General analysis of the rare $B_c \rightarrow D_s^* \ell^+ \ell^-$ decay beyond the standard model

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Abstract

The general analysis of the rare $B_c \rightarrow D_s^* \ell^+ \ell^-$ decay is presented by using the most general, model independent effective Hamiltonian. The dependencies of the branching ratios, longitudinal, normal and transversal polarization asymmetries for $\ell^-$ and the combined asymmetries for $\ell^-$ and $\ell^+$ on the new Wilson coefficients are investigated. Our analysis shows that the lepton polarization asymmetries are very sensitive to the scalar and tensor type interactions, which will be very useful in looking for new physics beyond the standard model.

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1 Introduction

Rare $B$ meson decays, induced by flavor-changing neutral current (FCNC) $b \to s, d$ transitions, are excellent places to search for new physics because they appear at the same order as the standard model (SM). In rare $B$ meson decays, effects of the new physics may appear in two different manners, either through the new contributions to the Wilson coefficients existing in the SM or through the new structures in the effective Hamiltonian which are absent in the SM.

There have been many investigations of the new physics through the study of rare radiative, leptonic and semileptonic decays of $B_{u,d,s}$ mesons induced by FCNC transitions of $b \to s, d$ [1] since the CLEO observation of $b \to s \gamma$ [2]. The studies will be even more complete if similar decays for $B_c$ are also included.

The study of the $B_c$ meson is by itself quite interesting too, due to its outstanding features [3]–[5]. It is the lowest bound state of two heavy quarks ($b$ and $c$) with explicit flavor that can be compared with the charmonium ($c\bar{c}$-bound state) and bottomium ($b\bar{b}$-bound state) which have implicit flavor. The implicit-flavor states decay strongly and electromagnetically whereas the $B_c$ meson decays weakly. The major difference between the weak decay properties of $B_c$ and $B_{u,d,s}$ is that those of the latter ones are described very well in the framework of the heavy quark limit, which gives some relations between the form factors of the physical process. In case of $B_c$ meson, the heavy flavor and spin symmetries must be reconsidered because both $b$ and $c$ are heavy.

From the experimental side, the running B factories in KEK and SLAC continue to collect data samples and encourage the study of rare $B$ meson decays. It is believed that in future experiments at hadronic colliders, such as the BTeV and LHC-B most of rare $B_c$ decays should be accessible.

One of the efficient ways in establishing new physics beyond the SM is the measurement of the lepton polarization [7]–[16]. In this work we present a study of the branching ratio and lepton polarizations in the exclusive $B_c \to D_s^*\ell^+\ell^-$ ($\ell = \mu, \tau$) decay for a general form of the effective Hamiltonian including all possible form of interactions in a model independent way without forcing concrete values for the Wilson coefficients corresponding to any specific model.

It is well known that the theoretical study of the inclusive decays is rather easy but their experimental investigation is difficult. However for the exclusive decays the situation is contrary to the inclusive case, i.e., their experimental detection is very easy but theoretical investigation has its own drawbacks. This is due to the fact that for description of the exclusive decay form factors, i.e., the matrix elements of the effective Hamiltonian between initial and final meson states, are needed. This problem is related to the nonperturbative sector of the QCD and and it can only be solved in framework of the nonperturbative approaches.

These matrix elements have been studied in framework of different approaches, such as light front, constituent quark models [5], and a relativistic quark model proposed in ref. [6]. In this work we will use the weak decay form factors calculated in ref. [6].

The paper is organized as follows. In section 2, we first give the effective Hamiltonian for the quark level process $b \to s\ell^+\ell^-$ and the definitions of the form factors, and then introduce the corresponding matrix element. In section 3, we present the model independent
expressions for the longitudinal, transversal and normal polarizations of leptons. We also give the lepton-antilepton combined asymmetries. Section 4 is devoted to the numerical analysis and discussion of our results.

2 Effective Hamiltonian

In the standard effective Hamiltonian approach, the $B_c \to D_s^* \ell^+\ell^-$ decay is described at the quark level by the $b \to s\ell^+\ell^-$ process, which can be written in terms of twelve model independent four-Fermi interactions as follows [11],

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left\{ C_{SL} s\bar{s}\sigma_{\mu\nu} \frac{q^\nu}{q^2} L b \bar{\ell} \gamma^\mu \ell + C_{BR} s\bar{s}\sigma_{\mu\nu} \frac{q^\nu}{q^2} R b \bar{\ell} \gamma^\mu \ell 
+ C_{LL}^\text{tot} s\bar{L}\gamma_\mu b_L \bar{\ell}_L \gamma_\mu \ell_L + C_{LR}^\text{tot} s\bar{L}\gamma_\mu b_L \bar{\ell}_R \gamma_\mu \ell_R + C_{RL}^\text{tot} s\bar{R}\gamma_\mu b_R \bar{\ell}_L \gamma_\mu \ell_L 
+ C_{RR}^\text{tot} s\bar{R}\gamma_\mu b_R \bar{\ell}_R \gamma_\mu \ell_R + C_{LRLL}^\text{tot} \bar{s}Lb_L \bar{\ell}_L \ell_R + C_{RLRR}^\text{tot} \bar{s}Rb_R \bar{\ell}_L \ell_R 
+ iC_{TE} \epsilon^{\mu\nu\alpha\beta} \bar{s}\sigma_{\mu\nu} b \bar{\ell}\sigma_{\alpha\beta} \ell \right\},$$

where the chiral projection operators $L$ and $R$ in (1) are defined as

$$L = \frac{1 - \gamma_5}{2}, \quad R = \frac{1 + \gamma_5}{2},$$

and $C_X$ are the coefficients of the four-Fermi interactions. The coefficients, $C_{SL}$ and $C_{BR}$, are the nonlocal Fermi interactions which correspond to $-2m_s C_{7}^{\text{eff}}$ and $-2m_b C_{7}^{\text{eff}}$ in the SM, respectively. The following four terms in Eq. (1) are the vector type interactions with coefficients $C_{LL}$, $C_{LR}$, $C_{RL}$ and $C_{RR}$. Two of these vector interactions containing $C_{LL}^\text{tot}$ and $C_{LR}^\text{tot}$ already exist in the SM in combinations of the form $(C_{9}^{\text{eff}} - C_{10})$ and $(C_{9}^{\text{eff}} + C_{10})$. Therefore we write

$$C_{LL}^\text{tot} = C_{9}^{\text{eff}} - C_{10} + C_{LL},$$
$$C_{LR}^\text{tot} = C_{9}^{\text{eff}} + C_{10} + C_{LR},$$

so that $C_{LL}^\text{tot}$ and $C_{LR}^\text{tot}$ describe the sum of the contributions from SM and the new physics. The terms with coefficients $C_{LRLL}$, $C_{RLRR}$, $C_{LRRL}^\text{tot}$ and $C_{RLRL}^\text{tot}$ describe the scalar type interactions. The remaining two terms with the coefficients $C_T$ and $C_{TE}$ describe the tensor type interactions.

After giving the general form of four-Fermi interaction for the $b \to s\ell^+\ell^-$ transition, we now need to estimate the matrix element for the $B_c \to D_s^* \ell^+\ell^-$-decay. These can be expressed in term of invariant form factors as follows:

$$\langle D_s^*(p_{D^*}, \varepsilon) | \bar{s}\gamma_\mu (1 \pm \gamma_5) b | B_c(p_{B_c}) \rangle = \mathcal{A}_+ \left[ \frac{2V(q^2)}{m_{B_c} + m_{D^*} \pm i\frac{\varepsilon^\mu (m_{B_c} - m_{D^*}) A_0(q^2)}{m_{B_c} + m_{D^*}} \mp iq^\mu \varepsilon^\nu A_-(q^2)}{m_{B_c} + m_{D^*}} \right].$$

2
\[
\langle D_s^*(p_{D^*}, \varepsilon)|\bar{s}i\sigma_{\mu\nu}q^\nu(1 \pm \gamma_5)b(B(p_B))\rangle = \\
2\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{\nu\lambda}p_{D^*}^\mu q^\sigma g(q^2) + i \left[ \varepsilon_\nu^*(m_{B_c}^2 - m_{D^*}^2) - (p_B + p_{D^*})_\mu (\varepsilon^* q) \right] a_0(q^2) \\
\pm i(\varepsilon^* q) \left[ q_\mu - (p_{B_c} + p_{D^*})_\mu \frac{q^2}{m_{B_c}^2 - m_{D^*}^2} \right] \frac{(m_{B_c}^2 - m_{D^*}^2)}{q^2} (a_+(q^2) - a_0(q^2)),
\]
\[
\langle D_s^*(p_{D^*}, \varepsilon)|\bar{s}\sigma_{\mu\nu}b(B(p_B))\rangle = \\
i\epsilon_{\mu\nu\lambda\sigma} \left[ -g(q^2)\varepsilon^\lambda(p_{B_c} + p_{D^*})^\sigma + \frac{1}{q^2}(m_{B_c}^2 - m_{D^*}^2)\varepsilon^\lambda q^\sigma(g(q^2) - a_0(q^2)) \right. \\
- \frac{2}{q^2} \left( g(q^2) - a_+(q^2) \right) (\varepsilon^* q)p_{D^*}^\sigma q^\nu 
\]
\[
\langle D_s^*(p_{D^*}, \varepsilon)|\bar{s}(1 \pm \gamma_5)b(B(p_B))\rangle = \\
\frac{1}{m_b} \left[ \mp (\varepsilon^* q)(m_{B_c} - m_{D^*}) \right. \\
+ A_0(q^2) - A_+(q^2) - \frac{q^2}{m_{B_c}^2 - m_{D^*}^2} A_-(q^2) \left] .
\]

where \( q = p_{B_c} - p_{D^*} \) is the momentum transfer and \( \varepsilon \) is the polarization vector of \( D_s^* \) meson. The matrix element \( \langle D_s^*|\bar{s}(1 \pm \gamma_5)b|B \rangle \) is calculated by contracting both sides of Eq. (2) with \( q^\mu \) and using equation of motion.

By using Eqs. (1)–(5), we can now write the matrix element of the \( B_c \to D_s^*\ell^+\ell^- \) decay as

\[
\mathcal{M}(B_c \to D_s^*\ell^+\ell^-) = \frac{G_F}{4\sqrt{2}\pi} V_{tb} V_{ts}^* \\
\times \left\{ \bar{\ell}\gamma^\mu(1 - \gamma_5)\ell \left[ -2A_1\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{\nu\lambda}p_{D^*}^\mu q^\sigma - iB_1\varepsilon^\mu_\mu + iB_2(\varepsilon^* q)(p_{B_c} + p_{D^*})_\mu + iB_3(\varepsilon^* q)q_\mu \right] \right. \\
+ \bar{\ell}\gamma^\mu(1 + \gamma_5)\ell \left[ -2C_1\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{\nu\lambda}p_{D^*}^\mu q^\sigma - iD_1\varepsilon^\mu_\mu + iD_2(\varepsilon^* q)(p_{B_c} + p_{D^*})_\mu + iD_3(\varepsilon^* q)q_\mu \right] \\
+ \bar{\ell}(1 - \gamma_5)\ell \left[iD_4(\varepsilon^* q)\right] + \bar{\ell}(1 + \gamma_5)\ell \left[iD_5(\varepsilon^* q)\right] \\
+ 4\bar{\ell}\sigma_{\mu\nu}\ell \left(iC_T\epsilon_{\mu\nu\lambda\sigma}\right) \left[ -2g\varepsilon^\lambda(p_{B_c} + p_{D^*})_\nu + B_6\varepsilon^* q^\nu - B_7(\varepsilon^* q)p_{D^*}^\lambda q^\nu \right] \\
+ 16C_T\bar{\ell}\sigma_{\mu\nu}\ell \left[ -2g\varepsilon^\mu(p_{B_c} + p_{D^*})^\nu + B_6\varepsilon^* q^\nu - B_7(\varepsilon^* q)p_{D^*}^\lambda q^\nu \right] \right\},
\]

where

\[
A_1 = (C_{LL}^{tot} + C_{RL}) \frac{V}{m_{B_c} + m_{D^*}^2} - (C_{BR} + C_{SL}) \frac{q}{q^2},
\]
\[
B_1 = (C_{LL}^{tot} - C_{RL})(m_{B_c} - m_{D^*^2})A_0 - (C_{BR} - C_{SL})(m_{B_c}^2 - m_{D^*}^2) a_0(q^2),
\]
\[
B_2 = \frac{C_{LL}^{tot} - C_{RL}}{m_{B_c} + m_{D^*}^2} A_+ - (C_{BR} - C_{SL}) a_+ \frac{q}{q^2},
\]
\[
B_3 = (C_{LL}^{tot} - C_{RL}) \frac{A_-}{m_{B_c} + m_{D^*}^2} + 2(C_{BR} - C_{SL})(a_+ - a_0) \frac{m_{B_c}^2 - m_{D^*}^2}{q^4},
\]

\[3\]
\[ B_4 = - (C_{LRL} - C_{RLR}) \frac{m_{B_c} - m_{D_s^*}}{m_b} (A_0 - A_\perp - \frac{q^2}{m_{B_c}^2 - m_{D_s^*}^2}) , \]

\[ B_5 = - (C_{LRL} - C_{RLR}) \frac{m_{B_c} - m_{D_s^*}}{m_b} (A_0 - A_\perp - \frac{q^2}{m_{B_c}^2 - m_{D_s^*}^2}) , \]

\[ B_6 = \left( m_{B_c}^2 - m_{D_s^*}^2 \right) \frac{g - a_0}{q^2} , \]

\[ B_7 = \frac{2}{q^2} (g - a_0 - a_+) , \]

\[ C_1 = A_1 (C_{LL}^{\text{tot}} \to C_{LR}^{\text{tot}} ; C_{RL} \to C_{RR}) , \]

\[ D_1 = B_1 (C_{LL}^{\text{tot}} \to C_{LR}^{\text{tot}} ; C_{RL} \to C_{RR}) , \]

\[ D_2 = B_2 (C_{LL}^{\text{tot}} \to C_{LR}^{\text{tot}} ; C_{RL} \to C_{RR}) , \]

\[ D_3 = B_3 (C_{LL}^{\text{tot}} \to C_{LR}^{\text{tot}} ; C_{RL} \to C_{RR}) . \] 

(7)

3 Lepton polarizations

We now like to calculate the final lepton polarizations for the \( B_c \to D_s^* \ell^+ \ell^- \) decay. For this we will use the convention followed by the earlier works, such as [10],[11] and define the following orthogonal unit vectors, \( S_i^{-\mu} \) in the rest frame of \( \ell^- \) and \( S_i^{+\mu} \) in the rest frame of \( \ell^+ \), for the polarization of the leptons along the longitudinal \( (i = L) \), transverse \( (i = T) \) and normal \( (i = N) \) directions:

\[ S_L^{-\mu} \equiv (0, \vec{e}_L^-) = \left( 0, \frac{\vec{p}_- \times \vec{p}_-}{|\vec{p}_-|^2} \right) , \]

\[ S_N^{-\mu} \equiv (0, \vec{e}_N^-) = \left( 0, \frac{\vec{p}_- \times \vec{p}_-}{|\vec{p}_-|^2} \right) , \]

\[ S_T^{-\mu} \equiv (0, \vec{e}_T^-) = \left( 0, \vec{e}_N \times \vec{e}_L^- \right) , \]

\[ S_L^{+\mu} \equiv (0, \vec{e}_L^+) = \left( 0, \frac{\vec{p}_+ \times \vec{p}_+}{|\vec{p}_+|^2} \right) , \]

\[ S_N^{+\mu} \equiv (0, \vec{e}_N^+) = \left( 0, \frac{\vec{p}_+ \times \vec{p}_+}{|\vec{p}_+|^2} \right) , \]

\[ S_T^{+\mu} \equiv (0, \vec{e}_T^+) = \left( 0, \vec{e}_N \times \vec{e}_L^+ \right) , \]

where \( \vec{p}_\pm \) and \( \vec{p} \) are the three momenta of \( \ell^\pm \) and \( D_s^* \) meson in the center of mass (CM) frame of the \( \ell^+ \ell^- \) system, respectively. The longitudinal unit vectors \( S_L^- \) and \( S_L^+ \) are boosted to CM frame of \( \ell^+ \ell^- \) by Lorentz transformation,

\[ S_{L,CM}^- = \left( \frac{|\vec{p}_-|}{m_\ell} \frac{E_\ell \vec{p}_-}{m_\ell |\vec{p}_-|} \right) , \]

\[ S_{L,CM}^+ = \left( \frac{|\vec{p}_-|}{m_\ell} \frac{E_\ell \vec{p}_-}{m_\ell |\vec{p}_-|} \right) , \]

while vectors of perpendicular directions are not changed by boost.
The differential decay rate of the $B_c \rightarrow D_s^* \ell^+ \ell^-$ decay for any spin direction $\vec{n}^{\pm}$ of the $\ell^\pm$ can be written in the following form

$$
\frac{d\Gamma(\vec{n}^{\pm})}{ds} = \frac{1}{2} \left( \frac{d\Gamma}{ds} \right) \left[ 1 + \left( P_L^L \overline{\ell}_L^+ \epsilon_L^+ + P_\perp^N \overline{\epsilon}_N^+ + P_T^T \overline{\epsilon}_T^+ \right) \cdot \vec{n}^{\pm} \right],
$$

(10)

where $\vec{n}^{\pm}$ is the unit vector in the $\ell^\pm$ rest frame and $s = q^2/m_{B_c}^2$. Here, the superscripts $+$ and $-$ correspond to $\ell^+$ and $\ell^-$ cases, the subscript 0 corresponds to the unpolarized decay rate, whose explicit form is given by

$$
\left( \frac{d\Gamma}{ds} \right)_0 = \frac{G^2 \alpha^2 m_{B_c}}{2^{14/3} \pi^5} |V_{tb}V_{ts}^*|^2 \sqrt{\lambda v} \times
\left\{ 32m_{B_c}^4 \lambda \left[ (m_{B_c}^2 s - m_{\ell}^2) \left( |A_1|^2 + |C_1|^2 \right) + 6m_{\ell}^2 \text{Re}(A_1C_1^*) \right] + 96m_{\ell}^2 \text{Re}(B_1D_1^*) - \frac{4}{r} m_{B_c}^2 m_{\ell} \lambda \text{Re}[B_1(-B_3^* + D_2^* + D_3^*)] + \text{Re}[D_1(B_2^* + B_3^* - D_3^*)] - \text{Re}(B_4B_5^*) \right\}
+ \left\{ \frac{8}{r} m_{B_c}^2 m_{\ell}^2 \lambda \left[ \text{Re}[B_1(-B_3^* + D_2^* + D_3^*)] + \text{Re}[D_1(B_2^* + B_3^* - D_3^*)] - \text{Re}(B_4B_5^*) \right] \right\} + \frac{8}{r} m_{B_c}^2 m_{\ell} \lambda (2 + 2r - s) \text{Re}(B_3D_2^* + \frac{2}{r} m_{B_c}^2 m_{\ell} \lambda (|B_4|^2 + |B_5|^2)
+ \frac{8}{r} m_{B_c}^2 m_{\ell} \lambda (2 + 2r - s) \text{Re}(B_3D_2^* + \frac{2}{r} m_{B_c}^2 m_{\ell} \lambda (|B_4|^2 + |B_5|^2)
+ \frac{8}{r} m_{B_c}^2 m_{\ell} \lambda (2 + 2r - s) \text{Re}(B_3D_2^* + \frac{2}{r} m_{B_c}^2 m_{\ell} \lambda (|B_4|^2 + |B_5|^2)
+ \frac{8}{r} m_{B_c}^2 m_{\ell} \lambda (2 + 2r - s) \text{Re}(B_3D_2^* + \frac{2}{r} m_{B_c}^2 m_{\ell} \lambda (|B_4|^2 + |B_5|^2)
+ \frac{256}{3r \lambda} \left[ |g|^2 |C_T|^2 m_{B_c}^2 \left( 4m_{\ell}^2 |\lambda(8r - s) - 12rs(2 + 2r - s) | + m_{B_c}^2 s \left[ \lambda(16r + s) + 12rs(2 + 2r - s) \right] \right) + \frac{1024}{3r \lambda} \left[ |g|^2 |C_T|^2 m_{B_c}^2 \left( 8m_{\ell}^2 |\lambda(4r + s) + 12rs(2 + 2r - s) | + m_{B_c}^2 s \left[ \lambda(16r + s) + 12rs(2 + 2r - s) \right] \right) \right] + \left[ 128 \left| m_{B_c}^2 m_{\ell} \lambda \left[ \lambda + 12r(1-r) \right] \text{Re}[B_1 + D_1](gC_T)^* \right| + \frac{128}{r} m_{B_c}^4 m_{\ell} \lambda (1 + 3r - s) \text{Re}[B_2 + D_2](gC_T)^* \right] + 512m_{B_c}^4 m_{\ell} \lambda \text{Re}[A_1 + C_1](gC_T)^* \right\}
$$

(11)
\[ + \frac{16}{3r} m_{B_e}^2 \left( 4(m_{B_e}^2 s + 8m_{B_e}^2) |C_{TE}|^2 + m_{B_e}^2 s v^2 |C_T|^2 \right) \times \left( 4(\lambda + 12rs) |B_6|^2 \right) \\
+ m_{B_e}^2 \lambda^2 |B_T|^2 - 4m_{B_e}^2 (1 - r - s) \lambda \text{Re}(B_6 B_7^* - 16 [\lambda + 12r(1 - r)] \text{Re}(gB_6^*) \\
+ 8m_{B_e}^2 (1 + 3r - s) \lambda \text{Re}(gB_7^*) \right) \right) 
\]

where \( \lambda = 1 + r^2 + s^2 - 2r - 2s - 2rs \), \( r = m_{D_3}^2 / m_{B_e}^2 \) and \( v = \sqrt{1 - 4m_{B_e}^2 / sm_{B_e}^2} \) is the lepton velocity.

The polarizations \( P_L^\pm \), \( P_T^\pm \) and \( P_N^\pm \) in Eq. (10) are defined by the equation

\[ P_i^\pm (q^2) = \frac{d}{dq^2} (\bar{\eta}^\pm = \bar{\epsilon}_{i}^\pm) - \frac{d}{dq^2} (\bar{\eta}^\pm = -\bar{\epsilon}_{i}^\pm) \]

for \( i = L, N, T \), i.e., \( P_L^\pm \) and \( P_T^\pm \) represents the charged lepton \( \ell^\pm \) longitudinal and transversal asymmetries in the decay plane, respectively, and \( P_N^\pm \) is the normal component to both of them. After some lengthy algebra, we get for the longitudinal polarization of the \( \ell^\pm \)

\[ P_L^\pm = \frac{4}{\Delta m_{B_e}} m_{B_e}^2 \left\{ \frac{1}{3r} \lambda^2 m_{B_e}^4 \left[ |B_2|^2 - |D_2|^2 \right] + \frac{1}{r} \lambda m_{\ell} \text{Re}[(B_1 - D_1)(B_4^* + B_5^*)] \right. \\
- \frac{1}{r} \lambda m_{B_e}^2 m_{\ell} (1 - r) \text{Re}[(B_2 - D_2)(B_4^* + B_5^*)] \mp \frac{8}{3} \lambda m_{B_e}^4 s \left[ |A_1|^2 - |C_1|^2 \right] \\
- \frac{2}{3r} \lambda m_{B_e}^2 (1 - r - s) \left[ \text{Re}(B_1 B_2^*) - \text{Re}(D_1 D_2^*) \right] \mp \frac{256}{3} \lambda m_{B_e}^4 m_{\ell} \left( \text{Re}[A_1^* (C_T \mp C_{TE})g] - \text{Re}[C_1^* (C_T \pm C_{TE})g] \right) \\
+ \frac{4}{3r} \lambda^2 m_{B_e}^4 m_{\ell} \left( \text{Re}[B_2^* (C_T \mp 4C_{TE})B_7] + \text{Re}[D_2^* (C_T \pm 4C_{TE})B_7] \right) \\
- \frac{8}{3r} \lambda m_{B_e}^2 m_{\ell} (1 - r - s) \left( \text{Re}[B_2^* (C_T \mp 4C_{TE})B_6] + \text{Re}[D_2^* (C_T \mp 4C_{TE})B_6] \right) \\
- \frac{4}{3r} \lambda m_{B_e}^2 m_{\ell} (1 - r - s) \left( \text{Re}[B_1^* (C_T \mp 4C_{TE})B_7] + \text{Re}[D_1^* (C_T \pm 4C_{TE})B_7] \right) \\
+ \frac{8}{3r} (\lambda + 12rs) m_{\ell} \left( \text{Re}[B_1^* (C_T \mp 4C_{TE})B_6] + \text{Re}[D_1^* (C_T \pm 4C_{TE})B_6] \right) \\
- \frac{16}{3r} m_{\ell} [\lambda + 12r(1 - r)] \left( \text{Re}[B_1^* (C_T \mp 4C_{TE})g] + \text{Re}[D_1^* (C_T \pm 4C_{TE})g] \right) \\
+ \frac{16}{3r} \lambda m_{B_e}^2 m_{\ell} (1 + 3r - s) \left( \text{Re}[B_2^* (C_T \mp 4C_{TE})g] + \text{Re}[D_2^* (C_T \pm 4C_{TE})g] \right) \\
+ \frac{16}{3r} \lambda^2 m_{B_e}^6 s |B_T|^2 \text{Re}(C_T C_{TE}^*) \\
+ \frac{64}{3r} (\lambda + 12rs) m_{B_e}^2 s |B_6|^2 \text{Re}(C_T C_{TE}^*) \right\} 
\]

(12)
where $\Delta$ is the term inside curly brackets of Eq. (11).

Similarly, we find for the transverse polarization $P_T^-$

$$P_T^- = \frac{\pi}{\Delta} m_{B_c} \sqrt{s} \lambda \left\{ -8 m_{B_c}^2 m_\ell \text{Re}[(A_1 + C_1)(B_1^* + D_1^*)] 
+ \frac{1}{r} m_{B_c}^2 m_\ell (1 + 3r - s) \left[ \text{Re}(B_1 D_2^*) - \text{Re}(B_2 D_1^*) \right] 
+ \frac{1}{r s} m_\ell (1 - r - s) \left[ |B_1|^2 - |D_1|^2 \right] 
+ \frac{2}{r s} m_{B_c}^2 (1 - r - s) \left[ \text{Re}(B_1 B_2^*) - \text{Re}(D_1 B_4^*) \right] 
- \frac{1}{r} m_{B_c}^2 m_\ell (1 - r - s) \text{Re}[(B_1 + D_1)(B_3^* - D_3^*)] 
- \frac{2}{r s} m_{B_c}^2 m_\ell^2 \lambda \left[ \text{Re}(B_2 B_5^*) - \text{Re}(D_2 B_4^*) \right] 
+ \frac{1}{r s} m_{B_c}^4 m_\ell (1 - r) \lambda \left[ |B_2|^2 - |D_2|^2 \right] + \frac{1}{r} m_{B_c}^4 m_\ell \lambda \text{Re}[(B_2 + D_2)(B_3^* - D_3^*)] 
- \frac{1}{r s} m_{B_c}^2 m_\ell [\lambda + (1 - r - s)(1 - r)] \left[ \text{Re}(B_1 B_2^*) - \text{Re}(D_1 D_2^*) \right] 
+ \frac{1}{r s} (1 - r - s)(2m_\ell^2 - m_{B_c}^2 s) \left[ \text{Re}(B_1 B_4^*) - \text{Re}(D_1 B_5^*) \right] 
+ \frac{1}{r s} m_{B_c}^2 \lambda (2m_\ell^2 - m_{B_c}^2 s) \left[ \text{Re}(D_2 B_5^*) - \text{Re}(B_2 B_4^*) \right] 
- \frac{16}{r s} \lambda m_{B_c}^2 m_\ell \text{Re}[(B_1 - D_1)(B_7 C_{TE})^*] 
+ \frac{16}{r s} \lambda m_{B_c}^4 m_\ell^2 (1 - r) \text{Re}[(B_2 - D_2)(B_7 C_{TE})^*] 
+ \frac{8}{r} \lambda m_{B_c}^4 m_\ell \text{Re}[(B_4 - B_5)(B_7 C_{TE})^*] 
+ \frac{16}{r} \lambda m_{B_c}^4 m_\ell^2 \text{Re}[(B_3 - D_3)(B_7 C_{TE})^*] 
+ \frac{32}{r s} m_\ell^2 (1 - r - s) \text{Re}[(B_1 - D_1)(B_6 C_{TE})^*] 
- \frac{32}{r s} m_{B_c}^2 m_\ell^2 (1 - r)(1 - r - s) \text{Re}[(B_2 - D_2)(B_6 C_{TE})^*] 
- \frac{16}{r} m_{B_c}^2 m_\ell (1 - r - s) \text{Re}[(B_4 - B_5)(B_6 C_{TE})^*] 
- \frac{32}{r} m_{B_c}^2 m_\ell^2 (1 - r - s) \text{Re}[(B_3 - D_3)(B_6 C_{TE})^*] \right\} ,$$

for $s > 0$. For $s < 0$, $P_T^+$ can be obtained by replacing $s$ with $-s$. The expressions involve terms with $m_{B_c}$, $m_\ell$, $\lambda$, and $\Delta$, corresponding to the terms inside the curly brackets.
\[
\begin{align*}
-16m_{Bc}^2 & \left(4m_T^2 \text{Re}(A_1^* (C_T + 2C_{TE}) B_6) - m_{Bc}^2 s \text{Re}(A_1^* (C_T - 2C_{TE}) B_6)\right) \\
+ 16m_{Bc}^2 & \left(4m_T^2 \text{Re}(C_1^* (C_T - 2C_{TE}) B_6) - m_{Bc}^2 s \text{Re}(C_1^* (C_T + 2C_{TE}) B_6)\right) \\
+ \frac{32}{s} & m_{Bc}^2 (1 - r) \left(4m_T^2 \text{Re}(A_1^* (C_T + 2C_{TE}) g) - m_{Bc}^2 s \text{Re}(A_1^* (C_T - 2C_{TE}) g)\right) \\
- \frac{32}{s} & m_{Bc}^2 (1 - r) \left(4m_T^2 \text{Re}(C_1^* (C_T - 2C_{TE}) g) - m_{Bc}^2 s \text{Re}(C_1^* (C_T + 2C_{TE}) g)\right) \\
+ \frac{64}{rs} & m_{Bc}^2 m_T^2 (1 - r) (1 + 3r - s) \text{Re}((B_2 - D_2) (g C_{TE})^*) \\
+ \frac{64}{rs} & m_{Bc}^2 m_T^2 (1 + 3r - s) \text{Re}((B_3 - D_3) (g C_{TE})^*) \\
+ \frac{32}{r} & m_{Bc}^2 m_T (1 + 3r - s) \text{Re}((B_4 - B_5) (g C_{TE})^*) \\
+ \frac{64}{rs} & m_{Bc}^2 (rs - m_T^2 (1 + 7r - s)) \text{Re}((B_1 - D_1) (g C_{TE})^*) \\
- \frac{32}{s} & (4m_T^2 + m_{Bc}^2 s) \text{Re}((B_1 + D_1) (g C_T)^*) \\
- 2048m_{Bc}^2 m_T \text{Re}((C_{T g}) (B_6 C_{TE})^*) \\
+ \frac{4096}{s} & m_{Bc}^2 m_T (1 - r) |g|^2 \text{Re}(C_T C_{T E}^*) \right) ,
\end{align*}
\]

and

\[
P_T^+ = \frac{\pi}{\Delta} m_{Bc} \sqrt{s} \lambda \left\{ -8m_{Bc}^2 m_T \text{Re}((A_1 + C_1)(B_1^* + D_1^*)) \\
- \frac{1}{r} m_{Bc}^2 m_T (1 + 3r - s) \left[ \text{Re}(B_1 D_2^*) - \text{Re}(B_2 D_1^*)\right] \\
- \frac{1}{rs} m_T (1 - r - s) \left[ |B_1|^2 - |D_1|^2\right] \\
+ \frac{1}{rs} (2m_T^2 - m_{Bc}^2 s) (1 - r - s) \left[ \text{Re}(B_1 B_5^*) - \text{Re}(D_1 B_4^*)\right] \\
+ \frac{1}{r} m_{Bc}^2 m_T (1 - r - s) \text{Re}((B_1 + D_1)(B_3^* - D_3^*)) \\
- \frac{1}{rs} m_{Bc}^2 \lambda (2m_T^2 - m_{Bc}^2 s) \left[ \text{Re}(B_2 B_5^*) - \text{Re}(D_2 B_4^*)\right] \\
- \frac{1}{rs} m_{Bc}^4 m_T (1 - r - s) \lambda \left[ |B_2|^2 - |D_2|^2\right] - \frac{1}{r} m_{Bc}^4 m_T \lambda \text{Re}((B_2 + D_2)(B_3^* - D_3^*)) \\
+ \frac{1}{rs} m_{Bc}^4 m_T (1 - r - s) \lambda (1 - r - s) \text{Re}((B_1 B_2^*) - \text{Re}(D_1 D_2)) \\
+ \frac{2}{rs} m_T^2 (1 - r - s) \text{Re}(B_1 B_3^*) - \text{Re}(D_1 B_3^*) \\
+ \frac{2}{rs} m_T^2 m_T \lambda \text{Re}(D_2 B_5^*) - \text{Re}(B_2 B_4^*) \\
+ \frac{16}{rs} m_{Bc}^2 m_T^2 \text{Re}((B_1 - D_1)(B_7 C_{TE})^*) \\
- \frac{16}{rs} m_{Bc}^4 m_T^2 (1 - r) \text{Re}((B_2 - D_2)(B_7 C_{TE})^*) \right\} ,
\]
Finally for normal asymmetries we get

\[
\begin{align*}
&- \frac{8}{r} \lambda m_{36}^4 m_{\ell} \text{Re}[(B_1 - B_5)(B_7 C_{TE})^*] \\
&- \frac{16}{r} \lambda m_{36}^4 m_{\ell}^2 \text{Re}[(B_3 - D_3)(B_7 C_{TE})^*] \\
&- \frac{32}{r s} m_{\ell}^2 (1 - r - s) \text{Re}[(B_1 - D_1)(B_6 C_{TE})^*] \\
&+ \frac{32}{r s} m_{36}^2 m_{\ell}^2 (1 - r)(1 - r - s) \text{Re}[(B_2 - D_2)(B_6 C_{TE})^*] \\
&+ \frac{16}{r} m_{36}^2 m_{\ell} (1 - r - s) \text{Re}[(B_1 - B_5)(B_6 C_{TE})^*] \\
&+ \frac{32}{r s} m_{36}^2 m_{\ell}^2 (1 - r - s) \text{Re}[(B_3 - D_3)(B_6 C_{TE})^*] \\
&+ 16 m_{36}^2 \left( 4 m_{\ell}^2 \text{Re}[A_1^*(C_T - 2 C_{TE}) B_6] - m_{36}^2 s \text{Re}[A_1^*(C_T + 2 C_{TE}) B_6] \right) \\
&- 16 m_{36}^2 \left( 4 m_{\ell}^2 \text{Re}[C_1^*(C_T + 2 C_{TE}) B_6] - m_{36}^2 s \text{Re}[C_1^*(C_T - 2 C_{TE}) B_6] \right) \\
&- \frac{32}{s} m_{36}^2 (1 - r) \left( 4 m_{\ell}^2 \text{Re}[A_1^*(C_T - 2 C_{TE}) g] - m_{36}^2 s \text{Re}[A_1^*(C_T + 2 C_{TE}) g] \right) \\
&+ \frac{32}{s} m_{36}^2 (1 - r) \left( 4 m_{\ell}^2 \text{Re}[C_1^*(C_T + 2 C_{TE}) g] - m_{36}^2 s \text{Re}[C_1^*(C_T - 2 C_{TE}) g] \right) \\
&- \frac{64}{r s} m_{36}^2 m_{\ell}^2 (1 - r)(1 + 3 r - s) \text{Re}[(B_2 - D_2)(g C_{TE})^*] \\
&- \frac{64}{r} m_{36}^2 m_{\ell}^2 (1 + 3 r - s) \text{Re}[(B_3 - D_3)(g C_{TE})^*] \\
&- \frac{32}{r} m_{36}^2 m_{\ell} (1 + 3 r - s) \text{Re}[(B_4 - B_5)(g C_{TE})^*] \\
&- \frac{64}{r s} [m_{36}^2 r s - m_{\ell}^2 (1 + 7 r - s)] \text{Re}[(B_1 - D_1)(g C_{TE})^*] \\
&- \frac{32}{s} (4 m_{\ell}^2 + m_{36}^2 s) \text{Re}[(B_1 + D_1)(g C_T)^*] \\
&- 2048 m_{36}^2 m_{\ell} \text{Re}[(C T g)(B_6 C_{TE})^*] \\
&+ \frac{4096}{s} m_{36}^2 m_{\ell} (1 - r) |g|^2 \text{Re}(C_T C_{TE}^*) \right) .
\end{align*}
\]

Finally for normal asymmetries we get

\[
P_N^- = \frac{1}{\Delta} \pi v m_{36}^3 \sqrt{s} \lambda \left\{ 8 m_{\ell} \text{Im}[(B_1^* C_1) + (A_1^* D_1)] \\
- \frac{1}{r} m_{36}^2 \lambda \text{Im}[(B_1^* B_4) + (D_2^* B_5)] \\
+ \frac{1}{r} m_{36}^2 m_{\ell} \lambda \text{Im}[(B_2 - D_2)(B_3^* - D_3^*)] \\
- \frac{1}{r} m_{\ell} (1 + 3 r - s) \text{Im}[(B_1 - D_1)(B_2^* - D_2^*)] \\
+ \frac{1}{r} (1 - r - s) \text{Im}[(B_1^* B_4) + (D_1^* B_5)] \right\} .
\]
\[
- \frac{1}{r} m_\ell (1 - r - s) \text{Im}[(B_1 - D_1)(B_3^* - D_3^*)] \\
- \frac{8}{r} m_{B_{ec}}^2 m_\ell \lambda \text{Im}[(B_4 + B_5)(B_7 C_{TE})^*] \\
+ \frac{16}{r} m_\ell (1 - r - s) \text{Im}[(B_4 + B_5)(B_6 C_{TE})^*] \\
- \frac{32}{r} m_\ell (1 + 3r - s) \text{Im}[(B_4 + B_5)(g C_{TE})^*] \\
- 16m_{B_{ec}}^2 s \left( \text{Im}[A_1^*(C_T - 2 C_{TE})B_6] + \text{Im}[C_1^*(C_T + 2 C_{TE})B_6] \right) \\
+ 32m_{B_{ec}}^2 (1 - r) \left( \text{Im}[A_1^*(C_T - 2 C_{TE})g] + \text{Im}[C_1^*(C_T + 2 C_{TE})g] \right) \\
+ 32 \left( \text{Im}[B_1^*(C_T - 2 C_{TE})g] - \text{Im}[D_1^*(C_T + 2 C_{TE})g] \right) \\
+ 512 m_\ell \left( |C_T|^2 - 4 |C_{TE}|^2 \right) \text{Im}(B_6^* g) \right\} ,
\]

and

\[
P_N^+ = \frac{1}{\Delta} \pi v m_{B_{ec}}^3 \sqrt{s} \lambda \left\{ - 8 m_\ell \text{Im}[(B_1^* C_1) + (A_1^* D_1)] \\
+ \frac{1}{r} m_{B_{ec}}^2 \lambda \text{Im}[(B_2^* B_5) + (D_2^* B_4)] \\
+ \frac{1}{r} m_{B_{ec}}^2 m_\ell \lambda \text{Im}[(B_2 - D_2)(B_3^* - D_3^*)] \\
- \frac{1}{r} m_\ell (1 + 3r - s) \text{Im}[(B_1 - D_1)(B_2^* - D_2^*)] \\
- \frac{1}{r} (1 - r - s) \text{Im}[(B_1^* B_5) + (D_1^* B_4)] \\
- \frac{1}{r} m_\ell (1 - r - s) \text{Im}[(B_1 - D_1)(B_3^* - D_3^*)] \\
+ \frac{8}{r} m_{B_{ec}}^2 m_\ell \lambda \text{Im}[(B_4 + B_5)(B_7 C_{TE})^*] \\
- \frac{16}{r} m_\ell (1 - r - s) \text{Im}[(B_4 + B_5)(B_6 C_{TE})^*] \\
+ \frac{32}{r} m_\ell (1 + 3r - s) \text{Im}[(B_4 + B_5)(g C_{TE})^*] \\
- 16m_{B_{ec}}^2 s \left( \text{Im}[A_1^*(C_T + 2 C_{TE})B_6] + \text{Im}[C_1^*(C_T - 2 C_{TE})B_6] \right) \\
+ 32m_{B_{ec}}^2 (1 - r) \left( \text{Im}[A_1^*(C_T + 2 C_{TE})g] + \text{Im}[C_1^*(C_T - 2 C_{TE})g] \right) \\
- 32 \left( \text{Im}[B_1^*(C_T + 2 C_{TE})g] - \text{Im}[D_1^*(C_T - 2 C_{TE})g] \right) \\
+ 512 m_\ell \left( |C_T|^2 - 4 |C_{TE}|^2 \right) \text{Im}(B_6^* g) \right\} .
\]

From Eqs. (12)-(16), we observe that for longitudinal and normal polarizations, the difference between \( \ell^+ \) and \( \ell^- \) lepton asymmetries results from the scalar and tensor type interactions. Similar situation takes place for transverse polarization asymmetries in the
where $m_\ell \to 0$ limit. From this, we can conclude that their experimental study may provide essential information about new physics.

Another source of useful information about new physics can be a combined analysis of the lepton and antilepton polarizations, since in the SM $P_L^- + P_L^+ = 0$, $P_N^- + P_N^+ = 0$ and $P_T^- - P_T^+ \approx 0$ \[1\]. Using Eqs. \((12)-(16)\) we get

\[
P_L^- + P_L^+ = \frac{4}{\Delta} m_{B_{c}} \sqrt{v} \left\{ \frac{2}{r} m_\ell \lambda \text{Re}[(B_1 - D_1)(B_4^* + B_5^*)] \right. \\
- \frac{2}{r} m_{B_{c}} m_\ell \lambda (1 - r) \text{Re}[(B_2 - D_2)(B_4^* + B_5^*)] \\
- \frac{1}{r} m_{B_{c}} s_\lambda \left( |B_4|^2 - |B_5|^2 \right) - \frac{2}{r} m_{B_{c}} m_\ell \lambda s_\lambda \text{Re}[(B_3 - D_3)(B_4^* + B_5^*)] \\
+ \frac{8}{3r} m_{B_{c}}^4 m_\ell \lambda^2 \text{Re}[(B_2 + D_2)(B_T C_T)^*] \\
+ \frac{32}{3r} m_{B_{c}}^6 s_\lambda^2 |B_T|^2 \text{Re}(C_T C_{TE}^*) \\
- \frac{8}{3r} m_{B_{c}}^2 m_\ell \lambda (1 - r - s) \text{Re}[(B_1 + D_1)(B_T C_T)^*] \\
- \frac{16}{3r} m_{B_{c}}^2 m_\ell \lambda (1 - r - s) \text{Re}[(B_2 + D_2)(B_6 C_T)^*] \\
- \frac{128}{3r} m_{B_{c}}^4 s_\lambda (1 - r - s) \text{Re}(B_6 B_T^*) \text{Re}(C_T C_{TE}^*) \\
+ \frac{16}{3r} m_\ell (\lambda + 12rs) \text{Re}[(B_1 + D_1)(B_6 C_T)^*] \\
+ \frac{128}{3r} m_{B_{c}}^2 s_\lambda (\lambda + 12rs) |B_6|^2 \text{Re}(C_T C_{TE}^*) \\
+ \frac{512}{3r} m_{B_{c}}^2 [\lambda (4r + s) + 12r (1 - r)^2] |g|^2 \text{Re}(C_T C_{TE}^*) \\
- \frac{512}{3r} m_{B_{c}}^2 s [\lambda + 12r (1 - r)] \text{Re}(g B_6^*) \text{Re}(C_T C_{TE}^*) \\
+ \frac{256}{3r} m_{B_{c}}^4 s_\lambda (1 + 3r - s) \text{Re}(g B_7^*) \text{Re}(C_T C_{TE}^*) \\
+ \frac{512}{3} m_{B_{c}}^2 m_\ell \lambda \text{Re}[(A_1 + C_1)(g C_{TE})^*] \\
- \frac{32}{3r} m_\ell [\lambda + 12r (1 - r)] \text{Re}[(B_1 + D_1)(g C_T)^*] \\
+ \frac{32}{3r} m_{B_{c}}^2 m_\ell \lambda (1 + 3r - s) \text{Re}[(B_2 + D_2)(g C_T)^*] \right\}.
\]

For the case of transverse polarization, it is the difference of the lepton and antilepton polarizations that is relevant and it can be calculated from Eqs. \((13)\) and \((14)\)

\[
P_T^- - P_T^+ = \frac{\pi}{\Delta} m_{B_{c}} \sqrt{s_\lambda \lambda} \left\{ \frac{2}{r s} m_{B_{c}}^4 m_\ell (1 - r) \lambda \left[ |B_2|^2 - |D_2|^2 \right] \\
+ \frac{1}{s} m_{B_{c}}^4 \lambda \text{Re}[(B_2 + D_2)(B_4^* - B_5^*)] \right\}
\]
+ \frac{2}{r} m_{B_c}^4 m_t \lambda \text{Re}[(B_2 + D_2)(B_3^* - D_3^*)] \\
+ \frac{2}{r} m_{B_c}^2 m_t (1 + 3r - s) \left[ \text{Re}(B_1 D_2^*) - \text{Re}(B_2 D_1^*) \right] \\
+ \frac{2}{r s} m_t (1 - r - s) \left[ |B_1|^2 - |D_1|^2 \right] \\
- \frac{1}{r} m_{B_c}^2 (1 - r - s) \text{Re}[(B_1 + D_1)(B_4^* - B_5^*)] \\
- \frac{2}{r} m_{B_c}^2 m_t (1 - r - s) \text{Re}[(B_1 + D_1)(B_3^* - D_3^*)] \\
- \frac{2}{r s} m_{B_c}^2 m_t \lambda (1 - r) (1 - r - s) \left[ \text{Re}(B_1 B_2^*) - \text{Re}(D_1 D_2^*) \right] \\
- \frac{32}{r s} m_{B_c}^2 m_t^2 \lambda \text{Re}[(B_1 - D_1)(B_7 C_{TE})^*] \\
+ \frac{32}{r s} m_{B_c}^4 m_t^2 \lambda (1 - r) \text{Re}[(B_2 - D_2)(B_7 C_{TE})^*] \\
+ \frac{16}{r} m_{B_c}^4 m_t \lambda \text{Re}[(B_4 - B_5)(B_7 C_{TE})^*] \\
+ \frac{32}{r} m_{B_c}^4 m_t^2 \lambda \text{Re}[(B_3 - D_3)(B_7 C_{TE})^*] \\
+ \frac{64}{r s} m_t^2 (1 - r - s) \text{Re}[(B_1 - D_1)(B_6 C_{TE})^*] \\
- \frac{64}{r s} m_{B_c}^2 m_t^2 (1 - r) (1 - r - s) \text{Re}[(B_2 - D_2)(B_6 C_{TE})^*] \\
- \frac{32}{r} m_{B_c}^2 m_t (1 - r - s) \text{Re}[(B_4 - B_5)(B_6 C_{TE})^*] \\
- \frac{64}{r} m_{B_c}^2 m_t^2 (1 - r - s) \text{Re}[(B_3 - D_3)(B_6 C_{TE})^*] \\
+ \frac{64}{r} m_{B_c}^4 m_t \lambda (1 + 3r - s) \text{Re}[(B_4 - B_5)(g C_{TE})^*] \\
- \frac{64 m_{B_c}^4 (1 - r) v^2 \text{Re}[(A_1 - C_1)(g C_T)^*] \\
+ \frac{128}{r s} [m_{B_c}^2 r s - m_t^2 (1 + 7r - s)] \text{Re}[(B_1 - D_1)(g C_{TE})^*] \\
+ \frac{128}{r s} m_{B_c}^2 m_t^2 (1 - r) (1 + 3r - s) \text{Re}[(B_2 - D_2)(g C_{TE})^*] \\
+ \frac{128}{r s} m_{B_c}^2 m_t^2 (1 + 3r - s) \text{Re}[(B_3 - D_3)(g C_{TE})^*] \right) .

In the same manner it follows from Eqs. (15) and (16)

\[ P_N^- + P_N^+ = \frac{1}{\Delta} \pi v m_{B_c}^3 \sqrt{s \lambda} \left\{ - \frac{2}{r} m_t (1 + 3r - s) \text{Im}[(B_1 - D_1)(B_2^* - D_2^*)] \\
- \frac{2}{r} m_t (1 - r - s) \text{Im}[(B_1 - D_1)(B_3^* - D_3^*)] \\
- \frac{1}{r} (1 - r - s) \text{Im}[(B_1 - D_1)(B_4^* - B_5^*)] \right\} .

12
were reported by CDF Collaboration [18]. Recently, CDF quoted a new value as well as the lepton-antilepton combined asymmetries $P_\ell$. The values of the individual Wilson coefficients that appear in the SM are listed in Table 4. We here present our numerical analysis about the branching ratios and averaged polarization asymmetries. As seen from the Eqs. (12)-(19), all the expressions of the lepton polarizations depend on both $s = q^2/m_B^2$ and the new Wilson coefficients. However, it may be experimentally easier to study the dependence of the polarizations of each lepton on the new Wilson coefficients only. For this reason we eliminate $s$ dependence by considering their averaged forms over the allowed kinematical region. The averaged lepton polarizations are defined as

$$\langle P_i \rangle = \frac{\int (1-m_D^*/m_B) dP_i dB ds}{\int (1-m_D^*/m_B)^2 dB ds}. \quad (20)$$

4 Numerical analysis and discussion

We here present our numerical analysis about the branching ratios and averaged polarization asymmetries $< P_L^- >$, $< P_T^- >$ and $< P_N^- >$ of $\ell^-$ for the $B_c \rightarrow D^{*+}\ell^-\ell^-$ decays with $\ell = \mu, \tau$, as well as the lepton-antilepton combined asymmetries $< P_L^- + P_L^+ >$, $< P_T^- - P_T^+ >$ and $< P_N^- + P_N^+ >$. We first give the input parameters used in our numerical analysis:

$$m_{B_c} = 6.50 GeV, \ m_{D^*} = 2.112 GeV, \ m_b = 4.8 GeV, \ m_\mu = 0.105 GeV, \ m_\tau = 1.77 GeV,$$

$$|V_{tb}V_{ts}^*| = 0.0385, \ \alpha^{-1} = 129, \ G_F = 1.17 \times 10^{-5} \text{GeV}^{-2},$$

$$\tau_{B_c} = 0.46 \times 10^{-12} \text{s}. \quad (21)$$

The values of the individual Wilson coefficients that appear in the SM are listed in Table I. The values for the mass and the lifetime of the $B_c$ meson given above in Eq. (21) were reported by CDF Collaboration [18]. Recently, CDF quoted a new value $m_{B_c} = 6.2857 \pm 0.0053 \pm 0.0012 \text{GeV}$ [19]. Also, D0 has observed $B_c$ and reported the preliminary results $m_{B_c} = 5.95^{+0.14}_{-0.13} \pm 0.34 \text{GeV}$ and $\tau_{B_c} = 0.45^{+0.12}_{-0.10} \pm 0.12 \text{[20]}$. However, we observed that our numerical results are not sensitive to the numerical values of $m_{B_c}$ more than 3–5%.
Table 1: Values of the SM Wilson coefficients at $\mu \sim m_b$ scale.

We note that the value of the Wilson coefficient $C^\text{eff}_9$ in Table (1) corresponds only to the short-distance contributions. $C^\text{eff}_9$ also receives long-distance contributions due to conversion of the real $\bar{c}c$ into lepton pair $\ell^+\ell^-$ and they are usually absorbed into a redefinition of the short-distance Wilson coefficients:

$$C^\text{eff}_9(\mu) = C_9(\mu) + Y(\mu),$$ (22)

where

$$Y(\mu) = Y_{\text{reson}} + h(y,s)[3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)]$$

$$- \frac{1}{2}h(1,s) [4C_3(\mu) + 4C_4(\mu) + 3C_5(\mu) + C_6(\mu)]$$

$$- \frac{1}{2}h(0,s) [C_3(\mu) + 3C_4(\mu)]$$

$$+ \frac{1}{2} (3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)),$$ (23)

with $y = m_c/m_b$, and the functions $h(y,s)$ arises from the one loop contributions of the four quark operators $O_1,...,O_6$ and their explicit forms can be found in [21]. It is possible to parametrize the resonance $\bar{c}c$ contribution $Y_{\text{reson}}(s)$ in Eq.(23) using a Breit-Wigner shape with normalizations fixed by data which is given by [22]

$$Y_{\text{reson}}(s) = -\frac{3}{\alpha_e^2} \frac{\kappa}{s} \sum_{V_i = \psi_i} \frac{\pi \Gamma(V_i \rightarrow \ell^+\ell^-)m_{V_i}}{m^2_{V_i} - m_{V_i}^2 + imV_i} \Gamma_{V_i}$$

$$\times \left[ (3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)) \right],$$ (24)

where the phenomenological parameter $\kappa$ is usually taken as $\sim 2.3$.

As for the values of the new Wilson coefficients, they are the free parameters in this work, but it is possible to establish ranges out of experimentally measured branching ratios of the semileptonic and also purely leptonic rare B-meson decays

$$BR(B \rightarrow K\ell^+\ell^-) = (0.75^{+0.25}_{-0.21} \pm 0.09) \times 10^{-6},$$

$$BR(B \rightarrow K^*\mu^+\mu^-) = (0.9^{+1.3}_{-0.9} \pm 0.1) \times 10^{-6},$$

reported by Belle and Babar collaborations [23]. It is now also available an upper bound of pure leptonic rare B-decays in the $B^0 \rightarrow \mu^+\mu^-$ mode [24]:

$$BR(B^0 \rightarrow \mu^+\mu^-) \leq 2.0 \times 10^{-7}.$$ (25)

Being in accordance with this upper limit and also the above mentioned measurements of the branching ratios for the semileptonic rare B-decays, we take in this work all new Wilson coefficients as real and varying in the region $-4 \leq C_X \leq 4$. 

| $C_1$  | $C_2$  | $C_3$  | $C_4$  | $C_5$  | $C_6$  | $C^\text{eff}_7$ | $C_9$  | $C_{10}$ |
|--------|--------|--------|--------|--------|--------|-----------------|--------|----------|
| -0.248 | +1.107 | +0.011 | -0.026 | +0.007 | -0.031 | -0.313          | +4.344 | -4.624   |
Among the new Wilson coefficients that appear in Eq. (1), those related to the helicity-flipped counter-parts of the SM operators, namely, $C_{RL}$ and $C_{RR}$, vanish in all models with minimal flavor violation in the limit $m_s \to 0$. However, there are some MSSM scenarios in which there are finite contributions from these vector operators even for a vanishing s-quark mass. In addition, scalar type interactions can also contribute through the neutral Higgs diagrams in e.g. multi-Higgs doublet models and MSSM for some regions of the parameter spaces of the related models. In literature there exists studies to establish ranges out of constraints under various precision measurements for these coefficients (see e.g. [25]) and our choice for the range of the new Wilson coefficients are in agreement with these calculations.

To make some numerical predictions, we also need the explicit forms of the form factors $A_0, A_+, A_-, V, a_0, a_+$ and $g$. In our work we have used the results of [6], in which $q^2$ dependencies of the form factors are given as

$$F(q^2) = \frac{F(0)}{(1 - as + bs^2)^2},$$

where the values of parameters $F(0), a$ and $b$ for the $B_c \to D_s^* \ell^+ \ell^-$ decay are listed in Table 2.

|       | $F(0)$ | $a$  | $b$  |
|-------|--------|------|------|
| $A_0$ | 0.279  | 1.30 | 0.149|
| $A_+$ | 0.156  | 2.16 | 1.15 |
| $A_-$ | -0.321 | 2.41 | 1.51 |
| $V$   | 0.290  | 2.40 | 1.49 |
| $a_0$ | 0.178  | 1.21 | 0.125|
| $a_+$ | 0.178  | 2.14 | 1.14 |
| $g$   | 0.179  | 2.51 | 1.67 |

Table 2: $B_c$ meson decay form factors in a relativistic constituent quark model.

We present the results of our analysis in a series of figures. Before the discussion of these figures, we give our SM predictions for the longitudinal, transverse and the normal components of the lepton polarizations for $B_c \to D_s^* \ell^+ \ell^-$ decay for $\mu$ ($\tau$) channel for reference:

$$< P_L^- > = 0.6211 (0.6321),$$
$$< P_T^- > = 0.0017 (0.0468),$$
$$< P_N^- > = -0.0837 (-0.17).$$

Figs. (1) and (2) give dependence of the integrated branching ratio (BR) on the new Wilson coefficients for the $B_c \to D_s^* \mu^+ \mu^-$ and $B_c \to D_s^* \tau^+ \tau^-$ decays, respectively. From
these figures we see that BR depends strongly on the tensor interactions and weakly on the vector interactions, while it is completely insensitive to the scalar type of interactions. It is also clear from these figures that dependence of the BR on the new Wilson coefficients is symmetric with respect to the zero point for the muon final state, but such a symmetry is not observed for the tau final state for the tensor interactions.

In Figs. (3) and (4), we present the dependence of averaged longitudinal polarization \(< P^\perp_L >\) of \(\ell^-\) and the combined averaged \(< P^\perp_L + P^\perp_T >\) for \(B_c \rightarrow D_s^* \mu^+ \mu^-\) decay on the new Wilson coefficients. We observe that \(< P^\perp_L >\) is more sensitive to the existence of the tensor type interactions while the combined average \(< P^\perp_L + P^\perp_T >\) is to that of scalar type interactions only. The fact that \(< P^\perp_L + P^\perp_T >\) does not exhibit any dependence on the vector type of interactions are already an expected result since vector type interactions are cancelled when the longitudinal polarization asymmetry of the lepton and antilepton is considered together. We also note that the values of \(< P^\perp_L >\) becomes substantially different from the SM value (at \(C_X = 0\)) as \(C_X\) becomes different from zero, which indicates that measurement of the longitudinal lepton polarization in \(B_c \rightarrow D_s^* \mu^+ \mu^-\) decay can be very useful to investigate new physics beyond the SM. From Fig. (3), we see that, the contributions coming from all types of interactions to \(< P^\perp_L >\) are positive and it is an increasing (decreasing) functions of both \(C_T\) and \(C_{TE}\) for their negative (positive) values. We observe from Fig. (4) that \(< P^\perp_L + P^\perp_T >\) becomes zero at \(C_X = 0\), which conforms the SM results, and its dependence on \(C_X\) is symmetric with respect to this zero value. It is also interesting to note that \(< P^\perp_L + P^\perp_T >\) is positive for all values of \(C_{LRLR}\) and \(C_{RLLR}\), while it is negative for remaining scalar type interactions.

Figs. (5) and (6) are the same as Figs. (3) and (4), but for \(B_c \rightarrow D_s^* \tau^+ \tau^-\). Similar to the muon case, \(< P^\perp_L >\) is more sensitive to the tensor interactions than others. Contributions to \(< P^\perp_L >\) from all type of interactions are positive for all values of \(C_X\) except for \(C_{TE}\): in region \(0.25 \leq C_{TE} < 4\), \(< P^\perp_L >\) changes the sign and becomes negative. As for the main interesting point in Fig. (6), although \(< P^\perp_L + P^\perp_T >\) for \(B_c \rightarrow D_s^* \mu^+ \mu^-\) decay depends only on scalar interactions, for \(B_c \rightarrow D_s^* \tau^+ \tau^-\) decay it is also very sensitively dependent on tensor type of interactions. It is also interesting to note that \(< P^\perp_L + P^\perp_T >\) changes sign: it takes positive (negative) values for the negative (positive) values of \(C_T\) and \(C_{TE}\). Thus, one can provide valuable information about the new physics by determining the sign and the magnitude of \(< P^\perp_L + P^\perp_T >\). We finally note that as in case of muon final state, in tau final state too, \(< P^\perp_L + P^\perp_T >\) becomes zero at \(C_X = 0\) and confirms the SM result.

In Figs. (7) and (8), we present the dependence of averaged transverse polarization \(< P_T^\perp >\) of \(\ell^-\) and the combined averaged \(< P_T^\perp - P_T^\perp >\) for \(B_c \rightarrow D_s^* \mu^+ \mu^-\) decay on the new Wilson coefficients. From these figures, it is seen that for the \(< P_T^\perp >\), there appears strong dependence on tensor and scalar interactions and also a weak dependence on vector interactions. On the other hand, vector contributions to the \(< P_T^\perp - P_T^\perp >\) is negligible and main contribution comes from the tensor interactions and \(C_{LRLR}\) and \(C_{RLLR}\) components of the scalar interactions. As seen from Figs. (7) and (8), both \(< P_T^\perp >\) and \(< P_T^\perp - P_T^\perp >\) are positive (negative) for the negative (positive) values of \(C_T\) and \(C_{LRLR}\), except in a region about the zero values of the coefficients, \(-1 \leq C_X \leq 1\), while their behavior with respect to \(C_{TE}\) and \(C_{LRLR}\) are opposite. Therefore, determination of the sign and magnitude of these observables can also give useful information about existence of new physics.

Figs. (9) and (10) are the same as Figs. (7) and (8), but for \(B_c \rightarrow D_s^* \tau^+ \tau^-\). We see
from Fig. (9) that the \( < P_T > \) is quite sensitive to all types of interactions and behavior of scalar interaction is identical for coefficients \( C_{LRRL}, C_{LRLR} \) and \( C_{RLLR}, C_{RLRL} \) in pairs. It can be seen from Fig. (10) that although tensor and scalar interactions are dominant for \( < P_T - P_T^+ > \), the dependence of vector interactions are also more sizable as compared with the case of muon final state. In addition, change in sign of \( < P_T > \) and \( < P_T^+ > \) are observed depending on the change in the tensor and scalar interaction coefficients, whose measure may provide useful tools for new physics.

In Figs. (11) and (12), we present the dependence of averaged normal polarization \( < P_N > \) of \( \ell^- \) and the combined averaged \( < P_N^- + P_N^+ > \) for \( B_c \rightarrow D_s^*\mu^+\mu^- \) decay on the new Wilson coefficients. We see from Fig. (11) that \( < P_N^- > \) strongly depends on the tensor interactions. Its dependence on the scalar type of interactions is moderate and identical for the coefficients \( C_{LRRL}, C_{LRLR} \) and \( C_{RLLR}, C_{RLRL} \) in pairs. As seen from Fig. (12), the behavior of \( < P_N^- + P_N^+ > \) is determined by the tensor interactions only. We also observe that \( < P_N^- + P_N^+ > \) is positive (negative) when \( C_T < 0 (C_T > 0) \) while its behavior with respect to \( C_{TE} \) is opposite. Further, \( < P_N^- + P_N^+ > \) becomes zero at \( C_X = 0 \) as expected in the SM.

Figs. (13) and (14) are the same as Figs. (11) and (12), but for \( B_c \rightarrow D_s^*\tau^+\tau^- \). We first note that as being opposite to the muon final state case, here \( < P_N^- > \) always takes the positive values except when \( C_{TE} \approx -0.25 \) and \( C_T \gtrsim 2 \). As seen from Fig. (14), \( < P_N^- + P_N^+ > \) depends only on the tensor interactions and its behavior is the same as that of the muon final state case.

In conclusion, we present the most general analysis of the lepton polarization asymmetries in the rare \( B_c \rightarrow D_s^*\ell^+\ell^- \) decay using the general, model independent form of the effective Hamiltonian. The dependence of the longitudinal, transversal and normal polarization asymmetries of \( \ell^- \) and their combined asymmetries on the new Wilson coefficients are studied. It is found that the lepton polarization asymmetries are very sensitive to the existence of the tensor and scalar type interactions. Moreover, \( < P_T > \) and \( < P_N > \) change their signs as the new Wilson coefficients vary in the region of interest. This conclusion is valid also for the combined polarization effects \( < P_T^- + P_T^+ >, \ < P_T^- + P_T^+ > \) and \( < P_N^- + P_N^+ > \) for the same decay channel. It is well known that in the SM, \( < P_T^- + P_T^+ > = < P_T^- - P_T^+ > = < P_N^- + P_N^+ > \approx 0 \) in the limit \( m_\ell \rightarrow 0 \). Therefore any deviation from this relation and determination of the sign of polarization is decisive and effective tool in looking for new physics beyond the SM.

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Figure 1: The dependence of the integrated branching ratio for the $B_c \rightarrow D_s^* \mu^+ \mu^-$ decay on the new Wilson coefficients.

Figure 2: The dependence of the integrated branching ratio for the $B_c \rightarrow D_s^* \tau^+ \tau^-$ decay on the new Wilson coefficients.
Figure 3: The dependence of the averaged longitudinal polarization $< P^-_L >$ of $\ell^-$ for the $B_c \rightarrow D^*_s \mu^+ \mu^-$ decay on the new Wilson coefficients.

Figure 4: The dependence of the combined averaged longitudinal lepton polarization $< P^-_L + P^+_L >$ for the $B_c \rightarrow D^*_s \mu^+ \mu^-$ decay on the new Wilson coefficients.
Figure 5: The same as Fig. (3), but for the $B_c \to D_s^* \tau^+ \tau^-$ decay.

Figure 6: The same as Fig. (4), but for the $B_c \to D_s^* \tau^+ \tau^-$ decay.
Figure 7: The dependence of the averaged transverse polarization $< P_T^- >$ of $\ell^-$ for the $B_c \to D^*_s \mu^+ \mu^-$ decay on the new Wilson coefficients.

Figure 8: The dependence of the combined averaged transverse lepton polarization $< P_T^- - P_T^+ >$ for the $B_c \to D^*_s \gamma \mu^+ \mu^-$ decay on the new Wilson coefficients.
Figure 9: The same as Fig. (7), but for the $B_c \to D_s^* \tau^+ \tau^-$ decay.

Figure 10: The same as Fig. (8), but for the $B_c \to D_s^* \tau^+ \tau^-$ decay.
Figure 11: The dependence of the averaged normal polarization $< P_N^- >$ of $\ell^-$ for the $B_c \rightarrow D_s^* \mu^+\mu^-$ decay on the new Wilson coefficients.

Figure 12: The dependence of the combined averaged normal lepton polarization $< P_N^- + P_N^+ >$ for the $B_c \rightarrow D_s^* \mu^+\mu^-$ decay on the new Wilson coefficients.
Figure 13: The same as Fig. (10), but for the $B_c \to D_s^* \tau^+ \tau^-$ decay.

Figure 14: The same as Fig. (11), but for the $B_c \to D_s^* \tau^+ \tau^-$ decay.