The DEA Method and Its Application Possibilities for Measuring Efficiency in the Public Sector—The Case of Local Public Employment Services

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Abstract: Public Employment Services (PES) are identified as important institutions in the process of improving the match between supply and demand in the labor market, which, despite their importance, still do not achieve the desired efficiency. The indicated problem is partly due to the lack of appropriate evaluation methods for the applied labor market policy instruments. This paper aims to verify the possibility of using the two-stage Data Envelopment Analysis (DEA) method in measuring the efficiency of public sector entities. The authors focused on 39 PES operating in Mazovia province, Poland in 2019. In the first stage, the model of technical efficiency of local PES included six variables (four inputs and two outputs). Only seven PES obtained full efficiency. The inefficiency of analyzed PES varied from about 1% to 80%. In the second stage, the attention focuses on the relationship between true unknown efficiency and its determinants (five environmental variables, both demand and supply oriented). Then, the regression coefficients and confidence intervals showed that three out of five variables influence the efficiency results, the share of the long-term unemployed, the share of the unemployed under 30, and the share of the unemployed over 50 in the total number of unemployed.

Keywords: efficiency; DEA method; public policy; labor market; public employment services

1. Introduction

In line with the trends observed in developed countries (including Poland), the state’s priority has become to abandon one-sided policies aimed primarily at mitigating the effects of unemployment and shifting the emphasis to active labor market activities—ALMP (Escudero 2018). An opinion can be formulated that passive public welfare, which usually boils down to cash benefits, is not able to effectively solve the life problems of citizens and lead to their return to the resources of working people (Kraft 1998; Romero and Kuddo 2019). Various studies (Lalive et al. 2002; Góra and Sztanderska 2006) prove that one of the basic dangers in the labor market is the relationship between broad access to passive methods of counteracting unemployment and the decline in job-seeking activity, which contributes to the extension of the period of unemployment. On the other hand, ALMP constitutes a deliberate intervention of the state in the labor market, aimed at improving its functioning, but their efficiency depends on various factors, including target groups, labor market characteristics, policy design, economic structure, and national institutional environment (Elezaj et al. 2019). The condition that should be considered as the boundary within the framework of the functioning of systemic solutions counteracting unemployment in the environment of limited resources is the identification of the desired scope.
of labor market policy tools financed from public funds, considering two main criteria—groups disadvantaged in the labor market that require special support and expected effects of activities. These dependencies result from the implemented policy of combating unemployment and the real financial possibilities of the state. The implementers of active labor market policy are public employment services—PES (Weishaupt 2010). At the same time, it can be concluded that public institutions need efficient management more than commercial entities. This is because they are not affected by market stimuli, which automatically would force them to act effectively and efficiently. Additionally, it is important to consider the specificity of the activities of entities involved in the implementation of the labor market policy as public institutions (Mihaiu et al. 2010) and the need to balance social aspects and economic profitability, which may be expressed in various units (López-Penabad et al. 2020). Accordingly, this policy, as an activity involving public funds, should be systematically assessed as to whether the incurred outlays bring about appropriate effects (Bánociová and Martinková 2017; Yun 2020). The key to the discussed problem is the clear understanding of the concept of efficiency and the proper selection of methods for its measurement.

Efficiency is an important economic category, and it is also a multi-dimensional concept that should be analyzed from a different perspective in the private and public sectors (Terziev 2018). While it seems that in the case of the assessment of business solutions, where profit is one of the most important (if not the most important) objectives, it was possible to develop a satisfactory basis for measurement (while still improving), in the field of implementing the public policy programs, it is often said about the need to conduct activities efficiently. However, following such statements, there are no specific criteria enabling verification of the level of efficiency. According to one of the most classical approaches, efficiency means using resources in the most efficient way (Samuelson and Nordhaus 2009). Efficiency action could therefore be understood as one whose net benefit (obtained benefit considering the costs incurred) related to its taking is not only positive but also the highest possible. As a result of this way of thinking, the ratio of input to output becomes the key in determining whether the efficiency criteria are met (Cicea 2020). This approach usually refers to technical efficiency, in which, assuming the optimal use of resources, results-oriented efficiency and input-oriented efficiency are distinguished (Farrell 1957). It can be clarified that the results-oriented efficiency analysis was more developed in other works (Färe et al. 1985).

To organize the considerations, the rest of the paper is arranged as follows. Section 2 presents the theoretical background and relevant literature that underpins the need to measure the efficiency of PES, the related difficulties and benefits resulting from the use of reliable methods in this area. Section 3 shows the DEA method employed, including the data set and variables used. Section 4 provides detail on our empirical findings. Section 5 contains a comparison and critical analysis of the results of this study with the results of other related studies. Finally, the main conclusions are given.

2. Literature Review

In general, there is high interest in Public Employment Services (PES) evaluating their efficiency of them. Work in this field can be classified into several areas. The first trend focuses on trying to find the most reliable method of measuring the effectiveness of these units, with the authors’ attention in particular to the DEA method and its developments or the Stochastic Frontier Analysis (SFA) method. Additionally, studies can be distinguished that are aimed at identifying the factors determining the effectiveness of Public Employment Services. Another line of considerations is devoted to comparative analyzes of PES offices or ALMP programs carried out in a specific region or, less frequently, throughout the country.

Starting from the research on technical efficiency mentioned in the introduction of this paper, different techniques have gradually been developed to calculate and estimate the efficient frontier against which to measure efficiency. These techniques can be divided in different ways, one of which distinguishes between parametric and non-parametric
approaches. In the parametric approach, the functional form of the efficient frontier is imposed a priori while in the non-parametric scheme the frontier is calculated based on the sample observations (Cesaro et al. 2009). Among the parametric methods, the most popular one is the Stochastic Frontier Analysis—SFA, and the most important method among the non-parametric ones is the Data Envelopment Analysis—DEA (Lampe and Hilgers 2015). Various articles in the literature compare not only both groups of methods but even the SFA and DEA methods. Considering the specificity of the analyzes and other works using the methods of measuring efficiency in relation to public entities (also in other countries), the DEA method was selected for this research. An additional advantage is that with DEA, multiple outputs and inputs can be considered simultaneously and they can be quantified using different units of measurement. This can be very useful in analyzing the efficiency of the PES (Honorati and de Silva 2018). Moreover, for example, Torgersen et al. (1996), Vassiliev et al. (2006), Kthiri et al. (2011), Agovino et al. (2013), Eskelinen and Keshvari (2017), and Sánchez-Sánchez et al. (2021) also successfully applied this method to the labor offices, active labor market policies, or labor market.

In all countries where PES are present, they connect job seekers with employers. Although the management of these entities varies, each time they aim to improve the match between supply and demand in the labor market using active labor market policies. In particular, ALMPs aim either at increasing the likelihood of re-employment for unemployed people or at reducing the likelihood of losing a job in the case of unemployed workers (Baños et al. 2019). Even though different methods have been applied across PES in the different member states of the European Union, it seems that no PES as yet has put together an integrated and comprehensive system for assessing efficiency (Ejler and Sidelmann 2015). At the same time, due to the role of PES, the methodology and the results of evaluation studies (Heckman et al. 1999; Ramirez and Vassilev 2007), which indicate that active labor market programs usually have an insufficient impact on the improvement of the professional situation of people covered by support and the chance of finding a job by the unemployed, it is even more important to develop a methodology measuring the efficiency. Looking at the problem of PES efficiency, it seems that a properly selected indicator of its measurement should be included in the assessment of the economic development of socio-economic systems. The literature expresses the need to conduct research aimed at proposing new ways to quantify the dynamics of this development (Ginevičius et al. 2020).

Despite the problems that cause the efficiency of activities in the field of active labor market programs to be burdened with methodological difficulties and the risk related to obtaining adequate reliability of the measurement, it provides important information about the situation on the labor market and the efficiency of activation measures. It is necessary to identify the situation in a specific area to change it; this may favor the formulation of recommendations that may solve localized problems, as well as the case of PES offices (Albuquerque et al. 2018). The observation of the various territorial units that shape a given region and the characteristics of each area is of great importance in determining the potential disparities between these units (Beccatini et al. 2003). Local labor markets are seen as regions that can be defined by a method that does not differ from core identification. Thus, their progress can be explained by their interactions and their geographic location (Feria et al. 2015). For example, comparing regions with each other and comparing them to the national average may form the basis for a decision to provide a given region with greater assistance in the form of substantive support or in the area of managing specific labor market instruments (O’Leary et al. 2001).

An additional important issue that cannot be ignored when considering the effectiveness of PES is the conditions in the country in which these units provide services. Poland is one of the countries in the Central and Eastern Europe (CEE) region which, due to the events of the past, is characterized by certain features characteristic of this area. The phenomenon of unemployment is also subject to this specificity. Unemployment in this European region was higher than in Western Europe due to post-socialism economic restructuring at the end of the 1980s. It can also be noticed that the problem of unemploy-
ment particularly affects several characteristic groups, referred to as people disadvantaged in the labor market. These are women, young people (under 30), people over 50, the long-term unemployed, migrants and ethnic minorities, people from rural areas, people with a low level of education, or people with disabilities. Depending on additional conditions, including cultural ones, in some countries of the CEE region, along with the particular vulnerability of given groups to unemployment, additional unfavorable phenomena also occur. An example can be the situation of unemployed Roma people in Slovakia (Kozubik et al. 2020) or in Romania (Crețan and O’brien 2019). On the other hand, in Poland, a much bigger problem is in particular long-term unemployment, unemployment of people under 30, and unemployment of people over 50.

With reference to the above considerations, the main purpose of this article is the verification of the possibility of using the two-stage DEA method in measuring the efficiency of public sector entities, in particular PES. The study confirmed that it is reasonable to distinguish particular groups in a special situation on the labor market from all the unemployed. The adoption of such criteria is justified because it allows the introduction of special solutions dedicated to these groups, which translates into an improvement in the efficiency of the activities carried out by public authorities.

The assumed hypothesis is as follows: Disparities in the efficiency level of local PES can be explained by environmental factors, therefore they should be included in the efficiency measurement method.

The procedure of evaluation of the efficiency of PES was based on the case study of one of the Polish provinces—Mazovia province. The reason for the selection of Polish PES is that those entities are evaluated in practice by effectiveness, not efficiency methods and measures (Rollnik-Sadowska 2019). Following the law in force in Poland, two main indicators are used to assess the efficiency (in fact—effectiveness) of PES: the employment efficiency index and the cost-efficiency index. The literature indicates that they do not reliably reflect the real situation in public sector entities and there is a need to strive to develop better methods of assessing efficiency (Bak et al. 2019; Gavurova et al. 2020). Moreover, as regional and local labor markets are extremely diverse in Poland (Ciżkowicz et al. 2016; Rollnik-Sadowska et al. 2020), it is important to study the influence of environmental factors on efficiency results of Polish PES.

3. Methodology

The methodology is applied to 39 local PES operating in Mazovia province, Poland. That region was selected as it represents the diversified economic situation among territorial units which creates the heterogeneous potential of those units in terms of labor demand, as well as labor supply (Cichowicz et al. 2018). The economic dominance of Warsaw—Polish capital creates the development disparity for other local units of Mazovia province as they meet diversified environmental conditions of PES activity. That is the reason for researching the influence of environmental variables on efficiency results.

In our research, we use the two-step approach proposed by Simar and Wilson (2007). We apply DEA to estimate the efficiency scores of each unit at the first stage and then we use truncated regression with double-bootstrapping to examine the influence of uncontrolled variables on efficiency scores. The Data Envelopment Analysis, introduced by Charnes et al. (1978), is a nonparametric method of efficiency analysis. It considers a group of units, called decision-making units (DMU), which apply some inputs to “produce” outputs. The production plans, consisting of the vectors of inputs and outputs of DMU units, define the production set \( \mathcal{P} \) and the efficient units are those on the frontier of \( \mathcal{P} \), which means that the ratio of their weighted outputs to weighted inputs is optimal (the concept of efficiency taken into account is the Farrell’s efficiency (Farrell 1957)). We should underline the fact, that we analyze the relative efficiency among the considered group of decision-making units (DMUs). It is assumed that we have \( n \) DMUs that produce \( k \) outputs by use of \( m \) inputs. Denote by \( x_{ri} \) the volume of the \( r \)-th input used by the \( i \)-th DMU and by \( y_{ti} \) the volume of the \( t \)-th output produced by the \( i \)-th DMU. Let \( X \) denote the \( m \times n \) matrix of
composed of $x_{ij}$ and $Y$ denote the $k \times n$ matrix with entries $y_{ij}$. The production set in the original model of Charnes, Cooper, and Rhodes (CCR model) is:

$$\mathcal{P} = \{(x, y) : x \geq X\lambda, y \leq Y\lambda, \lambda \geq 0\},$$  \hspace{1cm} (1)

where $x$ is an input vector and $y$ is an output vector.

We will call a pair $(x, y)$ a production plan. In particular, the CCR model assumes constant returns to scale, i.e., if $(x, y)$ belongs to $\mathcal{P}$, then $(ax, ay)$ is also in $\mathcal{P}$ for each positive $a$.

In this paper, we apply the Banker, Charnes, and Cooper (BCC) model (Banker et al. 1984), where the assumption of constant returns to scale is replaced by variable returns to scale. The production set in the BCC model is:

$$\mathcal{P} = \{(x, y) : x \geq X\lambda, y \leq Y\lambda, \lambda \geq 0, \sum_{i=1}^{n} \lambda_i = 1\}$$  \hspace{1cm} (2)

The only difference with the CCR model is the condition $\sum_{i=1}^{n} \lambda_i = 1$. Together with the nonnegativity of $\lambda$ this means that all convex combinations of available production plans are available too. The BCC model may be input-oriented, which means that we want to minimize the input given the output or output-oriented, i.e., the goal is to maximize the output given the input. In the output-oriented BCC model, the following problem is solved for each DMU$_o$ ($o = 1, 2, \ldots, n$) to determine its efficiency (Cooper et al. 2000):

$$\delta_o = \max \delta$$  \hspace{1cm} (3)

subject to

$$x_o \geq X\lambda$$  \hspace{1cm} (4)

$$\delta y_o \leq Y\lambda$$  \hspace{1cm} (5)

$$\sum_{i=1}^{n} \lambda_i = 1, \lambda \geq 0$$  \hspace{1cm} (6)

If a DMU$_o$ is efficient, then $\delta_o$ is equal to 1, otherwise $\delta_o > 1$, so we use the reciprocal of $\delta_o$ to obtain the efficiency measure between 0 and 1. Benchmarks for an inefficient DMU$_o$ are all DMU$_j$ such that the corresponding $\lambda_j$ in the solving of the optimization problem of DMU$_o$ is positive. The set of all benchmarks of DMU$_o$ is called the reference set. Any DMU that is a benchmark of a DMU$_o$ is efficient (Cooper et al. 2000).

In the research process, the BCC model was used assuming variable scale effects. As the PES’s inputs are rather difficult to control (they follow the legal regulations and the algorithm of PES financing included in the Act on the promotion of employment and labor market institutions from 20 April 2004 (with later amendments)), the output-oriented model seemed to be more appropriate for this study (Cichowicz et al. 2018).

The selection of variables included in the model was implied by the data availability, as well as their substantive significance for the PES activity. The data was downloaded from the resources of the Voivodship Labour Office in Warsaw and Mazovian Labour Market Observatory. Moreover, we considered various assumptions concerning the number of variables included in the model to get its good discriminatory power (Sarkis 2006). Golany and Roll (1989) establish the rule of thumb stating that the number of units should be at least twice the number of inputs and outputs considered. Bowlin (1998) mentions the need to have three times the number of DMUs as there are input and output variables. Dyson et al. (2001) recommend a total of two times the product of the number of input and output variables. The number of variables in our model satisfies all these rules.

In the first stage, the model of technical efficiency of local PES in Mazovia province included six variables (four inputs and two outputs).

As inputs, authors classified both financial and human resources:

- $I_1$: Limits of labor found,
- $I_2$: Expenditures from labor found,
- $I_3$: Number of job brokers in a county labor office,
I4: Number of employees at the key positions in a county labor office.

The outputs concerned the PES effectiveness in term of the employment of the unemployed:
O1: Outflow from unemployment due to starting work,
O2: Taking a job by the unemployed in a special situation on the labor market.

In the second stage we focus on the relationship between true unknown efficiency and its determinants given by the model:

$$\delta_j = \beta_0 + \sum_k \beta_k z_{kj} + \epsilon_j \quad (7)$$

where $z_{kj}$, $k = 1, \ldots, r$ are explanatory variables that we assume to influence the efficiency.

Following the algorithm proposed by Simar and Wilson (2007), we obtain bias-corrected estimates of efficiency scores and estimates of parameters $\beta_k$.

In the second stage, the influence of five environmental variables (both demand and supply oriented) on efficiency results was verified:
UI1: annual number of vacancies and places of professional activation,
UI2: the share of women in the total number of unemployed,
UI3: the share of long-term unemployed in the total number of unemployed,
UI4: the share of the unemployed under 30 in the total number of unemployed,
UI5: the share of the unemployed over 50 in the total number of unemployed.

Our results are obtained with the use of the rDEA package (Simm and Besstreminnaya 2016).

4. Results

In the first stage of the analysis, the efficiency of 39 local PES was evaluated using the DEA method based on the model presented in Figure 1.

**Figure 1.** Model of technical efficiency of local PES in Mazovia province. Source: own study.

Table 1 includes the result of the analysis carried out using the rDEA package. DMUs—county labor offices are organized in descending order, so the best performers in terms of technical efficiency are ranked at the top. The efficient units are ranked by counting the number of times they appear as benchmarks in the reference sets of inefficient units (an idea developed in the paper of Charnes et al. 1985).
Table 1. Ranking of PES technical efficiency in Mazovia province in 2019.

| DMU                          | Benchmark Frequency | DEA Score TE |
|------------------------------|---------------------|--------------|
| Węgrowski                    | 32                  | 1            |
| city Warszawa                | 13                  | 1            |
| Lipski                       | 9                   | 1            |
| Grodziski                    | 5                   | 1            |
| Radomski with city Radom     | 5                   | 1            |
| Łosicki                      | 4                   | 1            |
| Przasnyski                   | 2                   | 1            |
| Kozielicki                   | 0                   | 0.992508     |
| Ciechanowski                | 0                   | 0.975124     |
| Zachodni Warszawski          | 0                   | 0.947582     |
| Gostyniański                 | 0                   | 0.925927     |
| Płoński                      | 0                   | 0.836755     |
| Płocki                       | 0                   | 0.802101     |
| Grójecki                     | 0                   | 0.694958     |
| Białobrzeski                 | 0                   | 0.694917     |
| Sochaczewski                 | 0                   | 0.691146     |
| Ostrołęcki with city Ostrolęka | 0          | 0.677492    |
| Zyrardowski                  | 0                   | 0.676645     |
| Sokolowski                   | 0                   | 0.655014     |
| city Płock                   | 0                   | 0.623787     |
| Garwoliński                  | 0                   | 0.615        |
| Siedlecki with city Siedlice | 0                   | 0.612847     |
| Zwoleński                    | 0                   | 0.608197     |
| Szydłowiecki                 | 0                   | 0.60746      |
| Przysuski                    | 0                   | 0.598829     |
| Pułtuski                     | 0                   | 0.542641     |
| Sierpecki                    | 0                   | 0.53166      |
| Piaseczyński                 | 0                   | 0.525973     |
| Mławski                      | 0                   | 0.501256     |
| Makowski                     | 0                   | 0.49517      |
| Miński                       | 0                   | 0.48154      |
| Żurominski                   | 0                   | 0.481108     |
| Otwocki                      | 0                   | 0.479989     |
| Pruszkowski                  | 0                   | 0.478506     |
| Legionowski                  | 0                   | 0.430586     |
| Ostrowski                    | 0                   | 0.406        |
| Nowodworski                  | 0                   | 0.394114     |
| Wyszkowski                   | 0                   | 0.370204     |
| Wołomiński                   | 0                   | 0.200102     |

Source: own study.

In 2019, only seven out of 39 analyzed PES obtained full efficiency. The leader in terms of benchmark frequency was the PES in Węgrowski county. The specificity of that labor office is close cooperation with employers in terms of information about available labor market instruments, as well as support in their realization, which could have contributed to reaching full efficiency. Moreover, the Węgrowski labor office developed a monitoring procedure for the progress of the implemented forms of professional activation. The second position in terms of benchmark frequency was the PES located in the Polish capital, which is characterized by the highest labor demand in Poland. The third in the ranking was the PES from Lipski county, which as the Węgrowski labor office implemented and developed cooperation with employers (that labor office was recognized by the Ministry of Family, Labour and Social Policy as the best from the Mazovia province in 2018 for effective activation measures taken with the unemployed).

The inefficiency of analyzed PES varied from about 1% to 80%. The lowest rate was for the PES in Wołomiński county.
At the second stage of the analysis, the regression coefficients and confidence intervals showed that three out of five environmental variables influence the efficiency results—Table 2. Initially, a model with the above-mentioned five environmental variables was considered. The variables: UI1: annual number of vacancies and places of professional activation, UI2: the share of women in the total number of unemployed, occurred to be insignificant at each of the usual levels of significance. We could not reject the hypotheses that the coefficient corresponding to UI1, as well as to UI2, is equal to zero.

Table 2. Regression coefficients and confidence intervals.

| Variables                                      | Robust Coefficient of Truncated Regression | 5%       | 95%       |
|------------------------------------------------|-------------------------------------------|----------|----------|
| (Intercept)                                    | 32.49415                                  | 18.45998 | 48.57042 |
| Annual number of vacancies and places of professional activation (UI1) | 0.021634                                  | −0.26373 | 0.280835 |
| The share of women in the total number of unemployed (UI2) | −9.47104                                  | −20.2172 | 0.21394  |
| The share of long-term unemployed in the total number of unemployed (UI3) | −13.7336                                  | −22.1137 | −6.83528 |
| The share of the unemployed under 30 in the total number of unemployed (UI4) | −23.3806                                  | −38.1669 | −10.406  |
| The share of the unemployed over 50 in the total number of unemployed (UI5) | −48.133                                   | −76.0818 | −24.2125 |

Source: own study.

After removing insignificant variables, we got the following model:

$$\delta_j = 26.64226 - 13.08865z_{1j} - 23.57189z_{2j} - 45.69974z_{3j}$$

(8)

where:

- $z_1$ (UI3)—the share of the long-term unemployed in the total number of unemployed,
- $z_2$ (UI4)—the share of the unemployed under 30 in the total number of unemployed,
- $z_3$ (UI5)—the share of the unemployed over 50 in the total number of unemployed.

All environmental variables included in the model were significant at a significance level of 0.01—Table 3. They were supply oriented factors such as the share of long-term unemployed in the total number of unemployed, the share of the unemployed under 30 in the total number of unemployed, and the share of the unemployed over 50 in the total number of unemployed, which influence the efficiency results of the local PES in Mazovia province. Variables UI3, UI4, UI5 show a positive relationship with efficiency. It may be related to the specificity of labor market instruments available to Polish PES under active labor market policy. They are mostly dedicated to the elderly, young, or the long-term unemployed.
Table 3. Regression coefficients and confidence intervals.

| Variables                                             | Robust Coefficient of Truncated Regression | 0.5%       | 99.5%      |
|-------------------------------------------------------|-------------------------------------------|------------|------------|
| (Intercept)                                           | 26.6422592245564                         | 7.68587681635954 | 48.1253689814252 |
| The share of long-term unemployed in the total number of unemployed (UI3) | $-13.08864904$                          | $-25.5867$ | $-3.29742$ |
| The share of the unemployed under 30 in the total number of unemployed (UI4) | $-23.57189268$                          | $-47.5677$ | $-2.20158$ |
| The share of the unemployed over 50 in the total number of unemployed (UI5) | $-45.69973889$                          | $-90.0395$ | $-8.1376$  |

Source: own study.

Following the results of regression coefficients and confidence intervals, it can be stated that the third environmental variable—the share of the unemployed over 50 in the total number of unemployed seems to have the greatest impact on efficiency results.

5. Discussion

In the literature, there are examples of research on the efficiency of PES conducted based on the non-parametric DEA method. It is the most common non-parametric method, often used to analyze the efficiency of public institutions (Behrenz et al. 2013).

In our study, inputs selected for the model included financial resources (limits of labor found and expenditures from labor found), as well as human resources of PES (number of job brokers in a county labor office and number of employees at the key positions in a county labor office). Althin and Behrenz in their study of the efficiency of the 297 PES in Sweden (Althin and Behrenz 2004) included the wider scope of inputs such as human resources (number of employees), material resources (office space), and information resources (computer use measured as the number of subscriptions to network and computer connections). Andersson, Månsson, and Sund who researched 185 Swedish PES considered seven inputs in the model, including not only human and material resources but also the specificity of the job-seekers. The inputs covered: the number of office workers employed full-time, office space, the number of the registered unemployed, the number of individuals participating in employment programs, the number of newly registered unemployed, profiling measure taking into account the differences in job-seeker composition by employment offices, the number of days that job-seekers spent at private contractors (Andersson et al. 2014). The reason for the usage of the wider range of variables may be the greater number of DMUs included in the model.

The choice of outputs depends on the PES effectiveness assumptions in individual countries. In our study, both outputs (outflow from unemployment due to starting work and taking a job by the unemployed in a special situation on the labor market) concern the PES effectiveness in terms of the employment of the unemployed. Comparable attitudes in terms of outputs selection were represented by Althin and Behrenz in their study in Sweden, as the outputs were the number of the registered unemployed people who took up non-subsidized employment, the number of registered unemployed people who took up subsidized employment, and the number of beneficiaries of labor market services and instruments (Althin and Behrenz 2004). As a transition to regular education in Sweden is considered as a successful outcome (Andersson et al. 2014), other studies on the efficiency of PES for that country included in outputs not only the employment effectiveness of these institutions but also the educational effectiveness. Althin and Behrenz repeated the
efficiency studies using other variables for the model and selected for the outputs: the number of outflows from the registers due to taking up employment, the number of people who started education (other than internships), the number of deregistration’s from the employment office for other reasons (Althin and Behrenz 2005). Whereas, Andersson, Månsson, and Sund in their study included the outputs such as the number of registered individuals in programs with activity support (as intermediate output) and the number of individuals that got a job placement, and the number of individuals that transferred to outside education (final outputs) (Andersson et al. 2014).

Other studies also verified the influence of environmental variables on efficiency results. Our study conducted among the selected Polish PES proves that supply oriented factors such as the share of long-term unemployed in the total number of unemployed, the share of the unemployed under 30 in the total number of unemployed, and the share of the unemployed over 50 in the total number of unemployed influences the efficiency results of the local PES. Althin and Behrenz who applied regression analysis identified that the number of unemployed and the number of vacancies had a slight, but positive, impact on the efficiency results. On the other hand, the population of the commune in which a given PES operates did not show a statistical relationship with the efficiency of this entity (Althin and Behrenz 2004). Meanwhile, Sheldon conducted a two-stage evaluation study of 126 regional PES in Switzerland and showed that environmental factors that are beyond the control of PES (such as the structure of the unemployed served in terms of age, qualifications, origin, gender, the propensity to spatial mobility) significantly affect their efficiency. Moreover, the results of the study confirmed that career counseling is more effective than other active labor market services and instruments in increasing the efficiency of the analyzed PES (Sheldon 2003).

Research on the efficiency of PES activities using the DEA method shows that it can be used to assess the management of public funds by these entities and to conduct comparisons of institutions operating in a given country. However, the DEA method is mainly of interest to the academic community and has not been popularized in practice so far. It is only used to monitor the activities of the Swedish PES (Behrenz et al. 2013).

6. Conclusions

This paper proposes casting light on the efficiency of activity undertaken by PES in matching supply and demand in the labor market. Concerning the methodology currently used by public authorities in Poland which focuses on the indicator approach, the proposed tool and the selection of variables seem to be a more justified concept, as it provides more appropriate results concerning the efficiency of the compared labor offices. In particular, we examined that the employment offices included in the analysis had a significantly diversified level of efficiency as they operated under certain environmental circumstances. However, the research hypothesis has been partially positively verified. The obtained results indicated that disparities in the efficiency level of local PES can be explained by environmental factors but not all selected environmental variables will influence the efficiency results. There is a significant relationship between the level of efficiency achieved in analyzed local PES in Mazovia province and the share of the long-term unemployed in the total number of unemployed, share of the unemployed under 30 in the total number of unemployed, and share of the unemployed over 50 in the total number of unemployed. At the same time, the annual number of vacancies and places of professional activation, as well as the share of women in the total number of unemployed, did not appear to be significant for the efficiency of the analyzed PES.

The two-step approach presented in that study seems appropriate for the efficiency evaluation of selected public institutions as it allows comparisons of PES abilities of transformation of their inputs into outputs. Moreover, it identifies the efficiency determinants, that may support the authorities in shaping the labor market policy. The indicator approach, which is mainly used by public authorities to evaluate PES efficiency, does not allow to consider both inputs and outputs and it represents the effectiveness, not efficiency perspective.
It is recommended to use the DEA method in labor market policy practice especially when the evaluated units act in the diversified local conditions and the environmental variables influence the outputs they reach.

The limitation of the research conducted in that paper is presenting the method only for labor offices from one province in Poland. Moreover, the access to reliable data determined the variables selection. These studies could be carried out on a larger sample and a wider resource of variables could be selected.

The proposed model of technical efficiency of local PES is one of many possibilities. The presented research procedure contributes to further research, as there can be tested various DEA models with different sets of variables adapted to the peculiarities of local conditions. Moreover, future studies could include other regions, which would allow consideration of environmental variables related to their specificity.

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