STABILITY AND OUTPUT FEEDBACK CONTROL FOR SINGULAR MARKOVIAN JUMP DELAYED SYSTEMS

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Abstract. This paper is concerned with the admissibility analysis and control synthesis for a class of singular systems with Markovian jumps and time-varying delay. The basic idea is the use of an augmented Lyapunov-Krasovskii functional together with a series of appropriate integral inequalities. Sufficient conditions are established to ensure the systems to be admissible. Moreover, control design via static output feedback (SOF) is derived to achieve the stabilization for singular systems. A new algorithm is built to solve the SOF controllers. Examples are given to show the effectiveness of the proposed method.

1. Introduction. Singular systems, which refer to as implicit systems, descriptor systems, differential-algebraic systems, or semi-state systems, are more convenient than state-space systems in the description of physical systems. Since the regularity and impulse-free property need to be checked, the study of singular systems is often much more challenging [27]. Markovian jump systems (MJS), served as a special class of stochastic hybrid systems in which the transitions among different modes are manipulated by a Markovian chain, are widely considered. Application of this kind of systems ranges from manufacturing systems, chemical process, power systems, fault-tolerant systems, target tracking problems, to economic problems [1, 14, 22]. Compared with the singular systems and Markovian jump systems, singular Markovian jump systems (SMJS) are more general, and much attention has been paid on the study of the admissibility and stabilization for this kind of systems.

On the other hand, the stability issue and control design for linear time-delay systems has received considerable attention and various methods have been developed [5, 8–13, 18, 23, 24, 30, 31]. The stability criteria for time-delay systems can

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be obtained based on Lyapunov-Krasovskii functional (LKF) methods. To reduce conservatism of the criteria, two aspects can be focused on. One is to choose an appropriate LKF, such as the augmented LKF \cite{15–17} or the delay-partitioning-based LKF \cite{23,28,29}. The other is to use conservative inequalities to deal with the integrals which appear in the derivative of the LKF. Among various techniques, Jensen inequality \cite{7} is the most popular one. Recently, new inequalities called Wirtinger-based \cite{19} and auxiliary-function-based integral inequalities \cite{15} are proposed which include Jensen inequality as a special case.

Among various control schemes, static output feedback control has been attracted many researchers’ interest due to its low implementation price. But the SOF controller design criteria are bilinear matrix inequalities (BMI) and can not be equivalently transformed into LMIs. In \cite{6}, the cone-complement linearization algorithm is used to solve the SOF control problem for MJSs. An LMI-based iterative method is proposed in \cite{4} for singular systems with time delay. Regarding the finite-time SOF control for MJS, sufficient conditions involving restrict LMIs are presented in \cite{20}. To the best of the authors’ knowledge, no results have been reported on the SOF control for SMJSs with time-varying delay which motivates this study.

In this paper, we focus on the admissibility analysis and SOF control synthesis for a class of singular systems with Markovian jumps and time delay. We assume that the delay may be either constant or time-varying. By using an augmented Lyapunov-Krasovskii functional and linear matrix inequality technique, sufficient conditions are obtained to ensure the SMJSs with time delay to be admissible. Then, the conditions are extended to solve the SOF control design problems. Two sufficient conditions are established and an algorithm is built for the SOF control design.

This paper is organized as follows. Section II is the problem formulation and preliminaries. Section III gives the admissibility criteria. Section IV gives the method of SOF control design for SMJS with time-varying delay. Section V provides several numerical examples to show the merits and efficiency of the results. Section VI concludes this paper.

**Notation.** Throughout this paper, for real symmetric matrices $X$ and $Y$, the notation $X \leq Y$ (respectively, $X < Y$) means that the matrix $X - Y$ is negative semi-definite (respectively, negative definite). $I$ and $0$, respectively, are the identity matrix and zero matrix with appropriate dimensions. The superscripts $-1$ and $T$ denote the matrix inverse and transpose, respectively. The symbol * is used to denote a matrix which can be inferred by symmetry. $E[\cdot]$ and $\| \cdot \|$ are the expectation operator and the Euclidean norm, respectively. For any $A \in \mathbb{R}^{n \times n}$, we define $Sym\{A\} = A + A^T$. $e_i \in \mathbb{R}^{n \times 8n}$ is defined as $e_i = [0_{n \times (i-1)n}] I_n 0_{n \times (8-i)n}$ for $i = 1, 2,...,8$. $\bar{e}_i \in \mathbb{R}^{n \times 5n}$ is defined as $\bar{e}_i = [0_{n \times (i-1)n}] I_n 0_{n \times (5-i)n}$ for $i = 1, 2,...,5$.

2. **Problem formulation and preliminaries.** Consider the following continuous time-delay SMJSs:

$$
\begin{align*}
E \dot{x}(t) &= A(r_1)x(t) + A_d(r_1)x(t - \tau(t)) + B(r_1)u(t), \\
y(t) &= C(r_1)x(t), \\
x(t) &= \phi(t), \quad t \in [-\tau, 0],
\end{align*}
$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, and $y(t) \in \mathbb{R}^s$ is the measured output. The matrix $E$ may be singular, and assume that $rank(E) = r \leq n$. 


The state delay $\tau(t)$ is an unknown time-varying differentiable function which satisfies $0 \leq \tau(t) \leq \tau$ and $\dot{\tau}(t) \leq \mu < 1$, where $\tau \geq 0$ and $\mu$ are known constants.

The stochastic process $r_t, t \geq 0$, taking values in a finite set $\mathcal{S} = \{1, 2, \ldots, s\}$, governs the switching among different system modes with the following mode transition probabilities,

$$P_r\{r_{t+\Delta} = j \mid r_t = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & i \neq j \\ 1 + \pi_{ij}\Delta + o(\Delta), & i = j \end{cases}$$

where $\Delta > 0$, $\lim_{\Delta \to 0} (o(\Delta) / \Delta) = 0$, and $\pi_{jj} \geq 0$, $(i \neq j)$ is the transition rate from mode $i$ at time $t$ to mode $j$ at time $t + \Delta$, and it holds

$$\pi_{ii} = - \sum_{j=1, j \neq i}^s \pi_{ij} \leq 0.$$  

Considering the following unforced time-delay SMJSs without disturbance:

$$\begin{cases} E\dot{x}(t) = A(r_t)x(t) + A_d(r_t)x(t-\tau(t)), \\ x(t) = \phi(t), \ t \in [-\tau, 0], \end{cases}$$

we have the following definitions.

**Definition 2.1.** [27] For a given scalar $\tau > 0$, system (4) is said to be

(i) regular and impulse free if the pairs $(E, A(r_t))$ are regular and impulse-free for every $r_t \in \mathcal{S}$;

(ii) stochastically stable if there exists a scalar $M(r_0, \phi(\cdot))$ such that

$$\lim_{T \to \infty} \mathcal{E}\left[ \int_0^T \|x(t)\|^2 dt \mid r_0, x(s) = \phi(s) \right] \leq M(r_0, \phi(\cdot));$$

(iii) stochastically admissible if it is regular, impulse-free and stochastically stable.

In this paper, we are interested in designing the SOF controller

$$u(t) = Ky(t)$$

such that the resulting closed-loop system

$$\begin{cases} E\dot{x}(t) = (A(r_t) + B(r_t)K(r_t))C(r_t)x(t) + A_d(r_t)x(t-\tau(t)), \\ x(t) = \phi(t), \ t \in [-\tau, 0], \end{cases}$$

is stochastically admissible.

The following lemmas will be useful in the proof of our main results.

**Lemma 2.2.** [15] For a positive definite matrix $R \in \mathbb{R}^{n \times n}$, and a differentiable function $\{x(u) \mid u \in [a, b]\}$, the following inequality holds:

$$\int_a^b x^T(s)R x(s)ds \geq \frac{1}{b-a} \int_a^b x^T(s)ds R \int_a^b x(s)ds + \frac{3}{b-a} \Omega_1^T R \Omega_1 + \frac{5}{b-a} \Omega_2^T R \Omega_2,$$  

(8)
where
\[
\begin{align*}
\Omega_1 &= \int_a^b x(s)ds - \frac{2}{b-a} \int_a^b \int_a^b x(s)dsd\alpha, \\
\Omega_2 &= \int_a^b x(s)ds - \frac{6}{b-a} \int_a^b \int_a^b x(s)dsd\alpha + \frac{12}{(b-a)^2} \int_a^b \int_a^b \int_a^b x(s)dsd\alpha d\beta.
\end{align*}
\]

It should be mentioned that (8) reduces to the Wirtinger-based inequality in [19] when removing the last term \(\frac{3}{b-a} \Omega_2 R \Omega_2\) in (8):
\[
\int_a^b x^T(s)Rx(s)ds \geq \frac{1}{b-a} \int_a^b x^T(s)dsR \int_a^b x(s)ds + \frac{3}{b-a} \Omega_2^T R \Omega_2.
\]

**Lemma 2.3.** [16] For any vectors \(x_1, x_2\), matrices \(R > 0, S\), and real scalars \(\alpha \geq 0, \beta \geq 0\), satisfying \(\alpha + \beta = 1\) and \(\begin{bmatrix} R & S \\ ST & R \end{bmatrix} > 0\), the following inequality holds:
\[
-\frac{1}{\alpha} x_1^T Rx_1 - \frac{1}{\beta} x_2^T Rx_2 \leq - \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} R & S \\ ST & R \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.
\]

**Lemma 2.4.** [3] For matrices \(P, T, U\), and \(A\) with appropriate dimensions and a scalar \(\beta\), the following two inequalities are equivalent
\[
(i) \begin{bmatrix} T \\ \beta PT + UA - \beta U - \beta U^T \end{bmatrix} < 0,
\]
\[
(ii) T < 0, \quad T + A^T PT + PA < 0.
\]

**Lemma 2.5.** [2] For matrices \(P, T, U\), and \(N\) with appropriate dimensions and a scalar \(\beta\), the following two inequalities are equivalent
\[
(i) \begin{bmatrix} T \\ UN - \beta U - \beta U^T + \beta^2 P \end{bmatrix} < 0,
\]
\[
(ii) T < 0, \quad T + N^T PN < 0.
\]

**Lemma 2.6.** (Finsler’s Lemma) [21] Consider real matrices \(\Omega \in \mathbb{R}^{n \times n}\) and \(M \in \mathbb{R}^{m \times n}\) such that \(\Omega = \Omega^T\) and rank\((M) = r < n\). Then the following statements are equivalent:
1. There exists a vector \(\xi \in \mathbb{R}^n\) such that \(\xi^T \Omega \xi < 0\) and \(M \xi = 0\);
2. There exists a scalar \(\sigma \in \mathbb{R}\) such that \(\Omega + \sigma M^T M < 0\);
3. The following condition holds: \(M^T \Omega M < 0\).

3. Stochastically admissibility. Let \(V \in \mathbb{R}^{n \times (n-r)}\) be any matrix with full column rank satisfying \(E^T V = 0\). For System (1) with \(u(t) = 0\), we have the following criterion.

**Theorem 3.1.** For any delay \(\tau(t) > 0\), the time-delay SMJS (1) with \(u(t) = 0\) is stochastically admissible if there exist positive definite matrices \(P_i, R_i, Q_{1i}, Q_{2i}, S_i, X_i, Q, S, X\) and matrices \(G_{1i}, G_{2i}, \Phi_i, M_{1i}, M_{2i}\) such that for every \(i \in S\),
the following inequalities hold:

\[ \Xi_i < 0, \]
\[ \sum_{j=1}^{s} \pi_{ij} Q_{1j} - Q < 0, \]
\[ \sum_{j=1}^{s} \pi_{ij} (Q_{1j} + Q_{2j}) - Q < 0, \]
\[ \sum_{j=1}^{s} \pi_{ij} S_j - S < 0, \]
\[ \sum_{j=1}^{s} \pi_{ij} X_j - X < 0, \]
\[ \dot{X}_i = \begin{bmatrix} \text{diag}\{X_i, 3X_i, 5X_i\} \\ M_{1i} \end{bmatrix} > 0, \]

where

\[ \Xi_i = \text{sym}\{e_1^T E^T P_1 e_2 + \Pi_1^T R_i \Pi_4 + e_1^T \Phi_i V^T e_2 + (G_{1i} e_1 + G_{2i} e_2)^T (-e_2 + A_i e_1 + A_{di} e_3)\} + e_1^T \sum_{j=1}^{s} \pi_{ij} E_j \Pi_3 + e_1^T Q_{1i} e_1 - e_1^T Q_{1i} e_4 + e_1^T Q_{2i} e_1 \]
\[-(1 - \dot{\tau}(t))e_2^T Q_{2i} e_3 + \tau e_1^T Q_{1i} e_1 + \tau e_1^T E^T S E e_1 + \tau^2 e_2^T E^T S E e_1 + \tau^2 e_2^T X e_2 + \tau^2 e_2^T X e_2 - \Pi_1^T \dot{X}_i \Pi_1 \}
\[-\dot{\tau}(t)(e_2^T S e_1 + 3(e_3 - e_7)^T S(e_5 - e_7)) - (\tau - \dot{\tau}(t))(e_2^T S e_1 + 3(e_3 - e_7)^T S(e_5 - e_7)), \]

and

\[ \Pi_1 = \begin{bmatrix} E e_1 - E e_3 \\ E e_1 + E e_3 - 2 e_5 \\ E e_1 - E e_3 + 6 e_5 - 6 e_7 \\ E e_3 - E e_4 \\ E e_3 + E e_4 - 2 e_6 \\ E e_3 - E e_4 + 6 e_6 - 6 e_8 \end{bmatrix}, \]
\[ \Pi_2 = \begin{bmatrix} \tau(t) e_5 \\ (\tau - \dot{\tau}(t)) e_6 \\ \tau(t) e_7 \\ \frac{1}{2} (\tau - \dot{\tau}(t))^2 e_8 \end{bmatrix}, \]
\[ \Pi_3 = \begin{bmatrix} \dot{\tau}(t) e_5 + (\tau - \dot{\tau}(t)) E e_1 - (\tau - \dot{\tau}(t)) e_6 \end{bmatrix}. \]

**Proof.** First, we prove that System (1) with \( u(t) = 0 \) is regular and impulse free. Pre- and postmultiplying the left and right sides of (15) by

\[ \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ and its transpose yields} \]
\[ \begin{bmatrix} \text{sym}\{A^T G_{1i}\} + \sum_{j=1}^{s} \pi_{ij} E^T P_j E \\ + Q_{1i} + Q_{2i} + \tau Q + \tau E^T S E \\ + \frac{1}{2} E^T S E - 9 E^T X e_1 \end{bmatrix} < 0. \]

Since rank(\( E \)) = \( r \leq n \), there exist nonsingular matrices \( G \) and \( H \) such that

\[ \dot{E} = G E H = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \]

and \( G^{-T} V = \begin{bmatrix} 0 \\ U \end{bmatrix} \), where \( U \in \mathbb{R}^{(2n-r) \times (2n-r)} \) is a non-singular matrix. Let

\[ \tilde{A}_i = G A_i H = \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix}, \]
\[ G^{-T} P_i G^{-1} = \begin{bmatrix} P_{i11} & P_{i12} \\ P_{i21} & P_{i22} \end{bmatrix}, \]
\[ H^T \Phi_i = \begin{bmatrix} \Phi_{i11} & \Phi_{i12} \end{bmatrix}, \]

for every \( i \in S \). Pre- and postmultiplying the left and right sides of (18) by \( [H^T \ 0]^T \) and \( [H^T \ 0]^T \), respectively, we have \( \text{sym}\{A_{i22}^T U \Phi_{i2}\} < 0 \), which implies \( A_{i22} \) is nonsingular and thus the pair \((E, A_i)\) is regular and impulse-free for every \( i \in S \).
Then, by Definition 2.1, System (1) with \( u(t) = 0 \) is regular and impulse-free for every \( i \in \mathcal{S} \).

Next, we prove the stochastic stability of System (1) with \( u(t) = 0 \). Consider an LKF as

\[
V(x(t), r_t, t) = \sum_{m=1}^{4} V_m(x(t), r_t, t),
\]

where

\[
\begin{align*}
V_1(x(t), r_t, t) &= x^T(t)E^TP_r(t)Ex(t) + \xi_1^T(t)R_r(t)\xi_1(t) \\
V_2(x(t), r_t, t) &= \int_{t-\tau}^{t} x^T(s)Q_1(r_t)x(s)ds + \int_{t-\tau(t)}^{t} x^T(s)Q_2(r_t)x(s)ds \\
&\quad + \int_{-\tau}^{0} \int_{t+\beta}^{t} x^T(s)Qx(s)d\beta \\
V_3(x(t), r_t, t) &= \int_{-\tau}^{0} \int_{t+\beta}^{t} x^T(s)E^TS(r_t)Ex(s)d\beta \\
&\quad + \int_{-\tau}^{0} \int_{t+\beta}^{t} x^T(s)E^TSEx(s)d\beta \\
V_4(x(t), r_t, t) &= \tau \int_{-\tau}^{0} \int_{t+\beta}^{t} x^T(s)E^TX(r_t)Ex(s)d\beta \\
&\quad + \tau \int_{-\tau}^{0} \int_{t+\beta}^{t} x^T(s)E^TXEx(s)d\beta \\
\end{align*}
\]

and \( \xi_1(t) = \left[ \int_{t-\tau(t)}^{t} (Ex(s))^T ds \int_{t-\tau}^{t} (Ex(s))^T ds \int_{-\tau}^{0} \int_{t+\beta}^{t} (Ex(s))^T d\beta \right]^T \).

Let \( \mathfrak{F} \) be the weak infinitesimal generator of the random process \( \{ (x(t), r_t), t \geq 0 \} \). Then, for every \( i \in \mathcal{S} \), we have

\[
\begin{align*}
\mathfrak{F}V_1(x(t), r_t, t) &= Sym\{x^T(t)E^TP_r(t)Ex(t) + \xi_1^T(t)R_r(t)\xi_1(t)\} \\
&\quad + x^T(t) \sum_{j=1}^{s} \pi_{ij} E^TP_jEx(t) + \xi_1^T(t) \sum_{j=1}^{s} \pi_{ij} R_j \xi_1(t), \\
\mathfrak{F}V_2(x(t), r_t, t) &= x^T(t)Q_1x(t) - x^T(t-\tau)Q_1x(t-\tau) + x^T(t)Q_2x(t) \\
&\quad - (1-\tau(t))x^T(t-\tau(t))Q_2x(t-\tau(t)) \\
&\quad + \tau x^T(t)Qx(t) + \int_{t-\tau}^{t} x^T(s) \left( \sum_{j=1}^{s} \pi_{ij} Q_{1j} - Q \right) x(s)ds \\
&\quad + \int_{t-\tau(t)}^{t} x^T(s) \left( \sum_{j=1}^{s} \pi_{ij} (Q_{1j} + Q_{2j}) - Q \right) x(s)ds, \\
\mathfrak{F}V_3(x(t), r_t, t) &= \tau x^T(t)E^TS_1Ex(t) - \int_{t-\tau}^{t} x^T(s)E^TS_1Ex(s)ds + \frac{\tau^2}{2} x^T(t)E^TSEx(t)
\end{align*}
\]
\[ + \int_{-\tau}^{t} \int_{t+\beta}^{t} x^T(s)E^T \left( \sum_{j=1}^{s} \pi_{ij}S_{j} - S \right) E x(s) ds d\beta, \]

\[ \delta V_4(x(t), r_t, t) \]

\[ = \tau^2 \dot{x}^T(t)E^T X_t \dot{x}(t) - \tau \int_{t-\tau}^{t} \dot{x}^T(s)E^T X_t \dot{x}(s) ds \]

\[ + \frac{\tau^3}{2} \dot{x}^T(s)E^T X E \dot{x}(s) \]

\[ + \tau \int_{-\tau}^{0} \int_{t+\beta}^{t} \dot{x}^T(s)E^T \left( \sum_{j=1}^{s} \pi_{ij}X_{j} - X \right) E \dot{x}(s) ds d\beta, \]

where \( \dot{\xi}_1(t) = \begin{bmatrix} Ex(t) - (1 - \tau(t)) Ex(t - \tau(t)) \\ (1 - \tau(t)) Ex(t - \tau(t)) - Ex(t - \tau) \\ \tau(t) - 1 \int_{0}^{t} Ex(s) ds + \tau(t) Ex(t) \\ -\tau(t) \int_{t-\tau(t)}^{t} Ex(s) ds - \int_{t-\tau(t)}^{t-\tau} Ex(s) ds + (\tau - \tau(t)) Ex(t) \end{bmatrix} \).

By Lemma 2.2, the integral term in (22) can be expanded as

\[ - \int_{t-\tau}^{t} x^T(s)E^T S_t E x(s) ds \]

\[ = - \int_{t-\tau(t)}^{t} x^T(s)E^T S_t E x(s) ds - \int_{t-\tau(t)}^{t} x^T(s)E^T S_t E x(s) ds \]

\[ \leq - \frac{1}{\tau - \tau(t)} \left( \int_{t-\tau(t)}^{t} x^T(s)E^T ds \right) \int_{t-\tau(t)}^{t} E x(s) ds - \frac{2}{\tau - \tau(t)} \int_{t-\tau(t)}^{t} \int_{t+\beta}^{t} x^T(s)E^T ds d\beta \right) \times \]

\[ \int_{t-\tau}^{t} E x(s) ds - \frac{2}{\tau - \tau(t)} \int_{t-\tau}^{t} \int_{t+\beta}^{t} E x(s) ds d\beta \right) \]

\[ - \frac{1}{\tau(t)} \left( \int_{t-\tau(t)}^{t} x^T(s)E^T ds \right) \int_{t-\tau(t)}^{t} E x(s) ds \]

\[ - \frac{2}{\tau(t)} \int_{t-\tau(t)}^{t} \int_{t+\beta}^{t} E x(s) ds d\beta \right) \times \]

\[ \int_{t-\tau(t)}^{t} E x(s) ds - \frac{2}{(\tau(t))} \int_{t-\tau(t)}^{t} \int_{t+\beta}^{t} E x(s) ds d\beta \right) \]

\[ - \tau \int_{t-\tau}^{t} \dot{x}^T(s)E^T X_t X \dot{x}(s) ds \]

\[ = - \tau \int_{t-\tau(t)}^{t} \dot{x}^T(s)E^T X_t X \dot{x}(s) ds - \tau \int_{t-\tau(t)}^{t} \dot{x}^T(s)E^T X_t X \dot{x}(s) ds \]

\[ \leq - \tau \left( \frac{1}{\tau(t)} (x(t) - x(t - \tau(t)) \tau) E^T X_t E(x(t) - x(t - \tau(t))) \right) \]
\[ +3 \left( x(t) + x(t - \tau(t)) - \frac{2}{\tau(t)} \int_{t-\tau(t)}^{t} x(s) ds \right)^T E^T X_1 E \times \\
\left( x(t) + x(t - \tau(t)) - \frac{2}{\tau(t)} \int_{t-\tau(t)}^{t} x(s) ds \right) \\
+5 \left( x(t) - x(t - \tau(t)) + \frac{6}{\tau(t)} \int_{t-\tau(t)}^{t} x(s) ds \right) \\
- \frac{12}{\tau(t)^2} \int_{\tau(t)}^{0} \int_{t+\beta}^{t} x(s) ds d\beta \right)^T E^T X_1 E \times \\
\left( x(t - \tau(t)) + x(t - \tau(t)) - \frac{2}{\tau - \tau(t)} \int_{t-\tau}^{t-\tau(t)} x(s) ds \right) \\
+5 \left( x(t - \tau(t)) - x(t - \tau(t)) + \frac{6}{\tau - \tau(t)} \int_{t-\tau}^{t-\tau(t)} x(s) ds \right) \\
- \frac{12}{(\tau - \tau(t))^2} \int_{-\tau}^{-\tau(t)} \int_{t+\beta}^{t} x(s) ds d\beta \right)^T E^T X_1 E \times \\
\left( x(t - \tau(t)) + x(t - \tau(t)) - \frac{2}{\tau - \tau(t)} \int_{t-\tau}^{t-\tau(t)} x(s) ds \right) \\
+5 \left( x(t - \tau(t)) - x(t - \tau(t)) + \frac{6}{\tau - \tau(t)} \int_{t-\tau}^{t-\tau(t)} x(s) ds \right) \\
- \frac{12}{(\tau - \tau(t))^2} \int_{-\tau}^{-\tau(t)} \int_{t+\beta}^{t} x(s) ds d\beta \right) \right) \\
\leq -\xi_2^T(t) X_1 \xi_2(t), \quad (25) \]

where

\[ \xi_2(t) = \left[ (x(t) - x(t - \tau(t)))^T E^T, \ (x(t) + x(t - \tau(t)))^T E^T - \frac{2}{\tau(t)} \int_{t-\tau(t)}^{t} x(s) ds, \\
(x(t) - x(t - \tau(t)))^T E^T + \frac{6}{\tau(t)} \int_{t-\tau(t)}^{t} x(s) ds \right]^T E^T d\beta, \\
(x(t) - x(t - \tau(t)))^T E^T - \frac{12}{\tau(t)^2} \int_{-\tau(t)}^{0} \int_{t+\beta}^{t} x(s) ds d\beta, \ (x(t - \tau(t)) + x(t - \tau(t)))^T E^T - \frac{2}{\tau - \tau(t)} \int_{t-\tau(t)}^{t-\tau(t)} x(s) ds, \ (x(t - \tau(t)) - x(t - \tau(t)))^T E^T + \frac{6}{\tau - \tau(t)} \int_{t-\tau(t)}^{t-\tau(t)} x(s) ds \right]^T E^T d\beta \right]^T, \text{ and } (17) \text{ holds.} \]

Noting that

\[ Sym\{x^T(t)\Phi_1 V^T E \dot{x}(t)\} = 0, \quad (26) \]

and

\[ Sym\{(G_1 x(t) + G_2 E \dot{x}(t))^T (-E \dot{x}(t) + A_1 x(t) + A_2 x(t - \tau(t)))\} = 0. \quad (27) \]

Combining Eqs. (20)-(27), we obtain

\[ \mathcal{V}(x(t), r_1, t) < \xi_2^T(t) \Xi_4(t), \text{ for } i \in S, \quad (28) \]
and (16) hold, where
\[\xi_i(t) = \left[ x^T(t), \dot{x}^T(t), x^T(t-\tau(t)), x^T(t-\tau), \frac{1}{\tau-	au(t)} \int_{t-\tau(t)}^{t} (Ex(s))^T ds, \right.\]
\[\frac{1}{\tau-	au(t)} \int_{t-\tau(t)}^{t} (Ex(s))^T ds, \frac{2}{\tau-\tau(t)} \int_{t-\tau(t)}^{t} (Ex(s))^T dsd\beta, \]
\[\frac{2}{\tau-\tau(t)} \int_{t-\tau(t)}^{t} (Ex(s))^T dsd\beta \right]^T.\]

It follows from (28) that there exists a scalar \( \lambda > 0 \) such that for every \( i \in \mathcal{S} \), \( \mathcal{F}V(x(t), r_t, t) \leq -\lambda \| x(t) \|^2 \). Hence, by Dynkin’s formula, we obtain
\[\mathcal{E}[\mathcal{F}V(x(t), i, t)] - \mathcal{E}[\mathcal{F}V(x(\tau), r_{\tau}, \tau)] \leq -\lambda \mathcal{E} \left[ \int_{\tau}^{t} \| x(s) \|^2 ds \right],\]
for any \( t \geq \tau \), which yields \( \mathcal{E}[\int_{\tau}^{t} \| x(s) \|^2 ds \leq \lambda^{-1} \mathcal{E}[V(x(\tau), r_{\tau}, \tau)] \). Using a procedure similar to the proof as in [26], it is clear that there exists a scalar \( \bar{\beta} \) such that
\[\mathcal{E} \left[ \int_{0}^{t} \| x(s) \|^2 ds \right] = \mathcal{E} \left[ \int_{0}^{T} \| x(s) \|^2 ds \right] + \mathcal{E} \left[ \int_{\tau}^{t} \| x(s) \|^2 ds \right] \leq \bar{\beta} \mathcal{E}[\| \phi \|_2^2]. \quad (29)\]

Considering this and Definition 2.1, System (1) with \( u(t) = 0 \) is stochastically admissible.

Moreover, in case of constant delay, System (1) with \( u(t) = 0 \) reduces to the following form:
\[
\begin{cases}
E\dot{x}(t) = A(r_t)x(t) + A_d(r_t)x(t-\tau), \\
x(t) = \phi(t), \quad t \in [-\tau, 0].
\end{cases}
\]

Considering the LKF
\[V(x(t), r_t, t) = \sum_{m=1}^{4} V_m(x(t), r_t, t), \quad (31)\]
where
\[
\begin{align*}
V_1(x(t), r_t, t) &= x^T(t)E^T P(r_t) Ex(t) + \tilde{\xi}_1^T(t) R(r_t) \tilde{\xi}_1(t), \\
V_2(x(t), r_t, t) &= \int_{t-\tau}^{t} x^T(s)Q(r_t)x(s)ds + \int_{t-\tau}^{t} \int_{t-\tau}^{t} x^T(s)Qx(s)d\beta, \\
V_3(x(t), r_t, t) &= \int_{t-\tau}^{t} \int_{t-\tau}^{t} x^T(s)E^T S(r_t) Ex(s)d\beta, \\
&\quad + \int_{t-\tau}^{t} \int_{t-\tau}^{t} x^T(s)E^T Sx(s)d\beta d\beta, \\
V_4(x(t), r_t, t) &= \tau \int_{t-\tau}^{t} \int_{t-\tau}^{t} \dot{x}^T(s)E^T X(r_t)Ex(s)d\beta, \\
&\quad + \tau \int_{t-\tau}^{t} \int_{t-\tau}^{t} \dot{x}^T(s)E^T XEx(s)d\beta d\beta
\end{align*}
\]
and \( \tilde{\xi}_1(t) = \left[ \int_{t-\tau}^{t} (Ex(s))^T ds \int_{t-\tau}^{t} \int_{t-\tau}^{t} (Ex(s))^T dsd\beta \right]^T \).

By following the proof similar to that of Theorem 3.1, we can obtain the following corollary.

**Corollary 1.** For constant delay \( \tau > 0 \), the time-delay SMJS (30) is stochastically admissible if there exist positive definite matrices \( P_i, R_i, Q_i, S_i, X_i, Q, S, X \) and
matrices $G_{1i}$, $G_{2i}$, $\Phi_i$ such that for every $i \in S$, the following inequalities hold:

\[
\mathcal{Z}_i < 0,
\]

\[
\sum_{j=1}^s \pi_{ij} Q_j - Q < 0, \quad \sum_{j=1}^s \pi_{ij} S_j - S < 0,
\]

\[
\sum_{j=1}^s \pi_{ij} X_j - X < 0,
\]

where

\[
\mathcal{Z}_i = \text{sym}\{\tilde{e}_t^T P_i \tilde{e}_2 + \bar{\Pi}_3^T R_i \bar{\Pi}_4 + \tilde{e}_t^T \Phi_i V^T \tilde{e}_2 + (G_{1i} \tilde{e}_1 + G_{2i} \tilde{e}_2)^T (-\tilde{e}_2 + A_i \tilde{e}_1 + A_{di} \tilde{e}_3)\} + \tilde{e}_t^T \sum_{j=1}^s \pi_{ij} P_j \bar{e}_1 + \bar{\Pi}_3^T \sum_{j=1}^s \pi_{ij} R_j \bar{\Pi}_4 + \tilde{e}_t^T Q_i \tilde{e}_1 - \tilde{e}_3^T Q_i \tilde{e}_3 + \tau \tilde{e}_t^T S_i \tilde{e}_1 + \frac{\tilde{e}_t^T}{\tau^2} \tilde{e}_t^T S_i \tilde{e}_1 + \tilde{e}_t^T X_i \tilde{e}_2 + \frac{\tilde{e}_t^T}{\tau^2} X_i \tilde{e}_2 - (E \tilde{e}_1 - E \tilde{e}_3)^T X_i (E \tilde{e}_1 - E \tilde{e}_3) - 3(E \tilde{e}_1 + E \tilde{e}_3 - 2 \tilde{e}_4)^T X_i (E \tilde{e}_1 + E \tilde{e}_3 - 2 \tilde{e}_4) - 5(E \tilde{e}_1 - E \tilde{e}_3 + 6 \tilde{e}_4 - 6 \tilde{e}_5)^T X_i (E \tilde{e}_1 - E \tilde{e}_3 + 6 \tilde{e}_4 - 6 \tilde{e}_5) - \tau (\tilde{e}_t^T S_i \tilde{e}_4 + 3(\tilde{e}_4 - \tilde{e}_5))^T S_i (\tilde{e}_4 - \tilde{e}_5), \quad \text{and}
\]

\[
\bar{\Pi}_3 = \begin{bmatrix} \tau \tilde{e}_4 & \frac{\tilde{e}_t^T}{\tau^2} \tilde{e}_5 \end{bmatrix}, \quad \bar{\Pi}_4 = \begin{bmatrix} E \tilde{e}_1 - E \tilde{e}_3 & 0 \\ 0 & -\tau \tilde{e}_4 + \tau E \tilde{e}_1 \end{bmatrix}.
\]

**Remark 1.** The LKF in this paper is composed of non-integral, single integral, double integral, and triple integral terms which can be regarded as an augmented LKF. However, the quadruple integral term is not included in the LKF which is simpler than the LKF in [18]. It is worth mentioning that a new non-integral term $\xi_i^T(t) R_i(t) \xi_i(t)$ is introduced in the LKF. Moreover, inequality (8) is proved to be less conservative than Wirtinger-based inequality in [19] and Jensen inequality. Lemma 2.3 brings some adjustable slack variables into the inequality. So one can easily come to the conclusion that less conservative stability criteria can be derived by using these inequalities.

**Remark 2.** The augmented LKF brings the term depending on $\tau(t)$ which introducing more degrees of freedom. Also, the derivative of the LKF contains some terms about $\tau(t)$ and $\dot{\tau}(t)$ which can not be solved by an LMI toolbox. By convex combination theorem, we can come to the conclusion that $\mathcal{Z}_i < 0$ is equivalent to the following inequalities:

\[
\mathcal{Z}_i |_{\tau(t)=0, \dot{\tau}(t)=0} < 0, \quad \mathcal{Z}_i |_{\tau(t)=\tau, \dot{\tau}(t)=0} < 0, \quad \mathcal{Z}_i |_{\tau(t)=0, \dot{\tau}(t)=\mu} < 0, \quad \mathcal{Z}_i |_{\tau(t)=\tau, \dot{\tau}(t)=\mu} < 0.
\]

4. **Static output-feedback stabilization.** In this section, we will derive the stabilization criterion for System (1) on the basis of Theorem 3.1, and give the algorithms to solve the related nonlinear matrix inequalities in the stabilization criterion.

**Theorem 4.1.** The time-delay SMJS (1) is stochastically admissible with mixed $H_\infty$ and passivity performance $\gamma$ if there exist positive definite matrices $P_i$, $R_i$, $Q_{1i}$, $Q_{2i}$, $S_i$, $X_i$, $Q_i$, $S_i$, $X_i$ and matrices $K_i$, $G_{1i}$, $G_{2i}$, $\Phi_i$, $M_{1i}$, $M_{2i}$ such that for every $i \in S$, the following inequalities hold:

\[
\Xi_i + \Xi_{1i} < 0,
\]

\[
\sum_{j=1}^s \pi_{ij} Q_{1ij} - Q < 0, \quad \sum_{j=1}^s \pi_{ij} (Q_{1j} + Q_{2j}) - Q < 0, \quad \sum_{j=1}^s \pi_{ij} S_j - S < 0,
\]
\[ \sum_{j=1}^{s} \pi_{ij} X_j - X < 0, \quad (35) \]
\[ \dot{X}_i > 0, \quad (36) \]

where
\[ \Xi_{ii} = \text{sym}\{ (G_{1i} e_1 + G_{2i} e_2)^T (B_i K_i C_i e_1) \}, \]
and \( \Xi_i, \Pi_1, \Pi_3, \Pi_4, \bar{X}_i \) are defined in Theorem 3.1.

Proof. By replacing \( A_i \) with \( A_i + B_i K_i C_i \) in inequality (15) of Theorem 3.1, the proof can be easily completed. \( \square \)

It can be seen in Theorem 4.1 that \( \Xi_i \) is a linear term and \( \Xi_{ii} \) is bilinear. The following theorem will present a new linearization method to solve the above bilinear conditions.

**Theorem 4.2.** The time-delay SMJS (1) is stabilizable via static output feedback if there exist positive definite matrices \( P_i, R_i, Q_{1i}, Q_{2i}, S_i, X_i, Q, S, X, \) matrices \( G_{1i}, G_{2i}, \Phi_i, M_{1i}, M_{2i}, J_i, L_i, \) and scalars \( \sigma_{1i}, \sigma_{2i} \) such that for every \( i \in S \), the following inequalities hold:

\[ \Xi_i - \sigma_{1i} e_i^T C_i^T C_i e_1 < 0, \quad (37) \]
\[ \Gamma_i < 0, \quad (38) \]
\[ \sum_{j=1}^{s} \pi_{ij} Q_{ij} - Q < 0, \quad \sum_{j=1}^{s} \pi_{ij} (Q_{1ij} + Q_{2ij}) - Q < 0, \quad \sum_{j=1}^{s} \pi_{ij} S_j - S < 0, \quad (39) \]
\[ \sum_{j=1}^{s} \pi_{ij} X_j - X < 0, \quad \bar{X}_i > 0, \quad (40) \]

where
\[
\Xi_i = \begin{bmatrix}
\Xi_i - \text{sym}\{ (B_i^T (G_{1i} e_1 + G_{2i} e_2))^T Y_i \} & * & * \\
+ (B_i^T (e_1 + e_2))^T L_i \} + \sigma_{2i} Y_i^T Y_i & L_i + J_i^T (B_i^T (e_1 + e_2)) + \sigma_{2i} Y_i & - J_i - J_i^T & * \\
- (B_i^T (G_{1i} e_1 + G_{2i} e_2)) & 0 & - J_i - J_i^T + \sigma_{2i} I \\
\end{bmatrix}, \quad \Gamma_i = \begin{bmatrix}
0 \\
\end{bmatrix}, \quad Y_i, i \in S \]

are fixed matrices, and \( \Xi_i, \bar{X}_i \) are defined in Theorem 3.1.

Proof. By Lemma 2.5, (38) is transformed into

\[
\Xi_i - \text{sym}\{ (B_i^T (G_{1i} e_1 + G_{2i} e_2))^T Y_i \} + (B_i^T (e_1 + e_2))^T L_i \} + \sigma_{2i} Y_i^T Y_i + \sigma_{2i} (J_i^T L_i) + J_i^T L_i \}
\]
\[
L_i + J_i^T (B_i^T (e_1 + e_2)) + \sigma_{2i} Y_i - J_i - J_i^T \\
- B_i^T (G_{1i} e_1 + G_{2i} e_2) < 0. \quad (41) \]

Then, by Lemma 2.4, if (41) is satisfied, the following inequality holds.

\[ \Xi_i - \text{sym}\{ (B_i^T (G_{1i} e_1 + G_{2i} e_2))^T \hat{Y}_i \} + \sigma_{2i} \hat{Y}_i^T \hat{Y}_i < 0, \quad (42) \]

where \( \hat{Y}_i = Y_i + J_i^T L_i \). Letting \( \tilde{\sigma}_2 = \frac{1}{\sigma_{2i}} \), and noting that

\[ - \text{sym}\{ (B_i^T (G_{1i} e_1 + G_{2i} e_2))^T \hat{Y}_i \} + \sigma_{2i} \hat{Y}_i^T \hat{Y}_i \]
\[ \geq - \tilde{\sigma}_2 (B_i^T (G_{1i} e_1 + G_{2i} e_2))^T (B_i^T (G_{1i} e_1 + G_{2i} e_2)), \quad (43) \]
\(\Xi_i - \tilde{\sigma}_2(B_i^T (G_{1i}e_1 + G_{2i}e_2))^T (B_i^T (G_{1i}e_1 + G_{2i}e_2)) < 0.\) \(\text{(44)}\)

According to \((37), (44)\) and Lemma 2.6, we obtain
\[\Xi_i + \Xi_{11i} < 0.\] \(\text{(45)}\)

**Remark 3.** In Theorems 4.2, inequality \((34)\) in Theorem 4.1 is equivalently transformed into two inequalities \((37)\) and \((44)\) by Lemma 2.6. \((44)\) is nonlinear and needs to be treated. By introducing new slack matrices \(\tilde{Y}_i\), the nonlinear term \(-\tilde{\sigma}_2(B_i^T (G_{1i}e_1 + G_{2i}e_2))^T (B_i^T (G_{1i}e_1 + G_{2i}e_2))\) is represented by two terms in \((33)\). \(\tilde{Y}_i\) is divided into two parts \(Y_i + J_i^{-T}L_i\), where \(Y_i\) is fixed, and the terms in \((33)\) which contain \(J_i^{-T}L_i\) are treated by Lemmas 2.4 and 2.5. The linearization method in this paper is less conservative than that in \([4]\) in which \(Y_i\) is fixed.

By Theorem 4.1 and 4.2, the following algorithm for solving SOF control problem for System \((1)\) is obtained.

**Algorithm 1**

Step 1 Set \(l = 1, \tau = \tau_0, \gamma = \gamma_0, \theta = \theta_0,\) and \(Y_i = 0.\)

Step 2 Solve the following optimization problem with respect to \(P_i, R_i, Q_{1i}, Q_{2i}, S_i, X_i, Q, S, X, G_{1i}, G_{2i}, \Phi_i, M_{1i}, M_{2i}, J_i, L_i,\) and scalars \(\sigma_{1i}, \sigma_{2i}\) for every \(i \in S.\)

\[\text{OP1: } \min \alpha\]
\[\text{s.t. } P_i > 0, R_i > 0, Q_{1i} > 0, Q_{2i} > 0, S_i > 0, X_i > 0,\]
\[Q > 0, S > 0, X > 0,\]
\[\Gamma_i < \alpha I\]
\[\text{and } (37), (39), (40) \text{ hold.}\]

Step 3 Solve the resultant LMIs in Theorem 4.1. If the LMIs are feasible, stop, and \(K_i, i \in S\) are the stabilizing SOF gains.

Step 4 Set \(l = l + 1.\) If \(l < l_0,\) where \(l_0\) is the maximum number of iterations allowed, then let \(Y_i = Y_i + J_i^{-T}L_i,\) and go to Step 2. Else, stop with conclusion that our method fails finding out a stabilizing SOF gain.

5. **Numerical example.** In this section, two examples are given. Example 1 is with respect to the SOF controller design problems for SMJS with time delay.

**Example 1.** Consider the SMJS \((1)\) with two modes, e.g., \(S = \{1, 2\}\) and parameters

\[E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} -3 & 1 & 0 \\ 0.3 & -2.5 & -4 \\ -0.1 & 0.3 & -3.8 \end{bmatrix}, A_2 = \begin{bmatrix} -2.5 & 0.5 & -0.1 \\ 0.1 & -3.5 & 0.3 \\ -0.1 & 1 & -2 \end{bmatrix},\]

\[A_{d1} = \begin{bmatrix} -0.1669 & 0.0802 & 1.682 \\ -0.8162 & -0.9373 & 0.5936 \\ 2.0942 & 0.6357 & 0.7902 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.1053 & -0.1948 & -0.6855 \\ -0.1586 & 0.0755 & -0.2684 \\ 0.8709 & -0.5266 & -1.1883 \end{bmatrix},\]

\[B_1 = \begin{bmatrix} 1.5 \\ 1.0 \\ 1.0 \end{bmatrix}, B_2 = \begin{bmatrix} 1.0 \\ 0.5 \\ 1.5 \end{bmatrix}, C_1 = C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.\]
The mode switching is governed by the rate matrix

\[
P = \begin{bmatrix}
-0.8 & 0.8 \\
0.3 & -0.3
\end{bmatrix}.
\]

According to Corollary 1, it can be shown that the system is stochastically admissible for any constant time delay \( \tau \) satisfying \( 0 < \tau < 3.91 \) which is less conservative than the result in [23].

Next, we will consider the case of time-varying delay SOF control problems. When \( \tau = 1.5 \) and \( \mu = 0.3 \), the corresponding static output feedback controller gain are solved as \( K_1 = [-8.1305, -2.5741] \) and \( K_2 = [-0.9463, -1.4771] \). Fig. 1 shows the state response of \( x(t) \) with the obtained controller gains and Fig. 2 shows the possible jumping between modes during simulation.

**Figure 1.** The closed-loop response curves in Example 1

**Figure 2.** Jumping modes
When $E = I$, System (1) reduces to a regular system. In order to compare our admissibility results with the methods in [18,23,26], the following example is given.

**Example 2.** Consider the SMJS (1) with two modes, e.g., $S = \{1,2\}$ and $E = I$,

$$A_1 = \begin{bmatrix} -3.5 & 0.8 \\ -0.6 & -3.3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2.5 & 0.3 \\ 1.4 & -0.1 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} -0.9 & -1.3 \\ -0.7 & -2.1 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -2.8 & 0.5 \\ -0.8 & -1.0 \end{bmatrix}.$$  

The mode switching is governed by the rate matrix

$$\Pi = \begin{bmatrix} -0.4 & 0.4 \\ 0.8 & -0.8 \end{bmatrix}.$$  

The upper delay bounds obtained in papers [18,23,26] are listed in Table 1 together with maximum allowable upper bounds for constant delay $\tau$ with various $\pi_{11}$ obtained in this paper. It is clear that the delay-dependent conditions obtained in Corollary 1 yield better results than those obtained in [18,23,26]. Next, we consider the same parameters described above for the time-varying delays. Assume that $\mu = 0.3$. by applying Theorem 3.1, we obtain that the System (1) is stochastically stable for the upper bound of time-varying delay satisfying $\tau \leq 0.8894$.

### Table 1. Maximum allowable upper bounds of time delay $\tau$ for Example 1.

| $\pi_{11}$ | -0.4 | -0.55 | -0.7 | -0.85 | -1.00 |
|------------|------|-------|------|-------|-------|
| [26]       | 0.6078 | 0.5894 | 0.5768 | 0.5675 | 0.5603 |
| [23]       | 0.6322 | 0.6120 | 0.5981 | 0.5881 | 0.5805 |
| [18]       | 0.8181 | 0.7815 | 0.7597 | 0.7473 | 0.7377 |
| Corollary 1 | 0.9874 | 0.9312 | 0.8944 | 0.8684 | 0.8491 |

6. **Conclusion.** The stochastical admissibility and SOF control problems for singular Markovian jump systems with time delay have been discussed in this paper. Based on an augmented Lyapunov-Krasovskii functional and several integral inequalities, new delay-dependent sufficient conditions for the stochastically admissibility for SMJS are obtained. Furthermore, the conditions are extended to deal with the problems of static output feedback control. Two sufficient conditions are presented and an algorithm is built. Numerical examples demonstrate the effectiveness of the admissibility criterion and SOF stabilization method.

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