Thermal and gravitational chiral anomaly induced magneto-transport in Weyl semimetals

Kamal Das¹, * and Amit Agarwal¹, †

¹Dept. of Physics, Indian Institute of Technology Kanpur, Kanpur 208016, India

Quantum anomalies in Weyl semimetal (for either $\mathbf{E} \cdot \mathbf{B} \neq 0$ or $\nabla T \cdot \mathbf{B} \neq 0$) leads to chiral charge and energy pumping between the opposite chirality nodes. This results in chiral charge and energy imbalance between the Weyl nodes which manifests in several intriguing magneto-transport phenomena. Here, we investigate the role of electrical-, thermal-, and gravitational chiral anomaly on magneto-transport in Weyl semimetals. We predict the planar Ettinghausen and Righi-Leduc effect to be a distinct signature of these quantum anomalies. We also demonstrate a significant enhancement in the thermo-electric conductivity, Seebeck effect, Nernst effect and thermal conductivity with increasing temperature. Interestingly, this anomaly induced transport violates the Wiedemann-Franz law and Mott relation.

Massless chiral fluids in presence of a magnetic field exhibit quantum anomalies, which manifest as the non-conservation of chiral charge and energy densities [1–6]. Weyl semimetals (WSM) hosting a pair of Weyl nodes of opposite chirality, which also act as Berry curvature monopoles [7–12], offer an ideal platform to explore these. The non-conservation of chiral charge in presence of electric field ($\mathbf{E} \cdot \mathbf{B} \neq 0$) is known as electrical chiral anomaly (ECA) [1, 3, 13]. This leads to very interesting magneto-electric transport phenomena [3, 4, 14–21] in WSM. The non-conservation of chiral energy in presence of temperature gradient ($\nabla T \cdot \mathbf{B} \neq 0$) is a manifestation of the gravitational chiral anomaly (GCA) [2, 4, 5]. This leads to interesting signatures in magneto-thermal transport in WSM [5, 22–31]. Here, we demonstrate another kind of chiral anomaly associated with chiral charge pumping in presence of $\nabla T \cdot \mathbf{B} \neq 0$, the thermal chiral anomaly (TCA).

The origin of these quantum chiral anomalies can be traced to the magnetic field induced equilibrium chiral charge and energy current in WSM ($j_{c,eq}^s$ and $j_{E,eq}^s$, respectively, with $s = \pm 1$ being the chirality). These are

$$j_{c,eq}^s = -e (\mu C_0^s \mathbf{B} + T C_1^s \mathbf{B}) ,$$

$$j_{E,eq}^s = \mu \frac{C_0^s}{2} \mathbf{B} + \mu T C_1^s \mathbf{B} + T^2 C_2^s \mathbf{B} .$$

Here, $C_0^s$ (defined later) are the coefficients of different quantum chiral anomalies. Equation (1) generalizes the chiral magnetic effect in WSM to include finite temperature. In Eq. (1), $C_0^s$ is the ECA coefficient, while $C_1^s$ determines the charge pumping in presence of a finite $T$. It has not been explored earlier and we will refer to it as the coefficient of TCA. In Eq. (2) the first two terms simply denote the energy carried by chiral charge current, while the third term $\propto C_2^s$ captures the thermal component and it is known to be analogous to the GCA [2, 4, 5]. See Sec. S1 of the supplementary materials (SM) [32] for more details. In a non-equilibrium scenario, this chiral charge pumping is stabilized by inter-node scattering and results in chiral charge (different $\mu^s$) and energy imbalance (different $T^s$) in the two Weyl nodes (see

![Fig. 1](https://example.com/fig1.png)

**Fig. 1.** Schematic of the chiral chemical potential ($\delta \mu$) and chiral temperature ($\delta T$) imbalance in WSM for (a) $\mathbf{E} \cdot \mathbf{B} \neq 0$ and (b) $\nabla T \cdot \mathbf{B} \neq 0$. Both of these lead to quantum chiral anomalies, which pump chiral charge and energy from one Weyl node to the other.

Fig. 1). This charge and energy imbalance in the Weyl nodes, gives rise to several interesting effects in magneto-transport experiments.

In this letter, we present a unified framework for these three chiral quantum anomalies in the Boltzmann transport formalism. We explicitly calculate all the magneto-transport coefficients, and predict their magnetic field dependence, angular dependence (between $\mathbf{B}$ and $\mathbf{E}$ or $\nabla T$), and temperature scaling. In addition to the planar Hall and planar Nernst effect, we predict planar Ettinghausen and Righi-Leduc effect to be a manifestation of these anomalies. Remarkably, we find significant enhancement in the magnetic field induced thermo-electric conductivity, Seebeck and Nernst effects and thermal conductivity with increasing temperature. We also demonstrate that the quantum chiral anomaly induced transport coefficients violate the Wiedemann-Franz law, as well as the Mott relation in WSM.

The dynamics of charge carriers in a Weyl cone of a given chirality is described by the following equations for the carriers position ($r$) and momentum ($k$) [33, 34],

$$\dot{r}^s = \Delta^s \left[ \tilde{v}^s + \frac{e}{\hbar} \mathbf{E} \times \Omega^s + \frac{e}{\hbar} (\tilde{v}^s \cdot \Omega^s) \mathbf{B} \right] ,$$

$$\hbar \dot{k}^s = \Delta^s \left[ -e \mathbf{E} - e \tilde{v}^s \times \mathbf{B} - \frac{e^2}{\hbar} (\mathbf{E} \cdot \mathbf{B}) \Omega^s \right] .$$

Here, $' -e'$ is the electronic charge, $\Omega^s$ is the Berry cur-
Here, the carried by the chiral charge transfer. The quantum chiral anomalies discussed in this letter. This generalizes the semiclassical ECA and temperature $T^s$ for the $s = 1$ node, in the $\mu - T$ plane. The $\delta \mu^s$ induced by (a) $\mathbf{E} \parallel \mathbf{B}$ and (b) $\nabla T^s \parallel \mathbf{B}$ have opposite signs. The $\delta T^s$ induced by (c) $\mathbf{E} \parallel \mathbf{B}$ and (d) $\nabla T^s \parallel \mathbf{B}$, also have opposite signs. In all cases, the impact of the $\mathbf{E} \parallel \mathbf{B}$ term is larger. Furthermore, while the $\mathbf{E} \cdot \mathbf{B} \neq 0$ term increases the chiral $\delta \mu$ of the positive chirality node, it reduces the chiral temperature $\delta T$. Here we have chosen $v_F = 2 \times 10^6$ m/s, $\tau_v = 10^{-9}$ s, $B = 6$ T, sample length $l = 50$ nm, $|\mathbf{E}| = 1$ mV/l, and $|\nabla T| = 350$ mK/l [5].

Working in the linear response regime in $\mathbf{E}$ and $\nabla T$, we find that the imbalance of the chiral carriers and temperatures are small, i.e., $\delta \mu^s < \mu$, and $\delta T^s < T$. Solving for $\delta \mu^s$ and $\delta T^s$ (see Sec. S2 and S3 in SM [32] for more details) in this regime, we obtain

$$
\left( \frac{\delta \mu^s}{\delta T^s / T} \right) = -\frac{\tau_v}{2} \left( \Lambda_0^s \Lambda_1^s \Lambda_2^s \right) \left( \frac{e \mathbf{E}}{\nabla T / T} \right).
$$

Here, we have defined the generalized energy densities and the generalized energy velocities at the Fermi energy

$$
\left( \frac{D_n^s}{\Lambda_n^s} \right) = \int \frac{dk}{(2\pi)^3} \left( \tilde{\epsilon}^s - \mu \right) \left( -\partial_{\tilde{\epsilon}} f \right) \times \left( \mathbf{v} \times \frac{e}{h} (\mathbf{v} \cdot \mathbf{\Omega}^s) \mathbf{B} \right),
$$

with $n = \{0, 1, 2\}$. Here, $D_0^s$ defines the finite temperature density of states in presence of Berry curvature. $\Lambda_0^s$ and $\Lambda_1^s$ are the total chiral magnetic velocity and the total chiral energy velocity at the Fermi level, respectively. It is evident from Eq. (9) that both the electric field and the temperature gradient contribute to generate the chiral chemical potential and chiral temperature imbalance in the WSM. It turns out that in $\Lambda_n^s$, only the contribution of the chiral magnetic velocity survives and we have $\Lambda_2^s = \Lambda_2^s \mathbf{B}$, along with $\Lambda_n^s \propto s$.

The anomaly coefficients are explicitly given by

$$
\{C_0^s, C_1^s, C_2^s\} = \left\{ \Lambda_0^s \Lambda_1^s \Lambda_2^s \frac{1}{2T^2} \right\}.
$$
Clearly, in this semiclassical formalism, all these three anomalies arise from the presence of a finite Berry curvature and chiral magnetic velocity. However, while the chiral anomaly ($\propto C_0^s$) is well explored in WSM, the other two anomalies of a similar origin are relatively less explored [4 and 5]. Developing a common theoretical framework to explore all of them is one of the main highlights of this letter.

Below, we present results for a low energy model of a WSM with a single pair of Weyl nodes. The low energy Hamiltonian for a Weyl node is

$$\mathcal{H}^s = \hbar v_F \sigma \cdot \mathbf{k},$$

(13)

where $v_F$ is the Fermi velocity and $\sigma = \{\sigma_1, \sigma_2, \sigma_3\}$ is the set of Pauli spin matrices. The Berry curvature and the orbital magnetic moment for the conduction band of the Hamiltonian in Eq. (13), are $\Omega^s = -\mathbf{s}k/(2|k|^2)$ and $\mathbf{m}^s = -se v_F \mathbf{k}/(2|k|^2)$, respectively (see Sec. S5 of the SM [32] for more details).

For this model Hamiltonian, it is straightforward to calculate the quantities defined in Eq. (10). In the limit $\mu > k_B T$, where Sommerfeld expansion is valid, the generalized energy velocities can be calculated to be

$$\{\Lambda^s_0, \Lambda^s_1, \Lambda^s_2\} = -s e^{e\beta\mu}/4\pi^2 \hbar^2 \left\{ \mathcal{F}_0, \frac{1}{\beta^2} \mathcal{F}_1, \frac{1}{\beta^2} \mathcal{F}_2 \right\}.$$  

(14)

Here, $\mathcal{F}_i$'s are functions of $x = \beta \mu$ and we have defined, $\mathcal{F}_0(x) = 1/(1+e^{-x})$, $\mathcal{F}_1(x) = x/(1+e^{-x}) + \ln[1+e^{-x}]$ and $\mathcal{F}_2(x) = \pi^2/3 - x \left(\frac{x}{1+e^{-x}} + 2 \ln[1+e^{-x}] + 2 \text{Li}_2(-e^{-x})\right)$, with Li denoting the polylog function (see Fig. S1 in the SM [32] for their temperature dependence). Similarly we calculate the generalized energy density of states,

$$\{\mathcal{D}^s_0, \mathcal{D}^s_1, \mathcal{D}^s_2\} \approx \frac{\mu^2}{2\pi^2 \hbar^2 v_F^2} \left\{ \mathcal{F}_0, \frac{2}{\beta^2} \mathcal{F}_1, \frac{1}{\beta^2} \mathcal{F}_2 \right\}. $$

Note that in calculating the generalized energy densities, we have neglected the magnetic field correction which is
very small. In the limiting case of $\beta \mu \to \infty$, we have
\{F_0, F_1, F_2\} \to \{1, 0, \pi^2/3\}. In this limit, $C_1^* \to 0$ implying
that there is no TCA.

Using Eqs. (14)-(15) in Eq. (9), we evaluate $\delta \mu^s$ and
$\delta T^s$ in the $\beta \mu \to \infty$ limit to be
\[
\left( \frac{\delta \mu^s}{\delta T^s / T} \right) = \frac{\tau_0 k^3 v_F^3}{2 \pi^2} \frac{1}{2h} \left( \frac{e^2}{\mu} - \frac{e k_B \frac{\pi^2}{3}}{\mu} \right) \left( \mathbf{B} \cdot \mathbf{E} \right).
\]

(16)

For finite $\beta \mu$, their variation in the $\mu - T$ plane (for $s = 1$
node) is shown in Fig. 2, and analytical results are
presented in Sec. S4 of the SM [32]. The $\mathbf{B} \cdot \mathbf{E}$ and $\mathbf{B} \cdot \nabla T$
terms compete with each other to change both $\delta \mu^s$ as well as
$\delta T^s$.

The chiral anomaly induced transport coefficients
($\sigma, \alpha, \tilde{\alpha}$, and $\tilde{\kappa}$) can now be obtained from Eq. (12). In
the $\beta \mu \to \infty$ limit, these are given by
\[
\sigma_{ij} \equiv \frac{\tau_0 e^2 v_F^2}{2 \pi^2 \hbar^2 \mu} B^2 \left( \frac{2 e^2 \kappa \beta B T}{3 \beta} \right) \frac{2 \pi^2 \hbar v_F}{\pi^2 v_F} \left( \mathbf{B} \cdot \mathbf{E} \right).
\]

(17)

Here, $(\theta, \phi)$ denotes the polar and azimuthal angle of
the magnetic field. Note that in Eq. (17), $\alpha$, and $\kappa$ are $\propto T$
while $\tilde{\alpha} \propto T^2$. The angular dependence of all transport
coefficients is given by the matrix
\[
L(\theta, \phi) = \begin{pmatrix}
\frac{\sin^2 \theta \cos^2 \phi}{\sin \theta \sin 2\phi} & \frac{\frac{1}{2} \sin^2 \theta \sin 2\phi}{\sin \theta \sin 2\phi} & \frac{\frac{1}{2} \sin \theta \cos \theta}{\sin \theta \cos \theta} \\
\frac{\frac{1}{2} \sin \theta \sin \phi}{\sin \theta \sin \phi} & \frac{\frac{1}{2} \sin \theta \cos \phi}{\sin \theta \cos \phi} & \frac{\frac{1}{2} \sin \theta \cos \phi}{\sin \theta \cos \phi} \\
\frac{\frac{1}{2} \sin \theta \sin \phi}{\sin \theta \sin \phi} & \frac{\frac{1}{2} \sin \theta \cos \phi}{\sin \theta \cos \phi} & \frac{\frac{1}{2} \sin \theta \cos \phi}{\sin \theta \cos \phi}
\end{pmatrix}.
\]

(18)

Co-planar $\mathbf{E}$ and $\mathbf{B}$, give rise to the planar Hall effect
($\sigma_{xy}$) along with the planar Ettingshausen effect
($S_{xy} = TS_{xy}$). Co-planar $\nabla T$ and $\mathbf{B}$ result in the planar
Nernst effect ($S_{xy}$) and the the planar Righi-Leduc effect
($\kappa_{xy}$). We find that the angular dependence of $\kappa_{xy}$ and
$S_{xy}$ is identical ($\propto \sin 2\phi$), in contrast to $\rho_{xy}$ which
$\propto -\sin 2\phi$ as shown in Fig. 3(a).

To explore the impact of anomalies on longitudinal
magneto-resistance (MR), we define generic $MR = \gamma(B)/\gamma(0) - 1$, where $\gamma$
denotes the transport coefficients: $\sigma, \alpha, \tilde{\alpha}$, or $\kappa$. To evaluate the MR, we
need to add the Drude components to the respective
transport coefficients. Evaluating the Drude components
for each Weyl node [39], we have $\{\sigma_0, \alpha_0, \kappa_0\} = \frac{\tau_0 e^2 v_F^2}{2 \pi^2 \hbar^2 \mu} \left\{ v_F^2, 0, 0 \right\}$. In the $\beta \mu \to \infty$
limit, to all orders in $B$ we obtain the longitudinal
MR, $\{MR_{\sigma}, MR_{\alpha}, MR_{\kappa}\} = -\frac{3 \tau_0 e^2 \zeta^2}{2 \pi^2 \hbar v_F} \left\{ 0, 1, 2 \right\}$. Here $\zeta = e v_F^2 B / (2 \mu^2)$. Both of these show negative MR which
is quadratic for small $B$ and saturates with increasing $B$
values. This behaviour, along with $MR_\sigma / MR_\kappa = 1/2$, persists
even for finite $\beta \mu$ values, as shown in Fig. 3(b)
for an electronically doped system ($\mu > 0$). Negative saturating $MR_\sigma$ in WSM has been reported in several
experiments [16-18]. Negative $MR_\kappa$ has also been reported
in recent experiments on GdPtBi [24] and Cd$_3$As$_2$ [25].

For $\alpha$ and $\kappa$, in the $\beta \mu \to 0$ limit, we obtain
$\{MR_{\alpha}, MR_{\kappa}\} = \frac{3 \tau_0 e^2 \zeta^2}{2 \pi^2 \hbar v_F} \left\{ -1, 1 \right\}$. In contrast to $MR_\sigma$, $MR_S$ and consistent with calculations of Ref. [26], both of these
show a non-saturating behaviour with $MR_{\alpha}$ being negative,
and $MR_{\kappa}$ being positive. This trend persists even for finite
$\beta \mu$ as shown in Fig. 3(c)-(d). The observation of semi-classical negative $MR_\alpha$ (for $\mu > 0$) has
also been reported in the Weyl semimetal NbP [5] as well as
in GdPtBi [24]. A positive $MR_{\kappa}$ has been recently reported in Ref. [40]. We emphasize that the relatively larger MR
in $\alpha$ and $\kappa$ as compared to $MR_\sigma$, has its origin primarily
in the $C^*_1$ term or the GCA.

The temperature dependence the diagonal and off-
diagonal transport coefficients is shown in Fig. 4(a)-(b).
While the Drude Seebeck coefficient is negative for $\mu > 0$,
the quantum anomalies reverse its sign, making it positive.
This sign reversal is a distinct signature of quantum
anomalies in chiral fluids (See Fig S4 and Sec. S8
of SM [32] for details). The Drude thermal conductivity
is positive, and the chiral anomalies enhance it significantly
(by $\sim 6x$ at $T = 100$ in Fig. 4(a)]. This possibly
explains the exciting experimental report of magnetic
field induced 300% enhancement of the electronic
component of the longitudinal thermal conductivity in the
WSM phase of Bi$_8$Sb$_{11}$ [41]. We also find a similar
enhancement in $S_{xy}$ and $\kappa_{xy}$, as shown in Fig. 4(b),
and also in $\alpha$ while the relative change in $\rho$ is small (see Fig. S2
in SM [32]).
The violation of the Wiedemann-Franz law and the Mott relations in the anomaly induced transport coefficients is shown in Fig. 4(c)-(d). To highlight that this violation goes much beyond the breakdown of the Sommerfeld expansion, which can occur in normal metals as well, we plot the ratio, \(L(B,T)/L_0(T)\) and \(M(B,T)/M_0(T)\), where \(L_0\) and \(M_0\) are the corresponding Drude counterpart. See Sec. S7 of the SM [32] for more details.

To summarize, we have predicted a new quantum anomaly in chiral fluids, the thermal chiral anomaly, which pumps chiral charge from one node to other in presence of \(\nabla T \cdot \mathbf{B} \neq 0\). We have shown that in addition to planar Hall and planar Nernst effects, the planar Ettinghausen and Righi-Leduc effects also arise from these anomalies. We predict a significant enhancement in the magnetic field induced thermo-electric conductivity, Seebeck effect, Nernst effect and thermal conductivity with increasing temperature. We also demonstrate that the quantum anomaly induced magneto-transport in WSM violates the Wiedemann-Franz law and Mott relation at finite temperatures. Our work provides a robust framework for exploring and interpreting the impact of quantum anomalies in magneto-transport experiments in WSM.

AA acknowledges Science Education and Research Board (SERB) and Department of Science and Technology (DST) of government of India for financial support.

\* kamaldas@iitk.ac.in
\† amitag@iitk.ac.in

[1] H.B. Nielsen and Masao Ninomiya, “The adler-bell-jackiw anomaly and weyl fermions in a crystal,” Physics Letters B 130, 389 – 396 (1983)
[2] Karl Landsteiner, Eugenio Megas, and Francisco Pena-Benitez, “Gravitational anomaly and transport phenomena,” 107, 021601 (2009)
[3] D. T. Son and B. Z. Spivak, “Chiral anomaly and classical negative magnetoresistance of weyl metals,” Phys. Rev. B 88, 104412 (2013)
[4] Andrew Lucas, Richard A. Davison, and Subir Sachdev, “Hydrodynamic theory of thermoelectric transport and negative magnetoresistance in weyl semimetals,” Proceedings of the National Academy of Sciences 113, 9463–9468 (2016)
[5] Johannes Gooth, Anna C. Niemann, Tobias Meng, Adolfo G. Grushin, Karl Landsteiner, Bernd Gotsmann, Fabian Menges, Marcus Schmidt, Chandra Shekhar, Vicky düff, Ruben Hühne, Bernd Rellinghaus, Claudia Felser, Binghai Yan, and Kornelius Nielsch, “Experiments on the mixed axial-gravitational anomaly in the weyl semimetal nipi.” Nature 547, 324 (2017)
[6] M. N. Chernodub, Alberto Cortijo, and María A. H. Vozmediano, “Generation of a nernst current from the conformal anomaly in dirac and weyl semimetals,” Phys. Rev. Lett. 120, 206601 (2018)
[7] Xiangang Wan, Ari M. Turner, Ashvin Vishwanath, and Sergey Y. Savrasov, “Topological semimetal and fermiarc surface states in the electronic structure of pyrochlore iridates,” Phys. Rev. B 83, 205101 (2011)
[8] N. P. Armitage, E. J. Mele, and Ashvin Vishwanath, “Weyl and dirac semimetals in three-dimensional solids,” Rev. Mod. Phys. 90, 015001 (2018)
[9] Su-Yang Xu, Ilya Belopolski, Nasser Alioudist, Madhab Neupane, Guang Bian, Chenglong Zhang, Raman Sankar, Guoqing Chang, Zhujun Yuan, Chi-Cheng Lee, Shin-Ming Huang, Hao Zheng, Jie Ma, Daniel S. Sanchez, BaoKai Wang, Arun Bansil, Fangcheng Chou, Pavel P. Shibayev, Hsin Lin, Shuang Jia, and M. Zahid Hasan, “Discovery of a weyl fermion semimetal and topological fermi arcs,” Science 349, 613–617 (2015)
[10] Shin-Ming Huang, Su-Yang Xu, Ilya Belopolski, Chi-Cheng Lee, Guoqing Chang, BaoKai Wang, Nasser Alioudist, Guang Bian, Madhab Neupane, Chenglong Zhang, Shuang Jia, Arun Bansil, Hsin Lin, and M. Zahid Hasan, “A weyl fermion semimetal with surface fermi arcs in the transition metal monopnicptide taas class,” Nature Communications 6, 7373 (2015)
[11] Alexey A. Soluyanov, Dominik Gresch, Zhijun Wang, QuanSheng Wu, Matthias Troyer, Xi Dai, and B. Andrei Bernevig, “Type-ii weyl semimetals,” Nature 527, 495 (2015)
[12] M. Zahid Hasan, Su-Yang Xu, Ilya Belopolski, and Shin-Ming Huang, “Discovery of weyl fermion semimetals and topological fermi arc states,” Annual Review of Condensed Matter Physics 8, 289–309 (2017)
[13] Dam Than Son and Naoki Yamamoto, “Berry curvature, triangle anomalies, and the chiral magnetic effect in fermi liquids,” Phys. Rev. Lett. 109, 181602 (2012)
[14] A. A. Zyuzin and A. A. Burkov, “Topological response in weyl semimetals and the chiral anomaly,” Phys. Rev. B 86, 115133 (2012)
[15] S. K. Yip, “Kinetic equation and magneto-conductance for Weyl metal in the clean limit,” arXiv e-prints , arXiv:1508.01010 (2015)
[16] Jun Xiong, Satya K. Kushwaha, Tian Liang, Jason W. Krizan, Max Hirschberger, Wudi Wang, R. J. Cava, and N. P. Ong, “Evidence for the chiral anomaly in the dirac semimetal na3bi,” Science 350, 413–416 (2015)
[17] Xiaochun Huang, Lingxiao Zhao, Yujia Long, Peipei Wang, Dong Chen, Zhanhai Yang, Hui Liang, Mianqi Xue, Hongming Weng, Zhong Fang, Xi Dai, and Genfu Chen, “Observation of the chiral-anomaly-induced negative magnetoresistance in 3d weyl semimetal taas,” Phys. Rev. X 5, 031023 (2015)
[18] Hui Li, Hongtao He, Hai-Zhou Lu, Huachen Zhang, Hongchao Liu, Rong Ma, Zhiyong Fan, Shun-Qing Shen, and Jiannong Wang, “Negative magnetoresistance in dirac semimetal cd3as2,” Nature Communications 7, 10301 (2016)
[19] Qiang Li, Dmitri E. Kharzeev, Cheng Zhang, Yuan Huang, I. Pletikosic, A. V. Fedorov, R. D. Zhong, J. A. Schneeloch, G. D. Gu, and T. Valla, “Chiral magnetic effect in zrte5,” Nature Physics 12, 550 (2016)
[20] A. A. Burkov, “Giant planar hall effect in topological fermi arcs,” Phys. Rev. B 96, 041110 (2017)
[21] S. Nandy, Girish Sharma, A. Taraphder, and Sumanta Majumdar, “Weyl and dirac semimetals in three-dimensional solids,” Rev. Mod. Phys. 90, 015001 (2018)
als,” Phys. Rev. B 90, 165115 (2014)

[23] Ki-seok Kim, “Role of axion electrodynamics in a Weyl metal : Violation of Wiedemann-Franz law,” 121108, 1–4 (2014)

[24] Max Hirschberger, Satya Kushwaha, Zhijun Wang, Quinn Gibson, Sihang Liang, Carina A. Belvin, B. A. Bernevig, R. J. Cava, and N. P. Ong, “The chiral anomaly and thermopower of weyl fermions in the half-heusler?gdptbi,” Nature Materials 15, 1161 (2016)

[25] Zhenzhao Jia, Caizhen Li, Xinqi Li, Junren Shi, Zhimin Liao, Dapeng Yu, and Xiaosong Wu, “Thermoelectric signature of the chiral anomaly in cd3as2,” Nature Communications 7, 13013 (2016), article

[26] B. Z. Spivak and A. V. Andreev, “Magnetotransport phenomena related to the chiral anomaly in weyl semimetals,” Phys. Rev. B 93, 085107 (2016)

[27] Girish Sharma, Pallab Goswami, and Sumanta Tewari, “Nernst and magnetothermal conductivity in a lattice model of weyl fermions,” Phys. Rev. B 93, 035116 (2016)

[28] U Stockert, R D dos Reis, M O Ajeesh, S J Watzman, M Schmidt, C Shekhar, J P Heremans, C Felser, M Baenitz, and M Nicklas, “Thermopower and thermal conductivity in the weyl semimetal NbP,” Journal of Physics: Condensed Matter 29, 325701 (2017)

[29] Vladimir A. Zyuzin, “Magnetotransport of weyl semimetals due to the chiral anomaly,” Phys. Rev. B 95, 245128 (2017)

[30] Girish Sharma and Sumanta Tewari, “Transverse thermopower in Dirac and Weyl semimetals,” arXiv e-prints , arXiv:1812.10681 (2018), arXiv:1812.10681 [cond-mat.mes-hall]

[31] S. Nandy, A. Taraphder, and Sumanta Tewari, “Planar thermal hall effect in weyl semimetals,” Phys. Rev. B 100, 115139 (2019)

[32] Supplementary material detailing the following: 1) The chiral charge and energy current in equilibrium and coefficients of the quantum anomalies, 2) Local equilibrium approximation, 3) Chiral chemical potential and temperature imbalance, 4) Anomaly induced transport coefficients, 5) Berry curvature and magnetic moment, and 6) Temperature dependence of transport coefficients, and 7) Violation of Wiedemann Franz law and Mott relations 8) Sign reversal of α.

[33] Ganesh Sundaram and Qian Niu, “Wave-packet dynamics in slowly perturbed crystals: Gradient corrections and berry-phase effects,” Phys. Rev. B 59, 14915–14925 (1999)

[34] Di Xiao, Ming-Che Chang, and Qian Niu, “Berry phase effects on electronic properties,” Rev. Mod. Phys. 82, 1959–2007 (2010)

[35] Di Xiao, Junren Shi, and Qian Niu, “Berry phase correction to electron density of states in solids,” Phys. Rev. Lett. 95, 137204 (2005)

[36] N.W. Ashcroft and N.D. Mermin, Solid State Physics, HRW international editions (Holt, Rinehart and Winston, 1976)

[37] Kamal Das and Amit Agarwal, “Linear magnetochiral transport in tilted type-i and type-ii weyl semimetals,” Phys. Rev. B 99, 085405 (2019)

[38] Kamal Das and Amit Agarwal, “Berry curvature induced thermopower in type-i and type-ii weyl semimetals,” Phys. Rev. B 100, 085406 (2019)

[39] Clemens Schindler, Satya N. Guin, Walter Schnelle, Nitesh Kumar, Chenguang Fu, Horst Bormann, Chandra Shekhar, Yang Zhang, Yan Sun, Claudia Felser, Tobias Meng, Adolfo G. Grushin, and Johannes Gooth, “Observation of an anomalous heat current in a Weyl fermion semimetal,” arXiv e-prints , arXiv:1810.02300 (2018), arXiv:1810.02300 [cond-mat.mes-hall]

[40] Dung Vu, Wenjuan Zhang, Cüneyt Şahin, Michael Flatté, Nandini Trivedi, and Joseph P. Heremans, “Thermal chiral anomaly in the magnetic-field induced ideal Weyl phase of Bi89Sb11,” arXiv e-prints , arXiv:1906.02248 (2019), arXiv:1906.02248 [cond-mat.mtrl-sci]