The percolation model of relative conductivities and phase permeabilities

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Abstract. The universal mathematical model of relative conductivities of percolation clusters and phase permeabilities of oil-water-saturated rocks is presented. It is obtained on the basis of percolation theory, porous body physics and statistics. The model takes into account the influence of change in pore space surface properties and the nature of fluid flow on the studied characteristics and may be applied for comprehensive analysis and modeling of technological processes of oil production.

The paper is dedicated to the development of the model of relative conductivities of percolation clusters (RCPC) \( \Lambda \) [1, 2], which are used to predict the effective conductivity (heat and electrical conductivity) of heterogeneous environment [3], and relative phase permeabilities (RPP) \( K \) [4, 5], used to calculate the dynamics of oil production [6]. The object of the study is oil-water-saturated rocks; their micro-heterogeneous structure is modeled as a set of interpenetrating percolation (infinite) clusters (PC), consolidated from the structural elements (phases, particles, capillaries) of the same-name components and other elements not included in the PC.

Both studied quantities are structural-topological characteristics of the considered environments. Due to the similarity of the transfer processes, they can be presented as a function of the share of the active part of the percolation cluster \( \theta \), participating in the transfer process, on the dimensionless critical section \( \alpha_k \) of PC, and on the volume concentration of the component \( \theta \) [1, 2]. In the general case for a model of a two-component heterogeneous environment, the function of the RCPC and, hence, the RPP on the specified characteristics may be written as

\[
K(\theta) \sim \Lambda(\theta) = \theta^0.5 \alpha_k^0.5 \theta^0.5 = \theta^0.5 \theta^0.5 = \theta^0.5,
\]

(1)
where \( C=\alpha_k/\theta \) is the critical adjacency characterizing a degree of phase consolidation.

The quantities included in (1) have all the properties of probability and their values are in the range from 0 to 1, so they can be represented as a power-law relation on the corresponding volumetric content of the component \( \theta \). Critical adjacency can be expressed as the product of the probability of \( \theta \) appearance by the probability of preserving it - \( C = \theta Z \) (\( Z \) - phase consolidation coefficient, \( 0 \leq Z \leq 1 \)) [2]. For a two-component system the condition \( Z_1 + Z_2 = 1 \) is met, giving the relation that takes into account the interdependence of the critical adjacencies of the corresponding PCs \( C = \theta Z_0 (1 - Z_0) \).

For the case of two-component heterogeneous environment with equal probability of distribution of components’ phases (symmetric structure) the model of RCPC [2] is developed on the basis of
percolation theory, porous body physics and statistics and the following functions are received: $\Lambda'_{l(2)} = \vartheta_{l(2)} - \vartheta_c$ - for homogeneous and $\Lambda'_{l(2)} = \left(\vartheta_{l(2)} - \vartheta_c\right)'$ - for fractal and quasifractal PC regions, where $t = f\left(\vartheta_{l(2)} - \vartheta_c\right)$ and $n = f\left(\vartheta_{l(2)}\right)$ are critical variable conductivity indices for corresponding PC regions, and $\vartheta_c$ is the flow threshold. Taking into account these functions and representing the phase consolidation coefficient as

$$Z_{l(2)} = \vartheta_{l(2)} - \vartheta_c \leq 1,$$

it follows from (1) for symmetric structures that

$$\Lambda'_{l(2)} = \vartheta_{l(2)} - \vartheta_c \leq 1,$$

The RCPC model for symmetric structures is adapted for environments consisting of unequal components (asymmetric structure), using “oil-water” system in the pore space of oil-water-saturated reservoirs as an example. For such environments, the condition $0 \leq Z_{l(2)} \leq 1$ that takes into account the surface properties of the pore space must be fulfilled. It is assumed that the critical radius of capillaries $r_k = 2\xi \cos \Theta / \Delta P$ ($\xi$ - coefficient of interphase tension, $\Theta$ - wetting angle (contact angle), $\Delta P$ - pressure difference in the fluids [4]) determines the critical section $- \alpha_k \sim r_k^2$, and therefore the critical adjacency $C \sim r_k^2 / \Theta$. Then the ratio of the areas of the critical section of the fluid, respectively, at $0^\circ \leq \Theta \leq 90^\circ$ and $\Theta = 0$ gives the expression

$$\beta = (\cos \Theta)^2,$$

which was used to calculate the PC’s phase consolidation coefficient of both asymmetric and symmetric structures in the form $Z_{l(2)} = \vartheta_{l(2)}^{m(1-\beta)}$. It follows from (3) that at $\Theta = 90^\circ$ ($\beta = 0$) the value $Z_{l(2)}$ corresponds to neutral wettability, and hence to the symmetric RCPC model - $\vartheta_{l(2)} \leq Z_{l(2)} = Z^*_{l(2)} \leq 1$. For hydrophilic and hydrophobic cases (asymmetric system) the wetting fluid angle $0^\circ \leq \Theta_wf < 90^\circ$ is taken into account ($0 < \beta \leq 1$, $0 \leq Z_{l(2)} < 1$).

**Figure. 1.** Profiles of oil and water contact surface in the capillaries of hydrophilic (a, b) and hydrophobic (c, d) environment: a, c – drainage; b, d - impregnation.

Fig. 1 shows capillary profiles in the area of oil and water contact, where the wetting angles used to calculate by formula (3) for drainage and impregnation for hydrophilic and hydrophobic cases are shown.

From comparison of RCPC formulas (1) and (2) it follows that
\[ \Lambda_{l(2)} = \Lambda_{l(2)}^* \sqrt{\frac{Z_{l(2)}^*}{Z_{l(2)}^*}} = \Lambda_{l(2)}^* \Phi_{l(2)}^{0.5\beta/(1-\beta)}. \] (4)

The above dependences allow calculating the RCPC in the following sequence: the wetting fluid angle \( \Theta_{\text{wf}} \) is set; the value of \( \beta \) is defined by formula (3); for the wetting fluid - RCPC \( \Lambda_{\text{wf}} \) is calculated by formula (4) using the RCPC \( \Lambda_{\text{wf}}^* \) for the symmetric model and \( Z_{\text{wf}} = Z_{\text{wf}}^* \Phi_{\text{wf}}^{\beta/(1-\beta)} \) is calculated; for the non-wetting fluid - \( Z_{\text{nwf}} = 1 - Z_{\text{wf}} \) is found and the RCPC \( \Lambda_{\text{nwf}} \) is calculated by expression (4).

When applying the considered model to predict RPP, the relation of fluid-conductivity and the critical capillary radius characteristic of the Poiseuille flow (\( K \approx r_k^4 \)) is taken into account [7]. It follows that: \( \alpha_k = \alpha_{K_k} - r_k^4 \) (\( C_{K_k(2)} = \alpha_{K_k}/\Phi \sim r_k^4 \)) and \( C_{K_k(2)} = C_{K_k(2)}^* \Phi_{K_k(2)} \), which allows finding the correlation between RCPC and RPP according to expression (1)

\[ K_{l(2)} = \theta_{l(2)} \sqrt{C_{K_k(2)}^*} = \Lambda_{l(2)} \sqrt{C_{l(2)}^*} \Phi_{l(2)}. \] (5)

When analyzing the results obtained using the considered model for the process of two-phase (oil-water) filtration in the formation, on the basis of the formula (5), the emphasis is placed on prediction of RPP, for which there is sufficient information on experimental and theoretical data for verification [4, 5, 8]. Hereinafter, RCPC, RPP and other parameters for water are written with the index "w" (\( \Lambda_w, K_w, \Theta_w, \Phi_w \)), and for oil - "o" (\( \Lambda_o, K_o, \Theta_o, \Phi_o \)).

**Figure. 2.** Functions of RCPC and RPP for "oil-water" environment on water-saturation for strongly hydrophilic and strongly hydrophobic rocks, obtained using percolation model of RCPC and RPP:

1, 1' – RCPC, the rest – RPP, solid lines – drainage, dashed lines - impregnation; 1 - 6 - oil;
1' - 6' - water; 1, 1', 2, 2' - \( \Theta_w = 90^\circ \); 3, 3' - \( \Theta_w = 25^\circ \); 4, 4' - \( \Theta_w = 155^\circ \); 5, 5' - \( \Theta_w = 25^\circ \) and \( \chi = 0.7 \); 6, 6' - \( \Theta_w = 155^\circ \) and \( \chi = 0.3 \);
- corresponds to the condition \( \Lambda_w = \Lambda_o \), • - corresponds to the condition \( K_w = K_o \).
Fig. 2 shows the calculated functions of RCPC and RPP on water-saturation $S_w$ obtained using the proposed model for strongly hydrophilic ($\Theta_w = 25^\circ$), neutral ($\Theta_w = 90^\circ$) and strongly hydrophobic ($\Theta_w = 155^\circ$) rocks. The obtained functions confirm Craig's rule [9]: by condition $K_w = K_o$ water-saturation for strongly hydrophilic systems is $S_w > 0.5$, for strongly hydrophobic $S_w < 0.5$, for neutral $S_w = 0.5$; initial (critical) water-saturation is around 0.16 - 0.2 for strongly hydrophobic systems and above 0.25 for strongly hydrophilic systems. The RCPC curves (1, 1’) are higher than the RPP curves (2, 2’) corresponding to the same wetting angle ($\Theta_w = 90^\circ$), which follows from expression (5).

The presented model covers almost all possible variants of water-oil systems arising during drainage and impregnation for hydrophilic and hydrophobic systems, what allows taking into account the hysteresis phenomenon of RCPC and RPP arising at a change of direction of filtration flows [4, 5].

During drainage (displacement of the wetting fluid by the nonwetting fluid), for the hydrophilic case (Fig. 1a) the calculated angle of the wetting fluid (water) is $\Theta_w < 90^\circ$, for the hydrophobic one (Fig. 1b) the calculated angle of the wetting fluid (oil) is $\Theta_o < 180^\circ - \Theta_w$.

In case of impregnation (displacement of nonwetting fluid by wetting fluid), at a decrease of degree of capillary surface hydrophilicity, values of interphase contact degree for oil and water are calculated as at drainage, but with larger angle $\Theta_w$ for hydrophilic systems, and with larger angle $\Theta_o$ for hydrophobic ones. The calculated angle is $\Theta_o < 180^\circ - \Theta_w$ in case of surface state changing from hydrophilic to hydrophobic (Fig. 1c), and $\Theta_o < 90^\circ$ in case of changing from hydrophobic to hydrophilic (Fig. 1d). The differences in the drainage and impregnation processes disappear when $\Theta_w = \Theta_o = 90^\circ$ (curves 1, 1’, 2, 2’ in Fig. 2).

In the process of impregnation the surface properties of capillaries can change, which leads to microheterogeneous wettability. This phenomenon can be taken into account by means of approach [4] consisted in grouping capillaries into two groups: the first group includes capillaries with unchanged properties and has relative fraction $\chi$, and the second group includes capillaries with changed properties and, respectively, has fraction $1-\chi$. Fig. 2 shows RPP functions for impregnation process (5, 5’, 6, 6’) calculated considering division of capillaries into two groups with greatly differing surface properties: with wetting angles $\Theta_w = 25^\circ$ and $\Theta_w = 155^\circ$, respectively, with fractions $\chi$ and $1-\chi$. It is obvious that the system passes from strongly hydrophobic to strongly hydrophilic state in the limit case when $\chi$ changes from zero to one, herewith the value of water-saturation corresponding to $K_w = K_o$ increases from $S_w \approx 0.3$ to $S_w \approx 0.7$. 
For complex microheterogeneous wettability in the proposed model it is suggested to form capillaries with equal surface properties by groups in fractions of total number ($\chi_i$ - fraction of the $i$-th group, $\sum_{i=1}^{n} \chi_i = 1$), and to calculate RCPC and RPP of each group using appropriate calculated wetting angle $\Theta_i$. The resultant values of RCPC and RPP are obtained by summing the RCPC and RPP of all groups, taking into account the weight of each: $\Lambda_{w(o)} = \sum_{i=1}^{n} \chi_i \Lambda_{w(o)}(\Theta_i)$, $K_{w(o)} = \sum_{i=1}^{n} \chi_i K_{w(o)}(\Theta_i)$.

In order to more accurately approximate the calculated relations to the experimental ones, there is an extension of the model by introducing a correction that takes into account changes in the rheological properties of oil and water, occurring, for example, during RPP hysteresis. The correction is based on the character of fluid-conductivity dependence on critical capillary radius at flow of fluids with different properties. In view of this, the adjacency used in formula (5) is expressed as $C_{K(1/2)} = \Theta_{1/2}^{-1} \left[ C_{l(1/2)} \Phi_{l(1/2)} \right]^{1+k^{-1}}$, where $k$ is the flow index [5]).

The considered model is verified through fitting its parameters by comparing the obtained calculated values of RPP on water-saturation with the known experimental and calculated functions [4, 5, 8]. As an example, Fig. 3 shows experimental and calculated (by the proposed method) relations of RPP on water-saturation for dolomite and calcite samples, normalized to the absolute permeability of the environment for the same phase [5]. Given the level of reliability of the experimental data, the results of verification can be considered satisfactory.

The presented model shows a generality, as it allows comprehensively analyzing and predicting the RCPC and RPP by selecting optimal values of phase consolidation coefficients, used in calculations,
by comparing with experimental data. In consequence of similarity of RCPC and RPP in a number of cases within the presented model, it seems reasonable to use values of RPP obtained by experimental or analytical methods in calculations of effective thermal conductivity of oil-water-saturated rocks.

In contrast to the known percolation models of RPP hysteresis [4, 5], the advantages of the presented model are the simpler mathematical apparatus used without qualitative losses, the complexity of the study of RCPC and RPP, the prospect of taking into account various factors affecting the phase consolidation processes and the possibility of application to study other transfer properties.

References
[1] Kolesnikov B P Influence of fractal substructures of the percolating cluster on transferring processes in macroscopically disordered environments // Journal of Physics: Conference Series Volume 891 (2017) 012355
[2] Kolesnikov B P The unified approach to a definition of effective conductivity index of percolating cluster in macroscopically disordered environments // Journal of physics: conference series : All-Russian scientific conference with international participation "Thermophysics and Power Engineering in Academic Centers" (TPEAC-2019)
[3] Kolesnikov B P Prediction of effective conductivity of multicomponent macroscopically disordered environments by mathematical modeling // Electronic multidisciplinary journal “Scientific works of KubGTU” (2016) № 16 – p. 459-468
[4] Kadet V V, Galechyan A M Percolation model of relative phase permeability hysteresis // Applied Mechanics and Technical Physics (2013) V54 № 3 – p. 95-105
[5] Kadet V V, Galechyan A M Consideration of fluid rheology in the hydrophobization model of relative phase permeability hysteresis hysteresis // Applied Mechanics and Technical Physics (2017) V58 № 6 – p. 58-68
[6] Gudok N S, Bogdanovich N N, Martinov V G Determination of physical properties of oil-bearing rocks // Nedra (2007) – p. 592
[7] Landau L D, Livshic E M Theoretical Physics: // Textbook VI. Hydrodynamics. 3rd ed. Moscow (1986) – p. 736
[8] Moiseev V D Experimental determination of relative phase permeabilities for oil and gas reservoirs in the fields of northern West Siberia // Oil and Gas (2020) – p. 46-53
[9] Graig F F The reservoir engineering aspects of wettability on reservoir rocks and its evaluation // Prod. Monthly (April 1968) V 32 № 4 p/ 2-7