Medium-enhanced $c\bar{c}$ radiation

Maximilian Attems,$^{1, *}$ Jasmine Brewer,$^{1, †}$ Gian Michele Innocenti,$^{2, ‡}$ Aleksas Mazeliauskas,$^{1, §}$ Sohyun Park,$^{1, ¶}$ Wilke van der Schee,$^{1, † †}$ and Urs Achim Wiedemann$^{1, † † †}$

$^1$Theoretical Physics Department, CERN, 1211 Geneva 23, Switzerland
$^2$Experimental Physics Department, CERN, 1211 Geneva 23, Switzerland

We show that the same QCD formalism that accounts for the suppression of high-$p_T$ hadron and jet spectra in heavy-ion collisions predicts medium-enhanced production of $c\bar{c}$ pairs in jets.

Introduction. In ultra-relativistic heavy-ion collisions, heavy flavor quarks (charm and beauty) are produced in high-momentum transfer processes, which are calculable in perturbative QCD (pQCD) [1]. As these heavy quarks traverse the Quark Gluon Plasma (QGP) created in the collision, they probe its properties. Over the lifetime of the QGP, charm and beauty quarks are stable and can be tagged experimentally. This makes them ideally suited for studying quark propagation in the QCD plasma [2].

Theory predicts that energetic quarks and gluons (partons) lose energy due to medium-induced gluon radiation while traversing the QGP [3–5]. This jet quenching phenomenon is observed in heavy-ion collisions as a generic suppression of high-$p_T$ single inclusive hadron and jet spectra [6]. Here, we show that the same jet quenching formalism that accounts for these suppression phenomena leads to a medium-enhanced $c\bar{c}$ pair production within high-$p_T$ jets.

In hadronic collisions, charm is produced in $c\bar{c}$-pairs of squared invariant mass $Q^2 = (p_c + p_{\bar{c}})^2 = (p_c + p_{\bar{c}})_{\mu}^\mu$, bounded by the charm quark mass and the partonic center of mass energy, $4m_c^2 \leq Q^2 \leq \hat{s}$. Most $c\bar{c}$ pairs are produced at large relative pair momentum, $Q^2 \sim \mathcal{O}(\hat{s})$. As the QGP does not affect such large-$Q^2$ short-distance processes, the total charm yield is (almost) unmodified by the medium, although medium modification of the $c \to c g$ splitting softens the charm transverse momentum distribution [5]. Here, the “almost” refers to the small phase space region $Q^2 \ll \hat{s}$ that is dominated by the medium modification of $g \to c\bar{c}$ on which this Letter focuses.

In the collinear limit $Q^2 \ll \hat{s}$, partonic cross sections for $c\bar{c}$ production factorize. For instance, the cross section for $gg \to c\bar{c}X$ can be written as

$$\hat{s} = \frac{\alpha_s}{2\pi} Q^2 P_{g\to c\bar{c}}(z).$$

The $g \to c\bar{c}$ splitting function depends on the momentum fraction $z$ carried by the charm quark and the virtuality $Q$ of the gluon,

$$P_{g\to c\bar{c}}^{\text{vac}} = \frac{1}{2} \left( z^2 + (1-z)^2 + \frac{2m_c^2}{Q^2} \right).$$

We use $E_g$ to denote the gluon energy and $2k$ the relative $c\bar{c}$ pair momentum transverse to the direction of the gluon. For collinear splittings $k \ll zE_g, (1-z)E_g$, the squared gluon virtuality is $Q^2 = m_c^2(1-z)/(1-z^2)$.

For sufficiently high gluon energy, the gluon is boosted with respect to the QGP by a Lorentz factor $\gamma = E_g/Q$, and the formation time of the $c\bar{c}$ pair is delayed by $\gamma$. In this topology, $g \to c\bar{c}$ is the long-distance process that is modified by the medium, see Fig. 1. Our calculation will show that $c\bar{c}$ radiation is enhanced in the medium. Qualitatively, this can be understood in terms of gluons that require interactions with the QGP to overcome the mass threshold $Q^2 > 4m_c^2$ for splitting. While this is a power-suppressed contribution to the total charm yield, it can be singled out by searching for $c\bar{c}$ pairs within the same jet.

Medium-modified $g \to c\bar{c}$ splitting function. For the $q \to qg$ and $g \to gg$ leading-order splitting functions, medium modifications have been calculated in the BDMPS-Z formalism [3, 4, 7] and in several related set-ups [8, 9]. These calculations resum the effects of multiple interactions between the QCD plasma and the splitting process in a close-to-eikonal formulation, where the energy of the parent parton is much larger than any transverse momentum or mass scale. For the $g \to c\bar{c}$ splitting function, medium-modifications have been calculated to first order in opacity [10, 11] as well as in the BDMPS-Z
the path-integral formalism where they read \[ \frac{1}{Q^2} P_{g \rightarrow c\bar{c}} \equiv \frac{1}{Q^2} P_{g \rightarrow c\bar{c}}^\text{vac} + \frac{1}{Q^2} P_{g \rightarrow c\bar{c}}^\text{med} \]

\[ = 2 \Re \frac{1}{8 E_g} \int_0^\infty dt \int_t^\infty d\tilde{t} \int_0^\infty dr \]

\[ \times e^{-\frac{m^2_2}{2} (t-\tilde{t}) c[t]-c[\tilde{t}]} e^{-\frac{1}{4} \int_0^\infty d\xi \hat{q}(\xi, z) r^2 e^{-i \cdot \hat{r}}} \]

\[ \times \left[ \frac{m^2_2}{z(1 - z)} + \frac{z^2 + (1 - z)^2}{z(1 - z)} \frac{\partial}{\partial x} \frac{\partial}{\partial r} \right] \mathcal{K}[x = 0, t; r, \tilde{t}] . \]

This expression has the space-time interpretation of a gluon that splits into a $c\bar{c}$ pair at longitudinal positions $t$ ($\tilde{t}$) in amplitude (complex conjugate amplitude). In the multiple soft scattering approximation, $\mathcal{K}[x, t; r, \tilde{t}]$ is the path-integral of a harmonic oscillator with imaginary potential

\[ \mathcal{K}[x, t; r, \tilde{t}] = \int_{p(t)=x} \mathcal{D}p e^{i \int_{t}^{\tilde{t}} d\xi \left( \frac{E_g(1 - z)}{2} p^2 - \hat{q}(\xi, z) x^2 \right)} . \]

It describes how the $c\bar{c}$ dipole grows from transverse size $x = 0$ at $t$ to size $r$ at $\tilde{t}$. Equation (3) depends on the kinematics of the splitting and on a single medium property, the quenching parameter $\hat{q}(\xi, z)$ along the parton trajectory in the medium.

Both adjoint and fundamental color configurations contribute in a $z$-dependent way to the interaction of the $g \rightarrow c\bar{c}$ splitting with the QGP \[ [11] \]. This dependence can be factorized from the quenching parameter, $\hat{q}(\xi, z) \equiv (C_F - C_A z(1 - z)) \hat{q}$. For small $z$, the quenching parameter is hence $\hat{q} = C_F \hat{q}$, in contrast to $\hat{q}_A \equiv C_A \hat{q}$ for $q \rightarrow g g$ and $g \rightarrow g g$, since the leading medium modification of these splittings is from the scattering of the emitted gluon \[ [3, 4, 7] \]. To estimate $\hat{q}$ we considered recent constraints \[ [12–16] \] on $\hat{q}_A$ from heavy-ion data to determine the momentum transferred from the medium

\[ \langle q^2 \rangle^\text{med} = C_F \int_{\tau_f}^{\tau_i} d\xi \hat{q}(\xi) . \]

Here, $\tau_i/\tau_f$ denote the initial and final time within which jet-medium interactions occur. Explicit calculation of the line integrals for the jet quenching models in \[ [15] \] yields consistently $4 \text{ GeV}^2 < \langle q^2 \rangle^\text{med} < 8 \text{ GeV}^2$ for the $0$–$5\%$ most central PbPb collisions at $\sqrt{s_{NN}} = 5.02 \text{ TeV}$. To be conservative about possible model dependencies \[ [12–14, 16, 17] \], we consider the wider range $2 \text{ GeV}^2 < \langle q^2 \rangle^\text{med} < 8 \text{ GeV}^2$. We evaluate Eq. (3) for a static medium of length $L = \tau_f - \tau_i$ and $C_F \hat{q} L = \hat{q} L = \langle q^2 \rangle^\text{med}$ that leads to equivalent quenching (see Ref. \[ [11] \] for details). With $\hat{q} L$ fixed, the only additional dependence on $L$ is through the dependence on the gluon energy. For the phenomenological results presented here, we choose $L = 4$ fm.

Figure 2 shows numerical results of the medium-modification of Eq. (3) for a momentum transfer $\hat{q} L = 4 \text{ GeV}^2$ \[ [18] \]. Medium-induced transverse momentum broadening leads to a characteristic enhancement at the scale $k^2 \sim \hat{q} L$ while depleting the vacuum distribution of $c\bar{c}$ pairs at very small relative momenta $k^2$. This can be understood in terms of transverse Brownian motion of the charm quarks in the medium, which pushes the collinear pairs to larger $k^2$. Compared to the vacuum splitting function, the medium-modified splitting can be almost $100\%$ larger in a range of intermediate $k^2$ for $\hat{q} L = 4 \text{ GeV}^2$.

**Modified $c\bar{c}$ yield in parton showers.** To identify the $g \rightarrow c\bar{c}$ splitting in Eq. (1), one requires $Q^2 \ll \hat{s}$. It is a generic feature of QCD that this scale difference $Q^2 \ll \hat{s}$ leads to a large logarithm that requires resummation. The right-hand side of Eq. (1) is therefore written in terms of the cross section $\tilde{\sigma}^{gg \rightarrow g}$ for a gluon jet, and the $c\bar{c}$ pair can arise at any stage in the branching history of that gluon. This resummation is accounted for in a parton shower. The medium modification of $c\bar{c}$ pairs in jets depends both on the medium modification itself (Eq. (3)) and on how the parton shower distributes $c\bar{c}$ pairs in $z$, $k^2$, and $E_g$.

Parton showers are simulated by evaluating branching probabilities defined in terms of parton splitting functions. In principle, a medium-modified parton shower should include the modification of all splitting functions. Though a medium-modified parton shower based on the BDMPS-Z multiple soft scattering formalism exists \[ [19] \], it does not include mass effects and the modification of the $g \rightarrow c\bar{c}$ splitting. Since the $g \rightarrow c\bar{c}$ splitting is sufficiently rare and to leading $O(\alpha_s)$, we expect that the medium modification of $g \rightarrow c\bar{c}$ can be implemented by...
reweight $c\bar{c}$ pairs in a vacuum parton shower by the
phase-space differential weight factor [11]

$$1 + \frac{\left( \frac{1}{\kappa^2} \frac{1}{P_{g\rightarrow c\bar{c}}} \right)^{\text{med}} (E_g, \kappa^2, z)}{\left( \frac{1}{\kappa^2} \frac{1}{P_{g\rightarrow c\bar{c}}} \right)^{\text{vac}} (\kappa^2, z)}.$$  \hspace{1cm} (5)

This prescription for computing the medium modification of $c\bar{c}$ pairs assumes that the modification of other splitting functions (especially $g \rightarrow gg$) does not substantially modify the phase space for producing gluons that could produce $c\bar{c}$ pairs. In the Supplemental Material, we corroborate our main results by instead including the modification of $g \rightarrow q\bar{q}, g \rightarrow gg$ and $q \rightarrow gg$ splitting functions in a simplified parton shower.

For our main results we simulate $pp$ collisions at $\sqrt{s_{NN}} = 5.5$ TeV in Pythia 8.3 (Monash tune) [20] with initial state radiation (ISR) off and use FastJet [21] to reconstruct anti-$k_T$ jets with $R = 0.4$. We select jets with exactly one $D^0\bar{D}^0$ pair, which leads to a high-purity sample of jets in which the $D^0\bar{D}^0$ pair came from one $g \rightarrow c\bar{c}$ splitting. Including ISR increases the total jet yield but does not impact the ratio of $D^0\bar{D}^0$-tagged and inclusive jets on which our argument is based. The probability of having more than one $g \rightarrow c\bar{c}$ splitting in a $D^0\bar{D}^0$-tagged jet is below 1% for $p_T^{jet} < 100$ GeV. We compute the effect of medium modification by reweighting each $g \rightarrow c\bar{c}$ splitting by the factor Eq. (5).

A comment on momentum broadening. In the next section, we will elaborate on the phenomenological signatures of the modified $g \rightarrow c\bar{c}$ splitting in the enhanced yield of $c\bar{c}$ pairs. Here, we wish to comment briefly on other features of the $g \rightarrow c\bar{c}$ modification. The BDMPS-Z formalism generically predicts the broadening of the relative momentum $\kappa$, in this case of the $c\bar{c}$ pair, due to medium effects. We show in Fig. 3(a) the normalized probability distribution of $\kappa^2$ in vacuum (blue) and after medium modification (red). The $g \rightarrow c\bar{c}$ splitting could prove to be a clean process in which to access this characteristic broadening, which depletes the yield of $c\bar{c}$ pairs at small $\kappa^2$. We also note that the $E_g$-dependence of $P_{g\rightarrow c\bar{c}}$ can be understood in terms of formation time physics: with increasing $E_g$ the spatial position of the $g \rightarrow c\bar{c}$ vertex is boosted to larger distances/times where the $c\bar{c}$ pair sees a smaller part of the medium, and hence $P_{g\rightarrow c\bar{c}}$ decreases [11]. We show in Fig. 3(b) the normalized probability distributions for the angular separation $r_{c\bar{c}}$ between the $c$ and $\bar{c}$ with and without medium modification, for different jet $p_T$. In principle, changing the jet $p_T$ gives access to formation time-dependence, since higher jet $p_T$ generically accesses $g \rightarrow c\bar{c}$ with higher average $E_g$. Observing larger broadening for lower jet $p_T$ (smaller formation time) would be a signature of this effect, see also [22, 23]. However, the vacuum distribution of $r_{c\bar{c}}$ and $\kappa$ also change with $E_g$ (dashed curves in Fig. 3(b)), so robust signatures of formation time-dependence will require a more dedicated study. We postpone this to future work and focus the rest of this manuscript on the medium-induced enhancement of the $g \rightarrow c\bar{c}$ splitting.

**Tracing $g \rightarrow c\bar{c}$ via $D^0\bar{D}^0$ pairs in jets.** We now discuss one strategy for testing the enhanced $g \rightarrow c\bar{c}$ radiation experimentally. Charm quarks fragment into $D^0$ mesons. Figure 4(a) shows the yield of jets containing a single $D^0\bar{D}^0$ pair at mid-rapidity ($|\eta_{jet}| < 1.6$) expected for the projected luminosity ($L_{\text{int}} = 10 \text{ nb}^{-1}$) of the High-Luminosity Heavy-ion LHC (HL-LHC) [24]. Non-prompt contributions arising from $b$-quark fragmentation can be removed experimentally and are not considered in the following. The two-body decay $D^0 \rightarrow K^- \pi^+$ can be reconstructed in ultra-relativistic heavy-ion collisions, but it has a branching ratio of 3.96% [25] so only $1.6 \times 10^{-3}$ of all $D^0\bar{D}^0$ pairs decay via this channel. Figure 4(a) therefore indicates that $\mathcal{O}(1000)$ counts are pro-
duced in this particular channel for $80 \text{ GeV} < p_T^{\text{jet}} < 90 \text{ GeV}$, and larger yields are obtained for lower $p_T^{\text{jet}}$. While the experimental feasibility of such measurement requires further study, this suggests that such an effect may be measurable at the HL-HI-LHC.

In Fig. 4(b) (solid blue line), we show the fraction $N_{D^0 \bar{D}^0}/N_{\text{jets}}$ of jets containing exactly one $D^0 \bar{D}^0$ pair compared to inclusive jets as a function of $p_T^{\text{jet}}$. In the absence of a medium, this fraction ranges from 0.2% to 1% for jets between 20 and 200 GeV. In the presence of a medium, $N_{D^0 \bar{D}^0}/N_{\text{jets}}$ is modified due to several effects. It is enhanced due to the medium-modified $g \rightarrow c\bar{c}$ splitting, which is the effect we wish to access. As shown in red in Fig. 4(b), this enhancement can be determined by measuring independently $N_{D^0 \bar{D}^0}/N_{\text{jets}}$ in $pp$ and PbPb collisions. This is an observable signature of medium-enhanced $c\bar{c}$ radiation.

However, even in the absence of medium-modified $g \rightarrow c\bar{c}$, jets lose energy to the QGP so $p_T^{\text{jet}}$ decreases. We estimate the size of this effect by shifting both the $D^0 \bar{D}^0$-tagged and inclusive jet yields by the average fractional energy loss $\Delta p_T^{\text{jet}}/p_T^{\text{jet}} \approx 10\%$ of inclusive jets in central PbPb collisions [26] (blue dashed curve in Fig. 4(b)). This shift is a heuristic prescription and neglects possible differences in energy loss of $D^0 \bar{D}^0$-tagged and inclusive jets. In the BDMPS-Z formalism, jet energy loss occurs because of enhanced out-of-cone radiation from medium-modified splitting functions. In the Supplemental Material, we use a simple parton shower with medium modification of all splitting functions to demonstrate that the enhancement of $N_{D^0 \bar{D}^0}/N_{\text{jets}}$ shown in Fig. 4(b) is primarily due to the $g \rightarrow c\bar{c}$ enhancement. In this implementation, we find that the modification of $N_{D^0 \bar{D}^0}/N_{\text{jets}}$ from jet energy loss is small compared to that estimated from the 10% shift in jet $p_T$, reinforcing that the medium enhancement of $D^0 \bar{D}^0$ pairs in jets is significant compared to the shift of the baseline.

Figure 4 includes jets that contain a $D^0$ and a $\bar{D}^0$ meson of any $p_T$. Experimentally, such a measurement amounts to accessing the $D^0 \bar{D}^0$ yield down to arbitrarily soft momenta where reconstruction efficiency degrades and backgrounds are high. While experimental limitations require further study, they may not be critical since the $D^0$ and $\bar{D}^0$ mesons entering Fig. 4(b) typically carry a rather large fraction of the jet $p_T$ (data not shown) due to the hard fragmentation of gluons into heavy quarks.

**Conclusion and Outlook.** The detection of medium-enhanced $c\bar{c}$ production would be a qualitatively novel test that jet quenching arises from medium-modified parton splitting functions. The experimental strategy proposed here relies on the future HL-HI-LHC program. In the longer term, this physics will benefit from novel detector technologies that allow for highest rates and extreme signal purities in Run 5 and 6 at the LHC [27]. On the theory side, future work should embed $p_{\text{med}}^{g \rightarrow c\bar{c}}$ on par with all other medium-modified splittings in a state-of-the-art jet quenching Monte Carlo. This would make it possible to predict modified phase space distributions of $c\bar{c}$ radiation that could be tested with more differential measurements.

As alluded to in the discussion of Fig. 3, the $g \rightarrow c\bar{c}$ splitting may provide unique phenomenological opportunities beyond the scope of the present work. Jet substructure techniques tailored to $g \rightarrow c\bar{c}$ [28] could be used in the future to access the detailed kinematics of the $g \rightarrow c\bar{c}$ splitting from the hadron level. This could provide clean access to signatures of momentum broadening of the $c\bar{c}$ pair and of formation time dependence of quenching. Advances in tagging jets with a $c\bar{c}$ pair [29] may also make it possible to sample the entire $c\bar{c}$ statistics for the study of

![Figure 4(a)](image1.png)  
![Figure 4(b)](image2.png)
boosted $g \to c\bar{c}$ topologies. If statistics permits, accessing an unquenched proxy for the jet energy (for example, a $Z$ or isolated high-$p_T$ photon recoiling against the $c\bar{c}$-tagged jet) would eliminate uncertainties from out-of-cone radiation discussed in the context of Fig. 4(b).

**Acknowledgments:** We thank Leticia Cunqueiro, Alexander Huss, José Guilherme Milhano, Pier Francesco Monni, Andreas Morsch, Krishna Rajagopal, Ivan Vitev, Nima Zardoshti and Korinna Zapp for useful discussions. MA acknowledges support through H2020-MSCA-IF-2019 ExHolo 898223.

---

* maximilian.attems@cern.ch
† jasmine.brewer@cern.ch
‡ gian.michele.innocenti@cern.ch
§ aleksas.mazeliauskas@cern.ch
¶ sohyun.park@cern.ch
** wille.van.der.schee@cern.ch
†† urs.wiedemann@cern.ch

[1] M. Cacciari, S. Frixione, N. Houdeau, M. L. Mangano, P. Nason, and G. Ridolfi, JHEP 10, 137 (2012), arXiv:1205.6344 [hep-ph].
[2] A. Andronic et al., Eur. Phys. J. C 76, 107 (2016), arXiv:1506.03881 [nucl-ex].
[3] R. Baier, L. Dolshitzer, A. H. Mueller, S. Peigne, and D. Schiff, Nucl. Phys. B 483, 291 (1997), arXiv:hep-ph/9607355.
[4] B. G. Zakharov, JETP Lett. 63, 952 (1996), arXiv:hep-ph/9607440.
[5] Y. L. Dokshitzer and D. E. Kharzeev, Phys. Lett. B 519, 199 (2001), arXiv:hep-ph/0106202.
[6] M. Connors, C. Nattrass, R. Reed, and S. Salur, Rev. Mod. Phys. 90, 025005 (2018), arXiv:1705.01974 [nucl-ex].
[7] U. A. Wiedemann, Nucl. Phys. B 588, 303 (2000), arXiv:hep-ph/0005129.
[8] M. Gyulassy, P. Levai, and I. Vitev, Nucl. Phys. B 594, 371 (2001), arXiv:nucl-th/0006010.
[9] X.-N. Wang and X.-f. Guo, Nucl. Phys. A 696, 788 (2001), arXiv:hep-ph/0102230.
[10] Z.-B. Kang, F. Ringer, and I. Vitev, JHEP 03, 146 (2017), arXiv:1610.02043 [hep-ph].
[11] M. Attems, J. Brewer, G. M. Innocenti, A. Mazeliauskas, S. Park, W. van der Schee, and U. A. Wiedemann, (2022), arXiv:2203.11241 [hep-ph].
[12] K. M. Burke et al. (JET), Phys. Rev. C 90, 014909 (2014), arXiv:1312.5003 [nucl-th].
[13] C. Andrés, N. Armesto, M. Luzum, C. A. Salgado, and P. Zurita, Eur. Phys. J. C 76, 475 (2016), arXiv:1606.04837 [hep-ph].
[14] C. Andres, N. Armesto, H. Niemi, R. Paatelainen, and C. A. Salgado, Phys. Lett. B 803, 135318 (2020), arXiv:1902.03231 [hep-ph].
[15] A. Huss, A. Kurkela, A. Mazeliauskas, R. Paatelainen, W. van der Schee, and U. A. Wiedemann, Phys. Rev. C 103, 054903 (2021), arXiv:2007.13758 [hep-ph].
[16] S. Cao et al. (JETSCAPE), Phys. Rev. C 104, 024905 (2021), arXiv:2102.11337 [nucl-th].
[17] P. Caucau, E. Iancu, and G. Soyez, JHEP 04, 209 (2021), arXiv:2012.01457 [hep-ph].
[18] Since Eq. (3) depends on only two combinations of $\hat{q}, L$ and $E_\gamma$, only two dependencies need to be specified in this figure.
[19] N. Armesto, L. Cunqueiro, and C. A. Salgado, Eur. Phys. J. C 63, 679 (2009), arXiv:0907.1014 [hep-ph].
[20] T. Sjöstrand, S. Ask, J. R. Christiansen, R. Corke, N. De sai, P. Ilten, S. Mrenna, S. Prestel, C. O. Rasmussen, and P. Z. Skands, Comput. Phys. Commun. 191, 159 (2015), arXiv:1410.3012 [hep-ph].
[21] M. Cacciari, G. P. Salam, and G. Soyez, Eur. Phys. J. C 72, 1896 (2012), arXiv:1111.6097 [hep-ph].
[22] L. Apolinário, J. G. Milhano, G. P. Salam, and C. A. Salgado, Phys. Rev. Lett. 120, 232301 (2018), arXiv:1711.03105 [hep-ph].
[23] F. Domínguez, J. G. Milhano, C. A. Salgado, K. Tywoniuk, and V. Vila, Eur. Phys. J. C 80, 11 (2020), arXiv:1907.03653 [hep-ph].
[24] Z. Citron et al., CERN Yellow Rep. Monogr. 7, 1159 (2019), arXiv:1812.06772 [hep-ph].
[25] P. A. Zyla et al. (Particle Data Group), PTEP 2020, 083C01 (2020).
[26] S. Chatrchyan et al. (CMS), Phys. Rev. C 84, 024906 (2011), arXiv:1102.1957 [nucl-ex].
[27] C. ALICE, Letter of intent for ALICE 3: A next generation heavy-ion experiment at the LHC, Tech. Rep. (CERN, Geneva, 2022).
[28] P. Ilten, N. L. Rodd, J. Thaler, and M. Williams, Phys. Rev. D 96, 054019 (2017), arXiv:1702.02947 [hep-ph].
[29] S. Iwamoto, G. Lee, Y. Shadmi, and Y. Weiss, JHEP 09, 114 (2017), arXiv:1703.05748 [hep-ph].
[30] S. Höche, in Theoretical Advanced Study Institute in Elementary Particle Physics: Journeys Through the Precision Frontier: Amplitudes for Colliders (2015) pp. 235–295, arXiv:1411.4085 [hep-ph].
[31] S. Höche, “Introduction to Parton Shower and Matching,” (2019).
[32] S. Acharya et al. (ALICE), Phys. Rev. D 105, L011103 (2022), arXiv:2105.06335 [nucl-ex].
Supplemental material. To obtain the main results in Figs. 3 and 4(b), we have reweighted $g \rightarrow c\bar{c}$ splittings generated in PYTHIA 8.3 vacuum parton showers with Eq. (5). This reweighting procedure is correct up to $\mathcal{O}(\alpha_s)$ for sufficiently rare processes [11]. However, it neglects medium modifications of $g \rightarrow gg$ and $q \rightarrow qg$ that modify the distribution of gluons that can split into $c\bar{c}$, and that therefore may affect $c\bar{c}$ production at realistic couplings. A quantitative study of these effects in our formalism requires a parton shower that includes the medium modification of all splitting functions, including mass effects. Here we further corroborate our main conclusions with a simplified medium-modified parton shower.

Our starting point is the public implementation of a massless $\kappa$-ordered dipole shower with final-state radiation only [30, 31]. The shower is initialized with a color neutral $q\bar{q}$ or $gg$ pair of momenta

$$p_1^\mu = E(1,0,1,0), \quad p_2^\mu = E(1,0,-1,0),$$

and it is evolved in $t = \kappa$ from the initial scale $E^2$ down to the cut-off scale $t_0 = 1\text{GeV}^2$. Gluons and $N_f = 5$ massless quark flavor are considered in the evolution. For the medium modification, the $q \rightarrow gg$ and (massless) $g \rightarrow qg$ vacuum splitting functions are supplemented with additive BDMPS-Z corrections $P_{a\rightarrow bc}^{\text{tot}} = P_{a\rightarrow bc}^{\text{vac}} + P_{a\rightarrow bc}^{\text{med}}$ valid in the small-$z$ limit. These take the form

$$P_{a\rightarrow bc}^{\text{med}}(E, \kappa^2, z) \equiv P_{a\rightarrow bc}^{\text{vac}}(z)\mathcal{J} \left( \frac{\kappa^2}{q_c L^2} \frac{z E}{2 q_c L^2} \right) \bigg|_{z \ll 1}$$

(7)

for all splitting functions. Here, $\mathcal{J}$ is the universal modification factor

$$\mathcal{J} \left( \kappa^2, \tilde{\omega} \right) = \frac{\kappa^2}{2 \tilde{\omega}^2} \mathcal{R}_e \int_0^L dt \int \frac{d\tilde{t}}{\tilde{t}} \int \frac{d\tilde{r}}{\tilde{r}} \int d\tilde{r} \int d\tilde{x} e^{-|\tilde{t}| - |\tilde{r}|}$$

$$\times e^{-\frac{1}{\tilde{r}} \int_{\tilde{r}}^{\tilde{r}} d\tilde{r} \tilde{q}(\tilde{r})^{2} - i \kappa \frac{\tilde{r}}{\tilde{x}} \frac{\partial}{\partial \tilde{x}} K[x = 0, t, \tilde{r}, \tilde{r}]} \, ,$$

(8)

which depends on the rescaled variables $\kappa^2 \equiv \kappa^2/4 \tilde{L}^2$ and $\tilde{\omega} \equiv \tilde{\omega}/\sqrt{q_c L}$ with $\omega = zE$. The variable $z$ denotes the momentum fraction of the softer splittee, which is the gluon for $q \rightarrow gg$ and $g \rightarrow gg$ or the quark or antiquark for $g \rightarrow qg$. Correspondingly, $\tilde{q}_c = \tilde{q}_A = C_A \tilde{q}$ for $q \rightarrow qg$, $g \rightarrow gg$ and $q_c = C_F \tilde{q}$ for $g \rightarrow qg$.

For the $g \rightarrow gg$ and $q \rightarrow qg$ splitting functions, medium modifications of the form Eq. (7) have been implemented previously in the Q-PYTHIA medium-modified parton shower [19]. Extrapolating this technically simpler $z \ll 1$ approximation to all $z$ can be justified with the dominance of soft emission in $g \rightarrow gg$ and $q \rightarrow qg$. This argument cannot be paralleled for the case of $g \rightarrow q\bar{q}$. However, using the extrapolation of the small-$z$ expression Eq. (7) to the full $z$ range is sufficient for illustrating the possible impact of enhanced $g \rightarrow gg$ and $q \rightarrow qg$ splitting on $q\bar{q}$ pair production, which is the main aim of this supplement.

Figure 5 shows the universal modification factor $\mathcal{J} \left( \kappa^2, \tilde{\omega} \right)$. As the medium-modified parton shower treats vacuum splitting functions and their medium modification Eq. (7) on the same footing, the factor $\mathcal{J} \left( \kappa^2, \tilde{\omega} \right)$ is sampled only in the range $\omega > \kappa$ and $t_0 < \kappa^2 < t_{\text{init}}$ available for vacuum splittings. For two different choices of the quenching parameter and in-medium path length, these constraints are indicated in Fig. 5. Similar to Fig. 2, Fig. 5 shows for small $\kappa^2$ and larger $\omega$ a depletion due to transverse momentum broadening. Significant enhancement is found for small $\omega$ and $\kappa^2 \sim \tilde{q}L$. Motivated by the maximal enhancement of $\mathcal{J} \left( \kappa^2, \tilde{\omega} \right)$ within the kinematically-allowed region, we have oversampled the vacuum distribution with a factor $P_{\text{max}} = 3$ and use a veto algorithm to implement the modification factor $1 + \mathcal{J} \left( \kappa^2, \tilde{\omega} \right)$.

Model results for medium-modified jets. For each event we reconstruct anti-$k_t$ jets with $R = 0.4$. We define $q\bar{q}$-tagged jets as jets containing a single $q\bar{q}$ pair that originates from the same gluon splitting. We divide the total reconstructed inclusive and tagged jets by $2N_{\text{ev}}$, where $N_{\text{ev}}$ is the number of generated events. This defines the average inclusive and tagged jet spectra $dN_{i\rightarrow j}(E)/dp_T$ per parton $i$ (quark or gluon) with initial energy $E$. As we work with a simplified medium-modified parton shower that is not part of a hadronic event generator, we require additional information about the hard partonic production rate of these partons $i$ in hadronic collisions. For the relevant partonic cross-sections $d\sigma_i/dE$ we use leading order calculations of $d\sigma/dp_T dy$ averaged over rapidity window $y < 1$ at $\sqrt{s} = 5.02 \text{TeV}$ from Ref. [15] and we take $E = p_T$. The
jet $p_T$ spectra are then given by weighting average contributions from single initial parton jets with the respective leading-order partonic cross-section
\[ \frac{d\sigma}{dp_T} = \int dE \left( \frac{d\sigma_q}{dE} \frac{dN_{q\to\bar{q}j}(E)}{dp_T} + \frac{d\sigma_g}{dE} \frac{dN_{g\to\bar{q}j}(E)}{dp_T} \right). \] (9)

We show the resulting ratio of the $q\bar{q}$-tagged to inclusive jet yield in Fig. 6. One may multiply the results in Fig. 6 with the square of the $c \to D^0$ branching ratio ($\sim 0.4^2$ [32]) to estimate $N_{D_sD_0}/N_{\text{jets}}$. We caution, however, that this model setup does not account for effects of charm mass and lacks other potentially relevant effects that are included in a full hadronic event generator like PYTHIA 8.3. While a precise quantitative comparison with PYTHIA is hence not meaningful, strong commonalities between Fig. 6 and the PYTHIA pp baseline in Fig. 4(b) are noteworthy. In particular, both agree in shape and, taking the branching ratios into account, agree in magnitude within a factor of 2. To illustrate the interplay between medium modification of all $g \to q\bar{q}$, $q \to qg$ and $g \to q\bar{q}$ splitting functions, we therefore regard Fig. 6 as a suitable baseline.

We show the nuclear modification factor $R_{AA}$ in this simulation for inclusive and for $q\bar{q}$-tagged jets in Fig. 7(a). If the medium modification of all splitting functions is included, then $R_{AA}^{\text{inc}}$ for inclusive jets shows the significant suppression characteristic of jet quenching (blue solid). The $R_{AA}^{q\bar{q}}$ for $q\bar{q}$-tagged jets shows a significantly smaller suppression (blue hatched) since the effects leading to jet quenching in $R_{AA}^{\text{inc}}$ are partially compensated by a characteristic rate enhancement due to medium-modified $g \to q\bar{q}$. If only the $g \to q\bar{q}$ splitting function is medium-modified, there is little modification of inclusive jets (solid red) but substantial enhancement of $q\bar{q}$-tagged jets (hatched red). In Fig. 7(b) we show the double ratio $R_{AA}^{q\bar{q}}/R_{AA}^{\text{inc}}$, which helps to isolate the effects from medium-enhanced $q\bar{q}$ production. Irrespective of whether the parton shower includes medium modifications to all splitting functions (blue band) or only modifications to the $g \to q\bar{q}$ splitting function (red band), a significant enhancement of the double ratio is observed. This signals the dominant role of medium-enhanced $g \to q\bar{q}$ splitting in this enhancement.

In both panels of Fig. 7 we additionally show in green the results of the reweighting procedure applied to this parton shower. The reweighting is in reasonable agreement with the effect of enhancing only the $g \to c\bar{c}$ splitting.

Fig. 7 corroborates our main conclusions in several ways. First, within the bands of $qL$ variation, the enhancement of $R_{AA}^{q\bar{q}}/R_{AA}^{\text{inc}}$ obtained from the medium-modified dipole parton shower is comparable to the enhancement obtained from the reweighting procedure, thus justifying the use of the latter in the main pa-
per. Second, inclusion of medium-modified $g \to gg$ and $q \to qg$ splitting has a numerically small effect on the ratio $R_{AA}^{qg}/R_{AA}^{\text{inc}}$ (cf. Fig. 7(b)) although it affects the nuclear modification factors $R_{AA}^{\text{inc}}$ and $R_{AA}^{qg}$ significantly (cf. Fig. 7(a)). This supports our conclusion that the enhancement of $N_{D^0 \bar{D}^0}/N_{\text{jets}}$ signals medium-enhanced $c\bar{c}$ production. Third, since medium-modified $g \to gg$ and $q \to qg$ were not included in the simulation leading to Fig. 4(b), we had accounted for parton energy loss effects with a 10% $p_T$-shift of the $pp$ baseline. In the present model, this shift is not inserted ad hoc, but it is included via the calculation of $R_{AA}^{\text{inc}}$. We find that the $g \to q\bar{q}$-induced enhancement of $q\bar{q}$-tagged jets can be identified as an enhancement of $N_{D^0 \bar{D}^0}/N_{\text{jets}}$ even in the presence of energy loss.

The model study reported here does not replace the development of a full BDMPS-Z medium-modified parton shower (including $g \to q\bar{q}$ for massive quarks) that is combined with a state-of-the-art formulation of hard processes in hadronic collisions and with a modern formulation of the dynamically evolving medium with which the parton shower interacts. However, such a tool is currently not available and its development and phenomenological validation will require significant further effort. Given this, the present model study gives significant additional support that our main conclusion remains unchanged in refined formulations of the problem. It supports that $N_{D^0 \bar{D}^0}/N_{\text{jets}}$ will show a significant enhancement that is indicative of medium-enhanced $c\bar{c}$ production.