. 1a,b, Methods, and Extended Data Fig. 2).

**Ohmic flow**

We first examine the current flow in the Au films. Figure 1f shows $B_z(x, y)$ measured above the Au sample with $\theta = 180^\circ$, corresponding to a strip with two half-disc chambers. By inversion of the magnetic field $B_z(x, y)$ [51], we reconstruct the 2D current density $J(x, y)$ (Methods), with $J_x(x, y)$ and $J_y(x, y)$ components presented in Figs. 1d,g. The $x$ and $y$ directions, are defined in Fig. 1e. The longitudinal $J_y$ component demonstrates that the current flowing upwards in the central strip spreads out into the two chambers. The current flows into the right chamber through its lower half (red $J_x$ in Fig. 1g), circulates counterclockwise, and exits through the upper half (blue $J_x$ in Fig. 1g). The left chamber shows a mirrored flow pattern. The finite-element numerical simulations, performed using COMSOL (Methods), in the ohmic regime in Figs. 1e,h show good agreement with the experimental data, describing a laminar (non-vortical) flow pattern. The finite-element numerical simulations, performed using COMSOL (Methods), in the ohmic regime in Fig. 1i. Upon decreasing the aperture size, less current enters the chambers as seen by $J_y(x, y)$ in Fig. 1j for $\theta = 45^\circ$. The $J_x(x, y)$ in Fig. 1m shows qualitatively similar behavior, with current flowing counterclockwise in the right chamber. Numerical simulations of $J_y$ and $J_x$ in Figs. 1k,n agree with the experimental data; the streamlines in Figs. 1o show laminar flow.

We note that usually ‘laminar’ means no turbulence. Here, in a linear response regime, turbulence is not encountered. And yet, vortices may or may not appear depending on the dynamical phase of the electron fluid. Indeed, a vortex is a flow in which the streamlines form closed loops. Therefore, streamlines that go from source to drain without forming closed loops will be referred to hereafter as laminar.

**Vortical flow**

For large opening angle, $\theta = 120^\circ$, the current flow pattern in a WTe$_2$ sample (Fig. 2a) looks qualitatively similar to that of Au with the $J_y(x, y)$ component spreading substantially into the chambers. The corresponding $J_x(x, y)$ in Fig. 2d shows counter-clockwise (clockwise) flow in the right (left) chamber, similar to the laminar flow in Au in Figs. 1g,m. To quantify the expected behavior in the hydrodynamic regime, $l_{ee} < W < l_{mr}$, we conducted finite-element numerical simulations to solve the linearized Navier-Stokes equation with an ohmic term [1,30,47],

$$-D^2 \nabla^2 \mathbf{J} + \mathbf{J} = -\sigma \nabla \phi,$$

where $D$ is the Gurzhi length, usually defined as $D = \sqrt{l_{ee} l_{mr}}/2$. The resulting $J_y(x, y)$ and $J_x(x, y)$ in Figs. 2b,e show good agreement with the experimental data. The calculated current streamlines in Fig. 2f show a laminar flow resembling the ohmic regime in Figs. 1i,o.

The flow pattern changes drastically as the aperture size becomes smaller, as illustrated in Figs. 2g,j for the case of $\theta = 20^\circ$. While $J_y(x, y)$ remains large in the central strip and relatively small in the chambers (Fig. 2g), an essential difference between the flow in WTe$_2$ and the ohmic flow in Au is revealed in the transverse component $J_x(x, y)$. On approaching the aperture from below, $J_x$ is initially directed to the right (red) towards the right chamber. Yet, rather than maintaining its flow into the chamber as in Fig. 2d, $J_x$ switches its direction and flows to the left (blue). Similarly, in the top half of the chamber, $J_x$ flows out of the chamber near the aperture (blue), but into the chamber further away from the aperture (red). Near the aperture the current thus flows counterclockwise while in the interior of the chamber, the current circulates clockwise. Namely, a clockwise (anticlockwise) current vortex is formed in the right (left) chamber. The hydrodynamic finite-element simulations of $J_y(x, y)$ and $J_x(x, y)$ in Figs. 2h,k confirm this picture, with the vortices in the two chambers represented by closed-loop streamlines (blue) in Fig. 2l (see...
The presence of vortices can be also confirmed by direct inspection of $B_z(x, y)$ without relying on current reconstruction as shown in Extended Data Fig. 5. To the best of our knowledge, this constitutes the first direct observation of current vortices in an electron fluid.

Fig. 1. Ohmic electron flow in Au film. a-b, Schematic experimental layout showing the scanning SOT and the Au (a) and WTe$_2$ (b) samples with double-chamber geometry. The red curves indicate laminar (open-loop) current streamlines while the blue curves represent closed-loop vortex streamlines. c, Atomic force microscope (AFM) topography image of WTe$_2$ sample with $\theta = 45^\circ$. $W$, $R$, $\theta$, and $\Delta$ are the width of the central strip, radius of the circular chambers, aperture angle, and size of the aperture $\Delta = 2R \sin(\theta/2)$, respectively. d-i, Measurements and simulations of Au sample with $\theta = 180^\circ$. d, Current density $J_y(x, y)$ normalized by $I_0/W$ reconstructed from $B_z(x, y)$ in (f). The black contours mark the sample edges. e, Simulated $J_y(x, y)$ in the ohmic regime. f, $B_z(x, y)$ measured by the SOT above the Au sample under current $I_0 = 50$ µA at 4.5 K. g, $J_x(x, y)$ reconstructed from (f). The light blue texture outside the sample is an artifact of current reconstruction (Methods). h, Simulated $J_x(x, y)$. i, Simulated current streamlines. j-
Measurements and simulations of Au sample with $\theta = 45^\circ$. j, Current density $J_y(x, y)$ reconstructed from (l). k, Simulated $J_y(x, y)$ in the ohmic regime. I, $B_z(x, y)$ profiles in the Au sample carrying $I_0 = 50 \mu$A. m, $J_x(x, y)$ reconstructed from (l). n, Simulated $J_x(x, y)$. o, Simulated current streamlines.

Fig. 2. Laminar and vortical flows in WTe$_2$. a-f, Measurements of WTe$_2$ sample with $\theta = 120^\circ$ and corresponding simulations in the hydrodynamic regime (Eq. 1) with $D/W = 0.28$ and $\xi = 200$ nm. a, Current density $J_y(x, y)$ normalized by $I_0/W$ reconstructed from (c). b, Simulated $J_y(x, y)$. c, $B_z(x, y)$ measured by the SOT above the WTe$_2$ sample under current $I_0 = 50 \mu$A at 4.5 K. d, $J_x(x, y)$ reconstructed from (c). e, Simulated $J_x(x, y)$. f, Simulated current streamlines showing laminar flow. g-l, Measurements of WTe$_2$ sample with $\theta = 20^\circ$ and corresponding simulations in the hydrodynamic regime. g, Current density $J_y(x, y)$ reconstructed from (l). See Extended Data Figs. 2k-o in which the color scale is expanded so that the counterflow vortical current $J_y(x, y)$ is resolved. h, Simulated $J_y(x, y)$. i, $B_z(x, y)$ measured in the WTe$_2$ sample carrying $I_0 = 50 \mu$A. j, $J_x(x, y)$ reconstructed from (l). k, Simulated $J_x(x, y)$. l, Simulated current flow showing laminar (open-loop, red) and vortical (closed-loop, blue) streamlines.
Hydrodynamic and ballistic vortex stability phase diagram

Though the hydrodynamic simulations show good agreement with the experimental data, the question arises whether ballistic trajectories may create similar vortex patterns. Indeed, several studies have pointed out the difficulty in distinguishing the hydrodynamic and ballistic regimes using transport data [3,22,35,36,45]. Moreover, recent studies of current profiles in a WTe2 whisker suggest ballistic transport at low temperatures [11]. A key aspect that enables formation of vortices in both the hydrodynamic and ballistic regimes is system geometry [32]. Another key aspect is the boundary conditions, which has been an outstanding question in electron hydrodynamics from its early days [1,8–11,14,17,18,30,32,48,49,52,53]. The boundary conditions for the perpendicular and parallel current components are given by \( \mathbf{J}_{\perp} = \mathbf{J} \cdot \hat{n} = 0 \) and \( \mathbf{J}_{\parallel} = \xi \hat{n} \cdot \nabla \mathbf{J}_{\parallel} \), where \( \hat{n} \) is the unit vector perpendicular to the boundary and \( \xi \) is the slip length (Methods). The slip length can vary from \( \xi = 0 \) for no-slip \( (\mathbf{J}|_{\text{boundary}} = 0) \) to \( \xi = \infty \) for no-stress boundaries \( (\hat{n} \cdot \nabla \mathbf{J}_{\parallel}|_{\text{boundary}} = 0) \). A full treatment of the electron transport would require solving the Boltzmann kinetic equation, a challenging endeavor in an arbitrary 2D geometry [4,44]. However, for realistic parameters, Eq. 1 serves as a good model in the regime of interest, \( D/W \lesssim 1 \), as well as in the quasi-ballistic regime, \( D/W \gtrsim 1 \), provided no-stress boundary conditions are used [36] (see Methods).

To model vortex formation, we solve Eq. 1 in the two-chamber geometry and compute the total counterflow current \( I_v \) carried by the vortex, \( I_v = \int_{w/2}^{-w/2} \left( |J_y(x,0)| - J_y(x,0) \right) dx / 4 \), where \( w = W + 2R[1 + \cos(\theta/2)] \) is the width of the structure in its widest section. Figures 3a,b show the resulting vortex stability phase diagram as a function of the aperture angle \( \theta \) and the ratio of the Gurzhi length to the strip width, \( D/W \), for no-stress and no-slip boundary conditions (see Methods). The resulting phase diagrams are quite similar, predicting that in the quasi-ballistic regime \( D/W \gtrsim 1 \) the vortices feature large counterflow \( I_v \), are stable up to large angles \( \theta \), and show weak dependence on \( D/W \). In the hydrodynamic regime contrarily, the counterflow \( I_v \) is lower, the presence of vortices is limited to smaller \( \theta \), and the vortex stability is strongly dependent on \( D/W \), with a linear vortex-to-laminar (no-vortex) phase transition line \( \theta_t(D/W) \) (dashed green line), essentially independent of the boundary conditions. Nonetheless, the total circulating current \( I_v \) carried by the vortex strongly depends on the boundary conditions, showing an approximate four-fold reduction in the maximum \( I_v \) for no-slip boundaries.

These findings imply that by fabricating a series of samples with varying \( \theta \), one can pinpoint the transition angle \( \theta_t \). This allows to i) determine whether the flow is ballistic (large \( \theta_t \)) or hydrodynamic (small \( \theta_t \)), ii) evaluate the maximum \( I_v \), and iii) extract the value of \( D \) in the hydrodynamic case. Moreover, the derived \( I_v \) can provide an estimate of the electron slip length \( \xi \). Accordingly, we have fabricated six samples from a single WTe2 flake with a sequence of aperture angles \( \theta = 20^\circ, 35^\circ, 54^\circ, 72^\circ, 90^\circ, \) and \( 120^\circ \) (Extended Data Fig. 9a). The observed current flow patterns are presented in Figs. 2, 3, and 4 (see Methods and Extended Data Figs. 9 and 10 for additional samples and geometries). The six data points are mapped onto the two phase diagrams in Figs. 3a,b. A single vortex in each chamber is observed in the two samples with smallest \( \theta \) (marked by \( \bigcirc \)), a double-vortex is found in the \( \theta = 54^\circ \) sample (marked by \( \bigotimes \)), while laminar flow is found for the three largest \( \theta \) (marked by \( \times \)). As described below, the \( \theta = 54^\circ \) sample resides very close to the phase transition line, which allows us to identify the transition angle \( \theta_t \approx 54^\circ \). The small value of \( \theta_t \) clearly establishes the hydrodynamic nature of the observed current vortices.
Fig. 3. Vortex stability phase diagram. a-b, Vortex stability phase diagram showing the magnitude of the circulating vortex current, $I_v$, in the chambers vs. the aperture angle $\theta$ and the Gurzhi length scaled by the strip width, $D/W$, for no-stress, $\xi = \infty$ (a), and no-slip, $\xi = 0$ (b), boundary conditions. The dashed line indicates the vortical-to-laminar phase transition line $\theta_t(D/W)$ in the $D/W \ll 1$ limit. The symbols $\bigcirc$, $\bigotimes$, and $\times$ mark the parameters of the chambers that feature single vortex, double vortex, and no vortices, respectively. The double-vortex state at $\theta = 35^\circ$ is described in Fig. 5.

c-f, Measured $J_x(x, y)$ in WTe$_2$ sample with $\theta = 20^\circ$ (c) and the corresponding simulated $J_x(x, y)$ for $D/W = 0.28$ and electron slip length at the edges of $\xi = \infty$ (d), 200 nm (e), and 0 (f). g-j, Measured $J_y(x, y)$ in WTe$_2$ sample with $\theta = 35^\circ$ (g) and the corresponding simulated $J_x(x, y)$ for $\xi = \infty$ (h), 200 nm (i), and 0 (j).

Gurzhi length and boundary conditions

The obtained $\theta_t$ value translates into $D/W \approx 0.28$, which for $W = 550$ nm in our devices yields the Gurzhi length $D \approx 155$ nm, nearly independent of the boundary conditions in this range of parameters. The boundary conditions, however, strongly impact the vortex current $I_v$ as seen in Fig. 3. The experimentally derived $J_x(x, y)$ in $\theta = 20^\circ$ and $35^\circ$ samples are shown in Figs. 3c,g alongside simulated $J_x(x, y)$ for three values of the slip length, $\xi = 0$, 200 nm, and $\infty$. Figures 3f,j demonstrate that, for no-slip boundary conditions, the circulating current in the vortex is much weaker than the one measured experimentally. Circulating currents comparable to the experimental values in Figs. 3c,g can be achieved only for large $\xi \gtrsim 200$ nm, as shown in Figs. 3e,j. In this limit, the resulting current distribution is nearly identical to the one found for the no-stress boundary conditions, $\xi = \infty$, in Figs. 3d,h. The relatively low values of the
measured $J_x/(I_0/W) \cong 0.02$, as compared to the significantly higher values calculated for the ballistic regime, provide additional evidence for the hydrodynamic nature of the observed vortices.

Our finding of a large slip length $\xi \gtrsim 200$ nm in the hydrodynamic flow is consistent with several transport studies of graphene [1,30,32] and with theoretical models [53], but are in an apparent disagreement with recent spatially resolved studies of graphene [8–10] and WTe$_2$ [11], which have suggested diffuse or no-slip boundary conditions in the hydrodynamic regime. Note, however, that these studies are based on the analysis of current profiles in a strip geometry in which ballistic and hydrodynamic flows with large slip length result in essentially indistinguishable current profiles (see further discussion in Methods and Extended Data Fig. 3). The finding of a large slip length calls for further investigation of its microscopic origin.

**Transition from laminar to vortical flow**

We now examine more closely the transition between laminar and vortical flows. Figure 4 shows the experimental current distributions in samples with apertures $\theta = 90^\circ$, 72$^\circ$, 54$^\circ$, and 35$^\circ$. The first two geometries, $\theta = 90^\circ$ and 72$^\circ$, show laminar flow, in good agreement with the numerical results (Figs. 4a-j). In the sample with a smaller aperture $\theta = 35^\circ$, a well-resolved vortex is observed in each chamber—also in agreement with simulations (Figs. 4p-t). The intermediate $\theta = 54^\circ$ aperture shows, however, a markedly different $J_x(x,y)$ flow pattern (Fig. 4l). The corresponding numerical simulations (Fig. 4m-o) reveal that at the vortex-to-no-vortex phase transition ($\theta = \theta_t$), rather than vanishing continuously, the vortex elongates into an arc (see Methods and Supplementary Video 1) and eventually splits into two sub-vortices in the top and bottom parts of the chamber as shown by the streamlines in Fig. 4o. As a result, in each chamber, $J_x(x,y)$ shows two pairs of red-blue streaks, one for each sub-vortex (Fig. 4l), instead of a single pair of streaks as in Fig. 4q. The finite-element simulations show that the double-vortex flow occurs only in a narrow interval of parameters just below the $\theta_t(D/W)$ phase transition line, which allows us to determine $\theta_t \cong 54^\circ$ and $D/W \cong 0.28$ in our devices. Importantly, the double-vortex state can occur only in the hydrodynamic regime and is precluded in the ballistic transport (see Methods and Supplementary Video 2), thus providing additional strong evidence for the hydrodynamic origin of the observed vortical flow.

**Current dependence**

Our SOT microscope setup operation is limited to temperatures of about 4 K, but the electron temperature can be raised substantially by increasing the applied current [17,18]. Since both $l_{ee}$ and $l_{mr}$ decrease with increasing temperature, increasing the current $I_0$ is expected to reduce the Gurzhi length $D$. Figure 5 shows the evolution of $J_x(x,y)$ in the $\theta = 35^\circ$ sample upon increasing $I_0$ up to 400 $\mu$A. The normalized $J_x/(I_0/W)$ circulating in the vortex gradually decreases as $I_0$ grows, until the double-vortex state is formed at our highest applied current. This behavior is qualitatively consistent with the expected decrease of $D$ as indicated by the arrow in Fig. 3a. The degree of the reduction in $D$ is, however, surprising. By measuring the sample resistance as a function of $I_0$ and comparing it to the temperature dependence of the resistance, we infer that the electron temperature reaches about 18 K at our highest applied current. Based on theoretical estimates of $l_{ee}$ and $l_{mr}$ [11], such temperature increase should have reduced $D$ by more than an order of magnitude relative to the value at 4.5 K. In contrast, Fig. 3a indicates that $D$ has decreased by less than a factor of two. Note that in Ref. [11], the measured temperature evolution was also found to be weaker than predicted. This weak temperature dependence of the Gurzhi length provides important insight into the mechanism underlying hydrodynamics in our system, as discussed below.
**Discussion**

Perhaps our most unexpected finding is fluid-like momentum transport characterized by a Gurzhi length as small as $D \approx 155 \text{ nm}$, much smaller than $D = \frac{\sqrt{m_r l_{ee}^2}}{2}$ estimated from the bulk microscopic parameters. Indeed, transport measurements indicate that $l_{mr}$ values in our bulk samples are about 10 µm (Methods and Extended Data Fig. 1). Recent electronic structure calculations [11] predict $l_{ee}$ in WTe$_2$.
on the order of a few mm at 4 K. Accounting for phonon-mediated electron-electron scattering reduces $l_{ee}$ to about 100 µm [11]. By taking into account the compensated semimetal band structure of WTe$_2$, we show that the bare $l_{ee}$ can be of the order of 10 µm (Methods and Extended Data Fig. 8). Yet, even then $l_{ee}$ remains much larger than $W$, which should have resulted in ballistic transport, with $l_{mr}, l_{ee} \gg W$.

To gain insight into the origin of this surprising behavior, we recall the general derivation of the Ohm-Stokes law, Eq. 1. Kinetic theory links momentum relaxation to the decay rate $\gamma_1$ of the first angular harmonic of the nonequilibrium electron momentum distribution, giving $l_{mr} = v_F/\gamma_1$, whereas the kinematic viscosity $\eta = \frac{1}{4} v_F^2/\gamma_2$ is expressed through the decay rate $\gamma_2$ of the second harmonic of the momentum distribution, where $v_F$ is the Fermi velocity (Methods). This results in $D = \sqrt{\eta/\gamma_1} = v_F/\sqrt{4\gamma_1\gamma_2}$ (see also [31]). Importantly, this expression is valid for any microscopic momentum-scattering mechanism. $\gamma_1$, $\gamma_2$ can originate from a number of sources, including impurity scattering, phonons, and electron-electron collisions. Note though that momentum conserving processes do not enter into $\gamma_1$.

Usually, one would identify $\gamma_1 = v_F/l_{mr}$, $\gamma_2 = v_F/l_{ee}$, resulting in $D = \sqrt{l_{mr}l_{ee}}/2$. As discussed above, because of the large values of bulk $l_{mr}$ and $l_{ee}$, these relations are inconsistent with $D$ inferred from the observed behavior.

However, spatial diffusion of momentum can originate from effects unrelated to electron-electron scattering. One appealing alternative mechanism that could result in both the diffusion and relaxation of electron momenta is related to the finite thickness $d$ of the sample. The resistivity of WTe$_2$ flakes has been found to be strongly thickness dependent [54], a behavior attributed to surface oxidation. Our transport measurements show that the conductivity of the flakes is one to two orders of magnitude lower than that of the bulk crystals (see Methods), indicating a large reduction in $l_{mr}$ and an enhancement of $\gamma_1$ induced by the momentum relaxing scattering off the surfaces. However, an enhancement of $\gamma_1$ alone, such that $\gamma_1 > \gamma_2$, would of course lead to an ohmic transport with no vorticity, in contrast to the observed hydrodynamic flow, which requires $\gamma_2 > \gamma_1$. Yet, it has recently been pointed out [31] that an enhancement in $\gamma_1$ implies an enhancement in $\gamma_2$. Indeed, using the method described in [45], we show in Methods that small-angle scattering results in $\gamma_2 \approx 4\gamma_1$, giving rise to para-hydrodynamic transport with $D = v_F/\sqrt{4\gamma_1\gamma_2} = v_F/4\gamma_1 = l_{mr}/4$. Based on $D \approx 155$ nm derived from the vortex stability diagram, we arrive at the effective surface-induced $l_{mr} = 4D \approx 620$ nm in our samples, close to the effective $l_{mr} \approx 530$ nm derived independently from transport measurements of the conductivity in thin flakes (Methods).

The emerging picture is therefore as follows. For a fully specular surface scattering, the transport is ballistic. Small-angle scattering at the surfaces results in enhancement in momentum relaxation and in the lateral momentum diffusion. In this para-hydrodynamic regime, momentum diffusion occurs not through the usual momentum-conserving electron-electron scattering, but rather through close-to-specular scattering of individual particles at the top and bottom surfaces. The surface-induced para-hydrodynamics presents a unique opportunity to explore hydrodynamic phenomena in a wide range of high mobility materials without the hard-to-achieve strong bulk electron-electron interactions.

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Author contributions A.A., T.V., A.K. and E.Z. conceived the experiments. A.K.P. and M.H. grew and characterized the bulk WTe$_2$ crystals. A.K. fabricated and characterized the devices. T.V. and A.A. carried out the SOT magnetic imaging measurements and data analysis. I.R. and Y.M. fabricated the SOTs and the tuning fork feedback. A.Y.M. developed the current density reconstruction method. M.E.H. designed and built the SOT readout system. Y.W., T.H. and B.Y. carried out the band structure and the electron-electron scattering calculations. T.H. developed the vortex stability model. A.A. performed the finite-element numerical simulations. G.F., L.S.L. and E.Z. developed the para-hydrodynamic model. A.A., T.V., T.H., A.K.P, E.Z., G.F. and L.S.L. wrote the manuscript with contributions from the rest of the authors.

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Data availability The data that support the findings of this study are available from the corresponding authors on reasonable request.

Code availability The current reconstruction codes used in this study are available from the corresponding authors on reasonable request.
Methods

Synthesis of WTe₂ crystals

Dirac and Weyl semimetals have been shown to display a range of phenomena associated with hydrodynamic electron transport [11,21,55–59]. In addition, they display ultrahigh electron mobilities and electrical conductivities [55,60–70]. WTe₂, in particular, has the advantage of being a van der Waals material that allows exfoliation of thin flakes with very low sheet resistance, which allows application of relatively high current densities with Joule heating that is substantially lower than in 2D electron fluids in graphene and GaAs heterostructures. With this motivation, we have undertaken synthesis of high mobility WTe₂ crystals.

To obtain high quality WTe₂ single crystals we conducted a series of synthesis experiments using both chemical vapor transport (CVT) and the flux growth technique, as well as starting materials with different purity. These experiments led to progressively better crystals with increased $\text{RRR} = \frac{\rho(300 \text{ K})}{\rho(2 \text{ K})}$ and magneto resistance ratio $\text{MR} = \frac{\rho(9 \text{ T}) - \rho(0 \text{ T})}{\rho(0 \text{ T})}$, as summarized in Extended Data Fig. 1c, and are described here in chronological order. Eventually all devices for the hydrodynamic flow experiments were fabricated from our highest quality single crystals described last. The first crystals were grown by CVT [63,64] using elemental W (99.95%) and Te (99.99%) from Stanford Advanced Materials. Initially, polycrystalline WTe₂ was prepared by solid-state reaction in a vacuum-sealed quartz ampule at 750 °C. The obtained precursor material was then vacuum-sealed (1.33 × 10⁻⁵ mbar) in a 16 cm long quartz tube with a minute amount of TeBr₄ transport agent, and placed in a temperature gradient of 850 °C – 750 °C for several days. Few millimeters wide sheet-shaped single crystals were collected from the cold end of the ampule after cool down. However, we found that both $\text{RRR}$ and $\text{MR}$ of the CVT grown crystals were extremely low, despite testing a range of different growth conditions.

Much higher quality crystals were obtained using tellurium self-flux growth [65]. Elemental W and Te were mixed at a molar ratio of 1:30, loaded into frit-disc alumina crucibles, and sealed in a quartz ampule under vacuum. All steps of materials handling were performed in an Ar glove box with O₂ and H₂O concentration < 0.1 ppm. Quartz ampule, alumina crucibles, and quartz wool for cushioning were heat treated at 800 °C prior to the growth experiment. The tungsten tellurium mixture was heated in a box furnace to 1100 °C at a rate of 30 °C/h, followed by soaking at 1100 °C for 10 h. The metal solution was then slowly cooled down to 650 °C at a rate of 2 °C/h, followed by centrifuging to separate the Te flux from the crystals. To remove any trace of Te flux from the crystal surfaces, they were again vacuum sealed in a quartz ampule and placed on the hot side of a temperature gradient of 400 °C – 190 °C. Although these needle shaped crystals were of much higher quality than those grown by CVT, they were still inferior to the best crystals reported in literature [65]. Subsequent optimization of the growth parameters, such as a higher sample-to-flux ratio of W:Te = 1:50 and W:Te = 1:120, varying cooling rates, changing crucible arrangements, using pre-reacted WTe₂ and excess tellurium as starting materials, led to further improvements but the $\text{RRR}$ and $\text{MR}$ values were still unsatisfactory; see two points with lowest $\text{RRR}$ in Extended Data Fig. 1c.

The deciding factor that led to our best quality crystals was the use of higher purity W (99.999%) and Te (99.9999%) from Furuuchi Chemical Corporation. In addition the starting materials were loaded directly into the quartz tube, using a quartz wool filter instead of the frit-disc alumina crucibles. This allowed us to use larger amounts of starting materials, which should improve the volume to surface ratio of the melt in favor of a lower impurity density in the crystals from contact contamination with the quartz tube. Using our previously optimized mixing ratio of W:Te = 1:120, and a cooling rate of 2 °C/h from 1000 °C to 600 °C followed by centrifuging, we obtained crystals with excellent $\text{RRR}$ and $\text{MR}$ values on par with the best crystals reported in literature (see Extended Data Fig. 1c). We note that a cooling rate of 1 °C/h resulted in crystals with slightly lower quality.
Device fabrication

WTe$_2$ crystals were mechanically exfoliated onto an oxidized silicon wafer (290 nm of SiO$_2$) and suitable flakes were identified by optical microscopy. Standard nano-fabrication techniques were used for device fabrication: electron beam lithography (EBL), inductively coupled plasma etching (ICP), and electron gun metal deposition (E-gun). Separate EBL steps were used to define the mesa and the contact geometries. The WTe$_2$ mesa was etched with ICP using SF$_6$ (15 sccm) and O$_2$ (5 sccm), RF power of 20 W, resulting in an etch rate of $\sim$0.8 nm/s.

The Au reference samples and contacts to WTe$_2$ flakes were fabricated by E-gun deposition of Ti (2 nm) and Au (30 to 60 nm) followed by a lift-off procedure in acetone. Ar ion milling was used prior to the contact metal deposition for removal of the WTe$_2$ oxidation layer. For transport characterization, separate devices were fabricated in Hall bar geometry with width $W = 5 \mu m$, thickness $d \approx 40$ nm, and distance between the voltage contacts $L \approx 3.3 \mu m$.

Extended Data Fig. 1. Transport characterization of bulk WTe$_2$ single crystals. a, Resistivity, $\rho$, as a function of temperature of our highest purity crystal. At $T = 2$ K, the resistivity is $\rho = 0.23$ $\mu \Omega \cdot cm$ corresponding to $RRR \approx 3,250$. Inset: optical image of crystals from the optimized quality growth. b, Magnetoresistance, $MR = \frac{\rho(B) - \rho(0)}{\rho(0)}$, as a function of magnetic field at 2 K showing $MR \approx 62,000$ at 9 T. c, $MR$ vs. $RRR$ at $T = 2$ K and $B = 9$ T of our different crystals synthesized by flux growth (black dots) in comparison to reported values (open circles) in the literature [11,65–67]. The black line is a guide to the eye. d, Longitudinal and transverse conductivities $\sigma_{xx}$ and $\sigma_{xy}$ vs. magnetic field at 4.2 K and their fit to the two band model with resulting parameters $n_e = 2.4 \times 10^{19}$ cm$^{-3}$, $n_h = 2.3 \times 10^{19}$ cm$^{-3}$, $\mu_e = 5.1 \times 10^5$ cm$^2$/Vs, and $\mu_h = 2.7 \times 10^5$ cm$^2$/Vs.
Magnetotransport measurements

For bulk transport measurements, crystals with elongated geometry were selected with typical dimensions of width $W = 250$ to $350 \mu m$, thickness $d = 22$ to $240 \mu m$, and distance between the voltage contacts $L = 1$ to $3.7 \, mm$. Electrical contacts were made with conductive silver epoxy resin (EPO-TEK H20E) using $50 \mu m$ diameter gold wire. The epoxy contacts were cured at 150 °C under continuous N$_2$ flow. An optical image of a representative crystal with current and voltage contacts is shown in the inset of Extended Data Fig. 1b. The transport measurements (temperature and field dependence of resistivity) were carried out in a physical property measurement system (PPMS, Quantum Design) using a dc current of 1 to 5 mA for bulk samples and an ac current of 100 nA at frequency of $f = 11.51 \, Hz$ for crystal flakes. Transverse and longitudinal voltages were symmetrized and anti-symmetrized with respect to the magnetic field.

$\text{WTe}_2$ is a nearly compensated semimetal with electron and hole pockets contributing to transport [63,65–70]. We thus use a two-band conductivity model for the analysis of the magnetotransport:

$$
\sigma_{xx}(B) = e \left( \frac{n_e \mu_e}{1 + \mu_e B^2} + \frac{n_h \mu_h}{1 + \mu_h B^2} \right),
$$

$$
\sigma_{xy}(B) = e \left( \frac{n_h \mu_h B}{1 + \mu_h B^2} - \frac{n_e \mu_e B}{1 + \mu_e B^2} \right),
$$

where $e$ and $B$ are the elementary charge and applied magnetic field, and $n_e$, $n_h$, $\mu_e$, and $\mu_h$ are the electron and hole densities and mobilities, which are the fitting parameters.

Extended Data Fig. 1a shows the resistivity $\rho(T)$ measurement of a crystal from our best quality batch. The attained $\text{RRR} \approx 3,250$ slightly exceeds the values of $\text{RRR} = 900$ to 2500 reported in previous landmark studies [61,65–67]. Also, the magnetoresistance (MR) at 2 K (Extended Data Fig. 1b) shows an exceptionally high value of $\sim 62,000$ at 9 T, even exceeding the values of $\text{MR} \approx 42,000$ in WP$_2$ [62] and $\text{MR} \approx 17,500$ in WTe$_2$ [65], measured at the same field. At 14 T, our WTe$_2$ crystal attains $\text{MR} \approx 140,000$.

In Extended Data Fig. 1c, we compare the $\text{RRR}$ and MR values in some of our crystals from different batches with previously reported values. By fitting the conductivity at low fields to the two-band model, we obtained the electron and hole concentrations and their mobilities shown in Extended Data Fig. 1d. We took the average electron mobility in our bulk samples to be $\mu_e \approx 2.5 \times 10^5 \, cm^2/Vs$ at 4.2 K, from which we derived the electron momentum-relaxing mean free path $l_{mr} = \hbar k_F \mu_e / e \approx 20 \, \mu m$ ($k_F = 1.22 \, nm^{-1}$ is the Fermi wavelength [67]).

Transport measurements of our WTe$_2$ flakes in Hall bar geometry with thickness $d \approx 40 \, nm$ show typical conductivities of $\sigma_{\text{flakes}} \approx 8 \times 10^4 \, Ohm^{-1}cm^{-1}$ at 4.2 K, which are significantly lower than the bulk conductivities, $\sigma_{\text{bulk}} \approx 3 \times 10^6 \, Ohm^{-1}cm^{-1}$, leading to estimated effective $l_{mr} \approx 530 \, nm$ in our flakes.

SQUID-on-tip and magnetic imaging

Scanning SQUIDs have been used extensively for imaging vortices and their dynamics in superconductors [71–76]. Here, we have used Pb SOTs with diameters ranging from 120 nm to 140 nm fabricated following the methods described in Ref. [13]. The SOTs were protected from oxidation by deposition of 3 to 5 nm thick Ti films below and on top of the Pb film. The SOTs included integrated shunt resistors on the tip [77] and had magnetic sensitivity of approximately 50 nT/Hz$^{1/2}$ in applied magnetic field of 60 mT. The SOT readout was carried out using a cryogenic SQUID series array amplifier (SSAA) [78–80]. For height control, the SOT was attached to a quartz tuning fork as described in Ref. [81].

Magnetic imaging was carried out at 4.5 K in 25 µbar residual He pressure in the chamber. For the measurements in Figs. 1 to 4, an ac current of $I_0 = 50 \, \mu A$ at frequency $f = 186.4 \, Hz$ was applied to the WTe$_2$ or Au samples and the corresponding out-of-plane component of the Oersted field $B_x(x, y)$ was measured by a lock-in amplifier at a constant height of 50 nm above the sample surface. For the scans in Fig. 5, the ac current was varied between $I_0 = 100 \, \mu A$ and $I_0 = 400 \, \mu A$. The images were acquired with a pixel size of 13 nm, acquisition time of 40 ms/pixel, and image size of $430 \times 305$ pixels.
**Current density reconstruction**

For the reconstruction of the 2D current density $J(x, y)$ from the measured $B_z(x, y)$, we have used the inversion method described in detail in Ref. [51]. The procedure allows for correction of a small possible tilt of the SOT from the vertical axis and for its finite size, and takes into account the finite thickness $d$ of the sample.

The inversion, however, is an ill posed problem and as such is prone to various artifacts, including high sensitivity to noise and fluctuations, boundaries of the imaging window, fields arising from sources outside the imaging window, ringing at sharp edges due to scanning height related low-pass filtering, and high sensitivity to the assumed height of the sensor. To stabilize the solution with respect to fluctuations, filtering and regularization methods are required [51,82]. As a result, the qualitative features of the resulting $J(x, y)$ are well reproduced; however, the precise quantitative details and the fine structure are less reliable. In the following, we detail the artifacts arising due to ringing and sensor height.

For controlling the scanning height, the SOT is attached to a quartz tuning fork (TF) [81]. The TF is exited electrically at its resonance frequency $\sim 33$ kHz and the shift in its phase is monitored as a function of height $h$ upon approaching the sample surface. A threshold of 1° phase shift is defined as the “poking” height, $h = 0$. We then retract the SOT and scan at a nominal height of $h = 50$ nm. We note that the actual effective height of the magnetic imaging should be larger due to the low phase shift threshold, possible surface residues, and the finite thickness of the Ti/Pb/Ti film of the SOT. In addition, the accuracy of the calibration of the vertical displacement of the piezoelectric scanner is limited.

Extended Data Fig. 2 demonstrates the effect of the assumed effective sensor height $h$ on the current distribution reconstructed from the measured $B_z(x, y)$ in the $\theta = 35^\circ$ sample. It shows that the qualitative features of the current flow patterns, including the vortices in the chambers, are robust with respect to the assumed height in the range of $h = 20$ to 150 nm. Inspection of $J_y$ shows, as expected, that low $h$ of 20 and 50 nm results in some broadening of the derived current profiles (Extended Data Figs. 2g,h), whereas higher $h$ of 100 and 150 nm gives rise to enhanced ringing at the edges with negative current apparent outside the sample edges (light blue in Extended Data Figs. 2i,j). This ringing is much less pronounced in the $J_x$ distribution in Extended Data Figs. 2b-d and is noticeable in Extended Data Fig. 2e predominantly near the chamber apertures. The ringing in $J_x$ is less significant because the value of $J_x$ in the strip and in the chambers is comparable and its absolute value is a much lower than $J_y$ in the strip. To minimize the ringing in $J_y$, we therefore use the nominal height $h = 50$ nm for the current reconstruction in all of the figures in the main text.

The ringing at the sharp edges, however, is an unavoidable feature of the low-pass filtering of the inversion procedure and is one of the limiting factors in determining quantitatively the accurate current profiles. This is exemplified by taking a uniform current density $J_y$ in an infinite strip of width $W = 550$ nm and thickness $d = 48$ nm (as expected in a Au strip). We then calculate numerically $B_z(x)$ at $h = 150$ nm, and perform numerical inversion back to current. The resulting reconstructed $J_y$ (green dots in Extended Data Fig. 3a) deviates substantially from the original uniform current density (light green line). It shows ringing both inside and outside of the strip and finite slope at the edges. The ringing artifacts depend on the various parameters of the inversion procedure, including the pixel size, but they cannot be eliminated. Thus deriving precise current profiles from the measured magnetic field has always a limited accuracy. The black solid line in Extended Data Fig. 3a shows the current profile reconstructed from the experimentally measured $B_z(x)$ across the Au strip assuming effective height of 150 nm. It shows a qualitative agreement with the green dotted curve, consistent with a uniform current distribution in the ohmic regime in Au. Assuming a lower effective height results in apparent broadening of $J_y(x)$, while higher effective $h$ causes large oscillations.
Extended Data Fig. 2. Dependence of the reconstructed current densities on the assumed SOT scanning height. a, Numerical simulation of \( J_x(x, y) \) normalized by the average current density \( I_0/W \) in the strip in \( \theta = 35^\circ \) sample for \( D/W = 0.28 \) and \( \zeta = 200 \) nm. The span of the color scale is \( \pm 0.05 \). b-e, Current densities \( J_x(x, y) \) reconstructed from the inversion of the measured \( B_z(x, y) \) in WTe\(_2\) sample A with \( \theta = 35^\circ \) assuming effective SOT scanning heights of \( h = 20 \) nm (b), 50 nm (c), 100 nm (d) and 150 nm (e). The nominal scanning height was 50 nm. The span of the color scale is \( \pm 0.05 \). f-j, Same as a-e, but for \( J_y(x, y) \) on color scale of \( \pm 1 \). k-o, Same as f-j, but on expanded color scale of \( \pm 0.05 \). The \( J_y \) vortex counterflow current (light blue) is resolved in the chambers on a large artificial ringing background outside the strip edges.

In the strip geometry the current profile in the hydrodynamic regime is given by

\[
J_y(x) = J_0 \left[ \frac{1 + (\zeta/D) \tanh(W/2D) - \cosh(x/D) / \cosh(W/2D)}{1 + (\zeta/D - 2D/W) \tanh(W/2D)} \right],
\]

where \( \zeta \) is the slip length at the boundaries. For no slip conditions (\( \zeta = 0 \)), one obtains the familiar Poiseuille profile as shown by the light blue curve in Extended Data Fig. 3b. Even though the no-slip boundary conditions have been considered for the analysis of current profiles [8–11], it has been argued that they are not physical in electron fluids, and that \( \zeta \) of the order of \( l_{ee} \) should be expected even in the case of fully diffuse boundaries [53]. From the analysis of vortex intensity in Fig. 3, we conclude \( \zeta = 200 \) nm or larger in our samples. In this regime, the hydrodynamic current profiles in the strip geometry are almost indistinguishable from the ballistic profiles given by

\[
J_y(x) = \frac{I_0}{\pi} \int_{-\pi}^{\pi} d\theta \cos^2 \theta \left[ 1 - (1 - r) \frac{\cosh(csc \theta |W - 2xsgn(\theta)|/2l_{mr}) + \sinh(csc \theta |W - 2xsgn(\theta)|/2l_{mr})}{-r + \cosh(W \cdot csc |\theta|/l_{mr}) + \sinh(W \cdot csc |\theta|/l_{mr})} \right],
\]

where \( 0 < r < 1 \) is the reflectivity coefficient, with \( r = 0 \) corresponding to fully diffuse boundaries and \( r = 1 \) describing specular boundaries. Extended Data Figs. 3e-g compare the ballistic profiles for \( r = 0, 0.5, \) and 1 (red) with the hydrodynamic case with \( \zeta = 200 \) nm (blue). These results show that in a strip geometry, the difference between these cases based on the reconstructed \( J_y(x) \) (blue and red dots) is small and experimentally insignificant. In contrast, in the chamber geometry, the vortex stability differs greatly between ballistic and hydrodynamic cases and is hardly affected by the boundary conditions, as shown in Fig. 3.
**Extended Data Fig. 3. Current profiles in narrow Au and WTe₂ strips.**

**a,** A uniform current density \( J_y(x) \) in \( W = 550 \text{ nm} \) strip (light green line) from which \( B_z(x) \) is computed at a height \( h = 150 \text{ nm} \). The \( J_y(x) \) (green dotted symbols) is then reconstructed by inversion of the calculated \( B_z(x) \), showing the unavoidable distortions and ringing. The \( J_y(x) \) reconstructed from the experimental \( B_z(x) \) in the Au strip (black line) shows consistency with a uniform current distribution in the ohmic regime.

**b,** Same as (a) for a Poiseuille current profile (light blue) with \( D/W = 0.28 \) and no-slip boundary conditions. The reconstructed \( J_y(x) \) from the experimentally measured \( B_z(x) \) in WTe₂ strip (black) is inconsistent with the theoretically reconstructed \( J_y(x) \) (dotted blue) corresponding to the Poiseuille profile.

**c,** Same as (b) for hydrodynamic flow with \( D/W = 0.28 \) and slip length \( \xi = 200 \text{ nm} \) (light blue) showing good correspondence between the theoretically reconstructed \( J_y(x) \) (dotted blue) and the experimentally derived \( J_y(x) \) (black) in accord with the conclusions in the main text.

**d,** Same as (b) for hydrodynamic flow with \( D/W = 0.28 \) and no-stress boundary conditions (light blue). The reconstructed theoretical \( J_y(x) \) (dotted blue) underestimates the experimentally derived \( J_y(x) \) (black) supporting the conclusion of a finite slip length.

**e-g,** Comparison between theoretically calculated current profiles in the hydrodynamic regime with \( \xi = 200 \text{ nm} \) (light blue line) and in the ballistic flow (light red line) with boundary reflectivity coefficients of \( r = 0 \) (fully diffuse) (e), \( r = 0.5 \) (f), and \( r = 1 \) (specular) (g). The solid lines show \( J_y(x) \) calculated from Eqs. 2 and 3 while the dotted lines are the current profiles reconstructed from the calculated corresponding \( B_z(x) \). These results demonstrate the difficulty in using reconstructed current profiles in strip geometry for distinguishing between the hydrodynamic flow with finite slip length and the ballistic transport, in contrast to vastly different vortex stability phase diagrams in these two regimes.

Taking the above limitations into account, we now inspect \( J_y(x, y) \) more carefully within the chambers. Extended Data Fig. 2k shows the calculated \( J_y \) with color scale expanded 20 times, such that the color of the laminar current in the strip (red) is strongly saturated. On this expanded scale, the \( J_y \) counterflow becomes visible (light blue). Note that the density of the vortex current counterflow both in \( J_x \) and \( J_y \) is only about 1% of the laminar current density in the strip as seen in Extended Data Figs. 2a,k. By expanding the color scale of the experimental \( J_y(x, y) \) in Extended Data Figs. 2l-o by the same factor of 20, the counterflow in \( J_y \) (light blue) becomes visible. On this expanded scale the ringing in \( J_y \) is very pronounced even for \( h = 20 \text{ nm} \) and grows significantly with \( h \), but the enhanced light blue signal of \( J_y \) in the far side of the chambers is resolved at all values of \( h \) and shows little dependence on \( h \). Note that the experimental
$J_x$ in Extended Data Figs. 2b-e is presented on the same color scale as $J_y$ in Extended Data Figs. 2l-o, demonstrating that the magnitude of the backflow in $J_y$ matches the scale of $J_x$ in the chambers, providing an independent confirmation for the observation of a vortex. Since the ringing problem in $J_x$ is much less pronounced, the counterflow of the vortex current is readily resolved in $J_x$ despite being only about 1% of the driving current density.

**Detecting vortices by current reconstruction and through a direct observation from $B_z(x, y)$**

The goal of this section is to illustrate different methods of imaging vortical flows. One method is imaging the flow by current reconstruction, described in the main text. Here we illustrate it for two different materials, Au and WTe$_2$, patterned into the samples of the same geometry. These two systems allow to directly compare current flows in the ohmic regime (Au) and in the hydrodynamic and ballistic regimes (WTe$_2$).

Extended Data Fig. 4 shows the raw data for $B_z(x, y)$ and the current components reconstructed from it. For the aperture angle $\theta = 180^\circ$ both Au (top row) and WTe$_2$ (second row) show laminar flow. Notably, in the ohmic case of Au the current spreads out more uniformly over the chambers, whereas in the hydrodynamic flow the current remains more concentrated along the central strip as observed by the $J_y(x, y)$ distributions in panels (b) and (e). Consequently, the red and blue peaks in $B_z(x, y)$, which mark the position of sharp changes in current density, are pushed out to the outer edges of the chambers in panel (a), as compared to panel (d) where the peaks occurs within the chambers similarly to the $B_z(x, y)$ along the strip sections.

For the smaller aperture with $\theta = 45^\circ$ in the ohmic regime (third row), the $J_y$ that penetrates into the chambers, panel (h), has a gradual profile and essentially vanishes on approaching the outer edges of the chambers. Accordingly, no peaks in $B_z(x, y)$ are visible in panel (g) within the chambers. In WTe$_2$, to the contrary, the current remains focused in the central strip, resulting in clear red and blue peaks in $B_z(x, y)$ along the entire sample length in panel (j). The fact that vortices are formed in the chambers in this case, as evident by $J_x(x, y)$ in panel (l), has no readily visible signature in $B_z(x, y)$ in panel (j). This is because the vortex current is only about 1% of the applied current, and hence field generated by the vortex is masked by the much larger $B_z(x, y)$ originating from the laminar current flowing in the central strip.

An alternative method of detecting vortices relies on extracting their contribution to the measured magnetic field by subtracting the $B_z(x)$ trace measured across the strip away from the chambers. Using this approach, the magnetic field generated by the vortices can be visualized directly without invoking current reconstruction. This method is demonstrated in Extended Data Fig. 5. Panel (a) shows $B_z(x, y)$ in WTe$_2$ sample with $\theta = 20^\circ$. By subtracting $B_z(x)$ measured along the dashed lines, panel (b) reveals a red (positive) signal in the left chamber generated by a vortex with counterclockwise current circulation, while the right chamber shows a blue (negative) signal originating from a clockwise vortex. Panels (c) and (d) show the corresponding numerical simulation results. Similar behavior is observed in $\theta = 35^\circ$ sample (second row). The two vortex state is clearly resolved in panel (j) for $\theta = 54^\circ$ sample. For larger apertures of $\theta = 72^\circ$ and $\theta = 90^\circ$ the flow in the chambers is laminar with current circulating in the opposite direction, resulting in $B_z(x, y)$ of opposite sign in panels (n) and (r), as compared to panels (b) and (f) with vortical flow in the chambers. We therefore conclude that the results of this direct method are in good agreement with those found by current reconstruction.
Extended Data Fig. 4. Comparison of field and current profiles in Au and WTe$_2$ device C. a, $B_z(x,y)$ in Au sample with $\theta = 180^\circ$ (same as Fig. 1f). b, Reconstructed current density $J_y(x,y)$ normalized by $I_0/W$ (same as Fig. 1d). c, Reconstructed current density $J_x(x,y)$ (same as Fig. 1g). d, $B_z(x,y)$ in WTe$_2$ sample with $\theta = 180^\circ$. e, Reconstructed current density $J_y(x,y)$. f, Reconstructed current density $J_x(x,y)$. g, $B_z(x,y)$ in Au sample with $\theta = 45^\circ$ (same as Fig. 1l). h, Reconstructed current density $J_y(x,y)$ (same as Fig. 1j). i, Reconstructed current density $J_x(x,y)$ (same as Fig. 1m). j, $B_z(x,y)$ in WTe$_2$ sample with $\theta = 45^\circ$. k, Reconstructed current density $J_y(x,y)$. l, Reconstructed current density $J_x(x,y)$ (same as Extended Data Fig. 9n).
Extended Data Fig. 5. Visualizing vortices without current inversion in WTe$_2$ device A. a, $B_z(x, y)$ in WTe$_2$ sample with $\theta = 20^\circ$. b, Same data after subtraction of the average field along the dashed lines, $B_z(x, y) = [B_z(x, y = 1.22 \mu m) + B_z(x, y = -1.22 \mu m)]/2$, revealing a counterclockwise vortex in the left chamber generating a positive $B_z$ (red) and a clockwise vortex with negative $B_z$ (blue) in the right chamber. c-d, Corresponding numerical simulations in the hydrodynamic regime showing $B_z(x, y)$ after the background subtraction (c). e-t, Same as (a-d) for $\theta = 35^\circ$, $54^\circ$, $72^\circ$, and $90^\circ$ samples.

Vorticity in the vortical and laminar flows

As discussed in the main text, we use the term ‘laminar’ to describe current stream lines that flow from source to drain in contrast to vortical streamlines, which form closed loops. This topological distinction should not be confused with vorticity, $\omega = \nabla \times J$, which can be finite for both types of flows, since it is a local characteristic of the flow determining the rotation rate of an infinitesimal fluid element. Extended Data Fig. 6 shows numerically calculated vorticity and the stream lines for $\theta = 35^\circ$ and $\theta = 72^\circ$. The strongest vorticity (of the order of $I_0/W^2$) is present in the central strip where the flow is fully laminar. Similarly, finite vorticity is present in the chambers both in the presence and in the absence of a vortex. Finally, the location of the sign change in vorticity in the $\theta = 35^\circ$ chambers is uncorrelated with the crossover from laminar to vortical streamlines, confirming that vorticity per se is not a signature distinguishing laminar and vortical flows.
Extended Data Fig. 6. Comparison between vorticity and vortical stream lines. a, Numerical calculation of vorticity, $\omega = \nabla \times J$, normalized by $I_0/W^2$, in the hydrodynamic regime with $D = 155$ nm and $\xi = 200$ nm in $\theta = 35^\circ$ sample. The colors in the strip region, where the normalized vorticity is of the order of 1, are greatly saturated in order to show the vorticity in the chambers. b, Calculated laminar (red) and vortical (blue) streamlines in the same geometry. c-d, Same as (a-b) for $\theta = 72^\circ$ sample.

**Finite-element numerical simulations**

The 2D finite-element numerical simulation of an ohmic electron flow and of transport described by Eq. 1 for the ohmic, ballistic and hydrodynamic regimes, as discussed in the main text, were carried out using COMSOL Multiphysics 5.4. The shape function of the elements used in the simulations is Lagrangian of second order (quadratic). We used the Coefficients Form PDE module, which solves the general equation:

$$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot (-c \nabla u - au + \gamma) + \beta \cdot \nabla u + au = f,$$

where the field $u$ is:

$$u = \begin{pmatrix} \phi \\ J_x \\ J_y \end{pmatrix},$$

and the coefficients were chosen to match Eq. 1:

$$e_a = d_a = \gamma = f = 0,$$

$$c = \begin{pmatrix} 0 & 0 & 0 \\ 0 & D^2 & 0 \\ 0 & 0 & D^2 \end{pmatrix}, \quad a = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \beta = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \alpha = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

where $D$ is the Gurzhi length, $\sigma$ is the conductivity, and the source and drain are simulated by Dirichlet boundary conditions on the potential $\phi$. The resulting current density field $J(x, y)$ was normalized by the average current density $I_0 = \int W \int J(x, y) dx/W$ in the strip. A complete description of the problem is given by employing boundary conditions. The boundary conditions for the perpendicular current were $J_\perp = J \cdot \hat{n} = 0$ in all simulations. For the tangential current, $J_\parallel = (J - J \cdot \hat{n})$, we have used the three types of
boundary conditions described in the main text: (i) no slip, $\mathbf{J}_{\parallel} = 0$, (ii) no stress, $\mathbf{n} \cdot \nabla \mathbf{J}_{\parallel} = 0$, and (iii) a finite slip length, $\mathbf{J}_{\parallel} = \xi \mathbf{n} \cdot \nabla \mathbf{J}_{\parallel}$. The dimensions of the simulated devices were chosen to be equal to those of our samples, with the width of the central strip $W = 550$ nm in the dual chamber geometry and the disk radius of the chambers of $R = 900$ nm.

For derivation of the vortex stability phase diagram, a geometry depicted in Fig. 3 was used, with variable $D$ and $\theta$. The vortex current $I_v$ for each simulated geometry and $D$ was calculated according to

$$I_v(D, \theta) = \frac{1}{4} \int (|J_y(x, 0)| - J_0) \, dx,$$

where the integral was carried out along the cross-section through the central horizontal line connecting the centers of the disk chambers. For $D/W > 1$, the maximum of $I_v$ occurs at $\theta = 60^\circ$ when the aperture and the radius of the disk chambers form an equilateral triangle. An equilateral triangle in the context of hydrodynamic flow was also reported in Ref. [22]. Upon $D/W$ decreasing, the maximum point of $I_v$ shifts to lower $\theta$.

Supplementary Videos 1 and 2 show the evolution of the streamlines upon decreasing $\theta$ in the hydrodynamic and ballistic regimes, respectively. As $\theta$ increases, on approaching the vortex-to-laminar transition, the laminar streamlines (red) penetrate deeper into the chambers causing distortion of the vortex into a banana shape. In the ballistic case the vortex (blue streamlines) is pushed out of the chamber as a whole, while in the hydrodynamic regime it splits into two vortices at the top and bottom of the chambers. This enhanced stability of the two-vortex solution in the hydrodynamic regime is well captured by the analytical estimates presented in the theory section below.

**Theory**

In the last two decades, electron hydrodynamics has been intensively studied theoretically for both metals and semimetals regarding flow patterns, potentials, viscosity and magnetic field dependence, using variety of methods [19,27–50,52,53,83–94]. In the following, we will focus on the phenomenology of para-hydrodynamics and the key aspects of vortical flow.

**Angular scattering, diffusion along the Fermi surface and para-hydrodynamics**

This section aims to provide a microscopic justification of Eq. 1 which is used as a benchmark model in the main text. We consider a Fermi gas in two dimensions with a cylindrically symmetric dispersion and a circular Fermi surface. Transport in the system weakly perturbed away from equilibrium is described by a steady-state carrier distribution of the form $f(p, r) = f^\circ + \delta f(r, \varphi)$, with $f^\circ$ being the equilibrium distribution and $\delta f(r, \varphi)$ a perturbation, where $\varphi$ is the angle on the Fermi surface, $p$ is electron momentum and $r = (x, y)$. The steady-state distribution satisfies the linearized kinetic equation:

$$v_F \cos \varphi \frac{\partial \delta f}{\partial x} + v_F \sin \varphi \frac{\partial \delta f}{\partial y} - \hat{I} \delta f = -\mathbf{F} \cdot \nabla_p \bar{f} \quad (4)$$

Here $\hat{I}$ is the linearized collision operator of the elastic scattering and $\mathbf{F}$ is an external force. Equation 4 is valid for a collision operator of a general form; below, we apply it to describe scattering at the sample upper and lower surfaces, the process discussed in the main text. Following [45], we expand our perturbation over angular harmonics:

$$\delta f(y, \varphi) = \int \frac{d^2k}{(2\pi)^2} \sum_{m=-\infty}^{m=\infty} f_m(k) e^{im\varphi + ikr}.$$

Statistical isotropy of scattering means that the angular harmonics are eigenfunctions of the scattering operator: $\hat{I} f_m = -\gamma_m f_m$. Because of the $\cos \varphi$ and $\sin \varphi$ structure of the streaming term in Eq. 4, the harmonics $f_m(k)$ satisfy a tridiagonal system of linear equations. The contribution generated by the $m$-th harmonic of the right-hand side of Eq. 4 (denoted $B_m$) satisfies the equation:
\[ \gamma_m f_m + iA f_{m+1} + i\tilde{A} f_{m-1} = B_m, \]  

(5)

where \( A = (k_x - i k_y) v_F/2 \). Following [45], we solve Eq. 5 for the ratios \( \alpha_m = \frac{f_{m+1}}{f_m} \), which for \( n \neq m \) satisfy the recurrence relation

\[ \alpha_{n-1} = \frac{\tilde{A}}{\gamma_n + A \alpha_n}. \]

The solution of this problem is given in a closed form as a continued fraction

\[ \alpha_{n-1} = \frac{\tilde{A}}{\gamma_n + \frac{|A|^2}{\gamma_{n+1} + |A|^2 \gamma_{n+2} + \cdots}}. \]

Similarly for \( \beta_m = \frac{f_{m-1}}{f_m} \) we obtain the fraction running down:

\[ \beta_{n+1} = \frac{\tilde{A}}{\gamma_n + \frac{|A|^2}{\gamma_{n-1} + |A|^2 \gamma_{n-2} + \cdots}}. \]

Substituting into Eq. 5 yields a contribution to the \( m \)-th harmonic of perturbed distribution as follows:

\[ f_m = \frac{B_m}{\gamma_m + \frac{(kv_F/2)^2}{\gamma_{m-1} + (kv_F/2)^2 + \cdots \gamma_{m+1} + (kv_F/2)^2 \gamma_{m+2} + \cdots}}. \]

(6)

Considering long-wavelength limit, we retain only the terms up to quadratic in wavenumber, which gives dissipation and diffusion terms:

\[ \left( \gamma_m + \frac{(kv_F)^2}{4\gamma_{m-1}} + \frac{(kv_F)^2}{4\gamma_{m+1}} \right) f_m = B_m. \]

For \( m = 0 \), one usually has \( \gamma_0 = 0 \) due to charge conservation, so that we have pure diffusion. For a locally homogeneous electric field we have

\[ v_F \cos \varphi \frac{\partial \delta f}{\partial y} + v_F \sin \varphi \frac{\partial \delta f}{\partial x} - i \delta f = -\frac{\partial}{\partial \epsilon} \left( v_F \cos \varphi \frac{\partial \varphi}{\partial y} + v_F \sin \varphi \frac{\partial \varphi}{\partial x} \right) \]

\[ \gamma_m f_m + iA f_{m+1} + i\tilde{A} f_{m-1} = B \delta_{m,1} + B \delta_{m,-1}. \]

Here, \( 2B = e(E_x + iE_y) v_F \partial \tilde{f} / \partial \epsilon \), where \( \epsilon \) is electron energy. In this case, Eq. 6 is the current-field relation with a nonlocal conductivity:

\[ J_k = \sigma(k) E_k, \]

(7)

\[ \sigma(k) = \frac{ne^2/m}{\gamma_1 + \Gamma(k)}, \quad \Gamma(k) = \frac{(kv_F/2)^2}{\gamma_2 + (kv_F/2)^2 \gamma_3 + \cdots}. \]

(8)

In the long-wavelength limit set by the \( m = 3 \) harmonic decay rate, such that \((kv_F)^2 < 4\gamma_2\gamma_3\), Eqs. 7 and 8 give

\[ \frac{(kv_F)^2}{4\gamma_1\gamma_2} J_k + J_k = -\sigma E_k. \]

(9)

After applying a Fourier transform, we recover Eq. 1 from the main text with \( D = v_F/\sqrt{4\gamma_1\gamma_2} \):

\[ -D^2 \nabla^2 J + J = -\sigma \nabla \phi. \]

(10)
Notably, this equation is applicable not only in the hydrodynamic and ohmic regimes but also in the ballistic regime. This behavior is unique to the situation when \( \gamma_3 \gg \gamma_2 \), since in this case the condition for the length scales \((k v_F)^2 < 4\gamma_2\gamma_3\) used to derive Eq. 10 is valid in both the hydrodynamic and ballistic regimes. This is in contrast to the conventional electron fluids, where Eq. 10 is valid only in the hydrodynamic and ohmic regimes, but not in the ballistic regime. The extended validity range of Eq. 10 is a salient feature due to small-angle scattering.

The relation between our interpretation of the observed hydrodynamic behavior in terms of Eq. 10 and the assumption that the rates \( \gamma_1, \gamma_2, \gamma_3 \ldots \) are determined by the small-angle scattering on sample surfaces can be further substantiated as follows. The requirement for Eq. 10 to hold is \( \gamma_3 \gg \gamma_2 \gg \gamma_1 \). This condition can be approximately fulfilled for small-angle scattering that leads to angular diffusion. Indeed, in this case we have \( I \approx \gamma \frac{\partial^2}{\partial \theta^2} \) and therefore \( \gamma_m = \gamma_1 m^2 \). The condition for the long-wavelength limit is then \( k v_F < \sqrt{4\gamma_2\gamma_3} = 12\gamma_1 \). In terms of the effective momentum-relaxation length \( l_{mr} = v_F/\gamma_1 \), the condition takes the form \( kl_{mr} < 12 \), which is not too restrictive. Indeed, if we put \( k \approx 1/W \), the Eq. 10 with \( D = l_{mr}/4 \) is expected to work reasonably well for \( l_{mr} < 12W \), a regime well satisfied for our parameters. In contrast, for large-angle scattering, the decay rates for different harmonics take similar values, \( \gamma_3 \approx \gamma_2 \approx \gamma_1 \). In this case, depending on the size of \((k v_F)^2\), Eq. 8 can only be truncated at zeroth order or never, thus precluding a regime where Eq. 10 holds.

**Hydrodynamic vs. ballistic vortex formation: general considerations and scaling analysis**

The discussion in this section provides qualitative arguments in support of the hydrodynamic origin of the observed vortices, as suggested by our simulation results. In a system of size \( W \), the flow pattern depends on two dimensionless parameters, \( l_{ee}/W \) and \( l_{mr}/W \). The hydrodynamic regime occurs when \( l_{ee}/W \rightarrow 0 \) and \( l_{mr}/W \rightarrow \infty \), whereas the ballistic regime corresponds to \( l_{ee}/W \rightarrow \infty \) and \( l_{mr}/W \rightarrow \infty \), and the ohmic regime takes place when both \( D/W \rightarrow 0 \) and \( l_{mr}/W \rightarrow 0 \). To gain insight into the character of the flow patterns in all three limits we consider the dissipation in the flow: \( \int dx dy [\sigma \frac{\partial J}{\partial x_i} + \frac{\partial \sigma}{\partial x_i}] \). Here, the first term is the ohmic dissipation, while the second term is the viscous dissipation, determined by the stress \( \sigma_{ij} = \eta (\partial v_i/\partial x_j + \partial v_j/\partial x_i) \). The physical flow can be obtained by minimizing this dissipation functional supplemented with suitable boundary conditions, a procedure known as the principle of minimum entropy production.

The flow in the ohmic limit tends to minimize the current density everywhere, while the ballistic limit tends to minimize stress globally. Finally, the hydrodynamic limit minimizes stress locally, but not globally (Extended Data Fig. 7). As discussed below, the vortex stability phase boundary and the crossovers between different regimes can be obtained directly from the analysis based on the principle of minimum entropy production.

**Hydrodynamic-to-ohmic and ballistic-to-ohmic crossovers**

Since there is no vortex in the ohmic regime, the transition to this regime (upon a decrease in \( l_{mr} \), for instance) from either hydrodynamic or ballistic flow can be understood as a weakening of the vortex in the chamber. As vortex weakens, the laminar current \( j_l \) from the strip is expected to penetrate deeper and deeper into the chamber. As a result, the laminar streamlines superimpose onto the vortical streamlines in panels (b) or (c) in Extended Data Fig. 7. In the core of the ballistic or hydrodynamic vortex, the superimposed laminar flow points in the direction of the strip flow, which means that the combined flow pattern in the presence of a small laminar component has its vortex center (where the current vanishes) displaced further away from the aperture compared to the purely ballistic or hydrodynamic case.
Extended Data Fig. 7. Schematic streamlines for purely ohmic, hydrodynamic and ballistic flow. a, If the sample is purely ohmic, the current leaks into the chamber, forming a current dipole decaying as inverse distance squared. b, For a purely hydrodynamic flow, no laminar current (red) leaks from the strip into the chamber. Instead, a vortex forms in the vicinity of the aperture in the chamber (blue) in order to decrease the shear due to the gradient in the velocity profile. c, In a purely ballistic flow, only the geometry dictates the streamlines, producing a vortex (blue) whose center is positioned near the chamber center.

In the following analysis, we consider three components that comprise the total current density: laminar current $j_l$, hydrodynamic current $j_{hyd}$, and ballistic current $j_{bal}$. The aperture presents a small dipolar source and sink of the laminar component of the current, which leaks from the strip into in the chamber. Along the chamber boundary and as a function of the distance $\delta$ from the aperture, the density of the laminar component thus decays as $j_l \sim 1/\delta^2$ (for $l_{mr} > W$). In contrast, the vortical components (blue in Extended Data Fig. 7) do not flow into or out of the strip, but circulate entirely within the chamber. The ballistic vortex is almost rotationally symmetric and is positioned close to the center of the chamber. As a result, its density along the chamber boundaries is almost constant, $j_{bal} \sim \text{const}$. The hydrodynamic vortex, in contrast, forms directly outside the aperture, producing a flow that decays as $j_{hyd} \sim 1/\delta$ far outside the vortex core. Thus, if the laminar flow is strong enough to push the vortex core out to the far side of the chamber, it inevitably overpowers the flow in the upper and lower segments of the chamber, meaning that no vortical flow remains in either the hydrodynamic or the ballistic case. However, for hydrodynamic flow, the streamlines are not uniquely fixed by the geometry, and more than one vortex might form. This is indeed the case, as we demonstrate next: in the hydrodynamic regime, vortical flow can be stabilized by creating two vortex cores even if the single, large vortex is annihilated. In contrast, ballistic flow cannot form two vortex cores next to each other, because ballistic trajectories intersect each other without any effect, which means that the vortex cores do not repel each other, and instead merge into a single large vortex.

Principal components of hydrodynamic flow

To illustrate the basic physics behind the appearance and disappearance of vortices in the chamber, we develop a simple model which allows us to estimate dissipation and choose a state that minimizes it. We want to model the vortex that forms when current in a strip of width $W$ leaks into a large chamber or open space through an aperture in the strip wall at $x = 0$ of size $\Delta = 2R \sin(\theta/2)$, where $R$ is the chamber radius. The relevant current densities are $j_c$ in the strip, $j_v$ of the vortex, and $j_l$ the laminar current in the open space. These current densities are taken to be additive, where the relative amplitudes are determined by the requirement that the resulting flow minimizes the dissipation.

First we consider laminar flow, which can be modelled as a dipole describing current that flows out into the $x > 0$ half-plane in the interval $-\Delta/2 < y < 0$ and returns in the interval $0 < y < \Delta/2$. For concreteness, we take the profile of the outflow through the aperture to be parabolic of the form $\sim y(|y| - \Delta/2)$, with the $y$ dependence sign-changing with a zero net current. For weak ohmic dissipation, a laminar ballistic flow in a half plane injected through an aperture with such a profile yields
\[ j_1(x, y) = j_{10} \int_{-\Delta/2}^{\Delta/2} dy' (y' (|y' - \Delta/2| + x (y - y'))/ (y' - y)^2 + x^2 (y - y')), \]  

with the \( x \) component odd in \( y \) and the \( y \) component even in \( y \). This is a dipole source, and the integral can be done analytically. Expanding in large \( x \) for \( y = 0 \) yields \( |j_1(x, 0)| = \frac{|j_{10} \Delta^4}{6 \pi^2} \), meaning that the current decays inversely with distance squared at large distances from the aperture, as expected.

Next we consider a vortex positioned inside the chamber in proximity to the aperture. We assume that it has the shape of a Kaufmann vortex, with current density in polar coordinates given by \( j_v(r, \varphi) = j_{v0} r R_c / (r^2 + R_c^2) \hat{\varphi} \). Here, \( R_c \) is the size of the vortex core, and \( j_{10} \) and \( j_{v0} \) measure the strength of the laminar and vortical flows, respectively. We assume that the vortex is located at \( x = d_v \), and account for the boundary conditions close to the walls by adding a counter-rotating image vortex centered around \( x = -d_v \). We further impose a condition that the total current \( I_0 \) is conserved in any cross-section with a fixed \( y \), which leads to the condition that in the presence of the laminar outflow into the half-space, the current in the strip at \( y = 0 \) becomes \( I_0 - c_{cp} l \Delta \), where \( c_{cp} \) is a dimensionless geometrical factor.

Turning to the discussion of dissipation, we note that the decreased current density in the strip also reduces viscous dissipation, which we account for by subtracting the current densities from both the laminar current and the vortical flow. The current profile across the strip is parabolic if \( D \ll W \). However, in the hydrodynamic limit when \( D \ll W \), the current profile becomes very flat and the viscous dissipation is only relevant in a small boundary region. In this latter case, the term for the viscous dissipation acquires an additional factor of \( D \) in the denominator. We therefore write an approximate expression for the total dissipation in our system area which is affected by the chamber (i.e. for the chamber itself and for the strip section between \(-\Delta/2 \) and \( \Delta/2 \)), in which we treat the laminar and vortical flows as additive contributions. This gives

\[ P(J_1, J_v, R_c) = P_{\rho, W} + P_{v, W} + P_{\rho, R} + P_{v, R} + P_{\rho, mixed} + P_{v, mixed}. \]  

Here, ohmic dissipation \( P_{\rho} = P_{\rho, W} + P_{\rho, R} + P_{\rho, mixed} \) measures the local current density squared, integrated over the system area, while the viscous dissipation \( P_v = P_{v, W} + P_{v, R} + P_{v, mixed} \) measures the square of the current density gradients. For the central strip, the effective area is \( W \Delta \) for current in the strip, while it is \( \Delta^2 \) for the laminar flow that penetrates from the strip into the chamber, and \( R_c^2 \) for the vortical flow. For the viscous dissipation, we note that the gradients are usually smooth and essentially cancel the area integral. Only in the strip, for small \( D \ll W \), this is not true and the current density gradients are restricted to a narrow boundary region of size \( D \). Therefore, \( P_{\rho, W} = \frac{\rho}{2} \left( I_0 - c_{cp} l \Delta \right)^2 \frac{\Delta}{W} \) is the ohmic dissipation in the strip section, while \( P_{v, W} = \frac{\eta}{2} \frac{\rho I_0}{W} - c_{cn} (J_1 + J_v) \right)^2 \frac{\Delta}{\min[\Delta, W]} \) is the viscous dissipation in the same area (\( \rho \) is the sheet resistance). The leading term, proportional to \( \frac{\rho}{2} I_0^2 \), does not affect the competition between the vortex and no-vortex states. Analogously, in the chamber we find \( P_{\rho, R_c} = \frac{\rho}{2} (c_{lp} j_1^2 \Delta^2 + c_{vp} j_1^2 R_c^2) \) and \( P_{v, R_c} = \frac{\eta}{2} \left( c_{ln} I_1^2 + c_{vn} I_1^2 \right) \), up to logarithmic corrections in \( R/W \). Here, \( c_{cn}, c_{cp}, c_{lp}, c_{vp}, c_{ln}, c_{vn} \) are dimensionless form factors.

One implication of the construction of the flow in terms of \( j_1 \) and \( j_v \) is that for both ohmic and viscous dissipation, the mixed terms in Eq. 12 between both components are negligible, a property that greatly simplifies the analysis. From our general considerations, we know that the effective size of the vortex goes to zero as the chamber opening is made smaller, i.e. \( R_c (\Delta \to 0) = 0 \). As the first main observation, we thus find that \( P_{\rho, R_c} (\Delta \to 0) = 0 \). In contrast, the viscous dissipation \( P_{v, R_c} \) does not depend explicitly on the size of \( \Delta \) (and \( R_c \)), and therefore \( P_{\rho, R_c} / P_{v, R_c} \to 0 \), i.e. for small apertures, the ohmic dissipation is negligible compared to the viscous one. Consequently, for small opening sizes \( \Delta \), the solution is independent of \( P_{\rho, R_c} \). This suffices to find a unique solution to \( j_{10} \) and \( j_{v0} \) (cf. Eq. 11 and below), and subsequently construct an upper bound for the existence of a single vortex depending on whether the
total dissipation is smaller with rather than without vortical flow. The vortex is energetically favorable as long as

\[
\frac{\Delta_1}{W} < \frac{c_{liq} c_{vn}}{c_{vn}^2 (c_{liq} + 2c_{vn})} \min [D, W]
\]

This result provides the general form of the vortex stability phase diagram: for \( D/W \ll 1 \), the vortex to no-vortex phase transition line is linear with \( \Delta_1/W \approx \theta \), while for \( D/W > 1 \), \( \theta \) saturates at a finite value, consistent with the numerically derived phase diagram in Figs. 3a,b.

Using the same ansatz, but starting from two vortex cores, the same condition Eq. 13 is recovered for the limiting opening size \( \Delta_2 \), but where \( c_{vn} \) is replaced by \( 4c_{vn} \), (the factor 4 is due to the square of the current which enters in the dissipation) to account for the viscous dissipation from two cores. Since it holds that \( \Delta_2 > \Delta_1 \) this means that there is a narrow range of opening sizes where not one but instead two vortices can form. We note that by the same argument, even more vortex cores might be stabilized. However, at the same time, the positive effect of the vortex formation on the stress at the strip opening diminishes, making these more exotic solutions incompatible with the geometrical constraints imposed by the chamber. Numerically, all form factors are of order 1 and depend little on the particular choices of integration cutoffs. Specifically, for the dipole laminar flow and a Kaufmann vortex, we find that \( c_{liq} \approx 4.0 \) and \( c_{vn} \approx 2.0 + 0.8 \log (W/R_c) \). We further estimate that \( W/R_c \sim 3 \) and \( c_{vn} \sim 1/2 \), in which case \( \Delta_1 < 4.7 \text{min}[D, W] \) and \( \Delta_2 < 6.8 \text{min}[D, W] \). These numbers are reasonably close to both the numerical and experimental findings, even though the ratio of \( \Delta_2/\Delta_1 \) seems to be somewhat smaller in the simulations. In summary, for weakly ohmic flow, a single hydrodynamic vortex can form inside the chamber close to the aperture, with a size of vortex core that increases with the aperture size. For large apertures, this vortex becomes unstable, and we find a narrow range of parameters where it is favorable to form two vortices instead. Vortical flow disappears once the aperture size becomes sufficiently large to allow spreading of the laminal currents over the entire area of the chamber.

**Electron-electron scattering length in a compensated semimetal**

Electron-electron interactions which lead to hydrodynamic flow are usually designated as strong electron-electron interactions. However, to be precise let us point out that for the appearance of the hydrodynamic regime the electron-electron interaction must lead to a short quantum lifetime of the quasiparticle. Such is not necessarily the case for all types of strong electron-electron interactions. Here, we estimate the electron-electron scattering rate using the fermionic self-energy calculated in the random phase approximation (RPA). To that end, we consider three contributions that might be relevant in reducing \( l_{ee} \) below the values reported in [11]. They are all related to the compensated nature of WTe\(_2\), where the band edges of several bands are close to (but not at) the Fermi level.

(i) Firstly, the proximity of the band edges violates the core requirement of semiclassical estimates of the relaxation rate that the ratio of temperature over Fermi energy, \( T/E_F \ll 1 \), so that a fully quantum-mechanical treatment is needed. In this latter approach [95], the relaxation rate explicitly contains the occupation functions which account for both virtual and thermal fluctuations of the electron fluid. Given a dispersion \( \varepsilon_{mk} \) and eigenfunctions \( |u_{mk}\rangle \), where \( m \) is the band index and \( k \) the momentum, the imaginary part of the self-energy \( \Sigma \) assumes the form,

\[
\text{Im} \Sigma(q, \varepsilon_{nq}) = \sum_m \int \frac{d^3k}{(2\pi)^3} \text{Im} \left[ \frac{C(k - q)|u_{nq}|^2 u_{mk}^*}{1 - \Pi(k - q, \varepsilon_{mk} - \varepsilon_{nq})C(k - q)} \right] (\varepsilon_{mk} - \varepsilon_{nq}) + n(\varepsilon_{mk}) \text{.}
\]

Here \( \Pi \) and \( n \) denote the Bose and Fermi functions, respectively. The dielectric function is determined by the charge susceptibility \( \Pi \) in the RPA-approximation,

\[
\Pi(q, \omega) = \sum_{ij} \int \frac{d^3k}{(2\pi)^3} \left| u_{ik} u_{j,k+q}^* \right|^2 \left[ \frac{n(\varepsilon_{ik}) - n(\varepsilon_{j,k+q})}{i\omega + \varepsilon_{ik} - \varepsilon_{j,k+q}} \right]
\]

\(32\)
while the Coulomb interaction is

\[ C(k) = \frac{4\pi e^2}{|k|^2}. \]  

(16)

(ii) Secondly, the presence of band edges also precludes the extrapolation of relaxation rates obtained at high temperatures under the assumption of a simple Fermi-liquid \( T^2 \)-scaling, at least \textit{a priori}. We find in particular that the \( T^2 \)-dependence is violated for temperatures above 100 K.

(iii) Thirdly, the compensated nature of the material with both hole and electron Fermi surfaces requires a high-fidelity calculation with a fine momentum space grid to properly resolve the nesting between electron and hole Fermi surfaces. For example, we observe convergence of the obtained relaxation rates only for grid sizes larger than 100×50×7 in the \( x-y-z \) momentum volume.

Taking all these issues into account, we calculated \( l_{ee} \) for temperatures between 70 K and 300 K, checked for convergence in terms of grid resolution, and extrapolated to lower temperatures based on a power-law fit. To decrease runtime, as an approximation we restricted the effects of the Coulomb interaction to the first Brillouin zone and set the imaginary part of the dielectric function to zero for bands far from the chemical potential. Extended Data Fig. 8 shows the Fermi surface near the compensation point, and the scaling of \( l_{ee} \) with temperature, averaged for band 55 which forms one of the hole pockets, taking into account a total of 20 bands closest to the chemical potential. Comparable values are obtained for the other Fermi pockets. As is clearly visible, the low-temperature asymptotic temperature dependence sets in only below 100 K. We also confirmed the convergence of our calculation with respect to contributions from far bands by comparing the obtained relaxation rates for 4, 8, 12, 16 and 20 bands at an intermediate temperature of 145 K. Extrapolating our results to low temperatures under the assumption of a \( T^{-2} \) dependence, we obtain \( l_{ee} = 0.5 \) \( \mu \)m at \( T = 20 \) K and \( l_{ee} = 4 \) \( \mu \)m at \( T = 7 \) K. We note that these values are lower than the ones reported for the electron-electron mean free path in [11], which was calculated with a much smaller grid resolution. However, they are comparable to the latter’s estimate for the phonon-mediated interacting mean free path. While a full analysis is beyond the scope of this work, we point out that the combined momentum-conserving mean free path could thus be even slightly smaller. These estimated lower values of \( l_{ee} \) give credence to the mechanism described in the main text whereby a weakly ballistic flow can effectively become hydrodynamic in thin samples.

Extended Data Fig. 8. Fermi surface and electron-electron mean free path. a, Fermi surface cut for \( k_z = 0 \). Typical for a compensated semimetal, small electron and hole pockets appear close to the compensation point. If the hole density is slightly larger than the electron density, the Fermi surface features hole pockets.

Extended Data Fig. 8. Fermi surface and electron-electron mean free path. a, Fermi surface cut for \( k_z = 0 \). Typical for a compensated semimetal, small electron and hole pockets appear close to the compensation point. If the hole density is slightly larger than the electron density, the Fermi surface features hole pockets.
near the Gamma point (red) and electron pockets (blue). \( b \), \( l_{ee} \) as calculated from Eq. 14 for 20 bands as a function of temperature (red points). For \( T = 145 \) K, we also show the values for a smaller number of bands. The blue lines denote upper and lower estimates for the \( T^{-2} \) dependence of \( l_{ee} \), where the lower one corresponds to the low-temperature asymptotics.

**Transition from vortical to laminar flow in additional samples**

Analogously to the analysis in the main text (sample A), we show the transition from vortical to laminar flow in two additional samples, B and C. These samples provide additional insight into the dependence of the flow on the geometrical parameters, including the width of the central strip, \( W \), the chamber radius, \( R \), and the sample thickness, \( d \). Importantly, samples A and B were fabricated from the same batch of WTe\(_2\) crystals, while sample C was exfoliated from a different batch of lower quality. AFM images of samples A, B, and C are shown in Extended Data Figs. 9a-c.

Sample B is characterized by \( W = 350 \) nm, \( R = 450 \) nm, and \( d = 23 \) nm. To avoid current heating due to the narrower central strip, the excitation current was reduced to \( I_0 = 25 \) µA. Similar to the behavior in sample A in Fig. 4, the \( J_x(x, y) \) images in Extended Data Figs. 9d,g,j show a transition from single-vortex to two-vortex to no-vortex state upon increasing \( \theta \). This is in good agreement with simulations of \( J_x(x, y) \) (Extended Data Figs. 9e,h,k) as well as simulated current streamlines (Extended Data Figs. 9f,l,l). Here, the two-vortex state occurs at \( \theta_c = 60^\circ \) as compared to \( \theta_c = 54^\circ \) in sample A. This aperture angle allows us to deduce \( D/W = 0.35 \) and resulting \( D = 123 \) nm in sample B. Thus, the smaller strip width \( W \) of sample B results in an effective vertical upshift of the data points in Fig. 3a.

Extended Data Figs. 9m-r shows \( J_x(x, y) \) distributions in sample C, which is characterized by \( W = 770 \) nm and \( d = 30 \) nm. Here, the disc chambers of radius \( R = 950 \) nm show single-vortex state for aperture angles of \( \theta = 24^\circ \) (m), \( \theta = 45^\circ \) (n), and \( \theta = 60^\circ \) (o), and laminar flow for \( \theta = 180^\circ \) (p). Although, no chamber with \( \theta \) between \( 60^\circ \) and \( 180^\circ \) was available in this sample, the presence of a single vortex at \( \theta = 60^\circ \) indicates \( D/W > 0.35 \). The resulting \( D > 270 \) nm significantly exceeds the values derived for samples A and B. This may be attributed to the different batches of source material with different microscopic parameters or to variations in fabrication resulting in differences in the top and bottom surface quality.

In addition, single-vortex behavior was found in chambers with \( R = 725 \) nm and \( \theta = 45^\circ \) as well as for \( R = 500 \) nm and \( \theta = 60^\circ \) (Extended Data Figs. 9q,r), which shows the stability of the single-vortex state with respect to chamber radius.
Extended Data Fig. 9. AFM images of WTe$_2$ samples and additional vortical-to-laminar transitions. a, AFM image of sample A analyzed in the main text with $W = 550$ nm, $R = 900$ nm, $d = 48$ nm, and aperture angles $\theta = 20^\circ$, $35^\circ$, $54^\circ$, $72^\circ$, $90^\circ$, and $120^\circ$. b, Sample B used for Extended Data Figs. 9d-l with $W = 350$ nm, $R = 450$ nm, and $d = 23$ nm. c, Sample C with $W = 770$ nm and $d = 30$ nm, and $R = 950$, $725$, and $500$ nm (Extended Data Figs. 9m-r) and dual-drive geometry at the bottom part (Extended Data Figs. 10). d-I, Transition from single-vortex to two-vortex to laminar flow in sample B. d-f, Measurement of single vortex state in device B with $\theta = 40^\circ$ and corresponding simulations in the hydrodynamic regime with $D = 123$ nm and $\xi = 200$ nm. d, Measured current density $J_y(x,y)$ normalized by $I_0/W$ at $I_0 = 25$ $\mu$A. e, Simulated $J_y(x,y)$. f, Simulated current streamlines showing laminar (red) flow in the central strip and vortical flow (blue) in the chambers. g-i, Same as (d-f) for $\theta = 60^\circ$ showing banana-shaped vortex at the transition from a single to double-vortex state. j-l, Same as (d-f) for $\theta = 100^\circ$, showing laminar flow. m-r, Vortical flow in sample C with different geometrical parameters. Current density $J_x(x,y)$ in sample C with $W = 770$ nm and $d = 30$ nm and various chamber parameters: $\theta = 24^\circ$ and $R = 950$ nm (m), $\theta = 45^\circ$ and $R = 950$ nm (n), $\theta = 60^\circ$ and $R = 950$ nm (o), $\theta = 180^\circ$ and $R = 950$ nm (p), $\theta = 45^\circ$ and $R = 725$ nm (q) and $\theta = 60^\circ$ and $R = 500$ nm (r). Laminar flow is observed in (p), while vortical flow is present in all the rest of the geometries.

Dual-drive geometry

We have also studied an alternative geometry of a central disk with two apertures with $\theta = 44^\circ$ on opposite sides connected to two current-driven strips patterned in WTe$_2$ sample C with $d = 30$ nm, and in Au film of similar thickness, as shown in Extended Data Fig. 10. An ac current of $I_L = 50$ $\mu$A was applied to the left strip with source at the bottom and drain at the top. A lower excitation frequency of $f = 17.73$ Hz was used to reduce capacitive currents between the two strips. A separate floating current source was used to apply current to the right strip with three values, $I_R = 0, -50$ $\mu$A, and $50$ $\mu$A.

Panels a$_1$ and a$_2$ in Extended Data Fig. 10 show the $J_y(x,y)$ and $J_x(x,y)$ current distributions in Au sample for $I_R = 0$. In the ohmic regime, the current penetrates substantially into the central chamber similar to Fig. 1j. In the hydrodynamic case of WTe$_2$ (panel b$_1$), in contrast, the $J_y$ is mostly concentrated along the left strip with little protrusion into the chamber, analogous to Fig. 2g and consistent with the numerical simulations in panels g$_1$ and h$_1$. The transverse current, $J_x(x,y)$, reveals a laminar flow in Au (panel a$_2$) and a vortical flow in WTe$_2$ (panel b$_2$) in the chamber, consistent with the numerical simulations in panels g$_2$ and h$_2$, and with the simulated streamlines in panels g$_3$ and h$_3$. This configuration is equivalent to Figs. 1m and 2j of the main text.

Upon applying $I_R = -50$ $\mu$A, interesting flow patterns are observed in the central chamber. In the ohmic case, instead of flowing in and out of the chamber as observed for a single drive in panel a$_2$, part of the current traverses the chamber in its lower part (red $J_x$ in panel c$_3$), and then flows down to the bottom drain of the right strip as corroborated by the simulated streamlines in panel i$_1$. In the top half of the chamber, an opposite flow from top source of the right strip to the top drain on the left strip occurs (blue $J_x$), exchanging part of the currents from the two sources. Remarkably, in the hydrodynamic case, the shear forces of counter propagating currents in the two strips add up constructively and propel a single massive clockwise vortex in the entire chamber as observed in panel d$_2$ and simulated in panel j$_2$. The simulated streamlines in panel j$_2$ show that in this case the currents of the left and right sources do not mix: the laminar streams in the two strips are isolated by the whirlpool in the central chamber.

One would then expect that in the case of copropagating currents in the two strips (panel f$_1$), the opposing shear forces at the two apertures act destructively, annihilating the massive vortex. Instead, we find that a vortex–antivortex pair is nucleated in the chamber as visualized by $J_x(x,y)$ in panel f$_2$ and simulated in panels l$_2$ and l$_3$. 

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Extended Data Fig. 10. Vortex–antivortex formation in dual-drive geometry. a-f, Experimentally derived current densities $J_y(x, y)$ (top row) and $J_x(x, y)$ (bottom row) in Au and WTe$_2$ samples. a-b, Current $I_L = 50 \mu$A is driven in the up direction in the left strip with no current applied to the right strip resulting in a single vortex in the WTe$_2$ chamber in b. c-d, Counterpropagating currents $I_L = 50 \mu$A and $I_R = -50 \mu$A applied to the right and left strips, giving rise to a single massive vortex in d. e-f, Copropagating currents $I_L = 50 \mu$A and $I_R = 50 \mu$A applied to both strips which generates a vortex–antivortex pair in f. g-l, Numerical simulations of current densities $J_y(x, y)$ (top row), $J_x(x, y)$ (middle row) and the corresponding streamlines (bottom row) in the ohmic and hydrodynamic regimes for the three current configurations. The laminar streamlines are marked in red and the vortex streamlines in blue. The experimental data were acquired with pixel size of 10 nm, acquisition time of 40 ms/pixel, and image size of 600×350 pixels/image.
Captions of Supplementary Videos

Supplementary Video 1 | Simulations of vortical-to-laminar flow transition in the para-hydrodynamic regime vs. $\theta$. Numerical simulation of the current density $J_x(x, y)$ (top right) and the corresponding streamlines (bottom right) in the double-chamber geometry upon increasing the aperture angle $\theta$ for $D/W = 0.28$. The left panel shows the vortex stability phase diagram with no stress boundary conditions as presented in Fig. 3a. The purple dot marks the value of the varying $\theta$ along the $D/W = 0.28$ line. For $\theta \leq 54^\circ$, there is a single vortex in each chamber (blue streamlines). Upon increasing $\theta$ further, the laminar flow (red streamlines) splits the single vortex in each chamber into two vortices, which are stable up to $\theta \leq 60^\circ$. Finally, for $\theta > 60^\circ$, the laminar streamlines fill the entire area of the chambers.

Supplementary Video 2 | Simulations of vortical-to-laminar flow transition in the quasi-ballistic regime vs. $\theta$. Numerical simulation of the current density $J_x(x, y)$ (top right) and the corresponding streamlines (bottom right) in the double-chamber geometry upon increasing $\theta$ for $D/W = 1.5$. The left panel shows the vortex stability phase diagram with no stress boundary conditions as presented in Fig. 3a. The purple dot marks the value of the varying $\theta$ along the $D/W = 1.5$ line. With increasing $\theta$, the laminar streamlines (red) gradually penetrate deeper into the chambers, distorting the vortices (blue streamlines) and pushing them towards the outer boundaries. The vortices become extinct at $\theta \approx 150^\circ$ without splitting into double vortices as is the case in the hydrodynamic regime in Supplementary Video 1. For $\theta > 150^\circ$, the laminar streamlines fill the entire area of the chambers.