Effects of Planetary Dynamics on the Formation of Terrestrial Planets

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Abstract. Formation of terrestrial planets by agglomeration of planetesimals in protoplanetary disks sensitively depends on the velocity evolution of planetesimals. We describe a novel semi-analytical approach to the treatment of planetesimal dynamics incorporating the gravitational scattering by massive protoplanetary bodies. Using this method we confirm that planets grow very slowly in the outer Solar System if gravitational scattering is the only process determining planetesimal velocities, making it hard for giant planets to acquire their massive gaseous envelopes within \( \lesssim 10^7 \) yr. We put forward several possibilities for alleviating this problem.

1. Introduction

Current paradigm of planetary origin (Ruden 1999) assumes that terrestrial planets have formed in protoplanetary nebulae out of swarms of planetesimals — rocky or icy bodies with initial sizes of several kilometers. The same process is thought to account for the growth of solid cores of giant planets in the core instability scenario which postulates that huge gaseous envelopes of gas giants were acquired as a result of instability-driven gas accretion on preexisting cores made of solids (Mizuno 1980).

Our understanding of planetesimal accretion dates back to pioneering works by Safronov (1969) who (1) proposed to use the methods of kinetic theory for investigating the behavior of large number of planetesimals, and (2) included planetesimal dynamics in the picture of their gravitational agglomeration. Gravitational scattering between planetesimals tends to excite their random motions increasing the velocities with which they approach each other. This can have an important effect on their merging because of the phenomenon of gravitational focusing — an enhancement of collision cross-section of two bodies through the deflection of their orbits caused by their mutual gravitational interaction. Gravitational focusing increases collision cross-section by a factor \( 1 + \frac{v_{\text{esc}}^2}{v_{\text{rel}}^2} \) over its geometrical value \( \pi(R_1 + R_2)^2 \), where \( R_{1,2} \) are the physical radii of colliding planetesimals, \( v_{\text{esc}} \) is their mutual escape velocity, and \( v_{\text{rel}} \) is the relative velocity of planetesimals at infinity.

From this formula it can be seen that gravitational focusing is important only provided that \( v_{\text{rel}} \) is significantly below \( v_{\text{esc}} \). Safronov’s original assumption (1969) was that the biggest bodies in the system would be able to quickly increase the velocities of surrounding small-mass planetesimals to \( v_{\text{esc}} \) thus rendering...
further accretion of planetesimals by these massive protoplanets inefficient. As a result, typical timescale for forming the Earth at 1 AU from the Sun is very long — about $10^8 - 10^9$ yr. This timescale rapidly increases as one goes further out in the Solar System and reaches $\sim 10^{11}$ yr at 10 AU from the Sun (roughly present location of Saturn). This timescale is in stark contrast with the age of the Solar System (about 4.5 Gyr) implying that the Safronov’s assumption of $v_{rel} \sim v_{esc}$ is faulty.

Wetherill & Stewart (1989) pointed out that at least initially planetesimal velocities in protoplanetary disks are not that large ($v_{rel} \ll v_{esc}$) and are moderated by mutual planetesimal scattering rather than by a small number of very massive bodies (which contain too little mass). They showed that in this case planetesimal accretion by massive bodies proceeds in a self-accelerating manner when most massive objects exhibit fastest growth; as a result, a single massive object detaches itself from the continuous mass spectrum of planetesimals. This so called “runaway” accretion allows Moon or Mars sized objects to appear on rather short timescale (typically $10^4 - 10^5$ yr) in the terrestrial zone. It also seemed to have rescinded the timescale problem for the giant planets by enabling their solid cores to grow within the gaseous nebula lifetime of several Myr at 5 – 10 AU from the Sun thus allowing them to accrete gas.

The runaway growth scenario was challenged by Ida & Makino (1993) who demonstrated using N-body simulations that massive protoplanetary “embryos” are in fact able to locally couple dynamically to the planetesimal disk after reaching some threshold mass. The major results of their study were that (1) massive embryo can strongly “heat up” planetesimal velocities within several Hill radii of its orbit (dynamically “heated zone”), and (2) embryo tends to repel planetesimal orbits away from its own orbit thus decreasing the surface density of small bodies at its location. The former effect decreases the role of gravitational focusing while the latter lowers the amount of mass which can be accreted by the massive body. Both of them act to reduce the accretion rate of the embryo and this stops its rapid runaway growth. This accretion regime was termed “oligarchic growth” since in this picture one embryo would reign inside its own heated zone, while there can be many such embryos (and their corresponding heated zones) growing within the local patch of the disk.

Straightforward N-body simulations are neither very well suited for determining the threshold mass at which transition from runaway to oligarchic growth occurs as a function of planetesimal disk properties, nor they can follow the evolution of the system for long enough. Although they can treat gravitational interactions between planetesimals and the embryo directly, without simplifications, they are too time consuming and not very flexible. Thus it is important to come up with alternative approaches which would be better suited for treating this important problem.

2. Planetesimal dynamics in the vicinity of protoplanetary embryo

Given the large number of planetesimals present in protoplanetary disks it is natural to employ the statistical approach in studying their dynamics. At the same time, presence of inhomogeneities in the planetesimal disk induced by embryo’s
Figure 1. Planetesimal disk evolution driven by the presence of a protoplanetary embryo. The plots contain numerical (left row) and analytical (right row) time sequences of profiles of $\sigma_e (a,b)$, $\sigma_i (c,d)$, and dimensionless surface density normalized by its value at infinity $(e,f)$. See text for details [from Rafikov (2003b)].

Gravity calls for inclusion of spatial dimension into consideration, something which conventional one-zone coagulation simulation are lacking.

Rafikov (2003a, 2003b) came up with an analytical statistical prescription for treating planetesimal-planetesimal and embryo-planetesimal gravitational interactions. He assumed that the distribution function of planetesimal velocities in the disk has a Schwarzschild form, which allows one to considerably simplify the collision operator in the Boltzmann equation (describing the evolution caused by gravitational perturbations). In this approximation, planetesimal disk is unambiguously described by just three quantities for each mass population — surface number density $N$, dispersion of eccentricities $\sigma_e$, and dispersion of inclinations $\sigma_i$ — which are the functions of radial coordinate in the disk (all these quantities are assumed to be azimuthally averaged). A set of three integro-differential equations self-consistently describes the evolution of these quantities in time and space as mutual planetesimal perturbations and gravitational scattering by massive protoplanetary embryos cause them to evolve.

Equations significantly simplify in two regimes: (1) shear-dominated, when the planetesimal random velocities are small compared to the differential shear in the disk across the corresponding Hill radius, and (2) dispersion-dominated,
when these velocities are larger than the shear across Hill radius. First case can only be appropriate for the embryo-planetesimal interactions. When it is realized, scattering of planetesimals has a deterministic character which substantially simplifies its treatment. In the dispersion-dominated regime, appropriate for planetesimal-planetesimal scattering and in most cases for embryo-planetesimal scattering, Fokker-Planck expansion can be performed on the collision operator and evolution equations reduce to a set of partial differential equations. Intermediate regime, when planetesimal random velocities are comparable to the shear across the Hill radius can be treated by interpolation between the two limiting cases.

To check the performance of this approach we have compared the results of its application to studying the embryo-planetesimal scattering in different velocity regimes with the outcome of numerical orbit integrations (local) of the same problem (Rafikov 2003b). Representative results for the evolution of $\sigma_e, \sigma_i$ and $N$ in the dispersion-dominated case (initially $\sigma_e = \sigma_i = 3$ in Hill units) are shown in Figure 1. Curves of different colors represent different moments of time (displayed in panels (e) and (f) by corresponding color coding). One can see that analytical theory (right panels) is in excellent agreement with numerical simulations (left panels) — they follow each other with considerable quantitative accuracy even in minor details. This makes one confident that semi-analytical approach of Rafikov (2003a, 2003b) is quite robust. It is numerically inexpensive: analytical calculations displayed in Figure 1 took about 1 minute to run on a conventional desktop, while the orbit integrations (following several $10^5$ particles for several hundred approaches to the embryo to achieve enough accuracy) required about 1 month on the same hardware. Planetesimal-planetesimal scattering was neglected in this calculation (reasonable assumption for large embryo masses), and only a single mass planetesimal population was considered. One can clearly see both the local excitation of planetesimal velocities and the development of a gap in the surface density of planetesimal orbits near the embryo, in agreement with previous N-body simulations.

This semi-analytical approach thus represents powerful and efficient tool for studying the dynamics of planetesimal disks. It can be easily extended to include the planetesimal-planetesimal scattering (see below), planetesimal mass spectrum and its evolution, dynamics of several embryos, dissipative effects, etc.

3. Growth of an isolated embryo

As a particular application of this approach we have studied the growth of an isolated embryo in a single mass planetesimal disk evolving dynamically and spatially under the action of both embryo-planetesimal and planetesimal-planetesimal gravitational interactions (Rafikov 2003c). Planetesimal-planetesimal scattering acts as an effective viscosity in the planetesimal disk opposing the embryo’s tendency of clearing a gap. Masses of planetesimals in a disk are assumed not to vary, a simplification justified when embryo’s mass increases faster than planetesimals grow. This would be the case in the runaway growth regime, as well as in the oligarchic case (Ida & Makino 1993). At the same time, embryo’s mass was allowed to increase by accreting small planetesimals. Accretion rate was calculated analytically taking into account inhomogeneity of planetesimal
Figure 2. Growth of the embryo’s mass in the Jupiter region of proto-Solar nebula for different initial conditions. See text for details [from Rafikov (2003c)].

Several calculations for different initial starting embryo masses and planetesimal velocities were carried out and embryo’s mass as a function of time is displayed in Figure 2. Properties of the planetesimal disk are those to be expected at 3.6 AU with planetesimal mass set to $6 \times 10^{20}$ g ($M$ is embryo’s mass scaled by the planetesimal mass, $\tau$ is time scaled by the synodic period of planetesimals separated by a Hill radius; corresponding physical values are displayed on the right and upper axes). Thin solid lines display the mass evolution tracks which embryo follows if its dynamical effect on its surroundings is neglected. As expected, these tracks exhibit unimpeded runaway growth and embryo reaches very large mass on a rather short timescale of $\sim 10^6$ yr. Thick lines represent tracks with the same initial conditions but with embryo’s local perturbations taken into account. One can see that they initially follow the runaway tracks (solid portions). This is the result of small embryo’s mass, which makes it incapable of perturbing planetesimals around it: their velocity dispersions increase independently of embryo’s growth and planetesimal-planetesimal scattering is strong enough to smooth any inhomogeneities around the embryo. However, when embryo grows beyond some threshold mass ($\sim 10^{24}$ g in this case) it takes over the control of planetesimal dynamics around it (dashed portions of thick curves): planetesimals are being pushed away from the embryo’s orbit, clearing a gap (similar to the simple calculation described in §2), and their velocities increase in accord with the embryo’s growth. As a result, rapid runaway growth changes to a slower power-law increase of embryo’s mass with time, roughly linearly with $t$. Thus, it would require a considerably longer timescale (by a factor of $\sim 10$) to reach $1M_\oplus$ than simple runaway picture would predict. At 5 AU
from the Sun this would stretch the formation timescale of giant planet cores to \( \sim 10^8 \) yr which is unacceptable from the point of view of core instability scenario of giant planet formation given short (\( \lesssim 10^7 \) yr) lifetimes of gaseous nebulae.

4. Discussion

Simple problem described above clearly demonstrates the difficulty (encountered by conventional scenarios of planet formation) of producing solid cores of giant planets in the outer Solar System on reasonable timescales. The primary reason for this is the strong dynamical coupling between massive protoplanetary bodies and surrounding planetesimals, which causes their gravitational focusing to decrease with time making accretion less and less efficient. In conclusion we want to suggest several possibilities for curing this problem.

Embryos likely not have evolved in complete isolation — as they grow in mass their heated zones overlap and they start affecting each other’s environment. This would likely reduce the tendency for gap formation around embryo orbits, keeping planetesimal disks homogeneous enough to provide the steady supply of planetesimals.

Dissipative processes such as gas drag and inelastic collisions between planetesimals counteract the tendency of planetesimal velocities to increase under the action of embryo’s perturbations. And one does expect gas to be naturally present during the formation of solid cores of gas giants (and initial stages of core formation of ice giants). This damping would not allow embryos to go back to runaway growth, but it would still let them grow faster than if gravity were the only force affecting planetesimal velocities.

Fragmentation of planetesimals in energetic collisions can grind them down to small sizes in the vicinity of massive bodies. Planetesimals would then be strongly affected by dissipative processes and their velocities could be considerably reduced allowing embryos to grow faster.

Closer look at these processes would hopefully help us in resolving the issue of planet formation timescale in the outer Solar System.

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