High-Frequency Current Ripple Sample Anti-aliasing Strategy of Digitally Controlled Converter Based on Notch Filter

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ABSTRACT The digital controller of the grid-connected converter achieves closed-loop control by sampling key voltage and current signals. High-frequency commutation of power electronic switches introduces high-frequency ripples into the signals, which may cause sampling aliasing. Severe distortion may happen in the output current of the converter because of the sampling aliasing. This paper explains how the aliasing problem arises by analyzing the spectrum of signals for sampling control. The explanation shows the severity of the aliasing problem and the necessity of adopting anti-aliasing methods. Next, this paper analyzes the impact of commonly used anti-aliasing low-pass filters on current control stability. From the conclusions, adopting a low-pass anti-aliasing filter reduces the stability margin of the current loop, which needs improvement. So this paper proposes a novel hybrid anti-aliasing filter based on the combination of the notch and low pass analog filters. An experiment on a 6kVA prototype verifies the proposed anti-aliasing method. This paper shows distortion and step-response oscillation of the output current when proposed and traditional anti-aliasing methods adopted. The comparison shows the advantages of the proposed anti-aliasing method.

KEYWORDS grid-connected converter, high-frequency ripples, anti-aliasing strategy, stability margin.

I. INTRODUCTION

The number of digitally controlled converters is increasing in the industrial fields, such as new energy grid connection and power quality compensation [1][2]. Digital controllers achieve flexible functions by sampling, real-time calculation, and digital pulse-width modulation. Though digital controllers are flexible, some nonideal characteristics still exist. Digital control delay and sampling aliasing are two of them which are researched the most.

Due to the digital control delay and resonance peak of the passive filter, widely used LCL grid-connected inverters are prone to get into high-frequency oscillation. This high-frequency oscillation problem has been studied in detail, and many improving strategies have been proposed until now. Proper feedback of the voltage [3][4] or current [5]−[7] on the capacitor branch of the LCL passive filter is the solution of these strategies.

However, the sampling aliasing problem caused by high-frequency ripples is seldom considered together with stability problems. According to [9]−[13], spectrum components of high-frequency ripples are concentrated in the frequency range around integer multiples of switching frequency. The high-frequency side-band components have significant amplitude [8], which tends to cause severe sampling aliasing, resulting in distortion in the converter's output current [14].

Therefore, some sample anti-aliasing strategies [14]−[18] should be adopted to suppress the aliasing problem. Some of these anti-aliasing strategies, such as analog low pass filters [18], may introduce phase lag to the sampled signals. And the phase lag reduces the stability margin of the current control loop. Other strategies require digital controllers with higher calculation power, such as sampling moment scheduling [17] and oversampling.

Anti-aliasing strategy based on analog filters has the advantages of low cost and high robustness. But as told above, traditional low-pass analog filters affect signal components in the control frequency range too much, harming stability. So this paper proposes a new analog anti-
aliasing filter based on the combination of low pass and notch filters as an improved version.

This paper takes the three-level grid-connected converter with LCL-LC type passive filter [20] as an example for analysis. Inverter side inductor current and capacitor + LC trap current are analyzed in the frequency domain to explain the arising of sampling aliasing, clarifying the necessity of sampling anti-aliasing strategies. Then the disadvantages of the traditional low-pass filter are shown in two aspects: anti-aliasing effectivity and stability margin of the current control loop. Next, this paper introduces the proposed anti-aliasing method. Theoretical analysis shows advantages of the proposed method compared with the traditional low-pass filter in the above two aspects. This paper also explains parameter synthesis and hardware implementation of the proposed anti-aliasing filter. At last, the experiment on a 6kVA prototype verifies the anti-aliasing effectivity of the proposed strategy and compares it with the traditional low-pass filter.

II. ANALYSIS OF SAMPLING ALIASING MECHANISM OF GRID-CONNECTED CONVERTER

A. Modeling of the Investigated Grid-Connected Converter

This paper takes the 3 phase 4 wire three-level LCL-LC type grid-connected converter as an example to carry out the analysis. It can be decoupled into three single-phase subconverters with ignorance of voltage fluctuation on the DC side. FIGURE 1 shows the diagram of the single-phase subconverter in each phase.

As shown in FIGURE 1, the LCL-LC type passive filter is adopted to attenuate current ripple, with the neutral line of the grid, AC filter capacitor, and DC middle point connected. Inverter side current \( i_m \) is sampled and fed back for the current control loop. Capacitor branch + LC trap current \( i_c \) is sampled and fed back for active damping control to prevent the resonance peak of the passive filter from causing the current loop to oscillate.

The current and active damping controllers are considered proportional controllers with gains of \( K_p \) and \( K_c \). The grid voltage is fed forward to modulation wave \( m \) through the narrow band-pass filter \( G_{al}(s) \) [21] to suppress inrush current at the starting-up moment and harmonic current caused by grid voltage harmonics. This paper ignores the dynamic characteristic of \( G_{al}(s) \). Besides, DC side voltage variation is ignored for simplicity as well.

FIGURE 2 shows a block diagram of the control system, where \( Z_{in}(s) \) means the impedance of the filter capacitor in parallel with LC trap.

Switching frequency ripples in sampled signal \( i_m \) and \( i_c \) are excited by high-frequency switching of power electronic devices. And sample/control frequency of digital controllers is usually set as switching frequency [22]. So the high-frequency ripples are easy to be aliased to low-frequency range through the ’Sample’ process in FIGURE 2. The severity of the aliasing phenomenon depends on sampling moments and the effectivity of anti-aliasing filter \( H_i(s) \). Next, analyze the frequency spectrum of \( i_m \) and \( i_c \) to reveal the mechanism of sampling aliasing in the grid-connected converter.

B. Spectrum Analysis of Signals for Sampling Control

Converter side voltage \( v_m \) is a series of pulses. According to [9]-[12], it can be decomposed into the form of 2-dimensional Fourier series, as shown in (1).

\[
v_m(t) = A_{00} + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn} e^{j\omega_m t} + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} C_{mn} e^{j\omega_m t} + \sum_{m=0}^{\infty} C_{mn} e^{j(n \omega_m + m \omega_s) t} \]

In (1), \( \omega_0 \) and \( \omega_s \) are angular frequencies of the utility grid and pulse width modulation carrier. \( A_{00} \) and \( C_{mn} \) correspond to the DC component and harmonic coefficients of each order. They are obtained by two-dimensional Fourier decomposition [9], with math expression shown in (2).

\[
C_{mn} = \begin{cases} \frac{0.5MV_{dc}}{m \pi} , & n = 1 \\ 0 , & n \neq 1 \end{cases} \\
C_{mn} = \frac{2V_m}{m \pi} (1 -( -1)^n) \sum_{m=1}^{\infty} J_{mn} \left( \frac{m \pi M}{2k-1} \right) \sin \left( \frac{\pi (n+k)}{2} \right) - \sin \left( \frac{\pi (n-k)}{2} \right) 
\]

\[
C_{mn} = \frac{V_m}{m \pi} (1 -( -1)^n) \sum_{m=1}^{\infty} J_{mn} \left( \frac{m \pi M}{2k-1} \right) \sin \left( \frac{\pi (n+k)}{2} \right) - \sin \left( \frac{\pi (n-k)}{2} \right) 
\]

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According to (2), the spectrum of high-frequency ripples is mathematically described by trigonometric, sinc, and Bessel functions. And coefficients of each frequency component are all real numbers. From (2), three laws about \( V_m \) can be obtained. First, high-frequency components are concentrated around integer multiples of the switching frequency due to the attenuation characteristics of sinc and Bessel functions. (The coefficients decrease as index \( n \) increases.) Second, all high-frequency components are in phase or anti-phase with each other. Third, the side-band high-frequency components near each integer multiple of the switching frequency are symmetrical about the attached integer multiple of switching frequency point. The spectrum of \( V_m \) in FIGURE 3 verifies the above three laws. Table 1 shows the corresponding chosen parameters of the converter, and the grid voltage is 380V (line to line) and 50Hz.

High-frequency ripples in sampled signals \( i_m \) and \( i_c \) are all excited by inverter side voltage \( v_m \) according to FIGURE 2, so frequency spectrum of \( i_m \) and \( i_c \) can be obtained from \( v_m \). The transfer function (3) shows the dynamic relationships between \( i_m \), \( i_c \), and \( v_m \).

\[
Y_m(s) = \frac{i_m(s)}{v_m(s)} = \frac{L_ms + Z_{nat}(s)}{L_mL_s s^2 + Z_{nat}(s)(L_m + L_s)^s}, \quad Z_{nat}(s) = \frac{L_i C_s s^2 + 1}{L_i C_s s^2 + (C + C_i)s}
\]

(3)

Substituting parameters in Table 1 into (3), frequency characteristics of transfer function \( Y_m(s) \) and \( Y_c(s) \) can be obtained, as shown in the bode plot in FIGURE 4.

From FIGURE 4, phases of \( Y_m(s) \) and \( Y_c(s) \) are both -90° at integer multiples of switching frequency. As explained above, high-frequency side-band components of \( v_m \) concentrate around integer multiples of switching frequency and are in phase with each other. So high-frequency components of \( i_m \) and \( i_c \) are in phase just like in \( v_m \) and have -90° phase lag relative to corresponding components in \( v_m \). With parameters set according to Table 1, FIGURE 5 shows the frequency spectrum of \( i_m \) and \( i_c \).

**C. Impact of Sample Aliasing on Output Current**

The mechanism of sample aliasing is explained based on the above conclusions as follows. Synchronous sampling control strategy [22] is analyzed in detail in this paper for its wide application range. With this strategy adopted, \( i_m \) and \( i_c \) are sampled and fed back in switching frequency. The phase of the sampling moment relative to PWM carrier can be artificially modified to change control delay [4] and sampling aliasing characteristics [17]. For the convenience of analysis, sampling delay time \( T_{sam} \) and \( T_{ds} \) are defined to express phases of the sampling moments, as shown in FIGURE 6.

The sampling process is equivalent to the following two mathematical processes. First, the sampled signal is multiplied with an equally placed ideal unit pulse train \( \sum \delta(t - kT_s) \) in the switching period \( T_s \). Then the multiplication result is convoluted with a square pulse with the width of \( T_s \) and height of \( 1/T_s \). The process of the multiplication of the sampled signal and the pulse train causes the sampling aliasing phenomenon.

Multiplication of sampled signals and pulse train in the time domain equals convolution of their spectrum in the frequency domain. So if \( i_m \) and \( i_c \) don't have low frequency (several integer multiples of \( \omega_0 \)) harmonics in themselves,
The frequency spectrum of sampling results $i_{am}[kT_s]$ and $i_{ac}[kT_s]$ around grid fundamental frequency can be analytically expressed in the complex form of each component as (4).

$$I_s(k\omega_0) = I_s(\omega_s + k\omega_b)e^{jT_{am}\omega_s} + I_s(\omega_s - k\omega_b)e^{-jT_{am}\omega_s}$$

$$+ I_s(2\omega_s + k\omega_b)e^{2jT_{ac}\omega_s} + I_s(2\omega_s - k\omega_b)e^{-2jT_{ac}\omega_s}$$ (4)

In the expression, $\omega_s = \omega_m$ or $\omega_c$. $I_{am}(k\omega_0)$ and $I_{ac}(k\omega_0)$ are frequency components' coefficients of $i_{am}$ and $i_{ac}$ at frequency point $k\omega_0$. In (4), only aliasing effects caused by high-frequency ripple components near first and second-order switching frequencies are considered. This is because the low-pass characteristic of LCL-LC filter makes $i_m$ and $i_c$ have poor frequency components at more than twice the switching frequency. So components at higher frequency points could be ignored.

From (4), sampling aliasing causes low order harmonics in sampling results, and the amplitude of the low order harmonics depend on side-band components around integer multiples of sampling & switching frequency, as shown in FIGURE 5. Low harmonics' coefficients in sampling results $i_{am}$ and $i_{ac}$ caused by sampling aliasing can be calculated according to (2)(3)(4). The mechanism of how these low-order harmonics affect the grid-connected converter control system can be expressed in the block diagram in FIGURE 7.

In FIGURE 7, $i_{am}$ and $i_{ac}$ mean low order harmonic components caused by sampling aliasing in $i_m$ and $i_c$, which means $i_{am} = H(s)i_m$, $i_{ac} = H(s)i_c$. They are added to the current control loop at marked positions in red color, resulting in low harmonics in the current control loop and eventually causing low harmonic distortion in the converter's output current.

From FIGURE 7, the relationships between $i_g$ and $i_{am}$, $i_g$ and $i_{ac}$ can be obtained as (5).

$$F_{am}(s) = \frac{i_g(s)}{i_{am}(s)} = \frac{K_p e^{-sT_{am}}} {L_m L_g s^2 + Z_{am}(s)(L_m + L_g)s} + (K_p L_g s - K_c L_g s - K_p Z_{am}(s))e^{-sT_{am}}H_i(s)$$

$$F_{ac}(s) = \frac{i_g(s)}{i_{ac}(s)} = \frac{K_p e^{-sT_{ac}}} {L_m L_g s^2 + Z_{ac}(s)(L_m + L_g)s} + (K_p L_g s - K_c L_g s - K_p Z_{ac}(s))e^{-sT_{ac}}H_i(s)$$ (5)

Total low harmonic (2-10 multiples of the fundamental frequency of the utility grid) root-mean-square value in output current $i_g$ caused by sampling aliasing can be calculated according to (5) with different sampling delays $T_{am}$ and $T_{ac}$. Taking $T_{am}$ and $T_{ac}$ as independent variables.
and RMS value of low order harmonics in $i_g$ as the dependent variable, a two-dimensional histogram can be obtained as FIGURE 8.

According to FIGURE 8, low order harmonics in $i_g$ would become unacceptable with certain $T_{dsm}$ and $T_{dsc}$. The maximum RMS value of low order harmonics in $i_g$ exceeds 1A. Meanwhile, the rated current of the 6kVA converter analyzed in this paper is 10Arms, which means extremely high THD.

Although the arrangement of $T_{dsm}$ and $T_{dsc}$ can suppress sampling aliasing effectively [17], $T_{dsm}$ and $T_{dsc}$ are also tuned for stability margin improving sometimes. These two requirements often conflict with each other. Therefore, some anti-aliasing strategies should be taken to suppress sampling aliasing shown in FIGURE 8, making amplitudes of low harmonics lower.

III. PRINCIPLE AND DESIGNING OF PROPOSED ANTI-ALIASING METHOD

A. Limitations of Commonly Used Anti-aliasing Strategies

In the converter control system, commonly used anti-aliasing strategies can be classified into three types: sampling moment arrangement [16][17], oversampling-based digital filtering [14], analog filtering [18].

The principle of sampling moment arrangement strategy can be expressed by (4). That is adjusting sampling moments of $i_m$ and $i_c$ ($T_{dsm}$ and $T_{dsc}$) to cancel out the aliasing components caused by high-frequency side-band components around integer multiples of the switching frequency. This kind of strategy doesn't need extra hardware or higher computing power of the digital controller. However, as mentioned before, the adjusting of $T_{dsm}$ and $T_{dsc}$ often conflicts with other requirements. Besides, these strategies cannot work well in situations where the modulation waves have multiple harmonic components, such as active power filters. Because multiple harmonics in modulation waves influence the spectrum of high-frequency ripples in converter side voltage $v_m$, sampling moments should be changed according to harmonics in modulation waves to achieve efficient anti-aliasing. These two disadvantages limit the application range of this strategy.

The strategy based on oversampling and digital filtering fundamentally solve the sampling aliasing problem by improving the sampling frequency to several times of switching frequency. This kind of anti-aliasing strategy is very effective. But oversampling and high-frequency calculation for real-time control means this strategy requires more calculation power, limiting its application in cost-sensitive situations such as household power converters.

Compared with the above two kinds of strategies, the anti-aliasing strategy based on analog pre-filters has advantages of low cost and high robustness. So this kind of strategy is mainly discussed in this paper.

Low-pass filters are usually adopted for anti-aliasing. The Butterworth type second order filter [19] for example. The transfer function of it is shown in (6) and the Bode plot is shown in FIGURE 9.

$$H_{LF}(s) = \frac{a_0^2}{s^2 + \sqrt{2}a_0\omega_s + a_0^2} \quad (6)$$

According to FIGURE 9, Butterworth low-pass filter has high attenuation of -40dB/dec in stop band, which can attenuate high frequency ripples effectively. To achieve efficient anti-aliasing and guarantee THD of $i_g$ lower than 4%, this paper chooses $\omega_s=0.4\omega_i$ for ripple attenuation. Setting $\omega_i=0.4\omega_s$, the relationship between low-order harmonics' total RMS value of the grid-side current and $T_{dsm}$ & $T_{dsc}$ is calculated according to (2)–(5). The result of calculation is shown in FIGURE 10.

Comparison of FIGURE 10 and FIGURE 8 implies that amplitudes of low order harmonics are much lower when low-pass anti-aliasing filter is adopted. This means that the low-pass filter is efficient for anti-aliasing.

But from FIGURE 9, it can be found that the low-pass filter has significance phase lag in pass band, -25° at 0.3$\omega_i$, for example. Because the filter is in the current control loop shown as $H_i(s)$ in FIGURE 2, the pass-band phase lag may reduce stability margin of the current loop. This is detailed in FIGURE 11.

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below. 

Grid-connected converters with inverter side current feedback control and capacitor current feedback active damping are researched in this paper, whose parameters are given in Table 1. According to FIGURE 2, the return ratio of the current control loop can be obtained, as shown in (7). 

\[
T(s) = \frac{K e^{-j(1.5)T_{s}T_i}(s)}{L_{m}i_{g}L_{q}s^{2} - K_{L}sL_{m}i_{g}e^{-j(1.5)T_{s}T_{th}} + Z_{ad}(s)(L_{m} + L_{q})s} 
\]

(7)

In (7), the anti-aliasing filter \(H_{i}(s)\) appears in the numerator and the second term of the denominator. As a result, the adoption of the traditional low-pass anti-aliasing filter influences the stability of the converter control system in two ways. The \(H_{i}(s)\) in the numerator results in phase lag and reduces phase margin at the gain cross frequency of the current loop’s return ratio. On the other hand, the \(H_{i}(s)\) in the denominator influences active damping control that reduces gain margin at the passive filter’s resonance frequency.

To verify the above analysis, frequency characteristics of current loop return ratios \(T(s)\) with/without traditional second-order Butterworth low-pass anti-aliasing filters, at the premise of \(T_{dsm}=T_{dsc}=0\), are shown in FIGURE 11.

According to FIGURE 11, when the traditional low pass filter \(H_{i}(s)\) is adopted, the return ratio \(T(s)\) has the lower phase margin at the gain cross frequency (about 1kHz in FIGURE 11). And at the resonance frequency of the passive filter (about 3kHz in FIGURE 11), a gain-bulge appears in \(T(s)\), resulting in a poor gain margin of the current control loop. These can correspond to theoretical analysis based on (7).

In summary, if a traditional low-pass filter is adopted for anti-aliasing filtering in the converter control system, the filter will significantly reduce the stability margin of the current control loop to achieve efficient anti-aliasing. So the traditional low-pass anti-aliasing filter needs improving.

B. Principles of the Proposed Anti-aliasing Filter Based on Low-pass and Notch Filters

From the analysis above, the reason for reducing the stability margin is that the traditional low-pass filter introduces significant phase lag on feedback signals in the pass-band. This paper proposes a hybrid analog anti-aliasing filter for improvement, with the transfer function shown in (8).

\[
H_{i}(s) = \frac{\omega_{n}^{2}}{s^{2} + 2\zeta\omega_{n}s + \omega_{n}^{2}} 
\]

The proposed hybrid filter is composed of a traditional second-order Butterworth low-pass filter and a second-order notch filter. The low-pass part’s turning frequency is \(\omega_{n}\) and the damping factor equals 0.707. Whereas the notch part’s notch center is \(\omega_{n}\) and the damping ratio is \(\zeta\). Setting \(\omega_{n}=2\pi*10^{4}\)rad/s, \(\zeta=0.15\), bode plots of the proposed hybrid anti-aliasing filter and traditional low-pass filter with \(\omega_{0}=0.4\omega_{n}=2\pi*4*10^{4}\)rad/s are drawn together for comparison, as shown in FIGURE 12.

With the notch filter’s center frequency set at switching frequency, the proposed hybrid filter can attenuate high-frequency ripples in \(i_{m}\) and \(i_{c}\) sufficiently even if the low-pass part of it is set higher than the traditional low-pass filter. On the other hand, the low-pass part that dominates the pass-band characteristic has a higher turning frequency. So the proposed hybrid filter has a much lower phase lag than the traditional low pass filter in pass-band, which means less impact on the current loop return ratio \(T(s)\) in the pass-band.

Adopting the proposed anti-aliasing filter and setting \(\omega_{n}=2\pi*10^{4}\)rad/s, \(\zeta=0.15\), the relationship between the low-order harmonics’ total RMS value of the grid-side current \(i_{g}\) and the two sampling delays \(T_{dsm}, T_{dsc}\) can be expressed in FIGURE 13.

From FIGURE 13, the THD of the converter’s output current \(i_{g}\) is not greater than 4% theoretically, no matter how the two delays are selected. So the proposed hybrid
filter meets the requirements of anti-aliasing filtering, just like the traditional filter did.

As for the hybrid filter’s influence on the current control loop, FIGURE 14 shows the bode curves of return ratios with/without the hybrid filter.

By comparing FIGURE 11 and FIGURE 14, it can be found that the stability phase margin of the current loop at gain crossing frequency reduces with adopting a hybrid filter. Still, the reduction is significantly smaller than when the traditional low-pass filter is adopted. And the reduction of gain margin at resonance frequency is much smaller too.

Therefore, the proposed hybrid anti-aliasing filter can fully meet the anti-aliasing requirements and ensure sufficient stability margin of the current control loop at the same time.

C. Parameter Designing and Hardware Implementation of Proposed Hybrid Anti-aliasing Filter

As shown in (8), three parameters need designing in the proposed hybrid anti-aliasing filter. They are turning frequency \( \omega_n \) of the low-pass part, center frequency \( \omega_n \) of the notch filter, and damping factor \( \zeta_n \) of the notch filter. The designing procedure is shown below.

First, the center frequency of the notch filter is designed. According to FIGURE 12, the notch filter attenuates high-frequency components near switching frequency, so the turning frequency of the low pass part can be set higher to prevent a large phase lag in the pass-band. Therefore, the center frequency is chosen as \( \omega_n = \omega_i \) in this paper.

Next, the damping factor \( \zeta_n \) of the notch filter is designed. FIGURE 15 shows the frequency characteristic of the notch filter with different damping factors.

According to FIGURE 15, the higher the damping factor is, the wider the effective attenuating frequency range of the notch filter is, but the notch filter introduces more phase lag in the pass-band. Phase lag caused by the notch filter reduces the stability margin of the current control loop just as the traditional low pass filter does. So there is a trade-off when designing damping factor \( \zeta_n \). The design aims to fully suppress high-frequency side-band components around the switching frequency and make the notch’s phase lag in the pass-band as low as possible.

According to FIGURE 5, high-frequency side-band components of \( i_n \) and \( i_c \) around switching frequency mainly concentrate at six frequency points that are \( f_c \pm 2f_0 \), \( f_c \pm 4f_0 \), and \( f_c \pm 6f_0 \). So the notch part of the proposed hybrid filter should attenuate these frequency components effectively. Concerning the demand of 4\% lower THD of \( i_n \) mentioned in III.A, the notch filter's attenuation on the six high-frequency components must be below 0.2.

The transfer function of the notch filter can be simplified around center frequency \( \omega_n \) with assuming \( |\omega-\omega_n|<<\omega_n \). The simplified math expression of the notch filter around \( \omega_n \) is shown as (9).

\[
\frac{s^2 + \omega_n^2}{s^2 + 2\zeta_n \omega_n s + \omega_n^2} \quad \frac{s^2 + \omega_n^2}{s^2 + \omega_n^2 + j2\zeta_n \omega_n \omega} \quad \frac{s^2 + \omega_n^2}{s^2 + \omega_n^2 + j\zeta_n \omega_n \omega} \quad \frac{s^2 + \omega_n^2}{s^2 + \omega_n^2 + j\zeta_n \omega_n \omega}
\]

From (9), the frequency range where the notch filter’s gain is below 0.2 has the width of \( 2\times0.2\zeta_n\omega_n \), marked in...
FIGURE 15. To make this frequency range contain the six main high-frequency components, it should cover the frequency range of 6f0*2=600Hz. So the damping factor is set as \( \zeta_n=0.15 \). If \( \zeta_n \) is set larger, the attenuation range of the notch filter will be wider, but more phase lag will be introduced in the notch filter’s pass-band.

The turning frequency \( \omega_1 \) of the low-pass part of the proposed hybrid filter is designed at last. The lower \( \omega_1 \) is, the more attenuation can be realized for high-frequency components around and over \( 2\omega_1 \), but the more phase lag and attenuation in pass-band of the filter will have. To fully attenuate high-frequency components around and over \( 2\omega_1 \) and make sure low enough phase lag in pass-band, choose switching frequency as the turning frequency of the low-pass filter, that is \( \omega_1=\omega_0 \). From the Bode diagram of the second-order Butterworth low pass filter shown in FIGURE 9, its gain of when \( \omega_1=\omega_0 \) is \([H(j\omega_0)]=0.241\), which is low enough for anti-aliasing filtering.

With the three parameters set as mentioned above, the frequency characteristic of the hybrid filter shown in FIGURE 12 and the anti-aliasing effect shown in FIGURE 13 can be obtained. And FIGURE 14 shows its influence on return ratio \( T(s) \) of the current control loop. So the proposed hybrid filter meets the requirements of anti-aliasing filtering and stability margin protecting at the same time.

Hardware implementation of the proposed hybrid filter is explained below. The transfer function of the hybrid filter is the product of a low-pass part and a notch part, so its hardware has a two-level cascade structure. Both levels are implemented using a set of multiple feedback (MBF) second-order filters, as shown in FIGURE 16.

The relationship between the main parameters of the proposed hybrid filter (\( \omega_1, \omega_n, \zeta_n \)) and hardware circuit parameters of resistors and capacitors are explained in (10).

When carrying out designing work, properly choose values of capacitors \( C_1, C_2 \) and \( C \) first. Then substitute them and \( \omega_1, \omega_n, \zeta_n \) into (10) to calculate resistors’ values. If the obtained resistors’ values from the calculation are negative, choose another set of capacitance values and do the calculation again. Capacitors of chosen values must be the industrial mass production models. As for the op-amp, a general-purpose model with GBW not lower than 1MHz is proper.

After the designing work explained above, hardware implementation of the proposed hybrid anti-aliasing filter is accomplished. The RC parameters chosen by this paper are shown in Table 2.

In the proposed hybrid filter, the notch filter is based on the resonance effect, and the resonance point drifts as parameters of \( R \) and \( C \) drift with the temperature. So the center frequency of the notch filter is not always exact 10kHz in its working process.

However, this doesn’t affect the normal function of the notch filter because of two reasons. First, adopting precise capacitors (±0.3%) and resistors (±0.1%) can promise the resonance point drifting within tens of Hz when the temperature changes. Second, little drifting of the resonance point weakly affects the effective range of the notch filter, as FIGURE 17 shows.

IV. EXPERIMENTAL RESULTS

In this chapter, the experimental results verify the effectivity of the proposed sampling anti-aliasing filter from two aspects.

First, steady-state output current waveforms of the grid-connected converter under conditions of adopting none anti-aliasing filter and proposed hybrid filter are compared. The results confirm that the proposed hybrid filter can achieve a good anti-aliasing effect.
Second, to show the benefit of the proposed anti-aliasing filter for current loop stability, step-response dynamic current waveforms of adopting traditional low pass filter and proposed hybrid filter for anti-aliasing are compared. Current high-frequency oscillation means that the characteristic equation of the current control loop has poles near the image axis. In other words, the stability of the current control loop is poor. On the other hand, if the dynamic process is smooth, it reflects the sufficient stability margin of the current control loop.

The experiment is carried out on a 6kVA three-phase grid-connected prototype, shown in FIGURE 18. Its topology is I-type three-level voltage source converter with 3 phase 4 wire connection, with the control structure for each phase shown in FIGURE 1. Parameters of the LCL-LC passive filter are shown in Table 1. Besides, sampling delays are set as $T_{dsm}=T_{dsc}=0.15T_s=15\mu s$, and parameters of anti-aliasing filters are shown in Table 3. The meaning of each parameter in Table 3 can be found in (6) for the traditional low pass filter and (8) for the proposed hybrid filter.

Steady-state current waveforms under situations of adopting no anti-aliasing filter and proposed hybrid filter verify the sufficient effect of the proposed anti-aliasing filter, shown in FIGURE 19(a)(b).

From the waveforms, the converter’s output current with no anti-aliasing filter has more distortion than the one with the proposed filter. For numerically comparing, the spectrum of converter’s output current $i_g$ is shown in FIGURE 20. The spectrum is obtained by MATLAB, with the waveforms’ data exported from a telescope.

### Table 3: Parameters of Anti-aliasing Filters Used in Experiment

| Parameters of Traditional Low Pass Filter | Turning Frequency $\omega_l$ | $2\pi\times4000$ rad/s |
|-------------------------------------------|-----------------------------|-----------------------|
| Damping Factor $\zeta_l$                   | 0.707                       |

| Parameters of Proposed Hybrid Filter       | Low Pass Part Turning Frequency $\omega_{lp}$ | $2\pi\times10^4$ rad/s |
|-------------------------------------------|---------------------------------------------|-----------------------|
|                                           | Notch Part Center Frequency $\omega_n$     | $2\pi\times10^4$ rad/s |
|                                           | Notch Part Damping Factor $\zeta_n$        | 0.707                 |

![FIGURE 18 Photograph of Grid-connected Converter Experiment Platform](image)

![FIGURE 19 Steady State Current Waveforms with/without Proposed Hybrid Anti-aliasing Filter](image)

![FIGURE 20 Spectrum of Converter Grid Side Current $i_g$ with/without Proposed Hybrid Anti-aliasing Filter](image)
From the above spectrum, low harmonic total harmonic distortion rate (THD) of output currents in the two situations can be obtained. The THD of $i_g$ when the proposed anti-aliasing filter is adopted is 2.23%, whereas the THD of $i_g$ when none anti-aliasing filter is adopted is 10.88%. This significant difference verifies the effectiveness of the proposed hybrid anti-aliasing filter.

Next, the comparison of the traditional low-pass and proposed anti-aliasing filter is carried out using dynamic waveforms. The comparison conclusion shows the advantage of the proposed anti-aliasing filter in protecting the stability margin of the current control loop.

The dynamic waveforms are shown in FIGURE 21. From FIGURE 21(a), when a traditional low-pass anti-aliasing filter is adopted, the adjusting time of $i_g$ is nearly 6ms, and the peak value of the ringing component is nearly 10A in the dynamic process. From FIGURE 21(b), when the proposed filter is adopted, the dynamic adjusting time is 1ms, and the ringing current is lower than 5A. On the other hand, the two situations' low harmonic distortion in steady-state waveforms are nearly the same. This experimental result shows that the proposed hybrid anti-aliasing filter can guarantee more stability margin than the traditional low-pass filter, in the case of achieving the same anti-aliasing filtering effect.

V. CONCLUSION

This paper illustrates the necessity of sampling anti-aliasing in the grid-connected converter's digital control system by analyzing the frequency spectrum of the feedback control signals (converter side current $i_n$ and capacitor current $i_c$). Then through frequency domain analysis, the inherent defect of the traditional low-pass anti-aliasing filter that it reduces the current control loop's stability margin is explained. So a kind of hybrid filter based on the combination of low pass and notch filters is proposed for improvement.

Through theoretical analysis and experiment, the following conclusions can be obtained.

First, feedback control signals ($i_n$ and $i_c$) have the following characteristics in the frequency spectrum. High-frequency components are concentrated around integer multiples of switching frequency, forming side-band peaks. These side-band peaks around each integer multiple of switching frequency are approximately symmetrical about this switching frequency.

Second, the sampling aliasing effect is significantly related to the sampling moment of the digital controller when no anti-aliasing filter is adopted. Because the low-frequency harmonics introduced by sampling aliasing nearly cancel each other out when sampling moment and the PWM carrier wave are in a specific phase relationship. This phenomenon could be taken as a kind of anti-aliasing strategy, but its scope of application is not wide enough and not robust enough.

Third, suppose a traditional low pass filter is adopted as the anti-aliasing filter. In that case, it will result in attenuation and phase lag in the control frequency range, causing reduction of phase margin at gain crossing frequency of current loop return ratio and gain margin at passive filter's resonance frequency. So stability margin of the current control loop is reduced.

Forth, the proposed hybrid anti-aliasing filter composed of low pass and notch filter significantly reduces the influence of the filter on control frequency range signal, compared with traditional low pass filter. So it has less influence on the current loop stability margin.

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