Factorization of the forward-backward asymmetry and measurements of the weak mixing angle and proton structure at hadron colliders

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The forward-backward charge asymmetry ($A_{FB}$) at hadron colliders is sensitive to both the electroweak (EW) symmetry breaking represented by the effective weak mixing angle $\sin^2 \theta_{\text{eff}}^{\ell}$, and the proton structure information in the initial state modeled by parton distribution functions (PDFs). Due to their strong correlation, the precisions of determination on $\sin^2 \theta_{\text{eff}}^{\ell}$ and PDFs using the measured $A_{FB}$ spectrum are limited. In this paper, we define a set of structure parameters which factorize the unique proton information of the relative difference between quarks and antiquarks in the $A_{FB}$ observation. Other than the conventional way of extracting $\sin^2 \theta_{\text{eff}}^{\ell}$ from the convolution of PDF and EW calculations, we propose a new method to simultaneously determine the value of $\sin^2 \theta_{\text{eff}}^{\ell}$ and the proton structure terms by fitting to the observed $A_{FB}$ distribution, and point out the necessity of specifying additional observations to further reduce the uncertainties on the proton structure terms respectively, so that model-independent high precision $\sin^2 \theta_{\text{eff}}^{\ell}$ and proton structure measurements can be achieved at the future LHC experiments.

**INTRODUCTION**

The forward-backward asymmetry ($A_{FB}$) of the neutral current process $f_i \bar{f}_j \to Z/\gamma^* \to f_i \bar{f}_j$ describes the relative difference between cross sections of the forward and backward scattering. It arises from the vector and axial vector couplings of $Z$ to fermions, and is governed by the effective weak mixing angle $\sin^2 \theta_{\text{eff}}^{\ell}$, which has been measured first in the electron-positron collisions at LEP and the SLC [1], then in the proton-antiproton collisions at the Tevatron [2]. At the Large Hadron Collider (LHC), the initial fermions and antifermions of the corresponding Drell-Yan (DY) process $pp (q \bar{q}) \to Z/\gamma^* \to \ell^{+} \ell^{-}$ are quarks and antiquarks, acting as partons in the proton, with their momentum distributions modeled by the parton distribution functions (PDFs). Since quarks and antiquarks would come from either side of the proton beams, the observed asymmetry at hadron colliders, denoted as $A_{FB}^h$, is diluted from its original $A_{FB}$ value. The reduction in magnitude from $A_{FB}$ to $A_{FB}^h$ directly reflects the difference between quark and antiquark momentum fractions in the proton, and can be used to constrain PDFs. Some of the earlier studies of using $A_{FB}^h$ to constrain PDFs can be found in Refs. [3-6].

Though $A_{FB}^h$ can be precisely measured at the LHC, it is difficult to use this observable to constrain the PDFs and extract a more precise value of $\sin^2 \theta_{\text{eff}}^{\ell}$, due to the strong correlation between them. On the one hand, as noted in Ref. [6], directly using the $A_{FB}^h$ observable in a typical global PDF analysis may yield a large bias induced by the limited precision of $\sin^2 \theta_{\text{eff}}^{\ell}$, which should be considered as an additional uncertainty in constraining the PDFs. On the other hand, as reported by the ATLAS, CMS and LHCb collaborations [7-9], the PDF-induced uncertainty is becoming the most dominant issue in the $\sin^2 \theta_{\text{eff}}^{\ell}$ measurements, and thus prevents the measurements of $\sin^2 \theta_{\text{eff}}^{\ell}$ at the LHC to achieve a relative precision at $\mathcal{O}(10^{-4})$. Such precision is recently achieved in the $W$ boson mass measurement by the CDF collaboration [10], which is significantly deviated from the Standard Model (SM) prediction. If such deviation hints new physics beyond the SM, it should also appear as a shift on $\sin^2 \theta_{\text{eff}}^{\ell}$. Therefore, it is essential to have a strategy to decouple the parton information from the EW prediction in the $A_{FB}^h$ observation, so that $\sin^2 \theta_{\text{eff}}^{\ell}$ and parton information can be simultaneously determined using the LHC data instead of fitting for one and fixing the other in the conventional methods.

However, such simultaneous determination is challenging in the framework of PDF global analysis, with $\sin^2 \theta_{\text{eff}}^{\ell}$ as a free parameter. Many of the old experimental data sets included in the PDF analyses were performed at the leading order in EW interaction, and those data are still important for providing PDF information [11]. To achieve a high precision of $\sin^2 \theta_{\text{eff}}^{\ell}$ determination, all the Wilson coefficients of the scattering processes included in the PDF analysis have to be updated to higher-order in EW interactions. This feature is not yet available in the current PDF global analysis programs.

In this paper, we propose a new method which factorizes the proton information relevant to the $A_{FB}^h$ measurement into a set of structure parameters. They are defined to be generally independent of any specific PDF modeling, and thus can be viewed as some new exper-
imental observables. These parameters, together with $\sin^2 \theta^\ell_{\text{eff}}$, can be determined from one single measurement of the $A_{FB}^h$ at the LHC, through a carefully-designed fitting procedure, as to be described below. The correlation between the determination of $\sin^2 \theta^\ell_{\text{eff}}$ and proton structure is automatically taken into account in the proposed analysis.

**FACTORIZATION OF $A_{FB}^h$**

The $A_{FB}^h$ at hadron colliders is observed in the Collins-Soper (CS) frame [12], which is a center-of-mass frame with the $\hat{z}$-axis defined as the bisector of the angle formed by the direction of the momentum of one incoming hadron ($H_A$) and the negative direction of the other incoming hadron ($H_B$). To decouple the original asymmetry from the proton structure information, we define two scattering angles, i.e. $\cos \theta_q$ and $\cos \theta_{\bar{q}}$, with different choices of $H_A$ and $H_B$. For the $\cos \theta_q$, $H_A$ and $H_B$ are assigned according to the directions of $q$ and $\bar{q}$, respectively. The differential cross section of the DY process can be expressed in terms of $\cos \theta_q$ as:

$$
\frac{d\sigma}{d \cos \theta_q dY dM dQ_T} = \frac{3}{8} \sum_f \alpha_f(Y, M, Q_T) \times \left\{ (1 + \cos^2 \theta_q) \right. \\
+ \left. \left( \hat{4} \right) \cos \theta_q \right\},
$$

where $Y$, $M$ and $Q_T$ are the rapidity, invariant mass and transverse momentum of the Z boson. The index $f$ denotes the flavor, and the term $\alpha_f$ is proportional to the cross section of the quark-antiquark subprocesses (with flavor $f$). The terms $\hat{4}$ and $\hat{1}$ are the normalized polar angular coefficient functions, while other angular functions are cancelled when integrated over the azimuthal angle. Both $\hat{4}$ and $\hat{1}$, as a function of $Y$, $M$ and $Q_T$, can be calculated in the perturbative expansion of the electroweak and QCD couplings. In this work, they are computed using the ResBos [13] package at approximate next-to-next-to-leading order (NNLO) plus next-to-next-to-leading logarithm (NNLL) in QCD interaction.

The canonical scales [14, 15] are set to the invariant mass of the lepton pair in the DY events. The EW calculation of ResBos is similar to the effective born approximation used in Zfitter [16]. The DY events are categorized as forward ($F$) for $\cos \theta_q > 0$, and backward ($B$) for $\cos \theta_q < 0$. It is customary to define $A_{FB}$ as:

$$
A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}
$$

where $\sigma_F$ and $\sigma_B$ are the cross sections of $F$ and $B$ events. In this case, the parton densities of the initial state quarks and antiquarks contribute only to $\alpha_f$ terms, and $A_{FB}^0$, $A_{FB}^f$ and $A_{FB}$ are independent of proton structure information. However, such $A_{FB}$ cannot be experimentally observed because the directions of $q$ and $\bar{q}$ are practically unknown.

At the LHC, the DY process is observed in terms of $\cos \theta_h$, of which $H_A$ is defined as the hadron which points to the same hemisphere aligned to the reconstructed $Z$ boson of the dilepton final state. Since the $Z$ boson is boosted along with the direction of the parton which carries higher energy, $\cos \theta_h = \cos \theta_q$ when $q$ has larger energy than $\bar{q}$, and $\cos \theta_h = - \cos \theta_q$ when $\bar{q}$ has the larger energy. We introduce the dilution factor $D_f(Y, M, Q_T)$ to represent the probability of having $\cos \theta_h = - \cos \theta_q$, i.e., the antiquark has higher energy than the quark in the hard scattering subprocess. In this manner, the differential cross section in term of $\cos \theta_h$ can be expressed as:

$$
\frac{d\sigma}{d \cos \theta_h dY dM dQ_T} = \frac{3}{8} \sum_f \alpha_f(Y, M, Q_T) \times \left\{ (1 + \cos^2 \theta_h) + \frac{1}{2} A_f^0(Y, M, Q_T)(1 - 3 \cos^2 \theta_h) \right. \\
+ \left. \left[ 1 - 2D_f(Y, M, Q_T) \right] A_f^f(Y, M, Q_T) \cos \theta_h \right\}
$$

It is clear that only the $A_f^f$ term would be affected by the dilution effect. Accordingly, the experimental observed asymmetry $A_{FB}^h$ can be formulated as follow:

$$
A_{FB}^h(Y, M, Q_T) = \frac{\sum_f [1 - 2D_f(Y, M, Q_T)] \alpha_f(Y, M, Q_T) A_f^f(Y, M, Q_T)}{\sum_f \alpha_f(Y, M, Q_T)}
$$

where $A_f^f(Y, M, Q_T)$ denotes the asymmetry calculated using $\cos \theta_q$ for each $f \bar{f}$ subprocess, which is independent with proton structure.

The derivation of Eq. 4 is straightforward and provided in Sec. S1 of the Supplemental Material [17]. The above equation reveals the nature of the observed $A_{FB}^h$, which is an average of the asymmetry contributed from each $f \bar{f}$ subprocess, weighted by its relative cross section, and the associated dilution factor reflects our limited knowledge of the $q$ and $\bar{q}$ PDFs of the proton.

Here, we assume that the parton distributions of strange quark and antiquark of the proton are the same, $s(x) = \bar{s}(x)$, though a small violation can be generated at the NNLO via DGLAP parton evolution [18]. Similarly, $c(x) = \bar{c}(x)$ and $b(x) = \bar{b}(x)$, with $c$ and $\bar{b}$ denoting charm and bottom flavor, respectively. Consequently, the dilution factors $D_d$, $D_s$ and $D_b$ equal to 0.5 in all $Z$ boson kinematic configurations, and their contributions in the numerator of Eq. 4 vanish.

Based on the argument, we propose to factorize the observed $A_{FB}^h$ as products of the proton structure parameters and independent EW-dominant parts:

$$
A_{FB}^h(Y, M, Q_T) = \left[ \Delta_s(Y, M, Q_T) + P_s^0(Y, Q_T) \right] \cdot A_{FB}^s(Y, M, Q_T; \sin^2 \theta^\ell_{\text{eff}}) + \left[ \Delta_d(Y, M, Q_T) + P_d^0(Y, Q_T) \right] \cdot A_{FB}^d(Y, M, Q_T; \sin^2 \theta^\ell_{\text{eff}}).
$$

(5)
where the structure parameters are related to Eq. (4) via

$$P^u_0(Y, Q_T) = \int \left[ \frac{1 - 2D_u(Y, M, Q_T)\alpha_u(Y, M, Q_T)}{\sum_f \alpha_f(Y, M, Q_T)} \right] dM / \int dM$$

$$P^d_0(Y, Q_T) = \int \left[ \frac{1 - 2D_d(Y, M, Q_T)\alpha_d(Y, M, Q_T)}{\sum_f \alpha_f(Y, M, Q_T)} \right] dM / \int dM$$

$$\Delta_u(Y, M, Q_T) = \left[ \frac{1 - 2D_u(Y, M, Q_T)\alpha_u(Y, M, Q_T)}{\sum_f \alpha_f(Y, M, Q_T)} \right] - P^u_0(Y, Q_T)$$

$$\Delta_d(Y, M, Q_T) = \left[ \frac{1 - 2D_d(Y, M, Q_T)\alpha_d(Y, M, Q_T)}{\sum_f \alpha_f(Y, M, Q_T)} \right] - P^d_0(Y, Q_T)$$

(6)

In Eq. 5, the $\sin^2 \theta_{\text{eff}}$-dependence of the light quark asymmetry $A^{u/d}_{FB}(Y, M, Q_T; \sin^2 \theta_{\text{eff}})$ are explicitly written out, which can be precisely predicted by the SM electroweak calculation. The structure parameters $P^u_0/Y, M, Q_T$ represent the parton information averaged in a given mass range of $M$, while the parameters $\Delta_u/Y, M, Q_T$ describe the variation around the averaged behavior provided by $P^u_0$ and $P^d_0$, respectively, in that mass range. Both $P^u_0$ and $\Delta_f(M)$ parameters are structure parameters, representing the parton information. Since $A^b_{FB}$ is usually observed in a narrow mass window around the Z pole, the relevant proton structure information and the corresponding uncertainties are dominated by $P^f_0$ parameters.

As an example, we show in the upper panel of Figure 1 the distribution of $A^b_{FB}(M)$, in the mass region of $60 < M < 120$ GeV/$c^2$, for a pseudo-data sample of the DY process produced at the 13 TeV LHC, generated using the ResBos program with CT18 NNLO PDFs [19]. In the lower panel of Figure 1, the PDF-induced uncertainty of $A^b_{FB}$ for each invariant mass bin is decomposed according to Eq. 5, for illustration. As expected, the uncertainty is dominated by those of $P^u_0$ and $P^d_0$, while the contributions from $\Delta_u(M)$ and $\Delta_d(M)$ are quite small.

By its definition in Eq. 6, the $P^f_0$ parameters contain information of the dilution effects of the light quarks and their relative cross sections. Accordingly, the values of $P^f_0$ are expected to be increasing as a function of $Y$, for the dilution effects should be reduced in the kinematic region where the difference in the quark and anti-quark energy is large. At the LHC energies, both $(1 - 2D_u)$ and $(1 - 2D_d)$ approach to 0 when $Y$ is zero, and toward 1 as the magnitude of $Y$ increases. This trend is demonstrated in Figure 2, where the $P^f_0$ values predicted by CT18 PDFs are shown as a function of $Y$ and $Q_T$. Additionally, in the high $Y$ region where the dilution effects of the $u$ and $d$ quarks are similarly small, the separation of $P^u_0$ and $P^d_0$ could be attributed to the difference in the light quark parton abundance and their corresponding cross sections. Note that if the ratio of $P^u_0$ and $P^d_0$ is employed, the information on $s$, $c$, and $b$ cross sections cancels out. Therefore, the experimental observation on such ratio can directly probe the ratio of $u$ and $d$ (anti)quark parton densities of the proton as a function of $x$. These phenomenal conclusions are also checked using predictions of MSHT20 [20] and NNPDF3.1 [21], respectively, as shown in Sec. S2 of the Supplemental Material.

FIG. 1. The spectrum of $A^b_{FB}(M)$, and the corresponding uncertainties from $P^u_0$, $P^d_0$, $\Delta_u(M)$ and $\Delta_d(M)$ terms, respectively. The central values and of $A^b_{FB}$ and the uncertainties in each $M$ bin are predicted by CT18 PDFs. The uncertainties correspond to 68% C.L. The observables are averaged over $Y$ and $Q_T$ in this figure.

SIMULTANEOUS FIT OF $\sin^2 \theta_{\text{eff}}$ AND $P^f_0$

Some progresses on reducing the PDF and EW correlation and potential bias have been made in the past. In Ref. [6], it was found that excluding the $A^b_{FB}$ data around the Z pole region in a PDF global analysis could reduce the bias. To further reduce the bias, at the expense of sensitivity, a new observable was proposed later in Ref. [22], which uses the gradient information of $A^b_{FB}(M)$ spectrum to perform the PDF global fitting, while using its average value around the Z-pole to determine $\sin^2 \theta_{\text{eff}}$. However, the residual bias given by the above two methods is still sizable. Alternatively, Ref. [23] explored the impact of the imperfect knowledge of the proton structure on the determination of EW parameters (e.g. $M_W$) via nuisance parameter formalism, by including the bin-by-bin correlation of the kinematic distributions with respect to PDF variations. Though the analysis would effectively reduce the PDF uncertainty, it relies on the information of specific PDF sets, which are not automatically updated in the process of measur-
The theoretical uncertainties associated with the input $\Delta_f$ terms in the fit (labelled as “$\Delta_f$-induced unc.”) are estimated by repeating the fitting procedure with different $\Delta_f(Y, M, Q_T)$ predictions given by the CT18 PDF error sets, instead of the central set, with $P_0^f$ and $\sin^2 \theta_{\text{eff}}^f$ as free fitting parameters, and quoting the variations of their fitted values, respectively. For comparison, the variations of $P_0^f$ values predicted by different CT18 PDF error sets, and the PDF-induced uncertainty on the $\sin^2 \theta_{\text{eff}}^f$, extracted from the conventional method of setting $\sin^2 \theta_{\text{eff}}^f$ as the only fitting parameter, are also depicted (labelled as “Original PDF unc.”), respectively. The factorization method provides a new perspective for the issue of the correlation between PDFs and $\sin^2 \theta_{\text{eff}}$.

Firstly, the $\Delta_f$-induced uncertainties arise from the variation of proton structure information in a small region of the Bjorken variable $x$, while the original PDF ones are dominated by the average magnitude of the structure parameters. Secondly, we note that the $\Delta_f$ terms do not contribute large uncertainty on $A_{FB}^h$ distribution itself, cf. Figure 1. However, when $A_{FB}^h$ is the sole data included in the fit, the $P_0^f$ and $\sin^2 \theta_{\text{eff}}^f$ are highly (negatively) correlated. To improve the resolution power of the proposed factorization form of $A_{FB}^h$, we could either introduce new $\Delta_f$-sensitive observables in the PDF global fitting to reduce the uncertainty on $\Delta_f$ itself, or add additional $P_0^f$-sensitive data in the simultaneous fit, so that the correlation between $\sin^2 \theta_{\text{eff}}^f$ and $P_0^f$ can be reduced. Of course, this kind of new $\Delta_f$ or $P_0^f$-sensitive observables, preferably at the LHC, ought to be $\sin^2 \theta_{\text{eff}}$-independent.

**CONCLUSION**

The $A_{FB}^h$ measurement at hadron colliders not only is an ideal observable for the determination of $\sin^2 \theta_{\text{eff}}$, but also provides unique proton structure information on the relative difference between quarks and antiquarks. It is essential to factorize the proton structure information with well defined observables, so that both $\sin^2 \theta_{\text{eff}}$ and the parton information can be simultaneously determined. In this article, we proposed a novel method to do just that, which is based on the factorization property of Eq. (5), and demonstrated how the method works. The $A_{FB}^h$ factorization provides a novel method to handle the proton distribution uncertainties in the electroweak measurement at the LHC, in contrast to the conventional PDF error estimation. It should also be pointed out that by introducing other observables in the same LHC data, which are sensitive to either of the two types of factorized $\Delta_f$ or $P_0^f$ terms, the precision of the simultaneous fit based on the $A_{FB}^h$ factorization could be further improved, so that the proton structure information and

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**FIG. 2.** The $P_0^a$ and $P_0^d$ values as a function of $Q_T$ and $Y$ predicted by ResBos + CT18. For the $Y$ dependence, results in the plots correspond to the $P_0^a$ values averaged over $|Y|$ in $[0, 1.0]$, $[1.0, 2.0]$, $[2.0, 3.0]$ and $[3.0, 4.0]$ bins. For the $Q_T$ dependence, the $P_0^a$ values are averaged in three regions of $[0, 0.5]$, $[0.5, 2.5]$ and $[2.5, 4.0]$ GeV, as examples.
FIG. 3. Various uncertainties of $P_0^\ell$ (upper), $P_0^d$ (middle) and $\sin^2\theta_{\text{eff}}$ (lower), as detailed in the main text. Results are given in the $Q_T$ range of [5, 10] GeV, as an example.

The EW $\sin^2\theta_{\text{eff}}$ parameter could be determined model-independently in the future LHC experiment.

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