Modeling of non-Newtonian fluid flows in porous textile structures under RTM technologies

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Abstract. In this paper, the investigation is devoted to the application of homogenization methods to the transport theory of non-Newtonian shear-thinning fluids in porous fabric structures. Mathematical model is proposed for describing the local transfer of non-Newtonian shear-thinning fluid over a periodic cell, and the symmetry of the microstructure and periodic boundary conditions are used to significantly reduce the complexity of the problem and the computational burden. Solving local problems allows us to find accurate local distributions of velocities, pressure and viscosity inside a separate pore, and also to evaluate the permeability of the porous medium and the effective viscosity of the fluid when only the geometry of the pore is known. Finally, the numerical results for the local transport of pseudoplastic fluid in porous media are given.

1. Introduction

In the field of rheology, fluids with apparent viscosity reduction under shear strain are very common, and these fluids are often found in polymer and foam solutions; this characteristic exhibited is called shear-thinning or pseudo-plasticity [1,2,14]. There are many constitutive models used to describe the rheological properties of pseudo-plastic fluids, such as the classic empirical power law equation [2], the more complex Carreau model [5,9] or Carreau-Yasuda model, Cross model, etc [1]. In these models, the viscosity of the fluid is related to the shear rate raised to a certain power with $n$, via a coefficient called the flow consistency index. For example, when $n < 1$, the Carreau model describes the non-Newtonian behavior called pseudo-plasticity or shear-thinning in which the fluid viscosity decreases under shear strain. For $n = 1$, Newtonian behavior is restored.

In modern industrial production, some applications need to understand the behavior of pseudo-plastic fluid in permeable porous media. For example, the process of enhancing oil recovery in naturally fractured petroleum reservoirs can use polymer solutions in water injection to increase the amount of oil produced. In the RTM technology [4,6-8], which is widely used to manufacture reinforcing materials, polymer solutions or resins generally used as liquid binders are composed of macro-molecules and exhibit shear-thinning non-Newtonian behavior. Therefore, an accurate description of the flow of non-Newtonian resin in the porous matrix affects the quality and production efficiency of polymer products.

In recent years, the flow of porous media has been studied by several theoretical methods, for example [2,3,6-8,10,14]. Here we employ an asymptotic averaging method, similar to the form described in [12], to evaluate the effect of external loads on the macroscopic flow in the porous structure. In addition, there have been some studies on the permeability of non-Newtonian fluids in porous media based on homogenization methods, focusing on the effect of regular pore-scale structures upon a macroscopic flow.

These local problems obtained by applying the asymptotic homogenization method have some specific characteristics: they are system problems of the integral differential type and include periodic boundary conditions. There have been some studies that have attempted to numerically solve local problems on periodic elements (for example, see [10,12]), but they have not discussed the particularity of local
problems, including integral-differential problems and periodic boundary conditions. The purpose of this work is to study the local filtration problem of pseudo-plastic fluids over a periodic cell.

2. Main assumptions and governing equation

2.1. Main assumptions

A porous medium filled with incompressible pseudoplastic Carreau fluid \((n < 1)\) is considered. It is assumed that the porous medium \(V\) has a periodic structure (Fig. 1) so that periodic cells \(V_i\) (PC) can be separated. In the three-dimensional case, the domains \(V_g\) and \(V_s\) occupied by the liquid and solid phases are connected. Similar and more detailed assumptions are given in the work [10], [12]. In this work, the two-dimensional case is considered, where the pores are longitudinal channels along the coordinate axis, and the domain stops keeping one-connected.

![Figure 1. A scheme of periodic porous structure \(V\) and periodic cell \(V_\xi\).](image)

And the deformation of porous media and the mass forces of pseudoplastic fluid are not considered in this work.

2.2. Governing equation

Under the framework of the above assumption, the movement of pseudoplastic Carreau fluid in porous media is described by the following non-dimensional equations [3, 9, 10]:

\[
v_{ij} = 0
\]

\[
-p_j + \eta \sigma_{ij,j} = 0
\]

\[
\sigma_{ij} = 2\mu D_{ij}
\]

\[
D_{ij} = \frac{1}{\xi} \left( v_{ij} + v_{ji} \right)
\]

\[
\mu = \varepsilon + (1-\varepsilon)\left(1 + Cu^2 I_2^2 \right)^{n-1} , \quad n < 1 , \quad I_2 \left(D_{ij}\right) = \sqrt{2D_{ij} D_{ji}} \]

\[
v_{i} \bigg|_{\Sigma} = 0
\]
where $\varepsilon = 0.001$, $\eta = \frac{p_0 x_0}{\mu_0 v_0}$, $Cu = \frac{\lambda v_0}{x_0}$ - Carreau number, $v_i$ - dimensionless velocity of fluid; $p$ - dimensionless pressure; $\mu$ - dimensionless non-newtonian viscosity and $p_0$, $\rho_0$, $v_0$ are their typical magnitudes.

3. Statement of the local problem

3.1. Basic framework of asymptotic theory

In the framework of the asymptotic averaging method, let $l_0$ be the characteristic size of periodic cell $V_\xi$, and $x_0$ be the characteristic global size of the whole porous medium $V$. we introduce the small parameter $\kappa = l_0/x_0 << 1$.

Let us also introduce local coordinates $\xi = \vec{x}/\kappa$ varying within a periodicity cell $V_\xi$, where $\vec{x} = x/x_0$ are the dimensionless global Cartesian coordinates. Then all main functions $f$ describing the non-newtonian fluid flow in pores can be considered to be quasiperiodic (i.e. depending on $\xi$ and $\vec{x}$) and periodic in $\xi$. Differentiation of the functions is realized by the following rule:

$$\frac{\partial f(x, \xi)}{\partial x_j} \to \frac{\partial f}{\partial \xi_i} + \frac{1}{\kappa} \frac{\partial f}{\partial \xi_i}. \quad (11)$$

Below, the following notation for derivatives with respect to local and global coordinates will be also used:

$$\frac{\partial f}{\partial \xi_i} \equiv f_i, \quad \frac{\partial f}{\partial x_j} \equiv f_j. \quad (12)$$

According to the asymptotic averaging method [12], the solution of problem (1)-(6) can be expressed as an asymptotic expansion of small parameters:

$$v_i = v_i^{(0)}(x, \xi) + \kappa v_i^{(1)}(x, \xi) + \kappa^2 v_i^{(2)}(x, \xi) + \cdots \quad (7)$$

$$p = p^{(0)}(x) + \kappa p^{(1)}(x, \xi) + \kappa^2 p^{(2)}(x, \xi) + \cdots \quad (8)$$

$$\mu = \mu^{(0)}(x, \xi) + \kappa \mu^{(1)}(x, \xi) + \kappa^2 \mu^{(2)}(x, \xi) + \cdots \quad (9)$$

The operation of averaging the functions $\langle f \rangle$ over the area of periodic cell $V_\xi$ for quasiperiodic functions $f$ is introduced:

$$\langle f \rangle = \frac{1}{\varphi_p} \int_{V_\xi} f dV \quad (10)$$

where $\varphi_p = \int_{V_\xi} dV$ - volume fraction of fluid in the periodic cell (porosity), and $|V_\xi|$ - volume of periodic cell $V_\xi$. 
3.2. Statement of the local problem

After substituting the asymptotic expansion (7)-(9) into the original problem (1)-(6), local problems of incompressible non-Newtonian pseudoplastic fluids in a composite periodic porous multichannel structures are obtained:

\[ v_{ij}^{(0)} = 0 \]
\[ -p_{ji}^{(1)} + \eta_0 (2\mu^{(0)}D_{ij}^{(0)}) = p_j^{(0)} \]
\[ D_{ij}^{(0)} = \frac{1}{2} (v_{ij}^{(0)} + v_{ji}^{(0)}) \]
\[ \eta^{(0)} = \varepsilon + (1 - \varepsilon)(1 + Cu^2)Z^{(0)}\frac{\varepsilon-1}{2}, \ n < 1, \ Z^{(0)} = 2D_{ij}^{(0)}D_{ji}^{(0)} \]
\[ \|v^{(0)}\|_{E_{ij}} = 0, \ \|p^{(0)}\| = 0 \]
\[ v^{(0)}_{ij} = 0, \ \{p^{(0)}\} = 0 \]

where unknown \( v_{ij}^{(0)} \) and \( p_{ji}^{(0)} \), the symbol \([\ ]\) denotes the periodicity conditions. \( p_j^{(0)} \) is considered as “input data”. And \( \eta = \kappa^2 \eta_0 \).

Due to non-linearity \[11\] of the local problem (11)-(16), its solution can always be written formally as a non-linear function of input data, i.e. of

\[ p^{(i)}(x, \xi) = \sum_{\alpha=1}^{2} q^{(\alpha)}(\xi, p^{(0)}_{\alpha}), \ v^{(0)}_{ij}(x, \xi) = \sum_{\alpha=1}^{2} w^{(\alpha)}(\xi, p^{(0)}_{\alpha}), \ \mu^{(0)}(x, \xi) = \sum_{\alpha=1}^{2} \phi^{(\alpha)}(\xi, p^{(0)}_{\alpha}) \]

where functions \( q^{(\alpha)}, w^{(\alpha)} \) and \( \phi^{(\alpha)} \) depend on coordinate \( \xi \) and \( p^{(0)}_{\alpha} \).

After substituting expression (17) into the local problem (11)-(16), we obtain

\[ w^{(\alpha)}_{ij} = 0 \]
\[ -q^{(\alpha)}_{ji} + \eta_0 (2\phi^{(\alpha)}d^{(\alpha)}_{ij}) = p^{(0)}_{\alpha} \]
\[ d^{(\alpha)}_{ij} = \frac{1}{2} (v^{(\alpha)}_{ij} + w^{(\alpha)}_{ji}) \]
\[ \phi^{(\alpha)} = \varepsilon + (1 - \varepsilon)(1 + Cu^2)Z^{(\alpha)}\frac{\varepsilon-1}{2}, \ n < 1, \ Z^{(\alpha)} = 2d^{(\alpha)}d^{(\alpha)}_{ji} \]

According to the symmetric or antisymmetric extension of functions \( q^{(\alpha)}, w^{(\alpha)} \) and \( \phi^{(\alpha)} \), we can write the boundary conditions on the boundary planes of the 1/4 periodic cell, satisfying the conditions of periodicity of the system (11)-(16):
\[
\xi_j = 0 \text{ or } \xi_j = \frac{1}{2} \left\{ \begin{array}{l}
\tilde{\gamma}^{(\alpha)}(\delta_{ij} + \delta_{ji} - 2\delta_{ij}\delta_{ji}) + \frac{\partial \tilde{\gamma}^{(\alpha)}}{\partial \xi_j} \left[ 1 - (\delta_{ij} + \delta_{ji} - 2\delta_{ij}\delta_{ji}) \right] = 0 \\
\tilde{p}^{(\alpha)}\delta_{ij} + \frac{\partial \tilde{p}^{(\alpha)}}{\partial \xi_j} (1 - \delta_{ij}) = 0 \\
\end{array} \right. \\
, \alpha, i, j = \overline{1, 2} 
\]

(22)

Figure 2. Symmetric and antisymmetric extensions of the solution for the problem (8) – (12).

4. Macroscopic characteristics of filtration process

After the local transport parameters \( q^{(\alpha)}, w^{(\alpha)}_i \) and \( \phi^{(\alpha)} \) in \( V_\beta \) have been determined, the corresponding function can be found in the whole periodic cell. Substitute the functions into relationships (17) and average the obtained expressions over the periodic cell. Then we find:

\[
\left\langle v_i^{(0)} \right\rangle = \frac{K_i^{(\alpha)}(I_1^p, I_2^p, I_3^p)}{\eta_0} p^{(0)}_{\alpha} 
\]

where \( I_i^p = p^{(0)}_{\alpha} \) -scalar functions of pressure gradient invariants, and \( K_i^{(\alpha)}(I_1^p, I_2^p, I_3^p) = \left\langle w_i^{(\alpha)} \right\rangle \).

Similarly, we can also get the effective viscosity of non-Newtonian fluid filtration process:

\[
\left\langle \mu \right\rangle = \mu^{(\alpha)}(I_1^p, I_2^p, I_3^p) = \mu^{(\alpha)} p^{(0)}_{\alpha} 
\]
5. Numerical solution of the local problem

Next, the finite element method [13] will be used to solve local problems of non-Newtonian viscous fluid filtration processes (18)-(22). Here, the chemical industry raw material benzene is taken as an example for numerical study, so \( \eta_0 = 0.0652 \).

Next, in the orthogonal structure, the results of calculating the local problem \( L^{(1)} \) for pseudoplastic fluids (when \( n = 0.25 \)) at \( \rho^{(0)}_{\alpha x} = (1,0)^T \), showing the distributions of functions \( w_1^{(1)} \), \( w_2^{(1)} \), \( q^{(1)} \) and \( \mu^{(1)} \) components in 1/4 the periodic cells of composite periodic porous multichannel structures, respectively, follow.

### Figures

- **Figure 3.** Distribution of \( w_1^{(1)} \).
- **Figure 4.** Distribution of \( w_2^{(1)} \).
- **Figure 5.** Distribution of \( q^{(1)} \).
- **Figure 6.** Distribution of \( \mu^{(1)} \).

6. Conclusions

The asymptotic averaging method is applied to the numerical study for the filtration process of incompressible non-Newtonian shear-thinning fluids in porous fabric structures. The mathematical problem used to describe the local transport of the non-Newtonian pseudoplastic fluid flowing on the periodic cell of the porous structure is obtained. And based on the results of local problems, the macro characteristics of filtering are analyzed. The numerical results highlight the differences between non-Newtonian viscosity fluids and Newtonian fluids during filtration.
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