Influence of basic motion on dynamic balance accuracy

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Abstract. Due to the complex environment of rotating machinery, rotating machinery is inevitably affected by environmental excitation. The basic motion excitation in environmental excitation will affect the dynamic response of the rotor. In order to study the impact of basic motion on the judgment of the rotor imbalance, the establishment of rotor motion is adopted. Differential equations and finite element simulation methods for dynamic equilibrium experiments. The results show that when the frequency of the basic motion excitation is the same as the unbalanced excitation in the system, the basic motion excitation will affect the amplitude and phase of the power frequency, and then affect the judgment of the rotor imbalance amount, resulting in a certain balance error and impact. Dynamic balance effect, but unlike unbalanced excitation, the direction of the force generated by the basic excitation will not change, so when the vertical amplitude and phase of the generated force of the excitation is used as a reference, the impact caused by the basic motion can be avoided.

1. Introduction
Rotating machinery is widely used in power, petrochemical, metallurgical, aerospace and other industries. It is currently developing in the direction of large-scale and high-speed. Due to its complex working conditions and external environment, it is unavoidably affected by environmental incentives [1]. The most common excitation is the excitation of basic motion. With the further development of rotor dynamics theory, more and more people pay attention to the rotor system of basic motion.

Due to the high experimental cost and the randomness of the basic motion, most of the domestic and foreign studies on the impact of the basic motion on the rotor system are based on economical numerical simulation methods. Most foreign scholars have studied rotor systems that perform simple harmonic motion on the foundation. French scholar Dakel has analyzed the steady-state dynamic behavior of a single-disc rotor under the rigid support of a single disc rotor under the coupling of mass imbalance and basic motion. The effect of time on the critical speed of the rotor system and the axis trajectory [2] [3]. Indian scholar Das et al. Used electromagnetic devices to weaken the response under the coupling of basic excitation and rotor unbalanced excitation [4]. French scholar Duchemin has studied the dynamic characteristics of flexible rotors under basic motion excitation and unbalanced excitation [5]. Domestic scholars are more inclined to study the impact of basic motion in the form of impact on rotor motion, especially in the field of aerospace machinery and marine machinery. Wan Qiang et al. Analyzed the dynamic characteristics of a high-pressure rotor of a steam turbine under the fundamental excitation of the rotor in the vertical or vertical direction through the finite element modeling method [6]. Ji Shaojin et al. Used the concentrated mass method to establish the differential equation of motion of the rotor system, and analyzed the corresponding impact of the basic impact on the rotor dynamics [7]. He
Shaohua et al. Analyzed the response of a ship's steam turbine rotor under the impact of an explosion [8].

Previous researches have analyzed the dynamic behavior of the rotor system of basic motion, but basically have not considered the impact on the rotor dynamic response and dynamic balance accuracy when the frequency of the basic motion is the same as that of the rotating machine. Aiming at this problem, this paper takes the Jeffcott rotor model as the research object, theoretically analyzes the response of the rotor when the same frequency of basic motion excitation and unbalanced excitation exist simultaneously, and further uses the finite element software ANSYS to analyze the basic excitation of the resulting errors were simulated.

2. The response of rotor under combined excitation

2.1. Jeffcott rotor model

As shown in Figure 1, a disk of mass \( M \) mounted on a weightless elastic shaft, and the system has a stiffness of \( k \) and a damping of \( c \). This is the most basic model in rotor dynamics. The results obtained from analysis, calculation and simulation based on this model are the most basic, but they are accurate enough for simple rotating machinery [9]. Qualitative analysis is used to illustrate complex problems.

![Jeffcott rotor model](image)

**Figure 1.** Jeffcott rotor model

The differential equations of motion of the rotor in the \( x \) and \( y \) directions are:

\[
\begin{align*}
M \ddot{x} + c \dot{x} + k x &= F_x(t) \\
M \ddot{y} + c \dot{y} + k y &= F_y(t)
\end{align*}
\]

Suppose the response in the \( x \) and \( y \) directions is:

\[
\begin{align*}
x(t) &= B \cos(\omega t - \phi) \\
y(t) &= B \sin(\omega t - \phi)
\end{align*}
\]

\( B \) is the amplitude, \( \phi \) is the phase lag.

Let the natural frequency, frequency ratio, and damping ratio of the rotor system be \( \omega_n, \lambda, \zeta \), respectively, then there are \( \omega_n^2 = k/M, \lambda = \omega / \omega_n, \zeta = c / (2M \omega_n) = c / (2 \sqrt{Mk}) \).

Bring the response into the differential equation of motion of the rotor:

\[
\begin{align*}
B(1-\lambda^2) \cos(\omega t - \phi) - 2\zeta \lambda \sin(\omega t - \phi) &= F_x(t)/(M \omega_n^2) \\
B(1-\lambda^2) \sin(\omega t - \phi) + 2\zeta \lambda \cos(\omega t - \phi) &= F_y(t)/(M \omega_n^2)
\end{align*}
\]
Because the coupling stiffness is not considered, the responses in the x, y directions are similar. The following uses the analytical solution in the x direction as an example to explain. Simplify the rotor model in the x direction as shown below.

![Figure 2. Jeffcott simplified model](image)

The differential equation of rotor motion in the x direction is: \( M\ddot{x} + c\dot{x} + kx = F_x(t) \)

The dimensionless differential equation is:

\[
B(1-\lambda^2)\cos(\omega t - \varphi) - 2\zeta \lambda \sin(\omega t - \varphi) = F_x(t)/(M\omega_x^2)
\]

(4)

### 2.2. Rotor unbalanced excitation

The x-direction motion differential equation is: \( M\ddot{x} + c\dot{x} + kx = me\omega^2 \cos(\omega t - \varphi_m) \)

Set up \( x(t) = B\cos(\omega t - \varphi) \)

Bringing in the dimensionless differential equation in the x direction can be obtained

\[
\begin{align*}
B &= \frac{me}{M} \frac{\lambda^2}{\sqrt{(1-\lambda^2)^2 + (2\zeta\lambda)^2}} \\
\varphi &= \arctan \frac{\frac{1-\lambda^2}{2\zeta\lambda} \cos \varphi_m + \sin \varphi_m}{\cos \varphi_m - \frac{1-\lambda^2}{2\zeta\lambda} \sin \varphi_m}
\end{align*}
\]

(5)

### 2.3. Basic exercise motivation

There is incentive \( y(t) = X_g \cos(\omega t - \varphi_g) \) in the x direction. The differential equation of motion is:

\( M\ddot{x} + c\dot{x} + kx = F_x(t) \)

(6)

\[
F_x(t) = -cX_g \sin(\omega t - \varphi_g) + kX_g \cos(\omega t - \varphi_g)
\]

(7)

Sorting out the above formula can be obtained
Assume \( x(t) = B \cos(\omega t - \phi) \), the dimensionless motion differential equation with x direction can be obtained

\[
\left\{ \begin{array}{l}
F_g(t)/(M\omega_n^2) = a \cos(\omega t + \alpha - \varphi_g) \\
a = X_g \sqrt{1 + (2\zeta \lambda)^2} \\
\alpha = \arctan(2\zeta \lambda)
\end{array} \right.
\]  

(8)

The ratio of the system response amplitude to the fundamental motion amplitude is called the displacement transfer rate [10], and it has:

\[
T_d = \frac{1 + (2\zeta \lambda)^2}{\sqrt{(1 - \lambda^2)^2 + (2\zeta \lambda)^2}}
\]  

(10)

It can be seen from the formula that the displacement transfer rate has nothing to do with the amplitude of the excitation, but only the physical parameters of the rotor system. For a constant speed rotor, it is constant.

2.4. Analytical solution of the x-direction response when two stimuli coexist

The differential equation of motion in the x direction when two excitations exist simultaneously is:

\[
M\ddot{x} + c\dot{x} + kx = F_g(t) + F_m(t)
\]  

(11)

According to the principle of superposition [11], the magnitude and phase solutions of the x-direction response under the two kinds of excitations can be obtained

\[
\left\{ \begin{array}{l}
B = \sqrt{\frac{(a \cos(\varphi_g - \alpha) + \frac{mc}{M} \cos \varphi_m)^2 + (a \sin(\varphi_g - \alpha) + \frac{mc}{M} \sin \varphi_m)^2}{(1 - \lambda^2)^2 + (2\zeta \lambda)^2}} \\
\phi = \arctan \frac{1 - \frac{\lambda^2}{2\zeta \lambda}}{(a \cos(\varphi_g - \alpha) + \frac{mc}{M} \cos \varphi_m) - (a \sin(\varphi_g - \alpha) + \frac{mc}{M} \sin \varphi_m)}
\end{array} \right.
\]  

(12)
Take $\lambda = 0.34$, $\zeta = 0.13$. When there is only unbalanced excitation in the system, set $me/M = 6 \times 10^{-6}$m, $\varphi_n = 0$, $a = 0$ to get the response in the X direction shown in Figure $B \approx 8.03 \times 10^{-6}$m, $\varphi \approx 65.76^\circ$. Keeping the system parameters and imbalance related parameters unchanged, let $a = 5 \times 10^{-6}$m, and get the response in the X direction shown in Figure a, $B \approx 7.61 \times 10^{-6}$m, $\varphi \approx 30.36^\circ$. In both cases, the response of the rotor system in the X direction is a simple resonance with the same frequency and different amplitude and phase.

The frequency of the unbalanced excitation is the same as the power frequency of the rotor system. Generally, the amplitude and phase of the power frequency are used as a reference when performing dynamic balancing. However, when the system has the same basic motion excitation as the power frequency, the power frequency of the system in the X direction is both the amplitude and phase have changed. At this time, if the dynamic balance is still performed with the amplitude and phase of the rotor vibration frequency in the X direction as a reference, a certain error will be caused, and a good balance effect cannot be achieved.

3. Simulated equilibrium experiment

3.1. Rotor theory in finite elements

In order to further study the influence of the basic motion on the rotor dynamic balance results, ANSYS software was used to perform transient analysis to solve its steady-state eddy response. The response was filtered to obtain its power frequency response, and then the simulation balance experiment was performed [12].

The differential equation of motion of the rotor system model established by the finite element method is

![Figure 3. Time domain graph and frequency domain graph](image-url)
Where $[M], [K], [C]$ respectively represent the mass matrix, the damping matrix, the stiffness matrix, the $[G]$ is gyro matrix, the $[Q]$ is excitation force matrix, $\omega$ is the angular velocity of the rotor around the rotation axis, and the solutions of each node can be expressed as:

$$\{u\} = (r_x, r_y, \theta_x, \theta_y)^T$$

Among them $r_x, r_y, \theta_x, \theta_y$ are the displacements in the X and Y directions, respectively.

3.2. Finite element model and equilibrium results

In this simulation experiment, the COMIN214 element is used to simulate the support of the rotor, the BEAM188 beam element is used to simulate the shaft, and the mass disc is simulated using the MASS21 element. The relevant parameters of the MASS21 element are measured by mass disc modeling using 3D modeling software. For the established model, the steady-state response of each node is obtained by solving the equations by analyzing the transient response. The model is shown in the figure. The main setting parameters are shown in Table 1. There are 7 nodes in the rotor system. The nodes are represented by s1, s2 ..., s7. The disk is located at s4 and P is used. The two supports are located at s2, s6 positions.

Add an imbalance of $30g \cdot cm \angle 0^\circ$ to the mass disc. The simulated experimental balance process and results are shown in Tables 2 and 3. The results show that under the model built, there is a horizontal basic motion with a frequency of power frequency. During the excitation, using the amplitude and response of the power frequency vibration in the X direction as a reference will cause a mass error of 16.41% and an angular error of 22.78°, but using the vibration data in the Y direction as a reference can achieve a good balance effect, so when there is a steady basic excitation, the amplitude and phase of the power frequency vibration perpendicular to the excitation direction must be found as a reference direction to achieve a good balance effect.
Table 1. Rotor model and related parameters of transient analysis

| Name                                      | Parameter value                                                                 |
|-------------------------------------------|---------------------------------------------------------------------------------|
| Physical parameters of the shaft          | Shaft length 0.52m, diameter 0.01m, density 7800kg/m³, Young’s modulus 2.1×10¹¹[N/m²], Poisson’s ratio 0.30 |
| Physical parameters of mass disc          | Mass 1.566kg, Moment of inertia about Z axis 1081.87kg×mm²                        |
| Bearing support physical parameters       | Stiffness Kxx=Kyy=1×10⁶ N/mm, Cross stiffness Kxy=Kyx=0 N/m, Damping Cxx=Cyy=1000Ns/mm, Cross damping Cxy=Cyx=0 Ns/m |
| Transient analysis parameters             | Total analysis time 40s, Step size setting 0.001s, Gyro effect: Rotating speed:1500rpm |

Table 2. Equilibrium process data of simulation experiment (speed: 1600r/min)

| Vibration test plane | Unbalanced incentives | Imbalance + basic motion excitation (X direction) | Imbalance + Basic Motion Excitation (Y direction) |
|----------------------|-----------------------|-------------------------------------------------|-------------------------------------------------|
|                      | S3(μ m∠°)             | S3(μ m∠°)                                       | S3(μ m∠°)                                       |
| Original steady state vibration | 6.46 ∠53.85 | 12.10 ∠68.41 | 6.45 ∠143.90 |
| Steady state vibration after weight     | 12.48 ∠68.75 | 18.41 ∠73.79 | 12.50 ∠158.90 |

Note: The test weight in both cases is 10g ∠30°

Table 3. Data of simulation experiment results (speed: 1600r/min)

| Calculation results(g ∠°) | Unbalanced incentives | Imbalance + basic motion excitation (X direction) | Imbalance + Basic Motion Excitation (Y direction) |
|---------------------------|-----------------------|-------------------------------------------------|-------------------------------------------------|
|                          | 10.13 ∠0.08           | 8.3595 ∠22.78                                   | 9.94 ∠0.28                                       |
| Mass error (x100%)       | 1.3%                  | 16.41%                                          | 0.6%                                             |
| Phase error (°)          | 0.08                  | 22.78                                           | 0.28                                             |

4. Conclusion
In this paper, the response of the Jeffcott rotor model under single excitation and combined excitation is solved and the errors of the amplitude and phase solutions of the system when the unbalanced excitation and the two excitations are present are compared. The research results are as follows:

1. When there is a basic excitation with a power frequency in the environment, the amplitude and phase of the power frequency of the rotor system will change. Because the amplitude and phase of the basic excitation are related to the physical parameters of the rotor support, the basic excitation has different effects on rotor systems with different physical parameters.

2. If two kinds of excitations exist at the same time, when the rotor is dynamically balanced, the use of the influence coefficient method will produce a certain error. The magnitude of the error is related to the amplitude and phase of the basic excitation, and also to the direction of vibration testing. Because
the force generated by the basic excitation is a direction-invariant force, if the amplitude and phase of the power frequency vibration in the direction perpendicular to the basic motion excitation direction are used as a reference, the balance result will not be affected by the basic excitation.

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