The Heat Transfer Coefficient Predictions in Engineering Applications

Junchi Wan\textsuperscript{1a*}
\textsuperscript{1}Shandong University, Jinan, China, 250002
\textsuperscript{a*}201800180151@mail.sdu.edu.cn

Abstract—Most engineering applications have boundary layers; the convective transport of mass, momentum and heat normally occurs through a thin boundary layer close to the wall. It is essential to predict the boundary layer heat transfer phenomenon on the surface of various engineering machines through calculations. The experimental, analogy and numerical methods are the three main methods used to obtain convective heat transfer coefficient. The Reynolds analogy provides a useful method to estimate the heat transfer rate with known surface friction. In the Reynolds analogy, the heat transfer coefficient is independent of the temperature ratio between the wall and the fluid. Other methods also ignore the effect of the temperature ratio. This paper summarizes the methods of predicting heat transfer coefficients in engineering applications. The effects of the temperature ratio between the wall and the fluid on the heat transfer coefficient predictions are studied by summarizing the researches. Through the summary, it can be found that the heat transfer coefficients do show a dependence on the temperature ratio. And these effects are more obvious in turbulent flow and pointing out that the inaccuracy in the determination of the heat transfer coefficient and proposing that the conjugate heat transfer analysis is the future direction of development.

1. Introduction

A flow boundary layer is created near the surface when viscous fluid flows over a solid surface. Usually, there is a temperature difference between the main flow and the wall. Not only does the boundary layer have a drastically changing velocity gradient, but also the temperature changes drastically. Therefore, convective heat transfer occurs intensively in the boundary layer. This phenomenon occurs in many engineering applications, such as the blades of gas turbines, the walls of heat exchangers, and the heat dissipation surfaces of integrated circuits [1]. Common convective heat transfer can be divided into forced convection and natural convection. But in simplify, both types of convective heat transfer can divide the boundary layer flow into the laminar flow and turbulent flow. The upstream flow has a great influence on the heat transfer coefficient in a laminar boundary layer, including leading-edge conditions and wall temperature changes. In the turbulent boundary layer, the heat transfer coefficient is not sensitive to the slow flow direction change of the wall temperature [2].

This paper mainly predicts the heat transfer coefficient, including analogy, experimental, and numerical methods. This paper discusses these methods in detail to analyze the existing shortcomings. As the basis of convective heat transfer, these methods are used and studied in many research. However, many papers do not consider their accuracy. Through comparison, the influence of the temperature ratio on the heat transfer coefficient and the future development trend of the heat transfer coefficient prediction techniques are shown clearly.
2. Methods for predicting the heat transfer coefficient

Newton's cooling equation for calculating the rate of convective heat transfer $Q$ is expressed as

$$
Q = hA(T_w - T_\infty)
$$

(1)

Where $h$ is the heat transfer coefficient, $A$ is the area of heat transfer, $T_w$ is the wall temperature of solid, and $T_\infty$ is the temperature of the main fluid. In engineering calculations, the temperature difference $(T_w - T_\infty)$ is always positive.

Newton's cooling equation is only a definition formula of the heat transfer coefficient $h$. It does not reveal the connection between the heat transfer coefficient and the related factor that affect it. The prediction method of heat transfer coefficient is obtained based on Eq. (1) and these connections.

2.1. Analogy method

The analogy method refers to the method of obtaining the basic relationship between two different physical phenomena due to the similarity in the governing equations by measuring one of the phenomena. In boundary layer heat transfer, the most basic and most commonly used analogy is the Reynolds analogy. It is an analogy between heat and momentum transfer due to molecular motion. It provides an available tool to use the shear stress to calculate the heat transfer coefficient for both laminar and turbulent flows. The general expression of the Reynolds analogy is

$$
Nu = \frac{C_f}{2} Re Pr
$$

(2)

$C_f$ is the skin friction coefficient; it can be obtained from experiments, $Nu$ is the Nusselt number, $Re$ is the Reynolds number, $Pr$ is the Prandtl number. These three dimensionless numbers are defined as

$$
Nu = \frac{h l}{k}, \quad Re = \frac{U_{\infty} l}{v}, \quad Pr = \frac{v}{\alpha}
$$

(3)

Here $k$ is the thermal conductivity, $\nu$ is the kinematic viscosity, $u$ is the dynamic viscosity, $U_\infty$ is the fluid velocity, $c_p$ is the specific heat at constant pressure, $l$ is the characteristic length. Through combining Eq. (2) and Eq. (3), the expression of the heat transfer coefficient $h$ can be determined

$$
h = \rho U_\infty c_p \frac{C_f l}{2}
$$

(4)

So $h$ can be determined by Eq. (4), all the fluid properties are obtained at the reference temperature $T_m = (T_w + T_\infty)/2$ [3]. However, the Reynolds analogy has a limitation that the Prandtl number must be assumed as one. Although for most gas and water which the Prandtl number is close to one, Eq (2) is still valid, the error can be ignored. Later, Chilton and Colburn [4,5] modified the Reynolds analogy, known as the Reynolds-Colburn Analogy

$$
Nu = \frac{C_f}{2} Re Pr^{\frac{1}{3}} (0.6 < Pr < 60)
$$

(5)

Zeng et al. [6] found that the experimentally measured air-side heat transfer coefficients were consistent with the modified Reynolds analogy, verifying its applicability.

The modified Reynolds analogy and some other analogies have a large number of applications in engineering. Wang et al. used the heat-momentum transfer analogy to correlate the single-phase heat transfer data in microfin and showed good accuracy [7]. A semi-empirical model based on an analogy between heat and momentum transfer in turbulent flow was proposed by Arsenyeva et al. to predict the heat transfer for flows in channels of a plate heat exchanger [8]. Similarly, KWON et al. developed a model using a heat-momentum transfer analogy to estimate the evaporative heat transfer coefficient in complicated cooling systems [9]. Conversely, the measured heat transfer coefficient can be used to calculate the skin friction coefficient through the Reynolds analogy to study the effect of roughness, like Abuaf et al. did on turbine airfoils [10]. Also, in the turbine airfoil, the naphthalene sublimation technique based on a heat and mass transfer analogy was applied by Häring et al. to measure the heat transfer coefficient [11].

2.2. Experimental method

The calculation formulas of the heat transfer coefficient are obtained through the experiments, and they are the basis of engineering design. These formulas are also called correlations, in which the heat transfer coefficient to be determined included in the Nusselt number. Usually, dimensional analysis is used to
derive the non-dimensional number that reflects the physical parameters' inner relationship. For example, dimensional analysis is used on the forced convection heat transfer. It shows that the Nusselt number can be expressed as a function of Reynolds number and Prandtl number \[ Nu = f(Re, Pr) \] (6)

But the determination of the specific function form is empirical. Usually, people determine the constants in Eq. (6) through experimental data of different Reynolds numbers and Prandtl numbers, as Lienhard concluded in flat-plate boundary layers. Researchers are constantly conducting researches on correlations, and some new and more accurate correlations are proposed for the same type of problem [2].

However, correlations are only applicable to the convective heat transfer problems on the surface of some objects with simple and regular shapes, such as the outer surfaces of plates, cylinders and spheres, and the inner surfaces of circular tubes non-circular channels. For objects with more complicated shapes, like the turbine blades, it is so challenging to use correlations for calculation. In this case, it is necessary to back to the basic Newton's cooling equation (1). That the heat transfer coefficient is expressed as 

\[ h = \frac{q}{T_w - T_f} \] (7)

\(q\) is the heat flux. Andreini et al. experimented on an open-loop wind tunnel with a cooling system of a high-pressure gas turbine leading-edge model to determine the heat transfer coefficient distribution on the internal surface, using the Eq. (7) [12]. An electrically heated sheet applied on the internal surface was used to imposed a constant heat flux. The wall temperature was measured by wide band thermochromic liquid crystals, and the fluid temperature was measured by T-type thermocouples. Similarly, the heat transfer coefficient distribution behind the rib is determined by Davletshin et al., using constant-temperature heat flux generated from a printed circuit board and wall temperature measured by resistance thermometers [13]. Will et al. conducted similar experiments on spheres of various diameters [3]. It can be seen that the temperature obtained by experimental measurement is the basis. In addition to using Eq. (7), the energy conservation equations and the lumped parameter approximation were also used [6,10].

2.3. Numerical method
The numerical solution of the convective heat transfer problem refers to using various numerical methods to solve the governing equation, including the Navier-Stokes equation and energy conservation equation, etc. under fixed conditions. The fixed conditions include the velocity, pressure, temperature and other relevant conditions at the initial moment and on the boundary. Usually, the solution process needs to use computational fluid dynamics (CFD) software to obtain the temperature distribution of the fluid and the heat flux at the wall. Then the heat transfer coefficient predictions are obtained by Eq. (7) [14].

2.4. Discussion
Back to Eq. (4) derived from the Reynolds analogy, it can be seen that the heat transfer coefficient depends on the fluid properties obtained at the reference temperature \( T_m = (T_w + T_\infty)/2 \). If the temperature differences between the wall and free stream are small, the variations in these properties would be small. But when the temperature differences are relatively large, the accuracy of the predictions will decrease, so the effect of the temperature ratio between the wall and free stream should be considered. Eq. (7) is the key basis when using experiments or numerical methods for the heat transfer coefficient predictions. The fixed thermal boundary conditions are imposed, like constant heat flux, and the constant heat transfer coefficient is assumed. As shown in Fig. 1, the heat flux \( q \) is linear with the wall temperature; the slope is the result. But it should be pointed out that such an assumption is not accurate because this hypothesis set that the thermal boundary condition does not affect the heat transfer coefficient. In fact, the \( h \) would change with the changing wall temperature. For predicting the heat transfer coefficient more accurately in engineering applications, the dependence of the heat transfer coefficient on the wall-to-fluid temperature ratio is worth research.
3. Effects of the temperature ratio on the heat transfer coefficients

To figure out the temperature ratio effects, Fitt et al. proposed an equation $Nu = Nu_0(T_w/T_{\infty})^n$ under constant Reynolds number [15]. They conducted experiments of airflow on a flat plate with zero pressure gradient, using the transient technique to make sure the properties of tested airflow were constant. The Reynolds number per meter was $2.7 \times 10^7$ and the Mach number was 0.55. With fixed air temperature and changing wall temperature by forced convection, the range of the wall-to-fluid temperature ratio was 0.6 to 1.3. Within this range, with the help of a power law, the measured value of $n$ is between -0.28 and -0.21. Some numerical calculations were also done by them, shown similar results of turbulent flow compared to the experiments, but the effect of the temperature ratio in laminar flow is very small. It can be shown that the heat transfer coefficients in the turbulent boundary layer exist a dependency on the wall-to-fluid temperature ratio. The effect is proved; the next step is to quantify it. This work was done by Maffulli and He [16]. They used CFD with the Spalart-Allmaras turbulence model to analyze the external heat transfer coefficients on a 2D Nozzle Guide Vane profile under three different temperature ratios (0.95, 0.75 and 0.6). Both laminar and turbulent conditions were considered. The result was shown in Fig. 2, which shows that the values of $h$ at the trailing edge in 0.6 temperature ratio are about 25% higher than those in 0.95 temperature ratio.

![Fig. 1. The line under the constant heat transfer coefficient](image1.png)

![Fig. 2. Dependence of external heat transfer coefficient distributions on the temperature ratio](image2.png)

Obviously, the temperature ratios have fewer effects on the laminar flow area, the 0% to 40% of curve length. This result is consistent with the conclusions of Fitt et al. It can be confirmed that the effects of the temperature ratio on the heat transfer coefficient are much smaller in the laminar flow state than in the turbulent flow state.
4. Conclusion

Whether it is a flat plate with a simple shape or a steam turbine blade with a complex structure, the heat transfer coefficient of the surface should be accurately predicted to obtain the heat transfer characteristics. It plays a very important role in the design of various mechanical parts in engineering applications. Three different methods for determining the heat transfer coefficient and their applications have been summarized in this article. The factors that affect the accuracy of the results have been discovered from the basic principles, and the effects of the wall-to-fluid temperature ratio are proposed. By summarizing the research of other authors, it is concluded that the temperature ratio has a certain effect on the heat transfer coefficient. This influence is almost absent in laminar flow but more obvious in a turbulent flow. The difference in heat transfer coefficients can even reach 25% on the suction side of the blades at two temperature ratios with a relatively large difference. Fundamentally, the effect of the temperature ratio is caused by the fixed thermal boundary conditions in the calculation process. Because in the process of convective heat transfer, the thermal boundary conditions are affected by the interaction between the fluid and the wall, so it will change as the heat transfer process progresses. This problem can be solved if applying the coupled solid-fluid conjugate heat transfer analysis because the boundary conditions are taken as the calculation results. The rationality and accuracy of the simulation results of the heat transfer problem would be much improved. But for now, the conjugate heat transfer analysis is still in development, and the use cost is relatively expensive. So the non-conjugate methods are still most frequently used. In the future, the continuously developing conjugate heat transfer analysis techniques would better solve the heat transfer problem in engineering applications.

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