Infrared Gluon and Ghost Propagator Exponents From Lattice QCD

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The compatibility of the pure power law infrared solution of QCD and lattice data for the gluon and ghost propagators in Landau gauge is discussed. For the gluon propagator, the lattice data is well described by a pure power law with an infrared exponent $\kappa \sim 0.53$, in the Dyson-Schwinger notation. $\kappa$ is measured using a technique that suppresses finite volume effects. This value implies a vanishing zero momentum gluon propagator, in agreement with the Gribov-Zwanziger confinement scenario. For the ghost propagator, the lattice data seem not to follow a pure power law, at least for the range of momenta accessed in our simulation.

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I. INTRODUCTION AND MOTIVATION

The infrared properties of the Landau gauge gluon and ghost propagators in momentum space, respectively,

\[
D^{ab}_{\mu\nu} (q) = \delta^{ab} \left( \delta_{\mu\nu} - \frac{q_{\mu} q_{\nu}}{q^2} \right) D(q^2),
\]

\[
G^{ab} (q) = -\delta^{ab} G(q^2),
\]

are connected with gluon confinement mechanisms, namely the Kugo-Ojima scenario (KO) \cite{1} and the Gribov-Zwanziger horizon condition (GZ) \cite{2,3}. The GZ mechanism requires $D(0) = 0$ (which implies maximal violation of reflection positivity) and an enhanced ghost propagator, relative to the perturbative function. The KO confinement mechanism demands $1/(q^2 G(q^2)) = 0$ in the limit $q \to 0$. From the point of view of the KO and GZ confinement mechanisms, the requirements on $D(0)$ and $G(0)$ are necessary conditions and its violation immediately rules out these scenarios.

In the recent years there has been a renewed interest in the computation of gluon and ghost propagators in the pure gauge theory, due to progress on solutions of the Dyson-Schwinger equations (DSE) and lattice simulations which explore further the infrared region.

The DSE are an infinite tower of coupled nonlinear equations for the QCD Green’s functions. The computation of the gluon and ghost propagators within DSE requires defining a truncation scheme and parametrizing some of the QCD vertices. For the Landau gauge, recent solutions can be classified in two categories: solutions which are not compatible with the KO or GZ confining mechanism \cite{4,5,6,7} and solutions which do not rule out the two confining mechanisms \cite{8,9,10,11}. For the first class of solutions, $D(0)$ is finite and does not vanish. For the later class of solutions, in \cite{12} an analytical solution was found for the deep infrared region \cite{13}. The solution assumes infrared ghost dominance and connects the two propagators via a single exponent, $\kappa$,

\[
Z(q^2) = q^2 D(q^2) = \omega \left( \frac{q^2}{\sigma^2} \right)^{2\kappa},
\]

\[
F(q^2) = q^2 G(q^2) = \omega' \left( \frac{q^2}{\sigma^2} \right)^{-\kappa};
\]

$\sigma$ is a constant with dimension of mass. Moreover, DSE equations predict $\kappa = 0.595$, which, for the zero momentum, implies a null (infinite) gluon (ghost) propagator. Furthermore, renormalization group analysis \cite{14,15} restrict the possible values for $\kappa$ to $0.52 \leq \kappa \leq 0.595$. This result suggests a null (infinite) zero momentum gluon (ghost) propagator. In \cite{16} it was argued that the solution \cite{17-19} is the unique power law infrared solution compatible with DSE and functional renormalization group equations. In \cite{17} a similar analysis of the DSE within time-independent stochastic quantisation predicted the same behaviour and $\kappa = 0.52145$.

The computer simulations of 4D pure SU(3) and SU(2) gauge theories on a lattice \cite{20,21,22,23,24,25} show a finite non-vanishing zero momentum gluon propagator. Although $D(0)$ is a decreasing function of the lattice volume, from naive extrapolations to the infinite volume \cite{26,27,28}, it is not clear if the lattice result becomes compatible with the GZ confinement mechanism, i.e. if $D(0)$ approaches zero as $V \to \infty$. Moreover, recent 4D lattice QCD calculations of the gluon propagator on large volumes, $27$ fm$^4$ for SU(2) \cite{29}, and $13$ fm$^4$ for SU(3) \cite{29,30}, show a propagator which goes to a constant in the infrared limit, although not excluding a gluon propagator tending towards zero. However, to work out such huge lattice volumes, the simulations were carried out with the Wilson action and using a relatively large value for the lattice spacing, $a \sim 0.21$ fm for SU(2) and $a \sim 0.165$ for SU(3). It is not clear yet whether this have any influence on such propagator.

Recently, in \cite{30} rigorous upper and lower bounds for the zero-momentum gluon propagator were derived and its scaling behaviour with the volume analyzed for the SU(2) Yang-Mills theory. The authors found a finite and non-vanishing $D(0)$ for SU(2) in the infinite volume.
limit. In [30], the same analysis was performed but for the SU(3) Yang-Mills theory. It turns out that for SU(3), the infinite volume limit gives a $D(0) = 0$. The different behaviours in the infinite volume limit is a puzzling result and, presently, we have no understanding for such a difference.

In what concerns the lattice ghost propagator [20, 21, 22, 23, 31, 34, 35], the simulations show an enhanced propagator but not in agreement with that predicted by the solution [4]. The lattice data seems to be closer to the solution of [7]. Indeed, the lattice data shows a propagator which is almost identical to the perturbative propagator.

In order to try to understand the difference between the DSE solution which compatible with the GZ and KO confinement mechanisms, in [40] the DSE were solved on a 4D symmetric torus. According to the authors, the gluon and ghost propagators approach slowly the infinite volume value. Moreover, they claim that to observe the suppression of the gluon propagator one should go to volumes as large as $(10 \text{ fm})^4$. Note that recent lattice simulations [24, 25] have volumes well above the $(10 \text{ fm})^4$.

In [21, 27, 32, 35, 36] we have tried to measure both the gluon and ghost propagators using a set of large asymmetric lattices, i.e. $L^3 \times T$ with $T$ larger than $L$, to access the deep infrared region and to check the compatibility between the lattice data and the solution [3-4]. The lattices used are larger than 10 fm, by a factor of $\sim 2.5$, in the temporal direction and are much shorter, by a factor of $\sim 1/5$, in the spatial directions. For the $L^3 \times T$ lattices, besides the finite volume effects also observed in simulations with symmetric $L^4$ lattices, one has to care about how the asymmetry changes the gluon and ghost propagators data.

The finite lattice effects in $D(q^2)$ and $G(q^2)$ in the asymmetric lattices are, qualitatively, equal to the effects observed in the solutions of the DSE on a symmetric 4D torus [40] and on the simulation of 3D asymmetric SU(2) lattices [41].

For the gluon propagator, in [39], using the lattice dressing function $q^2 D(q^2)$, and excluding the zero momentum point from the analysis, we have demonstrated that the gluon lattice data is compatible with [3]. We would like to call the reader attention that we started by simulating $16^3 \times 128$ and $16^3 \times 256$ lattices for $\beta = 6.0$. For these two lattices, it turned out that the $16^3 \times 128$ data cannot be described by a pure power law. Only the fits to the $16^3 \times 256$ gluon data have acceptable $\chi^2 / \text{d.o.f.}$, i.e. $\chi^2 / \text{d.o.f.} < 2$. Furthermore, an attempt to extrapolate the lattice to infinite spatial volume suggests a $\kappa$ in the range 0.498 to 0.525. Despite this result, which gives some support to the KO and GZ confinement scenarios, before extrapolations the lattice data shows no suppression of the gluon propagator for small momenta, except for $D(0)$ when compared to the first non-zero momentum (see figure [1]).

In what concerns the ghost propagator computed using asymmetric lattices [21, 22], we have observed that $G(q^2)$ is enhanced, compared to the perturbative solution, in the infrared region (see figure [1]), but not as much as predicted by the DSE solution [3].

In this work we discuss a method which, assuming a pure power law behavior for the infrared propagators, aims to measure the infrared exponents without relying on data extrapolation. The measure relies on the definition of a convenient ratio of propagators which, in principle, suppresses the finite volume effects. The results reported show that, for the same asymmetric lattices used previously, the method provides estimates of the gluon propagator exponent which are stable against variation of the range of momenta and variation of the spatial lattice extent $L$. Furthermore, the measured $\kappa$ are compatible with the GZ confinement scenario, i.e. they predict a vanishing zero momentum gluon propagator. On the other hand, the data for the ghost propagator, although stable against variation of the lattice volume, is not compatible with a pure power law behaviour.

The paper is organized as follows. In section [II] we report on the asymmetric lattices used to investigate the infrared gluon and ghost propagators. In section [III] the method used to measure the infrared exponents is discussed together with the results for the gluon propagator. The section also includes a discussion of the finite volume and asymmetry effects. In section [IV] the method is applied to the ghost propagator and finally in section [V] we draw the conclusions.

### II. DETAILS OF THE SIMULATIONS

In this article we use Wilson action, $\beta = 6.0$, gauge configurations for the lattices reported in table [I]. The asymmetric lattices have a temporal extension of $T = 256$ ($\sim 26$ fm) which allow us to access momenta as low as 48 MeV. The main difference to [39] being the larger statistics for the largest lattices.

The propagators were computed in the minimal Landau gauge and the gauge fixing was performed using a Fourier accelerated steepest descent algorithm; see [39] for details and definitions.

In the following, we will discuss the infrared propagators and will consider only time-like momenta, defined

| Lattice | Therm. | Sep. | # Conf. |
|---------|--------|------|---------|
| $8^3 \times 256$ | 1500 | 1000 | 80 |
| $10^3 \times 256$ | 1500 | 1000 | 80 |
| $12^3 \times 256$ | 1500 | 1000 | 80 |
| $14^3 \times 256$ | 3000 | 1000 | 128 |
| $16^3 \times 256$ | 3000 | 1500 | 155 |
| $18^3 \times 256$ | 2000 | 1000 | 150 |
as
\[ q[n] = q_4[n] = \frac{2}{a} \sin \left( \frac{\pi n}{T} \right), \quad n = 0, 1, \ldots \frac{T}{2}, \]
where \( T \) is the time lattice extent. For the conversion to physical units we use \( a^{-1} = 1.943(47) \text{ GeV} \). The ghost propagator was computed with the method described in [43], for the smallest \( D \). For the gluon
\[ G \]
ghost propagators are reported in figure 1.

The gluon data for \( R_Z[n] \) as a function of \( R_g[n] \), see figure 2, shows a linear behavior for a surprising large range of momenta for all the lattices \( 8^3 \times 12^4 \times 256 \) and 1800 bootstrap samples for \( 14^3 \times 18^4 \times 256 \) lattices.

we get, for the gluon propagator,
\[ R_Z[n] = 2\kappa R_g[n]. \]

The gluon data for \( R_Z[n] \) as a function of \( R_g[n] \), see figure 2, shows a linear behavior for a surprising large range of momenta for all the lattices \( 8^3 \times 12^4 \times 256 \) and 1800 bootstrap samples for \( 14^3 \times 18^4 \times 256 \) lattices. Moreover, the slopes seem to be similar for all lattices. It seems that the corrections to \( \kappa \) are compatible with the ratios show up as a constant which adds up to \( (5) \). This hypothesis can be tested fitting the ratios to
\[ R_Z[n] = 2\kappa R_g[n] + C, \]
assuming that \( C \) is a constant. The fits of the lattice ratios to \( (5) \) using momenta up to \( \sim 400 \text{ MeV} \) are reported in table 11.

III. THE GLUON PROPAGATOR

For the measurement of the infrared exponents, it will be assumed that the lattice dressing functions \( Z(q^2) \) and \( F(q^2) \) are described by pure power laws \( (q^2)^\kappa \), as in \( (6) \) and (11), times a factor \( \Delta(q) \) which summarises the finite volume corrections and/or deviations from the pure power law. If these corrections are constant (small), they are eliminated (suppressed) by taking ratios of \( Z \) and \( F \) at consecutive lattice momenta, i.e.
\[ \ln \left[ \frac{G(q^2[n+1])}{G(q^2[n])} \right] = \alpha \ln \left[ \frac{q^2[n+1]}{q^2[n]} \right] + \cdots, \]
where for the gluon \( G(q^2) = Z(q^2) \) and, according to \( (6) \), \( \alpha = 2\kappa \). For the ghost \( G(q^2) = F(q^2) \) and \( \alpha = -\kappa \). In the infrared one expects that corrections to \( (6) \) are subleading. Defining
\[ R_Z[n] = \ln \left[ \frac{Z(q^2[n+1])}{Z(q^2[n])} \right], \]
\[ R_g[n] = \ln \left[ \frac{q^2[n+1]}{q^2[n]} \right], \]

The measured \( \kappa \)'s are stable against variation of the fitting range and spatial lattice size. Note that the \( \kappa \) measured with the smallest lattice \( 8^3 \times 256 \) is compatible with the \( \kappa \) measured using our largest volume \( 18^3 \times 256 \). Moreover, the central values for \( \kappa \) are clearly above 0.5 and typically, within one standard deviation, \( \kappa \geq 0.5 \). In particular, for the three largest volumes \( 14^3 \times 18^3 \times 256 \), which have the largest statistics, \( \kappa > 0.5 \) within one standard deviation. Statistical errors on \( \kappa \) decrease as the fitting range increases, because one is using a larger set of data. For the largest fitting range \( \sim 381 \text{ MeV} \) and for the three largest lattices, the \( \kappa \) is compatible with 0.5 only within 4 standard deviations. In this sense, in what concerns the infrared gluon propagator, the fits to \( (9) \) point towards \( \kappa \sim 0.53 \), suggesting a vanishing gluon propagator at zero momentum.

Note that the absolute value of the constant \( C \), in general, approaches zero as the lattice volume increases (see

FIG. 1: Bare gluon and ghost propagators for time like momenta. Note the logarithmic scale for the ghost propagator.

FIG. 2: Ratios of gluon (left) and ghost (right) dressing functions. The statistical errors were computed using 1000 bootstrap samples for \( 8^3 \times 12^4 \times 256 \) lattices and 1800 bootstrap samples for \( 14^3 \times 18^4 \times 256 \) lattices.
A fit of the ratios computed using the $16^3 \times 128$ data to \( \chi^2/d.o.f. = 0.01 \) with a \( \chi^2/d.o.f. = 0.01 \). The \( \kappa \) value is in excellent agreement with the corresponding figure for $16^3 \times 256$. For the \( C \) figures, it comes that $C_{128} \approx 2 \times C_{256}$. This result reinforces our interpretation of \( C \) as a measure of the finite volume corrections to the pure power law.

In what concerns the symmetric lattice data in figure \( 4 \) despite their relative large smallest momentum, in the infrared the slope follows essentially the slope of the asymmetric ratios. The reader should be aware that, for each of the symmetric lattices, one can estimate a \( \kappa \) directly from the two most right points which gives \( \kappa \approx 0.6 \) or higher. The main difference seems to be in the constant \( C \), which takes larger values for smaller lattices. In particular, note that \( \beta = 6.2, 64^4 \) and \( \beta = 6.0, 48^4 \) have essentially the same physical volume and their ratio data is, within statistical errors, indistinguishable.

Given the results summarized in figure \( 4 \) one can conclude that the ratios defined in (7) provide reliable estimates of the gluon infrared exponent, i.e. the estimates seem to be independent of the lattice volume and asymmetry.

IV. THE GHOST PROPAGATOR

In what concerns the ghost propagator, the data for the ratios of the dressing functions, see figure \( 2 \) do not show a linear behaviour as in the case of the gluon propagator. We have tried a number of functional forms to fit the data, but their \( \chi^2/d.o.f. \) was always too large. At most, the slope of ratios of the ghost dressing function suggests a negative value for the ghost infrared exponent.

In figure \( 5 \) we show the ratios of the ghost dressing functions for the three larger lattices, including the
TABLE II: Fitting the gluon ratios with equation \( \text{(10)} \) for \( L^3 \times 256 \) lattices. The first line is the maximum momentum used in the fit. \( \chi^2 \) stands for \( \chi^2/d.o.f. \). The errors in \( \kappa \) are statistical and were computed with the bootstrap method.

| \( q_{\text{max}} \) \( \times 10^3 \) MeV | 191 MeV | 238 MeV | 268 MeV | 286 MeV | 333 MeV | 381 MeV |
|---|---|---|---|---|---|---|
| \( L \) | \( \kappa \) | \( \chi^2 \) | \( \kappa \) | \( \chi^2 \) | \( \kappa \) | \( \chi^2 \) | \( \kappa \) | \( \chi^2 \) | \( \kappa \) | \( \chi^2 \) | \( \kappa \) | \( \chi^2 \) |
| 8 | \( \kappa \) | 0.526(27) | 0.12 | 0.531(19) | 0.11 | 0.531(13) | 0.08 | 0.522(16) | 0.48 | 0.527(12) | 0.54 |
| \( C \) | 0.179(54) | | \( -0.194(34) \) | | \( -0.193(19) \) | | \( -0.171(28) \) | | \( -0.184(18) \) | |
| 10 | \( \kappa \) | 0.511(35) | 0.69 | 0.525(14) | 0.98 | 0.525(21) | 0.74 | 0.523(17) | 0.56 | 0.527(16) | 0.50 |
| \( C \) | -0.114(66) | | \( -0.161(42) \) | | \( -0.146(30) \) | | \( -0.144(21) \) | | \( -0.150(19) \) | |
| 12 | \( \kappa \) | 0.509(31) | 0.11 | 0.517(21) | 0.16 | 0.508(18) | 0.33 | 0.521(18) | 0.84 | 0.530(14) | 1.03 |
| \( C \) | -0.094(56) | | \( -0.112(35) \) | | \( -0.094(25) \) | | \( -0.119(27) \) | | \( -0.138(18) \) | |
| 14 | \( \kappa \) | 0.536(24) | 0.33 | 0.540(19) | 0.20 | 0.548(16) | 0.39 | 0.545(12) | 0.34 | 0.542(11) | 0.34 |
| \( C \) | -0.114(44) | | \( -0.123(30) \) | | \( -0.140(21) \) | | \( -0.134(15) \) | | \( -0.127(12) \) | |
| 16 | \( \kappa \) | 0.539(22) | 1.77 | 0.528(17) | 1.24 | 0.534(12) | 0.96 | 0.536(12) | 0.78 | 0.539(11) | 0.68 |
| \( C \) | -0.125(43) | | \( -0.102(30) \) | | \( -0.112(19) \) | | \( -0.118(14) \) | | \( -0.123(12) \) | |
| 18 | \( \kappa \) | 0.529(20) | 0.39 | 0.516(16) | 0.77 | 0.523(14) | 0.85 | 0.536(11) | 1.79 | 0.559(95) | 1.58 |
| \( C \) | -0.099(36) | | \( -0.068(25) \) | | \( -0.085(19) \) | | \( -0.111(14) \) | | \( -0.119(13) \) | |

TABLE III: Lattice setup used to check finite volume and asymmetry effects. To convert to physical units we used the central values for the lattice spacing reported in \[12\]: \( a = 0.1016 \text{ fm} \) for \( \beta = 6.0 \) and \( a = 0.0728 \text{ fm} \) for \( \beta = 6.2 \). Volume is reported in \text{fm}. \( q_{\text{min}} \) (in MeV) is the smallest non-vanishing momentum for each lattice. For symmetric lattices only on-axis momenta are considered.

| \( \beta \) | \( \text{Lat} \) | \( \text{Volume} \) | \( q_{\text{min}} \) (in MeV) | \# Conf. |
|---|---|---|---|---|
| 6.0 | \( 16^3 \times 128 \) | \( (1.63)^3 \times (13.00) \) | 95 | 164 |
| 6.0 | \( 48^4 \) | \( (4.88)^4 \) | 254 | 104 |
| 6.0 | \( 64^4 \) | \( (6.50)^4 \) | 190 | 120 |
| 6.2 | \( 64^4 \) | \( (4.66)^4 \) | 266 | 99 |

curves

\[
R_F[n] = -\kappa R_q[n] + C ,
\]

where \( \kappa = 0.529 \) and \( C \) was adjusted to reproduce the ratio computed using our smallest momenta. The figure shows that either the data is still far from the linear behaviour or the infrared ghost propagator does not follow a pure power law.

V. CONCLUSIONS

In conclusion, in this article we discuss a method which does not rely on extrapolations to the infinite volume and i) allows to check if the infrared gluon and ghost propagators follow a pure power law; ii) measure the infrared exponents if the propagator is compatible with a power law.

For the gluon propagator the lattice data validates the power law behavior and the corresponding infrared exponent is measured. From the two large asymmetry lattices and the two smallest fitting ranges we have \( \kappa = 0.529(20) \) and \( 0.539(22) \) which gives a combined value of \( \kappa = 0.535(14) \). As discussed, the ratio method devised in this paper seems to be independent of the asymmetry and lattice volume, at least for the simulations reported here. This gives confidence on our estimate for \( \kappa \). Furthermore, the reported exponent \( \kappa = 0.535(14) \) is within the range of values allowed by the renormalization group analysis of \[14, 15\]. Given the results for the ghost propagator, it is not clear to the authors if this \( \kappa \) should or not in that range. More, the measured \( \kappa \) supports a vanishing zero momentum gluon propagator in agreement with the Gribov-Zwanziger gluon confinement scenario.

The reader should note that, on a finite lattice, simulations have produced always a finite and non-zero value for

![FIG. 5: \( R_F[n] \) as a function of \( R_q[n] \) for the three largest lattices. The dash lines show, for each lattice, the curve \( -\kappa R_q[n] + C \) where \( \kappa = 0.529 \) and \( C \) adjusted to reproduce the right end point in the graph.](image-url)
the gluon propagator at zero momentum. The method discussed here do not use directly \( D(0) \). Anyway, as mentioned before, the conclusions are in perfect agreement with the scaling analysis of \( D(0) \) as a function of the volume performed in [30], which gives a \( D(0) = 0 \) in the infinite volume limit. In this sense, the results of [30] reconcile the results of the lattice simulations with those reported in this work, i.e. that in the Landau gauge the continuum result is \( D(0) = 0 \).

In what concerns the ghost propagator, the lattice data suggests that the propagator does not follow a pure power law. In this sense, our simulation does not validate the infrared solution \( \Box \) of the DSE. The observed difference between the lattice and the DSE solution could be because either the lattices used here are not long enough or the ghost propagator does not follow a pure power law in the deep infrared region. The answer to this question requires further investigations.

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**APPENDIX A: FINITE VOLUME EFFECTS ON THE INFRARED GLUON DRESSING FUNCTION; AN EFFECTIVE PARAMETERIZATION**

The results on the ratios of the gluon dressing function, discussed in section III suggest that the finite volume and asymmetry corrections are summarized in \( C \). Let \( \Delta(q) \) be the multiplicative correction to the continuum dressing function \( Z(q^2) \), i.e let us assume that the lattice dressing function is given by

\[
Z_{\text{Lat}}(q^2) = Z(q^2) \Delta(q). \tag{A1}
\]

Then, \( \Delta(q[n + 1]) = \Delta(q[n]) e^{C} \) which allows to write

\[
\frac{d\Delta(q)}{dq} \sim \frac{\Delta(q[n + 1]) - \Delta(q[n])}{q[n + 1] - q[n]} \\
\sim \Delta(q) \frac{e^{C} - 1}{q} = \Delta(q) A \tag{A2}
\]

where \( A \) is a constant. The integration of the last equation gives \( \Delta(q) = \Delta_{0} \exp(Aq) \) where \( \Delta_{0} \) is a constant of integration. The reader should note that this form for \( \Delta(q) \) is only valid in the infrared region. Computing \( \Delta(p) \) for the full range of momenta is beyond the scope of this work. Nevertheless, for high momenta, where the propagator is well described by its perturbative behavior and the lattice gluon propagator does not seem to depend on the volume, one should have \( \Delta(p) \sim 1 \). In principle, this restriction allow us to fix \( \Delta_{0} \) via the normalization condition \( \Delta(q_{h}) = 1 \) where \( q_{h} \) is a sufficiently high momentum value. The present lack of knowledge on the functional form for \( \Delta(p) \) at momenta \( q > 400 \text{ MeV} \) prevent us to compute \( \Delta_{0} \). At best, one can estimate \( \Delta_{0} \) at the highest momentum value of \( q \sim 400 \text{ MeV} \) where one can still approximate \( \Delta(q) = \Delta_{0} \exp(Aq) \). Then, \( \Delta_{0} = \exp(-Aq_{h}) \) and

\[
\Delta(q) = \Delta_{0} \exp(Aq) \sim \exp\left(A(q - q_{h})\right) \tag{A3}
\]

for the infrared region.

For practical purposes, like fitting the lattice data, \( \Delta_{0} \) can be absorbed into the definition of \( \omega \), and we get an exponential correction to \( Z(q^2) \),

\[
Z_{\text{Lat}}(q^2) = \omega (q^2)^{2\kappa} e^{Aq}, \tag{A4}
\]

with the constant \( A \) parametrizing the finite volume effects and/or the corrections to the pure power law and \( \kappa \) is the continuum exponent. The results of fitting (A4) to the lattice gluon dressing function are reported in table IV.

The \( \kappa \) values in tables II and IV are essentially the same. The good agreement between the two results can be viewed as a cross-check of the original method discussed previously and, in this sense, gives further confidence in the final result. The reader should note that, like \( C \), takes negative values and approaches zero as the lattice volume increases.

In what concerns the function \( \Delta(q) \), see eq. (A3), it provides a correction to the gluon propagator which decreases with increasing momenta, as expected if it models the finite volume effects. Indeed, \( \Delta(q = 0) \sim \exp(-Aq_{h}) \gg 1 \) and \( \Delta(q_{h}) \sim 1 \) – see figure 6 where we have corrected the propagator using \( q_{h} = 400 \text{ MeV} \). The reader should not forget that we are only estimating the corrections due to finite volume/asymmetry. Figure 6 shows a corrected gluon propagator which is supressed in the infrared region. Moreover, the corrected data shows also a good agreement between all propagators, excluding our smallest lattice, for momenta below 200 MeV.

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TABLE IV: Fitting the gluon dressing functions to eq. (A3) for $L^3 \times 256$ lattices. The first line is the maximum momentum used in the fit. $\chi^2$ stands for $\chi^2/d.o.f.$ The errors are statistical and were computed with the bootstrap method. $A$ in GeV$^{-1}$.

| $q_{\text{max}}$ | 191 MeV | 238 MeV | 286 MeV | 333 MeV | 381 MeV |
|------------------|---------|---------|---------|---------|---------|
| $L$              | $\chi^2$ | $\chi^2$ | $\chi^2$ | $\chi^2$ | $\chi^2$ |
| 8 $\kappa$       | 0.526(26) | 0.90 | 0.533(19) | 0.12 | 0.534(11) | 0.08 | 0.523(10) | 0.62 | 0.524(9) | 0.51 |
| $A$              | $-3.75 \pm 1.1$ | -4.06(68) | $-4.11(34)$ | -3.69(28) | -3.73(23) |
| 10 $\kappa$      | 0.511(27) | 0.53 | 0.536(22) | 1.08 | 0.534(17) | 0.73 | 0.531(14) | 0.58 | 0.534(13) | 0.49 |
| $A$              | $-2.3 \pm 1.1$ | -3.40(69) | $-3.33(31)$ | -3.22(37) | -3.30(29) |
| 12 $\kappa$      | 0.508(31) | 0.07 | 0.515(22) | 0.12 | 0.507(15) | 0.24 | 0.520(12) | 0.84 | 0.537(9) | 1.94 |
| $A$              | $-1.9 \pm 1.2$ | -2.25(78) | $-1.92(46)$ | -2.40(36) | -2.96(23) |
| 14 $\kappa$      | 0.538(23) | 0.24 | 0.542(18) | 0.17 | 0.552(14) | 0.47 | 0.551(11) | 0.36 | 0.546(9) | 0.45 |
| $A$              | $-2.42(87)$ | -2.62(59) | $-3.00(41)$ | -2.96(29) | -2.80(21) |
| 16 $\kappa$      | 0.541(22) | 1.15 | 0.532(16) | 0.78 | 0.535(10) | 0.55 | 0.539(9) | 0.50 | 0.543(8) | 0.54 |
| $A$              | $-2.67(84)$ | -2.29(54) | $-2.39(31)$ | -2.53(24) | -2.66(18) |
| 18 $\kappa$      | 0.529(20) | 0.28 | 0.516(15) | 0.59 | 0.523(12) | 0.54 | 0.539(9) | 2.14 | 0.550(8) | 2.71 |
| $A$              | $-2.05(79)$ | -1.50(51) | $-1.75(33)$ | -2.31(24) | -2.66(20) |

FIG. 6: Corrected and uncorrected gluon propagator using eq. (A3) with $q_h = 400$ MeV.

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