A Sphericity Error Assessment Application Based on Whale Optimization Algorithm

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Abstract: In recent years, the sphericity error calculation has been increasingly favored by technicians. In order to further improve the calculation accuracy and iterative convergence speed of sphericity, the whale optimization algorithm is proposed to solve the sphericity calculation model. First of all, according to the standard, the mathematical model of sphericity is established under the minimum zone method. Secondly, the whale optimization algorithm is introduced. Finally, the algorithm is validated by the instance data. The results show that the algorithm is effective and advantageous in the calculation of sphericity.

1. Introduction

Due to the error in the parts manufacture, shape error has become an important technical indicator for controlling the quality of parts manufacturing. Among them, roundness, straightness, cylindricity and flatness have uniform standards and specifications. As a special shape factor, sphericity errors are also receiving more and more attention. As one of the geometrical elements in the revolving parts, scholars usually use the roundness error method to evaluate the sphericity error. At this stage, the sphericity error algorithm mainly includes the minimum circumscribed ball method, the maximum inbound ball method, the least squares ball method and the minimum area method. Among them, the minimum area method can meet the principle of minimum shape error in national standards, and the minimum area method has the highest precision. However, the minimum area method lacks a specific calculation formula. Therefore, many scholars at home and abroad have proposed various calculation methods.

There are many literature about the sphericity error evaluation algorithm. In 1992, An Libang et al. used linear saddle point programming to calculate the sphericity error. In 2000, Liu Wenwen et al. constructed a mathematical model for nonlinear sphericity error and he used the optimization theory to solve the problem. In 2011, Luo Jun et al. proposed an improved bee colony algorithm. In 2012, Hu Jie et al. used an improved particle swarm optimization algorithm to calculate the sphericity error. In 2015, Liu Fei et al. used the string intercept method to evaluate the sphericity error. All of the algorithms can accurately measure the sphericity error in accuracy and stability, but this method has certain defects, and this method is easy to be ended in the local optimal problem. In order to further improve the accuracy and iterative convergence speed of the algorithm, this paper introduces the new swarm optimization algorithm whale optimization algorithm into the sphericity error evaluation.

The Whale Optimization Algorithm (WOA) has many advantages, including less control parameters and good computational stability. This method has been favored by technicians in various fields. Scholars have applied this method to engineering fields (such as reservoir gate flow scheduling optimization and turbine output power optimization) and it has achieved good calculation results.
Therefore, in order to further improve the calculation accuracy and stability of the sphericity error, this paper applies the whale optimization algorithm to the sphericity error calculation, and uses the relevant data to verify the experiment, which shows the superiority of the algorithm.

2. Whale optimization algorithm

The Whale Optimization Algorithm (WOA) is a new group intelligent optimization algorithm proposed by Mirjalili. The algorithm simulates the search behavior, enveloping behavior and attack predation behavior of the whale. The calculation principle is easy to understand and the control parameters are easy to set. For the basic whale optimization algorithm, the number of whale populations is \( N \), the search space dimension is \( M \), and the position of the global optimal solution is \( X_{\text{best}} = (x_{\text{best,1}}, x_{\text{best,2}}, \ldots x_{\text{best,M}}) \), and the number of iterations is \( T \). The whale optimization algorithm has the following basic steps.

2.1 Wandering for food

During the wandering phase, humpback whales need to obtain information on prey through group cooperation. Therefore, humpback whales will close and track the prey near themselves, and through continuous search, they gradually obtains the optimized solution. The mathematical expression for the process is shown in (1)-(3).

\[
H = |(2 \cdot \text{rand}(0,1)) \cdot X_{\text{best}} - X(t)|
\]

\[
X(t + 1) = X_{\text{best}} - (2 \cdot p \cdot \text{rand}(0,1) - p) \cdot H
\]

\[
p = 2 - 2t/T
\]

Where \( H \) is the distance between the whale and the current prey; \( t \) is the number of iterations; \( X_{\text{best}} \) is the position of the optimal solution; \( X(t) \) is the position of the whale; \( p \) is the linear decreasing coefficient; \( \text{rand}(0,1) \) is the random number from 0 to 1, and \( T \) is the maximum number of iterations.

2.2 Surrounding the prey

In the process, which the author refers to it the spiral update strategy, it is assumed that the whale has a certain probability to carry out the strategy when it surrounds the prey, thus this paper further updates the position information of the humpback whale. The mathematical model in the process is shown in formula (4).

\[
X(t + 1) = |X_{\text{best}} - X(t)| \cdot e^{bq} \cdot \cos(2\pi q) + X_{\text{best}}
\]

Where \( b \) is a constant coefficient; \( q \) is a random number between (0, 1).

According to the description in the algorithm, the further update strategy mathematical model is shown in formula (5):

\[
X(t + 1) = \begin{cases} 
X(t) - (2 \cdot p \cdot \text{rand}(0,1) - p) \cdot H, p < 0.5 \\
|X_{\text{best}} - X(t)| \cdot e^{bq} \cdot \cos(2\pi q) + X_{\text{best}}, p \geq 0.5 
\end{cases}
\]

2.3 Predation behavior

In addition to the above-mentioned enveloping methods, humpback whales can also take random predation behaviors to search for prey. The principle is that humpback whales capture prey through information exchange between populations. The mathematical model in this method is shown in formula (6).

\[
X(t + 1) = X(t) - (2 \cdot p \cdot \text{rand}(0,1) - p) \cdot |(2 \cdot \text{rand}(0,1)) \cdot X_{\text{best}} - X(t)|
\]

3. Mathematical model of sphericity error

The minimum area principle in the sphericity error is the variation between the actual ball and the ideal ball. The magnitude of the variation is the containment area. When the area value of the area reaches the minimum, the specific value of the roundness error is obtained, as shown in Figure 1, \( f \) is the size of the roundness error sought. Suppose a set of sphericity measurement points is \( W = (a_i, b_i, \ldots c_i) \), \( k \) is the total number of measurement points, and \( O(a_0, b_0, c_0) \) is the spherical center coordinate. Then the
mathematical model in the minimum area $f$ can be obtained according to the relevant definition in the standard, which is shown in formula (7) and formula (8).

$$f(a_0, b_0, c_0) = \min (\max(r_i) - \min(r_i))$$  \hspace{1cm} (7)

$$r_i = \sqrt{(a_i - a_0)^2 + (b_i - b_0)^2 + (c_i - c_0)^2} \hspace{1cm} i=1,2,\ldots,n$$  \hspace{1cm} (8)

Figure 1 The diagrammatic sketch of sphericity error

4. Experiment and discussion

In order to verify the effectiveness of the whale optimization algorithm, the algorithm is first tested by the test function. The $\text{Sphere} = \sum_{i=1}^{n} x_i^2$ function is a commonly used function to test the iterative speed and precision of the algorithm. The image is shown in Figure 2. Since the sphericity error is a three-dimensional function, the whale optimization algorithm is tested in three-dimensional space and another new intelligent optimization algorithm (Dragonfly Algorithm, DA) is used for comparison. Set the population as 30 and the maximum number of iterations as 500. The function calculation times is 10. The minimum average value of WOA optimization is $4.3E-102$, the minimum value is $1.43E-105$, the maximum value is $3.85E-105$, and the standard deviation is $1.30E-201$. The iteration curve is shown in Figure 3. It can be seen that, compared to DA, WOA has higher precision.

Figure 2 The functional image of Sphere function

Figure 3 The figure of algorithm iterations
To test the algorithm in the sphericity error, the sphericity error measurement data in the relevant literature [12] is selected for experimental verification. The calculation results are compared with relevant literature. Figure 4 is an iteration curve for the sphericity error under WOA. It can be seen that after ten calculations, the average sphericity error calculated by WOA is 0.9667mm, and the spherical center coordinate of \( O(a_0, b_0, c_0) \) is (0.0035, -0.0033, -2.87e-04). Compared with the calculation data, the data is smaller than the algorithm in the literature, and compared with the results of the immune evolution algorithm in the literature [3] (IEA: 0.009966mm; IGA: 0.009678mm; EGA: Compared with 0.056038mm), the data is also smaller than the calculated error value. In addition, the average convergence algebra of the WOA algorithm is 60 generations, which can complete the calculation relatively quickly. It is a stable and accurate sphericity error calculation algorithm.

Table 1 Data of sphericity error [12] (mm)

| No | x    | y    | z    | No | x    | y    | z    |
|----|------|------|------|----|------|------|------|
| 1  | 12.6709 | 2.2467 | 48.3276 | 19 | -45.6577 | -20.8342 | 8.6855 |
| 2  | 10.6074 | 7.4263 | 48.7619 | 20 | -31.6754 | -47.6001 | 8.6893 |
| 3  | 6.4751 | 11.2152 | 48.3306 | 21 | -12.7544 | -48.5287 | 8.6889 |
| 4  | -4.4291 | 12.1689 | 48.3296 | 22 | 8.5599 | -45.5287 | 8.6889 |
| 5  | -9.1564 | 9.1564 | 48.3267 | 23 | 28.2626 | -40.3632 | 8.6884 |
| 6  | -12.1674 | 4.4286 | 48.3238 | 24 | 42.6737 | -24.6377 | 8.6886 |
| 7  | -12.9006 | -1.1287 | 48.3396 | 25 | 19.8699 | 7.2320 | -45.3458 |
| 8  | -11.2156 | -6.4753 | 48.3325 | 26 | 14.9521 | 14.9512 | -45.3439 |
| 9  | -7.4285 | -10.6090 | 48.3344 | 27 | 7.2323 | 19.8707 | -45.3476 |
| 10 | -2.2489 | -12.7539 | 48.3296 | 28 | 1.8430 | 21.0654 | -45.3476 |
| 11 | 3.3517 | -12.5086 | 48.3296 | 29 | -10.5727 | 18.3125 | -45.3467 |
| 12 | 8.3242 | -9.9204 | 48.3306 | 30 | -17.3203 | 12.1278 | -45.3439 |
| 13 | 42.6720 | 24.6367 | 8.6882 | 31 | -20.8217 | 3.6714 | -45.3412 |
| 14 | 28.2609 | 40.3608 | 8.6879 | 32 | -20.4250 | -5.4729 | -45.3467 |
| 15 | -12.7534 | 47.5963 | 8.6886 | 33 | -16.1994 | -13.5929 | -45.3494 |
| 16 | -31.6730 | 37.7464 | 8.6884 | 34 | -8.9374 | -19.1662 | -45.3412 |
| 17 | -44.6551 | 20.8230 | 8.6879 | 35 | 8.9365 | -19.1643 | -45.3467 |
| 18 | -49.2685 | 0.0000 | 8.6874 | 36 | 16.1987 | -13.5923 | -45.3476 |

Figure 4 Algorithm iterations figure of Sphericity Error

5. Conclusions
Sphericity is a special geometric shape factor, and evaluating sphericity error has been a hot topic research. In order to further improve the calculation accuracy and stability of the sphericity error, the new swarm intelligence optimization algorithm WOA is applied to evaluating sphericity error. By establishing the minimum regional mathematical model on sphericity error, substituting the measuring
data, and the experimental verification and analysis are carried out by the algorithm. The results show that the algorithm can be effectively applied to the sphericity error evaluation.

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