The critical parameters enhanced by microwave field in thin-film type-II superconductors

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Abstract. Here we give a results of experimental study of enhancement of superconductivity by microwave irradiation in superconducting films. An influence of the power, frequency of microwave irradiation, as well as temperature and width of superconducting films on behavior of experimental dependencies of enhanced critical current and the current at which a vortex structure of the resistive state vanishes and the phase-slip first line appears is analyzed. The experimental studies of films with different width reveal that the effect of superconductivity enhancement by microwave field is common and occurs in both the case of uniform (narrow films) and non-uniform (wide films) distribution of superconducting current over the film width. It is shown that enhancement of superconductivity in a wide film increases not only the critical current and the critical temperature, but also the maximum current at which the vortex state can exist in the film. The phenomenon of superconductivity enhancement by microwave irradiation in wide films can be described by the Eliashberg theory, which was used to explain the same phenomenon in narrow channels. It was experimentally found that the interval of irradiation power in which the superconductivity enhancement was observed, became narrower with increasing film width, what impairs the probability of its detection.

Keywords: enhancement of superconductivity, microwave irradiation, critical current, wide superconducting films, phase slip line (PSL)

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1. Introduction

For a long time there was a general opinion that effect of electromagnetic field on a superconductor should always lead to a reduction of the energy gap $\Delta$, the critical current $I_c$, the critical magnetic field $H_c$ and the critical temperature $T_c$. However, an increase of the critical current of a thin narrow superconducting bridge near its critical temperature under an influence of high-frequency electromagnetic field has been reported in 1966 in reference [1]. Later this phenomenon has been observed in almost all types of superconducting weak links. This effect has been explained in the Aslamazov – Larkin theory only in 1978 [2]. The phenomenon of enhancement of superconductivity has also been found in narrow superconducting channels (single-crystal filaments (whiskers) and thin (the thickness $d \ll \xi(T), \lambda_\perp(T)$), narrow (the width ($d \sim \xi(T), \lambda_\perp(T)$) films). Here, $\xi(T)$ and $\lambda_\perp(T) = 2\lambda^2(T)/d$ are the coherence length and the penetration depth of a weak magnetic field perpendicular to the film, respectively, $\lambda(T)$ the London penetration depth. In 1970 Eliashberg G.M. has proposed a microscopic theory [3], which considers the effect of electromagnetic irradiation on the energy gap $\Delta$ of a superconductor. The Eliashberg theory explained the phenomenon of enhancement in a superconducting channel, and this theory did not exclude a possibility of its existence in wide films. However, decades had passed, but this phenomenon could not been found in wide films. After the discovery of high-temperature cuprate superconductors (HTSC), which caused an increased research activity, there has been a series of works devoted to the experimental study of an effect of microwave irradiation on the superconducting properties of HTSC films. Reference [4] was among the first which mentioned the finding of the effect of superconductivity enhancement in HTSC films. That paper shows a family of I-V characteristics (IVC) for wide (w $\sim 10$ $\mu$m) and long (L $\sim 15$ $\mu$m) bridge of an epitaxial film of $YBa_2Cu_3O_7-x$.

In the IVC it is seen that for low ($\sim 10^{-8}$ W) power the critical current $I_c$ as well as the superconducting one $I_s$ increase compared to the values in the absence of microwave irradiation, indicating the enhancement of superconductivity by an electromagnetic field. With a further increase in power of microwave irradiation the critical superconducting current decreases, and harmonic and sub-harmonic steps of the current appear at voltages on the bridge $V_{mn}$ related to the frequency of an external electromagnetic field $f$ by the Josephson relation: $V_{mn} = (n/m)hf/2e$, where $m, n$ are integers, $h$ is Planck’s constant, $e$ is the electron charge. The authors of reference [4] suggest that the mechanism responsible for the increase of $I_c$ and $I_s$ in the investigated bridges of HTSC is an energy diffusion of quasiparticles in a contact area caused by a ”jitter” of the potential well due to microwave irradiation [2]. It is this mechanism of enhancement of superconductivity that was also found in cuprate HTSC by other experimenters. For example, in studying the dependence of the superconducting current on the irradiation power in $YBa_2Cu_3O_7-x$ samples [5]. It was found that it is the Aslamazov – Larkin mechanism of enhancement [2] characteristic of superconducting weak links that is responsible for increasing the superconducting current under microwave irradiation.
Unfortunately, preliminary studies of the effect of superconductivity enhancement in cuprate HTSC caused by the Aslamazov – Larkin mechanism \cite{2} have not been continued.

The phenomenon of superconductivity enhancement by microwave irradiation in quasi-one-dimensional films (narrow channels) already belongs to classical effects in the physics of superconductivity. The experimental manifestation of this effect in a narrow channel is an increase of its critical temperature $T_c$ and the Ginzburg-Landau critical current $I_{c_{GL}}^c(T)$. When a current flowing through a channel is greater than the current $I_{c_{GL}}^c(T)$, the narrow channel comes to the resistive current state caused solely by the appearance of phase-slip centers. In contrast, in high-quality superconducting wide films ($w \gg \xi(T), \lambda_{\perp}(T)$) in excess of the critical current $I_c(T)$ the vortex state appears, so-called flux flow regime. A wide film is in this regime until the transport current reaches the maximum current $I_{m}(T)$ at which in a wide film a vortex structure of the resistive state vanishes \cite{6, 7}, and the first phase slip line (PSL) appears. In 2001 it has been experimentally observed \cite{8}, that in response to a microwave field not only the critical current $I_c(T)$ increases but the maximum current of existence of the vortex resistivity $I_{m}(T)$ does so. In this connection, the problem of superconductivity enhancement in wide films is particularly interesting, as it requires consideration of behavior in a microwave field of both the critical current $I_c(T)$ and the maximum current of existence of the vortex resistive state $I_{m}(T)$.

2. A microscopic theory of superconductivity of films, enhanced by microwave irradiation

2.1. The effect of microwave irradiation on the energy gap of a superconductor

A microscopic theory of superconductivity enhancement of films, uniform in the order parameter, by a microwave field was proposed by Eliashberg \cite{3} and developed in references \cite{9, 12}. The theory applies to relatively narrow \cite{13} and thin ($w, d \ll \xi(T), \lambda_{\perp}(T)$) films in which the spatial distribution of microwave power and accordingly the enhanced gap are uniform over the film cross section. At the same time, the length of scattering of an electron by impurities $l_i$ should be small compared with the coherence length.

To understand properly behavior of superconductors with an energy gap in an alternating electromagnetic field it was necessary to take into account both the processes of absorption of electromagnetic energy by quasiparticles (electrons), and the inelastic processes of scattering of the absorbed energy.

To illustrate the physical nature of the effect of superconductivity enhancement, we turn to the basic equation of Bardeen – Cooper – Schrieffer theory (BCS) \cite{14}, which relates the energy gap $\Delta$ with the equilibrium distribution function of electrons

$$n(\varepsilon) = (\exp^{\varepsilon/kT} + 1)^{-1}$$
\[ \Delta = g \int_{\Delta}^{\hbar \omega_D} d\varepsilon \frac{\Delta}{\sqrt{\varepsilon^2 - \Delta^2}} [1 - 2n(\varepsilon)] \] (1)

In the theory [3], it was shown that if a superconductor with a uniform spatial distribution of \( \Delta \) is in an electromagnetic field whose frequency is lower than the frequency related with the energy gap by the ratio \( \hbar \omega = 2\Delta \), and higher than the inverse relaxation time of electrons \( \tau_\varepsilon \) (the relaxation time of inelastic collisions), then the equilibrium distribution function of electrons \( n(\varepsilon) \) is shifted to higher energies, which leads to a steady non-equilibrium state and an increase of the superconductor’s energy gap and consequently its superconducting properties. And the total number of excitations does not change. This shift in the electron distribution function, as seen in equation (1), results in an increase of the gap and thus enhances the superconducting properties. A change of \( n(\varepsilon) \) is proportional to the field intensity \( E^2 \) (for not too large \( E \)) and the relaxation time of energy excitations \( \tau_\varepsilon \).

It should be noted that in the presence of a microwave field the energy gap is variable in space and time. There is no coordinate dependence for sufficiently thin samples. And when \( \omega \tau_\varepsilon \gg 1 \) it turns out that temporal oscillations of \( \Delta \) can also be ignored. It was also assumed that the mean free path of electrons is less than the film thickness. Otherwise it would be necessary to consider peculiarities of reflection from walls.

If we restrict our consideration to not too high intensities of electromagnetic irradiation, an equation for the time-averaged \( \Delta \) is as follows:

\[ \left\{ \frac{T_c - T}{T_c} - \frac{7\zeta(3)\Delta^2}{8(\pi k_B T_c)^2} - \frac{\pi l_i v_F e^2}{6T_c \hbar c^2} \left[ A_0^2 + \frac{A_\omega^2}{2} - \frac{3\Delta \hbar c^2}{2\pi l_i v_F e^2} G \right] \right\} \Delta = 0 \] (2)

where \( T_c \) is the critical temperature, \( A_0 \) is the potential representing a static magnetic field or a direct current, \( A_\omega \) is the amplitude of an electromagnetic field, \( v_F \) is the Fermi velocity, \( l_i \) is the mean free path of electrons under the scattering, \( n_1(\varepsilon) \) is the non-equilibrium part of \( n(\varepsilon) \), \( \zeta(3)=1.202 \) is the particular value of the Riemann zeta function.

\[ G = -\frac{2T}{\Delta} \int_{\Delta}^{\infty} d\varepsilon \frac{\sqrt{\varepsilon^2 - \Delta^2}}{\sqrt{\varepsilon^2 + \Delta^2}} n_1(\varepsilon) \] (3)

At low power of an external electromagnetic field

\[ n_1(\varepsilon) = \frac{\alpha \omega}{\gamma 4T} \left( \frac{\varepsilon(\varepsilon - \omega) + \Delta^2}{\varepsilon \sqrt{(\varepsilon - \omega)^2 - \Delta^2}} \right) \theta(\varepsilon - \Delta - \omega) - \frac{\varepsilon(\varepsilon + \omega) + \Delta^2}{\varepsilon \sqrt{(\varepsilon + \omega)^2 - \Delta^2}} \theta(\varepsilon - \Delta) - \frac{2\varepsilon(\varepsilon - \omega) + \Delta^2}{\omega \sqrt{(\varepsilon - \omega)^2 - \Delta^2}} \theta(\varepsilon - \Delta) \theta(\omega - \Delta - \varepsilon) \] (4)
where \( \alpha = (1/3) v_F l e^2 A_0^2 / \hbar c^2 \) is proportional to the power of an external electromagnetic field, \( \gamma = \hbar / \tau_c \). Taking into account Equation (4), Equation (3) can be written as

\[
G = \frac{\omega^2 \alpha}{2 \Delta \gamma} \int_{-\Delta}^{\infty} d\varepsilon \frac{\varepsilon (\varepsilon + \omega + \Delta^2)}{(\varepsilon + \omega) \sqrt{\varepsilon^2 - \Delta^2 [(\varepsilon + \omega)^2 - \Delta^2]}} + \frac{1}{\Delta \gamma} \int_{-\Delta}^{\omega - \Delta} d\varepsilon \frac{\varepsilon (\varepsilon - \omega + \Delta^2)}{\sqrt{\varepsilon^2 - \Delta^2 [(\varepsilon - \omega)^2 - \Delta^2]}} \theta (\omega - 2\Delta)
\]

or

\[
G = \frac{\alpha \hbar \omega}{2 \gamma \Delta} f \left( \frac{\hbar \omega}{\Delta} \right)
\]

In limiting cases, the function

\[
f \left( \frac{\hbar \omega}{\Delta} \right) = \frac{\hbar \omega}{\Delta} \left[ \ln \left( \frac{8 \Delta}{\hbar \omega} \right) - 1 \right] \text{ at } \frac{\hbar \omega}{\Delta} \ll 1
\]

\[
f \left( \frac{\hbar \omega}{\Delta} \right) = \frac{\pi \Delta}{\hbar \omega} \text{ at } \frac{\hbar \omega}{\Delta} \gg 1
\]

Taking into account Equation (7), Equation (2) can be rewritten as follows:

\[
\frac{T_c - T}{T_c} = -\frac{7\zeta(3)\Delta^2}{8(\pi k_B T_c)^2} - \frac{\pi l_F \nu_F e^2 A_0^2}{6 k_B T_c \hbar c^2} \left[ A_0^2 + A_0^2 \left(1 - \frac{\hbar \omega}{2\pi \gamma} f \left( \frac{\hbar \omega}{\Delta} \right) \right) \right] = 0
\]

In Equation (8) there is no term accounting for the interaction of an electromagnetic field with excitations, located substantially above the gap edge, which has the form [9]

\[-0.11 \frac{\pi}{2} \left( \frac{\hbar \omega}{k_B T_c} \right)^2 \frac{\alpha}{\gamma} \]

Now we can write a complete equation of the microscopic theory of superconductivity, which takes into account basic mechanisms of the interaction of a superconductor with an external electromagnetic irradiation

\[
\frac{T_c - T}{T_c} = -\frac{7\zeta(3)\Delta^2}{8(\pi k_B T_c)^2} - \frac{\pi \alpha}{2 k_B T_c} - \frac{\pi l_F \nu_F e^2 A_0^2}{6 k_B T_c \hbar c^2} \left[ A_0^2 + A_0^2 \left(1 + 0.11 \frac{(\hbar \omega)^2}{\gamma k_B T_c} - \frac{(\hbar \omega)^2}{2\pi \gamma} \left[ \ln \left( \frac{8 \Delta}{\hbar \omega} \right) - 1 \right] \right) \right] = 0
\]

In this equation, the first two terms describe a temperature dependence of the equilibrium \((\alpha = 0)\) superconducting gap, and the third one is a contribution of a static magnetic field or a direct current. The fourth term of the equation describes a usual pair-breaking effect in an external microwave field, the fifth is a contribution of high-energy excitations, and the sixth term is a contribution of the interaction with an external electromagnetic field of quasiparticles located at the Fermi surface. It is this interaction that is responsible for the effect of superconductivity enhancement [3]. Effects of heating of a superconductor by an electromagnetic field are not considered in Equation (9).
It is important to note one more circumstance. In Equation (9) it is seen that with increasing the radiation frequency a contribution of the latter two terms increases, and an effect of second one leads to an increase of ∆(T, α) for a given electromagnetic field power α. ∆(T, α) is greater than ∆(T, α = 0) (superconductivity enhancement), when

\[ 1 + 0.11 \frac{(\hbar \omega)^2}{\gamma k_B T_c} - \frac{(\hbar \omega)^2}{2 \pi \gamma \Delta} \left( \ln \left( \frac{8 \Delta}{\hbar \omega} \right) - 1 \right) \leq 0 \] (10)

At not very high frequency of external irradiation the term

\[ \frac{0.11 (\hbar \omega)^2}{\gamma k_B T_c} \]

describing a contribution of high-energy excitations can be neglected, and with \( \ln(8\Delta/\hbar \omega) > 1 \) from Equation (10) we obtain an expression for the lower frequency limit of the superconductivity enhancement

\[ \omega_L^2 = \frac{2 \pi \gamma \Delta}{\hbar^2 \ln (8\Delta/\hbar \omega)} = \frac{2 \pi \Delta}{\hbar T_c \ln (8\Delta/\hbar \omega)} \] (11)

2.2. A non-equilibrium critical current of superconducting films in a microwave field

Theoretical studies [3, 9–12], considered superconductivity enhancement for narrow channels in which the equilibrium energy gap and the superconducting current density \( j_s \) are distributed uniformly over the sample cross section.

According to this theory, the effect of microwave irradiation on the energy gap of a superconductor, through which a constant transport current with density \( j_s \) flows, is described by Equation (9) which can be rewritten as

\[ \frac{T_c - T}{T_c} - \frac{7 \zeta(3) \Delta^2}{8 (\pi k_B T_c)^2} - \frac{2 k_B T_c \hbar}{\pi e^2 D A^4 N^2(0)} j_s^2 + M(\Delta) = 0 \] (12)

where \( N(0) \) is the density of states at the Fermi level, \( D = v_F l_i/3 \) is the diffusion coefficient, \( v_F \) is the Fermi velocity, and \( M(\Delta) \) is a non-equilibrium add-on due to the "non-equilibrium" of the distribution function of electrons [15–17]

\[ M(\Delta) = -\frac{\pi \alpha}{2 k_B T_c} \left[ 1 + 0.11 \frac{(\hbar \omega)^2}{\gamma k_B T_c} - \frac{(\hbar \omega)^2}{2 \pi \gamma \Delta} \left( \ln \left( \frac{8 \Delta}{\hbar \omega} \right) - 1 \right) \right] \] (13)

Often, an experimental study of the enhancement effect assumes the measurement of the critical current, rather than the energy gap. Using Equation (12) and Equation (13) one can derive an expression for the density of the superconducting current, \( j_s \), as a function of energy gap, temperature and microwave power

\[ j_s = \eta \Delta^2 \left\{ \frac{T_c - T}{T_c} - \frac{7 \zeta(3) \Delta^2}{8 (\pi k_B T_c)^2} - \frac{\pi \alpha}{2 k_B T_c} \right\} \times \left[ 1 + 0.11 \frac{(\hbar \omega)^2}{\gamma k_B T_c} - \frac{(\hbar \omega)^2}{2 \pi \gamma \Delta} \left( \ln \left( \frac{8 \Delta}{\hbar \omega} \right) - 1 \right) \right]^{1/2}, \] (14)

\[ \eta = e N(0) \sqrt{\frac{\pi D}{2 \hbar k T_c}} \]
The critical parameters enhanced by microwave field in thin films

The extremum condition for the superconducting current, \( \partial j_s/\partial \Delta = 0 \), at given temperature and irradiation power results in a transcendental equation for the energy gap \( \Delta_m \), for which the maximum value \( j_s \) is reached, i.e., the critical current density

\[
\frac{T_c - T}{T_c} = \frac{21\zeta(3)\Delta_m^2}{(4\pi k_B T_c)^2} - \frac{\pi \alpha}{2k_B T_c} 
\times \left[ 1 + 0.11 \frac{(\hbar \omega)^2}{\gamma k_B T_c} - \frac{(\hbar \omega)^2}{4\pi \gamma \Delta_m} \left( \frac{3}{2} \ln \frac{8\Delta_m}{\hbar \omega} - 1 \right) \right] = 0.
\]

Thus, substituting the solution \( \Delta_m \) of Equation (15) into Equation (14) we find an expression for the critical current in a microwave field [18]

\[
I_{Pc}(T) = \eta dw \Delta_m^2 \left( \frac{T_c - T}{T_c} - \frac{7\zeta(3)\Delta_m^2}{8(\pi k_B T_c)^2} - \frac{\pi \alpha}{2k_B T_c} \right) \times \left[ 1 + 0.11 \frac{(\hbar \omega)^2}{\gamma k_B T_c} - \frac{(\hbar \omega)^2}{2\pi \gamma \Delta_m} \left( \ln \frac{8\Delta_m}{\hbar \omega} - 1 \right) \right]^{1/2}.
\]

Without external microwave field (\( \alpha = 0 \)), Equation (16) is transformed into an expression for the equilibrium pair-breaking current

\[
I_c(T) = I_{GLc}(T) = \eta dw \Delta_0^2 \left[ \frac{T_c - T}{T_c} - \frac{7\zeta(3)\Delta_m^2}{8(\pi k_B T_c)^2} \right]^{1/2},
\]

In this case \( \Delta_m = \sqrt{2/3}\Delta_0 \), where

\[
\Delta_0(T) = \pi k_B T_c \sqrt{8(T_c - T)/7\zeta(3)T_c} = 3.062k_B T_c \sqrt{1 - T/T_c}
\]

is the equilibrium value of the gap at zero transport current.

Note that the use in Equation (17) of the Equation (14) with the density of states \( N(0) = m^2 v_F / \pi^2 \hbar^2 \), calculated in the free electron model, results in a significant difference between the theoretical and experimental values of the equilibrium critical current, indicating a relative roughness of such an estimation for a metal (e.g., tin) used in preparation of samples. At the same time, expressing the density of states in terms of the experimentally measured quantity, the resistance of the film on a square, \( R^2 = R_{4.2} w/L \), where \( R_{4.2} \) is the total film resistance at \( T = 4.2 K \) and \( L \) is the film length, we obtain an expression for \( \eta = (edR^2)^{-1} \sqrt{3\pi/2k_B T_c v_F l} \), which when substituted into Equation (17) leads to its good agreement with both experimental values of the equilibrium pair-breaking current and those calculated in the Ginzburg – Landau theory, \( I_{GLc}(T) \) (see Equation (19)) [19]. This expression for the parameter \( \eta \) will be used below in the formula (16) for enhanced critical current in its comparison with experimental results.

It is interesting to note that prior to reference [19], to our knowledge, the temperature dependencies of the enhanced critical current, arising from Equation (16), were not compared directly with experimental data for \( I_{Pc}(T) \). We point out, however, that attempts to compare, at least qualitatively, experimental dependencies \( I_{Pc}(T) \) with the Eliashberg theory have been made. For example, in reference [14] the authors presented the pair-breaking current of Ginzburg – Landau, using Equation (18) for the...
equilibrium gap, in the form
\[ I^c_{GL}(T) = \frac{c\Phi_0 w}{6\sqrt{3}\pi^2\xi(0)\lambda(0)}(1 - T/T_c)^{3/2} = K_1\Delta^3_0(T), \quad (19) \]
where \( \Phi_0 = hc/2e \) is the magnetic flux quantum. Since the temperature dependence of the enhanced critical current in a narrow channel appeared to be close to equilibrium in its shape, \( I^c_{P}(T) \propto (1 - T/T^c_P)^{3/2} \), where \( T^c_P \) is the superconducting transition temperature in a microwave field, the \( I^c_{P}(T) \) was approximated by an expression similar to Equation (19)
\[ I^c_{P}(T) = K_2\Delta^3_{P}(T). \quad (20) \]
where the stimulated energy gap \( \Delta_{P}(T) \) was calculated in the Eliashberg theory at zero superconducting current (Equation (12) at \( j_s = 0 \)). After that, assuming \( K_1 = K_2 \) and using the value of the microwave power as a fitting parameter, the authors of reference \[14\] fitted calculated values of \( I^c_{P}(T) \) to experimental data with a certain degree of accuracy.

It is clear that such a comparison of experimental results with the Eliashberg theory is only a qualitative approximation, and cannot be used to obtain quantitative results \[10\]. First of all, Equation (19) and Equation (20) contain a value of the gap in a zero-current regime \( (j_s = 0) \), which differs from that when there is a current. Second, the pair-breaking curves \( j_s(\Delta) \) in the equilibrium state \( (P = 0) \) and with a microwave field are very different \[18\]. Finally, as shown in references \[11, 14, 18\] for \( T \to T^c_P - 0 \) the enhanced order parameter \( \Delta_{P}(T) \) tends to a finite (but small) value \( \Delta_{P}(T^c_P) = (1/2)\hbar\omega \) and abruptly vanishes at \( T > T^c_P \), while the critical current vanishes continuously (without a jump) at \( T \to T^c_P \) and, therefore, cannot in general be satisfactorily described by the formula of the type (20). This is evidenced by a marked deviation of the dependence (20) from experimental points in the close vicinity of \( T_c \).

In this review, an analysis of experimental data is based on the exact formula (16) with using the numerical solution of Equation (15).

3. Enhancement of superconductivity by an external microwave irradiation in films of different widths

Upon enhancement of superconductivity by an electromagnetic field the superconducting gap \( \Delta \) is, strictly speaking, variable in space and time. However, for sufficiently thin and narrow superconducting samples the dependence of \( \Delta \) on coordinates can be neglected. Moreover, near \( T_c \) the relaxation time of the order parameter \( \tau_\Delta \simeq 1.2\tau_\varepsilon/(1 - T/T_c)^{1/2} \) is large compared to the inverse frequency of stimulating microwave radiation \( (\omega \tau_\Delta \gg 1) \), and the temporal oscillations of \( \Delta \) can also be neglected. Therefore, the microscopic theory of Eliashberg \[3\] included neither the time nor spatial variations of the order parameter in a sample.

To realize experimentally the case discussed in this theory of a spatially homogeneous non-equilibrium state of a superconductor in a high-frequency field it
was necessary to ensure the constancy of the energy gap over the sample volume \((w, d \sim \xi(T), \lambda_\perp(T))\). This is a purely technological problem. It was also important to ensure a uniform distribution of the transport current through the sample volume. Failure to do so resulted in a non-uniform distribution of \(\Delta\) due not to technological reasons, but because of the dependence of the energy gap on the transport current \(\Delta(I)\). Finally, it was important to provide an efficient heat transfer from a sample. It was shown that narrow superconducting film channels deposited on relevant substrates best met these requirements. The theory proposed in [3] has been fully confirmed in experimental studies of such samples (see, e.g., references [15] and [17]).

For wider films, electro dynamical changes of \(\Delta\) over the film width with a non-uniformly distributed current and with the presence of intrinsic vortices cannot be neglected. Therefore, the theory [3], strictly speaking, does not apply in the case of wide films. Yet, although it is difficult to develop a theory in the case of a non-uniform distribution of \(\Delta\) in a superconductor, in principle there should also be an effect of enhancement of superconductivity in this case.

In 2001, the enhancement of superconductivity by an external electromagnetic field has also been found in wide, \(w \gg \xi(T), \lambda_\perp(T)\), high-quality superconducting films of tin [8] with a non-uniform spatial distribution of \(\Delta(I)\) over the sample width. It has been experimentally shown that under an external electromagnetic field not only the critical current \(I_c\) increases but also the current of formation of the first phase-slip line (see figure 1) does so. In reference [8], this current is designated as \(I_{dp}^c\).

In reference [20] the temperature dependencies of the current \(I_{dp}^c\) was analyzed with taking into account the nontrivial distribution of the transport current and the density of vortices over a wide film. As a result, it was shown that the current \(I_{dp}^c\) is the critical pair-breaking current of Ginzburg – Landau \(I_{cGL}\), if a film corresponds to the parameters of a vortex-free narrow channel in the temperature region near \(T_c\). Far from \(T_c\) this current is the maximum current of existence of the vortex resistive state \(I_m\) in the Aslamazov – Lempitsky theory [6]. A physical meaning of the current \(I_m\) is that it is the maximum current, at which a steady uniform flow of intrinsic vortices of the transport current across a wide film is still possible. If it is exceeded, \(I > I_m\), the vortex structure collapses, and in its place there appears a structure of phase-slip lines [7]. It is the phase that determines the resistivity of a sample for a further increase in the transport current.

In this connection, the problem of superconductivity enhancement in wide films becomes particularly interesting, because it requires consideration of the behavior in a microwave field not only of both the critical current and the critical temperature. An important object of the investigation is the current \(I_m\), as well as its relation with \(I_c\) under an external electromagnetic irradiation of different frequencies \(f\) and power \(P\).

This section presents the results of studying a dependence of enhancement of the critical current \(I_c\) and the current of formation of the first PSL \(I_m\) on power and frequency of an electromagnetic field in thin (thickness \(d \ll \xi(T), \lambda_\perp(T)\)) superconducting films as a function of their width \(w\) [20]. To understand how the effect of
superconductivity enhancement manifests itself in a wide film, the authors of this work increased gradually the sample width starting from a narrow channel, and observed how the phenomenon of superconductivity enhancement was changing. As samples thin \((d \ll \xi(T), \lambda_\perp(T))\) tin films which were prepared as described in reference [7] were used. This original technology enabled to minimize defects in both the film edge and its volume. The critical current of these samples is determined by suppression of the barrier for entering vortices when the current density at the edge of the film is of the order of \(j_c^{GL}\), and reaches the maximum possible theoretical value \([6]\), indicating the absence of edge defects which create local lowering of the barrier and thus reduce the \(I_c\). IVC were measured by a four-probe method. In measuring the IVC samples were placed in a double screen of annealed permalloy. In the area of the sample the magnetic field was: \(H_\perp = 7 \times 10^{-4}\) Oe, \(H_\parallel = 6,5 \times 10^{-3}\) Oe. To supply an electromagnetic irradiation to the film sample it was placed in a rectangular wave-guide parallel to the electric field component in the wave-guide, or irradiated from the shorted end of a coaxial line, or the sample was connected to a 50 - ohm coaxial line through a separating capacitance (contact method). The parameters of the samples are shown in Table 1. The temperature was measured by the vapor pressure using mercury and oil pressure gauges. In doing so, an influence of the microwave field introduced into the cryostat during the experiment on measurements of temperature in the case of using electronic thermometers was excluded. The temperature stabilization (helium vapor pressure) was provided by a membrane manostat with accuracy better than \(10^{-4}\) K.

In reference [8] it was shown that long \((L \gg \xi(T), \lambda_\perp(T))\) and wide \((w \gg \xi(T), \lambda_\perp(T))\) superconducting films reveal the effect of increasing the critical current and the current \(I_m\) under an external microwave irradiation. Figure 1 shows a family of current-voltage characteristics of one of these films (SnW5) of the width of 42 \(\mu m\) for different power levels of microwave irradiation with a frequency \(f = 12.89\) GHz. Here, as in reference [7] the notation are introduced: \(I_c(T)\) is the current of voltage appearance across the sample as a result of entering vortices of its intrinsic magnetic flux current, \(I_m(T)\) is the maximum current of existence of a stable uniform flow of

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**Table 1.** Parameters of tin film samples: \(L\) is the length, \(w\) is the width, \(d\) is the thickness of the sample, \(l_i\) is the electron mean free path, and \(R^{\square} = R_{4.2}\frac{w}{L}\)

| Sample | \(L\), \(\mu m\) | \(w\), \(\mu m\) | \(d\), \(nm\) | \(R_{4.2}\), \(\Omega\) | \(R^{\square}\), \(\Omega\) | \(T_c\), \(K\) | \(l_i\), \(nm\) | \(R_{300}\), \(\Omega\) |
|--------|----------------|---------------|-------------|----------------|----------------|-------------|-------------|-----------|
| Sn1    | 64             | 1.5           | 90          | 3.05           | 0.071          | 3.834       | 174         | 59        |
| SnW5   | 92             | 42            | 120         | 0.14           | 0.064          | 3.789       | 145         | 2.270     |
| SnW6   | 81             | 17            | 209         | 0.185          | 0.039          | 3.712       | 152         | 3.147     |
| SnW8   | 84             | 25            | 136         | 0.206          | 0.061          | 3.816       | 148         | 3.425     |
| SnW10  | 88             | 7             | 181         | 0.487          | 0.040          | 3.809       | 169         | 9.156     |
3.1. The critical current

For a narrow channel Sn1 of the width of \( w = 1.5 \mu m \) at \( T/T_c = 0.994 \) and \( w/\lambda_\bot(T = 3.812 K) = 0.28 \) as a function of the reduced power \( P/P_c \) of microwave irradiation the relative magnitude of the effect of enhancement of the critical superconducting current \( I_c(P)/I_c(0) \) is shown in figure 2 for various frequencies of an external irradiation. Here, \( P_c \) is the minimum power at which \( I_c(P = P_c) = 0 \). The curve 3 corresponds to a low enough frequency of irradiation, 3.7 GHz, the curve 2 is plotted for the irradiation frequency of 8.1 GHz; the curve 1 corresponds to the frequency of 15.4 GHz. The arrows indicate the values of powers under which the maximal effect of enhancement \( I_c \) was observed for each of the irradiation frequencies. For the irradiation frequency \( f=3.7 \) GHz the reduced power of microwave irradiation, at which a maximum of the effect is observed, equals \( P/P_c=0.25 \). For the frequency \( f=8.1 \) GHz, \( P/P_c=0.51 \), and for \( f=15.4 \) GHz, \( P/P_c=0.61 \). It is seen that with increasing the irradiation frequency the reduced power, at which a maximum of the enhancement effect is observed, increases [20]. Unfortunately, in the enhancement theory [3] a reduction of the effect, after the maximum, with increasing the irradiation power is not considered. Therefore, a shift of maximal manifestation of superconductivity enhancement under
The critical parameters enhanced by microwave field in thin films

Figure 2. The dependence of the relative critical current \( I_c(P)/I_c(0) \) in the sample Sn1 on the reduced microwave irradiation power \( P/P_c \) at \( T = 3.812 \, \text{K} \) for different irradiation frequencies \( f \), GHz: 15.4 (▲), 8.1 (■), 3.7 (■) \( I_c(0) \) is the critical current of the film at \( P = 0 \); \( P_c \) is the minimum power of electromagnetic irradiation at which \( I_c(P) = 0 \).

Electromagnetic irradiation towards higher power with increasing the frequency the theory [3] cannot explain.

The reduced excess of the critical current as a function of the irradiation frequency for films of different width is shown in figure 3. It is seen that with increasing the frequency the effect of exceeding the critical current \( I_{\text{max}}(P) \) over \( I_c(P = 0) \) increases for both narrow (curves 1 and 2) and wide (curve 3) films. With further increase of the frequency this dependence passes through a maximum and then begins to decrease (not shown here). It should be noted that the frequency at which the maximum effect of enhancement of the critical current is observed, decreases with increasing the film width (for Sn1 the maximum frequency is about 30 GHz, and for SnW5 about 15 GHz). It is interesting to note that for the film Sn1 (see figure 3, curve 1) the calculation of the lower cut-off frequency of enhancement \( f_L \) from Equation (11) gives the value of 3 GHz (indicated in figure 3 by a symbol △), which corresponds well to the experiment, as was shown previously for narrow channels [15]. It is important to emphasize that for the calculation of the lower cut-off frequency of enhancement for the sample Sn1, the value \( \tau_c = 8.3 \times 10^{-10} \, \text{s} \) typical of this series of samples was used.

The dependencies of the reduced critical current \( I_c(P)/I_c(0) \) on the reduced power \( P/P_c \) of a microwave field for different irradiation frequencies for the widest sample SnW10 of the width of 7 \( \mu \text{m} \) at a temperature \( T = 3.777 \, \text{K} \) \( (T/T_c = 0.992) \) are shown in figure 4. At this temperature \( w/\lambda_{\perp} = 3.56 \), i.e., less than 4. As shown in references [7] and [13], at this temperature, the sample SnW10 is still a narrow channel and there is no resistive part, caused by the motion of Pearl vortices, in its
The critical parameters enhanced by microwave field in thin films

Figure 3. The reduced value of exceeding the maximum critical current $I_{\text{cmax}}(P)$ over $I_c(0)$ as a function of the irradiation frequency for the samples Sn1 (▲), SnW10 (■) and SnW5 (●) at $T/T_c \approx 0.99$: the values of lower cut-off frequencies of superconductivity enhancement calculated by Equation (11) for the samples Sn1 (△), SnW10 (□)) and SnW5 (○).

current-voltage characteristics. Indeed, the dependencies 1 and 2 in figure 4 do not differ qualitatively from those curves in figure 2. The arrows in figure 4 have the same meaning as in figure 2. In figure 4 it is seen that on increasing the irradiation frequency the reduced power at which the maximum effect of superconductivity enhancement is observed, increases as it was for a narrow channel. Moreover, the calculation of the lower cut-off frequency from Equation (11) gives the value of 4.8 GHz (denoted by a symbol □), which also agrees quite well with the experimental value $f_L$, as seen in figure 3 (curve 2). It is important to note that to calculate the lower cut-off frequency of enhancement for the sample SnW10, the value $\tau_\varepsilon = 4.3 \times 10^{-10}$ s typical of this series of samples was used.

In figure 4, the experimental dependence (▼) was obtained for the relatively low radiation frequency ($f = 0.63$ GHz). This frequency is below the cut-off frequency of the effect of superconductivity enhancement, $f_L$, so there is only a suppression of $I_c$ with increasing $P$. Since in these experimental conditions the sample SnW10 is a narrow channel, it is interesting to compare the experimental dependence (▼) and the theoretical curve 3. In reference [21] it is shown that for superconducting films, the critical current of which is equal to the pair-breaking current of Ginzburg–Landau, the following dependence of the critical current on the irradiation power of electromagnetic field is valid:

$$I_c(P, \omega)/I_c(T) = [1 - (P/P_c(\omega))]^{1/2}$$

$$\times [1 - (2P/(\omega\tau_\Delta)^2P_c(\omega))]^{1/2}$$

(21)
The critical parameters enhanced by microwave field in thin films

Figure 4. The dependence of the relative critical current $I_c(P)/I_c(0)$ in the sample SnW10 on the reduced microwave irradiation power $P/P_c$ at $T = 3.777\,\text{K}$ for different irradiation frequencies $f$, GHz: 12.91 ($\blacksquare$), 6.15 ($\bullet$), 0.63 ($\triangledown$); dashed curve 3 is the dependence $I_c(P)/I_c(0)(P/P_c)$ calculated by Equation (21). At $\omega\tau_\Delta \gg 1$. In our case $\omega\tau_\Delta \approx 24$ and the calculated dependence [21] is shown in figure 4 by a dashed curve 3. It is seen that it agrees quite well with the experimental dependence ($\triangledown$) and confirms the conclusion made in reference [7] that at $w/\lambda_\perp < 4$ films are narrow channels. On lowering the temperature of the sample SnW10 below the crossover temperature $T_{cross}$[7], the relation $w/\lambda_\perp$ increases and becomes slightly greater than 4. This is due to a gradual decrease of $\lambda_\perp(T)$ upon changing the temperature far away from $T_c$. As a result, the distribution of the transport current over the width of the film becomes non-uniform, but not enough to significantly affect the behavior of the film in an electromagnetic field, and consequently the form of $I_c(P)$. To observe significant differences it is necessary to lower significantly the temperature, but the effect of superconductivity enhancement decreases markedly in this case. This is due to a decrease in the number of excited quasiparticles above the gap [3, 15, 17]. Therefore, to further investigate the effect of superconductivity enhancement an initially wider film (the SnW5 sample of the width of 42 $\mu\text{m}$) should be taken. In figure 5 for this sample at $T/T_c = 0.988$ and $w/\lambda_\perp(T = 3.744\,\text{K}) = 20$ there are dependencies of the reduced critical current $I_c(P)/I_c(0)$ on the reduced power $P/P_c$ of a microwave field with different irradiation frequencies. The meaning of the arrows is the same as in figure 2 and figure 4. A figure 5 shows that the reduced power, at which the maximum effect of superconductivity enhancement is observed, increases with the irradiation frequency [20]. Moreover, it is seen that descending parts of the dependencies 1, 3–5 in figure 5 differ from those in figure 2 and figure 4 by a curvature sign: in figure 2 and figure 4 descending parts of the curves are convex, while in figure 5 they are...
The critical parameters enhanced by microwave field in thin films

Figure 5. The dependence of the relative critical current $I_c(P)/I_c(0)$ in the sample SnW5 on the reduced microwave radiation power $P/P_e$ at $T = 3.744$ K for different irradiation frequencies $f$, GHz: 15.2 (▲), 11.9 (●), 9.2 (■), 5.6 (▼). The dashed curve 2 is the dependence $(I_c(P)/I_c(0))(P/P_e)$ calculated by Equation (21).

concave. The curve 5 was obtained at the irradiation frequency $f = 5.6$ GHz, and in this case the enhancement effect was not observed. The dotted curve 2 shows the calculated dependence $I_c(P)$ by Equation (21) for the SnW5 film if the transport current in it was distributed uniformly over its width. It is seen that the curves 2 and 5 are significantly different from each other. Therefore, the concavity of the descending part of the experimental dependence 5 may well be attributed to the non-uniform current distribution across the width of the film. In figure 5 the dependencies 1, 3 and 4 were obtained for irradiation frequencies: 15.2, 11.9, and 9.2 GHz, respectively. The concavity of descending parts can be associated, as for the curve 5, with non-uniform current distribution over the sample width.

Interestingly, in the narrow film Sn1 the enhancement effect is already clearly seen at the irradiation frequency $f = 3.7$ GHz (see figure 2 curve 3), while in the film SnW5 it is not observed even at $f = 5.6$ GHz (see figure 5 curve 5). The calculation of $f_L$ for the film by the formula (11) gives the value of 5.1 GHz, which no longer corresponds to the experimental value of 8.0 GHz. It is important to emphasize that for calculation of the lower cut-off frequency of enhancement in the sample SnW5, the value $\tau_e = 4 \times 10^{-10}$ s typical of this series of samples was used.

The dependencies $I_c(P)$ in relative units for films of different widths for the same experimental conditions are shown in figure 6. The arrows in figure 6 have the same meaning as in figure 2 and figure 4. In figure 6 it is seen that with growth of the film width the ratio $P/P_e$, at which there is a maximum enhancement effect, is reduced, and the effect of enhancement of the critical current in wider films is observed at lower
radiation power, since in the microwave range the value of $P_c$ is practically independent of frequency [16, 21]. Figure 7 shows the dependence of power region of external irradiation, $P/P_c$, where the effect of enhancement of the critical current is observed, on the film width $w$ at a fixed irradiation frequency and temperature. From the data in the figure it follows that as the film width increases the power range $\Delta P$, where the effect of superconductivity enhancement is observed, is reduced. Therefore, one can assume that for rather wide tin films ($w > 100 \ \mu m$) the effect of superconductivity enhancement can be practically unrealisable in experiment as due to a very narrow power range of existence of this effect and because of its small size.

### 3.2. A maximum current of the existence of vortex resistivity

In subsection 3.1 we have found out how electromagnetic irradiation affects the critical current $I_c$ of films of different widths. Another important characteristic current of a wide film is the so-called maximum current of the existence of vortex resistivity $I_m$. Experimentally the current $I_m$ was studied in reference [7] and has the form [6]:

$$I_m(T) = C I_c^{GL}(T) \ln^{-1/2}(2w/\lambda_{\perp}(T))$$

(22)

To date there is no theory of superconductivity enhancement in wide films, and therefore, at present the results of experimental studies of enhancement of $I_m(T)$ cannot be compared with theoretical predictions. However, from Equation (22) obtained for the equilibrium (without external irradiation) current $I_m(T)$, it can be assumed that the behavior of $I_m(P, f)$ in an electromagnetic field is determined by the effect of the irradiation on $I_c^{GL}(T)$ and $\lambda_{\perp}(T)$. 

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**Figure 6.** The dependence of the relative critical current $I_c(P)/I_c(0)$ on the reduced microwave irradiation power $P/P_c$ with the frequency $f = 9.2 \ \text{GHz}$ at $T/T_c \approx 0.99$ in different samples: SnW5 (▲), SnW6 (●) and SnW10 (■).
The critical parameters enhanced by microwave field in thin films

Figure 7. A region of external irradiation power $P/P_c$, where the effect of enhancement of the critical current is observed, as a function of the film width $w$ for the frequency of 9.2 GHz at $T/T_c \approx 0.99$.

Upon enhancement of superconductivity $I_c^{GL}(T)$ increases, and $\lambda_\perp(T)$, according to general considerations (enhancement of $T_c$), must decrease. The reduction rate of $\lambda_\perp(T)$ also depends on the proximity of the operating temperature $T$ to $T_c$, other things being equal. Thus, it is clear qualitatively that the rate of increasing $I_m(P)$ must be lower than the growth rate of $I_c(P)$. Figure 8 shows the experimental dependencies $I_c(P)$ and $I_m(P)$ for the film SnW5 [22]. The inset shows the initial parts of the curves for a more visual representation of the growth rate of $I_c(P)$ and $I_m(P)$. In the figure it is seen that indeed with increasing the irradiation power of the film its critical current $I_c(P)$ increases faster than the current $I_m(P)$. The question arises whether there is enough change in $\lambda_\perp(T)$ under electromagnetic irradiation to suppress the growth of $I_m(P)$ in comparison with an increase of $I_c(P)$. An estimation of changes in $\lambda_\perp(P)$ at the relative temperature at which the curve in figure 8 was measured, indicates that they are sufficiently small, and in accordance with Equation (22) they cannot slow down significantly an increase of $I_m(P)$. Therefore, there must be another reason. An analysis of experimental data suggests that the reason may be the non-uniformity of the current distribution over the film width and the presence of resistive vortex background, which affects $I_m$. This background also depends on an external irradiation, what Equation (22) for the equilibrium current $I_m$ ignores. In this context, it is necessary to draw attention to the fundamental difference between $I_c$ and $I_m$. $I_c$ always appears against a background of the pure superconducting state. So due to the transverse Meissner effect it is always first achieved at the film edges. And the larger its width with respect to $\lambda_\perp$, the more inhomogeneous distribution of the transport current is in it. In contrast, the current $I_m$ is the maximum current at which a uniform flow of vortices across the film...
The critical parameters enhanced by microwave field in thin films

Figure 8. The dependence of the critical current $I_c$ (■) and the maximum current of existence of the vortex resistivity $I_m$ (○) for the sample SnW5 on the reduced microwave power $P/P_c$ with the frequency $f=12.89$ GHz at $T=3.748$ K. The inset shows an enlarged fragment of the above-mentioned dependencies.

is still possible. The presence of a moving vortex lattice makes the distribution of the superconducting current across the film more uniform, although specific [6]. Thus, in a wide film being in a vortex-free state at $I \leq I_c$ the current is always more non-uniformly distributed over the width than in the same film in the presence of intrinsic vortices for the currents $I_c < I \leq I_m$. Because of the above reasons, there is a need of new theory of non-equilibrium state of a wide film, which could take into account the non-uniform distribution of the transport current and the order parameter over the film width in calculating $I_c(P, f)$ and the presence of the vortex resistivity $R(P, f)$ when calculating $I_m(P, f)$.

4. Temperature dependencies of currents enhanced by microwave power in wide films

4.1. The critical current

This section presents results of systematic study of the critical current enhancement in wide superconducting films. It is established that the main properties of superconductivity enhancement in wide films with non-uniform current distribution over the cross section of a sample and those in narrow channels are very similar [19]. A relative moderation of the current non-uniformity in wide films near $T_c$ allowed for using, with a little change, the theory of superconductivity enhancement in spatially homogeneous systems to interpret experimental results in wide films.

Figure 9 shows experimental temperature dependencies of the critical current for the
The critical parameters enhanced by microwave field in thin films

Figure 9. The experimental temperature dependence of the critical currents $I_c(P = 0)$ (■), $I_c(f = 9.2 \text{ GHz})$ (●), and $I_c(f = 12.9 \text{ GHz})$ (▼) for the sample SnW10. The theoretical dependence $I_{cGL}(T) = 7.07 \times 10^2(1 - T/T_c)^{3/2}$ mA calculated by Equation (19) [23] (curve 1); calculated dependence $I_c(T) = 5.9 \times 10^2(1 - T/T_c)^{3/2}$ mA (curve 2); theoretical dependence $I_{cAL}(T) = 9.12 \times 10^1(1 - T/T_c)$ mA calculated by Equation (27) [6] (straight line 3); theoretical dependence $I_c(T) = 6.7 \times 10^2(1 - T/3.822)^{3/2}$ mA (curve 5); theoretical dependence $I_c(T) = 6.7 \times 10^2(1 - T/3.822)^{3/2}$ mA (curve 4); theoretical dependence $I_c(T) = 6.7 \times 10^2(1 - T/3.822)^{3/2}$ mA (curve 5); theoretical dependence $I_c(T) = 6.7 \times 10^2(1 - T/3.822)^{3/2}$ mA (curve 6); calculated dependence $I_c(T) = 5.9 \times 10^2(1 - T/3.818)^{3/2}$ mA (straight line 7).

sample SnW10 [19]. At first, a behavior of $I_c(T)$ without an external electromagnetic field (see figure 9 (■)) is considered. A width of the film SnW10 is relatively small ($w = 7 \text{ µm}$), so in the temperature range $T_{cros1} < T < T_c = 3.809 \text{ K}$ close enough to $T_c$, the sample behaves like a narrow channel, and the critical current is equal to the pair-breaking current of Ginzburg – Landau $I_{cGL}(T) \propto (1 - T/T_c)^{3/2}$ which indicates the high quality of the sample. The crossover temperature $T_{cros1} = 3.769 \text{ K}$ corresponds to the transition of the sample in the wide film regime: at $T < T_{cros1}$ there is a vortex part in the IVC. The temperature dependence $I_c(T)$ at $T < T_{cros1}$ initially retains the form $(1 - T/T_c)^{3/2}$, although the value of $I_c(T)$ turns out to be less than the pair-breaking current $I_{cGL}(T)$ due to the appearance of a non-uniform distribution of the current density and its decrease far away from the film edges. Finally, when $T < T_{cros2} = 3.717 \text{ K}$ the temperature dependence of the critical current becomes linear $I_c(T) = I_{cAL}(T) = 9.12 \times 10^1(1 - T/T_c)$ mA, which corresponds to the Aslamazov–Lempitsky theory [6]. The latter fact confirms our earlier conclusion about the high quality of the film sample SnW10.

For measurements in a microwave field [19] the irradiation power was chosen in such
The critical parameters enhanced by microwave field in thin films

a way that the critical current $I_c^P(T)$ was maximal. Consider the behavior of $I_c^P(T)$ in the sample SnW10 in a microwave field with a frequency $f=9.2$ GHz (figure 9 (●)). In the temperature range $(T_{cros1}(9.2 \text{ GHz}) < T < T_c^P(9.2 \text{ GHz}))$ ($T_{cros1}(9.2 \text{ GHz}) = 3.744$ K, $T_c^P(9.2 \text{ GHz}) = 3.818$ K) there is no vortex part in the IVC, i.e., the sample behaves as a narrow channel. Note that $T_{cros1}(9.2 \text{ GHz}) < T_{cros1}(P=0)$, whereas $T_c < T_c^P$, i.e., at the critical current $I_c$ becomes linear (figure 9, straight line 7).

Figure 9 also shows the temperature dependence of the highest enhanced critical current of the sample SnW10 at a higher irradiation frequency $f=12.9$ GHz (figure 9 (▼)) [19]. It can be seen that, as in a narrow channel, the highest enhanced critical current increases with the irradiation frequency. Note that at the given irradiation frequency there is no vortex part in the IVC over the entire temperature range investigated (up to temperatures $T = 3.700$ K and even a bit lower). In other words, in the temperature range $(T_{cros1}(12.9 \text{ GHz}) < T < T_c^P(12.9 \text{ GHz}))$ $T_{cros1}(12.9 \text{ GHz}) < 3.700$ K and it is not shown in figure 9. $T_c^P(12.9 \text{ GHz}) = 3.822$ K) the sample behaves as a narrow channel. Note that $T_{cros1}(12.9 \text{ GHz}) < T_c^P(12.9 \text{ GHz}) < T_c^P(9.2 \text{ GHz}) < T_{cros1}(P=0)$, whereas $T_c < T_c^P(9.2 \text{ GHz}) < T_c^P(12.9 \text{ GHz})$. Thus, in conditions of optimal enhancement of superconductivity the temperature range where the sample behaves as a narrow channel increases with the irradiation frequency [19].

It is also important to note that the experimental dependence $I_c^P(T)$ (▼) at $f= 12.9$ GHz is in good agreement with the theoretical one obtained in calculating the enhanced critical current by Equation (16) for a narrow channel (figure 9, curve 5) over the entire temperature range, and is well approximated by the dependence $I_c(T) = 6.7 \times 10^2 (1 - T/3.822)^{3/2}$ mA. Hence, it follows that the temperature of the transition to the wide film regime, where the vortex region appears in the IVC, as well as the deviation temperature $T^{**}$ of the experimental dependence from the dependence, calculated by Equation (16), decrease with increasing the irradiation frequency [19].
The critical parameters enhanced by microwave field in thin films

field. A width of the film is large enough ($w=25 \, \mu m$), so this sample is a narrow channel only in the immediate vicinity of $T_c = 3.816$ K, and at $T < T_{c\text{ross}} = 3.808$ K behaves as a wide film. At $T_{c\text{ross}} = 3.740 \, K < T < T_{c\text{ross1}}$, the temperature dependence of the critical current has the form $(1 - T/T_c)^{3/2}$, although the value of $I_c$ is less than $I_{c\text{GL}}$. At $T < T_{c\text{ross2}}$, the temperature dependence of the critical current becomes linear and corresponds to the Aslamazov – Lempitsky theory [6]

$\mathbf{I_{c}(T) = I_{c\text{AL}}(T) = 1.47 \times 10^2 (1 - T/3.835)^{3/2} \, mA.}$

In a microwave field with a frequency $f = 15.2 \, GHz$, similar to the narrow sample SnW10, there is an increase of the critical temperature $T_{c\text{ross}}(15.2 \, GHz)=3.835 \, K$ and a noticeable decrease of crossover temperatures: $T_{c\text{ross1}}^P = 3.738 \, K$ and $T_{c\text{ross2}}^P = 3.720 \, K$. At the same time, in order to achieve a good agreement between the experimental dependence of the highest enhanced critical current $I_{c}(T)$ and Equation [10], it is necessary to normalize this formula by the measured equilibrium ($P=0$) critical current $I_c(T) = 1.0 \times 10^3(1 - T/T_c)^{3/2} \, mA$ over the entire temperature range measured (figure 10, curve 3). In this temperature range, the critical current can be approximated by the dependence: $I_c(T) = 1.0 \times 10^3(1 - T/3.835)^{3/2} \, mA$. When $T < T_{c\text{ross2}}^P$ the temperature dependence $I_{c}(T)$ is linear (figure 10, straight line 4). From the data in figure 10 it follows that under enhancement of superconductivity by a microwave field even a fairly wide film behaves as a narrow channel down to low temperatures than that without irradiation ($T_{c\text{ross1}}^P < T_{c\text{ross1}}$); the vortex region in the IVC in this temperature range is also absent [10].

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure10.png}
\caption{The experimental temperature dependencies of the critical currents $I_c(P = 0)$ (■), $I_c(f = 15.2 \, GHz)$ (▲) for the sample SnW8: calculated dependence $I_c(T) = 1.0 \times 10^3(1 - T/T_c)^{3/2} \, mA$ (curve 1); theoretical dependence $I_{c\text{AL}}(T) = 1.47 \times 10^2(1 - T/T_c) \, mA$ calculated by Equation [27] (straight line 2); theoretical dependence $I_c(f = 15.2 \, GHz)$ calculated by Equation [10], normalized by the curve 1, and fitting dependence $I_c(T) = 1.0 \times 10^3(1 - T/3.835)^{3/2} \, mA$ (curve 3); calculated dependence $I_c(T) = 1.72 \times 10^2(1 - T/3.835) \, mA$ (straight line 4).}
\end{figure}
The critical parameters enhanced by microwave field in thin films

A qualitative similarity between obtained in reference [19] results of studies of wide films and experimental results of studies of narrow channels [15], and the ability to quantitatively describe the temperature dependence of $I_c^P(T)$ in wide films using equations of the Eliashberg theory suggest that the mechanism of the enhancement effect is common for both wide films and narrow channels [19]. It lies in increasing the energy gap, caused by the redistribution of nonequilibrium quasiparticles to higher energies under a microwave field [3]. This conclusion is not entirely obvious for wide films with a non-uniform current distribution across the sample width.

A similarity of enhancement mechanisms in narrow channels and wide films can be supported by the following observations. Despite an increase of the current density, near the edges of a wide film the main current, both transport and induced by a microwave field, is distributed over the entire film width. Thus, the nonequilibrium of quasiparticles in a wide film, as in a narrow channel, is excited by a microwave field within an entire volume of a superconductor, and therefore, the effect of enhancement in wide films undergoes a certain quantitative modification due to non-uniform current distribution. In this regard, we emphasize a significant difference in the conditions of formation of nonequilibrium under a microwave field in wide films and bulk superconductors, in which up to now the effect of enhancement was not observed. In the latter case, the total current is concentrated in a thin Meissner layer of the thickness of $\lambda$ near the surface of the metal, which leads to an additional relaxation mechanism: the spatial diffusion of nonequilibrium quasiparticles excited by the microwave field, from the surface into the equilibrium volume. An intensity of this mechanism is determined by time $\tau_D = \lambda^2(T)/D$ during which quasi-particles leave Meissner layer, which at typical temperature is three to four orders of magnitude less than the inelastic relaxation time. This high efficiency of the diffusion mechanism of relaxation, apparently, leads to suppression of the effect of enhancement in bulk superconducting samples [19].

The current state of a wide film, despite the semblance with the Meissner state of a bulk current-carrying superconductor is qualitatively different from the latter. While in a bulk superconductor the transport current is concentrated in a thin surface layer and decays exponentially at a distance of the London penetration depth $\lambda(T)$ away from the surface, in a wide film the current is distributed over its entire width $w$ according to approximating power law $[x(w - x)]^{-1/2}$, where $x$ is the transverse coordinate [6]. Thus, the characteristic length $\lambda_\perp(T) = 2\lambda^2(T)/d$ ($d$ is the film thickness), called in the theory of the current state of wide films usually as a penetration depth of perpendicular magnetic field, in fact defines not a spatial scale of the current decay for outgoing from the edges, but a magnitude of the edge current density, acting as a ”cut-off” factor in the above-mentioned law of the current density distribution at distances $x, w - x \sim \lambda_\perp$ away from the film edges [19].

Being based on a qualitative difference between the current states in bulk and thin-film superconductors, one can argue that moderate non-uniformity of the current distribution in wide films does not cause fatal consequences for the effect of enhancement, and that the diffusion of nonequilibrium quasiparticles excited in the whole bulk of a
film, making only minor quantitative deviations from the Eliashberg theory \[19\].

The authors of reference \[19\] used a modeling approach to account for these deviations by introducing a numerical form factor of the current distribution in Equation (16) for the enhanced critical current of Eliashberg. They evaluated this form factor by fitting the limiting case of Equation (16) at zero-power microwave irradiation, i.e., Equation (17) for measured values of the equilibrium critical current. They then used the obtained values of the form factor in Equation (16) for \( P \neq 0 \), to yield a good agreement with the experimental data (see figure 9).

Noteworthy is the question of how the Eliashberg’s mechanism ”works” in a wide film in narrow channels the superconductivity is destroyed due to the mechanism of homogeneous pair-breaking of Ginzburg-Landau, while in a wide film the superconductivity is destroyed due to the emergence of vortices. We believe that in this case the enhancement of the energy gap leads to a corresponding increase of the barrier for entering vortices, and that enhances the critical current in a wide film \[19\]. It is interesting to note that there are no significant features in the curves \( I_c(T) \), when upon decreasing the temperature the films go from the regime of a narrow channel to the regime of a wide vortex film. One can therefore conclude that the transition between the regimes of a uniform pair-breaking and vortex resistivity affect neither the value nor the temperature dependence of the critical current.

To complete the discussion of the effect of superconductivity enhancement, we attract an attention to the empirical fact that all the theoretical curves for \( I_c^P(T) \), derived from the equations of the Eliashberg theory, are well approximated by a power law \( (1 - T/T_{c}^P)^{3/2} \). This law is very similar to the temperature dependence of the pair-breaking current of Ginzburg-Landau, in which the critical temperature \( T_c \) is replaced with its enhanced value \( T_{c}^P \). The explicit expressions for such approximating dependencies with numerical coefficients are given in the captions to figure 9 and figure 10.

The other important result of these studies is significant expansion of temperature range near the superconducting transition temperature, where a film behaves as a narrow channel during enhancement of superconductivity: In a microwave field, the crossover temperature in the wide film regime \( T_{cros}^P \) is significantly reduced compared to its equilibrium value \( T_{cros} \), while \( T_{cros}^P > T_c \). At first glance, this result is somewhat contrary to the criterion of the transition between the different regimes of a superconducting film: \( w = 4\lambda_\perp(T_{cros}) \) \[13\] since an increase of the energy gap upon irradiation implies a reduction of the \( \lambda_\perp \) and, consequently, reduction of the characteristic size of vortices. This obviously makes easier the conditions for entry of vortices into a film. Consequently, the crossover temperature is expected to increase in a microwave field. However, it appears that the mechanism of an influence of microwave radiation on vortices is somewhat different. So, it was found that a wide film with a vortex region in IVC under a microwave field behaves like a narrow channel: The vortex region in IVC disappears (see, e.g., figure 9 (▼)). It should be noted that this kind of IVC may be in two cases. In the first case, under the influence of a microwave field there is delay in motion of
vortices up to the point of their termination, i.e., vortices appear, but under a microwave radiation, they do not move. In the second case the vortices do not appear at all. Turn back to figure 9. In the temperature range $T_{cros1} = 3.769 \text{ K} < T < T_c = 3.809 \text{ K}$ without microwave irradiation the sample SnW10 is a narrow channel. Under microwave field with the frequency $f=12.9 \text{ GHz}$ in this sample there is an increase of the critical current (enhancement of superconductivity) (see figure 9 (▼)). At the same time, it is important to emphasize that the temperature dependence of the enhanced critical current agrees well with the theoretical dependence $I_{cP}(T)$ (see figure 9 curve 5), plotted in accordance with the Eliashberg theory for a narrow superconducting channel with a uniform current distribution over the cross section of the sample. It is interesting to note that this theoretical dependence coincides well with experimental points (see figure 9 (▼)) not only in the temperature range $T_{cros1} = 3.769 \text{ K} < T < T_c = 3.809 \text{ K}$, in which the sample SnW10 is a narrow channel in the absence of microwave irradiation, but at much lower temperatures (up to $T < 3.700 \text{ K}$). This behavior of $I_{cP}(T)$ suggests that under the microwave field of $f = 12.9 \text{ GHz}$ in the temperature range $3.700 \text{ K} < T < T_{cP}$ there are no vortices in the sample SnW10. Otherwise, in figure 9 in the temperature range $3.700 \text{ K} < T < 3.769 \text{ K}$ the values of $I_{cP}(T)$ would be lower compared with the theoretical curve 5 calculated within the Eliashberg theory. Moreover, the crossover would be seen in the dependence $I_{cP}(T)$ upon entering vortices. The suppression of vortex resistivity in a wide film by a microwave field is discussed in more detail in reference [23].

Thus, referring to figure 9 one can say the following. For the irradiation frequency $f=12.9 \text{ GHz}$ the maximum value of $I_{cP}(T)$ in the sample SnW10 is realized at high ($P/P_c=0.45$) power of an external microwave field, which prevents the formation of vortices, so the sample behaves as a narrow channel in the temperature range from $T_{cP}$ up to $T < 3.700 \text{ K}$. In this case, the curve 5 in figure 9 plotted using Equation (16) of the Eliashberg theory for a narrow channel, and giving the pair-breaking current density of Ginzburg – Landau at $P=0$ is in a good agreement with the experimental curve $I_{cP}(T)$ (figure 9 (▼)). On this basis, it can be argued that in this case due to a microwave field the sample becomes a narrow channel (there is no vortex region in IVC and $I_{cP}(T)$ is fully consistent with the formula (16) of the Eliashberg theory, assuming a uniform distribution of the superconducting current over the cross section of the sample.

As the irradiation frequency ($f=9.2 \text{ GHz}$) decreases the power at which the maximum value of $I_{cP}(T)$ is realized and consequently its influence are reduced. This leads to a smaller decrease of $I_{cP\text{ros1}}$ with respect to $T_{cros1}$. It is important to note that in this case too, the experimental dependence $I_{cP}(T)$ (see figure 9 (●)) agrees quite well with the curve 4 in figure 9 plotted according to Equation (16) for a narrow channel, up to the temperature of $T^{*\ast} = 3.760 \text{ K} < T_{cros1} = 3.769 \text{ K}$. At temperatures $T_{cros1} < T < T^{*\ast}$ for the sample SnW10 there is no vortex region in IVC, but $I_{cP}(T)$ deviates downward from the theoretical curve 4 in figure 9 plotted for a narrow channel and normalized in such a way that it gives the pair-breaking current of Ginzburg – Landau at $P=0$. 
As can be seen from Equation (17) and Equation (18) in the Eliashberg theory, the expression for the critical current at $P=0$, enhanced by a microwave field, transforms to the formula for the pair-breaking current of Ginzburg–Landau. Like the whole theory, this is true only in the case of a narrow channel. At the same time, at temperatures $T < T_{cros1}$ the SnW10 film reveals itself as wide (there appear a vortex region in IVC), the distribution of the superconducting current over its cross section becomes non-uniform, and the critical current $I_c(T) = 5.9 \times 10^2(1 - T/T_c)^{3/2}$ mA of this film at $P = 0$ is less than the pair-breaking current $I_{cGL}(T) = 7.07 \times 10^2(1 - T/T_c)^{3/2}$ mA for the sample SnW10, although the temperature dependence is preserved. Interestingly that the ratio $I_c(T)/I_{cGL}(T) \approx 0.83$ for sample SnW10. In this case, it turns out that if a normalization factor in Equation (16) is introduced so that at $P=0$ it will give not $I_{cGL}(T)$ but $I_c(T)$, then using the formula one can plot a curve (see figure 9 curve 6), which is in a good agreement with the experimental dependence $I_P^*(T)$. It should be noted that in this case, the normalization factor is 0.83 [19]. An introduction of a universal normalization factor over the entire temperature range $T_{cros2} < T < T_{cros1}$ is possible due to the fact that the temperature dependence $I_c^P(T)$, described by Equation (16), although is quite complex, yet is numerically very close to the law $\propto (1 - T/T_{cP})^{3/2}$, which at $P=0$ transforms to the dependence $I_{c}(T) \propto (1 - T/T_c)^{3/2}$ for a wide film.

A similar situation is observed for a much wider film SnW8. This sample is a narrow channel only in the immediate vicinity of the $T_c$. Therefore, for temperatures $T < T_{cros1} = 3.808$ K Equation (16) gives the values of the enhanced critical current that do not coincide with the experimental values of $I_c^P(T)$. However, for normalization of Equation (16) to the equilibrium critical current $I_c(T) = 1.0 \times 10^3(1 - T/T_c)^{3/2}$ mA at $P=0$ there is also a good agreement between theory and experiment (figure 10, curve 3). Note that in this case too, the normalization factor of Equation (16) of the Eliashberg theory is the same as the ratio $I_c(T)/I_{cGL}(T)$.

Thus, we conclude that if the equilibrium critical current ($P=0$) of a wide film has the temperature dependence $I_c(T) \propto (1 - T/T_c)^{3/2}$, typical for a narrow channel, then using the formula of the Eliashberg theory, normalized to $I_c(T)$, one can well describe the experimentally measured dependencies of the enhanced critical current $I_c^P(T)$, which are numerically very close to $(1 - T/T_c^{cP})^{3/2}$. In the temperature range $T < T_{cros2}^P$, where the temperature dependence of the critical current for a wide film is linear, $I_c(T) \propto 1 - T/T_c$, the temperature dependence of the enhanced critical current is also linear: $I_c^P(T) \propto 1 - T/T_c^{cP}$. These facts, albeit indirectly, confirm the hypothesis that the mechanism of superconductivity enhancement in wide films is the same as in narrow channels.

4.2. The current of phase-slip processes

This section presents the results of experimental study of enhancement of the current $I_m(T)$, at which the first PSL is formed, in a wide temperature range under an external microwave irradiation of different frequencies [22].
Taking into account the fact that there is no theory of superconductivity enhancement in wide films, one can try at least qualitatively to describe the effect of microwave irradiation on the current $I_m(T)$ using the following considerations. In studies of the critical current enhancement in superconducting films with different width, the following experimental facts were obtained [19]. In narrow channel, the equilibrium critical current has the temperature dependence $I_{GL}^c(T) \propto (1 - T/T_c)^{3/2}$. At the same time, the enhanced critical current $I_{Pc}^e(T)$ of this channel, perfectly described by the Eliashberg theory [3,9–12] can be well approximated by the dependence $I_{Pc}^e(T) \propto (1 - T/T_{Pc})^{3/2}$ [14]. Here, $T_{Pc}$ is the enhanced critical temperature. In a wide (vortex) film near $T_c$ the temperature dependence of the equilibrium critical current is $I_c^e(T) \propto (1 - T/T_c)^{3/2}$ [7]. It turns out that the critical current enhanced by a microwave field in this case can also be well approximated by a similar dependence: $I_{Pc}^e(T) \propto (1 - T/T_{Pc})^{3/2}$ [19]. When $T < T_{cros2}$ in a wide film there is a linear temperature dependence of the equilibrium critical current [7]. Almost at the same temperatures the enhanced critical current can also be approximated by the linear dependence $I_{Pc}^e(T) \propto (1 - T/T_{Pc})$ [19]. Based on the above experimental facts, one can also try to approximate the temperature dependencies of the current $I_{Pm}^e(T)$, enhanced by a microwave field, by a dependence similar to Equation (22) for the equilibrium case.

Figure 11 shows the experimental temperature dependencies of the currents $I_{Pm}^e(T)$ in a microwave field and currents $I_m(T)$ in the absence of the field for the sample SnW5 [22]. For clarity, in figure 11(b) the results are shown for a narrower temperature range near $T_c$ than that in figure 11(a). A width of the film SnW5 is large enough ($w = 42 \mu m$), so even for temperatures $T < T_{cros2} = 3.740 K$ there is a linear temperature dependence of the critical current [7], what is close enough to $T_c$.

First, we consider the behavior of the current at which the first PSL appears, $I_m(T)$, without an external electromagnetic field (see figure 11 (•)). The solid curves 1 in these figures are calculations of $I_m(T)$ according to Equation (22) with taking into account the film parameters (see Table 1)

$$I_m(T) = 2.867 \times 10^3 (1 - T/T_c)^{3/2} \times 1.35[\ln(2 \times 42)$$

$$\times (1 - T/T_c)/0.02532)]^{-1/2} [mA].$$

As can be seen in figure 11, the experimental dependence $I_m(T)$ is in a good agreement with calculated one (see curve 1 ) [22]. The experimental dependence of the current $I_{Pm}^e(T)$ at the irradiation frequency $f = 9.2$ GHz (see figure 11 (▼)) is well approximated by the dependence

$$I_{Pm}^e(T) = 2.869 \times 10^3 (1 - T/T_{Pc})^{3/2} \times 1.44[\ln(2 \times 42)$$

$$\times (1 - T/T_{Pc}/0.02531)]^{-1/2} [mA].$$

similar to Equation (22) (figure 11 curve 2). Here and in the calculation of the pair-breaking current of Ginzburg – Landau, the enhanced critical temperature $T_{Pc} = 3.791 K$ was used. The experimental dependence of the current $I_{Pm}^e(T)$ at the frequency of an external electromagnetic field $f = 12.9$ GHz (not shown due to space limitations) is well
approximated by

\[
I_m^P(T) = 2.875 \times 10^3 (1 - T/T_c^P)^{3/2} \times 1.28 \ln(2 \times 42 \times (1 - T/T_c^P)/0.02529)]^{-1/2} \text{[mA]}. \quad (25)
\]

Here, the enhanced critical temperature \(T_c^P = 3.797 \text{ K}\) was also used. The experimental dependence of the current \(I_m^P(T)\) at the frequency of microwave field \(f = 15.2 \text{ GHz}\) (see figure 11 (▲)) is well approximated by

\[
I_m^P(T) = 2.877 \times 10^3 (1 - T/T_c^P)^{3/2} \times 1.28 \ln(2 \times 42 \times (1 - T/T_c^P)/0.02528)]^{-1/2} \text{[mA]}. \quad (26)
\]

similar to Equation (22) (figure 11(a), curve 3). Here, the stimulated critical temperature \(T_c^P = 3.799 \text{ K}\) was used. For measurements of the current \(I_m^P(T)\) of films in microwave field the irradiation power was chosen in such a way that the critical current \(I_m^P(T)\) was maximal. In this case the current \(I_m^P(T)\) was also maximal due to some correlation of these quantities [20].

Since the theory [6], in which the definition \(I_m(T)\) is introduced, assumes a linear temperature dependence of the critical current

\[
I_c^{AL}(T) = 1.5 I_c^{GL}(0) (\pi \lambda_\perp(0)/w)^{1/2} (1 - T/T_c).
\]

then, strictly speaking, Equation (22) should be applicable only in the temperature range \(T < T_{c\text{ro}}\), where such a dependence of the critical current is observed. However, as seen in figure 11, Equation (22) for the equilibrium dependence \(I_m(T)\) and Equation (24) – Equation (26) for the case of enhancement of \(I_m^P(T)\) by an electromagnetic field sufficiently well describe the experimental dependencies in the case of \(T > T_{c\text{ro}}\) too. This is obviously due to the fact that at \(T < T_{c\text{ro}}\) and \(T > T_{c\text{ro}}\) the resistive current states at \(I \sim I_m\) differ little from each other: both of these states are characterized by fairly uniform current distribution over the width of the sample due to the quite dense filling of the film by a lattice of vortices [22].

Thus, the experimental temperature dependencies of the enhanced current \(I_m^P(T)\) are well approximated by Equation (24) – Equation (26), similar to formula (22) for the equilibrium case of the Aslamazov – Lempitsky theory [6], where the critical temperature \(T_c\) is replaced by the enhanced critical temperature \(T_c^P\) [22].

Consider the behavior of \(I_m^P(T)\) of the sample SnW5 in a microwave field with the frequency \(f = 9.2 \text{ GHz}\) (figure 11 (▼)). It can be seen that upon irradiation of the film by microwave power there is the enhancement of \(I_m^P(T, f = 9.2 \text{ GHz})\) up to \(T = 3.708 \text{ K}\). At temperatures \(T < 3.708 \text{ K}\) the enhancement of \(I_m^P(T)\) was not observed. For the irradiation of the sample SnW5 by microwave field with the frequency \(f = 12.9 \text{ GHz}\) the enhancement of \(I_m^P(T, f = 12.9 \text{ GHz})\) is observed up to \(T = 3.690 \text{ K}\). At lower temperatures the enhancement of \(I_m^P(T)\) was not found. For the irradiation of the sample SnW5 by microwave field with the frequency \(f = 15.2 \text{ GHz}\) (figure 11 (▲)) enhancement of \(I_m^P(T, f = 15.2 \text{ GHz})\) is observed up to \(T = 3.655 \text{ K}\). At temperatures \(T < 3.655 \text{ K}\) the enhancement of \(I_m^P(T)\) was not observed. It should be noted that in figure 11 it is seen that \(I_m^P(T, f = 15.2 \text{ GHz}) > I_m^P(T, f = 12.9 \text{ GHz}) > I_m^P(T, f = 9.2 \text{ GHz})\).
The critical parameters enhanced by microwave field in thin films

Figure 11. The experimental temperature dependencies of the maximum current $I_m$ of existence of stationary uniform flow of intrinsic vortices of transport current across the film SnW5: $I_m(T, P = 0)$ (●), $I_m^P(T, f = 9.2 \text{ GHz})$ (▼) and $I_m^P(T, f = 15.2 \text{ GHz})$ (▲). Curve 1 is the theoretical dependence $I_m(T)$ (see Equation (23)); curve 2 is the calculated dependence $I_m^P(T, f = 9.2 \text{ GHz})$ (see Equation (24)); curve 3 is the calculated dependence $I_m^P(T, f = 15.2 \text{ GHz})$ (see Equation (26)).

Thus, an absolute value of $I_m^P(T)$ increases with the irradiation frequency, and the temperature region of enhancement of $I_m^P(T)$ is extended toward lower temperatures [22]. By the way, in the same way behaves a critical current $I_c^P(T)$ [19]. Let us try to find an explanation for this.

We take into account two factors. First of all, as already noted, in wide films at $I \approx I_m$ the current distribution across the width of the film is close to uniform. In this case it is wise to make use of the knowledge accumulated for narrow channels. Second,
The critical parameters enhanced by microwave field in thin films

Figure 12. The calculated dependencies of the equilibrium (curve 1) and enhanced by microwave field the gap for the sample SnW5 (curve 2, $f=9.2$ GHz; curve 3, $f=15.2$ GHz).

the source of enhanced critical parameters $I^P_m(T)$ and $I^P_c(T)$ of a superconductor is a non-equilibrium distribution function of quasiparticles over energy. In this case, first of all the value of the energy gap increases [3, 9–12].

With taking into account the above arguments, in figure 12 the curve 1 represents a temperature dependence of the equilibrium gap $\Delta_0(T)$ (in frequency units), and the curves 2 and 3 show temperature dependencies of the enhanced gap $\Delta^P_m(T)$ for the sample SnW5 at the irradiation frequencies of 9.2 and 15.2 GHz, assuming a uniform distribution of the transport current density over the cross section. In the figure it is seen that the upper branch of the temperature dependence of the energy gap enhanced in a superconductor, $\Delta^P_m(T)$, intersects with a similar dependence of the equilibrium gap $\Delta_0(T)$. Moreover, the higher the irradiation frequency, the lower in the temperature is the point of intersection of dependencies $\Delta^P_m(T)$ and $\Delta_0(T)$ ($T_{02} = 3.762 \text{ K} < T_{01} = 3.782 \text{ K}$), where the enhancement of the energy gap ceases.

It should be noted that the temperature ranges where the non-equilibrium values of the energy gap and the currents exceed the equilibrium values do not coincide. The reason for this lies in a significant difference between the curves of pair-breaking $I_s(\Delta)$ in equilibrium and non-equilibrium cases [15].

It is important to note that the maximum enhanced critical temperatures $T^P_{c1} = 3.791 \text{ K}$ at the irradiation frequency of 9.2 GHz and $T^P_{c2} = 3.799 \text{ K}$ at the frequency of microwave field 15.2 GHz, derived from theoretical curves 2 and 3 in figure 12, are in a good agreement with the values of $T^P_c$, obtained by fitting the experimental curves $I^P_m(T)$ (see figure 11).
5. Conclusion

In the present review, the behavior of the critical current \( I_c \) and the maximum current \( I_m \) at which in a wide film a vortex structure of the resistive state disappears and the first phase-slip line arises is analyzed in thin superconducting films of different width, located in a microwave field. A enhancement of superconductivity by an external electromagnetic field was found in wide \( w \gg \xi(T), \lambda_{\perp}(T) \), superconducting films [8] with a non-uniform spatial distribution of the current over the sample width. The superconductivity enhancement in a wide film increases not only the critical current \( I_c \), but also the maximum current at which there is a vortex resistive state, \( I_m \) [8]. In the framework of the Eliashberg theory an equation for the enhanced critical current was derived and expressed in terms of experimentally measured quantities [19]. A comparison of experimental temperature dependencies of the enhanced critical current with those calculated in the framework of the Eliashberg theory revealed a good agreement between them [19]. It was shown [19], that near the superconducting transition the temperature dependence of the enhanced critical current in not very wide films \( (w < 10\lambda_{\perp}(T)) \) appears to be numerically very close to the law \((1-T/T^{P}_{c})^{3/2}\) for the equilibrium pair-breaking critical current when \( T_c \) is replaced with the enhanced critical temperature \( T^{P}_{c} \). It was found [19], that for sufficiently wide films \( (w > 10\lambda_{\perp}(T)) \) the enhanced critical current has a linear temperature dependence \( I^{P}_{c}(T) \propto (1-T/T^{P}_{c}) \), similar to that in the equilibrium theory of Aslamazov  Lempitsky with replacement of \( T_c \) by \( T^{P}_{c} \). Experimental dependencies of the enhanced critical current \( I^{P}_{c} \) and the enhanced current of formation of the first PSL, \( I^{P}_{m} \), on power and frequency of microwave irradiation were obtained in thin (thickness \( d \ll \xi(T), \lambda_{\perp}(T) \)) superconducting films of different width \( w \) [19, 20, 22]. It was found experimentally that when the film width increases, the range of irradiation power, at which the effect of superconductivity enhancement is observed, shrinks abruptly, and hence the probability of its detection decreases [20]. This statement is an answer to the question on much delayed discovery of the enhancement effect in wide films. It is established that with an increase of the film width the ratio \( P/P_c \), at which there is a maximum enhancement effect, is reduced, and the effect of enhancement of the critical current in wider films is observed at lower irradiation power, since in the microwave range the value of \( P_c \) is practically independent of frequency [16, 21]. The power, at which the maximum effect of enhancement is observed, increases with the frequency of microwave irradiation. It was found that when the film width increases, the curvature sign of a descending section in the dependence \( I^{P}_{c}(P) \) is changed [20].

Studies of the critical current enhanced by a microwave irradiation confirmed an earlier conclusion, based on studies of the equilibrium critical current (in the absence of external irradiation) [7], that narrow channels are films which satisfy to the relation \( w/\lambda_{\perp} \leq 4 \). For them, according to a theory [3] the calculated values of lower boundary frequencies of superconductivity enhancement correspond to experimental values. In much wider films there appears a dependence of characteristic parameters of
The critical parameters enhanced by microwave field in thin films

enhancement effect on the film width. In this connection, to describe a non-equilibrium state of wide films \((w/\lambda_\perp > 4)\) in electromagnetic fields it is necessary to develop a theory, which in contrast to the Eliashberg theory \([3]\) initially takes into account a non-uniform current distribution and the presence of vortices of its own magnetic flux.

An unexpected effect of electromagnetic field on the current \(I_m\), which cannot be considered as a trivial influence of the irradiation on the \(I_{mGL}(T)\) and \(\lambda_\perp(T)\), was found \([22]\). The experimental temperature dependencies of the enhanced current \(I^P_m(T)\) are well approximated by formulas which are similar to the formula \(22\) for an equilibrium case of the Aslamazov – Lempitsky theory \([6]\), in which the critical temperature \(T_c\) is replaced by the enhanced critical temperature \(T^P_c\). It was found that an absolute value of \(I^P_m(T)\) increases with the irradiation frequency, and the temperature region of enhancement of \(I^P_m(T)\) is extended toward lower temperatures \([22]\).

One more important fact is worth mentioning. References \([8, 19, 20, 22, 23]\) present the main results of a study of superconductivity enhanced by microwave irradiation in wide films. In those works, the equilibrium critical current in wide films reached a maximum possible value - a value of the pair-breaking current, and corresponded to the critical current of the Aslamazov – Lempitsky theory. A significant excess of this pair-breaking critical current obtained in the Aslamazov – Lempitsky theory in the absence of external fields was observed under microwave irradiation. This indicates the existence of superconductivity enhancement in wide films and the negligible effect of overheating, if it occurs, including an overheating of an electronic system in a superconductor.

Thus, experimental studies of films with different width showed that the effect of superconductivity enhancement by microwave irradiation is common, and occurs in both the case of uniform (narrow films) and non-uniform (wide films) distribution of the superconducting current over the film width.

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