The effect of loop quantum gravitational rainbow functions on the formation of naked singularities

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Abstract

In this paper, we will investigate the consequences of loop quantum gravitational modifications on the formation of naked singularities. The loop quantum gravitational effects will be incorporated in the collapsing system using gravity’s rainbow. This will be done by using rainbow functions, which are constructed from loop quantum gravitational modifications to the energy momentum dispersion. It will be observed that this modification will prevent the formation of naked singularity. Thus, such a modification can ensures that the weak cosmic censorship hypothesis will hold for any collapsing system.

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I. INTRODUCTION

In the loop quantum gravity field variable is a self-dual connection, instead of the metric, and this new field variable is called the Ashtekar connection [1, 2]. The Wheeler-DeWitt equation is then reformulated in terms of the traces of the holonomies of the Ashtekar connection [3, 4]. In loop quantum gravity, the area and volume are represented by operators with discrete eigenvalues [5]. This discretization occurs near Planck scale, and this leads to a modification of the usual energy momentum dispersion relation to a modified dispersion relation at Planck scale [6, 7]. Even though this modified dispersion relation reduces to the usual dispersion relation in the IR limit, it considerably deviates from the usual energy momentum dispersion relation in the UV limit. This behavior of modification in the UV limit is also observed in the Horava-Lifshitz gravity, which is motivated by Lifshitz scaling between space and time [9, 10]. In fact, such Lifshitz scaling also produces a deformation of the standard energy momentum dispersion relation to modified dispersion relation in the UV limit of the theory [11, 12]. The Lifshitz deformation of supergravity theories in the UV limit has also been studied [13–16]. It is also possible to motivate a different form of modified dispersion relation [17–20] using the high energy cosmic ray anomalies [21, 22]. The modification of standard dispersion relation to modified dispersion relation has motivated the construction of double special relativity, where the Planck energy acts as another universal constant [23, 24]. This theory of double special relativity has been constructed using a non-linear modification of the Lorentz group. Such modification to the dispersion relation have also been obtained from string field theory [25–27]. Thus, along with the phenomenological reasons, there are strong theoretical reasons to modify the energy momentum dispersion relation [17–20].

It is possible to generalize double special relativity to curved space-time, and the resultant theory is called gravity’s rainbow [28, 29]. In this theory the metric depends on the energy of the probe used to analyze the geometry. The energy dependence is introduced into the metric using rainbow functions. As these rainbow functions depend on the energy of the probe, which in turn depend implicitly on the coordinates, they cannot be removed by rescaling [30–33]. In fact, this is expected as the gravity’s rainbow is related to Lifshitz deformation of geometries [30]. Here the energy of the probe is converted into the length scale at which the probe is investigating the geometry, and this in turn has upper bound.
Hence in gravity’s rainbow, we cannot probe the space-time below Planck scale, as it is not possible to obtain an energy greater than the Planck energy. The experimental constraints on the rainbow functions from various experiments have been proposed [34]. The effect of these rainbow functions on the black information paradox have also been investigated [35]. Such a modification of a higher curvature gravity from gravity’s rainbow has been studied, and used to analyze its quasinormal modes [36]. The gravity’s rainbow geometries have been used to study the modification to the physics of neutron stars [37]. In all these systems, the gravity’s rainbow only changes the UV Planck scale behavior of the system. However, the system behaves as the original un-deformed system, in the IR limit. Furthermore, the effect of rainbow function on a system depends on the kind of rainbow functions used to deform it [34]. Thus, in this paper, we will use the rainbow function obtained from the modification of energy momentum dispersion relation due to loop quantum gravity [38, 39].

It has been proposed that naked singularity will not form due to loop quantum gravitational effects, and this was done by analyzing the non-perturbative semi-classical modifications to a collapsing system near the singularity [40]. Here we will demonstrate that these results can be obtained by analyzing the modification of energy momentum dispersion relation from loop quantum gravity [28, 29]. To analyze such effect from loop quantum gravitational modifications on the formation of naked singularity, we will use gravity’s rainbow. It has been proposed that naked singularities cannot form due to the weak cosmic censorship conjecture [41, 42]. However, several violation of cosmic censorship conjecture have been studied, and thus it seems that it is possible to form naked singularity in space [43–46]. In fact, it has been argued that accretion properties of a collapsing system can distinguish between a naked singularity, a wormhole and a black hole [47]. It has been suggested that a naked singularity can form during a critical collapse of a scalar field [48]. The gravitational lensing by a strongly naked null singularity has been investigated [49]. It has been demonstrated that the nature of this divergence are not logarithmic. It has also been suggested that the formation of a naked singularity can be tested using astrophysical observation [50]. The shadow of a naked singularity without photon sphere has been analyzed [51]. It has been argued that the naked singularity cannot form due to quantum gravitational effects [52, 53]. Thus, it becomes important to analyze the effect of loop quantum gravity on the formation of naked singularities. As such modification to black hole geometries have already been studied [30–33], we will use gravity’s rainbow to analyze the
The effect of loop quantum gravity on the formation of naked singularities. The effect of rainbow functions on the formation of naked singularities has also been studied, and it was observed that rainbow deformation can violate the cosmic censorship conjecture [54]. However, we will argue that this cannot be the case, as due to the rainbow deformation, it is not possible to probe space-time below Planck scale. This presents the formation of naked singularities.

II. COLLAPSE IN GRAVITY’S RAINBOW

To properly analyze the formation of naked singularities, and weak cosmic censorship conjecture, we will first analyze the deformation of a solution with Einstein equation by rainbow functions, which are consistent with loop quantum gravity [6, 7]. It has been proposed that the Einstein equations depend on the energy due to rainbow deformation, $G_{\mu\nu}(E/E_p) = \mathcal{R}_{\mu\nu}(E/E_p) - \frac{1}{2}g_{\mu\nu}(E/E_p)\mathcal{R}(E/E_p) = 8\pi T_{\mu\nu}$ (with the standard unit $G = c = 1$). Here $E_p$ is Planck energy, and $E$ is the energy at which the system is probed. Thus, the geometry depends on the energy used to probe it. However, for $E \ll E_p$, we can neglect the rainbow deformation. It is possible to incorporate this rainbow deformation into the spherically symmetric line element in comoving coordinates $(t, r, \theta, \phi)$ as (with $(-,+,+)$ as the signature)

$$ds^2 = -\frac{e^{2\lambda(t,r)}}{f(E)^2}dt^2 + \frac{e^{2\psi(t,r)}}{g(E)^2}dr^2 + \frac{R(t,r)^2}{g(E)^2}d\theta^2 + \frac{R(t,r)^2\sin^2 \theta}{g(E)^2}d\phi^2$$

where $R(t,r)$ is the physical radius at time $t$ of the shell labeled by $r$. Here $f(E)$ and $g(E)$ are rainbow functions which make the metric depend on the energy of the probe $E$. As in the IR limit, the gravity’s rainbow has to reduce to the usual general relativity, we expect that these rainbow function satisfy

$$\lim_{E/E_p \rightarrow 0} f(E/E_p) = 1 \quad \lim_{E/E_p \rightarrow 0} g(E/E_p) = 1$$

It is possible to obtain a specific form of these rainbow function from loop quantum gravity [6, 7]. We will use these these specific rainbow functions, and demonstrate that they will prevent the formation of naked singularities. However, before that we observe that any rainbow functions will limit the scale to which we can probe the system. It is possible to translate the uncertainty $\Delta p \geq 1/\Delta x$ into a bound on energy of the probe $E$, as $E \geq 1/\Delta x$. Here $\Delta x$ would correspond to the scale to which any length scale in the system
can be measured. We cannot take this length to be the same order as the Planck length, as such a bound is restricted by black hole physics [55–57]. As there is this minimal length in the system, we have a bound on the maximum energy need to probe such a minimal length. Now if \( E < E_p \) is such a maximum energy needed to probe, then we have to analyze rainbow modification to the systems when \( E \) is of the same order as \( E_p \) (and can be neglected for \( E << E_p \)).

The formation of naked singularity from a collapsing spherically symmetric object has been already been studied [58, 59]. Here we will investigate the rainbow deformation of a collapsing spherically symmetric object. To explicitly analyze such a system, we express the energy momentum tensor for a spherically symmetric object in comoving coordinates as

\[
T^\mu_\nu = \text{diag}(-\rho, p_r, p_\theta, p_\theta)
\]

where \( \rho, p_r \) and \( p_\theta \), are functions of \( t \) and \( r \). After solving for different non-zero components of Einstein equations, and suitably deforming it by rainbow functions (using the OGRE package [60]), we get the following set of equations,

\[
G^t_t = -\frac{g(E)^2 \left(1 - e^{-2\psi} \left(R'^2 + 2RR' - 2RR'\psi'\right)\right) + e^{-2\lambda} f(E)^2 \left(\dot{R}^2 + 2R\dot{R}\dot{\psi}\right)}{R^2}
= -8\pi \rho \tag{3}
\]

\[
G^r_r = -\frac{e^{-2\psi} g(E)^2}{R^2} \left(e^{2\psi} - R'^2 - 2RR'\lambda'\right) - \frac{e^{-2\lambda} f(E)^2}{R^2} \left(\dot{R}^2 + 2R\dot{R} - 2R\dot{R}\lambda\right)
= 8\pi p_r \tag{4}
\]

\[
G^{\theta}_{\theta} = \frac{e^{-2\psi} g(E)^2}{R} \left(R'' + R' \left(\lambda' - \psi'\right) + R \left(\lambda'' + \lambda'^2 - \lambda'\psi'\right)\right)
- \frac{e^{-2\lambda} f(E)^2}{R} \left(\ddot{R} - \dot{R} \left(\dot{\lambda} - \dot{\psi}\right) + R \left(\ddot{\psi} + \dot{\psi}^2 - \dot{\lambda}\dot{\psi}\right)\right)
= 8\pi p_\theta \tag{5}
\]

\[
G^t_t = \frac{2e^{-2\psi} g(E)^2 \left(\dot{R}\lambda' + R' \dot{\psi} - \dot{R}'\right)}{R} = 0 \tag{6}
\]

where dot and prime are the derivatives with respect to time and radial coordinates respectively. Now using the definition of Misner–Sharp Mass \( F(t, r) \) as

\[
F(t, r) = 1 - (F(t, r) / R) = g^{\mu\nu} \nabla_\mu R \nabla_\nu R \quad [61],
\]

we can write a rainbow deformation of Misner–Sharp Mass as

\[
F(t, r) = R \left(1 + f(E)^2 e^{-2\lambda} \dot{R}^2 - g(E)^2 e^{-2\psi} R'^2\right) \tag{7}
\]

It would be useful to define \( j = 1 - g(E)^2 \). Using this, we observe \( F' = j R' + 8\pi \rho R^2 R' \) and \( \dot{F} = j \dot{R} - 8\pi p_r R^2 \dot{R} \). Now due to the conservation of energy momentum tensor \( T^\mu_{\nu\mu} = 0 \), we
obtain \( \dot{\rho} + (\rho + p_r) \dot{\psi} + 2 (\rho + p_\theta) \dot{R}/R = 0 \) and \( p'_\theta + \lambda' (\rho + p_r) + 2 (p_r - p_\theta) R'/R = 0 \). The acceleration equation can be obtained by defining \( h(t, r) = 1 - g(E)^2 e^{-2\psi} R^2 \). Here \( h(t, r) \) depends on the rainbow function \( g(E) \), hence is an energy dependent function. Thus, the initial density and velocity profile conditions of the collapsing dust \(^{58}\) would depend on the maximum energy of the system, and would get deformed in the UV limit. We observe that using \( h(t, r) \), we can obtain,

\[
F(t, r)/R = f(E)^2 e^{-2\lambda} \tilde{R}^2 + h(t, r)
\]

and \( \dot{h}/(1 - h) = -2\tilde{R}\lambda'/\tilde{R}' \). Now we can write the equation for acceleration as

\[
\tilde{R} = \frac{e^{2\lambda}}{f(E)^2} \left( \frac{j}{2\tilde{R}} - 4\pi R p_r - \frac{F}{2R^2} + \frac{(1 - h)}{f(E)^2 R'} \left( -\frac{p'_r}{\rho + p_r} + \frac{2R'(p_\theta - p_r)}{R(\rho + p_r)} \right) \right)
\]

We can define the proper time as \( d\tau = \frac{e^{\lambda}}{f(E)} dt \), and use it to obtain

\[
\frac{d^2 R}{d\tau^2} = \frac{(1 - f(E)^2)}{e^{2\lambda}} \tilde{R} + \frac{1}{f(E)^2} \left( \frac{j}{2\tilde{R}} - 4\pi R p_r - \frac{F}{2R^2} + \frac{(1 - h)}{f(E)^2 R'} \left( -\frac{p'_r}{\rho + p_r} + \frac{2R'(p_\theta - p_r)}{R(\rho + p_r)} \right) \right)
\]

This modified equation depends on the maximum energy of the system due to the rainbow functions \( f(E) \) and \( g(E) \). So, the conditions for the collapse from this equation will be implicitly energy dependent. This interesting energy dependence constraints can have important consequences for the formation of naked singularity.

Let us take an example of perfect fluid where the radial and tangential pressures are equal \((p_r = p_\theta = p)\), and write the equation of acceleration for this perfect fluid as

\[
\frac{d^2 R}{d\tau^2} = \frac{(1 - f(E)^2)}{e^{2\lambda}} \tilde{R} + \frac{1}{f(E)^2} \left( \frac{j}{2\tilde{R}} - 4\pi R p_r - \frac{F}{2R^2} - \frac{(1 - h)}{f(E)^2 R'} \frac{p'_r}{\rho + p_r} \right)
\]

Now by setting the acceleration and velocity equal to zero, we obtain the Oppenheimer–Volkoff equation for hydrostatic equilibrium \(^{62}\)

\[
-R^2 p' = \frac{f(E)^2 (\rho + p)}{1 - \frac{F}{R}} \left( \frac{F}{2} + 4\pi R^3 p - \frac{R(1 - g(E)^2)}{2} \right)
\]

where we have used \( h = F/r \) (after putting velocity equal to zero). The derivative of radial pressure appears in the acceleration equation. The fate of the collapse is determined by the radial and tangential pressure. It is noted from Eq.(10) that positive tangential pressure and
negative tangential pressure opposes and supports the collapse, respectively. The rainbow functions that appear in Eq.(9) and (10) could reduce the value of acceleration, but could not change the overall sign of these equations, because these rainbow functions are always less or equal to one ($f(E) \leq 1$ and $g(E) \leq 1$). Thus, they can only change the relative magnitude of these forces, and the effect these forces will have on the collapsing system.

III. CONDITIONS FOR COLLAPSE FROM ACCELERATION EQUATION

In this section, we will discuss different cases for the spherical collapse, and the effect of the rainbow functions on them. For the dust case, we set the tangential and radial pressures equal to be zero ($p_r = p_\theta = 0$). So, let us assume that collapse starts from rest at $t = 0$, and we set the initial conditions as $R(t, r)|_{t=0} = r$ and $F(t, r)|_{t=0} = F_c(r)$. Using these initial conditions in expression of $\dot{F}$, Misner-Sharp mass becomes

$$F(t, r) = F_c(r) + (1 - g(E)^2)(R - r)$$

(13)

Here $p_\theta = p_r = 0$ for the dust case and the $\lambda$ is only the function of $t$. Thus, we can redefine $t$ and set $\lambda = 0$. Similarly, for the dust case, $h(t, r)$ will be function of $r$ only. With these re-definitions of the variables, we can write the rainbow deformed metric for the dust case as

$$ds^2 = \frac{-dt^2}{f(E)^2} + \frac{R'^2}{1 - h(r)} dr^2 + \frac{R^2}{g(E)^2} d\Omega^2$$

(14)

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. Here $h(r)$ is less than 1, so that the coefficient of $dr^2$ remain spacelike.

From Eq.(8), we find the equations, governing the behaviour of dust in the framework of rainbow functions that are consistent with quantum gravity.

$$\frac{F_c(r) + j(R - r)}{R} = f(E)^2 \dot{R}^2 + h(r)$$

(15)

Eq.(15) is discussed in details in the next section.

For collapse to start, the acceleration has to be negative or inward. We will use this fact to derive the condition for collapse to discuss different cases of tangential pressure, radial pressure and perfect fluid.
A. Tangential Pressure

Now if the tangential pressure \( p_\theta \) is non-zero, and the radial pressure \( p_r \) is zero, a simplification of Eq.(10) occurs. This simplified equation of acceleration can be written as

\[
\frac{d^2 R}{d\tau^2} = \left( 1 - f(E)^2 \right) \frac{\dot{R} \lambda}{e^{2\lambda}} + \frac{1}{f(E)^2} \left( \frac{j}{2R} - \frac{F}{2R^2} + \frac{(1-h)}{f(E)^2 R} \frac{2R' p_\theta}{\rho R} \right)
\]

For collapse to begin, the acceleration has to be negative. Assuming the collapse to start from rest (\( \dot{R} = 0 \)), and from Eq.(8) we obtain \( h_0 = \frac{F}{r} \), then the condition for collapse to begin at \( t = 0 \) turns out to be,

\[
\frac{F_c}{2r} > \frac{j/2 + 2p_\theta}{1 + \frac{4p_\theta}{f(E)^2 \rho}} = p_\theta \text{ condition}
\]

If the condition of Eq.(17) is satisfied at \( t = 0 \) and holds at all later times, the collapse will begin. In this case, the singularity will form at \( r = 0 \), if acceleration is negative, throughout the later time evolution of the system. We also observe from Eq.(17), that this system depends on the energy because of the rainbow functions.

![FIG. 1. Plots of R.H.S. of Eq.(17) versus f(E) and g(E) for different of ratio of p_\theta/\rho.](image)

The dependence on the ratio \( p_\theta/\rho \) generally evolves with time. However, to examine the dependency of collapse on these rainbow functions, we assume the ratio \( p_\theta/\rho \), remains constant in time. For positive tangential pressure and density, we assume that this ratio lies in the range \( 0 \leq p_\theta/\rho \leq 1 \). In Fig.(1), we have plotted Eq.(17). It follows from both the graphs, as the energy of the system tends towards the Planck scale, \( f(E) \) and \( g(E) \) tends towards zero. This makes it difficult for collapse to happen, which in turn has consequences
for the singularity formation. Hence we can conclude that the rainbow functions modify the collapsing system. In the IR limit, \( f(E) = g(E) = 1 \), we get back the same condition as described in [63].

\[
\frac{F_c}{r} > \frac{4 p_0 / \rho}{1 + 4 p_0 / \rho}
\]  

(18)

**B. Radial Pressure**

Here we can analyze the effect of a non-zero radial pressure \( p_r \), with a vanishing tangential pressure \( p_\theta \). This assumption again leads to a simplification of Eq.(10), and this modified equation of acceleration is given by

\[
\frac{d^2 R}{d\tau^2} = \frac{(1 - f(E)^2)}{e^{2\lambda}} \frac{\dot{R} \dot{\lambda}}{f(E)^2} + \frac{1}{f(E)^2} \left( \frac{j}{2R} - 4\pi R p_r - \frac{F}{2R^2} - \frac{(1 - h)}{f(E)^2 R^2} \left( \frac{2R' p_r}{R (\rho + p_r)} + \frac{p_r'}{\rho + p_r} \right) \right)
\]  

(19)

Now the condition on radial pressure for the collapse to begin at \( t = 0 \) turns out to be,

\[
\frac{F_c}{2r} > \frac{j^2 - 4\pi r^2 p_r - \frac{r p_r + 4 p_r + 2 r p_r'}{f(E)^2 (\rho + p_r)}}{1 - \frac{r p_r + 4 p_r + 2 r p_r'}{f(E)^2 (\rho + p_r)}}
\]  

(20)

Again, in the IR limit \( f(E) = g(E) = 1 \), we obtain the condition for collapse which can be derived from general relativity [63]. This depends on the density \( \rho \), radial pressure and its derivative

\[
\frac{F_c}{2r} > \frac{4\pi r^2 p_r + \frac{r p_r + 4 p_r + 2 r p_r'}{\rho + p_r}}{1 - \frac{r p_r + 4 p_r + 2 r p_r'}{\rho + p_r}}
\]  

(21)

**C. The Perfect Fluid**

In this approximation, called the perfect fluid approximation both the radial and tangential pressures are set to zero, \( p_r = p_\theta = p \). In this case, Eq.(10) becomes,

\[
\frac{d^2 R}{d\tau^2} = \frac{(1 - f(E)^2)}{e^{2\lambda}} \frac{\dot{R} \dot{\lambda}}{f(E)^2} + \frac{1}{f(E)^2} \left( \frac{j}{2R} - 4\pi R p - \frac{F}{2R^2} - \frac{(1 - h)}{f(E)^2 R^2} \left( \frac{2R' p}{R (\rho + p)} + \frac{p'}{\rho + p} \right) \right)
\]  

(22)

For collapse to begin, the acceleration has to be negative. Now using Eq.(8), we obtain \( h = \frac{E}{r} \) and the condition for collapse to begin at \( t = 0 \) turns out to be,

\[
\frac{F_c}{2r} > \frac{j^2 - 4\pi r^2 p - \frac{r p + 4 p + 2 r p'}{f(E)^2 (\rho + p)}}{1 - \frac{r p + 4 p + 2 r p'}{f(E)^2 (\rho + p)}}
\]  

(23)
Here in the IR limit, \( f(E) = g(E) = 1 \), we obtain the condition which can be obtained from general relativity

\[
\frac{F_c}{2r} > \frac{4\pi r^2 p + \frac{r p'}{\rho + p}}{-1 + \frac{2r p'}{\rho + p}} \tag{24}
\]

We have investigate the dependence of this conditions for the different physical values of the pressure, such as when the tangential pressure or radial pressure are zero or both are set equal. We are interested in studying the case of dust and the effect of loop quantum gravitational modifications on the formation of naked singularity using gravity’s rainbow framework.

IV. THE DUST SOLUTION

The Tolman-Bondi dust collapse has been investigated in general relativity [58, 59, 63, 64]. The results for a marginally bound case and a non-marginally bound case have been analyzed in these studies. However, we will restrict our discussion here to only the marginally bound case i.e. \( h(r) = 0 \). The results for the non-marginally bound case can be derived using the same procedure. Now we will explicitly use the rainbow function motivated from loop quantum gravity [38, 65]

\[
f(E) = 1, \quad g(E) = \sqrt{1 - \frac{E}{E_p}} \tag{25}
\]

Using these rainbow functions, we can explicitly write the metric as

\[
ds^2 = -dt^2 + \hat{R}^2(t, r)dr^2 + \frac{R^2(t, r)}{g(E)^2}d\Omega^2 \tag{26}
\]

Here \( \hat{R} \) will also depend on the energy of the probe

\[
\hat{R} = -\sqrt{j + \frac{F_c(r) - rj}{\hat{R}}} \tag{27}
\]

We note that, along radial null geodesic, we can write

\[
\frac{\partial t}{\partial r} = R' \tag{28}
\]

Solving the above equation, at constant \( r \) using the boundary condition \( R_{t=0} = r \), we obtain
\[
t = \frac{\sqrt{rF_c}}{j} - \frac{F_c - rj}{j^{3/2}} \tanh^{-1} \left( \sqrt{F_c(rj)^{-1}} \right) - \frac{R}{j} \sqrt{j + \frac{F_c - rj}{R}} \]

\[
+ \frac{F_c - rj}{j^{3/2}} \tanh^{-1} \left( \sqrt{1 + \frac{F_c - rj}{R_j}} \right)
\]

(29)

Using the standard procedure, we introduce the auxiliary variables \(u, X\) [66],

\[
u = r^\alpha, \ \alpha > 0, \ X = \frac{R}{u} \]

(30)

In order for the singularity at \(r = 0\) to be naked, radial null geodesics should be able to propagate outwards, starting from the singularity. A necessary and sufficient condition for this to happen is that the area radius \(R\) increases along an outgoing geodesic, because \(R\) becomes negative in the unphysical region. Thus in the limit of approach to singularity, we write

\[
X_0 = \lim_{R \to 0, u \to 0} \frac{R}{u} = \lim_{R \to 0, u \to 0} \frac{dR}{du} = \lim_{R \to 0, r \to 0} \frac{1}{\alpha r^{\alpha - 1}} \left( R' + \frac{\partial t}{\partial r} \dot{R} \right)
\]

(31)

We can evaluate \(R'\) from Eq. (29) and in the resulting expression, substitute \(R = Xr^\alpha\). Then divide by \(r^{\alpha - 1}\), we obtain the following expression

\[
\frac{R'}{r^{\alpha - 1}} = X \left\{ - 2r^{3/2} A_1 A_2 F_c' \sqrt{j} + r \sqrt{F_c(A_2 + r^{\alpha/2} A_1 \sqrt{X})} \sqrt{j} + 2A_1 A_2 (\tanh^{-1} (\sqrt{F_c(rj)^{-1}})
\]

\[
- \tanh^{-1} A_1) \left( \sqrt{F_c} A_2 + r^{\alpha/2} A_1 \sqrt{X} \right) \left( r^{\alpha/2} A_1 \sqrt{X} \right) \right) \right\}^{-1}
\]

(32)

where \(A_1^2 = 1 - r^{-\alpha} (jr + F_c)(jX)^{-1}\) and \(A_2^2 = j(r - r^\alpha X) + F_c\). Assuming \(u = r^\alpha\) along the radial null geodesic, we can write the following

\[
\frac{dR}{du} = \frac{1}{\alpha r^{\alpha - 1}} \frac{dR}{dr} = \frac{1}{\alpha r^{\alpha - 1}} \left( R' + \frac{\partial t}{\partial r} \dot{R} \right)
\]

(33)

Using the expression from Eq. (28), the above equation takes the form,

\[
\alpha \frac{dR}{du} = \left( 1 - \sqrt{j + \frac{F_c - rj}{R}} \right) \frac{R'}{r^{\alpha - 1}} \]

(34)
here $\frac{R'}{r'}$ is given by Eq. (32). Roots analysis is done for the above case numerically as it is not possible analytically. To proceed further we consider the power series form of $F_c(r)$ as

$$F_c(r) = F_0 + F_1r + F_2r^2 + F_3r^3 + F_4r^4...$$ (35)

FIG. 2. Plots of $X$ versus $r$ for various values of Energy $E = 0, 0.1, 10$ and $100$ in Rainbow energy function $f(E) = 1, g(E) = \sqrt{1 - \frac{\eta}{E_p}}$. Here $E_p$ is taken as $10^{19}$ and $\eta = 10^{17}$

We have plotted the value of $dR/du = R/u$ versus $r$ to see how the plot behaves near $r \to 0$. This is done to investigate if $dR/du = R/u$ has a solution, in the limit $r \to 0$. As shown by red curves in Fig. (2) that, in general relativity there is a real and positive value of $X$ in the limit $r \to 0$, while in gravity’s rainbow, the curves departs farther from the vertical axis with increasing the value of $E$ as depicted from the plots itself. This suggest that there is no real and positive value of $X$ exists as $r \to 0$ due to the deformation by gravity’s rainbow as opposed to general relativity. In fact, one can check numerically that it gives the complex roots of equation $dR/du = R/u$ for all values of $r < r_0$ near $r = 0$. This can be checked for different values of the energy of the probe $E$. We observe as long as $E < E_p$, and of the same order (such that we cannot neglect $E/E_p$), the naked singularity will not form. This has been explicitly demonstrated for $E = 0.1, 10$ and $100$. For numerical analysis values of all model parameters are mentioned in plots. It can be concluded from above analysis that due to the deformation from the rainbow function which are motivated from loop quantum gravity [38, 65], the naked singularity will not form.
V. CONCLUSION

It is known that the energy momentum dispersion relation would be modified at Planck scale due to loop quantum gravitational effects. This loop quantum gravitational modified energy momentum dispersion relation can be used to obtain suitable rainbow functions. We have used those rainbow function to analyze the effect of loop quantum gravity on a collapsing system. We demonstrate that the modifications to the collapsing system by loop quantum gravity prevents the formation of a naked singularity. We comment that this was expected, as in loop quantum gravity the Planck scale structure of space-time is modified, and so we would expect the singularities would be removed due to these effects. This is explicitly demonstrated in this paper using gravity’s rainbow. The maximum energy of the system, which acts as a probe, is fixed using the uncertainty principle. Thus, the energy of the rainbow functions is expressed in terms of distance scale in the collapsing system.

It would be interesting to analyze the collapse in different modified theories of gravity using this Planck scale modification. Thus, we can analyze this system gravity with higher curvature terms, and then deform that system by gravity’s rainbow. We would also like to point out that the deformation by gravity’s rainbow depends on the rainbow functions. Here the rainbow functions were obtained using results from loop quantum gravity. However, it is possible to obtain rainbow functions from other motivations. It is expected that the formation of naked singularities would also depend critically on the kind of rainbow functions used to deform the system.

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