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Improving directional radiation quality based on a gradient amplitude acoustic leaky wave antenna

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Keywords: gradient amplitude, leaky wave antenna, directional radiation

Abstract

In this work, we show how to modify radiation amplitude with a leaky wave antenna to improve the quality of sound radiation. The designed gradient amplitude leaky wave antenna consists of a straight pipe with periodically loaded membranes, open channels and Helmholtz resonators. An equivalent acoustic composite right/left-hand transmission line that considers the effects of viscous-thermal and viscous-elastic losses is utilized to steer the radiation angle continually from backward to forward as a function of the incident frequency. The numerical results show that by appropriately selecting the structural parameters of the channel and Helmholtz resonator cavity, the quality of the directional radiation is improved based on the gradient distribution of the radiation amplitude and the near unitary phase. Compared with traditional antennas, the proposed gradient amplitude antenna incorporates a frequency scanning capability with gradient amplitude, which improves the directivity quality of the acoustic waves among the operated frequency band, and provides a new design method for acoustic leaky wave antennas.

1. Introduction

Acoustic directional radiation is crucial in many areas such as acoustic integrated and detection devices [1, 2]. Acoustic leaky wave antennas (ALWAs) as a type of passive antennas possess the capability of directional radiation scanning as a function of the incident frequency [3–7]. Although phased-array antennas are also effective at acoustic steering, the presence of numerous active elements leads to high costs of construction and significant power requirements. Guided wave metamaterial with periodic side holes constitutes a basic design for ALWAs. Generally, this kind of ALWA only supports right-hand (RH) propagation and realizes radiated beam scanning from broadside to forward [0:90°] [3]. However, because a composite right/left-hand transmission line (CRLH-TL) metamaterial can generate both RH and left-hand (LH) propagations simultaneously, it can be incorporated to create CRLH-TL ALWA. The ALWA constructed by CRLH-TL metamaterial will allow the wavenumbers to continuously vary from negative to positive, and the main radiation beam scans from backward to forward [−90°:90°] as a function of the incident frequency [4]. In previous studies, the CRLH-TL ALWAs have been designed using an array of side holes and membranes that were periodically loaded along the waveguide, where power could leak along the waveguide through the periodical side holes [5–7]. So far, acoustic antennas have attracted considerable attention because they exhibit various unique applications, such as single sensor direction-finding [7], acoustic dispersive prisms [8], sound source localization [9] and noise control [10–12]. Despite the tremendous prospect of these devices with acoustic steering, they will unavoidably bring many side lobes with the increase of unit cell number and size.

Acoustic metasurfaces as a new kind of planer profile metamaterials have open up a new avenue for acoustic wavefront manipulation [13]. Over the past few years, we have witnessed considerable interest in acoustic metasurfaces, which could modulate refracted/reflected waves by introducing discrete phase variations from 0 to 2π along their surface [14–16]. In general, the generalized acoustic Snell’s law is widely utilized to provide...
accurate phase response of the metasurface and explore the fascinating wavefront phenomena. By using an acoustic metasurface with gradient phase profile, various unique properties have been demonstrated, such as anomalous refraction/ reflection [17–19], flat focusing [20, 21], asymmetric propagation [22, 23], wave bending [24, 25] and propagation waves being converting into surface waves [26]. In addition to metasurface with gradient phase, another metasurface with gradient amplitude has recently been developed for the complete control of acoustic waves [27]. For example, the decoupled modulation of amplitude and phase of reflected sound by tailoring the leaky loss has been demonstrated by providing fine manipulation of three-dimensional sound fields. This method significantly improves the quality of spatial manipulation of sound fields relative to previous pure-phase metasurfaces [27]. Therefore, there is great potential in combining the concept of gradient amplitude or phase of an acoustic metasurface with an ALWA.

Inspired by the traditional ALWA and gradient metasurface, we present a gradient amplitude ALWA, which significantly suppresses side lobes and improves the quality of directional radiation relative to traditional ALWA. Moreover, because the traditional ALWA has the property of frequency scanning from backward to forward over a broad bandwidth, our gradient amplitude ALWA can also work over a certain bandwidth. The gradient amplitude ALWA consists of 39 unit cells chosen among nine different types. Each type of unit cells is comprised of a straight pipe with a membrane clamped inside, an open channel and a Helmholtz resonator (HR) loaded along one side. The width of the channel is designed to manipulate the radiated pressure amplitude, and the height of the HR cavity is used to compensate for the phase change caused by the channel width and maintain the balance condition. By appropriately selecting the width of the channel and the height of the HR cavity based on a viscous-thermal and viscous-elastic loss-inclusive CRLH-TL, the discrete amplitude profile of [0:1] and near unitary phase are exhibited for improving the quality of directivity. Our results provide a new method for the design of ALWAs.

2. Design and analysis of the gradient amplitude antenna unit cell

The gradient amplitude ALWA is comprised of 39 unit cells chosen from nine different types. Figure 1(a) shows the geometry of these nine types of unit cells in the $xy$-plane. The width of the channel and the height of the HR cavity are tunable for these nine units, such that the normalized radiated amplitude change could cover [0:1] and the phase is almost unitary. A schematic diagram of an individual unit cell is depicted in figure 1(b), which is comprised of a straight pipe with a clamped one-dimensional membrane, an open channel and a HR loaded along one side. The width and total height of the unit cell are $d = 15$ mm and $l = l_1 + w_3$, where $l_1$ and $w_3$ are the height of the channel and the width of the straight pipe, respectively. The one-dimensional membrane made of DuPont™ Kapton® FPC is characterized by its mass density $\rho_m = 1420$ kg m$^{-3}$, Young’s modulus $E = 2.758$ GPa, Poisson’s ratio $\nu = 0.34$, thickness $d_m = 0.125$ mm and width $w_3 = 7.25$ mm. $w_2$ is also the width of the straight pipe. The HR consists of a short neck with width $w_2 = 1$ mm and height $l_1 = 1.9$ mm and a cavity with fixed width $b = 11.5$ mm and tunable height $a$. The width and height of the channel are $w_1$ and $l_1 = 15$ mm, where the width $w_1$ is variable. The density and acoustic velocity of the background medium are $\rho_0 = 1.25$ kg m$^{-3}$ and $c_0 = 343$ m s$^{-1}$. In this study, a channel with a tunable width and an HR cavity with a tunable height are used to make the designed ALWA with a discrete radiation amplitude profile of [0:1] and a near unitary phase under the operated frequency range.

2.1. Composite right/left-hand transmission line

The equivalent acoustic CRLH-TL circuit could be adopted as a fundamental method to investigate ALWAs. The unit cell of the present gradient amplitude ALWA is analyzed using the equivalent acoustic CRLH-TL circuit shown in figure 1(c). The unit cell is separated into three parts: i.e. a straight pipe with a tight membrane, a shunted HR and a shunted channel. In the model, the narrow HR neck and channel are subjected to viscous-thermal losses [4, 28–30]. The corresponding viscous-thermal losses of the narrow HR neck and channel, and viscous-elastic losses of the membrane have been taken into account in figure 1(c) by adding acoustic resistances. It has been proven that an acoustic model of a one-dimensional membrane is equivalent to a series acoustic impedance $Z_{am} = R_{am} + j[\omega m_{am} - 1/(\omega C_{am})]$, which is equivalent to a series resonant circuit comprised of an acoustic resistance $R_{am}$, acoustic mass $m_{am}$ and acoustic capacitance $C_{am}$ [31, 32]. By considering air-membrane interactions, the air section of the straight pipe can be described by a circuit consisting of a series acoustic mass $m_0 = \rho_0 (d - d_m)/w_0$ and a shunt acoustic capacitance $C_0 = w_3 (d - d_m)/(\rho_0 c_0^2)$. When taking the viscous-thermal losses into consideration, the acoustic model of the shunted HR is equivalent to a shunt acoustic impedance $Z_{hr} = R_{hr} + j[\omega m_{hr} - 1/(\omega C_{hr})]$, which is equivalent to a series resonant circuit comprised of an acoustic resistance $R_{hr} = l_{eff}/(2w_3^2)\sqrt{2\eta_k \rho_0 c_0 \omega}$, acoustic mass $m_{hr} = \rho_0 l_{eff}/w_0$ and acoustic capacitance $C_{hr} = ab/(\rho_0 c_0^2)$, where $\eta_k = \eta [1 + (\gamma - 1)\sqrt{c_0/(\gamma\kappa\eta)}]$ is the total viscous-thermal losses coefficient of air, $\kappa$, $c_p$ and $\eta$ are the thermal conductivity, the specific heat at constant pressure and the viscosity

\[
\eta_k = \eta [1 + (\gamma - 1)\sqrt{c_0/(\gamma\kappa\eta)}]
\]
of air, respectively, \( \gamma = 1.4 \) is the heat capacity ratio of air and \( l_{\text{eff}} \) is the effective height of the short neck \([33–35]\). Additionally, for a channel with viscous-thermal losses, the acoustic model of the shunted channel is equivalent to a shunt acoustic impedance \( Z_{\text{sh}} = R_{\text{sh}} + j\omega m_{\text{sh}} \), which is equivalent to a series circuit comprised of an acoustic resistance \( R_{\text{sh}} = \frac{1}{2w_1^2} \sqrt{2n\rho_0\omega} \) and an acoustic mass \( m_{\text{sh}} = \rho_0 c_0/w_1 \tan(k_0 h_1)/\omega \) [4, 32].

The propagation constant \( \gamma \) of the acoustic wave along the \( x \) direction and the acoustic impedance \( Z_0 \) for the unit cell of ALWA can be defined as \([3]\)

\[
\gamma = \alpha + j\beta = \frac{1}{d} \text{arcosh} \left(1 - \frac{Z}{Y} \right),
\]

\[
Z_0 = \sqrt{\frac{Z}{Y}},
\]

where \( \alpha \) and \( \beta \) are the leakage constant and phase constant of the acoustic wave inside the straight pipe, respectively. \( Z \) and \( Y \) are the acoustic impedance in series connection and the acoustic admittance in parallel connection of the equivalent acoustic CRLH-TL circuit, which can be expressed as

\[
Z(\omega) = j \left( \frac{\omega m_{\text{sp}} + m_{\text{am}}}{\omega c_{\text{am}}} \right) = j\omega m_{\text{sp}} + Z_{\text{am}},
\]

\[
Y(\omega) = j \left( \frac{\omega C_{\text{sp}}}{\omega m_{\text{sh}}} \right) + \frac{1}{j[\omega m_{\text{hr}} - 1/(\omega c_{\text{hr}})]},
\]

From equations (2a) and (2b), the series resonant angular frequency and the parallel resonant angular frequency can be obtained as \( \omega_s = 1/\sqrt{(m_{\text{sp}} + m_{\text{am}})C_{\text{am}}} \) and \( \omega_p = \sqrt{(b_l + \sqrt{b_l^2 + 4a_l})}/2a_l \), where \( a_l = C_{\text{sp}}m_{\text{sh}}C_{\text{hr}} m_{\text{sp}} \) and \( b_l = m_{\text{sh}}C_{\text{sp}}C_{\text{hr}}m_{\text{sh}} + C_{\text{sp}}m_{\text{hr}} \). Here, the series resonant frequency \( f_s(\omega_s = 2\pi f_s) \) is controlled by membrane, and the parallel resonant frequency \( f_p(\omega_p = 2\pi f_p) \) is controlled by open channel and HR. These two resonant frequencies are expected to be almost identical (\( f_s \approx f_p \)), so that the band gap between the negative and positive index regions is close and a seamless transition from negative index to positive index is achieved under the balanced condition \([5]\). The nine types of unit cells of the gradient amplitude ALWA are all

Figure 1. Schematic diagram illustration of the gradient amplitude acoustic leaky wave antenna, where (a) corresponds to the geometry of the gradient amplitude leaky wave antenna consisting of nine types of units. The red lines refer to the one-dimensional membranes. The width of the channel and the height of the HR cavity are tunable for nine units. (b) and (c) Represent the schematic illustration and equivalent acoustic composite right/left-hand transmission line circuit of an individual unit.
designed with the balanced condition by tuning the channel width and the height of the HR cavity simultaneously.

Generally, the phase constant $b$ of the wave propagating along the $x$ direction inside the pipe is used to determine the main radiation angle $\theta_0$ for the ALWA, which is expressed as

$$\theta_0(\omega) = \arcsin(b(\omega)/k_0).$$

In addition, the leakage constant $\alpha$ is another important parameter for the ALWA. For an antenna without reflection, $\alpha$ is calculated by

$$\psi_2 = \psi_1 e^{-2\alpha L},$$

where $L$ is the total length of the antenna, $\psi_1$ and $\psi_2$ are the average power of the incident and radiated cross sections of the antenna, respectively. Equation (4) indicates that the leakage constant $\alpha$ is related to the ratio of the radiated power to the incident power. Because the sound power leakage happens through the channel with open termination, the value of leakage constant $\alpha$ is able to modulate by changing the width of the channel.

### 2.2. Radiating mode based on gradient distribution amplitude

In the following, the relationship between the radiation directivity and the amplitude distribution along the antenna is studied. The gradient amplitude ALWA is designed by the periodic arrangement of nine types of unit cells (as shown in figure 1(a)) along the $x$ direction. The channel of the unit cell with a deep subwavelength is designed to radiate sound energy toward the free space. Therefore, the ALWA can be regarded as an array of elements arrangement in a line. Figure 2 shows an array of elements that have a symmetrical distribution, which contains $m = 2n + 1$ unit cells.

Figure 2. Schematic representation of a leaky wave antenna. Considering the leaky wave antenna is designed by an array of elements that have a symmetrical distribution, which contains $m = 2n + 1$ unit cells.

The directivity factor is calculated as

$$D(\theta) = \frac{P_{rad}(r, \theta)}{P_{rad}(r, 0)} = \frac{\sum_{i=-n}^{n} a_ie^{-j\beta x_i}e^{-jk_0r_i}}{\sum_{i=-n}^{n} a_ie^{-j\beta x_i}}.$$
where $\sum_{i=1}^{n} q_i = A$ is a constant and we assume that $A = 1$. Here, we also assume that the directivity factor $D(\phi)$ of the periodic arrangement of unit cells is expressed as

$$D(\phi) = \begin{cases} 0 & |\phi| > \theta_h, \\ 1 & |\phi| \leq \theta_h, \end{cases}$$

where $\theta_h = k_0 \sin(\theta_b)$, and $\theta_b$ is defined as the maximum directivity sharpness angle. By substituting equations (9) into (8), the expression for the radiated sound pressure amplitude of the $i$th unit cell is rewritten as

$$a_i = \frac{d}{2\pi} \int_0^{\pi} D(\phi) e^{-i\phi \beta} d\phi = \frac{d}{2\pi} \int_{-\phi_h}^{\phi_h} e^{-i\phi \beta} d\phi = \frac{d\phi_h}{\pi \sin(\pi \phi_h)} \sin(x_i \phi_h) \cos(x_i \phi_h).$$

It is noted that when $x_i = 0$, the sound pressure amplitude of the origin unit cell is $a_0 \approx \frac{d\phi_h}{\pi \sin(\pi \phi_h)}$. Thus, the ratio of the sound pressure amplitude of the $i$th unit cell to the origin unit cell is $\sin(x_i \phi_h)/(x_i \phi_h)$. Moreover, for a fixed maximum directivity sharpness angle $\theta_h$, we could estimate how many unit cells must be needed for the construction of the gradient amplitude ALWA.

### 2.3. High quality directivity of the gradient amplitude ALWA

In order to design the gradient amplitude ALWA with the high quality of radiation directivity, we employ nine types of units as shown in figure 1(a), which could tailor the radiated waves with discrete pressure amplitude shifts that covered the range $[0:1]$. It should be noted that, under the specific frequency, these nine types of unit cells support the radiated sound pressure amplitude profile of $[0:1]$ and near unitary phase simultaneously. In previous studies, holey structured lossy acoustic metamaterials were widely introduced to modulate pressure amplitude [27, 36]. Relevant discussions have indicated that the leaky wave property of the ALWA depends on the leakage constant $\alpha$, which is decided by the width of the channel $w_1$. Therefore, we manipulate the pressure amplitude by changing the channel width $w_1$. Meanwhile, the change of channel width will unavoidably bring a phase change and break the balance condition, hence, the height of the HR cavity should be adjusted to compensate for the phase change caused by the channel (equation (2)). Here, the desirable gradient amplitude profile and unitary phase are achieved by a channel with a tunable width and a HR cavity with a tunable height simultaneously. To modify the radiated pressure amplitude along the $x$-axis of the gradient amplitude ALWA, the channel width $w_1$ varies from 1.6 mm at the center of the gradient antenna to 0.1 mm at the edge of the gradient antenna. The corresponding height of the HR cavity $h$ reduces from 12.5 to 11.0 mm. By combining equations (2a) and (2b), the corresponding series and parallel resonant frequencies of nine types of unit cells are calculated as $f_s \approx f_p = 3500 \text{ Hz}$, which means the gradient amplitude ALWA constructed by these nine types of units is under the balance condition. For the case of $\theta_h = 0.2 \text{ rad}$, the theoretical normalized pressure amplitude distributions of the ideal ALWA (black solid curve) and the gradient amplitude ALWA (red solid lines) for an incident frequency of $f = 3500 \text{ Hz}$ are illustrated in figure 3(a). From figure 3(a), it is estimated that the gradient antenna length must be at least 0.58 m, which is about 39 unit cells ($=0.585 \text{ m}$). Here, we adopt 39 unit cells chosen among nine different types to discretize the ideal amplitude profile into nine stepwise zones. Each zone is composed of two identical type unit cells, apart from the zones with normalized amplitude 1, 0.96 and 0.85, which are composed of three identical unit cells. The width of the channel $w_1$ and the height of the HR cavity $h$ utilized in these nine types of units are listed in table 1, which ensures that the series and parallel resonant frequencies $f_s \approx f_p = 3500 \text{ Hz}$ and the band gaps almost disappear in these nine units based on equations (2a) and (2b). Then, to verify that these nine types of units have the identical radiation phases, the data calculated from equation (3) for the main radiation beam angle $\theta_0$ as a function of the incident frequency is shown in figure 3(b) as solid curves. It is found that the specific frequency corresponds to the specific direction of the radiation, and the main beams of nine unit cells scan from the backward direction to the forward direction during the operated frequency range from 3300 to 3800 Hz without band gaps. Because the deviation of radiation beam angles in these nine unit cells is within about 5$^\circ$ among the operated frequency band, the frequency scanning capability over the operated frequency range for these nine unit cells is almost the same. For other frequencies outside the operated frequency band (i.e. $f < 3300 \text{ Hz}$ or $f > 3800 \text{ Hz}$), since the deviation of radiation beam angles in these nine unit cells is larger than 5$^\circ$, it will inevitably bring an impact on the directional radiation quality and the discussions of directional radiation quality in these frequencies are not presented here. Moreover, in order to fulfill the desired gradient amplitude distribution as illustrated by the red solid lines in figure 3(a), the pressure amplitude of the radiated waves for the nine units should be also calculated. Figure 3(c) plots the viscous-thermal and viscous-elastic loss-inclusive numerically calculated pressure amplitudes of the radiated waves for the nine units at the resonant frequency of 3504 Hz, which is similar to the results derived from figure 3(a) (shown as the red solid lines). Note that the simulated amplitude has been normalized and the
simulated resonant frequencies \( f_s \approx f_p = 3504 \text{ Hz} \) are little larger than the theoretical value of 3500 Hz, which is within the acceptable range. Therefore, these nine types of unit cells can be used to design the gradient amplitude ALWA for a gradient radiation amplitude profile of [0:1] and near unitary phase.

3. Numerical results and discussions

We now turn to numerical simulations to illustrate the effect of high-quality directivity. The viscous-thermal and viscous-elastic loss-inclusive finite element method based on COMSOL Multiphysics is used to verify the high-quality directivity of the gradient amplitude ALWA. The ‘Thermoviscous Acoustic Module’ is added for
The red curve in the ideal amplitude distribution derived from equation (6) with \( \theta_h = 0.2 \text{ rad} \) and \( f = 3504 \text{ Hz} \). The effects of viscous-thermal losses. In order to consider the effect of viscous-elastic losses of the membrane, an isotropic structural loss factor \( \eta_s \approx 0.004 \) (DuPont™ Kapton® FPC polyimide film) is considered at the operated frequency band [37, 38]. The corresponding discrete radiation amplitude profile is employed on the \( x \) direction of the gradient amplitude ALWA, as shown in figure 3(a). The perfectly matched layers are imposed on the radiation boundary and the terminus of the gradient amplitude ALWA. The simulated frequency dependence of the radiation beam angle is presented by the dots’ \(^*\) shown in figure 3(b). It is found that the theoretical beam angles of radiation for nine types of unit cells (solid curves) fit well with the simulated results. Thus, the gradient amplitude antenna still has the frequency scanning capability. Based on the radiation beam angle of the gradient amplitude antenna, the curve of figure 3(d) shows the simulated radiation efficiency of the gradient amplitude ALWA. We observe a radiation efficiency of around 25% apart from the frequency close to the resonant frequency, which is guaranteed for the operated frequency band. Figures 4(a) and (b) present the numerically calculated pressure amplitude distributions of the gradient amplitude antenna and conventional antenna for an incident frequency of \( f = 3504 \text{ Hz} \), respectively. The conventional antenna is designed by 39 identical unit cells with fixed channel width \( w_1 = 0.6 \text{ mm} \) and HR cavity height \( a = 11.5 \text{ mm} \). It is observed that the radiated beam formed by the gradient amplitude antenna has higher quality directivity than that formed by the conventional antenna. Additionally, the theoretical pressure field distribution calculated by equation (6) with \( \theta_h = 0.2 \text{ rad} \) and \( f = 3500 \text{ Hz} \) is applied to generate the high-quality directivity of radiated sound beam shown in figure 4(c), which is similar to that of the gradient amplitude antenna, and provides confirmation that the high-quality directivity sound antenna is realized by the proposed gradient amplitude ALWA. For a quantitative analysis of its high quality directivity characteristics, the normalized radiated sound pressure amplitude profile at 2.54 m (\( \approx 26\lambda \)) far away from the antenna is shown in figure 4(d). In figure 4(d), the black and blue curves refer to the pressure amplitude distributions of the conventional antenna and the gradient amplitude antenna, respectively. Compared to the conventional antenna, the gradient amplitude antenna provides efficient radiation pattering, and the side lobes are effectively suppressed. It should be noted that, since the side lobes are suppressed through the gradient amplitude ALWA with discrete amplitude profile of \([0:1]\), it will inevitably bring a little influence on the full width at half maximum (FWHM) of ALWA, and the FWHM of the antenna with gradient amplitude is slightly larger than that of the antenna with constant amplitude. The influence on FWHM caused by gradient amplitude has been controlled within acceptable range by setting appropriate value of \( \theta_h \). The red curve in figure 4(d) is the pressure amplitude distribution of the antenna surface with the ideal amplitude distribution derived from equation (6), which shows good agreement between the numerical and theoretical results. Thus, the gradient amplitude antenna is used to suppress the side lobes and
concentrate more radiated sound energy on the main lobe, which improves the quality of the directional radiation.

The present theoretical and numerical results have confirmed that the designed gradient amplitude ALWA is capable of high quality directivity at the resonant frequency $f = 3504$ Hz. Because the gradient amplitude ALWA has the characteristic of beam scanning with frequency, the high quality control of radiation direction still exists in the range of other working frequencies of the gradient amplitude antenna (3300–3800 Hz). Figures 5(a), (b) and (d), (e) show the pressure amplitude distributions of the gradient amplitude antenna and the conventional antenna for incident frequencies of 3300 Hz and 3800 Hz, respectively. Figures 5(c) and (f) are the normalized radiated sound pressure amplitude profiles at 2.54 m far away from antennas for incident frequencies of 3300 Hz and 3800 Hz, respectively. It is found that the gradient amplitude antenna also reduces the side lobes and concentrates more radiated sound energy on the main lobe than the conventional antenna among the operated frequency band. It is noted that the FWHM of the gradient amplitude antenna will be increased with the increase of radiation beam angle $\theta_0$. From figures 5(c) and (f), it is also observed that the radiation beam angles for incident frequencies of 3300 Hz and 3800 Hz are around $-41^\circ$ and $47^\circ$, respectively. Thus, the FWHM in figures 5(c) and (f) will be larger than that in figure 4(d). For the designed gradient amplitude ALWA, the smaller the value of the maximum directivity sharpness angle $\theta_0$ is, the smaller the value of the FWHW of gradient amplitude ALWA is. Therefore, we could reduce the radiation beam angle $\theta_0$ or the maximum directivity sharpness angle $\theta_0$ to alleviate the problem of FWHM the becoming larger.

4. Discussion

In this work, we have designed and demonstrated a gradient amplitude ALWA, which achieves the capability of frequency scanning and improves the quality of directional radiation by independently modulating the amplitude and phase of the radiated sound waves. The proposed antenna utilizes a channel with a tunable width and a HR cavity with a tunable height to generate the gradient radiation amplitude and near unitary phase. An equivalent acoustic CRLH-TL circuit is developed to interpret this phenomenon and maintain the gradient amplitude ALWA under the balanced condition. In contrast to the conventional ALWA, the gradient amplitude ALWA significantly improves the directional radiation quality via the gradient distribution amplitude, which suppresses the side lobes while guaranteeing the quality of the main lobe.

Acknowledgments

This work is supported by the National Key R&D Program (No. 2017YFA0303702), State Key Program of National Natural Science Foundation of China (No. 11834008), National Natural Science Foundation of China (No. 11474160, No. 61571222), State Key Laboratory of Acoustics, Chinese Academy of Sciences (No.
SKLA201809), Key Laboratory of Underwater Acoustic Environment, Chinese Academy of Sciences (No. SSHJ-KFKT-1701) and AQSIQ Technology R&D Program (No. 2017QK125).

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