A Game-Theoretic Approach for Detection of Overlapping Communities in Dynamic Complex Networks

Elham Havvaei and Narsingh Deo
Department of Computer Science
University of Central Florida
(Dated: March 3, 2016)

Complex networks tend to display communities which are groups of nodes cohesively connected among themselves in one group and sparsely connected to the remainder of the network. Detecting such communities is an important computational problem, since it provides an insight into the functionality of networks. Further, investigating community structure in a dynamic network, where the network is subject to change, is even more challenging. This paper presents a game-theoretical technique for detecting community structures in dynamic as well as static complex networks. In our method, each node takes the role of a player that attempts to gain a higher payoff by joining one or more communities or switching between them. The goal of the game is to reveal community structure formed by these players by finding a Nash-equilibrium point among them. To the best of our knowledge, this is the first game-theoretic algorithm which is able to extract overlapping communities from either static or dynamic networks. We present the experimental results illustrating the effectiveness of the proposed method on both synthetic and real-world networks.

Keywords: Community Structure; Dynamic Network; Extremal Optimization; Nash Equilibrium

I. INTRODUCTION

Community detection problem in complex networks has been the subject of intensive studies throughout the last decade [1]. There is no formal or universally accepted definition for the notion of community. Intuitively, a community can be seen as a dense group of nodes which has more edges within itself than to the nodes outside of the group. Community structure is the set of all such communities in a network. Being able to extract community structure within a network provides a deeper insight through the functionality of systems represented as networks. Most complex networks such as social or biological networks exhibit community structure properties. In social networks a community is interpreted as a group of people who may have common interests, ethnicity or geographic location. Protein-protein interaction network is an example of a biological network displaying community structure in which proteins in a community are likely to have the same functionality[2]. Most techniques for detecting community structures partition networks into disjoint communities. However, most networks inherently have overlapping community structure. In social networks, people may be a member of multiple social communities. In citation networks, researchers may collaborate with different research groups. In the protein-protein interaction network, a large fraction of proteins reside in multiple protein complexes, at the same time. Thus, in contrast to disjoint community detection, techniques for detecting overlapping community structure where a node is able to reside in multiple communities reveal a deeper feature of many real-world networks [3][5]. Fundamentally, most complex networks in the real world are highly dynamic. In each timestep, a dynamic network is subject to a series of changes where new nodes or edges either appear or some existing ones vanish. Tracking and monitoring changes happening in a community structure while the network is experiencing a series of events (including adding or removing nodes or edges), give an insight into the future functionality of the system. Therefore, pursuing the evolution of community structures over time in dynamic networks is more informative in comparison to its static counterparts, and it is also more challenging. An online community detection algorithm which is capable of updating the community structure at each timestep, should be able to exploit the history of the community structure instead of starting from scratch. Fundamentally, most complex networks in the real world are highly dynamic. In each timestep, a dynamic network is subject to a series of changes where new nodes or edges either appear or some existing ones vanish. Tracking and monitoring changes happening in a community structure while the network is experiencing a series of events (including adding or removing nodes or edges), give an insight into the future functionality of systems. Therefore, pursuing the evolution of community structure over time in dynamic networks is more informative in comparison to its static counterparts, and it is also more challenging. An online community detection algorithm which is capable of updating the community structure at each timestep, should be able to exploit the history of community structure instead of starting from scratch. A major advance in the study of community detection problem was made by Newman and Girvan who introduced a quality measure $Q$, called modularity [3]. Given a community structure $C = \{c_1, c_2, ..., c_k\}$ in a corresponding network $G = (V, E)$ with node set $V$ and the edge set $E (n = |V|, m = |E|)$. The modularity is com-
where the sum runs over all communities. \( L(c) \) is the number of edges inside community \( c \) and \( d(c) \) is the sum of degrees of nodes in \( c \). Modularity was designed to measure the strength of partition of a network into communities. High value of this measure implies a community structure with dense communities which has more number of intra-edges within communities than the expected value of such number in a random partition. Optimizing the modularity value is recognized as a NP-complete problem. Therefore, a vast number of works have been devoted to optimizing this measure by employing approximation and heuristic methods. The focus of this paper is on extracting overlapping as well as non-overlapping community structures from either static or dynamic complex networks. We aim at, maximizing the modularity value by establishing a game among the nodes of the networks as players. Initially, each player resides in a singleton community and their goal is to maximize their payoffs by choosing to reside in one or more communities. The ideal state of the game is to reach a Nash equilibrium point where no player has a tendency to deviate from her situation by taking a unilateral action. In our context, a Nash equilibrium point implies that players are not able to gain a higher payoff by changing their communities, unilaterally. Further, an extremal optimization method is employed in order to perform the search and direct the game to the Nash equilibrium point.

II. RELATED WORK

There exist various techniques in literature which try to extract good quality community structures from complex networks. Some methods reduce the community detection problem to the graph partitioning, in which the graph is divided into a predefined number of communities such that the number of inter-edges crossing the network is minimal. One drawback of graph partitioning is that, generally, the number of communities is unknown. Hierarchical clustering is another technique which considers a hierarchical structure for the underlying networks and applies an agglomerative or divisive method to build a hierarchy of clusters. Hierarchical clustering technique requires a definition for similarity measure to group similar nodes together. One advantage of this technique is that there is no need for preliminary knowledge about the size or number of clusters. However, since each level of hierarchy corresponds to one partition of the network, choosing the one as a true community structure without any quality function is problematic.

In principle, most techniques try to define a quality function and by maximizing it, obtain a high quality community structure. Modularity \( (Q) \) is the best known quality function used as the quality index of extracted community structure. Modularity maximization is NP-complete since the number of possible divisions of a network into communities exceeds any power of the network size (number of nodes). Therefore, many works have been proposed to approximate \( Q \) and obtain a fair value of this measure by using greedy, extremal optimization, simulated annealing, or spectral optimization methods. In most techniques in literature, the extracted community structure is a set of disjoint communities. However, it is more reasonable to have overlapping community structure where a node is not restricted to merely belonging to one single community. A very popular method to extract overlapping community structure is the clique percolation method (CPM) introduced in . CPM builds a community structure by percolating a k-clique, i.e., a complete graph of \( k \) nodes, over a network. In this context, a community structure is the union of all k-cliques that can be reached from each other through the percolation process. Trivially, since the k-cliques can share nodes, the resulting community structure is overlapping. One limitation of CPM is to make an assumption about the value \( k \). Besides, another critical problem with CPM is its failure networks which have fewer number of cliques.

Another well-known method for detection of overlapping communities includes Link Partitioning that considers communities as groups of links/edges rather than nodes. In this method, a node is overlapping if its incident edges belong to more than one community. Moreover, non-negative matrix factorization is another technique, borrowed from machine learning and used for overlapping community detection. This method, generally, possesses a high complexity due to matrix multiplication.

Despite the difficulty of static community detection where the nodes and edges are fixed, this problem has also been studied on dynamic networks. Several evolutionary clustering methods have been proposed to extract evolving clusters, in such a way that the changes occurring in each timestep are not dramatic and in a user point of view the transitions seem smooth. In another attempt, AFOS [29] tries to categorize changes happening in a network, and to take an appropriate action, accordingly, to update the community structure. As an example, adding a new edge inside an existing community is considered as one category.

A more recent approach for community detection involves game-theoretic techniques which considers nodes of networks as rational decision makers. In this line of work, nodes are considered as a set of players who decide to join or leave communities with respect to their payoffs. Typically, at the start of community detection games, players reside in singleton communities or are distributed randomly to some number of communities. The choice of payoff function plays an important role in the quality of extracted community structures. One possible payoff function is the contribu-
tion of each individual to the global modularity value. Therefore, by establishing a game among the nodes of networks as the players, instead of directly optimizing a global objective function, the players are appointed to take actions and maximize it as they seek to maximize their own payoffs. Since, generally, achieving the maximum possible payoff does not exist, Nash equilibrium is often employed as the solution concept in games with no cooperation among the players. Nash equilibrium is a situation where no player has the tendency to deviate from her strategy while the strategy of others are kept unchanged.

III. OVERLAPPING COMMUNITY DETECTION GAME

The main contribution of this paper is to set up a game with the ultimate goal of identifying overlapping communities which maximizes modularity $Q$ in the extracted communities using an extremal optimization method. We refer to our method as Overlapping Communities Extremal Optimization (OCEO). In this method, each node takes a role as a player. A pair of players are considered neighbors if there is an edge between them. The neighboring set of player $p_i$ is defined as

$$\mathcal{N}_{p_i} = \{ p_j \in V \mid \exists \, e(p_i,p_j) \in E \}. \quad (2)$$

The number of players is finite and payoff of each player is a function of chosen action from the player’s strategy space. The players can only act on their turn. The elements of the game are as follows:

- **Players**: $\{p_1, p_2, ..., p_n\}$ denotes the set of players. Each player corresponds to a node in the set $V$.

- **Strategies**: The strategy space available to each player $p_i$ is to join an adjacent community or leave one of her current communities. A community $c$ is considered adjacent to a player if at least one of the player’s neighbors resides in $c$. The set of all adjacent communities for the player $p_i$ is defined as

$$AC(p_i) = \{ c \in \mathcal{C} \mid (\exists \, p_j \in \mathcal{N}_{p_i}) \wedge (c \in \text{COM}(p_j)) \}, \quad (3)$$

where COM($i$) is the set of all communities that player $i$ resides in. The strategy spaces of players (the set of adjacent communities) are finite which leads to a finite game.

- **Payoffs**: As stated, modularity $Q$ is the global variable to optimize. The idea is to appoint players in the game to maximize it for us by defining payoff function of players proportional to their contributions to the modularity $Q$. In [14], the contribution of each individual $i$ to modularity $Q$, by residing in community $c$, is computed as

$$q_i(c) = L_i(c) - k_i \times \frac{d(c)}{2m}, \quad (4)$$

where $L_i(c)$ is the number of edges player $i$ has inside community $c$, $k_i$ is the degree of the player (degree of the associated node) and $d(c)$ is the sum of degrees of all players residing in $c$. The total contribution of player $i$ to the modularity is the sum of $q_i(c)$ over all communities that $i$ belongs to. Specifically,

$$q_i = \sum_{c \in \text{COM}_i} q_i(c). \quad (5)$$

Note that $Q = \frac{1}{2m} \sum_i q_i$, where $i \in V$. In our game, we define the payoff function of each player $i$ in community $c$ as the contribution of the mentioned player to the global modularity by residing in $c$, rescaled by the degree of the player:

$$u_i(c) = \frac{q_i(c)}{k_i} = \frac{L_i(c) - d(c)}{2m}. \quad (6)$$

Due to the rescaling, the payoff of each associated player would be relative to their own degree and also normalized in the interval [-1,1] with 1 being the highest payoff a player can gain. Therefore during the game as long as the players attempt to improve their payoffs, modularity $Q$ is optimized.

The total payoff of $p_i$ is the sum of all $p_i$’ payoffs in all communities $p_i$ resides in, which is computed as

$$u_i = \sum_{c \in \text{COM}(i)} u_i(c). \quad (7)$$

In order to obtain a high quality community structure which corresponds to a high value of modularity, a heuristic search, based on extremal optimization is employed to perform a local search. Extremal optimization algorithm, proposed by Boettcher and Percus [32], finds high quality solutions for hard optimization problems by successively improving the undesirable components of the sub-solution. In other words, extremal optimization works on a single candidate solution and performs a local modification to the worst components by treating each of the components as species evolving locally through the process [33]. In the context of our game, extremal optimization keeps two solutions, that each of which corresponds to a community structure. One community structure preserves the best community structure found so far, and the other evolves through some number of iterations by performing a modification in the strategy of worst-payoff players. OCEO is a non-cooperative game since the players’ decisions are made independently and there is no collaboration or coalition among the players.

One of the most fundamental ideas for non-cooperative games is the concept of Nash equilibrium introduced by Nash [34]. Nash equilibrium point of a game is a situation in which no player can gain a higher payoff by taking a unilateral action. Existence of Nash equilibrium depends on the choice of the payoff function in a game with finite number of players.
A. Existence of Nash Equilibrium in OCEO

In this section, we prove the existence of Nash equilibrium in OCEO by characterizing it as a potential game, introduced by Monderer and Shapley [35]. In every finite potential game, the existence of Nash equilibrium is guaranteed [36]. First, we define the notions required for the definition of potential games. Consider strategy profile \( \chi = (\chi_1, \chi_2, ..., \chi_n) \) as a set of players’ strategies in the game in which \( \chi_i \) denotes the strategy of player \( i \). In our game, \( \chi_i \) is interpreted as a set of communities that player \( i \) resides in and \( \chi \) is capable of revealing the community structure of the underlying network. A reduced strategy profile \( \chi_{-i} \) is the strategy profile \( \chi \) for all players excluding the strategy of player \( i \), i.e., \( \chi_{-i} = (\chi_1, \chi_2, ..., \chi_{i-1}, \chi_{i+1}, ..., \chi_n) \). We also use \((\chi_{-i}, \chi'_i)\) to denote a strategy profile where the strategy of player \( i \) is replaced with strategy \( \chi'_i \).

**Definition 1:** A game \( \Gamma(V, \chi_i, \{u_i\}_{i \in V}) \) with \( u_i : \chi \to R \), is called an exact potential game, if it possesses a potential function \( \phi : \chi \to R \) such that \( \forall i \in V, \forall x_{-i} \in \chi_{-i}, \forall x_i \in \chi_i: \)

\[
\phi(x_{-i}, x'_i) - \phi(x) = u_i(x_{-i}, x'_i) - u_i(x),
\]

which means the change in the payoff of player \( i \) by taking a unilateral action \( x'_i \) is equal to the change in the global potential function. There is a weaker class of potential games, called weighted potential which admits a \( \omega \)-potential function.

**Definition 2:** Let \( \omega = \{\omega_i\}_{i \in V} \) be a vector of positive weights. Game \( \Gamma \) is weighted potential if it admits a \( \omega \)-potential function \( \phi : \chi \to R: \)

\[
\phi(x_{-i}, x'_i) - \phi(x) = \omega_i(u_i(x_{-i}, x'_i) - u_i(x)).
\]

Note that the exact potential games are subset of weighted potential games with \( \omega_i = 1 \) for each player \( i \in V \). It is proved that in every finite exact or weighted potential games, the best response dynamics, i.e., dynamics in which each player chooses a strategy with highest payoff, given the strategies of the other players, converges to a Nash equilibrium in a finite number of iterations [36].

**Lemma 1:** OCEO is a weighted potential game.

Proof: By considering the choice of payoff function \( u \), defined in Equation [4] and assigning weight vector \( \omega = (\omega_i)_{i \in V} \) where \( \omega_i = \frac{1}{k_i} \) for each player \( i \in V \), it can be verified that OCEO accepts the following \( \omega \)-potential function:

\[
\phi(\chi) = \sum_{c \in X} L(c) - \frac{1}{2m} \sum_{\substack{\text{COM}(i) = \text{COM}(j) \in X}} k_i k_j.
\]

Therefore, OCEO possesses at least one Nash equilibrium point and we aim at finding such a point which corresponds to the highest value of modularity \( Q \). In order to incorporate the Nash equilibrium concept into OCEO, we use a domination relation, Nash ascendancy, introduced by Lung, et al. [37] to direct the search performed by extremal optimization towards the Nash Equilibrium point. This relation enables the comparison of two solutions in Nash sense and determines which solution is closer to an equilibrium. Further, the use of this relation reduces the computational complexity of the search in comparison with the deterministic Nash equilibrium relation [37]. In [27], Nash ascendancy relation is defined for two non-overlapping community structures. By extending this relation to overlapping community structures, community structure \( D \) precedes \( P \) in Nash sense, if there are more number of players who prefer \( D \) over \( P \):

\[
\zeta(D, P) > \zeta(P, D),
\]

where \( \zeta(D, P) \) is the number of players who prefer their communities assignments in community structure \( D \) over \( P \). More formally, \( \zeta(D, P) \) is defined as

\[
\zeta(D, P) = \left| \{p_i : u_i(D) > u_i(P)\} \right|
\]

where \( |S| \) denotes cardinality of set \( S \). In other words, if \( D \) Nash ascends \( P \), then \( D \) has more number of players \( p_i \) who can increase their payoffs by switching from \( \text{COM}_D(i) \) to \( \text{COM}_P(i) \) than vice-versa and as a result, \( D \) seems more stable than \( P \) in Nash-sense.

B. Extremal Optimization Over Overlapping Community Detection Game

Assume initially each player resides in a singleton community. Community structure \( D \) is initialized as the set of all these singleton communities. Our purpose is to evolve \( D \) through some number of iterations by employing extremal optimization. Intuitively by giving the turn to a player \( p_i \) with the least payoff in particular community \( c_i \in \text{COM}_P(i) \) to perform some actions, that player is provided a chance to improve her total payoff by leaving \( c_i \) or joining a new community available on her strategy space. We define pairs of \(<p_i, c_i>\), in which \( p_i \) is a player who belongs to community \( c_i \). In case of overlapping community structure, there are multiple pairs \(<p_i, c_i>\). In our method, we rank such pairs \(<p_i, c_i>\) according to their corresponding payoff \( \gamma \) and keep all the sorted pairs in a pairs-pool. Thus, the first multiple pairs correspond to a player who has the least payoff in the associated community. In each iteration, \( \gamma \) pairs are chosen based on a selection mechanism from the pool. For each selected pair \(<p_i, c_i>\), player \( p_i \) tries to improve her payoff by joining a new community or leaving \( c_i \). In our selection mechanism, we naturally favor pairs with the lower payoff. We apply truncation selection in which the \( \gamma \) top-ranked players are selected from the pool. Within the selected set of pairs there is no further selection and all corresponding players have a chance to take action. To avoid early convergence, when the best solution has not been updated for \( \eta \) iterations, we switch from truncation to tournament selection method for the next iteration which we call that impulse iteration. A tournament consists of picking \( \delta \) (known as tournament size)
pairs from the pool using a uniform distribution probability. The winner of the tournament is the one with the lower value of payoff. Choosing \( \gamma \) pairs requires running \( \gamma \) tournaments. The selection pressure is adjusted by the tournament size. Small assignment of \( \delta \) brings in more randomness in the selection process and the pure randomness occurs when \( \delta = 1 \). On the other hand, larger value of \( \delta \) results in more selection of worst-payoff pairs which contradicts with our initial purpose of employing tournament selection. In our method, \( \eta = 5 \) and \( \delta = 3 \) work the best. OCEO proceeds as follows to evolve the single community structure \( D \):

1. Community structures \( D \) and \( P \) are initialized as each player resides in a singleton community. Community structure \( P \) is used to store the best \( D \) found in each iteration.

2. The payoffs of players, in each community they reside, belonging to community structure \( D \) are computed.

3. Pairs of \(<i, c>\) are sorted according to the payoff of player \( i \) in \( c \) and are maintained in a pairs-pool.

4. \( \gamma \) pairs are chosen from the pairs-pool based on truncation selection. The selection method is switched to tournament selection if it is an impulse iteration. For each selected pair \(<i, c>\) player \( i \), performs the following actions:
   - Finds a community \( c' \) among her strategy space with the maximum payoff by joining it and joins \( c' \) if \( u_i(c') > \min(u_i(c), 0.2) \)
   - Leaves \( c \) if \( u_i(c) < 0.2 \)

The selected players, by performing actions, modify the community structure \( D \). Note that the proposed algorithm is also capable of detecting non-overlapping communities by forcing the players to leave their current communities upon joining one with higher payoff.

5. If the resulting \( D \) Nash-ascends \( P \), \( P \) is replaced by \( D \). This replacement implies that the resulting \( D \) is the best solution found so far. Otherwise, \( D \) keeps performing the search without being stored.

6. Steps 4-5 are repeated until either the maximum number of iterations is reached or \( P \) has not been updated for \( \psi \) iterations after the last update. Upon termination, highly overlapped communities in the community structure \( P \) will be merged together and it is returned as the solution. In our method, two communities are considered highly overlapped if the division of common nodes to the size of smaller community exceeds 70%.

In step 4, after players’ movements, payoffs of all involved players need to be recalculated. When a player \( p_i \) joins/leaves community \( c \), the payoffs of all players who reside in \( c \) should be updated according to Equation 6 and the pair \(<p_i, c>\) should be inserted/deleted to/from the pairs-pool. In each iteration, the average number of players whose payoffs are imposed to change is \( \gamma |c| \), where \( |c| \) is the average number of players in one community. In the worst case, \( \gamma |c| \) is of the order of \( O(|V|) \). Payoffs of all other players remain unchanged. In order to keep the pairs-pool sorted, there is no need to sort the whole pool after applying the updates. Instead, we sort the updated pairs and merge them into the pairs-pool which is already sorted. In our method, we limit the players to reside in at most \( \kappa \) communities at the same time. Consequently, the maximum size of pairs-pool is \( \kappa |V| \) and the complexity of keeping the pairs-pool updated, in each iteration, is of the order of \( O(\gamma |c| \log\frac{|c|}{\gamma} + \gamma |c| \log |V|) \). The pseudocode of the explained process is presented in Algorithm 1.

\[ \gamma = \frac{N}{k}. \]  (13)

C. Parameter Settings

There are a number of parameters whose values can have a considerable influence in the performance of the algorithm.

- Number of selected players (\( \gamma \)): This parameter determines the number of players who will be provided the chance to play simultaneously and change their strategies, in each iteration. The higher the size of a network, the greater number of players will be selected to play. Thus, parameter \( \gamma \) should be proportional to the size of network. In an ideal situation, no pair of selected players should be adjacent. For clarification, assume two selected players A and B are adjacent who reside in communities \( c_1 \) and \( c_2 \), respectively. Obviously, \( c_2 \) appears in the strategy space of the player A and may persuade A to join \( c_2 \). Further, assume upon the joining of A to \( c_2 \), in the meantime, B leaves \( c_2 \). In this case, however rare, A’s payoff estimate would be impaired, since A relies on the current strategy of her adjacent players. More nodes are adjacent to each other when the network possesses a high average degree. Thus, the number of selected players should be related to both size of network and also the average degree of nodes:

- Max-Iteration: As shown in Algorithm 1 the game is repeated until either the maximum number of iterations reaches or the best solution obtained, does not get updated for \( \psi \) iterations. We set the Max-Iteration and \( \psi \) to 2000 and 100, respectively.
D. OCEO Complexity

Let \( k \) be the average degree of players in a network. Then, the time complexity \( O(\gamma k) \) is required for \( \gamma \) players to search through their strategy spaces and find the best communities to join. As stated above, the complexity of efficiently updating the pairs-pool is of the order of \( O(|V| \log |V| + |V|^2 \log |V|) \). Further, in each iteration, for determining the Nash-ascendency relation, payoff of each involved player in the dummy and parent communities needs to be compared which results in \( O(|V|) \) comparisons. Therefore, in overall, the complexity of OCEO is of the order of \( O(\text{MAX-Iteration} \times (|V| \log |V| + |V|^2 \log |V|)) \).
nism suffers from resolution limit and fails to detect the true modular structure. It finds a partition in which modules are merged together into groups of two, leading to the maximum modularity value (represented with dotted lines). In our method, this issue is addressed and the algorithm is capable of identifying each module as a community. In this network, combining two or more modules together into one community would increase the modularity value. Therefore, there should exist some number of players who make more contribution to the modularity value and also gain a higher payoff upon the merger. Remember that the payoff of each player is the contribution of that individual rescaled by the degree of that player. Considering the two modules, in Figure 1b as separate communities, contribution of players \(\{2,3,6,7\}\) to modularity are all the same and equal to \(q(2,3,6,7) = 3 - \frac{3 \times 14}{2 \times 350} = 2.94\). Accordingly, for the players \(\{1,4,5,8\}\), it is equivalent to \(q(1,4,5,8) = 3 - \frac{4 \times 14}{2 \times 350} = 2.92\). By merging these two modules together, the first set of players including players 1 and 8 experience a drop in their contribution to the modularity (and also their payoffs) since non of them have an edge to another module while the sum of degree of the players in the resulted community has increased (an increase in the second term of Equation 1). In this case \(q(2,3,6,7) = 3 - \frac{3 \times 28}{2 \times 350} = 2.88\) and \(q(1,8) = 3 - \frac{4 \times 28}{2 \times 350} = 2.84\). On the other hand, two players 4 and 5 make a higher contribution due to the existence of an edge connecting them, which is equal to \(q(4,5) = 4 - \frac{4 \times 28}{2 \times 350} = 3.84\). The whole point is that the increase in the contribution of players 4 and 5 are greater than the loss of other players which leads to a higher global modularity and as a result, the merger of the two modules. However, OCEO is capable of detecting all modules as communities, despite the fact that true community structure has the lower modularity value. The mechanism to resolve this issue is embedded in Nash ascendancy relation where the preferences of all individuals are taken into account, regardless of magnitude of the change in their payoffs or contributions to the modularity value. According to Equation 12, the true community structure Nash ascends the one with the pairs of adjacent modules as communities since there are fewer number of players who prefer the false community structure. Fundamentally, the true partition is inherently more stable in Nash sense. Hence, the Nash ascendancy relation, in addition to directing the game to the Nash equilibrium point, resolves the resolution limit in OCEO by considering the preferences of majority of players.

F. OCEO on Dynamic Networks

The dynamic network \(G_d = \{G_1,G_2,...,G_t\}\) is defined as a set of network snapshots evolving over time. Each
$G_t$ is a snapshot of the network $G_d$ at timestep $i$. The problem of community detection in a dynamic network is to detect the community structure at each timestep by using the extracted one of the previous snapshot. Our approach for dealing with the dynamic aspect is the same as one proposed in [27]. At each timestep, when receiving the next snapshot, OCEO reinitializes the community structure $P$ such that each player resides in a singleton community while $D$ keeps performing the search. By keeping the $D$ unchanged, the information from the community structure of previous snapshot is utilized.

IV. EXPERIMENTAL RESULTS

In this section, we compare the effectiveness of OCEO on both synthetic and real-world networks with other community detection algorithms. In the following, we describe datasets, metrics and analysis.

A. Datasets

Synthetic Networks: Lancichinetti, et al., present a benchmark (LFR benchmark) for community detection algorithms [39]. The LFR benchmark generates static networks with built-in community structure. The configuration of generated networks depends on various user-specified parameters. Number of nodes is $N$, $k$ specifies the average degree of nodes and $k_{\text{max}}$ is the upper bound on degrees of nodes. The mixing parameter $\mu$ is the fraction of edges that a node has to the nodes outside of its community. Therefore, as we decrease $\mu$ we obtain a clear set of communities with fewer number of inter-edges. Later, the authors adapted the LFR benchmark to generate overlapping communities [40]. Parameter $O_n$ specifies the number of overlapping nodes and $O_m$ controls the number of membership of overlapping nodes.

Real-world Networks: We present the performance of OCEO on several real-world complex networks with the absent of ground-truth communities: Zachary’s karate club [41], Jazz musician network [42] and C. elegans metabolic network [13].

Metrics: Investigating the effectiveness of community detection algorithms involves defining a similarity measure between the extracted community structure and the partition one wishes to discover. The most popular measure to compare the similarity between the delivered community structure and the ground-truth communities is Normalized Mutual Information (NMI). We have used an implementation of NMI measure made available by McDaid, et al., for sets of overlapping communities [44]. Further, we use modularity value of obtained community structures and number of identified communities as additional measures when the ground-truth community structure is unknown. For each dataset, we run OCEO 20 times due to the non-deterministic aspect of the game and report the average values of NMI or modularity.

B. Analysis

In the first experiment we show the effectiveness of our method for resolving the resolution limit problem on the LFR benchmark graphs with implanted non-overlapping communities where mixing parameter varies. The networks consist of 1,000 nodes and 5,000 edges where degree of each node is exactly set to 10, and the size of each built-in community is 10. These networks meet the resolution limit condition wherein $d(c_1) \times d(c_2) < 2m$ for two adjacent communities, $c_1$ and $c_2$. A strict modularity optimization method fails to resolve the small communities which were unambiguously defined. We compare our method with three modularity optimization methods: EO [14], Game [30] and CNM [13]. The results in Figure 2a implies that EO and CNM fail to detect the important substructure of the network. In the meantime, OCEO achieves a significantly better result and also outperforms Game. Among all these methods EO achieves the highest modularity by merging most communities into groups of two.

For investigating the performance of OCEO in detecting non-overlapping communities, we contrast OCEO with the following methods: Game, AFOCS and CNM. The first two algorithms are also capable of detecting overlapping communities as well as non-overlapping ones. However for the purpose of this experiment the overlapping aspect is disabled. We compare these methods for the increasing range of mixing parameter values in networks of $N=5000$ where $\langle k \rangle = 50$. The size of implanted communities lies between 50 and 200. As can be seen in Figure 2b, EO and OCEO are very competitive and both are far better than Game in detecting ground-truth community structures.

Further, we extend our experiments to examine the efficiency of our method on identifying overlapping communities in static networks. Figure 2c displays the performance of OCEO, Game and AFOCS over networks of $n=1000$, $m=7368$ and $\langle k \rangle = 15$. A node can belong to at most three distinct communities. The x-axis presents fraction of nodes belonging to multiple communities in the corresponding networks with mixing parameter $\mu = 0.1$. OCEO and Game clearly outperforms AFOCS in detecting overlapping community structures. Our method is able to achieve NMI over 90% when the overlapping threshold is less than 0.2. However, as the fraction of overlapping nodes increases, OCEO and AFOCS tend to obtain a lower NMI. The reason is that these methods merge two communities together when the fraction of joint nodes is higher than a threshold and consequently an increment in the number of overlapping nodes leads to more merger of communities. In this experiment the overlapping threshold is set to 0.6 for both methods. It is worth mentioning that the running time of OCEO is significantly lower than the Game. The main reason behind Game’s long running time is the way it tries to reach a local equilibrium. It randomly picks a node and chooses the better operation among the possi-
ble strategies (join, leave or switch). This process repeats until no node can improve itself for a long number of iterations. The upperbound for this recurrence is of the order of $O(m^2)$ which explains the slow convergence of the Game.

We next observe the performance of OCEO on dynamic networks in comparison with AFOCS. The synthesized network used for this experiment is generated by LFR benchmark with $n = 5000$, $m = 147324$, wherein 10% of nodes are overlapping. The network is evolving through four snapshots where each of which comprises 25% of data. The results are presented in Figure [23]. As expected, both methods obtain increasing values of NMI as the network evolves and perform very well by achieving NMI values being above 90% when received the last snapshot.

In Table [1] we present the performance of OCEO on several real-world complex networks with the absence of ground-truth communities: the Zachary’s karate club, Jazz musician network and C. elegans metabolic. We have run OCEO and EO for 20 trials. According to these results, compared to EO, OCEO is capable of identifying a greater number of communities and at the same time achieving a high modularity value.

| Dataset   | Size  | $Q_{OCEO}$ | $\#C_{OCEO}$ | $Q_{EO}$  | $\#C_{EO}$ |
|-----------|-------|------------|--------------|-----------|------------|
| Zachary   | 34    | 0.47 ± 0.0 | 6 ± 0.0      | 0.41 ± 0  | 4 ± 0      |
| Jazz      | 458   | 0.44 ± 0   | 6 ± 0        | 0.44 ± 0  | 3 ± 0      |
| C. elegans| 453   | 0.42 ± 0   | 9 ± 1.33     | 0.43 ± 0  | 8.79 ± 1.16|

TABLE I: Modularity and number of communities obtained by OCEO and EO on different real-world complex networks

V. CONCLUSION

In this paper, we have proposed a game-theoretic method, OCEO, to detect communities in complex networks. To the best of our knowledge the proposed method is the first game-theoretic algorithm, capable of extracting overlapping as well as non-overlapping communities on either static or dynamic complex networks. Nodes as players try to maximize their payoffs by choosing one or more communities to join. The payoff of players, in their communities, is proportional to their contributions to the modularity value $Q$. Therefore, global modularity is optimized through the iterations, while players improve their payoffs, and the game is propelled to converge to a Nash-equilibrium point among the players. The choice of individuals’ payoff function and also the way the game drags itself to a Nash-equilibrium resolve the major issue of modularity optimization, resolution limit and distinguish our method from strict modularity optimization. Experimental results demonstrate the effectiveness of OCEO, in terms of obtaining high values of NMI and modularity from both synthetic and real-world networks in a reasonable time.

[1] S. Fortunato, Physics Reports 486, 75 (2010).
[2] J. Chen and B. Yuan, Bioinformatics 22, 2283 (2006).
[3] I. Psorakis, S. Roberts, M. Ebden, and B. Sheldon, Physical review E 83, 066114 (2011).
[4] J. Yang and J. Leskovec (ACM, 2013) pp. 587–596.
[5] S. Zhang, R.-S. Wang, and X.-S. Zhang, Physica A: Statistical Mechanics and its Applications 374, 483 (2007).
[6] M. E. Newman and M. Girvan, Physical review E 69, 026113 (2004).
[7] U. Brandes, D. Delling, M. Gaertler, R. Görke, M. Hoefer, Z. Nikoloski, and D. Wagner, arXiv preprint physics/0608255 (2006).
[8] W. Kernighan and S. Lin, Bell system technical journal 49, 291 (1970).
[9] M. E. Newman, Physical review E 69, 066133 (2004).
[10] F. Radicchi, C. Castellano, F. Cecconi, V. Loreto, and D. Parisi, Proceedings of the National Academy of Sciences of the United States of America 101, 2658 (2004).
[11] Z. Li, S. Zhang, R.-S. Wang, X.-S. Zhang, and L. Chen, Physical review E 77, 036109 (2008).
[12] M. Gong, B. Fu, L. Jiao, and H. Du, Physical review E 84, 056101 (2011).
[13] A. Clauset, M. E. Newman, and C. Moore, Physical review E 70, 066111 (2004).
[14] J. Duch and A. Arenas, Physical review E 72, 027104 (2005).
[15] R. Guimerà, M. Sales-Pardo, and L. A. N. Amaral, Physical Review E 70, 025101 (2004).
[16] D. He, J. Liu, D. Liu, D. Jin, and Z. Jia (IEEE, 2011) pp. 1151–1155.
[17] J. Reichardt and S. Bornholdt, Physical Review E 74, 016110 (2006).
[18] S. White and P. Smyth (SIAM, 2005) pp. 76–84.
[19] M. E. Newman, Physical Review E 88, 042822 (2013).
[20] I. Derényi, G. Palla, and T. Vicsek, Physical review letters 94, 160202 (2005).
[21] Y.-Y. Ahn, J. P. Bagrow, and S. Lehmann, Nature 466, 761 (2010).
[22] T. Evans and R. Lambiotte, Physical Review E 80, 016105 (2009).
[23] N. P. Nguyen, T. N. Dinh, Y. Xuan, and M. T. Thai (IEEE, 2011) pp. 2282–2290.
[24] D. Greene, D. Doyle, and P. Cunningham (IEEE, 2010) pp. 176–183.
[25] Y. Sun, J. Tang, J. Han, M. Gupta, and B. Zhao (ACM, 2010) pp. 137–146.
[26] D. S. Bassett, M. A. Porter, N. F. Wymbs, S. T. Grafton, J. M. Carlson, and P. J. Mucha, Chaos: An Interdisci-
plinary Journal of Nonlinear Science 23, 013142 (2013).

[27] R. I. Lung, C. Chira, and A. Andreica, PloS one 9, e86891 (2014).

[28] D. Chakrabarti, R. Kumar, and A. Tomkins (ACM, 2006) pp. 554–560.

[29] N. P. Nguyen, T. N. Dinh, S. Tokala, and M. T. Thai (ACM, 2011) pp. 85–96.

[30] W. Chen, Z. Liu, X. Sun, and Y. Wang, Data Mining and Knowledge Discovery 21, 224 (2010).

[31] P. J. McSweeney, K. Mehrotra, and J. C. Oh, in Proceedings of the 2012 International Conference on Advances in Social Networks Analysis and Mining (ASONAM 2012) (IEEE Computer Society, 2012) pp. 227–234.

[32] S. Boettcher and A. Percus, Artificial Intelligence 119, 275 (2000).

[33] S. Boettcher and A. G. Percus, arXiv preprint math/9904056 (1999).

[34] J. Nash, Annals of mathematics, 286 (1951).

[35] D. Monderer and L. S. Shapley, Games and economic behavior 14, 124 (1996).

[36] O. Candogan, A. Ozdaglar, and P. A. Parrilo, ACM Transactions on Economics and Computation 1, 11 (2013).

[37] R. I. Lung and D. Dumitrescu, Int. J. of Computers, Communications & Control 3, 364 (2008).

[38] S. Fortunato and M. Barthélemy, Proceedings of the National Academy of Sciences 104, 36 (2007).

[39] A. Lancichinetti, S. Fortunato, and F. Radicchi, Physical review E 78, 046110 (2008).

[40] A. Lancichinetti and S. Fortunato, Physical Review E 80, 016118 (2009).

[41] W. W. Zachary, Journal of anthropological research, 452 (1977).

[42] P. M. Gleiser and L. Danon, Advances in complex systems 6, 565 (2003).

[43] Https://github.com/aaronmcdaid/Overlapping-NMI.

[44] A. F. McDaid, D. Greene, and N. Hurley, arXiv preprint arXiv:1110.2515 (2011).