Design optimization of perforated plate matrix heat exchangers for cryogenic applications using Teaching Learning Based Optimization (TLBO) method

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Abstract. A perforated plate matrix heat exchanger (PPHE) is a bonded stack of alternately arranged plates and spacers. Because of their high surface area density and high effectiveness, PPHEs are highly suitable for cryogenic applications such as cryogenic refrigeration and liquefaction systems. The aim of this study is to arrive at the optimized shape and design variables of a PPHE that satisfy the specified constraints, namely effectiveness and pressure drop, while the volume of the heat exchanger (as objective function) is minimized. A Teaching-Learning-Based Optimization (TLBO) method is used for optimum sizing of the PPHE.

1. Introduction
A perforated plate matrix heat exchanger (PPHE), as shown in figure 1, is a compact heat exchanger which is made of alternately arranged high conductivity perforated plates and thermally insulated spacers. These types of heat exchangers, because of their high compactness and good performance, are suitable for cryogenic applications [1-6]. However, only limited information on the selection of flow channel geometry and design optimization of a PPHE is available in the literature. Pavan Kumar and Venkatarathnam [7] minimized the volume of PPHEs following a Lagrangian multiplier technique for a given heat duty and pressure drop. Bhanumurthy et al. [8] also used effectiveness and pressure drop as constraints and optimized the design of PPHEs for a minimum volume of the heat exchanger.

The aim of this study is to determine optimum values of the design variables that satisfy the required heat duty and pressure drop, while keeping the volume of the heat exchanger to a minimum value. Optimizations have been carried out considering four different types of flow channel geometries, as shown in figure 2. A Teaching Learning Based Optimization (TLBO) method is used for optimization. For a given problem, the solutions obtained by the proposed method are compared with the existing solutions as well as with those obtained by a Genetic Algorithm (GA) method.

2. Fixation of parameters prior to optimization

2.1 Material selection and thickness of plates and spacers
A combination of copper plates (CU) and stainless steel spacers (SS) is chosen for the present optimization study. In order to decide the domains for the thickness of the plates and spacers, the standard sizes or gauges [10-12] available in the market are considered.

2.2 Selection of wall width
The width of the fluid separating wall (s) and boundary wall (b) should be decided from the operating pressure and the minimum width that is required to ensure leak-proof joints. Higher wall width would
cause increased longitudinal heat conduction and performance deterioration. As per our experience, a minimum wall width of 2 mm is essential to obtain good bonding and leak-proof joints.

### 2.3 Selection of pore diameter (d)

Photo chemical milling (PCM) [13] is the best adaptable method for making both perforated plates and spacers. For photo chemical etching, the ratio of pore diameter to the plate thickness should be in between 1.1 and 1.5 [14,15]. For given values of Reynolds number (Re), porosity (p), plate thickness (t_p) and spacer thickness (t_s), the friction factors generally decrease with reduction of pore diameter, while heat transfer coefficients increase with the decrease of pore diameter [16]. Therefore, one should use smallest pore diameter (d) obtainable from a manufacturing point of view. Thus, we take

\[
d = 1.1 t_p
\]

### 2.4 Selection of porosity (p)

Pores in a perforated plate are usually arranged in hexagonal arrangement, as shown in figure 3. From a manufacturing point of view, the width of the material between two holes should be at least equal to plate thickness. From equation (1), the ratio t_p/d is 0.91. When t_p/d is greater than 0.5 the Colburn factor increases with porosity [16]. Also the friction factor decreases with increasing porosity. Therefore, the porosity should be chosen as the maximum permissible from a manufacturing point of view. Thus, we take the lowest possible value of w_h (see figure 3) i.e.

\[
w_h = t_p
\]

From figure 3 and equations (1) and (2),

\[
c_h = w_h + d = t_p + 1.1t_p = 2.1t_p
\]

Porosity for a hexagonal arrangement of holes [17] is

\[
p = \frac{2\pi}{\sqrt{3}} \left( \frac{d}{2c_h} \right)^2 = \frac{2\pi}{\sqrt{3}} \left( \frac{1.1t_p}{4.2t_p} \right)^2 = 0.25
\]

The ratio (k_e) of the effective thermal conductivity of plate to that of the plate material decreases almost linearly with increasing porosity. At porosity 0.25, the k_e is 0.6 [17], which is an acceptably high value.

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**Figure 1.** Construction of a perforated plate heat exchanger [9].

**Figure 2.** Various flow channel geometries for PPHEs (plates are shown with a spacer on top).

**Figure 3.** Hexagonal arrangement of holes.
3. Computations

The effective number of transfer unit (NTU<sub>eff,i</sub>) of PPHEs considering axial conduction through the inner wall and under unbalanced flow condition [18,19] is given by

$$\text{NTU}_{eff,i} = \frac{N (1 - \alpha_h)(1 - \alpha_n)\left(1 + \lambda_m f\right)}{N \lambda_m (1 - \alpha_h)(1 - \alpha_n) + 1 - \alpha_h \alpha_n + (1 - \alpha_h)(1 - \alpha_n)/\text{NTU}_{po}}$$

(5)

where N is the number of plates, \(\alpha\) = \exp(-NTU<sub>eff,i</sub>) (subscripts f=fluid, j=h or c depending upon hot or cold fluid), \(\lambda_m = \nu \lambda_{w,i}\) (\(\nu\) being the ratio of the minimum to maximum heat capacity of the fluids and \(\lambda_{w,i}\) is the axial conduction parameter for the inner wall) and \(\phi = (\lambda_m \text{NTU}/(1+\lambda_m \text{NTU}))^{1/2}\) in which NTU is the design NTU of the heat exchanger.

Axial conduction parameter through boundary wall (\(\lambda_{w,o}\)) is defined as, for rectangular geometry,

$$\lambda_{w,o} = k_i 2 b(W + H_0 + H_i + 2b + s)/\left(\left(\text{NTU}\right) N t_s (\text{m c}_p)_{\text{min}}\right)$$

(6)

where \(k_i\) is the thermal conductivity of the spacer material, while \(\text{m}\) and \(\text{c}_p\) refer to the mass flow rate and specific heat (at constant pressure) of the fluid with lower heat capacity, respectively. Similarly for Circular geometry with \(R_o\) as the inner radius of the outer wall we have

$$\lambda_{w,o} = k_i 2 \pi R_o b/(\left(\text{NTU}\right) N t_s (\text{m c}_p)_{\text{min}})$$

(7)

The effective NTU considering longitudinal heat conduction through fluid separating wall and boundary wall is calculated as follows [19]

$$\text{NTU}_{eff,o} = \frac{\text{NTU}_{eff,i}}{1 + \phi \lambda_{w,o} \nu NTU}$$

(8)

The parameter \(\phi\) in equation (8) depends on the design NTU and Eigen roots of the characteristic equation. The final expression for effectiveness of the heat exchanger is therefore

$$\varepsilon = \frac{1 - \exp[-\text{NTU}_{eff,o} (1 - \nu)]}{1 - \nu \exp[-\text{NTU}_{eff,o} (1 - \nu)]}$$

(9)

Pressure drop in each side of the fluid is computed from

$$\Delta P = 4f \frac{L G^2}{d 2 \rho} = 4f \frac{N(t_p + t_i) (\text{m c}_p)^2}{d 2 \rho}$$

(10)

where L is the length of the heat exchanger and \(\rho\) is the density of the fluid. Friction factor \(f\) is computed from the correlations given in [16].

4. Optimization of PPHEs using TLBO

The volume of the heat exchanger (V) is chosen as the objective function which is minimized through optimization. Constraints are specified for Effectiveness (\(\varepsilon\)) and pressure drop (\(\Delta P\)) in either channels as 0.975 and \(\leq 4000\) Pa respectively. Teaching-Learning-Based Optimization (TLBO) was first introduced by Venkata Rao [20]. Enhancing the knowledge of learners in a classroom through teaching is the main motivation in developing TLBO. In the TLBO algorithm, the number of students represents the population, the number of subjects represents the design variables and the grade of the students is analogous to the objective function, or fitness value.

4.1. Example problem

A problem is taken from the literature [8] for which design optimizations of PPHEs are carried out. Table 1 gives the details of the problem. Search domains of design variables are given in Table 2. The fluid is a mixture of nitrogen, methane, ethane and propane.

4.1.1. Solution. The procedure of optimum sizing of a PPHE for the present problem is presented with an assumed value of effectiveness. Figure 4 shows that convergence is achieved after 300 iterations. Optimized volumes of the various heat exchangers are presented graphically in figure 5. The figure shows that the volume of PPHEs using circular annular, rectangular three channels and rectangular five
channels geometries are close to each other. Among these, the volume for circular annular geometry is the minimum in most of the cases. Circular annular geometry also offers fabricational advantage over the other geometries.

Table 1. Fluid and material details of the problem

|                      | Hot stream | Cold stream |
|----------------------|------------|-------------|
| Mass flow rate, \(m\) | 2.0        | 2.0         |
| Inlet temperature, \(T\) (K) | 300        | 220         |
| Desired outlet temperature (K) | 226.8      | 298         |
| Density, \(\rho\) (kg/m\(^3\)) | 25.46      | 6.71        |
| Specific heat, \(c_p\) (J/kg-K) | 1504       | 1411        |
| Thermal conductivity, \(k\) (W/m-K) | 0.018       | 0.017       |
| Viscosity, \(\mu\) (\(\mu\)pa-s) | 11.4     | 9.69        |

**Material properties:**
- Thermal conductivity of the plate material, \(k_p\) (W/m-K) 400
- Thermal conductivity of the spacer, \(k_s\) (W/m-K) 15

Table 2. Search range of design variables

| Design variable | Search range |
|-----------------|--------------|
| Number of plates, \(N\) | 50 – 1000 |
| Channel width, \(W\) | 20 – 150 mm |
| Channel height, \(H\) | 2 – 20 mm |
| Outer radius, \(R_o\) | 10 – 50 mm |
| Inner radius, \(R_i\) | 5 – 30 mm |
| Plate thickness, \(t_p\) | 0.203 – 0.724 mm |
| Spacer thickness, \(t_s\) | 0.218 – 0.556 mm |

In order to compare the proposed TLBO method with other methods, an optimization has been carried out by using a Genetic Algorithm (GA) technique [21]. The results are shown comparatively in table 3. The table shows that the TLBO algorithm results in approximately a 12.6% smaller volume compared to the GA method. The results are also compared with those of [8], as shown in table 3. The optimized volume of the heat exchanger obtained by Bhanumurthy et al. is less than that obtained in the proposed method. However, this is mainly because of the use of manufacturing constraints - i.e. \(d=1.1.t_p\), \(p=0.25\) (see section 3.2 & 3.3) - in the proposed TLBO method which were not considered by Bhanumurthy et al. The TLBO algorithm is also applied without the manufacturing constraints and the optimized volume is found to be about 4% less than that obtained by Bhanumurthy et al. [8].

5. Conclusion

In this paper a TLBO method is discussed for optimizing PPHEs. Four different types of flow channel geometries are considered in the studies. The results obtained when applying a TLBO method to an optimisation problem from the literature [8] are compared with those obtained using a genetic algorithm (GA) and with the results reported in the original reference. The TLBO method is found better than the other methods. The optimized results indicate that circular annular geometry offers the minimum heat exchanger volume.
Table 3. Comparison of optimization results

|                  | Bhanumurthy et al. (2010) | TLBO (using j & f as used by Bhanumurthy et al. (2010)) | TLBO (d & p as dependent variables) | GA |
|------------------|---------------------------|--------------------------------------------------------|------------------------------------|-----|
| R (mm)           | 12.0                      | 15.24                                                  | 17.38                              | 16.7 |
| R₀ (mm)          | 17.1                      | 21.0                                                   | 22.87                              | 23.1 |
| t₀ (mm)          | 0.45                      | 0.398                                                  | 0.203                              | 0.217 |
| t₁ (mm)          | 0.32                      | 0.407                                                  | 0.297                              | 0.480 |
| d₀ (mm)          | 0.45                      | 0.2                                                    | 0.238                              | 0.238 |
| d₁ (mm)          | 0.45                      | 0.2                                                    | 0.238                              | 0.238 |
| p₀               | 0.26                      | 0.202                                                  | 0.25                               | 0.25 |
| p₁               | 0.26                      | 0.248                                                  | 0.25                               | 0.25 |
| N                | 322                       | 170                                                   | 345                                | 286 |
| ε (%)            | 97.5                      | 97.5                                                   | 97.5                               | 97.5 |
| ∆P₀ (Pa)         | 2403                      | 3962                                                   | 3988                               | 2221 |
| ∆P₁ (Pa)         | 4000                      | 4000                                                   | 4000                               | 3845 |
| volume, V (cc)   | 281.1                     | 227.9                                                  | 335.2                              | 395.2 |

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