Localized Intersecting Brane Solutions of $D = 11$ Supergravity

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Abstract

We present a class of two-charged intersecting brane solutions of the $D = 11$ supergravity, which contain the $M2$-brane, $M5$-brane, Kaluza-Klein monopole or Brinkmann wave as their building blocks. These solutions share the common feature that one charge is smeared out or uniform over all spatial directions occupied by the branes or waves, while the other charge is localized in them.

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1. Introduction

The bosonic part of 11 dimensional supergravity contains the metric tensor and a 3-form gauge potential whose field strength is a 4-form. It is known that there are four basic classical solutions in this theory: the $M_2$-brane, $M_5$-brane, Brinkmann wave and Kaluza-Klein monopole. The $M_2$-brane has a natural coupling to the 3-form gauge potential and carries electric 4-form charge. The $M_5$-brane is the magnetic dual of the $M_2$-brane and carries magnetic 4-form charge. The Brinkmann wave and Kaluza-Klein monopole, on the other hand, are purely gravitational, i.e., the 3-form gauge potential is identically zero for them. The four solutions share some common features: they are BPS states and preserve $1/2$ of the supersymmetries, the solutions are described by a harmonic function in the transverse space and parallel branes/waves can be superposed. For a review of these BPS branes, see [6].

One can further superpose the four basic objects to obtain various composite BPS solutions. Different components in the solution can be of the same type of objects or different types. We will refer to the relative transverse directions of a brane as those orthogonal to the brane but tangent to some other brane, and the common transverse directions as those orthogonal to all branes. Most of the solutions that have been discussed are the all smeared out solutions where different components of the composite are smeared out in their relative transverse directions. Such solutions are described by several independent harmonic functions, each one is associated with a different component, which means one can superpose different components in arbitrary way.

There has also been some discussions that generalize the all smeared out solutions to localized or partially localized ones. One such solution describes the superposition of $NS5$-brane and fundamental string in type II supergravity, with the fundamental string parallel to the $NS5$-brane and localized in the four relative transverse directions. The $NS5$-brane is described by the usual harmonic function and the equation describing the fundamental string is modified compared with the smeared out case. If we lift this solution (more precisely the type IIA one) up to 11 dimensions, it becomes the superposition of $M2$-brane and $M5$-brane. The $M2$-brane and $M5$-brane intersect at a line, the $M5$-brane is smeared out in its relative transverse direction (which is the direction of the dimensional reduction), while the $M2$-brane is localized in its relative transverse directions. This solution is in fact a special case of a more general solution
which describes two \textit{NS5}-branes intersecting at a line with fundamental strings superposed parallel to the intersecting line, the two \textit{NS5}-branes are each localized in their relative transverse space. Another solution that has been discussed is the superposition of \textit{D5}-brane, \textit{D1}-brane and gravitational waves traveling parallel to the \textit{D1}-brane \cite{17}.

In this paper, we study further such localized solutions in 11 dimensional supergravity. For simplicity, we will just consider solutions with two charges (i.e., with two components). The common feature of these solutions is that one charge is uniform or smeared out and the other charge is localized in its relative transverse directions. We will obtain the following four solutions:

(i) \(M2\)-brane + nonuniform wave (the wave is parallel to one of the spatial directions of the brane and localized in the other direction);

(ii) \(M2 \perp M2(0)\) with one \(M2\)-brane smeared out in its relative transverse directions and the other one localized in its relative transverse directions, the notation \(M2 \perp M2(0)\) means that the two \(M2\)-branes intersect at a point;

(iii) \(M5\)-brane + nonuniform wave;

(iv) Kaluza-Klein monopole + nonuniform wave.

These four solutions plus the \(M2 \perp M5(1)\) solution mentioned previously are basic, in that they are independent and can not be deduced from each other by dualities. From these basic solutions, we can derive other two-charged configurations by U-duality \cite{21}, which include \(M5 \perp M5(3)\), \(M2 \perp K.K.(2)\)(Kaluza-Klein monopole), \(M5 \perp K.K.(5)\). More specifically, we can start from the basic solutions, smear them out in extra dimensions if necessary, go down to 10 dimensions, perform the U-duality transformations in type II supergravity and lift them up back to 11 dimensions.

We should point out that although we are using the word “localized solution”, what we have obtained (and what has often been discussed before) is the form of the localized solution in terms of two functions which satisfy simple equations. We have not tried to find explicit analytical solutions to these equations in this paper. For all the solutions, the two equations associated with the two charges are similar to those of the \textit{NS5}-brane and fundamental string.

We present the four basic solutions in the Section II and discuss some relevant issues in Section III.
2. The Four Localized Two-Charged Solutions

In this section, we discuss in turn the supergravity solutions of M2-brane with nonuniform wave, M2-brane intersecting M2-brane, M5-brane with nonuniform wave and Kaluza-Klein monopole with nonuniform wave. We will consider the first two cases in detail and be brief about the latter two cases.

2.1. M2-brane with Nonuniform Wave

We start with the bosonic part of $D = 11$ supergravity

\[ S_{11} = \int d^{11}x \left[ \sqrt{-g} \left( R - \frac{1}{48} F^2 \right) + \frac{1}{6} F \wedge F \wedge A \right]. \] (2.1)

$A$ is a 3-form gauge potential with a gauge transformation $\delta A = d\Lambda$, $\Lambda$ is a 2-form. $F = dA$ is the corresponding 4-form field strength, in components $F_{\mu
u\rho\lambda} = 4 \partial_{[\mu} A_{\nu\rho\lambda]}$. The third term in the Lagrangian is invariant under the gauge transformation of the 3-form up to a total derivative. The equations of motion are

\[ R_{MN} = \tilde{T}_{MN}, \]
\[ \tilde{T}_{MN} = \frac{1}{12} F_{MPQR} F_N^{PQR} - \frac{1}{144} F^2 g_{MN}, \] (2.2)
\[ \nabla_Q F^{MNPQ} - \frac{1}{2(4!)} \varepsilon^{MNPQ}_{\quad \quad \quad 1} \varepsilon^{\quad R} F_{Q_1Q_2Q_3Q_4R_1R_2R_3R_4} = 0. \]

The second term in the last equation vanishes for the solutions considered in this section. The last equation is therefore simplified to be

\[ \partial_Q \left( \sqrt{-g} F^{MNPQ} \right) = 0 \] (2.3)

The $M2$-brane solution takes the form

\[ ds_{11}^2 = H_M^{-2/3} (dt^2 + dz^2 + dy^2) + H_M^{1/3} dx_i dx_i, \quad i = 1, 2, ..., 8, \]
\[ A_{tzy} = H_M^{-1}, \]
\[ H_M = H_M(x_i), \quad \partial_x^2 H_M = 0. \] (2.4)

The rest of the components of the 3-form potential are zero except the ones related to $A_{tzy}$ by symmetry. We can obtain solutions describing $n$ parallel $M2$-branes located at different points in the transverse space by choosing $H_M$ to be

\[ H = 1 + \sum_{k=1}^{n} \frac{a_k}{r_k^6}, \quad r_k = |\vec{x} - \vec{x}_k|, \] (2.5)
where $\vec{x}_k, k = 1, 2, ..., n$ are locations of the branes.

The Brinkmann wave solution is purely gravitational. It exists in gravitational theory in any dimensions and in 11 dimensions the solution is given by

$$ds^2_{11} = \left[-dt^2 + dz^2 + (H_W - 1)(dt - dz)^2\right] + dx_i dx_i, \quad i = 1, 2, ..., 9,$$

$$H_W = H_W(x_i), \quad \partial^2_x H_W = 0,$$

which describes the wave traveling in the $z$ direction.

The $M2$-brane can be superposed with a gravitational wave smeared along the brane, the metric of which takes the form

$$ds^2_{11} = H_M^{-2/3}\left[-dt^2 + dz^2 + dy^2 + (H_W - 1)(dt - dz)^2\right] + H_M^{1/3} dx_i dx_i,$$

$$i = 1, 2, ..., 8.$$

The 3-form potential and $H_M$ are still given by (2.4). $H_W$, however, is only a harmonic function of the eight common transverse directions, $H_W = H_W(x_i), \quad \partial^2_x H_W = 0$, which means the wave is uniform in the $y$ direction. Note we can superpose the $M2$-brane and the wave in an arbitrary way, in particular the wave does not have to live in the brane. This is associated with the fact that the solution preserves 1/4 of the supersymmetries.

We now show that this solution can be generalized to the case where the wave does depend on the $y$ coordinate. We make the ansatz that the metric and the 3-form potential are still given by (2.4) and $H_M = H_M(x_i)$, but with the modification that $H_W = H_W(x_i, y)$.

First we consider the equation of motion of the 3-form potential. The nonzero components of the 4-form field strength are $F_{tzyx_i}$, which depends on $H_M$ only. The nonzero components in the upper index form are also $F^{tzyx_i}$. Moreover, $F_{tzyx_i}$ also just depend on $H_M$. This can be understood from the fact that locally the presence of the $(H_W - 1)(dt - dz)^2$ term corresponds to an infinite boost in the $z$ direction, but the component values of $F_{tzyx_i}$ or $F^{tzyx_i}$ do not change under boosts in the $z$ direction. For the same reason, $\sqrt{-g}$ does not depend on $H_W$ neither. So the equation of motion of the 3-form potential is completely independent of $H_W$, irrespectively of whether $H_W$ depends on the $y$ coordinate or not. Thus it simply reduces to $\partial^2_x H_M = 0$, as in the pure $M2$-brane case.

Next we consider the equation of motion of the metric tensor. Straightforward calcu-
lation gives $\bar{T}_{\hat{M}\hat{N}}$ and $R_{\hat{M}\hat{N}}$ as follows

$$
\bar{T}_{ii} = \frac{1}{3H_M^{1/3}} \left( \frac{\partial_x H_M}{H_M} \right)^2,
$$

$$
\bar{T}_{\hat{x}\hat{x}} = -\frac{1}{3H_M^{1/3}} \left( \frac{\partial_x H_M}{H_M} \right)^2,
$$

$$
\bar{T}_{\hat{y}\hat{y}} = -\frac{1}{3H_M^{1/3}} \left( \frac{\partial_x H_M}{H_M} \right)^2,
$$

$$
\bar{T}_{\hat{x}_1\hat{x}_1} = \frac{1}{6H_M^{1/3}} \left[ \left( \frac{\partial_x H_M}{H_M} \right)^2 - 3 \left( \frac{\partial_{x_1} H_M}{H_M} \right)^2 \right],
$$

$$
\bar{T}_{\hat{x}_1\hat{x}_2} = -\frac{1}{2H_M^{1/3}} \left( \frac{\partial_{x_1} H_M}{H_M} \right) \left( \frac{\partial_{x_2} H_M}{H_M} \right),
$$

We have listed here only the necessary components of $\bar{T}_{\hat{M}\hat{N}}$ and $R_{\hat{M}\hat{N}}$. The unlisted components are either zero or can be obtained by symmetries of the tensors. We see that the terms in $R_{\hat{M}\hat{N}}$ involving products of first derivatives of $H_M$ are precisely canceled by $\bar{T}_{\hat{M}\hat{N}}$. The remaining terms in $R_{\hat{M}\hat{N}}$ are always linear combinations of $\partial_x^2 H_M$ and $(\partial_x^2 H_W + H_M \partial_y^2 H_W)$. This means the equations of motion for the 3-form potential and the metric are reduced to

$$
\partial_x^2 H_M = 0, \quad \partial_x^2 H_W + H_M \partial_y^2 H_W = 0.
$$

If we let $H_W$ to be independent of the $y$ coordinate, the solution is just the uniform wave case, the equations decouple and become linear. If $H_W$ depends on the $y$ coordinate,
the equation becomes nonlinear and in general the principle of superposition does not apply. The relation between \( H_M \) and \( H_W \) is asymmetric and we can think of \( H_M \) as being a background for \( H_W \).

2.2. \( M2 \)-brane Intersecting \( M2 \)-brane

The solution describing two orthogonal \( M2 \)-branes intersecting at a point with both branes smeared out in their relative transverse directions takes the form

\[
\begin{align*}
\text{(2.10)} \\
 ds^2_{11} &= \left( -H_1^{-2/3} H_2^{-2/3} dt^2 + H_1^{1/3} H_2^{-2/3} (dz_1^2 + dz_2^2) + H_1^{-2/3} H_2^{1/3} (dy_1^2 + dy_2^2) \\
 &\quad \quad + H_1^{1/3} H_2^{1/3} dx_i dx_i \right), \quad i = 1, 2, 3, 4, 5, 6, \\
 A_{ty_1y_2} &= H_1^{-1}, \quad A_{tz_1z_2} = H_2^{-1}, \\
 H_1 &= H_1(x_i), \quad H_2 = H_2(x_i), \\
 \partial^2_x H_1 &= 0, \quad \partial^2_x H_2 = 0.
\end{align*}
\]

The rest of the components of the 3-form potential are zero except the ones related to \( A_{tz_1z_2} \) and \( A_{ty_1y_2} \) by symmetries. The \( M2 \)-brane associated with \( H_1 \) extends in the \( y_1, y_2 \) direction and is smeared out in the \( z_1, z_2 \) direction and the \( M2 \)-brane associated with \( H_2 \) extends in the \( z_1, z_2 \) direction and is smeared out in the \( y_1, y_2 \) direction. The two kinds of \( M2 \)-brane can be superposed in an arbitrary way, in particular they do not have to intersect.

Now we generalize this solution to the case where one \( M2 \)-brane is smeared out in its relative transverse directions while the other \( M2 \)-brane is fully localized in its relative transverse directions. We make the ansatz that the metric and the 3-form potential are still given by (2.10) and \( H_1 = H_1(x_i) \), but with the modification that \( H_2 = H_2(x_i, y_1, y_2) \). So the solution describes the \( M2 \)-brane associated with \( H_1 \) being smeared out in the \( z_1, z_2 \) directions while the one associated with \( H_2 \) being localized in the \( y_1, y_2 \) directions.

Let us consider the equation of motion of the 3-form potential first. By the ansatz the 4-form field strength is given by

\[
\begin{align*}
 F_{t_y_1y_2x_i} &= \frac{\partial_{x_i} H_1}{H_1^2}, \\
 F_{t_z_1z_2x_i} &= \frac{\partial_{x_i} H_2}{H_2^2}, \quad F_{t_{z_1}z_2y_i} = \frac{\partial_{y_i} H_2}{H_2^2}, \quad (2.11)
\end{align*}
\]
or in upper index form

\begin{align*}
F^{ty_1y_2x_i} &= -\frac{\partial_{x_i} H_1}{H_1^{1/3} H_2^{1/3}}, \\
F^{tz_1z_2x_i} &= -\frac{\partial_{x_i} H_2}{H_1^{1/3} H_2^{1/3}}, \\
F^{t^2z_1z_2y_i} &= -\frac{H_2^{1/3} \partial_{y_i} H_2}{H_2^{1/3}},
\end{align*}

(2.12)

and also

\[ \sqrt{-g} = H_1^{1/3} H_2^{1/3}. \]

(2.13)

The equation of motion of the 3-form potential is readily seen to be reduced to

\[ \partial^2_x H_1 = 0, \quad \partial^2_x H_2 + H_1 \partial^2_y H_2 = 0. \]

(2.14)

Next we consider the equation of motion for the metric tensor. Straightforward calculation gives

\begin{align*}
\tilde{T}_{ii} &= \frac{1}{3H_1^{1/3} H_2^{1/3}} \left[ \left( \frac{\partial_x H_1}{H_1} \right)^2 + \left( \frac{\partial_x H_2}{H_2} \right)^2 + H_1 \left( \frac{\partial_y H_2}{H_2} \right)^2 \right], \\
\tilde{T}_{z_1z_1} &= \frac{1}{6H_1^{1/3} H_2^{1/3}} \left[ \left( \frac{\partial_x H_1}{H_1} \right)^2 - 2 \left( \frac{\partial_x H_2}{H_2} \right)^2 - 2H_1 \left( \frac{\partial_y H_2}{H_2} \right)^2 \right], \\
\tilde{T}_{y_1y_1} &= \frac{1}{6H_1^{1/3} H_2^{1/3}} \left[ -2 \left( \frac{\partial_x H_1}{H_1} \right)^2 + \left( \frac{\partial_x H_2}{H_2} \right)^2 + H_1 \left( \frac{\partial_y H_2}{H_2} \right)^2 - 3H_1 \left( \frac{\partial_y H_2}{H_2} \right)^2 \right], \\
\tilde{T}_{y_1y_2} &= -\frac{H_2^{1/3} \partial_{y_1} H_2 \partial_{y_2} H_2}{2H_2^{1/3}}, \\
\tilde{T}_{y_1x_1} &= -\frac{H_1^{1/6} \partial_{y_1} H_2 \partial_{x_1} H_2}{2H_2^{1/3}}, \\
\tilde{T}_{x_1x_1} &= \frac{1}{6H_1^{1/3} H_2^{1/3}} \left[ \left( \frac{\partial_x H_1}{H_1} \right)^2 + 3 \left( \frac{\partial_x H_1}{H_1} \right)^2 + \left( \frac{\partial_x H_2}{H_2} \right)^2 + H_1 \left( \frac{\partial_y H_2}{H_2} \right)^2 + 3 \left( \frac{\partial_x H_2}{H_2} \right)^2 \right], \\
\tilde{T}_{x_1x_2} &= -\frac{1}{2H_1^{1/3} H_2^{1/3}} \left[ \frac{\partial_{x_1} H_1 \partial_{x_2} H_1}{H_1^2} + \frac{\partial_{x_1} H_2 \partial_{x_2} H_2}{H_2^2} \right],
\end{align*}

(2.15)
\[ R_{\hat{t}\hat{t}} = \frac{1}{3H_1^{1/3}H_2^{1/3}} \left[ \left( \frac{\partial_x H_1}{H_1} \right)^2 - \frac{\partial_x^2 H_1}{H_1} + \left( \frac{\partial_x H_2}{H_2} \right)^2 + H_1 \left( \frac{\partial_y H_2}{H_2} \right)^2 - \frac{\partial_y^2 H_2}{H_2} \right] , \]
\[ R_{\hat{x}_1\hat{z}_1} = \frac{1}{6H_1^{1/3}H_2^{1/3}} \left[ \left( \frac{\partial_x H_1}{H_1} \right)^2 - \frac{\partial_x^2 H_1}{H_1} - 2 \left( \frac{\partial_x H_2}{H_2} \right)^2 - 2H_1 \left( \frac{\partial_y H_2}{H_2} \right)^2 + 2 \frac{\partial_y^2 H_2}{H_2} + 2H_1 \frac{\partial_y^2 H_2}{H_2} \right] , \]
\[ R_{\hat{y}_1\hat{y}_1} = -\frac{H_1^{2/3}}{2H_2^{1/3}} \frac{\partial_{y_1} H_2 \partial_{y_2} H_2}{H_2^2} , \]
\[ R_{\hat{x}_1\hat{x}_1} = -\frac{H_1^{1/6}}{2H_2^{1/3}} \frac{\partial_{x_1} H_2 \partial_{x_2} H_2}{H_2^2} , \]
\[ R_{\hat{x}_1\hat{x}_2} = -\frac{1}{2H_1^{1/3}H_2^{1/3}} \left[ \frac{\partial_{x_1} H_1 \partial_{x_2} H_1}{H_1^2} + \frac{\partial_{x_1} H_2 \partial_{x_2} H_2}{H_2^2} \right] . \]

(2.16)

We have listed only the necessary components of \( \hat{T}_{\hat{M}\hat{N}} \) and \( R_{\hat{M}\hat{N}} \). The unlisted components are either zero or can be obtained by symmetries of the tensors. We see the same pattern as in the case of \( M2 \)-brane with wave. The terms in \( R_{\hat{M}\hat{N}} \) involving product of first derivatives of \( H_1 \) and \( H_2 \) are precisely canceled by \( \hat{T}_{\hat{M}\hat{N}} \). The remaining terms in \( R_{\hat{M}\hat{N}} \) are always linear combinations of \( \partial_x^2 H_1 \) and \( (\partial_x^2 H_2 + H_1 \partial_y^2 H_2) \). Thus the ansatz is solved by (2.14).

Again the relation between \( H_1 \) and \( H_2 \) is asymmetric. We have tried to generalize the ansatz by letting \( H_1 = H_1(x, z_1, z_2) \), \( H_2 = H_2(x, y_1, y_2) \) and still assuming the metric and the 3-form potential are given by (2.10). We found that the most general solution is what we have obtained, i.e., we need one of the two \( M2 \)-branes to be smeared out in its relative transverse directions. To find the solution of both \( M2 \)-branes being localized needs to go beyond this ansatz.
2.3. M5-brane with Nonuniform Wave

The magnetic dual of the M2-brane is the M5-brane, which carries magnetic 4-form charge. The solution describing the M5-brane superposed with the Brinkmann wave smeared out along the brane takes the form

\[
d s_{11}^2 = H_M^{-1/3} \left[ -dt^2 + dz^2 + (H_W - 1)(dt - dz)^2 + dy_\alpha dy_\alpha \right] + H_M^{2/3} dx_i dx_i, \quad \alpha = 1, 2, 3, 4, \quad i = 1, 2, 3, 4, 5
\]

\[
F_{i_1 i_2 i_3 i_4} = \epsilon_{i_1 i_2 i_3 i_4 i_5} \partial_{i_5} H_M, \\
H_M = H_M(x_i), \quad H_W = H_W(x_i), \\
\partial_x^2 H_M = 0, \quad \partial_x^2 H_W = 0,
\]

where \(\epsilon_{i_1 i_2 i_3 i_4 i_5}\) is the flat 5th rank totally antisymmetric tensor of the transverse space and all other components of the 4-form field strength are zero. \(H_M\) and \(H_W\) are two independent harmonic functions associated with the M5-brane and the wave. The wave travels in the \(z\)-direction. If we set \(H_W\) equal to 1, the solution reduces to the pure M5-brane case.

Similar to the M2-brane case, to obtain the nonuniform wave solution, we let \(H_W = H_W(x_i, y_\alpha)\) and assume the metric and the 4-form field strength are given by (2.17). The equation of motion and the Bianchi identity of the 4-form field strength are untouched by this modification (or for that matter, they are untouched by adding the wave at all). The calculation of \(R_{\tilde{M} \tilde{N}}\) and \(\tilde{T}_{\tilde{M} \tilde{N}}\) shows the same pattern as before. The first derivative terms in \(R_{\tilde{M} \tilde{N}}\) are canceled by \(\tilde{T}_{\tilde{M} \tilde{N}}\) and the remaining terms in \(R_{\tilde{M} \tilde{N}}\) are linear combinations of \(\partial_x^2 H_M\) and \((\partial_x^2 H_W + H_M \partial_y^2 H_W)\). Thus the nonuniform wave solution is given by

\[
\partial_x^2 H_M = 0, \quad \partial_x^2 H_W + H_M \partial_y^2 H_W = 0.
\]

(2.18)

2.4. Kaluza-Klein Monopole with Nonuniform Wave

The Kaluza-Klein monopole solution in 11 dimensions takes the form

\[
d s_{11}^2 = -dt^2 + dy_\alpha dy_\alpha + H_K dx_i dx_i + H_K^{-1} (dx_5 + A_i dx_i)^2, \quad \alpha = 1, 2, ..., 6, \quad i = 1, 2, 3, \\
H_K = H_K(x_i), \quad \partial_x^2 H_K = 0, \\
\partial_{x_i} H_K = \epsilon_{ijk} \partial_{x_j} A_k,
\]

(2.19)
where $x_5$ is periodically identified. If we suppress the $y_\alpha$, $\alpha = 1, 2, ..., 6$, the solution describes magnetic monopoles in the $4 + 1$ dimensional Kaluza-Klein theory, where the $A_i$ is the gauge field of the monopoles. Adding back the $y_\alpha$’s, the monopoles become 6-branes in 11 dimensions.

Gravitational wave can be added to the Kaluza-Klein monopole along one of the spatial directions of the brane. The metric takes the form

$$ds_{11}^2 = -dt^2 + dz^2 + (H_W - 1)(dt - dz)^2 + dy_\alpha dy_\alpha + H_K dx_i dx_i$$

$$+ H_K^{-1}(dx_5 + A_i x_i)^2, \quad \alpha = 1, 2, 3, 4, 5, \quad i = 1, 2, 3,$$

where the wave travels in the $z$-direction and is smeared out along the $y_\alpha$’s. $H_W$ is the harmonic function of the wave, $H_W = H_W(x_i)$, $\partial_x^2 H_W = 0$. $H_K$ and $A_i$ are the same as in (2.19).

Now we modify $H_W$ to be $H_W = H_W(x_i, y_\alpha)$ and assume the metric is still given by (2.20). Since the solution is purely gravitational, the only equations of motion need to be satisfied are $R_{MN} = 0$. Upon computing the Ricci tensor of the metric (2.20), we find that the terms involving $H_W$ always come in the form $\partial_x^2 H_W + H_K \partial_y^2 H_W$ and $R_{MN} = 0$ is reduced to

$$\partial_x^2 H_W + H_K \partial_y^2 H_W = 0$$

(2.21)

plus the equations in (2.19).

3. Discussions

We have obtained a class of classical solutions to the 11 dimensional supergravity. These solution carry two charges and can be viewed as superposition of the basic components: the $M2$-brane, $M5$-brane, Brinkmann wave and Kaluza-Klein monopole. They share the common feature that one charge is localized in its relative transverse dimensions while the other is smeared out in its relative transverse dimensions (in the case of superposition of two branes) or uniform (in the case of superposition of a brane and a wave). It is likely that these solutions all preserve $1/4$ of supersymmetries, although we have not checked the supersymmetric variations for all of them.

Dimensionally reducing these solutions gives various solutions in type II supergravity. Let us consider some examples. If we start with the $M2 + \text{wave}$ solution and dimensionally reduce along the wave direction, we get the solution of D0-brane localized in
F-string (fundamental string). If we smear the $M2 + \text{wave}$ out in one more dimension and reduce in that dimension, we get $D2(D2-\text{brane}) + \text{wave}$. Start from the $M2 \perp M2(0)$ solution and reduce along one of spatial direction of the localized brane, we get $D2 + F$-string with the $D2$-brane smearing out along the $F$-string and the $F$-string localized in the brane. T-dualize it along the $F$-string, we end up with $D3 + \text{wave}$ in IIB. Starting from $M5 + \text{wave}$ and reducing along the wave direction gives us $D4 + D0$ with the $D0$-brane localized in the $D4$-brane. If we smear out $M5 + \text{wave}$ in one more dimension and reduce along it, we get $NS5 + \text{wave}$ in IIA. Also start from $M5 + \text{wave}$, if we let the wave to be independent of one of the spatial directions along the $M5$-brane and reduce along it, we get $D4 + \text{wave}$. Dimensionally reducing the $K.K. + \text{wave}$ along the $S^1$ of the Kaluza-Klein monopole gives us the $D6 + \text{wave}$. The $D5 + \text{wave}$ solution is U-dual to $NS5 + F$-string in IIA, which can be obtained from $M2 + M5(1)$ by dimensional reduction, as discussed in the introduction. So all $Dp$-branes with $p = 2, 3, 4, 5, 6$ can carry nonuniform wave.

As mentioned in the introduction, other two-charged solutions, such as $M5 \perp M5(3)$, $M2 \perp K.K.(2)$, $M5 \perp K.K.(5)(M2 \perp M5(1)$ with the $M2$ smeared out and $M5$-brane localized is also one), are not independent but can be obtained by U-duality. For example, the $M5 \perp M5(3)$ is equivalent to $D4 \perp D4(2)$ (in the sense of dimensional reduction), which in turn is $T$-dual to $D4 + D0$ smeared out in two more dimensions.

One can certainly try to generalize these localized solutions to more than two charges. In fact, certain multiply charged solutions have already been considered in the context of type II supergravity [17,20]. We expect that more of such solutions exist in 11 dimensional supergravity. A more important question is to find the fully localized solutions of brane intersection. It seems that to find them we need to go beyond the ansatz made in this paper.

**Note Added**

After the paper was submitted, I was informed of [22,23] which contain significant overlap with the results described here.

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