Langevin dynamics of the deconfinement transition for pure gauge theory

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We investigate the effects of dissipation in the deconfinement transition for pure SU(2) and SU(3) gauge theories. Using an effective theory for the order parameter, we study its Langevin evolution numerically. Noise effects are included for the case of SU(2). We find that both dissipation and noise have dramatic effects on the spinodal decomposition of the order parameter and delay considerably its thermalization. For SU(3) the effects of dissipation are even larger than for SU(2).

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I. INTRODUCTION

Recent results from lattice QCD [1], corroborated by experimental data from BNL-RHIC [2], indicate that strongly interacting matter under extreme conditions of temperature and pressure undergoes a phase transition to a deconfined plasma. Such extreme conditions are believed to have happened in the early universe, and might also be found in the core of neutron stars [3].

The process of phase conversion during the deconfinement transition can occur in different ways. For a pure gauge SU(N) theory, the trace of the Polyakov loop provides a well-defined order parameter [4,5,6], and one can construct an effective Landau-Ginzburg field theory based on this quantity [7,8]. The effective potential for $T \ll T_c$ has only one minimum, at zero, where the whole system is localized. With the increase of the temperature new minima appear ($N$ minima for $Z(N)$, the center of SU(N)). At the critical temperature, $T_c$, all the minima are degenerate, and above $T_c$, the new minima become the true vacuum states of the theory, so that the system starts to decay. In the case of SU(3), within a range of temperatures close to $T_c$ there is a small barrier, and the process of phase conversion will be guided by bubble nucleation. For larger $T$, the barrier disappears and the system explodes in the process of spinodal decomposition. For SU(2), the transition is second-order, and there is never a barrier to overcome.

In this paper, we consider pure SU(2) and SU(3) gauge theories, without dynamical quarks, that are rapidly driven to very high temperatures, well above $T_c$, and decay to the deconfined phase via spinodal decomposition. We are particularly interested in the effect of noise and dissipation on the time scales involved in this “decay process”. In what follows, we adopt an effective model proposed in Ref. [8] for the order parameter and the effective potential. Numerical calculations for the evolution of the order parameter are performed on a lattice, using a local Langevin equation.

The paper is organized as follows. Section II briefly describes the effective model for the order parameter. In Section III, we consider the Langevin evolution, discussing how to fix the dissipation coefficient from lattice simulations, and present our results for SU(2) and SU(3). Section IV contains our final remarks.

II. THE EFFECTIVE MODEL

The model proposed in [8] intends to provide a better representation of lattice results for the gluon plasma equation of state as compared to the usual bag model. It is obtained combining a few phenomenological inputs with $Z(N)$ symmetry and some known features of the perturbative equation of state. In particular, in a temperature range going from the deconfinement temperature $T_d$ to $5T_d$ the model gives reasonable results and exhibits a thermodynamic behaviour that is coherent with data obtained from lattice simulations.

In this approach, thermodynamic properties are determined by functions of the Polyakov loop, defined in Euclidean finite temperature gauge theories as [9]:

$$P(\vec{x}) = e^{\mathcal{T} \frac{1}{g} \int_0^{1/T} d\tau A_0(\vec{x}, \tau)},$$ (1)

where $\mathcal{T}$ denotes Euclidean time ordering, $g$ is the gauge coupling constant and $A_0$ is the time component of the vector potential. We work with SU(2) and SU(3), representing the color degrees of freedom. Consequently, we have a $Z(N)$ symmetry for the case of pure gauge theories that is spontaneously broken. It would be explicitly broken in the presence of quarks.

Working in the imaginary time framework, we have bosonic fields being periodic and fermionic fields being antiperiodic in the imaginary time $\tau$:

$$A_\mu(\vec{x}, \beta) = +A_\mu(\vec{x}, 0), \quad q(\vec{x}, \beta) = -q(\vec{x}, 0).$$ (2)

Any gauge transformation periodic in $\tau$ respects these boundary conditions. However, as demonstrated by 't Hooft [5], one can consider more general gauge transformations which

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are only periodic up to the center of the group: \( \Omega(\bar{x}, \tilde{\beta}) = \Omega \), \( \Omega(\bar{x}, 0) = 1 \).

Color adjoint fields are invariant under these transformations, while those in the fundamental representation are not:

\[
A^\Omega(\bar{x}, \tilde{\beta}) = \Omega^\dagger A^\mu(\bar{x}, \tilde{\beta}) \Omega^\mu = A^\mu(\bar{x}, \tilde{\beta}) = +A^\mu(\bar{x}, 0),
\]

where \( \psi = 0 \) represents confinement. It is interesting to connect the \( \psi \) used here and the trace of the Polyakov loop, used in Ref. \( [7] \) as the order parameter. For the diagonalized matrix we have the trace, according to our parametrization, as \( e^\theta + e^{-\theta} \), namely \( TrL = 2 \cos(\phi) \), or, as defined above, \( TrL = 2 \cos(\pi(1 - \psi)/2) \). So, when \( \psi = 0 \) we have \( TrL = 0 \), which represents confinement, and when \( \psi \to 1 \), then \( TrL \to 1 \), representing the deconfined state.

The phase transition in this case is second order, as expected \( [8] \). The value of \( M \) can be determined from the deconfinement temperature through the relation \( T_d = (3/2)^{1/2} M/\pi \approx 0.38985 M \), so that it is possible to extract the deconfining temperature from the lattice and then fix \( M \). The minimum of the potential occurs for

\[
\psi_0 = \sqrt{1 - \frac{3M^2}{2T^2 \pi^2}}.
\]

For \( SU(3) \) there are three eigenvalues: \( \phi_1 = \phi \), 0 and \( \phi_{-1} = -\phi \). The potential assumes the form:

\[
V = -T^3 \frac{8\pi^2}{45} + \frac{T^3}{6\pi^4} \left[ 8\phi^2 (\phi - \pi)^2 + \phi^2 (\phi - 2\pi)^2 \right]
+ \frac{2TM^2}{3} + \frac{TM^2}{2\pi^2} \left[ 2\phi (\phi - \pi) + \phi (\phi - 2\pi) \right].
\]

Again, it is useful to rewrite the potential in terms of a new variable \( \psi = 2\pi/3 - \phi \), so that one obtains

\[
V = \frac{8\pi^2 T^3}{405} + \left( \frac{3}{2\pi} TM^2 - \frac{2}{3} T^3 \right) \psi^2
- \frac{2}{3\pi} T^3 \psi^3 + \frac{3}{2\pi} T^3 \psi^4.
\]

Now, \( M \) and \( T_d \) are related as follows:

\[
T_d = \frac{9}{20\sqrt{10} M} \approx 0.45296 M,
\]

and the minimum is at

\[
\psi_0 = \frac{\pi T + 3\sqrt{T^2 \pi^2 - 2M^2}}{6T}.
\]

In this case, \( TrL = e^\theta + 1 + e^{-\theta} \) and the connection with \( \psi \) becomes \( TrL = \frac{2}{3} \cos (\frac{2\pi}{3} - \psi) + \frac{1}{3} \).

Immediately above the critical temperature, the \( SU(3) \) potential presents a barrier between the old and the new vacua. This barrier, however, is very small and quickly disappears with the increasing of the temperature. One should notice that above \( 2T_c \) the changes in the potential are negligible.

### III. LANGEVIN EVOLUTION

Let us now consider the real-time evolution of the order parameter for the breakdown of \( Z(N) \). We assume the system to be characterized by a coarse-grained free energy

\[
F(\phi, T) = \int d^3x \left[ \frac{B}{2} (\nabla \phi)^2 + V_{eff}(\phi, T) \right],
\]

where \( \psi = 0 \) represents confinement. It is interesting to connect the \( \psi \) used here and the trace of the Polyakov loop, used in Ref. \( [7] \) as the order parameter. For the diagonalized matrix we have the trace, according to our parametrization, as \( e^\theta + e^{-\theta} \), namely \( TrL = 2 \cos(\phi) \), or, as defined above, \( TrL = 2 \cos(\pi(1 - \psi)/2) \). So, when \( \psi = 0 \) we have \( TrL = 0 \), which represents confinement, and when \( \psi \to 1 \), then \( TrL \to 1 \), representing the deconfined state.
where $V_{\text{eff}}(\phi, T)$ is the effective potential obtained in the last section, and $B = \pi^2 T / g^2$ for $SU(2)$ and $B = 4T / g^2$ for $SU(3)$. The time evolution of the order parameter and its approach to equilibrium will be dictated by the following Langevin equation

$$B \left( \frac{\partial^2 \psi}{\partial t^2} - \nabla^2 \psi \right) + \Gamma \frac{\partial \psi}{\partial t} + V'_{\text{eff}}(\psi) = \xi, \quad (14)$$

where $g$ is the QCD coupling constant, and $\Gamma$ is the dissipation coefficient, which is usually taken to be a function of temperature only, $\Gamma = \Gamma(T)$. The function $\xi$ is a stochastic noise assumed to be gaussian and white so that $\langle \xi(\vec{x}, t) \rangle = 0$ and $\langle \xi(\vec{x}, t) \xi(\vec{x'}, t') \rangle = 2\Gamma \delta(\vec{x} - \vec{x'})\delta(t - t')$. The noise and dissipation terms are originated from thermal and quantum fluctuations resulting either from self-interactions of the Polyakov loop field or from the coupling to different fields (such as chiral fields). The case with only first-order time derivative was considered in Ref. [10].

This description is admittedly very simplified. A more complete analysis should consider different contributions of noise and dissipation terms and memory kernels instead of simple Markovian terms proportional to the first time derivative of the field $\xi(\vec{x}, t)$ [11, 12]. In general, one obtains a complicated dissipation kernel that simplifies to a multiplicative dissipation term which depends quadratically on the amplitude of the field as $\Gamma_1(T) \psi^2(\vec{x}, t) L(\xi, t)$ where $\Gamma_1$ is determined by imaginary terms of the effective action for $\psi$ and depends weakly (logarithmically) on the couplings. The fluctuation-dissipation theorem implies, then, that the noise term will also contain a multiplicative contribution of the form $\psi(\vec{x}, t) \psi(\vec{x}, t)$, and be in general non-Markovian. The white noise limit is reobtained only for very high temperatures.

For the $SU(2)$ case we have fixed $\Gamma$ in the following way. We have used pure-gauge Euclidean lattice Monte Carlo simulations in the line discussed in Ref. [13]. In this approach, spinodal decomposition is obtained on the lattice performing local heat-bath updates of gauge field configurations at $\beta = 4 / g^2 = 3$, after thermalizing the lattice at $\beta = 4 / g^2 = 2$. The critical value of $\beta$ for deconfinement is found to be $\beta_d \sim 2.3$. $\Gamma$ is then extracted by comparing the short-time exponential growth of the correlation function $\langle L(k, t)L(-k, t) \rangle$ predicted by the lattice simulations [13] and the Langevin description, assuming of course that both dynamics are the same. Making this comparison for the lowest lattice momentum mode, it is found that $\Gamma = 7.6 \times 10^3 T^3 / \mu$, where $\mu$ is a time scale relating Monte Carlo time and real time. Assuming that typical thermalization times are of the order of a few fm/c, we obtain $\Gamma \sim 10^3 \text{fm}^{-2}$.

In our numerical calculations we solve Eq. (14) on a cubic spacelike lattice with $64^3$ sites under periodic boundary conditions. We use a semi-implicit finite-difference scheme for the time evolution and a finite-difference Fast Fourier Transform for the spatial dependence [14]. For $SU(2)$, the critical temperature is $T_c = 302$ MeV [15] and we obtain $M = 775$ MeV. We took the average of several realizations with random initial configurations around $\psi \sim 0$. We consider the time dependence of the volume average of $\psi$

$$\langle \psi \rangle = \frac{1}{N^3} \sum_{ijk} \psi_{ijk}(t), \quad (15)$$

where $N$ is the number of lattice sites in each spatial direction, and $i, j, k = 1, \cdots, N$ are the lattice sites. In Fig. 2 we plot $\langle \psi \rangle / \psi_0$, where $\psi_0 > 0$ is the positive minimum of the bare effective potential, for three situations: no dissipation and no noise (dotted curve), no noise (dashed curve) and full solution (solid curve). When considering noise, we have added the appropriate counterterms to make the equilibrium solution independent of the lattice spacing [16]. All curves are for $T = 6.6 T_d$.

Clearly seen in Fig. 2 is the large effect of dissipation, which delays the rapid exponential growth of the order parameter due to spinodal decomposition. The retardation seen here for the deconfinement transition is substantially larger than the corresponding delay seen for the chiral condensate evolution in Ref. [17]. The effect of noise is also in the direction of delaying equilibration, as expected. Also expected, and clearly
shown in Fig. 2 is the effect of noise in the equilibrium value of $\psi$ which is larger than $\psi_0$.

For SU(3) one has $T_d = 263$ MeV \(^\text{[18]}\), so that $M = 580$ MeV. The results of our simulations at $T = 6.6 T_d$ are shown in Fig. 3. Here we are using the same lattice and same dissipation $\Gamma$ as for SU(2). As seen in this figure, the effect of dissipation is even more dramatic than for SU(2), with the proviso of course that we are using the value of $\Gamma$ extracted from SU(2) lattice simulations. As mentioned earlier, immediately above the critical temperature the SU(3) potential presents a barrier between a local minimum and an absolute minimum. However, this barrier has no effect on the delay seen in Fig. 3 since our simulations are done for high temperatures, $T \gg 2 T_d$. We have not investigated the effect of noise in this case because the appropriate renormalization counter-terms for an effective potential with a first-order transition are not yet available \(^\text{[16]}\).

IV. SUMMARY AND OUTLOOK

We have investigated the effects of dissipation and noise in the deconfinement transition of SU(2) and SU(3) pure gauge theories. We have used the effective model proposed in Ref. \cite{8}, which combines phenomenological inputs with Z(N) symmetry and some known features of the perturbative equation of state. The model provides a reasonable representation of lattice results for the pure-gluon plasma equation of state in the temperature range between $T_c$ and $5T_c$. We have performed numerical simulations for the evolution of the order parameter on a spatial cubic lattice using a local Langevin equation. We find that both dissipation and noise have dramatic effects on the spinodal decomposition of the SU(2) order parameter, delaying considerably its thermalization. Dissipation effects are even larger for SU(3).

The present work must be improved in several aspects. Perhaps the most important one is in the method used to extract the dissipation coefficient $\Gamma$ \(^\text{[13]}\). This was done using Euclidean lattice Monte Carlo simulations, in which spinodal decomposition of the order parameter is obtained performing local heat-bath updates of gauge field configurations above the deconfinement temperature. One of the major uncertainties in this approach is the relation between Monte Carlo updates and real time. Another source of uncertainties comes from a richer structure of noise and dissipation terms, including an evaluation of memory kernels. It is widely known that, in general, quantum corrections lead to complicated dissipation kernels that only in very special situations simplify to an additive noise term as used here. These issues will be considered in a future publication \cite{15}.

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