The simulation of the gas flow through the porous media and fences

Martin Kyncl1,* and Jaroslav Pelant1,**

1VZLÚ - Czech Aerospace Research Centre, Beranových 130, 199 05 Praha - Letňany, Czech Republic

Abstract. The paper is focused on the numerical simulation of the compressible gas flow through the porous media and fences. We work with the the non-stationary viscous compressible fluid flow, described by the RANS equations. The flow through the porous media is characterized by the loss of momentum. It is possible to use various methods for the simulation of such flow. Here we present the approach with the modification of the face flux. The original approach was presented recently by the authors analysing the modification of the Riemann problem with one-side initial condition, complemented with the Darcy’s law and added inertial loss. Another aim of this paper is the evaluation and estimate of the forces acting on the diffusible barrier (fence) with given parameters. The presented examples were obtained with the own-developed code for the solution of the compressible gas flow.

1 Introduction

The physical theory of the compressible fluid motion is based on the principles of conservation laws of mass, momentum, and energy. The mathematical equations describing these fundamental conservation laws form a system of partial differential equations (the Euler equations, the Navier-Stokes equations, the Navier-Stokes equations with turbulent models). We focus on the flow through the porous media and through the diffusible barriers (fences). We choose the well-known finite volume method to discretize the analytical problem, represented by the system of the equations in generalized (integral) form. To apply this method we split the area of the interest into the elementary elements. In order to compute these fluxes, various methods can be used. For example it is possible to use the solution of the Riemann problem for the construction of the face fluxes. Recently, in [1–3] we have shown the use of the analysis of this exact solution also for the discretization of the fluxes through the boundary edges/faces and on the edges/faces simulating the diffusible barrier (fences). Here we present other more simple method for the construction of the flux through the face representing the diffusible barrier. This method was implemented into own computational code, and used in the numerical examples. Another method for the numerical simulation of the porous media is described in [4].

2 Formulation of the Equations

We consider the conservation laws for viscous compressible turbulent flow of ideal gas with the zero heat sources in a domain $\Omega \in \mathbb{R}^3$, and time interval $(0, T)$, with $T > 0$. The system of the Reynolds-Averaged Navier-Stokes equations in 3D has the form

$$\frac{\partial \mathbf{w}}{\partial t} + \sum_{s=1}^{3} \frac{\partial f_s(w)}{\partial x_s} = \sum_{s=1}^{3} \frac{\partial R_s(w, \nabla w)}{\partial x_s} + S_{\Omega} = \Omega \times (0, T).$$

(1)

Here $x_1, x_2, x_3$ are the space coordinates, $t$ the time, $w$ is the state vector, $f_s$ are the inviscid fluxes, $R_s$ are the viscous fluxes, $S$ are additional sources. $\mathbf{v} = (v_1, v_2, v_3)^T$ denotes the velocity vector, $\rho$ is the density, $p$ the pressure, $\theta$ the absolute temperature, $E = \rho \theta + \frac{1}{2} \rho \mathbf{v}^2 + \rho \mathbf{k}$ the total energy.

$$w(x, t) = (\rho, \rho v_1, \rho v_2, \rho v_3, E)^T$$

$$f_s = (\rho v_s, \rho v_s v_1 + \delta_{1s} p, \ldots, \rho v_s v_3 + \delta_{3s} p, (E + p) v_s)^T$$

$$R_s = (0, \tau_{s1}, \tau_{s2}, \tau_{s3}, \sum_{i=1}^{3} \tau_{s} v_i + C_s \partial \theta / \partial x_s)^T$$

Here $\delta_{ij}$ denotes Kronecker’s delta, and $\tau_{s} = (\mu + \mu_T) S_{s} - \delta_{ij} S_{ij}$, with $S_{11} = \frac{1}{\gamma} \left( \frac{\gamma}{\gamma - 1} \right) \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{\gamma} \frac{\partial u_i u_j}{\partial x} \right), S_{12} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}, S_{13} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}, S_{21} = S_{12}, S_{22} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + 2 \frac{\partial u_i u_j}{\partial x} \right), S_{23} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}, S_{31} = S_{13}, S_{32} = S_{23}, S_{33} = \frac{1}{4} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + 2 \frac{\partial u_i u_j}{\partial x} \right)$, where $\mu$ is the dynamic viscosity coefficient dependent on temperature, $\mu_T$ is the eddy-viscosity coefficient. For the specific internal energy $e = c_p \theta$ we assume the caloric equation of state $e = p / (\gamma - 1)$, $c_p$ is the specific heat at constant volume, $\gamma > 1$ is called the Poisson adiabatic
constant. The constant $C_k$ denotes the heat conduction
coefficient $C_k = \left(\frac{1}{\gamma} + \frac{\nu}{\nu_T}\right)c\gamma$, and $P_r$ is laminar and $P_{tr}$
is turbulent Prandtl constant number. In our application of
flow in the gravitational field we set the source terms to
$S = S_g = (0, \varepsilon g_1, \varepsilon g_2, \varepsilon g_3, \varepsilon g \cdot \mathbf{v})$, where $\mathbf{g} = (g_1, g_2, g_3)$ is the
gravitational acceleration vector.

2.1 Porous media source term

The porous media is simulated using the modification of
the system of equations (1). The simple porous media can be
simulated via the new source term, written as

$$S_{PM} = \begin{pmatrix}
0 \\
-\frac{\eta}{\mu} v_1 - C_0 \frac{\mu}{\mu} |v_1| \\
-\frac{\eta}{\mu} v_2 - C_0 \frac{\mu}{\mu} |v_2| \\
-\frac{\eta}{\mu} v_3 - C_0 \frac{\mu}{\mu} |v_3| \\
-\frac{\eta^2}{\mu^2} - C_0 \frac{\mu}{\mu} |v^2|
\end{pmatrix}$$

(2)

Here $\alpha$ is the permeability coefficient, $C_p$ is the
pressure gradient coefficient. For the simulation of the thin
fence with small viscous effect we choose large value of
the coefficient $\alpha_{\text{alpha}} = 1e7$, which leads to negligible
viscous term $\frac{\eta}{\mu}$. Based on the work [5] we may
estimate the parameter $C_0 = 0.52(1 - \eta^3)/\eta^2$. Here $\eta$ is the porosity
parameter of the barrier. For the 30% barrier we choose
$\eta = 0.3$. For the barriers 15%,30%,42%,50%,70% we may estimate $C_0 = 2300, 525, 250, 150, 55$.

2.2 Model of turbulence

Here we assume the system (1) equipped with the two-
equation turbulent model $k - \omega$ (Kok), described in [6].
The effective turbulent viscosity is $\mu_T = \frac{\eta^2}{\mu}$.

$$\frac{\partial k}{\partial t} + \frac{\partial (\mu T k)}{\partial x_1} + \frac{\partial (\mu T k)}{\partial x_2} + \frac{\partial (\mu T k)}{\partial x_3} = P_k - \beta \frac{\eta^2}{\mu} k +$$

$$+ \frac{\partial}{\partial x_1} \left( \mu + \eta k \right) \frac{\partial k}{\partial x_1} +$$

$$+ \frac{\partial}{\partial x_2} \left( \mu + \eta k \right) \frac{\partial k}{\partial x_2} + \frac{\partial}{\partial x_3} \left( \mu + \eta k \right) \frac{\partial k}{\partial x_3}$$

(3)

$$\frac{\partial \omega}{\partial t} + \frac{\partial \omega}{\partial x_1} + \frac{\partial \omega}{\partial x_2} + \frac{\partial \omega}{\partial x_3} = P_\omega - \beta \frac{\eta^2}{\mu} \omega +$$

$$+ \frac{\partial}{\partial x_1} \left( \mu + \eta k \right) \frac{\partial \omega}{\partial x_1} +$$

$$+ \frac{\partial}{\partial x_2} \left( \mu + \eta k \right) \frac{\partial \omega}{\partial x_2} + \frac{\partial}{\partial x_3} \left( \mu + \eta k \right) \frac{\partial \omega}{\partial x_3}$$

(4)

where $k$ the specific turbulent kinetic energy and $\omega$ the
turbulent dissipation are functions of time $t$ and space coordinates $x_1, x_2, x_3$. The production terms $P_k$ and $P_\omega$ are given

by formulas

$$P_k = \tau_{11} \frac{\partial v_1}{\partial x_1} + \tau_{12} \frac{\partial v_1}{\partial x_2} + \tau_{13} \frac{\partial v_1}{\partial x_3} +$$

$$+ \tau_{21} \frac{\partial v_2}{\partial x_1} + \tau_{22} \frac{\partial v_2}{\partial x_2} + \tau_{23} \frac{\partial v_2}{\partial x_3} +$$

$$+ \tau_{31} \frac{\partial v_3}{\partial x_1} + \tau_{32} \frac{\partial v_3}{\partial x_2} + \tau_{33} \frac{\partial v_3}{\partial x_3}$$

$$P_\omega = \frac{\alpha_{\omega} \omega P_k}{k}$$

where functions $\tau$ are defined in (1) with $\mu = 0, \alpha_\omega = \frac{1}{\eta^2}$ and $\sigma_k = \frac{3}{2}, \beta = 0.09, \beta_k = 0.5, \kappa = 0.41$. The cross-diffusion term $C_D$ is defined as

$$C_D = \sigma_d \frac{\partial \omega}{\partial \omega} \left( \frac{\partial k}{\partial x_1} \frac{\partial \omega}{\partial x_1} + \frac{\partial k}{\partial x_2} \frac{\partial \omega}{\partial x_2} + \frac{\partial k}{\partial x_3} \frac{\partial \omega}{\partial x_3} \right)$$

(5)

where $\sigma_d = 0.5$ is constant.

3 Numerical method

For the discretization of the system we proceed as described in [7, 8]. We use either explicit or implicit finite volume method (FVM) to solve the systems sequentially. Other possible discretizations were shown in [16, 17]. Here we present the discretization of the system (1) using FVM. By $\Omega_i$ we let the domain the polygonal
approximation of $\Omega$. The system of the closed polyhedrons with mutually disjoint interiors $D_i = [D_i]_{i \in j}$, where
$J \subset Z^* = \{0, 1, \ldots\}$ is an index set and $h > 0$, will be called a finite volume mesh. This system $D_h$ approximates
the domain $\Omega$, we write $\partial D_h = \bigcup_{i \in j} D_i$. The elements $D_i \in D_h$ are called the finite volumes. For two neighboring
elements $D_i, D_j$ we set $\Gamma_{ij} = D_i \cap \partial D_j = \Gamma_{ji}$. Similarly, using the negative index $j$ we may denote the boundary faces.
Here we will work with the so-called regular meshes, i.e. the intersection of two arbitrary (different) elements is either
empty or it consists of a common vertex or a common edge or a common face (in 3D). The boundary $\partial D_i$ of each element $D_i$ is

$$\partial D_i = \bigcup_{\Gamma_{ij} \ni i} \Gamma_{ij}$$

(6)

Here the set $\Gamma_{ij}$, $\Gamma_{ij} \subseteq \partial D_j$ forms the boundary
$\partial D_j$. By $n_{i,j}$ let us denote the unit outer normal to $\partial D_j$ on $\Gamma_{ij}$. Let us construct a partition $0 = t_0 < t_1 \ldots$ of the
interval $[0, T]$ and denote the time steps $t_k = t_{k+1} - t_k$.
We integrate the system (1) over the set $D_j \times (k, k+1)$. With the integral form of the equations we can study a flow with
discontinuities, such as shock waves, too.

$$\int_{D_j} \frac{\partial \omega}{\partial t} \frac{\partial x}{\partial x} dt + \int_{\partial D_j} \int_{D_j} \frac{\partial f_i \omega}{\partial x_j} dx dt =$$

$$= \int_{D_j} \int_{\partial D_j} \frac{\partial R_i (\omega, \nabla \omega)}{\partial x_j} dx dt$$

(7)

Using the Green’s theorem on $D_j$ it is

$$\int_{D_j} \sum_{m=1}^{3} \frac{\partial f_i \omega}{\partial x_j} dx = \int_{\partial D_j} \sum_{m=1}^{3} R_i (\omega, \nabla \omega) \omega_i ds$$

(8)
Here \( n = (n_1, n_2, n_3) \) is the unit normal to \( \partial D_i \). Further we use (5), and we rewrite (6)

\[
\int_{D_i} (w(x, t_k + \Delta t) - w(x, t_k)) \, dx + \int_0^{\Delta t} \sum_{i=1}^3 \sum_{j=1}^3 (f_j(w) - R_i(w, \nabla w)) \, dS \, dt = 0
\]  

(8)

where \( w_i \) denotes the value of the approximate solution on \( D_i \) at time instant \( t_k \). We approximate the integral over the element \( D_i \)

\[
\int_{D_i} (w(x, t_k)) \, dx \approx |D_i| w_i^k
\]  

(9)

Further we proceed with the approximation of the fluxes. Usually the integral \( \sum_{i=1}^3 \sum_{j=1}^3 f_j(w)(n_{i,j}) \, dS \) is being approximated by a numerical flux at suitable time instant \( t_k \)

\[
\sum_{i=1}^3 \sum_{j=1}^3 f_j(w)(n_{i,j}) \, dS \approx H(w_i, w_{i,j}, n_{i,j})
\]  

(10)

with \( w_i, w_{i,j} \) denoting the approximate solution on the elements adjacent to the edge \( \Gamma_{ij} \) at the time instant \( t_k \). In the case of a boundary face the vector \( w_i \) has to be specified. Here we show the numerical flux based on the solution of the Riemann problem for the split Euler equations. By \( w_{i,j} \) let us denote the state vector \( w \) at the center of the edge \( \Gamma_{ij} \) at the time instant \( t_k \), and let us suppose \( w_{i,j} \) is known. Evaluation of these values will be a question of the further analysis, here we use them to approximate the integrals with the one-point rule

\[
\sum_{i=1}^3 \sum_{j=1}^3 f_j(w(x, t_k))(n_{i,j}) \, dS \approx |\Gamma_{ij}| \sum_{i=1}^3 f_j(w_{i,j})(n_{i,j})
\]  

(11)

Here \( \nabla w_{i,j} \) denotes the flux at the center of the edge \( \Gamma_{ij} \) at time instant \( t_k \). Now it is possible to approximate the system (8) by the following explicit finite volume scheme

\[
(w_i^{k+1} - w_i^k) + \Phi(w_i^k) = 0
\]  

(12)

with

\[
\Phi(w_i^k) = \frac{\Delta t}{|D_i|} \sum_{i,j=1}^3 |\Gamma_{ij}| \left( H(w_i^k, w_{i,j}^k, n_{i,j}) - \sum_{i=1}^3 R_i(w_{i,j}^k, \nabla w_{i,j}^k) n_{i,j} \right)
\]  

3.1 Porous media discretization

The simplest way to include the porous media within the FVM is to choose the elements (porous area), where the flux \( w_i^k \) is constant on each element \( D_i \), and \( \Gamma_i \) is the time instant. By \( w_i^k \) we denote the value of the approximate solution on \( D_i \) at suitable time instant \( \Gamma_i \). For the porous media discretization, we use (5), and we rewrite (6)

\[
\sum_{i=1}^3 \sum_{j=1}^3 f_j(w)(n_{i,j}) \, dS = \left( \int_{\Gamma_i} \sum_{j=1}^3 R_i(w, \nabla w) \, dS \right)_{t_k} dt = 0
\]  

(13)

where \( D_i \) is the time instant. By evaluat-

\[
\int_{\Gamma_i} \sum_{j=1}^3 R_i(w, \nabla w) \, dS 
\]

(14)

where \( D_i \) is the time instant. By evaluat-

\[
\int_{\Gamma_i} \sum_{j=1}^3 R_i(w, \nabla w) \, dS 
\]

(15)

Fur-

\[
\int_{\Gamma_i} \sum_{j=1}^3 R_i(w, \nabla w) \, dS 
\]

(16)

The Newton method.

We may proceed as shown in [9][page 216]. At each time level \( t_{k+1} \) the following iterative scheme is applied:

\[
w_i^{k+1,0} = w_i^k
\]  

(17)

\[
I + \frac{D\Phi(w_i^{k+1,r}, n_{i,j})}{Dw} (w_i^{k+1,r+1} - w_i^{k+1,r}) = 0
\]  

(18)

Here \( \frac{D\Phi(w_i^{k+1,r}, n_{i,j})}{Dw} \) is the Jacobian matrix of the mapping \( \Phi \). Using such process we obtain the sequence \( \{w_i^{k+1,r}\} \) converging to \( w_i^{k+1} \) as \( r \to \infty \).

The crucial problem of this discretization lies with the evaluation of the face fluxes \( H(w_i, w_{i,j}, n_{i,j}) \). One possibility is to use the linearization via the Taylor expansion of the vector function \( H(w, w_{i,j}, n_{i,j}) \), this was shown in [7].

Here \( |\Gamma_{ij}| \) is the volume of the non-existing (artificial) element adjacent to the face, representing the diffusible barrier (fence) with diameter \( d \).
4 Example

Let us suppose the gas flow with the total temperature $\theta_0 = 293.15K$, total pressure $p_0 = 101325$ Pa, and given velocity magnitude $u_\infty$ (freestream values). Let us place the fence with the known parameter $C_0$ into such flow. Our aim is to compute/estimate the force acting on such fence. The force is computed by integration of the source term $S_{PM}$ at the barrier. Figure 1 shows the graph of the force component $F_X$ depending on the barrier parameter $C_0$ and the velocity regime $u_\infty$.

![Figure 1. CFD simulations, force $F_X$ acting on the barrier depending on the regime velocity $u_\infty$, barriers 50%,42%,30%,15% (from top left), the computed values are fitted with the quadratic curves.](image)

Based on the CFD simulations, figure 1, it is possible to use rough approximation of the force acting on the barrier as

$$F_{XA} = \int_S A_1 u^2 + A_2 u \, dS, \quad \text{or} \quad F_{XB} = \int_S K u^2 \, dS.$$  \hfill (15)

To be more precise, the fluid density should be also included in these simplified relations

$$F_{XA} = \int_S B_1 u^2 + B_2 u \, dS, \quad \text{or} \quad F_{XB} = \int_S K_2 u^2 \, dS.$$  

The estimated values for the coefficients for the considered barriers are shown in table 1.

Further, let us suppose the gas flow above the plate with the 0.1m fence. Let us form the initial and the boundary conditions using the flat plate simulation with the regime $\theta_0 = 293.15K$, $p_0 = 101325$, $u_\infty = 15m.s^{-1}$, stationary state, cut at 70m from the plate start. The regime is shown in figure 2. The resulting velocity is shown in figure 3. The comparison of the CFD computed force and simplified estimation is shown in figure 4.

Further examples involve the body placed behind the fence in various distances. The aim was to estimate the reduction of the horizontal force acting on the body.

![Fig. 2. Initial condition 1570, velocity profile. CFD simulation, regime $u_\infty = 15m.s^{-1}$. Horizontal velocity profiles of the flow above the flat plate at given distances from the start of the plate (4.6,5.10,20,30,50,70,100,200,300,500,700). Data measured during the VZLU experiment are shown by the red boxes, black triangles approximate this profile by logarithmic curve. The chosen cut at 70m is close to this experimental simulation.](image)

Table 1. Coefficients for the quadratic estimation of the force (15) acting on the barrier (fence) with given parameters.

| BARRIER | $C_0$ | $K$   | $A_1$  | $A_2$   |
|---------|-------|-------|--------|---------|
| 15%     | 2300  | 0.784316 | 0.782287 | 0.101437 |
| 30%     | 525   | 0.667824  | 0.666403 | 0.071075 |
| 42%     | 250   | 0.57244   | 0.570988 | 0.0726   |
| 50%     | 150   | 0.473028  | 0.471984 | 0.0522125|
| C200A5e-9 | 200 | 0.646404  | 0.607705 | 1.93496  |
| C100A1e-8  | 100 | 0.51678   | 0.464194 | 2.62933  |
| C50A5e-8   | 50  | 0.284178  | 0.244203 | 1.99874  |

Table 1. Coefficients for the quadratic estimation of the force (15) acting on the barrier (fence) with given parameters.

![Table 1. Coefficients for the quadratic estimation of the force (15) acting on the barrier (fence) with given parameters.](image)

Conclusion

This paper is focused on the viscous compressible flow, with the focus on the porous media and the diffusible barrier. The boundary condition for the diffusible barrier based on the modification of the Riemann Problem was shown in [1–3], here we have shown other possibility. We have shown the simple estimate of the force acting on the fence, and further we used our CFD for the estimation of the forces acting on the body placed behind the barrier. All codes were implemented into the own-developed software. The numerical examples were presented.

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Fig. 3. CFD simulations of the flow through the barrier, regime 1570: isolines of the horizontal velocity component. From top: the used computational mesh, barriers with the porosity 50%, 42%, 30%, 15%.

Fig. 4. Regime 1570, the force acting on the barriers with porosity 50%, 42%, 30%, 15%, CFD simulations and simplified estimations based on the relation (15) and table 1.

Fig. 5. CFD simulations of the flow through the barrier, regime 1570 the force acting on the body in the distance $5H,10H,15H,20H$ behind the barrier with porosity 30% and height $H$.

Fig. 6. Regime 1570, the force acting on the body in the distance $5H,10H,15H,20H$ behind the barrier with porosity 50%, 42%, 30%, 15% and height $H$. 
References

1. M. Kyncl and J. Pelant, *EPJ WoC*, **45**, 01054, (2013)
2. M. Kyncl and J. Pelant, *EPJ WoC*, **180**, 02052, (2018)
3. M. Kyncl and J. Pelant, *ENGINEERING MECHANICS 2018*, **24**, p.469-472, (2018)
4. J. Česenek, *EPJ WoC* (to be published) (2019)
5. Reynolds, A. J., *J. Mech. Eng. Sci.*, **11**, p. 290–294, (1969)
6. C. J. Kok, *AIAA Journal*, **38**, 7, (2000)
7. M. Kyncl and J. Pelant, *Implicit method for the 3d RANS equations with the k-ω (Kok) Turbulent Model*. Technical report R5453, (2012)
8. M. Kyncl and J. Pelant, *Implicit method for the 3d Euler equations*. Technical report R5375, VZLÚ, (2012)
9. M. Feistauer, J. Felcman, and I. Straškraba, *Mathematical and Computational Methods for Compressible Flow*. Oxford University Press, Oxford, (2003)
10. M. Feistauer, *Mathematical Methods in Fluid Dynamics*. Longman Scientific & Technical, Harlow, (1993)
11. E. F. Toro, *Riemann Solvers and Numerical Methods for Fluid Dynamics*. Springer, Berlin, (1997)
12. M. Kyncl, *Numerical solution of the three-dimensional compressible flow*. Doctoral Thesis, Prague, (2011)
13. M. Kyncl and J. Pelant, *Proc. ECCOMAS 2014, ECFD VI*, 20.-25.7.2014, Spain (2014)
14. M. Kyncl and J. Pelant, *Appl. Mech. and Mat.*, **821**, 70-78, (2016)
15. M. Kyncl and J. Pelant, *Proc. ECCOMAS 2016*, 5.-10.6.2016, Greece (2016)
16. J. Česenek, *EPJ WoC* **114**, 02012 (2016)
17. J. Česenek, *EPJ WoC* **180**, 02016 (2018)