Effect of thermal conductivity of thin walls limiting the inclined liquid layer on non-stationary conjugate natural convective heat exchange and temperature fields in thin walls

K A Mitin¹, A V Mitina¹ and V S Berdnikov¹,²,*

¹Kutateladze Institute of Thermophysics SB RAS, Novosibirsk, Russia
²Novosibirsk State Technical University, Novosibirsk, Russia
*E-mail: berdnikov@itp.nsc.ru

Abstract. Non-stationary conjugate convective heat transfer in a rectangular cavity at an angle of 45 degrees is studied numerically in the conjugate formulation. A liquid layer with a Prandtl number of 25.66 is located in a cavity with thin metal or low-heat walls. The outer surface of the lower wall is suddenly heated, and the outer surface of the upper wall is maintained at the original temperature. The system of equations of non-stationary thermogravitation convection is solved in the Boussinesq approximation in a dimensionless form. The time evolution of convective flows and temperature fields in the liquid and in the walls is studied. It is shown that the thermal conductivity of walls significantly affects the spatial shape of convective flows and the regularities of conjugated convective heat exchange.

1. Introduction
The thermal state of thin-walled structures, such as aircraft during take-off and landing, at the initial stages of reaching the cruising speed significantly depends on the processes of non-stationary conjugated convective heat exchange in the fuel tanks and in the air layers of the fuselage. When flying at supersonic speeds, the processes of heating the aircraft skin are added. With the development of aviation technology, requirements for the quality of calculations of non-stationary temperature fields and thermal stresses in thin-walled elements of aircraft structures are noticeably increasing [1]. Similar problems are typical for many technical devices in the heating or cooling regimes on and off. In unevenly heated volumes of liquid located in the field of gravity natural convective flows develop. The spatial shape of convective flows has a significant influence on the laws of conjugate heat transfer. In turn, the shape of convective flows largely depends on the configuration of the cavity and the orientation of the heated and cooled walls relative to the gravity vector [1-5]. For adequate estimates of thermal stress fields in structures, it is necessary to know the local features of hydrodynamics and the features of local conjugate heat exchange generated by them, and as a result, the regularities of temperature fields dependence on time in thin walls. Reliable knowledge of the laws of non-stationary conjugate convective heat transfer is necessary when evaluating and accurately calculating thermal stresses and analyzing the overall stress-strain state of structures. This work continues the research conducted at the S.S. Kutateladze Institute of Thermophysics SB RAS and series of works aimed at studying the effect of conjugated natural convective heat transfer on the temperature distribution in thin walls [2-5].
2. Model

The study area is a two-dimensional rectangular cavity filled with liquid and inclined by 45 degrees. The ratio of layer length to thickness is 5:1. The liquid is bounded on all sides by thin walls with a thickness equal to 0.0133 of the layer thickness. Two types of walls are considered, with a thermal conductivity equal to that of the liquid and with a thermal conductivity 1041.3 times higher than that of the liquid. The outer surface of the upper wall is maintained at a constant temperature equal to the original temperature of the liquid filling the area. The outer surface of the lower wall suddenly warms up and is maintained at a constant set temperature. The outer surface of the side walls is adiabatic.

Numerical simulations are performed in a dimensionless form in a two-dimensional conjugate formulation in Cartesian coordinates. The thickness of the layer is $H$. The temperature scale $\Delta T = T_1 - T_2$, where $T_1$ and $T_2$ are the temperatures on the outer surface of the lower and upper walls, respectively, is chosen as the geometric size scales. The velocity scale is $\nu/H$, where $\nu$ is the kinematic viscosity of the liquid, and the time scale is $H^2/\nu$. A system of dimensionless equations of unsteady thermogravitation convection is solved in the Boussinesq approximation, written in terms of temperature, vortex, and stream function:

$$\left\{ \begin{array}{l}
\frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial y} = 1 \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \\
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega \\
\frac{\partial \omega}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \omega}{\partial y} = \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) + Gr \frac{\partial T}{\partial x} 
\end{array} \right.$$

where $Gr = g \beta H^3 \Delta T \nu^2 $ is the Grashof number, where $g$ is the acceleration of gravity, $\beta$ is the volume expansion coefficient of the liquid, $Pr = \nu/\alpha_l$ is the Prandtl number, $\alpha_l$ is the thermal diffusivity of the liquid, $T$ is the dimensionless temperature, $\omega$ is the dimensionless vortex, and $\psi$ is the dimensionless stream function.

Conductive heat transfer in solid walls is described by the heat equation:

$$\frac{\partial T}{\partial t} + \alpha_s \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = 0$$

where $\alpha_s$ is the thermal diffusivity of the solid wall material.

Numerical simulation is performed for the following parameters: $\rho_f = 819 \ [kg/m^3]$ is the fluid density; $\beta_f = 8.3\cdot10^{-4} \ [1/K]$ is the coefficient of volume expansion of the liquid; $\eta_f = 1.49\cdot10^3 \ [kg/(m \cdot s)]$ is the dynamic viscosity of the fluid; $\lambda_f = 0.1162 \ [W/(m \cdot K)]$ is the fluid thermal conductivity; $C_p = 2001.29 \ [j/(kg \cdot K)]$ is the heat capacity of the fluid; $\rho_s = 2780 \ [kg/m^3]$ is the density of the material of the walls; $\lambda_s = 121 \ [W/(m \cdot K)]$ is the solid thermal conductivity; $C_p = 921 \ [j/(kg \cdot K)]$ is the heat capacity of the material of the walls. In the simulation with walls equal to the liquid thermal conductivity as, the values $\rho_s$ and $\lambda_s$ are substituted by the values $\rho_f$ and $\lambda_f$, respectively.

The calculations are performed using the finite element method on an irregular triangular grid with thickening to solid walls. Linear basis functions on triangles are used. When calculating the slope layer, a unified grid with 23569 nodes is used. The coordinates of the nodes are rotated by the required angle using an affine transformation before the calculations. The dimensionless time step is $1/2500$. 
3. Results and discussion

The results of calculations for $Pr = 25.66$ and $Gr = 10^5$ for two wall materials with the ratio of the thermal conductivity of the wall materials to the thermal conductivity of the liquid $\lambda_s/\lambda_l = 1$ and $\lambda_s/\lambda_l = 1041.3$ are presented when the layer angle is 45 degrees.

Figure 1. Temperature dependence on time at point $(x, y) = (4, 0.0133)$ with:

a – $\lambda_s/\lambda_l = 1041.3$, b – $\lambda_s/\lambda_l = 1$.

Figure 1 shows the dependence of temperature on time at the point located on the inner surface of the lower wall. These dependences show when non-stationary boundary layers lose stability and when the system enters a steady turbulent flow regime. There are clearly visible differences in the scenarios of loss of stability by boundary layers and in the amplitudes of temperature pulsations on the surface of the lower wall, depending on the thermal conductivity of the wall material.

Figure 2. Estimation of the power spectrum of the temperature ripple at $(x, y) = (4, 0.0133)$ with:

a – $\lambda_s/\lambda_l = 1041.3$, b – $\lambda_s/\lambda_l = 1$. 
Figure 2 shows estimates of power spectra of temperature ripples based on 2048 last time steps of realization. To calculate the dimensional frequency of the spectrum, the dependence $f = k/T$ is used; here $k$ is the harmonic number (along the abscissa axis), and $T$ is the realization length (for 2048 steps over a dimensionless time of $4 \cdot 10^{-4}$). To get the size value, we use the formula $f = k \cdot \nu \cdot T^{-1} \cdot H^{-2}$. It is noticeable that  at $\lambda_s/\lambda_f = 1$, peaks at the 34th and 67th harmonics are distinguished in the spectrum, which at the height of the liquid layer equal to 100 mm will correspond to 0.0075 Hz and 0.0149 Hz, respectively. When $\lambda_s/\lambda_f = 1041.3$ peaks on the 34th and 67th harmonics are added to the peaks on the 10th, 19th, and 28th harmonics (0.0022 Hz, 0.0042 Hz, and 0.0062 Hz, respectively). In other words, powerful low-frequency vibrations are added.

Figures 3-4 show temperature fields and stream function isolines averaged over the time interval corresponding to the last 2048 time steps in the realizations for two values of thermal conductivity of the tank wall materials. To save space, layers that are tilted by 45$^\circ$ degrees are positioned horizontally. It can be seen that in both cases flow patterns (fields of stream function isolines) and isotherm fields close to symmetric ones are established.

In this case, the asymmetry is more pronounced at $\lambda_s/\lambda_f = 1041.3$ due to the fact that under the influence of high thermal conductivity of the tank wall material temperature distribution is close to linear.
and is established on the side walls (Figure 5). That is, in the left part, the liquid flow descends along a wall inclined at 45° with a linear temperature distribution, and in the right part, the liquid flow rises along a similar wall. A similar effect is described in [4] for laminar modes.

Figures 6-9 show the time evolution of the temperature fields and the stream function at different values of the thermal conductivity of the tank walls. It is clearly seen that depending on the thermal conductivity of the wall material, the scenarios of loss of stability by the boundary layer, the process of development of turbulent flow, and the shape of convective flows in general change significantly.

**Figure 6.** Evolution of the temperature field in time with $\frac{\lambda_s}{\lambda_f} = 1041.3$ at time: a – $t = 9000$, b – 9200, c – 9400, d – 9600.

**Figure 7.** Evolution of the temperature field in time with $\frac{\lambda_s}{\lambda_f} = 1$ at time: a – $t = 14000$, b – 14200, c – 14400, d – 14600.

Figures 7 and 9 show that at $\frac{\lambda_s}{\lambda_f} = 1$ on the lower wall in the area of $x = 3$, the boundary layer loses stability and secondary vortexes are generated and carried to the right wall.
Figure 8. Evolution of the stream function in time with $\lambda_s/\lambda_f = 1041.3$ at time: a – $t = 9000$, b – 9200, c – 9400, d – 9600.

On the upper wall, a similar process develops in the area of $x = 2$ almost independently, and the formed secondary vortexes are carried to the left wall. Figure 4b clearly shows that when averaging the stream function, two almost identical vortexes are visible in the center of the region.

Figure 9. Evolution of the stream function in time with $\lambda_s/\lambda_f = 1$ at time: a – $t = 14000$, b – 14200, c – 14400, d – 14600.

Figure 10 clearly shows the effect of boundary layer stability loss on the temperature field inside the wall. Due to the formation of secondary vortexes carried up the layer, a heat wave begins to run along the surface and inside the solid wall. It is also noticeable that the lower wall heats up inhomogeneously and asymmetrically, and a similar effect is described in [4] for laminar regimes.

Figure 10. Temperature profiles with $\lambda_s/\lambda_f = 1$ at $y = 0.01$ at time: 1 – $t = 14000$, 2 – 14300, 3 – 14600, 4 – 14900.

Figures 6 and 8 show the development of a more complex structure for $\lambda_s/\lambda_f = 1041.3$. At the same time, it is already difficult to distinguish secondary vortexes in the fields of stream function isolines. However, the evolution of the temperature field (Figure 6) and the temperature distribution inside the lower wall (Figure 11) clearly show the process of formation of pop-up thermals on the lower wall, which are formed at the left wall and are carried to the right wall. A similar process occurs in reverse order on the upper wall.

This is due to the oscillatory process in the lower left and upper right areas of the tank, generated by the flow of liquid on the wall tilted at 450 with a temperature distribution close to linear. As a result, the boundary layers on the lower and upper walls immediately lose stability, and low-frequency harmonics are added to the spectrum of their oscillations (Figure 2a).
Figure 11. Temperature profiles with $\lambda_s/\lambda_f = 1041.3$ at $y = 0.01$ at time: 1 – $t = 9000$; 2 – 9300; 3 – 9600; 4 – 9900.

In this case, the figure shows that when $\lambda_s/\lambda_f = 1041.3$ the lower wall heats up inhomogeneously and asymmetrically, a heat wave also runs along it, but due to the intense conductive heat exchange in the wall, the amplitude of the heat wave and the inhomogeneity of the temperature field inside the solid wall is significantly smoothed out.

Conclusions
Numerical studies of non-stationary convective heat transfer in the model of a thin-walled fuel tank at an angle of 45 degrees have been carried out in the conjugate formulation. The evolution of convective flows and temperature fields after sudden heat supply under the tank base at $\lambda_s/\lambda_f = 1041.3$ is studied and $\lambda_s/\lambda_f = 1$. The temperature fields in the liquid and in the tank walls have been calculated. It is shown that conjugated heat transfer has a significant influence on the loss of stability of boundary layers on inclined walls, scenarios for the development of turbulent flows, and temperature ripple spectra on the inner surface of the tank.

At $\lambda_s/\lambda_f = 1$, the processes of development and loss of stability of the boundary layers on the lower and upper walls are clearly visible. At $\lambda_s/\lambda_f = 1041.3$, these processes are superimposed by low-frequency oscillations originating in the lower left and upper right regions of the tank. In both cases, temperature pulsations are observed on the surface of solid walls that penetrate the walls, but at $\lambda_s/\lambda_f = 1041.3$, the amplitude of the pulsations is significantly lower than at $\lambda_s/\lambda_f = 1$, due to the high thermal conductivity of the wall and intense conductive heat exchange in it.

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