Students’ mathematical communication in discovering the concept of graphs of trigonometric functions through instructional videos

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**Abstract.** Mathematical communication is one of the critical aspects of learning mathematics. One way to improve mathematical communication skills is integrating technology in learning. The technology that can be employed is an instructional video, for instance, GeoGebra tutorial video. The video helps students discover the characteristics of graphs of trigonometric functions. Initially, students watched the instructional video before the lesson. In this way, they could imagine and illustrate the graphs of trigonometric functions. The purpose of this study was to describe students' mathematical communication in discovering concepts of trigonometric function graphs at senior high schools through instructional videos. This qualitative study utilized observation, interviews and student worksheets as instruments. The researchers selected three students as participants and observed and analyzed their written and oral communication skills using rubric guidelines. Students’ worksheets and teacher’s note were used for data analysis. The results of this study found that by watching instructional videos, students were able to master the problems that contained all indicators of mathematical communication. It can be concluded that, generally, students’ mathematical communication skills were considered good. Finally, this study implied that instructional videos could be used as an alternative/innovation in learning to improve students’ mathematical communication skills.

1. **Introduction**

Communication is one of the abilities that students need to have in learning mathematics. It is stated in the principles and standards of the National Council of Teacher of Mathematics (NCTM). Various sources also mention the vital role of communication in learning mathematics [1-3]. Based on NCTM, "Communication is an essential part of mathematics and mathematics education" [1].

"Communication is the transmission of information, ideas, emotions, skills, etc. by the use of the symbol" [4]. The act or process of transmission is called communication. Besides, mathematical communication skills are important for students to sharpen the way they think, build their abilities, improve their social and problem-solving skills, assess their understanding, organize and develop their mathematical knowledge, promote their reasoning, and establish mathematical communities [5]. Communication between teachers
and students is essential for developing students' mathematical potential in life, such as having curiosity, attention, and interest in learning mathematics, as well as being tenacious and confident in problem-solving.

On the other hand, some experts formulate mathematical communication in three ways: (1) Expressing mathematical ideas through speech, writing, demonstration, and describing them visually in different types; (2) Understanding, interpreting and evaluating ideas presented in written, oral, or visual forms; and (3) Constructing, interpreting and linking various representations of ideas and their relationships [6].

Since mathematical communication is crucial in learning mathematics, students’ mathematical communication skills should be improved. However, mathematical communication in learning mathematics is not optimal yet. Hence, it is necessary to have innovative efforts to solve this issue; one of them is by using technology.

Regulation of the Indonesian Ministry of Education and Culture Number 81A of 2013 (Permendikbud No. 81A, 2013) requires the integration of technology into the learning process to reduce learning difficulties caused by the abstract objects in mathematics [7]. One of the methods that can be developed to be media in learning mathematics is an instructional video. It can be watched and learned by students whenever and wherever they are.

The instructional video used in this study is tutorial videos of GeoGebra to help students discover the characteristics of graphs of trigonometric functions. GeoGebra is a dynamic mathematics software and an open-source (free) for learning and teaching mathematics at schools and aims to support the learning process. GeoGebra is one of the computer programs that can be used to teach students the concepts of geometry and algebra [8]. GeoGebra has multiple representations that include: 1) the appearance of algebra, 2) graphical display, and 3) numerical display. These three views are dynamically connected. Some studies have been conducted related to the use of GeoGebra in graphing trigonometric functions [9, 10]. Nevertheless, the studies have not studied the use of GeoGebra tutorials to improve students' mathematical communication related to graphs of trigonometric functions. Thus, this study intends to answer the following research question: How is mathematical communication of high school students in discovering the concepts of graphs of trigonometric function through GeoGebra tutorial videos?

2. Method

This research is a qualitative descriptive study that describes the tenth-grade senior high school students’ mathematical communication during the implementation of GeoGebra tutorial videos. The experiment was conducted in two meetings, involving 32 senior high school students in Banda Aceh. However, to observe and analyze the students’ mathematical communication, three students—student A, student B, and student C—who had good oral and written communication skills were selected. These students were chosen by looking at their criteria in expressing their opinion during the presentation and constructing graphs of the trigonometric functions on the worksheet. The data in this study were collected using instruments, i.e., student worksheets and self-reflection notes during the learning process. Students' abilities in presenting their work were assessed as oral communication ability.

The indicators of mathematical communication can be seen from:
1. The ability to express mathematical ideas in oral and written forms, and demonstrate and visualize them;
2. The ability to understand, interpret and evaluate mathematical ideas in written and in other visual forms;
3. The ability to use terms, mathematical notations and its structures to present ideas and describe relationships with situation models [1].

Figure 1 is part of the instructional video shown before the learning began. The graphs of functions $y = \sin x$, $y = \cos x$ and $y = \tan x$ were displayed in GeoGebra. The functions of sine x (in green colour), cosine x (in red colour) and tangent x (in blue colour) can be seen in Figure 1.
Rubrics for scoring students' answers were arranged according to three indicators by NCTM [1]. The rubric guidelines are presented in Table 1 and Table 2.

**Table 1. Rubric for written mathematical communication skills**

| Score | Written text | Drawing | Mathematical expression |
|-------|--------------|---------|-------------------------|
| 0     | No answer. If the answer is provided, it does not give any understanding of the concept; As such, the information provided is not useful. | Only a few pictures, diagrams, or tables are correct | Only a few mathematical approaches are correct |
| 1     | Only a few explanations are correct | Sketch a diagram, picture, or table, but it is not complete and correct | Create a mathematical approach correctly, but give an incorrect solution |
| 2     | Mathematical explanations are reasonable, but partially complete and correct | Sketch a diagram, picture, or table completely and correctly | Create a mathematical approach correctly, do calculations or get a solution completely and correctly |
| 3     | Mathematical explanations are not logically arranged, or a few language errors exist | Sketch a diagram, picture or table completely and correctly | |
| 4     | Mathematical explanations are reasonable, clear and logically arranged | Maximum score = 4 | Maximum score = 3 | Maximum score = 3 |

*Adapted from [11,12].

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**Figure 1.** The illustration of the instructional video using GeoGebra.
Table 2. Rubric for oral mathematical communication skills.

| No | Indicators/criteria       | Level 1                                      | Level 2                                      | Level 3                                      |
|----|---------------------------|----------------------------------------------|----------------------------------------------|----------------------------------------------|
| 1  | Concept mastery           | Not describe clearly/frequently look at notes| Describe clearly and rarely look at notes     | Describe clearly and never look at notes      |
| 2  | Intonation                | Not aligned with the meaning of the delivered expression | Slightly aligned with the meaning of the delivered expression | Aligned with the meaning of the delivered expression |
| 3  | Eye contact               | Always read notes and avoid eye contact with the audience | Frequently read notes and make eye contact with the audience | Never read notes and always make eye contact with the audience |
| 4  | Gestures                  | Gestures do not support the material presented. Gestures and facial expressions are stiff. | Gestures slightly support the material presented. Gestures and facial expressions seem slightly stiff or uncomfortable. | Gestures support the material presented. Gesture and facial expression do not look stiff and comfortable. |

3. Results and discussion

Based on students’ answers in the worksheet and presentation, the analysis of students’ mathematical communication will be discussed as follows.

Based on the observation on student A, after watching the instructional video, the student was able to graph trigonometric functions, \( y = 2 \sin x \) (in red colour) and \( y = 2 \sin x + 1 \) (in green colour), as shown in Figure 2.

Figure 2. Student A could construct the graphs of trigonometric functions.
Student B was also able to express mathematical ideas in other graphs of trigonometric functions, $y = \cos x$, $y = 2 \cos x$ and $y = 2 \cos x - 1$, as shown in Figure 3.

![Figure 3](image)

**Figure 3.** Student B could express mathematical ideas in the cosine graph.

By using the GeoGebra tutorial video, students could see graphical displays of the trigonometric functions. It helped them learn abstract objects of algebra. Student C made conclusions about the concepts presented in the instructional video, as in Figure 4.

![Figure 4](image)

**Figure 4.** Students C’s conclusions about the concepts presented in the GeoGebra instructional video.
After finding sine and cosine graphs from the GeoGebra instructional video, the students were asked to find the concept of a tangent graph. One of the students’ answers is shown in Figure 5. The students had a good understanding of the concepts; as such, they were able to express other graphs of trigonometric functions.

![Tangent Graph](image)

**Figure 5.** The student discovered the concept of a tangent graph.

Based on the students’ answers, scores of students’ mathematical communication are reported in Table 3 and 4.

**Table 3.** The analysis results of written mathematical communication skills.

| No | Name     | Observed aspects | Average |
|----|----------|------------------|---------|
| 1  | Student A| 2                | 2.67    |
| 2  | Student B| 3                | 2.50    |
| 3  | Student C| 3                | 2.83    |
|    | Total average |            | 2.67    |

*The second aspect was analyzed based on the students’ work in graphing trigonometric functions.

**Table 4.** The analysis results of oral mathematical communication skills.

| No | Name     | Observed aspects | Average |
|----|----------|------------------|---------|
| 1  | Student A| 2                | 2.75    |
| 2  | Student B| 3                | 2.50    |
| 3  | Student C| 3                | 2.75    |
|    | Total average |            | 2.553   |
To get further information about students’ mathematical communication, the researchers conducted an interview as presented in the following vignette.

Researcher : Which problem do you think the most difficult one?
Student B : Problem number 2.
Researcher : What does the problem ask?
Student B : Draw a graph.
Researcher : Then, what else does the problem ask?
Student B : I don’t know how to shift the graph.
Researcher : Afterwards, what did you do?
Student B : Finding the equation, but I could not solve it because I forgot how to shift the graph right, left, up, and down.
Researcher : What do you think about the instructional video?
Student B : Very helpful. I could review the video repeatedly at home until I understand.

Based on the analysis of the students' answers and the interview results, student A obtained a score of 2 for the first aspect. It was because she provided reasonable mathematical explanations, but they were partially complete and correct. However, the student was able to construct the graphs well for the given trigonometric functions and able to express the mathematical expressions. The way she presented her work indicated that her mastery of the concept was still limited because she frequently looked at her notes during the presentation, while the other aspects were in a good category. Generally, student A had good communication skills. On the other hand, student B was not able to construct the graph correctly because one step was missed. Consequently, the 'trace on' button did not show up. Regarding the presentation, student B gave intonation that slightly aligned with the meaning of the delivered expression, and she frequently read the notes. Overall, student B had good communication skills. Furthermore, student C performed good mathematical communication, both in oral and written forms. She could draw the graphs well and do calculations correctly.

The implementation of the instructional video by using GeoGebra in mathematics learning can foster students' mathematical communication. In general, students' mathematical communication skills were in a good category, both in written and oral forms. This finding is supported by a study that integrated technology in learning by using GeoGebra and also collaborated it with classical learning so that students' mathematical communication skills achieved a good category [13]. Another study found that using GeoGebra for students’ learning at secondary schools could improve mathematical communication and problem solving and support them in strengthening their abilities [14]. Meanwhile, a study reported that graduate students believed learning mathematics with GeoGebra could increase their motivation and mathematical communication [15].

4. Conclusion
Based on the pilot study conducted in the mathematics classroom and the data obtained from the teacher's self-reflection notes on the GeoGebra instructional video, the students' mathematical communication was considered good. For written communication, student B got a score of 2 on mathematical expression. Different from student B, student A and student C reached a score of 3. While for oral communication, they all achieved an average score. Regarding written communication, the students were able to provide mathematical explanations, although some of them were not logically arranged or a few language errors existed. They were able to draw graphs and create mathematical approaches correctly. Then, they did calculations and found the solutions completely and accurately. Regarding oral communication, the three students were able to present their work well in front of the class.
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