Modelling Lightning Initiation and Attachment to Aircraft

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Abstract. We present initial calculations of the formation of streamers on an aircraft. A two-dimensional model has been used to determine electric field strengths and charge densities around solids of various geometries and electrical conductivities. The calculations take into account the distortion of the background electric fields by the solid material and the production and motion of charged species. Detailed time-dependent visualizations of the streamer initiation and propagation are presented. The effects of size and aspect ratio of the gross features of the aircraft on the development of the streamers is discussed.

1. Introduction
In recent years, aircraft manufacturers have been motivated to produce aeroplanes largely made out of carbon composite materials rather than aluminium, due to the significant reduction in weight for comparable or superior mechanical properties. These materials however, have significantly reduced electrical conductivities, which makes lightning protection more difficult. Planes are struck by lightning about once every year on average, so a thorough understanding of the initiation of lightning by aircraft, and in particular which regions of the aeroplane are likely to be struck by lightning, is important.

There have been a number of studies in the past in which the development of streamers has been modelled \cite{1–5}. Earlier attempts used simplified one dimensional models with an assumed diameter of the lightning leader \cite{1–3}. In more recent years, there have been some studies in more complex chemical models (i.e. with nitrogen and oxygen species considered separately) have been calculated in two dimensions.\cite{4, 5}. However the ionization rates, which have an exponential dependence on the ratio of the electric field strength to the number density of the gas, were kept either constant or artificially low, possibly due to numerical instabilities and the method used to solve the equations.

A two-dimensional axisymmetric model has been developed to determine electric field strengths and charge densities within and around solids of various geometries and conductivities. The calculations include the distortion of background electric fields by the metal and dielectric materials in the aircraft and the initiation of corona discharges on the aircraft. A finite-difference approach incorporating a direct method is used to solve the system of equations in a coupled fashion. By solving all of the equations simultaneously, as opposed to using a segregated approach, we are able to overcome numerical instabilities associated with an ionization rate which grows exponentially with the electric field strength.
2. Model

In the atmosphere, there is a fairly continuous production of free electrons, produced for example by cosmic rays. We represent the effect by including a constant production term $S_e$. Assuming air has an extremely low conductivity in the absence of an electric field (e.g., $10^{-17} \text{ S/m}$) and knowing the mobilities of the charged species, one can estimate the equilibrium charge densities without the presence of an electric field; this is our initial condition. The computed solution is not sensitive to the initial conditions for the charged species densities for the parameters used in these calculations, as the peak electric field far exceed the breakdown value of air and the charged species density grows rapidly in the initial stages. Electrons in the atmosphere slowly attach to form negative ions, in particular $\text{O}^-$ charged species density grows rapidly in the initial stages. Electrons in the atmosphere slowly attach to form negative ions, in particular $\text{O}^-$ ions. The rate of this attachment is represented by $\eta/N$, where $N$ is the gas number density and the attachment coefficient $\eta$ is the fraction of electrons attached in moving a unit distance forward in the electric field. In regions of high electric field, electrons are of sufficient energy to ionize air molecules to form a new electron and a positive ion. This rate is represented by $\alpha/N$, where the ionization coefficient $\alpha$ is the fraction of the electrons that produce a new electron by ionization in moving a unit distance forward in the electric field. Thus the net increase in electrons in unit time at a given position is $\partial n_e/\partial t = n_e (\alpha - \eta) W_e$, where $W_e$ is the electron drift velocity. Additional terms are required in the electron continuity equation to account for the recombination of electrons and positive ions, and to represent the effects of convective flow of the charged particles. Lastly, a diffusion term is included, where the diffusion coefficient, $D_e$, is estimated by using the Einstein–Smoluchowski relation, i.e., $D_e/\mu = k_B T/e$, where $\mu = W_e/E$ is the electron mobility, $k_B$ is the Boltzmann constant, $e$ is the charge of an electron and $T$ is the electron temperature.

Predictions of the onset of corona (streamer) discharges at typical electric potentials are obtained by using a control volume approach to solve the equations

$$\nabla^2 \phi = \frac{e}{\epsilon_0} (n_+ - n_- - n_e)$$  \hspace{1cm} (1)

$$\frac{\partial n_e}{\partial t} = \alpha W_e n_e - \eta W_e n_e - \gamma n_e n_+ - \nabla \cdot (W_e n_e) + \nabla \cdot (D_e \nabla n_e) + S_e$$  \hspace{1cm} (2)

$$\frac{\partial n_+}{\partial t} = \alpha W_e n_e - \gamma n_+ (n_e + n_-) - \nabla \cdot (W_+ n_+) + \nabla \cdot (D_+ \nabla n_+) + S_+$$  \hspace{1cm} (3)

$$\frac{\partial n_-}{\partial t} = \eta W_e n_e - \gamma n_- n_+ - \nabla \cdot (W_- n_-) + \nabla \cdot (D_- \nabla n_-)$$  \hspace{1cm} (4)

where $t$ is the time, $n_x$ is the number density of electrons, positive and negative ions for $x = e, +$ and $-$ respectively and $\gamma$ is the recombination coefficient.

The ionization and attachment coefficients are each a function of the electric field; that is $\alpha/N$, $\eta/N$ and $W$ depend on $E/N$. The first two of these variables are shown in Fig. 1. The values are taken from the theoretical predictions of [6], and are consistent with other theoretical investigations and also with experimental values of both ionization and attachment coefficients [7]. Recombination coefficients and ion mobilities were taken from [6].

3. Results

The particular configuration of our calculations is illustrated in Fig. 2. The thundercloud is represented as imposing a potential of -100 MV at the upper plane of the computational domain. Frequently, thunderclouds have been calculated to be at higher potentials, for example at -500 MV. The height of this potential surface is taken to be at 2 km. Aircraft frequently fly at higher altitudes, but most lightning strikes to aircraft seem to occur as the aircraft approaches the earth. We choose the model aircraft to be a vertical cylinder of radius 25 cm and length 50 m. We choose the cylinder to be vertical as this will be a position for which there is maximum enhancement of the electric field from the cloud at the tips of the cylinder. The length of 50 m
is chosen to be representative of lengths of typical large aircraft. The radius of 25 cm is chosen to approximate the radii of curvature occurring at wing tips and the tail of aircraft. Then, the ratio of the cylinder’s length (measured from its centre to its tip) of 25 m, divided by its radius of 25 cm, gives an aspect ratio of 100.

There are three stages in the development of the electric field. First, there is the background field from the thundercloud of 50 kV/m produced from the 100 MV postulated for the cloud potential at a height of 2 km. Second, there is distortion by the cylinder or aircraft fuselage, as illustrated by Fig. 3, where as yet there are no electrical charges in the air at the high field tips of the cylinder. Third, there are distortions due to the space charge produced by the discharge.

Fig. 3 shows the electric field strength in the immediate surroundings of the conductive cylinder. The electric field strength inside the conductor is calculated to be less than 10 V/m, significantly lower than the average field strength of 50 kV/m. The electric fields in the gas in the immediate surroundings of the cylinder tips can be seen to be significantly enhanced, with field strengths of the order of 5 MV/m, in agreement with the approximate analytic prediction.

The left-hand side of Fig. 4 shows, in more detail, the electric field in the vicinity of the top tip of the cylinder shown in Fig. 3. The peak electric field in this figure exceeds 2.5 MV/m, thus streamer formation is expected as the electric field exceeds the breakdown field of air. The right-hand side of Fig. 4 shows typical calculated field distributions once the space charge effects have become important. This field distribution changes with time as the discharge develops and becomes larger, there being a significant maximum of the electric field at the leading edge of the discharge.

The distributions of electron, positive-ion, negative-ion and net-charge densities for the

Figure 1. Ionization and attachment coefficients, normalised to the number density, as a function of the ratio of electric field strength and number density in dry air. Top and right scales indicate values at 1 atmosphere.
Figure 2. Configuration of the calculations, with the aircraft represented by a cylinder of length 50 m and radius 25 cm halfway between the earth and a thundercloud at 100 MV and height 2 km. (Figure not to scale).

Figure 3. Electric field at an early stage (t ≈ 100 ns), showing distortion of the background field by the cylinder. The difference between cylinders of different conductivities cannot be distinguished at this magnification. Note the scale is logarithmic in this figure and the background field is 50 kV/m.

conditions of Fig. 4 are shown in Fig. 5. In these figures, the values of the densities in the metal are set to zero for clarity, as the electron and positive ion densities are several orders of
Figure 4. Left: Electric field at an early stage ($t \approx 100$ ns) near the top cap of a cylinder, showing distortion of the background field by the cylinder. Right: As with left at $t = 1.84 \mu$s.

The maximum electron densities on the left of Fig. 5 occur in the regions of maximum electric field strength in Fig. 4. When the field drops below 2.5 MV/m, there is a rapid transition from net electron production to net electron loss due to attachment of electrons to the neutral gas molecules. Electron attachment to neutrals will reduce the brightness of a stepped leader.

Figure 5. Electron density (left) and positive ion density near the top cap of the cylinder at $t = 1.84 \mu$s.

The right-hand side of Fig. 5 shows the positive-ion density distributions. The negative-ion density, which is not shown, has approximately the same distribution. These densities are generally larger than the electron density. Despite this fact, the net charge density is still large, as shown in Fig. 6. This indicates that there is a significant degree of distortion of the electric field by space charge.
Figure 6. Absolute value of net charge density near the top cap of the cylinder at $t = 1.84 \mu s$. The inset shows the net charge, indicating the regions which are negatively or positively charged.

The drift velocity of the electrons with the calculated electric fields is of the order of $10^4$ m/s. Within the time frame, $\sim 2 \mu s$ of Fig. 5, the streamers have grown to approximately 25 cm, which means the streamers grow at a rate of $10^5$ m/s, which is an order of magnitude larger than the drift velocity of the electrons (which in turn is two orders of magnitude larger than the drift velocity of the heavy ionized species). This indicates that the growth of the streamers is predominately due to the production of charges rather than the charged species rearranging themselves at a rapid pace. In areas where the electric field is large, the charged species production is also large which then quickly leads to a large density of charge species. Once the number of charged species is sufficiently large, the distance each charged species needs to move to make a significant differences to the net charge density (and therefore the electric field) is reduced and therefore the electric field within the streamers are significantly smaller than at the edges of the streamers, where there are fewer charged species to alter the field.

The results presented here were only for the top half (positive end) of the cylinder. The results on the other end are similar; however, the streamer grows at a steady rate on the negative end in contrast to the positive end, which has a stop–start behaviour.

4. Conclusion
We have simulated the initiation of streamer discharges from a capped cylinder, of dimensions similar to that of an aeroplane fuselage, in an electric field representing that between the thundercloud and ground. The calculations were made in two dimensions because of the available computational resources. This means that the streamers are effectively conical in shape. Nevertheless, we illustrate the interaction between electric fields of charged species that lead to corona initiation and streamer progress.

We have achieved significantly higher charge densities than were possible in a previous study [8], in which values of the ionization coefficient were capped to avoid numerical instabilities. Because such capping is no longer necessary with our new code, we are able to achieve higher charge densities, and accurately predict the faster formation of streamers that occurs in nature.

This model is and does not take into account the role of ice or water. Ice is widely believed...
to play a role in the formation of lightning. There are two mechanisms through which ice is believed to assist the formation of lightning. The first is that collisions between ice particles are believed to cause non-inductive charge transfer which then separate forming a large electric field [9]. The second is the fact that ice can have sharp edges which, combined with its relatively high electrical conductivity compared to air, can enhance the electric field locally around the ice particles. Neither of these mechanisms has been considered, as explaining the high potential of the cloud was outside the scope of the paper, whose main focus is investigating how lightning is triggered from the aircraft. Water in the form of vapour modifies both the ionization and attachment coefficients. The effect on the formation of streamers is complex and will be a subject of future work.

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