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On nonlinear classical and fractional order dynamical system addressing COVID-19

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ABSTRACT

This work studies a new SEIR type mathematical model for SARS-CoV-2. We show how immigration, protection, death rate, exposure, cure rate and interaction of infected people with healthy people affect the population. Our model has four classes including susceptible, exposed, infected and recovered respectively. Here, we find the basic reproduction number and local stability through jacobean matrix. Lyapunov function theory is used to calculate the global stability for the problem under investigation. Also an attempt is made to derive some numerical interpretation under fractional derivative by using fractional order nonstandard finite difference (NSFD) scheme. The graphical presentations are given for some real data.

Introduction

The biggest and most difficult task for us to control the diseases in our environment. However, for this reason, some instruction were issued. Also some guidelines are provided by government in this regard. Some limits are set so as not to disturb nature or environment beyond that. Epidemics are a basic and real threat to the human population and their economic and social conditions. However, unless there is a specific trace point of understanding of the disease. The society cannot be controlled over it. Implementing plans to prevent the spread of the pandemic has been diagnose as a major challenge. To deal with such and other diseases, mathematical modeling is an important tool to study of general and pandemic diseases in the current literature has broadened our understanding of how to detect and control such epidemics and other pathogens (see [8,11]).

Mathematical modeling is playing an important role in describing the epidemic of infectious diseases. The purpose of mathematical modeling is to represent different types of real world situation in the language of mathematics. A number of mathematical models are studies in the pervious literature (see [1,2,3,4,5,7,8,10,12]). Also SARS-CoV-2 is study by many researchers in current research literature (see [9,13]). We will study SARS-CoV-2 by developing SEIR model later on in this work.

To find out the different dynamics of a disease and therefore to overcome it at an early stage, mathematical modeling plays an important role there. The area dedicated to the investigation of biological pandemic and also epidemic models for recent diseases SARS-CoV-2 of research. Numerous examples of biological models have been found in the current literature. To studying the theory of the stability, the results of existence, and the theory of reform SARS-CoV-2, (see[6,19]), can be mediated, and its future behavior can be predicted. It is also possible to plan prevention. In addition, one can find a possible prevention strategy. Especially impressed with the excellent features of the SEIR model using non-linear saturated incidence rates (see[6,14,8]). In this paper, we are developed the SEIR model for SARS-CoV-2. The host population is divided into fours basic compartments, which are susceptible, exposed, infected and recovered compartments. For the dynamic non-linear...
Recently many authors have established numerous models for COVID-19 under different concept of fractional calculus. In this regards very useful models have been established, we refer some as [20–25]. As fractional order models are more comprehensive to investigate real world problems for global dynamics. Now to find exact analytical solutions to fractional order systems is quiet difficult job. Therefore for best approximate solutions, various numerical schemes have been established in literature. The concerned techniques have been evaluated for classical models very well. But in case of fractional differential equations the said methods have also utilized in plenty of ways. These scheme includes Adam Bash Forth method, Euler method and Non-standard finite difference (NSFD) scheme. Among the mentioned method NSFD method is one of the powerful tool to find approximate solutions to fractional order models, for some detail we refer [15–18].

To establish a numerical scheme usually some method of numerical will be used. Here we have used Nonstandard Finite Difference Scheme. For which first the derivative should be discredited this has been done by using here the difference scheme. Also we remark that in most situations, the solution of a fractional differential equation does not exist in terms of a finite number of elementary functions. Therefore it is needed to device numerical methods in order to practically evaluate approximate solutions by means of difference schemes or other alternative methods [29–33].

Model formulation

Here in this section, we present a mathematical system for the population at the time of SARS-CoV-2 and divide the entire population into four sections. Susceptible $S$, exposed $E$, infected $I$ and recovered $R$. The dynamics of the system form the following differential equations.

\[
\begin{align*}
\frac{dS(t)}{dt} &= b - a_1 S(t) I(t) \left( \frac{d_0 + \alpha}{1 + \gamma I(t)} \right) - (d_0 + a) S(t) \\
\frac{dE(t)}{dt} &= a_1 S(t) I(t) - a_2 E(t) I(t) - d_0 E(t) - a_3 E(t) I(t) \\
\frac{dI(t)}{dt} &= a_2 E(t) I(t) + a_1 S(t) I(t) - (d_0 + \mu + w) I(t) \\
\frac{dR(t)}{dt} &= w I(t) - d_0 R(t).
\end{align*}
\]

The dynamics of the model (1) above is present in the following chart.
3

\[
\frac{dX}{dt} = F - \mathcal{V},
\]

where

\[
F = \left( \frac{a_iS(t)I(t)}{1 + pI(t)} + a_3E(t)I(t) \right) \frac{0}{0}
\]

and

\[
\mathcal{V} = \left( \frac{\mu + d_i + w)I(t)}{0} \right).
\]

For the disease-free equilibrium Jacobian is given below,

\[
F = \left( a_iS^0 + a_3\beta E^0 \right) \frac{0}{0}
\]

and for the disease-free equilibrium Jacobian of \( \mathcal{V} \) is given

\[
V = \left( \mu + d_i + w \right) \frac{0}{0}.
\]

Hence

\[
V^{-1} = \frac{1}{d_i(\mu + d_i + w)} \left( \begin{array}{cc}
0 & 0 \\
0 & \mu + d_i + w
\end{array} \right).
\]

We have

\[
FV^{-1} = \left( \begin{array}{cc}
d_i(a_iS^0 + a_3\beta E^0) & 0 \\
0 & 0
\end{array} \right).
\]

Hence the required \( R_0 \) is given by

\[
R_0 = \frac{b(b_i\mu + a_3\beta)}{d_i(\mu + d_i + w)(\mu + d_i + w)}.
\]

To compute the basic reproduction number we obtained \( R_0 = 1.13213 \) from the parameters used in Table 2.

We have the following theorem on the basis of (4).

**Theorem 0.1.**

(i) There is no positive equilibrium of system, if \( R_0 \leq 1 \).

(ii) There is a unique positive equilibrium \( E^* = (S^*(t), E^*(t), R^*(t)) \) of the model (1), called the endemic equilibrium, if \( R_0 > 1 \).

**Local stability**

We reduced our model (1) to obtained the result for local stability as

\[
\frac{dS(t)}{dt} = b - \frac{a_iS(t)I(t)}{1 + pI(t)}(d_i + a)S(t) + \frac{bS(t)(1 + pI(t))}{d_i + a + a_1I(t)}(d_i + a + a_1I(t)) \frac{0}{0}
\]

\[
\frac{dE(t)}{dt} = aS(t) - d_iE(t) - \frac{a_3E(t)I(t)}{1 + pI(t)} \frac{0}{0}
\]

\[
\frac{dR(t)}{dt} = \frac{a_2E(t)I(t) + a_3S(t)I(t)}{1 + pI(t)} - (d_i + \mu + w)I(t).
\]

Subject to initial condition

\[
S(0) = S_0 \geq 0, E(0) = E_0 \geq 0, I(0) = I_0 \geq 0.
\]

we are going to present our coming theorem.

**Theorem 0.2.** “At \( E^0 \), the disease free equilibrium of the model (5) is locally asymptotically stable under the condition \( R_0 < 1 \).”

**Proof.** At \( E^0 \), we find Jacobian Matrix as

\[
J^0 = \left( \begin{array}{ccc}
-(\alpha + d_i) & 0 & a_iS^0 \\
\alpha & -d_i & \alpha_3E^0 \\
0 & 0 & R_0 - 1
\end{array} \right).
\]

The auxiliary equation of \( J^0 \) is given by

\[
\lambda^3 + \alpha^2c_1 + \alpha c_2 + c_3 = 0,
\]

\[
\lambda^3 + \alpha^2c_1 + \alpha c_2 + c_3 = 0.
\]
Table 3: Parameters and their description of the system 1.

| Variables       | Their physical description                                    | Exact numerical value  |
|-----------------|----------------------------------------------------------------|------------------------|
| $S(t)$          | Susceptible compartment                                      | 219.87904 in millions  |
| $E(t)$          | Exposed compartment                                           | 0                      |
| $I(t)$          | Infected compartment                                          | 0.521211 in millions    |
| $R(t)$          | Recovered compartment                                         | 0.486225 in millions    |
| $\beta$         | Reducing infection rate                                       | 0.0009                 |
| $d_s$           | Natural death rate                                            | 0.0019                 |
| $\mu$           | Covid death rate                                              | 0.019 [26]             |
| $r$             | Constant of saturation                                        | 0.0009601              |
| $b$             | Rate of recruitment                                           | 0.0003                 |
| $a_1$           | Contact rate of disease                                       | 0.0006. 0.0007. 0.0008. 0.0009 assumed |
| $w$             | Rate of infection recovery                                     | 0.0058                 |
| $a_2$           | Rate it which susceptible become exposed                      | 0.0006. 0.0007. 0.0008. 0.0009 assumed |
| $\alpha$        | Rate it which susceptible become exposed                      | 0.0728 assumed          |

We have

\[
\begin{align*}
1. & c_1 = d_0(\alpha + d_0)(\mu + d_0 + w)(1 - R_0) > 0 \\
2. & c_2 = d_0(\alpha + d_0)(\mu + d_0 + w)(1 - R_0) > 0 \\
3. & c_3 = d_0(\alpha + d_0)(\mu + d_0 + w)(1 - R_0) > 0.
\end{align*}
\]

(6) show that the “Routh-Hurwitz criteria is satisfied” as $k_1 > 0, k_2 > 0, k_3 > 0$ and $k_1k_2 - k_3 > 0$, if $R_0 < 1$, which show the system (1) is locally asymptotically stable at $C^0,s$.

**Theorem 0.3.** “Under the condition $R^0 > 1$, at $C^0(\tau)$ model (5) is locally asymptotically stable, otherwise unstable.”

**Proof.** The jacobian matrix of the system (5), at $C^0$ the endemic equilibrium is

After some operations on matrix $J_1$, we get

\[
J_1 = \begin{pmatrix}
-(\alpha + d_0) & -\alpha S'(t)/(1 + \beta E'(t)) & 0 \\
\alpha & -d_0 - \alpha S'(t)/(1 + \beta E'(t)) & -\alpha \beta E'(t) \\
\alpha S'(t)/(1 + \beta E'(t)) & \alpha \beta E'(t) & \alpha S'(t)/(1 + \beta E'(t)) + \alpha \beta E'(t) - (d_0 + \mu + w)
\end{pmatrix}
\]

We calculate trace and determinant of $J_1$

\[
\text{tra} J_1 = -d_0[\alpha + \alpha \beta (I'(t) + d_0E'(t)) + \alpha (d_0 + \mu + w)] - \frac{\alpha (d_0 + \mu + w)}{1 + \beta E'(t)} < 0
\]

and

\[
\text{det} J_1 = d_0[\alpha + \alpha \beta (I'(t) + d_0)]\left[\alpha \alpha \beta E'(t) + \frac{(d_0 + \mu + w)}{1 + \beta E'(t)}\right]
\]

\[
+ (d_0 \beta \alpha S'(t)/(1 + \beta E'(t)) + \alpha (d_0 + \mu + w))\left[\alpha \beta E'(t) - (d_0 + \mu + w)\right] > 0.
\]

The determinant of $J_1 > 0$ if $(\beta \alpha_2 - \alpha_1) > 0$. For the system (5) endemic equilibrium at $C^0(t)$ has negative real part. Thus, we conclude that the disease free endemic equilibrium at $C^0(t)$ of system (5) is stable local and asymptotically with condition $R^0 > 1$. 

\[
J_1 = \begin{pmatrix}
-d_0 & -d_0 & -(d_0 + \mu + w) \\
0 & d_0(\alpha + \alpha \beta I'(t) + d_0) & -d_0 \beta \alpha S'(t)/(1 + \beta E'(t)) - \alpha (d_0 + \mu + w) \\
d_0 \beta \alpha I'(t) - \frac{d_0 \alpha \beta E'(t)}{1 + \beta E'(t)} & d_0(\alpha \beta E'(t) + \alpha (d_0 + \mu + w)) - \frac{(d_0 + \mu + w)}{1 + \beta E'(t)}
\end{pmatrix}
\]
Global stability

In this section of our work, we study global stability of model (1). For global stability, we constructed “Lyapunov function” for diseases free and endemic equilibria.

**Theorem 0.4.** “The disease free equilibrium of the system (5) is globally asymptotically stable if $R_0 < 1$.”

**Proof.** To prove this required result, we construct a Lyapunov function as following:

$$L = z_1(S(t) - S_0) + z_2(E(t) - E_0) + z_3I(t),$$

such that $z_1, z_2, z_3 > 0$ are constant may be computed later. With respect to time $t$ taking derivative of (7) with, we have

$$\frac{dL}{dt} = z_1 \left( b - \frac{\alpha S(t)I(t)}{1 + \gamma I(t)} - (d_i + \alpha)S(t) \right) + z_2(\alpha S(t) - d_E E(t) - \beta a_2 E(t)I(t))$$

$$+ z_3 \left( \frac{\alpha S(t)I(t)}{1 + \gamma I(t)} + \beta a_2 E(t)I(t) - (d_i - \mu + w)I(t) \right).$$

We get
Let assume \( z_1 = z_2 = z_3 = 1 \), we get finally

\[
\frac{dL}{dt} = a_2 S(t) I(t) \left( z_3 - z_1 \right) + \beta_2 E(t) I(t) \left( z_3 - z_1 \right) - z_2 \beta - z_3 (d_0 + \alpha) + z_3 a S(t) - z_3 d_1 E(t) - z_3 (d_0 + \mu + w) I(t).
\]

Hence “globally asymptotically stable” for the considered model with \( R_0 < 1 \) has reached.

**Theorem 0.5.** “The endemic equilibrium \( \mathbf{E}^* \) of model (5) is stable globally asymptotically if \( R_0 > 1 \).”

**Proof.** On construction of Lyapunov function as

\[
\ell' = d_0 (S(t) - S'(t)) + \frac{d_0 (d_0 + \alpha)}{\alpha} E(t) + d_1 I(t).
\]  \hspace{1cm} (8)

Take the derivative (8) w.r.t. \( t \) along with system (5),

\[
\frac{d\ell'}{dt} = d_0 \frac{d_0 (d_0 + \alpha)}{\alpha} E(t) + d_1 \ell(t).
\]

Putting the values from (1) yields

\[
\frac{d\ell'}{dt} = -d_0 (\mu + d_1) (d_0 + \alpha) R_0 - \beta a d_0 (d_0 + \alpha) - a_1 d_0 (d_0 + \mu + w) I(t) < 0.
\]

Thus \( \frac{d\ell'}{dt} < 0 \), the endemic equilibrium \( \mathbf{E}^* \) of the system (5) is stable globally asymptotically, prove that \( R_0 > 1 \).

**Sensitivity analysis**

Keeping in view Section 4, The Threshold number \( R_0 \) is calculated for the SEIR model. For endemic equilibrium (4) makes it clear how many parameters are important for the transmission of the disease. Information are very important not for practical or theoretical design, but also for the integration of data and the lack of complex models [7]. This is a sensitivity analysis that is commonly used to determine the predictive power of a model about parameter values, since the data collected usually contains errors and perceived parameter values.

The sensitivity indices for the parameters for \( R_0 \) are given in 2. It is usually used to find the parameters that have the greatest effect on the Basic Reproduction Number. \( R_0 \) and the intervention strategy should be targeted. Clearly, sensitivity indicators allow us to examine the relative change in variables in relation to changes in parameters. Depending on the purpose, the normal forward synchronization index of a variable is used in relation of a parameter given, how which interpreted as the relation of the corresponding change of the given parameter. If such a variable difference is found in the parameter, the sensitivity index is described below (see Table 3).

**Remark 1.** Sensitivity index for \( R_0 \), which is normalized forward is define below, taking the derivative w.r.t \( \tau \)

\[
\Psi_{R_0} = \frac{\partial R_0}{\partial \tau} \frac{\tau}{R_0}.
\]  \hspace{1cm} (9)

The sensitivity values indices that for the numerical values of Table 1, we presented above in Table 2. The index of sensitivity depend upon more parameters in system (1). It also permanent independent upon the other parameters. i.e \( \Psi_{R_0} = + 1 \). means that a given percentage increase (less) increases an tau increases by eighty percent always happens. Sensitive parameters will be calculated very carefully, as a small error in this parameter lead us to relevant big changes. Furthermore, estimating a parameter rather than a small value for the sensitivity index does not require much attention, as a small mishap in this parameter gives us a small changes.

**Numerical interpretation for classical model (1)**

In this part of our manuscript, we study simulation for model (1) by using the values of Table 2. From first January up to next 300 days, we simulate the considered model (1) for Pakistan a.

Here, using NSFD [14,19] to rewrite the system in difference equations form. Hence from the first equation of system (1) via NSFD one has

\[
\frac{dS(t)}{dt} = b - \frac{a_1 S(t) I(t)}{1 + \beta I(t)} - (d_0 + \alpha) S(t).
\]  \hspace{1cm} (10)

Thank to NSFD, (10) yields

\[
\frac{S_{i+1} - S_i}{h} = b - \frac{a_i S(t) I(t)}{1 + \beta I(t)} - (d_0 + \alpha) S_i(t).
\]  \hspace{1cm} (11)

![Fig. 4. Dynamical behavior of Recovered R(t) class of the model at given different values](image-url)
Like (11), Using NSFD scheme, we decomposed the system (1) as follow

\[ S_{j+1} = S_j + h \left( b - a_1S_j(t)I_j(t) - (d_i + a)S_j(t) \right) \]

\[ E_{j+1} = E_j + h(aS_j(t) - a_2\beta E_j(t)I_j(t) - d_iE_j(t)) \quad (12) \]

\[ I_{j+1} = I_j + h \left( a_1S_j(t)I_j(t)I_j(t) + a_2\beta E_j(t)I_j(t) - (d_i + \mu + \omega)I_j(t) \right) \quad (13) \]

By using NSFD scheme, developed in (12). Using the values from 3, we plotted the system (1).

Using real data, we testify our system (1) for the real data of Pakistan with used Table 2 for the parameters values from 18the January 2021 for coming 90 days. Using the NSFD scheme. In Fig. 1–4, we investigate dynamic of transmission of SARS-CoV-2 in Pakistan at different values of \( a_i, i = 1, 2 \). Here for graphical interpretation, we take initial data as given

Fig. 5. Fractional Dynamical behavior of susceptible S class of the model at given different vales \( \omega \) by using fractional NFSD scheme.

Fig. 6. Fractional Dynamical behavior of exposed E class of the model at given different vales \( \omega \) by using fractional NFSD scheme.

Fig. 7. Fractional Dynamical behavior of infected I class of the model at given different vales \( \omega \) by using fractional NFSD scheme.
in [26] as \((S_0, E_0, I_0, R_0) = (219.78904, 0.0.521211, 0.486225)\) at 18 January 2021. Now we simulate the numerical results of different compartments corresponding to different values of \(a_i\), \(i = 1, 2\). As we increase the values of \(a_i\), \(i = 1, 2\), the susceptibility decay is faster as shown in Fig. 1. This indicates that increasing the contact rate will highly increase the susceptible population density. This causes the increase in exposed population density to infectious. The corresponding growth behavior of exposed at various values of \(a_i\) is given in Fig. 2. On the other hand these factor increase will result in increase of infectious. Greater change can be observed in the dynamics of respective compartment.

**Numerical interpretation of the Model (1) under concept of fractional calculus**

In this section we established a numerical scheme for the considered model (1) by using NFDS under the concept of fractional order derivative. We use Grünwald-Letnikov approximation for Caputo derivatives in order to simulate our proposed model. About this scheme we refer [27,28]. Here we state that a tedious difficulty in the numerical investigation of fractional order differential equations is the presence of the long and persistent memory due to the nonlocal nature of fractional order differential operators. This type of schemes are computationally expensive. But on the other hand at the same time preserving some of the main necessary physical properties of the solution including positivity, monotonicity and convergence towards a stable steady-state results.

**Definition 0.6.** Let \(Z \in C[0,T]\), then Caputo derivatives with fractional order \(\omega > 0\) is given by

\[
D^\omega Z(t) = \begin{cases} 
\int_0^t (t-\theta)^{\omega-1} Z(\theta)d\theta, & 0 < \omega < 1, \\
\frac{dZ}{dt}, & m = \omega = 1, 
\end{cases}
\]

with \(m = [\omega] + 1\). Further fractional order Riemann–Liouville integral operator is defined as

\[
P^\omega Z(t) = \frac{1}{\Gamma(\omega)} \int_0^t (t-\tau)^{\omega-1} Z(\theta)d\theta, \quad \omega > 0.
\]

To extend the classical NSFD method for fractional order model, we chose the Grünwald-Letnikov method approximation for the one-dimensional fractional order derivative as follows:

\[
D^\omega Z(t) = \lim_{h \to 0} \sum_{i=0}^{N} (-1)^i \binom{\omega}{i} Z(t-ih).
\]

where \(t = Nh\), where \(h\) is the step size. Here, we consider the following general problem as

\[
\begin{aligned}
D^\omega Z(t) &= f(t, Z(t)), \quad t \in [0,T], T < \infty, \\
Z(t_0) &= Z_0.
\end{aligned}
\]

Inview of (15), left side of (16) is discretized and hence one has

\[
\sum_{i=0}^{N} K^\omega Z(t_{m-1}) = f(t, Z(t)), \quad m = 1, 2, 3, \ldots
\]

where \(t_m = mh\) and \(K^\omega_i\) are the Grünwald-Letnikov coefficients defined as
Here we extend our considered model (1) to the fractional-order system by fixing the values of the parameter \( K \) as

\[
S(t_{n+1}) = \frac{1}{K^\alpha} \left[ -\sum_{j=1}^{n+1} K_j^\alpha S(t_{n+1-j}) + b \frac{aS(t_n)I(t_n)}{1+\gamma I(t_n)} - (d_0 + a)S(t_n) \right]
\]

\[
E(t_{n+1}) = \frac{1}{K^\alpha} \left[ -\sum_{j=1}^{n+1} K_j^\alpha E(t_{n+1-j}) + aS(t_{n+1}) - d_0 E(t_n) - a_2 b E(t_n)I(t_n) \right]
\]

\[
I(t_{n+1}) = \frac{1}{K^\alpha} \left[ -\sum_{j=1}^{n+1} K_j^\alpha I(t_{n+1-j}) + a_2 b E(t_{n+1})I(t_n) + \frac{a_1 S(t_{n+1})I(t_n)}{1+\gamma I(t_n)}(d_0 + \mu + w)I(t_n) \right]
\]

\[
R(t_{n+1}) = \frac{1}{K^\alpha} \left[ -\sum_{j=1}^{n+1} K_j^\alpha R(t_{n+1-j}) + wI(t_{n+1}) - d_3 R(t_n) \right].
\]

Now we simulate the results corresponding to the given initial data in Table 2 by fixing the values of \( \alpha = 0.0006 \) and taking different fractional order, we present numerical solutions through graphs as given in Figs. 5–8. We see that at different fractional order the dynamics is further glorified for understanding. We see that by increasing the order the decay process is becoming slow while the growth process of infectious, exposed and recovered class is rapid. As we enlarge the order the concerned behavior of various compartments tend to that of integer order. Here fractional calculus provides global dynamics which is help in understanding of transmission dynamics of COVID-19 for coming few months.

**Conclusion**

A dynamical model addressing the transmission of COVID-19 has been considered under classical as well as fractional order derivative. The model under consideration has been investigated for global and local stability via Lyapunov theory. Basic reproduction number has been computed and sensitivity analysis derived. Then on using NSFD method numerical interpretations for classical model were performed against real data for initial values. Then we have extended the NSFD method to fractional order and investigate the model under consideration for fractional order dynamics. From the plots 5–8, we concluded that fractional order model is more effective in understanding the transmission dynamics as compared to classical order model.

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**Authors Contributions**

All authors played their role equally.

**Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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