Constraints on vector mesons with finite momentum in nuclear matter

Bengt Friman\textsuperscript{1,2}, Su Houng Lee \textsuperscript{2,3}\textsuperscript{*} and Hungchong Kim\textsuperscript{4}
\textsuperscript{1} Institute fur Kernphysik, TU Darmstadt, D-64289 Darmstadt, Germany
\textsuperscript{2} GSI, Planckstr. 1, D-64291 Darmstadt, Germany
\textsuperscript{3} Department of Physics, Yonsei University, Seoul, 120-749, Korea
\textsuperscript{4} Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan

Abstract

Using the QCD operator product expansion, we derive the real part of the transverse and longitudinal vector-vector correlation function with the quantum numbers of the ρ and ω mesons to leading order in density and three momentum ($q^2$) for $\omega^2 \rightarrow -\infty$. The operator product expansion provides, through the Borel transformed energy dispersion relation, a model independent constraint for the momentum dependence of the vector meson spectral density in nuclear matter. Existing model calculations for the dispersion effect of the ρ, where the vector-meson nucleon scattering amplitude is obtained by resonance saturation in the s-channel, in general violate this constraint. We trace this to an inconsistent choice for the form factor of the $\Delta N \rho$ vertex. With a consistent choice, where both the form factor and the coupling constant are obtained from the Bonn potential, the contribution of the $\Delta$ is substantially reduced and we find good agreement with the constraint equation. We briefly comment on the implications of our result for attempts to interpret the enhancement of low-mass dileptons in heavy-ion collisions.

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\textsuperscript{*}Alexander von Humboldt fellow
I. INTRODUCTION

It is expected that the properties of vector mesons at finite density and/or temperature should be reflected in the dilepton spectrum observed in nuclear reactions. It has also been suggested that modifications of the vector-meson properties are related to the restoration of chiral symmetry in dense matter \[1,2\]. Indeed the enhancement of low-mass dileptons found in heavy-ion collisions \[3\] can be interpreted in terms of a non-trivial change of the vector meson spectral density in hot/dense matter. However, the dilepton spectra observed in heavy-ion collisions involve dileptons from different stages of the collision. Consequently, collision models have to be invoked to relate the spectra to the vector-meson spectral density at a given temperature and density. At present, the data can be qualitatively interpreted in terms of two scenarios; on the one hand, a scenario where the vector meson masses decrease in matter \[4–9\] and on the other hand, a scenario based on hadronic models where the \(\rho\)-meson spectrum exhibits a non-trivial dispersion effect (three momentum dependence) \[10,11\] and an enhanced width \[12,13\].

In spite of attempts to unify the theoretical descriptions of the vector meson spectral densities in matter \[14\], the characteristics of the two approaches still differ and the relation between them remains nebulous. In order to reduce the discrepancies and eventually approach a unified description, it is important to reduce the uncertainties of the theoretical models by constraining the parameters with experimental data \[15,16\] and results derived directly from QCD.

In this work, we derive in detail QCD constraints on the three-momentum dependence of the vector meson spectral density in nuclear matter. These constraints have been derived previously in the context of QCD sum rules \[17,18\] and shown to imply a small three-momentum dependence of the vector meson mass at finite density. Here, we give a detailed derivation of the constraints and test to what extent it is satisfied by existing hadronic model calculations. In the first stage we derive the Operator Product Expansion (OPE) and constructs constraints on the spectral density using the Borel transformed dispersion relation. In the second stage, model calculations are confronted with these constraints. As we will see, the relevant condensates that appear in the constraints for the dispersion effect are well known up to nuclear matter density and the corresponding QCD sum rule provides a solid reference point for model calculations \[17–19\].

An alternative approach is to model the phenomenological side by a simple functional form with a few parameters and obtain medium modifications of these parameters and thus of the spectral density. This procedure is in general not unique unless the form of the spectral density can be constrained by independent experimental or theoretical input. For the \(\rho\) meson, for instance, there is a trade-off between a reduction of the mass and an increase of the width. Thus, there is a band of possible combinations of mass and width, where the quality of the fit to the constraint is almost unchanged. Furthermore, there are uncertainties related to the medium expectation value of the four-quark operator, for which the ground state saturation hypothesis was used. Consequently, the resulting constraint \[4\] can be satisfied either by a large decrease in mass and a small increase in width or by a small decrease in mass and a large increase in width or something in between \[12,20\].

The paper is organized as follows. In section II, we derive the OPE relevant for the dispersion effect of the vector meson spectral density. In section III, we state the constraint,
which should be satisfied by model calculations. Finally in section IV, we discuss to what extent the existing hadronic model calculations satisfy this constraint.

II. OPERATOR PRODUCT EXPANSION

Consider the vector-current correlation function in nuclear matter

$$\Pi_{\mu\nu}(\omega, q) = i \int d^4xe^{iq \cdot x} \langle G|T[J_\mu(x)J_\nu(0)]|G\rangle,$$

where $J_\mu = \bar{q}\gamma_\mu q$, $|G\rangle$ is the nuclear ground state at rest and $q^\mu = (\omega, q)$. In what follows, when we give results for explicit vector mesons, we will use the currents $J_\rho,\omega^\mu = \frac{1}{2}(\bar{u}\gamma_\mu u \mp \bar{d}\gamma_\mu d)$ for the $\rho$ and $\omega$ mesons, respectively.

In general, because the vector current is conserved, the polarization tensor in eq. (1) has only two independent functions

$$\Pi_{\mu\nu}(\omega, q) = \Pi^T q^2 P^T_{\mu\nu} + \Pi^L q^2 P^L_{\mu\nu},$$

where we assume the nuclear medium to be at rest, such that $P^T_{00} = P^T_{0i} = P^T_{i0} = 0$, $P^T_{ij} = \delta_{ij} - q_i q_j / q^2$ and $P^L_{\mu\nu} = (q_\mu q_\nu / q^2 - g_{\mu\nu} - P^T_{\mu\nu})$. The longitudinal and transverse polarization functions are given by

$$\Pi_L = \frac{1}{q^2} \Pi_{00}, \quad \Pi_T = -\frac{1}{2}(\frac{1}{q^2} \Pi^\mu_\mu + \frac{1}{q^2} \Pi_{00}).$$

In the limit of $q \to 0$, the longitudinal and transverse polarizations are degenerate, i.e., $\Pi_L = \Pi_T$.

In this work, we address only the leading three momentum ($q$) dependence of the polarization functions. Consequently, we expand the correlation function in a Taylor series around $q = 0$ and examine the dispersion relation at fixed $q$.

$$\text{Re}\Pi_{L,T}(\omega^2, q^2) = \text{Re} \left( \Pi^0(\omega^2, 0) + \Pi^1_{L,T}(\omega^2, 0) q^2 + \cdots \right)$$

$$= \int_0^\infty du^2 \left( \frac{\rho^0_{L,T}(u, 0)}{(u^2 - \omega^2)} + \frac{\rho^1_{L,T}(u, 0)}{(u^2 - \omega^2)} q^2 + \cdots \right),$$

where $\rho(u, q) = (1/\pi)\text{Im}\Pi^R(u, q)$, and $R$ denotes the retarded correlation function. The OPE for $\Pi^0$ appropriately modified for the nuclear medium together with the corresponding energy dispersion relation provides constraints on the mass shift and spectral changes at $q = 0$ [6]. In this work, we will explore the three momentum dependence by studying the OPE in medium for $\Pi^1_{L,T}$.

In general, the OPE [22,23] for the polarization function at large $Q^2 = -\omega^2 + q^2$ is of the form,

$$\Pi_{\mu\nu}(\omega, q) = (q_\mu q_\nu - g_{\mu\nu} q^2) \left[ -c_0 \ln|Q^2| + \sum_n \frac{1}{Q^{2n} A^{n,n}} \right]$$

$$+ \sum_{\tau=2} \sum_k [g_{\mu\tau} q_{\mu k} + g_{\mu k} q_{\mu \tau} + g_{\mu k} q_{\tau \nu} + g_{\mu k} g_{\nu \tau} Q^2]$$
Here we have extracted the trivial $q^2$ dependence so that $A^{d, \tau}, C^{d, \tau}$ represent the residual Wilson coefficient times matrix element of an operator of dimension $d$ and twist $\tau = d - s$, where $s = 2k$ is the number of spin indices of the operator. The first set of terms is due to the OPE of scalar operators, while the second set originates from operators with spin, which have been written as a double sum in twist and spin. The coefficients $A, C$ represent the two linearly independent sums of operators, which reflect the two polarization directions.

The longitudinal and transverse polarization functions can be obtained by using eq. (5) in eq. (3). The $q$ dependence emerging from the first line of eq. (5), namely the contributions from the scalar operators, is hidden in $Q^2$. This we call the “trivial” $q$ dependence, where the momentum $q$ appears in the Lorenz invariant combination $\omega^2 - q^2$. Here, we are interested only in the “non-trivial” $q$ dependence of $\Pi_{L,T}$. Consequently the scalar operators do not contribute to $\Pi_{L,T}$ and only operators with spin contribute to the non-trivial $q$ dependence. This is done by expressing the polarization function in terms of $Q^2$ and $q^2$, i.e., $\Pi(Q^2, q^2)$ and extracting the term linear in $q^2$.

### A. Linear density approximation

To leading order in density

$$\langle G|A|G \rangle = \langle 0|A|0 \rangle + \frac{\rho_n}{2m} \langle p|A|p \rangle ,$$

where $|G\rangle$ denotes the ground state of nuclear matter. The first term on the right hand side is the vacuum expectation value, which vanishes for operators with spin. In the second term $|p\rangle$ denotes a nucleon state with the normalization $\langle p||p' \rangle = 2p_0\delta^3(p - p')$, $\rho_n$ the nuclear density and $m$ the nucleon mass. As in vacuum, we include operators up to dimension 6 in the OPE. This implies that in eq. (4) there are contributions from $(\tau, s) = (2, 2), (2, 4)$ and $(4, 2)$. The nucleon matrix elements of the $\tau = 2$ operators are very well determined.

The $\tau = 4$ matrix element appearing in the $\rho, \omega$ sum rule are similar to those appearing in deep inelastic scattering (DIS) of electrons off nucleons [24]. These matrix elements have been estimated [24,26] with an uncertainty of about $\pm 30\%$ using DIS data from CERN and SLAC.

The operators with spin indices are symmetric and traceless so that when the matrix elements are taken, the general form can be written as follows [27,28],

$$A_{\mu_1 \cdots \mu_{2k}} = \left( \sum_{j=0}^{k} a_j p^{2j} [g_{\mu_1, \mu_2} \cdots g_{\mu_{2j-1}, \mu_{2j}} p_{\mu_{2j+1}} \cdots p_{\mu_{2k}} + \text{permutations}] \right) A_{2k} ,$$

where there are $\frac{(2k)!}{(2k-2j)!(2j)!}$ terms for a given value of $k$ and $j$ and $a_j = (-1)^j \frac{(2k-j)!}{2^j(2k)!}$ (more details are given in Appendix A). For $s = 2$ and $s = 4$ one finds
\[ A_{\mu\nu} = (p_\mu p_\nu - \frac{1}{4} g_{\mu\nu} p^2) A_2, \]
\[ A_{\mu\nu\alpha\beta} = \left[ p_\mu p_\nu p_\alpha p_\beta - \frac{p^2}{8} (p_\mu p_\nu g_{\alpha\beta} + p_\mu p_\alpha g_{\nu\beta} + p_\mu p_\beta g_{\alpha\nu} + p_\nu p_\alpha g_{\mu\beta} + p_\nu p_\beta g_{\mu\alpha} + p_\alpha p_\beta g_{\mu\nu}) \right. \]
\[ \left. + \frac{p^4}{48} (g_{\mu\nu} g_{\alpha\beta} + g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha}) \right] A_4. \]

Substituting eq. (7) into eq. (5), we obtain the contributions of the spin dependent operators to linear order in density

\[ \Pi_{\mu\nu} = \frac{\rho_n}{2m} \left[ \sum_{\tau=2} \sum_{k=1} C_{2k,\tau} \sum_{j=0}^{k} \left( \frac{p^2}{Q^2} \right)^j \frac{1}{x^{2k-2j}} \frac{(2k-j)!}{(2k-j-1)(2k-2j-2)!} \right] + e_{\mu\nu} \left[ \sum_{\tau=2} \sum_{k=1}^{k} A_{2k,\tau} \sum_{j=1}^{k} \left( \frac{m^2}{Q^2} \right)^j \frac{1}{x^{2k-2j}} \frac{4 \cdot (2k-j) \cdot (2k-j)!}{(2k-1)(2k-2j)(j-1)!} \right], \]

where \( x = \frac{Q^2}{2 p^2} \), \( e_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu / q^2 \) and \( d_{\mu\nu} = -p_\mu p_\nu q^2 / (p \cdot q)^2 + (p_\mu q_\nu + p_\nu q_\mu) / p \cdot q - g_{\mu\nu} \). The latter two are the usual polarization tensors used in DIS [23]. The sum over \( k \) corresponds to a sum over the number of spin indices while the sum over \( j \) is a sum over the target mass corrections. It should be noted that in the Bjorken limit \( x \to \infty, Q^2 \to \infty \), the leading terms are twist-2 with any number of spins, while the higher twist terms and the target mass corrections are suppressed by powers of \( 1/Q^2 \).

The nontrivial contribution to \( \Pi_{L,T} \) can be obtained as follows. First, we use eq. (9) in eq. (3). Since we have assumed the ground state to be at rest, the nucleons are also at rest \( p = (m,0) \) so that \( c_{00} = -\frac{q^2}{2q^2}, d_{00} = \frac{q^2}{2q^2}, e_{\mu} = 3, d_{\mu} = -\frac{q^2}{\omega^2} - 2 \). We then extract the term proportional to \( \vec{q}^2 \) as discussed above. For the \( s = 2 \) terms we find

\[ \Pi_L^s = \frac{\rho_n}{2m} \sum_{\tau} (-1)^{\tau/2} \frac{m^2}{\omega^{\tau+4}} (4C_2^{2+\tau,\tau}), \]
\[ \Pi_T^s = \frac{\rho_n}{2m} \sum_{\tau} (-1)^{\tau/2} \frac{m^2}{\omega^{\tau+4}} (4C_2^{2+\tau,\tau} - 4A_2^{2+\tau,\tau}), \]

while for \( s = 4 \),

\[ \Pi_L^4 = \frac{\rho_n}{2m} \sum_{\tau} (-1)^{\tau/2+1} \frac{m^4}{\omega^{\tau+6}} (4A_4^{4+\tau,\tau} - 20C_4^{4+\tau,\tau}), \]
\[ \Pi_T^4 = \frac{\rho_n}{2m} \sum_{\tau} (-1)^{\tau/2+1} \frac{m^4}{\omega^{\tau+6}} (18A_4^{4+\tau,\tau} - 20C_4^{4+\tau,\tau}). \]

In general the contribution of operators with \( s \) spin indices is a sum of terms of order \( 1/\omega^{\tau+s+2} \).
B. Form of $A, C$ for $\tau = 2$ in the linear density approximation

The coefficients $A$ and $C$ can be determined by comparing eq. (9), in the Bjorken limit, with the expectation value of the polarization tensor (5) in a one-nucleon state [23,29],

$$i \int d^4x e^{i q x} \langle p | T [ J_\mu(x) J_\nu(0) ] | p \rangle = \sum_n \frac{1}{x^n} \left[ e_{\mu \nu} ( C_{L,n}^a A_n^a + C_{G,n}^a A_n^G ) + d_{\mu \nu} ( C_{2L,n}^a A_n^a + C_{2G,n}^a A_n^G ) \right] + O \left( \frac{1}{Q^2} \right),$$  \hspace{1cm} (12)

where in the MS scheme, the Wilson coefficients are given in Table (1). For a proton state normalized as $\langle p | p \rangle = (2\pi)^3 2 p_0 \delta^3(0)$, we have

$$A_n^a(\mu) = 2 \int_{0}^{1} dx \ x^{n-1} \left[ q(x, \mu) + ( -1 )^n \bar{q}(x, \mu) \right],$$

$$A_n^G(\mu) = 2 \int_{0}^{1} dx \ x^{n-1} G(x, \mu),$$  \hspace{1cm} (13)

where $q(x, \mu)$ ($G(x, \mu)$) is the quark (gluon) distribution function inside the proton at the scale $\mu^2$. Comparing eq. (12) multiplied by $\rho_n^2 m$ with the leading term in eq. (9) in the Bjorken limit, we find,

$$A_{2k}^{2k+2,2} = C_{2,2k}^a A_{2k}^a + C_{2,2k}^G A_{2k}^G ,$$

$$C_{2k}^{2k+2,2} = C_{L,2k}^a A_{2k}^a + C_{L,2k}^G A_{2k}^G .$$  \hspace{1cm} (14)

C. Form of $A, B$ for $\tau = 4$ in the linear density approximation

The contributions due to $\tau = 4$ operators have been computed in [24] and those due to the mass dependent $\tau = 4$ operator in [26]

$$i \int d^4x e^{i q x} \langle p | T [ J_\mu(x) J_\nu(0) ] | p \rangle \rightarrow$$

$$\frac{1}{x^2 Q^2} \left[ d_{\mu \nu} ( H^1 + \frac{5}{8} H^2 + \frac{1}{16} H^g - \frac{13}{4} H^m ) + e_{\mu \nu} ( \frac{1}{4} H^2 - \frac{3}{8} H^g - \frac{9}{2} H^m ) \right],$$  \hspace{1cm} (15)

where, for a current of $J^{Q \mu}_\mu = \bar{q} Q q \gamma_\mu q$, the $H'$s are the spin independent parts (such as the $A$ in eq. (8)) of the following operators,

$$H_{\mu \nu}^1 = \langle g^2 ( \bar{q} \gamma_\mu \gamma_5 \frac{\lambda^a}{2} Q q) ( \bar{q} \gamma_\nu \gamma_5 \frac{\lambda^a}{2} Q q) \rangle ,$$

$$H_{\mu \nu}^2 = \langle g^2 ( \bar{q} \gamma_\mu \frac{\lambda^a}{2} Q q) ( \sum_{q=u,d,s} \bar{q} \gamma_\nu \frac{\lambda^a}{2} q) \rangle ,$$

$$H_{\mu \nu}^3 = \langle ig ( \bar{q} \{ D_\mu, G_{\nu a} \} \gamma^n \gamma_5 Q_q^2 q) \rangle ,$$

$$H_{\mu \nu}^m = \langle \bar{q} D_\mu D_{\nu} m_q Q_q^2 q \rangle .$$  \hspace{1cm} (16)

Comparing eq. (15) multiplied by $\frac{\rho_n^2 m}{2m}$ with the $s = 2, \tau = 4$ part of eq. (8), we find,
\[ A_{2,4}^6 = H^1 + \frac{5}{8} H^2 + \frac{1}{16} H^g - \frac{13}{4} H^m, \]
\[ C_{2,4}^6 = \frac{1}{4} H^2 - \frac{3}{8} H^g - \frac{9}{2} H^m. \]  

(17)

The matrix elements of these operators are not all known. However, the values of two different combinations in the \( F_2 \) and \( F_L \) directions have been obtained by one of us \cite{25} by analyzing the DIS data at CERN and SLAC. In order to proceed, we make the following assumption \cite{25}:

\[ \langle \bar{d} \Gamma_\mu \Delta_\nu d \rangle = \langle \bar{u} \Gamma_\mu \Delta_\nu u \rangle \equiv \beta, \]  

(18)

where \( \Gamma_\mu \) is some gamma matrix and \( \Delta_\nu \) is an isospin singlet operator and we take \( \beta = 0.48 \).

Now, it is possible to uniquely determine the matrix elements appearing in the \( \rho, \omega \) meson sum rules. To see this, let us rewrite the scalar part of the matrix elements in eq. (16),

\[ H^1 = Q_u^2 K_{1u} + Q_d^2 K_{1d} - (Q_u - Q_d)^2 K_{ud}/2, \]
\[ H^2 = Q_u^2 K_{2u} + Q_d^2 K_{2d}, \]
\[ H^g = Q_u^2 K_{gu} + Q_d^2 K_{gd}, \]  

(19)

where we consider only the \( u, d \) sector and neglect the quark masses \( m_u, m_d \). The \( K \)'s are the matrix elements obtained from eq. (16),

\[ K_u^i = \frac{2}{m^2} \langle p | \bar{u} \Gamma^1_{+} \Delta^i_+ u | p \rangle, \quad i = 1, 2, \]
\[ K_u^g = \frac{2iq}{m^2} \langle p | \bar{u} \{ D_+ , *G_+ \} \gamma^\mu \gamma_5 u | p \rangle, \]
\[ K_{ud}^1 = \frac{2}{m^2} \langle p | 2(\bar{u} \Gamma^1_{+} u)(\bar{d} \Gamma^1_{+} d) | p \rangle. \]  

(20)

Here, \( \Gamma^1_\mu = \gamma_\mu \gamma_5 \frac{X}{2}, \Gamma^2_\mu = \gamma_\mu \frac{X}{2}, \Gamma_+ = (\Gamma_0 + \Gamma_3)/\sqrt{2} \) and \( \Delta^i_\mu = \bar{u} \Gamma^i_+ u + \bar{d} \Gamma^i_+ d \) is a flavor singlet operator.

Using the DIS data for \( F_L, F_2 \) on proton and neutron targets, one can now determine the three combinations

\[ Z_1 = K_{1u} + \frac{5}{8} K_{2u} + \frac{1}{16} K_u^g \simeq -0.06 \quad \text{GeV}^2, \]
\[ Z_2 = K_{2u} - \frac{3}{2} K_u^g \simeq 0.56 \quad \text{GeV}^2, \]
\[ Z_3 = K_{ud}/2 \simeq -0.04 \quad \text{GeV}^2. \]  

(21)

The operator product expansion for \( \Pi_{L,T} \) is then given by

\[ \Pi^1_L = \frac{m \cdot \rho_n}{\omega^8} \left[ \frac{1}{4} (1 + \beta) \left( \frac{1}{2} Z_2 \right) \right], \]
\[ \Pi^1_T = \frac{m \cdot \rho_n}{\omega^8} \left[ \frac{1}{4} (1 + \beta) \left( -2Z_1 + \frac{1}{2} Z_2 \right) + \frac{1}{2} Z_3 \right]. \]

(22)

In the last term in \( \Pi_{L,T} \), the + sign refers to the \( \rho \) and the − sign to the \( \omega \) meson.
D. Final form of the OPE

The final form of the OPE for $\Pi^1$ can now be obtained by using eqs. (10, 11, 14) and (22):

$$\Pi_{L,T}^1(\omega)/\rho_n = \frac{b^L_{2,T}}{\omega^6} + \frac{b^T_{3,T}}{\omega^8}. \quad (23)$$

For the longitudinal (L) and transverse (T) parts one finds [17, 18],

$$b^T_2 = \left(\frac{1}{2}C^q_{2,2} - \frac{1}{2}C^q_{L,2}\right)mA^u_2 + (C^G_{2,2} - C^G_{L,2})mA^G_2,$$

$$b^T_3 = \left(\frac{9}{4}C^q_{2,4} - \frac{5}{2}C^q_{L,4}\right)m^3A^u_4 + \left(\frac{9}{2}C^G_{2,4} - 5C^G_{L,4}\right)m^3A^G_4$$

$$+ \frac{1}{2}m \left[1 + \beta \right] \left(-Z_1 + \frac{1}{4}Z_2 + (1 \pm 1)Z_3 \right),$$

$$b^L_2 = -\frac{1}{2}C^q_{L,2}mA^u_2 - C^G_{L,2}mA^G_2,$$

$$b^L_3 = \left(\frac{1}{2}C^q_{2,4} - \frac{5}{2}C^q_{L,4}\right)m^3A^u_4 + \left(C^G_{2,4} - 5C^G_{L,4}\right)m^3A^G_4 + \frac{m}{8}(1 + \beta)Z_2, \quad (24)$$

where again the $\pm$ refers to the $\rho$ and $\omega$ mesons, respectively.

E. Infinite sum of twist-2 contribution

Since the matrix elements of all the $\tau = 2$ operators are known, one can sum the higher order $\frac{1}{\omega^n}$ ($n > 6$) twist-2 correction. Here we present the result to lowest order in $\alpha_s$, i.e., including only the quark part of eq. (14). By substituting this into eq. (3) and evaluating the sums we find,

$$\Pi^1_L = \frac{\rho_n m}{2\omega^6} \int_0^1 dx \frac{\left(1 + \frac{m^2x^2}{\omega^2}\right)^2}{\left(1 - \frac{m^2x^2}{\omega^2}\right)^2} [q(x) + \bar{q}(x)]$$

$$- \frac{\rho_n m}{2\omega^6} \int_0^1 dx \left[8 \left(1 + \frac{m^2x^2}{\omega^2}\right)^6 - 12 \left(1 + \frac{m^2x^2}{\omega^2}\right)^4 + 3 \left(1 + \frac{m^2x^2}{\omega^2}\right)^2 \right]$$

$$\times \int_1^x dy [q(y) + \bar{q}(y)]$$

$$+ \frac{\rho_n m}{2\omega^6} \int_0^1 dx \left[8 \left(1 + \frac{m^2x^2}{\omega^2}\right)^6 - 16 \left(1 + \frac{m^2x^2}{\omega^2}\right)^4 + 8 \left(1 + \frac{m^2x^2}{\omega^2}\right)^2 \right]$$

$$\times \int_1^x dy y^{-1} [q(y) + \bar{q}(y)],$$

$$\Pi^1_T = \frac{\rho_n m}{\omega^6} \int_0^1 dx \frac{\left(1 + \frac{m^2x^2}{\omega^2}\right)^2}{\left(1 - \frac{m^2x^2}{\omega^2}\right)^2} [q(x) + \bar{q}(x)]$$

$$- \frac{\rho_n m}{\omega^6} \int_0^1 dx \left[16 \left(1 + \frac{m^2x^2}{\omega^2}\right)^5 - 14 \left(1 + \frac{m^2x^2}{\omega^2}\right)^3 + \left(1 + \frac{m^2x^2}{\omega^2}\right) \right]$$

$$\times \int_1^x dy y^{-1} [q(y) + \bar{q}(y)].$$
\[ \int_1^x dy[q(y) + \bar{q}(y)] + \rho_n m^6 \int_0^1 dx x \left[ \frac{16(1 + \frac{m^2 x^2}{\omega^2})^5}{(1 - \frac{m^2 x^2}{\omega^2})^6} - \frac{20(1 + \frac{m^2 x^2}{\omega^2})^3}{(1 - \frac{m^2 x^2}{\omega^2})^4} + \frac{4(1 + \frac{m^2 x^2}{\omega^2})}{(1 - \frac{m^2 x^2}{\omega^2})^2} \right] \times \int_1^x dy y^{-1}[q(y) + \bar{q}(y)]. \]  

(25)

In the following section we use this partial summation to estimate the effect of higher dimensional \((n > 6)\) operators on the constraint.

III. CONSTRAINTS

We now make a Borel transformation of \(\omega^2\) times eq. (23) and examine the dispersion relation,

\[ \rho_n \left( \frac{b_2}{M^2} - \frac{b_3}{2M^4} \right) = \int d\omega^2 \rho^{1}(\omega^{2}) \omega^2 e^{-\omega^2/M^2} - \rho_n b_{\text{scatt}}. \]  

(26)

The reason for multiplying by \(\omega^2\) is to get rid of any possible subtraction constants proportional to \(1/\omega^2\). There could be further subtraction constants proportional to \(\rho_n b_{\text{scatt}}/\omega^4\), which we include. In a given model calculation, the corresponding value of \(b_{\text{scatt}}\) should be computed. The contribution from particle-hole intermediate states is \(b_{\text{scatt}} = 1/4m\) for the longitudinal \(\rho, \omega\) meson and zero for the transverse parts. (This is a non-trivial contribution in the sense discussed above.) After the Borel transformation, the contribution from dimension=\(n\) operators to the OPE is divided by \((n-2)!\). This improves the convergence of the expansion, and enhances the contribution of the low-mass region.

In Fig. 1, we show the OPE side of eq. (26) as a function of \(M^2\) for the \(\rho\) meson. The solid lines shows the full calculation including dimension 6 operators, while the dot-dashed line shows the OPE without the twist-4 contributions and the long dashed line without dimension 6 contributions. At \(M_{\text{min}}^2 \sim 0.8\) GeV\(^2\) (\(M_{\text{min}}^2 \sim 2.5\) GeV\(^2\)), the contributions of dimension 6 operators is approximately 40\% of those from dimension 4 operators for transverse (longitudinal) polarization. Therefore, the OPE should be reliable only for values of the Borel mass above these, where one expects the contributions from higher dimensional operators to be suppressed. We also show (short-dashed line) the OPE including the contributions from the infinite sum of twist-2 operators, eq. (25). The corresponding Borel transform is given in Appendix B. The fact that the contribution of twist 2 operators of dimension larger than 6 is very small for \(M^2 > M_{\text{min}}^2\), indicates that the truncated OPE is indeed reliable above the minimum Borel mass.

Thus, the constraints on the spectral density are given by eq. (26), for \(M^2 > M_{\text{min}}^2\). The Borel transformation also introduces an exponential weighting factor of the spectral density \(\exp(-s/M^2)\), which for small values of Borel mass, suppresses the contributions at large energy. This has the advantage that in practical calculation, one can concentrate on the change of the spectral density near and below the vector meson mass region, while the contribution from higher energies can be effectively taken into account by a continuum. This approximation will be valid for Borel masses below some maximum value \(M_{\text{max}}\). We estimate \(M_{\text{max}}\) by requiring that the continuum contribution should be less than that of the low energy region.
IV. COMPARISON WITH HADRONIC MODEL CALCULATIONS

A possible non-trivial momentum dependence of the vector-meson self energy in nuclear matter has recently been extensively discussed. Such a momentum dependence arises naturally through the energy dependence of the $\rho$-nucleon scattering amplitude. In this context $\rho$-nucleon interactions in p-wave channels play an important role \cite{10,11,31}. In a complementary approach, the momentum dependence due to in-medium modifications of the $\rho$-meson pion cloud has been explored \cite{30}. We note that in none of these models is the momentum dependence directly constrained by data. Therefore it is useful to confront them with the constraints from QCD sum rules. Here we explore models of the type studied in refs. \cite{10,31}, where the $\rho$-nucleon interaction is mediated by s-channel resonance exchanges.

To make a systematic comparison with our constraints, we have to first relate the correlation operator eq. (1) and the polarization functions eq. (2) to the $\rho$-meson self-energy $\Sigma_{L,T}(\omega, q)$, as computed in a hadronic model. To this end, we invoke vector-meson dominance \cite{32}

\begin{equation}
\Pi_{L,T}(\omega, q) = -\frac{1}{q^2 g^2} \frac{1}{g^2 - m^2 + i\Gamma m - \Sigma_{L,T}(\omega, q)},
\end{equation}

where $g^2 = 6.05$ is the $\rho\pi\pi$ coupling constant.

The next step is to extract the lowest order nontrivial $q^2$ dependence to linear order in density. To this order, the self energy is of the following generic form for p-wave resonances \cite{31},

\begin{equation}
\Sigma_T = q^2 \left( \frac{f_{RF}}{m_R} \right)^2 \left[ F(q) \right]^2 \frac{S}{2} \rho_n \frac{E_R(q) - m_N}{\omega^2 - (E_R(q) - m_N)^2}.
\end{equation}

In this section we consider only transversely polarized vector mesons; in the longitudinal channel the expected momentum dependence is much weaker \cite{17,31}. For s-wave resonances \cite{31} the leading $q^2$ dependence in (28) is replaced by a factor proportional to $\omega^2$,

\begin{equation}
\Sigma_T = \omega^2 \left( \frac{f_{RF}}{m_R} \right)^2 \left[ F(q) \right]^2 \frac{S}{2} \rho_n \frac{E_R(q) - m_N}{\omega^2 - (E_R(q) - m_N)^2},
\end{equation}

where the form factor is parametrized as

\begin{equation}
F(q) = \frac{\Lambda^2}{\Lambda^2 + q^2} \quad \text{with} \quad \Lambda = 1.5 \text{ GeV},
\end{equation}

and

\begin{equation}
E_R(q) = \sqrt{m_R^2 + q^2} - \frac{i}{2} \Gamma_R.
\end{equation}

Here $m_N, m_R$ are the masses of the nucleon and the resonances, and $\Gamma_R$ is the total width of the resonance and its momentum dependence will be modeled as in ref. \cite{11},

\begin{equation}
\Gamma_R(\omega, q) = \Gamma^0_R \left( \frac{p}{p_R} \right)^{2l+1}.
\end{equation}
In the phase-space factor, \( l = 0(1) \) for s- (p-)wave resonances and

\[
p = \sqrt{[(s - m_N^2 - m_\pi^2)^2 - 4m_N^2m_\pi^2]/4s},
\]

\[
p_R = \sqrt{[(m_R^2 - m_N^2 - m_\pi^2)^2 - 4m_N^2m_\pi^2]/4m_R^2},
\]

(33)

where \( s = (\omega + m_N)^2 - q^2 \). However, in the present context the effect of the momentum dependence of the width turns out to be weak.

As can be seen from eq. (28), the contribution from p-wave resonances is proportional to \( q^2 \). Hence its contribution to \( \text{Im}\Pi^1 \) is trivially obtained by neglecting the remaining \( q^2 \) dependence. In contrast, eq. (29) is proportional to \( \omega^2 \). The contribution of such terms to \( \text{Im}\Pi^1 \) is most easily obtained by replacing \( \omega^2 \) by \( q^2 + q^2 \) and expanding to leading order in \( q^2 \) keeping \( q^2 \) fixed. Finally, we obtain

\[
\text{Im}\Pi^1(\omega^2)_{\text{res}} = -\frac{1}{\omega^2 g_p^2} \text{Im} \left[ \frac{1}{(\omega^2 - m_p^2 + im_p\Gamma_p)^2} \times \right.
\]

\[
\left\{ \sum_{s,p-\text{wave}} \left( \frac{f_{RN_p}}{m_p} \right)^2 \rho_n \frac{m_R - m_N - \frac{i}{2}\Gamma_R}{\omega^2 - (m_R - m_N - \frac{i}{2}\Gamma_R)^2} \right.
\]

\[
+ \sum_{s-\text{wave}} \left( \frac{f_{RN_p}}{m_p} \right)^2 \rho_n \omega^2 \left( -\frac{1}{\Lambda^2} \frac{m_R - m_N - \frac{i}{2}\Gamma_R}{\omega^2 - (m_R - m_N - \frac{i}{2}\Gamma_R)^2} \right.
\]

\[
+ \frac{1}{2m_R} \left( \frac{1}{\omega^2 - (m_R - m_N - \frac{i}{2}\Gamma_R)^2} + \frac{1}{m_R} \frac{(m_N + \frac{i}{2}\Gamma_R)(m_R - m_N - \frac{i}{2}\Gamma_R)}{\omega^2 - (m_R - m_N - \frac{i}{2}\Gamma_R)^2} \right) \right\}. \] (34)

where we have neglected the small contribution due to the momentum dependence of \( \Gamma_R \).

The contribution from the higher energy region is modeled using the following assumption. In the vacuum QCD sum rule approaches, the spectral density is approximated by a pole and a continuum. Since we make a hadronic calculation of the changes in the pole, we will add the simplest change in the continuum and allow a non-trivial change in the continuum threshold,

\[
c_0\theta \left( q^2 - (s_0 + s'q^2) \right), \] (35)

where \( s' \) is a parameter to be determined. The parameter \( c_0 \), which characterizes the strength of the high-energy continuum, is determined by QCD duality. This excludes a simple \( q^2 \) dependence, such as \( c_0 \to c_0 + c'q^2 \). We thus find the following contribution to the spectral density,

\[
\text{Im}\Pi^1(\omega^2)_{\text{cont}} = -c_0 s' \delta(\omega^2 - s_0) \] (36)

The phenomenological spectral density \( \rho^1 \) is then given by the sum of eq. (34) and eq. (36), i.e., \( \rho^1 = (1/\pi)(\text{Im}\Pi^1_{\text{res}} + \text{Im}\Pi^1_{\text{cont}}) \). The parameter \( s' \) in eq. (34) is determined by minimizing the difference between the left and right hand side in eq. (28). We note that the parameter \( s_0 = 1.43 \text{ GeV}^2 \) is fixed by the QCD sum rule [4] at \( q = 0 \).
A. A ‘standard’ set of model parameters

We first explore the model employed by Peters et al. [31]. The corresponding parameters are given in Table 2. We choose this set, since it is representative for the sets used also by other groups, with similar values for the parameters (see e.g. refs. [11,16,33]). In Fig. 2 we compare the OPE with the phenomenological side, i.e., the left and the right hand sides of eq. (26), including both the resonance and continuum contributions in the phenomenological spectral density. The best fit was obtained in the Borel window 0.5 – 2.0 GeV², by adjusting the parameter s’ of the continuum. We find a huge discrepancy between the OPE and the phenomenological side. The major contributions to the phenomenological side are due to the nucleon as well as the ∆(1232), N(1520), N(1680), N(1720) and ∆(1905) resonances. For comparison we show the right hand side for only these six states (denoted as ‘selected subset’ in Fig. 2) as well as some of the individual contributions. Clearly, the very large effect of the ∆(1232) is mainly responsible for the disagreement. In the best fit, its contribution, which in itself is much larger than the OPE side, cannot be compensated by the continuum over a reasonable range of Borel masses. The fact that the continuum lies at a much higher energy than the ∆(1232), leads to a characteristic difference in the dependence on the Borel mass. Thus, in most of the Borel window, the best fit lies far outside the estimated boundary of uncertainty shown in Fig. 1.

In Fig. 3, we show the corresponding spectral density ρ. The contribution of a particular resonance is identified by extracting the corresponding term in the curly brackets in eq. (34). Also here the dominance of the ∆(1232) is apparent. Moreover, in the constraint equation (26), the contribution of the ∆(1232) is further enhanced compared to the other structures at higher energy due to the exponential factor. Thus, we conclude that the strong effective ∆Nρ coupling originally employed by Rapp et al. [11] is incompatible with the constraint from QCD sum rules.

B. A consistent model

As noted above, the most important contributions to the sumrule are due to the selected set of resonances. Consequently, in the following analysis, we will include only these baryon states. Let us first recall how the coupling constants f_{RNρ} are fixed. The couplings for the N(1720) and the ∆(1905) are determined from their decays into a nucleon and a ρ meson [10]. Hence, these couplings should be reliable for the process ρ + N → resonance considered here. If the mass of the resonance lies below the ρ N threshold, however, the corresponding coupling constant can clearly not be determined in this way. Consequently, in many models the ρ N couplings to the nucleon and the ∆(1232), are obtained e.g. from the Bonn potential [31]. An important point, which apparently has been overlooked, is that in the Bonn potential the coupling constant f_{RNρ} is multiplied by a form factor [34],

\[ F_ρ(q^2) = \left( \frac{\Lambda_ρ^2 - m_ρ^2}{\Lambda_ρ^2 + q^2} \right)^{n_ρ}. \]  

where Λ_ρ = 1.4GeV and n_ρ = 1 for the NNρ and n_ρ = 2 for the ∆Nρ vertex. Hence, the effective value of the coupling constant at q = 0 is reduced by 0.7 for the nucleon and by
0.49 for the \( \Delta(1232) \). Since the self energy is proportional to the square of the coupling constant and the corresponding form factor, the contribution of the nucleon \((\Delta(1232))\) is reduced by a multiplicative factor of 0.49 (0.24). This reduction of the effective coupling constant has been left out in previous calculations.

The resonances \( N(1520) \) and \( N(1680) \) are situated below the nominal threshold for the decay into a nucleon and a hypothetical zero-width \( \rho \) meson. Thus, they can decay into a nucleon and a \( \rho \) meson only because of the finite widths of the particles involved. Since in this case only the low-mass tail of the \( \rho \) meson is seen in the two-pion spectrum \cite{35}, a reliable identification of the \( \rho \) is very difficult. Consequently, the determination of the corresponding coupling constants is rather uncertain. In order to explore possible consequences of this uncertainty, we also show results for the case where the two coupling constants vanish.

Let us now confront this set of coupling constants with the QCD sum rules. For the nucleon and the \( \Delta(1232) \), we use the couplings of the Bonn potential, with the corresponding form factors, extrapolated into the time-like region

\[
F_c^\rho(q^2) = \begin{cases} 
\frac{\Lambda_\rho^2-m_\rho^2}{\Lambda_\rho^2-q^2}^{n_i} & \text{if } q^2 < m_\rho^2 \\
1 + \left(\frac{q^2-m_\rho^2}{\Lambda_\rho^2}\right)^{n_i} & \text{otherwise.}
\end{cases}
\tag{38}
\]

Thus, for \( q^2 < m_\rho^2 \) we use the covariant generalization of \cite{37}. However, close to the pole at \( q^2 = \Lambda_\rho^2 \) this form clearly makes no sense. For \( q^2 > m_\rho^2 \) we therefore employ the simple form shown in the second line of \cite{38}, with the exponent \( n_i = 0, 1 \) or 2. Fortunately, due to the exponential weighting, the QCD sum rule \cite{25} is insensitive to the precise form of \( F_c^\rho \) in this range of energies; the real parts of \( \Pi_1 \) obtained from the right-hand side of \cite{26} for different values of \( n_i \) are almost indistinguishable. Thus, for the nucleon and the \( \Delta(1232) \) we replace the coupling constant \( f_{RN\rho} \) in eq. \cite{34} by \( f^*_{RN\rho}(\omega^2) = F_c^\rho(\omega^2)f_{RN\rho} \).

For the \( \Delta(1905) \) and the \( N(1720) \) we use the coupling constants \( f_{RN\rho} \) determined in ref. \cite{10}. Since the corresponding form factors are normalized to unity for zero three-momentum, they do not affect the leading \( q^2 \) dependence. Furthermore, a possible frequency dependence can also be neglected, because, in contrast to the \( \Delta(1232) \), these resonances contribute significantly only in the kinematical region where the coupling constants are determined, i.e., for almost on-shell \( \rho \) mesons. The coupling constants used are given in Table III. Note that here we use the \( NN\rho \) and \( \Delta(1232)\rho \) coupling constants of ref. \cite{34}, which differ slightly from those used in ref. \cite{11}.

In Fig. 4 we show the effect of a consistent treatment of the \( \Delta N\rho \) form factor on the spectral density. As expected, the contribution due to the \( \Delta(1232) \) is substantially reduced. This reduction is also reflected in an improved agreement with the constraint equation, as shown in Fig. 5. There, the distance between the two solid lines, which correspond to the OPE with and without the dimension 6 operators, represent the uncertainty of the left-hand side of \cite{26}. The long-dashed line is the right-hand side without the form factor for the delta and the nucleon, while the dashed line shows the result including form factors of the form eq. \cite{15} with \( n_i = 0 \). (Since, as noted above, the dependence on \( n_i \) is minimal, we do not show results for \( n_i = 1, 2 \).) Furthermore, the dotted line shows the result of including the form factors but with vanishing couplings to the \( N(1520) \) and \( N(1680) \) resonances. Thus, the range between the dashed and dotted lines indicate the uncertainty due to the poorly known \( N(1520) \) and \( N(1680) \) coupling constants.
Using the criteria discussed in Sect. III, we obtain a Borel window, where the constraint equation can be applied, of \(0.8 \text{ GeV}^2 < M^2 < 1.8 \text{ GeV}^2\). Clearly, there is a good agreement between the OPE and the phenomenological calculation, when the \(\Delta N\rho\) form factor is consistently taken into account. We note that most of the disagreement is removed already with a static (frequency independent) form factor, i.e., by renormalizing the effective coupling strength by 0.7 for the nucleon and by 0.49 for the \(\Delta\). The resulting phenomenological side is shown by the dash-dotted line in Fig. 5. However, the frequency dependence, as parameterized in (38), leads to a further improvement (see Fig. 5).

Our work indicates that the \(\Delta N\rho\) coupling used in many model calculations is unphysical. The Bonn potential offers no justification for the very strong effective couplings used. Furthermore, such couplings violate the constraints obtained from QCD sum rules. Thus, it is necessary to re-analyze those calculations of the low-mass dilepton spectrum in heavy-ion collisions, where the \(\Delta N\rho\) coupling was treated inconsistently.

V. DISCUSSION

We have presented a model independent constraint for the three momentum dependence of the vector meson spectral function in a nuclear medium. We have shown that the momentum dependence of the spectral density obtained in model calculations [11,16,31,33] is too strong to satisfy the constraint derived from QCD sum rules. The main source of the discrepancy is the large contribution from the \(\Delta(1232)\). In these models, the coupling of the rho meson to the nucleon and the \(\Delta(1232)\) is not accounted for in a consistent manner. While the coupling constant is taken e.g. from the Bonn potential, the form factor is not; the normalization of the form factor differs considerably from that, which is used in the Bonn potential. This leads to an effective coupling strength which is approximately a factor two too large. When the original Bonn form factor is employed, the contribution of the \(\Delta(1232)\) is substantially reduced and we find good agreement with the constraint equation.

In view of this it would be important to reexamine the hadronic model calculations that are used to interpret the low-mass dilepton enhancement found in relativistic heavy-ion collisions, taking the form factors properly into account. The unphysically large coupling to the \(\Delta(1232)\) gives rise to a spurious enhancement of the vector meson spectral density for low invariant masses.

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APPENDIX A: COEFFICIENTS OF TRACE TERMS

Here we specify the trace terms in detail. As discussed in the text,
\[ A_{\mu_1 \ldots \mu_{2k}} = \left( \sum_{j=0}^{k} a_j p^{2j} [g_{\mu_1 \mu_2} \cdots g_{\mu_{2j-1} \mu_{2j}} p_{\mu_{2j+1}} \cdots p_{\mu_{2k}} + \text{permutations}] \right) A_{2k} \quad (A1) \]

where for a given value of \( k \) and \( j \) there are \( \binom{2k}{2j} \) different terms and \( a_j = (-1)^j \binom{2k-j}{j} \).

Now consider the two indices \( \mu_1, \mu_2 \). They can enter either as indices of the metric tensor \( g_{\mu \nu} \) or of the nucleon four momentum \( p_\mu \). Depending on how the two indices appear, we can distinguish four types of terms

\[
\begin{align*}
&g_{\mu_1 \mu_2} \times [(j - 1) \text{ factors } (g_{\mu \nu})] \times [2(k - j) \text{ factors } (p_\mu)] \\
g_{\mu_1 \kappa} g_{\mu_2 \lambda} \times [(j - 2) \text{ factors } (g_{\mu \nu})] \times [2(k - j) \text{ factors } (p_\mu)] \\
g_{\mu_1 \kappa} p_{\mu_2} \times [(j - 1) \text{ factors } (g_{\mu \nu})] \times [(2k - 2j - 1) \text{ factors } (p_\mu)] \\
p_{\mu_1} p_{\mu_2} \times [(j) \text{ factors } (g_{\mu \nu})] \times [(2k - 2j - 2) \text{ factors } (p_\mu)].
\end{align*}
\]

There are \( \binom{2k-2}{2j-1} \) number of terms of the first type, where the remaining indices are ordered differently. Of the second type there are \( \binom{2k-2}{2j-2} \) of the third \( \binom{2k-2}{2j-1} \) and of the fourth \( \binom{2k-2}{2j} \) number of terms. Using these results, the infinite series of twist-2 contributions can be summed.

**APPENDIX B: BOREL TRANSFORMATION OF THE INFINITE SUM**

The Borel transformations of eq. (25) can be obtained from the basic formula

\[
\text{B.T.} \left( \frac{1}{(\omega^2 - A)^s} \right) = (-1)^s \frac{1}{(s-1)!} \frac{1}{(M^2)^{s-1}} e^{-A/M^2} \quad (B1)
\]

We find,

\[
\frac{\text{B.T.}(\omega^2 \Pi^0)}{\rho_n m} = \int_0^1 dx \left[ \frac{1}{m^2 x^2} (1 - e^{-m^2 x^2/M^2}) + \frac{2}{M^2} e^{-m^2 x^2/M^2} \right] \left[ q(x) + \bar{q}(x) \right]
\]

\[
+ \int_0^1 dx \left[ -\frac{1}{m^2 x^2} (1 - e^{-m^2 x^2/M^2}) + e^{-m^2 x^2/M^2} \left( \frac{6}{M^2} - \frac{12 m^2 x^2}{M^4} \right)
\]

\[
+ \frac{8 m^4 x^4}{3 M^6} \right] \int_1^x dy [q(y) + \bar{q}(y)]
\]

\[
- \int_0^1 dx e^{-m^2 x^2/M^2} \left( \frac{8}{M^2} - \frac{12 m^2 x^2}{M^4} + \frac{8 m^4 x^4}{3 M^6} \right) \int_1^x dy y^{-1} [q(y) + \bar{q}(y)] , \quad (B2)
\]

\[
\frac{\text{B.T.}(\omega^2 \Pi^1)}{\rho_n m} = \int_0^1 dx \left[ \left( \frac{1}{2M^2} - \frac{2}{m^2 x^2} \right) + e^{-m^2 x^2/M^2} \left( \frac{2}{m^2 x^2} + \frac{2}{M^2} \right) \right] \left[ q(x) + \bar{q}(x) \right]
\]

\[
+ \int_0^1 dx \left[ \frac{6}{m^2 x^2} (1 - e^{-m^2 x^2/M^2}) + e^{-m^2 x^2/M^2} \left( -\frac{6}{M^2} - \frac{16 m^4 x^4}{M^6} \right)
\]

\[
+ \frac{32 m^6 x^6}{3 M^8} - \frac{32 m^6 x^8}{15 M^{10}} \right] \int_1^x dy [q(y) + \bar{q}(y)]
\]
\[ + \int_0^1 dx e^{-m^2x^2/M^2} \left( \frac{32 m^4x^4}{3 M^6} - \frac{32 m^6x^6}{3 M^8} + \frac{32 m^8x^8}{15 M^{10}} \right) \]
\[ \times \int_1^x dy y^{-1}[q(y) + \bar{q}(y)] , \quad \text{(B3)} \]

\[
\frac{\text{B.T.}(\omega^2\Pi^L_{1/2})}{\rho_n m} = \int_0^1 dx \left[ -\frac{1}{m^2x^2} + e^{-m^2x^2/M^2} \left( \frac{1}{m^2x^2} + \frac{2}{M^2} \right) \right] [q(x) + \bar{q}(x)] \\
+ \int_0^1 dx \left[ -\frac{3}{m^2x^2} \left( 1 - e^{-m^2x^2/M^2} \right) + e^{-m^2x^2/M^2} \left( -\frac{6}{M^2} + 36 \frac{m^2x^2}{M^4} - \frac{200 m^4x^4}{3 M^6} + 32 \frac{m^6x^6}{M^8} - \frac{64 m^8x^8}{15 M^{10}} \right) \right] \int_1^x dy [q(y) + \bar{q}(y)] \\
+ \int_0^1 dx e^{-m^2x^2/M^2} \left( -\frac{24 m^2x^2}{M^4} + \frac{176 m^4x^4}{3 M^6} - \frac{32 m^6x^6}{M^8} + \frac{64 m^8x^8}{15 M^{10}} \right) \]
\[ \times \int_1^x dy y^{-1}[q(y) + \bar{q}(y)] . \quad \text{(B4)} \]
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### TABLE I. Wilson coefficients for $\tau = 2$. $T(R) = \frac{f}{2}, C_2(R) = \frac{4}{3}$.

| Wilson coefficient | $n = 2$ | $n = 4$ |
|--------------------|---------|---------|
| $C_{2,n}^q$ | $1 + \frac{\alpha_s}{4\pi} B_{2,n}^{NS}$ | $1 + \frac{\alpha_s}{4\pi} (0.44)$ |
| $C_{2,n}^q$ | $\frac{\alpha_s}{4\pi} C_2(R) \frac{4}{n+1}$ | $\frac{\alpha_s}{4\pi} (\frac{16}{9})$ |
| $C_{2,n}^G$ | $\frac{\alpha_s}{4\pi} T(R) \frac{4}{(n+1)(n+2)} - \frac{4}{n+2} + \frac{1}{2} - \frac{n^2 + n + 2}{n(n+1)(n+2)(1 + \sum_{j=1}^{n+1} \frac{1}{j})}$ | $\frac{\alpha_s}{4\pi} (-\frac{1}{2})$ |
| $C_{L,n}^G$ | $\frac{\alpha_s}{4\pi} T(R) \frac{16}{n(n+1)(n+2)}$ | $\frac{\alpha_s}{4\pi} (\frac{4}{3} \frac{15}{180})$ |

### TABLE II. The parameters used in ref. [31].

| $I(J^P)$ | $\Gamma_{Total}$ (GeV) | $l_\rho$ | $f_{RN\rho}$ | $S_\Sigma$ |
|----------|-------------------------|----------|-------------|---------|
| N(940)   | $\frac{1}{2} (\frac{1}{2}^+)$ | 0        | 1           | 7.7     | 4       |
| N(1520)  | $\frac{1}{2} (\frac{1}{2}^-)$ | 120      | 0           | 7       | 8/3     |
| N(1650)  | $\frac{1}{2} (\frac{1}{2}^-)$ | 150      | 0           | 0.9     | 4       |
| N(1680)  | $\frac{1}{2} (\frac{3}{2}^+)$ | 130      | 1           | 6.3     | 6/5     |
| N(1720)  | $\frac{1}{2} (\frac{3}{2}^+)$ | 150      | 1           | 7.8     | 8/3     |
| $\Delta(1232)$ | $\frac{1}{2} (\frac{3}{2}^+)$ | 120      | 1           | 15.3    | 16/9    |
| $\Delta(1620)$ | $\frac{1}{2} (\frac{3}{2}^-)$ | 150      | 0           | 2.5     | 8/3     |
| $\Delta(1700)$ | $\frac{3}{2} (\frac{3}{2}^-)$ | 300      | 0           | 5.0     | 16/9    |
| $\Delta(1905)$ | $\frac{3}{2} (\frac{5}{2}^+)$ | 350      | 1           | 12.2    | 4/5     |

### TABLE III. Parameters used in a consistent model for the selected subset of resonances.

| $I(J^P)$ | $\Gamma_{Total}$ (GeV) | $f_{RN\rho}$ |
|----------|-------------------------|-------------|
| N(940)   | $\frac{1}{2} (\frac{1}{2}^-)$ | 0           | 8.1 |
| N(1520)  | $\frac{1}{2} (\frac{3}{2}^-)$ | 120         | 7   |
| N(1680)  | $\frac{1}{2} (\frac{3}{2}^+)$ | 130         | 6.3 |
| N(1720)  | $\frac{1}{2} (\frac{3}{2}^+)$ | 150         | 7.2 |
| $\Delta(1232)$ | $\frac{1}{2} (\frac{3}{2}^+)$ | 120         | 16  |
| $\Delta(1905)$ | $\frac{3}{2} (\frac{5}{2}^+)$ | 350         | 9.0 |
FIG. 1. The OPE in GeV$^{-2}$. The solid line shows the full calculation, while the short-dashed line includes the infinite sum of the twist-2 operators. The dashed line shows the OPE without dim-6 and the dash-dotted without twist-4 operators. All curves are at nuclear matter density.

FIG. 2. Real part of $\Pi^1$ in GeV$^{-2}$. The solid lines denote the OPE and the best fit including all resonances in the model of ref. [31]. The contributions from the nucleon, $N(1720)$, $\Delta(1232)$, $\Delta(1905)$ and the continuum are also shown. Furthermore, the contribution of the most important resonances (the selected subset) is shown for the parameters given in Table II.
FIG. 3. Imaginary part of $\Pi^1 = \rho^1$ in GeV$^{-2}$ as a function of the energy $\omega$.

FIG. 4. Imaginary part of $\Pi^1 = \rho^1$ in GeV$^{-2}$ for the selected subset (Table II) with and without form factors (38) for the nucleon and the $\Delta(1232)$. 
FIG. 5. Real part of $\Pi^1$ in GeV$^{-2}$ with the parameters given in Table III. The long-dashed line shows the phenomenological side without the form factor for the nucleon and the delta, while the dashed line shows the result including the form factors (38). The dotted line is obtained with the form factors but with zero coupling constants for the $N(1520)$ and $N(1680)$ resonances. Finally, the dash-dotted line shows the phenomenological side obtained when the frequency dependence of the form factors (38) is neglected.