Dependence of the Fractional Quantum Hall Edge
Critical Exponent on the Range of Interaction

Recent experiments on external electron tunneling into an edge of a fractional quantum Hall (FQH) system find a striking current-voltage power law behavior $I \propto V^\alpha$, with exponent $\alpha \approx 2.7$ on the $f = \frac{1}{3}$ FQH plateau [1]. Such power law $I$-$V$ characteristic with $\alpha = 3$ was predicted for electron tunneling into an edge channel at the boundary of the $f = \frac{1}{3}$ FQH system: the low-energy dynamics is effectively 1D, and field-theoretic descriptions of edge channels as chiral Luttinger liquids have been developed [2].

The Hilbert space is restricted by consideration of orbitals with angular momentum $\ell \leq \ell_{max}$ only. For the Haldane $V_1$ short-range interaction, the Laughlin $\rho_L(m)$ with $m > m_{max}$ is $3(N - 1)$ vanish identically; for Coulomb interaction good convergence of $\rho_C(m)$ is obtained by $m_{max} = m_{max}^L + 5$ [3]. For example, for $N = 12$, the largest $f = \frac{1}{3}$ FQH system studied, $M = 198$ and the size of the Hilbert space is 15,293,119 for $m_{max} = 35$. Details of this study will be published elsewhere [3]. As has been demonstrated by Wen [3], the critical exponent $\alpha$ is equal to the ratio of the occupation numbers

$$\alpha = \rho(m_{max}^L - 1)/\rho(m_{max}^L)$$

for the Laughlin state $\Psi_L$ on the disk. Wen has also argued that this relationship must hold for any interaction, so long as the FQH state at the same filling $f$ exists, and is not unique for the Laughlin wave function. In Fig. 1 we present the ratio of the occupation numbers for both $\Psi_L$ (short-range interaction) and $\Psi_C$ (true Coulomb interaction) for $N = 3$ to 12. As expected, we obtain $\alpha_L = 3$ to machine accuracy for $\Psi_L$. For Coulomb-interacting electrons, the ratio is always less than 3, and an extrapolation to the thermodynamic limit gives $\alpha_C = 2.62$.

While we do not know whether the extrapolation shown in Fig. 1 holds for $N > 12$, certain other systematic behavior present in the numerical data [3] allows us to project that in the $N \to \infty$ limit $2.58 \leq \alpha_C \leq 2.75$, and is definitely less than 3.

Thus we propose that the deviation of the experimental $\alpha$ from the predicted $\alpha_L$ values is not an artefact, and is not due to corrections such as finite bulk diagonal conductivity, disorder, or a variation of the electron density in the sample. Rather, we propose, the effect has a fundamental origin: the value of the critical $I$-$V$ exponent is not universal, but depends on the range of particular interaction.

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[6] The sum of $\rho_C(m)$ with $m > m_{max}$ is less than 0.01/$N$. This means that fixing total angular momentum $M = \frac{2}{3}N(N - 1)$ selects the $f = \frac{1}{3}$ state for Coulomb interaction too: it fixes average density on the disk $\langle \rho_C \rangle \approx \langle \rho_L \rangle$ (for $r < r_{edge}$) to better than $10^{-3}$ for any $N$. 

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FIG. 1. The ratio of the angular momentum occupation numbers $\rho(m)$ for $N$ interacting electrons on the disk. Shown are the Laughlin state $\rho_L$ for a short-range interaction, and the exact ground state for the Coulomb interaction $\rho_C$. 

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