New approaches for modelling vortex flow meters

Zhanat Dayev¹,³, Aiat Kairakbaev¹,⁴, Alexandra Duzheva² and Nail Sultanov³

¹ Baishev University, Aktobe, Kazakhstan
² Samara State Technical University, Samara, Russia
³ Orenburg State University, Orenburg, Russia
⁴ E-mail: kairak@mail.ru

Abstract. The paper proposes a new model of a vortex flow meter, which is developed on the basis of modification of the basic equations of incompressible fluid hydrodynamics. The article deals with the modification of the Euler equation and the continuity equation of the flow. In this paper, dependencies are obtained for estimating the final standard uncertainty of flow rate using a vortex transducer. A detailed analysis of the influence of incompressible fluid flow parameters and the geometric dimensions of the vortex transducer on the final uncertainty of flow rate is performed. Recommendations are given for improving the method of flow rate using vortex transducers.

1. Introduction

Measuring the flow rate of liquids and gases plays an important role in the organization of various production processes. Among a large number of flow measurement methods, vortex flow meters are of interest, the principle of operation of which are based on the registration of vortex shedding that occur behind the bluff body. The frequency of vortices \( f \) in the first approximation is proportional to the flow velocity and depends on the dimensionless Strouhal number, as well as the size of the bluff body. A large number of scientific papers have been devoted to the development of vortex flow meters. For example, works [1, 2, 3] are devoted to the study of processes behind the bluff body. In [4, 5], an experimental study of various types of bluff bodies and their influence on the flow measurement process is performed.

The advantages of vortex flowmeters are the absence of moving elements inside the pipeline, a fairly wide range of measurements, and the invariance of the measurement method with respect to the electrical properties and aggregate state of the moving medium [6, 7, 8].

Improving the accuracy and reliability of vortex flow meters is carried out in various directions. For example, in works [1, 6, 9], signals from flow transducers are processed, improving their accuracy. In [4, 5, 10, 11], studies are performed related to the study of bluff bodies and the structure of vortex flow meters. Due to the complex processes that occur in vortex flow transducers, this work is important in order to organize measurements of high accuracy and reliability.

It should be noted that the physical processes occurring in the pipeline behind the bluff body are very complex. Pulsations of pressure, temperature, speed of sound, and other physical parameters occur in the flow. Therefore, there is a large amount of work that focuses on modelling these processes, they are no less important for the development of vortex flow meters. Such works include researches [12 – 14]. Despite the rapid development of numerical methods for describing complex objects, there are still no satisfactory mathematical models of hydrodynamic processes occurring in vortex flow meters. There
are a large number of questions related to the spatial and temporal distribution of physical characteristics in a moving medium, depending on the speed, aggregate state, and viscosity of the medium. The bluff body under vortex formation experiences a complex stress-strain state, where there are both torsion and bending vibrations, and others. One thing is clear, that the processes occurring in vortex flow meters are quite complex, but for a complete understanding, simple tools are needed to model these processes. Therefore, in this work, the authors aim to obtain simple modelling methods for obtaining the characteristics of vortex flow meters. The proposed models will allow us to evaluate the influence of various parameters of the medium and geometric dimensions of the vortex flow transducer, as well as serve for the further development of vortex flow meters.

2. Model for non-stationary one-dimensional fluid flow

To model vortex flow meters, we use the following approach, which allows us to associate the Strouhal number with the Bernoulli equation, which was already used in the author's works to describe the non-stationary flow rate of liquids and gases through the Venturi tube in papers [15, 16, and 17]. Below we will try to describe this approach in detail, which will allow us to obtain a model of a vortex flow meter.

To do this, we write the Euler equation for the one-dimensional case in accordance with [18]:

\[
\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} = -\frac{1}{\rho} \frac{\partial \rho}{\partial x},
\]

(1)

where \( V(t,x) \) - the velocity of the fluid, \( \rho \) - pressure, \( \rho \) - liquid density, \( t \) - time, \( x \) - coordinate along the central axis of the flow.

To solve our problem, we will perform the following replacement for the flow rate:

\[
V = u(z),
\]

where \( z = f \cdot t + \frac{x}{L} \) - new dimensionless variable, \( f \) - the frequency of the flow pulsation, a value inversely proportional to time, \( L \) - the length of some characteristic body size in the stream (bluff body).

Now we need to express equation (1) in terms of the new velocity. To do this, perform the following transformations for each partial derivative in equation (1):

\[
\frac{\partial V}{\partial t} = \frac{du}{dz} \cdot \frac{\partial z}{\partial t} = \frac{fu}{dz},
\]

\[
\frac{\partial V}{\partial x} = \frac{du}{dz} \cdot \frac{\partial z}{\partial x} = \frac{1}{L} \cdot \frac{du}{dz};
\]

\[
dz = \frac{\partial z}{\partial t} dt + \frac{\partial z}{\partial x} dx = f \cdot dt + \frac{1}{L} dx;
\]

\[
\frac{\partial \rho}{\partial x} = \frac{dp}{dz} \cdot \frac{\partial z}{\partial x} = \frac{dp}{dz} \cdot \frac{f \cdot dt + \frac{1}{L} dx}{dx} = \frac{dp}{dz} \cdot \left( f \cdot \frac{dt}{dx} + \frac{1}{L} \right) = \frac{1}{L} \frac{dp}{dz}.
\]

In the last expression, the relation \( \frac{dt}{dx} = 0 \) is due to the mutual independence of the variables \( t \) and \( x \).

After that, we will perform substitution of the last expressions for partial derivatives in the original equation (1). Then we get the following equation:

\[
\frac{du}{dz} + \frac{u}{L} \frac{du}{dz} = -\frac{1}{\rho} \frac{dp}{dz}.
\]

Next, divide the two parts of the equation by the value \( \frac{1}{L} \) and integrate the new variable \( z \). In this case we get the following equation:

\[
L \cdot f \cdot u + \frac{u^2}{2} + \frac{p}{\rho} = \text{const}.
\]
Convert the last equality to the following form:

\[
\frac{u^2}{2} \left( \frac{2L \cdot f}{u} + 1 \right) + \frac{p}{\rho} = \text{const}
\]

where \( Sh = \frac{Lf}{V} \) - dimensionless Strouhal number.

Thus, we have obtained some modification of the Bernoulli equation. The value in brackets in equation (2) containing the Strouhal number. It can also play the role of the Coriolis coefficient, in the case of non-stationary and uneven fluid flow. In the case when the Strouhal number in equation (2) is zero, the expression (2) takes the form of the classical Bernoulli equation for the stationary motion of an incompressible fluid.

A similar modification can be performed for the continuity equation, which represents the law of conservation of mass of the fluid flow. Write down the continuity equation for a one-dimensional flow [18]:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho V) = 0.
\]

(2)

By performing similar actions with variables for the last equation, and assuming that the fluid is incompressible, we get a solution in the form of an equation for the fluid flow density in the following form:

\[
(Sh + 1)\rho V = \text{const}.
\]

(3)

If the Strouhal number is zero in the last equation (3), it becomes the classical solution for the continuity equation of the incompressible flow.

Thus, we obtained modified equations for the laws of conservation of momentum and matter for a one-dimensional unsteady fluid flow. Now, based on equations (2) and (3), we will try to perform a simulation of a vortex flow meter.

3. Model of a vortex flow meter

The principle of operation of a vortex flow meter is based on the formation of vortices behind the bluff body, installed in the flow part of the flow meter and their subsequent accounting, i.e. measurement. Figure 1 shows a photo from [19], which reflects the nature of the fluid movement around the bluff body. In the photo, water flows around the cylinder, and vortices form behind it, creating a Karman vortex street.

To build a model, we must specify the following conditions:

- The flow of liquid or gas to the bluff body moves steadily and evenly;
- The flow of a liquid or gas after the bluff body becomes unsteady, i.e. the flow becomes pulsating.

Figure 1. A Karman vortex street behind the bluff body.
The flow meter model will be built based on these two conditions, so for each of these conditions we will apply equations (2) and (3). For this purpose, we will draw a section of pipe with the vortex transducer. This section is shown in figure 2.

Figure 2 shows two pipe sections. The first section corresponds to a stationary flow of fluid. The figure shows that there is no pulsating flow in section 1, so the Strouhal number is zero at this point of the flow. The second section corresponds to a pulsating non-stationary section, disturbed due to the bluff body. At this point, all the parameters of the flow change, and the Strouhal number is not equal to zero. The geometric dimensions of the pipe and the cylindrical body of the flow section can be seen from figure 2.

Write down equations (2) and (3) for the pipeline shown in figure 2 for each section separately. In this case we have the following dependencies:

\[
\frac{V_1^2}{2} + \frac{p_1}{\rho} = \frac{V_2^2}{2} (2S_1 + 1) + \frac{p_2}{\rho} + \xi \frac{V_2^2}{2} (2S_1 + 1), \tag{4}
\]

\[
V_1 F_1 + (S_1 + 1)V_2 F_2, \tag{5}
\]

where \(\xi\) - coefficient of resistance in the area between the first and second sections, \(F_1\) - flow area in section 1, \(F_2\) - flow area in section 2.

Let’s find the velocity value in the first section from equation (5):

\[
V_1 = (S_1 + 1)V_2 \frac{F_2}{F_1} = (S_1 + 1)V_2 m, \tag{6}
\]

where \(m\) - the relative area of the vortex flow transducer, its value is equal to the following expression:

\[
m = \frac{2}{\pi} \left[ \arccos \beta - \beta \sqrt{1 - \beta^2} \right],
\]

where \(\beta = d/D\) - the diameter ratio, taken by analogy with the orifice plate.

If we substitute expressions (6) in equation (4), we get the following expression for the flow velocity in the second section:

\[
V_2 = \frac{1}{((S_1 + 1)^2(m^2 + 1 - \xi) + S_1^2(\xi - 1))^{1/2}} \cdot \frac{\sqrt{2\Delta p}}{\rho},
\]

where \(\Delta p = p_1 - p_2\) - pressure loss on the vortex transducer.

Substitute the last expression in the continuity equation of the flow and get the volume flow rate of the liquid:

\[
Q = (S_1 + 1)V_2 F_2 = \frac{(S_1 + 1)D^2 \left[ \arccos \beta - \beta \sqrt{1 - \beta^2} \right]}{2((S_1 + 1)^2(m^2 + 1 - \xi) + S_1^2(\xi - 1))^{1/2}} \cdot \frac{\sqrt{2\Delta p}}{\rho}. \tag{7}
\]
The last equation (7) takes into account the value for the second section. From this equation, we can see that the expression for the flow rate, which is determined using a vortex transducer, is a complex quantity that depends on the flow fluctuations, the geometric dimensions of the pipe and the transducer. Equation (7) also takes into account the amount of pressure loss, which is ignored in many models, and affects the resulting uncertainty of flow measurement. In [14], a simultaneous measurement of pressure losses on a vortex flow transducer with the main signal of the liquid pulsation frequency is proposed.

For example, figure 3 shows the relationship for the relative area of a vortex transducer. It is intuitively clear that only with certain ratios of the bluff body diameter and the pipe diameter will the optimal result be achieved. If the relative area value increases, it will be difficult or even impossible to measure the flow rate. The latter demonstrates the dependency in figure 3.

![Figure 3. Dependence of the relative area on the diameter ratio.](image)

We estimate the water flow rate in a wide range of Strouhal numbers, taking into account the constant value of pressure losses. It is also possible to estimate the optimal value of the relative diameter.

Now let's consider the flow behavior in a wide range of Strouhal numbers for different values of the diameter ratio of the vortex transducer.

The graphs in figure 4 show that to ensure a wide range of liquid flow rate measurement, it is necessary to select the optimal value of the diameter ratio. At small values of the relative diameter, as can be seen from the figure, the flow rate remains almost constant. To increase the dynamic range of flow rate measurements, it is necessary to increase the value of the diameter ratio, but as it follows from the graph in figure 3, it is impossible to increase it indefinitely due to the fact that the flow passage section will not be able to pass this amount of liquid and a reverse flow will occur, and measurement becomes impossible. The nature of the curves shown in figure 4 coincides with the dependencies of experimental studies.

![Figure 4. Dependence of the flow rate on the Strouhal number.](image)
Let’s analyze the uncertainty of flow rate for the model in the form of equation (7), which can form the basis of the vortex flow meter conversion function.

The formula for the relative standard uncertainty of the flow rate measurement result can be obtained from equation (7) in the following form:

\[ u_Q^2 = \frac{1}{4} a_1^2 \cdot u_f^2 + a_2^2 \cdot u_D^2 + a_3^2 \cdot u_d^2 + \frac{1}{4} (u_{\Delta p}^2 + u_{\rho}^2), \]  

(8)

where \( u_f \) - standard relative uncertainty of measurement of the flow pulsation frequency behind the bluff body, \( u_D \) - standard relative uncertainty of pipeline diameter measurement, \( u_d \) - standard relative uncertainty of measurement of the bluff body diameter, \( u_{\Delta p} \) - standard relative uncertainty of measurement of the differential pressure on the bluff body, \( u_{\rho} \) - standard relative uncertainty of liquid density measurement. Equation (8) also contains \( \alpha_1, \alpha_2, \alpha_3 \) - sensitivity coefficients, which will be described below.

The first sensitivity coefficient from equation (8) is represented by the following expression:

\[ \alpha_1 = \frac{Sh(3m^2(Sh + 1)^2 - (4Sh + 3)(\xi - 1))}{2(Sh + 1)(m^2(Sh + 1)^2 - (2Sh + 1)(\xi - 1))}. \]

The second sensitivity coefficient from equation (8) is represented by the following expression:

\[ \alpha_2 = 2 \left( 1 + \frac{\beta}{\pi m \sqrt{1 - \beta^2}} \left[ 2\beta^2 + \frac{m^2(1 - \beta^2)(Sh^2 + 1)}{m^2(Sh + 1)^2 - (2Sh + 1)(\xi - 1))} \right] \right). \]

The last sensitivity coefficient from equation (8) is represented by the following expression:

\[ \alpha_3 = \frac{2\beta}{\pi m \sqrt{1 - \beta^2}} \left( 2\beta^2 + \frac{m^2(1 - \beta^2)(Sh^2 + 1)}{m^2(Sh + 1)^2 - (2Sh + 1)(\xi - 1))} \right). \]

From the last expressions for the sensitivity coefficients, we see that they may implicitly depend on the flow rate.

4. Discussion of results
Let’s look at the behaviour of the flow measurement uncertainty, and also find out how individual values affect the final uncertainty. Similar experimental studies devoted to a detailed study of the uncertainty of flow rate measurement results were conducted in [20].

![Figure 5. The dependence on the uncertainty of flow rate measurement.](image-url)
Figure 5 shows a graph of the change in flow rate measurement uncertainty from fluid flow rate for different diameter ratio values. The dependencies are obtained in accordance with equation (8), where the conversion of the sensitivity coefficients is performed. At the same time the calculation uses a constant Strouhal number $Sh = 0.2$ and the following standard uncertainties are accepted:

$$u_f = 0.05\%, u_d = u_d = 0.01\%, u_{\Delta p} = 0.075\%, u_\rho = 0.01\%.$$  

The relationship in figure 5 is linear, and in accordance with equation (8), the relative uncertainty of flow rate measurement increases proportionally to the flow rate. The diameter ratio value, as seen in the figure, does not significantly change the final uncertainty of flow rate measurement, but the diameter ratio also increases the uncertainty. Therefore, it is necessary to select the optimal value of $\beta$ to achieve the best result of the uncertainty of flow rate measurement.

The sensitivity coefficients in equation (8) can only be expressed in terms of Strouhal numbers using equation (7). This will allow us to get dependencies that reflect the influence of the Strouhal number on the uncertainty of the flow rate measurement result. Figure 6 shows the dependence of the flow rate measurement uncertainty on the Strouhal number. From this dependence, it can be seen that in a wide range of Strouhal numbers, the uncertainty of flow measurement remains almost linear and does not change much. The latter phenomenon is quite consistent with experimental researches.

Figure 7 shows another relationship, which is the inverse relationship for the graph in figure 6. This relationship reflects the dynamics of changes in the uncertainty of flow rate measurement from the diameter ratio $\beta$. As can be seen from figure 7, as the values of $\beta$ increase, the uncertainty also increases, but the value of the Strouhal number has a minimal impact.

![Figure 6. Dependence of the flow rate measurement uncertainty on the Strouhal number.](image)

![Figure 7. Dependence of the flow rate measurement uncertainty on the diameter ratio of the flow transducer.](image)
Let’s consider the influence of uncertainties of measuring devices that are used during the measurement of liquid flow rate. Figure 8 shows the dependences for the flow rate measurement uncertainty on the measurement uncertainty of the liquid pulsation frequency and pressure drop.

![Figure 8. Dependence of flow measurement uncertainty on measurement instrument uncertainty.](image)

As we can see from the graphs in figure 8, the accuracy of the measuring instruments also affects the resulting uncertainty of flow rate measurement. The more accurately the pulsation frequency is measured, the more accurate the final results of measuring the flow rate are. It can also be concluded from this figure that choosing a more accurate differential pressure transmitter will provide better flow rate measurement results. Figure 8 shows that differential pressure transmitters with large uncertainty values significantly increase the overall uncertainty of fluid flow rate measurement.

The proposed vortex flow meter model also explains the need for straight sections of the pipeline in front of the flow transducer. The flow up to the vortex flow transducer must be stationary, i.e. in the flow up to the flow transducer, the Strouhal number must be zero. In [8, 21], it is proposed to install flow straighteners for the formation of a stationary flow to the bluff body. This circumstance is explained by the application of equation (4). Otherwise, if there are vortices in the incoming flow, they would need to be taken into account by the corresponding Strouhal number to the bluff body. Therefore, for the successful implementation of the vortex flow meter that would provide the necessary accuracy in accordance with the proposed model, it is necessary to ensure the conditions that are stipulated at the beginning of section 3 of this article.

Equation (8) must be multiplied by a coverage factor that corresponds to a certain confidence level to obtain an expanded uncertainty of fluid flow rate measurement.

5. Conclusions

Thus, within the framework of this article, the problem of modelling the operation of the vortex flow meter based on modified equations for the non-stationary fluid flows is solved. The article offers modified equations of motion of a pulsating liquid, a model of a vortex flow meter. The article analyses the obtained model of a vortex flow meter. The influence of flow parameters and geometric dimensions of the vortex transducer on the uncertainty of flow rate measurement is analysed. Dependencies and graphs that reflect data between patterns are presented.

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