Strangeness Enhancement due to String Fluctuations

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We study string fragmentation in high multiplicity proton-proton collisions in a model where the string tension fluctuates. These fluctuations produce exponential pion spectra which are fitted to the transverse momentum distributions of charged particles for different multiplicities. For each multiplicity the so obtained hadronic slope parameter defines the magnitude of the string fluctuations which in turn determines the produced ratio of strange to light quarks. Pythia string simulation are used to convert each ratio of strange to light quarks to the appropriate ratio of strange hadrons to pions.

I. INTRODUCTION

Recently hadronic spectra with strange and multistrange hadrons were measured in pp collisions at the LHC [1]. With increasing multiplicity a strong enhancement of strangeness was observed. This result suggests collective processes in pp collisions which have been advocated for heavy ion collisions since long time ago [2, 3]. In this paper we want to present an alternative approach in line with work focussing on the string dynamics in low momentum hadron production. Our work parallels other recent work on modifications of string dynamics [4, 5, 6].

Previously we have shown that in AA collisions where many strings are produced, flux tube dynamics can influence the azimuthal symmetry of the produced hadrons [7]. In this work we did not relate the Gaussian momentum distribution of quarks obtained from the Schwinger model [8] and the tunneling mechanism of Casher, Neuberger, and Nussinov [9] with the observed exponential hadron spectra. This difference has been discussed in the literature [10] and the idea is that the fluctuations of the string tension (the energy per unit length of the tube) can account for the “thermal” distribution of hadrons. The spectrum of primarily produced hadrons would then be close to the maximum entropy distribution [11]. There would be no need for further collisions between partons to obtain the final form of the observed distribution. Such a mechanism could explain early thermalization. Obviously, pp collisions present good examples to test this hypothesis, since in these collisions the available interaction volume and interaction time is limited.

In this paper we will work out the details of this idea in three stages. We first calculate the effect of string tension fluctuations on the Gaussian transverse momentum spectrum of produced quarks in the Schwinger model. Then we take into account that the mean transverse momentum of the produced hadron arises from the transverse momenta of the produced quark and antiquark, i.e., it is larger than the transverse momentum of the quarks. As a second step, we fit this theoretical form of the spectrum to the observed hadron spectra. The data clearly indicate that the fluctuations of the string tension become larger with increasing hadron multiplicity. In the third step we use the ratios of produced strange to light quarks as input to a Pythia calculation of string fragmentation. This way each strange and multistrange hadron is calculated from the string fragmentation as implemented in the standard Pythia 8.2 code. The fluctuations with higher average string tension naturally produce relatively more strange quarks. The result of the third stage is then calculated as a function of the total charged-hadron multiplicity and compared with data.

II. HADRONIC FRAGMENTATION
SPECTRUM FROM A FLUCTUATING FLUX TUBE

The transverse spectrum of quarks produced in a flux tube by tunneling from vacuum is given by [9]

\[
\frac{dn}{d^2 p} = e^{-\frac{m_q^2}{\kappa}} ,
\]

(1)

where \( \kappa \) is the string tension, the energy stored in the tube per unit length, and \( m_\perp = \sqrt{m_q^2 + p_\perp^2} \) is the transverse mass of the quark. Conventionally, the quark masses which enter this formula are the constituent masses, i.e., for the light u and d quark mass \( m_q \) and for the strange quark \( m_s \) we use,

\[
m_q = 0.3 \text{ GeV} , \quad m_s = 0.5 \text{ GeV}.
\]

(2)

These quark masses are approximate, and we did not try to optimize their values for a best fit to the data. The transverse flux tube size varies as \( r^2 \propto 1/\kappa \). Defining \( \lambda^2 \equiv \kappa \) and following the suggestion of Bialas [10] we let \( \lambda \) fluctuate according to a Gaussian distribution. More general weight functions are possible. In the context of Tsallis distributions it has been suggested that gamma distributions are appropriate as weight functions [12]. As an example, fluctuations of the inverse string tension \( 1/\kappa \) according to a gamma distribution are studied in the appendix. Here we proceed with Gaussian fluctuations:

\[
P(\lambda) \, d\lambda = \sqrt{\frac{2}{\pi \mu}} e^{-\frac{\lambda^2}{2\mu}} \, d\lambda .
\]

(3)
The average string tension is
\[ \langle \kappa \rangle \equiv \langle \lambda^2 \rangle = \mu = \int_0^\infty \lambda^2 P(\lambda) \, d\lambda. \tag{4} \]

Taking into account the Gaussian fluctuations as described by Eq. \[3\] one obtains an exponential transverse momentum distribution for the produced quark or antiquark:
\[ \frac{dN}{dp_{\perp}^2} \propto e^{-\frac{2\pi (m_\perp^2 + p_{\perp}^2/\langle \kappa \rangle)}{\langle \kappa \rangle}}. \tag{5} \]

Of course after the quark production a complicated hadronic formation process takes place. The coalescence of produced quarks and antiquarks contributes to low momentum produced hadrons. At large momenta gluon radiation and subsequent fragmentation would be more appropriate. Quark antiquark coalescence leads to an average meson momentum \(p_\perp^2\) approximately twice as large as the quark average momenta:
\[ \langle p_\perp^2 \rangle = \langle p_{q,\perp}^2 \rangle + \langle p_{\bar{q},\perp}^2 \rangle. \tag{6} \]

This leads to the following spectrum for pions:
\[ \frac{dN}{dp_\perp^2} = N_0 e^{-\frac{2\pi (m^2_\perp + p^2_\perp/\langle \kappa \rangle)}{\langle \kappa \rangle}}. \tag{7} \]

The mean transverse momentum squared of the meson of this distribution equals the sum of the mean transverse momenta squared of the quark and antiquark. The spectrum does obey \(m\perp\) scaling, but with a meson mass which is 1.5 times higher than the constituent quark mass. This spectrum does not take into account chiral symmetry associated with the small mass of the pion and therefore leads to a worse quality fit at small momenta. But we think that the slope of the spectrum dominated by high momenta is a good indicator of the quark fragmentation dynamics underlying it. In principle large differences of quark momenta are influenced by the wave function of the meson which is not taken into account in the simple formula above. A full calculation including the folding of the quark distributions with the wave function of the meson would make the consequences of string fluctuations less transparent, therefore we rely on the simplified procedure.

III. FITTING HADRONIC SPECTRA FOR HIGH MULTIPlicITIES IN PP COLLISIONS

Recently the ATLAS collaboration has obtained charged hadron transverse momentum spectra in pp collisions at a center-of-mass energy 8 TeV in different charged-hadron multiplicity classes \[13\], reaching high average multiplicities of \(dN_{ch}/d\eta \approx 30\) corresponding to more than 4–5 times the multiplicity in minimum bias pp collisions at this collision energy \[14\]. The charged-hadron transverse momentum spectra start at \(p_\perp \approx 0.55\, GeV/c\). In order to determine the average string tension for each of the four multiplicity classes we fit the function of Eq. \[7\] in the range \(0.5 < p_\perp < 1.4\, GeV/c\) to the spectra measured. Translating this parameter into the quark spectrum Eq. \[3\] for strange and light quarks we obtain the ratios of produced strange to light quarks \(s\bar{s}/(u\bar{u} + d\bar{d})\). To estimate the total average charged-hadron multiplicity of each class we extrapolate the measured spectrum to \(p_\perp = 0\) using a Tsallis function \[15\] \[16\]. The results are summarized in Table I.

\begin{table}[h]
\begin{tabular}{cccc}
(dN_{ch}/d\eta)_{\eta=0} & \langle \kappa \rangle in GeV^2 & s\bar{s}/(u\bar{u} + d\bar{d}) \\
7.92 & 0.21 & 0.237 \\
11.87 & 0.22 & 0.243 \\
18.8 & 0.25 & 0.258 \\
31.7 & 0.29 & 0.275 \\
\end{tabular}
\caption{Charged-hadron multiplicities at mid-rapidity in pp collisions at 8 TeV, mean string tension in GeV$^2$, and the resulting ratio $s\bar{s}/(u\bar{u} + d\bar{d})$.}
\end{table}

In Fig. 1 we show the corresponding distributions of \(\lambda\), the square root of the string tension, for the four multiplicity classes. The curve for the lowest multiplicity is the most narrow. For higher multiplicities the widths of the Gaussian fluctuations increase. A higher average string tension corresponds to a smaller average diameter of the flux tube. The Schwinger mechanism takes into account pair creation by tunneling in a time-independent electric field, which is proportional to the average string tension \[9\]. The field itself is determined by the charges at the end of the string and the transverse area of the flux tube.
FIG. 2. The ratio of strange to light quark production, \( s\bar{s}/(u\bar{u} + d\bar{d}) \), produced in pp collisions vs charged-particle multiplicity.

which can fluctuate [17].

Fluctuations of the string tension to larger values lead to an increase of multiplicity of all hadronic species, but the relative increase is larger for strange hadrons than for pions. Besides, in high multiplicity pp collisions multiple flux tubes are produced which repel each other. This leads to squeezing of the tubes, i.e., their transverse dimension is reduced. Thus, during the production process the area shrinks due to the high density of surrounding strings. A repulsive force between strings can be easily understood because a merged string with higher charges at the end has a higher energy than two isolated 33 strings, as outlined in our previous paper [7] on anisotropies in heavy ion collisions. The enhanced average string tension makes it easier to produce the more massive strange quarks which can explain the observed relative strangeness enhancement observed in high multiplicity collisions.

An increase of the central multiplicity \( dN_{ch}/d\eta \) of produced charged hadrons by a factor of four leads to an increase of the average string tension by about 30%. This increase generates more quarks, and strange quarks are more affected than light quarks. In Fig. 2 we show the relevant increase of strange \( s\bar{s} \) relative to \( u\bar{u} + d\bar{d} \) quark production obtained from the integration of Eq. 5 over transverse momentum, inserting as quark masses the strange quark mass and the light \((u,d)\) quark masses of Eq. 2. Of course, the result depends on the constituent quark masses for the two flavors. We take the values of Eq. 2.

It is important to realize that averaging the string tension changes the quadratic mass dependence of the conventional Schwinger model into an exponential mass dependence. This change and the increase of the average string tension leads a reasonable ratio of strange to light quarks. In string fragmentation as modeled by Pythia 8.2 [18, 19] the value for the ratio of strange to light quarks (which is called StringFlav:probStoUD in the program) is

\[
\frac{s\bar{s}}{u\bar{u} + d\bar{d}} = 0.217. \tag{8}
\]

In our model we find for the lowest multiplicity pp collisions a ratio

\[
\frac{s\bar{s}}{u\bar{u} + d\bar{d}}_{\text{model}} = 0.237. \tag{9}
\]

IV. STRANGE HADRON ENHANCEMENT FROM PYTHIA SIMULATIONS

Having obtained ratios of strange to light quark production from the Schwinger model with fluctuating string tension we simulate the complicated hadronization process using the conventional string fragmentation. For this purpose we repeatedly simulate with Pythia 8.235 the decay of only one string spanned by a \( u\bar{u} \) quark pair where each quark has an energy of 0.5 TeV. The yields of different hadrons normalized to the yields for \( \pi^+ \) are shown in Figure 3 as a function of the strange-to-light quark ratio. One can see that the relative increase of the hadron to pion ratio becomes stronger with increasing strange content of the hadron. This is in qualitative agreement with the observations of the ALICE experiment [1].

In Fig. 4 we show the relative production ratios for different strange particles obtained from the simplified Pythia simulation of one string, where the input value for the strange-to-light quark production ratio (StringFlav:probStoUD) has been modified. Comparing with the data one sees that the trend for the \( h/\pi \) ratios is reproduced over four orders of magnitude, but the theory underestimates the absolute ratios. Please note that our Pythia simulations do not take into account a change of diquark production. We were not sure how to parametrize the necessary \( uu, ud, dd, su, sd, \) and \( ss \) diquark masses and how to implement them consistently in the code. This may explain the discrepancies.

V. CONCLUSIONS

We have presented a simple and schematic model for strangeness enhancement in high multiplicity pp collisions. We modified the result of the Schwinger model by letting the string tension \( \kappa \equiv \lambda^2 \) entering the Schwinger formula fluctuate according a Gaussian distribution for \( \lambda \). Using as experimental input the observed charged-particle transverse momentum spectra of hadrons at various multiplicities we determined the effective average string tension entering the Gaussian string fluctuations at each multiplicity. Higher multiplicities correspond to...
higher average string tension. This modified Schwinger model gives definite $s\bar{s}/(u\bar{u} + d\bar{d})$ ratios which then serve as inputs into simulations string fragmentation code using Pythia 8.2. The hadronic rates coming out of the code reproduce qualitatively the hadron production rates measured in the ALICE experiment [1].

Quark production is always suppressed exponentially. In the Schwinger model the exponent depends quadratically on the quark mass. In our modified model with a fluctuating string tension the suppression is only linearly in the exponent, thus enhancing the production of heavier quarks relative to light ones. We note that a scenario with merging flux tubes [5, 20] leading to a higher string tension can also enhance strangeness. It appears to us, however, that phenomenological fits [4] favor the linear mass dependence we generate in our model. Finally, the quite successful thermal models always include
a linear suppression of heavier flavor states in the exponent. In the thermal picture it remains a challenge to find good theoretical reasons for quick apparent thermalization in spite of the small volume created in pp collisions. An attempt in this direction has been made in reference [21] where for extremely short times such a suppression is obtained in the bosonized Schwinger model. Recently, a preliminary investigation [22] using a time dependent electric field created by color sources with different transversal structure give rates which after integration over all times yield a linear mass suppression in the exponent. In the future the study of pp and pA collisions with shorter reaction times extended over smaller volumes make it an experimental and theoretical task to understand the phenomenon of early “thermalization.”

As another main phenomenological question remains the necessity to simulate the baryon rates adequately, i.e., to modify the diquark rates and implement them in the code.

Notice that high multiplicities are usually reached due to multiple interactions, producing multi-string final state configurations. In terms of Glauber-like models this correspond to large number of unitary cut Pomerons in the case of pp collisions [23], or to large number of participants in the case of nuclear collisions [24]. Coherence in such collisions leads to the effects of shadowing and saturation which are stronger for light than for heavy flavors. This is why, e.g., the production rate of $J/\psi$ rises steeply with light hadron multiplicity [25]. Some enhancement remains for strangeness production, although not as pronounced as for charm. For the sake of clarity we ignored here these additional mechanisms of strangeness enhancement, but concentrated on the single string dynamics.

Appendix: Comparison of different weight functions for the fluctuating string tension

Here we show fits of two functional forms to the ATLAS charged-hadron transverse momentum spectra [13]. The first function, shown in the top panel of Fig. 5, is the one of Eq. 7 which we use to obtain the mean string tension $\langle \kappa \rangle$ given in Tab. I. The second function, used in the bottom panel, corresponds to fluctuations of $1/\kappa$ following a gamma distribution.

1. Gaussian fluctuations of $\lambda = \sqrt{\kappa}$

Using Eq. 4 and Eq. 3 the resulting quark transverse momentum spectrum (Eq. 5) is obtained as

$$\frac{dn}{d^2p_{q,T}} \propto \int_0^\infty e^{-\frac{\lambda^2}{2x^2}} P(\lambda) d\lambda. \quad (A.1)$$

Fig. 5 shows that the corresponding expression for the pion transverse momentum spectrum gives a decent description of the data in the range $0.5 \lesssim p_\perp \lesssim 1.5 \text{GeV/c}$.

2. Fluctuations of $1/\kappa$ following a gamma distribution

A much better fit of the data points at transverse momenta up to $3 \text{GeV/c}$ and above is obtained when the inverse of the string tension $\nu := 1/\kappa$ fluctuates accord-
ing to the gamma distribution, i.e.,
\[ P(\nu) = \gamma(\nu; \alpha, \theta) = \frac{e^{-\nu/\theta}(\nu/\theta)^{\alpha-1}}{\Gamma(\alpha)}. \]  
(A.2)

The quark transverse momentum spectrum in this case is given by
\[ \frac{dN}{d^2p_{T,q}} \propto \int_0^\infty e^{-\pi\nu_{m^2}} P(\nu) d\nu = A \left(1 + \pi\theta(m^2_q + p^2_{q,T})\right)^{-\alpha}. \]  
(A.3)

This bears some resemblance to the convolution of a Boltzmann-Gibbs distribution \(\exp(-E/T)\) with fluctuations of \(1/T\) following a gamma distribution which yield a Tsallis distribution [26]. Analogous to Eq. 7 the corresponding pion transverse momentum distribution is
\[ \frac{dN}{d^2p_{T,\pi}} = A \left(1 + \pi\theta(m^2_q + p^2_{q,T}/2)\right)^{-\alpha}. \]  
(A.4)

The fits are shown in Fig. 5 and the fit parameters are given in Tab. [??]. Interestingly, the values for the ratio \(s/s/(u\bar{u} + d\bar{d})\) are similar to the ones shown in Tab. [??], i.e., to the one obtained for the case of Gaussian fluctuations of \(\lambda\).

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