Isocurvature fluctuations in the Affleck–Dine mechanism and constraints on inflation models

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Abstract. We argue that, in the Affleck–Dine mechanism, baryonic isocurvature fluctuations are generated in most inflation models in supergravity. The inflationary scale and a reheating temperature are constrained in order not to induce too large baryonic isocurvature fluctuations. In particular, high-scale inflation models with high reheating temperatures are disfavored in this context.

Keywords: cosmological perturbation theory, baryon asymmetry, inflation, physics of the early universe

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1. Introduction

The origin of baryon asymmetry in our universe remains a big mystery in modern cosmology. The Affleck–Dine (AD) mechanism provides one of the promising baryogenesis scenarios [1]. It utilizes a flat direction of the supersymmetric standard model, which possesses a non-zero baryon (or lepton) number. A flat direction responsible for the AD mechanism is referred to as the AD field. The AD field is assumed to develop a large expectation value during inflation, and it starts to oscillate after inflation when the cosmic expansion rate becomes comparable to its mass. The baryon number is effectively created at the onset of the oscillations. Finally, the AD field decays into the ordinary quarks, leaving the universe with the right amount of baryon asymmetry.

The scalar potential along the flat direction is crucial for the AD mechanism sketched above to work. The flat direction has a vanishing potential at the level of the renormalizable operators in the limit of supersymmetry (SUSY). In other words, the flat directions can be lifted by the non-renormalizable operators and the SUSY breaking effects. During inflation, SUSY is largely broken by the inflaton potential [2]. In particular, the radial component of the AD field generically acquires a mass of the order of the Hubble parameter, referred to as the Hubble-induced mass, due to supergravity effects in the F-term inflation models. The sign of the Hubble-induced mass is assumed to be negative for the AD field to develop a large field value during inflation.

The flat directions can be lifted also by non-renormalizable operators in a superpotential. In fact, the non-renormalizable operator not only lifts the potential at large scales, but also provides a baryon-number violation needed to generate the baryon asymmetry. Our concern here is the strength of the baryon-number violation during and after inflation. It is often claimed that, during inflation, there appears a baryon-number
violating $A$-term with a coefficient comparable to the Hubble parameter. Such a large $A$-term is referred to as the Hubble-induced $A$-term. If there were indeed a Hubble-induced $A$-term during inflation, the phase component of the AD field would acquire a mass of the order of the Hubble parameter, and hence would quickly settle down in one of the minima given by the Hubble-induced $A$-term. Thus, as long as the Hubble-induced $A$-term is not suppressed, the phase component of the AD field does not have any sizable fluctuations beyond the horizon scale during inflation.

In this paper we reconsider the AD mechanism and show that the Hubble-induced $A$-term is suppressed in most $F$-term inflation models. Since the Hubble-induced $A$-term is absent in the $D$-term inflation, this result is quite generic. As a result, the phase direction of the AD field is rather flat and therefore quantum fluctuations along that direction are generated during inflation. Those fluctuations turn into the baryonic isocurvature fluctuations, which are now tightly constrained by the observations of the cosmic microwave background (CMB). Assuming that the AD mechanism is responsible for generating the baryon asymmetry of the universe, we will show that both the inflation scale and the reheating temperature must satisfy a certain constraint.

Lastly, let us comment on the differences of the present paper from the works in the past. The baryonic isocurvature perturbations in the AD mechanism were discussed for instance in [4, 5], both of which focused on the $D$-term inflation model [6]. This is partly because the absence of the Hubble-induced $A$-term as well as the Hubble-induced mass term is obvious in the $D$-term inflation. It was also pointed out in [4] that the Hubble-induced $A$-term can be suppressed in a certain class of the $F$-term inflation models. However, they did not examine realistic inflation models such as the hybrid inflation model [7], and how generic the Hubble-induced $A$-term is suppressed was not clear. Indeed, it was often claimed that the presence of the Hubble-induced $A$-term was an unavoidable consequence of the $F$-term inflation in supergravity. The purpose of this paper is to examine representative inflation models and explicitly show that the Hubble-induced $A$-term is suppressed under certain conditions. Furthermore, by using the recent observational constraints on the isocurvature fluctuations, we put tight bounds on the inflationary scale and the reheating temperature for the first time.

2. Affleck–Dine mechanism

Let us here briefly review the AD mechanism. In particular we explain how the resultant baryon asymmetry is related to the baryon-number violating operators, which will be important later on for estimating the baryonic isocurvature fluctuations.

Flat directions are parameterized by composite gauge-invariant monomial operators such as $udd$ and $LH_u$, and the dynamics of a flat direction can be expressed in terms of a complex scalar field $\phi$. The flat directions of the minimal supersymmetric standard model are classified in [8]. We assume that $\phi$ has a non-zero baryon number in the following.

First let us consider the scalar potential of $\phi$ in a flat space time. We assume that the AD field $\phi$ has a non-renormalizable operator in the superpotential:

$$W(\phi) = \frac{\lambda}{n} \phi^n,$$

(1)

It was first pointed out in [3] that their amplitude could be observably large without discussion of Hubble-induced mass and $A$ terms.
where $\lambda$ is a numerical coefficient, and $n$ is an integer, $n = 4, 5, 6, 7$ and 9, which depends on flat directions. We set $\lambda$ to be real without loss of generality. We adopt the Planck unit, $M_P = 1$ (or $M_P = 2.4 \times 10^{18}$ GeV) here and in what follows unless explicitly stated otherwise. The above operator (1) lifts the flat direction at large scales. In addition to the non-renormalizable operator, the AD field has a soft SUSY breaking mass, $m_\phi$. Thus the relevant potential for the AD field is expressed as

$$V_0(\phi) = m_\phi^2 |\phi|^2 + \lambda^2 |\phi|^{2n-2} + V_A(\phi),$$

with

$$V_A(\phi) = a \lambda m_3/2 \phi^n + \text{h.c.}$$

Here we add the $A$-term, $V_A$, where $a$ is a numerical coefficient of order unity, and $m_3/2$ denotes the gravitino mass. The presence of $V_A$ is generic, and one of the contributions comes from the cross-term, between (1) and the constant term in the superpotential $W_0 \simeq m_3/2$. The $A$-term explicitly violates the baryon symmetry, which will play an important role in the AD mechanism as described below.

Let us make a comment that the $A$-term is proportional to the $R$ symmetry breaking, since one can assign the $R$ charge 2 on the operator (1), i.e., $R[\phi] = 2/n$. Note that the $R$ symmetry is necessarily violated by the constant term in the superpotential to make the cosmological constant (almost) vanish. Therefore the $A$-term $V_A$ naturally arises in a flat spacetime, picking up the $R$ symmetry breaking, $W_0$. As we will discuss in the next section, however, it depends on the inflation models whether there are larger $R$ breakings during and after inflation.

When the inflaton dominates the energy density of the universe, SUSY is largely broken by the potential energy of the inflaton, and the potential of $\phi$ receives corrections. We focus on the $F$-term inflation models. The scalar potential in supergravity is given by

$$V = e^K (D_i W g^{ij*} (D_j W)^* - 3|W|^2),$$

where we adopt the usual convention that a subscript $i$ denotes a derivative with respect to a scalar field $\phi_i$. The sum over the indices $i, j, \ldots$ is understood unless otherwise stated. $K$ is the Kähler potential, $W$ the superpotential, and $g^{ij*}$ the inverse of the Kähler metric $g_{ij}$. During inflation, the inflaton potential is related to the Hubble expansion rate:

$$V(I) \simeq 3H^2.$$ 

If there is a quartic coupling between the AD field and the inflaton in the Kähler potential, $K = c |\phi|^2 |I|^2$ with $c > 1$, the AD field has a negative Hubble-induced mass term. The

5 In the gauge-mediated SUSY breaking models [9], the potential coming from the SUSY breaking effects takes a different form and there might be another contribution to the $A$-term. However, it does not change the following arguments qualitatively.

6 This is actually the anomaly-mediated SUSY breaking effect [10], since the operator (1) explicitly breaks the scale invariance.

7 We are not interested in such inflation models as are inconsistent with the AD baryogenesis. For instance, if the Hubble parameter during inflation is smaller than $m_\phi$, the AD mechanism does not work. This is the case in the MSSM inflation [11], which uses the potential (2) with finely tuned parameters.
scalar potential of $\phi$ can be written as
\begin{equation}
V(\phi) \simeq -c_H H^2 |\phi|^2 + V_0(\phi),
\end{equation}
where $c_H \equiv 3(c-1)$ is a numerical coefficient of order unity. For simplicity we set $c_H = 1$ in the following. The expression (6) is actually valid as long as the inflaton dominates the energy density of the universe, and we can use (6) after inflation until the reheating is completed.

There might be a further correction called the Hubble-induced $A$-term, given by
\begin{equation}
V_{AH}(\phi) = b\lambda H \phi^n + \text{h.c.},
\end{equation}
which is similar to the $A$-term, $V_A$ (see (3)). Here $b$ is a numerical coefficient, and its size is important for the baryonic isocurvature fluctuations. In this section we drop the Hubble-induced $A$-term, and the condition for its appearance will be discussed in detail in the next section.

Let us now describe the dynamics of the AD field. To this end we decompose $\phi$ into the radial and phase components:
\begin{equation}
\phi = \frac{\varphi}{\sqrt{2}} e^{i\theta}.
\end{equation}
When $H \gg m_\phi$, the potential minimum of $\varphi$ is located at
\begin{equation}
\varphi_{\text{min}} \simeq \alpha_n \left(\frac{H}{\lambda}\right)^{1/(n-2)},
\end{equation}
with $\alpha_n \equiv (2^{n-2}/(n - 1))^{1/2(n-2)}$. During inflation, $\varphi$ stays at the minimum, $\varphi = \varphi_{\text{min}}$. After inflation, the Hubble parameter decreases with time, and so does the minimum. The radial component $\varphi$ continues to track $\varphi_{\text{min}}$ until $H \sim m_\phi$, since its effective mass is comparable to the velocity of the instantaneous minimum, i.e., $m_\phi^{(\text{eff})} = H \sim \varphi_{\text{min}}/\varphi_{\text{min}}$. On the other hand, the phase component $\theta$ has a rather flat potential coming only from $V_A$, and it is written as
\begin{equation}
V_A = \frac{|a|}{\sqrt{n-1}} m_3/2 H \varphi^2 \cos (n\theta + \text{arg}[a]),
\end{equation}
where we have used $\varphi = \varphi_{\text{min}}$. The mass of $\theta$ is of $O(\sqrt{m_3/2H})$, which is much smaller than $H$ before $\phi$ starts to oscillate. Thus $\theta$ is in general deviated from the minima of $V_A$.

When the Hubble parameter becomes comparable to $m_\phi$, the AD field starts to oscillate about the origin. The baryon asymmetry is effectively generated at that moment, and the field is kicked in the phase direction by the baryon-number violating potential, $V_A$. The baryon-number density $n_B$ is defined by
\begin{equation}
n_B \equiv i \left( \dot{\phi}\phi^* - \dot{\phi}^* \phi \right),
\end{equation}
where we assume that $\phi$ has an unit baryon number. The evolution of the baryon-number density is given by
\begin{equation}
\dot{n}_B + 3H n_B = -i \phi \frac{\partial V_A}{\partial \phi} + \text{h.c.},
\end{equation}
where $\theta$ that has mass dimension 1, rather than $\theta$. Note also that the mass of $\varphi$, $m_\phi$, is generically larger than or comparable to $m_3/2$. 

To be precise, it is $\varphi \theta$ that has mass dimension 1, rather than $\theta$. Note also that the mass of $\varphi$, $m_\phi$, is generically larger than or comparable to $m_3/2$. 

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where the dot denotes a derivative with respect to the time. The baryon-number density at the onset of the oscillations is estimated as

$$n_B \sim \frac{n}{m_\phi} |a| \lambda m_{3/2} \varphi^{n_{\text{osc}}} \sin (n\theta + \arg[a]),$$

where $\varphi_{\text{osc}}$ represents $\varphi$ evaluated at $H = m_\phi$ by using (9). The baryon-to-entropy ratio is then given by

$$\frac{n_B}{s} \sim \frac{m_{3/2}}{m_\phi^2} \varphi^{2}_{\text{osc}} T_{\text{RH}} \sin (n\theta + \arg[a]),$$

where $T_{\text{RH}}$ is the reheating temperature, and we assume that the reheating is not completed when the AD field starts the oscillations, since otherwise too many gravitinos would be produced by thermal scatterings.

From the arguments above, it is clear that the resultant baryon asymmetry is directly related to the baryon-number violating operator, $V_A$. Our estimate on the baryon asymmetry actually remains unchanged even if there is an unsuppressed Hubble-induced $A$-term. However, whether the isocurvature fluctuations in the baryon asymmetry is generated crucially depends on the presence of the Hubble-induced $A$-term. If it is unsuppressed, $\theta$ does not have any sizable fluctuations beyond the Hubble horizon scale during inflation. On the other hand, if the Hubble-induced $A$-term is suppressed, $\theta$ acquires quantum fluctuations of $O(H)$ during inflation, which will turn into the baryonic isocurvature fluctuations. Therefore, the strength of the baryon-number violation is a crucial issue, and we will discuss possible origins of the Hubble-induced $A$-term in the next section.

### 3. Hubble-induced $A$-terms

In this section we discuss conditions for the Hubble-induced $A$-terms to arise and will see that they are not met in most $F$-term inflation models. The Hubble-induced $A$-term is given by (7). The size of the numerical coefficient $b$ is important for estimating the baryonic isocurvature fluctuations. Assuming that the radial component $\varphi$ tracks the instantaneous minimum (9), one can express $V_{AH}$ in terms of $\varphi$ and $\theta$:

$$V_{AH} = \frac{|b|}{\sqrt{n - 1}} H^2 \varphi^2 \cos(n\theta + \arg[b]).$$

Therefore, if $b$ is of $O(1)$, the phase component $\theta$ has a mass comparable to the Hubble parameter. If this is the case, $\theta$ cannot have sizable fluctuations beyond the Hubble horizon scale, and the baryonic isocurvature fluctuations are absent.

What kind of conditions are necessary for the Hubble-induced $A$-term to arise? The $R$ symmetry must be largely broken during inflation in order to have sizable Hubble-induced $A$-terms, since they violate the $R$ symmetry by $\Delta R = 2$, as we will see. If there were not any $R$-breaking terms other than the constant term $W_0 \sim m_{3/2}$ in the superpotential, the Hubble-induced $A$-term could not arise; we would only have the ordinary $A$-term (10).

If the supergravity effects are negligible, the superpotential during inflation can be approximated by

$$W \simeq v^2 I,$$
where \( I \) is the inflaton, and the Hubble parameter during inflation is \( H \simeq |F_I|/\sqrt{3} \sim v^2 \), where \( F_I \simeq -(W_I)^* = -v^2 \) is the \( F \)-term of the inflaton. In the inflation models with multiple scalar fields, the superfield \( I \) in (16) may not be the inflaton which slow-rolls generating the adiabatic density perturbations. For definiteness we call such a field \( I \) that has a large \( F \)-term during inflation the inflaton. Most inflation models such as new, hybrid, and their variants fall into this category. We can then naturally assign the \( R \)-charge of the inflaton as \( R[I] = +2 \). In order to have the Hubble-induced \( A \)-term, the following operators should be unsuppressed [2,12]:

\[
\mathcal{O}_1 = \int d^4 \theta \left( I \cdot |\phi|^2 + \text{h.c.} \right),
\]

\[
\mathcal{O}_2 = \int d^2 \theta I \cdot \frac{\lambda}{n} \phi^n + \text{h.c.,}
\]

where both \( \mathcal{O}_1 \) and \( \mathcal{O}_2 \) violate the \( R \) symmetry by \( \Delta R = 2 \). The operator \( \mathcal{O}_1 \) induces a kinetic mixing, \( g_{I\bar{\phi}} = -g_{\phi I} = \phi \), and the Hubble-induced \( A \)-term arises from \( W_\phi g_{\phi I} (W_I)^* \) (see equation (4)). In the meantime, one can see that \( \mathcal{O}_2 \) generates the Hubble-induced \( A \)-term by noting \( |F_I| \sim H \).

If the operator \( \mathcal{O}_1 \) or \( \mathcal{O}_2 \), is not suppressed, the inflaton \( I \) is effectively a singlet; there is no reason to expect that the inflaton does not have any other operators violating the \( R \)-symmetry. This can cause two cosmological problems. One is that, since \( I \) can be treated as a singlet, it is hard to have a flat potential for successful inflation without severe fine-tunings, because all terms like \( I, I^2, I^3, I^4, \ldots \), which otherwise could be forbidden by symmetry, will appear in the inflaton potential. Thus, a severe fine-tuning will be necessary in general. The other is that the inflaton generically couples to the SUSY breaking sector, and so decays into the gravitinos with an intolerably large rate [13]–[17]. This is nothing short of a cosmological disaster unless the gravitino mass is extremely heavy or light. Therefore, in order to evade these problems, it is usually necessary to impose that the \( R \)-symmetry is a (relatively) good symmetry during inflation, which leads to suppression of both \( \mathcal{O}_1 \) and \( \mathcal{O}_2 \).

So far we have neglected the supergravity effects. In particular, the field value of the inflaton \( I \) is assumed to be smaller than the Planck scale: \( |I| \ll 1 \). This is the case for most inflation models except for the so-called large-scale inflation models. When the supergravity effects are important, the Hubble-induced \( A \)-term is generically generated if one of the following conditions is met:

\[
|I| \gtrsim 1,
\]

\[
|W(I)| \sim H.
\]

If we substitute (19) into (16), we obtain (20). Of course, this naive argument may not be applicable when the full supergravity effects are taken into account: in principle, there can be cancelation between several contributions to \( W \). If \( |I| \gtrsim 1 \), one can obtain the

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9 Another operator, \( \mathcal{O}_1' = \int d^3 \theta I^* \phi \cdot \phi + \text{h.c.} \), can also generate the Hubble-induced \( A \)-term. Such an operator is only allowed for \( H_uH_d \) and \( LH_u \) directions. However, the \( H_uH_d \) direction cannot create any baryon asymmetry of the universe, and the inflaton \( I \) must violate the lepton number for the \( LH_u \) direction.

10 One can obtain the operator \( \mathcal{O}_2 \) from \( \mathcal{O}_1 \), by rescaling \( \phi \) as \( (1+I)\phi \to \phi \) in the presence of the non-renormalizable operator (1).
operator $\mathcal{O}_1$ with a coefficient $\langle I \rangle \gsim 1$, from the quartic coupling $K = c|\phi|^2|I|^2$ needed for the negative Hubble-induced mass term, after expanding the inflaton about its field value. On the other hand, if $|W(I)| \sim H$, it gives rise to the Hubble-induced $A$-term as in the ordinary $A$-term, since $|W(I)|$ measures the breaking of the $R$-symmetry.

Our concern here is whether the condition (19) or (20) is satisfied in the realistic large-scale inflation models such as the chaotic inflation model. Although it is hard to construct a successful chaotic inflation model in supergravity, one was proposed in [18]. Actually, however, neither condition is met in this model. The superpotential is $W = mXY$, where $m \simeq 2 \times 10^{13}$ GeV is the inflaton mass. $Y$ is charged under a shift symmetry and has a large expectation value during inflation, $\text{Im}[Y] \gsim 1$. On the other hand, it is $X$ that has a large $F$-term, so we must consider $I = X$ in equations (19) and (20). During inflation, the $X$ stays at the origin, $X = 0$. Therefore neither (19) nor (20) is satisfied. Let us comment on other large-scale inflation models. The no-scale type chaotic inflation was constructed in [19]. Besides the problem as to the moduli stabilization and too large gravitino mass in this model, the no-scale nature leads to the vanishing Hubble-induced mass and $A$-terms. On the other hand, the natural inflation model was constructed in [20] with the use of a shift symmetry. In this model, Hubble-induced $A$-terms can be obtained, but the gravitino mass is comparable to the Hubble parameter during inflation. Therefore the Hubble-induced $A$-term is more or less comparable to the ordinary $A$-term. We do not consider such cases, and focus on those inflation models that have much larger Hubble parameter than the gravitino mass.

For completeness we consider whether the Hubble-induced $A$-term appears after inflation, because its appearance after inflation will diminish the amplitude of the fluctuations in the $\theta$-direction produced during inflation. The energy density of the universe after inflation is dominated by the inflaton oscillating about its potential minimum until it decays into radiation. Expanding the inflaton about its potential minimum, one can express the Kähler potential and the superpotential respectively as

$$K = |I|^2 + \cdots = I_{\text{min}}^* \hat{I} + I_{\text{min}} \hat{I}^* + |\hat{I}|^2 + \cdots, \quad (21)$$

$$W = \frac{1}{2} M (I - I_{\text{min}})^2 + \cdots = \frac{1}{2} M \hat{I}^2 + \cdots, \quad (22)$$

where $I_{\text{min}}$ denotes the minimum of the potential, and $\hat{I} \equiv I - I_{\text{min}}$ is the excitation of the inflaton about $I_{\text{min}}$. The Hubble parameter is related to the amplitude of the inflaton as

$$H^2 \sim |F_I|^2 \sim M^2 \left\langle \hat{I}^2 \right\rangle, \quad (23)$$

where $F_I = -M \hat{I}^*$ denotes the $F$-term of the inflaton. Since the inflaton oscillates around its potential minimum, the averaged values of $F_I$ vanishes. Therefore the Hubble-induced $A$-term is suppressed after inflation [21].

We therefore conclude that it is hard to generate the Hubble-induced $A$-terms in most, if not all, of supergravity $F$-term inflation without having severe fine-tunings or cosmological difficulties. As we will see in the next section, the absence of the unsuppressed Hubble-induced $A$-term leads to the baryonic isocurvature fluctuations.

In this respect, this chaotic inflation model can be classified into the category considered below equation (16). In fact, $R[X] = +2$ and $v^2 = m/I$. 

\[ \]
4. Baryonic isocurvature fluctuations

If there is no sizable Hubble-induced $A$-term, the phase of the AD field is effectively massless during inflation. Then it acquires quantum fluctuations,

$$\delta \theta = \frac{H_{\text{inf}}}{2\pi \varphi_{\text{inf}}},$$

where $\varphi_{\text{inf}}$ denotes the field value of the $\varphi$ during inflation, obtained by substituting $H = H_{\text{inf}}$ into (9). We neglect the tilt of the fluctuations and assume that the radial component does not have fluctuations beyond the horizon scale. Let us define the baryonic isocurvature fluctuation $S_{b\gamma}$ as

$$S_{b\gamma} \equiv \frac{\delta \rho_B}{\rho_B} - \frac{3}{4} \frac{\delta \rho_{\gamma}}{\rho_{\gamma}} = \delta \log \left( \frac{n_B}{s} \right),$$

where $\rho_B$ and $\rho_{\gamma}$ denote the energy densities of the baryons and photons, respectively, and we use $\rho_B \propto n_B$ and $\rho_{\gamma}^{3/4} \propto s$ in the second equality. Inserting equation (14) into (25), we obtain

$$S_{b\gamma} = n \cot \left( n \theta_{\text{inf}} + \text{arg}[a] \right) \delta \theta. \quad (26)$$

The observations of CMB have shown that the density perturbations are predominantly adiabatic, and therefore the isocurvature perturbations are tightly constrained [22]–[25]. The latest WMAP five-year data set puts an upper bound at 95% C.L. [26],

$$\left| \frac{\Omega_h}{\Omega_c} S_{b\gamma} \right| \lesssim \left( \frac{0.067}{1 - 0.067} \cdot 2.4 \times 10^{-9} \right)^{1/2} \simeq 1.3 \times 10^{-5}, \quad (27)$$

where the isocurvature fluctuation is taken to be uncorrelated with the adiabatic one, and $\Omega_h$ and $\Omega_c$ denote the density parameters of the baryon and the cold dark matter, respectively. With $\Omega_h \simeq 0.046$ and $\Omega_c \simeq 0.23$ [26], we arrive at

$$|S_{b\gamma}| \lesssim 6.6 \times 10^{-5}. \quad (28)$$

Using (14), (24), (26) and (28), we can rewrite the constraint as

$$T_{RH} \lesssim 1.7 \times 10^{-7} \frac{M_P^2}{n^2 m_{3/2}^2} \left( \frac{m_\phi}{H_{\text{inf}}} \right)^{2(n-6)/(n-2)} \left( \frac{n_B}{s} \right) \Theta, \quad (29)$$

with

$$\Theta \equiv \frac{\sin \left( n \theta_{\text{inf}} + \text{arg}[a] \right)}{\cos^2 \left( n \theta_{\text{inf}} + \text{arg}[a] \right)}, \quad (30)$$

where we eliminate $\varphi_{\text{inf}}$ by using $\varphi_{\text{inf}} \simeq (H_{\text{inf}}/m_\phi)^{1/(n-2)} \varphi_{\text{osc}}$. This is the main result of this paper.

The constraint (29) reads

$$T_{RH} \lesssim 6 \times 10^6 \text{GeV} \left( \frac{m_{3/2}}{1 \text{ TeV}} \right)^{-1} \left( \frac{m_\phi}{1 \text{ TeV}} \right) \left( \frac{H_{\text{inf}}}{10^{12} \text{ GeV}} \right)^{-1} \Theta, \quad (31)$$
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Figure 1. Upper bounds on inflation models from AD isocurvature fluctuations for \( n = 4 \) (red, solid lines) and 6 (green, dashed lines). We set \( \Theta = 1 \) and \( m_{3/2} = 1 \) TeV. The reheating is assumed to occur after the AD field starts to oscillate, which gives an upper bound on the reheating temperature, shown by the shaded region. The lower limits on \( T_{RH} \) are shown for the chaotic, new, and hybrid inflations by the purple square, light blue triangle, and gray dot–dashed line, respectively.

\[
T_{RH} \lesssim 80 \ \text{GeV} \left( \frac{m_{3/2}}{1 \ \text{TeV}} \right)^{-1} \left( \frac{m_\phi}{1 \ \text{TeV}} \right)^{3/2} \left( \frac{H_{\text{inf}}}{10^{12} \ \text{GeV}} \right)^{-3/2} \Theta, \quad (32)
\]

for \( n = 4 \), and

\[
T_{RH} \lesssim 80 \ \text{GeV} \left( \frac{m_{3/2}}{1 \ \text{TeV}} \right)^{-1} \left( \frac{m_\phi}{1 \ \text{TeV}} \right)^{3/2} \left( \frac{H_{\text{inf}}}{10^{12} \ \text{GeV}} \right)^{-3/2} \Theta, \quad (32)
\]

for \( n = 6 \), where we use \( n_B/s = 8.8 \times 10^{-11} \). We show the constraints (31) and (32) as red (solid) and green (dashed) lines, respectively, in figure 1. One can see that the upper bound on the reheating temperature becomes severer for larger \( H_{\text{inf}} \) and \( n \). Note also that, for \( \Theta \ll 1 \), the constraint on the reheating temperature becomes relaxed. This can be understood as follows. For \( \Theta \ll 1 \), the baryon asymmetry (14) becomes suppressed, and so we need to make \( \varphi_{\text{inf}} \) larger to compensate it. Then, the baryonic isocurvature perturbation gets suppressed (see (24)), which ameliorates the constraint on the reheating temperature.

A comment on Q-ball formation follows. It is known that non-topological solitons called Q balls may be produced associated with AD mechanism [27, 28]. As long as those Q balls decay eventually into the ordinary quarks, the estimates of the baryon number and the isocurvature fluctuations remain intact. This is the case for the gravity or anomaly mediation. In the gauge mediation, the amplitude of \( A \)-terms and the dynamics of the field are more model dependent. Also the Q balls can be stable, and only a small fraction of the baryon number may be released from them [29]. Then the constraint on the reheating temperature will be relaxed by a factor \( \Delta^{-(n-2)/(2n-6)} \), where \( \Delta \) denotes the fraction of the baryon number emitted from the Q ball. Note however that the lightest supersymmetric particle produced by the decay of the Q balls, or the Q balls themselves if stable, may contribute to the dark matter abundance. In this case, the constraint (29) may become severer, since the constraint on the dark matter isocurvature perturbations is more severe than that on the baryonic one.
5. Constraints on inflation models

There is an absolute upper bound on the reheating temperature obtained from $\rho_{\text{rad}}(T_{\text{RH}}) \leq 3H_{\text{inf}}^2M_p^2$,

$$T_{\text{RH}} \leq \left( \frac{90}{\pi^2g_*} \right)^{1/4} \sqrt{H_{\text{inf}}M_p} \sim 7.2 \times 10^{14} \text{ GeV} \left( \frac{g_*}{200} \right)^{-1/4} \left( \frac{H_{\text{inf}}}{10^{12} \text{ GeV}} \right)^{1/2}.$$ (33)

On the other hand, the inflaton inevitably decays into the standard-model sector through the top Yukawa coupling, if it has non-vanishing expectation value at the potential minimum, i.e., if $I_{\text{min}} \neq 0$ [16]. The lower bound reads

$$T_{\text{RH}} \gtrsim 1.9 \times 10^3 \text{ GeV} |Y_t| \left( \frac{g_*}{200} \right)^{-1/4} \left( \frac{I_{\text{min}}}{10^{15} \text{ GeV}} \right) \left( \frac{m_I}{10^{12} \text{ GeV}} \right)^{3/2},$$ (34)

where $Y_t$ is the top Yukawa coupling. The reason why such a decay proceeds is as follows. When $I_{\text{min}} \neq 0$, the inflaton has a linear term in the Kähler potential as in equation (21). Then the inflaton $I$ couples to any particles that appear in the superpotential. This is because it is a combination given by $e^{K/2W}$ that is more important in supergravity at tree level, instead of $K$ and $W$. In other words, using the Kähler transformation, one can remove the linear term of the inflaton from the Kähler potential, and obtain a new interaction, $W = (I_{*})^T \cdot Y_t TQH_u$, in the superpotential.

The reheating temperature is not known, due to our ignorance of the inflation and the reheating processes; it can a priori take any values between (34) and (33). The upper bound on $T_{\text{RH}}$, (29), therefore excludes large parameter spaces of the reheating temperature, which are otherwise allowed\footnote{There are other constraints on the reheating temperature, coming from, e.g., the thermal production of the gravitinos. Our constraint (29) is independent of those constraints. In fact, (29) gives a very stringent upper bound as (32), which cannot be obtained otherwise.}. Since the bounds (34) and (33) depend on the inflaton parameters such as the inflation scale, the mass and the field value at the potential minimum, let us consider the bounds for representative inflation models such as chaotic [18], new [30], and hybrid [31] inflation models, separately. The details of each of the models are given in the appendix.

For the chaotic inflation model without a $Z_2$ symmetry, we expect $I_{\text{min}} \sim 1$. The inflaton mass is determined to be $m_I \simeq 2 \times 10^{15} \text{ GeV}$, leading to $T_{\text{RH}} \gtrsim 2.4 \times 10^8 \text{ GeV}$ from (34). As can be seen in figure 1, the chaotic inflation is inconsistent with the constraint from the baryonic isocurvature fluctuations, unless we impose a $Z_2$ symmetry on the inflaton to suppress its expectation value as $|I_{\text{min}}| \ll 1$ [13]. Note that the constraint is actually more severe in the chaotic inflation, since the tensor modes will leave less room for the baryonic isocurvature fluctuations.

For the new inflation model, the inflaton parameters are $m_I \simeq 4 \times 10^9 \text{ GeV}$ and $I_{\text{min}} \simeq 3 \times 10^{15} \text{ GeV}$ for $m_{3/2} = 1 \text{ TeV}$. The lower bound on the reheating temperature is $T_{\text{RH}} \gtrsim 1.4 \text{ GeV}$, which can satisfy the bound (29). The inflation scale is relatively low, given by $H_{\text{inf}} \simeq O(10^5) \text{ GeV}$.

For the hybrid inflation model, for $\kappa \sim 10^{-1} - 10^{-5}$, we have $m_I \sim 10^{15} - 10^{10} \text{ GeV}$ and $I_{\text{min}} \simeq 10^{15} \text{ GeV}$ for $H_{\text{inf}} \sim 10^{11} - 10^{6} \text{ GeV}$. The lower bounds on the reheating temperature
read $T_{\text{RH}} \gtrsim 6.0 \times 10^7 - 1.9$ GeV. Therefore, large values of $\kappa$ are excluded by the bounds for isocurvature fluctuations.

6. Conclusion

We have reconsidered the Affleck–Dine mechanism and discussed the condition for the Hubble-induced $A$-term to arise. It has turned out that the $R$-symmetry needs to be largely broken during inflation. If this is the case, however, theoretical and cosmological difficulties arise; one needs fine-tunings to make the inflaton potential flat enough, and moreover, the inflaton decay produces too many gravitinos. Therefore, $R$-symmetry, under which the inflaton is charged, should be a good symmetry to some extent, which results in the suppression of the Hubble-induced $A$-term. Then the phase direction of the AD field is effectively massless during inflation, and its fluctuations contribute to sizable baryonic isocurvature perturbations, while the fluctuation in the radial direction is absent due to the Hubble-induced mass term in the $F$-term inflation models. Note that the Hubble-induced $A$-term is generally suppressed after inflation. Using the latest WMAP five-year data, we have derived tight constraints on the inflation models in terms of the Hubble parameter during inflation and the reheating temperature, based on the assumption that the AD mechanism is responsible for the baryon asymmetry of the universe. The constraint on the reheating temperature is very tight especially for the high-scale inflation models and higher values of $n$. We conclude that large-scale inflation models with high reheating temperatures are not preferred in the context of the AD mechanism.

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Appendix. Inflation models

In the appendix, we give brief explanations on the inflation models that we have considered in the text.

A.1. Chaotic inflation

The Kähler potential is expressed as [18]

$$K(I, I^\dagger) = \frac{1}{2}(I + I^\dagger)^2,$$

which is invariant under the shift $I \rightarrow I + iA$, where $A$ is a dimensionless real parameter. The inflaton field is identified as the imaginary part of $I$, i.e., $\sqrt{2}\text{Im} I$. The superpotential is written as

$$W(I, X) = mXI,$$
which breaks the shift symmetry a bit with a small mass scale $m \simeq 2 \times 10^{13} \text{GeV}$ ($\ll 1$). Here another chiral multiplet $X$ is introduced, whose $F$-term is responsible for the potential energy during and after inflation: $V \simeq |F_X|^2$.

A.2. New inflation

The Kähler potential and superpotential of the inflaton sector are written as [30]

$$K(I, I^\dagger) = |I|^2 + \frac{k}{4} |I|^4,$$

$$W(I) = v^2 I - \frac{g}{s+1} I^{s+1},$$

respectively, where $k$ and $g$ are constants, $s$ is an integer, and $v$ is the inflation energy scale. In order to explain the observed density fluctuations, we need $v \simeq 4 \times 10^{-7} (g/0.1)^{-1/2}$ and $k \lesssim 0.03$ for $s = 4$. After inflation, the vev of the inflaton becomes $I_{\text{min}} \simeq v^2 / g$ and it is given by $m_I \simeq sv^2 / I_{\text{min}}$. The gravitino mass in this model is related to $v$ as $m_3 \simeq sv^2 I_{\text{min}} / (s+1)$, because the inflaton induces a spontaneous $R$-symmetry breaking.

A.3. Hybrid inflation

The superpotential of the inflaton sector is written as [31]

$$W(I, \psi, \bar{\psi}) = I(v^2 - \kappa \psi \bar{\psi}),$$

where the two superfields $\psi(+1)$ and $\bar{\psi}(-1)$ are the waterfall fields, which are charged under a $U(1)$ gauge symmetry. $\kappa$ is a coupling constant and $v$ is the inflation energy scale. To account for the WMAP normalization on the density fluctuations, $v$ and $\kappa$ are related as $v \simeq 2 \times 10^{-3} \kappa^{1/2}$ for $\kappa \gtrsim 10^{-3}$, while $v \simeq 2 \times 10^{-2} \kappa^{5/6}$ for $\kappa \lesssim 10^{-3}$.

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