Multiplicity dependence of Bose-Einstein correlations in $\bar{p}p$ reactions: a discussion of possible origins

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The observed pronounced multiplicity dependence of correlation functions in hadron-hadron reactions and in particular of Bose-Einstein correlations provides information about underlying physics. We discuss in this contribution several interpretations, giving special attention to the string model for Bose-Einstein correlations of Andersson and Hofmann, as well as the core-halo picture of Csörgő, Lörstad and Zimanyi.

1. Introduction

It has been well known for years $^{[1–3]}$ that Bose-Einstein (BE) correlations measured in hadron-hadron reactions at sufficiently high energies exhibit a pronounced dependence on multiplicity in the form of the strength parameter $\lambda$. In $e^+e^-$ reactions, this behaviour is either absent $^{[4,5]}$ or at least much weaker, especially with a 2-jet selection $^{[6]}$. Heavy ion reactions, on the other hand, see $\lambda$ values decreasing with multiplicity at lower multiplicity densities (lower $A$ reactions) but increasing again $^{[7]}$ at higher multiplicity densities $^{[7]}$.

These findings may suggest a possible common explanation in terms of superposition of several sources or strings, where each of them symmetrizes separately according to the model of Andersson and Hofmann$^{[8]}$. No BE correlations are then expected between decay products of different strings $^{[8]}$. The following arguments could be used: In the framework of the Dual Parton Model for hadron-hadron and heavy ion reactions $^{[10]}$, it is expected that higher multiplicities correspond to a higher number of strings or chains. This can explain the multiplicity dependence in hadron-hadron reactions. In $e^+e^-$ reactions, with a selection of two-jet events, only one single string is formed and consequently one does not expect much dependence of BE correlations on multiplicity. In heavy ion reactions, there is again an increasing number of strings with increasing multiplicity density, but eventually the densely-packed strings coalesce until they finally form a large single fireball. This picture can qualitatively explain why the BE signal is decreasing at low $A$ and finally increasing again $^{[7]}$. A clean situation of a superposition of two strings occurs if two $W$-bosons are coproduced in $e^+e^-$ reactions and both decay hadronically. If there are no BE correlations between the pions from different strings, the $\lambda$ values are expected to be diminished. Unfortunately, these measurements are hampered by low statistics and experimental difficulties $^{[11 – 13]}$.

It should be stressed, however, that this is not the only possible explanation of experimental behaviour. In the case of hadron-hadron reactions, for example, several alternative explanations do
exist [14], among them the core-halo picture [13] which connects consistently the multiplicity dependence of correlation functions of like-charge with those of opposite-charge pion pairs.

From all these considerations, it should be clear that there is important information in the observed multiplicity dependencies, in particular when comparing different reaction types.

This contribution concentrates on a discussion of possible origins which could lead in the case of high energy hadron-hadron reactions to the observed multiplicity dependence. We first adress in Section 2 the question of the influence of jet production by using again pp collisions at \( \sqrt{s} = 630 \text{ GeV} \) measured in the UA1 detector [14,16]. Low-\( p_T \) and high-\( p_T \) subsamples are investigated. Using the low-\( p_T \) subsample where the influence of jets is removed to a large extent, we finally discuss in Section 3 possible underlying physics.

2. The influence of jets

It is well known [17] that in hadron-hadron reactions the probability of jet production rises with energy and multiplicity. The multiplicity dependence of correlation functions can be influenced by this hard subprocess. To investigate this influence, we used a data sample similar to that in [14] but with larger statistics: It consists of 2,460,000 non-single-diffractive pp reactions at \( \sqrt{s} = 630 \text{ GeV} \) measured by the UA1 central detector [14]. Only vertex-associated charged tracks with transverse momentum \( p_T \geq 0.15 \text{ GeV/c} \) and \( |\eta| \leq 3 \) have been used. We restrict the azimuthal angle to \( 45^\circ \leq |\phi| \leq 135^\circ \) (“good azimuth”). These cuts define a region in momentum space which we call \( \Omega \).

Since the multiplicity for the entire azimuthal range is the physically relevant quantity, we select events according to their uncorrected all-azimuth charged-particle multiplicity \( N \). The corrected multiplicity density is then estimated as twice \( n \), the charged-particle multiplicity in good azimuth: \( \langle dN_c/d\eta \rangle \simeq 2 \langle dn/d\eta \rangle \).

The measured quantities are \( K_2^Q \), the second order normalized cumulant correlation functions in several multiplicity intervals \( N \in [A, B] \); for a complete definition, refer to [14]. They are measured for pairs of like-sign (\( \ell s \)) and opposite-sign (\( os \)) charge separately as

\[
K_2^\ell s(Q|AB) = \frac{\pi_{\ell s}^2}{n(n-1)_{\ell s}} r_2^\ell s(Q|AB) - 1, \tag{1}
\]

\[
K_2^{os}(Q|AB) = \frac{\pi_+ \pi_-}{n_+ n_-} r_2^{os}(Q|AB) - 1, \tag{2}
\]

where \( \pi_{\ell s} = \pi_+ = \pi_- \), \( n_+ n_- \) and \( n(n-1)_{\ell s} \) are the mean numbers of positive or negative particles, \( os \) pairs and \( \ell s \) pairs respectively in the whole interval \( \Omega \) and in the multiplicity range \([A, B]\). The prefactors in front of the normalized density correlation functions \( r \) correct for the bias introduced by fixing multiplicity [14]. The functions \( r_2^{\ell s} \) and \( r_2^{os} \) are defined for \( \ell s \) and \( os \) pairs in the correlation integral description [18]

\[
r_2^{\ell s}(Q) = \frac{\rho_{\ell s}^+}{\rho_1 \otimes \rho_1^s(Q)} = \frac{\rho_{\ell s}^+}{\rho_1 \otimes \rho_1^s(Q)} = \frac{\rho_{\ell s}^+}{\rho_1 \otimes \rho_1^s(Q)}, \tag{3}
\]

\[
r_2^{os}(Q) = \frac{\rho_{os}^+}{\rho_1 \otimes \rho_1^o(Q)} = \frac{\rho_{os}^+}{\rho_1 \otimes \rho_1^o(Q)}, \tag{4}
\]

with \( Q = \sqrt{-(p_1 - p_2)^2} \) the spacelike four-momentum difference of the pion pairs.

Internal cumulants \([1] \) and \([2] \) are analysed in three samples as a function of pion transverse momentum \( p_T \) as follows:

(i) An all-\( p_T \) sample of all like-sign and opposite-sign pion pairs in \( \Omega \). This is shown in Fig. 1 for three representative multiplicity densities.

(ii) A low-\( p_T \) subsample containing only charged particles\( [3] \) with \( p_T \leq 0.7 \text{ GeV/c} \). Also removed from the subsample were entire events containing either a jet with \( E_T \geq 5 \text{ GeV} \) or at least one charged particle with \( p_T \geq 2.5 \text{ GeV/c} \). This is shown in Fig. 2 with the same three multiplicity selections. The prefactors entering Eqs. \([1] \).

\[2\] The pion sample contains about 15% kaons.
are calculated in this case by using only charged particles $0.15 \leq p_T \leq 0.7$ GeV/c.

(iii) A high-$p_T$ subsample where only charged particles with $p_T \geq 0.7$ GeV/c are considered. This is shown in Fig. 3. It has been demonstrated previously \cite{14} that all high-$p_T$ particles stem from jets or minijets.

From Figs. 1–3, we see that whereas the low-$p_T$ subsample behaves very similarly to the all-$p_T$ sample, there is a pronounced increase in the strength of correlation functions for the high-$p_T$ case (iii) (note the different scale on the plot!). This can be interpreted as the influence of jets, which are inherently spiky in nature. In this case, BE correlations are hence mixed up with correlations originating from jets, so that it would be difficult to measure them separately.

We note further that $\ell s$ functions in Fig. 3 reveal a crossover in the region $Q \approx 1$ GeV, making the determination of the multiplicity dependence of $K_2^\ell s$ highly $Q$-dependent. The fit to this high-$p_T$ sample using eq. (3) gives “radius parameters” $R$ which decrease with increasing multiplicity, in contrast to the samples (i) \cite{14} and (ii). Fig. 3 shows also a pronounced secondary peak after a minimum at $Q \approx 1.4$ GeV which can be attributed to the onset of local $p_T$ compensation with two back-to-back particles with at least $p_T = 0.7$ GeV/c corresponding to the cut applied in this subsample \cite{21}.

The fits performed to $\ell s$ pair data in Fig. 1–Fig. 3 are exponential,

$$K_2^\ell s(Q) = a + \lambda e^{-RQ}.$$  

The corresponding dependence of $\lambda$ on multiplicity is used in subsequent figures.

Fig. 4 compares the multiplicity dependence of the all-$p_T$ and low-$p_T$ samples in the small-$Q$ region ($Q = 0.1$ GeV). The two samples differ only slightly in their respective cumulants as well as their $\lambda$ values.

Fig. 5 shows the corresponding multiplicity dependence in the high-$p_T$ sample, once again on a larger scale in $K_2^\ell s$. The influence of jets shows up dramatically: all correlation functions are increased in height, and the $os$ functions do not show a multiplicity dependence for particle densities $dN_c/d\eta \leq 3.3$. A decrease like in (i) and (ii) is probably compensated by increasing jet activity.

The interplay of BE correlations and resonance production with the onset of jet production and the transition from soft to hard interactions are interesting questions in their own right and can be studied with samples like the high-$p_T$ one. We will, however, concentrate in the following on the low-$p_T$ subsample.

### 3. Multiplicity dependence of the low-$p_T$ sample

The results of a study with the all-$p_T$ sample (i) have been published in Ref. \cite{14}. In Fig. 6, we plot for the low-$p_T$ sample (ii) the same ratio

$$\frac{K_2^\ell s(Q \mid (dN_c/d\eta = 6.9))}{K_2^\ell s(Q \mid (dN_c/d\eta = 1.2))}$$

for like-sign and opposite-sign pairs respectively, while Fig. 7 shows the behaviour of the low-$p_T$ cumulants for fixed $Q$ but varying multiplicity. These figures show that the all-$p_T$ results found previously remain valid for the low-$p_T$ sample also, namely:

- The like-sign and opposite-sign cumulants have very similar multiplicity dependence when compared in the same $Q$ region.

- Cumulants behave distinctly differently at small and large $Q$: at small $Q$, the multiplicity dependence of both samples is weaker than $1/N_c$, while at large $Q$ ($\geq 2$ GeV) the cumulants are negative and follow roughly a $1/N_c$ law$^3$.

- A third region around $Q = 1$ GeV shows small and rapidly changing cumulants.

In the following, we discuss four possible explanations of these phenomena. It should be stressed that the LUND Monte Carlo model (PYTHIA) cannot reproduce the multiplicity dependence of correlation functions and in particular of the BE effect \cite{20}.

$^3$ For simplicity we write $N_c$ instead of $dN_c/d\eta$ here and below ($N_c = 6 \cdot dN_c/d\eta$).
3.1 Bose Einstein correlations are the result of symmetrization within individual strings only: When several strings are produced, each string symmetrizes separately and decay products of different strings would hence not contribute to BE correlations [3].

Because unnormalized cumulants of independent distributions combine additively, the independent superposition in momentum space of $\nu$ equal sources/strings, each with a $q$th order cumulant $\kappa_q(p_1, \ldots, p_q)$ and each with some multiplicity distribution (e.g. Poisson) results in an unnormalized combined cumulant $\kappa^{(\nu)}_q$ of

$$\kappa^{(\nu)}_q(p_1, \ldots, p_q) = \nu \kappa_q(p_1, \ldots, p_q), \quad (7)$$

while the combined single particle spectrum is given in terms of individual sources’ spectra by

$$\rho_1^{(\nu)}(p) = \nu \rho_1(p), \quad (8)$$

so that the $q$th order normalized cumulants for $\nu$ superimposed sources is given by

$$K^{(\nu)}_q(p_1, \ldots, p_q) = \frac{\nu K_q(p_1, \ldots, p_q)}{\rho_1(p_1) \ldots \rho_1(p_q)}. \quad (9)$$

Hence $K^{(\nu)}_q$ is inversely proportional to the number of sources. This remains true for the correlation integral $K^{(\nu)}(Q)$ also.

The above derivation is only for illustration. If $K^{(\nu)}_q \propto 1/\nu$, this would imply $K^{(1)}_q \propto 1/N$ only if $N \propto \nu$, i.e. for identical sources each of fixed multiplicity. In reality, the assumption of equal sources is probably not fulfilled. The following scenario might be more realistic: Fixing multiplicity does not necessarily mean fixing the number of sources. The sources (we will them define below) probably do possess a whole multiplicity distribution rather than a single fixed multiplicity. Our selected multiplicities range from 0.83 to 9.1, varying over about a factor 10. At small $Q$, however, the $K^{(1)}_q$ vary only by at most a factor 3. This suggests that at the highest selected multiplicity we would observe the superposition of only 3 sources, from which we are sampling their high multiplicity tails.” In the Dual Parton Model approach one source might be identified with the topology of one pomeron exchange [4]. If we select low-multiplicity events, we expect to select the case of one pomeron exchange. Multiparton collisions corresponding to multipomeron exchange are expected to contribute to higher-multiplicity events. Estimates in ref. [22] predict two- to three-pomeron exchanges at the highest multiplicities seen by UA1. The number of sources would increase correspondingly. This could explain the suppression of $\ell s$ (Bose-Einstein) functions in Fig. 7a. However, additional assumptions are needed to explain the similar behaviour of $\ell s$ and $os$ functions in Figs. 6a and 7a and their $1/N$ dependence at large $Q$ (Figs 6b, 7b). Resonance production and colour reconnection effects might be candidates (see sect. 3.3).

3.2 Quantum statistical approach: A chaoticity parameter $p$ decreasing with multiplicity would, in the quantum statistical approach [23, 24], decrease BE correlations at higher multiplicities. In this picture, however, the question arises what the physical nature of the subprocess causing increased coherence at higher multiplicities would be. Also, the similarity of the behaviour of $\ell s$ and $os$ correlation functions in Fig. 7 is not easily explained within this framework.

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4 This, however, implies no multiplicity dependence of correlation functions within one source, which still is a strong assumption.

5 In the DPM, the exchange of one pomeron corresponds already to the formation of two chains or strings. We consider this case here as “one source”.

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3.3 BE and resonance-induced correlations combined: One could hypothesise that the observed multiplicity dependence of $\ell s$ correlation functions is the result of two processes \cite{27}:

a) Bose-Einstein correlations in the classical sense which, being a global effect, are independent of multiplicity \cite{20} and

b) the production of higher mass-resonances or clusters decaying into two or more like-sign pions: $R^+ \rightarrow \pi^+ \pi^+ + X$ (as seen for example in $\eta'$ decay). If the unnormalized cumulants $\kappa_2^{\rho \os}$ and $\kappa_2^{\ell s}$ were wholly the result of resonance decays and if the number of resonances were proportional to the multiplicity $N_c$, then $\kappa_2^{\rho \os} \propto N_c$. Assuming $\rho_1(p | N_c) \propto N_c \rho_1(p)$ gives $\rho_1 \otimes \rho_1 \propto N_c^2$, and hence after normalization, the resonance-inspired guess yields $1/N_c$ behaviour,

$$K_2^{\ell s}(Q | N_c) \approx a(Q) + \frac{b(Q)}{N_c},$$

(11)

in agreement with the behaviour of $\ell s$ functions in Fig. 7a (straight line). The $1/N_c$ dependence in Fig. 7b at large $Q$ suggests the existence of a resonance/cluster component for both $\ell s$ and os pair production. The resonance contribution to the correlation functions would be concentrated mainly at the region around $Q \approx 1$ GeV as in the case of $\rho^0$ production, which is visible as a peak in the os functions in Figs. 1–3. This region is however difficult to investigate because there the $K_2^{\rho \os}$ are decreasing rapidly with increasing $Q$ while the $K_2^{\ell s}$ are already small. A $1/N_c$-dependence due to resonances or clusters in this dominant phase space region around 1 GeV could presumably cause the large-$Q$ region to follow suit via missing pairs, thus explaining the observed dependence there.

Once again, however, the similarity of $\ell s$ and os functions in the small-$Q$ region in Fig. 7a can hardly be explained by assuming only the two components a) and b).

One possible explanation could be the existence of global correlations for os pairs too. It would give a behaviour for $K_2^{\rho \os}$ similar to that of Eq. (11) and would explain its constant $a(Q)$ part by noting that the number of $+$– pairs that can be formed from $N_+$ positive pions and $N_-$ negative pions would be $N_+ N_-$. If each of these pairs was correlated (statistically speaking), i.e. if all $(N_+ + N_-)$ pairs were mutually correlated, the unnormalised $\kappa_2^{\rho \os}$ would be proportional to $N_c^2$ and hence after normalisation $K_2^{\rho \os}$ constant in $N_c$. Such a constant term signalizes maximum possible correlations in some events.

More generally and following the arguments leading to Eq. (10), we could also say: \textit{“If resonance production were to rise more quickly than $\propto N_c$, then $K_2^{\rho \os}$ would decrease more slowly than $1/N_c$.”}

3.4 The Core-Halo picture \cite{13}. This picture is currently the only one which connects the multiplicity dependence of $\ell s$ with that of os correlation functions. The core-halo picture is based on the fact that Bose-Einstein correlations of decay products from long-lived resonances are not observable by experiments because they occur below experimental resolution and hence by definition belong to the “halo” of resonances that decay at large distances. Examples are $1/\Gamma_{\eta'} = 23.5$ fm/c, $1/\Gamma_{\eta} = 986.5$ fm/c, $1/\Gamma_{\eta} = 164400$ fm/c, which are all unresolvable within the UA1 experiment which can resolve decay products for distances $\leq 6$ fm only, corresponding to $Q \geq 30$ MeV. Because of the halo, the $\lambda$ values of BE fits are in general reduced \cite{13},

$$\lambda(Q_{\text{min}}) = \frac{f_c^2}{P_{\text{core}}^2(p)} \frac{P_{\text{core}}^2(p)}{P_{\text{halo}}^2(p)},$$

(12)

\footnote{The observed multiplicity dependence in ref. \cite{20} is compensated by our prefactors in Eqs. (1) and (4).}
where the second factor describes the momentum dependence and $Q_{\text{min}}$ is the smallest $Q$ value accessible by the experiment.\footnote{In\cite{15} is assumed that $Q_{\text{min}} \simeq 10$ MeV and that the fit parameters (of Gaussian fits) are insensitive to the exact value of $Q_{\text{min}}$ in a certain restricted region.}

The relation between $\lambda$ and the fraction of pions emitted by the halo follows from Eq.\ (12) and from

$$f_c = \frac{\langle N_{\text{core}} \rangle}{\langle N \rangle} = 1 - \frac{\langle N_{\text{halo}} \rangle}{\langle N \rangle}, \quad (13)$$

where $\langle N_{\text{core}} \rangle$ is the mean multiplicity of directly produced “core” pions, $\langle N_{\text{halo}} \rangle$ is the mean multiplicity of halo pions and $\langle N \rangle = \langle N_{\text{core}} \rangle + \langle N_{\text{halo}} \rangle$. This means that

$$\lambda \propto \frac{\langle N_{\text{core}} \rangle^2}{\langle N \rangle^2} = \left( 1 - \frac{\langle N_{\text{halo}} \rangle}{\langle N \rangle} \right)^2. \quad (14)$$

From Eq.\ (14) it is evident that $\lambda$ remains independent of multiplicity only if $\langle N_{\text{core}} \rangle \propto \langle N \rangle$ and consequently $\langle N_{\text{halo}} \rangle \propto \langle N \rangle$. If however the number of halo-resonances increases faster than $\propto \langle N \rangle$, then also the number of their decay particles $\langle N_{\text{halo}} \rangle$. As a consequence the BE parameter $\lambda$ will decrease with multiplicity.

A second consequence emerges immediately (see last sentence in sect. 3.3): The fraction of $K_2^{\omega \pi}$ stemming directly from the decay of halo resonances will decrease less rapidly than $\propto 1/N$. A previous study\cite{28} revealed the $Q$ region where two-body $\pi^+\pi^-$ decay products of the halo resonances $\eta, \eta'$ and $\omega$ contribute, namely in $0.03 < Q \leq 0.55$ GeV. This is exactly the region where $\rho$ correlation functions indeed show a multiplicity dependence weaker than $\propto N^{-1}$ as shown in Fig. 6b (where the $1/N$ case is indicated by the dashed line).

So far, this discussion is purely qualitative. How the decrease of $\lambda$ with $N_c$ would compare to an effective slower-than-$1/N_c$ decrease of $K_2^{\omega \pi}$ is, of course, a quantitative question not answered by the above argument, both because $\lambda$ refers to the minimum $Q$-value and its stability against shifts is not yet tested, and because the experimental fraction $\langle N_{\text{halo}} \rangle / \langle N \rangle$ is known only sparsely, if at all. A more quantitative estimate has to be done in future, including the fact that the previously measured $K_2^{\omega}$ for the whole sample\cite{28} are already near unity at $Q \simeq 0.03$ GeV, even after subtracting the background contribution due to the non-Poissonian overall multiplicity distribution. Correcting for an additional halo contribution could finally cause $K_2^{\omega \pi}$ for $Q \to 0$ to be greater than 1.

The four different explanations considered above each have some merit. It is clearly desirable to shorten this list of candidates. We believe that higher order cumulants are suitable for this purpose: If symmetrization of individual strings is the right explanation, we can expect from Eq.\ (8) and Ref.\cite{29} that the higher-order cumulants would decrease much faster with multiplicity than $K_2$ (e.g. $K_4 \propto 1/\nu^2$). Since such a fast decrease is not predicted e.g. for the core-halo picture\cite{21}, the measurement of the multiplicity dependence of $K_2$ could probably decide between the two cases (3.1) and (3.4).

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Figure 1. Normalised $\ell s$ and $os$ internal cumulants for the all-$p_T$ sample (i) with multiplicity densities $dN_c/d\eta = 1.22$ (open circles), $dN_c/d\eta = 2.72$ (full circles) and $dN_c/d\eta = 6.85$ (diamonds). The fits in a) are exponential as in eq. (3).
Figure 2. Same as in Fig. 1, but for the low-$p_T$ sample (ii).

Figure 3. Same as Fig. 1, but for the high-$p_T$ sample (iii).
Figure 4. Comparison of the multiplicity dependence of the low-\(p_T\) sample (ii) with that of the all-\(p_T\) sample (i). Full lines are fits of the all-\(p_T\) sample to the \(1/N_c\) behaviour as in eq. (11); dashed lines are the corresponding fits for the low-\(p_T\) sample.
Figure 5. Multiplicity dependence of the high-$p_T$ sample (iii). The lines (full lines for $\ell s$ pairs, dashed line for $os$ pairs and for $1/(dN_c/d\eta) > 0.2$), pro-forma fits using (Eq. (11)), are clearly an inadequate representation of the data and are hence intended only to provide comparison to $1/N_c$ behaviour. Note that these high-$p_T$ cumulants exceed one by a considerable amount; high-$p_T$ data is hence clearly dominated by processes other than BE correlations.
Figure 6. The ratio of two $K_I^2(Q)$ corresponding to two selections of $dN_c/d\eta$ as indicated, for the low-$p_T$ sample (ii). The dashed line indicates the value of the ratio for the case that $K_I^2(Q) \propto 1/N_c$.

Figure 7. a) Multiplicity dependence of $K_{Is}^2$ (filled circles) and $K_{Os}^2$ (open circles), both at $Q = 0.1$ GeV, b) as in a) but for the large $Q = 7$ GeV-region. Note from Fig. 1 that the cumulants are negative at large $Q$. For better comparison of the respective dependencies on $dN_c/d\eta$, the absolute values have been scaled by constant factors. Dashed lines are best fits using Eq. (11).