Non-hermiticity in a kicked model: Decoherence and the semiclassical limit

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We study the effects of non-hermitian perturbation on a quantum kicked model exhibiting a localization transition. Using an exact renormalization scheme, we show that the critical line separating the extended and localized phases approaches its semiclassical limit as the imaginary part of the kicking parameter is steadily increased. Further, the metastability of the quantum states appears to be directly correlated with the deviation between the semi-classical and quantum results. This direct evidence of quantum-classical correspondence suggests that non-hermitian perturbations may be used to model decoherence.

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Decoherence, namely the loss of quantum coherence through perturbations from environmental degrees of freedom, is one of the most exciting and active area of research at the forefront of physics. In addition to its importance at technological fronts such as quantum computing [1], the subject is crucial to the field of quantum chaos. It is argued that decoherence is essential in establishing the correspondence principle for quantum systems with classically chaotic limits [2]. The subject has created theoretical and experimental challenges in modeling and controlling the interaction of quantum systems with the environment.

There are recent suggestions that decoherence and dissipation may be modeled as non-hermitian perturbations [3]. At the same time, there has been a considerable amount of interest in delocalization effects in non-hermitian systems, although these have not considered the connection to decoherence at all. These studies were triggered by the result [4] that a complex vector potential delocalizes wave functions otherwise localized in a random potential. This is reminiscent of the delocalization due to decoherence of dynamical localization in quantum systems with classically chaotic dynamics [1]. In this paper, we argue that effects of non-hermiticity may be understood as a special case of the general effect of quantum properties being destroyed by decoherence. Among other interesting questions, this opens up the issue of characterizing the transition from quantum to classical properties as a function of the strength of the complex perturbation.

Earlier studies relating non-hermitian perturbations and delocalization were confined to the complex vector potential. The non-hermitian vector potential was argued to be intrinsically different from non-hermitian scalar potentials [5] in that the imaginary vector potential singles out a direction in space, breaking the symmetry between the left and the right-moving particles, while the imaginary scalar potential can be understood as singling out a direction in time. However, our results indicate that transport characteristics are in general affected by non-hermiticity irrespective of whether the non-hermitian terms appear. This line of reasoning emerged from a recent study of non-hermitian lattice models exhibiting a localization-delocalization transition [6]. There, the non-hermitian scalar and vector potentials correspond to the non-hermitian diagonal and off-diagonal perturbations which are related by a Fourier transformation. That is, the effects on spatial localization due to the complex vector perturbation correspond to the effects on momentum space localization due to the scalar term. In particular, it was found that for a complex vector potential, the extended phase is accompanied by complex eigenenergies (as found earlier [7]) while for the complex scalar potential the same was true for the localized states. Thus, the main issue is the non-hermiticity itself rather than its source in the vector or the scalar potential. This perspective specifically interprets non-hermitian terms via their effects as decohering perturbations. In the following, we study a non-autonomous system, with non-hermitian kicking, exhibiting a localization-delocalization transition. Using a renormalization group(RG) technique, we study the effects of non-hermiticity on this transition. We show explicitly that as the non-hermitian perturbation is increased, the system’s localization-delocalization phase diagram monotonically approaches the semiclassically determined diagram.

Periodically kicked Hamiltonian systems such as

\[ H = T(p) + V(x) \sum \delta(t-n) \]  \hspace{1cm} \text{(1)}

with \( T(p) = p^2/2 \) (kicked rotor) and with \( T(p) = L \cos(p) \) (kicked Harper) are an important class of theoretical and experimental systems for studying the quantum dynamics of classically non-integrable systems [8]. Despite extensive study for almost two decades, questions of classical-quantum correspondence and dynamical localization in these systems remain open. When the
quasienergy states of the system are projected on the angular momentum basis, these models map onto lattice models \[3\]. However, in contrast to the lattice models for autonomous systems, kicked systems describe long-range interactions and hence are more difficult to study. A special class of kicked models with \[V(x) = 2\hbar \arctan(K \cos(x))\] are useful \[4\] as they can be represented by a nearest-neighbor (NN) tight-binding model (tbm) of the form

\[
\psi_{m+1} + \psi_{m-1} + 2/\bar{K} \tan[T(p) + \omega/2] \psi_m = 0 \tag{2}
\]

where \(\bar{K} = K/(2\hbar)\) and \(\omega\) is the quasi-energy. Here the lattice index \(m\) represents the angular momentum quantum number in the absence of the kicking term.

With \(T(p) = p^2/2\), this model corresponds to a lattice model with a pseudo-random potential exhibiting localization with the localization length equal to that of the Lloyd model \[1]\(4\). We study the model with \(T(p) = L \cos(p)\) which exhibits both extended and localized phases \[1]\(4\) for irrational \(\hbar/2\pi\). The system also describes the NN truncation of the kicked Harper model and for small \(K\) and \(L\), it reduces to the Harper equation \[24\] with \(E = \omega\). Finally, the system also describes the quantum spin-1/2 XY chain, kicked periodically by a transverse magnetic field \[22\]. This model is thus a good testing ground for investigating the effects of non-hermitian perturbations on the transport characteristics of non-autonomous systems.

Although this model has no nontrivial classical limit, there exists a nontrivial semiclassical limit. As \(\hbar \to 0\) the lattice representation of the model can be written as the quantum continuum Hamiltonian \[13\] \(14\), \(H_{cont} = [\bar{K} \cos(p) + \tan[\bar{L} \cos(x + \omega/2)] = 0\) where \(x = \hbar n\) and \([x, p] = i \hbar\). Remarkably, if this Hamiltonian is interpreted classically, the resulting orbits carry a signature of the localization-delocalization transition for the original lattice model. Specifically, unbounded phase-space trajectories correspond to extended states along \(x\) or \(p\) while a bounded orbit describes the localization-delocalization boundary. It is easy to show that for this continuum Hamiltonian, such a bounded orbit exists for \(\bar{K} = \tan(\bar{L})\), independent of \(\omega\). This is therefore the semiclassical condition for the critical line separating the extended and localized states. We show below that this semiclassical critical line is a reasonable approximation to the `exact’ quantum critical line, and for complex kicking, the exact critical line tends toward this semiclassical line. For the Harper equation, which describes an autonomous system, the semiclassical prediction for the critical line is exact.

The quantum phase diagram in \(K - L\) space is studied using a renormalization group (RG) approach for a fixed quasi-energy. We use dimer decimation \[16\] \(17\] which has conceptual and intuitive advantages over other RG schemes \[13\] \(14\] \(15\]. The key idea underlying the renormalization scheme is the simultaneous decimation of the two central sites of the doubly infinite lattice \(-\infty, ..., -2, -1, 0, 1, 2, ..., \infty\), namely \(\pm 1, \pm 2\) and so on, after we have eliminated the central site \(m = 0\). At the \(n\)th step where all sites with \(|m| < n\) have been eliminated, the tbm for \(m = \pm n\) reads

\[
\Phi_{n+1} + G^+(n)\Phi_{n} - E^- (n) \Phi_n = 0 \tag{3}
\]

\[
\Phi_{n-1} + G^-(n)\Phi_n - E^+ (n) \Phi_{n} = 0 \tag{4}
\]

where \(G^\pm(n)\) and \(E^\pm(n)\) respectively describe the renormalized coupling and the on-site potential term at the \(n\)th step of the renormalization. The \(+(-)\) correspond to the right (left) parts of the lattice. With the initial conditions determined by Eq. (2), the renormalized parameters are given by the exact RG flow \[13\]

\[
M_{n+1} = f_{n+1} + M_n^{-1} \tag{5}
\]

where \(M\) is a \(2 \times 2\) matrix defined as

\[
M = \begin{pmatrix} E^+ & G^- \\ G^+ & E^- \end{pmatrix} \tag{6}
\]

and \(f\) is a diagonal \(2 \times 2\) matrix, \(f_{n,n} = E(n)\). Asymptotically, the renormalized lattice can be viewed as a dimer where the transport characteristics are determined by the quantum interference between the two sites of the dimer. Interestingly, the extended phase corresponds to a rigid dimer while the localized phase is asymptotically a broken dimer. Therefore, the transport and localization properties are described by the effective coupling of the renormalized dimer, which is the ratio between the off-diagonal and the diagonal part of the renormalized tbm \(R(n) = GG^\dagger/EE^\dagger\). The scaling exponent \(\beta = \lim_{n \to \infty} \log R(n)/\log n\) effectively quantifies the transport properties, since extended states are described by (typically monotonic) convergence of \(\beta(n)\) \(\to 0\). For exponential localization, \(\beta(n) \to -\infty\). In contrast, the critical states are characterized by negative \(\beta\) and non-convergent, oscillatory behavior. This provides an extremely high precision method of obtaining a phase diagram for fixed quasienergy of systems exhibiting extended, localized and critical states.

Figure (1) shows the results, at almost machine precision, of this method applied to our system \[20\]. The important feature to note is that as the kicking parameter increases, the diagram has a narrow re-entrant phase (a peak) and a plateau. Interestingly, with the exception of the region near this peak, the phase diagram is more or less consistent with the semiclassical prediction. Further, Fig. (2) shows the transport characteristics for wavepackets in this system, and this global phase diagram corresponds closely to Fig. (1), which is for a pure quantum state. This is important, since RG tools to compute the phase diagram for \(\omega = 0\) require a small fraction of the computational time for computing the wavepacket transport characteristics.
FIG. 1. Phase diagram for the kicked model with $\hbar/2\pi = (\sqrt{5} - 1)/2$ and $\omega = 0$. The shaded regime is the extended phase. The solid line is the semiclassical critical line.

FIG. 2. Transport properties of a quantum wave packet. Using a plane wave initial condition, we compute $\langle p^2 \rangle$ after 1000 kicks. The lightly-shaded regime shows the parameter space where $\langle p^2 \rangle$ is greater than $10^4$. The darker regime corresponds to $\langle p^2 \rangle$ between $10^3 - 10^4$. The narrowness of this regime confirms that the computation is converging accurately to the localization boundary.

In Fig. (3) we show the effects of a complex perturbation, $K \rightarrow K_r + iK_i$, on the phase diagram. The focus is on the changes in the critical line as $K_i$ is increased. What is interesting is the manner in which the transition curve moves toward the semi classical critical line as the non-hermiticity of the perturbation is increased. The transition appears to happen in distinct stages: (a) the peak diminishes and then disappears, (b) the curve gets closer in shape to the semiclassical line while still maintaining a difference and finally (c) when the real part of $K$ is switched off, the curve is almost exactly on top of the semiclassical line. Also, note that the effects of non-hermiticity are consistent with earlier work [4], since the non-hermitian perturbation shifts the critical line in parameter space so as to increase the measure of parameter space corresponding to dynamical localization (in momentum space) and hence enhance the parameter space corresponding to delocalization in real space. This reinforces our earlier arguments on delocalization being a phenomenon that occurs independent of the source of non-hermiticity.

FIG. 3. The effects of complex perturbation on the critical line. The parameter $K$ is the absolute value of the complex kicking. The lines from top to bottom correspond to $K_i = 0, K_i = .1, K_i = .5, K_i = 1$, and finally with $K_r = 0$, respectively. The crosses show the semiclassical critical line.

The RG approach was also used to study the effects of non-hermiticity on the quasienergy spectrum [13]. It turns out that with the exception of the $\omega = 0$ state, all other quasienergies are complex [21]. The pure imaginary part of the spectrum exhibits a band structure, as shown in Fig. (4). We show only extended states that are easy to compute from the RG analysis [17]. The non-hermiticity thus associates continuum families of life-times with the state with $\omega = 0$. As we approach the localization-delocalization boundary, this band splits into sub-bands with the notable localization of the $\omega_i = 0$ band at the onset of localization as confirmed by further simulations. Therefore, the localization threshold is signalled by the $\omega_i = 0$ band degenerating to a point and the state is localized, with both the real and imaginary part of the
An intriguing feature of this pure imaginary part of the spectrum is the presence of relatively large values of $\omega_i$ in the parameter regime corresponding to the peak of the phase diagram Fig. (1). The results shown in Fig. (4) suggest that this regime, which corresponds to extended states, is more unstable than the rest of the delocalized phase. Since this part of the parameter space corresponds to greater deviation between the quantum and the semiclassical behavior, this relates the set of allowed life times of a quantum state to its proximity to the part of the phase diagram which is not described semiclassically. This is consistent with the intuition from the decoherence literature, and the statement that the part of the phase diagram which is not described by the semiclassical theory is most sensitive to the non-hermitian perturbation is arguably general.

Previous work has indicated that complex Hamiltonians may be associated with decoherence effects. The signature of decoherence is that the quantum system behaves increasingly classically as the decoherence parameter is increased, independent of the value of $\hbar$. Decoherence is in fact argued to be necessary for quantum-classical correspondence in chaotic systems. Our results are consistent with this, showing that results from decoherence may be used to understand complex Hamiltonians, and alternatively, that a complex Hamiltonian formulation may be used to model decoherence effects. This opens new possibilities in modelling the interaction between non-integrable systems and the environment, providing a simpler alternative to solving master equations. We hope that our study will stimulate further exploration of non-hermitian systems, and particularly of those displaying chaos.

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FIG. 4. The pure imaginary part of the spectrum for $K_r = 0$ as the localization transition is approached, for $K_l = 1.5$ (top) and $K_l = 2.5$ (bottom). The figure shows only extended states with $|\omega_i| < 3$; the spectrum is symmetric about $\omega_i = 0$. A comparison between the top and the bottom figures shows that peak part of the phase diagram is associated with shorter lifetimes and hence is more unstable.

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