De-noising of EEG signals with shift-based cycle spinning on wave atoms

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Accepted: 18 December 2021 / Published online: 8 March 2022
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Abstract
EEG signals offer qualitative insights during brain disorder analysis and help in the brief assessment of brain diseases. However, EEG signal acquisition is susceptible to noise of various kinds and renders the analysis phase difficult and ill-posed. Hence, an appropriate technique is necessary to reduce the impairment effect caused by the noise on analysis. Wavelet thresholding is one of the most widely used techniques to recover the EEG signals from the noise. However, all the variants of wavelet thresholding algorithms suffer from pseudo-Gibbs phenomenon leading to ringing effect. Wave atom is a novel multiscale–multidirectional transformation technique. However, this wavelet thresholding produces pseudo-Gibbs phenomena, which are visual distortions and oscillations in the area of signal processing. The widely appreciated wave atom transformation too fails from artifacts around the sharp edges. The proposed shift-based cycle spinning technique of wave atom transformation model estimates the thresholding parameter, in an unbiased manner, from the data and de-noises the signal in a translation-invariant manner. De-noising studies on the OpenNeuro EEG dataset indicate the usefulness of the suggested technique. The results are analysed based on the performance measurements SNR and MSE that establish an advantage of shift-based cycle spinning model in getting better results.

Keywords EEG · DWT · Wave atom (WA) · Cycle spinning · Anisotropy · SNR · MSE

1 Introduction
Electroencephalography (EEG) records the electrical signal on skull surface area of human brain. The analysis of EEG signal is required to detect the abnormal functionality of brain. However, various types of noise can corrupt the signal. So, the signal diagnosis becomes more difficult because of noise. Therefore, these noises need to be removed from the signal for better diagnosis. For de-noising EEG signals, DWT is the widely used among the techniques. Several works proposed the use of different wavelets and thresholding techniques of DWT (Biswas and kazi Rayadul Hasan 2015). Soft thresholding, hard thresholding, sigmoid thresholding and other thresholding techniques can be used (Sharma and Verma 2016). Researchers frequently use wavelet transforms to detect noise in EEG signals, but they are only sensitive at the signal’s edges (Candes and Demanet 2005; Candes and Donoho 2004).

Coifman and Donoho proposed cycle spinning mechanism as an improvement over the fundamental wavelet thresholding methods (Coifman et al. 1995). As a result, Coifman and Donoho proposed the cycle spinning technique as an upgrade to simple wavelet thresholding. The proposed method used shift-variant property of wavelet transform. The cycle spinning technique is primarily used prevent de-noising aberrations (Sahraeian et al. 2007; Sahraeian and Marvasti 2007). Furthermore, pseudo-Gibbs phenomena can be effectively controlled by the combination of cycle spinning technique with wave atoms. Noisy signals are represented by varying signal shifts when using the time-variant wavelet transform. By using this process, one can extract the original signal from the noisy one. Different errors are discovered in different shifts, and
signal averaging reduces them. In wave atoms of the given signal, there is a sparsity in oscillation function than that of wavelet, Gabor atoms (curve wave) (Demanet and Ying 2007a). Wave atoms capture the pattern through oscillation and vibration mode in contrast with curve wave. Pseudo-Gibbs invented the artificial visual distortion phenomenon to overcome the translation invariance in a noisy signal.

Now cycle spinning is proposed on wave atoms to overcome the limitations found at the edges of the noisy signal (Zhang et al. 2014; Plonka and Ma 2008). Hence, wave atom can produce better results using cycle spinning.

### 2 Literature survey

Form this literature survey, cycle spinning with appropriate number of shifts could give better results for removal of noise from EEG signals. Rodriguez-Hernandez (2016) proposed an algorithm to implement one of the undecimated wavelet transforms (UWTs) which is cycle spinning (CS) to analyse the signal. He applied a variant of cycle spinning to ultrasonic trace de-noising. He assured that his technique produced better results in de-noising the ultrasonic signal. Zhang et al. (2014) found that de-noising of signal (Zhang et al. 2014; Plonka and Ma 2008). Hence, de-noising of image by thresholding the coefficients may lead to pseudo-Gibbs phenomena. This is because of lack of translation invariance in wave atom transformation. Therefore, they use cycle spinning method to remove noise and remain edges by controlling pseudo-Gibbs phenomena efficiently. Alyson et al. (2002) introduced a new algorithm recursive cycle spinning. This algorithm repeatedly translates and de-noising the signal with wavelet de-noising technique translating back recursively and applied convergence properties of projections. Finally, the de-noising projections converge to the original signal.

Jiao (2019) suggested a method that use wave atoms with cycle spinning for removal of heavy background noise from an image. He concluded that wave atoms are a promising multiscale transformation available for the removal of image noise. Eddine and Hassene (2017) decided that the wave atom transformation uses more parameters like dimension, location, orientation, etc. Because of good orientation characteristic, wave atom transformation performs much better than wavelet and curvelet transforms. Eddine et.al. proposed algorithm is based on hard thresholding that optimizes the performance of wave atom transform. Shruthi and Kumar (2017) de-noised the images by decomposing them using wave atom transform and wavelet transforms. Constrained least square filtering is the algorithm employed, followed by a de-noising procedure. By separating the image into texture and cartoon sections, Sruthi and Kumar de-noised the image using curvelet transform. The texture part is de-noised using wave atom transform, while the cartoon part is de-noised using wavelet transform. Hard thresholding is applied to preserve edge details. John et al. (2017) proposed an algorithm which uses cuckoo search algorithm to optimize coefficients of wave atom transform. Optimization techniques are used to reduce noise. Adaptive thresholding method is applied to de-noise the signal.

### 3 Wave atom transform for de-noising

For the sake of completeness of the discussion, a brief discussion on the definition and characteristics is taken place.

#### 3.1 Introduction to wave atoms

The wave atoms \( \{ \psi_u(X) \} \) are the elements of wave packet framework that satisfies scaling and rotation-related properties of waves in frequency domain as formalized below.

- \( |\psi_u(X)| \leq CN 2^j (1 + 2|X - X_u|)^{-N} \)
- \( |\psi_u(X)| \leq CN 2^j (1 + 2|\omega - \omega_u| J^{-N} + CM 2^{-j} (1 + 2^{-j}|u - \omega| J^{-N} \)

where \( N > 0 \).

Only a qualitative description of wave atoms with spatial frequency location limitations is provided by the definition. The wave atom transform does not comply with the features of wavelets with reference to the frequency localization capabilities and thus resulted in translation variance. The intuitions of Villemose (2002) addressed this limitation by constructing the wave atoms as a tensor product of adequately chosen 1D wave packets. His trick involved designing of basic functions which are bounded in both the domains, time and frequency, which is not at the disposal of conventional multiresolution analysis such as wavelets. The cosine window-based frequency domain filtering had effectively minimized the aliasing effect in de-noising algorithms (Kolaczyk 1994).

#### 3.2 Signal de-noising

De-noising of signals is the pre-processing step for obtaining quality results in biomedical signal analysis. It is essential to develop the de-noising filters that work in either spatial or transformed domains. The wavelet packet-based transforms such as Gabor, ridgelets, curvelets and wave atoms are concisely specified using \( x \) and \( \beta \) parameter. The parameter \( x \) determines whether the separation of components is multistate (\( x \) is equal to 1) or not (\( x \) is equal to 0) and the parameter \( \beta \) determines whether the basic elements are regional and bounded. When \( \beta = 1 \), then it is poorly directional, also referred to as isotropic, and when \( \beta = 0 \), then it is fully directional or anisotropic. Thus,
we can classify the transform based on these parameters $\alpha$ and $\beta$ as follows. For wavelets, $\alpha$ is 1 and $\beta$ is 1, for ridgelets, $\alpha$ is 1 and $\beta$ is 0, for Gabor, $\alpha$ is 0 and $\beta$ is 0, for curvelets, $\alpha$ is 1 and $\beta$ is $\frac{1}{2}$, and for wave atom, $\alpha$ corresponds to $\frac{1}{2}$ and $\beta$ is $\frac{1}{2}$. The following figure summarizes this classification (Fig. 1).

Wave atoms are invented from one-dimensional wave packets. One-dimensional wave packets can be characterized as (Boutella and Serir 2013).

The wave packet framework elements are known as wave atoms:

$$|\varphi_{a}(x)| \leq C_{N}2^{j}(1 + 2^{j}|x - x_{u}|)^{-N}$$

$$|\varphi_{a}(x)| \leq C_{N}2^{j}(1 + 2^{j}|\omega - \omega_{u}|)^{-N} + C_{M}2^{-j}(1 + 2^{-j}|\omega + \omega_{u}|)^{-N}$$

where $N > 0$.

Wave atom transform captures the oscillatory patterns and finer textures present in the signals and offers a robust mechanism for de-noising. It conforms to the relation of parabolic scaling, i.e.

- wavelength $\sim$ (diameter)$^{2}$.
- $\psi_{u}(x)$, $j$ and $m \geq 0$, and $n \in \mathbb{Z}$.
- $u$ represents scale parameter, rotation and translation. For 1D systems, this $u$ will be $u=(d, m, n)$ and for 2D systems this $u$ will be $(j, m_{1}, m_{2}, n_{1}, n_{2})$ where $j, m_{1}, m_{2}, n_{1}, n_{2}$ are integers and index of point $(x_{j,n}, \omega_{j,m})$ in the phase space.
- $\pm \omega_{j,m} = \pm \pi 2^{j} m$: Frequency restrictions are defined as centre of frequency domain with $C_{1}2^{j} \leq m \leq C_{2}2^{j}$.
- The centre of space is defined as $X_{j,n} = 2^{j}n$.

Figures 2, 3, 4 represent wave atom tiling in spatial domain with increasing scale parameter. Figures 5, 6, 7 represent wave atom tiling in frequency domain with increasing scale parameter.

The pseudo-Gibbs phenomenon results in mistakes at edges and in texture elements because wave atoms lack

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Fig. 1 Various transforms of wave packet families

Fig. 2 One-dimensional wave atom in spatial domain for $j = 3, m = 3$

Fig. 3 One dimensional wave atom in spatial domain for $j = 4, m = 5$

Fig. 4 One dimensional wave atom in spatial domain for $j = 5, m = 8$
translation invariance. The cycle spinning approach on wave atoms (Misiti et al. 2015) is used to control edge distortion, which is a step forward in the literature. Thresholding is an important step, and the threshold should be chosen using a data-driven approach. For the sake of completeness, a discussion of several thresholding strategies is included here.

4 Proposed method

4.1 Shift-based cycle spinning

The intuitions behind de-noising method based on cycle spinning revolved around the preservation of edge characteristics. However, that method did not characterize the nature of the noise, and hence, it perceived it as white noise. But the signal generating modality might induce noise with some spatial characteristics. Hence, it is prudent to address the noise characteristics by modelling it well-proven kernel functions. For example, moving average scheme is the most computationally efficient and effective algorithm to handle first-order stationary noise. RBF, Gaussian and Spline kernels are capable of addressing the complex noise patterns over higher-dimensional space by casting them over a random fields. These spatial kernels are tunable for addressing isotropic and anisotropic noise models efficiently provided the hyper parameters are learnt from the data using either machine learning or computational intelligence algorithms (Raghava et al. 2016). The EEG signals are of one-dimensional signals, and the induced noise is also span in one-dimensional. Hence, the noise exhibits the spatial characteristics along time dimension and correspondingly dampens the signal. Such a 1-D influence can be efficiently handled using a moving average based technique. The careful integration of this intuition into cycle spinning technique will improve the quality parameters. In this technique, we can compute different estimates of unknown signal using different shifts of noisy signal and then linearly average them. For each
shift, the wave atom coefficients are obtained by performing cycle spinning technique. The application of de-noising is done through wave atom coefficients at analysis stage, over the coefficients $d_{s,j}$ and $a_{s,j}$ as shown in the following figure (Fig. 8).

The application of thresholding to these coefficients will produce de-noised new coefficients $d'_{s,j}$ and $a'_{s,j}$ as shown in the following figure, which made as input for the next synthesis stage. An inverse wave atom transformation and reverse cycle spinning are applied over these coefficients. Finally, de-noised signal can be obtained by averaging all these results (Fig. 9).

4.2 Proposed unbiased shift-based cycle spinning algorithm (SCS)

- **Step 1**: Perform cycle spinning on a noisy signal $u$ and obtained $S(u)$.
- **Step 2**: Wave atom transformation is applied on $S(u)$ and transformation coefficients are acquired.
- **Step 3**: Calculate an unbiased threshold value, $A_h$.
- **Step 4**: Apply the unbiased threshold on the cycle spinning-based wave atom coefficients.
- **Step 5**: Take inverse wave atom transformation for the coefficients obtained in step 4.
- **Step 6**: Apply reverse cycle spinning on the signal obtained in step 5.
- **Step 7**: Repeat Steps from 1 to 6, and compute the averaging of all shifted signals about $u$, resulting in a final de-noised signal.

4.3 Threshold selection

The steps of Algorithm 1 show that the thresholding is a critical step and the threshold must be picked from a data-driven approach. For the sake of completeness, a discussion of various thresholding mechanisms is provided below. The most widely used thresholding techniques involve universal threshold and minimax threshold.

The universal threshold value is computed as follows.
The minimax threshold can be calculated using the following formula,

\[ T_{s_j}^{\text{Min}} = \sigma_{s_j} \sqrt{2 \ln N_j} \]

where \( N_j \) is number of wave atom coefficients \( d_{s_j} \) at level \( j \) decomposition and \( \sigma_{s_j} \) is given as

\[ \sigma_{s_j} = \frac{\text{median} \left| d_{s_j} \right|}{0.6745} \]

The minimax threshold can be calculated using the following formula,

\[ T_{s_j}^{\text{Min}} = \sigma_{s_j} \lambda_0 (N_j) \]

The Stein’s unbiased risk estimation (SURE) of thresholding is more amenable with the proposed shift-based noised reduction and is established in the results presented in the next section. SURE can be formally expressed as below.

\[ \text{SURE}(T:X) = N_j - 2 \cdot \{ i : |X_i| \leq T \} \]

\[ + \sum_{i=1}^{N_j} \left( \min(|X_i|, T) \right)^2 \]

where \( \{ i : |X_i| \leq T \} \) is the cardinality of the set brackets. The risk function of wave atom coefficients is minimized when the value is as follows:

\[ T_{s_j}^{S} = \arg\min_{T \geq 0} \left\{ \text{SURE} \left( T, d_{s_j} / \sigma_{s_j} \right) \right\} \]

Table 1: Comparison of MSE and SNR values for various threshold values

| S. No | Threshold value   | Even shifts | Odd shifts |
|-------|-------------------|-------------|------------|
|       |                   | MSE   | SNR   | MSE   | SNR   |
| 1     | Universal threshold value | 1.2323 | 73.1364 | 1.2308 | 73.1369 |
| 2     | Minimax           | 0.3465 | 73.6874 | 0.3520 | 73.6806 |
| 3     | SURE              | 0.0033 | 75.7079 | 0.0033 | 75.7125 |
The upcoming section presents the performance of the proposed SCS algorithm on the Benchmark signals dataset (Salisbury et al. 2021)

| S. no | Author                          | Method                                     | Parameter | Value  |
|-------|---------------------------------|--------------------------------------------|-----------|--------|
| 1     | Shruthi and Satheesh Kumar (2017) | Wave atom and wavelet transforms | PSNR      | 31.59  |
| 2     | Eddine and Seddik (2017)        | Wave atom                                 | PSNR      | 27.47  |
|       |                                 | Wavelet transform                         | SSIM      | 0.999  |
|       |                                 |                                           | PSNR      | 25.04  |
|       |                                 |                                           | SSIM      | 0.989  |
| 3     | John et al. (2017)              | Wavelet transform                         | SNR       | 10.7701|
|       |                                 |                                           | MSE       | 0.0251 |
|       |                                 | Wave atom transform                       | SNR       | 20.2054|
|       |                                 |                                           | MSE       | 0.006853|
| 4     | Zhang et al. (2014)             | Wave atom transform                       | PSNR      | 28.41  |
|       |                                 | Wave atom with cycle spinning             | PSNR      | 29.53  |

| Table 3 | Results of adaptive filter with wavelet decomposition
|---------|--------------------------------------------------|
| S.no    | Step value | Haar MSE SNR | Db1 MSE SNR | Sym2 MSE SNR | Coif1 MSE SNR |
|---------|------------|--------------|-------------|--------------|---------------|
| 1       | 0.1        | 0.2492 73.8304 | 0.2497 73.8296 | 0.2491 73.8291 | 0.247 73.8304 |
| 2       | 0.2        | 0.0063 75.4445 | 0.0064 75.4449 | 0.0060 75.4441 | 0.0060 75.4445 |
| 3       | 0.3        | 7.3135e-4 76.3630 | 7.3381e-4 76.3615 | 7.3306e-4 76.3620 | 7.3087e-4 76.3632 |
| 4       | 0.4        | 2.0228e-4 76.9211 | 2.0272e-4 76.9202 | 2.0225e-4 76.9212 | 2.0329e-4 76.9190 |
| 5       | 0.5        | 8.8695e-5 77.2792 | 8.8917e-5 77.2781 | 8.9048e-5 77.2775 | 8.8897e-5 77.2782 |
| 6       | 0.6        | 501780e-5 77.5129 | 5.2174e-5 77.5096 | 5.1797e-5 77.5128 | 5.2002e-5 77.5111 |
| 7       | 0.7        | 407954e-5 77.5463 | 4.764e-5 77.5491 | 4.8121e-5 77.5448 | 4.7559e-5 77.5499 |
| 8       | 0.8        | 1.1255e-4 77.1757 | 1.1239e-4 77.1764 | 1.1252e-4 77.1759 | 1.1153e-4 77.1797 |
| 9       | 0.9        | 0.001 76.2219 | 0.001 76.2225 | 0.001 76.2219 | 0.001 76.2221 |
| 10      | 1.0        | 743 70.3561 | 754 70.3492 | 5.3931e4 68.4953 | 3.5943e5 67.6715 |

| Table 4 | Results of wavelet and curvelet thresholding for various threshold values
|---------|--------------------------------------------------|
| S. no   | Signal | Wavelet thresholding MSE | Curvelet thresholding SNR |
|---------|--------|--------------------------|--------------------------|
| 1       | EEG Signal 1 | 0.8440 68.8911 | 0.0405 70.2097 |
| 2       | EEG Signal 2 | 0.6783 70.3956 | 0.0401 71.6242 |
| 3       | EEG Signal 3 | 0.7563 70.0134 | 0.0401 71.2919 |
| 4       | EEG Signal 4 | 0.5254 69.8901 | 0.0396 71.0134 |
| 5       | EEG Signal 5 | 0.7205 69.9608 | 0.0396 71.2099 |

5 Results analysis

Original EEG signals were taken from the OpenNeuro Datasets (Salisbury et al. 2021), and a random noise is added using the routine available in MATLAB library. The
experiments conducted have reproduced the results of the native CS model (John et al. 2017). Wavelet and curvelet transforms and results are compared to study the performance of cycle spinning with various shifts. It is observed that SCS had outperformed the other methods in terms of SNR and MSE. Similar behaviour was observed while handling different noise parameters. The following figures present the saliency of the model and its relative performance. And it is found that noise at the edges is effectively removed while preserving the sharp characteristics of the edge. From the following tables and the figures, substantiate the efficacy of model. The SCS could produce the de-noising results with SNR value as 75.7125 and MSE value as 0.0033 for SURE threshold value.

Figure 10 presents original EEG signal taken from the OpenNeuro Dataset. The efficacy of the proposed algorithm is presented in Figs. 11 and 12. The quality parameters extracted from the results highlight that SCS algorithm could withstand the non-stationary noise resulted in better SNR and MSE values. Table 1 presents the performance of the SCS against the noise handling based on the various thresholding intuitions. It is evident from the row 3 that unbiased noise estimation model helped in improving the de-noising results (Fig. 13) (Tables 2, 3, 4).

6 Conclusion

In this paper, we have implemented a novel de-noising algorithm for improving the quality of EEG signal, based on stationarity characteristics of the noise. The error-damaged signal data stream was recovered using translation followed by an aggregation operation over the cycle spinning transform. The directional feature of the noise is enveloped into a bounded interval. Different types of thresholding mechanisms were employed for each type of shift, and results were analysed in terms of SNR and MSE. The results established that the spatial convolution operation can improve the performance of cycle spinning de-noising technique over wave atom transformation. Future work investigates various possibilities of handling the non-stationary noise.

Author contribution All of the authors participated equally and have read and approved the final version of the paper.

Funding No Funding.

Declarations

Conflict of interest There are no conflicts of interest declared by the authors.

Ethical approval Any of the authors’ investigations with human participants or animals are not included in this article.

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