Semileptonic Decay of $B$ and $D \rightarrow K^*_0(1430)\bar{\ell}\nu$

From QCD Sum Rule

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Abstract

We calculate $B_{(s)}$ and $D_{(s)}$ to $K^*_0(1430)$ transition form factors, and study semileptonic decays of $B_{(s)}$ and $D_{(s)} \rightarrow K^*_0(1430)\bar{\ell}\nu$ based on QCD sum rule. Measuring these semileptonic decays with high statistics will give valuable information on the nature of light scalar mesons.

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Semileptonic decays of $B$ and $D$ mesons are important for studying quark flavor mixing and extracting the Cabibbo-Kobayashi-Maskawa (CKM) matrix element, because strong interactions involved in these processes are simpler than that of hadronic decays. The strong binding effects can be parameterized as transition form factors, which can be calculated by Lattice QCD [1], QCD sum rule [2, 3, 4, 5], light-cone sum rule [6], or by quark model [7], light-front approach [8], and recently by large-energy and heavy-quark-effective theory [9]. Experimental data on semileptonic decay of $B$ and $D$ mesons can be used to test these theoretical method of treating nonperturbative dynamics. Transition form factors inducing semileptonic decays not only depend on dynamics of strong interactions between quarks in the initial and final hadrons, but also on the structure of the hadrons involved in the semileptonic decays.

The structures of scalar mesons are long-standing problems in particle physics. A large amount of scalar mesons have been found in experiment [10]. They include $\sigma$ [or $f_0(600)$], $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $a_0(980)$, $a_0(1450)$, $\kappa$, $K^*_0(1430)$, etc. [11]. At least they can be divided into two flavor nonets, one below or near 1GeV, the other above 1GeV. The structure of scalar mesons is still not well established theoretically [12]. In the literature, many suggestions are discussed such as $q\bar{q}$, $q\bar{q}q\bar{q}$ and meson-meson bound states. Among the controversy, almost every model on scalar states agrees that $K^*_0(1430)$ is dominated by $s\bar{u}$ or $s\bar{d}$ state. Recent analysis from QCD sum rule agrees with this.
The result of QCD sum rule favors that $K_0^*(1430)$ is mainly $s\bar{q}$ bound state. The predicted mass of $s\bar{q}$ $0^+$ scalar bound state is consistent with the mass of $K_0^*(1430)$.

To investigate the structure of scalar meson, a large amount of experimental data and theoretical studies are necessary. Scalar mesons are produced in $\pi N$ scattering, $p\bar{p}$ annihilation, decays of heavy flavor mesons, etc.. Recently, hadronic $D$ decays involving scalar mesons are studied in generalized factorization approach \[14\]. Hadronic decay of $D$ meson is complicated, it suffers from final-state interactions which are difficult to control theoretically. Compared with hadronic decays, semileptonic decays are more simpler. They are free from final-state interactions. All nonperturbative binding effects are parameterized in the transition form factors.

In this work, we shall study semileptonic decays of $B_s$ and $D_s$ meson involving $K_0^*(1430)$. In Ref.\[15\], semileptonic decays of $D$ meson into $\sigma, \kappa$ are studied in QCD sum rule, where the scalars $\sigma$ and $\kappa$ are treated as $q\bar{q}$ bound states, which is different from the picture that scalars below 1 GeV are predominantly multiquark states. According to our previous study in QCD sum rule, $K_0^*(1430)$ is dominantly the ground state of $s\bar{q}$ scalar channel. We treat $K_0^*(1430)$ as $s\bar{q}$ bound state in this work. The transition form factors of $B_s$ and $D_s \to K_0^*(1430)$ are calculated in QCD sum rule. Then they are used to study semileptonic $B$ and $D$ decays. The semileptonic decay modes of $B_s$ and $D_s$ mesons involving $K_0^*(1430)$ include $B_0^s \to K_0^*(1430)\bar{\ell}\nu$, $D^0 \to K_0^*(1430)\bar{\ell}\nu$, $D^+ \to K_0^*(1430)^0\bar{\ell}\nu$, and $D_s^+ \to K_0^*(1430)^0\bar{\ell}\nu$, where $\ell$ is the lepton $e$ or $\mu$. Compared to the large mass of $B$ and $D$ meson, the mass of the lepton $e$ or $\mu$ is dropped.

Figure 1: Semileptonic decays of $B_s$ and $D_s$ involving $K_0^*(1430)$. (a) for $B_0^s \to K_0^*(1430)^-\bar{\ell}\nu$, (b) for $D^0 \to K_0^*(1430)^-\bar{\ell}\nu$, (c) for $D^+ \to K_0^*(1430)^0\bar{\ell}\nu$, and (d) for $D_s^+ \to K_0^*(1430)^0\bar{\ell}\nu$

The Feynman diagrams for the semileptonic $B_s$ and $D_s$ decays into $K_0^*(1430)\bar{\ell}\nu$ are shown in Fig.1. In the limit of flavor SU(3) symmetry, the amplitudes of the three decay modes $D^0 \to K_0^*(1430)^-\bar{\ell}\nu$, $D^+ \to K_0^*(1430)^0\bar{\ell}\nu$ and $D_s^+ \to K_0^*(1430)^0\bar{\ell}\nu$ should be the same. However, flavor SU(3) symmetry is not an exact symmetry, because the mass of $s$ quark is much larger than that of $u$ and $d$ quarks. To investigate SU(3) symmetry breaking effect, the mass of $s$ quark $m_s$ is kept in our calculation.
The amplitude of $B$ and $D$ decaying into $K_0^*(1430)\bar{\ell}\nu$ is

$$A = \frac{G_F}{\sqrt{2}} V_{q'q}^* \bar{q}'\gamma_{\mu}(1 - \gamma_5)\ell\langle K_0^*(1430)|\bar{q}'\gamma^\mu(1 - \gamma_5)Q|H \rangle$$  \hspace{1cm} (1)$$

where $G_F$ is Fermi constant, $H$ is the heavy flavor meson $B$ or $D$, $Q$ is $b$ or $c$ quark, $q'$ can be light quark $u$, $d$, $s$, and $V_{q'q}$ is the relevant CKM matrix element. Strong interactions are contained in the hadronic matrix element $\langle K_0^*(1430)|\bar{q}'\gamma^\mu(1 - \gamma_5)Q|H \rangle$. By analyzing the parity property of the hadrons and the current, we can know that the vector current does not contribute to the pseudoscalar-scalar hadronic matrix element

$$\langle K_0^*(1430)|\bar{q}'\gamma^\mu Q|H \rangle = 0,$$  \hspace{1cm} (2)$$

therefore, only the axial current contributes. From the Lorentz invariance, the hadronic matrix element of axial current can be decomposed into

$$\langle K_0^*(1430)|\bar{q}'\gamma^\mu\gamma_5 Q|H \rangle = -i[(p_1 + p_2)^\mu F_+(q^2) + (p_1 - p_2)^\mu F_-(q^2)],$$  \hspace{1cm} (3)$$

where the parameters $F_+(q^2)$ and $F_-(q^2)$ are the transition form factors, which only depend on the momentum transfer squared. The momenta $p_1$ and $p_2$ are the momenta of the initial and final state mesons, respectively, and $q = p_1 - p_2$.

Next we shall calculate the transition form factors in QCD sum rule method. In the limit of the lepton mass $m_\ell \sim 0$, the form factor $F_-$ does not contribute to the semileptonic decays of $B$ and $D$ mesons. So we shall not consider $F_-$ in this work.

In the standard procedure of QCD sum rule for calculating transition form factors, all hadrons involved in the transition process are interpolated by currents with the appropriate quantum numbers relevant to the hadrons. The heavy flavor meson $H$ ($B$ or $D$) is interpolated by pseudoscalar current $J_H(x) = \bar{Q}(x)i\gamma^5q'(x)$, summation over the spinor and color indices being understood in this interpolating current. The final scalar meson $K_0^*(1430)$ is interpolated by the scalar current $J_K(x) = \bar{q}(x)s(x)$, here $q$ is up or down quark. Then the decay rate is obtained from the vacuum expectation value of time ordered product of the interpolating fields $J_H(x), J_K(x)$ and the weak axial current $J_\mu = \bar{q}'\gamma_\mu\gamma_5Q$. The vector part of the weak current is not presented here because its contribution to $B$ or $D \rightarrow K_0^*(1430)$ transition is zero. A three-point correlation function can be constructed from the vacuum expectation value of time ordered product of interpolating fields and axial weak current

$$\Pi_\mu(p_1^2, p_2^2, q^2) = i^2 \int d^4x d^4y e^{ip_2\cdot x - ip_1\cdot y}\langle 0|T\{J_K(x)J_{\mu}(0)J_H(y)\}|0 \rangle.$$  \hspace{1cm} (4)$$

Phenomenologically, one can insert a full set of intermediate states into the time ordered product and obtain the double dispersion relation, then the correlation function is expressed as a sum of the contributions of the lowest lying and excited states

$$\Pi_\mu(p_1^2, p_2^2, q^2) = \frac{\langle 0|\bar{q}_2'q_1'\langle K_0^*(1430)\rangle\langle K_0^*(1430)|\bar{q}'_1\gamma_\mu\gamma_5Q|H \rangle\langle H|\bar{Q}_i\gamma_5q_2'|0 \rangle}{(m_H^2 - p_1^2)(m_{K_0^*}^2 - p_2^2)}$$

$$+ \text{excited states},$$  \hspace{1cm} (5)$$

where $q'_1$ and $q'_2$ can be $u$, $d$, $s$ quarks, but they do not take the same flavor at the same time and there must be one to be $s$ quark in $q'_1$ and $q'_2$. By introducing $\langle 0|\bar{q}s|K_0^*(1430)\rangle = \ldots$
The coefficients $f_{K^*}$ and $m_{K^*}$ and $\langle H|\bar{q}_1\gamma_5q_2|0\rangle = \frac{m_H^2}{m_Q + m_{q_2}}f_H$, where $f_{K^*}$ and $f_H$ are decay constants, the correlation function is obtained as

$$\Pi_\mu(p_1^2, p_2^2, q^2) = \frac{m_H^2}{m_Q + m_{q_2}} \frac{f_H f_{K^*} m_{K^*}}{(m_H^2 - p_1^2)(m_{K^*}^2 - p_2^2)} \langle K_0^*(1430)|\bar{q}_1\gamma_\mu\gamma_5Q|H \rangle + \text{excited states}$$ (6)

Therefore the correlation function is related to the transition matrix element by eq. (5). If the correlation function $\Pi_\mu(p_1^2, p_2^2, q^2)$ can be calculated in QCD reliably, then the transition form factors can be extracted through eq. (5). In QCD, the three-point correlation function can indeed be evaluated by operator-product-expansion (OPE) method in the deep Euclidean region

$$p_1^2 \ll (m_Q + m_{q_2})^2, \quad p_2^2 \ll (m_s + m_q)^2.$$ (7)

Although it is in the deep Euclidean region of $p_1^2$ and $p_2^2$ that the correlation function can be calculated, eq. (6) clearly shows that the value of $\Pi_\mu(p_1^2, p_2^2, q^2)$ can still be used to extract the transition form factors which is typically a Minkowskian quantity, because the valuables $p_1^2$ and $p_2^2$ are clearly separated from the transition matrix element $\langle K_0^*(1430)|\bar{q}_1^*\gamma_\mu\gamma_5Q|H \rangle$ in the left hand of eq. (6). Next we shall evaluate the correlation function in QCD.

The time-ordered current operators in the three-point correlation function in eq. (4) can be expanded in terms of a series of local operators with increasing dimensions,

$$i^2 \int d^4xd^4ye^{ip_2x - ip_1y}T\{J_K(x)J_\mu(0)J_H(y)\} = C_{0\mu}I + C_{3\mu}\bar{\Psi}\Psi + C_{4\mu}G^{a\alpha\beta}G^{a\alpha\beta} + C_{5\mu}\bar{\Psi}\sigma_{\alpha\beta}T^{a}G^{a\alpha\beta}\Psi + C_{6\mu}\bar{\Psi}\Gamma\Psi\bar{\Psi}\Gamma'\Psi + \cdots,$$ (8)

where $C_i$'s are Wilson coefficients, $I$ is the unit operator, $\bar{\Psi}\Psi$ is the local Fermion field operator of light quarks, $G^{a\alpha\beta}$ is gluon strength tensor, $\Gamma$ and $\Gamma'$ are the matrices appearing in the procedure of calculating the Wilson coefficients. Considering the vacuum expectation value of the OPE of the interpolating field operator, we get the correlation function in terms of Wilson coefficients and condensates of local operators,

$$\Pi_\mu(p_1^2, p_2^2, q^2) = i^2 \int d^4xd^4ye^{ip_2x - ip_1y}\langle 0|T\{J_K(x)J_\mu(0)J_H(y)\}|0\rangle = C_{0\mu} + C_{3\mu}\langle 0|\bar{\Psi}\Psi|0\rangle + C_{4\mu}\langle 0|G^{a\alpha\beta}G^{a\alpha\beta}|0\rangle + C_{5\mu}\langle 0|\bar{\Psi}\sigma_{\alpha\beta}T^{a}G^{a\alpha\beta}\Psi|0\rangle + C_{6\mu}\langle 0|\bar{\Psi}\Gamma\Psi\bar{\Psi}\Gamma'\Psi|0\rangle + \cdots,$$ (9)

The Wilson coefficients $C_i$'s depend only on $p_1$ and $p_2$, according to the Lorentz structure of the correlation function, the result can be re-expressed by two parts in the “theoretical expression”

$$\Pi_\mu(p_1^2, p_2^2, q^2) = -if_+(p_1 + p_2) - if_-(p_1 - p_2),$$ (10)

The coefficients $f_\pm$ above collect all the contributions of perturbative and condensate terms

$$f_\pm = f_\pm^{pert} + f_\pm^{(3)} + f_\pm^{(4)} + f_\pm^{(5)} + f_\pm^{(6)} + \cdots$$ (11)
where $f_{\pm}^{\text{pert}}$ is the perturbative contribution of the unit operator, and $f_{i}^{(3)}, \ldots, f_{i}^{(6)}$ are contributions of condensates of dimension 3, 4, 5, 6, \ldots operators in the OPE. The perturbative contribution and gluon condensate contribution can be written in the from of dispersion integration,

$$f_{\pm}^{\text{pert}} = \int ds_1 ds_2 \frac{\rho_{\pm}^{\text{pert}}(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)}.$$  

$$f_{\pm}^{(4)} = \int ds_1 ds_2 \frac{\rho_{\pm}^{(4)}(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)}.$$

We can approximate the contribution of excited states as integrations over some thresholds $s_1^0$ and $s_2^0$ in the above equations, which is the assumption of quark-hadron duality [2, 16]. Then equate the “phenomenological” and “theoretical” expressions of the correlation function in Eq. (11 and 12), we can get an equation for the form factors. But such equation may highly depend on the approximation for the contribution of excited states and the condensate of higher dimensional operators in OPE. To improve such equation, one can make Borel transformation over $p_1^2$ and $p_2^2$ in both sides, which can suppress the contributions of excited states and condensate of higher dimensional operators. The definition of Borel transformation to any function $f(p^2)$ is

$$\hat{B}|_{p^2, M^2} f(p^2) = \lim_{n \to \infty} \frac{(-p^2)^n}{(n-1)!} \frac{\partial^n}{\partial (p^2)^n} f(p^2).$$

$$p^2 \to -\infty \quad \text{and} \quad -p^2/n = M^2$$

Some examples of Borel transformation is given in the following,

$$\hat{B}|_{p^2, M^2} \frac{1}{(s - p^2)^k} = \frac{1}{(k-1)!} \frac{1}{(M^2)^k} e^{-s/M^2},$$

$$\hat{B}|_{p^2, M^2} (p^2)^k = 0, \quad \text{for any} \quad k \geq 0.$$

Equating the two expressions of the correlation function, subtracting the contribution of excited states, and performing Borel transformation in both variables $p_1^2$ and $p_2^2$, we finally obtain the sum rules for the form factor

$$F_+(q^2) = \frac{m_Q + m_{q^2}}{f_{K_s^0, M_{\pi^0}}^2} M_1^2 M_2^2 e^{m_{q^2}^2/M_1^4} e^{m_{K_s^0}^2/M_2^4} \hat{B} f_+,$$

where $\hat{B} f_+$ denotes Borel transforming $f_+$ in both variables $p_1^2$ and $p_2^2$, $M_1$ and $M_2$ are Borel parameters. After subtracting the contribution of the excited states, now the dispersion integration for perturbative and gluon condensate contribution should be performed under the threshold,

$$f_{\pm}^{\text{pert}} = \int s_1^0 ds_1 \int s_2^0 ds_2 \frac{\rho_{\pm}^{\text{pert}}(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)},$$

$$f_{\pm}^{(4)} = \int s_1^0 ds_1 \int s_2^0 ds_2 \frac{\rho_{\pm}^{(4)}(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)}.$$

In this work we calculate the Wilson coefficients in OPE up to the contribution of the operator of dimension 6. The diagrams we considered here are similar to our previous
work in Ref. [5]. We find again that the contributions of the diagrams for the gluon-gluon condensate in fig. 2 exactly cancel in the sum in the case of pseudoscalar to scalar transition. Therefore, the gluon-gluon condensate does not contribute to $B$ and $D \to K^*_0(1430)$ transition form factors.

![Diagrams for contributions of bi-gluon operator.](image)

Figure 2: Diagrams for contributions of bi-gluon operator.

The results of Borel transformed coefficient $Bf_+$ in eq. (12) are given in the Appendix. Since we do not take into account radiative corrections in QCD, we choose the parameters at a fixed renormalization scale of about 1 GeV. The values of the condensates are taken as [2, 17],

$$\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3, \quad \langle \bar{s}s \rangle = m_0^2 \langle \bar{q}q \rangle,$$

$$g \langle \bar{\Psi} \sigma T G \Psi \rangle = m_0^2 \langle \bar{\Psi} \Psi \rangle, \quad \alpha_s \langle \bar{\Psi} \Psi \rangle^2 = 6.0 \times 10^{-5} \text{ GeV^6}, \quad (13)$$

$$m_0^2 = 0.8 \pm 0.2 \text{ GeV^2}.$$

The quark masses are taken to be $m_b = 4.7 \pm 0.1 \text{ GeV}, m_c = 1.3 \pm 0.1 \text{ GeV}, m_s = 150 \pm 10 \text{ MeV}$ and $m_u \sim m_d \sim 0$. The decay constants of $B(s), D(s)$ and $K^*_0(1430)$ are taken from the two-point QCD sum rule: $f_D = 200 \pm 20 \text{ MeV}, f_B = 180 \pm 30 \text{ MeV}$ [16, 18], and from the ratios $f_{D_s}/f_D = 1.19 \pm 0.08, f_{B_s}/f_B = 1.16 \pm 0.09$ [16], we can obtain $f_{D_s} = 238 \pm 29 \text{ MeV}, f_{B_s} = 209 \pm 38 \text{ MeV}$. For the decay constant of $K^*_0(1430)$, we get $f_{K^*_0} = 427 \pm 85 \text{ MeV}$ from two-point sum rule [13].

The threshold parameters $s_1^0$ and $s_2^0$ are also determined from the two-point sum rule. The parameter $s_1^0$ is for the threshold of $B$ and $D$ meson, they are taken to be $s_1^0 = 35 \pm 2 \text{ GeV^2}, s_2^0 = 6 \pm 1 \text{ GeV^2}$ [18]. $s_2$ is for the threshold of $K^*_0(1430)$, its value is $s_2^0 = 4.4 \pm 0.4 \text{ GeV^2}$ [13].

The Borel parameters $M_1$ and $M_2$ are not physical parameters. The physical result should not depend on them if the operator product expansion can be calculated up to infinite order. However, OPE has to be truncated to some finite orders in practice. Therefore, Borel parameters have to be selected in some “windows” to get the best stability of the physical results. We choose $M_1$ and $M_2$ in the region where 1) the contribution of the
excited states is effectively suppressed, which can ensure that the sum rule does not sensitively depend on the approximation for the excited states, and 2) the contribution of the condensates should not be too large, which can ensure that the contribution of the higher dimensional operators is small and the truncated OPE is effective. The contribution of the excited states to the three-point correlation function is in the form $e^{-m_{\text{excited}}^2/M_i^2}$, where $m_{\text{excited}}$ denotes the mass of the excited state, and the series in OPE generally depend on Borel parameters in the denominator $1/M_{1,2}$, therefore, for effectively suppressing the contributions of the excited states and the higher dimensional operators in OPE, the Borel parameters $M_{1,2}$ should be neither too large, nor too small. We find the optimal stability with the requirements shown in Table 1. The regions of Borel parameters which satisfies the requirements of Table 1 are shown in Fig.3 in two-dimensional diagram of $M_1^2$ and $M_2^2$. We find good stability of the form factors within these regions.

Table 1: Requirements to select Borel Parameters $M_1^2$ and $M_2^2$ for each form factors

| Form Factors | contribution of condensate | excited states of $H$ channel | excited states of $K_0^*(1430)$ channel |
|-------------|----------------------------|-----------------------------|--------------------------------------|
| $F_{+}^{B_s K^0_0}(0)$ | $\leq 42\%$ | $\leq 10\%$ | $\leq 42\%$ |
| $F_{+}^{D K^0_0}(0)$ | $\leq 30\%$ | $\leq 17\%$ | $\leq 24\%$ |
| $F_{+}^{D_s K^0_0}(0)$ | $\leq 28\%$ | $\leq 12\%$ | $\leq 14\%$ |

Figure 3: Selected regions of $M_1^2$ and $M_2^2$: (a) for $F_{+}^{B_s K^0_0}$; (b) for $F_{+}^{D K^0_0}$; (c) for $F_{+}^{D_s K^0_0}$.

In general the higher the dimension of the operators, the smaller the relevant contributions of the condensates. We find that, in the numerical analysis, the main contributions
to the form factors are from perturbative term and condensate of dimension-3 operator, the contributions of operator of dimension 6 are negligible.

The final results for the form factors at $q^2 = 0$ are

$$
F_{+}^{B_{s}K_{0}^{*}}(0) = 0.24 \pm 0.10,
$$

$$
F_{+}^{DK_{0}^{*}}(0) = 0.57 \pm 0.19,
$$

$$
F_{+}^{D_{s}K_{0}^{*}}(0) = 0.51 \pm 0.20.
$$

The error bars are estimated by the variation of Borel parameters, the variation of the threshold parameters $s_{1,2}^0$, the uncertainty of the condensate parameters, the variation of the quark masses and meson decay constants, and the possible $\alpha_s$ corrections to the three-point function. The main contribution comes from the uncertainties of the threshold parameters and decay constants, which is about 10% $\sim$ 20% of the central value, the other uncertainties are only a few percent. The $\alpha_s$ corrections to two-point functions of $f_B$, $f_D$ and $f_{K_0^*}$ are 10% to 30%, we estimate the $\alpha_s$ corrections to three-point function shall be the same order. We take 30% as the error caused by the $\alpha_s$ correction in this work.

Numerically, $F_{+}^{DK_{0}^{*}}$ and $F_{+}^{D_{s}K_{0}^{*}}$ are slightly different. The difference is completely caused by the SU(3) symmetry breaking effect. The numerical result shows that this effect is about 10%.

There have been studies of heavy flavor meson to $K_0^*(1430)$ transition form factors in the literature. In [14], $F_{+}^{DK_{0}^{*}}(0)$ is extracted from the data of the hadronic $D$ decay in the generalized factorization model, $F_{+}^{DK_{0}^{*}}(0) = 1.20 \pm 0.07$. It is also calculated in covariant light-front approach, $F_{+}^{DK_{0}^{*}}(0) = 0.48$ [8]. Compared with these values, our present result $F_{+}^{DK_{0}^{*}}(0) = 0.57 \pm 0.19$ is consistent with the light-front approach.

The physical region for $q^2$ in $B$ or $D \rightarrow K_0^*(1430)\ell\bar{\nu}$ decay extends from 0 to $(m_{B,D} - m_{K_0^*})^2$, which is not large because of the large mass of $K_0^*(1430)$. The $q^2$ dependence of the form factors in the physical range can be calculated directly from QCD sum rule, which has been discussed in detail in [4]. Within the physical range of $q^2$ in $B$ or $D \rightarrow K_0^*(1430)$ decays, there is no non-Landau-type singularity [4] with the threshold parameters $s_1^0$ and $s_2^0$ considered in this paper. From the sum rule for the form factors listed in the appendix, we see that the formulas are not singular at $q^2 = 0$, and $q^2$ appears in the numerator, therefore, if $|q^2|$ is too large, OPE will fail. So the condition for the OPE be effective is to keep $|q^2|$ in a moderate range. For the whole physical range of $q^2$ considered in this paper, within the selected window of the Borel parameters, the contribution of higher-dimensional operators is effectively small, therefore the OPE is effective.

The form factors as functions of $q^2$, normalized by the value at $q^2 = 0$ GeV$^2$, $F_{+}(q^2)/F_{+}(0)$’s are shown in Fig[4]. Within the physical range, the behavior of the form factor $F_{+}(q^2)$ is compatible with the pole-model,

$$
F_{+}(q^2) = \frac{F_{+}(0)}{1 - q^2/m_{\text{pole}}^2}.
$$

The result of the form factors from QCD sum rule can be fitted with the pole model. The fitted pole masses are,

$$
m_{B_{s}K_{0}^{*}}^{\text{pole}} = 4.80 \pm 0.12 \text{ GeV},
$$

$$
m_{DK_{0}^{*}}^{\text{pole}} = 2.9 \pm 0.3 \text{ GeV},
$$

$$
m_{D_{s}K_{0}^{*}}^{\text{pole}} = 1.96 \pm 0.12 \text{ GeV}.
$$
Next we shall use the form factors calculated in QCD sum rule to study the differential and total decay rates of $B$ and $D \rightarrow K^*_0(1430)\ell\nu$ decays. The differential decay rate is calculated to be

$$\frac{d\Gamma_L}{dq^2} = \frac{G_F^2 |V_{Qq}'|^2}{192\pi^3 m_H^2} F_+(q^2)^2 \left( (m_{H}^2 + m_{K_0^*}^2 - q^2)^2 - 4m_{H}^2 m_{K_0^*}^2 \right)^{3/2}.$$  \hspace{1cm} (16)

The values of the CKM matrix elements are taken in Wolfenstein parameterization, $A = 0.8$, $\lambda = 0.22$, $\rho = 0.20$, and $\eta = 0.34$ [10], which are relevant to $V_{cs} = 0.976$, $V_{cd} = -0.22$.

The differential decay widths as a function of momentum transfer squared $q^2$ are shown in Fig.4. The decay rate of $D_s^+ \rightarrow K_0^*(1430)^0 \ell\nu$ is one more order smaller than that of $D^0 \rightarrow K_0^*(1430)^0 \ell\nu$ and $D^+ \rightarrow \bar{K}_0^*(1430)^0 \ell\nu$ because of the CKM matrix element suppression of $V_{cd}$ over $V_{cs}$. After integration over $q^2$ in the whole physical region, we get the integrated decay widths

$$\Gamma(B_s^0 \rightarrow K_0^*(1430)^0 \ell\nu) = (1.6^{+1.7}_{-1.1}) \times 10^{-17} \text{ GeV},$$
$$\Gamma(D^0, \rightarrow K_0^*(1430)^0 \ell\nu) = (2.9^{+2.3}_{-1.6}) \times 10^{-16} \text{ GeV},$$
$$\Gamma(D_+^s \rightarrow K_0^*(1430)^0 \ell\nu) = (3.2^{+3.0}_{-2.0}) \times 10^{-17} \text{ GeV}.$$  \hspace{1cm} (17)

The branching ratio is defined by

$$Br = \frac{\Gamma}{\Gamma_{total}},$$  \hspace{1cm} (18)
where $\Gamma$ denotes the decay width of each decay mode, and $\Gamma_{total}$ the total decay width of $B$ or $D$ meson. The total decay width of one meson is related to its mean life time
Measuring these decay modes with high statistics will give valuable information on the 
$B_B$ collaboration gives an upper limit for the ratio, are rare. Measuring them in experiment needs high statistics, but these measurements

$\tau$ by $\Gamma_{total} = h/\tau$. The mean life times of $B$ and $D$ mesons are: $\tau_{B_s} = 1.46 \times 10^{-12}$ s, $\tau_{D^\pm} = 1.04 \times 10^{-12}$ s, $\tau_{D^0} = 0.41 \times 10^{-12}$ s, and $\tau_{D_s} = 0.49 \times 10^{-12}$ s [10]. We can obtain the branching ratios

$$
Br(B_s^0 \to K^*_0(1430)^-\ell\nu) = (3.6^{+3.8}_{-2.4}) \times 10^{-5},
Br(D^0 \to K^*_0(1430)^-\ell\nu) = (1.8^{+1.5}_{-1.0}) \times 10^{-4},
Br(D^+ \to K^*_0(1430)^0\ell\nu) = (4.6^{+3.7}_{-2.6}) \times 10^{-4},
Br(D_s^+ \to K^*_0(1430)^0\ell\nu) = (2.4^{+2.2}_{-1.5}) \times 10^{-5}.
$$
(19)

Although $\Gamma(D^0 \to K^*_0(1430)^-\ell\nu) = \Gamma(D^+ \to K^*_0(1430)^0\ell\nu)$ under the isospin symmetry, the branching ratios of these two decay modes are greatly different. This is due to the large difference of the total decay widths of $D^0$ and $D^\pm$ mesons.

The numerical result shows that the branching ratios of $B$ and $D$ to $K^*_0(1430)^0\ell\nu$ are rare. Measuring them in experiment needs high statistics, but these measurements will give valuable information on the nature of the light scalar mesons. Recently, Focus collaboration gives an upper limit for the ratio, $\frac{\Gamma(D^+\to K^*_0(1430)^0\mu^+\nu)}{\Gamma(D^+\to K^-\pi^+\mu^+\nu)} < 0.64\%$, in analyzing the mass spectrum of $D^+ \to K^-\pi^+\mu^+\nu$ decay [19]. By considering the branching ratio of $Br(D^+ \to K^-\pi^+\mu^+\nu) = 4.00 \pm 0.32\%$ given in PDG [10], one can get the branching ratio $Br(D^+ \to K^*_0^+(1430)^0\mu^+\nu) < 2.8 \times 10^{-4}$. Compared with this upper limit, our result $Br(D^+ \to K^*_0^+(1430)^0\ell\nu) = (4.6^{+3.7}_{-2.6}) \times 10^{-4}$ is compatible with it. Direct experimental measurement of these semileptonic branching ratios and more precise theoretical prediction are highly desired.

In summary, we have calculated the $B$ and $D$ to $K^*_0(1430)$ transition form factors $F_{+}^{B,K^*_0}(q^2), F_{+}^{D,K^*_0}(q^2), F_{+}^{D,J,K^*_0}(q^2)$ in QCD sum rule, and studied the semileptonic decays of $B_s^0 \to K^*_0(1430)^-\ell\nu, D^0 \to K^*_0(1430)^-\ell\nu, D^+ \to K^*_0(1430)^0\ell\nu, \text{ and } D_s^+ \to K^*_0(1430)^0\ell\nu$. Measuring these decay modes with high statistics will give valuable information on the structure of light scalar mesons.

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Appendix

The analytical results of the coefficients \( \hat{B}_f \) for the form factors are given here. Two cases are classified, one is for \( B_s \) and \( D_s \rightarrow K_0^*(1430) \) transitions, the other for \( B \) and \( D \) transitions.

(1) For \( B_s \) and \( D_s \rightarrow K_0^*(1430) \) transitions, the form factor in eq.(12) will be

\[
F_+(q^2)^{HsK_0^*} = \frac{m_Q + m_s}{f_{K_0^*} m_{K_0^*}} M_2^2 e^{m^2_s / M_2^2} e^{m^2_s / M_2^2} \hat{B}_{f+HsK_0^*},
\]

where \( H_s \) denotes \( B_s \) or \( D_s \), and

\[
\hat{B}_{f+HsK_0^*} = \hat{B}_{f+Hs}^{pert} + \hat{B}_{f+Hs}^{(3)} + \hat{B}_{f+Hs}^{(5)} + \hat{B}_{f+Hs}^{(6)}.
\]

The perturbative contribution is

\[
\hat{B}_{f+Hs}^{pert} = \int_{m_2^2}^{0} ds_1 \int_{s_1^f}^{0} ds_2 \frac{3}{8M_1^2 M_2^2 \pi^2 \lambda^3 / 2} e^{-s_1^f / M_1^2} e^{-s_2 / M_2^2} \{ q^2 (q^2 - s_1 - s_2) m_Q m_s \\
- (q^2 - s_1 + s_2) m_Q^2 m_s + m_Q^2 (q^2 + s_1 - s_2) s_2 + (q^2 - s_1 + s_2) m_s^2 \\
+ 2q^2 (-s_1 s_2 + (-q^2 + s_1 + s_2) m_s^2),
\]

where the lower integration limit of \( ds_1 \) is determined by the condition that all internal quarks in the perturbative diagram be on mass shell [20],

\[
s_1^f = \frac{m_Q^2}{m_Q^2 - q^2 s_2 + m_s^2},
\]

and \( \lambda = (s_1 + s_2 - q^2)^2 - 4s_1 s_2 \).

The condensate contributions are

\[
\hat{B}_{f+Hs}^{(3)} = -\frac{g(\bar{q} q T G q)}{4 M_1^2 M_2^4} e^{-m_Q^2 / M_1^2} \{ 2M_1^4 M_2^4 m_Q - M_2^4 (2M_2^2 + m_Q^2) m_s \\
+ M_2^4 m_Q [-M_1^2 q^2 + (M_1^2 + M_2^2) m_Q^2] m_s \},
\]

(23)

\[
\hat{B}_{f+Hs}^{(5)} = -\frac{g(\bar{q} q T G q)}{24 M_1^4 M_2^4} e^{-m_Q^2 / M_1^2} \{ 3M_1^2 m_Q [M_1^2 (M_1^2 + 2M_2^2 + q^2) - (M_1^2 + M_2^2) m_Q^2] \\
+ (4M_1^2 + m_Q^2) [-M_1^2 q^2 + (M_1^2 + M_2^2) m_Q] m_s \},
\]

(24)

\[
\hat{B}_{f+Hs}^{(6)} = -\frac{4\pi \alpha_s \langle \bar{q} q \rangle^2}{32 M_1^6 M_2^6 (m_Q^2 - q^2)} e^{-m_Q^2 / M_1^2} \{ M_1^2 \{ 2M_1^4 [M_1^2 (72M_2^2 - 17q^2) - q^2 (M_2^2) \\
+ 6q^2)] + M_1^2 [-38M_1^4 + q^2 (24M_2^2 + 5q^2) + 2M_1^2 (M_2^2 + 19q^2)] m_Q^2 \\
- 2[13M_1^4 - 2M_2^2 q^2 + M_2^2 (12M_2^2 + 5q^2)] m_Q^2 + (5M_1^2 - 4M_2^2) m_Q^6 \} \\
+ 36M_1^6 M_2^6 m_Q (3M_2^2 + q^2 - m_Q^2) m_s - 18M_1^6 (4M_2^2 - M_2^2 m_Q^2) m_s^2 \},
\]

(25)
(2) For $B$ and $D \to K_0^*(1430)$ transitions, the form factor in eq. (12) will be

$$F_+(q^2)^{H_q K_0^*} = \frac{m_Q}{f_{K_0^*} m_{K_0^*} m_{H_q}} M_1^2 M_2^2 e^{m_{H_q}^2 / M_1^2} e^{m_{K_0^*}^2 / M_2^2} \hat{B}_{f_H K_0^*},$$

(26)

where $H_q$ denotes $B$ or $D$, and

$$\hat{B}_{f_H K_0^*} = \hat{B}_{f_H}^{pert} + \hat{B}_{f_H}^{(3)} + \hat{B}_{f_H}^{(5)} + \hat{B}_{f_H}^{(6)}.$$  

(27)

The perturbative contribution is

$$\hat{B}_{f_H}^{pert} = \int_{m_s^2}^{q^2} ds_2 \int_{s_1}^{m_s^2} ds_1 \frac{-3}{8M_1^2 M_2^2 \pi^2 \lambda^{3/2}} e^{-\frac{s_1}{M_1^2} - \frac{s_2}{M_2^2}} \{s_2(-q^2 - s_1 + s_2) m_Q^2 + [(s_1 - s_2)^2 - q^2(s_1 + s_2)] m_Q m_s + (q^2 - s_1 + s_2) m_Q^2 m_s + s_1[2q^2 s_2 - (q^2 - s_1 + s_2) m_s^2]\},$$

(28)

The condensate contributions are

$$\hat{B}_{f_H}^{(3)} = -\frac{\langle \bar{q}q \rangle}{2M_1 M_2} e^{-\frac{m_Q^2}{M_1^2} - \frac{m_s^2}{M_2^2}} (m_Q - m_s),$$

(29)

$$\hat{B}_{f_H}^{(5)} = -\frac{g \langle \bar{\Psi} T \Gamma \Psi \rangle}{8M_1 M_2} e^{-\frac{m_Q^2}{M_1^2} - \frac{m_s^2}{M_2^2}} \left(-M_1^2 (M_1^2 + M_2^2) m_Q^2 - M_2^2 (2M_1^2 + M_2^2) + q^2 m_s + M_2^2 (M_1^2 + M_2^2) m_Q^2 m_s + M_1^2 m_Q M_2^2 (M_1^2 + 2M_2^2 + q^2) - (M_1^2 + M_2^2) m_s^2\right),$$

(30)

$$\hat{B}_{f_H}^{(6)} = -\frac{4\pi \alpha_s \langle \bar{\Psi} \Psi \rangle^2}{324M_1 M_2 (m_Q^2 - q^2)} e^{-\frac{m_Q^2}{M_1^2} - \frac{m_s^2}{M_2^2}} \left\{2M_1^4 [2M_2^2 (11M_2^2 - q^2) q^2 - M_1^2 q^2 (6M_2^2 + q^2) + M_2^2 (54M_2^2 + 23q^2)] m_Q m_s - 2[23M_1^6 + 7M_1^4 q^2 - 2M_1^4 (3M_2^2 + q^2) + M_1^2 (22M_2^4 - 5M_2^2 q^2)] m_s^3 m_s - 2(M_1^4 + 3M_2^2 (M_2^2 - 7M_1^4)] m_Q m_s + (5M_1^4 - 4M_2^2) m_Q^2 (m_2^2 - m_1^2) - M_1^2 m_Q^2 [-3M_1^2 (2M_2^2 + 5q^2) + 2M_1^2 (M_2^2 + 5q^2)] + 58M_1^4 + 6M_1^2 (2M_2^2 + 7q^2)] m_s^2\right\} + m_1^2 [-2M_2^2 (13M_1^4 - 2M_2^2 q^2 + M_1^2 (12M_2^2 + 5q^2)] + [29M_1^4 - 4M_2^2 q^2 + M_1^2 (27M_2^2 + 10q^2)] m_s^2] + M_1^2 \left\{2M_2^2 (M_1^2 (72M_2^2 - 17q^2) - q^2 (2M_2^2 + 6q^2)) + [-4M_1^2 (18M_2^2 - 19q^2) + q^2 (12M_2^2 + 13q^2)] m_s^2\right\}.$$

(31)

**References**

[1] A. Abada et al., Nucl Phys. B619 (2001) 565; C. Aubin et al., Phys. Rev. Lett. 94 (2005) 011601.

[2] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147 (1979) 385; 448.
[3] T.M. Aliev, V.L. Eletskii, and Ya. I. Kogan, Sov. J. Nucl. Phys. 40 (1984) 527; A.A. Ovchinnikov and V.A. Slobodenyyuk, Z. Phys. C44 (1989) 433; V.N. Baier and Grozin, Z. Phys. C47 (1990) 669; A.A. Ovchinnikov, Sov. J. Nucl. Phys. 50 (1989) 519; P. Colangelo and F. De Fazio, Phys. Lett. B 520 (2001) 78.

[4] P. Ball, V.M. Braun and H.G. Dosch, Phys. Rev. D44 (1991) 3567; P. Ball, Phys. Rev. D48 (1993) 3190.

[5] D.S. Du, J.W. Li and M.Z. Yang, Eur. Phys. J. C37 (2004) 173.

[6] A. Khodjamirian, R. Rückl, S. Weinzierl, C.W. Winhart, O. Yakovlev, Phys. Rev. D62 (2000) 114002.

[7] M. Wirbel, B. Stech and M. Bauer, Z. Phys. C29 (1985) 637; M. Bauer, B. Stech and M. Wirbel, Z. Phys. C34 (1987) 103; N. Isgur, D. Scora, B. Grinstein and M.B. Wise, Phys. Rev. D39 (1989) 799; D. Scora and N. Isgur, Phys. Rev. D52 (1995) 2783; D. Melikhov and B. Stech, Phys. Rev. Dd2 (2000) 114002.

[8] H.Y. Cheng, C.K. Chua and C.W. Hwang, Phys. Rev. D69 (2004) 074025.

[9] S. Fajfer and J. Kamenik, Phys. Rev. D71 (2005) 014020; and eprint hep-ph/0506051.

[10] S. Eidelman, et al., Particle Data Group, Phys. Lett. B592 (2004)1.

[11] S. Spanier and N.A. Törnqvist, Review article in Particle Data Group, Phys. Lett. B592 (2004)1.

[12] S. Godfrey and J. Napolitano, Rev. Mod. Phys. 71 (1999) 1411; F.E. Close and N.A. Törnqvist, J. Phys. G28 (2002) R249.

[13] D.S. Du, J.W. Li and M.Z. Yang, Phys. Lett. B619 (2005) 105.

[14] H.Y. Cheng, Phys. Rev. D67 (2003) 034024.

[15] H.G. Dosch, E.M. Ferreira, F.S. Navarra, and M. Nielsen, Phys. Rev. D65 (2002) 114002; M. Nielsen, F.S. Navarra, H.G. Dosch and E.M. Ferreira, Nucl. Phys. B (Proc. Suppl.) 121 (2003)110.

[16] For a recent review article, see, P. Colangelo and A. Khodjamirian, in: At the Frontier of Particle Physics / Handbook of QCD, edited by M. Shifman (World Scientific, Singapore, 2001), hep-ph/0010175

[17] S. Narison, *QCD Spectral Sum Rules* (World Scientific, Singapore, 1989).

[18] A. Khodjamirian, R. Rückl, S. Weinzierl, C.W. Winhart, and O. Yakovlev, Phys. Rev. D62 (2000) 114002; A. Khodjamirian, R. Rückl, eprint hep-ph/9801443, published in Adv. Ser. Direct. High Energy Phys. 15 (1998) 345.

[19] J.M. Link et al. (FOCUS collaboration)Phys. Lett. B621 (2005) 72.

[20] L.D. Landau, Nucl. Phys. 1 (1959) 181.