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Evaluation of linear and nonlinear system models in hierarchical model predictive control of HVAC systems

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Abstract. Buildings are responsible for one third of the global final energy consumption. Model predictive control (MPC) can reduce their energy consumption and improve thermal comfort. However, designing the required models can be time consuming. Splitting the control problem into smaller subproblems could make the modeling process more modular and therefore cheaper. A hierarchical MPC structure is proposed in this work, where the building model is divided into a lower layer consisting of the producer side and an upper layer consisting of the consumers. Linear and non-linear model equations as well as a cost-based and a control quality-based cost function for a building energy system are developed. In a simulation, the nonlinear controller outperforms the linear controller in both constraint satisfaction and energy costs.

1. Introduction
To combat global warming, increased energy efficiency and a reduction of CO₂ emissions are needed. Despite their large contribution to the global energy consumption, most buildings today adopt simple rule-based control strategies with limited energy saving capabilities [1]. Nowadays, advanced control strategies are becoming increasingly accessible, due to improvements in IT infrastructure and available computing resources. Recent work demonstrates that model predictive control (MPC) can offer energy savings between 15 % and 50 % compared to conventional control techniques, while improving on thermal comfort [1, 2]. In the model predictive control algorithm, the optimal control input of the system is computed by solving a trajectory optimization problem over a set prediction horizon. Contrary to classical controllers, MPC is able to act optimally in anticipation of future predictions.

However, practical implementation of system-wide MPC in building energy systems (BES) is challenging:

- A control model needs to be developed for the entire BES, including components for energy generation and distribution, as well as thermal zones. As every building is unique, this requires costly modeling for each building.
- Considering the dynamics of an entire building may result in a prohibitively large non-linear optimization problem.

By using a hierarchical approach, model development of smaller subsystems can be handled separately, lessening the aforementioned problems. According to Scattolini [3], different forms
of hierarchical MPC for multi layer systems can be distinguished. They can be divided into control of multi time scale systems, systems with an innate hierarchical structure and hierarchical control for plant-wide optimization. The latter two approaches can be used to split a large optimization model into simpler submodels. Rawlings et al. used hierarchical control for plant-wide optimization to control a large campus of 25 buildings [4]. Herrera et al. proposed an algorithm for systems with an innate hierarchical structure, where multiple consumers provide a selection of demand profiles to a producer, which in turn has to select the one with the lowest cost [5]. They considered nonlinear hybrid dynamics of the system, giving a glimpse into how such a hierarchical approach could work in practice.

In this work, we will examine a system that is divided into one producer system and multiple consumer systems. First, the consumer systems determine their optimal consumption profiles. Then, the producer calculates the optimal control sequence to satisfy the consumer systems demands while minimizing energy costs. Unlike Herrera et al., we only consider continuous dynamics. As a result, we can obtain optimal solutions with longer prediction horizons, allowing us to better exploit the inertia of the thermal capacities within the examined BES.

Generally, simplifying the control structure with a hierarchical approach instead of solving the monolithic optimization problem will at best result in an equal solution to the original problem. The goal of this work is to compare the performance of a hierarchical MPC to its corresponding central MPC and determine the influence of the model type on the control quality. To test the suitability of these methods for different control tasks, a simple tracking MPC and an economic MPC are implemented. Their performance is evaluated based on quality of control, and in the economic case, energy costs.

The remainder of this paper is structured as follows: In section 2, the controlled system is presented. Then, the control algorithm is outlined in section 3. In section 4, the results of the numerical simulations are presented. Finally, an evaluation of the results and an outlook are given in section 5.

2. Controlled plant

The controlled building system consists of a stratified storage tank, three different heat producers (boiler, heat pump, CHP) and two thermal zones each equipped with a heating system. It is modeled with Modelica, using AixLib [6]. The model used by the controller is described by a nonlinear continuous differential equation of the form

\[ \dot{X} = f(X, U, D). \] (1)

Here, \( X = [T_{stor,h}, T_{stor,c}, T_{u,1}, T_{u,2}, T_{x,1}, T_{x,2}, T_{w,1}, T_{w,2}]^T \) is the vector of states, consisting of the upper and lower storage temperatures, and the temperatures of the underfloor heating, zone interior and exterior wall for the two consumers. The vector of controlled inputs \( U = [u_b, u_h, u_c, p_1, p_2]^T \) consists of the relative power set points for the boiler \( u_b \), heat pump \( u_h \) and CHP \( u_c \), and the relative pump speeds for the two heating systems \( p_1 \) and \( p_2 \). Finally, \( D = [T_{amb}] \) is the vector of uncontrolled inputs, consisting of the ambient temperature. For hierarchical control, the system is split at the boundary between the room and the heating unit as shown in figure 1, resulting in a producer side and \( n \) consumers. As a result, the system formulations for the producer \( P \) and the consumers \( s \) read

\[ \dot{X}_s = f_s(X_s, U_s, D_s, V_s) \] (2)
\[ \dot{X}_P = f_P(X_P, U_P, D_P, V), \] (3)

where \( V = [V_1, ..., V_n]^T \) with \( V_s = [T_{u,s}, \dot{Q}_{u,s}] \) is the vector of shared variables between systems \( s \) and \( P \), consisting of the underfloor heating temperature \( T_{u,s} \), and the heat flow rate \( \dot{Q}_{u,s} \) between the heating and the room.
3. Control formulation

To find the optimal control input, the model predictive controller minimizes the cost function $\Phi$ subject to the system dynamics (Eq. (1) - (3)) and box constraints on the system states $X$ and controlled inputs $U$

$$X_{\text{min}} \leq X \leq X_{\text{max}}$$
$$U_{\text{min}} \leq U \leq U_{\text{max}}.$$

The resulting trajectory optimization problem is discretized with the direct collocation method, using a third order Legendre-Polynomial. The optimization code is written in Python using the CasADi framework [7] and the nonlinear solver IPOPT [8].

3.1. Centralized control problem

Two different objective functions are examined. As a simple base case, the tracking formulation serves to confirm the ability of the controller to stabilize the system around a constant set point.

The cost function in the tracking formulation $\Phi_{\text{tr}}$ is of the form

$$\Phi_{\text{tr}} = \sum_{i=k}^{k+N-1} \int_{t_i}^{t_{i+1}} \left[ \sum_{s=1}^{n} q_s \left( T_{z,s}(t) - T_{z,s}^{\text{ref}}(t) \right)^2 + \sum_{j=1}^{m} r_j u_j^2(t) \right] dt,$$

where $k$ is the current control step, $N$ is the prediction horizon, $t$ is the time, and $T_{z,s}^{\text{ref}}$ is the reference trajectory for the room temperature. The $n$ consumer systems are indexed by $s$, $m$ is the number of different heat producers, $u_j$ is the actuation of producer $j$, and $q_s$ and $r_j$ are weighting factors for the control offset and control effort respectively.

Instead of tracking a target value, the economic formulation enforces the comfort requirements through a soft constraint on the room temperature, introducing a slack variable $\varepsilon_s$ for each consumer

$$T_{z,s}^{\text{min}} \leq T_{z,s} + \varepsilon_s \leq T_{z,s}^{\text{max}}.$$

The usage of this slack variable and the energy costs of the producers are penalized in the cost function. The resulting cost function for the economic MPC $\Phi_{\text{ec}}$ takes the form

$$\Phi_{\text{ec}} = \sum_{i=k}^{k+N-1} \int_{t_i}^{t_{i+1}} \left[ \sum_{s=1}^{n} q_s \varepsilon_s^2(t) + \sum_{j=1}^{m} f_{r,j}(u_j(t)) \right] dt,$$

where $f_{r,j}$ is a function representing the energy costs of producer $j$ with regard to the actuation $u_j$. For the linear MPC, all quadratic terms in the cost function are kept, while nonlinear cost function terms and system dynamics are linearized at a fixed linearization point. The resulting quadratic optimization problem is solved with the same methods as the nonlinear optimization problem.

3.2. Hierarchical problem

The structure of the hierarchical algorithm is illustrated in figure 1. First, the consumer systems in the upper layer solve their local optimization problems. The cost functions for the consumers $s$ for the tracking formulation and economic formulation are

$$\Phi_{\text{tr},s} = \sum_{i=k}^{k+N-1} \int_{t_i}^{t_{i+1}} \left[ \sum_{s=1}^{n} q_s \left( T_{z,s}(t) - T_{z,s}^{\text{ref}}(t) \right)^2 \right] dt,$$

$$\Phi_{\text{ec},s} = \sum_{i=k}^{k+N-1} \int_{t_i}^{t_{i+1}} \left[ \sum_{s=1}^{n} q_s \varepsilon_s^2(t) + \rho_s \left( T_{u,s}(t) - T_{z,s}(t) \right)^2 \right] dt,$$
respectively. In the economic formulation, a quadratic term incentivizing lower energy usage is added. It consists of the underfloor heating temperature $T_{u,s}$, the room temperature $T_{z,s}$ and the small penalty factor $\rho_s$. Then, the consumers send the optimal trajectories of the shared variables to the producer system. They are incorporated into the cost function of the producer system through a quadratic coupling term

$$\Delta V(t) = \sum_{s=1}^{n} \left[ q_{T,s}(T_{u,s}(t) - T_{u,s}^*)(t))^2 + q_{Q,s}(\dot{Q}_{u,s}(t) - \dot{Q}_{u,s}^*)(t))^2 \right].$$

(11)

It ensures the coupling variables in the producer system $T_{u,s}$ and $\dot{Q}_{u,s}$ follow their optimal trajectories as determined by each consumer. The •* superscript denotes optimal values from the consumers, and $q_{T,s}$ and $q_{Q,s}$ are weighting factors. The resulting producer cost functions for the tracking and economic MPC read

$$\Phi_{tr,P} = \sum_{i=k}^{k+N-1} \int_{t_i}^{t_{i+1}} \left[ \sum_{j=1}^{m} r_j u_j^2(t) + \Delta V(t) \right] dt$$

(12)

$$\Phi_{ec,P} = \sum_{i=k}^{k+N-1} \int_{t_i}^{t_{i+1}} \left[ \sum_{j=1}^{m} f_{r,j}(u_j(t)) + \Delta V(t) \right] dt.$$  

(13)

![Figure 1. Illustration of the hierarchical algorithm on the studied energy system.](image)

### 4. Results

Simulations were performed over a period of 8 days in winter with real data for the ambient temperature. The controller always has perfect knowledge of the states and disturbances. The simulations are performed with a control horizon $N = 48$ with a time step $\Delta t = 1800 \text{ s}$, resulting in a 24 h prediction length. The linearization points are set based on the results of the nonlinear MPC. Figures 2 and 3 show a 24 h window of the simulations, after the system has settled. Figure 2 shows the temperature of one of the rooms using the tracking control objective, with
the reference room temperature trajectory set to 21°C. The solid lines show the temperature with the central linear (grey) and nonlinear (red) MPC, while the dash-dotted lines show the hierarchical linear (light red) and nonlinear (dark red) MPC. It can be seen that the controllers are able to stabilize the system. Only the linear hierarchical MPC shows some chattering. Figure 3 shows the case using the economic control objective. The lower bound on the room temperature is set to 21°C between 8am and 8pm and 17°C otherwise. The energy usage penalty $\rho_s$ for each consumer should be chosen with the weight on the comfort violation $q_s$ in mind. Here, we chose a value of 0.001 $q_s$. Setting this parameter too high will provoke comfort violations, while setting it too low increases the energy cost. Slight differences can be seen between all four variations, however the nonlinear controllers show significantly fewer constraint violations than the linear variants.

![Nonlinear hierarchical vs. Linear hierarchical](image1)

**Figure 2.** Room temperature over one day with the tracking objective.

![Nonlinear central vs. Linear central](image2)

**Figure 3.** Room temperature over one day with the economic objective. The dashed black line shows the lower soft boundary.

Figure 4 shows the energy costs and root mean squared constraint violation (RMSE) in the cases with an economic MPC. Looking at the graph on the left, one can see that the RMSE of the comfort violations is below 0.5 K with all controllers. However, the precision of the controller increases noticeably with nonlinear models. On the right, the energy costs in EUR per hour can be seen. Notably, the linear controllers have more than 60% higher energy costs than their nonlinear counterparts. With the underlying model assumptions, the CHP is the cheapest heat producer, while the air source heat pump operates under unfavorable conditions. Therefore, the controllers are strongly rewarded for planning ahead and maximizing CHP runtime by using the storage. However, using different temperature levels of the storage forces the linear MPC to leave its linearization point. This greatly reduces model accuracy, explaining the shortcomings of the linear MPC.
5. Conclusion
A hierarchical approach to model predictive control for buildings was presented. The controller was tested on a building energy system with a stratified storage tank, two rooms and a boiler, in addition to a CHP and a heat pump as heat producers. Results were compared to a central MPC of the system. Simulations showed that the proposed algorithm achieves a similar control quality compared to the centralized MPC for both constant and variable demand profiles. Using nonlinear system models considerably improves comfort satisfaction and reduces energy costs. With the present formulation, only the computed trajectories of the shared variables need to be communicated, allowing the subsystems some degree of autonomy. Additionally, the reduced system size eases computational burden compared to the monolithic system. There are two main shortcomings of the current algorithm. First of all, real HVAC systems incorporate multiple discrete decision variables, such as on/off switches, which were not considered in the present study. Additionally, the hierarchical algorithm loses the information flow from the producer to the consumer. This would impede advanced demand side management techniques, such as using the thermal inertia of the building envelope to shift loads. Addressing these challenges will be the topic of future work.

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