Two-loop SUSY QCD correction to the gluino pole mass

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We calculate the pole mass of the gluino as a function of the running parameters in the lagrangian, to $O(\alpha_s^2)$ in SUSY QCD. The correction shifts the pole mass from the running mass by typically 1–2 %. This shift can be larger than the expected accuracy of the mass determination at future colliders, and should be taken into account for precision studies of the SUSY breaking parameters. The effects of other corrections are briefly commented.

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1. Introduction

Extension of the standard model by supersymmetry (SUSY), with the breaking scale not much higher than the electroweak scale, has been studied as a very promising solution to the hierarchy problem between the electroweak scale and the Planck/grand unification scale. In these models, such as the minimal supersymmetric standard model (MSSM) \cite{1}, all particles in the standard model have their superpartners with masses below $O(1)$ TeV. These new particles are then expected to be produced at colliders in near future, such as the CERN Large Hadron Collider (LHC) and the International Linear Collider (ILC).

One of the main motivations for the experimental study of these new particles \cite{2}, the SUSY particles, is the determination of the soft SUSY breaking parameters \cite{3,4}, which gives an important information of the SUSY breaking mechanism in the unified theory. For example, the unification of three gaugino masses ($M_3, M_2, M_1$) at the same scale as that of the gauge couplings is a crucial test for the SUSY grand unified theory \cite{5,6} and superstring phenomenology \cite{7}.

For this purpose, in addition to the precise measurements of the physical parameters of the SUSY particles \cite{2}, one also needs precise prediction of the relations between these observables and parameters in the lagrangian. In some cases, we have to calculate the relations beyond the one-loop order to match expected experimental precision. For example, the two-loop mass corrections have been calculated for the top and bottom quarks \cite{8}, squarks in the first two generations \cite{9}, and Higgs bosons \cite{10}.

Here we focus our attention to the mass of the gluino $\tilde{g}$, the SU(3) gaugino, in the framework of the MSSM. At the LHC, gluino is, if it is sufficiently light, expected to be copiously produced \cite{11}. Its mass $m_{\tilde{g}}$ can be determined from the distributions of the decay products. A study \cite{12} shows that, for the SUSY parameter set SPS1a given in Ref. \cite{13} with $m_{\tilde{g}} \simeq 600$ GeV, $m_{\tilde{g}}$ can be determined to accuracy $\delta m_{\tilde{g}} = 8$ GeV from precision data at the LHC with 300 fb$^{-1}$, and even to $\delta m_{\tilde{g}} = 6.5$ GeV when combined with the data from the ILC. On the other hand, the one-loop QCD contribution to the difference between the pole mass $m_{\tilde{g}}$ and the running mass $M_3$ of the gluino \cite{14,15,16} is much larger, typically $O(10)$ %. One therefore naively expect the two-loop mass correction might be $O(1)$ %, similar to the experimental uncertainty. It is therefore important to examine whether higher-order corrections to the gluino mass is really relevant in the determination of the SUSY breaking parameters at future precision measurements.

In this talk we present the pole mass of the gluino as a function of the lagrangian parameters, including $O(\alpha_s^2)$ SUSY QCD correction obtained by diagrammatic calculation \cite{17}.

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2. Two-loop SUSY QCD mass correction

The pole mass $m_{\tilde{g}}$ of the gluino, which is defined by the complex pole $s_p = (m_{\tilde{g}} - i\Gamma_{\tilde{g}}/2)^2$ of the gluino propagator, is given at the two-loop order as

$$m_{\tilde{g}} = M_3 + \delta m_{\tilde{g}}^{(1)} + \delta m_{\tilde{g}}^{(2)},$$

where the corrections $\delta m_{\tilde{g}}^{(1,2)}$ are expressed in terms of the one-loop and two-loop parts of the gluino self energy $\Sigma(p) = \Sigma_K(p^2) + \Sigma_M(p^2)$ as

$$\begin{align*}
\delta m_{\tilde{g}}^{(1)} &= -\text{Re}[M_3 \Sigma_K^{(1)}(M_3^2) + \Sigma_M^{(1)}(M_3^2)], \\
\delta m_{\tilde{g}}^{(2)} &= -\text{Re}[M_3 \Sigma_K^{(2)}(M_3^2) + \Sigma_M^{(2)}(M_3^2)] \\
&\quad + \text{Re}\left[\lbrace M_3 \Sigma_K^{(1)}(M_3^2) + \Sigma_M^{(1)}(M_3^2)\rbrace\right. \\
&\quad \times \lbrace \Sigma_K^{(1)}(M_3^2) + 2M_3^2 \Sigma_M^{(1)}(M_3^2) \rbrace + 2M_3 \Sigma_M^{(1)}(M_3^2)\right].
\end{align*}$$

(2)

Here $M_3$ is the running tree-level gluino mass. The dot in Eq. (2) denotes the derivative with respect to the external momentum squared $p^2$.

The SUSY QCD contribution to $\Sigma(p)$ is generated by loops with the gluino, gluon, quarks, and squarks. Masses and couplings in the lagrangian are renormalized in the $\overline{\text{DR}}$ scheme at the scale $Q$. Here we ignore SU(2)×U(1) breaking effects in the loops, such as the quark masses and squark left-right mixings. This approximation is valid for the case where the gluino and squarks are sufficiently heavier than the quarks. Later we will briefly comment on the effects of these SU(2)×U(1) breakings. For simplicity, we also assume degenerate mass $m_{\tilde{g}}$ for squarks.

The one-loop correction $\delta m_{\tilde{g}}^{(1)}$ in our approximation is

$$\delta m_{\tilde{g}}^{(1)} = \frac{C_V \alpha_s}{4\pi} M_3 \left(5 - 6 \log \frac{M_3}{Q}\right) + \frac{\alpha_s}{\pi} N_q T_F M_3 B_1(M_3^2; 0, m_{\tilde{g}}).$$

(3)

where $C_V = 3$, $T_F = 1/2$, and $N_q = 6$ is the number of quarks. Parameters $(\alpha_s, M_3, m_{\tilde{g}})$ in Eq. (3) are the $\overline{\text{DR}}$ running ones at the renormalization scale $Q$. $B_1(p^2, m_1, m_2)$ is the one-loop function in the convention of Ref. [20].

The two-loop $O(\alpha_s^2)$ correction $\delta m_{\tilde{g}}^{(2)}$ consists of two parts, $\delta m_{\tilde{g}}^{(2)} = \delta m_{\tilde{g}}^{(2,1)} + \delta m_{\tilde{g}}^{(2,2)}$, where $\delta m_{\tilde{g}}^{(2,1)}$ is the contribution of the diagrams with only gluons and gluinos, while $\delta m_{\tilde{g}}^{(2,2)}$ is the remaining contribution including quark and squark loops. Two-loop diagrams for these contributions are shown in Fig. 1 and Fig. 2 respectively. In these figures, the wavy line, solid line without an arrow, solid line with an arrow, and dashed line with an arrow represent the gluon, gluino, quark, and squark, respectively.

![Figure 1. Two-loop $O(\alpha_s^2)$ contributions to $\delta m_{\tilde{g}}^{(2,1)}$, without quark and squark propagators.](image)

The contribution $\delta m_{\tilde{g}}^{(2,1)}$ is obtained by applying the formula of the $O(\alpha_s^2)$ QCD correction to the quark masses [21] in the $\overline{\text{DR}}$ scheme [22] to the SU(3) octet fermion. The result is

$$\delta m_{\tilde{g}}^{(2,1)} = \left[\frac{C_V \alpha_s}{4\pi}\right]^2 M_3 \left(-48 \log \frac{M_3}{Q} + 36 \log^2 \frac{M_3}{Q} + 26 + 5\pi^2 - 4\pi^2 \log 2 + 6\zeta_3\right),$$

where $\zeta_3 = \sum_{n=1}^{\infty} n^{-3} \approx 1.202$. We have verified Eq. (4) by explicit calculation of the diagrams. At $Q = M_3$, the correction [4] is $\delta m_{\tilde{g}}^{(2,1)}/M_3 \sim 31(\alpha_s/\pi)^2 \approx 0.03$.

The contribution $\delta m_{\tilde{g}}^{(2,2)}$ including quark and squark loops is calculated by decomposition into...
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Figure 2. Two-loop $O(\alpha_s^2)$ contributions to $\delta m_\tilde{g}^{(2,2)}$, with quark and squark propagators. Other diagrams obtained by charge conjugation are not shown.

two-loop basis integrals \cite{23,24} using the integration by parts technique \cite{23,25,26}, and their numerical evaluation by the package TSIL \cite{27}. We have analytically checked that the $Q$ dependence of $\delta m_\tilde{g}^{(2)}$ is consistent with the two-loop renormalization group equation \cite{14,28} of $M_3$.

The explicit form of $\delta m_\tilde{g}^{(2,2)}$ is rather long. Here we just show, for reference, the form in the limit of $m_\tilde{q} \gg M_3$:

$$\delta m_\tilde{g}^{(2,2)}(m_\tilde{q} \gg M_3) = \frac{\alpha_s^2 M_3}{(4\pi)^2} \left[ 72 \log^2 \frac{m_\tilde{q}}{Q} + 242 \log \frac{m_\tilde{q}}{Q} \right. $$

$$+ \log \frac{M_3}{Q} \left( 54 - 288 \log \frac{m_\tilde{q}}{Q} \right) - 172 + \frac{14}{3} \pi^2 \left. \right] $$

$$+ \frac{\alpha_s^2 M_3}{(4\pi)^2} N_q C V T_F \left( -8 \log^2 \frac{M_3}{Q} + \frac{52}{3} \log \frac{M_3}{Q} \right. $$

$$- \frac{37}{3} - \frac{4}{3} \pi^2 \left. \right).$$

The last term of Eq. \ref{eq:delta_m_g_22}, which is independent of $m_\tilde{q}$, comes from the diagram (a) in Fig. \ref{fig:diagrams}. We have checked that the $m_\tilde{q}$ dependence of Eq. \ref{eq:delta_m_g_22} is consistent with the two-loop running of the gluino mass in the effective theory where squarks are integrated out \cite{29}.

3. Numerical results

We present some numerical results of the $O(\alpha_s^2)$ pole mass of the gluino, for the running tree-level mass $M_3(M_3) = 580$ GeV which is close to the values in the SPS1a point. The strong coupling constant within the standard model is given as $\alpha_s(M_Z) = 0.12$.

We first show, in Fig. \ref{fig:dependence}, the residual dependence of the one-loop pole mass $m_\tilde{g}^{(1)}$ and two-loop pole mass $m_\tilde{g}^{(2)}$ on the renormalization scale $Q$, for the running squark mass $m_\tilde{q}(Q_0) = 800$ GeV at $Q_0 = 580$ GeV. All parameters in the formulas are evolved by $O(\alpha_s^2)$ renormalization group equations. For reference, the tree-level $M_3(Q)$ decreases from 589 GeV at $Q = 400$ GeV to 559 GeV at $Q = 1400$ GeV. We see that the $Q$ dependence slightly improves by including $\delta m_\tilde{g}^{(2)}$. One should however note that, contrary to naive expectation, $\delta m_\tilde{g}^{(2)}$ is much larger than the $Q$-dependence of the one-loop result $m_\tilde{g}^{(1)}$.

Figure 3. Dependence of the one-loop (dashed) and two-loop (solid) pole masses of the gluino on the renormalization scale $Q$. Mass parameters are $(M_3, m_\tilde{q}) = (580, 800)$ GeV at $Q = 580$ GeV.
4. Other two-loop corrections

Beyond one-loop, there are also mass corrections involving Yukawa and electroweak couplings. For example, diagrams involving Yukawa couplings $h_q$ of the Higgs bosons/higgsinos to quarks and squarks in the third generation, such as those in Fig. 5, give the $O(\alpha_s h_q^2)$ mass corrections. Analytic form of these contributions can be derived from general formulas of the two-loop corrections to the fermion pole masses in the approximation of massless vector bosons in the loops. As a numerical example, again in the SU(2)×U(1) symmetric approximation, the top Yukawa contribution for $M_3 = m_{\tilde{q}} = m_{\chi^0} = \mu$, $\tan \beta = 10$ is

$$\frac{\delta m_{\tilde{g}}^{(2),h_q}}{M_3} \sim (2, 10) \times \alpha_s h_q^2 / (4\pi)^3$$

for $A_t = (M_3, -M_3)$, respectively, which is much smaller than the $O(\alpha_s^2)$ contribution. We expect that the smallness of the Yukawa contribution of squarks. Since these parameters break the SU(2)×U(1) gauge symmetry, their contributions to $m_{\tilde{g}}$ should be suppressed by factors $m_q^2/m_{\tilde{q}}^2$ or $m_q^2/m_{\tilde{q}}^2$ compared to the gauge symmetric contribution shown here. In addition, they only modifies contributions involving quarks and squarks in the third generation, while all generations contribute to the SU(2)×U(1)-symmetric part of $\delta m_{\tilde{g}}^{(2,2)}$ with equal weight. We therefore do not expect that these SU(2)×U(1)-breaking contributions are numerically relevant in future realistic studies of the SUSY particles. However, detailed study in cases of light gluino and/or squarks is necessary for definite conclusion.

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for $A_t = (M_3, -M_3)$, respectively, which is much smaller than the $O(\alpha_s^2)$ contribution. We expect that the smallness of the Yukawa contribution
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would also hold in more general cases.

5. Conclusions

We have calculated the two-loop SUSY QCD contribution to the gluino pole mass, ignoring SU(2) × U(1) symmetry breakings in the loops. The $O(\alpha_s^2)$ correction to the gluino mass has been shown to be typically 1 – 2 %. For the case of $M_3(M_3) = 580$ GeV, this correction is similar to, or larger than, the expected uncertainty in the mass determination from precision measurements at future colliders. The two-loop correction would be therefore important in the extraction of $M_3$ from experimental data and the determination of the SUSY breaking at the unification scale.

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