Stimulated Brillouin Scattering in integrated photonic waveguides: forces, scattering mechanisms and coupled mode analysis

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Recent theoretical studies of Stimulated Brillouin Scattering (SBS) in nanoscale devices have led to an intense research effort dedicated to the demonstration and application of this nonlinearity in on-chip systems. The key feature of SBS in integrated photonic waveguides is that small, high-contrast waveguides are predicted to experience powerful optical forces on the waveguide boundaries, which are predicted to further boost the SBS gain that is already expected to grow dramatically in such structures because of the higher mode confinement alone. In all recent treatments, the effect of radiation pressure is included separately from the scattering action that the acoustic field exerts on the optical field. In contrast to this, we show here that the effects of radiation pressure and motion of the waveguide boundaries are inextricably linked. Central to this insight is a new formulation of the SBS interaction that unifies the treatment of light and sound, incorporating all relevant interaction mechanisms — radiation pressure, waveguide boundary motion, electrostriction and photoelasticity — from a rigorous thermodynamic perspective. Using this formalism, we study the limitations on different technologically important structures, discuss the effect of inelastic contributions to the forces, and show how finite-length effects can be used to either increase or reduce the SBS gain in nanoscale devices. Our approach also clarifies important points of ambiguity in the literature, such as the nature of edge-effects with regard to electrostriction, and of body-forces with respect to radiation pressure. This new perspective on Brillouin processes leads to physical insight with implications for the design and fabrication of SBS-based nanoscale devices.

I. INTRODUCTION

Stimulated Brillouin Scattering (SBS) is a nonlinear process by which light interacts coherently with acoustic vibrations in an optically transparent medium [1]. Predicted by Brillouin in 1922 [2], SBS was first experimentally demonstrated by Chiao, Townes and Stoicheff soon after the invention of the laser [3], and thereafter became one of the standard techniques for measuring the mechanical properties of materials at high frequencies [4]. SBS has historically often been regarded as a non-desirable side-effect that must be either suppressed or accommodated, for example in fiber optics, where it can strongly deplete narrow-band pumps. However in recent years there has been a remarkable resurgence of interest in guided wave SBS [5], driven largely by the ability to harness the effect in modern nanophotonics experiments [6–10]. As well as forming the basis for the investigation of fundamental physical effects such as slow-light [11] and non-reciprocity [12], these experiments have led to a number of interesting SBS-based applications, such as narrow-linewidth tunable sources [13], as well as on-chip processing of optical [14] and radio-frequency [15, 16] signals.

The SBS interaction arises from a pair of physical mechanisms that transfer energy back and forth between the electromagnetic field and the mechanical stresses and strains of the material [1, 17]. These mechanisms can be categorized as scattering (or forward-action) processes, by which the acoustic modes scatter light from one state to another, and back-action (or force-like) processes, by which the optical field generates mechanical motion via optical forces and pressures. Informed by the understanding of fiber and bulk systems, it was thought until very recently that SBS was driven entirely by the photoelasticity and electrostriction. These are inverse scattering and back-action processes respectively, linked by the thermodynamics of dielectric materials under mechanical strain. However these mechanisms are not the only possibilities for optomechanical interaction: in particular in the field of nano-optomechanics it is known that the back-action of radiation pressure can be very large in small, high-refractive-index-contrast devices, with the effect playing a central role in the interaction between optical and acoustic resonators in a number of seminal experiments in this field [18–20]. In recent work Rakich et al. [21] showed that radiation pressure can also make a significant contribution to the SBS gain: this is the result of large optical forces that act on the boundaries of suspended waveguides and resonantly excite acoustic modes of the free-standing structure. This prediction has important implications for the harnessing of SBS in CMOS-compatible materials such as silicon, in which electrostriction is relatively weak. Radiation pressure is strongest for small, high index contrast waveguides, and radiation-pressure-induced SBS has recently
been observed in a silicon/silicon nitride hybrid waveguide [22].

In this recent work and in the associated literature [21, 24], the SBS gain is obtained directly from the phonon generation rate, which can be calculated by summing the optical forces due to both radiation pressure and electrostriction acting on the waveguide. In this treatment the scattering of the optical mode by the acoustic field is not considered directly; instead the effect of all forward-action processes is implicit in the equations for particle-conservation. While this approach is valid for simple interactions between two optical modes, scattering mechanisms are important for any experiment in which the stimulated acoustic wave acts on other optical fields that are also present in the waveguide, as occurs in the generation of frequency combs via SBS [22] or in SBS-based optical isolation [23]. Perhaps more critically, scattering forms an important part of the physics of SBS, and a full description of the scattering mechanisms is necessary to complete our understanding of Brillouin processes in integrated photonic waveguides. At the same time, there exist some points of confusion or ambiguity in the literature regarding the precise application of optical forces needed to compute the SBS gain. Specifically, it is unclear whether electrostriction results in a pressure term that acts on the boundary of the waveguide [22], and whether radiation pressure gives rise to a body force-density within the waveguide’s interior [21, 23]. A rigorous formalism that examines the combined effects on SBS of back-action processes, such as radiation pressure and electrostriction, and scattering processes, such as photoelasticity and waveguide boundary motion, would resolve these issues.

Here we present a new formalism for SBS that considers all relevant interaction mechanisms in a unified way. We treat optical scattering by the motion of the waveguide boundary rigorously and show that this mechanism forms the inverse process to the radiation pressure between lossless dielectrics, just as the photoelastic effect is associated with the reversible contribution to the electrostrictive force. We derive coupled mode equations that include all of these effects in a consistent framework. To the best of our knowledge this is the first such treatment in the literature. Furthermore, we observe that optical losses are related to inelastic forces using the case of radiation pressure as an example; this breaks the aforementioned symmetry between the scattering and back-action processes. Finally, we analytically solve the modal equations for two important limiting cases: A very long waveguide, such as an optical fiber or cm-scale integrated waveguide, and a waveguide that is so short that the pump beam is not depleted and acoustic propagation effects play a vital role. The latter is of great interest in the context of integrating functional devices based on the SBS-effect into small scale optical circuits, especially in the context of CMOS-compatible integrated optics. We find that the effective SBS-gain in short waveguides can be drastically reduced by finite-length effects: this is be-

cause the acoustic field requires a finite distance to build up to its maximum value, this distance being determined by the acoustic decay length. This leads to the somewhat counter-intuitive result that, for short waveguides, the gain is not increased by reducing acoustic losses. This has important implications for the design of SBS-based systems in which sections of waveguide are cascaded to increase the overall SBS gain.

II. PRELIMINARIES

We consider the interaction between optical and acoustic fields in waveguides having the general form depicted in Fig. 1 with a material cross section that is invariant along the z-axis. Although a specific waveguide cross-section is shown here, the results apply to waveguides of arbitrary cross section and composition.

FIG. 1. Schematic of the forward (Stokes field represented by the red arrow) and backward (Stokes field in blue) SBS interactions in a waveguide aligned along the z-axis. Although a specific waveguide cross-section is shown here, the results apply to waveguides of arbitrary cross section and composition.
A. Electromagnetic part

We describe the evolution of the optical fields by the electromagnetic wave equation in terms of the electric field

\[ \nabla \times \nabla \times \mathbf{E} = -\mu_0 \partial_t^2 \mathbf{D}; \quad \mathbf{D} = \varepsilon \mathbf{E}; \quad \varepsilon = \varepsilon_0 \varepsilon_r. \tag{1} \]

Here, \( \mathbf{E} \) and \( \mathbf{D} \) are the electric field and electric induction field respectively, and \( \partial_t \) denotes the partial derivative with respect to time. In the context of acousto-optics, we prefer the term “electric induction field” over “electric displacement field” to avoid confusion with the mechanical displacement field. Later on, we also use the magnetic field \( \mathbf{H} \). The dielectric function \( \varepsilon(\mathbf{r}) = \varepsilon(x, y) \) is isotropic and homogeneous in the \( z \)-direction; \( \mathbf{r} = (x, y, z)^T \) is the position vector. We assume that the electromagnetic fields can be approximated as a superposition of two propagating optical eigenmodes

\[ \mathbf{E} = \mathbf{E}^{(1)} + \mathbf{E}^{(2)}, \tag{2} \]

where

\[ \mathbf{E}^{(i)}(\mathbf{r}, t) = \mathbf{e}^{(i)}(\mathbf{r}, t) a^{(i)}(z, t) + \text{c.c.}, \tag{3} \]

and the mode functions factor as

\[ \mathbf{e}^{(i)}(\mathbf{r}, t) = \mathbf{e}^{(i)}(x, y) \exp(i \beta^{(i)} z - i \omega^{(i)} t), \tag{4} \]

with frequencies \( \omega^{(i)} \) and wave vectors \( \mathbf{k}^{(i)} = \hat{z} \beta^{(i)} \). Note that the propagation constants \( \beta^{(i)} \) may be both positive or negative. The dimensionless envelope functions \( a^{(i)} \) are assumed to change only slowly over an optical wavelength and the time scale of an optical cycle. The other fields \( \mathbf{D} \) and \( \mathbf{H} \) are likewise expanded. The basis functions \( \mathbf{e}^{(i)} \) are bound solutions to the 2D-eigenproblem

\[ (\nabla_\perp + i \beta \hat{z}) \times (\nabla_\perp + i \beta \hat{z}) \times \mathbf{e} = \varepsilon \mu_0 \omega^2 \mathbf{e}, \tag{5} \]

where \( \nabla_\perp \) is the nabla operator in the \( x, y \)-plane. There is no need to normalize these modes, although there may be practical advantages in doing so in numerical simulations. Since we neglect dispersion, the average electromagnetic energy density per unit length of the waveguide and the corresponding energy flux carried by the unnormalized mode functions are \([27]\)

\[ \mathcal{E}^{(i)} = 2 \int d^2 r \ \varepsilon [\mathbf{e}^{(i)}]^* \cdot \mathbf{e}^{(i)}; \tag{6} \]

\[ \mathcal{P}^{(i)} = 2 \int d^2 r \ \dot{\mathbf{z}} \cdot ([\mathbf{e}^{(i)}]^* \times \mathbf{h}^{(i)}), \tag{7} \]

where the integration is across the whole transverse plane. Using Maxwell’s equations, the latter can be recast to an expression that only involves the electric field; the energy transport velocity of the mode (which in the lossless case here is equal to its group velocity) \([27]\) is given by the ratio of the energy flux and energy densities:

\[ \mathcal{P}^{(i)} = \frac{1}{-i \omega^{(i)} \mu_0} \int d^2 r \ \{ [\mathbf{e}^{(i)}]^* \cdot [\mathbf{e} \times (\nabla_\perp \times \mathbf{e}^{(i)})] \}
\]

\[ + [\mathbf{e}^{(i)}]^* \cdot [(\nabla_\perp \times \mathbf{e}^{(i)})] \} \]

\[ \dot{v}^{(i)} = \mathcal{P}^{(i)}/\mathcal{E}^{(i)}. \tag{8} \]

B. Acoustic part

The fundamental equation for the mechanical part of the problem is the acoustic wave equation \([28]\) for the (mechanical) displacement field \( U \)

\[ -\rho \partial_t^2 U_i + \sum_{jkl} \partial_j [c_{ijkl} + \eta_{ijkl} \partial j] \partial k U_i = -F_i, \tag{9} \]

where \( \rho \) is the density, \( \mathbf{c} \) is the stiffness tensor and \( \eta \) is the viscosity tensor. Here, \( \partial_j \) denotes the spatial derivative in the \( j \)-th spatial direction along \( j \), where \( j \in \{x, y, z\} \). The source term \( \mathbf{F} \) on the right hand side is the driving external force field per unit volume through which the coupling to the electromagnetic field will be introduced. Assuming first that the acoustic losses are weak, we express the displacement field in terms of a solution \( \mathbf{u} \) with carrier \( \exp(it) \) to the lossless wave equation (i.e. with \( \eta = 0 \)) with a corresponding dimensionless envelope function \( b \):

\[ \mathbf{u}(\mathbf{r}, t) = (U(r, t) = \mathbf{u}(\mathbf{r}, t) b(z, t) + \text{c.c.} \); \tag{10} \]

\[ \mathbf{u}(\mathbf{r}, t) = \mathbf{u}(\mathbf{r}, t) \exp(i q z - i \Omega t); \tag{11} \]

where \( \mathbf{u} \) is an eigenmode of the equation

\[ \rho \Omega^2 \mathbf{u} + \sum_{jkl} (\nabla_j + i q \hat{z})_j c_{ijkl} (\nabla_k + i q \hat{z})_k \mathbf{u} = 0. \tag{12} \]

Note that we need not assume that the acoustic modes are strictly bound, since significant SBS can occur with leaky acoustic modes \([29]\). However, we assume that the acoustic propagation loss is small so that it appears to first approximation in the equation of motion for \( b \). The first main advantage of this approach (i.e. moving the loss into the dynamics) is that the set of functions from which we choose \( \mathbf{u} \) is formed by eigenfunctions to a Hermitian operator resulting in convenient orthogonality relations. Second, \( |b|^2 \) is related to the amplitude of the acoustic field in a straightforward way throughout the whole system and so is directly related to the acoustic energy density. Again, there is no need to normalize the acoustic basis function. For what follows, the acoustic wave is chosen to be phase-matched with the beat between the two optical modes, i.e. we specify for the rest of this paper that

\[ \Omega = \omega^{(2)} - \omega^{(1)} \quad \text{and} \quad q = \beta^{(2)} - \beta^{(1)}. \tag{14} \]

Note that such conditions are not in general automatically satisfied if the two electric fields are chosen freely.
There must also be an appropriate resonant, phase-matched acoustic mode to provide the coupling. This strict phase-matching condition is used to select appropriate basis functions for the subsequent modal expansion.

The energy density of the acoustic field is the sum of the kinetic and the elastic energy. For a traveling wave, we focus on the time-averaged total energy per unit length of the waveguide. For an acoustic mode with unit envelope $b = 1$ it thus reads

$$\mathcal{E}_b = \frac{1}{2} \left\langle \int d^2r \rho |\partial_t U|^2 + \sum_{ijkl} S_{ij} c_{ijkl} S_{kl} \right\rangle_{T_{ac}},$$

(15)

where $S_{ij} = \frac{1}{2} (\partial_t u_j + \partial_j u_t)$ is the strain tensor and the subscript $T_{ac}$ indicates that the average is taken over a time window that is much longer than one acoustic cycle but shorter than the time scale for any relevant slower process. The transverse integral extends over the interior cross-section of the waveguide. If, as is typically the case, the waveguide’s total momentum and angular momentum are both zero, the average kinetic energy is equal to the average elastic energy and we may simplify:

$$\mathcal{E}_b = \left\langle \int d^2r \rho |\partial_t U|^2 \right\rangle_{T_{ac}} = 2\Omega^2 \int d^2r \rho |U|^2.$$

(16)

The time-averaged energy flux that traverses the waveguide cross section $\mathcal{P}_b$ is given as the normal projection of the product between the velocity field and the stress tensor $\mathbf{T}$; the mode’s energy transport velocity $v_b$ is defined as in Eq. (9):

$$\mathcal{P}_b = -\left\langle \int d^2r \tilde{z} \cdot (\partial_t U) \cdot \mathbf{T} \right\rangle_{T_{ac}},$$

(17)

$$= 2\Omega \int d^2r \sum_{ikl} c_{ikl} u_i^* \partial_k u_l,$$

(18)

$$v_b = \mathcal{P}_b / \mathcal{E}_b.$$

(19)

Finally, we assume that the coupling mechanism between optical and mechanical modes is reversible, i.e. that energy is lost only by the propagation of modes but not by the conversion between them. This means we neglect for example the optical force that occurs due to absorption of light. We comment on this in Section VII.

### III. OPTICAL MODAL EQUATIONS

In this section, we derive the equations of motion for the optical envelope functions $a^{(i)}$ and examine the two main effects by which a sound wave can scatter energy from one optical mode into the other.

#### A. Dynamic equations

The mechanical deformation affects the electromagnetic field in two ways. First, it changes the value of the permittivity. This is known as the photoelastic effect. Second, the material boundaries can be displaced and do work on the fields, an effect for which no familiar name seems to exist, but which we refer to as moving boundary scattering. In either case, the deformation leads to time-dependent changes $\Delta \mathbf{E}$, $\Delta \mathbf{D}$ in the electric field and induction, as illustrated by Fig. 2. Furthermore, we allow for an additional magnetization $\Delta \mathbf{H}$ due to the motion of polarized particles. In formulating the problem in this way, we go beyond similar prior descriptions of SBS.

The distorted fields are still solutions of Maxwell’s equations and so satisfy the wave equation

$$\nabla \times \nabla \times (\mathbf{E} + \Delta \mathbf{E}) + \mu_0 \partial_t^2 (\mathbf{D} + \Delta \mathbf{D}) + \mu_0 \partial_t \nabla \times \Delta \mathbf{H} = 0.$$

(20)

The field perturbations contain contributions with all possible sum and difference frequencies, but only those perturbations that simultaneously match the spatial and temporal frequency of a basis function (i.e. are phase-matched to a basis function) are relevant for what follows. Thus, we neglect all but the phase-matched contributions

$$\Delta \mathbf{E}(\mathbf{r}, t) = \Delta e^{(1)}(\mathbf{r}, t) a^{(2)}(z, t) b^*(z, t) + \Delta e^{(2)}(\mathbf{r}, t) a^{(1)}(z, t) b(z, t) + c.c.$$

(21)

Each one of the thus far unspecified patterns $\Delta e^{(i)}$ contains the corresponding optical wave carrier:

$$\Delta e^{(i)}(\mathbf{r}, t) = \Delta e^{(i)}(x,y) \exp(i\beta^{(i)} z - i\omega^{(i)} t),$$

(22)

The other field perturbations $\Delta \mathbf{D}$ and $\Delta \mathbf{H}$ are treated likewise. We have explicitly assumed that the field perturbations that are phase-matched to one optical mode stem from the interaction between the other optical mode and the sound wave and that they are linear in both fields. To proceed, we evaluate the contribution from the first mode to Eq. (20):
The second optical mode is treated likewise. We end up with the final equations for the optical envelope functions, where \( v^{(1,2)} \) are the respective (potentially negative) group velocities:

\[
\begin{align*}
\partial_z a^{(1)} + \frac{1}{v^{(1)}} \partial_t a^{(1)} &= - \frac{i \omega^{(1)} a^{(2)} b^*}{P^{(1)}} Q_1, \quad (27) \\
\partial_z a^{(2)} + \frac{1}{v^{(2)}} \partial_t a^{(2)} &= - \frac{i \omega^{(2)} a^{(1)} b^*}{P^{(2)}} Q_2. \quad (28)
\end{align*}
\]

Here

\[
Q_1 = \int d^2 r \left[ \left( e^{(1)} \right)^* \cdot \Delta d^{(1)} - \left( d^{(1)} \right)^* \cdot \Delta e^{(1)} - \mu_0 h^{(1)} \cdot \Delta h^{(1)} \right],
\]

\[
Q_2 = \int d^2 r \left[ \left( e^{(2)} \right)^* \cdot \Delta d^{(2)} - \left( d^{(2)} \right)^* \cdot \Delta e^{(2)} - \mu_0 h^{(2)} \cdot \Delta h^{(2)} \right],
\]

are works per unit length associated with the couplings. In a transparent solid insulator, the two effects that lead to the perturbations \( \Delta e^{(1)} \) and \( \Delta d^{(1)} \) are the common photoelastic effect and field perturbations caused by...
the changing continuity conditions when a dielectric interface is shifted. The perturbation $\Delta h^{(i)}$ is caused by an effective dynamic magnetic photoelastic effect. We now discuss these in more detail.

### B. Electric photoelastic effect

The term photoelasticity refers to the effect that the electric susceptibility of matter changes if it is subject to strain. In a solid and for small deformations, this can be phenomenologically described by a fourth rank tensor $\mathbf{p}$. This tensor can be derived from quasi-static or acousto-optic experiments:

$$\chi_{ijkl}^{(ePE)} = \varepsilon_0^2 \sum_{kl} p_{ijkl} \partial_k u_l,$$  \hspace{1cm} (31)

where $\mathbf{S}$ is the strain tensor and we have exploited the symmetry $p_{ijkl} = p_{ijlk}$ of the photoelastic tensor. From Eq. (31) follow the photoelastic parts of the overlap integrals in Eq. (27) and Eq. (28):

$$Q_1^{(ePE)} = \int_A d^2 r \ [e(1)]^* \cdot \Delta d^{(1)}  \hspace{1cm} (32)$$

$$= \varepsilon_0 \varepsilon_r^2 \int_A d^2 r \ \sum_{ijkl} [e_{ij}^{(1)}]^* e_{kl}^{(2)} p_{ijkl} \partial_k u_l^*, \hspace{1cm} (33)$$

By interchanging the optical mode labels, we find $Q_1^{(ePE)} = [Q_2^{(ePE)}]^*$. Again, the integral is only to be taken over the interior of the waveguide’s material cross section $A$. Although the displacement field is discontinuous at the waveguide boundary, the strain field goes from a finite value to zero and the photoelastic change of the permittivity remains finite. Boundary effects that are caused by a displacement of the material boundary are treated in Section III D.

### C. Magnetic photoelastic effect

It may be surprising that deformation can lead to a magnetic polarization in a body that was explicitly assumed to have no magnetic susceptibility. In fact, the absence of magnetic material response only guarantees that a static deformation cannot cause such an effect. A changing mechanical displacement field creates a temporary magnetic polarization that is proportional to the electric polarization, because the latter describes the dipole moment density of a microscopic charge separation. When the material is deformed, the separated charges are forced to move and form two separated, counter-directed microscopic currents that induce a magnetic field at the position of the moving dipole. The effective magnetic polarization is

$$\Delta \mathbf{H} = (\partial_t \mathbf{U}) \times \mathbf{P}. \hspace{1cm} (34)$$

The phase-matched terms are

$$\Delta h^{(1)} = i \Omega \varepsilon_0 (\varepsilon_r - 1) \mathbf{u}^* \times e^{(2)}, \hspace{1cm} (35)$$

and the overlap product with the magnetic induction field $\mu_0 \mathbf{h}$ is after a permutation of the triple product:

$$Q_1^{(mPE)} = i \Omega \mu_0 \varepsilon_0 (\varepsilon_r - 1) \int_A d^2 r \ \mathbf{u}^* \cdot (e^{(2)} \times [h^{(1)}]^*). \hspace{1cm} (36)$$

As before, a permutation of mode labels leads to $Q_1^{(mPE)} = [Q_2^{(mPE)}]^*$.

This term has not been well-appreciated in the recent literature on SBS. For optical modes that resemble plane waves, the magnetic induction field in the overlap integral Eq. (36) can be expressed in terms of the optical wave vector and the electric field (see Appendix B) and can be incorporated into the electric photoelastic tensor, where it appears as a dispersive, anti-symmetric contribution. This is the situation e.g. in conventional optical fibers but clearly not in integrated waveguides such as silicon nanowires. As a consequence, care must be taken when predicting the SBS-coefficients of small waveguides using photoelastic tensor elements that were measured with high-frequency deformation fields e.g. in acousto-optic or SBS experiments. We will resume this discussion in Section V B.

### D. Boundary term

The second important coupling mechanism is caused by the displacement of the material interfaces of the waveguide as the sound wave propagates along it. This leads to a strong change in the fields over a very small area exactly at the waveguide surface. This is in contrast to the photoelastic effect, which causes small field changes over the full waveguide cross section. As a consequence, this effect becomes more relevant as the waveguide cross-sectional area is decreased.

Clearly, this type of field perturbation appears at every dielectric interface; between different solids or liquids and between condensed materials and gases or vacuum. For the sake of illustration, we discuss this using the example of a rectangular nanowire with permittivity $\varepsilon_a$ surrounded by a domain with another permittivity $\varepsilon_h$. Consider a section of the waveguide outline that is displaced outward as illustrated by Fig. 4. We choose the interface normal vector to point outwards. Maxwell’s equations require that the normal component of the induction field and the in-plane components of the electric fields are continuous across the interface. Thus, the electric and induction fields in the space between the old and
the displaced boundary are modified according to

\[ \begin{align*}
E'(\text{before}) &= E_{\parallel} + \varepsilon^{-1}_b \varepsilon_0^{-1} D_{\perp} \\
E'(\text{after}) &= E_{\parallel} + \varepsilon^{-1}_a \varepsilon_0^{-1} D_{\perp}, \\
D'(\text{before}) &= \varepsilon_b \varepsilon_0 E_{\parallel} + D_{\perp} \\
D'(\text{after}) &= \varepsilon_a \varepsilon_0 E_{\parallel} + D_{\perp},
\end{align*} \]

where the subscripts \( \perp \) and \( \parallel \) refer to the normal and the in-plane parts of the field vectors, respectively. Thus, we find for the field perturbations in this small area:

\[ \begin{align*}
\Delta E &= (\varepsilon^{-1}_b - \varepsilon^{-1}_a) \hat{n} (\hat{n} \cdot D), \\
\Delta D &= (\varepsilon_a - \varepsilon_b) (-\hat{n} \times \hat{n} \times E).
\end{align*} \]

Essentially, this is the action of the deformation-related perturbation operator on the stress-free solution. These formulae have already been discussed at length by Johnson et al. in the context of perturbation theory for cavity eigenfrequencies \([31]\).

Assuming that the boundary displacement is so small that the field perturbations are homogeneous in the normal direction, we may replace the integration over the whole of the transverse plane in Eqs. \([24]\) and \([25]\) with a line integration only over all boundary contours \( \mathcal{C} = \{ \mathcal{C}_i \} \) of the waveguide cross section. In terms of time-harmonic wave patterns, we find for the coupling coefficients due to the moving boundary

\[ Q_1^{(\text{MB})} = \int_{\mathcal{C}} \mathbf{d}r \left( \mathbf{u}^* \cdot \hat{n} \right) \left[ (\varepsilon_a - \varepsilon_b) (\hat{n} \times \mathbf{e}^{(1)})^* (\hat{n} \times \mathbf{e}^{(2)}) \\
- (\varepsilon^{-1}_b - \varepsilon^{-1}_a) (\hat{n} \times \mathbf{d}^{(1)})^* (\hat{n} \times \mathbf{d}^{(2)}) \right], \]

where the factor \( (\mathbf{u}^* \cdot \hat{n}) \) is simply the distance by which the interface is displaced. Again, we find by re-labeling optical mode designators that \( Q_2^{(\text{MB})} = [Q_1^{(\text{MB})}]^* \). Finally the total coupling in Eqs. \([24]\) and \([25]\) is the sum of the photoelastic and radiation pressure effects:

\[ Q_i = Q_i^{(\text{P})} + Q_i^{(\text{R})} + Q_i^{(\text{MB})}. \]

IV. ACOUSTIC MODAL EQUATIONS

After our description of the dynamics of the optical envelope functions, we now turn to the acoustic part of the problem. First, we derive the dynamic equations and then show how the driving term is related to the respective driving terms of the optical modal equations already found Section II.

A. Dynamical equations

We start with Eq. \([10]\), where the driving force densities \( F_i \) are due to the electromagnetic fields. In analogy to the deriving terms in the optical wave equation, we assume that the phase-matched part of the driving force density \( F \) is linear in the two optical envelope functions:

\[ F(r, t) = f(r, t) \left[ a^{(1)}(z, t) \right]^* a^{(2)}(z, t) + c.c. \cdot (42) \]

Substituting this ansatz into equation \([10]\) and dropping higher order terms eventually yields

\[ -i\Omega \sum_{jkl} \left[ (c_{ijkl} \partial_k + \partial_j c_{ijkl}) u_l \partial_z b - 2i\Omega \rho u_i \partial_t b \right. \]

\[ \left. + (\partial_j \eta_{ijkl} \partial_k u_l) b + [a^{(1)}]^* a^{(2)} f_j \right] + c.c. = 0. \]

After projecting onto the mode \( u \), we end up with the acoustic mode equation

\[ \partial_z b + \frac{1}{v_b} \partial_t b + ab = -\frac{i\Omega [a^{(1)}]^* a^{(2)}}{P_b} Q_b \]

\[ \alpha = \frac{Q^2}{P_b} \left\{ \int d^2 r \sum_{jkl} u_j^* \partial_j \eta_{ijkl} \partial_k u_l \right\}; \]

\[ Q_b = \int d^2 r \mathbf{u}^* \cdot \mathbf{f}, \]

where the coupling parameter \( Q_b \) is again explicitly a work linear density, and \( 1/\alpha \) is the effective dissipation length for the acoustic mode. This approach to the acoustic part of the SBS process differs significantly from the treatment in previous works \([21–23]\). The main difference is the fact that we regard the sound field as a moving wave rather than a localized oscillator. In Section VI A we show how to obtain expressions that are consistent with the literature by assuming that neither the optical nor the acoustic amplitude vary over the mean acoustic propagation length — an approximation that is well justified in long waveguides. In very short waveguides, however, the propagation of the sound wave can no longer be neglected and a treatment like ours is necessary. In Section VI B we discuss how the acoustic propagation reduces the total SBS-gain of a short waveguide relative to the prediction of the traditional long-waveguide approximation.
Regarding the limitations of our treatment, it should be stressed that the slowly varying envelope approximation is not necessarily justified for the acoustic part of the problem, because it requires that the acoustic wavelength is much smaller than the length scale on which the envelope function varies. This means that \( q \) has to be much larger than the damping constant \( \alpha \) and the beat length \( \pi/q \) much less than the free propagation length along the waveguide. This is usually the case for backward-SBS and forward-SBS between different branches of the optical dispersion relation unless they are nearly degenerate. However, an example for a SBS-setup where the SVEA (and therefore our equations) is formally not justified can be found in Ref. [22]. In this work, the waveguide consists of a series of forward-type SBS-active suspended regions with a length of 100 \( \mu \)m each. This is clearly the maximum free propagation length for the acoustic wave. The beat length between the optical modes, on the other hand, is given by the SBS Stokes shift and the optical phase velocity, leading to an acoustic wavelength of the order of centimeters. Here, the suspended regions resemble localized harmonic oscillators and a treatment along the lines of Ref. [20] seems appropriate.

B. Optical forces and thermodynamic considerations

We now come to a key part of the analysis, identifying the optical force density from the optical scattering integral Eq. (29). There are two common approaches to derive the force that is caused by an optical field. The first way is via the Lorentz force. To this end, the material response is expressed in terms of microscopic charges and currents which interact with the incident field. We basically follow this path in Section V.B. The second way is via a thermodynamic potential, see e.g. Ref. [32] for such a discussion of the connection between electrostriction and the photo-elastic effect. While less familiar in the photonics community, the latter approach is attractive for our problem, provided that the change in the entropy is known.

As part of our assumptions, we neglect optical loss and inelastic coupling effects. The only source of entropy is the mechanical loss, which can be neglected because of the very small acoustic amplitudes in SBS. If optical loss were to be included, the entropic contribution to the thermodynamic potential could become appreciable. However, the most common experimental situation is a steady state where temporally constant optical and acoustic intensities vary spatially along the waveguide. In this case, the temperature would approach an equilibrium distribution that can be controlled via the properties of a heat sink; the temperature is therefore the natural choice for an independent variable.

Typically, the mechanical contribution to the free energy of a solid body is separated into boundary terms (surface pressures) and interior density-like terms (internal stress) [33]. However, such a distinction is not convenient for our problem because of the complexity associated with the moving waveguide boundary. Accordingly, we adopt a picture based on a displacement field \( \mathbf{U} \) and a driving force density field \( \mathbf{F} \) that yields both body force densities and boundary pressures. Furthermore, the magnetic part to the photoelastic effect depends on \( \partial_t \mathbf{U} \), so a Lagrangian picture is best suited to the problem. Finally, the electromagnetic continuity conditions force us to decompose the electric fields in an unusual way.

The variation in the free energy density \( \mathcal{F} \) of a waveguide satisfies

\[
\delta \mathcal{F} = -S \delta T + \delta E^{(\text{mech})} + \delta E^{(\text{opt})},
\]

where \( \delta E^{(\text{mech})} \) and \( \delta E^{(\text{opt})} \) are the changes in mechanical and optical energy, respectively, and \( S \) and \( T \) denote entropy per unit length and temperature. The latter are not of great importance in this context as we assume a thermodynamically inert process. We can therefore neglect them. For the two other terms we assume:

\[
E^{(\text{mech})} = \frac{\rho}{2} |\partial_t \mathbf{U}|^2 + \Phi,
\]

\[
E^{(\text{opt})} = \frac{1}{2} \left( \mathbf{E} : \mathbf{D} + \mathbf{H} : \mathbf{B} \right)_{\text{opt}},
\]

where \( \Phi(\mathbf{U}) \) is the stored elastic energy and (recognizing that the mechanical system cannot follow the rapid electromagnetic oscillations,) the electromagnetic energy density is averaged over a time window \( T_{\text{opt}} \) that includes many optical cycles but is much smaller than one acoustic cycle. Note that in this context we distinguish between \( \mathbf{B} \) and \( \mu_0 \mathbf{H} \), because we assign the effect of the sound wave to one of them (\( \mathbf{H} \)) while the other one (\( \mathbf{B} \)) is kept fixed as an independent variable. Next, we perform a Legendre transformation to obtain the Lagrangian for the opto-mechanical system. Here, it comes as a great convenience that the electric fields only depend on \( \mathbf{U} \) (hence are potential-like) while the magnetic field only depends on \( \partial_t \mathbf{U} \) and effectively provides a correction to the kinetic energy:

\[
\mathcal{L} = \frac{\rho}{2} |\partial_t \mathbf{U}|^2 - \Phi - \frac{1}{2} \left( \mathbf{E} : \mathbf{D} - \mathbf{B} : \mathbf{H} \right)_{\text{opt}}.
\]

The first two terms lead to the acoustic wave equation, whereas the last two terms correspond to the optical driving force \( F \). By separating these two types of terms in the Euler-Lagrange equations, we find

\[
\mathbf{F} = \rho \partial_t^2 \mathbf{U} + \frac{\partial \Phi}{\partial \mathbf{U}} = - \frac{1}{2} \left( \frac{\partial (\mathbf{E} : \mathbf{D})}{\partial \mathbf{U}} \right)_{\text{opt}} - \frac{1}{2} \frac{d}{dt} \left( \mathbf{B} : \frac{\partial \mathbf{H}}{\partial (\partial_t \mathbf{U})} \right)_{\text{opt}}.
\]

At this stage it is not yet clear whether the electric part to the electromagnetic energy is best expressed with respect to the induction field or the electric field as the
independent variable. The aim must be to formulate the optical field in terms that are not influenced by the mechanical displacement field and, thus, allow us to describe the optical power independently from the acoustic excitation. In fact, when the waveguide is deformed as described by $U$, both $E$ and $D$ may be perturbed. A key observation, however, is that there is always a composition of field components of $E$ and $D$ that is not changed by the deformation. This is most easily seen for perturbations of any boundary between different dielectrics (see Section III D). According to the continuity conditions, this unchanged composition consists of the normal part of $D$ and the tangential part of $E$; for the photoelastic effect, which can be described as a change in permittivity (see Section III B), the unchanged quantity is $E$. Such compositions of $E$ and $D$ are independent of the presence of a weak sound wave and determined only by the choice of the waveguide optical modes excited. We denote them with a subscript “indep”:

$$\frac{\partial E_{\text{indep}}}{\partial U_i} = \frac{\partial D_{\text{indep}}}{\partial U_i} = 0.$$  (53)

The dependent variables (subscript “dep”) then are what remains, i.e. the difference between perturbed fields and the independent parts:

$$E_{\text{dep}} = E + \Delta E - E_{\text{indep}},$$  (54)
$$D_{\text{dep}} = D + \Delta D - D_{\text{indep}}.$$  (55)

By construction, they completely contain the deformation-dependence of the optical fields:

$$\frac{\partial E_{\text{dep}}}{\partial U_i} = \frac{\partial \Delta E}{\partial U_i}, \quad \frac{\partial D_{\text{dep}}}{\partial U_i} = \frac{\partial \Delta D}{\partial U_i}.$$  (56)

If we assume that the continuity conditions at material discontinuities determine the decomposition of the fields into dependent and independent quantities, we can furthermore say that

$$E_{\text{indep}} \cdot D_{\text{indep}} = E_{\text{dep}} \cdot D_{\text{dep}} = 0.$$  (57)

This is also true for electrostriction, because in this case $D_{\text{indep}} = E_{\text{dep}} = 0$. In order to calculate the optical forces, we thus perform another Legendre transformation:

$$\tilde{\mathcal{L}} = \mathcal{L} - \left\langle \int d^2r \ E_{\text{dep}} \cdot D \right\rangle_{T_{\text{opt}}}.$$  (58)

Its electric part satisfies the form

$$\left\langle \frac{\partial (E \cdot D)}{\partial U} \right\rangle_{T_{\text{opt}}} = \left\langle E_{\text{indep}} \cdot \frac{\partial D_{\text{dep}}}{\partial U} - D_{\text{indep}} \cdot \frac{\partial E_{\text{dep}}}{\partial U} \right\rangle_{T_{\text{opt}}}
+ \left\langle E_{\text{indep}} \cdot \frac{\partial D_{\text{indep}}}{\partial U} - D_{\text{indep}} \cdot \frac{\partial E_{\text{indep}}}{\partial U} \right\rangle_{T_{\text{opt}}}
+ \left\langle E \cdot \frac{\partial D}{\partial U} - D \cdot \frac{\partial E}{\partial U} \right\rangle_{T_{\text{opt}}}.$$  (59)

where we used Eq. (57) in the second step. With this, the optical force density at position $r$ with the illumination of the waveguide held constant becomes

$$\mathbf{F}(r) = \frac{1}{2} \left\langle \mathbf{E} \cdot \frac{\partial (\Delta \mathbf{D})}{\partial U} - \mathbf{D} \cdot \frac{\partial (\Delta \mathbf{E})}{\partial U} \right\rangle_{T_{\text{opt}}}
- \frac{1}{2} \frac{d}{dt} \left\langle \mathbf{B} \cdot \frac{\partial (\Delta \mathbf{H})}{\partial (\partial U)} \right\rangle_{T_{\text{opt}}}. \quad (61)$$

Next, we note that the total deformation-induced field perturbations $\Delta \mathbf{E}$, $\Delta \mathbf{D}$ and $\Delta \mathbf{H}$ are to leading order proportional to the displacement field $U$ or its time derivative, respectively. This follows because we evaluate the derivative at the point $U = 0$ and any higher order dependence would be to no effect. Then, the force is independent of the displacement amplitude and the total work density becomes

$$W(U) = - \int d^2r \ U \cdot \mathbf{F}$$  (62)

$$= \frac{1}{2} \int d^2r \sum_i U_i \left\langle \mathbf{D} \cdot \frac{\Delta \mathbf{E}}{U_i} - \mathbf{E} \cdot \frac{\Delta \mathbf{D}}{U_i} \right\rangle_{T_{\text{opt}}}
+ \left( \partial_i U_i \right) \left\langle \mathbf{B} \cdot \frac{\Delta \mathbf{H}}{(\partial U)} \right\rangle_{T_{\text{opt}}}
- \frac{d}{dt} \left\langle \mathbf{U} \cdot \left[ \mathbf{B} \cdot \frac{\Delta \mathbf{H}}{\partial U} \right] \right\rangle_{T_{\text{opt}}}.$$  (63)

$$= \frac{1}{2} \int d^2r \left[ \mathbf{D} \cdot (\Delta \mathbf{E}) - \mathbf{E} \cdot (\Delta \mathbf{D})
+ \mathbf{B} \cdot (\Delta \mathbf{H}) \right]_{T_{\text{opt}}}
- \frac{d}{dt} \left\langle \mathbf{U} \cdot \left[ \mathbf{B} \cdot \frac{\Delta \mathbf{H}}{\partial U} \right] \right\rangle_{T_{\text{opt}}}.$$  (64)

It is important to point out that the last term oscillates and averages out to zero over an acoustic cycle. With this in mind, Eq. (64) is a significant result which confirms that the energy that appears as mechanical work per acoustic cycle is precisely the change in the average optical energy density.

Using this result, we can evaluate the mechanical overlap integral from Eq. (44). The coupling integral is supposed to drive the acoustic envelope function $b(z,t)$, which we assumed to vary only slowly compared to one acoustic cycle. So what we really need is a time-average of the mechanical work

$$\left\langle -W(U) \right\rangle_{T_{\text{ac}}} = \left\langle \int d^2r \ U \cdot \mathbf{F} \right\rangle_{T_{\text{ac}}} = a_1^* a_2 b^* \int d^2r \mathbf{u}^* \cdot \mathbf{f} + \text{c.c.}, \quad (65)$$

where all terms that oscillate with at acoustic frequencies

$$\text{c.c.}$$
average to zero. From Eq. (64), we find
\[
\langle -\mathcal{W}(U) \rangle_{t_0} = \frac{a_1^* a_2 b^*}{2} \int d^2 r \left\{ [e(1)]^* \cdot \Delta d(1) + e(2) \cdot [\Delta d(2)]^* - [d(1)]^* \cdot \Delta e(1) - d(2) \cdot [\Delta e(2)]^* - \mu_0 [h(1)]^* \cdot \Delta h(1) - \mu_0 h(2) \cdot [\Delta h(2)]^* \right\} + \text{c.c.} . \tag{67}
\]
By comparing with Eqs. (49) and (50) and identifying terms with the same dependence on the envelope functions, we find in the absence of irreversible scattering processes
\[
Q_b = \frac{Q_1 + Q_2^*}{2} . \tag{68}
\]
Thus, we have managed to formulate the mechanical driving integral Eq. (40) in terms of the field perturbations, and have found a first connection between the scattering strength of the sound wave for optical modes and the optical forces that are exerted on the waveguide.

In Section III, we observed that \( Q_1 = Q_2^* \), which further simplifies Eq. (68). Next, we show that this is not a coincidence but, instead, a consequence of our initial assumptions and modal approximation.

V. DISCUSSION

In this and the following section, we discuss some aspects of our model more closely. We do so based on the dynamic equations and coupling terms that we have derived up to now:
\[
\begin{align*}
\partial_z a(1) + \frac{1}{v(1)} \partial_t a(1) &= - \frac{i \omega (1)^* Q_1}{\rho(1)} a(2) b^* , \tag{69} \\
\partial_z a(2) + \frac{1}{v(2)} \partial_t a(2) &= - \frac{i \omega (2)^* Q_2}{\rho(2)} a(1) b , \tag{70} \\
\partial_z b + \frac{1}{v_b} \partial_t b + ab &= - \frac{i \Omega Q_b}{\rho_b} [a(1)]^* a(2) , \tag{71}
\end{align*}
\]

First, we discuss the impact of (approximate) conservation of energy on the coupling coefficients and make with a few remarks about inelastic (i.e. lossy) forces.

A. Conservation of energy

The main finding Eq. (61) of the previous discussion is not restricted to the expansion into guided modes. It could be possible that the approximation that comes with such a modal expansion spoils the conservation laws. This justifies a check under which conditions the modal equations Eq. (27), Eq. (28) and Eq. (44) themselves conserve energy.

The total energy that is stored in the optical and acoustic modes is
\[
U = \int_{-\infty}^{\infty} dz [E(1)|a(1)|^2 + E(2)|a(2)|^2 + E_b|b|^2] . \tag{75}
\]

Energy is conserved if its time-derivative vanishes:
\[
0 = \partial_t U = \int_{-\infty}^{\infty} dz \partial_t \left[ E(1)|a(1)|^2 + E(2)|a(2)|^2 + E_b|b|^2 \right]
\]
\[
\begin{align*}
&= \int_{-\infty}^{\infty} dz \left\{ E(1)[a(1)]^* \left[ -v(1) \partial_z a(1) + \frac{i \omega (1)^* Q_1}{E(1)} a(2) b^* \right] + E(2)[a(2)]^* \left[ -v(2) \partial_z a(2) + \frac{i \omega (2)^* Q_2}{E(2)} a(1) b \right] \\
&\quad + E_b b^* \left[ -v_b (\partial_z + a) b + \frac{i \Omega Q_b}{E_b} [a(1)]^* a(2) \right] \right\} + \text{c.c.} . \tag{76}
\end{align*}
\]

Clearly, energy can only be conserved in the absence of inelastic forces and of propagation loss, i.e. we have to
assume $\alpha = 0$ and $Q_b^{\text{(inelastic)}} = 0$. After collecting terms with identical combinations of envelopes, we find a condition for the coupling integrals:

$$\omega^{(1)}Q_1 - \omega^{(2)}Q_2 + \Omega Q_b = 0. \quad (78)$$

In conjunction with Eq. (14), this yields

$$Q_b = \frac{\omega^{(1)}Q_1 - \omega^{(2)}Q_2}{\omega^{(1)} - \omega^{(2)}} \quad (79)$$

which with Eq. (88) implies

$$Q_1 = Q_2^* = Q_b. \quad (80)$$

We find that energy can only be conserved if all coupling constants are equal modulo complex conjugation. This means in particular that $Q_1 = Q_2^*$, an equality that we have found to be true for the coupling terms in Section III. This illustrates that the modal theory is consistent. At first glance, this conclusion can no longer be drawn for $\alpha \neq 0$. However, if the coupling mechanism is reversible, loss parameters such as the viscosity tensor cannot explicitly appear in the coupling integrals; the $Q_\alpha$ can depend on the loss parameters only through the basis functions. In Section III, we chose them to be solutions to the lossless wave equations, because we assumed loss to be so weak that its impact on the eigenmode pattern can be ignored. Thus, introducing a non-zero $\alpha$ does not affect the coupling coefficients within our approximations and Eq. (88) is valid for weak propagation loss. On the other hand, this provides a sanity check to identify situations where the modal expansion is no longer justified.

### B. Identification of optical forces

In the previous section, we showed that the coupling coefficients $Q_\alpha$ for the excitation of the optical and acoustical modes are identical (apart from complex conjugation) if energy is conserved in a weak sense, i.e. if the coupling processes conserve entropy and propagation losses are so weak that the differences between the actual eigenmodes and those of the lossless wave equations are negligible. In Section III we derived expressions for these coupling coefficients from the perturbation of the optical eigenmodes caused by the deformation of the waveguide. They include all contributions to the scattering due to boundary movement and strain in the material. As a consequence, we can describe SBS without referring to optical forces.

Prior formulations of SBS relied on optical forces, namely on electrostriction and radiation pressure. The former is the reciprocal effect to the electric photoelastic term discussed in Section III B, however the latter can lead to some confusion, especially in the waveguide interior. Sometimes, the radiation pressure force density is assumed to be the divergence of Maxwell’s stress tensor

$$\mathbf{T}^{\text{(opt)}} = \mathbf{E} \otimes \mathbf{D} + \mathbf{H} \otimes \mathbf{B} - \frac{1}{2}(\mathbf{E} \cdot \mathbf{D} + \mathbf{H} \cdot \mathbf{B}), \quad (81)$$

where the symbols $\otimes$ and $\ll$ denote the tensor product of two vectors and the unit tensor, respectively. This is not generally correct; for mono-atomic fluids for example, the radiation pressure can be expressed \[34\] as

$$\mathbf{F} = \nabla \cdot \mathbf{T}^{\text{(opt)}} - \partial_t \mathbf{G}^{\text{(opt)}},$$

where $\mathbf{G}^{\text{(opt)}}$ is the momentum density of the optical wave. Both the interpretation of the quantities $\mathbf{T}^{\text{(opt)}}$ and $\mathbf{G}^{\text{(opt)}}$, and their exact expressions have been under debate for about a century. The two best-known versions of $\mathbf{G}^{\text{(opt)}}$ were proposed by Minkowski ($\mathbf{D} \times \mathbf{B}$) and Abraham ($\varepsilon_0 \mu_0 \mathbf{E} \times \mathbf{H}$), where electrostatic and quasi-static experiments were in agreement with the latter \[34\]. For completeness we note that Minkowski and Abraham also ended up with differences in the Maxwell stress tensor, but that those differences are only relevant for anisotropic materials, which we neglect here. A third expression for the momentum density

$$\mathbf{G}^{\text{(opt)}} = \varepsilon_0 \mathbf{E} \times \mathbf{B} \quad (82)$$

has been derived by several authors including Nelson \[35\] and is indistinguishable from Abraham’s expression under our restrictions and assumptions. Furthermore, Nelson showed that the stress induced by the optical fields and by mechanisms within the solid cannot be separated. Although the question of optical forces has been mainly resolved in recent years, it is prone to subtle mistakes. This is a key motivation for our decision to base our formulation on field perturbations, which can be deduced from directly measurable quantities. As we have shown in Section V A this is sufficient for reversible interactions and weak losses.

In the remainder of this section, we attempt to clarify the issue of body force densities related to radiation pressure. We base our discussion on Nelson’s treatment \[32\]. The central insight is that light inside a material can be considered to consist of two separate waves. The first wave is formed by the fields $\mathbf{E}$ and $\mathbf{B} = \mu_0 \mathbf{H}$ and carries the momentum in Eq. (82). The remainder of the total light wave is a polarization wave inside the material. This wave is caused by collective excitations such as optical phonons (more precisely: the corresponding polaritons) and does not carry any momentum. Instead, it carries a pseudo-momentum (also known as quasi-momentum or crystal momentum) associated with the solid’s lattice coordinates rather than the coordinates of the global coordinate system. The macroscopic body force appears in conservation laws for both quantities, because it can accelerate the solid’s overall center of mass as well as excite a sound wave, which only carried pseudo-momentum. The latter happens if the force is oscillatory and the local momentum of each piece of material assumes various values over time but averages to zero. Thus, we are free to choose from which momentum-like quantity to derive the body force. We favor the continuity equation for the “proper” momentum:

$$-\rho \partial_t \mathbf{U} = \nabla \cdot (\mathbf{T}^{\text{(mech)}} + \mathbf{T}^{\text{opt}} + \mathbf{T}^{\text{(ES)}}) - \partial_t \mathbf{G}^{\text{(opt)}}, \quad (83)$$
where $G^{(\text{opt})}$ is the expression from Eq. (82) and $T^{(\text{mech})}$ and $T^{(\text{opt})}$ are the strain-induced mechanical stress and Maxwell's stress tensor, respectively. The last contribution $T^{(ES)}$ is a partially material-induced nonlinear stress, which we identify as the electrostrictive stress. The derivation of Eq. (83) is quite complex and requires different notation to the remainder of this section. It can be found in Appendix A. The left hand side of Eq. (83) and $\nabla \cdot T^{(\text{mech})}$ form the acoustic wave equation. Along the lines of Eq. (72), the optical force is the rest:

$$
F = \nabla \cdot (T^{(\text{opt})} + T^{(ES)}) - \partial_t G^{(\text{opt})}. 
$$

(84)

Next, we match optical force terms to the scattering processes identified in Section III. To this end, we first notice that in the absence of macroscopic charges and current the divergence of $T^{(\text{opt})}$ is equal to $\partial_t (D \times B)$ up to a surface term [34]. However, $G^{(\text{opt})} = \varepsilon_0 \sigma E \times B$. Thus, there is a dynamic body force density $\nabla \cdot T^{(\text{opt})} - \partial_t G^{(\text{opt})} = \partial_t (P \times B)$, whose overlap with the displacement field is

$$
i \Omega \varepsilon_0 \varepsilon_r (\varepsilon_r - 1) \int_A d^2 r \ u^* \cdot (e^{(2)} \times |h^{(1)}|^*)^*, \quad (85)
$$

which exactly matches Eq. (86). We would like to point out two things regarding this body force: First, it is neither zero nor $\nabla \cdot T^{(\text{opt})}$, although the difference from the latter (being a factor of $\varepsilon_r^{-1}$) is small for high index materials such as silicon. Thus, previous gain calculations including body forces are technically incorrect but the errors are unlikely to exceed a few percent. Second, the force term in question is a pure body force because it does not involve a spatial derivative that could cause a singularity at a material boundary. Thus, the boundary pressure is the boundary part of $\nabla \cdot T^{(\text{opt})}$ alone and ends up to be $\frac{1}{2} \left[ E \cdot (\nabla \times D) - D \cdot (\nabla \times E) \right]$, whose overlap with the displacement field is

$$
\int_C d^2 r \ (u^* \cdot \hat{n}) \left[ (\varepsilon_a - \varepsilon_b) (\hat{n} \times e^{(1)})^* (\hat{n} \times e^{(2)})
- (\varepsilon_b^{-1} - \varepsilon_a^{-1}) (\hat{n} \cdot d^{(1)})^* (\hat{n} \cdot d^{(2)}) \right],
$$

(86)

which matches the moving boundary scattering effect of Eq. (11). Finally, the electrostrictive effect is connected to the electric photoelastic effect via a Maxwell relation and this connection has been investigated e.g. in Ref. 32. With this, we have managed to pair up all optical force terms with their corresponding contributions to Eq. (29) and thus again showed consistency with Eq. (68). It should be noted that the stress $T^{(ES)}$ identified to describe electrostriction in Appendix A does not depend on $\Omega$. As a consequence, we can conclude that the photoelastic tensor in Eq. (53) should be taken from a measurement under static stress.

\section*{C. Inelastic forces}

Our discussion of forces and scattering effects has certain limitations. Most importantly, the identities Eq. (68) and Eq. (80) no longer hold for irreversible coupling mechanisms. The key feature of such mechanisms is the creation of entropy, i.e. the connection to loss of some sort. Although such terms can in principle be generated by the absorption of phonons (for example, the mechanical friction could separate charges that contribute to the scattering of light waves), it is likely that the optical force caused by the absorption of light is most important among inelastic coupling effects.

Consider a waveguide composed of an optically lossy material. The loss may either be due to absorption or to diffuse scattering, e.g. Rayleigh scattering. As light is absorbed or diffusely scattered, its momentum is not lost but transferred to the solid, leading to a longitudinal radiation pressure force, which can be described as a Lorentz force:

$$
F = \left< J \times H \right>_{T_{opt}} = \sigma \left< E \times H \right>_{T_{opt}}.
$$

(87)

Here, both absorption and the time-averaged force are caused by that part of the current density $J$ that is in phase with the electric field, i.e. that can be expressed as $J = \sigma E$ with some real-valued conductivity $\sigma(\omega) = 3 \{\omega \varepsilon\}$. However, this force will not be balanced by a corresponding term in the optical mode evolution equations, because it is not reversible. More precisely, the mechanism that causes the force increases the total entropy, either by heating the crystal lattice or by inflating the phase space volume of the radiation, and thus breaks Eq. (63).

The occurrence of irreversible contributions is not restricted to radiation pressure. It has been reported that the electrostrictive effect of semiconductors in a quasi-static electric field is dominated by an irreversible process [30]. In this case, the finite conductivity leads to both a dissipative current and a redistribution of charge carriers in reciprocal space that energetically favors a distorted crystal lattice.

\section*{VI. EXAMPLES}

We now turn to two simple examples in contrasting situations. Specifically, we focus on approximations for steady-state experiments in two types of waveguides. The first example is a very long waveguide with weak SBS (e.g. silica fiber) and we obtain the well-known result for the stimulated Brillouin gain $G$ by making a local-response approximation to the acoustic field. The second example is a short waveguide with a strong SBS effect, but where non-local acoustic effects cannot be neglected. This discussion is especially motivated by the proposal of suspended silicon nanowires as CMOS-compatible SBS-active waveguides [21].
A. Long waveguides in steady state

In many applications, SBS is a weak process (though still possibly the strongest nonlinearity present) and the length scale on which the optical power changes is larger than the decay length of the acoustic wave. Furthermore, SBS is often investigated in a quasi-static setting where all mode power levels are in equilibrium. In this subsection, we show how the dynamic equations can be approximated for this situation and that we obtain the expected Lorentzian resonance behavior. To this end, we need to allow for weak detuning of the laser fields. This means that the phase-matching conditions Eq. (11) can no longer both be met at the same time. However, it is still possible to find modes that fulfill one of them and the detuning can be expressed either by a frequency difference \( \delta \omega \) or a wave vector difference \( \kappa = \delta \omega / v_b \). We assume that the detuning is so small that the eigenmodes, frequencies, powers and the decay parameter are effectively unchanged. These assumptions are typically justified except at band edges. As we are aiming for a steady-state solution, it is advisable to retain the frequency condition \( \Omega = \omega(2) - \omega(1) \). Consequently, we can express the detuning in terms of a wave vector mismatch \( \kappa = q - \beta(2) + \beta(1) \), which we incorporate as a slowly varying, harmonic relative phase between the envelope functions along the waveguides.

First, we impose the steady-state condition, which means that we neglect any time-derivative in the dynamic equations:

\[
\begin{align*}
\partial_z a^{(1)} &= -\frac{i\omega(1)Q_1}{P^{(1)}} a^{(2)*} b, \\
\partial_z a^{(2)} &= -\frac{i\omega(2)Q_2}{P^{(2)}} a^{(1)} b, \\
\partial_z b + a b &= -\frac{i\Omega b}{P_b} [a^{(1)}]^* a^{(2)}.
\end{align*}
\]

Whether for forwards or backwards SBS, the acoustic wave evolves towards positive \( z \), and we solve the last equation by means of its Green’s function:

\[
b(z) = -\frac{i\Omega b}{P_b} \int_0^\infty dz' \left\{ [a^{(1)}(z-z')]^* a^{(2)}(z-z') \right\} \exp(-\alpha z').
\]

Next, we use the assumption that the optical powers vary on a length scale that is much larger than \( \alpha^{-1} \):

\[
[a^{(1)}(z-z')]^* a^{(2)}(z-z') \approx [a^{(1)}(z)]^* a^{(2)}(z) \exp(i\kappa z'),
\]

with some detuning parameter \( \kappa \) that expresses a violation of the phase matching condition. We find:

\[
b(z) \approx -\frac{i\Omega b}{P_b} [a^{(1)}(z)]^* a^{(2)}(z) \times \int_0^\infty dz' \exp[-(\alpha + i\kappa)z']
\]

\[
= -\frac{i\Omega b}{P_b} [a^{(1)}(z)]^* a^{(2)}(z) L(\kappa),
\]

where \( L(\kappa) = (\alpha - i\kappa)^{-1} \) is a Lorentzian resonance that defines the bandwidth of the SBS process. With this and approximating \( \omega(1) \approx \omega(2) = \omega \), we obtain simplified equations:

\[
\begin{align*}
\partial_z a^{(1)} &= \pm GP^{(2)} |a^{(2)}|^2 a^{(1)}, \\
\partial_z a^{(2)} &= -GP^{(1)} |a^{(1)}|^2 a^{(2)},
\end{align*}
\]

\[
G = \frac{\omega Q_1^2}{P^{(1)}P^{(2)}P_b(\alpha - i\kappa)}.
\]

with the conventional SBS gain parameter \( G \). In the presence of inelastic force terms, the gain is modified:

\[
G = \frac{\omega Q_1^2}{P^{(1)}P^{(2)}P_b(\alpha - i\kappa)(Q_1^2 + Q_b^{(inelastic)})^*}.
\]

The sign of the Stokes mode power \( P^{(1)} \) distinguishes between propagation in positive and negative \( z \)-direction, and hence between forward and backward SBS. In practice, it is often more convenient to express the light field in terms of transmitted powers \( p^{(i)} = (|a^{(i)}|^2P^{(i)}) \) rather than the complex envelope functions themselves. To this end, we apply the \( z \)-derivative to the power carried by each mode and insert Eq. (99):

\[
\partial_z p^{(1)} = ((a^{(1)}*a^{(1)}) + a^{(1)}a^{(1)}*a^{(1)})P^{(1)}
= \pm 2\Re\{G\}p^{(1)}p^{(2)},
\]

\[
\partial_z p^{(2)} = -2\Re\{G\}p^{(1)}p^{(2)}.
\]

Thus, the SBS-gain relating optical power levels is

\[
\Gamma = 2\Re\{G\} = \frac{2\alpha}{\alpha^2 + \kappa^2}.
\]

B. Acoustic pile-up in short waveguides

One of the ultimate goals of the study of SBS in integrated waveguides is the incorporation of SBS-active devices in a CMOS-chip. To this end, the waveguides should be as short as possible with as strong an interaction as possible. Typical candidates are silicon nanowires in free air that make full use of radiation pressure effects, but these can only be suspended over a distance of about 100 µm. This is comparable to or even less than the acoustic decay length that can be estimated from the acoustic wave vector and the mechanical quality factor. We can illustrate this using the example of backward
SBS in a waveguide with an optical effective mode index of 2 and a mechanical quality factor of 1000 at a vacuum wavelength of 1550nm. The resulting decay length would be $\alpha^{-1} \approx 62\mu$m. One can hope to reach significant total gain by cascading a large number of such waveguide sections. However, we may expect a degradation of the gain because in each segment the acoustic amplitude requires some length of the order of $\alpha^{-1}$ to build up to its nominal value. In the following, we investigate this problem.

As before, we assume steady state dynamics. Furthermore, we assume that the pump amplitude does not change: $\partial_z a_2 = 0$. The simplified dynamical equations now read

$$\partial_z a^{(1)} = -\frac{i\omega^{(1)} Q_1}{\mathcal{P}^{(1)}} a^{(2)} b^*, \quad \partial_z b + (\alpha - i\kappa) b = -\frac{i\Omega Q_b}{\mathcal{P}_b} (a^{(1)})^* a^{(2)}.$$  

We suppose the waveguide section starts at $z = 0$ and assume the initial condition $b(0) = 0$. This leads to the solution for the acoustic part of the problem:

$$b(z) = B_0 \sinh(\Lambda z) \exp(-\lambda z).$$  

where $B_0$ is a constant that is ultimately determined by the input power of the Stokes mode. Figure 4 shows the behavior of this function on resonance ($\kappa = 0$) for forward SBS ($\mu > 0$) and backward SBS ($\mu < 0$). The optical amplitude $a^{(1)}$ could be calculated by integrating Eq. (103) with an appropriate boundary condition. However, it is easier to obtain it directly from Eq. (104):

$$[a^{(1)}]^* = A_0 \left[ \cosh(\Lambda z) + \frac{\Lambda}{\lambda} \sinh(\Lambda z) \right] \exp(-\lambda z),$$  

where $A_0$ is the Stokes amplitude at $z = 0$ (ie. the input amplitude for forward SBS, and the output amplitude for backward SBS). Figure 4 shows the behavior of this function on resonance both for forward and backward SBS. Clearly, the Stokes amplitude is reduced in comparison to the simple exponential growth that would be expected from the results of Section VI A (dotted lines in Fig. 4b). Finally, one would expect that the SBS resonance broadens as the effective gain decreases. We can see this from Eq. (109) by taking the logarithm and dividing it by the factor $\mu z$. This quantity $L = (\mu z)^{-1} \ln[a^{(1)}/A_0]$ corresponds to the Lorentzian function $L(\kappa)$ in Section VI A. Its real part describes the spectral distribution of gain, whereas its imaginary part is related to the evolution of the optical
phase. We can see in Fig. 4, that the gain reduction is rather robust with respect to the coupling strength $\mu$, so we may expect this to be true for the resonance shape as well. This justifies that we assume the term $\mu/\lambda^2$ to be small and approximate to leading order:

$$
\tilde{L} \approx \ln \left\{ \left(1 - \frac{\mu}{2\lambda^2}\right)^{\frac{\mu z}{\lambda}} \sinh \left(\lambda z + \frac{\mu z}{2\lambda}\right) + \cosh \left(\lambda z + \frac{\mu z}{2\lambda}\right) \exp(-\lambda z) \right\} 
$$

(110)

$$
\approx \frac{1}{2\lambda} \left( 1 - \frac{1}{2\lambda} \right) + \frac{1}{4\lambda^2 z} \exp(-2\lambda z), \quad (111)
$$

where we have used $\ln(1 + x) \approx x$ in the second step. The leading term $1/(2\lambda) = 1/(\alpha - i\kappa)$ is exactly the Lorentzian resonance that we expect for an infinitely long waveguide (Section VI A). For a sufficiently long waveguide, the exponential remains negligible and the effective waveguide length is essentially just reduced by one acoustic decay length $1/\alpha$. For shorter waveguides, the waveform term broadens the resonance and distort it into a non-Lorentzian shape as depicted in Fig. 3. Since the asymptotic behavior of the curves in Fig. 4c, the gain of a short waveguide is not increased by reducing acoustic losses. Instead, the resonance line is deformed into a non-Lorentzian shape with a number of side bands and the total gain saturates at a maximum value.

VII. CONCLUSION

In this paper, we have formulated the dynamics of SBS in a coupled mode framework. Based on energy considerations, we have established connections between the nonlinear coupling coefficients that mediate the interplay of optical and acoustic eigenmodes in a waveguide. In this context, the connection between scattering of optical eigenmodes due to the motion of dielectric interfaces in conjunction with the electromagnetic continuity conditions on the one hand and the transverse radiation pressure on the other hand is a finding of some interest. We have also pointed out that inelastic forces are bound to appear in lossy waveguides and that these coupling terms form a qualitatively different contribution to the SBS gain. This finding may become relevant in the near future as people start to begin to consider metals in SBS-devices.

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Appendix A: Conservation of momentum

In this appendix, we show how to obtain Eq. S32 from the continuity equation for the ordinary momentum inside a solid. This discussion is meant to bridge the gap between the present manuscript and Nelson’s paper [35]. As such, it requires the reader to be familiar with that reference. The following discussion is carried out in the notation employed by Nelson and we refer the reader to Ref. 35 for an explanation of the symbols that appear here. Finally, we refer equations from that paper with a letter ‘N’ in front of the equation number.

The continuity equation is given by equation (N34):

$$
\frac{\partial}{\partial t}(\rho \dot{\mathbf{x}} + \varepsilon_0 \mathbf{E} \times \mathbf{B})_i - \frac{\partial}{\partial z_i} (t_{ij}^E + m_{ij} - \rho \dot{x}_i \dot{x}_j) = 0, \quad (A1)
$$

where $t_{ij}^E$ and $m_{ij}$ are given by equations (N30) and (N33). Next, we identify the force density as $f = \rho \dot{x}$. Furthermore, we set $\mathbf{B} = \mu_0 \mathbf{H}$, neglect the magnetization and electric quadrupolarization terms and finally we add and immediately subtract the term $E_i P_j - \delta_{ij} E_k P_k/2$ to find:

$$
f_i = -\frac{\partial}{\partial t} (\varepsilon_0 \mathbf{E} \times \mathbf{B})_i + \frac{\partial}{\partial z_i}\left[ x_{i,A} x_{i,B} \frac{\partial (\rho \Sigma)}{\partial E_{AB}} - \rho \dot{x}_i \dot{x}_j \right.

\left. + E_i D_l + H_i B_l - \frac{1}{2} (E_k D_k + H_k B_k) \delta_{il} \right.

\left. + \sum_{\nu} \rho_{\nu} \dot{y}_i^\nu \dot{y}_j^\nu - E_i P_j + \frac{1}{2} E_k P_k \delta_{ij} \right] \quad (A2)
$$

The first line inside the square bracket is the conductive and advective momentum flux inside the solid. The second line is Maxwell’s stress tensor as expected inside a material. However, the last line requires some further treatment to allow for a better interpretation. To this end, we use equations (N16) and (N21), where we again neglect magnetization and quadrupole terms, to find:

$$
\sum_{\nu} \rho_{\nu} \dot{y}_i^\nu \dot{y}_j^\nu

= -\frac{1}{J} \sum_{\nu} \left( \frac{\partial (\rho \Sigma)}{\partial y_i^\nu} + q_{\nu} \epsilon_{ijk} \dot{x}_j B_k \right) \dot{y}_j^\nu + E_i P_j. \quad (A3)
$$

Next, we assume that the displacement field $\mathbf{x}$ is weak, which clearly is the case in SBS. Consequently, we drop all terms that involve $\mathbf{x}$ or derivatives as their overlap with the displacement field would be non-linear. We cannot make such an assumption for the polarization-related
The fields $y^\nu$, because the optical fields are strong:

$$f_i = -\frac{\partial}{\partial t} (\varepsilon_0 \mathbf{E} \times \mathbf{B})_i + \frac{\partial}{\partial z} \left[ \frac{x_i A x_i B}{J} \frac{\partial (\rho^\Sigma)}{\partial z} \mathbf{T}^{(mech)} + E_i D_i + H_i B_i - \frac{1}{2} (E_k D_k + H_k B_k) \delta_{ik} \mathbf{T}^{(opt)} \right] + \frac{1}{J} \sum_{\nu} \frac{\partial (\rho^\Sigma)}{\partial y_{i \nu}} y_i T_{\nu} \mathbf{T}^{(ES)} \right],$$

(A4)

where we annotated which terms give rise to the individual contributions to the total stress tensor in Eq. \(\mathbf{T}^{(ES)}\). The first term in the last line is the electrostrictive stress for an isotropic, easily compressible system such as a gas [Eq. (9.2.7) together with Eq. (8.3.12) in Boyd’s book [1]]. The second term in the last line is a modification of this basic expression due to the internal anharmonicity and non-linear coupling between the internal degrees of freedom $y^\nu$ and the displacement field. This coupling stems from higher order terms in the expansion shown in the first equation in the appendix of Nelson’s paper [25], specifically from the terms $M^{\mu\rho}_{ABCD}$ and $M^{\mu\rho}_{AB}$. The last line as a whole can be identified as the electrostrictive stress of a dielectric insulator under static conditions.

**Appendix B: Estimate of magnetic photoelastic effect for plane waves**

In this appendix, we estimate the relevance of the magnetic coupling term described in Section [11]. We base this on the expressions for the reciprocal optical force contributions as identified in Section [V1]. The contribution from the magnetic coupling depends heavily on details of the structure, especially on the geometry and mode symmetry. Thus, a discussion of its influence on the SBS-properties of specific designs for integrated optical waveguides is beyond the scope of this paper. On the other hand, the problem of plane waves can be easily solved and is important to estimate the contribution of the magnetic part in measurements of the photoelastic parameters of a material based on acousto-optic methods.

As we assume quasi-plane waves, we are dealing only with the bulk material, which we furthermore assume to be isotropic. Finally, we assume backward-type SBS caused by the longitudinal acoustic wave. The force caused by the reciprocal process of the conventional electric photoelastic effect and by the reciprocal process of the dynamic, effective magnetic photoelastic effect are:

$$\mathbf{F}^{(ePE)} = \nabla \cdot \mathbf{T}^{(ES)},$$

(B1)

$$\mathbf{F}^{(mPE)} = \partial_i (\mathbf{P} \times \mathbf{B}).$$

(B2)

Both terms oscillate at optical frequencies. The appropriate time-averages are:

$$\langle \mathbf{F}^{(ePE)} \rangle_{T_{opt}} = \partial_z \left[ \varepsilon_0 \varepsilon_r^2 p_{xxzz} |\mathbf{E}|^2 \right],$$

(B3)

$$= - i \Omega \varepsilon_0 \varepsilon_r^2 c_p |\mathbf{E}|^2,$$

(B4)

$$\langle \mathbf{F}^{(mPE)} \rangle_{T_{opt}} = - i \Omega (\mathbf{P} \times \mathbf{B}),$$

(B5)

$$= - i \Omega \varepsilon_0 (\varepsilon_r - 1) \sqrt{\varepsilon_r} |\mathbf{E}|^2,$$

(B6)

where $c$ and $c_p$ are the vacuum speed of light and the speed of the longitudinal acoustic wave, respectively. The
ratio between both forces is
\[
\frac{\langle F^{(\text{mPE})} \rangle_{T_{\text{opt}}}}{\langle F^{(ePE)} \rangle_{T_{\text{opt}}}} = \frac{c_{\text{ac}}}{c} \cdot \frac{(\varepsilon_r - 1)}{\varepsilon_r^{3/2} p_{xxzz}}.
\] (B7)

The first factor is proportional to $\Omega/\omega$, the second factor is of the order of 1. This means that the magnetic coupling is a very weak effect for plane waves.

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