THE MECHANISM OF SWING ABSORPTION OF FAST MAGNETOSONIC WAVES IN INHOMOGENEOUS MEDIA

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ABSTRACT

The recently suggested swing interaction between fast magnetosonic and Alfvén waves (Zaqarashvili & Roberts, 2002a) is generalized to inhomogeneous media. We show that the fast magnetosonic waves propagating across an applied non-uniform magnetic field can parametrically amplify the Alfvén waves propagating along the field through the periodical variation of the Alfvén speed. The resonant Alfvén waves have half the frequency and the perpendicular velocity polarization of the fast waves. The wavelengths of the resonant waves have different values across the magnetic field, due to the inhomogeneity in the Alfvén speed. Therefore, if the medium is bounded along the magnetic field, then the harmonics of the Alfvén waves, which satisfy the condition for onset of a standing pattern, have stronger growth rates. In these regions the fast magnetosonic waves can be strongly ‘absorbed’, their energy going in transversal Alfvén waves. We refer to this phenomenon as ‘Swing Absorption’. This mechanism can be of importance in various astrophysical situations.

Key words: MHD waves; wave coupling.

1. INTRODUCTION

Wave motions play an important role in many astrophysical phenomena. Magnetohydrodynamic (MHD) waves may transport momentum and energy, resulting in heating and acceleration of an ambient plasma. A variety of waves have recently been detected in the solar atmosphere using the SOHO and TRACE spacecraft. Hence, an understanding of the basic physical mechanisms of excitation, damping and the interaction between the different kinds of MHD wave modes is of increasing interest (Roberts, 2004). Formally speaking, there is a group of direct mechanisms of wave excitation by external forces (e.g. turbulent convection, explosive events in stellar atmospheres, etc.) and wave dissipation due to non-adiabatic processes in a medium (such as viscosity, thermal conduction, magnetic resistivity, etc.). There is a separate group of wave amplification and damping processes due to resonant mechanisms (Goossens, 1991; Poedts, 2002). This means that particular wave modes may be damped (amplified) due to energy transfer (extraction) into or from other kinds of oscillatory motions, even when wave dissipation is excluded from consideration.

Recently, a new kind of interaction between different MHD wave modes, based on a parametric action, has been suggested (Zaqarashvili, 2001; Zaqarashvili & Roberts, 2002a,b; Zaqarashvili et al., 2002, 2004). In this case the mechanism of wave interaction originates from a basic physical phenomenon known in classical mechanics as ‘parametric resonance’, occurring when an external force (or oscillation) amplifies the oscillation through a periodical variation of the system’s parameters. The mechanical analogy of this phenomenon is a mathematical pendulum with periodically varying length. When the frequency of the length variation is twice the frequency of the pendulum oscillation, the amplitude of the oscillation grows exponentially in time. Such a mechanical system can consist of a pendulum (transversal oscillations) with a spring (longitudinal oscillations). A detailed description of such a mechanical system is given in Zaqarashvili & Roberts (2002a) (hereinafter referred to as Paper I).

Here we present a brief report on our recent results on swing interactions of fast and Alfvén modes in inhomogeneous media (Shergelashvili et al. (2004)).

2. BASIC EQUATIONS AND EQUILIBRIUM MODEL

Consider a magnetized medium with zero viscosity and infinite conductivity, where processes are assumed to be adiabatic. Then the macroscopic dynamical behaviour of this medium is governed by the ideal magnetohydrodynamic (MHD) equations:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0, \]
3. FAST MAGNETOSONIC WAVES

Now let us consider the propagation of ’pure’ fast magnetosonic waves across the magnetic flux surfaces, i.e. along the \(x\) axis, taking \(\mathbf{u} = (u_x, 0, 0)\) and \(\partial / \partial y \equiv 0, \partial / \partial z \equiv 0\). The equation governing the dynamics of fast magnetosonic waves propagating across the magnetic field lines in the inhomogeneous medium can be obtained from equations \(\text{I}-\text{IV}\) as (Roberts, 1981, 1991):

\[
\frac{d}{dx} \left[ \left( \gamma p_0(x) + \frac{B_0^2(x)}{4\pi} \right) \frac{du(x)}{dx} \right] + \omega^2 \rho_0(x) u(x) = 0. 
\]

(7)

where \(u_x\) is the velocity perturbation.

The solution of this equation for different particular equilibrium conditions can be obtained either analytically or numerically. Equation (7) describes the propagation of a fast wave with speed \((C^2_s + V_A^2)^{1/2}\), where \(C_s = (\gamma p_0 / \rho_0)^{1/2}\) is the sound speed and \(V_A = (B_0^2 / 4\pi \rho_0)^{1/2}\) is the Alfvén speed.

We study Eq. (7) in accordance with the boundary conditions:

\[
u(0) = u(L_x) = 0,
\]

(8)

and \(u(x)\) are fast waves bounded by walls located at \(x = 0\) and \(x = L_x\). These boundary conditions make the spectrum of fast modes discrete: Eq. (7) has a nontrivial solution only for a discrete set of frequencies,

\[
\omega = \omega_n, \omega_1, \ldots. 
\]

(9)

In this case the solutions for different particular quantities can be represented as

\[
\begin{align*}
u_x &= \alpha v(x) \sin(\omega_n t), & \rho_1 &= \alpha r(x) \cos(\omega_n t), \\
\beta_x &= \alpha h(x) \cos(\omega_n t), \end{align*}
\]

(10)

where the density and magnetic field perturbations are related to the velocity perturbations through

\[
\begin{align*}
\alpha v(x) &= \frac{1}{\omega_n^2} \left( v(x) \frac{d\rho_0}{dx} + \rho_0 \frac{dv(x)}{dx} \right), \\
\alpha h(x) &= \frac{1}{\omega_n} \left( v(x) \frac{dB_0}{dx} + B_0 \frac{dv(x)}{dx} \right).
\end{align*}
\]

(11-12)

Here (and subsequently) we use a subscript \(n\) to denote the frequency of a given standing fast mode.

4. SWING AMPLIFICATION OF ALFVÉN WAVES

Now consider Alfvén waves that are linearly polarized in the \(y\) direction and propagate along the magnetic field (see Figure 1). In the linear limit these waves are decoupled from the magnetosonic waves and the equation governing their dynamics is:

\[
\frac{\partial^2 b_y}{\partial t^2} - V_A^2(x) \frac{\partial^2 b_y}{\partial x^2} = 0, 
\]

(13)

where \(V_A(x)\) is Alfvén speed. It is clear from Eq. (13) that the phase speed of this mode depends on \(x\) parametrically. Therefore, an Alfvén wave with a given wave length propagates with a ‘local’ characteristic frequency. Each magnetic flux surface can evolve independently in this perturbation mode.
Let us now address the non-linear action of the fast magnetosonic waves, considered in the previous section, on Alfvén waves. We study the weakly non-linear regime. This means that the amplitudes of the fast magnetosonic waves are considered to be large enough to produce significant variations of the environment parameters, which can be felt by propagating Alfvén modes, but too small to affect the Alfvén modes themselves. Hence, the magnetic flux surfaces can still evolve independently. Therefore, as in paper I, the non-linear terms in the equations arising from the advective derivatives $u_x \partial b_y / \partial x$ and $(\rho_0 + \rho_1) u_x \partial b_y / \partial x$ are assumed to be negligible. Under these circumstances the governing set of equations takes the form (see Paper I):

\[
\frac{\partial b_y}{\partial t} = \left( B_0 + b_z \right) \frac{\partial u_y}{\partial z} - \frac{\partial u_x}{\partial x} b_y, \quad (14)
\]

\[
(\rho_0 + \rho_1) \frac{\partial u_y}{\partial t} = \frac{B_0 + b_z}{4\pi} \frac{\partial b_y}{\partial z}. \quad (15)
\]

These equations describe the parametric influence of fast magnetosonic waves propagating across the magnetic field on Alfvén waves propagating along the field. An analytical solution of Eqs. (14) and (15) is possible for a standing pattern of fast magnetosonic waves, the medium being assumed bounded in the $x$ direction.

Combining Eqs. (14) and (15) we obtain the following second order partial differential equation:

\[
\frac{\partial^2 b_y}{\partial t^2} + \left[ \frac{\partial u_x}{\partial x} - \frac{1}{B_0 + b_z} \frac{\partial b_z}{\partial t} \right] \frac{\partial b_y}{\partial t} + \left( \frac{\partial^2 u_x}{\partial t \partial x} - \frac{1}{B_0 + b_z} \frac{\partial b_z}{\partial t} \frac{\partial u_x}{\partial x} \right) b_y - \frac{(B_0 + b_z)^2}{4\pi (\rho_0 + \rho_1)} \frac{\partial^2 b_y}{\partial z^2} = 0. \quad (16)
\]

Writing

\[
b_y = h_y(z, t) \exp \left[ -\frac{1}{2} \int \left( \frac{\partial u_x}{\partial x} - \frac{1}{B_0 + b_z} \frac{\partial b_z}{\partial t} \right) \, dt \right], \quad (17)
\]

we obtain

\[
\frac{\partial^2 h_y}{\partial t^2} + \frac{1}{2} \left[ S_1(x, t) - S_2(x, t) \right] h_y - S_3(x, t) \frac{\partial^2 h_y}{\partial z^2} = 0, \quad (18)
\]

where,

\[
S_1 = \frac{\partial^2 u_x}{\partial t \partial x} + \frac{1}{B_0 + b_z} \frac{\partial^2 b_z}{\partial t^2}, \quad (19)
\]

\[
S_2 = \frac{1}{(B_0 + b_z)^2} \left( \frac{\partial b_z}{\partial t} \right)^2 + \frac{1}{2} \left[ \frac{\partial u_x}{\partial x} + \frac{1}{B_0 + b_z} \frac{\partial b_z}{\partial t} \right]^2, \quad (20)
\]

\[
S_3 = \frac{(B_0 + b_z)^2}{4\pi (\rho_0 + \rho_1)}. \quad (21)
\]

Finally, applying a Fourier analysis with respect to the $z$ coordinate,

\[
h_y(z, t) = \int \tilde{h}_y(k_z, t) e^{ik_z z} \, dk_z, \quad (22)
\]

and neglecting the second and higher order terms in $\alpha$, we obtain the following Mathieu-type equation:

\[
\frac{\partial^2 \tilde{h}_y}{\partial t^2} + k_z^2 V_A^2 \left[ 1 + \alpha F(x) \cos(\omega_n t) \right] \tilde{h}_y = 0, \quad (23)
\]

where

\[
F(x) = \frac{2}{B_0} \frac{h(x)}{\rho_0} - \frac{v(x)}{2k_z^2 V_A^2} B_0 \frac{d B_0}{dx}. \quad (24)
\]

It should be noted that the expression (20) for $S_2(x, t)$ consists only of terms of second order and higher in $\alpha$, and so can be neglected directly for the case of weakly non-linear action addressed here.

Equation (28) has a resonant solution when the frequency of the Alfvén mode $\omega_A$ is half of $\omega_n$:

\[
\omega_A = k_z V_A(x) \approx \frac{1}{2} \omega_n. \quad (25)
\]

This solution can be expressed as

\[
\tilde{h}_y(k_z, t) = \tilde{h}_y(k_z, t = 0) e^{\frac{i\omega_A t}{2}} \left[ \cos\frac{\omega_n t}{2} - \sin\frac{\omega_n t}{2} \right], \quad (26)
\]

where

\[
\delta(x) = \alpha k_z^2 V_A^2(x) F(x). \quad (27)
\]

The solution has a resonant nature within the frequency interval

\[
\left| \omega_A - \frac{\omega_n}{2} \right| < \frac{\delta}{\omega_n}. \quad (28)
\]

Similar expressions have been obtained in the Paper I for a homogeneous medium. In that case, the Alfvén speed is constant and, therefore, the fast magnetosonic waves amplify the Alfvén waves with the same wavelength everywhere. In the case of an inhomogeneous Alfvén speed, the resonance condition (26) implies that the wavelength of the resonant harmonics of the Alfvén waves depends on $x$. This means that the fast magnetosonic waves now amplify Alfvén waves with different wavelengths (but with the same frequency) in different magnetic flux surfaces (i.e., different $x$-values).

### 4.2 Standing Alfvén waves

When we consider a system that is bounded in the $z$ direction, the boundary conditions along the $z$ axis introduce an additional quantization of the wave parameters. In particular, in this case each spatial harmonic of the Alfvén mode can be represented as

\[
\tilde{h}_y^m = \tilde{h}_y(k_m, t) \cos(k_m z), \quad (29)
\]
where \( k_m = \pi m / L_z \) (\( m = 1, 2, \ldots \)) and \( L_z \) is the characteristic length of the system in the \( z \) direction. This then leads to a further localization of the spatial region where the swing transfer of wave energy from longitudinal to transversal oscillations is permitted. The resonant condition \( 24 \) implies that

\[
k_m V_A(x_{n,m}) \approx \frac{1}{2} \omega_n. \quad (30)
\]

Therefore, the resonant areas are concentrated around the points \( x_{n,m} \) for which condition \( 24 \) holds. Within these resonant areas the longitudinal oscillations damp effectively and their energy is transferred to transversal oscillations with wave numbers \( k_z = k_m \) satisfying the resonant conditions. These resonant areas are localized in space and can be referred to as regions of swing absorption of the fast magnetosonic oscillations of the system. The particular feature of this process is that the energy transfer of fast magnetosonic waves to Alfvén waves occurs at half the frequency of the fast waves.

5. NUMERICAL SIMULATION

In this section we consider in detail the process of swing absorption of fast waves into Alfvén waves. We consider a numerical study of equation \( 24 \), subject to the boundary conditions \( 8 \). We study, as an example, the case of a polytropic plasma when both the thermal and magnetic pressures are linear functions of the \( x \) coordinate:

\[
p_0 = p_{00} + p_{01} \frac{x}{L_x}, \quad (31)
\]

\[
p_0 = C^2 \left( p_{00} + p_{01} \frac{x}{L_x} \right)^\frac{1}{2}, \quad (32)
\]

\[
B_0 = \sqrt{h_{00} + h_{01} \frac{x}{L_x}}, \quad (33)
\]

where, \( p_{00}, p_{01}, h_{00}, h_{01} \) and \( C \) are constants, and \( L_x \) denotes the length of the system along the \( x \) direction. The pressure balance condition \( 8 \) immediately yields

\[
p_{01} = -\frac{h_{01}}{8 \pi}. \quad (34)
\]

The solution of the wave equation depends on the values of the above set of constant parameters. In general, different equilibrium regimes can be considered including those corresponding to different limits of the plasma \( \beta: \beta \ll 1, \beta \approx 1 \) and \( \beta \gg 1 \). The values of all constant parameters are given in Table \( 1 \). We took arbitrary values of parameters, but they are somewhat appropriate to the magnetically dominated solar atmosphere (say the chromospheric network). In Figure 2 we show the profiles of \( \omega v(x) \) for the standing wave solutions, for two cases with different modal ‘wavelength’. Panels A and B, respectively, correspond to the characteristic frequencies: \( \omega_1 \approx 1.87 \cdot 10^{-2} \text{ s}^{-1} \) (period 5.61 min) and \( \omega_2 \approx 5.66 \cdot 10^{-2} \text{ s}^{-1} \) (period 1.85 min).

| \( L_x \text{ km} \) | \( h_{00} \text{ G}^2 \) | \( h_{01} \text{ G}^2 \) |
|----------------|------------------|------------------|
| 100           | -70              | 10^4             |
| 15000         | 6.5 \cdot L_x    | 10^{-6}          |

Table 1. Values of the constant parameters used in the calculation of our illustrative solutions. The dimension of \( C \) is \( g^t/2 \text{ cm}^{(2-3\gamma)/2} \text{/dyn}^{1/2} \text{\gamma} \).

For the configuration described by the equilibria profiles \( 41 - 43 \) the resonant condition \( 24 \) yields the areas of swing absorption (located along the \( x \) axis) as the solutions of the following equation:

\[
\omega_n C (p_{00} + p_{01} \frac{x_{n,m}}{L_x})^\frac{1}{2} - \frac{k_m}{\sqrt{\pi}} \sqrt{h_{00} + h_{01} \frac{x_{n,m}}{L_x}} = 0. \quad (35)
\]

In Figure 3 (panels A1 and A2) we plot

\[
F_1 = \frac{\omega_n^2}{2} - k_m V_A \quad (36)
\]

(solid line) and

\[
F_2 = \frac{\delta(x)}{\omega_n} \quad (37)
\]

(dotted line) against the normalized coordinate \( x/L_x \). These curves correspond to the zeroth-order harmonic of the fast magnetosonic mode shown in Figure 2 (panel A) and the standing Alfvén mode with wave numbers \( m = 3 \) (panel A1) and \( m = 4 \) (panel A2).

In order to examine the validity of the approximations we made during the analysis of the governing equation \( 15 \), we performed a direct numerical solution of the set of equations \( 14 - 16 \) and obtained the following results. The Alfvén mode \( m = 3 \) is amplified effectively close to the resonant point \( x/L_x \approx x_{0.3}/L_x = 0.7967 \). This is shown on panel C1 of Figure 3. Far from this resonant point, the swing interaction is weaker, as at the point \( x/L_x \approx x_{0.3}/L_x = 0.0767 \) (panel B1). For the Alfvén mode \( m = 4 \) we have the opposite picture: the area of ‘swing absorption’ is situated around the point \( x_{0.4} \) (see panel B2, Figure 3) and the rate of interaction between modes decreases far from this area, as at point \( x_{0.3} \) (panel C2, Figure 3). In these calculations we took \( \alpha = 0.015 \).

Similar results are obtained for the fast magnetosonic mode shown in panel B of Figure 2 corresponding to \( \alpha = 0.03 \). In this case, the fast magnetosonic mode effectively amplifies four different spatial harmonics of Alfvén modes, viz. \( m = 8, 10, 11 \) and 12 (for details see Shergelashvili et al. 2004). A similar analysis can be performed for the case of any other equilibrium configuration and corresponding harmonics of the standing fast magnetosonic modes.
Figure 2. Sample solutions of the standing fast magnetosonic modes. Panel A: zeroth-order harmonic $n = 0$, period $T = 5.6069$ min., $\alpha = 0.015$; Panel B: second-order harmonic $n = 2$, period $T = 1.8514$ min., $\alpha = 0.03$.

Figure 3. The swing absorption of the fast mode $n = 0$ by the standing Alfvén modes $m = 3$ and $4$. 
6. DISCUSSION AND CONCLUSION

The most important characteristic of swing absorption is that the velocity polarization of the amplified Alfvén wave is strictly perpendicular to the velocity polarization (and propagation direction) of fast magnetosonic waves. This is due to the parametric nature of the interaction. For comparison, the well-known resonant absorption of a fast magnetosonic wave can take place only when it does not propagate strictly perpendicular to the magnetic flux surfaces and the plane of the Alfvén wave polarization. In other words, the energy in fast magnetosonic waves propagating strictly perpendicular (i.e. \( k_\perp = 0 \)) to the magnetic flux surfaces cannot be resonantly ‘absorbed’ by Alfvén waves with the same frequency polarized in the perpendicular plane. This is because the mechanism of resonant absorption is analogous to the mechanical pendulum undergoing the direct action of an external periodic force. This force may resonantly amplify only those oscillations that at least partly lie in the plane of force. On the contrary, the external periodic force acting parametrically on the pendulum length may amplify the pendulum oscillation in any plane. A similar process occurs when the fast magnetosonic wave propagates across the unperturbed magnetic field. It causes a periodical variation of the local Alfvén speed and thus affects the propagation properties of the Alfvén waves. As a result, those particular harmonics of the Alfvén waves that satisfy the resonant conditions grow exponentially in time. These resonant harmonics are polarized perpendicular to the fast magnetosonic waves and have half the frequency of these waves. Hence, for standing fast magnetosonic waves with frequency \( \omega_n \), the resonant Alfvén waves have frequency \( \sim \omega_n / 2 \).

In a homogeneous medium all resonant harmonics have the same wavelengths (see Paper I). Therefore, once a given harmonic of the fast and Alfvén modes satisfies the appropriate resonant conditions (Eqs. (23) and (25) in Paper I), then these conditions are met within the entire medium. Thus, in a homogeneous medium the region where fast modes effectively interact with the corresponding Alfvén waves is not localized, but instead covers the entire system. However, when the equilibrium is inhomogeneous across the applied magnetic field, the wavelengths of the resonant harmonics depend on the local Alfvén speed. When the medium is bounded along the unperturbed magnetic field (i.e. along the \( z \) axis), the resonant harmonics of the standing Alfvén waves (whose wavelengths satisfy condition (23) for the onset of a standing pattern) will have stronger growth rates. This means that the ‘absorption’ of fast waves will be stronger at particular locations across the magnetic field. In the previous section we showed numerical solutions of standing fast magnetosonic modes for a polytropic equilibrium (\( p_0 \sim \rho_0^\gamma \)) in which the thermal pressure and magnetic pressure are linear functions of \( x \). Further, we performed a numerical simulation of the energy transfer from fast magnetosonic waves into Alfvén waves at the resonant locations, i.e. the regions of swing absorption.

The mechanism of swing absorption can be of importance in a variety of astrophysical situations.

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