Connections between the Seesaw and Dark Matter Searches

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In some dark matter models, the coupling of the dark matter particle to the standard model Higgs determines the dark matter relic density while it is also consistent with dark matter direct detection experiments. On the other hand, the seesaw for generating the neutrino masses probably arises from a spontaneous symmetry breaking of global lepton number. The dark matter particle thus can significantly annihilate into massless Majorons when the lepton number breaking scale and hence the seesaw scale is near the electroweak scale. This leads to an interesting interplay between neutrino physics and dark matter physics and the annihilation mode has an interesting implication on dark matter searches.

PACS numbers: 95.35.+d, 14.60.Pq, 14.80.Bn

I. INTRODUCTION

The existence of non-baryonic cold dark matter [1] indicates the necessity of supplementing the $SU(3)_c \times SU(2)_L \times U(1)_Y$ standard model (SM) with new ingredients. This has led to many interesting dark matter models. For example, the dark matter particle may be a scalar field [2]. In this case, the dark matter scalar can have a quartic coupling to the SM Higgs doublet. Through the t-channel exchange of the Higgs boson, there will be elastic scattering of the dark matter scalar by nucleons. This opens a window for dark matter direct detection experiments [3]. In the case where the dark matter particle is not a scalar, but a vector or a fermion, it can indirectly couple to the SM Higgs. It does this by coupling directly to a non-SM Higgs, which mixes with the SM Higgs in presence of the Higgs portal [4]. Direct detection experiments give strict upper bounds on the dark matter-nucleon cross section. It should also be noted that a dark-matter-Higgs coupling may also be responsible for and hence constrained by the relic density of dark matter.

On the other hand, neutrino oscillation experiments prove that neutrinos have masses and mixings [4] which also requires new physics beyond the SM. The cosmological bound shows that neutrino masses should be in the sub-eV range [1]. The small neutrino masses can be naturally explained in the seesaw [5] extension of the SM. The seesaw requires, however, some generic lepton number violation as the neutrinos are assumed to be Majorana particles. This lepton number violation can arise from a spontaneous symmetry breaking in some more fundamental theories. The simplest possibility is, for example, to consider the singlet Majoron model [6], where the lepton number is a global symmetry and its breaking will leave a massless Nambu-Goldstone boson – the Majoron. The global lepton number breaking scale can be as low as the electroweak scale [11, 12]. In this case the right-handed neutrinos can be at an accessible scale which is is testable at colliders [13]. At the same time the dark matter particle can have a sizable coupling with the Majoron.

In this paper, we will study interplay of the dark-matter-Majoron coupling on the dark-matter-Higgs coupling. For illustration, we will focus on the simplest dark matter candidate, a real SM-singlet scalar. This model proposed by Silveira and Zee [2] has been studied before [3, 4]. In these works, dark matter annihilation is determined by the dark-matter-Higgs coupling as the SM couplings are well known. Thus, direct detection and relic density will both constrain the dark-matter-Higgs coupling for a given dark matter mass. In the presence of a low lepton number breaking scale, we can have a sizable dark-matter-Majoron coupling besides an accessible seesaw scale. The dark matter thus could significantly annihilate into the Majorons. In this case, the dark-matter-Majoron coupling can affect the relic density in addition to the dark-matter-Higgs coupling and the SM couplings. As a result, a smaller dark-matter-Higgs coupling is required. This has an interesting implication on dark matter direct detection experiments.

II. THE MODEL

In the singlet Majoron model [10, 1], the right-handed neutrinos $N_R(1,1,0)$ have no Majorana masses, which explicitly break the lepton number. Instead, they have the following Yukawa couplings with a complex singlet scalar $\sigma(1,1,0)$,

$$\mathcal{L} \supset -\frac{1}{2} f \sigma \bar{N}_R^c N_R + \text{H.c.} ,$$

(1)

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1 Alternatively, one can also consider the spontaneous symmetry breaking of global lepton number in other seesaw models with or without right-handed neutrinos [14, 15].
which exactly conserves the lepton number. After the complex singlet $\sigma$ develops a vacuum expectation value (VEV) to spontaneously break the lepton number, the right-handed neutrinos $N_R$ can obtain their Majorana masses,

$$\mathcal{L} \supset -\frac{1}{2} m_N \overline{N_R} N_R + \text{H.c.} \text{ with } m_N = f(\sigma).$$

Consequently the seesaw for generating the small neutrino masses is available as the right-handed neutrinos also have the Yukawa couplings with the SM lepton and Higgs doublets,

$$\mathcal{L} \supset -y_L \bar{l}_L \sigma_N R + \text{H.c.} .$$

Here $l_L(1, 2, -\frac{1}{2})$ and $\phi(1, 2, \frac{1}{2})$ denote the SM lepton and Higgs doublets, respectively.

We now extend the singlet Majoron model with a real singlet scalar $\chi(1,1,0)$. The full scalar potential should be

$$V = \frac{1}{2} \mu_+^2 |\chi|^2 + \mu_\chi |\sigma|^2 + \mu_{\phi}^2 |\phi|^2 + \frac{1}{4} \lambda_\chi |\chi|^4 + \lambda_\phi |\phi|^4 + \alpha |\phi|^2 |\sigma|^2 + \beta |\phi|^2 |\phi|^2 + \gamma |\sigma|^2 |\phi|^2 .$$

Here we have imposed a $Z_2$ discrete symmetry under which only the real singlet $\chi$ is odd while other SM and non-SM fields all carry an even parity. The $Z_2$ symmetry is required to hold at any energy scales. So, the real singlet $\chi$ should be stable.

After the global and gauge symmetry breaking, we can describe the singlet $\sigma$ and the doublet $\phi$ by

$$\sigma = \frac{1}{\sqrt{2}} (v' + h') e^{i \varphi}, \quad \phi = \begin{bmatrix} \frac{1}{\sqrt{2}} (v + h) \\ 0 \end{bmatrix} ,$$

where $\rho$ is the massless Majoron whereas $v'$ and $v$ are the vacuum expectation value (VEV),

$$v' = \sqrt{-\mu_+^2 + \frac{2 \Gamma}{\lambda_\chi}} , \quad v = \sqrt{-\mu_{\phi}^2 + \frac{4 \lambda_\phi}{\lambda_\chi}} \approx 246 \text{ GeV} .$$

We thus can obtain the following masses and interactions of the physical bosons,

$$V \supset \frac{1}{2} m_\chi |\chi|^2 + \frac{1}{2} m_\chi^2 h'^2 + \frac{1}{2} m_h^2 h^2 + m_{h'h'}^2 h'h' + \alpha v' h'^2 + \beta v h^2 .$$

Here the masses have been defined by

$$m_\chi^2 = \mu_\chi^2 + \alpha v'^2 + \beta v^2 , \quad m_h^2 = 2 \lambda_\phi v'^2 , \quad m_{h'h'}^2 = \gamma v' v .$$

The Higgs bosons $h'$ and $h$ now mix together. The mass eigenstates should be given by

$$h_1 = h' \cos \vartheta - h \sin \vartheta , \quad h_2 = h' \sin \vartheta + h \cos \vartheta .$$

with

$$\vartheta = \frac{1}{2} \arctan \frac{\gamma v' v}{\lambda_\phi v'^2 - \lambda_\chi v^2} .$$

and

$$m_{h_1}^2 = \lambda_\phi v'^2 + \lambda_\chi v^2 + \sqrt{(\lambda_\phi v'^2 - \lambda_\chi v^2)^2 + \gamma^2 v'^2 v^2} ,$$

$$m_{h_2}^2 = \lambda_\phi v'^2 + \lambda_\chi v^2 - \sqrt{(\lambda_\phi v'^2 - \lambda_\chi v^2)^2 + \gamma^2 v'^2 v^2} .$$

From the kinetic term, it is also easy to derive the trilinear coupling of the non-SM Higgs boson $h'$ to the Majoron $\rho$,

$$\mathcal{L}_K \supset (\partial^\mu \sigma)^*(\partial^\mu \sigma) \rightarrow \frac{1}{v} h' \partial_{\rho} \partial^\mu \rho .$$

The existence of the massless Majoron will result in some phenomenological implications. For example, at one-loop order the right-handed neutrinos will mediate the lepton flavor violating decays including $\mu \rightarrow e \nu$, $\tau \rightarrow \rho e$, and $\tau \rightarrow \rho \mu$. The Majoron will also have implications on astrophysics, such as the cooling rates of white dwarfs, the helium ignition process in red giants, and the energy emission of neutron stars. Furthermore, the Majoron will contribute to the relativistic degrees of freedom which has been stringently constrained by Primordial Big-Bang Nucleosynthesis (BBN). After taking all of the experimental limits into account, we can still expect the global symmetry of lepton number to spontaneously break near the electroweak scale \cite{11, 12}. This is also consistent with the stability and triviality bounds \cite{11}. For such a lepton number breaking scale, the seesaw can be detected at colliders \cite{13}.

In the following, we shall simply assume the right-handed neutrinos $N_R$ are heavier than the stable scalar $\chi$ . Therefore the right-handed neutrinos can decouple from the discussions on the dark matter property.

### III. DARK-MATTER-NUCLEON SCATTERING

The real SM-singlet $\chi$ has a trilinear coupling with the SM Higgs boson $h$, see Eq. (7). The t-channel exchange of $h$ will result in an elastic scattering of $\chi$ by nucleons and hence a nuclear recoil. The spin-independent cross section of the elastic scattering would be

$$\sigma_{\chi N \rightarrow \chi N} = \frac{1}{\pi} \left[ \left( -\frac{v'}{2v} \alpha \sin 2\vartheta + \beta \sin^2 \vartheta \right) \frac{1}{m_{h_2}^2} + \left( \frac{v'}{2v} \alpha \sin 2\vartheta + \beta \cos^2 \vartheta \right) \frac{1}{m_{h_1}^2} \right] \times \frac{\mu^2_{\chi} f^2 m_N^2}{m_{\chi}^2} .$$
where $m_N$ is the nucleon mass, $\mu = m_N m_N / (m_R + m_N)$ is the reduced mass, the factor $f$ in the range $0.14 < f < 0.66$ with a central value $f = 0.30$ [10] parameterizes the Higgs to nucleons coupling from the trace anomaly, $f m_N \equiv (N) \sum q \bar{q} q | N \rangle$. With a small mixing angle $\vartheta$, which can be naturally achieved for $\nu' = \mathcal{O}(100 \text{ GeV} - 1 \text{ TeV})$ and $\gamma = \mathcal{O}(0.1 - 1)$, we can approximate $h_4$ to be the SM Higgs boson $h$ and then simplify the above formula,

$$\sigma_{\chi N \rightarrow \chi N} = \frac{\beta^2}{\pi} \frac{\mu^2}{m^2_{\chi} m_h} f^2 m_N^2. \quad (14)$$

In the following, we shall focus on this simplified case. If $\chi$ is the dark matter particle, the scattering cross section should be stringently constrained by the dark matter direct detection experiments. For example, we can obtain

$$\sigma_{\chi N \rightarrow \chi N} = 3.8 \times 10^{-44} \text{ cm}^2 \left( \frac{\beta}{0.06} \right)^2 \left( \frac{70 \text{ GeV}}{m_\chi} \right)^2 \times \left( \frac{120 \text{ GeV}}{m_h} \right)^4 \left( \frac{f}{0.3} \right)^2, \quad (15)$$

which is consistent with bound from the recent CDMS II result [20].

**IV. DARK MATTER ANNIHILATION**

The real SM-singlet $\chi$ could provide the dark matter relic density if its annihilation decouples at an appropriate freeze-out temperature, which is determined by the thermally averaging cross section [21] [22].

$$\langle \sigma_A v_{\text{rel}} \rangle = \frac{\int_{4m^2_\chi}^{\infty} s \sqrt{s - 4m^2_\chi} K_1 \left( \frac{\sqrt{s}}{T} \right) \sigma_A v_{\text{rel}} ds}{\int_{4m^2_\chi}^{\infty} s \sqrt{s - 4m^2_\chi} K_1 \left( \frac{\sqrt{s}}{T} \right) ds}, \quad (16)$$

where

$$v_{\text{rel}} = 2 \left( 1 - \frac{4m^2_\chi}{s} \right)^{\frac{1}{2}}, \quad (17)$$

is the relative velocity with $s$ being the squared center of mass energy. The total cross section $\sigma_A v_{\text{rel}}$ could be conveniently divided into two parts,

$$\sigma_A v_{\text{rel}} = \sigma_{\chi \chi \rightarrow ff v_{\text{rel}}} + \sigma_{\chi \chi \rightarrow \nu \nu v_{\text{rel}}} \quad (18)$$

with $f$ being the SM fermions. Here we have assumed the SM gauge and Higgs bosons and the other fields for the seesaw are heavier than $\chi$ so that these heavy fields will only give a negligible contribution to the thermal averaging cross section.

We calculate

$$\sigma_{\chi \chi \rightarrow ff v_{\text{rel}}} = \frac{\beta^2}{\pi} \frac{1}{(s - m^2_h)^2 + m^2_h \Gamma_h^2} \times \sum_f N_f^2 m_f^2 \left( 1 - \frac{4m^2_f}{s} \right)^{\frac{1}{2}}, \quad (19)$$

$$\sigma_{\chi \chi \rightarrow \nu \nu v_{\text{rel}}} = \frac{\alpha^2}{8\pi} \frac{m_\nu^2}{s} \left( s - m^2_\nu \right)^2 + m^2_h \Gamma_h^2. \quad (20)$$

Here $N_f = 1$ for the leptons while $N_f = 3$ for the quarks. For $m_\nu \ll m_\chi < m_W$, we can take $s = 4m^2_\chi$ to read

$$\langle \sigma_A v_{\text{rel}} \rangle = \langle \sigma_{\chi \chi \rightarrow ff v_{\text{rel}}} \rangle + \langle \sigma_{\chi \chi \rightarrow \nu \nu v_{\text{rel}}} \rangle = \frac{3\beta^2}{(4m^2_\chi - m^2_h)^2} \frac{m_\nu^2}{s} \pi \frac{m^2_\chi}{2\pi (4m^2_\chi - m^2_h)^2} \sigma_0, \quad (21)$$

By analytically solving the Boltzmann equations [22], we can determine the frozen temperature,

$$x_f = \frac{m_\chi}{T_f} \simeq \ln [0.038g_*^{-1/2}m_\nu m_\chi \sigma_0] - 0.5 \ln \{\ln [0.038g_*^{-1/2}m_\nu m_\chi \sigma_0]\}, \quad (22)$$

and then the relic density

$$\Omega_\chi h^2 = 1.07 \times 10^9 \frac{x_f \text{ GeV}^{-1}}{(g_* S/\sqrt{\pi}) m_\nu \sigma_0} \quad (23)$$

Here $m_\nu \simeq 1.22 \times 10^{19} \text{ GeV}$ is the Planck mass and $g_* S \simeq g_* \approx 100$ is the relativistic degrees of freedom. For $m_\chi = 70 \text{ GeV}$, we need

$$\sigma_0 = 1.47 \times 10^{-9} \text{ GeV}^{-2} \quad \text{and then} \quad x_f = 18.5 \quad (24)$$

to generate the desired relic density,

$$\Omega_\chi h^2 = 0.11. \quad (25)$$

If the dark-matter-Majoron coupling is absent, the thermal cross section is only related to the dark-matter-Higgs coupling. For $m_h = 120 \text{ GeV}$ and $m_\chi = 70 \text{ GeV}$, we can determine $\beta = 0.0486$ for generating a right relic density. The induced dark-matter-nucleon scattering cross section is slightly smaller than the experimental bound. Now the dark-matter-Majoron coupling can significantly contribute to the dark matter annihilation. For example, if we take $m_\chi = 70 \text{ GeV}$, the annihilation of the dark matter into the Majorons can account for the dark matter relic density when $m_\nu$ decreases from 865 GeV to 120 GeV while $\alpha$ decreases from 1 to 0.00714. Therefore, to make the dark matter annihilation not too fast, the dark-matter-Higgs coupling must be reduced in the presence of a significant annihilation of the dark matter into the Majorons. In consequence, we will get a smaller dark-matter-nucleon scattering cross section.
V. SUMMARY

In some interesting dark matter models, the SM Higgs boson could be an unique messenger between dark and visible matters. In such models, usually the dark-matter-Higgs coupling fully determines the dark matter relic density as the SM couplings are well known. So, the relic density and direct detection will both constrain the dark-matter-Higgs coupling. We considered the singlet Majoron model \[10\], where the right-handed neutrinos obtain their Majorana masses after the global symmetry breaking there will be a massless Majoron. If the seesaw is expected to detect at colliders, the symmetry breaking scale could not be much heavier than the electroweak scale. In this case the dark matter particle could sizably couple to and hence significantly annihilate into the Majoron. This implies a smaller dark-matter-Higgs coupling and hence a smaller dark-matter-nucleon scattering cross section. We demonstrated this possibility in the simplest dark matter model where a real SM-singlet scalar acts as the dark matter. Our conclusion could be applied to other dark matter models (for instance, see \[7, \[23\]).

Acknowledgement: AA and ML are supported by the Sonderforschungsbereich TR 27 of the Deutsche Forschungsgemeinschaft. PHG is supported by the Alexander von Humboldt Foundation.

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