Asymmetry bandwidth of stock market index time series as a warning signal

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Abstract. The time series of the S&P 500 index from 2003 to 2018 and the stochastic time series are considered. The asymmetry is called the change in the complexity Cmp of the time series when time direction is changing, and the values of the asymmetry of the substrings are in the interval of a certain width. The asymmetry bandwidth of the S&P 500 series is minimal for 2005, and stochastic strings can have such a small width. The 2007-2008 crisis seems to have been preceded by a rush of heightened stochasticity (a fall in predictability) that has led to a decline in investor optimism. By calculating asymmetry bandwidth, a warning signal could be obtained for a possible stock market crash.

1. Introduction
Financial market is a complex system where risk tolerance of individuals is crucial for system resilience. Throughout history, many concepts have emerged that have attempted to predict the decline of the stock market, the relationship between the variables that determine trading and stock market turnover. In traditional approach to problems in economy a equilibrium is assumed – there’s no adaptation, no innovation or structural change. In 1987 a conference The Economy as an Evolving Complex System is organized by Nobel laureate in economy Kenneth Arrow and Nobel laureate in physics Philip Anderson. This started a new approach to economy as a complex system that is evolving [1]. Various methods for stock market crash indicator have been proposed. Quax et al. propose that analysis of financial time series could provide a signal indicating upcoming Lehman Brothers bankruptcy that started global financial crisis [2]. Saracco et al. analyse global trading network and finds that it was possible to have indication of global financial crisis [3]. Bouchard et al. claim that classical economics lacks methods for predicting and avoiding crises and that it needs a scientific revolution [4]. Lu et al. propose one simplified dynamic model on networks of investors and stocks and relation between price limit and critical market confidence [5]. Stavrogoula et al. propose a method based on symbolic dynamics, which probes beneath the surface of abstract causality and unveils the nature of causal interactions [6].

Here we present novelty method of stock market crash indicator. By taking values of S&P 500 stock market index time series for years 2003-2018, asymmetry is calculated. It is presumed that real time series element is a sum of deterministic and stochastic terms. Increase of time series stochasticity yields narrow band of asymmetry values.

2. Cmp and asymmetry
We divide time series $A_1, A_2, ..., A_{110}$ into eleven subseries. First ten subseries are
where \( j = 1, 2, ..., 10 \). The eleventh subseries we represent as linear combination of previous ones.

\[
F_{1,j} = A_j, \quad F_{2,j} = A_{j+10}, \quad ..., \quad F_{10,j} = A_{j+90} \quad (1)
\]

We now make permutations inside first ten subseries and get new subseries

\[
F'_{1,1} = A_{10}, \quad F'_{1,j} = A_{j-1} \\
F'_{2,1} = A_{20}, \quad F'_{2,j} = A_{j+9} \\
F'_{3,1} = A_{30}, \quad F'_{3,j} = A_{j+19} \\
... \\
F'_{10,1} = A_{100}, \quad F'_{10,j} = A_{j+89} \quad (3)
\]

where \( j = 2, 3, ..., 10 \).

The equations

\[
A_{101} = \sum_{i=1}^{10} c_i F'_{i,1}, \quad A_{102} = \sum_{i=1}^{10} c_i F'_{i,2}, \quad ..., \quad A_{110} = \sum_{i=1}^{10} c_i F'_{i,10} \quad (4)
\]

yield new constants of the linear combination. Using 2-norm of vectors, we define complexity:

\[
Cmp = -\ln \frac{\|c'_1 c'_2 ... c'_{10} - (c_1 c_2 ... c_{10})\|}{\|c'_1 c'_2 ... c'_{10}\|} \quad (5)
\]

Asymmetry is:

\[
Cmp_+ - Cmp_- \quad (6)
\]

where \( Cmp_+ \) is complexity for time series \( A_1, A_2, A_3, ..., A_{110} \) and \( Cmp_- \) is complexity for same time series observed in reverse \( A_{110}, A_{109}, A_{108}, ..., A_1 \).

For a completely stochastic time series it would be expected that asymmetry value is zero. For a computer generated random time series a very small value of asymmetry is obtained. If a very small deviation from zero is obtained when calculating asymmetry for a real time series, it is presumed that this is caused by very high level of stochasticity of that time series.

3. Asymmetry of S&P 500 index time series

Real time series representing daily values of S&P 500 stock market index was analyzed [7]. It is presumed that S&P 500 stock market index value is representing a current state of economy. Asymmetry was calculated for 16 S&P 500 time series, corresponding to years 2003-2018.
Asymmetry values are plotted on graphs (Figures 1-3), with every graph containing 142 curves. Every curve is related to one value of shift. Shift determines subseries of observed series of values of S&P 500 stock market index. Increasing shift value is consistent with change of S&P 500 stock market index forward in time. Asymmetry is calculated for real time series and for similar trajectories of original time series. This was performed to examine stability of the solutions. Real time series corresponds to K=0. Values of K that are different from 0 correspond to generated trajectories of S&P 500 stock market index that are slightly different than the original time series. Small change in trajectory of time series isn’t affecting value of asymmetry drastically.

Close trajectories of S&P stock market index are describing same change in a state of economy. When observing multiple curves, band emerges. Narrow band correlates with highly stochastic time series.
Figure 5. Asymmetry values for year 2007

Figure 6. Asymmetry values for year 2008

Figure 7. Asymmetry values for year 2012

Figure 8. Asymmetry values for year 2014

Figure 9. Asymmetry values for year 2016

Figure 10. Asymmetry values for year 2018
4. Conclusion
During the historical development of the society, there were many systems that predicted movements in the stock market. Complex systems such as stock markets demonstrate the ability of a system to adjust its activity to maintain its core functionality when errors, breakdowns and environmental changes appear. It is very difficult to predict the stock market crash. Many scientists and financial workers have dealt with this issue. There are various models trying to explain what is causing the stock market crash. In this paper, an analysis of data from the period 2003-2018 is presented. The proposed model shows increase in stochasticity in the period before the stock market crash (Figures 2. and 3.). During the stock market crash and after stock market crash no characteristic narrow band is visible. Narrowing of the asymmetry bandwidth can be a warning signal for market crash.

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