Bound States in Gauge Theories as the Poincaré Group Representations

A.Yu. Cherny,¹ A.E. Dorokhov,¹ Nguyen Suan Han,² V.N. Pervushin,¹,∗ and V.I. Shilin¹,³

¹Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Russia
²Department of Theoretical Physics, Vietnam National University
³Moscow Institute of Physics and Technology, Dolgoprudnyy, Russia

The bound state generating functional is constructed in gauge theories. This construction is based on the Dirac Hamiltonian approach to gauge theories, the Poincaré group classification of fields and their nonlocal bound states, and the Markov-Yukawa constraint of irreducibility. The generating functional contains additional anomalous creations of pseudoscalar bound states: para-positronium in QED and mesons in QCD in the two gamma processes of the type of $\gamma + \gamma = \pi_0 + \text{para-positronium}$. The functional allows us to establish physically clear and transparent relations between the perturbative QCD to its nonperturbative low energy model by means of normal ordering and the quark and gluon condensates. In the limit of small current quark masses, the Gell-Mann-Oakes-Renner relation is derived from the Schwinger-Dyson (SD) and Bethe-Salpeter (BS) equations. The constituent quark masses can be calculated from a self-consistent non-linear equation.

Is dedicated to the 60-th anniversary of the birth of Professor S.I. Vinitsky

1. INTRODUCTION

At the beginning of the sixties of the twentieth century Feynman found that the naive generalization of his method of construction of QED fails in the non-Abelian theories. The unitary S-matrix in the non-Abelian theory was obtained in the form of the FP path integral by the brilliant application of the theory of connections in vector bundle [1]. Many physicists are of opinion that the FP path integral is the highest level of quantum description of the...
gauge constrained systems. Anyway, the FP integral at least allows us to prove the both renormalizability of the unified theory of electroweak interactions and asymptotic freedom of the non-Abelian theory. However, the generalization of the FP path integral to the bound states in the non-Abelian theories still remains a serious and challenging problem.

The bound states in gauge theories are usually considered in the framework of representations of the homogeneous Lorentz group and the FP functional in one of the Lorentz-invariant gauges. In particular, the “Lorentz gauge formulation” was discussed in the review [2] with almost 400 papers before 1992 on this subject being cited. Presently, the situation is not changed, because the gauge-invariance of the FP path integral is proved only for the scattering processes of elementary particles on their mass shells in the framework of the “Lorentz gauge formulation” [3].

In this paper, we suggest a systematic scheme of the bound state generalization of FP-functional and S-matrix elements. The scheme is based on irreducible representations of the nonhomogeneous Poincaré group in concordance with the first QED description of bound states [4, 5]. This approach includes the following elements.

i) The concept of the in- and out-state rays [6] as the products of the Poincaré representations of the Markov-Yukawa bound states [7–10].

ii) The split of the potential components from the radiation ones in a rest frame.

iii) Construction of the bound state functional in the presence of the radiation components. This functional contains the triangle axial anomalies with additional time derivatives of the radiation components.

iv) The joint Hamiltonian approach to the sum of the both standard time derivatives and triangle anomaly derivatives.

All these elements together leads to new anomalous processes in the strong magnetic fields. One of them is the two gamma para-positronium creation accompanied by the pion creation of the type of $\gamma + \gamma = \pi_0 +$ para-positronium.

Within the bound state generalization of the FP integral, we establish physically clear and transparent relations between the parton QCD model and the Numbu-Jona-Lasinio ones [11–13]. Below we show that it can be done by means of the gluon and quark condensates, introduced via the normal ordering.

The paper is organized as follows. Sec. 2 regards the Poincaré classification of in- and out-states. In Sec. 3, the Dirac method of gauge invariant separation of potential and radiation
variables is considered within QED. Sec. 4 is devoted to the bound state generalization of the Faddeev-Popov generating functional. In Sec. 5, we discuss the bound state functional in the presence of the radiation components, which contains the triangle axial anomalies with additional time derivatives of these radiation components. Sec. 6 is devoted to the axial anomalies in the NJL model inspired by QCD. In Appendix A, the bound state functional in the ladder approximation is considered. In Appendix B, the BS equations are written down explicitly and discussed.

2. BOGOLIUBOV-LOGUNOV-TODOROV RAYS AS IN-, OUT-STATES

According to the general principles of quantum field theory (QFT), physical states of the lowest order of perturbation theory are completely covered by local fields as particle-like representations of the Poincaré group of transformations of four-dimensional space-time.

The existence of each elementary particle is associated with a quantum fields $\psi$. These fields are operators defined in all space-time and acting on states $|\mathcal{P}, s\rangle$ in the Hilbert space with positively defined scalar product. The states correspond to the wave functions $\Psi_\alpha(x) = \langle 0|\psi_\alpha(x)|\mathcal{P}, s\rangle$ of free particles.

Its algebra is formed by generators of the four translations $\hat{P}_\mu = i\partial_\mu$ and six rotations $\hat{M}^{\mu\nu} = i[x_\mu\partial_\nu - x_\nu\partial_\mu]$. The unitary and irreducible representations are eigen-states of the Casimir operators of mass and spin, given by

$$\hat{P}^2|\mathcal{P}, s\rangle = m_\psi^2|\mathcal{P}, s\rangle, \quad (1)$$
$$-\hat{\omega}_\rho^2|\mathcal{P}, s\rangle = s(s + 1)|\mathcal{P}, s\rangle, \quad (2)$$
$$\hat{\omega}_\rho = \frac{1}{2}\varepsilon_{\lambda\mu\nu\rho}\hat{P}^\lambda\hat{M}^{\mu\nu}. \quad (3)$$

The unitary irreducible Poincaré representations describe wave-like dynamical local excitations of two transverse photons

$$A_{(b)}^T(t, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} \sum_{\alpha = 1, 2} \frac{1}{\sqrt{2\omega(k)}} \varepsilon^{(b)\alpha} [e^{i(\omega_k t - k\mathbf{x})}A_{b, \alpha}^+ + e^{-i(\omega_k t - k\mathbf{x})}A_{b, \alpha}^-]. \quad (4)$$

Two independent polarizations $\varepsilon^{(b)\alpha}$ are perpendicular to the wave vector and to each other, and the photon dispersion is given by $\omega_k = \sqrt{k^2}$.

The creation and annihilation operators of photon obey the commutation relations

$$[A_{b, \alpha}^-, A_{b', \beta}^+] = \delta_{\alpha, \beta}\delta(\mathbf{k} - \mathbf{k'}).$$
The bound states of elementary particles (fermions) are associated with bilocal quantum fields formed by the instantaneous potentials (see [7–9])

$$M(x, y) = M(z|X) = \sum_H \int \frac{d^3 \mathcal{P}}{(2\pi)^3 \sqrt{2\omega_H}} \int \frac{d^4 q e^{iq\cdot z}}{(2\pi)^4} \left\{ e^{i\mathcal{P}\cdot X} \Gamma_H(q^\perp|\mathcal{P}) a_H^+(\mathcal{P}, q^\perp) + e^{-i\mathcal{P}\cdot X} \Gamma_H(q^\perp|\mathcal{P}) a_H^- (\mathcal{P}, q^\perp) \right\},$$

where $\mathcal{P} \cdot X = \omega_H X_0 - \mathcal{P} X, \mathcal{P}_\mu = (\omega_H, \mathcal{P})$ is the total momentum components on the mass shell (that is, $\omega_H = \sqrt{M_H^2 + \mathcal{P}^2}$), and

$$X = \frac{x + y}{2}, \quad z = x - y.$$  \hspace{1cm} (6)

are the the total coordinate and the relative one, respectively. The functions $\Gamma$ belongs to the complete set of orthonormalized solutions of the BS equation [4] in a specific gauge theory, $a_H^\pm (\mathcal{P}, q^\perp)$ are coefficients treated in quantum theory as the creation (+) and annihilation operators (see Appendix B).

The irreducibility constraint, called Markov-Yukawa constraint, is imposed on the class of instantaneous bound states

$$z^\mu \hat{P}_\mu M(z|X) \equiv iz^\mu \frac{d}{dz^\mu} M(z|X) = 0.$$  \hspace{1cm} (7)

In Ref. [6] the in- and out- asymptotical states are the “rays” defined as a product of these irreducible representations of the Poincaré group

$$\langle \text{out} | = \prod_j \mathcal{P}_j, s_j |, \quad | \text{in} \rangle = | \prod_j \mathcal{P}_j, s_j \rangle.$$  \hspace{1cm} (8)

This means that all particles (elementary and composite) are far enough from each other to neglect their interactions in the in-, out- states. All their asymptotical states $\langle \text{out} |$ and $| \text{in} \rangle$ including the bound states are considered as the irreducible representations of the Poincaré group.

These irreducible representations form a complete set of states, and the reference frames are distinguished by the eigenvalues of the appropriate time operator $\hat{\ell}_\mu = \frac{\mathcal{P}_\mu}{M_J}$

$$\hat{\ell}_\mu | \mathcal{P}, s \rangle = \frac{\mathcal{P}_\mu}{M_J} | \mathcal{P}, s \rangle,$$  \hspace{1cm} (9)

where the Bogoliubov-Logunov-Todorov rays (9) can include bound states.
3. SYMMETRY OF S-MATRIX

The S-matrix elements are defined as the evolution operator expectation values between in- and out- states

\[
\mathcal{M}_{\text{in, out}} = \langle \text{out} | \hat{S}[\ell] | \text{in} \rangle, \tag{10}
\]

where the abbreviation “G-inv”, or “gauge-invariant”, assumes the invariance of S-matrix with respect to the gauge transformations.

The Dirac approach to gauge-invariant S-matrix was formulated at the rest frame \( \ell_0^\mu = (1, 0, 0, 0) \) [14–16]. Then the problem arises how to construct a gauge-invariant S-matrix in an arbitrary frame of reference. It was Heisenberg and Pauli’s question to von Neumann: “How to generalize the Dirac Hamiltonian approach to QED of 1927 [14] to any frame?” [10, 15, 16, 27]. The von Neumann reply was to go back to the initial Lorentz-invariant formulation and to choose the comoving frame

\[
\ell^\mu_0 = (1, 0, 0, 0) \rightarrow \ell^\text{comoving}_\mu = \ell_\mu, \quad \ell_\mu \ell^\mu = \ell \cdot \ell = 1 \tag{11}
\]

and to repeat the gauge-invariant Dirac scheme in this frame.

Dirac Hamiltonian approach to QED of 1927 was based on the constraint-shell action [14]

\[
W_{\text{Dirac}}^{\text{QED}} = W_{\text{QED}} \bigg|_{\delta W_{\text{QED}} \delta A_0 = 0}, \tag{12}
\]

where the component \( A_0^\mu \) is defined by the scalar product \( A_0^\mu = A \cdot \ell \) of vector field \( A_\mu \) and the unit time-like vector \( \ell_\mu \).

The gauge was established by Dirac as the first integral of the Gauss constraint

\[
\int^t dt \frac{\delta W_{\text{QED}}}{\delta A_0^\mu} = 0, \quad t = (x \cdot \ell). \tag{13}
\]

In this case, the S-matrix elements (10) are relativistic invariant and independent of the frame reference provided the condition (9) is fulfilled [9, 18].

Therefore, such relativistic bound states can be successfully included in the relativistic covariant unitary perturbation theory [18]. They satisfy the Markov-Yukawa constraint (7).

This framework yields the observed spectrum of bound states in QED [5], which corresponds to the instantaneous potential interaction and paves a way for constructing a bound state generating functional. The functional construction is based on the Poincaré group representations (51) with \( \ell^0 \) being the eigenvalue of the total momentum operator of instantaneous bound states.
4. QED

4.1. Split of potential part from radiation one

Let us formulate the statement of the bound state problem in the terms of the gauge-invariant variables using QED. It is given by the action \[16\]

\[ W[A, \psi, \bar{\psi}] = \int d^4x \left[ -\frac{1}{4}(F_{\mu\nu})^2 + \bar{\psi}(i\nabla(A) - m^0)\psi \right] , \] (14)

\[ \nabla_\mu(A) = \partial_\mu - ie A_\mu, \quad \nabla = \nabla_\mu \gamma^\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu . \] (15)

Dirac defined these gauge-invariant variables by the transformations

\[ \sum_{a=1,2} e_a A^D_a = A^D[A] = v[A] \left( A_k + \frac{1}{e} \partial_k \right) (v[A])^{-1}, \] (16)

\[ \psi^D[A, \psi] = v[A] \psi, \] (17)

where the gauge factor is given by

\[ v[A] = \exp \left\{ ie \int^t dt' a_0(t') \right\} \] (18)

\[ a_0[A] = \frac{1}{\Delta} \partial_i \partial_0 A_i(t, x) . \] (19)

Here the inverse Laplace operator acts on arbitrary function \( f(t, x) \) as

\[ \frac{1}{\Delta} f(t, x) \overset{\text{def}}{=} -\frac{1}{4\pi} \int d^3y \frac{f(t, y)}{|x - y|} \] (20)

with the kernel being the Coulomb potential.

Using the gauge transformations

\[ a^A_0 = a_0 + \partial_0 \Lambda \Rightarrow v[A^A] = \exp[ie\Lambda(t_0, x)]v[A] \exp[-ie\Lambda(t, x)] , \] (21)

we can find that initial data of the gauge-invariant Dirac variables (16) are degenerated with respect to the stationary gauge transformations

\[ A^D_i[A^A] = A^D_i[A] + \partial_i \Lambda(t_0, x), \quad \psi^D[A^A, \psi^A] = \exp[ie\Lambda(t_0, x)]\psi^D[A, \psi] . \] (22)

The Dirac variables (16) as the functionals of the initial fields satisfy the Gauss law constraint

\[ \partial_0 \left( \partial_i A^D_i(t, x) \right) \equiv 0 . \] (23)
Thus, explicit resolving the Gauss law allows us to remove two degrees of freedom and to reduce the gauge group into the subgroup of the stationary gauge transformations (22).

We can fix a stationary phase \( \Lambda(t_0, \mathbf{x}) = \Phi_0(\mathbf{x}) \) by an additional constraint in the form of the time integral of the Gauss law constraint (23) with zero initial data

\[
\partial_i A_i^D = 0 \rightarrow \Delta \Phi_0(\mathbf{x}) = 0. \tag{24}
\]

Dirac constructed the \textit{unconstrained system}, equivalent to the initial theory (14)

\[
W^* = W|_{\delta W/\delta A_0=0}[A^D_a = A^*_a, \psi^D = \psi^*] \tag{25}
\]

\[
= \int d^4x \left[ \frac{(\dot{A}^*_i)^2 - B_i^2}{2} + \frac{1}{2} \dot{j}^*_0 \frac{1}{\Delta} j^*_0 - j^*_i A^*_i + \bar{\psi}^*(i\dot{\hat{\mathcal{D}}} - m)\psi^* \right],
\]

where

\[
\dot{A}^*_i = \sum_{a=1,2} \partial_0 A^*_a e^a_i, \tag{26}
\]

\[
B_i = \varepsilon_{ijk} \partial_j A^*_k \tag{27}
\]

are the electric and magnetic fields, respectively.

In three-dimensional QED, there is a subtle difference between the model (25) and the initial constrained system (14). This is the origin of the current conservation law. In the initial constrained system (14), the current conservation law \( \partial_0 j_0 = \partial_i j_i \) follows from the equations for the gauge fields, whereas a similar law \( \partial_0 j^*_0 = \partial_i j^*_i \) in the \textit{equivalent unconstrained system} (25) follows only from the classical equations for the fermion fields. This difference becomes essential in quantum theory. In the second case, we cannot use the current conservation law if the quantum fermions are off mass-shell, in particular, in a bound state. What do we observe in an atom? The bare fermions, or \textit{dressed} ones (16)? Dirac supposed [14] that we can observe only \textit{gauge invariant} quantities of the type of the \textit{dressed} fields.

4.2. Bilocal fields in the Ladder Approximation

The constraint-shell QED allows us to construct the relativistic covariant perturbation theory with respect to radiation corrections [19]. Recall that our solution of the problem of relativistic invariance of the nonlocal objects is the choice of the time axis as a vector operator with eigenvalues proportional to the total momenta of bound states [20, 21]. In this
case, the relativistic covariant unitary S-matrix can be defined as the Feynman path integral

\[ Z_\hat{\eta}[s, \bar{s}, J^*] = \langle \ast \mid \int D\bar{\psi}^* D\psi^* e^{iW_\hat{\eta}[\bar{\psi}^*, \psi^*] + iS^*} \mid \ast \rangle, \tag{28} \]

where

\[ W_\hat{\eta}[\psi, \bar{\psi}] = \int d^4x [\bar{\psi}(x) (i\partial - ieA^* - m^0)\psi(x) + \frac{1}{2} \int d^4y (\psi(y)\bar{\psi}(x))K(\ell^\perp | X)(\psi(x)\bar{\psi}(y))], \tag{29} \]

and the symbol

\[ \langle \ast \mid \ldots \mid \ast \rangle = \int \prod DA_j^* e^{iW_\hat{\eta}^*[A^*]} \ldots \tag{30} \]

stands for the averaging over transverse photons. Here by definition \( \partial = \partial^\mu \gamma_\mu \), and \( K^{(\ell)} \) is the kernel

\[ K^{(\ell)}(z^\perp | X) = \ell V(z^\perp)\delta(z \cdot \ell), \tag{31} \]

\[ \ell = \ell^\mu \gamma_\mu = \gamma \cdot \ell, \quad z^\perp_\mu = z_\mu - \ell_\mu(z \cdot \ell), \]

where \( z \) and \( X \) are the relative and total coordinates \( (6) \). The potential \( V(z^\perp) \) depends only on the transverse component of the relative coordinate with respect to the time axis \( \ell \). The requirement for the choice of the time axis \( (9) \) in bilocal dynamics is equivalent to Markov - Yukawa condition \( (7) \).

Apparently, the most straightforward way for constructing a theory of bound states is the redefinition of action \( (29) \) in terms of the bilocal fields by means of the Legendre transformation \[ (22) \]

\[ \int d^4xd^4y(\psi(y)\bar{\psi}(x))K(x, y)(\psi(x)\bar{\psi}(y)) = \]

\[ = -\frac{1}{2} \int d^4xd^4yM(x, y)K^{-1}(x, y)M(x, y) + \]

\[ + \int d^4xd^4y(\psi(x)\bar{\psi}(y)), M(x, y) \tag{32} \]

where \( K^{-1} \) is the inverse kernel \( K \) given by Eq. \( (31) \). Following Ref. \[ (23) \], we introduce the short-hand notation

\[ \int d^4xd^4y(\psi(y)\bar{\psi}(x))(i\partial - ieA^* - m^0)\delta^{(4)}(x - y) = (\psi\bar{\psi}, -G^{-1}_\Lambda), \tag{33} \]

\[ \int d^4xd^4y(\psi(x)\bar{\psi}(y)), M(x, y) = (\psi\bar{\psi}, M). \tag{34} \]
After quantization over $N_c$ fermion fields (here $N_c$ is the number of colors equal to 3), the functional (28) takes the form

$$Z_{\eta}^*[s, \bar{s}^*, J^*] = \langle \ast | \int \prod \mathcal{D} \mathcal{M} e^{i W_{\text{eff}}[\mathcal{M}] + i S_{\text{eff}}[\mathcal{M}] | \ast} \rangle,$$

(35)

where

$$W_{\text{eff}}[\mathcal{M}] = \text{tr} \left[ \log(-G^{-1}_A + \mathcal{M}) \right] - \frac{1}{2} (\mathcal{M}, \mathcal{K}^{-1}_A \mathcal{M})$$

(36)

is the effective action, and

$$S_{\text{eff}}[\mathcal{M}] = (s^* \bar{s}^*, (G^{-1}_A - \mathcal{M})^{-1})$$

(37)

is the source term. The effective action can be decomposed as

$$W_{\text{eff}}[\mathcal{M}] = -\frac{1}{2} (\mathcal{M}, \mathcal{K}^{-1}_A \mathcal{M}) + i \sum_{n=1}^{\infty} \frac{1}{n} \Phi^n.$$

(38)

Here $\Phi \equiv G_A \mathcal{M}, \Phi^2, \Phi^3,$ etc. mean the following expressions

$$\Phi(x, y) \equiv G_A \mathcal{M} = \int d^4 z G_A(x, z) \mathcal{M}(z, y),$$

$$\Phi^2 = \int d^4 x d^4 y \Phi(x, y) \Phi(y, x),$$

$$\Phi^3 = \int d^4 x d^4 y d^4 z \Phi(x, y) \Phi(y, z) \Phi(z, x),$$

etc.

As a result of such quantization, only the contributions with inner fermionic lines (but not the scattering and dissociation channel contributions) are included in the effective action, since we are interested only in the bound states constructed as unitary representations of the Poincaré group.

4.3. The anomalous creation of Para-Positronium in QED

The effective bound state functional in the presence of radiation fields contains a triangle anomaly decay of positronium $\eta_P$ with an additional time derivative of these fields

$$W_{\text{eff}} = W(A^*) + W(\eta_P)$$

$$W(A^*) = \int d^4 x [C_P \eta_P \dot{A}_i B_i + \frac{\dot{A}_i^2 + B_i^2}{2}]$$

$$W_{\eta} = \int d^4 x \left\{ \frac{1}{2} \left[ \eta_P^2 - M^2_P \eta_P^2 - (\partial_i \eta_P)^2 \right] \right\},$$
where $B_i$ are the magnetic field component (27), and the parameter of the effective action is given by

$$C_P = \frac{2\alpha}{m_e} \left( \frac{\psi_{\text{Sch}}(0)}{m_e^{3/2}} \right) = \sqrt{\pi} \alpha^{5/2} = \frac{\alpha \sqrt{2}}{F_P \sqrt{\pi}},$$

(40)

where $\alpha = 1/137$ is the QED coupling constant, and

$$F_P = \frac{\psi_{\text{Sch}}(0) m_e}{m_e^{3/2} \sqrt{2\pi}} = \frac{m_e \sqrt{2}}{\alpha^{3/2} \pi},$$

(41)

is the positronium analogy of the pion weak-coupling constant $F_\pi$ discussed in the next Section.

The product $C_P \hat{A}_i B_i$ is obtained from the triangle diagram shown in Fig. 1.

![Figure 1](image1.png)

**Figure 1.** The standard triangle diagram, used for calculating $C_P$, the parameter of the effective action.

The Hamiltonian of this system is the sum of the Hamiltonians of the free electro-magnetic fields and the positronium ones $\eta_P(x)$ and the interaction

$$W_{\text{eff}} = \int dt d^3x \left\{ E_i \dot{A}_i + P_\eta \dot{\eta}_P - \mathcal{H} \right\},$$

$$\mathcal{H} = \mathcal{H}_\eta + \mathcal{H}_A + \mathcal{H}_{\text{int}},$$

$$\mathcal{H}_\eta = \frac{1}{2} (\dot{\eta}_P - M_P \eta_P^2 - (\partial_i \eta_P)^2),$$

$$\mathcal{H}_{\text{int}} = C_P^2 \eta_P E_i B_i + \frac{C_P^2 \eta_P^2}{2} B_i^2.$$  

(42)

The anomalous processes of creation of the positronium pairs in the external magnetic field at the photon energy value $E_\gamma \simeq m_e$ (see Fig. 2) are described by the differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^4 E_\gamma^2}{\pi 128 F_P^4} \sqrt{1 - \left( \frac{2m_e}{E_\gamma} \right)^2} = \frac{\pi \alpha^{10} E_\gamma^2}{512 m_e^4} \sqrt{1 - \left( \frac{2m_e}{E_\gamma} \right)^2}. $$

(43)
Figure 2. The new diagram for the anomalous processes of creation of the positronium pairs in the external magnetic field that follows from the Hamiltonian anomaly, given by $\mathcal{H}_{\text{int}}$ in Eqs. (42).

### 4.4. The Schwinger QED$_{1+1}$

The Schwinger two dimensional QED$_{1+1}$ was considered in the framework of the Dirac approach to gauge theories distinguished by the constraint-shell action [24].

This constraint-shell action has an additional time derivative term of the gauge field that goes from the fermion propagator in the axial anomaly. This anomalous time derivative term changes the initial Hamiltonian structure of the gauge field action

\[
W_{\text{Scwinger}} = \int dt dx \left\{ \frac{1}{2} \dot{\eta}^2_S + C_S \eta_S \dot{A} + \frac{\dot{A}^2}{2} \right\} \tag{44}
\]

\[
= \int dt dx \left[ P_S \dot{\eta}_S + E \dot{A} - \frac{E^2}{2} + C_S \eta_S E - C_S^2 \frac{\dot{\eta}_S^2}{2} \right], \tag{45}
\]

\[
C_S = \frac{e}{2\pi}. \tag{46}
\]

Finally, an additional Abelian anomaly given by the last term in Eq. (45) enables us to determine the mass of the pseudoscalar bound state [24]. In QED$_{(1+1)}$, it is the well-known mass of the Schwinger bound state

\[
\Delta M^2 = \frac{e^2}{\pi}.
\]

The Schwinger model justifies including of the similar additional terms in the 4-dimensional QED.
5. NON-ABELIAN DIRAC HAMILTONIAN DYNAMICS IN AN ARBITRARY FRAME OF REFERENCE

In order to demonstrate the Lorentz-invariant version of the Dirac method [14] given by Eq.(12) in a non-Abelian theory, we consider the simplest example of the Lorentz-invariant formulation of the naive path integral without any ghost fields and FP-determinant

\[ Z[J, \eta, \bar{\eta}] = \int \left[ \prod_{\mu,a} dA^a_\mu \right] d\psi d\bar{\psi} e^{iW[A,\psi,\bar{\psi}]+iS[J,\eta,\bar{\eta}]} \quad (47) \]

We use standard the QCD action \( W[A,\psi,\bar{\psi}] \) and the source terms

\[ W = \int d^4x \left[ -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \bar{\psi}(i\gamma^\mu(\partial_\mu + \hat{A}_\mu) - m)\psi \right], \quad (48) \]
\[ F^a_{0k} = \partial_0 A^a_k - \partial_k A^a_0 + g f^{abc} A^b_0 A^c_k \equiv \hat{A}^a_k - \nabla^a_k A^b_0, \quad (49) \]
\[ S = \int d^4x \left[ A_\mu J^\mu + \bar{\eta}\psi + \bar{\psi}\eta \right], \quad \hat{A}_\mu = ig \frac{\lambda^a A^a_\mu}{2}. \quad (50) \]

There are a lot of drawbacks of this path integral from the point of view of the Faddeev-Popov functional [1]. They are the following:

1. The time component \( A_0 \) has indefinite metric.
2. The integral (47) contained the infinite gauge factor.
3. The bound state spectrum contains tachyons.
4. The analytical properties of field propagators are gauge dependent.
5. Operator foundation is absent [25].
6. Low-energy region does not separate from the high-energy one.

All these defects can be removed by the integration over the indefinite metric time component \( A_\mu \ell^\mu \equiv A \cdot \ell \), where \( \ell^\mu \) is an arbitrary unit time-like vector: \( \ell^2 = 1 \). If \( \ell^0 = (1, 0, 0, 0) \)
then \( A_\mu \ell^\mu = A_0 \). In this case

\[
Z[\ell^0] = \int \left[ \prod_{x,j,a} dA_j^a(x) \right] e^{iW_{YM}(L^a) \left[ \det (\nabla_j(A^*))^2 \right]^{-1/2}} Z_\psi, \quad (51)
\]

\[
L^a = \int dt \nabla_i^{ab}(A^*) \dot{A}_i^{ab} = 0, \quad (52)
\]

\[
W_{YM}^* = \int d^4x (\dot{A}_j^a)^2 - (B_j^a)^2, \quad (53)
\]

\[
Z_\psi = \int d\psi d\bar{\psi} e^{-\frac{i}{2}(\bar{\psi}\gamma, \gamma_\psi \psi) - (\bar{\psi} G^{-1}_{A^a}) + is[j^a,j^b]} \quad (54)
\]

\[
(\bar{\psi}\gamma, G^{-1}_{A^a}) = \int d^4x \bar{\psi} \left[ i\gamma_0 \partial_0 - \gamma_j (\partial_j + \dot{A}_j^*) - m \right] \psi, \quad (55)
\]

\[
(\bar{\psi}\gamma, K \gamma \psi) = \int d^4x d^4y j^a_0(x) \left[ \frac{1}{(\nabla_j(A^*))^2} \delta^4(x - y) \right]^{ab} j^b_0(y). \quad (56)
\]

The infinite factor is removed by the gauge fixing (52) treated as an antiderivative function of the Gauss constraint. \( A_i^{*a} \) denotes fields \( A_i^a \) under gauge fixing condition (52). It becomes homogeneous \( \nabla_i^{ab}(A^*) \dot{A}_i^{ab} = 0 \) because \( A_i^a \) is determined by the interactions of currents (56).

It is just the non-Abelian generalization [10, 26–28] of the Dirac approach to QED [14]. In the case of QCD there is a possibility to include the nonzero condensate of transverse gluons

\[
\langle A_i^{*a} A_i^{*b} \rangle = 2C_{\text{gluon}} \delta_{ij} \delta^{ab}. \quad (58)
\]

The Lorentz-invariant bound state matrix elements can be obtained, if we choose the time-axis \( \ell \) of Dirac Hamiltonian dynamics as the operator acting in the complete set of bound states (9) and given by Eqs.(6) and (7). This means the von Neumann substitution (11) given in [15]

\[
Z[\ell^0] \to Z[\ell] \to Z[\hat{\ell}] \quad (57)
\]

instead of the Lorentz-gauge formulation [1].

6. AXIAL ANOMALIES IN THE NJL MODEL INSPIRED BY QCD

6.1. Formulation of the NJL model inspired by QCD

Instantaneous QCD interactions are described by the non-Abelian generalization of the Dirac gauge in QED

\[
S_{\text{inst}} = \int d^4x \bar{q}(x) \left( i\partial - m^0 \right) q(x) - \frac{1}{2} \int d^4x d^4y j^a_0(x) \left[ \frac{1}{(\nabla_j(A^*))^2} \delta^4(x - y) \right]^{ab} j^b_0(y). \quad (58)
\]
where \( j_{0}^{a}(x) = \bar{q}(x) \frac{\lambda^{a}}{2} \gamma_{0} q(x) \) is the 4-th component of the quark current, with the Gell-Mann color matrices \( \lambda^{a} \) (see the notations in Appendix A). The symbol \( \hat{m}^{0} = \text{diag}(m_{u}^{0}, m_{d}^{0}, m_{s}^{0}) \) denotes the bare quark mass matrix.

The normal ordering of the transverse gluons in the nonlinear action (56) \( \nabla^{db} A^{b}_{0} \nabla^{dc} A^{c}_{0} \) leads to the condensate of gluons

\[
g^{2} f^{ba1d} f^{da2c} \langle A_{1}^{a1} A_{2}^{a2} \rangle = 2g^{2} [N_{c}^{2} - 1] \delta^{bc} \delta_{ij} C_{\text{gluon}} = M_{g}^{2} \delta^{bc} \delta_{ij},
\]

where

\[
\langle A_{j}^{a} A_{i}^{b} \rangle = 2C_{\text{gluon}} \delta_{ij} \delta^{ab}.
\]

This condensate yields the squared effective gluon mass in the squared covariant derivative \( \nabla^{db} A^{b}_{0} \nabla^{dc} A^{c}_{0} =: \nabla^{db} A^{b}_{0} \nabla^{dc} A^{c}_{0} + M_{g}^{2} A^{d}_{0} A^{d}_{0} \) of constraint-shell action (56) given in Appendix A. The constant

\[
C_{\text{gluon}} = \int \frac{d^{3}k}{(2\pi)^{3} 2\sqrt{k^{2}}}
\]

is finite after substraction of the infinite volume contribution, and its value is determined by the hadron size like the Casimir vacuum energy [29]. Finally, in the lowest order of perturbation theory, this gluon condensation yields the effective Yukawa potential in the colorless meson sector

\[
V(k) = \frac{4}{3} g^{2} \frac{1}{k^{2} + M_{g}^{2}}
\]

and the NJL type model with the effective gluon mass \( M_{g}^{2} \). While deriving the last equation, we use the relation

\[
\sum_{a=1}^{a=N_{c}^{2}-1} \lambda_{1,1'}^{a} \lambda_{2,2'}^{a} = \frac{4}{3} \delta_{1,2'} \delta_{2,1'}
\]

in the colorless meson sector.

Below we consider the potential model (58) in the form

\[
S_{\text{inst}} = \int d^{4}x \bar{q}(x) (i \partial - \hat{m}^{0}) q(x) - \frac{1}{2} \int d^{4}x d^{4}y j_{\xi}^{a}(x) V(x^{+} - y^{+}) \delta((x - y) \cdot \xi) j_{\xi}^{a}(y)
\]

with the choice of the time axis as the eigenvalues of the bound state total momentum, in the framework of the ladder approximation given in Appendix A.
6.2. Schwinger-Dyson equation: the fermion spectrum

The equation of stationarity (A6) can be rewritten from the SD equation

\[ \Sigma(x - y) = m^0 \delta^{(4)}(x - y) + i K(x, y) G_\Sigma(x - y). \]  

(63)

It describes the spectrum of Dirac particles in bound states. In the momentum space with

\[ \Sigma(k) = \int d^4 x \Sigma(x) e^{i k \cdot x} \]

for the Coulomb type kernel, we obtain the following equation for the mass operator (\( \Sigma_0 \))

\[ \Sigma(k) = m^0 + \frac{i}{2} \int \frac{d^4 q}{(2\pi)^4} V(k^\perp - q^\perp) G_\Sigma(q \ell), \]  

(64)

where \( G_\Sigma(q) = (\hat{q} - \Sigma(q))^{-1}, V(k^\perp) \) is the Fourier representation of the potential, \( k^\perp_\mu = k_\mu - \ell_\mu (k \cdot \ell) \) is the relative transverse momentum. The quantity \( \Sigma_0 \) depends only on the transverse momentum \( \Sigma(k) = \Sigma(k^\perp) \), because of the instantaneous form of the potential \( V(k^\perp) \). We can put

\[ \Sigma_a(q) = E_a(q) \cos 2\nu_a(q) \equiv M_a(q). \]  

(65)

Here \( M_a(q) \) is the constituent quark mass and

\[ \cos 2\nu_a(q) = \frac{M_a(q)}{\sqrt{M_a^2(q) + q^2}} \]  

(66)

determines the Foldy - Wouthuysen type matrix

\[ S_a(q) = \exp[(q \gamma / q) v_a(q)] = \cos v_a(q) + (q \gamma / q) \sin v_a(q) \]  

(67)

with the vector of Dirac matrices \( \gamma = (\gamma_1, \gamma_2, \gamma_3) \) and some angle \( v_a(q) \). The fermion spectrum can be obtained by solving the SD equation (64). It can integrated over the longitudinal momentum \( q_0 = (q \cdot \ell) \) in the reference frame \( \ell^0 = (1, 0, 0, 0) \), where \( q^\perp = (0, q) \). By using Eq. (67), the quark Green function can be presented in the form

\[ G_{\Sigma_a} = [q_0 \ell - E_a(q^\perp) S_a^{-2}(q^\perp)]^{-1} = \]  

(68)

\[ = \left[ \frac{\Lambda^{(\ell)}_{(\pm)a}(q^\perp)}{q_0 - E_a(q^\perp) + i \epsilon} + \frac{\Lambda^{(\ell)}_{(-a)}(q^\perp)}{q_0 + E_a(q^\perp) + i \epsilon} \right] \ell, \]

where

\[ \Lambda^{(\ell)}_{(\pm)a}(q^\perp) = S_a(q^\perp) \Lambda^{(\ell)}_{(\pm)a}(0) S_a^{-1}(q^\perp), \quad \Lambda^{(\ell)}_{(\pm)}(0) = (1 \pm \ell) / 2 \]  

(69)
are the operators separating the states with positive \((+E_a)\) and negative \((-E_a)\) energies. As a result, we obtain the following equations for the one-particles energy \(E\) and the angle \(\nu\) with the potential given by Eq. (61)

\[
E_a(k^\perp) \cos 2\nu_a(k^\perp) = m^0_a + \frac{1}{2} \int \frac{d^3q^\perp}{(2\pi)^3} V(k^\perp - q^\perp) \cos 2\nu_a(q^\perp). \tag{70}
\]

In the rest frame \(\ell^0 = (1, 0, 0, 0)\) this equation takes the form

\[
M_a(k) = m^0_a + \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} V(k - q) \cos 2\nu_a(q). \tag{71}
\]

By using the integral over the solid angle

\[
\int_0^\pi d\vartheta \sin \vartheta \frac{2\pi}{M_g^2 + (k - q)^2} = \int_{-1}^{+1} d\xi \frac{2\pi}{M_g^2 + k^2 + q^2 - 2kq\xi} = \frac{\pi}{kq} \ln \frac{M_g^2 + (k + q)^2}{M_g^2 + (k - q)^2}
\]

and the definition of the QCD coupling constant \(\alpha_s = 4\pi g^2\), it can be rewritten as

\[
M_a(k) = m^0_a + \frac{\alpha_s}{3\pi k} \int_0^\infty dq \frac{qM_a(q)}{\sqrt{M_g^2(q) + q^2}} \ln \frac{M_g^2 + (k + q)^2}{M_g^2 + (k - q)^2}. \tag{72}
\]

The suggested scheme allows us to consider the SD equation (71) in the limit when the bare current mass \(m^0_a\) equals to zero. Then the ultraviolet divergence is absent, and, hence, the renormalization procedure can be successfully avoided.

This kind of nonlinear integral equations was considered in the paper [30] numerically. The solutions show us that in the region \(q \ll M_g\) the function \(\cos 2\nu_a\) is almost constant \(\cos 2\nu_a \simeq 1\), whereas in the region \(q \gg M_g\) the function \(\cos 2\nu_a(q)\) decays in accordance with the power law \((M_g/q)^{1+\beta}\). The parameter \(\beta\) is a solution of the equation

\[
\alpha_s \cot(\beta\pi/2) = \frac{3}{2}, \tag{73}
\]

lying in the range \(0 < \beta < 2\). This equation has two roots for \(0 < \alpha_s < 3/\pi\), the first, belonging to the interval \(0 < \beta_1 < 1\), and the second, related to the first one by \(\beta_2 = 2 - \beta_1\). At \(\alpha_s = 3/\pi\), the two solutions merges into \(\beta = 1\), and there is no root for larger values of the coupling constant. Equation (73) can be obtained by means of linearization of Eq. (71) within the range \(q \gg M_g\), because in this range \(M_a(q) \ll q\). Thus, the solution for \(\cos 2\nu_a(q)\) is a reminiscent of the step function. This result justifies the estimation of the quark and meson spectra in the separable approximation [21] in agreement with the experimental data. Currently, numerical solutions of the nonlinear equation (72) are under way, and the details of computations will be published elsewhere.
6.3. Spontaneous chiral symmetry breaking

As discussed in the previous section, the SD equation (71) can be rewritten in the form (72). Once we know the solution of Eq. (72) for $M_a(q)$, we can determine the Foldy-Wouthuysen angles $\nu_a$, $(a = u, d)$ for u-, d- quarks with the help of relation (66). Then the BS equations in the form (B10)

$$M_\pi L_2^\pi (p) = [E_u(p) + E_d(p)]L_1^\pi (p) - \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} V(p - q)L_1^\pi (q)[c^-(p)c^-(q) + \xi s^-(p)s^-(q)],$$  (74)

$$M_\pi L_1^\pi (p) = [E_u(p) + E_d(p)]L_2^\pi (p) - \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} V(p - q)L_2^\pi (q)[c^+(p)c^+(q) + \xi s^+(p)s^+(q)]$$  (75)

yield the pion mass $M_\pi$ and wave functions $L_1^\pi (p)$ and $L_2^\pi (p)$. Here $m_u, m_d$ are the current quark masses, $E_a = \sqrt{p^2 + M_a^2(p)}$, $(a = u, d)$ are the u-, d- quark energies, $\xi = (p \cdot q)/pq$, and we use the notations

$$E(p) = E_a(p) + E_b(p),$$  (76)

$$c^\pm(p) = \cos [\nu_a(p) \pm \nu_b(p)],$$  (77)

$$s^\pm(p) = \sin [\nu_a(p) \pm \nu_b(p)].$$  (78)

The model is simplified in some limiting cases. Once the quark masses $m_u$ and $m_d$ are small and approximately equal, then Eqs. (71) and (74) take the form

$$m_a = M_a(p) - \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} V(p - q) \cos 2\nu_u(q),$$  (79)

$$\frac{M_\pi L_2^\pi (p)}{2} = E_u(p)L_1^\pi (p) - \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} V(p - q)L_1^\pi (q).$$  (80)

Solutions of equations of this type are considered in the numerous papers [31–35] (see also review [30]) for different potentials. One of the main results of these papers was the pure quantum effect of spontaneous chiral symmetry breaking. In this case, the instantaneous interaction leads to rearrangement of the perturbation series and strongly changes the spectrum of elementary excitations and bound states in contrast to the naive perturbation theory.

In the limit of massless quarks $m_a = 0$ the left-hand side of Eq. (79) is equal to zero. The nonzero solution of Eq. (79) implies that there exists a mode with zero pion mass $M_\pi = 0$ in accordance with the Goldstone theorem. This means that the BS equation (80), being the equation for the wave function of the Goldstone pion, coincides with the the SD equation (79) in the case of $m_a = M_\pi = 0$. Comparing the equations yields

$$L_1^\pi (p) = \frac{M_a(p)}{F_\pi E_u(p)} = \frac{\cos 2\nu_u(p)}{F_\pi},$$  (81)
where the constant of the proportionality $F_\pi$ in Eq. (81) is called the weak decay constant. In the more general case of massive quark $m_u \neq M_\pi \neq 0$, this constant is determined from the normalization condition (B17)

$$1 = \frac{4N_c}{M_\pi} \int \frac{d^3q}{(2\pi)^3} L_2 L_1 = \frac{4N_c}{M_\pi} \int \frac{d^3q}{(2\pi)^3} L_2 \frac{\cos 2\nu_u(q)}{F_\pi}$$

(82)

with $N_c = 3$. In this case the wave function $L_1^\pi(p)$ is proportional to the Fourier component of the quark condensate

$$C_{\text{quark}} = \sum_{n=1}^{n=N_c} \langle q_n(t, \mathbf{x}) \bar{q}_n(t, \mathbf{y}) \rangle = 4N_c \int \frac{d^3p}{(2\pi)^3} \frac{M_u(p)}{\sqrt{p^2 + M_u^2(p)}}.$$

(83)

Using Eqs. (66) and (81), one can rewrite the definition of the quark condensate (83) in the form

$$C_{\text{quark}} = 4N_c \int \frac{d^3q}{(2\pi)^3} \cos 2\nu_u(q).$$

(84)

Let us assume that the representation for the wave function $L_1$ (81) is still valid for non-zero but small quark masses. Then the subtraction of the BS equation (80) from the SD one (79) multiplied by the factor $1/F_\pi$ determines the second meson wave function $L_2$

$$\frac{M_\pi}{2} L_2^\pi(p) = \frac{m_u}{F_\pi}.$$

(85)

The wave function $L_2^\pi(p)$ is independent of the momentum in this approximation. Substituting the equation $L_2 = \text{const} = 2m_u/(M_\pi F_\pi)$ into the normalization condition (82), and using Eqs. (81) and (84), we arrive at the Gell-Mann-Oakes-Renner relation [36]

$$M_\pi^2 F_\pi^2 = 2m_u C_{\text{quark}}.$$

(86)

Our solutions including the GMOR relation (86) differ from the accepted ones [30–35], where $\cos 2\nu_a(q)$ is replaced by the sum of two Goldstone bosons, the pseudoscalar and the scalar one [$\cos 2\nu_a(q) + (q/q) \sin 2\nu_a(q)$]. This replacement can hardly be justified, because it is in contradiction with the Bethe-Salpeter equation (B16) for scalar bound state with nonzero mass.

The coupled equations (71), (74), and (75) contain the Goldstone mode that accompanies spontaneous breakdown of chiral symmetry. Thus, in the framework of instantaneous interaction we prove the Goldstone theorem in the bilocal variant, and the GMOR relation
directly results from the existence of the gluon and quark condensates. Strictly speaking, the postulate that the finiteness of the gluon and quark condensates are finite implies that QCD is the theory without ultraviolet divergence. They can be removed by the Casimir type subtraction [29] with the finite renormalization [37].

6.4. New Hamiltonian interaction inspired by the anomalous triangle diagram with a pseudoscalar bound state

It was shown [22, 23] that the Habbard-Stratanovich linearization of the four fermion interaction leads to an effective action for bound states in any gauge theory. We include here an effective action describing the direct pion-positronium creation

\[
W_{\text{eff}} = \int d^4x \left\{ \frac{\alpha}{\pi} \left( \frac{\pi_0}{F_\pi} + \frac{\eta p}{F_P} \right) \dot{A}_i B_i + \frac{\dot{A}_i^2 + B_i^2}{2} \right\},
\]

where \(\alpha = 1/137\) is the QED coupling constant, and \(F_P\), contained in Eq. (40), plays a role of the pion weak coupling parameter \(F_\pi = 92\) GeV. The first term \(\frac{\alpha}{\pi} \left( \frac{\pi}{F_\pi} + \frac{\eta p}{F_P} \right) \dot{A}_i B_i\) comes from the triangle diagram (i.e., the anomalous term). This term describes the two \(\gamma\) decay of pseudoscalar bound states \(P_{bs}\).

For each bound state one can obtain the corresponding two-photon anomalous creation cross section from Eq. (43). In the case of the process \(\gamma + \gamma = P_{bs} + P_{bs}\) we repeat Eq. (43)

\[
\frac{d\sigma}{d\Omega} = \frac{\alpha^4 E_\gamma^2}{\pi 128 F_{P_{bs}}^4} \sqrt{1 - \left( \frac{2m_e}{E_\gamma} \right)^2},
\]

where \(P_{bs}\) is the \(F_\pi\) analogy. In the case of the process \(\gamma + \gamma = P_\pi + P_{pos}\) we obtain

\[
\frac{d\sigma}{d\Omega} = \frac{\alpha^4 E_\gamma^2}{\pi 32 F_{P_{bs}}^2 F_{P_{pos}}^2} \sqrt{1 - \left( \frac{2m_e}{E_\gamma} \right)^2}.
\]

The Hamiltonian of this system is the sum of the energy of the free EM fields, the pseudoscalar Hamiltonians and their interactions

\[
W_{\text{eff}} = \int dt d^3x \left\{ E_i \dot{A}_i - \frac{E_i^2 + B_i^2}{2} - \mathcal{H}_{\text{int}} \right\},
\]

\[
\mathcal{H}_{\text{int}} = \frac{\alpha}{\pi} \left( \frac{\pi_0}{F_\pi} + \frac{\eta p}{F_P} \right) E_i B_i + \frac{\alpha^2}{2\pi^2} \left( \frac{\pi}{F_\pi} + \frac{\eta p}{F_P} \right)^2 B_i^2.
\]

This action contains the additional terms in comparison with the standard QED. They leads to the additional mass of the pseudoscalar bosons [38] and the anomalous processes of the
creation of the bound state pairs in the external magnetic field. The last term of the effective action (91) yields cross-sections of creation of both the two positronium atoms and the pion and the positronium together.

The creation of two positronium atoms is $\alpha^3$ times less than the creation of the pion and the positronium together. In this case, one can speak about the pion catalysis of the positronium creation.

7. SUMMARY

In this paper we obtain the bound state functional by Poincaré-invariant generalization of the FP path integral based on the Markov-Yukawa constraint for description of both the spectrum equations and the S-matrix elements. The axiomatic approach to gauge theories presented here allows us to construct the bound state functional in both QED and QCD on equal footing of the Poincaré group representations.

It is shown that the Poincaré S-matrix, as compared with the Lorentz one, contains

1. Creation of bound states inspired by the anomalous (triangle) diagram within the Hamiltonian approach.

2. This additional anomalous contribution includes the processes like $\gamma + \gamma \to Ps + Ps$, $\gamma + \gamma \to \pi_0 + Ps$, $\gamma + \gamma \to \pi_0 + \pi_0$ (where $Ps$ – a pseudoscalar para-positronium).

This raises the problem of physical consequences of these additional processes.

The bound state generating functional (51), where the time-axis is chosen as eigenvalue of the total momentum operator of instantaneous bound states (57), has a variety of properties. It describes spontaneous breakdown of chiral symmetry, the bilocal variant of the Goldstone theorem, and the direct derivation of the GMOR relation directly from the fact of existence of the finite gluon and quark condensates introduced by the normal ordering of the QCD action. The postulate of the finiteness of the gluon and quark condensates implies that both the QED and QCD can be considered on equal footing as the theory without ultraviolet divergences. They can be removed by the Casimir type substriction [29] with the finite renormalization [37].
Acknowledgments

The authors would like to thank A.B. Arbuzov, B.M. Barbashov, A.A. Gusev, A.V. Efremov, O.V. Teryaev, R.N. Faustov, S.I. Vinitsky, and M.K. Volkov for useful discussions. NSH is grateful to the JINR Directorate for hospitality.

APPENDIX A: LADDER APPROXIMATION

The generating functional (54) can be presented by means of the relativistic generalization of the Hubbard-Stratonovich (HS) transformation [22]

\[
\exp[-ax^2/2] = [2\pi a]^{-1/2} \int_{-\infty}^{+\infty} dy \exp[-i\phi y - y^2/(2a)].
\] (A1)

The basic idea of the HS transformation is to reformulate a system of particles interacting through two-body potentials into a system of independent particles interacting with a fluctuating field. It is used to convert a particle theory into its respective field theory by linearizing the density operator in the many-body interaction term of the Hamiltonian and introducing a scalar auxiliary field [22]

\[
Z_\psi = \int d\psi d\bar{\psi} e^{-\frac{i}{2} (\bar{\psi} K \psi) - (\bar{\psi} G A^{-1}) + i S[J, \psi^\dagger]} \]

\[
= \int \left[ \prod_{x,y,a,b} dM_{ab}^{xy} \right] \exp\{iW_{\text{eff}} [\mathcal{M}, A^\ast] + i(\eta^\dagger, G A) \}. \] (A2)

The effective action in Eq. (A3) can be decomposed in the form

\[
W_{\text{eff}} [\mathcal{M}, A^\ast] = -\frac{1}{2} N_c (\mathcal{M}, K^{-1} \mathcal{M}) + i N_c \sum_{n=1}^{\infty} \frac{1}{n} \Phi^n. \] (A4)

Here \( \Phi \equiv G A^\ast \mathcal{M}, \Phi^2, \Phi^3 \) etc. mean the following expressions

\[
\Phi(x, y) \equiv G A^\ast \mathcal{M} = \int d^4 z G A^\ast (x, z) \mathcal{M} (z, y),
\]

\[
\Phi^2 = \int d^4 x d^4 y \Phi(x, y) \Phi(y, x),
\]

\[
\Phi^3 = \int d^4 x d^4 y d^4 z \Phi(x, y) \Phi(y, z) \Phi(z, x), \text{ etc.}
\] (A5)

The first step to the semi-classical quantization of this construction [39] is the determination of its minimum of the effective action

\[
N_c^{-1} \frac{\delta W_{\text{eff}}(\mathcal{M})}{\delta \mathcal{M}} = -K^{-1} \mathcal{M} + \frac{i}{G A^\ast - \mathcal{M}} = 0. \] (A6)
We denote the corresponding classical solution for the bilocal field by \( \Sigma(x - y) \). It depends only on the difference \( x - y \) at \( A^* = 0 \) because of translation invariance of vacuum solutions.

The next step is the expansion of the effective action around the point of minimum \( \mathcal{M} = \Sigma + \mathcal{M}' \),

\[
\begin{align*}
W_{\text{eff}}(\Sigma + \mathcal{M}') &= W_{\text{eff}}^{(2)} + W_{\text{int}}; \\
W_{\text{eff}}^{(2)}(\mathcal{M}') &= W_Q(\Sigma) + N_c[-\frac{1}{2} \mathcal{M}'K^{-1}\mathcal{M}' + \frac{i}{2}(G_\Sigma \mathcal{M}')^2]; \\
W_{\text{int}} &= \sum_{n=3}^{\infty} W^{(n)} = iN_c \sum_{n=3}^{\infty} \frac{1}{n} (G_\Sigma \mathcal{M}')^n, \quad (G_\Sigma = (G_{A^*}^{-1} - \Sigma)^{-1}), \quad (A7)
\end{align*}
\]

and the representation of the small fluctuations \( \mathcal{M}' \) as a sum (5)

\[
\begin{align*}
\mathcal{M}(x, y) &= \mathcal{M}(z|X) \\
&= \sum_H \int \frac{d^3P}{(2\pi)^3\sqrt{2}\omega_H} \int \frac{d^4q}{(2\pi)^4} \{ e^{iP \cdot X} \Gamma_H(q^\perp|P)a_H^+(P) + e^{-iP \cdot X} \bar{\Gamma}_H(q^\perp|P)a_H^+(P) \},
\end{align*}
\]

over the complete set of orthonormalized solutions \( \Gamma \), of the classical equation

\[
\frac{\delta^2 W_{\text{eff}}(\Sigma + \mathcal{M}^{'})}{\delta \mathcal{M}^2} \cdot \Gamma = 0 \quad (A9)
\]

with a set of quantum numbers \( (H) \) including masses \( M_H = \sqrt{P^2} \) and energies \( \omega_H = \sqrt{P^2 + M_H^2} \). The bound state creation and annihilation operators obey the commutation relations

\[
\begin{align*}
\left[ a_H^+(P'), a_H^-(P) \right] = \delta_{H'H} \delta^3(P' - P). \quad (A10)
\end{align*}
\]

The corresponding Green function takes the form

\[
\begin{align*}
\mathcal{G}(q^\perp, p^\perp|P) &= \sum_H \left\{ \frac{\Gamma_H(q^\perp|P)\bar{\Gamma}_H(p^\perp|P) - \bar{\Gamma}_H(p^\perp|P)\Gamma_H(q^\perp|P)}{(P_0 - \omega_H - i\varepsilon)2\omega_H} \right\} . \quad (A11)
\end{align*}
\]

To normalize vertex functions, we can use the "free" part of the effective action (A7) for the quantum bilocal meson \( \mathcal{M}' \) with the commutation relations (A10). The substitution of the off-shell \( \sqrt{P^2} \neq M_H \) decomposition (5) into the "free" part of effective action defines the reverse Green function of the bilocal field \( \mathcal{G}(P_0) \)

\[
W_{\text{eff}}^{(0)}[\mathcal{M}] = 2\pi\delta(P_0 - P'_0) \sum_H \int \frac{d^3P}{\sqrt{2}\omega_H} a_H^+(P) a_H^-(P) \mathcal{G}_H^{-1}(P_0) \quad (A12)
\]

where \( \mathcal{G}_H^{-1}(P_0) \) is the reverse Green function which can be represented as a difference of two terms

\[
\mathcal{P}_H^{-1}(P_0) = I(\sqrt{P^2}) - I(M_H^b(\omega)) \quad (A13)
\]
where $M_{H}^{ab}(\omega)$ is the eigenvalue of the equation for small fluctuations (A9) and

$$I(\sqrt{p^2}) = iN_c \int \frac{d^4q}{(2\pi)^4} \times \text{tr} \left[ G_{\Sigma_0}(q + \frac{p}{2}) \Gamma_{ab}^{H}(q^k| - p) G_{\Sigma_0}(q + \frac{p}{2}) \Gamma_{ab}^{H}(q^k|p) \right]$$

where

$$G_{\Sigma}(q) = \frac{1}{\beta - \Sigma(q)} , \quad \Sigma(q) = \int d^4x \Sigma(x)e^{iqx} \quad \text{(A14)}$$

is the fermion Green function. The normalization condition is defined by the formula

$$2\omega = \frac{\partial G^{-1}(p_0)}{\partial p_0} \big|_{p_0 = \omega} = \frac{dM(p_0)}{dp_0} \frac{dI(M)}{dM} \big|_{p_0 = \omega} . \quad \text{(A15)}$$

Finally, we get that solutions of equation (A9) satisfy the normalization condition [40]

$$iN_c \frac{d}{dp_0} \int \frac{d^4q}{(2\pi)^4} \text{tr} \left[ G_{\Sigma}(q - \frac{p}{2}) \Gamma_H(q^k| - p) G_{\Sigma}(q + \frac{p}{2}) \Gamma_H(q^k|p) \right] = 2\omega_H . \quad \text{(A16)}$$

The achievement of the relativistic covariant constraint-shell quantization of gauge theories is the description of both the spectrum of bound states and their S-matrix elements.

It is convenient to write the relativistic-invariant matrix elements for the action (A7) in terms of the field operator

$$\Phi'(x, y) = \int d^4x G_{\Sigma}(x - x_1) M'(x_1, y) = \Phi'(z|X)$$

Using the decomposition over the bound state quantum numbers $(H)$

$$\Phi'(z|X) = \sum_H \int \frac{d^3p}{(2\pi)^3/2\sqrt{2\omega_H}} \int \frac{d^4q}{(2\pi)^4} \times \left\{ e^{ip\cdot x} \Phi_H(q^k|p) a_H^+(p) + e^{-ip\cdot x} \Phi_H(q^k| - p) a_H(p) \right\} , \quad \text{(A17)}$$

where

$$\Phi_{H(ab)}(q^k|p) = G_{\Sigma_0}(q + p/2) \Gamma_{H(ab)}(q^k|p) , \quad \text{(A18)}$$

we can write the matrix elements for the interaction $W^{(n)}$ (A7) between the vacuum and the n-bound state [20]

$$\langle H_1 p_1, ..., H_n p_n | iW^{(n)} | 0 \rangle =$$

$$-i(2\pi)^4 \delta^4 \left( \sum_{i=1}^{n} p_i \right) \prod_{j=1}^{n} \left[ \frac{1}{(2\pi)^3 2\omega_j} \right]^{1/2} M^{(n)}(p_1, ..., p_n) , \quad \text{(A19)}$$
\[ M^{(n)} = \int \frac{id^4q}{(2\pi)^4} \sum_{\{i_k\}} \Phi_{H_{i_1}}^{a_1,a_2}(q|\mathcal{P}_{i_1}) \times \]
\[ \Phi_{H_{i_2}}^{a_2,a_3}(q - \frac{\mathcal{P}_{i_1} + \mathcal{P}_{i_2}}{2}|\mathcal{P}_{i_2}) \Phi_{H_{i_3}}^{a_3,a_4}(q - \frac{2\mathcal{P}_{i_2} + \mathcal{P}_{i_1} + \mathcal{P}_{i_3}}{2}|\mathcal{P}_{i_3}) \times \]
\[ ... \Phi_{H_{i_n}}^{a_n,a_1}(q - \frac{2(\mathcal{P}_{i_2} + ... + \mathcal{P}_{i_{n-1}}) + \mathcal{P}_{i_1} + \mathcal{P}_{i_n}}{2}|\mathcal{P}_{i_n}) , \] (A20)

where \( \{i_k\} \) denotes permutations over \( i_k \).

Expressions (A11), (A17), (A19), and (A20) represent Feynman rules for the construction of a quantum field theory with the action (A7) in terms of bilocal fields.

**APPENDIX B: BETHE - SALPETER EQUATION**

Equations for the spectrum of the bound states (A9) can be rewritten in the form of the Bethe-Salpeter (BS) one [5]

\[ \Gamma = i\mathcal{K}(x,y) \int d^4z_1 d^4z_2 G_\Sigma(x - z_1)\Gamma(z_1,z_2)G_\Sigma(z_2 - y) . \] (B1)

In the momentum space with

\[ \Xi(q|\mathcal{P}) = \int d^4x d^4y e^{ix\cdot q} e^{i(x-y)\cdot \mathcal{P}} \Gamma(x,y) \]

we obtain the following equation of the vertex function (\( \Xi \))

\[ \Gamma(k,\mathcal{P}) = i \int \frac{d^4q}{(2\pi)^4} V(k^\perp - q^\perp)\ell \left[ G_\Sigma(q + \frac{\mathcal{P}}{2})\Gamma(q|\mathcal{P})G_\Sigma(q - \frac{\mathcal{P}}{2}) \right] \ell \] (B2)

where \( V(k^\perp) \) means the Fourier transform of the potential, \( k_\mu^\perp = k_\mu - \ell_\mu(k \cdot \ell) \) is the relative momentum transversal with respect to \( \ell_\mu \), and \( \mathcal{P}_\mu \) is the total momentum.

The quantity \( \Xi \) depends only on the transversal momentum

\[ \Xi(k|\mathcal{P}) = \Xi(k^\perp|\mathcal{P}), \]

because of the instantaneous form of the potential \( V(k^\perp) \) in any frame.

We consider the Bethe - Salpeter equation (B1) after integration over the longitudinal momentum \( q_0 \). The vertex function takes the form

\[ \Gamma_{ab}(k^\perp|\mathcal{P}) = \int \frac{d^3q^\perp}{(2\pi)^3} V(k^\perp - q^\perp)\ell \Psi_{ab}(q^\perp)\ell, \] (B3)
where the bound state wave function $\Psi_{ab}$ is given by

$$
\Psi_{ab}(q^\perp) = \frac{1}{\ell} \left[ \frac{\bar{\Lambda}_{(+)}a(q^\perp \Gamma(q^\perp |P_\Lambda) \Lambda_{(+)b}(q^\perp)}{E_T - \sqrt{P^2} + i\epsilon} + \frac{\bar{\Lambda}_{(-)}a(q^\perp \Gamma(q^\perp |P_\Lambda) \Lambda_{(-)b}(q^\perp)}{E_T + \sqrt{P^2} - i\epsilon} \right] \ell \tag{B4}
$$

$E_T = E_a + E_b$ means the sum of one-particle energies of the two particles $(a)$ and $(b)$ defined by (70) and the notation (69)

$$
\bar{\Lambda}(\pm)(q^\perp) = S^{-1}(q^\perp) \Lambda(\pm) (0) S(q^\perp) = \Lambda(\pm) (-q^\perp) \tag{B5}
$$

has been introduced.

Acting with the operators (69) and (B5) on equation (B3) one gets the equations for the wave function $\psi$ in an arbitrary moving reference frame

$$
(E_T(k^\perp) + \sqrt{P^2}) \Lambda_{(\pm) a}(k^\perp) \psi_{ab}(k^\perp) \Lambda_{(\mp) b}(-k^\perp) =
\Lambda_{(\pm) a}(k^\perp) \int \frac{d^3q^\perp}{(2\pi)^3} V(k^\perp - q^\perp) \psi_{ab}(q^\perp) \Lambda_{(\mp) b}(-k^\perp). \tag{B6}
$$

All these equations (B3) and (B6) have been derived without any assumption about the smallness of the relative momentum $|k^\perp|$ and for an arbitrary total momentum

$$
P_\mu = (\sqrt{M_A^2 + P^2}, P \neq 0).$$

We expand the function $\Psi$ on the projection operators

$$
\Psi = \Psi_+ + \Psi_- , \quad \Psi_{\pm} = \Lambda_{\pm}^{(\ell)} \Psi \Lambda_{\pm}^{(\ell)} . \tag{B7}
$$

According to Eq. (B4), $\Psi$ satisfies the identities

$$
\Lambda_{\pm}^{(\ell)} \Psi \Lambda_{\pm}^{(\ell)} = \Lambda_{\mp}^{(\ell)} \Psi \Lambda_{\mp}^{(\ell)} \equiv 0 , \tag{B8}
$$

which permit the determination of an unambiguous expansion of $\Psi$ in terms of the Lorentz structures:

$$
\Psi_{a,b\pm} = S_a^{-1} \left\{ \gamma_5 L_{a,b\pm}(q^\perp) + (\gamma_\mu - \ell_\mu) N_{a,b\pm} \right\} \Lambda_{\mp}^{(\ell)} (0) S_b^{-1} , \tag{B9}
$$

where $L_{\pm} = L_1 \pm L_2$, $N_{\pm} = N_1 \pm N_2$. In the rest frame $\ell_\mu = (1, 0, 0, 0)$ we get

$$
N^\mu = (0, N^i) ; \quad N^i(q) = \sum_{a=1,2} N_a(q) e^i_a(q) + \Sigma(q) \hat{q}^i .
$$

The wave functions $L, N^\alpha, \Sigma$ satisfy the following equations.

1. Pseudoscalar particles.
\[ M_L^0 \odot \beta (p) = E_L^0 \odot \beta (p) - \int \frac{d^3q}{(2\pi)^3} V(p-q)(c^-_pc^-_q - \xi s^-_p s^-_q) \odot \beta \odot \alpha (q) ; \]

\[ M_L^0 \odot \alpha (p) = E_L^0 \odot \alpha (p) - \int \frac{d^3q}{(2\pi)^3} V(p-q)(c^+_pc^+_q - \xi s^+_p s^+_q) \odot \beta \odot \alpha (q) . \]

Here, in all equations, we use the following definitions

\[ E(p) = E_a(p) + E_b(p) , \]

\[ c^\pm(p) = \cos[v_a(p) \pm v_b(p)] , \]

\[ s^\pm(p) = \sin[v_a(p) \pm v_b(p)] , \]

\[ \eta = \hat{p}_i \cdot \hat{q}_i , \]

where \( E_a, E_b \) are one-particle energies and \( v_a, v_b \) are the Foldy-Wouthuysen angles of particles (a,b) given by Eqs.(71) and (72).

**2. Vector particles.**

\[ M_N^0 \odot \beta (p) = E_N^0 \odot \beta (p) - \int \frac{d^3q}{(2\pi)^3} V(p-q)\{(c^-_pc^-_q - \xi s^-_p s^-_q) \odot \beta \odot \alpha (q) \} ; \]

\[ M_N^0 \odot \alpha (p) = E_N^0 \odot \alpha (p) - \int \frac{d^3q}{(2\pi)^3} V(p-q)\{(c^+_pc^+_q - \xi s^+_p s^+_q) \odot \beta \odot \alpha (q) \} . \]

**3. Scalar particles.**

\[ M_S^0 \odot \beta = E_S^0 \odot \beta - \int \frac{d^3q}{(2\pi)^3} V(p-q)\{(c^-_pc^-_q + \xi s^-_p s^-_q) \odot \beta \odot \alpha (q) \} ; \]

\[ M_S^0 \odot \alpha = E_S^0 \odot \alpha - \int \frac{d^3q}{(2\pi)^3} V(p-q)\{(c^+_pc^+_q + \xi s^+_p s^+_q) \odot \beta \odot \alpha (q) \} . \]
The normalization of these solutions is uniquely determined by equation (A16)

\[
\frac{2N_c}{M_L} \int \frac{d^3 q}{(2\pi)^3} \left\{ L_1(q) L_2^*(q) + L_2(q) L_1^*(q) \right\} = 1, \tag{B17}
\]

\[
\frac{2N_c}{M_N} \int \frac{d^3 q}{(2\pi)^3} \left\{ N_1^\mu(q) N_2^{\mu*}(q) + N_2^\mu(q) N_1^{\mu*}(q) \right\} = 1, \tag{B18}
\]

\[
\frac{2N_c}{M_\Sigma} \int \frac{d^3 q}{(2\pi)^3} \left\{ \Sigma_1(q) \Sigma_2^*(q) + \Sigma_2(q) \Sigma_1^*(q) \right\} = 1. \tag{B19}
\]

If the atom is at rest (\( P_\mu = (M_A, 0, 0) \)) equation (B6) coincides with the Salpeter equation [4]. If one assumes that the current mass \( m^0 \) is much larger than the relative momentum \( |q^\perp| \), then the coupled equations (B3) and (B6) turn into the Schrödinger equation. In the rest frame (\( P_0 = M_A \)) equation (70) for a large mass \( (m^0/|q^\perp| \to \infty) \) describes a nonrelativistic particle

\[
E_a(k) = \sqrt{(m_a^0)^2 + k^2} \simeq m_a^0 + \frac{1}{2} \frac{k^2}{m_a^0},
\]

\[
\tan 2\nu = \frac{k}{m_a^0} \to 0; \quad S(k) \simeq 1; \quad \Lambda(\pm) \simeq \frac{1 \pm \gamma_0}{2}.
\]

Then, in equation (B6) only the state with positive energy remains

\[
\Psi \simeq \Psi(+) = \Lambda(+) \gamma_5 \sqrt{4\mu} \psi_{\text{Sch}}, \quad \Lambda(-) \psi \Lambda(+) \simeq 0,
\]

where \( \mu = m_a \cdot m_b / (m_a + m_b) \). And finally the Schrödinger equation results in

\[
\left[ \frac{1}{2\mu} k^2 + (m_a^0 + m_b^0 - M_A) \right] \psi_{\text{Sch}}(k) = \int \frac{d^3 q}{(2\pi)^3} V(k - q) \psi_{\text{Sch}}(q), \tag{B20}
\]

with the normalization \( \int d^3 q |\psi_{\text{Sch}}|^2 / (2\pi)^3 = 1 \).

For an arbitrary total momentum \( P_\mu \) equation (B20) takes the form

\[
\left[ -\frac{1}{2\mu} (k^\perp)^2 + (m_a^0 + m_b^0 - \sqrt{P^2}) \right] \psi_{\text{Sch}}(k^\perp) = \int \frac{d^3 q^\perp}{(2\pi)^3} V(k^\perp - q^\perp) \psi_{\text{Sch}}(q^\perp), \tag{B21}
\]

and describes a relativistic atom with nonrelativistic relative momentum \( |k^\perp| \ll m_{a,b}^0 \). In the framework of such a derivation of the Schrödinger equation it is sufficient to define the total coordinate as \( X = (x + y)/2 \), independently of the magnitude of the masses of the two particles forming an atom.

1. L. Faddeev, V. Popov, Phys. Lett. B 25 (1967) 29.
2. R.W. Haymaker, La Rivista del Nuovo Cimento 14 (1991) 1-89.
3. L. D. Faddeev, Theoretical and Mathematical Physics, 1969, 1:1, 1Π13
4. E.E. Salpeter and H.A. Bethe, Phys. Rev. 84 (1951) 1232.
5. E.E. Salpeter, Phys. Rev. D 87 (1952) 328.
6. N.N. Bogoliubov, A.A. Logunov, A. I. Oksak, I.T. Todorov, General Principles of Quantum
   Field Theory, Springer (1989).
7. M.A. Markov, J. Phys. (USSR) 3 (1940) 453;
   H. Yukawa, Phys. Rev. 77 (1950) 219.
8. A.A. Logunov, A.N. Tavkhelidze, Nuovo Cim. 29 (1963) 380;
   V.G. Kadyshevsky, R.M. Mir-Kasimov and N.B. Skachkov, Sov.J.Part.Nucl. 2-3 (1973) 69.
9. J. Lukierski, M. Oziewicz, Phys. Lett. B 69 (1977) 339.
10. V.N. Pervushin, Nucl. Phys. B 15 (Proc. Supp.) (1990) 197.
11. Y. Nambu and G. Jona - Lasinio, Phys. Rev. 122 (1961) 345 ;
    Phys. Rev. 124 (1961) 246.
12. D. Ebert and M.K. Volkov, Z. Phys. C16 (1983) 205;
    M.K. Volkov, Ann. Phys. 157 (1984) 282.
13. D. Ebert and H. Reinhardt, Nucl. Phys. B271 (1986) 188.
14. P.A.M. Dirac, Proc. Roy. Soc. A 114 (1927) 243-249;
    Can. J. Phys. 33, (1955) 650-661.
15. W. Heisenberg, W. Pauli, Z. Phys. 56 (1929) 1;
    Z. Phys. 59 (1930) 166.
16. I.V. Polubarinov, Phys. Part. Nucl. 34, 741 (2003).
17. V.N. Pervushin, Phys. Part. Nucl. 34, 348 (2003).
18. L.Y. Kalinovsky, et al., Few Body Systems 10 (1991) 87.
19. S. Love, Ann. Phys., 113, 153 (1978).
20. Yu.L. Kalinovsky, W. Kallies, B.N. Kuranov, V.N. Pervushin, N.A. Sarikov, Sov. J. Nucl. Phys.
    49 (1989) 1709.
21. Yu.L. Kalinovsky, L. Kaschluhn, V.N. Pervushin, Phys. Lett. B 231 (1989) 288;
    Fortsch. Phys. 38 (1990) 353.
22. D. Ebert, V. N. Pervushin, Conference on High Energy Physics, Vol. I (Dubna, 1976) C125;
    Dynamical Breakdown of Chiral Symmetry and Abnormal Perturbation Expansions. JINR Com-
23. V.N. Pervushin, H. Reinhardt, D. Ebert, Sov. J. Part. Nucl. 10, 1114 (1979).
24. S. Gogilidze, Nevena Ilieva, V.N. Pervushin, Int. J. Mod. Phys. A 14 (1999) 3531.
25. J. Schwinger, Phys. Rev. 127 (1962) 324.
26. N. Ilieva, Nguyen Suan Han, V.N. Pervushin, Sov. J. Nucl. Phys. 45 (1987) 1169.
27. V.N. Pervushin, Phys. Part. Nucl. 34, (2003) 348.
28. V. N. Pervushin, Nucl. Phys. (Proc. Supp.), 15, (1990) 197.
29. A. A. Actor, Fortsch. Phys. 43 (1995) 141;
   M. Bordag, G. L. Klimchitskaya, U. Mohideen, and V.M. Mostepanenko, \textit{Advances in the Casimir Effect}, Oxford University Press Inc., New York (2009).
30. I.V. Puzynin, et al. Phys. Part. Nucl. 30 (1999) 87.
31. T. Kunihiro and T. Hatsuda, Phys. Lett. B206 (1988) 385.
32. V. Bernard, R.L. Jaffe and U.-G. Mei\ssner, Nucl. Phys. B308 (1988) 753.
33. V. Bernard and U.-G. Mei\ssner, Nucl. Phys. A489 (1988) 647.
34. H. Reinhardt and R. Alkofer, Phys. Lett. B207 (1988) 482.
35. A. Le Yaouanc et al., Phys. Rev. D31 (1985) 137.
36. Kurt Langfeld, Christiane Kettner, Mod. Phys. Lett. A11 (1996) 1331.
37. M.K. Volkov, D.I. Kazakov, V.N. Pervushin, T.M.F. 28 (1976) 46.
38. D. Blaschke, H.-P. Pavel, V.N. Pervushin, G. Röpke and M.K. Volkov, Phys. Lett. B 397 (1997) 129.
39. V.N. Pervushin, H. Reinhardt, D. Ebert, Sov. J. Part. Nucl. 10 (1979) 1114-1155;
   D. Ebert, V. Pervushin, JINR D-2-2004-66, Dubna, (2004) 131-136.
40. N. Nakanishi, Suppl. Prog. Theor. Phys. 43 (1969) 1.