Hadron-Gluon Interactions and the Pomeron

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Abstract

An interaction term is added to the QCD Lagrangian involving hadron and gluon fields. An effective vertex is calculated for such interactions through exchanges of reggeized gluons. This gives rise to an effective coupling for hadron-gluon elastic scattering in the t-channel, which is used in an inclusive hadron-hadron interaction from which the Pomeron intercept $\alpha(0)_P$ is calculated.

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I. INTRODUCTION

The QCD Pomeron and its Regge trajectory remains a formidable challenge in particle physics to this day. Perturbative QCD has led to the BFKL equation [1] for which solutions have been constructed to obtain Regge behavior in particular the intercept of the Pomeron trajectory $\alpha(0)_P$. With this approach the scattering is done at the quark level requiring that a minimum of two gluons be exchanged to first order, and that all exchanges be done between the same two quarks with a color singlet projected from the $S$ matrix in each order. One may refer to this treatment of a singlet exchange as a ‘hard’ Pomeron [7] because the non-perturbative region of QCD where $q < \Lambda_{QCD}$ is left out. In hadron physics this means that one assumes the separation of the scattering quarks to be much smaller than hadronic lengths. Realistically, the dynamics of hadron scattering is such that IR effects play a crucial role in predicting the ‘soft’ Pomeron [7] behavior for which an observed [6] trajectory gives $\alpha_P(0)$ at approximately 1.08. In a sense the Pomeron is an admission that perturbation has its limits in describing the compositeness of hadrons from constituent quarks, and it remains a challenge to formulate the interplay between the two. A more recent approach [3] has emerged in which it has been shown that QCD in the IR region is a diffeomorphism like interaction in which two gluon fields when contracted with their color indices make up a tensor $G_{\mu\nu} = \delta^{ab}A^a_{\mu}A^b_{\nu}$. This tensor has properties similar to that of a metric tensor in gravity, which suggests new interaction terms to be included in Lagrangians involving both gluons and hadron matter fields. This approach assumes no information on the scattering mechanism of the quarks inside the hadron, but ensures that the hadron remains color neutral.

In the following, ideas from both approaches mentioned above will be incorporated to treat the problem using perturbation (in which case $q > \Lambda_{QCD}$) at the hadron level by introducing a term involving two gluon fields with derivatives, coupled to hadron fields. Although a BFKL equation will not be formulated at this stage, we will consider reggeized gluon exchanges which will ultimately lead to Regge behavior of the $S$ matrix. By introducing such interactions the model is an effective theory which circumvents the problem of specifying which quarks participate in the interactions, and thus there is no need to assume that scattering is done only between the same two quarks. The only stringent constraint is that the color neutrality of the hadron is preserved in the scattering process.

Our starting point is with the following Lagrangian density:

$$\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_{HG}$$

where the first term on the right denotes the full QCD Lagrangian, while the second term denotes an interaction term given by:

$$\mathcal{L}_{HG} = \frac{\lambda}{2} \phi \delta^{ab} (\partial^\mu A^{\mu a} - \partial^\nu A^{\mu a}) (\partial_\mu A^b_\nu - \partial_\nu A^b_\mu)$$

In Eq. (2)[2] $\phi$ denotes a hadron matter field, $\lambda$ is a coupling with dimension $M^{-D}$ (where for example a scalar meson has $D = 2$ while a spin $\frac{1}{2}$ baryon has $D = 3$), $A^{\mu a}$ is a gluon field.

1Greek letter indices will be used to denote space-time, while small case Latin letter indices will
and $\delta^{ab}$ is the $SU(N)$ color metric. Since our ultimate goal is to investigate the Pomeron behavior, the interaction term given above is the minimal interaction of gluons coupled to a color neutral field which posses a Ward identity. This will ensure current conservation, and the neutrality of the hadron in the scattering process. Since the coupling $\lambda$ of this interaction has a negative mass dimension, one should expect that at high energy $M_D^\lambda$ will decay. In the language of renormalization group \[11\] $\mathcal{L}_{HG}$ consists of irrelevant operators which die away at relatively high energy. Since we are dealing with hadron interactions for which QCD is the underlying theory then we should follow the energy scale defined by this theory which is $\Lambda_{QCD}$. Thus for perturbation to be useful we require that $\Lambda_{QCD}^\lambda$ is small.

II. HADRON-GLUON VERTEX

The Feynman rule for the hadron-gluon interaction follows from Eq. (2):

$$V^{\mu\nu ab} = i\lambda \delta^{ab} \triangle^{\mu\nu}$$

(3)

where

$$\triangle^{\mu\nu} = g^{\mu\nu} k \cdot k' - k'_{\nu} k^\mu$$

(4)

$V^{\mu\nu ab}$ is the vertex shown in figure (1). For simplicity the hadrons in (2) are chosen to be scalars, though this could be extended to any hadron of any spin without changing the tensorial structure of the vertex. As was mentioned above one can see that this vertex obeys a Ward identity; $k_{\mu} V^{\mu\nu ab} = 0$. This is crucial for ensuring the color neutrality of the hadron.

Our next step is to evaluate corrections to this vertex in the regge limit ($t_s \ll 1$) using perturbation theory where in an s-channel process of two gluons annihilating to two hadrons, a reggeized QCD octet exchange \[2\] between the two incoming gluons takes place (figure 2). This octet exchange denoted by $F$ is given by \[7\]:

$$F = PT^a \otimes T^a 8\pi \alpha_s k^2 \left( \frac{s}{k^2} \right) \alpha_G(t) \frac{1 - e^{i\pi \alpha_G(t)}}{2},$$

(5)

where $P$ denotes the contracted on-shell gluon polarization vectors, $T^a$ are the generators of the color group in the adjoint representation, and $\alpha_G(t)$ is the regge trajectory of the gluon. Expanding the second factor in powers of $\alpha_G(t)$, the zeroth order reggeized amplitude is real, and is given by an exchange of a reggeized gluon \[2,5,4\] in the t-channel between the two incoming gluons (see figure 3, the vertical line indicates that this is a reggeized gluon). This amounts to replacing the gluon propagator in the t-channel (Feynman gauge) with:

$$-ig^{\mu\nu}_{q^2} \Rightarrow ig^{\mu\nu}_{q^2} \left( \frac{s}{k^2} \right) \epsilon(q),$$

(6)

where

be used to denote color of the gauge group. Also indices pertaining to symmetries such as flavor have been suppressed.
\[ \epsilon(q) = -\frac{N}{4\pi^2} \alpha_s \int \frac{d^2k}{k^2(k-q)^2}, \]  
\[ (7) \]
and \( \mathbf{k} \) is a typical momentum scale at which gluons reggeize with the following condition:
\[ \sqrt{s} \gg |\mathbf{k}| > \Lambda_{QCD}. \]

We proceed to evaluate the first order amplitude using s-channel unitarity which give rise to the Cutkosky cutting rules [12]. According to these rules the imaginary part of an amplitude is given by:
\[ \text{Im}(A_{ab}) = \frac{1}{2} (2\pi)^4 \delta \left( \sum_a p_a - \sum_b p_b \right) \sum_c A_{ac} A_{cb}^\dagger \]  
\[ (8) \]
where the summations in the brackets are over in and out states momenta respectively, and the third summation is over intermediate states denoted by a cut in the diagrams where those states appear. To obtain the real part of the amplitude one can utilize the analytical properties of the S matrix [7] in which it can be shown that if in an s-channel cut the imaginary part of the amplitude to \( n \)th order is given by \( A(ln s)^n \) in leading logs, then by using the identity \( \ln(-s) = \ln(s) - i\pi \) the real part of the amplitude is given by \( -\frac{A}{\pi(n+1)}(\ln s)^{n+1} \).

For the s-channel process at hand the zeroth order amplitude (figure 1) is purely real. The imaginary part of the amplitude of the first order correction is given according the cutting rules (figure 4):
\[ \text{Im}(A_{HG})_1 = \frac{1}{2} \sum_G \int \frac{d^4u}{(2\pi)^3} \frac{d^4u'}{(2\pi)^3} \delta(u^2) \delta(u'^2) (2\pi)^4 \delta^4(k+k'-u+u') A_{GG}^0 \delta^4(-s\beta-q^2) \delta(s\alpha-q^2), \]  
\[ (9) \]
where \((A_{GG})_0\) is the amplitude left of the cut, and \((A_{GH})_0\) is the amplitude right of the cut, and the sum is over intermediate gluon polarizations. In the limit where the t-channel gluon with momentum \( q \) is mostly transverse and small compared to the center of mass energy, it is convenient to introduce Sudakov parameters and write \( q^\mu \) as:
\[ q^\mu = \alpha k_1^\mu + \beta k_2^\mu + q_\perp \]  
\[ (10) \]
with \( \alpha \), and \( \beta \) being much less than unity. Thus to a good approximation it follows that:
\[ q^2 = -q^2. \]

With these simplifications the phase space of the integral in Eq. (9) becomes:
\[ \int \frac{d^4u}{(2\pi)^3} \frac{d^4u'}{(2\pi)^3} \delta(u^2) \delta(u'^2) (2\pi)^4 \delta^4(k+k'-u+u') = \frac{s}{8\pi^2} \int d\alpha d\beta d^2q \delta(-s\beta-q^2) \delta(s\alpha-q^2), \]  
\[ (11) \]
where one delta function has been absorbed, and second order terms in $\alpha$, and $\beta$ have been dropped. The two remaining delta functions in the integrand above give the condition that $\alpha = -\beta = \frac{q^2}{s}$. Keeping in mind that $q$ is small the tree level amplitudes (figure 4) is given by:

$$ (A_{GG})_0 = 8\pi s \alpha_s g^{\mu A} g^{\nu B} \varepsilon_{\mu}^{+} \varepsilon_{\nu}^{-} d^{b} d^{e} d^{c} d^{d} $$

$$ \times f^{abc} f^{ade} \left( \frac{1}{q^2} \right) \left( \frac{s}{k^2} \right)^{\epsilon(q)} $$

$$ (A_{HG})_0 = \lambda \delta^{\lambda e} \Delta^{\nu \mu} \varepsilon_{\lambda}^{+} \varepsilon_{\nu}^{+} \varepsilon_{\mu}^{-} \varepsilon_{\nu}^{-} $$

with $f^{abc}$ being the structure constants of the color group. Putting this together with Eq. (11) into Eq. (9), and summing over gluon polarizations we get the following expression for the imaginary part of the first order correction:

$$ \text{Im}(A_{HG})_1 = 2N \alpha_s \frac{1}{4\pi} \frac{\lambda \delta^{\lambda e} \Delta^{\nu \mu} \varepsilon_{\lambda}^{+} \varepsilon_{\nu}^{+} \varepsilon_{\mu}^{-} \varepsilon_{\nu}^{-}}{q^2} \epsilon(q) $$

The exponent in the integral of Eq. (14) is given by Eq. (7). The integral in the latter is infra-red divergent, but can be regularized [7] to give:

$$ \epsilon(q) = -\frac{N}{2\pi} \left( \frac{1}{\epsilon - \gamma_E + \ln 4\pi + \ln q} \right) $$

The dependence of $\epsilon(q)$ on regularization parameters can be eliminated through scaling the expression above by $\Lambda_{QCD}$. This divergence appears at low energies where gluons are confined, and QCD becomes non-perturbative. This region can be avoided by defining $\epsilon(q)$ to exist only where $q > \Lambda_{QCD}$. Thus, we redefine $\epsilon(q)$ as:

$$ \epsilon(q) \rightarrow \epsilon(q) - \epsilon(\Lambda_{QCD}) = -\frac{N}{2\pi} \alpha_s \ln \left( \frac{q}{\Lambda_{QCD}} \right). $$

Including the running $\alpha_s$ [1] in the expression for $\epsilon(q)$ Eq. (19) we get

$$ \epsilon(q) = -\frac{N}{b_0}, $$

where $b_0$ is the familiar constant given by

$$ b_0 = 11 - \frac{2}{3} n_f, $$

The running of $\alpha_s$ can be included on the condition that the integration over $q$ in Eq. (14) be limited to the perturbative region of QCD; namely from $\Lambda_{QCD}$ and up. One cannot include the running in Eq. (8) since there $q$ can take on any values including those not included in perturbative QCD, and therefore would make $\alpha_s$ diverge which would spoil the reggeization of the gluon.
and $n_f$ is the number of flavors. The $q$ dependence of $\epsilon$ has been eliminated, and Eq. (14) reads:

$$\text{Im}(A_{HG})_1 = -\pi \epsilon M(k_1, k_2) \left( \frac{s}{k^2} \right)^\epsilon \int dq \frac{2\lambda}{q \ln \left( \frac{\Lambda_{QCD}}{q} \right)} \left( \frac{s}{k^2} \right)^\epsilon \lambda_{QCD}.$$  

(18)

$M(k_1, k_2)$ is the tensorial product of the vertex with gluon polarization vectors times a color factor, and $\lambda_{QCD}$ is the corrected coupling of the original vertex due to the running of $\alpha_s$. Since we are in the perturbative region of QCD away from $\Lambda_{QCD}$, the integral above is well behaved as long as its lower limit does not include this point. Since we also anticipate that in the perturbative region of QCD $\lambda_{QCD}$ will be small compared to the momenta of the gluons involved and fairly constant, we impose the condition:

$$\lambda = \lambda_{QCD},$$  

(19)

where now $\lambda$ is assumed to be constant. This is just a normalization condition on $\lambda_{QCD}$ which says that QCD corrections at high energies don’t contribute much to the running of $\lambda$ due to its negative mass dimension. According to Eq. (19) it follows that:

$$\int_{\xi_o}^{\sqrt{s}} \frac{dq}{q \ln \left( \frac{\Lambda_{QCD}}{q} \right)} = \frac{1}{2},$$  

(20)

where the lower limit is given by a new scale $\xi_o$ and the upper limit is bounded by $\sqrt{s}$ since this is the maximum momenta of the reggeized gluons. The integral in Eq. (20) can be integrated, and the value of $\xi_o$ can be obtained by the following:

$$\xi_o = \Lambda_{QCD} \left( \frac{\sqrt{s}}{\Lambda_{QCD}} \right)^{\frac{1}{2s}}.$$  

(21)

As expected $\xi_o$ is well above $\Lambda_{QCD}$, meaning we are still in the perturbative region of QCD.

With this the real part of the amplitude $A_{HG}$ is given by:

$$\text{Re}(A_{HG}) = M(k_1, k_2) \lambda \left( -1 + \left( \frac{s}{k^2} \right)^\epsilon \right).$$  

(22)

Adding this to the zeroth term, the amplitude to first order correction is given by

$$A_{HG_{s-ch''0+1}} = M(k_1, k_2) \lambda \left( \frac{s}{k^2} \right)^\epsilon (1 - i\pi \epsilon).$$  

(23)

The second factor on the right of the equation above looks like it contains the first two terms in a series of a multi-regge exchange, suggesting that the entire amplitude in the $s$-channel takes the form:

$$A_{HG_{s-channel}} = M(k_1, k_2) \lambda \left( -\frac{s}{k^2} \right)^\epsilon.$$  

(24)
To verify this we use crossing symmetry, and note that in the t-channel where the process is the elastics scattering of a hadron by a gluon (figure 3) in which \( t \) is negative, there are no cuts to be made in a multi-regge exchange between the in-going and out-going gluons, and the amplitude is purely real. Thus according to Eq. (23) this amplitude up to first order in \( \epsilon \) is given by:

\[
A_{HG_{t-ch'0+1}} = M(k_1, k_2) \lambda \left( \frac{|t|}{k^2} \right)^\epsilon (1 - i\pi \epsilon)
\]

However, as mentioned before the t-channel amplitude is real to all orders in \( \epsilon \) suggesting that Eq. (24) is the entire multi-regge exchange amplitude for the process initially considered.

In fact we may replace the vertex (3) with the following:

\[
V^{\mu\nu ab} = i\lambda \delta^{ab} \triangle^{\mu\nu} \left( -\frac{Q^2}{k^2} \right)^\epsilon,
\]

where \( Q \) now is a measure of the gluons four momenta. Such corrections to the vertex are considered to be structure functions of the hadron which become significant as the magnitude of \( Q \) decreases due to \( \epsilon \) being negative.

### III. INCLUSIVE INTERACTIONS AND THE POMERON

Using the results obtained so far we now turn to treat the problem of hadron-hadron scattering. Our goal is to obtain the regge intercept for such a process by calculating the amplitude to second order in \( \lambda \) together with the effective vertices found in the previous section.

We consider an inclusive hadronic interaction by which two hadrons with a center of mass energy \( \sqrt{s} \) scatter through an exchange of a gluon in the t-channel, plus two outgoing gluons which ultimately give rise to jets (figure 5). Thus the interaction is of the type:

\[
h_1 + h_2 \rightarrow h'_1 + h'_2 + 2g,
\]

where \( h, h' \) are the initial and final hadronic states respectively (these could include resonances), and \( g \) denotes the out-going gluons which would ultimately give rise to the emission of two jets. Thus the process could very well describe the initial stages of hadron diffraction dissociation in which the scattering hadrons have a large rapidity gap.

Total momentum conservation of this process gives the following:

\[
p_1 + p_2 = p'_1 + p'_2 + k_1 + k_3,
\]

where \( p_1, p_2 \) and \( p'_1, p'_2 \) are the momenta of the in-coming and out-going hadrons respectively, and \( k_1, k_3 \) are the momenta of the out-going gluons. Conserving momenta at each vertex (figure 5) gives the following:

\[
p_1 - k_2 = k_1 + p'_2
\]

\[
p_2 + k_2 = p'_2 + k_3,
\]
where \( k_2 \) is the momentum of the gluon exchanged in the t-channel which is assumed to be soft, and mostly transverse as prescribed by the regge limit. Therefore we may assume that its \( t \) and \( z \)-components are negligible, so that to a good approximation we have:

\[
\begin{align*}
  k^\mu &\rightarrow k_\perp^\mu, \\
  \frac{k_2^2}{s} &\ll 1.
\end{align*}
\]  

(30a) 

(30b)

Working in the center of mass frame (assuming again that our hadrons are scalars) in the massless limit where we have \( p_1 = (\sqrt{s}/2, \sqrt{s}/2, 0, 0), p_2 = (\sqrt{s}/2, -\sqrt{s}/2, 0, 0) \) it is again convenient to introduce the **Sudakov parameters** and write the following vectors as:

\[
  k_i^\mu = \alpha_i p_1^\mu + \beta_i p_2^\mu + k_i^\perp,
\]

(31)

where \( \alpha_2 = \beta_2 = 0 \).

In specifying the kinematic regime for the process at hand, we note that since the reggeon exchanged (in this case it is a gluon field) between the two hadrons is soft relative to the incoming energy, the process is plagued with non-perturbative effects (the exchange is done at long distances). This requires the knowledge of parton distribution functions in this region for which at this time we don’t have a way of computing from first principles. Never the less, for perturbative purposes we will assume that \( k_2 \gg m_h \approx \Lambda_{\text{QCD}} \) which ensures that

\[
  \frac{1}{k_2} \approx \Delta_{\text{int}} \ll \Delta_{\text{hadron}} \approx \frac{1}{m_{\text{hadron}}}.
\]

In doing so we adopt the scheme [4] that a hadron is a composite of fields that have a life time much greater than their interaction time; such that the interaction described (figure 5) is just the first step in hadron diffraction dissociation where an off-shell gluon field probes the hadron in a way that initially leaves it in tact, though eventually will lead to it’s break up. Since gluons emitted in the first stages of the brake up will eventually hadronize, there is no real way to distinguish them from the out-going hadrons in terms of their momenta after the break up has occurred. Since an inclusion of these gluons is a must in order to keep the color neutrality of the hadron, we could very well consider them as a partons carrying some fraction of the in-coming energy. We assume this fraction is small such that:

\[
  1 \gg \alpha_1 > \alpha_3 \gg \frac{k_2^2}{s},
\]

(32a)

\[
  1 \gg \beta_3 > \beta_1 \gg \frac{k_2^2}{s},
\]

(32b)

The orderings above conform to the requirement that initially the hadrons momenta are not altered much (in these first stages the hadrons are still intact), meaning there is a high rapidity gap in this stage, which enforces the strong ordering on the left of (32). Now since the out-going gluons are on shell we have the condition \( s\alpha_1\beta_1 = k_1^2 \) (with a similar one for the second on-shell gluon with momentum \( k_3 \)). Due to this and (29) we see that \( k_{1\perp}, k_{3\perp} \approx k_{2\perp} \), and thus at most \( \alpha_i\beta_i \approx \frac{k_i}{\sqrt{s}} \) which justifies the strong ordering on the right of (32).

Having specified the kinematics, Eq. (27) can be used to evaluate the zeroth order amplitude of hadron-hadron scattering with two out-going gluons to give:
\[ A_{(HGG)} = \delta^{ab} \lambda' \triangle^{\mu \lambda} \left( \frac{-q_1^2 k^2}{k_1^2} \right) \left( \frac{g_{\lambda \sigma}}{k_1^2} \right) \times \triangle^{\sigma \nu} \left( \frac{-q_2^2}{k^2} \right) \varepsilon^a \varepsilon^b \]

Since this is a tree level amplitude which is real meaning there are no cuts to be made at all on the diagram, we have the momenta \( q_1^2 = -(k_2 + k_1)^2 \), and \( q_2^2 = -(k_2 - k_3)^2 \) that are obtained from each vertex in figure (3) respectively. Using the on-shell condition for both \( p_1' \), and \( p_2' \) in the massless limit we get:

\[
\begin{align*}
(-q_1^2)^\epsilon &= s \epsilon_3^\beta_1 \\
(-q_2^2)^\epsilon &= s \epsilon_3^\beta_1.
\end{align*}
\]

The tensorial product in Eq. (33) is given by:

\[
g_{\lambda \sigma} \triangle^{\mu \lambda} \triangle^{\sigma \nu} = \left( g^{\mu \nu} (k_1 \cdot k_2) (k_2 \cdot k_3) - (k_1 \cdot k_2) k_2^\mu k_3^\nu - (k_2 \cdot k_3) k_2^\mu k_1^\nu + (k_1 \cdot k_3) k_2^\mu k_2^\nu \right).
\]

Again by use of (34), and (32) in the massless limit, keeping only second order terms in Sudakov parameters the expression above is reduced to:

\[ g_{\lambda \sigma} \triangle^{\mu \lambda} \triangle^{\sigma \nu} = -\frac{g^{\mu \nu}}{4} \alpha_3 \beta_1 s^2 \]

From the discussion at the end of section II the correction to the hadron-gluon vertex given by the factor in Eq. (27) can be associated with hadron structure functions due to reggeized gluon exchanges in the perturbative region of QCD. In light of this and Eq. (34) it is useful to define the normalized longitudinal gluon distribution functions in terms the Sudakov parameters given by:

\[ f_i = \psi_i^\dagger (\alpha_i) \psi_i (\alpha_i) = (1 + \epsilon) \alpha_i \]

where \( \psi(\alpha) \) is the amplitude of a gluon with a fractional longitudinal momentum \( \alpha \). Thus the probability of finding a gluon with a longitudinal momentum between \( \alpha \), and \( d\alpha \) is given by

\[ dP = f_i d\alpha_i. \]

Using (34), (35), and (37) the amplitude (33) may be written as:

\[ A_{HHG} = \varepsilon_a \varepsilon_b \left[ < \alpha_3 > < \beta_1 > > \frac{g^2 g^{\mu \nu}}{k_2^2} \left( \frac{s}{k^2} \right)^{2+2\epsilon} \right], \]

where \( < \alpha_3 >, < \beta_1 > \) are the expectation values of the gluons longitudinal fractional momenta \( k_3 \) and \( k_1 \) respectively, and \( g^2 \) is a dimensionless coupling given by \( g^2 = \frac{\lambda' \kappa^4}{4(1+\epsilon s)} \). Thus the amplitude (38) is factored into two parts; the first dealing purely with the out-going gluon momenta and their polarizations, while the second is a reggeized propagator which describes an exchange of a reggeon between two hadrons with a regge trajectory given by:
\[ \alpha(t) = 2 + 2 \epsilon. \quad (39) \]

This trajectory is independent of \( t \) though this should be expected since we have considered only the tree level amplitude in our scheme. We expect that in a completely elastic process where the amplitude (33) is a cut of an imaginary amplitude of the type \( h_1 + h_2 \rightarrow h'_1 + h'_2 \) with both s,t-channel gluons appearing in the intermediate states, higher order corrections will lead to a trajectory with \( t \) dependence. Since the amplitude (38) is color neutral meaning our S-matrix describes transitions between in and out states with no net change in color nor in flavor, we conclude that the trajectory in (39) is nothing more than \( \alpha_P(0) \) which is the regge intercept of the Pomeron. It is well known from regge theory \[10\] that at high energy for a particular reggeon exchange it is \( \alpha_R(0) \) which dominates the interaction, and gives the asymptotic behavior of the total cross section namely:

\[ \sigma_{\text{total}} \sim s^{\alpha_R(0) - 1}. \]

For \( \alpha_R(0) > 1 \) the Pomeranchuk theorem \[8\] predicts regge exchanges of particles carrying quantum numbers of the vacuum (gluons), and a rising cross section. Eq. (39) can be written as:

\[ \alpha(0)_P = 2 + 2 \epsilon = 2 - \frac{2N}{11 - \frac{2}{3} n_f}. \quad (40) \]

Putting \( N = 3 \), and \( n_f = 6 \) in Eq. (40) we get in \( \alpha_p(0) = 1.14 \), giving a rising cross section in accord with the Pomeranchuk theorem. A peculiar feature of this expression is that it does not depend on the strong coupling \( \alpha_s \). We stress that this comes about because the scale chosen in Eq. (16) was precisely the scale at which non-perturbative effects become important namely \( \Lambda_{QCD} \). This gave a cancellation of the coupling up to a constant. Had a different scale \( M \) been chosen which coincides with a particular hadron mass, such a cancellation would not have occurred, and an explicit dependence on the coupling would have appeared in the expression for \( \alpha_P(0) \). For light hadrons the scale \( \Lambda_{QCD} \) is a good approximation for describing their mass scale, and in doing so we have simplified the calculation of \( \alpha_P(0) \).

**IV. CONCLUSIONS**

In the calculation for \( \alpha_P(0) \) above it seems that a significant contributing factor for it’s value comes from knowing the corrected vertices Eq. (27) which provide knowledge on hadron structure functions in the regge region. It is important to stress that these functions are in the region where \( q > \Lambda_{QCD} \) and that complete knowledge of hadron structure functions in the non-perturbative region are still not at hand. In our model a ‘soft’ Pomeron is an exchange of a single gluon in the t-channel with two out-going gluons forming a color singlet in their final state. The term ‘soft’ is used because \( q \) is low (unlike the ‘hard’ Pomeron in which scattering is treated at the parton level between two partons requiring high \( q \)), and also because this model treats the hadron as an effective field in which color singlet exchanges are more likely to happen between different partons since the exchange is a soft one (long distances), and the non locality of the hadron is taken into account through it’s structure functions. The measured value of \( \alpha_P(0) \) is \( 1.08 \) \[4\], so we conclude that the
structure functions in the non-perturbative region play a crucial role in understanding the ‘soft’ Pomeron intercept.

The dependence of $\alpha_P(0)$ (Eq. (40)) on the number of flavors has an interesting implication. As the number of flavors increases $\alpha_P(0)$ drops. In fact increasing the number of flavors to seven would cause it to drop below 1. Having $\alpha < 1$ would clash with Pomeranchuk’s theorem which guarantees a rising cross with $s$ as long as the exchanges are of particles which carry quantum numbers of the vacuum. In other words if we are to uphold the theorem even at very high energies, six flavors seems to be a crossroads in our model, since with the addition of one more flavor some new physics should appear to compensate for the drop in $\alpha$. Whatever this new physics is (Supersymmetry?) it should involve particles that reggeize, and that play a major role in hadron interactions.

Finally the tensor given in (2) of gluons and their derivatives before contraction on its Lorenz indices has the form $R_{\mu\nu\rho\sigma}$. This tensor is antisymmetric on the exchange of $\mu$ and $\nu$, or $\rho$ and $\sigma$, but is symmetric with the exchange of $\mu\nu$ with $\rho\sigma$. These algebraic properties of a tensor describe a particle of spin 2 [13]. Eq. (40) would have given $\alpha = 2$ had we not incorporated loop corrections in our amplitude. This conforms to the model given in [3] which predicts a spin 2 behavior in QCD with no matter fields.

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FIG. 1. Tree level amplitude for hadron-gluon scattering.
FIG. 2. Multi-Regge amplitude between two gluons.

FIG. 3. A reggeized gluon exchange between gluons.
FIG. 4. Imaginary part of the first order amplitude for s-channel gluon-hadron scattering.

FIG. 5. Hadron-Hadron scattering with two out-going gluons.