NONLEPTONIC DECAYS OF CHARMED MESONS
INTO TWO PSEUDOSCALAR MESONS

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Abstract
We investigate the nonleptonic decay of charmed meson into two pseudoscalar mesons using the vector–dominance model, and compare the results with those obtained from the factorization model. In particular, we discuss the role of the annihilation diagrams in the two models.
Much of what we understand about nonleptonic decays of strange particles comes from current algebra and the soft pion technique. Unfortunately, this technique cannot be applied to the decay of heavy mesons carrying the $c$ or the $b$ quark, since the emitted pions are generally not soft. In this case a completely different method has been developed based on the assumption that the hadronic matrix element of currents factorizes$^1$.

Historically, the nonleptonic decays of strange particles have also been discussed in terms of a dynamical model$^2$ based on the idea of vector dominance. In particular, it is well–known that for the $K \to 2\pi$ decays this model provides a satisfactory description. It would be of interest to see how vector dominance fares in describing also the heavy meson decays. In this paper we deal with this question and, as an extension of $K \to 2\pi$, discuss the decay of $D$ and $D_S$ mesons into two pseudoscalars. Indeed such an analysis was attempted several years ago$^3$. The present work obviously benefits from the availability of better data. More importantly, however, we undertake a detailed comparison of the vector dominance model and the factorization model. In particular, we discuss the role of the so–called annihilation amplitudes in the two models.

For nonleptonic decay of charm, the effective weak Hamiltonian in the current–current form may be taken to be$^4$

$$H_W = \frac{G_F}{\sqrt{2}} [a_1(\bar{u}d')(\bar{s}c)_\mu + a_2(\bar{s}d')(\bar{u}c)_\mu]$$  \hspace{1cm} (1)

where $(\bar{q}_\beta q^\alpha)_\mu$ are color–singlet V–A currents

$$i\bar{q}_\beta \gamma_\mu (1 + \gamma_5) q^\alpha = (V_\mu)^\alpha_\beta + (A_\mu)^\alpha_\beta \hspace{1cm} (\alpha, \beta = 1, 2 \ldots 4)$$  \hspace{1cm} (2)

and $a_1$, $a_2$ are real coefficients which we treat as phenomenological parameters. The primed quark fields are related to the unprimed ones by the usual Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix. We shall ignore the Penguin–type contributions. For the nonleptonic $D$ and $D_S$ decays into two mesons, the Hamiltonian (1) leads to two main classes of quark–model diagrams, the spectator and the annihilation diagrams shown in Figs. 1(a)
and 1(b) respectively. It is well-known that the annihilation diagram contribution in the quark model is helicity suppressed.

The factorization model assumes that the matrix element for the decay \( D \to P_1 P_2 \) \((P_1 \text{ and } P_2 \text{ are light mesons})\) can be written in the factorized form

\[
< P_1(q_1)P_2(q_2)|H_W|D(q) > \\
\sim \frac{G_F}{\sqrt{2}} [ a_1 < P_2(q_2)|(\bar{u}d')_\mu|0 > < P_1(q_1)| (\bar{s}'c)_\mu|D(q) > \\
+ a_2 < P_1(q_1)|(\bar{s}'d')_\mu|0 > < P_2(q_2)| (\bar{u}c)_\mu|D(q) > ]
\]  

The two terms on the right can easily be seen to correspond to the quark-model spectator diagrams of Fig. 1(a). An important feature of the factorization approximation is that the annihilation diagram can be neglected\(^1\). To see this, note that the annihilation diagram corresponds to the factorization

\[
\frac{G_F}{\sqrt{2}} a_2 < 0|(\bar{u}c)_\mu|D(q) > < P_1(q_1)P_2(q_2)| (\bar{s}'d')_\mu|0 >
\]  

if the decaying particle is a neutral \( D^0 \). For the charge-carrying \( D^+ \) or \( D_S^+ \) a similar factorization can be written down involving the \( a_1 \) term in the Hamiltonian. In these factorized forms, the charmed meson is connected to vacuum by an appropriate current. From Lorentz invariance, this matrix element is proportional to \( q_\mu = (q_1 + q_2)_\mu \). Multiplied by the other matrix element in (4), we see that the annihilation contribution is then proportional to the matrix element of the divergence of the current \( (\bar{s}'d')_\mu \) formed from light quarks, and is thus proportional to the masses of the light quarks. This is the analogue of helicity suppression in the quark-model annihilation diagram. By comparison, the spectator contribution (3) is proportional to quark masses involving the heavy charmed quark.

There is another argument that shows an additional suppression of the annihilation contribution. To appreciate this, note that the structure of the matrix element

\[
< P_1(q_1)P_2(q_2)|(\bar{s}'d')_\mu|0 >
\]  
in (4) can be written from Lorentz invariance in terms of form-factors to be evaluated at the momentum transfer \((q_1 + q_2)^2 = q^2 = -m_D^2\). In
the standard pole-dominated form, the form-factor is then expected to lead to a further suppression of the annihilation amplitude. By contrast, again, the form-factors appearing in the spectator amplitude (3) are to be evaluated at the low momentum transfer $(q - q_1)^2 = q_2^2 = -m_2^2$ or $(q - q_2)^2 = q_1^2 = -m_1^2$, and thus need not be suppressed.

Consider now the vector-dominance model. Here we take the currents in $H_W$ to be the hadronic currents given by the field-current identities ($\alpha, \beta = 1, 2, \ldots 4$)

\begin{align*}
(V_\mu)_{\alpha\beta}^\alpha &= \sqrt{2} g_V (\phi_\mu)^\alpha_{\beta} \\
(A_\mu)_{\alpha\beta}^\alpha &= \sqrt{2} f_P \partial_\mu P^\alpha_{\beta}
\end{align*}

where $(\phi_\mu)^\alpha_{\beta}$ and $P^\alpha_{\beta}$ are the field operators of the vector and the pseudoscalar mesons respectively, and $g_V$ and $f_P$ are the corresponding decay constants. The nonleptonic weak interaction (1) can then be represented by a two-meson vertex. For the parity violating decay $D \to P_1 P_2$, the Cabibbo allowed and once-suppressed weak vertices are listed in Figs. 2(a) and 2(b) respectively.

The Feynman diagrams for the decay $D \to P_1 P_2$ are depicted in Fig. 3. It is easily seen that the diagrams in Fig. 3(a) and 3(b) are respectively the analogues of the spectator and the annihilation diagrams of the quark model. The annihilation amplitude in Fig. 3(b) is proportional to

\begin{equation}
A_a \propto q_\mu \frac{\delta_{\mu\nu} + q_\mu q_\nu / m_V^2}{q^2 + m_V^2} (q_1 - q_2)_\nu
\end{equation}

In the pole term, if we set $q^2 = -m_D^2$, at first sight it seems to lead to a suppression in the annihilation amplitude. However it is trivial to see that the pole actually cancels, and one obtains

\begin{equation}
A_a \propto \frac{m_2^2 - m_1^2}{m_V^2}
\end{equation}

The annihilation amplitude does depend on the masses $m_1, m_2$ of the light mesons. Again, this is the analogue of the helicity suppression in the quark model. However, it is easy to see that $V$ is a light vector meson in this case, so the annihilation amplitude is actually proportional to the ratio of light meson mass squares. By contrast, for the spectator
diagram of Fig. 3(a), the amplitude is proportional to

$$A_s \propto (q + q_1)_\mu \frac{\delta_{\mu\nu} + q_2\mu q_2\nu/m_V^2}{q_2^2 + m_V^2} q_2\nu = \frac{m_1^2 - m_D^2}{m_V^2}$$

This time since $V$ is a charmed vector meson $D^*$ or $D_S^*$, we find that the spectator contribution while containing a piece proportional to the charmed meson mass term $m_D^2$, is actually determined by the ratio of heavy meson mass squares. Thus, there is no apriori reason why $A_a$ and $A_s$ may not be of the same order of magnitude, and we are not justified in neglecting the annihilation amplitude.

We now use the vector dominance model to compute the decays $D, D_S \to P_1 P_2$, taking into account contributions from both the Feynman diagrams in Fig. 3. We follow Bauer et al.\(^1\) (BSW) and first determine the parameters $a_1, a_2$ by confronting our model to the data on $D \to K\pi$ decays. As in BSW, we have to take the final state interaction into account, and we do this by considering only elastic scattering in the final $K\pi$ state. In terms of the isospin amplitudes, we have

$$A(D^o \to K^-\pi^+) = \frac{1}{\sqrt{3}} A_{3/2} + \sqrt{\frac{2}{3}} A_{1/2}$$

$$A(D^o \to \bar{K}^0\pi^o) = \sqrt{\frac{2}{3}} A_{3/2} - \frac{1}{\sqrt{3}} A_{1/2}$$

$$A(D^+ \to \bar{K}^0\pi^+) = \sqrt{3} A_{3/2}$$

where

$$A_I = |A_I| e^{i\delta_I}$$

is the amplitude in the isospin state $I$ and $\delta_I$ is the phase shift in that channel. Using the data from the recent particle properties data\(^5\) booklet, we obtain

$$|A_{1/2}| = 2.94 \times 10^{-6} \text{ GeV}$$

$$|A_{3/2}| = 7.37 \times 10^{-7} \text{ GeV}$$

$$\delta_{1/2} - \delta_{3/2} = 93.4^\circ$$

We now use the vector dominance model to compute these amplitudes. Since we do not know all the strong interaction coupling constants, we use flavor SU(4) symmetry to
relate these to $g_{\rho\pi\pi}$, which is known from the $\rho \to 2\pi$ decay ($g_{\rho\pi\pi} \simeq 4.0$). The $D \to K\pi$ amplitudes can then be written down from the Feynman diagrams of Fig. 3 to be

$$A(D^o \to K^-\pi^+) = \gamma \left[ -a_2 g_{K^*} f_D \frac{m_\pi^2 - m_K^2}{m_{K^*}^2} + a_1 g_{D^*_S} f_{\pi} \frac{m_K^2 - m_{D^*}^2}{m_{D^*_S}^2} \right]$$

$$A(D^o \to \bar{K}^o\pi^0) = \frac{\gamma}{\sqrt{2}} \left[ a_2 g_{K^*} f_D \frac{m_\pi^2 - m_K^2}{m_{K^*}^2} + a_2 g_{D^*} f_K \frac{m_\pi^2 - m_{D^*}^2}{m_{D^*}^2} \right]$$

$$A(D^+ \to K^o\pi^+) = \gamma \left[ a_2 g_{D^*} f_K \frac{m_\pi^2 - m_{D^*}^2}{m_{D^*}^2} + a_1 g_{D^*_S} f_{\pi} \frac{m_K^2 - m_{D^*}^2}{m_{D^*_S}^2} \right]$$

where

$$\gamma = i \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} 2g_{\rho\pi\pi}$$

The $V$'s are the CKM matrix elements and the couplings $g_V$ and $f_P$ in (12) are defined in (5). For the vector meson decay constant $g_V$, we use the spectral function sum–rule based on asymptotic SU(4), which predicts identical values of $g_V/m_V$ for $V = \rho, K^*, D^*$, and $D^*_S$. For the $\rho$–meson, we extract from $\rho \to \ell\bar{\ell}$ decay, $g_{\rho}/m_{\rho} \simeq 0.15$ GeV. For the pseudoscalar meson decay constant $f_P$, we take $f_{\pi} = 0.093$ GeV, $f_K \simeq 1.2 f_{\pi} = 0.112$ GeV and choose $f_D \simeq f_{D_S} \simeq 0.136$ GeV as determined from the sum–rule technique in quantum chromodynamics. The isospin amplitudes in the vector dominance model can then be computed to be

$$|A_{1/2}| = 1.07 \left|(a_1 - 1.03a_2)\right| \times 10^{-6} \text{ GeV}$$

$$|A_{3/2}| = 7.58 \left|(a_1 + 1.34a_2)\right| \times 10^{-7} \text{ GeV}$$

Using the values (11) for these amplitudes obtained from the data, we find two solutions (only the relative sign of $a_1$ and $a_2$ is important)

I. $a_1 = 1.98 \quad , \quad a_2 = -0.75$  \hspace{1cm} (15)

II. $a_1 = 1.13 \quad , \quad a_2 = -1.57$  \hspace{1cm} (16)

It should be emphasized that the annihilation contribution has been included in this analysis. We find however that in the $D \to K\pi$ decays, this contribution is numerically small compared with the spectator contribution.
Isospin analyses similar to the one above can also be performed for the Cabibbo suppressed decay modes $D \to K\overline{K}$ and $D \to \pi\pi$. In the $D \to \pi\pi$ decays, only $D^0 \to \pi^+\pi^-$ has a measured branching ratio at present. It is not hard to see that this experimental value is already sufficient to rule out the solution II. Also the surviving solution I is consistent with the data with negligible final state interaction. In the $D \to K\overline{K}$ decays, on the other hand, we find that the data require sizable final state interaction. In this case, while neither of the solutions can be ruled out, we find that I satisfies the data better. We do not present the details of these analyses here, but accept the conclusion that the present data favors the solution I over II.

It is of interest to compare our solution to the one obtained by BSW in the factorization approach. Using the values (11) of the isospin amplitudes obtained from the recent $D \to K\pi$ data, we find that the BSW solution is given by

$$a_1 = 1.09 \ , \ a_2 = -0.48$$

(17)

Our individual values of $a_1$ and $a_2$ in solution I are somewhat larger, although the ratio is not very different from the one in (17).

We have calculated the various allowed and suppressed decays of the type $D, D_S \to P_1P_2$, and have listed the results of the vector dominance model in Table 1. Also listed in the table are the results from the factorization model, and the experimental data, where available.

In conclusion, we would like to emphasize that the vector dominance model affords a natural way of taking into account the annihilation contribution. While this contribution is small for the $D, D_S \to PP$ decays, one would expect it to be larger in decays involving the heavier vector mesons as in $D, D_S \to PV$ and $D, D_S \to VV$. In these cases, the vector dominance model has to be extended to include pseudoscalar and other meson poles. This work will be reported elsewhere.

This work was supported in part by the U.S. Department of Energy Grant No. DE–FG–02–91ER40685.
References and Footnotes

1. M. Bauer, B. Stech and M. Wirbel, Z. Phys. C – Particles and Fields 34, 103 (1987).
2. J. J. Sakurai, Phys. Rev. 156, 1508 (1967).
   G. S. Guralnik, V. S. Mathur and L. K. Pandit, Phys. Rev. 168, 1866 (1968).
3. V. S. Mathur, Proceedings of a Conference on Quark Confinement and Field Theory, Rochester, NY 1976, edited by D. R. Stump and D. H. Weingarten (published by John Wiley & Sons).
4. K. Jagannathan and V. S. Mathur, Nucl. Phys. B171, 78 (1980) and ref. 1.
5. Review of Particle Properties, Phys. Rev. D45, Part 2 (June 1992). We have ignored the errors quoted in the data for our analysis here.
6. S. Weinberg, Phys. Rev. Lett. 18, 507 (1967).
   T. Das, V. S. Mathur and S. Okubo, Phys. Rev. Lett. 18, 761 (1967) and ref. 3.
7. V. S. Mathur and M. T. Yamawaki, Phys. Rev. D29, 2057 (1984).
8. BSW have used slightly different values of the couplings $f_P$ and $g_V$ than those used by us.
| Branching Ratio | Vector–dominance Model | Factorization Model¹ | Experiment⁵ |
|-----------------|------------------------|----------------------|-------------|
| BR($D^o \rightarrow K^- \pi^+$) | 4.9% | 5.0% | (3.65 ± 0.21)% |
| BR($D^o \rightarrow \bar{K}^- \pi^0$) | 0.8% | 0.7% | (2.1 ± 0.5)% |
| BR($D^+ \rightarrow \bar{K}^- \pi^+$) | 2.7% | 2.7% | (2.6 ± 0.4)% |
| BR($D^+_s \rightarrow \bar{K}^- K^+$) | 1.2% | 1.3% | (2.8 ± 0.7)% |
| BR($D^+_s \rightarrow \eta \pi^+$) | 3.7% | 2.6% | (1.5 ± 0.4)% |
| BR($D^o \rightarrow \bar{K}^- \eta$) | 0.1% | 0.3% | <2.3% |
| BR($D^o \rightarrow K^+ K^-$) | $2.9 \times 10^{-3}$ | $3.7 \times 10^{-3}$ | $(4.1 \pm 0.4) \times 10^{-3}$ |
| BR($D^o \rightarrow K^o \bar{K}^o$) | 0 | 0 | $(1.1 \pm 0.4) \times 10^{-3}$ |
| BR($D^+ \rightarrow K^+ \bar{K}^o$) | $7.4 \times 10^{-3}$ | $9.6 \times 10^{-3}$ | $(7.3 \pm 1.8) \times 10^{-3}$ |
| BR($D^o \rightarrow \pi^+ \pi^-$) | $2.9 \times 10^{-3}$ | $2.6 \times 10^{-3}$ | $(1.63 \pm 0.19) \times 10^{-3}$ |
| BR($D^o \rightarrow \pi^o \pi^0$) | $0.8 \times 10^{-4}$ | $2.5 \times 10^{-4}$ | <$4.6 \times 10^{-3}$ |
| BR($D^+ \rightarrow \pi^+ \pi^0$) | $1.4 \times 10^{-3}$ | $1.1 \times 10^{-3}$ | <$5.3 \times 10^{-3}$ |
| BR($D^+_s \rightarrow K^o \pi^+$) | $4.6 \times 10^{-3}$ | $2.3 \times 10^{-3}$ | <$6 \times 10^{-3}$ |
| BR($D^+_s \rightarrow K^+ \pi^o$) | $5.4 \times 10^{-4}$ | $2.3 \times 10^{-4}$ | |
**Figure and Table Captions**

Fig. 1 Quark–model diagrams for the nonleptonic decay of $D$ and $D_S$ into two mesons. 1(a) describe the spectator diagrams and 1(b) the annihilation diagrams.

Fig. 2 Parity–violating weak vertices in vector–dominance model. 2(a) describe the Cabibbo–allowed vertices and 2(b) the once–suppressed ones.

Fig. 3 Feynman diagrams for the decay $D, D_S \rightarrow P_1 P_2$ in vector–dominance model. The weak vertex is represented by a black dot and the strong vertex by an open circle.

Table 1 Branching ratios of the Cabibbo–allowed and once–suppressed decays of the type $D, D_S \rightarrow P_1 P_2$. The calculated values in the table ignore the final state interactions. For the vector–dominance model, we have used the values of $a_1$ and $a_2$ given in solution I. For the factorization model, we have used the BSW values given in (17).