Lossy Source Transmission over the Relay Channel

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Abstract—Lossy transmission over a relay channel in which the relay has access to correlated side information is considered. First, a joint source-channel decode-and-forward scheme is proposed for general discrete memoryless sources and channels. Then the Gaussian relay channel where the source and the side information are jointly Gaussian is analyzed. For this Gaussian model, several new source-channel cooperation schemes are introduced and analyzed in terms of the squared-error distortion at the destination. A comparison of the proposed upper bounds with the cut-set lower bound is given, and it is seen that joint source-channel cooperation improves the reconstruction quality significantly. Moreover, the performance of the joint code is close to the lower bound on distortion for a wide range of source and channel parameters.

I. INTRODUCTION

In many sensor network applications, the goal is to obtain a high fidelity reconstruction of an underlying physical phenomenon at the access point. While sensors might each try to send their own observations directly to the access point, this might result in poor reconstruction quality due to the power limitations of the sensor nodes. Cooperative transmission, in which the nearby nodes help each other’s transmissions, has been analyzed extensively in the literature as a promising technique to overcome the power limitation [1], [2]. However, in many sensor network applications, the sensor observations are highly correlated. It is natural to exploit correlated source coding together with cooperative communication, and this might lead to better reconstruction at the access point than using these two techniques independently. Finding the optimal strategy in terms of the end-to-end reconstruction fidelity in a general network is a very difficult problem. In fact, we do not even know the necessary and sufficient conditions for lossless transmission over simple network components such as multiple access or broadcast channels with correlated sources [3], [4]. There are some special cases for which the optimal performance has been characterized [5]-[7]. However, it is likely that the optimal performance requires joint source-channel coding (e.g. [7], [8]), as Shannon’s source-channel separation theorem does not apply to multi-terminal scenarios in general.

In this paper, we consider cooperative transmission in which the relay has correlated side information. The goal is to reconstruct the observation of the source terminal at the destination with the least possible distortion. In this scenario, the relay terminal can help the source by both improving the channel transmission rates and by reducing the source compression distortion. In general, the problem is a joint source-channel coding problem, and characterization of the necessary and sufficient conditions is an open problem. We considered this problem for lossless transmission in [6] and gave the necessary and sufficient conditions for some special cases. In [9], we focused on the Gaussian relay channel with quadratic Gaussian sources, and provided achievable schemes and comparisons with the joint source-channel cut-set bound. The proposed joint source-channel relaying schemes in [9] are grouped into three types. In the first group, called channel cooperation, the relay simply ignores its side information, and applies one of the well-known cooperative transmission schemes such as decode-and-forward (DF) or compress-and-forward (CF) [1]. In the second group, called source cooperation, the relay ignores its received signal and uses only its side information either by uncoded transmission followed by MMSE estimation at the decoder, or by a separation-based scheme in which the source and the relay first compress their sources at rates specified by one-helper source coding problem [11] and then transmit the compressed information separately by multiple access channel coding [9]. Finally, in the third group, called hybrid cooperation, the relay sends a superposition of channel and source cooperation codewords by a suitable power allocation. It is shown in [9] that hybrid cooperation performs better than either of the other two types, and approaches the cut-set lower bound closely for various source-channel settings.

In this paper, we extend the results of [9] by using the techniques of [6], which were originally developed for lossless transmission. Specifically, we propose a joint source-channel decode-and-forward (jDF) scheme for discrete memoryless sources and channels. In jDF, the relay uses its side information to improve the relay transmission rates through a joint source-channel coding technique. The results are then generalized to the Gaussian case. In addition to jDF, for the Gaussian case, we propose a joint source-channel partial DF protocol as well as a hybrid version of jDF, in which the relay cooperates for both source compression and channel transmission. Then we propose a generalized source-channel...
cooperation protocol combining all of the above techniques, which reduces to the previous protocols as special cases. We compare the squared-error distortion achieved by the proposed schemes and the cut-set lower bound for various source-channel parameters. Compared to the classical DF scheme, jDF improves the end-to-end distortion significantly, especially when the relay side information quality is high. Further improvement is obtained with the generalized protocol which dominates the other known schemes for a wide range of source-channel parameters.

II. SYSTEM MODEL

In this section, we introduce the system model for discrete memoryless (d.m.) sources and d.m. channels with a general distortion measure and input cost constraint. Later in Section III we concentrate on the Gaussian source and channel case and provide a comparison of the proposed schemes for this case. Let \( \{S_1,k, S_2,k, S_3,k\}_{k=1}^{\infty} \) be a sequence of independent drawings of the correlated random variables \( S_1, S_2 \) and \( S_3 \) having joint distribution \( P_{S_1, S_2, S_3} \) on the set \( S_1 \times S_2 \times S_3 \).

The source \( S_1 \) is to be transmitted over a discrete memoryless relay channel specified by the conditional distribution \( P_{Y_1|X_1, X_2} \) over the set \( X_1 \times X_2 \times Y_1 \times Y \), where \( X_1, X_2 \) are the input alphabets for the source and the relay, respectively, and \( Y_1, Y \) are the relay and the destination output alphabets, respectively. The relay terminal has access to the correlated side information \( S_2 \), and the destination has access to the side information \( S_3 \). The goal is to reproduce \( S_1 \) at the destination according to the single letter distortion measure \( d: S_1 \times S_1 \rightarrow [0, \infty] \).

The source encoder observes \( S_1^m = (S_{1,1}, \ldots, S_{1,n}) \) and maps it to the transmitted codeword \( X_n^m = (X_{1,1}, \ldots, X_{1,n}) \) using a joint source-channel encoding function \( f_{1,x}^{(m,n)} \). At time instant \( k \), the received signals at the relay and the destination are \( Y_1,k, Y,k \), respectively. The relay encoder is \( f_{2,x}^{(m,n)} = (f_{2,1}^{(m,n)}, \ldots, f_{2,n}^{(m,n)}) \) where we have \( X_{2,k} = f_{2,k}^{(m,n)}(Y_{1,k}, \ldots, Y_{1,k-1}, S_2^m) \), for \( 1 \leq k \leq n \). We denote by \( b \overset{m}{=} n/m \) the rate of the code.

We assume separate cost constraints \( \Gamma_1, \Gamma_2 \) at the source and the relay, respectively. The average cost function is given as \( \gamma_i(X_n^m) = \frac{1}{n} \sum_{j=1}^{n} \gamma_i(X_{i,j}) \), where \( \gamma_i : X_i \rightarrow [0, \infty) \) is the cost of the input signal for \( i = 1, 2 \). The destination decoder observes the received vector \( Y^m = (Y_1, \ldots, Y_n) \) and its own correlated side information \( S_3^m \) and outputs its estimate of the source \( \hat{S}_1^m = (\hat{S}_{1,1}, \ldots, \hat{S}_{1,m}) = g^{(m,n)}(Y^m, S_3^m) \), where \( g^{(m,n)} : Y^m \times S_3^m \rightarrow S_1^m \) is the reconstruction function.

Definition 2.1: For given \( (b, \Gamma_1, \Gamma_2) \), we say that the average distortion \( D \) is achievable if, for any \( \epsilon > 0 \), there exist sufficiently large \( m, n \) with \( n/m = b \), and encoding and decoding functions \( f_{1,x}^{(m,n)}, f_{2,x}^{(m,n)}, g^{(m,n)} \) satisfying the input cost constraints \( \gamma_i(f_{1,x}^{(m,n)}) \leq \Gamma_i, \) for \( i = 1, 2 \), while the average distortion is bounded as

\[
\frac{1}{m} E \left[ \sum_{i=1}^{m} d(S_{1,i}, \hat{S}_{1,i}) \right] \leq D + \epsilon. \tag{1}
\]

The minimum achievable distortion \( D_{\text{min}} \) for given \( (b, \Gamma_1, \Gamma_2) \) is defined as \( D_{\text{min}} = \inf\{ D : D \) is achievable \}. We want to find \( D_{\text{min}} \) for a given source-channel model. In this paper, we provide upper and lower bounds on \( D_{\text{min}} \).

III. JOINT SOURCE-CHANNEL DECODE-AND-FORWARD (jDF)

This scheme is based on the well-known decode-and-forward relaying scheme; however, we use the side information at the relay to achieve higher rates over the source-relay channel. We use a joint source-channel relaying technique [6] which was introduced for lossless transmission of a source over the relay channel with correlated relay and destination side information. In the lossy case, the source terminal encoder generates a quantization of the source \( S_1 \) based on an auxiliary random variable \( Z \) satisfying the Markov chain relationship \( Z - S_1 - (S_2, S_3) \), and transmits a codeword to the destination with the help of the relay. For transmission, we use block Markov encoding. At the destination backward decoding specialized to this joint source-channel coding is used. We have the following achievable result for the lossy scenario, which we give without proof due to lack of space. We refer the reader to [6] for the details of the scheme.

**Theorem 3.1:** Given a relay channel \( p(y, y_1|x_1, x_2) \) and source and side information with joint distribution \( P_{S_1, S_2, S_3} \), then distortion \( D \) with respect to some distortion measure \( d(\cdot, \cdot) \) is achievable for a given triplet \( (b, \Gamma_1, \Gamma_2) \) if there exists an auxiliary random variable \( Z \) and input distributions \( P_{X_1} \) and \( P_{X_2} \) at the source and the relay, respectively, satisfying the following conditions:

1) \( Z - S_1 - (S_2, S_3) \) form a Markov chain,
2) \( E[d(S_1, h(Z, S_3))] \leq D \) for some reconstruction function \( h : Z^n \times S_3^m \rightarrow S_1^m \), and
3) \( X_i \) satisfy the input cost constraints \( E[\gamma_i(X_i)] \leq \Gamma_i \),
4) \( I(S_1; Z|S_2) \leq bI(X_1; Y_1|X_2), \) and \( \tag{2} \)
5) \( I(S_1; Z|S_3) \leq bI(X_1; X_2|Y), \) \( \tag{3} \)
   where \( p(s_1, s_2, s_3, z, x_1, x_2, y_1, y) = p(s_1, s_2, s_3) p(z|s_1) \)
   \( p(x_1, x_2)p(y_1, y|x_1, x_2) \), and the cardinality bound \( |Z| \leq |S_1| + 2 \) is satisfied.

IV. THE QUADRATIC GAUSSIAN CASE

Here we focus on the quadratic Gaussian problem, in which the source and the side information are jointly Gaussian while the channel is an additive white Gaussian noise (AWGN) channel. For clarity of presentation, we assume that the destination does not have side information, i.e., \( S_3 = \emptyset \), although our achievability schemes can be extended to the case with side information as well. Without loss of generality, we assume that the sequences \( \{S_{1,i}\} \) and \( \{S_{2,i}\} \) are independent and identically distributed (i.i.d.), zero mean and jointly Gaussian with covariance

\[
C_{S_1 S_2} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}, \tag{4}
\]
Then we have

$$Z \sim N(0, N)$$

**A. Joint Source-Channel Decode-and-Forward (jDF)**

The minimum quantization noise is given by

$$\text{var}(Z) = \min \{ \text{var}(S), \text{var}(X) \}$$

where, as before, the quadratic Gaussian case is given as [9]:

$$h = h_s - h_c - h_z = \min \{ I(S; X), I(X; Y), I(Y; Z) \}$$

Without loss of generality, we assume $|h_1|^2 = 1$, and define $|h_2|^2 \triangleq \alpha$ and $|h_3|^2 \triangleq \beta$ and let $N = N_1 = 1$ (see Fig. 1). We assume one channel use per source sample, i.e., $b = 1$.

The joint source-channel cut-set lower bound [12] in the quadratic Gaussian case is given as [9]:

$$D_{\text{min}} \geq \min_{0 \leq \xi \leq 1} \max \{ 1 - \rho^2(1 + (1 - \xi^2)(1 + \alpha)P_1)^{-1}, \}

\sigma_i^2(1 + \rho + \xi P_2 + 2\xi \sqrt{\beta P_1 P_2})^{-1} \}

in which $\xi$ is the correlation coefficient between the source and the relay codewords.

**B. Hybrid Joint Source-Channel Decode-and-Forward (hjDF)**

In the hybrid joint source-channel DF (hjDF) protocol, the relay divides its power between sending its observation to the destination directly (source cooperation) [9] and the joint source-channel DF protocol. It then transmits a superposition of the two codewords obtained using the two strategies. Suppose the relay reserves $\gamma P_2 (0 \leq \gamma \leq 1)$ for the hjDF protocol above, and $(1 - \gamma)P_2$ for transmitting a quantized version of $S_2$ to the destination. The destination first decodes the quantized $S_2$, treating the hjDF codeword as noise. Then for hjDF, this quantized $S_2$ can be used as side information at the destination. Finally, the destination combines the side information received from the relay and the information received from the hjDF codeword to obtain a reproduction of the source.

Let $Z_h = S_2 + V$, where $S_2 \perp V$ and $V \sim N(0, \sigma_v^2)$. The rate at which the relay can send $Z_h$ is

$$R_h(\gamma) = \frac{1}{2} \log \left( 1 + \frac{\beta (1 - \gamma)P_2}{1 + P_1 + \beta \gamma P_2} \right). \quad (10)$$

Then we have

$$\sigma_v^2 \geq \frac{1 + P_1 + \gamma \beta P_2}{\beta (1 - \gamma)} \quad (11)$$

For hjDF, we set $Z = S_1 + W_1$ with $S_1 \perp W_1$ and $W_1 \sim N(0, \sigma_w^2)$ in Theorem [5]. We need to have $I(S_1; Z|S_2) \leq I(X_1; Y_1|X_2)$ and $I(S_1; Z|Z_h) \leq I(X_1, X_2; Y)$, where $X_2$ is the codeword of the relay forwarding $Z_h$. We have

$$I(S_1; Z|Z_h) = \frac{1}{2} \log \left( 1 + \frac{1 - \rho^2 + \sigma_v^2}{\sigma_w^2 (1 + \sigma_v^2)} \right). \quad (12)$$

Other mutual information expressions can be calculated as in Section [V-A] by replacing $P_2$ with $\gamma P_2$. For fixed $\xi$, the minimum quantization noise is found as

$$\sigma_w^2 = \max \left\{ \frac{1 - \rho^2}{(1 - \xi^2) \alpha P_1}, \frac{1 - \rho^2 + \sigma_v^2}{(1 + \sigma_v^2)(P_1 + \beta \gamma P_2 + 2\xi \sqrt{\beta P_1 P_2})} \right\}, \quad (13)$$

and the minimum distortion for hjDF is found as

$$D_{jDF_h} = \min_{0 \leq \gamma \leq 1} \min_{0 \leq \xi \leq 1} \left( \frac{\sigma_w^2 - \rho^2}{1 + \sigma_v^2} + \frac{1}{\sigma_w^2} \right)^{-1}. \quad (14)$$

**C. Joint Source-Channel Partial DF**

In the partial decode-and-forward protocol [1], only a portion of the source message is transmitted using the relay, while the rest is transmitted directly from the source. It is shown in [13] that, for AWGN channels the partial DF protocol picks the best protocol between direct transmission (DT) and decode-and-forward (DF), but does not improve the achievable rate of DF. However, as we will show, in the presence of correlated side information at the relay terminal, partial joint source-channel DF (pjDF) has the potential for achieving lower distortion.
We have a need to signal is subtracted before decoding the direct information. We utilize the side information at the destination. Note that the relayed signal portion is decoded by treating the direct transmission as a superimposed codewords. As in partial DF [1], first the IV-B portion is decoded by treating the direct transmission as jDF. In this section, we compare the average distortion achieved by the strategies in Section IV-B for various source and channel conditions we also provide comparisons with [9]. We assume \( \sigma_1^2 = 1 \) and measure the S−R, S−D and R−D link qualities by \( P_1, \alpha P_1 \) and \( \beta P_2 \), respectively.

We first analyze the improvement of jDF over the well-known channel cooperation schemes DF and compress-and-forward (CF), where the relay side information is ignored. We set S−D and R−D link qualities to 5 dB and 10 dB, respectively. We see in Fig. 2 that, as expected, jDF improves upon the usual DF even with very low quality relay side information, i.e., for \( \rho = 0.3 \). As the side information quality increases, i.e., for \( \rho = 0.9 \), jDF outperforms both DF and CF, at all channel conditions under consideration. CF performs better than DF when the S−R quality is low.

Next, in Fig. 3 we compare the different joint source-channel cooperation schemes introduced in Section IV. We set S−D and R−D link qualities to 5 dB and 10 dB, respectively and the correlation coefficient to \( \rho = 0.5 \). Note that pjDF improves upon jDF when the S−R link quality is below the S−D link quality. On the other hand hjDF improves upon jDF for a wide range of S−R link SNR’s. This improvement increases with the quality of side information, \( \rho \), as sending this high quality side information to the destination further helps the reconstruction at the destination. We also include the advanced CF (aCF) protocol, introduced in [9], in the plot as well. This is the best CF-based protocol where private information from the source to relay is sent to improve the relay’s helper quality while the relay uses both source and channel cooperation through the hybrid strategy. We observe transmission of its source samples. The remaining power at the source and the relay is used for cooperation through the jDF protocol. We call this scheme hybrid partial jDF (hpjDF). The minimum distortion expression for this scheme can be derived as in the previous subsections.

V. COMPARISON AND DISCUSSION OF THE RESULTS

In this section, we compare the average distortion achieved by the strategies in Section IV-B for various source and channel conditions. We also provide comparisons with [9]. We assume \( \sigma_1^2 = 1 \) and measure the S−R, S−D and R−D link qualities by \( P_1, \alpha P_1 \) and \( \beta P_2 \), respectively.

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that aCF outperforms other protocols when the $S - R$ quality is relatively low, but jDF-based schemes surpass aCF with increasing $S - R$ quality. hpjDF performs very close to aCF even when the S-R link is weak, and the gap between the upper and lower bounds tightens with increasing $S - R$ quality.

Next, we fix both S-D and S-R SNRs at 0 dB, and consider high quality side information ($\rho = 0.9$). Along with hpjDF we also plot aCF and source cooperation, in which we ignore the received signal at the relay and choose the best among uncoded transmission and a separation based scheme detailed in [9]. In Fig. 4 we see that source cooperation performs much better than aCF for low R-D qualities. Due to high correlation and a relatively weak S-R link, it is more important to transmit relay’s side information to the destination. hpjDF outperforms all other protocols up to large R-D SNR values, after which aCF starts to dominate. This is in accordance with the fact that the CF strategy performs better than DF in terms of achievable rates as the S-R link becomes the bottleneck for DF [2].

Finally, we fix the average received SNRs for S-D, S-R and R-D links at 4, 10 and 4 dB, respectively, and compare achievable distortions for increasing side information quality at the relay. For low values of $\rho$ values, aCF performs better than the other protocols. This is due to the relatively high quality of the R-D link which favors CF transmission as seen in Fig. 4 as well. However, with increasing side information quality, both pure source cooperation and hpjDF outperform aCF. Note that, with even $\rho = 1$, i.e., when the relay has the exact source information, aCF falls short of the lower bound as it does not utilize its side information for beamforming to the destination as the DF based schemes.

VI. CONCLUSION

We have considered joint source-channel coding over the relay channel in the presence of correlated side information at the relay and the destination terminals. We have proposed a joint source-channel decode-and-forward (jDF) protocol which improves upon the usual decode-and-forward scheme by using the correlated side information at the relay. Then we have considered the Gaussian relay channel with jointly Gaussian source and relay side information and analyzed the achievable squared-error distortion at the destination. We have proposed a hybrid partial jDF protocol where the relay allocates its power between sending helper information and jDF while the source allocates its power between direct transmission and jDF. Currently, we are generalizing our joint source-channel relaying scheme to incorporate compress and forward [1] type forwarding at the relay as well.

REFERENCES

[1] T. M. Cover and A. El Gamal, “Capacity theorems for the relay channel,” IEEE Trans. Inf. Theory, vol. 25, no. 5, pp. 572-584, Sep. 1979.
[2] G. Kramer, M. Gastpar and P. Gupta, “Cooperative strategies and capacity theorems for relay networks,” IEEE Trans. Inf. Theory, vol. 51, no. 9, pp. 3037-3063, Sept. 2005.
[3] T. M. Cover, A. El Gamal and M. Salehi, “Multiple access channels with arbitrarily correlated sources,” IEEE Trans. Inf. Theory, vol. 26, no. 6, pp. 648-657, Nov. 1980.
[4] T. S. Han and M. H. M. Costa, “Broadcast channels with arbitrarily correlated sources,” IEEE Trans. Inf. Theory, vol. 33, no. 5, pp. 641-650, Sep. 1987.
[5] T. S. Han, “Stein-Wolf-Cover theorem for a network of channels,” Inform. and Control, vol. 47, no. 1, pp. 67-83, 1980.
[6] D. Guinduz and E. Erkip, “Reliable cooperative source transmission with side information,” Proc. IEEE Inf. Theory Workshop, Bergen, Norway, July 2007.
[7] D. Guinduz and E. Erkip, “Interference channel and compound MAC with correlated sources and receiver side information,” Proc. IEEE Int’l. Symp. on Inf. Theory, Nice, France, June 2007.
[8] M. Gastpar, “Uncoded transmission is exactly optimal for a simple Gaussian sensor network,” Proc. Inf. Theory and App. Works., San Diego, CA, Jan. 2007.
[9] D. Guinduz, C. T. K. Ng, E. Erkip and A. J. Goldsmith, “Source transmission over relay channel with correlated relay side information,” Proc. IEEE Int’l. Symp. on Inf. Theory, Nice, France, June 2007.
[10] A. D. Wyner and J. Ziv, “The rate-distortion function for source coding with side information at the decoder,” IEEE Trans. Inf. Theory, vol. 22, no. 1, pp. 1-10, Jan. 1976.
[11] Y. Oohama, “Gaussian multiterminal source coding,” IEEE Trans. Inf. Theory, vol. 43, no. 6, pp. 1912-1923, Nov. 1997.
[12] M. Gastpar, “Cut-set arguments for source-channel networks,” Proc. IEEE Int’l. Symp. on Inf. Theory, Chicago, IL, June 2004.
[13] S. Zahedi, On Reliable Communication over Relay Channels, Ph.D. Thesis, Stanford University, Stanford, CA, 2005.