Statistical analysis on experimental calibration data for flowmeters in pressure pipes

Alessandro Lazzarin, Enrico Orsi, Umberto Sanfilippo

Dipartimento di Ingegneria Civile e Ambientale (DICA), Politecnico di Milano
Piazza Leonardo da Vinci 32, 20133 Milano, Italy, settore-portate@polimi.it

Abstract. This paper shows a statistical analysis on experimental calibration data for flowmeters (i.e.: electromagnetic, ultrasonic, turbine flowmeters) in pressure pipes. The experimental calibration data set consists of the whole archive of the calibration tests carried out on 246 flowmeters from January 2001 to October 2015 at Settore Portate of Laboratorio di Idraulica “G. Fantoli” of Politecnico di Milano, that is accredited as LAT 104 for a flow range between 3 l/s and 80 l/s, with a certified Calibration and Measurement Capability (CMC) - formerly known as Best Measurement Capability (BMC) - equal to 0.2%. The data set is split into three subsets, respectively consisting in: 94 electromagnetic, 83 ultrasonic and 69 turbine flowmeters; each subset is analysed separately from the others, but then a final comparison is carried out. In particular, the main focus of the statistical analysis is the correction $C$, that is the difference between the flow rate $Q$ measured by the calibration facility (through the accredited procedures and the certified reference specimen) minus the flow rate $Q_M$ contemporarily recorded by the flowmeter under calibration, expressed as a percentage of the same $Q_M$.

1. Introduction

For any hydraulic application, like in every engineering field, it is very important to develop and to adopt reliable procedures and devices to measure the physical quantities of interest (i.e.: water depths, pressures, velocities, flows and so on). Moreover, the performances of each single measuring device should be monitored and ensured by means of proper maintenance and periodical calibration. This requirement is valid in general, for both laboratory researches, industrial plants and field surveys.

About calibration, it could be interesting to have also an overview about the probability distributions of the corrections that accredited calibration laboratories provide by tests results for specific kinds of measuring devices, of course hiding customer name and device brand.

To this aim, the present paper shows a statistical analysis on experimental calibration data for flowmeters (i.e.: electromagnetic, ultrasonic, turbine flowmeters) in pressure pipes. The experimental calibration data set consists of the whole archive of the calibration tests carried out on 246 flowmeters from January 2001 to October 2015 at Settore Portate of Laboratorio di Idraulica “G. Fantoli” of Politecnico di Milano, that is accredited as LAT 104 for a flow range between 3 l/s and 80 l/s, with a certified Calibration and Measurement Capability (CMC) - formerly known as Best Measurement Capability (BMC) - equal to 0.2% [9].

The study presented in this paper extends the analyses carried out by the Authors and previously published about the part of the current database that was already available in 2008 [1]. Moreover, now the topic is issued with an enhanced method of analysis.
2. Flowmeters calibration test facility

Figure 1 represents the scheme of the calibration facility at Settore Portate of Laboratorio di Idraulica “G. Fantoli” of Politecnico di Milano (LAT 104), which has been accredited since 2002 [4] by the Italian branch (formerly SIT, now ACCREDIA) of the European Accreditation chain [7], [8].

The adopted calibration method is quite simple, as it is based on the comparison between on the one hand the discharge $Q_M$ measured by the instrument and on the other hand the discharge $Q$ that is really flowing in the pipe on which the instrument is installed.

The real discharge $Q$ is obtained measuring the volume stored, during a fixed time interval, in a prismatic calibrated tank having a capacity of 9.37 m³ and a maximum water depth of 1.15 m. This sample tank is referred to a certified primary specimen of unitary volume (that is 1 m³).

The calibration test phases are the following, according to the scheme reported by Figure 1:

- the discharge that is wanted to flow in the pressure pipe on which the instrument is installed, is diverted from the supplying constant-level tanks;
- the discharge flowing in the pressure pipe is adjusted by a valve placed downstream of the flow meter;
- moving the “start” lever, the flow-diverter starts to divert the flow towards the calibrated tank; before this operation, the discharge went directly to the recirculation system;
- at the same time of moving the “start” lever, a chronometer starts;
- various readings of the discharge indicated by the flowmeter display are carried out;
- at the end of a fixed time interval, the “stop” lever of the flow diverter is moved, in the way that the flow is directed towards the recirculation system; the chronometer stops automatically;
- the tank level raising, due to the water volume flowed into the tank during the fixed time interval, is measured through a staff gauge placed into an appropriate stilling well, and, knowing the tank surface, the real discharge $Q$ is calculated.

Figure 1. Scheme of the calibration facility at Settore Portate of Laboratorio di Idraulica “G. Fantoli” of Politecnico di Milano.

The test facility makes use of certificated instruments (tank, flow-diverter, chronometer, staff gauge, thermometer), that are subjected to periodic checks and calibrations.
During each flowmeter calibration, the performance of the gauge under test is usually evaluated for 5 different discharge values at least, comparing the $Q_M$ value showed by the flowmeter with the correspondent real $Q$ evaluated by the calibration tank. So, the correction $C = Q - Q_M$ is calculated.

Positive corrections ($C > 0$) indicate that the discharge measured by the instrument underestimates the real discharge, while negative corrections ($C < 0$) indicate a flow overestimation.

Each calibration certificate indicates flowmeter data and calibration environmental conditions, the values of the discharges $Q_M$ and $Q$ and of the correction $C$ that testify the various calibration points and, last but not least, the adimensional ratio $C/Q_M$ and the extended uncertainties [2] for the corrections $C$, both absolute $U(C)$ and adimensional $U(C/Q_M)$.

### 3. Experimental database

As already said, the experimental calibration data set consists of the whole archive of the calibration tests carried out on 246 flowmeters from January 2001 to October 2015 at Settore Portate of Laboratorio di Idraulica “G. Fantoli” of Politecnico di Milano, that is accredited as LAT 104 for a flow range between 3 l/s and 80 l/s.

Just few of those flowmeters sustained a calibration test more than once from January 2001 to October 2015, but anyway all of their tests were considered exactly like the others, under the hypothesis that random alterations of their performances might have occurred because of the generally quite long period of time passed from one test to another.

Such a data sample have been split into three subsets, one per each of the three different kind of devices normally tested at that laboratory, respectively consisting in: 94 electromagnetic, 83 ultrasonic and 69 turbine flowmeters; then, each subset has been analysed separately from the others. Indeed, no other kind of flow meters (such as differential pressure flowmeters, Coriolis flowmeters, etc.) has been tested so far at LAT 104 [6].

### 4. Data analysis method

For each single calibrated device, the following analyses on the test results have been considered about the values of the correction $C$, that is - as already said - the difference between the flow rate $Q$ measured by the calibration facility (through the accredited procedures and the certified reference specimen) minus the flow rate $Q_M$ contemporarily recorded by the flowmeter under calibration), expressed as the adimensional ratio $C/Q_M$:

- calculation of the mean value of the adimensional correction (with sign) $m_i(C/Q_M)$ for each generic $i$-th tested flowmeter, as the arithmetic mean of all the values of the adimensional correction $C/Q_M$ for the test points (usually 5 points, that is 5 different flow rates for each tested flowmeter);
- screening and filtering of the mean values of the adimensional correction (with sign) $m_i(C/Q_M)$ through the Chauvenet criterion [5], in order to discard anomalous flowmeters of each one of the three subsets;
- calculation of both the mean value and the standard deviation of the mean values of the adimensional correction (with sign) $m(m_i(C/Q_M))$ and $s(m_i(C/Q_M))$ just for not discarded flowmeters;
- frequency distribution plots about the mean values of the adimensional correction (with sign) $m(m_i(C/Q_M))$ for not discarded flowmeters;
- implementation of the above described procedure also to the values of the adimensional correction expressed as absolute value $|C|/Q_M$ (that is without sign).
In particular, the Chauvenet criterion has been applied, for each one of the three data subsets separately (respectively about: electromagnetic, ultrasonic and turbines), as follows:

- a first estimation of the mean $m(m_i(C/Q_M))$ and the standard deviation $s(m_i(C/Q_M))$ of the $N$ values of the elements $m_1(C/Q_M)$, $m_2(C/Q_M)$, ..., $m_i(C/Q_M)$, ..., $m_N(C/Q_M)$, where – as already said – each $m_i(C/Q_M)$ represents the global mean results of a specific calibration certificate;
- individuation of the maximum absolute difference between $m(m_i(C/Q_M))$ and each single value of $m_i(C/Q_M)$, that is $t = \max \{|m_i(C/Q_M) - m(m_i(C/Q_M))|\}$;
- calculation of the probability value $P(t)$, that is equal to the cumulated probability of occurrence of $t$ under the hypothesis that the quantity $(m_i(C/Q_M) - m(m_i(C/Q_M)))/s(m_i(C/Q_M))$ is just a standard Gaussian random variable;
- calling $P(t) = (100\% - P'(t))$, the datum of $m_i(C/Q_M)$, that is the calibration certificate, which corresponds to $\max \{|m_i(C/Q_M) - m(m_i(C/Q_M))|\}$ must be discarded from the sample if $N \cdot P < 0.5$;
- if this happens, then such a datum is discarded from the data sample, that therefore becomes made of just $N - 1$ left elements; so, new values of mean $m(m_i(C/Q_M))$ and standard deviation $s(m_i(C/Q_M))$ must be recalculated.
- such a procedure is iterated untill no further datum has to be rejected (maybe even the first turn could be already the right one, i.e. when no datum has to be discarded);
- after that, the same Chauvenet filtering criterion is applied also to each one of the three data subsets separately (respectively about: electromagnetic, ultrasonic and turbines) of the adimensional correction expressed as absolute value $|C/Q_M|$ (that is without sign), in a similar way.

5. Results

Table 1, Table 2 and Table 3 show the summary of the results for the experimental data analysis carried out on the calibration certificates for respectively electromagnetic flowmeters, ultrasonic flowmeters and turbine flowmeters, for both the adimensional correction expressed with sign $C/Q_M$ and the adimensional correction expressed as absolute value $|C/Q_M|$. In particular:

- the first column reports the number of actually considered certificates $N_i$, as the difference between the total number of certificates $T$ minus the number of the discarded certificates $D$ because of the Chauvenet filtering;
- the second column reports the mean value and the standard deviation of the mean values of the adimensional correction:
  - with sign: $m(m_i(C/Q_M))$ and $s(m_i(C/Q_M))$;
  - as absolute value: $m(m(|C/Q_M|))$ and $s(m(|C/Q_M|))$;
- the third column reports, in addition, also the mean value of the standard deviation (i.e. standard deviation $s_i$ of the outcomes of the adimensional correction in a generic $i$-th certificate) and the standard deviation of the standard deviation:
  - with sign: $m(s_i(C/Q_M))$ and $s(s_i(C/Q_M))$;
  - as absolute value: $m(s_i(|C/Q_M|))$ and $s(s_i(|C/Q_M|))$;

Moreover, Figure 1, Figure 2 and Figure 3 show the summary of frequency distribution plots about the statistical analysis of the mean $m_i$ of the adimensional corrections in the $N$ actually considered certificates.
Table 1. Summary of the results for the experimental data analysis carried out on the calibration certificates for the electromagnetic flowmeters tested by Settore Portate of Laboratorio di Idraulica “G. Fantoli” of Politecnico di Milano (LAT 104).

| Number of actually considered certificates | Statistical analysis of the mean $m_i$ of the adimensional corrections in the $N$ actually considered certificates | Statistical analysis of the st. dev. $s_i$ of the adimensional corrections in the $N$ actually considered certificates |
|-------------------------------------------|-------------------------------------------------|-------------------------------------------------|
| $88 = 94 - 6$                             | Mean $m(m(C/Q_U)) = -0.18\%$                    | Mean $m(s_i(C/Q_U)) = 0.66\%$                   |
|                                           | St. Dev. $s(m(C/Q_U)) = 1.79\%$                | St. Dev. $s(s_i(C/Q_U)) = 0.68\%$              |
| $87 = 94 - 7$                             | Mean $m(m_i(C/Q_U)) = 1.24\%$                  | Mean $m(s_i(C/Q_U)) = 0.54\%$                  |
|                                           | St. Dev. $s(m_i(C/Q_U)) = 1.19\%$              | St. Dev. $s(s_i(C/Q_U)) = 0.55\%$              |

Table 2. Summary of the results for the experimental data analysis carried out on the calibration certificates for the ultrasonic flowmeters tested by Settore Portate of Laboratorio di Idraulica “G. Fantoli” of Politecnico di Milano (LAT 104).

| Number of actually considered certificates | Statistical analysis of the mean $m_i$ of the adimensional corrections in the $N$ actually considered certificates | Statistical analysis of the st. dev. $s_i$ of the adimensional corrections in the $N$ actually considered certificates |
|-------------------------------------------|-------------------------------------------------|-------------------------------------------------|
| $82 = 83 - 1$                             | Mean $m(m(C/Q_U)) = +1.09\%$                    | Mean $m(s_i(C/Q_U)) = 1.65\%$                   |
|                                           | St. Dev. $s(m(C/Q_U)) = 4.97\%$                | St. Dev. $s(s_i(C/Q_U)) = 1.50\%$              |
| $81 = 83 - 2$                             | Mean $m(m_i(C/Q_U)) = 3.68\%$                  | Mean $m(s_i(C/Q_U)) = 1.41\%$                  |
|                                           | St. Dev. $s(m_i(C/Q_U)) = 3.04\%$              | St. Dev. $s(s_i(C/Q_U)) = 1.36\%$              |

Table 3. Summary of the results for the experimental data analysis carried out on the calibration certificates for the turbine flowmeters tested by Settore Portate of Laboratorio di Idraulica “G. Fantoli” of Politecnico di Milano (LAT 104).

| Number of actually considered certificates | Statistical analysis of the mean $m_i$ of the adimensional corrections in the $N$ actually considered certificates | Statistical analysis of the st. dev. $s_i$ of the adimensional corrections in the $N$ actually considered certificates |
|-------------------------------------------|-------------------------------------------------|-------------------------------------------------|
| $69 = 69 - 0$                             | Mean $m(m(C/Q_U)) = -0.11\%$                    | Mean $m(s_i(C/Q_U)) = 0.70\%$                   |
|                                           | St. Dev. $s(m(C/Q_U)) = 2.41\%$                | St. Dev. $s(s_i(C/Q_U)) = 0.74\%$              |
| $69 = 69 - 0$                             | Mean $m(m_i(C/Q_U)) = 1.73\%$                  | Mean $m(s_i(C/Q_U)) = 0.57\%$                  |
|                                           | St. Dev. $s(m_i(C/Q_U)) = 1.75\%$              | St. Dev. $s(s_i(C/Q_U)) = 0.63\%$              |
Figure 2. Frequency distribution plots about the results for the experimental data analysis carried out on the calibration certificates for the electromagnetic flowmeters tested by Settore Portate of Laboratorio di Idraulica “G. Fantoli” of Politecnico di Milano (LAT 104) from January 2001 to October 2015: a) for adimensional values with sign \(\frac{C}{Q_m}\); b) for adimensional absolute values \(|\frac{C}{Q_m}|\).
Figure 3. Frequency distribution plots about the results for the experimental data analysis carried out on the calibration certificates for the ultrasonic flowmeters tested by Settore Portate of Laboratorio di Idraulica “G. Fantoli” of Politecnico di Milano (LAT 104) from January 2001 to October 2015:

a) for adimensional values with sign $C/Q_M$;

b) for adimensional absolute values $|C|/Q_M$.
Figure 4. Frequency distribution plots about the results for the experimental data analysis carried out on the calibration certificates for the turbine flowmeters tested by Settore Portate of Laboratorio di Idraulica “G. Fantoli” of Politecnico di Milano (LAT 104) from January 2001 to October 2015:

a) for adimensional values with sign $C/Q_M$;

b) for adimensional absolute values $|C|/Q_M$. 

Turbine Flowmeters
$N = 69$
$m = -0.11\%$
$s = 2.41\%$

Turbine Flowmeters
$N = 69$
$m = 1.73\%$
$s = 1.75\%$
6. Conclusions
The following conclusions can be drawn from the results of the statistical analysis presented here.

- The Chauvenet filtering criterion has discarded a significant number of certificates for electromagnetic flowmeters (6 out of 94 for $C/Q_M$ and 7 out of 94 for $|C|/Q_M$), very few certificates for ultrasonic flowmeters (1 out of 83 for $C/Q_M$ and 2 out of 83 for $|C|/Q_M$) and none for turbine flowmeters (0 out of 69 for both $C/Q_M$ and $|C|/Q_M$); this implies that the performances among the different electromagnetic flowmeters that have been tested look less homogeneous in comparison to the other two kind of devices.

- Nevertheless, once the worst certificates have been the discarded by the Chauvenet filtering criterion, so just considering the remaining data, the standard deviation of the mean values for the tested electromagnetic flowmeters (1.79% for $C/Q_M$ and 1.19% for $|C|/Q_M$) is slightly lower than the one for the tested turbine flowmeters (2.41% for $C/Q_M$ and 1.75% for $|C|/Q_M$), while the highest standard deviation of the mean values is by far for the tested ultrasonic flowmeters (4.97% for $C/Q_M$ and 3.04% for $|C|/Q_M$); this implies that the performances among just the electromagnetic flowmeters of the not discarded certificates look more homogeneous in comparison to what happens to the other two kind of devices (indeed especially in comparison to ultrasonic flowmeters).

- The mean value of the mean values of the adimensional correction (with sign) $C/Q_M$, again considering just not discarded flowmeters, comes out actually (as expected) close to 0% for the tested electromagnetic flowmeters (-0.18%) and turbine flowmeters (-0.11%), but not negligible for the tested ultrasonic flowmeters (+1.09%); but this crucial issue should be investigated with a larger data set than the available one, because it could be caused either by the unlucky contribute of some bad performing tested devices (although not discarded by the Chauvenet filtering criterion as the standard deviation for ultrasonic flowmeters is quite high) or by a really systematic problem.

- Frequency distribution plots about the mean values of the adimensional correction (with sign) $C/Q_M$, just for not discarded flowmeters, looks Gaussian for each one of the three subsets of tested flowmeters.

- In terms of the mean values of the absolute values of the corrections (that is $|C|/Q_M$), the best average performances seem to be the one of the tested electromagnetic flowmeters (1.24%), followed by the tested turbine flowmeters (1.73%) and finally by the tested ultrasonic flowmeters (3.68%).

- Frequency distribution plots about the mean values of the adimensional correction (as absolute values) $|C|/Q_M$, just for not discarded flowmeters, looks exponentially decreasing for each one of the three subsets of tested flowmeters.

- Both the mean of the standard deviation and the standard deviation of the standard deviation are quite low for the tested electromagnetic flowmeters (the mean of the standard deviation is 0.66% for $C/Q_M$ and 0.54% for $|C|/Q_M$, while the standard deviation of the standard deviation is 0.68% for $C/Q_M$ and 0.55% for $|C|/Q_M$) and for the tested turbine flowmeters as well (the mean of the standard deviation is 0.70% for $C/Q_M$ and 0.57% for $|C|/Q_M$, while the standard deviation of the standard deviation is 0.74% for $C/Q_M$ and 0.63% for $|C|/Q_M$); but they are significantly higher for the tested ultrasonic flowmeters (the mean of the standard deviation is 1.65% for $C/Q_M$ and 1.41% for $|C|/Q_M$, while the standard deviation of the standard deviation is 1.50% for $C/Q_M$ and 1.36% for $|C|/Q_M$); this suggest that the performances of each one of the tested ultrasonic flowmeters are more variable through the different test points (that is for the different flow rates) in comparison to the tested electromagnetic and turbine flowmeters.
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References
[1] Camnasio E., Lazzarin A., Orsi E., Sanfilippo U., “Experimental Data on the Reliability of Flowmeters for Pressure Pipes”, International Conference “IGHEM 2008”, Innovation on Hydraulic Efficiency Measurement, 3-6 September 2008, Milan, Italy.
[2] European Accreditation, “Evaluation of the Uncertainty of Measurement in Calibration”, EA-4/02 M: 2013.
[3] Gorla F., Trentin M., Analisi statistica su dati di taratura relativi a misuratori di portata per correnti in pressione, Thesis, Politecnico di Milano, 2016.
[4] Orsi E., Sanfilippo U., Taratura di misuratori di portata con metodo volumetrico e certificazione di qualità, in “Il Giornale dell'Ingegnere”, N. 19, p. 11, Italy, 15 nov 2002.
[5] Taylor J.R., An Introduction to Error Analysis – The Study of Uncertainties in Physical Measurements, Second Edition, University Science Books, Sausalito, CA, United States,1997.
[6] https://en.wikipedia.org/wiki/Flow_measurement – Summary of flowmeters technologies.
[7] www.accredia.it – ACCREDIA official website.
[8] www.european-accreditation.org – European Accreditation official website.
[9] www.qualita.polimi.it/index.php?option=com_sppagebuilder&view=page&id=33 – Settore Portate of Politecnico di Milano LAT 104 official website.