Search for $X(3872)$ and $X(3915)$ decay into $\chi c\eta^0$ in $B$ decays at Belle

V. Bhardwaj, S. Jia, I. Adachi, H. Aihara, D. M. Asner, T. Ashevi, R. Ayad, V. Babu, I. Badhrees, S. Bhalipati, V. Bansal, P. Behera, C. Beleón, M. Berger, B. Bhuyan, T. Bilka, J. Biswal, A. Bobrov, A. Bondar, G. Bonvicini, A. Boyeck, M. Bracco, T. E. Browder, M. Campajola, L. Cao, D. Červenkov, P. Chang, V. Chekleidian, A. Chen, B. G. Cheon, K. Chilikin, H. E. Cho, K. Cho, S.-K. Choi, Y. Choi, S. Choudhury, D. Cinabro, S. Cunliffe, S. Di Carlo, Z. Dołęga, T. V. Dong, S.-K. Eldelman, D. Epifanov, J. E. Fast, T. Ferber, B. G. Fulsom, R. Garg, V. Gaur, A. Garmash, A. Giri, P. Goldenzweig, D. Greenwald, O. Grzymkowska, J. Haba, T. Hara, K. Hayasaka, H. Hayashi, W.-S. Hou, C.-L. Hsu, T. Iijima, K. Inami, A. Ishikawa, R. Itoh, M. Iwasaki, W. W. Jacobs, J. Jinn, D. Joffe, K. K. Joo, T. Julius, A. B. Kaliyar, G. Karyan, Y. Kato, T. Kawasaki, C. Kiesling, C. H. Kim, D. Y. Kim, S. H. Kim, K. Kinoshita, P. Kodyš, S. Korpar, D. Kotchetkov, P. Križan, R. Kroeger, P. Krokovny, T. Kuhr, R. Kulasiński, R. Kumar, Y.-J. Kwon, K. Lalwani, J. S. Lange, I. S. Lee, J. K. Lee, S. C. Lee, J. L. Li, B. Li, L. Giö, J. Libby, D. Liventsev, P.-C. Lu, J. MacNaughton, C. MacQueen, Masuda, T. Matsuda, D. Matvienko, M. Merola, K. Miyabayashi, R. Mizuk, G. B. Mohanty, T. Mori, R. Mussa, M. Nakao, J. N. K. Nisar, S. Nishida, S. Ogawa, H. Ono, O. Onuki, P. Pakhlov, G. Pakhlova, B. Pai, S. Pardi, H. Park, S.-H. Park, S. Patra, S. Paul, T. K. Pedlar, R. Pestotnik, L. E. Pilonen, V. Popov, E. Precinc, P. K. Resmi, M. Ritter, A. Rostomyan, G. Russo, Y. Sakai, M. Salehi, S. Sandilya, L. Santelj, T. Sanuki, V. Savinov, O. Schneider, G. Schnell, C. Schwanda, Y. Seino, K. Senyo, O. Seon, M. E. Sevor, C. P. Shen, J.-G. Shiu, B. Shwartz, F. Simon, A. Sokolov, E. Solovieva, M. Starić, Z. S. Strottler, M. Sumihana, T. Sumiyoshi, W. Sutcliffe, M. Takizawa, T. Tamponi, K. Tanida, F. Tenchini, K. Trabelsi, M. Uchida, S. Uehara, T. Uglow, S. Uno, U. Urquijo, R. Van Tonder, G. Varner, B. Wang, C. H. Wang, M.-Z. Wang, P. Wang, X. L. Wang, M. Watanabe, S. Watanuki, E. Won, S. B. Yang, H. Ye, J. Yelton, J. H. Yim, J. Zhang, Z. P. Zhang, V. Zhilich, V. Zhukova, V. Zhulanov, V. Babu.

(\text{The Belle Collaboration})

1 University of the Basque Country UPV/EHU, 48080 Bilbao
2 Beihang University, Beijing 100191
3 Brookhaven National Laboratory, Upton, New York 11973
4 Budker Institute of Nuclear Physics SB RAS, Novosibirsk 630090
5 Faculty of Mathematics and Physics, Charles University, 121 16 Prague
6 Chonnam National University, Kwangju 660-701
7 University of Cincinnati, Cincinnati, Ohio 45221
8 Deutsches Elektronen–Synchrotron, 22607 Hamburg
9 University of Florida, Gainesville, Florida 32611
10 Key Laboratory of Nuclear Physics and Ion-beam Application (MOE) and Institute of Modern Physics, Fudan University, Shanghai 200443
11 Justus-Liebig-Universität Gießen, 35392 Gießen
12 Gifu University, Gifu 501-1193
13 Il. Physikalisches Institut, Georg-August-Universität Göttingen, 37073 Göttingen
14 Sokendai (The Graduate University for Advanced Studies), Hayama 240-0193
15 Gyeongsang National University, Chinju 660-701
16 Hankyong University, Seoul 133-791
17 University of Hawaii, Honolulu, Hawaii 96822
18 High Energy Accelerator Research Organization (KEK), Tsukuba 305-0801
19 J-PARC Branch, KEK Theory Center, High Energy Accelerator Research Organization (KEK), Tsukuba 305-0801
20 Forschungszentrum Jülich, 52425 Jülich
21 IKERBASQUE, Basque Foundation for Science, 48013 Bilbao
22 Indian Institute of Science Education and Research Mohali, SAS Nagar, 140306
23 Indian Institute of Technology Bhubaneswar, Satya Nagar 751007
24 Indian Institute of Technology Guwahati, Assam 781039
25 Indian Institute of Technology Hyderabad, Telangana 502285
26 Indian Institute of Technology Madras, Chennai 600036

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The $X(3872)$ state was observed for the first time by the Belle collaboration in 2003 via its decay to $J/\psi\pi^+\pi^-$. The mass ($3871.69 \pm 0.17$) MeV/c$^2$ is consistent with a conventional charmonium state. However, the measured width ($20 \pm 10$ MeV) [2], and other properties suggest it to be a non-conventional $c\bar{c}$ state. The $X(3872)$ has also been seen in other decay modes: $D^0\bar{D}^0$, $J/\psi\gamma$, $\psi(2S)\gamma$, and $J/\psi\pi^+\pi^-\pi^0$ [3–7]. Very recently, a new decay mode, $\chi_{c1}\pi^0$, was reported by BESIII [8] in $e^+e^- \rightarrow \chi_{c1}\pi^0\gamma$. According to their measurement, $R_{X(3872)}^{\chi_{c1}/\psi} : B(X(3872) \rightarrow \chi_{c1}\pi^0) / B(\psi(2S) \rightarrow J/\psi\pi^+\pi^-) = 3.66 \times 10^{-3}$.

If the $X(3872)$ structure is dominated by a charmonium $\chi_{c1}(2P)$ component, we expect the branching fraction for the pionic transition, $X(3872) \rightarrow \chi_{c1}\pi^0$, to be very small due to isospin breaking by the light quark masses [9], significantly suppressed compared to that for $X(3872) \rightarrow \chi_{c1}\pi^+\pi^- (R \approx 4.0\%)$. The BESIII result disfavors the $\chi_{c1}(2P)$ interpretation of the $X(3872)$ and suggests instead a tetraquark or molecular state with a significant isovector part in its wave function, which results in an enhanced single-pion transition [9].

In the search for $X(3872) \rightarrow \chi_{c1}\pi^+\pi^- [10]$, the Belle Collaboration determined the branching fraction $B(B^+ \rightarrow X(3872)\bar{K}^+) \times B(X(3872) \rightarrow \chi_{c1}\pi^+\pi^-)$ to be less than $1.5 \times 10^{-6}$ at 90% confidence level (C.L.). In addition, the Belle Collaboration observed $B^+ \rightarrow \chi_{c1}\pi^0 K^+$ and published the background-subtracted $\sigma_P$ plot [11] distribution for $M_{\chi_{c1}\pi^0}$, which showed no structure at the $X(3872)$ mass. We use a similar technique to provide a limit on $R_{X(3872)}^{\chi_{c1}/\psi}$.

The $X(3915)$ was first observed, via its decay to $J/\psi\omega$, by the Belle Collaboration in $B \rightarrow J/\psi\omega K$ decay [12]. The quantum numbers of $X(3915)$ were identified to be $J^{PC} = 0^{-+} [13]$, suggesting it may be $\chi_{c0}(2P)$. If $X(3915)$ is $\chi_{c0}(2P)$, its width should be larger [14]. However, the measured width ($20 \pm 5$ MeV/c$^2$) [2] is significantly narrower than theoretical expectations ($> 100$ MeV/c$^2$). The $J/\psi\omega$ is also expected to be suppressed by the Okubo-Zweig-Iizuka (OZI) rule in the $\chi_{c0}(2P)$ scenario [15]. A $J^{PC} = 2^{++}$ assignment is also consistent with our observation [16]. If $X(3915)$ is a non-conventional $c\bar{c}$ state, then one may expect the single pion transition to be enhanced in $X(3915)$ decays as compared to charmonium, where it is suppressed due to isotopic symmetry breaking.

In the study reported here, we reproduce the previous result for $B^+ \rightarrow \chi_{c1}\pi^0K^+ [10, 17]$, search for the intermediate states $X$ (denotes $X(3872)$ and $X(3915)$), and measure the product branching fraction $B(B^+ \rightarrow X(3872)K^+) \times B(X(3872) \rightarrow J/\psi\pi^+\pi^-) = 0.88^{+0.33}_{-0.27} \times 0.10$, where the first uncertainty is statistical and the second is systematic. In comparison with conventional charmonium, this ratio seems to be large; e.g., $B(\psi(2S) \rightarrow J/\psi\pi^0) / B(\psi(2S) \rightarrow J/\psi\pi^+\pi^-) = 3.66 \times 10^{-3}$.

We use a sample of $772 \times 10^6 B\bar{B}$ events collected with the Belle detector at the KEKB asymmetric-energy $e^+e^-$ collider, operating at the $Y(4S)$ resonance [19]. The Belle detector is a large-solid-angle spectrometer, which includes a silicon vertex detector (SVD), a 50-layer drift chamber (DCD), an array of aerogel threshold Cherenkov counters (ACC), a time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter (ECL) comprised of 8736 CsI(Tl) crystals located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux return yoke located outside the coil is instrumented to detect K$^0_L$ mesons and identify muons. The detector is described in detail elsewhere [18]. Two inner detector configurations were used. A first sample of $152 \times 10^6 B\bar{B}$ events was collected with a 2.0-cm-radius beam pipe and a 3-layer SVD, and the remaining $620 \times 10^6 B\bar{B}$ pairs were collected with a 1.5-cm-radius beam pipe, a 4-layer SVD and a modified CDC [20].

We use EVTGEN [21] with QED final-state radiation by PHOTOS [22] for the generation of Monte Carlo (MC) simulation events. GEANT3-based [23] MC simulation is used to model the response of the detector and determine the efficiency of the signal reconstruction. Signal MC is used to estimate the efficiency and selection criteria for reconstructing $B^+ \rightarrow X(\rightarrow \chi_{c1}\pi^0)K^+$ decay.

We reconstruct the $B^+ \rightarrow \chi_{c1}\pi^0K^+$ decay mode with the same selection criteria as those used in the previous analysis [10]. To suppress continuum background, we require the ratio of the second to the zeroth Fox-Wolfram moment [24] to be less than 0.5. Charged tracks are required to originate from the vicinity of the interaction.
point (IP): the distance of closest approach to the IP is required to be within 3.5 cm along the beam direction and within 1.0 cm in the plane transverse to the beam direction. An ECL cluster is treated as a photon candidate if it is isolated from the extrapolated charged tracks, and its energy in the lab frame is greater than 100 MeV. We reject a photon candidate if the ratio of energy deposited in the central 3×3 square of cells to that deposited in the enclosing 5×5 square of cells in its ECL cluster is less than 0.85. This helps to reduce photon candidates originating from neutral hadrons.

The $J/\psi$ meson is reconstructed via its decay to $\ell^+\ell^-$ ($\ell = e$ or $\mu$) and selected by the invariant mass of the $\ell^+\ell^-$ pair ($M_{\ell\ell}$). For the dimuon mode, $M_{\ell\ell}$ is the invariant mass $M_{\mu^+\mu^-}$; for the dielectron mode, the four-momenta of all photons within 50 mrad cone of the original $e^+$ or $e^-$ direction are absorbed into the $M_{\ell\ell} \equiv M_{e^+e^-(\gamma)}$ to reduce the radiative tail. The reconstructed invariant mass of the $J/\psi$ candidates is required to satisfy $2.95 \text{ GeV}/c^2 < M_{\ell^+\ell^-} < 3.13 \text{ GeV}/c^2$ or $3.03 \text{ GeV}/c^2 < M_{\mu^+\mu^-} < 3.13 \text{ GeV}/c^2$. For the selected $J/\psi$ candidates, a vertex-constrained fit is applied to the charged tracks and then a mass-constrained fit is performed to improve the momentum resolution. The $\chi_{c1}$ candidates are reconstructed by combining a $J/\psi$ candidate with a photon. To reduce background from $\pi^0 \rightarrow \gamma \gamma$, a likelihood function is employed to distinguish isolated photons from $\pi^0$ daughters using the invariant mass of the photon pair, photon energy in the laboratory frame and the polar angle with respect to the beam direction in the laboratory frame [25]. We combine the candidate photon with any other photon and then reject both photons of a pair whose $\pi^0$ likelihood is larger than 0.8. For further analysis, we keep the $\chi_{c1}$ candidates with a reconstructed invariant mass satisfying $3.467 \text{ GeV}/c^2 < M_{J/\psi\gamma} < 3.535 \text{ GeV}/c^2$, which corresponds to $[-4.5\sigma, +2.8\sigma]$ about the nominal mass of the $\chi_{c1}$ [2], where $\sigma$ is the $\chi_{c1}$ mass resolution from the fit to the MC simulated $J/\psi\gamma$ mass distribution. To improve the momentum resolution a mass-constrained fit is applied to the selected $\chi_{c1}$ candidates.

Particle identification is performed using specific ionization information from the CDC, time measurements from the TOF, and the light yield measured in the ACC. Charged kaons and pions are identified using the $K$ likelihood ratio, $R_K = L_K / (L_K + L_\pi)$, where $L_K$ and $L_\pi$ are likelihood values for the kaon and pion hypotheses [26]. Kaon tracks are correctly identified with an efficiency of 94.9%, whereas the probability of misidentifying a pion as a kaon is 10.1% for $B^+ \rightarrow X (3872) (\rightarrow \chi_{c1}\pi^0)K^+$. Photon pairs are kept as $\pi^0$ candidates whose invariant mass lies in the range 120 MeV/$c^2 < M_{\gamma\gamma} < 150$ MeV/$c^2$ ($\pm 3\sigma$ about the nominal mass of $\pi^0$). To reduce combinatorial background, the $\pi^0 \rightarrow \gamma \gamma$ candidates are also required to have an energy balance parameter $|E_1 - E_2|/(E_1 + E_2)$ smaller than 0.8, where $E_1$ ($E_2$) is the energy of the first (second) daughter photon in the laboratory frame. For each selected $\pi^0$ candidate, a mass-constrained fit is performed to improve its momentum resolution.

To identify the $B$ meson, two kinematic variables are used: the beam-energy-constrained mass $M_{bc}$ and the energy difference $\Delta E$. The former is defined as $\sqrt{E_{\text{beam}}^2/c^2 - (\sum_i p_i)^2/c}$ and the latter as $\sum_i E_i - E_{\text{beam}}$, where $E_{\text{beam}}$ is the beam energy and $p_i$ and $E_i$ are the momentum and energy of the $i$-th daughter particle in the center-of-mass (CM) frame; the summation is over all final-state particles used to reconstruct the $B$ candidate. We reject candidates having $M_{bc}$ less than 5.27 GeV/$c^2$ or $|\Delta E| > 120$ MeV. After the reconstruction, an average of 1.24 $B$ candidates per event is found. When there are multiple $B$ candidates in one event, we retain only the candidate with the the lowest $\chi^2$ value defined as:

$$\chi^2 = \chi^2_V + \chi^2_{\pi^0} + \frac{(M_{\chi_{c1}} - m_{\chi_{c1}})^2}{\sigma_{\chi_{c1}}} + \frac{(M_{B} - m_B)^2}{\sigma_{M_{bc}}}$$

where $\chi^2_V$ is the reduced $\chi^2$ returned by the vertex fit of all charged tracks, $\chi^2_{\pi^0}$ is the reduced $\chi^2$ for the $\pi^0$ mass-constrained fit, $M_{\chi_{c1}}$ is the reconstructed mass of the $\chi_{c1}$, and $m_{\chi_{c1}}$ and $m_B$ are the nominal masses of the $\chi_{c1}$ and $B$ mesons, respectively. This method has 95% efficiency for selecting the true candidate.

We extract the signal yield from an unbinned extended maximum likelihood (UML) fit to the $\Delta E$ distribution. The signal probability density function (PDF) is modeled by a sum of a Gaussian function and a logarithmic Gaussian function [27]. The mean and width of the core Gaussian with larger fraction are floated and the remaining parameters of tail distribution are fixed from studies of MC simulation.

To study the background from events with a $J/\psi$, we use MC-simulated $B \rightarrow J/\psi X$ sample corresponding to 100 times the integrated luminosity of the data sample. Possible peaking backgrounds from the feed-across of $B^+ \rightarrow \chi_{c2}\pi^0 K^+$ are found in the $\Delta E$ distribution around −50 MeV, which are due to the mass-constrained fit to $\chi_{c1} \rightarrow J/\psi\gamma$ candidates; we estimate that only five such events are expected in real data. Thus, we fix this peaking background contribution in the fit. The PDF for the peaking background is modeled by an asymmetric Gaussian distribution for which the parameters are fixed according to MC simulation after MC/data correction (using the signal events whose mean and sigma of the core Gaussian are floated).

The rest of the background is combinatorial and modeled by using a first-order Chebyshev polynomial. The fit to the $\Delta E$ distribution for $B^+ \rightarrow \chi_{c1}\pi^0 K^+$ is shown in Fig. 1(a). We obtain 806 ± 69 signal events for the $B^+ \rightarrow \chi_{c1}\pi^0 K^+$ decay mode, which is consistent with our previous study [10]. In order to improve the resolution on the invariant mass of the combined $\chi_{c1}$ and $\pi^0$ candidates ($M_{\chi_{c1}\pi^0}$), we scale the energy and momentum of the $\pi^0$, such that $\Delta E$ (defined below) is equal to zero while the $M_{\pi^0}$ is kept constant to its already mass-constrained value. This corrects for the incomplete energy measurement of the $\pi^0$ detection. The corrected
The four-momentum of the \( \pi^0 \) is then used to improve the invariant mass \( M_{\chi_c \pi^0} \) and \( M_{K^+ \pi^0} \).

To search for the \( X \), we examined the background-subtracted \( M_{\chi_c \pi^0} \) distribution produced with the \( sPlot \) technique [28] for the range \( (3.75 \text{ GeV}/c^2 < M_{\chi_c \pi^0} < 4.05 \text{ GeV}/c^2) \) as shown in Fig. 1(b). Figure 1(c) shows the \( M_{K^+ \pi^0} sPlot \) distribution in the range of interest \((3.75 \text{ GeV}/c^2 < M_{\chi_c \pi^0} < 4.05 \text{ GeV}/c^2)\), where most events come from the \( K^* \) decays.

In order to extract the \( X \) signal yield, we use the \( M_{\chi_c \pi^0} \) distribution within the signal-enhanced window of \(-30 \text{ MeV} < \Delta E < 20 \text{ MeV} \) for \( B^+ \to (\chi_c \pi^0) K^+ \) candidates. We veto events from \( B^+ \to \chi_c K^+ \) decay by rejecting events with \( 791.8 \text{ MeV}/c^2 < M(K^+ \pi^0) < 991.8 \text{ MeV}/c^2 \). This requirement reduces the background by 32% with a signal efficiency of 84%. We extract the signal by performing a 1D UML fit to the \( M_{\chi_c \pi^0} \) distribution. The signal PDFs for both \( X(3872) \) and \( X(3915) \) are modeled by the sum of two Gaussians. All the PDF parameters are fixed from the MC simulation after a MC/data correction estimated from the \( B^+ \to \psi(2S) \to \chi_c \gamma ) K^+ \) sample is applied [29] (the mean and sigma of the core Gaussian were fixed after scaling, while the tail parameters were fixed from signal MC).

The efficiency (\( \epsilon \)) is estimated to be 5.35% and 5.37% for \( B^+ \to X(3872) \to (\chi_c \pi^0) K^+ \) and \( B^+ \to X(3915) \to (\chi_c \pi^0) K^+ \) using the MC simulations, respectively. This efficiency has been calibrated by the difference between MC simulation and data, as described later. A fit to the data shown in Fig. 2 results in a signal yield of \( 2.7 \pm 5.5 \) (42 \pm 14) events having significance of 0.3 \( \sigma \) (2.3 \( \sigma \)) for the \( B^+ \to X(3872) \to (\chi_c \pi^0) K^+ \) \((B^+ \to X(3915) \to (\chi_c \pi^0) K^+) \) decay mode. The systematic uncertainty (explained later) has been included in the significance calculation.

With the absence of any significant signal, we estimate an upper limit (U.L.) at 90% C.L. We apply a frequentist method that uses ensembles of pseudoexperiments. For a given signal yield, sets of signal and background events are generated according to their PDFs and fits are performed. The C.L. is determined from the fraction of samples that give a yield larger than that of data. We estimate the branching fraction according to the formula \( B = Y^{U.L.}/(\epsilon \times B_s \times N_{B\bar{B}}) \); here \( Y^{U.L.} \) is the estimated U.L. yield at 90\% C.L., \( \epsilon \) is the reconstruction efficiency, \( B_s \) is the product of secondary branching fraction taken from Ref. [2], and \( N_{B\bar{B}} \) is the number of \( B\bar{B} \) mesons in the data sample. Equal production of neutral and charged \( B \) meson pairs in the \( \Upsilon(4S) \) decay is assumed. For this assumption, an uncertainty of 1.2% is added to the total systematics.

We estimate the U.L. on the product of branching fractions \( B(B^+ \to X(3872) K^+) \times B(X(3872) \to \chi_c \pi^0) \) directly from the above MC pseudoexperiment samples. The limit includes the systematic uncertainties from efficiency, particle identification, and signal extraction method into the yield obtained by smearing the assumed values by their uncertainties. Along with that we also
smear the $N_{B\bar{B}}$ and secondary branching fraction by adding their systematic uncertainties as a fluctuation of the value used to calculate the branching fraction. Using the MC pseudoexperiment samples we estimate the U.L. (90% C.L.) on the product branching fraction as:

$$B(B^+ \to X(3872)K^+) \times B(X(3872) \to \chi_{c1}\pi^0) < 8.1 \times 10^{-6}$$
$$B(B^+ \to X(3915)K^+) \times B(X(3915) \to \chi_{c1}\pi^0) < 3.8 \times 10^{-5}$$

To measure the $R_{\chi_{c1}/\psi}^X$, we use the previous Belle measurement of $B(B^+ \to X(3872)K^+) \times B(X(3872) \to J/\psi\pi^+\pi^-) = (8.63 \pm 0.82{\text{(stat.)}} \pm 0.52{\text{(syst.)}}) \times 10^{-6}$ [30]. Some of the systematic uncertainties cancel, such as lepton identification, $B(J/\psi \to \ell\ell)$, some tracking systems, and kaon identification. The U.L. on $R_{\chi_{c1}/\psi}^X$ is estimated in the same manner as that on $B(B^+ \to X(3872)K^+) \times B(X(3872) \to \chi_{c1}\pi^0)$. We remove the cancelled systematic uncertainties and smear the pseudoexperiments with the remaining ones. We further smear $B(B^+ \to X(3872)K^+) \times B(X(3872) \to J/\psi\pi^+\pi^-)$ by its statistical uncertainty and uncancelled systematic uncertainties. For each toy sample, $R_{\chi_{c1}/\psi}^X$ is estimated for the generated $R_{\chi_{c1}/\psi}^X$. The C.L. value is then determined from the fraction of samples of pseudoexperiments having $R_{\chi_{c1}/\psi}^X$ larger than the central value of data. We estimate the U.L. to be $R_{\chi_{c1}/\psi}^X < 0.97$ at 90% C.L.

**TABLE I:** Summary of the systematics uncertainties for the $B(B^+ \to X K^+) \times B(X \to \chi_{c1}\pi^0)$ and $R_{\chi_{c1}/\psi}^X$.

| Source               | $B$ (%) | $R_{\chi_{c1}/\psi}^X$ (%) |
|----------------------|---------|---------------------------|
|                      | $X(3915)$ | $X(3872)$                  |
| Lepton identification| 2.3     | 2.2                       |
| Kaon identification  | 1.0     | 1.0                       |
| Efficiency           | 0.5     | 0.5                       |
| $BB$ pairs           | 1.4     | 1.4                       |
| B production         | 1.2     | 1.2                       |
| Tracking             | 1.1     | 1.1                       |
| $\gamma$ identification| 2.0    | 2.0                       |
| $\pi^0$ veto         | 1.2     | 1.2                       |
| $\pi^0$ reconstruction| 2.2    | 2.2                       |
| Signal extraction    | 3.0     | 3.0                       |
| Secondary $B$        | 44.4    | 44.4                      |
| Total                | 20.2    | 20.2                      |

Table I summarizes systematic uncertainties for the measured product branching fraction $B(B^+ \to X K^+) \times B(X \to \chi_{c1}\pi^0)$ and the ratio $R_{\chi_{c1}/\psi}^X$. A correction for the small difference in the signal detection efficiency between MC and data is applied for the lepton identification requirements, which are determined from $e^+e^- \to e^+e^-\ell^+\ell^-$ and $J/\psi \to \ell^+\ell^-$. $\ell = e$ or $\mu$ samples. Dedicated $D^{*+} \to D^{0}(K^-\pi^+)\pi^+$ samples are used to estimate the kaon (pion) identification efficiency correction. The uncertainty on the efficiency due to limited MC statistics is 0.5%, and the uncertainty on the number of $B\bar{B}$ pairs is 1.4%. The uncertainty on the track finding efficiency is found to be 0.35% per track by comparing data and MC for $D^* \to D^0\pi^-$ decay, where $D^0 \to K_S^0\pi^+\pi^-$ and $K_S^0 \to \pi^+\pi^-$. The uncertainty on the photon identification is estimated to be 2.0% from a sample of radiative Bhabha events. The systematic uncertainty associated with the difference of the $\pi^0$ veto between data and MC is estimated to be 1.2% from a study of the $B^+ \to \chi_{c1}(\to J/\psi\gamma)K^+$ sample. For $\pi^0$ reconstruction, the efficiency correction and systematic uncertainty are estimated from a sample of $\pi^- \to \pi^-\pi^0\nu_{\tau}$ decays. The errors on the PDF shapes are obtained by varying all fixed parameters by $\pm \sigma$ and taking the change in the yield as the systematic uncertainty. The largest uncertainty in the PDF parameterization for $X(3872)$ ($X(3915)$) is 30% ($^{+15}\%$) from fixing the mass (width) of the $X(3872)$ ($X(3915)$) to the value reported in Ref. [2]. In order to estimate the uncertainty coming from the background shape, we used a third-order polynomial and took the difference as the uncertainty. Further, we also used large fitting range and added the difference in quadrature to the uncertainty coming from signal extraction procedure. The uncertainties due to the secondary branching fractions are also taken into account. Assuming all the sources are independent we add them in quadrature to obtain the total systematic uncertainties.

To summarize, in our searches for $X(3872)$ and $X(3915)$ decaying to $\chi_{c1}\pi^0$, we did not find a significant signal. We obtained $2.7 \pm 5.5$ (42 $\pm 14$) events, with a signal significance of $0.3 \sigma$ ($2.3 \sigma$) for the $B^+ \to X(3872) \to \chi_{c1}\pi^0K^+(B^+ \to X(3915) \to \chi_{c1}\pi^0)K^+$ decay mode. We determine an U.L. on the product branching fractions $B(B^+ \to X(3872)K^+) \times B(X(3872) \to \chi_{c1}\pi^0) < 8.1 \times 10^{-6}$ and $B(B^+ \to X(3915)K^+) \times B(X(3915) \to \chi_{c1}\pi^0) < 3.8 \times 10^{-5}$ at 90% C.L. The null result for our search is compatible with the interpretation of $X(3872)$ as an admixture state of a $D^0D^{*0}$ molecule and a $\chi_{c1}(2P)$ charmonium state [9]. One can further estimate $R_{\chi_{c1}/\psi}^X < 0.97$ at 90% C.L. Our U.L. does not contradict the BESIII result [8]. This information can be used to constrain the tetraquark/molecular component of the $X$ states.

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The logarithmic Gaussian is parameterized as:

\[ Z = 0.5 \sqrt{2 \ln 2} \alpha + \mu \quad \text{and} \quad \sigma = \sqrt{\ln(2)}/2 \sigma_0 \]

where \( \alpha \) is the asymmetry, \( \sigma \) is the standard deviation, \( \mu \) is the mean, and \( \sigma_0 \) is the normalization.