Nonlocal transport mediated by nonlocal Hikami boxes: Condensation of evanescent quasiparticles injected into the superconducting gap

S. Duhot and R. Mélin

Institut NEEL, CNRS & Université Joseph Fourier, BP 166, F-38042 Grenoble Cedex 9

Evanescent quasiparticles entering a superconductor and propagating over distances larger than the coherence length give rise to intermediate states induced by double Andreev scattering on disorder. The resulting effective attractive interaction between evanescent quasiparticles is retarded at the extremely slow frequency of the applied bias voltage. The out-of-equilibrium mesoscopic superconductor with a fluctuating phase variable is compatible with a recent experiment [S. Russo et al., Phys. Rev. Lett. 95, 027002 (2005)]. Microscopic theory is discussed in the random phase approximation.

PACS numbers: 74.45.+c,74.78.Na,74.78.Fk

I. INTRODUCTION

The developments in nanotechnology allow to envision the realization of a source of entangled pairs of electrons, the electronic counterpart of a source of Einstein, Podolsky Rosen pairs of photons, with possible applications to quantum information and to a test of non locality of quantum mechanics with electrons. Recent experiments on three terminal devices by Beckmann et al., Russo et al. and more recently by Cadden-Zimansky and Chandrasekhar realize an important step in this direction by probing crossed Andreev reflection, in which Cooper pairs from a superconductor give rise to pairs of correlated electrons in two different ferromagnetic or normal electrodes.

Andreev reflection, the mechanism by which an electron from a normal electrode is reflected as a hole at a normal metal-superconductor (NS) interface while a pair is transmitted into the superconductor, takes place in a region of size ξ, the coherence length of the disordered superconductor. Crossed Andreev reflection (CAR) in a three terminal device normal metal-insulator-superconductor-normal metal (N_a,ISIN_b) trilayer corresponds to an Andreev process such that an electron from N_b is transmitted as a hole in N_a over a distance of order ξ, leaving a pair in the superconductor. Elastic cotunneling (EC), the other competing channel, amounts to transporting an electron from N_b to N_a without changing its spin. The two possibilities are shown schematically on Figs. 1 and 2. The nature of the dominant crossed transport channel can be controlled by the relative spin orientation of strongly polarized ferromagnets in a ferromagnet-insulator-superconductor-insulator-ferromagnet (F_aISIF_b) structure. EC (CAR) dominates with (anti)parallel spin orientations, as it can be seen from the spin of the electron or hole transmitted in electrode “a” (see Fig. 1). On the other hand, EC dominates for normal metals with highly transparent interfaces. Crossed transport dominated either by EC or by CAR is obtained in the three recent experiments mentioned above.

More precisely, the crossed conductance of a three terminal device measures the sensitivity of the current \( I_a(V_a, V_b) \) flowing through electrode “a” with voltage \( V_a \) to the voltage \( V_b \) applied on electrode “b”:

\[
\mathcal{G}_{a,b}(V_a, V_b) = \frac{\partial I_a(V_a, V_b)}{\partial V_b} \tag{1}
\]

(see the circuit on Fig. 3a). The voltage \( V_a \) may be arbitrary but, according to available experiments, we consider that electrode “a” is grounded (\( V_a = 0 \)), as the superconductor.

Theoretically, the crossed conductance defined by Eq. (1) vanishes to lowest order in the tunnel amplitudes for a N_a,ISIN_b trilayer because the electron and hole transmitted by EC and CAR have opposite charges, and because of a symmetry of crossed transport: CAR and EC have identical transmission coefficients (see Fig. 2). This property does not hold for a single channel.
II. QUANTUM INTERFERENCE EFFECT IN SUPERCONDUCTORS

A. Transmission modes related to evanescent waves

Weak localization enhances the return probability and the phase fluctuations of a normal metal\(^{24}\). A superconducting condensate is on the contrary delocalized and has a well defined phase. Superconductivity and weak localization are related to each other because the Cooper pairs of a superconductor can participate to the Cooperons of weak localization. Smith and Ambegaokar show indeed that the phase stiffness of a superconductor is reduced by weak localization\(^{25}\). Weak localization is however strongly modified in subgap transport through a superconductor by the following property of evanescent wave functions.

Electron tunneling through a normal metal involves the simultaneous forward propagation in time of an electron and the backward propagation in time of a hole, forming a diffuson. The terminology “transmission mode” is used here for describing the effect of disorder on subgap tunneling. For evanescent states, a hole propagating backward in time can be replaced in a transmission mode by an electron propagating forward in time: the wave-function of a normal electron in a box of size \(\xi\) being real valued, is equal to the time reversed particle wave-function in the absence of a magnetic field. Transmission modes of range \(\xi\) in subgap transport through a superconductor can thus be made also either of a pair of electrons or of a pair of holes propagating forward in time\(^{26}\). In a superconductor, a pair of electrons propagating forward in time can be obtained from a Cooper pair in the condensate.

B. Electron-hole conversion in transmission modes

As discussed in the preceding section, transmission modes in a disordered superconductor correspond to two quasiparticles scattering on the same sequence of impurities. Transmission modes are described in a standard way on the basis of ladder diagrams (see for instance Smith and Ambegaokar\(^{27}\) in the context of localization in a superconductor). As a direct consequence of Ref. 25, intermediate states with electron-hole conversion from one impurity to the next such as on Fig. 3b appear as the result of disorder scattering, as opposed to usual Andreev reflection at a normal metal-superconductor interface\(^{27}\) being induced by spatial variations of the superconducting pair potential. Intermediate states with electron-hole conversion in the disordered case are of course not due to anomalous electron-hole scattering on impurities, but to electron-hole conversion during propagation in between two impurities.

For instance, transmission of an electron from electrode \(N_b\) as an electron in electrode \(N_a\), and transmission of an electron in electrode \(N_b\) as a hole in electrode \(N_a\).
are shown on Figs. 3a and b respectively. The extreme case of a sequence of electron-hole conversions due to disorder, resulting in net transmission from electrode Na to electrode Nb in the electron-electron channel is shown on Fig. 3b. Another case of net transmission of an electron (a) and (c)) and for crossed Andreev reflection with transmission of a hole ((b) and (d)). “↑” corresponds to a spin-up electron and “↓” to a hole in the spin-down band. For clarity, the ↑ and ↓ symbols of electrons and hole are not shown for the diffuson.

C. Self-crossings of transmission modes

Self-crossings of transmission modes\(^{28,29}\) are the building blocks of weak localization in the normal case\(^{24,30}\) (see Fig. 4a). The distinguishing features of diffusion self-crossings in subgap transport by evanescent wave functions (see Figs. 1b and 5b) were already pointed out in Ref. 17. The necessity of accounting for “advanced-advanced” or “retarded-retarded” transmission modes was already discussed by Altland and Zirnbauer\(^{26}\) for an Andreev quantum dot. Weak localization-like loops such as on Figs. 4b and 5b contain, among all possibilities, intermediate states corresponding to double Andreev processes in which an evanescent electron-like quasiparticle (an AR transmission mode made of an advanced and a retarded Green’s function for an electron propagating forward in time and a hole propagating backward in time) transforms by a first Andreev process at \(x_1\) in an evanescent hole-like quasiparticle (another AR transmission mode for a hole propagating forward in time and an electron propagating backward in time) and the transmission of a pair (an AA transmission mode made of two advanced Green’s function for a pair of electrons propagating forward in time), that recombine at \(x_2\) by another Andreev process after a propagation over \(|x_2 - x_1| \sim \xi^2\). The intervention of the condensate in such loop processes is obvious from noting that the contribution of weak localization-like processes to the crossed conductance couples to a phase gradient in the superconductor\(^{21}\). These processes result in an attractive interaction of strength \(v\) per carrier injected in the gap, in units of the Fermi energy \(\epsilon_F\). This interaction due to quantum interference effects has all the features of a very slow phonon exchange: it is attractive, and retarded by a time \(2\pi/\epsilon V_b\) (because weak localization-like processes induces transitions only at energy \(\hbar \omega = \epsilon V_b\)).

As discussed previously, a transmission mode in the ladder approximation induces electron-hole conversion due to scattering on disorder, having the elastic scattering time as a characteristic time/inverse energy scale. Weak localization-like loops such as on Fig. 4b correspond to the transport of pairs over a distance comparable to the coherence length \(\xi\), and have thus the superconducting gap \(\Delta\) as a characteristic energy\(^{15}\). Similar double Andreev processes were discussed in Ref. 17 in a NISIN three-terminal device, and, interestingly, were introduced previously by Jacobs and Kümmer\(^{32}\) for the time evolution of wave-packets. As we show now, even smaller energy scales are obtained from many interact-
III. SLOW ATTRACTIVE INTERACTION IN A SUPERCONDUCTOR

A. Energy scale within the gap

Changing the phase by $\pm 2\pi$ during a time interval $2\pi/\omega_c$ requires the time $2\pi/\omega_c$ to be smaller than $2\hbar/eV_b$, the retardation in the interactions that prevent the phase from fluctuating. The BCS case with a phase constant in time corresponds to retarded interactions with a Debye energy $\hbar\omega_D$, orders of magnitude larger than the superconducting gap $\Delta$. In the opposite limit $eV_b < \hbar\omega_c$, that we consider, the interaction $v$ has to visit and to fix the phase of a number of state $N(0)/\hbar\omega_c$ in the normal metal to rotate the phase by $2\pi$ within a delay $2\pi/\omega_c$, with $N(0) = 2\pi L_\perp \frac{\hbar}{eV_b}/\lambda_F^2\epsilon_F$ the number of normal states per unit energy, and with $\epsilon_F$ the Fermi energy and $\lambda_F$ the Fermi wave-length (see $L_x$, $L_y$ and $L_z$ on Fig. 5). By the uncertainty relation $\epsilon_F \Delta t = \hbar$, the effective number of interactions in a time interval $2\pi/\omega_c$ is $N_{int}(\omega_c) = 2\pi/\omega_c \Delta t = 2\pi \epsilon_F/\hbar\omega_c$. The characteristic energy $\hbar\omega_c$ is obtained by equating $N_{int}(\omega_c) = N(0)\omega_c$:

$$\frac{\hbar\omega_c}{\epsilon_F} = \sqrt{\frac{e\lambda_F^2}{2\hbar L_x L_y}}.$$  \hspace{1cm} (2)

The decoupling between $\hbar\omega_c$ and the states at energy larger than $\Delta$ is justified if $\hbar\omega_c < \Delta$, which we suppose in the following. The superconducting phase fluctuates by $\pm 2\pi$ with a frequency $\omega_c$, but its average value over a time window larger than the retardation of interactions $2\hbar/eV_b$ does not fluctuate. The Andreev current from the superconductor to electrode $N_a$ vanishes for $eV_b < \hbar\omega_c$ because the superconductor chemical potential fluctuates between typical values $\pm \hbar\omega_c$. The crossed conductance is then dominated by EC, as it can be seen from second order perturbation theory in the tunnel amplitudes (33) (see Fig. 7).

B. Qualitative behavior of the crossed conductance

We consider now the case $eV_b \gtrsim \hbar\omega_c$. The proximity of the characteristic energy $\hbar\omega_c$ induces pair correlations among evanescent quasiparticles injected in the superconductor at bias voltage energies $eV_b \gtrsim \hbar\omega_c$. CAR and EC processes are described schematically (33): (b), (c) and (d) in the insert of Fig. 5 correspond to acting twice with the tunnel Hamiltonian, starting from the superconductor in the BCS ground state for (b) and with a correlated pair in the initial state for (c) and (d). The crossed signals due to the EC and CAR processes (c) and (d) cancel with each other because (c) involves an extra anticommutation of fermions compared to (d). The EC process (c) cancels with a CAR process (not shown on Fig. 5) in which two electrons at energies $-eV_b$ and $eV_b$ in electrodes $N_a$ and $N_b$ enter the superconductor as a pair. Fluctuations by one quasiparticle with spin-$\sigma$ in the superconductor do not couple to the crossed signal because EC has an opposite sign for the two orientations of $\sigma$. The remaining contribution to the crossed signal (not shown on Fig. 5) is due to tunneling of the two electrons of a correlated pair from the superconductor to electrode $N_a$, leading to a negative differential crossed resistance, as for local Andreev reflection above the superconducting gap in a NIS junction. From these arguments, we conclude that a change of sign from a positive EC crossed resistance for $eV_b < \hbar\omega_c$ to a negative Andreev reflection crossed resistance for $eV_b \gtrsim \hbar\omega_c$ occurs, and that the crossed signal disappears at higher energies, as in experiments.
FIG. 6: (Color online.) Schematic representation the link between Secs. III and IV. A characteristic energy scale $\hbar \omega_c$ is obtained from symmetry breaking between elastic cotunneling and crossed Andreev reflection ($\epsilon \sim 1$) in Eq. (3) and (4). The energy scale $\hbar \omega_c$ is identified to the characteristic frequency for phase fluctuations in the presence of a very slow attractive interaction, which, in turns, favors a symmetry breaking in favor of elastic cotunneling.

FIG. 7: (Color online.) Schematic representation the lowest order processes for superconductor chemical potentials $\hbar \omega_c$ larger than $eV_b$ ((a) and (b)), or smaller than $eV_b$ ((c) and (d)). The crossed Andreev reflection (CAR) processes (a) and (c) lead to opposite crossed currents for superconductor chemical potentials $\pm \hbar \omega_c$ while the elastic cotunneling (EC) processes (b) and (d) result in additive crossed currents. The crossed conductance with a superconductor chemical potential fluctuating between $\pm \hbar \omega_c$ is thus dominated by EC.

IV. RANDOM PHASE APPROXIMATION FOR WEAK LOCALIZATION-LIKE PROCESSES

A. Method and algorithm

1. Crossed conductance and resistance

Now, we start from a static superconducting gap (see Fig. 6) and determine from microscopic theory the analog of Eq. (2) for a dirty superconductor. We denote by $T_{CAR}(R, \omega)$ and $T_{EC}(R, \omega)$ the CAR and EC dimensionless transmission coefficients of a superconductor at distance $R$ and energy $\hbar \omega$ below the gap corresponding respectively to electron and hole transmission (see Figs. 1 and 2) described by transmission modes in the ladder approximation such as on Fig. 3.

We enforce a finite crossed current by breaking the symmetry $T_{EC}(R, \omega) = T_{CAR}(R, \omega)$ according to

$$T_{EC}(R, \omega) = T_0(R, \omega)(1 + \epsilon) \quad (3)$$

$$T_{CAR}(R, \omega) = T_0(R, \omega)(1 - \epsilon). \quad (4)$$

The crossed conductance per conduction channel

$$G_{a,b}(R, \omega) = \frac{e^2}{h} \left[ T_{2} T_{ch}(R, \omega) \right]$$

generalizing Eq. (1), where the entries $a_i$ and $a_j$ in Eq. (7) correspond to the normal electrode labels “a” and “b”. We deduce from Eq. (6) the off-diagonal matrix element of the crossed resistance matrix:

$$R_{a,b} = \frac{G_{a,b}}{G_{a,a} G_{b,b} - G_{a,b} G_{b,a}} \quad (8)$$

with

$$G_{a_i, a_j}(V_a, V_b) = \frac{\partial I_{a_i}}{\partial V_{a_j}}(V_a, V_b) \quad (7)$$

corresponds to processes of lowest order $T^2$ in an expansion in the dimensionless interface transmission coefficient $0 < T < 1$, and the charge transmission coefficient of the superconductor $T_{ch}(R, \omega) = T_{EC}(R, \omega) - T_{CAR}(R, \omega)$ accounts for the different carriers transmitted by EC and CAR in electrode “a”. The resulting crossed resistance per channel is obtained as the inverse of the extra diagonal element of the inverse of the crossed conductance matrix:

$$\begin{bmatrix} R_{a,a} & R_{a,b} \\ R_{b,a} & R_{b,b} \end{bmatrix}^{-1} = \begin{bmatrix} G_{a,a} & G_{a,b} \\ G_{b,a} & G_{b,b} \end{bmatrix}^{-1}, \quad (6)$$
approximated as
\[ R_{a,b} \simeq -\frac{G_{a,b}}{G_{a,a}G_{b,b}}, \] (9)

because of the damping of crossed processes over the coherence length \( \xi \). The dependence on \( V_0 \) and \( V_0 \) is not explicit in Eqs. [8] and [9]. We obtain from Eq. [5]:
\[ R_{a,b}(R,\omega) \simeq \left( \frac{\hbar}{2e^2} \right)^2 T^{-2} T_{eh}(R,\omega), \] (10)
much larger than for our previous approach for higher order processes at the interface15,17.

2. Algorithm

The excitations of the condensate, integrated out by the random phase approximation (RPA) for the transmission modes, lead to a new value of the crossed conductance corresponding to replacing \( \epsilon \) by \( \tilde{\epsilon}(R,\omega,\epsilon) \). We identify the characteristic energy \( \hbar\omega_c \) as the divergence in the energy dependence of \( \tilde{\epsilon}(R,\omega,\epsilon) \), as for a gap edge singularity. The symmetry breaking parameter \( \epsilon_n(R,\omega,\epsilon) \) with \( n \) localization loops is obtained by inverting
\[ \tilde{T}_{EC}(n,\epsilon) = \tilde{T}_0(n) + \tilde{\epsilon}_n(\epsilon) T_0 \] (11)
\[ \tilde{T}_{CAR}(n,\epsilon) = \tilde{T}_0(n) - \tilde{\epsilon}_n(\epsilon) T_0, \] (12)
with \( n \) the number of weak localization-like loops, and where the dependence on \( R \) and \( \omega \) is not explicit in Eqs. [11] and [12]. We deduce the value of
\[ F_n(q,\omega,\epsilon) \equiv \frac{\tilde{\epsilon}_n(q,\omega,\epsilon)}{\epsilon} - 1, \] (13)
where we changed variable from \( R \) to the wave-vector \( q \) by a Fourier transform.

An algorithm generates the topologically inequivalent higher order RPA diagrams for the transmission mode with the tree structure shown on Figs. 4: d, e, f. These diagrams maximizing the number of imbricated loops for a given number of branchings are the most relevant for describing intermediate states with a proliferation of Anderson reflections internal to the superconductor for \( \omega \simeq \omega_c \). In order to reduce the computation time, we first specialize to a single channel by restricting to the transverse components \((q'_x, q'_y, q'_z) = (0, 0, 2\pi n_z/L)\) of the wave-vector \( q' = (q'_x, q'_y, q'_z) = (2\pi n_x/L_x, 2\pi n_y/L_y, 2\pi n_z/L) \) in the weak localization-like loop, with \( n_x, n_y \) and \( n_z \) three integers (see Fig. 5). The scaling with the number of longitudinal components is restored afterwards.

B. Results

Now, we present the result of RPA resummation. The \( q \)-dependence of the transmission coefficients is, up to an overall rescaling, the same as for the bare transmission coefficient, because of the properties of the convolution of exponentials in the corresponding transmission coefficients in real space. We thus evaluate only the rescaling factor \( F_n(0,\omega,\epsilon) \) that collapses on master curves (see Fig. 5) when plotted as a function of the dimensionless parameter
\[ \lambda_\omega \epsilon^2 = k_F L \left( \frac{\hbar\omega}{e_F} \right)^2 (k_F l_e)^{2\alpha}, \] (14)
according to \( F_n(0,\omega,\epsilon) \equiv F_n(\lambda_\omega \epsilon^2) \), with \( l_e \) the elastic mean free path such that \( 2 \leq \xi / l_e \leq 14 \), and with \( \alpha = 1.1 \pm 0.1 \). Summing the RPA diagrams leads to \( \tilde{\epsilon} \), the new value of \( \epsilon \) modified by the response of the condensate:
\[ \tilde{\epsilon}(\omega,\epsilon) = \epsilon (1 + F_\infty(\lambda_\omega \epsilon^2)) = \frac{\epsilon}{1 - \lambda_\omega \epsilon^2}. \] (15)

V. IDENTIFICATION OF THE ENERGY SCALES

A. Characteristic energy of a disordered superconductor

To take into account the large longitudinal dimensions of the junction, we note that convolutions of the exponential envelope of the transmission coefficients lead to

FIG. 8: (Color online.) Scaling plot of \( F_n(\lambda_\omega \epsilon^2) \) as a function of \( \lambda_\omega \epsilon^2 \) for diagrams with \( n = 1 \) (●), \( n = 2 \) (▲) and \( n = 3 \) (■) weak localization-like loops. The energy \( \hbar\omega \) and the dimensionless symmetry breaking parameter \( \epsilon \) vary over 3 orders of magnitude. The values of the elastic mean free path \( l_e \), such that \( 2 \leq \xi / l_e \leq 14 \) (as compared to \( \xi / l_e = 5 \pm 7 \) in the experiment15), vary from the dirty limit to the cross-over with the ballistic limit. For clarity, not all calculated points are shown on the figure. (a) shows one value \( 2\pi / \omega_c \) of the superconductor chemical potential. (b), (c) and (d) show EC and CAR processes for \( eV_0 \geq \hbar\omega_c \) with residual pair correlations among evanescent quasiparticles.
an enhancement of $\lambda_c e^2$ by a factor $k_F^2 L_x L_y$ for extended interfaces. More precisely, combining diffusion modes for 0 to $x$ and from $x$ to $L$ leads to convolutions of the form

$$
\int_0^L \exp (-x/\xi) \exp (-L - x)/\xi) dx = L \exp (-L/\xi),
$$

where the one dimensional case with two evanescent diffusion modes is considered for simplicity.

We deduce from Eq. (13) the characteristic energy $\hbar \omega_c$ of a dirty superconductor:

$$
\frac{\hbar \omega_c}{\epsilon_F} = \frac{1}{|\epsilon|} \left( \frac{1}{(k_F l_\varphi)^2} \right) \frac{1}{\sqrt{k_F^2 L_x L_y}} \tag{17}
$$

The parameter of interactions $v$ in Eq. (2) is independent on $\Delta$, as expected for a mechanism due to electron-hole conversion below the gap $e_F^2/\Delta^2$, the lowest of the characteristic energy $\hbar \omega_c$ and the voltage $e V_b$, rather than $v e_F$, is an indicator of the strength of weak localization-like quantum interference effect in subgap transport. The form of the transmission coefficients puts the constraint $|\epsilon| < 1$. The value $\epsilon = 1$ favoring EC with respect to CAR and minimizing $\hbar \omega_c$ in Eq. (17) for a static superconducting gap is compatible with the picture of a fluctuating chemical potential (see Fig. 6), and with the large EC crossed signal measured experimentally at zero bias in Ref. 4.

**B. Comparison with experiments**

We use $l_\varphi \approx 2$ nm from Ref. 4, $\epsilon_F \approx 5.3$ eV, $k_F \approx 1 \text{Å}^{-1}$ for Nb. In experiments, $\hbar \omega_c$ is limited by $l_\varphi \approx 0.1 \div 0.2 \mu m$ (as obtained from the inelastic electron-electron scattering time $\tau_{ee} \approx 1 \text{ns}$), instead of the sample dimensions $L_x$ and $L_y$ in Eq. (17), leading to $\hbar \omega_c \approx 21 \div 10 \mu eV, 11 \div 6 \mu eV$ and $6 \div 3 \mu eV$ for $L \approx 15, 50, 200$ nm respectively. Russo et al. find experimentally $\hbar \omega_c \approx 270, 50 \mu eV$, for $L \approx 15, 50, 300$ nm, and $\hbar \omega_c$ below the resolution threshold for $L \approx 200$ nm. The theoretical approximation underestimates the experimental $\hbar \omega_c$ in the regime $L \gtrsim \xi$ ($L = 15, 50$ nm and $\xi = 10 \div 15 \text{nm}$ in experiments). The energy scale $\Delta$ is associated to objects of size $\lesssim \xi$ formed by the accommodated weak localization-like loops for $L \approx \xi$. The energy scale $\hbar \omega_c$ is thus enhanced by a cross-over to perturbative effects of weak localization-like loops, with $\hbar \omega_c$ reaching $\Delta$ for $L \approx \xi$, as in the absence of weak localization-like loops. The cross-over is not captured by Eq. (17) with $L \gg \xi$.

**VI. CONCLUSIONS**

In summary, intermediate states due to double Andreev processes induced by a weak localization-like quantum interference effect in subgap transport are at the root of a very slow attractive interaction between evanescent quasiparticles. The interaction arises in an out-of-equilibrium situation if quasiparticles are forced to travel through the superconductor over distances exceeding the coherence length $\xi$, which does not apply to usual Andreev reflection limited by $\xi$. The phase of the mesoscopic superconductor fluctuates with a characteristic time $2\pi/\omega_c$ if the bias voltage energy $eV_b$ is smaller than $\hbar \omega_c$, because of the large retardation $\hbar eV_b$ in double Andreev processes due to weak localization-like loops, as opposed to interactions retarded over the Debye frequency for a BCS superconductor. The model explains the following experimental facts: i) the existence of an energy scale $\hbar \omega_c$ within the superconducting gap, decaying to zero as the superconductor thickness $L$ increases; ii) a change of sign from EC for $eV_b < \hbar \omega_c$ to CAR for $eV_b > \hbar \omega_c$, and a disappearance of the crossed signal for $\hbar \omega_c$ larger than a few $\hbar \omega_c$; and iii) the coupling of weak localization-like quantum interference effect to a magnetic field. A challenging issue is to account for the interplay between weak localization-like quantum interference effect discussed here and the coupling to the electromagnetic field.

**Acknowledgments**

The authors acknowledge crucial discussion with B. Douçot, D. Feinberg, M. Houzet, F. Pistolesi, J. Raninger, and thank H. Courtois, S. Florens and K. Matho for useful remarks on the manuscript.

1. M.S. Choi, C. Bruder and D. Loss, Phys. Rev. B 62, 13569 (2000); P. Recher,E. V. Sukhorukov and D. Loss, Phys. Rev. B 63, 165314 (2001).
2. G. B. Lesovik, T. Martin and G. Blatter, Eur. Phys. J. B 24, 287 (2001); N. M. Chtchelkatchev, G. Blatter, G.B. Lesovik and T. Martin, Phys. Rev. B 66, 161320(R) (2002).
3. D. Beckmann, H.B. Weber and H.v. Löhneysen, Phys. Rev. Lett. 93, 197003 (2004).
4. S. Russo, M. Kroug, T.M. Klapwijk and A.F. Morpurgo, Phys. Rev. Lett. 95, 027002 (2005).
5. P. Cadden-Zimansky and V. Chandrasekhar, Phys. Rev. Lett. 97, 237003 (2006).
6. J. M. Byers and M. E. Flatté, Phys. Rev. Lett. 74, 306 (1995).
7. G. Deutsch and D. Feinberg, App. Phys. Lett. 76, 487 (2000);
8. G. Falci, D. Feinberg and F. Hekking, Europhysics Letters 54, 255 (2001).
Rev. Lett. 91, 157002 (2003).
10 E. Prada and F. Sols, Eur. Phys. J. B 40, 379 (2004).
11 P.K. Polináik, C.J. Lambert, J. Koltai and J. Cserti, Phys. Rev. B 74, 132508 (2006).
12 T. Yamashita, S. Takahashi and S. Maekawa, Phys. Rev. B 68, 174504 (2003).
13 D. Feinberg, Eur. Phys. J. B 36, 419 (2003).
14 R. Mélin and D. Feinberg, Phys. Rev. B 70, 174509 (2004).
15 R. Mélin, Phys. Rev. B 73, 174512 (2006).
16 A. Levy Yeyati, F.S. Bergeret, A. Martin-Rodero and T.M. Klapwijk, [cond-mat/0612027].
17 S. Duhot and R. Mélin, Eur. Phys. J. B 53, 257 (2006).
18 J.P. Morten, A. Brataas and W. Belzig, Phys. Rev. B 74, 214510 (2006).
19 F. Giazotto, F. Taddei, F. Beltram and R. Fazio, Phys. Rev. Lett. 97, 087001 (2006).
20 A. Brinkman and A.A. Golubov, Phys. Rev. B 74, 214512 (2007).
21 A.F. Andreev, Sov. Phys. JETP 19, 1228 (1964).
22 C.J. Lambert and R. Raimondi, J. Phys.: Condens. Matter 10, 901 (1998).
23 F.J. Jedema, B.J. van Wees, B.H. Hoving, A.T. Filip and T.M. Klapwijk, Phys. Rev. B 60, 16549 (1999).
24 E. Akkermans and G. Montambaux, Physique méscopique des électrons et des photons, EDP Sciences/CNRS Editions (2004).
25 R. A. Smith and V. Ambegaokar, Phys. Rev. B 45, 2463 (1992).
26 A. Altland and M.R. Zirnbauer, Phys. Rev. B 55, 1142 (1997).
27 G.E. Blonder, M. Tinkham and T.M. Klapwijk, Phys. Rev. B 25, 4515 (1982).
28 L.P. Gorkov, A. Larkin, and D.E. Khmelnitskii, Pis’ma Zh. Eksp. Teor. Fiz 30, 248 (1979) [JETP Lett. 30, 228 (1979)].
29 S. Hikami, Phys. Rev. B 24, 2671 (1981).
30 A Hikami box in subgap transport contains an “AA” diffusion made of two advanced Green’s functions, ending by two crossings between three diffusons12.
31 S. Duhot and R. Mélin, cond-mat/0701748.
32 A. Jacobs and R. Kümmel, Phys. Rev. B 64, 104515 (2001).
33 B.I. Spivak and S.A. Kivelson, Phys. Rev. B 43, 3740 (1991).
34 P.W. Anderson, Phys. Rev. 112, 1900 (1953).