Proxy Discrimination* in Data-Driven Systems
Theory and Experiments with Machine Learnt Programs

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ABSTRACT
Machine learnt systems inherit biases against protected classes, historically disparaged groups, from training data. Usually, these biases are not explicit, they rely on subtle correlations discovered by training algorithms, and are therefore difficult to detect. We formalize a notion of proxy discrimination in data-driven systems, a class of properties indicative of bias, as the presence of protected class correlates that have causal influence on the system’s output. We evaluate an implementation on a corpus of social datasets, demonstrating how to validate systems against these properties and to repair violations where they occur.

KEYWORDS
indirect discrimination, proxy

1 INTRODUCTION
Utility of machine learning has spurred adoption of automated systems in many areas of life. Systems, from credit and insurance assessors[45] to recidivism predictors[5], have significant impact on the affected individuals’ future. Machine learnt systems, however, are constructed on the basis of observational data from the real world, with its many historical or institutionalized biases. As a result, they inherit biases and discriminatory practices inherent in the data. Adoption of such systems leads to unfair outcomes and the perpetuation of biases.

Examples are plentiful: race being associated with predictions of recidivism [5]; gender affecting displayed job-related ads [18]; race affecting displayed search ads [57]; Boston’s Street Bump app focusing pothole repair on affluent neighborhoods [53]; Amazon’s same day delivery being unavailable in black neighborhoods [34]; and Facebook showing either “white” or “black” movie trailers based upon “ethnic affiliation” [56].

Various instances of discrimination are prohibited by law. In the United States, for example, Title VII of U.S. Civil Rights act prohibits making employment decisions on the basis of race, sex, and other protected attributes [1]. Further legislation makes similar restrictions on the use of protected attributes for credit [24] and housing decisions [37]. Other law establish similar protections in other jurisdictions [3].

In the United States, legal arguments around discrimination follow one of two frameworks: disparate treatment or disparate impact [6]. Disparate treatment is the intentional and direct use of a protected class for a prohibited purpose. An example of this type of discrimination was argued in McDonnell Douglas Corp. v. Green [48], in which the U.S. Supreme Court found that an employer fired an employee on the basis of their race. An element of disparate treatment arguments is an establishment of the protected attribute as a cause of the biased decision [17].

Discrimination does not have to involve a direct use of a protected class; class memberships may not even take part in the decision. Discrimination can also occur due to correlations between the protected class and other attributes. The legal framework of disparate impact [49] addresses such cases by first requiring significantly different outcomes for the protected class, regardless of how the outcomes came to be. An association between loan decisions and race due to the use of applicant address, which itself is associated with race, is an example [33] of this type of discrimination, as are most of the examples cited earlier in this introduction. The association requirement is not causal and thus further arguments must be made to establish that the cause of the observed disparate impact can be attributed to use of an attribute correlated with the protected class, which cannot be excused due to business necessities.

Discrimination arising due to use of features correlated to protected classes is referred to as discrimination by proxy in U.S. legal literature or indirect discrimination in other jurisdictions such as the U.K. [3]. In this paper we will use the term “proxy” to refer to a feature correlated with a protected class whose use in a decision procedure can result in indirect discrimination*. This terminology*

*This work is a companion paper to an earlier paper[19] where the same techniques presented here were applied to formalizing and enforcing privacy restrictions.
is consistent with its use in recent works in the field of fairness in machine learning [4, 38, 58].

In the context of machine learnt systems, non-human decision makers, direct discrimination as in disparate treatment is not difficult to recognize: a feature indicating the protected class is present and used. Usage can be determined by inspecting the system code or determined experimentally [20]. Indirect discrimination like disparate impact can also be computed experimentally. Such a check, however, does not establish cause which 1) is an element of legal arguments, 2) underlies business necessity claims which excuse certain types of disparate outcomes, and 3) is suggestive of remedies for the repair of bias.

In this work we formalize a notion of proxy discrimination that captures use of proxies of protected information types (e.g., race, gender) in data-driven systems. Further we design, implement, and apply algorithms for detecting these types of indirect discrimination and for removing them from machine learnt models.

**Proxy use** A key technical contribution in this paper is a formalization of proxy use of features in programs, the formal models for machine learnt systems. The formalization relates proxy use to intermediate computations obtained by decomposing a program. We begin with a qualitative definition that identifies two essential properties of the intermediate computation (the proxy): 1) its result perfectly predicts the protected information type in question, and 2) it has a causal affect on the final output of the program.

We arrive at this program-based definition after a careful examination of the space of possible definitions. In particular, we prove that it is impossible for a purely semantic notion of intermediate computations to support a meaningful notion of proxy use as characterized by a set of natural properties or axioms (Theorem 1). The program-based definition arises naturally from this exploration by replacing semantic decomposition with decompositions of the program. An important benefit of this choice of restricting the search for intermediate computations to those that appear in the text of the program is that it supports natural algorithms for detection and repair of proxy use. Our framework is parametric in the choice of a programming language in which the programs (e.g., machine learnt models) are expressed and the population to which it is applied. The choice of the language reflects the level of white-box access that the analyst has into the program.

Every instance of proxy use does not constitute a case for discrimination by proxy, and the distinction is a normative judgement. For example, in the U.S., voluntary attributes such as hair style are not considered proxies even when they are highly correlated with race [54]. Further, business necessities may excuse the use of even involuntary (race, gender, etc.) proxies. As a result, our theory relies on a normative judgement oracle to decide whether a particular proxy use is acceptable. Section 2 discusses the role of normative judgement in our theory.

**Closely related work** A wide body of work addresses the problem of finding discrimination in machine learning systems and avoiding violations with adjustments to training data, training algorithms, or trained models (see Section 6 for a brief overview). Threats to fairness from proxy use are recognized in the literature [1, 4, 22, 38]. Our treatment differs significantly from the prior work.

Tramèr et al. developed a system for discovering associations, or proxies in observational data [58]. Their work emphasizes need for differing association metrics and provides a means of discovering unwarranted associations in sub-populations of individuals. Both right metrics and right sub-populations are necessary for discovering and tracking down subtle biases. These elements are complementary to our goals and methods.

Adler et al. [4] describe a method for estimating the indirect influence of a protected class on a model’s outcome by computing that model’s accuracy on a dataset in which proxies of the protected class have been obscured. They argue that the difference between this accuracy and accuracy on the un-obscured data is a measure of a proxy’s influence in a model and can determine whether it is a cause of disparate outcomes arising from indirect use of protected classes. Their technique is designed not to rely on white-box access to the models but in order to obscure proxies it assumes that the relationship between potential proxies and the other attributes can be learned by a chosen set of algorithms. Our setting and assumptions differ in that we make no assumptions about the proxy-attributes relationship (and our notion of association is information theoretic) though we require white-box access. We also provide repair algorithms that can strip proxy use from previously learnt models.

Kalibertus et al. [38] follow Pearl’s work on the discrimination [52] by describing indirect/proxy discrimination in terms of causal graphs. They also discuss algorithms for avoiding such discrimination in some circumstances. Our work does not require access to a causal graph that specifies causal relationships between features.

**Contributions** We make the following contributions:

- An conception of proxy discrimination in data-driven systems that restricts the use of protected classes and some of their proxies (i.e., strong predictors) in automated decision-making systems (Section 2).
- A formal definition of proxy use—the key building block for proxy discrimination—and an axiomatic basis for this definition (Section 3).
- An evaluation of the techniques to several use-cases based on real-world datasets.

This paper is a companion to our earlier work [19] where we motivated and applied proxy use for formalizing and enforcing privacy restrictions. We replicate here the motivating and definitional aspects of the earlier work from a discrimination perspective. We summarize the algorithms for detection and repair of proxy use but leave off implementation details and proofs justifying their suitability. The earlier work includes an evaluation over privacy-related use-cases while in this paper we conclude with discrimination-based use cases. In the earlier paper [19] we describe:

- Algorithmic details of the detection procedure and proof that it is sound and complete relative to our proxy use definition.
- Algorithmic details of a repair algorithm and proof that it removes violations of the proxy use instantiation in a machine learning model that are identified by our detection algorithm and deemed inappropriate by a normative judgment oracle.
We model the data processing system as a program $p$. We formalize the notions of proxy and use, preliminaries to the definition. The definition itself is presented in §3.3 and §3.4. Finally, in §3.5, we provide an axiomatic characterization of the notion of proxy use that guides our definitional choices. We note that readers may skip §3.5 without loss of continuity.

2 PROXY DISCRIMINATION

We model the data processing system as a program $p$. The proxy discrimination constraint governs a protected class $Z$. Our definition of proxy discrimination makes use of two building blocks: (1) a function that given $p$, $Z$, and a population distribution $P$ returns a witness $w$ of proxy use of $Z$ in a program $p$ (if it exists); and (2) a normative judgment oracle $O(w)$ that given a specific witness returns a judgment on whether the specific proxy use is appropriate (true) or not (false).

Not all instances of proxy use of a protected class are inappropriate. For example, business necessity allows otherwise prohibited uses in some cases. Our theory of proxy discrimination makes use of a normative judgment oracle that makes this inappropriateness determination for a given instance.

**Definition 1 (Proxy Discrimination).** Given a program $p$, protected class $Z$, normative judgment oracle $O$, and population distribution $P$, a program $p$ exhibits proxy discrimination if there exists a witness $w$ in $p$ of proxy use of $Z$ in $P$ such that $O(w)$ returns false.

In this paper, we formalize the computational component of the above definition, by formalizing what it means for a model to use a protected class directly or through proxies ($§3$), and designing algorithms to detect proxy uses in programs and remove inappropriate uses ($§4$). We assume that the normative judgment oracle is given and use it to identify inappropriate proxy uses. In our experiments, we illustrate our analysis and repair algorithms to identify proxies and repair ones deemed inappropriate by the oracle ($§5$).

The normative oracle separates computational considerations that are mechanically enforceable and ethical judgments that require input from human experts. This form of separation exists also in some prior work on fairness [21] and privacy [28].

3 PROXY USE: A FORMAL DEFINITION

We now present an axiomatically justified, formal definition of proxy use in data-driven programs. Our definition for proxy use of a protected class involves decomposing a program to find an intermediate computation whose result exhibits two properties:

- **Proxy:** strong association with the protected class
- **Use:** causal influence on the output of the program

In §3.1, we present a sequence of examples to illustrate the challenge in identifying proxy use in systems that operate on data associated with a protected class. In doing so, we will also contrast our work with closely-related work in privacy and fairness. In §3.2, we formalize the notions of proxy and use, preliminaries to the definition. The definition itself is presented in §3.3 and §3.4. Finally, in §3.5, we provide an axiomatic characterization of the notion of proxy use that guides our definitional choices. We note that readers keen to get to the discussion of the detection and repair mechanisms may skip §3.5 without loss of continuity.

3.1 Examples of Proxy Use

Prior work on detecting use of protected information types [16, 25, 40, 58] and leveraging knowledge of detection to eliminate inappropriate uses [25] have treated the system as a black-box. Detection relied either on experimental access to the black-box [16, 40] or observational data about its behavior [25, 58]. Using a series of examples demonstrating redlining[], we motivate the need to peek inside the black-box to detect proxy use.

**Example 3.1.** (Explicit use, Fig. 1a) A bank explicitly uses race in order to evaluate loan eligibility.

This form of explicit use of a protected information type can be discovered by existing black-box experimentation methods that establish causal effects between inputs and outputs (e.g., see [16, 20, 40]).

**Example 3.2.** (Inferred use, Fig. 1b) Consider a situation where applicants’ zip-code is indicative of their race. The bank can thus use zip-code in place of race to evaluate loan eligibility as in Figure 1b.

This example, while very similar in effect, does not use race directly. Instead, it infers race via associations and then uses it. Existing methods (see [25, 58]) can detect such associations between protected classes and outcomes in observational data.

**Example 3.3.** (No use, Fig. 1c) The bank uses some uncorrelated selection of zip-codes to determine eligibility. In Figure 1c, the zip-codes $w_1, b_1$ could designate suburban areas that as a category are not associated with race.

In this example, even though the bank could have inferred race from the data available, no such inference was used in loan evaluation. As associations are commonplace, a definition of use disallowing such benign use of associated data would be too restrictive for practical enforcement.

**Example 3.4.** (Masked proxy use, Fig. 1d) Consider a more insidious version of Example 3.2. To mask the association between the outcome and race, the bank offers loans to not just the white population, but also those with low expressed interest in loans, the people who would be less likely to accept a loan were they offered one. Figure 1d is an example of such an algorithm.

While there is no association between race and outcome in both Example 3.3 and Example 3.4, there is a key difference between them. In Example 3.4, there is an intermediate computation based on zip-codes that is a predictor for race, and this predictor is used to make the decision, and therefore is a case of proxy use. In contrast, in Example 3.3, the intermediate computation based on zip-code is uncorrelated with race. Distinguishing between these examples by measuring associations using black box techniques is non-trivial. Instead, we leverage white-box access to the code of the classifier to identify the intermediate computation that serves as a proxy for race. Precisely identifying the particular proxy used also aids the normative decision of whether the proxy use is appropriate in this setting.

3.2 Notation and Preliminaries

We assume individuals are drawn from a population distribution $P$, in which our definitions are parametric. Random variables $W, X,$
A function

\[ (X, A) \in \mathcal{P} \]

A model, which is a function \( A \) used for prediction, operating on random variables \( X \), in population \( \mathcal{P} \)

\( X \) A random variable

\( p \) A program

\[ [p_1/X]p_2 \] A substitution of \( p_1 \) in place of \( X \) in \( p_2 \)

\( X \) A sequence of random variables

Table 1: Summary of notation used in the paper

Y, \( \ldots \) are functions over \( \mathcal{P} \), and the notation \( W \in \mathcal{W} \) represents that the type of random variable is \( W : \mathcal{P} \to \mathcal{W} \). An important random variable used throughout the paper is \( X \), which represents the vector of features of an individual that is provided to a predictive model. A predictive model is denoted by \( (X, A) \in \mathcal{P} \), where \( A \) is a function that operates on \( X \). For simplicity, we assume that \( \mathcal{P} \) is discrete, and that models are deterministic. Table 1 summarizes all the notation used in this paper, in addition to the notation for programs that is introduced later in the paper. Though we formalize proxies in terms of distributions and random variables, in practice we will operate on datasets. Datasets are samples of a population and approximate that population’s distribution. This point is further discussed in Section 7.2.

3.2.1 Proxies. A perfect proxy for a random variable \( Z \) is a random variable \( X \) that is perfectly correlated with \( Z \). Informally, if \( X \) is a proxy of \( Z \), then \( X \) or \( Z \) can be interchangeably used in any computation over the same distribution. One way to state this is to require that \( \Pr(X = Z) = 1 \), i.e. \( X \) and \( Z \) are equal on the distribution. However, we require our definition of proxy to be invariant under renaming. For example, if \( X = 0 \) whenever \( Z = 1 \) and vice versa, we should still identify \( X \) to be a proxy for \( Z \). In order to achieve invariance under renaming, our definition only requires the existence of mappings between \( X \) and \( Z \), instead of equality.

Definition 2 (Perfect Proxy). A random variable \( X \in \mathcal{X} \) is a perfect proxy for \( Z \in \mathcal{Z} \) if there exist functions \( f : \mathcal{X} \to \mathcal{Z}, g : \mathcal{Z} \to \mathcal{X} \), such that \( \Pr(f(Z) = X) = \Pr(g(Z) = X) = 1 \).

While this notion of a proxy is too strong in practice, it is useful as a starting point to explain the key ideas in our definition of proxy use. This definition captures two key properties of proxies, equivalence and invariance under renaming.

Equivalence. Definition 2 captures the property that proxies admit predictors in both directions: it is possible to construct a predictor of \( X \) from \( Z \), and vice versa. This condition is required to ensure that our definition of proxy only identifies the part of the input that corresponds to the protected attribute and not the input attribute as a whole. For example, if only the final digit of a zip code is a proxy for race, the entirety of the zip code will not be identified as a proxy even though it admits a predictor in one direction. Only if the final digit is used, that use will be identified as proxy use.

The equivalence criterion distinguishes benign use of associated information from proxy use as illustrated in the next example. For machine learning in particular, this is an important pragmatic requirement: given enough input features one can expect any protected class to be predictable from the set of inputs. In such cases, the input features taken together are a strong associate in one direction, and prohibiting such one-sided associates from being used would rule out most machine learnt models.

Example 3.5. Recall that in Figure 1, zip-codes \( w_1, w_2 \) indicate white populations and \( b_1, b_2 \) indicate black populations. Consider Example 3.3 (No use), where zip-code is an influential input to the program that determines loan offers, using the criterion \( \text{zip-code} \in \{ w_1, b_1 \} \). According to Definition 2, neither race nor this criterion are proxies, because race does not predict zip-code (or specifically the value of the predicate \( \text{zip-code} = \{ w_1, b_1 \} \)). However, if Definition 2 were to allow one-sided associations, then \( \text{zip-code} \) would be a proxy because it can predict race. This would have the unfortunate effect of implying that the benign application in Example 3.3 has proxy use of race.

Invariance under renaming. This definition of a proxy is invariant under renaming of the values of a proxy. Suppose that a random variable evaluates to 1 when the protected information type is 0 and vice versa, then this definition still identifies the random variable as a proxy.

3.2.2 Influence. Our definition of influence aims to capture the presence of a causal dependence between a variable and the output of a function. Intuitively, a variable \( x \) is influential on \( f \) if it is possible to change the value of \( f \) by changing \( x \) while keeping the other input variables fixed.

Definition 3. For a function \( f(x, y) \), \( x \) is influential if and only if there exists values \( x_1, x_2, y \), such that \( f(x_1, y) \neq f(x_2, y) \).

In Figure 1a, race is an influential input of the system, as just changing race while keeping all other inputs fixed changes the...
3.3 Definition

We use an abstract framework of program syntax to reason about programs without specifying a particular language to ensure that our definition remains general. Our definition relies on syntax to reason about decompositions of programs into intermediate computations, which can then be identified as instances of proxy use using the concepts described above.

Program decomposition We assume that models are represented by programs. For a set of random variables \( X, (X, p_1, p_2) \) denotes the assumption that \( p \) will run on the variables in \( X \). Programs are given meaning by a denotation function \( \llbracket p \rrbracket_X \) that maps programs to functions. If \( (X, p_1, p_2) \), then \( \llbracket p \rrbracket \) is a function on variables in \( X \), and \( \llbracket p \rrbracket(X) \) represents the random variable of the outcome of \( p \), when evaluated on the input random variables \( X \). Programs support substitution of free variables with other programs, denoted by \( [p_1/X][p_2] \), such that if \( p_1 \) and \( p_2 \) programs that run on the variables \( X \) and \( X \), respectively, then \([p_1/X][p_2] \) is a program that operates on \( X \).

A decomposition of program \( p \) is a way of rewriting \( p \) as two programs \( p_1 \) and \( p_2 \) that can be combined via substitution to yield the original program.

Definition 4 (Decomposition). Given a program \( p \), a decomposition \( (p_1, X, p_2) \) consists of two programs \( p_1, p_2 \), and a fresh variable \( X \), such that \( p \) \( = [p_1/X][p_2] \).

For the purposes of our proxy use definition we view the first component \( p_1 \) as the intermediate computation suspected of proxy use, and \( p_2 \) as the rest of the computation that takes in \( p_1 \) as an input.

Definition 5 (Influential Decomposition). Given a program \( p \), a decomposition \( (p_1, X, p_2) \) is influential iff \( X \) is influential in \( p_2 \).

Main definition

Definition 6 (Proxy Use). A program \( (X, p) \) \( \tau \) has proxy use of \( Z \) if there exists an influential decomposition \( (p_1, X, p_2) \) of \( (X, p) \), and \( \llbracket p_1 \rrbracket(X) \) is a proxy for \( Z \).

Example 3.6. In Figure 1d, this definition would identify proxy use using the decomposition \( (p_1, U, p_2) \), where \( p_2 \) is the entire tree, but with the condition \( (a_1, a_2 \in \text{zip-code}) \) replaced by the variable \( U \). In this example, \( U \) is influential in \( p_2 \), since changing the value of \( U \) changes the outcome. Also, we assumed that the condition \( (b_1, b_2 \in \text{zip-code}) \) is a perfect predictor for race, and is therefore a proxy for race. Therefore, according to our definition of proxy use, the model in 1d has proxy use of race.

3.4 A Quantitative Relaxation

Definition 6 is too strong in one sense and too weak in another. It requires that intermediate computations be perfectly correlated with a protected class, and that there exists some input, however improbable, in which the result of the intermediate computation is relevant to the model. For practical purposes, we would like to capture imperfect proxies that are strongly associated with an attribute, but only those whose influence on the final model is appreciable. To relax the requirement of perfect proxies and non-zero influence, we quantify these two notions to provide a parameterized definition. Recognizing that neither perfect non-discrimination nor perfect utility are practical, the quantitative definition provides a means for navigating non-discrimination vs. utility tradeoffs.

\( \epsilon \)-proxies We wish to measure how strongly a random variable \( X \) is a proxy for a random variable \( Z \). Recall the two key requirements from the earlier definition of a proxy: (i) the association needs to be capture equivalence and measure association in both directions, and (ii) the association needs to be invariant under renaming of the random variables. The variation of information metric \( d_{\text{var}}(X, Z) = H(X|Z) + H(Z|X) \) [15] is one measure that satisfies these two requirements. The first component in the metric, the conditional entropy of \( X \) given \( Z \), measures how well \( X \) can be predicted from \( Z \), and \( H(Z|X) \) measures how well \( Z \) can be predicted from \( X \), thus satisfying the requirement for the metric measuring association in both directions. Additionally, one can show that conditional entropies are invariant under renaming, thus satisfying our second criteria. To obtain a normalized measure in \([0, 1]\), we choose \( 1 - \frac{d_{\text{var}}(X, Z)}{H(X, Z)} \) as our measure of association, where the measure being 1 implies perfect proxies, and 0 implies statistical independence. Interestingly, this measure is identical to normalized mutual information [15], a standard measure that has also been used in prior work in identifying associations in outcomes of machine learning models [58].

Definition 7 (Proxy Association). Given two random variables \( X \) and \( Z \), the strength of a proxy is given by normalized mutual information.

\[
d(X, Z) \overset{\text{def}}{=} 1 - \frac{H(X|Z) + H(Z|X)}{H(X, Z)}
\]

where \( X \) is defined to be an \( \epsilon \)-proxy for \( Z \) if \( d(X, Z) \geq \epsilon \).

\( \delta \)-influential decomposition Recall that for a decomposition \( (p_1, X, p_2) \), in the qualitative sense, influence is interference which implies that there exists \( x, x_1, x_2, \) such that \( p_2(X, x_1) \neq p_2(X, x_2) \). Here \( x_1, x_2 \) are values of \( p_1 \), that for a given \( x \), change the outcome of \( p_2 \). However, this definition is too strong as it requires only a single pair of values \( x_1, x_2 \) to show that the outcome can be changed by \( p_1 \) alone. To measure influence, we quantify interference by using Quantitative Input Influence (QII), a causal measure of input influence introduced in [20]. In our context, for a decomposition \( (p_1, X, p_2) \), the influence of \( p_1 \) on \( p_2 \) is given by:

\[
i(p_1, p_2) \overset{\text{def}}{=} \mathbb{E}_{X, X', p_1, p_2} \Pr \left( \left[ p_2 \right](X, [p_1](X)) \neq \left[ p_2 \right](X, [p_1]'(X')) \right).
\]

Intuitively, this quantity measures the likelihood of finding randomly chosen values of the output of \( p_1 \) that would change the outcome of \( p_2 \). Note that this general definition allows for probabilistic models though in this work we only evaluate our methods on deterministic models.

Definition 8 (Decomposition Influence). Given a decomposition \( (p_1, X, p_2) \), the influence of the decomposition is given by the QII of \( X \) on \( p_2 \). A decomposition \( (p_1, X, p_2) \) is defined to be \( \delta \)-influential if \( i(p_1, p_2) > \delta \).
Now that we have quantitative versions of the primitives used in Definition 6, we are in a position to define quantita
tive proxy use (Definition 9). The structure of this definition is the same as before, with quantitative measures substituted in for the qualitative assertions used in Definition 6.

Definition 9 ((ε, δ)-proxy use). A program (X, p)p has (ε, δ)-proxy use of random variable Z iff there exists a δ- influential decomposition (p₁, X, p₂), such that ¹p²p(X) is an ε-proxy for Z.

This definition is a strict relaxation of Definition 6, which reduces to (1,0)-proxy use.

3.5 Axiomatic Basis for Definition

We now motivate our definitional choices by reasoning about a natural set of properties that a notion of proxy use should satisfy. We first prove an important impossibility result that shows that no definition of proxy use can satisfy four natural semantic properties of proxy use. The central reason behind the impossibility result is that under a purely semantic notion of function composition, the causal effect of a proxy can be made to disappear. Therefore, we choose a syntactic notion of function composition for the definition of proxy use presented above. The syntactic definition of proxy use is characterized by syntactic properties which map very closely to the semantic properties.

Property 1. (Explicit Use) If Z is an influential input of the model (⟨X, Z⟩, A)p, then ⟨⟨X, Z⟩, A⟩p has proxy use of Z.

This property identifies the simplest case of proxy use: if an input to the model is influential, then the model exhibits proxy use of that input.

Property 2. (Preprocessing) If a model (⟨X, A⟩, A)p has proxy use of random variable Z, then for any function f such that Pr(f(X) = Z) = 1, let A( x )= A(x, f(x)). Then, (⟨X, A⟩, A)p has proxy use of Z.

This property covers the essence of proxy use where instead of being provided a protected information type explicitly, the program uses a strong predictor for it instead. This property states that models that use inputs explicitly and via proxies should not be differentiated under a reasonable theory of proxy use.

Property 3. (Dummy) Given (⟨X, A⟩, A)p, define A’ such that for all x , x’, A’ (x, x’) = A(x), then (⟨X, A⟩, A)p has proxy use for some Z iff ⟨⟨X, X⟩, A’⟩p has proxy use of Z.

This property states that the addition of an input to a model that is not influential, i.e., has no effect on the outcomes of the model, has no bearing on whether a program has proxy use or not. This property is an important sanity check that ensures that models aren’t implicated by the inclusion of inputs that they do not use.

Property 4. (Independence) If X is independent of Z in P, then (⟨X, A⟩, A)p does not have proxy use of Z.

Independence between the protected information type and the inputs ensures that the model cannot infer the protected information type for the population P. This property captures the intuition that if the model cannot infer the protected information type then it cannot possibly use it.

While all of these properties seem intuitively desirable, it turns out that these properties can not be achieved simultaneously.

Theorem 1. No definition of proxy use can satisfy Properties 1-4 simultaneously.

See our companion paper [19, Appendix A] for a proof of the impossibility result and a discussion. The key intuition behind the result is that Property 2 requires proxy use to be preserved when an input is replaced with a function that predicts that input via composition. However, with a purely semantic notion of function composition, after replacement, the proxy may get canceled out. To overcome this impossibility result, we choose a more syntactic notion of function composition, which is tied to how the function is represented as a program, and looks for evidence of proxy use within the representation.

We now proceed to the axiomatic justification of our definition of proxy use. As in our attempt to formalize a semantic definition, we base our definition on a set of natural properties given below. These are syntactic versions of their semantic counterparts defined earlier.

Property 5. (Syntactic Explicit Use) If X is a proxy of Z, and X is an influential input of (⟨X, X⟩, p)p, then (⟨X, X⟩, X)p has proxy use.

Property 6. (Syntactic Preprocessing) If (⟨X, X⟩, p₁)p has proxy use of Z, then for any p₂ such that Pr(⟨p₂⟩(X) = Z) = 1, (⟨X, p₂/X⟩p₁)p has proxy use of Z.

Property 7. (Syntactic Dummy) Given a program (⟨X, p⟩, p)p, (⟨X, p⟩, p)p has proxy use for some Z iff (⟨X, X⟩, p)p has proxy use of Z.

Property 8. (Syntactic Independence) If X is independent of Z, then (⟨X, p⟩, p)p does not have proxy use of Z.

Properties 5 and 6 together characterize a complete inductive definition, where the induction is over the structure of the program. Suppose we can decompose programs p into (p₁, X, p₂) such that p = [p₁/X][p₂]. Now if X, which is the output of p₁, is a proxy for Z and is influential in p₂, then by Property 5, p₂ has proxy use. Further, since p = [p₁/X][p₂], by Property 6, p has proxy use. This inductive definition where we use Property 5 as the base case and Property 6 for the induction step, precisely characterizes Definition 6. Additionally, it can be shown that Definition 6 also satisfies Properties 7 and 8. Essentially, by relaxing our notion of function composition to a syntactic one, we obtain a practical definition of proxy use characterized by the natural axioms above.

4 DETECTION AND REPAIR OF PROXY USE

In this section, we summarize algorithms for 1) identifying proxy use of specified variables in a given machine-learning model and 2) repairing those models so that the proxy use is removed. Details of these algorithms are presented in the companion paper [19]. There the reader can also find proofs of the theorems noted in this section as well as various optimizations that are part of our implementations.

4.1 Environment Model

The environment in which our detection algorithm operates is comprised of a data processor, a dataset that has been partitioned into analysis and validation subsets, and a machine learning model.
4.2 Models as expression programs

Our techniques are not tied to any particular language, and the key ideas behind them apply generally. For our implementation work we focused on a simple expression (functional) language that is rich enough to support commonly-used models such as decision trees, linear and logistic regression, Naive Bayes, and Bayesian rule lists. Programs denote functions that evaluate arithmetic expressions, which are constructed from real numbers, variables, common arithmetic operations, and if-then-else constructs.

Boolean expressions, which are used as conditions in if-then-else expressions, are constructed from the usual connectives and relational operations. Finally, we use λ-notation for functions, i.e., λx.e denotes a function over x which evaluates e after replacing all instances of x with its argument. Details of this language and how machine learning models such as linear models, decision trees, and random forests are translated to this expression language are discussed in the companion paper [19, B.2]. The consequences of the choice of language and decomposition in that language are further discussed in Section 7.

Distributed proxies Our use of program decomposition provides for partial handling of distributed representations, the idea that concepts can be distributed among multiple entities. In our case, influence and association of a protected class can be distributed among multiple program points. First, substitution (denoted by [p1/X]p2) can be defined to replace all instances of variable X in p2 with the program p1. If there are multiple instances of X in p2, they are still describing a single decomposition and thus the multiple instances of p2 in p1 are viewed as a single proxy. Further, implementations of substitution can be (and is in our implementation) associativity-aware: programs like x1 + x2 + x3 can be equivalent regardless of the order of the expressions in that they can be decomposed in exactly the same set of ways. If a proxy is distributed among x1 and x3, it will still be considered by our methods because x1 + (x2 + x3) is equivalent to (x1 + x3) + x2, and the sub-expression x1 + x3 is part of a valid decomposition.

4.3 Analyzing Proxy Use

Algorithm 1 describes a general technique for detecting (ε, δ)-proxy use in expression programs. In addition to the parameters and model expression, it operates on a description of the distribution governing the feature variables X and Z. In practice this will nearly always consist of an empirical sample, but for the sake of presentation we simplify here by assuming the distribution is explicitly given. In

```
Algorithm 1 Detection for expression programs.

Require: association (d), influence(ε) measures

procedure ProxyDetect(p, X, Z, ε, δ)
    P ← ∅
    for each subprogram p1 appearing in p do
        for each program p2 such that [p2/u]P1 = p do
            if δ(p1, p2) ≥ δ ∧ d([p1][X], Z) ≥ ε then
                P ← P ∪ {(p1, p2)}
        return P
```

In Section 4.3.1, we describe how the algorithm can produce estimates from empirical samples.

The algorithm proceeds by enumerating sub-expressions of the given program. For each sub-expression ε appearing in p, ProxyDetect computes the set of positions at which ε appears. If ε occurs multiple times, we consider all possible subsets of occurrences as potential decompositions. It then iterates over all combinations of these positions, and creates a decomposition for each one to test for (ε, δ)-proxy use. Whenever the provided thresholds are exceeded, the decomposition is added to the return set. This proceeds until there are no more subterms to consider. While not efficient in the worst-case, this approach is both sound and complete with respect to Definition 9. It is important to mention, however, that our definitions are not meant to capture all types of indirect discrimination; completeness and soundness here is therefore only in relation to an incomplete definition.

Theorem 2 (Detection soundness). Any decomposition (p1, p2) returned by ProxyDetect(p, X, ε, δ) is a decomposition of the input program p and had to pass the ε, δ thresholds, hence is a (ε, δ)-proxy use.

Theorem 3 (Detection completeness). Every decomposition which could be a (ε, δ)-proxy use is enumerated by the algorithm. Thus, if (p1, p2) is a decomposition of p with δ(p1, p2) ≥ δ and d([p1][X], Z) ≥ ε, it will be returned by ProxyDetect(p, X, ε, δ).

Our detection algorithm considers single expressions in its decomposition. Sometimes a large number of syntactically different proxies with weak influence might collectively have high influence. A stronger notion of program decomposition that allows a collection of multiple different expressions to be considered a proxy would identify such a case of proxy use but will have to search over a larger space of expressions. Exploring this tradeoff between scalability and richer proxies is an important topic for future work.

4.3.1 Estimating influence and association. It is rarely the case that one has access to the precise distribution from which data is drawn. Instead, a finite sample must be used as a surrogate when reasoning about random variables. We describe how the two primary quantities used in ProxyDetect, influence and association, are estimated from such a sample. The use of a sample in place of a distribution is discussed in Section 7.2.

Quantitative decomposition influence Given a decomposition (p1, u, p2) of p, Algorithm ProxyDetect first calculates the influence of p1 on p2’s output to ensure that the potential proxy

---

This occurs often in decision forests.
Algorithm 2 Witness-driven repair.

Require: association (d), influence (i), utility (u) measures, oracle (O)

procedure REPAIR(p, X, Z, ε, δ)
    \( P \leftarrow \{ d \in \text{PROXYDETECT}(p, X, Z, ε, δ) : \text{not } O(d)\} \)
    if \( P \neq \emptyset \) then
        \((p_1, p_2) \leftarrow \text{element of } P\)
        \( p' \leftarrow \text{PROXYREPAIR}(p_1, p_2, X, Z, ε, δ) \)
    else return REPAIR(p', X, Z, ε, δ)
\return \( p \)

quantity is relevant to the model’s output. Recall that this influence is defined as:

\[
i(p_1, p_2) \equiv \frac{1}{|D|} \sum_{x \in D} \sum_{X \in D} \mathbb{I}(p_2(X, [p_1]X) \neq [p_2](X, [p_1]X'))
\]

Assuming deterministic models and given a dataset \( D \) drawn from \( P \), we estimate this expectation by aggregating over the rows:

\[
i(p_1, p_2) \equiv \mathbb{E}_{X, X'} \mathbb{P}(\{p_2\}(X, [p_1]X) \neq [p_2](X, [p_1]X'))
\]

The quadratic cost of this computation makes it infeasible when \( D \) is large, so in practice we take a sample from \( D \times D \). By Hoeffding’s inequality [32], we select the subsample size \( n \) to be at least \( \log(2/\beta)/\alpha^2 \) to ensure that the probability of the error \( i(p_1, p_2) - i(p_1, p_2) \) being greater than \( \beta \) is bounded by \( \alpha \). An additional optimization follows if we introduce a notion of reachability for subexpressions. An input \( X \) reaches a sub-expression \( p_1 \) inside \( p \) if the evaluation of \( p \) on \( X \) requires evaluating \( p_1 \). Using reachability, we improve the computation of influence by realizing that if an input \( X \) does not reach \( p_1 \), there is no value we can replace \( p_1 \) with that will change the outcome of \( p \) evaluated on \( X \). When estimating influence, we take advantage of the optimization by conditioning our sampling on the reachability of the decomposed subexpression \( p_1 \).

Association As discussed in Section 3, we use mutual information to measure the association between the output of a subprogram and \( Z \). This quantity can be estimated from a sample in time \( O(|D| + k|Z|) \), where \( k \) is the number of elements in the range of \( p_1 [46] \). For each \((x, z)\) in the dataset, the procedure computes \( p_1 \) on \( x \) and builds a contingency table indexed by \(([p_1][x], z)\). The contingency table is used to compute the required conditional entropies.
A particular concern while estimating associations is associations appearing by random chance on a particular sample. In the companion paper [19, Appendix B.3] we discuss how to mitigate the reporting of such spurious associations.

4.4 Removing Proxy Use Violations

Our approach for removing proxy use violations has two parts: first (REPAIR, Algorithm 2) is the iterative discovery of proxy uses via the PROXYDETECT procedure described in the previous section and second (PROXYREPAIR, Algorithm 3) is the repair of the ones found by the oracle to be violations. Our repair procedures operate on the expression language, so they can be applied to any model that can be written in the language. Further, our violation repair algorithm does not require knowledge of the training algorithm that produced the model. The witnesses of proxy use localize where in the program violations occur. To repair a violation we search through expressions local to the violation, replacing the one which has the least impact on the accuracy of the model that at the same time reduces the association or influence of the violation to below the \((\epsilon, \delta)\) threshold.

At the core of our violation repair algorithm is the simplification of sub-expressions in a model that are found to be violations. Simplification here means the replacement of an expression that is not a constant with one that is. Simplification has an impact on the model’s performance hence we take into account the goal of preserving utility of the machine learning program we repair. We parameterize the procedure with a measure of utility \( v \) that informs the selection of expressions and constants for simplification. We briefly discuss options and implementations for this parameter later in this section.

The repair procedure (PROXYREPAIR) works as follows. Given a program \( p \) and a decomposition \((p_1, p_2)\), it first finds the best simplification to apply to \( p_2 \) that would make \((p_1, p_2)\) no longer a violation. This is done by enumerating expressions that are local to \( p_1 \) in \( p_2 \). Local expressions are sub-expressions of \( p_2 \) as well as \( p_1 \) itself and if \( p_1 \) is a guard in an if-then-else expression, then local expressions of \( p_2 \) also include that if-then-else’s true and false branches and their sub-expressions. Each of the local expressions corresponds to a decomposition of \( p \) into the local expression \( p_1' \) and the context around it \( p_2' \). For each of these local decompositions we discover the best constant, in terms of utility, to replace \( p_1' \) with. We then make the same simplification to the original decomposition \((p_1, p_2)\), resulting in \((p_1', p_2')\). Using this third decomposition we check whether making the simplification would repair the original violation, collecting those simplified programs that do. Finally, we take the best simplification of those found to remove the violation (Line 8). Details on how the optimal constant is selected is described in the companion paper [19, Appendix C.1].

Two important things to note about the repair procedure. First, there is always at least one subprogram that will fix the violation, namely the decomposition \((p_1, p_2)\) itself. Replacing \( p_1 \) with a constant in this case would disassociate it from the protected class. Secondly, the procedure produces a model that is smaller than the one given to it as it replaces a non-constant expression with a constant. These two let us state the following:

**Theorem 4.** Algorithm LOCALREPAIR terminates and returns a program that does not have any \((\epsilon, \delta)\)-Proxy Use violations (instances of \((\epsilon, \delta)\)-Proxy Use for which oracle returns false).
5 EVALUATION

In this section we empirically evaluate our definition and algorithms on use-cases based real datasets. We demonstrate a detection and repair scenario in Section 5.2 and present two additional use-cases of our theory and algorithms in Section 5.3. We describe our findings of interesting proxy uses and demonstrate how the outputs of our detection tool would allow a normative judgment oracle to determine the appropriateness of proxy uses. We begin by noting some details regarding our implementation and the datasets.

**Models and Implementation** Our implementation currently supports linear models, decision trees, random forests, and rule lists. Note that these model types correspond to a range of commonly-used learning algorithms such as logistic regression, support vector machines [14], CART [9], and Bayesian rule lists [41]. Also, these models represent a significant fraction of models used in practice in predictive systems that operate on personal information, ranging from advertising [13], psychopathy [31], criminal justice [7, 8], and actuarial sciences [27, 29]. In this paper we evaluate our methods on decision trees while discussion of results on other modes can be found in the companion paper [19]. Our prototype implementation was written in Python, and we use scikit-learn package to train the models used in the evaluation. Our implementation is available at https://sites.google.com/site/proxynondiscrimination.

### 5.1 Datasets

**Adult** The UCI Adult dataset is widely used in the privacy and fairness literature as a benchmark for evaluating new techniques. It contains roughly 48,000 instances consisting of demographic information and a classification of the individual as making more or less than $50,000 per year, which we can interpret as a loan decision (predicting income of greater than $50,000 corresponds to an accepted loan, while less is a rejection). To maintain consistency with prior work using this benchmark [25, 36], we treat gender as the protected attribute in our scenario.

**Japanese Credit** The Japanese Credit dataset was collected in 1992 from examples of 125 individuals who placed consumer credit applications. It contains a number of demographic and financial attributes for each individual, and is available in the UCI repository [42] in two formats. The first is contains fifteen features whose names and values have been randomly chosen to protect privacy, as well as a binary classification variable corresponding to an accepted or rejected credit application. The second format is a set of fifteen attributes for each individual, and is available in the UCI repository.

5.2 Detection and Repair Scenario

In this scenario, a bank uses income prediction data such as the UCI Adult dataset to train a model for determining whether to accept loans. The bank uses the predicted income from this model. Then, using our detection procedure, they can check that in

![Figure 2: Example domain theory rule from the Japanese Credit dataset. This rule encodes that unemployed, unmarried females should be denied credit.](image)

Lisp predicates over row indices with descriptive names (purchase-item, jobless, male, female, unmarried, problematic-region, age, deposit, monthly-payment, num-months, num-years-in-company), and an accompanying domain theory provided by an expert. An example rule from the domain theory is shown in Figure 2, which reflects a policy of denying credit to jobless, unmarried females; the remark on the discriminatory nature of this rule is taken verbatim from the original file. The row predicates describe attribute values for each individual, whereas the domain theory is a set of rules written in Lisp that operate over the row predicates to determine a credit decision. We extracted a row-structured dataset from the second format, removed redundant attributes, and treated gender as the protected attribute in our experiments.

**Ricci v. DeStefano** This dataset comes from the U.S. District Court of Connecticut’s decision on the Ricci v. DeStefano case [2]. It contains the oral, written, and combined promotion exam scores, as well as the race (Black, White, or Hispanic) and position (Captain or Lieutenant), of 118 New Haven firefighters. The fire department’s policy at the time stipulated that any applicant with a combined score of 70% or above is eligible for promotion, and that whenever n promotions are available, applicants must be selected from among the top $n + 2$ scorers. However, the department decided not to promote anyone in this case because too few minorities matched this criteria. A subset of the test-takers filed a reverse-discrimination lawsuit, and the Supreme Court eventually ruled in their favor. We examined the latter decision rule for proxy usage, using race as the protected feature. As the case questioned the treatment of all minorities alongside that of non-minorities, we collapsed the Black and Hispanic labels into a single minority label. The decision tree corresponding to the latter policy is shown in Figure 3.

![Figure 3: Decision tree used to determine promotion eligibility in the Ricci v. DeStefano case.](image)
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Figure 4: The association and influence of the expressions composing a decision tree trained on the UCI Adult dataset. Original tree expressions are denoted by • while repaired tree expressions are designated by ×. Dark area in the upper-left designates the thresholds used in repair. Narrow lines designate the sub-expression relationship. Marker size denotes the relative size of the sub-expressions pictured.

In the model they train, there are no proxy uses of gender that have high impact on the decision.

We construct a decision tree model from this dataset as a representative of the kind of model the bank could use, and analyze the potential proxy uses of gender that could be present. Figure 4 visualizes all of the expressions making up the model (marked as •), along with their association and influence measures. In decision trees, sub-expressions like these coincide with decompositions in our proxy use definition; each sub-expression can be associated with a decomposition that cuts out that sub-expression from the tree, and leaves a variable in its place.

The point labeled A in Figure 4 is the predicate relationship ≤ 0.5 and has significant correlation with gender and influence (it is the predicate of the root note of the tree). On further examination, the relationship status in this dataset encodes gender in most cases as husband and wife are two of its possible values. This use would most likely be deemed inappropriate as modeled by the normative oracle and thus we will remove it. The ideal solution to this problem is to remove gender indicators from from the relationship attribute but for the sake of this demonstration, we instead use our repair algorithm.

We determine that any proxies with association and influence exceeding the thresholds indicated by the shaded area in Figure 4 are too strong; any decomposition or sub-expression in that region is unacceptable. This area includes the problematic predicate as well as the root of the model, indicating that the decision procedure’s outcome is itself associated with gender to a significant enough degree.

Applying our repair procedure to this model, with the association and influence thresholds as indicated in the figure, we produce another tree. This model is designated with × in Figure 4. Note that this repaired version has no sub-expressions in the prohibited range while a lot of the tree remains unchanged (the • and × markers largely coincide). Interestingly, the problematic predicate is still in the model, but now at point A’, which has much lower influence than it had in the un-repaired tree. This occurred because the repair procedure did not replace the predicate itself but instead it replaced one of the deeper predicates (education_num ≤ 11.5) in one of the branches of the root node. This replacement also reduced the association of the whole tree with gender.

In general repair comes with a cost of utility, or the accuracy of the repaired model as related to the original. The techniques we presented here and specifically the parameterized \((\epsilon, \delta)\)-proxy use definition must therefore navigate the trade-off between fairness and utility.

5.3 Other Use Cases

5.3.1 Exam score proxy in Ricci v. DeStefeno. We analyzed the models used in the Ricci v. DeStefeno case to understand whether the phenomenon documented in the case is reflected as proxy usage. The association between race and outcomes in this data is apparently paradoxical: an analysis of the test scores reveals a substantial

\[ \text{Note that we pre-processed nominal features into numeric ones for our experiments.} \]

\[ \text{Repair can improve test accuracy as it can serve as a regularizer. For some training algorithms, repair can occasionally have no train accuracy impact.} \]
difference between minority and non-minority applicants, but an analysis of the passing and promotion eligibility rates (as defined by the model) shows no appreciable difference. Recalling Figure 3, we found that the entire tree showed low association scores ($\epsilon = 0.044$). However, both subtrees showed a larger association ($\epsilon = 0.051$ and $\epsilon = 0.060$). That is, both the lieutenant and captain decisions were more correlated with race than the procedure as a whole.

The reason for this is simple. For the lieutenant test, there were zero non-minorities who met the criteria, and only three for the captain test. This means that both subtrees signaled non-minority status when it output a positive classification, using race as a proxy for that particular outcome. Note, however, that the association strength implies that the outcome of this subtree is not a perfect proxy; while a passing lieutenant grade perfectly predicts non-minority status, a failing one does not. Although the absolute association numbers may appear small, the difference in association strength between the final output and the intermediate computations reveals the contentious issue in the case, and the seemingly paradoxical associations. Without looking into the model, we would have no way of identifying this underlying phenomenon.

5.3.2 Purchase proxy in Japanese Credit. As discussed previously, the domain theory for the Japanese Credit dataset refers to gender as a critical factor in some decisions. We trained a decision tree model on the Japanese Credit dataset after removing gender to understand whether the learning algorithm introduces proxy usage of gender to compensate for this missing data. In this case, the entire model had low association with gender, showing ($\epsilon = 0.005$, $\delta = 0.25$)-proxy usage. The predicate used in on the root of this tree, jobless $\leq 0.5$, has a much stronger association ($\epsilon = 0.020$). Joblessness could, however, be excused as a business necessity in evaluating credit worthiness.

6 RELATED WORK

6.1 Discrimination Definitions

The literature on use restrictions has typically focused on explicit use of protected information types, not on proxy use (see Tschantz et al. [59] for a survey and Lipton and Regan [43]). Recent work on discovering personal data use by black-box web services focuses mostly on explicit use of protected information types by examining causal effects [18, 40]; some of this work also examines association effects [39, 40]. Associational effects capture some forms of proxy use but not others as we argued in Section 3.

A number of definitions of information use have been proposed in prior work. We categorize these definitions into two types: (i) associative notions, which measure the association between inputs or outputs of the system, and the attribute under consideration, or (ii) explicit use notions, which identify the causal effect of the attribute under consideration on the outcomes of a system. Our formalization of proxy usage can be viewed as a synthesis of these two notions where a proxy (measured by associations) has a causal influence on the outcome (measured by causal influence).

**Associative Notions** Disparate impact, a legal term introduced in Griggs v. Duke Power Co., 1971 [49], measures the difference in the statistical outcomes of different groups under the protected attribute. This formulation of discrimination has been adopted by a number of technical approaches to preventing discrimination [11, 36, 62]. Disparate impact is a special case of an association measure for outcomes that is generalized by Tramèr et al. [58] who provide a framework for finding conditional associations in sub-populations. In a different approach, Feldman et al. [25] restrict the association between the inputs that are provided to the model with the protected attribute, as such a restriction is guaranteed to restrict an association with the outcome for any model. We argue in §1 that the presence of association with outcomes is not necessary for the presence of proxy usage, while the presence of association with inputs is not sufficient for the presence proxy usage. In this sense, our definition of proxy usage is not subsumed by any of these prior works.

**Explicit Use Notions** An alternate approach to defining information use is by identifying an explicit causal influence of a protected attribute on the outcome [16, 40]. Another explicit use notion is Quantitative Input Influence – a family of causal influence measures [20] that we use in this paper to quantify the influence of a sub-computation. The explicit use approach requires the protected attribute to be an actual input to the model under scrutiny. Thus, it does not account for proxy usage in machine learning applications. Dwork et al. [23] define fairness in terms of Lipschitz continuity. Their definition states that similar people are treated similarly, which ensures that irrelevant inputs have no explicit causal influence on the outcomes. Irrelevant inputs are encoded in the choice of a domain specific distance metric. In principle, appropriate distance metrics could rule out proxy usage. However, they do not provide a method for constructing such distance metrics. They leave it to future work. The follow-up work of Zemel et al. [62] mentioned above ensures that there is no disparate impact but does not eliminate proxy use.

**Causal Indirect Use** Adler et al. [4] describe a method for estimating the indirect influence of a protected class on a model’s outcome by computing that model’s accuracy on a dataset in which proxies of the protected class have been obscured. They argue that the difference between this accuracy and accuracy on the un-obscured data is a measure of the protected class’s influence and can thus determine whether it is a cause of disparate outcomes. Their technique does not rely on white-box access to the models but assumes that proxies-class relationship can be learned by a given set of algorithms. Our setting and assumptions differ in that we make no assumptions about the proxy-class relationship though we require white-box access. We also provide repair algorithms that can strip proxy usage from previously learnt models.

Kilbertus et al. [38] follow Pearl’s work on the discrimination [52] by describing indirect/proxy discrimination in terms of causal graphs. They also discuss algorithms for avoiding such discrimination in some circumstances. Our work does not rely on presence of a causal graph to specify the causal relationships between features.

6.2 Detection and Repair Methods

Our detection algorithm operates with white-box access to the prediction model which is a stronger access assumption.

**Access to observational data** Detection techniques working under an associative use definition [25, 58] usually only require access to observational data about the behavior of the system.
Access to black-box experimental data Detection techniques working under an explicit use definition of information use [18, 40] typically require experimental access to the system. This access allows the analyst to control some inputs to the system and observe relevant outcomes.

The stronger white-box access level allows us to decompose the model and trace an intermediate computation that is a proxy. Such traceability is not afforded by the weaker access assumptions in prior work. Thus, we explore a different point in the space by giving up on the weaker access requirement to gain the ability to trace and repair proxy use.

Tramèr et al. [58] solve an important orthogonal problem of efficiently identifying populations where associations may appear. Since our definition is parametric in the choice of the population, their technique could allow identifying relevant populations for further analysis using our methods.

Repair Techniques for the repair of fairness violations are as varied as fairness definitions. Repair mechanisms that operate solely on the population dataset, removing unfairness inherent in it, include variations that relabel the class attribute [44], modify entire instances while maintaining the original schema [26], and transform the dataset into an alternate space of features [22, 63]. Several approaches function instead on the training algorithm employed, or rather introduce variations that ensure produced models respect fairness constraints. Repair in such approaches means replacing a standard algorithm with a fairness-aware one, and requires access to the training data and the learning pipeline (e.g., [10, 12, 35]). Adjustments to Naive Bayes [12] and trainers amiable to regularization [35] are examples.

7 DISCUSSION

Several design decisions and setting assumptions dictate where and how our methodologies can be applied. We do not address the adversarial setting as our definition of proxy use relies on strict program decomposition which can be subverted by an intentional adversary. Further, our metrics of influence and association are based on a distribution correlating program inputs with protected classes. What this distribution represents and how we to obtain it are both points warranting discussion.

7.1 Beyond strict decomposition

Theorem 1 shows that a definition satisfying natural semantic properties is impossible. This result motivates our syntactic definition, parameterized by a programming language and a choice of program decomposition. In our implementation, the choice of program decomposition is strict. It only considers single expressions in its decomposition. However, proxies may be distributed across different terms in the program. As discussed in Section 4.1, single expressions decompositions can also deal with a restricted class of such distributed proxies. Our implementation does not identify situations where each of a large number of syntactically different proxies have weak influence but together combine to result in high influence. A stronger notion of program decomposition that allows a collection of multiple terms to be considered a proxy would identify such a case of proxy use.

The choice of program decomposition also has consequences for the tractability of the detection and repair algorithms. The detection and repair algorithms summarized in this paper currently enumerate through all possible subprograms in the worst case. Depending on the flexibility of the language chosen and the model being expressed there could be an exponentially large number of subprograms, and our enumeration would be intractable.

Important directions of future work are therefore organized along two thrusts. The first thrust is to develop flexible notions of program decompositions that identify proxy uses for other kinds of machine learning models, including deep learning models that will likely require new kinds of abstraction techniques due to their size. The second thrust is to identify scalable algorithms for detecting and repairing proxy use for these flexible notions of program decomposition.

7.2 Distributions and datasets

As we noted starting in Section 3.2, our formalism is written in terms of distributions whereas our practical implementation operates on datasets. Though it is best to think of a distribution as a sample of some real-world population and thus an approximation of reality, we do not address this assumption in our work. Disparity between reality and the analyzed datasets introduces concerns regarding the conclusions drawn from our methods.

If the analyzed dataset does not exhibit the associations establishing proxies that do exist in the real world then we can no longer rely on our methods to discover and repair real-world discriminatory practices. On the other hand, if a dataset introduces proxies that are not present in the real world, subtle philosophical and legal questions arise: does apparent discrimination still count as discrimination if it does not apply to the real-world? In the latter case of false positives, the ethical oracle in our formalism can be used in place of philosophical or legal assessments. The former possibility of a false negative, however, cannot be resolved in our formalism as there are no regards for human intervention without a proxy-use witness.

We believe that dataset disparity with reality is a problem orthogonal to the issues we address in this work; our conclusions are only as good as the accuracy of the datasets to which we apply our methods.

7.3 Data requirements

Our definitions and algorithms require access to datasets that contain both necessary inputs to execute a model and the attributes indicating protected class. The latter may not be explicitly collected or inferred for use in the models being analyzed. Therefore, to discover unwanted proxy uses of protected classes, an auditor might need to first infer the class from the collected data to the best extent available to them. If the protected class is sensitive or private information in the specific context, it may seem ethically ambiguous to infer it in order to (discover and) prevent its uses. However, this is consistent with the view that privacy is a function of both information and the purpose for which that information is being used.
We develop a theory of proxy discrimination in data-driven systems. Distinctively, our approach to use constraints not only the direct use of protected class but also their proxies (i.e. strong predictors), unless allowed by exceptions justified by ethical considerations.

We formalize proxy use and summarize a program analysis technique for detecting it in a model. In contrast to prior work, our analysis is white-box. The additional level of access enables our detection algorithm to provide a witness that localizes the use to a part of the algorithm. Recognizing that not all instances of proxy use of a protected class are inappropriate, our theory of proxy discrimination makes use of a normative judgment oracle that makes this appropriateness determination for a given witness. If the proxy use is deemed inappropriate, our repair algorithm uses the witness to transform the model into one that does not exhibit proxy use.

Using a corpus of social datasets, our evaluation shows that these algorithms are able to detect proxy use instances that would be difficult to find using existing techniques, and subsequently remove them while maintaining acceptable classification performance.

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