How Li & Paczyński Model of Kilonova Fits GW170817 Optical Counterpart

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ABSTRACT

The original Li & Paczyński model of kilonova was compared with the observed bolometric optical light curve of the GW170817 electromagnetic counterpart. Perfect agreement is obtained for early observations up to 1.5 days since the time of merger.

1 Introduction

Multi-messenger observations of the gravitational waves source GW170817 (Abbott et al. 2017) have definitely confirmed the existence of kilonova as a new kind of cosmic objects. The existence of such objects has been postulated twenty years ago by Li & Paczyński (1998, therein LP98).

It has been known at that time that due to the merger tidal tails, a part of neutron stars mass is expected to be converted by r-process to radioactive elements and isotopes (Lattimer and Schramm 1974, 1976)

Considering the merger of two neutron stars or a single neutron star with a black hole LP98 omitted details of ejecting some neutron star matter out of a merging binary system. They started with produced by r-process radioactive elements and isotopes sufficiently rich for feeding thermonuclear explosion in the accelerated to subrelativistic velocity radioactive cloud, which expands its size by a factor up to around $10^8$ and more.

For an early, including the point of maximal luminosity interval of time, they obtained an analytic solution for the bolometric luminosity dependence on time expressed in terms of Dawson function.

This paper is devoted to the comparison of LP98 model with the observational data on the GW170817 electromagnetic counterpart compiled by Waxman et al. (2017).

2 Basic properties of the Li & Paczyński model

In this section we shall present specifications of LP98 model and show the relations between the model parameters and the observational properties of the related optical transient.

An important assumption, suggested by David Spergel and adopted by LP98, is the approximate shape of the time dependence of the heat generation per gram
per second $\epsilon$

$$\epsilon = \frac{fc^2}{t},$$  \hspace{1cm} (1)

where $c$ is the speed of light and the energy input is parametrized by $f$. This expression can be written in more general form $\epsilon \propto t^{-\alpha}$. In LP98 model it is assumed that $\alpha = 1$.

The average opacity is $\kappa$. The opacity caused by electron scattering is $\kappa_e = 0.2 \text{ cm}^2\text{g}^{-1}$ and it is used as the reference value of opacity.

The critical time $t_c$ at which the expanding sphere became optically thin is

$$t_c = \left(\frac{3\kappa M}{4\pi V^2}\right)^{1/2},$$  \hspace{1cm} (2)

where $V = \beta c$ is the expansion speed at the surface and $M$ is mass of the ejected matter.

Using Equation (1), LP98 have obtained following approximate expression for the time dependence of the bolometric luminosity $L$

$$L \approx L_0 \sqrt{\frac{8\beta}{3}} Y\left(\sqrt{\frac{3}{8\beta}} \tau\right),$$  \hspace{1cm} (3)

where $L_0 = fMc^2/(4\beta t_c)$, $\tau = t/t_c$, Dawson function $Y$ is

$$Y(x) = e^{-x^2} \int_0^x e^{s^2} ds.$$  \hspace{1cm} (4)

When building a simple model for a specific object one need to know 5 parameters $\alpha$, $\beta$, $\kappa$, $f$ and $M$. After fixing the value of $\alpha$, approximate values of the time from the beginning of expansion to the peak luminosity $t_m$, the peak luminosity $L_m$, and the effective temperature at the peak luminosity $T_{\text{eff},m}$ can be expressed as functions of the remaining four parameters

$$t_m \approx 0.98 \text{ days} \left(\frac{M}{0.01M_\odot}\right)^{1/2} \left(\frac{3V}{c}\right)^{-1/2} \left(\frac{\kappa}{\kappa_e}\right)^{1/2},$$  \hspace{1cm} (5)

$$L_m \approx 2.1 \times 10^{44} \text{ ergs s}^{-1} \left(\frac{f}{0.001}\right) \left(\frac{M}{0.01M_\odot}\right)^{1/2} \left(\frac{3V}{c}\right)^{1/2} \left(\frac{\kappa}{\kappa_e}\right)^{-1/2},$$  \hspace{1cm} (6)

$$T_{\text{eff},m} \approx 2.5 \times 10^4 \text{ K} \left(\frac{f}{0.001}\right)^{1/4} \left(\frac{M}{0.01M_\odot}\right)^{-1/8} \left(\frac{3V}{c}\right)^{-1/8} \left(\frac{\kappa}{\kappa_e}\right)^{-3/8}.$$  \hspace{1cm} (7)

LP98 do not propose any strict predictions for the parameters $\beta$, $\kappa$, $f$ and $M$. Only the plausible reference values of these parameters are used in Equations (5)-(7).

LP98 noticed that in the optically thin region the produced and emitted energies are equal and consequently

$$L(t) \approx M \times \epsilon(t) = \frac{Mf c^2}{t}.$$  \hspace{1cm} (8)
In addition, LP98 give an equation that serves as a prediction. They give an estimate of time needed for luminosity to drop by factor of 3 from its peak value

\[ \Delta t = 2.27 \times t_m \text{ days.} \]  

\( \text{(9)} \)

### 3 Li & Paczyński model fitted to the GW170817 optical counterpart

The photometric observations of the GW170817 optical counterpart were published in several papers. The compilation of Waxman et al. (2017) seems to suit best for our comparison. We have used the integrated luminosities \( L_{\text{int}} \) from Table 3 of their paper. These measurements of bolometric luminosity have been confronted with Equation (3). Using the Levenberg-Marquardt algorithm (Press et al. 2007) applied to the 6 earliest data points we obtain the following values of \( t_m \) and \( L_m \)

\[ t_m = 0.600 \pm 0.008 \text{ days,} \quad L_m = (7.92 \pm 0.11)e + 41 \text{ ergs/s.} \]  

\( \text{(10)} \)

We can read from the same Table 3 in Waxman et al. (2017) the value of temperature at the maximum luminosity

\[ T_{\text{eff,m}} = (10809 \pm 712)K, \]  

\( \text{(11)} \)

so we have estimates for the left hand sides of Equations (5)-(7).

Figure 1 presents the comparison of the measurements of bolometric luminosity with the LP98 model fitted to the six earliest data points. The open circle corresponds to the test point suggested by LP98. The earliest six observations fit surprisingly well to the used by LP98 Dawson function. Next four observations seem to lie along a straight line whose slope agrees with Equation (8).

We shall try to understand the general picture as follows: The basic quantity is a total cloud heat. In LP98 model the total cloud heat is fed by the radioactive heat input and it is a source of the cloud bolometric luminosity. LP98 model describes the interrelations between these three components in the time range between 0.5 and 1.5 days. There should exist in that time range a moment \( t_a \), when originally increasing total cloud heat attains its maximum value. At that moment the emitted bolometric luminosity is equal to the energy produced by the radioactive process. The time dependences of both these quantities follow Equation (8). The location of this moment can be obtained by requiring the local fulfilment of the Equation (8) soon after the bolometric luminosity maximum. The result is

\[ t_a = 0.97563 \text{ days,} \quad L_a = 6.25943e + 41 \text{ ergs/s,} \]  

\( \text{(12)} \)

and it is marked in the Figure 1 with a star symbol.

Let us assume, that a time interval between 2.5 and 5.5 days since merger is contained in the optically thin region. Therefore in that interval, all the
produced heat is equal to the heat radiated. Starting from point $t_a$, we shall draw in Figure 1 a line with its slope equal to $-1.0$ and this line shows very good agreement with four data points inside time interval $[2.5, 5.5]$, what seems to confirm our interpretation of the point $t_a$ and also our assumption that 2.5-5.5 days is inside the optically thin time region.

We shall not consider the behavior of the luminosity vs time dependence inside the time intervals $1.5 < t < 2.5$ and $t > 5.5$ days.

The basic results of LP98 model are presented by Equations (5)-(7) which describe the model with help of 4 parameters $V, M, \kappa$ and $f$ while we have only three equations. However we can solve these Equations with a reduced set of unknowns, namely $V, M \times f$ and $\kappa/f$ or $M \times \kappa$.

\[
\left(\frac{3V}{c}\right) \approx \left(\frac{L_m}{2.1 \times 10^{44} \text{ ergs s}^{-1}}\right)^{1/2} \left(\frac{T_{\text{eff},m}}{2.5 \times 10^4 \text{ K}}\right)^{-2} \left(\frac{t_m}{0.98 \text{ days}}\right)^{-1}, \quad (13)
\]
\[
\left(\frac{M}{0.01 M_\odot}\right) \left(\frac{f}{0.001}\right) \approx \left(\frac{L_m}{2.1 \times 10^{44} \text{ ergs s}^{-1}}\right) \left(\frac{t_m}{0.98 \text{ days}}\right), \quad (14)
\]
\[
\left(\frac{\kappa}{\kappa_e}\right)^{-1} \left(\frac{f}{0.001}\right) \approx \left(\frac{L_m}{2.1 \times 10^{44} \text{ ergs s}^{-1}}\right)^{1/2} \left(\frac{T_{\text{eff},m}}{2.5 \times 10^4 \text{ K}}\right)^2. \quad (15)
\]
Looking closer on Equation (15) we can notice that the left hand side of this equation is based on the atomic data, while we have the observational data on the right hand side. Therefore it can be used for checking the quality of the atomic data.

Using Equations (10) and (11) we obtain

\[ V \approx (0.179 \pm 0.024) \, \text{c}, \]  
(16)

\[ M \times f \approx (2.307 \pm 0.044) \times 10^{-8} \, M_\odot, \]  
(17)

and

\[ f/\kappa \approx (5.75 \pm 0.76) \times 10^{-5} \, \text{cm}^{-2} \, \text{g}. \]  
(18)

or

\[ M \times \kappa \approx (4.01 \pm 0.31) \times 10^{-4} \, M_\odot \, \text{cm}^{2} \, \text{g}^{-1}. \]  
(19)

The lower limit for \( \kappa \) may be set as

\[ \kappa > 0.2 \, \text{cm}^{2} \, \text{g}^{-1}, \]  
(20)

and that makes it possible to get the following inequalities

\[ M < 2.0 \times 10^{-3} \, M_\odot, \]  
(21)

\[ f > 1.15 \times 10^{-5}. \]  
(22)

4 Conclusions

LP98 is a spherically symmetric, homogeneous, single component model. In spite of being simple, LP98 model has presented a new look on the observable remnants of the neutron star merging with black hole or another neutron star. They have realized that the presence of r-process nuclei will be followed by a continuous nuclear explosion in expanding with sub-relativistic velocity cloud which, after being enlarged by factor \( \approx 10^8 \) can be sufficiently big and bright to be observed in the optical spectral region. All the following papers describing this kind of explosion have their roots in Li & Paczyński model. The name for this model “kilonova” has been coined by Valé Petrosian (Metzger et al. 2010).

LP98 have noticed, assuming correctness of Equation (1), that the overall heat balance leads to the generic equation for Dawson function. This has resulted in obtaining a simple analytical solution for the earliest and also brightest part of the luminosity-time dependence. They have noticed also that later on, in the optically thin region, the luminosity may follow equation (8), forming a straight line on a log-log luminosity vs time diagram. Figure 1 shows surprisingly good, though possibly fortuitous, agreement of these features with observations.
Such a fine agreement between model and observations need confirmation from data on similar objects. After such confirmation LP98 model may be useful at early selection of kilonovae among optical transits. That may help to find orphian kilonovae at the absence of gravitational and/or gamma ray counterparts. It may also suggest early approximate limits for the object parameters what can be useful at constructing more sophisticated models.

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