NOTHING HAPPENS ON CLOSED CAUSAL CURVES

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Abstract. We prove in a semi-classical setting that in the context of the Events, Trees, Histories (ETH) approach to Quantum Theory points on closed causal curves are physically indistinguishable. Therefore there is no observation that could be made by an observer to tell any two points on a closed causal curve apart. We thus conclude that closed causal curves have no physical significance and one can assume that time travel will forever remain a fantasy.

Contents

1. Introduction 1
2. Background 2
  2.1. Causal Structure in General Relativity 2
  2.2. “Events Trees Histories” Approach to Quantum Theory 2
3. The Semi-Classical Set-Up 6
4. Closed Causal Curves in the Semi-Classical Regime 6
5. Conclusion 8
References 8

1. Introduction

Few theoretical curiosities have inspired the collective human imagination as much as the possibility of time travel, the idea that one might be able to go back in time. The idea of a time machine was popularized by H. G. Wells' 1895 novel “The Time Machine” and the genre endures a lasting popularity. This lasting popularity was aided by the fact that General Relativity admits solutions with closed timelike curves [9, 16]. This suggests that time travel might, at least in principle, be physically possible. However, due to the fact that it has never been observed and the many logical paradoxes associated with time travel, such as the grandfather paradox, time travel is by many considered to be an artefact in the theory. (See for example [10] for a philosophical discussion of the problem). Nevertheless there is a vast scientific literature discussing closed causal curves in various contexts.

The present work was inspired by Rovelli’s recent paper [15] where he claims that a full thermodynamical treatment along the closed causal curves manages to resolve the apparent paradoxes without resorting to Quantum Theory (QT). In the present paper, on the contrary, we will argue that instead of resorting to thermodynamic and arrow of time type arguments, considering closed causal curves in the context of a new consistent formulation of QT [6] resolves the problem already in the semi-classical
regime. We first collect those definitions from Fröhlich’s Events, Trees, Histories (ETH) approach to QT which are relevant to the argument in this paper. In this context we compare the causal structure with the theory of “Causal Sets” [4], another approach to fundamental physics. This comparison shows that closed causal chains can not exist in the relativistic formulation of the ETH approach, essentially by definition of the causal structure. However, it is not entirely clear yet how the classical spacetime emerge in a suitable limit. Therefore one often works in a semi-classical setting with a QT living on a fixed background spacetime. We will demonstrate that already in this setup no events, according to the definition of the ETH approach, happen along a closed causal curve. Therefore all the points are physically indistinguishable and hence closed causal curves, if they were to exist in the background spacetime, are physically irrelevant. This is a first practical example of a (pseudo) passive state in the ETH approach.

Overview of the paper. In Section 2 we briefly recall the definition of the causal structure in General Relativity. Then in Section 2.2 we collect the relevant definitions for the ETH approach of QT and give a short comparison to the theory of “Causal Sets” and how this prevents closed causal chains in the full QT. In Section 3 we discuss the semi-classical set up and the underlying assumptions and finally in Section 4 we prove Proposition 4.1 which is the main result of this paper.

2. Background

In this section we collect the definitions of the causal structure in General Relativity and the structures of the ETH approach to QT which are relevant for the argument in the paper.

2.1. Causal Structure in General Relativity. As it is central for the argument in this paper we shortly recall the definition for the causal structure in General Relativity. The central postulate of General Relativity is a 4-dimensional time orientable Lorentzian manifold equipped with a metric $(M,g)$. Here we choose the signature of $g$ to be $(-,+,+,+)$. A parametrized curve $\gamma$ at a point $p$ in the manifold $M$ can either be time-, space-, or lightlike/null depending on whether

$$\epsilon(p) = -g_{\rho\sigma} \dot{\gamma}^\rho \dot{\gamma}^\sigma \big|_p$$

is positive, negative, or zero. Here $\dot{\gamma}^\rho$ is the tangent vector of the curve $\gamma$ at the point $p$. This definition is then used to introduce a causal structure on the spacetime $(M,g)$. An event at one point on the manifold can influence events at another point on the manifold if there exists a causal, future directed curve from the first to the second point. A causal curve is one that is everywhere timelike or null. The causal future of a set of points denoted by $S$ on a manifold $M$ is denoted by $J^+(S)$. It consists of all points in $M$, that can be reached from a point in $S$ by a future directed, causal curve. The causal past $J^-(S)$ is defined analogous to its future counterparts.

2.2. “Events Trees Histories” Approach to Quantum Theory. Here we only collect those structures and definitions of the ETH approach that are relevant to the argument in the later sections. For a reader unfamiliar with the approach the recent review by Fröhlich [9] or the discussion in [5] is a good place to start. For technical details on the non-relativistic formulation see [8]. Here, we limit the discussion to the relativistic formulation and closely follow [7] and [5] but restricting the exposition to
a minimum of what is needed to understand the arguments in the present paper. We will use the occasion to point out similarities between the relativistic formulation of the ETH approach and the theory of “Causal Sets” [4] and use this to argue that the full QT prohibits the existence of closed causal chains.

In the ETH formulation of relativistic QT, a model of an isolated physical system \( S \) is defined by specifying the following data:

\[
S = \{M, E, \mathcal{H}, \{E_P\}_{P \in M}, >\},
\]

where \( M \) is a model of spacetime, \( E \) is a \( C^* \)-algebra represented on a Hilbert space \( \mathcal{H} \), \( \{E_P\}_{P \in M} \) is a family of von Neumann algebras associated to every point in \( M \) and \( > \) is the relation on \( M \) induced by the “Principle of Diminishing Potentialities” for timelike separated points which is given by the following definition.

**Definition 2.1.** A spacetime point \( P' \) is in the future of a spacetime point \( P \), written as \( P' \succ P \) (or, equivalently, \( P \) is in the past of \( P' \), written as \( P \prec P' \)) if

\[
E_{P'} \supsetneq E_P, \quad E_{P'} \cap E_P \text{ is an } \infty \text{-dim. non-commutative algebra}
\]

(2.2)

In fact, this definition holds as a theorem in an axiomatic formulation of quantum electrodynamics in four-dimensional Minkowski space proposed by Buchholz and Roberts [3].

**Definition 2.2.** If a spacetime point \( P' \) is neither in the future of a spacetime point \( P \) nor in the past of \( P \) we say that \( P \) and \( P' \) are space-like separated, written as \( P \sim P' \).

At this point it is interesting to remark that the strict partial order defined on \( M \) by Definition 2.1 is compatible with the first two axioms of Causal Set Theory [4]. In Causal Set Theory a causal set is defined to be a locally finite partially ordered set. This means a set \( C \) together with a relation \( \prec \), called precedes, which satisfy the following axioms:

(a) if \( x < y \) and \( y < z \) then \( x < z \), \( \forall x, y, z \in C \) (transitivity);
(b) if \( x < y \) and \( y < x \) then \( x = y \), \( \forall x, y \in C \) (non-circularity);
(c) for any pair of fixed elements \( x \) and \( z \) of \( C \), the set \( \{y | x < y < z\} \) of elements lying between \( x \) and \( z \) is nite.

If we consider the spacetime \( M \) as our set of interest, then it is clear that the relation in Definition 2.1 satisfies both axiom (a) and axiom (b) of Causal Set Theory. It then follows immediately from axiom (b) that no closed causal chain of points in \( M \) can exist. Before we can talk about axiom (c) in the context of the ETH approach we need to define under which conditions an actual event happens.

We denote by \( \Omega \) the density matrix on \( \mathcal{H} \) representing the actual state of the system \( S \). We use the notation

\[
\omega(X) := \text{tr}(\Omega X), \quad \forall X \in \mathcal{L}(\mathcal{H}),
\]

to denote the expectation value of the operator \( X \) in the state \( \omega \) determined by \( \Omega \). We next define the centralizer and the center of an algebra for a given state.

**Definition 2.3 (Centralizer).** Given a \( \ast \)-algebra \( A \) and a state \( \omega \) on \( A \), the centralizer, \( \mathcal{C}_\omega(A) \), of the state \( \omega \) is the subalgebra of \( A \) spanned by all operators \( Y \) in \( A \) with the property that

\[
\omega([Y, X]) = 0, \quad \forall X \in A,
\]
\[
\mathcal{C}_\omega(A) := \{ Y \in \mathcal{A} | \omega([Y,X]) = 0, \; \forall X \in \mathcal{A} \}.
\]

**Definition 2.4 (Center of the Centralizer).** The center of the centralizer, denoted by \(\mathcal{Z}_\omega(A)\), is the abelian subalgebra of the centralizer consisting of all operators in \(\mathcal{C}_\omega(A)\) which commute with all other operators in \(\mathcal{C}_\omega(A)\), i.e.

\[
\mathcal{Z}_\omega(A) := \{ Y \in \mathcal{C}_\omega(A) | [Y,X] = 0 \; \forall X \in \mathcal{C}_\omega(A) \}.
\]

With these definitions at hand an event is defined in the following way.

**Definition 2.5 (Event).** An event \(\{\pi_\xi, \xi \in \mathcal{X}\} \subset \mathcal{E}_P\), with \(\{\pi_\xi, \xi \in \mathcal{X}\}\) not contained in \(\mathcal{E}_P^o\), for \(P' < P\), starts happening in the future of \(P\) if \(\mathcal{Z}_{\omega_1}(\mathcal{E}_P)\) is non-trivial,

\[
\{\pi_\xi, \xi \in \mathcal{X}\} \text{ generates } \mathcal{Z}_{\omega_P}(\mathcal{E}_P),
\]

and

\[
\omega_P(\pi_{\xi_j}) \text{ is strictly positive, } \xi_j \in \mathcal{X}, \; j = 1,2,\ldots,n,
\]

for some \(n \geq 2\).

Here \(\mathcal{X}\) denotes a Hausdorff topological space of orthogonal projections, \(\pi_\xi\), on \(\mathcal{H}\) with the properties

\[
\pi_\xi \cdot \pi_\eta = \delta_{\xi\eta} \pi_\xi \text{ for all } \xi, \eta \in \mathcal{X}, \text{ and } \sum_{\xi \in \mathcal{X}} \pi_\xi = 1.
\]

It is expected that events usually have a finite extent in spacetime (see [1]). This implies that the operators \(\{\pi_\xi \xi \in \mathcal{X}\}\) representing a potential event in the future of the point \(P\) would be localized in a compact region of spacetime contained in the future of \(P\) (the future light-cone with apex at \(P\)). Accordingly, if we only look at the events in \(\mathcal{M}_E(\omega) \subset \mathcal{M}\), hence if we eliminate all points from \(\mathcal{M}\) except those for which, given a state \(\omega\) an event begins in their immediate future, then it seems reasonable to expect, that \(\mathcal{M}_E(\omega)\) satisfies axiom (c) of Causal Set Theory. However at this point we need to emphasize that the question regarding the “size” of events is an open problem. Though it seems plausible to assume that an event is no smaller than the Planck scale.

Now that we know when an event to occurs, let us see what it means for an event to happen. Let \(\omega_P\) be the state of an isolated system \(S\) right before the spacetime point \(P\). Let us suppose that an event \(\{\pi_\xi, \xi \in \mathcal{X}\}\) generating \(\mathcal{Z}_{\omega_P}(\mathcal{E}_P)\) begins happening in the future of \(P\). The ETH approach requires the following Axiom.

**Axiom 1.** The actual state of the system \(S\) right after the event \(\{\pi_\xi, \xi \in \mathcal{X}\}\) starting at \(P\) has happened is given by one of the states

\[
\omega_{P,\xi_s}(\cdot) := [\omega_P(\pi_{\xi_s})]^{-1} \omega_P(\pi_{\xi_s}(\cdot)\pi_{\xi_s}),
\]

for some \(\xi_s \in \mathcal{X}\) with \(\omega_P(\pi_{\xi_s}) > 0\). The probability for the system \(S\) to be found in the state \(\omega_{P,\xi_s}\) right after the event \(\{\pi_\xi, \xi \in \mathcal{X}\}\) starting at \(P\) happened is given by Born’s Rule, i.e., by

\[
\text{prob}\{\xi_s, P\} = \omega_P(\pi_{\xi_s}).
\]
“repeated measurements” while in the ETH approach the projection, by Axiom 1 is naturally part of the dynamics, and there is no need to invoke an observer performing measurements.

Finally we consider the evolution of the state in the relativistic formulation of the ETH approach to QT in a subset $\mathcal{F}$ of spacetime $\mathcal{M}$. Let $P, P'$ be points in $\mathcal{F}$, let $\mathcal{Z}_{\omega_P}$ denote the center of the centralizer of the state $\omega_P$ on the algebra $\mathcal{E}_P$, which describes the event $\{\pi^P_\xi | \xi \in \mathcal{A}^P\}$ happening in the future of $P$, and let $\mathcal{Z}_{\omega_{P'}}$ be the algebra describing the event happening in the future of the point $P'$. We now introduce the following axiom.

**Axiom 2.** *(Events in the future of space-like separated points commute):* Let $P \times P'$. Then all operators in $\mathcal{Z}_{\omega_P}$ commute with all operators in $\mathcal{Z}_{\omega_{P'}}$. In particular,

$$[\pi^P_\xi, \pi^P_{\eta'}] = 0, \; \forall \xi \in \mathcal{A}^P \text{ and all } \eta \in \mathcal{A}^{P'}.$$

Following [7], this axiom may be one reflection of what people sometimes interpret as the fundamental non-locality of quantum theory: projection operators representing events in the future of two space-like separated points $P$ and $P'$ in spacetime are constrained to commute with each other! This can be seen as the equivalent condition of what Dowker calls “discrete general covariance” in the context of Causal Set Theory [4] namely that the probability of a particular finite partial causal set does not depend on the order in which the elements are “born”. As we will discuss next, in the context of the ETH approach Axiom 2 guarantees that the probability of a particular event happening does not depend on the “order” in which spacelike separated events occur.

Assume that $\mathcal{F}$ contains a space-like hypersurfaces $\Sigma_0$ on which there exists an initial state $\omega_{\Sigma_0}$ (for a detailed construction see [7]). Denote by $V^-_P(\mathcal{F})$ all points in $\mathcal{F}$ that lie in the past of $P$. Let $\{P_i | i \in \mathcal{I}(\mathcal{F})\}$ denote the subset of points in $V^-_P(\mathcal{F})$ in whose future events happen, and let

$$\{\pi^{P_i} | i \in \mathcal{I}(\mathcal{F})\} \subset \mathcal{E}_{\Sigma_0}$$

be the actual events that happen in the future of the points $P_i, i \in \mathcal{I}(\mathcal{F})$ (here $\mathcal{I}(\mathcal{F})$ is a set of indices labelling the points in $V^-_P(\mathcal{F})$ in whose future events happen; it is here assumed to be countable). Fröhlich [7] then defines a so-called “History Operator”

$$H(V^-_P(\mathcal{F})) := \prod_{i \in \mathcal{I}(\mathcal{F})} \pi^{P_i}_{\xi_i}, \quad \text{(2.6)}$$

where the ordering in the product $\prod$ is such that a factor $\pi^{P_i}_{\xi_i}$ corresponding to a point $P_i$ stands to the right of a factor $\pi^{P_j}_{\xi_j}$ corresponding to a point $P_j$ if and only if $P_j < P_i$ (i.e., if $P_j$ is in the past of $P_i$). But if $P_i \times P_j$, i.e., if $P_i$ and $P_j$ are space-like separated, then the order of the two factors is irrelevant thanks to Axiom 2!

The state on the algebra $\mathcal{E}_P$ relevant for making predictions about events happening in the future of $P$ is then given by

$$\omega_P(X) = \omega^\mathcal{F}_P(X) = \left[N^\mathcal{F}_P \right]^{-1} \omega_{\Sigma_0} (H(V^-_P(\mathcal{F}))^* X H(V^-_P(\mathcal{F}))) \ , \ X \in \mathcal{E}_P, \quad \text{(2.7)}$$

where the normalization factor $N^\mathcal{F}_P$ is given by

$$N^\mathcal{F}_P = \omega_{\Sigma_0} (H(V^-_P(\mathcal{F}))^* \cdot H(V^-_P(\mathcal{F}))).$$

The quantities $N^\mathcal{F}_P$ can be used to equip the tree-like space (what Fröhlich in [6] refers to as the “non-commutative spectrum” of $S$) of all possible histories of events
in the future of $\Sigma_0$ with a probability measure. It would be interesting to investigate how the “non-commutative spectrum” in the ETH approach relates to the “law of growth” in the context of Causal Set Theory. However this is beyond the scope of this paper.

This are all the details one needs to know about the ETH approach with regards to the following considerations.

3. The Semi-Classical Set-Up

For the argument in this paper we consider a semi-classical setup in the following sense. We assume there exists a classical time orientable spacetime manifold $(M, g)$ as a background on which we study a quantum system following the ETH dynamics. This is similar to the setup used in the proof of the “Principle of Diminishing Potentialities” by Buchholz and Roberts in the context of four-dimensional Minkowski space. We assume the causal structure provided by the ETH approach to QT to be compatible with the causal structure of the background manifold. In particular this translates to the following semi-classical assumptions.

**Assumption 3.1.** If $J^+(x) \subset J^+(y)$ then $E_x \subset E_y$

**Assumption 3.2.** If $J^-(y) \subset J^+(x)$ then there exists an admissible ordering of the history operators $H(V_x^-(\mathfrak{S}))$, $H(V_y^-(\mathfrak{S}))$ such that $H(V_y^-(\mathfrak{S}))$ is a factor of $H(V_x^-(\mathfrak{S}))$.

In particular we have

$$H(V_x^-(\mathfrak{S})) = \prod_{i \in [J_x(\mathfrak{S}), J_y(\mathfrak{S})]} \pi_{\xi_i} \cdot H(V_y^-(\mathfrak{S})).$$

(3.1)

Assumption 3.1 is motivated in two ways. First this is a necessary assumption for the causal structure of the ETH approach to QT to be compatible with the causal structure of the background spacetime. Classically $J^+(x) \subset J^+(y)$ implies that $x$ is in the future of $y$. In the ETH approach to QT $E_x \subset E_y$ is compatible with the statement that $x$ is in the future of $y$. The second motivation comes from a recent paper comparing the ETH approach to QT with the Causal Fermion Systems theory (see for an introduction). In the CFS setting the relevant future algebras are constructed explicitly from the future light cone.

Assumption 3.2 is a compatibility assumption and an application of Axiom 2. Essentially it is stating, that we can write all the projections corresponding to events that happen in the spacetime region $J^-(x) \setminus J^-(y)$ to the left of those projections corresponding to events that happen in $J^-(y)$. This is true because points in $J^-(x) \setminus J^-(y)$ are never in the past of points in $J^-(y)$ and thus either have to (in the future) or can (if spacelike separated) be written on the left of those in $J^-(y)$.

4. Closed Causal Curves in the Semi-Classical Regime

We will now discuss the physical nature of closed causal curves in the above defined semi-classical regime. Because we aim to discuss closed causal curves we will deviate slightly from the formulation of the relativistic QT in the ETH approach in that we will assume a general state $\omega$ on the global algebra $E$ to be a normalized, positive linear functional on $E$. This is in contrast to Section 2.2 where we defined a state $\omega_{\Sigma_0}$ with respect to a particular Cauchy surface. Accordingly we replace $\mathfrak{S}$ with $\mathcal{M}$ in the definition of the history operator and we assume the events in the past of every point to be countable. This set up is a simplification to avoid a technical discussion about
the choice of a relevant initial data surface in a spacetime with closed causal curves. We can then prove the following Proposition.

**Proposition 4.1.** Let \( x \) and \( y \) be two arbitrary distinct points on a closed causal curve. Given Assumption 3.1 and Assumption 3.2 we have that

\[
\omega_x(X) = \omega_y(X), \quad X \in \mathcal{E}_x = \mathcal{E}_y. \tag{4.1}
\]

**Proof.** We start by observing that if \( x \in J^+(y) \), therefore \( J^+(x) \subset J^+(y) \), and \( y \in J^+(x) \), therefore \( J^+(y) \subset J^+(x) \), and thus by Assumption 3.1 we have

\[
\mathcal{E}_x \subset \mathcal{E}_y \text{ and } \mathcal{E}_y \subset \mathcal{E}_x, \tag{4.2}
\]

which can be combined to give \( \mathcal{E}_x = \mathcal{E}_y \).

Next, we have that \( x \in J^-(y) \), therefore \( J^-(x) \subset J^-(y) \), and \( y \in J^-(x) \), therefore \( J^-(y) \subset J^-(x) \), and thus by Assumption 3.2 we have

\[
H(V_x^-(\mathcal{M})) = \prod_{\iota \in [\mathcal{J}_x(\mathcal{M}) \setminus \mathcal{J}_y(\mathcal{M})]} \pi_{\xi_\iota}^{P_{\iota}} \cdot H(V_y^-(\mathcal{M})) \tag{4.3}
\]

and

\[
H(V_y^-(\mathcal{M})) = \prod_{\iota \in [\mathcal{J}_y(\mathcal{M}) \setminus \mathcal{J}_x(\mathcal{M})]} \pi_{\xi_\iota}^{P_{\iota}} \cdot H(V_x^-(\mathcal{M})). \tag{4.4}
\]

Plugging (4.3) into the right hand side of (4.4) and vice versa gives

\[
\prod_{\iota \in [\mathcal{J}_x(\mathcal{M}) \setminus \mathcal{J}_y(\mathcal{M})]} \pi_{\xi_\iota}^{P_{\iota}} \cdot \prod_{\iota \in [\mathcal{J}_y(\mathcal{M}) \setminus \mathcal{J}_x(\mathcal{M})]} \pi_{\xi_\iota}^{P_{\iota}} = \tag{4.5}
\]

\[
\prod_{\iota \in [\mathcal{J}_x(\mathcal{M}) \setminus \mathcal{J}_y(\mathcal{M})]} \pi_{\xi_\iota}^{P_{\iota}} \cdot \prod_{\iota \in [\mathcal{J}_y(\mathcal{M}) \setminus \mathcal{J}_x(\mathcal{M})]} \pi_{\xi_\iota}^{P_{\iota}} = 1 \tag{4.6}
\]

observing, that both \( \prod_{\iota \in [\mathcal{J}_x(\mathcal{M}) \setminus \mathcal{J}_y(\mathcal{M})]} \pi_{\xi_\iota}^{P_{\iota}} \) and \( \prod_{\iota \in [\mathcal{J}_y(\mathcal{M}) \setminus \mathcal{J}_x(\mathcal{M})]} \pi_{\xi_\iota}^{P_{\iota}} \) are products of projection operators and the identity is the only projection operator that is invertable we get that the sets \( \mathcal{J}_x(\mathcal{M}) \setminus \mathcal{J}_y(\mathcal{M}) \) and \( \mathcal{J}_y(\mathcal{M}) \setminus \mathcal{J}_x(\mathcal{M}) \) have to be empty and thus we get immediately

\[
H(V_x^-(\mathcal{M})) = H(V_y^-(\mathcal{M})). \tag{4.7}
\]

Finally using the general state \( \omega \) in (2.10) completes the result. \( \Box \)

By Proposition 4.1 all points that lie along the same closed causal curve share the same future algebra and the same state. Therefore not a single event will happen along a closed causal curve and as a consequence all points on these curves are physically indistinguishable. This means that there does not exist a single observation that any kind of observer could carry out to distinguish between different points along a closed causal curve.

The result discussed in this section can be understood as a *quasi passive state*. A passive state, as introduced in [1] for the non relativistic formulation of the ETH approach, is a state such that for any time \( t > t_0 \) the center of the centralizer is trivial and hence no further events occur after the time \( t_0 \). What was showed here is that no event can occur within any region that features closed causal curves. However, in principle one can imagine, that a causal curve can leave the region where closed causal curves exist. Apriori there is nothing preventing events from happening at points along the causal curve that lie in the future but outside of the region where closes causal curves exist. Therefore the states discussed here are *quasi passive* in the sense that there exists a macroscopic region where no events occur but, in principle, events can occur in the future of this region.
5. Conclusion

In the present paper we demonstrated with a simple semi-classical argument relying on the ETH approach to QT, that points on closed causal curves are physically indistinguishable and therefore closed causal curves are physically irrelevant already in the semi-classical regime. Therefore we showed that in a region where closed causal curves exist, all states are quasi passive states.

(Quasi) passive states as discussed in the present paper are an interesting aspect of the ETH approach to QT as, according to Fröhlich [6], thermal equilibrium states are passive states. Particularly in the cosmological setting thermal (quasi) equilibrium states might play a role if, along the lines of [11, 12, 13] one thinks of the evolution of the universe as a sort of transition between a (quasi) thermal state with a large cosmological constant $\Lambda$ and a thermal state with a small cosmological constant. Ideas along this line have also been explored in [13]. Thus the properties of passive states in the ETH approach to QT should be studied in more detail.

Finally, the semi-classical argument presented here is consistent with the fact that in the full relativistic formulation of the ETH approach to QT closed causal chains do not exist. Hence, it can be expected that closed causal curves in General Relativity are a pathology stemming from the fact that the theory fails to take quantum effects into account. It thus seems that a complete quantum theory of gravity will not admit time travel as a physical phenomenon.

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