Concept of Lie Derivative of Spinor Fields. A Geometric Motivated Approach

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In this paper using the Clifford and spin-Clifford bundles formalism, which permits to give a meaningful representative of a Dirac-Hestenes spinor field (even section of $\mathcal{C}\ell_{\text{Spin}^\frac{1}{2}}(M, g)$) in the Clifford bundle $\mathcal{C}\ell(M, g)$, we give two distinct geometrical motivated definitions for possible Lie derivative of spinor fields in a Lorentzian structure $(M, g)$ where $M$ is a manifold such that $\dim M = 4$, $g$ is Lorentzian of signature (1, 3). These Lie derivatives, called the spinor Lie derivative ($\overset{s}{\mathcal{L}}$) and the $g$-Lie derivative ($\overset{g}{\mathcal{L}}$) are given by identical formulas when applied to spinor fields, but whereas in general $\overset{s}{\mathcal{L}} \xi g \neq 0$ (unless $\xi$ is a Killing vector field) we always have for any arbitrary differentiable vector field $\xi$ that $\overset{g}{\mathcal{L}} \xi g = 0$. We compare our definitions and results with the many others appearing in (a vast and confused) literature on the subject.