On the nature of $\Xi_c(2930)$

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The single charmed excited $\Xi_c(2930)$ state was discovered many years ago by BABAR collaboration and recently confirmed by Belle experiment. However, both of these experiments, unfortunately, could not fix the quantum numbers of this particle and its nature is under debates. In the present study, we calculate its mass and width of its dominant decay to $\Lambda_c K$. To this end we consider $\Xi_c(2930)$ state once as angularly excited and then radially excited single charmed baryon in $\Xi_c$ channel. Comparison of the obtained results with the experimental data suggests assignment of $\Xi_c(2930)$ state as the angular excitation of the ground state $\Xi_c$ baryon with quantum numbers $J^P = \frac{3}{2}^-$.

I. INTRODUCTION

With impressive developments of experimental techniques many new conventional and exotic states have been discovered [1]. These discoveries have opened a new direction in hadron physics. Indeed, the heavy baryon spectroscopy receives special attention as heavy baryons represent a very suitable place for testing the ground of heavy quark symmetry and provide us with deep understanding of the details of the strong interaction.

While many new excited charmed baryons are discovered in different experiments and many theoretical works are devoted to establishing their quantum numbers, still their nature is not evident, and many open questions remain on their internal structure and quark organization. For instance, there have been made many suggestions on the structures of the newly observed five narrow resonances $\Omega_c(3000), \Omega_c(3050), \Omega_c(3066), \Omega_c(3090), \Omega_c(3119)$ by LHCb Collaboration in the $\Xi_c^+ K^-$ invariant mass spectrum [2]: some authors have treated them as usual three-quark resonances [3-7], while some others have interpreted them as new penta-quark states [8]. At present, there is unfortunately no any phenomenological model, which can successfully describe the properties of such complicated systems [10, 11]. For this, more experimental and theoretical attempts are needed to understand dynamics of these new systems.

New some excited states at $\Xi_c$, $\Sigma_c$ and $\Lambda_c$ channels have also been discovered that are of great importance and deserve investigations with the aim of clarification of their nature and internal structure. The observation of the charmed-strange baryon $\Xi_c(2930)$, which is the subject of the present study, has a long history in the experiment. This state was firstly observed by BABAR Collaboration in 2008 with mass $m = [2931 \pm 3\text{(stat.)} \pm 5\text{(syst.)}] \text{MeV}/c^2$ and width of $\Gamma = [36 \pm 7\text{(stat.)} \pm 11\text{(syst.)}] \text{MeV}$ as an intermediate resonance in the decay $B^- \to \Lambda_c^+ \Lambda_c^- K^-$. Note that the Belle Collaboration had before measured the branching ratios of the decays $B^+ \to \Lambda_c^+ \Lambda_c^- K^+$ and $B^0 \to \Lambda_c^+ \Lambda_c^- K^0$ in 2006 [13] but could not find any intermediate charmed resonances. After observation of $\Xi_c(2930)$ by BABAR, the state $\Xi_c(2930)$ was investigated in the framework of different theoretical models like constituent quark model [14, 15], chiral quark model [16], QCD sum rules [17], etc.

Very recently, Belle Collaboration performed an updated measurement on $B^- \to K^- \Lambda_c^+ \Lambda_c^- K^-$ decay and observed the $\Xi_c(2930)$ state in the $K^- \Lambda_c^+$ invariant mass with a significance of $5.1\sigma$ [18]. The measured mass and width is:

$$m = [2928.9 \pm 3\text{(stat.)} \pm 0.8\text{(syst.)}] \text{MeV}/c^2$$
$$\Gamma = [19.5 \pm 8\text{(stat.)} \pm 5\text{(syst.)}] \text{MeV},$$

respectively. However, both of the experiments could not, unfortunately, fix the quantum numbers of $\Xi_c(2930)$ state. This automatically suggests more experimental and theoretical efforts on the properties of this resonance.

We aim to calculate some parameters of $\Xi_c(2930)$ state in the present study to clarify its nature and fix its quantum numbers. To this end, we assume it once as angularly excited negative parity ($\Xi_c^{-}$) and the second as radially excited positive parity ($\Xi_c^{+}$) spin-1/2 baryon at $\Xi_c$ channel. In quark model’s notations these states are represented by $1^2P_{1/2}$ and $2^2S_{1/2}$, respectively. For customary, in next discussions, we will denote these states as $J^P = 1/2^-$ and $J^P = 1/2^+$, respectively. We evaluate the widths of the strong decays $\Xi_c \to \Lambda_c K$ and $\Xi_c \to \Lambda_c K$. For this, firstly we compute the mass and residue of the ground state, first angularly and radially excited $\Xi_c$ baryons as well as the couplings of the strong $\Xi_c \Lambda_c K$ and $\Xi_c \Lambda_c K$ vertices allowing us to find the required decay widths. For calculation of the masses and residues we employ QCD two-point sum rule, whereas in the case of the strong couplings we apply the technique of QCD light-cone sum rule (LCSR). Note that using $J^P = 1/2^0$ model the authors in [19, 20] have concluded that the resonance $\Xi_c(2930)$ may be P-wave, D-wave or 2S-wave excitation of the ground state $\Xi_c$ baryon with different
quantum numbers, $J^P = \frac{3}{2}^+, \frac{5}{2}^+$ or $\frac{5}{2}^-$ by analyses of different excitations of charmed strange baryons. In order to distinguish among these possibilities, they have suggested measurements of some ratios of the branching fractions associated to some possible decay modes of the $\Xi_c(2930)$ state.

The article is organized in the following way. In section II, the mass sum rules for $\Xi_c$ baryons including its first angular and radial excitations are calculated, and the values of the masses and residues are found. Section III is devoted to the calculation of the strong coupling constants defining the $\Xi_c\Lambda_cK$ and $\Xi_c\Lambda_cK$ vertices. We estimate the widths of the decay channels under consideration and compare the results obtained on the masses and widths with the experimental data with the aim of fixing the quantum numbers of the $\Xi_c(2930)$ resonance. The last section is reserved for summary and concluding remarks.

II. MASSES AND POLE RESIDUES OF THE FIRST ANGULARLY AND RADially EXCITED $\Xi_c$ STATES

As we noted, the $\Xi_c(2930)$ has been seen as a peak in the $\Lambda_c^+K^-$ invariant mass distribution. But unfortunately, its quantum numbers have not established yet. In present work, we consider two possible scenarios for it: a) The $\Xi_c(2930)$ is considered as the radial excitation of the ground state $\Xi_c(2467)$. In other words it carries the same quantum numbers as $\Xi_c(2467)$, i.e. $J^P = \frac{1}{2}^+$.

b) The $\Xi_c(2930)$ is treated as the first angular excitation of the $\Xi_c(2467)$, that is negative parity baryon with $J^P = \frac{1}{2}^-$. Note that, in the following, we will consider in more details the second scenario. The results for the first scenario will be obtained by some replacements that will be mentioned later. Here we should also note that in [21] the P-wave heavy baryon masses are calculated with QCD sum rules in the framework of the heavy quark effective theory.

In order to calculate the mass and residue of $\Xi_c$ baryon, we start with the following two point correlation function:

$$\Pi(q) = i \int d^4xe^{iq\cdot x} \langle 0|T\{\eta_{\Xi_c}(x)\eta_{\Xi_c}(0)\}|0\rangle,$$

where $\eta_{\Xi_c}(x)$ is the interpolating current for $\Xi_c$ state with spin-parity $J^P = \frac{1}{2}^+$ and $T$ indicates the time ordering operator. The general form of the interpolating current for the heavy spin-$\frac{1}{2}$ $\Xi_c$ baryon belonging to antitriplet representations of $SU(3)$ can be written as:

$$\eta_{\Xi_c} = \frac{\epsilon_{abc}}{\sqrt{6}} \left\{ \frac{2}{q_1^aTq_2^b} \gamma_5c\bar{c} + 2\frac{\beta}{q_1^aT\gamma_5q_2^b} c\bar{c} \right\} + \frac{\epsilon_{abc}}{q_1^aTc\bar{b}} \gamma_5q_2^c + \beta \left( q_1^aT\gamma_5c\bar{b} \right) q_2^c + \left( c^aTq_2^b \right) \gamma_5q_1^c + \beta \left( c^aT\gamma_5q_2^b \right) q_1^c \right\},$$

where $a, b, c$ are the color indices, $C$ is the charge conjugation operator and $\beta$ is an arbitrary parameter with $\beta = -1$ corresponding to the lotope current. $q_1$ and $q_2$ are $u(d)$ and $s$ quarks for $\Xi_c^+(\Xi_c^0)$ baryon, respectively. Here some details about the above current are in order. According to the quark model, $\Xi_c$ belongs to the antitriplet representation of the $SU(3)$, i.e. the current describing this state should be antisymmetric with respect to the exchange of the light quarks' fields. The interpolating current must also be a color singlet. Therefore, its general form satisfying both these conditions can be written as:

$$\eta_{\Xi_c} \sim \epsilon_{abc} \left\{ \left( q_1^aTc\bar{b} \right) \gamma_5c\bar{c} + \left( q_1^aTc\bar{b} \right) \gamma_5q_2^c - \left( q_1^aTc\bar{b} \right) q_2^c \right\},$$

where $\Gamma, \tilde{\Gamma} = 1, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu$ or $\sigma_{\mu\nu}$. We need to determine $\Gamma$ and $\tilde{\Gamma}$. To this end let us first consider the transpose of the term $\epsilon_{abc}(q_2^aTq_1^b)$:

$$\left[ \epsilon_{abc} q_1^aTc\bar{b} \right]^T = \epsilon_{abc} q_2^aT C(CTC^{-1}) q_1^b,$$

where $CTC^{-1}$ is equal to $\Gamma$ for $\Gamma = 1, \gamma_5$ or $\gamma_5\gamma_\mu$ and it is equal to $-\Gamma$ for $\Gamma = \gamma_\mu$ or $\sigma_{\mu\nu}$. The transpose of a one by one matrix should be equal to itself, i.e.

$$\epsilon_{abc} q_1^aTc\bar{b} = -\epsilon_{abc} q_2^aTc\bar{b},$$

for $\Gamma = 1, \gamma_5$ or $\gamma_5\gamma_\mu$. In result, indeed $\epsilon_{abc} q_1^aTc\bar{b}$ is antisymmetric for the quark $q_1 \leftrightarrow q_2$ replacement if $\Gamma = 1, \gamma_5$ or $\gamma_5\gamma_\mu$.

The simplest way is to take the $\Xi_c(2930)$ to have the same total spin and spin projection as the charm quark. Thus the spin of the diquark formed by light quarks is zero. This implies $\Gamma = 1$ or $\gamma_5$. Therefore, the two possible forms of the interpolating current can be written as:

$$\eta_1 = \epsilon_{abc} \left( q_1^aTc\bar{b} \right) \gamma_5c\bar{c}$$

$$\eta_2 = \epsilon_{abc} \left( q_1^aTc\bar{b} \right) \gamma_5q_2^c.$$

The forms of $\Gamma_1$ and $\Gamma_2$ are determined through the Lorentz and parity considerations. Since $\eta_1$ and $\eta_2$ are Lorentz scalars, one must have $\Gamma_1 = 1, \Gamma_2 = 1$ or $\gamma_5$. The parity transformation leads to the result that $\Gamma_1 = \gamma_5$ and $\Gamma_2 = 1$. Therefore the two possible forms of the interpolating current are:

$$\eta_1 = \epsilon_{abc} \left( q_1^aTc\bar{b} \right) \gamma_5c\bar{c}$$

$$\eta_2 = \epsilon_{abc} \left( q_1^aTc\bar{b} \right) \gamma_5q_2^c.$$

Obviously their arbitrary linear combination can better represent the baryon under consideration, i.e.

$$\eta \sim \epsilon_{abc} \left[ \left( q_1^aTc\bar{b} \right) \gamma_5c\bar{c} + \beta \left( q_1^aTc\bar{b} \right) q_2^c \right],$$
where we introduced the general parameter $\beta$ to obtain the most general form of $\eta$. Performing similar analyses for the second and third terms in Eq. (1) and with the combinations presented in Eq. (3) we get the most general form of the interpolating current for $\Xi_c(2930)$.

To derive the mass sum rules for the $\Xi_c(2930)$ baryon we calculate the correlation function using two languages: hadronic, in terms of the masses and residues called the physical side and QCD, in terms of the fundamental QCD degrees of freedom called the QCD or theoretical side. By equating these two representations, one can get the QCD sum rules for the physical quantities of the baryons under consideration. The physical side of the correlation function is obtained by inserting the complete sets of intermediate states with both parities:

$$\Pi^{\text{Phys}}(q) = \frac{\langle 0|\eta_\Xi_c|\Xi_c(q,s)\rangle \langle \Xi_c(q,s)|\eta_\Xi_c \rangle}{m^2 - q^2} + \frac{\langle 0|\tilde{\eta}_\Xi_c|\tilde{\Xi}_c(q,\tilde{s})\rangle \langle \tilde{\Xi}_c(q,\tilde{s})|\tilde{\eta}_\Xi_c \rangle}{\tilde{m}^2 - q^2} + \ldots , \tag{10}$$

where $m$, $\tilde{m}$ and $s$, $\tilde{s}$ are the masses and spins of the ground and first angularly excited $\Xi_c$ baryons, respectively. The dots denote contributions of higher resonances and continuum states. In Eq. (10) the summations over the spins $s$ are $\tilde{s}$ are implied.

The matrix elements in Eq. (10) are determined as

$$\langle 0|\eta_\Xi_c|\Xi_c(q,s)\rangle = \lambda u(q,s),$$
$$\langle 0|\tilde{\eta}_\Xi_c|\tilde{\Xi}_c(q,\tilde{s})\rangle = \tilde{\lambda} \gamma_5 u^-(q,\tilde{s}). \tag{11}$$

Here $\lambda$ and $\tilde{\lambda}$ are the residues of the ground and first angularly excited $\Xi_c$, baryons, respectively. Using Eqs. (10) and (11) and carrying out summations over the spins of corresponding baryons, we obtain

$$\Pi^{\text{Phys}}(q) = \frac{\lambda^2(q + m)}{m^2 - q^2} + \frac{\tilde{\lambda}^2(q - \tilde{m})}{\tilde{m}^2 - q^2} + \ldots . \tag{12}$$

Performing Borel transformation of this expression we have

$$\mathcal{B}\Pi^{\text{Phys}}(q) = \lambda^2 e^{-\frac{m^2}{2\Lambda}}(q + m) + \tilde{\lambda}^2 e^{-\frac{\tilde{m}^2}{2\Lambda}}(q - \tilde{m}). \tag{13}$$

The QCD side of the aforementioned correlation function is calculated in terms of the QCD degrees of freedom in deep Euclidean region. After inserting the explicit form of the interpolating current given by Eq. (3) into the correlation function in Eq. (2) and performing contractions via the Wick’s theorem, we get the QCD side in terms of the light and heavy quarks propagators. By using light and heavy quark propagators in the coordinate space and performing the Fourier and Borel transformations, as well as applying the continuum subtraction, after lengthy calculations for the correlation function we obtain

$$\mathcal{B}\Pi^{\text{QCD}}(q) = \mathcal{B}\Pi^{\text{QCD}}_1(q) + \mathcal{B}\Pi^{\text{QCD}}_2(q), \tag{14}$$

| Parameters | Values |
|------------|--------|
| $m_c$      | (1.28 ± 0.03) GeV [1] |
| $m_s$      | 96.3 ± 5.4 MeV [1] |
| $m_{\Xi_c(2467)}$ | (2467.8 ± 0.30) MeV [1] |
| $\langle q\bar{q} \rangle$ | $-0.24 ± 0.01$ GeV$^3$ |
| $\langle s\bar{s} \rangle$ | $0.8 · (-0.24 ± 0.01)^3$ GeV$^3$ |
| $\langle \pi q Gq \rangle$ | $m_0^2(q\bar{q})$ |
| $\langle \pi Gq Gs \rangle$ | $m_0^2(\bar{s}s)$ |
| $m_0^2$ | $0.8 ± 0.1$ GeV$^2$ |
| $\langle \bar{u} s d \rangle^2$ | $0.012 ± 0.004$ GeV$^4$ |

To perform analysis of the sum rules for the masses and residues of the angularly and radially excited state it is enough to make replacement $\tilde{m} \to -m'$ and redefine the residue $\tilde{\lambda}$ as $\tilde{\lambda}'$ in expressions of Eq. (16).
of the Ξ_c baryon as well as the residue of the ground state we need some inputs which are presented in Table I. The
mass of the ground state $\Xi_c$ is also taken as an input parameter. Besides the input parameters, QCD sum rules contains three auxiliary parameters namely the continuum threshold $s_0$, Borel parameter $M^2$ and an arbitrary mixing parameter $\beta$. The working windows of these parameters are determined by demanding that the physical quantities under consideration are roughly independent of these parameters. To assess the working interval of the Borel parameter $M^2$ one needs to consider two criteria: convergence of the series of operator product expansion (OPE) and adequate suppression of the higher states and continuum. Consideration of these criteria in the analysis leads to the following working interval of $M^2$:

$$3 \text{ GeV}^2 \leq M^2 \leq 5 \text{ GeV}^2. \quad (17)$$

To determine the working region of the continuum threshold, we impose the conditions of the pole dominance and OPE convergence. This leads to the interval

$$3.1^2 \text{ GeV}^2 \leq s_0 \leq 3.3^2 \text{ GeV}^2. \quad (18)$$

In order to explore the sensitivity of the obtained results on the Borel parameter $M^2$ and continuum threshold $s_0$, as examples, in Figs. 1-4 we depict the mass of the $\Xi_c$ baryon and residues of the ground state $\Xi_c$, $\Xi_c'$ and $\Xi_c''$ baryons as functions of these parameters at fixed value of $\beta = -0.75$. From these figures, we see weak dependence of the quantities under consideration on $M^2$ and $s_0$, satisfying the requirements of the method used.

To find the working region of $\beta = \tan \theta$, as examples, in Fig. 5 we present the dependence of the $\Xi_c$’s mass and residue on $\cos \theta$ at average values of $M^2$ and $s_0$. From this figure we see that the results show relatively weak dependence on the variations of $\cos \theta$ when it varies in the regions

$$-1 \leq \cos \theta \leq -0.5, \quad 0.5 \leq \cos \theta \leq 1. \quad (19)$$
TABLE II: The sum rule results for the masses and residues of the first angularly and radially excited $\Xi_c$ baryon as well as residue of the ground state. Note that the masses of the angularly and radially excited states are obtained to be exactly the same.

| $\Xi_c$ (MeV) | $\Xi_c'$ (MeV) |
|-------------|-------------|
| 2922 ± 83  | 2922 ± 83  |

The errors coming from the variations of the results with respect to the variations of the auxiliary parameters remain within the limits allowed by the method used and they are included in final results.

We depict the numerical results of the masses and residues of the first angularly and radially excited $\Xi_c$ baryons as well as the residue of the ground state particle obtained using the above-presented working intervals for the auxiliary parameters in table II. Note that we get the same mass for the first angularly and radially excited $\Xi_c$ baryons. The errors in the presented results are due to the uncertainties in the determination of the working regions for the auxiliary parameters as well as the errors of other input parameters. The values presented in table II will be used as inputs in next section.

III. $\Xi_c$ AND $\Xi_c'$ TRANSITIONS TO $\Lambda_c K$

In this section we calculate the strong coupling constants $g_{\Xi_c \Lambda_c K}$ and $g_{\Xi_c' \Lambda_c K}$, which are necessary to calculate widths of the decays $\Xi_c \rightarrow \Lambda_c K$ and $\Xi_c' \rightarrow \Lambda_c K$. For this aim we introduce the correlation function

$$T(p, q) = i \int d^4x e^{i p x} \langle K(q) | T \{ J_{\Lambda_c}(x) \bar{J}_{\Xi_c}(0) \} | 0 \rangle,$$  

(20)

where $J_{\Lambda_c}(x)$ is the interpolating current for the $\Lambda_c$ baryon which can be obtained from Eq. (14) with $q_1 = u$ and $q_2 = d$.

Firstly let consider the $\Xi_c \rightarrow \Lambda_c K$ transition. Before calculations we note that the interpolating current for $\Lambda_c$ interact with both positive and negative parity $\Lambda_c$ baryons. Taking into consideration this fact, inserting complete sets of hadrons with the same quantum numbers as the interpolating currents and isolating the ground states, we obtain

$$T^{\text{Phys}}(p, q) = \frac{\langle 0 | J_{\Lambda_c^+}(p, s) \rangle}{p^2 - m^2_{\Lambda_c^+}} (K(q) \Lambda_c^+(p, s) | \Xi_c(p', s') \rangle \times \frac{\langle \Xi_c(p', s') | \bar{J}_{\Xi_c}(0) \rangle}{p^2 - m^2_{\Xi_c}} + \frac{\langle 0 | J_{\Lambda_c^-}(p, s) \rangle}{p^2 - m^2_{\Lambda_c^-}} (K(q) \Lambda_c^-(p, s) | \Xi_c(p', s') \rangle \times \frac{\langle \Xi_c(p', s') | \bar{J}_{\Xi_c}(0) \rangle}{p^2 - m^2_{\Xi_c}} + \ldots, \quad (21)$$

where $p' = p + q$, $p$ and $q$ are the momenta of the $\Xi_c$, $\Lambda_c$ baryons and $K$ meson, respectively. $\Lambda_c^+$ and $\Lambda_c^-$ are the positive and negative parity baryons in spin-1/2 $\Lambda_c$ channel. In this expression $m_{\Lambda_c}$ is the mass of the $\Lambda_c$ baryon. The dots in Eq. (21) stand for contributions of the higher resonances and continuum states.

The matrix elements in Eq. (21) are parameterized as

$$\langle 0 | J_{\Lambda_c^+}(p, s) \rangle = \lambda_{\Lambda_c^+} u(p, s),$$  

$$\langle 0 | J_{\Lambda_c^-}(p, s) \rangle = \lambda_{\Lambda_c^-} \gamma_5 u(p, s),$$  

$$\langle K(q) \Lambda_c^+(p, s) | \Xi_c(p', s') \rangle = g_{\Xi_c \Lambda_c^+ K} u(p', s'),$$  

$$\langle K(q) \Lambda_c^-(p, s) | \Xi_c(p', s') \rangle = g_{\Xi_c \Lambda_c^- K} \gamma_5 u(p', s'),$$  

where $g_i$ are the strong coupling constants for corresponding transitions.

Using the matrix elements given in Eq. (22) and performing summations over spins of $\Lambda_c$ and $\Xi_c$ baryons and applying the double Borel transformations with respect $p^2$ and $q^2$, for the physical side of the correlation func-
tion, we get

\[ BT^{\text{Phys}}(p, q) = g_{\Xi_c^+ K} \lambda_{\Xi_c^+} \lambda_K e^{-m_{\Xi_c}^2 / M_1^2} e^{-m_{\Lambda_c}^2 / M_2^2} \times \left( \phi + m_{\Lambda_c} \right) \gamma_5 \left( \phi + q + m_{\Xi_c} \right) \]

\[ - g_{\Xi_c^+ K} \lambda_{\Xi_c^+} e^{-m_{\Xi_c}^2 / M_1^2} e^{-m_{\Lambda_c}^2 / M_2^2} \times \gamma_5 \left( \phi + m_{\Lambda_c} \right) \gamma_5 \left( \phi + q + m_{\Xi_c} \right) \gamma_5 \]

\[ + g_{\Xi_c^+ K} \lambda_{\Xi_c^+} e^{-m_{\Xi_c}^2 / M_1^2} e^{-m_{\Lambda_c}^2 / M_2^2} \times \left( \phi + m_{\Lambda_c} \right) \left( \phi + q + m_{\Xi_c} \right) \gamma_5 \]

\[ + g_{\Xi_c^+ K} \lambda_{\Xi_c^+} \left( \phi + m_{\Lambda_c} \right) \left( \phi + q + m_{\Xi_c} \right), \quad (23) \]

where \( M_1^2 \) and \( M_2^2 \) are the Borel parameters.

From Eq. (23) follows that we have different structures which can be used to derive the sum rules for the strong coupling constants for \( \Xi_c^+ \to \Lambda_c^+ K \) channel. We have four couplings (see Eq. (21)), and in order to determine the coupling \( g_{\Xi_c^+ K} \), we need four equations. Therefore we select the structures \( \gamma_5 \gamma_7, \gamma_5 \gamma_5, \gamma_5 \gamma_7 \) and \( \gamma_5 \). Solving four algebraic equations for \( g_{\Xi_c^+ K} \), we obtain

\[ g_{\Xi_c^+ K} = \frac{m_{\Xi_c}^2}{m_{\Xi_c}^2} e^{-m_{\Xi_c}^2 / M_1^2} \left[ \lambda_{\Xi_c^+} \lambda_{\Lambda_c^+} \left( m_{\Xi_c} + m_{\Lambda_c} \right) \right] \times \left[ T_1^{\text{QCD}} \left( m_{\Xi_c}^2 + m_{\Lambda_c} - m_{\Xi_c} \right) \right] \]

\[ + T_2^{\text{QCD}} \left( m_{\Xi_c}^2 - m_{\Lambda_c} - m_{\Xi_c} \right) \]

\[ + T_3^{\text{QCD}} \left( m_{\Xi_c}^2 - m_{\Xi_c} \right) - T_4^{\text{QCD}}, \quad (24) \]

where \( T_1^{\text{QCD}}, T_2^{\text{QCD}}, T_3^{\text{QCD}} \) and \( T_4^{\text{QCD}} \) are the invariant amplitudes corresponding to the structures \( \gamma_5 \gamma_7, \gamma_5 \gamma_5, \gamma_5 \gamma_7 \) and \( \gamma_5 \), respectively.

The general expressions obtained above contain two Borel parameters \( M_1^2 \) and \( M_2^2 \). In our analysis we choose

\[ M_1^2 = M_2^2 = 2M^2, \quad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}. \quad (25) \]

which is traditionally justified by the fact that masses of the involved heavy baryons \( \Xi_c \) and \( \Lambda_c \) are close to each other. The sum rules corresponding to the coupling constant defining the \( \Xi_c^+ \to \Lambda_c^+ K \) transition can be easily obtained from Eq. (24), by replacing \( m_{\Xi_c} \to -m_{\Xi_c} \) and \( \lambda_{\Xi_c} \to -\lambda_{\Xi_c} \). The QCD side of the correlation function for \( \Pi^{QCD}(p, q) \) can be obtained by contracting out the quark fields using Wick’s theorem and inserting into the obtained expression the relevant quark propagators. For obtaining nonperturbative contributions in light cone QCD sum rules, which are described in terms of the \( K \)-meson distribution amplitudes, one can use the Fierz rearrangement formula

\[ \pi_{a}^{i} u_{\beta} = \frac{1}{4} \Gamma^{i} \left( \pi \Gamma^{i} u_{\beta} \right), \]

where \( \Gamma^{i} = 1, \gamma_5, \gamma_\mu, i\gamma_5 \gamma_\mu, \sigma_{\mu\nu} / \sqrt{2} \) is the full set of Dirac matrices. Sandwiched between the \( K \)-meson and vacuum states, these terms as well as the ones generated by insertion of the gluon field strength tensor \( G_{\alpha\beta}(uv) \) from quark propagators, give these distribution amplitudes (DAs) of various quark-gluon contents in terms of wave functions with definite twists. The DAs are main nonperturbative inputs of light cone QCD sum rules. For \( K \)-meson they are derived in [22 24], which will be used in our numerical analysis. All these steps summarized above result in a lengthy expression for the QCD side of correlation function. In order not to overwhelm the study with overlong mathematical expressions, we prefer not to present them here. Apart from parameters in the distribution amplitudes, the sum rules for the couplings depend also on numerical values of the \( \Lambda_c^+ \) baryon’s mass and pole residue. In numerical calculations we utilize

\[ m_{\Lambda_c} = 2286.46 \pm 0.14 \text{ MeV}, \lambda_{\Lambda_c} = 0.038 \pm 0.009 \text{ GeV}^3, \]

\[ (26) \]

where the value for the residue \( \lambda_{\Lambda_c} \) has been extracted from the corresponding mass sum rules in the present study and we use the mass of \( \Lambda_c \) state from PDG [1]. The working regions of the Borel mass \( M^2 \), threshold \( s_0 \) and \( \beta \) parameters for calculations of the relevant strong couplings are chosen the same as the mass sum rules analyses.

Using the couplings \( g_{\Xi_c^+ K} \) and \( g_{\Xi_c^+ \Lambda_c^+ K} \) we can easily calculate the width of \( \Xi_c^+ \to \Lambda_c^+ K \) and \( \Xi_c^+ \to \Lambda_c^+ K \) decays. After some computations we obtain:

\[ \Gamma \left( \Xi_c^+ \to \Lambda_c^+ K \right) = \frac{g_{\Xi_c^+ K}^2}{16m_{\Xi_c}^2} \left[ (m_{\Xi_c}^2 + m_{\Lambda_c}^2)^2 - m_K^2 \right] \]

\[ \times \lambda_{\Xi_c}^{1/2} \left( m_{\Xi_c}^2, m_{\Lambda_c}^2, m_K^2 \right), \quad (27) \]

and

\[ \Gamma \left( \Xi_c^+ \to \Lambda_c^+ K \right) = \frac{g_{\Xi_c^+ \Lambda_c^+ K}^2}{16m_{\Xi_c}^2} \left[ (m_{\Xi_c}^2 - m_{\Lambda_c}^2)^2 - m_K^2 \right] \]

\[ \times \lambda_{\Xi_c}^{1/2} \left( m_{\Xi_c}^2, m_{\Lambda_c}^2, m_K^2 \right), \quad (28) \]

In expressions above the function \( \lambda(x^2, y^2, z^2) \) is given as:

\[ \lambda(x^2, y^2, z^2) = x^4 + y^4 + z^4 - 2x^2y^2 - 2x^2z^2 - 2y^2z^2. \]

Numerical values obtained from our analyses for coupling constants and decay widths are presented in Table [III]. The obtained central value for the decay width of the \( \Xi_c^+ \to \Lambda_c^+ K \) case is in nice consistency with
the central value of the experimental data, \(19.5 \pm 8.4\text{(stat.)}^{+5.4}_{-7.7}\) MeV \([18]\). In order to make a definite conclusion about the nature of \(\Xi_c(2930)\) state, more new and refined experimental data with small errors are needed.

IV. SUMMARY AND CONCLUDING REMARKS

We performed a QCD sum rule based analysis on the mass and width of the \(\Xi_c(2930)\) considering it as first angularly/ radially excited charmed-strange baryon in \(\Xi_c\) channel. We obtained the same mass for both the angularly and radially excited states, and in excellent agreement with the experimental value by Belle Collaboration, preventing us to assign any of these possibilities for the structure of this state. In next step, we considered the dominant decay of \(\Xi_c(2930)\) to \(\Lambda_c K\) in both scenarios. The obtained central value for the width is nicely consistent with the central value of the experimental data of BELLE Collaboration, when we consider \(\Xi_c(2930)\) state as the angular excitation of the ground state charmed-strange baryon. This suggests the assignment of a spin-parity \(J^P = \frac{1}{2}^-\) for this state. However, to make a final decision about the nature of \(\Xi_c(2930)\) state more new and refined data with small statistical and systematical errors are needed.

In Ref. [17] and some other studies the orbitally excited single charmed baryons are classified into the \(\rho\) and \(\lambda\) modes according to the quark model. When we compare our results and assignment on \(\Xi_c(2930)\) state with the results presented, for instance, in Ref. [17], we observe that the \(\Xi_c(2930)\) state is close to the \(\lambda\) mode.

Appendix: The QCD side of the correlation function

In this appendix we present the explicit expressions of the functions \(B\Pi_1^{\text{QCD}}(q)\) and \(B\Pi_2^{\text{QCD}}(q)\) used in mass sum rules:

\[
\begin{align*}
B\Pi_1^{\text{QCD}}(q) &= \int_{m_c^2}^{s_0} e^{-\frac{1}{26\pi^2 s^2}} \left( \frac{5b^2 + 2\beta + 5}{32\pi^2} \right) \left( 8m_c^6 s - m_c^8 - 8m_c^2 s^3 + s^4 + 12m_c^2 s^2 \log \left( \frac{s}{m_c^2} \right) \right) - (\bar{s}s) + (\bar{d}d)
\right)
\times \frac{m_c(m_c^2 - s)^2}{3} (5b^2 - 4\beta - 1) + \left( \frac{g_s^2GG}{48\pi^2} \right) [3sm_c^2(1 + \beta)^2 - 8m_c^4(1 + \beta + \beta^2) + s^2(5 + 2\beta + 5\beta^2)]
\left. + \frac{m_c^2}{12} (\bar{s}s) + (\bar{d}d) \right) m_c(\beta - 1) [m_c^2(\beta + 7) - 6s(\beta + 1)] - \frac{\langle g_s^2GG \rangle (\bar{s}s) + (\bar{d}d)}{6} m_c(\beta^2 - 1)
\times e^{-\frac{m_c^2}{2\pi^2}} \left( \frac{\bar{s}s)(\bar{d}d)}{72} (11\beta^2 + 2\beta - 13) + \frac{g_s^2GG}{864\pi^2 m_c^2} (\bar{s}s) + (\bar{d}d) \right) (\beta^2 + \beta - 2) - \frac{g_s^2GG}{221184\pi^4 M^2}
\times (13\beta^2 + 10\beta + 13) - \frac{m_c^2(\bar{s}s)(\bar{d}d)}{288 M^4} (\beta - 1) \left[ m_c^2(26 + 22\beta + M^4(25 + 23\beta)] - \frac{\langle g_s^2GG \rangle m_c^2 (\bar{s}s) + (\bar{d}d)}{55296\pi^2 m_c M^4}
\times (\beta - 1) \left[ m_c^2(\bar{s}s) + (\bar{d}d) \right] m_c^2 (11\beta^2 + 2\beta - 13) + \frac{\langle g_s^2GG \rangle m_c^2 (\bar{s}s) + (\bar{d}d)}{10368 M^{10}}
\times m_c^2(2M^2)(11\beta^2 + 2\beta - 13),
\end{align*}
\]
BII^{QC}(q) = \int_{m_c^2}^{s_0} e^{-\frac{q^2}{\pi^2}} \frac{1}{3 \cdot 2^6 \pi^2 s} \left\{ m_c (11 \beta^2 + 2 \beta - 13) \left[ s^3 + 9 m_c^2 s^2 - 9 m_c^4 s - m_c^6 - 6 m_c^2 (s + m_c^2) \log \frac{s}{m_c^2} \right] \right.
\left. - (2 \langle \bar{s}s \rangle + \langle \bar{d}d \rangle) \right) \left[ (5 \beta^2 - 4 \beta - 1)(m_c^2 - s^2) + \frac{\langle g_s^2 GG \rangle (\beta - 1)}{3 \cdot 2^4 \pi^2 m_c} \left( (s - m_c^2) (s(11 \beta + 13) + m_c^2(67 \beta + 53)) \right) \right.
\left. - 3 m_c^2 s (13 \beta + 11) \log \frac{s}{m_c^2} \right\] + \frac{m_c^2 (\langle \bar{s}s \rangle + \langle \bar{d}d \rangle)}{2^2} \left( \beta - 1 \right) \left( 6 s (\beta + 1) - m_c^2 (\beta + 5) \right) \right.
\left. + e^{-\frac{m_c^2}{\pi^2}} \left[ \frac{1}{24} \langle \bar{s}s \rangle \langle \bar{d}d \rangle m_c (5 \beta^2 + 2 \beta + 5) + \frac{\langle g_s^2 GG \rangle \left( \langle \bar{d}d \rangle + \langle \bar{s}s \rangle \right)}{3^3 \cdot 2^7 \pi^2} \left( \beta^2 - 8 \beta + 7 \right) - \frac{\langle g_s^2 GG \rangle}{3^3 \cdot 2^{13} \pi^2 M^2} m_c \right] \times \left( m_c^2 - 2 M^2 \right) (13 \beta^2 - 2 \beta - 11) + \frac{m_c^2 (\langle \bar{s}s \rangle + \langle \bar{d}d \rangle)}{3^2 \cdot 2^4 M^4} \left[ M^2 (\beta^2 - 1)^2 - 3 m_c^2 (5 \beta^2 + 2 \beta + 5) \right]
\left. + \frac{\langle g_s^2 GG \rangle m_c^2 \langle \bar{s}s \rangle \langle \bar{d}d \rangle + \langle \bar{d}d \rangle \langle \bar{d}d \rangle + \langle \bar{s}s \rangle \langle \bar{s}s \rangle + \langle \bar{s}s \rangle \langle \bar{d}d \rangle}{3 \cdot 2^1 \pi^2 M^4} \left[ m_c^2 - 3 M^2 \right] (5 \beta^2 + 2 \beta + 5) \right)
\left. + \frac{\langle g_s^2 GG \rangle m_c^2 (\langle \bar{s}s \rangle + \langle \bar{d}d \rangle) m_c}{3^3 \cdot 2^7 M^4} \left[ m_c^2 (5 \beta^2 + 2 \beta + 5) \right]. \right\}
\right.

(A.2)

In calculations we set \( m_s = m_d = 0 \) but \( m_s \neq 0 \). To shorten the above expressions, the terms proportional to \( m_s \) have not been presented, but their contributions have been taken into account when performing the numerical analysis.

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[1] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018).
[2] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 118, no. 18, 182001 (2017).
[3] S. S. Agaev, K. Azizi and H. Sundu, EPL 118, no. 6, 61001 (2017).
[4] S. S. Agaev, K. Azizi and H. Sundu, Eur. Phys. J. C 77, no. 6, 395 (2017).
[5] T. M. Aliiev, S. Bilmis and M. Savci, [arXiv:1704.03439 [hep-ph]].
[6] M. Karliner and J. L. Rosner, Phys. Rev. D 95, no. 11, 114012 (2017).
[7] W. Wang and R. L. Zhu, Phys. Rev. D 96, no. 1, 014024 (2017).
[8] G. Yang and J. Ping, Phys. Rev. D 97, no. 3, 034023 (2018).
[9] H. Huang, J. Ping and F. Wang, Phys. Rev. D 97, no. 3, 034027 (2018).
[10] V. Crede and W. Roberts, Rept. Prog. Phys. 76, 076301 (2013).
[11] H. Y. Cheng, Front. Phys. 10, 101406 (2015).
[12] B. Aubert et al. (BABAR Collaboration), Phys. Rev. D 77, 031101(R) (2008).
[13] N. Gabyshev et al. (Belle Collaboration), Phys. Rev. Lett. 97, 202003 (2006).
[14] K. L. Wang, Y. X. Yao, X. H. Zhong and Q. Zhao, Phys. Rev. D 96, no. 11, 116016 (2017).
[15] B. Chen, K. W. Wei, X. Liu and T. Matsuki, Eur. Phys. J. C 77, no. 3, 154 (2017).
[16] L. H. Liu, L. Y. Xiao, and X. H. Zhong, Phys. Rev. D 86, 034024 (2012).
[17] H. X. Chen, Q. Mao, W. Chen, A. Hosaku, X. Liu, and S. L. Zhu, Phys. Rev. D 95, 094008 (2017).
[18] Y. B. Li et al. [Belle Collaboration], Eur. Phys. J. C 78, no. 3, 252 (2018).
[19] D. D. Ye, Z. Zhao and A. Zhang, Phys. Rev. D 96, no. 11, 114003 (2017).
[20] D. D. Ye, Z. Zhao and A. Zhang, Phys. Rev. D 96, no. 11, 114009 (2017).
[21] Q. Mao, H. X. Chen, W. Chen, A. Hosaku, X. Liu and S. L. Zhu, Phys. Rev. D 92, no. 11, 114007 (2015).
[22] P. Ball, V. M. Braun and A. Lenz, JHEP 0605, 004 (2006).
[23] V. M. Belyaev, V. M. Braun, A. Khodjamirian and R. Ruckl, Phys. Rev. D 51, 6177 (1995).
[24] P. Ball and R. Zwicky, Phys. Rev. D 71, 014015 (2005).