Scaling Law and Irreversibility Fields in High Temperature Superconductors

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Abstract. Theoretical analysis of the irreversibility fields $B_{irr}$ was obtained from the flux creep theory based on a depinning mechanism caused by thermally activated flux creep. Near the critical temperature $T_c$, the calculated $B_{irr}$ depends on power of $(1 - (T/T_c)^2)$. The measured $B_{irr}$, however, increases more rapidly than the power law at low temperature. This enhancement from the power law in low temperatures and high fields has been ascribed to a different pinning mechanism. The experimental temperature dependence of $B_{irr}$ is obtained from the AC susceptibility measurements in Hg-based composites of Ag$_x$(HgBa$_{1.9}$Bi$_{0.1}$Ca$_2$Cu$_3$O$_{8+\delta}$)$_{1-x}$ ($x = 0.1$ and $0.3$) superconductors. AC and DC magnetizations were measured using a SQUID magnetometer and a PPMS susceptometer at temperature range 5-150 K under magnetic fields up to 14 T. The irreversibility fields $B_{irr}$ are estimated from the peaks of imaginary parts of AC susceptibilities and shown to agree well with the numerical estimation of the original equation. The magnetization characteristics are also successfully obtained from the pinning parameters.

1. Introduction
Superconductors cannot carry nonresistive transport current outside the irreversibility line. A high irreversibility field $B_{irr}$ is a necessary condition for transporting large current densities in high magnetic fields. Many research works on the irreversibility field $B_{irr}$ and the temperature $T_{irr}$ are performed in high-$T_c$ superconductors [1, 2].
Matsushita introduced an expression to describe the irreversibility field $B_{irr}$ based on the thermally activated fluxoids (flux creep) model \cite{3} as
\[
B_{irr} \approx \left( \frac{K}{T} \right)^2 \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]^{\gamma \delta - \gamma} \left( 1 - \frac{B_{irr}}{B_{c2}} \right)^d,
\]
where $K$ is approximately constant determined by the voltage criterion of the irreversibility. Parameters $m$, $n$, and $p$ are introduced by the assumed temperature and magnetic field dependence in the scaling law of creep-free critical current density $J_c$:
\[
J_c = J_0 \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]^{\gamma \delta - \gamma} \left( 1 - \frac{B_{irr}}{B_{c2}} \right)^d.
\]
The empirical temperature dependence of the upper critical field: $B_{c2}(T)$ is given by
\[
B_{c2}(T) = B_{c2}(0)(1 - (T/T_c)^2).
\]
In high-$T_c$ superconductors $B_{irr}$ is small compared to $B_{c2}$ and Eq. (1) reduces to
\[
B_{irr}(T) = B_{irr}(0) \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]^n,
\]
where $B_{irr}(0) = (K/T)^{\gamma}$, and index $n$ and $p$ are
\[
n = \frac{2(m - \gamma)}{3 - 2\gamma}, \quad p = \frac{4}{3 - 2\gamma}.
\]
At high temperatures of $T \sim T_c$, Eq. (3) is further transformed to
\[
B_{irr}(T) = B_{irr}(0) \left[ 1 - \left( \frac{T}{T_c} \right)^{2\gamma} \right]^n.
\]
This expression is widely used to discuss the irreversibility line and the gradient of the logarithmic plot of $B_{irr}$ and $(1-(T/T_c)^2)$ gives the index $n$.

2. Theoretical curves
The temperature characteristic of $B_{irr}$ is plotted in Fig. 1 as a function of $(1-(T/T_c)^2)$ for three cases of (i) high $T$ ($\sim T_c$) with Eq. (5) as dash lines, (ii) low $B$ ($<< B_{c2}$) with Eq. (3) as dotted lines and (iii) general cases with Eq. (1) as solid lines, where in Fig.1(a) the ratio between the irreversibility field $B_{irr}(0)$ and the upper critical field $B_{c2}(0)$ is varied as 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1. All lines agree well in the range of $(1-(T/T_c)^2) \leq 0.1$ (i.e. $T/T_c \geq 0.95$) and deviate significantly below $T/T_c \leq 0.9$. When $B_{irr}(0)$ is sufficiently larger than $B_{c2}(0)$, Eq. (3) gives the same results by Eq. (1). It was reported that temperature dependence of $B_{irr}$ of high-$T_c$ superconductors is well described using Eq. (3) at low temperature range down to $T/T_c = 0.2$ \cite{4}. However, $B_{irr}$ computed with Eq. (3) diverges as $T$ tends to 0. Numerically estimated values of Eq. (1) deviate from the results of Eq. (3) near $1-(T/T_c)^2 = 0.4$ and converge to $B_{irr}(0)/B_{c2}(0)$ at $T = 0$. Although the parameter $m$ shifts the curve entirely as shown
Figure 1. Theoretical irreversibility field $B_{irr}(T)$ as a function of $1-(T/T_c)^2$. The dotted lines for the low $B$ case of (ii) show the computed values of Eq. (3) and the solid lines for the general (iii) indicate the numerically estimated results of Eq. (1), which converge to $B_{irr}(0)/B_{c2}(0)$ at $T = 0$. In (a) $B_{irr}(0)/B_{c2}(0)$ is varied between 0.1 and 1, and in (b), parameter $m = 2-8$.

in Fig. 1(b), the parameter gives substantial effect only at low temperatures as in Fig. 2(a). By increasing the parameter as 0, 1, 2, 3, 4 and 5, the computed curves begin to decrease from that of Eq. (3) and all curves converge to $B_{irr}(0)/B_{c2}(0) = 5$ at $T = 0$, as displayed in Fig. 2(b). The wide availability of Eq. (1) for the low temperature MgB$_2$ superconductor was reported in the preceding paper [5].

Figure 2. Theoretical irreversibility field $B_{irr}(T)$ as a function of $1-(T/T_c)^2$. The dotted lines for the case of (ii) show the computed values of Eq. (3) and the solid lines for (iii) indicate the numerically
estimated results of Eq. (1), which converge to $B_{irr}(0)/B_{c2}(0) = 5$ at $T = 0$. Parameter $\delta$ is $1 \sim 5$ and the enlarged characteristics at $T = 0$ is shown in (b).

3. Results and discussion
The irreversibility field $B_{irr}$ of Hg-based composites of $\text{Ag}(\text{HgBa}_{1+x}\text{Bi}_{0.1}\text{Ca}_{2}\text{Cu}_{3}\text{O}_{8+\delta})_{1-x} (x = 0.1: \text{Ag-01 and 0.3: Ag-03})$ superconductors was reported [6, 7], where $B_{irr}$ is estimated from the peaks of the imaginary part $''$ of the AC susceptibilities and shown as solid circles in Fig. 3 as a function of $(1-(T/T_c)^2)$. The calculated curve of the case (i) is plotted with the dash line, where $T_c = 131$ K, $m = 2.82$, $\gamma = 0.28$, $\delta = 2$ and $B_{irr}(0)/B_{c2}(0) = 4.6 \times 10^{-3}$. At low temperatures, $B_{irr}$ increases more rapidly and has been attributed so far to the result of different pinning sites. The dotted line shows the computed result of case (ii) by Eq. (3) which diverges when $T$ approaches to 0. In high temperature superconductor the effect of $\delta$ up to $14$ T is covered by a large magnitude of $B_{c2}(0)$ and no difference is recognized between the cases of (ii) and (iii). The parameters for the sample Ag-03 are the same and shows similar characteristics as Ag-01.

Figure 3. Temperature dependence of the irreversibility field $B_{irr}$ in Ag-01 estimated from the peaks of the imaginary part $''$ of the AC susceptibilities.
The magnetization width $\Delta M$ proportional to the critical current density is calculated using the scaling law of Eq. (2) with the above-estimated parameters and a pinning penetration field $B_p(=\mu_0 dJ_c$, $d$ is sample width). Critical current density of Eq. (2) is that of the creep-free case and is difficult to estimate experimentally. Since the pinning energy well is substantially reduced by the thermally activated energy, the upper critical field $B_{c1}$ is replaced by the irreversibility field $B_{irr}$ and $\Delta M$ are plotted in Fig. 4. The theoretical results of Eq. (2) agree well with the measured magnetization width by adjusting the parameter whose values are difficult to evaluate in the $B_{irr}(T)$ measurement. The calculated values at $T = 0$ and $T = 77$ K plotted as the dotted line Fig. 4(a) show a large deviation from the observed data. The parameter is shown to be effective at low temperatures and high fields. Small $B_p$ in Ag-03 indicates the small superconducting volume fraction due to an excess addition of Ag [7].

4. Conclusions

Availability of the irreversibility field $B_{irr}$ based on flux creep was discussed. In high temperature superconductor $\text{Ag}(\text{HgBa}_{1.9}\text{Bi}_{0.1}\text{Ca}_2\text{Cu}_3\text{O}_{8+\delta})_{1-x}$, Eq. (3) is applicable up to the applied magnetic field of 14 T in the case of low magnetic field.

Figure 4. Dependence of the magnetization width $\Delta M$ of (a) Ag-01 and (b) Ag-03 on magnetic field at $T = 20$-80 K. Dotted line in (a) shows the calculated curves at $T = 0$ and $T = 77$ K.
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