Electromagnetic Channel Model for Near Field MIMO Systems in the Half Space

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Abstract—In most multiple-input multiple-output (MIMO) communication systems, the amount of information that can be transmitted reliably depends on the effective degrees of freedom (EDoF) of the wireless channel. Conventionally, one can model the channel matrix and study the EDoF, based on an electromagnetic (EM) channel model that is built with the free space Green function. However, the EDoF of free-space channel model may not fit the practical scenario when EM waves only transmit above the ground. In this letter, we analyze the EDoF for both discrete and continuous aperture MIMO systems in the half space. We also propose an approach to quickly calculate the Green function in the half space from the Sommerfeld identity. Simulation results show that the difference between the EDoF in the half space and that in the free space is non-negligible for the near field communications, which indicates that the ground exerts noticeable influence on EDoF. The proposed study establishes a fundamental electromagnetic framework for MIMO wireless communication in the half space.

Index Terms—MIMO, electromagnetic models, effective degrees of freedom, green function.

I. INTRODUCTION

MULTIPLE-INPUT-MULTIPLE-OUTPUT (MIMO) technology using spatial multiplexing has been developed to enhance the channel capacity of modern wireless communications. Different electromagnetic (EM) modes, as orthogonal bases, have been employed in MIMO systems. Typically, the channel matrix is modeled based on the scalar Green function for MIMO systems in free space [1].

Recently, the number of single input single output (SISO) subchannels defined as the effective degrees of freedom (EDoF) has aroused much interest, because the EDoF fixes the maximum achievable capacity in the MIMO system [2], [3]. The evaluation of the EDoF is a complicated problem, which can be studied based on two kinds of models, i.e., the conventional channel model and the EM model.

Based on the conventional channel model of the MIMO system in the free space, [4] calculates EDoF by studying the spectral efficiency of MIMO systems with different shapes of arrays. In [5], the authors estimate the EDoF of the MIMO system when the channel matrix is subject to empirical distribution. However, the conventional channel model is less accurate than the EM model of the MIMO system.

Based on the EM model of the MIMO system in free space, [6] discusses the method to calculate EDoF, the limit of EDoF and the optimal number of sources and receivers. Furthermore, an EM model for the hologram MIMO has been proposed in [7], where the channel matrix is built with plane-wave expansion. The EM model is useful for linking EDoF with the capacity limit of the MIMO system.

However, the channel models in [5], [6], [7], and [8] are all built in the free space scenario, while in the near field, the free space does not fit the reality well because the EM waves can only transmit above the ground [8]. Hence, the boundary condition on the ground should be considered to generate the more accurate channel model, namely, more accurate channel model should be established in the half space, which could help to predict the channel capacity more precisely [6].

In this letter, we analyze the EDoF for both discrete and continuous aperture MIMO systems in the half space. We also propose an approach to quickly calculate the Green function in the half space from the Sommerfeld identity. Simulation results show that the difference between the EDoF in the half space and that in the free space is significant for the near field communications, which indicates that the ground exerts noticeable influence on the channel capacity. With the increase of the antenna number, the EDoF for discrete MIMO systems converges to that for continuous MIMO systems.

II. DISCRETE-APERTURE MIMO SYSTEM

Suppose the source is equipped with N antennas and the receiver is equipped with M antennas. The channel vector from the source to the mth antenna of the receiver is defined as $h_m \in \mathbb{C}^N \quad (m = 1, \ldots, M)$, which can be written as [6]

$$h_m = [G_{m1}, \ldots, G_{mN}]^T,$$

(1)

where $G_{mn}$ is the scalar Green function linking the position of the $m$th transmit antenna and the $n$th receiver antenna. The overall channel $H \in \mathbb{C}^{N \times M}$ is obtained by merging $h_m \quad (m = 1, \ldots, M)$ as

$$H = [h_1, \ldots, h_M]^T.$$

(2)

Define the correlation matrix as $R = HH^H$. The EDoF of the discrete-aperture MIMO system is a function of $R$ represented...
as \( \Xi(\mathbf{R}) \), and can be approximately calculated as [4], [9]

\[
\Xi(\mathbf{R}) = \left( \frac{\text{tr}(\mathbf{R})}{\|\mathbf{R}\|_F} \right)^2 = \frac{\left( \sum_{i=1}^{N} \sigma_i \right)^2}{\sum_{i=1}^{N} \sigma_i^2}, \quad (3)
\]

where \( \sigma_i \) is the \( i \)th eigenvalue of \( \mathbf{R} \). In the far-field communications, the leading eigenvalue of \( \mathbf{R} \) is significantly larger than the other eigenvalues, and the EDoF is close to one corresponding to the only communication mode where a plane wave travels from the source to the receiver [10].

III. CONTINUOUS-APERTURE MIMO SYSTEM

Recently, the study of continuous-aperture MIMO systems has aroused much interest, because the continuous-aperture MIMO systems can sufficiently exploit physical properties of spatial electromagnetic waves, leading to extreme spatial resolution, high spectrum efficiency, and high energy efficiency [11]. The formulation of the EDoF in discrete-aperture MIMO systems can be extended to continuous-aperture MIMO systems with the help of auto-correlation kernel function.

Suppose the continuous-aperture source and the continuous-aperture receiver are uniform linear arrays (ULA) whose lengths are \( L_s \) and \( L_r \), respectively. Let \( S \) denote the region of the source and \( R \) denote the region of the receiver. Define the Green function relating two arbitrary locations \( \mathbf{r}_s \in S, \mathbf{r}_r \in R \) as \( G(\mathbf{r}_s, \mathbf{r}_r) \), and define the auto-correlation kernel \( K(\mathbf{r}_s, \mathbf{r}_r') \) that correlates two locations in the source region \( \mathbf{r}_s \in S, \mathbf{r}_r' \in S \) as

\[
K(\mathbf{r}_s, \mathbf{r}_r') = \int_R G^H(\mathbf{r}_r, \mathbf{r}_s) G(\mathbf{r}_r, \mathbf{r}_r') d\mathbf{r}_r. \quad (4)
\]

The channel correlation matrix \( \mathbf{R} \) is then derived from \( \mathbf{R} = H^H \mathbf{H} \) under the condition \( M \to \infty, N \to \infty \) while the sizes of the source and the receiver are fixed. The \( (n_1, n_2) \)th element of \( \mathbf{R} \) corresponds to the channel correlation between the \( n_1 \)th and \( n_2 \)th antennas of the source, and has the asymptotic representation:

\[
\mathbf{R}_{(n_1, n_2)} = | \sum_{m=1}^{M} G^H_{mn_1} G_{mn_2} |^2 \to \frac{M^2}{L_r^2} | K(\mathbf{r}_s, \mathbf{r}_r') |^2. \quad (5)
\]

Then the following equations hold:

\[
\|\mathbf{R}\|_F^2 = \sum_{n_1=1}^{N} \sum_{n_2=1}^{N} | \sum_{m=1}^{M} G^H_{mn_1} G_{mn_2} |^2 \\
\to \frac{N^2}{L_s^2} \int_S \sum_{m=1}^{M} \frac{| K(\mathbf{r}_s, \mathbf{r}_r') |^2}{L_r^2} d\mathbf{r}_s d\mathbf{r}_r' \quad (6)
\]

\[
\text{tr}(\mathbf{R}) = \sum_{m=1}^{M} \sum_{n=1}^{N} | G_{mn} |^2 \to \frac{MN}{L_s L_r} \int_S \int_R | G(\mathbf{r}_s, \mathbf{r}_r) |^2 d\mathbf{r}_s d\mathbf{r}_r, \quad (7)
\]

where the asymptotic representation comes from the fact: \( d\mathbf{r}_s \sim \frac{L_s}{N}, \quad d\mathbf{r}_r' \sim \frac{L_r}{M}, \quad \) and \( d\mathbf{r}_r \sim \frac{L_r}{N} \). Since \( \|\mathbf{R}\|_F^2 \) is \( O(M^2N^2) \) and \( \text{tr}(\mathbf{R}) \) is \( O(MN) \), \( \Xi(\mathbf{R}) \) is \( \left( \frac{\text{tr}(\mathbf{R})}{\|\mathbf{R}\|_F} \right)^2 \).

Fig. 1. The ground is in the \( xy \) plane where \( z = 0 \). The images of the source are composed of a single quasi-static image and several images with complex positions.

converges when \( M \to \infty, N \to \infty \). The EDoF of the continuous-aperture MIMO system is then computed as

\[
L(\mathbf{R}) = \lim_{M,N \to \infty} \Xi(\mathbf{R}) = \lim_{M,N \to \infty} \left( \frac{\text{tr}(\mathbf{R})}{\|\mathbf{R}\|_F} \right)^2 = \frac{(\int_S \int_R | G(\mathbf{r}_s, \mathbf{r}_r) |^2 | d\mathbf{r}_s d\mathbf{r}_r')^2}{\int_S \int_R | K(\mathbf{r}_s, \mathbf{r}_r') |^2 d\mathbf{r}_s d\mathbf{r}_r'}. \quad (8)
\]

IV. GREEN FUNCTION IN THE HALF SPACE

Note that \( \Xi(\mathbf{R}) \) in (3) as well as \( L(\mathbf{R}) \) in (8) relies on the Green function, which will be computed in this section.

A. The Closed Form Representation of the Green Function

Since the EM waves can only transmit above the ground, the boundary condition on the ground where \( z = 0 \) should be considered when calculating the EM field, which is written in the coordinate in Fig. 1 as

\[
\frac{\partial \phi}{\partial z} + i k_0 \beta \phi = 0, \quad z = 0, \quad (9)
\]

where \( \phi \) is the electric field, \( k_0 \) is the wave number, and \( \beta \) is the normalized admittance of the ground. Then the reflection coefficient for the ground is defined as

\[
\tilde{C}(k) = \frac{k_z - k_0 \beta}{k_z + k_0 \beta}, \quad (10)
\]

where \( k_z \) is the wave number in \( z \) direction with the form

\[
k_z = \sqrt{k_0^2 - k^2}. \quad (11)
\]

In the half space, the exact expression of the Green function for the Helmholtz equation relating \( \mathbf{r}_r = [x_r, y_r, z_r]^T \) and

\[
\mathbf{r}_s = [x_s, y_s, z_s]^T \quad \text{can be represented by the method of images as} \quad [12]
\]

\[
G(\mathbf{r}_r, \mathbf{r}_s) = \frac{i}{4\pi} \int_0^\infty \frac{e^{ikz_s} | z_r - z_s | k J_0(k\rho) dk}{k_z} + \frac{i}{4\pi} \int_0^\infty \frac{e^{ikz_s} (z_r + z_s) k J_0(k\rho) \tilde{C}(k) dk}{k_z}, \quad (12)
\]

where the horizontal distance in the \( xy \) plane is \( \rho = \left( (x_r - x_s)^2 + (y_r - y_s)^2 \right)^{1/2} \) and \( J_0 \) is the 0th-order Bessel
function of the first kind. The integral in (12) is referred to as Sommerfeld integral whose analytical solution is not discovered so far [13]. To simplify the numerical evaluation of the Sommerfeld integral, we can use the Sommerfeld identity:

$$\frac{e^{ikR}}{R} = i \int_0^\infty J_0(kp)e^{ik_z(z+z')} k_z dk,$$

where $R = \sqrt{\rho^2 + (z + z')^2}$. Note that when $k_0 = 0$, $\tilde{C}(k)$ is constantly equal to 1. The image corresponding to $\tilde{C}(k) = 1$ is named as the quasi-static image. Using the Sommerfeld identity to extract the quasi-static term of the image, we can rewrite (12) as

$$G(r, r_s) = \frac{\exp(ik_0D_1)}{4\pi D_1} + \frac{\exp(ik_0D_2)}{4\pi D_2} + \frac{i}{4\pi} \int_0^\infty \frac{e^{ik_z(z+z')}J_1(\kappa p)}{k_z} \left(\tilde{C}(k) - 1\right) dk,$$

where $D_1 = \left[\rho^2 + (z_r - z_s)^2\right]^{1/2}$, $D_2 = \left[\rho^2 + (z_r + z_s)^2\right]^{1/2}$.

The last integral in (14) is hard to calculate straightforwardly due to the oscillation of the integrand. Thus, we apply the exponential expansion method. The function $\tilde{C}(k) - 1$ can be expanded with high accuracy in regard of $k_z$ as [14]

$$\tilde{C}(k) - 1 \approx \sum_{q=1}^{Q} a_q e^{-b_q k_z},$$

where $a_q$ and $b_q$ are complex numbers and $Q$ is an integer. Utilizing (15) to simplify (14), we obtain a closed-form approximation of $G(r, r_s)$ as

$$G(r, r_s) \approx \frac{\exp(ik_0D_1)}{4\pi D_1} + \frac{\exp(ik_0D_2)}{4\pi D_2} + \frac{\sum_{q=1}^{Q} a_q e^{ik_0R_q}}{4\pi R_q},$$

where $R_q$ is a complex distance with positive real part and is given by

$$R_q^2 = \rho^2 + (z_r + z_s + b_q i)^2.$$

The three terms in (16) are regarded as the contributions from the original source, its quasi-static image, and several images with complex positions, respectively. Note that (16) applies for sources and receivers located anywhere within the half-space above the ground, for a single set of constants $a_q$ and $b_q$. The Green function in the free space is the first term in (16), i.e.,

$$G_{free}(r, r_s) = \frac{\exp(ik_0D_1)}{4\pi D_1}.$$  

B. Obtaining the Coefficients $a_q$ and $b_q$

The closed form expression for the Green function is quite simple. However, obtaining the coefficients $a_q$ and $b_q$ for (16) is not straightforward. Note that the integrand in (14) has a pole as well as a branch point at $k = k_0$. To avoid the rapid variation near the branch point, we select a deformed path of integral in (14), defined as [15]

$$k_z = k_0 \left[ i \xi + \left( 1 - \frac{\xi}{T} \right) \right], \quad 0 \leq \xi \leq 1,$$

where $T$ is adjustable and controls the real axis intercept of the path. For near-field computation, choosing $T = 10$ can ensure high accuracy of the approximation in (16), as is proven in [16].

On the deformed path in (19), $\tilde{C}(k) - 1$ can be approximated by an exponential expansion in regard of $\xi$ as

$$\tilde{C}(k) - 1 \approx K(\xi) \Delta = \sum_{q=1}^{Q} A_q e^{B_q \xi}$$

Comparing the coefficients in (15) and (20), $a_q$ and $b_q$ can be written in terms of $A_q$ and $B_q$ as

$$a_q = A_q, \quad b_q = B_q \frac{T}{k_0(1 - iT)}.$$  

Thus, we translate the task of computing $a_q$ and $b_q$ into the task of computing $A_q$ and $B_q$, which will be discussed in the next subsection.

C. The Sampling Method to Obtain $A_q$ and $B_q$

In order to obtain $A_q$ and $B_q$ such that $\tilde{C}(k) - 1$ can be approximated by $K(\xi)$, we apply the modified Prony method. Denote $W$ as the number of sampling points for $K(\xi)$ in the range of $0 \leq \xi \leq 1$ and define the uniformly sampling points $F(w) \Delta = K(\frac{\xi}{W}) = \sum_{q=1}^{Q} A_q \exp \left( \frac{B_q}{W} \frac{\xi}{W} \right)(w = 1, \ldots, W)$.

Define the polynomial $f(x)$ with roots $\zeta_q = \exp \left( \frac{B_q}{W} \right)$ as

$$f(x) = \prod_{q=1}^{Q} (x - \exp \left( \frac{B_q}{W} \right)) = x^Q + C_{Q-1} x^{Q-1} + \cdots + C_0.$$  

Thus, $y_q(w) \Delta = \exp \left( \frac{B_q}{W} \right)$ ($q = 1, \ldots, Q$) satisfies the $Q$th-order linear difference equation with the characteristic equation as $f(x)$:

$$y_q(w + Q) + C_{Q-1} y_q(w + Q - 1) + \cdots + C_0 y_q(w) = 0$$

Since each $F(w)$ is the linear combination of $y_q(w)$, $q = 1, \ldots, Q$, $F(w)$ satisfies the difference equation of the same form. Define

$$\mathbf{A} = \begin{bmatrix} F(1) & F(2) & \cdots & F(Q) \\ F(2) & F(3) & \cdots & F(Q) \\ \vdots & \vdots & \ddots & \vdots \\ F(W - Q) & F(W - Q + 1) & \cdots & F(W - 1) \end{bmatrix},$$

$$\mathbf{g} = \begin{bmatrix} C_0 \\ C_1 \\ \vdots \\ C_{Q-1} \end{bmatrix}, \quad \mathbf{b} = - \begin{bmatrix} F(Q + 1) \\ F(Q + 2) \\ \vdots \\ F(W) \end{bmatrix}.$$
Then, there is
\[ \eta = 0.3 - 0.1 i. \]
Thus we have \( \beta = 1/\eta = 3 + 1i \). We choose \( T = 10 \), \( Q = 5 \), and \( W = 10 \) to calculate Green function in the half space, which is proved to be accurate in near field scenario [16]. We set the wave length \( \lambda = 0.1m \) and define the height of the source and the receiver as \( z_s \) and \( z_r \) respectively. The source and the receiver are both equipped with extremely long ULAs which are parallel to each other with \( L_r = 4m \) and \( L_s = 12m \). The great lengths of ULAs are utilized to prevent communication happening in the far-field regime where the EDoF is constantly close to one [10].

A. The EDoF Versus the Number of Antennas
For the discrete aperture, we suppose the numbers of the antennas at the source side and at the receiver side are the same, i.e., we set \( M = N \). We set \( z_r = 1m \) and explore the change of EDoF with the increase of \( N \) when \( \rho = 10m \) as is shown in Fig. 4. It is seen that, \( \Xi \) increases with fluctuation until a certain number, which is defined as the optimal number for MIMO systems, and \( \Xi \) slowly decreases afterwards. Interestingly, the half-space and free-space EDoF are not distinguished for small number of antennas, because before \( M \) and \( N \) surpass the optimal number, all \( \sigma_i \) in (3) are of the same order of magnitude [10], and thus \( \Xi \) is constantly close to \( N \). However, after \( M \) and \( N \) surpass the optimal number, the exceeding \( \sigma_i \) corresponding to the exceeding antennas will become too small to contribute to the EDoF, and EDOF will reach its maximum value [6]. The exceeding \( \sigma_i \) slightly increases the imbalance between large and small eigenvalues in (3), and thus \( \Xi \) slowly decreases afterwards. Moreover, \( \Xi \) converges to \( L \) with the increase of \( M \), which verifies the convergence analysis in Section III.

Both \( \Xi \) and \( L \) in the half space are different than \( \Xi \) and \( L \) in the free space. While the optimal numbers for MIMO systems are the same both in the half space and in the free space. The difference between half space and free space when \( z_s = 10m \) is more significant than the difference when \( z_s = 50m \), because when the source is more close to the ground, the influence on the EM field by the images is more remarkable.

V. SIMULATION RESULTS AND ANALYSIS
In the simulations, the normalized surface impedance value is chosen to be \( \eta = 0.3 - 0.1 i. \)

This value is obtained from the gray clay loam of San Antonio.
VI. Conclusion

In this letter, we analyze the EDoF for MIMO systems in the half space by the method to quickly calculate the Green function in the half space. Simulation results show that the difference between the EDoF in the half space and that in the free space is pronounced for the near field communications, which indicates that the ground exerts significant influence on the EDoF.

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