Stochastic-Based Reliability Analysis of Stainless-Steel Beams Under Bending

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Abstract. The presented article studies the bending resistance of a stainless steel hot-rolled profile UPE 80, which is stressed by bending around the minor principal axis. Resistance is studied as the random output variable, which is a function of input material and geometric characteristics. The paper deals with the stochastic analysis of this static resistance. The computational model is created on the basis of the finite element method using geometric and materially nonlinear solution. The Ansys software with 4-node Shell 181 element is used. The input random variables of the stainless steel are taken from previous research aimed at identifying the material mechanical properties based on experimental research of austenitic chromium-nickel stainless steel 1.4307 / AISI 304 L. Statistical analysis is performed using the Latin Hypercube Sampling method. The probability of achieving standard design resistance is estimated and compared with the reliability level in standard EN1990 given by the reliability factor beta 3.8. The article discusses the need for a larger number of samples for reliable estimates of design resistances and for the verification of partial reliability factors, which are a challenge for further research.

1. Introduction
The use of stainless steel for bridge structures is relatively recent, 20 to 25 years. In addition to the traditional use as architectural features, guardrails and handrails, stainless steel is increasingly being used for structural components of decks or in suspension systems, as well as in anchorage components. Material savings in stainless steel structures are just as desirable as in carbon steel structures. Economical design of light and slender systems is a general trend. However, design codes and standards for new stainless steel structures are not as sophisticated as for carbon steel structures. In particular, the long-term behaviour of stainless steel in bridge systems has not been sufficiently examined.

Stainless steel has a number of advantages that make it increasingly popular in architectural and construction applications. The main advantage of structural stainless steel is its strength in bridges and low thermal conductivity in civil buildings. For bridge structures, stainless steel is used for a relatively short time and thus both design conditions and long-term behaviour and degradation processes under repeated loading are still the subject of research [1, 2]. The priority issue is safe design, which must respect design differences from carbon steel structures, whose design criteria are known and have been verified for a long time. The objective of this paper is the statistical analysis of the bending resistance of a hot-rolled cross section UPE80 made from austenitic chromium-nickel stainless steel 1.4307 / AISI 304 L.
Great attention has been paid to diagnostic methods of corrosion in previous periods [3-5]. Most of these problems are eliminated when stainless steels are used for the load-bearing structural system and other important parts of the structure exposed to environmental degradation processes. Computational models of stainless steel load-bearing elements are usually based on geometrically and materially nonlinear solutions, similar to simulations for carbon steel structures [6]. In civil engineering, computational models can generally be divided into deterministic [7, 8] and stochastic [9-11]. The goal of stochastic computational models is statistical [12], sensitivity [13, 14] and probabilistic analysis [15]. Another useful extension of computational models are algorithms based on reliability functions and operational research [16, 17]. These and other methods can be adapted to study the limit states of stainless steel structures, which are designed on the basis of similar principles as existing structures. Although the price of stainless steel is higher, easy maintenance and long service life are clear advantages that make it important to examine the limit states and service life of stainless steel structures. The aim of the presented article is the ultimate limit state, which is one of the important states in the design of load-bearing structures.

2. Stochastic model based on finite elements and nonlinear solution

The static resistance of the UPE80 stainless steel beam subjected to bending around the z-axis is studied, see Figure 1. The static analysis was performed on the model of a steel member using the geometrical and materially nonlinear FEM calculation in the ANSYS program [18]. The ANSYS program is declared by the manufacturer as an "open system", which allows control of the computation, definition of own material models, etc. When entering input data via text files (macros), it is possible to use library functions for output control, mathematical, search and other functions. The possibility of effectively repeating the calculation with modified input values gives space for the development of parametric studies.

Figure 1. The geometry of hot-rolled cross section UPE80.

The UPE80 rolled steel bar was modelled using shell elements Shell 181 [18]. It can be noted that a similar modelling approach has been used effectively in nonlinear analyses [19, 20] and has lower CPU time requirements than modelling using the Solid finite element [21, 22]. The length of the bar is considered to be two meters. The Shell 181 element used here is a 4-node element with six degrees of freedom at each node - 3 translations and 3 rotations. This shell element is suitable for solving plasticity tasks, large displacements and large strain. The density of division of the structure by finite elements was set so that the ratio of the lengths of the sides of the elements is as close as possible to the ideal ratio of 1:1, see Figure 3.
Unlike standard carbon steel, the study of stainless steel has no sharp yield point and possess a rounded stress-strain curve, with higher ductility. The stress vs strain curve identified in [23] was used in the material model, see Figure 2. This curve has been simply replaced by a parabolic-rectangular working curve, which is linear up to $0.5\sigma_{0.2}$ and then continues with a quadratic parabola up to $\sigma_!$, where $\sigma_{0.2}$ is the material 0.2% proof stress. Although this approach is used for concrete structures, it has also been shown to be usable for stainless steel structures.

The boundary conditions at the member end have free displacements in the direction of the member axis. The left end has fixed displacements in the direction of both axes in the cross-sectional plane at all nodes of the web of the cross-section. The right end has fixed displacements in the cross-sectional plane at all nodes of the web of the cross-section, but only in the direction perpendicular to the web of the cross-section. Bending is introduced into the cross section by pairs of forces on each flange. The last row of elements at both ends has a modulus of elasticity fifty percent higher than the intermediate part of the beam. This stiffening ensures that the bending moment is introduced into the cross section without local failure that influences the output static resistance.

The mean value of $\sigma_{0.2}$ is considered as 257 MPa according to research [23]. The coefficient of variation is considered as 0.06 according to [2]. Other geometric characteristics are introduced as deterministic. According to the results of research [24-29] the flange thickness $t_2$ is the second most influential variable, which should be considered as random. The statistical characteristics of flange thickness $t_2$ are introduced according to experimental research [30, 31]. The mean value of flange thickness $t_2$ is introduced with the nominal value 7 mm. The variation coefficient of flange thickness $t_2$ is introduced with the value of 0.046 and thus the standard deviation is 0.323 mm. The third variable, which is considered random, is the web thickness $t_1$. The mean value of $t_1$ is considered to be 4 mm. The variation coefficient of web thickness $t_1$ is introduced with the value of 0.039 and thus the standard deviation is 0.156 mm. All three input random variables are introduced using Gauss probability density functions.
3. **Stochastic analysis results**

The stochastic analysis was performed using the numerical Latin Hypercube Sampling LHS simulation method \([32, 33]\). Ten thousand runs of the LHS method were used to evaluate the resistance of hot-rolled steel members from stainless steel, see Figure 4.

![Figure 3](image1.png)

**Figure 3.** Finite element model of UPE80 made from stainless steel 1.4307.

![Figure 4](image2.png)

**Figure 4.** Finite element model of UPE80 made from stainless steel 1.4307.

The statistical characteristics of random resistance obtained using ten thousand runs in the Ansys software based on a geometrically and materially nonlinear analysis in each run are as follows. Valid observations: 10000 runs, minimum: 2494.6 kN, Maximum: 4105.6 kN, range: 1611.1 kN, median: 3234.9 kN, arithmetic mean: 3236.5 kN, geometric mean: 3229.3 kN, mean square: 46213 kN, variance: 46218 kN, stand. deviation: 214.98 kN, coef. of variation: 0.066425, standard skewness: 0.059311, standard kurtosis: 3.0409. The relatively small values of standard skewness and standard kurtosis close to three indicate the possibility of approximation by Gauss probability density function. The Chi-square distribution test was selected as the testing method. One hundred classes were used in the test. The
resistance samples were compared with a Gauss probability distribution function using the significance level of 0.37. The critical significance level was 0.05. The test concluded that the hypothesis should not be rejected. Gauss probability distribution function can be used as a suitable approximation of the static resistance. Repeating the same test for the lognormal probability distribution function concluded that the hypothesis should be rejected and therefore the lognormal probability distribution function is not suitable for approximating the static resistance.

The EN 1996-1-4 standard specifies the static resistance of the UPE80 beam under bending as 2860 kNm. Four percent of all random realizations are below the standard (design) resistance level of 2860 kNm, see Figure 4. This is a relatively high probability, provided that the design resistance is 0.1 percentile for the reliability index $\beta=3.8$, which is provided by the EN1990 standard [34]. An alternative approach may compare the design resistance estimated as 0.1 percentile from the statistical analysis of the static resistance with the design resistance according to EN 1996-1-4 rather than comparing the achieved and required probability of not achieving the optimal design value [35, 36]. The goal of all approaches is to verify the reliability and achieve optimal design parameters.

4. Conclusions
Hot-rolled beams made of austenitic chromium-nickel stainless steel 1.4307 / AISI 304 L are a relatively new product range on the market with great prospects for load-bearing structures exposed to corrosion. The material and geometrical characteristics of these steel have not yet been studied as thoroughly as carbon steel, so it is extremely important to collect statistical characteristics from experimental research so that they are useful inputs of stochastic computational models. The presented article showed that the reliability of a load-bearing element made of austenitic chromium-nickel stainless steel 1.4307 / AISI 304 L may not be sufficient. This conclusion was reached by estimating the probability of attaining the design quantile from the design standard using random realizations of the static resistance calculated by the finite element method. Although the results refer to one UPE80 rolled bar, they indicate low reliability and the need for further verification of reliability parameters using stochastic models and experimental research.

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