Low Carbon Production Inventory Model for Imperfect Quality Deteriorating Items with the Screening Process

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Abstract
Carbon emissions play an important role in global warming. Currently, some countries are alert for reducing the carbon emissions. Basically, many countries use carbon cap & tax policies to reduce emission. Observance this issue in mind, we considered a production inventory model for a manageable carbon emissions rate. In this paper, we consider an imperfect production inventory model for decaying product with carbon emission and the defective rate is product dependent. To select the perfect and imperfect items have been inspected the produced items. The defective goods are sold in an only one consignment at the stop of the 100% screening process at a lower cost.

The planned process is validated with the help of a numerical example. A solution process is given to establish the most favorable ordering policy. Finally, Sensitivity analysis of different parameters has been approved.

Keywords: Inventory, Deteriorating items, Carbon emission, Imperfect production process, screening process.

1. Introduction
Global warming is a big environmental issue all over the world. Our environment becoming hot day by day by rise in the level of atmospheric carbon emission. Fossil fuels like coal and oil are increase the level of carbon dioxide. Many researchers, industries, and other organizations are presenting supply chain management including greening and low carbon emission, to keep the issue of the environmental protection in mind. Basically, carbon tax and cap policy are the key mechanism to reduce carbon emission in many countries. Carbon emissions occur during the three stages of the model: production, holding, and removal of imperfect goods. The first emanation is the production setup that emits constant amounts of CO2 throughout the production cycle. The second emanation is the production process which may include melting, assembling, chemical reactions, etc. The third emanation is basic machining operations that depend on the time during which the production process is run. Emissions, during the time in which the inventory is stored and disposed, depend on the inventory level and time, and depend on the inventory quantity of disposed product respectively. S.R. Singh [1] established a paper for decaying products under the effect of inflation and considered inventory level depended demand. Hua et al. [2] represented a green approach to stock control that the impacts of a carbon cost related to the ordering and holding cost. This model is the extension of the inventory model with carbon emissions under the cap and trade system. Wahab et al. [3] established a model to consider the carbon related cost from the onward and backward between the retailer and the consumer. Singh and Saxena [4] developed a reverse logistics model for decaying products and for a single item with two different quality standards with remanufacturing. The supplier reproduces only those used materials, whose quality level exceeds the required level required by the market, and it overcame the remaining products. Toptal et al. [5] developed an inventory model with three reductions (1. carbon cap; 2. tax; and 3. cap-and-trade policies) of carbon emission speculation policy. Qin et al. [6] presented an inventory policy under carbon tax, and carbon cap and trade policy in which demand depend on the permissible delay period. They found from their model that a carbon related variable have a negative effect on the permissible delay period. Hovelaque et al. [7] presented a model with carbon emissions considered
emissions and price dependent demand. Sarkar et al. [8] established a model for defective products with carbon emissions. Singh and Jawla [9] developed a reverse logistic inventory model for the defective product with conservation technology investment under the learning effect and inflation. Bazan et al. [10] developed a vendor-managed inventory with carbon emission & energy cost, consignment stock policy. Tiwari et al. [11] presented a model for decaying product for two-echelon supply chain including carbon emission. The authors assumed that the buyer performs a complete screening procedure, and the decay items are withdrawn. Kung et al. [12] established an EPQ model for decaying product considered machine breakdown, inspection, and partial back-ordering. The authors produced imperfect products with different rates before and after inspection. Bai et al. [13] developed the supply chain inventory model considered the effect of carbon emission reduction policy. Carbon cap and trade policy has been used to reduce that carbon emissions are being generated due to production and holding. In this paper, we consider an imperfect production model for decaying items. The defective rate is production dependent and carbon emission are also considered. To select perfect and imperfect items we apply screening process and then inspection. Section 2 &3 describes notations and assumptions, mathematical modeling of this paper respectively. The solution procedure are given in section 5. The model has been illustrated with numerical solution which is represented in section 6. The effect on the model from some influential parameters is represented on section 7 through sensitivity analysis. Observations of this paper are shown in section 8. Finally, conclusions and scope for future research are given in section 9.

2. Assumptions and Notations

2.1 Assumptions
1. The demand rate for buyer and the production rate for supplier are known and constant.
2. The demand rate is lower than the screening process rate.
3. The unit cost of production depends on produced items, wear–tear and raw material, progress costs and is of the form, \( C_P = C_r + \frac{K}{P} + NP \); Where \( C_r \) is raw material charge, \( K \) is labor charge, and \( N \) is tool wear and tear charge respectively.
4. It is supposed that the defective rate is production depended \( \theta = \theta_0 - \theta_1 P \). Where \( \theta_0 \), \( \theta_1 \) > 0.

2.2 Notations

\[
\begin{align*}
D & \quad \text{Demand rate} \\
P & \quad \text{Production rate, } P > D \\
C_s & \quad \text{Set up cost per cycle} \\
C_P & \quad \text{Production cost per unit} \\
h_c & \quad \text{the cost to a hold the inventory ($/unit/time) in production center} \\
h_w & \quad \text{the cost to a hold of perfect quality items ($/unit/time)} \\
h_i & \quad \text{the cost to a hold of imperfect quality items ($/unit/time)} \\
C_d & \quad \text{Deteriorating cost of the inventory ($/unit/time) in production center} \\
C_d^p & \quad \text{Deteriorating cost of perfect quality product ($/unit/time)} \\
C_d^i & \quad \text{Deteriorating cost of imperfect quality product ($/unit/time)} \\
C_s & \quad \text{Screening cost per unit item} \\
S & \quad \text{Selling price for perfect item ($/unit/time)}
\end{align*}
\]
3. **Mathematical modeling:**

We consider a production system (Figure 1 and 2) that produces both perfect and defective items up to a time $t_1$. Here manufacture perform the screening process to select perfect and imperfect items up to time $t_2$ with a screening rate $(1-x)$. Per unit time $(1-x)p$ inventory is to be gathering in screening center until a period $t_2$.

3.1 **Mathematically representation of model in production center**

At time $t = 0$, production starts and continues up to $t_1$. The produced goods incessantly are sending into the screening cell with the rate $P_x$.

The behavior of inventory level w.r.t. to time can be described by the differential equation
\[ I'(t) = P - xP, \quad 0 \leq t \leq t_1 \]  
\[ I'(t) = -xP, \quad t_1 \leq t \leq t_2 \]  

With the b. c. \( I(0) = 0 \), \( I(t_1) = q \) and \( I(t_2) = 0 \). The solutions of the given equation are as follows 

\[ I(t) = (1 - x)tP \]  
\[ , 0 \leq t \leq t_1 \]  
\[ I(t) = xP(t - t_1) \]  
\[ , t_1 \leq t \leq t_2 \]  

Using the condition \( I(t_2) = q \) from (5) and (6), We get 

\[ q = (1 - x)tP_t = xP(t_2 - t_1) \Rightarrow t_1 = 

3.2 Mathematically representation of model for perfect and imperfect quality item in a screening center 

In the screening cell, at time \( t=0 \) screening starts and continues up to time \( t_2 \). After the screening process, the holding rate of the faultless item is \( (1 - \theta)P \) which is larger than the demand of consumer. After satisfying the demand of consumer, the additional inventory \( (1 - \theta)P - D(t) \) per unit time is to being stock in best quality item center until period \( t_1 \). After the screening process stock becomes zero, due to demand and deterioration at time \( T \). At times \( t_2 \) imperfect items are sold in only one consignment with price \( S' \).

So, the variation of perfect quality items \( q_i(t) \) w.r.t. time can be described by the following differential equation:

\[ I_1'(t) = (1 - u)xP - \theta I_1(t) - D, \quad 0 \leq t \leq t_2 \]  
\[ I_1'(t) = -\theta I_2(t) - D, \quad t_2 \leq t \leq T \]  

Subject to the conditions that \( I_1(0) = 0 \), \( I_1(t_2) = I_2(0) \), \( I_2(T) = 0 \)

\[ \frac{dI_1(t)}{dt} = uxP - \theta I_1(t), \quad 0 \leq t \leq t_2 \]  
\[ \text{s.t.} \quad I_1(0) = 0 \]  
\[ I_1(t) = \left[ \frac{(1-u)xP-D}{\theta} \right] (1-e^{-\theta t}) \]  
\[ I_2(t) = \frac{D}{\theta} (e^{\theta(t-t_1)} - 1) \]  
\[ I_3(t) = \frac{Pux}{\theta} (1-e^{-\theta t}) \]  

From continuity

\[ \left[ \frac{(1-u)xP-D}{\theta} \right] (1-e^{-\theta t_2}) = \frac{D}{\theta} (e^{\theta(t-t_1)} - 1) \Rightarrow t_2 = \frac{DT \left[ 1 + \frac{\theta T}{2} \right]}{(1-u)xP - D} \]  

(14)
4. Cost components of this system:

4.1. Production cost (PC): \[ (C_p + C_{PE}) \int_0^T P \, dt = (C_p + C_{PE}) P_T \]

4.2. Screening cost (SC): \[ C_s \int_0^T x P \, dt = C_s P x t_2 \]

4.3. Set up cost (SUC): \[ C_s \]

4.4. Sales Revenue cost (SR): The amount of money which obtained after selling inventory is known as total revenue cost or sales revenue cost. The formula for the cost is given as follows:

\[ S \int_0^T D(t) \, dt + S \int_0^T u x P \, dt = SDT + S P x t_2 \]

4.5. Holding cost (HC): The amount of money which needs to hold the inventory is known as holding cost. In this model we considered holding cost as time dependent as the expression for finding holding cost is as follows:

\[ \frac{h_s + h_d}{2} \left( 1-x \right) P_T + x P \left( \frac{t_2-t_1}{2} \right)^2 \left( h_m + h_d \right) \left[ \frac{(1-u) x P - D}{\theta^2} \left( \theta t_2 + \left( e^{\theta t_2} - 1 \right) \right) + \left( h_m + h_d \right) \right] + \frac{h_s + h_d}{2} \left( e^{\theta t_2} - 1 \right) \left( \theta t_2 + \left( e^{\theta t_2} - 1 \right) \right) \]

4.6. Deterioration cost (DC): The amount of money which is lost due to deterioration or damage of inventory is known as deterioration cost. The expression for deteriorating cost is given as follows:

\[ \left( C_d + C_{de} \right) \int_0^T I_1(t) \, dt + \left( C_d + C_{de} \right) \int_0^T I_2(t) \, dt + \left( C_d + C_{de} \right) \int_0^T I_3(t) \, dt \]

\[ = \left( C_d + C_{de} \right) \left[ \left( 1-u \right) x P - D \right] \theta t_2 + \left( e^{\theta t_2} - 1 \right) + \left( C_d + C_{de} \right) \frac{D}{\theta^2} \left[ \left( e^{\theta (T-t_2)} - 1 \right) - \theta (T-t_2) \right] + \left( C_d + C_{de} \right) u x P \left[ \theta t_2 + \left( e^{\theta t_2} - 1 \right) \right] \]

Total Profit = \[ SR - PC - SC - HC - DC - SUC \]

\[ \left( SDT + S P x t_2 \right) - C_s P x t_2 - C_s - (C_p + C_{PE}) P x t_2 - \left( h_s + h_d + C_d + C_{de} \right) P x t_2 - \left( C_d + C_{de} \right) \left[ \left( 1-u \right) x P - D \right] \left( t_2 \right) + \left( h_s + h_d + C_d + C_{de} \right) \frac{D}{2} (T - t_2)^2 \]

5. Solution methodology
For the function to be concave, \( \frac{\partial^2 AP}{\partial T^2} = 0 \), and \( \frac{\partial^2 AP}{\partial t_2^2} = 0 \) at least one point \( T \) & \( t_2 \geq 0 \) at which the conditions \( \left( \frac{\partial^2 AP}{\partial T^2} \right)^2 - \left( \frac{\partial^2 AP}{\partial t_1^2} \right) \left( \frac{\partial^2 AP}{\partial T \partial t_1} \right) < 0 \) must be satisfied. Which shows the optimality condition of total cost function.

6. Numerical examples:
Here, we use MATHEMATICA 9 for numerical examples. We then study the sensitivity analysis of the optimal solution w.r.t. some important parameters.

\[ C_s = 300 \text{ /unit}, \ T_x = 61.8 \text{ /tonCO}_2, \ e_c = 4 \text{ KWh / m}^3, \ E_e = 0.2 \text{tonCO}_2 / \text{KWh}, \ \theta = 0.2 / \text{units}, \ \theta_1 = 0.23 / \text{units}, \ h_1 = 0.5 / \text{units}, \ h_2 = 0.13 / \text{units}, \ h_w = 0.25 / \text{units}, \ C_s = 0.015 / \text{units}, \ C_3 = 0.045 / \text{units}, \ C_4 = 0.36 / \text{units}, \ h_s = 0.58 / \text{units}, \ C_w = 12 / \text{units}, \ C_{de} = 0.06 / \text{units}, \ \epsilon = 20 / \text{kwh}, \ P = 46 \text{units/year}, \ \therefore \text{optimal sol is } T = 1.16754 \text{month}, \ \text{Average profit= } 3461.97 / \text{month}, \ t_1 = 0.276452 \text{month}, \ t_2 = 0.368603 \text{ month.}

Figure 3 shows that the behavior of the optimal cost function between cycle length period and screening period.

7. Sensitivity analysis:
Table 1: Sensitivity analysis w.r.t. different parameter

| Variable | % vary of parameter | Value of parameter (t<sub>p</sub> (production run time)) | t<sub>x</sub> | T(cycle length) | AP (average profit) |
|----------|---------------------|----------------------------------------------------------|-------------|----------------|-------------------|
| x        | -20                 | 0.52                                                      | 0.2209380   | 0.514512       | 1.13329           | 3405.73           |
|          | -15                 | 0.5525                                                   | 0.224964    | 0.469856       | 1.14114           | 3417.89           |
|          | -10                 | 0.585                                                    | 0.229358    | 0.43122        | 1.14949           | 3431.21           |
|          | -5                  | 0.6175                                                   | 0.234179    | 0.397736       | 1.15832           | 3445.84           |
|          | 5                   | 0.6825                                                   | 0.245602    | 0.34312        | 1.17694           | 3479.81           |
|          | 10                  | 0.715                                                    | 0.252264    | 0.32078        | 1.18617           | 3499.6            |
|          | 15                  | 0.7475                                                   | 0.259595    | 0.300955       | 1.19454           | 3521.6            |
|          | 20                  | 0.78                                                     | 0.267546    | 0.283254       | 1.20086           | 3546.09           |
| e<sub>p</sub> | -20                | 16                                                       | 0.221162    | 0.368603       | 1.16754           | 5085.27           |
|          | -15                 | 17                                                       | 0.221162    | 0.368603       | 1.16754           | 4679.44           |
|          | -10                 | 18                                                       | 0.221162    | 0.368603       | 1.16754           | 4273.62           |
|          | -5                  | 19                                                       | 0.221162    | 0.368603       | 1.16754           | 3867.8            |
|          | 5                   | 21                                                       | 0.221162    | 0.368603       | 1.16754           | 3056.15           |
|          | 10                  | 22                                                       | 0.221162    | 0.368603       | 1.16754           | 2650.32           |
|          | 15                  | 23                                                       | 0.221162    | 0.368603       | 1.16754           | 2244.5            |
|          | 20                  | 24                                                       | 0.221162    | 0.368603       | 1.16754           | 1838.67           |
| T<sub>ε</sub> | -20                | 49.44                                                    | 0.264705    | 0.407239       | 1.28992           | 3647.69           |
|          | -15                 | 51.53                                                    | 0.259885    | 0.399823       | 1.26643           | 3600.9            |
|          | -10                 | 55.62                                                    | 0.251186    | 0.38644        | 1.22404           | 3542.32           |
|          | -5                  | 58.71                                                    | 0.245177    | 0.377196       | 1.19476           | 3508.02           |
|          | 5                   | 64.89                                                    | 0.234378    | 0.360581       | 1.14213           | 3416.2            |
|          | 10                  | 67.18                                                    | 0.230733    | 0.354974       | 1.12437           | 3370.74           |
|          | 15                  | 71.07                                                    | 0.22492     | 0.34603        | 1.09604           | 3311.07           |
|          | 20                  | 74.16                                                    | 0.22061     | 0.3394         | 1.07504           | 3280.83           |
| e<sub>ε</sub> | -20                | 3.2                                                      | 0.264705    | 0.407239       | 1.28992           | 3510.91           |
|          | -15                 | 3.4                                                      | 0.257673    | 0.39642        | 1.25565           | 3498.16           |
|          | -10                 | 3.6                                                      | 0.251186    | 0.38644        | 1.22404           | 3485.78           |
|          | -5                  | 3.8                                                      | 0.245177    | 0.377199       | 1.19476           | 3473.72           |
|          | 5                   | 4.2                                                      | 0.234378    | 0.360581       | 1.14213           | 3450.5            |
|          | 10                  | 4.4                                                      | 0.229499    | 0.353076       | 1.11836           | 3439.3            |
|          | 15                  | 4.6                                                      | 0.22492     | 0.34603        | 1.09604           | 3428.34           |
|          | 20                  | 4.8                                                      | 0.22061     | 0.3394         | 1.07504           | 3417.61           |
|        | -20                 | 0.16                                                     | 0.264705    | 0.407239       | 1.28992           | 5134.21           |
|        | -15                 | 0.17                                                     | 0.257673    | 0.39642        | 1.25565           | 4715.64           |
|        | -10                 | 0.18                                                     | 0.251186    | 0.38644        | 1.22404           | 4297.43           |
8. Result and discussion:
   i. If we increase the screening rate then average profit (AP), cycle length (T), production run time (t₁) also increase because average profit is directly proportional to screening rate from (15) and production run time (t₁) is also directly proportional to screening rate from (7).
   ii. As increase average electricity consumption for production (eₚ), then average profit (AP) decreases but cycle length (T), production run time (t₁) remains the same.
   iii. As an increase Production rate (P), then average profit (AP), cycle length (T) and production run time (t₁) are increase because average profit is directly proportional to Production rate.
   iv. As an increase Deterioration rate (θ), then average profit (AP), cycle length (T) and production run time (t₁) are decrease.
v. As an increase in carbon tax \( T_e \), then average profit (AP), cycle length (T) and production run time \( t_i \) are decreases.

vi. As increase standard emissions for electricity generation \( E_e \), then average profit (AP), cycle length (T) and production run time \( t_i \) are decrease.

vii. As increase in average electricity consumption per warehouse space unit \( e_s \), then average profit (AP), cycle length (T) and production run time \( t_i \) are decrease.

viii. As increase space occupied by a unit product(v), then average profit (AP), cycle length (T) and production run time \( t_i \) are decrease.

9. Conclusion:
In this paper, we study an EPQ model with a screening process considering carbon emissions. The objective is to reduce carbon emission costs and maximize profit. This model includes the effect of deterioration, defective products, and carbon emissions. We also examine the effect of energy consumption, and the result confirms the effect of carbon tax regulation in reducing carbon emissions. This paper can be extended by considering technology investment, an adjustable production rate to reduce the possibility of defective and deteriorating goods. Further we can extend included trade credit policy and carbon cap policy with variable parameters.

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