A POSSIBLE EXPLANATION FOR THE SIZE DIFFERENCE OF RED AND BLUE GLOBULAR CLUSTERS

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\section*{ABSTRACT}

Most observations of the projected half-light radii of metal-rich globular clusters in a variety of galaxies have shown them to be $\sim 20\%$ smaller than those of their metal-poor counterparts. We show using multi-mass isotropic Michie-King models that the combined effects of mass segregation and the dependence of main sequence lifetimes on metallicity can account for this difference, under the assumption that clusters with similar central potentials have the same distribution of half mass radii. If confirmed, this would represent a new constraint on theories of globular cluster formation and evolution.

\textit{Subject headings:} galaxies: elliptical and lenticular, cD — galaxies: star clusters — globular clusters: general

\section{1. INTRODUCTION}

The last decade has seen rapid progress in the characterization of globular cluster (hereafter GC) systems in external galaxies. An important task is to disentangle the properties of GCs which are universal form those that are correlated with other of their properties or those of their host galaxies. The \textit{Hubble Space Telescope} can partially resolve the spatial profiles of GCs well beyond the Local Group, and thus study their structural parameters. The recovery of these parameters requires modeling of the point-spread function, and this has been carried out using different methods and instruments by various groups, using a range of galaxies including spirals and ellipticals (Kundu \& Whitmore 1998; Kundu et al. 1999; Puzia et al. 1999; Larsen, Forbes \& Brodie 2001; Barnby, Holland \& Huchra 2002; Larsen et al. 2001; Harris et al. 2002; Jordán et al. 2004). Most of these studies have revealed that metal-rich (red) GCs appear to have half-light radii $\sim 20\%$ smaller than their metal-poor (blue) counterparts.

Larsen \& Brodie (2003) have advanced the only plausible explanation so far for this size difference. They argue that the observed difference can arise as a projection effect, resulting from combination of a correlation between galactocentric distance and size and the differing spatial distributions of the GC subpopulations. Assuming that GCs in all galaxies follow a relation between galactocentric distance and size similar to that of Galactic GCs, they find that this mechanism is able produce the observed size difference, albeit with some fine tuning.

In this Letter we propose a simple explanation for the observed difference. We propose that the difference is a consequence of mass segregation and the fact that lower metallicity stars have longer lifetimes for a given mass. Assuming that the average half mass radius does not depend on metallicity, we model the observed light profiles with Michie-King multi-mass models and stellar isochrones, and show that a size difference of the observed magnitude arises naturally.

\section{2. MODELS}

We model GCs using Michie-King multi-mass isotropic models. The formalism is described in Gunn \& Griffin (1979). In what follows, we repeat some of their expressions restricted to the isotropic case to set the notation.

Each mass class is labeled by an integer $j$ and is assumed to have a distribution function of the form $f(E) = e^{A_j E} - 1$, where $E = \frac{1}{2} \psi^2 + \psi$ and $\psi$ is the potential energy. Due to the short relaxation times at the core, we assume thermal equilibrium there, which demands that $A_j = \beta \bar{m}_j$, where $\beta$ is a constant and $\bar{m}_j$ is the mean mass of the $j$-th class. Choosing a characteristic radius $r_c$ and velocity variance $v_0^2$, and letting $W = -\psi/v_0^2$ and $\xi = r/r_c$, the Poisson equation reads $\nabla^2 W = -9\sigma$, where $\sigma = \rho/\rho_0$, $\rho = \sum_j \rho_j$ is the mass density and $\rho_0 \equiv \rho(0)$.

If we define $\alpha_j = \rho_{0j}/\rho_0$ to be the fractional density contribution of mass class $j$ at the center, a model is completely specified by a value for $W$ at the center, $W_0$, and the $\{\alpha_j\}$. With these quantities specified, the Poisson equation is integrated until $W = 0$, at which point the tidal limit is reached. The normalized mass densities for class $j$, $\sigma_j(r)$, are then calculated. The models have mass segregation, and there is no expression relating the $\{\alpha_j\}$ to the total mass in class $j$, which is the quantity we would like to specify via a global initial mass function $\zeta(m)$, defined such that the total number of stars in the GC with initial masses between $m$ and $m + dm$ is $\zeta(m) dm$, and a relation between initial and present mass, $m_f(m)$. The problem is solved by iteration until the $\{\alpha_j\}$ and the total mass in class $j$, $M_f = \int m_f(m) \zeta(m) dm/\sum_i M_i$, are self-consistent. The space densities are finally projected to obtain the observed mass densities.

Determining the initial mass function in GCs is a difficult task because of the need to take into account the effects of mass segregation. Using Michie-King multi-mass models to account for dynamical effects, Paresce \& De Marchi (2000) find that the mass functions of a dozen globulars are well described by a lognormal distribution with a mean $m_c = 0.33$ and dispersion $\sigma = 0.34$. We will adopt this as the mass function for GCs for...
$m \leq 0.9M_\odot$. At higher masses, it is found that the IMF is well described by a power law with an exponent close to Salpeter (Chabrier 2003). Thus, we adopt $\zeta(m) \propto m^{-x}$ for $m > 0.9M_\odot$. In what follows we assume $x = 2$, as this allows us to obtain a good match to the observed size difference in M87. While this value is certainly consistent with the observations, there is otherwise no fundamental reason for our choice. Below we comment the effect of varying $x$.

We construct the mass classes as follows. We take the minimum mass in the initial mass function to be 0.1$M_\odot$ and the maximum to be 30$M_\odot$. If $m_{\text{to}}$ is the mass at the main sequence turnoff, $m_{\text{trgb}}$ the mass at the tip of the red giant branch and $N = [(m_{\text{to}} - 0.4)/0.1] \equiv (m_{\text{to}} - 0.4)/\Delta m$, the limits of the mass classes are $(0.1, 0.2, 0.3, 0.4, 0.4 + \Delta m, \ldots, 0.4 + (N - 1)\Delta m, m_{\text{to}}, m_{\text{trgb}}, 2, 3, 4, 5, 6, 7, 8, 30)$. The masses in each class are obtained differently depending on whether the value of the mass is greater than $m_{\text{trgb}}$. We ignore in what follows the evolutionary stages between the tip of the red giant branch and a white dwarf. This should not affect greatly the derived light profiles, as those stars will have density profiles close to that of stars with $m_{\text{to}}$, and thus will just add light to the already dominating component. For $m < m_{\text{trgb}}$, we have simply $M_j \propto \int_j m\zeta(m)dm$. We assume that stars with $m_{\text{trgb}} < m < 8$ end their lives as white dwarfs and that stars with $8 < m < 30$ end as neutron stars with a mass of 1.4$M_\odot$. For the masses $m_{\text{trgb}} < m < 8$, we determine the relation between the white dwarf mass and the star’s initial mass, $m_{\text{wd}}(m)$, by interpolating linearly the data presented in Table 3 of Weidemann (2000). The mass of the white dwarf bins are given then by $M_j \propto \int_j m_{\text{wd}}(m)\zeta(m)dm$. Neutron stars are observed to be born with velocity kicks of hundreds of km s$^{-1}$ (e.g. Lyne & Lorimer 1994), and so most of them should escape from their host GCs. Of course, the presence of millisecond pulsars and low mass X-ray binaries in GCs means that some of the neutron stars must be retained. Here we will assume that a fraction $f_{ns} = 0.05$ of neutron stars are retained by a typical GC. The mass in the neutron star mass class is then $M_{ns} \propto 1.4f_{ns}\int_{m_{\text{to}}}^{30} \zeta(m)dm$. The adopted value for $f_{ns}$ is consistent with the results of a comprehensive study of neutron star retention in GCs by Pfalz, Rappaport & Podsiadlowski (2002). Our results are not very sensitive to the precise value adopted for $f_{ns}$ and remain essentially unchanged when varying $f_{ns}$ in the range 0.01 - 0.1.

The final ingredient to obtain the observed light profiles is stellar isochrones, from which we obtain the mass-luminosity function $L(m)$. We used the Y$^2$ isochrones (Yi et al. 2003). Observational evidence points to most GCs in early-type galaxies being an old and coeval population (see, e.g., Jordán et al. 2002). We will thus assume that GCs have an age of 13 Gyr, the mean age inferred for the GC system of M87 using spectroscopic line indices (Cohen, Blakeslee & Ryzhov 1998). We interpolated stellar populations with [Fe/H] in $\{−2, −1.75, −1.5, −1.25, −1, −0.75, −0.5, −0.25, 0\}$, an age of 13 Gyr and alpha enhancement $\alpha = 0.3$. Note that the stellar isochrones have a minimum mass of 0.4$M_\odot$. To obtain the luminosities of lower mass stars we used the zero-age main sequence mass-luminosity relation of Tout et al. (1996), ensuring continuity with the luminosity given in the Y$^2$ isochrones for $m = 0.4M_\odot$. With a given isochrone in hand, we determined $m_{\text{to}}$, $m_{\text{trgb}}$, and constructed the corresponding Michie-King multi mass model. The projected densities of each of the mass classes with $m < m_{\text{trgb}}$ were then multiplied by the mean V-band luminosity $L_vj$ obtained from the isochrones as $L_vj = \int_j L(m)\zeta(m)dm/\int_j \zeta(m)dm$.

As described above, the models constructed are specified by a value of $W_0$ and the set of $\{\alpha_j\}$. There remain two arbitrary scale factors, which correspond to setting the scale for the spatial coordinates and the overall mass of the system. In order to compare the models against each other, we set the half mass radius $r_{hm}$ to the same physical length for all models. This assumption would follow if the overall structure of young GCs is determined by processes mostly independent of metallicity, and if GCs were subjected afterwards on average to the same dynamical effects. For each model, we then recorded the projected half light radius $r_{hl}$ in units of $r_{hm}$ (for each metallicity and $W_0$).

3. RESULTS AND DISCUSSION

In Figure 1 we show mass and light profiles for two models with $W_0 = 9$ and [Fe/H] equal to $−1.5$ and $−0.5$. The half light radii are indicated in the figure, and it can be seen that $r_{hl}$ is smaller for the metal-rich model by 14%. This is because the mass of the most luminous stars becomes larger as the metallicity increases, and thus their density profile is more concentrated. In Figure 2 we show $r_{hl}$ as a function of [Fe/H] for various values of $W_0$. It is evident from the figure that for a given value of $W_0$, $r_{hl}$ gets smaller as the metallicity increases. Note that the half light radii are roughly half the corresponding half-mass radii. In order to see directly the size of the effect, we also plot the same curves normalized to their values.
at [Fe/H] = −1.5, typical of metal-poor GCs. A typical metal-rich GC in early-type galaxies will have [Fe/H] \sim −0.3. The size of the effect at that metallicity is in the range \sim 5−25\%, the exact value depending on the value of W_0. We will take a W_0 = 9 model to be representative of a typical GC, as the light profile of such a model will have \log(r_t/r_{cl}) \sim 1.58, where r_t is the tidal radius and we have defined the core radius of the projected light profiles r_{cl} in a way akin to single mass King models, which for a concentration c \sim 1.5 is 1.05 times the radius at which the luminosity is half the central value. Thus, this concentration measure is comparable to a typical concentration of a Galactic GC (Harris 1996). From Figure 2 we see that a typical metal-rich GC in an early-type galaxy will be observed to be \sim 20\% smaller than a corresponding metal-poor GC. This is consistent with the observations, and thus our models can explain them by assuming that GCs have universal physical properties and the combined effects of mass segregation and stellar evolution.

We can go beyond the mean difference in size, as our procedure gives a definite prediction for the behavior of the half light radius as a function of [Fe/H]. The proper way of comparing with the observations would be to input the distribution of W_0, and then get the predicted behavior for r_{hl}. As we don’t have the distribution of central potentials available to us, we will assume as above that the population of GCs is well represented by a model with W_0 = 9. In Figure 3 we show the measured half light radii (average of g_{475} and z_{475} measurements) with uncertainties less than 0.5 pc for GCs in M87, using data from the ACS Virgo Cluster Survey (Côté et al. 2004). These measurements form part of a systematic investigation of structural parameters for GCs in Virgo galaxies (Jordán et al., in preparation). The solid line is a robust smoothing of the data done with the Lowess (Cleveland 1979) method. While we show in the figure only GC candidates with 1 pc < r_{hl} < 4 pc, there was no restriction in r_{hl} for the analysis. The values of [Fe/H] are obtained from (g_{475} − z_{475}) as described in Jordán et al. (2004). The dashed line is the predicted behavior of our models for W_0 = 9, where we have set the normalization such that the curves coincide at [Fe/H] = −1.5. The agreement is very good, especially when considering the crudeness of comparing with a single value of W_0.

The results above depend on the assumption that, given a value of W_0, the average half mass radius does not depend on [Fe/H]. This assumption is appealing in that it points to a universality in the formation and evolution process of GCs. If clusters observed today with a certain central potential were formed with the same average half mass radius and were subjected, on average, to the same dynamical effects from the potential field of their galaxy and internal mass loss processes, they should have on average the same half mass radii. Individual GCs might of course been subjected to quite different histories. The process of GC formation is yet to be fully understood, and thus there are few theoretical handles that would let us assess the plausibility of assuming a constant average half-mass radii with metallicity. Some proposed formation mechanisms determine the overall scale of the proto-GCs by mechanisms that should be largely independent of the metal content, such as cosmological reionization compression of subgalactic halos (Cen 2001), formation out of dense cores of supergiant molecular clouds (Harris & Pudritz 1994) or on the high mass and pressure clouds of gas partitioned by supersonic turbulence (Elmegreen & Efremov 1997). Observationally, Larsen (2004) finds no evidence for variations on the average sizes of young stellar clusters in a sample of nearby spirals. Overall, the assumption seems certainly plausible in light of our current understanding of GC formation.

**Fig. 2.—** (Top) Projected half light radius r_{hl} in units of the half mass radius as a function of [Fe/H]. The different curves correspond to different values of the central potential W_0, ranging from W_0 = 5 (upper curve) to W_0 = 13 (lower curve) in steps of \Delta W = 1. (Bottom) This panel shows the same set of curves normalized to their value at [Fe/H] = −1.5, a typical metallicity for a metal-poor GC.

**Fig. 3.—** The dots are the projected half light radius for a sample of GCs in M87 measured using ACS images. The solid curve represents a robust estimate of the mean half light radius as a function of [Fe/H], and the dashed line is the predicted behavior of this quantity for a model with W_0 = 9 which is normalized to the observed value at [Fe/H] = −1.5.
There are a number of factors that can affect the predicted behavior of our models. An obvious one is the shape of the mass function, as this will change the derived \( \{ \alpha_j \} \). As an example, changing the power law exponent for the high mass part of our assumed mass function to \( x = 2.35 \) reduces the magnitude of the difference by a factor of \( \sim 0.86 \). Another factor which should have an effect insofar as it will change the evolutionary state of the stellar populations is age. If the populations are coeval, changing the age from 13 to 11 Gyr reduces the magnitude of the difference by a factor of \( \sim 0.86 \). If there is an age difference between the subpopulations they would have evolved dynamically for a different total time and the assumption of them having the same half mass radius would be less warranted. Predicting the effect of varying the age properly would thus require knowledge of how the average half mass radii evolves through dynamical effects. This caveat notwithstanding, the fact that most models show \( r_{hm} \) to be a rather stable quantity over the cluster’s evolution (e.g., Aarseth & Heggie 1998) make it reasonable to use the present models and the assumption of constant half-mass radius to get an estimate of the effect of an age difference. If clusters with \( \text{[Fe/H]} \geq -1 \) are 3 Gyr younger than their metal-poor counterparts, the magnitude of the difference would be increased by a factor of \( \sim 1.5 \). At any rate, most determinations of the relative ages of GCs in early-type galaxies, where most of the size difference observations have been made, are consistent with the GCs being roughly coeval (see, e.g., Jordán et al. 2002 and references therein). The relation \( m_{wd}(m) \) for white dwarf remnants also plays an important role in determining the magnitude of the difference as they contribute an appreciable fraction (\( \sim 20\% \)) to the cluster mass.

Variations of the distributions of \( r_{hm} \) and \( W_0 \) with \( \text{[Fe/H]} \) or galactocentric radius can potentially be important, but the main point to be made from our results is that the observations can comfortably be reproduced without resorting to intrinsic differences in the sizes of the GCs as a function of metallicity. The use of Michie-King multi mass models is well-suited to this task, but for a more precise determination of the expected size difference, and the dependence of it with variations in the input ingredients, it would be very useful that this effect be followed with N-body simulations that take into account the effects of stellar evolution and of the gravitational potential of the host galaxy (e.g., Baumgardt & Makino 2003). With more detailed models in hand, the dependencies of the predicted half light radii could perhaps be used with other observed properties to simultaneously constraint factors such as age and the form of the initial mass function.

Our models let us predict that if the GCs are roughly coeval, then the size of the observed average half light radius difference should scale with the mean metallicity of the metal-rich GCs. This is known to correlate with galaxy luminosity (Brodie & Huchra 1991), so the average size difference should scale with it. In contrast with the proposal of Larsen & Brodie (2003) our models do not predict a change in the size difference with projected radius. Larsen & Brodie (2003) argue that some pointings away from the central regions in M87 do not show a significant size difference between the GC subpopulations. Although a dilution of the size difference with radius could be introduced in our models by radial variations in other factor such as the mean \( \text{[Fe/H]} \) of the metal-rich subpopulation, it is unlikely that the effect will disappear at large radii without some fine tuning. Thus, a larger number of observations of size difference at large galactocentric radii will be very useful in discriminating between the models. It is possible that the overall effect results as a combination of projection and mass segregation effects, the former disappearing at large galactocentric radius. We stress that large samples of GCs are needed to investigate this issue, as GCs will have a distribution of intrinsic radii and we need an accurate determination of the average behavior. So while no size difference was reported in NGC 5128 by Harris et al. 2002, the low number of clusters they observed, as they note, precludes the drawing of strong constraints.

We suggest in light of the models presented here that the size differences observed so far are consistent with GCs having half mass radii distributions that do not depend on metallicity. If true, GCs would present us with another universal property, such as the shape of their luminosity function (Harris 2001) or their formation efficiencies (Blakeslee et al. 1997; McLaughlin 1999), which can hold an important clue to their formation and subsequent evolution. And even if the effects of mass segregation do not account entirely for the observed size difference, it is clear that its contribution must be included when interpreting the observations and their implications for GC formation and evolution.

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