Large Eddy Simulation of wind turbine wakes: detailed comparisons of two codes focusing on effects of numerics and subgrid modeling

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Abstract. In this work we report on results from a detailed comparative numerical study from two Large Eddy Simulation (LES) codes using the Actuator Line Model (ALM). The study focuses on prediction of wind turbine wakes and their breakdown when subject to uniform inflow. Previous studies have shown relative insensitivity to subgrid modeling in the context of a finite-volume code. The present study uses the low dissipation pseudo-spectral LES code from Johns Hopkins University (LESGO) and the second-order, finite-volume OpenFOAM code (SOWFA) from the National Renewable Energy Laboratory. When subject to uniform inflow, the loads on the blades are found to be unaffected by subgrid models or numerics, as expected. The turbulence in the wake and the location of transition to a turbulent state are affected by the subgrid-scale model and the numerics.

1. Introduction
During recent years large-eddy simulation (LES) has become one of the main tools for performing highly resolved numerical studies of wind turbine wakes and wind farms [1, 2, 3]. The main models for representing wind turbines in LES are the actuator disk and, for more highly resolved LES the Actuator Line Model (ALM). In ALM the turbines are represented as forces along the blades which enter the filtered Navier-Stokes equation as body forces [4, 5]. In this work we study the effects of subgrid-scale modeling in LES of a wind turbine wake using the ALM. Simulations of uniform inflow past a wind turbine are performed with two different codes, a finite-volume based solver, and a low dissipation pseudo-spectral solver. Different subgrid-scale (SGS) models of the Smagorinsky type are compared. Previous studies have shown insensitivity to the SGS model when the turbine is subject to turbulent inflow [6]. In this study we consider the more simplified case and compare the effect of SGS modeling and numerical method on simulations with uniform inflow.

2. Numerics
The present study compares two codes, the Simulator for Wind Farm Applications (SOWFA) package [7] from the National Renewable Energy Laboratory (NREL) and LESGO, the Johns Hopkins University (JHU) Turbulence Research Group’s code [8]. The codes have different
numerical implementations in order to solve the filtered Navier-Stokes equations with an SGS turbulence model. The effect of the numerics is addressed by comparing both codes with the same SGS model. Moreover, different SGS models are compared for each code, specifically a detailed comparison is presented between standard Smagorinsky SGS models and Dynamic type models. The table below describes the cases simulated for each code. All models represent the deviatoric part of the SGS stress tensor, $\tau_{ij}$, using a Smaroginsky type SGS eddy viscosity according to

$$
\tau_{ij} = -2\nu_{SGS}\tilde{S}_{ij} = -2(\text{Cs}\Delta)^2\vert\tilde{S}\vert\tilde{S}_{ij},
$$

where $\text{Cs}$ is the Smagorinsky coefficient and $\tilde{S}_{ij}$ is the symmetric part of the resolved velocity gradient tensor [9]. Dynamic models calculate the $\text{Cs}$ coefficient as a function of space and time making use of the Germano identity [10, 11, 12].

The turbine used in the present study is the NREL 5MW Reference [13]. The turbine diameter is $D = 126m$ and the dimensions of all the simulations are as follows:

- $L_x = 24D$
- $L_y = 6D$
- $L_z = 6D$
- $N_x = 768$
- $N_y = 192$
- $N_z = 2N_y = 384$
- turbine location: 3 D
- Inflow: $U_\infty = 8\text{ m/s}$
- $\rho = 1.0\text{ kg/m}^3$

When smearing the actuator line forces onto the computational grid, a smoothing parameter $\epsilon$ is introduced as the smearing length-scale [4, 14]. This value is related to the geometry of the blades, but there is not a general guideline based on the physics of the problem to establish its size [5]. From prior experience using a finite volume implementation the known numerical limit of the value under uniform inflow is related to the grid size as $\epsilon = 2\Delta x$ [5]. For the current case a value of $\epsilon = 10m$ was used, which is on the safe side with $\epsilon = 2.5\Delta x$. 

![Figure 1: Computational domain.](image-url)
2.1. Simulator for Wind Farm Applications (SOWFA)
SOWFA is an LES solver meant for wind farm simulations implemented under the OpenFOAM framework. It is a finite-volume code with second-order numerics in both space and time. An ALM implementation is used to represent the turbines. The SGS models are of the standard Smagorinsky type with fixed $C_S$ coefficient and a dynamic model with Lagrangian averaging [11].

2.2. LESGO
LESGO is the pseudo-spectral LES code from the Turbulence Research Group at Johns Hopkins University (JHU) [8]. It is based on the early work of Moeng’s [15] and Albertson and Parlange [16]. It is pseudo-spectral in two directions with the third direction using second-order centered finite differencing. Time advancement is done using a second order Adams-Bashforth scheme. The SGS models are of the standard Smagorinsky type with fixed $C_S$ coefficient and a scale dependent dynamic model (see Bou-Zeid [12]).

For the JHU LESGO code, the x and y directions are spectral while the z direction uses centered finite differencing. It is not known a priori which resolution to use for the spectral and finite difference directions. Initial simulations were carried out at the same resolutions for both and it was observed that the Reynolds stresses differed in both directions. Based on the modified wave number, it seems like a good choice to have the finite difference resolution to be twice the spectral resolution, since the modified wavenumber up to 1/2 of the cutoff wavenumber limits the attenuation to about 33% only. With this value the Reynolds stress profiles look now very similar for the two different directions.

3. Results
3.1. Comparisons to Blade Element Momentum
In this section simulation results are compared to Blade Element Momentum Theory (BEM), which assumes inviscid flow and a zero velocity component in the radial direction [17, 18]. When non-turbulent uniform inflow passes through the rotor, the flow does not break down until several diameters downstream. This can be seen in Figure 2, where the lines for all turbulence models lie exactly on top of each other. This means that the turbulence model has negligible impact in this portion of the flow domain. There are differences between the LES simulations and the predictions of BEM. There are several reasons for these differences. The first being that there is a blockage effect in the computational box. The current domain is 6D by 6D in the spanwise directions. In order to eliminate the blockage effect the domain would have to be greater than 10D by 10D which is constrained by the computational resources available. The other difference, which is more noticeable is the induction. It is known that the induction created by the turbine is affected by the way the forces are smeared onto the domain [5]. In our case, the width of the Gaussian kernel is quite large compared to the chord length scale. This causes a difference in the axial velocity and thus, affects the angle of attack and hence the lift and drag. The differences in angle of attack seem to be small, but these differences amplify when calculating the lift and drag coefficients. These differences could be further reduced, but it requires many more grid points, exceeding our computational resources. Another important difference between the BEM and LES-ALM is that the former assumes locally 2D pressure distributions around the blade. The radius-dependent foil sections induce radial changes that the real pressure field will smooth out in the radial direction, while the BEM assumes locally constant 2D conditions to prevail with sudden jumps in axial velocity and forces which cannot happen in reality. Next, we study the behavior of the wakes, where we look into the numerics and the SGS modeling.
Figure 2: Comparison with Blade Element Momentum Theory. Lift and Drag have been non-dimensionalized using the diameter $D$, inflow velocity $U_\infty$, density $\rho$ and width $l$.

3.2. Wake Profiles
The wake of a turbine in uniform inflow is characterized by two main regions. A mostly non-turbulent ‘smooth’ region, the near wake, and a turbulent region where the breakdown happens and the components of the Reynolds stress tensor have much higher values. Figure 3 shows a volume rendering of the instantaneous streamwise velocity component, where the two regions of the wake are clearly shown. The contour plot shows the streamwise velocity component on a plane across the wake. Volume rendering shows the streamwise velocity component in the wake.
In order to study the differences in the wakes, contours of streamwise velocity component are shown for all cases in Figure 4. The wake distributions are very similar, specially in the near wake. This suggests that the SGS model has very little effect in this region of the flow. Greater differences are observed as we move downstream, where the breakdown of the wake occurs. The cases for the same SGS model but different codes show the effects of the different numerics of each code. The contours of the dynamic models and the Smagorinsky case with $C_S = 0.08$ are very similar. These similarities are because the $C_S$ values obtained from the Dynamic type models are closer on average to $C_S = 0.08$.

It can be observed that the Smagorinsky model case with $C_S = 0.16$ from the SOWFA package differs from the rest. There are streamwise inhomogeneities that look like structures in the center not present in the other cases. Figure 5 shows the velocity component normal to the plane for the Smagorinsky model case with $C_S = 0.16$. The flow on the outside part of the wake keeps separating from the center. To conserve angular momentum the fluid in the inner region of the wake starts counter rotating. The reason for this behavior in these simulations is still unclear. The most noticeable aspect for this case is that it has the highest dissipation from the turbulent model and numerical diffusion as a result of the second-order finite volume discretization.
Contours of the streamwise normal $\overline{w'w'}/U_{\infty}^2$ component of the Reynolds stress tensor in Figure 6 show these differences more clearly. It can be concluded quite clearly that the Smagorinsky model with a coefficient of $C_s = 0.16$ (derived value for homogeneous isotropic turbulence [9]) predicts a delayed transition of the wake. This difference is caused by a higher SGS viscosity ($\nu_{SGS}$) that damps the flow structures in the shear layer.

Figure 4: Streamwise velocity $\overline{u}/U_{\infty}$ contours of the wake with flow going from left to right.

Figure 5: Velocity component normal to the plane $\overline{w}/U_{\infty}$ contours of the wake with flow going from left to right.
Differences can also be observed when the same SGS model is used in both codes. The biggest difference is a delayed transition to turbulence in the Finite-Volume SOWFA framework. This is expected since the second order numerics will introduce more dissipation. This will delay the breakdown of the shear layer in the wake. There are also some differences in the Scale Dependent Lagrangian Dynamic Model simulation (LESGO) and the Dynamic Model without scale dependence (NREL). The comparison shows a delayed transition of the wake in the Dynamic (NREL) case. These differences are caused by the fundamental differences in the models, but as in the other cases it is due mostly to the numerics which add more dissipation.

Figures 7 and 8 show mean velocity profiles of streamwise velocity $u/U_\infty$ and Reynolds stress $u'w'/U_\infty^2$ profiles along lines across the wake 1, 4, 7, and 9 diameters downstream. This clearly shows how the velocity field is unaffected by the SGS model for the first 4 diameters downstream and noticeable differences are observed past 4 diameters downstream. More noticeable differences can be seen for the Reynolds stress component. The main feature to note is that for profiles downstream, the Reynolds stress components for Smagorinsky $C_s = 0.16$ in both codes differ from the rest of the cases. The Dynamic models and Smagorinsky $C_s = 0.08$ cases show very similar profiles. This suggests that adjusting the Smagorinsky coefficient will give results very close to what a dynamic model will predict as long as one knows what correct value to use for $C_s$. 

Figure 6: Streamwise normal component of the Reynols stress $u'w'/U_\infty^2$ contours of the wake with flow going from left to right.
Figure 7: Streamwise velocity $u/U_\infty$ across the wake 1, 4, 7 and 9 diameters downstream.
4. Conclusions

An ALM implementation has been tested using detailed comparisons of results obtained using two LES codes with different numerical schemes and BEM. Subgrid scale modeling makes no significant difference in the loads and flow field near the turbine when running simulations under uniform inflow. The SGS model makes a difference further downstream where there is a breakdown of the wake. The numerics in LES of wind turbine wakes under uniform inflow play a very important role. The dissipation added by second order finite difference numerics damp the turbulence which delays the breakdown of the wake. A properly chosen value of the Smagorinsky coefficient $C_S$ can provide very similar results as a Dynamic model, but with a reduced computational cost. It is hard to predict an exact $C_S$ value for each case, in our cases similar results were obtained with $C_S = 0.08$ (i.e. about half of the derived value for homogeneous isotropic turbulence) to those using Lagrangian type models.

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