0.1 Invariance for change of local coordinates

0.1.1 Introduction

In early global geometry, global means invariant for changes of local coordinates. We refer here to a global regular model, when the model has only regular approximations. Later, in global geometry, existence of regular approximations was discussed, we refer here to global very regular models. It has been proved ([17]) that a global regular model is independent of choice of local coordinates, given a strong topology. We will discuss dependence of choice of coordinates in some other cases.

Assume \( U \in G \) a change of coordinates in a Lie group \( G \) and \( T \in H \), a space of distributions. We start with \( \langle T, U_j I \rangle = \langle U^\ast j T, I \rangle \), where we assume \( U^\ast H \subset H \). Given \( \Sigma f_0 * \tau_j \delta_0 \) represents a function in \( L^1 \) and when \( \langle,\rangle \) is given by convolution, the scalar product can be used to generate \( H = L^1 \) through \( \langle Uf, I \rangle = \int fdU^\ast \) and \( dU^\ast \perp I \) in \( \infty \). We assume a separation condition, for instance \( \{ U \phi < \lambda \} \subset \subset B \), for constants \( \lambda \). A Schwartz type separation condition ([11]) gives that given \( U \) locally reduced, \( U^\ast \) is locally surjective. Given \( dU \to dU^\perp \) is given by a subnuclear mapping, convexity is preserved. When we assume \( E' \) is generated by \( G \), through a Stieltjes integral, we must assume \( E' \) is nuclear and \( \langle Uf, \phi \rangle = \langle f, dU >, \phi \rangle \).

0.1.2 Order of change of local coordinates

Assume \( dU \) BV (of bounded variation) on \( \Omega = \{(x, y) \mid \alpha \} \), a neighborhood of 0, for some constant \( \lambda \) and consider the convex hull \( T(\Omega_\lambda) \). We assume there exists a restriction \( d\tilde{U} \) with \( \tilde{\Omega}_\lambda = \{ (x, y) \mid d\tilde{U} \} \subset \subset \Omega_\lambda \). Thus we have on \( \tilde{\Omega}_\lambda \), that \( d\tilde{U} \) is invertible and Lebesgue is absolute continuous relative \( d\tilde{U} \), that is \( d\tilde{U} = dx/\alpha \), where \( \alpha \in L^1(d\tilde{U}) \). Schwartz condition means that \( \tilde{\Omega}_\lambda \to T(\Omega_\lambda) \) is compact. Assume \( U, U^\perp \in G \), where \( U^\perp \) is the result of \( \psi : U \to U^\perp \) with respect to \( \psi : U \to \tilde{U} \), completed to \( H \). We will discuss \( \psi \in \mathcal{L}_c(E, F) \), for locally convex and separated topological vector spaces \( E, F \). The topology is given by uniform convergence over convex, compact equilibrated sets ([16]). Consider \( W = \{ \phi \mid 0 < |U - U^\perp| (\phi) < \epsilon \} \), for a small positive \( \epsilon \) and the convex hull \( T(W) : \{ \phi \mid |U - U^\perp| (\phi) < \epsilon \} \). Assume existence of a regular approximation, for instance \( I \in \mathcal{L}_c(T(W), F) \), but \( I \notin \mathcal{L}_c(W, F) \). Given \( (U, U^\perp) \) preserves reducedness, we have that \( U^\perp \prec \prec U \), that is \( U \not= U^\perp \) in \( \infty \) and \( I \notin \mathcal{L}_c \) in \( \infty \). Note that, given \( U - U^\perp \) analytic over \( \phi \), we can consider...
\( \phi(u, u') \), as change of local coordinates. Example: given \( P \) corresponds to a partially hypoelliptic operator and \( \mathcal{L}_0 \) is the topology for convergence relative \( dU \) BV over convex and radical domains. If further \( K \) corresponds to a parametrix to \( P \), we have \( K(\langle P \phi \rangle) = 0 \) is non-trivial, but \( K_N(\langle P^N \phi \rangle) = 0 \) is trivial, that is \( \phi \in C^\infty \). Thus, \( I \in \mathcal{L}_{BV,convex,equilibrated} \) and \( I \notin \mathcal{L}_{BV,convex} \).

**Maximal Order 0.1** Given \( f, g \in \mathcal{H} \), consider a chain \( L_g(f, g) \) between \( f \) and \( g \). The number of movements that define a chain is referred to as the order of the chain. For instance, consider for \( G \in \mathcal{H} \), \( G(u_1, \ldots, u_k) \) over \( L_g \), then given \( \frac{SG}{u_j} \neq 0 \) for some \( j \), we have \( dG \neq 0 \). The chain has maximal order, \( k \), if \( \mathcal{H} \) can be generated by chains of order \( k \).

Assume \( A \) are chains in \( \mathcal{G} \) maximal relative \( \mathcal{H} \) and (I), an ideal of functions regular and reduced over \( A \). Assume in particular \( A \) algebraic over (I). Since \( A \cap N(I) \) is algebraic, we have an analytic complement. We can assume for invariant sets in the complement, that they can be given by \( dU_j = 0 \), \( U_j \in \mathcal{G} \). Thus, for \( \text{rad} \) (I) we have partial algebraicity and the approximation property holds partially. Consider for instance \( f(u_1, \ldots, u_k) \), with \( dU_k = \alpha_k dU_1 \) and \( \alpha_k \in L^1(dU_1) \), then existence of regular changes of coordinates is implied by the strict approximation property. Note that precise change of coordinates through a maximal chain with invariant sets, is sufficient for \( f \) to be non-reduced. On the other hand, if \( \mathcal{H} \) is generated by \( \mathcal{G} \) over a base \( f_a \in L^1 \) and \( \mathcal{G} \) can be reduced to order 0, it is sufficient to consider translation.

Consider \( U^+ = (U - I) + V \) completed to \( L^1 \). We have that \( U \in \mathcal{G} \) implies \( U^{-1} \in \mathcal{G} \), but it is not necessarily reduced. Assume \( U^+ = 0 \) implies \( U = I - V \) with \( R(V) \subset C^\infty \), then we have that \( U \) is topologically reduced in \( \mathcal{D}^F \), that is locally reduced modulo \( C^\infty \).

Given two uniform spaces \( X, Y \) (\( \mathbb{R} \)), we have \( Id : X \to Y \) uniformly continuous iff \( X \subset Y \). Consider \( (U, U^+) \), where we assume for some open set \( \Omega \), that neighborhoods of \( R(U) \cap \Omega \), are formed by movements of the same character as \( U, U^+ \). Assume \( X^+_1 = R(U)^+, R(U^+)^+ = X_2 \) are uniform then \( \psi : X^+_1 \to X_2 \) uniformly continuous iff \( X^+_1 \subset X_2 \), that is \( U^+ \) surjective. In particular, we have this if \( R(U) \) has the approximation property and we consider analytic functions \( \tilde{g} \) with compact sub-level surfaces \( \sqcap R(U) \).

Assume \( \Delta(f) \) is the lineality, that is the set of translation invariant lines for \( f \). Invariant sets for \( f \) can be derived from invariant sets for \( |f| \) using uniformities. Assume \( \psi : \Delta(f) \to \Delta(|f|) \) with image a uniformity. Assume \( B \) the set where \( \Delta(f) \simeq \Delta(|f|) \), where \( B \) is the uniformity that is given by the inverse to \( \psi \). Through Hurwitz (I), given the mapping is analytic, we have \( \Delta(f) = \Delta(|f|) \) over \( B \). We note that in the plane \( (|f|, |f|) \), then \( \mathcal{G} \) can be given by at most 8 movements (II). Note that given \( U - I \) analytic over \( f \) and \( U \) absolute continuous with \( (Uf)' = Vg' \), then we have \( dU(f) = dI(f) \) iff \( Vg' = g' \).

Given \( dU^+ = -\rho dU \), with \( \rho \in L^1(dU) \) and \( U \to U^+ \) subnuclear , we have that \( dU^+ \) is absolute continuous relative \( dU \). When \( F \) is ac and symmetric with respect to \( U, U^+ \), that is \( F \sim \int dF(U, U^+) \), we have that \( dF = (1 - \rho)dU^+F \), that is \( dF \equiv 0 \) over spirals (\( F = \text{const} \)).

Note that linear independence is not necessary for a global model, it is sufficient with a global pseudobase (III), such that for instance \( \text{ord} \{f_1 = f_2 \} \) finite. If \( df_2 = p_1 df_1 \), given \( p_1 \in L^1 \) and Schwartz type separation conditions,
we have that $p_1 = 1$ is removable. Note that $dU_j = p_j dU_1$, why we do not have necessarily that $U_j \rightarrow U$ preserve order (cf. $< f, \tilde{g} >$).

Assume $U, V$ linearly independent on a domain $\Omega$, where both movements are analytic. Assume $U f_j = V f_j$ with uniform convergence on compact sets, then we have through Hurwitz ([11], $(U - V) f_j \equiv f_j (u) - f_j (v)$ and $f (u) = f (v)$ or $f (u) \neq f (v)$ locally in $(u, v)$. Given monodromy, in the first case, we can identify $u - v = 0$ locally. More generally, if $U + V = I$ in $\mathcal{L}_{ac}$ (over absolute continuous functions) and $d(U + V) = (\alpha + \beta) dx$, $(\alpha + \beta) f \in \mathcal{L}^1_{ac}$, we can conclude that $U \simeq I - V$ in $\mathcal{L}_{ac}$.

### 0.1.3 Regular continuation

Consider $dU = \alpha dV$, with $\alpha \rightarrow 0$ in $\infty$, this defines a convergence domain over lines. Given $U$ monotonous, we can consider $U_{ac} + U_{sng}$, where we assume that $dU_{sng} = 0$ defines a regular complement. Assume $\left. U \right| T \simeq \left. VT \right|$, where $V, U$ have the same character. We then have, given $U$ algebraic, that $\left. U \right| I = I U$, that is $\left. I \right| = \left. U \right|$. Sufficient for this is analyticity and linear independence. Existence of $\left. dU \right| = (\alpha + \beta) dx$, $(\alpha + \beta) f \in \mathcal{L}^1_{ac}$, we can conclude that $\left. U \right| T \simeq \left. V \right|$ and the approximation property, the conclusion is that $L^p$ is generated by translation and rotation outside the polar.

Assume $dU = \alpha dV$, such that $1/\alpha \rightarrow 0$ iff $R(U) < R(V)$. According to Riesz-Thorne: ([10]) given $\frac{1}{\alpha} = 1 - \frac{q}{p}$, we have that $\forall f \in L^p$, there is $g \in L^1$ and $h \in L^2$, with $f = g + h$, that is $L^p \subset L^1 + L^2$. Assume $d(A + B) = (\alpha + \beta) dU_1$, where $1/(\alpha + \beta) \in L^1(dU_1)$. When we assume $(A + B)^{-1} : L^1 \rightarrow L^p$, we must have $1/(\alpha + \beta)p \rightarrow 0$. Conversely, given that $(A + B) : L^2 \rightarrow L^1$, we have that $(A + B)^{-1} g = (C + D) f = f \in L^2$, that is we can choose $C = I - D$. Assume $1/(\alpha + \beta)p \rightarrow 0$ in $\infty$. Sufficient for this is that $\alpha = p/q$ and $\beta = q/p$ under the condition that $\alpha \rightarrow 0$ or $\beta \rightarrow \infty$, for instance that $p/q \in \text{rad} (I_{reg})$. In particular, consider $(\alpha + \beta)p \rightarrow 0$ in $\infty$, such that we have existence $(\alpha \beta > 0)$.

Assume $\hat{R}(U)^{\perp} \cup R(U_j)^{\perp}$, with $R(U_j) \subset R(U)$, that is the leafs are defined by $U_j^{\perp}$. Assume further that the completion to $L^p$ preserves character, sufficient for this is analyticity and linear independence. Existence of $U_j^{\perp}$, can be motivated by annihilator theory. Considering iterations, we assume $U^{\perp} = V f$, for a conjugated $V$. In this manner $U^{\perp}$ is defined as multivalued.

Concerning monotropy, assume $\log f \in L^1$, such that we have existence $f_x \in C^\infty$ with $| f_x - f | < \epsilon$. Then $U$ is algebraic over $f_x$. Further, given $d\log f = \alpha \in C^\infty$, then we have that given $d^2 f = 0$ and $\alpha' = 0$ on closed curves $\sim 0$, that $ad f$ defines a closed form. Assume $dU^{\perp} = \gamma dU$ with $\gamma \in D_L(dU)$, given $\gamma' = 0$ denotes max points on $R(d U)$, we have that $| dU^{\perp} | \leq C | dU |$ on $R(dU)$. Given the property of approximation through truncation, $\gamma$ can be used as a continuous deformation, that is $U^{\perp} \sim U$. Existence of $U$ such that $\forall f, g \in H, U f = g$ is a lifting principle, that is $L^1$ has a lifting principle relatively $U_1, U_2$. Assume $\{dU\} \simeq \tilde{E} \simeq \mathcal{G}$, where $\tilde{E}$ is a domain for $U$ analytic in $\mathcal{G}$, that is $\tilde{E}$ Hausdorff, we have $\overline{\mathcal{P}(E)} \simeq L^p(\tilde{E})$ ([16]). We can consider the continuation to $F$ such that $U$ is algebraic over $F$, in particular $U = I$ $U$, that is $P_1 \leq U \leq P_2$ locally over $F$. 

3
0.1.4 Conjugated invariant sets

Assume $F$ symmetric with respect to the reflection axes $L_1, L_2$ and that $L_1 \rightarrow L_2$ is bijective, then we have that $F$ is symmetric. More precisely $(x, y)L_1(x_1, y_1)L_2(y, x)$, for instance $z \rightarrow z^* \rightarrow z^*$, where $F$ is separately symmetric, that is $F(z) = F(z^*)$ and $= F(z^*)$. In the case where we do not have a bijection $L_1 \rightarrow L_2$, for instance $z \rightarrow 1/z$ is not bijective, we use hypo continuous $F$. Under a hypo continuous mapping, the image of products of equi continuous sets, is relatively compact (16)). Consider $\sigma - \mu$ decomposable sets. For instance $d\mu(L_2) = \alpha(L_1 \rightarrow L_2)d\sigma(L_1)$ (17) Lie factorization over conjugated transformations). In particular $L_2 = L_1^*$, that is $d\mu(x^*) = \alpha(x \rightarrow x^*)d\sigma(x)$ (9) contact transforms, Lie). Note that given $T - I$ nuclear, we have that $IT - I = IT - I$. If also $I$ is nuclear, then $IT$ is nuclear.

**Conjugated semi-norm 0.1** Assume $p, q$ semi-norms, with $q \leq p$. Assume existence of a semi-norm $h$, with $h^2 = 0$ algebraic and $\mid p(x) - h(1/x) \mid < \epsilon$ in $\infty$, (if $p$ preserves constant value in $\infty$). Then we have given $U^\perp = (I - U)^+V$, that $q(U^\perp) \leq \Sigma_{h(U^j)} + p(V) + \epsilon$, as long as $h(U^j) \rightarrow 0$, when $j \uparrow \infty$ and we have $\mid q(U^\perp) - h((I - U)^{-1})\mid < \epsilon$, given $p(V) = 0$.

When $U \rightarrow U^\perp$ is considered as conjugation, we assume $dU^\perp/dU \neq 0$. Assume further, for $T \in E'$, that $\Delta$ is given in $\mathcal{G}$, with $< T, \varphi > \gg \Delta T, \psi >$, for $\varphi, \psi \in \mathcal{B}$ (16). Given $\Delta$ subnuclear, we have $\triangledown \Delta \psi \in \mathcal{B}$, given $\Delta$ nuclear, we have $\triangledown \Delta \psi \in \mathcal{B}$. Thus we can chose $T$ such that $< T, \varphi > = 0$, in $\mathcal{D}'_L$. Note that according to Lie, relative conjugation it is sufficient to consider $< > \sim const (5)$, for the orthogonal to be defined.

We can define an annihilator theory relative first surfaces: $X^* = \{ S \mid (\psi, S) = 1 \ S \in X^* \ \psi \in X \}$ and $X = \{ \psi \mid (\psi, S) = 1 \ S \in X^* \ \psi \in X \}$ (11), given isolated singularities, we can prove that closedness for $X$, means $\circ X^* = X$, for instance $X = L^1, H$. Given $S$ is scalarly absolute continuous, $d(S, \psi) = 0$ implies $S \in X^*$. Finally, consider $X = \{ \psi \mid d \mid U \mid (\psi) = 0 \}$, that is a domain over which $\mid U \mid$ is analytic. Given $\mid U \mid$ is absolute continuous (that is isolated singularities), we get the annihilator theory as above.

0.1.5 Integral representation

Assume $\mathcal{H}$ normal, nuclear with $\gamma$-topology (17), prop 4, pg 41) and the strict approximation property, assume the strong dual is nuclear and quasi-complete. We can then define a scalar product, hypocontinuous with respect to bounded sets in $\mathcal{H}(E)$ and compact sub sets in $\mathcal{H}(F)$.

**Global regular model 0.2** Under the conditions in prop. 4 (17), we assume $< T, e' > (\varphi) = < T(\varphi), e' >$, that is $\mathcal{H}$ is in the algebraic dual to $E'$ and $E'$ is generated by $\mathcal{G}$. As long as $UT \in \mathcal{H}$, when $T \in \mathcal{H}$, we have that when $\mathcal{H}$ has the property of approximation by regularization and truncation, the representation is independent of choice of local coordinates in $\mathcal{G}$.

(17), prop. 5, pg. 43. Assume $E' \simeq \mathcal{G}$ and that for instance the mapping $U \rightarrow U^\perp$, maps $E' \rightarrow F'$. Given $U$ reflexive, but not projective, we have for $U^\perp = (I - U)^+V$, that $V^\perp = V$. For instance, starting from $\mathcal{D} \rightarrow E$ finite dimensional, we can choose $E = F^\perp$. When $F = R(A)$, we have that $E$ is polar.
to $A$. Given $V, V ^\perp$ are analytic, we have $\Gamma = \{ V = V ^\perp \} = \{ 0 \}$. If they are only linearly independent continuous movements, we can have segments in $\Gamma$.

In particular given $(V - V ^\perp) f < \lambda$, then $| x | \to \infty$.

$\text{Schwartz criterion for } \mathcal{D}_L^\mathcal{g}$: For instance given $T \in L^{1}_{\text{loc}}$, we can give $< T, dU > \in \mathcal{D}_L^\mathcal{g}$, as long as the coefficients to $dU$ preserves integrability. Alternatively, given $L^\mathcal{g}$ is generated by $U \in \mathcal{g}$, we can define $\mathcal{D}_L^\mathcal{g}$.

Given $E'$ is defined by derivatives and $\mathcal{H} = L^1$, we have $\mathcal{H}(E) \subset \mathcal{D}_L^\mathcal{g}$, or given $T \in L^1$, we have $< T, e' > \in \mathcal{D}_L^\mathcal{g}$, in this case we assume $\phi \in \hat{B}$. $\forall H \in \mathcal{D}_L^\mathcal{g}$ we have existence of a vector $\mathcal{T}_\alpha \in L^1$, such that $< \mathcal{T}_\alpha, e' >= H$. Given $\phi \in L \subset \hat{B}$, we have existence of $H \in \mathcal{D}_L^\mathcal{g}$, such that $H(\phi) = 0$, when $L$ is closed we have that $L^\phi = \{ H \in \mathcal{D}_L^\mathcal{g} \ H(L) = 0 \}$ and given $\phi \in \hat{B}$, we have $\phi \in L$.

Given $E'$ is very regular $\forall j$, we have $(I - E)^{-1}$ is very regular (a fixed singularity). Given $TE = I$ (modulo $C^\infty$), then $TE$ can modulo $C^\infty$ be considered as of type 0, symbols that constitute a convolution algebra. Given $TE = I$ (modulo $C^\infty$) implies $E = I$ (modulo $C^\infty$), we have $T$ has type 0.

Assume $G$ a fundamental solution relative $I = I_{\text{red}}$ with $R(I_{\text{red}})^{1} = R(V) \subset C^\infty$, then $G$ is parametrix outside the polar. Given further $V(\phi) = 0$ implies $\phi \in C^\infty$, then a very regular $G$ can be continued to very regular in $\mathcal{H}(E)$.

A disk-neighborhood can be generated by $(U_1, U_2)$. Assume $dU = \beta dU_1 \beta dU_2$. The domain for $(U, U ^\perp)$ is assumed to satisfy the condition that $(\alpha, \beta) \in L^1(dU_1, dU_2)$. Note that if we restrict $U$ to acting on a disk, we have $UI = IU$, implies $P_1 \subset U \subset P_2$, that is $\{ U = dU = 0 \}$ has Lebesgue measure zero. Note also $R(U) = \{ U f - f \ H \} \simeq \mathcal{H}(E)$.

Assume $L(f, g)$ a connected path between $f$ and $g$, then we have, given $f, g \perp \psi$ with $\psi$ in a nuclear space, that $L(f, g, \psi) = 0$. In particular, given $R(U)^{1} = R(V)$, for some movement $V$, we have that $V$ can be chosen as subnuclear.

Assume $L(\varphi, \psi)$ path between $\varphi, \psi$ and $\Delta : \mathcal{H} \to \mathcal{H}^{\perp}$ subnuclear, that is preserves convexity, then there is a corresponding path in the orthogonal. More precisely, assume $< U \varphi, \phi_1 > < U \varphi, \phi_2 > 0$ implies existence of path $U ^\perp$ between $\phi_1$ and $\phi_2$, under the conditions that $\phi_1, \phi_2 \in R(U) ^{1}$ and $U \to \mathcal{U}$ continuous. Assume $dU = \beta dU$. The condition $\alpha > 0$ locally, means that the movement does not change orientation locally. The condition $\alpha' > 0$ implies in particular that $U$ is absolute continuous locally.

Assume $B(f, f')$ bilinear, and that a regular chain $L$ between $f, g$ defines a leaf for $B$. Assume $L(f', g') \perp L(f, g)$ with respect to $B$, and further that $(f', g') \sim 0$, that is $L$ gives a projective decomposition that can be reduced continuously to a trivial polar, that is we have “surjectivity”. Given $(f, g) \sim (f', g')$ through conjugation (duality), we have that every chain between $f, g$ has a corresponding chain between $f', g'$. Given $(f', g') \in R(V) \subset C^\infty$ we have a regular chain. Given $U ^{1} = (I - U) + V$ with $R(V) \subset C^\infty$ and $U$ pseudo local, we have that $(I - U)^{-1} U ^{1} - I \subset C^\infty$, where $N(U ^{1})$ corresponds to a 1-polar. Note that
$$(I - U)^{-1}(I - U)f = 0$$ means that $f(0) = 0$.

0.2 Maximum principle

0.2.1 Introduction

The criterion we use for inclusion $\mathcal{H}_1(E) \subset \mathcal{H}_2(E)$, related to weighted $L^p$-spaces, is that the quotient of weights tend towards 0 in $\infty$.

Starting from $< T(\varphi), e' > = < T, e' > (\varphi)$, where $T \in \mathcal{H} \subset E'$, we see that $E_1' \subset E_2'$ induces the inclusion $E_1' \supset E_2'$, for instance $dU_1 = adU_2$, with $\alpha \in B$. But since $dU_1 = 0$ does not imply $dU_2 = 0$, the order of $E_2'$ is greater than the order of $E_1'$.

Given $Q$ a semi-norm on a separated space $E$, we can define $N_{Q,p}(f) = (\int_Q Q(f(x))^p d\mu(x))^{1/p}$ \cite{17} and $\Delta_p(E)$ functions finite with respect to $N$. The closure of $\Delta_p(E) \cap E'$ is not separated. The associated separated space is denoted $\mathcal{T}'(E)$. We have $\mathcal{T}'(E) \subset \mathcal{T}(E)$, when $p \leq q$. Given $p,q$ semi-norms, corresponding to $dU_1$ and $dU_2$, for instance $p(f) = |\int f dU_1| = q^{(a)}(f)$, where $a \to 0$ in $\infty$, we assume that $p/q \to 0$ in $\infty$.

Assume $|< p(f), dU >| = |< f, d | V >|$, for a semi-norm $p$. Thus when $U$ is analytic it can be related to an harmonic measure. For example, given $\frac{\delta f}{\delta x} = \beta_1 \frac{\delta \varphi(f)}{\delta x} + \beta_2 \frac{\delta \varphi(f)}{\delta y}$ are well defined. Assume the coefficients to $d U$ are $(\xi, \eta)$ and the ones to $d | V |$ are $(\xi_1, \eta_1)$, so that the movements can be related through $\frac{\delta \varphi(f)}{\delta \eta_1} = \frac{\xi}{\eta}$. Further, we can construct a semi-norm $q$, such that $\int p(f) dU_T \simeq \int q^{(U_T f)} dT$, for a parameter $T$, given $q(U_T f) = 0$ on boundary to the domain. According to the above, $q''(\cdot) \in L^1$ implies $q''(\cdot) \in L^\infty$, but $q(dU_T(f)) = 0$ does not imply $U_T$ analytic over $f$. Given a completing domain, where the norm is given by $q$, $U_T$ can be seen to be analytic under these conditions. Under the condition on the mean, $M(q''(\frac{d}{dT}(U_T f))) \leq q''(U_T f)$, we have that convergence for $U_T f$, with respect to $q''$, implies convergence in $L^1(E)$.

0.2.2 Maximum principle

The concept of dimension assumes linear independence for base and precesum of $0$, since $0 \notin \mathcal{C}$, cylinder web, we prefer to consider order. In particular, for $f,g \in \mathcal{H}$ and $U \in \mathcal{G}$, we consider regular chains on the form $Uf = \Sigma_{j} f_j = g$, where $f_j \in \mathcal{H}$ constitutes a base for the change of coordinates. The movement is regular, if it is regular over base elements.

Maximum principle 0.3 It is to determine hypoellipticity, sufficient to consider chains of maximal order. More precisely, if $\mathcal{F}'$ are chains of maximal order between $f,g$ that are given by $U$ and if $E'$ are shorter chains and if $E$ has the approximation property, given $I \in \mathcal{L}(E) = \mathcal{(E,F)}$, then $F$ has the approximation property, that is $U$ is algebraic.

\cite{15}, Proposition 2, pg. 7, the closure is in $\mathcal{L}$ with topology induced by $\mathcal{L}_c$.

Note that $U$ is locally such that $dU \sim \Sigma_{j \in \mathcal{F}} \epsilon_j dU_j \sim \Sigma_{j \in \mathcal{F}'} \epsilon_j dU_j$, for some finite set of indexes and where $dU_j$ can have higher distributional order. A necessary condition for $(U,U^\perp)$ to preserve hypoellipticity is that $U^\perp \prec \prec U$ \cite{11}. We assume also that the real part of the symbol does not change sign on a connected set $\subseteq \infty$.

Assume $T \in \mathcal{H}$. We denote with $\text{ord} < T(x), e' >$, the number of movements in $E'$, that are necessary to generate $\mathcal{H}(E)$ locally. We assume $T^* \varphi \in L^1$
implies \( \text{ord } \mathcal{H}(E) = 0 \), that is, is generated by analytic poly cylinders, on the support of \( T \). Further, \( T \ast \varphi \in L^2 \) implies \( \text{ord } \mathcal{H}(E) > 0 \). Note that over \( \mathcal{D}_{L^2} \), we have that \( U \) acts algebraically, that is assume \( U \mid T \mid = VT \) with \( | T | \in \text{ rad } (I)_{L^1} \), then we have that \( U \) can be limited to translations, that is \( VT \mid N \in L^1 \) is of order 0 outside the zero-space.

Further, since \( | VT | - | T | \leq | VT - T | \), we have that \( U \mid T \mid^2 = | T |^2 \) does not imply \( VT = T \), that is \( | T |^2 \) reduced in \( L^1 \) does not imply \( T \) reduced in \( L^2 \). Since \( L^2 \) is not nuclear, \( V \) can not be determined from \( U \) uniquely, and given a suitable separation condition, we do not have that \( V \) is surjective in \( L^1 \).

Assume \( \phi \) real and does not change sign on connected components \( \exists \infty \). Assume \( \Omega = \{ \phi > 0 \} \sim \{ F < \lambda \} \) is semi-algebraic \( \exists \infty \). Then \( \Omega \ni z \rightarrow 1/z \in \Omega \) continuous, does not preserve compactness for \( \Omega \). Given \( F(z) < \lambda \) implies \( | z | < C \), we do not simultaneously have that \( | 1/z | < C \). But, given \( | z | + | 1/z | \leq F(z,1/z) \), we have that \( F \) is downward bounded separately in \( z,1/z \). Thus \( F \) is \( \text{“hypo-reduced”} \). Assume further \( F = G \), then we have that the inequality implies \( (1+| z |^2) \leq | zF | \), thus given a condition on existence of \( j \) such that \( \delta^G j \) is hypoelliptic (cf. very regular boundary), we have that \( F_j(z,1/z) \) is hypo-reduced as above. Consider \( \phi(z) = | z | + | 1/z | \), as long as \( | z | \) is a monotonous deformation, \( \phi \) has relatively compact sub level surfaces.

When \( L_\chi(\phi) = \langle \chi, \varphi \rangle \sim \text{Id}(\varphi) \) we have that \( L_\chi : \chi \rightarrow \tilde{\chi} \), that is the symmetry operator \( [10] \). Note that \( \text{Id} \) is optimally non-reduced, that is corresponds to a symmetric polar. On \( D \) we have that \( \varphi \equiv 0 \) implies \( \varphi^* \) (an algebraic dual) symmetric on \( D^0 \) (\( \varphi \rightarrow \varphi^* \)). Note that determination of the spiral clustersets when \( D^0 \) is the polar, is in some cases unsolved. An algebraic polar implies \( \varphi \) not identically zero on a disk (a necessary condition, given \( \varphi \) is bounded and analytic, is that \( \varphi \) is not identically zero on a line).

Given \( f \) convex, we have \( \int f'dI = \int f''dx = f' \), that is \( I = I_{\text{red}} \). When \( f'' = \alpha f' \), we have \( adx = dI \). Given \( f \in X \), are convex functions, we have \( \int f' dx = f \), that is \( I=I_{\text{RED}} \), as opposed to evaluation in a point. \( \delta_s(\varphi) = 0 \) if \( \varphi(x) = 0 \), but \( \delta_x \) has a translation invariant symbol, that is we have presence of a polar. Let \( \tilde{X} \) be a a convex continuation of \( X \), then we have \( \int_X f(I) = \int_X f dI \), for instance \( tf + (1-t) \int f'dI \). Thus \( I(f) \) can be deformed continuously to \( f \), analogous with an approximative identity. On \( T(X) \supset X \), for example \( \int_{T(X)} f dI = \int_X f'dx + \int_X f dx \), that is given \( f = 0 \) over \( \Gamma \), we have \( I = f \). Consider \( I \) completed to \( I - \gamma : D' \rightarrow D'_{L^1} \). then for suitable \( \gamma \), we have convex \( (L^1_{ac}) \), ramified neighborhoods.

0.3 Invertible movements

0.3.1 Resolution of identity

Resolution of identity in this article, assumes a regular covering. Assume \( S_j \) planes complementary to \( R(U) \). Assume \( S_j^1 \perp S_j \). Assume \( S_j^1 \) intersects a tangent \( dU \). The system \( R(U) \) does not necessarily have the property of approximation, but \( R(dU) \) can be completed to a system with this property. More precisely, \( \{ dU \} \cap S_j^1 \neq \emptyset \) that is we have continuation of tangents \( dU \) to \( S_j^1 \), sufficient for this is that \( R(dU)^1 \) has a projective decomposition, that is \( S_j \cap S_j^1 = \{ p \} \), \( p \) point. When \( R(U) \) is given by (integration of) \( R(\Sigma dU_j) \), we can get an approximation principle for the completed system.
Consider \( \lim_{y \to x} \int \varphi(x, y) dy \), for \( x \) fixed on \( S_j^I \) and \( y \in S_I \). Given a normal tube (regular covering), we have that the limit is not dependent of choice of starting point. Assume \( R(dU) \) completing and \( dU \cap S_j^I \) a closed segment and \( dU(0) = 0 \), then we have \( E \cap \delta U \) is a Banach-space. Assume \( S_j^I + S_j = I \) absolute continuous, then the condition for projectivity (local) is \( dS_j \) scalarly invertible. Note that \( \varphi \) is non-trivial. Assume further the strength is very regular, that is \( \varphi \) is analytic over \( D \), then \( \varphi \) is invertible over \( D \).

**0.3.2 Invertibility**

Under the condition \( T \to 0 \) in \( \mathcal{L}(\mathcal{H}, \mathcal{H}) \), we have \( < T, \varphi > \to 0 \) on convex, equilibrated, compact sets. In the same manner, \( T \to I \), given \( < (T-I), \varphi > \to 0 \) in the same topology. Note that this does not imply that \( T \) is invertible, that is \( T = I \) does not imply \( \varphi = 0 \). The condition \( T(\varphi) \to 0 \) and further \( T \to I \) implies \( \varphi \to 0 \), is sufficient for \( T \) to have a continuous inverse.

Assume \( \Sigma V_\lambda = \Sigma_N + A \), where \( A \) is absolute continuous, that is \( N(V_\lambda - I) \subset N(V_{\lambda+1} - I) \), and for \( \lambda, \lambda' \) finite \( j \). Assume \( V_\lambda(\varphi) = V(\varphi) \), that is \( V_\lambda \) are absolute continuous over \( (I) \) if \( V \) is absolute continuous over \( (I)_j \), we then have a domain for absolute continuity. Alternatively, assume \( V_\lambda - I \in C^\infty \) implies \( V_{\lambda+1}^J - I \in C^\infty \), that is \( V_\lambda \) are very regular in \( D_{\lambda} \).

For using instance the spectral mapping theorem, given an algebraic change of local coordinates (over \( D_{\lambda} \)), invariant sets are preserved. Consider \( I = (I_0) \oplus (I_1) \oplus (I_2) \), where \( (I_0) = \{ \varphi \text{ \, absolute continuous over } \varphi \} \), \( (I_1) = \{ \varphi \text{ \, absolute continuous over } \varphi \} \), that is \( \text{rad} (I_0) \) and \( (I_2) \) what is remaining. Then we have \( (I - U)^{-1} \sim \Sigma_N + R \), where we assume \( \text{Rac over} (I) \).

Obviously, \( (I_0) \subset (I_1) \). Since \( (I - U)(\Sigma_N + R) = I \), when we assume \( dR(\varphi) = 0 \) implies \( U^{N+1}(\varphi) = 0 \), we have \( (I - U)^{-1} \sim \Sigma_j \) a finite sum, that is, given \( \varphi \in (I_1) \), with \( dR(\varphi) = 0 \), we have that \( (I - U) \) is invertible.

Assume \( U^J = (U - I) + V \), where \( U, V \in G \).

**Invertible radical 0.4** Assume some iterate of \( U \) is absolute continuous and \( U \) is analytic over \( \varphi \) and that the polar is given by \( V \), of the same strength as \( U \), then \( U^J \) is invertible over \( \varphi \).

Consider \( (I - U_{\varphi}^J) \sim \Sigma(V_{\lambda} - I) \). Given \( U^J \) is absolute continuous, when \( j > N \) and \( dU^J = 0 \), we then have \( U_{\varphi}^J = 0 \), when \( V_{\lambda} \neq 0 \), that is it is necessary that \( V \) is non-trivial. Assume further the strength is \( U - I \sim V \), in the sense that \( (U - I)/V \to 0 \) in \( \infty \), then we can derive the convergence for \( V^{-1}(I - U_{\varphi}^J)^{-1} \) from the conditions above.

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0.3.3 Parametrices

In $\mathcal{D}'(\mathbb{R}^n)$ the polynomials with constant coefficients are surjective, that is given $Uf \simeq P(D)f_0$ in $\mathcal{D}'^F$, with $f_0$ very regular, we have existence (modulo $C^\infty$) of parametrices, that is $\exists V$ such that $VU \sim I$ in $\mathcal{D}'^F$. Given $P(D)$ hypoelliptic, the parametrices can be chosen as very regular, that is modulo $\mathcal{C}$, with trivial kernel. Consider $E : \mathcal{D}' \to \mathcal{D}'^F$, a parametrix to a polynomial operator. Given $E$ a projection operator (modulo $C^\infty$), that is $E^2 \simeq E$, we can assume $E^\perp \sim I - E$. Sufficient for this is that $P(D)$ hypoelliptic.

Assume $u : \mathcal{D} \to \mathcal{D}$ and simultaneously $u^\perp; \mathcal{D}' \to \mathcal{D}'^m$, then we can have clustersets for $U$, even when $U^\perp$ has relatively compact sub level surfaces. Given $Uf = \int fdu$, we have $dU < \lambda dx$ implies $Uf < \lambda$, when $f \in L^1$ (normalized), that is $dU$ is reduced, if $U$ is reduced over $L^1$. In the same manner for $\mathcal{D}'_{L^1}$, even when $U$ is algebraic over $\mathcal{D}'_{L^1}$, we can have non removable zero-sets for $U$.

Partial hypoellipticity 0.5 Assume $f(u_1, \ldots, u_k)$ hypoelliptic over a non maximal chain. Assume $f \in \text{rad } (I_{\text{red}})$ relative maximal chains. Assume for a maximal chain, $U^\perp = (I - U) + V$, where $I + V$ is algebraic, then $V$ can be represented as analytic, that is $dV = 0$.

Note that the condition that $f \in \text{rad}(I_{\text{red}})$ relative maximal chains, implies that given an approximation property, $f$ is partially hypoelliptic, which in this context means that it is reduced relative translation.

0.3.4 Maximal rank

Assume $PG = I_{\text{red}}$, for a hypoelliptic symbol $P$, and continue to $I_{\text{red}}$, with corresponding $G$, parametrix to $P$. Then, $R(G) \subset \mathcal{D}'^F$, that is we have an approximation property.

Starting from a covering $\{\Omega_j\}$, where we assume $A_jf \in \mathcal{O}^*(\Omega_j)$, that is analytic without invariant surfaces on $\Omega_j$ and $f$, further $B_jA_j^{-1}f \in \mathcal{O}^*(\Omega_j \cap \Omega_j)$ and $\Omega_j \cap \Omega_j \neq \emptyset$, then we have existence of a global $U$, such that $UA_j^{-1}f \in \mathcal{O}^*(\Omega_j \cap \Omega_j)$ $\forall j$. In particular, when $dU = odU_1$ and $\alpha \to 1$ regularly (without invariant sets), we have Oka’s property ([13]). Over log $f$, we have for instance given $dI BV$, $dI = dU + dV$, with $dU, dV$ BV and monotonous. Using Fourier duality, when $\Omega \ni 0$, we have a corresponding $\Omega^* \ni \infty$ and the condition $(UA_j^{-1}f)^*g \in \mathcal{B}(\Omega^*)$. For analytic movements, we can assume $G(u_1, \ldots, u_k) \in B_1 \times \ldots \times B_k$ and we can determine a domain for $ac$ and corresponding polar, for instance $I_{\text{red}} + V$ with $R(V) \subset C^\infty$.

Dependence of topology 0.6 Assume $X$ a domain for existence of a very regular fundamental solution $G$ and consider $(U + W)$, with $U, W \in \mathcal{G}$ and $I \prec U$. Then we have that, $G^*(U + W)^{-1}$ corresponds to a very regular fundamental solution, only if $W \prec \prec U$ on $X$ and $W/U \in \mathcal{E}^{(0)}(X)$.

Given $G-I$ of type $-\infty$, that is corresponding to regularizing action, we have $e^{c|x|} | G - I | \prec C$, for some positive constant $c$, when $| x | \to \infty$. Thus the condition $e^{c|x|} | \frac{d}{dx} G | \prec C$, when $| x | \to \infty$ is necessary and we do not necessarily have that $G$ is very regular under a change of coordinates, that generate the range. For parametrices $E$, given $P$ is hypoelliptic, we have that, if
$PE$ is of type $-\infty$ outside the diagonal, the same holds for $E$ (Nullstellensatz). When $E$ is Fredholm, we have that $\ker E$ is of type $-\infty$, and the condition is satisfied.

**Maximal rank 0.7** Assume $P$ symbol to a d.o. hypoelliptic over chains of maximal order, assume the symbol $G$ to a fundamental solution is in $H(X)$, of order $k$. Assume $(U+V)$ a change of variables of order $k$, then the resulting $G$ is not necessarily very regular. Assume $X$ a poly cylinder in $L^{1}$, maximal for which $G$ corresponds to a very regular distribution and that $I \in \mathcal{L}_{c}(\hat{X},X)$, where $\hat{X}$ is a strict extension of $X$, then $G$ is not very regular on $X$.

Assume $H \ni f \to Uf \in H$ preserves order. When $(U+V)f = 0$ with $V \ll U$, then we must have $U \neq V$ in $\infty$, that is we do not have $U^{-1}Vf = f$ in $\infty$. Presence of a non-trivial kernel $Vf = 0$, that is continuation with 0, corresponds to presence of invariant sets (for $W = V + I$). Assume $R((U+V)^{-1}) = D(U+V)$, that is $(U + V)$ preserves order, when $\text{ord}D(U+V) = \text{ord}R(U+V)$, which means absence of non-trivial kernels. In $L^{1}_{ac}$, this means $\text{ord}(U+V)^{-1} = 0$ and when $H$ has the approximation property, $(U + V)^{-1}$ can be reduced to order 0. Otherwise, we must consider completing sets.

Consider $PG = I$ in $L_{ac}$, that is $(U+V)I(U+V)^{-1} = I$ in $L^{1}_{ac}$. Assuming the approximation property, the equation can be extended from $L_{ac}$ to $L_{c}$. Consider $d(U + V) = (\alpha + \beta)du_{1}$ and a semi-norm $q$, such that $e^{C|x|}q(U_{1}G) \leq C'$, as $|x| \to \infty$ for some $C > 0$, that is $U_{1}G$ has negative type and corresponds to regularizing action. For the extension to $L_{c}$, we assume $q((U+V)^{-1}G) \leq e^{A|x|}q(G)$, as $|x| \to \infty$ for some $A > 0$. Thus, $e^{(A+C)|x|}q(G) \leq C'e^{A|x|}$, as $|x| \to \infty$, is not necessarily of negative type and we do not necessarily have regularizing action.

On the boundary for convergence we do not necessarily have two-sided regularity, that is not regular surjectivity in $H$ for $(U,V)$. Note that precense of essential singularities, means that hypo continuity is not sufficient for existence of two-sided limits. Assume pluri complex convergence radius, that is $L$ radius to $A$ completing, $A \subset \tilde{A}$ and $\tilde{A} \cap L$ a closed segment, then we can assume that $(U+V)^{-1}$ is divergent on $L$, for instance $V/U \ll c$ on $L$. Here $L$ corresponds to eigen vectors for $(U+V)$. In particular when $(U+V)^{-1} : \mathcal{D}' \to \mathcal{D}'$, then loss of derivatives and divergence is given for instance by $V/U \in \mathcal{E}^{\infty}$. A discontinuous convergence is given by $(V/U)^{X} \in \mathcal{E}^{(0)}$, when $N \geq N_{0}$, that is $V/U$ improves regularities.

When $X$ is a symbol to an operator that does not have maximal rank, there is a set for multivalentness in $X'$. Relative norm, we assume $X^{0} = \{x' \in X' : x' < x, x > 0\}$. Given that dimension is relative $C^{\infty}$, we consider $R(X)^{0}$ as generated by an “algebraic” functional, which is implied by the approximation property. Given $\{F = e\}$ is given by a holomorphic function, there is a corresponding domain for multivalentness $\mathcal{D}$. Consider $f = e^{\phi}$, then we have $\phi \in C^{\infty}$ does not imply $f \in C^{\infty}$ and given $\phi$ has negative type, it does not imply $f$ with negative type (regularizing action). Further, $\delta^{2}e^{\phi} = (\delta^{2}\phi + (\delta\phi)^{2})e^{\phi}$ (cf. convexity outside the polar). A sufficient condition, for $\phi$, complete with respect to maximal chains, to imply $f$ complete with respect to maximal chains, is that $G$ acts algebraically. Note that $\{f = \text{const}\}$ on sets $\{\phi = 0\}$ and $\{\phi = \text{const}\}$, but constant surfaces for $\phi$, do not have the same property as zero sets. However, given Schwartz separation condition and $\phi \in L^{1}$, the sets can be considered as
equivalent. Note that invariant sets for \( f \) corresponds to invariant sets for \( \phi \),
that is \( U e^\phi - e^\phi = 0 \) iff \( e^{U\phi - \phi} = 0 \).

Starting with \( \int U^\perp df = \int f dU^\perp - \int_T U^\perp f \). Assume \( \Gamma \) a line in the polar
\( U^\perp \) is algebraic over the tangent space, we have outside the polar
and over non trivial \( f \), that \( U^\perp f \neq 0 \) a.e. Given \( R(U^\perp) \) convex \( \subset H \)
a Banach space, the polar defines a completing set. Consider now \( \{U^\perp \varphi\} - \{U \varphi\}^1 \).
Assume for instance \( X \) is defined by \( \mathcal{G} \), such that \( \{U \varphi\}^1 \simeq \{V \varphi\} \),
where \( V \in \mathcal{G} \), is not necessarily unique. Further, \( dU^\perp \sim p dx \) and \( dU \sim q dx \), where \( p \)
is reduced and \( p/q \rightarrow 0 \) in \( \infty \). Given \( p/q \) algebraic, there is not precense of a
spiral. \( p \sim q \) implies possible precense of a spiral. Assume \( H = R(U) \bigoplus R(U)^1 \)
and that \( U \) is monotonous with \( dU = adI \), \( a > 0 \) locally, that is \( \alpha_{ac} + \alpha_s > 0 \)
locally. Given \( \int \alpha_w df = I_{red}(f) \) locally, we have \( U f = I_{red}(f) + U_s(f) \), with
\( dU_s(f) = 0 \) locally, note that \( U_s \) is dependent of the topology. To conclude that
\( I\phi = 0 \) nbd \( 0 \), implies \( \phi \equiv 0 \) (\( \delta \ast \phi = 0 \) implies \( \phi(0) = 0 \)), it is necessary that
\( \dim_{C^\infty} R(I) = 0 \).

0.4 Topological convergence

0.4.1 Boundary condition

Assume a very regular boundary (\( \mathcal{H} \)), that is precense of regular approxima-
tions, in particular \( \exists \varphi \in \mathcal{D}, T\ast \varphi \rightarrow T \) with \( T \in \mathcal{H} \), where \( \mathcal{H} \) is normal.
The boundary can be defined through the condition \( \{F^{(U)} = const\} \) isolated
points, for some analytic function \( F \), which excludes precense of a spiral. Consider
\( f \in X/C^\infty \), that is \( f \sim \delta - \gamma \) with \( \gamma \in \mathcal{D} \), where \( \gamma \rightarrow 0 \), it is then
sufficient to consider \( f \) of type \( 0 \) (real type). Given \( f \in \mathcal{D}' \), we have that
\( f \sim \Sigma D^\alpha f_\alpha \), where \( f_\alpha \in L^1 (\mathbb{R}) \), that is given \( f_\alpha \) has distributional order
\( 0 \), we have that \( f \) has finite distributional order . Assume \( \delta f = cf \) (that is
\( \delta \log f = c \)) on segments, but \( \delta^2 f \sim cf \) implies isolated points, then we have
\( \delta^2 \phi f \sim (1 + \delta \phi)(1 - \delta \phi)f \). Sufficient for this is twosided regular limits and we
have a decomposition \( (\delta \phi)^2 \bigoplus \delta^2 \phi = 1 \), where \( f \neq 0 \).

Given \( dv \in \mathcal{D}'^m \), we have existence of \( dm \in \mathcal{D}'^0 \), such that \( dm = adv \), where
we choose \( \alpha \sim 1/P \) (modulo \( C^\infty \)), for a polynomial \( P \), such that \( \alpha : \mathcal{D}' \rightarrow \mathcal{D}'^F \)
injective, (\( \mathbb{R} \)) the property (\( \varepsilon \)). Alternatively, consider \( dv \sim dv \) (modulo
\( C^\infty \)), for instance \( adv(\varphi) = 0 \) implies \( \varphi \in C^\infty \). In particular, given \( \alpha \) very
regular, we can associate \( dm \) to \( dv \in \mathcal{D}'^m \).

Assume \( bdA = \{U = \lambda I\} \), that is given \( U \) is absolute continuous we can write
\( bdA = \{dU = 0\} \). Given \( U \) analytic over \( bdA \), the boundary can be
given by a connected set. When \( U \) is analytic over \( A \), we have \( U \leq \lambda \), that
is maximum is reached on the bd A. Given \( \Omega \) pseudo convex, we have that \( bd \Omega \)
is of order \( 0 \) (cylindrical). Consider \( T \in \mathcal{E}'(\Omega) \), such that \( T |_{bd\Omega} \in \mathcal{L} \) and
\( \Omega = \{(x,y) \leq P(x,y) < \lambda\} \). For instance \( S \circ (P,\overline{P})(x,y) = T(x,y) \), where \( P < \lambda \)
iff \( \overline{P} < \lambda \) and \( S \in \mathcal{E}'(\Omega) \). In this case we have \( P = S^{-1}T \) with respect to an
involution condition.

Every differential operator on the real line, corresponds to a reduced
measure, in particular when \( \lambda \in \mathbb{R} \), we can assume \( P(D\lambda)f_\lambda = 0 \) implies \( f \in C^\infty(\lambda) \),
that is for finite values on \( \lambda \), we can assume an analytic dependence of \( \lambda \).
0.4.2 Orthogonal decomposition

Given $U = I$, we have that $U^\perp$ is not necessarily uniquely determined. In the plane there are finitely many leaves. In particular, given I is separately continuous on $(U, U^\perp)$ and equi continuous on leafs of BV, then $I(U, U^\perp)$ is considered as hypo continuous. Note that, given $(U, U^\perp) \simeq (U, \psi(U))$ gives a closed graph, under a Schwartz type separation condition, we have that $\psi$ is continuous. Further, that given isolated singularities, we can we identify $\{U = I\}$ with $\{U = 0\}$, for instance given $U\phi \in L^1$, we have $\{e^{U\phi} \sim I\}$ iff $\{U\phi \sim 0\}$.

Consider $\psi : dU \to dU^\perp$, with $\psi$ very regular, that is $\psi(dU) = dU^\perp + dR$, with $dR$ regularizing, that is $dU - dU^\perp = dR \in C^\infty$ but $dR(\varphi) = 0$ does not imply $\varphi \in C^\infty$. Given that R is very regular and can be chosen downward bounded, we have $(R - I)(\phi) \in C^\infty$ and $R(\phi) \in C^\infty$ implies $\phi \in C^\infty$. Note, given R is absolute continuous, $dR(\phi) = 0$ (modulo $C^\infty$) implies $(R - I)(\phi) \in C^\infty$.

Given duality $L \to L'$ and $M \to M'$, we can continue $T : L' \to M \to M'$, for instance $\phi \in L'$ has existence of $\Omega$.

For completing B, we have $(L \otimes M)^\circ = \{(L \otimes M)', <(\xi, m(\xi)>(_M) = 0\}$, that is $\xi = (\xi) \perp m(\psi)$. Thus $L^\circ \otimes M^\circ \subseteq (L \otimes M)^\circ$. Given $L', M'$ reduced, we have however $\xi(L \otimes M)^\circ = \{0\}$. Further, given $UT \in C^\infty$ and $U^{-1}$ pseudo local, we have $T \in C^\infty$, that is $U$ is algebraic.

Assume $K$ very regular with respect to $(U, U^\perp)$ and a norm $\| \cdot \|$, then the condition $\|U - U^\perp\| = 0$, gives precence of a spiral, but $\|U\| = 1$, $\|U^\perp\| = 0$ implies Parseval (the movement is considered as an operator). Further, $\|dU - dU^\perp\| = 0$, given $U$ absolute continuous, means $(U - U^\perp) = I$, that is $U^\perp = I - U$ and we have projectivity.

Assume for spaces of distributions, $\mathcal{H}, K$, that $\Omega = \text{nbhd} 0$ in $\mathcal{H}$ and that we have existence of $\Omega' = \text{nbhd} 0$ in $\mathcal{K}$, with $\Omega' \subset \subset \Omega$ and $\psi : \Omega \to \Omega'$ (restriction), for instance $L^1 \to L^1_{\text{loc}}$. In particular $\psi(dU) = \alpha dU_{\psi}$, with $\alpha \in L^1(dU_1)$ and $\alpha \to 0$ in $\infty$ or $dU' = \alpha dI$ with $\alpha \to 0$ in $\infty$. Then over a regular chain in $(L^1_{\text{loc}})$, we have that I is reduced. Given instead $\alpha$ independent of some variables, the separations condition above is not satisfied. Assume conversely $\psi$ an extension and $\Omega', \Omega$ disks, with $\psi(\Omega') \subset \Omega$, then $\psi$ is sub-nuclear and for instance $\beta dU' = dI$, with $\beta$ downward bounded (I is invertible relative the representation). Finally, consider $T(f) = \int f dU$, where $\mathcal{H}(E)$ is generated by $E' \simeq dU, dV$. Given $T(f) = \text{const}$. when V is varying, we have for the corresponding regular chain, precence of a polar (multivalentness).

0.4.3 Orthogonal decomposition

When E is quasi-complete, we have $\forall B$ bounded $\subset E$, that B is completing ([17]). For completing B, we have $F_B$ is a Banach-space, that is we can discuss orthogonal decompositions. Note, given the parallelogram law $2(U, U^\perp) = \|U - U^\perp\|^2$, that is $(U, U^\perp)(\phi) = 0$ iff $U\phi = U^\perp\phi$. Note in particular when $U^\perp = (I - U) + V$, we have $\|I + V\|^2 + \|I - V\|^2 = \|U\|^2 + \|U^\perp\|^2$. However an orthogonal decomposition is not necessary for a global model (pseudo-bases).

Consider $U^\perp(\varphi) = 0$, with $\varphi_1 \otimes \ldots \otimes \varphi_n$ dense in $\varphi$, for instance over a domain where $U^\perp$ is analytic over $\varphi$ (pluri complex zero). Then $U^\perp$ can be represented through $(V_{n1}^\perp, \ldots, V_{n^2}^\perp)$, such that $U^\perp \varphi \simeq (V_{n1}^\perp \varphi_1, \ldots, V_{n^2}^\perp \varphi_n)$. In particular, given $\varphi_1 \perp \varphi_2$, there is a decomposition through conjugated $V_{j}^\perp$, $(W, W^\perp)$.
The condition of existence of $U \in \mathcal{G}$ analytic over a chain to $f$, means that
$
\delta_1 \ldots \delta_k f(u_1, \ldots, u_k) = 0.
$ Note that maximal order with respect to $\mathcal{G}$, does not assume that all elements are analytic (Oka’s property).

**Absolute continuous representation** 0.8 Assume $U$ absolute continuous. Given $dU = 0$ can be decomposed into linearly independent measures, $\cap \{dU_j = 0\}$, $\cap N(dU_j) = \{0\}$ and given $R(U)$ has the approximation property, then $U$ has a projective decomposition over regular chains.

Assume $dU = \xi \eta + n_{\eta} \delta \eta$ and $\frac{\delta \eta}{\eta} = -\xi$ and $\frac{\delta \xi}{\xi} = \eta$. Then we have $< dU(f), \phi > = < f, (\eta \delta - \xi \xi) \phi > + < f, dU(\phi) >$. Given $G$ regular, $\{G\}$ locally 1-1 iff $\{G\}$ surjective, gives the approximation property through regularization. Given $\Delta G = 0$ we have that $dU = d^U$. In order to apply the partial Fourier transform, we must assume $\xi, \eta$ algebraic in $x, y$. In this case we have $(x, y) \rightarrow (\xi, \eta)$ subnuclear. Given also $f$ is subnuclear, we have that the coefficients are in a nuclear space. Assume, starting from $\{f, G\} = 0$, that $R(df)$ is generated by $U \times U^\perp$, then $R(dG)$ is generated by $^t(U \times U^\perp)$. Given reflexivity, $^t(U, U^\perp) \simeq (U^\perp, U)$ that does not affect the order (dimension).

0.4.4 **Polars**

Assume $\Sigma = \{\alpha \beta = 1\}$, where $dU = \alpha dI$ and $dW = \beta dU$, where $\alpha, \beta \in B$. Assume $L$ an oriented chain of maximal order, then we have given the approximation property, that $\Sigma$ gives invertibility. Note that $dW(f) \in C^\infty$ implies $Wf \in C^\infty$. Given $U, W$ pseudo local, then $Uf \in C^\infty$ implies $W^{-1}$ pseudo local over $\Sigma_{\alpha \beta}$ and $W$ algebraic over $\Sigma_{\alpha \beta}$.

Assume $A \subset B$ a disk neighborhood, where $B$ is defined through $|\xi| > d(x, \Gamma) < c$ (weighted disk neighborhood), where we assume $\Gamma$ is defined as analytic. Then we have that $B \setminus \Gamma$ is analytic. Starting from hypoellipticity, we can define $\Gamma$ through chains of maximal order, that do not contribute micro locally, that is $\Sigma_{\alpha \beta \gamma}$ are trivial. Note for shorter chains, $\Sigma_{\alpha}$ trivial, does not imply $\Sigma_{\alpha \beta}$ trivial.

**Ramified polar set** 0.9 Given $f/g = \alpha$ analytic, with $\alpha \rightarrow 0$ on a radius starting from the origin, then we have that $< \alpha g, g > \equiv 0$, that is $f \perp Lg$. Assume $\Omega$ is completing, then we can motivate $f \perp \Omega g$. Assume $U^\perp \in \mathcal{R}_I(U)$, then we have $U^\perp \simeq U^N$ (Nullstellensatz). The condition $U^N + U^\perp = 1$ gives a “rectifiable” change of variables, or a ramified polar.

Using for instance Lindelöf’s theorem, given $\alpha \in B_{u_1, \ldots, u_k}$, we have that given $\alpha = 0$ has a positive measure, on a line through 0, then $\alpha$ is zero on a disk. If we assume $\alpha = \alpha_1 \ldots \alpha_k$, we have motivated $f \perp \Omega g$. Given $H$ a space of distributions, we have $\mathcal{L}_c(H', E) \simeq \mathcal{H}(E)$. The topology for polars is given relative $\mathcal{L}_c$, that is uniform convergence on convex, compact and equilibrated sets. We can for instance assume on polars, that $f^2 \in K$ does not imply $f \in K$. Or if $T$ is in $\mathcal{L}_c$ on $0 < |U - U^\perp| < c$, but not in $\mathcal{L}_c$ on $|U - U^\perp| < c$, that is $T$ is not uniformly continuous on the spiral, then we have that $T$ is in the polar relative $\mathcal{L}_c$.

Assume $E'$ is generated by $\mathcal{G}$, $UT(\varphi)(\zeta) = T(\varphi)(\zeta_T)$. Note that we do not need a global model for $\zeta_T$. It is sufficient that $^tU \varphi$ is surjective and that locally $^tU \varphi(\zeta) \rightarrow \zeta_T$ continuous.
Assume $dV = \alpha dU$ continuous, where $\alpha \to 0$ in $\infty$. Given $\varphi \in B$, we have $\varphi_0 \in \hat{B}$, why $dV \in D_L^T$ (with approximation property), that is given very regular boundary, where $\varphi \to 0$ in some direction, we have a decomposition of variables. Assume $\phi \in (\hat{B})'$ and completed to $\phi - I \in C^\infty$, then $\phi$ can be given by a hypoelliptic operator outside the polar.

**Preservation of character 0.10** Assume $dU$ Stieltjes measures, defined according to Lie, with $C^\infty \supset R(U) \to R(U') \subset C^\infty$ continuous. Assume $\Gamma = \{ f \mid d(U^{-1}U)(f) = 0 \}$, that is the movement preserves character over $\Gamma \subset \mathcal{H}$. Assume $\Gamma_0 = \{ f \mid I \in T_c(Uf \to U'f) \}$. then we have $\Gamma_0 \subset \Gamma$.

Consider the condition $\eta_\alpha - \xi_\alpha = 0$ according to $\Delta G = 0$. Given $U^{-1}U$ absolute continuous, we have that $\Gamma_0 \neq \Gamma$ on non trivial sets. Note $I K = K I$ implies $P_1 \prec K \prec P_2$, for polynomials $P_j$. More precisely, assume $K$ has symbol in $H$ (holomorphic) and $K(e^\phi) \sim e^{K(\phi)}$, with respect to Exp-norm in the Fourier-Borel symbol space. Thus $K(\phi^1) \sim K(\phi)j$, which means that the zero space to the symbol to $K$, can be given as geometrically equivalent with the zero space to polynomials. Thus, we have existence of $P_1, P_2$, as above.

Consider $\Delta T = S$, for instance $T = S$ or $T^1 = S$. Starting from completing sets, with $F_0 = G E$, we can define $F_0^\uparrow$ through a Banach-norm. Assume $F_1 = (I - U)E + VE$, where $F_1 = F_2 + P$ with $P = R(V)$, then we have $F_0 + F_2 = E$ and $F = E + P$. Note in particular, given $U \in G$ then $U^{-1} \in G$, but when $U^{-1} = (I - U) + V \in G$, we do not have $\exists (I - U)^{-1} \in G$. Consider $E' \to F$ and $F^0 = \{ x' \prec x, x' > = 0 \}$, $x \in F$. Given $F$ is closed, we have $\{ x' \prec x \} \to E' = \prec x', x' > = 0 \} \subset F$.

Consider the polar as an envelop, for instance $M \subset I(\otimes d\mu)$, that is an ideal that is generated by tangents, for instance that the polar is defined as a zero space to Lie-movements (analytic movements). Note $< X(x, y), f(x)f(y) >$, means that given $X$ symmetric, that $I_X(f) = 0$ iff $I_X(f_0) = 0$. Concerning linear independence, $< I, \varphi >= 0$ iff $\varphi(0) = 0$, that is we do not have $\varphi = 0$ nbd $0$. Assume $\varphi \in Z$, then we have $T(\varphi) = 0$ implies $T(\varphi) = (E')^c$, implies $< T, e' > \in Z^c$. Note that given $T \in \mathcal{H}$, that is generated by $E'$ and $< T, e' >$ $(\varphi) = 0$ implies $< T(\varphi), e' > = 0$, then the implication $T(\varphi) = 0$ means that we do not necessarily have a global regular model for $\mathcal{H}$.

**0.4.5 Decomposable sets**

Assume $B \subseteq L \otimes M$, where $L = R(dU), M = R(dU^\perp)$. As long as $I \neq B$, B can be represented through a projection operator. Assume $B \subseteq L_{ac}^*(d\mu)$, Assume $dv = \alpha d\mu$, with $\alpha \in L^1(d\mu)$. We then have $\frac{d\alpha}{d\mu} \geq 0$ iff $\frac{d\alpha}{d\mu} \geq -\frac{d\alpha}{d\mu} \log \frac{d\mu}{d\mu}$. Given $dv - 1dx \sim d\mu$, means $d\mu = 1/(1-\alpha)dx$, given $\alpha |< 1$ is invertible.

Assume $\psi : dv \to dv$, for instance according to Radon-Nikodym (2), $dv = \alpha d\mu$ with $\alpha \in L^1(d\mu)$. Assume $L \otimes M \subseteq L_{ac}^*(d\mu)$, for instance $dv$ absolute continuous with respect to $d\mu$ and $v$ absolute continuous, then we have that the integral over $L \otimes M$ is defined. We can assume on bounded sets, that $\psi$ is defined modulo analytic action, that is $\psi U = U^\perp + V$, where $dV = 0$, thus modulo regularizing action.

Consider $V : ST=1$ and define $\tilde{V} = \{ y : T(y) = T^{-1}(x) \} \ x \in V \}$. Note that $S(x) = 0$ implies $T(y) = 0$, that is $x \notin V$, but $T(x) = 1$ implies $T(y) = 1$, that is we consider conjugated surfaces $x \sim y$, that is $\exists x \to x_0$ with $T(x) = 1$ and $\exists y \to x_0$ with $T(y) \neq 1$ (very regular boundary).
The condition \( T(x)T(y) = I \), can be written \( e^t + t^{-1} = I \). Assume \( t + t_1 = I \), then we have \( I = t_1 - t^{-1} \). Given projectivity \( R(t^{-1}) = R(t_1) \). The condition \( N(t^{-1}) = \{ \phi, t^{-1}(\phi) = 0 \} \subset C^\infty \) corresponds to a algebraic (removable) set. Given analyticity for \( T \), we can consider \( x(T)y(T) = I \), that is \( y(T) = 1/x(T) \).

0.4.6 Discontinuous movements

Given \( \mathcal{H} \) scalarly of order \( m \) with property \((e)\) and \( E \) 1-dimensional, then \( \mathcal{H} \subset \mathcal{D}' \) is of finite order \([16]\). In the case of \( \dim E > 1 \), there are topologies involving a spiral, where the limit can be undetermined. I can be approximated by \( I - \gamma \), of finite order, when \( \gamma \in \mathcal{D} \). Note that modulo \( C^\infty \), operators of finite type can be considered as of type 0 (real type).

When \( \hat{T} = T \circ \phi = 0 \) implies \( \phi \perp R(T) \), we note that given \( \hat{T} \to T \), we do not have \( \delta \perp R(T) \). Sufficient for convergence is \( \hat{T} \) downward bounded, as \( \hat{T} \to T \). Assume for \( \Omega, T(\Omega) \) the closed convex hull, that is the barrel to \( \Omega \). Assume \( W \) a barrel \( \subset \Omega \). Assume \( p_1 \) a semi-norm to \( U \mid_W \), \( p_2 \) to \( U \mid_\Omega \) and \( p_3 \) to \( U \mid_{T(\Omega)} \). A condition equivalent with inclusion (related to \( L^p \)) is \( p_3/p_1 \to 0 \) in \( \infty \), for instance \( p_1/p_2 \to 0 \), where \( p_2/p_1 \) is bounded. Given \( p_1, p_3 \) algebraic (removable zero sets), we have hypo continuous convergence in \( L_c(\Omega, E) \).

Necessary for hypo continuity is \( I \circ \epsilon' = \epsilon' \circ I \), that is topological algebraicity. Assume \( F : E \to E \), according to \( F(| \epsilon |) \to F(\epsilon) \) | equicontinuous. We have given \( \mid F \mid (| x |, | y |) \in E^{(0)} \), that \( F \in \mathcal{E}^{(0)} \). For an equicontinuous mapping \( E \ni (x, y) \to dU(x, y) \), we have that \( \mid dU \mid < 1 \) corresponds to \( (x, y) \ni nbhd0 \), for instance \( \{(x, y) \mid \mid dU \mid < 1 \} \subset C \mathcal{E}^{(0)} \). Assume existence of \( U \), such that \( T \ast U \in \mathcal{E}^{(0)} \), that has the approximation property. Further, \( U_1 I \ast T + U_2 I \ast T \in \mathcal{E}^{(0)} \) and given \( U + U^{1} = I \) (outside the polar), we have \( T \in \mathcal{E}^{(0)} \). In particular, when \( (U, U^{1})f_0 \) generates \( L^1 \) (outside the polar) we have that \( (U, U^{1/2}) \) is locally 1-1 (outside the polar).

Discontinuous limits 0.11 Spirals are discontinuous with respect to \( L_c \).

Given \( dU BV \), we have a determined tangent, even when \( dU \) is discontinuous \((2) \). A parameterization of spirals does not have a determined tangent, in particular when \( (x, y) = (tx_0, t^k y_0) \), we have \( dU(x, y) = tdU(x_0, y_0) \), that is \( \mid dU \mid = 1 \) implies \( t=1 \).

0.4.7 Semi-regularity

Assume \( B(\phi, \psi) \) a bilinear form, then we have that the zero space \( N = \{(\phi, \psi) \} \) defines orthogonals. Assume \( B \) absolute continuous, then we have \( d \) \( \subset \Omega \) implies \( B = const \). B considered over a convex set, is separately absolute continuous. Consider for this reason \( T(N) \) and \( N \) completing over the spiral. Assume \( (\phi, \psi) \) decomposable and \( p \) a semi-norm defined relative \( B \). Given \( 0 \in T(N) \) and \( p \) continuous in \( 0 \), then \( p \) defines a norm on \( T(N) \). We assume a separation condition, for instance \( \{ B < \lambda \} \subset \subset \Omega \). Given \( B \) separately locally 1-1 (with respect to norm), we have \( tB(\phi, \psi) = B(\psi, \phi) \) separately locally surjective. Note that \( B(\phi + \psi, \phi - \psi) = B(\phi, \phi) - B(\psi, \psi) \), gives \( B(\phi, \psi) \equiv B(\psi, \phi) \) locally.

Assume \( I = R_1 R_2 \), where \( R_j \) denotes reflection with respect to a bilinear form \( B(f, g) \), for instance \( R_1 B(f, g) = B(-f, g) = -B(f, g) = BR_1(f, g) \). B can then be considered as semi algebraic, if for instance \( BR_1(f, g) = R_2 B(f, g) \).
We assume $\langle f, dU \rangle = \langle f, dU \rangle$ and $\langle g, dU \rangle = \langle g, dU \rangle$, then we can define $\langle f, g, dU \rangle$. Cf. the lifting principle, that is over analytic polycylinders, for instance over $f, g \in \mathbb{L}^1$, we have existence of $U$ with $\langle f, dU \rangle = 0$ and existence of $U'$ with $\langle f, dU \rangle = \langle U, g \rangle$.

$\phi, \psi$ is holomorphic or $U = I$, with respect to uniform convergence on compact sets.

In analogy with homology, we can write $dV = \sum_{\alpha} dU_\alpha$, where the right hand side traces a polycylinder in the domain. We assume $dV$ absolute continuous relative $dU$. Given $V = I$ on the cylinder web, the limit is on one or several of the leaves.

0.4.8 Symmetry

Assume $I_x(\varphi) - \gamma_x \in C^\infty$ and $I_y(\varphi) - \gamma_y \in C^\infty$, then we have $\varphi(x) - \varphi(y) \in C^\infty$. Given this implies $\varphi(x - y) \in C^\infty$, we have that separately regular with respect to $x, y$ implies very regular. In particular, consider $\varphi(x) - \varphi(y) \in \hat{B}$, when $x, y \to \infty$, implies $\varphi(x - y) \in \hat{B}$.

The symmetry operator $T(\varphi) = T(s^{-1} \varphi)$, does not imply $s^2 = |I|$. Assume for instance $S$ a multivalued domain with leaves $\{S_i\}$, then we have that reflection on every leaf such that $s^2 = |I|$, implies symmetry on $S$. Further, assume $U = I$ symmetric iff $U^\perp$ symmetric, where $S$ is generated by $U$. Assume $U$ absolute continuous, is defined by $X_U(f) = 0$, then according to Hurwitz, we have that $U$ is holomorphic or $U = I$, with respect to uniform convergence on compact sets.

Given $U$ projective $U^\perp = I - U$, we can define $(U, U^\perp) = s(U^\perp, U)$.

$\phi, \psi$ is holomorphic or $U = I$, with respect to uniform convergence on compact sets.

In analogy with homology, we can write $dV = \sum_{\alpha} dU_\alpha$, where the right hand side traces a polycylinder in the domain. We assume $dV$ absolute continuous relative $dU$. Given $V = I$ on the cylinder web, the limit is on one or several of the leaves.

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Assume $s = s_1 s_2$, for instance $s_1$ harmonic conjugation and $s_2$ complex conjugation. Then we have $z \to \overline{z} \to \overline{\overline{z}}$ gives $(x, y) \to (y, x)$ and $s^2 = id$ that is reflection through the diagonal. When $s^2 = s$, that is $s$ a projection operator, we get the diagonal (the spiral when $x, y$ is $u_1, u_2$). Starting from the two mirror model, where $s_j$ is reflection through $L_j$, $j = 1, 2$, we have that $F$ is symmetric with respect to $L_1, L_2$ and $L_1 \to L_2$ projective, implies $F$ symmetric outside the diagonal. Assume $s = s_1 s_2$, such that $s^2 = s$, $s_1^2 = -s_2^2 = -1$, then we have $s_2 s_1 = -I$. Assume $s_1(A) = B$, so the mapping over the axis $L_1$, then $s_2 s_1$ gives a normal model. When $s$ is projective in the plane, we can through $s_0$ compare with a transversal model. Necessary for the two mirror model is a separated space, that is points can be chosen outside the reflection axes.

Consider $y/x \to x/y$ regular, that is $y(x) \to x(y)$ invertible. Assume $F$ subnuclear and consider $F_1(x, \hat{z})$ with support on half the disk, in the same manner $F_2(y, \hat{z})$ with support on the other half. Given $F = F_1 + F_2$ and $IF_1 = F_2, IF_2 = F_1 I$, we have $IF = FI$.

### 0.4.9 Sub-nuclear mappings

Assume $d \mid U \mid (| \varphi |) = 0$ implies $|U \mid (| \varphi |) = |\varphi |$. Then $|U \mid - |I||U - I|$, that is $U$ with constant variation does not imply $U$ absolute continuous, even when $|U \mid$ absolute continuous. We assume that $|U \varphi - \varphi \mid \to 0$ implies $U(| \varphi | \mid \to |\varphi |$. Note that $d \mid U \mid (| \varphi |) = 0$ implies $dU(\varphi) = 0$ implies $U$ analytic over some $\varphi$. Further, $\int_{\Omega} d \mid U \mid |U|^2 = 0$, implies $|U|^2|_{\Omega} = 0$, but $N(U)$ can have a positive measure (2).

Let $U^\perp = (I - U)^{-1}$, then we have for instance $U_{\overline{1}f} = U_{\overline{2}f} = ((I - U_{\overline{1}}) + V)\hat{f}$. Over the lineality, we have $(I - U_{\overline{1}})\hat{f} = 0$ and $V$ rotation. Assume $\mathcal{H} \oplus \mathcal{H}_1 = \mathcal{K}$ and $\Delta$ a restriction, with $\Delta + (I - \Delta) = I$ and $R(I_{I - \Delta}) = \mathcal{H}_1$. Assume $R(I_{\Delta}) \subset C^\infty$. Given $\Delta + I - I_\Delta = I_\Delta + I_{\Delta - I}$, assume $\Delta \in C^\infty(x, y)$ symmetric, with $\int \Delta(x, y) f(y) dy \equiv 0$, when $f \to \delta$, then we have $\Delta(0) = 0$. Given $f \in D$, we have for all derivatives, $\Delta(D^n f) = 0$ in $C^\infty$ and in the same manner for $^t\Delta$, why $\Delta = 0$ (nbhd 0).

### 0.4.10 Convexity

Assume $\Omega = \{dU(f) = 0\}$ and construct $B$, the absolute continuous closure of $\Omega$. Then we have that $B$ can be defined by $\{U = I\}$. When $\Omega$ contains a 1-polar, we have that $B$ has a 1-polar, but not conversely, that is if $U$ is projective on $\Omega$, it is not necessarily projective on $B$. Assume $U$ 1-homogeneous and consider $z(t) = tf + (1 - t)g$, then we have $Uz(t) = tuf + (Ug - t Ug)$, thus convexity is preserved. But over the set where $Utg = g$ (polar set), this is not the case. Thus, given $U$ absolute continuous, we have that $U$ is not necessarily subnuclear over continuations to $g$, such that $dU(g) = 0$. On $B$ we have that $\int f' = f$ and given $f = 0$ on $bdB$, we have $<f, dU > = \int (U f')dx$. Thus, given $^t(U f)$ absolute continuous, we have $<f, dU > = ^tU f$. Given $U$ subnuclear, when $U \to I$, $B$ is nuclear.

Assume $R(U)$ is defined by $dU$ and that $(dU)^\perp$ is given by $R(dU^\perp)$. Assume the polar is given by $dV$, where $V \in G$ and $dV$ of distributional order 0. Assume $V = W_1 W_2$, where $W_1$ are not spiral and $V^\perp = V$, then must we have $W_1^\perp \neq W_1$, why $W_1^\perp = W_2$. Note that for the spiral to be well defined, the orientations
for $W_1, W_2$ must be compatible. Given $U, U^\perp$ absolute continuous, we have $d(U - U^\perp)(T) = 0$ implies $U^\perp = (I - U)$, that is $U$ projective over $R(T)$.

I is optimally non-reduced, I preserves convex functions, given I preserves character over tangents, for instance $U \to I$ regularly, with $U f' = (U f)'$. Assume $L$, a connected line between $f, g$, $L(U)(f, g) = L(U)(f, g)$, that is sufficient for a connected line to be preserved under $U \to L$, is that $L \to L$ is continuous.

**The Lifting principle 0.12** Assume $\Gamma$ chains of maximal order, without invariant sets and $\Gamma_0$ chains of order 0, that is cylindrical sets. On $\Gamma$ the composition of movements is not necessarily invertible. Through the lifting principle over $\Gamma_0$, we have existence of $U$, analytic over $f$, such that $g = U f$.

Given $\Gamma_0$ has the approximation property and $I \in \mathcal{L}_c(\Gamma_0)$, then $\Gamma$ has the approximation property ([19], prop. 2), that is $\Gamma_0$ has convex (analytic) continuation to $\Gamma$.

### 0.5 Discontinuous continuation

#### 0.5.1 Equilibrated sets

For an equilibrated set, we have $e^{\lambda y} \sim e^y$, when $|\lambda| < 1$, that is $(I)(U) \supset e^{\lambda y} = I(e^{\lambda y})$ if $I(\phi^y) = \phi^y \in (\tilde{I})$, that is we can assume this ideal is radical. Note that Mackey topology does not imply absence of a spiral (over the polar), that is $(L'_0)' \simeq R$ is reflexive but does not imply projectivity.

Note that a closed contour can be compared with $z \to 1/z$ bijective. Assume $R_0 \ s = 1, 2$, such that $R_0 R_0 = I$ and $R_0 U = U R_0$. Note $| I U | = | R_0 U | = | R_2 U | = | I U |$, that is with respect to hypo continuity, $| U |$ is algebraic. Given $d | U | = d | U_1 |$, with $\phi \in L^1$, where $| U_1 |$ is analytic, $| U |$ is harmonic, that is has a decomposition into conjugated movements. Note that $\{ d | U | < \lambda \} \subset G \subset \Omega$, that is the separation condition implies hypo continuity, in the sense that $l \in f | U | (f))$ is closed. Assume $| F(\phi) : G(\phi) \in L^1(\phi))$, where $G$ continuous over $| \phi |$, then $F$ is $\sim$ hypo continuous. Given $F \to I$, we have $G \to I$. Assume $F = AB$ and $dF$ absolute continuous with respect to $d B$, when $F \to I$, then $A$ is invertible with respect to $B$. When $I$ is subnuclear, we have existence of $A$ with $\frac{dF}{dB} \in L^1(dB)$, such that $d F$ is absolute continuous with respect to $dA$.

When $u \otimes v$ is an absolute continuous function in $E^*$, then we have given $d(u \otimes v) = 0$, that is we can define $x \sim y$. Given $u \otimes v(x, y) = \Phi(x, y)$ with $\Phi = 0$ outside $x = y$, we must have $\Phi = \delta_x$. Note that since $D_A \times D_B = D_{A \times B}$ and when $u \otimes v(x, y) = \Phi(x, y)$, with $\Phi$ according to above, we can determine $x \sim y$ uniquely.

Assume $\Omega = \{ x \ f(x) \in L^1 \}$ and consider $\Omega_{ac} = \{ x \ f(x) \text{ absolute continuous} \}$ $\subset \Omega$ and $\Gamma_{ac} = \{ f \ f \in L^1(\Omega_{ac}) \}$. Given $f \in L^1$ there is $\Omega_{ac}(f)$, such that $f \in \Gamma_{ac}$. Over $\Omega_{ac}$ we have $f d\gamma \sim \int f dx$. Given a maximum-principle on $\Gamma_{ac}$ and $I \in \mathcal{L}_c(\Gamma, \Gamma_{ac})$, we have that $G$ has the approximation property.

Assume $T = P(D)d\mu$, where $d\mu \in \mathcal{E}^{(0)}$, then we have $T \in D^{m}$, given $P$ of degree $m$. Assume $\Gamma$ boundary for a pseudo convex domain, that is of order 0 with $(\Gamma)$ of finite order. This means that $(\Gamma) \subset D^{m}(\mathcal{E})$ can be generated by chains of finite order. Given $d\mu$ of finite type, when we consider measures modulo $C^\infty$, $d\mu$ is of type 0. Further, $d\mu \sim \int \nu \delta V^2$ is of finite order on $(\Gamma)$. We assume invertibility for $T$ only on $\Gamma$. 

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Given $dv =< f, e'> dx$, that is $\frac{dv}{dx} =< f, e'>$, then through partial integration $\int \frac{dv}{dx} \varphi dx = \int < f, e'> \varphi(x) dx$. Note $< f, dU > =< f, dU^1 >$ does not imply $dU = dU^1$, when the space is not nuclear. Note that when $< f, dU_1 > < f, dU_2 > (\varphi) =< f(\varphi), dU_1 \otimes dU_2 > = 0$ implies $f(\varphi) \in C^\infty$, this does not give the same implication for movements separately. For instance, $(f \otimes g)^\phi =< X, f \otimes g >= 0$ does not imply $X(f) = 0$ or $X(g) = 0$, for instance $e^p - I = 0$ iff $P = 0$, that is when $S \sim e^P$ is projective, $S^2$ must be locally of finite order.

### 0.5.2 Separated spaces

**Hypocontinuous convolution algebras 0.13** Assume $H(G)$ a discontinuous convolution algebra, $UT \in H$ implies $U^{-1} T \in H$, $IT \in H$ implies $(I - U) T \in H$ but does not imply $(I - U)^{-1} T \in H$. A hypocontinuous convolution algebra, implies that $(U, U^2)$ has relatively compact sub level surfaces, given $U, U^2 \in G$ equicontinuous.

More precisely, assume $H \ni UT \rightarrow U^\perp T \in H$, where given $H$ has the approximation property, we can define $U^\perp$ through a bilinear form $B$. Assume $p, q$ semi-norms over $H$ relative $U, U^\perp$. Assume $r$ a semi-norm relative $(U, U^\perp)$. Then hypo continuity means (19), that $r(T(u, u^\perp)) \leq p(T(u))q(T(u^\perp))$. Assume $U^\perp = (I - U) + V$, then we have $d(I + V) = dU + dU^\perp = (1 + \beta)dU$. Given $(1 + \beta)^{-1} \rightarrow 0$, when $|x|, |y| \rightarrow \infty$, for instance $(1 + \beta)^{-1} \in \mathcal{D}_{L^1}$, then we have $\{\beta < \lambda\} \subset \subset \Omega$, where $\Omega$ an open set. Note if $U$ is algebraic over $\phi$ and $U^\perp = \overline{V \phi}$, we assume $U \sim V$, that is of the same character. With these conditions $U$ is determined through dependence of the polar.

Assume $p$ a semi-norm on $D'$, $q$ a semi-norm on $H$, then $H \subset D'$ means that $p \leq q$. The property $(\epsilon)$ means that $p \sim q$. (16), Prop. 13). Assume $H_p \subset H_q \subset H_p$, where the weights are related to $L^p-$ spaces. Given $p, q$ polynomials (algebraic), it is not necessary that $w$ is algebraic. However, given the property $(\epsilon)$, $w$ must be algebraic. Note that given $pq \leq p^2 + q^2$, we can have a “spiral”. In particular, presence of spiral according to $(I) \subset (J)^1$ and $(J) \subset (I)^1$, when $(I) = R(U)$ and $(J) = R(U^\perp)$, that is $(I) = (J)^\perp$. Note $|< f, \tilde{g} >| \leq \|f\| \|\tilde{g}\|$, that is given either $\|Uf\| = 0$ or $\|U^\perp \tilde{g}\| = 0$, we have $Uf \perp U^\perp g$, that is $R(U^\perp) \subset R(U^\perp)$. Given $U \neq 0$ and $U$ locally 1-1, we have that $U^\perp$ surjective over $\tilde{g}$ and $0 \neq U^\perp \tilde{g} \perp UF$, for some $\tilde{g}$.

Consider the completion to a closed curve, according to $\int_{\tilde{c}} dc = \int_{\tilde{c}} d\tilde{c}$, where $\tilde{g}$ a closed curve. Assume $\gamma$ a spiral (not $\sim 0$), with $\tilde{c} = 1$ over $\gamma$ (absolute continuous), given the spiral intersects a closed curve, we have that $\exists dc$ a closed form associated to $\tilde{g}$. Every curve on the cylinder web, can be intersected by a spiral. But we do not have that $\tilde{c} \sim 0$

### 0.5.3 Uniformity

When $E$ is a barrel and $M' \subset E'$, we have $M' \subset E'$ iff $M'$ equi-bounded (16, Prop. 2). Assume for instance $R(dU^\perp) \subset \subset R(dU^\perp)$. When $M' = \{P < \lambda\}$ and $E' = \{Q < \lambda\}$, the condition means $Q/P \rightarrow 0$ in $\infty$. 

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The condition means that \( |x|^2 \frac{\phi}{\sigma} < c \), for some \( \sigma > 0 \), \( |x| \to \infty \), that is the condition can be written \( \{ P < Q \} \) relatively compact.

Assume \( d\mu \) BV on \( L \) completed and \( M = \{ \varphi \in L \mid \int |d\mu| (\varphi) = const \} \). Assume \( dV = d\mu \mid \) and \( dV \) absolute continuous relative d I on M, then we have \( V \) absolute continuous on M. That is, \( dV = \alpha dI \), where \( \alpha \in L^1_{ac} \) on M and \( V = \text{const}.I \) on M.

Assume \( dF \subset dU \otimes dU^\perp \), with \( (dU)^\perp \subset (dU^\perp) \). Given \( F \) absolute continuous, we have projectivity over analytic chains. Given \( dU \otimes U^\perp \sim 0 \), implies \( U + U^\perp = I \) outside the polar, where for instance \( I(U \to U^\perp) \notin L_{ac} \). Over absolute continuous functions, we have thus \( dU \sim dI \), that is \( dU \) approximates \( dI \) regularly. Note that precence of a regular chain is necessary for reduction to absolute continuous functions.

Given a barrel \( T \), there is a semi-norm \( p \), such that the unit ball can be given by \( \{ p \leq 1 \} \). \( p \) is continuous iff \( 0 \in T \) (19). Thus, the semi-norm corresponding to the cylinder web is not continuous. The approximation property through regularizing and truncation on a normal space implies the strict approximation property (10), Prop. 3, pg. 10).

Assume \( W_c = \{ (x, y) \mid dU(x, y) = c \} \) compact and \( (W_c) = \{ (x, y) \mid dU(x, y) < c \} \). Given \( (W_c) \subset \subset \Omega \), with \( \infty \in \Omega \), then \( |dU| \) can be given as locally reduced. When \( |dU| = \beta dI \), where \( \beta \) is a continuous function, maximum is assumed on \( W_c \). Note that \( dU \) is not necessarily locally reduced.

Assume \( << T, \varphi, e >, e' > = 0 \) for arbitrary \( \varphi \in D^{1,1}_c \), then using the approximation property for \( D^{1,1}_c \) and relative topology for uniform convergence on compact sets, we have existence of \( dF \) closed locally, with \( \int < T, e > \varphi dx = \int \varphi dF \). In particular, when \( e' \) defines an analytic movement, \( \int < T, e > \varphi dx \) is on closed contours locally independent of choice of local coordinates.

### 0.5.4 Discontinuous continuations

Consider \( (U, U^\perp) \) as a continuation element, that is \( R(U) = R(U, 0) \subset R(U, U^\perp) \). Given \( (U, U^\perp) \) locally 1-1, we have given Schwartz separation condition, \((U, U^\perp) \simeq (U^\perp, U^\perp) \). Given \( U \) reflexive, then \( (U, U^\perp) \simeq U + U^\perp \) can be considered as projective. Given \( U^\perp = (I - U) + V \) and \( U \) absolute continuous, we have \( dU = 0 \) implies \( U^\perp \equiv V \) on the polar. Further, when \( dU^\perp = 0 \), we have \( V^\perp = V \) on the polar.

Assume \( u \to v \) decomposable, in the sense that we have corresponding seminorms \( p_u \geq q_v \). Given \( d \mid U^\perp = \beta d \mid U \mid \) and \( dU^\perp = \alpha dU \), we have that \( \beta = \text{const} \) does not imply \( \alpha = \text{const} \). Given \( V(U, U^\perp) \) the polar, we have that \( R(V(U, U^\perp)) \) is decomposable does not imply \( R(V(U, U^\perp)) \) decomposable. Assume \( K_1, K_2 \in \mathcal{H} \). Consider \( K_1(l, u)K_2(m, v) \simeq K(l, m)K_0(u, v) \), where \( K_0 \) regulates the change of variables \( u \to v \), for instance \( K_2(v, m) = v^* \otimes \text{inv}^I K_2(u, m) \) (17). The condition \( \frac{\mu}{\pi} \in L^1(du) \) gives an integrable change of variables. Given \( \mathcal{H} \) has the approximation property through truncation, we can define a continuous deformation.

In particular for instance \( K_1(u, u)K_2(u^*, w) \simeq K(u, u^*)K_0(u, w) \), where \( u, w \) are in complementary sets. Assume \( f, \tilde{g} \) analytical and \( f\tilde{g} \equiv 0 \) in nbhd \( u_0 \), then \( N(f\tilde{g}) = N(f) \cup N(\tilde{g}) \). When \( \{ f\tilde{g} < \lambda \} \) is semi-algebraic with \( \tilde{g}(u) = h(u^*) \) and when the set is decomposable, we can assume \( f\tilde{g} \sim f(u)^P(1/u) \), for some polynomial \( P \). More generally, assume \( dI \) BV considered over \( ac \) functions, then
we have \( dU + dV = dI \), where \( dU, dV \) are BV measures, \( dU \) non-decreasing and \( dV \) non-increasing. We can extend the domain to \( \alpha dI \), where \( \alpha \in L^1(dx) \), to motivate an integrable change \( u^* \to u \).

Given \( (I_1) \subset (I_2) \), ideals of analytic functions, we have \( N(I_2) \subset N(I_1) \), but first surfaces to \( I_1 \) are not necessarily first surfaces to \( I_2 \). For instance \( (I_1) \) has the strict approximation property, but \( (I_2) \) has only the approximation property. Assume \( (I_1) \subset (I_2) \) according to \( p/q \to 0 \) in \( \infty \) for the corresponding semi-norms. Given \( T_j \in (I_1) \) implies \( T_j \in (I_2) \), that is \( T_j \to T \) in \( (I_1) \), does not imply \( T_j \to T \) in \( (I_2) \); for instance, \( T_j \to T + c \) in \( (I_2) \).

\( L^1 \to L^2 \) is not bijective, \( dU^2 \) locally 1-1 (absolute continuous) does not imply \( dU \) locally 1-1 (absolute continuous). According to Lie ([7] chapter 12), we have \( dU \neq 0 \) implies \( dU \), the condition for invertibility \( dU_1 = \gamma dV \), is \( \gamma = \Sigma j \beta_j/\alpha_j \to 0 \) in \( \infty \).

Assume \( H = H_0 + H_1 \), given \( I \in \mathcal{L}_c(H, H_1) \), where \( H_0 \) has the approximation property, we have the same for \( H \) ([16]). Note \( I \in \mathcal{L}_c \) does not imply that \( I \) is locally reduced. Assume \( H_0 \) is defined by \( (U, U^\perp) \), with \( U \neq U^\perp \). Note \( U = U^\perp \) defines the web, convex with respect to spirals. The condition \( F(U, U^\perp) = 0 \) implies \( U = U^\perp \), that is \( F \) is reduced over \( (U, U^\perp) \), can be written \( \int \phi dF = 0 \) for \( \phi \in (I) \) implies \( (I) \subset C \) \( (\phi \neq 0) \). Given \( dF = 0 \) implies \( F = I \), that is a constant surface, means that \( U^\perp \) is not uniquely determined outside the polar.

Assume \( U = \alpha dV \), with \( \alpha \to 0 \) in \( \infty \), for instance \( \alpha \in B \). We then have inclusion between the corresponding ideals, further \( \mathcal{L}_c(dV) \to \mathcal{L}_c(dU) \) is continuous, that is we have topology induced by truncation. Thus, existence of an uniformly convergent subsequence in \( dV \) implies existence of an uniformly convergent subsequence in \( dU \).

Assume \( U^\perp = U + V \). Given \( \{ V = 0 \} \) removable, we have regular approximations. Assume \( V \) is defined through completion to \( L^1 \), in the sense that \( dx \) is absolute continuous relative \( dV \). Given \( dV = 0 \) implies \( -V = I \), then we have \( f \) absolute continuous, that the polar is removable. For a normal space and the closure of \( U^\perp \) in \( L^1 \), we have that it is sufficient for a removable continuation, that \( U^\perp \neq U \) close to \( U = I \).

Consider hypo density, that is \( | P |^2 \) polynomial does not imply \( P \) polynomial. If \( T \) is downward bounded by \( P_1 P_2 \), we do not necessarily have separate density. Consider for instance \( y = y(x) \), why \( P_1(x)P_2(y) \) polynomial in \( (x, y) \) does not imply \( P_1 P_2 \) polynomial in \( x \).
0.6.2 Completing sets

Given $\mathcal{K}$ normal with the approximation property through regularization, truncation, given $\mathcal{T}$ saturated and completing, with $S = i\Delta T$ and $i\Delta : \mathcal{H} \to \mathcal{K}$, then the scalar product is independent of $T$ (\cite{17}, Prop 20), for instance given $\Delta$ Fourier dual, then $< \varphi, T >$ can be given through only $N_{a}$ (\cite{12}), $\exp_{t||\Lambda,1}$ (\cite{11}), when the movement is defined through action in phase (\cite{9})

Given the conditions in prop. 20 (\cite{17}), we can reduce the condition for hypoellipticity, from the plane of $|f|, |\hat{f}|$ to plane of $f$, for instance it is necessary that $f$ is algebraic over the spiral (polar).

Necessary condition for hypoellipticity 0.14 Without the condition on the approximation property, it is necessary in the condition for hypoellipticity over maximal chains, to consider the spiral to a completing set

(\cite{6}). Given $T \in \mathcal{D}(F, \beta_{0})$ has support in $A$, then we have $(\mathcal{D}_{A})' \cong \mathcal{D}'/(\mathcal{D}_{A})^{0}$, where $\mathcal{D}_{A}^{0}$ is orthogonal to $\mathcal{D}_{A}$ in $\mathcal{D}'$. When $U^{\perp}$ is defined starting from the scalar product, it is sufficient to define $U^{\perp}$ over $\widehat{\varphi}$, the product is not dependent of $F, \mathcal{H}, \mathcal{K}$. Given $U^{0}$ the polar to $U$, we have $\mathcal{K}_{U^{0}} \cong (\mathcal{K}_{U})'$ (\cite{17}).

Hilbert spaces have the metric approximation property. When $F_{B}$ a Banach-space (B completing), we have given the parallelogram law, that $F_{B}$ is Hilbert. When $\parallel dU - dU^{\perp} \parallel = 0$ implies a measures zero set, then $dU$ is projective and $dU^{\perp}$ generates $(dU)^{\perp}$. Consider $I = \{dU = dU^{\perp}\}$, given $d$ locally reduced, then $(dU)^{\perp}$ does not leave space for a spiral. Let $\hat{U} = (U, U^{\perp})$.

Note if $R(\hat{U})$ completing set, then we have through change of variables to $(U_{1}, U_{2})$, that the radius $L$ can be seen as representing a spiral, as long as the change of variables preserves decomposability.

We consider $d(U, U^{\perp}) = (\alpha, \beta)d(U_{1}, U_{2})$, envelop to $R(\hat{U})$ generated by $(U_{1}, U_{2})$. Assume $p, q$ semi-norms relative $U, U^{\perp}$, consider $r$ a semi-norm relative the polar $V = U^{\perp} + U - I$. Assume $r(V) \leq p(U)q(U^{\perp})$. Over a convex set of $f$, given $r$ defines a norm, we have that $r(V(f)) = 0$ implies $V(f) = 0$, that is projectivity for $(U, U^{\perp})$. Note that when the domain is defined by $(U_{1}, U_{2})$, we have for the polar $V$ that $I + V \cong U_{1} + U_{2} = U_{2} + U_{1} = I + V^{\perp}$, where we assume reflexivity over the domain.

Given existence of $\limsup |U| = +\infty$, we do not have necessarily existence of $\limsup U$ (for instance essential singularities). Sufficient, to guarantee two-sided limits, is that $U$ is acting algebraically in phase (preserves constant value in $\infty$) and $U(\frac{1}{u}) = U^{-1}(\varphi) \in L^{1}$. Note that $I = I_{\text{red}}$, implies $I(f)I(1/f) = I(1) = I$. $\varphi$ can be seen as a closed contour, given $z \to 1/z \to z$ injective, that is $\psi(z) = 1/z$ and $\psi^{2} = I$. Assume $dU = adU_{1}$ with $\alpha \in L^{1}(dU_{1})$.

Let $\Sigma_{1} = \{dU_{1} = 0\}$ and $\Sigma_{1} = \{dU = 0\}$. Further, $\Sigma_{1}^{|0} = \{\Delta U_{1} = 0\}$ and $\Sigma_{1}^{0} = \{\Delta U = 0\}$ (Laplace operator). Associated to $\Sigma_{1}^{0}$, we have an conjugated resolution of identity. Assume $\Omega = \{\alpha = \text{const}\}$ and the closure $\bar{\Omega} = \{\alpha \in L^{0}_{\text{loc}}\}$. Note that a resolution starting from $\Omega$ has eigenvectors $\mathcal{H}$ (through linear independence). Given $\Omega \cap \Sigma = \emptyset$, we have $\Omega \cap \Sigma = \emptyset$, where $\Sigma = \text{envelope to } \Sigma^{|0}$ (\cite{5}). Given (nbhd $\Omega$ - preimage $P=0$, for a polynomial $P$) convex, we can relate to a regular covering. Assume $dU^{\perp} = \beta dU^{\hat{U}}$ (harmonic conjugate) and that the mapping corresponding to $\Sigma^{0} \to \Sigma^{1}$, maps lines on lines 1-1 and maps closed forms on closed forms 1-1. Thus, where $dU, dU^{\perp}$ are analytic, $dU$ is harmonic.
0.6.3 The spiral

An approximation with sequential intersection of the spiral, has a discrete intersection with the spiral. The cylinder web is convex for spirals, every sequential movement in \((U_1, U_2)\) is in the complement to the spiral. The spiral is a purely complex figure in \((u_1, u_2)\), transversals purely real, that is \(F(u_1, u_2)\) on spirals is complete. Spirals give presence of a complete approximation, in the case of holomorphic leaves \([14]\). Puiseux approximation.

Assume \(S = \bigoplus \cdots \bigoplus G_n\). Given \(\Omega\) a domain, such that \(U_j\) is analytic over \(\Omega\) for all \(j\), we have a covering. This is not evident for a very regular boundary, where only some \(U_j\) is analytic.

Assume \(\hat{I}(\Omega) = I(T(\Omega))\). Then we have \(\int \phi \, dt = \int \frac{d}{dt} (Uf) \, dt\) and given \(\int \phi \, dt\) is absolute continuous, \(\simeq \int \phi \, dt\) (boundary condition \(\int \phi \, dt = 0\) on bd \(T (\Omega)\)). Assume \(\Omega\) analytic over \(f\), that is \(\log f\) convex. Assume \(\hat{I} \simeq \text{rad} I\), then we have that \(\phi_N \simeq \phi_1\), where \(\phi_N\) is the phase corresponding to \(f^N\) and \(\phi_N\) is convex iff \(\phi_1\) convex.

Convexity with respect to path: Assume \(dU = \alpha dV\), where \(\alpha \in \hat{B}\), then we have that \(U\) is convex with respect to \(V\), given \(\frac{d}{dt} \geq 0\). Given \(L(f, g) = tf + (1 - t)g\), we have that \(\frac{d}{dt} = f - g = \int d\mu\). Given \(\frac{d}{dt} = 0\), we have that \(L\) defines a closed path. Define a continuous deformation, \(Uf + U^\bot g = W_I(f, g)\), where \(Uf \to t\) iff \(U^\bot \hat{g} = 0\), then given \(f, g \in R(W)\) locally convex, we have \(W_I(f, g) \subset R(W)\) and we have presence av intermediate values \(h \in R(W)\) under the condition \(R(U)^\bot \subset R(U^\bot)\).

Starting from \(U^\bot = (I - U) + V\), with \(U\) reflexive but not projective, we have that when \(U \to I\), \(U^\bot\) approximates a spiral and \(V = V^\bot\) approximates \(U^\bot = I\). The mapping \(U \to \hat{U}\) assumes regularity, if \(Uf \sim e^\phi\), then \(\{\phi = -\infty\}\) is mapped on the set \(\{\phi = 0\}\), that is \((U - I)f = 0\). Given \(\phi\) locally algebraic, the polar must be the preimage of a locally algebraic set.

Given \(E\) is separable, then \(T\) is locally summable iff we have existence of \(f \in E'\), such that \(UT(\varphi) = \int f(x) \, d^\alpha x > \varphi(x) \, dx\). \([2]\), Radon -Nikodym.

Consider \(wT\) locally bounded in \(\mathcal{H}\), where \(w\) is a multiplier or weight. When \(w\) is reduced, we have that \(T\) is locally bounded. Alternatively, \(wT \to 0\), defines \(T\) relative the \(L^1\)-norm. Note that \(\{\alpha = 0\}\) has normals of the type \(\{\alpha = \text{const}\}\), given the separations condition (isolated singularities). For instance, assume \(S\) a measure on \(\Gamma\), such that \(S - I\) is reduced modulo \(\Gamma\), with \(S \in E'(\mathcal{H}((\Gamma))\), that is \((S - I)(\phi) = 0\) implies \(\phi \in \Gamma\). Given \(S\) reduced and convex (absolute continuous) on \(\Gamma\), it is sufficient that \(\text{d}S = 0\) on \(\Gamma\). Note that the cylinder web \(C\) is convex with respect to \(dU\) and \(dUS\varphi \to \varphi\) implies \(\varphi \in C\), but \(dUS \to 0\), that is \(0 \notin C\). However 0 \(\in T(C)\), the convex disk closure. When \(dUS\) is a measure on \(C\) and \(dUS\) a subnuclear measure on \(T(C)\), we have that \(dUS\varphi \to \varphi\) implies \(\varphi \in T(C)\). Note \(dUS = \xi\delta_x + \eta \delta_y\), has \(\xi, \eta \to 0\), when \((x, y) \to 0\).

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