MATHMATICAL MODEL OF RHYTHMOCARDIOSIGNAL IN VECTOR VIEW OF STATIONARY AND STATIONARY-RELATED CASE SEQUENCES

Abstract. The paper deals with the substantiation of the mathematical model of rhythmocardiogram with high resolution in the form of a vector of stationary and stationary related random processes. The structure of probabilistic characteristics of this model for analysis of cardiac rhythm in modern cardiodiagnostic systems is investigated. Analysis of the heart rhythm makes it possible to evaluate not only the state of the cardiovascular system, but also the state of the adaptive capacity of the whole human body. Most modern systems of automated heart rate analysis are based on statistical analysis by a rhythmocardiogram, which is an ordered set of durations of R-R intervals in a registered electrocardiogram, be able to explore its temporal dynamics. To take into account the temporal dynamics of the rhythmocardiogram with high resolution, it is necessary to use a mathematical apparatus of the theory of random sequences, namely, to consider it as a vector of discrete random sequences. The purpose of this work is to solve the scientific and practical task of creating a mathematical model of rhythmocardiogram with high resolution in the form of a vector of stationary and stationary related random processes. The object of the study is information technology for the diagnosis and assessment of the status of the rhythmocardiogram to analyze the heart rhythm in modern cardiodiagnostic systems. The mathematical model of rhythmocardiogram with high resolution in the form of a vector of stationary and stationary related random sequences is substantiated. The structure of probabilistic characteristics of this model for analysis of cardiac rhythm in modern cardiodiagnostic systems is investigated.

Keywords: vector of steady-state and stationary-related random sequences; electrocardiogram; rhythm; cardiac rhythm.

Introduction

Analysis of the heart rhythm makes it possible to evaluate not only the state of the cardiovascular system, but also the state of the adaptive capacity of the whole human body. Most modern systems for automated cardiac rhythm analysis are based on statistical analysis by rhythmocardiogram, which is an ordered set of durations of R-R intervals in a registered electrocardiogram [1-8]. However, this approach is uninformative, since the R-R intervals reflect only the change in the duration of the cardiac cycles and not the totality of the time intervals between single-phase values of the electrocardio signal for all its phases.

Among the many varieties of environmental impacts, dust pollution from atmospheric air, which is formed as a result of receipt from sources of emissions at industrial enterprises (primary) and by physical and chemical processes in places of storage of pulverized wastes of production (secondary), among which special finely dispersed (<100 μm) saw dust is the place of disposal

Actuality of theme

In [9, 10], a new approach to its analysis of cardiac rhythm was developed on the basis of high resolution rhythmocardiogram. As noted in these papers, the classical rhythmocardiogram is embedded in a high-resolution rhythmocardiogram, which is the basis for increasing the level of informativeness of the heart rhythm analysis in modern computer systems of functional diagnostics of the human heart state with the increased rhythmocardiogram.

In [9, 10] it is justified to use a vector of random variables as a mathematical model of rhythmocardiogram with high resolution. However, this model is a relatively poor mathematical model of rhythmocardiogram with high resolution, since it does not allow to study its temporal dynamics. To take into account the temporal dynamics of the rhythmocardiogram with high resolution, it is necessary to use a mathematical apparatus of the theory of random sequences, namely, to consider it as a vector of discrete random sequences.

Purpose and tasks of the work. The purpose of this work is to solve the scientific and practical task of creating a mathematical model of rhythmocardiogram with high resolution in the form of a vector of stationary and stationary related random processes. The object of the study is information technology for the diagnosis and assessment of the status of the rhythmocardiogram to analyze the heart rhythm in modern cardiodiagnostic systems, presenting main material.

Main part

One of the simplest stochastic models that takes into account the dynamics of high resolution rhymocardial is the vector xed and stationary random sequences. In this vector, the index indicates the cycle number of the electrocardio signal, and the index indicates the reference number of the electrocardio signal within its cycle. The number of counts per cycle of the electrocardio signal determines the resolution of the rhythm cardio signal and sets the number of phases in the cycle of the electrocardio signal that can be separated by methods of segmentation and detection in solving the problem of automatic formation of the rhythm cardio signal from the electrocardio signal.

Let us now proceed to justify the probabilistic characteristics of a random sequence vector $\Xi_I(o', m)$. One of the simplest stochastic models that takes into account the dynamics of high resolution rhymocardial is the vector $\Xi_I(o', m) = \{\xi_l(o', m), o' \in 0, l = 1, \ldots, m \in Z\}$ fixed and stationary random sequences. First of all, we note that the vector $\Xi_I(o', m)$ of stationary and stationary coupled random sequences, in the particular
case if its constituents are stationary sequences with independent values, that is, white noises given on the set of integers, is a known rhythmocardiogram signal model, in the form of a random variable vector developed in [9, 10]. However, in practice, the hypothesis of the independence or non-correlation of the rhythm cardiogram counts is not true, requiring a stochastic dependence between the rythocardiogram counts with higher resolution, and hence the use of a more complex and general mathematical model \( \mathbf{E}_L \) stationary and stationary random sequences, then it has equality:

\[
F_{p_{\mathbf{m}_1}, \mathbf{m}_p} (x_1, ..., x_p, m_1, ..., m_p) =
\]

of the family \( p \ (p \in \mathbb{N}) \) of stationary distribution functions of stationary and stationary related random sequences, the following equality holds:

\[
F_{p_{\mathbf{m}_1}, \mathbf{m}_p} (x_1, ..., x_p, m_1, ..., m_p) =
\]

\[
= F_{p_{\mathbf{m}_1}, \mathbf{m}_p} (x_1, ..., x_p, m_1 + k, ..., m_p + k),
\]

\( x_1, ..., x_p \in \mathbb{R}, m_1, ..., m_p \in \mathbb{Z}, l_1, ..., l_p \in \{1, L\}, k \in \mathbb{Z}. \)

Distribution function

\[
F_{p_{\mathbf{m}_1}, \mathbf{m}_p} (x_1, ..., x_p, m_1, ..., m_p)
\]

when \( l_1 = l_2 = ... = l_p = l \) is distribution function

\[
F_{p_{\mathbf{m}_1}, \mathbf{m}_p} (x_1, ..., x_p, m_1, ..., m_p),
\]

is a distribution function \( T_l (\omega', m) \), vector \( \mathbf{E}_L \) stationary and stationary related random sequences, the following equality holds:

\[
F_{p_{\mathbf{m}_1}, \mathbf{m}_p} (x_1, ..., x_p, m_1, ..., m_p) =
\]

\[
= F_{p_{\mathbf{m}_1}, \mathbf{m}_p} (x_1, ..., x_p, m_1 + k, ..., m_p + k),
\]

\( x_1, ..., x_p \in \mathbb{R}, m_1, ..., m_p \in \mathbb{Z}, l_1, ..., l_p \in \{1, L\}, k \in \mathbb{Z}. \)

In the case where equality \( l_1 = l_2 = ... = l_p = l \) is not executed then the distribution function

\[
F_{p_{\mathbf{m}_1}, \mathbf{m}_p} (x_1, ..., x_p, m_1, ..., m_p)
\]

is a compatible distribution function for several (at least two) stationary components of a vector \( \mathbf{E}_L \) stationary and stationary related random sequences, describing the time distances between single-phase electrocardiogram readings for it \( l \) phases. In particular, if \( p = 1 \), then we will have one-dimensional \( F_{1_l} (x, m) \) autofunction of stationary random sequence distribution \( T_l (\omega', m) \).

In practice, for analysis of high-resolution rhythm, it is reasonable to use mixed high-order moment functions, namely, mixed second-order initial moment functions - covariance functions and mixed second-order central moment functions - correlation functions. In this case, the initial second-order moment functions for the vector \( \mathbf{E}_L \) stationary and stationary related random sequences are represented as a matrix of covariance functions:

\[
C_T =
\]

\[
= \begin{bmatrix}
  c_{2T_{\mathbf{m}_1} (m_1, m_2)} & c_{2T_{\mathbf{m}_1} (m_1, m_2)} \\
  c_{2T_{\mathbf{m}_1} (m_1, m_2)} & c_{2T_{\mathbf{m}_1} (m_1, m_2)} \\
  & \vdots & \ddots & \vdots \\
  c_{2T_{\mathbf{m}_1} (m_1, m_2)} & c_{2T_{\mathbf{m}_1} (m_1, m_2)}
\end{bmatrix}
\]
which can be more compactly submitted as follows:

\[
C_T = \left[ c_{2\hat{\eta}_1\hat{\eta}_2} (m_1,m_2), l_1,l_2 = 1,L \right].
\] (5)

where each of its elements is a covariance function
\[ c_{2\hat{\eta}_1\hat{\eta}_2} (m_1,m_2), \] which is given so:

\[
c_{2\hat{\eta}_1\hat{\eta}_2} (m_1,m_2) = M \left\{ T_{\hat{\eta}_1} (\omega', m_1) \cdot T_{\hat{\eta}_2} (\omega', m_2) \right\},
\]

\[
m_1,m_2 \in \mathbb{Z}, l_1,l_2 \in \left\{ 1,L \right\}.
\] (6)

Since the components of a random sequence vector \( \Xi_L (\omega', m) \) are stationary and stationary related sequences, their covariance functions are functions of only one integer argument \( u \), which is equal. Therefore, the covariance matrix of this random vector can be represented as follows:

\[
C_T = \left[ c_{2\hat{\eta}_1\hat{\eta}_2} (u), l_1,l_2 = 1,L \right],
\] (7)

where each of its elements is a covariance function
\[ c_{2\hat{\eta}_1\hat{\eta}_2} (u) \] which is equal to:

\[
c_{2\hat{\eta}_1\hat{\eta}_2} (u) = c_{2\hat{\eta}_1\hat{\eta}_2} (m_1-m_2),
\]

\[
u, m_1,m_2 \in \mathbb{Z}, l_1,l_2 \in \left\{ 1,L \right\}.
\] (8)

Provided that \( l_1 = l_2 = l \), the covariance function
\[ c_{2\hat{\eta}_1\hat{\eta}_2} (u) \] is an autocovariance function \( l \) stationary components \( T_l (\omega', m) \) vector \( \Xi_L (\omega', m) \), which describes the time distances between single-phase electrocardiogram readings for it \( l \). If \( l_1 \neq l_2 \), then the covariance function \( c_{2\hat{\eta}_1\hat{\eta}_2} (u) \) function for two stationary components of a vector \( \Xi_L (\omega', m) \), describing the time distances between single-phase electrocardiogram readings for \( l_1 \) and \( l_2 \) its phases. Mixed central second-order moment functions for a vector \( \Xi_L (\omega', m) \) stationary and stationary related random sequences are presented as a matrix of correlation functions:

\[
R_T = \begin{bmatrix}
  r_{\hat{\eta}_1\hat{\eta}_2} (m_1,m_2) & \cdots & r_{\hat{\eta}_1\hat{\eta}_2} (m_1,m_2) \\
  r_{\hat{\eta}_2\hat{\eta}_2} (m_1,m_2) & \cdots & r_{\hat{\eta}_2\hat{\eta}_2} (m_1,m_2) \\
  \vdots & \ddots & \vdots \\
  r_{\hat{\eta}_p\hat{\eta}_1} (m_1,m_2) & \cdots & r_{\hat{\eta}_p\hat{\eta}_p} (m_1,m_2)
\end{bmatrix},
\] (9)

which can be presented more compactly so:

\[
R_T = \left[ r_{\hat{\eta}_1\hat{\eta}_2} (m_1,m_2), l_1,l_2 = 1,L \right].
\] (10)

where each of its elements is a correlation function
\[ r_{\hat{\eta}_1\hat{\eta}_2} (m_1,m_2), \] which is given so:

\[
c_{2\hat{\eta}_1\hat{\eta}_2} (m_1,m_2) =
\]

Because the components of the vector \( \Xi_L (\omega', m) \) random sequences are stationary and stationary sequences, their correlation functions are functions of only one integer argument \( u \), which is equal to \( u = m_1 - m_2 \). Therefore, the correlation matrix of this random vector can be represented as follows:

\[
R_T = \left[ r_{\hat{\eta}_1\hat{\eta}_2} (u), l_1,l_2 = 1,L \right],
\] (12)

where each of its elements is a correlation function
\[ r_{\hat{\eta}_1\hat{\eta}_2} (u) \] which is equal to:

\[
r_{\hat{\eta}_1\hat{\eta}_2} (u) = r_{\hat{\eta}_1\hat{\eta}_2} (m_1-m_2),
\]

\[
u, m_1,m_2 \in \mathbb{Z}, l_1,l_2 \in \left\{ 1,L \right\}.
\] (13)

Provided that \( l_1 = l_2 = l \), correlation function
\[ r_{\hat{\eta}_1\hat{\eta}_2} (u) \] is an autocorrelation function \( l \) stationary components \( T_l (\omega', m) \) vector \( \Xi_L (\omega', m) \), which describes the time distances between single-phase electrocardiogram readings for him \( l \) phases. If \( l_1 \neq l_2 \), then the correlation function \( r_{\hat{\eta}_1\hat{\eta}_2} (u) \) is a reciprocal correlation function for two stationary components of a vector \( \Xi_L (\omega', m) \), describing the time distances between single-phase electrocardiogram readings for \( l_1 \) and \( l_2 \) phases.

Fig. 1-4 shows the results of statistical processing of the rhythm cardio signal with high informativeness, by statistical evaluation of its corresponding statistical characteristics.

**Fig. 1. Several cycles of the investigated electrocardio signal**

**Conclusions**

The mathematical model of rhythmocardiogram with high resolution in the form of a vector of stationary and stationary related random sequences is substantiated. The structure of probabilistic characteristics of this model for analysis of cardiac rhythm in modern cardiodiagnostic systems is investigated.
Heart rate variability: measurement standards, indicators, features of the method

- The first component $T_1(\omega', m)$ and the second component $T_2(\omega', m)$, describing duration respectively:
  - $P$, electrocardio signal intervals;
  - $R$, electrocardio signal intervals

- The first component $T_{1u}(m)$ and the second component $T_{2u}(m)$, describing duration respectively:
  - $P$, electrocardio signal intervals;
  - $R$, electrocardio signal intervals

**Fig. 2.** Schedule of sales $T_{1u}(m)$, $T_{2u}(m)$ component of the vector
rhythmocardiogram of the first component $T_1(\omega', m)$ and the second component $T_2(\omega', m)$, describing duration respectively:
- $P$, electrocardio signal intervals;
- $R$, electrocardio signal intervals

**Fig. 3.** Implementation histograms $T_{1u}(m)$, $T_{2u}(m)$ component of the vector rhythmocardiogram of the first component $T_1(\omega', m)$ and the second component $T_2(\omega', m)$, describing duration respectively:
- $P$, electrocardio signal intervals;
- $R$, electrocardio signal intervals

**Fig. 4.** Schedule of sales $\hat{r}_{2N}(u)$ statistical estimates of autocorrelation functions $r_{2N}(u)$, describing duration respectively:
- $P$, electrocardio signal intervals;
- $R$, electrocardio signal intervals

**REFERENCES**

1. Baevsky R.M., Kirillov O.I. and Kletskin S.Z. (1984), *Mathematical analysis of changes in heart rate during stress*, Science, Moscow, 222 p.
2. (1999), “Heart rate variability. Standards of measurement, physiological interpretation and clinical use. Working Group of the European Cardiology Society and the North American Society of Stimulation and Electrophysiology”, *Bulletin of Arrhythmology*, Issue 11, pp. 52-77.
3. Resurrection A.D. and Wentzel M.D. (1974), “Statistical analysis of cardiac rhythm and indicators of hemodynamics in physiological studies”, *Problems of space biology*, Moscow, p. 42.
4. Zarubin, V.F. (1998), “Heart rate variability: measurement standards, indicators, features of the method”, *Bulletin of arrhythmology*, Issue 10, pp. 25-30.
5. Kalakutsky, L.I. and Manelis E.S. (2001), *Monitoring of heart rate variability parameters in critical state medicine*, Engineering-Medical Center “New Devices”, Samara Medicine, Pharmacy # 14.
6. Ragozin A.N. (2020), “Spectral analysis of heart rate variability on the plane of complex frequencies”, *Ural Cardiology Journal*, Vol. 1, pp. 18-25.
7. Ragozin A.N. and Kononov D.Yu. (1999), “Analysis of the spectral structure of multichannel physiological signals”, *Digital radioelectronic systems*, electronic journal, Issue 3, available to: [http://www.prima.tu-chel.ac.ru/drj/](http://www.prima.tu-chel.ac.ru/drj/)
8. Ryabkyina G.V. and Sobolev A.V. (1998), *Heart rate variability*, Starco, Moscow.
9. Lupenko, S., Lutsyk, N., Yasnyi, O. and Sobaszek, L. (2018), “Statistical analysis of the human heart with increased informativeness”, *Acta mechanica et automatica*, vol. 12, pp. 311–315.
10. Lupenko, S., Lutsyk, N., Yasnyi, O. and Zozulia A. (2019), “Modeling and Diagnostic Features in Computer Systems of Heart Rhythm Analysis with Increased Informativeness”, *2019 9th International Conference on Advanced Computer Information Technologies (ACIT)*, IEEE, pp. 121-124.

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ABOUT THE AUTHORS / ВІДОМОСТІ ПРО АВТОРИВ

Литвиненко Ярослав Володимирович – доктор технічних наук, доцент, доцент кафедри комп’ютерних наук, Тернопільський національний технічний університет імені Івана Пулюя, Тернополь, Україна;

Литвиненко Ярослав Володимирович – Doctor of technical sciences, Associate Professor, Associate Professor of Computer Science Department, Ternopil Ivan Puliyu National Technical University, Ternopil, Ukraine;
e-mail: lytvynenko@tntu.edu.ua; ORCID ID: [http://orcid.org/0000-0001-7311-4103](http://orcid.org/0000-0001-7311-4103)

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Лупенко Сергій Анатолійович – доктор технічних наук, професор, професор кафедри комп’ютерних систем та мереж, Тернопільський національний технічний університет імені Івана Пулюя, Тернопіль, Україна;
Sorhei Lupenko – Doctor of technical sciences, Professor, Professor of Computer Systems and Networks Department, Ternopil Ivan Pulyuy National Technical University, Ternopil, Ukraine;
e-mail: lupenko.san@gmail.com; ORCID ID: http://orcid.org/0000-0002-6559-0721

Ониськік Петро Анатолійович (Петро Ониськік) – аспірант кафедри комп’ютерних наук, Тернопільський національний технічний університет імені Івана Пулюя, Тернопіль, Україна;
Petro Onysyk – postgraduate of Computer Science Department, Ternopil Ivan Pulyuy National Technical University, Ternopil, Ukraine;
e-mail: petro.onysyk95@gmail.com; ORCID ID: http://orcid.org/0000-0002-9717-4538

Триснюк Василь Миколайович – доктор технічних наук, старший науковий співробітник, завідувач відділу досліджень навколишнього середовища, Інститут телекомунікацій і глобального інформаційного простору, Київ, Україна;
Vasyl Trysnyuk – Doctor of technical sciences, Senior Research, Head of the environmental research department, Institute of Telecommunications and Global Information Space of NAS of Ukraine, Kyiv, Ukraine;
e-mail: trysnyuk@ukr.net; ORCID ID: http://orcid.org/0000-0001-9920-4879

Зозуля Андрій Миколайович – науковий співробітник, Інститут телекомунікацій і глобального інформаційного простору, Київ, Україна;
Andrіy Zozуля – Research Associate, Institute of Telecommunications and Global Information Space of NAS of Ukraine, Kyiv, Ukraine;
e-mail: bestguru@gmail.com; ORCID ID: http://orcid.org/0000-0003-1582-3088

Математична модель ритмокардиосигналу у вигляді вектора стационарних та стационарно пов’язаних випадкових послідовностей
Я. В. Литвиненко, С. А. Лупенко, П. А. Ониськів, В. М. Триснюк, А. М. Зозуля

Анотація. Робота присвячена обґрунтуванню математичної моделі ритмокардиосигналу із підвищеною роздільною здатністю у вигляді вектора стационарних та стационарно пов’язаних випадкових процесів. Досліджено структуру ймовірнісних характеристик цієї моделі для аналізу серцевого ритму у сучасних системах кардіодіагностики. Метою роботи є розв’язання науково-практичного завдання створення математичної моделі ритмокардиосигналу з підвищеною роздільною здатністю у вигляді вектора стационарних та стационарно пов’язаних випадкових процесів. Об’єкт дослідження є інформаційні технології для діагностики і оцінки стану ритмокардиосигналу для аналізу серцевого ритму у сучасних системах кардіодіагностики. Для досягнення цієї мети використано структурно умовні характеристики цієї моделі для аналізу серцевого ритму у сучасних системах кардіодіагностики.

Ключові слова: вектор стационарних та стационарно-пов’язаних випадкових послідовностей; електрокардиосигнал; ритмокардиосигнал; серцевий ритм.

Математическая модель ритмокардиосигнала в виде вектора стационарных и стационарно связанных случайных последовательностей
Я. В. Литвиненко, С. А. Лупенко, П. А. Ониськів, В. М. Триснюк, А. Н. Зозуля

Аннотация. Работа посвящена обоснованию математической модели ритмокардиосигнала с повышенной разрешающей способностью в виде вектора стационарных и стационарно связанных случайных процессов. Исследована структура вероятностных характеристик этой модели для анализа сердечного ритма в современных системах кардиодиагностики. Объект исследования является информационные технологии для диагностики и оценки состояния ритмокардиосигнала для анализа сердечного ритма в современных системах кардиодиагностики. Для достижения этой цели использовалась структурно условные характеристики этой модели для анализа сердечного ритма в современных системах кардиодиагностики.

Ключевые слова: вектор стационарных и стационарно-связанных случайных последовательностей; электрокардиосигнал; ритмокардиосигнал; сердечный ритм.