FROM CARBON-TRANSITION PREMIUM TO CARBON-TRANSITION RISK

SURYADEEPTO NAG *  SIDDHARTHA P. CHAKRABARTY †  SANKARSHAN BASU ‡

Abstract

Investor awareness about impending regulations requiring firms to reduce their carbon footprint has introduced a carbon transition risk premium in the stocks of firms. On performing a cross-section analysis, a significant premium was estimated among large caps in the US markets. The existence of a risk premium indicates investor awareness about future exposure to low-carbon transition. A new measure, the Single Event Transition Risk (SETR), was developed to model the maximum exposure of a firm to carbon transition risk, and a functional form for the same was determined, in terms of risk premia. Different classes of distributions for arrival processes of transition events were considered and the respective SETRs were determined and studied. The trade-off between higher premia and higher risks was studied for the different processes, and it was observed that, based on the distributions of arrival times, investors could have a lower, equal or higher probability of positive returns (from the premium-risk trade-off), and that despite a fair pricing of the carbon premium, decisions by investors to take long or short positions on a stock could still be biased.

Keywords: Carbon transition; Climate risk; Carbon premium; Transition risk; Ergodicity economics

1 INTRODUCTION

A continuous increase in the global carbon footprint, along with a concurrent impact of climate change, has intensified the focus, as well as concerns, about the possible consequent legislative and regulatory changes, that would potentially make it incumbent upon firms, to transition away from the current levels of their carbon footprint. Accordingly, such a change is likely to result in a fundamental paradigm shift, in case of firms, bearing high carbon footprint, triggering an element of considerable risk for investors in such firms.

The narrative on financial risk, contingent on climate-related factors, and its consequent management, greatly relies upon prudent measurement, as well as transparent disclosure of such risk exposures, on the part of the firms [4]. Pursuant to this, the final report of the Task Force on Climate Related Financial Disclosures (TCFD) was released in 2017, to motivate climate-related disclosures (through setups such as Carbon Disclosure Project (CDP), Climate Disclosure Standards Board (CDSB) and Sustainability Accounting Standards Board (SASB)) and bring about a greater clarity about the financial exposure, resulting from such risks. With compulsory TCFD disclosures being mandated only in a handful of countries, concerns about insufficiency in reporting have been repeatedly expressed by TCFD. In absence of a regulatory framework being in place, the collation of information had to be accomplished through tedious manual exercise, or more recently through the usage of Artificial Intelligence (AI). This motivated the development of ClimateBERT, a context-based algorithm, which extracts information from the TCFD reports. The results reported in [4] show a pattern of selective reporting in TCFD, primarily on non-material information on climate risk, prompting the recommendation that the practice of voluntary reporting be

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*Indian Institute of Science Education and Research Pune, Pune-411008, Maharashtra, India, e-mail: suryadeepto.nag@students.iiserpune.ac.in
†Department of Mathematics and Mehta Family School of Data Science and Artificial Intelligence, Indian Institute of Technology Guwahati, Guwahati-781039, Assam, India, e-mail: pratim@iitg.ac.in, Phone: +91-361-2582606, Fax: +91-361-2582649
‡Department of Finance and Accounting, Indian Institute of Management Bangalore, Bengaluru-560076, Karnataka, India, e-mail: sankarshan.basu@iimb.ac.in
replaced with a formal regulatory framework. Bolton and Kacperczyk [10] examined the impact of disclosures (both voluntary and mandatory) of carbon emissions by a firm, on their key parameters such as returns, risk and turnover. The article relies on the tractability in quantification of greenhouse gas (GHG) emissions in general and CO\(_2\) emissions in particular. The CO\(_2\) emission measurement is carried out through a well-formulated protocol and can be estimated, even in absence of the protocol. Further, the mandatory disclosure requirements in a few countries (such as the UK) enables to make a distinction between the impact of voluntary disclosure, and those disclosures that are mandated. It was observed that firms making voluntary disclosures of Scope 1 emissions, offer lower returns on investments, as compared to firms not disclosing, along with a concurrent and enhanced divestment by institutional investors, resulting from such voluntary disclosures. In case of UK, however, the requirement of mandatory disclosures, did result in lower volatility in the stock valuation. In addition, the practice of mandating disclosures did impact other markets within the sector and those in the geo-economic vicinity.

In [8], Bolton and Kacperczyk, present the conclusions from an exhaustive and global empirical analysis to estimate the risk premium, resulting from a global transition from fossil fuel to renewable sources of energy. The results of this extensive study, led to a narrative from two perspectives, namely, the overall transition risk, and the specific risk components. It was observed, that higher carbon footprint, resulted in higher returns for investors, independent of sectors as well as geographical location of the firms. Further, the transition risk was short-term and long-term, in case of countries more dependent on traditional fossil fuel and countries with stricter regulations, respectively. In another article [9], the same authors carry out an extensive study of the impact of carbon emission across a cross-section of stocks in the US market. One key factor that is typically not included in cross-sectional patterns is the carbon emission incurred by the firms. This, however, needs to be addressed in the context of its impact on the cross-section of returns on stock holdings of firms. An interesting question that needs to be examined is whether carbon emission constitutes a systematic risk factor. The analysis carried out in the paper, determined that investors of firms with higher carbon footprints, end up getting higher returns. This, however, cannot be merely attributed to either unexpected sources of profit or some other inherent risk factors, which leads to an obvious likely explanation that investors in such firms are increasingly demanding greater returns, as a manner of compensation for their exposure to these firms with high carbon emission levels. The impact of regulatory framework for carbon emission on the performance of stocks of firms in China is quantitatively analyzed in [36]. The key interesting conclusion of the work was that the establishment of a trading market for carbon emission, in China, had the consequence of achieving excess returns in case of participating firms. Further, the carbon premium has seen gains post-establishment of the carbon emission trading market. Finally, the positive coefficient of the carbon risk factor can be attributed to the greater participation by firms with higher carbon exposures, in the carbon market.

In a 2019 working paper, Delis et. al. [12] set forth to address the question pertaining to the risk quantification of firms holding on to fossil fuel reserves. Accordingly, the data was manually collated on a global basis for fossil fuel reserve held by firms and the rate for credit charged to these firms, which was then compared with the rates of syndicated loans. Eventually in this paradigm (along with exposure resulting from climate policies) a comparative analysis was executed between such firms and non-fossil fuel firms. The empirical analysis led to the identification of 2015 as the point in time, wherein banks initiated the pricing of exposure resulting from climatic policy of the firms. More specifically, post-2015, there was an observed increase of 1.6 percent in the cost of credit for fossil fuel firms. In a related observation, the authors determined that there was credence to the belief that the so-called “green-banks” tend to extend marginally higher credit rates to credit seekers from the fossil fuel firms. The interdependent issues of carbon risk, its awareness and impact on credit cost is addressed in [16]. The authors investigated whether creditor’s decision on lending rates takes into account the carbon risk exposures of the firm seeking credit. Further, they also analyzed whether the firms can offset the additional cost of borrowing through the demonstration of awareness for their own carbon risks. The key observation of the
empirical analysis was that there existed a positive inter-dependency between the cost of credit and carbon risk, in case of the firm’s failure to act on CDP survey. In numerical terms, this is translated to one standard deviation increase in carbon risk being mapped to a range of 3.8 percent to 6.2 percent increase in cost of securing credit. On the same note, this penalty in terms of cost of credit is offset in case of firms being aware of their carbon risk, thereby highlighting the importance of carbon awareness on the part of the firms, as well as creditors to such firms. The valuation of carbon risk in case of electric utilities in Europe is studied in [18]. Accordingly, a group of stocks from 20 European electric utilities firms for the period of 2005-2010, were studied to ascertain the impact of carbon risk on firm-specific cost of credit. It was demonstrated that for most of the firms (mainly low emission) the carbon risk exposure was not significant, and hence they do not really benefit in terms of reduced cost of credit. However, a handful of firms (with high-emitting fuel mix) do exhibit risk exposure, resulting in higher cost of credit, as well as erosion in the value of equity. In addition to the role of fuel mix, two other factors, namely, permit allocation process and replacement cost, plays a direct role in reduction of exposure for the firm’s strategies, adopted towards reduction of carbon exposure, leading to arrest in the erosion of equity values. An analysis of the impact of climate risk on a portfolio of credit extended by Chinese banks to energy infrastructure is presented in [25]. The authors note the growing recognition within development banks to consider the impact of climate action on the performance of projects funded by them, and the economic and financial constraints set forth by fossil-fuel based energy firms towards transition to a low-carbon economy. The authors developed an approach for stress testing of a pool of loans extended to firms involved in energy infrastructure projects, and implemented it in case of overseas loans extended by policy banks in China. It was estimated that adverse impact will mostly emanate from projects in coal and oil sectors in the range of 4.2 percent to 22 percent of the total loan principals, which when accounted for in the paradigm of the large leveraged position of policy banks in China, could potentially have serious macroeconomic implications. Aspects of expedited de-carbonation of economies driven by the necessity of mitigating climate risk are discussed in [32]. Such a logistically massive structural transition can be achieved primarily through rapid reduction in the proportional share of both production, as well as consumption of fossil-fuel industries. This is likely to have an immediate impact in terms of credit defaults and bubbles in what the authors term as “sunset” and “sunrise” industries, respectively, a concept that is presented in detail by the authors. Finally, a theoretical setup is developed to encompass and analyze the consequences of phasing out firms with high carbon risk, in the economic and financial setup, and on policy measures that can mitigate the implications of this transition. The necessity of hedging against carbon risk, thereby safeguarding the interests of both customers as well as financial stakeholders, from consequences of carbon emissions, are discussed in detail in [7]. The authors, recognize that any investment in fossil-fuel industries today will continue to pose carbon risk for the succeeding three to four decades. Accordingly, such utility firms are increasingly evaluating and incorporating future cost of carbon emission, so as to minimize the cost of resource input. Bearing in mind that greater regulations are likely to be introduced during the lifespan of fresh investments in this sector, the utility firms are already putting in place (both voluntarily and mandatorily) long term strategies to deal with the cost resulting from such regulatory mechanisms being put in place. One aspect of this is the experience driven strategy adopted by utility firms so as to decouple investments made today from the future costs to customers and shareholders alike.

Andersson et. al. in their article on hedging against climate risk [2], presented a basic dynamic model for investment strategy, designed for passive investors (such as large institutional investors, pension funds, banks and sovereign funds) to hedge their investments against climate risk, while retaining acceptable return levels. For this purpose, a benchmark index and low-carbon index were chosen, with the carbon footprint for the latter being 50 percent less than the former. Further, this low-carbon index was obtained by reduction in holdings or exclusion of certain stocks having a high carbon footprint. The resulting “carbon-reduced” or “de-carbonized” portfolio, was demonstrated to exhibit very low (to the extent of being considered eliminated) tracking error.
vis-a-vis the benchmark index, which amounts to what the authors call as a “free option on carbon”. The main appeal of this de-carbonized portfolio is that, until the regulatory changes are introduced, it will offer a return similar to that of the benchmark, and whenever the changes come into effect, it will start outperforming the benchmark. The disinvestment from investment in carbon bubble and its limitations from the perspective of institutional investors is detailed in [28]. With the policy driven reduction in consumption of fossil fuel, it is inevitable that the energy firms will need to undergo fundamental structural changes to comply with the new regulatory regimen. An important consequence of this is the uncertainty in the values of securities being held by investors in fossil-fuel driven industry. This risk termed as “Carbon Bubble” (as a direct consequence of stranded/unused fossil-fuel resulting from new regulatory regimen) has intensified the push for disinvestment of holdings in fossil-fuel firms, by institutional investors. The authors examined the entire issue from the perspective of disinvestment, in conjunction with Carbon Bubble risk. In conclusion, the authors opined that the extent to which an institutional investor can divest their holdings in carbon risk firms, is somewhat limited, resulting in them still holding on to a significant level of carbon risk linked funds. A mathematical framework that encompasses Environmental, Social and Governance (ESG) criterion for sustainable investment is discussed in [26]. The model looks at the investment perspective under two scenarios, namely, equilibrium and stock impact to ESG factors. While in case of the former, the investors have come to expect lower returns, which also acts as a hedge against climate risk, in case of the latter, there is a shift in terms of customer preferences moving towards greener products and consequent investor shift towards green investments. The comparative analysis of the difference between ESG and non-ESG portfolios are presented in detail. The four key conclusions include, shift of equity prices resulting from ESG, three-fund separation in case of such portfolios, more ESG investments result in more dispersion from market portfolio, and the sustainable investment driven social impact.

In an earlier policy paper [29], the authors deliberated on the key question of the timing of transition to a low-carbon economy. The authors opined that the pace of the required transition has been extremely slow, and they attribute this to a variety of factors, such as the belief that the change is expensive and overrated, the attitude of stakeholder participation and active interest group at play. However, the authors make a positive note about the progress made in terms of a growing awareness about the extent of risks, and recognition that crucial transition to a low-carbon economy is feasible, even in case of the heavily-invested energy sector. Finally, the authors emphasize that a level playing field in terms of accessing sustainable development, is an imperative prerequisite to the dynamic partnership between developed and developing economies, in achieving the goals of carbon-risk reduction. The preceding narrative, highlights the necessity of building a carbon neutral portfolio that will minimize the reduction in returns, until the new regulations are introduced and the valuation of these firms fall.

Building on the existing literature, in this paper, we first test for the existence of a carbon transition risk premium for S&P500 firms. We then, develop a risk measure, the Single Event Transition Risk (SETR), in order to gauge the maximum exposure of the share of a firm to carbon transition at any given time, and then formulate a mathematical relationship between the carbon transition risk premia and the SETR. We analyze the SETR functions for different arrival processes of the carbon transition, and evaluate if it is beneficial for investors to hold a short or long position in the firms on a probabilistic basis.

2 Data

To study the presence of a carbon risk premium in stock returns, we consider data from S&P500 companies. For data on carbon emissions we rely on publicly available disclosures made by firms to external organizations or those declared on their sustainability reports. For company financials and stock prices data, we use the publicly available data on Yahoo Finance. The data on carbon emissions was compiled manually and the financial data (Stock prices, Income Statements and Balance Sheets) was collected for firms which had carbon emissions data available for the period of 2015-2020. Eventually, the emissions data was found for 208 firms. Consequently, the
financial data was found for these firms and those firms which had both emissions data and data for all controls used in regression (after removing the outliers) available, were eventually considered. This resulted in a dataset of 197 firms being finally assembled and considered. We use this data to first investigate the existence of a “carbon risk premium” in the American market, and then subsequently quantify the price of the carbon transition risk on the prices of shares of firms at the time of transition, and then ask how this drives investor decisions about investment in the firm.

The dataset for firm-wise GHG emissions was constructed in accordance with the Greenhouse Gas Protocol (GHGP), a partnership between the World Research Institute (WRI) and the World Business Council for Sustainable Development (WBCSD), which sets the standards for corporate carbon accounting. According to GHGP standards, carbon emissions are classified into Scope 1, Scope 2 and Scope 3 emissions. Scope 1 emissions account for direct emissions by the firm. These are all emissions that happen from sources that are either owned by the company or are in its control, including all emissions from fuel combustion. Scope 2 emissions account for indirect emissions arising from purchased electricity, heat, and steam. These are emissions that are caused due to the company’s use of resources but the emissions are produced at other sources. Scope 3 emissions include other indirect emissions such as emissions from purchased products or materials, waste disposal, transport in vehicles not owned by the firm, and other outsourced activities. In our study, however, we look at only Scope 1 and Scope 2 emissions, since Scope 3 emissions are declared less often, and using only those firms which have all three Scopes listed would reduce the size of our dataset. Therefore we use the aggregate of Scope 1 and Scope 2 emissions and consider it to be the total emissions of the firm. The entire data has been manually collected from publicly available sources, from companies who publish their response to CDP’s questionnaire under corporate social responsibility or from the annual ESG report of the firms. The data for emissions accounts for emissions from carbon-dioxide and other greenhouse gases and are reported by firms in carbon-dioxide equivalents of units of mass (most commonly metric tons (MT) CO$_2$-eq). In our dataset, we use MT CO$_2$-eq to measure the firm-wise emissions. The average annual Scope 1 emissions from a firm (of 208 firms) is 5.23 million metric tons of CO$_2$-eq with a standard deviation of 14.82 million metric tons of CO$_2$-eq. For Scope 2, the annual average stands at 1.25 million metric tons of CO$_2$-eq with a standard deviation of 4.16 million metric tons of CO$_2$-eq. We determined that, out of the 208 firms in our dataset, 149 of them had reduced their carbon footprint while only 59 of them had increased their footprint, in the 2016-2019 period. This caused a total reduction of 94.61 million metric tons of CO$_2$-eq out of an average annual (2015-2020) 1.78 billion metric tons of CO$_2$-eq or 7.02% (all companies weighed equally), which is still a fairly slow reduction, given the 5-year long period. However, in the year 2020 and even early 2021 most economies in the world were forced into having a lower carbon footprint, which may aid the journey towards a cleaner economy as the revival of the economies of the world may involve a restructuring towards sustainability over the next few years (34).

For the remaining data, we use the publicly available database from Yahoo Finance. We collect stock data from 2016-2021. This corresponds to the time for which both the financials are available on Yahoo Finance and Carbon emissions data of the previous year is available. We use numbers on two variables under Income Statement, namely, Total Revenue, and Total Expenses, and four variables under Balance Sheet, namely, Total Capitalization, Invested Capital, Total Debt and Tangible Book Value, as listed under Yahoo Finance. We use these variables either directly or in combination with each other, as control variables in our linear regression model to study the effect of carbon emissions on stock returns. Following the norm, we choose the logarithm of annualized stock returns as the dependent variable. The annual returns are measured from stock data by considering the period for which we have data on financials. The average log-return of a firm in our set is 0.0095 and the standard deviation is 0.3223. The logarithm of the total GHG emissions (Scope1 + Scope 2) is the primary variable in the regression. In our sample it has a mean of 14.71 and standard deviation of 2.25. For control variables, we use the logarithm of the total capitalization at the end of the reporting year (with an average of 23.57 and a standard
deviation of 1.10), the logarithm of the ratio of the total debt-to-tangible book value (with an average of -0.50 and a standard deviation of 1.24), the ratio of invested capital-to-tangible book value (with an average of 2.35 and a standard deviation of 7.06), the ratio of tangible book value-to-total capitalization (with an average of 0.77 and a standard deviation of 0.52), the logarithm of the total revenue (with an average of 23.37 and a standard deviation of 1.18), and the logarithm of the total expenses (with an average of 22.31 and a standard deviation of 1.71). Along with these, we use two more control variables whose values are calculated from historical data. These are average momentum and average historical volatility. Momentum at a day \( t \) is assumed to be the average of ten days (\( t - 10 \) through \( t - 1 \)) of daily stock returns, which is then averaged annually (as we use annual returns in the regression). Historical volatility is assumed to be the standard deviation of ten days of daily returns, which is averaged over one year to give us the average historical volatility for the year being considered. For the stocks in our data, the mean of average momentum is found to be 1.000 and the standard deviation is found to be 0.001 and the corresponding estimates for average historical volatility are 0.023 and 0.013. These averages and standard deviations are for the data being used in the regression \( i.e., \) for 197 firms and 602 data-points, each corresponding to one year of one firm over the period considered. As a result, the averages may be biased towards firms based on the availability of emissions data in the period.

3 Is There a Carbon Risk Premium?

Before we attempt to model the price of carbon transition risk, it is important to examine if investor awareness about such a risk has been factored into stock prices at all. While Oestreich and Tsiakas [20] and Bolton and Kacperczyk [8, 9] report evidence of the existence of a carbon risk premium, Gorgen et. al. [14] do not report finding a carbon premium, although they do report higher returns for “Brown firms” when compared to “Green firms”. Our analysis is restricted to S&P500 firms and therefore we do not aim to assert or deny the universality of the existence of a carbon risk premium. We aim to estimate the risk premium (if any) for large caps in the American market and use the estimate of the risk premium to determine the price of carbon transition risk for the firms in our sample. Thus we begin by choosing a suitable definition of the risk premium. In the context of social cost of carbon, Kousky, Kopp and Cooke [19] define risk premium as the maximum willingness-to-pay of a risk-averse individual, for the reduction in risk. While for stock returns, Oestreich and Tsiakas [20] define carbon premium as the “abnormal excess returns of the “dirty-minus-clean” portfolio”. In line with these, we define the carbon risk premium \( p_{i,T} \) for firm \( i \) in the year \( T \) as,

\[
p_{i,T} = \rho_{i,T} - \rho_{i,T}^0.
\]

Here \( \rho_{i,T} \) is the total annual stock returns in the year \( T \) and \( \rho_{i,T}^0 \) is the risk-free stock returns of the same year \( i.e., \) the returns the shares of the company would have had in the absence of an impending carbon transition risk.

For our analysis, we use a linear regression model similar to that employed by Bolton and Kacperczyk [8], based on the model presented by Daniel and Titman [11], with some changes. While the former employs the logarithm of Scope 1, Scope 2 and Scope 3 emissions as the primary variables whose regression coefficients the authors are interested in, we use a simpler model with the logarithm of the total GHG emissions instead, as described in the preceding section. Apart from some differences in the control variables, a major difference is the use of log-returns as the dependent variable, as in Oestreich and Tsiakas [20]. Accordingly, we use the following model:

\[
r_{i,T} = \alpha_0 + \alpha_1 \ln (1 + E_{i,T-1}) + \alpha_2 \cdot C_{i,T} + \epsilon, \quad (1)
\]

where, \( E_{i,T-1} \) refers to the total carbon emissions (Scope 1+Scope 2) of firm \( i \) reported at the end of the year \( (T - 1) \), \( C_{i,T} \) is an 8-dimensional vector of the values of the control variables, namely, log-total capitalization, log-ratio of debt-to-book value, ratio of invested capital-to-book value, ratio of book value-to-total capitalization,
average momentum, average historical volatility, log-revenue and log-expenses — of firm \( i \) reported (or calculated) at the end of the year \( T \),

\[ r_{i,T} \equiv \log \left( \frac{S_{i,T}}{S_{i,T-1}} \right) \]

(where \( S_{i,T} \) denotes the price of stock \( i \) at time \( T \)) i.e., the logarithm of the annual stock returns in the year \( T \), and \( \epsilon \) is the idiosyncratic error. Scalars \( \alpha_0, \alpha_1 \) and the vector \( \alpha_2 \) are the constants we wish to estimate, particularly, \( \alpha_1 \), which shall give us insights about the existence and value of the carbon risk premium.

There are three reasons why we choose log returns instead of simply returns as the dependent variable. Firstly, it gives a better fit. The linear regression of the variables with log-returns returns an \( R \)-squared (adjusted for multiple regression) of 0.9538 compared to the corresponding value of 0.9373 for the case of simply using returns. The second reason is rooted deeper in the mathematics of price movements. Called the ”standard model of finance” by Voit [35], most existing models of stock prices, from as early as Osborne in 1959 [22] and Black and Scholes’ landmark article [5], either directly use the notion of a geometric Brownian motion or build on it. Such a random walk causes for the stock prices to be distributed log-normally. Assuming stock prices to follow a log-normal distribution, we arrive at the following expression for \( r_{i,T} \) as defined above:

\[ r_{i,T} = \ln \left( \frac{S_{i,T}}{S_{i,T-1}} \right) = \left( \mu_i - \frac{\sigma_i^2}{2} \right) + W, \] (2)

where the first term is the drift and \( W \) is a normally distributed random variable. We can see that equation (2) bears a strong resemblance with equation (1). Here \( W \) corresponds to the idiosyncratic error \( \epsilon \) and the drift term corresponds to the remaining linear expression. In essence, by the estimation of parameters in the linear regression, we can arrive at an expression for an annual average of this drift term for the stock returns of firm \( i \).

The final reason for opting for log-returns as the dependent variable is based on the nature of the risk premium for carbon. It is also related to the choice of \( \ln(1 + E_{i,T-1}) \) instead of simply \( \ln(E_{i,T-1}) \). Based on an intuitive understanding of the risk premium, we would prefer a model wherein the value of the premium for a firm would have a positive correlation with the changes in the amount of emissions of the firm. If the firms’ emissions increase over time, the value of the premium should go up, and if the amount of Carbon emissions falls, so should the premium. However, we would not expect a linear relationship between the amount of GHG emissions, because if a firm increases their carbon footprint drastically over the course of the next few years, it would be unrealistic to expect the premium to increase linearly because such a linear increase may lead to the premium becoming too large. Similarly we would expect the value of the premium to become very small (nearing zero) as firms reduce their footprint drastically. We can see that our model is in line with such intuitive expectations about the properties of the carbon risk premium. Exponentiating both sides of equation (1), we get

\[ \rho_{i,T} = e^{\left( \alpha_0 + \alpha_1 \ln(1 + E_{i,T-1}) + \alpha_2 C_{i,T} + \epsilon \right)} . \]

Bringing out the Carbon term we get,

\[ \rho_{i,T} = e^{\alpha_1 \ln(1 + E_{i,T-1})} \rho_{i,T}^0 . \]

Therefore,

\[ p_{i,T} = \rho_{i,T} - \rho_{i,T}^0 = \rho_{i,T}^0 \left( e^{\alpha_1 \ln(1 + E_{i,T-1})} - 1 \right) . \] (3)

The first inference that we can draw from equation (3) is that as the total emissions of a firm goes to 0, so does the carbon risk premium. Secondly, we can see that the \( p_{i,T} \) scales sub-linearly (Figure [1]).

It is also useful to look at the variation of the risk premium with changes in firm-wise carbon emissions over time. Since we have firms that have increased their footprint in the period considered, as well as those which have reduced theirs, it is useful to look at the theoretical implications that this model may have on the rate of change of premium with respect to the carbon footprint for different (both positive and negative) values for \( \dot{E} \). If we examine
the time derivative of equation (3), we get,

\[ \dot{p}_{i,T} = \rho_{i,T} \cdot \frac{\alpha_1}{1 + \mathcal{E}} e^{\alpha_1 \ln(1 + \mathcal{E})} \dot{\mathcal{E}} \]

From equation (4) we can see that the expression scales as \( \frac{1}{\mathcal{E}} \), as the emission dependent expression in the numerator is approximately equal to 1 for any reasonable \( \mathcal{E} \), unless there is a very drastic change in the firm-specific emissions in a short interval of time. Therefore, a prediction of this model is that for firms with a large carbon footprint, the risk-premium is almost invariant with small changes in the total emissions (whether positive or negative). A consequence of the prediction would be that highly polluting firms would have to undergo drastic operational changes in order to mitigate the high carbon transition risk they face. These predictions, while agreeing with our intuitive understanding of carbon emissions’ effect on a firm’s stock return, are difficult to quantitatively validate, especially given the size of our dataset. Thorough empirical studies involving much longer durations of time will be required to confirm or refute the validity of these predictions.

Before performing the cross-section analysis, it is important study how GHG emissions vary with all other financial variables (Figure 2). There are two motivations to this. The first obvious motivation is that we would like to know if there are specific variables that vary in a certain way depending on the GHG emissions. But another reason is to ensure that our linear regression model does not involve control variables which are very highly correlated with our primary variable, as then the premium we would be estimating would not strictly be a carbon premium. Ideally, the variables should be completely independent, as is the nomenclature. However, realistically, some correlation is inevitable, and we do not correct for the weak correlations.
Figure 2: **Correlation of log-total GHG Emissions with financial parameters** I: The x-axis displays the distribution of log-total GHG Emissions and the y-axis shows the distribution of the financial variables.

From the 2-dimensional hexbin scatter plots, we can look at how the log-GHG emissions vary with the other variables. We see that there happens to be a moderately weak correlation that the log-GHG emissions have with
Table 1: Pearson Correlation Coefficient of Dependent Variable, Primary Variable and Control Variables with Each other

| Variables                  | Log-Returns | log(1 + E) | Total-cap | Log debt-to-book | Invested capital-to-book | Book-to-total cap | Momentum | Volatility | Log-Revenue | Log-Expenses | Log-Returns | log(1 + E) | Total-cap | Log debt-to-book | Invested capital-to-book | Book-to-total cap | Momentum | Volatility | Log-Revenue | Log-Expenses |
|----------------------------|-------------|------------|-----------|------------------|--------------------------|-------------------|-----------|------------|-------------|-------------|-------------|------------|----------|-----------|------------------|--------------------------|-------------------|-----------|------------|-------------|-------------|
| Log-Returns                | 1.0         | -0.0893    | 1.0       |                  |                          |                   |           |            |              |              |              |            |         |           |                  |                          |                   |           |            |              |              |
| log(1 + E)                 | -0.0191     | 0.3875     | 1.0       |                  |                          |                   |           |            |              |              |              |            |         |           |                  |                          |                   |           |            |              |              |
| Total-cap                  |             |           |           |                  |                          |                   |           |            |              |              |              |            |         |           |                  |                          |                   |           |            |              |              |
| Log debt-to-book           |             |           |           |                  |                          |                   |           |            |              |              |              |            |         |           |                  |                          |                   |           |            |              |              |
| Invested capital-to-book   | 0.0033      | 0.1969     | 0.1391    | 1.0              |                          |                   |           |            |              |              |              |            |         |           |                  |                          |                   |           |            |              |              |
| Book-to-total cap          | 0.00319     | -0.0587    | 0.1178    | 0.5045           |                          |                   |           |            |              |              |              |            |         |           |                  |                          |                   |           |            |              |              |
| Momentum                   | -0.0421     | 0.0345     | -0.2148   | -0.4363          | -0.4439                 |                   |           |            |              |              |              |            |         |           |                  |                          |                   |           |            |              |              |
| Volatility                 | -0.2224     | -0.0347    | -0.0465   | -0.0244          | 0.0286                  | 0.0371            |           | 1.0s      |              |              |              |            |         |           |                  |                          |                   |           |            |              |              |
| Log-Revenue                | -0.0051     | 0.3027     | 0.6971    | 0.0315           | 0.0223                  | -0.1163           | 0.0071    | 1.0        | -0.0091    | -0.0261     | -0.1201     | 0.6048     | 1.0       |         |           |                  |                          |                   |           |            |              |              |
| Log-Expenses               | 0.0514      | 0.1318     | 0.3749    | 0.1244           | 0.0363                  | -0.1232           | -0.0       | -0.1201   | 0.6048     | 1.0         |              |            |         |           |                  |                          |                   |           |            |              |              |

Table 2: Estimated Coefficients in Cross-Section Analysis

| Variables                  | $\alpha_0$   | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | $\alpha_4$ | $\alpha_5$ | $\alpha_6$ | $\alpha_7$ | $\alpha_8$ | $\alpha_9$ |
|----------------------------|---------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| Coefficients               | -279.98658    | 0.001351   | 0.00628    | 0.00091    | 0.00012    | -0.00846   | 280.14561  | -11.03386  | -0.00914   | 0.00112    |
| Standard error             | 2.57792       | 0.00144    | 0.00380    | 0.00086    | 0.00049    | 0.00642    | 2.57739    | 0.22190    | 0.00393    | 0.00211    |
| 95% CI lower limit         | -272.42448    | 0.00562    | 0.01728    | 0.01065    | 0.00143    | 0.01201    | 287.58325  | -10.42770  | 0.00398    | 0.00779    |

log-total capitalization and log-total revenue, with Pearson correlation coefficients 0.3875 and 0.3027 respectively and corresponding p-values for testing non-correlation are $5.74 \times 10^{-23}$ and $3.18 \times 10^{-14}$. The null hypothesis of no correlation is rejected at the 0.25% level (We choose a low threshold for correlation to filter only significant correlations), implying definitive correlation, although weak. A fairly weaker positive correlation is observed with the log-ratio of total debt-to-tangible book value at 0.1969 and a p-value of $1.13 \times 10^{-6}$, and with the log-Expenses with a correlation coefficient of 0.1318 and a p-value of 0.0012. No other variables show a definitive correlation at the 0.25% level. Although the p-value test is approximate for our data as it assumes normality of the distributions, which is an approximation at best, the extremely small values allow us to still reject the null hypothesis.

![Histogram of Risk Premia](image1.png)  
(a) Histogram of Risk Premia

![Variation of Carbon Risk Premium with log-emissions](image2.png)  
(b) Variation of Carbon Risk Premium with log-emissions

Figure 3: (a) Histogram of Risk Premia: The figure shows the distribution of 196 firms over 5 years (595 data-points) with the carbon risk premium in stock returns. (b) Variation of Carbon Risk Premium with log-emissions: While there is a clear positive correlation, there is a significant variation in the distribution due to varying risk-free returns due for different firms and different years in consideration.
A possible explanation for the correlation with the log-total capitalization and log-total revenue and log-total expenses is that GHG emissions scale with the size of the organizations, as do these other financial variables. The observation of Log-expenses having a significantly lower correlation coefficient than those of total cap and log revenue, may be because of expenses scale sub-linearly with size and would show a smaller correlation. However, a more interesting inference would be that transition is expensive and comes from newer technology and innovation, and firms with a higher carbon footprint would not have incurred too much of these costs. The overall correlation matrix of all the variables of the cross section analysis is reported in Table 1.

On carrying out the linear regression, we observe that there exists a significant carbon risk premium in the stock returns of American large cap firms. On our sample, we estimate $\alpha_1 = 1.51 \times 10^{-3}$. From Figure 3a we see that the distribution of firms about their risk premia is a unimodal distribution with a slight positive skew, centered around the average of 2.31% with a standard deviation of 0.75%. However, there the range of the risk premia overall firms and all years was large, with the smallest being almost negligible at 0.20% and the largest being 7.08% (an outlier).

In order to analyze the errors in the linear regression, we use the method of parametric bootstrap, in particular that of residual bootstrap, where we generate a large number (100,000) of ”bootstrapped samples” by randomly sampling with replacement from the residual vector of the least squares estimate, and adding them to the best fit prediction. On carrying out the entire parameter estimation process on the 100,000 samples, we get a distribution of each coefficient measured in the regression. Using this we determine the standard error of estimation in each coefficient and the 95% confidence intervals.

4 What will the Risk of Transitioning Amount to?

The existence of a significant carbon transition risk premium implies that investors are already aware of the risks of investing in firms with higher carbon footprints. However, the risk of transition itself is difficult to model, and there is a paucity of literature on how low-carbon transition of firms will affect the overall operations and future business prospects of firms. The risk itself can be due to many factors, and a long term higher pricing of carbon, as has been suggested by Baranzini et. al. [3], can cause brown firms to lose revenue continuously and which would imply lesser returns for investors on the long term. More radical government policy, would magnify this risk for investors. Sustainable climate policies [30] that rely on systemic changes, large reductions in emissions and rapid innovations, could even force carbon dependent firms to go out of business or sustain large losses on their existing infrastructure. Therefore, for investors, having an idea of the magnitude of their exposure to carbon-transition is essential. To fulfill this purpose we develop a risk measure called the ”Single Event Transition Risk” (SETR) that gives investors the price of the expected risk on each share of a polluting firm, if a single event were to force a firm with high carbon dependence into transitioning. The SETR is a measure of the maximum exposure of the price of a stock to low-carbon transition.

4.1 The Single Transition Event

A transition event can be thought of as any event that causes the market to factor in the ”price” of transitioning into the value of the stock of a firm. It can be a firm specific event where a firm transitions by making conscious decisions to lower its carbon footprint or a larger, general transition where the entire economy transitions to become cleaner in terms of greenhouse gas emissions. However, such transition events occur irregularly [24], with only an inconsistent yet small amount of transition risk being factored into the value of the stock each time. Because of the irregularity in the arrival of such information in the market and even more importantly, because of the variation in the magnitude of the pricing each time, such transition events are difficult to model, especially since we are interested in the overall cost of transitioning. It is therefore, useful, for modelling purposes, to
replace all the small independent transition events by a single large transition event that factors in the entire price of transitioning into the value of the stock instantaneously (or in a very short interval of time). This event could be a hypothetical company decision to radically reduce their carbon dependence in a very short time, at all costs, or a government policy decision which forces regulations on firms that make it imperative for them to transition to a low carbon footprint. The transition event itself can be modelled by a stochastic risk process \( R(t) \) defined by,

\[
R(t) = \begin{cases} 
1, & \text{The transition event occurs at } t, \\
0, & \text{The transition event does not occur at } t.
\end{cases}
\]

Let \( P[(t_A, t_B)] \) denote the probability of the transition event occurring in some interval \((t_A, t_B)\) i.e., the probability that,

\[
\exists \ t \in (t_A, t_B) \text{ such that } R(t) = 1.
\]

Therefore, given an initial time \( t_0 \), such that low-carbon transition is yet to happen, we can model the arrival process by some probability density function \( \tau(t) \in [0, \infty) \) \( \forall t \in (t_0, \infty) \) such that,

\[
P[(t_A, t_B)] = \int_{t_A}^{t_B} \tau(t) dt \ \forall \ t_0 \leq t_A \leq t_B < \infty.
\]

Since \( \tau(t) \) describes a probability density function, we additionally know that,

\[
\int_{t_0}^{\infty} \tau(t) dt = 1.
\]

4.2 Model for Pricing Single Event Transition Risks

Consider an investor who takes a long position in the stock of price \( S_i \) of a firm \( i \), such that the firm has some carbon footprint \( \epsilon_i \). We know from Section 3 and existing literature that investors receive higher premia from stocks of firms with higher Carbon footprint. Investors continue to invest in said firms under the assumption that said premium mitigates the risk of a fall in the prices of the stock when Carbon risk is appropriately priced. In an arbitrage-free, frictionless economy, we expect that the “fair-value” of the premium that would keep investors invested in the firm would be such that the expectation of the premium earned by the investor would equal to the expectation of the risk when the price of the stock falls. In order to mathematically measure the fall in the prices of the stocks, given the carbon risk premium on the returns till date, we use an approach inspired from pricing Credit Default Swaps [6]. Here the single transition event can be thought of to be analogous to a default. The investor can be thought of as a “protection seller” who, for timely payments of annual risk premia, chooses to hold on to the shares of the risky stock. However, both systems are significantly different in certain aspects and therefore cannot be thought of to be perfectly analogous.

Consider an investor who buys a stock at time \( t_0 \). Note that, to buy a stock and to hold a position in a stock is equivalent to the scenario that an investor will only be interested in profit opportunities in the future. Without loss of generality, we can assume the pre-risk price of every share at \( t = t_0 \) to be \( S_i(t_0) = 1 \) (Note that even though \( S_i(t) \) is accounted for the premia, it is not accounted for any future transition risk). We can subsequently define a real valued function \( P_i(t) : [t_0, \infty) \to \mathbb{R} \) that describes the potential fall of the price of each share of firm \( i \), on any day \( t \) on which the transition event occurs. Suppose, that the time at which the transition event occurs is \( t_e \) i.e., \( \mathcal{R}(t_e) = 1 \). Since there is only a single transition event, we know that \( t_e \) is unique i.e., if the transition event takes place at time \( t_e \), it will not take place at any other time. Therefore we can model the price of a stock generally as

\[
S_i^*(t) = S_i(t) - P_i(t)\mathcal{R}_i(t) \ \forall \ t \in [t_0, t_e].
\]
Based on this, we expect that the existence of the risk premia for the shares of firms is motivated by the functions $P_i(t)$ and the process $R_i(t)$ (and therefore implicitly by the density function $\tau(t)$). Therefore, given $\tau(t)$, we seek to find the relationship between the risk premium $p_i(t)$ of the shares of a firm and the SETR function $P_i(t)$.

In order to model the risk premium process, we choose a simple constant risk premium,

$$p_i^t = p_i^{t_0}.$$  

If an investor expects to receive a constant flow of premium $p_i^{t_0}$ from time $t_0$ through $t_e$, then the amount an investor expects to receive in premium is,

$$A_i = \int_{t_0}^{\infty} \int_{t_0}^{t_e} \tau(t_e) p_i^t \, ds \, dt_e.$$  

Here, the inner integral gives us the total amount that an investor expects to earn from risk premia above the risk free returns, for a given $t_e$, while the outer integral averages over all possible times for the transition event with respect to the arrival process. On simplifying this further by assuming a constant rate of risk premia, we obtain,

$$A_i = \int_{t_0}^{\infty} \tau(t_e) \int_{t_0}^{t_e} p_i^t \, ds \, dt_e.$$  

Therefore,

$$A_i = p_i^{t_0} \left[ \int_{t_0}^{t_e} \tau(t_e) \, dt_e - t_0 \int_{t_0}^{t_e} \tau(t_e) \, dt_e \right].$$  

We can see that the first integral is the expected time of arrival of the risk and the second integral is 1. Therefore,

$$A_i = p_i^{t_0} [\mathbb{E}(t_e) - t_0].$$  

This is an interesting result, because this tells us that the amount an investor can expect to earn from risk premia does not depend on the form of the arrival process at all and only depends on the expected time of arrival. Now, for a fair pricing of the premium, this surplus gain from risk premia should equal the expected losses incurred by a fall in the price of the stock at $t_e$, i.e.,

$$p_i^{t_0} [\mathbb{E}(t_e) - t_0] = \int_{t_0}^{\infty} P_i(t_e) \tau(t_e) \, dt_e.$$  

Furthermore, we see that the expected value of the risk grows linearly with the amount of premium and the expected arrival time of the transition event. This, however is a consequence of assuming a constant expected annual risk premium. A better calculation could use a continuously growing risk premium as was obtained by Bolton and Kacperczyk [8]. However, due to the dearth of available data on risk premia over sufficiently long periods of time and models that explicitly show how the risk premia vary with time, we assume a constant risk premia.

The expected value of the risk however, does not tell us much about the form of $P_i(t_e)$, and is only an average. However, if we assume (and reasonably so) that $P_i(t_e)$ is a well behaved, bounded function that does not fluctuate wildly, then we can get an approximate value of $P_i(t_e)$ by averaging over a short interval of time. Let us consider some later time $t = t_0 + \Delta t$, such that the transition event has still not occurred. Then,

$$p_i^{t_0 + \Delta t} [\mathbb{E}(t_e|t_e > t_0 + \Delta t) - t_0 - \Delta t] = \int_{t_0 + \Delta t}^{\infty} P_i(t_e) \tau(t_e) \, dt_e.$$  

13
where the factor $C$ has been included for normalizing the conditional probability distribution such that $t_e > t_0 + \Delta t$, and is equal to reciprocal of the conditional probability of the transition event taking place after $t_0 + \Delta t$ given that it takes place after $t_0$. Thus,

$$C = \frac{\int_{t_0}^{\infty} \tau(t_e)dt_e}{\int_{t_0+\Delta t}^{\infty} \tau(t_e)dt_e} = \frac{1}{\int_{t_0+\Delta t}^{\infty} \tau(t_e)dt_e}.$$  

From this, we can derive an expression for the expected price of the risk, if the transition event happens in the small interval $(t_0, t_0 + \Delta t)$. Accordingly,

$$p_i^{t_0+\Delta t} \left[ \mathbb{E}(t_e|t_e > t_0 + \Delta t) - t_0 - \Delta t \right] = C \int_{t_0}^{\infty} P_i(t_e)\tau(t_e)dt_e - C \int_{t_0}^{t_0+\Delta t} P_i(t_e)\tau(t_e)dt_e.$$  

Making the substitution from equation (5), we obtain,

$$p_i^{t_0+\Delta t} \left[ \mathbb{E}(t_e|t_e > t_0 + \Delta t) - t_0 - \Delta t \right] = C r_{t_0} \left[ \mathbb{E}(t_e) - t_0 \right] - C \int_{t_0}^{t_0+\Delta t} P_i(t_e)\tau(t_e)dt_e.$$  

Let $t_0 + \Delta t = t'$. Therefore, making this substitution in equation (6), we obtain,

$$\int_{t_0}^{t'} P_i(t_e)\tau(t_e)dt_e = p_{t_0} \left[ \mathbb{E}(t_e) - t_0 \right] - \frac{p_{t_0+\Delta t}}{C} \int_{t'}^{t} \tau(t_e)dt_e.$$  

In order to determine the explicit form of $P_i(t_e)$ we now take the partial derivative of both sides with respect to $t'$, to obtain,

$$\frac{\partial}{\partial t'} \left( \int_{t_0}^{t'} P_i(t_e)\tau(t_e)dt_e \right) = \frac{\partial}{\partial t'} \left[ p_{t_0} \left[ \mathbb{E}(t_e) - t_0 \right] - \frac{p_{t_0+\Delta t}}{C} \int_{t'}^{t} \tau(t_e)dt_e \right].$$  

$$\therefore P(t') \tau(t') = \frac{p_{t_0}}{\tau(t')} \left\{ t' \tau(t') + \int_{t'}^{t} \tau(t_e)dt_e \right\}. $$  

We can therefore write the value of the SETR explicitly in terms of the arrival process $\tau$, as follows:

$$P(t') = \frac{p_{t_0}}{\tau(t')} \int_{t'}^{\infty} \tau(x)dx.$$  

One caveat of using this approach of SETR, to price carbon risk, is that it implicitly assumes that the “fair” value of the carbon premium has been realized for the “correct” arrival process. It is entirely possible that investors do not have a good enough idea about the value of the risk for each firm. However, the results derived in this article still remain both valid and useful as they can be used to determine a firm-wise fair price for the carbon transition premium, once the SETR has been estimated at a firm level by looking into the firm’s operations, assets, liabilities and other fundamentals. This would help investors hedge their risk and make prudent decisions with regards to divesting from carbon.
4.3 Some Special Arrival Processes

In this subsection, we seek to explicitly determine the value of the SETRs, for some examples of arrival processes, and then evaluate the SETRs, for the firms in our data for different parameter values. We study arrival processes of the form of exponential distributions, Gamma distributions and uniform distributions. The motivation behind studying these functions are their popular usage for service times in queuing theory. It is intuitive to use models of service times, and waiting time of queues for modelling the arrival of a transition event as the event that is most likely to come from a government policy, which can be thought of analogous to a server, and the time taken for the policy being enacted therefore becomes the service time. Alternatively, the transition event can be thought of as a law waiting to be passed in a queue of laws, thus making the time to the law being passed analogous to the waiting time in a queue.

4.3.1 Exponential Distributions

Exponential distributions have been used most frequently in queuing theory to model service times [17, 23]. While it is a special case of the Gamma distribution, we analyze it separately as the results are unique (special case of the result in Gamma distributions) and the integrals easy to evaluate exactly. An exponential distribution with the parameter \( \lambda \) is given by

\[
\tau(t) = \begin{cases} 
\lambda e^{-\lambda t} & t \geq t_0, \\
0 & t < t_0.
\end{cases}
\]

This distribution has the mean \( \mu_{\exp} = \frac{1}{\lambda} \) and the variance \( \sigma_{\exp}^2 = \frac{1}{\lambda^2} \). Let \( t_0 = 0 \) (without loss of generality). Substituting for \( \tau \) in equation (7), we obtain,

\[
P_{\exp}(S_i, t') = \frac{p_i^{t'}}{\lambda e^{-\lambda t'}} \int_{t'}^{\infty} \lambda e^{-\lambda x} \, dx = \frac{p_i^{t'}}{\lambda e^{-\lambda t'}} \left[ -e^{-\lambda x} \right]_0^{\infty} = \frac{p_i^{t'} e^{-\lambda t'}}{\lambda e^{-\lambda t'}}.
\]

Therefore,

\[
P_{\exp}(S_i, t') = \frac{p_i^{t'}}{\lambda}.
\]

We find that the value of the SETR is independent of the time when the transition event occurs and only depends on the value of the premium at the time of the transition. This happens because of the memoryless nature of the exponential distributions.
Figure 4: Variation of SETR with the parameters for average, minimum and maximum premia in the year 2020: (a) Variation of SETR with expectation value of the time of transition (b) Variation of SETR with $\lambda$.

In 2020, the average firm in our dataset had a carbon transition risk premium of 2.06%, thus we expect the value of the average SETR to be 2.06%, 20.65% and 41.30% of the initial (at the time of purchase) value of the stock, for the expected arrival times 1 year, 10 years and 20 years respectively. The corresponding SETR values for the stock with the maximum premium are 4.50%, 45.04% and 90.08% for the expected arrival times 1 year, 10 years and 20 years respectively.

4.3.2 Uniform Distributions

Uniform distributions are among the simplest two-parameter distributions and have at times been used in modelling waiting times in queuing models [33]. Uniform distributions are also common as the arrival times of events that are Poisson distributed (in time) are uniformly distributed [31]. A uniform distribution may be particularly useful in modelling the arrival of transition events, as transition events are often contingent on the environmental policies of parties in democratic governments. Therefore it would be reasonable to model the arrival process as being uniformly distributed in the period for which a party with a “progressive” stance on regulating emissions is elected to power, after which the probability density is 0. A uniform distribution with parameters $\theta_{\min}$ and $\theta_{\max}$ can be defined as

$$\tau(t) = \begin{cases} 0 & t < \theta_{\min} \\ (\theta_{\max} - \theta_{\min})^{-1} & \theta_{\min} \leq t \leq \theta_{\max} \\ 0 & t > \theta_{\max} \end{cases}$$

and has the mean $\mu_{\text{uni}} = \frac{1}{2}(\theta_{\max} + \theta_{\min})$ and variance $\sigma_{\text{uni}}^2 = \frac{1}{12}(\theta_{\max} - \theta_{\min})^2$. While, it may be such that $\theta_{\min} < t_0$, however, there would then not be much use of calculating $P(t_e)$ for $t_e \in (t_0, \theta_{\min})$ as the probability of the event taking place in such an interval would be 0. We also only consider the case where $t' \in (\theta_{\min}, \theta_{\max})$ as outside this interval there would be no probability of the transition event taking place at time $t'$. Therefore, we
consider the case that $t_0 = \theta_{\text{min}}$ and $t' \leq \theta_{\text{max}}$ and substitute for $\tau$ in equation (7).

$$P_{\text{uni}}(S_i, t') = \frac{p_i^t}{(\theta_{\text{max}} - \theta_{\text{min}})^{-1}} \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \frac{1}{\theta_{\text{max}} - \theta_{\text{min}}} dx$$

Therefore,

$$P_{\text{uni}}(S_i, t') = p_i^t (\theta_{\text{max}} - t').$$

(9)

Figure 5: Variation of SETR with $\theta_{\text{max}}$ and time $t'$: We see that the surface that describes the variation of SETR with the parameters is a plane whose height increases with $\theta_{\text{max}}$ but decreases with $t'$.

Unlike the case with the exponential distribution, we observe here that the SETR is linear in the time at which the transition event takes place, but we find that the value of the SETR increases linearly with the expectation value of the time of arrival, akin to the exponential case. In our dataset, for $\theta_{\text{max}} = 4$ years and $\theta_{\text{max}} = 8$ years, respectively (choice of $\theta_{\text{max}}$ based on terms of American elected governments), we find the average SETRs to be 4.12% and 8.24%, and the maximum SETRs are 9% and 18% for the same choices of $\theta_{\text{max}}$.

4.3.3 Gamma Distributions

Gamma distributions, although have only two parameters, are extremely versatile and can be used to model distributions of various shapes. Gamma distributions have been used for modelling waiting times in queues [1][13]. We consider the Gamma arrival process with parameters $a$ and $b$,

$$\tau(t) = \begin{cases} 
0 & t < t_0 \\
\frac{b^a}{\Gamma(a)} t^{a-1} e^{-bt} & t \geq t_0 
\end{cases}$$
This has the expected time of arrival $\mu = \frac{a}{b}$ and variance $\sigma^2 = \frac{a}{b^2}$. For $a = 1$ and $b = \lambda > 0$, we get the exponential distribution. Let $t_0 = 0$ (without loss of generality). Rewriting equation (7) for the Gamma distribution we get,

$$P(S, t') = \frac{p_i t' \Gamma(a)}{b a t^{a-1} e^{-bt'}} \int_b^t \frac{b^a}{\Gamma(a)} x^{a-1} e^{-x} dx. \tag{10}$$

If we make the substitution $y = bx$, we obtain,

$$P(S, t') = \frac{p_i t' \Gamma(a)}{b a t^{a-1} e^{-bt'}} \int_{bt'}^\infty y^{a-1} e^{-y} dy. \tag{11}$$

The integral in equation (10) is the upper incomplete Gamma function, and often denoted by $\Gamma(a, bt')$.

Equation (11) gives us the value of the SETR for a gamma distributed arrival process. Substituting $a = 1$ in equation (11) reduces back to equation (8), as the exponential distribution corresponds to the Gamma distribution with $a = 1$.

For computational simplicity, it may be useful to also find an approximate expression for the SETR. For this purpose, we write this in terms of the Gamma function $\Gamma(a)$ and the lower incomplete gamma function $\gamma(a, bt')$ that can be defined as,

$$\gamma(a, bt') = \int_0^{bt'} z^{a-1} e^{-z} dz = \Gamma(a) - \Gamma(a, bt').$$

Therefore rewriting equation (10) with this substitution, we obtain,

$$P(S, t') = \frac{p_i t' \Gamma(a)}{b a t^{a-1} e^{-bt'}} [\Gamma(a) - \gamma(a, bt')]$$

As stated in [21] and proved in [15], we use the power series expansion for $\gamma(a, bt')$.

$$\gamma(a, bt') = \sum_{k=0}^{\infty} \frac{(bt')^k e^{-bt'}}{a(a+1) \ldots (a+k)} = (bt')^a e^{-bt'} \Gamma(a) \sum_{k=0}^{\infty} \frac{(bt')^k}{\Gamma(a+k+1)}.$$

Therefore,

$$P(S, t') = \frac{p_i t' \Gamma(a)}{b a t^{a-1} e^{-bt'}} \left[ \Gamma(a) - (bt')^a e^{-bt'} \Gamma(a) \sum_{k=0}^{\infty} \frac{(bt')^k}{\Gamma(a+k+1)} \right].$$

Hence we arrive at the following expression:

$$P(S, t') = \frac{p_i t' \Gamma(a)}{b a t^{a-1} e^{-bt'}} - p_i t' \Gamma(a) \sum_{k=0}^{\infty} \frac{(bt')^k}{\Gamma(a+k+1)} = \frac{p_i t' \Gamma(a)}{b a t^{a-1} e^{-bt'}} \left[ 1 - b^a t^a e^{-bt'} \sum_{k=0}^{\infty} \frac{(bt')^k}{\Gamma(a+k+1)} \right]. \tag{12}$$

Since the power series converges for all $a > 0$ and $bt' > 0$, we can use the following expression to approximate the value of the SETR, by ignoring terms of sufficiently high orders.
Figure 6: Variation of SETR (% of price of stock at purchase) with time for $p_t = 1\%$: (a) Different values of $a$ for $b = 1$ (b) Different values of $a$ for $b = 0.01$ (c) Different values of $b$ for $a = 0.5$ (d) Different values of $b$ for $1.25$

5 Are the Premia Worth the Risk?

The estimation of the value of the SETR in Section 4 has been carried out under the assumption that the expected returns of the risk premium is equal to the expected value of the SETR. However, this does not necessarily mean that the probability of an investor making positive or negative returns from the premium-SETR offset is equal. Instead what it means is that the expectation of returns, over all possible cases is 0. But as has been pointed out by Ole Peters [27], taking an ensemble average of possibilities “doesn’t reflect the situation of individual decision makers”. Therefore, in order to answer the question of whether it is in an investor’s best interests, it will be useful to quantify the probability of positive returns without weighing it by the size of the return. Let us define this quantity as $q$. Since a constant expected rate of the risk premium has been assumed in our model, the expected return from the risk premia for stock $i$ with premia $r_i$ till the time of transition is $p_t^{t_0} (t_e - t_0)$. Therefore, we need to find the area under the graph of the density function of the arrival process $\tau(t)$ over all intervals in $[t_0, \infty]$ such
that for any \( t \) in that interval, the expected returns is more than the value of SETR at \( t \) i.e.,

\[
p_i(t_e - t_0) > \frac{p'_i}{\tau(t')} \int_{t'}^{\infty} \tau(x)dx.
\]

Let \( t_0 = 0 \) for the calculations in this section. For an exponential arrival process with parameter \( \lambda \), we know from equation (8) that the value of the SETR is a constant, and, therefore, we know that the investor will make positive returns for transitions occurring at all such times \( t \) such that \( t \in \left[ \frac{1}{\lambda}, \infty \right] \) i.e., \( t > \frac{1}{\lambda} \). Hence,

\[
q_{\exp} = \int_{\frac{1}{\lambda}}^{\infty} \lambda e^{-\lambda x} dx = \left[ -e^{-\lambda x} \right]_{\frac{1}{\lambda}}^{\infty} = e^{-1} < 0.5.
\]

Therefore, despite the expectation of the returns being 0, the probability of getting a positive return is less than half i.e., more often than not, an investor will make losses from carbon-risk, by holding on to a polluting stock, if the arrival process of the single transition event is exponential.

For the uniform distribution, the SETR is not constant in time and is constantly varying, and, accordingly, the criterion for making profits is \( p_i t_e > p_i(\theta_{\max} - t_e) \).

\[
\therefore t_e = \frac{\theta_{\max}}{2}.
\]

Hence,

\[
q_{uni} = \int_{\frac{\theta_{\max}}{2}}^{\theta_{\max}} \frac{1}{\theta_{\max}} dx = \frac{1}{2}.
\]

Therefore, for a uniform arrival process, holding on to a polluting stock is probabilistically risk-neutral, as not only does the expected premium offset the expected risk, but the probability of making and losing money from the trade is also equal.

For a Gamma distribution, the criterion is,

\[
p_i t_e > \frac{p_i}{b^a t_e^a e^{-bt_e}} \Gamma(a, bt_e).
\]

Rearranging the terms, we finally obtain,

\[
b^a t_e^a e^{-bt_e} > \Gamma(a, bt_e).
\]

To determine the probability of positive returns, we generate large ensembles of \( t_e \) drawn from a \( \Gamma(a, b) \) distribution for various combinations of \( a \) and \( b \) and estimate the probability in each case, by finding the proportion of cases in which the criterion holds.
(a) Variation of $q_\gamma$ with $\mu_\gamma, \sigma^2_\gamma$

(b) Variation of $q_\gamma$ with $a, b$

Figure 7: **3D Scatter plot depicting the variation of $q_\gamma$:** (a) With $a$ and $b$ (b) With $\mu_\gamma$ and $\sigma^2_\gamma$ (Some of the extreme points with high $\mu_\gamma$ and $\sigma^2_\gamma$ have not been shown in the plot, to get a better representation of the area that had a size-able $q_\gamma$)

On executing the simulations, we find that the dependence of $q_\gamma$ has a sigmoid appearance on both $a$ and $b$, with both having a positive correlation with the probability of positive returns (Figure 7a). As the mean and variance of the expected time of transition are inversely proportional to $b$ and $b^2$ respectively, despite being linear functions of $a$, both of them show a negative correlation with $q_\gamma$ (Figure 7b). However, as we can see from Figure 7, the value of $q_\gamma$ can be both higher and lower than 0.5 depending on the arrival process, unlike for exponential and uniform distributions of arrival processes, where $q \leq 0.5$ is independent of the parameters of the distribution.

6 Conclusion

A hope inducing observation that we can make from our dataset, is that most big firms are on a path to reducing their carbon footprint, as we have verified for the firms in the cross-section analysis, for the period of 2016-2020. The financial year 2021 would have seen an even more significant drop in carbon emissions, in light of the economic disruption caused by the COVID-19 pandemic. However, to truly be able to control accelerated levels of global warming, firms must significantly reduce their carbon footprint. Firms not being dynamic enough in reducing their carbon footprint faster, opens doors for government regulation and a forced acceleration in terms of carbon transition. Such action brings great risks to firms with a high or moderate dependency on fossil fuels. A rapid transition, forced upon firms could significantly shrink the value of the firm, i.e., diminish the utilities of its assets, increase its debt and in the most extreme cases, send the firms into bankruptcy, thereby sending the prices of the shares of the firm plummeting downwards. These risky shares would have attracted fewer and fewer investors with time, in the absence of a carbon transition risk premium. In Section 3 of this article, we perform a cross section analysis, wherein we confirm the existence of a significant carbon transition risk premium in the American large-cap market. The existence of such a premium implies investor awareness about an impending low-carbon transition of the economy. In Section 4, we attempt to quantify the value of this risk. However, since the knowledge on the nature of the transition is speculative, it is impossible to be able to find out the exact magnitude of the risk. Therefore we expect to measure the maximum exposure of the share of a firm to a single-big transition event, or the Single Event Transition Risk (SETR). For this, we assume that the expected returns that an investor can get from the risk-premium, offsets the expected risk of transition. Although the value and time of arrival of
the risks are what drive the premium, we go backwards, as the value of the risk is unknown, while premia can be estimated. Much like the value of the transition risk, the probability distribution for when the transition event will take place, is largely speculative. Therefore, without making any restrictive assumptions about the arrival process of the transition event, *i.e.*, for any given arrival process, our problem reduces to finding the value of the risk for which the fairly priced transition premia would be those found in the market. We also find the exact form of the SETR for some possible classes of arrival processes, namely, the exponential, uniform and Gamma arrival processes. Finally, we demonstrate that the existence of a fairly priced premium does not necessarily make it a neutral decision (neither advantageous nor disadvantageous) for individuals to invest in these firms, and that based on the arrival process, it would be possible for investors to invest in a firm with greater or lower probability, despite the expected returns from the transition risk premium offsetting the expected transition risk.
REFERENCES

[1] Adesina, O.S., 2018. Modelling Queuing System With Inverse Gamma Distribution: A Spreadsheet Simulation Approach.

[2] Andersson, M., Bolton, P. and Samama, F., 2016. Hedging climate risk. Financial Analysts Journal, 72(3), pp.13-32.

[3] Baranzini, A., Van den Bergh, J.C., Carattini, S., Howarth, R.B., Padilla, E. and Roca, J., 2017. Carbon pricing in climate policy: seven reasons, complementary instruments, and political economy considerations. Wiley Interdisciplinary Reviews: Climate Change, 8(4), p.e462.

[4] Bingler, J.A., Kraus, M. and Leippold, M., 2021. Cheap Talk and Cherry-Picking: What ClimateBert has to say on Corporate Climate Risk Disclosures. Available at SSRN.

[5] Black, F. and Scholes, M., 2019. The pricing of options and corporate liabilities. In World Scientific Reference on Contingent Claims Analysis in Corporate Finance: Volume 1: Foundations of CCA and Equity Valuation (pp. 3-21).

[6] Bluhm, C., Overbeck, L. and Wagner, C., 2005. An introduction to credit risk modeling. JOURNAL-OPERATIONAL RESEARCH SOCIETY, 56(12), p.1453.

[7] Bokenkamp, K., LaFlash, H., Singh, V. and Wang, D.B., 2005. Hedging carbon risk: Protecting customers and shareholders from the financial risk associated with carbon dioxide emissions. The Electricity Journal, 18(6), pp.11-24.

[8] Bolton, P. and Kacperczyk, M., 2021. Global pricing of carbon-transition risk (No. w28510). National Bureau of Economic Research.

[9] Bolton, P. and Kacperczyk, M., 2021. Do investors care about carbon risk?. Journal of Financial Economics.

[10] Bolton, P. and Kacperczyk, M.T., 2020. Signaling through carbon disclosure. Available at SSRN 3755613.

[11] Daniel, K. and Titman, S., 2005. Evidence on the characteristics of cross-sectional variation in stock returns (pp. 317-352). Princeton University Press.

[12] Delis, M.D., de Greiff, K. and Ongena, S., 2019. Being stranded with fossil fuel reserves? Climate policy risk and the pricing of bank loans. Climate Policy Risk and the Pricing of Bank Loans (April 21, 2019). Swiss Finance Institute Research Paper, (18-10).

[13] Ghoshal, A., 1962. Queues in series. Journal of the Royal Statistical Society. Series B (Methodological) , 1962, Vol. 24, No. 2 (1962), pp. 359-363

[14] Görgen, M., Jacob, A., Nerlinger, M., Riordan, R., Rohleder, M. and Wilkens, M., 2020. Carbon risk. Available at SSRN 2930897.

[15] Jameson, G.J.O., 2016. The incomplete gamma functions. The Mathematical Gazette, 100(548), p.298.

[16] Jung, J., Herbohn, K. and Clarkson, P., 2018. Carbon risk, carbon risk awareness and the cost of debt financing. Journal of Business Ethics, 150(4), pp.1151-1171.
[17] Kendall, D.G., 1953. Stochastic processes occurring in the theory of queues and their analysis by the method of the embedded Markov chain. The Annals of Mathematical Statistics, pp.338-354.

[18] Koch, N. and Bassen, A., 2013. Valuing the carbon exposure of European utilities. The role of fuel mix, permit allocation and replacement investments. Energy Economics, 36, pp.431-443.

[19] Kousky, C., Kopp, R.E. and Cooke, R.M., 2011. Risk premia and the social cost of carbon: a review. Economics: The Open-Access, Open-Assessment E-Journal, 5.

[20] Oestreich, A.M. and Tsiakas, I., 2015. Carbon emissions and stock returns: Evidence from the EU Emissions Trading Scheme. Journal of Banking & Finance, 58, pp.294-308.

[21] Olver, F.W., Lozier, D.W., Boisvert, R.F. and Clark, C.W. eds., 2010. NIST handbook of mathematical functions hardback and CD-ROM. Cambridge university press.

[22] Osborne, M.F., 1959. Brownian motion in the stock market. Operations research, 7(2), pp.145-173.

[23] Medhi, J., 1975. Waiting time distribution in a Poisson queue with a general bulk service rule. Management Science, 21(7), pp.777-782.

[24] Meinerding, C., Schüller, Y.S. and Zhang, P., 2020. Shocks to Transition Risk. Available at SSRN 3654155.

[25] Monasterolo, I., Zheng, J.I. and Battiston, S., 2018. Climate transition risk and development finance: A carbon risk assessment of China’s overseas energy portfolios. China & World Economy, 26(6), pp.116-142.

[26] Pástor, Ž., Stambaugh, R.F. and Taylor, L.A., 2020. Sustainable investing in equilibrium. Journal of Financial Economics.

[27] Peters, O., 2019. The ergodicity problem in economics. Nature Physics, 15(12), pp.1216-1221.

[28] Ritchie, J. and Dowlatabadi, H., 2015. Divest from the carbon bubble? Reviewing the implications and limitations of fossil fuel divestment for institutional investors. Review of Economics & Finance, 5(2), pp.59-80.

[29] Romani, M., Rydge, J. and Stern, N., 2012. Recklessly slow or a rapid transition to a low-carbon economy? Time to decide. Policy Paper, December, Centre for Climate Change Economics and Policy and Grantham Research Institute on Climate Change and the Environment.

[30] Rosenbloom, D., Markard, J., Geels, F.W. and Fuenfschilling, L., 2020. Opinion: Why carbon pricing is not sufficient to mitigate climate change—and how “sustainability transition policy” can help. Proceedings of the National Academy of Sciences, 117(16), pp.8664-8668.

[31] Ross, S.M., Kelly, J.J., Sullivan, R.J., Perry, W.J., Mercer, D., Davis, R.M., Washburn, T.D., Sager, E.V., Boyce, J.B. and Bristow, V.L., 1996. Stochastic processes (Vol. 2). New York: Wiley.

[32] Semieniuk, G., Campiglio, E., Mercure, J.F., Volz, U. and Edwards, N.R., 2021. Low-carbon transition risks for finance. Wiley Interdisciplinary Reviews: Climate Change, 12(1), p.e678.

[33] Shastrakar, D.F. and Pokley, S.S., 2018. Application of binomial distribution and uniform distribution to study the finite queue length multiple server queuing model. International Journal of Pure and Applied Mathematics, 120(6), pp.10189-10205.
[34] Steffen, B., Egli, F., Pahle, M. and Schmidt, T.S., 2020. Navigating the clean energy transition in the COVID-19 crisis. Joule, 4(6), pp.1137-1141.

[35] Voit, J., 2005. The statistical mechanics of financial markets. Springer Science & Business Media.

[36] Wen, F., Wu, N. and Gong, X., 2020. China’s carbon emissions trading and stock returns. Energy Economics, 86, p.104627.