Field-Theoretic Realization of Heavy Meson as Composite Particle

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We construct a realistic field-theoretic model for the structure of a heavy meson in the heavy quark limit. The model is fully covariant and satisfies heavy quark symmetry. The Isgur-Wise function, the decay constant, and the axial vector coupling constant are studied. This model overcomes the limitations caused by non-covariance of the light-front quark model, and provides an ideal framework to systematically evaluate the heavy quark symmetry breaking effects caused by the $1/m_Q$ correction terms in the heavy quark effective theory of quantum chromodynamics.

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It is well known that in the infinite quark mass limit, quark-gluon interaction becomes flavor and spin independent, leading to considerable simplification in the description of heavy hadron physics. However, even in this hypothetical limit, low energy quark-gluon dynamics is nonperturbative, and the problem of calculating heavy hadron bound states directly from QCD remains unsolved. Consequently, in order to quantitatively study heavy hadron physics, one still has to model the structures of the hadrons phenomenologically. In order to study the transition properties of hadrons at arbitrary momentum transfers, it is essential to have a fully Lorentz covariant model for the bound state structure of hadrons, otherwise one often runs into ambiguities and may obtain incorrect results. Unfortunately, none of the available quark models meets this requirement.

Among the various quark models, the light-front quark model comes close to being truly relativistic. However, the fact that it is not fully covariant severely limits its usefulness. In a previous paper, we studied a covariant light-front model of heavy mesons, which provided a partial solution to the Lorentz covariance problem. However, the approach taken there is not systematic, and light-quark currents are not considered. In this work, we propose to construct a realistic heavy meson bound state in a field theoretic approach, so that covariance is automatically guaranteed. This model satisfies heavy quark symmetry (HQS), and at the same time has the simplicity of the quark model picture. Calculations of hadronic matrix elements in this model reduce to computing Feynman diagrams which are finite. With this model, we can systematically and quantitatively study the HQS breaking effects caused by the $1/m_Q$ correction terms in the heavy quark effective theory (HQET) of quantum chromodynamics (QCD), which is important for a thorough understanding of heavy hadron physics.

In this letter we will present the basic formalism, and study the Isgur-Wise function, the decay constant, and the axial vector coupling constant. Furthermore, connection will be established with our previous work on covariant light-front model. Evaluation of HQS breaking effects will however be left to a forthcoming longer paper.

To begin, we represent a pseudoscalar heavy meson effectively by a quantum field operator $\Phi$, with the familiar free Lagrangian

$$\mathcal{L}_0^M = \partial_\mu \Phi^\dagger \partial^\mu \Phi - m_M^2 \Phi^\dagger \Phi, \quad (1)$$

where $m_M$ is the meson mass. In order to be consistent with HQET, we remove the heavy quark mass $m_Q$ from $\mathcal{L}_0^M$ by redefining $\Phi$ as

$$\Phi(x) = \frac{1}{\sqrt{m_M}} e^{-im_Q v \cdot x} \Phi_v(x), \quad (2)$$

where $v$ is the velocity of the heavy meson (or its constituent heavy quark). In terms of the new field $\Phi_v$, $\mathcal{L}_0^M$ becomes ($m_Q \to \infty$)

$$\mathcal{L}_0^M = 2\Phi_v^\dagger (iv \cdot \vec{\partial} - \bar{\Lambda}) \Phi_v \quad (3)$$

where $\bar{\Lambda} = m_M - m_Q$, and $\vec{\partial} = \frac{1}{2} (\vec{\partial} - \frac{v}{v} \partial)$. Thus $\Phi_v$ corresponds to a particle with mass $\Lambda$.

The pseudoscalar heavy meson ($\Phi_v$) couples to the heavy ($\psi Q$) and light ($\psi q$) quarks via

$$\mathcal{L}_0^M = -G_0 \bar{\Phi}_v \gamma_5 F(-iv \cdot \vec{\partial}) \psi_q + h.c., \quad (4)$$

where $G_0$ is the coupling constant,

$$h_v(x) = \frac{1 + \not{v}}{2} e^{im_Q v \cdot x} \psi Q(x) \quad (5)$$

is the familiar reduced heavy quark field operator in HQET, and $F$ is a vertex structure function related to the heavy meson bound state wave function, on which we shall impose the constraint that the heavy meson does not decay into $Q$ and $\bar{q}$ physically. Note that $F$ is independent of the residual momentum of the heavy quark so that the heavy quark charge is not spread out, otherwise the Isgur-Wise function will not be properly normalized.

To study the compositeness of the the heavy meson, we shall assume that, after summing the spin-flavor independent part of the interaction between a heavy quark and gluons in HQET, the effective coupling between the heavy quark and a light quark can be written as
\[ \mathcal{L}_{Qq}^M = g_0 \bar{h}_v i\gamma_5 f(-iv \cdot \partial) \psi_q \cdot f(iv \cdot \partial) \tilde{\psi}_q i\gamma_5 h_v \]  \hfill (6)

in the pseudoscalar channel, where \( g_0 \) is the coupling constant, and \( f \) is a form factor, whose presence is expected for an effective interaction resulting from non-perturbative QCD dynamics. \( \mathcal{L}_{Qq}^M \) can be considered as a generalized four-fermion coupling model \[ \text{[8]} \] inspired by QCD in the heavy quark limit.

Connection between the meson picture and the quark picture is established by demanding that \( \mathcal{L}_{Qq}^M \) and \( \mathcal{L}_{Qq}^q \) give the same heavy-light quark scattering amplitude as depicted in Fig. 1. In the meson-quark interaction picture, the renormalized scattering amplitude is given by

\[ A_M = \frac{iG^2 F(v \cdot p) F(v \cdot p')}{2(v \cdot k - \Lambda) - G^2 \Pi_F(v \cdot k)}, \]  \hfill (7)

where \( \Pi_F(v \cdot k) \) is the meson self-energy from the heavy-light quark loop shown in Fig. 1, which has been expanded around the mass shell \( v \cdot k = \Lambda \):

\[ \Pi_F(v \cdot k) = \Pi_F(\Lambda) + \Pi_F'(\Lambda)(v \cdot k - \Lambda) + \Pi_F''(v \cdot k), \]  \hfill (8)

and the renormalized quantities are defined by

\[ \tilde{\Lambda} = \Lambda_0 + \frac{1}{2} G^2 \Pi_F(\Lambda), \]  \hfill (9)

\[ G = \sqrt{Z_3 G_0}, \]  \hfill (10)

\[ Z_3 = 1 + \frac{1}{2} G^2 \Pi_F'(\Lambda). \]  \hfill (11)

The corresponding scattering amplitude in the quark-quark interaction picture is,

\[ A_{Qq} = g_0 f(v \cdot p') f(v \cdot p) \frac{i}{1 - g_0 \Pi_f(v \cdot k)}, \]  \hfill (12)

where \( \Pi_f \) is the same as \( \Pi_F \), except that the structure function \( F \) is replaced by \( f \). If we demand that the interaction is strong enough to produce a bound state of mass \( \Lambda \), then \( A_{Qq} \) should have a pole at \( v \cdot k = \Lambda \), which implies that \( g_0 = 1/\Pi_f(\Lambda) \), and

\[ A_{Qq} = \frac{-if(v \cdot p') f(v \cdot p)}{\Pi_f(\Lambda)(v \cdot k - \Lambda) + \Pi_f'(v \cdot k)}, \]  \hfill (13)

where \( \Pi_f(v \cdot k) \) has been expanded as in Eq. \[ \text{[8]} \]. From Eqs. \[ \text{[8]} \] and \[ \text{[13]} \], it is seen that for \( A_M = A_{Qq} \), we must have \( F = f \), so that \( \Pi_F = \Pi_f \), and

\[ G^2 = -2/\Pi'(\Lambda). \]  \hfill (14)

Hence the compositeness condition for the heavy meson fixes the strength of the \( (\Phi_v Qq) \)-coupling vertex through Eq. \[ \text{[14]} \]. As we shall see later, Eq. \[ \text{[14]} \] is related to the wave function normalization condition in the light-front quark model. The above discussion can be readily generalized to include heavy vector mesons \( (\Phi_v^+ \Phi_v^-) \); by heavy quark spin symmetry, the vertex structure function \( F \) must be the same for pseudoscalar and vector heavy mesons. Due to the lack of space, we will skip the details here.

We have now constructed an effective relativistic quantum field theory for quarks and heavy mesons. The complete Lagrangian is given by

\[ \mathcal{L} = \mathcal{L}_0^M + \mathcal{L}_M^q + \mathcal{L}_{QCD}^q + \mathcal{L}_{HQET}^{L/m_Q}, \]  \hfill (15)

where the notations are self evident. Within this framework, hadronic matrix elements are calculated via standard Feynman diagrams. The Feynman rules are the same as in QCD and HQET, except the meson-quark vertex, for which the Feynman rule is

\[ (\Phi_v Qq) - \text{vertex} = -iGF(v \cdot p) \Gamma_M, \]  \hfill (16)

where \( p \) is the momentum of the light quark, and \( \Gamma_M = \Gamma(\Phi_v^+ q)^0(\Phi_v^- q)^0 \). This model is simpler to work with than the ordinary light-front quark model, moreover it can be used to calculate hadronic form factors at arbitrary momentum transfers.

In the limit \( m_Q \to \infty \), flavor symmetry is trivially satisfied in our model. To demonstrate that spin symmetry also holds, we calculate the transition matrix element between heavy mesons due to the external heavy quark current (see Fig. 2a)

\[ J_\mu(x) = \bar{h}_v^\nu(x) \gamma_\mu h_v^\nu(x), \]  \hfill (17)

\[ \langle M'(v')|J_\mu|M(v)\rangle = -iG^2 \int \frac{d^4 p}{(2\pi)^4} F(v' \cdot p)F(v \cdot p) \frac{Tr \left[ (-\not{p} + m_q)\Gamma_M \frac{1+i\gamma_5}{2} \Gamma_M \frac{1-i\gamma_5}{2} \Gamma_M \right]}{(\Lambda - v' \cdot p + ie)(\Lambda - v \cdot p + ie)(p^2 - m_q^2 + ie)}, \]  \hfill (18)

where \( |M(v)\rangle \) stands for a heavy meson (pseudoscalar or vector) state, with the normalization \( \langle M(v')|M(v)\rangle = (2\pi)^3 2\delta(\Lambda v' - \Lambda v) \). By Lorentz covariance,

\[ \not{p} \to a\not{\gamma} + b\not{\gamma}', \]  \hfill (19)

with

\[ a = \frac{v \cdot p - v' \cdot pv \cdot v'}{1 - (v \cdot v')^2}, \quad b = \frac{v' \cdot p - v \cdot pv \cdot v'}{1 - (v \cdot v')^2}. \]  \hfill (20)

Moreover, since \( v \cdot \epsilon = v' \cdot \epsilon' = 0 \), and \( \gamma_5 \epsilon = -\gamma_5 \epsilon' \), Eq. \[ \text{[18]} \] can be rewritten as

\[ \langle M'(v')|J_\mu|M(v)\rangle = \left[ -\xi(v \cdot v') Tr \left[ \frac{1}{4} \Gamma_M (1 + \gamma_5 \gamma_5') (1 + \gamma_5) \Gamma_M \right] \right], \]  \hfill (21)

where

\[ \xi(v \cdot v') = iG^2 \int \frac{d^4 p}{(2\pi)^4} F(v' \cdot p)F(v \cdot p) \frac{(a + b + m_q)}{(\Lambda - v' \cdot p + ie)(\Lambda - v \cdot p + ie)(p^2 - m_q^2 + ie)}, \]  \hfill (22)
is called the Isgur-Wise function. Thus we have proved that in the $m_q \to \infty$ limit, transitions between heavy mesons due to external heavy quark currents are described by only one independent form factor $\xi(v \cdot v')$.

Similarly, the heavy meson self-energy $\Pi(v \cdot k)$ can be easily calculated, which in turn yields

$$G^{-2} = i \int \frac{d^4p}{(2\pi)^4} \frac{F^2(v \cdot p)(v \cdot p + m_q)}{(\Lambda - v \cdot p + i\epsilon)^2(p^2 - m_q^2 + i\epsilon)},$$  \tag{23}

through Eq. (14). Thus we see that when $v = v'$, $\xi(1) = 1$, as required by HQS.

For the heavy meson decay constant, we calculate the Feynman diagram shown in Fig. 2b. The result is

$$\langle 0| \bar{q}_A t^a Q| M_{Qq}(v) \rangle = F_M Tr \left[ \frac{1}{4} \Gamma_\mu (1 + \gamma_i) \Gamma_M \right],$$  \tag{24}

where $\Gamma_\mu = \gamma_\mu$ or $\gamma_\mu \gamma_5$, and

$$F_M = i2\sqrt{3}G \int \frac{d^4p}{(2\pi)^4} \frac{F(v \cdot p)(v \cdot p + m_q)}{(\Lambda - v \cdot p + i\epsilon)(p^2 - m_q^2 + i\epsilon)}$$  \tag{25}

is the decay constant in the heavy quark limit, which is the same for pseudoscalar and vector heavy meson. $F_M$ is related to the usual meson decay constant $f_M$ by $F_M = \sqrt{m_M f_M}$.

Finally, we consider the axial vector coupling constant ($g$) by evaluating matrix elements of the axial vector current $A^a_\mu = \bar{q}_A \gamma_\mu \gamma_5 \gamma^a q$. $g$ is related through PCAC to the strength of interactions between heavy mesons and Goldstone bosons ($\phi^0$) \cite{5}. In Ref. \cite{5}, we have shown how to conform with covariance in calculating $\xi(v \cdot v')$ and $F_M$ in a light-front quark model, however doing the same with $g$, which involves a purely light-quark current, remains a problem unsolved. With the field-theoretic approach proposed in the present work, Lorentz covariance is automatically guaranteed for heavy-quark and light-quark currents alike. This means the axial vector coupling constant extracted in this model should be more reliable. The calculation is by now straightforward, and the answer is (see Fig. 2c)

$$\langle M_{Qq}(v) | A^a_\mu | M_{Qq}(v) \rangle = g \ Tr \left[ \frac{1}{4} \gamma_\mu \gamma_5 \Gamma_{M'} (1 + \gamma^i) \Gamma_M \right],$$  \tag{26}

where we have omitted the SU(3) matrix element $\chi_{M'}^\dagger \chi^a \chi_M$, and the axial vector coupling constant $g$ is given by

$$g = -G^2 \int \frac{d^4p}{(2\pi)^4} F^2(v \cdot p) \frac{p^2 + 3m_q^2 + 6m_q v \cdot p + 2(v \cdot p)^2}{(\Lambda - v \cdot p + i\epsilon)(p^2 - m_q^2 + i\epsilon)^2}.$$  \tag{27}

To explicitly evaluate $\xi(v \cdot v')$, $F_M$, and $g$, we need to specify the structure function $F$, which is unfortunately not calculable at present. Nevertheless, from the constraints that (1) $F$ does not depend on the heavy quark momentum and (2) it forbids on-shell dissociation of the heavy meson to $Q\bar{q}$, a plausible form for $F$ is

$$F(v \cdot p) = \varphi(v \cdot p)(\bar{\Lambda} - v \cdot p),$$  \tag{28}

where the function $\varphi$ does not have a pole at $v \cdot p = \bar{\Lambda}$. The integrations over $p^0$ or $p^-$ in Eqs. (22), (23), and (25) can be easily performed. To facilitate comparison with light-front quark model results, we carry out the $dp^-$ integrations, and obtain

$$\xi(v \cdot v') = G^2 \int \frac{dp^+ dp^\perp}{(2\pi)^3 2p^+} \varphi(v' \cdot p) \varphi(v \cdot p) \cdot (a + b + m_q)$$  \tag{29}

$$F_M = 2\sqrt{3}G \int \frac{dp^+ dp^\perp}{(2\pi)^3 2p^+} \varphi(v \cdot p)(v \cdot p + m_q),$$  \tag{30}

$$g = -G^2 \int \frac{dp^+ dp^\perp}{(2\pi)^3 2p^+} \left\{ \varphi^2(v \cdot p) (\bar{\Lambda} - v \cdot p) \frac{1}{p^+ \frac{\partial}{\partial p^-}} \{ \varphi^2(v \cdot p) (\bar{\Lambda} - v \cdot p) \cdot [p^2 + 3m_q^2 + 6m_q v \cdot p + 2(v \cdot p)^2] \} \right\},$$  \tag{31}

and

$$G^{-2} = \int \frac{dp^+ dp^\perp}{(2\pi)^3 2p^+} \varphi^2(v \cdot p)(v \cdot p + m_q),$$  \tag{32}

where $p^+ \geq 0$ and $p^2 = m_q^2$.

It is interesting to observe that, if

$$\varphi(v \cdot p) = \frac{e^{-v \cdot p/\omega}}{\sqrt{v \cdot p + m_q}},$$  \tag{33}

then $\xi(v \cdot v')$ and $F_M$ are the same as those obtained in the covariant light-front quark model \cite{5}, and $G$ equals the wave function normalization constant. We note that $e^{-v \cdot p/\omega}$ is just the covariant light-front wave function proposed in \cite{5}, and the factor $\sqrt{v \cdot p + m_q}$ is originated from the Melosh transformation.

However, $e^{-v \cdot p}$ is not bounded when $p$ is off the mass shell, so that we must use $e^{-|v \cdot p|}$ in a four dimensional integral, which, together with the square root sign in $\xi(v \cdot v')$, is repugnant in a field theoretic formalism. Hence we will choose instead the more well behaved form,

$$\varphi(v \cdot p) = \frac{e^{-|v \cdot p|/2\omega^2}}{v \cdot p + m_q - i\epsilon},$$  \tag{34}

which yields very reasonable results both in the heavy quark limit and for $1/m_q$ corrections. If we take $F_M \approx F_B$, $f_B = 0.18$ GeV \cite{5}, and $m_Q = 0.25$ GeV, then Eq. (34) gives $\omega = 0.60$ GeV. The resulting Isgur-Wise function is almost indistinguishable from that of
Ref. [5] HQET analyses of inclusive semileptonic $B$ and $D$ decays find $\Lambda \simeq 0.45$ GeV [10], which implies that $g = 0.32 (35)$ in the heavy quark limit. Eq. (35) is consistent with the constraint of $g D - D \pi < 0.7$ obtained from $D^*$-meson decay width [11]; it also compares well with the QCD sum rules results of $g = 0.21 \lesssim 0.39$ [12]. Clearly, in order to make more precise physical predictions, we must also take into account $1/m_Q$ corrections, which will be treated in another publication.

We have now completed the specification of our field-theoretic model for the bound state structure of a heavy meson. The heavy meson bound state we have constructed, which obeys HQS, can be considered as one due to the lowest order Hamiltonian in the HQET. Hence it provides an appropriate basis for studying systematically the HQS breaking $1/m_Q$ effects. For example, Fig. 3 shows an $1/m_Q$ correction to the Isgur-Wise function $\xi(\nu \cdot \nu')$ in this model, which can be easily evaluated.

In summary, we have developed a realistic field-theoretic model for heavy meson as a composite particle. Lorentz covariance allows us to extract hadronic transition form factors at arbitrary momentum transfers without running into ambiguities; hence it overcomes a severe drawback caused by non-covariance in the light-front quark model. Our model yields physically realistic results, and provides an ideal framework in which $1/m_Q$ corrections to HQS can be evaluated quantitatively and systematically. Results for $1/m_Q$ effects and further extension of the model will be published elsewhere.

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[1] N. Isgur and M.B. Wise, Phys. Lett. B 232, 113 (1989); 237, 527 (1990).
[2] For a review, see M. Neubert, Phys. Rep. 245, 261 (1994).
[3] W.M. Zhang, Phys. Rev. D 56, 1528 (1997).
[4] H.Y. Cheng, C.Y. Cheung, and C.W. Hwang, Phys. Rev. D 55, 1559 (1997).
[5] H.Y. Cheng, C.Y. Cheung, C.W. Hwang, and W.M. Zhang, Phys. Rev. D (to appear), hep-ph/9709412.
[6] E. Eichten and B. Hill, Phys. Lett. B 234, 511 (1990); H. Georgi, Phys. Lett. B 240, 447 (1990).
[7] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1955).
[8] T.M. Yan, H.Y. Cheng, C.Y. Cheung, G.L. Lin, Y.C. Lin, and H.L. Yu, Phys. Rev. D 46, 1148 (1992); M.B. Wise, Phys. Rev. D 45, R2188 (1992); G. Burdman and J. Donoghue, Phys. Lett. B 280, 287 (1992).
[9] C.W. Bernard, J.N. Labrenz, and A. Soni, Phys. Rev. D 49, 2536 (1994).
[10] A.F. Falk, M. Luke, and M.J. Savage, Phys. Rev. D 53, 6316 (1996).
[11] ACCMOR Collaboration, S. Barlag et al, Phys. Lett. B 278, 480 (1992).
[12] See, e.g., V.M. Belyaev, V.M. Braun, A. Khodjamirian, and R. R"uckl, Phys. Rev. D 51, 6177 (1995).

Fig. 1 Heavy-light quark scattering in (a)Meson-quark coupling picture, and (b)Quark-quark coupling picture.

(a): $k - p - p' - p$

(b):

Fig. 2 Feynman diagrams for (a)Isgur-Wise function, (b)Decay constant, and (c)Axial vector coupling constant.

(c): $\Lambda v \Gamma^\mu \Lambda v' \otimes A_\mu - p$

Fig. 3 $\mathcal{O}(1/m_Q)$ correction to Isgur-Wise function.