Efficient User Scheduling for Uplink Hybrid Satellite-Terrestrial Communication

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Abstract—Due to increasing demands of seamless connection and massive information exchange across the world, the integrated satellite-terrestrial communication systems develop rapidly. To shed lights on the design of this system, we consider an uplink communication model consisting of a single satellite, a single terrestrial station and multiple ground users. The terrestrial station uses decode-and-forward (DF) to facilitate the communication between ground users and the satellite. The channel between the satellite and the terrestrial station is assumed to be a quasi-static shadowed Rician fading channel, while the channels between the terrestrial station and ground users are assumed to experience independent quasi-static Rayleigh fading. We consider two cases of channel state information (CSI) availability. When perfect CSI is available, we derive the instantaneous achievable sum rate of all ground users and formulate an optimization problem to maximize the sum rate. When only channel distribution information (CDI) is available, we derive a closed-form expression for the outage probability and formulate another optimization problem to minimize the outage probability. Both optimization problems correspond to scheduling algorithms for ground users. For both cases, we propose low-complexity user scheduling algorithms and demonstrate the efficiency of our scheduling algorithms via numerical simulations.

Index Terms—Hybrid satellite-terrestrial communication systems, decode-and-forward, user scheduling, outage probability, space-air-ground integrated networks.

I. INTRODUCTION

To expand communication to a broader coverage area and meet the demand of higher throughput [1], [2], satellite communication systems have been extensively studied and utilized across the world. Due to heavy shadowing caused by obstacles when the user moves into buildings or vegetation areas, the line-of-sight (LOS) link between the satellite and terrestrial users might be unavailable [3]. To combat the performance degradation, hybrid satellite-terrestrial relay systems are proposed to improve coverage and reliability of satellite communication with the aid of terrestrial relays [4], [5], [6]. Most current hybrid satellite-terrestrial systems adopt either amplify-and-forward (AF) or decode-and-forward (DF) [7], [8] relaying technique.

For a hybrid satellite-terrestrial system, the performance is strongly related to the design of multiuser access schemes [8], relay selection methods [9] and power allocation strategies [10]. For satellite-terrestrial systems with a single relay, multiuser access schemes, especially user scheduling algorithms, are critical and should be optimized jointly with power allocation schemes. Furthermore, the development of integrated space-terrestrial information network (SIN) and the 6th Generation (6G) communication architecture enforces the system to allow access of massive number of devices. To meet the extremely high throughout and massive connectivity demands of future communication systems, it is vital to comprehensively study efficient user scheduling strategies with low complexity for hybrid satellite-terrestrial communication systems.

To shed lights on the design of such a system, we study user scheduling algorithms for the hybrid satellite-terrestrial model with a single satellite, a single terrestrial station and multiple ground users. The channel from the terrestrial station to the satellite is assumed to be a quasi-static shadowed Rician fading channel, and the channel from each user to the terrestrial station is assumed to be a quasi-static Rayleigh fading channel. Our choice of quasi-static fading is validated by information theoretical analysis of low-latency communication in [11] and [12]. In particular, under quasi-static fading channels, the asymptotic notions of outage capacity and outage probability provide tight approximations to the non-asymptotic performance at low latency. We choose shadowed Rician fading since it provides a good approximation for the land-mobile satellite communication channel [13]. For the links between each ground user and the relay, we choose Rayleigh fading because it is widely used in wireless communications, and is accurate for most terrestrial environments with rich scatterers [14].

A. Main Contributions

We study the hybrid satellite-terrestrial model with either instantaneous channel state information (CSI) or channel
distribution information (CDI) available at the terrestrial station and the satellite. When instantaneous CSI is available, the terrestrial station is informed of the exact channel fading levels of each user, and the satellite is informed of the exact fading level from the terrestrial station. When CDI is available, the terrestrial and the satellite are only provided with the distribution of respective fading models. We study user scheduling algorithms for both cases. Our contributions are summarized as follows.

When instantaneous CSI is available, combining capacity results for the multiple access channel and the relay channel [15], we derive the optimal instantaneous achievable rate region of all ground users. Subsequently, under a homogeneous target rate constraint, we formulate an optimization problem to maximize the achievable sum rate via user scheduling algorithms. The optimal solution requires an exhaustive search and is too complicated for practical use. As a compromise, we first derive theoretical upper and lower bounds on the maximum achievable rate and then propose low-complexity scheduling algorithms. In particular, we propose a greedy algorithm that achieves close-to-optimal performance and a further simplified sub-optimal algorithm that has almost linear complexity. The efficiency, computational complexity and stability of our scheduling algorithms are analytically demonstrated and numerically verified.

When CDI is available, the instantaneous achievable rate region can not be obtained. Instead, under a homogeneous target rate constraint for all ground users, we derive the outage probability of the uplink communication as a function of the rate constraint and CDI. Subsequently, we formulate an optimization problem to minimize the outage probability and derive a theoretical lower bound that serves as the benchmark. Furthermore, to obtain a feasible solution to the optimization problem, we propose a user scheduling algorithm using the idea of alternative optimization (AO) [16], [17]. Via numerical simulations, we find that the outage probability obtained from our user scheduling algorithm converges to our derived benchmark after a small number of iterations. We also numerically illustrate the optimality and computational complexity of our scheduling algorithm by comparing with the optimal exhaustive search method.

B. Related Works

The study on hybrid satellite-terrestrial communication systems has a long history. In contrast to our setting, most papers focused on the downlink model, where a satellite transmits messages to a group of ground users, e.g., [18], [19], [20], [21], [22]. In [18], for a multiuser hybrid AF satellite-terrestrial cooperative network, Yuan et al. proposed an opportunistic user scheduling algorithm using signal-to-noise ratio (SNR)-threshold based feedback. In [19], for a multiuser AF hybrid satellite-terrestrial relay network with opportunistic scheduling, An et al. derived the analytical bounds on the ergodic capacity. For a multiuser satellite-terrestrial downlink model with a multi-antenna satellite and a single-antenna AF relay, Bankey and Upadhyay [20] analyzed the ergodic capacity for opportunistic user scheduling with outdated CSI. However, the spectrum efficiency of the orthogonal opportunistic user scheduling algorithms of the above studies is very low since only one user is allowed to access the channel at a time slot. To improve the throughput and spectrum efficiency of the downlink hybrid system, subsequent works apply non-orthogonal multiple access (NOMA) into hybrid satellite-terrestrial systems to support multiple users simultaneously in a time slot, e.g., [21], [22]. However, studies on downlink NOMA-based integrated terrestrial-satellite network are significantly different from our uplink analysis in this paper.

The more related works to ours is the studies of the uplink hybrid satellite-terrestrial system, e.g., [23], [24], [25]. In [23], Yang et al. derived an upper bound on the ergodic capacity for a multi-beam geostationary earth orbit mobile satellite communication system. In [24], Huang et al. proposed a space division multiple access (SDMA) scheme to maximize the average sum rate of a hybrid satellite-terrestrial AF relaying network, where a high-altitude platform is deployed as a relay to assist the transmission from ground users to the satellite. In [25], Arti studied a two-way AF satellite system with only two ground users and one terrestrial relay and designed beamforming and combining vectors for the satellite.

However, none of the above works use Shannon theory [26] to study the critical problem of efficient user scheduling algorithms to achieve the full potential of the uplink hybrid satellite-terrestrial system. Although above works considered multiple antennas, the analysis is mostly achievable without optimality guarantee. Specifically, these studies used dimensional advantage of multiple antennas to improve the throughput with SDMA or a certain beamforming design. Note that the capacity is the maximal rate achievable of any communication scheme and analysis based on specific coding scheme without optimality guarantee is far from satisfactory.

In this paper, to complement the existing literature in the uplink hybrid satellite-terrestrial system, we use information theoretical results to study efficient user scheduling algorithms and demonstrate close-to-optimality performance of our proposed algorithms. For simplicity, we present results for the single antenna terminals in this paper. Our results can be generalized to the multi antenna terminals by using corresponding information theoretical results for the multiple input and multiple out channels [27], [28].

II. PROBLEM FORMULATION

Notation

We use \( \mathbb{R} \), \( \mathbb{R}_+ \) and \( \mathbb{N} \) to denote the set of real numbers, positive numbers and integers respectively. Denote by \( \mathcal{A} \setminus \mathcal{B} \) the minus between set \( \mathcal{A} \) and \( \mathcal{B} \). For any \( x \in \mathbb{R} \), we use \( \lfloor x \rfloor \) to denote the maximum integer that is less than or equal to \( x \). For any \( (a, b) \in \mathbb{N}^2 \), we use \( [a : b] \) to denote the set of integers between \( a \) and \( b \) and use \( [a] \) to denote \( [1 : a] \). Furthermore, for any two integers \( (a, b) \in \mathbb{N}^2 \) such that \( a < b \), denote by \( [b]_a \) the set of vectors that contain exactly \( a \) unique elements of \( [b] \) and by \( \mathcal{F}(a, b) \) the set of bijective mappings from \( [b]_a \) to \( [b]_a \).

A. The Uplink Communication Model

We consider an uplink model for the hybrid satellite-terrestrial communication system as shown in Fig. 1. There are
$N$ ground users with indices $\mathcal{N} = [N]$, one terrestrial relay, and one satellite, all assumed to have a single antenna. Each user wishes to transmit messages to the satellite with the assist of the relay. Since the resource of the satellite is finite, we assume that at most $K$ users are allowed to transmit messages in a time slot simultaneously. Let $\mathbf{u} = \{u_1, \ldots, u_K\} \in [N]_K$ denote the vector of the indices of the selected users by a scheduling algorithm.

At each time slot, after user scheduling, the communication between the selected $K$ users and the satellite proceeds in the following two phases. In the first phase, for each $k \in [K]$, the selected user $u_k$ aims to send a message $M_{u_k} \in [1 : 2^{nR_{u_k}}]$ to the relay with power $P_1$, rate $R_{u_k}$ and blocklength $n$. Each message $M_{u_k}$ is then encoded into a length-$n$ codeword $X_{u_k}^n$. The received signal at the terrestrial relay is

$$Y_R^n = \sum_{k \in [K]} h_{u_k} X_{u_k}^n + Z_{u_k}^n,$$  

where $h_{u_k}$ is the channel fading coefficient between the user $u_k$ and the relay, and $Z_{u_k}^n$ is the independent additive white Gaussian noise (AWGN) with zero mean and unit variance. Using $Y_R^n$ and CSI for all $K$ users, the terrestrial relay decodes the messages of each user as $\{\hat{M}_{u_k}\}_{k \in [K]}$. In the second phase, the relay encodes its decoded messages $\{\hat{M}_{u_k}\}_{k \in [K]}$ into a combined codeword $X_R^n$ with power $P_2$ and sends it to the satellite. The received signal at the satellite is

$$Y_{\hat{M}}_R^n = h_{D,R} X_R^n + Z_{D,R}^n,$$  

where $h_{D,R}$ is the channel fading coefficient between the relay and the satellite, and $Z_{D,R}^n$ is the independent AWGN with zero mean and unit variance. Finally, using the noisy outputs $Y_{\hat{M}}_R^n$ and CSI on the fading coefficient $h_{D,R}$, the satellite decodes the message $\{\hat{M}_{u_k}\}_{k \in [K]}$.

### B. Fading Channels and Assumptions

We assume that the channel between each ground user $i \in [N]$ and the terrestrial relay is an AWGN with Rayleigh fading, i.e., the fading coefficient satisfies $h_i \sim \mathcal{CN}(0, 2\sigma_i^2\mathbf{1})$. The fading random variables for different ground users are independent. Furthermore, we assume that the channel gain $|h_{D,R}|$ for the channel from the relay to the satellite follows a Shadowed-Rician (SR) distribution with parameters $(\Omega, b_0, m_s)$, i.e., $|h_{D,R}| \sim \text{SR}(\Omega, b_0, m_s)$, where $\Omega$, $b_0$ and $m_s$ are positive real numbers. Here, $\Omega$ denotes the average power of the LOS component, $m_s$ is the fading order, and $2b_0$ represents for the average power of the scattered component.

To evaluate the fundamental limits of the uplink hybrid satellite-terrestrial system, depending on the availability of CSI, we study the instantaneous sum rate as well as the outage probability and formulate corresponding optimization problems to schedule the transmission of ground users. Specifically, we consider the following two cases.

- **Case 1: Perfect CSI**
  Similar to most existing works, e.g., [29], [30], we assume that perfect CSI of all channels are available. The CSI can be obtained by channel estimation during each transmission time slot via a backhaul channel [21]. In this setting, we derive the achievable instantaneous sum rate and propose efficient user scheduling algorithms to maximize the sum rate.

- **Case 2: Statistical CSI**
  The assumption of perfect CSI can be impractical for practical satellite communication systems. To address this concern, we also study the case where only CDI is available. Specifically, the relay only knows the distribution of fading coefficients for channels from each ground user and the satellite only knows the distribution of the fading coefficient for the channel from the relay. In practical systems, CDI can be estimated via channel parameter estimation methods, e.g., the MUSIC algorithm in [31]. Under this setting, we derive the outage probability and propose user scheduling algorithms to minimize the outage probability.

### III. MAIN RESULTS FOR THE CASE WITH PERFECT CSI

In this section, we analyze the instantaneous achievable rate for the given system and formulate an optimization problem to maximize the sum rate. A solution to the optimization problem is equivalent to a user scheduling algorithm. The optimal solution requires an exhaustive search, which is highly complicated and renders its impossible for practical communication systems. To solve this problem, we first derive theoretical lower and upper bounds and then propose low-complexity user scheduling algorithms that can exploit the tradeoff between the complexity and performance.

#### A. Analysis of the Instantaneous Achievable Sum Rate

For each $k \in [K]$, let $S_{u_k} := P_1|\hat{h}_{u_k}|^2$ denote the received SNR for the channel between user $u_k \in [N]$ and the relay and let $S_{D,R} := P_2|h_{D,R}|^2$ denote the SNR for the channel
between the relay and the satellite. Using cut-set bounds for the relay channel and the multiple access channel in [15, Sec. 19.1, Th. 19.1], we conclude that given selected users \( u = (u_1, \ldots, u_K) \), the instantaneous rate of each user \( R_{u_j} \) is upper bounded by

\[
\sum_{j \in [K]} R_{u_j} \leq \min \left\{ C \left( \sum_{j \in [K]} S_{u_j} \right), C(S_{D,R}) \right\}, \tag{3}
\]

where \( C(a) = \log(1+a) \) is the capacity of an AWGN channel with SNR \( a \in \mathbb{R}_+ \).

In a full-duplex DF relaying system with perfect CSI, the sum rate upper bound given by (3) can be achieved via block Markov coding and backward decoding [15, Sec. 16.4.4 and Table 16.2], when superposition encoding and successive cancellation decoding are used at the relay and the satellite. For subsequent analysis, we consider the case where the rate tuples \( \{R_{u_k}\}_{k \in [K]} \) lies at a corner point on the rate region and thus maximal sum rate is achieved. In particular, let \( u := (u_1, \ldots, u_K) \in [N]^K \) and let \( u_1 \rightarrow u_2 \rightarrow \ldots \rightarrow u_K \) be the decoding order of successive cancellation decoding at the relay, then the signal-to-interference plus noise ratio (SINR) for user \( u_k \) at the relay satisfies

\[
\gamma_{u_k} = \frac{S_{u_k}}{\sum_{i \in [k+1:K]} S_{u_i} + 1}, \quad k \in [K-1] \quad \text{and} \quad \gamma_{u_K} = S_{u_K}. \tag{4}
\]

Since the decoding order of successive cancellation at the satellite can be different from that at the relay, we use a bijective mapping \( \Psi \in \mathcal{F}(K, N) \) and the decoding order \( u \) at the relay to describe the decoding order at the satellite. Specifically, for any \( k \in [K] \), when \( u_k \) is the \( k \)-th decoded message at the relay, then it is the \( \Psi(u_k) \)-th decoded message at the satellite. Thus, the decoding order \( u \) and the bijective mapping \( \Psi \in \mathcal{F}(K, N) \) suffice to describe the set of selected users and the decoding orders at both the terrestrial station and the satellite. Note that the transmission power \( P_2 \) is split among all \( K \) messages and thus the SINR for each message from the relay to the satellite satisfies

\[
\gamma_{\Psi(u_k)} = \begin{cases} \frac{\alpha_{\Psi(u_k)} S_{D,R}}{\sum_{i \in [\Psi(u_k)+1:K]} \alpha_i + S_{D,R}} & \text{if } \Psi(u_k) \neq K, \\ \alpha_{\Psi(u_k)} S_{D,R} & \text{otherwise}, \end{cases} \tag{5}
\]

where \( \alpha_{\Psi(u_k)} \) is the power splitting ratio of the message from user \( u_k \) satisfies \( \sum_{k \in [K]} \alpha_{\Psi(u_k)} = 1 \). Therefore, given \( u \) and \( \Psi \), the instantaneous achievable rate of user \( u_k \) satisfies

\[
R_{u_k} = \min \left\{ C(\gamma_{u_k}), C(\gamma_{\Psi(u_k)}) \right\}, \quad k \in [K]. \tag{6}
\]

B. Maximize the Instantaneous Achievable Sum Rate

Under the assumption that at most \( K \) users are allowed to transmit in a time slot, given a per user rate constraint \( R_k \in \mathbb{R}_+ \), we formulate the optimization problem in Problem 1 to maximize the achievable instantaneous sum rate as follows (7)–(16), shown at the bottom of the next page.

The objective function of Problem 1 is the maximum achievable sum rate and we need to optimize over all possible user decoding orders \( u \) at the terrestrial station, all possible decoding orders at the satellite determined by \( u \) and the bijective mapping \( \Psi \), all power allocation vectors \( \{\alpha_{\Psi(u_1)}, \ldots, \alpha_{\Psi(u_K)}\} \) at the relay. The first constraint in (8) makes sure that every scheduled user satisfies the rate constraint \( R_{\text{target}} \), the second constraint in (9) is the power constraint at the relay to make sure the total transmission power at the relay is \( P_2 \), and the third constraint in (10) guarantees that the decoding order at the relay: \( u_1 \rightarrow u_2 \rightarrow \ldots \rightarrow u_K \) is in the descend order of received SINRs. To solve the problem, we need to jointly optimize the scheduled users \( u \) at the terrestrial station, the decoding order \( \{\Psi(u_k)\}_{k \in [K]} \) at the satellite and power allocation vector \( \{\alpha_{\Psi(u_k)}\}_{k \in [K]} \) at the relay.

We remark that Problem 1 can be decomposed into two sub-problems. This is because \( C(\sum_{u \in U} S_{u}) \) and \( C(S_{D,R}) \) depend on different optimized parameters. Specifically, we have sequential sub-problems Problems 2(a) and Problem 2(b), where Problem 2(b) uses the solution of Problem 2(a) to proceed.

By solving Problem 2(a), we obtain the set of scheduled users and its decoding order \( u \) at the relay that maximizes the sum rates of \( K \) users satisfying the rate constraint \( R_{\text{target}} \). Given \( u \), we can then solve Problem 2(b) for power allocation at the relay to maximize the achievable rate from the relay to the satellite that satisfies \( R_{\text{target}} \) for each scheduled user \( u_k \). Note that the objective function of Problem 2(b) only depends on the fading coefficient \( h_{D,R} \) between the satellite and the relay and is thus independent from user scheduling and power allocation strategies.

With the constraint in (15), the power allocation vector \( \{\alpha_{\Psi(u_k)}\}_{k \in [K]} \) satisfies \( \sum_{k \in [\Psi(u_k)+1:K]} \alpha_i S_{D,R} \geq 2R_{\text{target}} - 1 \) for \( \Psi(u_k) \in [K-1] \) and \( S_{D,R} \alpha_K \geq 2R_{\text{target}} - 1 \). Thus, \( \alpha_{\Psi(u_k)} \geq \frac{2R_{\text{target}} - 1}{S_{D,R}} \alpha_K \geq 2R_{\text{target}} - 1 \). Combined with the power constraint that \( \sum_{k \in [K]} \alpha_{\Psi(u_k)} = 1 \), we conclude that Problem 2(b) is feasible and we can always find a feasible power allocation vector \( \{\alpha_{\Psi(u_k)}\}_{k \in [K]} \) if the number of users supported with rate \( R_{\text{target}} \) satisfies

\[
K \leq \frac{\log(S_{D,R} + 1)}{R_{\text{target}}}. \tag{17}
\]

Note that (17) also implies that \( S_{D,R} \) should be larger than \( 2R_{\text{target}} - 1 \) to make sure that at least one user can be served with desired rate, i.e., \( K \geq 1 \).

C. Upper and Lower Bounds on the Instantaneous Achievable Sum Rate

To obtain an exact solution of Problem 1, we need to use an exhaustive search method over all possible user scheduling \( U \in [N]^K \) and find the largest achievable sum rate \( R_{\text{sum}} \). However, such a method is infeasible, especially when the total number of users \( N \) is large. As a compromise, we derive closed-form upper and lower bounds on the instantaneous achievable sum rate \( R_{\text{sum}} \), which can be used as benchmarks for low complexity user scheduling algorithms.

We assume that (17) holds so that Problem 2(b) is feasible and the achievable rate between the relay and the satellite is
Then, to bound $R^*_\text{sum}$, it suffices to derive upper and lower bounds for the optimal value of Problem 2(a). For any decoding order $u \in [N]_K$, let $S_{\max} := \max_{k \in [N]} S_k$ and $S_{\min} := \min_{k \in [N]} S_k$. Furthermore, let $S = \{S_i : i \in [N]\}$, given any $a \in \mathbb{R}^+_+$. Let $\phi(a, \mathcal{A}) := \min_{S_i \in \mathcal{A}} : S_i \geq a S_i$. Finally, given any $R_{\text{target}} \in \mathbb{R}^+_+$, define the following quantities:

\begin{align*}
\gamma_{\text{target}} &:= 2R_{\text{target}} - 1, \\
\hat{S}_{uk} &:= \phi(\gamma_{\text{target}}, S), \\
\hat{S}_{ub} &:= \phi(\gamma_{\text{target}}, \sum_{i \in [k+1:K]} \hat{S}_{ui} + 1), \\
S_{\max} &:= \max_{i \in [K]} S_i, \\
S_{\min} &:= \min_{i \in [K]} S_i, \\
S_{\hat{S}_{ub}} &:= \frac{S_{\max}}{(\gamma_{\text{target}} + 1)^{-k - 1}}, \\
S_{ub} &:= \frac{S_{\max}}{(\gamma_{\text{target}} + 1)^{-k - 2}} - 1, \\
\end{align*}

where $k \in [K - 1]$. Theorem 1: Given any target rate $R_{\text{target}}$ and corresponding $\gamma_{\text{target}}$, the following results hold.

(i) Problem 2(a) is feasible if and only if

\begin{equation}
\frac{\gamma_{\text{target}}}{S_{\max}} \leq \min \left\{ 1, \frac{1}{\sum_{i \in [k+1:K]} \hat{S}_{ui} + 1} \right\}.
\end{equation}

(ii) If Problem 2(a) is feasible, then

\begin{equation}
R^*_\text{sum} \geq \min \left\{ C \left( \sum_{k=1}^K S_{ub}^k \right), C(S_{D,R}) \right\},
\end{equation}

where $S_{ub}^k = \hat{S}_{ub}^k$ for $k \in [2, K]$ and $S_{ub}^1 = S_{\max}$. (iii) $R^*_\text{sum}$ is upper bounded by

\begin{equation}
R^*_\text{sum} \leq \min \left\{ C \left( \sum_{k=1}^K S_{ub}^k \right), C(S_{D,R}) \right\}.
\end{equation}

The proof of Theorem 1 is available in Appendix A.

D. A Greedy Iterative User Scheduling Algorithm

To efficiently solve Problem 2(a) with low complexity, we propose a user scheduling algorithm that corresponds to a solution of Problem 2(a) and achieves a near optimal performance of exhaustive search. Specifically, our algorithm proceeds in two phases: given CSI, we first derive the maximum number of users $K$ that can be served by the system and then propose a greedy iterative user scheduling algorithm to select $K$ users to transmit. Details are as follows.

1) Determine the Maximum Number of Supported Users $K$: Given an $R_{\text{target}}$ and the terrestrial SNR values $S$, we should first determine the number of supported users. This is because to ensure reliable communication of a multiple-access system, individual rate decreases with the increasing of user numbers [15]. In previous analysis, we derive an upper bound on $K$ in (17) that depends on the SNR $S_{D,R}$ of the satellite-relay link. However, $K$ is also related with channel qualities of users. Combined with effects of both, we determine $K$ via the following steps:

\begin{align*}
\text{Problem 1:} \\
R^*_\text{sum} := \max_{\mathcal{A}} \min \left\{ C \left( \sum_{j \in [K]} S_{uj} \right), C(S_{D,R}) \right\},
\end{align*}

s.t. \hspace{1cm}

\begin{align*}
R_{uk} &\geq R_{\text{target}}, \\
\sum_{k \in [K]} \alpha_{\phi(u_k)} &= 1, \\
S_{ui} &\geq S_{uj} \text{ for } (i, j) \in [K]_2 \text{ and } i > j.
\end{align*}

\begin{align*}
\text{Problem 2(a):} \\
\max_{u \in [N]_K} \left( \sum_{u_k \in \mathcal{U}} C \left( \sum_{u_k \in \mathcal{U}} S_{uj} \right) \right),
\end{align*}

s.t. \hspace{1cm}

\begin{align*}
C(\gamma_{uk}) &\geq R_{\text{target}}, k \in [K], \\
S_{ui} &\geq S_{uj} \text{ for } (i, j) \in [N]_2 \text{ and } i > j.
\end{align*}

\begin{align*}
\text{Problem 2(b):} \\
\max_{\psi \in \mathcal{F}(K,N), \alpha(\psi(u_k)) \in [0,1]^K} \left( \sum_{k \in [K]} C(S_{D,R}) \right),
\end{align*}

s.t. \hspace{1cm}

\begin{align*}
C(\gamma_{\psi(u_k)}) &\geq R_{\text{target}}, k \in [K], \\
\sum_{k \in [K]} \alpha_{\psi(u_k)} &= 1.
\end{align*}
Let $L$ follows. For each $S_i \in [N]$, calculate $S_i$ for $i \in [K]$, and $S_{\text{max}}, S_{\text{min}}, \gamma_{\text{target}}$. Choose the first user as $u_1^* = \arg \max_{i \in [N]} S_i, k = 2$, $\mathcal{U}_2 = \mathcal{U}_1 \backslash \{u_1^*\}$, while $k \leq K$ do

- Calculate the set $\mathcal{V}_k^*$ from (29)
- if The set $\mathcal{V}_k^*$ is empty then $k \leftarrow k - 1$
- else
  - Select the $k$-th user as $u_k^* = \arg \max_{i \in \mathcal{V}_k^*} S_i$
  - $\mathcal{U}_k \leftarrow \mathcal{U}_k \backslash \{u_k^*\}$, $\mathcal{U}_{k+1} \leftarrow \mathcal{U}_k$
  - $k \leftarrow k + 1$
- end if
- end while

### E. A Low Complexity User Scheduling Algorithm

The complexity of the user scheduling scheme in Algorithm 1 is strongly related to the channel qualities of terrestrial users. It scales polynomially with the total number of users $N$ and scales exponentially with the selected number of users $K$ in the worst case. Such a complexity might still not be feasible for power limited terrestrial stations.

In claim (ii) of Theorem 1, we derive a lower bound on the achievable sum rate using $\{S_{\text{lb}}^k\}_{k \in [K]}$. Inspired by this idea, we propose a lower-bound based user scheduling algorithm (LBUS) by choosing all users to our theoretical derivations $\{S_{\text{lb}}^k\}_{k \in [K]}$ except $u_K^*$. The user scheduling scheme is summarized in Algorithm 2.

### F. Numerical Results

In this section, we present simulation results of our proposed algorithms, i.e., the greedy algorithm GIUS and the lower complexity algorithm LBUS, under the assumption of knowledge of perfect CSI for channels. We assume that $\{S_i\}_{i \in [N]}$ are generated i.i.d. from a Rayleigh distribution with the variance $P_i \sigma^2 = 10$. To better evaluate the performance of different user scheduling schemes, we assume that the channel between the satellite and the relay is strong enough so that the sum rate is only restricted by user scheduling schemes.

1) Achievable Sum Rate: In Figure 2, we plot the simulated performances of our proposed user scheduling algorithms GIUS and LBUS, versus the optimal exhaustive search, a time division multiple access scheme (TDMA) where each ground user acquires the channel by a equal time duration, an opportunistic user scheduling scheme where the system always selects the user with the strongest channel gain [19], and our derived upper and lower bounds in Theorem 1. For each user scheduling algorithm, we obtain each data point by averaging the simulated results of 5000 independent simulations. Figure 2 shows that our GIUS user scheduling algorithm achieves sum rate with negligible loss compared
with exhaustive search and thus demonstrates the efficiency of our GIUS algorithm. Comparing the results in Fig. 2(a) and (b), we conclude that the gap between the performance of GIUS and the upper bound becomes smaller as the total number of users $N$ increases. This phenomenon implies that GIUS tends to be “optimal” for large-scale user groups. It is reasonable because the received SNRs of selected users will tend to the optimal solution $\{S_{uk}\}_{k \in [K]}$ when $N$ is large enough.

Besides, Fig. 2 also shows that the performance of LBUS with nearly-linear complexity has worse performance than GIUS, but still better than TDMA and opportunistic user scheduling. This implies that LBUS algorithm, although simple, is still an effective non-orthogonal user scheduling algorithm. Finally, we remark that as $R_{\text{target}}$ increases, the performance of all user scheduling algorithms becomes identical. This is because when $R_{\text{target}}$ is sufficiently large, only one user can be served with the desired target rate and the user scheduling reduces to the orthogonal case.

2) Computational Complexity: In this section, we discuss the computational complexity of our user scheduling algorithms with respect to the total number of users $N$ and the scheduled number of users $K$. From the analysis in Section III-E, we conclude that the user scheduling algorithm LBUS has nearly linear complexity with respect to $N$ as it only needs to select the $K$-th user. The computational complexity of GIUS is more complicated. Without loss of generality, assume that the transmit SNR is normalized as 1 and assume that the channel from each user to the terrestrial station is subject to Rayleigh fading with the same variance $\sigma^2$, i.e., $h_i \sim CN(0, 2\sigma^2I)$ for all $i \in [N]$. For each $k \in [K]$, let the number of candidates of GIUS be $N_k$ and its statistical average with respect to channel statistics is $E(N_k) = \int_{S_{\min}}^{S_{\max}} \left( T(k-1)-L_{\min}(k)-S_{\min} \right) e^{-S/\sigma^2} dx$. Specifically, we have (31)–(34), shown at the bottom of the next page.

For $N$ large enough, we have $S_{uk-1} \approx S_{ub}$ as the received SNRs of selected users will tend to the theoretical optimal solution given by (22) in this case. If $\frac{S_{\min}}{\gamma_{\text{target}}(\gamma_{\text{target}}-1)} - L_{\max}(k) - 1 > S_{\min}$, it follows from (31)-(34) that $E(N_k) \leq N e^{-\frac{S_{\min}}{2\sigma^2}} \left( 1 - e^{-S_{\min}/\sigma^2} \right) \frac{x_{\min}}{x_{\min} - 1}$.
which is further upper bounded by

\[ \mathbb{E} (N_k) \leq Ne^{-\frac{x_{\min}}{2\sigma^2}} \left( 1 - e^{-\frac{1}{2\sigma^2} (L_{\min}(k) + 1 + S_{\min})} \right), \quad (35) \]

when \( k \) is large enough. Thus, the average number of candidate users \( \mathbb{E} (N_k) \) decrease exponentially with \( k \), which is the total number of selected users so far. This implies that the set of candidate users shrinks significantly after selecting each user and this is the reason why GIUS has much lower complexity compared to exhaustive search.

Next, we numerically simulate the complexity of user scheduling algorithms. To do so, we run 2000 independent simulations of exhaustive search, GIUS, and LBUS algorithms with the target rates \( R_{\text{target}} = 0.6/1.2 \) bit/s/Hz. We then calculate the average computing times of each user scheduling algorithm. The simulated results are plotted in Figure 3, where Fig. 3(a) demonstrates the running time and Fig. 3(b) plots the achievable sum rate.

Fig. 3(a) shows that when \( R_{\text{target}} = 1.2 \) bit/s/Hz, the complexity of exhaustive search is much higher than our proposed algorithms GIUS and LBUS. Further, the gaps between the computational complexity of exhaustive search and our algorithms grow when the total number of users grow. On the other hand, Fig. 3(b) shows that for the same case, the achievable sum rates of our GIUS algorithm and exhaustive search are almost the same. This implies that GIUS achieves close-to-optimal performance with much reduced complexity.

When the target rate decreases to \( R_{\text{target}} = 0.6 \) bit/s/Hz, the number of selected users \( K \) grow. In this case, it is out of computing power to simulate exhaustive search and we only compare the performance of our proposed algorithms. As expected, LBUS has a much reduced complexity, especially for \( K \geq 20 \). Surprisingly, the loss of achievable rate is tolerable and implies that our linear complexity algorithm LBUS is very useful when the number of selected users is small.

To further compare the computational complexity of our proposed algorithms GIUS and LBUS, we simulate both algorithms for different number of selected users \( K \) by changing the target rates \( R_{\text{target}} \), when the total number of users \( N = 40 \). The average computing times are plotted in Figure 4. From Figure 4, we find that the computational complexity of GIUS is much more sensitive to the number of selected users \( K \) (and thus the target rate \( R_{\text{target}} \)) while LBUS roughly changes linearly with the number of selected users. In particular, the average computing times of GIUS grows quickly and reaches its maximum point roughly for \( K = 20 \), while LBUS has much smaller running time for almost all cases.

We also compare the stability of computational complexity of our algorithms. To do so, by setting \( N = 40 \) and \( R_{\text{target}} = 0.4 \) bit/s/Hz, we simulate both algorithms 200 times independently and record the running time. The results are plotted.

\[
\mathbb{E} (N_k) = N \int_{S_{\min}}^{\min \{ T(k-1) - L_{\min}(k) - 1, S_{\min} \}} \frac{1}{2\sigma^2} e^{-\frac{x}{2\sigma^2}} dx
\]

\[
\leq N \int_{S_{\min}}^{T(k-1) - L_{\min}(k) - 1} \frac{1}{2\sigma^2} e^{-\frac{x}{2\sigma^2}} dx
\]

\[
= N \left[ e^{-\frac{x_{\min}}{2\sigma^2}} - e^{-\frac{T(k-1) - L_{\min}(k) - 1}{2\sigma^2}} \right]
\]

\[
\leq N \left( e^{\frac{x_{\min}}{2\sigma^2}} - e^{\frac{T(k-1) - L_{\min}(k) - 1}{2\sigma^2}} \right)
\]

\[
\leq N \left( e^{\frac{x_{\min}}{2\sigma^2}} - e^{\frac{1}{2\sigma^2} \left( S_{\max}/\gamma_{\text{target}} - \sum_{i=2}^{k-1} S_{\min,i} - L_{\min}(k-1) \right)} \right).
\]
For simplicity, given each \( k \in [M] \), we use \( \mathcal{M}_k \) to denote users in group \( k \), whose channel fading coefficients follow Rayleigh distribution with parameter \( \sigma^2_k \), i.e., \( h_j \sim \mathcal{CN}(0, 2\sigma^2_k I) \) for all \( j \in \mathcal{M}_k \) and \( \sigma^2_k \) is the CDI of users in group \( \mathcal{M}_k \). Note that when only CDI is available, user selection is essentially user group selection. For each time slot, \( K \leq M \) users are selected to transmit messages. Let \( \mathbf{w} := \{w_1, \ldots, w_M\} \in [M]^M \) be the set of the indices of selected user groups and let \( \mathbf{v} := (v_1, \ldots, v_K) \in [N]^K \) be the index vector of the selected users where \( v_k \in \mathcal{M}_{w_k} \). Each user \( v_k \) in the group \( \mathcal{M}_{w_k} \) is selected in a time division manner.

The communication proceeds in the two phases as described in Section II. After user scheduling, \( K \) users \( \mathbf{v} \), each from a group are selected. For each \( k \in [K] \), user \( v_k \) sends a message \( \mathbf{M}_{v_k} \) to the satellite with power \( P_1 \) with the aid of the terrestrial relay, who works in full-duplex DF mode with power \( P_2 \). Successive cancellation decoding is done at the relay and the satellite. The decoding order at the relay is given by \( v_1 \rightarrow v_2 \rightarrow \ldots \rightarrow v_K \), which satisfies \( \sigma^2_{v_1} > \sigma^2_{v_2} > \ldots > \sigma^2_{v_K} \). This assumption simplifies the theoretical analysis in the following section. The decoding order at the satellite is related with that at the relay via the bijective mapping \( \Psi' \in \mathcal{F}(K, M) \).

### A. Expressions of Outage Probabilities

When only CDI is available, we consider the outage probability, which is the probability that the transmission at a given rate is not supported, as the performance criterion. Given \( \mathbf{v} = (v_1, \ldots, v_K) \), the outage event of user \( v_j \) occurs if any one of the first \( j-1 \) user suffers an outage event. Specifically, given a target rate \( R_{\text{target}} \) and thus corresponding \( \gamma_{\text{target}} \) (cf. (19)), for each \( k \in [K] \), the outage probability for user \( v_k \) during the first phase satisfies that for each \( k \in [K-1] \)

\[
\mathbb{P}_{v_k, \text{out}} = 1 - \mathbb{P}
\left( S_{v_j} \geq \left( \sum_{i \in [j+1:k]} S_{v_i} + 1 \right) \gamma_{\text{target}}, \quad j \in [k] \right),
\]

and

\[
\mathbb{P}_{v_K, \text{out}} = 1 - \mathbb{P}
\left( S_{v_j} \geq \sum_{i \in [j+1:k]} S_{v_i} + 1 \gamma_{\text{target}}, \quad \forall j \in [K-1], \quad \text{and} \quad S_{v_K} \geq \gamma_{\text{target}} \right),
\]

where \( S_{v_k} = P_1 |h_{v_k}|^2 \) is the received SNR of the signals from user \( v_k \).

Recall that DF is used at the relay and a power allocation vector is used to split the power \( P_2 \) at the relay among \( K \) messages of users. Recall that \( \gamma_{\Psi(v_k)} \) is the received SNR of user \( v_k \) defined in (5). For the second phase, the outage probability of each user \( v_k \) is

\[
\mathbb{P}_{v_k, \text{out}} = 1 - \mathbb{P}
\left( \gamma_{\Psi(v_k)} \geq \gamma_{\text{target}}, \quad j \in [k] \right), \quad k \in [K].
\]

Note that (38) is subject to the power allocation \( \{\alpha_{\Psi(v_k)}\}_{k \in [K]} \) and the received SNR \( S_{\text{D,R}} \) for the satellite-relay channel. From arguments around (17), if the number of selected users
satisfies $K \leq \log(S_{\text{sat}}+1)/\bar{R}_{\text{target}}$, we can always find a power allocation scheme that guarantees $\gamma_{\text{target}}$ is larger than the threshold $\gamma_{\text{target}}$. Thus, the outage probability of each user at the second hop satisfies $P_{\text{out}}^{(2)}_{v_k,\text{out}} = 1 - \mathbb{P}\left(K \leq \log(S_{\text{sat}}+1)/\bar{R}_{\text{target}}\right)$ for $k \in [K]$, i.e., $P_{\text{out}}^{(2)}_{v_k,\text{out}} = P(S_{\text{D},R} < 2^K \bar{R}_{\text{target}} - 1)$ for $k \in [K]$.

In Theorem 2, we derive a closed-form expression for $P_{v_k,\text{out}}$, the outage probability for user $v_k$ at the relay. As we shall show, our user scheduling algorithm relies only on $P_{v_k,\text{out}}$.

Theorem 2: The following claims hold.

(i) Given the selected users $v = \{v_1, v_2, \ldots, v_K\}$ and the corresponding CDI $\{\sigma_{v_k}^2\}_{k \in [K]}$ for their terrestrial channels to the relay, let $\lambda_{v_k} := 1/2P_{v_k}^2$ for $k \in [K]$.

The outage probability $P_{v_k,\text{out}}$ of user $v_k$ during the first phase of uplink communication satisfies

$$P_{v_k,\text{out}}^1 = 1 - \frac{\lambda_{v_k}A_{K-1}}{\gamma_{\text{target}}B_{K-1} + \lambda_{v_k}} e^{-\gamma_{\text{target}}((1+\gamma_{\text{target}})B_{K-1} + \lambda_{v_k})},$$

where

$$A_1 = 1, B_1 = \lambda_{v_1},$$

$$A_k = \frac{\lambda_{v_k}A_{k-1}}{\gamma_{\text{target}}B_{k-1} + \lambda_{v_k}},$$

$$B_k = (1 + \gamma_{\text{target}})B_{k-1} + \lambda_{v_k}, k \in [2, K-1].$$

(ii) The outage probability $P_{\text{out}}^2$ for the second phase communication satisfies

$$P_{\text{out}}^2 = \left(\frac{2b_0 s_m}{2b_0 s_m + \Omega}\right)^m \sum_{n=0}^{\infty} \frac{(m)_n}{(n)!^2} \left(\frac{\Omega}{2b_0 s_m + \Omega}\right)^n \Gamma\left(n+1, \frac{2^K \bar{R}_{\text{target}} - 1}{2b_0 P_2}\right),$$

where $(x)_n$ is the Pochhammer symbol and $\Gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt$ is the unnormalized incomplete Gamma function.

(iii) Combining the above two claims, the outage probability for user $u_K$ satisfies $P_{u_K,\text{out}} = 1 - (1 - P_{v_k,\text{out}}^1)(1 - P_{\text{out}}^2)$.

The proof of Theorem 2 is available in Appendix B.

C. Minimize the Outage Probability

Note that the outage probability $P_{\text{out}}^2$ is a fixed function of the SNR of the satellite-relay channel and does not rely on user scheduling in the first phase. Thus, to minimize the outage probability of uplink communication, it suffices to minimize the outage probability for the first phase. We then have the following optimization problem.

Problem 3:

$$P_{\text{out}}^* := \min_{v \in [N], w \in [M], k \in [K]} \max_{v_k \in M_{v_k}, k \in [K]} \{P_{v_k,\text{out}}\}$$

subject to $P_{v_k,\text{out}} = 1 - (1 - P_{v_k,\text{out}}^1)(1 - P_{\text{out}}^2)$,

$$\lambda_{v_i} < \lambda_{v_j}, \text{ for } (i, j),$$

$$\lambda_{v_i} \geq \lambda_{\text{min}}, \text{ where } \lambda_{\text{min}} := \min_{k \in [M]} \{\lambda_k\},$$

and $\lambda_k := \frac{1}{2P_{v_k}^2}$ for each $k \in [M]$. The relaxed optimization problem is as follows.

Problem 4:

$$\min_{\lambda_{v_i} \geq \lambda_{\text{min}}, k \in [K]} P_{v_k,\text{out}}$$

subject to $P_{v_k,\text{out}} = 1 - (1 - P_{v_k,\text{out}}^1)(1 - P_{\text{out}}^2)$.

Theorem 3: Problem 4 has unique optimal solution $\{\lambda_{v_k}^\text{op}\}$, which satisfies

$$\lambda_{v_1}^\text{op} = \frac{1}{2\sigma_{w_k}^2},$$

$$\lambda_{v_k}^\text{op} = \frac{\gamma_{\text{target}}B_{k-1}}{2} + \sqrt{\frac{\gamma_{\text{target}}B_{k-1}}{2} + 4B_{k-1}},$$

$$\lambda_{v_k}^\text{op} = \left\{\begin{array}{ll}
\frac{1}{\lambda_{v_k}^\text{op} + D_{k,i}} + \gamma_{\text{target}}\left(1 + \gamma_{\text{target}}\right)^{K-k}, & \text{for } i \in [k+1 : K],
\end{array}\right.$$
Algorithm 3 AO Iterative User Group Selection Algorithm (AOIUS)

**Input:** User group numbers $M$, user group indices $\mathcal{M}_k (k \in [M])$, CDI parameters $\{\sigma_k^2\}_{k \in [M]}$, the transmit power $P_i$, the target SINR $\gamma_{\text{target}}$, the threshold $\delta$.

**Output:** Scheduled user groups $w^* = \{w_1^*, w_2^*, \ldots, w_K^*\}$.

1. Calculate $\lambda_k = \frac{1}{2P_i\sigma_k^2}$ for $k \in [M]$; Choose the first user group as $w_1^* = \text{arg min}_{k \in [M]} \{\lambda_k\}$.
2. Initialize $\mathcal{W} = \{w_1^*, w_2^*, w_3, \ldots, w_K^*\}$ by randomly selecting from $[M]$;
3. Set $\mathbb{P}^1_{\text{out}}(0) = 1$. Calculate the outage probability $\mathbb{P}^1_{\text{out}}(1)$ according to (39); $i = 1$.

   **while** $|\mathbb{P}^1_{\text{out}}(i) - \mathbb{P}^1_{\text{out}}(i-1)| > \delta$ **do**

   **for** $k = 2, \ldots, K$ **do**

   1. $\mathcal{W} \leftarrow \mathcal{W} \setminus w_k^{i-1}$.
   2. Calculate the zero point $z_{w_k^i}$ of $h(\lambda_{w_k^i})$.
   3. Calculate $w_k^i$ by $w_k^i = \text{arg min}_{j \in [M]} \left\{\mathcal{W} \setminus \{\lambda_j - z_{w_k^i}\} \mid \lambda_{k-1} < \lambda_j < \lambda_{k+1}\right\}$.
   4. $\mathcal{W} \leftarrow \mathcal{W} \cup \{w_k^i\}$.

   **end for**

   **end while**

   Calculate $w_k^i$ by $w_k^i = \text{arg min}_{j \in [M]} \left\{\mathcal{W} \setminus \{\lambda_j - z_{w_k^i}\} \mid \lambda_{k-1} < \lambda_j < \lambda_{k+1}\right\}$,

   where $z_{w_k^i} = -\frac{\gamma_{\text{target}} B_{k-1}}{2} + \frac{\gamma_{\text{target}} B_{k+1} + 4B_{k-1}}{2}$.

   **end while**

Note that Theorem 3 provides an optimal solution $\{\lambda_{w_k^i}^0\}_{k \in [K]}$ to Problem 4, a relaxed version of Problem 3. To obtain a feasible solution, our user scheduling algorithm proceeds by choosing the $\lambda_{w_k^i}$ close to $\lambda_{w_k^i}^0$. As stated before, the user selection is essentially user group selection.

We select user groups $w^* = \{w_1^*, \ldots, w_K^*\}$ as follows. The first user group $w_1^*$ is selected as the one with the maximal CDI $\sigma_k^2$, i.e., $w_1^* = \text{arg min}_{k \in [M]} \{\lambda_k\}$. When only two users are allowed, the second one is selected as $w_2^* = \text{arg min}_{k \in [M]} \{\lambda_k - z_{w_2^i}\}$, where $z_{w_2^i} := -\frac{\gamma_{\text{target}} \lambda_{w_1^*}}{2} + \frac{\gamma_{\text{target}} \lambda_{w_1^*}^2 + 2\lambda_{w_1^*}}{2}$.

When more than two users are allowed, i.e., $K > 2$, the selection of user groups is more involved and we propose an iterative user group selection scheme based on alternate optimization that is summarized in Algorithm 3 and named AOIUS. We remark that AOIUS converges because the outage probability monotonically decreases during iteration rounds.

Specifically, for $i \in \mathbb{N}$, in the $i$-th iteration, for each $k \in [2 : K]$, the $k$-th user (group) is allowed, i.e., $\sigma_k^2$ is updated in the current iteration and obtained in the last iteration. Therefore, the outage probability converges quickly after the first round of the iteration, and slows down as the iteration progresses.

E. Numerical Results

To demonstrate the efficiency of our proposed algorithm, we systematically simulate the performance of AOIUS in terms of outage probability and computational complexity. The channel between the relay and the satellite are assumed to be under frequent heavy shadowing, whose parameters are given by $(b_0, m_s, \Omega) = (0.063, 0.739, 8.97 \times 10^{-4})$ [13]. In order to eliminate random effects, we assume that $P_1\sigma_k^2$ of group $k$ is uniformly distributed between $(-\infty, 20 \text{ dB})$, based on which average outage probability is computed after 200 times of average channel parameters realizations. The transmit power at the relay is given by $P_2 = 30 \text{ dB}$. Note that the effects of the noise and large-scale fading of the terrestrial and the satellite channels are not considered during simulation, as they can be included in the transmit power $P_1$ and $P_2$ and normalized.

1) Convergence: To illustrate the convergence of our user scheduling algorithm AOIUS, we simulate AOIUS and plot the results in Fig. 6. Without loss of generality, assume that the channel gain between the satellite and the relay is strong enough so that $\mathbb{P}^2_{\text{out}} \approx 1$. This assumption doesn’t affect the convergence analysis. The terrestrial average received SNR $P_1\sigma_k^2$ of group $k$ is assumed uniformly distributed over $(-\infty, 20 \text{ dB})$. The target rate is set as $R_{\text{target}} = 0.02$ bit/s/Hz and the total number of user groups $M$ is assumed to be 7000. The threshold $\delta$ for the stopping criterion is set as zero, which means the algorithm stops only when the final outage probability doesn’t change. We simulate AOIUS when the number of allowed users $K = 5, 10, 15$. For each $K$, we simulate AOIUS five times for independent channel realizations and plot calculated outage probabilities versus iterations. From Fig. 6, we find that using AOIUS, the outage probability
of scheduled users monotonically decreases with iteration times and finally converges to a fixed value, which is the theoretical lower bound on the outage probability obtained from Theorem 3. This implies that our user scheduling algorithm is close to optimal in this numerical example.

2) Outage Probability: To demonstrate the efficiency of our proposed algorithm, we simulate AOIUS for the cases of $K=2,3$ and compare the results with the optimal exhaustive search strategy. We set $M=10$, and $\delta=0$ as the stopping criterion. The results are plotted in Fig. 7. The difference between theoretical and simulation results is that the theoretical results correspond to simulations of user schedule algorithms where the outage probability is calculated from (55) and numerical correspond to Monte Carlo simulation of the outage probability, which is calculated as the ratio of outage event for $10^4$ independent realizations of fading channels. For both user scheduling algorithms, the matching of theoretical and numerical simulation results verify the correctness of the calculation of the outage probability in (55). Furthermore, since exhaustive search is optimal, the almost negligible gap between the outage probabilities of AOIUS and exhaustive search implies that our proposed user scheduling algorithm is efficient in minimizing the outage probability. Finally, we comment that the outage probability increases with number of selected users $K$. This is because interference among users is more severe when more users transmit messages simultaneously.

3) Computational Complexity: The optimal exhaustive search is not practical since its complexity scales exponentially as the number of selected users grows. The complexity of AOIUS depends on channel distribution, the target rate, the total number of user groups $M$ and the number of selected user groups $K$. To illustrate the complexity of AOIUS, we simulate the computing times of AOIUS for different $K$ and target rates. The complexities of AOIUS and exhaustive search are shown in Fig. 8, where each data point corresponds to the average computing time of a single run of user scheduling algorithms over 100 realizations of $\{\sigma_i\}_{i\in[M]}$. In this case, we set $M=40$ and $R_{\text{target}} = 0.02\text{bit/s/Hz}$. From Fig. 8, we find that the complexity of AOIUS is much lower and stable, especially for a large number of selected users $K$.

Finally, to explore the effect of the target rate $R_{\text{target}}$ and the selected number of user groups $K$ on the computational complexity of AOIUS, we simulate AOIUS for $M=5000$ user groups and target rates $R_{\text{target}} = 0.04/0.1/0.4/1\text{bit/s/Hz}$ respectively and plot the results in Fig. 9. From Fig. 9, we observe that the computation complexity of AOIUS roughly grows linearly with the number of selected users when $R_{\text{target}} = 0.04/0.1\text{bit/s/Hz}$. However, when $R_{\text{target}} = 0.4/1\text{bit/s/Hz}$, the computational complexity does not scale much with $K$. This is because when the target rate is relatively high, the outage event occurs frequently. In extreme case, AOIUS stops at the very beginning of the algorithm as the outage probability is always equals to 1. Thus, the computational complexities for the cases of $R_{\text{target}} = 0.4/1\text{bit/s/Hz}$ drop down when $K$ is larger than a certain threshold (denoted by $K_0$), and then grows very slowly as $K$ tends to infinity. This implies that the system cannot accommodate as more than $K_0$ users with the desired target rate. However, the exact characterization of $K_0$ is challenging and left as future work.

V. CONCLUSION

We studied the hybrid uplink satellite-terrestrial communication model with a single satellite, a terrestrial station as a relay and finitely many ground users. Under mild conditions on the channel models, we studied user scheduling algorithms to maximize the sum rate when perfect CSI of all channels...
are available and to minimize the outage probability when only CDI of all channels are available. Specifically, when perfect CSI is available, we first derived theoretical upper and lower bounds on the instantaneous sum rate and then proposed a greedy iterative user scheduling algorithm that achieves a near optimal performance with lower complexity. Furthermore, we also proposed a user scheduling algorithm based on our lower bound that has worse performance than the greedy algorithm but also achieves much lower complexity, especially if one wishes to schedule a large number of users in a time slot. When only CDI is available, we first derived analytical expressions for the outage probabilities of two-phase communication and then proposed an alternative optimization based user scheduling algorithm that strikes a balance between performance and complexity. Our results in this paper could shed light on and inspire the design of user scheduling algorithms for uplink communication of hybrid satellite-terrestrial models.

There are several avenues for future research. Firstly, one can generalize our results to the case where the satellite, the relay and even all the grounds users are equipped with multiple antennas. To do so, one need to use corresponding information theoretical results for the multiple input and multiple output channels [27], [28]. Secondly, one could generalize our results to the more practical communication models with multiple terrestrial relays and multiple satellites. Such analysis requires novel ideas of terrestrial station selection from the perspective of the satellite. Finally, one can also consider block fading models and derive the corresponding results for both CSI and CDI cases [35], [36].

APPENDIX

A. Proof of Theorem 1

Note that the optimizing over $u$ is equal to that over SNRs since we select users and decoding orders using SNR.

(i) The if part is easy. If (23) holds, $\{\hat{S}_{ui}^b\}_{k \in [K]}$ is a feasible solution and Problem 2(a) is feasible. Conversely, the only if part is proved through the following two steps. Firstly, we show that if Problem 2(a) is feasible, then any feasible solution $\{S_{uk}\}_{k \in [K]}$ satisfies $S_{uk} \geq \hat{S}_{ui}^b$ for each $k \in [K]$, which implies that $\{\hat{S}_{ui}^b\}_{k \in [K]}$ lower bounds any feasible solutions to Problem 2(a). Secondly, we use the above lower bound to conclude a contradiction if (23) fails to hold and Problem 2(a) is feasible.

We first show that $\{\hat{S}_{ui}^b\}_{k \in [K]}$ lower bounds any feasible solutions. If $\{\hat{S}_{ui}^b\}_{k \in [K]}$ is feasible to Problem 2(a) but satisfies $S_{ui} < \hat{S}_{ui}^b$ for some $i \in [K]$, we have that if $i = K$

$$\gamma_{\text{target}} \leq S_{uk} < \phi(\gamma_{\text{target}}, S)$$

and otherwise

$$\gamma_{\text{target}} \left( \sum_{j \in [i+1:K]} \hat{S}_{uj}^b + 1 \right) \leq S_{ui}$$

$$< \phi \left( \gamma_{\text{target}} \left( \sum_{j \in [i+1:K]} \hat{S}_{uj}^b + 1 \right), S \setminus \{\hat{S}_{ui}^b\}_{t \in [i+1:K]} \right)$$



However, from the definition of $\phi(\cdot)$, no such $S_{ui}$ exists. Therefore, any feasible solution $\{S_{uk}\}_{k \in [K]}$ satisfies $S_{uk} \geq \hat{S}_{ui}^b$ for $k \in [K]$. Secondly, if (23) fails to hold, then either $\gamma_{\text{target}} > S_{\text{max}}$ or there exists an index $i \in [K - 1]$ that satisfies $\gamma_{\text{target}} \left( \sum_{j \in [i+1:K]} \hat{S}_{uj}^b + 1 \right) > S_{\text{max}}$. For the former case, if Problem 2(a) is feasible, then from our claim above, each feasible solution $\{S_{uk}\}_{k \in [K]}$ satisfies $S_{uk} \geq \hat{S}_{ui}^b > S_{\text{max}}$. This is impossible from the definition of $S_{\text{max}}$. For the latter case, if Problem 2(a) is feasible, then our above claim implies that $\hat{S}_{ui}^b > \gamma_{\text{target}} \left( \sum_{j \in [i+1:K]} \hat{S}_{uj}^b + 1 \right)$ and each feasible solution $\{S_{uk}\}_{k \in [K]}$ satisfies $S_{ui} \geq \hat{S}_{ui}^b > S_{\text{max}}$. Again, this is impossible.

Therefore, we have now showed that Problem 2(a) cannot be feasible if (23) fails to hold and thus the only if part is proved.

(ii) The lower bound follows since the chosen user scheduling vector $u$ corresponding to $\{\hat{S}_{ui}^b\}_{k \in [K]}$ is a feasible solution to Problem 2(a).

(iii) The upper bound is obtained by relaxing the constraints in Problem 2(a) and solving the following new linear programming problem

Problem 2(a) LP :

$$\max_{S_{uk} \leq S_{\text{max}}} C \left( \sum_{k \in [K]} S_{uk} \right)$$

s.t. $C(\gamma_{uk}) \geq R_{\text{target}}, k \in [K],$

$$S_{ui} > S_{uj}, (i,j) \in [N]_2$$

$$i \neq j.$$  

where the optimization parameters $S_{R,uk}$ are assumed continuous and satisfy $S_{uk} \leq S_{\text{max}}$. Note that claim (iii) holds since the optimization region of SNRs is enlarged.

B. Proof of Theorem 2

We only provide proofs for claim (i) and (ii) in Theorem 2, as the last claim is straightforward.

(i) For Rayleigh fading channels, the probability density function (pdf) of the received SNR $S_{R,k}$ is $P_{S_{R,k}}(x) = \lambda_k e^{-\lambda_k x}$, where $\lambda_k = \frac{1}{2\pi\sigma_k^2}$. Using the properties of exponential distribution, we can derive the close-form of the outage probability during the first hop as (55)-(57), shown at the top of the next page.

(ii) The outage probability during the second transmission phase is given by $P_{\text{out}} = P(S_{D,R} < 2K R_{\text{target}} - 1) = \int_0^{2K R_{\text{target}} - 1} p_{S_{D,R}}(s) ds$, where $p_{S_{D,R}}(s)$ is the pdf of $S_{D,R}$. Using [13] (Eq. (3)), the pdf of $S_{D,R}$ is derived as (58), shown at the top of the next page, where $\text{F}(a,b,x)$ is the confluent hyper-geometric function [34]. The proof is done by using the fact that $\text{F}(a,b,x)$ can be simplified as

$$\text{F}(a,b,x) = \sum_{n=0}^{\infty} \frac{a^n}{n! (b)_n} x^n$$  

[37].
C. Proof of Theorem 3

Note that \( P_{u_{K, \text{out}}} \) is a function of \( \lambda_{u_1}, \ldots, \lambda_{u_K} \). For subsequent analysis, let \( f(\lambda_{u_1}, \ldots, \lambda_{u_K}) := \log(1 - P_{u_{K, \text{out}}}) \).

This way, the minimization of \( P_{u_{K, \text{out}}} \) is equivalent to the maximization of \( f(\cdot) \).

For each \( k \in [2, K-1] \), the partial derivation of \( f(\cdot) \) with respect to \( \lambda_{u_k} \) satisfies

\[
\frac{\partial f(\cdot)}{\partial \lambda_{u_k}} = \frac{\gamma_{\text{target}} B_{k-1}}{\lambda_{u_k} (\gamma_{\text{target}} B_k + \lambda_{u_k})} - \gamma_{\text{target}} (1 + \gamma_{\text{target}})^{K-k}
\]

\[+ \sum_{i=k+1}^{K} \frac{\gamma_{\text{target}} B_{i-1}^{i-k-1}}{\gamma_{\text{target}} B_i + \lambda_{u_i}}\]

\[
= \frac{1}{\lambda_{u_k}} - \sum_{i \in [k:K]} \frac{1}{\lambda_{u_k} + D_{k,i}} - \gamma_{\text{target}} (1 + \gamma_{\text{target}})^{K-k},
\]

(59)

We then prove that (59)-(60) only has one zero point, which implies \( f(\cdot) \) only has one global maximum value. The second partial derivative of \( f(\cdot) \) with respect to \( \lambda_{u_k} \) for each \( k \in [2, K-1] \) is

\[
\frac{\partial^2 f(\cdot)}{\partial \lambda_{u_k}^2} = -\frac{1}{\lambda_{u_k}^2} + \sum_{i \in [k:K]} \frac{2 \lambda_{u_i} D_{k,i}}{\left(\lambda_{u_k} + D_{k,i}\right)^2} = \frac{g(\lambda_{u_k})}{\lambda_{u_k}^2},
\]

where \( g(\lambda_{u_k}) := -1 + \sum_{i \in [k:K]} \frac{\lambda_{u_i} + D_{k,i}}{\lambda_{u_k} + D_{k,i}} \).

Note that \( g(\lambda_{u_k}) \) monotonically increases with \( \lambda_{u_k} \), and the extreme values satisfy \( \lim_{\lambda_{u_k} \to 0} \sum_{i \in [k:K]} g(\lambda_{u_i}) = -1 \), \( \lim_{\lambda_{u_k} \to \infty} g(\lambda_{u_k}) = K - k \). Thus, there only exists one zero point \( \lambda_{0}^{(u_k)} \) such that \( \frac{\partial^2 f(\cdot)}{\partial \lambda_{u_k}^2} \geq 0 \) if and only if \( \lambda_{u_k} \geq \lambda_{0}^{(u_k)} \), i.e., \( \frac{\partial f(\cdot)}{\partial \lambda_{u_k}} \) first decreases in \( \lambda_{u_k} \) and then increases in \( \lambda_{u_k} \) with the critical point of \( \lambda_{0}^{(u_k)} \). Further, we have \( \lim_{\lambda_{u_k} \to 0} \frac{\partial f(\cdot)}{\partial \lambda_{u_k}} = +\infty \) and \( \lim_{\lambda_{u_k} \to \infty} \frac{\partial f(\cdot)}{\partial \lambda_{u_k}} = -\gamma_{\text{target}} (1 + \gamma_{\text{target}})^{K-k} \). Thus, there is only one zero point \( \lambda_{0}^{(u_k)} \) of the first derivative \( \frac{\partial f(\cdot)}{\partial \lambda_{u_k}} \) and \( f(\cdot) \) increases in its parameters first and then decreases with the critical point \( \lambda_{0}^{(u_k)} \). As a result, \( f(\cdot) \) has only one global maximum value for any \( k \in [2, K-1] \). The cases of \( k = 1, K \) are straightforward so the detailed analysis are omitted.

The solution to Problem 4, \( \lambda_{0}^{(u_k)} \) \( k \in [K] \) meets (49). The theorem is proved.

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