Josephson current in unconventional superconductors through an Anderson impurity

Y. Avishai\(^1\)\(^*,\) and A. Golub\(^2\)

\(^1\)Department of Physics, Ben-Gurion University of the Negev, Beer-Sheva, Israel
\(^2\)Institute for Solid State Physics, University of Tokyo, 7-22-1 Roppongi, Minato-ku, Tokyo 106, Japan

(Received November 19, 2018)

Josephson current for a system consisting of an Anderson impurity weakly coupled to two unconventional superconductors is studied and shown to be driven by a surface zero energy (mid-gap) bound-state. The repulsive Coulomb interaction in the dot can turn a π junction into a 0-junction. This effect is more pronounced in p-wave superconductors while in high-temperature superconductors with \(d_{x^2-y^2}\) symmetry it can exit for rather large artificial centers at which tunneling occurs within a finite region.

PACS numbers:74.50.+r,73.40.Gk,74.60.Jg

Recently, the Josephson effect in unconventional superconductors has attracted a considerable attention \(^1\)\(^,\)\(^2\). Measurements of direct Josephson current yield valuable information on the symmetry of the order parameter which is essential for understanding the mechanisms of superconductivity in these complex materials. Phase interference experiments \(^1\)\(^2\) definitely suggest the presence of \(d\)-wave symmetry of the order parameter in high-\(T_c\) superconductors, while the recent discovery of superconductivity in \(Sr_2RuO_4\) \(^3\) implies the existence of a peculiar system for which the pair potential has a triplet \((p\)-wave\) symmetry \(^1\)\(^2\).

\(p\) and \(d\)-wave symmetries of the order parameter have in common a property which reflects the variation of the pair potential across the Fermi surface. This results in a strong sensitivity to inhomogeneities which, in turn, influences the Josephson effect. In particular, an anomalous temperature dependence of a single Josephson junction at low temperatures \(^3\)\(^,\)\(^4\) and an induced crossover from a usual \(0\)-junctions to a \(\pi\)-junction on approaching the critical temperature were observed.

Tunneling in a Josephson junction consisting of conventional \((s\)-wave\) superconductors and a dynamical impurity \((\text{Anderson and Kondo})\) was considered sometime ago \(^1\)\(^2\)\(^,\)\(^5\)\(^,\)\(^6\). In a recent work \(^1\)\(^7\) the tunneling current was calculated at zero temperature and was shown to be strongly dependent on the Coulomb interaction which, in some cases, may cause a sign change of the current. The experimental observations described above necessarily suggest that one must now go further and study the same device albeit with unconventional superconductors at finite temperatures. This missing part is addressed below. As will be demonstrated, the underlying physics is remarkably different.

The main focus here is the influence of Coulomb interaction on the low temperature behavior of the current in a 2D Josephson junction consisting of left (L) and right (R) superconductors with either \(p\) or \(d_{x^2-y^2}\) symmetry of the order parameter) weakly coupled to a quantum dot \((\text{via identical hoping matrix elements} t_L = t_R = t)\). The dot is represented by a finite \(U\) Anderson impurity whose energy \(\epsilon_0 < 0\) lies below the Fermi energy. Usually, the inequalities \(U > -\epsilon_0 > 0\) are maintained so that the ground state of the disconnected \((t = 0)\) dot is singly occupied. We use the non-perturbative scheme suggested in Ref. \(^8\) (extended for finite temperature) and elucidate the low temperature behavior of the Josephson current and its dependence on \(U\), \(t\) and the phase of the order parameter \(\Delta\).

Since \(\Delta\) is not isotropic, it is useful at this point to specify the underlying geometry. Each superconductor has the shape of half a plane defined as \(-\infty < y < \infty\) and \(x < 0\) \((x > 0)\) for the left (right) superconductor. The dot is located at the origin \(r = 0\) and tunneling is described by zero-range hoping between the impurity and the superconductors along the \(x\) axis.

For \(d\)-wave superconductors we choose the nodal line of the pair potential on the Fermi surface to coincide with the tunneling direction, such that \(\Delta = \psi_0 \rho_F \sin 2\alpha\) where \(\rho_F\) is the Fermi momentum. For spin-triplet superconducting states the order parameter is an odd vector function of momentum and a \(2 \times 2\) matrix in spin space. We chose to represent it by the time reversal symmetry breaking state \(^8\) which is off-diagonal in spin indices, that is, \(\Delta = \Delta_0 \exp i\alpha\). This state is a probable candidate for describing the recently discovered superconductor \(Sr_2RuO_4\) \(^8\).

In both cases, the pair potential has different values for electron like excitations which move at an angle \(\alpha\) and hole like excitations propagating along the direction \(\pi - \alpha\). This fact significantly affects the scattering process and causes the formation of a zero energy (mid-gap) bound state centered at the boundary.

To obtain the Josephson current we compute the partition function \(Z = \int D[\psi \overline{\psi} \phi] \exp(-S)\) and the corresponding free energy of the system. The functional integration is performed over Grassmann fields in the superconductors and the quantum dot (see below for a precise definition). The Euclidean action can be written as a sum \(S = S_L + S_R + S_{\text{int}} + S_U\), where, with obvious notations,
\[ S_L = T \sum_{\omega} \int dx dy \bar{\psi}_{L\omega}(xy) (i\omega + H_L^{BDG}(\hat{\rho}_x, \hat{\rho}_y)) \psi_{L\omega}(xy), \]

(1)

\[ S_{int} = -T \sum_{\omega_i=L,R} (t_i \bar{\psi}_{\omega i}(0) \tau_3 c_{\omega i} + t'_i \bar{c}_{\omega i} \tau_3 \psi_{\omega i}(0)), \]

(2)

\[ S_U = \int d\tau [\bar{c} \partial_\tau \bar{c} + \bar{c} \tau_3 c - U(\bar{c}c)^2/2], \]

(3)

where \( \bar{\psi} \) and \( \psi \) are Grassmann variables. The occurrence of \( \omega \) in the denominator. Performing the integration over the boundary fields \( \bar{\psi}_{Lk_y}(0), \psi_{Lk_y}(0) \) in the partition function is now straightforward. The remaining integrations over the \( c, \bar{c} \) fields is done by decoupling the Hubbard interaction using a Hubbard-Stratonovich transformation. Since the impurity operators do not depend on \( k_y \), the auxiliary fields \( \gamma \) depend only on \( \omega \). We then obtain

\[ Z = C \int d\omega \exp(-\frac{\gamma^2}{2UT} - \frac{\bar{\omega}}{T} - Q). \]

(7)

Here \( Q = -T \sum_{\omega} \ln[|detR(\omega)|] \) and \( C \) is a constant. The matrix \( R \) encodes the coupling between the superconducting (half) planes connected and the impurity, and reads

\[ R(\omega) = \bar{\epsilon} \tau_3 + \gamma_\omega - i\omega - \frac{\pi N(0)}{\omega} <|t|^2(r_L(\omega) + r_R(\omega))>, \]

(8)

where \( N(0) \) is the density of states at the Fermi level, and \( <O> = \pi^{-1} \int \frac{\pi}{\gamma} d\omega \omega |O(\omega)|. \) The averaging procedure thus defined is related to a possible dispersion of the transmission matrix element \( t_{k_y} \). For a point junction with constant \( t \), only the component of the Josephson current perpendicular to the interface is relevant. In this case, for even (d-wave symmetry) superconductors the mid-gap zero energy bound state does not contribute to the Josephson energy. Contrary, for p-wave potential this state defines the main contribution at low temperatures \( [13] \) (which is the temperature domain of our interest here). Intuitively, occurrence of dispersive tunneling matrix elements \( t_{k_y} \) correspond to deviation of the impurity from a point-like defect. Such a case can be realized e.g. by artificially - induced defects \([16]\). The spectroscopy of \( Bi_2Sr_2CaCu_2O_{8+\delta} \) surfaces indicates that such defects appear to be more extended in STM imaging. In this case one can expect non-zero contribution from the mid-gap level in d-waves superconductors as well.

With this point in mind we now proceed and consider the p-wave case. The functional integral in (7) is approximated by the saddle point method. The appropriate optimum solutions \( \gamma_\omega \) should then minimize the free energy

\[ F = -T \sum_{\omega>0} \ln[A^2(\omega) + 4\gamma_\omega^2 \omega^2(1 + a(\omega))^2] + \frac{\gamma^2}{2UT} + \bar{\omega}. \]

(9)

Here we use the notations:

\[ A(\omega) = \bar{\epsilon}^2 + \omega^2(1 + a(\omega))^2 - \frac{\gamma^2}{4} - b^2(\omega)|\Delta|^2 \cos^2\left(\frac{\phi}{2}\right), \]

(10)
\[ a(\omega) = \Gamma \sqrt{\omega^2 + |\Delta|^2}, \]  

where \( \Gamma = 2\pi t^2 N(0) \) is the bare impurity level width, \( b(\omega) = \Gamma / \omega \), and \( \phi \) is the phase difference between the two superconductors. The self-consistent equation for \( \gamma \omega \) is

\[ \frac{1}{2U} - 2T \sum_{\omega > 0} \frac{2\omega^2(1 + a(\omega))^2 - A(\omega)}{A^2(\omega) + 4\gamma^2_\omega \omega^2(1 + a(\omega))^2} = 0. \]

Once the solutions are defined we can calculate the current \( J = \frac{(2e/h)}{\partial F/\partial \phi} \) and the impurity occupancy \( n = \partial F/\partial \epsilon_0 \),

\[ J = -(2e/h) \sin(\phi)T \sum_{\omega > 0} \frac{|\Delta|^2 b^2(\omega) A(\omega)}{A^2(\omega) + 4\gamma^2_\omega \omega^2(1 + a(\omega))^2}, \]

\[ n = 1 - 4T \sum_{\omega > 0} \frac{\epsilon A(\omega)}{A^2(\omega) + 4\gamma^2_\omega \omega^2(1 + a(\omega))^2}. \]

We now analyze the main results of the present study. All the parameters having the dimension of energy \( (\epsilon, U, \Gamma, T) \) are expressed in units of \(|\Delta|\) and the current is given in units of \(|\Delta|^2 e/h\).

In Fig.1a,b, the current is displayed versus temperature in the low temperature region and for coupling strengths \( \Gamma \) ranging between 0.001 and 0.0882. At small Hubbard interaction \( (U=2.1, \text{Fig.1a}) \) the junction is in a '\( \pi \)'-state for which the current (within the present geometry of the tunneling direction) is negative. The current is strongly dependent on temperature, which is markedly distinct from the classical Ambegaokar-Baratoff formula. It is typical for superconducting systems for which the zero energy mid-gap bound state plays the major role. At higher values of \( U \) (Fig. 1b) the pattern is inverted: The current for most coupling strengths is now positive and the '\( \pi \)' junction is transformed into a '0' junction with slightly weaker temperature dependence. A similar situation takes place also in s-wave superconductors [12-14], where it is attributed to the single occupancy of the impurity which then becomes a degenerate magnetic moment. We have thus presented another example for this scenario, though the temperature dependence is quite different.

FIG. 1. Dependence of the Josephson current on temperature for \( U = 2.1 \) (a) and \( U = 3.0 \) (b) for different transparencies \( \Gamma \). Note the sign reversal for strong \( U \). Here \( \epsilon_0 = -2.0 \) and \( \phi = \pi/12 \) are fixed. The units of all quantities are explained in the text.

FIG. 2. Dependence of the Josephson current on the phase difference for (a) small transparencies and intermediate temperature \((T = 0.01)\) and (b) large transparencies and very low temperature \((T = 0.001)\). Other parameters are \( \epsilon_0 = -2.0 \) and \( U = 2.6 \).

Now let us discuss the behavior of the current \( J(\phi) \) as function of the phase difference as displayed in Figs.2a,b for fixed \( U \) and \( T \) and for numerous transparencies \( \Gamma \). Once again it is clear that as the transparency is increased, the sign of the current is reversed. Moreover, at
lower temperatures (Fig.2b) when $\Gamma/\Delta \geq 0.4$ it becomes irregular with jumps at certain values of $\phi$ at which the impurity is nearly incompressible. This effect can manifest itself when the flowing current $I > I_c$. In the frame of the RSJ model the averaged voltage $\bar{V}$ on the junction is related to the current $I$ and the resistance $R$ as,

$$R = \bar{V} \int_0^{2\pi} \frac{d\phi}{2\pi(I - J(\phi))}.$$  \hspace{1cm} (15)

The irregular pattern of $J(\phi)$ results in a deviation of the $\bar{V}(I)$ characteristic from its classical expression $I = \sqrt{J_c^2 + (\bar{V}/R)^2}$. Such deviation is seen on Fig.3 where the difference is mainly exhibited at low voltage.

![Graph](image)

**FIG. 3.** $V(I)$ characteristics of the $p$ wave junction (upper curve) compared with the classical square root expression (lower curve). The relevant parameters are $\epsilon_0 = -2.0$, $U = 3.0$ and $\Gamma = 0.66$.

To summarize, we have solved the problem of transport between two unconventional superconductors through an impurity and traced the dependence of the current on temperature, Coulomb interaction barrier transparency and phase of condensates. The essentially non-perturbative, self-consistent approach we have used yields a finite value for the current without adding relaxation terms that should be included if perturbation approach is adopted. It is mandatory at this final stage to point out the peculiarities related to the physics of non $s$-wave superconductors.

1. The contribution to the current in our case originates principally from the surface bound state which is related to the asymmetry of the pair potential, and has no analog for $s$-wave superconductors. The contribution of this state in low transparency junctions results in a large current, that is, $|J_p/J_s| \sim \sqrt{\Delta/\Gamma}$. It is also marked by a stronger temperature dependence especially at low temperatures.

2. For unconventional superconductors, the Josephson tunneling through an interacting quantum dot can serve as an indicator to distinguish odd parity superconductors from even parity ones. Superconductors with both $p$ and $d$-wave symmetry of the order parameter have surface bound state which contributes to the current (mainly at low temperatures). Yet, as we indicated above, for even-symmetry ($d$-wave) superconductors, the current vanishes in the limit of point-like impurity. In sharp distinction, under the same conditions, the current is maximal for odd-symmetry ($p$-wave) superconductors.

**Acknowledgment:** This research is supported by grants from the Israeli Science Foundation (Center of Excellence and Non-Linear Tunneling), The German-Israeli DIP foundation and the US-Israel BSF foundation.

* email: yshai@bgumail.bgu.ac.il
** email: agolub@bgumail.bgu.ac.il
[1] C. R. Hu, Phys.Rev.Lett. 72,1526 1994
[2] Y. Tanaka and S. Kashiwaya , Phys. Rev. Lett. 72, 1526 (1994)
[3] Yu.S. Barash, H Burkhardt, and D. Rainer, Phys. Rev. Lett. 77, 4070 (1996)
[4] Y. Tanaka and S. Kashiwaya, Phys. Rev. B56, 892 (1997)
[5] M.P. Samanta and S. Datta, Phys. Rev. B55, 8689 (1997)
[6] R.A. Riedel and P.F. Bagwell, Phys. Rev. B57, 6084 (1998)
[7] D.J. Van Harlingen, Rev.Mod.Phys. 67, 515(1995)
[8] Y. Maeno et.al, Nature, 372, 532(1994)
[9] T. M. Rice and M. Sigrist, J. Phys.:Condensed matter, 7, L643 (1995)
[10] H. Shiba and T. Soda, Prog.Theor.Phys. 41, 25 (1969)
[11] L.I. Glazman and K.A. Matveev, Pis’ma ZhETF 49, 570 (1989) [JETP Lett. 49, 659 (1989)]
[12] B. J. Spivak and S.A. Kivelson, Phys. Rev. B43 , 3740 (1991)
[13] A.A. Golub, Phys. Rev. B54, 3640 (1996)
[14] A.V. Rozhkov and D.P. Arovas, Phys. Rev. Lett. 82, 2788 (1999)
[15] Y. Tanaka, T. Hirari, K. Kusakabe, and S. Kashiwaya, cond-mat 14 June (1999)
[16] A. Yazdani et al, Phys. Rev. Lett. 83, 176 (1999)