Warranty cost analysis for a multi-component product protected by lemon laws

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Abstract. We study warranty cost analysis for a multi-component product protected by the lemon laws. The product is repairable and consists of critical and non-critical components. The failed product under warranty due to failure of any critical or non-critical component is minimally repaired. It is considered that the product turns to be a lemon when either the number of critical or noncritical component failures reaches \( k \). We obtain the expected warranty cost and the optimal period of lemon laws which are of interest to the manufacturer when the product is protected by the lemon laws, and provide numerical examples to illustrate the expected warranty cost and the optimal lemon law period.

1. Introduction

All durable products are sold with warranty in any marketplace. For instance, home appliances such as air conditioning, water heater, refrigerator, dishwasher, microwave are warranted for 1-3 years, a car is warranted for three years or 100,000km, whichever comes first. This warranty policy only requires the manufacturer to repair if the product fails during the warranty period. The manufacturer has no obligation to replace or refund the full purchase price if the product repeatedly fails in short time under warranty or a long repair time to get the car fixed. This in turn results in a significant inconvenience to the consumer.

With lemon laws, consumers are protected against a defective product that does not conform to standards of quality and performance. The lemon laws provide the consumers the right to have a new product or a full refund when the vehicle is considered a defective product (or declared to be a lemon). The lemon laws had been enacted since 1982 in state of Connecticut US and in the next five-year times, all states in US had implemented the lemon laws protecting new-car buyers from defective automobiles. Before the lemon laws was effective, the consumers who bought a defective product had bad experiences due to repeated failures with frustration, delays, expenses, and uncertainty to get the failed car repaired [1].

Nowadays, Canada, Europe, Australia, Singapore, and South Korea have adopted the lemon laws to provide protection to the customers. Initially, the lemon laws aimed at protecting automobile products but today other products have been protected by these lemon laws. For instances, in Singapore, all durable products are protected by lemon laws. The lemon laws for automotive products are described as follows. “A car turns to be a lemon if either (i) the car has been returned to the dealer four times to
have the same problem fixed, but the dealer was unable to repair the problem satisfactorily, or (ii) the
car has been out of service more than 30 days due to one or more defects.

The lemon laws give more protection not only to the customers buying a brand new product, but
also to those who buy a second hand product. The lemon laws, in one side, provide protection to the
consumers against repeat failures occurring during the warranty period. But, in the other side, the
lemon laws require the manufacturer or dealer to replace a defective product or refund its purchase
price should the product is declared a lemon. As the result, products protected by the lemon laws may
result in a significant additional cost to the manufacturer/dealer for rectifying a defective product, and
this in turn affects its profits.

Study of lemon law warranties has received less attention in literature. [2] studied automobile
lemon laws to estimate the value of lemon protection to consumers. [3] examined tractor lemon laws
and make comparison between automobile and tractor lemon laws. They used a principal agent to
model the economic efficiency of lemon laws. [4] learned a renewable minimal repair–replacement
warranty policy in which the rectification to replace or to repair the failed product during warranty
period depends on the repair time. It is replaced when the time needed to repair the failed product
exceeds the threshold, otherwise it is minimally repaired. This condition is quite similar to the
condition in the lemon law warranty. [5] explored cost analysis of lemon law warranties for one
dimensional warranty in which a product turns to be a lemon when the number of failures is greater
the threshold value (i.e. \( k = 3 \)) and examined to cases - refund and replacement cases. This work has
been extended to two dimensional warranties in [6]. And in [7] and [8], they extended warranty cost
analysis for remanufactured and used product considering lemon law. All of the works on lemon law
warranty consider a product as a single component system and use a black box approach to modelling
failures. Actually, the product is a multi-component system.

In this paper, we study warranty cost analysis for a multi-component product protected by lemon
laws, and obtain the expected warranty cost and the optimal period of lemon laws which are of interest
to the dealer when the product is protected by the lemon laws.

The paper is organized as follows. Section 2 deals with model formulation which present modelling
failure and ways of a lemon occurring. In Section 3, we give the expected warranty cost for a refund
case and obtain the optimal period of the lemon laws for a fixed warranty period in Section 4. Section
5 presents a numerical example for illustrating the expected warranty cost and the optimal lemon law
period. Finally, we present conclusions and a brief discussion of future research topics.

2. Model Formulation

Notation:

\[ W \] : Warranty period
\[ W_L \] : The lemon law period
\[ N_i(t)|N_2(t) \] : The number of natural failures of C [NC] components over \([0,t)\)
\[ \lambda_1(t)|\lambda_2(t) \] : Intensity function of a C [NC] component
\[ U_n|V_n \] : The time of the \( n^{th} \) failure of a C [NC] component \((n \geq 1)\)
\[ g_n(t)|h_n(t) \] : The density function for \( U_n|V_n \)
\[ G_n(t)|H_n(t) \] : The distribution function for \( U_n|V_n \)
\[ C_r \] : Average repair cost
\[ C_c \] : Average Critical-component repair cost
\[ C_{nc} \] : Average Non Critical-component repair cost
We consider that a repairable product consisting of critical and noncritical components (i.e., a multi-component system) is sold with a warranty period $W$ in which lemon laws are enforceable in the period $(0, W)$. In general, $W$ is equal to $W$ but in some cases, $W$ is less than $W$, for instance, in Singapore, the period is 6 months after purchase (which is less than the warranty period, 12 months or more). The product is declared a lemon if either of two conditions are met: (i) the product has been returned to the dealer $k$ times to have the failure fixed or (ii) the product has been out of service more than $\tau$ unit time (e.g., 30 days) due to one or more failures.

We look at the case where the lemon law only deals with number of failures and not the repair time (or out of service). The case where the lemon laws take into account the number of failures as well as the downtime will be considered as a topic for further studies.

Furthermore, as the product is repairable, the failed product due to any component failure is always minimally repaired. All repair times are small compared to the times between component failures and hence can be ignored. If $N_i(t)[N_s(t)]$ is the number of failures of the critical (C) and noncritical components (NC) over $[0, t]$ and each failure is fixed by a minimal repair, then $N_i(t)[N_s(t)]$ is a Nonhomogeneous Poisson process with intensity function $\lambda_i(t)[\lambda_s(t)]$.

We consider that both the critical and non-critical components can turn the product to be a lemon. Let $k_1$ and $k_2$ be the failure number thresholds for the critical and noncritical components under the lemon law, respectively, with $k_1 = k_2 = k$. This implies that when either the number of critical component failures reaches $k$ or the number of noncritical component failures reaches $k$, then the product turns to be a lemon and the manufacturer has to refund the sale price to the customer or replace the failed item with a new item together with a new warranty policy at the time of the $k^{th}$ failure of either C component or NC component.

Consequently, we have two rectification actions - i.e. refund or replace the failed item. In this paper, we consider a case with refund and a case with replacement can be viewed as a further research topic.

Let $W_k$ denote the period where the lemon law is enforceable. Two cases will be studied - Case (i) $W_k = W$ and Case (ii) $W_k < W$. Let $U_n[V_n]$ denote the time of the $n^{th}$ failure of a C [NC] component $(n \geq 1)$, and let $G_n(t)[H_n(t)]$ denote the distribution function for $U_n[V_n]$. The probability of $n$ successive failures of C [NC] components over (and hence minimal repairs) in $(0, t]$ is given by

$$\Pr \{ N_1(t) = n \} = G_n(t) - G_{n+1}(t) \left[ \Pr \{ N_2(t) = n \} = H_n(t) - H_{n+1}(t) \right]$$

where $P(U_n \leq t) = G_n(t) = 1 - \sum_{i=0}^{n-1} e^{-\lambda(t)} \frac{\lambda(t)^i}{i!}$ and $P(V_n \leq t) = H_n(t) = 1 - \sum_{i=0}^{n-1} e^{-\lambda(t)} \frac{\lambda_s(t)^i}{i!}$, $\Lambda_1(t) = \int_0^t \lambda_1(\tau) d\tau$, and $\Lambda_2(t) = \int_0^t \lambda_2(\tau) d\tau$.

Define, $\Gamma_k = \text{Min} \{ U_k, V_k \}$. $\Gamma_k$ can be less or greater than $W$, and a lemon is declared when $\Gamma_k \leq W$.

Figure 1 represents a lemon due to the C-component, and the NC-component for $k_1 = k_2 = 4$. 
Figure 1. A lemon due to the C-component, and the NC-component for $k_1 = k_2 = 4$.

We need to consider the ways in which $\Gamma_k$ can occur - (i) due to C-component failures, $(V_k, N_2 = k, N_1 \leq k - 1)$ and (ii) due to NC-component failures, $(V_k, N_2 = k, N_1 \leq k - 1)$. Hence, we need to obtain distribution functions $P(U_k \leq W, V_k > U_k)$, $P(V_k \leq W, U_k > V_k)$ and $P(U_k > W, V_k > W)$.

3. Expected Warranty Cost

The manufacturer is required to refund the sale price to the customer when a product turns to be a lemon. The expected warranty cost due to the critical and noncritical component failures can be expressed as.

\[
\text{The expected warranty cost} = E[\text{No - Refund}] + E[\text{Refund}],
\]

where

\[
E[\text{No - Refund}] = E[\text{Repair cost of the C component}] + E[\text{Repair cost of the NC component}],
\]

and

\[
E[\text{Refund}] = E[\text{Due to the C component}] + E[\text{Due to the NC component}].
\]

We obtain the expressions for the expected warranty cost for two cases - (i) Case (i) $W_L = W$ and (ii) Case (ii) $W_L < W$.

Case (i) $W_L = W$

If the item is declared a lemon under warranty (or $\Gamma_k \leq W$), then the manufacturer refund all or part of the purchasing price to the buyer, and the life cycle of the product is terminated. Let $C_i(W; k)$ be the cost to the manufacturer to service the warranty for the case of refund. Let $\Gamma_k$ denote the time instant where a product is declared a lemon.

\[
\Gamma_k = \text{Min}\{U_k, V_k\}
\]  

(2)

Conditioning on $\Gamma_k$, we have the conditional expected cost given by


\[ E[C_1(W;k)|\Gamma_k] = \begin{cases} E[C_1^a(W;k)|\Gamma_k], & \text{if } \Gamma_k = U_k \leq W, V_k > U_k \\
E[C_1^b(W;k)|\Gamma_k], & \text{if } \Gamma_k = V_k \leq W, U_k > V_k \\
E[C_1^c(W;k)|\Gamma_k], & \text{if } \Gamma_k > W \ (\text{or } U_k, V_k > W) 
\end{cases} \tag{3} \]

where,

\[
E[C_1^a(W;k)|\Gamma_k] = \left[ c_v(k - 1) + c_r + \sum_{m=1}^{k-1} mP\{N_2(W) = m\}c_m \right]
\]

\[
E[C_1^b(W;k)|\Gamma_k] = \left[ c_u(k - 1) + c_r + \sum_{m=1}^{k-1} mP\{N_i(W) = m\}c_m \right]
\]

and \( E[C_1^c(W;k)|\Gamma_k] = \left[ \sum_{n=1}^{k-1} mP\{N_1(W) = m\}c_n + \sum_{m=1}^{k-1} mP\{N_2(W) = m\}c_m \right] \)

Then, removing the conditioning yields

\[
E[C_1(W;k)] = \left[ c_v(k - 1) + c_r + \sum_{m=1}^{k-1} mP\{N_2(W) = m\}c_m \right] \int_0^W \tilde{G}_k(x)h_v(x)dx \\
+ \left[ c_u(k - 1) + c_r + \sum_{m=1}^{k-1} mP\{N_i(W) = m\}c_m \right] \int_0^W \tilde{H}_k(x)g_u(x)dx \\
+ \left[ \sum_{n=1}^{k-1} mP\{N_1(W) = m\}c_n + \sum_{m=1}^{k-1} mP\{N_2(W) = m\}c_m \right] \tilde{G}_k(W)\tilde{H}_k(W) \tag{4} \]

where \( P(U_k \leq W, V_k > U_k) = \int_0^W \tilde{H}_k(x)g_u(x)dx \), \( P(V_k \leq W, U_k > V_k) = \int_0^W \tilde{G}_k(x)h_v(x)dx \), and \( P((U_k > W, V_k \leq W)) = \tilde{G}_k(W)\tilde{H}_k(W) \).

**Case (ii)** \( W_k < W \)

Let \( \Gamma_k \) denote the time instant where a product is declared a lemon. Then, \( \Gamma_k = \text{Min}\{U_k, V_k\} \). Here, the item is declared a lemon under warranty if \( \Gamma_k \leq W_k \), then the manufacturer will refund the full price to the customer, and the warranty is terminated. Let \( C_2(W,W_k;k) \) be the cost to the manufacturer to service the warranty for the Case (ii). We obtain \( C_2(W,W_k;k) \) by conditioning on \( \Gamma_k \). Conditioning on \( \Gamma_k \), we have the conditional expected cost given by

\[
E[C_2(W,W_k;k)|\Gamma_k] \]

is equal to

\[
E[C_2(W,W_k;k)|\Gamma_k] = \begin{cases} E[C_2^a(W,W_k;k)|\Gamma_k], & \text{if } \Gamma_k = U_k \leq W, V_k > U_k \\
E[C_2^b(W,W_k;k)|\Gamma_k], & \text{if } \Gamma_k = V_k \leq W, U_k > V_k \\
E[C_2^c(W,W_k;k)|\Gamma_k], & \text{if } \Gamma_k > W \ (\text{or } U_k, V_k > W) 
\end{cases} \tag{5} \]

where,

\[
E[C_2^a(W,W_k;k)|\Gamma_k] = \left[ c_v(k - 1) + c_r + \sum_{m=1}^{k-1} mP\{N_2(W_k) = m\}c_m \right]
\]

\[
E[C_2^b(W,W_k;k)|\Gamma_k] = \left[ c_u(k - 1) + c_r + \sum_{m=1}^{k-1} mP\{N_i(W_k) = m\}c_m \right]
\]
Removing the conditioning gives,
\[ E[C_z^c(W,W_L;k)] = \left( \sum_{m=1}^{k-1} mP\{N_1(W_L) = m \} c_m + \sum_{m=1}^{k-1} mP\{N_2(W_L) = m \} c_m \right) + c_e \int_{w_L}^{w} \lambda(t)dt + c_s \int_{w_L}^{w} \lambda_2(t)dt \]

4. Optimization
The warranty ceases either at \( \Gamma_k \), if \( \Gamma_k \leq W_L \) (when a lemon is declared and the manufacturer will refund a full price to the customer) or at \( W \), if \( \Gamma_k > W_L \) (when no refund occurs under warranty).

Let \( L_w \) be the length of warranty services, then \( L_w \) is given by
\[ L_w = \begin{cases} \Gamma_k & \text{if } \Gamma_k \leq W_L \\ W & \text{if } \Gamma_k > W_L \end{cases} \]

As a result, we have the expected length of warranty services is
\[ E[L_w] = \int_0^{w_L} s f_t(s)ds + W \bar{H}_k(W_L) \bar{H}_k(W_L) \]
where
\[ F_t(s) = P(\Gamma_k < s) = \int_0^{s} \bar{H}_k(x)g_k(x)dx + \int_0^{s} \bar{G}_k(x)h_k(x)dx \]
and
\[ f_t(s) = \frac{\partial F_t(s)}{\partial s} \]

Let \( EWR(W,W_L;k) \) be the warranty cost rate. Then
\[ EWR(W,W_L;k) = \frac{E[C_z(W,W_L;k)]}{E[L_w]} \]

The optimal of the lemon law period, \( W_L^* \), which minimizes the warranty cost is the solution of the following optimization problem.
\[ \text{Min} \{ EWR[C_z(W,W_L;k)] \} \]

Since equation (7) involved complex integral equations, we use a numerical computation to obtain the value of \( W_L^* \).

5. Numerical Examples
We consider that failure distributions for critical components and noncritical components are given by Weibull distribution with failure rate functions are
\[ \lambda_c(t) = \frac{\beta_c}{\alpha_c} \left( \frac{t}{\alpha_c} \right)^{\beta_c-1} \]
\[ \lambda_n(t) = \frac{\beta_n}{\alpha_n} \left( \frac{t}{\alpha_n} \right)^{\beta_n-1} \]
The parameter values used are \( C_c = 100, C_n = 0.5C_c, C_m = 0.5C_c, W=1 \) and \( \beta = 2 \).
\( k = 3 \) for \( W^*_{\text{L}} = W = 0.63 \) \([652.123]\)

\( k = 4 \) for \( W^*_{\text{L}} = W = 0.65 \) \([772.443]\)

\( k = 5 \) for \( W^*_{\text{L}} = W = 0.69 \) \([850.002]\)

\( k = 3 \) for \( W^*_{\text{L}} = 0.587 \) \([967.132]\)<\( W \)

\( k = 4 \) for \( W^*_{\text{L}} = 0.63 \) \([975.416]\)<\( W \)

\( k = 5 \) for \( W^*_{\text{L}} = 0.69 \) \([979.039]\)<\( W \)

**Figure 2.** Curve of warranty cost rate with \( W_{\text{L}} = W \) and \( W_{\text{L}} < W \)

**Table 1.** Results for refund cases with \( W_{\text{L}} = W \) and \( W_{\text{L}} < W \)

| \( k \) | \( W_{\text{L}} = W \) | \( W_{\text{L}} < W \) |
|-------|----------------|----------------|
| \( W^*_{\text{L}} \) | \( EWR(W_{\text{L}}; k) \) | \( W^*_{\text{L}} \) | \( EWR(W_{\text{L}}; k) \) |
| 3     | 0.63           | 652.123        | 0.587         | 967.132        |
| 4     | 0.65           | 772.443        | 0.63          | 975.416        |
| 5     | 0.69           | 850.002        | 0.69          | 979.039        |

Table 1 shows the optimal of the lemon law period \( W^*_{\text{L}} \) and the warranty cost rate \( EWR(W_{\text{L}}; k) \) for \( k = 3, 4 \) and \( 5 \) with \( \alpha_c = 0.4 \) and \( \alpha_{mc} = 0.7 \). For refund case with \( W_{\text{L}} = W \), the optimal of the lemon law period \( W^*_{\text{L}} \) and the expected warranty servicing cost increases when the failure number thresholds for the critical and noncritical components \( k \) increases from 3 to 5. This is as expected since higher number of failures of the product means the longer the lemon period will increase the expected warranty servicing cost. For refund case with \( W_{\text{L}} < W \), the effect of the failure
number thresholds for the critical and noncritical components $k$ to the expected warranty servicing cost is similar.

6. Conclusions
We have studied warranty cost analysis for a multi-component product where the lemon laws are enforceable during the warranty period, and obtained the expected warranty cost and the optimal period of lemon laws which are of interest to the manufacturer (or dealer) when the product is protected by the lemon laws. This paper only considers the case of refund whilst the case of replacement would be one of further research. The other topics are that the lemon laws invoked by both the number of failures as well as the downtime, and the extension of this study to the case of a two-dimensional warranty. The research of the further topics is on the ways.

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