Vortices in exciton-polariton condensates with polarization splitting

M Toledo Solano and Yuri G Rubo
Centro de Investigación en Energía, Universidad Nacional Autónoma de México,
Temixco, Morelos, 62580, Mexico
E-mail: ygr@cie.unam.mx

Abstract. The presence of polarization splitting of exciton-polariton branches in planar semiconductor microcavities has a pronounced effect on vortices in polariton condensates. We show that the TE-TM splitting leads to the coupling between the left and right half-vortices (vortices in the right and left circular components of the condensate), that otherwise do not interact. We analyze also the effect of linear polarization pinning resulted from a fixed splitting between two perpendicular linear polarizations. In this case, half-vortices acquire strings (solitons) attached to them. The half-vortices with strings can be detected by observing the interference fringes of light emitted from the cavity in two circular polarizations. The string affects the fringes in both polarizations. Namely, the half-vortex is characterized by an asymmetric fork-like dislocation in one circular polarization; the fringes in the other circular polarization are continuous, but they are shifted by crossing the string.

1. Introduction
Half-integer vortices (HV) are elementary topological defects in superfluids and Bose-Einstein condensates with multicomponent order parameter \([1]\). They are characterized by half-quantum change of the phase of the condensate, i.e., the phase is changed by \(\pm \pi\) after encircling the point of singularity. In condensates of exciton-polaritons in planar semiconductor microcavities, where the order parameter has two components, namely, the left and the right circular-polarization components, the vortex, in general, is described by two winding numbers \((k, m)\), where \(k\) is for the polarization angle, and \(m\) is for the phase. The HV is characterized by simultaneous change of both angles by \(\pm \pi\), so that \(k, m = \pm 1/2\) and there are four different HV’s \([2]\). Higher-order vortex entities can be considered as superpositions of several HV. For example, the integer phase vortex \((0, 1)\) is the superposition of two HV, \((1/2, 1/2)\) and \((-1/2, 1/2)\). Both the integer phase vortices \([3]\) and the half-integer vortices \([4]\) have been discovered recently in exciton-polariton condensates.

The experimental observations \([3, 4]\) revealed the importance of polarization splitting of polariton branches and, in particular, the presence of pinning of the condensate polarization to a specific crystallographic direction. The HV can be observed only when this splitting is sufficiently small. The goal of this paper is to present the brief theoretical discussion of the effects of weak polarization splitting on the properties of HV’s. In what follows we discuss two different types of splitting, the wave-vector dependent TE-TM splitting and the wave-vector independent splitting due to the cavity asymmetry.
2. Vortices in presence of TE-TM splitting

Near the bottom of lower polariton branch the kinetic energy density of polariton condensate is

\[ \mathcal{E} = \frac{\hbar^2 n}{2m^*} \frac{k^2 + m^2}{r^2} + \frac{\hbar^2 n}{4} \left( \frac{1}{m_t} - \frac{1}{m_l} \right) \frac{k^2 - m^2}{r^2} \cos[2(k - 1)\phi], \]

where \( m_t \) and \( m_l \) are the transverse (TE) and longitudinal (TM) effective masses of polaritons. The two-dimensional vector \( \psi = \{ \psi_x, \psi_y \} \) is the condensate wave function normalized to the condensate concentration \( n = |\psi|^2 \). The presence of TE-TM splitting does not destroy HV’s, but it results in the warping of their polarization texture. More importantly, it changes qualitatively the HV energies and HV interactions. We consider these effects below for the case of uniform exciton-polariton condensate without any potential disorder.

In the elastic region, where the condensate concentration is constant, the order parameter of linearly polarized condensate can be written as \( \psi = \sqrt{n} e^{i\theta} \{ \cos \eta, \sin \eta \} \), where \( \theta \) and \( \eta \) are the phase and polarization angles, respectively. In the case of small difference \( |m_l - m_t| \), one can take these angles as in unperturbed case. Namely, in the polar coordinates \((r, \phi)\) these angles depends on the azimuth as \( \theta = m \phi \) and \( \eta = k \phi \). The kinetic energy density then reads

\[ \mathcal{E} = \frac{\hbar^2 n}{2m^*} \left( \frac{1}{m_t} - \frac{1}{m_l} \right) \frac{k^2 - m^2}{r^2} \cos[2(k - 1)\phi], \]

\[ \frac{1}{m^*} = \frac{1}{2} \left( \frac{1}{m_t} + \frac{1}{m_l} \right). \]

The energy of single vortex in logarithmic approximation is found by integrating this expression over the microcavity plane. The integral should be cut at small \( k \) and for large \( r \) by the excitation spot radius \( R \). The core radius is \( a = h/\sqrt{2m^* \mu} \), where the chemical potential \( \mu \) can be measured by the blue-shift of the emission line of the polariton condensate. The value of \( a \) is typically about a few microns. Due to the oscillating factor, the second term in the integration contributes only for \( k = 1 \), and the energy of the vortex is

\[ E_{(k,m)} = \frac{\pi \hbar^2 n}{2} \left[ \left( \frac{1}{m_t} + \frac{1}{m_l} \right) (k^2 + m^2) + \left( \frac{1}{m_t} - \frac{1}{m_l} \right) (1 - m^2) \delta_{1,k} \right] \ln \left( \frac{R}{a} \right). \]

It is seen, that the energies of most of the vortices are defined by \( m^* \), but the energy of polarization “hedgehog” vortex \((1, 0)\) is defined by the pure longitudinal mass \( m_t \). This result can be visualized by consideration of polarization texture of integer vortices shown in figure 1. For the \((0, \pm 1)\) and \((-1, 0)\) vortices [figure 1(a-c)] the polarization field is longitudinal in some areas and transverse in others. For the \((1, 0)\) vortex [figure 1(d)] the field is longitudinal everywhere.

Without TE-TM splitting there is no long-range interaction between the left and right HV’s, that is between HV’s with the different sign of the product \( km \). The TE-TM splitting leads to weak coupling for a particular pair, the \((1/2, 1/2)\) and \((1/2, -1/2)\) half-vortices. This can be easily understood since when these HV’s are put together they form the “hedgehog” vortex with the energy \( E_{(1/0)} = (\pi \hbar^2 n/m_t) \ln(R/a) \). When this pair are well separated the energy of the system is equal to the doubled energy of the single HV, that is to \( (\pi \hbar^2 n/m^*) \ln(R/a) \). The interaction energy of these vortices separated by distance \( r_{12} \) is

\[ V_{12} = \frac{\pi \hbar^2 n}{2} \left( \frac{1}{m_t} - \frac{1}{m_l} \right) \ln \left( \frac{r_{12}}{a} \right). \]

The long-range interaction between left and right HV’s can be important for the analysis of the Berezinskii-Kosterlitz-Thouless (BKT) transitions (without this interaction there are two decoupled BKT transitions \([2, 5]\)). It should be noted also that the interrelation between \( m_t \) and \( m_l \) depends on the detuning of the frequency of the cavity photon mode from the center of the stop-band of the distributed Bragg mirror \([6]\). So, one can have both weak attraction and weak repulsion of the \((1/2, 1/2)\) and \((1/2, -1/2)\) half-vortices.
3. Half-vortices with strings and interference fringes

The TE-TM splitting is absent for position-independent order parameter. In this section we consider the effects of the other type of polarization splitting observed in exciton-polariton condensates. In this case the energy of a uniform condensate becomes dependent on the orientation of polarization. The corresponding term in the condensate energy density is [7]

\[ \mathcal{H}_\epsilon = \frac{\epsilon}{2} \left( n - |\psi_x|^2 + |\psi_y|^2 \right) = \epsilon n \sin^2(\eta), \quad (6) \]

where \( \epsilon \) is the energy of the splitting and we have chosen the \( x \)-axis along the direction of polarization pinning (the easy axis for polarization), so that the energy of the condensate is minimized for the horizontal polarization. Below we consider the case of weak pinning, when the energy of splitting \( \epsilon \ll \mu \) and the characteristic length \( b = \hbar / \sqrt{2m^*\epsilon} \gg a \).

In conditions of cw excitation one can expect that the HV will be formed with the polarization texture that minimizes the total elastic energy. The minimization of \( \int (\mathcal{T} + \mathcal{H}_\epsilon) d^2r \) for the case \( m_t = m_l \) gives the equations for the phase and polarization angles,

\[ \Delta \theta = 0, \quad 2b^2 \Delta \eta = \sin(2\eta). \quad (7) \]

The solution to the first equation describes the same uniform change of the phase angle with the azimuth, \( \theta = m\phi \) with \( m = \pm 1/2 \). However, the polarization angle \( \eta \) changes from 0 to \( \pm \pi \) only in a narrow region \( \sim b \) when one encircles the core far away from the center. This region of rapid change of \( \eta \) defines a string or soliton attached to the HV (see figure 16.1 in [1]). Formally, the string is defined by the line where polarization becomes vertical.

The numerical solution to the sine-Gordon equation (7) is shown in figure 2(a). It was calculated with the boundary condition \( d\eta/d\phi = k \) near the core \( (r \ll b) \) with the winding number \( k = 1/2 \). The axes are chosen such that \( x \)-axis is directed to the right and \( y \)-axis is directed downwards, so that the azimuth \( \phi \) increases for the clockwise rotation. For large \( r \gg b \), the gradient of \( \eta \) is zero almost everywhere, except the region of large negative \( y \) and \( |x| \sim b \), where the angle changes according to the known kink solution \( \eta = 2\tan^{-1} \left[ \exp(x/b) \right] \). This way the solution in figure 2(a) describes the HV with the string going upwards.

In general, any orientation of the string is possible and HV’s with the strings oriented differently have the same energy for the case \( m_t = m_l \). Note that the string carries the energy proportional to its length. In finite size condensates the string can terminate on the boundary. In this case the HV will be attracted by the boundary and can relax towards it. The string can terminate also in another HV with the opposite sign of the winding number \( k \), and this leads to the interaction of left and right HV’s. The interaction energy between HV’s with opposite signs
Figure 2. (a) The contour plot showing the change of the polarization angle $\eta$ for HV with the string going vertically up. The solid lines are drawn for the values of $\eta$ changing from 0 to $\pi$ (on the line going downwards) with the step $\pi/12$. Parts (b) and (c) show the interference fringes (see text) in the left-circular ($\sigma^-$) and in the right-circular ($\sigma^+$) polarizations, respectively.

of $k$ coupled by the string becomes proportional to the length of the string and grows linearly with the distance.

The behavior of the interference fringes in two circular polarizations is shown in figure 2(b,c) for the string terminating into the (1/2, 1/2) half-vortex. These fringes appear when one observes the interference of the signal emitted from the polariton condensate in a circular polarization with plane wave of the same intensity propagating in the downward direction. Experimentally [3, 4], this plane wave originates from the same condensate but from a different place, where the order parameter is approximately constant. The signal emitted in $\sigma^\pm$ polarization is $\propto \exp\{i(\theta \mp \eta)\}$, respectively, and the fringes in figure 2(b,c) were calculated as $|e^{i(\theta \mp \eta)} + e^{i\kappa y}|^2$. One can see that the half-vortex with the string is characterized by an asymmetric fork-like dislocation in one circular polarization. Moreover, the fringes that cross the string are shifted by half wave-length in both circular polarizations. The choice of orientation of the plane wave-vector to be collinear with the string (so that the fringes are perpendicular to it) is the best to observe the string in the interference pattern.

Acknowledgments
We are grateful to Alexey Kavokin, Jonathan Keeling, and Igor Luk’yanchuk for discussions. This work was supported in part by the grant IN107007 of DGAPA-UNAM.

References
[1] Volovik G E 2003 The Universe in a Helium Droplet (The International Series of Monographs on Physics no 117) (Oxford: Clarendon Press)
[2] Rubo Y G 2007 Phys. Rev. Lett. 99 106401
[3] Lagoudakis K G, Wouters M, Richard M, Baas A, Carusotto I, André R, Dang L S and Deveaud-Plédran B 2008 Nature Physics 4 706–10
[4] Lagoudakis K G, Ostatnický T, Kavokin A V, Rubo Y G, André R and Deveaud-Plédran B 2009 Science in press
[5] Keeling J 2008 Phys. Rev. B 78 205316
[6] Panzarini G, Andreani L C, Armitage A, Baxter D, Skolnick M S, Astratov V N, Roberts J S, Kavokin A V, Vladimirova M R and Kaliteevski M A 1999 Phys. Rev. B 59 5082–89
[7] Kasprzak J, André R, Dang L S, Shelykh I A, Kavokin A V, Rubo Y G, Kavokin K V and Malpuech G Phys. Rev. B 75 045326