Constraints on the Mass Accretion Rate onto the Supermassive Black Hole of Cygnus A Using the Submillimeter Array

Wen-Ping Lo1,2, Keiichi Asada2, Satoki Matsushita2, Masanori Nakamura2,3, Hung-Yi Pu2,4,5, Chihyin Tseng1,2, Kazunori Akiyama6,7,8,9,13, Juan Carlos Algabe10,11, Geoffrey C. Bower12, Ramprasad Rao12, Jun Yi Koay2, and Makoto Inoue2

Abstract

We present the first detailed polarimetric studies of Cygnus A at 230 GHz with the Submillimeter Array (SMA) to constrain the mass accretion rate onto its supermassive black hole. We detected the polarized emission associated with the core at a fractional polarization of 0.73 ± 0.15%. This low fractional polarization suggests that the polarized emission is highly depolarized. One of the possible explanations is due to a significant variance in the Faraday rotation measure within the synthesized beam. By assuming the Faraday depolarization caused by inhomogeneous column density of the magnetized plasma associated with the surrounding radiatively-inefficient accretion flow within the SMA beam, we derived the constraint on the mass accretion rate to be larger than 0.15 $M_\odot$ yr$^{-1}$ at the Bondi radius. The derived constraint indicates that an adiabatic inflow–outflow solution or an advection-dominated accretion flow should be preferable as the accretion flow model in order to explain the jet power of Cygnus A.

1. Introduction

Active galactic nuclei (AGNs) are ultra-luminous sources in the universe, powered by the mass accretion processes onto their central supermassive black holes (SMBHs) causing a variety of dynamic phenomena. Unveiling the nature of the mass accretion process onto the SMBHs is one of the primary interests of modern high energy astrophysics. The mass accretion rate ($\dot{M}$) is a crucial parameter to describe the mass accretion process, significantly affecting the physical properties of the accretion flow (e.g., Yuan & Narayan 2014).

Low-luminosity AGNs (LLAGNs) are a subclass of AGNs that can be characterized by their bolometric luminosity $L_{bol} < 10^{2\pm L_{Edd}}$, where $L_{Edd}$ is the Eddington luminosity. Accreting materials around the central region of SMBHs in LLAGNs are considered to be hot, optically thin, and have geometrically thick radiatively-inefficient accretion flow (RIAF, see, e.g., Yuan & Narayan 2014).

The Faraday rotation measure (RM), a tracer of the magnetized plasma density, is a powerful tool to investigate the mass accretion process in the vicinity of the SMBHs of LLAGNs (Bower et al. 2003; Marrone et al. 2006; Kuo et al. 2014; Plambeck et al. 2014; Bower et al. 2017). Based on several physical assumptions, such as equipartition in magnetic energy density and particle thermal energy (see the references described above), the RM method is capable of constraining the electron number density, which is a critical parameter to derive the mass accretion rate.

Currently, there are only limited observations at millimeter/submillimeter (mm/submm) wavelengths despite the importance of the RM observations toward the LLAGNs. RMs have successfully constrained the mass accretion rate for the Galactic center Sgr A* (Marrone et al. 2006; Bower et al. 2018) and several LLAGNs such as 3C 84 (Plambeck et al. 2014) and M 87 (Kuo et al. 2014). While RM studies were used to probe the mass accretion rate onto SMBH, we note there are other interpretations for the origin of the RMs, such as a turbulent accretion flow model (Pang et al. 2011) or from the sheath or boundary layer of the jet (Jorstad et al. 2007; Plambeck et al. 2014; Lico et al. 2017; Park et al. 2019). Recent observations at mm wavelengths toward nearby LLAGN M 84 and M 81 provide no detection of the polarized emission (Bower et al. 2017). To further investigate the origin of the RM, it is...
necessary to extend the current studies to other sources hosting a potential RIAF with a powerful jet.

In this paper, we report the first detailed polarimetry observation toward Cygnus A at mm wavelengths. Cygnus A is a radio galaxy at a distance of 244.7 Mpc and possesses a SMBH at its center with mass $M_{\text{BH}} \approx (2.5 \pm 0.7) \times 10^6 M_\odot$ (Tadhunter et al. 2003). The corresponding angular distance of Cygnus A is $1'' \sim 1 \text{kpc} \sim 3.3 \times 10^6 r_s$, where $r_s = 2GM_{\text{BH}}/c^2$ is the Schwarzschild radius. Cygnus A is a marginal sub-Eddington source with $L_{\text{bol}} / L_{\text{Edd}} \sim 10^{-2}$ (Vasudevan et al. 2010), which is believed to be a low-luminosity AGN. It has not been clear if Cygnus A hosts either a cold and geometrically thin accretion disk or a RIAF. It is also possible that these two kinds of accretion flows may co-exist (Honma 1996; Esin et al. 1997; Yuan & Narayan 2014). Besides the accretion properties, Cygnus A is a well-studied radio galaxy at multi-wavelengths (e.g., Carilli & Barthel 1996).

The jet power of Cygnus A was measured to be $L_{\text{jet}} \approx 3.9 \times 10^{45} \text{ erg s}^{-1}$ (McNamara et al. 2011), while the jet power of M87 and 3C84, whose mass accretion rates were constrained by RMs with mm polarimetric observations, is $L_{\text{jet}} \sim 10^{42–43} \text{ erg s}^{-1}$ (Fujita et al. 2014). Since the jet power of Cygnus A is $10^5$ times larger than the other LLAGNs, the RM measurements will be critical to investigate the connections between inflow and outflow for a more powerful source.

The polarized emission of Cygnus A was intensively studied for arcsecond-scale hotspots and radio lobes in the 1980s and 1990s with radio interferometers and revealed highly polarized structure (e.g., Perley et al. 1984; Dreher et al. 1987; Carilli & Barthel 1996). Wright & Birkinshaw (2004) detected the core and the hotspots at 230 GHz with mm interferometry. Ritacco et al. (2017) report the polarimetric observations of Cygnus A at 150 and 260 GHz, which are non-detection of the polarized flux around the nuclear region. We note that their observations are for calibrations on the NIKA camera of the IRAM 30 m telescope. Very long baseline interferometry (VLBI) observations at 15.4 GHz show the fractional polarization to be less than 1% toward its nuclei region (Middelberg 2004), while Lopez-Rodriguez et al. (2014, 2018) show infrared (IR) polarimetric observations from $8.7 \mu m$ to $89 \mu m$, measuring the fractional polarization of $\sim 10\%$. Because of the large uncertainties in the observed polarization angles, IR measurements alone are insufficient to derive RM (see Section 4 for more discussion). In addition to that, the IR polarization emissions presumably originate from the aligned dust grains far outside the core region ($\sim 350$ pc). RM constrained by the IR observations may not be relevant to investigate the mass accretion process onto SMBH. Therefore, it is necessary to have dedicated polarimetric observations at mm wavelength in order to constrain the mass accretion rate through RM studies.

Also, the IR measurements are probing a region that is $\sim 100$ times larger than the 230 GHz core. In this paper, we present the first detailed polarimetric observations of Cygnus A at mm wavelengths.

This paper is organized as follows. In Section 2, we summarize our Submillimeter Array (SMA, see Ho et al. 2004) observations together with our data calibration procedures. In Section 3, we show our main results. The discussions and physical explanations of the results are provided in Section 4 and are summarized in Section 5.

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2. Observations and Data Calibrations

2.1. Polarimetric Observations in 2015

Cygnus A was observed with the SMA at 230 GHz on 2015 May 20 and 21. The observations utilized seven 6 m antenna in the compact array configuration in the polarization mode. The total on-source time for Cygnus A was 11 hr over two days. The SMA receivers operated in double-sideband mode with each sideband having a bandwidth of 4 GHz. Initially, we split each sideband into two 2 GHz wide intermediate frequencies (IFs) sub-bands and treated each one as an independent band to derive the RM. The center frequencies of those IFs are at 229.1 and 234.1 GHz in the upper sideband (USB) and 215.9 and 220.9 GHz in the lower sideband (LSB), respectively. The SMA polarimeter utilizes positionable quartz and sapphire quarter-wave plates to observe all four polarized correlations (LL, LR, RL, and RR) by changing the polarization between the left-hand and right-hand circularly polarized feeds in a period 16 Walsh function patterns. More detailed discussion on the SMA polarimetry system can be found in Marrone (2006).

Data were calibrated using the MIR-IDL data reduction packages, and included data flagging, absolute flux, bandpass, and gain calibrations. Neptune was used for the flux calibration, while the gain phases and amplitudes were calibrated by the quasars 2015 + 371, and the bandpass was calibrated using the quasar 3C 273.

The polarization calibration and imaging were performed in the MIRIAD (Wright & Sault 1993) software. We observed 3C 273 and 3C 279 as the polarization calibrators with extensive coverage of parallactic angles to solve the polarization leakage solutions (the instrumental imperfections, so-called D-terms). We derived three different sets of leakage solutions by using 3C 273, 3C 279, and Cygnus A for each solution, respectively. The D-terms were in excellent agreement among these three sources. The mean values of the leakage solutions by different polarization calibrators were approximately 1%–2% in the upper sideband and 5%–6% in the lower sideband. To validate the appropriateness of the polarization leakages, we estimated the difference between the leakage terms derived by 3C 273 and 3C 279 to be $\sim 0.1\%$. We applied the solutions derived with 3C 273 and 3C 279 to Cygnus A independently, and further confirmed the consistencies of the results. We also calibrated the polarization without splitting into sub-bands (hereafter, merged data set) to achieve a higher sensitivity.

Stokes I, Q, U, and V images were obtained for the four sidebands and also for the merged data set for better sensitivity. We conducted the deconvolution to the Stokes I images by using the CLEAN algorithm in MIRIAD using natural weighting. The synthesized beam for the continuum images is $1''3 \times 3''1$, with position angle $-50.6^\circ$.

2.2. Continuum Flux Measurements in 2017

We also conducted photometric observations toward Cygnus A at 230/240 GHz and 240/345 GHz simultaneously using the new wide-band SMA (wSMA) on 2017 September 13 and 15, respectively. The observed bandwidth is 8 GHz per receiver, centered at 217.5 GHz and 233.5 GHz for the 230 GHz receiver, 225.5 GHz and 241.5 GHz for the 240 GHz receiver, and 337 GHz and 353 GHz for the 345 GHz receiver.

The calibration procedures are similar to the steps mentioned above in Section 2.1 except for the polarization leakage...
calibrations. The synthesized beams are $1\arcsec 1 \times 1\arcsec 1$ at 230 GHz, and $0\arcsec 8 \times 0\arcsec 8$ at 345 GHz. The peak flux densities are measured, and the spectral indices, $\alpha$ (defined by $F \sim \nu^\alpha$), are fitted to data. We quote a nominal 20% flux density error due to the flux calibration using planets (e.g., Liu et al. 2017). The systematic error can explain the apparent flux variations between two observing days due to flux calibrations.

3. Results

3.1. Polarization Percentage

The Stokes images of the individual sidebands are shown in Figure 1. We summarize the flux densities for Stokes $I$, $Q$, $U$, $V$, $P$, and $m$ at the position of the peak in the Stokes $I$ image at each sideband frequencies and the associated noise levels in Table 1. $P$ and $m$ are the linear polarized flux densities and fractional polarization, which are defined as,

$$P = \sqrt{Q^2 + U^2},$$

$$m = \frac{P}{I}.$$  (1)

Cygnus A is detected as a point source with a typical peak flux density of $820 \text{ mJy beam}^{-1}$ in the Stokes $I$ images at frequencies ranging from 215.9 GHz to 234.1 GHz, where the noise levels are typically around $2.6 - 2.7 \text{ mJy beam}^{-1}$, as is presented in Figure 1. On the other hand, the polarized emission at Stokes $Q$ and $U$ at each sub-band are significantly weaker than those in Stokes $I$. Those are typically less than $7 \text{ mJy beam}^{-1}$, which correspond to 2.0–4.3$\sigma$. To enhance the sensitivity, we further merged the data sets of the individual sidebands to suppress the noise level of images (see Figure 2). By adding four sideband images, the noise levels are suppressed to be around $\sim 1.3 \text{ mJy/beam}$ for the image of linear polarization. As a result, weakly polarized emissions are seen toward the peak of the Stokes $I$ images with linear polarized flux density of $\sim 6 \text{ mJy}$ in the band-merged data set (see also Table 1).

We expect that the noise distribution of linear polarized flux density follows the Rayleigh distribution if no significant polarized emission is detected. In this case, the polarized flux density distribution will follow,

$$f(P; \sigma_{QU}) = P/\sigma_{QU}^2 \exp(-P^2/(2\sigma_{QU}^2)),$$

where $\sigma_{QU}$ is the standard deviation of noises in $Q$ and $U$ maps. In Figure 3, we show the histogram of polarized brightness of the merged image on a semi-log scale. While the observed distribution is in good agreement with the Rayleigh distribution, we see a clear excess above $5 \text{ mJy beam}^{-1}$, where the signal-to-noise ratio is $3.8$. This excess is associated with the region where we have the peak in Stokes $I$ image.

To evaluate if this weak polarized emission is due to imperfect D-term calibration or not, we performed Monte-Carlo calculations to simulate the imperfect instrumental polarimetric calibration (see the Appendix). The expected uncertainty of instrumental polarimetric calibration can cause only $\sim 0.1 \text{ mJy}$ change in polarized flux, which is significantly smaller than the thermal noise ($\sim 1 \text{ mJy}$). In addition to that, while the prominent polarized emission is observed in only one sideband at 220.9 GHz, the marginally detected polarized emission is seen in the other three bands with the signal-to-noise ratio greater than 2.0 at the peak location of Stokes $I$ image. Therefore, we concluded that we detected linear polarized emission toward Cygnus A. The estimated fractional polarization at 230 GHz for the merged data set is $0.73 \pm 0.15\%$, which is derived by averaging the fractional polarization calibrated by two independent polarization calibrators. In the following section, we will use the fractional polarization to constrain the accretion model and discuss the mass accretion rate of Cygnus A.

3.2. Spectral Index Fitting

We show the photometric observations results in Table 2. The simultaneous observations at 230 and 345 GHz bands in 2017 enabled us to estimate the spectral index as $\alpha = -0.73 \pm 0.12$ (see also Figure 4). The fitting is conducted with the least square method assuming flux densities are sampled from Gaussian distribution with the mean as the average observed flux and the standard deviation as the measured errors. The measurement errors are the 20% of the measured flux, which are described in Section 2.2. We represent the fitted $\alpha$ with the mean and the standard deviation derived from the iterative procedures after it converges. We also conducted the fitting, including measured flux density in 2015 at 230 GHz, and it gives the spectral index $\alpha = -0.82 \pm 0.12$. Negative spectral indices derived from both fittings indicate that the source is dominated by optically-thin synchrotron emission in this wavelength.

4. Discussion

4.1. Possible Depolarization Schemes

The peak flux density of Cygnus A is detected at around $800 \text{ mJy at} 230 \text{ GHz in} 2015$ and $600-700 \text{ mJy in} 2017$ at $230 \text{ GHz}$, while previous measurements (Wright & Birkinshaw 2004) showed a flux density associated with the nucleus to be $390 \text{ mJy as observed with the BIMA array}$. Their observations were done in 2000 and 2001, which were 15 yr before ours (mid-2015) with the angular resolution of $\sim 1\arcsec$, which is comparable to ours. The intrinsic variability of the AGN can explain the difference between the peak flux measurements. This suggests that most of the flux measured by the SMA is most likely associated with the AGN activity. Based on our flux measurements, we therefore argue that the AGN core emission is coming from the jet base that is synchrotron radiation dominated and should be intrinsically highly polarized.

Our SMA observations show a low level of fractional polarization with $0.73 \pm 0.15\%$ at $230 \text{ GHz}$. The significantly lower fractional polarization at $230 \text{ GHz}$ cannot be explained by optically-thick synchrotron emission, which has an intrinsically low polarization fraction (Rybicki & Lightman 1986), since the measured spectral index between 230 and 345 GHz indicates optically-thin emission. Therefore, the small polarization fraction is most likely attributed to a strong depolarization at $230 \text{ GHz}$.

The depolarization scheme can help to explain the results and constrain the physical parameters such as mass accretion rate. The depolarization is assumed to be caused by various mechanisms or a combination of them. The mechanism that affects the polarization of emitting photons can be either an external emission screen or intrinsic depolarization mechanisms. Polarized emissions associated with the jet are detected with VLA observations (Carilli & Barthel 1996), which
Figure 1. Stokes $I$, $Q$, $U$, and $P$ images of Cygnus A for separated sidebands in grayscale. The contour images are the Stokes $I$ image of each sideband, and the contour levels are 200, 400, 600, 800, 1000, 1200, 1400 times $2.6 \text{ mJy}$, which is the noise level. The $P$ image is defined by Equation (1). The grayscale bar of each image is in the unit of Jy. We show two images for the same sidebands to demonstrate the consistency for the choice of the difference instrumental polarization calibrators (3C 273 or 3C 279). The corresponded Stokes image, sideband, and polarization calibrators are labeled in each image’s captions, and the frequencies can be referenced in Table 1.
indicates that the intrinsic mechanisms are less likely to depolarize the emission. Externally, there are two ways that may cause the depolarization in the frequency or spatial domains. The former one refers to the bandwidth depolarization, and the latter one refers to beam depolarization. Both of these are the cancellation of polarized properties due to structures within the observing kernel.

For a Faraday rotator screen with dense electron density, the rapidly changing polarization angle with frequency may cause the smearing of polarization fraction within the wide observing bandwidth. With the 2 GHz bandwidth at 230 GHz in our SMA observations, the foreground screen has to have a Faraday rotation measure screen greater than $10^3$ rad m$^{-2}$ (Bower et al. 2017) to suppress the polarized emission via bandwidth depolarization. While this effect is physically possible, the beam depolarization can provide tighter constraints on RM and the mass accretion rate, which we describe next.

By observing an extended source, with non-uniformly distributed polarization angles that is unresolved with the synthesized beam, its net polarized signal will be canceled out. This is the scheme of the beam depolarization. In the case that this non-uniformly distributed polarization angle is derived by Faraday rotation associated with the non-uniform distributed foreground medium, it is called Faraday beam depolarization. If we take the uncertainty on the formula for Faraday rotation of polarization position angles $\rho = \alpha$ leads to depolarization, $\sigma_{\alpha} = \sigma_{\rho} \sqrt{\rho^{2} + \theta^{2}}$, with $\rho$ being the observed polarization angle, $\rho_{0}$ and RM as two free parameters, and $\lambda$ is the observed wavelengths. We assume that the observed $\rho$ follows a Gaussian distribution with the mean and standard deviation described in Lopez-Rodriguez et al. (2018). The calculations are performed with 10,000 realizations and the mean and 1-sigma error of RM are derived upon it. This provides a limit of $(4 \pm 7\%) \times 10^2$ rad m$^{-2}$. The average value of RM $(4 \times 10^2$ rad m$^{-2})$ is higher than $1.8 \times 10^6$ rad m$^{-2}$, which does not rule out the possibility of beam depolarization in 230 GHz, if we assume the plasma properties do not differ much in timescales of years and also in the frequency domain.

The Faraday rotation measure associated with the jet and hotspots of Cygnus A is suggested to be $\sim 10^5$ rad m$^{-2}$ (Dreher et al. 1987; Carilli & Barthel 1996). Snios et al. (2018) provide the detailed electron density measurements at the arcsecond scale with Chandra observations, estimated to be $10^{-2}$ to $10^{-3}$ cm$^{-3}$. This implies RM is around $10^5$ to $10^6$ rad m$^{-2}$ with a typical magnetic field strength value of $10 \mu$G (e.g., Carilli & Barthel 1996). This RM value is too small to be the origin of the beam depolarization of the nuclear region. Therefore, an additional foreground Faraday rotation measure screen is necessary to provide RM $\sim 10^6$ rad m$^{-2}$.

### 4.2. Constraints on $M$ with a Beam Depolarization Scenario and RIAF model

In this section, we consider a beam depolarization by assuming a RIAF model as the foreground medium based on Marrone et al. (2006) in order to constrain the mass accretion rate. We consider the case that polarized emission originates along the jet and consider the Faraday beam depolarization is caused by the Faraday screen of the magnetized plasma associated with the accretion flow.

In Figure 5, we show a schematic plot of the assumed structure. The jet is considered to be the source for the polarized emission. $z$ is the distance from the nuclei along the jet axis, and a viewing angle of the jet is assumed to be 80$^\circ$ (Krichbaum et al. 1998). We assume the jet is surrounded by the RIAF and its outermost radius, $r_b$, corresponds to the Bondi radius of the disk at $z = z_0$.
The RM integrated along the line of sight (LOS) toward each jet position $z$ is expressed as,

$$\text{RM}(z) \propto \int_{z}^{\infty} n(s) B_{\text{LOS}}(s) ds.$$  \hspace{1cm} (2)

We assume that the physical size of the Bondi radius corresponding to the sphere of gravitational influence, and we adopted $2.6 \times 10^5 r_s \sim 78$ pc (Fujita et al. 2014) in this work.

By considering a simple RIAF model (e.g., Marrone et al. 2006), the electron number density distribution of the accretion flow follows the radial profile as,

$$n(r) = n_0 \left( \frac{r}{r_s} \right)^{-\beta},$$  \hspace{1cm} (3)

where $\beta$ is the power-law indices indicating different sub-models of RIAF: $\beta = 1.5$ is for advection-dominated accretion flow (ADAF: Narayan & Fabian 2011), 0.5 for convection-
dominated accretion flow (CDAF: Igumenshchev & Narayan 2002), and between these two values are adiabatic inflow–outflow solutions (ADIOS: Blandford & Begelman 1999) and \( n_s \) is the number density at the Schwarzschild radius.

We use the velocity profile of the accretion flow from a self-similar solution of ADAF (e.g., Narayan & Yi 1995; Yuan & Narayan 2014),

\[
v(r) \simeq 0.04 \left( \frac{r}{r_s} \right)^{-1/2},
\]

where we assume the viscosity parameter as 0.1 to derive from Equation (8) of Yuan & Narayan (2014).

The mass accretion rate is a function of the number density (Equation (3)) and velocity distribution (Equation (4)), which can be written as

\[
\dot{M}(r) = 4\pi r^2 m_{H} n(r) v(r),
\]

where \( m_{H} \) is the mass of the proton. We assume that the magnetic field along the LOS also follows a power-law distribution,

\[
B_{\text{LOS}}(r) = B_{r} \left( \frac{r}{r_s} \right)^{-\kappa},
\]

where \( B_{r} \) is the magnetic field strength at the Schwarzschild radius. We note that if the system is in equipartition with particle kinetic energy, gravitational energy, and magnetic field energy (Melia 1992), then,

\[
\kappa = \frac{\beta + 1}{2}, \quad B_{r} = \sqrt{4 \pi c^2 m_{H} n_{s}},
\]

We note that adopting an equipartition assumption oversimplifies the system, and may result in a nonphysical \( B \)-field that does not obey \( \nabla \cdot B = 0 \) for some cases as recognized by Melia (1992). It may require processes such as dissipation by the magnetic reconnection, or particular choice for the power-law indices of \( \kappa \) and \( \beta \). With this configuration, RM at each jet position can be expressed as a function of the power-law indices indicating different sub-model (\( \beta \)), the mass accretion rate at the Bondi radius (\( \dot{M}_{\text{in}} \)), and the innermost radius where the electron temperature becomes relativistic, and then contribution on the RM is negligible (\( r_{\text{in}} \)). We numerically derived RM toward each jet position.

Following the brightness temperature arguments in Kuo et al. (2014), we estimate the physical size of the innermost region to be \( \theta \approx 0.09 \) mas, which corresponds to \( r_{\text{in}} = 350 \) \( r_s \) using the core flux (204.8 \pm 0.15 mJy) of 86 GHz VLBI observations (Boccardi et al. 2016). In addition to that, if we estimate \( r_{\text{in}} \) by extrapolating the gas temperature at the Bondi radius of 0.66 keV \( \sim 7.4 \times 10^6 \) K (Fujita et al. 2014) with \( T \propto r^{-1} \) (Yuan & Narayan 2014), it gives the gas temperature of \( T \approx 5 \times 10^6 \) K at 350 \( r_s \), which is in good agreement with the expected innermost radius based on brightness temperature arguments.

Since the polarization angle (PA) of the polarized emission will be rotated when passing through the accretion flow and the amount of rotation should be different at each jet position, we expect beam depolarization and constrain the polarized flux density with the RM distributions. Assuming the initial polarization angle is 0 and the rotated one will follow the equation PA (\( z \)) = RM (\( z \)) \( \lambda^2 \). Therefore, we construct the Stokes Q and U linear density as a function of \( z \), and represent them as (Thompson et al. 2017):

\[
Q(z) = P_0 \cos[2PA(z)] = P_0 \cos[2RM(z)\lambda^2],
\]

\[
U(z) = P_0 \sin[2PA(z)] = P_0 \sin[2RM(z)\lambda^2],
\]

where \( P_0 \) is the polarimetric linear density, which is assumed to be uniformly distributed with the upper limit of the intrinsic polarization of 70% (Rybicki & Lightman 1986). This is equivalent to suppose that the magnetic field in the emitting region is well aligned, thus also giving tighter constraints on the lower bound of the mass accretion rate. Therefore, \( P_0 \) can be related to the total flux linear density \( I_0 \) with polarization fraction \( \delta = P_0/I_0 \). The observed \( I_{\text{obs}}, Q_{\text{obs}}, \) and \( U_{\text{obs}} \) corresponding to the integration toward \( I_0, Q(z), \) and \( U(z) \) within the polarized region \([-r_{\text{out}}, +r_{\text{out}}] \) are,

\[
I_{\text{obs}}(\beta, \dot{M}_{\text{in}}) = \int_{-r_{\text{out}}}^{+r_{\text{out}}} I_0 dz,
\]

\[
Q_{\text{obs}}(\beta, \dot{M}_{\text{in}}) = \int_{-r_{\text{out}}}^{+r_{\text{out}}} Q(z) dz = \delta I_0 \int_{-r_{\text{out}}}^{+r_{\text{out}}} \cos[2RM(z; \beta, \dot{M}_{\text{in}}, r_{\text{in}})\lambda^2] dz,
\]

\[
U_{\text{obs}}(\beta, \dot{M}_{\text{in}}) = \int_{-r_{\text{out}}}^{+r_{\text{out}}} U(z) dz = \delta I_0 \int_{-r_{\text{out}}}^{+r_{\text{out}}} \sin[2RM(z; \beta, \dot{M}_{\text{in}}, r_{\text{in}})\lambda^2] dz,
\]

where \( r_{\text{out}} \) corresponds to the outskirt of the emitting region of Cygnus A jet that is unknown at 230 GHz. The integrations are from \(-r_{\text{out}}\) to \(+r_{\text{out}}\) since we consider both the approaching and the counter jets. Here, we adopt the detected region of the Cygnus A jet in Boccardi et al. (2016) at 86 GHz, which is \( r_{\text{out}} = 5 \) mas \( \sim 16,000 r_s \).

Under the assumption of the beam depolarization (Section 4.1), the polarized flux density will be canceled out inside the synthesized beam. With the polarization fraction
being 0.73% in our SMA observation, the mass accretion rate can be constrained. Since the SMA synthesized beam is three orders of magnitude larger than the VLBA beam at 86 GHz, the observed polarization fraction can be represented by smearing all over the emitting jet region, derived as,

$$ P_{\text{obs}}(\beta, M_{\text{rb}}) = \frac{\sqrt{Q_{\text{obs}}^2 + U_{\text{obs}}^2}}{I_{\text{obs}}} = 0.73\%. $$

We perform the computations based on the above model with fixed $r_{\text{in}} = 350 r_s$ and $r_{\text{out}} = 16000 r_s$. The calculations are conducted numerically since there is no analytic solution when combining Equations (2), (9), and (10). The integral scheme we adopted is the composite Simpson’s rule (using SciPy package), and the integration spacing is $10^7$ data points for $z$ in $[-r_{\text{out}}, r_{\text{out}}]$. The spacing is adopted with the considerations of computational resources and required resolutions. We perform the calculations with different spacing to see whether it converges or not and spacing of $10^7$ is acceptable. The left panel of Figure 6 demonstrates that the observed polarization fraction varies with $\beta$ the RIAF parameter. The right panel of Figure 6 shows the $M_{\text{rb}}$ with different parameters ($\beta$) indicates a different state of RIAF. To have the fractional polarization as 0.73%, we require $M_{\text{rb}}$ to be 0.15 $M_\odot$ yr$^{-1}$ for $\beta = 1.5$ and 2.25 $M_\odot$ yr$^{-1}$ for $\beta = 0.5$. The derived $M_{\text{rb}}$ is consistent in the order of magnitudes with that in Fujita et al. (2014), which is 0.1 to 3 $M_\odot$ yr$^{-1}$.

Zamaninasab et al. (2014) estimated the accretion power of Cygnus A to be $L_{\text{acc}} \simeq L_{\text{disk}}/\eta = 9.28 \times 10^{44}$ erg s$^{-1}$ assuming the conversion efficiency of accretion power into the disk luminosity ($\eta$) to be 0.4. If we compare it with the jet power estimated with the PdV work of $3.9 \times 10^{45}$ erg s$^{-1}$ (McNamara et al. 2011), the feedback efficiency between the jet power and the accretion power exceeds $L_{\text{jet}}/L_{\text{acc}} \simeq 419\%$. A feedback efficiency greater than 419% is far beyond the gravitational binding energy ($\sim 30\%$) released by the standard thin disk around an extremely spinning black hole ($a = 0.998$, Thorne 1974), and only explainable solution so far is the magnetically arrested disk model with RIAF (MAD, see e.g., Tchekhovskoy et al. 2011). In this scenario, the spin of the black hole plays a crucial role to provide sufficient power for the jet. The quasi-symmetric limb-brightened jet emission nature (Boccardi et al. 2016) also suggest the possibility of the spinning black hole (Takahashi et al. 2018).

Further, we use the scaling relation between $\dot{M}$ and the distance to the black hole in the RIAF together with the jet power derived by McNamara et al. (2011). Based on the RIAF solution, we expect a substantial amount of the decrease of $\dot{M}$ for some of the RIAF sub-models toward the innermost region as follows (see also Figure 7),

$$ M(r; \beta, M_{\text{rb}}) = M_{\text{rb}} \left( \frac{r}{r_B} \right)^{1.5 - \beta}. $$

In the context of the MAD scenario, the mass accretion rate decreases via a power law as approaching the black hole, and is suppressed around $10 r_s$ (see, e.g., Sadowski et al. 2013; Nemmen & Tchekhovskoy 2015; Yuan et al. 2015). Based on the MAD, up to 300% of accretion power ($M c^2$) at $10 r_s$ can be converted to be the jet power (see e.g., Tchekhovskoy et al. 2011). With the jet power of $1.2 \times 10^{45}$ erg s$^{-1}$ and the conversion factor of 300%, the accretion rate should be larger than $2.25 \times 10^{-2} M_\odot$ yr$^{-1}$ at $10 r_s$. On the other hand, with constraint on the mass accretion rate ($0.15 \lesssim \dot{M}_{\text{rb}} \lesssim 2.25 M_\odot$ yr$^{-1}$) derived by the beam depolarization model, the expected $M$ of $2.25 \times 10^{-2} M_\odot$ yr$^{-1}$ at $10 r_s$ gives a constraint on the RIAF parameter to be $1.3 \leq \beta \leq 1.5$ (see also Figure 7). Park et al. (2019) constrained the RIAF parameter $\beta$ for the case of M 87, and revealed that the gas density profile is $n \propto r^{-1}$ (i.e., $\beta = 1$ in Equation (3)). By Equation (7), it implies $\kappa = 1$, suggesting the toroidal magnetic field is dominant in the hot accretion flows as expected (e.g., Hirose et al. 2004) and, interestingly, the condition of $\nabla \cdot B = 0$ is satisfied. In this case, it does not require local process, such as dissipation by the magnetic reconnection to maintain the equipartition condition locally. In addition to that, $\beta \simeq 1$ is favored with the magneto-hydrodynamical simulations (e.g., Pang et al. 2011, the $\beta \simeq 0.9 - 1.3$). In the case of Cygnus A, $\kappa$ between...
1.25 and 1.35 is suggested based on our depolarization analysis. The constraint of $\beta$ favors the ADIOS or ADAF model, and the CDAF model ($\beta = 0.5$) is less likely to be the case. In the case of ADIOS, the existence of wind is expected, which agrees with Boccardi et al. (2016). The wind causes a substantial mass loss during the accretion process (Blandford & Begelman 1999; Yuan et al. 2018), and also possibly plays the role of confining the jet to form a parabolic shape (Nakamura et al. 2018).

5. Summary

We present the first detailed polarimetric observation toward Cygnus A at 230 GHz. We detect polarized emission at 230 GHz with fractional polarization of $0.73 \pm 0.15\%$. We also show the results of measurements on the total flux density of Cygnus A at mm wavelength together with the spectral index. The observed steep spectrum of $-0.73 \pm 0.12$ indicates that the optically-thin synchrotron jet component dominates the emission. The RM variance is consistent with Faraday rotation caused by the surrounding RIAF with the mass accretion rate $M \geq 0.15 M_\odot$ yr$^{-1}$ at the Bondi radius. Derived constraints on the mass accretion rate are in good agreement with ADIOS or ADAF, while CDAF is less likely to be the case. We also compare the constraint of the mass accretion rate with the jet power of Cygnus A, suggesting the jet power can be explained by the mass accretion power associated with RIAF.

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Appendix

Polarization Leakage Impacts Simulations

The appendix provides a detailed analysis of the investigation on the marginal detection of the polarized flux. As we see in Section 3.1, we see an excess of the polarized flux from the Rayleigh distribution at the high end (see also Figure 4). This excess is associated with the region where we see the peak in Stokes $I$ images.

To evaluate the impact of polarization calibration error on this excess, we performed Monte-Carlo calculations to simulate the imperfect instrumental polarimetric calibration. In each Monte-Carlo sampled realization, each antenna’s D-terms are randomly generated from the Gaussian distribution with the mean of the original calibrated value and 0.1% (Marrone 2006) as the standard deviation and formulate Stokes $I$, $Q$, $U$, and polarimetric images. The iterations are 1000 realizations. We divided Stokes $Q$ and $U$ images into nine blocks (see also Figure 8), and derived the distributions of polarized brightness toward each block for each realization. We compared with the Rayleigh distribution, $f(P; \sigma_{QU}) = \frac{1}{\sigma_{QU}} \exp(-P^2/(2\sigma_{QU}^2))$, where $P$ is defined in Equation (1) and $\sigma_{QU}$ is the standard deviation of noises in $Q$ and $U$ maps. In Figure 8, we show the averaged distribution of polarized brightness toward each block normalized with the Rayleigh distribution. The error bar is derived from the Monte-Carlo sampled pool. The derived distribution match with the Rayleigh distribution, except toward block 5 where we see the peak in Stokes $I$ images. For block 5, we see the clear excess, which is consistent with Figure 3. It demonstrates that the excess we see in Figure 3 should be primarily located in the central part of the image (block 5). Therefore, we conclude that we see the excess of the

![Figure 7. Mass accretion rate (left label) and accretion power (right label) as a function of the distance from the black hole. The accretion power is derived with $Mc^2$. The mass accretion rate at the Bondi radius (Bondi rate) is $0.15-2.25 \times 10^{-6} M_\odot$ yr$^{-1}$, which is derived from the beam depolarization scenario (see Figure 6). The red dotted and light blue dashed lines indicate the ADAF and CDAF solutions with associated boundary Bondi rate values based on Equation (11). The light green dotted lines are the mass accretion rate distribution extrapolated from the Bondi rate toward the black hole with varying RIAF $\beta$ from 0.5 to 1.5 with 7 samples. The blue solid line indicates the minimum required mass accretion rate to support jet power with conversion factor of 300%, which is suggested for the MAD case. The orange dashed-dotted line represents the solution corresponding to the MAD contexts.](image-url)
polarized flux toward block 5, and it cannot be associated just with the thermal noise.

Then, we also calculate the signal-to-noise ratio distributions across the Monte-Carlo sampled polarization images as shown in Figure 9. The signal-to-noise ratio of polarized fraction is consistent among different Monte-Carlo sampled leakages terms. The expected uncertainty of instrumental polarimetric calibration can cause only ~0.1 mJy, which is derived by the standard deviation of Monte-Carlo sampled peak polarized flux pools. This error is one order of magnitude smaller than the thermal error (~1 mJy), and it is significantly smaller than the detected weak polarized emission. The analysis demonstrates the robustness of weak polarization features in the underlying images. We conclude that the instrumental polarization leakage’s nominal noise does not affect our primary results.

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Figure 8. Pixel-wise data to fitted model ratio. Note that the derived best-fit parameter of $\sigma_{uv} = 1.33 \pm 0.16 \text{ mJy beam}^{-1}$ (errors are 99% confidence interval) is consistent with the noise level derived from the Stokes $Q$ and $U$ images separately.

Figure 9. Signal-to-noise ratio of polarization fraction via Monte-Carlo sampled instrumental polarization leakages. The $x$-axis shows the sub-band label, and the $y$-axis shows the signal-to-noise ratio of the polarization fraction. Blue and orange colors demonstrate the different polarization calibrators. The marginal detection of polarized flux with a signal-to-noise ratio of 4.5 originates from only one band, while those at the other bands are 1.7–3.0.
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