The $\eta_c\gamma$ Transition Form Factor

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Abstract

The $\eta_c\gamma$ transition form factor is calculated within a perturbative approach. It is shown that the $Q^2$ dependence of the form factor is well determined in the region where experimental data is expected in the near future.

1. Introduction

Pseudoscalar meson-photon transition form factors (see Fig. 1) at large momentum transfer $Q^2$ have attracted the interest of many theoreticians during the last years, stimulated by the CLEO measurements. At the upper end of the measured $Q^2$ range the CLEO data for the $\pi\gamma$ form factor only deviate by about 15% from the limiting value $\sqrt{2f_\pi}/Q^2$ which is predicted by QCD. The data allow for a rather precise determination of the pion’s light-cone wave function, and we find, within the modified hard scattering approach (mHSA), a value of $-0.01 \pm 0.1$ for the expansion coefficient $B_2$ at the scale $\mu = 1$ GeV (see Fig. 1). The situation is more complicated for the $\eta\gamma$ and the $\eta'\gamma$ form factors due to the mixing and $SU(3)_F$ flavor symmetry breaking. A determination of the decay constants and the mixing angle from the $\eta\gamma$ and $\eta'\gamma$ transition form factors is also possible.

Figure 1: Meson-Photon transition form factors in $e^+e^-$ collisions (left). The $\pi\gamma$ form factor: experimental data and mHSA fit (right).

There is a fourth form factor of the same type, namely the $\eta_c\gamma$ form factor which is neither experimentally nor theoretically known. Since a measurement of that form factor up to a momentum transfer of about 10 GeV$^2$ seems feasible, a theoretical analysis and prediction of it is desirable and has been performed by us recently.

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2. The perturbative approach

In analogy to the $\pi\gamma$ case we employ a perturbative approach on the basis of a factorization of short- and long-distance physics. Observables are then described as convolutions of a perturbatively calculable hard scattering amplitude $T_H$ and a universal (process-independent) hadronic light-cone wave function $\Psi$ of the $\eta_c$'s leading $c\bar{c}$ Fock state which embodies soft non-perturbative physics,

$$F_{\eta_c\gamma}(Q^2) = \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \Psi(x, \vec{k}_\perp) T_H(x, \vec{k}_\perp, Q). \quad (1)$$

Here $x$ is the usual meson's momentum fraction carried by the $c$ quark, and $\vec{k}_\perp$ denotes its transverse momentum. In the present case the mass of the charm quarks already provides a large scale, which allows the application of the perturbative approach even for zero virtuality of the probing photon, $Q^2 \to 0$; and for heavy quarks a Sudakov factor in (1) can be ignored. The hard scattering amplitude in leading order is easily calculated. With one photon being almost on-shell ($q_1^2 \approx 0$) and the virtuality of the second photon denoted as $q_2^2 = -Q^2$, this leads to (with $\bar{x} = (1-x)$)

$$T_H(x, \vec{k}_\perp, Q) = \frac{e_c^2 2\sqrt{6}}{x Q^2 + m_c^2 + x \bar{x} M_{\eta_c}^2 + \vec{k}_\perp^2} + (x \leftrightarrow \bar{x}) + O(\alpha_s) \quad (2)$$

where $M_{\eta_c} (= 2.98 \text{ GeV})$ is the mass of the $\eta_c$ meson, and $m_c \simeq M_{\eta_c}/2$ is the charm quark mass. The charge of the charm quark in units of the elementary charge is denoted by $e_c$. For the $\eta_c$ wave function,

$$\Psi(x, \vec{k}_\perp) = \frac{f_{\eta_c}}{2\sqrt{6}} \phi(x) \Sigma(\vec{k}_\perp), \quad (3)$$

we use a form adapted from Bauer, Stech and Wirbel. Here $f_{\eta_c}$ is the decay constant (corresponding to $f_\pi = 131$ MeV), and $\phi(x)$ is the quark distribution amplitude which is parameterized as

$$\phi(x) = N_\phi \ x \bar{x} \ exp\left[-a^2 M_{\eta_c}^2 (x - x_0)^2\right]. \quad (4)$$

The normalization constant $N_\phi$ is determined from the usual requirement $\int_0^1 dx \phi(x) = 1$. The distribution amplitude (3) exhibits a pronounced maximum at $x_0$ and is exponentially damped in the endpoint regions. Furthermore, $\Sigma$ is a Gaussian shape function which takes into account the finite transverse size of the meson,

$$\Sigma(\vec{k}_\perp) = 16\pi^2 a^2 \exp[-a^2 \vec{k}_\perp^2]. \quad (5)$$

The decay constant of the $\eta_c$ meson is not accessible in a model-independent way at present. Usually, one estimates $f_{\eta_c}$ in a non-relativistic approach which provides a connection between $f_{\eta_c}$ and the well-determined decay constant of the $J/\psi$, $f_{\eta_c} \simeq f_{J/\psi} = 409$ MeV. However, the $\alpha_s$ corrections are large, and the relativistic corrections are usually large and model-dependent.

\[1\] Higher Fock state contributions to the $\eta_c\gamma$ form factor are suppressed by powers of $\alpha_s/m_c^2$. However, higher Fock states can be important in other decays of heavy quarkonia.
The parameters entering the wave function are further constrained by the Fock state probability $P_{cc}$. One expects $0.8 \leq P_{cc} < 1$ for a charmonium state (for smaller values of $P_{cc}$ one would not understand the success of non-relativistic potential models for these states). Since the perturbative contribution to the $\eta_c\gamma$ form factor only mildly depends on the value of $P_{cc}$, we use $P_{cc} = 0.8$ as a constraint for the transverse size parameter $a$. For $f_{\eta_c} = 409$ MeV this leads to a reasonable value $a = 0.97$ GeV.

The two photon decay width $\Gamma[\eta_c \to \gamma\gamma]$, the experimental value of which still suffers from large uncertainties\textsuperscript{16}, can be directly related to the $\eta_c\gamma$ transition form factor at $Q^2 = 0$

$$\Gamma[\eta_c \to \gamma\gamma] = \frac{\pi a^2 M_{\eta_c}^3}{4} |F_{\eta_c\gamma}(0)|^2$$

One may use this decay rate as a normalization condition for $F_{\eta_c\gamma}(Q^2 = 0)$ and present the result in the form $F_{\eta_c\gamma}(Q^2)/F_{\eta_c\gamma}(0)$. In this way the perturbative QCD corrections at $Q^2 = 0$ to the $\eta_c\gamma$ transition form factor are automatically included, and also the uncertainties in the present knowledge of $f_{\eta_c}$ do not enter our predictions.

3. Results and Conclusions

![Graph](image)

Figure 2: The predictions for $Q^2 F_{\eta_c\gamma}(Q^2)$ scaled to $\Gamma[\eta_c \to \gamma\gamma] = 6$ keV in the leading order of the perturbative approach (for $P_{q\bar{q}}=0.8$). The dashes indicate the $Q^2$ region where QCD corrections may alter the predictions slightly.

In Fig. 2 we present the result for the transition form factor $Q^2 F_{\eta_c\gamma}$ scaled to a partial width $\Gamma[\eta_c \to \gamma\gamma]$ of 6 keV. In order to discuss the qualitative features of our result in a rather simple fashion, one can restrict oneself to first order corrections to the collinear ($\vec{k}_\perp^2 \simeq 0$) and peaking approximation ($x \simeq x_0$), which can be expressed by the small quantity $\langle k_\perp^2 \rangle = 1/2a^2 \ll M_{\eta_c}^2$. For $Q^2 \leq M_{\eta_c}^2$ one then obtains the following approximation\textsuperscript{10}

$$F_{\eta_c\gamma}(Q^2) \simeq \frac{4 e^2 f_{\eta_c}}{Q^2 + M_{\eta_c}^2 + 2 \langle \vec{k}_\perp^2 \rangle} \simeq \frac{F_{\eta_c\gamma}(0)}{1 + Q^2/(M_{\eta_c}^2 + 2 \langle \vec{k}_\perp^2 \rangle)}$$

which reveals that, to a very good approximation, the predictions for the scaled $\eta_c\gamma$ form factor are rather insensitive to the details of the wave function. Only the mean transverse momentum following from it is required, leading to an effective pole mass of $\sqrt{M_{\eta_c}^2 + 2 \langle k_\perp^2 \rangle} = 3.15$ GeV which is very close to the value of the $J/\psi$ mass that one
would have inserted in the vector meson dominance model. The deviation from the full result amounts only to 4% at $Q^2 = 10$ GeV$^2$, which is likely smaller than the expected experimental errors in a future measurement of the $\eta_c\gamma$ form factor. These considerations nicely illustrate that the $Q^2$ dependence of the $\eta_c\gamma$ form factor is well determined. The main uncertainty of the prediction resides in the normalization, i.e. the $\eta_c$ decay constant or the value of the form factor at $Q^2 = 0$.

Let us briefly discuss, how $\alpha_s$ corrections may modify the leading order result for the $\eta_c\gamma$ form factor: One has to consider two distinct kinematic regions. First, if $Q^2 < \sim M^2_{\eta_c}$ one can neglect the evolution of the wave function, and one is left with the QCD corrections to the hard scattering amplitude $T_H$, which have been calculated in the peaking and collinear approximation to order $\alpha_s$. For the scaled form factor the $\alpha_s$ corrections at $Q^2$ and at $Q^2 = 0$ cancel to a high degree, and even at $Q^2 = 10$ GeV$^2$ the effect of the $\alpha_s$ corrections is less than 5%.

Secondly, for $Q^2 \gg M^2_{\eta_c}$ one can neglect the quark and meson masses and arrives at the same situation as for the pions. The $\alpha_s$ corrections to the hard scattering amplitude and the evolution of the wave function with $Q^2$ are known. For very large values of $Q^2$ the asymptotic behavior of the transition form factor is completely determined by QCD, since any meson distribution amplitude evolves into the asymptotic form $\phi(x) \to \phi_{as}(x) = 6x \bar{x}$,

$$F_{\eta_c\gamma}(Q^2) \to \frac{2e^2 f_{\eta_c}}{Q^2} \int_0^1 dx \frac{\phi(x)}{x} \to \frac{8 f_{\eta_c}}{3 Q^2} \cdot (\ln Q^2 \to \infty) \quad (8)$$

A precise measurement of the strength of the $\eta_c\gamma$ transition form factor may serve to determine the decay constant $f_{\eta_c}$ (see (4)). Though attention must be paid to the fact that the obtained value of $f_{\eta_c}$ is subject to large QCD corrections (about of the order 10-15% for $Q^2 \lesssim 10$ GeV$^2$) which should be taken into account for an accurate extraction of the $\eta_c$ decay constant.

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