We propose a protocol for conditional quantum logic between two 4-state atoms inside a high Q optical cavity. The process detailed in this paper utilizes a direct 4-photon 2-atom resonant process and has the added advantage of commonly addressing the two atoms when they are inside the high Q optical cavity.

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Quantum information science has rapidly developed into a major theme of modern research in recent years. Various physical implementations have been proposed for studying quantum communications and, in particular, for controlling quantum logic operations between individual qubits inside a high Q optical cavity [1]. The strongly coupled cavity QED system is unique as it is among the selected few where coherent dynamics at the level of a single quanta (electron, atom, photon, or phonon) have already been observed. Furthermore, photons represent one of the best choices for quantum information distribution and communication, the prospect of inter-converting quantum information between light and matter as afforded in the cavity QED system has led to imaginations of quantum information networks in the future [2].

The first proposal for quantum computing with atoms inside a high Q optical cavity appeared in 1995 [3], when Pellizzari et al. discovered that conditional logic between two atomic qubits can be achieved with the use of the common cavity mode as a quantum data-bus. The protocol of Ref. [3] was based on adiabatic passage making use of dark state structures in the combined atom-cavity system. It is robust against both atomic and cavity decays and also immune to noise from externally applied lasers. While much progress has been made on the experimental side [4–8], the successfully implementation of this atom-cavity protocol between two atoms has yet to be achieved. The main difficulties are: 1) precisely localizing each atomic motional wave packet (this is needed to ensure the Lamb-Dicke limit); 2) obtaining a double A-type, 6-state, level diagram for each atom; and 3) individually addressing each atom during the dynamic gate operation when both atoms are inside the cavity. Recent successes in combining an ion trap with a cavity [4,5] and in realizing trapped atoms inside a cavity [6,7,9] have raised the hope that the first difficulty will be eliminated. It is therefore highly desirable to develop protocols that could potentially overcome the latter two limitations. We note a similar scenario appeared with the ion trap based quantum computing implementation [10], where the development of the commonly addressing protocol [11] has lead to the first deterministic generation of 4-atom maximally entangled state [12].

In this paper, we suggest a protocol for quantum logic between two 4-state atoms that requires only common
addressing when atoms are inside the high Q optical cavity [13]. The system of two 4-level atoms inside a single mode high Q optical cavity is described by

$$H = H_A + H_B + H_C,$$

$$H_\mu = \hbar \omega_1 |1\rangle_\mu \langle 1| + \hbar \omega_e |e\rangle_\mu \langle e| + \hbar \omega_a |a\rangle_\mu \langle a|,$$

$$+ \left[ \frac{1}{2} \hbar \Omega_\mu(t) e^{-i \omega_L t} |e\rangle_\mu \langle 1| + \hbar g_\mu \langle \vec{r}_\mu | c |e\rangle_\mu \langle a| + h.c. \right],$$

$$H_C = \hbar \omega_C c^\dagger c,$$

(1)

where $H_\mu = A/B$ and $H_C$ are respectively the Hamiltonian for atom (A/B) and for the single mode cavity. The atomic level scheme is as shown in Fig. 1. $\Omega_\mu$ is the Rabi frequency due to an external laser field ($|e\rangle \leftrightarrow |1\rangle$) at frequency $\omega_L$ and $g_\mu$ (assumed real) is the single photon coherent coupling ($|e\rangle \leftrightarrow |a\rangle$) rate with the cavity mode. Similar models were considered earlier [13–15].

![FIG. 1. Two 4-state atoms interacting with a common cavity mode field.](image)

Before presenting the physical mechanism of our protocol, we change to the interaction picture with

$$U(t) = e^{-i(\omega_1 + \omega_L) t} \sum_\mu |e\rangle_\mu \langle e| - i \hbar \Omega_\mu(t) \sum_\mu |1\rangle_\mu \langle 1|$$

$$- e^{-i \omega_a t} \sum_\mu |a\rangle_\mu \langle a| - i \hbar \omega_C - (\omega_a - \omega_1) \hbar c^\dagger c.$$

(2)

The resulting effective system dynamics is then governed by [16]

$$H_{\text{eff}} = -\hbar \Delta_L \sum_\mu |e\rangle_\mu \langle e| - \hbar \delta_C c^\dagger c$$

$$+ \sum_\mu \left[ \frac{1}{2} \hbar \Omega_\mu(t) |e\rangle_\mu \langle 1| + \hbar g_\mu |e\rangle_\mu \langle a| + h.c. \right],$$

(3)

where we have defined $\Delta_L = \omega_L - (\omega_e - \omega_1)$, $\delta_C = (\omega_L - \omega_C) - (\omega_a - \omega_1)$, and used the notations $\Delta_L = \Delta_L + i \Gamma/2$, $\delta_C = \delta_C + i \kappa$ that include the dissipative dynamics in their standard form. $\Gamma$ denotes the atomic spontaneous emission rate and $\kappa$ is the cavity decay rate (of each side). We have neglected the position dependence in the cavity coupling by assuming the Lamb-Dicke limit.

In our model, $|0\rangle$ and $|1\rangle$ are long lived atomic states, thus are ideal candidates for atomic qubits. The auxiliary
atomic state \(|a\rangle\) (also assumed to be long lived) is coupled
to the excited state \(|e\rangle\) through the single mode cavity
field. Our protocol starts with a Raman pulse (from two
classical laser fields) on atom A between states \(|1\rangle\) and
\(|a\rangle\) via \(|e\rangle\). After a time corresponds to a \(\pi\) pulse of the
effective Raman interaction, the state \(|1\rangle_A\) is mapped
onto \(|a\rangle_A\). Although this step requires the individual
addressing of atom A, it can be affected when the atom
is outside the tightly confined optical cavity.

The required conditional phase gate dynamics then read
\[
(\alpha|0\rangle + \beta|a\rangle)_A \otimes (\mu|0\rangle + \nu|1\rangle)_B \otimes |0\rangle_C
\rightarrow (\alpha|0\rangle + \beta|a\rangle)_A \otimes \mu|0\rangle_B \otimes |0\rangle_C
+ (\alpha|0\rangle - \beta|a\rangle)_A \otimes \nu|1\rangle_B \otimes |0\rangle_C,
\]
with the use of notation \(|i\rangle_A \otimes |j\rangle_B \otimes |k\rangle_C = |i,j,k\rangle\).
This conditional phase gate gives a relative phase shift of
\(\pi\) between states \(|0,1,0\rangle\) and \(|a,1,0\rangle\). Any overall phase
of the two states can be compensated for by single bit
operations on atom B [17].

When atoms A and B are in the cavity, the second step
consists of using a common laser field to drive atomic
transitions \(|1\rangle \leftrightarrow |e\rangle\). The atom-atom interaction re-
quired for conditional dynamics (4) arises from the emis-
sion and absorption of a common cavity photon. By op-
erating in the far off-resonant limit for all intermediate
one atom processes when \(|\Delta_L| \gg \Omega_\mu, |\delta_C| \gg |g_\mu|\), and
when the cavity is initially empty (of photons), we real-
ize an effective 4-photon resonant process in the following
order \(|a,1,0\rangle \rightarrow |a,e,0\rangle \rightarrow |a,a,1\rangle \rightarrow |e,a,0\rangle \rightarrow |1,a,0\rangle\).
Upon elimination of all intermediate states, we end up with an effective 4-photon 2-atom coupling between
atomic states \(|a,1\rangle\) and \(|1,a\rangle\). Naively we expect a 4-
photon Rabi frequency being \(\propto \Omega_A \Omega_B/(4\Delta_L^2) \times g_{AB}/\delta_C\)
when \(|\Delta_L| \gg |g_\mu^2/\delta_C|\) is also satisfied. Therefore, by driv-
ing a \(2\pi\) pulse on this 4-photon resonant transition, we
gain a factor of \((-1)\) in front of the state \(\beta\nu|a,1\rangle\). As
long as there is a large detuning \(\delta_C\), no other real trans-
itions are possible. Hence the conditional phase gate
(4). For atom B, its state \(|1\rangle\) also experiences a light
shift of order \(|\Omega_B|^2/(4\Delta_L)\) due to the classical field when
\(|\Delta_L| \gg |g_\mu^2/\delta_C|\) is also satisfied. The transition from
\(|1\rangle_B|0\rangle_C \rightarrow |a\rangle|1\rangle_C\) simply does not happen if the detun-
ing \(\Delta_L\) is made large. The level shift becomes much more
complicated if \(|\Delta_L| \gg |g_\mu^2/\delta_C|\) is not satisfied.

Now, we perform a detailed investigation of the above
envisioned 4-photon 2-atom transition. We find the rele-
vant dynamics by expanding the state in the basis \(|i,j,k\rangle\)
with similarly indexed coefficients \(C'\)'s to arrive at the fol-
lowing Schrodinger equation
\[
\begin{align*}
    i\dot{C}_{1a0} &= \frac{1}{2} \Omega_A C_{ea0}, \\
    i\dot{C}_{a10} &= \frac{1}{2} \Omega_B C_{ae0}, \\
    i\dot{C}_{ea0} &= \frac{1}{2} \Omega^*_A C_{1a0} - \tilde{\Delta}_L C_{ea0} + g_A C_{aa1}, \\
    i\dot{C}_{ae0} &= \frac{1}{2} \Omega^*_B C_{a10} - \tilde{\Delta}_L C_{ae0} + g_B C_{aa1},
\end{align*}
\]
\[ i \dot{C}_{aa1} = -\tilde{\delta}_C C_{aa1} + g_A C_{e0} + g_B C_{e0}. \]  

(5)

First we adiabatically eliminate the state \(|a, a, 1\rangle\) assuming \(|\delta_C| \gg |g_\mu|\). Then the two states \(|e, a, 0\rangle\) and \(|a, e, 0\rangle\) can be eliminated as long as \(|\Delta_L| \gg |g_\mu|, |g_\mu^2/\delta_C| \) or \(|\Delta_L - |g_\mu|^2/\delta_C| \gg |g_\mu|\). Finally we obtain the 4-photon resonant process between states \(|1, a, 0\rangle\) and \(|a, 1, 0\rangle\)

\[ i \dot{C}_{1a0} = \delta C_{1a0} + \frac{1}{2} \Omega_{\text{eff}} C_{a10}, \]

\[ i \dot{C}_{a10} = \frac{1}{2} \Omega_{\text{eff}}^* C_{1a0} + \delta C_{a10}, \]  

(6)

with

\[ \delta = \frac{1}{2} \frac{\Omega_{\text{eff}}^2}{4 \Delta_L} \left( \frac{1}{1 - 2s} + 1 \right), \]

\[ \Omega_{\text{eff}} = \frac{\Omega_{\text{eff}}^2}{4 \Delta_L} \left( \frac{1}{1 - 2s} - 1 \right). \]  

(7)

\[ s = g^2/(\Delta_L \delta_C). \] For simplicity we have assumed \(g_A = g_B = g\) and \(\Omega_A = \Omega_B = \Omega\) in the above consistent with the common addressing requirement. We note that an overall phase shift for the two states given by \(\Theta(t) = \int_0^t \tilde{\delta}(t') dt'/2\) can be simply absorbed to yield the solution to Eq. (6)

\[ C_{1a0}(t) e^{i \Theta(t)} = C_{1a0}(0) \cos \theta(t) - i C_{a10}(0) \sin \theta(t), \]

\[ C_{a10}(t) e^{i \Theta(t)} = C_{a10}(0) \cos \theta(t) - i C_{1a0}(0) \sin \theta(t), \]

(8)

with the effective pulse area \(\theta(t) = \int_0^t \Omega_{\text{eff}}(t') dt'/2\).

The other group of coupled states is the single atom off-resonant process on atom B. It can be studied with

\[ |\psi(t)\rangle = C_{101}(t)|0, 1, 0\rangle + C_{0e0}|0, e, 0\rangle + C_{0a1}|0, a, 1\rangle, \]

and the corresponding Schrödinger equation

\[ i \dot{C}_{010} = \frac{1}{2} \Omega_B C_{e0}, \]

\[ i \dot{C}_{0e0} = \frac{1}{2} \Omega_B C_{010} - \tilde{\Delta}_L C_{e0} + g_B C_{a0}, \]

\[ i \dot{C}_{0a1} = -\tilde{\delta}_C C_{0a1} + g_B C_{e0}. \]  

(9)

Following the same adiabatic elimination procedure as used above, we arrive at

\[ i \dot{C}_{010} = \delta' C_{0e0}, \]

\[ \delta' = \frac{|\Omega|^2}{4 \Delta_L (1 - s)}. \]  

(10)

i.e. a purely phase shift \(C_{010}(t) = C_{010}(0)e^{-i \delta'(t)}\) with \(\Theta' = \int_0^t \delta'(t') dt'.\)

In the limit when \(|\Delta_L| \gg |\Omega|, |\delta_C| \gg |g|, \) and \(|\Delta_L \delta_C| \gg g^2\), we find that

\[ \delta \approx \frac{|\Omega|^2}{4 \Delta_L} (1 + s) \approx \delta', \]

\[ \Omega_{\text{eff}} \approx \frac{\Omega^2}{2 \Delta_L}. \]  

(11)
Although $|\Omega_{\text{eff}}| \ll |\delta|, |\delta'|$, the 4-photon Rabi frequency $\Omega_{\text{eff}}$ does affect a net $(-1)$ phase shift upon completion of its $(2\pi)$ pulse since $\Theta(t) \approx \Theta'(t)$.

Figure 2 shows a numerical simulation without dissipations ($\Gamma = \kappa = 0$). To satisfy the adiabatic limit, we have used $\Omega = (2\pi) 20$ (MHz), $\Delta_L = (2\pi) 100$ (MHz), $g = (2\pi) 10$ (MHz), and $\delta_C = (2\pi) 50$ (MHz). In the top panel, the oscillating line denotes the probability amplitude of state $|a, 1, 0\rangle$, which returns to its initial value upon completion of the $2\pi$ pulse. The constant line denote the probability of state $|0, 1, 0\rangle$ which remains at unity as expected. The thickening of the lines is due to the rapid secular oscillations. In the lower panel, the saw-teeth like curve is the absolute phase of the amplitude of state $|a, 1, 0\rangle$, while the other solid line is for the relative phase between states $|a, 1, 0\rangle$ and $|0, 1, 0\rangle$. As expected it settles down to $\pm \pi$.

Surprisingly, we find that the conditional dynamics persists even beyond the limit when adiabatic elimination is valid. For instance, in Fig. 3 we display results for $\Omega = (2\pi) 10$ (MHz), $\Delta_L = (2\pi) 30$ (MHz), $g = (2\pi) 3$ (MHz), and $\delta_C = (2\pi) 8.75$ (MHz). Apparently, the fact that both atoms share the same cavity field data-bus is enough for establishing an effective interaction between them.
FIG. 3. The same as in Fig. 2, but not in the adiabatic limit.

Now we discuss effects of the dissipation/decoherence due to both the cavity loss (κ) and the atomic decay (Γ). As with any proposal for quantum computing implementation, ultimately its success depends on being able to complete many coherent dynamics during the decoherence time. In our case, as long as |Ω_{eff}| ≫ Γ, κ, we would expect essentially the same results as illustrated in Figs. 2 and 3. On the other hand, this condition is difficult to achieve because the 4-photon 2-atom resonant transition is a relatively weak process due to large off-resonant detunings for all of its intermediate states. In this respect, we find the second set of parameters as used for Fig. 3 more interesting as it points to the use of longer cavities with smaller κ and g, as well as the use of atoms with weaker transitions, thus smaller Γ. During numerical simulations that include Γ and κ, we find that in addition to a reduced success rate and a slightly reduced fidelity of the logic gate, the rapid secular oscillation is also suppressed. Overall, it seems that atomic loss Γ, rather than cavity decay κ, is the main cause of failure as in the adiabatic passage protocol [3]. For instance, with Γ = (2π)0.03 (MHz) and κ = (2π)0.1 (MHz), the result for Fig. 3 remains essentially the same with a success rate close to 0.9. For the parameters of Fig. 2, a success rate larger than 0.9 is achieved when Γ = (2π)0.05 (MHz) and κ = (2π)0.1 (MHz).

We note that there is wide regime of choices for the external laser parameters Ω and Δ_L. In fact, similar outcomes are expected as long as their ratio is maintained. This points to prospects of perhaps using a higher order longitudinal mode for the classical light such that it can also be sent through the cavity directly [6].

Finally we want to stress that realizations of all existing cavity QED based quantum logic protocols remain challenging because of the technological limit of the Fabry-Perot optical cavity. Realistically, successful implementation of our protocol requires g^2 ≈ 10^4 Γκ, sim-
ilar to that required as in Ref. [19], but more stringent than Ref. [13] which requires $g^2 \sim 10^2 \Gamma \kappa$. Using the whispering gallery mode of a high Q optical sphere, the parameter sets may be met if it is possible to integrate with trapped atoms or ions [20].

In conclusion, we have suggested a new protocol for conditional quantum phase gate between two atoms inside a high Q cavity using a 4-photon 2-atom resonant process. Our protocol eliminates the difficult task of individual addressing of atoms while they are inside the cavity and, therefore, becomes easier to implement. Furthermore, only 4-state atoms are used which opens a wider opportunity of experimental choices. Cavity QED based systems are usually deemed desirable, because the possibility of converting quantum information from atoms to photons for distribution and communication, and because of the potentially high clock cycle as afforded in the strong coupling limit. In the latter respect, similar to the recently proposed environment induced decoherence free space idea [13,19] our protocol does not offer much advantage as the inherent 4-photon 2-atom resonance at large intermediate detunings results in relatively slow dynamics.

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