Two–pion light–cone distribution amplitudes from the instanton vacuum

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Abstract

We calculate the two–pion light–cone distribution amplitudes in the effective low–energy theory based on the instanton vacuum. These generalized distribution amplitudes describe the soft (non-perturbative) part of the process $\gamma^*\gamma \to \pi\pi$ in the region where the c.m. energy is much smaller than the photon virtuality. They can also be used in the analysis of exclusive processes such as $\gamma^*p \to p + 2\pi, 3\pi$ etc.

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Hadron production in photon–photon collisions at low invariant masses has become a subject of great interest recently [1]. Under certain conditions such processes are amenable to a partonic description, if the virtuality of the photon, $Q^2$, is much larger than the squared c.m. energy, $W^2$. The simplest such process, which has intensively been studied [2], is $\gamma^*\gamma \rightarrow \pi^0$, which provides a unique framework for studying the pion light–cone distribution amplitude. Recently, Diehl et al. have argued that also production of a pair of hadrons, $\gamma^*\gamma \rightarrow h\bar{h}$, can be described in factorized form [3]. The amplitudes for these processes contain new non-perturbative functions describing the exclusive fragmentation of a quark–antiquark pair into two hadrons. In addition to the usual parton momentum fraction with respect to the total momentum of the hadronic final state, these functions depend also on the distribution of longitudinal momentum between the two hadrons, as well as on the invariant mass of the produced system, $W^2$. In particular, the authors of Ref.[3] consider the production of two pions, the soft part of which is contained in a generalized two–pion distribution amplitude. In contrast to the one–pion distribution amplitude, which has been studied using QCD sum rules and other non-perturbative methods [3, 5], the two–pion distribution amplitude is an entirely unknown object. Quantitative estimates of this function are urgently needed for the computation of the cross section for such processes.

In this note we calculate the two–pion distribution amplitude at a low normalization point in the effective low–energy theory based on the instanton vacuum [7]. This approach has recently been used to study the pion and the photon distribution amplitudes [6, 8]. The pion distribution amplitude was found to be close to the asymptotic one and consistent with the recent CLEO measurements [9].

By crossing symmetry the process $\gamma^*\gamma \rightarrow h\bar{h}$ is related to virtual Compton scattering, $\gamma^*h \rightarrow \gamma h$, which can be factorized into a hard photon–parton scattering amplitude and an off–forward parton distribution (OFPD), which in addition to the light–cone momentum fraction, $x$, depends also on the longitudinal component of the momentum transfer to the hadron, $\xi$. The off–forward isosinglet quark/antiquark distributions in the nucleon have been recently been computed in the effective low–energy theory based on the instanton vacuum [12]. It was found that at $x = \pm \xi/2$ the OFPD exhibits characteristic discontinuities. Actually, these would–be discontinuities, which would violate factorization, are reduced to sharp but continuous crossovers by the fact that the dynamical quark mass derived from the instanton vacuum is momentum dependent. Given the intimate connection between the OFPD and the generalized two–particle distribution amplitudes we expect characteristic discontinuities also in the latter. We indeed find such would–be discontinuities in the two–pion amplitude considered here. However, contrary to virtual Compton scattering, in the $\gamma^*\gamma \rightarrow h\bar{h}$ case they do not seem to lead to violations of the factorization assumptions.

Here we perform a rough model estimate of the two–pion distribution amplitude at a low scale ($\sim 600$ MeV), following the approach of Refs.[6, 8]. Our intention is to discuss qualitative features of these new objects, such as their dependence on the momentum distribution.

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1Similar functions have previously been introduced to describe multi–hadron production at large invariant masses [4].

2Recently, Radyushkin has discussed the behavior of the off–forward parton distribution near $x = \pm \xi/2$ in more general terms using double distributions [10].
between the pions, the invariant mass, and isospin, and to calculate the convolution with the tree–level hard scattering amplitude. An account of a more extensive investigation will be published elsewhere [11].

*Generalized two pion distribution amplitudes.* Our aim is to compute the generalized two–pion distribution amplitudes (GDA’s), which are defined as [3]

\[
\Phi(z, \zeta, W^2) = \frac{1}{4\pi} \int dx^+ e^{-\frac{i}{2} x^+ p^+ x^-} \langle \pi^a(p_1) \pi^b(p_2) | \bar{\psi}(x) \hat{n} T \psi(0) \rangle \bigg|_{x^+=0, x^\perp=0}.
\]

Here, \( n \) is a light–like vector \((n^2 = 0)\), which we take as \( n^\mu = (1, 0, 0, 1) \). For any vector, \( V \), the “plus” light–cone coordinate is defined as \( V^+ \equiv (n \cdot V) = V^0 + V^3 \); the “minus” component as \( V^- = V^0 - V^3 \). The outgoing pions have momenta \( p_1, p_2 \), and \( P \equiv p_1 + p_2 \) is the total momentum of the final state. Finally, \( T \) is a flavor matrix \((T = 1 \text{ for the isosinglet, } T = \tau_3 \text{ for the isovector GDA})\). For brevity we suppress the isospin indices \( a, b \) on the function \( \Phi \).

The generalized distribution amplitude, Eq. (1), depends on the following kinematical variables: the quark momentum fraction with respect to the total momentum of the two–pion state, \( z \); the variable \( \zeta \equiv p_1^+/P^+ \) characterizing the distribution of longitudinal momentum between the two pions, and the invariant mass (c.m. energy), \( W^2 = P^2 \).

In what follows we shall work in the reference frame where \( P^\perp = 0 \). In this frame

\[
P^- = \frac{W^2}{P^+}, \quad p_1^- = \frac{W^2(1 - \zeta)}{P^+}, \quad p_2^- = p_1^- = p_2^- = W^2 \zeta(1 - \zeta) - m_\pi^2.
\]

From these relations one obtains the following kinematical constraint:

\[
\zeta(1 - \zeta) \geq \frac{m_\pi^2}{W^2}.
\]

We shall compute the GDA, Eq. (1), using the effective theory of pions interacting with massive “constituent” quarks, which has been derived from the instanton model of the QCD vacuum [7]. The coupling of the pion field to the quarks, whose form is restricted by chiral invariance, is described by the action

\[
S_{\text{int}} = \int d^4x \, \bar{\psi}(x) \sqrt{M(\partial^2)} U^\gamma_5(x) \sqrt{M(\partial^2)} \psi(x),
\]

where

\[
U^\gamma_5(x) = e^{i\gamma_5 \tau^a \pi^a(x)/F_\pi} = 1 + \frac{i}{F_\pi} \gamma_5 \pi^a(x) \tau^a - \frac{1}{2 F_\pi^2} \pi^a(x) \pi^a(x) + \ldots.
\]

Here, \( F_\pi = 93 \text{ MeV} \) is the weak pion decay constant.

The momentum–dependent dynamical quark mass, \( M(p) \), plays the role of an UV regulator. Its form for Euclidean momenta was derived in Ref. [7]. The momentum dependent mass cuts loop integrals at momenta of order of the inverse average instanton size, \( \bar{\rho}^{-1} \approx 600 \text{ MeV} \). One should note that the value of the mass at zero momentum, \( M(0) \), is parametrically small compared to \( \bar{\rho}^{-1} \); the product \( M(0) \bar{\rho} \) is proportional to the packing fraction of the instantons.
in the medium, which is a small parameter fundamental to this picture. Numerically, a value $M(0) = 345$ MeV was obtained in Ref. [7].

**Computation of GDA in the effective chiral theory.** With the simple action Eq. (4) we can proceed to compute the matrix element defining the GDA, Eq. (1). In the leading order of the $1/N_c$–expansion the result for $\Phi(z, \zeta, W^2)$ is given by the sum of the two Feynman graphs shown in Fig. 1. [The quark propagator here contains the dynamical quark mass.] The first graph, Fig. 1a, evidently contributes only to the isosinglet part of the GDA ($T = 1$), whereas the second graph, Fig. 1b, contributes to both the isosinglet and isovector part ($T = \tau^3$).

We first consider the contribution from Fig. 1a. By straightforward calculation we obtain

$$
\Phi^{(1)}(z, \zeta, W^2) = \frac{4i N_c P^+}{F_\pi^2} \delta^{ab} \text{Tr}(T) \int \frac{d^4k}{(2\pi)^4} \delta(k^+ - zP^+) \sqrt{M(k)M(k-P)} \frac{z M(k-P) - (1-z) M(k)}{[k^2 - M(k)^2 + i0][(k-P)^2 - M(k-P)^2 + i0]}.
$$

From this we immediately see that $\Phi^{(1)}(z, \zeta, W^2)$ is independent of $\zeta$, and also that

$$
\Phi^{(1)}(z, W^2) = -\Phi^{(1)}(1-z, W^2),
$$

a property which actually follows from $C$–invariance [3].

Let us first evaluate the integral in Eq. (6) neglecting for a moment the momentum dependence of the constituent quark mass. In this case the result takes a simple form,

$$
\Phi^{(1)}(z, W^2) = -\frac{N_c M_0^2}{\pi F_\pi^2} \delta^{ab} \text{Tr}(T) \theta[z(1-z)](2z-1) \int \frac{d^2k_\perp}{(2\pi)^2} \frac{1}{k_\perp^2 + M_0^2 - W^2 z(1-z)},
$$

where $M_0 \equiv M(0)$ and $\theta$ denotes the step function. This expression is nonzero at the endpoints, $z \to 0$ and 1. Such behavior of the distribution amplitude would violate the factorization theorem for the process $\gamma^* \gamma \to \pi\pi$, because the amplitude for this process contains the integral

$$
I(\zeta, W^2) = \int_0^1 dz \frac{2z - 1}{z(1-z)} \Phi(z, \zeta, W^2),
$$

which would be divergent if $\Phi$ were nonzero at the endpoints. However, such conclusion would be premature, as one can easily see that the momentum dependence of the constituent quark mass becomes crucial at the end point (see the discussion of this point in Refs. [12, 6]). This means that when computing the isosinglet GDA one cannot neglect the momentum dependence of the constituent quark mass. For the numerical estimates we can employ a simple numerical fit to the momentum dependent constituent quark mass obtained from the instanton model of the QCD vacuum,

$$
M(-p^2) = \frac{M_0}{(1 + 0.5 p^2 \bar{\rho}^2)^2}.
$$
Results of a computation of the distribution are shown in Fig. 2, where we have plotted the contribution $\Phi^{(1)}$ to the isosinglet GDA as a function of $z$ at various values of $W^2$. [Remember that this contribution is $\zeta$–independent]. It is useful to expand the isosinglet GDA in Gegenbauer polynomials of index $3/2$, which are the eigenfunctions of the evolution kernel \[13],

$$
\Phi^{(1)}(z, W^2) = \delta^{ab} \text{Tr}(T) \ 6z(1-z) \sum_{n \text{ odd}} a_n(W^2) C_n^{3/2}(2z - 1).
$$

(11)

Due to the antisymmetry with respect to $z \rightarrow 1 - z$ only polynomials of odd degree appear in the expansion of the isosinglet part. The numerical results for the first few coefficients can in the momentum range $0 \leq W^2 \leq 4M_0^2$ be approximated by the forms\[5]

$$
a_1(W^2) = -\frac{0.5556}{1 - \frac{W^2}{0.75}},
$$

$$
a_3(W^2) = -\frac{0.036(1 - 6.2W^2)}{\left(1 - \frac{W^2}{0.82}\right)^2},
$$

$$
a_5(W^2) = -\frac{0.0036}{\left(1 - \frac{W^2}{0.5}\right)^{3/2}},
$$

(12)

where $W^2$ is taken in GeV$^2$. One sees that the coefficients of the expansion Eq. (11) decrease rapidly with increasing the order of Gegenbauer polynomials. We also give the result for the contribution of $\Phi^{(1)}$ to the isosinglet part of the integral of Eq. (9). With good accuracy the numerical results can be fitted by

$$
I^{(1)}(W^2) = -\delta^{ab} \text{Tr}(T) \frac{3.58}{1 - \frac{W^2}{1.0}}.
$$

(13)

Let us now turn to the second contribution to the GDA, given by the graph in Fig. 3b. One can easily convince oneself that the neglection of the momentum dependence of the constituent quark mass in this case does not lead to any violation of factorization. Therefore, in order to make the discussion more transparent we shall in the following neglect the momentum dependence of the constituent quark mass in the contribution of graph Fig. 3b. The logarithmic divergence of the loop integrals can always be absorbed in the pion decay constant, $F_\pi$, which is defined by the logarithmically divergent integral (written here as an integral over Euclidean momenta)

$$
F^2_\pi = 4N_c \int \frac{d^4k}{(2\pi)^4} \frac{M_0^2}{(k^2 + M_0^2)^2}.
$$

(14)

\[3\] The functions given here should be considered as purely numerical fits. In particular, the powers in the denominators have no physical meaning, but are simply chosen such as to represent the numerical results in the most compact form.
All formulae below can easily be generalized to the case of a momentum–dependent mass.

Computing the Feynman integral corresponding to graph Fig. 1b we obtain for this contribution to the GDA

\[
\Phi^{(2)}(z, \zeta , W^2) = \frac{1}{2} \text{Tr}(T[\tau^a, \tau^b]) \left[ \phi^{(2)}(z, \zeta , W^2) - \phi^{(2)}(z, 1 - \zeta , W^2) \right]
\]

\[+ \text{Tr}(T) \delta^{ab} \left[ \phi^{(2)}(z, \zeta , W^2) + \phi^{(2)}(z, 1 - \zeta , W^2) \right], \tag{15}\]

where

\[
\phi^{(2)}(z, \zeta , W^2) \quad z \leq \zeta \quad \left( \frac{M_0^2 N_c}{\pi F_\pi^2} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{z}{k_\perp^2 + M_0^2 - W^2 z(1 - z)} \right.
\]

\[\times \left. \left\{ 1 + \frac{\vec{k}_\perp \cdot \vec{p}_\perp + m_\pi^2 z - W^2 z(1 - \zeta)}{(k_\perp^2 + M_0^2)\zeta - 2\vec{k}_\perp \cdot \vec{p}_\perp z - m_\pi^2 z + W^2(1 - \zeta) z^2} \right\} \right\}; \tag{16}\]

the value at \( z > \zeta \) can be obtained from the above expression as

\[
\phi^{(2)}(z, \zeta , W^2) \quad z \geq \zeta \quad -\phi^{(2)} (z < \zeta) (1 - z, 1 - \zeta , W^2). \tag{17}\]

Some comments are in order here. The expressions for the GDA given by Eqs. (8) and (15, 16, 17) are valid in the parametrically wide region of invariant c.m. energies \( 4m_\pi^2 < W^2 < 4M_0^2 \). For \( W^2 > 4M_0^2 \) the graphs of Fig. 1 develop an imaginary part which is related to the presence of a two-quark threshold in this model. This singularity is not physical, because for momenta of order \( \bar{\rho}^{-1} \sim 600 \text{ MeV} \) degrees of freedom not accounted for in the effective theory (e.g. heavy resonances) start to play significant role. These contributions can in principle be estimated phenomenologically by adding the coupling of the resonances to constituent quarks to the effective action, Eq. (4). Note also that the GDA’s computed in this model are real at low \( W^2 \) only in the leading order of the \( 1/N_c \)–expansion; an imaginary part appears in the next–to–leading order due to pion loop contributions, which can be estimated using chiral perturbation theory techniques.

The contribution to the GDA given by Eqs. (15, 16, 17) exhibits discontinuities at \( z = \zeta \) and \( z = 1 - \zeta \). The nature of these discontinuities is the same as those found in the non-diagonal parton distributions computed in this model [12]. As was shown in Ref. [12], the inclusion of the momentum dependence of the constituent quark mass smears this discontinuity. However, in contrast to the case of non-diagonal parton distributions, the discontinuities in the two–pion GDA do not lead to violation of factorization, so we can be negligent and evaluate Eqs. (15, 16) with a constant quark mass.

Let us study the result Eq. (17) in the chiral limit, \( m_\pi = 0 \), and expand in powers of \( W^2 \). In this way we obtain an analytic expression for the GDA,

\[
\phi^{(2)}(z, \zeta , W^2) \quad z \leq \zeta \quad \left\{ 1 + \frac{N_c W^2 z [2\zeta(1 - z) - (1 - \zeta)]}{8\pi^2 F_\pi^2 \zeta} + \ldots \right\},
\]

\[
\phi^{(2)}(z, \zeta , W^2) \quad z \geq \zeta \quad -(1 - z) \left\{ 1 + \frac{N_c W^2 (1 - z) [2(1 - \zeta)z - \zeta]}{8\pi^2 F_\pi^2 (1 - \zeta)} + \ldots \right\}. \tag{18}\]
From this expression we find for the lowest two moments

\[ \int_0^1 dz \Phi^{(2)}(z, \zeta, W^2) = \frac{1}{2}(2\zeta - 1) \text{Tr}(T[\tau^a, \tau^b]) \left\{ 1 + \frac{N_c}{24\pi^2 F^2_\pi} W^2 + \ldots \right\}, \quad (19) \]

\[ \int_0^1 dz \; (2z - 1)\Phi^{(2)}(z, \zeta, W^2) = \delta^{ab} \text{Tr}(T) \left\{ \frac{1}{3} - 2\zeta(1 - \zeta) + \frac{N_c W^2}{120\pi^2 F^2_\pi} (1 - 5\zeta(1 - \zeta)) + \ldots \right\}. \quad (20) \]

The first moment, Eq. (19), corresponds to the expansion of the pion electromagnetic form factor for small time–like momenta. The pion charge radius read off from Eq. (19) is

\[ \langle r^2 \rangle_{\text{e.m.}} = \frac{N_c}{4\pi^2 F^2_\pi}, \quad (21) \]

which coincides with the result obtained previously in Ref. [7]. Note that only the isovector part of the GDA contributes to the first moments, as it should be on grounds of C–parity.

Of \( \Phi^{(2)} \) only the isosinglet part contributes to the integral Eq. (9). This contribution to the integral is given by

\[ I^{(2)}(\zeta, W^2) = 4\delta^{ab} \text{Tr}(T) \left\{ -\left( 1 + \frac{1}{2}\log[\zeta(1 - \zeta)] \right) + \frac{N_c W^2}{48\pi^2 F^2_\pi} \left( 4\zeta(1 - \zeta) + 3\zeta^2 \log[\zeta(1 - \zeta)] + 3(1 - 2\zeta) \log(1 - \zeta) \right) \right\}. \quad (22) \]

Note that the R.H.S. is symmetric with respect to \( \zeta \rightarrow 1 - \zeta \).

The total result for the integral Eq. (9) is given by the sum of Eqs. (13) and (22)). In Fig. 3 we plot the total result as a function of \( \zeta \) at several values of \( W^2 \). We see that the absolute value of this integral for intermediate values of \( \zeta \) increases with \( W^2 \), and that the dependence on \( \zeta \) is strong only for small values of \( \zeta(1 - \zeta) \), a region which is difficult to access because of the kinematical constraint, Eq. (3).

Finally, in Fig. 4 we show the total result for the isosinglet GDA, given by the sum of \( \Phi^{(1)} \), Eq. (3), and \( \Phi^{(2)} \), Eq. (18), for a value of \( \zeta = 0.25 \). Note the different symmetry of the contributions with respect to \( z \rightarrow 1 - z \), as can be seen also from the analytic expressions.

We emphasize once more that the discontinuities in \( \Phi^{(2)} \) are smeared out when taking into account the momentum dependence of the dynamical quark mass.

**Asymptotic behavior of the GDA.** The scale dependence of the GDA is governed by the usual Efremov–Radyushkin–Brodsky–Lepage evolution [13, 3]. This is obvious when one notes that the hard–scattering kernel of the amplitude for \( \gamma^* \gamma \rightarrow \pi\pi \) is the same as that for a process with only one meson in the final state (with appropriate quantum numbers). The evolution equations can thus be solved, in analogy to those for the usual pion wave function [3], by expanding in eigenfunctions of the evolution kernel. In the isosinglet part of the GDA one has to take into account the mixing with gluon operators [14]; the evolution of the isosinglet GDA calculated in the effective low–energy theory will be considered elsewhere [11]. However, one can easily establish the scale dependence of the isovector part of the
GDA, which does not mix with gluons. This function does not enter in the amplitude for \( \gamma^*\gamma \to \pi\pi \), but it can appear in exclusive processes of the type \( \gamma^*p \to Xp \), where \( X \) is a two– (or more) pion state, cf. the discussion below. The asymptotic behavior of the isovector GDA is given by

\[
\Phi(z, \zeta, W^2) = 3 \text{Tr} \left( T[r^a, r^b] \right) z(1 - z)(2\zeta - 1) F_\pi(W^2),
\]

where \( F_\pi(W^2) \) is the pion electromagnetic form factor in the time–like domain.

**Conclusions.** In this note we have given a first model estimate of the two–pion GDA’s at a low scale. A number of qualitative features of the two–particle distribution amplitudes have emerged, which are consequences of the general structure of the effective low–energy theory and should thus be general, even given the crudeness of the quantitative approximations made.

First, the GDA’s generally exhibit discontinuities at \( z = \zeta \) and \( 1 - \zeta \), which are in complete analogy to the discontinuities found in the off–forward parton distribution functions\(^4\). We have also noted that, as in the OFPD’s, the momentum dependence of the dynamical quark mass derived from the instanton vacuum turns these discontinuities into continuous but sharp crossovers. However, since the discontinuities of the GDA’s are not at odds with the factorization of the amplitude for \( \gamma^*\gamma \to hh \) we have been negligent about implementing the momentum–dependent mass and estimated the amplitude with the discontinuous GDA’s.

Second, we have found that the isosinglet and –nonsinglet GDA’s for the two–pion final state computed at the low scale show different symmetry properties with respect to \( z \to 1 - z \).

Our results concerning the \( W^2 \)–dependence of the GDA’s and the total amplitude should be regarded as crude estimates. In particular, the imaginary part of the functions at \( W^2 > 4M_0^2 \) should not be regarded as physical, since it occurs in a region where the effective theory is no longer applicable. A more conservative approach would be to calculate the GDA’s in an expansion in the external pion momenta and the pion mass, adopting, so to speak, the philosophy of the chiral Lagrangian. The physical imaginary part, which is due to final state interactions, appears in the next order of the \( 1/N_c \) expansion and can be estimated using the methods of chiral perturbation theory. This approach will be developed in a forthcoming paper\(^5\).

We note that the studies reported in this note can easily be generalized to distribution amplitudes for \( n > 2 \) pions in the final state. Such multi–pion GDA’s play a role in the analysis of exclusive processes of the type \( \gamma^*p \to Xp \), where \( X = 2\pi, 3\pi \) etc.\(^6\). For instance, the two–pion isovector GDA computed here can be used to analyze the \( 2\pi \) background in the diffractive lepton production of rho mesons.

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\(^4\)In Ref.\(^1\) only the OFPD’s of the nucleon were computed. The OFPD’s in the pion at the low scale exhibit would–be discontinuities similar to those in the nucleon. A complete study of the OFPD’s in the pion will be published shortly\(^6\).

\(^5\)We are grateful to L. Frankfurt and M. Strikman for discussion of this point.
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Figure 1: The diagrams defining the two contributions to the two–pion distribution amplitude in the effective low–energy theory, $\Phi^{(1)}$ and $\Phi^{(2)}$. The solid line denotes the quark propagator with the dynamical quark mass, the solid points the vertices defined by Eq. (4).
Figure 2: The contribution $\Phi^{(1)}(z, W^2)$ to the isosinglet two-pion distribution amplitude, Eq. (6), as a function of $z$ for various values of $W^2$. This contribution is independent of $\zeta$. Solid line: $W = 0.3$ GeV; dashed line: $W = 0.5$ GeV, dotted line: $W = 2M_0 = 0.69$ GeV.
Figure 3: The integral determining the $\gamma^*\gamma \rightarrow \pi\pi$ amplitude, Eq. (9), given by the sum of Eqs. (13) and (22), as a function of $\zeta$ for various values of $W^2$. Solid line: $W = 0.3$ GeV; dashed line: $W = 0.5$ GeV, dotted line: $W = 2M_0 = 0.69$ GeV.
Figure 4: The total result for the isosinglet two–pion distribution amplitude, $\Phi(z, \zeta, W^2)$, as a function of $z$, for $\zeta = 0.25$ and $W^2 = 0.25$ GeV$^2$. *Dashed line:* contribution $\Phi^{(1)}$, Eq. (6); *dotted line:* contribution $\Phi^{(2)}$, Eqs. (15, 16, 17); *solid line:* total.