Amplitude Decaying Characteristic in a One-Dimensional Continuous Microcantilever Probe Model

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Abstract

In this study, we numerically investigate the oscillatory behavior of the cantilever probe in a dynamic AFM, which is approximated by a one-dimensional continuous model. In particular, we investigate the amplitude characteristics of the fundamental oscillation mode and two higher oscillation modes.

1. Introduction

Atomic force microscope (AFM) is a fundamental measurement instrument used for observing the shape of micro/nano scale-size samples [1]. The detection of oscillation of the microcantilever probe used in the sensor part in an AFM plays an essential role in realizing its function [2]. In an AFM operated in the dynamic mode, nonlinear problems such as destabilization of probe oscillation may occur due to the interaction force between the probe and the sample. In particular, if the distance between the probe and the sample is small, the nonlinear phenomenon becomes essential. Recently, the oscillation mode analysis of the cantilever probe has been attracting considerable research interest not only for fundamental oscillation mode but also for higher oscillation modes, because the higher oscillation mode has the potential for improving the scanning speed in dynamic AFM [2, 3, 4, 5].

In this study, we numerically investigate the oscillatory behavior of the cantilever probe in a dynamic AFM, which is approximated by a one-dimensional continuous model. The continuous model has infinite number of eigen frequencies, which enables us to investigate the oscillation state including higher oscillation modes. Therefore, by assuming Euler-Bernoulli approximation, we use the equation of motion of the cantilever probe written in the one-dimensional continuous model [3]. In this study, we assume that both the interaction and the external sinusoidal forces are injected into the probe and investigate the amplitude characteristic of the probe oscillation when the interaction force between the probe and the sample surface increases. In particular, we show that monotonically decaying amplitude characteristics appear in the higher oscillation modes, which are qualitatively consistent with the properties of the fundamental oscillation mode [6].

Figure 1: Normalized one-dimensional cantilever beam model on a sample surface

2. One-Dimensional Continuous Cantilever Model

The cantilever probe with a length $L$ [m], which is fixed at the end $(x' = 0)$ and is free at the opposite end $(x' = L)$, is approximated by a micro-beam. The probe is set on an objective sample with a distance $z_0$ [m] in the $z'$ direction corresponding to the orthogonal axis from the spatial variable of the beam $(x')$, and is driven by an external force, which is typically a harmonic force, in the dynamic AFM. We assume that $E$ [Pa], $I$ [m$^4$], $C$ [kg/(s · m)], and $\mu$ [kg/m] correspond to elastic modulus, moment of inertia, viscous damping constant, and mass per unit length of the cantilever beam, respectively.
The normalized version of the dynamical model of the cantilever beam is written by the following Euler-Bernoulli beam approximation [3]

\[
\frac{\partial^4 v}{\partial x^4} + \frac{1}{Q} \frac{\partial v}{\partial \tau} + \frac{\partial^2 v}{\partial \tau^2} = 0
\]  

(1)

where the following normalized variables and parameters are used

\[
x = \frac{x'}{L}, \quad v = \frac{v'}{\sigma}, \quad \tau = \sqrt{\frac{EI}{\mu L^4}} t
\]

(2)

\[
Q = \sqrt{\frac{EI\mu}{L^2 C}}, \quad \varepsilon = \frac{z_0}{\sigma}
\]

The constant \(\sigma\) [m] in Eq. (2) corresponds to the distance between atoms in equilibrium and is set to be \(0.25 \times 10^{-9}\) by assuming that the probe and the sample surface are composed of silicon. In this study, the parameters for the cantilever beam \(L, E, C, I, \) and \(\mu\) are fixed as \(250 \times 10^{-6}, 130 \times 10^6, 1.48684 \times 10^{-4}, 3.57 \times 10^{-23},\) and \(1.8607 \times 10^{-7}\), respectively [2]. Furthermore, we assume that both the interaction force \(f_{at}(v + \varepsilon)\) and the external sinusoidal force \(f_e(\tau)\) are injected into the probe tip at \(x = 1\). Then, the boundary conditions are written as follows:

\[
v(0, \tau) = \frac{\partial v(0, \tau)}{\partial x} = 0
\]

\[
\frac{\partial^2 v(1, \tau)}{\partial x^2} = f_{at}(v + \varepsilon) + f_e(\tau)
\]

\[
\frac{\partial^3 v(1, \tau)}{\partial x^3} = 0
\]

(3)

where the interaction and the external forces are

\[
f_{at}(v + \varepsilon) = \mu_1 \left\{ \frac{1}{4}(v + \varepsilon)^{-10} - \frac{1}{2}(v + \varepsilon)^{-4} \right\}
\]

\[
+ \mu_2 \left\{ (v + \varepsilon)^{-11} - (v + \varepsilon)^{-5} \right\}
\]

(4)

\[
f_e(\tau) = a_e \sin \Omega \tau
\]

The schematic setup of the cantilever beam is shown in Fig. 1. In the following results, we numerically investigate the microcantilever probe model in Eq. (1) with the boundary conditions in Eq. (3). The spatial and time discretizations \(\Delta x\) and \(\Delta \tau\) are set as \(\Delta x = 150\) and \(\Delta \tau = 10^{-5}\), respectively. In the following results, we fix the parameters \(a_e = 0.25\) and \(Q = 100\) by assuming that the operation is under atmosphere conditions with relatively small amplitude of the external periodic force. In addition, we also fix the parameters \(\mu_1 = 58.44\) and \(\mu_2 = 0.4469\). This condition is derived by assuming that both the probe tip and the observed sample are composed of silicon as reported in [3]. Furthermore, we employ the tip-sample distance \(\varepsilon\) as a control parameter.

### 3. Amplitude Decaying Characteristics

Because the dynamics in Eq. (1) are based on a spatially continuous model, under the boundary conditions in Eq. (3), there are infinite oscillation modes for \(C = 0\) with the eigen frequencies in the following equation:

\[
\omega_i = \frac{\lambda_i^2}{L^2} \sqrt{\frac{EI}{\mu}}, \quad (i = 1, 2, 3, \ldots)
\]

(5)

where \(\lambda_1 = 1.875, \lambda_2 = 4.694, \lambda_3 = 7.855,\) and the remaining \(\lambda_i, i \geq 4,\) are determined by Eq. (3) without any interaction or external force. Each oscillation mode with the angular frequency \(\omega_i\) is called “oscillation mode \(i\)”.

First, we investigate the case of oscillation mode 1, where the frequency of the external force \(\Omega\) is fixed near \(\Omega = \lambda_1\). Figure 2: Initial conditions for the three oscillation modes
\( v(1, \tau) \) for three values of \( \varepsilon \). Figure 3 shows the time series of oscillations at the probe tip as a function of the tip-sample distance. In this figure, for smaller \( \varepsilon \), we use the notation \( v_p \) to represent the amplitude of oscillation at the steady-state. This amplitude property, which depends on the tip-sample distance agrees with the experimental results, and the results in the models are represented by one degree of freedom [6]. It is confirmed that a qualitatively similar monotonical decay of amplitude is observed in our continuous model as shown in Fig. 4.

The continuous model used in this study has infinite oscillation modes. Therefore, it is possible to obtain the amplitude characteristic curves for the higher oscillation modes in addition to the result of oscillation mode 1. Figure 5 shows the amplitude characteristic curves for oscillation mode 2, where the initial condition is set to realize this oscillation mode as shown in Fig. 2 (b), and their derivatives are set to zero. Then, the frequency of the external force \( \Omega \) is fixed near \( \Omega = \sqrt{(\mu L^4)/(EI)\omega_1} \). The parameter regime in terms of \( \varepsilon \), in which the nearly constant \( v_p \) is obtained, becomes larger than that for oscillation mode 1, because the influence from the sample becomes weak due to the smaller steady-state amplitude. Nevertheless, the monotonically decaying amplitude characteristic is also obtained in oscillation mode 2. Moreover, by employing the initial condition in Fig. 2 (c) with \( \Omega = \sqrt{(\mu L^4)/(EI)\omega_3} \), a qualitatively similar result is confirmed for oscillation mode 3 as shown in Fig. 6, where the steady-state amplitude showing nearly free oscillation is
smaller than those of the oscillation modes 1 and 2.

Figures 7 (a) and (b) show the amplitude decaying characteristics for three values of $a_e$ of oscillation modes 2 and 3, respectively. In this parameter regime of $a_e$, qualitatively similar characteristic curves except for the steady-state amplitude of the nearly free oscillation are obtained for the higher oscillation modes.

Figure 7: Amplitude decaying characteristics for three values of $a_e$

4. Conclusions

By using a one-dimensional continuous microcantilever probe model, the amplitude properties of the fundamental oscillation mode and two higher oscillation modes were obtained. As a result, it was confirmed that the amplitude of oscillation of the higher modes monotonically decayed as a function of the tip-sample distance, which was qualitatively consistent with the result of the fundamental oscillation mode [6]. In our previous work, by using a simpler model described with one degree of freedom, we observed non-resonant frequency components, which corresponded to the frequency of transient beats superimposed on the stable steady-state solution, around the fundamental oscillation mode. The non-resonant frequency components are influenced by a coexisting distinctive solution, which intermittently contacts with the sample surface, and appears in the parameter regime where the monotonically decaying amplitude property of amplitude begins to occur. Because qualitatively similar results are obtained for the higher oscillation modes in this study, it is expected that such non-resonant frequency components appear in the higher oscillation modes.

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