Angular Power Spectrum of B-mode Polarization from Cosmic String Wakes

Robert Brandenberger, Nick Park and Grant Salton

Department of Physics, McGill University, Montréal, QC, H3A 2T8, Canada

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I. INTRODUCTION

Cosmic microwave background (CMB) polarization is emerging as a new observational window to probe the primordial universe. CMB polarization is produced by Thompson scattering of the CMB off of free electrons. Anisotropies of CMB polarization are produced by the scattering of the CMB quadrupole off of density inhomogeneities which contain free electrons (see [2] for an introduction to the physics of CMB polarization). Polarization is described by the polarization tensor which can be decomposed into two modes - E-mode and B-mode. E-mode polarization has already been observed (see e.g. [3]), and several dedicated experiments to probe polarization at improved sensitivity are in construction or in planning (see e.g. [4]) which will reach the required sensitivity to detect primordial B-mode polarization. In fact, very recently the B-mode polarization signal induced by gravitational lensing has been observed for the first time [5].

Of specific interest here is B-mode polarization. In cosmologies with Gaussian adiabatic fluctuations, no B-mode polarization arises at linear order in cosmological perturbation theory. B-mode polarization on large angular scales can be generated by primordial gravitational waves, and on smaller angular scale by lensing of E-mode polarization [6]. The search for B-mode polarization has in particular attracted a lot of attention because of the promise to use the results to detect primordial gravitational waves such as those produced by a period of inflation in the very early universe [7] (but see [8] for a discussion of other sources of gravitational waves in the early universe).

In this paper we study the angular power spectrum of CMB polarization (in particular B-mode polarization) induced by a scaling solution of cosmic string wakes. Cosmic strings (see [9, 10] for reviews on cosmic strings) are linear topological defects which are predicted to form during a phase transition taking place in the very early universe. They arise in a wide set of quantum field theory models beyond the “Standard Model” of particle physics. The important point is that if our microwaves is described by any model which contains cosmic strings, a network of such strings will inevitably form [12] in a symmetry breaking phase transition and persist to the present time. Cosmic strings carry energy and hence lead to gravitational effects on space-time which can be searched for in cosmological observations. Finding evidence for cosmic strings would be an exciting discovery, but even non-observation of effects predicted by strings would be interesting since it would provide new constraints on particle physics beyond the “Standard Model”. In fact, searching for strings in cosmological observations is a way to probe particle physics which is complementary to accelerator tests in the sense that the cosmological tests will probe particle physics at the high energy end of the current region of ignorance whereas accelerators provide probes at the low energy end.

Cosmic strings are characterized by their mass per unit length \( \mu \) which is usually given in terms of the dimensionless quantity \( G \mu \) (where \( G \) is Newton’s gravitational constant, and we use natural units in which the speed of light \( c \) and Planck’s constant are set to one). At the present time, the best constraint on \( G \mu \) comes from analyses of the angular power spectrum of CMB temperature maps and yields the constraint \( G \mu < 10^{-7} \) [13, 14] (see also [15] for previous limits based on older data) \(^1\). Other windows to probe cosmic strings include high redshift galaxy surveys (see [16] for a recent study and for references to previous works) and 21cm redshift surveys \(^2\) (see also [18]). In this paper, we will focus on signatures of strings in CMB polarization maps. In contrast to earlier work [19], we here focus on the effects of strings between the surface of last scattering and the present...

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\(^1\) Note that the exact value of the upper bound depends on features of the distribution of the string network which are drawn from numerical string evolution simulations and have systematic uncertainties attached to them.

\(^2\) See [20] for a review on how to use new observational windows to probe for the possible existence of cosmic strings.
time.

Cosmic strings are either infinite or else loops. Causality tells us that a network of infinite strings must be present at all times after the phase transition which generates them \[^{12}\]. We can view the network of infinite string as a system of random walks with mean step length (i.e. curvature radius) and separation given by a correlation length \( \xi(t) \). Causality tells us that the correlation length must be smaller or equal to the horizon distance \( t \) \[^{12}\], and a Boltzmann equation which describes the energy loss of the long string network due to expansion and string loop production indicates that \( \xi(t) \) cannot be much smaller than \( t \) (see e.g. \[^{11}\]). In this paper we will focus on the signatures of the long strings.

Due to their tension, cosmic strings acquire relativistic velocities. Due to the conical geometry of space perpendicular to a straight string segment (see the review in the following section), such a string segment moving with velocity \( v_s \) will produce a wedge behind it within which the initial density is twice the background value. This is the string wake \[^{22}\].

Since they are overdense regions of matter (including a fraction of free electrons), string wakes are sites of enhanced Thompson scattering and hence lead to CMB polarization anisotropies. The special geometry of a string wake in position space leads to a distinctive signal in CMB polarization maps \[^{1}\]. In particular, a string wake will produce direct B-mode polarization with an amplitude which is statistically (averaged over all wake orientations relative to the CMB quadrupole) equal to that of the induced E-mode polarization. Hence, since at linear order in cosmological perturbation theory Gaussian adiabatic fluctuations do not induce B-mode polarization, searching for B-mode CMB polarization appears as a promising window to detect or constrain cosmic strings.

As in much of the analytical work on cosmic strings and structure formation (see e.g. \[^{20}\] for a recent short review), we will be working in terms of a toy model \[^{21}\] in which we break up the long string network into a set of straight segments of length \( \xi(t) \) (the typical curvature radius of the long strings) which each live for one Hubble expansion time (the typical time interval between intersections of long string segments). The number density \( n \) of string segments per Hubble volume at time \( t \) is fixed by the scaling solution

\[
n = n_w t^{-3},
\]

where \( n_w \) is a constant which according to numerical simulations of cosmic string networks is in the range 1 < \( n_w < 10 \). \[^{24}\] The set of string segments are taken to be uncorrelated in different Hubble time steps.

In a previous paper \[^{1}\], the position space signal of a single cosmic string wake was studied. In this paper we work out the angular power spectrum of a scaling distribution of string wakes. This means, in particular, that we must sum the contributions over all Hubble time steps \( t_i \) when string wakes are formed, and over all times \( t \) when our past light cone intersects the string wake.

Since the string signals are characterized by edges in position space maps, good angular resolution is more important than full sky coverage. Hence, having in mind application to telescopes such as SPTPol \[^{25}\] and ACT-Pol \[^{20}\], we work with patches of the sky for which the “flat sky approximation” (see e.g. \[^{27}\]) is applicable.

We find that the angular power spectrum of B-mode polarization due to cosmic strings has a very similar shape as that of the gravitational lensing contribution. For values of \( G \mu \) comparable to the current upper bound, the amplitude of the power spectrum is smaller than the lensing signal caused by the Gaussian perturbations. On the other hand, the amplitude is sufficiently large such that the string signals (which are highly non-Gaussian) ought to be visible in position space analyses on B-mode maps.

II. POLARIZATION SIGNAL OF A COSMIC STRING WAKE

Wakes arise as a consequence of the geometry of space perpendicular to a long straight string. Space is locally flat, but globally it corresponds to a cone with deficit angle \[^{28}\]

\[
\alpha = 8 \pi G \mu ,
\]

with the tip of the cone coinciding with the location of the string.

On scales comparable to the Hubble length \( t \), the long string network is curved, which induces relativistic transverse velocities of the string. A moving string yields a velocity kick towards the plane in the wake of the string, which in turn leads to a region behind the string with twice the background density. The length of the wake is set by the length of the cosmic string segment, i.e. by the curvature radius of the long string, and is \( c_1 t_i \) for a string passing through the gas at time \( t_i \). Here, \( c_1 \) is a numerical constant of order 1. The depth of the wake is determined by the string velocity \( v_s \) and is \( v_s \gamma_s t_i \), where \( \gamma_s \) is the relativistic gamma factor associated with the velocity \( v_s \). The mean initial thickness of the string wake is \( 4 \pi G \mu \gamma_s t_i \).

The planar dimensions of the wake are fixed in comoving coordinates. The wake thickness, on the other hand, increases due to gravitational accretion from above and below onto the string wake \[^{5}\]. This accretion process can

\[^{3}\] We are making use of the “one-scale” model of the string network where the curvature radius and mean separation of the long strings are the same \[^{22}\].

\[^{4}\] See also \[^{23}\] for earlier work on CMB polarization from cosmic defects.

\[^{5}\] Note that once formed, the wake will persist even after the string
be studied \cite{29} by means of the Zel’dovich approximation \cite{30} with the result that the comoving distance \( q_{nl}(t, t_i) \) from the central plane of the wake which is beginning to fall in at time \( t > t_i \) is given by \cite{1}

\[
q_{nl}(t, t_i) = \frac{24\pi}{5} G \mu v_s \gamma_s(z(t_i) + 1)^{-1/2} t_0 \left( \frac{t}{t_i} \right)^{2/3}, \tag{3}
\]

where \( z(t) \) is the cosmological redshift at time \( t \), and \( t_0 \) is the present time. The last factor corresponds to the growth factor from linear cosmological perturbation theory. Note that in the case of wakes formed before the time \( t_{eq} \) of equal matter and radiation this formula needs to be modified in two ways: firstly the linear growth factor must be replaced by \( \frac{1}{t_{eq}} \) since the accretion of cold dark matter starts only at \( t_{eq} \). Secondly, the formula

\[
(z(t) + 1)t = (z(t) + 1)^{-1/2} t_0 \tag{4}
\]

must be modified to read

\[
(z(t) + 1)t = (z(t) + 1)^{-1} (z(t_{eq}) + 1)^{1/2} t_0. \tag{5}
\]

To summarize this discussion: the comoving size to which a wake formed at time \( t_i \) grows by time \( t \) is

\[
c_i t^c \times v_s \gamma_s t^c \times \frac{24\pi}{5} G \mu v_s \gamma_s t^c \left( \frac{t}{t_i} \right)^{2/3}, \tag{6}
\]

where \( t^c \) indicates the comoving distance corresponding to \( t \):

\[
t^c = (z(t) + 1)t, \tag{7}
\]

and where, as mentioned above, for \( t_i < t_{eq} \) the gravitational growth factor involves the time \( t_{eq} \) instead of the time \( t_i \).

Since the string wake is a region of enhanced free electrons, CMB photons emitted at the time of recombination acquire extra polarization when they pass through a wake. Since the quadrupole direction is statistically uncorrelated with the tangent vector of the string and its velocity vector, statistically an equal strength E-mode and B-mode signal is produced.

In general, polarization can be described by a magnitude \( P \) and a direction \( \alpha \). In terms of the Stokes parameters \( Q \) and \( U \), the amplitude \( P \) and angle \( \alpha \) are given by

\[
\begin{align*}
P & = \sqrt{Q^2 + U^2}, \\
\alpha & = \frac{1}{2} \arctan \left( \frac{U}{Q} \right). \tag{8}
\end{align*}
\]

The Stokes parameters in turn determine the \( E \) and \( B \) modes of the polarization.

\[\dots\]

segment which has seeded it is no longer present.

FIG. 1: Sketch of the polarization signal on the sky of a single string segment. The amplitude of the polarization at any point on the rectangle which experiences extra CMB polarization due to the string wake is indicated by the length of the arrow, the direction of polarization is indicated by the arrow.

The strength of the polarization signal is determined by the CMB quadrupole \( Q_{quad} \), the column density of free electrons which the CMB photons cross when passing through the wake, and by the Thompson cross section \( \sigma_T \). The column density of free electrons is given by the ionization fraction \( f(t) \) of the wake multiplied by the baryon column density, which in turn is determined by the baryon fraction \( \Omega_B \), the proton mass \( m_p \) and the wake thickness. The polarization signal of a single wake segment has a characteristic pattern which is depicted in Fig. 1. The polarization strength is smallest at the position of the string (since this is the tip of the wake wedge), and it is largest at the trailing edge. The polarization direction is roughly constant across the wedge as long as wakes are considered which cover a small fraction of the sky (and we will see that it is small wakes which dominate the angular power spectrum). We see that the wake produced by a string segment will lead to a rectangular region in the sky with extra polarization.

The mean strength of the polarization signal \( P \) of a wake produced at time \( t_i \) and intersected by our past light cone at time \( t_i \), averaged across the polarization rectangle in the sky, is \cite{1}

\[
P_{\text{quad}} \approx \frac{24\pi}{25} \left( \frac{3}{4\pi} \right)^{1/2} \sigma_T f(t) G \mu v_s \gamma_s \times \Omega_B \rho_c(t_0) m_p^{-1} t_0 (z(t) + 1)^2 (z(t_i) + 1)^{1/2} \tag{9}
\]

where \( \rho_c(t_0) \) is the energy density of a spatially flat universe today. As we will see in the following section, the wakes which contribute the largest amount to the angular B-mode polarization power spectrum are those created at around the time \( t_{eq} \) of equal matter and radiation for which our past light cone intersects the wake at a time \( t \) just after recombination. Hence, to get a feeling for the order of magnitude of the polarization signal of a string
wake we insert the numerical values for the dimensional constants $\rho_c(t_0), m_p, \sigma_T$ and $t_0$ and normalize the redshifts at a value of $10^3$ to obtain

$$\frac{P}{Q_{\text{quad}}}(t, t_i) \sim f(t) G \mu v_s \gamma_s \Omega_B \left(\frac{z(t) + 1}{10^3}\right)^2 \left(\frac{z(t_i) + 1}{10^3}\right)^{1/2} 10^7. \quad (10)$$

For a value of $G \mu = 10^{-7}$ comparable to the current upper bound on the string tension, then we find for $t_i \simeq t \simeq t_{\text{rec}}$

$$\frac{P}{Q_{\text{quad}}} \sim \Omega_B, \quad (11)$$

where we have inserted the value $v_s \gamma_s \simeq 1$ and used the fact that just after recombination $f \sim 1$. We thus see that in position space, the polarization signal of a cosmic string wake has a large amplitude.

The distribution of cosmic strings is highly non-Gaussian. More specifically, string wakes only cover a fraction of about $G \mu$ of the volume of space at $t_{eq}$. Hence, when computing the power spectrum we expect the non-Gaussianness of the string distribution to lead to an amplitude of the power spectrum of $P/Q_{\text{quad}}$ which is suppressed by this small factor compared to unity, and thus we expect that the string signal is much harder to identify in the power spectrum.

We end this section with a discussion of the formula for polarization which will underly our study. The starting point is the fact that the linear polarization of the CMB is described by two Stokes parameters $Q(x)$ and $U(x)$, where $x$ are Cartesian coordinates on a patch of the celestial sphere. The Stokes parameters determine the E and B mode polarization functions. In the flat sky approximation, these are given by

$$Q(x) = \int \frac{d^2 l}{(2\pi)^2} \left[ E(l) \cos(2\phi l) - B(l) \sin(2\phi l) \right] e^{i l \cdot x}
$$

$$U(x) = \int \frac{d^2 l}{(2\pi)^2} \left[ E(l) \sin(2\phi l) + B(l) \cos(2\phi l) \right] e^{i l \cdot x}
$$

(12)

where $I = l(\cos(\phi l), \sin(\phi l))$. These formulas can be inverted to give $E$ and $B$ as functions of $U$ and $Q$, which are then determined in turn by the total amplitude $P$ and the angle $\alpha$ via $\mathcal{S}$. In the flat sky Fourier space the inverted formulas are

$$E(l) = \tilde{Q}(l) \cos(2\phi l) + \tilde{U}(l) \sin(2\phi l)
$$

$$B(l) = -\tilde{Q}(l) \sin(2\phi l) + \tilde{U}(l) \cos(2\phi l), \quad (13)$$

where $\tilde{Q}$ and $\tilde{U}$ are the Fourier coefficients of $Q$ and $U$, respectively.

In the following section we use the above basic equations (12) to compute the angular power spectrum of both E-mode and B-mode polarization, starting with the expression (11) for the polarization of a single wake, and summing over all wakes which are crossed by the past light cone.

### III. computation of the angular power spectrum

We have performed two independent numerical computations of the angular power spectrum of CMB polarization produced by a scaling distribution of cosmic string wakes [32, 33]. Since good angular resolution is more important to identify cosmic string signals than full sky coverage, we have applications to experiments such as the South Pole Telescope [34] and the Atacama Cosmology Telescope [35] in mind which map out portions of the sky to which the flat sky approximation can be applied. Hence, in both of our computations we make use of this approximation.

Our starting point is the formula (9) for the amplitude of polarization from a single string wake created at time $t_i$ which is being crossed by our past light cone at time $t > t_i$. The angular power spectra of E and B mode polarization obtain contributions from each string wake which is crossed by the past light cone. Since the centers and orientations of the string wakes are uncorrelated, the contributions of all strings are independent, and hence the power spectra are obtained by integrating the power spectra produced by a single string wake over the times $t_i$ and $t$.

According to the string scaling solution, there are a fixed number $n_w$ of string segments per Hubble volume. They live for a Hubble expansion time. Thus, to take care of the integration over the formation time $t_i$, we divide the time interval from $t_{eq}$ to the present time into Hubble expansion times, pick the initial value $t_i$ in each interval, place a number $N$ of string wakes at random in each Hubble volume at time $t_i$, and sum over $t_i$. The reason for only considering formation times after $t_{eq}$ is the fact that earlier stringers are smaller but start to accrete cold dark matter only at the time $t_{eq}$.

For each time $t_i$, we have to determine how many string wakes formed at that time are crossed by the past light cone at time $t$. We again break up the time interval for $t$ into Hubble time steps, and compute the number of wakes formed in the $k$’th Hubble time step centered at $t_k$ crossing the past light cone in the Hubble time step centered at the discrete values of $t$. The formula (11) then can be used to compute the contribution of each such string wake to the power spectra. Finally, we must integrate over $t > t_j$. Obviously the condition $t > t_{\text{rec}}$ must be imposed (since the polarization we consider is produced by the decoupled CMB radiation being scattered in the wake).

Based on the statistical independence of the positions and orientations of string wakes, the angular power spec-

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6 Note the typo in the second to last factor of Eq. (19) in [1].
trum of a scaling solution of strings is
\[ C_l = \sum_{t_i} \sum_{t > t_i} N_w(t, t_i) C_l(t, t_i), \quad (14) \]
where \( N_w(t, t_i) \) is the number of wakes laid down at time \( t_i \) (within one Hubble expansion time) which intersect the past light cone in a Hubble expansion time about time \( t \), and \( C_l(t, t_i) \) is the angular power spectrum of a string wake produced at time \( t_i \) which intersects the past light cone at time \( t \).

The number of string wakes created in a Hubble expansion time around \( t_i \) and intersecting the past light cone of the observing area in a Hubble time about \( t \) is obtained by computing how many comoving Hubble volumes at \( t_i \) are contained in the comoving past light cone (PLC) of the observing volume at time \( t \), multiplied by the number \( n_w \) of strings per Hubble volume:
\[ N_w(t, t_i) = n_w Vol(PLC)_{\text{com}}(t) \frac{\text{Vol(Hubble)}_{\text{com}}(t_i)}{\text{Vol(Hubble)}_{\text{com}}(t)}, \quad (15) \]
where the notation is explained in the text above.

The comoving Hubble volume at time \( t_i \) is given by
\[ \text{Vol(Hubble)}_{\text{com}}(t) = \frac{9\pi}{2} t_i^3, \quad (16) \]
and the volume of the past light cone of the observation area of angular sizes \( \theta_1 \) and \( \theta_2 \) is
\[ \text{Vol(PLC)}_{\text{com}}(t) = 18\theta_1 \theta_2 t_i^3 (1 - a(t)^{1/3})^2 a(t)^{1/2}, \quad (17) \]
where we have normalized the scale factor to be one at the present time \( t_0 \). Inserting these results into (15) we get
\[ N_w(t, t_i) = n_w \frac{4\theta_1 \theta_2}{\pi} \left( \frac{t_i}{t_0} \right)^{1/3} \left( 1 - \left( \frac{t}{t_0} \right)^{1/3} \right)^2. \quad (18) \]

Next we need to compute the angular power spectrum \( C_l(t, t_i) \) of a string wake created at time \( t \) intersecting the past light cone at time \( t_i \). Since the angular power spectrum involves an average over space and an average over angles we can - without loss of generality - set the center of the wake to be at the origin of the flat sky coordinates, and take the string which creates the wake to be moving along the x-axis. Thus, the polarization amplitude is a linearly increasing function of \( x \) and is independent of \( y \), and its average magnitude \( P(t, t_i) \) is given by (9):
\[ P(x, y, t, t_i) = P(t, t_i) \Theta(|x| < \frac{d}{2}) \Theta(|y| < \frac{w}{2} \left( \frac{d}{2} x + 1 \right)), \quad (19) \]
where \( d = d(t, t_i) \) and \( w = w(t, t_i) \) give the angular dimensions of the wake seen in the sky. These angular dimensions are given by
\[ d(t, t_i) = v_s \gamma_s \omega(t, t_i), \]
\[ w(t, t_i) = c_1 \omega(t, t_i), \quad (20) \]
where, as discussed in Section 2, \( v_s \) is the velocity of the string, \( \gamma_s \) is the associated relativistic gamma factor, and \( c_1 \) is a constant of order one describing the coherence length of the string network in units of \( t \). The quantity \( \omega(t, t_i) \) is the angular scale of the comoving Hubble radius at \( t_i \) as seen at time \( t > t_i \). It is given by
\[ \omega(t, t_i) = \frac{(t_i/t_0)^{1/3}}{2(1 - (t/t_0)^{1/3})}. \quad (21) \]

The magnitude \( \tilde{P}(l) \) of the polarization of the string wake is given by the flat space Fourier integral of the position space polarization \( P(x, y, t) \). This integral can be performed exactly analytically, with the result
\[ \tilde{P}(l) = P(t, t_i) \times \frac{4\sin(l_y w/2)(l_x d/2 + 2i + l_x d)\sin(l_x d/2))}{l_x^2 l_y d}, \]
where \( P(t, t_i) \) is given by (9).

To evaluate the above result it is necessary to make use of the ionization fraction \( f(t) \). We have used an analytical fit to the time dependence of \( f(t) \) determined in [30]. At the time of recombination (corresponding to a redshift of \( z = 10^5 \)), the ionization fraction is unity. By a redshift of \( z = 500 \) the fraction has decreased to \( f(t) \sim 10^{-3} \). This effect increases the importance of wakes crossing the Hubble radius within the first Hubble expansion time after \( t_{\text{rec}} \) relative to those crossing at later times. After reionization \( f(t) \) jumps back up to close to unity.

In the flat sky approximation, the angular power spectrum \( C_l^{XX} \) of the quantity \( X \) (where \( X \) can stand for temperature \( T \) or polarizations \( E \) and \( B \) ) is given by
\[ C_l^{XX} = \langle X(1) X^*(1) \rangle, \quad (23) \]
where \( l \) is the magnitude of \( l \) and the angular brackets indicate averaging over angles of \( l \) and over the random variables describing the position of the wake and the orientation with respect to the CMB quadrupole, the angle called \( \alpha \) earlier in the text. The cross-correlation functions are described by a similar formula
\[ C_l^{XY} = \langle X(1) Y^*(1) \rangle, \quad (24) \]

7 This result is obtained by computing the comoving distance light travels from \( t \) to \( t_0 \), taking the square of this quantity, multiplying with the comoving Hubble time at \( t \) and finally multiplying by \( \theta_1 \times \theta_2 \).

8 Note that the definition of the angular correlation functions involves averaging over the entire sky. In the case of surveys and theoretical simulations involving only a fraction of the sky, the result obtained by integrating over the solid angle \( \Omega \) for which data is present must be multiplied by \( 4\pi/\Omega \). In our case, \( \Omega \) is given by \( \Omega = \theta_1 \theta_2 \). As a consequence, the factor \( \theta_1 \theta_2 \) in (15) is replaced by \( 4\pi \).
FIG. 2: The angular power spectrum $\sqrt{l(l+1)C_l/2\pi}$ of B-mode polarization (in units of $Q_{\text{quad}}$) for a scaling solution of cosmic strings, for a value of $G\mu = 10^{-7}$, for $c_1 = 1$, $v_s = 1$. The vertical axis is $\sqrt{l(l+1)C_l/2\pi}$, the horizontal axis is $l$. Both axes are logarithmic. We have normalized to the number $n_w$ of wakes per Hubble volume.

First we show that the angular correlation functions of E and B mode polarization are the same

$$C_l^{BB} = C_l^{EE},$$

and that the cross-correlation function between E and B mode polarization vanishes

$$C_l^{EB} = 0.$$  \hspace{1cm} (26)

To see this, we make use of the random orientation of the string motion relative to the CMB quadrupole to write

$$Q = P\cos(2\alpha)$$

$$U = P\sin(2\alpha),$$

where $\alpha$ is a random angle. Inserting (27) into (13) and the result of that into the expressions (23) and (24) we find that upon averaging over $\alpha$ the expressions for the E and B mode polarization power spectra are the same, and that the cross-correlation function vanishes. More specifically, we obtain the following expression for the contribution of a string wake created at time $t_i$ and intersecting the past light cone at time $t$

$$C_l^{EE}(t, t_i) = C_l^{BB}(t, t_i) = \frac{1}{4\pi} \int_0^{2\pi} |\tilde{P}(t, t_i)|^2 d\phi$$ \hspace{1cm} (28)

where $\tilde{P}(t, t_i)$ is given by (22).

The final power spectrum of B and E mode polarization is obtained by inserting (28) and (18) into (14) and performing the double sum. It is straightforward to see that (even before taking into account the time dependence of $f(t)$) the sum is dominated by the earliest times $t_i$ and by the earliest times $t$ consistent with $t > t_{\text{rec}}$. The time dependence of $f(t)$ further increases the importance of wakes with early $t$ relative to those with later $t$. The sum in (14) was computed numerically, taking into account the time dependence of $f(t)$. The results are shown in Figures 2 and 3. Figure 2 shows $\sqrt{l(l+1)C_l/(2\pi)}$, Figure 3 $C_l$ alone (showing the large angle plateau).

The amplitude of the angular power spectrum can be estimated analytically. Combining the formulas for the ingredients in (14) and restricting the sum to the dominant wakes, those with $t_i = t_{\text{eq}}$ and $t = t_{\text{rec}}$, we find

$$C_l^{BB} \sim N_{\text{eq}}^{-2} P^2(t_{\text{eq}}, t_{\text{rec}}).$$

Inserting the value of the local polarization of the dominant wakes from (11) for the value $G\mu = 10^{-7}$ we obtain an amplitude of the order $10^{-11} Q_{\text{quad}}$ \hspace{1cm} (29)

which agrees well with the numerical results. Note that the amplitude of $\sqrt{l(l+1)C_l}$ is linear in $G\mu$.

Since it is wakes created at time $t_{\text{eq}}$ intersecting the past light cone at time $t = t_{\text{rec}}$ which dominate the angular correlation function, the position of the peak in the angular power spectrum in Figure 2 corresponds to the angular size of these dominant wakes. From the analytical formula (22), it follows that in the low $l$ limit, the amplitude of $C_l$ is constant. This explains the slope of the power spectrum of $C_l$ at small values of $l$. The decrease in the angular power spectrum for values of $l$ larger than that corresponding to the peak position can be argued for from the Riemann-Lebesgue lemma. However,

We have used the value $\Omega_{\text{B}} = 0.022$.\hspace{1cm} (22)
IV. DISCUSSION AND CONCLUSIONS

We have computed the power spectrum of E and B mode polarization produced by a scaling distribution of cosmic strings. As already realized in [1], cosmic strings produce B mode polarization at leading order.

We find that the contribution to the E and B mode power spectra is dominated by the earliest wakes (those created at around $t_{eq}$, and those which are intersected by our past light cone closest to the time $t_{rec}$ of last scattering. For values of the string tension $G\mu = 10^{-7}$ close to the current observational bound, the predicted polarization signal in position space has an amplitude $P/Q_{quad} \sim \Omega_B$ [1]. Cosmic string wakes, however, correspond to a very non-Gaussian distribution of density enhancements. Hence, the string signal in the power spectrum is greatly suppressed. For $G\mu = 10^{-7}$ we find an amplitude of the power spectrum $\sqrt{l(l+1)C_l} P/Q_{quad}$ which is (for values of $l$ smaller than that corresponding to the peak position) of order $l_{eq}^{-3/2} \Omega_B$, much smaller than the position space signal, but only one order of magnitude smaller than the predicted B-mode signal from gravitational lensing produced by the dominant Gaussian fluctuations. A rough way of understanding the suppression of the signal in the power spectrum compared to the signal in position space is to realize that the string wakes which dominated the polarization signal occupy a small fraction of space, and that hence a suppression by this factor is to be expected. The magnitude of the polarization signal scales linearly in $G\mu$.

Note that the power spectrum of B mode polarization from string wakes has the same shape as that of the lens-
We thank Gil Holder for discussions on this point.

M. B. Hindmarsh and T. W. B. Kibble, “Cosmic strings and radiation. The slope of the lensing maps. The slope of the lensing signal on large angular scales can be understood via the Poisson distribution of the small but dominant lensing kicks. The peak position of the lensing signal is related to the scale where the matter power spectrum turns over and hence is comparable to the peak position of the string signal.

The fact that the amplitude of the angular power spectrum of string-induced B-mode polarization is (for a value of $G\mu = 10^{-7}$) an order of magnitude smaller than that of the B-mode polarization induced by lensing will make it hard to see the string-induced signal in Fourier space. However, the fact that the position space signal has a specific geometry will make it easy to detect the string signal above the lensing noise using a position space analysis of the lensing maps.

We conclude that searches for cosmic strings in B-mode polarization must be done in position space. We must search for the distinctive geometrical patterns on the sky which string wakes predict. For example, one could use edge detection algorithms like the Canny algorithm to search for the distinctive edges in the B mode sky produced by string wakes, in a similar way that this algorithm was used to search for cosmic string wake signals in CMB temperature maps. The studies of in fact showed that the string signals can be dug out of a Gaussian noise with a much larger amplitude. In current work, we are studying the prospects for the application of the Canny algorithm to polarization maps.

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