Bayesian Optimisation for Premise Selection in Automated Theorem Proving (Student Abstract)

Agnieszka Słowik, Chaitanya Mangla, Mateja Jamnik, Sean B. Holden, Lawrence C. Paulson
University of Cambridge, Department of Computer Science and Technology
William Gates Building, 15 JJ Thomson Ave, Cambridge CB3 0FD, UK
agnieszka.slowik@cl.cam.ac.uk

Abstract
Modern theorem provers utilise a wide array of heuristics to control the search space explosion, thereby requiring optimisation of a large set of parameters. An exhaustive search in this multi-dimensional parameter space is intractable in most cases, yet the performance of the provers is highly dependent on the parameter assignment. In this work, we introduce a principled probabilistic framework for heuristic optimisation in theorem provers. We present results using a heuristic for premise selection and the Archive of Formal Proofs (AFP) as a case study.

Introduction
Theorem provers use heuristics at various points in their operation, such as in search control and premise selection. These heuristics often have parameters that greatly influence the practical performance of a prover. Existing approaches to selecting such parameters require human supervision, rules of thumb or extensive testing (Hoder and Voronkov 2011). Such testing is often conducted on large theory sets, and is thus computationally expensive. For instance, Open CYC (Matuszek, Cabral, and Wirbrock 2006) contains over 3 million axioms while each of the problems has a proof involving up to 20 premises. An alternative to the exhaustive search is to sparsely navigate the multi-dimensional space of parameters. We argue that probabilistic search enables efficient and automated optimisation of parameterised heuristics in theorem proving.

Here, we explore Bayesian Optimisation (Mõckus 1975) with Gaussian Processes (GPs) (Rasmussen and Williams 2005) as a general solution to efficient heuristics tuning in automated theorem proving. We conduct a case study in premise selection using a state-of-the-art heuristic Sumo Inference Engine (SInE) (Hoder and Voronkov 2011). Our framework based on GPs takes at most nine minutes to find the optimal set of parameters in ten AFP articles. The premises recommended by the optimised SInE were sufficient to prove 85.3% of the conjectures using Sledgehammer (Böhme and Nipkow 2010).

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Thank you.
Table 1: Premise selection results on the AFP articles.

| AFP article          | Nr of goals | Proofs found [%] | Time [s] | Optimal parameters |
|----------------------|-------------|------------------|----------|--------------------|
| Polynomials          | 135         | 87%              | 57s      | t: 16.3, g: 58, k: 131 |
| AbstractHoareLogics  | 793         | 63%              | 249s     | t: 17.6, g: 57, k: 130 |
| Completeness         | 475         | 89%              | 151s     | t: 18.9, g: 63, k: 134 |
| FinFun               | 263         | 95%              | 73s      | t: 19.6, g: 57, k: 132 |
| HeardOf              | 716         | 93%              | 331s     | t: 19.5, g: 57, k: 131 |
| InductiveConfidentiality | 1425       | 82%              | 451s     | t: 19.6, g: 58, k: 130 |
| RefineMonadic        | 1509        | 95%              | 522s     | t: 14.7, g: 64, k: 123 |
| MiniML               | 345         | 84%              | 104s     | t: 19.1, g: 58, k: 131 |
| RecursionTheory      | 656         | 85%              | 205s     | t: 19, g: 57, k: 130  |
| SortEncodings        | 776         | 80%              | 437s     | t: 14, g: 64, k: 123  |

Experiments

Dataset

AFP (Jaskelioff and Merz 2005) is a collection of proofs formalised in Isabelle (Nipkow, Wenzel, and Paulson 2002). We used a parsed version of the dataset that meets the input requirements of MaSh (Kühlwein et al. 2013), the machine learning premise selector currently implemented in Isabelle. Here, we report the results on 10 articles containing various theories and of sizes ranging from around 100 to around 1500 conjectures. Each conjecture was paired with a history of premises extracted from Sledgehammer logs that were used to determine which lemmas are needed to prove a goal.

Evaluation

In premise selection, it is acceptable to provide more premises than necessary to prove a conjecture in order to minimise the risk of missing a key lemma. However, the main purpose of filtering is to lower the cost of considering irrelevant lemmas, and so an efficient algorithm should minimise the number of unnecessary recommendations. To let this trade-off guide the optimisation process, we use a metric based on precision and recall. At the testing stage we evaluate the algorithm based on the number of conjectures that would be proved in practice by Sledgehammer using the premises recommended by SnE. We assume that all of the premises used by Sledgehammer are necessary to prove the conjecture whereas in practice the prover might be able to find an alternative solution that requires a different set of premises. Consequently, this testing metric will tend to underestimate the number of conjectures proved using the SnE recommendations.

Analysis

Preliminary results (see Table 1) suggest that the framework is efficient in finding the optimal parameter combination across different theories. This allows us to explore a wider range of parameters and produce an offline heuristic recommendation to a theorem prover.

Future Work

A possible future direction involves reproducing our premise selection experiments on a larger set of conjectures, and applying the Bayesian Optimisation framework to another bottleneck in automated theorem proving, for example strategy scheduling.

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