A investigates the behaviour and stiffness of reinforced concrete slabs subjected to torsion

M C T Nguyen1,2 and P T Pham1,3
1 Faculty of Civil Engineering, Hanoi Architectural University, Km 10 Nguyen Trai Street, Thanh Xuan District, Hanoi, Vietnam
2 Faculty of Engineering and Technology, Quy Nhon University, 170 An Duong Vuong Street, Quy Nhon, Vietnam

Email: phamphutinh@yahoo.com

Abstract. This paper presents an investigation on RC slab under torsion, by both experiment and finite element analysis. The torsion tests were done on three similar square RC slabs with dimensions of 1900×1900×150 mm. The behaviour of slabs at pre-cracking and post-cracking of concrete phases were investigated, via Load-displacement, twisting moment-curvature relationships, and torsional stiffness of slabs. The experimental results are compared with the FEA and the results in literatures. The torsional stiffness of slab at the phase of concrete cracked and steel yield is about 1/25 of the stiffness at the pre-cracking phase.

Keywords: Torsional stiffness, RC slab, FEA.

1. Introduction
Reinforced concrete slabs are widely used in the concrete constructions. In structural analysis, the torsional stiffness of slab is ignored in common. When this stiffness is taken into account, the exact theory of bending of elastic plates shows that the twisting moment relieve the bending moments about 25 percent [1, 2].

Among the researchers on torsional stiffness of RC slabs, there were Nielsen [3], Marti and Kong [4]. Their study showed the analytical solution for torsional stiffness of RC slab at pre-cracking and post-cracking of concrete phases. Gudmund-Høyer [5] has investigated the torsional and bending stiffness of RC slab under flexural and membrane effect. All of these researches presented the formulae, which consists of elastic modulus of concrete and steel, thickness of slab, and steel ratio. At pre-cracking phase, torsional stiffness of RC slab are quite close to torsional stiffness of elastic and homogeneous slab. Huber introduced the torsional stiffness of RC slab based on bending stiffness, but his formula is only suitable for pre-cracking phase, this formula was cited in Timoshenko [1].

By experimental study, Marti et al. [6] had tested nine RC slabs under pure torsional moment in the reinforcement directions. Torsion was introduced by two couples of equal and opposite forces, applied at four corners of specimens. Extensive strain on the specimen surfaces and displacement at slab corners were measured. Crack spacings, crack widths, magnitudes of strains, collapsed mechanisms, nonlinear responses of load-deformation, moment-average strains in reinforcement were shown. The ultimate moments from Marti’s tests were between 5-46% below yield-line theory predictions, between 8-117% greater than American code predictions, 3-41% greater than Canadian code.
predictions, these codes are applied to wide beams.

Lopes et al. [7] based on the test model of Marti with some modifications. Torsion was introduced by one force, applied at one corner of specimen, where other corners were restrained from displacement. Displacements and forces (reactions) at four corners were measured. Load-displacement, twisting moment-curvature relationships, torsional stiffness of slab at two stages, pre-cracking and post-cracking of concrete had been presented. This research showed that torsional stiffness of RC slabs in the cracked phase is about 1/17-1/15 of the stiffness in the elastic phase.

In this work, three similar RC slabs were tested. The test model was based on Lopes’s one, with some modifications. The torsion was introduced as the same manner. Displacements at four points on top surface of each specimen and strain of reinforcing steel were measured. Test results and test interpretations were presented in section 4, and validated by finite element analysis.

2. Experimental program

2.1. Purpose
To study the behaviour of slab under torsion, include occurring and developing of cracks in concrete, stress developing in steel bars, the limit load at some states: start of cracking in concrete, start of yielding in steel, ultimate state, load-displacement, twisting moment-curvature relationship, torsional stiffness of slab at pre-cracking and post-cracking stages.

2.2. Specimens
Three similar square slabs, named S1, S2 and S3, with the dimensions of 1900×1900×150 mm, are tested. The sample slabs are reinforced by two identical reinforcing steel meshes, at top and bottom surface. Within each mesh, the steel bars are arranged isotropically, means they are the same in two directions. To study the stress developing in steel bars, there are eight strain gages were glued to the steel bars, see Figure 1. The locations of strain gages are determined with the help of FEA. The mechanical properties of concrete, by cubic test results at 28 days are: mean value of compressive strength $f_{cm}$ is 38 MPa, modulus of elasticity $E_c$ is 28 GPa, and of steel are: modulus of elasticity $E_s$ is 194 GPa, the yield strength $f_y$ is 468 MPa, the ultimate strength $f_u$ is 562 MPa. Details of the samples are shown in Table 1.

![Strain gages at the top mesh of reinforcement](image1)
![Strain gages at the bottom mesh of reinforcement](image2)
![A picture for more detail](image3)

Figure 1. The location of Strain gages in three specimens.
Table 1. The characteristics of the test specimens.

| Name | Thickness (cm) | Steel Mesh | Age at test (days) | $f_{cm}$ (MPa) |
|------|----------------|------------|-------------------|----------------|
| S1   | 15.4           | $\phi10a$200$(X,Y)$ | 96               | 40.21          |
| S2   | 15.5           | $\phi10a$200$(X,Y)$ | 103              | 41.76          |
| S3   | 15.2           | $\phi10a$200$(X,Y)$ | 107              | 40.83          |

2.3. Experimental procedure

The vertical downward displacement at corners $C3$ and $C4$, and the vertical upward displacement at corner $C2$ are restrained, the test load was applied on corner $C1$, see Figure 2.

The slabs were applied a concentrated force on corner $C1$, the test load is increased from zero until slab collapsed or the electromechanical actuator reaches its maximum displacement, which was set up 75 mm. The capacity of electromechanical actuator is 500 kN. The loading is controlled by displacement, with the velocity is 0.05 mm per second.

Load cell with a maximum capacity of 500 kN and a LVDT was used to measure the force and displacement at the corner $C1$. Four LVDTs are used to measure displacements at points $d1$, $d2$, $d3$, $d4$ on the top surface of slab. Distance between them are 500 mm. All LVDTs, Load cell and Strain gages were connected with a DRA-30A Data Logger to read and record the data from the instruments, see Figure 3. The test results are shown in Figure 7 to 10, in Table 2 and Table 3, in the section 4 of this paper.

Figure 2. General scheme of the test.

Figure 3. General view of the test.
3. Finite element analysis

3.1. Element, mesh and boundary conditions
The numerical simulations have been done for the slab by the help of ANSYS, V15.0. The slab was
discreted by finite elements, where element SOLID65 was applied for concrete, SOLID185 was
applied for steel plates at the corners, and LINK180 was applied for reinforcing bars, and the “discrete”
model was used.

The FE mesh is shown in Figure 4, where, by the thickness, the slab was discreted into four
elements, by two other sides, it was discreted into 22 elements. This mesh was checked to give the
stable results, means the mesh refinement is good enough.

The static and kinematic boundary conditions are the same with the experimental conditions. That
means, the nodes on steel plate at C3 and C4 corners were restrained from downward displacement,
the nodes on steel plate at C2 corner were restrained from upward displacement, and the load was
applied on nodes at corner C1.

![FE mesh](image)

*Figure 4. FE mesh.*

3.2. Material laws
The Kachlakev law was applied for compression concrete [8], expressed by equations from (1), (2)
and (3) [9, 10], and drawn in Figure 5a, and for concrete in tension, the material law was available in
ANSYS [11], see Figure 5b. The elastic perfectly-plastic law was applied for reinforcing steel, Figure
6. Elastic modulus and modulus of rupture of concrete was calculated from equation (4) and (5) [2, 12,
13].

\[
f = \frac{E_c \varepsilon}{1 + \left(\frac{\varepsilon}{\varepsilon_0}\right)^2}
\]

\[
\varepsilon_0 = \frac{2f_c}{E_c}
\]

\[
E_c = \frac{f}{\varepsilon}
\]

\[
E_c = 0.043 \rho^{1.8} \sqrt{f_c}
\]

\[
f_c = K \sqrt{f_c}
\]

where \(K = 8 \div 12\) as suggestion in [2], and \(K = 9 \div 13\) as suggestion in [13]. In this study, we take \(K = 9.5\).

\(f\) is the compression stress in concrete at strain \(\varepsilon\); \(\varepsilon_0, f_c, E_c\) are the ultimate strain, the compression...
strength, and the elastic modulus of concrete respectively, \( f_r \) is rupture modulus, \( \rho \) is the concrete density.

The material properties were collected from our material tests. Since, slab S1, S2, S3 are different age at tested date, but their concrete strength are nearly equal, see Table 1, then the average value of concrete strength was calculated.

![Figure 5a. Concrete law in uniaxial compression [8].](image1)

![Figure 5b. Concrete law in uniaxial tension [11].](image2)

![Figure 6. Reinforcing steel law in compression and tension [13].](image3)

By FEA, position and the moment when steel bars start to yield are easily investigated. The positions where the steel stress reaches yield stress are the same positions from the experiment, where the strain gages were glued. The FEA results are very close to the experiment results, and have been shown in Figure 9 and 10.

4. Results

4.1. Crack, deformed shape, failure mode of the slabs, and stress in the reinforcement

Crack patterns and failure mode from the present experiment and FEA, as well as from Marti’s experiment are very close to each other.

Cracks developed at approximate 45\(^\circ\) to the edges of the slab. At the top surface, the first crack occurred at corners C3 and C4, started from the edge of slab, and propagated into slab centre. Sequential cracks are also started from the edge of slab, parallel to the first one, and propagated in direction 2, see Figure 7a, 7b.

At the bottom surface, the occurring and the propagating of cracks are the same manner to that at the top surface, but starting from corners C1 and C2, instead of C3 and C4, see Figure 7c, 7d.

The slab was collapsed by crushing of compression concrete at the top and bottom surfaces, at corner C3 or C4, Figure 8. By the FEA results, slab was collapsed by crushing of compression concrete at corners C3 and C4 simultaneously, but in this tests, the slabs were collapsed at either corner C3 or C4, since the imperfection and not absolutely symmetry of the specimens.

(a) Cracks at the top surface of slab S2

(b) Cracks at the bottom surface of slab S2
The stresses in reinforcements were calculated from the strain values, which were collected from strain gages, at two states: \( \sigma_{s,cr} \) at the point that concrete start to crack, and \( \sigma_{s,max} \) at ultimate load. These stresses are presented in Table 2. In the table, sign (-) means the strain gage did not work. At ultimate load, all reinforcement bars where strain gages were glued, reached yield, except strain gage number 7 in specimen S2.

### Table 2. Stress in reinforcements.

| Specimen | S1 \( \sigma_{s,cr} \) (MPa) | S1 \( \sigma_{s,max} \) (MPa) | S2 \( \sigma_{s,cr} \) (MPa) | S2 \( \sigma_{s,max} \) (MPa) | S3 \( \sigma_{s,cr} \) (MPa) | S3 \( \sigma_{s,max} \) (MPa) |
|-----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1         | 30.9            | 492.3           | 52.0            | 533.5           | 35.4            | 474.1           |
| 2         | 18.1            | 472.1           | 3.5             | 474.6           | -               | -               |
| 3         | 11.6            | 471.3           | 23.8            | 473.4           | 6.1             | 470.5           |
| 4         | 5.9             | 474.6           | 3.6             | 473.2           | 1.5             | 471.8           |
| 5         | 5.0             | 472.4           | -               | -               | 6.0             | 472.3           |
| 6         | 5.1             | 471.5           | 4.5             | 471.7           | 5.9             | 472.8           |
| 7         | 23.1            | 485.1           | 25.8            | 372.2           | 36.3            | 472.5           |
| 8         | 9.0             | 472.3           | 2.7             | 474.3           | -               | -               |

#### 4.2. Load-displacement and twisting moment-curvature diagram

From the load value at corner \( C1 \) and displacements at four points \( d1, d2, d3, d4 \), twisting moment, \( m_{sys} \), curvature \( \kappa_{sys}, \kappa_{cr} \) will be calculated by equations (6), (7) hereafter.
\[ m_{xy} = \left( \frac{P}{2} + \frac{W}{8} \right) \frac{l}{b} \]  

(6)

\[ \kappa_{xy} = \kappa_{yx} = \frac{\partial \phi}{\partial y} \]  

(7)

where, \( m_{xy} \) is twisting moment per unit length; \( \kappa_{xy}, \kappa_{yx} \) are twisting curvatures in the direction x-y, and y-x respectively; \( P, W \) is the applied load at corner \( C1 \) and self-weight of slab respectively, \( l = 1.75 \) m, \( b = 1.8 \) m are the spans of slab, with the consideration for concrete cover.

From Figure 9 and Figure 10, it can be seen that the test curves of three slabs, are very convergent, and in good comparison with FEA results.

4.3. Torsional stiffness of the slabs

The torsional stiffness of slab in the stage I, \( D_{xy,I} \), and in the stage II, \( D_{xy,II} \), were calculated by equation (8) and (9)

\[ D_{xy,I} = \frac{m_{xy,cr}}{\kappa_{xy,cr}} \]  

(8)

\[ D_{xy,II} = \frac{m_{xy,y} - m_{xy,cr}}{\kappa_{xy,y} - \kappa_{xy,cr}} \]  

(9)

where, \( m_{xy,cr} \) and \( \kappa_{xy,cr} \) is twisting moment and twisting curvature respectively, at the beginning of concrete crack, \( m_{xy,y} \) and \( \kappa_{xy,y} \) is twisting moment and twisting curvature respectively, at the moment of reinforcing steel start to yield.

The torsional stiffness of slab also can be calculated from formulae introduced by Nielsen. The formulae are rewritten in equations (10) and (11), as below. The more detail can be seen in [3].

- Uncracked slab:

\[ D_{xy,I} = \frac{1}{12} E_c h^3 \]  

(10)

- Cracked slab:

\[ D_{xy,II} = E_c h^3 \left( \frac{2}{3} (n\phi)^2 \right)^2 + \frac{1}{4} n\phi \frac{2}{3} (n\phi) \frac{2}{3} \sqrt{(n\phi)^2} + 2n\phi - \frac{2}{3} n\phi \sqrt{(n\phi)^2} + 2n\phi \]  

(11)

where \( \phi = \frac{A}{bh} \) and \( n = \frac{E_c}{E} \).

All results are shown in Table 3.
Table 3. Torsional stiffness of the slab.

| Slab | Present, experiment | Present, FEA | Nielsen [3] |
|------|---------------------|--------------|-------------|
|      | $D_{xy,I}$ (kNm$^2$) | $D_{xy,II}$ (kNm$^2$) | $D_{xy,I}$ (kNm$^2$) | $D_{xy,II}$ (kNm$^2$) |
| S1   | 7891                | 309          | 25.6        | 8231        | 273        | 30.1      |
| S2   | 7470                | 288          | 26.0        | 7455        | 297        | 25.1      | 8388        | 275        | 30.5      |
| S3   | 7718                | 326          | 23.7        | 8294        | 274        | 30.3      |

Table 3 shows that the present results from experiment, FEA, are very closed with the Nielsen’s solutions.

5. Conclusion

The paper presents an investigation on the behaviour and torsional stiffness of reinforced concrete slab under torsion, by both experiment and finite element analysis. The test results, FEA results and results in literature are close to each other.

Cracks occur firstly on the top surface, then on the bottom surface. Cracks on the top surface and cracks on the bottom surface are orthogonal.

At ultimate load, all reinforcement bars where strain gages were glued, reached yield.

Torsional stiffness of RC slabs in the cracked phase is about 1/25 of the stiffness in the elastic phase for the steel ratio is 0.0032.

References

[1] Timoshenko S and Woinowsky-Krieger S 1959 Theory of Plates and Shells 2nd Ed. (New York: McGraw-Hill Companies) pp 118-120
[2] Nilson A H, Darwin D and Dolan C W 2010 Design of Concrete Structures 14th Ed. (New York: McGraw-Hill Companies) pp 432-434
[3] Nielsen N J 1920 Beregninger af Spændinger i Plader (København: Doktorafhandling ved Den Polytekniske Læreanstalt)
[4] Matri P and Kong K 1987 Response of reinforced concrete slab elements to torsion Journal of Structural Engineering 113(5) 976-993
[5] Gudmand-Høyer T 2004 Stiffness of Concrete Slabs Ph.D Thesis Technical University of Denmark 4 pp 1-57
[6] Marti P, Leesti P and Khalifa W U 1987 Torsion test on reinforced concrete slab elements Journal of Structural Engineering 113(5) 994-1010
[7] Lopes A V, Lopes S M R and Carmo R N F D 2014 Stiffness of reinforced concrete slabs subjected to torsion Materials and Structures 47 227-238
[8] Kachlakiev D, Miller T, Yim S, Chansawat K and Potisuk T 2001 Finite element modeling of reinforced concrete structures strengthened with frp laminates Research Project Work Plan SPR 316 (Washington: Oregon Department of Transportation) pp 11-12
[9] Desayi P and Krishnan S 1964 Equation for the stress-strain curve of concrete Journal of the American Concrete Institute 61(3) 345-350
[10] Gere J M 2004 Mechanics of Materials 6th Ed. (New York: McGraw-Hill) pp 23-24
[11] ANSYS 2013 Inc Theory reference Release 15.0 Documentation for ANSYS
[12] AS 3600-2009 2009 Australian Standard: Concrete Structures (Sydney: The Council of Standards Australia) p 37
[13] Park R and Paulay T 1974 Reinforced Concrete Structures (New York: Wiley-Interscience Publication) p 16