Dirac fermions in the Rindler spacetime with background electromagnetic field

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Abstract

In this article, we have solved the Dirac equation explicitly and performed the field quantization in the Rindler spacetime in the presence of background electromagnetic fields of constant strengths, and in and out modes are computed. We next consider the full Rindler right and left wedges and construct the local and global modes and their Bogoliubov transformations. Using the squeezed state expansion obtained from the Bogoliubov transformation, the spectra of created particles are computed. We also discuss some applications of this result in the context of quantum entanglement. We forge on the interpretation of pair creation in the Rindler spacetime with background electromagnetic fields of constant strengths. Our chief motivation is that the astrophysical black holes are often endowed with such background fields due to the accretion of plasma. The model considers here serves as a very simple toy model to address such a scenario.

1 Introduction

Quantum entanglement is one of the essential features of quantum physics, which is a concept necessary to quantum information theory, technology, and related topics. Quantum entanglement is rapidly gaining a distinction in discussions of quantum field theory in curved spacetime, as demonstrated by the Unruh and the Hawking effect. Relativistic quantum information is procreated from combing different and abundant branches of physics. Namely, general relativity, quantum field theory, and quantum information theory. Research into these effects of quantum fields in curved spacetime might hint at unifying the gravity theory and quantum mechanics.

Entanglement emerging due to quantum statistics of identical particles, either bosonic or fermionic is receiving attention and raising several fundamental issues [1, 2] whereas it can be quantified uniquely for pure states by the Von-Neumann entropy [3, 4] and for mixed states several measures have been proposed such as entanglement cost, distillable entanglement, and logarithmic negativity [5, 6, 7]. Logarithmic negativity is a valuable measure to quantify entanglement for a mixed bipartite state [8, 9, 10] and satisfies the property of entanglement monotone [11, 12]. Experimentally how to compute entanglement of a mixed many-body system is explained in [13]. Entanglement for the non-inertial observer is calculated in terms of logarithmic negativity in [14, 15, 16, 17]. Various experiments are performed to check that whether a uniformly accelerated object coupled to a quantum field would emit radiation or not, known as the Unruh-deWitt effect.
detector [18, 19] and shows the experimental observation of acceleration-induced thermality [20]. There are a couple of distinct relativistic sectors where entanglement properties of quantum fields emerge very naturally due to the creation of entangled particle pairs. Here we wish to study entanglement between the particles generated by the background electric field and due to non-zero acceleration of the Rindler observer.

Unruh showed that a uniformly accelerated detector moving in flat space perceives the Minkowski vacuum to be thermally populated at temperature \( T_U = \frac{a}{2\pi} \) here \( a \) is the acceleration of the uniformly accelerated observer. This effect is known as the Unruh effect [21, 22, 23, 24] and the nature of the interaction between a quantum field and an accelerating particle detector is analyzed from the point of view of an inertial observer in [25]. Whereas on the other hand, the quantum field theory vacuum is unstable in the presence of a background electric field, which leads to pair production, and this process is known as the Schwinger effect [27]. Under the influence of strong backgrounds, the vacuum may spontaneously break down due to quantum fluctuations [28, 29, 30], and virtual pairs can be separated either by the energy of the fields or the causality of spacetime. The charged particle in a constant electric field experiences a uniform acceleration due to the non-zero electric field and non-zero acceleration of the Rindler observer. One motivation is to use these two phenomena to study the near horizon geometry of non-extremal black holes endowed with an electromagnetic field, whose near horizon limit is the Rindler spacetime [31, 32, 33, 34, 35, 36, 37]. And thermal radiation that is theorized to be released outside a black hole’s event horizon because of relativistic quantum effects is known as Hawking radiation. Property of quantum radiation is based on quantum entanglement; entanglement-based description of the Minkowski vacuum is important to understand theoretical quantum radiation generated due to the Unruh and the Schwinger effect. The near-horizon geometry of charged black hole can be approximated by the Rindler space with the acceleration computed by the surface gravity. Thus, the quantum electrodynamics appearance in the Rindler space utilizes astrophysics and gravity. In this article, we obtain a straightforward toy model for the quantization of a fermionic field in the Reissner-Nordstrom black hole [38] geometry. The charged black hole provides a fascinating model to inspect the quantum nature of black holes because of the Hawking radiation and the Schwinger pair production of charges from the electric field around the black hole. The astrophysical black holes are often endowed with background electromagnetic fields due to the accretion of plasma. Therefore it is one of the motivations to investigate charged particle dynamics in the presence of the external electromagnetic fields near astrophysical black holes. Here in this paper, we are studying the effect of the magnetic field on the quantum correlations between the particles created by the Hawking radiation and the Schwinger effect. The Schwinger effect in near-extremal static black holes in an arbitrary \( D \)-dimensional asymptotically flat and AdS or dS spacetime is studied by [39].

The study of the degradation of correlations between the particles created by the Unruh effect was studied earlier by [40, 41, 42, 43, 44]; this phenomenon is known as the Unruh decoherence. In contrast, the study of the correlation between the particles created by the Schwinger effect is studied by [45, 46, 47, 48, 49, 50]. The impact of background magnetic field on the correlations between the particles generated by the Schwinger effect in the Minkowski spacetime studied in [51], whereas in the inflationary scenario is studied in [52], in both cases, electric and magnetic fields opposes each other. The Schwinger’s mechanism of complex scalars was first studied in the 2-dimensional Rindler spacetime by Gabriel, and Spindel [46], in which the vacuum persistence and the mean number of charged pairs were completely calculated. The quantum electrodynamics vacuum polarization and the vacuum persistence amplitude, and the Schwinger pair creation in an accelerating frame when a constant electric field exists in the Minkowski spacetime. A curious theoretical model is an accelerating observer in de Sitter space, in which the Unruh effect and the Gibbons-Hawking radiation are intersecting and point to an effective temperature of the geometric mean of the Unruh and the
Hawking temperature [53]. In this work, our main goal is to quantify entanglement between the particles created due to the Schwinger and the Unruh effect in the right (R) and left (L) wedge. We want to see how the magnetic field is affecting it. This paper considers the Dirac field in the Rindler spacetime with background electromagnetic fields of constant strength. We are interested in studying the entanglement-degradation between the particles in R and L regions with varying electromagnetic fields. Earlier the Dirac field entanglement in de Sitter with background electromagnetic field is studied by [52, 55, 56, 57], whereas entanglement for the Dirac field in the Rindler spacetime is studied in [22, 41, 43, 59, 60, 61, 62, 63].

The rest of the paper is organized as follows. In Section 2, the Dirac field is quantized in the R and L Rindler wedge explicitly, as well as the Bogoliubov relation between the local modes is shown explicitly. In Section 3, Appendix A and Appendix B, global modes are constructed by the superposition of the local modes in the right and left Rindler wedges. The Bogoliubov transformation is computed between the local and global creation and annihilation operators. Further, in Section 4 firstly number density for local number operator with respect to global vacuum is computed and its variation with parameter is plotted in Fig. 2, secondly logarithmic negativity between the particle-antiparticle in region R and L is computed and is variation with parameter \( \Delta = \frac{m^2 + (2n+1)eB}{eE} \) defined in Eq. (35) is shown in Fig. 3. Finally, we concluded in Section 5.

We shall work with the mostly positive signature of the metric in (3 + 1)-dimensions and will set \( c = k_B = \hbar = 1 \) throughout.

2 The Dirac modes

The Rindler coordinate transformations divide the Minkowski space into two patches, denoted hereafter by the labels R and L. On each of these patches the coordinate transformations between Minkowski \((\tau, \rho, y, z)\), and Rindler \((t, x, y, z)\) coordinates are given as

\[
\tau = \frac{1}{a} e^{ax} \sinh at_R, \quad \rho = \frac{1}{a} e^{ax} \cosh at_R \quad (R); \quad \tau = \frac{1}{a} e^{ax} \sinh at_L, \quad \rho = -\frac{1}{a} e^{ax} \cosh at_L \quad (L)
\]

here for each of these quadrants, the respective Rindler coordinates run from \(-\infty\) to \(\infty\) shown in Fig. 1. On R and L, the vector field \(\partial_t\) is timelike. The world lines of uniformly accelerated observers in the Minkowski coordinates correspond to hyperbolas to the left and right of the origin, which are bounded by lightlike asymptotes constituting the Rindler horizon, so the two Rindler regions are causally disconnected from each other. An observer undergoing uniform acceleration remains constrained to either the Rindler region R or L and has no access to the other sector. In Fig. 1, \(I^-_{R,L}\) and \(I^+_{R,L}\) are the past and future null infinities, whereas \(H^-_{R,L}\) and \(H^+_{R,L}\) are the past and future horizons whereas \(u\) and \(v\) are lightlike coordinates defined as \(u = t - x\) and \(v = t + x\). Under the transformation Eq. (1), the line element takes the form

\[
ds^2 = e^{2ax}(-dt^2 + dx^2) + dy^2 + dz^2,
\]

where \(a\) is the acceleration parameter, the metric is the (3+1)-Rindler spacetime metric. Let us now focus on the fermionic field theory coupled to external or background electromagnetic fields in the four-dimensional Rindler spacetime.

The Dirac equation for the \(\psi\) in curved spacetime is given as

\[
(i\gamma^\mu D_\mu - m)\psi(x) = 0
\]
Here the gauge cum spin covariant derivative is defined as

\[ D_\mu = \partial_\mu + iqA_\mu + \Gamma_\mu \]  

ensuring the local gauge symmetry and the general covariance. Here \( \Gamma_\mu \) is the spin connection and its only non-zero component is \( \Gamma_0 \) written as

\[ \Gamma_0 = \frac{a}{2} \gamma^{(1)}\gamma^{(0)} \],

where \( \gamma^{(a)} \)'s are the flat spacetime gamma matrices. Next, we introduce the tetrads \( e_\mu^a \) \( (a, b, c, \ldots = 0, 1, 2, 3) \) are indices for the local Lorentz transformation and the Greek indices \( \mu, \nu \cdots \) are for spacetime), the tetrad field is related to the metric in curved spacetime with the help of the four-dimensional Minkowski metric as

\[ g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab} \].

Following from Eq. (6) and Eq. (2) we choose the tetrads for the Rindler metric Eq. (2) as

\[ e_\mu^a = \text{diag}(e^{-ax}, e^{-ax}, 1, 1) \]

Defining a new variable \( \tilde{\psi} = e^{\frac{2a}{\omega}} \psi \) in equation Eq. (3), it becomes

\[ (ie_\mu^a \gamma^{(a)} \partial_\mu - q e_\mu^a \gamma^{(a)} A_\mu - m) \tilde{\psi}(x) = 0 \]

Substituting next

\[ \tilde{\psi}(x) = (ie_\mu^a \gamma^{(a)} \partial_\mu - q A_\mu e_\mu^a \gamma^{(a)} + m) \chi(x) \]
in Eq. (8) gives
\[
\left( \frac{1}{a} \partial_x^2 + (\partial_t + i a A_0)^2 + a \partial_x^2 - \partial_x^2 \right) - \partial_y^2 - \partial_z^2 - (\partial_x + i q A_3)^2 + \frac{\gamma(1) \gamma(0)}{e^{2a_x}} (a \partial_t + i q \partial_x A_0) - i q \partial_y A_3 \gamma(2) \gamma(3) - m^2 \right) \chi(x) = 0
\]
(10)
We choose the gauge which gives us constant electric and magnetic fields along \( x \)–axis as
\[
A_\mu = \frac{E e^{2a_x}}{2a} \delta_\mu + B y \delta_\mu
\]
(11)
where \( E, B \) and \( a \) are the constants. We consider the ansatz \( \chi(x) = e^{-i \omega t} e^{i k_z z} \zeta_s(x,y) \epsilon_s \) (no sum on \( s \)) in Eq. (10), we have
\[
\left( -\frac{1}{a} \partial_x^2 + (\omega - q E e^{2a_x})^2 + \frac{a}{e^{2a_x}} \partial_x^2 - \partial_y^2 + (k_z + q B y)^2 - \frac{\gamma(1) \gamma(0)}{2a_x} (a \omega) + q E e^{2a_x} - i q B \gamma(2) \gamma(3) - m^2 \right) \zeta_s(x,y) \epsilon_s = 0
\]
(12)
where \( \epsilon_s \) are the simultaneous eigenvectors of \( \gamma(1) \gamma(0) \) and \( \gamma(2) \gamma(3) \), such that it has the following eigenvalue equations with \( \gamma(1) \gamma(0) \) and \( \gamma(2) \gamma(3) \), \( \gamma(1) \gamma(0) \epsilon_1 = -\epsilon_1, \gamma(1) \gamma(0) \epsilon_2 = -\epsilon_2, \gamma(1) \gamma(0) \epsilon_3 = \epsilon_3, \gamma(1) \gamma(0) \epsilon_4 = \epsilon_4, \gamma(2) \gamma(3) \epsilon_1 = -i \epsilon_1, \gamma(2) \gamma(3) \epsilon_2 = i \epsilon_2, \gamma(2) \gamma(3) \epsilon_3 = -i \epsilon_3 \) and \( \gamma(2) \gamma(3) \epsilon_4 = i \epsilon_4 \) whereas the explicit form of \( \epsilon_s \) are as follows
\[
\epsilon_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \quad \epsilon_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \epsilon_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \end{pmatrix} \quad \text{and} \quad \epsilon_4 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}.
\]
By using the eigenvalue equations and separation of variables as done in [52], it gives us two differential equations as follows
\[
\left( \partial_x^2 - a \partial_x + \omega^2 - \frac{q E \omega}{a} e^{2a_x} - i \omega a - \frac{q E e^{2a_x}}{2a} \right) \zeta_s(x) = 0
\]
(13)
and
\[
\left( \partial_y^2 - (k_z + q B y)^2 + q B - S_s \right) H_s(y) = 1
\]
(14)
where
\[
S_1 = -2 n_L q B \quad \text{and} \quad S_2 = -(2 n_L + 1) q B
\]
(15)
are the separation constant and \( n_L \) corresponds to the Landau level. The general solution of Eq. (13) for \( s = 1 \), i.e. \( \zeta_1(x) \) are \( e^{i \lambda_1 x} e^{-\frac{q E e^{2a_x}}{4a_x} + e^{i \omega x} U(\lambda_1, \nu, \xi)} \) and \( e^{i \lambda_2 x} e^{-\frac{q E e^{2a_x}}{4a_x} + e^{i \omega x} L(-\lambda_1, \nu - 1, \xi)} \), where \( U \) and \( L \) are the confluent hypergeometric and the generalized Laguerre functions respectively whose explicit form are given in [64], whereas solution of Eq. (14)
\[
H_s(y) = \left( \frac{\sqrt{q B}}{2^{n+1} \pi (n+1)!} \right)^{1/2} e^{-\frac{\bar{y}^2}{2} + H_n(\bar{y})} = h_n(\bar{y}) \quad \text{(say)}
\]
(16)
here \( \lambda_1, \lambda_2 \) and \( \nu \) are the parameters defined as
\[ \lambda_1 = \lambda_3 = \frac{i(m^2 + S_1) + 2qE}{2qE}, \quad \lambda_2 = \lambda_4 = \frac{i(m^2 + S_2) + 2qE}{2qE}, \quad \nu = \frac{3}{2} + \frac{i\omega}{a} \tag{17} \]

whereas the variables \( \tilde{y} \) and \( \xi \) are defined as

\[ \xi = -\frac{iqEe^{2ax}}{2a^2}, \quad \tilde{y} = \left( \sqrt{qB}y + \frac{k_s}{\sqrt{qB}} \right) \tag{18} \]

Let us now find out the in modes for right wedge (R), where at \( x \to \infty \) and \( x \to -\infty \), that corresponds to \( I_R^+ \) and \( H_{R-} \) respectively Fig. 1. Mode emerging from \( H_{R-} \) is moving towards \( I_R^+ \) and the relevant part of mode proportional to \( e^{-i\omega u} \)

\[ \zeta_s(x) \sim e^{ax} e^{-\frac{iqEe^{2ax}}{4a^2}} e^{-i\omega(t-x)} \xi^{-\lambda_s}, \quad s = 1, 2 \]

Similarly, modes emerging from \( I_R^- \) are moving towards \( H_{R-}^+ \) and the relevant part of mode is proportional to \( e^{-i\omega u} \)

\[ \zeta_s(x) \sim e^{ax} e^{\frac{iqEe^{2ax}}{4a^2}} e^{-i\omega(t+x)} \frac{\Gamma(-\lambda_s^+ + \nu^+)}{\Gamma(\nu^+)\Gamma(-\lambda_s^+)} \xi^s, \quad s = 1, 2 \]

Putting these together we have four modes from which two corresponds to \( I_R^- \) and two to \( H_{R-}^- \) written as

\[ \chi(x)_{H_{R-},s} = e^{-i\omega(t-x)} e^{ik_s z} e^{ax} e^{-\frac{iqEe^{2ax}}{4a^2}} U(\lambda_s, \nu, \xi) H_s(y) \xi_s, \tag{19} \]

\[ \chi(x)_{I_{R-},s} = e^{-i\omega(t+x)} e^{ik_s z} e^{ax} e^{\frac{iqEe^{2ax}}{4a^2}} (L(-\lambda_s, \nu - 1, \xi)^s H_s(y) \xi_s \tag{20} \]

here \( s = 1, 2 \) in Eq. (19) and Eq. (20).

### 2.1 Quantization on the right wedge (R)

For computing full modes we need to substitute \( \chi(x) \) in Eq. (9) and then using the definition \( \psi = e^{-\frac{qE}{a}\tilde{y}} \psi \) the final particle in modes are given as

\[ U_{s,n}(x)_{H_{R-}} = e^{-\frac{ax}{N_s}} (ie_{t_a}^{\mu_s}(a) \partial_\mu - qA_\mu e_{t_a}^{\mu_s}(a) + m)e^{-i\omega(t-x)} e^{ik_s z} e^{ax} e^{-\frac{iqEe^{2ax}}{4a^2}} U(\lambda_s, \nu, \xi) H_s(y) \xi_s, \tag{21} \]

\[ U_{s,n}(x)_{I_{R-}} = e^{-\frac{ax}{M_s}} (ie_{t_a}^{\mu_s}(a) \partial_\mu - qA_\mu e_{t_a}^{\mu_s}(a) + m)e^{-i\omega(t+x)} e^{ik_s z} e^{ax} e^{\frac{iqEe^{2ax}}{4a^2}} (L(-\lambda_s, \nu - 1, \xi)^s H_s(y) \xi_s, \tag{22} \]

here \( s = 1, 2 \) for Eq. (21) and Eq. (22) and the parameter \( \xi \) is defined in Eq. (18), they are the positive-frequency modes with respect to a future-directed time like Killing vector \( \partial_t \), whereas negative energy modes are given as

\[ V_{s,n}(x)_{H_{R-}} = e^{-\frac{ax}{P_s}} (ie_{t_a}^{\mu_s}(a) \partial_\mu - qA_\mu e_{t_a}^{\mu_s}(a) + m)e^{i\omega(t-x)} e^{ik_s z} e^{-\frac{iqEe^{2ax}}{4a^2}} \xi U(\nu - \lambda_s, \nu, \xi) \xi_s, \tag{23} \]

\[ V_{s,n}(x)_{I_{R-}} = e^{-\frac{ax}{R_s}} (ie_{t_a}^{\mu_s}(a) \partial_\mu - qA_\mu e_{t_a}^{\mu_s}(a) + m)e^{i\omega(t+x)} e^{ik_s z} e^{ax} e^{\frac{iqEe^{2ax}}{4a^2}} \xi^{1-\nu^+} (L(\nu - \lambda_s - 1, 1- \nu, \xi)^s \xi_s \tag{24} \]
here $s = 3, 4$ for Eq. (23) and Eq. (24), and $N_s$, $M_s$, $P_s$ and $R_s$ are the normalization constants obtained by normalizing modes at constant $u$ and $v$ surfaces, which are shown in the Appendix A explicitly. Orthonormality relations of these modes are given by

$$
(U_{s,n}(x)_{H_R^+}, I_R^+; U_{s',n'}(x)_{H_R^+}, I_R^+) = \delta(k_z - k'_z)\delta(\omega - \omega')\delta_{nn'}\delta_{ss'} \quad (25)
$$

Next, we have computed another set of orthonormal outmodes corresponding to the regions $I_R^+$ and $H_R^+$ of Fig. 1. Since the $\text{in}$-basis functions contain a particle plus a pair in the future, they are not useful to describe quantization in terms of single quanta in the future. Their time reversed versions are given by following definition $U_{s,n}(t, \bar{x})_{I_R^-} = (U_{s,n}(-t, \bar{x})_{I_R^-})^*$ analogous to [36, 37], in which they have used the same definition to compute out modes for scalar field in the $(1+1)$-Rindler spacetime, similarly for negative frequency out modes we have $V_{s,n}(t, \bar{x})_{I_R^+} = (V_{s,n}(-t, \bar{x})_{I_R^+})^*$ (explicit form of the modes and the calculation of normalization constants are shown in the Appendix A). We now make the field quantization on $R$ in terms of the modes on them as follows

$$
\psi_R(x) = \sum_{n,s} \int \frac{d\omega dk_z}{2\pi} \left[ a_0(\omega, k_z, s, n)_{H_R^-} U_{s,n}(x; \omega, k_z)_{H_R^-} + b_0(\omega, k_z, s, n)_{H_R^-} V_{s,n}(x; \omega, k_z)_{H_R^-} \right] \\
+ \sum_{n,s} \int \frac{d\omega dk_z}{2\pi} \left[ a_0(\omega, k_z, s, n)_{H_R^+} U_{s,n}(x; \omega, k_z)_{H_R^+} + b_0(\omega, k_z, s, n)_{H_R^+} V_{s,n}(x; \omega, k_z)_{H_R^+} \right] \\
+ \sum_{n,s} \int \frac{d\omega dk_z}{2\pi} \left[ a_0(\omega, k_z, s, n)_{I_R^-} U_{s,n}(x; \omega, k_z)_{I_R^-} + b_0(\omega, k_z, s, n)_{I_R^-} V_{s,n}(x; \omega, k_z)_{I_R^-} \right] \\
+ \sum_{n,s} \int \frac{d\omega dk_z}{2\pi} \left[ a_0(\omega, k_z, s, n)_{I_R^+} U_{s,n}(x; \omega, k_z)_{I_R^+} + b_0(\omega, k_z, s, n)_{I_R^+} V_{s,n}(x; \omega, k_z)_{I_R^+} \right] 
$$

here the creation and annihilation operators are assumed to satisfy the usual canonical anti-commutation relations. Using the relation between confluent hypergeometric functions $[64]$, we can write the Bogoliubov relation between $\text{in}$ and $\text{out}$ modes as

$$
U_{s,n}(x)_{H_R^-} = \alpha_s^* U_{s,n}(x)_{I_R^-} + \beta_s^* V_{s,n}(x)_{I_R^-}^*, \\
V_{s,n}(x)_{H_R^+} = \alpha_s^* V_{s,n}(x)_{I_R^+} + \beta_s^* (U_{s,n}(x)_{I_R^+})^*
$$

here $s = 1, 2$ in Eq. (30) and $\alpha_s$ and $\beta_s$ are the Bogoliubov coefficients given as

$$
\alpha_s = \frac{N_s \Gamma(1 - \lambda_s) \sin \pi (\lambda_s - \nu)}{M_s \sin \pi \nu}, \quad \beta_s = \frac{N_s \sin \pi \lambda_s \Gamma(\nu - \lambda_s)}{R_s \sin \pi \nu} \quad (31)
$$

whereas the Bogoliubov transformation between the creation and annihilation operators are given as

$$
a(\omega, k_z, s, n)_{H_R^-} = \alpha_0 a(\omega, k_z, s, n)_{I_R^-} + \beta_0^* b^\dagger(-\omega, -k_z, s, n)_{I_R^-}, \quad (32) \\
b(\omega, k_z, s, n)_{H_R^-} = \alpha_0 b(\omega, k_z, s, n)_{I_R^-} + \beta_0 a^\dagger(-\omega, -k_z, s, n)_{I_R^-}, \quad (33)
$$

The canonical anti-commutation relations ensure, $|\alpha_s|^2 + |\beta_s|^2 = 1$. The coefficient $\beta_s$ is responsible for pair production, and the quantity $|\beta_s|^2$ is the mean number density of particles

$$
|\beta_s|^2 = \frac{\sinh^2 \pi \Delta}{e^{\pi \Delta} \cosh^3 \pi (\Delta - \frac{\nu}{a}) + \sinh^3 \pi \Delta}, \quad (34)
$$
here the parameter $\Delta$ is defined as,

$$\Delta = \text{Im}(\lambda) = \frac{m^2 + S_s}{2qE}$$  \hfill (35)

$|\beta_s|^2$ is independent of momentum. For $\Delta \to \infty$, that corresponds to zero electric field ($E \to 0$) or electric charge ($q \to 0$) the number density for local vacuum leads to zero $|\beta_s|^2 \to 0$ and this behaviour is similar to the usual Minkowski vacuum in the presence of background electromagnetic field computed in [51] for complex scalar field in the Minkowski spacetime.

### 2.2 Quantization on the left wedge (L)

The field equations and their solutions are the same on the left and right wedges. The only difference between L and R wedge is that on L, the vector field $\partial_t$ points near the past whereas $\partial_t = -\partial_t$ plays the role of future-directed Killing vector as explained in [25], which implies that the sign of the charges and the in and out labels have to interchange concerning their values on the R region. Hence, the complete set of in and out modes for L wedge are given as

$$U_{s,n}(x_L)_{H^-_L} = e^{-\frac{aq_L}{N_s}}(ie_a^\mu \gamma^\mu(\alpha)\partial_\mu - q A_\mu e_a^\mu(\alpha) + m)e^{-i\omega(t_L + x_L)e^{-\frac{iqE_{2a}t_L}{4m}}L(-\lambda_s, \nu - 1, \xi_L)H_\nu(y)\epsilon_s,}$$  \hfill (36)

$$U_{s,n}(x_L)_{I^-_L} = e^{-\frac{aq_L}{M_s}}(ie_a^\mu \gamma^\mu(\alpha)\partial_\mu - q A_\mu e_a^\mu(\alpha) + m)e^{-i\omega(t_L - x_L)e^{-\frac{iqE_{2a}t_L}{4m}}L(-\lambda_s, \nu - 1, \xi_L)H_\nu(y)\epsilon_s,}$$  \hfill (37)

$$V_{s,n}(x_L)_{H^-_L} = e^{-\frac{aq_L}{P_s}}(ie_a^\mu \gamma^\mu(\alpha)\partial_\mu - q A_\mu e_a^\mu(\alpha) + m)e^{i\omega(t_L + x_L)e^{-\frac{iqE_{2a}t_L}{4m}}(\xi_LU(\nu - \lambda_s, \nu, \xi_L))H_\nu(y)\epsilon_s,}$$  \hfill (38)

$$V_{s,n}(x_L)_{I^-_L} = e^{-\frac{aq_L}{R_s}}(ie_a^\mu \gamma^\mu(\alpha)\partial_\mu - q A_\mu e_a^\mu(\alpha) + m)e^{i\omega(t_L - x_L)e^{-\frac{iqE_{2a}t_L}{4m}}(\xi_LU(\nu - \lambda_s, \nu, \xi_L))H_\nu(y)\epsilon_s,}$$  \hfill (39)

where $s = 1, 2$ for Eq. (36), Eq. (37) and $s = 3, 4$ for Eq. (38), Eq. (39), the parameter $\xi$ is defined by Eq. (18), and they are the positive and negative energy modes with respect to $\partial_t$ respectively. Now, by using the definition for out modes defined in previous section, we can find the out modes for L wedge also whereas the Bogoliubov transformation as well as coefficients will remain similar as that of right (R).

### 3 The global modes and Bogoliubov coefficients

Since R and L are just two patches of the Minkowski spacetime, they don’t cover the whole Minkowski spacetime; therefore, we form global modes to cover the whole Minkowski spacetime. The set of modes on $I_R^+ \cup H_R^-$ and $I_L^+ \cup H_L^+$ are disconnected in Fig. 1, therefore to cover the whole Minkowski spacetime we construct global modes having support in $R \cup L$ using Unruh’s prescription as used in [35, 55]. For the construction of global in modes, we take the linear combination of the modes on R and L wedges by comparing their asymptotic limit behavior of modes shown explicitly in Appendix B. According to that the set of global in modes are constructed by the superposition of $U_{1,n}(x)_{H^-_R}, V_{1,n}(x)_{H^-_R}, U_{1,n}(x)_{I^-_L}$ and
\[ V_{1,n}(x)_{I_L} \text{, are as follows} \]

\[
\phi_G^x(x) = \frac{1}{\sqrt{2 \cosh \frac{\pi \hbar}{a}}} \left( e^{\frac{\pi \hbar}{a}} U_{1,n}(x)_{H^+_R} + e^{-\frac{\pi \hbar}{a}} V_{1,n}(x)_{I_L} \right) \tag{40}
\]

\[
\phi_Z^x(x) = \frac{1}{\sqrt{2 \cosh \frac{\pi \hbar}{a}}} \left( e^{\frac{\pi \hbar}{a}} U_{1,n}(x)_{I_L} + e^{-\frac{\pi \hbar}{a}} V_{1,n}(x)_{H^+_R} \right) \tag{41}
\]

\[
\phi_S^x(x) = \frac{1}{\sqrt{2 \cosh \frac{\pi \hbar}{a}}} \left( e^{\frac{\pi \hbar}{a}} V_{1,n}(x)_{H^+_R} - e^{-\frac{\pi \hbar}{a}} U_{1,n}(x)_{I_L} \right) \tag{42}
\]

\[
\phi_F^x(x) = \frac{1}{\sqrt{2 \cosh \frac{\pi \hbar}{a}}} \left( e^{\frac{\pi \hbar}{a}} V_{1,n}(x)_{I_L} - e^{-\frac{\pi \hbar}{a}} U_{1,n}(x)_{H^+_R} \right) \tag{43}
\]

Therefore, Eq. (40), Eq. (41), Eq. (42) and Eq. (43) are the global \textit{in} modes, whereas the global \textit{out} modes can be obtained by substituting the local modes Bogoliubov transformation from Eq. (30) in Eq. (40), Eq. (41), Eq. (42) and Eq. (43). Further, we write the field quantization of the Dirac field \( \psi \) in \( R \cup L \) in terms of the local modes in left (L) and right (R) as well as in terms of global modes (we have suppressed the subscript for \( s = 1 \) from now onwards), given as follows

\[
\psi(x) = \sum_n \int \frac{d\omega dk_z}{2\pi} \left[ a(\omega, k_z, n)_{I_L} U_n(x; \omega, k_z)_{I_L} + b^\dagger(\omega, k_z, n)_{I_L} V_n(x; \omega, k_z)_{I_L} \
+ a(\omega, k_z, n)_{I_L} U_n(x; \omega, k_z)_{I_L} + b^\dagger(\omega, k_z, n)_{I_L} V_n(x; \omega, k_z)_{I_L} \right] 
\]

\[
= \sum_n \int \frac{d\omega dk_z}{2\pi} \left[ a(\omega, k_z, n)_{H^+_R} U_n(x; \omega, k_z)_{H^+_R} + b^\dagger(\omega, k_z, n)_{H^+_R} V_n(x; \omega, k_z)_{H^+_R} \
+ a(\omega, k_z, n)_{H^+_R} U_n(x; \omega, k_z)_{H^+_R} + b^\dagger(\omega, k_z, n)_{H^+_R} V_n(x; \omega, k_z)_{H^+_R} \right] 
\]

\[
\text{in terms of local modes, whereas in terms of global modes, it is as follows} 
\]

\[
\psi(x) = \sum_n \int \frac{d\omega dk_z}{2\pi} \left[ c_1(\omega, k_z, n)\phi_G^x(x) + d_1^\dagger(\omega, k_z, n)\phi_Z^x(x) + c_2(\omega, k_z, n)\phi_S^x(x) + d_2^\dagger(\omega, k_z, n)\phi_F^x(x) \right] \tag{44}
\]

Comparing Eq. (45) and Eq. (44), we obtain the Bogoliubov relations,

\[
c_1 = \frac{1}{\sqrt{2 \cosh \frac{\pi \hbar}{a}}} \left( e^{\frac{\pi \hbar}{a}} a_{H^+_R} (\omega, k_z, n) - e^{-\frac{\pi \hbar}{a}} b_{I_L}^\dagger (\omega, k_z, n) \right), 
\]

\[
d_1^\dagger = \frac{1}{\sqrt{2 \cosh \frac{\pi \hbar}{a}}} \left( e^{\frac{\pi \hbar}{a}} a_{I_L} (\omega, k_z, n) - e^{-\frac{\pi \hbar}{a}} b_{H^+_R}^\dagger (\omega, k_z, n) \right) \tag{46}
\]

Now, using Eq. (32) and Eq. (46) we can find out the relation between global and local \textit{out} operators which gives

\[
c_1 = \frac{1}{\sqrt{2 \cosh \frac{\pi \hbar}{a}}} \left( e^{\frac{\pi \hbar}{a}} \alpha_1 a_{I_L} - e^{\frac{\pi \hbar}{a}} \beta_1 b_{I_L}^\dagger - e^{-\frac{\pi \hbar}{a}} \alpha_1^\dagger b_{H^+_R}^\dagger + e^{-\frac{\pi \hbar}{a}} \beta_1 a_{H^+_R} \right) \tag{47}
\]
Similarly, there will be another set of creation and annihilation operator \( (c_2, d_2^\dagger) \) corresponding to other set of local vacua. The global vacuum can therefore be defined as \( |0\rangle^{1}_{R} \otimes |0\rangle^{2}_{L} \), where \( |0\rangle^{1} \) is annihilated by \( (c_1, d_1) \) and \( |0\rangle^{2} \) is annihilated by \( (c_2, d_2) \). We will work with only \( |0\rangle^{1} \) only as the other will have similar results. Using the Bogoliubov relations we can write \( |0\rangle^{1} \) in terms of the local 'out' R-L vacuum. We are now ready to compute the particle creation and R-L entanglement.

## 4 The number density and logarithmic negativity

The local in vacua \( |0\rangle^{I}_{R}^{\pm}, |0\rangle^{I}_{L}^{\pm} \) are defined as,

\[
a_{I_{R}}|0\rangle^{I}_{R}^{\pm} = b_{I_{R}}|0\rangle^{I}_{R}^{\pm} = 0, \quad a_{I_{L}}|0\rangle^{I}_{L}^{\pm} = b_{I_{L}}|0\rangle^{I}_{L}^{\pm} = 0
\]

whereas local out vacua \( |0\rangle^{H}_{R}^{\pm}, |0\rangle^{H}_{L}^{\pm} \) as,

\[
a_{H_{R}}|0\rangle^{H}_{R}^{\pm} = b_{H_{R}}|0\rangle^{H}_{R}^{\pm} = 0, \quad a_{H_{L}}|0\rangle^{H}_{L}^{\pm} = b_{H_{L}}|0\rangle^{H}_{L}^{\pm} = 0
\]

The in and out vacuums on right \( R \) wedge are related by

\[
|0\rangle^{H}_{R} = \alpha_{1}|0_{k0_{-k}}\rangle^{I}_{R}^{+} + \beta_{1}|1_{k1_{-k}}\rangle^{I}_{R}^{+}
\]

The global vacuum is defined as

\[
ce_{\sigma}|0\rangle^{\sigma} = d_{\sigma}|0\rangle^{\sigma} = 0
\]

where \( \sigma = 1, 2 \). From the Bogoliubov relation of the preceding section Eq. (47), we express global in vacua in terms of out local vacua as follows

\[
|0\rangle^{1} \equiv |0_{k0_{-k}}\rangle^{I} = \frac{1}{(1 + e^{-\pi\omega})^{2}} \left( \alpha_{1}^{2}|0_{k0_{-k}}0_{k0_{-k}}\rangle^{I}_{H_{R}}^{+};H_{L}^{+} + \beta_{1}^{2}|1_{k1_{-k}}1_{k1_{-k}}\rangle^{I}_{H_{R}}^{+};H_{L}^{+} + \alpha_{1}\beta_{1}^{*}(|1_{k1_{-k}}0_{k0_{-k}}\rangle^{I}_{H_{R}}^{+};H_{L}^{+} + |0_{k0_{-k}}1_{k1_{-k}}\rangle^{I}_{H_{R}}^{+};H_{L}^{+})e^{-\frac{2\pi\omega}{\pi}}|1_{k0_{-k}}0_{k0_{-k}}\rangle^{I}_{H_{R}}^{+} \right)
\]

here the first two entries corresponds to \( R \), whereas the last two corresponds to \( L \) and \( 1 \langle 0\rangle^{1} = 1 \). The Hilbert space \( \mathcal{H} \) is constructed by the tensor product, \( \mathcal{H} = \mathcal{H}_{k}^{R} \otimes \mathcal{H}_{k}^{L} \otimes \mathcal{H}_{k}^{L} \), where \( \mathcal{H}_{k}^{R} (\mathcal{H}_{k}^{L}) \) and \( \mathcal{H}_{k}^{L} (\mathcal{H}_{k}^{L}) \) are the Hilbert spaces of the modes of the particle and the antiparticle, respectively and the superscript \( R \) and \( L \) corresponds to right and left wedge respectively. We find the spectra of pair creation

\[
\rho_{N} = 1 \langle 0_{k0_{-k}} \vert a_{I_{R}}^{+}a_{I_{R}}^{\dagger} \vert 0_{k0_{-k}} \rangle^{1} = \frac{\vert \beta_{1} \vert^{2} e^{2\pi\omega}}{1 + e^{2\pi\omega}} + \frac{1}{1 + e^{2\pi\omega}}
\]

where \( \vert \beta_{1} \vert^{2} \) is given by Eq. (34) in terms of variable \( \Delta \) defined in Eq. (18). In Eq. (53), the first term on the right-hand side depends on the parameter \( \Delta \), whereas the second term is independent of parameter \( \Delta \) and it reproduces the usual fermionic black body spectrum depending on the acceleration and energy of the Rindler observer only, e.g. [26, 56] and for \( \Delta \rightarrow \infty \) (i.e., \( E \rightarrow 0 \) or \( q \rightarrow 0 \), which corresponds to Eq. (53) is reduced to

\[
\rho_{N} = \frac{1}{1 + e^{\pi\omega}}
\]
Figure 2: Number density of global vacuum, $|0\rangle^1$. As we have discussed in main text we have plotted Eq. (53) (blue curve) and Eq. (54) (red curve) variation with respect to the parameter $\Delta = m^2 + (2n + 1)q_B/q_E$, where we have taken $\frac{\omega_a}{a} = 1$. For a given mode, the number density first monotonically increases with increase in $\Delta$ then reaches to a plateau, after that decreases monotonically with increasing $\Delta$. Whereas for $\Delta \to \infty$ Eq. (54) becomes independent of $\Delta$, i.e. $\rho_N \approx 0.002$.

The result Eq. (54) is a fermionic Planck spectrum with temperature $T_U = a/2\pi$, which is the usual Unruh temperature observed by an observer moving with uniform acceleration through the Minkowski vacuum observes a thermal spectrum of particles. We have plotted number density Eq. (53) vs $\Delta$ in Fig. 2 (blue curve), $\rho_N$ increases monotonically with increase in $\Delta$ (up to $\Delta = 1$) and then reaches to a plateau after that it monotonically decreases with increase in $\Delta$. It shows that up to $\Delta = 1$, the strength of $(m^2 + (2n + 1)q_B)$ is comparatively same as $q_E$, so the strength of the magnetic field is comparable to $(m^2 + (2n + 1)q_B)$, and is not much to compensate the affect electric field, whereas for large $\Delta$ which corresponds to comparatively large $(m^2 + (2n + 1)q_B)$ from $q_E$, pair creation decreases with increase in $\Delta$ that is due to the Lorentz force applied by magnetic on the particles created by electric field similar thing happens for scalars in the Minkowski spacetime in [51]. For $\Delta \to \infty$, $\rho_N \approx 0.06$ (shown by red curve in Fig. 2) which corresponds to the same value of $\rho_N$ that one can obtain from Eq. (54) (for $\frac{\omega_a}{a} = 1$), whereas for scalars in the Minkowski spacetime it is zero, here this non-zero contribution is due to the acceleration of Rindler observer, similarly for de Sitter also we have obtained this non-zero contribution due to spacetime curvature [52].

Next, we wish to compute entanglement between the particles and antiparticles in the R and L regions, respectively. The state, which represents the particle-antiparticle of the R and L regions, is characterized by a mixed state density matrix given by Eq. (55). Logarithmic negativity is a good measure to compute entanglement for a mixed state, therefore we computed logarithmic negativity for Eq. (55).

Even for mixed states, there is a measure of the entanglement of bipartite states [8, 9], called the entanglement negativity, defined as $N(\rho_{AB}) = \frac{1}{2} \left( \| \rho_{A|B}^{TA} \|_1 - 1 \right)$, where $\rho_{A|B}^{TA}$ is the partial transpose of $\rho_{AB}$ with respect to the subspace of $A$, i.e., $(|i\rangle_A|n\rangle \otimes |j\rangle_B\langle\ell|)^{TA} := |n\rangle_A|\ell\rangle \otimes |j\rangle_B\langle i|$. Here, $\| \rho_{A|B}^{TA} \|_1$ is the trace norm, such that $\| \rho_{A|B}^{TA} \|_1 = \sum_{i=1}^{\text{all}} |\mu_i|$, where $\mu_i$ is the $i$-th eigenvalue of $\rho_{A|B}^{TA}$. The logarithm of $\| \rho_{A|B}^{TA} \|_1$ is called the logarithmic negativity, which can be written as $L_N(\rho_{AB}) = \log (1 + 2N(\rho_{AB}))$. These quantities are entanglement monotones that do not increase under local operations and classical communications. These quantities measure a violation of the positive partial transpose (PPT) in $\rho_{AB}$. The PPT criterion can be stated as follows. If $\rho_{AB}$ is separable, the eigenvalues of $\rho_{A|B}^{TA}$ are non-negative. Hence, if $N \neq 0$ ($L_N \neq 0$), $\rho_{AB}$ is an entangled state. On the other hand, if $N = 0$ ($L_N = 0$), we cannot judge the existence of the entangle-
ment from this measure since there exist PPT and entangled states in general. However, the logarithmic negativity can be helpful since it is a calculable measure, and more discussions on it can be found in e.g. [12].

The total density operator \( \rho_{\text{global}} = |0\rangle^1 {}^1 \langle 0| \). We thus obtain the reduced density operator for particles of R region and antiparticles of L region by tracing out antiparticles of R region and particles of L region

\[
\rho_{p; a}^{R; L} = \frac{1}{1 + e^{-\frac{2\pi}{\omega} a}} \left( |\alpha_1|^4 |00\rangle \langle 00| + (|\beta_1|^4 + e^{-\frac{2\pi}{\omega} a}) |11\rangle \langle 11| + |\alpha_1|^2 |\beta_1|^2 (|10\rangle \langle 10| + |01\rangle \langle 01|) + e^{-\frac{2\pi}{\omega} (\alpha_1^2 |10\rangle \langle 10| + \alpha_1^* |01\rangle \langle 01|)} \right) \tag{55}
\]

Now using the definition of logarithmic negativity \( (L_N) \) given above, it is given as

\[
L_N = \log_2 \left[ 1 + e^{-\frac{\pi}{\omega} a} \right] \tag{56}
\]

for \( \rho_{p; a}^{R; L} \) given in Eq. (55). For \( \Delta \to \infty \),

\[
L_N = \log_2 \left[ 1 + \frac{2}{e^{\frac{\pi}{\omega} a} + e^{-\frac{\pi}{\omega} a}} \right] \tag{57}
\]

We have plotted \( L_N \) vs \( \Delta \) in Fig. 3, where we observed that the logarithmic negativity decreases monotonically with an increase in \( \Delta \) up to \( \Delta \approx 1 \) then it reaches to a plateau, whereas further increases monotonically with the increase in \( \Delta \). The behavior of logarithmic negativity of \( \rho_{p; a}^{R; L} \), is different from the behaviour of the number desity of \( \rho_N \), which can be due to the mixed structure of density matrix.

![Figure 3: Logarithmic negativity of global vacuum, |0\rangle^1.](image)

As we have discussed in main text we have plotted Eq. (56) (blue curve) and Eq. (57) (red curve) variation with respect to the parameter \( \Delta = \frac{m^2 + (2n+1)qB}{qE} \), where we have taken \( \frac{q}{E} = 1 \). For a given mode, the number density first monotonically decreases with increase in \( \Delta \) then reaches to a plateau, after that increases monotonically with increasing \( \Delta \). Whereas for \( \Delta \to \infty \) Eq. (57) becomes independent of \( \Delta \), i.e. \( L_N \approx 0.12 \).

5 Summary and outlook

In this work, we have computed the entanglement of the fermionic field for two casually disconnected R-L regions in the Rindler spacetime with a constant electromagnetic fields. The main characteristic of this
problem is that it involves two acceleration parameters: the acceleration of the Rindler observer \(a\) and the natural acceleration of the charged quanta \((qE/m)\), along with this acceleration magnetic field is opposing the effect of the electric field. So we obtain a toy (but explicitly solvable) model for the quantization of a fermionic field in the Reissner-Nordstrom black hole geometry, in the same way as the Unruh detector mimics the physics around a Schwarzschild black hole. We have find the in and out local modes explicitly for both right \((R)\) and left \((L)\) regions, and computed the Bogoliubov transformations between the for both the regions Section 2. Further, in Section 3 we have computed the global in and out modes using the asymptotic form of local modes and found the Bogoliubov relation between global in and local out modes operators. Using that, we have written the squeezed state expansion of global vacua in terms of out local vacua. In Section 4, using the squeezed state expansion of global vacua, we have computed the number density of the global vacua and entanglement entropy between the particle-antiparticle of the regions \(R\) and \(L\) respectively.

Only magnetic field alone cannot create vacuum instability, but in the presence of background electric field and non-zero acceleration, it affects pair creation. We find out that the magnetic field does not affect the pair creation due to the non-zero acceleration of the Rindler observer but opposes the effect of pair creation due to the electric field. From Eq. (34) it is clear that there is no contribution of the magnetic field in the absence of an electric field, whereas when the magnetic field strength is much larger than the electric field strength, then the pair creation due to electric field is diminished but still there is a non-zero contribution in number density at \(\Delta \to \infty\), due to the acceleration of Rindler observer. Further, in Fig. 2, we have taken into account the variation of Eq. (53) concerning the parameter \(\Delta\) defined in Eq. (18). Next, we find out logarithmic negativity for particles-antiparticles of region \(R\) and \(L\) respectively. In Fig. 3 we have taken in account the variation of logarithmic negativity concerning parameter \(\Delta\), it quantifies the entanglement between the particles and antiparticles corresponding to regions \(R\) and \(L\) respectively and its behavior is non-monotonic, and it depends up on the choice of sector that we obtained from full density operator, \(\rho_{\text{global}}\).

The above analysis can be attempted to extend in different scenarios. Such as, it can be used to analyze the geometry near the black hole horizon as well as to study the quantum correlations between the particles-antiparticles near the black hole. This can be further extended with time-dependent electromagnetic fields. Also, we wish to extend this work to study the correlation between different sectors of an initially entangled state constructed by two fields (which can be differentiated by mass, charge, quantum number).

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**A Explicit form of the mode functions and normalizations**

\[
U_{1,n}(x)_{H_R} = \frac{1}{N_1} e^{\frac{\pi i}{4} \frac{qE}{m} \left( \frac{\epsilon_1}{e^{ax}} \partial_t - \frac{\epsilon_4}{e^{ax}} \partial_x \right)} e^{-\frac{qE}{m} \left( \frac{\epsilon_2}{2a} \partial_t - \frac{\epsilon_3}{2a} \partial_x \right)} e^{\frac{qE}{m} \left( \frac{\epsilon_4}{2a} + qBye^{ax} + m\epsilon_4 \right)}
\times e^{-i\omega (t-x)} e^{-ik_z e^{ax} e^{-\frac{qE e^{ax}}{m}} U(\lambda_1, \nu, \xi) H_1(y)}
\]

(58)
\[ U_{1,n}(x)_{I_R} = \frac{1}{M_1 e^{\frac{\beta}{2}}} \left( \frac{ie_4}{e^{ax}} \partial_t - \frac{ie_4}{e^{ax}} \partial_x - e_2 \partial_2 - i e_2 \partial_3 - \frac{qEe^{ax}}{2a} e_4 + qBye_2 + mc_1 \right) \]

\[ U_{2,n}(x)_{H_R} = \frac{1}{N_2 e^{\frac{\beta}{2}}} \left( \frac{ie_3}{e^{ax}} \partial_t - \frac{ie_3}{e^{ax}} \partial_x - e_1 \partial_2 - i e_1 \partial_3 - \frac{qEe^{ax}}{2a} e_3 + qBye_1 + mc_2 \right) \]

\[ U_{2,n}(x)_{I_R} = \frac{1}{M_2 e^{\frac{\beta}{2}}} \left( \frac{ie_3}{e^{ax}} \partial_t - \frac{ie_3}{e^{ax}} \partial_x - e_1 \partial_2 - i e_1 \partial_3 - \frac{qEe^{ax}}{2a} e_3 + qBye_1 + mc_2 \right) \]

\[ V_{1,n}(x)_{H_R} = \frac{1}{P_1 e^{\frac{\beta}{2}}} \left( - \frac{ie_2}{e^{ax}} \partial_t + \frac{ie_2}{e^{ax}} \partial_x - e_4 \partial_2 - i e_4 \partial_3 + \frac{qEe^{ax}}{2a} e_2 + qBye_4 + mc_3 \right) \]

\[ V_{2,n}(x)_{H_R} = \frac{1}{P_2 e^{\frac{\beta}{2}}} \left( - \frac{ie_1}{e^{ax}} \partial_t + \frac{ie_1}{e^{ax}} \partial_x - e_3 \partial_2 - i e_3 \partial_3 + \frac{qEe^{ax}}{2a} e_1 + qBye_3 + mc_4 \right) \]

\[ V_{2,n}(x)_{I_R} = \frac{1}{R_2 e^{\frac{\beta}{2}}} \left( - \frac{ie_1}{e^{ax}} \partial_t + \frac{ie_1}{e^{ax}} \partial_x - e_3 \partial_2 - i e_3 \partial_3 + \frac{qEe^{ax}}{2a} e_1 + qBye_3 + mc_4 \right) \]

\[ \text{here in Eq. (62), Eq. (63), Eq. (64) and Eq. (65) } y_\approx = \left( \sqrt{qBy} - \frac{k}{\sqrt{qB}} \right) \text{ and } H_1(y_\approx) = H_2(y_\approx) = \left( \frac{2\pi}{\sqrt{\pi}(n+1)} \right)^{1/2} e^{-y^2/2} H_n(y_\approx). \]

The normalisation constants, \( N_1, N_2, M_1 \) and \( M_2 \) are given by previous section. We shall explicitly evaluate \( N_1 \) below, for which we choose constant time hypersurface with normal vector \( n_\mu = e^{-ax}\lambda_\mu \)

\[ \int g \gamma(0) \gamma(0) (U_{s,n}(x) H_R)^{-1} U_{s,n}(x) H_R = \frac{1}{|N_s|^2} \int \left[ \left( \frac{\omega}{e^{ax}} - \frac{qE}{2a} - i e^{ax} \left( a + \frac{q^2e^{2ax}}{2a} - i\omega - 2a\lambda_1 \right) \right) H_1'(y) H_1(y) + (\partial_\gamma + y + \sqrt{qB}) H_1'(y)(\partial_\gamma + y + \sqrt{qB}) H_1(y) + m_2^2 \right] \]

\[ \times e^{i(\omega - \omega')(t - x)} e^{i(k_z - k'_z)z} \left( \frac{2a^2}{\sqrt{qB}} \right)^{\lambda'} e^{-2ax} \]

\[ = \frac{1}{|N_s|^2} \left( \frac{a^2}{E^2} e^{-\pi a^2 \frac{z^2 + b^2}{4a^2}} \right) \delta_{nn'} \delta(\omega - \omega') \delta(k_z - k'_z) \]

Here we have used the asymptotic limit of \( U(\lambda, \nu, \xi) \) function at \( x \to \infty (|\xi| \to \infty) \), i.e. \( U(\lambda, \nu, \xi) \approx \xi^{-\lambda} \).

Note that the normalization of \( U_{s,n}(x)_{I_R} \) can be done in the same way as of \( U_{s,n}(x)_{H_R} \) for which we have
used the asymptotic form of $L(\lambda, \nu, \xi)$ at $x \to -\infty$ ($|\xi| \to 0$), i.e. $L(\lambda, \nu, \xi) = \frac{\Gamma(\nu+1)}{\Gamma(\nu+1)}$ which gives $M_s$, likewise we can normalize all other in modes. Set of out modes are given as follows

\[
U_{1,n}(x)_{H^+} = \frac{1}{N_1 e^{\frac{\pi}{4}}}( - \frac{i\epsilon_4}{e^{ax}} \partial_t + \frac{i\epsilon_4}{e^{ax}} \partial_x - \epsilon_2 \partial_2 + i\epsilon_2 \partial_3 - \frac{qe^{ax}}{2a} \epsilon_4 + qB\epsilon_2 + m\epsilon_1 ) \times e^{-i\omega(t+x)}e^{-ik_z e^{ax}}e^{-\frac{iq\epsilon_{2ax}}{4a^2}}(U(\lambda_1, \nu, \xi))^*H_1(y)
\]

(66)

\[
U_{1,n}(x)_{I^+} = \frac{1}{M_1 e^{\frac{\pi}{4}}}( - \frac{i\epsilon_4}{e^{ax}} \partial_t + \frac{i\epsilon_4}{e^{ax}} \partial_x - \epsilon_2 \partial_2 + i\epsilon_2 \partial_3 - \frac{qe^{ax}}{2a} \epsilon_4 + qB\epsilon_2 + m\epsilon_1 ) \times e^{-i\omega(t-x)}e^{-ik_z e^{ax}}e^{-\frac{iq\epsilon_{2ax}}{4a^2}}L\lambda_1, \nu - 1, \xi)H_1(y)
\]

(67)

\[
U_{2,n}(x)_{H^+} = \frac{1}{N_2 e^{\frac{\pi}{4}}}( - \frac{i\epsilon_3}{e^{ax}} \partial_t + \frac{i\epsilon_3}{e^{ax}} \partial_x - \epsilon_1 \partial_2 + i\epsilon_1 \partial_3 - \frac{qe^{ax}}{2a} \epsilon_3 + qB\epsilon_4 + m\epsilon_2 ) \times e^{-i\omega(t+x)}e^{-ik_z e^{ax}}e^{-\frac{iq\epsilon_{2ax}}{4a^2}}(U(\lambda_2, \nu, \xi))^*H_2(y)
\]

(68)

\[
U_{2,n}(x)_{I^+} = \frac{1}{M_2 e^{\frac{\pi}{4}}}( - \frac{i\epsilon_3}{e^{ax}} \partial_t + \frac{i\epsilon_3}{e^{ax}} \partial_x - \epsilon_1 \partial_2 + i\epsilon_1 \partial_3 - \frac{qe^{ax}}{2a} \epsilon_3 + qB\epsilon_4 + m\epsilon_2 ) \times e^{-i\omega(t-x)}e^{-ik_z e^{ax}}e^{-\frac{iq\epsilon_{2ax}}{4a^2}}L\lambda_2, \nu - 1, \xi)H_2(y)
\]

(69)

\[
V_{1,n}(x)_{H^+} = \frac{1}{P_1 e^{\frac{\pi}{4}}}( \frac{i\epsilon_2}{e^{ax}} \partial_t - \frac{i\epsilon_2}{e^{ax}} \partial_x - \epsilon_4 \partial_2 + i\epsilon_4 \partial_3 + \frac{qe^{ax}}{2a} \epsilon_2 + qB\epsilon_4 + m\epsilon_3 ) \times e^{-i\omega(t+x)}e^{ik_z e^{ax}}e^{-\frac{iq\epsilon_{2ax}}{4a^2}}(e^{U(\nu - \lambda_1, \nu, \xi))^*H_1(y-)}
\]

(70)

\[
V_{1,n}(x)_{I^+} = \frac{1}{R_1 e^{\frac{\pi}{4}}}( \frac{i\epsilon_2}{e^{ax}} \partial_t - \frac{i\epsilon_2}{e^{ax}} \partial_x - \epsilon_4 \partial_2 + i\epsilon_4 \partial_3 + \frac{qe^{ax}}{2a} \epsilon_2 + qB\epsilon_4 + m\epsilon_3 ) \times e^{i\omega(t-x)}e^{ik_z e^{ax}}e^{-\frac{iq\epsilon_{2ax}}{4a^2}}\xi^{1-\nu}L\nu - \lambda_1 - 1, 1 - \nu, \xi)H_1(y-)
\]

(71)

\[
V_{2,n}(x)_{H^+} = \frac{1}{P_2 e^{\frac{\pi}{4}}}( \frac{i\epsilon_1}{e^{ax}} \partial_t - \frac{i\epsilon_1}{e^{ax}} \partial_x - \epsilon_3 \partial_2 + i\epsilon_3 \partial_3 + \frac{qe^{ax}}{2a} \epsilon_1 + qB\epsilon_3 + m\epsilon_4 ) \times e^{-i\omega(t+x)}e^{ik_z e^{ax}}e^{-\frac{iq\epsilon_{2ax}}{4a^2}}(e^{U(\nu - \lambda_2, \nu, \xi))^*H_2(y-)}
\]

(72)

\[
V_{2,n}(x)_{I^+} = \frac{1}{R_2 e^{\frac{\pi}{4}}}( \frac{i\epsilon_1}{e^{ax}} \partial_t - \frac{i\epsilon_1}{e^{ax}} \partial_x - \epsilon_3 \partial_2 + i\epsilon_3 \partial_3 + \frac{qe^{ax}}{2a} \epsilon_1 + qB\epsilon_3 + m\epsilon_4 ) \times e^{i\omega(t-x)}e^{ik_z e^{ax}}e^{-\frac{iq\epsilon_{2ax}}{4a^2}}\xi^{1-\nu}L\nu - \lambda_2 - 1, 1 - \nu, \xi)H_2(y-)
\]

(73)

Remaining modes for $L$ region are as follows

\[
U_{1,n}(xL)_{H^+} = \frac{1}{N_1 e^{\frac{\pi}{4}}}( \frac{i\epsilon_4}{e^{ax}} \partial_t - \frac{i\epsilon_4}{e^{ax}} \partial_x - \epsilon_2 \partial_2 + i\epsilon_2 \partial_3 - \frac{qe^{axL}}{2a} \epsilon_4 + qB\epsilon_2 + m\epsilon_1 ) \times e^{-i\omega(tL-xL)}e^{ik_z e^{axL}}e^{-\frac{iq\epsilon_{2axL}}{4a^2}}U(\lambda_1, \nu, \xi)LH_1(y)
\]

(74)

\[
U_{1,n}(xL)_{I^+} = \frac{1}{M_1 e^{\frac{\pi}{4}}}( \frac{i\epsilon_4}{e^{ax}} \partial_t - \frac{i\epsilon_4}{e^{ax}} \partial_x - \epsilon_2 \partial_2 + i\epsilon_2 \partial_3 - \frac{qe^{axL}}{2a} \epsilon_4 + qB\epsilon_2 + m\epsilon_1 ) \times e^{-i\omega(tL+xL)}e^{ik_z e^{axL}}e^{-\frac{iq\epsilon_{2axL}}{4a^2}}(L(\nu - \lambda_1, \nu - 1, \xi)L)^*H_1(y)
\]

(75)
\[ U_{2,n}(x_L)_{H}^{+} = \frac{1}{N_s e^{\frac{\pi}{2}}} \left( \frac{ie_3}{e^{ax_L}} \partial_{x_L} - \frac{ie_3}{e^{ax_L}} \partial_x - e_1 \partial_2 - ie_1 \partial_3 - \frac{q E e^{ax_L}}{2a} \epsilon_3 + q B \epsilon_1 + m_2 \right) \times e^{-i\omega(t-x)L} e^{ik_z e^{ax_L} e^{-iqE e^{ax_L}} U(\lambda_2, \nu, \xi)L} H_2(y) \]  

(76)

\[ U_{2,n}(x_L)_{I}^{+} = \frac{1}{M_s e^{\frac{\pi}{2}}} \left( \frac{ie_3}{e^{ax_L}} \partial_{x_L} - \frac{ie_3}{e^{ax_L}} \partial_x - e_1 \partial_2 - ie_1 \partial_3 - \frac{q E e^{ax_L}}{2a} \epsilon_3 + q B \epsilon_1 + m_2 \right) \times e^{-i\omega(t+x)L} e^{ik_z e^{ax_L} e^{-iqE e^{ax_L}} (L(-\lambda_1, \nu - 1, \xi))^* H_2(y) \]  

(77)

\[ V_{1,n}(x_L)_{H}^{+} = \frac{1}{P_s e^{\frac{\pi}{2}}} \left( - \frac{ie_2}{e^{ax_L}} \partial_{x_L} + \frac{ie_2}{e^{ax_L}} \partial_x - e_4 \partial_2 - ie_4 \partial_3 + \frac{q E e^{ax_L}}{2a} \epsilon_2 + q B \epsilon_4 + m_3 \right) \times e^{i\omega(t-x)L} e^{-ik_z e^{ax_L} e^{-iqE e^{ax_L}} e^{i\frac{\pi}{4}\nu} U(\nu - \lambda_1, \nu, \xi)L} H_1(y^-) \]  

(78)

The normalization constants are \( N_s = e^{-\frac{\pi}{2}} \sqrt{\frac{\pi}{a}} \left( \frac{\sinh \pi \mu}{\cosh \pi (\mu - \frac{1}{2}) \mu + e^{-\pi \mu} \sinh \pi \mu} \right)^{\frac{1}{2}} \), \( M_s = e^{-\frac{\pi}{2}} \sqrt{\frac{\pi}{\mu}} \), \( P_s = \left( \frac{n_0+1}{e^B + m^2} \right)^{\frac{1}{2}} \) and \( R_s = \sqrt{\pi} \).

\section*{B Limits of modes near null infinities}

\[ U_{s,n}(x_L)_{H}^{+} = \frac{1}{N_s e^{\frac{\pi}{2}}} \left( \frac{ie_3}{e^{ax_L}} \partial_{x_L} - q A \partial_{x_L} + \frac{q E e^{ax_L}}{2a} \epsilon_3 + q B \epsilon_1 + m_2 \right) \times e^{-i\omega(t-x)L} e^{-ik_z e^{ax_L} e^{-iqE e^{ax_L}} \left( - \frac{i E}{2a} - \frac{e^{-\pi a^2/\mu}}{\pi} \right)} H_s(y) \]  

(82)

\[ V_{s,n}(x_L)_{H}^{+} = \frac{1}{P_s e^{\frac{\pi}{2}}} \left( \frac{ie_3}{e^{ax_L}} \partial_{x_L} - q A \partial_{x_L} + \frac{q E e^{ax_L}}{2a} \epsilon_3 + q B \epsilon_1 + m_2 \right) \times e^{i\omega(t+x)L} e^{ik_z e^{ax_L} e^{-iqE e^{ax_L}} \left( - \frac{i E}{2a} + \frac{e^{-\pi a^2/\mu}}{\pi} \right)} H_s(y) \]  

(83)

\[ U_{s,n}(x_L)_{I}^{+} = \frac{1}{M_s e^{\frac{\pi}{2}}} \left( \frac{ie_3}{e^{ax_L}} \partial_{x_L} - q A \partial_{x_L} + \frac{q E e^{ax_L}}{2a} \epsilon_3 + q B \epsilon_1 + m_2 \right) \times e^{-i\omega(t+x)L} e^{ik_z e^{ax_L} e^{-iqE e^{ax_L}} \left( - \frac{i E}{2a} + \frac{e^{-\pi a^2/\mu}}{\pi} \right)} \frac{\Gamma(-\lambda_1^* + \nu^*)}{\Gamma(-\lambda_1 + 1)} H_s(y) \]  

(84)

\[ V_{s,n}(x_L)_{I}^{+} = \frac{1}{R_s e^{\frac{\pi}{2}}} \left( \frac{ie_3}{e^{ax_L}} \partial_{x_L} - q A \partial_{x_L} + \frac{q E e^{ax_L}}{2a} \epsilon_3 + q B \epsilon_1 + m_2 \right) \times e^{i\omega(t+x)L} e^{ik_z e^{ax_L} e^{-iqE e^{ax_L}} \left( - \frac{i E}{2a} + \frac{e^{-\pi a^2/\mu}}{\pi} \right)} \frac{\Gamma(-\lambda_1^* + \nu^*)}{\Gamma(-\lambda_1 + 1)} H_s(y) \]  

(85)
References

[1] G. Ghirardi, L. Marinatto and T. Weber, Entanglement and Properties of Composite Quantum Systems: A Conceptual and Mathematical Analysis, J. Stat. Phys. 108, 49-122 (2002) [arXiv:quant-ph/0109017]

[2] H. Barghathi, C. M. Herdman, A. D. Maestro, R’enyi Generalization of the Accessible Entanglement Entropy, Phys. Rev. Lett. 121, 150501 (2018) [arXiv:1804.01114]

[3] M. A. Nielsen and I. L. Chuang (2010), Quantum Computation and Information Theory (Cambridge university press).

[4] A. Agarwal, D. Karabali and V. Nair, Gauge-invariant Variables and Entanglement Entropy, Phys. Rev. D 96, no.12, 125008 (2017) [arXiv:1701.00014 [hep-th]].

[5] D. Bruss, For a review on entanglement, J. Math. Phys. 43, 4237, (2002) [arXiv:quant-ph/0110078].

[6] M. B. Plenio and S. Virmani, An Introduction to entanglement measures, Quant. Inf. Comput. 7, 1 (2007) [quant-ph/0504163].

[7] H. S. Dhar, A. K. Pal, D. Rakshit, A. S. De and U. Sen, Monogamy of quantum correlations - a review, (2016) [arXiv:1610.01069v1 [quant-ph]].

[8] K. Zyczkowski, P. Horodecki, A. Sanpera and M. Lewenstein, Volume of the set of separable states, Phys. Rev. A 58, 883 (1998) [arXiv:quant-ph/9804024].

[9] G. Vidal and R. F. Werner, Computable measure of entanglement, Phys. Rev. A 65, 032314 (2002) [arXiv:quant-ph/0102117].

[10] X. Wang and M. M. Wilde, α-logarithmic negativity, Phys.Rev.A 102 (2020) 3, 032416 [arXiv:1904.10437 [quant-ph]].

[11] G. Vidal, Entanglement monotones, J. Mod. Opt. 47, 355 (2000) [arXiv:quant-ph/9807077].

[12] R. Horodecki, P. Horodecki, M. Horodecki and K. Horodecki, Quantum entanglement, Rev. Mod. Phys. 81, 865 (2009) [quant-ph/0702225].

[13] J. Gray, L. Banchi, A. Bayat and S. Bose, Machine Learning Assisted Many-Body Entanglement Measurement, Phys. Rev. Lett. 121, 150503 (2018) [arXiv:1709.04923 [quant-ph]].

[14] I. Fuentes-Schuller and R. B. Mann, Alice falls into a black hole: Entanglement in non-inertial frames, Phys. Rev. Lett. 95, 120404 (2005) [arXiv:quant-ph/0410172].

[15] Y. Ling, S. He, W. Qiu and H. Zhang, Quantum Entanglement of Electromagnetic Fields in Non-inertial Reference Frames, J. Phys. A 40:9025-9032,2007 [arXiv:quant-ph/0608209].

[16] M. Shamirzai, B. N. Esfahani and M. Soltani, Tripartite Entanglements in Non-inertial Frames, Int.J.Theor.Phys. 51 (2012) 787-804 [arXiv:1103.0258 [quant-ph]].

[17] E. Martin-Martinez, L. J. Garay and J. Leon, Unveiling quantum entanglement degradation near a Schwarzschild black hole, Phys. Rev. D 82, 064006 (2010) [arXiv:1006.1394 [quant-ph]].
[18] P. Chen and T. Tajima, Testing Unruh Radiation with Ultraintense Lasers, Phys. Rev. Lett. 83, 256 (1999).

[19] R. Schutzhold, G. Schaller, and D. Habs, Tabletop Creation of Entangled Multi-keV Photon Pairs and the Unruh Effect, Phys. Rev. Lett. 100, 091301 (2008).

[20] M. H. Lynch, E. Cohen, Y. Hadad and I. Kaminer (Technion), Experimental Observation of Acceleration-Induced Thermality, Phys. Rev. D 104 (2021) 2, 025015 [arXiv: 1903.00043 [gr-qc]].

[21] W. G. Unruh, Notes on black-hole evaporation, Phys. Rev. D 14 (1976), 870.

[22] L. C. B. Crispino, A. Higuchi, and G. E. A. Matsas, The Unruh effect and its applications, Rev. Mod. Phys. 80, 787 (2008) [arXiv:0710.5373 [gr-qc]].

[23] L. C. N. Santos and C. C. Barros, Fermions in the Rindler spacetime, Int. J. Geom. Meth. Mod. Phys. 16, no. 09 (2019) [arXiv: 1704.00408 [quant-ph]].

[24] W. Rindler, Kruskal Space and the Uniformly Accelerated Frame, Am. J. Phys. 34, 1174 (1966).

[25] W. G. Unruh and R. M. Wald, What happens when an accelerating observer detects a Rindler particle, Phys. Rev. D 29, no. 6, 1047 (1984).

[26] D. Roy, The Unruh thermal spectrum through scalar and fermion tunneling, Phys. Lett. B 681, 185-189, (2009) [arXiv:0908.3149 [hep-th]].

[27] J. S. Schwinger, On gauge invariance and vacuum polarization, Phys. Rev. 82, 664 (1951).

[28] J. Abedi and H. Arfaei, Obstruction of black hole singularity by quantum field theory effects, JHEP 03, 135, (2016) [arXiv: 1506.05844 [gr-qc]].

[29] L. E. Parker and D. J. Toms, Quantum Field Theory in Curved Spacetime: Quantized Field and Gravity, Cambridge Univ. Press (2009).

[30] D. Karabali, S. Kurkcuoglu and V. Nair, Magnetic Field and Curvature Effects on Pair Production II: Vectors and Implications for Chromodynamics, Phys. Rev. D 100, no. 6, 065006 (2019) [arXiv:1905.12391 [hep-th]].

[31] G. W. Gibbons, Vacuum polarization and the spontaneous loss of charge by black holes, Commun. Math. Phys. 44, 245 (1975).

[32] R. Wald, On particle creation by black holes Commun. Math. Phys. 45, 9 (1975).

[33] S. Hawking, Particle Creation by Black Holes, Commun. Math. Phys. 43, 199 (1975).

[34] S. W. Hawking, Black Holes and Thermodynamics, Phys. Rev. D 13 (1976) 191–197.

[35] M. Socolovsky, Rindler Space and Unruh Effect, [arXiv:1304.2833 [gr-qc]].

[36] R. Brout, S. Massar, R. Parentani, Ph. Spindel, A Primer for black hole quantum physics, Phys.Rept. 260 (1995) 329-454 [arXiv:0710.4345 [gr-qc]].

[37] R. Brout, S. Massar, S. Popescu, R. Parentani, P. Spindel, Quantum back reaction on a classical field, Phys.Rev.D 52 (1995) 1119-1133 [arXiv: hep-th/9311019 [hep-th]].
[38] M. R. R. Good and Y. C. Ong, Particle spectrum of the Reissner–Nordström black hole, The European Physical Journal C volume 80, Article number: 1169 (2020)

[39] R. G. Cai, C. M. Chen, S. P. Kim and J. R. Sun, Schwinger effect in near-extremal charged black holes in high dimensions, Phys. Rev. D 101 (2020) 10, 105015 [arXiv: 2004.00735 [hep-th]]

[40] I. Fuentes-Schuller and R. B. Mann, Alice Falls into a Black Hole: Entanglement in Noninertial Frames, Phys. Rev. Lett. 95, 120404 (2005) [arXiv:quant-ph/0410172 [quant-ph]].

[41] P. M. Alsing, I. F. Schuller, R. B. Mann, T. E. Tessier, Entanglement of Dirac fields in non-inertial frames, Phys. Rev. A 74, 032326 (2006) [arXiv:quant-ph/0603269].

[42] E. Martin-Martinez, L. J. Garay and J. Leon, Phys. Rev. D 82, 064028 (2010) [arXiv:1007.2858 [quant-ph]].

[43] B. Richter, Y. Omar Degradation of entanglement between two accelerated parties: Bell states under the Unruh effect, Phys.Rev.A 92, 022334 (2015) [arXiv:1503.07526 [quant-ph]].

[44] E. Martin-Martinez, Luis J. Garay and Juan Leon, Entanglement and quantum clocks in curved spacetime, Phys.Rev.D 82 (2010) 064006.

[45] P. T. Grochowski, K. Lorek and A. Dragan, Two-mode Gaussian quantum states measured by collinearly and noncollinearly accelerating observers, Phys. Rev. D 100, 025007 (2019) [arXiv:1902.03205v3 [quant-ph]].

[46] C. Gabriel and P. Spindel, Quantum Charged Fields in (1+1) Rindler Space, Annals Phys. 284 (2000), 263-335 [arXiv: 9912016 [gr-qc]].

[47] Z. Ebadi and B. Mirza, Entanglement Generation by Electric Field Background, Annals Phys. 351, 363 (2014) [arXiv:1410.3130 [quant-ph]].

[48] Y. Li, Y. Dai and Y. Shi, Pairwise mode entanglement in Schwinger production of particle-antiparticle pairs in an electric field, Phys. Rev. D 95, no.3, 036006 (2017) [arXiv:1612.01716 [hep-th]].

[49] D. C. Dai, State of a particle pair produced by the Schwinger effect is not necessarily a maximally entangled Bell state, Phys. Rev. D 100, no.4, 045015 (2019) [arXiv:1908.01005 [hep-th]].

[50] Y. Li, Q. Mao and Y. Shi, Schwinger effect of a relativistic boson entangled with a qubit, Phys. Rev. A 99, no.3, 032340 (2019) [arXiv:1812.08534 [hep-th]].

[51] S. Bhattacharya, S. Chakraborty, H. Hoshino and S. Kaushal, Background magnetic field and quantum correlations in the Schwinger effect, Phys. Lett. B 811, 135875 (2020)[arXiv:2005.12866 [hep-th]].

[52] M. S. Ali, S. Bhattacharya, S. Chakraborty and S. Kaushal, Fermionic Bell violation in the presence of background electromagnetic fields in the cosmological de Sitter spacetime, (2021)[arXiv:2102.11745v1 [hep-th]].

[53] C. M. Kim and S. P. Kim, Unruh effect and Schwinger pair creation under extreme acceleration by ultraintense lasers, [arXiv: 1712.02477 [hep-ph]].

[54] S. Deser and O. Levin, Accelerated detectors and temperature in (anti-) de Sitter spaces, Class. Quant. Grav. 14, L163 (1997).
[55] S. Bhattacharya, S. Chakrabortty and S. Goyal, Dirac fermion, cosmological event horizons and quantum entanglement, Phys.Rev.D 101 (2020) 8, 085016, arXiv:1912.12272 [hep-th].

[56] S. Bhattacharya, H. Gaur and N. Joshi, Some measures for fermionic entanglement in the cosmological de Sitter spacetime, Phys.Rev.D 102, 045017 (2020) [arXiv: 2006.14212 [hep-th]].

[57] I. Fuentes, R. B. Mann, E. Martin-Martinez and S. Moradi, Entanglement of Dirac fields in an expanding spacetime Phys. Rev. D 82, 045030 (2010) [arXiv:1007.1569 [quant-ph]].

[58] E. Bavarsad, S. P. Kim, C. Stahl, S-S. Xue, Effect of a magnetic field on Schwinger mechanism in de Sitter spacetime, Phys.Rev.D 97, no. 2, 025017 (2018) [arXiv: 1707.03975 [hep-th]].

[59] P. M. Alsing, I. Fuentes-Schuller, R. B. Mann, and T. E. Tessier, Entanglement of Dirac fields in noninertial frames, Phys. Rev. A 74, 032326 (2006) [arXiv:quant-ph/0603269].

[60] B. Richter, K. Lorek, A. Dragan, Y. Omar, Effect of acceleration on localized fermionic Gaussian states: from vacuum entanglement to maximally entangled states, Phys.Rev.D 95, 076004 (2017) [arXiv:quant-ph/0211063].

[61] N. Friis, P. Köhler, E. Martin-Martinez, R. A. Bertlmann, Residual entanglement of accelerated fermions is not nonlocal, Phys.Rev.A 84, 062111 (2011) [arXiv:1107.3235 [quant-ph]].

[62] K. Ueda, A. Higuchi, K. Yamamoto, A. Rohim and Y. Nan Entanglement of the Vacuum between Left, Right, Future, and Past: Dirac spinor in Rindler spaces and Kasner spaces, Phys. Rev. D 103, 125005 (2021) [arXiv:2104.06625 [gr-qc]].

[63] J. Foo and T. C. Ralph, Continuous-variable quantum teleportation with vacuum-entangled Rindler modes, Phys.Rev.D 101 (2020) 8, 085006 [arXiv: 2001.03387 [quant-ph]].

[64] M. Abramowitz and I. Stegun, Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, National Bureau of Standards (USA) (1964).

[65] W. P. Yao, B. Qiao, Z. Xu, H. Zhang, Z. H. Zhao, H. X. Chang, C. T. Zhou, S. P. Zhu, and X. T. He, Transport of relativistic jet in the magnetized intergalactic medium, The Astrophysical Journal 876 (2019) 10.3847 [arXiv:1809.07925 [physics.plasm-ph]].