Identifying Cover Songs Using Information-Theoretic Measures of Similarity

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Abstract

This paper investigates methods for quantifying similarity between audio signals, specifically for the task of cover song detection. We consider an information-theoretic approach, where we compute pairwise measures of predictability between time series. We compare discrete-valued approaches operating on quantised audio features, to continuous-valued approaches. In the discrete case, we propose a method for computing the normalised compression distance, where we account for correlation between time series. In the continuous case, we propose to compute information-based measures of similarity as statistics of the prediction error between time series. We evaluate our methods on two cover song identification tasks using a data set comprised of 300 Jazz standards and using the Million Song Dataset. For both datasets, we observe that continuous-valued approaches outperform discrete-valued approaches. We consider approaches to estimating the normalised compression distance (NCD) based on string compression and prediction, where we observe that our proposed normalised compression distance with alignment (NCDA) improves average performance over NCD, for sequential compression algorithms. Finally, we demonstrate that continuous-valued distances may be combined to improve performance with respect to baseline approaches. Using a large-scale filter-and-refine approach, we demonstrate state-of-the-art performance for cover song identification using the Million Song Dataset.

1 Introduction

In the field of music content analysis, quantifying similarity between audio signals has received a substantial amount of interest [1]. Owing to the proliferation

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of music in digital formats, there exists potential for applications using music similarity techniques, in a wide range of domains. At the level of individual tracks, these domains span audio fingerprinting \[2\], cover song identification \[3\], artist identification \[4, 5\] and genre classification \[6\]. Applications can be distinguished according to their degree of specificity \[1\], referring to the level of granularity required for retrieving audio tracks from a collection, given a query track. For example, in audio fingerprinting, the required specificity is high, since the set of possible tracks corresponding to a particular recording is typically small, in relation to the data set. In contrast, genre classification requires low specificity, since the set of tracks sharing a common genre is potentially large, in relation to the data set.

A cover song may be defined as a rendition of a previously recorded piece of music \[7\]. Cover song identification is deemed to have mid-level, diffuse specificity, since cover songs may differ from the original song in various musical facets, including rhythm, tempo, melody, harmonisation, instrumentation, lyrics and musical form. Correspondingly, cover song identification remains a challenging problem \[3\].

In this work, we investigate methods for cover song identification that are based on quantifying pairwise predictability between sequences. From a music-psychological perspective, the significance of predictability has been reflected on by Hanslick \[8\] and Meyer \[9\], who consider the aesthetic experience when listening to a piece of music to be informed by responses to fulfilled and unfulfilled expectations, as the piece of music unfolds in time. More recently, expectation processes have been proposed as a model of melodic cognition \[10\]. In a growing body of work, statistical learning is implicated in forming expectations \[11\]. Based on Shannon’s information theory \[12\], a successful approach to quantifying expectations in response to an unfolding stream of musical events, involves computing measures of predictive uncertainty \[13\].

Based on our previous work \[14\], we consider an information-theoretic approach to quantifying similarity between feature vector sequences. One possible approach based on the non-Shannon information measure of Kolmogorov complexity \[15\], the normalised compression distance (NCD) \[16\], quantifies similarity between two strings in terms of joint compressibility. The NCD has been applied successfully across a range of problem domains \[16–19\], including music content analysis \[20–25\].

For our chosen task of cover song identification, we interpret the NCD as a measure of pairwise predictability. Using our information-theoretic framework, we compare the NCD to alternative predictability measures based on Shannon information. We provide an evaluation of competing information-theoretic approaches and identify issues concerning their implementation.

The remainder of this paper is organised as follows: Section \[2\] discusses related work on audio-based cover song identification and methods for determining musical similarity. Section \[3\] introduces the pairwise similarity methods evaluated in this work. Section \[4\] describes our experimental procedure. Finally, in Sections \[5\] and \[6\] we present results and conclusions.
2 Related Work

2.1 Musical Similarity

Methods for characterising similarity between sequences of audio features can be distinguished based on whether the temporal order of features is discarded or retained [1]. In the former so-called ‘bag-of-features-approach’, a widespread method involves estimating distributions of features obtained from time-frequency representations of musical audio [20, 21]. The bag-of-features approach is unable to model the temporal aspect of music, in which rhythmic, harmonic and melodic objects exhibit sequential structure and in which repetition and variation are of importance [32]. Casey and Slaney [33] emphasise the role of sequences for music similarity applications, whereas Aucouturier et al. [29] discuss the relative limitations of the bag-of-features approach in a comparison of musical and non-musical audio modelling. Sequential approaches have been utilised in music structure analysis, for identifying repeated and contrasting sequences and their boundaries within a single piece of music [34], in addition to cover song identification.

2.2 Cover Song Identification

Owing to the importance of tonal content in determining whether a song is a cover of another, recent cover song identification approaches typically extract representations of the tonal content using chroma features [35, 36].

Chroma features quantify energy distributions across octave-folded bands, using pitch classes in the chromatic scale to map frequency bands to chroma bins.

A variety of cover song identification approaches are based on computing pairwise alignments between continuous-valued chroma sequences. Following Foote’s [37] method of applying dynamic time warping (DTW) to a similarity matrix constructed from spectral energy features, Gómez and Herrera [38] propose a DTW approach using chroma features. Serrà et al. [7] propose to compute binarised similarity matrices, substituting DTW with an alternative local alignment approach. The cross-recurrence approaches proposed by Serrà et al. [39] extend the notion of similarity matrices considered in the preceding investigations, in that time-lagged chroma vectors are combined to form higher-dimensional temporal features. In an alternative approach, Serrà et al. [40] utilise the previously described method of representing chroma features in combination with non-linear time series prediction techniques, using the cross-prediction error as a measure of similarity.

Using a signal processing approach, Ellis and Poliner [41] apply component-wise cross-correlation as a measure of similarity between chroma features. Jensen [42] computes the Euclidean distance between two-dimensional autocorrelations of chroma sequences. More recently, Bertin-Mahieux [43] proposes a key-invariant approach based on applying the two-dimensional Fourier transform to chroma sequences.
An alternative approach involves computing similarities between discrete-valued representations of musical content. Tsai et al. [44] apply DTW to discrete-valued sequences, using spectral peak-picking for predominant melody extraction. Bello [45] and Lee [46] perform chord estimation with hidden Markov models, using mappings of model states to chords. The resulting sequences are then aligned using DTW. Martin et al. [47] heuristically select chroma bin maxima to determine triads, before locally aligning sequences.

With particular regard to this work, a number of approaches are based on applying the NCD to discrete-valued sequences. Using symbolic musical representations directly, Cilibrasi et al. [21] apply hierarchical clustering to pairwise distances between pieces of music, performing an analysis of clusters with respect to musical genres, musical works and artists. Li and Sleep apply the NCD to genre classification of symbolic musical representations [20] and musical audio [22].

For audio-based cover song identification, Ahonen [24] obtains discrete-valued representations of frame-based chroma features by applying a hidden Markov model (HMM) to perform chord transcription. Predicted chord sequences are then converted to a differential representation, before computing pairwise distances between tracks using the NCD based on different compression algorithms. Ahonen [48] further proposes to compute multiple discrete-valued representations using additional HMMs and by computing chroma differentials, before averaging separately obtained pairwise distances using the NCD based on PPM compression. In addition, Ahonen [48] investigates continuous-valued chroma-derived representations which are then quantised using the $K$-medians algorithm and compressed using BW compression. Bello [25] applies the NCD to recurrence plots computed on individual tracks, as a measure of structural similarity between pieces of music. Finally, Tabus [49] proposes a similar approach to Ahonen based on quantising chroma-derived representations, observing that an alternative compression-based similarity measure outperforms the NCD. Additionally, Silva et al. [50] propose a measure of structural similarity based on video compression, observing superior performance using an alternative compression-based measure. Our work extends the above investigations, in that we examine and propose the use of alternative information-theoretic similarity measures to the NCD. Furthermore, we perform an extensive comparison of methods for estimating the NCD and related similarity measures, while proposing approaches which do not require quantising audio features.

A number of recent investigations are concerned with cover song identification using large-scale music collections containing millions of tracks. For such collections, it is typically infeasible to perform computationally expensive pairwise comparisons between a query and every track in the collection. Casey et al. [51] compute Euclidean distances between windowed chroma sequences. Pairwise similarity is then quantified as the number of distances falling below a threshold. Such an approach may be combined with locality-sensitive hashing [52] for retrieval with sub-linear time complexity, with respect to a single query. Using a similar approach, Bertin-Mahieux and Ellis [53] propose to identify salient ‘landmark’ chroma vectors in individual tracks by applying a threshold-
ing scheme. Identified landmark vectors are then encoded as an integer, thus the collection may be represented as a lookup table. Given a query, the same authors envisage that obtained results are re-ranked using a computationally expensive approach, as proposed by Khadkevich and Omologo [54]. In this work, we apply such a filter-and-refine approach [55], using information-theoretic similarity measures in the refinement stage.

2.3 Information-Theoretic Methods

Information-theoretic similarity measures between time series have been proposed in a variety of domains. The idea of jointly compressing two discrete-valued sequences is due to Loewenstein et al. [56] in the context of nucleotide sequence clustering. By parsing sequences using the Lempel-Ziv (LZ) algorithm [57], Ziv and Merhav [58] propose a method for comparing sequences by compressing one sequence using a model estimated on the other sequence. An alternative approach is considered by Benedetto et al. [59] for building language trees, where sequences are jointly compressed. Cilibrasi et al. [60] motivate their approach of jointly compressing sequences as an approximation of the normalised information distance [10].

3 Approach

We denote with \( X = (x_1, x_2, \ldots, x_N) \), \( Y = (y_1, y_2, \ldots, y_M) \) two multivariate time series, each representing a sequence of feature vectors extracted from a piece of musical audio. If we assume that both \( X, Y \) consist of independent and identically distributed realisations generated respectively by stochastic processes \( X, Y \), one possible means of quantifying dissimilarity between time series involves the Kullback-Leibler (KL) divergence, defined as

\[
D_{KL}(p_X \parallel p_Y) = \int p_X(u) \log \left( \frac{p_X(u)}{p_Y(u)} \right) du
\]

where \( p_X(u), p_Y(u) \) denote the probability density of observation \( u \) generated by \( X, Y \), respectively. Viewed in terms of Shannon information and taking the logarithm to base 2, recall that the KL divergence quantifies the expected number of additional bits required to represent observations generated by information source \( X \), given a code for observations generated by information source \( Y \). The KL divergence has been widely applied as a ‘bag-of-features’ approach for low-specificity music content analysis tasks [1].

To account for temporal structure in musical audio, we may use the NCD as a measure of musical dissimilarity between sequences of quantised feature vectors [22,24,49]. Given two strings \( x = (x_1, x_2, \ldots, x_N), y = (y_1, y_2, \ldots, y_M) \), the NCD is defined as

\[
NCD(x, y) = \frac{\max\{C(xy) - C(x), C(yx) - C(y)\}}{\max\{C(x), C(y)\}}
\]
where $C(\cdot)$ denotes the number of bits required to encode a given string, using a compressor such as the Lempel-Ziv (LZ) algorithm [57]. Similarly, $C(xy)$ denotes the number of bits required to encode the concatenation of strings $x, y$. The NCD is an approximation of the normalised information distance (NID) [16], defined as

$$\text{NID}(x, y) = \frac{K(x, y) - \min\{K(x), K(y)\}}{\max\{K(x), K(y)\}}$$

where the uncomputable function $K(\cdot)$ denotes algorithmic information content (AIC), also known as Kolmogorov complexity. The AIC of a given string is the length in bits of the shortest program which outputs the string and then terminates [15]. Similarly, $K(x, y)$ denotes the length of the shortest program which outputs $x, y$, in addition to a means of distinguishing between both output strings [15]. Thus, AIC quantifies the number of bits required to represent specified input strings, under maximally attainable compression. Furthermore, the NID characterises dissimilarity using the transformation under which input strings most closely resemble each other [16].

We are interested in examining the performance of the NCD as an approximation of the NID, where the choice of compressor determines the feature space used to compute similarities [61] in the NCD.

Furthermore, note that the choice of concatenation in $C(xy)$ to approximate $K(x, y)$ represents an additional heuristic [16]. In the following sections, we describe our contribution.

### 3.1 Quantifying Time Series Dissimilarity Using Shannon Information

We approach the problem of quantifying dissimilarity from the perspective of Shannon information. We assume finite-order, stationary Markov sources $X, Y$. We denote with $X_{1:N}$ the sequence of random variables $(X_1, \ldots, X_N)$ emitted by source $X$ at times $1, \ldots, N$. We denote with $H_\mu(X)$, $H_\mu(X, Y)$, $H_\mu(X|Y)$ the entropy rate, joint entropy rate and conditional entropy rate, respectively defined as

$$H_\mu(X) = \lim_{n \to \infty} \frac{1}{n} H(X_1, X_2, \ldots, X_n)$$

$$H_\mu(X, Y) = \lim_{n \to \infty} \frac{1}{n} H((X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n))$$

$$H_\mu(X|Y) = H_\mu(X, Y) - H_\mu(Y).$$

The entropy rate $H_\mu(X)$ defined in (4) quantifies the average amount of uncertainty about $X_n$, while accounting for correlation between $X_n$ for all $n$. Analogously, the joint entropy rate $H_\mu(X, Y)$ defined in (5) quantifies the average amount of uncertainty about the pair $(X_n, Y_n)$ emitted by sources $X, Y$, while in addition accounting for correlation between the sources. For the conditional
entropy rate $H_\mu(X|Y)$ we have

$$H_\mu(X|Y) = \lim_{n \to \infty} \frac{1}{n} H(X_{1:n}, Y_{1:n}) - H(Y_{1:n})$$

(7)

$$= \lim_{n \to \infty} \frac{1}{n} H(X_{1:n}|Y_{1:n}).$$

(8)

From (8) we may interpret $H_\mu(X|Y)$ as quantifying the average amount of uncertainty about a given emission $X_n$, while taking into account correlation between observations generated by $X$ and given knowledge of observations generated by $Y$.

For $N$ observations emitted from source $X$, up to an additive constant the expectation $E[K(X_{1:N})]$ may be approximated using the entropy [62],

$$E[K(X_{1:N})] \approx H(X_{1:N}).$$

(9)

Using (4), (5), we assume further approximations

$$E[K(X_{1:N})] \approx N H_\mu(X)$$

(10)

$$E[K(X_{1:N}, Y_{1:N})] \approx N H_\mu(X, Y)$$

(11)

where $E[K(X_{1:N}, Y_{1:N})]$ denotes the expected value of $K(\cdot, \cdot)$ for $N$ observations emitted from sources $X, Y$. In terms of Shannon information, following [63] we use (6) and estimate the NID as

$$\text{NID}(X, Y) \approx \max\{H_\mu(X|Y), H_\mu(Y|X)\} - \min\{H_\mu(X), H_\mu(Y)\}.$$  

(12)

### 3.2 Normalised Compression Distance with Alignment

As given in (12), the NID utilises the joint entropy rate $H_\mu(X, Y)$, which accounts for correlation between sources. In contrast, the approach of compressing concatenated strings to estimate $K(x, y)$ may be inadequate for compressors based on Markov sources, since correlation is not accounted for [63]. To address this possible limitation, we propose the normalised compression distance with alignment (NCDA), defined as

$$\text{NCDA}(x, y) = \frac{C((x, y)) - \min\{C(x), C(y)\}}{\max\{C(x), C(y)\}}.$$  

(13)

where $(a, b)$ performs alignment as a means of maximising correlation between strings $a, b$. We generate equal-length strings by padding the shorter of the two strings with the most common value of the longer string. Then, we determine the lag which maximises cross-correlation between circularly shifted $a$ and $b$. Finally, we interleave strings using the obtained lag value.
3.3 Predictive Modelling

As previously described, the NCD and NCDA rely on determining the number of bits required to encode strings, using a specified compression algorithm. As an alternative approach, we consider the relation between predictability and compressibility \[64, 65\] and perform sequence prediction. We illustrate our method for the case of continuous-valued observations. First, recall that the entropy rate \( H_\mu(X) \) is given as

\[
H_\mu(X) = \lim_{n \to \infty} -\frac{1}{n} \int p_X(u_{1:n}) \log p_X(u_{1:n}) \, du_{1:n}
\]

(14)

where \( p_X(u_{1:n}) \) denotes the probability of observing \( X_{1:n} = (u_1, \ldots, u_n) \). We may interpret the quantity \(-\log p_X(u_{1:n})\) as the number of bits required to represent \( u_{1:n} \), assuming an optimal code. \( H_\mu(X) \) thus quantifies the expected number of bits required to represent a single observation generated by \( X \), while accounting for correlation between observations. Assume that we have an empirical estimate \( \hat{p}_X(\cdot) \) of the distribution \( p_X(\cdot) \), based on finite observations \( x_{1:N} \).

Following [66], we estimate \( H_\mu(X) \) using average log-loss \( \ell(\hat{p}_X, x_{1:N}) \), defined as

\[
\ell(\hat{p}_X, x_{1:N}) = -\frac{1}{N} \sum_{i=1}^{N} \log \hat{p}_X(x_i | x_{1:i-1})
\]

(16)

where \( \hat{p}_X(x_i | x_{1:i-1}) \) denotes the estimated probability of observing \( x_i \), given preceding context \( x_{1:i-1} \). Using (16), we thus compute average log-loss by performing a series of predictions based on increasingly long contexts \( x_{1:i-1} \). Since \( \hat{p}_X \) is an estimate of \( p_X \), the described process is termed self-prediction [40].

We denote with \( p_Y(u_{1:n}) \) the probability of observing \( u_{1:n} \) from source \( Y \). A measure of disparity between sources \( X, Y \) is the cross entropy rate \( H^\times_\mu(X, Y) \),

\[
H^\times_\mu(X, Y) = \lim_{n \to \infty} -\frac{1}{n} \int p_X(u_{1:n}) \log p_Y(u_{1:n}) \, du_{1:n}
\]

(17)

quantifying the expected number of bits required to represent observations emitted by source \( X \), given a code for source \( Y \). We estimate \( H^\times_\mu(X, Y) \) by computing the average log-loss \( \ell(\hat{p}_Y, x_{1:N}) \) based on iterated prediction, where \( \hat{p}_Y \) denotes an estimate of \( p_Y \) based on observations \( y_{1:M} \). Since \( \hat{p}_Y, \hat{p}_X \) represent disparate sources, the described process is termed cross-prediction [40]. To obtain a symmetric distance between sources \( X, Y \) based on cross entropy, we compute the quantity

\[
D^\times(X, Y) = \frac{H^\times_\mu(X, Y) + H^\times_\mu(Y, X)}{H_\mu(X) + H_\mu(Y)}
\]

(18)
Figure 1: Evaluated prediction strategies. Sequences $x_{1:N}, y_{1:M}$ serve as model inputs, observation context $x_{1:n}$ forms basis of prediction $\hat{x}_{n+1}$. Quantity $\epsilon_{n+1}$ denotes prediction error.

where in (18) the denominator serves as a normalisation factor, analogous to the denominator in (2) and where we use self-prediction to estimate $H_\mu(X), H_\mu(Y)$.

To obtain a prediction-based estimate of the NID in (12), we estimate $H_\mu(X), H_\mu(Y)$ again using self-prediction. Furthermore, we estimate the conditional entropy rate $H_\mu(X|Y)$ using the distribution $\hat{p}_{X|Y}$, referring to the estimated distribution of observations emitted by $X$, given knowledge of observations $y_{1:M}$ generated by $Y$. Analogous to self-prediction and cross-prediction, we define the quantity $\ell(\hat{p}_{X|Y}, x_{1:N}, y_{1:M})$,

$$\ell(\hat{p}_{X|Y}, x_{1:N}, y_{1:M}) = -\frac{1}{N} \sum_{i=1}^{N} \log \frac{\hat{p}_{X|Y}(x_i|y_{1:M})}{\hat{p}_{X|Y}(x_i|x_{1:i-1}, y_{1:M})}.$$  \hspace{1cm} (19)

We refer to the process used to compute (19) as conditional self-prediction.

### 3.4 Continuous-Valued Approach

One means of computing the quantities described in Section 3.3 involves quantised feature vectors \([22, 24, 31, 49]\). As an alternative, we propose an approach requiring no prior quantisation.

As used in \([40]\), in our approach we utilise non-linear time series prediction. In contrast to \([40]\), we are concerned with evaluating distance measures which we compute as statistics of prediction errors. Therefore, we use a comparatively straightforward nearest-neighbours approach. Given the sequence of feature vectors $C$, consider first the process of time-delay embedding \([67]\), which yields
the vector sequence $S^c$, whose elements $s^c_r$ are defined as
\[ s^c_r = \text{vec} (c_r, c_{(r-1)\tau}, \ldots, c_{(r-d+1)\tau}). \] (20)
According to (20), each element $s^c_r$ aggregates feature vector $c_r$ along with its preceding temporal context $(c_{(r-1)\tau}, \ldots, c_{(r-d+1)\tau})$. The amount of temporal context is controlled by parameters $d, \tau$, respectively referred to as embedding dimension and time delay. Operator vec denotes vectorisation.

Our method of predicting features is based on determining nearest neighbours in time-delay embedded space. We first illustrate our method for the case of cross-prediction, depicted schematically in Fig. 1 (a). Given sequence $y_{1:M}$, we denote with $\tilde{x}_{t+h}$ the estimated successor of sequence $x_{1:t+h-1}$,
\[ \tilde{x}_{t+h} = y_q(t) + h \] (21)
where $h$ denotes the predictive horizon (how far into the future we predict), and where we define $q(t)$ as
\[ q(t) = \arg \max_{k \in [d \ldots M-h]} \text{corr}(s^y_k, s^x_t) \] (22)
with $\text{corr}(\cdot, \cdot)$ denoting Pearson’s correlation coefficient. We motivate use of correlation coefficients as an alternative to the Euclidean distance, following [68].

Depicted schematically in Fig. 1 (b), to perform self-prediction we set $Y = X$. Since features may be slowly-varying, when forming prediction $\tilde{x}_{t+h}$ we disregard observations in the immediate past of time step $t$. Thus we define
\[ \tilde{x}_{t+h} = x_{q'(t)} + h \] (23)
with $q'(t)$ defined as
\[ q'(t) = \arg \max_{k \in [d \ldots N-h], |k-t| > R} \text{corr}(s^x_k, s^x_t) \] (24)
and where $R$ denotes the radius below which observations are disregarded.

Finally, to perform conditional self-prediction, we use both time-delay embedded spaces $s^y, s^x$. Given predictions $y_{q(t)+h}, x_{q'(t)+h}$, respectively obtained using cross-prediction and self-prediction, we compute the linear combination
\[ \tilde{x}_{t+h} = y_{q(t)+h} \alpha + x_{q'(t)+h} (1 - \alpha). \] (25)
Similar to the approach given in [69], in (25) for weighting coefficient $\alpha$ we use
\[ \alpha = \frac{\text{MSE}_\text{self}}{\text{MSE}_\text{self} + \text{MSE}_\text{cross}} \] (26)
where $\text{MSE}_\text{cross}, \text{MSE}_\text{self}$ respectively denote cross-prediction and self-prediction mean squared errors. Fig. 1(c) depicts conditional self-prediction schematically.
Given the sequence of predictions $\tilde{x}_{1:N}$, we denote with $\epsilon_n$ the rescaled prediction error, whose $i$th component $\epsilon_{i,n}$ is given by

$$\epsilon_{i,n} = \frac{\tilde{x}_{i,n} - x_{i,n}}{s_i}$$

where $s_i$ denotes the sample variance of the $i$th component $x_{i,n}$ in $x_n$. We contrast our approach with the component-wise normalised mean squared error (NMSE) based on cross-prediction used in [40], which may be applied as an alternative measure of dissimilarity between time series. Our approach is based on assuming that the prediction error may be represented using a normally distributed random variable $Z$ with samples $Z_{1:N} = \epsilon_{1:N}$. Using the samples, we estimate the prediction error entropy $H(Z)$ parametrically. In the case of self-prediction, we assume the identity $H(Z) = H_\mu(X)$; analogously in the case of cross-prediction and conditional self-prediction, we assume respective identities $H(Z) = H_\mu(X,Y)$, $H(Z) = H_\mu(X|Y)$. Assuming normality, we estimate $H(Z)$ using the equation

$$H(Z) = \frac{1}{2} \log(2\pi e)^k |\Sigma|$$

where $\Sigma$ denotes the sample covariance. In our continuous-valued approach, using the prediction methods depicted in Fig. 1, we thus estimate information-based measures as statistics of the prediction error sequence.

4 Experimental Method

We first evaluate our proposed methods using a set of 300 audio recordings of Jazz standards.

We assume that two tracks are a cover pair if they possess identical title strings. Thus, we assume a symmetric relation when determining cover identities. The equivalence class of tracks deemed to be covers of one another is a cover set. The Jazz data set comprises 97 cover sets, with average cover set size 3.06 tracks.

Furthermore, we perform a large-scale evaluation based on the Million Song Dataset (MSD) [70]. This dataset includes meta-data and pre-computed audio features for a collection of $10^6$ Western popular music recordings. We use a pre-defined evaluation set of 5,236 query tracks partitioned into 1,726 cover sets, with average cover set size 3.03 tracks. Following [43], for each query track, we seek to identify the remaining cover set members contained in the entire $10^6$ track collection.

4.1 Feature Extraction

For the Jazz dataset, as a representation of musical harmonic content, we extract 12-component beat-synchronous chroma features from audio using the method and implementation described in [41].

\[http://www.eecs.qmul.ac.uk/~peterf/jazzdataset.html\]
\[http://labrosa.ee.columbia.edu/millionsong/secondhand\]
Assuming an equal-tempered scale, the method accounts for deviations in standard pitch from 440Hz, by shifting the mapping of FFT bins to pitches in the range of ±0.5 semitones. Following chroma extraction, beat-synchronisation is achieved using the method described in [71]. First, onset detection is performed by differencing a log-magnitude Mel-frequency spectrogram across time and applying half-wave rectification, before summing across frequency bands. After high-pass filtering the onset signal, a tempo estimate is formed by applying a window function to the autocorrelated onset signal and determining auto-correlation maxima. Varying the centre of the window function allows tempo estimation to incorporate a bias towards a preferred beat rate (PBR).

The tempo estimate and onset signal are then used to obtain an optimal set of beat onsets, by using dynamic programming. Chroma features are averaged over beat intervals, before applying square-root compression and normalising chroma features with respect to the Euclidean norm. Based on our previous work [14], we evaluate using a PBR of 240 beats per minute (bpm).

The MSD includes 12-component chroma features alongside predicted note and beat onsets [72], which we use in our evaluations. In contrast to the beat-synchronous features obtained for the Jazz dataset, MSD chroma features are initially aligned to predicted onsets. Based on preliminary evaluations, as an additional processing step we resample predicted beat onsets to match a rate of 240bpm. We then average chroma features over resampled beat intervals. Finally, we normalise features as described for the Jazz dataset.

4.2 Key Invariance

To account for musical key variation within cover sets, we transpose chroma sequences using the optimal transposition index (OTI) method [7]. Given two chroma vector sequences \( \mathbf{X} \) and \( \mathbf{Y} \), we form summary vectors \( \mathbf{h}_X \) and \( \mathbf{h}_Y \) by averaging over entire sequences. The OTI corresponds to the number of circular shift operations applied to \( \mathbf{h}_Y \) which maximises the inner product between \( \mathbf{h}_X \) and \( \mathbf{h}_Y \),

\[
\text{OTI}(\mathbf{h}_X, \mathbf{h}_Y) = \arg \max_i \mathbf{h}_X \cdot \text{circshift}(\mathbf{h}_Y, i)
\]  

(29)

where \( \text{circshift}(\mathbf{h}_Y, i) \) denotes applying \( i \) circular shift operations to \( \mathbf{h}_Y \). We subsequently shift chroma vectors \( \mathbf{Y} \) by OTI(\( \mathbf{h}_X, \mathbf{h}_Y \)) positions, prior to pairwise comparison.

4.3 Quantisation

For discrete-valued similarity measures, we quantise chroma features using the K-means algorithm. We cluster chroma features aggregated across all tracks, where we consider codebook sizes in the range [2..48]. To increase stability, we execute the K-means algorithm 20 times. We then select the clustering which minimises the mean squared error between data points and assigned clusters. The described quantisation method performs similarly to an alternative based
on pairwise sequence quantisation; for a detailed discussion we refer to our previous work [14].

4.4 Distance Measures

We summarise the distance measures evaluated in this work in Table 1 where for each distance measure, we list our estimation methods.

We utilise the following algorithms to compute distance measures by compressing strings: Prediction by partial matching (PPM) [73], Burrows-Wheeler (BW) compression [74] and Lempel-Ziv (LZ) compression [57], implemented respectively as PPMD [1], BZIP2 [2] and ZLIB [3].

In all cases, we set parameters to favour compression rates over computation time.

We use the described compression algorithms to determine the length in bits of compressed strings and compute NCD, NCDA distances. In a complementary discrete-valued approach, we use string prediction instead of compression. Using average log-loss, we compute NCDA using the formula

$$\ell \left( \hat{p}_{(X,Y)}, (x, y) \right) - \min \left\{ \ell \left( \hat{p}_X, x \right), \ell \left( \hat{p}_Y, y \right) \right\} \max \left\{ \ell \left( \hat{p}_X, x \right), \ell \left( \hat{p}_Y, y \right) \right\}$$  \hspace{1cm} (30)

where $\hat{p}_{(X,Y)}$ denotes the joint distribution estimated on aligned sequence $(x, y)$. We compute the NCD analogously by predicting concatenated strings. In addition, we use string prediction to estimate distance measure $D^\times$, as defined in (18). We perform string prediction using Begleiter’s [66] implementations of PPMC and LZ78 algorithms.

Note that the KL divergence given in (1) is non-symmetric. To obtain symmetry, we compute the Jensen-Shannon (JS) divergence $D_{JS}(p_X \parallel p_Y)$, defined as

$$D_{JS}(p_X \parallel p_Y) = D_{KL}(p_X \parallel p_A) + D_{KL}(p_Y \parallel p_A)$$  \hspace{1cm} (31)

where $p_A$ denotes the mean of $p_X, p_Y$,

$$p_A = \frac{1}{2} (p_X + p_Y).$$  \hspace{1cm} (32)

As a baseline method, we compute the JS divergence between symbol histograms normalised to sum to one.

We evaluate continuous-valued prediction using parameters $h \in \{1, 4\}, d \in \{1, 2, 4\}, \tau \in \{1, 2, 4, 6\}$, setting the exclusion radius in (24) to $R = 8$ based on preliminary analysis. We compute distance measure $D^\times$ using cross-prediction to estimate the numerator in (18). In a complementary approach, we estimate the NID using conditional self-prediction to estimate the numerator in (12). For $D^\times$ and NID, we use self-prediction to estimate the denominator in (18), (12), respectively.

\[^1\text{http://compression.ru/ds/}\]
\[^2\text{http://bzip2.org}\]
\[^3\text{http://zlib.org}\]
Distance Definition Estimation method

|   |   |   |
|---|---|---|
| NCD | Eqn. 2 | String compression (LZ, BW, PPM) Discrete prediction (LZ, PPM) |
| NCDA | Eqn. 13 | String compression (LZ, BW, PPM) Discrete prediction (LZ, PPM) |
| $D^\infty$ | Eqn. 18 | Discrete prediction Continuous prediction |
| $D_{JS}$ | Eqn. 31 | Normalised symbol histograms |
| NID | Eqn. 12 | Continuous prediction |

Table 1: Summary of evaluated distance measures.

Finally, to compensate for cover song candidates consistently deemed similar to query tracks, we normalise pairwise distances using the method described in [75]. We apply distance normalisation as a post-processing step, before computing performance statistics.

### 4.5 Large-scale cover song identification

For music content analysis involving large datasets, algorithm scalability is an important issue. Since the approaches in this work involve non-metric pairwise comparisons between tracks, retrieving result candidates for a given query track requires a linear scan through the dataset, which may be infeasible for large datasets. We use a scalable approach for our evaluations involving the MSD. Following [54] and similar to the method proposed in [76], we incorporate our methods into a two-stage retrieval process. By using a metric distance to determine similarity in the first retrieval stage, we allow for the potential use of indexing or hashing schemes, as proposed in [51,55]. We then apply non-metric pairwise comparisons in the second retrieval stage.

In the first stage, we quantise as described in Section 4.3 and represent each track with a normalised codeword histogram. Given a query track, we then rank each of the $10^6$ candidate tracks using the $L_1$ distance. To account for key variation, for each candidate track we minimise $L_1$ distance across chroma rotations. We then determine the top $L = 1000$ candidate tracks, which we re-rank in the second stage using our proposed methods. After both retrieval stages, we normalise pairwise distances as described in Section 4.4. We report performance based on the final ranking of all $10^6$ candidate tracks, across query tracks.

### 4.6 Performance Statistics

As used in [25], we quantify cover song identification accuracy using mean average precision (MAP).

Following [25], we use the Friedman test [77] to compare accuracies among distance measures. The Friedman test is based on ranking across queries each
distance measure according to average precision. We combine the Friedman test with Tukey’s range test \cite{78} to adjust for Type I errors when performing multiple comparisons.

4.7 Combining distance measures

To determine if combining distance measures improves cover song identification accuracy, we obtain pairwise distances as described in Section 4.4. We denote with $d_{i,j}^k$ the pairwise distance between the $i$th query track and the $j$th result candidate, obtained using the $k$th distance measure in our evaluation. We transform $d_{i,j}^k$ by computing the inverse rank $d_{i,j}^{k'}$, 

$$d_{i,j}^{k'} = 1 - \frac{\text{rank}(d_{i,j}^k)}{\text{rank}(d_{i,j}^k)}$$  \hspace{1cm} (33)

where rank($d_{i,j}^k$) denotes the rank of $d_{i,j}^k$ among all distances obtained with respect to query track $i$, given the $k$th distance measure. We apply this transformation to protect against outliers, while ensuring that distance decreases rapidly for track pairs deemed highly similar, for decreasing distance. Note that since our distance transformation preserves monotonicity and MAP itself is based on ranked distances, performance of unmixed distance measures is unaffected by transformation. Finally, we combine distances $d_{i,j}^{k'}$, $d_{i,j}^{m'}$ by computing a weighted average of distances pooled using max and min operators,

$$\max\{d_{i,j}^{k'}, d_{i,j}^{m'}\} \beta + \min\{d_{i,j}^{k'}, d_{i,j}^{m'}\} (1 - \beta)$$  \hspace{1cm} (34)

where we optimise weight $\beta$ with respect to MAP.

4.8 Baseline approaches

In addition to the JS divergence and cross-prediction NMSE baselines, we include an evaluation of the method and implementation described in \cite{41} based on cross-correlation. As a random baseline, we sample pairwise distances from a normal distribution.

5 Results

In Fig. 2 (a)–(c), based on the Jazz dataset we examine the performance of discrete-valued NCD and NCDA distance measures, combined with LZ, BW and PPM algorithms. For the LZ algorithm, NCDA yields a relative performance gain of 38.6\%, averaged across codebook sizes. In contrast, for PPM, with the exception of small codebook sizes in the range [2..8], NCDA yields no consistent improvement over NCD, however averaged across codebook sizes we obtain a mean relative performance gain of 11.0\%. Finally, the effect of using NCDA is reversed for BW compression, where performance decreases by an average of 21.8\%. 

15
Examining results for the MSD in Fig. 2 (c)–(g), we observe similar qualitative results for LZ and BW algorithms. For the LZ algorithm, NCDA yields an average relative performance gain of 10.1%, whereas for BW compression we observe an average relative performance loss of 6.5%. In contrast to the Jazz dataset, for PPM we observe an average relative performance loss of 1.5%.

For both datasets, NCDA appears to be most advantageous combined with LZ compression, whereas BW yields the least advantageous result. Note that BW compression is block-based in contrast to LZ and PPM compressors, both of which are sequential.

We attribute this observation to performance differences among compressors, since the assumptions made in Section 3.2 rely on assuming Markov sources. Noting differences in relative performance gains between datasets, following [54], we conjecture that chroma feature representation affects the performance of the evaluated distance measures.

We examine the performance of JS divergence between normalised symbol histograms, as displayed in Fig. 2 (d), (h). Surprisingly, for the Jazz dataset and for $K > 8$, JS divergence outperforms compression-based methods, with maximum MAP score 0.289 obtained for $K = 48$. This result is contrary to our expectation that NCD approaches should outperform the bag-of-features approach, by accounting for temporal structure in time series. In contrast, for the MSD and for optimal $K$, both NCD and NCDA outperform JS divergence across all evaluated compression algorithms. We attribute this disparity to differences in problem dimensionality between datasets, where for the Jazz dataset the problem size may be sufficiently small to amortise advantages of using NCD, NCDA compared to JS divergence.

In Fig. 3, we consider the performance of distance measures based on string prediction. For the Jazz dataset, comparing log-loss estimates of NCD and NCDA using the LZ algorithm, averaged across codebook sizes NCDA outperforms NCD; we obtain a mean relative performance gain of 105.1% (Fig. 3 (a)). For the PPM algorithm, although NCD maximises performance (MAP 0.140), we obtain a mean relative performance gain of 19.3% using NCDA over NCD (Fig. 3 (b)). Importantly, for both LZ and PPM the cross-prediction distance $D^\times$ consistently outperforms NCD and NCDA; for $K = 16$ and combined with PPM compression, we obtain MAP 0.329. For the MSD and using LZ compression, in contrast to the Jazz dataset we observe a mean relative performance loss of 1.8% when comparing $D^\times$ with NCDA. For both LZ and PPM, NCDA compared to NCD yields mean relative performance gains of 17.6% and 19.3%, respectively.

Table 2 displays the performance of continuous-valued prediction approaches. Note that for $d = 1$, parameter $\tau$ may be set to an arbitrary integer following (20). We consider results obtained for the Jazz dataset (Table 2 (a)–(c)). Using conditional self-prediction to estimate the NID, maximised across parameters $h, d, \tau$ we obtain MAP 0.346. In comparison, cross-prediction distance $D^\times$ yields MAP 0.454. As a baseline, we determine the cross-prediction NMSE, where maximising across parameters we obtain MAP 0.459. Table 2 (a)–(c) displays performance against evaluated parameter combinations. Examining results for
Table 2: Mean average precision scores for distances based on continuous prediction. In each subfigure, parameters $h$, $\tau$, $d$ denote predictive horizon, time delay and embedding dimension, respectively. Results displayed in subfigures (a)–(c), (d)–(f) for Jazz and MSD datasets, respectively.

| $h$ | $\tau = 1$ | $\tau = 2$ | $\tau = 4$ | $\tau = 6$ |
|-----|------------|------------|------------|------------|
|     |            |            |            |            |
| 1   | $0.282$    | $0.282$    | $0.282$    | $0.282$    |
| 2   | $0.308$    | $0.311$    | $0.293$    | $0.312$    |
| 4   | $0.327$    | $0.332$    | $0.318$    | $0.318$    |
|     |            |            |            |            |
| 1   | $0.243$    | $0.243$    | $0.243$    | $0.243$    |
| 2   | $0.262$    | $0.273$    | $0.291$    | $0.284$    |
| 4   | $0.307$    | $0.313$    | $0.321$    | $0.321$    |

(a) NID estimate; conditional self-prediction (Jazz)

| $h$ | $\tau = 1$ | $\tau = 2$ | $\tau = 4$ | $\tau = 6$ |
|-----|------------|------------|------------|------------|
|     |            |            |            |            |
| 1   | $0.347$    | $0.347$    | $0.347$    | $0.347$    |
| 2   | $0.412$    | $0.403$    | $0.390$    | $0.403$    |
| 4   | $0.454$    | $0.446$    | $0.432$    | $0.423$    |
|     |            |            |            |            |
| 1   | $0.321$    | $0.321$    | $0.321$    | $0.321$    |
| 2   | $0.362$    | $0.375$    | $0.390$    | $0.379$    |
| 4   | $0.417$    | $0.450$    | $0.446$    | $0.459$    |

(b) $D^X$ estimate; cross-prediction (Jazz)

| $h$ | $\tau = 1$ | $\tau = 2$ | $\tau = 4$ | $\tau = 6$ |
|-----|------------|------------|------------|------------|
|     |            |            |            |            |
| 1   | $0.0451$   | $0.0451$   | $0.0451$   | $0.0451$   |
| 2   | $0.0476$   | $0.0477$   | $0.0479$   | $0.0475$   |
| 4   | $0.0489$   | $0.0494$   | $0.0494$   | $0.0489$   |
|     |            |            |            |            |
| 1   | $0.0465$   | $0.0465$   | $0.0465$   | $0.0465$   |
| 2   | $0.0470$   | $0.0480$   | $0.0484$   | $0.0487$   |
| 4   | $0.0478$   | $0.0488$   | $0.0498$   | $0.0491$   |

(c) NMSE; cross-prediction (Jazz)

| $h$ | $\tau = 1$ | $\tau = 2$ | $\tau = 4$ | $\tau = 6$ |
|-----|------------|------------|------------|------------|
|     |            |            |            |            |
| 1   | $0.0191$   | $0.0191$   | $0.0191$   | $0.0191$   |
| 2   | $0.0230$   | $0.0222$   | $0.0239$   | $0.0250$   |
| 4   | $0.0238$   | $0.0275$   | $0.0303$   | $0.0295$   |
|     |            |            |            |            |
| 1   | $0.0250$   | $0.0250$   | $0.0250$   | $0.0250$   |
| 2   | $0.0208$   | $0.0239$   | $0.0236$   | $0.0260$   |
| 4   | $0.0228$   | $0.0276$   | $0.0303$   | $0.0301$   |

(d) NID estimate; conditional self-prediction (MSD)

| $h$ | $\tau = 1$ | $\tau = 2$ | $\tau = 4$ | $\tau = 6$ |
|-----|------------|------------|------------|------------|
|     |            |            |            |            |
| 1   | $0.0447$   | $0.0447$   | $0.0447$   | $0.0447$   |
| 2   | $0.0489$   | $0.0494$   | $0.0494$   | $0.0489$   |
| 4   | $0.0465$   | $0.0465$   | $0.0465$   | $0.0465$   |
|     |            |            |            |            |
| 1   | $0.0470$   | $0.0480$   | $0.0484$   | $0.0487$   |
| 2   | $0.0478$   | $0.0488$   | $0.0498$   | $0.0491$   |

(e) $D^X$ estimate; cross-prediction (MSD)

| $h$ | $\tau = 1$ | $\tau = 2$ | $\tau = 4$ | $\tau = 6$ |
|-----|------------|------------|------------|------------|
|     |            |            |            |            |
| 1   | $0.0341$   | $0.0341$   | $0.0341$   | $0.0341$   |
| 2   | $0.0404$   | $0.0420$   | $0.0431$   | $0.0437$   |
| 4   | $0.0447$   | $0.0474$   | $0.0478$   | $0.0465$   |
|     |            |            |            |            |
| 1   | $0.0431$   | $0.0431$   | $0.0431$   | $0.0431$   |
| 2   | $0.0450$   | $0.0457$   | $0.0467$   | $0.0471$   |
| 4   | $0.0466$   | $0.0494$   | $0.0499$   | $0.0494$   |

(f) NMSE; cross-prediction (MSD)
the MSD in Table 2 (d)–(f), we obtain qualitatively similar results with maximum MAP values 0.0303, 0.0498 and 0.0499 for NID, $D^\times$ and NMSE, respectively. For both datasets, we observe that increasing the value of $d$ consistently improves performance. In contrast, we observe no such effect for parameters $\tau, h$.

5.1 Summary of results and comparison to state of the art

Fig. 4 (a), (b) displays the result of significance testing as described in Section 4.6, where we assume 95% confidence intervals and where we maximise across evaluated parameter spaces. Table 3 displays a corresponding summary of MAP scores. As baselines we include Ellis and Poliner’s cross-correlation approach [41], in addition to randomly sampled pairwise distances.

For both Jazz dataset and MSD, we observe that continuous-valued approaches based on cross-prediction consistently outperform discrete-valued approaches. Moreover, with the exception of NCD combined with PPM-based string compression and for the MSD, using continuous-valued cross-prediction significantly outperforms discrete-valued approaches. For approaches based on string compression, we note that using NCDA with BW compression significantly decreases performance with respect to NCD. Similarly, using NCDA decreases MAP scores for PPM. Although we do not observe a significant performance gain using NCDA over NCD for LZ compression, performance improves consistently across datasets. For the Jazz dataset, we observe that the JS divergence baseline significantly outperforms the majority of string-compression approaches. In contrast, for the MSD the majority of string-compression approaches significantly outperform the JS divergence baseline. Whereas PPM with distance $D^\times$ consistently outperforms all discrete-valued approaches for the Jazz dataset, PPM with compression-based NCD consistently outperforms all discrete-valued approaches for the MSD and significantly outperforms the JS divergence baseline.

In a comparison of continuous-valued approaches, we observe that cross-prediction using either distance $D^\times$ or NMSE competes with cross-correlation for the Jazz dataset. In contrast, the same cross-prediction approaches significantly outperform cross-correlation for the MSD.

Examining continuous-valued approaches further, for both Jazz dataset and MSD, we observe a significant disadvantage in using our conditional self-prediction based estimate of NID, over cross-prediction based distances $D^\times$ and NMSE. The relatively poor performance of NID for the MSD suggests a limitation of our prediction approach when used with MSD chroma features. However, considering results for both datasets suggests that cross-prediction yields more favourable results than conditional self-prediction generally.

To facilitate further comparison, we consider the approaches proposed by Bertin-Mahieux and Ellis [43], Khadkevich and Omologo [54], who report MAP scores of 0.0295, 0.0371, respectively. Based on such a comparison, we obtain state-of-the-art results. Note that the stated approaches do not report any distance normalisation procedure as described in Section 4.4; we found that
normalisation improved our results.

Finally, using the method described in Section 4.7, we combine distances obtained using continuous-valued prediction. We display results in Table 3 and Fig. 4 (c), (d). Compared to using the baseline cross-prediction NMSE alone, combining NMSE with $D^\times$ significantly improves performance for both the Jazz dataset and MSD; we obtain relative MAP performance gains of 8.1% and 3.4% respectively. We obtain no performance gain by further combining NID estimates with NMSE and $D^\times$. Using the combination NMSE and $D^\times$, we obtain MAP scores 0.496 and 0.0516 for Jazz dataset and MSD, respectively, consistently outperforming the remaining distance measures considered in this work.

### Table 3: Summary of mean average precision scores.

| Dataset | Jazz | MSD |
|---------|------|-----|
| **Method** | NCD | NCD | NCD | NCD |
| PPM | 0.220 | 0.249 | 0.0460 | 0.0487 |
| BW | 0.143 | 0.220 | 0.0428 | 0.0480 |
| LZ | 0.196 | 0.168 | 0.0457 | 0.0438 |
| **PPM; $D^\times$** | 0.329 | 0.0428 |
| **LZ; $D^\times$** | 0.288 | 0.0415 |
| **JS divergence** | 0.289 | 0.0412 |
| **$D^\times$ (continuous)** | 0.454 | 0.0498 |
| **NID (continuous)** | 0.346 | 0.0303 |
| **NMSE (continuous)** | 0.459 | 0.0499 |
| Ellis and Poliner [11] | 0.465 | 0.0404 |
| Random | 0.026 | 0.0006 |
| $D^\times$ & NMSE (cont.) | 0.496 | 0.0516 |
| $D^\times$ & NID & NMSE (cont.) | 0.432 | 0.0463 |

We have evaluated measures of pairwise predictability between time series for cover song identification. We consider alternative distance measures to the NCD: We propose NCDA, which incorporates a method for obtaining joint representations of time series, in addition to methods based on cross-prediction. Secondly, we attend to the issue of representing time series: We propose continuous-valued prediction as a means of determining pairwise similarity, where we estimate compressibility as a statistic of the prediction error. We contrast methods requiring feature quantisation, against methods directly applicable to continuous-valued features.

Results indicate that the method of determining pairwise similarity significantly affects cover song identification performance. Firstly, the proposed continuous-valued approach outperforms discrete-valued approaches and competes with evaluated continuous baseline approaches. Secondly, we draw attention to using cross-prediction as an alternative approach to the NCD, where
we observe superior results in both discrete and continuous cases for Jazz cover song identification, and for the continuous case for cover song identification using the Million Song Dataset. Thirdly, using NCDA, we are able to mitigate differences in performance between evaluated discrete compression algorithms. We view the previous three points as evidence that using information-based measures of similarity, there exist significant practical issues related to representing and modelling features, in addition to the choice of distance measure. We argue that due to the ubiquity of time series similarity problems, our results are relevant to application domains extending beyond the scope of this work. Finally, in the context of cover song identification, we have demonstrated state-of-the-art performance using a large-scale dataset. We have shown that our distances based on continuous-valued prediction may be combined to improve performance relative to the baseline.

For future work, we aim to evaluate alternative time series models to those presently considered. To this end, further investigations might involve causal state space reconstruction [79] or recurrent neural networks such as the long short term memory architecture [80]. For future work, we aim to evaluate ensemble techniques for combining distances in greater detail.

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Figure 2: Effect of codebook size and distance measure on mean average precision (MAP). Results displayed for Lempel-Ziv (LZ), Burrows-Wheeler (BW) and prediction by partial matching (PPM) algorithms in subfigures (a)–(c), (e)–(g), for Jazz and MSD datasets respectively. Subfigures (d), (h) display results for Jensen-Shannon divergence baseline (JS), for Jazz and MSD datasets respectively.
Figure 3: Effect of codebook size and distance measure on mean average precision (MAP). Results obtained using string prediction approach, displayed for Lempel-Ziv (LZ) (subfigures (a), (c)) and prediction by partial march (PPM) (subfigures (b), (d)), for Jazz and MSD datasets respectively.
Figure 4: Mean ranks of average precision scores obtained using Friedman test. Error bars indicate 95% confidence intervals obtained using Tukey’s range test [78]. Higher mean ranks indicate higher performance. Results displayed for Jazz and MSD datasets in subfigures (a) and (b), respectively, with results for combined distances displayed in subfigures (c) and (d).