CP Asymmetries (& T Violation) in Known Matter – and beyond

From Roman history about data: Caelius (correspondent of Cicero) had taken a pragmatic judgment of who was likely to win the conflict and said: Pompey had the better cause, but Caesar the better army, and so I became a Caesarean.

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Abstract
Finding CP violation (CPV) in 1964 produced a real revolution in the fundamental dynamics, although the community did not understand it right away. The paper by Kobayashi & Maskawa [1] appeared in 1973 to describe CPV in classes of three (or more) families of quarks, non-minimal Higgs’ dynamics and/or charged right-handed currents. The Standard Model is defined now with three families of quarks. It can describe the measured CP & T violation in kaon and B mesons at least as the leading source. None has been found in baryons, charm mesons, top quarks and EDMs. We have failed the explain our matter vs. anti-matter huge asymmetry. Even when there is no obvious connections with that asymmetry, it makes sense to probe CPV for the signs of New Dynamics & their features. Furthermore we have to measure regional CPV in multi-body final states with accuracy. Finally I emphasize CPV in leptonic dynamics, connections with Dark Matter and possible axion’s impact of cosmology.

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Obviously I am a theorist working with the tools we got from & about quantum mechanics & quantum field theories. I cannot express better – in one sentence – about the connection with the works of experimenters and theorists, as you see in the Fig.1.

Prologue

This is a short review about CP violation and with some comments about the complex scenario of time reversal. The goal is to remember ‘mature’ readers what they have heard before; for the ‘young’ ones it should show the ‘roads’ where one can learn from references in details. Furthermore we have to use tools based on local gauge symmetries. One can

2Not all colleagues are so polite to give such credit.
Figure 1: "To be honest, I never would have invented the wheel if not for Urg’s groundbreaking theoretical work with the circle." [A long time ago I had found this cartoon on an in-flight journal of the United Airlines.]

see the difference about local vs. discrete symmetries in real world on the Fig.2, namely scenarios of physics vs. chemistry. There is a very long history on our planet. It has been

Figure 2: Running with discrete symmetry

the goal to understand fundamental dynamics: first about ‘elements’ and then ‘elementary particles’ in more & more refined versions. Afterwards we have used the practical words of ‘high energy physics’ (HEP) instead. Somewhat recently our community realized that
we might barking at the wrong tree; instead we have to think about ‘symmetries’ like local ones starting with QED and later about weak and strong forces. There is another class of symmetries, namely \textit{discrete} ones; there are subclasses: (i) Parity (P), charge conjugation (C) and time reversal (T); (ii) chiral symmetry; (iii) flavor symmetry. Those discrete symmetries are correlated due to dynamics in important ways as discussed below in some details.

I focus on CP violation; on the other hand (broken) chiral and flavor symmetries have great impact on CP asymmetries. I assume that the reader knows about basic tools for quantum mechanics & quantum field theories including non-abelian ones, Einstein-Podolsky-Rosen (EPR) correlations from quantum mechanics and the impact of symmetries like $P/C$\footnote{All animals are equal, but some animals are more equal than others!” G. Orwell, ‘Animal Farm’.}. Operators $P$ and $C$ are unitary, namely $P^{\dagger}[C^{\dagger}] = P^{-1}[C^{-1}]$. However the situation for the anti-unitary operator $T$ is more subtle\footnote{There is a well-known analogy (or allegory) about time reversal from classical physics: parking a car in a very limited space is a true challenge (in particular in Paris or Rome), while joining the traffic is much easier.}; one of the reasons changes initial $\leftrightarrow$ final states:

$$\langle A|T^{\dagger}T|B\rangle = \langle B|A\rangle$$

\textit{CPT} invariance has assumed as usual for very good reasons.

On the other hand it can help to understand the underlying dynamics, namely Kramers’ degeneracy:

$$T^2|x_1, s_1; \ldots; x_n, s_n\rangle = (-1)^n|x_1, s_1; \ldots; x_n, s_n\rangle ;$$

i.e., $T^2 = -1$ applies to a system with an odd number of same fermions and thus ‘usable’. It also helps to understand the rules of ‘detailed balance’.

There are more general comments about CP & T asymmetries:

- This article focus on the weak decays of kaon, charm and beauty mesons.
- I also mention weak decays of top quarks, but also their production in the connection with other states at very high energy collisions like Higgs states.
- We have to probe CP & T asymmetries in flavor independant transitions like in electric dipole moments (EDM) although non-zero number have not been found yet.
- With neutrino oscillation having been established we have a subtle, but wide landscape for New Dynamics (ND) to probe. It needs long time to achieve the goal – but there is a needed ‘price’ to reach the ‘prize’.
- History shows us there was a difference between nuclear and high energy dynamics; however I think that is not the best road now. In particular in Europe there are groups working between nuclear forces and HEP, namely called Hadrodynamics. We can see the connections between the tools produced in one section and than applied to others. It say it with different words: there are excellent reasons to...
probe fundamental forces at much higher energies – but also to go from accuracy to precision at lower energies with different tools. It seems to me that we are still at the beginning of this latter road.

1 History of CP violation & preview of the future

It is possible that we live on one of many, many universes (or multi-verses). However ours one is very special not only about the huge asymmetry of matter vs. anti-matter. The data tell us that baryonic (or known) matter produces around $\sim 4.5\%$, dark matter $\sim 26.5\%$ and vacuum (or dark) energy $\sim 69\%$ in our universe. Those ratios are not very close zero or 100% as you might guess, but sizable; therefore we have to deal with surprising landscape. Furthermore we have candidates for dark matter (like several versions of Super-Symmetry (=SUSY)). On the other hand we have hardly ‘realistic’ candidates for dark energy; at least I am too old to spend daytime to think about vacuum energy.

There is excellent evidence about the asymmetry of matter vs. anti-matter – namely ‘our existence’ on our earth. It was a real surprise to find that parity $P$ and charge conjugation $C$ are broken in charged weak forces. Our community quickly recovered that $\tau$ and $\theta$ – called then – are the same state: $K^\pm$ mesons decay to both parity even and odd final states. Furthermore we have neutral ones – $\bar{K}^0$ and $K^0$ – produce two mesons that are differentiated by their lifetimes: $K_S$ and $K_L$. Therefore $K_L$ was seen as parity odd mesons. Not only $P$ & $C$ violations were found, but also in maximal ways, namely charged weak mesons coupling only to left-chiral quarks. Also $\nu_L$ & $\bar{\nu}_R$ were found, but not $\nu_R$ & $\bar{\nu}_L$ for massless neutrinos. It is fine in a simple realization of CPT invariance.

A true revolution happened in 1964: it was found that $K_L$ that usually decays into three pions, can also – rarely – to two pions. At first it was suggested to introduce non-linear terms into the Schrödinger equation or new unobserved neutral particle $U$ with $K_L \rightarrow K_S + U \rightarrow 2\pi$ rather than giving up on CP symmetry. More data and more thinking showed we have found CPV in the data. Then Wolfenstein gave a paper about what is called super-weak CPV. In my view it is not even a model; instead it is a classification for models of CPV. In 1973 Kobayashi & Maskawa gave a published paper, where CPV can come from three classes: three (or more) families of quarks or/and charged Higgs states or/and more weak bosons with spin-one couplings to right-chiral bosons. At first some colleagues suggested the source of CPV comes from charged Higgs; afterwards we knew that we need (at least) three families of quarks for hadronic dynamics. Now we know that the CKM matrix produces at least the leading source of the measured CP asymmetries in the decays of kaons and $B$ meson; that is a tested part of the SM.

No CPV has been established (yet) in the dynamics of charm hadrons, baryons in general (except our existence) and in the productions & decays of top quarks (before they can produce top hadrons).

\[5\] Actually Okun stated in his book ‘Weak Interactions of Elementary Particles’ published in 1963 in Russian – i.e., clearly before the discovery of CP violation (CPV) in 1964 – it is crucial to probe CP asymmetries.
In the SM CP landscape is simple for leptons: when the three neutrinos are massless, one defines their leptonic flavor numbers by their couplings with charged leptons; furthermore e, μ and τ cannot show CP asymmetries in their decays – except in \( \tau^- \to \nu \bar{K}^0 \pi^- \ldots \) → \( \nu K_S \pi^- \ldots \) piggyback riding on \( \bar{K}^0 - K^0 \) oscillations.

Very short summary about past experience and prediction for the future:

1. CP violation are not always tiny as we found out first in neutral kaons. In the transitions of \( B \) mesons are large, even somewhat close to 100 %. The SM produces at least the leading source for that, but our understanding of the impact of non-perturbative QCD still is limited quantitatively.

2. CPV was found and established in neutral kaon decays, namely indirect and direct ones in \( \epsilon_K \) and \( \epsilon'/\epsilon_K' \), respectively. However the ‘job’ has not been finished yet about fundamental dynamics. When one looks at the triangles from the CKM matrix, one seems the impact of our understanding of \( \epsilon_K \) (like in Fig.5 below). I was told there is ‘soon’ a chance that progress in lattice QCD will show the impact also on \( \epsilon' \) in different situation.

3. On the other hand the CKM dynamics has nothing to do with huge asymmetries in matter vs. anti-matter. Therefore we still have to think & work about this source.

4. Measurable CP asymmetries need interferences at least two amplitudes. Never mind that we have failed to understand matter vs. anti-matter in our universe. The interference can linearly depend on the amplitude of ND and thus allows with much more sensitivity.

5. Asymmetries beams of \( e^+e^- \) collisions and new technologies for detectors with precision had entered a new era with the experiment LHCb at CERN and will continue with the experiment Belle II at KEK (Japan). It is a real challenge to analyze huge amount of data. It is crucial to probe correlations between different final states – including multi-body FS in charm & beauty decays.

6. On the theoretical side new tools with more accuracy to probe fundamental dynamics including operator product expansion, heavy quark expansion and lattice QCD. While the source of CPV is weak forces, their impact depends on strong forces – i.e., nonperturbative QCD.

7. Flavor independent CPV have been probed, in particular for EDMs in very different landscapes from elementary leptons to very complex states like nuclei or molecules – and we have to continue.

8. The landscape of CPV is very ‘complex’ (unless there is no asymmetry). For example: what does it mean ‘maximal’ CPV: in particular the phases of fermion fields can be changed by applying different parameterization. For three families of quarks one describes CPV in six triangles with very different patterns; however they give the same area.

9. Based on CPT invariance CPV & T are well connected. Of course one wants to probe CPT invariance; it is discussed in another contribution in this book. Usually it is assumed that EPR correlations are perfect.

10. There are two classes of CP asymmetries:

   (i) ‘Indirect’ CPV that can happen only on neutral mesons and need oscillations; these
\textbf{CP} asymmetries depend on the time of decay; observables are defined by the initial FS, namely: $K^0$ (or $K_L$), $D^0$, $B_d$ or $B_s$.

(ii) ‘Direct’ \textbf{CPV} can be seen in the decays of any hadron (and possibly also in some of that in production in connection of other states). Its impact depend on the FS and does not depend on the time of decay.

(iii) In neutral mesons one sees the interferences with both classes of \textbf{CPV}. Their impact depends on strong final state interaction (FSI) or re-scattering based on quantum theory amplitudes. It can be described in the world of hadrons or quarks.

(11) \textbf{CPT} invariance tells us that \textbf{CPV} is described by complex phases. You might say that ‘maximal’ \textbf{CPV} means a phase is 90°. However such a statement is fallacious; one can change the phase of the quark field of a given CKM matrix element and rotate it away; of course it will re-appear in other matrix elements. For example $|s\rangle \to e^{i\delta_s}|s\rangle$ leads to $V_{qs} \to e^{i\delta_s}V_{qs}$, with $q = u, c, t$.

(12) Penguin diagrams are defined in the world of quarks, gluons and weak bosons. Fig.3 (a) sees Feynman diagrams with gluon & $W$ gauge bosons and also $b$ quarks in the initial state instead; part (b) describes wave lines for gauge bosons; it is assumed that non-perturbative QCD complete the FS. Sometimes art helps somewhere. However there is a real challenge, namely to connect amplitudes in the world of quarks with those in the world of hadrons that are and can be measured.

This is a complex one on several levels. In the collisions of (anti-)baryons at low energies one hardly care about them being boundstates of three constituent (anti-)quarks – unless one describes their EDMs, where discuss the difference between current vs. constituent ones. When one talk about non-leptonic decays of hadrons, it is crucial to use current quarks. We know how to describe inclusive decays; however for probing \textbf{CPT} violation the landscapes are much more complex, and we need more subtle tools to describe also multi-body FS. We cannot focus only on two-body FS.

There is a general comment: it is one thing to draw Feynman diagrams, but understanding the underlying forces is another thing; one needs more thinking and use correlations with other transitions. One shows the impact of penguin diagrams in $K \to \pi\pi$ decays, although loop diagrams are usually suppressed. On the other hand their impact enhanced by chiral symmetry for two pions FS and somewhat for three pions one. However this does not work for multi-body FS in decays of charm or beauty hadrons.

(13) A general comment: usually one compares the predictions from models with the information gotten by fitting best the data. There is a good reason to say that the analyses are model insensitive. However it is only the first (and second) step; in particular when one has a good candidate or a real theory, one have to focus whether these predictions come around within two sigma or so and think & probe \textit{correlations} with other data. Theoretical uncertainties are systemetic at best; often ‘predictions’ follow the fashion.

It shows the connection of \textbf{CPV} with the violation of \textbf{T} reversal (TV), where one might say it is more obvious in $e^{i\phi_t}$ to reach the same goal going down a different road. On the other hand, the landscape of \textbf{T} reversal is very complex. It depends on its definition. For

\footnote{CKM phase like the "Scarlet Pimpernel: Sometimes here, sometimes there, sometimes everywhere".}
example, we know that it happens already in classical physics: it is much easier to get ‘down’ than ‘up’ – i.e., the different scenarios of initial and final states. In this article I will discuss TV in fundamental forces.

(14) There is a short comment about ‘oscillations’ vs. ‘mixing’. Of course ‘mixing’ covers more items in dynamics than ‘oscillation’. However I see no reason to be happy to use the same word for different regions of dynamics.

- ‘Oscillations’ needs $\Delta F = 2$ forces, and their impact depends on the time of decay in well-know and measurable way.

- It depends in the initial neutral decaying hadrons like $K^0$ or $B_d$.

- Oscillation is a much more narrow meaning by focus on indirect CPV; oscillation is a crucial step to probe CPV, but it can happen without CPV – as we know so
far about $D^0$ decays.

- I prefer to use the word of ‘mixing’ in narrow situations like $s \leftrightarrow d$ about the Cabibbo angle or in general:
  - It shows the connection of quarks with mass states with quarks that couple to weak charged bosons leading to the CKM matrix.
  - Likewise for ‘massive’ neutrinos: they couple with charged leptons leading to the Pontecorvo–Maki–Nakagawa–Sakata matrix \([2]\) (and maybe with Dark Matter).

2 \ CP asymmetries in hadrons’ decays

\textbf{CPV} in neutral kaons and $B$ mesons have been established; it depends on our quantitative understanding \emph{quark flavor} dynamics including non-perturbative QCD. None have been found (yet) in charm hadrons; so far we have not enough rate to probe for top quarks.

Of course one first focus on the transitions of neutral mesons with richer landscapes. \textbf{CPV} was found in $K_L$ or $\bar{K}^0/K^0$ and $\bar{B}^0/B^0$ decays. We have many examples, and the future data will produce show more. Also theorists like to discuss that using their tools based on quantum mechanics, quantum field theories, the correlations between different FS and differentiate the impact of ND and its features sometimes in subtle ways, in particular about two-body FS. Of course, it needs much more work & analyze. However I will emphasize the impact the informations we get from \textit{multi-body} FS about direct \textbf{CPV} with accuracy and will discuss that about baryon and charged mesons decays below.

$\Delta F \neq 0$ forces connect neutral flavor mesons $P^0$ and $\bar{P}^0$. Therefore mass eigenstates are described linear amplitudes based on \textbf{CPT} \([3]\):

\begin{align}
|P_1\rangle &= p|P^0\rangle + q|\bar{P}^0\rangle \\
|P_2\rangle &= p|P^0\rangle - q|\bar{P}^0\rangle
\end{align}

are mass & width eigenstates with eigenvalues & their differences\(^7\):

\begin{align}
M_1 - \frac{i}{2}\Gamma_1 &= M_{11} - \frac{i}{2}\Gamma_{11} + \frac{q}{p}(M_{12} - \frac{i}{2}\Gamma_{12}) \quad (5) \\
M_2 - \frac{i}{2}\Gamma_2 &= M_{11} - \frac{i}{2}\Gamma_{11} - \frac{q}{p}(M_{12} - \frac{i}{2}\Gamma_{12}) \quad (6) \\
M_2 - M_1 &= -2\text{Re} \left( \frac{q}{p}(M_{12} - \frac{i}{2}\Gamma_{12}) \right), \quad \Gamma_2 - \Gamma_1 = +4\text{Im} \left( \frac{q}{p}(M_{12} - \frac{i}{2}\Gamma_{12}) \right) \quad (7) \\
\left( \frac{q}{p} \right)^2 &= \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}, \quad \frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \quad (8)
\end{align}

\(^7\)There are opposite signs of $q/p$; using negative sign is equivalent to interchanging labels $1 \leftrightarrow 2$. 

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\( q/p \) itself is not an observable. One can change the phase of anti-particles: \(|P^0 \rangle \rightarrow e^{i\xi}|P^0 \rangle\) will modify the off-diagonal elements of \( M \) \& \( \Gamma \) and thus \( q/p \rightarrow e^{-i\xi}q/p \). However both \(|q/p| \) \& \( \frac{q}{p}(M_{12} - \frac{i}{2}\Gamma_{12}) \) are invariant and observable in different ways:

- \( P_1 \) \& \( P_2 \) states in general are not orthogonal to each other: \( \langle P_1 | P_2 \rangle = |p|^2 - |q|^2 \neq 0 \).
- This situation can be measured in semi-leptonic rates using CPT invariance with \(|A|^2 = |A(l^+)|^2 = |\bar{A}(l^-)|^2\):

\[
\Gamma(P^0 \rightarrow l^- + X^+) \propto e^{-\Gamma t} \left| \frac{q}{p} \right|^2 |A|^2
\]

\( \Gamma(\bar{P}^0 \rightarrow l^+ + X^-) \propto e^{-\Gamma t} \left| \frac{p}{q} \right|^2 |A|^2
\]

One probes CPV based on \( P^0 - \bar{P}^0 \) oscillations, however it is independent of time:

\[
A_{SL}(P^0) \equiv \frac{\Gamma(P^0 \rightarrow l^- + X^+) - \Gamma(\bar{P}^0 \rightarrow l^+ + X^-)}{\Gamma(P^0 \rightarrow l^- + X^+) + \Gamma(\bar{P}^0 \rightarrow l^+ + X^-)} = \frac{1 - |p/q|^2}{1 + |p/q|^2}
\]

It depends on the initial state with only indirect CPV with \( \Delta F = 2 \), namely \( K^0 \), \( B_d \), \( B_s \) and \( D^0 \) transitions.

- Basically quantum mechanics tell us about the two mass eigenstates \( P_1 \) \& \( P_2 \) of the \( P^0 - \bar{P}^0 \) oscillations using the Schwartz inequality arrive at

\[
|\langle P_2 | P_1 \rangle| \leq \sqrt{\frac{\sum_{f} 4\Gamma_f^I \Gamma_f^F}{(\Gamma_1 + \Gamma_2)^2 + 4(M_1 - M_2)^2}}
\]

This inequality is numerically relevant for kaons due to \( \Gamma_L \ll \Gamma_S \simeq 2\Delta M_K \):

\[
|\langle K_L | K_S \rangle| \leq \sqrt{\frac{2\Gamma_L}{\Gamma_S}} \sim 0.06
\]

as a very conservative bound: \( K_L \) and \( K_S \) are close to be odd and even CP states. Does it mean that we are just being lucky with \( 3m_\pi < M_K < 4m_\pi \) or these is a deep reason?

- The landscape is more complex with non-leptonic decays with impact of indirect and direct CPV \& their interferences even \( f \neq \bar{f} \) with obvious or subtle reasons:

\[
\Gamma(P^0(t) \rightarrow f) \propto \frac{1}{2} e^{-\Gamma \tau} |A(f)|^2 \cdot G_f(t)
\]

\[
G_f(t) = a + be^{\Delta \tau t} + ce^{\Delta \tau t/2}\cos\Delta M t + de^{\Delta \tau t/2}\sin\Delta M t
\]

\[a = \frac{1}{2} \left[ 1 + \left| \frac{q}{p} \bar{\rho}(f) \right|^2 \right] + \text{Re}\left( \frac{q}{p} \bar{\rho}(f) \right)
\]
\[
b = \frac{1}{2} \left(1 + \left|\frac{q}{p} \bar{\rho}(f)\right|^2\right) - \text{Re} \left(\frac{q}{p} \bar{\rho}(f)\right)
\]

\[
c = 1 - \left|\frac{q}{p} \bar{\rho}(f)\right|^2, \quad d = -2\text{Im} \frac{q}{p} \bar{\rho}(f)
\]

\[
\Gamma(\bar{P}^0(t) \to \bar{f}) \propto \frac{1}{2} e^{-\Gamma t} |\bar{A}(\bar{f})|^2 \cdot \bar{G}_f(t)
\]

\[
\bar{G}_f(t) = \bar{a} + \bar{b} e^{\Delta \Gamma t} + \bar{c} e^{\Delta \Gamma t/2} \cos \Delta M t + \bar{d} e^{\Delta \Gamma t/2} \sin \Delta M t
\]

\[
\bar{a} = \frac{1}{2} \left(1 + \left|\frac{p}{q} \rho(\bar{f})\right|^2\right) + \text{Re} \left(\frac{p}{q} \rho(\bar{f})\right)
\]

\[
\bar{b} = \frac{1}{2} \left(1 + \left|\frac{p}{q} \rho(\bar{f})\right|^2\right) - \text{Re} \left(\frac{p}{q} \rho(\bar{f})\right)
\]

\[
\bar{c} = 1 - \left|\frac{p}{q} \rho(\bar{f})\right|^2, \quad \bar{d} = -2\text{Im} \frac{p}{q} \rho(\bar{f})
\]

\[
\bar{\rho}(f) = \frac{\bar{A}(f)}{A(f)}, \quad \rho(\bar{f}) = \frac{\bar{A}(|\bar{f}|)}{A(\bar{f})}
\]

It is important to remember that \((q/p)\bar{\rho}(f)\) and \((p/q)\rho(\bar{f})\) do not depend on the definition of the phases and therefore observable, while singly \((q/p)\& \bar{\rho}(f)\) are not.

Here you can see the general situation. In our world we have \(|q/p| \sim 1\) and \(\Delta \Gamma / \Gamma \sim 0\) (except \(\Delta \Gamma(K_L)/\Gamma(K_S) \simeq 0.49\)). \(\Delta \Gamma = 0\) happens only due to a miracle; however \(\Delta \Gamma\) can be smaller or larger than expected from SM values; the impact of ND can hide in the experimental and/or theoretical uncertainties.

Sometime the situation is simpler, when the FS are even/odd CP eigenstates, when one gets

\[
\bar{\rho}(f_{\pm}) = \pm \frac{1}{\rho(f_{\pm})}.
\]

For charged \(P\) mesons the landscapes look much simpler\(^8\) – but not very much in reality:

\[
\Gamma(P \to f_a) \propto e^{-\Gamma t} |A(f_a)|^2
\]

\[
\Gamma(\bar{P} \to f_b) \propto e^{-\Gamma t} |\bar{A}(f_b)|^2
\]

There are several important statements, although they are not always obvious:

- Time depending data show the impact of indirect vs. direct CPV in neutral mesons. The amplitudes of indirect ones depend in the initial state – \(K^0\), \(B_{d,s}\) and \(D^0\). These can be probed in two-body FS.

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\(^8\)One can easily connect the expressions given for \(P^0\) vs. \(\bar{P}^0\) with \(\Delta \Gamma = 0 = \Delta M\).
• Direct CPV affect differently FS in the decays of hadrons. The point is that it is not enough to understand the dynamics with two-body FS; it is crucial to measure three- and four-body FS and not as a back-up information.

• The impact of strong re-scattering is crucial as discussed below. It happens in the world of hadrons and of quarks as we know as indicated in $f_a$ vs. $f_b$; however it is a true challenge to describe them quantitatively with subtle theoretical tools based on data at low energies.

3 The landscapes of K & B & D meson decays

3.1 Kaon decays – first ‘affair’

As stated above the existence CPV was first found in 1964 by $K_L \rightarrow \pi^+\pi^-$ i.e., the $K_L$ amplitude has a small non-zero CP odd component due to $K^0 - \bar{K}^0$ oscillations with $\Delta M_K/(\Gamma_S + \Gamma_L) \approx \Delta M_K/\Gamma_S \approx 0.49$:

$$\frac{\Gamma(K_L \rightarrow \pi^+\pi^-)}{\Gamma(K_S \rightarrow \pi^+\pi^-)} = [(2.0 \pm 0.4)] \times 10^{-3}\textsuperscript{2}$$  (28)

The existence of this small rate is connected with the asymmetry in $\bar{K}^0 \rightarrow l^+\nu\pi^-$ vs. $K^0 \rightarrow l^-\bar{\nu}\pi^+$ in the SM (and basically beyond) i.e., indirect CPV:

$$A_L = \frac{\Gamma(K_L \rightarrow \pi^+\nu\pi^-) - \Gamma(K_L \rightarrow \pi^-\bar{\nu}\pi^+)}{\Gamma(K_L \rightarrow \pi^+\nu\pi^-) + \Gamma(K_L \rightarrow \pi^-\bar{\nu}\pi^+)} = (3.32 \pm 0.06) \cdot 10^{-3}.$$  (29)

While $A_L$ comes from oscillations, this asymmetry does not depend on the time of the decays. The scenarios of non-leptonic decays are more complex also for $K_L$ decays: weak forces produce $K_L \rightarrow \pi^+\pi^-/\pi^0\pi^0$, which are calibrated by $K_S$ decays:

$$\eta_{++} \equiv \frac{\langle \pi^+\pi^-|H_W|K_L \rangle}{\langle \pi^+\pi^-|H_W|K_S \rangle}, \eta_{00} \equiv \frac{\langle \pi^0\pi^0|H_W|K_L \rangle}{\langle \pi^0\pi^0|H_W|K_S \rangle}$$  (30)

We differentiate indirect vs. direct CPV:

$$\eta_{++} = \epsilon_K + \epsilon' , \eta_{00} = \epsilon_K - 2\epsilon' ,$$  (31)

where $\epsilon_K$ is produced by oscillations, while $\epsilon'$ show the differences between different FS. Present data show [4]:

$$|\epsilon_K| = (2.228 \pm 0.011) \cdot 10^{-3}.$$  (32)

The response from the theoretical community about CPV was slow. It was suggested by Wolfenstein that we have a ND, namely super-weak one with $\epsilon' = 0$. However there was not a real theory, but a classification of theories for CPV.
3.2 New Standard Model ‘then’

Most HEP people know about three quarks, namely $u, d, s$; also they thought of them as a mathematical entities. Some outliers told about the fourth quark, namely $c$. To understand to underlying dynamics of CPV Kobayashi & Maskawa published a paper in 1973 that there are three classes of theories beyond the SM then, namely at least three families of quarks or right-handed charged currents or charged Higgs. Now we know that at least the leading source comes from three families with $(u, d), (c, s)$ and $(t, b)$ with weak forces $SU(2)_L \times U(1)$.

We have to deal with somewhat different landscapes, namely we can probe data based on hadrons and predict transitions based on quantum field theories with quarks & gluons & spin-one bosons. This connection comes from the word of ‘duality’ in different levels; some are obvious, others are subtle.

For the SM one gets an unitary CKM matrix for three families with six charged quarks in pairs $(u, d), (c, s)$ and $(t, b)$:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

It is described by six triangles with 12 parameters. However one gets only four observables, namely three angles and one phase. Their patterns are quite different – but their have the same area. The general parameterization of flavor dynamics is not obvious.

3.2.1 Wolfenstein’s original parameterization & refined ones

However Wolfenstein suggested a very good ‘usable’ one based on the expansion in the Cabibbo angle $\lambda = \sin\theta_C$ with $A$, $\bar{\rho}$ and $\bar{\eta}$ of the order of unity [5]:

$$V_{CKM} \simeq \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda, A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} - i\eta A^2\lambda^4, A\lambda^2(1 + i\eta\lambda^2) \\ \lambda A^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

\( V_{old} \)

‘Old’ triangle I.1 : $V_{ud}V_{us}^* [O(\lambda)] + V_{cd}V_{cs}^* [O(\lambda)] + V_{td}V_{ts}^* [O(\lambda^5)] = 0$ (35)

‘Old’ triangle I.2 : $V_{cd}^*V_{ud} [O(\lambda)] + V_{us}^*V_{cs} [O(\lambda)] + V_{tb}^*V_{ts} [O(\lambda^5)] = 0$ (36)

‘Old’ triangle II.1 : $V_{us}^*V_{ub} [O(\lambda^4)] + V_{cs}^*V_{cb} [O(\lambda^2)] + V_{ts}^*V_{tb} [O(\lambda^5)] = 0$ (37)

‘Old’ triangle II.2 : $V_{cd}^*V_{td} [O(\lambda^4)] + V_{cs}^*V_{ts} [O(\lambda^2)] + V_{cb}^*V_{tb} [O(\lambda^5)] = 0$ (38)

‘Old’ triangle III.1 : $V_{ud}^*V_{ub} [O(\lambda^3)] + V_{cd}^*V_{cb} [O(\lambda^3)] + V_{td}^*V_{tb} [O(\lambda^3)] = 0$ (39)

‘Old’ triangle III.2 : $V_{ad}^*V_{td} [O(\lambda^3)] + V_{us}^*V_{ts} [O(\lambda^3)] + V_{ub}^*V_{tb} [O(\lambda^3)] = 0$ (40)

The pattern is obvious, in particular about indirect CPV, namely very larger in $B_d - \bar{B}_d$ oscillations. It has been successful in describing the ‘golden’ triangle in $B_{d,u}$ decays in Fig[7] where triangle III.1 shows that the sizes of the three angles are quite similar. The angles $\phi_1, \phi_2, \phi_3$ are opposite the sides with $\bar{u}u, \bar{c}c, \bar{t}t$; other people name angles $\beta, \alpha, \gamma$.  

13
As I have said before (and will again): the SM with three families of quarks produces four observables in the CKM matrix with one phase and three angles. It is crucial to probe the correlations in the triangles. Fig. 5 shows that large CPV in $B_d \to \psi K_S$ is connected with very small one in $K_L \to \pi\pi$ transitions & the ratio of $B_d - \bar{B}_d$ & $B_s - \bar{B}_s$ oscillations
due to $\Delta M_{B_d}/\Delta M_{B_s}$: i.e., those observables (mostly) come from three triangles.

That successful description still has some weak points. There are some 'tensions' about these measured data and the expected predictions. Furthermore measured decays of $B_{d,u}$ mesons give us $\bar{\eta} \simeq 0.34$ and $\bar{\rho} \simeq 0.13$, which are not very close to unity to see it politely. With three families of quarks one gets six triangles to decays of kaons, charm & beauty decays and top quarks. Four of those six ones one can probe directly. The patterns of these triangles are very different; however the area are the same in the SM.

We know that the SM produces at least the leading source of CPV in $K_L \to 2\pi$ and $B$ decays with good accuracy. In our search of ND we need even precision and to measure the correlations with other suppressed FS.

\[
\begin{pmatrix}
1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8} - \frac{\lambda^6}{16}, & \lambda, & \bar{\lambda} + \lambda^4 e^{-i\delta_{\text{QM}}}, & \bar{h}\lambda^4 e^{-i\delta_{\text{QM}}}, & f\lambda^3, & -f\lambda^2 - \bar{h}\lambda^3 e^{-i\delta_{\text{QM}}}, & 1 - \frac{\lambda^2}{2} f^2 - \bar{h}\lambda^5 e^{-i\delta_{\text{QM}}}, & -\frac{\lambda^4}{2}\bar{h} e^{-i\delta_{\text{QM}}}, & + O(\lambda^7)
\end{pmatrix}
\]

(41)

Thus the landscape of the CKM matrix is more subtle than it is usually said [6]; it is described by six triangles that are different in subtle ways but still with the same area:

- Triangle I.1 : $V_{ud}V_{us}^* [O(\lambda)] + V_{cd}V_{cs}^* [O(\lambda)] + V_{td}V_{ts}^* [O(\lambda^5)] = 0$ (42)
- Triangle I.2 : $V_{ud}^*V_{us} [O(\lambda)] + V_{cs}^*V_{cs} [O(\lambda)] + V_{ub}^*V_{cb} [O(\lambda^6)] = 0$ (43)
- Triangle II.1 : $V_{us}V_{ub} [O(\lambda^5)] + V_{cs}V_{cb}^* [O(\lambda^2)] + V_{ts}V_{tb}^* [O(\lambda^7)] = 0$ (44)
- Triangle II.2 : $V_{cd}V_{td} [O(\lambda^4)] + V_{cs}^*V_{ts}^* [O(\lambda^2)] + V_{cb}^*V_{tb}^* [O(\lambda^3)] = 0$ (45)
- Triangle III.1 : $V_{ud}V_{ub}^* [O(\lambda^3)] + V_{cd}V_{td}^* [O(\lambda^3)] + V_{ts}V_{tb} [O(\lambda^4)] = 0$ (46)
- Triangle III.2 : $V_{cd}V_{td}^* [O(\lambda^4)] + V_{ub}^*V_{tb} [O(\lambda^4)] + V_{ub}^*V_{tb}^* [O(\lambda^4)] = 0$ (47)

The pattern in flavour dynamics is less obvious for CPV in hadron decays as stated before [7]; the situation has changed: we have to measure the correlations between four triangles, not focus on the 'golden triangle'. Some of the important points are emphasized:

- We have to probe triangle III.1 with precision in $B_{d,u}$ transitions.
- Triangle II.1 has sizable impact on $B_s$ amplitudes and connect with other $B_{d,u}$ decays.
- Class II.2 can produce CP asymmetries in SCS & DCS D decays.
- Triangle I.1 can be probed in tiny $K \to \pi\nu\bar{\nu}$ decays with small theoretical uncertainties.
- Again: one has to focus on correlations with several triangles with accuracy [9].

---

[9] I see a connection of ‘correlations’ for a well-known joke: "In a circus an artist put tables & chairs
3.3 Kaon decays – second ‘affair’

As said before $\epsilon'$ describes channel dependent CPV, while $\epsilon_K$ characterizes CPV phase in $K^0 - \bar{K}^0$ oscillations; therefore direct CPV is expressed through the ratio:

$$\text{Re} \frac{\epsilon'}{\epsilon_K} = \frac{1}{6} \frac{|\eta_{+-}|^2 - |\eta_{00}|^2}{|\eta_{+-}|^2} = (1.66 \pm 0.23) \cdot 10^{-3}$$ (48)

The measured values of $|\epsilon_K|$ gives somewhat small experimental uncertainty; the challenge is to connect with the CKM parameters as shown in Fig.5, namely mostly the impact of triangle I.1 on the golden one in triangle III.1.

These values do not do justice to the experimental achievement. The sensitivity achievement becomes more transparent [3]:

$$\frac{\Gamma(K^0 \to \pi^+\pi^-) - \Gamma(\bar{K}^0 \to \pi^+\pi^-)}{\Gamma(K^0 \to \pi^+\pi^-) + \Gamma(\bar{K}^0 \to \pi^+\pi^-)} = (5.16 \pm 0.71) \cdot 10^{-6}$$ (49)

There is no surprise that Re$(\epsilon'/\epsilon_K)$ is small with the large top quark mass including the impact of penguin diagrams, see Fig.3. However the experimental uncertainty is sizable, and now Re$(\epsilon'/\epsilon_K)$ gives no usable constraint on CKM parameters. On the positive side of more data and refined analyses of $K \to \pi\gamma\gamma$ & $K \to \pi\pi\gamma\gamma$ allow deeper probes of chiral symmetry and in general better treatment of long-distance dynamics. It gives tests of LQCD as a subtle tool. Furthermore the LQCD community might be able to show that Re$(\epsilon'/\epsilon_K)$ gives sizable (& novel) impact on the correlations with the golden triangle [8].

3.4 Future of very rare kaon decays

There is still an important point about understanding fundamental dynamics, namely to measure the rates of $K^+ \to \pi^+\bar{\nu}\nu$ vs. $K_L \to \pi^0\nu\bar{\nu}$ and probe $V_{td}$ with SM prediction with 5% vs. 2% uncertainties, respectively. There only challenge is to get enough data with refined analyses. Thus they could act as "standard candles" in the (soon) future – maybe.

3.5 CP asymmetry in the decays of charged mesons & baryons

While ‘only’ direct CPV can effect the decays of baryons and charged mesons, one might think that the landscape is less complex. The opposite is (mostly) true:

(i) Direct CPV depend on FS and on the classes of decaying hadrons.
(ii) CP asymmetries do not depend on the times of the decays. Measuring time depending asymmetries is a very powerful tool.
(iii) One has to focus even more importantly on regional CPV where one needs at least three pseudo-scalar ones in the FS. As discussed below large ones have been established in $B^\pm$.

put together & higher and jump to the top with a head-stand using broom-stick to produce balance – and play with a fiddle. One of the man watching that said to his wife: He is not like Haifetz (famous fiddler)."
3.6 Effective transition amplitudes

Strong re-scatterings happen all the time. Can we control them quantitatively? My response is ‘no’ due to the impact of non-perturbative QCD. However it helps to deal with this challenge with tools following constraints coming from symmetries (broken or not). It has great impact on direct CP asymmetries with CPT invariance as discussed in Refs.[9],[10],[11]; it is given in Sect. 4.10 of Ref.[3] with much more details:

\[ T(P \rightarrow f) = e^{i\delta_f} \left[ T_f + \sum_{f \neq a_j} T_{a_j} i T_{a_j f}^{\text{resc}} \right] \]
\[ T(\bar{P} \rightarrow \bar{f}) = e^{i\delta_f} \left[ T_f^* + \sum_{f \neq a_j} T_{a_j}^* i T_{a_j f}^{\text{resc}} \right] , \]

where amplitudes \( T_{a_j f}^{\text{resc}} \) describe FSI between \( f \) and intermediate on-shell states \( a_j \) that connect with this FS. Thus one gets regional CP asymmetries:

\[ \Delta \gamma(f) = |T(\bar{P} \rightarrow \bar{f})|^2 - |T(P \rightarrow f)|^2 = 4 \sum_{f \neq a_j} T_{a_j f}^{\text{resc}} \text{Im} T_{a_j f}^* T_{a_j} \]

CP asymmetries have to vanish upon summing over all such states \( f \) using CPT invariance between subclasses of partial widths:

\[ \sum_f \Delta \gamma(f) = 4 \sum_f \sum_{f \neq a_j} T_{a_j f}^{\text{resc}} \text{Im} T_{a_j f}^* T_{a_j} = 0 , \]

since \( T_{a_j f}^{\text{resc}} \) & \( \text{Im} T_{a_j f}^* T_{a_j} \) are symmetric & antisymmetric, respectively, in the indices \( f \) & \( a_j \).

These Eqs. (50,51) apply to amplitudes in general, whether for hadrons or quarks or \( \bar{q}_i q_j \) boundstates in between\(^{10}\). In which way one can connect the landscapes in hadronic and quark amplitudes – it depends. In the world of quarks one can describe them by refined tree, penguin etc. diagrams. Those give weak phases. Furthermore penguin diagrams coming from non-local operators produce \( \Delta \Gamma \) for \( B_{s,d} \) mesons and somewhat for \( D^0 \) one. Those give also imaginary part that one needs for FSI – however the situations are very ‘complex’ there. The ‘roads’ are quite different depending on the FS.

3.7 Impact of non-perturbative QCD

The scenarios of the weak decays of beauty and charm hadrons are more complex than in kaon ones. There are several reasons:

- Two-body FS produce only small parts of CKM suppressed of \( D_{(s)} \) decays and tiny in \( B_{(s)} \) ones. There is no reason why two-body FS give us all the information that we need to understand dynamics and even less for only charged two-body ones.

\(^{10}\)In principle one has to include baryons \( q_i q_j q_k \), but I will not discuss that in this article.
• The worlds of hadrons and quarks are different. One can hide that by using ‘consti-
tute’ quarks, which works fine for spectroscopy (in particular for strange hadrons),
but not for weak forces. Current quarks are based on theories, not just models.
However they are connected in subtle ways, and we (= theorists) have to apply
refined tools.

• In the world of quarks one can describe inclusive FS in beauty & charm hadrons,
where we have to use ‘duality’ often in subtle ways. CPT invariance produces
strong constraints. To connect finite data of hadrons with quarks descriptions one
has to use tools like based on chiral symmetry, broken U-spin symmetry, dispersion
relations \[12\] etc. and insist on correlations with other transitions.

• Probing CPV in multi-body FS one measures first averaged one and then regional
ones with accuracies. It is not a good idea to just follow the best fits; it is much
more important to understand the landscapes and their informations given to us.
Of course the analysis has to be very acceptable – but not giving the best fits.
Judgment helps significantly how to define regional asymmetries.
Furthermore it is crucial not to ignore the correlations with other transitions.

• One measures three-body FS for several reasons with a long history \[13\]. We know
how to probe Dalitz plots with a long history including regional ‘morphologies’; it
has been emphasized not only use ‘fractional’ asymmetries, but also about different
tools \[14\] and compare their results. However in my view it is not the final step; we
have to use more subtle theoretical tools like dispersion relations that depend also
on data about low energy collisions of hadrons \[12\] – and some judgment.

The landscapes are very different already qualitatively between $\Delta B \neq 0$ and $\Delta C \neq 0$.

3.7.1 Case I: Broken U-spin symmetry

With quarks one describes mostly inclusive transitions. ‘Currents’ quarks with $m_u < 
m_d << m_s$ are based on theory. I-, U- & V-spin symmetries deal with $u \leftrightarrow d, d \leftrightarrow s$
& $u \leftrightarrow s$. These three symmetries are obviously broken on different levels, and these
violations are connected in the SM. The operators producing inclusive FS depend on
their CKM parameters and the current quark masses involved there. However the real
scale for inclusive decays is given by the impact of QCD, namely $\Lambda \sim 1$ GeV as discussed
many times. \[11\] Thus the violations of U- & V-spin symmetries are small, and tiny for
I-spin one. We can deal with inclusive rates and asymmetries of beauty and maybe charm
hadrons using effective operators in the world of quarks.

The connections with inclusive with exclusive hadronic rates are not obvious at least,
in particular about quantitative ways. The violations of I-, U-(& V-)spin symmetries in
the measurable world of hadrons are expected to scale by the differences in pion and kaon

\[11\] For good reasons one uses different and smaller $\Lambda_{QCD} \sim 0.1 – 0.3$ GeV for describing jets in collisions.
masses, which are not small compared to $\bar{\Lambda}$ (or $[m_{K}^{2} - m_{\pi}^{2}]/(m_{K}^{2} + m_{\pi}^{2})]$). This is even more crucial about direct CPV and the impact of strong re-scattering on amplitudes.

Going back to the history: Lipkin had suggested that U-spin violations in $B$ decays are of the order of 10% [15] in CKM favoured ones. They might be larger in suppressed ones. One reason is that suppressed decays in the world of hadrons consist with larger numbers of states in the FS, where strong FSI have great impact with opposite signs. Furthermore the worlds of hadrons (or ‘constitute’ quarks) are controlled by FSI due to non-perturbative QCD; they show the stronger impact on exclusive ones. For good reasons it has been stated that violation of U-spin symmetry is around $O(10\%)$ in inclusive decays. It can be seen in the sum of exclusive ones in large ratios that go up and down much more sizably. The paper [16] suggests one can probe U-spin symmetry with three-body FS with small theoretical uncertainties and even with only charged hadrons in the FS; I quite disagree on both: ‘Effective transition amplitudes’ or re-scattering as discussed above (see Sect. 3.6) produce large impact. I suggest to think about the informations gotten from Sect. 3.8.2 using CPT invariance about their subtle morphologies discussed below.

3.7.2 Case II: Impact of penguin operators vs. diagrams

Penguin diagrams, Fig.3, were introduced for kaon decays where is little differences between exclusive vs. inclusive decays. The impact of penguin operators in CKM suppressed decays of beauty hadrons are enhanced by chiral symmetry in their amplitudes, in particular for two body FS with pions and somewhat for kaons. However in charm hadron transitions the leading source of penguin diagrams is not given by local or even short-distance dynamics.

3.8 $B_s$ decays

The SM gives at least the leading source of CP asymmetries in $B$ transitions [with the still possible exception in $B_s \rightarrow \psi \phi, \psi f_0(980)$ ones]. Now we are probing for impact of ND in CP asymmetries and its features.

3.8.1 Indirect CPV in $B^0 - \bar{B}^0$ oscillations

Using $\Delta \Gamma_{B_d} \ll \Delta M_{B_d} \sim \Gamma_{B_d}$ as expected due to the large top quark mass, one describes:

$$\Gamma(B_d[\bar{B}_d] \rightarrow \psi K_S) \sim e^{-\Gamma_{B_d}t}G_{\psi K_S}[\bar{G}_{\psi K_S}]$$ (54)

$$G_{\psi K_S} = |A(\psi K_S)|^2 \left[ 1 - \text{Im} \left( \frac{q}{p} \bar{\rho}(\psi K_S)\sin \Delta M_{B_d} t \right) \right]$$ (55)

$$\bar{G}_{\psi K_S} = |A(\psi K_S)|^2 \left[ 1 + \text{Im} \left( \frac{q}{p} \bar{\rho}(\psi K_S)\sin \Delta M_{B_d} t \right) \right].$$ (56)

While CPV represents by $\text{Im} \left( \frac{q}{p} \bar{\rho}(\psi K_S) \right)$, it is been measured due to $\Delta M_{B_d} \neq 0$, depends on the time of decay: $\frac{d}{dt}(G_{\psi K_S}/G_{\psi K_S}) \neq 0$; actually in a special way: $\sin \Delta M_{B_d} t$, which
shows the connection with ‘odd’ $T$ symmetry. Furthermore in the SM it is defined by one angle in the ‘golden’ triangle, namely

$$\text{Im} \left( \frac{q}{p} \bar{\rho} (\psi K_S) \right) \simeq \sin 2\phi_1 [\beta] = 0.676 \pm 0.021$$

Refined parameterization of the CKM matrix show that the maximal value possible in the SM is $\sim 0.72$, not really close to unity due to correlations with other transitions.

The situation is different about $B_s - \bar{B}_s$ oscillations with $\Delta M_{B_s} \simeq 26.9$ and $y_s \sim 0.07$: very fast oscillation has been established, but no CPV has been found (yet):

$$\phi_{c\bar{c}s} = (0.01 \pm 0.07 \pm 0.01) \text{ rad (measured)} \text{ vs. } \phi_{c\bar{c}s} = (-0.0363^{+0.0019}_{-0.0015}) \text{ rad (SM)}$$

These data are close to SM values, but also consistent with ND’s sizable contributions – even leading source there – or with the opposite sign. It is interesting that recent LHCb data about $B_s \to \psi \pi^+ \pi^- \Rightarrow \psi f(980)$ see no obvious contribution from scalar $\sigma \Rightarrow \pi^+ \pi^-$. 

### 3.8.2 Direct CPV in $B$ decays

The situations of decays of $B_d$, $B_s$ and $B^+$ (and even $B_c$) are complex (for optimistic physicists); they are ‘rich’ where one can find the impact of ND or at least important lessons about non-perturbative forces from QCD. Again first one focus on (quasi-)two-body FS about sizable asymmetries in $B^+ \to D_{CP} K^+$, which has impact of measuring the angle $\phi_3 / \gamma$. Furthermore penguin diagrams contribute to CPV in $B_d \to K^+ \pi^-$, $K^*(892)^+ \pi^-$, $B_s \to \pi^+ K^-$ and $B^+ \to \eta K^+$ on different levels\footnote{Here one has also interference with indirect CPV in $B_d \to \pi^+ \pi^-$.}. The real challenge is to establish the impact of ND as a non-leading source. In the world of quarks one can show the ways to connect with hadronic FS with penguin diagrams due ‘duality’, which is a true challenge in a quantitative way.

Probing CPV in the SM suppressed decays one gets only a number in two-body FS. Of course, to connect the information we get from the data with the fundamental dynamics is not trivial – but it is not enough about forces: we have to probe three- & four-body FS etc. We describe three-body FS due to two-dimensional Dalitz plots. The first step is to measure averaged CPV which also give numbers, but still connected with two-body ones. However it is crucial to probe regional asymmetries. I give recent examples about the power and the tools including CPT invariance.

### 3.8.3 CP asymmetries in $B^{\pm}$ decays

In this article I focus on charged three-body FS, although I will talk also about the general landscape including CPT. LHCb data of CKM suppressed $B^+$ decays to charged three-body FS give small rates, which are expected:

$$\text{BR}(B^+ \to K^+ \pi^- \pi^+) = (5.10 \pm 0.29) \cdot 10^{-5}, \text{BR}(B^+ \to K^+ K^- K^+) = (3.37 \pm 0.22) \cdot 10^{-5}$$
LHCb data also show sizable CP asymmetries averaged over the FS with correlations [17]:

\[
\begin{align*}
\Delta A_{CP}(B^\pm \to K^\pm \pi^+\pi^-) &= +0.032 \pm 0.008_{stat} \pm 0.004_{syst}[\pm 0.007_{\psi K^\pm}] \\
\Delta A_{CP}(B^\pm \to K^\pm K^+K^-) &= -0.043 \pm 0.009_{stat} \pm 0.003_{syst}[\pm 0.007_{\psi K^\pm}].
\end{align*}
\]

It is not surprising that these CP asymmetries come with opposite signs due to the road to CPT invariance. Furthermore it shows ‘regional’ CP asymmetries:

\[
\begin{align*}
A_{CP}(B^\pm \to K^\pm \pi^+\pi^-)|_{\text{regional}} &= +0.678 \pm 0.078_{stat} \pm 0.032_{syst}[\pm 0.007_{\psi K^\pm}] \\
A_{CP}(B^\pm \to K^\pm K^+K^-)|_{\text{regional}} &= -0.226 \pm 0.020_{stat} \pm 0.004_{syst}[\pm 0.007_{\psi K^\pm}].
\end{align*}
\]

To define ‘regional’ CPV one needs some judgment with finite data, as used by the LHCb collaboration. One needs to disagree with them; one has to remember that scalar resonances (like \(f_0(500)/\sigma \& \kappa\)) produce broad ones that are not described by Breit-Wigner parametrization; instead they can be described by dispersion relations [12] (or other ways). At the qualitative level one should not be surprised. Probing the topologies of Dalitz plots with accuracy one might find the existence of ND. Most of the data come along the frontiers, while the centers are practically empty. Therefore interferences happen on few places, and regional asymmetries are much larger than averaged ones – but so much?

One looks at even more CKM suppressed three-body FS:

\[
\begin{align*}
\text{BR}(B^+ \to \pi^+\pi^-\pi^+) &= (1.52 \pm 0.14) \cdot 10^{-5}, \quad \text{BR}(B^+ \to \pi^+K^-K^+) &= (0.52 \pm 0.07) \cdot 10^{-5}
\end{align*}
\]

LHCb has shown these averaged and ‘regional’ CP asymmetries [18]:

\[
\begin{align*}
A_{CP}(B^\pm \to \pi^\pm\pi^+\pi^-) &= +0.117 \pm 0.021_{stat} \pm 0.009_{syst}[\pm 0.007_{\psi K^\pm}] \\
A_{CP}(B^\pm \to \pi^\pm K^+K^-) &= -0.141 \pm 0.040_{stat} \pm 0.015_{syst}[\pm 0.007_{\psi K^\pm}],
\end{align*}
\]

\[
\begin{align*}
\Delta A_{CP}(B^\pm \to \pi^\pm\pi^+\pi^-)|_{\text{regional}} &= +0.584 \pm 0.082_{stat} \pm 0.027_{syst}[\pm 0.007_{\psi K^\pm}] \\
\Delta A_{CP}(B^\pm \to \pi^\pm K^+K^-)|_{\text{regional}} &= -0.648 \pm 0.070_{stat} \pm 0.013_{syst}[\pm 0.007_{\psi K^\pm}].
\end{align*}
\]

Again it is not surprising that these asymmetries come with opposite signs. We need more data (those will appear ‘soon’), find other regional asymmetries and work on correlations with other FS. Importantly we need more thinking to understand what the data tell us about the underlying dynamics including the impact of non-perturbative QCD. It seems that the landscape is even more complex as said before and show the impact of really broad resonances. It is not surprising that the central part of Dalitz plots and that interferences happens mostly at the corner. It is surprising that the impact on regional asymmetries are so large. Still we need thinking – model insensitive analyses are not always an excellent idea. At least we will learn about non-perturbative strong forces.

### 3.9 T violation with & without EPR correlations

Once one has established CPV directly, one has found T violation indirectly with CPT invariance. However the situation is more subtle due to EPR correlation; actually it is a
‘blessing in disguise’. People are not fair fans of history prefer the name of ‘entanglement’

For a special situation one has a pair of neutral B mesons who are produced in single coherent quantum state with spin-one & C odd where their oscillations are highly correlated with each other as done at BaBar & Belle experiments: $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B_d\bar{B}_d$. This pair cannot transmogrify itself into a $B_dB_d$ or $\bar{B}_d\bar{B}_d$. To say it in different ways. Using the neutral mass eigenstates $B_1$ & $B_2$ one gets only $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B_1B_2$, but not $F_1$ with $B_1B_1$ or $B_2B_2$. The simplest and best measured asymmetry gives $\Upsilon(4S) \rightarrow (l^-X)_B(\psi K_S)_B$ vs. $\Upsilon(4S) \rightarrow (l^+\bar{X})_B(\psi K_S)_B$ in the asymmetry of $e^+e^-$ collisions. One can measure the differences in the semi-leptonic & non-leptonic decays. Those are depend on $\Delta t$, but also very consistent with $\sin[\Delta M_{B_d}\Delta t]$ as expected. However there is more information, namely $\Delta t = 0$ within the experimental uncertainties. One has assumed CPT only for semi-leptonic decays, not non-leptonic one, as pointed out last century, shown on the Fig.6. The landscape of CPT violation has been probed with more details in Ref.[19] – but still assumes perfect EPR correlations.

3.10 $D_{(s)}$ decays

3.10.1 Indirect CPV in $D^0 - \bar{D}^0$ oscillation

$D^0 - \bar{D}^0$ oscillations have been established with $x_D \equiv \frac{\Delta M_D}{\Gamma_D} = (0.39^{+0.17}_{-0.18})\%$ and $y_D \equiv \frac{\Delta \Gamma_D}{2\Gamma_D} = (0.65^{+0.07}_{-0.09})\%$ due also interferences with favoured & DCS ones. The situation is fuzzy whether the SM can produce total or only the leading source. So far no CPV has been found in SCS transitions, where the SM should contribute around on the level on $O(0.001)$. The impact of ND can be seen mostly in $x_D$ due to local operator; the situation about $y_D$ is much more complex [20].

3.10.2 Direct CPV in SCS decays

In the world of hadrons strong re-scattering connect first steps of $D \rightarrow \pi\pi\pi$ with $D \rightarrow \pi K K$ and back. For very good reasons one describes three-body FS with amplitudes with quasi-two body FS and their interferences; however scalar resonances often are described by broad ones where one cannot use Blatt-Wigner parametrization. Furthermore the Dalitz plots are not empty; therefore interferences happens at many locations. No averaged CPV has been found in $D^{\pm} \rightarrow \pi^{\pm}\pi^{\pm}\pi^{-}$ or $D^{\pm} \rightarrow \pi^{\pm}\pi^{\pm}\pi^{-}$ or real data about regional asymmetry in $D^{\pm} \rightarrow \pi^{\pm}\pi^{\pm}\pi^{-}$. The SM is expected to produce averaged values for SCS decays $O(0.001)$ and larger values for regional asymmetries. The questions are: how much, where and about the impact of CPT invariance on subclasses with only charged hadrons or not. One has to probe averaged & regional ones in $D^{\pm}_s \rightarrow K^{\pm}K^{\pm}K^{-}$, $K^{\pm}\pi^{\pm}\pi^{-}$ and to think about correlations with $D^{\pm}$ decays.

13‘Entanglement’ seems to push out ‘EPR correlations’ more and more recently in the literature; for me it is not only unfair, but worse by ignoring the history of quantum mechanics; furthermore it ignores to establish large CP asymmetries in $e^+e^- \rightarrow B_d\bar{B}_d$. 

22
Chiral symmetry is a very good tool for $3\pi$ FS; however the power of that is decreased for FS with $K\pi\pi$, $K\bar{K}\pi$ and $3K$. Again – how much?

3.10.3 Basically zero CP asymmetries in DCS decays

The refined parameterization of the CKM matrix gives basically zero direct CPV in DCS in $D^\pm \rightarrow K^{\pm}\pi^+\pi^- / K^{\pm}K^+K^-$ and in exotic decay $D_s^\pm \rightarrow K^{\pm}K^{\pm}\pi^\mp$. The first step is to establish averaged CPV in $D_s$ decays, then the second one is to probe regional ones. Again it needs some judgment to define regional asymmetries with finite data. While the rates are very small, there is no ‘background’ from the SM. Furthermore the ‘exotic’ $D_s$ decays should be more standing out due to $\Delta S = 2$ in the FS; at least they give us unusual lesson about QCD.
4 Intermezzo – EDMs

EDMs are discussed in other articles in this book in details. I just add a few comments:

- We have found large CPV flavour dynamics, but it has nothing to do with the truly huge observed asymmetry in known matter vs. anti-matter of baryons.
- No EDM has been found yet. On the other hand we might found in the future the dynamics that produced this asymmetry.
- QCD faces the challenge to solve the problem with basically zero contributions of the operator $G \cdot \tilde{G} \equiv i\epsilon_{\mu\alpha\beta}G_{\mu\nu}G^{\alpha\beta}$ and the gateway for traditional and ‘new’ axions.
- There might be a connection of known vs. dark matter.
- A very general statement: to understand fundamental dynamics needs a lot of time, new tools – and thinking & ideas.

Direct test of T invariance comes for single particle static transitions. The energy shift of a system due to external small electric field can be described in powers of $\vec{E}$:

$$\Delta E = \vec{d} \cdot \vec{E} + \mathcal{O}(|\vec{E}|^2) = d \vec{j} \cdot \vec{E} + \mathcal{O}(|\vec{E}|^2)$$

The linear vector $d_i$ is called the electric dipole moment (EDM). A non-zero value of $d_i$ show the violation both discrete P and T symmetries. The crucial point is not ‘elementar-ity’ at all, but ‘non-degeneracy’ of the impact of the dynamics. It is a well-known example to compare the neutron $d_N$ with water molecules or ‘dumb-bells’ based on classical forces.

In quantum field theory EDMs are described by an operator in the Lagrangian

$$\mathcal{L}_{\text{EDM}} = -\frac{i}{2} d \bar{\psi} \sigma_{\mu\nu} \gamma_5 \psi F^{\mu\nu}$$

with dimension five; therefore its dimensionful coefficient $d$ can be calculated as a finite quantity in general.

Based on quark diagrams for neutrons, deuterons, molecules and also for e, mu & tau, it is clear that in the SM one gets values that are clearly beyond what one reach. Therefore it is a rich landscape for the existence of ND and its features, if you are patient enough to make the efforts that are needed with thinking & ideas.

On the other hand the situation it is even more subtle: QCD can produce CPV in flavor independent transitions, namely EDMs in hadrons. Actually it was realized that QCD with vector bosons have a problem used ‘$U(1)_A$ problem’. Let us look at QCD with only one family with u & d quarks. With massless quarks – which is very close to $m_u, m_d \ll \bar{\lambda}$ – one might think that QCD possesses a global $U(2)_L \times U(2)_R$ symmetry. Indeed the vectorial component $U(2)_{L+R}$ is conserved even after QM corrections and axial $SU(2)_{L-R}$ also in subtle ways (to give masses to $W^\pm$ and $Z^0$). However about $U(1)_{L-R}$? Axial currents are conserved in the classical symmetry due to chiral invariance for massless
quarks; however they are not conserved called ‘quantum anomaly’ (or ‘triangle anomaly’) due to one-loop corrections with internal quarks:

$$\partial_{\mu}J_{5}^{\mu} = \frac{g_{S}^{2}}{32\pi^{2}}G \cdot \tilde{G}$$

(65)

The resolution of the $U(1)_{L-R}$ due to complex structure of the QCD ‘vacuum’ comes with a price, namely the ‘Strong CP Problem’. The $U(1)_{L-R}$ & Strong CP is actually intertwined, when one includes the weak dynamics

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}} + \frac{\tilde{\theta} g_{S}^{2}}{32\pi^{2}} G \cdot \tilde{G}$$

(66)

namely $\tilde{\theta} = \theta - \arg \det M$ describing the mixing matrix of $U=(t,c,u)$- and $D=(b,s,d)$-quarks. Photon can couple neutrons with internal virtual protons & pions. One of the two effective pion nucleon operators couple by ordinary QCD, while to other one are due $G \cdot \tilde{G}$. A guess tell us:

$$d_{N} \sim \mathcal{O}(10^{-16}\tilde{\theta}) \text{e cm}$$

(67)

The limit from data gives:

$$\tilde{\theta} < 10^{-10}$$

(68)

While it is possible or worse ‘accidentally’, but very un-natural.

4.1 Traditional & novel axion scenarios

Most members of our community agree that one needs organizing principle to produce the needed required cancellations. The best known tool is some kind of symmetry. A real intriguing ansatz is to assume that a physical quantity usually used as a constant is re-interpret as a dynamics degree of freedom. In this case it was suggested by Peccei & Quinn [21], namely to add the SM a global $U(1)_{PQ}$ as a Nambu-Goldstone boson which is axial with following properties:

- it is a classical symmetry;
- it is subject to an axial anomaly; and
- it is broken spontaneously as well and
- possesses a huge vacuum expectation value (VEV) $v_{PQ} >> v_{EW}$.

Previously we thought there are two classes, namely (A) ‘visible’ axion with $m_{a} \sim \mathcal{O}(1 \text{MeV})$, and (B) ‘invisible’ axion with $m_{a} \ll 1 \text{MeV}$. It seems there is no chance that class (A) axion can exist. ‘Invisible’ axion might be found using coupling of axion with two photons. The name ‘invisible’ is obvious, namely due to the tiny axion mass the

\footnote{Often our community is sloppy with the names understanding the connections; other examples below: KSVZ axion or DFSZ axion.}
lifetime is larger than the age of our universe, and couplings to other fields are so minute
that they would not betray their presence under ‘ordinary’ circumstances. The best tool
might be by conversion an axion into a photon in a strong magnetic field $B$ [22]:

$$\text{axion} \xrightarrow{B} \text{photon}$$

(69)

It is still probed in our present world (including solar system due to astrophysical indirect
information).

Later ‘old’ cosmology enters the ‘scene’: it gave lower bound on the mass:

$$m_a > 10^{-6} \text{ eV}$$

(70)

Then connections of dark matter suggest stronger bounds:

$$m_a > 2 \cdot 10^{-5} \text{ eV}$$

(71)

Does it mean that the ‘dawn’ of axions go the their ‘dusk’?

Maybe the landscape of axion dynamics is even more subtle; namely actually PQ
symmetry can be broken not only in QCD anomaly, but also in the UV region in many
ways (and ideas) due to connection with gravity, gravitational waves, string theoretical
realization of the QCD axion etc. etc. Axions produced in the very early universe, can
be part of the Dark Matter (and maybe also in Dark Energy) in the present universe
and can be tested experimentally and directly. For example, it was described with more
observables in the Refs. [23], [24], [25] about the PLANCK & BICEP2 data.

I am not convinced (yet) by some comments; however even if those projects will not
be realised, they show the active situation in fundamental physics, which is wonderful in
my view: a true ‘Renaissance’ from an excellent idea about the impact of symmetries.

5 Probing CP asymmetries in leptonic transitions

In the SM the landscape of CPV in hadrons and leptons are quite different. There
the charged leptons and neutrinos are elementary with no original CPV. This century
data showed us that neutrinos are not massless due to oscillations. Some of us think
that CPV in neutrino oscillations can show the road to understand the huge difference
between matter vs. antimatter. It also shows we need a very long time efforts to make
progress there.

The SM landscape of leptonic dynamics about CPV is not very complex with massless
neutrinos and $e$, $\mu$ and $\tau$ transitions. One can see it as not very interesting – or opposite,
since there is hardly SM background on the theoretical side.

5.1 $\tau$ Cabibbo suppressed decays

Present data about CPV in SCS $\tau$ decays $\tau^- \to \nu K_S(\pi...)^-\pi^-\pi^-$ show one can compare SM
prediction due to well-known $K^0 - \bar{K}^0$ oscillation with a difference of 2.9 sigma:

$$A_{\text{CP}}(\tau^+ \to \bar{\nu} K_S \pi^+) |_{\text{SM}} \approx 0.36 \pm 0.01\% \quad 26$$

$$A_{\text{CP}}(\tau^+ \to \bar{\nu} K_S \pi^+ [+\pi^0/s]) |_{\text{BaBar2012}} \approx -(0.36 \pm 0.23 \pm 0.11)\% \quad 27$$

(72)

(73)
one can note the sign. One can probe CPV decays like \( \tau^- \to \nu K^0 \), \( \nu K^- \pi^+ \pi^- \) etc. and think about correlations due to CPT. We have to probe CPV in several FS like \( \tau^- \to \nu K^- \pi^0 \), \( \nu K^- \pi^+ \pi^- \), \( \nu K_S \pi^+ \pi^- \).

Now available data probe only integrated CP asymmetries. It is important to probe regional CP asymmetries in \( \tau^- \to \nu[S = -1] \) FS; we have to wait for Belle II (and Super-Tau-Charm Factory if & when it exists). Furthermore one has to compare regional data from \( \tau^- \to \nu[S = 0] \) FS like \( \tau^- \to \nu \pi^- \pi^0 \), \( \nu \pi^- \eta \), \( \nu \pi^- \pi^+ \pi^- \), \( \nu \pi^- \pi^0 \pi^0 \) etc. with accuracy. It is a test of experimental uncertainties; it would be a miracle to show CPV there.

It is important (as pointed out two years ago) to measure the correlations with \( D^+ \to K^+ \pi^+ \pi^- / K^+ K^- \) etc. [28]. Furthermore we have to look for regional asymmetries and spin correlations in the pairs of \( \tau^+ \tau^- \), in particular with polarized \( e^+e^- \) beams if we can use them.

### 5.2 CPV in neutrino oscillations

PMNS matrix very different than CKM matrix already in qualitative ways. In the world of quarks and also charged leptons masses they follow the catholic hierarchy. The situation is quite different about neutrino masses and angles. Furthermore neutrinos might have be partly Majoran. In general the three angles of the PMNS matrix \[2\] differ sizably from zero.

It is a very long time project to probe CPV in neutrino oscillations, which are affected by the environment of very mostly baryons rather than anti-baryons.

### 6 Summary and about the future

Now we have entered the era where ‘accuracy’ has been changed into ‘precision’ with better tools including much better understanding of strong forces – and the possible connection with dark matter.

#### 6.1 Present situations in hadronic forces

Up to now CPV basically have been probed and measured in two-body FS in kaons and \( B \) mesons. It is crucial to probe multi-body FS in kaons, \( D(s) \) and \( B(s) \) – and in baryons anywhere & EDMs in nuclei & molecules. Furthermore we have to use CPT as a tool to connect informations with different FS and regional CPV.

#### 6.2 Future(s) in leptonic dynamics including EDMs

No CPV or TV has been found in leptonic dynamics with the small limits so far. However we have to continue with precision, not only by understanding the underlying dynamics, but have a chance to find the source of the huge asymmetry in baryons vs. anti-baryon. Finally we know that the SM is not enough due to dark matter existence &
neutrino oscillations. Therefore we have to probe \textbf{CPV} in neutrino oscillations, although it needs long time project.

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