A short review of reliability-based design optimization

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Abstract. Reliability-based design optimization (RBDO) has become an important branch of reliability engineering after decades of development. In recent years, reliability optimization design methods considering uncertain factors have emerged in an endless stream. This article summarizes the research status and basic principles of RBDO problems. According to the different RBDO methods, the mathematical models of the double-loop method, single-loop method, and decoupling method and their respective advantages and disadvantages are detailed from the perspective of the optimization process. It also focuses on the analysis of several kinds of RBDO bottleneck problems, and discusses the existing related methods and possible solutions.

1. Introduction

In many disciplines, especially engineering disciplines, structural optimization design is common. Good optimization design results can not only save the economic cost of engineering applications, but also get better performance. But multi-source uncertainty in engineering practice is common, such as: material properties, boundary conditions, geometric dimensions, loads, etc. Therefore, the influence of uncertain factors must be fully considered in the design process to obtain the best results [1]. Although many existing engineering methods use existing traditional methods and introduce simple corrections (including the application of extreme values and safety factors, assumptions of deterministic models) to take into account uncertainty, such methods are only simplify, because different types of uncertainty will lead to varying degrees of performance and size changes in the final design [2].

In recent years, with the rapid development of computing technology, many optimization design methods that take into account uncertainties have shined in aerospace, shipbuilding, construction and other fields. Among them, reliability-based design optimization (RBDO) is considered to be a suitable and most potential design strategy. Such methods can obtain the best design that meets the expected reliability [3]. RBDO usually consists of inner reliability evaluation and outer optimization design. Among them, the reliability assessment is based on the concept of safety design, which analyzes the reliability of the designed structure or system and evaluates it with quantitative safety degree, so as to ensure that the structure or system can meet the specified work requirements. Although sufficient reliability is the basic goal when designing a system or constructing components, excessive reliability brings huge costs. Therefore, the combination of optimization design and reliability evaluation is an effective means to improve economic efficiency. At present, the optimization design problem is
usually solved by gradient-based optimization methods, such as sequential quadratic programming (SQP) and generalized reduced gradient algorithm [4]. Such related algorithms rely heavily on gradient information, and the obtained optimized solutions are mostly local optimal solutions, and the optimal design scheme cannot be obtained when the gradient information is missing. Therefore, they are only suitable for continuous variable optimization problems. In contrast, global optimization methods such as genetic algorithm (GA), differential evolution (DE), and particle swarm optimization (PSO) can search for the best solution without gradient information throughout the design space. This kind of method can not only obtain the global optimal solution, but also easy to deal with the optimization problem with discrete variables. Due to these advantages, the application of related optimization techniques and traditional reliability design methods to solve RBDO problems is becoming more and more extensive [5,6].

This article is organized as follows. The second section outlines the RBDO problem and its mathematical model; the section starts from the algorithm flow and details the research status of the RBDO method; the third section discusses the bottleneck problems of RBDO; the fourth section summarizes the full text and prospects the future research direction of RBDO.

2. Problem description of RBDO

When facing the optimization design problem of structure or system, it can be solved according to the following two design principles. One is deterministic optimization design, that is, the known or unknown quantities in the design problem are treated as non-statistics; the second is reliability-based optimization design, that is, some or all of the design problems are treated as random variables [7].

A typical RBDO model can be expressed as:

\[
\min_{d \neq x} f(d, \mu_x)
\]

\[
\text{s.t. } \begin{cases}
\Pr\{G_i(d, X) \leq 0\} \leq P_i^T (i = 1, 2, \ldots, n) \\
C_k(d) \leq 0 (k = 1, 2, \ldots, m)
\end{cases}
\]

where \(d\) is the deterministic design vector, the mean value \(\mu_x\) of random vector \(X\) is uncertain design vector, \(f(d, \mu_x)\) is the objective function, \(G_i(d, X) \leq 0\) is used to represent the structural failure, \(P_i^T\) is the allowable failure probability, \(C_k(d) \leq 0\) is the deterministic constraint, \(\Pr[\cdot]\) is the probability operator, \(n\) is the number of uncertain constraints, \(m\) is the deterministic constraint Number. In order to facilitate understanding, figure 2 gives a schematic diagram of the RBDO problem.

As can be seen from figure 2, the RBDO problem is a two-level nested optimization problem, and the reliability analysis loop is nested in the outer optimization loop. When solving, the reliability analysis of the inner layer needs to obtain the failure probability and other information, usually the failure probability is obtained by irregularly shaped high-dimensional integration. However, due to the complexity of the probability density function and the integration space, it is more difficult to solve directly by integration. At present, for this problem, two feasible methods are to convert the failure probability into a reliable index or performance function. The corresponding methods are called reliability index approach (RIA) [8] and performance measurement approach (PMA) [9]. Youn [10] and Yi [11] found that PMA is more efficient, stable, and insensitive to the type of probability distribution compared to RIA. In order to take full advantage of the solution advantages of RIA and PMA, Meng et al. [12] proposed an optimization method combining RIA and PMA, which has good accuracy, efficiency and robustness.
3. Algorithm flow of RBDO

Reviewing the development process of the RBDO method, according to the different algorithm flow, it can be divided into three categories: double-loop method, single-loop method and decoupling method.

3.1. Double-loop method

The double loop method is a traditional method to solve the RBDO problem, that is, nested reliability analysis in the optimization loop, and its optimization process is shown in figure 2. The common expressions of probability constraints in the inner loop of this method are RIA and PMA.

3.1.1. Optimization Model Based on RIA

Probability constraints are described as reliability indicators not less than target reliability indicators. This method of describing constraints is called reliability index method. The mathematical model of the RBDO double-loop algorithm using RIA is as follows:

$$\min_{d, \mu_x} f(d, \mu_x)$$

s.t. $$\beta_i(d, X) \geq \beta^T (i = 1, 2, \cdots, n)$$

$$C_k(d) \leq 0 (k = 1, 2, \cdots, m)$$

where $$\beta_i(d, X)$$ is the reliable index corresponding to the design vector, and $$\beta^T$$ is the allowable reliable index.

3.1.2. Optimization model based on PMA

PMA can be regarded as the reverse process of RIA. Under the condition that the distance between the design check point and the origin is equal to the reliability index, the minimum value of the state equation is obtained, and then whether the structure meets the reliability constraints is judged by
comparing whether the minimum value is greater than zero. The mathematical model of the RBDO double-loop algorithm using PMA is as follows:

$$\min_{d, \mu} f(d, \mu)$$

(5)

such that

$$G_p(d, X) \geq 0 (i = 1, 2, \ldots, n)$$

$$C_k(d) \leq 0 (k = 1, 2, \ldots, m)$$

(6)

where $G_p(d, X)$ is the minimum function value obtained by searching on the sphere of target reliability index in standard normal space. The double-loop method has the advantages of clear concept, simple solution and high stability, but Enevoldsen and Sorensen pointed out that the double-loop optimization method requires multiple performance function evaluations, especially when the optimized structure has material and geometric nonlinearity [13]. Due to the disadvantages such as low efficiency, this type of method can no longer meet the actual engineering needs.

![Figure 2. Optimization flow chart of double-loop method.](image)

3.2. single-loop method

Faced with the problem of low efficiency of the double-loop method, scholars introduced approximate equivalent conditions instead of reliability constraints, thereby avoiding the reliability analysis cycle in the optimization cycle. Such a method is called the single-loop method. The optimization process of the single-loop method is shown in figure 3.

The improved single-cycle method proposed by Liang et al. [14] uses KKT (Karush-Kuhn-Tucker) optimal conditions to convert reliability optimization design into deterministic optimization design, which greatly improves efficiency. As a typical representative of the single cycle method, its optimization model is as follows:
\[ \min_{d \neq x} f(d, \mu_x) \]  
\[ \text{s.t.} \quad G_i(d^k, X_i^k) \geq 0 (i = 1, 2, \ldots, n) \]  
\[ C_k(d^k) \leq 0 (k = 1, 2, \ldots, m) \]

\[ X_i^k = \mu^k + \alpha^k \beta^k \]  
\[ \alpha_i^k = \frac{\sigma_X \nabla_X G(d^k, X_i^{k-1})}{\| \sigma_X \nabla_X G(d^k, X_i^{k-1}) \|} \]

where \( X_i^k \) is the approximate minimum functional target point of the \( i \)-th performance function at the \( k \)-th iteration, which is obtained from the mean value \( \mu^k \), standard deviation \( \sigma_X \), target reliability index \( \beta^k \), and sensitivity of the random vector \( \alpha^k \).

![Data initialization](image)

Calculate the objective function and probability constraints

Solved approximate equivalent reliability information

Solve the current deterministic optimization

Whether to converge

Yes

No

End

**Figure 3.** Optimization flow chart of single-loop method.

The single loop and single vector (SLSV) method based on the quantile approximate limit state function proposed by Chen et al. [15] introduced the concept of single loop optimization to the RBDO problem for the first time, and many scholars then expanded on such methods in-depth study. For example, Zhou et al. [16] proposed a two-stage reliability optimization design method based on sequential approximation, which improved the optimization solution efficiency again. Based on reliable design space, Li [17] completely transformed the RBDO problem into a deterministic optimization problem, which significantly reduced the amount of calculation. However, the author clearly states that this method usually fails to retain the most worrying of the original problem. Although the single-cycle method can greatly improve the efficiency, it also has difficulty in solving
problems involving multiple reliability constraints and highly nonlinear performance function in practical engineering.

3.3. Decoupling method

After comparative analysis, Liao et al. [18] found that when solving the RBDO problem of complex engineering structures, the double-loop method is expensive due to the large amount of calculation and the single-loop method also has the disadvantages of poor robustness and low accuracy. Based on the traditional optimization algorithm, Royse et al. [19] proposed the decoupling method using the concept of semi-infinite programming (SIP). This method completely decouples the reliability analysis and optimization design links, using certainty Constraints replace reliability constraints, and the problems after reorganization can be solved by SIP and probability calculation methods. The optimization process of the decoupling method is shown in figure 4.

**Figure 4.** Optimization flow chart of decoupling method.

Du and Chen [20] determined the minimum functional target point through reverse reliability analysis, introduced the concept of offset vector to ensure that the point falls within the feasible region, and transformed the original probability constraint problem into a series of deterministic constraint sequential solutions, thus establishing sequence optimization and reliability assess method (SORA). The mathematical model of this method is as follows:

\[
\min_{d, \mu_x} f(d, \mu_x) \tag{10}
\]

subject to

\[
G_i(d, \mu_x - s^{(k)}) \geq 0 \quad (i = 1, 2, \ldots, n) \tag{11}
\]

\[
C_j(d) \leq 0 \quad (j = 1, 2, \ldots, m)
\]

where \(k\) is the current number of cycles, \(s\) is the offset vector.
SORA is the most potential decoupling method to solve the RBDO problem. Many scholars have developed many related algorithms based on this method. For example, Yi et al. [21] based on the concept of SORA, used the approximate minimum functional target point and the approximate probabilistic performance measure (PPM) in reliability evaluation, thereby transforming reliability optimization design into deterministic optimization design. In view of the situation that SORA is easy to obtain a local optimal solution and depends on the initial test points, Ho-Huu et al. [22] combined SORA with an improved constrained differential evolution (ICDE) method, which avoids the disadvantages of traditional SORA and improves applicability. Torii et al. [23] also proposed a RBDO algorithm using fitted offset vectors based on the SORA method to overcome the limitations of decoupling method for reliability analysis method selection. In addition to the research and development on the basis of SORA, Zhao Weitao et al. [24] proposed a decoupling optimization method for structural reliability based on threshold factors, which is insensitive to the method of disassembly of optimized variables, and is effective and inefficient. Both can obtain satisfactory optimization results, and the calculation efficiency is significantly higher than traditional methods. As a key method for solving problems in the RBDO field, the decoupling method has technical difficulties that need to be further addressed, such as highly nonlinear performance function.

4. The bottleneck problem of RBDO
There are many bottlenecks in solving the RBDO problem, mainly focused on: the variable distribution type is non-normal, the correlation between different variables, the performance function is highly nonlinear, and the system RBDO has multiple failure modes. Scholars at home and abroad are actively developing relevant algorithms based on the three major optimization processes, and are committed to overcoming the above problems.

4.1. Non-normal variables
The bottleneck problem of RBDO with random variables of non-normal distribution type is ultimately the bottleneck of the reliability analysis problem. In the process of structural reliability optimization design, intuitive and practical reliability indicators are often used to approximate probability constraints. Because they are defined under the condition that the performance function follows a normal distribution, there is an accurate correspondence with the failure probability. When the random variables are not normally distributed and the form of the performance function is not fixed, the performance function usually cannot meet the condition of normal distribution. At this time, the reliability index cannot be directly calculated. The relevant approximate calculation method needs to be developed to meet the needs of reliability analysis and optimal design.

The case of non-normal distribution of random variables can be divided into two types of treatment: independent non-normal random variables and non-normal random variables with correlation. The first type usually only needs to be normalized and converted into normal variables, and then the problem can be solved using first-order reliability methods. Figure 5 illustrates the equivalent normal distribution transformation of the non-normal distribution.

Figure 5. Schematic diagram of equivalent normal distribution of non-normal distribution.
In the figure 5, $x$ is a random variable, $f$ is the probability density of $x$, $\mu$ is the normalized variable mean, and $x^*$ is a specific point used for equivalent transformation. Commonly used normalization methods are equal probability transformation method [25], JC method [26], simplified weighted quantile method [27], etc. When the random variables are non-normally distributed and statistically related, the above normalization method may only use the marginal distribution of the variables for normalization, which may cause changes in the correlation of the variables and increase analysis errors. Another kind of edge transformation, Nataf transformation [28], can consider the change of variable correlation caused by variable transformation, and has higher solution accuracy. It is a better way to solve the RBDO non-normal bottleneck problem. Rosenblatt transformation [29] is a typical method for converting non-normal and related random variables into independent normal distributions, but it requires a joint cumulative distribution function of known variables, so it has greater limitations.

Non-normal problems can also be handled without using the normalization method. Zhang et al. [30] solved the reliability of mechanical parts with non-normally distributed random variables using the random perturbation method and the fourth-order moment technique. Zhou Jinyu et al. [31] introduced the generating function method in the structural reliability analysis of multi-state systems. This method is not affected by the distribution type of random variables and can obtain a higher reliability analysis accuracy. When solving the RBDO problem with non-normal random variables, whether it is equivalent normalization or other methods, the reliability analysis error cannot be avoided, and even the optimization results are not ideal. This problem still puzzles researchers in this field.

4.2. Correlated Variables
Existing methods mostly assume that random variables are independent of each other when solving RBDO problems. However, in practical engineering problems, there is a general correlation between different variables. Research by TANG et al. [32] shows that the correlation of variables may have a greater impact on the design optimization results and structural reliability, especially when the variables have tail correlations.

In recent years, a variety of methods have been developed to deal with such statistically relevant problems, such as: orthogonal transform [33], Rosenblatt transform, Nataf transform, generalized random space [34], etc. Nataf transform is the mainstream method to deal with variable-related problems, but because it can only describe the linear correlation of variables, large analysis errors may occur when dealing with non-linear related problems, which limits its application to a certain extent.

In recent years, an important mathematical tool, the Copula function, has been mined and used to deal with the problem of correlation in the field of uncertainty analysis, because it can describe the different types of nonlinearities between variables, and can capture such as tail correlation. Important information such as sex, so a more accurate joint probability distribution function of random variables can be established. Commonly used Copula function categories are Gaussian Copula (equivalent to Nataf transform), T Copula, Clayton Copula, Gumbel Copula and Frank Copula. Based on the Copula function, Wang Qianrong et al. [35] proposed an RBDO decoupling algorithm based on the Copula function to make up for the shortcomings of large analysis errors when using the Nataf transform to solve nonlinear related problems. Jiang Chao [36] et al. derived the correlation angle between random variables based on generalized random space, and proposed a probability-correlation-interval mixed uncertainty model and structural reliability analysis method that can deal with variables between variables. Relevance of mixed reliability analysis problems.

4.3. Highly nonlinear performance function
When the performance function of the RBDO problem is highly nonlinear, no matter whether it is a double-cycle, single-cycle or decoupling method, all kinds of problems will occur, such as: low efficiency and non-convergence of optimization results. Therefore, research on such problems is
particularly important. At present, for different optimization processes, many algorithms have been proposed one after another to solve the highly nonlinear limit state function.

The double-loop method is mainly based on RIA and PMA for solution. For RIA, the checkpoint method or HL-RF iterative method is widely used because of its simplicity and efficiency. For this method, the RBDO problem with a high degree of nonlinearity in the performance function often fails due to the inability of the optimization results to converge. For example, Gong et al. [37] proposed a step adjustment method, which improved the convergence performance of the algorithm by controlling the iterative step size of the HL-RF method, and proved that HL-RF is a specific case of the proposed algorithm. Kiureghian et al. [38] improved the convergence of the algorithm by introducing a benefit function. In addition, proxy models such as response surface, Kriging model, etc. are also used to solve reliable indicators of complex engineering structures. For PMA, the typical solution methods are: improved mean value (AMV) method [39], conjugate mean value (CMV) method, mixed mean value (HMV) method, but these methods can not effectively apply the performance function is highly nonlinear RBDO problem. Yang Dixiong and Yi Ping [40] proposed the chaos control (CC) method based on the chaos control principle. This method uses the stable conversion method in the chaos control principle to converge the non-convergence phenomenon in the AMV and CMV methods to obtain stability. Convergence solution. Although this method has good convergence for highly nonlinear problems, its solution efficiency is very low. Hao et al. [41] proposed an enhanced step size adjustment method (ASSA) for PMA. This method promotes the iterative process in terms of efficiency and robustness, and based on the relative position of the direction vector and the negative gradient direction and their position between each iteration point, a new strategy is established to identify the oscillations in the iterative process And redefine the iteration step. ASSA has wide applicability to nonlinear performance functions, and has the advantages of high efficiency and good robustness.

For the decoupling method, the solution of the offset vector is the key, because they are used to determine the position of the limit state constraint, which indirectly affects the accuracy of the overall optimization result. Figure 6 is a schematic diagram of the iterative decoupling method, which shows the influence of different degrees of nonlinearity of the performance function on the offset vector and the optimal solution. The performance function of figure 6(a) is linear or low-order nonlinearity, and the performance function of figure 6(b) is highly nonlinear. [42]

In figure 6(a), $X^k$ is the point on the $\beta_k$ circle that meets the minimum performance function value at the $k$-th iteration, and the offset vector $s^{k+1}$ at the $k+1$-th iteration is obtained by the difference between the design point $\mu^k$ and the point $X^k$ obtained at the $k$-th iteration. It can be found from the figure that when the performance function is linear or low-order nonlinear, the curve $G(X)=G^k$ and $G(X)=0$ are approximately parallel. At this time, the original probability constraint curve is translated along the direction of the offset vector to obtain the offset probability constraint curve. The distance between the $k+1$-th design point obtained at this time and the original limit state curve is the reliable index value of this point, so it satisfies the reliability constraint.

In figure 6(b), when the performance function is highly non-linear, the shapes of curves $G(X)=G^k$ and $G(X)=0$ differ greatly and are not parallel to each other. At this time, the calculated offset vector cannot provide the correct direction to translate the original probability constraint. From the figure, it can be found that the distance between the $k+1$-th iteration design point and the original probability constraint is less than $\beta_k$, so the design point cannot meet reliability constraint.
In response to the above problem of highly nonlinear limit state functions, Chen et al. [42] proposed an optimal displacement vector (OSV) method based on the hypersphere design space. This method establishes a new reliability analysis model that uses a performance function instead of a specific performance function with a minimum value to search for the offset vector. Therefore, even if the function is highly non-linear, a better design point can be obtained using the offset vector obtained by the new reliability analysis model, and the overall number of iterations required is less than the existing decoupling method.

For the single-cycle method, because the approximate deterministic constraint is used instead of the original probability constraint, this method can obtain a better optimized solution only when the performance function is linear or low-order nonlinear. When the function is highly nonlinear, most single-cycle methods often fail because they cannot converge. Lim et al. [43] proposed an efficient semi-single cycle method, which incorporates some reliability evaluation links, but has a complete single-cycle structure. For highly nonlinear problems, the proposed method has higher efficiency and can obtain approximate probability constraints with high precision, for low-degree nonlinear problems, almost no reliability analysis is required.

The simulation method represented by the MCS method can accurately perform reliability analysis because it is not affected by the nonlinear limit state function. It is one of the means to solve the highly nonlinear bottleneck problem, but such methods often have a heavy calculation burden. Various secondary developments for sampling methods have improved optimization efficiency and increased the practicality of sampling methods. For example, Yuan et al. [44] proposed a weighted importance sampling method combined with a decoupling method framework for RBDO, Nader et al. [45] proposed an improved weighted average simulation method, which significantly reduced the performance function required during the RBDO. The number of evaluations ensures the optimization accuracy while improving the solution efficiency.

4.4. Multiple failure modes and system RBDO

For large mechanical structure systems, due to the large number of components, the reliability optimization design of each component cannot be performed independently, and multiple failure modes must be considered comprehensively to optimize the reliability design. However, the probability constrained optimization model of the structural system must involve a variety of boundary load conditions, and there may be statistical correlation between the multiple failure modes. Therefore, it is an urgent problem to propose an optimization design method for the overall reliability of a multi-element, multi-failure mode structure that allows the calculation accuracy and efficiency to meet the actual requirements of the project.

System RBDO involves multiple problems such as quantification of multi-source mixed uncertainty, high-precision reliability evaluation under complex loads, multi-state system reliability
evaluation, and high-efficiency reliability optimization design. Therefore, the method used for system RBDO Numerous, Kuo et al. [46,47] made a comprehensive review of system RBDO, this article does not make too much redundant introduction.

Existing methods are not always effective in solving RBDO problems, although they show good performance in solving certain problems. Through the research of the above bottleneck problems, it is found that various types of bottleneck problems often do not appear separately, and they will appear in a problem at the same time in more cases. For example: random variables have statistical correlation and are non-normal. The failure modes are correlated, have multiple limit state functions and are highly nonlinear. When various errors and inefficiencies accumulate, we do not know what state the problem solving process and results will take, so the bottleneck problem of RBDO urgently needs the concerted efforts of researchers in the field of reliability!

5. Summary and Prospect

Whether it is a traditional RBDO method, or a RBDO method based on intelligent algorithms and fuzzy models that has emerged in recent years. The proposal or application of these methods shows that the research on RBDO is continuing, but the existing methods still have many bottleneck problems, which is also the focus of our follow-up research. For example, when the performance function is highly nonlinear and the variables are non-normal, the optimization results cannot converge or the accuracy of the convergence solution is too low; when the random variables are multivariable and correlated, the RBDO problem is difficult or impossible to solve. In this article, we summarize the progress of RBDO-related research, and discuss commonly used reliability analysis methods and RBDO solution strategies. By studying the existing methods, it can help us further explore the theoretical research and engineering application of RBDO.

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