Partition layout inside a muffler integrated with a thermoelectric generator: Multi-physics analysis and optimal design

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Abstract

A multi-physics-analysis-based topology optimization (TO) method is proposed to optimally design the internal partition layout of a muffler integrated with a thermoelectric generator (TEG). The basic equations governing the acoustical behavior, heat transfer, and fluid flow in the muffler are introduced, and their interaction is designated for exact numerical analysis in terms of acoustics, heat transfer, and fluid mechanics. To implement density-based TO, one design variable is assigned to each finite element in the design domain, and interpolation functions suitable for each physics phenomenon are employed. In the TO problem formulation, the sum of the squared acoustic pressures at the outlet of the muffler for multi-target frequencies is selected as an objective function to achieve broadband noise attenuation. The temperature of the TEG and the pressure drop are constrained for high energy recovery efficiency and fluid passage, respectively. The optimization problem formulated for the muffler design is solved for various design conditions. Optimal partition layouts are obtained depending on the location and length of the TEG, the upper limit value of the pressure drop, and the number of target frequencies in the same frequency band. The noise attenuation performances of each partition layout are compared, and their expected recovery energies are calculated. One optimal partition layout is discussed in terms of acoustics, heat transfer, and fluid mechanics. The numerical results strongly support the validity of our proposed method for the optimal design of a muffler integrated with a TEG.

Keywords

Topology optimization, muffler design, multi-physics analysis, thermoelectric generator, noise attenuation performance, pressure drop

Introduction

A muffler for duct noise reduction is integrated with a thermoelectric generator (TEG) in an exhaust system for vehicle waste heat recovery and power generation used to open and close an active valve.\textsuperscript{1-4} In general, the efficiency of a TEG is proportional to the difference between the temperatures inside and outside the muffler,\textsuperscript{1} and the noise attenuation performance of a muffler is strongly affected by the partition layout inside the muffler.\textsuperscript{5} A fluid with high thermal energy enters the muffler and exits from it with a relatively low temperature. Considering that the outside temperature is almost constant and the partition is a thermal conductor, the partitions should be optimally placed depending on the position of the TEG for significant noise reduction and high energy recovery efficiency. To this end, the inside temperature distribution should be accurately predicted using multi-physics analysis consisting of acoustical, thermal, and flow analyses. In addition, a muffler

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design method based on multi-physics analysis should be developed to simultaneously improve the noise reduction performance and the efficiency of the TEG.

Various optimal design methods have been developed based on single-physics analysis. In acoustical optimization, shape optimizations have been conducted to enhance the noise attenuation performance of a muffler by parameterizing the geometric design variables. For example, acoustical shape optimization is presented for a horn. Wadbro and Berggren designed an acoustic horn using topology optimization (TO), and Dühring et al. optimized the internal topology of an acoustic wall to achieve a high sound absorption coefficient. Lee and Kim solved the acoustical TO problem to improve the transmission loss (TL) value at a target frequency, and Lee conducted an experimental acoustic validation for topologically optimized mufflers. Lee et al. investigated the noise evaluation indices in acoustical TO problems for muffler design. Kook et al. optimally designed a soundproof wall using TO with Zwicker’s loudness model, and Oh and Lee proposed the sequential method for shape and TO to enhance the noise attenuation performance of a suction muffler. Additionally, reliability and metamaterial concepts have been combined with acoustical TO for muffler design. In thermal optimization, studies have been performed to determine the optimal heat path to achieve maximum thermal expansion on a simplified plate model considering thermal conductivity, as well as the heat sink and source boundary conditions. Similarly, another study was presented for a plate model considering both conduction and convection phenomena. For fluid flow optimization, Borrvall and Petersson proposed a topological duct configuration for Stokes flow under the assumption of very slow speed. Based on this, Guest and Prévost obtained the optimal solutions for various duct compositions. In addition, a TO study presented the optimal duct configuration considering the inertial characteristics of a fluid and pressure drop, and a commercially available software associated with a previous study was created. Subsequently, several advanced fluid flow TO studies have been conducted for unsteady incompressible flow, as well as turbulent flow TO.

On top of the excellent results of the previous studies, two-physics analysis based design results have been reported to reflect a coupling behavior or to increase calculation accuracy for industrial application. For example, based on static and acoustical analyses, Yoon and Lee obtained the optimal topology that simultaneously improves the stiffness and noise attenuation performance of a high-speed train panel, and Yoon et al. solved a simple bulkhead design problem by formulating a TO problem for a hybrid structural-acoustic coupling system. Kontoleontos et al. formulated and solved viscous flow TO problems including heat transfer. Lee and Jang optimized the internal partition layout of a muffler using flow and acoustical TO methods to improve both the pressure drop and noise attenuation performance. Oh and Lee performed TO to obtain an optimal muffler considering acoustical and heat transfer phenomena. In addition, thermo-acoustics studies were carried out to investigate the acoustical damping effect due to the small and narrow zones in the Helmholtz resonator. Of course, simply increasing the number of physics phenomena to consider do not always improve the outcome of design. However, increasing the number of physics phenomena in developing a muffler design method has yielded interesting design results, which could not be obtained without multi-physics analysis.

In this study, a muffler design method based on a three-physics analysis is developed to optimize the internal partition layout of a muffler integrated with a TEG to simultaneously enhance its noise attenuation performance and power generation. The TEG may be replaced by a convection-driven thermoacoustic engine with a piezoelectric generator. The partitions are regarded as thermal conductors, and the acoustical, thermal, and flow analyses are systematically connected in the problem formulation. The structure of this paper is as follows: The fundamental physical properties considered in the muffler optimal design problem formulation are introduced with the governing equations and boundary conditions in terms of acoustics, heat transfer, and fluid mechanics. The interaction between these three physical properties is presented. Then a TO problem based on multi-physics analyses is formulated for the design of a muffler integrated with a TEG. The objective function and constraints are defined with the design variables. After that, the formulated TO problem is solved for various design conditions. The optimal topologies are discussed based on the geometric parameters of the TEG, target frequency, and allowed pressure drop value. Finally, the conclusion is presented in the last section.

Three physics for muffler analysis and design

In general, a fluid with high thermal energy flows into the muffler and exits from its outlet, and noise enters the muffler together with the fluid and is transmitted to the outlet. Rigid partitions are inserted inside the muffler to reduce the level of the transmitted noise for the frequency range of interest. Due to the partitions, an increase in the fluid resistance results in a pressure drop, which is the pressure difference between the inlet and outlet. In addition, a temperature gradient occurs inside the muffler because of the thermal resistance change resulting from the partitions and heat energy exchange through the muffler cover panel. Therefore, the three physics (acoustics, heat transfer, and fluid mechanics) should be carefully evaluated to accurately predict the performance of a muffler when the vibration of the panel and partition is ignored.
**Figure 1.** An illustration of the muffler integrated with a thermoelectric generator.

Figure 1 shows a muffler integrated with a TEG. The muffler consists of an inlet, an expansion chamber, and an outlet, the centers of which do not coincide with one another. The TEG is commonly used to convert thermal energy into electrical energy. It consists of a heat source plate, sink plate, in-between metals, and thermocouples as shown in the inset of Figure 1. The heat source plate is attached to the cover panel of the expansion chamber of the muffler. It is assumed that the fluid flowing into the muffler is air and the fluid inside the muffler exchanges heat energy with the fluid outside the muffler through the cover panel. The vibration of the muffler is not considered in this work. The depth of the muffler is sufficiently small, compared with its length and height, such that the muffler can be regarded as a 2-dimensional muffler integrated with a TEG, as shown in Figure 2, which is a 2-dimensional finite element model.

**Governing equations and boundary conditions**

According to fluid mechanics theory, the continuity equation and Navier–Stokes momentum equation for calculating the fluid velocity \( u^f \) and pressure \( p^f \) are expressed as equations (1) and (2), respectively, where the superscript “\( f \)” denotes fluid-dependent variables. In addition, the energy equation for calculating the temperature \( T \) is given as equation (3), which is fully coupled with equations (1) and (2) due to the flow velocity

\[
\nabla \cdot \mathbf{u}^f = 0 \quad (1)
\]

\[
-\rho \nabla \cdot \left( \nabla u^f + (\nabla u^f)^t \right) + \rho \left( \nabla u^f \cdot (\nabla u^f)^t \right) + \nabla p^f = \mathbf{b} \quad (2)
\]

where the superscript “\( t \)” denotes the transpose of a vector or a matrix, and \( \mu, \rho, \mathbf{b} \) denote the viscosity coefficient, density, and body force, respectively

\[
-\nabla \cdot (k \nabla T) + \rho C_p u^f \cdot \nabla T = 0 \quad (3)
\]

where \( T \) is the temperature and \( k \) and \( C_p \) are the thermal conductivity coefficient and specific heat capacity at constant pressure, respectively.

The Dirichlet boundary conditions are assigned to the inlet \( (\Gamma_{in}) \), outlet \( (\Gamma_{out}) \), and wall \( (\Gamma_{wall}) \), as shown in Figure 2, for flow analysis using equations (4)–(6).
\( u_f = u_f^{in} \) on \( \Gamma_{in} \) 
\( p_f = p_f^{out} \) on \( \Gamma_{out} \) 
\( u_f = u_f^{wall} \) on \( \Gamma_{wall} \)

where \( u_f^{in}, p_f^{out}, \) and \( u_f^{wall} \) represent the velocity vector at the inlet, the pressure at the outlet, and the velocity vector on the wall, respectively. In the heat transfer analysis, the Dirichlet and Neumann boundary conditions are expressed as equations (7)–(9)

\[ T = T_{in} \quad \text{for} \quad \Gamma_{in} \]
\[ \nabla T \cdot n = q_{out}/k_{out} \quad \text{for} \quad \Gamma_{out} \]
\[ \nabla T \cdot n = h_{wall}(T - T_{ext}) \quad \text{for} \quad \Gamma_{wall} \]

where \( T_{in}, q_{out}, k_{out}, h_{wall}, \) and \( T_{ext} \) denote the temperature of the inlet, the heat flux at the outlet, the heat transfer coefficient at the outlet, the heat transfer coefficient of the wall of the muffler, and the temperature outside the muffler, respectively, and \( n \) is the normal vector.

The acoustic pressure \( (p^a) \) is governed by the Helmholtz equation as expressed in equation (10). In addition, the vibration velocity \( (u^a) \) of a particle in an acoustic medium is obtained with equation (11), which is called a linearized Euler equation. In all equations, the superscript “\( ^a \)” denotes acoustics-dependent variables

\[ \nabla \cdot \left( \rho^{-1} \nabla p^a \right) + \omega^2 p^a / B = 0, \quad B = \rho c^2 \]
\[ \nabla p^a = -j\omega\rho u^a \]

where \( B, \rho, \) and \( \omega \) denote the bulk modulus, the speed of sound, and the angular frequency, respectively. The Neumann boundary conditions and Robin boundary conditions are expressed as equations (12)–(14)

\[ \nabla p^a \cdot n = ( -j\omega \rho a u^a) \cdot n \quad \text{for} \quad \Gamma_{in} \]
\[ p^a / (\nabla p^a \cdot n) = j c_{out} / \omega \quad \text{for} \quad \Gamma_{out} \]
\[ p^a / (\nabla p^a \cdot n) = ( -j\omega \rho a u^a_{wall}) \cdot n \quad \text{for} \quad \Gamma_{wall} \]

where \( \rho_{in} \) and \( c_{out} \) represent the density at the inlet and the speed of sound at the outlet, respectively, and \( \rho \) is the density of air inside the muffler. Note that the density \( \rho \) is affected by only temperature distribution as introduced in equation (2). \( u^a_{in} \) and \( u^a_{wall} \) represent the vibration velocity of the particles in an acoustic medium at the inlet and the muffler wall, respectively.

**Figure 2.** A 2-dimensional muffler model.
Interaction between the three physics

This subsection explains the interaction mechanism of the three physical properties (acoustics, heat transfer, and fluid mechanics) to properly analyze and design a muffler in Figure 2. It is assumed that the partitions are made of 409 stainless steel, which is a thermal conductor. The acoustic properties, bulk modulus \( B_{\text{air}} \), and the speed of sound \( c_{\text{air}} \) affect the acoustic wave transmission are functions of temperature \( T \):

\[
B_{\text{air}}(T) = \rho_{\text{air}}(T) c_{\text{air}}(T)^2
\]

\[
\rho_{\text{air}}(T) = 372.7 \times \frac{1}{C_0^{1.011}}
\]

\[
c_{\text{air}}(T) = 331.3 \times \sqrt{T/273.15}
\]

where the unit of temperature is Kelvin (K). Therefore, the temperature distribution must be accurately determined by heat transfer analysis before an acoustical analysis can be conducted for the muffler. \(^{34}\)

In addition, the material properties \( B_{\text{steel}}, \rho_{\text{steel}}, \) and \( k_{\text{steel}} \) of 409 stainless steel also change with temperature as expressed in equations (15)–(17), which are only valid in the temperature range from 200 K to 1600 K\(^{31}\):

\[
k_{\text{steel}}(T) = 24.21475 + 0.00452 \times T
\]

\[
B_{\text{steel}}(T) = \frac{E_{\text{steel}}(T)}{3(1-2\nu)}
\]

\[
E_{\text{steel}}(T) = 2.49144 \times 10^{11} - 1.417075 \times 10^{9} \times T
\]

\[
+ 177140.6 \times 10^2 \times T^2 - 111.1717 \times T^3
\]

\[
\rho_{\text{steel}}(T) = 7770.253 - 0.2324425 \times T
\]

\[
- 2.488097 \times 10^{-5} \times T^2
\]

\[
k_{\text{air}}(T) = -0.0022758 + 1.1548 \times 10^{-4} \times T - 7.9025 \times 10^{-8} \times T^2
\]

\[
+ 4.1170 \times 10^{-11} \times T^3 - 7.4386 \times 10^{-15} \times T^4
\]

where a Poisson’s ratio of \( \nu = 0.33 \) is used. The governing equations in equations (1)–(3) for flow and thermal analyses are coupled due to the term of \( u' \) and must be carefully solved because the material properties \( C_p \) and \( \mu \) as well as \( k \) and \( \rho \) are functions of temperature \( T \): \( C_p = C_p(T) \) and \( \mu = \mu(T) \).

The multi-physics analysis procedure is summarized in the flow chart in Figure 3, where the thermal and flow analyses are followed by the acoustical analysis. First, equation (3) is solved for the initial values \( (u'_{\text{ini}}, T_{\text{ini}}) \) of a velocity vector and temperature distribution, and a temporary temperature distribution \( (T_{\text{temp}}) \) is obtained. Then \( u' \) and \( \rho' \) are calculated with equations (1) and (2) for \( T_{\text{temp}} \), and a temporary velocity vector \( (u'_{\text{temp}}) \) and temporary pressure \( (\rho'_{\text{temp}}) \) are obtained. Using \( T_{\text{temp}} \) and \( u'_{\text{temp}} \) as the new initial values of \( T_{\text{ini}} \) and \( u'_{\text{ini}} \), respectively, equation (3) is solved again. This process is
The muf
Density-based TO problem formulation

The partition layout of the muf

repeated until the convergence criteria in equation (19) is satisfied, and a temperature distribution \( T_f \) is determined finally after repetitive calculation

\[
\varepsilon_u' \leq \varepsilon_u^{\text{crit}} \quad \text{and} \quad \varepsilon_T \leq \varepsilon_T^{\text{crit}} \tag{19a}
\]

\[
\varepsilon_u' = \max_{\Omega_f} \left| \mathbf{u}_f - \mathbf{u}_f^{\text{temp}} \right| \tag{19b}
\]

\[
\varepsilon_T = \max_{\Omega_f} \left| T_{\text{ini}} - T_{\text{temp}} \right| \tag{19c}
\]

where \( \varepsilon_u' \) is the maximum value among differences between \( \mathbf{u}_f \) and \( \mathbf{u}_f^{\text{temp}} \) calculated at all nodes of the finite element model in Figure 2, and \( \varepsilon_T \) is calculated in the same way. \( \varepsilon_u^{\text{crit}} \) and \( \varepsilon_T^{\text{crit}} \) represent the allowed upper limits of \( \varepsilon_u' \) and \( \varepsilon_T \), respectively. For the final temperature distribution \( (T_f) \), \( \rho \), and \( c \) are determined to solve the Helmholtz equation (equation (10)). Then, the acoustical analysis is performed to calculate \( p^\alpha \) for the determined values of \( \rho \) and \( c \).

**Muffler design goal**

The partition layout of the muffler integrated with the TEG in Figure 2 must be carefully designed to maximize its harvested energy as well as its noise attenuation performance as described.\(^3\)\(^-\)\(^4\) It is well known that the internal partition layout strongly affects the noise attenuation performance of a muffler.\(^1\)\(^2\)\(^3\)\(^4\) The electric power generated by the TEG is a result of the Seebeck effect,\(^4\) which develops across two points of an electrically conducting material due to a temperature difference between the two points. Because the heat sink plate of the TEG on the outside of the muffler is cooled by natural or forced convection, the temperature of the heat source plate should increase. In addition, the energy efficiency of the TEG depends on not only the inside partition layout but also its location on the cover panel of the muffler because the rigid metal partition can increase the temperature of the heat source plate of the TEG. Therefore, the TEG should be optimally positioned on the cover panel of a muffler to maximize harvested energy.

Furthermore, a severe pressure drop should be avoided when designing a muffler. The inside partitions of a muffler play both positive and negative roles: they induce acoustic impedance mismatch to attenuate incoming noise but usually increase flow resistance, increasing the pressure drop of the muffler. Therefore, a muffler should be optimally designed considering not only the acoustical characteristics but also the fluidic characteristics. In summary, in this paper, a muffler integrated with a TEG is optimally designed to enhance its noise attenuation performance considering thermal and fluidic effects.

**TO problem formulation**

**Density-based TO**

The muffler analysis model is divided into the design and non-design domains (\( \Gamma_D \) and \( \Gamma_{ND} \)) as shown in Figure 2. For numerical analysis, the analysis model is converted into a finite element model. Then, one design variable (\( \chi_r \)) is assigned to each finite element in the design domain for the implementation of the density-based TO. The design variable changes continuously between “0” and “1” and determines the extent of the acoustic wave transmission, heat transfer, and fluid flow in the corresponding finite element. Instead of a solid isotropic material with the application of the penalization (SIMP) function,\(^4\) which is widely used in structural TO, the rational approximation of material properties functions\(^4\) are used in the acoustical and thermo-fluid TO. The functions are called interpolation functions, and their effectiveness in acoustical, thermal, and flow TO problems have been sufficiently validated in previous studies.\(^1\)\(^2\)\(^3\)\(^4\)\(^5\)\(^6\)\(^7\)\(^8\)\(^9\)\(^10\)\(^11\)\(^12\)\(^13\)\(^14\)\(^15\)\(^16\)\(^17\)\(^18\)\(^19\)\(^20\)\(^21\)\(^22\)\(^23\)\(^24\)\(^25\)\(^26\)\(^27\)\(^28\)\(^29\)\(^30\)\(^31\)\(^32\)\(^33\)\(^34\)\(^35\)\(^36\)\(^37\)\(^38\)\(^39\)\(^40\)\(^41\)\(^42\)\(^43\)\(^44\)

The density \( (\rho_r) \) and bulk modulus \( (B_r) \) of the \( r \)-th finite element in the design domain for the Helmholtz equation are determined by the interpolation functions given as equation (20)

\[
\rho_r(\chi_r, T_r) = \frac{1}{\rho_{\text{air}}(T_r)} + \chi_r \left( \frac{1}{\rho_{\text{steel}}(T_r)} - \frac{1}{\rho_{\text{air}}(T_r)} \right) \tag{20a}
\]

\[
B_r(\chi_r, T_r) = \frac{1}{B_{\text{air}}(T_r)} + \chi_r \left( \frac{1}{B_{\text{steel}}(T_r)} - \frac{1}{B_{\text{air}}(T_r)} \right) \tag{20b}
\]

where \( T_r \) is the temperature of the \( r \)-th element, and the subscripts “air” and “steel” denote the air and steel elements, respectively. The steel elements are constructed as rigid partitions.\(^1\)\(^2\) The thermal conductivity \( (k_r) \) of the \( r \)-th finite element in the energy equation is determined by the interpolation function given as equation (21)
where \( q_k \) denotes the penalization factor. Unlike acoustical and thermal TO, a body force (b) is interpolated because it is difficult to define the constitutive equations for the intermediate materials in between the fluid-elastic states of the Navier–Stokes equation for the flow (equation (2)). This interpolation is based on Darcy’s law, which expresses the volumetric force term as the extent of penetration of the flow:

\[
b = \begin{cases} 
-\alpha \mathbf{u} & \text{for } \Gamma_D \\
0 & \text{for } \Gamma_{ND}
\end{cases}
\]

where \( \alpha \) represents permeability. As \( \alpha \) increases, the number of fluid particles passing through the associated element decreases, and vice-versa. The interpolation functions for the permeability \( (\alpha_r) \) of the \( r \)-th finite element is expressed as:

\[
\alpha_r(\chi_r) = \alpha_0 + \chi_r(\alpha_1 - \alpha_0)(1 - \chi_r + q_\alpha)^{-1}
\]

where \( q_\alpha \) denotes the penalization factors for the permeability, and \( \alpha_0 \) and \( \alpha_1 \) denote the permeability value of the permeable and impermeable finite elements in the design domain, respectively.

### Physical properties characterizing each physics

The physical properties to be used in the objective functions and constraints are defined for the TO formulation. The noise attenuation performance of the muffler is evaluated with the squared absolute values \( (\Phi_n) \) of the acoustic pressure at a single frequency \( (f_n) \). The value of \( \Phi_n \) is obtained by squaring and summing the real and imaginary parts of the acoustic pressure \( (p_{out}^a) \) at the outlet as expressed in equation (24) with (25):

\[
\Phi_n = \Phi_n^R + \Phi_n^I = \left\{ \text{Re}(p_{out}^a(f_n)) \right\}^2 + \left\{ \text{Im}(p_{out}^a(f_n)) \right\}^2
\]

\[
p_{out}^a(f_n) = \left( \int_{\Gamma_{out}} p^a(f_n)d\Gamma_{out} \right)/m_{out}
\]

where \( m_{out} \) is the number of nodes at the outlet boundary of the muffler. In addition, the pressure difference \( (p_{PD}^f) \), which is expressed as equation (26), is defined as the flow characteristics of the muffler:

\[
p_{PD}^f = |p_{in}^f - p_{out}^f| = \left| \left( \int_{\Gamma_{in}} p^i d\Gamma_{in} \right)/m_{in} - \left( \int_{\Gamma_{out}} p^i d\Gamma_{out} \right)/m_{out} \right|
\]

where \( m_{in} \) is the number of nodes at the inlet boundary of the muffler. Finally, the thermal performance of the TEG is characterized by the temperature \( (T_{TEG}) \) of the heat source plate of the TEG, which is defined as:

\[
T_{TEG} = \left( \int_{\Gamma_{TEG}} T d\Gamma_{TEG} \right)/m_{TEG}
\]

where \( \Gamma_{TEG} \) is the boundary indicating the location of the TEG and is a part of the muffler wall boundary \( (\Gamma_{TEG} \subset \Gamma_{wall}) \), and \( m_{TEG} \) is the number of nodes at the boundary of the muffler attached to the TEG.

### Objective function and constraints

Using the physical properties defined above, the muffler TO design problem is formulated to increase energy harvesting, improve the noise attenuation performance, and avoid severe pressure drop as follows:

\[
\min_{0 \leq \chi_r \leq 1} \Phi
\]

subject to \( p_{PD}^f \leq p_{upp}^f \) \hspace{1cm} \( T_{TEG} \geq T_{low}^{TEG} \)
$\sum_{i=1}^{R} x_i / R \leq V_r$ \hfill (29c)

where $p_{\text{up}}'$ and $T_{\text{low}}^{\text{TG}}$ are the upper and lower limit values for the allowed pressure drop and TEG temperature, respectively. In equation (29c), $R$ is the total number of finite elements in the design domain and $V_r$ is the allowed volume ratio, which is the ratio between the allowed number of steel elements and $R$. The objective function ($\Phi$) is defined using the weighting factor method,\textsuperscript{45} as well as the scaling scheme\textsuperscript{46,47} as follows

$$\Phi = \sum_{n=1}^{N} w_n \Phi_n$$ \hfill (30a)

$$w_n = \frac{\Phi_n^{\text{old}}}{\sum_{n=1}^{N} \Phi_n^{\text{old}}}$$ \hfill (30b)

where the superscript “old” implies the value at the former iteration during the optimization process.

### Numerical results

In this section, the formulated problem is solved for various design conditions. The dimensions of the analysis model in Figure 2 are summarized in Table 1, and the specific values in equations (4)–(9) and (12)–(14) are summarized in Table 2.

The numerical analysis and optimal design are performed using a commercial finite element analysis program, COMSOL Multi-physics with Matlab (Ver. 5.2).\textsuperscript{41} The analysis model is discretized by the square element with a node distance of 0.005 m. The linear shape function is selected for the Helmholtz, energy, and continuity equations for each finite element except for the Navier–Stokes momentum equation that employs the quadratic shape function. The values of $\varepsilon_{\text{crit}}^u$ and $\varepsilon_{\text{crit}}^T$ in equation (19) are set to $10^{-6}$, that is, $\varepsilon_{\text{crit}}^u = 10^{-6}$ and $\varepsilon_{\text{crit}}^T = 10^{-6}$, respectively. A gradient-based optimization algorithm, the method of moving asymptotes, is used to update the design variables in the optimization process.\textsuperscript{48} The values of $\alpha_0$ and $\alpha_1$ in equation (23) are set to $10^{-2}$ and $10^2$, respectively, that is, $\alpha_0 = 10^{-2}$ and $\alpha_1 = 10^2$, referring to previous literature.\textsuperscript{23–27} The values of $q_\alpha$ and $q_k$ in equations (21) and (23) are set to $10^{-3}$ and $10^{-2}$, respectively, that is, $q_\alpha = 10^{-3}$ and $q_k = 10^{-2}$. The optimal partition layouts inside the muffler are obtained depending on the TEG location, TEG size, the upper limit value of the pressure drop, and target frequencies.

### Case I: Optimal topologies depending on TEG location

The TO problem formulated in equations (28)–(30) was solved for various locations of a single TEG: three locations at the upper side of the muffler ($l_{\text{TEG}}^{\text{upp}} = 0.18$ m in Case I-1, $l_{\text{TEG}}^{\text{upp}} = 0.26$ m in Case I-2, and $l_{\text{TEG}}^{\text{upp}} = 0.34$ m in Case I-3) and three locations at the bottom side of the muffler ($l_{\text{TEG}}^{\text{low}} = 0.18$ m in Case I-4, $l_{\text{TEG}}^{\text{low}} = 0.26$ m in Case I-5, and $l_{\text{TEG}}^{\text{low}} = 0.34$ m in Case I-6). In this case study, only one TEG was used and its size ($d_{\text{TEG}}$) was 0.02 m. Three target frequencies ($N = 3$) were selected in the frequency range of interest (450–950 Hz); $f_1 = 450$ Hz, $f_2 = 700$ Hz, and $f_3 = 950$ Hz. In equation (29), $p_{\text{up}}'$, $T_{\text{low}}^{\text{TG}}$, and $V_r$ were set to 0.0015 Pa, 450 K, and 0.05 (5%), respectively, which corresponded to $R_0 = 44$.
Because the TEG size

\[ \text{Case III: Optimal topologies depending on the allowed pressure drop value} \]

In this case study, the effect of TEG size is investigated. In the previous case study, the TEG size was fixed at 0.02 m. Because the TEG size \((d_{\text{TEG}})\) may affect the harvesting performance, several cases with TEG sizes from 0.04 m to 0.08 m are considered, the results of which are compared with that of Case I-2 with a TEG size of 0.02 m; specifically, \(d_{\text{TEG}} = 0.04\) m in Case II-1, \(d_{\text{TEG}} = 0.06\) m in Case II-2, and \(d_{\text{TEG}} = 0.08\) m in Case II-3. The TEG location is identical to that of Case I-2 \((R_{\text{TEG}} = 0.26\) m). The remaining optimization conditions are the same as in Case I-2 for the target frequencies \((f_1, f_2, f_3)\), pressure drop upper limit \((p_{\text{upp}}')\), TEG temperature lower limit \((T_{\text{low}})\), and volume ratio \((V_r)\): \(f_1 = 450\) Hz, \(f_2 = 700\) Hz, \(f_3 = 950\) Hz, \(p_{\text{upp}}' = 0.0015\) Pa, \(T_{\text{low}}^{\text{TEG}} = 450\) K and \(V_r = 0.05\) (5%), which corresponds to \(R_0 = 44\).

The optimal topologies for Case II-1 to Case II-3 are shown with the TEG indicated as red boxes in Figure 7. The TEG temperature values for these mufflers were 450.01 K, 450.00 K, and 452.39 K, and the pressure drop values were all 0.0015 Pa. It was observed that the partitions around the heat source plate of the TEG became thicker compared with Case I-2. This is because the thicker partitions can convey more energy to the TEG for increased energy harvesting. Furthermore, considering that the simulation is conducted in the 2-D domain, the harvesting trend will be exponentially increased in the 3-D domain because the area of the heat source plate and its number of thermocouples will increase accordingly. Figure 8 compares the TL curves of the optimal mufflers for Case II and Case I-2. The TL values at the target frequencies in Cases II-1 and II-2 were lower than that in Case I-2. For increased energy harvesting, the noise attenuation performances of the two cases were sacrificed. However, it is interesting to note that the TL value in Case III-3 was comparable to that in Case I-2. These results imply that increased energy harvesting does not always require sacrificing the noise attenuation performance.

\[ \text{Case III: Optimal topologies depending on the allowed pressure drop value} \]

In this case study, the effect of \(p_{\text{upp}}'\) in equation (29a) on an optimal topology is investigated. In addition to the previous value of \(p_{\text{upp}}' = 0.0015\) Pa for Case I-2), the three new values were used to solve the TO problem formulated in equations (28)-(30): \(p_{\text{upp}}' = 0.0013\) Pa in Case III-1, \(p_{\text{upp}}' = 0.0017\) Pa in Case III-2, and \(p_{\text{upp}}' = 0.0019\) Pa in Case III-3.

**Table 2.** Specific values assigned to the boundary conditions for the multi-physics analysis.

| Symbol | Value |
|--------|-------|
| Flow analysis | \(u_{\text{in}}\) | \(\{0.05, 0\}\) m/s |
| | \(p_{\text{in}}\) | 0 Pa |
| | \(u_{\text{out}}\) | \(\{0, 0\}\) m/s |
| Heat transfer analysis | \(T_{\text{in}}\) | 473.15 K |
| | \(q_{\text{out}}\) | 0 W/m² |
| | \(h_{\text{wall}}\) | 0.02 W/m²K |
| | \(T_{\text{ext}}\) | 293.15 K |
| Acoustic analysis | \(u_{\text{in}}\) | \(\{10^{-3}, 0\}\) m/s |
| | \(c_{\text{out}}\) | \(c(T_{\text{out}})\) |
| | \(u_{\text{wall}}\) | \(\{0, 0\}\) m/s |

Figure 4 shows the history of the objective function with several topologies during the optimization process for Case I-2. Figure 5 shows the optimal topologies for Case I-1 to I-6, where the red boxes on the top and bottom represent a TEG. Each optimal topology had one large steel partition and several small steel branch partitions. The large partition started from the location of each TEG. To show the noise attenuation performance of each optimal topology, the TL was calculated using equation (31)³

\[
TL(f) = 10 \log_{10} \left( \frac{|p_{\text{out}}'(f)|^2}{|p_{\text{wall}}'(f)|^2} \right) \quad (31)
\]

where \(p_{\text{out}}'(f)\) were calculated similarly to equation (25). Considering that \(p_{\text{wall}}'(f)\) in equation (31) is assumed to be constant at all frequencies, a small value of \(|p_{\text{wall}}'(f)|\) represents superior noise attenuation performance at the target frequency. Figure 6 compares the TL curves of the optimal mufflers in Figure 5, and they all show the increased TL values compared with the nominal muf-fer (= empty muf-fer) in the frequency range of interest. The values of \(T_{\text{TEG}}\) for the optimal topologies are summarized in Table 3. It is worth noting that \(T_{\text{TEG}}\) was higher at the top of the muf-fer than at its bottom for the same distance from the left end and increased as the location of the TEG approached the inlet.

**Case II: Optimal topologies depending on TEG size**

In this case study, the effect of TEG size is investigated. In the previous case study, the TEG size was fixed at 0.02 m. Because the TEG size \((d_{\text{TEG}})\) may affect the harvesting performance, several cases with TEG sizes from 0.04 m to 0.08 m are considered, the results of which are compared with that of Case I-2 with a TEG size of 0.02 m; specifically, \(d_{\text{TEG}} = 0.04\) m in Case II-1, \(d_{\text{TEG}} = 0.06\) m in Case II-2, and \(d_{\text{TEG}} = 0.08\) m in Case II-3. The TEG location is identical to that of Case I-2 \((R_{\text{TEG}} = 0.26\) m). The remaining optimization conditions are the same as in Case I-2 for the target frequencies \((f_1, f_2, f_3)\), pressure drop upper limit \((p_{\text{upp}}')\), TEG temperature lower limit \((T_{\text{low}})\), and volume ratio \((V_r)\): \(f_1 = 450\) Hz, \(f_2 = 700\) Hz, \(f_3 = 950\) Hz, \(p_{\text{upp}}' = 0.0015\) Pa, \(T_{\text{low}}^{\text{TEG}} = 450\) K and \(V_r = 0.05\) (5%), which corresponds to \(R_0 = 44\).

The optimal topologies for Case II-1 to Case II-3 are shown with the TEG indicated as red boxes in Figure 7. The TEG temperature values for these mufflers were 450.01 K, 450.00 K, and 452.39 K, and the pressure drop values were all 0.0015 Pa. It was observed that the partitions around the heat source plate of the TEG became thicker compared with Case I-2. This is because the thicker partitions can convey more energy to the TEG for increased energy harvesting. Furthermore, considering that the simulation is conducted in the 2-D domain, the harvesting trend will be exponentially increased in the 3-D domain because the area of the heat source plate and its number of thermocouples will increase accordingly. Figure 8 compares the TL curves of the optimal mufflers for Case II and Case I-2. The TL values at the target frequencies in Cases II-1 and II-2 were lower than that in Case I-2. For increased energy harvesting, the noise attenuation performances of the two cases were sacrificed. However, it is interesting to note that the TL value in Case III-3 was comparable to that in Case I-2. These results imply that increased energy harvesting does not always require sacrificing the noise attenuation performance.
Except for $p_{upp}^{f}$, the same design conditions as in Case I-2 were used. In the optimal results for the three cases, the TEG temperature and pressure drop were 453.49 K, 453.31 K, 450.34 K and 0.0013 Pa, 0.0017 Pa, 0.0019 Pa, respectively. Figure 9 compares the optimal topologies depending on $p_{upp}^{f}$. In all optimal topologies, the steel partitions started from the location of the TEG. As $p_{upp}^{f}$ increased, the steel partitions tended to invade the imaginary short flow path between the inlet and the outlet. While the internal partition in Figure 9(a) was horizontally extended so as not to block the short flow path as much as possible, Figure 9(b) and (c) show a partition layout that blocked the main flow. The noise attenuation performances of the optimal mufflers are represented as TL curves in Figure 10. The TL curves of all optimal mufflers were comparable to that of Case I-2.

**Case IV: Optimal topologies depending on target frequencies**

To verify the effectiveness of the proposed methodology for the broadband noise attenuation performance, the TO problem formulated in equations (28)–(30) was solved with an increasing number of target frequencies for the same frequency range of interest as in Case I-2. The target frequencies were selected with a frequency step of 150 Hz and 100 Hz in Case IV-1 and Case IV-2, respectively, that is, $f_1 = 500$ Hz, $f_2 = 650$ Hz, $f_3 = 800$ Hz, and $f_4 = 950$ Hz in Case IV-1; $f_1 = 500$ Hz, $f_2 = 600$ Hz, $f_3 = 700$ Hz, $f_4 = 800$ Hz, $f_5 = 900$ Hz and $f_6 = 1000$ Hz in Case IV-2. Case IV considers the same design parameters as in Case I-2 except for the target frequencies.
The optimal design results for Case IV satisfied all constraints on the thermal harvesting performance (TEG temperature) and pressure drop: $T_{TEG} = 452.24$ K and $\rho_{PD} = 0.00146$ Pa for Case IV-1; $T_{TEG} = 453.29$ K and $\rho_{PD} = 0.0015$ Pa for Case IV-2. Figure 11 shows the distribution of the absolute values of the acoustic pressures for the optimal mufflers at the target frequencies. The red and blue colors represent the highest and lowest values of the absolute value of acoustic pressure, respectively. All the acoustic pressures at the outlet were well minimized by the optimization. Even if Case IV-1 and IV-2 had the same frequency range of interest, the optimal topologies were different depending on the selected frequency interval. The smaller the frequency interval, the more diverse the partitions were distributed. In addition, the partition starting from the top of the muffler was thinner in Case IV-2 compared with Case IV-1. The TL curves of Case IV and Case I-2 are compared in Figure 12. In this figure, the TL values for Case IV at approximately 700–900 Hz were higher than those

### Table 3. The TEG temperature of the optimal topologies obtained for Case I.

| Case name | TEG temperature ($T_{TEG}$), K |
|-----------|-------------------------------|
| Case I-1  | 456.18                        |
| Case I-2  | 455.03                        |
| Case I-3  | 451.09                        |
| Case I-4  | 452.92                        |
| Case I-5  | 453.77                        |
| Case I-6  | 450.00                        |

Figure 6. Comparison of transmission loss curves of optimal mufflers for Case I with that of a nominal muffler: (a) Case I-1, Case I-2, Case I-3; (b) Case I-4, Case I-5, Case I-6.

Figure 11. The distribution of the absolute values of the acoustic pressures for the optimal mufflers at the target frequencies. The red and blue colors represent the highest and lowest values of the absolute value of acoustic pressure, respectively. All the acoustic pressures at the outlet were well minimized by the optimization. Even if Case IV-1 and IV-2 had the same frequency range of interest, the optimal topologies were different depending on the selected frequency interval. The smaller the frequency interval, the more diverse the partitions were distributed. In addition, the partition starting from the top of the muffler was thinner in Case IV-2 compared with Case IV-1. The TL curves of Case IV and Case I-2 are compared in Figure 12. In this figure, the TL values for Case IV at approximately 700–900 Hz were higher than those
in Case I-2 because the target frequency interval was smaller in Case IV than in Case I-2. Therefore, the target frequency interval should be carefully selected for the same target frequency range.

**Discussion**

To provide full insight into the harvesting performance by the TEG based on the temperature difference ($\Delta T = T_{TEG} - T_{ext}$), the power evaluation equation is introduced as equation (32)

$$P_{TEG} = \frac{\phi \beta^2 \Delta T^2 R_L}{(1 + R_{th,c} K_g + R_{th,h} K_g)^2 \left[ R_L + \phi \left( \frac{R_{th,c} T_{TEG} + R_{th,h} T_{ext}}{1 + R_{th,c} K_g + R_{th,h} K_g} \right) \right]^2}$$  \hspace{1cm} (32)

where $\phi$ and $\beta$ denote the number of thermocouples and the Seebeck coefficient of the thermocouple, respectively. In addition, $R_L$, $R_g$, $K_g$, $R_{th, c}$, and $R_{th, h}$ represent the electrical resistance of the circuit, electrical resistance of the TEG cell, thermal conductivity of the TEG, thermal resistances on the cold side of TEG, and thermal resistances on the hot side of the TEG. 

![Figure 7. Optimal topologies obtained for Case II: (a) Case II-1; (b) Case II-2; (c) Case II-3.](image)

![Figure 8. Comparison of transmission loss curves of optimal mufflers for Case II and Case I-2.](image)
TEG, respectively. In the simulations, the TEG specification is assumed as “TEG-127-150-9”. Note that the external temperature ($T_{\text{ext}}$) is already assumed as 293.15 K, as previously shown in Table 2. The parameters for calculating “TEG-127-150-9” are as follows: $\phi = 15$ except for Case II ($\phi = 30, \phi = 45$, and $\phi = 60$ for Case II-1, Case II-2, and Case II-3, respectively), $\beta = 0.05$ V/K, $R_g = 3.4$ Ohm, $R_L = 4$ Ohm, $R_{\text{th}, c} = 6$ K/W, $R_{\text{th}, h} = 0.1$ K/W, and $K_g = 2.907$ W/K. The evaluated TEG power values for the optimal mufflers are listed in Table 4.

In the partition layouts obtained by the TO, the main partitions all began at the side of the muffler, where the TEG was attached, and evaded the short flow path from the inlet to the outlet. The main partitions had several branches. To investigate the effect of each branch on one of the three physical properties (noise attenuation, pressure drop, and TEG temperature), one of the branches was partially removed. The main partition in Case I-2 was divided into three branches as shown in Figure 13: Branches A, B, and C. Each case was distinguished with a subscript. For example, Case I-2\textsubscript{A} implies removing branch A, and Case I-2\textsubscript{BC} implies removing branches B and C. The topological configurations for the six combinations depending on the removed branch are shown in Figure 14. Table 5 compares $P_{\text{PD}}$ and $T_{\text{TEG}}$ for all cases. Most of the cases satisfied the constraint on the pressure drop, but the following violated the constraint on the TEG temperature: Case I-2\textsubscript{B}, Case I-2\textsubscript{C}, and Case I-2\textsubscript{BC}. This implies that Branches B and C played a vital role in delivering heat to the attached TEG (recall that the material for the partition is stainless steel). Figure 15 compares the TL curves of the six cases with that of

Figure 9. Optimal topologies obtained for Case III: (a) Case III-1; (b) Case III-2; (c) Case III-3.

Figure 10. Comparison of transmission loss curves of optimal mufflers for Case III and Case I-2.
Case I-2. While two TL curves agreed fairly well when Branch A was removed, the obvious change in the TL curve was observed when Branch B or C was removed. That is, Branches B and C played a more important role in the acoustical attenuation performance and heat transfer compared with Branch A.

Overall, the well-formulated TO achieved the design goals for the TL, pressured drop, and TEG temperature values. In addition, it could be observed from the harvested power values in Table 4 that the power do not aid in the running of the vehicle engine but only in turning on the lights, and this could attract criticism regarding the practical aspects of a vehicle design. However, this methodology provides a new way to topologically design a device in a vehicle considering multiple purposes based on multi-physics. To further develop this design method, a thermo-acoustics analysis should be included when the geometry has a narrow region such as the neck area of Helmholtz resonator because of the viscous damping.

**Figure 11.** Distribution of the absolute values of the acoustic pressure of the optimal mufflers for Case IV at the target frequencies: (a) Case IV-1; (b) Case IV-2.

**Figure 12.** Comparison of transmission loss curves of optimal mufflers for Case IV and Case I-2.
Table 4. The TEG power values calculated using equation (32) for the optimal mufflers in all cases.

| Case name | $\Delta T = T_{\text{TEG}} - T_{\text{ext}}, K$ | TEG power ($P_{\text{TEG}}$), W |
|-----------|----------------------------------|-----------------|
| Case I-1  | 163.03                           | 0.0465          |
| Case I-2  | 161.88                           | 0.0459          |
| Case I-3  | 157.94                           | 0.0437          |
| Case I-4  | 159.77                           | 0.0447          |
| Case I-5  | 160.62                           | 0.0452          |
| Case I-6  | 156.85                           | 0.0431          |
| Case II-1 | 156.86                           | 0.0462          |
| Case II-2 | 156.85                           | 0.0472          |
| Case II-3 | 159.24                           | 0.0492          |
| Case III-1| 160.34                           | 0.0450          |
| Case III-2| 160.16                           | 0.0449          |
| Case III-3| 157.19                           | 0.0433          |
| Case IV-1 | 159.09                           | 0.0443          |
| Case IV-2 | 160.14                           | 0.0449          |

Figure 13. Three branches of the optimal muffler for Case I-2.

Figure 14. Partial modification of the optimal topology for Case I-2 for further investigation: (a) Case I-2A; (b) Case I-2B; (c) Case I-2C; (d) Case I-2AB; (e) Case I-2AC; (f) Case I-2BC.
effect in viscous and thermal boundary layers on the wall. In a similar way, an aero-acoustics analysis should be considered when the flow-induced noise is expected by turbulent flow such as vortex shedding at the trailing edge. Those phenomena

Table 5. Comparison of the pressure drop and TEG temperature values of the partially modified optimal mufflers for Case I-2.

| Case name       | Pressure drop ($p_{PD}$), Pa | TEG temperature ($T_{TEG}$), K |
|-----------------|------------------------------|-------------------------------|
| Case I-2A       | 0.0014820                    | 451.07                        |
| Case I-2B       | 0.0014733                    | 449.39                        |
| Case I-2C       | 0.0013477                    | 449.81                        |
| Case I-2AB      | 0.0014760                    | 450.58                        |
| Case I-2AC      | 0.0013469                    | 450.87                        |
| Case I-2BC      | 0.0011295                    | 448.40                        |

Figure 15. Comparison of transmission loss curves of partially modified mufflers in Figure 14: (a) Case I-2A; (b) Case I-2B; (c) Case I-2C; (d) Case I-2AB; (e) Case I-2AC; (f) Case I-2BC.
may happen because of the viscous damping around the narrow regions caused by partitions and the high-speed flow coming from the exhaust system.

**Conclusions**

In this study, a muffler design method was proposed using multi-physics-analysis-based TO and applied to an optimal design problem for a muffler integrated with a TEG. The centers of the inlet and outlet of the muffler were offset from each other, and a TEG was attached to the cover panel of the muffler. The design goals included enhancing the noise attenuation performance in the target frequency range, as well as the recovery energy efficiency of the TEG. A TO problem based on acoustics, heat transfer, and fluid mechanics was formulated, and the interaction between the governing equations for the three physics was properly designated for exact numerical analysis. The temperature-dependent material interpolation functions for parameterizing the 0 and 1 topological states of each finite element were defined in the design domain. The sum of the squared acoustic pressures at the outlet was selected as the objective function, and the lower limit value of the TEG temperature and the upper limit value of the pressure drop were constrained. The formulated TO problem was solved for various design conditions. The noise attenuation and heat transfer performances of the optimal mufflers were evaluated through their TL curves and recovery energies, respectively. The effects of the location and size of the TEG on the optimal internal partition layout were investigated, and the optimal partition layouts were compared depending on the upper limit value of the pressure drop and the number of target frequencies in the same frequency range of interest. The optimal partition layout of the muffler was discussed in terms of physical properties. The numerical results strongly supported the validity of the muffler design method based on the multi-physics-analysis-based TO process proposed in this paper.

As future work, a verification experiment should be carried out to verify the effectiveness of our design results. It requires a lot of experience, know-how and expensive experimental equipment for thermal fluid experiments to generate a uniform flow while maintaining a high temperature at the inlet of a muffler. In addition, microphones that can withstand high temperatures are required for acoustic pressure measurement, and an equipment to measure the harvested power of a TEG is also required. The experimental results would certainly contribute to development of a muffler design method.

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