Possible Vortex Fluid to Supersolid Transition in Solid $^4$He below $\sim$75 mK

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A detailed torsional oscillator (TO) study on a stable solid $^4$He sample at 49 bar with $T_o \sim$0.5 K, is reported to $T$ below the dissipation peak at $T_p$. We find both the shift of period and dissipation hysteretic behavior below $T_c \sim$75 mK, with changes of AC excitation amplitude $V_{ac}$. The derived difference of non-linear rotational susceptibility $\Delta N L R S(T)_{hys}$ across the hysteresis loop under systematic conditions is analyzed as a function of $V_{ac}$ and $T$. We propose that $\Delta N L R S(T)_{hys}$ is the non-classical rotational inertia fraction, $N C R I F$ itself, and it is actually the supersolid density $\rho_{ss}$ of the 3D supersolid state below $T_c$. $\rho_{ss}$ changes linearly with $T$ down to $\sim$60 mK and then increases much more steeply, approaching a finite value towards $T=0$. We find a characteristic AC velocity $\sim$40$\mu$m/s beyond which the hysteresis starts at $T < T_c$ and a "critical AC velocity" $V_c \sim$ 10 mm/s, above which $\rho_{ss}$ is completely destroyed. We obtain $\xi_0$ and $V_c = h/(m_4 \xi_0 \pi) = \sim$6-12 mm/s.

The supersolid state, which can be characterized as a solid with a lattice structure yet simultaneously having superfluid properties, has been one of the most interesting topics in condensed matter physics. Discussions started in the 1960’s (see recent reviews[1, 2]) and the experimental search for such a state in quantum solids followed for 40 years till now. Unusual properties of solid $^4$He have been reported for some time. The first convincing claim was made by Kim and Chan[3], in a report of non-classical rotational inertia ($N C R I$) in a phenomenon predicted long ago by Leggett[4]. This experimental observation with a torsional oscillator (TO) has been confirmed by other groups including the present authors’ group[5, 6, 7, 8, 9]. There were, however, a few fundamental problems in the identification of $N C R I$ in the reported experimental results by that time. The problems were discovered following newer observations, namely, the reported onset temperature $T_o$ of 0.1-0.5 K[2] is too high for the known number density of imperfections in solid $^4$He samples[10] to expect Bose-Einstein Condensation (BEC) on one hand, while the reported amount of the $N C R I$ fraction goes up to $\sim$20% of the total $^4$He mass[7] in conflict with Leggett’s original ideas about the origin of supersolidity as BEC in the imperfections. Furthermore, the failure to observe superflow[11, 12] is suggestive of some other phenomenon, which is actually responsible. Anderson proposed a picture of a vortex fluid (VF) state above a real $T_o$[13, 14] for the reported observations for the solid $^4$He. He took the absence of superflow into account and the sensitivity to the amount of disorder[7] and proposed the reported TO response is the "nonlinear rotational susceptibility". $N L R S$ of the VF state, instead of $N C R I$ for the real supersolid state and the latter should exist at some lower temperature below $T_c$. Actually another independent picture had been proposed by Shevchenko long ago[13, 10], considering a 1D dislocation core as an origin of superfluidity. It describes a real superfluid state in solid He below some $T_c$ and still they expect something like superfluid response above $T_c$ because of the dynamic properties. We do not consider this picture now because we do not find a direct connection with the experimentally found phase transition.

It is well-known that the VF is a state without superflow, or without 3D macroscopic coherence in the case of underdoped (UD) cuprates[17]. It is also argued to be characterized by almost constant amplitude of the macroscopic wave function through the real 3D macroscopic super-conducting(-fluid) transition at $T_o$, but phase fluctuations break down the macroscopic coherence[13, 18] at $T_o$, where quantized vortices (probably in a low D subsystem) start to appear thermally. As a result of increased low D coherence length $\xi_D$ (which increases towards $T = 0 K$), a real 3D supersolid transition should occur at some low $T$. A sharp peak in specific heat, which would indicate a 3D real phase transition, has been reported around 75 mK[19].

On the other hand, there also have been some attempts to explain features of observations in terms of classical dislocation motion trapped by $^3$He in connection to the response of solid $^4$He to shear motion apart from supersolid properties[20]. Actually, a similar shear modulus increase below about 200 mK has been observed not only in hcp solid $^4$He, but also in hcp solid $^3$He. But a TO response anomaly, $N L R S$ or $N C R I$ has been observed only in hcp $^4$He[21].

In a recent publication we have shown[9] experimental evidence which supports the VF picture in solid $^4$He samples at 32 bar as well as 49 bar below a common $T_o \sim 0.5 K$. $T_o$ was determined for the first time for solid $^4$He from a detailed study of the AC excitation $V_{ac}$ dependence change at this temperature[9]. We argued for $V_{ac}$ and $T$ dependent responses of the pre-existing thermal fluctuations of the phase of a mesoscopic scale wave function in the VF state. Stronger AC excita-
tions, which would cause formation of straight vortex lines, could suppress these thermal fluctuations of the VF state. Such suppression appears not only in NLRS, but also in the energy dissipation \( \text{NLRS} \). The unique \( \log(V_{ac}) \) linearly dependent suppression of the NLRS and its unique temperature dependence suggest that what we observed was not NCRI, expected for the 3D super-solid, but what is being proposed by Anderson \cite{22} as NLRS of the VF state. All the observations, especially above the energy dissipation peak, can be well described by the properties of the VF state \cite{9, 13, 14}. The VF state may have some features in common with superfluid turbulence, where the “polarization” of the vortex tangle under rotation may be described as an ensemble of vortex loops and its “polarizability” under rotation could show similar behavior to that represented in the Langevin function \cite{23}. Similar suppression of fluctuations by an external magnetic field has been reported for the VF state in layered superconductors \cite{24}, and cuprate high \( T_c \) superconductors \cite{17}. 3D superfluidity and 3D vortex lines are realized in a series of 3D connected He monolayer systems, where 3D connectivity of the superfluid is provided by the 3D connected surface of a porous substrate \cite{25}. The critical velocity of this system for destruction of the 3D superfluidity seems to be characterized by \( V_c \sim h/m \alpha \pi \), above which 3D superfluidity is destroyed and 2D superfluid features appear \cite{26}, where \( \alpha \) is the 3D vortex core size, or the minimum size of the 3D superfluid.

In the present paper we describe further a detailed TO study on the same stable solid \(^4\text{He}\) sample at 49 bar \cite{3}, but extended to \( T \) lower than the dissipation peak at \( T_p \). The first observation at the lower \( T \) is that the \( \log V_{ac} \) linear slope’s \( T \) dependence deviates from the reported \( 1/T^2 \) dependence at higher \( T \) for the VF state \cite{9}. To make the physical meaning clearer we consider the value of NLRS extrapolated to \( V_{ac}=0 \). It would represent the full amount of NLRS of the VF state. Fig. \( \text{1} \) indicates NLRS(\( T \)) for \( V_{ac} \) extrapolated to 0 as a function of \( 1/T^2 \). It also deviates from the \( 1/T^2 \) linear dependence at lower \( T \) and was found to follow the Langevin function quite well with \( x = 1/T^2 \) and it approaches a finite value towards \( T=0 \), 0.088% of the total solid \( ^4\text{He} \) mass. All the data in Fig. 1 inset were measured under a certain procedure, where the \( V_{ac} \) is set to a maximum value at \( T \gtrsim 0.5 \text{K} \) and cooled down to desired \( T \) and \( V_{ac} \) was swept downwards stepwise after equilibrium at each \( V_{ac} \). After completion of the measurement at a \( T \) another set is repeated for another \( T \). We call this procedure “measurement under equilibrium condition”. The \( T \) dependence may indicate that the VF state is best described by behavior of an ensemble of vortex loops as was discussed for superfluid turbulence in \cite{23}. Loops can be polarized in a manner similar to dipole systems, where instead of \( 1/T^2, 1/T \) appears.

In this lower \( T \) range where deviation from the \( 1/T^2 \) linear dependence occurs, we find a new feature, namely hysteretic behavior, when an appropriate procedure is followed. It appears below a certain temperature \( T_c \). We will propose that we have observed a transition from a vortex fluid state\(^6\) to a new state occurring below \( T_c \), which is characterized by the appearance of the hysteretic behavior and of a critical velocity of the order \( \sim 1 \text{ cm/s} \), beyond which the hysteretic component of NLRS is suppressed to zero. Actually hysteretic behavior itself has been reported by Kojima and his group at very low temperatures\(^{27} \) and by Chan’s group\(^{28} \), as well as by Reppy’s group\(^{29} \); however, none of them discussed the hysteresis in connection to the transition from the vortex

FIG. 1: \( \text{NLRS}(T) \) at \( V_{ac} \to 0 \), is displayed as a function of \( 1/T^2 \). The solid line through the data points is the Langevin function \( f(x) = e^{-a} \{e^{b} + e^{b(1-x)/x)}\} / \{e^{b} - e^{b(1-x)/x)}\} \) with \( a = 0.0878 \pm 0.0011 \), and \( b = 0.0148 \pm 0.0004 \). Inset shows the \( V_{ac} \) dependence for data at each \( T \leq 300 \text{ mK} \) and we can safely extrapolate to \( V_{ac} \to 0 \).

FIG. 2: The \( \log V_{ac} \) dependence of \( \text{NLRS} \) of solid \(^4\text{He} \) sample at 49 bar at constant \( T \)’s as given in the figure, obtained from the measurement of period change of TO.
fluid (VF) state. The VF state has been recently experimentally clarified by our report of the unique $V_{ac}$ and $T$ dependent TO responses, effects which are absent in known superfluid transition responses.

We describe here the first systematic study of the hysteretic component of the solid $^4$He TO responses as a function of $V_{ac}$ as well as $T$ below $T_c$ and discuss a possible order parameter of the supersolid (SS) state below $T_c$. In Fig. 2 the TO period change as a function of $V_{ac}$ is shown for various $T$. All the data points for each $T$ down to 80 mK follow a single line as was the case for our data in a previous publication above $T_c$. The data points below 75 mK actually follow two lines. These two lines were produced as follows: The first measurements were performed at $V_{ac}$=maximum set at 500 mK, as ”equilibrium” condition measurements. Then they were prepared by cooling down the sample to the lowest $T \sim 48$ mK, and then warmed up to the desired $T < T_c$ and measurements were performed by changing $V_{ac}$ step wise downwards over a long enough time, typically 12 hours or longer for the whole sweep one way, to allow for any relaxation at this constant $T$. Then the other series of measurements were prepared by reversing the excitation $V_{ac}$ change upwards to the measuring excitation step-wise after equilibrium is reached at each step up to the maximum $V_{ac}$. After completing measurements at a $T$ then another set of measurements at a different $T$ was performed in a similar manner. We observe in Fig. 2 that the hysteretic component (the difference between the two passages of $V_{ac}$) has a unique $V_{ac}$ dependence and increases towards lower $T$.

Fig. 3 shows much more clearly the hysteretic components of both dissipation and NLR$S$ as a function of $V_{ac}$ presented on a logarithmic horizontal scale. An interesting observation is that the hysteretic component appears only above a characteristic AC velocity $\sim 40 \mu m/s$ and it reaches a maximum at $\sim 200 \mu m/s$. Furthermore, at higher AC velocity $\sim 500 \mu m/s$ it starts to decrease. It looks as if it follows a linear relation passing through $\Delta NLR_{hys}=0$ in the range of $V_{ac}$, $8 \pm 2 \mu m/s$, signaling the total depression of the hysteretic NLR$S$ component. The upper column of Fig. 3 shows the energy dissipation change across the hysteresis loop $\Delta Q_{hys}^{-1}$ as $V_{ac}$. A most impressive thing is that while $\Delta Q_{hys}^{-1}$ is negative $\Delta NLR_{hys}$ is increasing over a considerable range of $V_{ac}$. We suggest this is evidence for a shielding current preventing penetration of vortex lines as well as thermally excited vortices into the volume as in the Landau state of superfluids or Meissner state of superconductors. At higher AC velocity above $500 \mu m/s$ introduction of vortex lines would cause a $logV_{ac}$ linear decrease of supersolid density $\rho_{ss}$.

Fig. 4 indicates the $T$ dependence of the hysteretic component of $\Delta NLR_{hys}(200 \mu m/s)$ together with ”equilibrium” NLR$S(V_{ac}=0)$. The former appears below about 75 mK and increases almost linearly toward lower $T$ to $\sim 60$ mK, then further increases more steeply as $T$ lowers. We have tried to fit it with the expected behavior for a 3D supersolid density, $\rho_{ss} = (1 - T/T_c)^{\gamma} = T^\gamma$ with $\gamma=2/3$ and it yields an extrapolation to 0 K ($t=1$). We can safely estimate ” the supersolid density extrapolated to 0 K” within some error bar. It would be $0.065 \pm 0.025\%$ of the total solid He sample mass with $T_c=56.7$ mK. The $\rho_{ss}(T=0)$ is less than but on the same order as the expectation value of NLR$S(T=0)$,
which increases towards 0 K following a Langevin function, as discussed. From the absolute value of the $\rho_{ss}$, we can evaluate supersolid coherence length $\xi$, following a common practice based on consideration of Josephson’s length [30]. We could fit rather well with the $t^{-2/3}$ dependence expected for a 3D superfluid, obtaining an extrapolated $\xi$ value at $T=0$, $\xi_0=25$ to 50 nm as shown in Fig.5 assuming $m_4$ to be the atomic $^4$He mass. We find $V_c=h/(m_4\xi_0\pi)\sim6-12$ mm/s, for $\xi_0=50-25$ nm, a surprising coincidence. It is very interesting considering the microscopic origin of the 3D macroscopic supersolid phenomenon as suggested by the present work. Actually there have been various observations of the size of $NLR_S$, but nobody else discussed the transition from the VF to the SS state except the present authors. A similar hysteresis phenomenon was already experimentally observed for a $^4$He sample with 30 ppm $^3$He long ago in an acoustic experiment by Iwasa and Suzuki in 1980[31], but without noticing the relationship to a vortex state.

We have found the start of the hysteretic behavior below $T_c$, and evaluated the hysteretic component $\Delta NLR_S$ as a function of $V_{ac}$ and discussed it as supersolid density $\rho_{ss}(T)$ and discussed the 3D coherence length $\xi$ and a consistent critical AC velocity $V_c\sim1$ cm/s. So far we have neglected the anisotropy of the hcp crystal of $^4$He and treated it as an isotropic supersolid. The linear $T$ dependence of $\rho_{ss}(T)$ between 75 and $\sim60$ mK may have some connection to this problem.

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