D–term Inflation in Type I String Theory

Edi Halyo

Department of Physics
Stanford University
Stanford, CA 94305

ABSTRACT

D-term inflation realized in heterotic string theory has two problems: the scale of the anomalous D-term is too large for accounting for COBE data and the coupling constant of the anomalous $U(1)$ is too large for supergravity to be valid. We show that both of these problems can be easily solved in D-term inflation based on type I string theory or orientifolds of type IIB strings.
1. Introduction

Early attempts to incorporate inflationary cosmology in string theory (or supergravity) were not very successful due to the inflaton mass problem, i.e. the mass of the inflaton is generically as large as the Hubble constant when the vacuum energy arises from F–terms\[1\]. In this case, inflation cannot take place since the slow–roll condition is violated.

An elegant solution to the inflaton mass problem in string theory (or supergravity) is D-term inflation\[2\]. In this scenario, the vacuum energy needed for inflation is dominated by D-terms rather than F-terms. Thus, the inflaton mass problem is trivially solved since the dangerous contribution to the inflaton mass due to the F–terms is vanishing (or negligible). This can be easily realized in heterotic string theories since generically there is an anomalous D-term arising from an anomalous $U(1)_A$ which can contribute to the vacuum energy\[4\]. In this scenario, the inflaton $\sigma$ is neutral under the anomalous $U(1)_A$ but has tree level couplings to other fields $\phi, \bar{\phi}$ in the superpotential, i.e. $W = \lambda \sigma \phi \bar{\phi}$. The fields $\phi, \bar{\phi}$ have $\pm 1$ charges under $U(1)_A$ and behave as the trigger fields in hybrid inflation models\[5\]. The scalar potential including the anomalous D-term is\[2\]

$$V = |\lambda \sigma|^2 (|\phi|^2 + |\bar{\phi}|^2) + |\lambda \phi \bar{\phi}|^2 + \frac{g^2}{2} (|\phi|^2 - |\bar{\phi}|^2 + M^2)$$ \hspace{1cm} (1)$$

where $\lambda \sim O(1)$ is a Yukawa coupling, $g$ is the gauge coupling of $U(1)_A$ and $M$ is the scale of the anomalous D-term. For heterotic strings it is given by\[6\]

$$M^2 = \frac{1}{192\pi^2} g^2 (TrQ_A) M_P^2$$ \hspace{1cm} (2)$$

Here $M_P \sim 2 \times 10^{18} \text{GeV}$ is the reduced Planck scale, $g \sim 1/2$ from gauge coupling unification and the trace is over the whole massless spectrum of the string theory giving generically $TrQ_A \sim 100$. Hybrid inflation occurs for large values of $\sigma$.

\* For earlier work on D–term inflation see \[3\].
σ ∼ M << M_P which gives a positive mass squared to φ, ¯φ and forces them to have vanishing VEVs. Then there is a constant nonzero vacuum energy $V_0 = g^2 M^2 / 2$ resulting in a period of inflation. Supersymmetry is broken by $V_0$ and as a result a one–loop potential for σ is generated[2]

$$V_{\text{one-loop}}(\sigma) = \frac{1}{2} g^2 M^4 \left[ 1 + \frac{g^2}{8 \pi^2} \log \left( \frac{\lambda \sigma}{\Lambda} \right) \right]$$

above Λ is the renormalization scale which does not affect the physics. Due to this potential the inflaton rolls slowly to its minimum during inflation. There is a critical value $\sigma_{cr} = gM/\lambda$ after which the mass squared of ¯φ becomes negative and it rapidly falls to its new minimum at ¯φ = M. This ends inflation and restores supersymmetry.

It has been noted that the above D-term inflation scenario has two problems when it is realized in heterotic string theory[7,8]. The first problem arises from the magnitude of density fluctuations obtained from COBE data which requires[8,10]

$$\left( \frac{V_0}{\epsilon} \right) = 6.7 \times 10^{16} \text{ GeV}$$

where $\epsilon$ is one of the slow–roll parameters of inflation; $\epsilon = \frac{1}{2} M_P^2 (V'/V)^2$. For inflation with $N (\sim 60)$ e-folds one needs

$$M \sim 8.5 \times 10^{15} \text{ GeV} \times \left( \frac{50}{N} \right)^{1/4}$$

However in heterotic string models with an anomalous D-term eq. (2) above gives a scale too large to account for COBE data.

The second problem arises from the fact that inflation can come to an end before the inflaton reaches its critical value if the second slow–roll parameter $\eta =

\[†\] For D-term inflation in explicit string models see [3,9].
$M_P^2|V''/V|^2$ becomes of order one (since slow–roll requires $\eta \ll 1$). This means that\cite{8,10}

$$\eta(\sigma) = \sqrt{\frac{\alpha}{2\pi} \frac{M_P}{\sigma}}$$

(6)

where $\alpha = g^2/4\pi$. We see that $\eta \sim 1$ when

$$\sigma_f \sim \sqrt{\frac{\alpha}{2\pi} M_P}$$

(7)

This gives $\sigma_f \sim M_P/10$ for the final value of $\sigma$ at the end of inflation which is much larger than $\sigma_{cr}$. Moreover, it can be shown that the initial value of the inflaton $\sigma_i$ needs to be

$$\sigma_i \sim \sqrt{\frac{\alpha N}{\pi} M_P}$$

(8)

which gives $\sigma_i \sim 0.8 M_P$. This is problematic because for Planckian values of the inflaton one cannot use the effective low–energy supergravity approximation. Actually, this problem is probably less severe than it seems because the criterion should not be $\sigma < M_P$ but rather that the one–loop corrections should be smaller than the tree level results (when the D–term is the source of vacuum energy and the corrections to the Kahler potential are not relevant). For example, for $V_{\text{one–loop}}$ we see that even for $\lambda \sim 1$, $\sigma$ can be much larger than $M_P$ due to the factor $g^2/8\pi^2$.$^\dagger$

In this letter, we show that both of these problems can be naturally solved in D–term inflation in the framework of type I string theory rather than heterotic string theory. The first problem is solved because in type I string theory the scale of the anomalous D–term is not fixed; it is given by the VEV of a modulus which can be of the required order of magnitude. Moreover, unlike the heterotic string case in type I theory there are gauge groups which can have rather small coupling constants. As a result, the coupling of $U(1)_A$ can be small enough to solve the second problem.

$^\dagger$ We thank Andrei Linde for clarifying this point.
In the next section we briefly review the features type I strings (orientifolds of IIB string theory) which are relevant for D-term inflation. In section 3, we show how these features can be used to solve the two problems that arise in heterotic string theories. We discuss our results and conclude in the last section.

2. Type I String Theory or Orientifolds of Type IIB Strings

Type I string compactifications with $N = 1$ supersymmetry can be obtained by orientifolds of type II strings[11]. We start with a type IIB string theory in $D = 10$ and mode it out by the world-sheet parity transformation $\Omega$. This gives a type I string theory in $D = 10$ with gauge group $SO(32)$. The gauge group arises from the 32 D9 branes required for tadpole cancellation. This type I string theory is further compactified on an orbifold of $T^6$ (i.e. on $T^6/\Gamma$ where $\Gamma$ is a discrete group such as $Z_n$) resulting in a $D = 4$ theory with $N = 1$ supersymmetry and chiral matter content. The above construction has only D9 branes but by considering more elaborate orientifolds one can obtain models with two types of branes in the theory. There can be two sets of D-branes: either D9 and D5 branes or D3 and D7 branes[11]. The number of each kind of brane is fixed again by tadpole cancellation. On the two different kinds of branes ($Dp$ and $Dp'$) there are gauge fields from strings with both ends on the same kind of brane (i.e. $pp$ or $p'p'$ strings), giving two gauge groups, $G_p$ and $G_{p'}$. There are also matter fields which arise from strings with ends on different kinds of branes (i.e. $pp'$ strings).

For our purposes it is enough to consider the simplest case, a type IIB orientifold with D3 branes and one set of D7 branes compactified on an orbifold of $T^6$ with radius $R_c$. The D3 branes are along $X_{1,2,3}$ and the D7 branes are along $X_1, \ldots, X_7$. (Our results apply equally well to the cases with more than one set of D7 branes or to the case with D9 and D5 branes.) These models have two gauge groups; $G_3$ arising from D3 branes and $G_7$ from D7 branes with couplings[12]

$$\alpha_3 = \frac{g_I}{2}, \quad \alpha_7 = \frac{g_I}{2M^4_I R_c^4}$$

(9)
Here \( g_I \) is the type I string coupling constant and \( M_I = \alpha_{str}^{-1/2} \) is the string scale. Newton’s constant is given by

\[
G_N = \frac{1}{M_P^2} = \frac{g_I^2}{8M_I^8 R_c^8}
\]  

(10)

We see that contrary to the heterotic string case \( M_P \) and \( M_I \) do not need to be of the same order of magnitude. From eqs. (9) and (10) we get

\[
\frac{\alpha_p M_P}{\sqrt{2}} = \frac{1}{M_I^{(p-7)} R_c^{(p-6)}} = 3.5 \times 10^{17} \text{ GeV}
\]  

(11)

assuming for one set of branes \( \alpha_p \sim \alpha_U \sim 1/25 \), the unified value of the Standard Model coupling constants. For the other set of branes we get

\[
\alpha_p = \frac{g_I}{2} \frac{1}{(M_I R_c)^{(p-3)}}
\]  

(12)

Note that there is freedom in the scales of \( M_I \) and \( R_c \) subject to eq. (11). This should be compared to the heterotic case for which \( M_h = \sqrt{\alpha_U/8} M_P \) is fixed to be close to \( M_P \) and independent of the compactification radii.

The other important feature of type I string theory is related to the anomalous D–term. In these models the scale of the anomalous D–term is not fixed (compare to the heterotic case with eq. (2)) but is given by the VEV of some twisted moduli with a coefficient of \( O(1) \) [13]. These moduli VEVs are related to the blowing up of the orbifold which smooths out the singularities of the compact space. They are fixed only after supersymmetry is broken but it is safe to assume that the same mechanism that fixes the radii of the compactification torus also fixes them. (Note that these radii are given by VEVs of the untwisted moduli.) Thus, we assume that untwisted moduli VEVs are of order \( R_c^{-1} \).

\* Here the situation is different from the one considered in [14] since there are D branes in the vacuum.
The matter content of these models arises from strings stretched between different branes, i.e. 33, 37, 73 and 77 strings. We denote these fields by \( M^{33}, M^{37}, M^{73}, M^{77} \). \( M^{33} \) and \( M^{77} \) are in the adjoint representation of the respective gauge groups, \( G_3 \) and \( G_7 \), whereas \( M^{37} \) and \( M^{73} \) are in the bifundamental representation. In realistic models the gauge groups will be broken down by Wilson lines and there will be gauge singlets coming from either \( M^{33} \) or \( M^{77} \) (or both). The tree level superpotential generically contains the terms\[12\]

\[
W = g_3(M^{33}M^{73}M^{37}) + g_7(M^{77}M^{37}M^{73})
\] (13)

Note that the Yukawa couplings are given by the two gauge couplings in eq. (9).

3. D–term Inflation in Type I String Theory

We now consider a type I string model such as the one above. This model has a gauge singlet \( M^{33} \) (after symmetry breaking by Wilson lines) which we identify with the inflaton field \( \sigma \). \( M^{33} \) has tree level couplings to other gauge nonsinglets such as \( M^{37}, M^{73} \) given by eq. (13). These play the role of the trigger fields \( \phi, \bar{\phi} \). Note that they are charged under \( G_7 \) so we assume that the anomalous \( U(1)_A \) comes from this sector. Also \( M^{37} \) and \( M^{73} \) are conjugates so their \( U(1)_A \) charges are opposite (which we take to be \( \pm 1 \)). \( g_3 \sim 1/2 \) gives the Yukawa coupling, i.e. \( \lambda \) in eq. (1). Thus, this simple model has all the ingredients for D–term inflation such as the inflaton and trigger fields, an anomalous D–term and the correct superpotential.

As mentioned above, in type I models the scale of the anomalous D–term is not fixed but given by the VEV of a twisted modulus \( M_t \). This VEV should be of the order of magnitude of other moduli VEVs such as compactification radii. On the other hand, we saw that in type I compactifications there is some freedom in \( M_I \) and \( R_c \) subject to eq. (11). If we embed the Standard Model inside the D3 branes...
(i.e. inside $G_3$) we get from eq. (12) $M_I \sim 2 \times 10^{16} \text{GeV}$ and $R^{-1}_c \sim 8 \times 10^{15} \text{GeV}$. Then, $M_t \sim R^{-1}_c$ is of the correct order of magnitude to account for COBE data. This is the solution of the first problem mentioned in the introduction.

In addition, we saw that the gauge group arising from one set of D branes (in our case $G_7$) can have a relatively small coupling given by eq. (9). With the above values for $M_I$ and $R_c$ we get for the coupling of the $U(1)$ (which comes from the D7 branes or $G_7$) $\alpha_7 \sim \alpha \sim 5 \times 10^{-4}$ which is a rather small value. Substituting this into eq. (7) we find that the initial value for the inflaton should be $\sigma_i \sim 0.1 M_P$. This is still much larger than the critical value $\sigma_{cr}$ but small enough for the supergravity approximation to string theory to be valid. In this case, the final value of the inflaton is close to the critical value, $\sigma_f \sim \sigma_{cr}$. More complicated models with more than two sets of D–branes and unisotropic tori can give smaller $\alpha$ and therefore smaller $\sigma_i$. In these cases, if $U(1)_A$ arises from a set of D7 branes with two large (i.e. larger than $M^{-1}_I$) dimensions from eq. (9) we see that $\alpha$ can be quite small resulting in $\sigma_i$ as small as $\sigma_{cr}$.

4. Discussion and Conclusions

In this letter, we showed that the two problems which are generic to D–term inflation in heterotic string models are absent in type I string models. The first problem related to the magnitude of the density fluctuations is solved by the low scale of the anomalous D–term in these models. The string scale and the compactification radii of type I strings are not fixed and may be much smaller than those of the heterotic ones. The scale of the anomalous D–term is of the same order of magnitude which is about the scale needed to accommodate the COBE data. The second problem related to the very large field values of the inflaton is also absent due to the very small gauge coupling of $U(1)_A$. This is possible in type I models since the gauge group arises from two types of D branes independently. One set of branes gives the Standard Model group (with $\alpha_U \sim 1/25$) whereas $U(1)_A$ can arise from the other set of D branes. In this case, for $R^{-1}_c > M_I$, the $U(1)_A$ gauge
coupling can be much smaller than $\alpha_U$. Then from eq. (7) we find that the initial value of the inflaton is at most $M_\text{P}/10$ which is small enough for the supergravity approximation to string theory to be valid.

In this letter, we considered the simplest possible type I string model with two sets of D branes on an isotropic $T^6$. This can be easily generalized to more complicated models with four sets of D branes (one set of D3 and three sets of D7 branes or D9 and D5 branes) and a torus with different compactification radii. All of our results will also hold in these cases. However, due to the extra fields and gauge symmetries present in these cases some other requirements such as reheating may be more easily met. We also mentioned that a smaller initial value for the inflaton is possible in these cases. In discussing the D–term inflation scenario above we made a few generic assumptions such as the presence of gauge singlet fields and the properties of $U(1)_A$. It would be interesting to build realistic $D = 4$ type I string models and see if these are in fact realized. We think it is quite encouraging to find that the generic problems of D–term inflation in heterotic string theory are easily solved in type I string models.

Acknowledgements

We would like to thank Ignatios Antoniadis for drawing my attention to refs. [11-13] and Andrei Linde for reading the manuscript.
REFERENCES

1. E. Copeland, A. Liddle, D. Lyth, E. Stewart and D. Wands, *Phys. Rev.* **D49** (1994) 6410; M. Dine, L. Randall and S. Thomas, *Phys. Rev. Lett.* **75** (1995) 398.

2. E. Halyo, *Phys. Lett.* **B387** (1996) 43, hep-ph/9606423; P. Binetruy and G. Dvali, *Phys. Lett.* **B388** (1996) 241, hep-ph/9606342.

3. J. A. Casas and C. Munoz, *Phys. Lett.* **B216** (1989) 37; J. A. Casas, J. Moreno, C. Munoz and M. Quiros, *Nucl. Phys.* **B328** (1989) 272.

4. M. Dine, N. Seiberg and E. Witten, *Nucl. Phys.* **B289** (1987) 585.

5. A. D. Linde, *Phys. Lett.* **B259** (1991) 38; *Phys. Rev.* **D49** (1994) 748.

6. J. Atick, L. Dixon and A. Sen, *Nucl. Phys.* **B292** (1987) 109.

7. C. Kolda and J. March–Russell, hep-ph/9802358.

8. C. Kolda and D. Lyth, hep-ph/9812234.

9. J. Espinoza, A. Riotto and G. Ross, *Nucl. Phys.* **B531** (1998) 461; hep-ph/9804214.

10. D. Lyth and A. Riotto, hep-ph/9807278.

11. G. Aldazabal, A. Font, L. Ibanez and G. Violero, hep-th/9804026.

12. L. Ibanez, C. Munoz and S. Rigolin, hep-ph/9812397.

13. L. Ibanez, R. Rabadan and A. Uranga, hep-th/9808139.

14. J. March–Russell, *Phys. Lett.* **B437** (1998) 318, hep-ph/9806426.