Negative Index of Refraction and Distributed Bragg Reflectors

Jaline Gerardin\textsuperscript{1} and Akhlesh Lakhtakia\textsuperscript{1}

\textsuperscript{1} CATMAS — Computational and Theoretical Materials Sciences Group
Department of Engineering Science and Mechanics
Pennsylvania State University, University Park, PA 16802–6812

\textbf{ABSTRACT:} The Bragg regime shifts when conventional materials in a multilayer distributed Bragg reflector (DBR) are replaced by artificial materials with the so-called negative index of refraction. This provides an avenue for characterizing the latter class of materials.

\textbf{Keywords:} Distributed Bragg reflector, Negative index of refraction
1 INTRODUCTION

Artificial materials with the so–called negative index of refraction at $\sim 10 \text{ GHz}$ frequency have garnered much recent interest [1]–[6]. These isotropic materials, with supposedly negative real permittivity and negative real permeability, were the subject of a mid–1960s paper of Veselago [7], who predicted certain unusual electromagnetic properties and coined the unclear term left–handed materials for them. Handed these materials are not [8], issue can be taken on the isotropy of their first samples [3], and strictly non–dissipative no materials can be [9]. Yet the only available experimental result [3] conclusively shows that these materials are different from their conventional counterparts (i.e., those with positive real permittivity and positive real permeability). As better realizations are probably in the wings, in colloquial terms, the business of macroscopic electromagnetics can no longer be as usual.

The manufacturing process of the subject artificial materials delivers samples in the form of slabs. Even a cursory perusal of key papers [1]–[3] reveals that characteristic observable properties of these materials supposedly are manifested most clearly when plane waves are obliquely incident on planar interfaces with their conventional counterparts. Indeed, the only experimental confirmation of their unusual characteristics exploited oblique incidence on a planar interface with air [3]. In contrast, we present here a configuration wherein the incidence is normal and yet the subject artificial materials can be easily distinguished from their conventional counterparts as follows.
2 ANALYSIS

Distributed Bragg reflectors (DBRs) are commonly used planar devices in optics [10]–[14]. A multilayer DBR is a stack of layers of two alternating materials with different indexes of refraction as well as low absorption, exhibiting very high reflectance in the so-called Bragg regime. If $P$ denotes the thickness of a unit cell comprising two adjacent layers, and $\bar{n}$ is some effective index of refraction, then the Bragg regime is located about the (free-space) wavelength [11]–[13]

$$\lambda^{Br}_0 = 2P\bar{n}.$$  \hfill (1)

(Bragg regimes are also possible around integer submultiples of this $\lambda^{Br}_0$, but it must be borne in mind that indexes of refraction are wavelength–dependent, in general.)

The parameter $\bar{n}$ depends on the indexes of refraction and the volumetric proportion of the two constituent materials.

Suppose the region $0 \leq z \leq L$ is occupied by a multilayer DBR, as shown in Figure 1. The multilayer DBR comprises $N$ unit cells, each made of two layers labeled $a$ and $b$, with relative permittivities $\epsilon_{a,b}$ and relative permeabilities $\mu_{a,b}$. With $P = L/N$ as the thickness of the unit cell, the thickness of layer $a$ is equal to $qP$, $0 \leq q \leq 1$.

A plane wave is normally incident on the DBR from the vacuous half–space $z \leq 0$, with $\lambda_0$ denoting its wavelength. Therefore, a reflected plane wave also exists in the half–space $z \leq 0$, and a transmitted plane wave in the vacuous half–space $z \geq L$.

The corresponding electric field phasors are given by

$$\mathbf{E}(z, \lambda_0) = u_x \begin{cases} a \exp(ik_0z) + r \exp(-ik_0z), & z \leq 0 \\ t \exp[ik_0(z - L)], & z \geq L \end{cases},$$ \hfill (2)

where $k_0 = 2\pi/\lambda_0$ is the free–space wavenumber; $a$, $r$ and $t$ are the amplitudes of the incident, reflected and transmitted plane waves, respectively; while $(u_x, u_y, u_z)$ is
the triad of cartesian unit vectors. An \( \exp(-i\omega t) \) time–dependence is implicit, where \( \omega = k_0/(\epsilon_o \mu_o)^{1/2} \) is the angular frequency, while \( \epsilon_o \) and \( \mu_o \) are the permittivity and the permeability of free space, respectively.

The amplitudes \( r \) and \( t \) must be determined in terms of \( a \). This is best done by setting up the \( 2 \times 2 \) matrix equation [15]

\[
\begin{bmatrix}
1 & 0 \\
\eta_0^{-1} & 1
\end{bmatrix} = \left( \exp \left( i\omega (1 - q) P \begin{bmatrix}
0 & \mu_o \mu_b \\
\epsilon_o \epsilon_b & 0
\end{bmatrix} \right) \right) \left( \exp \left( i\omega q P \begin{bmatrix}
0 & \mu_o \mu_a \\
\epsilon_o \epsilon_a & 0
\end{bmatrix} \right) \right)^N \begin{bmatrix}
(a + r) \\
\eta_0^{-1}(a - r)
\end{bmatrix}, \tag{3}
\]

where \( \eta_0 = (\mu_o/\epsilon_o)^{1/2} \) is the intrinsic impedance of free space. This equation has to be numerically solved, which we did. The principle of conservation of energy requires that \( |r|^2 + |t|^2 \leq |a|^2 \), with the equality holding only if the both constituent materials in the DBR are non–dissipative at the particular value of \( \lambda_o \). Our algorithm satisfied the conservation principle.

3 NUMERICAL RESULTS

Figure 2 shows the computed reflectance \( |r/a|^2 \) as a function of \( \lambda_o/P \) for three values of \( q \) when \( \epsilon_a = 4(1 + i0.001) \), \( \mu_a = 1.02(1 + i0.001) \), \( \epsilon_b = 1 \), \( \mu_b = 1 \) and \( N = 20 \). The Bragg regime is clearly observable via the rectangular feature with an almost flat top and two vertical sides in all three plots. The full–width–at–half–maximum bandwidth \( \Delta \lambda_o \) of the Bragg regime is typically \( < 1.25 P \) for all \( q \). Predictably, the Bragg feature vanishes for \( q = 0 \) and \( q = 1 \). As \( q \) increases, so does \( \lambda_o^{Br}/P \) (at the center of the Bragg feature in each plot); which amounts to an increase in \( \bar{n} \), as shown
Ideal Bragg features do not emerge for all values of \( q \in [0.2, 0.5] \), when \( \epsilon_a = 4(-1 + i0.001), \mu_a = 1.02(-1 + i0.001), \epsilon_b = 1, \mu_b = 1 \) and \( N = 20 \). Thus, the Bragg feature for \( q = 0.5 \) is not flat–topped in Figure 3, although it is well–developed for \( q = 0.6 \) and \( q = 0.7 \). Calculated values of \( \bar{n} \) as functions of \( q \) are shown in Table 1.

We note from the presented and related results that \( \bar{n} > 1 \) for all values of \( q \in (0, 1) \) when \( \text{Re}[\epsilon_a, \mu_a] > 0 \). In contrast, \( 0 < \bar{n} < 1 \) for \( q \gtrsim 0.67 \) when \( \text{Re}[\epsilon_a, \mu_a] < 0 \). The reduction of \( \bar{n} \) below the unit index of refraction of material \( b \) could suggest that the real part of the index of refraction is negative for the subject artificial materials, but that suggestion does not appear to be supported by the values of \( \bar{n} > 1 \) for \( q \gtrsim 0.67 \) when \( \text{Re}[\epsilon_a, \mu_a] < 0 \). Anyhow, in conjunction with Figure 3, Table 1 confirms that wave–material interaction in the subject artificial materials is intrinsically different from that in their conventional counterparts.

Our results also show that the Bragg regime would shift to shorter wavelengths, if a conventional dielectric/magnetic constituent of a multilayer DBR were to be replaced by its analog of the subject variety. Consequently, measurements of \( \bar{n} \) would illuminate the issue of the negative index of refraction, and could also help in the characterization of the subject artificial materials. At the same time, multilayer DBRs made with the subject artificial materials could be useful in wavelength regimes that are inaccessible with DBRs made with only conventional materials.

References
[1] D.R. Smith and N. Kroll, Negative refractive index in left–handed materials, Phys Rev Lett 85 (2000), 2933–2936.

[2] J.B. Pendry, Negative refraction makes a perfect lens, Phys Rev Lett 85 (2001), 3966–3969.

[3] R.A. Shelby, D.R. Smith and S. Schultz, Experimental verification of a negative index of refraction, Science 292 (2001), 77–79.

[4] I.V. Lindell, S.A. Tretyakov, K.I. Nikoskinen and S. Ilvonen, BW media — Media with negative parameters, capable of supporting backward waves, Microw Opt Technol Lett 31 (2001), 129–133.

[5] R.W. Ziolkowski and E. Heyman, Wave propagation in media having negative permittivity and permeability. Phys Rev E 64 (2001), 056625.

[6] J. Wang and A. Lakhtakia, On reflection from a half–space with negative real permittivity and permeability, Microw Opt Technol Lett 33 (2002) (accepted for publication).

[7] V.S. Veselago, The electrodynamics of substances with simultaneously negative values of $\epsilon$ and $\mu$, Sov Phys Usp 10 (1968) 509–514.

[8] A. Lakhtakia, Beltrami fields in chiral media, World Scientific, Singapore, 1994.

[9] W.S. Weiglhofer and A. Lakhtakia, On causality requirements for material media, Arch Elektr Übertr 50 (1996) 389–391.

[10] H.A. Macleod, Thin–film optical filters, Adam Hilger, London, UK, 1969, pp. 94–100.
[11] P.G. de Gennes and J. Prost, The Physics of Liquid Crystals, Clarendon Press, Oxford, UK, 1993, Sec. 6.1.2.

[12] A. Ghatak and K. Thyagarajan, Optical electronics, Cambridge University Press, Cambridge, UK, 1989, Sec. 18.6.

[13] A. Othonos, Fiber Bragg gratings, Rev Sci Instrum 68 (1997) 4309–4341.

[14] M. Guden and J. Piprek, Material parameters of quaternary III–V semiconductors for multilayer mirrors at 1.55 μm wavelength, Modelling Simul Mater Sci Eng 4 (1996) 349–357.

[15] A. Lakhtakia, Linear optical responses of sculptured nematic thin films (SNTFs), Optik 106 (1997) 45–52.
Table 1: Values of $\bar{n} = \lambda^{Br}_0/2P$ computed for different values of $q$; $N = 20$, $\epsilon_b = 1$ and $\mu_b = 1$.

| $q$ | $\epsilon_a = 4(1 + i0.001)$ | $\mu_a = 1.02(1 + i0.001)$ | $\epsilon_a = 4(-1 + i0.001)$ | $\mu_a = 1.02(-1 + i0.001)$ |
|-----|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| 0.1 | 1.14                        | 0.69                        |                             |                             |
| 0.2 | 1.26                        | 0.37†                       |                             |                             |
| 0.3 | 1.38                        | 0.09†                       |                             |                             |
| 0.4 | 1.47                        | 0.23†                       |                             |                             |
| 0.5 | 1.58                        | 0.56†                       |                             |                             |
| 0.6 | 1.68                        | 0.82                        |                             |                             |
| 0.7 | 1.78                        | 1.09                        |                             |                             |
| 0.8 | 1.87                        | 1.37                        |                             |                             |
| 0.9 | 1.96†                       | 1.68†                       |                             |                             |

† Bragg feature has a curved top — exemplified in the top plot of Figure 3 — which begins to flatten as $N$ increases. There exists also a certain degree of arbitrariness in the identification of the Bragg feature for some values of $q$ between $\sim 0.25$ and $\sim 0.42$ when $\text{Re}[\epsilon_a] = -4$ and $\text{Re}[\mu_a] = -1.02$. 
Figure 1: Schematic of the boundary value problem. The distributed Bragg reflector comprises \( N \) unit cells, each of thickness \( P \) and containing one layer each of two different materials labeled \( a \) and \( b \).
Figure 2: Reflectance $|r/a|^2$ of a DBR as a function of $\lambda_0/P$ for different values of $q$; $N = 20$, $\epsilon_a = 4(1 + i0.001)$, $\mu_a = 1.02(1 + i0.001)$, $\epsilon_b = 1$ and $\mu_b = 1$. 
Figure 3: Same as Figure 2, except $\epsilon_a = 4(-1 + i0.001)$ and $\mu_a = 1.02(-1 + i0.001)$. 