Abstract: A space vector pulse-width modulation (PWM) technique for a two-level asymmetrical six-phase drive is proposed for the first time in this study. The PWM technique is developed based on the modified vector space decomposition approach. The details regarding the process of choosing the switching sequences and the calculation of dwell times are presented. The PWM technique is verified through simulation using PLECS software and is further validated experimentally. Obtained results show the validity of the proposed PWM technique.

1 Introduction

In recent years, the research in the area of multiphase drives has increased significantly. The advantages offered by these drives, when compared to the conventional three-phase drives, such as in fault tolerant operation, contribute to their popularity [1]. Although the number of machine phases can be in range of 5 to more than 15 and if fact may be considered to be arbitrary, multiphase machines comprising of multiple three-phase sets (e.g. six- and nine-phase machine) are the most common. This is because they can be obtained through simple modification of the standard three-phase machines.

This paper deals with six-phase machine which in fact consists of two three-phase sets. Related to the angular displacement between the sets, asymmetrical six-phase machine (with 30° displacement), is proven to yield better torque ripple performance comparing to the other displacement angles [2]. Due to the switching device limitations at that time, the inverters operated in six-step mode and the low-order harmonics can be observed in current waveforms. Necessity to properly model machines of this type leads to introduction of vector space decomposition (VSD) and if fact may be considered to be arbitrary, multiphase machines. Therefore, this paper presents a new space vector pulse-width modulation (PWM) technique for a two-level asymmetrical six-phase drive with single neutral point configuration.

2 Space vector algorithm

The analysed drive topology is shown in Fig. 1. The machine is assumed to have sinusoidally distributed stator windings and, thus, it is desirable to drive the machine with sinusoidal phase voltage waveforms. Therefore, it can be said that the aim of the proposed space vector algorithm in this paper, is to realise sinusoidal phase voltage, $v_{ph}$, waveforms. These waveforms can be realised by applying the space vectors within the chosen switching sequences in accordance to their corresponding calculated dwell times. Furthermore, the space vectors of the chosen switching sequences are determined based on the leg voltage, $v_{LEG}$, space vectors.

The sinusoidal reference phase voltage ($v_{ph}$) waveforms of the analysed drive topology can be defined as

$$v_{ph} = V_f \cdot \cos(\omega t - k \cdot \pi/6)$$

where $k = 0, 1, 4, 5, 8$ and $9$, which also corresponds to phases $a$ to $f$, respectively. The reference phase voltage waveforms are shown in Fig. 2. In terms of the output leg voltages, phase voltages can be defined as

$$v_{ph} = v_{LEG} = \frac{1}{6} \sum_{k=1}^{6} v_{LEG} \cdot$$

where the second term in (2) represents common mode voltage (CMV).

Fig. 1 Two-level asymmetrical six-phase drive

Fig. 2 Sinusoidal reference phase voltage ($v_{ph}$) waveforms for two-level asymmetrical six-phase drive
The order-per-sector law states that the projected phase voltage space vectors in each sector in $a - \beta$ plane (including space vectors located on the borders of the bounding sectors, as well as at the origin) must satisfy the order of the reference phase voltage ($v_{\phi}^{\alpha}$) waveforms for the corresponding sector in the time domain. As already mentioned, the projections of the phase and leg voltage space vectors in $a - \beta$ plane are the same. Therefore, the order-per-sector law can be implemented by simply comparing the order of the normalised leg voltage levels, i.e. switching states represented in six-digit binary representation, with the order of $v_{\phi}^{\alpha}$ in the time domain. As an example, the conditions for the order-per-sector law for both sections of the first sector (S1-A and S1-B) are: $v_{\phi}^{\alpha} \geq v_{\phi}^{b} \geq v_{\phi}^{c} \geq v_{\phi}^{d} \geq v_{\phi}^{e} \geq v_{\phi}^{f}$ and $v_{\phi}^{d} \geq v_{\phi}^{e} \geq v_{\phi}^{f} \geq v_{\phi}^{c} \geq v_{\phi}^{b} \geq v_{\phi}^{a}$, respectively (refer to Fig. 2). By using VSD transformation matrix (4), one can see that switching state 54 (110110) is projected on the border of S1-A and S1-B sectors. However, one can see that it does not satisfy either of these conditions. Hence, it is discarded.

2.2 Reduction of the number of space vectors

Not all phase voltage space vectors can be used in order to realise the sinusoidal phase voltage waveforms. The unnecessary space vectors can be eliminated by implementing the order-per-sector law [6].

$$V_{\phi} = \frac{2}{\sqrt{3}} \left[ \begin{array}{c} \cos(\alpha) \\ \sin(\alpha) \\ \cos(5\alpha) \\ \sin(5\alpha) \\ \cos(8\alpha) \\ \sin(8\alpha) \\ \cos(12\alpha) \\ \sin(12\alpha) \\ \cos(9\alpha) \\ \sin(9\alpha) \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{array} \right] V_{\phi}$$

where $V_{\phi}$ is the phase voltage space vector, and the projection of $V_{\phi}$ onto $x-y$ plane can be obtained by

$$v_{\phi} = \left[ \begin{array}{c} \cos(\alpha) \\ \sin(\alpha) \\ \cos(5\alpha) \\ \sin(5\alpha) \\ \cos(8\alpha) \\ \sin(8\alpha) \\ \cos(12\alpha) \\ \sin(12\alpha) \\ \cos(9\alpha) \\ \sin(9\alpha) \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{array} \right] \times \left[ \begin{array}{c} v \[6] \\ v \[9] \\ v \[12] \\ -v \[12] \\ -v \[6] \\ 1 \end{array} \right]$$

In the analysed, standard two-level inverter topology, the upper and lower switches complement each other. Hence, there are two possible switching combinations per inverter leg, which also leads to two possible inverter leg voltage levels, 0 and $V_{dc}$. In addition, by normalising leg voltage levels by $V_{dc}$, they can be further denoted by 0 and 1, respectively. Thus, one finds that there are $2^6 = 64$ switching states (denoted from 0 to 63). These switching states can also be represented as 000000 to 111111 using normalised leg voltage levels.

2.1 Projection of space vectors

The projections of the possible switching states onto the three orthogonal planes ($\alpha - \beta$, $x-y$ and $0^\circ - 0^\circ$), can be obtained by using VSD approach [3], as in (see (3)) where $\alpha = \pi/6$. These projections in fact represent leg voltage space vectors. Similarly, from (2), phase voltage space vectors can also be obtained. In addition, it can be shown that the phase voltage voltage space vector projections are identical to the leg voltage space vector projections in $a - \beta$ and $x-y$ plane, but are different in $0^\circ - 0^\circ$ plane. The number of leg voltage and phase voltage space vectors is 64 and 63, respectively. This is because the projections of the phase voltage space vectors corresponding to the switching states 0 and 63 are identical.

The difference between the leg and phase voltage space vector projections in $0^\circ - 0^\circ$ plane becomes a problem for calculating the dwell times later on [4]. In order to solve the problem, a rotational transformation for $-\pi/4$ is applied onto $0^\circ - 0^\circ$ plane. The VSD transformation matrix now becomes (see (4)) One can see that the newly defined VSD matrix is unchanged for $a - \beta$ and $x-y$ plane, while it has new form for $0^\circ - 0^\circ$ plane. By using this transformation matrix, the projections of the leg and phase voltage space vectors into $0^\circ - 0^\circ$ plane are still not the same, but they have now identical projections onto the $0^\circ$ axis. In fact, the $0^\circ$ axis now represents CMV [5], which is normally not considered in space vector algorithms. Therefore, by not considering the $0^\circ$ axis, the projections of phase voltage and leg voltage space vectors are now identical. Hence, it can be said that it is advantageous to use (4) instead of (3) to develop the algorithm for the analysed drive topology. As a result, the realisation of sinusoidal phase voltage waveforms ($v_{\phi}^{\alpha}$) through application of leg voltage ($V_{LEGM}$) space vectors is now possible.

In addition, the rotational transformation of $0^\circ - 0^\circ$ plane also does not affect either the mapping of the low order harmonics nor the number of leg and phase voltage space vectors. Only, the low order harmonics of the order of $12k \pm 1$ ($k = 0, 1, 2, 3, \ldots$) which map into $a - \beta$ plane contribute to the machine's torque production. The low order harmonics of the order of $12k \pm 5$ ($k = 0, 1, 2, 3, \ldots$) and $3k$ ($k = 1, 3, 5, \ldots$), which map into $x-y$ and $0^\circ - 0^\circ$ plane, respectively, contribute to the machine losses [3]. Therefore, in the developed space vector algorithm, it is required that the projection of the chosen space vectors onto $x$, $y$ and $0^\circ$ axis (note that the $0^\circ$ axis is not being considered anymore) is zero on average within the switching period.
Normally, the subsequent space vectors, i.e. corresponding switching states, for every switching sequence are chosen within their own respective sectors in $\alpha - \beta$ plane [7–10]. However, this is not possible for the analysed drive topology. As an example, one can see the number of the remaining space vectors in sector S1-A in Fig. 3a is only four (corresponding switching states are: 0 or 63, 32, 48 and 57). Therefore, in order to have six space vectors per switching sequence, two additional space vectors have to be chosen outside S1-A sector. As a result, only potential switching sequence comprising of switching states 0, 32, 48, 49, 57, 59 and 63 i.e. $[000000–100000–110000–110001–111001–111011–111111]$ is able to satisfy the previously stated requirements and desirable conditions for sector S1-A. The leg voltage transitions of the chosen space vectors for S1-A are denoted by different-colour arrows in Fig. 3.

The potential switching sequences for the other sectors can be similarly determined which results in eighteen switching sequences (sequences for the sectors: S1-A, S1-B, S2, S3-A, S3-B, S4 etc.). These switching sequences produce their own unique leg voltage transition patterns in $\alpha - \beta$ and $x–y$ planes as well as on the 0$−$ axis. Thus, it can be said that there is no switching sequence redundancy for the analysed drive topology.

2.4 Calculation of dwell times

As the reference phase voltage ($v_{ph}^*$) space vector transverses through different sectors in $\alpha - \beta$ plane (S1-A, S1-B, S2, S3-A etc.), corresponding switching sequences should be applied. The times of application for each space vector within the sequence can be easily determined using:

$$
\begin{align*}
T_1 & = [V_{a,1} V_{b,2} V_{c,3} V_{a,4} V_{b,5} V_{c,6} ]^T \\
T_2 & = [V_{a,1} V_{b,2} V_{b,3} V_{a,4} V_{b,5} V_{b,6} ]^T \\
T_3 & = [V_{a,1} V_{a,2} V_{b,3} V_{a,4} V_{b,5} V_{b,6} ]^T \\
T_4 & = [V_{a,1} V_{a,2} V_{a,3} V_{a,4} V_{a,5} V_{a,6} ]^T \\
T_5 & = [ 1 1 1 1 1 1 ]^T \\
T_6 & = [ V_{0,-1} V_{0,+2} V_{0,+3} V_{0,+4} V_{0,+5} V_{0,+6} ]^T
\end{align*}
$$

where $T_k (k = 1, 2, \ldots, 6)$ are the dwell times of the applied space vectors. Since the first and the seventh switching state correspond to the same space vector, $T_1$ is equally shared between the two [11]. Note that due to the 0$+$ axis is not being considered in the algorithm, the fifth row of (4) is replaced by time balancing equation, as in (5). This ensures that the sum of the calculated dwell times is equal to the switching period ($T_{sw}$). The $x$, $y$ and 0$−$ reference voltages ($V_x^*$, $V_y^*$ and $V_{0-}^*$) are set to zero to meet the previously stated requirement. As for the $\alpha$ and $\beta$ reference voltages ($V_\alpha^*$ and $V_\beta^*$), they are set to $V_f^*\cos(\omega t)$ and $V_f^*\sin(\omega t)$, respectively. In fact, all these reference voltages are projections of (1) onto $\alpha$, $\beta$, $x$, $y$ and 0$−$ axes.
The simulation is run in open loop until the machine is controlled using Table 1. The inverter is developed based on [14] with parameters as listed in Table 1. The inverter is assumed ideal and supplied with frequency is chosen to be 2 kHz. The machine model is calculated using (5). If the solution of (5) exists, the location of the corresponding space vector is denoted by a dot in α−β plane. The regions of application for the switching sequences {0 32 48 49 57 59 63} and {0 16 48 56 60 61 63}, which correspond to S1-A and S2 sectors, are shown in Fig. 4. It can be seen that the solutions for the dwell times do not exist within the whole sector, but are enclosed within certain limit. In fact, this limit can also indicate the maximum modulation index (m) for linear operation of the analysed drive topology, which is m = 1.035. This is also in agreement with [13]. The m is defined as

\[ m = \frac{V_i^*}{\sqrt{3}V_{dc}} \]  

(6)

2.5 Determination of regions of application

Although the switching sequences have been successfully chosen for each sector, the dwell times, i.e. solutions of (5) do not necessary exist throughout the whole region in their respective sector. Hence, the regions of application where the solutions of (5) exist need to be properly determined first. This is done through visualisation of the location of \( v_{bh} \) space vector in α−β plane [12]. Thus, by gradually increasing the magnitude of \( v_{bh} \) space vector \((V_j')\) from zero up to \( \sqrt{2+\sqrt{3}}V_{dc} \) (the circumference of the largest polygon in α−β plane), the dwell times of each switching sequence is repetitively calculated using (5). If the solution of (5) exists, the location of the corresponding \( v_{bh} \) space vector is denoted by a dot in α−β plane. The regions of application for the switching sequences [0 32 48 49 57 59 63] and [0 16 48 56 60 61 63], which correspond to S1-A and S2 sector, are shown in Fig. 4. It can be seen that the solutions for the dwell times do not exist within the whole sector, but are enclosed within certain limit. In fact, this limit can also indicate the maximum modulation index \( m_i \) for linear operation of the analysed drive topology, which is \( m_i = 1.035 \). This is also in agreement with [13]. The \( m_i \) is defined as

\[ m_i = \frac{V_i^*/\sqrt{3}V_{dc}}{2} \]  

3 Simulation and experimental results

A PWM modulator based on the proposed space vector algorithm is developed and simulated in PLECS software. The inverter is assumed ideal and supplied with \( V_{dc} = 300 \text{ V} \). Its switching frequency is chosen to be \( f_{sw} = 2 \text{ kHz} \). The machine model is developed based on [14] with parameters as listed in Table 1. The machine is controlled using \( V/f \) and operated at no load. At the machine’s rated frequency (50 Hz), i.e. at \( m_i = 1 \), the \( V_i^* \) is chosen to be 150 V. The simulation is run in open loop until the machine has reached steady state for full linear modulation index range from \( m_i = 0.1 \) up to 1.035. The performance of the proposed algorithm is determined through the phase voltage and current total harmonic distortion (THDv and THDi) analysis, [15]. They are calculated as

\[ \text{THD}_v = \sqrt{\sum_{k=1}^{9} V_k^2/V_1^2} \]  

\[ \text{THD}_i = \sqrt{\sum_{k=1}^{7} I_k^2/I_1^2} \]  

where \( V_k \) and \( I_k \) represent the \( k \)th components of the voltage and current in the spectrum, while \( V_1 \) and \( I_1 \) are the fundamental values of the phase voltage and current, respectively. Harmonic components up to the value of \( h \) that corresponds to 21 kHz, i.e. first 10 sidebands, are included in the THD calculation. In addition, the proposed algorithm is also validated experimentally. Used experimental setup is shown in Fig. 5. Unlike the ideal inverter which was used in simulations, the real inverter has built-in dead time which is set to 6 μs. Moreover, the dead-time was not compensated. Obtained simulation results of phase ‘a’ at \( m_i = 1 \) are shown in Fig. 6. One can see that the fundamental phase ‘a’ voltage (|\( V_{a\text{fund}} | \)) is 150 V, which is in agreement with (6). In addition, it can be seen that the magnitude of the phase voltage low order harmonics, which map onto x, y and 0° axes (the values of the 3rd, 5th, 7th and 9th harmonic only, are shown in Fig. 6) are negligible comparing to \( V_{a\text{fund}} \). This is because, the \( V_x, V_y \) and \( V_0 \) are set to zero. One can also see that the phase voltage component of 0° axis is zero, which is expected since the sum of the phase voltages is zero. Furthermore, it can also be seen that the low order harmonics in the phase ‘a’ current spectrum also do not exist. In fact, the magnitude for the third harmonic is zero. Nevertheless, higher order harmonics do exist in the side bands around multiples of the switching frequency, i.e. around \( k \cdot f_{sw} (k = 1, 2, 3, \ldots) \).

The experimental results are shown in Fig. 7. Obtained \( |V_{a\text{fund}} | \) is also in agreement with (6), which further validates the developed algorithm. Measured value is slightly below 150 V because of the uncompensated dead-time effect. Although \( V_x, V_y \) and \( V_0 \) are set to zero, to ensure that their respective low order harmonics do not exist, they are still present in both phase voltage and current spectra. However, these harmonics are fairly small. The appearance of these low odd order harmonics is the consequence of the uncompensated inverter dead time [16, 17]. Calculated THDv and THDi of the proposed algorithm for full linear range of modulation index (m) are shown in Fig. 8. One can

| Parameter          | Value          | Parameter          | Value          |
|--------------------|----------------|--------------------|----------------|
| resistance \( R_t \) | 13.75 Ω        | resistance \( R_s \) | 5.775 Ω        |
| leakage inductance  | \( L_a = 5.3 \text{ mH} \) | leakage inductance  | \( L_b = 12.7 \text{ mH} \) |
| mutual inductance   | \( L_{ab} = 296.5 \text{ mH} \) | pole-pairs        | 3              |

![Fig. 4 Regions of application of S1-A and S2](image_url)

![Fig. 5 Experimental setup](image_url)
see that THD\textsubscript{v} and THD\textsubscript{i} characteristics of the obtained simulation and the experimental results are in a very good agreement. Small discrepancies are expected because the dead time is not modelled in the simulation.

4 Conclusion

In this paper, a space vector algorithm for two-level asymmetrical six-phase drive is proposed. A modified VSD transformation matrix is used in order to make the realisation of sinusoidal phase voltage waveforms through application of leg voltage space vectors possible. Furthermore, it is observed that the space vectors of the chosen switching sequences are not necessarily located within the same sector. The proposed algorithm is verified through simulation (ideal case) and experimentally for the full linear range of modulation index. Obtained simulation results show that the output phase voltage and current waveforms do not contain low order harmonics. Yet, due to uncompensated inverter dead time, these...
harmonics can be observed in the experimental results, though small in value.

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