Ambiguity of gamma-ray tracking of ‘two-interaction’ events

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Abstract

Tracking of gamma-ray interactions in germanium detectors can allow reconstruction of the photon paths, and is useful for many applications. Scrutiny of the kinematics and geometry of gamma rays which are Compton scattered only once prior to full absorption reveals that there are cases where even perfect spatial and energy resolution cannot resolve the true interaction sequence and consequently gamma-ray tracks cannot be reconstructed. The photon energy range where this ambiguity exists is from 255 keV to around 700 keV. This is a region of importance for nuclear structure research where two-point interactions are probable.

Keywords: Gamma-ray tracking, Compton scattering, Segmented germanium detectors

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1 Introduction

Experimental nuclear structure research will increasingly involve the use of radioactive ion beams to produce nuclei far from stability. Due to the technical difficulties associated with accelerating such beams, the anticipated intensities are much lower than those currently achieved with stable-ion beams. This will expose the efficiency shortcomings of the current generation of gamma-ray spectrometers, such as Gammasphere. Consequently, both in the U.S. [1] and in Europe [2], much attention has been focused on the development of gamma-ray-tracking detectors which can provide much greater efficiency and excellent doppler correction facility. The proposed devices take the form of a spherical shell of highly-segmented germanium detectors read out through

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digital electronics with sophisticated pulse-shape analysis. Despite the difficulties in employing this technology, a ‘proof of principle’ has been published for the proposed U.S. device Greta [3] and progress continues to be made [4]. Gamma-ray tracking is based upon the principle that when a photon interacts within one segment of a germanium detector, the transient signals produced in neighboring segments can be utilized to locate the interaction point with great precision. Matching excellent spatial resolution with the energy resolution afforded by germanium detectors, it is hoped that the full interaction path, or track, can be deduced for incident gamma rays. However, in discussion of this principle it has been reported that given perfect spatial and energy resolution, the tracking algorithm employed to decipher the events can identify the gamma-ray path exactly [3]. In this paper we show that there are exceptions to this rule. We focus upon photons which are scattered only once prior to full absorption to demonstrate that a general feature of Compton scattering leads to an ambiguity in the tracking of particular events. As we perceive there to be a general misconception that tracking ambiguities arise purely from experimental uncertainties in energy or position, we hope that this work will help to clarify the issue.

2 Tracking gamma rays involved in two interactions

We restrict our discussion to gamma rays which undergo a single Compton interaction prior to photoelectric absorption. For such events, the process of identifying the gamma-ray track is a matter of deducing the Compton scattering angle \( \theta \). If the spatial origin of the gamma ray is known, as it is approximately when studying decay from a radioactive source or a target bombarded with ions, then this problem is further reduced to a determination of which interaction occurred first. As the intrinsic timing of germanium detectors is inadequate for this task, precedence must be evaluated based upon a consideration of the measured energies and the Compton scattering formula [5],

\[
\frac{1}{E'_\gamma} - \frac{1}{E_\gamma} = \frac{1}{m_0c^2} (1 - \cos \theta),
\]

where \( E'_\gamma \) is the incident photon energy, \( E_\gamma \) is the energy of the scattered photon and \( \theta \) is the angle through which the scattering occurred. To illustrate, we analyse an event where both interaction points and the associated energies are known exactly. Labelling the interaction points \( A \) and \( B \), and the partial energies deposited at each point \( E_A \) and \( E_B \) respectively, the incident energy is determined by

\[
E_\gamma = E_A + E_B.
\]
However, as it is not known whether A or B is the point at which the scattering occurred, $E'_\gamma$ could be either $E_A$ or $E_B$. Rewriting Eq.(1) in terms of the angle $\theta$, and substituting for $E_\gamma$ and $E'_\gamma$, we obtain two possibilities,

$$\theta_{Ak} = \cos^{-1} \left[ 1 - 511 \left( \frac{E_A}{E_B(E_A + E_B)} \right) \right], \quad (3)$$

or

$$\theta_{Bk} = \cos^{-1} \left[ 1 - 511 \left( \frac{E_B}{E_A(E_A + E_B)} \right) \right], \quad (4)$$

where $E_A$ and $E_B$ are in units of keV and the subscript $k$ signifies that these angles correspond to purely kinematic solutions. Below a certain energy threshold, which we will deduce shortly, only one physical solution exists and the gamma-ray track is uniquely determined from consideration of the energies alone. However, above the threshold, both Eq.(3) and Eq.(4) may give physical solutions and the kinematic approach is insufficient, hence the need for ‘tracking’. The two known interaction points, in addition to the origin, define a triangle with geometry as shown in Fig.1. Solving this triangle yields the two ‘geometric’ solutions $\theta_{Ag}$ and $\theta_{Bg}$.

Therefore, if A is the point at which the incident photon was scattered, the Compton scattering angle is given by,

$$\theta_{Ag} = 180^\circ - \phi_A = \cos^{-1} \left[ (c^2 - a^2 - b^2)/2ab \right], \quad (5)$$

whereas the alternative solution is given by,

$$\theta_{Bg} = 180^\circ - \phi_B = \cos^{-1} \left[ (a^2 - b^2 - c^2)/2bc \right]. \quad (6)$$

Clearly, though there are two kinematic solutions and two geometric solutions, the true scattering angle $\theta$ should be a solution common to both approaches. If the two remaining solutions are not alike, then they can be rejected and the tracking is known exactly. However, if $\theta_{Ag} = \theta_{Ak}$ and $\theta_{Bg} = \theta_{Bk}$, then both solutions are equally valid and the gamma-ray track is ambiguous. To test whether such cases exist, we assume that $\theta = \theta_{Ak} = \theta_{Ag}$, and attempt to identify solutions for which $\theta_{Bk} = \theta_{Bg}$. For convenience, we introduce the shorthand terms,

$$X = \left[ 1 - 511 \left( \frac{E_A}{E_B(E_A + E_B)} \right) \right], \quad (7)$$
and

\[ Y = \left[ 1 - 511 \left( \frac{E_B}{E_A(E_A + E_B)} \right) \right], \quad (8) \]

such that \( \theta_{A_k} = \cos^{-1} X \) and \( \theta_{B_k} = \cos^{-1} Y \). Rearranging Eq.(5) in terms of \( c \), replacing \( \theta_{A_g} \) with \( \theta_{A_k} = \theta \), and substituting for \( c \) in Eq.(6) yields,

\[ \theta_{B_g} = \cos^{-1} \left[ \frac{-\left(2b^2 + 2baX\right)}{2b\sqrt{b^2 + a^2 + 2baX}} \right]. \quad (9) \]

Equating \( \theta_{B_k} \) with \( \theta_{B_g} \) gives, after further simplification,

\[ b + aX + Y\sqrt{b^2 + a^2 + 2baX} = 0, \quad (10) \]

which is quadratic in \( b \). Of the two solutions for \( b \), one corresponds to the situation where \( \theta_{B_g} = 180^\circ - \theta_{B_k} \). The other, for which \( \theta_{B_g} = \theta_{B_k} \) is given by,

\[ \begin{align*}
   b &= a \left[ -\cos \theta - \frac{E_\gamma}{511} \left( 1 - \cos \theta - \left( \frac{511}{E_\gamma} \right)^2 \right) \right. \\
   &\quad \left. \sqrt{\frac{1 - \cos^2 \theta}{2(1 - \cos \theta) - \left( \frac{511}{E_\gamma} \right)^2}} \right]. \quad (11)
\end{align*} \]

In physical terms, a positive solution to Eq.(11) indicates that for a photon of energy \( E_\gamma \), which travels a distance \( a \) and is scattered through an angle \( \theta \) for a further distance \( b \) prior to absorption, the gamma-ray track cannot be deduced from the locations of the two interaction points and the energies deposited. The limits for which such solutions exist are given by,

\[ \frac{1}{\sqrt{2}\sqrt{1 - \cos \theta}} \leq E_\gamma \leq \frac{1}{\sqrt{1 + \cos \theta}} \frac{511}{\sqrt{1 - \cos \theta}}. \quad (12) \]

3 Discussion

In Fig.2 the solutions to Eq.(11) for \( b/a \) are plotted against \( \theta \) for \( E_\gamma = 300 \rightarrow 700 \) keV. From a study of this figure and the conditions of Eq.(12), certain features are evident. Clearly, as the energy increases beyond 511 keV, the range of angles for which a tracking ambiguity may exist is reduced to larger and smaller \( \theta \) simultaneously. According to Eq.(12), solutions exist up to infinite energy as \( \theta \) approaches 180\(^\circ\). At energies below 511 keV, the range of ambiguity is increasingly restricted to higher \( \theta \). This trend occurs rapidly; below 255 keV there are no angles for which an ambiguity exist.
Taking a single gamma-ray event in isolation, there are two primary physical consequences of an ambiguity in the gamma-ray track. Firstly, the angle of gamma-ray emission, needed to perform Doppler correction and to measure angular distributions, cannot be deduced from two possibilities, which in Fig. 1 correspond to the angles subtended by \( a \) and \( c \) to \( z \). Secondly, the linear polarisation of the gamma ray cannot be ascertained, as the two possibilities \( \theta_{A_g} \) and \( \theta_{B_g} \) exist for the Compton scattering angle. Except for cases where only the energy of the incident gamma ray is of interest, the ambiguous events must presumably be identified and then suppressed, resulting in a reduction of efficiency.

Though there are too many factors affecting the efficiency of a tracking array for the scope of this paper, we can make some qualitative statements regarding this effect. Firstly, if we make the likely assumption that the tracking array is roughly spherical with a radius of \( \sim 10 \text{ cm} \), then solutions for which \( b/a \geq 1 \) can reasonably be expected to have a low probability, as they correspond to scattering lengths much greater than the mean free path of a gamma ray in germanium. Secondly, gamma rays with energies beyond 700 keV are unlikely to be involved in only one interaction prior to scattering and will therefore generate few ambiguous events. In addition, the probability of backward scattering reduces dramatically with increasing energy and therefore solutions for \( \theta > 90^\circ \) are improbable at energies above around 700 keV. These features suggest that it is roughly the energy range from 400 keV to 500 keV which is particularly susceptible to tracking ambiguity. Notably, this is an energy region of prime importance to nuclear structure research.

Our discussion has been based upon the principle of infinite spatial and energy resolution. In practice neither of these conditions exist, nor is the gamma-ray origin known with precision. This compounds the problem, effectively broadening each curve of Fig. 2. Additionally, it is at small values of \( a \) for which the effect is most liable to become a problem. Unfortunately the cost of building an array of segmented detectors is a strong motivation to bring the detectors as close to the source as possible which increases the probability of ambiguous events. Finally, the effect we have described may lead to higher order problems when multiple gamma rays are involved. Much work on the tracking algorithms has been dedicated to the identification of separate gamma-ray tracks. This added uncertainty may make such a task more difficult and needs to be considered when appraising the overall response of a tracking device or the overall efficiency of tracking algorithms. The analytical solutions and boundary conditions derived herein should prove valuable in this task.
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Fig. 1. A figurative representation of a photon emitted from a known origin O and subsequently interacting twice within matter at points A and B. The two possible scattering angles, $\theta_A$ and $\theta_B$, can be found from the geometry of the OAB triangle.
Fig. 2. Solutions to Eq.(11) given in terms of the ratio $b/a$. Negative values, denoted by the shaded region, correspond to non-physical solutions.