The phenomenon of voltage controlled switching in disordered superconductors

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Abstract
The superconductor-to-insulator transition (SIT) is a phenomenon occurring in highly disordered superconductors and may be useful in the development of superconducting switches. The SIT has been demonstrated to be induced by different external parameters: temperature, magnetic field, electric field, etc. However, the electric field induced SIT (ESIT), which has been experimentally demonstrated for some specific materials, holds particular promise for practical device development. Here, we demonstrate, from theoretical considerations, the occurrence of the ESIT. We also propose a general switching device architecture using the ESIT and study some of its universal behavior, such as the effects of sample size, disorder strength and temperature on the switching action. This work provides a general framework for the development of such a device.

Keywords: disordered superconductors, switching device, superconducting electronics
(Some figures may appear in colour only in the online journal)

1. Introduction

Superconducting switches have been in development for the past 60 years. The first attempt was the development of the cryotron, which provided magnetic field driven switching of a superconductor [1]. There have also been several attempts to generate FET architecture using superconductors [2, 3].

The discovery of the superconductor-to-insulator transition (SIT) for disordered superconductors [4] opened the doors to a new switching mechanism. The SIT was particularly attractive because in comparison with the metal-to-superconductor transition, it provided a much larger change of the current with one phase being superconducting, and hence having zero resistance, and the other being insulating, and hence having infinite resistance (ideally). However, such a transition was driven by a magnetic field or disorder modification or temperature change [5], making it unsuitable for application in integrated circuits.

Quite recently there have been a few demonstrations of an electric field driven SIT. Though these works hold the promise of leading to a further development of superconducting electronics, all of them are demonstrated for very specific materials and no microscopic analysis of such processes has been given [3, 6, 7].

In this work, we first demonstrate strong fluctuation of the superconducting pair amplitude with the electron density (number of electrons per lattice site), for a strongly disordered superconductor system. We start with a negative $U$ Hubbard model, equation (1), describing a disordered superconductor. Using this model, we then show the strong dependence of the superconducting pair amplitude, an internal parameter governing the superconductivity of a sample, on the average density of electrons per lattice site. We then demonstrate that such strong fluctuations can lead to a SIT, through phase correlation calculations. On the basis of this phenomenon, we then propose a general architecture for a superconducting switch, a device capable of yielding switching from a superconducting state, with effectively zero resistance, to an insulating state, with resistance of the order of $10\,k\Omega$ [5]. While there are few realizations of such a device, most of them require a large change of electron density to bring about a change of phase, as is evident from the high values of the voltage needed to switch such a system. The device that we are proposing is driven by a quantum phenomenon [8–10], where small changes of electron density can lead to a change of phase, hence requiring a small amount of voltage change, as compared to current experimental devices.
Finally, we study some universal properties, namely the effects of sample size, disorder strength and temperature on the behavior of the device.

2. The model and methods

We model the disordered superconductor using a negative-$U$ Hubbard Hamiltonian on an $L \times L$ square lattice. The Hamiltonian is given by

$$H = -t \sum_{\langle ij \rangle, \sigma} (C_{i\sigma}^\dagger C_{j\sigma} + C_{j\sigma}^\dagger C_{i\sigma}) + \sum_{i, \sigma} (V_i - \mu) C_{i\sigma}^\dagger C_{i\sigma} - U \sum_i C_{i\uparrow}^\dagger C_{i\downarrow} C_{i\downarrow} C_{i\uparrow}. \tag{1}$$

Here $C_{i\sigma}^\dagger$ ($C_{i\sigma}$) is the creation (annihilation) operator for an electron at site $i$ with a spin $\sigma$. $t$ represents the hopping energy, $V_i$ is a site dependent random potential with a uniform distribution from $-V$ to $+V$, $\mu$ is the chemical potential and $U$ is the strength of the attractive interaction between two electrons of opposite spins at the same site.

In this model, $t$ represents the kinetic energy of the electrons and all other parameters are scaled with $t$. $U$ represents the same site interaction between electrons of opposite spins and represents the Cooper attraction giving rise to superconductivity.

The partition function for this model is given by

$$Z = \int D[C_i, C_i^\dagger] \exp \left( -\int_0^\beta \sum_{\sigma} C_{i\sigma}^\dagger C_{i\sigma} \, d\tau \right. \times (-\partial_\tau + V_i - \mu) C_{i\sigma} \, d\tau + \left. i \sum_{\langle ij \rangle, \sigma} (C_{i\sigma}^\dagger C_{j\sigma} + \text{h.c.}) \right)$$

$$- U \sum_i C_{i\uparrow}^\dagger C_{i\downarrow} C_{i\downarrow} C_{i\uparrow}) \right), \tag{2}$$

where h.c. is the Hermitian conjugate and $\beta$ is the inverse temperature in units where the Boltzmann constant is unity.

We introduce the Hubbard–Stratonovich transformation with the Boltzmann constant is unity. The Hubbard–Stratonovich field ($\Delta_i e^{i\theta_i}$) can be obtained by applying the Hubbard–Stratonovich Gennes approximation (BdG). In the BdG approximation the partition function is evaluated at the saddle point. Under this approximation, we obtain an effective Hamiltonian given by

$$H' = -t \sum_{\langle ij \rangle, \sigma} (C_{i\sigma}^\dagger C_{j\sigma} + C_{j\sigma}^\dagger C_{i\sigma}) + \sum_{i, \sigma} (V_i - \bar{\mu}_i) C_{i\sigma}^\dagger C_{i\sigma}$$

$$+ \sum_i \Delta_i (e^{i\theta_i} C_{i\uparrow}^\dagger C_{i\downarrow} + e^{-i\theta_i} C_{i\downarrow}^\dagger C_{i\uparrow}). \tag{4}$$

The Hamiltonian has to follow two self-consistent relations, namely, $\Delta_i e^{i\theta_i} = -U(C_{i\uparrow} C_{i\downarrow})$ and $n_i = \sum_{\sigma} (C_{i\sigma}^\dagger C_{i\sigma})$. $H'$ is diagonalized by a Bogoliubov transformation: $\gamma_{\lambda\sigma} = \sum_{\sigma} (\bar{u}_\lambda (i) C_{i\sigma} + \sigma \bar{v}_\lambda (i) C_{i\sigma})$. The local Bogoliubov amplitudes are obtained from the following equation:

$$\left( \hat{\gamma}^\dagger \Delta_{\lambda \sigma} e^{i\theta_i} \gamma - \Delta_{\lambda \sigma} \right) \begin{pmatrix} u_{\lambda i} \\ v_{\lambda i} \end{pmatrix} = E_{\lambda} \begin{pmatrix} u_{\lambda i} \\ v_{\lambda i} \end{pmatrix}. \tag{5}$$

Here $\hat{\gamma}$ represents the single-particle contribution of $H'$ and the $E_{\lambda}$ are the eigenvalues [11]. The self-consistent relations in terms of the Bogoliubov amplitudes are

$$\Delta_i e^{i\theta_i} = U \sum_{\lambda} u_{\lambda i}^\dagger v_{\lambda i} \tag{6}$$

$$n_i = 2 \sum_{\lambda} |v_{\lambda i}|^2. \tag{7}$$

Starting from some initial guess values, we self-consistently obtain the values of $\Delta_i e^{i\theta_i}$ and $n_i$ for each lattice site $i$. We define the spatial average of $\Delta_i$ as $\Delta_{\text{op}}$ given by

$$\Delta_{\text{op}} = \frac{1}{N} \sum_i \Delta_i. \tag{8}$$

The average electron density per lattice site is defined as

$$n = \frac{1}{N} \sum_i n_i. \tag{9}$$

We can change the average electron density ($n$) in the sample by controlling the chemical potential ($\mu$). The BdG approximation gives the saddle point solution for $Z$. However, it has completely missed the fluctuations of the phase $\theta_i$, due to its mean-field nature.

2.2. Fluctuations around the saddle point

To incorporate phase fluctuations we go beyond the BdG approximation and introduce a newly developed method [12, 13] which allows us to calculate classical phase fluctuations while ignoring the time dependence of the order parameter (quantum fluctuations). Under this approximation the partition function becomes

$$Z = \int D[\Delta_i] D[\theta_i] \exp \left( -\frac{\beta}{U} \sum_i \frac{\Delta_i^2}{2} \right) \text{Tr}\left[\exp(-\beta H')\right]. \tag{10}$$
In terms of eigenvalues of equation (5), the partition function reads

\[
Z = \int \mathcal{D}[\Delta_i] \mathcal{D}[\theta_i] \exp \left( -\frac{\beta}{U} \sum_i \Delta_i^2 \right) \times \prod_{i=1}^{2N} (1 + \exp(-\beta E_i)). \tag{11}
\]

For obtaining the fluctuations around the saddle point, we relax the self-consistent constraint on \{\Delta_i, \theta_i\} and calculate the value of \(E_i\) for all possible values of \{\Delta_i, \theta_i\} through equation (5) (for a particular disorder realization). Using the values of \(E_i\) thus obtained, we can evaluate the values of \(Z\) and the expectation value of any observable \(\mathcal{O}\) given by

\[
\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[\Delta_i] \mathcal{D}[\theta_i] \mathcal{O}(\Delta_i, \theta_i) \exp \left( -\frac{\beta}{U} \sum_i \Delta_i^2 \right) \times \prod_{i=1}^{2N} (1 + \exp(-\beta E_i)). \tag{12}
\]

We have checked that for the temperature range that we are interested in, the values of \(\langle \Delta_i \rangle\) are practically the same as its value evaluated in the BdG approximation. However, \(\theta_i\) has a strong dependence on the temperature. Therefore we replace \(\Delta_i\) in equation (12) by its BdG value and hence we only need to integrate over \(\theta_i\). The integration over \(\theta_i\) is performed using the Monte Carlo method [12, 13]. The energy eigenvalues \(E_i \gg T\) can be ignored while calculating equation (12).

Using the partition function \(Z\), it has been shown that for weak disorder, the system shows small fluctuations of pair amplitudes while preserving long range phase correlation. Strong disorder, on the other hand, leads to strong fluctuations of the pair amplitudes because of the formation of superconducting islands and also destroys the long range phase correlation. This phenomenon has also been recently experimentally observed [14, 15].

3. The electric field driven SIT (ESIT)

At strong disorder, \(\Delta_{op}\) is strongly dependent on the value of \(n\), as is demonstrated in figure 1. This strong fluctuation of \(\Delta_{op}\) with \(n\) is due to rapid change in local density of states around the small window near the Fermi surface [8]. The key feature of this fluctuation is that the fluctuation only takes place when the sizes of the superconducting islands become comparable to the superconducting coherence length, \(\xi_0\). Our main motivation is to induce a SIT by controlling the size and distribution of the islands, which can be achieved by tuning \(n\).

The tuning of the electron density can be achieved by applying suitable electric fields. For a first-order calculation, we assume a classical dependence. The electric field inside the two electrodes separated by a distance \(d\) is \(V_g/d\), where \(V_g\) is the applied voltage. Therefore the charge density on the surface is \(e V_g/d\) where \(e\) is the dielectric constant. Because of the condition of equilibrium in a metal, the additional charge density on the surface of the superconductor is exactly equal to \(e V_g/d\). To convert this charge density into the electron density per lattice, we divide it by \(e/a^2\) where \(e\) is the electronic charge and \(a\) is the lattice constant on the superconducting plane. Therefore, we obtain the dependence of the electron...
Figure 2. This figure demonstrates the switching behavior in a region where \( \Delta_{op} \) changes significantly. The edge-to-edge phase correlation \((D)\) indicates whether the state is insulating or superconducting. Part (a) shows the region of variation of \( \Delta_{op} \) with \( V_g \) (unit: volts; through equation \((13)\)) where we demonstrate the switching phenomenon. This region is chosen because of the large change of \( \Delta_{op} \) for a small change of \( n \). The temperature dependence of the switching process is demonstrated across parts (b)–(d). Increase of the temperature \( T \) (scaled with the critical temperature) leads to a decrease of the gap between the switching states. It should be noted that for the temperature regime that we are working in, the temperature dependence of \( \Delta_{op} \) is insignificant. This demonstration of switching is for the same sample (same disorder realization and parameter values) as in figure 1. The change in density due to the applied voltage is from 0.42 \((V_g = -8.5)\) to 0.78 \((V_g = -2.0)\); the parameters used in equation \((13)\) are \( n(0) = 0.87 \) and \( \alpha = 0.053 \).

4. Device construction using an ESIT

The strong dependence of \( \Delta_{op} \) on the electron density and control of the electron density using an electric field open up the possibility of developing a voltage control device which switches between insulator and superconductor states. Because of this strong dependence, a small change of the electron density \( (\Delta n \sim 0.1) \) can drive the system from insulator to superconductor and vice versa. This, in turn, implies that a small voltage change (from equation \((13)\)) is required to drive this switching operation.

To demonstrate the switching action, we calculate the edge-to-edge phase correlation for a given sample as a function of the electron density. The edge-to-edge phase correlation is defined as \[ D = \frac{1}{Z} \int D[\Delta_i]D[\theta_i] \left( \sum_{m,n} \cos(\theta_m - \theta_n) \right) \times \exp \left( -\frac{\beta}{U} \sum_i |\Delta_i|^2 \right) \prod_{i=1}^{2N} (1 + \exp(-\beta E_z)). \] Here \( m \) and \( n \) correspond to site indices at the two opposite edges of the lattice. We have used an \( L \times L \) lattice on the \( xy \) plane with the edges from \( x = 1 \) to \( L \) and \( y = 1 \) to \( L \). For calculating the edge-to-edge phase correlation, we sum over all the lattice sites on the edge. This is because in an actual device, the phase correlations between all the lattice sites on the edge will contribute. We have assumed a periodic boundary condition along the \( x \) axis and an open boundary condition along the \( y \) axis. We assume the current to be flowing in the \( y \) direction and hence we need to measure the phase correlation along the \( y \) axis.

A non-zero value of the edge-to-edge phase correlation implies a superconducting state and effectively zero-resistance current flow \[16\]. Lack of such correlation, even in the presence of superconducting islands, is a typical signature of insulating states associated with the SIT \[15, 17–19\]. Such states have much higher resistance \( (\sim 10 \, \text{k\Omega}) \) compared to the superconducting states and a sample in such a state can effectively work like an open circuit. Figure 2 demonstrates the switching phenomenon.

We can now use this switching phenomenon to construct a device which can act as a voltage controlled electronic switch. The basic architecture is shown in figure 3. The switching takes place on the \( z = 0 \) plane whereas the control field acts in the \( z \) direction.
5. The effect of the sample size, disorder strength and temperature on the switching

The operational efficiency of the device depends on the strength of the disorder ($V$), the sample size of the device, compared to the coherence length ($L/\xi_0$), and the operating temperature ($T$).

5.1. The effect of the sample size

Figure 4 demonstrates the effect of sample size on the fluctuation of the superconducting pair amplitude. If $L/\xi_0 \sim 1$, then the fluctuations of $\Delta_{op}$ with $n$ would increase and the stability of the switch would be affected. On the other hand, for $L/\xi_0 \rightarrow \infty$, the fluctuation reduces and the switching property can be suppressed. The switching property arises because of the strong fluctuation of $\Delta_{op}$ with $n$. As shown in figure 1, for a particular value of $n$, we have large and closely spaced superconducting islands, corresponding to large values of $\Delta_{op}$, whereas for another value of $n$, we have large non-superconducting regions, corresponding to smaller values of $\Delta_{op}$. This is true for a system size comparable to the coherence length. However, for $L/\xi_0 \gg 1$, despite the regions being of size comparable to $\xi_0$, we have strong pair amplitude fluctuation with $n$; on the scale of the system size, change of electron density merely rearranges the positions of the superconducting islands, and hence the global properties like $\Delta_{op}$ and the edge-to-edge phase correlation do not show a significant dependence on $n$, leading to the suppression of the switching property. Thus for efficient operation, a suitable sample size must be selected, depending on the coherence length of the material used.

5.2. The effect of the disorder strength

In the weak disorder regime, the effect of the electron density $n$ on the value of $\Delta_{op}$ is insignificant. However, the stronger the disorder, the greater this effect, as is demonstrated in figure 5. This happens because of the rapid change of the superconducting landscape of a sample with a small change of the electron density in the strong disorder regime. A strong change of the local density of states depending on the value of $n$ leads to this changing landscape, as is described in [8].

Sample size and disorder strength give us two handles for controlling the change of electron density required for performing the switching action. By changing the sample size and the disorder strength, we can change the fluctuation of $\Delta_{op}$ with $n$ and hence we can control the change of density required for the system to switch from the insulating state to the superconducting state and vice versa. For example, if it is necessary to decrease the change of $n$ required for the switching, one can increase the disorder strength or reduce the sample size or both.

5.3. The effect of the temperature

Within the range in which we are working, finite temperature will have an insignificant effect on the superconducting
Figure 6. This figure explores the role of the temperature ($T$) in the behavior of the device. Increase of temperature reduces the gap in the values of $D$ (and hence the conductivity) between the two states. The variation with $T$ also depends on the strength of the disorder.

landscape ($\Delta_p$) and hence $\Delta_{up}$. However, the edge-to-edge phase correlation will have a strong dependence on the temperature. This, in turn, will affect certain properties of the switching action, such as the switching point (the voltage or the electron density about which the system undergoes a transition from insulator to superconductor), the switching gap (the difference in the value of $D$ across the switching point), etc. As can be seen in figure 2, with change of temperature, the difference between the switching states changes. This, in turn, can affect the stability of the switch. Figure 6 demonstrates the effect of temperature on the switching gap. However, the switching point ($V_c$), i.e., $D(V_c) = (D_{up} - D_{down})/2$, remains independent of the temperature, where $D_{up}$ and $D_{down}$ are the values of $D$ for the superconducting and insulating states respectively. The invariance of the switching point with the temperature can be attributed to the fact that for low temperatures, the landscape of the pair amplitude across the sample remains almost invariant. On the other hand, the change of the switching gap arises because of the decrease of the phase correlation in the superconducting state with increase of the temperature.

6. Discussion and conclusions

The strong dependence of the pair amplitude on the average electron density has thus enabled us to postulate a device, capable of switching between insulator and superconductor states, driven by very small change of the electron density. The requirement of small change of the electron density in turn implies that a small voltage change is required to drive the switching mechanism. In the example that we have shown (figure 2), the total change of the electron density is 0.34 along the $x$ axis. However, the change of the electron density across the transition point can be as low as 0.1.

We have used the following values of the material properties for equation (13): $a = 3$ Å, $d = 1$ μm and $\epsilon = 3 \times 10^3 \epsilon_0$, where $\epsilon_0$ is the free space permittivity. For these parameters, we need a voltage change of 2 V (density change = 0.1) for switching. However, by controlling the disorder and sample size, we can modify the switching voltage appropriately.

A typical coherence length is of the order of 50 nm. Therefore a typical system size can be of the order of 0.1 μm. However, one can change the size of a typical device by using materials having different coherence lengths. For example, by using aluminum, one can construct devices with size of the order of tens of microns, whereas by using materials such as alloys of Nb and Sn, one can construct devices with size of the order of tens of nanometers. Also the architecture provided is a very basic FET structure. But in a realistic situation, the architecture might be completely different and material dependent.

Individual characteristics of a device, such as the change in electron density for the transition, the electron density about which the transition occurs, the switching gap, etc, are strongly sample dependent. Different samples with different disorder realizations will have different switching points, switching gaps, etc. However, the phenomenon of the strong dependence of $\Delta_p$ on $n$, and the ESIT, are independent of the disorder landscape. Averaging over different disorder realizations will erase the fluctuations, but since a single sample will contain a particular disorder realization, disorder averaging is less informative in this context.

The architecture of this device gives us a distinct advantage over the previous attempts, since it can provide a possible adaptation of superconducting switches in integrated circuits. Also, the theoretical treatment allows us to claim in a generalized manner that such a switch can be developed, though the exact material, optimal for application, can only be determined experimentally. With an appropriate system, such a design can potentially usher in superconducting electronics, which could improve the efficiency and capability of large computational systems.

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