Probabilistic finite modeling of stochastic estimation of image inter-frame geometric deformations

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Abstract. The paper proposes an approach to probabilistic finite modeling of the process of stochastic estimation of the parameters of interframe geometric deformations of images. As the quantities characterizing the state of the stochastic procedure at each iteration of the estimation, the demolition probabilities of the formed parameter estimates (improvements, deterioration, or no change in the vector of estimates in the parameter space) are used. At the same time, the models of images and noises are given by probability distribution densities and autocorrelation functions. This made it possible to simplify the description of a complex set of factors influencing the formation of estimates. The use of adaptive limitation of the parameter space used for modeling reduces the computational cost several times while maintaining the adequacy of the model. In practice, probabilistic modeling can be used to find the accuracy and probability characteristics of stochastic algorithms for estimating interframe deformations of images for a given number of iterations.

1. Introduction

Estimation of geometric deformation parameters of image sequences remains one of the actual tasks of image representation and processing [1-4]. One of the approaches to solving this problem is the recurrence stochastic evaluation [5-8], the implementation of which, as a rule, leads to iterative procedures [9-11]. Stochastic procedures are effective in processing of digital image sequences [12-15] and digital signals [16, 17]. Stochastic approximation procedures that are asymptotically optimal in terms of convergence rate have been developed [18, 19]. They possess the maximum possible convergence rate but require a complete priori information and do not answer the question about the errors of deformation parameters estimation at the finite number of iterations due to the fact that the precision capabilities of procedures of this class have been studied mainly in asymptotics. The correct methods of translating asymptotic results to a finite number of iterations don’t exist.

There are approaches to the improvement (acceleration) and analysis of the accuracy of estimates of the algorithms of stochastic approximation at the finite number of iterations [20]. Acceleration is usually related to a priori information about the optimal solution, which is given by the final probability density (PDF). In the absence of such a priori information, the algorithms of this class on the finite iterations may lead to estimates of image deformation parameters that are far from optimal.
At the same time, the amount of studies devoted to probabilistic analysis of accuracy at the finite number of iterations is obviously not enough, which determined the direction of this work.

When modelling the process of deformation parameter estimation, we have to deal with the presence of a rather complex set of disturbing factors, such as temporal and spatial heterogeneity of image characteristics and interference, heterogeneity of sensor sensitivity, impulse interference, etc. By their nature, these factors are of a random, so that both parametric and nonparametric a priori uncertainty is almost always present when processing real images. In conditions of a priori uncertainty, relay stochastic adaptive procedures [21] of the type

\[
\mathbf{\tilde{a}}_{t+1} = \mathbf{\tilde{a}}_t - \mathbf{A}_t \mathbf{\text{sign}}(\mathbf{Q}(Z(0), Z(2)), \mathbf{\tilde{a}}_t))
\]

where \( \mathbf{a} = (a_1, a_2, ..., a_m)^T \) - the vector of geometric strain parameters to be evaluated; \( \mathbf{A}_t \) - gain matrix, \( \mathbf{\beta}_t = (\beta_1, \beta_2, ..., \beta_m)^T \) - pseudo gradient of the objective function (OF) \( \mathbf{Q}(Z(0), Z(2)) \) of the quality of evaluation; \( t \) - iteration number.

Further on we will consider that the images have additive noise:

\[
z_j = x_j(\mathbf{a}) + \theta_j(\mathbf{a}), \quad x_j(\mathbf{a}) = \mathbf{X}(1), \quad \theta_j(\mathbf{a}) = \Theta(1), \quad z_j = x_j + \theta_j(\mathbf{a}), \quad \mathbf{X}(1), \quad \Theta(1) - \text{independent Gaussian random fields with zero mean and equal variance } \sigma_\theta^2; \ j = (j_x, j_y) \in \Omega - \text{coordinates of the reference grid node } \Omega, \text{ of the images}; \ x, y - \text{basic image axes.} \]

At the same time, the image \( \mathbf{X}(2) \) is deformed \( \mathbf{X}(1) \) with the parameters \( \mathbf{a} \) of the assumed deformation model. In particular, we will consider the similarity model [22] which includes parallel shift \((h_1, h_2)\), rotation angle \( \nu \) and scale factor \( k \) as parameters, i.e. \( \mathbf{a} = (h_1, h_2, \nu, k)^T \).

The fact that a large number of factors, in particular, the characteristics of the examined images and disturbing noises, the choice of the objective function of the evaluation quality, as well as the parameters of the procedure influences the error of the deformation parameters' estimations formed by the procedure (1) complicates the modelling of the stochastic evaluation process. Probabilistic characteristics of images and disturbing noise can be set by the PDF and autocorrelation function (ACF). The stochastic procedure can be characterized by the type of pseudo gradient, gain matrix, number of iterations and initial approximation of the estimations of the required parameters.

2. Calculation of the probability of demolition of estimates

Assuming the model of the observed images, the factors that do not depend on the parameters of the procedure (1), are the PDF and ACF of the images \( \mathbf{X}(1) \), and the disturbing noise \( \Theta \). This also includes the type of target function \( \mathbf{Q} \). The characteristics of the procedure, which can be influenced by its implementation, include the method of calculating the pseudo gradient \( \mathbf{\beta} \), gain matrix \( \mathbf{A}_t \), the number of iterations \( T \) and the initial approximation \( \mathbf{\tilde{a}}_0 \) of parameter estimates \( \mathbf{a} \). To simplify the implementation of the modelling procedure, it is reasonable to use a minimum set of values characterizing independent factors enough to find the probabilistic estimates of parameters as a function, \( \mathbf{\beta}_t, \mathbf{A}_t, t \) and \( \mathbf{\tilde{a}}_0 \). As such values in the present \( \mathbf{X}(2) \) paper we used the probability of demolition of estimates (PDE) \( \rho_{ij} = (\rho_{ij}^+, \rho_{ij}^-, \rho_{ij}^0)^T \) [23, 24], where \( \rho_{ij}^+ - \text{the probability of change in the parameter's estimation } \alpha_i \text{ towards the optimal value } \alpha_i^*; \rho_{ij}^- - \text{from } \alpha_i^*; \rho_{ij}^0 - \text{no change in the estimation.} \)

It should be noted that if in accordance with the criteria of optimality of the OF, it is maximized, and \( (\mathbf{a}_i - \alpha_i^*) > 0 \), then \( \rho_{ij}^+ \) corresponds to the negative projection \( \beta_i \) of the OF pseudo gradient:

\[
\rho_{ij}^+(\mathbf{a}_i) = P(\beta_i < 0) = \int_{-\infty}^{0} w(\mathbf{Z}_i, \tilde{a}_{t=0}) \mathbf{I} \mathbf{\beta}_i ;
\]

(2)
where $\beta_i(Z_t, \hat{a}_{j+1})$ – projection PDF $\beta_i$ on an axis $\alpha_i$; $\varepsilon_i = \hat{a}_i - a^*$ – misalignment of a vector of estimations on the $t$-th iteration; $Z_t = \{z^{(1)}_p, z^{(2)}_p\}$ – two-dimensional local samples $z^{(2)}_p \in \mathbb{Z}^{(2)}$ and $z^{(1)}_p = \bar{z}^{(1)}(j, \hat{a}_j) \in \hat{Z}^{(1)}$, $j \in \Omega_t \in \Omega$, on which the pseudo gradient is calculated; $\hat{Z}^{(1)}_t$ – the re-sampled image $Z^{(1)}$ obtained using some approximation on current estimations $\hat{a}_i$ of parameters of used model of deformations.

Assuming that the images $Z^{(1)}$ and $Z^{(2)}$ have a Gaussian distribution of brightness with a zero mean with a known ACF $R_{j}$, we have found expressions for the calculation of PDE for the cases when using mean square of the interframe difference as the objective function with the estimation on the $t$-th iteration:

$$q_t = \mu^{-1} \sum_{j \in \Omega_t} \left( z^{(2)}_p - \bar{z}^{(1)}_p \right)^2,$$

and correlation coefficient:

$$q_i = \left( \mu \hat{\sigma}^{(1)}_Z \hat{\sigma}^{(2)}_Z \right)^{-1} \left( \sum_{j \in \Omega_t} z^{(2)}_p - \bar{z}^{(1)}_p \right) - \mu^{-1} \sum_{j \in \Omega_t} z^{(2)}_p \sum_{j \in \Omega_t} \bar{z}^{(1)}_p,$$

where $\mu$ - sample size $Z_t$; $\hat{\sigma}^{(1)}_Z$ and $\hat{\sigma}^{(2)}_Z$ - variance estimations of images $Z^{(1)}$ and $Z^{(2)}$.

According to (2) for the calculation $\rho^{\pm}_i(\bar{z})$ it is necessary to find the PDF $w(\beta_j)$ of the pseudo gradient projection $\beta_j$:

$$\beta_j = \frac{2}{\mu} \sum_{j \in \Omega_t} \left( z^{(2)}_p - \bar{z}^{(1)}_p \right) \left( \frac{d \bar{z}^{(1)}_p}{dx} \gamma_i + \frac{d \bar{z}^{(1)}_p}{dy} \zeta_i \right),$$

where $\gamma_i$ and $\zeta_i$ – functions depend on the parameters of the assumed model of interframe geometric deformations. Thus, we obtain for the parameters of similarity model:

$$\gamma_{h1} = 1, \gamma_{h2} = 0, \gamma_{v} = -k((a_i - j_{1o}) \sin \nu + (b_i - j_{2o}) \cos \nu), \gamma_{v} = (a_i - j_{1o}) \cos \nu - (b_i - j_{2o}) \sin \nu,$$

$$\zeta_{h1} = 0, \zeta_{h2} = 1, \zeta_{v} = k((a_i - j_{1o}) \cos \nu + (b_i - j_{2o}) \sin \nu), \zeta_{v} = (a_i - j_{1o}) \sin \nu - (b_i - j_{2o}) \cos \nu,$$

where $(j_{1o}, j_{2o})$ – rotation centre coordinates.

Since the dependence $\bar{z}^{(1)}_p$ on $a$ and $j$ a priori unknown in real conditions, we can use the estimates of derivatives $d\bar{z}^{(1)}_p/dx$ and $d\bar{z}^{(1)}_p/dy$ through finite differences. Then, at the second order approximation:

$$\frac{d \bar{z}^{(1)}_p}{dx} \approx -\frac{\bar{z}^{(1)}(j_{1i}, j_{2i}, a_{1}) - \bar{z}^{(1)}(j_{1i}, j_{2i} + 1, a_{1})}{2}, \quad \frac{d \bar{z}^{(1)}_p}{dy} \approx -\frac{\bar{z}^{(1)}(j_{1i}, j_{2i}, a_{1}) - \bar{z}^{(1)}(j_{1i}, j_{2i} + 1, a_{1})}{2},$$

Analysis show that the value of $\beta_j$ is quickly normalized with increasing $\mu$. Thus, already at $\mu = 2$ the expression (6) contains not less than sixteen similar summands. Then, considering the PDF $\beta_j$ close to Gaussian, according to (2) the probability $\rho^{\pm}_i$, $i = 1, m$ can be found as follows:

$$\rho^{\pm}_i = 1 - F\left( \frac{M[\beta_i]}{\sigma[\beta_i]} \right),$$

where $F(.)$ – the Laplace function.

When using the mean square of the interframe difference (3) as the objective function for the synthesis of the stochastic procedure, for the mean $M[\beta_i]$ and variance $\sigma^2[\beta_i]$ of the stochastic gradient we obtain:

$$M[\beta_i] = -\sum_{j=1}^{\mu} \sigma^2_i \left( (R(a_i - 1; b_j) - R(a_i + 1, b_j)) \gamma_i + (R(a_i; b_j - 1) - R(a_i; b_j + 1)) \zeta_i \right),$$

or
\[ \sigma^2[\beta_j] = 4 \sum_{i=1}^{n} \sigma^4 \left( \gamma_i^2 + \zeta_i^2 \right) \left( (1 - R(a_i; b_i))((1 - R(2)) + g^{-1}(2 - R(a_i; b_i) - R(2) + g^{-1})) + \right. \\
+ \left. \left( \gamma_i \left( R(a_i - 1; b_i) - R(a_i + 1; b_i) \right) + \zeta_i \left( R(a_i; b_i - 1) - R(a_i; b_i + 1) \right) \right)^2 \right). \] (9)

When using correlation coefficient (4):

\[ M[\beta] \approx -0.5(1 - \mu^{-1}) \sum_{i=1}^{n} \left( \left( \frac{R(a_i + 1; b_i) - R(a_i - 1; b_i)}{\gamma_i} \right) + \left( \frac{R(a_i; b_i + 1) - R(a_i; b_i - 1)}{\zeta_i} \right) \right), \] (10)

\[ \sigma^2[\beta_j] \approx 0.5(1 - \mu^{-1})^2 \left( \gamma_i^2 + \zeta_i^2 \right) \left( (1 - R(2)) + (\mu - 1)^{-1} \left( 1 + 2\mu^{-1} \sum_{i=1}^{n} R^2(a_i; b_i; j_1; j_2) \right) \right) + \\
+ g^{-1}(2 - R(2) + g^{-1}) + 0.25 \gamma_i \left( R(a + 1; b) - R(a - 1; b) \right) + \zeta_i \left( R(a; b + 1) - R(a; b - 1) \right)^2. \] (11)

where \( R(a; b) \) is the normalized ACF of the image; \( a_i \) and \( b_i \) - is the distance of mismatch between the coordinates of the samples \( z[2]_y \) and \( z[1]_y \) the axes \( x \) and \( y \) correspondingly. Similar expressions can be obtained for the probabilities \( \rho_{\varepsilon}^- \) and \( \rho_{\varepsilon}^+ \).

For example, Figure 1a shows the function’s graphs \( \rho_{\varepsilon_{xy}}^{+} \) from the mismatch (estimation error) \( \varepsilon_x = \hat{h}_x - h_x \) for the situation when the estimation parameter is a parallel shift of the image \( Z[2] \) along the coordinate \( x \). Graphs are calculated using the expressions (8) and (9) at \( \mu = 1 \) (curve 1), \( \mu = 4 \) (curve 2) and \( \mu = 10 \) (curve 3). Figure 1b shows the plots obtained using the expressions (10) and (11) at \( \mu = 2 \) (curve 1), \( \mu = 4 \) (curve 2) and \( \mu = 10 \) (curve 3). Calculation was made for images with Gaussian ACF with correlation radius 5 and signal-to-noise ratio \( g = \sigma_x^2 / \sigma_y^2 = 10 \). The experimental results (crosses) obtained by statistical modeling on simulated images with similar parameters are also shown there. The images were synthesized with use of wave model [25]. Experimental results are averaged over 150 implementations. One can see that already at \( \mu = 4 \) approximation of the pseudo gradient PDF \( \beta_i \) by Gaussian law gives satisfactory results.

\[ \text{Figure 1. Examples of PDE.} \]

3. Determination of PDF parameter estimates at a given iteration of estimation

Based on the method of calculating PDE described above, we developed a method of probabilistic finite modeling of the process of pseudo gradient estimation of parameters of interframe deformations of a sequence of images. The technique is aimed at calculating the PDF of estimated parameters and other probabilistic characteristics. The peculiarity of the method is that it allows instead of multidimensional PDF parameter estimates to store only the probability distributions of individual estimates. This made it possible to significantly reduce the requirements for the memory required for the modeling.

When forming arrays of PDE, we used the discretization of the parameter definition area. Moreover, with regular discretization of parameters in the domain of their definition, we obtain an
already irregular grid of points in the parameter space at which we calculate the values of the drift probabilities. To find the PDE values at intermediate points, a linear approximation of this irregular grid was used. The adopted approach has significantly reduced computational costs.

Another feature of the simulation is the adaptive limitation of the boundaries of the used region of the parameter space at each estimation iteration. At the same time, in order to preserve accuracy, we propose to use the PDE correction at the nodes of the sampled definition domain near the borders of the simulation window.

Figure 2 shows examples of the calculation of PDFs for the error in estimating the parallel shift of images at different iterations of relay estimation. Figure 2a obtained for the case of a varying element of the gain matrix corresponding to the shift parameter:

$$\lambda_\sigma = \sigma = \frac{\lambda_0}{1 + kt},$$

where $\lambda_0$ and $k$ are constant coefficients depending on the ACF of the images. Figure 2b corresponds to the situation $\lambda_\sigma = \text{const}$. In both cases, the initial shift offset was 5 sample grid steps, the volume of the local sample $\mu = 4$. One can see from the figures that $\lambda_\sigma = \text{const}$ starting from approximately 520 iterations the estimation process stabilizes. After that, a further increase in the number of iterations does not lead to an increase in the estimation accuracy. This allows for a given class of images to find the values of the elements of the gain matrix, which provide the required accuracy of estimation, as well as the number of iterations for the vector to achieve estimates of the stabilization region. When $\lambda_\sigma = \text{var}$ the process of formation of PDF does not have an equilibrium state and the variance of estimation theoretically constantly decreases. The accuracy of the generated estimates in this case depends on the number of iterations and the parameter $k$ reduction factor $\lambda_\sigma$. Figure 2c illustrates this by showing the dependences of the variances $\sigma^2_\varepsilon$ of estimates on the number of iterations for the same simulation conditions. Here curve 1 corresponds $\lambda_\sigma = \text{var}$, curve 2 - $\lambda_\sigma = \text{const}$. One can see that with $\lambda_\sigma = \text{const}$ the variance stabilizes, and when $\lambda_\sigma = \text{var}$ monotonously decreases with an increase in the number of iterations.

**Figure 2.** PDF and variance of parallel shift estimate at constant and decreasing step of estimation.

4. Conclusion
When modeling the process of pseudo gradient estimation of parameters of interframe geometric deformations of images, one has to take into account a complex set of influencing factors. In particular, the factors independent of the parameters of the pseudo gradient procedure are the PDF and ACF of the images and interfering noises, as well as the type of the objective function of the quality of estimation. The characteristics of the procedure that can be influenced include the method of calculating the pseudo gradient, the gain matrix, the number of iterations, and the initial approximation of the parameter estimates vector. For the feasibility of the simulation procedure, it is advisable to use a minimum set of values characterizing independent factors sufficient to find probabilistic parameter estimates as a function of the controlled characteristics of the procedure. As such values in the work used PDE.
To obtain the calculated expressions of the PDE parameters, the normalization of the pseudo gradient of the objective function is used with an increase in the volume of the local sample in which it is located. Expressions are obtained for situations where pseudo gradient estimation of the mean square of the interframe difference and the interframe correlation coefficient are used as the objective functions.

In modeling the pseudo-gradient estimation process, at each iteration, an adaptive limitation of the bounds of the simulation window in the parameter space was applied. At the same time, in order to preserve the accuracy of the simulation, a correction of the drift probabilities is performed at the nodes of the sampled parameter definition area near the borders of the simulation window. This consideration of the likelihood of finding estimates outside the simulation window made it possible to reduce the computational cost several times while maintaining the adequacy of the model.

The proposed method of probabilistic finite modeling can be used to find the accuracy and probability characteristics of stochastic algorithms for estimating interframe geometric deformations of images for a given number of iterations.

5. References

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