Strongly Nano Generalized Closed Sets in Nano Topological Spaces

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Abstract: The basic objective of this paper is to introduce and investigate the properties of Strongly nano generalized closed sets in Nano Topological Spaces which is the extension of Nano generalized closed sets introduced by Lellis Thivagar.

Keywords: Nano closed set, Nano open set, Generalized closed set, Nano generalized closed set, Strongly Nano generalized closed set.

I. INTRODUCTION

Levine[2] introduced the class of generalized closed sets, a super class of closed sets in 1970. This concept was introduced as a generalization of closed sets in Topological spaces through which new results in general topology were introduced. Lellis Thivagar[1] introduced Nano topological space with respect to a subset X of a universe which is defined in terms of lower and upper approximations of X. The elements of Nanotopological space are called Nano open sets. He also introduced the weak forms of Nano open sets namely Nano – α open sets, Nano semi open sets and Nano semi open sets. Nano generalized closed and nano strongly generalized closed was introduced by K. Bhuvaneswari[5,6]. In this paper some properties of strongly nano generalized closed sets in Nano topological spaces are studied.

II. PRELIMINARIES

1) Definition 2.1: A subset A of a topological space (X, τ) is called a generalized closed set (briefly g-closed) if Cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

2) Definition 2.2: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation of U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X ⊆ U

a) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is denoted by L_R(X). That is L_R(X) = U {R(x) : R(x) ⊆ X} where R(x) denotes the equivalence class determined by x.

b) The upper approximation of X with respect to U is the set of all objects which can be possibly classified as X with respect to R and it is denoted by U_R(X). That is U_R(X) = U {R(x) : R(x) ∩ X ≠ ∅}.

c) The boundary region of X with respect to R is the set of all objects, which can be classified neither X nor as not X with respect to R and it is denoted by B_R(X). That is B_R(X) = U_R(X) – L_R(X).

3) Property 2.3: If (U, R) is an approximation space and X, Y ⊆ U, then

a) L_R(X) ⊆ Y ⊆ U_R(X)

b) L_R(ϕ) = U_R(ϕ) = U_R(U) = U_R(U)

c) U_R(X ∪ Y) = U_R(X) ∪ U_R(Y)

d) U_R(X ∩ Y) ⊆ U_R(X) ∩ U_R(Y)

e) L_R(X ∩ Y) = L_R(X) ∩ L_R(Y)

f) L_R(X) ⊆ L_R(Y) and U_R(X) ⊆ U_R(Y) whenever X ⊆ Y

g) U_R(X^C) = [L_R(X)]^C and L_R(X^C) = [U_R(X)]^C

h) U_R U_R (X) = L_R U_R (X) = U_R (X)

i) L_R L_R (X) = L_R U_R (X) = L_R (X)

4) Definition 2.4: Let U be the universe, R be an equivalence relation on U and τ_R(X) = {U, ϕ, L_R(X), U_R(X), B_R(X)} where X ⊆ U. Then by property 2.3, τ_R(X) satisfies the following axioms:

a) U and ϕ belongs to τ_R(X).

b) The union of the elements of any sub collection of τ_R(X) is in τ_R(X).
The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets. The elements of $(\tau_R(X))^C$ are called as nano closed sets.

K. Remark 2.5: If $\tau_R(X)$ is the Nano topology on U with respect to X, then the set $B = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

**Definition 2.6:** If $(U, \tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then the nano interior of the set $A$ is defined as the union of all Nano open subsets contained in A and it is denoted by $NInt(A)$. That is $NInt(A)$ is the largest Nano open subset of A. The Nano closure of the set A is defined as the intersection of all Nano closed sets containing A and it is denoted by $NCI(A)$. That is $NCI(A)$ is the smallest Nano closed set containing A.

### III. STRONGLY NANO GENERALIZED CLOSED SET

Throughout this paper $(U, \tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$ and $R$ is an equivalence Relation on U, $U/R$ denotes the family of equivalence classes of U by R.

**Definition 3.1:** Let $(U, \tau_R(X))$ be a Nano topological space. A subset $A$ of $(U, \tau_R(X))$ is called Strongly Nano generalized closed set (briefly $SNG-\text{closed}$) if $NCI(NInt(A)) \subseteq V$ where $A \subseteq V$ and $V$ is Nano open.

**Example 3.2:** Let $U = \{w, x, y, z\}$ with $U/R = \{\{w\}, \{x\}, \{y, z\}\}$ and $X = \{w, y\}$. Then the Nanotopology $\tau_R(X) = \{U, \emptyset, \{w\}, \{x, y, z\}, \{x, \{w, x, y, z\}\}\}$. Nano closed sets are $\{\emptyset, U, \{x, y, z\}, \{x, \{w, x, y, z\}\}\}$. Let $V = \{x, y\}$ and $A = \{x\}$. Then $NCI(A) = \{x\} \subseteq V$. That is $A$ is said to be $SNG-\text{closed}$ in $(U, \tau_R(X))$.

**Theorem 3.3:** A subset $A$ of $(U, \tau_R(X))$ is $SNG-\text{closed}$ if $NCI(NInt(A)) \subseteq V$ and $V$ is Nano open.

**Proof:** Suppose if $A$ is $SNG-\text{closed}$, then $NCI(NInt(A)) \subseteq V$ where $A \subseteq V$ and $V$ is Nano open. Let $Y$ be a Nanoclosed subset of $NCI(NInt(A))$. Then $A \subseteq V$ implies $NCI(A) \subseteq V$ and $V$ is Nano open. Since $A$ is $SNG-\text{closed}$, $NCI(NInt(A)) \subseteq Y$ or $Y \subseteq [NCI(NInt(A))]^C$. That is $Y \subseteq NCI(NInt(A))$ and $Y \subseteq [NCI(NInt(A))]^C$ implies that $Y \subseteq \emptyset$. So $Y$ is empty.

**Theorem 3.4:** If $A$ and $B$ are $SNG-\text{closed}$, then $A \cup B$ is $SNG-\text{closed}$.

**Proof:** Let $A$ and $B$ are $SNG-\text{closed}$ set. Then $NCI(NInt(A)) \subseteq V$ where $A \subseteq V$ and $V$ is Nano open and $NCI(NInt(B)) \subseteq V$ where $B \subseteq V$ and $V$ is Nano open. Since $A$ and $B$ are subsets of $V$, $(A \cup B)$ is a subset of $V$ and $V$ is Nano open. Then $NCI(NInt(A \cup B)) = NCI(NInt(A)) \cup NCI(NInt(B)) \subseteq V$ which implies that $A \cup B$ is $SNG-\text{closed}$.

**Remark 3.5:** The intersection of two $SNG-\text{closed}$ sets is again an $SNG-\text{closed}$ set which is shown in the following example.

**Example 3.6:** Let $U = \{a, b, c, d\}$, $X = \{a, b\}$, $U/R = \{\{a\}, \{c\}, \{b, d\}\}$. Then $\tau_R(X) = \{U, \emptyset, \{a, b, d\}, \{b, d\}\}$. Let $A = \{a, b, c\}$, $B = \{a, c, d\}$ and $A \cap B = \{a, c\}$. Here $NCI(NInt(A \cap B)) \subseteq V$ when $(A \cap B) \subseteq V$ and $V$ is Nano open.

**Theorem 3.7:** If $A$ is $SNG-\text{closed}$ and $A \subseteq B \subseteq NCI(NInt(A))$, then $B$ is $SNG-\text{closed}$.

**Proof:** Let $B \subseteq V$ where V is Nano open int$_R(X)$. Then $B$ implies $A \subseteq V$. Since A in $SNG-\text{closed}$, $NCI(NInt(A)) \subseteq V$. Also $B \subseteq NCI(NInt(A))$ implies $NCI(NInt(B)) \subseteq NCI(NInt(A))$. Thus $NCI(NInt(B)) \subseteq V$ and so $B$ is $SNG-\text{closed}$.

**Theorem 3.8:** Every Nano closed set is Nano generalized closed set.

**Proof:** Let $A \subseteq V$ and $V$ is Nano open in $\tau_R(X)$. Since $A$ is Nano closed, $NCI(NInt(A)) \subseteq V$. Hence $A$ is Nano strongly Nano generalized closed set. The converse of the above theorem need not be true as seen from the following example.

**Example 3.9:** Let $U = \{a, b, c, d\}$ with $X = \{a, c\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$. Then $\tau_R(X) = \{U, \emptyset, \{a, b, d\}, \{c, d\}\}$. Nano closed sets are $U, \emptyset, \{b, c, d\}, \{a, b\}$. Here $\{b, c\}$ is strongly Nano generalized closed set but it is not Nano closed set.

**Theorem 3.10:** A $SNG-$ closed set $A$ is Nano closed if and only if $NCI(NInt(A)) \subseteq A$ is Nano closed.

**Proof:** (Necessity) Let $A$ is Nano closed. Then $NCI(NInt(A)) = A$ and so $NCI(NInt(A)) \subseteq A$ which is Nano closed.

(Sufficiency) Suppose $NCI(NInt(A)) \subseteq A$ is Nano closed. Then $NCI(NInt(A)) = A$ since $A$ is Nano closed. That is $NCI(NInt(A)) = A$ or $A$ is Nano closed.

**Theorem 3.11:** Suppose that $B \subseteq A \subseteq U$, $U$ is a $SNG-$closed set relative to $A$ and that $A$ is an $SNG-$closed subset of $U$. Then $B$ is $SNG-$closed relative to $U$.

**Proof:** Let $B \subseteq V$ and suppose that $V$ is Nanoopen in $U$. Then $B \subseteq A \cap V$. Therefore $NCI(NInt(B)) \subseteq A \cap V$. It follows then that $A \cap NCI(NInt(B)) \subseteq A \cap V$ and $B \subseteq V$ and $NCI(NInt(B))$. Since $A$ is $SNG-$ closed in $U$, we have $NCI(NInt(A)) \subseteq V$ and $NCI(NInt(B))$. Therefore $NCI(NInt(B)) \subseteq V$ and $NCI(NInt(B)) \subseteq V$. Then $B$ is $SNG-$closed relative to $V$. 

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12) Corollary 3.12: Let A be a SNg-closed set and suppose that F is a Nano closed set. Then A ∩ F is an SNg-closed set which is given in the following example.

13) Example 3.13: Let \( U = \{a,b,c,d\} \) with \( X = \{a,b\} \) with \( U/R = \{\{a\},\{c\},\{b,d\}\} \). \( \tau_R(X) = \{U, \emptyset, \{a\},\{a,b,d\},\{b,d\}\} \). Nano closed sets are \( U, \emptyset, \{b,c,d\},\{c\},\{a,c\} \). Let \( A = \{a,b,c\} \) and \( F = \{b,c,d\} \). Then \( A ∩ F = \{b,c\} \) is an SNg closed set.

14) Theorem 3.14: For each \( a \in U \), either \( \{a\} \) is Nano closed (or) \( \{a\}^c \) is Strongly Nano generalized closed in \( \tau_R(X) \).

Proof: Suppose \( \{a\} \) is not Nano closed in \( U \). Then \( \{a\}^c \) is not nano open and the only nano open set containing \( \{a\}^c \) is \( V \subseteq U \). That is \( \{a\}^c \subseteq U \). Therefore \( NCl(NInt(\{a\}^c)) \subseteq U \) which implies \( \{a\}^c \) is Strongly Nano generalized closed set in \( \tau_R(X) \).

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