Sensorless Control for PMSM Based on Multi-Innovation Two-Stage Extended Kalman Filter Algorithm

Fei Wu
Shanghai University, 99 Shangda Road, Baoshan District, Shanghai, China
364109006@qq.com

Abstract. Permanent magnet synchronous motors (PMSM) have been widely used in robotics, CNC machine tools, and aerospace, aerospace and marine fields due to its small size, high efficiency and high power density, and good operating performance. With the development of sensorless control technology, sensorless control has been widely used in many special areas. This paper is aiming at the sensorless control of permanent magnet synchronous motor, the sensorless control system of permanent magnet synchronous motor based on Multi-innovation Two-Stage Extended Kalman Filter (MI-TSEKF) algorithm is studied and implemented.

1. Introduction
The permanent magnet synchronous motor has the characteristics of simple structure, small size, light weight, low loss and high efficiency, and has been widely concerned in the early stage of its publication. Compared with DC motors, it has no commutator and brush, and has higher reliability[1]. Compared with asynchronous motors, it does not require reactive excitation current and has higher efficiency. Compared with ordinary synchronous motors, PMSM eliminates The excitation device has a flexible shape and size.

The installation of the position sensor brings many problems, and the position sensorless control technology can solve these problems well. The principle is to estimate the rotor position and speed by detecting the physical quantity and voltage of the motor, and avoid the mechanical sensor. The drawbacks increase the reliability of the system and reduce the system cost. Therefore, the position sensor technology of permanent magnet synchronous motors has been widely concerned by scholars at home and abroad for nearly two decades.

2. Sensorless Control based on MI-TSEKF algorithm
The first-order truncated Taylor expansion in MI-EKF does not consider the influence of high-order terms on approximate linearization, and reduces the accuracy of EKF algorithm for parameter estimation[2]. Aiming at this problem, this paper introduces multi-innovation theory into EKF algorithm and design a MI-EKF algorithm to improve the estimation accuracy of EKF algorithm[3]. At the same time, in order to reduce the computational complexity of the MI-EKF algorithm, a two-stage extended Kalman filter algorithm which is mathematically equivalent to the EKF algorithm is combined with the multi-innovation theory, and the MI-TSEKF algorithm is proposed and used. Position sensorless control of surface-mount permanent magnet synchronous motors.

2.1. Multi-innovation theory
The multi-innovation identification theory is a more accurate identification method based on the single innovation identification theory[4]. Innovation is useful information that improves the accuracy of state
parameter estimation. The system identification theory is a method to describe the motion law of the system by constructing a mathematical model. The main idea is to correct the current time parameters by the state estimation parameter of the output at the moment and the product of the innovation and the gain vector[5]. At present, most identification algorithms only use a single innovation to correct the parameters of the system. This identification algorithm is called a single innovation identification algorithm.

In a scalar system of linear regression models

\[ y(t) = \varphi(t)x(t) + v(t) \]  

(1)

Among them, the input of the system is \( \varphi(t) \), the output of the system is \( y(t) \), and \( x(t) \) is the parameter to be identified, \( v(t) \) is random noise and expects zero. Solving the parameters to be tested of Equation (1) by using the stochastic gradient algorithm or the least squares method:

\[ \dot{x}(t) = \dot{x}(t-1) + K(p,t)e(p,t) \]  

(2)

\[ e(t) = y(t) - \varphi(t)x(t) \]  

(3)

Where, \( \dot{x}(t) \) is the parameter estimation vector at time \( t \), \( K(p,t) \) is the gain vector of the algorithm, and \( e(t) \) is the innovation at time \( t \).

The multi-innovation identification algorithm is an extension of the single innovation identification algorithm, which promotes the number of single innovations to multiple, and promotes the single innovation \( e(t) \) to the multiple innovation vector \( e(p,t) \) and promotes the gain vector \( K(p,t) \) to the multi-innovation gain vector \( K(p,t) \), then the multi-innovation identification algorithm can be expressed as:

\[ \dot{x}(t) = \dot{x}(t-1) + K(p,t)e(p,t) \]  

(4)

\[ e(p,t) = \left[ e(t) \quad e(t-1) \quad \cdots \quad e(t-p+1) \right]^T \]  

(5)

Where, \( p (p \geq 2) \) is the length of the innovation vector.

The multi-innovation identification algorithm can be simply described as \( \dot{x}(t) \) product equal to \( \dot{x}(t-1) \) plus the modified parameter is the product of \( e(p,t) \) and \( K(p,t) \), which is called multi-incident identification theory.

2.2. Multi-innovation Two-Stage Extended Kalman Filter algorithm

In order to improve the estimation accuracy of the extended Kalman filter algorithm for state parameters, we extend the single update of the extended Kalman filter algorithm is extended to the MI-EKF algorithm. The MI-EKF algorithm not only calculates the state vector of the current time, but also considers the useful state vector of the past time[6], which improves the estimation accuracy of the EKF algorithm.

However, this algorithm also increases the complexity and computational complexity of the algorithm and reduces the response speed of the system. Aiming at this problem, a multi-innovation two-segment extended Kalman filter (MI-TSEKF) algorithm is proposed. Based on the MI-EKF algorithm, the 4-dimensional state variables in the MI-EKF system are decomposed into two. The two-dimensional state variable, that is, the full-order state variable \( X_k = [i_\alpha \quad i_\beta]^T \) and the augmented state variable \( \lambda_k = [\omega_k \quad \theta]^T \). Then the two 2-dimensional state variables are re-integrated into a new discrete system state variable \( X_k = [X_k \quad \lambda_k]^T \), the system input signal \( u_k = [u_\alpha \quad u_\beta]^T \), and the system output is \( Y_k = [i_\alpha \quad i_\beta]^T \), then the discrete state equation of the system in the two-phase stationary coordinate system can be expressed as:

\[
\begin{align*}
X_{k+1} &= A_k X_k + B_k u_k + W_k \\
Y_{k+1} &= C_k X_{k+1} + V_k
\end{align*}
\]  

(6)
The matrix in the equation is partitioned, and the matrix after the partition is expressed as:

\[
A_k = \begin{bmatrix} A_k & D_k \\ 0 & G_k \end{bmatrix}, \quad \overline{A}_k = \begin{bmatrix} 1 - \frac{R}{L_s} T & 0 \\ 0 & 1 - \frac{R}{L_s} T \end{bmatrix}, \quad D_k = \begin{bmatrix} \psi_x \sin \theta & 0 \\ \frac{\psi_x \cos \theta}{L_s} & 0 \end{bmatrix}, \quad G_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},
\]

\[
B_k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \overline{B}_k = \begin{bmatrix} T \\ \frac{\psi_x}{L_s} \\ 0 \end{bmatrix}, \quad C_k = [\overline{C}_k \ 0], \quad \overline{C}_k = [1 \ 0], \quad W_k = \begin{bmatrix} W_k^i \\ W_k^j \end{bmatrix}
\]

Substituting the new discrete state equation (6) into the MI-EKF algorithm, the formula can be expressed as:

\[
X_{k|k-1} = A_{k-1} X_{k-1} + B_{k-1} u_{k-1} \quad (7)
\]

\[
P_{k|k-1} = \Phi_{k-1} P_{k-1} \Phi_{k-1}^T + Q \quad (8)
\]

\[
K_k = P_{k|k-1} C_k^T (C_k P_{k|k-1} C_k^T + R)^{-1} \quad (9)
\]

\[
X_k = X_{k|k-1} + K(p,k) e(p,k) \quad (10)
\]

\[
P_k = (I - K_k C_k) P_{k|k-1} \quad (11)
\]

\[
K^T(p,k) = \begin{bmatrix} K_k \\ \vdots \end{bmatrix}, \quad e(p,k) = \begin{bmatrix} Y_k - C_k X_{k|k-1} \\ Y_{k-1} - C_k X_{k-1|k-2} \\ \vdots \\ Y_{k-p+1} - C_k X_{k-p+1|k-p} \end{bmatrix} \quad (12)
\]

Where, Q and R are the covariance matrices of the system noise W_k and the measurement noise V_k.

According to the formula for calculating the covariance, it can be concluded that:

\[
Q(\cdot) = \begin{bmatrix} Q'(\cdot) & 0 \\ 0 & Q'(\cdot) \end{bmatrix}, \quad P(\cdot) = \begin{bmatrix} P'(\cdot) & P^{ij}(\cdot) \\ (P^{ij}(\cdot))^T & P^{ij}(\cdot) \end{bmatrix} \quad (13)
\]

It is found by the above formula that the algorithm formula involves the operations of addition, multiplication, transposition, and inversion of the matrix operation[7]. In the matrix operation, the higher the number of dimensions, the more the number of operations of the matrix, and the number of operations and the matrix dimension are presented. Geometric multiples increase.

It is also observed that the calculation of \( P^{ij}(\cdot) \) and \( (P^{ij}(\cdot))^T \) in the error covariance matrix \( P(\cdot) \) in the algorithm increases with the increase of the system dimension, and the term can be eliminated by separating the state variables. Therefore, the multi-innovation two-segment extended Kalman algorithm will be based on the elimination of this item. To this end, a coordinate transformation matrix \( T[\cdot] \) is introduced. By this coordinate transformation, the error covariance matrix is changed into a new diagonal matrix \( \tilde{P}(\cdot) \). It is worth noting that the transformation matrix \( T[\cdot] \) is non-singular, which is a prerequisite for coordinate transformation.

\[
T[J] = \begin{bmatrix} I & J \\ 0 & I \end{bmatrix}, \quad \tilde{P}(\cdot) = \begin{bmatrix} \tilde{P}'(\cdot) & 0 \\ 0 & \tilde{P}'(\cdot) \end{bmatrix} \quad (14)
\]

We let \( M_k = P^a_{k|k-1} \left(P^a_{k|k-1}\right)^{-1}, N_k = P^a_{k|k} \left(P^a_{k|k}\right)^{-1} \), it can be concluded that:
\[
T[M_k] = \begin{bmatrix} I & M_k \\ 0 & I \end{bmatrix}, T[N_k] = \begin{bmatrix} I & N_k \\ 0 & I \end{bmatrix}
\]  \hspace{1cm} (15)

It is not difficult to see that the two transformation matrices are also non-singular, then you can use these two matrices to do the following transformation:

\[
\begin{align*}
\tilde{X}_{k/k-1} &= T[-M_k]X_{k/k-1} \\
\tilde{P}_{k/k-1} &= T[-M_k]P_{k/k-1}T[-M_k]^T \\
\bar{X}_k &= T[-N_k]X_k \\
\bar{K}_k &= T[-N_k]K_k \\
\bar{P}_k &= T[-N_k]P_kT[-N_k]^T
\end{align*}
\]  \hspace{1cm} (16)

In order to decompose the multi-innovation extended Kalman filter into a full-order filter and an augmented filter, the recursive substitution is performed, and the related parameter calculation formulas involved in the obtained MI-TSEKF algorithm are summarized as follows:

\[
\begin{align*}
& m_{k-1} = (A_{k-1}N_{k-1} + D_{k-1} - M_k G_{k-1}) \tilde{X}_{k-1} \\
& J_k = C_k \bar{P}_{k/k-1} (C_k)^T + R \\
& M_k = \tilde{M}_k \left( I - Q_{k}^T (\tilde{P}_{k/k-1})^{-1} \right) \\
& \tilde{M}_k = (A_{k-1}N_{k-1} + E_{k-1}) G_{k-1}^{-1} \\
& N_k = M_k - S_k \bar{K}_k \\
& M_0 = P_0^d (P_0^d)^T^{-1} \\
& \tilde{X}_0 = X_0' - M_0 \tilde{X}_0 \\
& \tilde{r}_0 = r_0 \\
& \bar{P}_0^d = P_0^d - M_0 P_0^d M_0^T \\
& \bar{r}_0 = P_0^d \\
& \tilde{P}_0 = P_0^d
\end{align*}
\]  \hspace{1cm} (17)

Mathematical induction can prove that the MI-TSEKF algorithm is mathematically equivalent to the MI-EKF algorithm[8], so the MI-TSEKF algorithm has higher parameter estimation accuracy than the EKF algorithm. Increasing the number of new interest rates will improve the estimation accuracy of the algorithm, but it will also increase the complexity and computational complexity of the MI-TSEKF algorithm. In order to make the MI-TSEKF algorithm achieve high estimation accuracy, it can also reduce the computational burden. The number of new interest is very important.

2.3. Theoretical analysis of algorithm operation
The calculation of the filter is realized by addition and multiplication, so the number of operations can be measured by the number of operations of multiplication[9]. Let \( a \) be the dimension of the state variable \( X_k \), \( b \) the dimension of the output variable \( Y_k \), and \( c \) the dimension of the input signal \( u_k \). For example, to calculate \( X_{k/k-1} = A_{k-1}X_{k-1} + B_{k-1}u_{k-1} \) in the extended Kalman filter, \( a = 4 \), \( b = c = 2 \), then the number of addition operations is \( a^2 - a + ac = 20 \), and the number of multiplication operations is \( a^2 + ac = 24 \), other formulas The number of calculations is analogized by this[10]. Table 1 lists the MI-TSEKF algorithm calculations statistics table.

It can be seen from the statistics in the above table that when the new interest rate \( p=2 \), the number of addition calculations of the MI-TSEKF algorithm is about 20.5% less than the EKF algorithm, and
the number of multiplication operations is about 12.8% less. It can be seen that the MI-TSEKF algorithm can effectively reduce the amount of computation and reduce the level of hardware requirements. If you continue to increase the number of new interest rates, the MI-TSEKF algorithm is almost no longer advantageous in terms of the amount of computation, so the new interest rate p=2 is selected.

| State variables | Addition amount | Multiplication amount |
|-----------------|-----------------|-----------------------|
| $X_{k/k-1}$     | $a^2 - a + ac = 20$ | $a^2 + ac = 24$ |
| $P_{k/k-1}$     | $2a^3 - a^2 = 112$ | $2a^3 = 128$ |
| $K_k$           | $a^2b + 2ab^2 + b^3 - 2ba = 56$ | $a^2b + 2ab^2 + b^3 = 72$ |
| $X_k$           | $2ba + ac = 24$ | $2ba + ac = 24$ |
| $P_k$           | $a^2b + a^3 - a^2 = 80$ | $a^2b + a^3 - a^2 = 96$ |
| Total           | 292             | 344                   |

3. Sensorless Control Simulation of PMSM Based on MI-TSEKF

In this paper, the simulation model of the position sensorless control system of permanent magnet synchronous motor based on MI-TSEKF is constructed by MATLAB/Simulink toolbox[11-13], as shown in Figure 1.

![Control system simulation model based on MI-TSEKF.](image)

The motor parameters in the simulation are set to be follows: number of pole pairs $p = 1$, stator inductance $L_d = L_q = 4.88 mH$, stator resistance $R_s = 1.703\Omega$, flux linkage $\psi_r = 0.171 Wb$, moment of Inertia $J = 0.6 kg\cdot m^2$. The simulation model of the MI-TSEKF estimation module is built, as shown in Figure 2.

Given the speed $n = 2500 rpm$, with the load $T = 0.1 \cdot N \cdot m$ start, after entering the steady state, a load torque disturbance is suddenly added at $t = 0.25 s$ to verify the correctness of the MI-TSEKF algorithm. The specific simulation results are shown as Figure 2. The simulation results are shown in Figure 3-7.
Figure 2. Simulation model of MI-TSEKF estimation module.

From the simulation results, the MI-TSEKF algorithm can well follow the actual angle and speed. The estimated current of the \( \alpha-\beta \) axis is basically the same as the actual \( \alpha-\beta \) current waveform. When the motor starts or the load of motor is abrupt and stable, we find that the sensorless control system based on the MI-TSEKF algorithm works well and proves the correctness and effectiveness of the algorithm. It can also be seen from the figure that the angle estimation error of the EKF algorithm is rad, the speed estimation error is rpm, and the angle estimation error of the MI-TSEKF algorithm is rad, and the speed estimation error is rpm. It can be seen that the steady-state estimation accuracy of the estimated angle of the MI-TSEKF algorithm is 31\% higher than that of the EKF algorithm, and the steady-state estimation accuracy of the estimated speed is 35\% higher than that of the EKF algorithm. Since the calculation amount is smaller than the EKF algorithm, MI-TSEKF calculates the tracking speed faster than the EKF algorithm. The simulation results prove the validity of the theoretical analysis in the previous section, and also reflect the advantages of the MI-TSEKF algorithm compared with the EKF algorithm.

Figure 3. Actual angle and estimated angle curve.
Figure 4. Angle error curve.

Figure 5. Actual speed and estimated speed curve.

Figure 6. A-axis actual current and estimated current curve.

Figure 7. B-axis actual current and estimated current curve.
4. Conclusion
Based on the comparison of the advantages and disadvantages of various position sensorless control algorithms, this paper starts from the control performance and the difficulty of engineering implementation. Through the improvement of the extended Kalman filter algorithm, the multi-innovation double-segment expansion Kalman is obtained. In order to reduce the computational complexity of MI-EKF algorithm, we propose a multi-innovation two-segment extended Kalman filter algorithm is proposed. The extended Kalman filter and multi-innovation two-segment extended Kalman filter are built by MATLAB/Simulink. The simulation platform of the permanent magnet synchronous motor without position sensor control compares the MI-TSEKF algorithm with the EKF algorithm through simulation experiments, and verifies the correctness and effectiveness of the MI-TSEKF algorithm.

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