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Optimal control and sensitivity analysis for transmission dynamics of Coronavirus

Chernet Tuge Deressa, Yesuf Obsie Mussa, Gemechis File Duressa *

Department of Mathematics, College of Natural Sciences, Jimma University, Ethiopia

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ABSTRACT

Analysis of mathematical models designed for COVID-19 results in several important outputs that may help stakeholders to answer disease control policy questions. A mathematical model for COVID-19 is developed and equilibrium points are shown to be locally and globally stable. Sensitivity analysis of the basic reproductive number (R₀) showed that the rate of transmission from asymptomatically infected cases to susceptible cases is the most sensitive parameter. Numerical simulation indicated that a 10% reduction of R₀ by reducing the most sensitive parameter results in a 24% reduction of the size of exposed cases. Optimal control analysis revealed that the optimal practice of combining all three (public health education, personal protective measure, and treating COVID-19 patients) intervention strategies or combination of any two of them leads to the required mitigation of transmission of the pandemic.

Introduction

According to Eykhoff [1], 'Mathematical models are representations of essential aspects of a system to be constructed which presents knowledge of that system in a usable form'. Mathematical models are applicable in disciplines such as epidemiology, physics, biology, electrical engineering, economics, sociology, political science, and several other areas. Mathematical models can take several forms which include differential equations, dynamical systems, and game models. Studies related to mathematical models involving differential equations have also different forms based on whether ordinary derivatives or fractional derivatives are used in the models; even though the second is the generalization of the first. There are many publications of studies involving mathematical models with fractional derivatives namely Caputo fractional derivatives, Caputo–Fabrizio fractional derivatives, Atangana– Baleanu fractional derivatives, and fractal-fractional derivatives. For detail concepts related to these studies involving fractional derivatives and their application, one can refer [2-11] and the references therein.

One of the central parts in the study of the epidemiology of infectious diseases is mathematical modeling and its analysis. Analysis of mathematical models designed for given infectious diseases or a pandemic like COVID-19 results in several important outputs including transmission trajectories, critical levels requiring intervention, the peak time of the infection, expected size of infection and recognizing best working intervention strategies. This helps governments or stakeholders to answer disease control policy questions.

Coronavirus (COVID-19) is an infectious disease declared a global pandemic on March 11, 2020 by World Health Organization [12]. Since then it has claimed the life many thousands of peoples and posed a huge threat to public health all over the world [13,14].

As a response to this risk, many scholars around the world tried to conduct research to understand the transmission dynamics and to analyze the effect of non-pharmaceutical intervention strategies in lessening the spread of coronavirus.

Some of the researchers are mathematical modelers. Since January 2020 these scholars around the world have developed mathematical models to understand the transmission dynamics of [15-18] Coronavirus. Mathematical models developed were mainly used to investigate the effects of different Non-pharmaceutical intervention strategies via simulation using different computing soft-wares [19].

Different mathematicians used different types of mathematical models in their analysis. For instance, an SEIR model was introduced by Wu et al. [14] for estimating the spread of the pandemic and approximated the basic reproductive number to 2.68 based on reported data. Tang et al. [16] used a deterministic model to estimate the basic reproductive number to be as high as 6.47 and they concluded that contact tracing followed by quarantining and then isolation can mitigate...
the spread of the pandemic via reducing the basic reproductive number. Kieszha et al. [19] used an SEIR mathematical model to simulate the outbreak of Coronavirus in Wuhan and indicated that restricting the Wuhan people’s movement could help in delaying the peak time of the pandemic. Roda et al. [20] used the SIR model to predict the COVID-19 epidemic in Wuhan and predicted the potential of a second outbreak after the return-to-work in the city.

Pang et al. used SEIHR mathematical model to investigate the effectiveness of quarantine measures applied in Wuhan city and factors affecting its effectiveness [21].

In this study, we used a mathematical model called SEIAHR, where the state variables S, E, I, A, H, and R represent susceptible, exposed, symptomatically infected, asymptomatically infected, isolated, or hospitalized, and Recovered/immune cases respectively. We have divided the infected cases into two groups: symptomatic and asymptomatic cases. Since most of the patients of COVID-19 are either asymptomatic or symptomatic, it seems reasonable to consider these two groups in developing a model and analysis.

Moreover, some of the studies considered above didn’t consider optimal control analysis of multiple intervention strategies. In this study, we designed three non-pharmaceutical control strategies named public health education, use of personal protective measures, and treatment of the patients. We have made both analytical analysis and numerical simulation of the effect of these strategies in mitigating the transmission of the pandemic.

In general, the rest of this article is organized as follows: In the second part of the work, the mathematical model is formulated; equilibrium points and the basic reproductive number are calculated. The third part deals with the local and global stability analysis of both the equilibrium points followed by numerical simulations. In the fourth part sensitivity analysis of the basic reproductive number is conducted followed by numerical simulations. In the fifth part of the article, optimal control analysis is made analytically followed by numerical simulations. Lastly, discussions and conclusions are provided.

The model

A compartmental approach is used to develop the mathematical model for COVID-19 transmission dynamics. The total population N is divided into six compartments named S, E, I, A, H, and R as explained in the introduction part. The flow chart of the model is shown in Fig. 1.

In the mathematical model developed in this study, humans get into the suspected group S at the rate of α and infected with Coronavirus as a result of contact with individuals in the group of A or I. The exposed group E gains population from infection induced by the Coronavirus.

A proportion α3 (0 < α3 < 1) of the members of the group E advance to the asymptomatic group A and the remaining proportion 1 − α3 progresses to the symptomatic group I. People in the group I and A progress either to the Hospitalization group H or recovery group R at the rates indicated in the Fig. 1 and Table 1. In the construction of the mathematical model, the exposed compartment E is included because people who are contracted with the virus don’t get infectious immediately; there is an incubation period for the virus to get infectious. The groups I and A are included in the model, as people infected with Coronavirus are either symptomatic or asymptomatic. COVID-19 induced death rate α11 is also considered in the model. As a result, the authors are convinced that the model considered in this study named SEIAHR model incorporates all essential components of COVID-19 to study its transmission dynamics, in agreement with the definition of a mathematical model in [1]. The parameters and their corresponding values used in the model are indicated in Table 1.

Corresponding to the aforementioned discussion and the flow chart in Fig. 1 the mathematical model used in this study called SEIAHR model is shown in (1),

\[
\begin{align*}
\dot{S} &= α - \frac{α_1(A + I)S}{N} - α_5S, \\
\dot{E} &= \frac{α(A + I)S}{N} - (α_4 + α_5)E, \\
\dot{I} &= (1 - α)α_5E - (α_3 + α_4 + α_5)I, \\
\dot{A} &= α_3α_5E - (α_3 + α_5)I, \\
\dot{H} &= α_4I + α_3A - (α_5 + α_6 + α_7)H, \\
\dot{R} &= α_5I + α_7A + (α_11 + α_8)H - α_7R.
\end{align*}
\]

The parameters indicated in Table 1, used in the model (1) are all assumed to be non-negative.

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**Table 1**

| Name of the parameter | symbol | Value | Source |
|-----------------------|--------|-------|--------|
| Influx rate           | α      | 136.89/day | assumed |
| Rate of transmission from A to S cases | α₁ | 0.25 | [22] |
| Rate of transmission from I to S cases | α₂ | 1 | assumed |
| The proportion of A cases. | α₃ | 0.80 | assumed |
| The incubation period of Coronavirus | α₄ | 1.9233/days | [23] |
| The rate at which I cases are transferred to H cases | α₅ | 0.6000/day | [24] |
| The cure rate of I cases | α₆ | 0.05/day | [25] |
| The cure rate of A cases | α₇ | 0.0714/day | [25] |
| Natural mortality rate | α₈ | 0.00004563/day | [25] |
| The rate at which H cases are transferred to R cases | α₉ | 0.04255/day | [22] |
| The rate at which A cases are transformed into H cases | α₁₀ | 0.03 | assumed |
| Coronavirus induced death rate | α₁₁ | 0.0018 | [25] |

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**Fig. 1.** Flow diagram of the model.
existence and uniqueness of solution

The mathematical model (1) needs to be biologically valid in the sense that the solutions of the model must be positive and bounded for all time $t$. The proof is given in the following lemmas.

Lemma 1. (Positivity of solutions): If the initial conditions $S(0) > 0$, $E(0) > 0, I(0) > 0, A(0) > 0, H(0) > 0, R(0) > 0$, and $t_0 > 0$, then for all $t \in [0, t_0]$, $S(t), E(t), I(t), A(t), H(t)$ and $R(t)$ remain positive in $\mathbb{R}_+^5$.

Proof: Since all the parameters in the model are assumed positive it is possible to set a lower bound for each of the equations in (1) as follows:

\[ S(t) > -\left(\frac{a_1(a+1)}{N} + a_0\right) t + \frac{a_1(a+1)}{N}, \]
\[ E(t) > -(a + a_0) t + \frac{a_1(a+1)}{N}, \]
\[ I(t) > (a + a_0) t, \]
\[ A(t) > -(a_0 + a) t + \frac{a_1(a+1)}{N}, \]
\[ H(t) > -(a_0 + a) t + \frac{a_1(a+1)}{N}, \]
\[ R(t) > -a_0 t. \]

Solving the above inequalities respectively leads to,

\[ S(t) > \frac{a_1(a+1)}{N} t, \]
\[ E(t) > \frac{a_1(a+1)}{N} t, \]
\[ I(t) > \frac{a_1(a+1)}{N} t, \]
\[ A(t) > \frac{a_1(a+1)}{N} t, \]
\[ H(t) > \frac{a_1(a+1)}{N} t, \]
\[ R(t) > \frac{a_1(a+1)}{N} t. \]

Thus, for all $t \in [0, t_0]$, $S(t), E(t), I(t), A(t), H(t)$, and $R(t)$ are positive in $\mathbb{R}_+^5$.

Lemma 2. (Boundedness of solutions): For the functions $S(t), E(t), I(t), A(t), H(t)$, and $R(t)$ of the mathematical model (1) there exists positive constants $s_m, e_m, i_m, a_m, h_m, r_m$ such that $\limsup_{t \to \infty} S(t) \leq s_m$, $\limsup_{t \to \infty} E(t) \leq e_m$, $\limsup_{t \to \infty} I(t) \leq i_m$, $\limsup_{t \to \infty} A(t) \leq a_m$, $\limsup_{t \to \infty} H(t) \leq h_m$, $\limsup_{t \to \infty} R(t) \leq r_m$, for all $t \in [0, t_0], t_0 > 0$.

Proof: If we add all the equations in the model (1) we obtain $\frac{d}{dt}(\alpha A + \alpha H + \alpha I + \alpha E + \alpha R) = 0$.

It then follows that $\frac{d}{dt} < 0$, for $N > \frac{a_1\alpha}{a_0}$.

Thus, solving $\frac{d}{dt} < 0$ by applying Gronwall’s inequality leads to $N(t) \leq N(0) e^{-\frac{a_0}{a_1\alpha} t} + \frac{a_0}{a_1\alpha} e^{\frac{a_0}{a_1\alpha} (1 - e^{-\frac{a_0}{a_1\alpha} t})}$.

In particular, for $N(0) \leq \frac{a_0}{a_1\alpha}$.

Now, choosing $s_m = e_m = i_m = a_m = h_m = r_m = \frac{a_0}{a_1\alpha} t_0 > 0$ it can be concluded that $S, E, I, A, H, R$ are all bounded since $S(t), E(t), I(t), A(t), H(t), R(t) \leq N(0) \leq \frac{a_1\alpha}{a_0}$.

Thus, the region

\[ \Omega = \left\{ (S, E, A, I, H, R) \in \mathbb{R}_+^6 : S(t) + E(t) + A(t) + I(t) + H(t) + R(t) \leq N(t) \leq \frac{a_1\alpha}{a_0} \right\} \]

Moreover, by the fundamental existence and uniqueness theorem [26] and Lemma 1 and 2 proved above, there exists a unique, positive, and bounded solution for the system of the differential Eq. (1) in $\mathbb{R}_+^6$.

Equilibrium points

From model (1) two equilibria points are obtained:

1. Disease-free equilibrium point (DFE): $N_0 = \left( \frac{\alpha}{a_0} 0, 0, 0, 0 \right)$
2. Endemic equilibrium point (EPP): $N^* = (S^*, E^*, I^*, A^*, H^*, R^*)$, where

\[ S^* = \frac{\alpha}{a_0} + \frac{l_1}{l_3} E^*, \]
\[ I^* = \frac{l_2}{l_3} E^*, \]
\[ A^* = -\frac{a_1\alpha_1}{l_3} E^*, \]
\[ H^* = \frac{l_1}{l_3} \left( \frac{a_1\alpha_2}{l_3} + \frac{a_1\alpha_3}{l_4} \right) E^*, \]
\[ R^* = \frac{l_1}{l_3} \left( \frac{a_1\alpha_4}{l_5} + \frac{a_1\alpha_5}{l_6} \right) + \frac{a_1\alpha_6}{l_6} \]

and $E^* = \frac{\alpha_1}{a_0} + \frac{a_0 N - a_0}{a_0 N}$, which is a solution of a quadratic equation $m_1 E^2 + m_2 E = 0$ where,

\[ m_1 = -\frac{l_1}{l_3} R_0, \]
\[ m_2 = \frac{l_1}{l_3} (-a_0 N + a_0 R), \]
\[ R_0 = l_1 + l_2, \]
\[ l_1 = \frac{a_1\alpha_1}{l_3}, \]
\[ l_2 = \frac{a_1\alpha_2}{l_3} \]
\[ l_3 = \frac{a_1\alpha_3}{l_5}, \]
\[ l_4 = \frac{a_1\alpha_4}{l_5} \]
\[ l_5 = \frac{a_1\alpha_5}{l_6}, \]
\[ l_6 = \frac{a_1\alpha_6}{l_6} \]

As a result, a positive endemic equilibrium point exists only for $R_0 > 1$ taking into account the assumption that $a = a_0 N$.

In the next subsection, it will be made clear that $R_0$ is the basic reproductive number.

The basic reproductive number

The basic reproductive number, denoted by $R_0$ is obtained by establishing the next generation matrix (27) as a spectral radius of the matrix $TV^{-1}$ at $N_0$. The matrices $T$ and $V^{-1}$ are obtained by linearizing the mathematical model (1) about DFE, which results in the Jacobian matrix $J_{DFE}$ given in (2).

\[ J_{DFE} = \begin{bmatrix}
-a_0 & 0 & -a_1 a_2 & -a_1 & 0 & 0 \\
0 & -(a_0 + a_1) & a_2 & a_3 & 0 & 0 \\
0 & (a_0 + a_1) a_3 & -(a_0 + a_1 + a_2) & 0 & 0 & 0 \\
0 & a_3 a_4 & 0 & -(a_0 + a_1 + a_2) & 0 & 0 \\
0 & a_4 & a_0 & -(a_0 + a_1 + a_2) & 0 & 0 \\
0 & a_0 & a_0 & a_0 & -(a_0 + a_1 + a_2) & 0
\end{bmatrix} \]
From the matrix, $J_{EE}$ we construct a matrix $M$ such that $M = T + V$, where

$$M = \begin{bmatrix}
-(a_3 + a_4) & a_1 & a_2 & 0 \\
(1 - a_1)a_4 & -(a_1 + a_3 + a_4) & 0 & 0 \\
0 & a_5 & 0 & -(a_1 + a_3 + a_4) \\
0 & (a_1 + a_4 + a_3) & a_6 & -(a_1 + a_3 + a_4)
\end{bmatrix},$$

$$T = \begin{bmatrix}
0 & a_1 & a_2 & a_1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}, \text{ and}$$

\[
V^{-1} = \begin{bmatrix}
\frac{1}{a_1 + a_3} & 0 & 0 & 0 \\
\frac{(1 - a_1)}{a_1 + a_3} & \frac{a_1a_4}{(a_1 + a_3)(a_1 + a_3 + a_4)} & \frac{a_1a_4}{(a_1 + a_3)(a_1 + a_3 + a_4)} & \frac{a_1}{(a_1 + a_3)(a_1 + a_3 + a_4)} \\
\frac{a_1}{(a_1 + a_3)(a_1 + a_3 + a_4)} & \frac{1}{a_1 + a_3} & 0 & 0 \\
0 & \frac{a_1}{(a_1 + a_3)(a_1 + a_3 + a_4)} & \frac{1}{a_1 + a_3} & 0 \\
0 & 0 & \frac{1}{a_1 + a_3} & 0 \\
\end{bmatrix}
\]

The basic reproductive number is the spectral radius $\rho(TV^{-1})$ and is given by

$$R_0 = \frac{a_1a_2(1 - a_1)}{(a_1 + a_3)(a_1 + a_3 + a_4)} + \frac{a_1a_4}{(a_1 + a_3)(a_1 + a_3 + a_4)},$$

$R_0$ can be written as $R_0 = R_1 + R_2$, $R_1 = \frac{a_1a_2}{b_0b_3}$, $R_2 = \frac{a_1a_4}{b_0b_3}$, where $l_i, i = 1, \ldots, 6$ are defined above.

**Stability analysis of DFE and EEP**

**Local stability analysis of DFE and EEP**

**Theorem 1.** The DFE of (1), $N_0 = \left(\frac{a_1}{a_3}, 0, 0, 0, 0\right)$ is locally asymptotically stable for $R_0 < 1$.

**Proof:** Three of the eigenvalues of (2) are given by $\lambda_1 = \frac{a_1}{a_3} = -a_8$ and $\lambda_3 = -(a_1 + a_3 + a_8)$ which are all negative. The sign of the real part of the remaining three eigenvalues is determined by the Routh-Hurwitz stability criteria from the characteristic Eq. (4).

$$\phi(\lambda) = \lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0, \quad (4)$$

where

$$a_2 = -(l_1 + l_2 + l_3), \quad a_1 = l_1l_2l_3(l_l - l_2) + l_1l_2l_4 > 0 \quad \text{for} \quad R_0 < 1 \quad \text{as} \quad R_1 \text{ and } R_2 \text{ are positive and } R_1 + R_2 = R_0.$$

Besides, $a_0 = -l_1l_2l_4 + a_1a_4a_5l_3 + a_1a_2l_2l_4 = l_1l_2l_4(R_0 - 1) > 0 \quad \text{for} \quad R_0 < 1.$

**Necessary Condition:**

From (4), we have $a_2 > 0$ and $a_1 = l_1l_2l_3(l_1 - l_2) + l_1l_2l_4 > 0 \quad \text{for} \quad R_0 < 1 \quad \text{as} \quad R_1 \text{ and } R_2 \text{ are positive and } R_1 + R_2 = R_0$.

Besides, $a_0 = -l_1l_2l_4 + a_1a_4a_5l_3 + a_1a_2l_2l_4 = l_1l_2l_4(R_0 - 1) > 0 \quad \text{for} \quad R_0 < 1.$

**Sufficient condition**

All the eigenvalues of the characteristic equation $\phi(\lambda) = \lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0$, have a negative real part by Routh-Hurwitz stability criteria as it can easily be shown that $a_0 - a_2a_1 < 0$.

That is, $a_0 - a_1a_2 = l_1l_2l_3(R_0 - 1) - \left[-(l_1 + l_2)(l_1l_3(1 - R_1) + l_1l_4(1 - R_2) + l_1l_4)\right]$

$$= 2l_1l_2l_3(l_1 + l_2)(l_1l_3(1 - R_1) + (l_1 + l_2)(l_1l_4(1 - R_2) + (l_1 + l_2)(l_1l_4) < 0$$

for $R_1 < 1$, $R_2 < 1$, $R_0 < 1$.

Thus, the diseases free equilibrium point $N_0$ is locally asymptotically for $R_0 < 1$.

**Local stability analysis of EEP**

**Theorem 2.** If $R_0 > 1$, then the EEP of (1) $N^* = (S^*, E', I', A^*, H', R^*)$ is locally asymptotically stable.

**Proof:** Suppose $R_0 > 1$ so that the EEP exists.

Now the Jacobian matrix $J_{EE}$ evaluated at the EEP is given by

$$J_{EE} = \begin{bmatrix}
-l_1 - a_4 & 0 & -a_1l_5 & -l_5 & 0 & 0 \\
l_1 & l_1 & a_2l_4 & l_4 & 0 & 0 \\
0 & l_2 & l_2 & 0 & 0 & 0 \\
0 & a_3 & 0 & l_3 & 0 & 0 \\
0 & 0 & a_5 & a_7 & l_5 & a_5 \\
0 & 0 & 0 & -a_7 & l_6 & -a_7
\end{bmatrix},$$

where

$l_1 = -(a_1 + a_3), l_2 = (1 - a_3)a_4, l_3 = -(a_1 + a_3 + a_4), l_4 = -(a_1 + a_5 + a_1), l_5 = -(a_1 + a_3 + a_8), l_6 = (a_1 + a_3), l_7 = a_2\sqrt{a_1 + a_3}.$

Two of the eigenvalues of the matrix $J_{EE}$ are $\lambda_1 = -a_6, \lambda_2 = \lambda_5 = -(a_1 + a_3 + a_8)$ which are both negative. The remaining four eigenvalues are determined if they have a negative real part or not by the method of Routh-Hurwitz stability criteria from a characteristic Eq. (5) given below.

$$\phi(\lambda) = \lambda^4 + b_1\lambda^3 + b_2\lambda^2 + b_1\lambda + b_0,$$

where

$$= 2l_1l_2l_3(l_1 + l_2)(l_1l_3(1 - R_1) + (l_1 + l_2)(l_1l_4(1 - R_2) + (l_1 + l_2)(l_1l_4) < 0$$

for $R_1 < 1$, $R_2 < 1$, $R_0 < 1$.
\[ b_1 = a_1 - l_1 - l_1 - l_1 + l_1, \]
\[ b_2 = a_1 l_1 + l_1 l_1 - l_1 l_1 - l_1 l_1 - a_1 l_1 - a_1 l_1 - a_1 l_1 - a_1 l_1, \]
\[ b_3 = a_1 l_1 + a_1 l_1 - l_1 l_1 - l_1 l_1 + l_1 l_1 - a_1 l_1 - a_1 l_1 - a_1 l_1 + a_1 l_1 + a_1 l_1, \]
\[ b_4 = a_1 a_1 l_1 - a_1 l_1 l_1 - l_1 l_1 l_1 + a_1 a_1 l_1 l_1. \]

**Necessary condition**

The coefficient \( b_2 \) can easily be shown to be positive and \( b_2, b_1, b_0 \) are also positive as shown below:

\[ b_2 = ((l_1 l_1 R_2 + l_1 l_1 R_1)/R_0) + l_1 l_1 - l_1 l_1 - l_1 l_1 - l_1 l_1 > 0, \]
\[ b_1 = a_1 l_1 l_1 (R_2 / R_0) + a_1 l_1 l_1 (R_1 / R_0) + a_1 l_1 - 2 l_1 l_1 l_1 + l_1 l_1 l_1 + l_1 l_1 l_1 l_1 > 0, \]
\[ b_0 = a_1 a_1 a_1 a_1 l_1 - a_1 l_1 a_1 a_1 l_1 - l_1 l_1 l_1 l_1 + a_1 a_1 a_1 a_1 l_1 l_1 = -l_1 l_1 l_1 l_1 > 0. \]

**Sufficient condition**

Since it is not difficult to show that \( b_0 b_2^2 b_3^2 - b_2 b_3 < 0 \), one can be concluded that, all the eigenvalues of the characteristic Eq. (5) have a negative real part. Therefore, the EEP \( N^* = (S^*, E^*, I^*, A^*, H^*, R^*) \) defined in section 2.1 is globally asymptotically stable in the region \( \Omega \).

It must be noted that EEP exists if and only if \( R_0 > 1 \) as shown in section 2.2.

**Global stability analysis of DFE and EEP**

**Global stability of DFE**

**Theorem 3.** The DFE is globally asymptotically stable for \( R_0 < 1 \).

**Proof:** Consider a Lyapunov function candidate \( F(S, E, I, A, H, R) \) defined by

\[ F(S, E, I, A, H, R) = \left( S - S_0 - S_0 \log \frac{S}{S_0} \right) + E + I + A + H + R \]

Differentiating \( F(S, E, I, A, H, R) \) with respect to time in the direction of the solution of (1), and then substituting the appropriate values from (1) and \( S_0 = \frac{S}{\theta} \) leads to,

\[ \dot{F} = a\left(2 - \frac{S_0}{S} - \frac{S_0^2}{S_0}\right) + \lambda_0 \left(\frac{S_0}{S}\right) - a_1 (E + I + A + H + R), \]

where \( \lambda_0 = \frac{a_1 (A + a_1 I) S}{N} \).

Since \( \lambda_0 \left(\frac{S_0}{S}\right) \) is non-negative we have,

\[ \dot{F} \leq a\left(2 - \frac{S_0}{S} - \frac{S_0^2}{S_0}\right) - a_1 (E + I + A + H + R) \]
\[ = a\left(2S_0 - \left(\frac{S_0^2}{S_0}\right)\right) - a_1 (E + I + A + H + R) \]
\[ = -a\left(\frac{2S_0 - \left(\frac{S_0^2}{S_0}\right)}{S_0}\right) - a_1 (E + I + A + H + R) \]

Note that, by the inequality of arithmetic and geometric means, we have

\[ 2S_0 - \left(\frac{S_0^2}{S_0}\right) \leq 0. \]

Thus, we have proved that \( F \) is a Lyapunov function and \( \dot{F} \leq 0. \)

Moreover, \( \dot{F} = 0 \) if and only if \( S_0 = S, E = I = A = H = R = 0. \)

Therefore, it follows that the largest invariant set in \( \{ (S, E, A, I, H, R) \in \Omega : \frac{\dot{F}}{F} \leq 0 \} \) is \( N_0 = \left\{ \left(\frac{S_0}{\theta}\right) = 0, 0, 0, 0, 0 \right\} \). Thus, by LaSalle’s invariance principle, the DFE is globally asymptotically stable.

**Global stability analysis of EEP**

The global stability of EEP is explored by proving Theorem 4.

**Theorem 4:** If \( R_0 > 1 \), then the EEP given by \( N^* = (S^*, E^*, I^*, A^*, H^*, R^*) \) defined in section 2.1 is globally asymptotically stable in the region \( \Omega \).

**Proof:** Suppose the basic reproductive number \( R_0 > 1 \) so that the EEP exists. Consider a Lyapunov function candidate \( I \) defined by,

\[ \Phi(S, E, I, A, H, R) = \left( S - S^* - S^* \log \frac{S}{S^*} \right) + \left( E - E^* - E^* log \frac{E}{E^*} \right) \]
\[ + \left( I - I^* - I^* log \frac{I}{I^*} \right) + \left( A - A^* - A^* log \frac{A}{A^*} \right) + \left( H - H^* - H^* log \frac{H}{H^*} \right) \]
\[ + \left( R - R^* - R^* log \frac{R}{R^*} \right). \]

Differentiating \( \Phi \) in the direction of the solution of model (1) results in,

\[ \frac{d\Phi}{dt} = \left( \frac{S - S^*}{S} \right) \dot{S} + \left( \frac{E - E^*}{E} \right) \dot{E} + \left( \frac{I - I^*}{I} \right) \dot{I} + \left( \frac{A - A^*}{A} \right) \dot{A} + \left( \frac{H - H^*}{H} \right) \dot{H} + \left( \frac{R - R^*}{R} \right) \dot{R}. \]

Replacing \( \dot{S}, \dot{E}, \dot{I}, \dot{A}, \dot{H}, \dot{R} \) from (1) leads to,

\[ \frac{d\Phi}{dt} = \left( \frac{S - S^*}{S} \right) \left( a - \frac{a_1 (A + a_1 I) S}{N} - a_5 S \right) \]
\[ + \left( \frac{E - E^*}{E} \right) \left( \frac{a_1 (A + a_1 I) S}{N} - (a_4 + a_5) E \right) \]
\[ + \left( \frac{I - I^*}{I} \right) \left( (1 - a_3) a_1 I E - (a_4 + a_5) I \right) \]
\[ + \left( \frac{A - A^*}{A} \right) \left( a_1 a_5 E - (a_1 + a_5) A \right) \]
\[ + \left( \frac{H - H^*}{H} \right) \left( a_1 I + a_5 A - (a_1 + a_5) H \right) \]
\[ + \left( \frac{R - R^*}{R} \right) \left( a_1 I + a_5 A - (a_1 + a_5) R \right) \]

\[ = \left( \frac{S - S^*}{S} \right) \left( a - \frac{a_1 (A + a_1 I) (S - S^*)}{N} - a_5 (S - S^*) \right) \]
\[ + \left( \frac{E - E^*}{E} \right) \left( \frac{a_1 (A + a_1 I) (S - S^*)}{N} - (a_4 + a_5) (E - E^*) \right) \]
\[ + \left( \frac{I - I^*}{I} \right) \left( (1 - a_3) a_1 I E - (a_4 + a_5) (I - I^*) \right) \]
\[ + \left( \frac{A - A^*}{A} \right) \left( a_1 a_5 E - (a_1 + a_5) (A - A^*) \right) \]
\[ + \left( \frac{H - H^*}{H} \right) \left( a_1 I + a_5 A - (a_1 + a_5) (H - H^*) \right) \]
\[ + \left( \frac{R - R^*}{R} \right) \left( a_1 I + a_5 A - (a_1 + a_5) (R - R^*) \right) \]

where

\[ a = a_5. \]
Thus, by LaSalle

\[ R_{\infty} = \frac{\alpha}{H} \left( a_1 + 2a_2 + a_3 \right) + \frac{\alpha}{E} \left( a_1 + 2a_2 + a_3 \right) \]

Since all the parameters used in the model (1) are non-negative we have \( \frac{\partial R_{\infty}}{\partial t} \leq 0 \) for \( t \rightarrow 2 \), and \( \frac{\partial R_{\infty}}{\partial t} = 0 \) if and only if \( \gamma_1 = \gamma_2 \) which in turn implies that \( \frac{\partial R_{\infty}}{\partial t} = 0 \) and only if \( S = S', E = E', I = I', A = A', H = H', R = R' \). Thus, by LaSalle’s invariance principle the EEP is globally asymptotically stable.

As a result of Theorem 4, the pandemic persists in society whenever \( R_0 > 1 \) irrespective of the initial size of the different compartments of the model.

In this section, numerical simulation as a verification of the analytical proofs for local and global stability of the DFE and EEP is given. Runge-Kutta method and MatLab R2018a were used for the simulations.

Firstly, let us consider the cases where \( R_0 < 1 \) obtained from a parameter value \( \alpha_1 = 0.025 \) for which the corresponding basic reproductive number \( R_0 = 0.2048 < 1 \). A constant number of the total population, \( \alpha_5 = 2.0778 \) and the natural mortality rate equals the birth rate of susceptible cases \( \alpha = \alpha_6 \) is assumed for the numerical simulation. The other required parameter values used are from Table 1 and the simulation result for the different initial sizes of cases in the compartments is shown in Fig. 2.

It can be seen from the Figure that the number of cases in the compartments \( E \) and \( I \) starts decreasing to zero from the onset of the pandemic whereas the number of cases in the compartments \( A \) and \( H \) surges for about the first five days and then decreases down to zero provided that the initial number of cases are in the region \( \Omega \). The simulation results verify that the pandemic dies out from society irrespective of the initial population for the basic reproductive number \( R_0 < 1 \). This is also agreement with the analytical proof made above.

Secondly, consider the case where \( R_0 = 2.0479 > 1 \) obtained from \( \alpha = 136.98, \alpha_1 = 0.25 \) and all other required parameter values in Table 1. A constant number of population \( N = 1 \) is considered. The corresponding EEP is calculated to be \( \mathcal{N}' = (S', E', I', A', H', R') \) = (712.1554, 0.4883, 42.1347, 1080, 1299, 22998800). The simulation result is shown in Fig. 3. As can be seen from the figures the number of cases in each of the compartments converges to their corresponding EEP as time increase. The simulation result verifies the fact that the pandemic persists in society when \( R_0 > 1 \) regardless of the initial size of
cases in the different compartments of the model.

Sensitivity analysis of the reproductive number

The purpose of this section is to perform a sensitivity analysis of the basic reproductive number. Sensitivity analysis of the basic reproductive number can be used to design a mitigation strategy to slow the spread of the pandemic by reducing $R_0$. Sensitivity analysis [28] for the basic reproductive number mainly helps to discover parameters that have a high impact on the values of $R_0$ and hence should be targeted for designing intervention strategy. Moreover, sensitivity analysis helps to determine the level of change necessary for input parameters to find the desired value of a predictor parameter (see 3rd column of Table 2).

Definition 1: Normalized forward sensitivity index of $R_0$ which is differentiable with respect to a given parameter $\omega$ is defined as [29].

$$\phi_\omega = \frac{\omega}{R_0} \frac{\partial R_0}{\partial \omega}$$

Using this definition, the sensitivity indices of the parameters of the basic reproductive number are given in Table 2.

As shown in Table 2, the highly sensitive parameter of the basic

| Parameter | Sensitivity indices | Corresponding % changes | Parameter values after changes made to reduce $R_0$ by 1% |
|-----------|---------------------|-------------------------|-----------------------------|
| $\alpha_1$ = 0.2500 | 1.0 | –1% | 0.2475 |
| $\alpha_2$ = 1000 | 0.0406 | –24.63 | 0.7537 |
| $\alpha_3$ = 0.80 | 0.7972 | –1.25 | 0.79 |
| $\alpha_4$ = 0.1923 | 0.00023723 | –4215.32 | – |
| $\alpha_5$ = 0.6000 | –0.0406 | 24.63 | 0.7478 |
| $\alpha_6$ = 0.0500 | –0.000000 | 0 | – |
| $\alpha_7$ = 0.0714 | –0.6753 | 1.48 | 0.0725 |
| $\alpha_8$ = 0.00004563 | –0.000066894 | 1494.90 | – |
| $\alpha_{10}$ = 0.0300 | –0.2837 | 3.52 | 0.0311 |
reproductive number is the rate of transmission from asymptotically infectious to suspected individuals denoted by $\alpha_1$ with the sensitivity index of 1. That is, to increase/decrease the value of $R_0$ by 1% it is required to increase/decrease the value of $\alpha_1$ by 1%. The other sensitive parameters in order of decreasing are $\alpha_3, \alpha_7, \alpha_{10}, \alpha_5$ with their corresponding sensitivity index shown in Table 2.

In the next part of this section, the effect of the sensitivity of some of the relatively sensitive parameters on the spread of the pandemic is shown using numerical simulation. The parameters considered are $\alpha_1, \alpha_3, \alpha_5, \alpha_{10}$.

We intended to stimulate different values of the selected parameters that reduce the basic reproductive number by 0%, 1%, 5%, and 10%. Accordingly, Fig. 4 shows the effect of various values of the most sensitive parameter $\alpha_1$ on the number of exposed cases at the peak.

As it can be seen from Fig. 4, the number of cases at the peak without reducing the basic reproductive number ($R_0 = 2.0479$) is 175,500 on 193.9th day, whereas after reducing $R_0$ by 10% using $\alpha_1 = 0.2250$ the number of cases reduced to133,600 on 227.3th day and the graph gets flattened as the peak day changes from 193.9 to 227.3. This amounts to reducing the number of exposed cases by about 24%.

It can be inferred from Fig. 5 that, the number of cases at the peak without reducing the basic reproductive number ($R_0 = 2.0479$) is 175,500 on 193.9th day, whereas after reducing $R_0$ by 10% using $\alpha_5 = 2.0778$, the number of cases reduced to 162,500 on 203th day of the pandemic and the graph gets flattened as the peak day is moved from 193.9 to 203. This is equivalent to reducing the number of exposed cases by 7.4%.

As it can be seen from Fig. 6, the number of cases at the peak without reducing the basic reproductive number is ($R_0 = 2.0479$) is 175,500 on 193.9th day, whereas after reducing $R_0$ by 10% using $\alpha_3 = 0.0410$ the number of cases reduced to135,100 on 224.2th day and the graph gets flattened as the peak day moved from 193.9 to 224.2. This amounts to reducing the number of exposed cases by about 23%.

From Fig. 7, the number of cases at the peak without reducing the basic reproductive number is ($R_0 = 2.0479$) is 175,500 on 193.9th day, whereas after reducing $R_0$ by 10% using $\alpha_3 = 0.0410$ the number of cases reduced to135,100 on 224.2th day and the graph gets flattened as the peak day moved from 193.9 to 224.2. This amounts to reducing the number of exposed cases by about 23%.
cases reduced to 145,300 on 212.1th day and the graph gets flattened as the peak day changes from 193.9 to 212.1. This amounts to reducing the number of exposed cases by about 17.2%.

It can also be observed upon comparing the simulation results in Figs. 4 to 7 that, the effect of reducing the reproductive number by 1% is consistent with the sensitivity index shown in Table 2. That is, reducing the most sensitive parameter by 1% is most effective in reducing the number of exposed cases, and the effect on reducing the number of cases goes in order of their sensitivity index in absolute value. The same can be said about the effect of the parameters in reducing the number of cases in all the remaining compartments not simulated in this section.

Optimal control analysis

In this section, an optimal control analysis of three proposed control strategies is conducted. They are denoted by $c_1(t)$, $c_2(t)$, $c_3(t)$ respectively representing public health education, personal protective measures (wearing facemask, regular hand washing, and social distancing), and treatment of COVID-19 patients in hospitals to minimize the suffering from the diseases. Consequently, the mathematical model (1) modified to incorporate the control variables is as shown in (6):

\[
\begin{align*}
\dot{S} &= a - \frac{\alpha_3(\alpha_2 I + A) S}{N} - (\alpha_5 + c_1) S, \\
\dot{E} &= \frac{\alpha_3(\alpha_2 I + A) S}{N} - (\alpha_4 + \alpha_5) E, \\
\dot{I} &= (1 - \alpha_3) \alpha_4 E - (\alpha_6 + \alpha_4 + c_2) I, \\
\dot{A} &= \alpha_3 \alpha_4 E - (\alpha_7 + \alpha_6 + c_2) A, \\
\dot{H} &= \alpha_5 I + \alpha_{10} A - (\alpha_8 + \alpha_6 + \alpha_8 + c_2 + c_3) H, \\
\dot{R} &= (\alpha_9 + c_2) I + (\alpha_7 + c_2) A + (\alpha_8 + \alpha_6 + c_2 + c_3) H - \alpha_4 R.
\end{align*}
\]

\tag{6}

The objective is to find an optimal control for the three control strategies while reducing their relative coasts. We used Pontryagin’s maximum principle [30] to establish necessary and sufficient conditions for the existence of optimal control. The objective function in the time interval $[0, t_f]$ is defined as in (7):

\[
\int_0^{t_f} \left( \lambda_1 \dot{S} + \lambda_2 \dot{E} + \lambda_3 \dot{I} + \lambda_4 \dot{A} + \lambda_5 \dot{H} + \lambda_6 \dot{R} \right) dt
\]

\tag{7}
The target is to develop an optimal control \( p_{\text{OPT}} \) such that
\[
J_{\text{OPT}}(c_i) = \int_0^t (k_E(t) + k_I(t) + k_A(t) + k_H(t)) + \frac{p_1^2}{2} c_1^2 + \frac{p_2^2}{2} c_2^2 + \frac{p_3^2}{2} c_3^2 dt,
\]
where \( k_E, k_I, k_A, k_H \) are positive weights to balance the factors and \( p_1, p_2, p_3 \) measures the relative cost of the intervention strategies under consideration. The target is to develop an optimal control \( c_i^* \), \( i = 1, 2, 3 \) such that
\[
J_{\text{OPT}}(c_1^*, c_2^*, c_3^*) = \min_{c_i \in C} J_{\text{OPT}}(c_i(t), c_i(t), c_i(t)),
\]
where the control set is given by \( C = \{ c_i(t) : 0 \leq c_i(t) \leq 1, 0 \leq t \leq t_f, i = 1, 2, 3 \} \) subjected to the constraints in (6).

Pontryagin’s Maximum Principle converts (6) and (7) into a problem of minimizing point-wise a Hamiltonian \( H_0 \) with respect to \( c_i(t) \), where \( H_0 \) is defined as
\[
H_0(y, c_1(t), c_2(t), c_3(t), \lambda_1(t), \lambda_2(t), \lambda_3(t), \lambda_4(t), \lambda_5(t), \lambda_6(t), \lambda_7(t)) = (k_E(t) + k_I(t) + k_A(t) + k_H(t)) + \frac{p_1^2}{2} c_1^2 + \frac{p_2^2}{2} c_2^2 + \frac{p_3^2}{2} c_3^2 + \dot{\lambda}_1(t) d_1(t) + \dot{\lambda}_2(t) d_2(t) + \dot{\lambda}_3(t) d_3(t) + \dot{\lambda}_4(t) d_4(t) + \dot{\lambda}_5(t) d_5(t) + \dot{\lambda}_6(t) d_6(t),
\]
where
\[
\begin{align*}
d_1 &= a - \frac{a_i(a_I(t) + A)}{N} - (a_1 + a_2) S, \\
d_2 &= \frac{a_i(a_I(t) + A)}{N} - (a_1 + a_2) E, \\
d_3 &= (1 - a_2) a_I E - (a_1 + a_2 + a_3) I, \\
d_4 &= a_I a_E - (a_1 + a_2 + a_3) E, \\
d_5 &= a_I A - (a_1 + a_2 + a_3 + c_2) A, \\
d_6 &= a_1 I + a_2 D - (a_1 + a_2 + a_3 + c_2 + c_3) H, \\
d_7 &= (a_1 + c_2) I + (a_1 + c_2) A + c_3 S + (a_1 + a_2 + c_2 + c_3) H - a_8 R.
\end{align*}
\]

The corresponding adjoint variables \( \lambda_j, j \in \{ S, E, I, A, H, R \} \) are given by (10):
\[
\begin{align*}
\dot{\lambda}_S &= \lambda_3(c_1 + a_1 + a_2) / N - \lambda_2 c_1 - (a_1 \lambda_3(A + a_2 I)) / N, \\
\dot{\lambda}_E &= \lambda_4(k_E - \lambda_3(k_E - k_E) - \lambda_4 a_2 A, \\
\dot{\lambda}_I &= \lambda_5(k_I - \lambda_3(k_I - k_I) - \lambda_6 a_2 E, \\
\dot{\lambda}_A &= \lambda_6(k_A - \lambda_3(k_A - k_A) - \lambda_7 a_2 A, \\
\dot{\lambda}_H &= \lambda_7(k_H - \lambda_3(k_H - k_H) - \lambda_8 a_2 H, \\
\lambda_k &= \lambda_k(a_1 + a_2 + a_3 + c_2 + c_3) k_3 - \lambda_2 k_8(a_1 + a_2 + c_2 + c_3), \\
\lambda_k &= \lambda_k(a_1 + a_2 + c_2 + c_3) k_3 - \lambda_2 k_8(a_1 + a_2 + c_2 + c_3).
\end{align*}
\]

As a result, the optimal controls and the optimality conditions are given respectively by (12) and (13):
\[
\begin{align*}
c_1^*(t) &= \frac{\lambda_1 - \lambda_2}{b_1} S, \\
c_2^*(t) &= \frac{\lambda_6 H + \lambda_4 I + \lambda_2 A - \lambda_6(A + I + H)}{b_2}, \\
c_3^*(t) &= \frac{\lambda_5 - \lambda_2}{b_3} H, \\
c^*_1(t) &= \max \left[ \min \left( \frac{1}{b_1}, \frac{(\lambda_1 - \lambda_2)}{b_1} S \right), 0 \right], \\
c^*_2(t) &= \max \left[ \min \left( \frac{1}{b_2}, \frac{\lambda_6 H + \lambda_4 I + \lambda_2 A - \lambda_6(A + I + H)}{b_2} \right), 0 \right], \\
c^*_3(t) &= \max \left[ \min \left( \frac{1}{b_3}, \frac{\lambda_5 - \lambda_2}{b_3} H \right), 0 \right].
\end{align*}
\]

The optimality system includes the state Eq. (6), the adjoint Eq. (9), the characterization of the optimal control (13), and the transversality condition (11).

Having the analytical behaviors of the optimal control described above, the corresponding numerical simulation is detailed in the next section.
Numerical simulation

A numerical solution for the optimality system is presented in this section. The initial size used in the simulation is $S(0) = 3000000$, $E(0) = 20$, $I(0) = 1$, $A(0) = 0$, $H(0) = 0$, $R(0) = 0$, $N = 3000021$, while the parameter values are indicated in Table 2.

The forward–backward sweep scheme MatLab code used in simulating the optimality system of this work is adapted from Martcheva [31].

The following six scenarios were considered for numerical simulation.

Case. I: $c_1(t) = c_2(t) = c_3(t) = 0$. In this case, model (6) is simulated without any of the control strategies being optimally practiced. The result of the simulation is depicted in Fig. 8.

It can be seen from Fig. 8 that, the peak is in the range of 194–215 days of the pandemic. There are about 252,600 hospitalized, 250,800 asymptotic, 10,390 symptomatic, and 176,000 exposed cases at the peak.

Case. II: Optimal practice of the three control strategies. In this case $(c_1(t) \neq 0, c_2(t) \neq 0, c_3(t) \neq 0)$. The simulation results of this case are
shown in Figs. 9 and 10 for the cumulative number of cases in the compartments and optimal control functions respectively.

From Fig. 10 we can see that, the practice of personal protective measures\(c_1(t)\), may not be required to be practiced at the maximum level from the onset of the pandemic but could be kept to more than 80% for the first few days before it begins to slowly minimize to the lower bound. Using Public health education about the pandemic \(c_1(t)\) need to be maintained at the maximum level for about the first 5 days before it slowly gets down to the lower bound whereas treating COVID-19 patients must be continued up to the end of the pandemic with the maximum intensity as in Fig. 10.

According to the simulation result shown in Fig. 9, if the intervention strategies are applied optimally as in Fig. 10, from the onset of the pandemic, then their effect in reducing the number of COVID-19 cases would have been as indicated in Fig. 9. Practicing the three control strategies together at the maximum level, from the onset of the pandemic reduced the number of cases in the compartments significantly to the extent that there are almost no cases as shown in Fig. 9.

Case. III: \((c_1(t) = 0, c_3(t) \neq 0, c_2(t)\neq0)\). In this case, optimal personal protective measure and optimal treatment of hospitalized cases with the absence of public health education is considered. The effect of these two control strategies in reducing the number of COVID-19 cases is the same as in Fig. 10. The optimal functions \(c_2(t)\)and\(c_3(t)\) are shown in Fig. 11.

Upon comparing Figures from cases II and III, we can say that, combining control strategies \(c_2(t)\)and\(c_3(t)\) leads to the same result as combining the three control strategies in mitigating the transmission of COVID-19, but for case III, the full intensity of using personal protective measures has to be prolonged as compared to case II, before it slowly gets reduced to the lower bound at the end of the pandemic.

Note that, cases where \((c_2(t) = 0, c_1(t) \neq 0, c_3(t) \neq 0)\) and \((c_3(t) = 0, c_1(t) \neq 0, c_2(t) \neq 0)\), have similar effects as cases III in reducing the number of cases in each of the compartments.

Case. IV: \(c_1(t) = 0, c_3(t) \neq 0, c_2(t)\neq0\). In this case, only optimal treatment of hospitalized cases is considered. The simulation result is shown in Fig. 12 and the control function is the same as \(c_3(t)\) Fig. 10.

From Fig. 12, the optimal practice of treating hospitalized COVID-19 cases without the optimal practice of the other two strategies couldn’t reduce the number of cases in the compartments \(E,A\) and \(I\). since the cumulative number of cases in Fig. 12 is the same as the cumulative.

![Fig. 12. Number of cases in the compartments with the control strategy \(c_1(t)\).](image)

![Fig. 13. Optimal control profile with the control strategy \(c_1(t)\).](image)
number of cases in Fig. 8. However, this intervention strategy significantly reduced the number of hospitalized cases as it can be seen by comparing the hospitalized cases in Figs. 8 and 12.

Case V: \( c_1(t) = 0, c_3(t) = 0 \). This is the case where the optimal practice of public health education alone is considered. The simulation result of the effect of optimally practicing this control strategy is shown in Fig. 13 for the optimal profile of \( c_1(t) \) and Fig. 14 for the number of cases in different compartments.

It can be inferred from Fig. 14 that, the optimal practice of public education alone from the onset of the pandemic is almost as effective as combining the three control strategies.

Case VI: \( c_1(t) = 0, c_3(t) = 0 \). In this case, the optimal practice of personal protective measures alone is considered. The simulation result of the control profile is shown in Fig. 15. It is found that the simulation result of the effect of the optimal practice of this intervention strategy in reducing the number of cases in the compartments is the same as shown in Fig. 9.

It can be inferred from Fig. 15 that, the usage of personal protective measures need to be optimally practiced for about the first 42 days of the duration of the pandemic. Upon comparing cases II and VI we can say that both cases independently lead to the required result of mitigating the transmission of the pandemic. However, in case VI the practices of personal protective measures need to be applied optimally for a relatively prolonged time as compared to case II.

Discussion and conclusion

In this study, a mathematical model for transmission dynamics of COVID-19 is developed and qualitative analysis including the existence and uniqueness of positive solutions, local and global stability analysis of the diseases-free, and endemic equilibrium points have been shown. Numerical simulation for verification of global stability analysis showed that the analytical proofs and the simulation results are in agreement.

The result of the sensitivity analysis showed that the most sensitive parameter of the reproductive number is the rate of transmission from asymptotically infected cases to suspected individuals; \( \alpha_1 \). Numerical simulation of the parameters of the basic reproductive number showed that reducing the value of the most sensitive parameter reduced the number of exposed cases more than the relatively less sensitive parameters thereby the simulation is in perfect agreement with the sensitivity index of the parameters shown in Table 2.
Optimal control analysis of the model to assess the effect of public health education, the effect of personal protective measures and the effect of treating hospitalized cases in mitigating the transmission of COVID-19 was conducted. The result showed that the optimal practice of the combination of all three intervention strategies significantly reduces the number of exposed, symptomatic, asymptomatic, and hospitalized cases (see Fig. 9).

Likewise, optimal usage of personal protective measures alone led to the required decreases in the number of cases in the compartments except that the optimal application of the control measure needs to be maintained relatively for a longer period (see Fig. 15). It is also found that combining control strategies; personal protective measures, and treatment of hospitalized cases (Case III) is as good as combining the three strategies (case II) in combating the deadly COVID-19 pandemic.

In general, it can be concluded that the optimal combination of the three strategies or optimal combination of any two of the strategies or optimal practice of personal protective measures alone reduced the number of COVID-19 cases in the compartments as shown in the simulation results.

The result of this study can be used as a policy input for different countries with COVID-19 pandemic. WHO and countries in the world need to put in place a policy that makes personal protective measures a mandatory practice throughout the pandemic period. Personal protective measures with the maximum effort possible can significantly decrease the disturbing effect of COVID-19 and safeguard the people of the world from the lethal coronavirus of our generation.

Limitations of the model

Since mathematical models are approximations of reality, they are inherently inaccurate. The parameter values are obtained from observations and experiments by using different numerical methods of computing software and hence are uncertain. We have used parameter values from different pieces of literature and made assumptions to some of them and hence, there is a possibility that the mathematical model used in the work overestimates or underestimates the pandemic at a later period. Therefore, readers of this manuscript need to take these limitations into account while interpreting the findings of this research.

Ethics approval

Ethical approval or individual consent do not apply to this study.

Availability of data and materials

NA

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Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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