Strong Phases in the Decays $B$ to $\pi\pi$

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Abstract

Two sources of strong phases in the decays $B$ to $\pi\pi$ are identified: (1) “quasi-elastic scattering” corresponding to intermediate states like $\pi\pi$ and $\rho\rho$, (2) “$c\bar{c}$” corresponding to intermediate states like $D\bar{D}$ and $D^*\bar{D}^*$. Possibilities of using data to identify these two sources are discussed and illustrated. Present data suggests both sources may be significant.
The decay of $B \to \pi\pi$ may be considered as due to the effective interaction

$$\lambda_u d(u\bar{u} - t\bar{t})b + \lambda_c d(c\bar{c} - t\bar{t})b,$$  \hspace{1cm} (1)

where

$$\lambda_u = V_{ub}^* V_{ud} = e^{i\gamma} A\lambda^3 \frac{\sin\beta}{\sin\alpha},$$

$$\lambda_c = V_{cb}^* V_{cd} = -A\lambda^3.$$  

The decay amplitudes may be written, neglecting the small electroweak penguins and assuming isospin invariance,

$$-A(\pi^+\pi^-) = Te^{i\delta_T} e^{i\gamma} + Pe^{i\delta_P},$$  \hspace{1cm} (2)

$$-\sqrt{2}A(\pi^0\pi^0) = Ce^{i\delta_C} e^{i\gamma} - Pe^{i\delta_P},$$  \hspace{1cm} (3)

$$-\sqrt{2}A(\pi^+\pi^0) = (Te^{i\delta_T} + Ce^{i\delta_C})e^{i\gamma},$$  \hspace{1cm} (4)

where $\delta_T$, $\delta_C$, and $\delta_P$ are phases due to the strong final state interaction. In terms of the isospin analysis of the $\lambda_u$ terms, $T$ and $C$ may be replaced by $A_2$ and $A_0$ and $(\delta_C, \delta_T)$ by $(\delta_2, \delta_0)$

$$Te^{i\delta_T} = e^{i\delta_0}(A_2 e^{i(\delta_2 - \delta_0)} + A_0),$$  \hspace{1cm} (5)

$$Ce^{i\delta_C} = e^{i\delta_0}(2A_2 e^{i(\delta_2 - \delta_0)} - A_0).$$  \hspace{1cm} (6)

The $T$ term is often referred to as the tree amplitude corresponding to the $b$ quark decay into $u\bar{u}d$, while $P$ is called the penguin corresponding to a loop diagram dominated by the virtual $t$ quark. However, as can be seen from Eq (1) there is also a $t\bar{t}$ loop contributing presumably in a small way to $T$. This is because we are using what is called the $c$ convention in contrast to the $t$ convention that separates off the $t$ loop [1][2]. Note that $P$ makes a contribution to the $I = 0$ final state so that the complete amplitude for $I = 0$ (sometimes given the notation $A_0$) is here a sum of the $A_0$ and $P$ terms.

If the final state scattering were purely elastic the phases $\delta_2$ and $\delta_0$ would be $\pi\pi$ scattering phases in accordance with the Watson theorem. In fact the final state scattering is expected to be very inelastic and described by the $N \times N$ $S$ matrix at the center-of-mass energy equal to the $B$ mass and $J = 0$. The simple Watson theorem can be applied only to final states that are eigenstates of the $S$ matrix. In general for any weak interaction operator $O$ the phase due to the strong interaction for the amplitude $A_f$ is determined from

$$\text{Im}A_f = \text{Im} \sum_i \langle f | S^{1/2} | i \rangle \langle i | O | B \rangle.$$  \hspace{1cm} (7)
In the case of $A_0$ and $A_2$ or, equivalently $C$ and $T$, determining the strong phases ($\delta_2, \delta_0$) or ($\delta_C, \delta_T$) the intermediate states $i$ are $u\bar{u}d(\bar{q})$ state where $\bar{q}$ is the spectator. These include $\pi\pi$, $\rho\rho$, and many others. We refer to these as “quasi-elastic” since the intermediate states include $\pi\pi$ and arise from the same quark set.

In the case of $\delta_P$ there are two classes of intermediate states: once again there is a phase due to “quasi-elastic” rescattering which would yield a phase $\delta_{0P}$, but also intermediate states of the form $c\bar{c}d(\bar{q})$ such as $DD$ or $D^*D^*$ which would yield a phase $\delta_D$. Thus the imaginary part of $P$, $\text{Im}P$, can be written

$$\text{Im}P = \text{Im}(\text{quasi–elastic}) + \text{Im}(c\bar{c}). \tag{8}$$

In the limit that $\text{Im}(c\bar{c})$ vanishes we have

$$\sin\delta_P = \sin\delta_{0P} \equiv \frac{\text{Im}(\text{quasi–elastic})}{|P|}. \tag{9}$$

In general $\delta_{0P}$ and $\delta_0$ need not be equal although they involve the same final $S$ matrix because the contributions of different states $i$ to the sum in Eq (7) may not be the same for penguin and tree operators. There is also a contribution of the form $s\bar{s}d(\bar{d})$ corresponding to states such as $K\bar{K}$; we expect this to be very small but include it in $\delta_{0P}$.

We turn to the question of theoretical expectations for these two types of strong phases. It is often stated that the outgoing $\pi\pi$ pair do not scatter thus ruling out the quasi-elastic source[4]. This is clearly wrong since reasonable estimates give a significant value for the $\pi\pi$ cross-section at the energy 5.3 GeV. It can be argued that the multi-particle states which dominate the $u\bar{u}d(\bar{q})$ final states in $B$ decay are not likely to rescatter into the $\pi\pi$ state. However since the final $\pi\pi$ states are less than one in a thousand of the final states even a small rescattering can yield a significant phase. It can be argued that different terms in the sum in Eq (7) cancel, but statistical analysis[5] allows for a significant final phase in spite of this.

At first one might expect that the $c\bar{c}$ states would be unimportant since scattering from these states to $\pi\pi$ is expected to be small by the Zweig rule. Furthermore the total branching ratio into these states is expected to be no more than a factor of 2 larger than the total rate for the $u\bar{u}$ states so the quasi-elastic would be much larger than the $c\bar{c}$ contribution. However two-body states are much more common among the $c\bar{c}$ states and these are expected to rescatter more readily into $\pi\pi$ than the multi-particle states that dominate $u\bar{u}$. For example, the branching ratio to $D^*D^*$ is 30 to 40 times larger than that to $\rho\rho$. Thus a number of papers have suggested that this should dominate the strong phase [6].

We now turn to what we can learn from experimental results. We assume the standard model and that the phase $\gamma$ has been determined from other experiments. The experimental
Table 1:

|                | \(Br[10^{-6}]\) | \(C_{\pi\pi}\) | \(S_{\pi\pi}\) |
|----------------|------------------|----------------|----------------|
| \(B^0 \to \pi^+\pi^-\) | 4.8 ± 0.5        | -0.37 ± 0.10   | -0.50 ± 0.12   |
| \(B^+ \to \pi^+\pi^0\)  | 5.6^{+0.9}_{-1.1} |               |                |
| \(B^0 \to \pi^0\pi^0\)  | 1.51 ± 0.28      |               |                |

results are the branching fraction ratios for the three \(\pi\pi\) decays and the asymmetries \(C_{\pi\pi}\) and \(S_{\pi\pi}\); recent experimental results are summarized in Table 1[7]. The values of \(C_{\pi\pi}\) and \(S_{\pi\pi}\) can be used to determine \(P/T\) and \(\delta_{PT} \equiv \delta_P - \delta_T\). In the approximation that \(P/T\) is small and \(\beta + \gamma < 90^\circ\)

\[
\tan(\delta_{PT}) \approx \cos(2(\beta + \gamma)) \frac{C_{\pi\pi}'}{S'}
\]

where \(S' = -(S + \sin(2(\beta + \gamma)))\). Exact results are shown in Table 2 for the central values in Table 1 and three values of \(\gamma\). As \(\gamma\) becomes smaller \(\delta_{PT}\) passes through \(-90\) degrees. Phases with magnitudes greater than \(90^\circ\) would be interpreted as a final state strong phase less than \(90^\circ\) with a reversal of the sign of \(P/T\).

The three \(\pi\pi\) rates can now be used to determine \(C/T\) and \(\delta_{CT} \equiv \delta_C - \delta_{T}\), or, equivalently, \(A_2/A_0\) and \(\delta_{20} \equiv \delta_2 - \delta_0\). This is illustrated in Table 2 for the central values in Table 1. Using these values and Eq (5) we determine \(\delta_T - \delta_0\) and then from \(\delta_{PT}\) we obtain \(\delta_{P0} \equiv \delta_P - \delta_0\). Two solutions for \(\delta_{CT}\) with opposite signs are shown. The positive sign leads to lower and more reasonable value for \(\delta_{P0}\); also if the difference between \(\delta_2\) and \(\delta_0\) is due to isovector exchange in the rescattering one obtains the positive sign. The two solutions may be distinguished by the values of \(C_{00}\); limited data available so far favors the negative sign for the central value of \(\gamma\).

The values for \(\delta_{20}\) are entirely due to “quasi-elastic” rescattering. The value is seen to depend significantly on the \(\pi^0\pi^0\) rate as shown in Fig 1 and is quite large for the present central value. If we assume that \(\delta_{0P} \sim \delta_0\) then in the absence of a \(c\bar{c}\) contribution \(\delta_P \sim \delta_0\) (Eq (9)); in this case the large values for \((\delta_P - \delta_0)\) shown in Table 2 would be evidence for \(c\bar{c}\) states contributing to \(\delta_P\). However it is important to note that the experiments cannot determine \(\delta_0\) or \(\delta_{0P}\) or \(\delta_0 - \delta_{0P}\); the isospin-independent quasi-elastic strong phase in \(T\) cannot be determined.

In conclusion we have tried to show what can be learned as to the origin of the strong phases simply using data on \(B\) to \(\pi\pi\) decays. A more ambitious attempt including \(B\) to \(K\pi\) and using \(SU(3)\) has been made by Christopher Smith[8]. Similar results to those shown in Table 2 have been given in a number of papers[9] devoted to determining \(\gamma\). Here we do not try to analyze
Table 2: Results for the central values in Table 1.

|   | 47°    | 57°    | 67°    |
|---|--------|--------|--------|
| $\gamma$ | 47°    | 57°    | 67°    |
| $\gamma$ | 47°    | 57°    | 67°    |
| $\delta_P - \delta_T$ | $-117.37^\circ$ | $-68.97^\circ$ | $-42.23^\circ$ |
| $\delta_C - \delta_T$ | $0.587$ | $0.768$ | $0.906$ |
| $\delta_P - \delta_0$ | $-102.64^\circ$ | $-51.33^\circ$ | $-23.37^\circ$ |
| $C_{00}$ | 0.108  | 0.488  | 0.746  |

the data in detail but simply try to illustrate what strong phase information can be obtained. Our conclusions are:

(1) Definite information on quasi-elastic strong phases in the tree amplitude can be obtained and present data points to a value $\delta_{20}$ of order 25° or larger but this is very sensitive to the $\pi^0\pi^0$ branching ratio.

(2) A second strong phase ($\delta_P - \delta_0$), which appears to be quite large, is associated with the penguin and has in general both quasi-elastic and $c\bar{c}$ contributions. If ($\delta_0P - \delta_0$), which represents the difference between the quasi-elastic $I = 0$ phase for the penguin and that for the tree, is small then there must be a significant $c\bar{c}$ term. This represents the contribution of rescattering from $c\bar{c}$ states like $D\bar{D}$ or $D^*\bar{D}^*$. Thus present data suggests that both sources of strong phases may be significant.

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Figure 1: The two solutions for the strong phase difference $\delta_{20}$ as a function of the $\pi^0\pi^0$ branching ratio using central values for the other observables and two values of $\gamma$ (Solid line $\gamma = 67^\circ$, Dashed line $\gamma = 47^\circ$).