The Dihadron fragmentation functions way to Transversity

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Abstract. Observations of the transversity parton distribution based on an analysis of pion-pair production in deep inelastic scattering off transversely polarized targets are presented. This extraction relies on the knowledge of dihadron fragmentation functions, which are obtained from electron-positron annihilation measurements. This is the first attempt to determine the transversity distribution in the framework of collinear factorization.

Keywords: transversity, proton structure
PACS: 13.87.Fh, 11.80.Et, 13.60.Hb

The distribution of quarks and gluons inside hadrons can be described by means of parton distribution functions (PDFs). In a parton-model picture, PDFs describe combinations of number densities of quarks and gluons in a fast-moving hadron. The knowledge of PDFs is crucial for our understanding of QCD and for the interpretation of high-energy experiments involving hadrons.

In the Bjorken limit, the partonic structure of the nucleon is described in terms of only three collinear PDFs: the unpolarized, $f^q_1(x)$, and helicity, $g^q_1(x)$, distribution functions, and the transversity distribution function $h^q_1(x)$, whose extraction we present here. Transversity measures the transverse polarization of quarks with flavor $q$ and fractional momentum $x$ in a transversely polarized nucleon.

There is no transversity for gluons in a nucleon, and $h^g_1$ has a pure non-singlet scale evolution [1]. Transversity is a chiral-odd function and appears in cross sections combined with another chiral-odd function, e.g., the Collins fragmentation function in the case of single-particle-inclusive DIS. The latter convolution gives rise to a specific azimuthal modulation of the cross section. The amplitude of the modulation has been measured by the HERMES and COMPASS collaborations [2, 3]. The first-ever extraction of $h^q_1$ arose from a simultaneous analysis of SIDIS data and the $e^+e^- \rightarrow \pi\pi X$ data from Belle [4], from which the Collins function can be determined [5].

As for single-particle-inclusive DIS, there are two main issues: the convolution should be analyzed in the transverse-momentum-dependent factorization framework; QCD evolution from Belle’s scale to HERMES’s has to be applied coherently to that framework.

Here we extract transversity in an independent way requiring only standard collinear factorization, i.e. by considering the semi-inclusive deep-inelastic production of two hadrons with small invariant mass, where the above complications are absent [6]. That is, in this case, the transversity distribution function is multiplied by a chiral-odd Dihadron Fragmentation Function (DiFF), denoted as $H^{\pm q}_1$ [7], which describes the cor-
relation between the transverse polarization of the fragmenting quark with flavor $q$ and 
the azimuthal orientation of the plane containing the momenta of the detected hadron 
pair. Contrary to the Collins mechanism, this effect survives after integration over quark 
transverse momenta and can be analyzed in the framework of collinear factorization. 
This process has been studied from different perspectives in a number of papers, e.g. [8].
The only published measurement of the relevant asymmetry has been presented by the 
HERMES collaboration for the production of $\pi^+\pi^-$ pairs on transversely polarized protons [9]. 
Preliminary measurements have been presented by the COMPASS collaboration [10].

Similarly to the single-hadron case, $H_{1,q}^{< q}$ has to be independently determined by 
looking at correlations between the azimuthal orientations of two pion pairs in back-to-
back jets in $e^+ e^-$ annihilation [11, 12, 13]. The measurement of this so-called Artru–
Collins azimuthal asymmetry has recently become possible thanks to the Belle collaboration [14].

The $A_{UT}^{\sin(\phi_R + \phi_S)} \sin \theta$ measured by HERMES [9] can be interpreted as [15]

$$A_{UT}^{\sin(\phi_R + \phi_S)} \sin \theta (x, Q^2) = -C_y \frac{\sum_q e_q^2 h_1^q(x, Q^2) n_q^+(Q^2)}{\sum_q e_q^2 f_1^q(x, Q^2) n_q(Q^2)}, \quad (1)$$

where we consider only the $x$ (the momentum fraction of the initial quark) binning 
as our interest here lies on the transversity distribution. The depolarization factor is 
$C_y \approx \frac{1-(y)}{1+(y)\pi/2}$; and

$$n_q(Q^2) = \int dz dM_h^2 D_1(z, M_h^2, Q^2); \quad n_q^+(Q^2) = \int dz dM_h^2 |R| H_{1,sp}(z, M_h^2, Q^2), \quad (2)$$

with $|R|/M_h = \sqrt{1/4 - m^2_{\pi}/M_h^2}$. $D_1$ is the unpolarized DiFF describing the hadronization 
of a quark $q$ into a $\pi^+ \pi^-$ pair plus any number of undetected hadrons, averaged over quark 
polarization and pair orientation. Finally, $H_{1,sp}^{< q}$ is a chiral-odd DiFF, and denotes 
the component of $H_1^{< q}$ that is sensitive to the interference between the fragmentation 
amplitudes into pion pairs in relative $s$ wave and in relative $p$ wave.

Isospin symmetry and charge conjugation allow for the assumptions [15]:

$$D_1^u = D_1^\bar{q} = D_1^\bar{\bar{u}}, \quad D_1^d = D_1^\bar{q} = D_1^\bar{\bar{d}}, \quad D_1^s = D_1^\bar{s}, \quad H_{1,sp}^{< u} = -H_{1,sp}^{< d} = -H_{1,sp}^{< \bar{d}} = H_{1,sp}^{< \bar{u}}, \quad (3)$$

the rest being 0. We also assume $D_1^u \equiv N_u D_u^u$ and we consider the two scenarios $N_u = 1$ 
and $N_u = 1/2$. The second choice is suggested by the output of the PYTHIA event 
generator. The difference between these 2 scenarios defines the theoretical error of our 
result.

The above assumptions allow us to turn Eq. (1) into the following simple relation 
(neglecting charm quarks)

$$x h_1^{u-v}(x, Q^2) - \frac{1}{2} x h_1^{d-v}(x, Q^2) = -\frac{A_{DIS}(x, Q^2)}{C_y} \frac{n_u(Q^2)}{n_u^+(Q^2)} \sum_{q=u,d,s} \frac{e_q^2 N_q}{e_u^2} x f_1^{q+\bar{q}}(x, Q^2), \quad (4)$$
where $N_u = N_d = 1$ and $f_1^{q+\bar{q}} = f_1^q + f_1^{\bar{q}}$, $h_1^{q\bar{q}} = h_1^q - h_1^{\bar{q}}$. Our goal is to derive from data the difference between the valence up and down transversity distributions by computing the r.h.s. of the above relation.

The unpolarized PDFs in Eq. (4) can be estimated using any parametrization of the unpolarized distributions. We chose to employ the MSTW08LO PDF set [16].

The only other unknown term on the r.h.s. of Eq. (4) is the ratio $n_{u\uparrow}/n_{u\downarrow}$. We extract this information from the recent measurement by the Belle collaboration of the Artru–Collins azimuthal asymmetry $A_{\cos(\phi_R+\phi_\theta)}$ [14]. For this purpose we need $n_{u\uparrow}/n_{u\downarrow}$ at the experimental values of $\langle Q^2 \rangle$ and integrated over the HERMES invariant-mass range $0.5 \leq M_h \leq 1$ GeV. We will get to this number in two steps.

First, the ratio $n_{u\uparrow}/n_{u\downarrow}$ is integrated over $0.5 \leq M_h \leq 1$ GeV at the Belle scale (100 GeV$^2$). The relevant variables for DiFFs in electron-positron annihilation are $(z, M_h, \bar{z}, \bar{M}_h)$. Belle’s data are differential in different pairs of variables. We consider the Belle asymmetry integrated over $(z, \bar{z})$ and binned in $(M_h, \bar{M}_h)$. We weight each relevant bin by the unpolarized cross section, which, in this process, is approximated by the inverse of the statistical error squared. By summing over all bins in the considered range, we get the total asymmetry

$$A_{\cos(\phi_R+\phi_\theta)} = \frac{-\langle \sin^2 \theta_2 \rangle \langle \sin \theta \rangle \langle \sin \theta \rangle 5 (n_{u\uparrow})^2}{\langle 1 + \cos^2 \theta_2 \rangle (5 + N_s^2) n_{u\downarrow}^2 + 4 n_s^2} = -0.0307 \pm 0.0011 ,$$

which leads to, \(^1\)

$$n_{u\uparrow}/n_{u\downarrow}(100 \text{ GeV}^2) = -0.273 \pm 0.007_{\text{ex}} \pm 0.009_{\text{th}} , \tag{6}$$

where the second error comes from using the two different values of the $s$–quark normalization $N_s$. \(^2\)

Second, since the Belle scale is very different from the HERMES one, DiFFs must be connected from one scale to the other via their QCD evolution equations [17]. In order to do this, we need to know the $z$ dependence of $H_1^{<u\downarrow}$ and $D_1^q$ for each $M_h$ value. We start from a parametrization of both DiFFs at $Q_0^2 = 1$ GeV$^2$, evolving at LO using the HOPPET code [18] and fitting Belle’s data. The fitting procedure will be further explained elsewhere. \(^3\) By integrating the extracted DiFFs in the HERMES range $0.5 \leq M_h \leq 1$ GeV and $0.2 \leq z \leq 1$, we can calculate the evolution effects on $n_{u\uparrow}/n_{u\downarrow}$ at each $\langle Q^2 \rangle$. It turns out that the ratio is decreased by a factor $0.92 \pm 0.08$, where the error takes into account the difference of $Q^2$ in the HERMES experimental bins as well as the uncertainty related to different starting parametrizations at $Q_0^2 = 1$ GeV$^2$. In conclusion, for the extraction of transversity in Eq. (4) we use the number

$$n_{u\uparrow}/n_{u\downarrow} = -0.251 \pm 0.006_{\text{ex}} \pm 0.023_{\text{th}} . \tag{7}$$

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1 Useful values extracted from Belle analysis are given in Ref. [6].

2 To verify the reliability of this procedure, we repeated the calculation estimating the denominator of the asymmetry using the PYTHIA event generator without acceptance cuts.

3 We checked that the final results are affected in a negligible way by the gluonic component $D_1^q(z, M_h; Q_0^2)$. 

Our result for the four HERMES data points is shown in Fig. 1. The central line represents the best fit for the combination $xh_{1}^{uu} - xh_{1}^{dd}/4$, as deduced from the most recent parametrization of $h_{1}^{uu}$ and $h_{1}^{dd}$ extracted from the Collins effect [19].

In summary, we have presented a determination of the transversity parton distribution in the framework of collinear factorization by using data for pion-pair production in deep inelastic scattering off transversely polarized targets, combined with data of $e^+e^-$ annihilations into pion pairs. The final trend of the extracted transversity seems not to be in disagreement with the transversity extracted from the Collins effect [19]. More data are needed to clarify the issue.

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