Applications of ferroelectric materials for dark matter axions detection haloscopes

J.M. García Barceló,*a A. Álvarez Melcón,a S. Arguedas Cuendis,b A. Díaz-Morcillo,a B. Gimeno,c A. Kanareykin,d A.J. Lozano-Guerrero,a P. Navarro,a and W. Wuensch*e

a Department of Information and Communications Technologies, Technical University of Cartagena, 30203 - Cartagena, Spain
b Institut de Ciències del Cosmos, Universitat de Barcelona (UB-IEEC), 08028 - Barcelona, Catalonia, Spain
c Instituto de Física Corpuscular (IFIC), CSIC-University of Valencia, 46071 - Valencia, Spain
d Euclid Techlabs, LLC, Bolingbrook, IL
e European Organization for Nuclear Research (CERN), 1211 Geneva 23, Switzerland

* Corresponding author
E-mail: josemaria.garcia@upct.es

ABSTRACT: Tuning is an essential requirement for the search of dark matter axions employing haloscopes, since its mass is not known yet to the scientific community. At the present day, most haloscope tuning systems are based on mechanical devices, which can lead to failures due to the complexity of the environment in which they are used. However, the electronic tuning making use of ferroelectric materials can provide a path that is less vulnerable to mechanical failures and thus complements and expands current tuning systems. In this work, we present and design several ideas for using the ferroelectric $K_TaO_3$ material as a tuning element in haloscopes based on coupled cavities. On the other hand, the structures used in the Relic Axion Detector Exploratory Setup (RADES) group are based on several cavities that are connected by metallic irises, which act as interresonator coupling elements. In this article, we also show how to use these $K_TaO_3$ films as interresonator couplings between cavities, instead of inductive or capacitive metallic windows. These two concepts represent a crucial upgrade over the current systems employed in the axion community. The theoretical results demonstrate the interest of the novel concepts proposed for the use of this kind of ferroelectric media in the search for dark matter axions.
1 Introduction

In recent years there has been a high interest for the search of axions and other particles compatible with the Standard Model which could be part of dark matter. Axions, particles predicted by Weinberg [1] and Wilczek [2] could also solve the strong Charge Conjugation-Parity problem [3, 4].

Several experimental groups have developed in the last 30 years structures for the detection of such particles [5], based on the inverse Primakoff effect [6]. This kind of detectors are divided also into several types: laboratory experiments or light-shining-through wall, helioscopes, which search for axions from the Sun (plasma photons in the solar inner layers), and haloscopes, which search for axions in the galactic halo, both using the axion-photon conversion driven by the action of a powerful external static magnetic field. For the latter, this coupling is enhanced when it occurs in a resonant device like a microwave resonant cavity, as described in [7].

In the last five years, the RADES group has worked with this concept, studying, designing, manufacturing, and taking data with axion detectors (haloscopes), searching for dark matter axions of masses around ∼ 34 µeV, although the Ultra High Frequency or UHF band is now also being investigated for new haloscopes design. The first haloscope developed by RADES, working at ∼ 8.4 GHz, is based on an array of five copper-coated stainless steel cavities connected by four inductive windows (coupling irises) a concept commonly employed in microwave filters [8]. The size of an individual cavity sets the working frequency (and the axion mass to be explored) since the resonance is determined by the cavity
dimensions. The advantage of this kind of haloscopes is that we can increase the volume of the structure without decreasing the working frequency (which is a common problem in the axion community), just by adding more cavities to the structure. The theoretical foundation of the detection principle can be found in [9].

The whole system is based on several components. First, due to the extremely low axion-photon coupling a cryogenic environment (few Kelvin) is needed to decrease the thermal noise. Second, the RADES haloscopes are connected to a receiver which amplifies, filter and down-convert the received radio frequency (RF) power with very low noise levels. And third, the receiver realizes the Analog-Digital conversion and the Fast Fourier Transform for the post-processing of the data taking.

The main objectives for an efficient axion detection are to maximize the power detected from the axion-photon coupling in order to increase the analysed axion mass range (and its scanning frequency rate) as well as to optimize the haloscope sensitivity. The RF power detected depends on properties intrinsic to the axion and on experimental cavity parameters, namely [10]:

\[
P_d = \kappa g_{a\gamma}^2 \frac{\rho_a}{m_a} B_e^2 C V Q_l\tag{1.1}
\]

where \(\kappa\) is the coupling to the external receiver (ideally \(\kappa = 0.5\) for critical coupling operation regime), \(g_{a\gamma}\) the unknown axion-photon coupling, \(\rho_a\) the dark matter density, \(m_a\) the axion mass, \(B_e\) the external static magnetic field (depends on the magnet used for the experiment), \(C\) the form factor, \(V\) the volume of the cavity and \(Q_l\) its loaded quality factor. The form factor, which measures the coupling between the external magnetostatic field and the RF electric field induced by the axion-photon conversion, can be expressed as:

\[
C = \frac{\left| \int_V \vec{E} \cdot \vec{B}_e \, dV \right|^2}{\int_V |\vec{B}_e|^2 \, dV \int_V \varepsilon_r |\vec{E}|^2 \, dV} \tag{1.2}
\]

where \(\varepsilon_r\) is the relative permittivity within the cavity of volume \(V\). A measure of the sensitivity of the haloscope is the axion-photon coupling that can be detected for a given signal to noise ratio \((\frac{S}{N})\), which can be obtained by

\[
g_{a\gamma} = \left( \frac{S}{N} \frac{k_B T_{sys}}{\kappa \rho_a C V Q_l} \right)^{\frac{1}{2}} \frac{1}{B_e} \left( \frac{m_a^2}{Q_a \Delta t} \right)^{\frac{1}{4}} \tag{1.3}
\]

where \(k_B\) is the Boltzmann constant, \(T_{sys}\) the noise temperature of the system, \(\Delta t\) the data taking time and \(Q_a\) the quality factor of the axion resonance [10]. In summary, the parameters that can be controlled in the haloscope design are \(\kappa, C, V\) and \(Q_l\).

The tuning in a haloscope is an extremely important feature because the axion mass is unknown. The data taking will be based on scanning a specific mass range of the whole spectrum, so a frequency shifting procedure will be needed. This work is focused on a system for improving the tuning in order to be able to scan a specific axion mass region.
easily using ferroelectric media.

Most of tuning mechanisms employed by haloscope experiments are based on mechanical systems. The ADMX [11, 12], HAYSTAC [13] and IBS/CAPP [14] groups use cylindrical cavities with one or more rods. The tuning is accomplished by rotating these metallic rods inside the cavities, which affects to the electromagnetic field of the resonance modifying the resonant frequency of the measured mode. The rotational movement of the rod system is obtained by a series of gears connected to a driven motor or by means of piezoelectric materials. In the case of the QUAX collaboration [15], they use movable sapphire shields to change the resonant frequency of the cavity. On the other hand, the CAST-CAPP/IBS group use two movable dielectric sapphire plates placed in parallel and symmetrically at the cavity sides [16]. Also the RADES group has employed a mechanical tuning mechanism based on splitting the haloscope in two identical halves and moving them symmetrically to increase the effective width dimension of the cavities thus modifying the axion frequency search [17].

In this paper we propose novel electrical tuning systems for axion dark matter searches studied in the RADES project scenario. In contrast to mechanical tuning, electrical tuning can provide an avenue that is less prone to mechanical failures (it avoids movable parts in a cryogenic environment, for example) and thus complements and expands existing techniques. Another advantage of this kind of tuning for multi-cavity systems is the ability to independently adjust the different cavity frequencies and coupling values to maintain the correct mode structure and electromagnetic field pattern (which could improve also the form factor). In addition, the mechanical tuning does not behave well in scalability when we evolve our haloscopes to higher frequencies (for example from X-band to Ku-band). This issue might be solved with the electrical tuning proposed in this work.

Different tunable technologies have been studied by the authors to implement an electrical tuning system in our haloscopes: dielectrics, ferromagnetics, ferroelectrics, liquid crystals, piezoelectrics, microelectromechanical systems (MEMS), composite ceramics based on mixtures of ferroelectrics, and semiconductors (varactors). However, due to the high requirements imposed in an axion detection system (cryogenic temperatures, high static magnetic field level, and low losses), the most promising is the ferroelectric technology. This kind of dielectrics provides a permittivity change with temperature or bias voltage, so the idea is to load our haloscopes with such ferroelectrics and then modify its relative electrical permittivity \( \varepsilon_r \) to produce a frequency shift. From a qualitative point of view we can use the equation of the resonant frequencies of a completely filled rectangular cavity:

\[
f_{mnl} = \frac{c}{2} \sqrt{\frac{\varepsilon_r}{\mu_r}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{l}{d}\right)^2},
\]

where \( f_{mnl} \) is the resonant frequency for a \( TE_{mnl} \) or \( TM_{mnl} \) mode in a rectangular cavity, \( c \) the speed of light in vacuum, \( \mu_r \) the relative magnetic permeability, and \( a, b, \) and \( d \) the dimensions of the rectangular cavity. Usually the \( TE_{101} \) fundamental resonance has been
used in RADES. As it can be seen, the resonant frequency is inversely proportional to the square root of the relative permittivity \( f_{\text{res}} \propto \frac{1}{\sqrt{\varepsilon}} \), so the higher the ferroelectric permittivity, the lower the haloscope frequency (and the axion mass to be search).

Ferroelectrics are non-linear dielectrics with high relative permittivity values widely employed in high-density commercial decoupling capacitors, acousto-electronic transducers, or MEMS [18]. Its permittivity can be modified by applying an external static electric field (or also varying the temperature), which is the most important attribute of some perovskites that make them attractive for agile microwave components (such as varactors, tunable RF filters, or phase shifters).

There are more than 300 ferroelectric materials available, but the most widely used is the Barium Strontium Titanate (\( Ba_xSr_{1-x}TiO_3 \) or BST) which operates in paraelectric phase at room temperatures (\( \sim 297 \) K) [19, 20]. For cryogenic temperatures, the Strontium Titanate (\( SrTiO_3 \) or STO) was used previously by the axion research group of ADMX-Fermilab that has investigated ferroelectrics as a tuning element for the ADMX project [21]. The Fermilab’s project [22, 23] focused on exploiting the novel electronic properties of non-linear dielectric materials such as strontium titanate to build more sensitive detectors for axion dark-matter particles. The group has produced and tested a number of thick-film STO samples on quartz and sapphire substrates using several film deposition techniques. The frequency shifts and the corresponding dielectric permittivity changes were not of the expected magnitude [22, 23].

In 2018, Euclid Techlabs proposed to implement a KTO ferroelectric tuning system [24, 25] for axion dark matter searches in the RADES [9, 10] and ADMX [21] projects. Euclid proposed using \( KTaO_3 \) (KTO) as a tuning element, a ferroelectric crystal that exhibits excellent tuning parameters and very low loss factor (\( \sim 10^{-5} \) at X-band) in the cryogenic temperature range 0.1 - 10 K. Euclid carried out electromagnetic simulations to study the KTO permittivity value and its change inside the 8 GHz test cavity [25]. Euclid is currently working to manufacture a small prototype of the test cavity with a ferroelectric element, characterize the small test cavity in the 2 - 10 K range in order to verify the permittivity value, its tuning range, and its loss tangent [25].

In section 2 we prove the effectiveness of the KTO ferroelectrics in several haloscopes alleviating the issues displayed in [22, 23]. In addition, we show how a high form factor is also maintained despite the huge relative permittivity values.

Regarding the use of ferroelectrics for inter-couplings between adjacent cavities based on existing literature, no solution has been found to the problem of cutting the metallic irises necessary for its manufacture. In section 3 the use of ferroelectric materials as interresonator couplings is also demonstrated, leading to an reconfigurable haloscope. To continue, in section 4 we comment the technical considerations for the correct biasing of the ferroelectrics. Finally, in section 5 we expose our conclusions and discussion of future
prospects.

2 The KTO ferroelectric as a tuning element

There are two operation regimes of the ferroelectric media depending on the Curie temperature $T_c$:

- $T > T_c$: When the operation temperature is higher than $T_c$ the lattice of the ferroelectric crystals acquires a cubic structure (the polar to non-polar transition). In this state the material has no spontaneous polarization and its permittivity obtains a high dependence on static electric field, temperature and strain. Figure 1a sketches the behaviour of the permittivity as a function of the temperature in a ferroelectric. In this case the crystal phase obtains a paraelectric behaviour manifesting a bell shape with the external static electric field (see Figure 1b).

- $T < T_c$: When the working temperature is lower than $T_c$ the crystal acquires a spontaneous polarization due to the charge shift. In this phase the crystal has a polar behaviour and its polarization shows an hysteresis performance (see Figure 1c). In addition, the permittivity exhibits a butterfly shape with the static electric field. This case is very useful for memory function. Ferroelectric capacitors are usually used for RAM memories in computing or for RF identification tags, for example.

The first case where $T > T_c$ is more desirable for tunable microwave applications because it avoids the hysteresis, making easier the permittivity shift with the voltage/temperature change [18]. That is the reason why we have looked for a material with very low $T_c$, since we will take data at cryogenic temperatures. As a consequence, the cryogenic operation temperature has to satisfy this requirement.

![Figure 1](image)

**Figure 1**: Response of a ferroelectric material. Relative permittivity versus temperature (a), and polarization (right y-axis) and permittivity (left y-axis) versus the external static electric field for $T > T_c$ (b) and $T < T_c$ (c). $P_r$ is the remanent polarization, $P_s$ is the spontaneous polarization, and $E_c$ is the coercive field.
The KTO material is an incipient ferroelectric with properties very similar to those of STO and Calcium Titanate (CaTiO$_3$ or CTO). Although KTO and STO have many similarities, the first material maintains the cubic structure at very low temperatures just decreasing its loss tangent between room temperature and 5 K. This is not the case for the STO material [26, 27]. The losses of KTO are lower than those of STO [27]. These properties make KTO single crystals interesting for the development of microwave tuning at cryogenic temperatures.

It should be noted that KTO single crystals have not been comprehensively studied in the 1 - 4 K and mK temperature ranges [27]. In [27], the microwave dielectric properties of single-crystal incipient quantum ferroelectrics have been measured at cryogenic temperatures using cylindrical sample as $TE_{0n1}$ and quasi-$TE_{011}$ dielectric resonators. No ferroelectric phase transition was observed, and KTO remains paraelectric down to 5.4 K, which is consistent with theoretical predictions. The results show that the dielectric constant is greatly reduced in the presence of a DC electric field. At zero bias, the dielectric constant increases to about 4500 on cooling, but appears to saturate at liquid helium temperatures [27]. No dielectric relaxation is observed in single crystals below the soft-mode frequency, which lies in the THz range.

In summary, ferroelectric materials (as KTO) at cryogenic temperatures could allow the development of a remarkably high tuning range system with very low losses. In [27] we can find the most accurate measurements to date of the Potassium Tantalate material at cryogenic temperatures. Crystals of 99.99 % purity are commercially available in [28].

Once we have clarified the properties of this material, the next step is to find the best allocation of KTO objects (with a particular geometry) inside the haloscope cavities in order to provide a good frequency tuning. The search of the optimal position of such ferroelectric objects has to be performed trying to avoid the concentration of all the electromagnetic fields inside the KTO (which would lead to a poor form factor and the lost of the $TE_{101}$ mode), which is a great challenge due to the very high relative permittivity value. For example, if we set a ferroelectric pill at the middle of a rectangular waveguide cavity (see Figure 2a) the electric field is mainly absorbed by the dielectric easily for high values of the relative permittivity (see Figures 2b and 2c). In Table I we can see the resonant frequency, the unloaded quality factor $Q_0$ and the form factor $C$ for several relative permittivity values in this system. Note how the form factor for $\varepsilon_r = 1$ is equal to the one for the unloaded cavity without dielectrics for the $TE_{101}$ mode: $C = 64/\pi^4 = 0.657$, which can be obtained from equation 1.2. Despite the good frequency shift achieved for $\varepsilon_r = 50$ it can be observed how the quality and form factors are quite reduced, which leads to reject this kind of position and shape for the KTO, which has permittivities close to 4500 leading to much worse results.

Another problem that is found in the electromagnetic analysis of ferroelectric-loaded haloscopes is the computational time due to the high meshing required for large dielectric
Figure 2: Cavity loaded with a dielectric pill at the center. 3D view (a), and electric field in a cross-section for $\varepsilon_r = 1$ (b) and $\varepsilon_r = 50$ (c).

| $\varepsilon_r$ | $f_r$ (GHz) | $Q_0$ | $C$  |
|---------------|-------------|-------|------|
| 1             | 8.406       | 45319 | 0.657|
| 2             | 8.249       | 45223 | 0.624|
| 5             | 8.070       | 44923 | 0.589|
| 10            | 7.972       | 44569 | 0.573|
| 20            | 7.903       | 44004 | 0.554|
| 30            | 7.865       | 43005 | 0.540|
| 40            | 7.819       | 40110 | 0.509|
| 50            | 7.710       | 30135 | 0.386|

Table I: Characteristics of a rectangular cavity (with $a = 22.86$ mm, $b = 10.16$ mm and $d = 28.5$ mm) loaded with a dielectric pill ($r_{pill} = 4$ mm, $h_{pill} = 2$ mm) at the middle, for several relative permittivity values.

In this work, an electromagnetic simulator based on finite element method (CST [29]) has been employed, where the higher the relative permittivity $\varepsilon_r$, the lower the wavelength, and a higher number of mesh cells is needed for a proper meshing in such region. The solution for this issue was the use of selective meshing and/or simulating with FEST3D, a software which employs an integral equation technique efficiently solved by the Method of Moments and the Boundary Integral-Resonant Mode Expansion method [30].
After studying a multitude of geometrical shapes and positions within the cavity (see examples in Figure 3), we finally arrived at a tuning system concept based on thin (∼ 500 microns) ferroelectric films that provides good results in quality factor, form factor and tuning frequency range. The features of this kind of system are explained in the next subsections.

Figure 3: Examples of geometrical shapes and positions studied for the KTO in our tuning system. These configurations provide non-desired results.

2.1 Rectangular haloscope tuning with ferroelectrics

Next we are going to demonstrate the viability of using KTO films as a tuner element in rectangular cavities. To show this, we analyze the model from Figure 4a\(^1\) where a single cavity with two KTO films separated 1.8 mm from the side walls is shown. For this example, we have used the WR-90 rectangular waveguide (\(a = 22.86 \text{ mm} \) and \(b = 10.16 \text{ mm}\)), which works at X-band frequencies (our frequency region for this study).

The key idea for using this kind of system is based on working with the \(TE_{301}\) cavity mode, where two of the three lobes in the width axis are narrowed thanks to the high permittivity value of the KTO films, while the central lobe is stretched at the vacuum section of the waveguide as depicted in Figure 4b. The thickness of the ferreoelectric films will be set for holding the waveguide half-wavelength using the following equation [31]:

\[
\lambda_d = \frac{\lambda_{g}^{KTO}}{2} = \frac{\lambda_0}{2 \sqrt{\varepsilon_r} \sqrt{1 - \left(f_c^{KTO} / f_0\right)^2}} = \frac{c}{2 f_0 \sqrt{\varepsilon_r} \sqrt{1 - \left(f_c^{KTO} / f_0\right)^2}},
\]

\(^1\)Note here how the metallic housing has been displayed (being the vacuum or air medium the rest of the space), while for the rest of the models in the previous figures it is omitted, assuming the metallic sections at the external boundaries of the air or dielectric solids.
where $\lambda_{gKTO}$ is the guided wavelength at the KTO region, $\lambda_0 = c/f_0$ the free-space wavelength, $f_{cKTO} = c/(2a\sqrt{\varepsilon_r})$ the cut-off frequency of the $TE_{10}$ at the KTO region, and $f_0$ the working frequency.

The variation of the KTO permittivity value changes slightly the position of the lateral lobes providing a good tuning range as we will prove at the next subsection, and avoiding the reduction of the form factor due to the small negative lobe region. The permittivity range that we will use in this work is from $\varepsilon_r = 3000$ to 5000. However, this concept is scalable for any range. The material permittivity could vary within a tolerance from the used in this work, so once it is measured a new design could be done for such material, providing similar results. Also, in section 1 we commented that the KTO losses are $\tan\delta = 10^{-5}$, however we use in the simulations $\tan\delta = 10^{-4}$ to be more conservative in order to contemplate future extra losses in real measurements. For the electric conductivity of the metallic housing we have used copper at cryogenic temperatures $\sigma = 2 \times 10^9$ S/m, which is also the one employed at the axion data campaigns [32].

2.2 Results for the KTO placed at sides

As example, we have employed the model from Figure 4a at the operation frequency $f_0 = 8.5$ GHz. This leads us using equation 2.1 for a KTO thickness from $l_d = 250$ $\mu$m (for $\varepsilon_r = 5000$) to $l_d = 322$ $\mu$m (for $\varepsilon_r = 3000$). On the other hand, the length of the cavity $l_c$ is set to have a $f_0 = 8.5$ GHz resonance of the $TE_{101}$ mode in this model without
ferroelectrics, that is, $l_c = 26.97$ mm.

Now, in order to select the best $l_d$ option and separation from the walls ($l_w$), we make an optimization to find the higher tuning range with the best quality and form factors. For the cases close to $l_d = \lambda_g/2$ the tuning is very low (of a few hundred KHz). However, for slightly higher or lower values, the tuning range improves significantly (of the order of hundreds of MHz). The best option will be to choose a lower value of $l_d$ in order to avoid the reduction of the quality and form factors. After a manual optimization process based on increase both factors and the tuning range, we have obtained an optimum point at $l_d = 235$ $\mu$m and $l_w = 1.88$ mm, where a 700 MHz frequency tuning range can be achieved with good quality and form factors.

In Figure 5 the tuning range, quality factor, form factor and figure of merit ($FM = Q_0 \times C^2$) parameters are shown for the previous configuration. For the FM we have used only the quality factor and form factor variables; the rest (like volume) remains unchanged. In addition, the form factor affects with the square since the scanning rate $\frac{dm_a}{dt}$ is generally used for the performance of a haloscope, which can be obtained from equation 1.3 [10]:

$$\frac{dm_a}{dt} = Q_0 Q_t \kappa^2 g_{\gamma}^4 \left( \frac{\rho_a}{m_a} \right)^2 B_c^4 C^2 V^2 \left( \frac{S}{N k_B T_{sys}} \right)^{-2}$$

(2.2)

As it can be seen, the tuning range achieved is 7.2 % (698 MHz), which is a very good tuning frequency range for the requested electronic tuning in realistic haloscopes. Also the form factor remains high ($C \simeq 0.52$) for all the permittivity range. However, the quality factor drops rapidly when the permittivity increases. This is the reason why the FM follows the same behaviour as the quality factor. Figure 6 shows the dependence of the quality factor with the frequency: the higher the frequency change (lower frequencies), the lower the quality factor.

If needed, the system can be reduced to the range $\varepsilon_r = (3000 - 4200)$, which provides a tuning of 2.23 % (216 MHz) with a quality factor higher than 20700 which is a good value for axion searches at this frequency. Despite the previous good results, more KTO geometries are under study for improving the tuning system, specially for increasing the $Q_0$ parameter.

Furthermore, an all-inductive four-subcavities system has been simulated with the developed ferroelectric tuning. In this case, we have employed in all subcavities a KTO thickness of $l_d = 250$ $\mu$m with $l_w = 1.5$ mm, obtained again from a manual optimization procedure based on increase $Q_0$, $C$ and the tuning range. Figure 7 shows the model used in these simulations and the tuning obtained with a small change in the relative permittivity, proving that this system can be used for the design of real haloscopes.

For this relative permittivity range ($\varepsilon_r \in [3000, 3600]$) we have obtained a frequency variation of $f = [9.0482 - 9.0117]$ (so a tuning range of 37 MHz), a quality factor change of $Q_0 = [46066 - 38369]$ and a form factor deviation of $C = [0.509 - 0.49]$. This permittivity
Figure 5: Parameters of the model from Figure 4 as a function of the relative permittivity. (a) Tuning range. (b) Quality factor. (c) Form factor. (d) Figure of Merit.

range provides in the first example (model of one cavity) tuning parameters of the same order.

Finally, in Figure 8\(^2\) we observe the electric field pattern at \(\varepsilon_r = 3000\) for both examples: one cavity and four-subcavities models. As it can be seen, the axion mode is the \(TE_{301}\) one for both cases. For higher permittivity values, the electric field is more concentrated at the ferroelectric material. This small change in the electric field provides the frequency change in the \(TE_{301}\) mode and as a consequence the requested frequency tuning.

In section 4 we expose the technical considerations for an appropriate biasing system in order to change the relative permittivity value of the ferroelectrics without high losses.

\footnote{In these images a strong scaling have been applied to correctly appreciate the negative electric field at the ferroelectric area. Without this scaling, a zero electric field level is observed at the inductive couplings, indicating that this is the correct mode configuration.}
Figure 6: Dependence of the unloaded quality factor $Q_0$ with the frequency in the model of Figure 4.

Figure 7: (a) Model based on an all-inductive four-subcavities haloscope with a coaxial port in each end cavity. The length of the two end cavities is 26.97 mm, the length of the two inner cavities is 26 mm, and the thickness and width of the three inductive irises are 2 and 9 mm, respectively. (b) $S_{21}$ scattering parameter magnitude as a function of the frequency for two different values of the relative permittivity.

3 The KTO ferroelectric as a coupling element

In the RADES group [10] rectangular waveguide cavities connected by iris couplings have been used for the haloscope designs. The coupling between cavities can be obtained by inductive or capacitive irises (see Figure 9). For example, in the previous section, we studied a four-subcavities haloscope that employs three inductive irises to connect the subcavities.

The size of the irises determines the coupling between adjacent subcavities. This inter-resonator coupling is used mainly for improving the mode separation between the axion one and its neighbours. Next we will see how the ferroelectric KTOs can be used as coupling
Electric field (vertical component) for $\varepsilon_r = 3000$ in the models (a) one-cavity and (b) four-subcavities. Zoom at the KTO area to observe the positive/negative transition of the $TE_{301}$ mode inside the one-cavity model.

Two types of iris couplings: inductive window (a), and capacitive window (b). The pictures show the symmetric half of each one, being the dashed regions the symmetry planes.

films instead of irises, which often cause problems in the quality factor when manufacturing, due to misalignment.

3.1 Modelling

Figure 10 shows the structure that is used to examine the effect of KTO ferroelectrics as interresonator couplings in rectangular waveguides. It consists on two waveguide sections of length $l_{port}$, connected by a KTO film of thickness $l_d$. 

---

---
Figure 10: Characterization of a ferroelectric KTO as a coupling element. The system is based on three transmission lines: the first and last ones are hollow waveguides ($\varepsilon = \varepsilon_0$, which is the vacuum permittivity), and the second one is the KTO ($\varepsilon = \varepsilon_0 \varepsilon_r$) filling the height and width dimensions. The waveguide dimensions are based on the WR-90 standard rectangular waveguide. The length of the KTO ($l_d$) and its relative permittivity ($\varepsilon_r$) control the type of coupling. $l_{port}$ is the distance to the reference plane of the port.

To analyze this system we can develop a simple single-mode transmission line model (see first circuit of Figure 11). The hollow waveguide regions are represented by the first (1) and the last (3) transmission lines filled with vacuum ($\varepsilon = \varepsilon_0$), while the KTO film is depicted by the central transmission line, representing a medium with permittivity $\varepsilon = \varepsilon_0 \varepsilon_r$. Due to the high relative permittivity change between the KTO and the hollow waveguide, a high impedance step between ferroelectric and vacuum appears. In fact, the characteristic impedances of media (1) and (3) are very large as compared to the characteristic impedance of medium (2). Therefore, the transmission line representing medium (2) can be considered to be loaded on both sides by open-circuits. In this situation, this transmission line behaves as a resonator when its length is close to half guided wave-length ($l_d = \lambda_{g^{KTO}}/2$) \[31\]. This can be represented with a parallel lumped LC resonator as shown in the second circuit of Figure 11, which resonates at frequency $f_{KTO}$.

For lower frequencies $f < f_{KTO}$, the impedance of the inductor ($Z_L$) is small and the impedance of the capacitor is very large ($Z_C \to \infty$). Therefore, the KTO acts as inductive coupling, since the capacitor behaves as an open circuit, as shown in the third circuit of Figure 11. Analogously, for $f > f_{KTO}$, the impedance of the capacitor ($Z_C$) is small and the impedance of the inductor is very large $Z_L \to \infty$, so the KTO acts as capacitive coupling (see fourth circuit in Figure 11). On the other hand, note that the relative permittivity of the KTO can be used to easily change its resonant frequency $f_{KTO}$. This can be used to conveniently adjust the frequency regions where the KTO behaves as inductive or as capacitive coupling. This will be elaborated further in the next section, when we characterize the KTO as coupling element.

Taking the model of Figure 10, we have characterized its behaviour analyzing the first circuit of Figure 11, from where the scattering parameters can be extracted with the...
Figure 11: Analysis of the system with transmission lines, using a single mode representation. From top to bottom: three transmission lines, KTO as LC resonator, KTO as inductive coupling (for low frequencies, where the capacitor is an open circuit) and KTO as capacitive coupling (for high frequencies, where the inductor is an open circuit). \( Z_i \) and \( \beta_i \) are the characteristic impedance and the phase constant of the fundamental mode \((TE_{10})\), respectively in each region \((i = 0, d\) for vacuum and dielectric filled waveguide regions, respectively).

Following equations [31]:

\[
S_{11} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}
\tag{3.1}
\]

\[
S_{21} = \frac{(1 + S_{11})(1 + \rho_2)e^{-j\beta_d l_d}}{1 + \rho_1},
\tag{3.2}
\]

where \( S_{11} \) and \( S_{21} \) are the reflection and transmission parameters, respectively, \( Z_{in} \) is the input impedance from line (1) to line (2), as shown in the first circuit of Figure 11, \( Z_0 \) is the characteristic impedance of lines (1) and (3), \( \rho_2 \) is the reflection coefficient of line (3) referred to line (2) (see first circuit of Figure 11), \( j = \sqrt{-1} \) is the complex imaginary unit, \( \beta_d \) is the propagation constant of medium (2), and \( \rho_1 \) is the reflection coefficient at the input referred to line (2) (see first circuit of Figure 11). The phase constant in medium (2) is defined as \( \beta_d = (2\pi) / \lambda_g^{KTO} \). \( Z_{in} \) is calculated using transmission line theory [31], as
follows:

\[ Z_{in} = Z_d \frac{Z_0 + jZ_d \tan (\beta_d l_d)}{Z_d + jZ_0 \tan (\beta_d l_d)} \]  

(3.3)

For \( \rho_2 \) and \( \rho_1 \) we obtain:

\[ \rho_2 = \frac{Z_0 - Z_d}{Z_0 + Z_d} \]  

(3.4)

\[ \rho_1 = \frac{Z_{in} - Z_d}{Z_{in} + Z_d} \]  

(3.5)

The characteristic impedances for the three lines (\( Z_0 \) and \( Z_d \)) are the modal impedances of the fundamental mode (\( TE_{10} \)) in each region, given by:

\[ Z_i = Z_{TE_{10}}^{10} = \frac{\eta/\sqrt{\varepsilon_{r_i}}}{\sqrt{1 - (f_{ic}/f_0)^2}}, \]  

where \( f_{ic} \) is the cut-off frequency of the \( TE_{10} \) mode in each region and \( \eta \approx 377 \ \Omega \) the free space characteristic impedance.

### 3.2 Results for the KTO as coupling element

Using the equations from subsection 3.1, and taking as example \( \varepsilon_r = 4000 \) and \( l_d = \lambda_g/2 = 279 \ \mu m \) for a KTO resonance at \( f_{KTO} = 8.5 \ \text{GHz} \), we obtain the results showed in Figure 12.

Note here how, in contrast to section 2, we will employ input/output ports in waveguide technology, instead of coaxial ports (see Fig. 10).

![Figure 12](image-url)

**Figure 12**: Results of the Scattering Parameters magnitude from the model of Figure 10 using equations 3.1 and 3.2. The KTO film behaves as inductive coupling in the region \( f < f_{KTO} \) while it acts as capacitive coupling in the region \( f > f_{KTO} \).
For $f < 8.5$ GHz, the transmission factor $|S_{21}|$ has a positive slope with frequency, which corresponds with an inductive coupling, as we explained in the previous subsection. Similarly, for $f > 8.5$ GHz, $|S_{21}|$ acquires a negative slope with frequency, which coincides with the response of a capacitive coupling. It is evident that by adjusting the KTO resonant frequency $f_{KTO}$, the frequency regions where the films acts as inductive or capacitive coupling can be easily adjusted. The adjustment of the KTO resonant frequency can be easily achieved by changing its thickness ($l_d$) or and the relative permittivity ($\varepsilon_r$).

For the RADES designs, the extraction of the physical coupling $k$ value is usually needed. This value is obtained with a model based on two resonant cavities connected with one interresonator coupling, which will be implemented in this case with the dielectric KTO slab. We have employed the physical model of Figure 13, where inductive windows have been selected to implement the input/output couplings.

![Figure 13](image-url)

**Figure 13:** Model based on two resonant cavities connected by an interresonator coupling implemented by the dielectric KTO film. The input/output couplings are implemented with standard inductive windows. The picture shows the symmetric half, being the dashed region the symmetry plane. The dimensions are the same as the model from Figure 10, but using a $l_{port}$ of 8 mm. The length of the cavities $l_c$ is set for a desired working frequency $f_0 = 8.42$ GHz, while the thickness of the KTO $l_d$ is adjusted to have its resonant frequency $f_{KTO}$ above or below the working frequency, to implement either inductive or capacitive coupling. The input/output iris width is fixed small ($a_i = 5$ mm) for reducing the load effect.

Using this structure, the $k$ value can be obtained as $[8]$:

$$k = \frac{f_{\text{even}}^2 - f_{\text{odd}}^2}{f_{\text{even}}^2 + f_{\text{odd}}^2}$$

(3.7)

where, $f_{\text{even}}$ and $f_{\text{odd}}$ are the even (or magnetic) and odd (or electric) frequencies, respectively.
Following this procedure, we want to characterize the KTO as a coupling element at a working frequency of $f_0 = 8.42$ GHz. First, we will extract the KTO couplings with inductive type. For this case, the KTO resonance is adjusted for a frequency higher than 8.42 GHz. In our test we have selected a KTO resonant frequency of $f_{KTO} = 9.5$ GHz, which corresponds with a length $l_d = 250 \, \mu m$ for a relative permittivity of $\varepsilon_r = 4000$. We will assume a KTO permittivity range from 3000 to 5000. In a second step, we characterize the KTO couplings for the capacitive case. For this, we select $l_d = 316 \, \mu m$, which leads to a KTO resonant frequency of $f_{KTO} = 7.5$ GHz for $\varepsilon_r = 4000$, lower than the target frequency of $f_0 = 8.42$ GHz. In Figure 14 we plot the values of the calculated couplings as a function of the permittivity range of the KTO for both inductive and capacitive cases.

A typical realizable value of $|k|$ for metallic irises is 0.02 [32]. To prove the effectiveness of the KTO as coupling, we show how we can obtain this value with a ferroelectric film. As can be observed in Figure 14, the $|k| = 0.02$ value is obtained at $\varepsilon_r = 4806$ for the inductive case, and at $\varepsilon_r = 3295$ for the capacitive case.

Finally, this example can be used to conceive a more practical structure with the design of four-subcavities connected with three alternating KTO couplings (two of them capacitive and one inductive) and coaxial input/output ports, as shown in Figure 15a. Employing the previous coupling value ($|k| = 0.02$) for all the couplings, we have to use the obtained KTO parameters, $l_d = 316 \, \mu m$, $\varepsilon_r = 3295$ for the capacitive coupling and $l_d = 250 \, \mu m$, $\varepsilon_r = 4806$ for the inductive coupling. The coaxial probes are optimized for a weak input/output coupling. In Figure 15b we observe the transmission response obtained with these KTO films. Also, in Figure 15c we can see the electric field of the axion mode, where the negative electric field value can be observed inside the KTOs (blue color), as expected. It is interesting to note that the design has been very effective, since the field is perfectly synchronous in all the cavities, as it is required to maximize the axion form factor [32]. The quality and form factor values for this mode are estimated in $Q_0 = 31668$ and $C = 0.586$, respectively, demonstrating that this idea can be used for the design of real haloscopes.

Moreover, as proved, any type of coupling (inductive or capacitive) can be implemented in haloscopes with ferroelectric KTOs. The RADES alternating coupling structures [32] can be designed by alternating low and high permittivities, which will give each coupling its necessary properties. As discussed in that paper, this would reduce the mode-mixing (non-desired modes close to the axion resonant frequency), and increase the tuning range. In addition, this system can provide a required final adjustment for the couplings, in case of manufacturing errors or if the response needs to be modified for any reason. This can simply be done by adjusting the KTO permittivity values using the mechanisms discussed in the next section. Finally, the use of ferroelectrics as couplings would avoid the need to manufacture irises or metal windows, which have caused many alignment problems for the haloscopes in previous RADES implementations, with subsequent degradation in quality factors.
4 Electrical biasing of ferroelectrics

As we observed in the previous sections, the ferroelectric permittivity can be changed with a static voltage or by modifying the operation temperature. For the voltage option, we can design an asynchronous system where each ferroelectric film can be tuned individually. On the other hand, for the temperature option the whole haloscope will be cooled/warmed, so all the ferroelectrics would have the same $\varepsilon_r$ value. In the last option, the implementation of an alternating coupling system (inductive + capacitive couplings) with ferroelectrics needs to take into account some details. The idea would be to have long enough $l_d$ for the ferroelectrics that we want to behave like capacitive couplings, making the frequency $f_{KTO}$ at

Figure 14: Physical coupling value vs. KTO permittivites for (a) inductive behaviour ($l_d = 250 \mu m$) and (b) capacitive behaviour ($l_d = 316 \mu m$). In both cases coupling values are calculated at the working frequency of $f_0 = 8.42$ GHz.
Figure 15: (a) Model based on a four-subcavities haloscope with coaxial input/output ports. The length of the two end cavities is 29.05 mm, the length of the two inner cavities is 27.34 mm, the thickness and relative permittivity of the first and last KTO films (capacitive couplings, pink colour) is $l_{d1} = 316 \, \mu\text{m}$, $\varepsilon_{r1} = 3295$, while the thickness and relative permittivity of the middle one (inductive coupling, yellow colour) is $l_{d2} = 250 \, \mu\text{m}$, $\varepsilon_{r2} = 4806$. (b) Transmission $S_{21}$ parameter magnitude. (c) Electric field (vertical component) of the axion mode (at $f_0 = 8.42 \, \text{GHz}$).

which the inductive/capacitive behaviour change occurs well below the operating frequency, so the coupling value would change if the permittivity decreases, but ensuring always a capacitive performance. Similarly, for the ferroelectrics as inductive couplings, we can set a
small enough $l_d$ to ensure that the behaviour switching is quite high in frequency, so the coupling value would change if the permittivity increases, but ensuring always an inductive performance. In the voltage case, $l_d$ could be the same for all the coupling ferroelectrics (although the behaviour of each class can be preserved if we change a bit the lengths by the method explained before).

Since the haloscope should operate at the minimum temperature, an increment of this will reduce the sensitivity of the experiment since $T_{sys}$ will increase. So the practical variation of permittivity is probably through electrical biasing. We will have to check which one is the best solution taking into account the compromise "Spurious free range - C factor - Tuning range".

If we want to apply voltage to these films, the position of the electrodes have to be considered carefully. For the fixing of the ferroelectric films, small slits at the haloscope walls are needed, making also internal access possible for the biasing cables, thus avoiding high losses due to parasitic elements that could be introduced in other regions with higher electromagnetic fields. Leakage through biasing holes can be considered negligible. The electrodes would be positioned at the two closest surfaces.

DC bias contacts and deposition technology strongly depend on the test cavity design [25]. If the operational mode of the design has no electric field components parallel to the KTO crystal surface, the DC contacts can be made either with high conductivity or with superconducting materials. In this case, there are no additional currents along the KTO crystal surface, thus providing no Q factor degradation of the cavity. In the case of normal conducting electrodes, high conductivity materials can be deposited, and adhesion/thermal expansion matching layers applied to ensure low microwave losses in the electrodes. An alternative option can be superconducting contact deposition. In [18], a resonator was fabricated on single-crystal (100) KTO discs $0.5 \times 10$ mm, and epitaxial-grade polished. Double-sided superconducting YBCO (or Yttrium Barium Copper Oxide) films were deposited using the co-evaporation technique [18].

In many currently considered haloscope cavity designs, the operating TM modes have their field components parallel to the DC contact surface for the KTO crystals at an optimal place inside the cavities. In this case, to prevent Q-factor degradation, the conducting but high resistivity DC bias contacts have to be deposited on the crystal surface [25].

5 Conclusions and prospects

Tuning capability in haloscope systems is an elementary necessity since the mass of the axion is unknown. This aspect has been a challenge for several groups searching for dark matter axions due to the complexity of the systems involved, under extreme conditions of temperature and magnetic fields. This paper exhibits for the first time the possibility of employing electronic tuning systems based on ferroelectric materials. Powerful simulation
results are shown for the elaboration of a tunable haloscope sweeping a considerable spectral region of axion masses.

In addition, the use of ferroelectric films as interresonator couplings has been demonstrated, which avoids the manufacturing of metallic iris windows and could improve the quality factor in the haloscope designs. The principal future research line in this work is the measurement of these KTO films in real prototypes for proving all these studies. Also, the combination of both ideas (tuning and coupling with ferroelectrics) is being investigated.

Acknowledgments

This work was performed within the RADES group. We thank our colleagues for their support. In addition, this work has been funded by the grant PID2019-108122GB-C33, funded by MCIN/AEI/10.13039/501100011033/ and by "ERDF A way of making Europe". JMGB thanks the grant FPI BES-2017-079787, funded by MCIN/AEI/10.13039/501100011033 and by "ESF Investing in your future". Also, this project has received partial funding through the European Research Council under grant ERC-2018-StG-802836 (AxScale).

References

[1] S. Weinberg, A new light boson?, Phys. Rev. Lett. 40 (1978) 223.
[2] F. Wilczek, Problem of strong P and T invariance in the presence of instantons, Phys. Rev. Lett. 40 (1978) 279.
[3] R. Peccei and H. Quinn, CP conservation in the presence of pseudoparticles, Phys. Rev. Lett. 38 (1977) 1440.
[4] R. Peccei and H. Quinn, Constraints imposed by CP conservation in the presence of pseudoparticles, Phys. Rev. D 16 (1977) 1791.
[5] I.G. Irastorza and J. Redondo, New experimental approaches in the search for axion-like particles, Prog. Part. Nucl. Phys. 102 (2018) 89 [1801.08127].
[6] H. Primakoff, Photoproduction of neutral mesons in nuclear electric fields and the mean life of the neutral meson, Phys. Rev. 81 (1951) 899.
[7] P. Sikivie, Experimental Tests of the Invisible Axion, Phys. Rev. Lett. 51 (1983) 1415.
[8] R.J. Cameron, C.M. Kudsia and R.R. Mansour, Microwave filters for communication systems: fundamentals, design, and applications., Wiley, second ed. (2018).
[9] A. Álvarez Melcón, S. Arguedas-Cuendis, C. Cogollos, A. Díaz-Morcillo, B. Döbrich, J.D. Gallego et al., Axion searches with microwave filters: the rades project, Journal of Cosmology and Astroparticle Physics 040 (2018) 1.
[10] A. Díaz-Morcillo, J.M. García Barceló, A.J. Lozano Guerrero, P. Navarro, B. Gimeno, S. Arguedas Cuendis et al., Design of new resonant haloscopes in the search for the dark matter axion: A review of the first steps in the rades collaboration, Universe 8 (2022) .
[11] I. Stern, A.A. Chisholm, J. Hoskins, P. Sikivie, N.S. Sullivan, D.B. Tanner et al., Cavity design for high-frequency axion dark matter detectors, Rev. Sci. Instrum. 86 (2015) 123305.
[12] C. Boutan, M. Jones, B.H. LaRoque, N.S. Oblath, R. Cervantes, N. Du et al., 
Piezoelectrically Tuned Multimode Cavity Search for Axion Dark Matter, 
Phys. Rev. Lett. 121 (2018) 261302.

[13] L. Zhong, S. Al Kenany, K.M. Backes, B.M. Brubaker, S.B. Cahn, G. Carosi et al., Results from phase 1 of the HAYSTAC microwave cavity axion experiment, 
Phys. Rev. D 97 (2018) 092001.

[14] J. Choi, S. Ahn, B. Ko, S. Lee and Y. Semertzidis, CAPP-8TB: Axion dark matter search experiment around 6.7 µev, 
Nucl. Instrum. Methods. Phys. Res. B 1013 (2021) 165667.

[15] D. Alesini, C. Braggio, G. Carugno, N. Crescini, D. D' Agostino, D. Di Gioacchino et al., 
Realization of a high quality factor resonator with hollow dielectric cylinders for axion searches, 
Nucl. Instrum. Methods. Phys. Res. B 985 (2020) 164641.

[16] L. Miceli, Haloscope axion searches with the cast dipole magnet: the CAST-CAPP/IBS detector, in 11th Patras Workshop on Axions, WIMPs and WISPs, pp. 164–168, 2015, DOI.

[17] S. Arguedas-Cuendis, A. Álvarez Melcón, C. Cogollos, A. Díaz-Morcillo, B. Döbrich, J.D. Gallego et al., The 3 Cavity Prototypes of RADES: An Axion Detector Using Microwave Filters at CAST, in Microwave Cavities and Detectors for Axion Research. Springer Proceedings in Physics, vol. 245, 2020, DOI.

[18] S. Gevorgian, Ferroelectrics in Microwave Devices, Circuits and Systems: Physics, Modeling, Fabrication and Measurements, Springer-Verlag London, first ed. (2009), 10.1007/978-1-84882-507-9.

[19] A. Ahmeda, I.A. Goldthorpe, and A.K. Khandani, Electrically tunable materials for microwave applications, 
Appl. Phys. Rev. 2 (2015) 011302.

[20] A. Kanareykin, E. Nenasheva, S. Kazakov, A. Kozyrev, A. Tagantsev, V. Yakovlev et al., 
Ferroelectric Based Technologies for Accelerators, in AIP Conference Proceedings, vol. 1086, pp. 380–385, 2009, DOI.

[21] ADMX collaboration, Extended search for the invisible axion with the axion dark matter experiment, 
Phys. Rev. Lett. 124 (2020) 101303.

[22] D. Bowring, T. Connolly, M. Kang, M. Ortega, S. Priya, C. Salemi et al., Dielectric tuning of cavities, in 3rd Workshop on Cavities and Detectors for Axion Research, 2018.

[23] “Fermilab LDRD Annual Report 2017. 
https://ldrd.fnal.gov/subdir/LDRD_AnnualReport_FY2017.pdf.”

[24] A. Kanareykin. Private communication.

[25] “https://www.euclidtechlabs.com/all-products.”

[26] V. Skoromets, C. Kadlec, H. Němec, D. Fattakhova-Rohlfing and P. Kužel, Tunable dielectric properties of KTaO₃ single crystals in the terahertz range, 
J. Phys. D: Appl. Phys. 49 (2016) 065306.

[27] R.G. Geyer, B. Riddle, J. Krupka and L.A. Boatner, Microwave dielectric properties of single-crystal quantum paraelectrics KTaO₃ and SrTiO₃ at cryogenic temperatures, 
Journal of Applied Physics 97 (2005) 104111.

[28] “http://www.crystal-material.com/Substrate-Materials/KTaO3-substrate.html.”

[29] “https://www.3ds.com/products-services/simulia/products/cst-studio-suite/.”

[30] “https://www.3ds.com/products-services/simulia/products/fest3d/.”
[31] D.M. Pozar, *Microwave Engineering*, John Wiley and Sons, Inc., fourth ed. (2012).

[32] A. Álvarez Melcón, S. Arguedas-Cuendis, C. Cogollos, A. Díaz-Morcillo, B. Döbrich, J.D. Gallego et al., *Scalable haloscopes for axion dark matter detection in the 30 µeV range with RADES*, *Journal of High Energy Physics* **084** (2020) 1.