Transverse Fierz-Pauli symmetry.

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Abstract

We consider some flat space theories for spin 2 gravitons, with less invariance than full diffeomorphisms. For the massless case, classical stability and absence of ghosts require invariance under transverse diffeomorphisms (TDiff), $h_{\mu\nu} \mapsto h_{\mu\nu} + 2\partial_\nu(\xi_\mu)$, with $\partial_\mu\xi^\mu = 0$. Generic TDiff invariant theories contain a propagating scalar, which disappears if the symmetry is enhanced in one of two ways. One possibility is to consider full diffeomorphisms (Diff). The other (which we denote WTDiff) adds a Weyl symmetry, by which the Lagrangian becomes independent of the trace. The first possibility corresponds to General Relativity, whereas the second corresponds to “unimodular” gravity (in a certain gauge). Phenomenologically, both options are equally acceptable. For massive gravitons, the situation is more restrictive. Up to field redefinitions, classical stability and absence of ghosts lead directly to the standard Fierz-Pauli Lagrangian. In this sense, the WTDiff theory is more rigid against deformations than linearized GR, since a mass term cannot be added without provoking the appearance of ghosts.
1 Introduction

It has long been known that, in theories containing a massless spin 2 graviton propagating in flat space, unitarity requires invariance under “transverse” diffeomorphisms. The argument runs as follows [1]. Consider a graviton of momentum $k$ travelling in the $z$ direction. The “little group” of Lorentz transformations which leave $k$ invariant has three generators. One of them, $I_z$, corresponds to rotations in the $x, y$ plane. The other two, $I_{0x}$ and $I_{0y}$, correspond to transverse boosts combined with rotations in the $x, z$ and $y, z$ plane\(^2\). This little group is isomorphic to the Euclidean group in 2 dimensions. The standard helicity polarizations of the graviton\(^3\) $h^\pm = i h^x$, transform unitarily under $I_z$ (picking up phases $\exp \pm 2i\theta$ under rotations of angle $\theta$) cfr. [1, 2]. But unitary representations of the non-compact “translations” $I_{0i}$ would be infinite dimensional, leading to an infinite number of polarizations for given $k$. This catastrophic degeneracy is avoided by declaring the equivalence of polarizations which are related to one another by standard gauge transformation $h_{\mu\nu} \mapsto h_{\mu\nu} + 2k_{(\mu} \xi_{\nu)}$. It can then be shown that the effect of $I_{0i}$ on the standard helicity eigenstates is “pure gauge”, and in this sense $I_{0i}$ act trivially, producing no new states of momentum $k$.

The interesting point, however, is that the trace $h = \eta^{\mu\nu} h_{\mu\nu}$ is Lorentz invariant (and hence invariant under $I_{0i}$), and therefore it is sufficient to consider the equivalence under “transverse” gauge transformations, which don’t affect the trace,

$$k_{\mu} \xi^\mu = 0. \quad (1)$$

These form a subgroup which we shall refer to as transverse Fierz-Pauli symmetry, or transverse diffeomorphisms (TDiff).

This paper is devoted to the study of some flat space theories containing spin two, that have less invariance than the full diffeomorphisms (Diff) of the standard Fierz-Pauli Lagrangian. We start our discussion, in Section II, with the most general

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\(^2\)These rotations correct for “aberration” of $k$ under the respective boosts.

\(^3\)Up to overall normalization, $h^+ \equiv dx \otimes dx - dy \otimes dy$ and $h^x \equiv dx \otimes dy + dy \otimes dx$. 
Lorentz invariant Lagrangian for a massless graviton. We show that a simple requirement of classical stability and absence of ghosts leads directly to TDiff invariance (and no more invariance than that).

In the non-linear regime, TDiff symmetry is realized in the so called unimodular theories. The best known example is Einstein’s 1919 theory (cf. [2] for a recent reference), which corresponds to the traceless part of the usual Einstein’s equations. This theory can be obtained from an action which is still generally covariant, but where the determinant of the metric is not dynamical [3]. The equations of motion are obtained from a restricted variational principle where

\[ \delta \sqrt{|g|} = 0. \tag{2} \]

The trace-free equations enjoy the property that a cosmological term in the matter Lagrangian is irrelevant. Nevertheless, the trace of Einstein’s equations can be recovered with the help of the Bianchi identity, and then a cosmological term reappears in the form of an integration constant.

Alternatively, we may start with a variational principle where \( \delta \sqrt{|g|} \) is unrestricted, and with an action which is invariant only under TDiff [4]. Generically, this leads to scalar-tensor theories, where the determinant of the metric plays the role of a new scalar. As we shall see, this new degree of freedom can be eliminated by imposing an additional Weyl symmetry (by which the action becomes independent of the determinant of the metric). Thus, Einstein’s unimodular theory can be thought of as the theory of massless spin two fields which is invariant under TDiff plus certain Weyl transformations.4

Section III is devoted to the massive theory. Mass terms which preserve TDiff would give mass to the new scalar, but not to the spin 2 polarizations. Graviton

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4Also, in the absence of this additional Weyl symmetry, the new scalar can be given a mass (which may be expected from radiative corrections, since it is not protected by any symmetry). If the mass is large enough, this leads us back to the situation where the extra scalar does not propagate at low energies, which is effectively equivalent to the scenario described by Eq. 2.
mass terms necessarily break all invariance under diffeomorphisms. For massive particles, the little group is that of ordinary spatial rotations $O(3)$, which leads to finite dimensional unitary representations without the need of invoking any gauge symmetry. Naively, one might think that this would increase the arbitrariness in the choice of the kinetic term. Nevertheless, as we shall see, the only theory with massive gravitons which is free from tachyons or ghosts is equivalent to the Fierz-Pauli theory, where the kinetic term is invariant under Diff (not just TDiff), and the mass term is of the standard form $m^2(h_{\mu\nu}h^{\mu\nu} - h^2)$ (see also [5]). In Section IV we consider the propagator for TDiff invariant theories, and the coupling to conserved matter sources. We conclude in Section V.

Throughout this paper we will follow the Landau-Lifshitz time-like conventions; the $n$-dimensional flat metric in particular, reads $\eta_{\mu\nu} = \text{diag}(1, -1, \ldots, -1)$. Lagrangians are written in momentum space as well as in configuration space, depending on the context. It is usually trivial to shift from one language to the other.

## 2 Massless theory

Let us begin our discussion with the most general Lorentz invariant local lagrangian for a free massless symmetric tensor field $h_{\mu\nu}$,

$$L = L^I + \beta L^{II} + a L^{III} + b L^{IV},$$

where we have introduced

$$L^I = \frac{1}{4} \partial_\mu h^{\nu\rho} \partial^\mu h_{\nu\rho}, \quad L^{II} = -\frac{1}{2} \partial_\mu h^{\mu\rho} \partial_\nu h^\nu_\rho,$$

$$L^{III} = \frac{1}{2} \partial^\mu h \partial^\rho h_{\mu\rho}, \quad L^{IV} = -\frac{1}{4} \partial_\mu h \partial^\mu h.$$  

The first term is strictly needed for the propagation of spin two particles, and we give it the conventional normalization. Before proceeding to the dynamical analysis, which will be done in Subsection 2.4, it will be useful to consider the possible symmetries of (3) according to the values of $\beta$, $a$ and $b$. 

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2.1 TDiff and enhanced symmetries.

Under a general transformation of the fields $h_{\mu\nu} \mapsto h_{\mu\nu} + \delta h_{\mu\nu}$, and up to total derivatives, we have

$$
\begin{align*}
\delta L^{I} &= -\frac{1}{2} \delta h_{\mu\nu} \Box h^{\mu\nu}, \\
\delta L^{II} &= \delta h_{\mu\nu} \partial^\rho \partial^{(\mu} h_{\nu)}^\nu, \\
\delta L^{III} &= -\frac{1}{2} \left( \delta h \partial^\mu \partial^\nu h_{\mu\nu} + \delta h_{\mu\nu} \partial^\mu \partial^\nu h \right), \\
\delta L^{IV} &= \frac{1}{2} \delta h \Box h.
\end{align*}
$$

(5)

It follows that the combination

$$
L_A \equiv L^I + L^{II}
$$

(6)

is invariant under restricted gauge transformations

$$
\delta h_{\mu\nu} = 2 \partial_{(\mu} \xi_{\nu)},
$$

(7)

with

$$
\partial_{\mu} \xi^{\mu} = 0.
$$

(8)

Since $L^{III}$ and $L^{IV}$ are (separately) invariant under this symmetry, the most general TDiff invariant Lagrangian has $\beta = 1$, and arbitrary coefficients $a$ and $b$:

$$
L_{TDiff} \equiv L_A + a L^{III} + b L^{IV}.
$$

(9)

An enhanced symmetry can be obtained by adjusting $a$ and $b$ appropriately. For instance, $a = b = 1$ corresponds to the Fierz-Pauli Lagrangian \[6\], which is invariant under full diffeomorphisms (Diff), where the condition \(8\) is dropped. In fact, a one parameter family of Lagrangians can be obtained from the Fierz-Pauli one through non-derivative field redefinitions,

$$
h_{\mu\nu} \mapsto h_{\mu\nu} + \lambda h \eta_{\mu\nu}, \quad (\lambda \neq -1/n)
$$

(10)
where \( n \) is the space-time dimension and the condition \( \lambda \neq -1/n \) is necessary for the transformation to be invertible. Under this redefinition, the parameters in the Lagrangian (9) change as

\[
a \mapsto a + \lambda (an - 2), \quad b \mapsto b + 2\lambda(nb - a - 1) + \lambda^2(bn^2 - n(2a + 1) + 2).
\]

Starting from \( a = b = 1 \), the new parameters are related by

\[
b = \frac{1 - 2a + (n - 1)a^2}{(n - 2)}.
\]

It follows that Lagrangians where this relation is satisfied are equivalent to Fierz-Pauli, with the exception of the case \( a = 2/n \), which cannot be reached from \( a = 1 \) with \( \lambda \neq -1/n \).

A second possibility is to enhance TDiff with an additional Weyl symmetry,

\[
\delta h_{\mu\nu} = \frac{2}{n} \phi \eta_{\mu\nu},
\]

by which the action becomes independent of the trace. In the generic transverse Lagrangian \( \mathcal{L}_{TDiff}[h_{\mu\nu}] \) of Eq. (9), replace \( h_{\mu\nu} \) with the traceless part

\[
h_{\mu\nu} \mapsto \hat{h}_{\mu\nu} \equiv h_{\mu\nu} - (h/n)\eta_{\mu\nu}.
\]

This is formally analogous to (10) with \( \lambda = -1/n \), but cannot be interpreted as a field redefinition. As such, it would be singular, because the trace \( h \) cannot be recovered from \( \hat{h}_{\mu\nu} \). The resulting Lagrangian

\[
\mathcal{L}_{WTDiff}[h_{\mu\nu}] \equiv \mathcal{L}_{TDiff}[\hat{h}_{\mu\nu}],
\]

is still invariant under TDiff [the replacement (14) does not change the coefficients in front of the terms \( \mathcal{L}^I \) and \( \mathcal{L}^{II} \)]. Moreover, it is invariant under (13), since \( \hat{h}_{\mu\nu} \) is. Using (11) with \( \lambda = -1/n \), we immediately find that this “WTDiff” symmetry corresponds to Lagrangian parameters

\[
a = \frac{2}{n}, \quad b = \frac{n + 2}{n^2}.
\]
This is the exceptional case mentioned at the end of the previous paragraph. Note
that the densitized metric \( \bar{g}_{\mu\nu} = g^{-1/n}g_{\mu\nu} \approx \eta_{\mu\nu} + \hat{h}_{\mu\nu} \) enjoys the property that \( \bar{g} = 1 \).
This is the starting point for the non-linear generalization of the WTDiff invariant
theory, which is discussed in Subsection 2.5.

It is easy to show that Diff and WTDiff exhaust all possible enhancements of
TDiff for a Lagrangian of the form (3) (and that, in fact, these are its largest possible
gauge symmetry groups). Note first, that the variation of \( L^I \) involves a term \( \Box h^{\mu\nu} \).
For arbitrary \( h_{\mu\nu} \), this will only cancel against other terms in (5) provided that the
transformation is of the form

\[
\delta h_{\mu\nu} = 2\partial_{(\mu}\xi_{\nu)} + \frac{2\phi}{n} \eta_{\mu\nu},
\]

for some \( \xi^\mu \) and \( \phi \). The vector can be decomposed as

\[
\xi_{\mu} = \eta_{\mu} + \phi_{\mu} \psi
\]

where \( \partial_{\mu}\eta^{\mu} = 0 \). Using (5) we readily find

\[
\delta L = \eta_{\nu}(\beta - 1)\Box(\partial_{\mu}h^{\mu\nu}) \\
+ \frac{\psi}{2} [(b - a)\Box^2 h + (2\beta - a - 1)\Box(\partial_{\mu}\partial_{\nu}h^{\mu\nu})] \\
+ \frac{\phi}{n} [(bn - a - 1)\Box h + (2\beta - na)\partial_{\mu}\partial_{\nu}h^{\mu\nu}].
\]

TDiff corresponds to taking \( \beta = 1 \), with arbitrary transverse \( \eta^{\mu} \) and with \( \phi = \psi = 0 \).
This symmetry can be enhanced with nonvanishing \( \phi \) and \( \psi \) satisfying the relation

\[
n(a - 1)\Box \psi = 2(2 - an)\phi,
\]

provided that

\[
b = \frac{1 - 2a + (n - 1)a^2}{(n - 2)}.
\]

Eq. (20) ensures the cancellation of the terms with \( \partial_{\mu}\partial_{\nu}h^{\mu\nu} \), and Eq. (21) eliminates
terms containing the trace \( h \). Eq. (21) agrees with (12), and therefore the Lagrangian
with the enhanced symmetry is equivalent to Fierz-Pauli, unless \( a = 2/n \), which corresponds to WTDiff\(^5\).

### 2.2 Comparing Diff and WTDiff

Let us briefly consider the differences between the two enhanced symmetry groups. A first question is whether the Fierz-Pauli theory \( \mathcal{L}_{\text{Diff}} \) is classically equivalent to \( \mathcal{L}_{\text{WTDiff}} \). Since Diff includes TDiff, we can use (15) to obtain

\[
\frac{\delta S_{\text{WTDiff}}[h]}{\delta h_{\mu\nu}} = \frac{\delta S_{\text{Diff}}[\hat{h}]}{\delta h_{\rho\sigma}} \left( \delta_{(\rho} \delta_{\sigma)}^{\mu\nu} - \frac{1}{n} \eta_{\rho\sigma} \eta^{\mu\nu} \right).
\]

Hence, the WTDiff equations of motion are traceless

\[
\frac{\delta S_{\text{WTDiff}}[h]}{\delta h_{\mu\nu}} \eta_{\mu\nu} \equiv 0.
\]

In the WTDiff theory, the trace of \( h \) can be changed arbitrarily by a Weyl transformation, and we can always go to the gauge where \( h = 0 \). Likewise, in the familiar Diff theory we can choose a gauge where \( h = 0 \). Then, \( h_{\mu\nu} = \hat{h}_{\mu\nu} \), and the WTDiff equations of motion (e.o.m.) are just the traceless part of the Fierz-Pauli e.o.m. Differentiating Eq. (22) with respect to \( x^\mu \) and using the Bianchi identity

\[
\partial_\rho \left( \frac{\delta S_{\text{Diff}}[h]}{\delta h_{\rho\sigma}} \right) = 0,
\]

one easily finds that \( \delta S_{\text{WTDiff}}[h]/\delta h_{\mu\nu} = 0 \) implies

\[
\frac{\delta S_{\text{Diff}}[h]}{\delta h_{\rho\sigma}} \eta_{\rho\sigma} = \Lambda.
\]

Hence, the trace of the Fierz-Pauli e.o.m. is also recovered from the WTDiff e.o.m. (in the gauge \( h = 0 \)), up to an arbitrary integration constant \( \Lambda \) which plays the role of a cosmological constant\(^6\). Thus, the two theories are closely related, but they are not quite the same.

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\(^5\)Incidentally, it may be noted that for \( n = 2 \) both possibilities coincide, since in this case the symmetry of the Fierz-Pauli Lagrangian is full diffeomorphisms plus Weyl transformations.

\(^6\)Here we assume \( \Lambda = O(h) \).
Let us now consider the relation between the corresponding symmetry groups. Acting infinitesimally on $h_{\mu\nu}$ they give

$$\delta^D h_{\mu\nu} = 2\partial(\mu \xi_\nu) = 2\partial(\mu \eta_\nu) + \partial_\mu \partial_\nu \psi$$  \hspace{1cm} (23)

$$\delta^{WTD} h_{\mu\nu} = 2\partial(\mu \bar{\eta}_\nu) + \frac{2}{n} \phi \eta_{\mu\nu}$$  \hspace{1cm} (24)

where $\partial_\mu \eta^\mu = \partial_\mu \bar{\eta}^\mu = 0$. In (23) we have decomposed $\xi_\nu = \eta_\nu + \partial_\nu \psi$ into transverse and longitudinal part. The intersection of Diff and WTDiff can be found by equating (23) and (24)

$$2\partial(\mu \eta_\nu) + \partial_\mu \partial_\nu \psi = 2\partial(\mu \bar{\eta}_\nu) + \frac{2}{n} \phi \eta_{\mu\nu}.$$  \hspace{1cm} (25)

Taking the trace, we have

$$\Box \psi = 2\phi.$$  \hspace{1cm} (26)

The divergence of (25) now yields

$$\Box (\bar{\eta}_\mu - \eta_\mu) = \frac{n-1}{n} \Box \partial_\mu \psi.$$  \hspace{1cm} (27)

Taking the divergence once more, we have

$$\Box \phi = 0.$$  \hspace{1cm} (28)

Taking the derivative of (27) with respect to $\nu$, symmetrizing with respect to $\mu$ and $\nu$, and using (25) and (26), we have $(n-2)\partial_\mu \partial_\nu \Box \psi = 0$. For $n \neq 2$ this implies $\partial_\mu \partial_\nu \phi = 0$, i.e.

$$\phi = b_\mu x^\mu + c,$$

where $b_\mu$ and $c$ are constants. Hence, not every Weyl transformation belongs to Diff, since only the $\phi$’s which are linear in $x^\mu$ qualify as such. Conversely, the subset of Diff which can be expressed as Weyl transformations are the solutions of the conformal Killing equation for the Minkowski metric [7],

$$\partial(\mu \xi^{CD}_\nu) = \frac{1}{n} \phi \eta_{\mu\nu},$$  \hspace{1cm} (29)

where $\phi = \partial^\rho \xi^{CD}_\rho$ (and, as shown above, $\phi$ has to be a linear function of $x^\mu$). These solutions generate the so called conformal group, which we may denote by CDiff. In
conclusion, the enhanced symmetry groups Diff and WTDiff are not subsets of each other. Rather, their intersection is the set of TDiff plus CDiff.

2.3 Traceless Fierz-Pauli and WTDiff

An alternative route to the WTDiff invariant theory is to try and construct a Lagrangian which will yield the traceless part of Einstein’s equations.

It is clear, however, that we can only obtain traceless equations of motion from a Lagrangian which is invariant under Weyl transformations. If the e.o.m. are traceless, then \( \delta S = 0 \) for variations of the form for \( \delta h_{\mu\nu} \propto \eta_{\mu\nu} \). This symmetry is not included in Diff, and therefore the traceless part of Einstein’s equations cannot be recovered from this Lagrangian in every gauge. Rather, we should look for a Lagrangian which will yield the traceless part of Einstein’s equations in some gauge.

Let us consider the Diff e.o.m. in momentum space

$$\frac{\delta S_{\text{Diff}}[h]}{\delta h_{\rho\sigma}} = K_{\text{Diff}}^{\rho\sigma\mu\nu} h_{\mu\nu},$$

where

$$8K_{\text{Diff}}^{\mu\nu\rho\sigma} = k^2 (\eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\mu\sigma} \eta^{\nu\rho} - 2\eta^{\mu\nu} \eta^{\rho\sigma}) -$$

$$\left(k^{\mu} k^{\rho} \eta^{\nu\sigma} + k^{\mu} k^{\sigma} \eta^{\nu\rho} + k^{\nu} k^{\rho} \eta^{\mu\sigma} + k^{\nu} k^{\sigma} \eta^{\mu\rho} - 2k^{\mu} k^{\nu} \eta^{\rho\sigma} - 2k^{\rho} k^{\sigma} \eta^{\mu\nu}\right).$$

We can also define the traces

$$\text{tr} K_{\text{Diff}}^{\mu\nu} = \eta_{\rho\sigma} K_{\text{Diff}}^{\rho\sigma\mu\nu} = \frac{n-2}{4} (k_{\rho} k_{\sigma} - k^2 \eta_{\rho\sigma}),$$

$$\text{tr tr} K_{\text{Diff}}^{\mu\nu \rho\sigma} = \eta_{\mu\nu \rho\sigma} K_{\text{Diff}}^{\rho\sigma\mu\nu} = -\frac{(n-1)(n-2)}{4} k^2.$$  

The traceless part of the \( K_{\text{Diff}}^{\rho\sigma\mu\nu} \),

$$8K_{\text{Diff}}^{t} = 8 \left(K_{\text{Diff}} - \frac{1}{n} \eta^{\mu\nu} \text{tr} K_{\text{Diff}}^{\rho\sigma}\right),$$

cannot be derived from a Lagrangian as it is not symmetric in the indices \((\rho\sigma)\) vs. \((\mu\nu)\). Nevertheless, we can still define traceless symmetric Lagrangians. One might
think of substituting $\eta^{\mu\nu}$ in the previous expression by $\text{tr} K_{\text{Diff}}^{\mu\nu}$, and dividing by its trace. However, this would be nonlocal.

For a local Lagrangian which is still invariant under TDiff, we must restrict to deformations which correspond to changes in the parameters $a$ and $b$ in (3). The most general symmetric Lagrangian with these properties is of the form

$$K_{t\text{Diff}}^{\mu\nu\rho\sigma} \equiv K_{\text{Diff}}^{\mu\nu\rho\sigma} - \eta^{\mu\nu} M^{\rho\sigma} - M^{\mu\nu} \eta^{\rho\sigma},$$

with $M_{\rho\sigma}$ a symmetric operator at most quadratic in the momentum. Asking that the result be traceless leads to:

$$M^{\mu\nu} = \frac{1}{n} \left( \text{tr} K_{\text{Diff}}^{\mu\nu} - (\text{tr} M) \eta^{\mu\nu} \right),$$

which implies

$$\text{tr} M = \frac{1}{2n} \text{tr tr} K_{\text{Diff}}.$$

Therefore

$$M^{\mu\nu} = \frac{1}{n} \left( \text{tr} K_{\text{Diff}}^{\mu\nu} - \frac{1}{2n} (\text{tr tr} K_{\text{Diff}}) \eta^{\mu\nu} \right),$$

and we can write

$$8 K_{t\text{Diff}}^{\mu\nu\rho\sigma} = k^2 (\eta_{\mu\rho} \eta_{\nu\sigma} + \eta_{\mu\sigma} \eta_{\nu\rho}) - (k_{\mu} k_{\rho} \eta_{\nu\sigma} + k_{\nu} k_{\sigma} \eta_{\mu\rho} + k_{\mu} k_{\sigma} \eta_{\nu\rho} + k_{\nu} k_{\rho} \eta_{\mu\sigma})$$

$$- \frac{2(n + 2)}{n^2} k^2 \eta_{\mu\nu} \eta_{\rho\sigma} + \frac{4}{n} (k_{\mu} k_{\nu} \eta_{\rho\sigma} + k_{\rho} k_{\sigma} \eta_{\mu\nu}).$$

Moving back to the position space, this corresponds to the WT Diff Lagrangian, i.e. the case $a = \frac{2}{n}$ and $b = \frac{n+2}{n^2}$ in (9). As shown before, this yields the traceless part of the Fierz-Pauli e.o.m. in the gauge $h = 0$.

A similar analysis could be done for the massive case. However, as we shall see in the next section, the corresponding Lagrangian has a ghost.

### 2.4 Dynamical analysis of the general massless Lagrangian.

The little group argument mentioned in the introduction indicates that the quantum theory is not unitary unless the Lagrangian is invariant under TDiff. In fact, in
the absence of TDiff symmetry the Hamiltonian is unbounded below. This leads to pathologies such as classical instabilities or the existence of ghosts.

To show this, as well as to analyze the physical degrees of freedom of the general massless theory (3), it is very convenient to use the “cosmological” decomposition in terms of scalars, vectors, and tensors under spatial rotations $SO(3)$ (see e.g. [8]),

$$h_{00} = A$$
$$h_{0i} = \partial_i B + V_i$$
$$h_{ij} = \psi \delta_{ij} + \partial_i \partial_j E + 2\partial_i F_j + t_{ij}$$

(39)

where $\partial^i F_i = \partial^i V_i = \partial^i t_{ij} = t^i_i = 0$. The point of this decomposition is that in the linearized theory the scalars ($A, B, \psi, E$), vectors ($V_i, F_i$) and tensors ($t_{ij}$) decouple from each other. Also, we can easily identify the physical degrees of freedom without having to fix a gauge (see Appendix A).

The tensors $t_{ij}$ only contribute to $\mathcal{L}^I$, and one readily finds

$$^{(t)}\mathcal{L} = -\frac{1}{4} t_{ij} \Box t_{ij}$$

(40)

The vectors contribute both to $\mathcal{L}^I$ and $\mathcal{L}^{II}$. Working in Fourier space for the spatial coordinates and after some straightforward algebra, we have

$$^{(v)}\mathcal{L} = \frac{1}{2} k^2 \left( V^i - \dot{F}^i \right)^2 + \frac{1}{2} (\beta - 1) \left( k^2 F^i + \dot{V}^i \right)^2.$$  

(41)

For $\beta = 1$, corresponding to TDiff symmetry, there are no derivatives of $V^i$ in the Lagrangian. Variation with respect to $V^i$ leads to the constraint $V^i - \dot{F}^i = 0$, which upon substitution in (41) shows that there is no vector dynamics.

Other values of $\beta$ lead to pathologies. The Hamiltonian is given by

$$^{(v)}\mathcal{H} = \frac{(\Pi_F + k^2 V)^2}{2k^2} - \frac{[\Pi_V + (1 - \beta) k^2 F]^2}{2(1 - \beta)} + \frac{(1 - \beta) k^4 F^2}{2} - \frac{k^2 V^2}{2},$$

(42)

where the momenta are given by $\Pi_F = k^2 \left( \dot{F} - V \right)$ and $\Pi_V = (\beta - 1) \left( k^2 F + \dot{V} \right)$, and we have suppressed the index $i$ in the vectors $F$ and $V$. Because of the alternating signs in Eq. (42), the Hamiltonian is not bounded below. Generically this leads to a
classical instability. The momenta satisfy the equations \( \dot{\Pi}_F = k^2 \Pi_V \) and \( \dot{\Pi}_V = -\Pi_F \).

These have the general oscillatory solution

\[
|k|\Pi_V + i \Pi_F = C \exp i(|k|t + \phi_0),
\]

where \( C \) and \( \phi_0 \) are real integration constants. On the other hand, \( V \) and \( F \) satisfy

\[
\ddot{V} + k^2 V = \frac{-\beta}{(\beta - 1)} \Pi_F, \tag{43}
\]

\[
\ddot{F} + k^2 F = \frac{\beta}{(\beta - 1)} \Pi_V. \tag{44}
\]

For \( \beta \neq 0 \) these are equations for forced oscillators. For large times, the homogeneous solution becomes irrelevant and we have

\[
V + i|k|F \sim \left( \frac{\beta Ct}{(\beta - 1)|k|} \right) \exp i(|k|t + \phi_0),
\]

whose amplitude grows without bound, linearly with time. This classical instability is not present for \( \beta = 0 \). However, in this case \( F \) and \( V \) decouple and we have

\[
L_{\beta=0}^{(s)} = \frac{1}{2} k^2 (\partial_\mu F^i)^2 - \frac{1}{2} (\partial_\mu V^j)^2,
\]

so \( V_i \) are ghosts.

Hence, the only case where the vector Lagrangian is not problematic is \( \beta = 1 \), corresponding to invariance under TDiff. The scalar Lagrangian is then given by\(^7\)

\[
L_{\text{TDiff}}^{(s)} = \frac{1}{4} \left[ (\partial_\mu A)^2 - 2k^2 (\partial_\mu B)^2 + N(\partial_\mu \psi)^2 - 2k^2 \partial_\mu \psi \partial_\mu E + k^4 (\partial_\mu E)^2 \right] - \frac{1}{2} \left[ (\dot{A} + k^2 B)^2 - k^2 \dot{B}^2 - k^2 \psi^2 + 2k^4 E \psi - k^6 E^2 + 2k^2 \dot{B} (\dot{\psi} - k^2 E) \right] + \frac{a}{2} \left[ (\dot{A} - N \dot{\psi} + k^2 E)(\dot{A} + k^2 B) - k^2 (A - N \psi + k^2 E)(\dot{B} - \psi + k^2 E) \right] - \frac{b}{4} \left[ \partial_\mu (A - N \psi + k^2 E) \right]^2, \tag{45}
\]

where \( N = n - 1 \) is the dimension of space. It is easy to check that \( B \) is a Lagrange multiplier, leading to the constraint

\[
(N - 1) \psi = (a - 1) \hbar, \tag{46}
\]

\(^7\)The equivalent expression in terms of gauge invariant combinations is given in Appendix A.
where $h = A - N\psi + k^2E$ is the trace of the metric perturbation. Substituting this back into the scalar action (45) we readily find

$$^{(s)}\mathcal{L}_{\text{TDiff}} = -\frac{\Delta b}{4}(\partial_\mu h)^2,$$

(47)

where

$$\Delta b \equiv b - \frac{1 - 2a + (n - 1)a^2}{n - 2}.$$  

(48)

Hence, the scalar sector contains a single physical degree of freedom, proportional to the trace. Whether this scalar is a ghost or not is determined by the parameters $a$ and $b$. For $b = (1 - 2a + (n - 1)a^2)/(n - 2)$, corresponding to the enhanced symmetries which we studied in the previous subsection, the scalar sector disappears completely, and we are just left with the tensor modes.

### 2.5 Nonlinear theory

Non-linear generalizations of TDiff invariant theories have been discussed in [4] (see also [9]). The basic idea is to split the metric degrees of freedom into the determinant $g$, and a new metric $\hat{g}_{\mu\nu} = |g|^{-1/n}g_{\mu\nu}$, whose determinant is fixed $|\hat{g}| = 1$. Note that $\hat{g}_{\mu\nu}$ is a tensor density, and under arbitrary diffeomorphisms [for which $\delta_{\xi}g_{\mu\nu} = 2\nabla_{(\mu}\xi_{\nu)}$] it transforms as

$$\delta_{\xi}\hat{g}_{\mu\nu} = 2\hat{g}_{\lambda(\mu}\hat{\nabla}_{\nu)}\xi^\lambda - \frac{2}{n}\hat{g}_{\mu\nu}\hat{\nabla}\lambda\xi^\lambda,$$

(49)

where $\hat{\nabla}$ denotes covariant derivative with respect to $\hat{g}_{\mu\nu}$. Next, one defines transverse diffeomorphisms as those which satisfy

$$\hat{\nabla}_\mu\xi^\mu = \partial_\mu\xi^\mu = 0,$$

(50)

where in the first equality we have used $|\hat{g}| = 1$. Under such TDiff, the new metric transforms as a tensor

$$\delta_{\xi}\hat{g}_{\mu\nu} = 2\hat{g}_{\lambda(\mu}\hat{\nabla}_{\nu)}\xi^\lambda,$$

while $g$ transforms as a scalar

$$\delta_{\xi}g = \xi^\lambda\partial_\lambda g.$$
Moreover, the only tensors under TDiff which can be constructed from $\hat{g}_{\mu\nu}$ are the geometric ones, such as $R_{\mu\nu\rho\sigma}[\hat{g}]$ and its contractions. It follows that the most general action invariant under TDiff which contains at most two derivatives of the metric takes the form

\[ S = \int \left( -\frac{\chi^2[g,\psi]}{2} R[\hat{g}_{\mu\nu}] + L[g,\psi,\hat{g}_{\mu\nu}] \right) d^nx. \quad (51) \]

Here, $\chi$ is a scalar made out of the matter fields $\psi$ and $g$. Thus, the TDiff invariant theories can be seen as “unimodular” scalar-tensor theories, where $g$ plays the role of an additional scalar. These are very similar to the standard scalar-tensor theories, except for the presence of an arbitrary integration constant in the effective potential.

Following [4], we may go to the Einstein frame by defining $\bar{g}_{\mu\nu} = \chi^2 \hat{g}_{\mu\nu}$, and we have

\[ S = -\frac{1}{2} \int \sqrt{-\bar{g}} \left( \bar{R} \bar{g}_{\mu\nu} \right) d^nx + S_M + \int \Lambda d^nx, \quad (52) \]

where

\[ S_M = \int \sqrt{-\bar{g}} \left[ \frac{(n-1)(n-2)}{2\chi^2} \bar{g}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi + \chi^{-n} L[\chi,\psi,\bar{g}_{\mu\nu}] - \chi^{-n} \Lambda \right] d^nx. \quad (53) \]

Here, we have first eliminated $g$ in favor of $\chi$, and we have then implemented the constraint $\bar{g} = \chi^{2n}[g,\psi]$ through the Lagrange multiplier $\Lambda(x)$. Note that the invariance under full diffeomorphisms which treat $\bar{g}_{\mu\nu}$ as a metric and $\chi$ and $\Lambda$ as scalars is only broken by the last term in (52). In particular, $S_M$ is Diff invariant, and since $\delta_\xi \Lambda = \xi^\mu \partial_\mu \Lambda$, it is straightforward to show that if the equations of motion for $\psi$, $\chi$ and $\Lambda$ are satisfied, then

\[ \left| \bar{g} \right|^{1/2} \bar{\nabla}^\mu T_{\mu\nu} = \partial_\mu \Lambda. \]

Here, we have introduced $T_{\mu\nu} = -2\bar{g}^{-1/2} \delta S_M / \delta \bar{g}_{\mu\nu}$. On the other hand, the Einstein’s equations which follow from (52) imply the conservation of the source $\tilde{\nabla}^\mu T_{\mu\nu} = 0$, and therefore we are led to

\[ \Lambda = const. \]

This is the arbitrary integration constant, which will feed into the equations of motion as an extra term in the potential for $\chi$, corresponding to the last term in Eq. (53). In
general, this will shift the height and position of the minima of the potential for the scalar fields on which $\chi$ depends. In the particular case where we have $\chi[g,\psi] = 1$ in Eq. (51), the effect is just an arbitrary shift in the cosmological constant.

Diff invariance is recovered when all terms in $S_M$, given in Eq. (53), except for the last one, are independent of $\chi$. In that case, $\chi$ is a Lagrange multiplier which sets $\Lambda = 0$, so the freedom to choose the height (or position) of the minimum of the potential is lost.

Likewise, if the action (51) does not depend on $g$, then the symmetry is WTDiff. The situation is exactly the same as in the TDiff case, where now $\chi = \chi[\psi]$. For instance the simple action

$$S_{\text{WTDiff}} = -\frac{1}{2} \int d^n x \, R[\hat{g}_{\mu\nu}],$$

which has $\chi = 1$, leads to the equations of motion

$$\hat{R}_{\mu\nu} - \frac{1}{2} \hat{g}_{\mu\nu} = \Lambda \hat{g}_{\mu\nu},$$

with arbitrary integration constant $\Lambda$ (note that in this case $\hat{g}_{\mu\nu} = \bar{g}_{\mu\nu}$). This coincides with the standard Einstein’s equations in the gauge $|g| = 1$. The same action can be expressed in terms of the “original” metric $g_{\mu\nu}$ as

$$S_{\text{WTDiff}} = -\frac{1}{2} \int d^n x (-g)^{1/n} \left( R[g_{\mu\nu}] + \frac{(n-1)(n-2)}{4n^2} \partial^\mu \ln g \partial_\mu \ln g \right).$$

This is invariant under Weyl transformations

$$g_{\mu\nu} \rightarrow \Omega^2(x)g_{\mu\nu},$$

since $\hat{g}_{\mu\nu}$ is unaffected by these. Of course, it is also invariant under transverse diffeomorphisms and provides, therefore, an example of a consistent non-linear completion of a pure spin two Lagrangian (namely the WTDiff Lagrangian which we considered in Section 2), which is different from GR.

Note that the equations of motion can be derived in two different ways: directly from (54) under restricted variations of $\hat{g}_{\mu\nu}$ (since by definition $|\hat{g}| = 1$), or from
under *unrestricted* variations of $g_{\mu\nu}$. Whichever representation is used may be a matter of convenience, but there seems to be no fundamental difference between the two. In the latter case, the equations of motion will be completely equivalent to (55), although they will only take the same form in the gauge $|g| = 1$.

It is worth mentioning that equations of the form (55) with an arbitrary $\Lambda$ can also be derived under *unrestricted* variations of an action which is not invariant under (57). An example is given by

$$S = -\frac{1}{2} \int \left[ \sqrt{-g} R + f(g) \right] d^nx, \quad (58)$$

Here, the second term breaks Diff to TDiff, and there is no Weyl invariance. However, the equations of motion will give

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \sqrt{-g} f'(g) g_{\mu\nu},$$

and from the Bianchi identities it follows that $g$ is an arbitrary constant (except in the Diff invariant case when $f \propto \sqrt{-g}$), a situation identical to (55). It is unclear whether the action (58) is of any fundamental significance, since the remaining TDiff symmetry does not forbid an arbitrary function of $g$ in front of $R$, and additional kinetic terms for $g$. Nevertheless, Lagrangians similar to (58) do arise in the context of certain bigravity theories where the interaction term between two gravitons breaks Diff $\times$ Diff to the diagonal Diff times a TDiff symmetry [10].

To conclude, it should be stressed that it seems to be very difficult to determine from experiment whether Diff, WTDiff or just TDiff is the relevant symmetry. The difference between WTDiff and the rest of TDiff theories is just the absence of the extra scalar. However, this scalar may well have a mass comparable to the cut-off scale, and in this case it would not be seen at low energies. Also, at the classical level, the WTDiff differs only from Diff in that the cosmological constant is arbitrary. Of course the measurement of this constant does not reveal too much about its origin. Therefore, the only “observable” differences between both theories may be in the quantum theory [2, 3, 11, 12, 13, 14].
3 Massive fields.

Let us now turn our attention to the massive case. The most general mass term takes the form

\[ \mathcal{L}_m = -\frac{1}{4} m_1^2 h_{\mu\nu} h^{\mu\nu} + \frac{1}{4} m_2^2 h^2. \]

First of all, let us note that for \( m_1 = 0 \), this mass term is still invariant under TDiff. The term \( m_2^2 h^2 \) gives a mass to the scalar \( h \), but not to the tensor modes \( t_{ij} \), which are traceless. Hence, the analysis of the previous Section remains basically unchanged. If \( -m_2^2 > 0 \) is larger than the energy scales we are interested in, the extra scalar effectively decouples, and we are back to the situation where only the standard helicity polarizations of the graviton are allowed to propagate.\(^9\)

When \( m_1 \neq 0 \), we must repeat the analysis\(^10\). With the decomposition (39), the Lagrangian for the tensor modes becomes

\[ (t) \mathcal{L} = -\frac{1}{4} t^{ij} \left( \Box + m_1^2 \right) t_{ij}, \]

and in order to avoid tachyonic instabilities we need \( m_1^2 > 0 \). For the vector modes, and for \( \beta \neq 1 \), the potential term

\[ \Delta \mathcal{H}_v = \frac{m_1^2}{2} \left[ k^2 (F^i)^2 - (V^i)^2 \right], \]

\(^8\)Here, we are disregarding the possibility of Lorentz breaking mass terms, which has been recently considered in [10, 15].

\(^9\)Note also that the addition of the term \( m_2^2 h^2 \) to both the Diff or the WTDiff Lagrangian does not change the propagating degrees of freedom of the theory. The analogous statement in a non-linear context is illustrated by the Lagrangian (58), where a “potential” \( f(g) \) is added to a Diff invariant Lagrangian (something does change, though, by the addition of the potential, since the new theory does have the arbitrary integration constant \( \Lambda \)). Hence, one may in principle construct classical Lagrangians which propagate only massless spin 2 particles, and whose symmetry is only TDiff, although in this case radiative stability is not guaranteed (i.e. we may expect other terms, such as kinetic terms for the determinant \( g \), which are not protected by the symmetry, to be generated by quantum corrections).

\(^{10}\)For a similar analysis in terms of spin projectors see [4].
is added to (12). The contribution proportional to $V^2$ is negative definite. Hence, to avoid ghosts or tachyons we must take $\beta = 1$. In this case, $\dot{V}^i$ does not appear in the Lagrangian and $V^i$ can be eliminated in favor of $\dot{F}^i$. This leads to

$${}^{(v)}\mathcal{L} = -\frac{1}{2} \left( \frac{k^2 m_1^2}{k^2 + m_1^2} \right) F^i \left( \Box + m_1^2 \right) F^i. \quad (60)$$

Out of the $(N^2 + N - 2)/2$ polarizations of the massive graviton in $N + 1$ dimensions, $(N^2 - N - 2)/2$ of these are expressed as transverse and traceless tensors $t_{ij}$, and $N - 1$ are expressed as transverse vectors $F^i$. The remaining one must be contained in the scalar sector. The scalar Lagrangian can be written as

$${}^{(s)}\mathcal{L} = {}^{(s)}\mathcal{L}_{\text{TDiff}} + {}^{(s)}\mathcal{L}_m, \quad (61)$$

where the first term is given by (45) and the second is given by

$${}^{(s)}\mathcal{L}_m = -\frac{m_1^2}{4} (A^2 - 2k^2B^2 + N\psi^2 - 2k^2\psi E + k^4E^2) + \frac{m_2^2}{4} (A - N\psi + k^2E)^2. \quad (62)$$

Variation with respect to $B$ leads to the constraint

$$m_1^2 B = (1 - a)(\dot{A} + k^2\dot{E}) - (1 - aN)\dot{\psi}. \quad (62)$$

To proceed, it is convenient to eliminate $E$ in favor of the trace $h$,

$$k^2E = h + N\psi - A,$$

and to further express $A$ and $\psi$ in terms of new variables $U$ and $V$,

$$(N - 1) A = (aN - 1) h + [2(N - 1)k^2 - Nm_1^2] U,$$

$$(N - 1) \psi = (a - 1) h - m_1^2 (U - V). \quad (63)$$

With these substitutions, and after some algebra, we find

$${}^{(s)}\mathcal{L} = -\frac{\Delta b}{4} h^2 + \frac{[Nm_1^2 - 2(N - 1)k^2]m_1^2}{4(N - 1)} \left( \dot{V}^2 - \dot{U}^2 \right) + \frac{W(h, U, V)}{4(N - 1)^2}, \quad (64)$$

where $\Delta b$ is given by (48) and

$$W \equiv \left\{ (N - 1)^2(k^2\Delta b + m_2^2) - [1 + (1 - 4a + a^2)N + a^2N^2]m_1^2 \right\} h^2.$$
\[+(N - 1)m_1^4 [(N - 2)k^2 - Nm_1^2] V^2\]
\[-m_1^2 [4(N - 1)^2k^4 + (2 + N - 3N^2)m_1^2k^2 + N(N + 1)m_1^4] U^2\]
\[+4(N - 1)m_1^2k^2[Nm_1^2 - (N - 1)k^2] UV\]
\[+2m_1^2 [(N + 1)a - 2] [(Nm_1^2 - (N - 1)k^2) U - (N - 1)k^2 V] h. \quad (65)\]

For \(2(N - 1)k^2 < Nm_1^2\) the variable \(U\) has negative kinetic energy, whereas for \(2(N - 1)k^2 > Nm_1^2\) the same is true of \(V\). Thus, the Hamiltonian is unbounded below, unless

\[\Delta b = 0. \quad (66)\]

In this case, \(h\) is non-dynamical, and it will implement a constraint between \(U\) and \(V\) provided that the coefficient of \(h^2\) in \(W\) vanishes identically. This requires

\[m_2^2 = \left(1 + \left(\frac{1 - 4a + a^2}{N + 1} \right) N + a^2N^2 \right) m_1^2. \quad (67)\]

As discussed in Section 2, as long as \(a \neq 2/(N + 1)\), all kinetic Lagrangians with \(\Delta b = 0\) are related to the Fierz-Pauli kinetic term by the field redefinition \([10]\). Thus, there are only two possibilities for eliminating the ghost: either the kinetic term is invariant under Diff or it is invariant under WTDiff.

### 3.1 Diff invariant kinetic term

Without loss of generality, we can take \(a = b = 1\), and from \([67]\) we have the usual Fierz-Pauli relation

\[m_1^2 = m_2^2. \]

Variation with respect to \(h\) leads to the constraint

\[(N - 1)k^2 V = [Nm_1^2 - (N - 1)k^2] U. \quad (68)\]

In combination with \([63]\), this yields

\[(N - 1)k^2 \psi = m_1^2 [Nm_1^2 - 2(N - 1)k^2] U. \quad (69)\]
Substituting (68) in the Lagrangian, and using (69) we obtain

\[ (s) \mathcal{L} = -\frac{N}{4(N-1)} \psi(\Box + m_1^2) \psi, \]  

(70)

which is the remaining scalar degree of freedom of the graviton.

The tensor, vector and scalar Lagrangians (59), (60) and (70) are not in a manifestly Lorentz invariant form, and the actual form of the propagating polarizations is obscured by the fact that the components of the metric must be found from \( F^i \) and \( \psi \) with the help of the constraint equations. Nevertheless, once we know that the system has no ghosts and all polarizations have the same dispersion relation, it is trivial to repeat the analysis in the rest frame of the graviton, \( k = 0 \). In this frame, the metric is homogeneous \( \partial_i h_{\mu
u} = 0 \) and we may write

\[ h_{00} = A, \quad h_{0i} = V_i, \quad h_{ij} = \psi \delta_{ij} + t_{ij}, \]

where \( t_i^i = 0 \). The Lagrangian for tensors becomes

\[ (t) \mathcal{L} = -\frac{1}{4} t_{ij} (\Box + m_1^2) t_{ij}, \]  

(71)

Vectors contribute to \( \mathcal{L}^I \) and \( \mathcal{L}^{II} \), giving

\[ (v) \mathcal{L} = \frac{1}{2} (\beta - 1) \dot{V}_i^2 + \frac{1}{2} m_1^2 V_i^2, \]  

(72)

which is non-dynamical in the present case because \( \beta = 1 \). Likewise, it can easily be shown that the scalars \( A \) and \( \psi \) are non-dynamical. Therefore, in the graviton rest frame the propagating polarizations are represented by the \([N(N+1)/2] - 1\) independent components of the symmetric traceless tensor \( t_{ij} \).

### 3.2 WTDiff invariant kinetic term

For \( a = 2/n = 2/(N+1) \), the last term in Eq. (65) disappears, and \( U \) and \( V \) do not mix with \( h \). Because of that, there are no further constraints amongst these variables and the ghost in the kinetic term in (64) is always present for \( m_1^2 \neq 0 \). This means
that the WTDiff theory cannot be deformed with the addition of a mass term for the graviton without provoking the appearance of a ghost.

Note that this is so even in the case of a mass term compatible with the Weyl symmetry, i.e. $m_1^2 = nm_2^2$. This relation causes $h$ to disappear from the Lagrangian, but of course it does nothing to eliminate the ghost.

4 Propagators and coupling to matter

In this section we shall consider the propagators and the coupling to external matter sources, for the different “trouble-free” Lagrangians which we have identified in the previous Sections.

On one hand, we have the standard massless and massive Fierz-Pauli theories, which have been thoroughly studied in the literature. There are also the generic ghost-free TDiff theories, which satisfy the condition

$$\Delta b \equiv b - \frac{1 - 2a + (n - 1)a^2}{n - 2} < 0.$$  \hspace{1cm} (73)

These may include a mass term of the form $m^2 h^2$, which affects the scalar mode but does not give a mass to the tensor modes. The WTDiff invariant theory completes the list of possibilities.

Throughout this Section, we will make use of the spin two projector formalism of [16], which is very useful in order to invert the equations of motion. The properties of these projectors are summarized in Appendix B.

4.1 Gauge Fixing.

As noted in [2], for the TDiff gauge symmetry there is no linear covariant gauge fixing condition which is at most quadratic in the momenta. This is in contrast with the Fierz-Pauli case, where the harmonic condition contains first derivatives only. The basic problem is that a covariant gauge-fixing carries a free index, which leads
to \( n \) independent conditions. This is more than what transverse diffeomorphisms can handle, since these have only \((n - 1)\) independent arbitrary functions. To be specific, let us consider the most general possibility linear in \( k \),

\[
M_{\alpha\beta\gamma} h^{\beta\gamma} = 0, \tag{74}
\]

where

\[
M_{\alpha\beta\gamma} = a_1 \eta_{\alpha(\beta k_{\gamma})} + a_2 \eta_{\beta\gamma} k_\alpha. \tag{75}
\]

In order to bring a generic metric \( h_{\mu\nu} \) to this gauge by means of a TDiff, we have

\[
M_{\alpha\beta\gamma} h^{\beta\gamma} = M_{\alpha\beta\gamma} \partial^\beta \xi^\gamma. \tag{76}
\]

However, deriving the r.h.s. of the previous expression with respect to \( x^\alpha \) and summing in \( \alpha \), this terms cancels, which implies that the integrability condition

\[
\partial^\alpha M_{\alpha\beta\gamma} h^{\beta\gamma} = 0, \tag{77}
\]

must be satisfied. This simply means that the gauge condition cannot be enforced on generic metrics.

It is plain, however, that the transverse part of the harmonic gauge (which contains only \( n - 1 \) independent conditions) can be reached by a transverse gauge transformation. The corresponding gauge fixing piece is obtained by projecting the harmonic condition with \( k^2 \eta_{\mu\nu} - k_\mu k_\nu \equiv k^2 \theta_{\mu\nu} \):

\[
\mathcal{L}_{gf} = \frac{1}{2M^4} (\partial_\alpha \partial^\mu \partial^\nu h_{\mu\nu} - \Box \partial^\mu h_{\alpha\mu})^2 \tag{78}
\]

The gauge fixing parameter is now dimensionful, and this has been explicitly indicated by denoting it by \( M^4 \). A study of this kind of term and its associated FP ghosts and BRST transformations can be found in [13].

By contrast, in the case of WTDiff, the additional Weyl symmetry allows for the use of gauge fixing terms which are linear in the derivatives (such as the standard harmonic gauge).
4.2 Propagators

The generic Lagrangian can be written in Fourier space as
\[
\mathcal{L} = \mathcal{L}^I + \beta \mathcal{L}^I + a \mathcal{L}^{II} + b \mathcal{L}^{IV} + \mathcal{L}_m + \mathcal{L}_{gf} = \frac{1}{4} h_{\mu \nu} K^{\mu \nu \rho \sigma} h_{\rho \sigma} - \\
\frac{1}{4} h_{\mu \nu} \left\{ (k^2 - m_1^2) P_2 + [(1 - \beta) k^2 - m_1^2 + \lambda^2(k)] P_1 + a_s P_0^s + a_w P_0^w + a_\times P_0^\times \right\} ^{\mu \nu \rho \sigma} h_{\rho \sigma},
\]
where \( P_1 \) and \( P_2 \) are the projectors onto the subspaces of spin 1 and spin 0 respectively, while the operators \( P_0^s, P_0^w \) and \( P_0^\times \equiv P_0^{sw} + P_0^{ws} \) project onto and mix the different spin 0 components. The definitions and properties of these operators are discussed in Appendix B. The coefficients in front of the spin 0 projectors are given by
\[
a_s = [1 - (n - 1)b]k^2 - m_1^2 + (n - 1)m_2^2, \\
a_w = (1 - 2\beta + 2a - b)k^2 - m_1^2 + m_2^2, \\
a_\times = \sqrt{n - 1} [(a - b)k^2 + m_2^2].
\]

In (79), we have included the term \( \lambda^2(k)P_1 \) which can be used to gauge fix the TDiff symmetry whenever it is present. Indeed, (78) can be written as
\[
\mathcal{L}_{gf} = \lambda^2(k) h_{\mu \nu} P_1^{\mu \nu \rho \sigma} h_{\rho \sigma}.
\]

where \( \lambda^2(k) = (1/4M^4)k^6 \). Even though we are primarily interested in the TDiff Lagrangian (which corresponds to \( \beta = 1 \)), we have kept generic \( \beta \) throughout this subsection. This can be useful to handle the cases with enhanced symmetry, since a generic \( \beta \) arises, for instance, from the conventional harmonic gauge fixing term (as we shall see below). When invertible, the previous Lagrangian yields a propagator \( \Delta \equiv K^{-1} \),
\[
\Delta = \frac{P_2}{k^2 - m_1^2} + \frac{P_1}{(1 - \beta) k^2 - m_1^2 + \lambda^2(k)} + \frac{1}{g(k)} \left( a_w P_0^s + a_s P_0^w - a_\times P_0^\times \right),
\]
where,
\[
g(k) = a_s a_w - a_\times^2.
\]
Consider a generic coupling of the form

\[ L_{int}(x) = \frac{1}{2} (\kappa_1 T^{\mu\nu} + \kappa_2 T^\eta_{\mu\nu}) h_{\mu\nu} \equiv \frac{1}{2} T_{\text{tot}}^{\mu\nu} h_{\mu\nu}. \]  

(83)

For conserved external sources,

\[ \partial_\mu T^{\mu\nu} = 0, \]  

(84)

this coupling is invariant under TDiff for all values of \( \kappa_1 \) and \( \kappa_2 \). Moreover, it is Diff invariant when \( \kappa_2 = 0 \), and WTDiff invariant for the special case \( \kappa_1 = -n\kappa_2 \). The interaction between sources is completely characterized by [17]

\[ S_{int} \equiv \frac{1}{2} \int d^n k L_{int}(k) = \frac{1}{2} \int d^n k T_{\text{tot}}(k)^\mu_\nu \Delta^{\mu\rho\sigma} T_{\text{tot}}(k)_{\rho\sigma}. \]  

(85)

From the properties of the projectors \( P_i \) listed in Appendix B, it is straightforward to show that

\[ L_{int}(k) = \kappa_1^2 T^*_\mu\nu \left( \frac{P^{\mu\rho\sigma}}{k^2 - m_1^2} \right) T_{\rho\sigma} + P_0 |T|^2, \]  

(86)

where the operator

\[ P_0 = \frac{1}{g(k)} \left[ \kappa_1^2 a_w \right] \right] + 2\kappa_1\kappa_2 \left( a_w - \frac{a_x}{\sqrt{n-1}} \right) + \kappa_2^2 \left[ (n-1)a_w + a_s - 2\sqrt{n-1}a_x \right] \]  

(87)

encodes the contribution of the spin 0 part. We are now ready to consider the different particular cases, which we present by order of increasing symmetry.

### 4.3 Massive Fierz-Pauli

In this case the parameters in the Lagrangian are given by \( \beta = a = b = 1 \) and \( m_1^2 = m_2^2 \). From (82), we have

\[ g(k) = -(n-1) m_2^4, \]

which does not depend on \( k \). Because of that, the denominator of the operator \( P_0 \) does not contain any derivatives. Its contribution to Eq. (86) corresponds only to contact terms, which do not contribute to the interaction between separate sources.
We are thus left with the spin 2 interaction, which ignoring all contact terms, can be written as
\[
\mathcal{L}_{\text{int}} = \kappa_1^2 T_{\mu\nu}^* \left( \frac{P_{\mu\nu}^{\rho\sigma}}{k^2 - m_1^2} \right) T_{\rho\sigma} = \frac{\kappa_1^2}{k^2 - m_1^2} \left[ T_{\mu\nu}^* T^{\mu\nu} - \frac{1}{(n-1)} |T|^2 \right].
\] (88)

The factor \(1/(n - 1)\) is different from the familiar \(1/(n - 2)\) which is encountered in linearized GR, and produces the well known vDVZ discontinuity in the massless limit [18, 19, 20].

4.4 TDiff invariant theory

In this case, we set \(m_1^2 = 0\) and \(\beta = 1\). Note that the gauge fixing term (81) will not play a role, since the term proportional to \(P_1\) does not contribute to the interaction between conserved sources. With these values of the parameters we have
\[
g(k) = (n - 2)(\Delta b \, k^2 - m_2^2) \, k^2,
\] (89)
which is quartic in the momenta. The terms proportional to \(\kappa_2\) in the numerator of Eq. (87) are also proportional to \(k^2\), so this factor drops out and we obtain the propagators for an ordinary massive scalar particle (provided that \(\Delta b < 0\), in agreement with our earlier dynamical analysis).

However, for the first term in Eq. (87) (the one proportional to \(\kappa_1^2\)) there is no global factor of \(k^2\) in the numerator, and we must use the decomposition
\[
\frac{1}{g(k)} = \frac{-1}{(n - 2)m_2^2} \left( \frac{1}{k^2} - \frac{1}{k^2 - m_1^2} \right).
\] (90)

Substituting in (87), and disregarding contact terms, we obtain
\[
P_0 = - \left( \frac{\kappa_1^2}{(n - 1)(n - 2)} \right) \frac{1}{k^2} - \left( \kappa_2 + \frac{1 - a}{n - 2} \kappa_1 \right)^2 \frac{1}{\Delta b k^2 - m_2^2}.
\] (91)

Substituting in (86) and adding the contribution of \(P_2\) for \(m_1^2 = 0\), which can be read off form (88), we have
\[
\mathcal{L}_{\text{int}} = \kappa_1^2 \left[ T_{\mu\nu}^* T^{\mu\nu} - \frac{1}{(n-2)} |T|^2 \right] \frac{1}{k^2} - \left( \kappa_2 + \frac{1 - a}{n - 2} \kappa_1 \right)^2 \frac{|T|^2}{\Delta b \, k^2 - m_2^2}.
\] (92)
Note that the massless propagator in (91) combines with the second term in the spin 2 part to give the factor $1/(n-2)$ in front of $|T|^2$. Eq. (92) shows that the massless interaction between conserved sources is the same as in standard linearized General Relativity.

In addition, there is a massive scalar interaction, with effective mass squared

$$m_{eff}^2 = \frac{m_2^2}{\Delta b} > 0.$$  \hfill (93)

(note that both parameters $m_2^2$ and $\Delta b$ must be negative, according to our earlier analysis), and effective coupling given by

$$\kappa_{eff}^2 = -\frac{1}{\Delta b} \left( \kappa_2 + \frac{1-a}{n-2} \kappa_1 \right)^2.$$  \hfill (94)

These are subject to the standard observational constraints on scalar tensor theories. If the scalar field is long range, then the strength of the new interaction has to be very small $\kappa_{eff} \lesssim 10^{-5} \kappa_1$ \cite{21, 22}. Alternatively, the interaction could be rather strong, but short range, shielded by a sufficiently large mass $m_{eff} \gtrsim (30 \mu m)^{-1}$ \cite{21, 22, 23}.

### 4.5 Enhanced symmetry

From general arguments, the interaction between sources in the WTDiff theory is expected to be the same as in standard massless gravity, since both theories only differ by an integration constant but have the same propagating degrees of freedom.

In fact the result for WTDiff can be obtained from the analysis of the previous Section by setting $\Delta b = 0$. In this case, the term $m_2^2 h^2$ can be thought of as the additional gauge fixing which removes the redundancy under the additional Weyl symmetry. With $\Delta b = 0$ the second term in (92) becomes a contact term, and we recover the same result as in the standard massless Fierz-Pauli theory \cite{17}, \hfill (95)

$$L_{int} = \kappa_1^2 \left[ T_{\mu\nu} T^{\mu\nu} - \frac{1}{(n-2)} |T|^2 \right] \frac{1}{k^2}.$$  

\hfill (95)

\hfill Note also that the WTDiff invariant coupling to conserved sources requires $\kappa_1 = -n \kappa_2$. Using this and $a = 2/n$ in (94) we have $\kappa_{eff} = 0$, which again eliminates the scalar contribution.
as expected.

Note that in the Diff and WTDiff invariant theories, there is a different possibility for gauge fixing. Rather than using the term \[ (81) \] in order to take care of the TDiff part of the symmetry, and then the \( m_2^2 h^2 \) to take care of the Weyl part, we can gauge fix the entire symmetry group with a standard term of the form

\[
L_{gf} = \frac{\alpha}{4} (\partial_\alpha h^{\alpha\mu} + \gamma \partial^\mu h)^2 ,
\]

where \( \alpha \) and \( \gamma \) are arbitrary constants. This can be absorbed in a shift of the parameters \( a, b \) and \( \beta \)

\[
a \mapsto a + \alpha \gamma, \quad b \mapsto b - \frac{\alpha \gamma^2}{2}, \quad \beta \mapsto \beta - \frac{\alpha}{2} .
\]

With these substitutions, the propagator becomes invertible, even if it is not for the original values of \( a, b \) and \( \beta \) which correspond to Diff or to WTDiff. Needless to say, the result calculated in this gauge coincides with (95).

5 Conclusions

In this paper we have expanded somewhat the classification of flat space spin 2 Lagrangians given by van Nieuwenhuizen in [5]. For the massless theory, we have explicitly shown that unless the TDiff symmetry is imposed, the Hamiltonian is unbounded below and a classical instability generically develops. We have also presented in some detail a few potentially interesting theories which are invariant under this symmetry.

Generic massless TDiff theories contain a propagating scalar proportional to the trace \( h \) (note that this is “gauge invariant” under TDiff), which disappears when the symmetry is enhanced in one of two ways. The standard choice is to consider the full group of diffeomorphisms Diff. Another possibility (which we call WTDiff) is to impose an additional Weyl symmetry, by which the action depends only on the traceless part of the metric \( \hat{h}_{\mu\nu} = h_{\mu\nu} - (1/n) \eta_{\mu\nu} \). This theory is equivalent to one in
which the determinant of the metric $\hat{g}_{\mu\nu} = \eta_{\mu\nu} + \hat{h}_{\mu\nu}$ is kept fixed to unity. In practice, however, it may be convenient to use the formulation in which this extra symmetry is present, since it can make the covariant gauge fixing somewhat simpler. Nonlinear extensions of the TDiff theory have been discussed in [4], and they correspond to scalar-tensor theories with an integration constant. The nonlinear extension of the WTDiff theory is a particular case where the additional scalar is not present.

It is sometimes claimed that General Relativity is the only consistent theory for spin 2 gravitons. Such discussions [24, 25, 26, 27, 28], however, always assume linearized Diff invariance as an input. In the light of the present discussion, it seems that linearized TDiff or WTDiff invariances should be just as good starting points.

In fact, it may be very difficult to distinguish between the various options experimentally. The additional scalar may be very heavy, in which case it can be integrated out leaving no distinctive traces at low energies. As for the cosmological constant, it seems clear that we cannot tell from its measurement whether it corresponds to an integration constant or to a fixed parameter in the Lagrangian.

An interesting difference between linearized GR and TDiff invariant theories is that, as we have shown, the latter cannot be extended with a mass term for the graviton without provoking the appearance of a ghost. In this sense, the WTDiff theory is more rigid against small deformations than the standard linearized GR.

It is at present unclear whether Diff (rather than TDiff, or WTDiff) is a fundamental symmetry of Nature. Even in string theory, the connection with GR is on-shell, which does not seem to exclude the fundamental symmetry from being WTDiff (see however [29] for a discussion in the context of closed string field theory). Classically the two theories are almost identical, but there may be important differences in the quantum theory [3, 11, 12, 2, 13]. These deserve further exploration, and are currently under research [14].
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Appendix A. TDiff lagrangians in terms of gauge invariant quantities.

As the lagrangian of \( L_{TDiff} \), is invariant under transverse transformations, one should be able to write it in terms of invariants under this transformations (for the Diff case see e.g. [8, 30, 31]). It is easy to see that under a general transformation \( h_{\mu\nu} \mapsto h_{\mu\nu} + 2\partial_{\mu}\xi_{\nu} \) the fields of the cosmological decomposition transform as

\[
\begin{align*}
t_{ij} & \mapsto t_{ij}, \quad V_i \mapsto V_i + \partial_0 \xi_i^T, \quad F_i \mapsto F_i + \xi_i^T, \quad A \mapsto A + 2\partial_0 \xi_0, \\
B & \mapsto B + \partial_0 \eta + \xi_0, \quad E \mapsto E + 2\eta, \quad \psi \mapsto \psi,
\end{align*}
\]

where \( \xi_i = \xi_i^T + \partial_i \eta \), with \( \partial^i \xi_i^T = 0 \). Whereas for a Weyl transformation \( h_{\mu\nu} \mapsto h_{\mu\nu} + \frac{1}{n} \phi \eta_{\mu\nu} \) only \( A \) and \( \psi \) change as

\[
A \mapsto A + \frac{\phi}{n}, \quad \psi \mapsto \psi - \frac{\phi}{n}.
\]

For general transverse transformations the only gauge invariant combinations are

\[
t_{ij}, \quad w_i = V_i - \partial_0 F_i,
\]

(97)
in the tensor and vector sectors respectively and
\[
\Phi = A - 2\partial_0 B + \partial_0^2 E, \quad \psi, \quad \Theta = (A - \Delta E), \quad (98)
\]
for the scalars. In terms of these combinations, the tensor, vector and scalar part of
the lagrangian (9) can be written as (we write also the mass term \(L^V = -m^2 h^2\))
\[
(\text{t}) L_{\text{TDiff}} = -\frac{1}{4} t^{ij} \Box t_{ij}, \quad (\text{v}) L_{\text{TDiff}} = -\frac{1}{2} w^i \Box w^i,
\]
\[
(\text{s}) L^I = \frac{1}{4} \left( -\dot{\Theta}^2 - \Theta \Delta (\Theta - 2\Phi) - 2\Delta \psi (\Phi - \Theta) + (n - 3)\psi \Delta \psi + (n - 1)\dot{\psi}^2 \right),
\]
\[
(\text{s}) L^{II} = \frac{a}{4} \left( (\Theta - (n - 1)\psi)(\Delta (\Theta - \psi - \Phi) - \ddot{\Theta}) \right),
\]
\[
(\text{s}) L^{III} = \frac{b}{4} \left( (\dot{\Theta} - (n - 1)\psi)^2 + (\Theta - (n - 1)\psi)\Delta (\Theta - (n - 1)\psi) \right),
\]
\[
(\text{s}) L^{IV} = -\frac{m^2}{4} (\Theta - (n - 1)\psi)^2,
\]
where \(\Delta = \sum_i \partial_i \partial_i = -\partial^i \partial_i\). From this decomposition we easily see that \(\Phi\) is a
lagrange multiplier whose variation yields the constraint
\[
\Delta ((1 - (n - 1)a)\psi - (1 - a)\Theta) = 0. \quad (99)
\]

In the case of general diffeomorphisms \((a = b = 1)\), only two scalar combinations
are gauge invariant, namely \(\Phi\) and \(\psi\). Thus, the lagrangian for the scalar part can
be expressed as
\[
(\text{s}) L_{\text{Diff}} = \frac{(2 - n)}{4} \left( -2\Phi \Delta \psi + (n - 1)\dot{\psi}^2 + (n - 3)\psi \Delta \psi \right). \quad (100)
\]

Concerning the Weyl transformations, we can write only two scalar invariants
which are also scalars for transverse transformations,
\[
\Xi = \Phi + \psi, \quad \Upsilon = \Theta + \psi. \quad (101)
\]
Thus, for the Weyl choice \(a = \frac{2}{n}, b = \frac{n+2}{n^2}\), we can write the lagrangian as
\[
(\text{s}) L_{\text{WTDiff}} = \frac{1}{4n^2} \left( (n - 2)(2n\Xi - (n - 1)\Upsilon) \Delta \Upsilon - (2 - 3n + n^2)\dot{\Upsilon}^2 \right). \quad (102)
\]
Thus, varying the lagrangian with respect to \(\Xi\) we find the constraint
\[
\Delta \Upsilon = 0. \quad (103)
\]
Besides, the mass term can be written as

\[(s)\mathcal{L}^V = -\frac{m^2}{4} (\Upsilon - n\psi)^2.\]  

(104)

**Appendix B. Barnes-Rivers Projectors.**

A useful tool for analyzing the lagrangians involving two component tensors is provided by the Barnes and Rivers projectors [16] (see also [5]). We start with the usual transverse and longitudinal projectors

\[\theta_{\alpha\beta} \equiv \eta_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2},\]

\[\omega_{\alpha\beta} \equiv \frac{k_\alpha k_\beta}{k^2}.\]  

(105)

and then define projectors on the subspaces of spin two, spin one, and the two different spin zero components, labelled by \((s)\) and \((w)\). We introduce also the convenient operators that map between these two subspaces.

\[P_2 \equiv \frac{1}{2} (\theta_{\mu\rho}\theta_{\nu\sigma} + \theta_{\mu\sigma}\theta_{\nu\rho}) - \frac{1}{(n-1)}\theta_{\mu\nu}\theta_{\rho\sigma},\]

\[P_0^s \equiv \frac{1}{(n-1)}\theta_{\mu\nu}\theta_{\rho\sigma},\]

\[P_0^w \equiv \omega_{\mu\nu}\omega_{\rho\sigma},\]

\[P_1 \equiv \frac{1}{2} (\theta_{\mu\rho}\omega_{\nu\sigma} + \theta_{\mu\sigma}\omega_{\nu\rho} + \theta_{\nu\rho}\omega_{\mu\sigma} + \theta_{\nu\sigma}\omega_{\mu\rho})\]

\[P_0^{sw} \equiv \frac{1}{\sqrt{(n-1)}}\theta_{\mu\nu}\omega_{\rho\sigma},\quad P_0^{ws} \equiv \frac{1}{\sqrt{(n-1)}}\omega_{\mu\nu}\theta_{\rho\sigma}.\]  

(106)

These projectors obey

\[P_i^a P_j^b = \delta_{ij} \delta^{ab} P_i^b,\]

\[P_i^{ab} P_j^{cd} = \delta_{ij} \delta^{bc} \delta^{ad} P_i^a,\]

\[P_i^a P_j^{bc} = \delta_{ij} \delta^{ab} P_i^{ac},\]

\[P_i^{ab} P_j^{bc} = \delta_{ij} \delta^{bc} P_i^{ac}.\]  

(107)
And the traces:

\[
\begin{align*}
\text{tr} P_2 &\equiv \eta^{\mu\nu}(P_2)_{\mu\nu\rho\sigma} = 0, \quad \text{tr} P_0^s = \theta_{\rho\sigma}, \quad \text{tr} P_0^w = \omega_{\rho\sigma} \\
\text{tr} P_1 &= 0, \quad \text{tr} P_0^{sw} = \sqrt{n-1}\omega_{\rho\sigma}, \quad \text{tr} P_0^{ws} = \frac{1}{\sqrt{n-1}}\theta_{\rho\sigma} \quad (108)
\end{align*}
\]

Apart from the previous expressions, these projectors satisfy

\[
P_2 + P_1 + P_0^{sw} + P_0^{ws} = \frac{1}{2}(\delta_{\mu\nu}\delta_{\rho\sigma} + \delta_{\rho\sigma}\delta_{\mu\nu}) \quad (109)
\]

and any symmetric operator can be written as

\[
K = a_2 P_2 + a_1 P_1 + a_w P_0^{sw} + a_s P_0^{ws} + a_{sw} P_0^x \quad (110)
\]

where \( P_0^x = P_0^{sw} + P_0^{ws} \). The inverse of the previous operator is easily found from (107) to be

\[
K^{-1} = \frac{1}{a_2} P_2 + \frac{1}{a_1} P_1 + \frac{a_s}{a_s a_w - a_{sw}} P_0^{sw} + \frac{a_w}{a_s a_w - a_{sw}} P_0^{ws} - \frac{a_{sw}}{a_s a_w - a_{sw}} (P_0^{ws} + P_0^{sw}) \quad (111)
\]

provided that the discriminant \( a_s a_w - a_{sw}^2 \) never vanishes.

References

[1] J. J. van der Bij, H. van Dam and Y. J. Ng, Physica 116A, 307 (1982).

[2] E. Alvarez, JHEP 0503, 002 (2005) [arXiv:hep-th/0501146].

[3] W. G. Unruh, Phys. Rev. D 40 (1989) 1048.

[4] W. Buchmuller and N. Dragon, Phys. Lett. B 207 (1988) 292.

[5] P. Van Nieuwenhuizen, Nucl. Phys. B 60, 478 (1973).

[6] M. Fierz and W. Pauli, Proc. Roy. Soc. Lond. A 173 (1939) 211.

[7] R. M. Wald, “General Relativity”, Univesity of Chicago Press (Chicago, 1984).

[8] V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, Phys. Rept. 215 (1992) 203.
[9] J. B. Pitts and W. C. Schieve, Gen. Rel. Grav. 33 (2001) 1319
arXiv:gr-qc/0101058.

[10] D. Blas, C. Deffayet and J. Garriga, In preparation.

[11] M. Kreuzer, Class. Quant. Grav. 7 (1990) 1303.

[12] N. Dragon and M. Kreuzer, Z. Phys. C 41 (1988) 485.

[13] E. Álvarez, J. J. López-Villarejo To appear in Proceedings of the Spanish Relativity Meeting 2005, ERE05.

[14] E. Álvarez, D. Blas, J. Garriga and E. Verdaguer. In preparation.

[15] V. A. Rubakov, arXiv:hep-th/0407104

[16] R. J. Rivers, Nuov. Cim. 34 (1964) 386.

[17] D. G. Boulware and S. Deser, Phys. Rev. D 6 (1972) 3368.

[18] H. van Dam and M. J. G. Veltman, Nucl. Phys. B 22, 397 (1970).

[19] V.I. Zakharov, JETP lett 12,312 (1970).

[20] A. I. Vainshtein, Phys. Lett. B 39 (1972) 393.

[21] C. M. Will, arXiv:gr-qc/0510072

[22] C. M. Will, Living Rev. Rel. 4 (2001) 4 arXiv:gr-qc/0103036.

[23] E. G. Adelberger, B. R. Heckel and A. E. Nelson, Ann. Rev. Nucl. Part. Sci. 53 (2003) 77 arXiv:hep-ph/0307284.

[24] V. I. Ogievetsky and I. V. Polubarinov, Ann. Phys. B 35, 167 (1965).

[25] S. Deser, Gen. Rel. Grav. 1 (1970) 9 arXiv:gr-qc/0411023.

[26] D. G. Boulware and S. Deser, Annals Phys. 89 (1975) 193.

[27] R. M. Wald, Phys. Rev. D 33 (1986) 3613.

[28] N. Boulanger, T. Damour, L. Gualtieri and M. Henneaux, Nucl. Phys. B 597 (2001) 127 arXiv:hep-th/0007220.

34
[29] D. Ghoshal and A. Sen, Nucl. Phys. B 380, 103 (1992) arXiv:hep-th/9110038.

[30] J. M. Bardeen, Phys. Rev. D 22 (1980) 1882.

[31] J. M. Stewart, Class. Quant. Grav. 7 (1990) 1169.