Numerical simulation of double-diffusion natural convection in rectangular square cavity with porous enclosure

Hanqing WANG 1,2,3, Yue ZHAO 1,2,3*, Ling TIAN 1,2,3

1College of Civil Engineering University of South China, Hengyang Hunan, China, 421001
2Engineering Lab of Hunan for Building Environment Control, Hengyang Hunan, China, 421001
3Key Lab of Hengyang for the Building Energy Efficiency and Environmental Control, Hengyang Hunan, China, 421001

Abstract: The double-diffusion mixed convection in a rectangular cavity with a porous structure was studied using numerical simulation. The influence of the Rayleigh number $Ra$, the porosity of the porous structure, the buoyancy ratio $N$ and the distance of the porous structure from the left wall surface $X_p$ on the flow characteristics were discussed by streamline, heatline, massline, Nu and Sh. It is found that the low floating-to-lift ratio has little effect on the heat transfer. When the buoyancy ratio is 5, the Nu number is linearly positively correlated with the porosity. When $N<-1$, the heat and mass transfer intensity is inversely proportional to the lift-up ratio.

1. Introduction
The heat and mass transfer problems of porous structure are widely found in water treatment, evaporative drying processes, food dehydration, solar heat storage and chemical contaminant dispersion[1-4], and it has guiding significance for practical engineering applications. Domestic and foreign scholars have conducted in-depth research on the natural convection of porous media. Yang Wei et al.[5] used the local heat balance hypothesis of porous media to analyze the influence of Ra number and Darcy number on the natural convection characteristics in the rectangular cavity with porous media. Zhao et al.[6-10] used numerical simulation to study the simultaneous transfer of heat and moisture through conjugate natural convection in partial shell with solid walls. The results show that the effect of heat and mass transfer was largely dependent on the wall material, size and Rayleigh number. Nikbakhti et al.[11] conducted a numerical study of double-diffusion natural convection in a porous cavity with partially moving walls. Mondal[12] analyzed the influence of buoyancy ratio under non-uniform boundary conditions on the double-diffused natural convection heat transfer characteristics in porous media cavities. Hadidi et al.[13] studied the double-diffusion natural convection in a tilted rectangular collector with two parallel porous layers, and discussed the influence of heat and solute sources on flow patterns and heat and mass transfer. Tian et al.[14] discussed the influence of heating frequency on the Nusselt number in a tilted square shell with a porous media whose internal heat source obeys the variation of the cosine function. Xu Hongtao[15-18] conducted an in-depth numerical simulation study of double-diffusion mixed convection in a built-in heating tube to analyze Richardson number, opening position and flow characteristics under unsteady conditions. Rout et al.[19] studied the effects of radiation and chemical reactions on the natural convection of...
porous media under the action of a transverse magnetic field, and observed that magnetic field parameters and radiation parameters significantly affected the flow in the flow field. Nield et al.[20] applied linear stability theorem to analyze the double-diffusion convective flow characteristics in porous media with two horizontal layers. Focusing on the influence of permeability, fluid conductivity, solute diffusion, solid conductivity, interphase heat transfer coefficient and porosity on heat and mass transfer. Jha et al.[21] deduced the analytical solution of natural convective heat and mass transfer in vertical annular non-Darcy porous media under steady state conditions, and found that the existence of Soret effect reduced the volume velocity, the dependence of total heat transfer rate and total mass transfer rate on internal radius, clearance ratio and improved darcy number. Kishan et al.[22] numerically simulated the natural convection in a rectangular cavity embedded with a double-layer non-darcy porous media, and discussed the effect of double stratification of porous media on dimensionless heat and mass transfer coefficient.

In summary, the current research focuses on the double-diffusion mixed convection filled with porous media in the cavity, but there is little research on the double-diffusion natural convection in the local porous media. However, in practical buildings, the local porous medium is sometimes located in the middle of the air domain, and the heat conduction inside the porous structure has a certain influence on the double-diffusion natural convection. In this paper, brinkman-darcy model is adopted and SIMPLE algorithm is used for coupling to conduct numerical research on double-diffusion natural convection in adjacent rectangular cavities. The influences of Rayleigh number, porosity, buoyancy ratio and porous media location on heat and mass transfer are analyzed, and the variation rules and flow characteristics of the temperature field and the concentration field are obtained. The research results can provide theoretical reference for the treatment of pollutants and residual heat removal systems in buildings.

2. Calculation model and mathematical description

2.1 Physical model

The physical model is presented in figure 1. The side length of the two-dimensional closed cavity is $L \times L$, and the width of the porous enclosure is $0.2L$. The length of the heat source on the left side is $0.5L$, which is the constant temperature boundary $T_h$ and constant concentration boundary $C_h$. The temperature and concentration on the right side are constant at $T_c$, $C_c$ ($T_h > T_c$, $C_h > C_c$). The other walls are adiabatic and impermeable to the wall surface, and the speeds are all without slip boundary conditions. The gravitational acceleration is $g$, and the number of Pr of the fluid in the cavity is 0.71. All thermophysical parameters of the air mixture are regarded as constants, but the density changes with temperature and concentration follow the Boussinesq hypothesis. Namely:

$$\rho = \rho_c [1 - \beta_T (T - T_c) - \beta_C (C - C_c)]$$  \hspace{1cm} (1)

Where, $T$ is the temperature; $C$ is the concentration; $\rho_c$ is the fluid density corresponding to $T_c$; $\beta_T$ is the coefficient of volume expansion caused by temperature; $\beta_C$ is the coefficient of volume expansion caused by concentration.
2.2 Control equation

The double-diffusion mixed convection non-dimensional governing equation is:

**Continuity equation:**
\[ \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \]  
\[ (2) \]

**Momentum equation:**
\[ \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial X} + \sqrt{\frac{Pr}{Ra}} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \]  
\[ (3) \]

**Energy equation:**
\[ \frac{\partial T}{\partial X} + \frac{\partial V}{\partial Y} = \frac{1}{\sqrt{Ra Pr}} \left( \frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right) \]  
\[ (4) \]

**Mass transfer equation:**
\[ \frac{\partial C}{\partial X} + \frac{\partial V}{\partial Y} = \frac{1}{Le \sqrt{Ra Pr}} \left( \frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2} \right) \]  
\[ (5) \]

The finite volume method is used to solve the governing equation discretely. The discrete method of the equation adopts the second-order upwind style, and the SIMPLE algorithm is used for the speed-pressure coupling calculation.

The dimensionless variable of the above equation is:
\[ (X, Y) = (x, y) / L, (U, V) = (u, v) / \alpha, \]
\[ P = p / \rho(\alpha / L)^3, T = (T_h - T_c) / \Delta T, \]
\[ C = (C_h - C_c) / \Delta C \]  
\[ (7) \]

Where, \( u, v \) for the horizontal direction and vertical direction of the speed component; \( T, p, C \) respectively for temperature, pressure and concentration; \( \alpha, \rho \) respectively for thermal diffusion coefficient and density; \( P \) for dimensionless pressure; \( L \) for the space length.

Dimensionless parameters:
\[ Pr = \nu / \alpha, Ra = g \beta \Delta TL^3 / \nu \alpha, Le = \alpha / D, \]
\[ N = \beta \Delta C / \beta T \Delta T, Da = K / L^2 \]  
\[ (8) \]

Where, \( \nu, g \) and \( D \) respectively for kinematic viscosity, gravitational acceleration and mass diffusion coefficient; \( Pr \) is Prandtl number; \( Ra \) is Rayleigh number; \( Le \) is Lewis number; \( Da \) is Darcy number, and \( K \) is permeability.

2.3 Model verification

The same model was established as the literature (as shown in Figure 2 below) to verify the correctness...
of the numerical solution method in this paper. The left side of the cavity is a porous structure filled with fluid, and the right side is a fluid region. Table 1 shows the results of the comparison between the average Nusselt number calculated in this paper and the literature, in which the maximum error is 0.45%. Proving the reliability of the running results of this program.

![Figure 2 Model used in literature [23]](image)

Table 1 Comparison of average Nu number with literature calculations

| Ra   | This article | Reference[23] | Relative error |
|------|--------------|---------------|----------------|
| 103  | 1.116        | 1.118         | 0.18%          |
| 104  | 2.242        | 2.235         | 0.31%          |
| 105  | 4.511        | 4.509         | 0.04%          |
| 106  | 8.813        | 8.853         | 0.45%          |

3. Results analysis and discussion

3.1 Effect of porosity on natural convection heat transfer

Fig. 3(a) shows that $Nu_{ave}$ and $Sh_{ave}$ vary with porosity and $Ra$. When $Ra$ is small, the porosity $\varepsilon$ has little effect on convective heat transfer. At the same porosity, the $Nu$ number increases and the $Sh$ number decreases as $Ra$ increases. The $Nu$ number first decreases and then increases as porosity increases when $Ra$ is equal to $10^6$, and the minimum value appears at $\varepsilon$ is equal to 0.3. Because the porosity appears in the convection term of the governing equation, and the value of the convection term is relatively small; it has little effect on heat and mass transfer, so the porosity nearly has no effect on the natural convection of the adjacent cavity.

Fig. 3(b) shows that $Nu_{ave}$ and $Sh_{ave}$ vary with porosity and $N$, and porosity nearly has no effect on mass transfer. Under the three kinds of buoyancy ratios, the $Sh$ number is always a horizontal straight line. Porosity has little influence on heat transfer under low buoyancy ratio. When the buoyancy ratio is increased to 5 or more, the $Nu$ number is linearly positively correlated with porosity. The larger the buoyancy ratio, the larger the slope.
3.2 Influence of buoyancy ratio on natural convection heat transfer

Fig. 4 shows the changes of the streamlines, hot lines and mass lines in the rectangular cavity under different buoyancy ratios. When $N$ is equal to -10, the buoyancy direction is caused by the temperature gradient and the concentration gradient is opposite; the two buoyancy forces weaken each other. A counterclockwise flow is formed in the cavity with the mass-lifting force as the main force. The maximum flow function $|\psi|$ is equal to 11, is located in the right air field near the cold source; when $N$ is increased to -5, the flow pattern of the entire flow field in the rectangular cavity is unchanged and the maximum flow function is reduced. When $N$ is equal to 0, there is no mass-lifting force and mass transfer is caused by the flow driven by heat transfer. That is, the temperature field is coupled to the flow field but not to the concentration field. The streamline is roughly symmetrical along the vertical centerline and flows in an elliptical clockwise direction; the maximum flow function $|\psi|$ is equal to 5.8. When $N$ is greater than zero, the fluid in the cavity flows clockwise under the synergistic effect of heat-lifting force and mass-lifting force; the maximum flow function $|\psi|$ is equal to 12. As $N$ increases, the core part of the stream-line central vortex increases.

Fig. 5 shows the $Nu_{ave}$ and $Sh_{ave}$ on the left high-temperature and high-concentration wall surface vary with $N$. When $N$ less than -1, the mass-lifting force plays a leading role and the fluid in the cavity flows counterclockwise. With the increase of buoyancy ratio, the heat and mass transfer intensity decreases gradually, because at this time, the direction of heat-lifting force and mass-lifting force is opposite, which weakens the effect of heat and mass transfer. When $N$ is equal to -1, the heat-lifting force and mass-lifting force reach a dynamic equilibrium; the flow intensity, heat and mass transfer intensity of the fluid are the weakest. The average $Nu$ and the average $Sh$ reach the minimum value. When $N$ is greater than -1, the heat-lifting force plays a leading role. The direction of heat-lifting force is the same as mass-lifting force. With the increase of the buoyancy ratio, the heat and mass transfer intensity increase gradually.
3.3 Influence of porous media position on natural convective heat transfer

It can be seen from figure 6 that when $X_p$ is equal to 0.1, the maximum value of the flow function appears in the fluid area on the right side; is equal to 7.5. Single vortex exists in the fluid region, and the streamline in the fluid region is obviously denser than that in the porous structure. That is, the porous
structure hinders the flow. The isotherm of the fluid region on the right is of "S" shape with an average temperature of 0.4. As the porous structure moves to the right, the flow function gradually decreases until the middle position reaches the minimum; the heat and mass boundary layer near the upper boundary of the wall is gradually thinner and the variation rules of streamline, hot line and quality line are basically the same. The temperature gradient in the fluid is greater than that in the porous structure, and the isotherm distribution in the fluid domain on both sides of the porous structure is basically the same. Both sides are dense near the wall surface, and the heat transfer effect in the fluid region is more sufficient than that in the porous structure.

It can be seen from Fig. 7(a) that when the porous media is close to the heat source wall, Vx reaches its maximum value; with the right shift of the porous media, the Vx value in the right flow domain gradually decreases. When Xp is equal to 0.2 and 0.3, a double vortex structure is formed and other positions form a single vortex structure only in a large area of the flow domain. According to Fig. 7(b), as the porous structure is far away from the heat source on the left, the average Nu number decreases and the average Sh number increases; indicating that the heat transfer of the fluid weakens and the mass transfer of the fluid enhances. When Xp is equal to 0.3, the Nu number is the smallest and the porous media has the greatest hindrance to heat transfer. When the Xp is between 0.3 and 0.4, the Nu number and the Sh number have a mutation process. At this time, the heat transfer effect in the rectangular cavity is the weakest and the mass transfer effect is the best; the Nu number and the Sh number remain basically unchanged after the mutation process. When Le is greater than or equal to 1, Le is negatively correlated with Sh and has little influence on Nu.
4. Conclusion

In this paper, the numerical simulation method is used to analyze the double-diffusion mixed convection problem in adjacent rectangular cavity. The following conclusions are obtained:

(1) Porosity $\varepsilon$ has little effect on convective heat transfer when $Ra$ is small. At the same porosity, the $Nu$ number increases and the $Sh$ number decreases as $Ra$ increases. Under the condition of low buoyancy ratio, porosity has little influence on heat transfer. When the buoyancy ratio is greater than or equal to 5, $Nu$ number is linearly positively correlated with porosity. The larger the buoyancy ratio, the larger the slope.

(2) The maximum flow function appears in the right air domain. As $N$ increases, the maximum flow function gradually moves up from the lower side of the cold wall. As the $|N|$ increases, the core portion of the centerline vortex increases and the heat and mass transfer effect is enhanced.

(3) When the $X_p$ is between 0.3 and 0.4, there exists a best position in the rectangular cavity that the heat transfer effect is the weakest and the mass transfer effect is the best; the $Nu$ number and the $Sh$ number remain basically unchanged after the mutation process. When $Le$ is greater than or equal to 1, $Le$ is negatively correlated with $Sh$ and has little influence on $Nu$.

The calculation results of this paper provide a theoretical basis for understanding the law of heat and mass transfer when porous media partition exists.

Main symbol table

| English symbol | Symbol |
|----------------|-------|
| $C$            | Dimensionless concentration |
| $\Delta C$     | Concentration difference    |
| $Pr$           | Prandtl number               |
| $Ra$           | Rayleigh number              |
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