Stock Return, Volume and Volatility in the EGARCH model

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I use EGARCH model to study the asymmetric impact of negative and positive shocks on stock return volatility. I find the asymmetric effects exist and the impact on volatility of a negative shock is greater than that of a positive shock. Furthermore, I examine the dynamic relationship between returns, volume and volatility of stock index by introducing trading volume as an exogenous variable into the EGARCH model. The results indicate that trading volume contributes some information to the returns processes of stock indexes. However, the persistence of volatility remains even after incorporating lagged volume effects, which are proxies for information flow. Granger causality tests demonstrate stronger evidence of returns causing volume than volume causing returns.

Key words: EGARCH models, Volatility Persistence, Trading volume, Information flow

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I. Introduction

Prior research has studied the effect of volatility persistence and changing equity premium in the stock market (Harris (1986); Karpoff (1987); Lamoureux and Lastrapes (1994); Chou (1988)). For example, Chou (1988) uses a univariate GARCH-M model to study stock return volatility persistence and its relationship with market fluctuations. The parameter estimates and the non-stationary test results suggest high persistence of shocks to the stock return. It is argued that shocks to volatility have to persist for a very long time in order for volatility to have a significant impact on stock prices. While volatility persistence in stock market is well documented by prior literature, the asymmetric impact of negative and positive shocks on stock return volatility, is not well understood.

To allow for possible asymmetry in the impact of good and bad news, I use the EGARCH (1,1)-M model. EGARCH has two advantages over GARCH. First, by using the exponential formulation, the restrictions of positive constraints on the estimated coefficients in ARCH and GARCH are no longer necessary. Second, a weakness of the GARCH model is that the conditional variance depends on the magnitude of the disturbance term, but not its sign. GARCH fails to capture the negative asymmetry apparent in many financial time series. The EGARCH model lessens this problem by modeling the standardized residual as a moving average (MA) regressor in the variance equation while preserving the estimation of the magnitude effect. This is potentially important as, ever since Black (1976), researchers have been aware of the possibility that the effect of shocks on the conditional volatility may depend on their sign. I therefore use EGARCH model to test if there is an uneven but persistent flow of information to stock market.
Glosten, Jagannathan, and Runkle (1993) use GJR-GARCH model to capture the asymmetric impacts of negative and positive shocks. This paper differs in that I use an EGARCH model to study the persistence of volatility after incorporating the trading volume effects, which are proxies for information flow. One of the main advantages of EGARCH is that it models logarithm of volatility. Therefore, during the estimation, there is no need for parameter restrictions. On the contrary, when estimating a GJR-GARCH model, it is common that alpha and beta are restricted by the estimation procedure to be larger than zero. No such a restriction is needed in the EGARCH model. EGARCH model best fit this paper by accommodating volatility persistence and leverage effect.

Secondly, according to the Mixture of Distribution hypothesis, price volatility and trading volume should be positively correlated because they jointly depend on a common underlying variable. This variable could be interpreted as the rate of information flow to the market. In other words, both the price and trading volume change contemporaneously in response to new information. To investigate the hypothesis that the flow of information to the market helps explain the volatility of returns, I use trading volume as a proxy for information innovations. To do this, I introduce detrended trading volume into the standard EGARCH model and examine if the positive relationship exists. In addition to study the relation between trading volume and conditional volatility, I use Granger causality test to examine the casual relation between trading volume and price changes (return).

1) The mixture of distribution hypothesis model presented in the seminal paper of Tauchen and Pitts (1983) offers an appealing explanation for the positive relation between trading volume and volatility of returns. In their specification, the information flow is the unobserved mixing variable responsible for moving both volumes and volatility. In this study, I analyze trading volume as information flows. The separation between volume and volatility implies an asymmetric behavior in stock prices and a leverage effect depending on unexpected trading volume.
Chiang, Qiao, and Wong (2010) find strong bi-directional nonlinear Granger causality between volume and volatility. Day and Lewis (1992), using S&P 100 index options, find that the implied volatility contains useful information in forecasting volatility for both EGARCH and GARCH models. This paper differs in that I study the dynamic relationship between return, volume, and volatility using a trading volume-augmented EGARCH model. Additionally, I use volume as a proxy for information arrival to examine if a positive or negative relationship exist between volume and stock return volatility.

The paper makes several contributions to the existing literature. First, using daily NYSE index data, I use exponential GARCH (EGARCH) model allow for asymmetry in the volatility, which may be present as a result of leverage effects. Second, I respond to evidence of two-way causality between volume and return (and return volatility) by introducing trading volume as an exogenous variable into the standard EGARCH(1,1) model. My results suggest the existence of asymmetry effect and the impact on volatility of a negative shock is greater than that of a positive shock. I find that trading volume contributes some information to the returns process. The results also show persistence in volatility even after I incorporate contemporaneous and lagged volume effects. Granger causality test indicates stronger evidence of return causing volume than volume causing return.

2) In a financial market, if bad news has a more pronounced effect on volatility than good news of the same magnitude, such asymmetry has typically been attributed as Leverage effect, and then the symmetric specification such as GARCH is not appropriate and could not capture the asymmetric effect, since the GARCH model assumes same effect for good and bad news. But, the fact of financial volatility is that negative shocks tend to have larger impact on volatility than positive shocks. The main drawback of the symmetric GARCH model is that the conditional variance is unable to respond asymmetrically to rise and fall in the stock returns. Hence to examine the asymmetric effect of the financial time series data, I use an Exponential GARCH (EGARCH) model in order to account for the leverage effect observed in return series of stocks.
II. Data

The data used in the paper are daily NYSE value-weighted price index and trading volume series from July 1, 1990 to Dec. 31, 2013. Following Chou (1988), the daily stock returns are calculated as the logarithmic first difference of the price index. I add volume series to the original dataset. The volume data are collected from Standard and Poor’s. Standard & Poor’s Statistical Service: Security Price Index Record reports daily NYSE share trading volume. My sample does not include dates when trading volume is not available. I match all series of indexes and trading volume.

(Table 1) Variables Definitions

| Series | Description (Source) | Sample Period, Size |
|--------|-----------------------|---------------------|
| $R_t$  | Daily returns of NYSE value-weighted index (CRSP, $R_t = (\ln p_t - \ln p_{t-1})/100$ where $p_t$ is the stock index price in period $t$) | 7/1990-12/2013 T=5903 |
| $H_t$  | NYSE daily raw trading volume (Standard & Poor’s Statistical Service: security Price Index Record) | 7/1990-12/2013 T=5903 |
| $h_t$  | Detrended NYSE trading volume (the detrend method is addressed in section 3) | 7/1990-12/2013 T=5903 |
III. Methodology

1. EGARCH modeling:

GARCH(1,1)-M provides a good technique in estimating the persistence of volatility of stock returns, however, it does not consider the asymmetric impact of shocks on volatility. I use the EGARCH(1,1)-M estimation technique. EGARCH has two well-known advantages over GARCH. First, no parameter restrictions are needed to ensure that the implied conditional variance of the return is always positive. Second, it allows for possible asymmetry in the impact of good and bad news respectively. I will therefore use AR(1)-EGARCH(1,1)-M model to see if there is an uneven but persistent flow of information to stock market. The EGARCH specification is as the follows:

\[ R_t = r_f + \delta R_{t-1} + e_t \quad \text{with } e_t \mid I_{t-1} \sim N(0, V_t) \]

\[ \ln V_t = \alpha_0 + \alpha \left( \frac{e_t}{\sqrt{V_{t-1}}} \right) + \gamma \left( \frac{e_t}{\sqrt{V_{t-1}}} - u \right) + \beta (\ln V_{t-1}) \]

where \( \mu = E(\frac{e_t}{\sqrt{V_{t-1}}}) = \left( \frac{2}{\pi} \right)^{0.5} \) (for a normal distribution)

In the above formulation, persistence of volatility is measured by \( \beta \). The asymmetric effect of negative and positive shocks is captured by \( \alpha \) and \( \gamma \). \( \alpha \) measures the sign effect and \( \gamma \) measures the size effect. We expect to find \( \gamma > 0 \), implying that shocks of above-average size (in absolute terms) increase volatility, other things being equal. If \(-1 < \alpha < 0\), the impact on volatility of a negative shock is greater than that of a positive shock. If \( \alpha < -1 \), a positive shock actually reduces volatility, while a negative shock causes it to increase. Either result could be attributed to a leverage effect, according to which
negative shocks have a magnified impact on stock values because they reduce the value of equity relative to debt and thereby increase the risk to equity holders. Therefore, I will use EGARCH to estimate the parameters $\alpha$, $\beta$ and $\gamma$, testing their magnitude as well as signs.

2. Examine the dynamic relationship between return, volume and volatility of stock indexes:

The Mixture of Distribution hypothesis predicts a positive relationship between price volatility and trading volume because they jointly depend on a common underlying variable. This variable could be interpreted as the rate of information flow to the market. Furthermore, the volatility persistence should become negligible if volume is serially correlated and is a good proxy for the flow of information to the market. To test this hypothesis, I introduce detrended trading volume into my EGARCH model and examine if the positive relationship exists. In addition to study the relation between trading volume and conditional volatility, it would be interesting to check if volatility persistence will be reduced as a result of this introduction. My estimation steps are as the follows:

Step 1: Trend and Unit Root Tests:

I use daily NYSE price index and trading volume series from July 1990 to Dec. 2013 obtained from CRSP and S&P. Trend stationary in trading volume is tested by regressing the series on deterministic function of time. To allow for a nonlinear time trend and a linear trend, I include a quadratic trend term:

$$H_i = \alpha + \beta_1 t + \beta_2 t^2 + \varepsilon_i$$  \hspace{1cm} (2)
Where $H_t$ is the raw trading volume. Here I use trading volume adjusted for both linear and nonlinear time trends. The detrended trading volumes are the residuals from the above regression.

To test for a unit root of the return and detrended trading volume series, I employ both the augmented Dickey–Fuller (D–F) test (1979) and the Phillips–Perron (P–P) test (1988). The difference between the two unit root tests lies in their treatment of any ‘nuisance’ serial correlation. The P–P test tends to be more robust to a wide range of serial correlations and time–dependent heteroskedasticity. In these tests, the null hypothesis is that a series is nonstationary (i.e., difference stationary): $\rho = 0$ and $\alpha = 1$:

$$A D F: \quad \Delta X_t = \rho_0 + \rho_1 X_{t-1} + \sum_{i=1}^{n} \delta_i X_{t-i} + \varepsilon_t$$

$$P h i l l i s - P e r r o n: \quad X_t = \alpha_0 + \alpha X_{t-1} + u_t$$

Where $X_t$ is the return or detrended trading volume. The lag length in the ADF and (P–P) regression is chosen by Akaike’s information criteria (AIC).

**Step 2: Trading Volume and stock price changes (return)**

To examine the contemporaneous correlation between detrended trading volume and stock return, I run the following regressions using two alternative forms of price change (return):

$$h_t = a + bR_t + u_t$$

$$h_t = a + b | R_t | + u_t$$

By examining the coefficient, results will tell me whether there is a positive contemporaneous returns–volume relation fit the data.
Step 3: Causal relation between detrended trading volume and stock price changes (return)

To test whether trading volume precedes stock returns, or vice versa. I use the causality test in Granger (1969). I use the following bivariate autoregressions to test for causality between the two variables detrended trading volume and stock returns:

\[
    h_t = \alpha_0 + \sum_{i=1}^{5} \alpha_i h_{t-i} + \sum_{j=1}^{5} \beta_j R_{t-j}
\]

\[
    R_t = \alpha_0 + \sum_{i=1}^{5} \gamma_i R_{t-i} + \sum_{j=1}^{5} \delta_j h_{t-j}
\]

Where \( h_t \) is detrended trading volume and \( R_t \) is return at time \( t \). For the estimation of the vector autoregression (VAR), I use five lags based on both the Akaike information criterion (AIC) and the Schwarz criterion. These lags amount to allowing for week-long information in the regression.

If the \( \beta_j \) coefficients are statistically significant, then including both past values of return and past history of volume yields a better forecast of future volume. Therefore, returns cause volume. If a standard F-test does not reject the hypothesis that \( \beta_j = 0 \) for all \( j \), then returns do not cause volume. If causality runs from volume to returns, then the \( \delta_j \) coefficients will jointly be different from zero. If both \( \beta \) and \( \delta \) are different from zero, there is a feedback relation between returns and trading volume.

Step 4: Detrended trading volume and conditional volatility in the EGARCH model:

To examine the hypothesis that the flow of information to the market helps
explain the volatility of returns, I use trading volume as a proxy for information innovations. I choose daily trading volume as a measure of the amount of daily information that flows into the market. The following AR(1)-EGARCH(1,1)-M model is extended with detrended trading volume:

\[ R_t = r_f + \delta R_{t-1} + e_t \quad \text{with} \quad e_t \mid I_{t-1} \sim N(0, V_t) \]

\[ \ln V_t = \alpha_0 + \alpha \left( \frac{e_t}{\sqrt{V_{t-1}}} \right) + \gamma \left( \frac{e_t}{\sqrt{h_{t-1}}} \right) - \mu + \beta \ln V_{t-1} + \lambda h_{t-1} \]

(7)

where \( \mu = E(\frac{e_t}{\sqrt{V_{t-1}}}) = (\frac{2}{\pi})^{0.5} \) (for a normal distribution)

Where in the conditional variance equation above, I use the lagged detrended trading volume \( h_{t-1} \) as an instrument for contemporaneous volume to avoid the problem of simultaneity since lagged values of endogenous variables are classified as predetermined.

The mixture of distribution hypothesis predicts that \( \lambda > 0 \). Furthermore, in the presence of volume with \( \lambda > 0 \), if daily volume is serially correlated, \( \beta \) will be small and statistically insignificant. The persistence of variance as measured by \( \beta \) should become negligible if volume is serially correlated and is a good proxy for the flow of information to the market. However, in the case where trading activity does not fully capture the rate of information arrival and other exogenous directing variables affect the variance equation, EGARCH effects, although reduced, will remain.
IV. Results

1. Trend and unit root tests:

Table 2 presents the basic statistics for the NYSE stock index returns and raw daily trading volume. Return is defined as log differences of the index levels. As can be seen the return series is positively skewed and leptokurtic compared to the normal distribution. Although the skewness statistics are not large, the positive skewness of the return series implies a higher probability of earning positive returns. The kurtosis value is larger than three and implies that the distribution of returns have fat tails compared with the normal distribution. The Ljung-Box Q(36) statistic for 36th order autocorrelation is statistically significant, while the Ljung-Box test statistic $Q^2(36)$ (for the squared data) indicates the presence of conditional heteroskedasticity.

(Table 2) Summary Statistics of Daily Stock Index Returns and Raw Trading Volume:
1990.07~2013.12 (N=5903)

|            | NYSE stock index returns | NYSE raw trading volume (million) |
|------------|---------------------------|-----------------------------------|
| Mean       | 0.039485                  | 29.753                            |
| Median     | 0.051                     | 16.830                            |
| Maximum    | 5.070                     | 236.565                           |
| Minimum    | -3.859                    | 0.337                             |
| Std. Dev   | 0.775                     | 30.857                            |
| Skewness   | 0.177                     | 1.805                             |
| Kurtosis   | 5.683                     | 6.062                             |
| Jarque-Bera| 1800.566*                 | 5509.846*                         |
| Ljung-Box Q(36) | 346.55*                 | 171287*                           |
| Ljung-Box $Q^2(36)$ | 4182.2*                | 116725*                           |

Note: stock return is calculated as $R_t=(\ln P_t-\ln P_{t-1})\times 100$ where $P_t$ is the stock index price in period $t$.

* indicates statistical significance at 1% level.
Previous studies report strong evidence of both linear and nonlinear time trends in trading volume series (e.g., Gallant, Rossi and Tauchen, 1992). As such, trend stationarity in trading volume is tested by regressing the series on a deterministic function of time. To allow for a nonlinear time trend as well as a linear trend, I include a quadratic time trend term:

To test for a unit root (or the difference stationary process), I employ both the augmented Dickey-Fuller (D-F) test (1979) and the Phillips-Perron (P-P) test (1988). The P-P test tends to be more robust to a wide range of serial correlations and time-dependent heteroskedasticity. In these tests, the null hypothesis is that a series is nonstationary (i.e., difference stationary): $\rho = 0$ and $\alpha = 1$ (see Table 3) respectively.

Test results are reported in Table 3. Panel A of Table 3 shows that the coefficients (with t-ratios in parentheses) of regressing trading volumes on a linear time trend alone. When a quadratic time trend term is added, the coefficients are very significant and the model fit is high. Therefore, I use trading volume adjusted for both linear and nonlinear trends for all volume.

Panel B of Table 3 shows that the null hypothesis that the stock return series and detrended trading volume series are nonstationary (i.e., have a unit root) is strongly rejected whether we allow for three lags or seven lags. This confirms that detrended trading volume and stock return series are both stationary, and we do not have to consider the possible cointegration problem associated with these variables. The lag length in the ADF and (P-P) regression is chosen by Akaike’s information criteria (AIC).
Panel A: Linear and nonlinear trend tests in trading volume

\[ H_t = \alpha + \beta_1 t + \beta_2 t^2 + \epsilon_t \]  
(Where \( H_t \) is the raw trading volume)

| Year       | \( \alpha \)  | \( \beta_1 \)  | \( \beta_2 \)  | \( R^2 \)  |
|------------|---------------|---------------|---------------|------------|
| 1990.07~2013.12 | -13.746       | 0.015         |               | 0.662      |
|            | (-29.444)*    | (107.585)*    |               |            |
|            | 15.801        | -0.015        | 5.09E-06      | 0.846      |
|            | (33.353)*     | (-41.247)*    | (83.662)*     |            |

Panel B: Unit root tests for stock returns and detrended trading volume

(a) Augmented Dickey-Fuller regression:
\[ \Delta X_t = \rho_0 + \rho_1 X_{t-1} + \sum_{i=1}^k \delta X_{t-i} + \epsilon_t \]

(b) Phillips-Perron regression:
\[ X_t = \alpha_0 + \alpha X_{t-1} + u_t \]

| Variable \((X_t)\) | Lags \((k)\) | \(\tau(\rho)\) | Lags \((k)\) | \(Z(\tau_\alpha)\) |
|-------------------|-------------|--------------|-------------|------------------|
| Return \((R_t)\)  | 3           | -63.210*     | 3           | -61.310*         |
| Detrended volume \((h_t)\) | 7           | -38.553*     | 7           | -28.670*         |

Note: Numbers in parentheses are \(t\)-statistics.
* denotes significant at the 1% level.

2. Trading Volume and stock price changes (return)

To examine the contemporaneous returns-volume relations, I regress detrended trading volume on returns as well as absolute stock returns. Table 4 shows the results of these regressions, where the dependent variable \((h_t)\) is detrended trading volume and independent variable is the natural logarithm of the price relative or its absolute value. The results suggest a positive contemporaneous relation between volume and return during July 1990-Dec. 2013. In panel A and B, the coefficients are statistically significant at 1% level.
Panel A: Regression of detrended daily trading volume on stock returns
\[ h_t = a + bR_t + u_t \]

| Detrended volume \((h_t)\) | \(a\) | \(b\) | \(R_t^2\) |
|--------------------------|-----|-----|-----|
| -0.092 \((-0.588)\) | 2.327 \((11.555)^*\) | 0.022 |

Panel B: Regression of detrended daily trading volume on absolute stock returns
\[ h_t = a + b|R_t| + u_t \]

| Detrended volume \((h_t)\) | \(a\) | \(b\) | \(R_t^2\) |
|--------------------------|-----|-----|-----|
| -1.720 \((-7.523)^*\) | 3.040 \((10.317)^*\) | 0.018 |

Note: Numbers in parentheses are \(t\)-statistics.
* denotes significant at the 1% level.

3. Causal relation between detrended trading volume and stock price changes (return)

Table 5 presents the causal relation test on the bivariate vector autoregression (VAR) model discussed in equation (6) of section 3. Panel A shows the results of the test of the null hypothesis that returns do not Granger-cause volume. The \(F\)-statistic is significant at the 1% level for both the full and sub-sample periods. Thus we reject the null hypothesis and find strong evidence for stock return causing trading volume. Panel B shows that in the test of the null hypothesis, volume does not Granger-cause returns. The \(F\)-statistics is significant at 10% level for the sub-sample period July 1990–Dec.2001. For other periods the \(F\)-statistics is insignificant. In addition, in Panel B all adjusted \(R^2\) values are very low, which indicates volume may have little predictive power for future returns. Overall, Granger causality tests demonstrate stronger evidence of returns causing volume than volume causing returns.
### Table 5: Granger Causality Tests Return and Detrended Volume

| Panel A: Tests of causality from returns to volume: | Panel B: Tests of causality from volume to return: |
|--------------------------------------------------|--------------------------------------------------|
| $h_t = \alpha_0 + \sum_{i=1}^{\delta} \alpha_i h_{t-i} + \sum_{j=1}^{\delta} \beta_j R_{t-j} + \epsilon_t$ | $R_t = \alpha_0 + \sum_{i=1}^{\delta} \gamma_i R_{t-i} + \sum_{j=1}^{\delta} \delta_j h_{t-j} + \epsilon_t$ |
| July 1990-Dec. 2013 | July 1990-Dec. 2001 | Jan. 2002-Dec. 2013 | July 1990-Dec. 2013 | July 1990-Dec. 2001 | Jan. 2002-Dec. 2013 |
|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| $\alpha_0$ | 0.001 (0.433) | 0.001 (0.442) | 0.001 (0.203) | $\alpha_0$ | 0.031 (3.131)* | 0.022 (1.864)*** | 0.040 (2.549)* |
| $\alpha_1$ | -0.419 (-31.547)* | -0.386 (-20.222)* | -0.437 (-23.556)* | $\gamma_1$ | 0.226 (16.991)* | 0.289 (15.121)* | 0.194 (10.397)* |
| $\alpha_2$ | -0.393 (-27.661)* | -0.378 (-18.706)* | -0.404 (-20.236)* | $\gamma_2$ | -0.045 (-3.265)* | -0.084 (-4.246)* | -0.026 (-1.378) |
| $\alpha_3$ | -0.288 (-19.584)* | -0.275 (-13.162)* | -0.297 (-14.394)* | $\gamma_3$ | 0.023 (1.661)*** | 0.036 (1.798)*** | 0.022 (1.146) |
| $\alpha_4$ | -0.153 (-10.862)* | -0.136 (-6.751)* | -0.163 (-8.259)* | $\gamma_4$ | -0.005 (-0.373) | 0.021 (1.044) | -0.026 (-1.370) |
| $\alpha_5$ | -0.026 (-3.409)* | -0.064 (-6.751)* | -0.001 (-0.074) | $\gamma_5$ | -0.001 (-0.046) | 0.005 (0.022) | -0.005 (-0.285) |
| $\beta_1$ | 0.0311 (10.386)** | 0.010 (1.954)*** | 0.042 (11.051)* | $\delta_1$ | 0.007 (0.125) | -0.027 (-0.369) | 0.036 (0.392) |
| $\beta_2$ | -0.015 (-4.956)* | -0.011 (-2.068)** | -0.017 (-4.228)* | $\delta_2$ | 0.118 (1.870)*** | 0.196 (2.500)* | 0.046 (0.473) |
| $\beta_3$ | 0.012 (3.724)* | 0.015 (2.961)* | 0.010 (2.646)* | $\delta_3$ | 0.090 (1.373) | 0.036 (0.445) | 0.135 (1.346) |
| $\beta_4$ | -0.008 (-2.675)* | -0.016 (-3.042)* | -0.005 (-1.236) | $\delta_4$ | 0.073 (1.159) | 0.007 (0.909) | 0.122 (1.274) |
| $\beta_4$ | -0.013 (-4.342)* | -0.002 (-0.349) | -0.019 (-4.974)* | $\delta_5$ | 0.141 (2.409)* | 0.120 (1.639) | 0.159 (1.788)*** |
| F-statistics | 30.615* [0.0000] | 3.836* [0.0018] | 32.7558* [0.0000] | F-statistics | 1.628 [0.1490] | 2.17145*** [0.0546] | 0.81534 [0.5385] |
| Adjusted R-square | 0.501 | 0.478 | 0.529 | Adjusted R-square | 0.051 | 0.081 | 0.039 |

Note: t-statistics are in parentheses and p-values are in brackets.
*, ** and *** indicates statistical significance at the 1%, 5% and 10% level respectively.
4. Detrended trading volume and conditional volatility in the EGARCH model:

(Panel A) EGARCH estimation of stock returns without trading volume:
\[ R_t = r_f + \delta R_{t-1} + e_t \quad \text{with} \quad e_t \mid I_{t-1} \sim N(0, \sigma_t^2) \]
\[ \ln \sigma_t^2 = \alpha_0 + \alpha \left( \frac{e_t}{\sqrt{\sigma_{t-1}^2}} \right) + \gamma \left( \frac{e_t}{\sqrt{\sigma_{t-1}^2}} - u \right) + \beta (\ln \sigma_{t-1}^2) \]
where \( \mu = E(\left| \frac{e_t}{\sqrt{\sigma_{t-1}^2}} \right|) = \left( \frac{2}{\pi} \right)^{0.5} \) (for a normal distribution)

|       | July 1990-Dec. 2013 | July 1990-Dec. 2001 | Jan. 2002-Dec. 2013 |
|-------|---------------------|---------------------|---------------------|
| \( \gamma_f \) | 0.025 (3.300)*      | 0.019 (2.179)**     | 0.0319 (2.423)**    |
| \( \delta \)   | 0.230 (18.390)*     | 0.286 (15.646)*     | 0.183 (9.999)*      |
| \( \alpha_0 \) | -0.108 (-16.355)*   | -0.147 (-12.031)*   | -0.079 (-9.153)*    |
| \( \gamma \)   | -0.0788 (-14.896)*  | -0.140 (-13.948)*   | -0.0345 (-5.432)*   |
| \( \beta \)    | 0.128 (16.107)*     | 0.149 (11.222)*     | 0.095 (9.071)*      |

|       | July 1990-Dec. 2013 | July 1990-Dec. 2001 | Jan. 2002-Dec. 2013 |
|-------|---------------------|---------------------|---------------------|
| \( \gamma_f \) | -0.003 (-0.391)    | 0.002 (0.177)       | -0.024 (-1.852)**   |
| \( \delta \)   | 0.208 (15.948)*    | 0.273 (14.891)*     | 0.155 (8.231)*      |
| \( \alpha_0 \) | -0.102 (-12.819)*  | -0.124 (-9.740)*    | -0.080 (-7.293)*    |
| \( \alpha \)   | -0.099 (-17.277)*  | -0.158 (-15.174)*   | -0.059 (-8.103)*    |
| \( \beta \)    | 0.113 (12.263)*    | 0.110 (7.958)*      | 0.093 (7.109)*      |

(Panel B) EGARCH estimation of stock returns with trading volume:
\[ R_t = r_f + \delta R_{t-1} + e_t \quad \text{with} \quad e_t \mid I_{t-1} \sim N(0, \sigma_t^2) \]
\[ \ln \sigma_t^2 = \alpha_0 + \alpha \left( \frac{e_t}{\sqrt{\sigma_{t-1}^2}} \right) + \gamma \left( \frac{e_t}{\sqrt{\sigma_{t-1}^2}} - \mu \right) + \beta (\ln \sigma_{t-1}^2) + \lambda v_{t-1} \]
where \( \mu = E(\left| \frac{e_t}{\sqrt{\sigma_{t-1}^2}} \right|) = \left( \frac{2}{\pi} \right)^{0.5} \) (for a normal distribution)

|       | July 1990-Dec. 2013 | July 1990-Dec. 2001 | Jan. 2002-Dec. 2013 |
|-------|---------------------|---------------------|---------------------|
| \( \gamma_f \) | -0.003 (-0.391)    | 0.002 (0.177)       | -0.024 (-1.852)**   |
| \( \delta \)   | 0.208 (15.948)*    | 0.273 (14.891)*     | 0.155 (8.231)*      |
| \( \alpha_0 \) | -0.102 (-12.819)*  | -0.124 (-9.740)*    | -0.080 (-7.293)*    |
| \( \alpha \)   | -0.099 (-17.277)*  | -0.158 (-15.174)*   | -0.059 (-8.103)*    |
| \( \beta \)    | 0.113 (12.263)*    | 0.110 (7.958)*      | 0.093 (7.109)*      |
| Ljung-Box Q (26) | 29.232 [0.380] | 14.639 [0.497] | 19.2767 [0.379] | Ljung-Box Q (26) | 13.734 [0.744] | 15.280 [0.760] | 22.089 [0.313] |
| Ljung-Box $Q^2(26)$ | 18.827 [0.533] | 18.028 [0.586] | 23.328 [0.273] | Ljung-Box $Q^2(26)$ | 14.906 [0.628] | 13.449 [0.857] | 21.769 [0.419] |
| Log-likelihood | -5918.428 | -2276.289 | -3593.588 | Log-likelihood | -5717.145 | -2197.374 | -3469.915 |
| Likelihood ratio test | 8.324*** | 6.223** | 7.740*** | Likelihood ratio test | 14.764*** | 9.352*** | 10.236*** |

Note: Z-statistics are in parentheses and p-values are in brackets. *, ** and *** indicates statistical significance at the 1%, 5% and 10% level respectively.
Panel A of Table 6 reports the results of the EGARCH model in equation (1) of section 3. The parameter estimates are obtained by maximizing the log-likelihood using the Berndt, Hall, Hall, and Hausman (1974) algorithm. The estimation results demonstrate, first, the volatility persistence, measured by $\beta$, is high but less than one indicating high but stationary persistence. Second, asymmetry is present since $\alpha$ is found to be statistically significant. Since $-1 < \alpha < 0$, the impact on volatility of a negative shock is greater than that of a positive shock. As far as the effect of shocks on the variance is concerned, notice that all the $\gamma$ are positive and significant, as we should expect, implying that above-average shocks increase conditional volatility, other things being equal. Third, as a model specification test the Ljung-Box statistics for 26th order serial correlation in the level and squared standardized residuals are reported. Both Ljung-Box statistics indicate that the residuals do not show any significant serial correlation. Thus, the estimated models fit the data well. Finally, the log-likelihood statistics are very large. This result implies that the EGARCH model is an attractive representation of daily return behavior that successfully captures the temporal dependence of return volatility. Likelihood ratio test between EGARCH models and their conventional Gaussian counterparts is also reported in Table 6. It demonstrates that an EGARCH model specification is more fit in the sample data than GARCH model under student-$t$ distribution.

The results when trading volume is included in the conditional variance of EGARCH(1,1)-M model are reported in Panel B of Table 6. Various points can be made. First, the coefficient of lagged trading volume is positive and statistically significant, which is consistent with the predications of the mixture of distribution hypothesis. The significant coefficient on volume indicates that volume is an exogenous variable in the system, and there is a
positive association between return variance and lagged trading volume. Second, the EGARCH effect remains when lagged volume is included in the model. However, the persistence in volatility as measured by $\beta$ is marginally smaller when we do so. Trading volume as a proxy for information innovations does not reduce the importance of $\beta$ in explaining persistence in volatility of stock returns. The results suggest that volume provides information about the quality of information signals, rather than representing the information signal itself.

V. Conclusion

I use EGARCH(1,1)-M model to study the asymmetric impact of negative and positive shocks on stock return volatility. The results suggest that the EGARCH models reflect an appropriate representation of the returns in stock index data. Asymmetric effects exist and the impact on volatility of a negative shock is greater than that of a positive shock. Furthermore, the EGARCH model is extended with trading volume to examine the dynamic relationship between returns, volume and volatility of stock index. The results indicate that trading volume contributes some information to the returns processes of stock indexes. However, the persistence of volatility remains even after incorporating lagged volume effects, which are proxies for information flow. Granger causality tests demonstrate stronger evidence of returns causing volume than volume causing returns.

Additional work could be done to test whether the effect of trading volume on volatility is homogeneous by separating volume into its expected and unexpected components and allowing each component to have a separate
effect on observed price volatility. By examining whether the expected and unexpended components of trading volume have different effects on the conditional variance, more can be learned about the stock market through the dynamics of returns and volume.
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