Neutrino Mass Matrix Model with a Bilinear Form

Yoshio Koide\textsuperscript{a} and Hiroyuki Nishiura\textsuperscript{b}

\textsuperscript{a} Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan
E-mail address: koide@het.phys.sci.osaka-u.ac.jp

\textsuperscript{b} Faculty of Information Science and Technology, Osaka Institute of Technology, Hirakata, Osaka 573-0196, Japan
E-mail address: nishiura@is.oit.ac.jp

Abstract

A neutrino mass matrix model with a bilinear form $M_\nu = k_\nu (M_D M_R^{-1} M_D^T)^2$ is proposed within the framework of the so-called yukawaon model, which has been proposed for the purpose of a unified description of the lepton mixing matrix $U_{PMNS}$ and the quark mixing matrix $V_{CKM}$. The model has only two adjustable parameters for the PMNS mixing and neutrino mass ratios. (Other parameters are fixed from the observed quark and charged lepton mass ratios and the CKM mixing.) The model gives reasonable values $\sin^2 \theta_{12} \simeq 0.85$, $\sin^2 \theta_{23} \sim 1$ and $\sin^2 \theta_{13} \sim 0.09$ together with $R_\nu \equiv \Delta m^2_{21}/\Delta m^2_{32} \sim 0.03$. Our prediction of the effective neutrino mass $\langle m \rangle$ in the neutrinoless double beta decay takes a sizable value $\langle m \rangle \simeq 0.0034$ eV.

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1 Introduction

Many particle physicists have searched for models which provide a unified description of the mass spectra and mixing patterns of quarks and leptons, the Cabibbo-Kobayashi-Maskawa mixing matrix $V_{CKM}$ \cite{1} and the Pontecorvo-Maki-Nakagawa-Sakata mixing matrix $U_{PMNS}$ \cite{2}. As one of such models, the so-called “yukawaon” model \cite{3} \cite{4} \cite{5} \cite{6} has been proposed. The model is a kind of “flavon” model \cite{7}.

In this model, Yukawa coupling constants $Y_f$ ($f = u, d, e, \cdots$) in the standard model are understood as vacuum expectation values (VEVs) of scalars (“yukawaon”) with $3 \times 3$ components, i.e. by $y_f \langle Y_f \rangle / \Lambda$, where $\Lambda$ is an energy scale of the effective theory. Our policy in building the yukawaon model is as follows: (i) We consider that the hierarchical structures of the effective Yukawa coupling constants can be understood only based on the charged lepton masses. For the moment, we do not ask for the origin of the charged lepton mass spectrum. (For an attempt to understand the origin of the charged lepton mass spectrum, for example, see Ref.\cite{8}.) (ii) We assume a $U(3)$ (or $O(3)$) family symmetry and $R$ charge conservation. Structures of yukawaon VEVs $\langle Y_f \rangle$ are obtained from SUSY vacuum conditions for a given superpotential, so that the VEV matrices are related to other yukawaon VEVs. (As stated in (i), the charged lepton mass
values are inputs for the moment, we do not discuss a mechanism which gives the observed charged lepton masses.) The first task in the yukawaon model is to search a superpotential form which gives reasonable mass spectra and mixings (in other words, to search for fields with suitable representations of $U(3)$ and $R$ charges. (iii) Effect of SUSY breaking depends on a SUSY breaking scenario. For the moment, we do not consider the SUSY breaking effects for yukawaon sector. We assume that the SUSY breaking in the quark and lepton sectors is induced by gauge mediation (this “gauge” means the conventional $SU(3)_c \times SU(2)_L \times U(1)_Y$ symmetries). (iv) At present, our aim is to search for a mass matrix model which can give a reasonable fit to whole of quark and lepton mass ratios and $V_{CKM}$ and $U_{PMNS}$ mixing matrices with parameters as few as possible. At present, our concern is in the construction of phenomenological mass matrix relations, not of a field theoretical model, i.e. neither in economizing of the yukawaon fields nor in making the superpotential compact. It is our next step to search for a model with more economical fields and with concise structure of superpotential.

The yukawaon model is in the process of research and development at present. In the yukawaon model, there are, in principle, no family-number-dependent parameters except for the charged lepton mass matrix $M_e$. Regrettably at present, we need a phase matrix $P_u$ (or $P_d$) with two phase parameters in order to obtain reasonable values of quark mixing matrix $V_{CKM}$. However, the final goal of our model is to remove such family dependent parameters.

The yukawaon model is constructed by using fundamental VEV matrices of scalar fields. In earlier yukawaon models $[3]$, the mass matrices are directly related to a fundamental VEV matrix matrix $\Phi_e \equiv \text{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$, while in recent yukawaon models, even the charged lepton mass matrix $M_e$ is given by a more fundamental VEV matrix $\Phi_0$. Here, we define VEV matrices which are associated with the mass matrix for up, down quarks, and charged leptons by a common form

$$\Phi_f = k_f \Phi_0 (1 + a_f X_3) \Phi_0,$$

where $f = u, d, e$. Here, for convenience, we have dropped the notations “(” and “)” on the VEV matrices. We will assign $\Phi_0$ to $(3^*, 3)$ of $U(3)\times U(3)'$ in the next section, so that we will denote $\Phi_0$ as $\bar{\Phi}_0$. In the present section in which we discuss the VEV matrices, for simplicity, we do not distinguish between $\Phi_0$ and $\bar{\Phi}_0$ (and also between $Y_f$ and $\bar{Y}_f$, and so on). $X_3$ and $1$ are also VEV matrices of other scalar fields. The matrices $\Phi_0$, $X_3$ and $1$ are defined by

$$\Phi_0 = \begin{pmatrix} x_1 & 0 & 0 \\ 0 & x_2 & 0 \\ 0 & 0 & x_3 \end{pmatrix}, \quad X_3 = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad 1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$  \quad (1.2)

Here, we have assumed that there is a basis in which the VEV matrix $\Phi_0$ takes a diagonal form and the VEV matrix $X_3$ takes a democratic form. Our mass matrix model is described on the premise that there can be such the flavor basis. The values of $(x_1, x_2, x_3)$ with $x_1^2 + x_2^2 + x_3^2 = 1$ are
fixed by the observed charged lepton mass values under the given value of $a_e$. The form $(1 + a_e X_3)$ is due to a family symmetry breaking $U(3) \to S_3$ \[9\] as we discuss later. The coefficients $a_f$ play an essential role in obtaining the mass ratios and mixings, while the family-number independent coefficients $k_f$ do not.

In this paper we propose a new model which improves the neutrino mass matrix. As far as mass matrices $M_\ell$, $M_d$ and $M_u$ of the charged leptons and down- and up-quarks are concerned, we assume the same VEV structures as those in the previous yukawaon model \[4, 5, 6\]:

$$ M_\ell = \Phi_\ell = \Phi_d = \Phi_u, \quad M_u = \Phi_u \Phi_u. \quad (1.3) $$

(Such the form $M_u = \Phi_u \Phi_u$ was suggested by a phenomenological fact $M_u^{\text{diag}} \sim (M_d^{\text{diag}})^2$).

Here and hereafter, we omit family-number independent coefficients ($k_f$ in Eq.(1.1) and so on), because we are interested only in family structures of $3 \times 3$ matrices. What is new in the present model is in the neutrino mass matrix $M_\nu$: we assume that $M_\nu$ takes the following form

$$ M_\nu = \Phi_\nu \Phi_\nu, \quad (1.4) $$

which is motivated by the up-quark mass matrix form $M_u = \Phi_u \Phi_u$ given in Eq. (1.3) and by the correspondence between quark and lepton mass matrices $M_\ell \leftrightarrow M_d$ and $M_\nu \leftrightarrow M_u$. The newly introduced VEV matrix $\Phi_\nu$ in Eq. (1.4) is given by

$$ \Phi_\nu = M_D M_R^{-1} M_D^T. \quad (1.5) $$

Here we take

$$ M_D = \Phi_D = \Phi_0^T (1 + a_D X_2) \Phi_0, \quad (1.6) $$

$$ M_R = \Phi_u \Phi_e + \Phi_e \Phi_u. \quad (1.7) $$

Though we use the notations $M_D$ and $M_R$ in Eqs.(1.5) - (1.7), they have no meaning of the Dirac or the right handed Majorana neutrino mass matrices differently from the previous model [see Eq.(2.3) later]. Note also that the form of $M_D$ given by Eq. (1.6) is different from that of other VEV matrices given by Eq. (1.1). Here, the matrix form $X_2$ \[9\] is defined by

$$ X_2 = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (1.8) $$

which will be discussed in Section 2.

Let us stress the difference of the form for the neutrino mass matrix between the present model and the previous one. In the previous yukawaon model \[4, 5, 6\], the neutrino mass matrix
where $\xi_\nu$-term was an additional term which was brought in order to fit neutrino mixing parameters $\sin^2 \theta_{23}$ and $\sin^2 \theta_{12}$. However, the model could not give reasonable fit for $\sin^2 \theta_{13}$. On the other hand, the mass matrix (1.4) with (1.7) in the new model has no such the $\xi_\nu$-term. Nevertheless, we can fit whole the observed mixing values $\sin^2 \theta_{23}$, $\sin^2 \theta_{12}$ and $\sin^2 \theta_{13}$ together with the ratio of neutrino mass-squared difference $R_\nu = \Delta m^2_{21}/\Delta m^2_{32}$ by using (1.5), as stated in Section 3. (The big drawback in the previous yukawaon models was that the model could not give the observed large value $^{[10]}$ of $\sin^2 2\theta_{13} \sim 0.09$.)

In Sec.2, we give VEV matrix relations in the new model. In Sec.3, we discuss parameter fitting of observed values only for the PMNS mixing and neutrino mass ratios because we revised the model only in the neutrino sector. The parameter values in the down-quark sector are effectively unchanged, so that we can obtain the same predictions for the down-quark mass ratios and CKM matrix parameters without changing the successful results in the previous paper $^{[9]}$.

### 2 VEV matrix relations

We assume that a would-be Yukawa interaction is given as follows:

$$W_Y = \frac{y_e}{\Lambda} \bar{\nu}_{e} Y_{e} \ell_{j} H_{d} + \frac{y_\nu}{\Lambda} \bar{\ell}_{j} H_{u} \bar{\ell}_{j} + \frac{y_d}{\Lambda} \bar{u}_{j} Y_{q} q_{j} H_{d} + \frac{y_u}{\Lambda} \bar{u}_{j} Y_{q} q_{j} H_{u}, \quad (2.1)$$

where $\ell = (\nu_L, e_L)$ and $q = (u_L, d_L)$ are SU(2)$_L$ doublets. Assignments of these fields to family symmetries U(3)×U(3)′ are given in Table 1. We denote the yukawaons with $(6^*, 1)$ and $(6, 1)$ as $\tilde{Y}$ and $Y$, respectively. Note that in Eq.(2.1) there are no SU(2)$_L$ singlet neutrinos. We have straightforwardly defined the neutrino mass matrix $M_\nu$ by the second term in Eq.(2.1). Although we denoted in Eq.(1.6) as if the matrix $M_D$ is a Dirac neutrino mass matrix, the matrix $M_D$ does not have a meaning of the Dirac mass matrix [see Eq.(2.3) later]. Under the definition of $\tilde{Y}_{\ell}$ ($Y^q$) in Eq.(2.1), the quark mixing matrix $V_{CKM}$ and the lepton mixing mixing matrix $U_{PMNS}$ are given by $V_{CKM} = U^d_{ij} U^u_{ij}$ and $U_{PMNS} = U^\dagger_{ij} U_{ij}$, respectively, where $U_f$ are defined by $U^d_{ij} M^d_{ij} U^u_{ij} = D^d_f$ ($D_f$ are diagonal). Here and hereafter, sometimes, we denote $\tilde{Y}_{\ell}$ and $Y^q$ as $Y_f$ for simplify. In order to distinguish each yukawaon from others, we assume that $Y_f$ have different $R$ charges from each other under consideration of $R$ charge conservation. (Of course, the $R$ charge conservation is broken at the energy scale $\Lambda$.)

We obtain VEV matrix relations from the superpotential which is invariant under the family symmetries U(3)×U(3)′ and is $R$ charge conserving. In the yukawaon model, the VEV matrix relations are phenomenological ones, and they are dependent on the $R$ charge assignments. Since
Table 1: SU(2)×SU(3)c×U(3)×U(3)' assignments and R charges

|   | \(\ell\) | cc | q | uc | dc |   | \(H_u\) | \(H_d\) | \(Y_e\) | \(Y_\nu\) | \(Y^d\) | \(Y^u\) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SU(2)_L | 2 | 1 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 1 | 1 |   |
| SU(3)_c | 1 | 1 | 3 | 3 | * | 3 | * | 1 | 1 | 6 | * | 6 | 6 |
| U(3) | 3 | 3 | 3 | * | 3 | * | 1 | 1 | 1 | 1 | 1 |   |
| U(3)' | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |   |   |   |
| R  | r_\ell | r_{ec} | r_q | r_{uc} | r_{dc} | r_{H_u} | r_{H_d} | \bar{r}_{Y_e} | \bar{r}_{Y_\nu} | r_{Y_d} | r_{Y_u} |   |

derivations of the VEV matrix relations are essentially similar to those in the previous papers [3, 4, 5, 6, 9], although the U(3)×U(3)' assignments and R charges are different. Besides, we must consider a complicated superpotential form in order to derive the desirable mass matrix relations. The purpose of the present paper is not to derive those mass matrix relations uniquely, but to investigate a possibility that the neutrino mass matrix \(M_\nu\) is given by a form

\[
M_\nu = (M_{D}M_{R}^{-1}M_{D}^T)^2,
\]

from the phenomenological point of view. Therefore, in this section, we present only the results of the mass matrix relations, the derivation of which is discussed in Appendix:

\[
\langle \bar{Y}_e \rangle = \langle \Phi_{\nu} \rangle = \langle \bar{\Phi}_{0} \rangle \left( 1 + a_{e}XX^{T} \right) \langle \bar{\Phi}_{0}^{T} \rangle, \tag{2.2}
\]

\[
\langle \bar{Y}_\nu \rangle = \langle \Phi_{\nu} \rangle \langle \bar{\Phi}_{\nu} \rangle, \tag{2.3}
\]

\[
\langle \bar{Y}_D \rangle = (\bar{Y}_D)(\bar{Y}_R)_{-1}(\bar{Y}_D), \tag{2.4}
\]

\[
\langle \bar{Y}_R \rangle = \langle \bar{Y}_e \rangle \langle \Phi^u \rangle + \langle \Phi^u \rangle \langle \bar{Y}_e \rangle, \tag{2.5}
\]

\[
\langle Y^u \rangle = \langle \Phi^u \rangle \langle \bar{\Phi}^u \rangle, \tag{2.6}
\]

\[
\langle \bar{P}_d \rangle \langle Y^d \rangle \langle \bar{P}_d \rangle = \langle \bar{\Phi}_{0} \rangle \left( 1 + a_{d}XX^{T} \right) \langle \Phi_{0}^{T} \rangle + \zeta_{d}^{0} \mathbf{1} \tag{2.7}
\]

Here, the fields \(\bar{\Phi}_{0}^{\alpha}\) and \(X_{\alpha i}\) are assigned to \((3^*, 3^*)\) and \((3, 3)\) of U(3)×U(3)', respectively. The field X has phenomenologically been introduced in the previous model [9], the VEV of which has the form

\[
\frac{1}{v_{X}} (X)_{\alpha i} = \frac{1}{2} \begin{pmatrix}
1 & 1 & 0 \\
1 & 1 & 0 \\
1 & 1 & 0
\end{pmatrix}_{\alpha i} \tag{2.10}
\]

The form (2.10) leads to

\[
(\langle X \rangle \langle X^{T} \rangle)_{\alpha \beta} = \frac{3}{2} (X_{3})_{\alpha \beta}, \quad (\langle X^{T} \rangle \langle X \rangle)_{ij} = \frac{3}{2} (X_{2})_{ij}, \tag{2.11}
\]
together with \( \langle X \rangle \langle X \rangle = \langle X \rangle \), where \( X_3 \) and \( X_2 \) is defined by Eqs. (1.2) and (1.8), respectively. Here, for simplicity, we have put \( v_X = 1 \) because we are interested only in the relative ratios among the family components. At present, there is no idea for the origin of the form (2.10). We may speculate that this form is related to a breaking pattern of \( U(3) \times U(3)' \) (for example, discrete symmetries \( U(3) \times U(3)' \rightarrow S_2 \times S_3 \)). In the present paper, the form (2.10) is only ad hoc assumption. However, as seen later, we can obtain a good fitting for the neutrino mixing angle \( \sin^2 2\theta_{13} \) due to this assumption.

3 Parameter fitting

We again summarize our mass matrix model as follows:

\[
M_e = \bar{Y}_e = \Phi_0 (1 + a_e X_3) \Phi_0^T, \quad (3.1)
\]
\[
M_\nu = \bar{Y}_\nu = \Phi_0 \Phi_\nu, \quad (3.2)
\]
\[
\Phi_\nu = \bar{Y}_D \bar{Y}_R^{-1} \bar{Y}_D, \quad (3.3)
\]
\[
M_D = \bar{Y}_D = \Phi_0^T (1 + a_D e^{i\alpha D} X_2) \Phi_0, \quad (3.4)
\]
\[
M_R = \bar{Y}_R = (\bar{Y}_u \Phi^u + \Phi^e \bar{Y}_e), \quad (3.5)
\]
\[
M_u = Y_u = \Phi^u \Phi^u, \quad (3.6)
\]
\[
\Phi^u = \Phi_0 \left(1 + a_u e^{i\alpha_u} X_3\right) \Phi_0^T, \quad (3.7)
\]
\[
P_d Y^d P_d = \Phi_0 (1 + a_d X_3) \Phi_0^T + \xi_0^d 1, \quad (3.8)
\]

where, for convenience, we have dropped the notations “(” and “)”. In numerical calculations, we use dimensionless expressions \( \Phi_0 = \text{diag}(x_1, x_2, x_3) \) (with \( x_1^2 + x_2^2 + x_3^2 = 1 \)) and \( P_d = \text{diag}(e^{-i\phi_1}, e^{-i\phi_2}, 1) \). The parameters are re-refined by Eqs. (3.1)-(3.8). In Eqs. (3.7) and (3.4), we have denoted \( a_u \) and \( a_D \) as \( a_u e^{i\alpha_u} \) and \( a_D e^{i\alpha_D} \), respectively, since we assume that the parameters \( a_e \) and \( a_d \) are real, while \( a_u \) and \( a_D \) are complex in our \( M_D \leftrightarrow M_u \) and \( M_e \leftrightarrow M_d \) correspondence scheme.

In this model, we have two parameters \( (a_D, \alpha_D) \) for neutrino sector, four parameters \( a_D, \xi_0^d \) and \( (\phi_1, \phi_2) \) for down-quark mass ratios and \( V_{CKM} \), and three parameters \( a_e, (a_u, \alpha_u) \) for charged lepton mass ratios and up-quark mass ratios as shown in Table 2. Especially, it is worthwhile noticing the neutrino mass ratios and \( U_{PMNS} \) are functions of only two parameters after \( a_e \) and \( (a_u, \alpha_u) \) have been fixed from the observed CKM mixing and up-quark mass ratios. There is effectively no change in the mass matrix structures except for \( Y_\nu \) from the previous paper [9], so that we can use the same parameter values for \( a_e \) and \( (a_u, \alpha_u) \) as those in the previous study [9], which are given by

\[
a_e = 7.5, \quad (a_u, \alpha_u) = (-1.35, 7.6^\circ). \quad (3.9)
\]

Therefore, as far as PMNS mixing and neutrino mass ratios are concerned, we have only two free parameters \( (a_D, \alpha_D) \) in the present neutrino mass matrix model.

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Table 2: Process for parameter fitting. Since the parameters listed in each step can slightly affect predictions listed in the other steps, we need fine tuning after the 5th step. New parameter fitting in the present paper starts from the 5th step.

| Step | Inputs | $N_{mp}$ | Parameters | $N_{par}$ | Predictions |
|------|--------|----------|------------|----------|-------------|
| 1st  | $m_1$, $m_2$, $m_3$, $m_4$, $m_5$, $m_6$, $m_7$ | 4 | $x_1$, $x_2$, $x_3$, $x_4$, $x_5$, $x_6$, $x_7$ | 4 | $V_{ud}$, $\delta_{CP}$ |
| 2nd  | $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$ | 3 | $\alpha_u$, $\phi_1$, $\phi_2$ | 3 | $\alpha_u$, $\phi_1$, $\phi_2$ |
| 3rd  | $m_1$, $m_2$, $m_3$, $m_4$, $m_5$, $m_6$, $m_7$ | 1 | $a_d$ | 1 | $\sin^2 2\theta_{13}$, $\delta_{CP}$ |
| 4th  | $m_1$, $m_2$, $m_3$, $m_4$, $m_5$, $m_6$, $m_7$ | 1 | $m_0$ | 1 | $\sin^2 2\theta_{13}$, $\delta_{CP}$, 2 Majorana phases |
| 5th  | $\sin^2 2\theta_{12}$, $R_\nu$ | 2 | $a_D$, $\alpha_D$ | 2 | $\sin^2 2\theta_{13}$, $\delta_{CP}$, 2 Majorana phases |
| option | $\Delta m^2_{atm}$ | 1 | $m_\nu$ | 1 | $(m_\nu_1, m_\nu_2, m_\nu_3)$, $(m)$ |
| $\sum N_{\ldots}$ | | 12 | | | 12 |

At present, the observed values [11] are as follows:

$$\sin^2 2\theta_{12}^{\text{obs}} = 0.857 \pm 0.024, \quad \sin^2 2\theta_{13}^{\text{obs}} > 0.95, \quad \sin^2 2\theta_{13}^{\text{obs}} = 0.098 \pm 0.013,$$

(3.10)

$$R_\nu^{\text{obs}} \equiv \frac{(\Delta m^2_{atm})^{\text{obs}}}{(\Delta m^2_{\odot})^{\text{obs}}} = \frac{(7.50 \pm 0.20) \times 10^{-5}}{(2.32^{+0.12}_{-0.08}) \times 10^{-3}} \text{ eV}^2 = (3.23^{+0.14}_{-0.19}) \times 10^{-2}.$$  

(3.11)

Since the parameters $(a_D, \alpha_D)$ are sensitive to the observables $\sin^2 2\theta_{12}^{\text{obs}}$ and $R_\nu^{\text{obs}}$, we use the observed values of $\sin^2 2\theta_{12}$ and $R_\nu$ in order to fix our parameter values $(a_D, \alpha_D)$. In Fig.1, we illustrate an allowed parameter region of $(a_D, \alpha_D)$ obtained from the observed values of $\sin^2 2\theta_{12}^{\text{obs}}$ and $R_\nu^{\text{obs}}$. As seen in Fig.1, the observed values uniquely fix the parameter values $(a_D, \alpha_D)$ as

$$(a_D, \alpha_D) = (8.7, 12^\circ).$$

(3.12)

It is worthwhile noticing that the parameter values (3.12) uniquely give a prediction of $\sin^2 2\theta_{13} \simeq 0.09$. For reference, in Fig.2, we illustrate behaviors of $\sin^2 2\theta_{12}$ and $R_\nu$ versus $\alpha_D$ in the case of $a_D = 8.7$. We find that the choice $\alpha_D = 12^\circ$ gives excellent fittings to the observed values of $\sin^2 2\theta_{12}$ and $R_\nu$ simultaneously:

$$\sin^2 2\theta_{12} = 0.8544, \quad R_\nu = 0.0331.$$  

(3.13)

Then, we obtain our predictions for $\sin^2 2\theta_{23}$ and $\sin^2 2\theta_{13}$ using (3.12) as follows:

$$\sin^2 2\theta_{23} = 0.9962, \quad \sin^2 2\theta_{13} = 0.0907,$$  

(3.14)
Figure 1: Allowed parameter region in \((a_D, \alpha_D)\) plane. The solid and dashed curves indicate the border and center curves of the allowed region which are obtained from the observe values of \(\sin^2 2\theta_{12}^{\text{obs}}\) and \(R_{\nu}^{\text{obs}} \times 10\), respectively. The dot-dashed curves represent contour curves of \(\sin^2 2\theta_{13}^{\text{obs}}\) for some typical values, \(\sin^2 2\theta_{13} = 0.08, 0.09,\) and \(0.10\).

which are in excellent agreement with the observed values given in Eq.\((3.10)\).

The fixing of the parameters \((a_D, \alpha_D)\), Eq.\((3.12)\), leads to the prediction of the \(CP\) violating phase parameter in the lepton sector too:

\[
\delta_{CP}^\ell = 127^\circ \quad (J^\ell = 2.74 \times 10^{-2}), \tag{3.15}
\]

where \(\delta_{CP}^\ell\) is the \(CP\) violating phase in the standard expression and \(J^\ell\) is the rephasing invariant \cite{12}. We can also predict neutrino masses:

\[
m_{\nu 1} = 0.00061 \text{ eV}, \quad m_{\nu 2} = 0.00899 \text{ eV}, \quad m_{\nu 3} = 0.05011 \text{ eV}, \tag{3.16}
\]

by using the input value \cite{13} \(\Delta m^2_{32} = 0.00243 \text{ eV}^2\). (Note that, in the present model, we cannot obtain an inverted neutrino mass hierarchy, because the hierarchies of the mass matrices are related to the hierarchy of the charged lepton mass hierarchy, i.e. to the VEV matrix \(\langle \Phi_0 \rangle\).)

We also predict the effective Majorana neutrino mass \cite{14} \(\langle m \rangle\) in the neutrinoless double beta decay as

\[
\langle m \rangle = |m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2| = 0.0034 \text{ eV}. \tag{3.17}
\]

This predicted value is considerably larger than those in other models with normal hierarchy \cite{15}.

Finally, we list the predicted values of the CKM mixing parameters and down-quark mass ratios, although they are essentially the same as those in the previous model \cite{9}:

\[
|V_{us}| = 0.2271, \quad |V_{cb}| = 0.0394, \quad |V_{ub}| = 0.00347, \quad |V_{td}| = 0.00780, \tag{3.18}
\]
Figure 2: Lepton mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and the ratio $R_\nu$ versus the phase parameter $\alpha_D$ for $a_D = 8.7$. The horizontal lines denote observed values (the center and $1\sigma$ values) of $\sin^2 2\theta_{12}^{\text{obs}}$, $\sin^2 2\theta_{13}^{\text{obs}} \times 10$ and $R_\nu^{\text{obs}} \times 10$. Our predicted value for $\sin^2 2\theta_{23}$ is well satisfied the obtained experimental bound of $\sin^2 2\theta_{23}^{\text{obs}}$.

\[ \delta_{CP}^q = 59.6^\circ \quad (J^q = 2.6 \times 10^{-5}), \]  
\[ r_{12}^u = \sqrt{\frac{m_d}{m_s}} = 0.00465, \quad r_{23}^u = \sqrt{\frac{m_d}{m_b}} = 0.0614. \]  
\[ r_{12}^d = \frac{m_d}{m_s} = 0.0569, \quad r_{23}^d = \frac{m_d}{m_b} = 0.0240. \]  

Here, we have used $a_d = 25$, $\xi_0^d = 0.0115$, and $(\phi_1, \phi_2) = (177.0^\circ, 197.4^\circ)$. The observed values are as follows: $|V_{us}| = 0.2252 \pm 0.0009$, $|V_{cb}| = 0.0409 \pm 0.0011$, $|V_{ub}| = 0.00415 \pm 0.00049$, $|V_{td}| = 0.0084 \pm 0.0006$, $J^q = (2.96^{+0.20}_{-0.16}) \times 10^{-5}$ \[\Box\], and $r_{12}^u = 0.045^{+0.013}_{-0.010}$, $r_{23}^u = 0.060 \pm 0.005$, $r_{12}^d = 0.053^{+0.005}_{-0.003}$, $r_{23}^d = 0.019 \pm 0.006$ \[\Box\].

4 Concluding remarks

In conclusion, we have proposed a new neutrino mass matrix form within the framework of the Yukawaon model, in which we have only two adjustable parameters, $(a_D, \alpha_D)$, for PMNS mixing and neutrino mass ratios. We have been able to remove the unnatural term $[\xi_\nu \text{ term in Eq.(1.9)}]$ in the previous model. Nevertheless, we can obtain reasonable results for PMNS mixing and neutrino mass ratios as shown in Eqs.(3.13) - (3.17) for the parameter values $(a_D, \alpha_D) = (8.7, 12^\circ)$. As seen in Fig.2, it is worthwhile noticing that only when we choose a reasonable value of $R_\nu \simeq 0.033$, we can obtain a reasonable value of $\sin^2 2\theta_{13} \simeq 0.09$. Also, note that our prediction gives a sizable value of $\langle m \rangle \simeq 0.0034$ eV among normal mass hierarchy models. Of
course, we have also obtained reasonable results for CKM mixing and quark mass ratios as same as those in the previous paper \cite{9}.

Such the phenomenological success is essentially based on the following assumptions: (i) We have assumed that only $Y_D$ takes the mass matrix form with $X_2$ (not $X_3$), while others $Y_f$ ($\Phi_f$) take the form with $X_3$ as given in Eq.(1.1). In Ref.\cite{6}, the form $X_3$ has been understood by a symmetry breakdown $\text{U}(3) \times \text{U}(3)' \rightarrow \text{U}(3) \times S_3$. However, for the form $X_2$, the model is still in a phenomenological level. (ii) We have the bilinear form of the neutrino mass matrix, $M_\nu = \Phi_\nu \Phi_\nu$, as well as the up-quark mass matrix $M_u = \Phi_u \Phi_u$. From the theoretical point view, there is no reason for the bilinear forms. We merely assigned $R$ charges so that bilinear forms are realized for $M_u$ and $M_\nu$.

In spite of such the phenomenological success, the model still leave some basic problems: (i) The model is not economical. At present, we need many flavons in order to prepare reasonable VEV matrix relations. Since the purpose of the present paper is to investigate phenomenological relations among mass matrices, the structure of the superpotential given in Appendix is a temporal one. The superpotential will be improved in our future work. (ii) We have not discuss scales of yukawaons. The present model is based on an effective theory with an energy scale $\Lambda$. The scale $\Lambda$ must be, at least, larger than $10^7$ TeV from the observed $K^0-\bar{K}^0$ mixing (and also $D^0-\bar{D}^0$ mixing) \cite{11}. In earlier version of the yukawaon model, it was considered to be $\Lambda \sim 10^{15}$ GeV. However, VEVs of individual yukawaons depend on parameters in the superpotential ($\mu_f$ in mass terms and couplings $\lambda_f$). We do not fix those scales in the present paper, although we expect that effects of those flavons are visible. (iii) We did not discuss SUSY breaking effects. As we stated in Section 1, for the time being, we assume that the SUSY breaking effects do not affect yukawaon sector. (iv) Our goal is to understand the hierarchical structures of all quark and lepton mass matrices on the basis of only the observed charged lepton masses. However, in the present model, we are still obliged to introduce flavon $\bar{P}_d$ whose VEV matrix includes flavor-dependent parameters $\phi_1$ and $\phi_2$ as seen in (A.11).

Generally speaking, the yukawaon model suggests that our direction to unified understanding of the flavor problems is not wrong, although we have many problems in the yukawaon model. By leaving the settlement of the problems to our future tasks, the yukawaon model will be improved step by step.

**Appendix**

In this Appendix, we discuss a derivation of the mass matrix relations (2.2)-(2.9) from superpotential. We assume the following superpotential $W = W_e + W_\nu + W_R + W_D + W_u + W_d$:

\[
W_e = \left\{ \mu_e \bar{Y}_e^{ij} + \frac{\lambda_e}{\Lambda} (\Phi_0)^{i\alpha} \left( \sum_{\alpha} E_{\alpha\beta} + \frac{a_e}{\Lambda^2} X_{ak} \bar{E}_{kl} X^{T, T}_{l\beta} \right) (\Phi_0^T)^{\beta j} \right\} \Theta_{ji}, (A.1)
\]
\[
W_\nu = \frac{1}{\Lambda} \left[ \lambda_\nu (E')^\alpha_k (E'^T)^\beta_l + \lambda_\nu (\Phi^T_\nu)^{\alpha \gamma} E''^\alpha_\nu \Phi^\delta_\nu \right] \Theta^\nu_{\beta \alpha} + \left[ \mu_\nu \Phi^\alpha_\nu + \frac{\lambda_\nu}{\Lambda} \Phi^\alpha_\nu \Phi^R \Phi^\delta_\nu \right] \Theta^\nu_{\beta \alpha}, \quad (A.2)
\]

\[
W_D = \left[ \mu_D \tilde{Y}^{\alpha \beta} + \frac{\lambda_D}{\Lambda} (\bar{\Phi}^T_D)^{\alpha k} \left( E_{kl} + \frac{\lambda'_D}{2} X^{T}_{kl} (E'')^{\beta \gamma} X^{-1}_{il} \right) \bar{\Phi}^{\beta \beta}_{\bar{D}} \right] \Theta^D_{\beta \alpha}, \quad (A.3)
\]

\[
W_R = \left[ \frac{\lambda_R}{\Lambda} \tilde{Y}^{ik}_R (E')^i_k \Phi^R_{\gamma \alpha} + \mu_R (E')^\alpha_\gamma \right] (\Theta R)^\alpha_i + \left[ \mu_R \tilde{Y}^{ij}_R + \frac{\lambda_R}{\Lambda} \left( \tilde{Y}^{ik}_R (E')^i_k + (E'' \Phi^R)^{i k} \bar{\Phi}^{k j}_{\bar{R}} \right) \right] \Theta^R_{ji}, \quad (A.4)
\]

\[
W_u = \left( \mu_u Y''_{ij} + \frac{\lambda_u}{\Lambda} \Phi^u_{ik} \tilde{E}^{ki} \Phi^u_{ij} \right) \tilde{\Theta}^{\alpha}_{ij}
\]

\[
+ \frac{1}{\Lambda} \left[ \lambda'_u \tilde{E}^{ik} \Phi^u_{ik} \tilde{E}^{ij}_{\alpha} + \lambda''_u (\Phi^u_0)^{\alpha} \left( (E''_{\alpha \beta} + \frac{\alpha''_u}{\Lambda^2} X_{a k} \tilde{E}^{ki} X^T_{i3} \left( \Phi^u_0 \right)^{\beta j} \right) \tilde{\Theta}^{\alpha}_{ij} \right], \quad (A.5)
\]

\[
W_d = \left[ \frac{\lambda_d}{\Lambda} \lambda_d \tilde{P}^{ij}_d Y''_{kl} \tilde{P}^{kl}_d + \frac{\lambda'_d}{\Lambda} (\Phi^d_0)^{\alpha} \left( (E''_{\alpha \beta} + \frac{\alpha''_d}{\Lambda^2} X_{a k} \tilde{E}^{ki} X^T_{i3} \left( \Phi^d_0 \right)^{\beta j} \right) \tilde{\Theta}^{\alpha}_{ij} + \mu_d \tilde{E}^{ij}_d \right] \Theta^d_{ji}, \quad (A.6)
\]

The VEV matrix relations (2.2) - (2.9) are obtained from SUSY vacuum conditions, \( \partial W / \partial \Theta_A = 0 \) (\( A = e, \nu, \cdots \)). Since we assume that all \( \Theta \) fields take \( \langle \Theta \rangle = 0 \), SUSY vacuum conditions with respect to another fields do not lead to meaningful relations, because such conditions always contain, at least, one \( \langle \Theta \rangle \).

In Eqs. (A.5) and (A.6), we have introduced fields \( E''_{\alpha \beta}, E''_{\nu}, E_{\nu}, \) and \( E_{\alpha} \) in addition to \( E'' \) and \( E \) in order to distinguish the \( R \) charges of \( \tilde{\Theta}^{\alpha} \) and \( \Theta^d \) from that of \( \Theta^e \). All VEV matrices \( \langle E \rangle \) are given by the forms \( \langle E \rangle \propto 1 \) as seen in (A.10). The VEV matrix relations (2.2) - (2.9) have already been presented by replacing \( \langle E \rangle \rightarrow 1 \).

We list the SU(2)_L \times SU(3)_c \times U(3) \times U(3)' assignments and \( R \) charges for additional fields in Table 3. The assignments of \( R \) charges are done so that the total \( R \) charge of the superpotential term is \( R(W) = 2 \). We have 17 constraints on the \( R \) charges of the fields from Eqs. (2.1) and (A.1) - (A.6), while we have 34 fields even except for \( \Theta \) fields in Tables 1 and 3. Therefore, we cannot uniquely fix \( R \) charge assignments of those fields. Here, let us give only typical constraints:

\[
2r_X = r''_E - \tilde{r}_E = r_E - \tilde{r}_E = r''_{E u} - \tilde{r}_{E u} = r''_{E d} - \tilde{r}_{E d}, \quad (A.7)
\]

\[
2r_0 = \tilde{r}_Y - r''_E = \tilde{r}_Y - r_E + 2 \tilde{r}_{E u} - r''_{E u} = \tilde{r}_Y + 2 \tilde{r}_{P d} - r''_{E d}. \quad (A.8)
\]

From Eq. (A.7), we obtain \( r'' + \tilde{r}_E = r_E + \tilde{r}_E \). When we take \( R(E'') + R(\tilde{E}'') = R(E) + R(\tilde{E}) = R(P^d) + R(\tilde{P}^d) = 1 \), we can introduce the following superpotential:

\[
W_{E,P} = \frac{\lambda_1}{\Lambda} \text{Tr}[\tilde{E}E\tilde{P}_d P_d] + \frac{\lambda_2}{\Lambda} \text{Tr}[\tilde{E}E]\text{Tr}[\tilde{P}_d P_d], \quad (A.9)
\]
Table 3: SU(2)$_L$×SU(3)$_C$×U(3)×U(3)$'$ assignments and $R$ charges

| $\Phi_{\nu}$ | $Y_D$ | $Y_R$ | $\Phi^D$ | $\Phi^u$ | $\Theta^e$ | $\Theta^\nu$ | $\Hat{\Theta}^D$ | $\Theta^R$ | $\Hat{\Theta}^u$ | $\Theta^d$ |
|--------------|------|------|---------|--------|---------|----------|-------------|--------|----------|--------|
| SU(2)$_L$   | 1    | 1    | 1       | 1      | 1       | 1        | 1           | 1      | 1        | 1      |
| SU(3)$_C$   | 1    | 1    | 1       | 1      | 1       | 1        | 1           | 1      | 1        | 1      |
| SU(3)       | 1    | 1    | $6^*$   | 1      | 6       | 6        | 1           | 6      | 1        | 6      |
| U(3)$'$     | $6^*$| $6^*$| 1       | 6      | 1       | 1        | 1           | 1      | 1        | 1      |

$\Phi_0$ | $X$ | $E$ | $\bar{E}$ | $E'$ | $E''$ | $E'''$ |
|------|-----|-----|---------|------|-------|-------|
| 1    | 1   | 1   | 1       | 1    | 1     | 1     |
| 1    | 1   | 1   | 1       | 1    | 1     | 1     |
| $3^*$| 3   | 6   | $6^*$   | 3    | 3     | 1     |
| $3^*$| 3   | 1   | 3      | 3    | 6     | $6^*$ |

$r_0 \frac{1}{\pi} (r_E + r_{E''} - 1)$

$E_u$ | $E_u$ | $E_d$ | $E_d$ | $E_u''$ | $E_u'$ | $E_u'''$ | $E_d''$ | $P_d$ | $P_d$ |
|-----|------|------|------|--------|-------|----------|--------|-----|-----|
| 1   | 1    | 1    | 1    | 1      | 1     | 1        | 1      | 1   | 1   |
| 1   | 1    | 1    | 1    | 1      | 1     | 1        | 1      | 1   | 1   |
| 6   | $6^*$| 6    | $6^*$| 1      | 1     | 1        | 6      | 6   | $6^*$|
| 1   | 1    | 1    | 1    | $6^*$  | $6^*$ | 1        | 1      | 1   | 1   |

$r_{E_u}$ | $1 - r_{E_u}$ | $r_{Ed}$ | $1 - r_{Ed}$ | $r_{E_u''}$ | $1 - r_{E_u''}$ | $r_{Ed''}$ | $1 - r_{Ed''}$ | $P_d$ | $1 - r_{P_d}$

from which we obtain relations $\langle E \rangle \langle \bar{E} \rangle \propto 1$ and $\langle P_d \rangle \langle \bar{P}_d \rangle \propto 1$. We assume following specific solutions of those relations:

$$\frac{1}{v_E} \langle E \rangle = \frac{1}{v_{\bar{E}}} \langle \bar{E} \rangle = 1, \quad (A.10)$$

$$\frac{1}{v_P} \langle P_d \rangle^\dagger = \frac{1}{v_{\bar{P}_d}} \langle \bar{P}_d \rangle = \text{diag}(e^{-i\phi_1}, e^{-i\phi_2}, 1), \quad (A.11)$$

as the explicit forms of $\langle E \rangle$, $\langle \bar{E} \rangle$ and $\langle P_d \rangle$. We assume similar superpotential forms for $(E, \bar{E})$, $(E_u, \bar{E}_u)$, $(E_d, \bar{E}_d)$, $(E''', \bar{E}''')$, $(E''u, \bar{E}''u)$, $(E''d, \bar{E}''d)$ and $(E', \bar{E'})$.

The term $\mu_d E_d$ in Eq.(A.6) has been introduced in order to adjust the down-quark mass ratio $m_d/m_s$ as seen in Sec.3. Additional terms like $\mu_d E_d$ in the lepton and up-quark sectors do not appear, because we take $R(E) \neq R(E_d)$ and $R(E_u) \neq (E_d)$.
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