Cosmological parameter estimation from CMB and X-ray cluster after Planck

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Abstract. We investigate constraints on cosmological parameters in three 8-parameter models with the summed neutrino mass as a free parameter, by a joint analysis of CCCP X-ray cluster data, the newly released Planck CMB data as well as some external data sets including baryon acoustic oscillation measurements from the 6dFGS, SDSS DR7 and BOSS DR9 surveys, and Hubble Space Telescope H0 measurement. We find that the combined data strongly favor a non-zero neutrino masses at more than 3σ confidence level in these non-vanilla models. Allowing the CMB lensing amplitude A_L to vary, we find A_L > 1 at 3σ confidence level. For dark energy with a constant equation of state w, we obtain w < -1 at 3σ confidence level. The estimate of the matter power spectrum amplitude σ8 is discrepant with the Planck value at 2σ confidence level, which reflects some tension between X-ray cluster data and Planck data in these non-vanilla models. The tension can be alleviated by adding a 9% systematic shift in the cluster mass function.

Keywords: neutrino masses from cosmology, cosmological parameters from CMBR, galaxy clusters, dark energy theory

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1 Introduction

Recently the Planck team released the initial observational data [1, 2] based on the first 15.5 months operations. The results strongly support the standard 6-parameter ΛCDM model, hereafter the vanilla model. And the cosmological parameter constraints are greatly improved, including a highly significant deviation from scale invariance of the primordial power spectrum of curvature perturbations. However, as exposed by the Planck team [2], there are some potential tensions among the various observables in the standard 6-parameter ΛCDM model, such as the Hubble constant $H_0$, matter density parameter $\Omega_m$ and amplitude of matter perturbations $\sigma_8$, compared to other measurements. For example, the Planck results are discrepant with recent direct measurements from Hubble Space Telescope (HST) Key Project [3] and Type Ia supernovae observations via the magnitude-redshift relation, such as the Union2.1 compilation [4], while they are in excellent agreement with geometrical constraints from several baryon acoustic oscillation (BAO) surveys [5–7]. Very recently the authors of [8] re-analyzed the Planck primary CMB data and found that the 217 GHz × 217 GHz detector set spectrum used in the Planck analysis might be responsible for the tensions. In order to reveal or reconcile the tensions between low redshift geometric and Planck CMB measurements, some efforts have been made [9–20].

Except for CMB observations, Large Scale Structure (LSS) surveys on various scales, such as galaxies and clusters of galaxies, could also provide us lots of cosmological information. A better understanding of the structure of our universe asks for the agreement between theoretical predictions and observations on various spatial and temporal scales. The cosmological information encoded in the CMB map is mainly on large spatial scales and at deep redshift. As complementary observations, the distribution of LSS tells us the history of structure formation due to the instability of gravity on relatively small scales and at low redshifts. As the most massive virialized structures in the universe, the clusters of galaxies are perfect probes of the matter distribution on large scales. Within the framework of dark matter structure formation scenario, baryonic matter traces the distribution of dark matter halos. When the baryonic gas falls into the gravitational potential wells of the dark matter halos, it could be heated up to $10^7$K so that X-rays will be emitted. Via this mechanism, galaxy clusters could be identified through their X-ray fluxes. Chandra Cluster Cosmology Project (CCCP) [21–24] utilizes the X-ray indicator to observe the galaxy clusters. A cluster catalog was compiled from a new Rontgensatellite (Rosat) PSPC survey [25] covering 400
| Parameter   | Prior range     | Baseline | Definition                                      |
|-------------|-----------------|----------|------------------------------------------------|
| $\Omega_b h^2$ | [0.005, 0.1]    | –        | Baryon density today                           |
| $\Omega_c h^2$ | [0.001, 0.99]   | –        | Cold dark matter density today                 |
| $100\theta_{MC}$ | [0.5, 10.0]    | –        | Angular scale of the sound horizon             |
| $n_s$        | [0.9, 1.1]      | –        | Scalar spectral index                          |
| $\ln(10^{10} A_s)$ | [2.7, 4.0]   | –        | Amplitude of primordial curvature perturbations |
| $\Sigma m_\nu [eV]$ | [0.5]        | –        | The sum of neutrino masses                     |
| $N_{\text{eff}}$ | [0.05, 10.0]   | 3.046    | Effective number of neutrino species           |
| $w$          | [−3.0, −0.3]    | −1       | Equation of state of dark energy               |
| $\Omega_K$   | [−0.3, 0.3]     | 0        | Curvature parameter today                      |
| $A_L$        | [0, 10]         | 1        | Amplitude of the lensing power spectrum        |

Table 1. Cosmological parameters and prior ranges used in our analysis.

square degrees sky area. Thanks to the high resolution of the Chandra X-ray observation, high-quality X-ray data of the resulting samples up to redshift $z = 0.9$ are obtained, which can be used to determine the galaxy cluster mass function and hence to estimate the cosmological parameters. The CCCP X-ray cluster data in combination with CMB data can provide precise constraints on the summed neutrino mass [12, 26].

The detection of solar and atmospheric neutrino oscillations indicates that neutrinos have non-zero masses, but it cannot tell us the absolute masses of neutrinos. Cosmological observations can provide significantly strong constraints on the summed neutrino mass through the cosmological effects of massive neutrinos. Neutrino masses affect the CMB power spectrum mainly through the early integrated Sachs-Wolfe effect, the BAO by changing the late-time expansion rate of the universe, and the abundance of galaxy clusters by smearing out a fraction of the mass over the neutrino free streaming scale [27]. The Planck team assumes a minimal-mass normal hierarchy for neutrino masses, i.e., $\sum m_\nu = 0.06 \text{ eV}$ in the baseline model and finds a significant discrepancy between the Planck data and the abundance of galaxy clusters [2]. This suggests that some tensions might be relaxed by increasing the summed neutrino mass because massive neutrinos effectively reduce the density fluctuations on scales smaller than the free-streaming scale. Therefore, in our analysis the summed neutrino mass is always assumed to be a free parameter. Note that for the $\Lambda$CDM+$\sum m_\nu$ model, combining Planck+WP and the high-$l$ experiments gives an upper limit on the summed neutrino mass of $\sum m_\nu < 0.66 \text{ eV} (95\%)$ [2]. In this paper we will study the cosmological parameter estimate for three one-parameter extensions of the $\Lambda$CDM+$\sum m_\nu$ model listed in table 1, by using the Planck+WP+BAO+HST data in combination with CCCP X-ray clusters.

This paper is organized as follows. In section 2 we will describe the data sets and model. In section 3 we will present our results for three non-vanilla models and reveal some tensions between X-ray cluster data and Planck data. Section 4 is devoted to our conclusions.

2 Data and models

The total Planck CMB temperature power-spectrum likelihood is divided into low-$l$ ($l < 50$) and high-$l$ ($l \geq 50$) parts. This is because the central limit theorem ensures that the distribution of CMB angular power spectrum $C_l$ in the high-$l$ regime can be well approximated by Gaussian statistics. However, for the low-$l$ part the $C_l$ distribution is non-Gaussian. For this reason the Planck team adopts two different methods to build the likelihood. In
detail, for the low-\(l\) part, the likelihood exploits all \textit{Planck} frequency channels from 30 to 353 GHz, separating the cosmological CMB signal from diffuse Galactic foregrounds through a physically motivated Bayesian component separation technique. For the high-\(l\) part, the \textit{Planck} team employs a correlated Gaussian likelihood approximation, based on a fine-grained set of angular cross-spectra derived from multiple detector combination between the 100, 143, and 217 GHz frequency channels, marginalizing over power-spectrum foreground templates. In order to break the well-known parameter degeneracy between the reionization optical depth \(\tau\) and the scalar spectral index \(n_s\), the \textit{Planck} team adopts the low-\(l\) WMAP polarization likelihood (denoted by WP) \cite{28}. We refer to this CMB data combination as \textit{Planck}+WP.

As stated above, in this paper we will study the effect of the X-ray cluster data \cite{29–31} on the cosmological parameters. The \textit{CCCP} project measures the cluster mass function by using a high-redshift (0.4 < \(z\) < 0.9) subsample of 37 galaxy clusters selected from the 400 square degree ROSAT PSPC galaxy cluster survey, and a low-redshift (\(z\) < 0.2) subsample of 49 galaxy clusters selected from the ROSAT all sky survey. The methodology of likelihood construction follows the standard derivation of the Poisson distribution of cluster mass \cite{32}. The likelihood function implicitly depends on the cosmological parameters through the cluster mass function (which reflects the growth, normalization and shape of the density perturbation power spectrum), through the cosmological volume-redshift relation which determines the survey volume, and through the distance-redshift as well as the masses-temperature relation. The details of likelihood construction and systematic uncertainty control are discussed in \cite{29}.

Furthermore, in order to break the cosmological parameter degeneracies, we will also use some external data sets, including BAO measurements from the 6dFGS \cite{5}, SDSS DR7 \cite{6} and BOSS DR9 \cite{7} surveys, and HST Key project \cite{3} \(H_0\) measurement.

Recently ref. \cite{12} found that adding X-ray cluster data could give the non-zero detection of the active or sterile neutrino mass (\(\sum m_{\nu}\) or \(m_s\)) with great statistical significance for various 8-parameter models, including the active/sterile neutrino mass as well as effective number of neutrino species \(N_{\text{eff}}\). Moreover, adding X-ray data set could also lead to a significant deviation of \(\sigma_8\), the matter power spectrum amplitude on the 8\(h^{-1}\)Mpc scale, from the \textit{Planck} result \cite{2}. In other words, \textit{Planck} data favors a higher value of \(\sigma_8\), while adding X-ray cluster data gives a lower value. Therefore, in this paper we explore the tension between \textit{Planck} and \textit{CCCP} X-ray cluster data sets with several 8-parameter models, including effective neutrino number \(N_{\text{eff}}\), constant dark energy equation of state \(w\), present spatial curvature \(\Omega_K\) and lensing amplitude \(A_L\). In particular, we will pay attention to the neutrino mass constraint, so that in our baseline model the summed neutrino mass is always set to be a free parameter. To give a better constraint, we restrict ourselves to one-parameter extensions to the baseline model of \(\Lambda\text{CDM} + \sum m_{\nu}\), as listed in table 1.

We compute the CMB angular power spectra and matter power spectra by using the public Einstein-Boltzmann solver, \textit{CAMB} \cite{33}, and explore the cosmological parameter space with a Markov Chain Monte Carlo sampler, \textit{CosmoMC} \cite{34}. For \textit{Planck} data we use the Planck Likelihood Code (PLC/clik) \cite{35} which is available in the Planck Legacy Archive \cite{36}, and for \textit{CCCP} X-ray data our analysis is based on the likelihood grids presented in \cite{29}, which can be download from the website \cite{37}.

## 3 Results

In this section we will make a joint analysis for the data sets described above. For convenience, we denote \(CL_{X-\text{ray}}\) for X-ray cluster data. We will first investigate the constraints on the
Table 2. Constraints on three 8-parameter models from the PWBH+CLX-ray data.

| Model                  | AC$\Lambda$CDM+$\Sigma m_\nu$+$N_{\text{eff}}$ | AC$\Omega$CDM+$\Sigma m_\nu$ | AC$\Lambda$CDM+$\Sigma m_\nu$+$\Omega_K$ |
|------------------------|-------------------------------------------------|--------------------------------|-------------------------------------------|
|                        | best fit 68% limits                               | best fit 68% limits               | best fit 68% limits                        |
| 100$h^2$               | 2.285 ± 0.027                                     | 2.205 ± 0.026                    | 2.224 ± 0.031                             |
| $\Omega_m h^2$        | 0.1242 ± 0.0044                                   | 0.1181 ± 0.0016                  | 0.1172 ± 0.0026                           |
| $100\theta_{\text{MC}}$ | 1.04085 ± 0.0082                                 | 1.04139 ± 0.0059                 | 1.04190 ± 0.0065                         |
| $\tau$                | 0.095 ± 0.015                                      | 0.086 ± 0.013                    | 0.089 ± 0.013                             |
| $n_s$                 | 0.9925 ± 0.0098                                   | 0.9595 ± 0.0059                  | 0.9654 ± 0.0077                           |
| ln($10^{10}A_s$)      | 3.107 ± 0.031                                     | 3.075 ± 0.025                    | 3.083 ± 0.026                             |
| $\Sigma m_\nu$ [eV] | 0.47 ± 0.12                                        | 0.56 ± 0.10                      | 0.41 ± 0.12                               |
| $N_{\text{eff}}$      | 3.80 ± 0.29                                        | 3.704 ± 0.29                     | —                                            |
| $w$                   | —                                                 | —                                | —                                            |
| $\Omega_K$            | —                                                 | —                                | 0.00695 ± 0.00411                         |
| $\Omega_m$            | 0.2989 ± 0.012                                     | 0.2693 ± 0.013                   | 0.3027 ± 0.0131                           |
| $H_0$                 | 71.36 ± 1.5                                       | 73.68 ± 2.1                      | 68.93 ± 1.0                               |
| $\sigma_8$            | 0.7461 ± 0.0151                                   | 0.7902 ± 0.0208                  | 0.7434 ± 0.0164                           |
| $\chi^2_{\text{min}}/2$| 4911.581                                          | 4908.152                         | 4912.452                                  |

3.1 $\sum m_\nu$

First, let us study the summed neutrino mass $\sum m_\nu$. The solar and atmospheric neutrino oscillations observations give a lower bound on the summed neutrino mass, $\sum m_\nu \geq 0.06$ eV [38]. Besides the local observations, we can also obtain neutrino mass information via the indirect measurements on cosmological scales. Generally speaking, there exist mainly two ways. One is through the secondary CMB anisotropies generated in the deep matter-dominated epoch, such as weak lensing effect. However, these anisotropies are so small, compared with the primordial signal, that the current CMB experiments could only give a very loose upper bound, such as $\sum m_\nu < 0.66$ eV [2] from Planck+WP+ACT [39]+SPT [40–42]. The other is to utilize the large scale structure tracers, such as matter power spectrum, selected cluster counting and cosmic shear, etc. Compared with the bounds obtained by CMB observations, these tomographic measurements could give a relatively stringent constraint. For example, including X-ray cluster data [12] or SZ-selected cluster data from Planck [15, 43] and the SPT survey [27] can lead to the non-zero detection of the summed neutrino mass with quite significant evidence. Galaxy surveys such as WiggleZ [44] can improve the upper bound on the neutrino masses.

Given the above facts, in what follows we will use CLX-ray data to do the joint analysis of the summed neutrino mass $\sum m_\nu$ with several related parameters, such as effective neutrino number $N_{\text{eff}}$, dark energy equation of state $w$ as well as spatial curvature $\Omega_K$. For comparison, both the results with and without CLX-ray are listed in table 2 and table 3, respectively. And the corresponding 2D posterior distributions are shown in figure 1.
Table 3. Constraints on three 8-parameter models from the PWBH data.

| Model | $\Lambda$CDM+$\Sigma m_\nu+N_{\text{eff}}$ best fit | $\Lambda$CDM+$\Sigma m_\nu+N_{\text{eff}}$ 68% limits | $\Lambda$CDM+$\Sigma m_\nu+\Omega_K$ best fit | $\Lambda$CDM+$\Sigma m_\nu+\Omega_K$ 68% limits |
|-------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| $100\Omega_0 h^2$ | 2.225, 2.249±0.027 | 2.183, 2.193±0.026 | 2.214, 2.210±0.031 | 2.214, 2.210±0.031 |
| $\Omega_M h^2$ | 0.1241, 0.1275±0.0048 | 0.1225, 0.1209±0.0022 | 0.1188, 0.1192±0.0028 | 0.1188, 0.1192±0.0028 |
| $100\theta_0 h^2$ | 0.091, 0.097±0.0014 | 0.086, 0.089±0.013 | 0.092, 0.091±0.013 | 0.092, 0.091±0.013 |
| $n_s$ | 0.9743, 0.9830±0.0097 | 0.9552, 0.9571±0.0065 | 0.9628, 0.9619±0.0077 | 0.9628, 0.9619±0.0077 |
| $\ln(10^{10} A_s)$ | 3.102, 3.122±0.030 | 3.087, 3.090±0.025 | 3.092, 3.091±0.031 | 3.092, 3.091±0.031 |
| $\Sigma m_\nu$ [eV] | 0.039 < 0.34 (95% CL) | 0.17 < 0.61 (95% CL) | 0.018 < 0.38 (95% CL) | 0.018 < 0.38 (95% CL) |

Figure 1. *Left:* likelihood contours (68% CL and 95% CL) in the $\Sigma m_\nu-N_{\text{eff}}$ plane for the Planck+WP+BAO+HST+CL−ray (red) and Planck+WP+BAO+HST (blue) data combinations. *Middle:* $\Sigma m_\nu-w$ likelihood contours. *Right:* $\Sigma m_\nu-\Omega_K$ likelihood contours.

From table 2 one can see a more than 3σ detection of the summed neutrino mass for the three non-vanilla models once CL−ray data are taken into account

$$\sum m_\nu = 0.46 \pm 0.12 \ (68\% \ ; \ \Lambda CDM + N_{\text{eff}} : \text{PWBH + CL}_{X-ray}) ,$$

$$\sum m_\nu = 0.55 \pm 0.10 \ (68\% \ ; \ \Lambda CDM + wCDM : \text{PWBH + CL}_{X-ray}) ,$$

$$\sum m_\nu = 0.45 \pm 0.12 \ (68\% \ ; \ \Lambda CDM + \Omega_K : \text{PWBH + CL}_{X-ray}) .$$

Note that Hou et al. use a sample of 100 SZ-selected clusters from the SPT survey, in combination with CMB+BAO+HST, and find $\sum m_\nu = (0.32 \pm 0.11) \text{eV}$ [27], while Ade et al. use a sample of 189 SZ-selected clusters from Planck, in combination with the Planck CMB data and BAO, and obtain $\sum m_\nu = (0.22 \pm 0.09) \text{eV}$ [43]. Compared to these SZ-selected clusters, the CCCP X-ray selected clusters used in our analysis prefer a higher value of the summed neutrino mass. On the other hand, in the case without including CL−ray data, the preferred neutrino masses drop significantly while the uncertainties increase. Hence, the
In this subsection, we study the two cosmological parameters, CMB lensing power amplitude $A_L$ and matter power spectrum amplitude $\sigma_8$. Here the former can be viewed as one of the indicators of extra source of cosmic shear if it’s value deviates from unity, such as what happens in dynamical dark energy or modified gravity models. As shown in figure 2, there exists only a tiny correlation between $\sum m_\nu$ and $A_L$, so that the constraint on $A_L$ in $\Lambda$CDM+$\sum m_\nu$ and $\Lambda$CDM+$A_L$ models are very close (see table 5). For example, using PWBH+$CL_{X-ray}$ we get

$$A_L = 1.37 \pm 0.11 \ (68\%; \Lambda$CDM $+ A_L :$ PWBH $+ CL_{X-ray}) .$$

### 3.2 $A_L$ and $\sigma_8$

| Model               | $\Lambda$CDM | $w$CDM+$\sum m_\nu$ | $w$CDM+$\sum m_\nu$+$A_L$ |
|---------------------|---------------|---------------------|--------------------------|
|                     | best fit      | 68% limits          | best fit                 | 68% limits               | best fit                 | 68% limits               |
| $100h^2$            | 2.219         | 2.237±0.024         | 2.213                    | 2.214±0.024              | 2.226                    | 2.243±0.027              |
| $\Omega_m^2$       | 0.1142        | 0.1142±0.0011       | 0.1178                   | 0.1170±0.0013            | 0.1182                   | 0.1166±0.0014            |
| $100\theta$        | 1.04162       | 1.04185±0.00054     | 1.04175                  | 1.04143±0.00056          | 1.04129                  | 1.04165±0.00059          |
| $\tau$             | 0.077         | 0.077±0.011         | 0.086                    | 0.089±0.013              | 0.081                    | 0.087±0.013              |
| $n_s$              | 0.9678        | 0.9707±0.0052       | 0.9656                   | 0.9648±0.0054            | 0.9665                   | 0.9685±0.0057            |
| $\ln(10^{10} A_s)$ | 3.047         | 3.050±0.021         | 3.077                    | 3.080±0.025              | 3.069                    | 3.076±0.024              |
| $\sum m_\nu$ [eV]  | —             | —                   | 0.40                     | 0.39±0.09                | 0.42                     | 0.38±0.09                |
| $A_L$              | —             | —                   | —                        | —                        | —                        | —                        |
| $\sigma_8$         | 0.7940        | 0.7951±0.0072       | 0.8109                   | 0.7977±0.0170            | 0.8031                   | 0.7639±0.0198            |
| $\chi^2_{\text{min}}/2$ | 4912.286     | 4907.444            | 4903.368                 |                          |                          |                          |

Table 4. Constraints on cosmological parameters from PWBH+$CL_{X-ray}$+9% Mass.

Constraints on the summed neutrino mass listed in table 3 are consistent with vanishing mass at 95% confidence level. Moreover, we see that in that case, the estimated values of $\sigma_8$ are higher than those from the PWBH+$CL_{X-ray}$ data. There is therefore some tension between the PWBH and $CL_{X-ray}$ data, as shown in figure 1. It is possible that the tension results from some residual systematics. As discussed in [31], there might exist a systematical error in hydrostatic mass measurements $\delta M/M \simeq 0.09$ in $CL_{X-ray}$ data sets. This is due to the mass measurements used in CCCP project are based on hot intracluster gas temperature and masse, which are calibrated using hydrostatic measurements of their total gravitational masses in nearby clusters [29]. The resulting 9% systematical error of hydrostatic mass estimation was obtained from the comparison with the cluster mass measurements using weak lensing data taken from [45]. Taking this mass function correction into account, we obtain the corresponding results listed in table 4. Thanks to this correction, we see that the mean values of summed neutrino mass get reduced but the standard deviation still stays

$$\sum m_\nu = 0.39 \pm 0.09 \ (68\%; \ w$CDM : PWBH $+ CL_{X-ray}$ + 9%Mass). \ (3.4)$$

$$A_L = 1.37 \pm 0.11 \ (68\%; \Lambda$CDM $+ A_L :$ PWBH $+ CL_{X-ray}) . \ (3.5)$$
Figure 2. Triangle posterior distribution of matter spectrum amplitude $\sigma_8$, summed neutrino mass $\sum m_\nu$ and CMB lensing amplitude $A_L$ with PWBH+$CL_X$–ray data sets.

![Triangle distribution](image)

| Model | $\Lambda$CDM+$\Sigma m_\nu+A_L$ | $\Lambda$CDM+$A_L$ | $\Lambda$CDM+$A_L$ (without HST+$CL_X$–ray) |
|-------|---------------------------------|------------------|----------------------------------|
|       | best fit 68% limits             | best fit 68% limits | best fit 68% limits                 |
| $100\Omega_b h^2$ | 2.292 2.276±0.027 | 2.304 2.285±0.025 | 2.254 2.250±0.028 |
| $\Omega_c h^2$ | 0.1132 0.1131±0.0011 | 0.1125 0.1124±0.0010 | 0.1167 0.1166±0.0017 |
| $100\theta_{MC}$ | 1.04279 1.04224±0.00056 | 1.04202 1.04224±0.00056 | 1.04156 1.04186±0.00058 |
| $\tau$ | 0.086 0.087±0.013 | 0.075 0.073±0.011 | 0.092 0.087±0.013 |
| $n_s$ | 0.9763 0.9768±0.0053 | 0.9807 0.9784±0.0051 | 0.9703 0.9698±0.0059 |
| $\ln(10^{10}A_s)$ | 3.069 3.068±0.025 | 3.046 3.039±0.021 | 3.091 3.078±0.025 |
| $\Sigma m_\nu$ [eV] | 0.25 0.28±0.08 | — | — |
| $A_L$ | 1.42 1.36±0.10 | 1.44 1.37±0.11 | 1.24 1.22±0.10 |
| $\Omega_m$ | 0.2907 0.2942±0.0108 | 0.2696 0.2693±0.0057 | 0.2940 0.2929±0.0099 |
| $H_0$ | 69.12 68.7±0.9 | 71.09 71.0±0.5 | 68.97 69.1±0.8 |
| $\sigma_8$ | 0.7538 0.7550±0.0139 | 0.7894 0.7867±0.0070 | 0.8218 0.8162±0.0119 |
| $\chi^2_{\text{min}}/2$ | 4907.083 | 4910.408 | 4903.236 |

Table 5. Constraints on cosmological parameters from PWBH+$CL_X$–ray. For comparison, in the third column we give constraints on the $\Lambda$CDM+$A_L$ model from the Planck+WP+BAO data.

For comparison, we list the results of $\Lambda$CDM+$A_L$ model without $CL_X$–ray in the third column of table 5, which are consistent with the Planck results [2]. It shows that without $CL_X$–ray
Figure 3. Left: marginalized posterior distribution of $\sigma_8$. Right: 2D posterior distribution between the lensing power amplitude $A_L$ and matter power spectrum amplitude $\sigma_8$.

and HST data\(^1\)

$$A_L = 1.22 \pm 0.10 \; (68\%; \Lambda\text{CDM} + A_L : \text{PWB}) . \quad (3.6)$$

Comparing (3.5) with (3.6), it is clear that adding $CL_{X-ray}$ data gives a significant detection of a deviation of $A_L$ from unity, the value in the vanilla model. Furthermore, in the third column of table 4, we can see that after considering 9\% mass correction, the tension in (3.5) with the vanilla model could be mildly reconciled:

$$A_L = 1.28 \pm 0.10 \; (68\%; w\text{CDM} + \sum m_{\nu} + A_L : \text{PWBH} + CL_{X-ray} + 9\%\text{Mass}) . \quad (3.7)$$

Next we turn to the matter power spectrum amplitude $\sigma_8$. As shown in figure 2, there exists a significant anti-correlation between $\sigma_8$ and $\sum m_{\nu}$. The reason is that the non-relativistic, massive, weakly-interacting neutrinos behave qualitatively as a species of warm dark matter, suppressing fluctuations on scales smaller than their thermal free-streaming length. Consequently, this correlation will lead to a relatively low value of $\sigma_8$ when $\sum m_{\nu}$ is allowed to vary (see figure 3 and table 5). For example, by using the data of PWBH+$CL_{X-ray}$ our analysis gives:

$$\sigma_8 = 0.7894 \pm 0.0070 \; (68\%; \Lambda\text{CDM} + A_L : \text{PWBH} + CL_{X-ray}) , \quad (3.8)$$

$$\sigma_8 = 0.7550 \pm 0.0139 \; (68\%; \Lambda\text{CDM} + \sum m_{\nu} + A_L : \text{PWBH} + CL_{X-ray}) . \quad (3.9)$$

In the 6-parameter $\Lambda$CDM model, Planck team [43] gives $\sigma_8 = 0.77 \pm 0.02$ by using SZ-selected galaxy clusters from Planck, in combination with BAO and BBN data. This result is consistent with ours here. On the other hand, we notice from figure 3 that the result with $CL_{X-ray}$ (3.9) (blue curve) is in a 2$\sigma$ tension with the one in the case without $CL_{X-ray}$ data (3.10) (red curve)

$$\sigma_8 = 0.8162 \pm 0.0119 \; (68\%; \Lambda\text{CDM} + A_L : \text{PWB}) . \quad (3.10)$$

\(^1\)In order to compare with the Planck results [2] we also remove HST data. As a background geometric measurements, HST data sets should be nearly blind to dynamical structure formation information on perturbation level. Hence, we should expect no significant change in $A_L$ value after removing HST data.
Of course, as in the $A_L$ case, with a 9% cluster mass correction we can also reconcile the tension with the data compilation without $C L_{X\text{-ray}}$:

$$\sigma_8 = 0.7639 \pm 0.0198 \ (68\%; \ wCDM + \sum m_{\nu} + A_L : PWBH + C L_{X\text{-ray}} + 9\%\text{Mass}) \ . \quad (3.11)$$

### 3.3 $H_0$ and $w$

In this subsection, we discuss two cosmological parameters, Hubble constant $H_0$ and a constant equation of state $w$ of dark energy. We can see from table 2 that without 9% mass correction in $C L_{X\text{-ray}}$ data, a larger $H_0$ value is favored. For example, for $wCDM + \sum m_{\nu}$ model, one has

$$H_0 = 74.0 \pm 2.1 \ (68\%; \ wCDM : PWBH + C L_{X\text{-ray}}) \ . \quad (3.12)$$

However, adding this correction gives a lower value of $H_0$ for the same model:

$$H_0 = 72.0 \pm 1.3 \ (68\%; \ wCDM : PWBH + C L_{X\text{-ray}} + 9\%\text{Mass}) \ . \quad (3.13)$$

Instead of the spatial curvature $\Omega_K$, once $w$ is allowed to vary, a larger $H_0$ can give a good fit to data (see the top left panel of figure 4). This is due to the $H_0 - w$ correlation illustrated in the top right panel of figure 4. For comparison we also show the results from joint analysis of $W M A P 7$ and $C L_{X\text{-ray}}$ data: the top right panel of figure 4 clearly shows that there exists a 2σ tension in the parameter plane between the $P l a n e k + W P + B A O + H S T + C L_{X\text{-ray}}$ (blue) and $W M A P 7 + B A O + H S T + C L_{X\text{-ray}}$ (green) data. In order to demonstrate this discrepancy more clearly, in the bottom panel of figure 4 we show three most discrepant parameters in these two data compilations: cold dark matter density contrast $\Omega_c h^2$, the equation of state $w$ of dark energy and the present Hubble constant $H_0$. We list their marginalized statistics as follows.

$$\Omega_c h^2 = 0.1176 \pm 0.0016 \ (68\%; \ wCDM : PWBH + C L_{X\text{-ray}}) \ , \quad (3.14)$$

$$\Omega_c h^2 = 0.1080 \pm 0.0015 \ (68\%; \ wCDM : W M A P 7 + B H + C L_{X\text{-ray}}) \ , \quad (3.15)$$

$$w = -1.39 \pm 0.12 \ (68\%; \ wCDM : PWBH + C L_{X\text{-ray}}) \ , \quad (3.16)$$

$$w = -1.00 \pm 0.08 \ (68\%; \ wCDM : W M A P 7 + B H + C L_{X\text{-ray}}) \ , \quad (3.17)$$

$$H_0 = 74.00 \pm 2.08 \ (68\%; \ wCDM : PWBH + C L_{X\text{-ray}}) \ , \quad (3.18)$$

$$H_0 = 70.94 \pm 2.41 \ (68\%; \ wCDM : W M A P 7 + B H + C L_{X\text{-ray}}) \ . \quad (3.19)$$

From the above results, we can clearly see that for the parameters $\Omega_c h^2$ and $w$, there exist 3σ and 2σ discrepancies, respectively, between $P l a n e k + W P + B A O + H S T + C L_{X\text{-ray}}$ and $W M A P 7 + B A O + H S T + C L_{X\text{-ray}}$ data compilation. Among these discrepancies, $\Omega_c h^2$ is the most controversial one, while the tension in $H_0$ and $w$ might be propagated from the one in $\Omega_c h^2$ via their degeneracies.

Due to the negative degeneracy between $H_0$ and $w$, a large value of $H_0$ will lead to a phantom-like equation of state of dark energy \[46\]

$$w = -1.39 \pm 0.12 \ (68\%; \ wCDM : PWBH + C L_{X\text{-ray}}) \ . \quad (3.20)$$

Here we would like to emphasize that this phantom-like value of equation of state is not resulted in by including $C L_{X\text{-ray}}$ data. From the middle column of table 3, one can see that without X-ray cluster data the equation of state still deviates from $-1$ more than 2σ:

$$w = -1.33 \pm 0.15 \ (68\%; \ wCDM : PWBH) \ . \quad (3.21)$$
Furthermore, if we further remove HST data, i.e., just use Planck+WP+BAO, there will be no significant deviation from $w = -1$ as
\begin{equation}
  w = -1.31 \pm 0.23 \ (68\%; \ w_{CDM} : \ PWB).
\end{equation}
Because of the well-known $H_0 - w$ anti-correlation, we think that this result partially shows the tension between HST and Planck on Hubble constant $H_0$ as found in [2]. In addition, we notice that the 9\% mass correction can cause $w$ more close to $-1$:
\begin{equation}
  w = -1.23 \pm 0.06 \ (68\%; \ w_{CDM} : \ PWB + CL + 9\%Mass).
\end{equation}
Finally, let us notice from table 4 that there exist relatively large differences of $- \ln \mathcal{L}_{\text{max}}$ between $\Lambda$CDM and $w$CDM models by using Planck+WP+BAO+HST+CL+X-ray+9\%Mass data. It reflects the fact that this data compilation might favor the non-vanilla model, so
Model $w_{CDM} + \sum m_\nu$ $w_{CDM} + \sum m_\nu + A_L$

| $\Delta BIC = BIC(\text{Model}) - BIC(\Lambda CDM)$ | 8.30 | 9.14 |

Table 6. BIC differences for PWBH+$CL_{X-ray}$+9% Mass.

we compute the Bayesian Information Criterium (BIC) to estimate its significance. The BIC definition is given by (for a recent discussion see [47])

$$BIC = -2 \ln L_{max} + k \ln N,$$  \hspace{1cm} (3.24)

where $k$ is the number of free parameters and $N$ denotes for the number of data points in the fits. As described in the section 2, the $2 \leq \ell \leq 49$ part of the Planck power spectrum is derived from all the channels between 30GHz and 353GHz, and the $\ell \geq 50$ part of the CMB temperature power spectrum is obtained from 100, 143 and 217GHz channels. Since Planck team does not publish the polarization data, in order to get sensible constraint on the parameters which are sensitive to the polarization signal, such as the reionization optical depth $\tau$, Planck team utilizes $2 \leq \ell \leq 32$ multiples of WMAP9 TT,TE,EE,BB spectra, which are obtained by using Ka-, Q- and V-band maps. We emphasize here we do not use WMAP9 BB spectra in our analysis. For the cluster mass function likelihood, the $CL_{X-ray}$ data set includes a high-redshift ($0.4 < z < 0.9$) 37 points as well as low-redshift ($z < 0.2$) 49 points. Besides, we use 4 data points from BAO and HST measurement. As a result we totally use $N = 8058$ data points in our analysis by taking into account the different channels of CMB data. In table 6 we give the differences of BIC between two different $w_{CDM}$ and $\Lambda CDM$ models, i.e., $\Delta BIC = BIC(w_{CDM}) - BIC(\Lambda CDM)$. It shows that although there are relatively large reductions in the likelihood of $w_{CDM}$ models, due to the large number of data sets we used, the penalty term in eq. (3.24) dominates in the BIC value. This leads to the fact that Planck+WP+BAO+HST+$CL_{X-ray}$+9%Mass data still strongly favor ($6 \leq \Delta BIC \leq 10$) the 6-parameter vanilla model.

4 Conclusions

We have presented constraints on cosmological parameters in three 8-parameter non-vanilla models using Planck, BAO, HST and CCCP X-ray data sets, in particular, paying attention to the constraint of the sum of neutrino mass. We have found that X-ray cluster data sets strongly favor a non-zero summed neutrino mass with more than 3$\sigma$ confidence level in these models, which is in good agreement with the results derived in [12]. The presence of massive neutrinos inhibits the growth of structures below the neutrino thermal free-streaming scale during structure formation, leading to a lower value of $\sigma_8$, which could improve consistency with X-ray cluster data. On the other hand, we have also revealed some tensions among different data compilations in the cosmological parameters, such as the matter power spectrum amplitude $\sigma_8$, lensing amplitude $A_L$, constant equation of state $w$ of dark energy as well as the Hubble parameter $H_0$.

For the matter power spectrum amplitude $\sigma_8$, adding X-ray cluster data favor a relatively low value compared to the case without X-ray cluster data. Because of the $\sigma_8 - \sum m_\nu$ degeneracy, this difference could be beyond 2$\sigma$ confidence level when the summed neutrino mass $\sum m_\nu$ is allowed to vary. For the CMB lensing amplitude $A_L$, the addition of X-ray cluster data makes its deviation from unity and results in more than 3$\sigma$ discrepancy.
The Planck+WP+BAO+HST+CLX-ray compilation prefers a large Hubble constant and quite negative equation of state of dark energy. However, these tensions/discrepancies could be reduced in some sense by making a 9% shift in the cluster mass function. We also have computed the Bayesian evidence (BIC) between $\omega$CDM and $\Lambda$CDM models by using Planck+WP+BAO+HST+CLX-ray+9%Mass data compilation. And the result shows that this data sets still favor the $\Lambda$CDM model. The resolution of these intensions will likely require either the identification of a currently-unknown systematic effect in at least one of these data sets or new physics.

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