The Case for Muon-based Neutrino Beams

Patrick Huber
Center for Neutrino Physics, Department of Physics, Virginia Tech, Blacksburg, VA 24061, USA

Alan Bros and Mark Palmer
Fermi National Accelerator Laboratory, Batavia, IL 60510, USA
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I. INTRODUCTION

For the foreseeable future, high energy physics accelerator capabilities in the US will be deployed to study the physics of the neutrino sector. This thrust for the US domestic program was confirmed by the recent Particle Physics Project Prioritization Panel (P5) report [1]. In this context, it is useful to explore the sensitivities and limiting systematic effects of the planned neutrino oscillation program, so that we can evaluate the issues that must be addressed in order to ensure the success of these efforts. It is only in this way that we will ultimately be able to elucidate the fundamental physics processes involved. We conclude that success can only be guaranteed by, at some point in the future, being able to deploy muon accelerator capabilities. Such capabilities provide the only route to precision neutrino beams with which to study and mitigate, at the sub-percent level, the limiting systematic issues of future oscillation measurements. Thus this analysis argues strongly for maintaining a viable accelerator research program towards future muon accelerator capabilities.

II. SHORT-BASELINE PHYSICS

Most models of neutrino mass generation involve right-handed, and hence sterile, neutrinos, however, the mass of those right-handed neutrinos is not well constrained. In principle, the mass scale of the right-handed neutrino can range from sub-eV up to the Planck scale and only a few regions of parameter space have been probed in detail. Thus, the existence of sterile neutrinos with a mass around the eV-scale seems plausible. Experimental results from LSND [2] indicate a $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ flavor conversion at the level of about $0.003$. Given the baseline and mean energy of neutrinos in this experiment, oscillation involving a neutrino with a mass squared difference $\Delta m^2 \sim 1 \text{ eV}^2$ is a possible and straightforward explanation. More recently, the MiniBooNE experiment [3, 4] has seen hints of flavor transitions in both $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and $\nu_\mu \rightarrow \nu_e$, which appear to be consistent with the oscillation interpretation of the LSND result.

Atmospheric neutrino oscillations imply $\Delta m^2_{31} \sim 2 \times 10^{-3} \text{ eV}^2$ and solar neutrino oscillations require $\Delta m^2_{21} \sim 7 \times 10^{-5} \text{ eV}^2$. Therefore, at least four neutrinos are required to allow for another $\Delta m^2$ to be of order $\text{eV}^2$. From the invisible decay-width of the Z-boson, it is known that there are only 3 active, light neutrinos and hence, the extra neutrino to explain LSND and MiniBooNE has to be sterile.

Any oscillation from one active neutrino into another active neutrino mediated by a sterile neutrino requires that the sterile neutrino mixes with both the initial and final active neutrino flavor. As a consequence, any appearance signal, as potentially observed by LSND and MiniBooNE, implies the existence of a corresponding disappearance signal. This correspondence can be made quantitative: the energy averaged oscillation probabilities obey the following inequality, irrespective of the number of sterile neutrinos,

$$\langle P_{\nu_\mu \rightarrow \nu_e} \rangle \leq 4 \left( 1 - \langle P_{\nu_\mu \rightarrow \nu_\mu} \rangle \right) \left( 1 - \langle P_{\nu_e \rightarrow \nu_e} \rangle \right).$$

An analogous expression holds for antineutrinos, noting that the energy averaged disappearance probabilities for neutrinos and antineutrinos are equal.

Somewhat more recently, the reactor antineutrino anomaly has been noted [5], which indicates a 6% deficit of $\bar{\nu}_e$ from nuclear reactors at distances of 10-100 m. Approximately one half of the effect, that is 3% of the deficit, are due to the re-evaluation of reactor antineutrino fluxes [6], which has been independently confirmed by one of the authors [7]. The error budget of the reactor antineutrino flux calculations is a difficult subject in its own right, due to the poorly understood impact nuclear structure might have [8] and the fact that a feature, the so-called 5 MeV bump,
in the measured spectrum of reactor antineutrinos has recently been found. This feature is not predicted by the flux calculations \[6, 7\]; for a summary on the 5 MeV bump see Ref. \[9\]. Taken at face value, the 6% neutrino deficit can be interpreted as disappearance of $\bar{\nu}_e$ at a level and with a $\Delta m^2$ consistent with the LSND and MiniBooNE results and their respective interpretation as sterile neutrino oscillation.

Support for an eV-scale neutrino also comes from the radioactive source calibrations of the radiochemical gallium solar neutrino experiments, GALLEX and SAGE. In order to verify the operation of these experiments, electron capture sources based on either chromium-51 \[10, 11\] or argon-37 \[12\] were used to expose the detectors to a well known, mono-energetic $\nu_e$ flux. The resulting number of germanium atoms stayed below the expectation by about 25% and this result can be interpreted as the disappearance of $\nu_e$ with oscillation parameters consistent with the previously mentioned evidence \[13\].

In combination, these indications have led to a renewed interest in the question of sterile neutrinos at the eV-scale \[14\] and, as a result, there is a plethora of newly proposed experiments. At the same time, the fact that no $\nu_\mu$ disappearance at the relevant value of $L/E$ has been observed and many other searches have produced null results is a source of significant tension in global fits, see for instance Ref. \[15\]. Also, cosmological observables are sensitive to the presence of a eV-mass sterile neutrino and while some authors claim considerable tension, other authors find acceptable compatibility, e.g. \[16\]. It appears that cosmology so far remains inconclusive for this problem.

To make any progress on the question of a light sterile neutrino new experiments are necessary and in recognition of this, recommendation 15 of the P5 report states:

> Select and perform in the short term a set of small-scale short-baseline experiments that can conclusively address experimental hints of physics beyond the three-neutrino paradigm. Some of these experiments should use liquid argon to advance the technology and build the international community for LBNF at Fermilab.

The key word here is “conclusively” and the question arises what it takes to provide a conclusion to the sterile neutrino saga. The P5 report further recommends that nuSTORM be not part of this (or any other domestic) portfolio. Conclusively testing the sterile neutrino interpretation will require sharpening the experimental results on both sides of Eq. \[1\] by simultaneously pursuing appearance and disappearance searches in both neutrino and antineutrino modes. Table \[1\] lists all possible oscillations channels and enumerates the experiments which can access a given channel. “SBL” summarizes all possible experiments which can be performed in a pion decay in-flight beam and includes all experiments proposed within the Fermilab short-baseline program. Pion decay in-flight beams contain an intrinsic $\nu_e$ component at the level of 1% which is about 3-10 times larger than the expected signal. It appears doubtful whether precision measurements in the case of a discovery are possible in this environment. Due to the poorly known primary beam flux, any credible disappearance search will require a near and far detector comparison, which in practice is difficult to achieve at high accuracy because of the different geometric acceptance of the near and far detector. OscSNS \[17\] is the proposal for an experiment exploiting neutrinos from pion decay-at-rest at the Spallation Neutron Source at Oak Ridge National Laboratory. OscSNS would use the same process to generate and to detect neutrinos as LSND did, at the same energy and baseline. This constitutes the most direct test possible of the original LSND result. Atmospheric neutrinos already provide stringent limits on $\nu_\mu$ and $\bar{\nu}_\mu$ disappearance and new experiments like low-energy extensions of IceCube, see for instance \[18\], as well as the ICAL detector at the Indian Neutrino Observatory are expected to significantly improve these limits. SOX is the proposal to deploy radioactive sources under the Borexino detector \[19\], currently two different types of sources are considered. One possibility is a 10 MCl exposure to a chromium-51 source, which provides a mono-energetic low-energy $\nu_e$ flux detected by elastic $\nu_e$-scattering. Another possibility is a 75 kCi cerium-144 source, which provides a relatively high-energy beta-spectrum type flux of $\bar{\nu}_e$, detected by inverse beta decay. The cerium source is pursued as an entirely European project and data taking may start by the end of 2015, whereas the chromium source would profit from U.S. involvement, specifically irradiation of chromium-50 in the High-Flux Reactor (HIFR) at Oak Ridge. Both sources are too low in energy to allow for appearance searches so this would constitute a disappearance search in the electron channel. IsoDAR \[20\] also exploits beta decay as its neutrino source. In this case it is the high-energy beta decay of lithium-8, which is produced online using neutrons from a spallation target driven by a 600 kW beam of 60 MeV $\mathrm{H}_3^+$ ions. The detection reaction is inverse beta decay and ideally multi-kiloton detectors based on either water or liquid organic scintillator are used. All source experiments provide a well-characterized source flux and spectrum \[1\] however the energy of the neutrino is entirely determined by nuclear physics. As a result, the accessible $L/E$-range is limited. The SOX configurations suffer from somewhat limited statistics. For IsoDAR, the question of cost looms large since a significant accelerator component is required which, in conjunction with the necessary shielding and decommissioning at the end.

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1 Perhaps with the exception of the cerium-144 source, where a few percent of emitted neutrinos stem from forbidden beta decay branches and hence are subject to considerable nuclear structure effects.
FIG. 1. Sensitivity of nuSTORM to the $\nu_e \rightarrow \nu_\mu$ appearance oscillation due to the presence of sterile neutrinos assuming a (3+1) model with anticipated and inflated systematics, compared to 99% confidence contours from global fits to the evidence for sterile neutrinos and to all available appearance experiments generated by Kopp et. al. [15] (filled contours) and limits set by ICARUS [23]. Figure and caption adapted from Ref. [22].

of the experiment, will require a careful assessment of the full lifetime cost. PROSPECT [21] is one of many proposed reactor short-baseline experiments aimed at directly confronting the reactor antineutrino anomaly. The key is to use the near/far detector concept employed very successfully by Daya Bay and RENO at a much shorter distance, meters instead of 100’s of meters. The resulting challenge lies in dealing with backgrounds from reactor operation and surface deployment. Statistical errors can be made quite small thanks to the enormous flux of reactor antineutrinos, however the $L/E$-range is limited by the reactor antineutrino spectrum.

It is quite plausible that a combination of a number of these experiments will be able to provide a conclusive test of the sterile neutrino interpretation of the anomalies listed at the beginning. Most of these experiments will have significant difficulties in going beyond a simple yes or no answer. In particular precision studies aimed at discerning the number of sterile neutrinos involved or studying the question of potential CP violation for 2 or more sterile neutrinos are beyond their reach. However, it also is plausible that the sum total of these experiments would constitute a large investment at a level similar to nuSTORM, but, as we will argue in the following paragraph, with significantly less overall capability.

The nuSTORM experiment would provide simultaneous access to all of the oscillation channels listed in Tab. I, a unique feature among the proposed facilities; for a detailed description see Ref. [22]. The absolute beam normalization and spectrum will be known to better than 1% based on beam instrumentation. Storing either $\mu^+$ or $\mu^-$ allows for the production of precisely controlled CP-conjugate beams. Combined with the right suite of detectors, appearance and disappearance measurements at an unprecedented and unrivaled accuracy become possible, potentially including a very precise neutral current disappearance search. The sensitivity in the golden appearance mode $\nu_e \rightarrow \nu_\mu$, the CPT conjugate of the original LSND signal, is shown in Fig. 1. Clearly, a test at very high confidence level is possible. What is more important is having both sufficiently large statistics and sufficiently small systematics to explore any possible signal in great detail, ultimately pinning down the underlying physics.

| Oscillation Channel | Experiment |
|---------------------|------------|
| $\nu_\mu \rightarrow \nu_\mu$ | atmospheric, SBL |
| $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$ | atmospheric, SBL |
| $\nu_e \rightarrow \nu_e$ | SOX |
| $\bar{\nu}_e \rightarrow \bar{\nu}_e$ | PROSPECT, isoDAR, SOX |

TABLE I. List of possible oscillation channels and experiments which can access each channel. “SBL” refers to all short-baseline pion decay in-flight neutrino beams.
FIG. 2. Value of the CP asymmetry $A$ for different choices of $\delta$ as a function of the baseline.

III. LONG-BASELINE PHYSICS

With the start of the LHC in 2010 and the shutdown of the Tevatron in 2011, the focus of the domestic experimental high-energy physics program has shifted from the Energy Frontier to the Intensity Frontier. In order to ensure a vital program, DOE is committed to making a very significant investment (at the level of a billion dollars) in new experiments at the Intensity Frontier over the next decade. This plan has recently been endorsed by the report of the P5 sub-panel of HEPAP. Specifically, recommendation 13 of the P5 report reads:

Form a new international collaboration to design and execute a highly capable Long-Baseline Neutrino Facility (LBNF) hosted by the U.S. To proceed, a project plan and identified resources must exist to meet the minimum requirements in the text. LBNF is the highest-priority large project in its timeframe.

The P5 report further stipulates that the minimum requirement is to have a sensitivity to discover CP violation for at least 75% of all CP phases at the $3\sigma$ confidence level. This requirement translates directly into an upper limit on the acceptable systematic uncertainty. The CP asymmetry, $A$, is defined as

$$A = \frac{\langle P \rangle - \langle \bar{P} \rangle}{\langle P \rangle + \langle \bar{P} \rangle},$$

(2)

where $\langle P \rangle$ is the energy averaged oscillation probability for $\nu_\mu \rightarrow \nu_e$ and $\bar{P}$ is the corresponding quantity for antineutrinos. The energy average is taken over the range defined by having one half of the peak probability around the first oscillation maximum. A rough approximation of the problem at hand is provided by stating that a measurement of the CP phase is equivalent to a measurement of the asymmetry $A$, since $A \propto \sin \delta$, with $\delta$ being the CP phase. For $\delta \sim 0$ or $\pi$ the error on $\sin \delta$ and $\delta$ are approximately the same. In Fig. 2 the value of the asymmetry for different choices of $\delta$ is shown as a function of the baseline. Note that, even for a CP-conserving value of $\delta = 0$, there is a non-vanishing asymmetry due to matter effects (this figure assumes a normal hierarchy). For baselines below 1500 km, the genuine CP asymmetry is at most $\pm 25\%$, whereas for 75% of the parameter space in $\delta$, the genuine CP asymmetry can be as small as $\pm 5\%$. That is, a $3\sigma$ evidence for CP violation in 75% of parameter space requires a $\sim 1.5\%$ measurement of the $P - \bar{P}$ difference. Assuming that the statistical and systematic error contribute at the same level, a 1% systematic error is required.

Of course, experiments neither directly measure $\langle P \rangle$ nor the probability $P$. Instead they measure event rate distributions, $R$, as a function of the visible energy:

$$R_{\beta}^\alpha(E_{\text{vis}}) = N \int dE \Phi_\alpha(E) \sigma_\beta(E, E_{\text{vis}}) \epsilon_\beta(E) P(\nu_\alpha \rightarrow \nu_\beta, E),$$

(3)

where $\alpha$ is the initial neutrino flavor and $\beta$ the corresponding final flavor. Furthermore, $N$ is the overall normalization (fiducial mass), $\Phi_\alpha$ is the flux at the detector of $\nu_\alpha$, $\sigma_\beta$ is the cross section for $\nu_\beta$, and $\epsilon_\beta$ is the detection efficiency for $\nu_\beta$. Note that $\sigma_\beta \epsilon_\beta$ always appear in that combination, hence we can define an effective cross section $\delta_\beta := \sigma_\beta \epsilon_\beta$.
\(\sigma_\beta\) depends on both the true neutrino energy \(E\) and the visible energy \(E_{\text{vis}}\), since a neutrino of a given energy \(E\) can produce a range of visible energy depending on the underlying event. In practice the situation is complicated by the detector response which will translate the visible energy into the reconstructed energy. Even if we ignore all energy dependencies of efficiencies, cross sections etc., we generally cannot expect to know any of the fluxes \(\phi\) or any of the effective cross sections \(\bar{\sigma}\) at the required percent level of accuracy. There generally are no constraints at the percent level on flux ratios between flavors and/or neutrinos and antineutrinos. For cross section ratios, it is clear that neutrino/antineutrino ratios are known only at very high energy exceeding the 10’s of GeV. At low energy, there is no guarantee that the nuclear cross sections of \(\nu_e\) and \(\nu_\mu\) scattering are the same. At the quark level, lepton universality ensures the same coupling strength at the vertex, but the lepton mass impacts the phase space and thus the momentum transfer to the nucleus. The response of the nucleus depends sensitively, and in an essentially not well known manner, to small changes in \(Q^2\). Neutrino experiments cannot measure the actual \(Q^2\) value in an event and thus, effectively only see the total interaction rates fully integrated over the kinematically allowed \(Q^2\)-range. A detailed discussion based on an analysis of form factors associated with the corresponding hadronic and leptonic currents has been presented by Day and McFarland [24]. In their analysis, differences of several percent in the \(\nu_e\) to \(\nu_\mu\) cross sections at energies below 1 GeV appear possible. Furthermore, even if we may be able to determine \(\sigma_e/\sigma_\mu\) from theory, we will not know the corresponding ratio of efficiencies \(\epsilon_e/\epsilon_\mu\).

The problem that the neutrino flux or cross section is not known with sufficient accuracy has been encountered many times in neutrino physics. A proven solution is the use of a near detector to measure the un-oscillated event rate. In the ratio of far to near detector data many uncertainties will cancel. In practice, this requires that near and far detectors are very well understood in their response and geometrical acceptance. Assuming that the detector response is identical between near and far detectors and that only total rates matter, the far/near ratio is given by

\[
\frac{R_{\alpha}^{\text{far}}(L)^2}{R_{\alpha}^{\text{near}}} = \frac{N_{\text{far}} \Phi_{\alpha} \bar{\sigma}_{\alpha} P(\nu_{\alpha} \rightarrow \nu_{\alpha})}{N_{\text{near}} \Phi_{\alpha} \bar{\sigma}_{\alpha} 1} = \frac{N_{\text{far}}}{N_{\text{near}}} P(\nu_{\alpha} \rightarrow \nu_{\alpha}).
\]

This method has been applied very successfully in the Daya Bay experiment to measure \(\theta_{13}\) [25], where the conditions were quite ideal: near and far detectors employ the same material, size and design, and thus are functionally identical with sub-percent precision [26]; near and far detectors see the reactor cores as point sources; the inverse beta-decay cross section is independently known; and the initial and final flavor are the same.

To extrapolate this experience to future beam experiments, the following factors need to be considered. First, seeing the neutrino source as point-like will require a not-so-near detector, which for baselines of more than a few hundred km requires prohibitively expensive tunneling. Second, due to the enormous size of the far detector, the near and far detectors cannot be truly identical. Third, in a neutrino beam the energy spread is sufficiently large that several different interaction mechanisms will contribute to the event sample and thus energy dependencies can be not neglected. For a disappearance measurement, MINOS provides a good real-world example of the arising issues and practical methods to deal with them [27]. Even in disappearance mode and employing a very sophisticated target-horn configuration to systematically cross check primary neutrino production, the overall systematic error remains at the level of 1-3% in neutrino mode [28] and at a higher level in the antineutrino mode [29].

For an appearance measurement, the initial and final flavor are different. Thus, everything else being equal, the cancellation in Eq. (4) will be less complete, and a term of the form \(\bar{\sigma}_{\beta}/\bar{\sigma}_{\alpha}\) will remain. Even under the assumption that the initial flux \(\phi_{\alpha}\) would be well known, a measurement of \(\bar{\sigma}_{\beta}\) in a pure beam of flavor \(\alpha\) is obviously not possible. A realistic pion decay-in-flight neutrino beam will contain a small admixture of \(\nu_e\) but the size of this admixture is not well constrained across the whole energy range. Also, the small size of this admixture will limit the statistical accuracy in the near detector. T2K made an attempt to exploit the \(\nu_e\) component in a \(\nu_\mu\) beam for a cross section measurement [30]. The total systematic error is about 16\% dominated by the \(\nu_e\) beam flux uncertainty and the detector response. At this point it is unclear how one could reduce the systematic error budget to the percent level. The situation for antineutrinos is likely to be considerably more difficult. In general, it is far from obvious how to achieve a 1\% cross section measurement in a beam for which the flux is only known to 5\%.

To illustrate the quantitative effect on the ability to measure CP violation, a T2HK-like setup was studied in Ref. [31], where a total of more than 20 nuisance parameters, including total cross section uncertainties, were considered. One of the main results is shown in Fig. [3] where the sensitivity to CP violation is shown both for statistical errors only as well as for the full systematic error budget. It is obvious that a constraint on the \(\bar{\sigma}_{\mu}/\bar{\sigma}_{e}\) ratio is the most efficient, and only, way to recover the desired statistical sensitivity.

Up to this point the whole discussion has been framed in terms of energy independent quantities, but looking at Eq. (5) it is obvious that the relation between the true neutrino energy \(E\) and the visible energy \(E_{\text{vis}}\) enters prominently into this problem. For the moment, we will neglect detector effects stemming from finite energy resolution. Under this simplifying assumption, the problem stems from the fact that detectors are made of nuclei and not free protons (or deuterium). This gives rise to a number of significant systematic effects, which we will summarily refer to as nuclear effects:

\[\text{(nuclear effects)}\]
FIG. 3. CP violation sensitivity at 3σ for a certain choice of of systematical errors according and for statistical errors only (curves delimiting the shaded region). We show also the sensitivity if certain constraints on the product of cross sections times efficiencies $\sigma$ are available: 1% accuracies on $\sigma_\mu$ and $\sigma_e$ for neutrinos and antineutrinos, and 5%, 2%, 1% accuracies on the ratios $\sigma_\mu/\sigma_e$ for neutrinos and antineutrinos. Figure and caption adapted from Ref. [31].

- Initial state momentum distribution where, due to being in a bound state, each nucleon has a non-zero kinetic energy and momentum.

- Nuclear excitations where, in an interaction between a neutrino and nucleus, some energy is transferred into the nuclear system, resulting in, for instance, de-excitation gamma rays.

- Reaction products leaving the nucleus where, when the primary vertex is deep inside the nucleus, the reaction products have to traverse the nuclear medium resulting in a modified energy distribution or complete absorption. It is, for instance, possible that a proton is produced but strikes a neutron and the final state therefore contains this neutron.

- Higher order interactions where, given the nuclear bound state, there are significant correlations between individual nucleons resulting in interactions with more than one nucleon.

As a function of $Q^2$ these effects are flavor blind, but $Q^2$ is not measured in a typical long-baseline experiment. As explained previously, the lepton masses affect the available phase space and thus the $Q^2$ distribution. Hence there are flavor effects in the total cross section. None of these nuclear effects is expected to be the same (or even similar) for neutrinos and antineutrinos. This is exemplified by the very different $y$-distributions of neutrinos and antineutrino in deep-inelastic scattering, where $y$ measures the degree of inelasticity. Despite the fact that the electroweak sector of the Standard Model is extremely well understood, it is very hard to perform precise computations of the neutrino-nucleus interactions. Since existing neutrino beams are subject to large intrinsic uncertainties, measurements of the neutrino nucleus cross sections have errors in the 10-30% range, with the exception of a few fully inclusive channels. Obtaining a significantly better quantitative understanding of neutrino cross sections on nuclear targets is a very hard theoretical problem, since nuclear structure for large nuclei such as argon is not well understood. Multi-nucleon correlations as well as final state interactions have to be correctly included to provide reliable neutrino cross sections. A number of techniques to derive systematic approximations exist, but many calculations so far cover only limited portions of the relevant kinematic regions. A considerable effort is currently being devoted to the development of theoretical models capable of providing a fully quantitative description of neutrino-nucleus interactions in the kinematic regime relevant to the MicroBooNE, LBNE, ArgoNeut and Captain experiments. On the other hand, for light nuclei up to carbon, detailed calculations of the wave function exist, which reproduce measured energy levels quite accurately. Most of the advances made in our theoretical understanding of neutrino-nucleus interactions are not available to experiments,
Arguably, the conceptually simplest example is provided by quasi-elastic interactions (QE) where the reconstructed energy distributions of true QE events and those with the same final state are shown in the left panel of Fig. 4, as for the solid blue line. In this analysis, data is generated with GiBUU and can then be fitted with GENIE. In Fig. 5, the solid allowed region shows the case where GiBUU is used both to generate and to fit the data. The open regions show where GENIE is used for the fitting process. The resulting bias is very significant. The left hand figure assumes that the energy scale is fixed. Given the offset in energies seen in the event rate distributions between the two generators, see Fig. 4, the resulting $\chi^2$ is quite poor. In the right hand panel, the energy scale is allowed to shift by 5%, which is enforced by the near detector data. In this case the bias is strongly reduced and the overall $\chi^2$ becomes much better. Nonetheless, the best fit data point is still more than 1 $\sigma$ away from the actual value.

We can identify two distinct, but related, problems: $\nu_e/\nu_\mu$ cross section ratios in a narrow band beam at low energy, like T2K, and the question of energy reconstruction for both water Cerenkov detectors and liquid argon TPCs. There are a number of steps which can be taken to improve the situation:

A common technique to estimate theory errors is to compare results obtained with different methods by different groups and to use the spread in results as an indicator of the associated uncertainty. Fig. 5 shows the results of such an analysis, utilizing a T2K-like setup, of the disappearance channel measurement of the atmospheric parameters $\theta_{23}$ and $\Delta m^2_{31}$. In this analysis, data is generated with GiBUU and can then be fitted with GENIE. In Fig. 5, the solid allowed region shows the case where GiBUU is used both to generate and to fit the data. The open regions show the results where GENIE is used for the fitting process. The resulting bias is very significant. The left hand figure assumes that the energy scale is fixed. Given the offset in energies seen in the event rate distributions between the two generators, see Fig. 4, the resulting $\chi^2$ is quite poor. In the right hand panel, the energy scale is allowed to shift by 5% which is enforced by the near detector data. In this case the bias is strongly reduced and the overall $\chi^2$ becomes much better. Nonetheless, the best fit data point is still more than 1 $\sigma$ away from the actual value.
FIG. 5. Impact on the results if a different generator is used to compute the true and fitted rates in the analysis. The shaded areas show the confidence regions at 1, 2 and 3σ that would be obtained in the \( \theta_{23} - \Delta m^2_{31} \) plane if the true and fitted rates are generated using the same set of migration matrices (obtained from GiBUU, with oxygen as the target nucleus). The colored lines show the same confidence regions if the true rates are generated using matrices produced with GiBUU, but the fitted rates are computed using matrices produced with GENIE. Both sets of matrices are generated using oxygen as the target nucleus. The red dot indicates the true input value, while the black triangle shows the location of the best fit point. The value of the \( \chi^2 \) at the best fit is also shown, together with the number of degrees of freedom. In the left panel no energy scale uncertainty is considered, while for the right panel an energy scale uncertainty of 5% is assumed, see text for details. Figure and caption adapted from Ref. [33].

- Better theory – there is considerable room for improvement, in particular, closing the gap between event generators and theory, see for instance a recent implementation of the spectral functions for GENIE [36].

- More electron scattering data – any theoretical model of the electroweak nuclear response should be able to reproduce the somewhat simpler electromagnetic nuclear response, which can be measured in electron-nucleus scattering. There is a recently approved experiment at Jefferson Lab to collect electron scattering data on argon [37].

- High resolution near detector – this is a very important ingredient, since a fully resolved vertex can greatly reduce model dependencies, but the question of flavor effects and energy containment remain.

- Better flux predictions – unlikely to reach percent level accuracy with superbeam sources.

A good sense of what is needed and what is believed to be achievable can be obtained from the LBNE science document [38]. In particular Tab. 4.5 lists the various systematic uncertainties achieved in past \( \nu_\mu \to \nu_e \) appearance searches and shows plausible extrapolations to how these might look for LBNE. This table includes all near/far cancellations and focuses almost entirely on rate-only effects. Furthermore, certain crucial cancellations in the table rely on the assumption of a valid three-flavor oscillation framework. Finally, nearly flawless hadron calorimetry performance is also assumed. Interestingly, even with this level of assumptions, the analysis barely reaches the required 1% goal.

The issue is then to determine what experimental strategies are available to improve our data on cross sections so that sufficient accuracy in long-baseline experiments becomes possible. Cross section measurements at the percent level of accuracy will ultimately require better neutrino sources, since a cross section measurement can, at best, be only as precise as the accuracy with which the beam flux is known. For a detailed understanding of detector response many exclusive, differential cross sections have to be measured as well, for instance the energy spectrum of neutral pions produced in neutral current interaction plays a central role in identifying \( \nu_e \) charged current events. A CP violation measurement relies to a large degree on the comparison of neutrino and antineutrino data and therefore,
TABLE II. The expected event rates for $\nu_\mu$ and $\nu_e$ for both $\mu^+$ and $\mu^-$ circulating beams and a 100 fiducial ton detector located 50 m from the straight. $\nu_\mu$ and $\bar{\nu}_\mu$ rates from pion decay in the nuSTORM production straight are also included. The exposure is $10^{21}$ POT. Table and caption adapted from Ref. [39].

|         | $\mu^+$                  | $\mu^-$                  |
|---------|---------------------------|---------------------------|
| $\bar{\nu}_\mu$ NC | 1,174,710                | $\bar{\nu}_\mu$ NC      | 1,062,240                |
| $\nu_\mu$ NC       | 1,817,810                | $\nu_\mu$ NC             | 2,074,930                |
| $\bar{\nu}_\mu$ CC | 3,030,510                | $\bar{\nu}_\mu$ CC      | 2,519,840                |
| $\nu_\mu$ CC       | 5,188,050                | $\nu_\mu$ CC             | 6,060,580                |
| $\pi^+$            |                           | $\pi^-$                  |                           |
| $\nu_\mu$ NC       | 14,384,192               | $\nu_\mu$ NC             | 6,986,343                |
| $\nu_\mu$ CC       | 41,053,300               | $\nu_\mu$ CC             | 19,939,704               |

any future facility to measure cross sections has to be able to provide neutrino and antineutrino beams at comparable levels of precision. As explained previously, flavor effects are non-negligible at lower energies and therefore both $\nu_\mu$ and $\nu_e$ cross sections need to be measured with similar accuracy. This results in the following list of requirements for a neutrino source for precision cross section studies:

- Sub-percent beam flux normalization
- Very high beam flux
- Neutrinos and antineutrinos
- $\nu_\mu$ and $\nu_e$

The only neutrino source which can deliver all of these characteristics is a muon storage ring, such as nuSTORM [39]. nuSTORM will deliver a beam with an equal number of muon and electron neutrinos with a beam flux known to better than 1%. Storage of $\mu^-$ and $\mu^+$ allows production of CP-conjugate beams. The very high beam intensity makes it possible to collect a sufficient number of events within a few years in a 100 t near detector. The number of events for the various reaction modes is given in Tab. II. Each event sample will at least comprise 1,000,000 events and thus statistical errors will be sufficiently small to fully exploit the very high systematic precision offered by nuSTORM.

The P5 report mandates a $3\sigma$ CP violation discovery capability over 75% of the parameter space. This implies that systematic uncertainties at the 1% level are necessary for a successful future long-baseline program. In particular, as can be see in Fig. 6, degradation of the systematic uncertainty to the 5% level corresponds to an exposure exposure increase of roughly 200-300% in a very non-linear fashion. Current efforts to reduce systematic errors can only credibly guarantee the upper end of the 1 − 5% range. No method other than a muon storage ring such as nuSTORM has been shown to reliably reach the lower end of this range. Given the $1-2$ billion scale of long-baseline experiments, investing in precise cross section measurements would provide a very good return on investment.

IV. SUMMARY

We have made the argument that, for short-baseline physics and the search for sterile neutrinos, a muon storage ring such as nuSTORM would provide a unique facility of unrivaled capability. It would provide access to all possible oscillation channels involving muon and electron neutrinos and, in the case of a sterile neutrino discovery, would allow detailed exploration of the underlying physics model.

Furthermore, in the context of the future long-baseline program, we discussed the need for an improved understanding of neutrino-nucleon cross sections. In order to achieve the P5 goal of $3\sigma$ discovery potential over 75% of all CP phases, a stringent systematic uncertainty goal of $\leq 1\%$ must be met. Currently, no method other than the precision neutrino beams from a muon storage ring can guarantee this level of accuracy with a high degree of certainty. The impact of a precise cross section determination on various long-baseline experiments and their ability to measure the CP phase $\delta$ is shown in Fig. 7.

It has been recognized for many years that a neutrino factory will be the ultimate tool in exploring neutrino oscillations and this statement remains true even when $\theta_{13}$ is large. With a view to the long-term evolution of the global and domestic neutrino programs, a staging scenario was developed [46], which exploits the unique facilities which will be created for LBNF in the best possible way. This leads to the NuMAX concept [43] and corresponding sensitivities are shown in Fig. 7. A neutrino factory, like NuMAX+ would enable CP violation to be measured in the neutrino sector with the same accuracy as has been achieved in the quark sector.
FIG. 6. Shown is the 75% CP violation reach of LBNE at 3σ confidence level as a function of the total exposure and its change under variations of the systematic error budget on signal normalization and background normalization respectively. Figure courtesy of M. Bass.

Muon-based neutrino beams offer the precision measurement capabilities required to ensure success in meeting the neutrino-related science goals as outlined by P5. In the near- to mid-term, a muon storage ring, such as nuSTORM, would provide the capabilities required to mitigate the otherwise substantial risk that LBNF might not achieve its sensitivity goal, as set by P5, for discovering CP violation in the neutrino sector. Furthermore, such a ring would enable a truly definitive search for sterile neutrinos. In the longer term, beyond LBNF, more advanced muon accelerator capabilities, such as NuMAX, would provide the tools required for precision studies of CP violation in the neutrino sector:

- NuMAX – precision CP phase
- NuMAX+ – high precision CP phase and unitarity

This leads to the conclusion that a high priority should be placed on maintaining the research effort towards muon accelerator capabilities as part of the ongoing accelerator R&D portfolio.

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FIG. 7. Expected precision for a measurement of $\delta$ at future long baseline oscillation experiments. Results are shown as a function of the fraction of possible values of $\delta$ for which a given precision (defined as half of the confidence interval at 1$\sigma$, for 1 d.o.f.) is expected. All oscillation parameters are set to their present best fit values, and marginalization is performed within their allowed intervals at 1$\sigma$, with the exception of $\theta_{13}$ for which marginalization is done within the allowed interval expected at the end of the Daya Bay run. Matter density is set to the value given by the PREM profile, and a 2% uncertainty is considered. The hierarchy is assumed to be normal, and no sign degeneracies are accounted for. Systematic uncertainties are implemented as in Ref. [42]. All facilities include an ideal near detector, and systematics are set to their ‘default’ values from Tab. 2 in Ref. [10]. The nominal running time is 10 years for each experiment. The different lines correspond to the following configurations. LBNF corresponds to the P5 endorsed version of the restructured LBNE project. The LBNE CDR [41] beam flux has been used. The detector performance has been simulated as in Ref. [41], using migration matrices for NC backgrounds from Ref. [40]. The baseline and off-axis angle are the same as for T2K. The detector performance has been simulated as in Ref. [40]. nuSTORM, the curves with an arrow pointing to them, assume that nuSTORM has delivered a 1% measurement of the relevant cross sections. NuMAX corresponds to a low-luminosity neutrino factory obtained from the decay of 5 GeV muons, simulated as in Ref. [43]. The beam luminosity is set to $1.9 \times 10^{20}$ useful muon decays of each polarity per $10^7$ s, and the flux is aimed to a 40 kton magnetized LAr detector placed at 1300 km from the source. NuMAX+ corresponds to $5 \times 10^{20}$ useful muon decays of each polarity per $10^7$ s. All calculations are performed with GLoBES [44, 45].
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