MHD models of stellar core collapse with GenASiS

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Abstract. We present magnetohydrodynamic (MHD) models of stellar core collapse computed with our astrophysical MHD code GenASiS. Microphysics in the models include a realistic equation of state and a simple parametrization of deleptonization during collapse. We give a brief description of some numerical results, and discuss requirements for capturing the physics of the magnetorotational instability in the context of core collapse supernovae.

1. Introduction

Core collapse supernovae represent the final evolutionary stage of a massive star 8-25 times more massive than the Sun. As the iron core reaches the Chandrasekhar mass and loses its pressure support, it collapses in its own gravitational field. Collapse proceeds to nuclear densities when the supernova shock wave is launched which ultimately disrupts the star. The details of the mechanism behind these cosmic explosions remain elusive.

Magnetic fields were considered, for the first time, in the context of stellar core collapse by LeBlanc and Wilson [8] in the early 1970s. Later MHD models appeared only sporadically in the literature, and the work of Symbalisty [12] remained for a long time the most sophisticated MHD model. The requirement of rapid rotation and strong fields may have prematurely deemed magnetic fields as insignificant to the explosion mechanism of core collapse supernovae because the required levels seemed unattainable.

After the magnetorotational instability (MRI) [2] was introduced into the context of core collapse supernovae [1], the study of magnetorotational collapse has become an active field of research (see [11] and references therein), and there is now a reemerging consensus that MHD effects may be important. The MRI provides a promising mechanism for generating dynamically significant fields on a time scale that is comparable to the explosion time scale (\(\leq 1\) s). The criterion for the MRI to operate is likely satisfied in the post-bounce-pre-explosion supernova environment, and MHD effects may prove to be a necessary part of a realistic model. Magnetic fields are only now being included in models with neutrino transport [3]. Detailed neutrino transport and the requirement of high spatial resolution in multiple dimensions make the problem computationally expensive. However, we expect to see progress in the near future as petascale computing platforms become available.
Table 1. Parameters summarizing our numerical models: Initial rotation period, $P_i$, maximum initial field strength, $|B|_{\text{max}}$, and the ratio of initial magnetic to rotational energy, $\beta_i$. The time of bounce is denoted $t_b$. We also list the maximum magnetic field, and ratio of magnetic to rotational energy 2 ms after bounce: $|B|_f^{\text{max}}$ and $\beta_f$, respectively.

| Model | $P_i$ [s] | $|B|_{\text{max}}$ [G] | $\beta$ | $t_b$ [ms] | $|B|_f^{\text{max}}$ [G] | $\beta_f$ |
|-------|-----------|-----------------|--------|--------|-----------------|--------|
| A     | 4.0       | $2 \times 10^{12}$ | $2.8 \times 10^{-3}$ | 108    | $8.7 \times 10^{14}$ | $5.1 \times 10^{-4}$ |
| B     | 4.0       | $2 \times 10^{13}$ | $2.8 \times 10^{-1}$ | 108    | $8.3 \times 10^{15}$ | $5.1 \times 10^{-2}$ |
| C     | 1.0       | $2 \times 10^{13}$ | $1.7 \times 10^{-2}$ | 131    | $6.7 \times 10^{15}$ | $1.2 \times 10^{-2}$ |

2. Physical model and numerical solution

We have computed axisymmetric non-relativistic MHD models of stellar collapse and early post-bounce evolution for the 13 solar mass pre-collapse progenitor of [10]. Rotation is added to the progenitor by a simple parametrization for the angular velocity [11]. The parameter controlling the degree of differential rotation is set to 1000 km. The initial configuration is threaded by a dipole-like magnetic field computed from a magnetic vector potential that scales with the density distribution of the progenitor [11]. In order to close the system of MHD equations we use a tabulated version of the equation of state (EoS) of [7]. Neutrino transport is important from the onset of collapse, and in order to approximately include the effects of neutrino physics in our models we use the simple, but very efficient, parametrization given by [9]. The parametrization is based on state-of-the-art collapse simulations employing multigroup Boltzmann neutrino transport in spherical symmetry. It is able to reproduce the dynamical response to the neutrino thermalization-diffusion process during collapse, up to core bounce and shock formation, making it useful for exploratory, multidimensional MHD models. The approximation breaks down shortly after core bounce, when non-local neutrino transport effects are important. We run our model somewhat beyond its validity in order to study the early post bounce evolution of the magnetic field and test our MHD code.

For the numerical solution the equations of MHD are cast into a conservative form, allowing the use of efficient shock capturing methods. In GenASiS we have implemented a second order, semi-discrete central scheme [6] in spherical coordinates, using the method of constrained transport [5] to preserve the divergence-free condition on the magnetic field. We use linear reconstruction inside computational cells for spatial differencing, and an explicit two-step Runge-Kutta integrator for the temporal evolution. The linear system arising from the finite volume discretization of Poisson’s equation is solved twice every time step for the gravitational potential using routines from the PETSc library.

Our computational domain covers 0 to 1400 km in radius and 0 to $\pi$ in co-latitude. Our grid resolution is chosen to $N_r \times N_\theta = 512 \times 64$. The mesh is equidistant in the polar angle while the radial zone size increases with radius, starting with an innermost zone with $\Delta r = 0.3$ km. This radial resolution is necessary in order to resolve the centrally condensed post bounce structure.

3. Results

Key parameters distinguishing our models (labeled A, B and C) are listed in table 1. The initial parameters are chosen in order to illustrate the combined effect of rotation and magnetic field.

All three models exhibit qualitatively similar evolution during collapse and early post bounce: Due to angular momentum conservation, the angular velocity of the homologously contracting inner core increases as $\Omega \propto \rho^{2/3}$. After core bounce, the angular velocity is nearly constant

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in the proto-neutron star (PNS), falling off as $\propto r^{-3/2}$ at larger radii. The magnetic field is “frozen-in” to the fluid, and the poloidal magnetic field increases as $B \propto \rho^{2/3}$ during collapse. Due to wrapping of the poloidal field by differential rotation the toroidal field, $B_\varphi$, increases somewhat more strongly with density; see upper panel in Figure 1. As the inner core reaches nuclear density a compression wave is formed which turns into a shock wave at the boundary between the homologously collapsing inner core and the supersonically collapsing outer core. The radially propagating shock weakens, stalls, and turns into an accretion shock around 100-200 km from the center, some 10-20 ms after bounce. In the lower panel of Figure 1, we plot the time evolution of the average magnetic field (model A only) during the first 35 ms post bounce. Field amplification due to compression has stopped: $B_r$ and $B_\theta$ stay constant at a level of about $2.5 \times 10^{14}$ G and $3.0 \times 10^{14}$ G, respectively. The toroidal component is increasing linearly with time on the rotation time scale due to differential rotation near the surface of the PNS.

The inner region settles quickly into quasi-hydrostatic equilibrium, defining the compact PNS. The central density is $\sim 3 \times 10^{14}$ g cm$^{-3}$. The PNS has an initial mass and radius of about 0.7 $M_\odot$ and 20 km, respectively, rotating with a period of a few milliseconds ($P \sim 8.5$ ms for models A and B, and $P \sim 3$ ms for model C). The rotational energy is about $3 \times 10^{50}$ erg, 2 ms after bounce, for models A and B, and $\sim 4 \times 10^{51}$ erg for model C. Magnetic energies and field strengths at this time are listed in Table 1. The magnetic energy must reach a significant fraction (\sim 0.05 – 0.1) of the rotational energy in order for magnetic stresses to have an impact on the rotational dynamics. Figure 2 illustrates that a strong magnetic field can effectively extract rotational energy from the PNS. In Figure 3 we compare models A and C about 40 ms after bounce: Model A differs very little from models without magnetic field, while the outflow along the polar axis in model C is strongly magnetized.

The magnetic field of pre-collapse progenitors, although unknown, is believed to be weaker
Figure 3. Plot of entropy per baryon 40 ms after bounce for model A (left panel) and model C (right panel). Contours are drawn for $\rho = [10^{12}, 10^{13}, 10^{14}]$ g cm$^{-3}$. The emerging outflow along the polar axis in the right panel is strongly magnetized.

than those used in our initial models. Even after the few orders of magnitudes increase achieved during collapse, a time longer than the explosion time may be needed to reach dynamically significant fields from winding due to differential rotation; especially if the progenitor star is rotating significantly slower than considered here. The MRI seems to be the most promising mechanism generating fields relevant to the supernova explosion.

The MRI was discovered as a powerful field amplification mechanism, present in weakly magnetized shear flows [2]. The instability operates quite generally in regions with negative gradient in the angular velocity. The region between the surface of the proto-neutron star and the stalled supernova shock wave is where the MRI is believed to operate [1]. The growth time for the fastest growing mode is $\tau_{\text{MRI}}^{\text{max}} \sim \frac{4}{3}P$, proportional to the rotation period and independent of the magnetic field strength. The wavelength of the fastest growing mode is $\lambda_{\text{MRI}}^{\text{max}} \sim v_A P$, where $v_A$ is the Alfvén speed. We find that the criterion for instability is satisfied over extended regions in our models, immediately after bounce. By calculating the characteristic time and length scales governing the fastest growing unstable mode we find that $\tau_{\text{MRI}}^{\text{max}}$ in the region between the surface of the PNS and the shock wave varies roughly between 10 ms to about 100 ms. The characteristic length scale of the fastest growing mode extends down to about 0.1 km around $r \sim 15$ km for the model with the weakest magnetic field and is increasing with radius. Beyond 50 km from the center all models show length scales larger than 1 km, with $\lambda_{\text{MRI}}^{\text{max}}$ varying in the range of about 1 km to about 20 km. However, $\lambda_{\text{MRI}}^{\text{max}}$ scales with the local field strength and will be smaller for weaker magnetic fields. We do not see any clear evidence of the MRI operating in our models. This is most likely due to limited spatial resolution, but perhaps also because in regions where the resolution may be sufficient the time scale is longer than our simulation time ($\sim t_b + 40$ ms). In order to fully resolve the MRI, unstable modes must be resolved by $\sim 10$ grid points [4]. A reasonable estimate of the saturation field is obtained by equating the Alfvén speed with the local rotation speed; $B_{\text{sat}} \sim (\mu_0 \rho)^{1/2} v_\phi$. Our models indicate saturation fields ranging from
10^{14} \text{ G}, to well above 10^{16} \text{ G}. Such strong fields could certainly have an impact on the dynamics inside the shocked cavity at the center of an exploding star.

4. Outlook
The high spatial resolution required in order to capture the MRI in supernova models, bearing in mind that each spatial cell carries a 3D momentum space grid for neutrino transport, is sobering. However, valuable insight into how the MRI operates in the early post-bounce supernova environment can be gained from three dimensional calculations similar to those presented here. Moving to three spatial dimensions is important in order to break free from the artificial constraint imposed by axisymmetry. In particular, the toroidal magnetic field generated by differential rotation is not permitted to be converted back into a poloidal magnetic field prohibiting a full dynamo to operate, excluding potentially important physical effects. The MRI is local in its nature, requiring efficient use of adaptive mesh refinement.

Evolving 10^{10} dynamically distributed zones for about 10^6 time steps with a flop count of 10^4 per zone per time step will require about four days on a petascale system, assuming a sustained 30 percent efficiency of peak performance. This will allow us to gain a deeper understanding of what role magnetic fields play as the stellar core reaches explosive conditions.

About 99% of the energy released during collapse is carried away by neutrinos that interact with matter and largely determine the physical conditions in the proto-neutron star “magnetosphere”. A realistic model must ultimately include angle dependent spectral neutrino transport at the required resolution. For this a Boltzmann neutrino transport solver is currently being developed and integrated into GenASiS. Petascale computing platforms will enable high resolution radiation magnetohydrodynamic simulations with neutrino transport in two spatial dimensions, which will bring a new level of realism to supernova modeling.

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