Topological phase states of the \(SU(3)\) QCD

To cite this article: Alexander P Protogenov et al 2014 J. Phys.: Conf. Ser. 482 012035

View the article online for updates and enhancements.
Topological phase states of the $SU(3)$ QCD

Alexander P Protogenov$^{1,2}$, Evgueni V Chulkov$^{2,3,4}$, Jeffrey C Y Teo$^5$

$^1$Institute of Applied Physics, Nizhny Novgorod 603950, Russia
$^2$Donostia International Physics Center (DIPC), 20018 San Sebastián/Donostia, Spain
$^3$Departamento de Física de Materiales UPV/EHU; Centro de Física de Materiales CFM - MPC and Centro Mixto CSIC-UPV/EHU, Facultad de Ciencias Químicas, Universidad del Pais Vasco/Euskal Herriko Unibertsitatea, 20080 San Sebastián/Donostia, Spain
$^4$Tomsk State University, Tomsk 634050, Russia
$^5$Department of Physics, University of Illinois at Urbana-Champaign, Urbana IL 61801, USA

E-mail: alprot@appl.sci-nnov.ru

Abstract. We consider the topologically nontrivial phase states and the corresponding topological defects in the $SU(3)$ d-dimensional quantum chromodynamics (QCD). The homotopy groups for topological classes of such defects are calculated explicitly. We have shown that the three nontrivial groups are

$$
\pi_n SU(3) = \mathbb{Z}, \quad \pi_5 SU(3) = \mathbb{Z}, \quad \text{and} \quad \pi_6 SU(3) = \mathbb{Z}_6 \quad \text{if} \quad 3 \leq d \leq 6.
$$

The latter result means that we are dealing exactly with six topologically different phase states. The topological invariants for $d=3,5,6$ are described in detail.

Introduction. - Topological invariants of field configurations are the fundamental objects in the quantum field theory and condensed matter physics, which classify topological defects and possible phase states [1, 2]. The well-known examples of topological field distributions are vortices, hedgehogs, and instantons. They are a direct consequence of the nontrivial homotopy groups $\pi_n S^n = \mathbb{Z}$ for the spheres $S^n$ with $n = 1, 2, \pi_3 SU(2) = \mathbb{Z}$, for the spatial dimensions $d = 1, 2, 3$, respectively. Here, $\mathbb{Z}$ is a group of integers. Recent progress in the theory of topologically ordered phase states [3, 4] is associated with the classification of the systems, in which the D-dimensional surface $S^D$ surrounds a defect in d-dimensional topological insulators or superconductors [5, 6, 7, 8] and $D \neq d$. In this case, the first nontrivial example $\pi_3 S^2 = \mathbb{Z}$ is the well-known Hopf mapping of the three-dimensional sphere $S^3$ into the two-dimensional one $S^2$. The corresponding topological invariant $Q$ is called the helicity in magnetohydrodynamics or the Abelian Chern-Simons action in the $(2 + 1)$-dimensional topological field theory. The integer $Q$ means the knotting degree of the field distributions and determines, in particular, the lower bound [9, 10] of the energy of the two-component Ginzburg-Landau model expressed in the form of the Skyrme-Faddeev-Niemi model [12, 13]. In this $O(3)$ $\sigma$-model, the $U(1)$ two-form $dA = n [dn \wedge ndn]$ is parametrized the Hopf invariant $Q = \frac{1}{16\pi^2} \int_{S^3} A \wedge dA \in \mathbb{Z}$ by the unit 3d vector $n$ which maps the base space $S^3$ into $S^2$. The target sphere $S^2$ of the map is topologically equivalent to the coset $SU(2)/U(1) \cong S^2$. The $n$-field is also a relevant on-shell variable [15, 14, 16, 18] in the infrared limit of the $SU(2)$ QCD.

In this paper, we use the $SU(3)$ group instead of the $SU(2)$ one. The change in the value of the rank is due to several reasons. Primarily, to the three colors of the QCD. From the point of view of the knot theory, this choice is also due to an attempt to extend the low-dimensional topology of the standard knot theory to higher dimensions of the $SU(3)$ QCD target space.
One approach to this problem is to use many-valued functionals [20] in accordance with the conjecture given in Ref. [21]. Another elegant method is based on the results obtained in Ref. [22]. However, it is more expedient to describe the target spaces of the $SU(2)$ target sphere $S^2$. The remains freedom of the maps is the dimension of the base space having a nontrivial homotopy group. For simplicity, we restrict our consideration to the spheres $S^n$ as the base spaces. Therefore, we focus on such $n$ of the homotopy groups $\pi_nF_2$, which yields nontrivial results. For comparison, in addition to the maximal torus $U(1)^2$ of $SU(3)$ that results in the general orbit $F_2$ [25], we calculate the homotopy groups of the degenerate orbit $CP^2$, which are equivalent to the coset space $SU(3)/U(2) = SU(3)/(SU(2) \times U(1)) = CP^2$.

It should be noted that we are restricted to the framework of the homotopy group approach. Therefore, we would like to determine the constraints on the type of the possible topological phase states and topological defects only. We will describe the geometry of the flag space $F_2$ and topological features in the last two sections. The results of our calculations are presented in two tables.

It is seen in Table I that the nontrivial homotopy groups for $d \leq 5$ are $\pi_2F_2$, $\pi_3F_2$, $\pi_5F_2$, and $\pi_6F_2$.

(i) It is known [24] that nontriviality of $\pi_2F_2 = \mathbb{Z} \times \mathbb{Z}$ accounts for the presence of two different monopoles in the theory (cf. $\pi_2CP^2 = \mathbb{Z}$ in Table II which means that we deal with a monopole of one type). The second homotopy group is nontrivial due to the fact that the simply connected flag space $F_2$ is a compact symplectic manifold.

(ii) The integers in the RHS of $\pi_3F_2 = \mathbb{Z}$ have the meaning of $SU(3)$ instanton topological charges because $\pi_3F_2 = \pi_3SU(3) = \pi_3SU(2)$.

(iii) The integers in the RHS of $\pi_5F_2 = \mathbb{Z}$ describe some textures and the corresponding phases. The nature of these textures is difficult to guess now.

(iv) The most interesting answer $\pi_6F_2 = \mathbb{Z}_6$ means that there are only six phase states with the labels \{0, 1, \ldots, 5\}. They are usually ordered as three quark doublets. We can topologically distinct the quark states because of the gauge invariant coupling of the fermions to the gauge potential. This takes place on the scales where we can consider the six-dimensional base space as the sphere $S^6$. Note that some additional parameters of the $(3 + 1)d$ gauge theory can add dimensions in order to have finally 6 dimensions of the base space [31]. We encounter these phenomena in some topologically ordered phases of condensed matter [4]. In our case, the best natural choice for the interpretation of the base space $S^6$ corresponds to the standard six-dimensional space-momentum phase space. We are free also to interpret the six-dimensional compact space $S^6$ as a complement to the $(3 + 1)$-dimensional space-time, but the previous suggestion is much better.

*Gauge fields on the flag space.* - Let us describe the flag space $F_2$ in detail to explain in particular at the end, why we addressed to homotopy theory approach. It is a compact Kähler

| $d$ | $\pi_dF_2$ |
|-----|------------|
| 0   | 0          |
| 1   | $\mathbb{Z} \times \mathbb{Z}$ |
| 2   | $\mathbb{Z}$ |
| 3   | $\mathbb{Z}$ |
| 4   | $\mathbb{Z}$ |
| 5   | $\mathbb{Z}$ |
| 6   | $\mathbb{Z}$ |
| 7   | $\mathbb{Z}$ |
| 8   | $\mathbb{Z}$ |
| 9   | $\mathbb{Z}$ |
| 10  | $\mathbb{Z}$ |

Table 1. A list of the homotopy groups $\pi_dF_2$ for dimensions $d \leq 10$.

| $d$ | $\pi_dCP^2$ |
|-----|-------------|
| 0   | 0           |
| 1   | $\mathbb{Z}$ |
| 2   | $\mathbb{Z}$ |
| 3   | $\mathbb{Z}$ |
| 4   | $\mathbb{Z}$ |
| 5   | $\mathbb{Z}$ |
| 6   | $\mathbb{Z}$ |
| 7   | $\mathbb{Z}$ |

Table 2. A list of the homotopy groups $\pi_dCP^2$ for dimensions $d \leq 7$. 

[17]
manifold which is a homogeneous nonsymmetric space of dimension \( \dim F_2 = 6 \). Since the flag manifold \( F_2 \) is the \( \text{Kähler} \) one, it possesses the complex local coordinates \( w_\alpha, \alpha = 1, 2, 3, \) the Hermitian Riemannian metric, \( ds^2 = g_{\alpha\bar{\beta}}dw^\alpha d\bar{w}^\beta, \) and the closed two-form (field strength) \( \Omega_K = ig_{\alpha\bar{\beta}}dw^\alpha \wedge d\bar{w}^\beta, \) i.e., \( d\Omega_K = 0 \). Here, \( d = \partial + \bar{\partial} = dw_\alpha \frac{\partial}{\partial w_\alpha} + d\bar{w}_\beta \frac{\partial}{\partial \bar{w}_\beta} \) denotes the exterior derivative, while the operators \( \partial \) and \( \bar{\partial} \) are called the Dolbeault operators.

According to the Poincaré lemma, any closed form \( \Omega_K \) is \( \text{locally} \) exact, i.e., \( \Omega_K = dw, \) where \( \omega \) is the gauge potential. The condition \( d\Omega_K = 0 \) is equivalent to \( g_{\alpha\bar{\beta}} = \frac{\partial}{\partial w^\alpha} \frac{\partial}{\partial \bar{w}^\beta} K, \) where \( K = K(w, \bar{w}) \) is the \( \text{Kähler} \) potential:

\[
K(w, \bar{w}) = \ln[|\Delta_1(w, \bar{w})|^m|\Delta_2(w, \bar{w})|^n],
\]

\[
\Delta_1(w, \bar{w}) = 1 + |w_1|^2 + |w_2|^2,
\]

\[
\Delta_2(w, \bar{w}) = 1 + |w_3|^2 + |w_2 - w_1 w_3|^2.
\]

By means of three complex variables \( w_\alpha, \) the flag space \( F_2 \) is realized as a set of triangular matrices of the form

\[
\begin{pmatrix}
1 & w_1 & w_2 \\
0 & 1 & w_3 \\
0 & 0 & 1
\end{pmatrix}^t \in F_2 = SU(3)/U(1)^2.
\]

The \( \text{Kähler} \) one-form and the two-form are \( \omega = \frac{i}{2}(\partial - \bar{\partial})K, \Omega_K = i\partial\bar{\partial}K. \) The explicit forms of the gauge potential \( \omega \) and the field strength \( \Omega_K \) are given by

\[
\omega = im\frac{w_1 d\bar{w}_1 + w_2 d\bar{w}_2}{\Delta_1(w, \bar{w})} + in\frac{w_3 d\bar{w}_3}{\Delta_2(w, \bar{w})} + in\frac{w_2 - w_1 w_3)(d\bar{w}_2 - \bar{w}_1 d\bar{w}_3 - \bar{w}_3 d\bar{w}_1}{\Delta_2(w, \bar{w})},
\]

\[
\Omega_K = dw = im(\Delta_1)^{-2}(1 + |w_1|^2)dw_2 \wedge d\bar{w}_2 - \bar{w}_2 w_1 dw_1 \wedge d\bar{w}_1
- w_2 \bar{w}_1 dw_1 \wedge d\bar{w}_1 + (1 + |w_2|^2)dw_1 \wedge d\bar{w}_1
+ in(\Delta_2)^{-2}|\Delta_1|d\bar{w}_3 \wedge d\bar{w}_3
- (w_1 + \bar{w}_3 w_2)dw_3 \wedge (d\bar{w}_2 - \bar{w}_3 d\bar{w}_1)
- (\bar{w}_1 + w_3 \bar{w}_2)(dw_2 - w_3 dw_1) \wedge d\bar{w}_3
+(1 + |w_3|^2)(dw_2 - w_3 dw_1)
(d\bar{w}_2 - \bar{w}_3 d\bar{w}_1).
\]

Calculation of the Poincaré polynomial \( P_{F_2}(t) = \sum_{i=0}^6 b_i t^i \) of \( F_2 \) (see [26, 27]) with the Betti numbers \( b_i \) yields \( P_{F_2}(t) = 1 + 2t^2 + 2t^4 + t^6, \) i.e., \( b_0 = b_6 = 1, b_2 = b_4 = 2. \) We see that the cohomology class is not zero because all even Betti numbers are nonzero.

The \( \text{Kähler} \) potential for \( CP^2 \) is given by

\[
K(w, \bar{w}) = \ln[|\Delta_1|^m],
\]
we also have shown a list of the homotopy groups are in accord with previous studies [28] (see also Ref. [29]). For completeness and comparison, where we cannot define the gauge connection $! \in \text{SU}(2)$ calculating the homotopy groups of $F$, topological invariants in the defined on the manifold. This is the reason why it is difficult to directly determine the Hopf-like $K$ is an element of the second cohomology group of $F$. Therefore, two-form (6) is closed, but not globally exact. One can say that $\Omega_K$ is the four-dimensional feature inside the six-dimensional flag space $F_2$. This means that we cannot define the gauge connection $\omega$ everywhere in $F_2$, because the one-form $\omega$ is not well defined on the manifold. This is the reason why it is difficult to directly determine the Hopf-like topological invariants in the $F_2$ case.

Topological invariants . - Let us proceed with the analysis of topological invariants by calculating the homotopy groups of $SU(3)$. The flag space $F_2 = SU(3)/U(1)^2$ is the base space of the $U(1)^2$-fiber bundle $SU(3) \rightarrow F_2$. We have the following exact sequences:

$$ 0 \rightarrow \pi_d(U(1)^2) \rightarrow \pi_d SU(3) \rightarrow \pi_d F_2 \rightarrow 0, \quad \text{for } d \geq 3, \quad (11) $$

$$ 0 \rightarrow \pi_2 SU(3) \rightarrow \pi_2 F_2 \rightarrow \pi_1(U(1)^2) \rightarrow \pi_1 SU(3) \rightarrow \pi_1 F_2 \rightarrow 0, \quad (12) $$

where $\pi_1(U(1)^2) = \mathbb{Z} \times \mathbb{Z}$ and $\pi_1 SU(3) = \pi_2 SU(3) = 0$. Thus, we have

$$ \left\{ \begin{array}{l}
\pi_0 F_2 = \pi_1 F_2 = 0, \\
\pi_2 F_2 = \mathbb{Z} \times \mathbb{Z}, \\
\pi_d F_2 = \pi_d SU(3), \quad \text{for } d \geq 3.
\end{array} \right. \quad (13) $$

We summarized the results in Table I. It presents the nontrivial homotopy groups of $F_2$, which are in accord with previous studies [28] (see also Ref. [29]). For completeness and comparison, we also have shown a list of the homotopy groups $\pi_d CP^2$ for $d \leq 7$ in Table II.
Table I is proved by the results of Ref. [30] and references therein. In particular, Ref. [30] presents two theorems that account for the 5th and 6th homotopy groups of SU(3). Theorem 1: $\pi_{2n-1}U(N) = \mathbb{Z}$ for $N \geq n$ and Theorem 2: $\pi_{2n}U(n) = \mathbb{Z}_n!$ for $n \geq 2$. We will study now the 3rd, 5th, and 6th homotopy groups in more detail.

1. The 3rd homotopy group of SU(3). The exact sequence of the fibration $SU(3) \to SU(3)/SU(2) \cong S^5$ is

$$
\pi_{d+1}S^5 \to \pi_d SU(2) \xrightarrow{i} \pi_d SU(3) \to \pi_d S^5.
$$

Let $d = 3$ and, since $\pi_4 S^5 = \pi_3 S^5 = 0$, let the inclusion $i: SU(2) \to SU(3)$ induce an isomorphism $i_*: \pi_3 SU(2) \cong \pi_3 SU(3)$. A generator for $\pi_3 SU(2)$ is given by

$$
g_2(r) = r^0 1 + ir^j \sigma_j,
$$

for $r = (r^0, r^1, r^2, r^3)$ and $|r| = 1$,

$$
g_3(r) = \begin{pmatrix}
1 & 0 & 0 \\
0 & r^0 + ir^3 & ir^1 + r^2 \\
0 & ir^1 - r^2 & r^0 - ir^3
\end{pmatrix}.
$$

Given any continuous function $g: S^3 \to SU(3)$, the topological invariant, i.e., the winding degree $[g] \in \pi_3 SU(3) = \mathbb{Z}$, is determined by the integral formula

$$
[g] = \frac{1}{24\pi^2} \int_{S^3} \text{Tr}[(g^*g)^3] =
$$

$$
\frac{1}{24\pi^2} \int_{S^3} d^3x \varepsilon_{\mu\nu\lambda} \text{Tr}(g^4 \partial_\nu g^j \partial_\mu g^g \partial_\lambda g).
$$

2. The 5th homotopy group of SU(3). Using exact sequence (14), we have

$$
\pi_5 SU(2) \to \pi_5 SU(3) \to \pi_5 S^5
$$

$$
\to \pi_4 SU(2) \to \pi_4 SU(3).
$$

It is known that $\pi_5 SU(2) = \pi_5 S^3 = \mathbb{Z}_2$ and $\pi_5 SU(3) = \mathbb{Z}$ (from theorem 1), and thus the first arrow must be the zero map ($\mathbb{Z}_2 \to \mathbb{Z}$). We also know that $\pi_4 SU(3) = 0$, as the homotopy group $\pi_4 SU(N) = 0$ stabilizes after $N \geq 3$. Finally, we need $\pi_4 SU(2) = \pi_4 S^3 = \mathbb{Z}_2$. We now have

$$
0 \to \pi_5 SU(3) \xrightarrow{x_2^2} \pi_5 S^5 \to \mathbb{Z}_2 \to 0.
$$

This means that given $g: \pi_5 S^5 \to SU(3)$,

$$
g(r) = \begin{pmatrix}
\mathbf{u}_1(r) \\
\mathbf{u}_2(r) \\
\mathbf{u}_3(r)
\end{pmatrix}, \quad \text{for } r \in S^5,
$$
the vector \( u_1 : S^5 \to S^5 \) has an even winding degree, namely, the winding degree \([u_1] = 2 \times \text{winding degree}[g]\).

The exact sequence for the fibration \( SU(N + 1) \to SU(N + 1)/SU(N) = S^{2N+1} \) shows that \( \pi_5 SU(N) = Z \) stabilizes after \( N \geq 3 \). Thus, the winding degree of \( g \) can be also deduced by the usual formula

\[
[g] = \frac{1}{480\pi^2} \int_{S^5} \text{Tr} \left( (gdg^\dagger)^5 \right). \tag{22}
\]

A particular generator of \( \pi_5 SU(3) \) can be found in Ref. [30].

3. The 6th homotopy group of \( SU(3) \). Exact sequence (14) yields

\[
\pi_7 S^5 \to \pi_6 SU(2) \to \pi_6 SU(3) \to \pi_5 SU(2), \tag{23}
\]

where \( \pi_5 S^5 = Z_2 \) and \( \pi_6 SU(2) = Z_{12} \). It turns out that \( \pi_6 SU(3) = Z_{d_4} = Z_6 \). A generator for \( \pi_6 SU(3) \) can be found in [30] in page 6.

Conclusion. - In conclusion, we focus on the nontrivial homotopy groups for \( d \leq 6 \pi_2 F_2, \pi_3 F_2, \pi_5 F_2, \) and \( \pi_6 F_2 \) considered so far for the spheres \( S^2 \) as the base space. The generalization \( S^2 \to T^n \), where \( T^n \) is the \( n \)-dimensional torus, is an interesting and more complicated extension even in the \( SU(2) \) case. The result of calculations [32] of the mapping class groups in the last case with \( T^3 = S^1 \times S^1 \times S^1 \) leads to the linear superposition of the topological invariants beginning from the first Chern class to the Hopf invariant (see also [12]). Similar behavior also takes place in the case of \( T^3 = S^2 \times S^1 \). The classification problems of the mappings \( T^n \to F_2 \) are still totally open.

Up to now, we did not pay any attention to the relation between the existence of strong interaction in the system and homotopy group results. It is well known in the condensed matter community that nontrivial answers \( Z_2 \) or \( Z \) for the topological invariant of non-interacting systems change drastically in particular to \( Z_8 \) in the case of the interacting system [33]. Considering from this point of view the result \( \pi_6 (F_2) = Z_{d_4} \), one can say that we deal here with the significant interaction as it takes place in our QCD system.

Thorough understanding of the role of the flag space \( F_2 \) in the \( SU(3) \) gauge theory is related to the search for an analog of the Hopf number, i.e., the linking number of pullbacks on a space \( M \) of two arbitrary ”points” on a target space \( N \) of the map \( M \to N \). Such an analog can have the form of pre-images of the target points in the codimension two [22]. This could take place if \( M = S^3 \) and \( N \) is the 2d complement of \( CP^2 \) with respect to the whole space \( F_2 \). This is an open question, which is difficult to answer without knowing the details of the map. We leave the problem of describing the details of this map for future work.

Acknowledgments

We are grateful to V.I. Arnold, J.E. Avron, J. Bernatska, L.D. Faddeev, P.G. Grinevich, C.L. Kane, Y. Hatsugai, A.W.W. Ludwig, I.A. Taimanov, G. E. Volovik for fruitful discussions. The coauthors (A.P., J.T.) especially thank the organizers of the 28th Jerusalem Winter School in Theoretical Physics A. Stern and S.-C. Zhang for hospitality at the Israel Institute for Advanced Studies where a part of the work was done. This work was supported in part by the RFBR Grant No. 13-02-12110.

[1] Mermin N D 1979 Rev. Mod. Phys. 51 591
[2] Volovik G E 2003 The Universe in a Helium Droplet (Clarendon: Oxford)
[3] Teo J.C.Y. and Kane C L 2010 Phys. Rev. Lett. 104 046401
[4] Teo J C Y and Kane C L 2010 Phys. Rev. B 82 115120
[5] Hasan M Z and Kane C L 2010 Rev. Mod. Phys. 82 3045
[6] Qi X-L and Zhang S-C 2011 Rev. Mod. Phys. 83 1057
[7] Schnyder A P, Ryu S, Furusaki A and Ludwig A W W 2008 Phys. Rev. B 78 195125; AIP Conf. Proc. 2009 1134 10; New J. Phys. 2010 12 065010
[8] Kitaev A. 2009 AIP Conf. Proc. 1134 22 (2009)
[9] Vakulenko A F and Kapitanskii L V 1979 Sov. Phys. Dokl. 24 433
[10] Protogenov A P and Verbus V A 2002 JETP Lett. 76 53
[11] Babaev E, Faddeev L. D. and Niemi A J 2002 Phys. Rev. B 65 100512
[12] Protogenov A P 2006 Physics-Uspekhi 49 667
[13] Jäykkä J, Hietarinta J and Salo P 2008 Phys. Rev. B 77 094509
[14] Cho Y M 1980 Phys. Rev. D 21 1080; 1981 Phys. Rev. Lett. 46 302
[15] Duan Y S and Ge M L 1979 Sinica Sci. 11 1072
[16] Faddeev L D and Niemi A J 1999 Phys. Rev. Lett. 82 1624
[17] The problem in this form has been formulated by L. Faddeev in Ref. [18] (see also Ref. [19]).
[18] Faddeev L 2001 Philos. Trans. R. Soc. London A 359 1399
[19] Bolokhov T A and Faddeev L D 2004 Theor. and Math. Phys. 139 679
[20] Novikov S P 1984 Uspekhi Mat. Nauk 39 97
[21] V.I. Arnold has suggested in 2004 to one (AP) of us to use the Novikov’s analytic generalized Hopf invariant [20] for description of the knots in the SU(3) case.
[22] A. Ranicki 1998 A High-dimensional Knot Theory. Algebraic Surgery in Codimension 2 (New York: Springer)
[23] Picken R F 1990 J. Math. Phys. 31 616
[24] Kondo K-I and Taira Y 2000 Prog. Theor. Phys. 104 1189
[25] Bernatska J and P. Holod P 2008 Geometry and topology of coadjoint orbits of semisimple Lie groups Preprint math.RT/0801.2913
[26] Borel A 1955 Bull. Am. Math. Soc. 67 397
[27] Boya L J, Perelomov A M and M. Santander M 2001 J. Math. Phys. 42 5130
[28] Mimura M and Toda H 1963 J. Math. Kyoto Univ. 3 217
[29] Deligne P, Griffiths Ph, Morgan J and Sullivan D, 1975 Invent. Math. 29 245
[30] Puettmann T and Rigas R 2003 Comment. Math. Helv. 78, 648
[31] Zubkov M A and Volovik G E 2012 Nucl. Phys. B 860 295
[32] Pontrjagin L S 1941 Mat. Sbornik (Recueil Mathematique N. S. ) 9 331
[33] Fidkowski L and Kitaev A The effects of interactions on the topological classification of free fermion systems Preprint cond-mat/0904.2197