A robust and fast control technology of AC power conditioning for high-speed micromachining

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Abstract. In this paper, a robust and fast control technology is used to AC power conditioning, thus increasing the performance of the high-speed micromachining. The robust and fast control technology is made up of a robust sliding function (RSF) and a computationally fast grey forecasting model (GFM). The RSF without singularity problem admits system state converged to zero within finite time so that the output-voltage with low harmonic distortion in AC power conditioning is obtained. Nevertheless, while a severe non-linear loading is applied to high-speed micromachining, the needless chattering around robust sliding function occurs. The chattering results in thermal breakdown and serious voltage distortion in AC power conditioning output, and the reliability and stability of the high-speed micromachining will be worsened. Therefore, the GFM with Fourier series is introduced as a computationally fast and algorithmically easy means of removing the chattering existing in RSF when the system uncertainty bound is overestimated. By using this presented control technology, the AC power conditioning provides a high-quality AC output-voltage with accurate steady state and fast transience under various loading conditions, thus obtaining the excellent reliability and stability of the high-speed micromachining. Experimental results are performed in support of the proposed control technology.

1 Introduction

The AC power conditioning has widely been applied in high-speed micromachining [1-4]. High performance AC power conditioning must supply the output AC voltage to the reference sinusoidal with low total harmonics distortion (THD) and fast transient response. To minimize THD, some control schemes have been proposed for AC power conditioning. PI control is frequently used in industry due to simple control structure and ease of design. But, PID control cannot give good control performance as the controlled plant is severe non-linear and uncertain [5], [6]. Deadbeat control can improve the shortcoming of the PI control, but it is highly dependent on the accuracy of the parameters [7]. The repetitive control and mu-synthesis approach can overcome system uncertainties. However, they have implementation difficult and algorithm complexity [8], [9]. Sliding mode control (SMC) is being given attention due to its robustness characteristics [10-13]. A number of SMC associated with AC power conditioning have been reported [14-18]. A fixed switching frequency sliding mode controlled AC power conditioning is presented in [14]. However, the control design uses a typical SMC and cumbersome analog implementation, and thus incurs distorted output voltage during steady-state operation with a non-linear load. In [15], the multiple-sliding-surface is suggested to improve the incomplete system dynamics of classic sliding surface. Though the system performance is improved, the proposed methodology has time-consuming operation in algorithms. The control scheme based on fixed-frequency SMC has also been applied to the design of grid-connected AC power conditioning. In this case, the resulting output voltage makes a concession between steady-state and transience [16]. Reference [17] employs an integral SMC law to achieve AC power conditioning. But, the system trajectory could not hit the desired sliding surface fast and accurately. The noticeable distortion exists in the output voltage waveform. A modified SMC with the elimination of the disturbances for AC power conditioning is developed by [18]; this technology has complicated hardware design and a chattering problem. As mentioned by [14-18], linear sliding surface is adopted. Its characteristic is that the system tracking error converges to zero asymptotically. A robust sliding function (RSF), which employs nonlinear sliding surface is developed instead of linear sliding surface. Compared with linear sliding-surface-based control, the RSF can drive the system tracking error to converge to zero in finite time and there is no singular problem [19-23]. From the point of view in practical high-speed micromachining application, if the load disturbance is a severe non-linear condition, the chattering around RSF occurs. The chattering leads to thermal breakdown and serious voltage distortion in AC power conditioning output, thus deteriorating the reliability and stability of the high-speed micromachining. A computationally fast grey forecasting model (GFM) with Fourier series is employed to describe and analyze the future trend of sequence numbers according to the past and nowadays data for

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dynamic system. The GFM improves the accuracy of the basic grey forecasting model through the Fourier series, and has been successfully applied in many areas of engineering [24-28]. Thus, a mathematically simple and accurately forecasted GFM is employed to eliminate the chattering while the system uncertainty bounds are overestimated. Combining RSF with GFM, the proposed control technology yields a closed-loop AC power conditioning with low total harmonic distortion and fast transience under different types of loading, thus increasing the performance of the high-speed micromachining. Experimental are shown to certify the performance of the proposed control technology.

2 System description

Fig. 1 shows a commonly used AC power conditioning. The $L_fC_f$ filter and resistive load $R$ can be regarded as a plant. Choosing the state variables $x_1 = v_o$ and $x_2 = \dot{v}_o$, where $v_o$ is the output voltage and $\dot{v}_o$ is its derivative, the state equation yields

$$\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = -a_1x_1 - a_2x_2 + b_1u
\end{cases}$$

(1)

where $a_1 = 1/L_fC_f$, $a_2 = 1/RC_f$, $b_1 = K_{pm}/L_fC_f$ and $u$ is the control signal. Owing to the unpredictability of the load condition, the parameters $a_2$ is almost impossible to know exactly, however to have reasonable approximations, the following assumptions are made as $a_2 = \bar{a}_2 + \Delta a_2$, where the $\bar{a}_2$ represents nominal parameters of the system, and $\Delta a_2$ is parameter uncertainties. The bound of the $a_2$ can be given by $a_{2\text{min}} \leq a_2 \leq a_{2\text{max}}$. If the switching frequency is much higher than the fundamental frequency of the AC output, $K_{pm}$ is regarded as a proportional gain of a pulse-width modulation (PWM) full-bridge converter and equals $V_s/\bar{v}_n$; $\bar{v}_n$ is the amplitude of the triangular wave $v_T$ in the PWM. Once $u$ is determined, by comparing $u$ with $v_T$, the PWM gating signals is produced and controls the power switches. Based on the state-space averaging and linearization technique, the product of $u$ and $K_{pm}$ is equivalent to output voltage $v_o$ of the full-bridge converter. In equation (1), the output voltage $v_o$ is desired to be maintained as close as possible to a sinusoidal reference voltage $v_{re}$. Generally, $v_{re}(t) = \sqrt{2}V_{rms}\sin(2\pi f_r t)$ in which $V_{rms}$ and $f_r$ are the root-mean-square and frequency values of the desired sinusoidal reference voltage, respectively. Therefore, the tracking requirement $v_{o}(t) \rightarrow v_{re}(t)$, $t \rightarrow \infty$ must be maintained, the design problem of the AC power conditioning can be regarded as a path-following control problem.

Define the tracking errors as

$$\begin{cases}
e_1 = x_1 - v_{re} \\
e_2 = x_2 - \dot{v}_{re}
\end{cases}$$

(2)

From (1) and (2), the error state equations of the AC power conditioning can be derived as

$$\begin{cases}
e_1 = e_2 \\
e_2 = -a_1e_1 - a_2e_2 + b_1u - f
\end{cases}$$

(3)

where $f = a_1\dot{v}_{re} + a_2\ddot{v}_{re} + \dot{v}_{re}$ is the disturbance.

3 Control technology design

For the error dynamics (3), the robust sliding function is defined as

$$\sigma(t) = e_1(t) + \frac{p}{\varepsilon}e_2^2(t)$$

(4)

where $\varepsilon > 0$, $p > q$, $p$ and $q$ are positive odd numbers $(1 < p/q < 2)$, and a sliding-mode reaching equation

$$\dot{\sigma} = -k_1\sigma^{p/q}\text{sign}(\sigma) - k_2\sigma^{q/p}\text{sign}(\sigma)$$

is constructed.

Then, the control law $u$ can be expressed as

$$u(t) = u_c(t) + u_s(t)$$

(5)
with
\[
    u_e(t) = -\left[ a_1 e_1 + a_2 e_2 - \frac{b}{p} \frac{q}{p} \right] ^{2-\frac{p}{q}}
\]
and
\[
    u_s(t) = -b^2 \left[ k_1 e^\beta \text{sign}(\sigma) + k_2 e^\beta \text{sign}(\sigma) \right]
\]
where \( k_1, k_2 > 0 \), \( \alpha > 0 \) and \( \beta > 0 \), \( u_e \) denotes the 

\text{equivalent control with non-singularity, and } u_s \text{ displays the sliding 

control for compensating the perturbation influences. Thus, the system will be driven to the sliding 

mode } \sigma = 0 \text{ and converged within finite time, and the perturbation } f(t) \text{ is bounded as } \| f(t) \| < \psi, \forall t \geq 0. 

Proof: Let us use the following Lyapunov candidate function:
\[
    V = 0.5 \cdot \sigma^2
\]

Along the trajectory of the dynamic system (3) with the control law (5), and using (4), the time derivative of 

\( V \) is given by
\[
    \dot{V} = \sigma \dot{\sigma}
\]

\[
    = \sigma \left( e_1 + \frac{1}{p} e_2 \right)
\]

\[
    \leq -\sigma \left( \frac{p}{q} e_2 \right) + k_1 e^\beta \text{sign}(\sigma) + k_2 e^\beta \text{sign}(\sigma)
\]

\[
    \leq -\sigma + k_1 e^\beta \text{sign}(\sigma) + k_2 e^\beta \text{sign}(\sigma)
\]

Since \( e_2 \) > 0 holds, \( \dot{V} \leq 0 \), which infers that the 

FCSMC manifold \( \sigma \) in (4) converges to zero within 

finite time. On the other hand, when \( \sigma = e_1 + \frac{1}{p} e_2 \) is 

reached, then the states of the system (3) will converge 

to zero within finite time. However, FCSMC has chattering in AC power conditioning system design. This 
is because of the changeable load, so once the loading is a severe uncertain condition, the system (3) will 

not provide accurate tracking performance. Thus, the control 

signal \( u(t) \) (5) is modified by the addition of the Fourier 
multiplier grey control ( \( u_{gm} \) ), which eliminates 

chattering in AC power conditioning system. The 

modeling steps of the GFM are described below.

Step 1: Input the original sample data sequence 

Letting the original data sequence be denoted as 
\[
    x^{(i)} = \left( x^{(i)}(k), \ k = 1, 2, \ldots, n \right)
\]

where \( x^{(i)} \) stands for the set of \( n \) original sample data. 

Step 2: Accumulated generating operation (AGO) 

By taking the AGO on \( x^{(i)} \), the following first- 

order AGO sequence is expressed as 
\[
    x^{(i)} = \sum_{i=1}^{\text{AGO}} x^{(i)}(k), \ k = 1, 2, \ldots, n
\]

Step 3: Grey model 

Based on the accumulated data sequence, \( x^{(i)} \), a first- 

order ordinary differential grey model, GM(1,1) is 

formed as 
\[
    \frac{dx^{(i)}}{dt} + ax^{(i)} = b
\]

where \( a \) and \( b \) stand for model coefficients, and need 

to be decided.

By employing MEAN generating operation to \( x^{(i)} \), 

namely \( \text{MEAN}(x^{(i)}) = (x^{(i)}(k+1) + x^{(i)}(k))/2 \), the (12) is 

restated as 
\[
    x^{(i)}(k+1) = -0.5(x^{(i)}(k) + x^{(i)}(k)) \left( \begin{array}{c} a \\ b \end{array} \right)
\]

where \( x^{(i)}(k+1) = x^{(i)}(k) - x^{(i)}(k) \).

Letting \( k = 1, 2, \ldots, n-1 \), and (13) is represented as 
\[
    x^{(i)}(0) = \left( \begin{array}{c} x^{(i)}(2) \\ \vdots \\ x^{(i)}(n) \end{array} \right) = \left( \begin{array}{c} -0.5(x^{(2)}(1) + x^{(1)}(1)) \\ \vdots \\ -0.5(x^{(n)}(1) + x^{(1)}(n-1)) \end{array} \right)
\]

The following matrices can be presumed as 
\[
    M = \left( \begin{array}{c} x^{(i)}(2) \\ \vdots \\ x^{(i)}(n) \end{array} \right), \quad N = \left( \begin{array}{c} -0.5(x^{(2)}(1) + x^{(1)}(1)) \\ \vdots \\ -0.5(x^{(n)}(1) + x^{(1)}(n-1)) \end{array} \right), \quad \Psi = \left( \begin{array}{c} a \\ b \end{array} \right)
\]

We can solve the estimated parameters \( \Psi \) via means 
of least square method below.
\[
    \Psi = \left( N^T N \right)^{-1} N^T M
\]

To solve the grey differential equation, substituting 
(15) into (12), and the forecast output is computed as 
\[
    \hat{x}^{(i)}(k+1) = \left( x^{(i)}(1) - \frac{b}{a} e^{-ak} + \frac{b}{a} \right)
\]

where “-” denotes forecasted value, and \( \hat{x}^{(i)}(k+1) \) 
symbols the approximation solution of differential 

equation in (16).

Step 4: Inverse accumulated generating operation (IAGO) 

By the use of the IAGO, the data sequence \( x^{(i)}(k) \) 
can be estimated as 
\[
    x^{(i)}(k) = x^{(i)}(k) - x^{(i)}(k-1)
\]

\[
    (1-e^{-a}) \left( x^{(i)}(1) - \frac{b}{a} e^{-a(k-1)} \right)
\]

Let \( k = 1, 2, \ldots, n \), the forecasted value yields 
\[
    \hat{x}^{(i)}(k) = \left( x^{(i)}(1), x^{(i)}(2), \ldots, x^{(i)}(n) \right)
\]
To improve the accuracy of forecasting models, the Fourier series is used in modifying the residuals in grey forecasting model GM(1,1) so that a Fourier modified grey model (FGM(1,1)) can be obtained.

Step 5: Get the residual series from GM(1,1)

Based on the forecasted series, a residual series is defined as

\[ e_r = \left[ e_r(2), e_r(3), ..., e_r(n) \right]^T \]  \hspace{1cm} (19)

where \( e_r(k) = x(k) - \hat{x}(k) \).

Step 6: Defining Fourier modified grey model (FGM(1,1))

The Fourier series can approximate the residual series as

\[ e_r = \frac{1}{2} a_0 + \sum_{i=1}^{k_f} a_i \cos \left( \frac{i \cdot 2 \pi}{T} k \right) + b_i \sin \left( \frac{i \cdot 2 \pi}{T} k \right) \]  \hspace{1cm} (20)

where \( k = 2, 3, ..., n \), \( T = n - 1 \) and \( k_f = \frac{n-1}{2} - 1 \).

Therefore, the residual series is restated as

\[ e_r = PC \]  \hspace{1cm} (21)

where

\[ P = \begin{bmatrix} \cos \left( \frac{\theta}{T} \right) & \sin \left( \frac{\theta}{T} \right) \\ -\sin \left( \frac{\theta}{T} \right) & \cos \left( \frac{\theta}{T} \right) \end{bmatrix} \]

and \( C = [a_0, a_1, b_1, a_2, b_2, ..., a_{k_f}, b_{k_f}]^T \).

The parameters \( a_0, a_1, b_1, a_2, b_2, ..., a_{k_f}, b_{k_f} \) are obtained through least squares method as follows:

\[ C = (P^T P)^{-1} P^T e_r \]  \hspace{1cm} (22)

While the parameters are computed, the modified residual series is fulfilled as

\[ \hat{e}_r = \frac{1}{2} a_0 + \sum_{i=1}^{k_f} a_i \cos \left( \frac{i \cdot 2 \pi}{T} k \right) + b_i \sin \left( \frac{i \cdot 2 \pi}{T} k \right) \]  \hspace{1cm} (23)

Step 7: Correct original forecast series

The original forecast series of FGM (\( x_f \)) can be corrected as

\[ \hat{x}_f(k) = \hat{x}(0)(k) + \hat{e}_r(k), \quad k = 2, 3, ..., n \]  \hspace{1cm} (24)

Therefore, the control law of (5) is rewritten as

\[ u(k) = u_g(k) + u_{gm}(k) \]  \hspace{1cm} (25)

where the added compensation component is Fourier modified grey control, \( u_{gm} \) that can eliminate the chattering.

\[ u_{gm}(k) = \begin{cases} 0 & \hat{\sigma}(k) < \delta \\ -K \hat{\sigma}(k) \sin \{ \hat{\sigma}(k) \} & \hat{\sigma}(k) \geq \delta \end{cases} \]  \hspace{1cm} (26)

where \( \hat{\sigma}(k) \) represents the forecasted value of \( \sigma(k) \), \( K \) is a constant, and \( \delta \) symbols the system boundary.

4 Experimental results

To evaluate the performance of the proposed control technology, the results of the proposed control technology are compared with the results of the classic SMC. The system parameters are listed in Table 1.

| Table 1. Parameters of the AC power conditioning. |
| :---: | :---: | :---: |
| Filter inductor | \( L = 0.2 \) mH |
| Filter capacitor | \( C = 5 \) \( \mu \)F |
| DC supply voltage | \( V_s = 200 \) V |
| Full resistive load | \( R = 12 \) \( \Omega \) |
| Output voltage and frequency | \( V_o = 110 \) V, \( f_o = 60 \) Hz |
| Switching frequency | \( f_s = 15 \) kHz |

Figure 2 and Fig. 3 show the output voltage and the load current of the AC power conditioning with the proposed control technology and the classic SMC, respectively, under full resistive load. Their output-voltages are close to sinusoidal waveforms. In order to verify the control technology under transient circumstances, step load change with linear resistive load is explored. Figure 4 shows the waveform obtained using the proposed control technology under step load change from no load to full load. Note that the transient behaviour is satisfactory, i.e., the output voltage dip is small and the recovery time is very speedy. On the contrary, the waveform obtained using the classic SMC, displayed in Fig. 5, has a significant voltage dip and a slow recovery time at the firing angle. Under rectifier load shown in Fig. 6, the output-voltage waveform with the proposed control technology is almost sinusoidal (%THD=1.72%), but that with the classic SMC shown in Fig. 7 has a high %THD of 10.51%. Also, Fig. 8 plots the error convergence time and it clearly demonstrates that the proposed control technology does give for reaching \( \sigma(k) = 0 \) in finite time and therefore reduces the distortion of the waveform.
5 Conclusions

This paper describes a high-performance AC power conditioning controlled high-speed micromachining by associating RSF with GFM. Classic SMC is intrinsically robust against internal parameter variations and external disturbances, but it will undergo infinite system-state convergence time. The RSF guarantees finite system-state convergence time and is singularity-free. But, while the system uncertainty bounds are overestimated, the chattering may occur. A computationally fast and algorithmically easy GFM is employed to resolve the chattering problem. Experimental results display that THD, transient response and chattering elimination results from an AC power conditioning under the presented system exceed the results achieved under the SMC system with various loads.

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