Chiral logic computing with twisted antiferromagnetic magnon modes

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Antiferromagnetic (AFM) materials offer an exciting platform for ultrafast information handling with low cross-talks and compatibility with existing technology. Particularly interesting for low-energy cost computing is the spin wave-based realization of logic gates, which has been demonstrated experimentally for ferromagnetic waveguides. Here, we predict chiral magnonic eigenmodes with a finite intrinsic, magnonic orbital angular momentum $\ell$ in AFM waveguides. $\ell$ is an unbounded integer determined by the spatial topology of the mode. We show how these chiral modes can serve for multiplex AFM magnonic computing by demonstrating the operation of several symmetry- and topology-protected logic gates. A Dzyaloshinskii–Moriya interaction may arise at the waveguide boundaries, allowing coupling to external electric fields and resulting in a Faraday effect. The uncovered aspects highlight the potential of AFM spintronics for swift data communication and handling with high fidelity and at a low-energy cost.

**RESULTS AND DISCUSSION**

**Chiral antiferromagnetic magnons**

As a magnonic waveguide we consider a cylindrical wire made of a G-type AFM$^{29}$ with an axis aligned along with the AFM easy $(e_z)$ direction (cf. Fig. 1). In practice, the waveguide can be deposited or imprinted on a substrate or be part of integrated magnonic circuits. For capturing the low-energy AFM dynamics, it is adequate to start from the Heisenberg Hamiltonian

$$\mathcal{H} = \frac{J}{2} \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} - \frac{K_z}{2} \sum_i (\mathbf{S}_i \cdot e_z)^2, \quad (1)$$

where $J > 0$ stands for the strength of the (uniform) AFM exchange coupling between neighboring spins $\mathbf{S}_i$ localized at lattice sites $i$ with a lattice distance $a$. The uniaxial anisotropy energy $K_z$ is unbounded and associated with the spatial topology or the "twist" of the eigenmode extending over the whole waveguide. We demonstrate how the twisted modes serve for realizing a class of symmetry-protected logic gates and for multiplex data transfer. The operation of the gates is robust, for the twisted beams are found as eigenmodes of the waveguide with a dispersion allowing for forming signals as fast as AFM magnonics. Waveguide boundaries can host a Dzyaloshinskii–Moriya$^{27,28}$ interaction (DMI) due to the break of inversion symmetry. We find DMI is useful for triggering and steering twisted modes. The analytic predictions are ubiquitous, meaning that twisted AFM magnons should appear in conventional AFM, SyAF, or van-der-Waals-AF cylindrical waveguides. We present a general theory and demonstrate with full-numerical simulations the character and the functionalization of twisted modes for a prototypical NiO AFM waveguide for a demonstration.

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contribution (with strength $\alpha K_z > 0$) is large enough to suppress quantum fluctuations enforcing so a collinear (Néel) ground state aligned with the easy axis (cf. Fig. 1). The spin dynamics follows Heisenberg’s equation of motion $d\mathbf{S}_i/dt = -i/\hbar \mathcal{H}_i$ where $\mathcal{H} = \mathbf{S} \times (K_S \mathbf{S} - \sum_j J_s \mathbf{S}_j)$. Low-energy excitations, meaning AFM spin waves are described by introducing the classical dimensionless unit vector field $\mathbf{s}_i = \mathbf{S}/S$, where $|\mathbf{s}_i| = S$ for all sites. The spin waves are then transversal excitations propagating according to

$$ih \frac{d\mathbf{s}_i}{dt} = -\left(K_S \mathbf{s}_i - \sum_{j \neq i} J_s \mathbf{s}_j\right) \mathbf{s}_i^\perp - \sum_{j \neq i} J_s \mathbf{s}_j^\perp,$$

with $s_i^\perp := s_i^\parallel \pm i s_i^\parallel$, $K_S = K_S h$, $J_s = J_S h$ are respectively the anisotropy and exchange energies. Using a plane-wave ansatz $s_i^\parallel(t) \sim e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$, one infers the dispersion relation for spin waves as $E_k^\parallel = \hbar^2 k^2 = (K_S + \xi_J)^2 - (\xi_J \mathbf{v}_k)^2$ where $\xi$ is the coordination number ($\xi = 2, 4, 6$ for respectively one-dimensional spin chain, two-dimensional square lattice, and a three-dimensional cubic lattice). $v_k = \sum_{j \neq i} e^{i \mathbf{k} \cdot \mathbf{r}_j} / \xi$. The same dispersion follows from a Holstein–Primakoff approach [30,31].

**AFM magnonic Dirac dynamics**

The physics of AFM low-energy excitations is most transparent upon a mapping onto two sub-lattices denoted A and B with antiparallel spins (cf. Fig. 1a). The ordering in each sublattice is FM but a translation by a lattice vector transforms $\mathbf{S}(\mathbf{r} + \mathbf{a}) \rightarrow -\mathbf{S}(\mathbf{r})$, meaning that the translational invariance is broken. Thus, the AFM Heisenberg system is mappable onto an antiferromagnetic (AFM) CP model [36]. Note, the AFM Hamiltonian is still invariant under the combined time-reversal ($\mathcal{T}$) and sub-lattice exchange ($\mathcal{I}$), a fact underlying the degeneracy of the two chiral magnon modes.

Equation (1) is, in addition, invariant under global spin rotation around the z-axis, and thus the z-component of the total spin is a good quantum number. For a further insight, let us follow Haldane [32] (see also [33,35,36]) and consider small sublattice-dependent fluctuations around the one-dimensional Néel ground state. To do so, one introduces $\mathbf{s}_i = (a_+ + \sqrt{1 - a_+^2} \mathbf{e}_x)$ and $\mathbf{s}_0 = (b_+ + \sqrt{1 - b_+^2} \mathbf{e}_x)$, where $|a_+| \ll 1$ and $|b_+| \ll 1$. Linearizing Eq. (2) and noting that $a_+ = a_+ + (r_j \cdot \nabla) a_+ + i/2 \mathbf{e}_z \nabla^2 a_+ + \ldots$ (and proceeding similarly for $b_+$) we arrive at the continuum Hamiltonian (valid up to the second-order derivatives)

$$ih \frac{d\psi^\parallel}{dt} = \mathcal{H}_D \psi^\parallel,$$

where $\psi^\parallel(z,t) = (a_+^\parallel, b_+^\parallel)^T$ is the two-component Dirac spinor, and $(\partial_z := 2a \partial / \partial z)$

$$\mathcal{H}_D = \begin{bmatrix} K_S + 2J_S & 2J_S - J_S \partial_z + J_S \partial_z^2 / 2 \\ -2J_S - J_S \partial_z - J_S \partial_z^2 / 2 & -K_S - 2J_S \end{bmatrix}.$$

The first-order derivative ($-J_S \partial_z$) in $\mathcal{H}_D$ is a parity-breaking term and as such is not invariant under sublattice exchange (A ↔ B) in the G-type AFMs. This term introduces a finite precessional phase difference between the sublattice A and B. In the long-wavelength limit of plane-wave ansatz, the eigenenergies of the Dirac Hamiltonian Eq. (3) are $E_k = \pm \left[\xi_J^2 k_z^2 + K_S (K_S + 4J_S)\right]^{1/2}$ with the corresponding eigenmodes

$$\psi_+^\parallel = \begin{bmatrix} \cosh \frac{\vartheta}{2} \\ -2e^{i\vartheta} \sinh \frac{\vartheta}{2} \end{bmatrix} \quad \text{and} \quad \psi_-^\parallel = \begin{bmatrix} \sinh \frac{\vartheta}{2} \\ e^{i\vartheta} \cosh \frac{\vartheta}{2} \end{bmatrix},$$

where $\cosh \theta = (K_S + 2J_S)/|E_k|$ and $\tan \phi = J_S k_z/(2J_S - J_S k_z^2/2)$ [37].

**Fig. 1** Schematic representation of twisted magnon beams in AFM waveguide. a Twisted magnon beams propagating along a cylindrical AFM waveguide with two magnetic sublattices A (up arrows) and B (down arrows). b Degenerate right- and left-handed spin wave modes, |L⟩ and |R⟩ dominated by the sublattice A and B, respectively. The small magnetization (green arrows) is defined as $\mathbf{m} = (\mathbf{S}_A + \mathbf{S}_B)/2S$. 
ψ_ℓ (ψ_j) with eigenfrequency ω_ℓ > 0 (ω_j < 0) describes the left-circularly (right-circularly) polarized modes dominated by the precession in A (B) sublattice, as illustrated in Fig. 1b. The opposite holds true for the complex conjugate ψ_ℓ^* = (a^*_ℓ, b^*_j)^T. In the following, we take the positive eigenfrequency modes, |L⟩ ≡ ψ_ℓ and |R⟩ ≡ ψ_j as the chirally complete basis for AFM magnons. Although Λ_0 is not Hermitian, the chiral basis can be easily normalized by applying a momentum-dependent factor, such that (L|L)/N_g = (R|R)/N_g = 1 with N_g = cosh θ and the averaged (L|R) = −(sink θ) = 0. Considering the spin precession around the z-axis, we define a chirality (chiral charge) as C_ℓ = (L|a_z L) = 1 and C_j = (R|a_z R) = −1, clearly, |L⟩ and |R⟩ have particle-hole symmetry. Note, the AFM magnon chirality is intrinsic and independent of the propagation direction of magnons. Its origin stem from the symmetry of the AFM system. Hence, it is useful to employ the magnon chirality for a non-volatile encoding of information, as explicitly demonstrated below.

**Magnonic Klein–Gordon dynamics**

When applying Haldane’s mapping procedure, no apparent parity-breaking exchange term appears in the continuum energy function. The parity-breaking term is important to correctly capture the intrinsic magnetization, as evident in the continuum limit of the free energy of AFM magnons. Let us employ the staggered field n = (S_a - S_b)/2S) and the intrinsic magnetization m = (S_a + S_b)/2S). For large and isotropic AFM exchange J > K_0, the total Lagrangian density reads

\[ \mathcal{L} = \rho_s m \cdot (\partial_t (\mathbf{n} \cdot \mathbf{n}) - \frac{m}{\chi_m} - \frac{1}{2} \partial_t \mathbf{n} \cdot \partial_t \mathbf{n}) \]

where m = 2S is the magnitude of the staggered spin angular momentum per unit cell, χ_m is the magnetic susceptibility, A = 2α/α^2 is the exchange stiffness, and ε_ℓ = 2α/α^2 signifies the amplitude of the parity-breaking term. In the absence of a strong external magnetic field, the spin density field m is a slave variable (m ≪ 1) that follows the temporal and spatial evolution of the staggered AFM order as

\[ m/(\chi_m = \rho_s (\partial_t (\mathbf{n} \cdot \mathbf{n}) - \xi_2 \partial_t \mathbf{n}). \]

Eliminating m we obtain the two-component Klein–Gordon equation for (ψ_ℓ, ψ_j) = (n_x+i_n_y, n_x-i_n_y) as

\[ \left( \frac{\partial^2}{\partial z^2} - \frac{1}{\epsilon^2} \right) \psi_\ell = K_z \psi_\ell, \]

where κ = 2αJ/h is the spin wave velocity, and K_z = K_z(4J_z/4J_s + J_s)/4J_s determines the spin wave gap. Obviously, ψ_ℓ (i.e., n_x+i_n_y) and ψ_j (i.e., n_x-i_n_y) are associated with the degenerate left- and right-handed chiral magnons, respectively. Conventional magnons without a spatial phase structure (i.e., without OAM) exhibit opposite intrinsic magnetization of the left- and right-handed magnons. Upon time averaging, we find

\[ \langle m^2 \rangle \propto \chi_\ell |\psi_\ell \partial_t \psi_\ell|^2 = \omega_\ell \rho_s \psi_\ell, \]

\[ \langle m^2 \rangle \propto \chi_\ell |\psi_j \partial_t \psi_j|^2 = -\omega_j \rho_s \psi_j, \]

which clarifies the chiral character of the modes.

Now we show the existence of eigenmodes characterized (additionally to their chirality) by a definite amount of magnonic (meaning akin to the quasiparticles, magnons) OAM. We term these helical modes as twisted AFM magnons.

**Twisted AFM magnons**

In an extended cylindrical AFM tube and in cylindrical coordinates r → (r, ϕ, z), the above Klein–Gordon equation admits the non-diffractive Bessel solutions

\[ \psi_\ell(r, t) \sim J_\ell(k_\ell r) \exp(i\ell \phi + ik_\ell z) \exp(-i\omega t), \]

with \( \ell \equiv 0, \pm 1, \pm 2, \ldots \) J_\ell(x) is the Bessel function of the first kind with order \( \ell \). For a cylindrical waveguide which is narrow compared to the magnon wavelength one may apply the paraxial approximation k^2 ≫ (k^2 − k_x^2), i.e., the transverse wave number k_x is small. One finds then that \( \partial_\ell z ≃ k^2 + 2ik_\ell \) leading to the Schrödinger-type equation for the spin waves

\[ i \partial_{\psi_\ell} / \partial z \approx -\left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{2} \frac{\partial^2}{\partial \ell^2} + k^2 \right] \psi_\ell, \]

where z = z/2k can be interpreted as the independent “time-like” variable. We conclude that in the AFM waveguides the modes are transversely confined Laguerre-Gaussian (LG) beams

\[ \psi_\ell = \left( \frac{r}{\ell!} \right) L_\ell^{(\ell)} \left( \frac{2r^2}{\ell^2 z^2} \right) \exp \left( - \frac{r^2}{\ell^2 z^2} \right) \exp \left( i\ell \phi + i\eta(z) \right), \]

where L_\ell^{(\ell)} are the generalized Laguerre polynomials, \( \ell = 0, \pm 1, \pm 2, \ldots \); \( \eta = 0, 1, 2, \ldots \) is the radial quantum number, w(z) = w_0(\sqrt{1 + z^2}/z_0^2) is the beam width depending on z due to diffraction, R(z) = \sqrt{1 + z^2}/2 is the radius of curvature of the wave fronts, and \( \eta(z) = \arctan(z/\ell_0) \) the last exponential factor \( 2z + |\ell| + 1 \eta(z) \) is related to the Gouy phase yielding an additional phase delay on the beam propagation. LG modes have well-defined azimuthal and radial wavefront distributions (quantified by \( \ell \) and \( \ell_0 \)) and they form an orthogonal and complete basis in terms of which an arbitrary function can be represented. The existence and utility of twisted modes for various systems ranging from electrons and neutron to acoustic waves are documented. In the AFM the underlying symmetries, free-energy density and the generic equations of motions are different from known cases, yet twisted AFM modes carrying OAM do exist under appropriate setting. To inspect the properties of the canonical OAM, we consider the continuity equation for spin angular momentum transfer in AFMs

\[ \mathbf{j}_m(r) = -\rho_s \partial_t \mathbf{m} - \nabla \times \mathbf{A} = \nabla \times \mathbf{A} = 0. \]

The spin current along the μ spatial direction reads

\[ \mathbf{j}_m = -\rho_s \partial_t \mathbf{m} - \mathbf{A} = \nabla \times \mathbf{A} = 0. \]

The first temporal term on the r. h. s. is determined by the precession of the staggered field m and thus carries the magnon chirality of any type of magnon beams. The second term is intimately related to the spatial topology of the (helical) magnon wave. In particular, the spatial phase modulations of the spin wave whose twist is characterized by the integer \( \ell \) is decisive. For example, in terms of the left-handed Bessel/LG beams, one finds the conserved z-component of the magnonic spin current

\[ \mathbf{j}_m = -\rho_s \partial_t \mathbf{m} = -\mathbf{A} = \mathbf{A} = \mathbf{A}, \]

where \( \rho_s \) is the magnon density of twisted, left-hand chiral magnons. \( \ell \) characterizes the z-component of the intrinsic OAM of the respective mode, \( L_\ell - \ell k \) that is independent on the choice of the coordinate origin and different \( \ell \) correspond to different modes, implying that these modes may serve as multiplex information channels. We note that \( \ell \) is unbounded and akin to the geometry and topology of the waveguide, presenting an additional twist that is independent of the magnon chirality. Due to (Gilbert) damping of magnetization precession, the magnon density is a time-decaying function. OAM \( \ell \) however, is robust to damping, meaning when functionalized \( \ell \) for information transmission, the signal becomes weaker with time but the information content is preserved. Further micromagnetic
simulations show that $\ell$ is a global property of the waveguide and is less affected by reasonable variations of deformation in the shape.

**DMI coupling, Faraday effects, and electric field control**

For controlling twisted AFM magnons with electric means, the electric-field-tunable interfacial DMI can be utilized to twisted spin wave computing. The break of the inversion symmetry at the cylindrical surface along the radial direction $e_z$ allows for a DMI of the following form, $\mathcal{H}_{\text{DM}} = -D_\phi \cdot (S \times S)_{\parallel}$. The DMI vector reads $D_\phi = D(e_z \times r)$. The DMI adds to the Lagrangian density $\mathcal{L}_{\text{DM}} = D_\phi \cdot (n \cdot (\nabla \times m) + \nabla \cdot (n \times m) - m \cdot (\nabla \times m))$, where $D_\phi = D_{Z\parallel} \nabla$ and $\nabla = (e_z \times \nabla_z)$. The second term on the r. h. s. is a parity-breaking term and has a structure of a total derivative. Hence, it does not affect the local dynamics. Dropping the small last higher-order term on the r. h. s. ($m \ll 1$), we arrive at in-plane DMI density (note, $r_i \in xy$-plane)

$$L_{xy}^{\text{DM}} = -D_\phi / r (n_i \partial_j n_j - n_j \partial_i n_i).$$

(16)

Otherwise, $r_i \parallel e_z$ results in the longitudinal DMI density

$$L_{zz}^{\text{DM}} = -D_\phi / r (n_i \partial_j n_j - (n_i \cos \phi + n_j \sin \phi)).$$

(17)

Up to the second order in the transversal fluctuations about the equilibrium of $n$, $L_{xy}^{\text{DM}}$ does not affect the magnon dynamics but pins Walker-type domain walls to the right-Néel type.

The Euler–Lagrange dynamics for the staggered field, Eq. (6) is augmented by an additional term in the presence of DMI. To a leading order in the density $L_{xy}^{\text{DM}}$, the magnon dynamics is governed by

$$\left[ \partial^2_\mu - \frac{1}{c^2} \partial^2_\mu + \iota_0 D_\phi \partial_\phi \right] \psi_n = K_\parallel \psi_n,$$

(18)

where $D = \frac{D_\parallel}{R}$ with $R$ being the radius of the AFM tube. Note, the thickness scaling as $r/R$ of the interface-induced DMI (i.e., $D_\phi \sim r/R$) has been used to derive Eq. (18). In the absence of DMI ($D = 0$), the topological charge $\ell$ is independent of the helicity of spin waves and the left-handed ($\psi_{\ell+}$) and right-handed ($\psi_{\ell-}$) chiral modes with arbitrary topological charges $\ell$ are degenerate. This infinite degeneracy is however lifted by the introduction of $L_{xy}^{\text{DM}}$, which behaves as a fictitious electric field that couples to the magnons via the Aharonov–Casher effect. For Bessel/LG modes as solutions for Eq. (18) we infer the modified dispersion relation of the chiral magnons

$$\omega_{\pm}^2 = c^2 (k_x^2 + k_z^2 \pm \Delta \ell + K_\parallel).$$

(19)

The energy dispersion is now dependent on the topological charge $\ell$. The spin wave gap is softened by the DMI, meaning that even below the AFM resonance point at $\omega_R = c \sqrt{K_{\parallel z}}$, twisted magnons can be excited. Equation (19) indicates that the chiral degeneracy of left- and right-handed magnons ($\psi_{\ell \pm}$) with same topological charge $\ell$ is lifted, however, the twofold topological $\pm \ell$ degeneracy survives in the presence of interfacial DMI. These degenerate twisted waves with opposite topological charge are decoupled from each other. The OAM-balanced superposition $|\psi_{\ell \pm}^{\Omega}\rangle$ is thus robust to the surface-induced DMI. The Bloch circles and the interference patterns are rotated during propagation along the waveguide resulting in a Faraday effect (cf. Fig. 2). In the paraxial approximation with relatively small transverse kinetic energy, the allowed wave numbers are approximated as $k_x \approx k + \delta k_{x}^2$, with $\delta k_{x}^2 = ± \pm (D_{Z\parallel} / 2k) + K_{\parallel z} / 2k)$. The DMI modifies the longitudinal wave vector resulting in an additional phase difference $\delta \phi(z) \sim (D_{Z\parallel} / k)$. Let $l$ be the length of the AFM tube, then $\delta \phi(l) = \pi$ can be realized since arbitrarily large values of $\ell$ are possible allowing so for any desired phase shift even at very weak DMI and the twisted magnon beams exhibit so a rich magneto-electric interference pattern. It is worth noting that such electrical control of the flow of OAM-carrying magnon beams can be directly realized by applying an external electric field through the Aharonov–Casher (AC) effect.

**OAM-based information coding and AFM logic gates**

A conventional way to read/detect the magnetically encoded data is to convert the signal back to electronic signals via the combination of two physical effects: spin pumping and the inverse spin Hall effect (ISHE). In terms of the staggered AFM order parameter $n$, there are two types of pumping effect generated by the spin current: the pumped spin density $\psi_{\ell,0}^\prime$ of the state $\cos \theta / |R|$ by the $\ell$-resolved ISHE. At the waveguide end, the $z$-polarized spins (green arrows) radially pumped into a narrow Pt layers results in an inverse spin Hall voltage $V_\parallel \sim \cos^2 \theta / |R|$. With this we arrive at a key result: The helical or twisted chiral magnons $\psi_{\ell,0}^\prime$ can be realized since arbitrarily large values of $\ell$ is present. However, a $\ell$-resolved analysis evidences a spatial distribution of pumped spin density. For instance, the state $\cos \theta / |R|$ gives rise to

$$I_\ell^z \propto \omega_{\parallel} \cos^2 \theta / |R| \quad \text{and} \quad I_\ell^\parallel \propto \ell \omega_{\parallel} \sin 2\theta / |R|.$$
additional degree of freedom, the topological charge chiralities are thus fully realized in the AFM waveguides with a particular chiral magnon state $i \chi_i$.

We may rewrite the above superposition into two Bloch circles as, $|\psi_x\rangle = e^{i\delta_1}|R\rangle + e^{i\delta_2}|L\rangle$,

$$|\psi_x\rangle = \cos \theta_i |R\rangle + e^{-i\delta_1} |L\rangle,$$

$$|\psi_y\rangle = \sin \theta_i + e^{i\delta_1} |L\rangle.$$  (22)

We may rewrite the above superposition into two Bloch circles as, $|\psi_x\rangle = |\psi_x^+\rangle + |\psi_x^+\rangle$ with $|\psi_x^+\rangle = \cos \theta_i |R\rangle - i \sin \theta_i |L\rangle$,

$$|\psi_x^+\rangle = \cos \theta_i |R\rangle + i \sin \theta_i |L\rangle.$$  (23)

Twisted wave packet propagation

To demonstrate the twisted wave packets propagation, we consider a waveguide with 100 × 100 × 300 unit cells (u. c.) excited at one end by a twisted magnetic field. For the frequency of 0.5 THz, the wavelength of the plane-wave mode is $\lambda \approx 30.5$ nm. The boundaries are modeled such that they absorb most of the incoming magnons by introducing an exponentially increasing damping parameter in a tube shell with a 20 u. c. thickness. This helps avoiding reflection effects in order to focus on the core features of twisted magnon beams. Because of the discrete rotational symmetry characterized by the OAM, twisted magnon modes are protected against perturbations including defects and finite size effects as discussed for the case of a FM wave guide in ref. 49. An example of a few-cycle wave packet propagating along the NiO wire is shown in Fig. 5a.

For further insight, let us continue with different wave packets propagating along the wire, especially focusing on the group velocity, broadening, and signal decay.

In Fig. 5b, few-cycle twisted AFM pulses are propagated along the wire, and the position of maximum magnon density is tracked. For this calculation, only one line of magnetic moments parallel to the y-axis is analyzed. The offset to the center is 25 u. c. (=10 nm).
Cartesian components of pumped spin density cross section of the NiO wire. The resulting patterns of the three lattice and subsequently calculate the ISHE via calculating the staggered magnetization from the full G-type AFM calculated from the spin dynamics calculations. This we do by induced electronic spin density at the surface of the tube can be In addition to the propagation of AFM twisted magnon beams, the Spin dynamics induced by twisted AFM beams

In addition to the propagation of AFM twisted magnon beams, the induced electronic spin density at the surface of the tube can be calculated from the spin dynamics calculations. This we do by calculating the staggered magnetization from the full G-type AFM lattice and subsequently calculate the ISHE via $I_z \sim n \cdot \partial_t n$. The three panels display the different components of the induced spin-current showing that only the $z$-component delivers a non-zero value after integration over the whole cross section. The staggered magnetization is obtained from full-numerical spin dynamics simulations.

The group velocities are different for different winding numbers $\ell$. The linear regressions give velocities of $v_{\ell=1} = 9.09 \text{ km s}^{-1}$, $v_{\ell=2} = 8.36 \text{ km s}^{-1}$ and $v_{\ell=3} = 7.35 \text{ km s}^{-1}$. Also, the signal amplitude decays more rapidly when increasing the topological number.

Spin dynamics induced by twisted AFM beams

In addition to the propagation of AFM twisted magnon beams, the induced electronic spin density at the surface of the tube can be calculated from the spin dynamics calculations. This we do by calculating the staggered magnetization from the full G-type AFM lattice and subsequently calculate the ISHE via $I_z \sim n \cdot \partial_t n$ for a cross section of the NiO wire. The resulting patterns of the three Cartesian components of pumped spin density $I_z$ for a beam with $\ell = 2$ are shown in Fig. 6. All signals are normalized with respect to the maximum value for each component. As predicted, only the $z$-component has a completely positive signal, delivering a finite signal when integrating over time. In contrast to that, the $x$- and $y$-components possess a twofold distribution with alternating positive and negative areas eventually summing up to a total induced spin density of zero.

The characteristic features of the $z$-component for different chiralities and topological charges are presented in Fig. 7 for a transversal profile through the center of the wire. Generally, the maximum pumped spin density can be found in the region where the highest magnon amplitudes are expected. This also means that for higher topological charges $\ell$, the maximum intensity distance increases (cf. dark red and bright green markers). The diminished intensity is explainable by the higher damping of the magnon amplitudes, which is proportional to the magnon density.

In addition, the sign of the pumped spin density is determined by the chirality of the magnon polarization, meaning that a change from left-handed to right-handed magnons exclusively changes the sign of the signal and not its distribution. Thus, the pumped spin current due to the ISHE delivers a fingerprint for different twisted magnon modes determined by both the chirality and the topological charge of the spin wave. Current efforts are focused on utilizing the twisted beam for driving AFM structures where we expect the topology of these beams to add an additional twist on existing AFM magnonics.

Superseded beams—proof of gate operations

In principle, the spin current density profiles shown in Figs. 6 and 7, generated by different types of magnon modes indicate the possibility of an OAM specific detection of signals. In order to close the gap between these results and the concept of parallel information transport and logic gate operations, we now proceed with numerical simulations of superpositions of different magnon modes. We perform spin dynamics calculations as before, but the cylindrical system is now excited with superimposed magnetic fields, each exciting an individual twisted magnon mode.

As an example, the superposition of beams with $\ell = \pm 1$ and positive helicity are excited in the same NiO cylinder as before. The time resolved pumped spin density is shown in Fig. 8 with the reference signal for a single $\ell = 1$ magnon in Fig. 8a. All results have in common, that the excited beams need a finite time to reach the measurement slice at 150 u.c. distance, therefore...
showing no spin accumulation at the first snapshot after 2.41 ps. Next, let’s consider two monochromatic beams (b, $f_1 = f_2 = 0.5\, \text{THz}$) and slightly detuned beams (c, $f_1 = f_2/1.1 = 0.5\, \text{THz}$). Because of the fixed phase difference in case of the monochromatic beam composition, a steady pattern in the ISHE signal is present in Fig. 8a, showing the areas of constructive and destructive interference composition, a steady pattern in the ISHE signal is present in Fig. of the

Spin dynamics simulations

The predictions are generic. For a material-specific demonstration we used NiO with a material parameters as determined experimentally. For the experimental guidance, we used the open-source, GPU-accelerated software package $\mu$max3\textsuperscript{13} for the micromagnetic simulations. Input data and more details are provided in the Supplementary Methods.

DATA AVAILABILITY

All data needed to reach the conclusions in the paper are present in the paper and/or the Supplementary Methods and may be requested from the authors.

CODE AVAILABILITY

The codes developed in this study are available from the authors upon reasonable request. The open-source software package $\mu$max3 is freely available.

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REFERENCES

1. Chumak, A. V., Vasyuchka, V. I., Serga, A. A. & Hillebrands, B. Magnon spintronics. Nat. Phys. 11, 453–461 (2015).
2. Wang, Q. et al. A magnonic directional coupler for integrated magnonic half-adders. Nat. Electron 3, 765–774 (2020).
3. Dieny, B. et al. Opportunities and challenges for spintronics in the microelectronics industry. Nat. Electron 3, 446–459 (2020).
4. Kampfrath, T. Coherent Terahertz Control of Antiferromagnetic Spin Waves. Nat. Photonics 5, 31–34 (2011).
5. Jungwirth, T., Martí, X., Wadleby, P. & Wunderlich. J. Antiferromagnetic spintronics. Nat. Nanotechnol. 11, 231–241 (2016).
6. Gomonay, O., Baltz, V., Brataas, A. & Tserkovnyak, Y. Antiferromagnetic spin textures and dynamics. Nat. Phys. 14, 213–216 (2018).
7. Gomonay, E. V. & Loktev, V. M. Spintronics of antiferromagnetic systems (Review Article). Low. Temp. Phys. 40, 17–35 (2014).
8. Smekal, L., Mokrousov, Y., Yan, B. & MacDonald, A. H. Topological antiferromagnetic spintronics. Nat. Phys. 14, 242–251 (2018).
9. Jungwirth, T. et al. The multiple directions of antiferromagnetic spintronics. Nat. Phys. 14, 200–203 (2018).
10. Baltz, V. Antiferromagnetic spintronics. Rev. Mod. Phys. 90, 015005 (2018).
11. Fukami, S., Lorentz, V. O. & Gomonay, O. Antiferromagnetic spintronics. J. Appl. Phys. 128, 070401 (2020).
12. Duine, R. A., Lee, K.-J., Parkin, S. S. P. & Stiles, M. D. Synthetic antiferromagnetic spintronics. Nat. Phys. 14, 217 (2018).
13. Lavrijsen, R. et al. Magnetic ratchet for three-dimensional spintronic memory and logic. Nature 493, 647 (2013).
14. Yang, S.-H., Ryu, K.-S. & Parkin, S. S. P. Domain-wall velocities of up to 750 m s$^{-1}$ driven by exchange-coupling torque in synthetic antiferromagnets. Nat. Nanotechnol. 10, 221 (2015).
15. Huang, B. Layer-dependent ferromagnetism in a van der Waals crystal down to the monolayer limit. Nature 546, 270 (2017).
16. Gong, C. Discovery of intrinsic ferromagnetism in two-dimensional van der Waals crystals. Nature 546, 265 (2017).
17. Xing, W. Electric field effect in multilayer Cr$_2$Ge$_2$Te$_6$: a ferromagnetic 2D material. 2D Mater. 4, 024009 (2017).
18. Wang, X. Raman spectroscopy of atomically thin two-dimensional magnetic iron phosphorus trisulfide (FePS$_3$) crystals. 2D Mater. 3, 031009 (2016).
19. Lee, J.-U. Ising-Type Magnetic Ordering in Atomically Thin FePS$_3$. Nano Lett. 16, 7433 (2016).
20. Xing, W. Magnon Transport in Quasi-Two-Dimensional van der Waals Antiferromagnets. Phys. Rev. X 9, 011026 (2019).
21. Kittel, C. Theory of Antiferromagnetic Resonance. Phys. Rev. 82, 565 (1951).
22. Keffer, F. & Kittel, C. Theory of Antiferromagnetic Resonance. Phys. Rev. 85, 392 (1952).
23. Willner, A. E. Optical communications using orbital angular momentum beams. Adv. Opt. Photon. 7, 66 (2015).
24. Bozinovic, N. Terabit-Scale Orbital Angular Momentum Mode Multiplexing in Fibers. Science 340, 1545 (2013).
25. Jia, C. L., Ma, D. C., Schäffer, A. F. & Berakdar, J. Twist magneton beams carrying orbital angular momentum. Nat. Commun. 10, 2077 (2019).
26. Chen, M., Schäffer, A. F., Berakdar, J. & Jia, C. L. Generation, electric detection, and orbital-angular momentum tunneling of twisted magnons. Appl. Phys. Lett. 116, 172403 (2020).
27. Dzyaloshinsky, I. A thermodynamic theory of “weak” ferromagnetism of anti-ferromagnetics. J. Phys. Chem. Solids 4, 241 (1958).
28. Moriya, T. Anisotropic Superexchange Interaction and Weak Ferromagnetism. Phys. Rev. 120, 91 (1960).
29. Liu, Y., Sellmyer, D. J. & Shindo, D. Handbook of Advanced Magnetic Materials (Springer, 2006).
30. Auerbach, A. Interacting Electrons and Quantum Magnetism (Springer-Verlag, Inc. 1994).
31. Nakata, K., Kim, S. K., Klinovaja, J. & Loss, D. Magnonic topological insulators in antiferromagnets. Phys. Rev. B 96, 224414 (2017).
32. Haldane, F. D. M. Nonlinear Field Theory of Large-Spin Heisenberg Antiferromagnets: semiclassically Quantized Solitons of the One-Dimensional Easy-Axis Néel State. Phys. Rev. Lett. 50, 1153–1156 (1983).
33. Ivanov, B. A. & Kolezhuk, A. K. Solitons with Internal Degrees of Freedom in 1D Heisenberg Antiferromagnets. Phys. Rev. Lett. 74, 1859 (1995).
34. Ivanov, B. A. Spin dynamics of antiferromagnets under action of femtosecond laser pulses (Review Article). Low. Temp. Phys. 40, 91–105 (2014).
35. Tveten, E. G., Muller, T., Linder, J. & Brataas, A. Intrinsic magnetization of antiferromagnetic textures. Phys. Rev. B 83, 104408 (2016).
36. Delfino, F., Pelissetto, A. & Vicari, E. Three-dimensional antiferromagnetic CPN-1 models. Phys. Rev. E 91, 052109 (2015).
37. Cheng, R., Okamoto, S. & Xiao, D. Spin Nernst Effect of Magnons in Collinear Antiferromagnets. Phys. Rev. Lett. 117, 217202 (2016).
38. Allen, L., Beijersbergen, M. W., Spreeuw, R. J. C. & Woerdman, J. P. Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes. Phys. Rev. A 45, 8185 (1992).
39. Yao, A. M. & Padgett, M. J. Orbital angular momentum: origins, behavior and applications. Adv. Opt. Photon. 3, 161 (2011).
40. Uchida, M. & Tonomura, A. Generation of electron beams carrying orbital angular momentum. Nature 464, 737 (2010).
41. Verbeeck, J., Tian, H. & Schattschneider, P. Production and application of electron vortex beams. Nature 467, 301 (2010).
42. Clark, C. W. Controlling Neutron Orbital Angular Momentum. Nature 525, 504 (2015).
43. Greenshields, C., Stamps, R. L. & Franke-Arnold, S. Vacuum Faraday effect for electrons. N. J. Phys. 14, 103040 (2012).
44. Tveten, E. G., Qaiumzadeh, A. & Brataas, A. Antiferromagnetic Domain Wall Motion Induced by Spin Waves. Phys. Rev. Lett. 112, 147204 (2014).
45. Tadic, M., Nikolic, D., Panjan, M. & Blake, G. R. Magnetic properties of NiO (nickel oxide) nanoparticles: blocking temperature and Néel temperature. J. All. Comp. 647, 1061 - 1068 (2015).
46. Cheng, R., Xiao, J., Niu, Q. & Brataas, A. Spin Pumping and Spin-Transfer Torques in Antiferromagnets. Phys. Rev. Lett. 113, 057601 (2014).
47. Nielsen, M. A. & Chuang I. L. Quantum Computation and Quantum Information (Cambridge University Press, 2010).
48. Coey, J. M. D. Magnetism and magnetic materials (Cambridge University Press, 2010).
49. Jia, C., Ma, D., Schaffer, A. F. & Berakdar, J. Twisting and tweezing the spin wave: on vortices, skyrmions, helical waves, and the magnonic spiral phase plate. J. Opt. 21, 124001 (2019).
50. Cheng, R., Daniels, M. W., Zhu, J.-G. & Xiao, D. Antiferromagnetic Spin Wave Field-Effect Transistor. Sci. Rep. 6, 24223 (2016).
51. Lan, J., Yu, W. & Xiao, J. Antiferromagnetic domain wall as spin wave polarizer and retarder. Nat. Commun. 8, 178 (2017).
52. Qaiumzadeh, A., Kristiansen, L. A. & Brataas, A. Controlling chiral domain walls in antiferromagnets using spin-wave helicity. Phys. Rev. B 97, 020402(R) (2018).
53. Vansteenkiste, A. et al. The design and verification of MuMax3. AIP Adv. 4, 107133 (2014).

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