The Initial Weight Estimation of New Aircraft Design Solutions

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Abstract. The work is devoted to the rapid assessment of the weight of the aircraft and its design. This problem occurs when creating new aircraft or in the process of constructive improvement of existing ones. To determine the mass of the entire aircraft, the method of "mass change coefficients" (growth coefficients) is used, which is based on the equation of the existence of the aircraft. To analyze the mass of initial design changes, the Komarov weight formula based on the concept of "load-carrying factor" is used. The article considers a numerical example of estimating changes in the mass of an aircraft due to the use of composite materials in the fuselage structure.

1. Introduction

Flight of any flight vehicle (FV) is the process of countering the gravity force with aerostatic, aerodynamic or rocket-dynamic ones. Therefore, for any FV its weight (mass) is a paramount parameter. The mass evaluation of FV from the very beginning of the project, during its consistent refinement in the design process and constant monitoring throughout the life cycle of the FV is the most important task. In the struggle to reduce the weight of the FV, their creators are looking for new technologies that are associated with both design and production.

For the design evaluation of the mass of FV and its aggregates, semi-empirical weight formulas are usually used [1-3].

The proposed work considers the problem of rapid assessment of the total change in the mass of an existing aircraft a prepared project due to a possible local change in the mass of its individual components, including its design due to new design solutions, for example, as a result of the use of a new material. In this case, there are generally three scenarios for the implementation of such changes:

– the target return remains unchanged, for which the weight of the airframe structure, engines and fuel are changed accordingly;
– the target load, power plant, fuel mass and geometric dimensions of the aircraft do not change, but due to changes in the mass of the structure, the take-off weight and flight properties of the aircraft change, which means that the target return changes;
– the change in the mass of the structure is compensated only by the change in the mass of the target load.

In this paper we are talking about the first scenario.

In the struggle to reduce the weight of FV, their creators are always striving for new designs and technologies. The last major achievements in this direction are related to the use of composite materials
(CM). In the design of the most modern mainline aircraft (Boing 787, Airbus A350), about 50% is accounted for by CM. This allows you to reduce the cost of a mile-flight by up to 15% and consequently reduce operating costs. However, the proportion of CM in the fuselages of these aircraft is significantly less than in the wing. In this regard, the study of the use of CM in the fuselage is certainly an urgent and important topic.

2. Initial estimation of the mass of FV

The aircraft is considered as a system having a certain hierarchical structure and its mass is represented as the sum of the masses of the following five parts: target load \( m_e \), airframe structure \( m_{str} \), power plant \( m_f \), fuel system \( m_c \), equipment and control system \( m_{eq} \)

\[
m = m_e + m_{str} + m_c + m_f + m_{eq} = \sum m_i.
\]  

Goal load is a subsystem whose elements are determined by the purpose of the aircraft: commercial (paid) or combat load; crew, equipment; equipment of passenger and baggage-cargo facilities, weapons.

The airframe design is a subsystem, the elements of which are determined by the implementation of the corresponding principle of flight in terms of reliable perception of operating loads. For aircraft, this wing, tail, fuselage and landing gear.

Power plant is a subsystem whose elements provide the creation of the required thrust: engines and devices for their installation.

Fuel system is a subsystem whose elements ensure the operation of engines during a given flight time on the provided flight modes: fuel and devices for its placement and pumping.

The equipment and control system ensures reliable operation of the aircraft its control during the flight.

Representation of the mass of the aircraft in the form of (1) is in accordance with the functional isolation of the five subsystems listed, each of which has its own specifics of mass formation. The target load is usually specified or determined independently of the total mass. The other subsystems depend on \( m \), and the dependencies \( m_{str}(m) \), \( m_c(m) \) are quite close to directly proportional. Dividing equation (1) by the mass of the aircraft yields an equation in the form of relative masses \( \bar{m}_i = m_i/m \)

\[
1 = \sum \bar{m}_i = \bar{m}_e + \bar{m}_{str} + \bar{m}_{en} + \bar{m}_t + \bar{m}_{eq}.
\]  

Equation (2) determines the balance of relative masses and the conditions of existence FV in terms of weight. With the help of equation (2) and a given mass of the target load \( m_e \) we can find the mass of the whole FV

\[
m = m_e / (1 - \sum \bar{m}_i).
\]  

To determine the mass in the first approximation, statistical data are usually used with a refined estimate of the relative mass of the fuel, based on a given range and speed, aerodynamic quality and specific fuel consumption (Breguet formula, adjusted for non-cruising modes). According to statistics, the relative masses of subsonic passenger aircraft are: \( \bar{m}_{str} = 0.25-0.32; \bar{m}_{en} = 0.08-0.14; \bar{m}_t = 0.18-0.4; \bar{m}_{eq} = 0.09-0.12. \)

3. Estimation of initial mass changes by the total mass of the aircraft

Let as a result of some changes there was a small initial change of mass \( \Delta m_{t0} \), then from (1) follows

\[
\Delta m_{t0} = m - (m_e + m_{str} + m_{en} + m_t + m_{eq}).
\]  

According to the chosen scenario, we must preserve the mass and layout of the target load, which means that the geometry of the fuselage of the passenger aircraft, which houses the target load, remains unchanged. To fulfill this condition, the mass components related to \( m_{en} \) and \( m_t \) are written in the form suggested in [4]

\[
m_{en} + m_t = (m_{en} + m_t) C_{D_{fus}} / C_D + (m_{en} + m_t) (1 - C_{D_{fus}} / C_D)
\]  

where \( C_{D_{fus}} \), \( C_D \) are the drag coefficients of the fuselage and the entire aircraft.
The first term of the right part (5) determines the outlay of thrust and fuel for the transportation of the fuselage of the original dimensions in cruising flight and when we have $\Delta m_0$, they will not change. Taking into account this remark $\Delta m_0$ can be written as follows (index old means the unchanged value)

$$\Delta m_0 = m - m_{\text{str}} + \Delta m_{\text{m}} + \Delta m_{\text{p}} m_{\text{old}} + (m_{\text{m}} + m_{\text{p}}) (C_{\text{ fus}} / C_{\text{ fus}} m_{\text{old}}) + (m_{m} + m_{p}) (1 - C_{\text{ fus}} / C_{\text{ fus}}) m_{\text{old}} m.$$  

(6)

We take the partial derivative of (6) by mass

$$\frac{\partial (\Delta m_0)}{\partial m} = 1 - \frac{m_{\text{str}} - \Delta m_{\text{m}} - (m_{\text{m}} + m_{\text{p}}) (1 - C_{\text{ fus}} / C_{\text{ fus}}) m}{m_{\text{m}} m_{\text{old}} m}.$$  

(7)

From the expression (7) we can find the inverse derivative, which is called the mass change factor (MCF) of the aircraft

$$\mu_m = \frac{\partial (m)}{\partial \Delta m_0} = \frac{1}{1 - \frac{m_{\text{str}} + \Delta m_{\text{m}} + (m_{\text{m}} + m_{\text{p}}) (1 - C_{\text{ fus}} / C_{\text{ fus}})}{m_{\text{m}} m_{\text{old}} m}}.$$  

(8)

MCF shows how many times the total mass change of the aircraft greater than the partial change $\Delta m_0$. If to pass to final changes (Figure 1), then at initial small change of weight on $\Delta m_0$

$$\Delta m = \mu_m \Delta m_0.$$  

(9)

Taking into account (2), the MCF can be presented in the form

$$\mu_m = 1 / \left[ \frac{m_0 - \Delta m_{\text{m}} + m_{\text{p}} + (m_{\text{m}} + m_{\text{p}}) C_{\text{ fus}} / C_{\text{ fus}}} {m_{\text{m}} m_{\text{old}} m} \right].$$  

(10)

### 4. Assessment of initial weight changes as a result of constructive measures

#### 4.1. Basics of a new weight estimation method

To calculate the initial mass change, we use the approach proposed by V. A. Komarov [5], which is based on the so-called integral criterion "load-carrying factor" (G-factor), which is generally written as

$$G = \int V \sigma^{\text{Eqv}} dV.$$  

(11)

where $\sigma^{\text{Eqv}}$ is equivalent stress acting in the structure, $V$ is the volume of the material of the load-carrying structure. The discretization of the design, in particular in the FEM variant, gives us

$$G = \sum V i \sigma_{i}^{\text{Eqv}} V_i.$$  

(12)

where $\sigma_{i}^{\text{Eqv}}, V_{i}$ are the equivalent stresses and the volume of the $i^{th}$ element in which $\sigma_{i}^{\text{Eqv}}$ is assumed constant.

When using the same type of material with specific strength $\bar{\sigma} = \sigma_e / \rho$, and in the case of reaching the maximum permissible stresses in each element $\sigma_{p}$, the mass of the structure can be estimated through the G-factor

$$m = \varphi G / \bar{\sigma}.$$  

(13)

The coefficient $\varphi$ in this expression takes into account various really existing additional mass gains (joints and non-force elements in the structure, deviations from the optimal distribution of material in favor of simplicity and manufacturability of structures, and much more), which are usually not taken into account in the design evaluation or which is difficult to take into account in initial design steps. So if the coefficient $\varphi = 1$, we have a so-called theoretical mass $m^{\text{Theor}}$.

As a result, all factors affecting the load-carrying mass are divided into three fairly independent groups [6]:

- level of constructive and technological perfection and target filling of a design ($\varphi$);
- design and power circuit and operating loads ($G$);
- physical and mechanical properties of the material ($\bar{\sigma}$).

For convenience, the parameter G can be represented as three cofactors: the dimensionless coefficient $C_{K}$, the characteristic load $P$ and the characteristic dimensional parameter $L$. 

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The image contains a document discussing the assessment of initial weight changes as a result of constructive measures, including the calculation of mass change factors and the use of integral criteria. The text is formatted with mathematical equations and expressions. The context involves the evaluation of material changes in structures, focusing on the calculation of mass change factors and the use of integral criteria for weight estimation. The document integrates various mathematical formulations and practical considerations in the field of structural engineering.
$G = C_K P L$  \hspace{1cm} (14)

As a result the final form of the weight formula of V. A. Komarov will have the form

$m = \varphi C_K P L / \bar{\sigma}$  \hspace{1cm} (15)

One of the main advantages of this approach is that the $C_K$ coefficient in the design process can serve as a dimensionless criterion of the weight perfection of structures, and its reduction will contribute to the reduction of mass.

Figure 2 shows the simplest example to explain this property: a symmetric two-rod truss loaded in the first case (solid line) by a horizontal force $P_1$, in the second – by a vertical force $P_2$ (dashed line); and the values of the coefficients $C_K$ for these cases as a function of the angle $\alpha$. The minimum values of $C_K$, and accordingly the minimum mass, will be achieved in the first case of loading at $\alpha=0^\circ$ ($C_K=1$) and $\alpha=45^\circ$ in the second ($C_K=2$).

**5. Example on the weight estimation of fuselage**

To combine approaches using MCF and G-factor, consider the problem of approximate design evaluation of the change in weight of the aircraft as a result of the replacement of the skin and longitudinal fuselage set from aluminum alloy to CM. The fuselage structure is represented as a cylindrical shell with length $l_{fus}$, width $b_{fus}$, and height $h_{fus}$; with a longitudinal-transverse set and a skin. The fuselage is reduced to a structurally axisymmetric with a diameter $D=(b_{fus}+h_{fus})/2=2R$. Figure 3 shows the plots of typical loads acting on the fuselage of a passenger aircraft: generalized axial $N_x$ and shear $Q_y$ forces, bending $M_z$ and torque $M_x$ moments, as well as excessive internal pressure $p$. By this the fuselage is considered fixed by the cross section of the location of the main wing attachment points and is divided into two cantilever compartments: the front with a length of $l_{fus,f}$ and back – $l_{fus,b}$. In the design evaluation we will compare the mass ratio of the metal variant with the composite one only by two main load factors which need the main part of the load-carrying mass: excessive internal pressure and bending moment (they are highlighted by contour lines).

**5.1. Metal fuselage.**

For design analysis, we use the thin-walled shell with the generalized thickness $\delta$, includes thickness of really skin and distributed area of the longitudinal set. The excessive internal pressure generates the
circumferential $\sigma_{\theta} = p R/\delta$ and meridional linear stresses $\sigma_{\alpha} = p R/(2\delta)$. By Mises the expression for equivalent stresses by the momentless theory by these stresses will be

$$\sigma_{\text{equiv}} = (\sigma_{\alpha}^2 + \sigma_{\theta}^2 - \sigma_{\alpha}\sigma_{\theta})^{1/2} = \frac{3}{2} p R/\delta \leq \sigma_{\text{equiv}}.$$  \hfill (16)

The theoretical mass ($\varphi = 1$) of such shell by its nominal thickness will be

$$m_p = \pi R^2 l \rho \frac{3\pi}{2} p V/\sigma_{\text{equiv}} = \frac{3}{2} p V/\sigma$$  \hfill (17)

where $V$ is the shell volume.

Comparing the expressions (17) and (15) it is obvious that if we take as a characteristic load overpressure $P = p$, and the characteristic size of the volume of the fuselage pressurized compartment $L = V$, and assuming $\varphi = 1$, than the coefficient of force factor $C_K = \sqrt{3}$.

The second main load is the bending moment. Consider the front part of the fuselage. From its action in the cross section of the fuselage there are normal stresses $\sigma = M \cos \beta/\delta$, where $\beta$ – the angle between the design point in cross-section and the $z$ axis, $I$ is moment of inertia, for a thin-walled circular ring – $I = \pi R^4/2$. From these relations, the reduced thickness of the shell is determined for the perception of bending stresses $\delta(\beta) = M \cos \beta/(\pi R^2 \sigma_{\text{equiv}})$. Bend moment taking into account its variability, is represented as $M = M_{\text{max}} F(x)$, where $M_{\text{max}}$ is the maximum moment value, $F(x)$ is the function that determines a moment distribution on the fuselage. $x$ is the relative coordinate, for nose part it is $x = x/l_{\text{fus,f}}$. Then the mass of the construction for perception of $M_i$ is

$$m_{M_i} = 2 \int_{x=0}^{1} \int_{\beta=-\pi}^{\pi} M \cos \beta/(\pi R^2) \sin \beta/\delta_\text{met} = 8\pi \int_{x=0}^{1} F_{M_i} \sin \beta/\delta_\text{met} = C_K \frac{P L}{\delta_\text{met}}$$  \hfill (18)

Here $\int_{x=0}^{1} F_{M_i} \sin \beta/\delta_\text{met} = 1/3$ since the bending moment is characterized by a parabolic dependence of the change in the length of the cantilever half of the fuselage; $P = M_{\text{max}}$; $L = l_{\text{fus,f}} = l_{\text{fus,f}}$ $/D$; $C_K = 8/(3\pi)$.

We can do the similar reasonings for the tail part of the fuselage.

5.2. Fuselage from CM.

The design of the fuselage from CM has a lot of specifics [7-10]. The composite fuselage consists from unidirectional structures (the longitudinal ribs: stringers, shelves of longitudinal beams, and shelves of frames), and a skin and walls which are given a laminate structure with the optimization of the directions of reinforcing elements (fibers, threads, tapes).

For a CM structure with annular layers of total thickness $\delta_R$ and spiral layers of thickness $\delta_S$ with angles $\pm \varphi$, we calculate the mass of the equal-strength structure. In this case, the layered structure is considered an equivalent homogeneous shell. Internal longitudinal and annular forces flows are related to the stresses in the spiral and annular layers by the ratios $N_{\alpha} = \sigma_{1S} \delta_S \cos^2 \varphi$, $N_{\beta} = \sigma_{1S} \delta_S \sin^2 \varphi + \sigma_{1R} \delta_R$, hence

$$\sigma_{1S} = p R/(2 \delta_S \cos^2 \varphi), \quad \sigma_{1R} = p R/(2 \delta_R)(2 - \tan^2 \varphi).$$  \hfill (19)

If the allowable stresses reach in all layers $\sigma_{1S}^2 + \sigma_{1R} = \sigma_{f,\text{p}}$, then the thickness of the fiber package at which the mass will be minimal

$$\delta_f = \delta_S + \delta_R = p R/(2 \sigma_{\text{f,p}}) = \delta_S(1 - f)/f.$$  \hfill (20)

At the same time, in the general package of CM of thickness $\delta_{CM}$, it is necessary to take into account the presence of a matrix, which according to the accepted model does not participate in the work. If $f$ is the volume content of fibers in CM $f = \delta_f/\delta_{CM}$, then the thickness of the matrix will be $\delta_m = \delta_f(1 - f)/f$.

The mass of the cylindrical shell from CM, which works only on the internal overpressure will be

$$m_p = 2 \pi R l (\rho_f \delta_f + \rho_m \delta_m) = 3 \rho V/(\sigma_{\text{CM}}) \left[1 + \rho_m (1 - f)/(\rho_f f)\right] = 3 \rho V/(\sigma_{\text{CM}})$$  \hfill (21)

where $\rho_m$, $\rho_f$ are density of matrix and fibers; $\sigma_{\text{CM}}$ is the specific strength of CM.
From (21) it follows that regardless of the angles of laying layers coefficient of force factor \( C_F = 3 \).
Analyzing the work of the cylindrical shell from CM from the bend and assuming that the reinforcing fibers are directed along the longitudinal \( x \) axis of the fuselage, we obtain a similar formula like (18), but with the corresponding specific strength \( \sigma_{CM} \).

6. Numerical example
Let’s make an approximate estimate of the weight of the fuselage structure using a traditional metal structure (for testing the technique) and compare it with what will give the transition to a composite one.

We do this on the example of a conventional medium-haul aircraft similar in characteristics to the Airbus A-320 aircraft and the new Russian MC-21 being developed, for which we will take the following values of the parameters necessary for calculating: \( m=80 \) t; \( l_{fus} = 40 \) m; \( D_{fus} = 4 \) m; \( T=240 \) kN; \( m_{eq}=0.25; \) \( m_{eq}=0.3; \) \( m_{eq}=0.1; \) \( m_{eq}=0.1; \) \( m_{eq}=0.25 \); the ratio of the aerodynamic drag coefficients is \( C_D_{fus}/C_D = 0.3 \). From \( m_{eq}=0.3 \) the third part is accounted for the design of the fuselage \( – m_{fus} < 80 \) t. 0.1 \( = 8 \) t. The maximum operational load factor is assumed \( n=2.5 \). Permissible stresses must be set taking into account the resource (100,000 flight hours), flutter, survivability, lack of loss of stability, residual strength after corrosion: specific strength for aluminum alloy elements \( \sigma_{net} = 0.3 \) GPa/2700 kg/m\(^3\); for carbon composites with \( \sigma_{net} = 2 \) GPa, with \( J = 0.5 \) and a polymer matrix \( \sigma_{CM} = 0.7 \) GPa/1500 kg/m\(^3\). The minimum safety factor is 1.5. Additional values of the safety coefficient: for compartments working under pressure – 3.5; for structures made of CM – 1.5. To simplify the problem, we also assume that the plane of attachment of the front and rear fuselage is located in the middle \((l_{fus,1}=l_{fus,2}=l_{fus}/2=20 \) m). We obtained theoretical load-carrying mass from two types of loads \( m_{str}=3.5 \) t and \( m_{CM}=2 \) t.
Performing the analysis for the remaining loads, as we did in sections 5.1 and 5.2, gives a small clarification of the theoretical mass of the load-carrying elements: \( m_{str}=4.2 \) t and \( m_{CM}=2.4 \) t.

We will assume that not bearing mass of the fuselage, which for the metal version was \( m_{fus}=m_{fus,\text{Theor}} = 8 \) t – 4.2 t = 3.8 t, and that it has not changed in the case of the composite version. Then we have \( \Delta m_{str} = m_{fus,\text{str}} – m_{fus,\text{CM}} \approx 1.8 \) t. The paper [11] contained an assessment of such a transition from aluminum load-carrying elements to carbon-composite ones. A reduction in the weight of the fuselage was by 2.5 t, but this value significantly depends on the amount of permissible stresses, which were not resulted.

Let’s analyze the decrease in the total weight of the aircraft with these parameters due to the transition from a metal fuselage to a composite one. According (10) we have \( \mu_{str,str}=2.2 \), then
\[
\Delta m = \mu_{str,str} \Delta m_{str,0} \approx 3.5 \text{ t.} \tag{22}
\]

7. Conclusion
The considered method of combining two approaches, which are based on the application of the mass change factor and the load-carrying factor, allowed to develop a method of operative\[ design evaluation of the aircraft mass due to the initial change in the mass of any elements while maintaining the flight performance properties of the aircraft.

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