Abstract: Channel estimation plays a very important role on the performance of wireless communication systems. Channel estimation can be carried out in different ways: with or without the help of a parametric model, using frequency and/or time correlation properties in the wireless channel, blind methods or those based on training pilots, adaptive or non-adaptive methods. As more antennas are added to Multiple Input Multiple Output (MIMO) system more computational resources and power are required. There is a need to address this problem of the overhead in training symbol based models for channel estimation. Compressed Sensing (CS) algorithms are beneficial in addressing these limitations in the system to increase the spectral efficiency can help free up resources and prevent additional taxing on the hardware. Compressed sensing the pilot symbols by using CS algorithms like OMP, SP, and CoSaMP algorithms. The channel coefficients are obtained through LS, MMSE and LMS techniques. Then, a very small amount of frequency-domain orthogonal pilots are used for the accurate channel estimation. The traditional algorithms like LS, MMSE and LMS combined with CS algorithms have better performance at low SNRs compared to conventional techniques alone in terms of computational time complexity and Normalised Mean Square Error (NMSE) performance. There is a halving of pilot symbols used for training by using the CS algorithm. MSE performance is increased with increase in sparsity level. CoSaMP performs better than SP and OMP at low SNRs. With increase in sparsity level after 50K, the performance of SP is comparable to that of CoSaMP. OMP is simple to implement but MSE performance is less and computational time is more compared to SP and CoSaMP.

Keywords: Compressed Sensing, Orthogonal Matching Pursuit, Least Square, Compressive Sampling Matching Pursuit, Subspace pursuit, Least mean square, Multiple Input Multiple Output, Minimised Mean Squared Error, Orthogonal Frequency Division Multiplexing.

I. INTRODUCTION

With increasing data rates and more users, advanced technologies like MIMO systems and OFDM modulation techniques are necessary. The requirements for wireless network technology have increased over the years. Data shows that over the last few years, the wireless network data transfer rates have almost grown from 10 Kbps to 10 mbps. However the major challenges for wireless communication systems are Inter Symbol Interference (ISI) and Inter Carrier Interference (ICI) affect the quality of service and transmission rates.

MIMO-OFDM is a technology in which multiple antennas are used at both the transmitter and the receiver in order to increase data throughput, but at the same time, countering ISI and other adversaries. It does it with elegance by increasing the symbol duration of each stream such that the multipath delay spread is only a small fraction of the symbol duration. Computational requirements grow only slightly faster than linearly with data rate or bandwidth. Since, OFDM allows for usage of MIMO for the reason it converts a frequency selective broad band channel into several narrowband flat fading channels. However, mostly due to relatively high sampling rates and multipath delay spread, the channel estimation can be done with compressed sensing algorithms combined with linear estimation techniques like least square (LS)[3].

Pilots positions are considered in the channel at some intervals to get channel estimates[1]. By placing known sequences at these positions we can get the channel impulse response, we interpolate the rest of the subcarriers using these interpretations. Channel impulse response has a large number of taps with many of them zero. One key is to adopt the channel sparsity as a priori. In this paper, we consider channel estimation as a sparse recovery problem. Pilot assisted channel estimates are computed using CS algorithms like Orthogonal Matching, Pursuit (OMP), Subspace Pursuit (SP), Compressive Sampling, Matching Pursuit (CoSaMP) and compare it with traditional techniques like least squares. The computational time complexity of CoSaMP is O(m.N) and the computational time complexity of SP is O(m.N.log(K)) where m is the number of pilots or rows in the measurement matrix. And N is the number of columns in the measurement matrix. K is the sparsity level.
II. MIMO-OFDM SYSTEM MODEL

In MIMO systems, there are multiple antennas at the transmitter as well as at the receiver side. This helps in increasing the data rate by transmitting several data streams at the same time and frequency which is called as multiplexing gain. Advanced DSP algorithms can be used to recover the original data streams. These types of systems are called as MIMO systems.

![Block diagram of MIMO-OFDM System](image)

MIMO-OFDM is a technology that combines OFDM with MIMO technology to provide mobile and wireless broadband access. It is regarded as a major and potential choice for future wireless technologies due to its support for high-speed wireless broadband access, minimal complexity (in terms of equalisation at the receiver), and spectral efficiency and flexibility.

The MIMO-OFDM system is similar to the OFDM system, with the exception that it must deal with multiple antennas at both the transmitter and receiver side. Before up-converting the Radio frequency transmission, each branch conducts serial-parallel conversion, followed by pilot insertion, IFFT, and adding of Cyclic Prefix at the transmitter side. Before broadcasting, each antenna undergoes channel encoding and digital modulation. After signal is received at the receiver side, using the pilot subcarriers the receiver must estimate the channel. The received signal is the aggregate of all the signals sent out by various branches. Using the preamble, the receiver must rectify frequency offsets and achieve synchronisation. After removing Cyclic Prefix, an N-point FFT is conducted at the receiver side. Pilot symbols in the frequency domain are used to estimate the channel. The remaining subcarriers are detected using the predicted channel coefficients. MIMO detection is done on a subcarrier basis. After detecting the bits, decoding and demodulation are performed. Each branch's received signal is a composite of all transmitted signals from Mt transmitted antennas.

Consider a MIMO-OFDM system with Mt transmit antennas and Mr receive antennas. The conditions of the MIMO channel requirement is that $Mt \leq Mr$. The system transmits frames of OFDM symbols. A frame is an OFDM block of data symbols.

The MIMO-OFDM System can be represented as

$$y(k, n) = H(k, n)s(k, n) + z(k, n)$$

where $y(k, n)$ is received signal, $H(k, n)$ is the MIMO channel matrix associated with subcarrier index $n$ and frame index $k$, $s(k, n)$ is a frame of OFDM symbol vector and $z(k,n)$ is additive white Gaussian noise.

III. COMPRESSED SENSING

Sampling theorem states that any bandlimited time-varying signal with highest frequency $f$ Hertz can be perfectly reconstructed by sampling the signal at regular intervals of $1/2f$ seconds. This theorem was proved by Shannon in 1949. We sample data at Nyquist rate and do signal processing before transmitting.

![The basic idea of traditional data sampling vs compressive sensing](image)
The ‘n-m’ samples is discarded by compressing signal to ‘m’ samples. Decompression, which is opposite of compression is applied at the receiver to get back the original ‘n’ samples from compressed ‘m’ samples[7].

Compressive sensing or sampling hypothesis states that the signal can be recovered from far less samples. As, this is opposed to Nyquist criteria, compressive sensing constructs the signal with only few non-zero coefficients in basis or dictionary. Depending on the specific reconstruction algorithm, number of samples can be chosen for the exact recovery of the signal.

The dot product of the input sparse signal and the measurement matrix gives the measurement vector[8].i.e fig.4. The equation for measurement vector is given by:

\[ Y = A \times x \]

Where, \( A \) is Measurement matrix; \( x \) is Sparse input signal; \( Y \) is Measurement vector. The compressed signal will go through the sparse recovery algorithms namely SP, OMP and CoSaMP to reconstruct the original signal[10].

**IV. CHANNEL ESTIMATION**

In an OFDM system, the transmitter modulates the message bit sequence into PSK/QAM symbols, performs IFFT on the symbols to convert them into time-domain signals, and sends them out through a (wireless) channel[4].

The received signal is usually distorted by the channel characteristics. In order to recover the transmitted bits, the channel effect must be estimated and compensated in the receiver.

In order to choose the channel estimation technique for the OFDM system under consideration, many different aspects of implementations, including the required performance, computational complexity and time-variation of the channel must be taken into account[5].

The orthogonality allows each subcarrier component of the received signal to be expressed as the product of the transmitted signal and channel frequency response at the subcarrier. Thus, the transmitted signal can be recovered by estimating the channel response just at each subcarrier.

In general, the channel can be estimated by using a preamble or pilot symbols known to both transmitter and receiver, which employ various interpolation techniques to estimate the channel response of the subcarriers between pilot tones. In general, data signal as well as training signal, or both, can be used for channel estimation[6].
V. SPARSE RECOVERY ALGORITHMS

Using sparse recovery algorithms to deal with compressed sensing is another possibility. The signal’s support is first determined repeatedly. The resultant sparse signal is then multiplied by the pseudo-inverse of the sensing or measurement matrix to reconstruct the original signal. The quickness of this process is a reasonable point of favour, but it also poses new challenges.

A. Orthogonal Matching Pursuit

The orthogonal matching pursuit (OMP) was created as an upgraded version of the matching pursuit method to alleviate the method’s limitations. The notion of orthogonalization underpins OMP. It calculates the inner product of the residue and the measurement matrix, then finds the maximum correlation column’s index and extracts it (in each iteration). The extracted columns are added to the atoms in the selected set. The OMP next executes orthogonal projection over the subspace of previously selected atoms, resulting in a new approximation vector that may be used to update the residual. Because the residual is always orthogonal to the columns of the CS matrix, no columns will be selected twice, and the number of picked columns will grow as the iterations progress[9].

Input: $A, b$

Result: $x_k$

Initialization $r_0 = b$, $\Lambda_0 = \emptyset$
- Normalize all columns of $A$ to unit $L_2$ norm;
- Remove duplicated columns in $A$ (make $A$ full rank);

for $k = 1, 2, ...$ do
  Step-1. $\lambda_k = \arg\max | \{a_j, r_{k-1} \} |$;
  Step-2. $\Lambda_k = \Lambda_{k-1} \cup \{\lambda_k\}$;
  Step-3. $x_k(i \in \Lambda_k) = \arg\min ||A_{\Lambda_k} x - b||_2$, $x_k(i \notin \Lambda_k) = 0$;
  Step-4. $b^*_{k} = A_{\lambda_k}$;
  Step-5. $r_k \leftarrow b - b^*_{k}$;
end

B. Subspace Pursuit

SP is another greedy technique that takes less time to compute and has a better BER performance. S columns are iteratively refined from the dictionary matrix using the LS method in the SP algorithm until the stop condition is satisfied. It picks S columns at each step, whereas MP and OMP only select one column. The downside of SP is that it requires prior knowledge of S before it can be used. As a result, it’s necessary to extend SP to cases when the sparsity is unknown. O($mN \log(K)$) is the overall computational time of SP. Because batch selection is used instead of one, the computational time complexity of SP is lower than that of OMP.

1) Inputs
   a) Initial channel common support $T_0$
   b) Channel sparsity level $K$;
   c) Received signal $M = U_i$
   d) Measurement matrix = $A$

2) Output: The K-sparse estimate $H \doteq H_i$

   Initial Configuration:
   $\Pi \leftarrow T_0$;
   $H^{0} \leftarrow 0$;
   $H^{0} \leftarrow A_{\Pi}^{-1} M$;
   $R \leftarrow M - AH^{0}$

   Iteration:
   For $k=1:K$ do $T_{k0} = 0$
   $p \leftarrow A^H R$;
\[ \Pi \leftarrow U \{ \arg \max_i \sum |jx_i| \}; \]
\[ H^{(k)} \leftarrow 0; \]
\[ H^{(k)} = A_i + M; \]
\[ R = M AH^{(k)}; \]
end for
\[ H = H^{(k)}; \]
\[ x \leftarrow 0, tX_{i} \leftarrow X_{\text{temp}} \]

C. CoSaMP

CoSaMP uses a matching filter to identify 2K (where K is the sparsity level) atoms, which is then merged with the support matrix or set calculated in the previous iteration. The estimated atoms from the matching filter are referred to as the union-set, which is the union of the support-set and candidate set computed in the previous phase. CoSaMP uses least-squares to find a new K-dimensional subspace from the union-set that decreases the sparse signal's reconstruction error. CoSaMP's total computational time is determined by O(m.N). In comparison to OMP, this is a very modest number.

Inputs \( A_{M \times N}, B_{N \times 1}, K \)
\[ r \leftarrow B, \Pi \leftarrow \emptyset \]
While stopping criterion not met do
\[ c \leftarrow A^H r \]
\[ \Pi \leftarrow \Pi \cup \text{findLargestIndices(applyModel(|c|)}^2),2K) \]
\[ X_{\text{temp}} \leftarrow \arg \min_x \| A_{\text{support}} x_{\text{support}} - y \|^2 \]
\[ \Pi \leftarrow \text{findLargestIndices(applyModel(|X_{\text{temp}}|)}^2),K) \]
\[ r \leftarrow r - A_{\text{support}} X_{\text{temp}} \]
end while
\[ x \leftarrow 0, X_{\text{support}} \leftarrow X_{\text{temp}} \]
\[ A = \text{measurement Matrix(dictionary)} \]
\[ B = \text{received Signal(samples obtained)} \]
\[ K = \text{Sparsity required} \]

VI. SIMULATION RESULTS

This Paper shows the Simulation results on Sparse Recovery algorithms for MIMO Channel using MATLAB and LABVIEW Software. The Sparse Recovery algorithms are applied on the MIMO Channel and the performance of algorithms are evaluated and compared in terms of NMSE vs SNR.

Figure 3 gives the performance comparison of LMS channel estimation combined with SP algorithm for 2x2 MIMO OFDM system. First, the LMS, channel coefficients are calculated and then these coefficients are subjected for sparse approximation through SP algorithms. In SP only once the batch selection is done with K largest magnitudes. From the above plot we can conclude that using LMS, the NMSE is reduced compared to LMS only. With SP, the performance can be improved further combined with LMS and MMSE.
Figure 4. MIMO-OFDM Estimation with LS, LMS and MMSE tools.

Figure 4, shows the graph of LS, LMS and MMSE for SNR vs NMSE. It shows a similar trend for LMS as it is an adaptive type algorithm that is better performing than MMSE and LS tools. Its BER is better even at lower SNR's. This simulation was done with 4x4 MIMO-OFDM with 16-QAM. A typical 4x4 MIMO system with QAM size of 16 and a Rayleigh-Rician fading channel with OFDM modulation is simulated. Here the bits generated are random at every iteration. The BER is shown on the top right corner. It also presents with a transmitter and receiver side constellation. At the receiver side is a constellation affected by the channel noise, modelling a noisy channel for the MIMO system.

Figure 5. Comparison of CoSaMP with traditional estimation techniques.

Figure 5, shows the NMSE vs SNR plot of CoSaMP compared with MMSE and LS. It is also combined with LS and MMSE. From the graph it is clearly noticeable that CoSaMP-MMSE performed better than CoSaMP-LS with a better NMSE. Compressed sensing algorithms perform better with the traditional algorithms.
VII. CONCLUSION
This project implements sparse algorithms for the reconstruction of the MIMO-OFDM pilot assisted channel estimation. For reducing the number of pilots or receiving compressed signals to reconstruct the original signal is the essence of the project. The channel is manipulated to be a sparse problem. CS theory has been used for sparse approximation of the signal. The compressed sensing of the pilot assisted channel and the channel coefficients were done in order to process it as a sparse system in a certain domain to linearly and the solution to recover the original signal. The channel coefficients obtained from conventional techniques like LS, MMSE and LMS were subjected to sparse approximation using CS algorithms. With the usage of CS algorithms it is observed that 24 pilots can be reduced to 12 pilots for estimation of signal. There is a 50% reduction in pilots usage from traditional tools to the CS algorithms. The other conclusion from this project is that with increasing QAM constellation size and its mean energy of the constellation is to remain the same, the points must be closer together and are thus more susceptible to noise, this results in a higher bit error rate and so higher-order QAM can deliver more data less reliably than lower-order QAM, for constant mean constellation energy. With the increase in antennas a better data rate and reliable data reception can be expected and increasing the constellation size thereby increasing the number of bits modulated at a time without compromising error rates. Designing a less complex algorithm and experimenting with other greedy algorithms can be the future scope of this project.

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