DOA estimation for coherent distributed sources based on common spectral peak search

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Abstract. Most of the existing DOA estimation algorithms are based on uniform array, whose estimation performance is limited by the shortcoming of small aperture of uniform array, and it is difficult to apply sparse array to DOA estimation of distributed sources. In order to solve the above problems, this paper introduces interqualitative array into DOA estimation of distributed sources, and proposes a DOA estimation algorithm for coherent distributed sources. Because the array element spacing is larger than half wavelength, the direct estimation will produce false peak. In order to eliminate the ambiguity, this paper proves that DOA estimation of two submatrices is performed separately, and its common value is unique, that is, the true incidence Angle. According to this proof, DOA estimation of two submatrices is carried out separately for the one-dimensional DOA estimation scenario of distributed sources, and the common part of the two sets of estimation results is taken as the final estimation result. The complexity analysis and performance verification of this method are also given. Simulation experiments show that the performance of the proposed method is better than that of the DSPE algorithm and TLS-ESPRIT algorithm under the same array number in the one-dimensional DOA estimation scenario.

1. Introduction
In the field of array signal processing, Direction-Of-Arrival (DOA) estimation is an important component [1][2][3]. Now, subspace algorithm has been quite mature, but these methods are for point source model. However, because of the scattering and reflection of channel environment, mass of multipaths are generated, making spatial expandability on signal. Thus the signal is called distributed source and it is of great importance to study it.

In a non-Gaussian environment, the approximate rotation-invariant relation of the fourth-order cumulant is constructed by using the double parallel linear array, and the central DOA of the coherent distribution source is solved by eigenvalue decomposition of the rotation-invariant matrix, which has better performance under the conditions of low SNR, small number of snapshots and different angle expansion [4]. A DOA estimation algorithm is proposed for coherent distributed source in MIMO system under the condition of circular and non-circular signals co-existing which improves the performance but the complexity is high. For coherent distributed non-circular signals, a DOA estimation algorithm is proposed which does not require the prior conditions of angular distribution function [5]. Based on the special geometrical characteristics of L-shaped uniform linear array, the improved propagator method is used to obtain the two-dimensional angle parameters independently for coherent distributed non-circular signals in literature [6].
The above documents [1-6] are all based on uniform arrays. Sparse arrays are worthy of further application in DOA estimation of distributed source due to the advantage of large array aperture which typically represented by co-prime arrays [7] and nested arrays [8]. For DOA estimation of distributed sources, a sparse representation model under the constraint of L1 nuclear norm is established in literature [9]. Because the traditional spatial smoothing method is no longer suitable for distributed sources, a new spatial smoothing method is proposed according to the characteristics of distributed sources in literature [10], which realize the estimation of source number and DOA under underdetermined conditions. Based on non-uniform L array, a continuous GESPRIT (S- GESPRIT) algorithm is proposed to obtain the 2D DOA estimation of coherent distributed sources, which reduces the computational complexity of the algorithm.

In this paper, a distributed source parameter estimation algorithm based on common spectral peak search in a mutual mass array is proposed. The proposed algorithm improves the DOA estimation performance by utilizing array spatial sparsity and distributed source Angle distribution characteristics for one-dimensional scenes respectively. This algorithm firstly proves that DOA estimation of two submatrices is performed separately, and its common value is unique, that is, the true incidence Angle. According to this proof, for the one-dimensional DOA estimation scenario of distributed sources, DOA estimation is carried out separately for the two submatrices, and the common part of the two sets of estimation results is taken as the final estimation result, which improves the estimation accuracy compared with the existing one-dimensional DOA estimation algorithm.

Table 1 shows the key notations and explanations in this paper.

| Notations | Explanation |
|-----------|-------------|
| (·)*      | Conjugate of matrix |
| (·)T      | Transpose of matrix |
| (·)H      | Conjugate transpose of matrix |
| \(E\{\cdot\}\) | Statistical expectation operator |
| diag(·)   | Diagonalization operation |
| \([·\]_{m,n}\) | \((m,n)\)-th entry of a matrix |
| \(\text{tr}(\cdot)\) | Matrix trace |
| ⊗          | Kornecker product |
| \(\mathbb{P}\) | Set of the locations of physical elements |
| \(\text{angle}(\cdot)\) | Phase acquired operation |

2. Signal model

The co-prime linear array consists of two uniform linear subarrays. The element numbers are \(M_1\) and \(M_2\), and the spacing of elements within subarrays are \(M_1\lambda/2\) and \(M_2\lambda/2\), respectively. \(M_1\) and \(M_2\) are mutual primes, \(\lambda\) represents the wavelength. The structure of co-prime linear array is displayed in Figure 1.

![Figure 1. Structure of co-prime linear array.](image)
Assuming that there are $K$ coherent distributed sources transmitting plane waves from center DOA $\theta=(\theta_1,\theta_2,\ldots,\theta_k)$ to the receiving array, which is composed of $M=M_1+M_2-1$ array elements. Then the signal output vector can be expressed as

$$\mathbf{x}_1 = \sum_{i=1}^{K} \mathbf{a}_i(\theta)d(\theta,\mu_i)d\theta + \mathbf{n}_1$$  \hspace{1cm} (1)

$$\mathbf{x}_2 = \sum_{i=1}^{K} \mathbf{a}_2(\theta)d(\theta,\mu_i)d\theta + \mathbf{n}_2$$  \hspace{1cm} (2)

where,

$$\mathbf{a}_1(\theta) = [e^{i\eta_1 \sin\theta} e^{i2\eta_1 \sin\theta} \ldots e^{i(M_2-1)\eta_1 \sin\theta}]$$  \hspace{1cm} (3)

$$\mathbf{a}_2(\theta) = [e^{i\eta_2 \sin\theta} e^{i2\eta_2 \sin\theta} \ldots e^{i(M_1-1)\eta_2 \sin\theta}]$$  \hspace{1cm} (4)

$$\eta_1 = \frac{2\pi d_1}{\lambda} = M_2\pi$$  \hspace{1cm} (5)

$$\eta_2 = \frac{2\pi d_2}{\lambda} = M_1\pi$$  \hspace{1cm} (6)

$s_i(\theta,t;\mu_i)$ is the angular signal density function and $\mu_i = (\theta,\sigma_i)$ is the angle parameter where $\theta_i$ and $\sigma_i$ respectively represent center DOA and angle spread.

For the coherent distributed source, $s_i(\theta,t;\mu_i)$ can be written as,

$$s_i(\theta,t;\mu_i) = s_i(t)g_i(\theta,\mu_i)$$  \hspace{1cm} (7)

where $s_i(t)$ and $s_i(\theta,t;\mu_i)$ reflects the time characteristics and the spatial distribution characteristics of $s_i(\theta,t;\mu_i)$.

Then, define $\mathbf{b}_1(\mu_i)$ and $\mathbf{b}_2(\mu_i)$ as follows,

$$\mathbf{b}_1(\mu_i) = \int \mathbf{a}_1(\theta)g_i(\theta,\mu_i)d\theta$$  \hspace{1cm} (8)

$$\mathbf{b}_2(\mu_i) = \int \mathbf{a}_2(\theta)g_i(\theta,\mu_i)d\theta$$  \hspace{1cm} (9)

Further, formula (1) and (2) can be rewritten as,

$$\mathbf{x}_1 = \sum_{i=1}^{K} \mathbf{b}_1(\theta)s_i(t)d\theta + \mathbf{n}_1$$  \hspace{1cm} (10)

$$\mathbf{x}_2 = \sum_{i=1}^{K} \mathbf{b}_2(\theta)s_i(t)d\theta + \mathbf{n}_2$$  \hspace{1cm} (11)

Thus, the received signal can be written as matrix form

$$\mathbf{X}_1 = \mathbf{B}_1\mathbf{S} + \mathbf{N}_1$$  \hspace{1cm} (12)

$$\mathbf{X}_2 = \mathbf{B}_2\mathbf{S} + \mathbf{N}_2$$  \hspace{1cm} (13)

Consider the $k$th element of $\mathbf{b}(\mu_i)$,

$$[\mathbf{b}(\mu_i)]_k = \int \mathbf{a}(\theta)g_i(\theta,\mu_i)d\theta$$

$$= \int e^{i\eta k \sin\theta} g_i(\theta,\mu_i)d\theta$$  \hspace{1cm} (14)

Let $\theta = \theta_i + \tilde{\theta}$, then

$$[\mathbf{b}(\mu_i)]_k = \int e^{i\eta k \sin(\theta_i+\tilde{\theta})} g_i(\theta_i+\tilde{\theta},\mu_i)d\tilde{\theta}$$  \hspace{1cm} (15)

Now, for small values of $\tilde{\theta}$, the function $e^{i\eta k \sin(\theta_i+\tilde{\theta})}$ can be approximated by the first terms in the Taylor series expansions. We obtain

$$e^{i\eta k \sin(\theta_i+\tilde{\theta})} \approx e^{i\eta k (\sin \theta_i + \tilde{\theta} \cos \theta_i)}$$  \hspace{1cm} (16)

thus, we can rewrite (15) as

$$[\mathbf{b}(\mu_i)]_k \approx e^{i\eta k \sin \theta_i} G_i(\mu_i)$$  \hspace{1cm} (17)
where

\[ G_i(\mu) = \int e^{j\phi \sin \theta} g_i(\theta_i + \tilde{\theta}, \mu) d\tilde{\theta} \] (18)

When the angle spread function is a gauss distribution, namely,

\[ g_i(\theta_i, \mu) = \frac{1}{\sqrt{2\pi \sigma_i^2}} e^{-i(\theta_i - \mu_i)^2 / 2\sigma_i^2} \] (19)

Using the integral formula

\[ \int e^{-t^2} e^{j(\theta_i + \tilde{\theta})^2} d\theta = \sqrt{\pi} e^{j\theta_i^2 / 2\sigma_i^2} (e^{j\theta_i^2 / 2\sigma_i^2}) \] (18),

which in turn results in

\[ G_i(\mu) = \frac{1}{\sqrt{2\pi \sigma_i^2}} e^{-0.5\sigma_i^2 k_i^2 \sin \theta_i} \] (20)

In a conclusion,

\[ [b(\mu)]_k \approx \frac{1}{\sqrt{2\pi \sigma_i^2}} e^{j\phi \sin \theta} e^{-0.5\sigma_i^2 k_i^2 \cos \theta_i} \] (22)

3. Proposed algorithm

A. Ambiguity problem

Let

\[ b_1(\mu) = e^{j\phi \sin \theta} \quad \text{and} \quad b_2(\mu) = \frac{1}{\sqrt{2\pi \sigma_i^2}} e^{-0.5\sigma_i^2 \phi^2 \sin \theta} \] (19),

where \( b_1(\mu) \) represents the phase weighted and is the same as the steering vector of point source model based on uniform linear array \( a(\theta) \), \( b_2(\mu) \) represents the amplitude weighted of the steering vector of distributed source model.

Thus,

\[ b(\mu) = b_1(\mu) * b_2(\mu) \] (23)

Firstly, the ambiguity phenomenon of \( b_1(\mu) \) is analysed. Literature [11] indicates that, the relationship between DOA ambiguity value and true value is as follows,

\[ b_1(\mu) = b_1(\mu') \] (24)

where \( \mu \) is the true value, \( \mu' \) is the ambiguity value. Namely,

\[ e^{j\phi \sin \theta} = e^{j\phi \sin \theta'} \] (25)

\[ 2\pi d_i \sin \theta / \lambda - 2\pi d_i \sin \theta' / \lambda = 2k\pi \quad i = 1, 2 \] (26)

\[ \sin \theta - \sin \theta' = \frac{2k}{M_i} \] (27)

where \( d_i = M_i \lambda / 2 \), because the range of one-dimensional direction is \(-90^\circ \leq \theta \leq 90^\circ\), thus,

\[ |\sin \theta - \sin \theta'| < 2 \] (28)

Then we can obtain

\[ k \in \{-(M_i - 1), \ldots, (M_i - 1)\} \] (29)

When \( M_i = 1 \), \( k \) can only be equal to zero. At this time, there is only one estimation result, that is, DOA estimation result in the traditional array form. When \( M_i > 1 \), the submatrices of the mutual matrix will produce \( M_i \) ambiguity values, which contain the corresponding true estimation of DOA.

Second, the ambiguity phenomenon of \( b_2(\mu) \) is analysed. Due to \( e^{-0.5\phi^2 / \sigma^2} \) is a monotone decreasing function, thus there is no ambiguity in parameter \( \phi \).
B. Ambiguity elimination

Assuming that $\hat{\theta}_1$ is the same estimation result of the two subarrays, and suppose besides $\hat{\theta}_1$, there also exists a $\hat{\theta}'_1$, which is also a peak in the same position of the DSPE spectrum of both the two decomposed subarrays. For the subarray with $M$ elements, from (27), we have

$$\sin \theta - \sin \theta' = \frac{2k_1}{M}$$

(30)

where $k_1 \in \{-M_1, \ldots, M_1\}$

In a similar way

$$\sin \theta - \sin \theta' = \frac{2k_2}{M_1}$$

(31)

where $k_2 \in \{-M_2, \ldots, M_2\}$

Thus

$$\frac{2k_1}{M} = \frac{2k_2}{M_1}$$

(32)

Since $M_1$ and $M_2$ are co-prime integer number, there exist no $\{k_1, k_2\}$ pair that fulfills the condition of (32). Thus, $\hat{\theta}'_1$ does not exist. In addition, it has been proved that there is no ambiguity in parameter $\sigma$. In conclusion, we can obtain the DOA estimation of distributed source by combining the DSPE results of the corresponding two decomposed uniform linear arrays.

4. Simulation results

Simulation 1:
In this part, we draw the parameter coordinates estimated by each Monte Carlo experiment into a scatter diagram, and describe the effectiveness of the estimation by its aggregation degree. The sub-arrays $M_1 = 3$, $M_2 = 4$. The DOA of distributed sources are -20° and 20°, and the parameter $\sigma = 2$, $\sigma = 3$, respectively. The step length of searching is $\Delta \sigma = 0.01$ and the SNR=10db. In Fig.2, The black dots are the results of estimation and the red cross is the true value. We obtain that the Monte Carlo results are distributed around true value, thus the proposed algorithm can obtain the accurate estimations.

![Figure 2. Scatter diagram of parameter estimation.](image)

Simulation 2:
The DOA of distributed source is 20°. The number of snapshots is set as $T = 500$. The sub-arrays $M_1 = 3$, $M_2 = 4$ in the proposed algorithm. To show the superiority of the proposed algorithm, the number of array is set as 6 in DSPE algorithm and the distance between adjacent sensors is $0.5\lambda$. The SNR ranges from -4dB to 20dB and the number of Monte Carlo experiments is 500. It is clear from Fig3
that the estimation accuracy of the proposed method is higher than the DSPE algorithm and the LS-ESPRIT algorithm under the same SNR.

![Figure 3. RMSE of DOA estimates versus SNRs.](image)

5. Conclusion
This paper introduces interqualitative array into DOA estimation of distributed sources, and proposes a DOA estimation algorithm for coherent distributed sources. Because the array element spacing is larger than half wavelength, the direct estimation will produce false peak. In order to eliminate the ambiguity, this paper proves that DOA estimation of two submatrices is performed separately, and its common value is unique, that is, the true incidence Angle. According to this proof, DOA estimation of two submatrices is carried out separately for the one-dimensional DOA estimation scenario of distributed sources, and the common part of the two sets of estimation results is taken as the final estimation result. Numerical simulations are provided to demonstrate the superiority of the proposed algorithm.

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