2-D Interpretation of Minamikayabe MT Data Set
by the Controlled Random Search Technique

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Within the MT-DIW3 project of the comparison of various MT interpretation techniques,
the controlled random search procedure was applied to the 2-D interpretation of the MT
data from the Minamikayabe MT polygon in Japan. Three NE-SW profiles together with
one perpendicular profile were 2-D modelled, simultaneously for both polarizations of the
MT field. Although the 2-D approximation seems to be rather far from the real resistivity
conditions, we could obtain relatively consistent interpretation results particularly for data
within the short period range, up to 1 s. Better fit was achieved for apparent resistivities
than for phases. For all NE-SW profiles, a near-surface conductive body is indicated below
the central part of the polygon. At both sides of this body two deeper conductive zones are
obvious. As for the perpendicular profile k6*s06, a large conductive zone is required below
the SE margin of the profile.

1. Introduction

For the comparison of various techniques of the magnetotelluric data interpretation, de-
veloped or employed by different teams all over the world, the MT-DIW3 working group has
recently suggested a new test set of MT data, specifically a 2-D array data set from the region
of Minamikayabe in the south of Hokkaido, Japan, obtained as a part of ‘Geothermal Devel-
opement Promotion Survey’ (New Energy and Industrial Technology Development Organization,
1988) by the Department of R&D, Geothermal Energy Research and Development Co., Ltd. The
uniqueness of that data set is given by the enormous number of sites measured, as well as by the
exceptional density of the measurements.

The Minamikayabe data set consists of measurements at 209 MT sites, from which 161
stations were arranged, except the northwesternmost part, on a regular grid of 13 × 13 lines,
distance 100 m from one another. The grid is rotated by 30° counterclockwise from north, to
fit the local topography. The lines running NE-SW are lettered ‘a’ to ‘m’, starting from the SE,
whereas the perpendicular lines are numbered ‘0’ to ‘12’ from the NE to the SW. The remaining
48 MT sites are scattered around that regular grid, covering an area of about 3800 × 4300 m²,
elongated slightly in the NNW-SSE direction.

At each of the stations a complete set of the MT impedances and geomagnetic transfer
functions was obtained over a broad band of periods, starting from about 10⁻⁴ s to several
hundreds of seconds. In the high-frequency range, typically up to about 1 s, the data seem to be
of very high quality. For longer periods, the MT functions are sometimes rather scattered, most
likely due to the noise contamination of the time series.

If the density of the measurements is compared with the period range covered, massive
overlap of the zones of the inductive influence for the individual stations is evident. Takasugi
(1992) and Takasugi et al. (1992) analyzed the network MT data in detail and concluded that
the dense network measurements may resolve spatially fine resistivity structures.
Another significant result of their analysis refers to the background effect of the sea. As the distance of the sounding point nearest to the sea is only 1 km from the coast, distorting influence of the coast effect on the MT parameters is likely to take place. Takasugi et al. (1992) showed, moreover, that not only the shape of the coast line, but also the distribution of the sea depth to a certain distance should be taken into consideration in designing geoelectrical models beneath the test area. A series of numerical experiments, carried out in the above paper, proved that the coast effect is a considerable factor for the distribution of the induction arrows across the network test area for periods longer than 1 s. Apparent resistivities seem to be less affected.

For the purpose of the comparison of the MT interpretation techniques within the MT-DIW3 activities, two variants of the analysis have been proposed—either to interpret the complete data set, by using 3-D procedures, or to restrict oneself to the 2-D interpretation of the two central profiles within the regular grid, labeled \( kbgs^* \) and \( kb^*s06 \) in Fig. 1, with special attention to the structural differences indicated beneath the crossing of those lines, after interpreting identical MT curves as different polarizations.

In this paper, we present results of several 2-D interpretation experiments, with an enlarged data set as compared to that proposed for the 2-D modelling above. Besides the basic data from the profiles \( kbgs^* \) and \( kb^*s06 \), we considered two more NE-SW profiles, \( kbcs^* \) and \( kbks^* \) (Fig. 1). After a preliminary inspection of the MT and geomagnetic induction parameters, similar to that illustrated by pseudosections and surface plots of various MT functions in (Takasugi, 1992) and (Takasugi et al., 1992), we applied a 2-D inversion procedure independently to the data from each of the profiles selected. Subsequently, we tried to correlate the geoelectrical sections along the parallel profiles, as well as to check the differences of the model structures obtained at the cross points of the perpendicular profiles involved.

Fig. 1. Layout of the reduced set of the Minamikayabe MT stations within the regular grid of 13 x 13 lines. The four profiles indicated by the grey arrow-lines are interpreted in this paper.
2. Method of the 2-D Interpretation

The method we used to interpret the data is a 2-D version of the controlled random search algorithm. This procedure was first described in (Price, 1977) and suggested as a general algorithm for the global optimization. In electromagnetics, it was successfully used by Martinez (1988) to invert MT curves for the parameters of a 1-D layered structure.

In general, the controlled random search algorithm belongs to the class of global optimization procedures, which combine the random search within the space of the model parameters with a certain strategy of continuing the search process, to speed up the convergence of the procedure. Well known algorithms like the simulated annealing or the genetic algorithm are other representatives of the same class of optimization techniques. The difference between all those methods consists mainly in different strategies proposed for directing the iteration process faster towards the solution.

Specifically, the controlled random search algorithm involves features of three classical optimization techniques—(i) the random generation of the test model set within the space of the model parameters, adopted from the Monte Carlo method, (ii) the application of a mutation and selection criteria to the test models, adopted from the evolution strategy, and (iii) the use of a largely reduced subset of the whole test model pool to carry out the subsequent iteration step of the procedure, which is an idea of the simplex algorithm. The main steps of the algorithm can be characterized as follows (Martinez, 1988):

1. Physically substantiated limits are set to constrain the parameters to be optimized within the parameter space.
2. A class of test models is generated randomly, with a number of models large enough to cover the whole parameter space with a sufficiently high density.
3. An iteration process is started, with the following steps carried out within each iteration:
   (a) A simplex is created, which consists of \( m + 1 \) models, \( r_1, r_2, \ldots, r_m, r_{m+1} \), selected randomly from the current test model pool, \( m \) being the number of the optimized model parameters.
   (b) A mutation step is performed by generating a new test model, \( p \), from the simplex, \( p = 2g - r_{m+1} \), where \( g \) is the 'centre of mass' of the first \( m \) models of the simplex, \( g = \frac{1}{m} \sum_{i=1}^{m} r_i \).
   (c) The selection criterion is applied to the new model \( p \), which results in replacing the currently worst-fit model in the test model pool by the new model \( p \) if the following two conditions are true:
      i. The parameters of the new model \( p \) lie within the pre-defined parameter limits of the parameter space.
      ii. The misfit of the new model is less than the maximum misfit within the current test model pool.
4. The iterations are repeated until the misfit of all the models within the test model pool is less than a pre-defined threshold, mostly defined by the \( \chi^2 \)-criterion.

The particular version of the algorithm used here for the 2-D inversion of the magnetotelluric data includes some specific features, given by the character of the problem solved. We use 2-D block models, with a fixed geometrical configuration throughout the interpretation process. The parameter space is defined by the conductivities of the individual blocks. The forward modelling is carried out numerically, by using the finite difference approximation. The target function is defined via the \( \chi^2 \)-criterion, i.e.

\[
\Phi(p) = \frac{1}{N} \sum_{i=1}^{N} \left[ \frac{f_i(p) - f_i^{exp}}{\delta f_i^{exp}} \right]^2,
\]  

(1)
where \( f_i(p), f_i^{exp}, \) and \( \delta f_i^{exp} \), \( i = 1, 2, \ldots, N \), are the \( i \)-th component of the model response, the \( i \)-th component of an experimental MT function, and the corresponding error of the experimental estimate, respectively. Additional weighting factors can be introduced into the process, making it possible to either emphasize or weight down some of the experimental data items. No smoothing is used, as the extent of the parameter space, i.e. the number of blocks with variable conductivity within the model, is usually relatively small, typically a few tens at the most.

The most time consuming part of the algorithm is the direct solution of the 2-D MT problem in each iteration step for the whole set of periods involved. This limits the possibilities of the algorithm considerably, as only several thousand iteration steps can be performed within one run of the procedure in a reasonable time, with only a PC available. 1-D experiments have shown, however, that the number of iterations higher by one order of magnitude, at least, would be required for the solution to get close enough to the \( \chi^2 \)-threshold, especially if fixed geometry is assumed. Therefore, a possibility of manual corrections of the parameter space is considered in our version of the algorithm, which allows us to constrain less significant parameters during the interpretation process. These interventions make, however, the process less general, as they additionally restrict the space for the random search.

In view of the excessive computing time requirements of the controlled random search algorithm, the question of price versus effect of its practical application is of primary importance. Our earlier experience with linearization optimization methods for practical MT data interpretations shows that these methods give results largely dependent on the initial approximation to the solution, the resulting models being often rather far from the true solution (Červ and Pek, 1981; Pek, 1987). With a relatively small number of model parameters, and with no a priori structural information available, it is often impossible to fit the experimental data within acceptable limits. A better agreement between the model and experimental data can be obtained by over-parametrizing the problem, like in RRI (Smith and Booker, 1991), Occam inversion (de Groot-Hedlin and Constable, 1990), or ABIT (Uchida, 1993). Uniqueness of the solution in those methods is forced by imposing additional constraints, e.g. smoothness, on the solution. Thus, minimum structures are generated which fit the data within statistically acceptable tolerance. Despite the large number of model parameters involved, and all the computational consequences implied, at present the inversion procedures for minimum structures undoubtedly represent the most efficient inversion tools for MT interpretations.

As increasing the number of model parameters slows down prohibitively the global minimum search in procedures like the controlled random search used here, these methods can hardly cope with underdetermined inverse problems. Moreover, with a fine discretization of the regularization grid laid over the structure, the global minimum of the target function (1) gives as a rule not much useful information on the structure, as models with excessive roughness usually result from overfitting the LSQ target.

For problems parametrized with a relatively small number of parameters, the multi-extremality of the target function (1) results not only from a complex dependence of this function on the model parameters, but also from errors in the experimental data, outliers in particular, and from improper parametrization of the structure. The latter factors can change the shape of the target function substantially, add or remove the local minima, or change their position in the parameter space. Nevertheless, 1-D experiments with the controlled random search procedure have allowed us to hypothesize the following features of the method:

a) The global minimum of the LSQ target function (1) is sufficiently robust as to its position in the parameter space within relatively broad limits of data errors and parametrization deficiencies.

b) After a relatively small number of search steps, the models of the test model pool tend to cluster around the most pronounced local minima of the target function. Thus, useful information on the topography of the target function may be derived from the test model
distribution at the initial stage of the search process.

c) The global minimum of the target function may provide a significant clue as to the conductivity distribution even in case of the target function being far from the $\chi^2$ termination limit. The final distribution of the test models may help in re-parametrizing the structure in a more adequate way, and in assessing the experimental data as to their contribution to the misfit.

3. 2-D Modelling Results and Their Discussion

We modelled the structure beneath three NE-SW profiles, $kbc^*$, $kbg^*$, and $kbk^*$, and along one perpendicular profile, $kb^*s06$, across the centre of the grid in the SE-NW direction (Fig. 1). As far as the two-dimensionality is concerned, the selection of the profiles may be only partly justified by the directional MT parameters and geomagnetic transfer functions, especially in certain segments for higher frequencies. In general, however, the spatial and frequency variability of both the induction arrows (Fig. 2) and Swift's principal directions (Fig. 3) indicate a typical complex 3-D character of the structure investigated, and make the 2-D approach only a rough first approximation of the real conditions. Moreover, while the induction arrows at higher frequencies seem to follow a certain regular pattern, though variable with site and frequency, the principal

![Unit arrow](image)

**Fig. 2.** Distribution of the real (black) and imaginary (grey) induction arrows at four frequencies of the MT field on the regular grid of MT sites across the Minamikayabe MT polygon.
MT directions are more scattered throughout the grid, most likely due to local static distortions. For longer periods, specifically for $T = 1.33$ s in Fig. 2, the influence of the coast effect seems to dominate the distribution of the induction arrows.

For the 2-D inversion along each of the selected profiles, we used all the MT data available, though at some points side effects of lateral inhomogeneities outside the profile must be expected. The only selection criterion, applied to the data, was based on the error parameter, $\Delta = \sqrt{\delta^2 Z_{xy} + \delta^2 Z_{yz}}/|Z_{xy} - Z_{yz}|$. Data items with $\Delta > 0.3$ were not considered. The experimental data, presented by the authors of the measurements in the grid-bound coordinate system (i.e. N30°W-E30°N) for the appropriate modes, were not rotated further.

The inversion process was carried out with the same fixed geometrical configuration of blocks for each profile, i.e. we tried to determine the conductivity of different blocks under a pre-selected geometrical parameterization. The effect of the topography was not considered. To simulate the influence of the sea in models along the NE-SW profiles, a highly conductive surface layer ($\rho_{sea} = 0.2$ $\Omega$m) was included 1.5 km away from the easternmost MT site. The depth of the sea was chosen 25 m near the coast, and 100 m starting from 1 km away from the coast line.

The interpretation was carried out simultaneously for 10 periods of the MT field, distributed log-regularly within the range of 256 Hz to 512 s. The short periods, specifically below 1 s, were
put a slightly greater weight on. Nevertheless, the larger scatter of the long-period MT functions sometimes caused the interpretation to fail even at high frequencies, as the LSQ misfit criterion was largely influenced by the outliers.

At the initial stage of the inversion, when the random search is a dominant factor of the modifications within the test model pool, we chose rather large limits for the variable resistivities. When a certain pattern of the distribution of the variables seemed to stabilize, we tried to speed up the convergence by narrowing the limits of the parameter space according to the most frequent resistivity values and by eliminating the obviously insignificant parameters.

In Fig. 4, we present, as grey plots, the four models obtained by the 2-D inversion procedure beneath the individual profiles studied. The models are oriented in accord with the arrow lines in Fig. 1, i.e. the models C, G, and K from the NE to the SW, and the profile 06 from the SE to the NW. The respective polarizations are determined by the orientation of the profiles with respect to the measuring axes. We inverted the apparent resistivities and phases for both modes simultaneously. The corresponding fit of the respective model responses to the experimental data is shown in Figs. 5, 6, 7, and 8 for the profiles $k_{bcs}^*$, $k_{bgs}^*$, $k_{bks}^*$, and $k_{b}s06$, respectively. They are displayed as joint plots of the model MT curves and the respective experimental data at four representative sites along each of the profiles. In the plots, only those experimental data items are shown which were actually used in the inversion procedure. The missing points, especially at long periods, were eliminated by the error criterion mentioned earlier in this section.

For most sites along all the profiles interpreted we could achieve a relatively good fit of the model curves to the experimental data within the short period range, typically for periods shorter than about 1 s. At longer periods, the data are either scattered, which refers in particular to the MT phases, or display large differences between the polarizations, which we failed to interpret by

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Fig. 4. Model structures beneath the individual profiles (labeled C, G, K, and 06 for simplicity) interpreted by the 2-D controlled random search algorithm. The grey plots show only the best fit model for each profile. The cross lines of each profile with the other profiles are indicated by dashed lines.
a 2-D model.

Regarding the structural features of the models, generally higher resistivities are required beneath the NE sections of the NE-SW profiles, i.e. close to their contact zone with the sea. In the centre of the region a more conductive body is indicated immediately below the surface. At both sides of this body two more conductive deeper zones are obvious, most clearly developed in the central model G. A highly conductive zone is observed below the stations kbs11 and kbks11, just beneath a continuous zone of very small induction arrows in the NW section of the polygon (see Fig. 2). This conductive zone may be continuous, and can be related to conductive fracture zones running approximately N-S and confirmed by Takasugi (1992). In this zone, however, a characteristic divergence of the E and B-mode curves takes place, which could not be explained satisfactorily by our 2-D models.

Regarding the perpendicular profile, kb*s06, the polarization modes are swapped with respect to those along the previous three profiles. As the model for this profile had given, in terms of misfit, the least satisfactory results of the inversion in terms of misfit, we repeated the calculations for this profile with a reduced set of good quality data within the range of shorter periods only ($10^{-3} \sim 2$ s). The results of the interpretation did not, however, improve substantially. We could
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Fig. 6. The fit of the model responses to the experimental data for the model G along the profile kbgs*. For explanation of the plots see Fig. 5.

decrease the misfit of the B-mode data, but for the E-mode the fit become slightly worse. The course of the polar impedance diagrams (see Fig. 3) and a similar behaviour of both the B and E-mode MT profile curves, with rapid changes in both modes, indicate, however, that along the profile kb*560 the conditions of two-dimensionality are seriously disturbed. The only structural consequence of cutting off the data for longer periods is that the extremely low resistivity (~0.1 Ωm) of the conductor below the SE end of the profile increases to more consistent values of a few Ωm.

When comparing the individual 2-D models, we paid special attention to the cross-points of the profiles, from the point of view of the conductivity sections interpreted in different directions. In Fig. 9, we show the experimental MT curves at all the cross-points of the profiles investigated, along with the respective model curves from both the model sections involved. The model curves are labeled by the label of the corresponding model, by which they were generated, as well as by the indicator of the polarization mode, E or B, with respect to that model. At high frequencies, the E and B curves are mostly coincident, and both of them fit the experimental data rather satisfactorily. For longer periods, specifically above 0.1 to 1 s, the curves often diverge, the respective B-mode curve being obviously closer to the experimental data.

In Fig. 10, a comparison of the resistivity sections beneath the crossing points of the profiles is shown. At the crossing of the profiles G and 06, and K and 06, the structures are qualitatively
very similar, the data from the profile K giving systematically larger conductivity contrasts. The most serious discrepancy of the resistivity sections is indicated at the crossing of the profiles C and 06, where, in addition a large lateral conductivity contrast between the stations kbcs06 and kbds06 can be observed.

4. Conclusion

The application of the controlled random search algorithm to the 2-D approximate interpretation of four profiles from the Minamikayabe MT polygon is presented. Although the 2-D modelling seems to be only a very rough approximation to the real resistivity conditions of the region, it can give a first idea regarding the influence of the lateral inhomogeneities within the structure on the experimental data. With the 2-D modelling experiments, we could achieve a relatively good fit to the experimental MT data in the short period range, specifically for periods shorter than about 1 s. At longer periods, the fit is not so good, partly due to rather noisy data, and partly due to the divergence of the experimental MT curves for different polarizations, which we failed to interpret by our 2-D models. In general, better coincidence was achieved for the apparent resistivities than for the phases. The character of the data indicates, however, that a
3-D modelling is inevitable to explain all the features of the experimental data.

From the point of view of the 2-D interpretation, the method of the controlled random search is rather time consuming, as many thousands 2-D forward solutions are required for the algorithm to converge sufficiently close to the minimum of the target function. On the other hand, the algorithm does not require any initial approximation to the solution to be given, and, moreover, it carries out, via the random search mechanism involved, a more detailed mapping of the parameter space, with the indication to possible multiple solutions to the optimization problem. There are potential possibilities of increasing the efficiency of the procedure, e.g. by extending the system of variables by the geometrical parameters of the models, or by optimizing some of the control parameters of the algorithm, e.g. the size of the test model pool. Other improvements, like combining the present algorithm with more deterministic search procedures, or refreshing the test model pool by a new random generation periodically at a certain stage of the process (A. Schultz, personal communication), are still under investigation.

This paper is meant as a contribution to a series of 2-D and 3-D interpretation experiments within the MT-DIW3 project. We have, therefore, intentionally avoided any more detailed geological discussion and interpretation of the geoelectrical results obtained by the algorithm presented.
Fig. 9. Comparison of the model and experimental MT curves at the crossing points of the NE-SW profiles with the perpendicular profile kbc0506. To each experimental curve, the corresponding model curves of the two crossing profiles are shown for the respective field modes (E-mode—full line, B-mode—dashed line).

Fig. 10. Comparison of the conductivity structures interpreted by the 2-D modelling at the crossings of the three NE-SW profiles, C, G, and K, with the perpendicular profile 06. Short sections of the respective models in the immediate vicinity of the crossing point are displayed. The labels beside the cross sections indicate the resistivities in $\Omega\cdot$m.
The present results can be only partly compared with the earlier results obtained in the region by Takasugi (1992) and Takasugi et al. (1992), as they interpreted data from a different profile, which runs under 45° to all profiles studied here. Nevertheless, if comparing the electrical parameters at the crossing points of the profiles, the resistivity sections resulting from both interpretations do not contradict each other.

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