On numerical modelling of gas flows through axisymmetric porous object with heterogeneous combustion sources under forced filtration

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Abstract. The time-dependent gas flows through axisymmetric porous objects with heterogeneous combustion sources are considered. The mathematical model, which describes the process, and the numerical method are proposed. The regimes of heterogeneous combustion wave propagation in axisymmetric porous object with various location of ignition zones under forced filtration are investigated. It is shown that the gas tends to bend around the heated zone and flows through colder regions as in the case of two-dimensional plane flows.

1. Introduction

Many natural and technological processes, such as peat fires, spontaneous combustion of solid waste landfills, burning of solid fuels, associated with heterogeneous combustion in porous media. Heterogeneous combustion is one of the types of filtration combustion, which occurs when an oxidizer from the gas phase and a condensed combustible component from the solid phase interact.

Basic principles of heterogeneous combustion are briefly described in [1] for forced and free filtration. There are a huge number of papers which devoted to solid porous media combustion, in particular [2-5]. Computational modeling is one of the methods of investigation of heterogeneous combustion in porous media. A numerical model for investigating the time-dependent processes of one-dimensional combustion wave propagation in porous objects with unknown gas flow rate and gas velocity at object boundaries was proposed in [6]. In [7, 8] two-dimensional unsteady gas flows through the porous media with heterogeneous combustion sources under forced and natural convection are considered. The original numerical method, proposed in these papers, is the development of numerical algorithm for computation of the gas flow through the porous objects with heat sources when the gas pressure at object boundaries is known [9, 10].

In the present work the time-dependent gas flows through the axisymmetric porous object with heterogeneous combustion sources under forced convection are investigated. The gas velocity, the solid temperature, the gas pressure and the degree of conversion of the solid combustible component were analyzed at various gas pressure at object bottom and various locations of the ignition zone.

2. Mathematical model and numerical method

The axisymmetric porous object opened at the top and the bottom with impermeable non-heat conducting side walls is considered. A cold gas flows into the porous object through its bottom due to...
forced filtration, the gas flows through porous medium and flows out through object top. Mathematical model is constructed on the model of two interactive interpenetrative continua [11] and consists of the equations of energy of gas and solid phase, the equation of motion, the continuity equation, the equation of concentration, the equation of state for gas. In the energy equation of solid phase we take into account the intensity of interphase heat exchange, which is assumed to be proportional to the rate of chemical reaction. In the energy equation of gas we take into account a thermal conductivity of gas and intensity of interphase heat exchange. The equation of motion for gas is the momentum conservation equation for porous media. The equation of motion for solid media degenerates, because the solid phase is immovable. It is shown in [12] that taking into account the temperature dependence of the gas dynamic viscosity can affect the solution of the problem of time-dependent gas flow through porous media not only quantitatively but also qualitatively, so in the present work the dynamic viscosity of gas depends on temperature according to Sutherland’s formula.

So, the system of equations is the following:

$$\begin{align*}
\left( \rho_0 c_{\sigma} + \rho_s c_{\sigma} + \rho g c_{\sigma} \right) \frac{\partial T}{\partial t} &= -\alpha (T_e - T_s) + Q \rho_{\sigma \sigma} W + (1-a_g) \lambda_g \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right), \\
\rho_s \left( \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) &= \alpha (T_e - T_s) + a_g \lambda_g \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right), \\
\rho_g \left( 1 + \chi (1-a_g) \right) \left( \frac{\partial v}{\partial t} + \mathbf{v} \cdot \nabla v \right) &= -a_g \frac{\partial p}{\partial r} - a_g^2 \frac{\mu g}{k} v - (1-\mu_g) \rho_{\sigma \sigma} W, \\
\rho_g \left( 1 + \chi (1-a_g) \right) \left( \frac{\partial v}{\partial t} + \mathbf{v} \cdot \nabla v \right) &= -a_g \frac{\partial p}{\partial z} - \rho_g g - a_g^2 \frac{\mu g}{k} v_z - (1-\mu_g) \rho_{\sigma \sigma} W, \\
\frac{\partial (\rho_g v)}{\partial t} + \frac{\partial (\rho_g v)}{\partial r} + \frac{v}{r} \frac{\partial (\rho_g v)}{\partial z} &= (1-\mu_g) \rho_{\sigma \sigma} W, \\
\rho \left( \frac{\partial C}{\partial t} + \mathbf{v} \cdot \nabla C \right) &= \frac{\partial (\rho_0 D_0) \frac{\partial C}{\partial r}}{\partial r} + \frac{\partial (\rho_s D_s) \frac{\partial C}{\partial z}}{\partial z} + \rho_s D_s \left( \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2} \right) - \\
&- \mu_p \rho_{\sigma \sigma} W - (1-\mu_g) \rho_{\sigma \sigma} W, \\
W &= (1-\eta) C k e^{E \frac{E}{RT}}, \\
- \frac{\partial p}{\partial r} &= W, \\
\rho_{\sigma \sigma} &= (1-\eta) \mu_{\sigma \sigma}, \\
\mu_{\sigma \sigma} &= \mu_p \rho_{\sigma \sigma} W, \\
a_g &= a_{g0} + (a_{g0} - a_{gEnd}) \eta, \\
D_g &= D_0 \left( \frac{T_g}{273} \right), \\
\mu &= \frac{T_g^{1.5}}{c_s + T_g}.
\end{align*}$$

Here $a$ is the porosity, $b$ is the exponent in the expression for diffusion coefficient, $C$ is the mass concentration of the oxidizer, $c$ is the heat capacity, $c_{11}$ and $c_{12}$ are the constant in Sutherland’s formula, $D_g$ is the diffusion coefficient of gas, $E$ is the activation energy, $g$ is the gravity acceleration, $k$ is the pre-exponential factor in the expression for the rate of reaction, $p$ is the gas pressure, $Q$ is the heat release of reaction, $R$ is the universal gas constant, $t$ is the time, $T$ is the temperature, $v_g$ is the gas velocity, $W$ is the rate of chemical reaction, $\alpha$ is the constant determining the interphase heat transfer intensity, $\eta$ is the degree of conversion of the combustible component of solid medium, $\chi$ is the thermal conductivity, $\mu$ is the dynamic viscosity of gas, $\rho$ is the effective density, $\nabla$ is the nabla operator and $\Delta$ is the Laplace operator. The subscripts are as follows: ‘0’ denotes the initial moment, ‘c’...
denotes the condensed phase, ‘End’ denotes the end point, ‘i’ denotes the inert component, ‘f’ denotes the fuel, ‘g’ denotes the gas and ‘p’ denotes the product.

At the inlet to the porous object the gas pressure, temperature of gas and mass concentration of oxidizer are known. At the output of the object the gas pressure is known, as the gas flows into the open space. At the inlet and outlet of the porous object and on the impermeable walls the heat exchange conditions are also known. The gas velocity and gas flow rate at the outlet of the object are unknown and have to be found from the solution of the problem. So, we can write the boundary conditions in the following form:

$$
\left. P \right|_{x \in G_1} = P_0, \quad \left. \frac{\partial T}{\partial n} \right|_{x \in G_1} = \frac{\beta H}{\lambda_c} \left( T_g \big|_{x \in G_1} - T_e \big|_{x \in G_1} \right),
$$

$$
T_g \big|_{x \in G_1} = T_{g0} \quad \text{и} \quad C \big|_{x \in G_1} = C_0, \quad \text{если} \quad v_g \big|_{x \in G_1} n \big|_{x \in G_1} \leq 0,
$$

$$
\left. \frac{\partial T}{\partial n} \right|_{x \in G_2} = 0 \quad \text{и} \quad \left. \frac{\partial C}{\partial n} \right|_{x \in G_2} = 0, \quad \text{если} \quad v_g \big|_{x \in G_2} n \big|_{x \in G_2} \geq 0,
$$

where $G_1$ is the object boundary opened to an atmosphere, $G_2$ is the impermeable boundary of the object, $n$ is the outward vector directed normally to $G_1$ or to $G_2$, $C_0$, $p_0$ and $T_{g0}$ are the mass concentration of the oxidizer, the gas pressure and the temperature of gas at the inlet of the object, $\beta$ is the heat removal coefficient.

A numerical method based on combination of explicit and implicit finite difference schemes was used to solve the problem. This method is a result of development of numerical method used for plane gas flows through porous media with heterogeneous combustion sources [7, 8]. According to the method the energy equations, equation of concentration and momentum conservation equation are transformed into the explicit finite difference equations. From these equations we find the temperature of solid media, gas temperature, the velocities of gas and mass concentration of oxidizer. The continuity equation is transformed into implicit finite difference equation. From this equation we find the gas pressure taking into account the equation of gas state using Thomas algorithm [13].

3. Results

We consider the axisymmetric porous object with heterogeneous combustion sources when ignition zone was located in the centre of object bottom, in the centre of object and near side walls at the object bottom. The figures 1-3 shows the field of the gas velocity, the temperature of solid medium, the gas pressure and the degree of conversion of the solid combustible component in some time after ignition when the ignition zone is located in the center of the object, in the center of the object bottom and near side walls at the object bottom respectively with the problem parameters used in the work [8]. We can see that as in the plane case, the gas tends to bend around the heated zone and prefers to flow through colder areas.

The figure 4 shows the degree of conversion of the solid combustible component at the different pressure at the object inlet when the ignition zone is located in the centre of the object bottom. We can see that the highest degree of conversion is observed at the lowest pressure.
Figure 1. The field of the gas velocity (a), the temperature of solid medium (b), the gas pressure (c) and the degree of conversion of the solid combustible component (d) in 5 hours after ignition when the ignition zone is located in the center of the object.

Figure 2. The field of the gas velocity (a), the temperature of solid medium (b), the gas pressure (c) and the degree of conversion of the solid combustible component (d) in 7 hours after ignition when the ignition zone is located in the center of the object bottom.
Figure 3. The field of the gas velocity (a), the temperature of solid medium (b), the gas pressure (c) and the degree of conversion of the solid combustible component (d) in 7 hours after ignition when the ignition zone is located near side walls at the object bottom.

Figure 4. Degree of conversion of the solid combustible component after the end of combustion when the ignition zone is located in center of the object bottom and the pressures at the inlet is equal to 105 kPa (a), 110 kPa (b) and 120 kPa (c).
4. Conclusion
The gas flows through axisymmetric porous object with heterogeneous combustion sources under forced filtration were investigated. The various locations of ignition zone and various gas pressure at object bottom were considered. It is shown that the main features coincide with the plane case: the gas tends to go around the heated zone and flow through colder areas, the size of burn-out part of the porous object strongly depends on the location of the ignition zone and gas pressure at the object inlet.

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