Optimization of Amplitude-Frequency Characteristic of Broadband Voltage Divider Intended for Measurement of Power Quality Parameters

Об’єктом дослідження є схема широкосмугового ємнісно-омічного подільника напруги з послідовно-паралельним з’єднанням її резистивних та ємнісних елементів. Дові його застосування подільників напруги обмежувалася вимірюванням різних напруг в умовах високовольтних лабораторій. Однак подільники напруги, у порівнянні із трансформаторами напруги, характеризуються більш широким діапазоном пропускання, тому вони стали розглядалися як один з основних засобів вимірювання напруг у високовольтних електричних мережах. Одним з критеріїв вибору технічних прийомів вилучення шумів електронних схем є метод оптимізації амплітудно-частотних характеристик. Інтерес до цього напряму зростає через зростання стандартизаційної поглиначісність широкосмугових подільників напруги.

Завдання оптимального коректування низьковольтного плеча подільника напруги було вирішено за допомогою застосування елементів лінійного програмування для дослідження функції систематичної похибки. Розрахунки були здійснені в плоскостях рівеньній, чітко визначеної граничної та відносної похибок. Результати відображають зростання коефіцієнта ділення досліджуваного подільника напруги, які відображають його систематичну похибку. Розрахунки показують, що відповідний розподіл величини питомих похибок може бути ефективною стратегією для підвищення якості електричної енергії.

1. Introduction

Measurement of electric power quality parameters is necessary during energy generation, distribution and consumption. In addition, measurements are necessary to ensure the possibility of controlling the quality of electricity as a type of product. For this purpose, technical and organizational measures are being implemented to achieve compliance of
electric power quality parameters with the requirements of international standards, for example, IEC 61000-4-30:2015. Under modern conditions, attention to the electric power quality is constantly growing, since electric power quality determines in many cases the ability to function of many complicated devices, critical equipment and entire systems. This task requires the development of measuring instruments with an extremely low error and the ability to measure voltages over a wide frequency range. Among high-voltage scale transducers, one should mention voltage transformers and voltage dividers. For many years, the use of voltage dividers was limited to measuring various voltages [1, 2] in high-voltage laboratories. However, voltage dividers, compared to voltage transformers, are characterized by a wider bandwidth. Therefore, many researchers began consider them as one of the main means for voltage measurement in high-voltage electric networks [3–5]. The authors also believe that voltage dividers have much greater potential for improvement than instrument transformers. In this connection, studies on the possibility of using voltage dividers to measure electric power quality parameters [6–8] were started at the Department of Theoretical Electrical Engineering of the National Technical University of Ukraine «Igor Sikorsky Kyiv Polytechnic Institute» (Ukraine). The work being performed contains both experimental and theoretical studies on the possibility of using voltage dividers instead of voltage transformers for measuring the electric power quality parameters. Thus, the object of research is the circuit diagram of a broadband capacitive-resistive voltage divider with a series-parallel connection of its resistive and capacitive components. In its turn, the main aim of the article is to study the adjustment of the amplitude-frequency characteristics of the voltage divider, which is aimed at reducing its measurement error.

2. Methods of research

The voltage divider, constructed according to principle of voltage distribution over the complex impedances, is usually called a mixed capacitive-resistive voltage divider with a series-parallel connection of its resistive and capacitive components. A generalized equivalent circuit for a voltage divider of this type, which also gives a general idea of the components. A generalized equivalent circuit for a voltage divider [9], the amplitude-frequency characteristic of the voltage divider is determined by the expression:

$$A(\gamma) = \frac{1}{K} A_1(\gamma), \%$$

(1)

According to the theory of voltage dividers [9], the amplitude-frequency characteristic of the voltage divider is determined by the expression:

$$A_1(\gamma) = \frac{K}{K-1} \left\{ \frac{1}{1 + \left( \frac{\gamma}{1 + \delta^2 \Theta} \right)^2} \right\} \left\{ \frac{1}{1 + \left( \frac{\gamma}{1 + \delta^2 \Theta} \right)^2} \right\}$$

(2)

$$a = 1 + f + \frac{1}{K - 1} \left( \frac{\gamma}{1 + \delta^2 \Theta} \right)^2$$

(3)
The parameter $\alpha$ is determined by the following expressions:

$$\gamma = \alpha R C_0; \quad R_0 = \frac{1}{n} \sum_{i=1}^{n} R_i; \quad C_0 = \frac{1}{n} \sum_{i=1}^{n} C_i,$$

where $R_0$ and $C_0$ are average values of resistive and capacitive components of the high-voltage arm of the voltage divider, respectively; $n$ is the total number of these components.

It is shown in [6] that the non-identity of the resistive components of the high-voltage arm of the voltage divider is negligible compared to the non-identity of the capacitive components.

Then in the expression (3) the function $f$ is determined by the ratio:

$$f = \frac{\gamma'}{1 + \gamma'} \frac{1}{n} \sum_{i=1}^{n} \alpha_i^2 \left( \gamma'(1 + \alpha_i) - 1 \right).$$

In its turn, in expression (4), the function $\delta$ is determined by the expression:

$$\delta = \frac{\gamma'}{1 + \gamma'} \frac{1}{n} \sum_{i=1}^{n} \alpha_i^2 \left( \gamma'(1 + \alpha_i) - 3 - \alpha_i \right).$$

In the two above expressions parameter $\alpha_i$ depends on the capacitance values of the high-voltage arm of the voltage divider. The selection of the low-voltage arm components is being performed according to the common expressions:

$$r = \frac{n R_0}{K - 1}, \quad c = \frac{C_0}{n}(K - 1).$$

what corresponds to the value of $\Theta = 0$ in formulas (2)–(4). Herewith, this case corresponds to the absence of the low-voltage arm adjustment of the voltage divider.

The value of $\Theta = 1$ in formulas (2)–(4) corresponds to the maximum (or, in other words, ultimate) adjustment of the low-voltage arm capacitance of the voltage divider, which takes the value:

$$c' = \frac{C_0}{n} \frac{K - 1}{1 + \delta'}.$$

As preliminary calculations show that the use of the maximum adjustment of the low-voltage arm capacitance allows reducing the maximum value of the systematic error of the voltage divider by more than 2 times:

$$\Delta_1 = \left( A_i - 1 \right) \cdot 100^\%.$$

However, this maximum value $\Delta_1$, in its turn, can again be reduced by almost 2 times by choosing the optimal parameter value $0 < \Theta_{opt} < 1$. The task of the work is to search for this optimal value $\Theta_{opt}$, as well as to study the dependence of the «minimized» error of the voltage divider on a number of factors.

It should be noted that the optimized value of the low-voltage arm capacitance of the voltage divider can be determined by the ratio:

$$C_{opt} = \frac{C_0}{n} \frac{K - 1}{1 + \Theta_{opt} \delta'},$$

and its amplitude-frequency characteristic is determined by the expressions (1)–(4) when substituting $\Theta = \Theta_{opt}$.

As a model of capacitive components' non-identity of the high-voltage arm of the voltage divider, a symmetric «triangular» distribution is used [6]. For this distribution the maximum deviations of the capacitances $C_i$ from $C_0$ are characterized by the ratio:

$$C_i = C_0 (1 \pm \Delta_i).$$

Program for optimizing the amplitude-frequency characteristic

$$\Theta := K_1 \left\{ \begin{array}{l}
\Theta \leftarrow 0.63 \\
\Theta \leftarrow 0.63 \\
\Theta \leftarrow 0.63
\end{array} \right.,$$

$$\Theta \leftarrow +0.01$$

while $\frac{-\max A}{\min A} < 0.9999$

for $i \in 1..10000$

$$\gamma_i \leftarrow 0.01 \cdot i$$

$$A_{res_i} \leftarrow \left( A(1 - K_1 \cdot \gamma_i \cdot \delta_{opt} / \Theta) - 1 \right) \cdot 100$$

$$\max A < A_{res_i}$$

for $i \in 1..10000$

$$\max A \leftarrow A_{res_i}$$

$$\min A \leftarrow A_{res_i}$$

for $i \in 1..10000$

$$\min A \leftarrow A_{res_i}$$

$$\Theta \leftarrow \Theta + 1 \cdot 10^{-4}$$

return $\Theta$

**Fig. 2.** Program for optimizing the amplitude-frequency characteristic.
Programming is performed with a help of the Mathcad software [10]. The program works as follows. First, the program searches for the maximum of the systematic error curve. Then, the program searches for the minimum of the systematic error curve. The program then compares the absolute values of the maximum and minimum. If these values are different, the variable $\Theta$ increments 0.0001 and the search loop repeats. The loop will end when equality is reached between the absolute value of the maximum and minimum of the studied function. The program allows finding the value of $\Theta_{opt}$ accurate to the fourth digit after the decimal point. The results of calculations obtained with a help of this program are given in the next section.

3. Research results and discussion

The calculations were performed for various values of the divider’s voltage ratio ($K=10^1; 10^2; 10^3; 10^4; 10^5$) and for various values of the maximum deviation of the high-voltage arm capacitances from the average value ($\Delta C = 0.01…0.20$). Under such conditions, authors obtained surface graphs of the variable $\Theta_{opt}$ (refer to Fig. 3) and the systematic error $\Delta_A$ (refer to Fig. 4).

![Fig. 3. Three-dimensional graph of the variable $\Theta_{opt}$](image)

![Fig. 4. Three-dimensional graph of the systematic error $\Delta_A$ (%) of the capacitive-resistive voltage divider](image)

In Fig. 3 and Fig. 4, for the divider’s voltage ratios $K$ the logarithmic scale is used (the orders of magnitude of the divider’s voltage ratio are plotted along the axis).

The surface graph in Fig. 3 depicts what value a variable $\Theta_{opt}$ should have so that for given $K$ and $\Delta C$ the absolute values of the maximum and minimum of the systematic error are the same.

The surface graph in Fig. 4 depicts that with an increase in the $\Delta C$ parameter value, almost independently of the value of the divider’s voltage ratio $K$, the value of the amplitude error $\Delta_A$ increases in a parabolic dependence on $\Delta C$. The graphs in Fig. 3 and Fig. 4 summarize a huge array of computational data.

Let’s show in more details one of the results of optimizing the amplitude-frequency characteristics of the voltage divider in Fig. 5. This graph shows the dependence of the systematic error $\Delta_A$ (%) on the generalized parameter $\gamma$. The shape of the curve is practically the same in the entire studied range of parameters $\Delta C$, $K$, $\Theta_{opt}$. Only the absolute values of the maximum and minimum differ.

![Fig. 5. Example of the systematic error graph $\Delta_A$ (%) obtained after optimization by adjusting the low-voltage arm of the voltage divider](image)

Functional dependence in Fig. 5 is obtained for $\Delta C = 0.05$ and $K=10^2$. Using the above search algorithm, a value $\Theta_{opt} = 0.656$ is obtained. With this optimal value the absolute values of the maximum and minimum amplitude errors are the same and equal to 0.01495 %.

The further development of the theory of voltage dividers is promising due to the fact that this category of high-voltage scale transducers has the potential to become mandatory for determining the quality parameters of electric energy directly at high voltage. One of the catalysts for this may be the intensive development of the Smart Grid concept, which requires new, more advanced means of monitoring the quality of electric power [11, 12]. Therefore, experimental and theoretical studies aimed at reducing the error of broadband voltage dividers are important.

4. Conclusions

As a result of the performed research, the possibility of optimizing the amplitude-frequency characteristics of a broadband capacitive-resistive voltage divider by varying the value of its low-voltage arm capacitance is shown.

The optimization parameter of the low-voltage arm capacitance value of the voltage divider in the entire range of the studied parameters $K=10^1–10^5$, $\Delta C = 0–0.2$ can be characterized by a constant value $\Theta_{opt} = 0.656$. 

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The dependence of the systematic error of the optimized voltage divider on the dimensionless frequency parameter $\gamma$ is universal in form with the difference in the maximum ultimate values of $\Delta A$.

For the values of divider's voltage ratio in the range $1 < K < 10$, an additional research is required to optimize the amplitude-frequency characteristic.

References

1. Harada, T., Wakimoto, T., Sato, S., Saki, M. (2000). Development of Japan’s National Standard Class 300 kV Lightning Impulse Voltage Divider. 2000 IEEE Power Engineering Society Winter Meeting. Conference Proceedings (Cat. No.00CH37077), 3, 1564–1568. doi: http://doi.org/10.1109/pewm.2000.847575

2. Prochazka, R., Hlavacek, J., Knenicky, M., Mahmoud, R. (2016). Determination of Frequency Characteristics of High Voltage Dividers in Frequency Domain. 17th International Scientific Conference on Electric Power Engineering (EPE), 1–4. doi: http://doi.org/10.1109/epe.2016.7521821

3. Muscas, C. (2010). Power quality monitoring in modern electric distribution systems. IEEE Instrumentation & Measurement Magazine, 13 (3), 19–27. doi: http://doi.org/10.1109/mim.2010.5585070

4. Pawelek, R., Wasiak, I. (2014). Comparative measurements of voltage harmonics in transmission grid of 400 kV. 2014 16th International Conference on Harmonics and Quality of Power (ICHQP), 606–610. doi: http://doi.org/10.1109/ichqpn.2014.6842763

5. Blajszczak, G. (2011). Resistive Voltage Divider for Higher Harmonics Measurement in 400 kV Network. 11th International Conference on Electrical Power Quality and Utilisation, 1–4. doi: http://doi.org/10.1109/equin.2011.6128053

6. Anokhin, Y. L., Brzhezitsky, V. O., Haran, Y. O., Masliuchenko, I. M., Protsenko, O. P., Trotsenko, Y. O. (2017). Application of high voltage dividers for power quality indices measurement. Electrical Engineering & Electromechanics, 6, 53–59. doi: http://doi.org/10.20998/2074-272x.2017.0.08

7. Trotsenko, Y., Brzhezitsky, V., Protsenko, O., Haran, Y., Chumalack, V. (2018). Calculation of High Voltage Divider Accuracy Using Duhamel’s Integral. 2018 IEEE 17th International Conference on Mathematical Methods in Electromagnetic Theory (MMMET), 213–216. doi: http://doi.org/10.1109/mmmet.2018.8460314

8. Trotsenko, Y., Brzhezitsky, V., Protsenko, O., Haran, Y. (2019). Experimental Laboratory Equipped with Voltage Dividers for Power Quality Monitoring. 2019 IEEE International Conference on Modern Electrical and Energy Systems (MEES), 270–273. doi: http://doi.org/10.1109/mees.2019.8896471

9. Brzhezitsky, V. O., Isakova, A. V., Rudakov, V. V.; Brzhezitsky, V. O., Mykhailov, V. M. (Eds.) (2005). Tehnika i elektrofizika vysokikh napryag. Kharkiv: NTU «KhPI»-Tornado, 930.

10. Makarov, E. G. (2005). Inzhenernye raschety v Mathcad. Saint Petersburg: Piter, 448.

11. Gharavi, H., Ghafurian, R. (2011). Smart Grid: The Electric Energy System of the Future. Proceedings of the IEEE, 99 (6), 917–921. doi: http://doi.org/10.1109/jproc.2011.2124210

12. Rahmatian, F. (2010). High-Voltage Current and Voltage Sensors for a Smarter Transmission Grid and Their Use in Live-Line Testing and Calibration. IEEE PES General Meeting, 1–3. doi: http://doi.org/10.1109/pes.2010.5590212