Antimatter spectra from a time-dependent modeling of supernova remnants

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We calculate the energy spectra of cosmic rays (CR) and their secondaries produced in a supernova remnant (SNR), taking into account the time-dependence of the SNR shock. We model the trajectories of charged particles as a random walk with a prescribed diffusion coefficient, accelerating the particles at each shock crossing. Secondary production by CRs colliding with gas is included as a Monte Carlo process. We find that SNRs produce less antimatter than suggested previously: The positron/electron ratio \( F_{e^+/e^{+}\text{−}} \) and the antiproton/proton ratio \( F_{\bar{p}/p} \) are a few percent and few \( \times 10^{-5} \), respectively. Both ratios do not rise with energy.

Introduction—Measurements of the antimatter fraction of cosmic rays (CR) provide not only insight into CR physics itself \( [1] \), as e.g. their propagation in the galaxy, but are also valuable probes for cosmology and particle physics. In particular, the annihilation of dark matter (DM) leads to an equal injection rate of matter and antimatter particles into the Galaxy, while the CR flux from astrophysical sources is matter-dominated. A possible way to detect DM is therefore to estimate carefully the expected antimatter fluxes from astrophysical sources and to search then for any excess \( [2] \).

The PAMELA collaboration presented recently results of their measurement of the positron fraction in CRs, which is rising rapidly from 10 to 100 GeV \( [3] \). At the same time, the antiproton ratio measured by PAMELA declines above 10 GeV \( [4] \), consistent with expectations. The conventional estimate for antimatter fluxes from astrophysical sources uses as only production mechanism of antimatter the scattering of CRs on interstellar gas \( [1] \). As discussed e.g. in Ref. \( [5] \), the energy dependence of the Galactic diffusion coefficient, \( D \propto E^3 \) with \( \delta = 0.5 - 0.6 \), is inconsistent with an increase of the antimatter fraction with energy. By contrast, the spectral shape of fragmentation functions leads quite naturally to such a rise in the case of DM annihilations.

The DM interpretation of the PAMELA excess faces however several difficulties \( [2] \): First, the required rate of positron production is larger than expected for a stable thermal relic. As a consequence, either the annihilation rate has to be enhanced, or the DM particle should be unstable with the appropriate life-time. Second, in gauge boson or quark fragmentation, positron, antiproton and photon production are tied together and thus one has to postulate a DM particle annihilating only into electrons and muons. More importantly, assuming antimatter production by diffusing CRs as the only astrophysical source for antimatter falls short: Since electrons lose fast energy, the high-energy part of the \( e^{-} + e^{+} \) spectrum should be dominated by local sources as nearby pulsars, as pointed out already 20 years ago \( [6] \). Moreover, electromagnetic pair cascades in pulsars result naturally in a large positron fraction together with a “standard” anti-proton flux.

More recently, supernova remnants (SNR) were put forward as an alternative astrophysical explanation for a rising positron fraction \( [7] \): Positrons created as secondaries of hadronic interactions in the shock vicinity participate in the acceleration process and, according to Ref. \( [7] \), should thus have a flatter energy spectrum than primary electrons. It was estimated that the resulting positron fraction can explain the PAMELA excess and rise up to 50% at higher energies \( [7] \), while subsequently a similar mechanism for antiprotons was suggested in Ref. \( [8] \). Since shock acceleration in SNR is expected to be the main source for Galactic CRs \( [9] \), such a scenario has also important consequences for the interpretations of CR data as, e.g., the boron-to-carbon ratio \( [10] \).

The present work examines the production of secondary \( \bar{p} \) and \( e^{+} \) in SNRs, improving on previous studies \( [7, 8, 11] \) in two respects: First, we use a Monte Carlo (MC) approach calculating the trajectory of each particle individually in a random walk picture. This makes it easy to include interactions and the production of secondaries. Second, our approach allows us to include the time (and spatial) dependence of relevant parameters describing the evolution of a SNR as, e.g., the shock radius and its velocity, the magnetic field or the CR injection rate and to test their influence on the CR spectra. We should also stress what are not the aims of the present work: We do address neither the problem of acceleration from a microscopic point of view nor consider any feedback of CRs on the shock or the magnetic field. Although the latter processes are important to obtain accurate CR escape fluxes, we shall show that our simplified treatment leads to an upper limit on the secondary fluxes.

Simulation procedure—Shocks around SNRs are supposed to be collisionless, with charged particle scattering mainly on inhomogeneities of the turbulent magnetic field. We model such trajectories by a random walk in three dimensions with step size \( l_0(E) \) determined by an energy-dependent diffusion coefficient \( D \). Diffusion close to the shock is usually assumed to proceed in the Bohm regime with the mean free path \( l_0 \) proportional to the Larmor radius \( R_L \). Thus \( D(E) = c_0/3 = e^2 p/(3 e f_B B) \),
where \( f_B \) denotes the ratio of the energy density in the turbulent and in the total magnetic field. We neglect the coupling between CRs and the turbulent magnetic field, assuming that a layer with Bohm diffusion extends far enough into the up-stream region. For a constant magnetic field, CRs do not escape but are confined in the SNR, corresponding to an “age-limited” scenario for the CR flux from SNRs.

We describe the evolution of the shock in the rest frame of the SNR. Then the (yet unshocked) up-stream region is at rest, \( v_1 = 0 \), and has the density of the surrounding interstellar medium (ISM), \( \rho_1 = \rho_{ISM} \). Assuming a strong shock with Mach number \( M \gg 1 \), the shocked down-stream region flows with the velocity \( v_2 = 3v_{sh}/R \) and has the density \( \rho_2 = R\rho_1 \). Here, \( R \) denotes the compression ratio \( R = (\gamma + 1)/(\gamma - 1) = 4 \) for a mono-atomic gas with \( \gamma = 5/3 \). We account for the flow, adding in the down-stream region on top of the random walk an ordered movement of the particle with velocity \( v_2 \) that is directed radially outwards. Thus a particle trajectory evolves during the time step \( \Delta t = l_0/c \) as

\[
x(t + \Delta t) = x(t) + v_2 \Delta t \vartheta(r_{sh} - r) e_r + l_0,
\]

where \( l_0 \) denotes a random step, \( r_{sh} \) the position of the shock, and \( \vartheta(x) \) the step function.

Crossing the shock front, particles are accelerated. We neglect that the relative energy gain \( \xi = (E_{k+1} - E_k)/E_k \) per cycle \( k \) depends on the angle of the trajectory to the shock front, and use for simplicity that on average for a non-relativistic shock \( \xi = \frac{2}{3}(v_2 - v_1) = v_{sh}/c \). For the position \( r_{sh} \) and the velocity \( v_{sh} \) of the SNR shock we use the \( n = 0 \) case of the analytical solutions derived in Ref. [12]. These solutions connect smoothly the ejecta-dominated phase with free expansion \( r_{sh} \propto t \) and the Sedov-Taylor stage \( r_{sh} \propto t^{2/5} \). The acceleration of CRs is assumed to cease after the transition to the radiative phase at the time \( t_{max} \).

As the injected particles diffuse, electrons lose energy via synchrotron radiation and inverse Compton scattering, while protons can scatter on gas of the ISM producing secondaries that include antiprotons and positrons. Cross sections and the final state of pp-interactions are simulated using QGSJET-II [13], while we use SIBYLL 2.1 [14] for decays of unstable particles.

The last ingredient for our simulation procedure is an injection model. To ease the comparison with the results of [7], we fix the electron/proton ratio \( K_{ep} \) at injection to \( K_{ep} = 7 \times 10^{-3} \). As injection energy we use \( E_0 = 10 \text{ GeV} \). In the first model used, the injection rate

\[
\dot{N} \propto r_{sh}^2 \nu_{sh} \delta(E - E_0) \delta(r - r_{sh})
\]

is proportional to the volume swept out per time by the shock, i.e. \( \alpha = 1 \) (thermal leakage model [15]). In the second model, the injection rate \( \dot{N} \) is proportional to the CR pressure [16] and \( \alpha = 3 \). In the case of model 2, the fraction of particles injected very early is significantly larger than in model 1.

\[\text{FIG. 1: Proton spectra (black) as function of energy for two different injection models and } f_B B = 1 \mu \text{G. Additionally the contribution of protons staying until } t_{max} \text{ in the up-stream (red) and of protons in down-stream region (blue) are shown.}\]
The total flux of antiprotons produced in the acceleration zone and inside the SNR, i.e. the sum of A and B, should be simply the proton flux scaled down by a constant factor. In particular, the secondary flux of the species i is bounded by the proton interaction depth $\tau$ and the (spectrally averaged) energy fraction $\langle z_i \rangle$ transferred to i. The maximal conversion rate during the life-time of a SNR is with $\sigma_{inel} = 30 \text{mb}$ as inelastic pp cross section at 100 GeV given by $\tau = c t_{max} R_{ISM} \sigma_{inel} \approx 3 \times 10^{-3}$.

The mean energy fraction of antiprotons (plus antineutrons) is $\langle z_p \rangle \approx 0.02$, so we may expect a maximal ratio of $F_\bar{p}/F_{\bar{p}+p} \sim \langle z_p \rangle \tau \approx 6 \times 10^{-5}$. The obtained $F_\bar{p}/F_{\bar{p}+p}$ ratio shown in Fig. 2 is indeed close to this estimate.

The relative size of the partial contributions A and B can be understood considering the relation between the time $t_{acc}$ spent by protons in A, their final energy $E \propto t_{acc}$ and thus the interaction depth $\tau_A$ in A as function of energy, $\tau_A \propto t_{acc} \propto E$. In particular, it takes all the life-time $t_{max}$ of the SNR to accelerate protons to the highest energies, cf. the up-stream component in Fig. 1. For the component B, the optical depth $\tau_B$ of the parent proton is $\tau_B \propto (t_{max} - t_{acc})$, which explains why the two components A and B sum up to a flat spectrum. Note that the relative normalization of component A and B in the stationary approach of Refs. [7, 8, 11] has to be imposed by hand, since $B$ is formally infinite.

The same discussion applies to the case of positrons, with the sole exception that the primary electron flux is scaled down by the factor $K_{bp}$ and that the energy fraction transferred to positrons is $\langle z_{e^+} \rangle \approx 0.05$. The results of our simulation are shown in Fig. 3, confirming with the maximal value of $F_{e^+}/F_{e^+} \leq \approx \approx \approx \approx 10\%$ this simple picture. Note that while above $\approx 100 \text{GeV}$ the ratio $F_{e^+}/F_{e^+} \approx \approx \approx \approx 10\%$ at 10 GeV in the PAMELA data.

Up to now, we have discussed only our numerical results for constant $f_B B = 1 \mu\text{G}$ and one may wonder if a “better” choice of parameters can increase the antimatter fluxes. In particular, the analytical formula of Ref. [8] seems to imply that the contribution A increases for weaker diffusion, i.e. larger D. However, the term $D/v^2$ regulating the importance of A limits also via $t_{acc} \propto D/v^2$ the maximal proton energy. Using a constant value $f_B B = 1/20$ as in Ref. [8], this reduces $E_{max}$ by the same factor. In the stationary approach, however, $E_{max}$ is an external parameter and by increasing $E_{max}$ relative to its natural value given by $t_{acc} = t_{max}$ one enlarges the relative contribution of A. This approach has been justified as a method to account in an effective way for amplification and damping of the magnetic field.

In our time-dependent approach, we test this suggestion considering a time-dependent magnetic field: assuming that non-linear effects amplify magnetic fields in the early phase [17], with $f_B B = 100 \mu\text{G}$ before the transition to the Sedov-Taylor phase at $t_\tau = 240 \text{yr}$, while in the late stage magnetic fields are damped, $f_B B = 1/20 \mu\text{G}$.
Protons that were injected early are accelerated up to \(10^{12}\) eV and positron spectra using the injection model 2.

\[ \frac{F_{e^+}/F_{e^++e^-}}{\xi} \lesssim 1\% \text{ and } \frac{F_{\bar{p}}/F_{p+p}}{\xi} \lesssim 6 \times 10^{-5} \]

will become stronger, since also the time for interactions in the acceleration zone will be shortened. Since our maximal values of \(F_{\bar{p}}/F_{p+p}\) and \(F_{e^+}/F_{e^++e^-}\) depend only on \(t_{\text{max}}\), which is lower in escape-limited than in age-limited models, we conclude that the contribution of SNR to the observed antimatter in CRs does not lead to rising antimum fractions and is smaller than estimated in earlier works.

Summary—We calculated the energy spectra of CRs and their secondaries produced in a supernova remnant using a simple random walk picture. In contrast to a previous prediction that the positron fraction \(F_{e^+}/F_{e^++e^-}\) can rise up to 40%-50% for \(K_{ep} = 7 \times 10^{-3}\), we found that the ratio levels off at a few percent. This value corresponds to the expectation combining the interaction depth \(\tau \approx 3 \times 10^{-3}\) of a proton during the life-time of a SNR with the energy fraction \(z \approx 0.05\) transferred to positrons. Similarly, the antiproton ratio \(F_{\bar{p}}/F_{p+p}\) does not rise beyond few \(10^{-5}\). Our results suggest that antiproton production in SNRs cannot explain the rise of the positron fraction observed by PAMELA. Since a rising antiproton fraction is neither expected from CR interactions with the ISM nor from pulsars, such a measurement could be used as a signature for dark matter.

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Note also that, in contrast to the assumptions of Refs. [9, 11], the average energy fraction per single antiproton (or positron) \(\xi_i = \langle z_i/n_i \rangle\) decreases strongly with energy, since the multiplicity \(n_i\) in pp interaction increases fast.