Quantum simulation and ground state preparation for the honeycomb Kitaev model

Tatiana A. Bespalova$^1$ and Oleksandr Kyriienko$^2,*$

$^1$ITMO University, St. Petersburg, 197101, Russia
$^2$Department of Physics and Astronomy, University of Exeter, Stocker Road, Exeter EX4 4QL, United Kingdom

(Dated: September 29, 2021)

We propose a quantum protocol that allows preparing a ground state (GS) of the honeycomb Kitaev model. Our approach efficiently uses underlying symmetries and techniques from topological error correction. It is based on the stabilization procedure, the developed centralizer ansatz, and utilizes the vortex basis description as the most advantageous for qubit-based simulations. We demonstrate the high fidelity preparation of spin liquid ground states for the original Kitaev model, getting the exact GS for $N = 24$ spins using 230 two-qubit operations. We then extend the variational procedure to non-zero magnetic fields, studying observables and correlations that reveal the phase transition. Finally, we perform dynamical simulation, where the ground state preparation opens a route towards studies of strongly-correlated dynamics, and a potential quantum advantage.

Introduction.—Strongly-correlated materials are described by models of interacting electrons and spins [1], where an efficient classical description is inapplicable due to the sign problem and long-range entanglement [2]. An area where strong correlations represent a formidable obstacle is magnetism [3]. When couplings between spins compete with each other, frustration leads to exotic phases of matter [4, 5], a striking example is a quantum spin liquid (QSL) phase [3, 6], where strong quantum fluctuations persist even at low temperature $T$. QSL is a potential candidate for modelling high-$T_c$ superconductivity, and sheds light on unconventional magnetic materials [7, 8]. They include honeycomb iridates (Na$_3$IrO$_3$, Li$_2$IrO$_3$) [9–11], 4d transition metal-based materials ($\alpha$-RuCl$_3$) [12–14], and herbertsmithite [15]. From the theoretical perspective QSL materials are often described by spin lattice models with bond-dependent Heisenberg coupling [3, 6]. The corresponding models for honeycomb lattices are Kitaev models [16–18]. In many cases spin liquid physics cannot be accessed by efficient classical approaches, as this requires studying low energy behavior and calls for an exact diagonalization (ED) [19–21]. The density matrix renormalization group can push this boundary [22, 23] at the expense of a truncated wave function. Here, quantum simulation (QS) offers an alternative solution [24–26], where strongly-correlated models can be studied on inherently quantum devices at a scale inaccessible classically [27, 28].

Quantum algorithms for materials and chemistry allow low-$T$ properties of a many-body Hamiltonian to be accessed using a ground state preparation protocol (GSP) [29–31]. They can use different principles. Some favor superior scaling, while leading to increased gate and qubit counts and targeting a fault-tolerant implementation [30, 32–36]. Others exploit quantum dynamics and overlap measurements to learn effectively the low-$T$ physics on mid-scale quantum simulators [37–42]. For near-term operation, the leading approaches are based on variational principles with a hybrid quantum-classical loop as introduced in the variational quantum eigensolver (VQE) [43–45]. This variational GSP relies on optimizing a parameterized quantum circuit (ansatz) that prepares a minimal energy state at optimal parameters [47–52]. This operation mode is favored experimentally [44–46, 53–55], as the circuit depth is typically reduced.

The efficiency of VQE crucially depends on the ansatz choice [56–59]. The performance limit is posed by the non-convex multiparameter optimization, where regions of vanishing gradients (barren plateaus) hinder efficient optimization at large circuit depth [60, 61]. Correctly parametrized circuits can perform GSP in reasonable depth for cases where deep circuits with unsuitable structures fail. In chemistry, ansatze guided by the coupled cluster theory can offer a good convergence [55, 62–64], though at the expense of depth due to the fermion-qubit mappings. Hardware efficient ansatze (HEA) VQE use shallow circuits that are easy-to-run [45], while missing the symmetries and rapidly approaching 2-designs [65]. Various ansatz search techniques were proposed based on adaptive generator screening [66–69], evolutionary and pruning approaches [70–76]. Here, crucial advantage is offered by symmetry-preserving circuits [77–81]. For spin systems, many works target simpler models corresponding to transverse Ising, XXZ, and Heisenberg chains [45, 82–86]. Recent developments comprise of the Hamiltonian variational ansatz for XXZ [87], natural gradient optimization for the Ising model [88], dynamical QS for 2D XY model [89], and tensor network ansatze for square Heisenberg [90] and $J_1$-$J_2$ models [91]. Many more challenging spin models remain unexplored.

In this Letter, we propose a quantum protocol for studying the physics of quantum spin liquids. Developing an initialization technique and an ansatz motivated by symmetries, we simulate the ground state preparation at increasing scale, with an advantage of being verifiable. Utilizing digital QS, we study the dynamics of QSL in the strongly-correlated regime. Noting that developed techniques align with the roadmap towards quantum error-
correction, we consider the simulation of spin liquids as a natural step towards advantage in material science.

**The model.**—We study the Kitaev model of coupled spin-1/2s on a honeycomb lattice [see sketch in Fig. 1(a)]. Being exactly solvable, it represents a prototypical QSL example of quantum spin liquids [3]. However, in the presence of a local magnetic field the analytical solution is infeasible, and the system requires a numerical solution.

The field-free honeycomb Kitaev Hamiltonian reads [16]

$$\hat{H}_0 = J^x \sum_{\langle i,j \rangle \in \mathcal{X}} \hat{X}_i \hat{X}_j + J^y \sum_{\langle i,j \rangle \in \mathcal{Y}} \hat{Y}_i \hat{Y}_j + J^z \sum_{\langle i,j \rangle \in \mathcal{Z}} \hat{Z}_i \hat{Z}_j,$$

where $\hat{X}_i$, $\hat{Y}_i$, $\hat{Z}_i$ are Pauli operators $\hat{P}^\alpha_i (\alpha = x, y, z)$, acting at site $i$ (or vertex or qubit). $J^\alpha$ are dimensionless interaction constants. $\mathcal{X}$, $\mathcal{Y}$, $\mathcal{Z}$ are sets of pairs of nearest neighbor sites (bonds), where $XX$, $YY$, and $ZZ$ bonds alternate on a hexagon. They are shown in Fig. 1(a) by links of different color. We consider the Kitaev model put on a torus, such that the lattice is periodic in both spatial dimensions. The lattice consists of $N$ vertices and $n = L_x \times L_y = N/2$ hexagonal plaquettes, where $L_x$ ($L_y$) is the number of plaquettes in horizontal (vertical) direction. The toric boundary is arranged by adding two extra vertices, which for the 18-qubit toric lattice are labeled as 17 and 18 [Fig. 1(a)]. The local magnetic field $\mathbf{h}$ with Cartesian components $h^\alpha$ is described by the bias terms

$$\hat{H}_m = \sum_{i=1}^{N} h_i^x \hat{X}_i + \sum_{i=1}^{N} h_i^y \hat{Y}_i + \sum_{i=1}^{N} h_i^z \hat{Z}_i,$$

where the field may be inhomogeneous. The full Kitaev Hamiltonian then reads $\hat{H} = \hat{H}_0 + \hat{H}_m$. In this work we concentrate on the isotropic Kitaev model $J^x, J^y, J^z = J$, while noting that our approach is also applicable to the anisotropic cases. While the Kitaev model in the absence of magnetic fields described by $\hat{H}_0$ is exactly solvable, its solution relies on describing the system in terms of Majorana fermions [16, 92]. Specifically, after performing the Jordan-Wigner transformation the Kitaev model can be reduced to free fermions in a static $\mathbb{Z}_2$ gauge field [93]. This was recently used for the simulation of braiding for Kitaev models in the momentum basis [94]. This mainly allows for predicting properties in the thermodynamic limit and calculating the spectrum [95], but does not provide a recipe for preparing the GS in the finite system. Similarly to Hartree-Fock procedures in chemistry, free-fermion models have polynomial complexity, but if one wants to use their ground states as a stepping stone for correlated methods they do require a complex GSP [46]. Moreover, since mapping between qubits and fermions is non-local, this creates an overhead that inflates the QS budget of seemingly native spin-1/2 model. Instead, we show how the effective use of symmetries can yield a successful and scalable GSP protocol and open the route to dynamical simulations and QSL in non-zero field.

**Symmetries.**—The key insight for the efficient Kitaev model description comes from considering the vortex basis [95–97]. First, we can assign operators for each plaquette, where bond operators are multiplied counter-clockwise along the chosen plaquette loop (e.g. see the plaquette operator $\hat{w}_1 = \hat{X}_1\hat{Z}_2\hat{Y}_3\hat{X}_4\hat{Z}_5\hat{Y}_6$ in Fig. 1(a)). The expectation value of the plaquette operator defines the presence or absence of a vortex associated to each plaquette, as denoted by $-1$ and $+1$ expectations, respectively ($\langle \hat{w}_1^2 \rangle = 1$). We note that plaquette operators commute with others, $[\hat{w}_p, \hat{w}_{p'}] = 0 \ (p, p' = 1, \ldots, n)$. They also commute with the Kitaev Hamiltonian, $[\hat{w}_p, \hat{H}_0] = 0 \ \forall \ p$, and so does their sum, meaning that the number of vortices is conserved in the system (this symmetry is broken for $\hat{H}_m$). We also have a constraint $\prod_p \hat{w}_p = 1$ such that vortices appear in pairs. The Hilbert space is thus fractured into separate manifolds labeled by the total number of vortices $W_{\text{tot}}$, that is described by the expected value of the operator $\langle \hat{w}_1 \rangle = (n^2 - \sum_{p=1}^n \hat{w}_p)/2$.

Other two integrals of motion arise as loop operators $\hat{\ell}_{x,y} = 0$ being the products of bond operators along two closed loops $\mathcal{L}_{x,y}$ on the torus. For example, we show the loop operators in Fig. 1(a) as solid blue lines corresponding to $\hat{\ell}_x = \prod_{i \in \mathcal{L}_x} \hat{Z}_i$ and $\hat{\ell}_y = \prod_{i \in \mathcal{L}_y} \hat{Y}_i$ strings [99]. The honeycomb Kitaev model can be seen

![Fig. 1: (a) Kitaev lattice with $N = 18$ spins arranged on a torus. Heisenberg couplings of $ZZ$, $YY$, and $XX$ type act on bonds around hexagonal tiles (plaquettes). Plaquette operators (pink hexagons, $\hat{P}^{\alpha}_i$) and loop operators (blue solid snakes) denote the conserved quantities. (b) Quantum protocol consisting of the stabilizer circuits $\hat{S}_{\text{plq}}$ and $\hat{S}_{\text{str}}$ that prepare an initial state with fixed number of vortices, which for the 18-qubit toric lattice are extra vertices, which for the 18-qubit toric lattice are shown in Fig. 1(a) by bonds around hexagonal tiles (plaquettes). Plaquette operators (pink hexagons, $\hat{P}^{\alpha}_i$) and loop operators (blue solid snakes) denote the conserved quantities.](image-url)
as a topological subsystem code [100], where no logical qubits are encoded and loop operators are responsible for gauge fixing [98, 99]. We can define the Abelian stabilizer group $\mathcal{S} = \{\hat{w}_p\}_{p=1}^v \cup \{\hat{\ell}_x, \hat{\ell}_y\}$ where all elements $\hat{S}_j \in \mathcal{S}$ pairwise commute. The physical state space of the Kitaev model corresponds to the codespace of $\mathcal{S}$ formed by eigenstates $\hat{S}_j|\psi\rangle = \pm|\psi\rangle$. Our goal is to utilize the symmetries when performing GSP.

Variational search.—To reach a ground state $|\psi_{\text{GS}}\rangle$ of a strongly-correlated Hamiltonian we need to design a unitary $\hat{U}_{\text{GS}}$ that when acting on some initial state $|\psi_0\rangle$ prepares $\hat{U}_{\text{GS}}|\psi_0\rangle = |\psi_{\text{GS}}\rangle$. For this we use a variational quantum protocol. During the variational search we train an ansatz circuit $\hat{U}_{\theta}$ parametrized by a vector $\theta$ such that at optimal parameters $\theta_{\text{opt}} = \arg\min_\theta \langle \psi_0 | \hat{U}_{\theta}^\dagger \hat{H} \hat{U}_{\theta} | \psi_0 \rangle$ the energy of the system is minimized, and $\hat{U}_{\theta_{\text{opt}}} \approx \hat{U}_{\text{GS}}$.

In the following, we compose an ansatz with $d$ layers of unitaries $\hat{U}_{\theta} = \prod_k^{d} \hat{U}_{\theta_k}^{(i)} \hat{U}_{\theta_k}^{(c)}$. Here, $\hat{U}_{\theta}^{(c)}$ is a unitary operator that preserves symmetries and $W_{\text{tot}}$, essential property for fast GSP in the absence of the magnetic field. The unitary operator $\hat{U}_{\theta_k}^{(w)}$ introduces processes that do not conserve vortices, and is added when $h \neq 0$.

State preparation.—For an efficient variational approach we need to start in a suitable vortex number eigenspace. In the thermodynamic limit the set of degenerate ground states of $\hat{H}_0$ corresponds to the zero-vortex sector ($W_{\text{tot}} = 0$), and this can differ for small systems. To initialize the system we prepare a lattice of qubits as one of the eigenstates of the stabilizer group $\mathcal{S}$. The procedure originates from techniques used in topological quantum computing, showing that for topological subsystem codes based on the Kitaev model the stabilization can be performed by two-qubit projections [98]. This resembles GHZ state preparation in the simple 2-qubit case, and 4-qubit stabilization for a rectangular grid of the surface code [101]. To apply projectors we use an ancillary qubit register with $N_0$ qubits. The initialization circuit is shown in Fig. 1(b), where unitary operators $\hat{S}_{\text{ph}}$ and $\hat{S}_{\text{str}}$, composed of CNOTs and Hadamards, are followed by ancilla measurements. After stabilization we can additionally apply $\hat{A}_w$ based on single-qubit rotations that enable pairwise creation and annihilation of vortices, adjusting $W_{\text{tot}}$ to zero (or any other sector), and prepare $|\psi_0\rangle$ (see details in SM, sec. A).

Ansatz construction.—To remain in the relevant subspace we need to compose an ansatz circuit using generators that obey the symmetries of the Kitaev model, yet do not commute with $\hat{H}_0$ and thus can change the energy. Such group operators are known as centralizers $\mathcal{C}(\mathcal{S})$ of the stabilizer group $\mathcal{S}$. Namely, centralizers $\hat{C}_\alpha \in \mathcal{C}(\mathcal{S})$ are bounded Pauli strings that commute with all the elements of $\mathcal{S}$. For the honeycomb Kitaev lattice these are bond operators $\hat{K}_\alpha^{(i,j)} \in \mathcal{A} = \mathcal{X}, \mathcal{Y}, \mathcal{Z}$, correspondingly), such that $[\hat{K}_\alpha^{(i,j)} | \hat{w}_p] = 0$ for all $(i, j)$ and $p$. In fact, $\hat{K}_\alpha^{(i,j)}$ form only one possible subgroup of centralizers, and we can obtain others by multiplication of $\hat{K}_\alpha^{(i,j)}$.

For the non-zero magnetic field the number of vortices is not conserved, and the GS of $\hat{H}$ possesses a fractional number of vortices. Yet, it is convenient to use the vortex basis such that additional variational operations $\hat{U}_{\theta_{\alpha}}$ are introduced as vortex creation and annihilation, as well as conditional vortex NOT (see SM, sec. B). As we show below, this ansatz is able to capture the physics of interacting vortices [97, 102, 103] even for $h \sim J$.

Results: $h = 0$.—First, we consider a pure Kitaev model with zero magnetic field. We perform variational GSP using the proposed workflow, and compare the prepared state $|\psi_\theta\rangle = \hat{U}_{\theta}|\psi_0\rangle$ with $|\psi_{\text{GS}}\rangle$ obtained from imaginary time propagation [104]. We consider four different lattice sizes with $N = \{8, 12, 18, 24\}$ qubits, and prepare initial states $\{|\psi_0\rangle\}_N$ with $\langle \psi_0 | W_{\text{tot}} | \psi_0 \rangle = \{4, 0, 0, 0\}$, respectively, such that the ground state can be reached by the centralizer-based ansatz. We use a fast Julia-based full state simulator realized in Yao [90, 105–107] and GPU acceleration for the GSP of a 24-qubit lattice. The variational search is performed using the gradient-based optimizer Adam. The results are shown in Fig. 2. As quality metrics we use an infidelity $1 - F = 1 - |\langle \psi_{\text{GS}} | \psi_\theta \rangle|^2$ and a GS energy distance $\Delta E$.

In Fig. 2(a) the infidelity is shown at a function of $d$ for different $N$. The proposed GSP shows an
excellent performance, converging to the exact ground state (up to numerical precision) already at \( d \sim N/6 \). In Fig. 2(b) we present the energy distance, showing perfect correspondence with the infidelity. Plotting the training as infidelity vs epoch number [Fig. 2(c)], we observe exponential convergence and high-quality GSP even for the largest \( N = 24 \) lattice, noting the absence of barren plateaus. Comparing results with HEA, we see that the latter struggles to prepare an approximate GS even for the minimal Kitaev lattice (see SM, sec. C). The convergence to relevant GS energies \( E \) is shown in Fig. 2(d).

Results: \( h \neq 0 \).—Next, we prepare the Kitaev model GS with a non-zero uniform magnetic field, choosing \( h^z \neq 0 \). The magnetic field lifts the GS degeneracy and leads to quantum phase transitions between QSL and a trivial polarized phase [19]. Due to the broken vortex conservation symmetry GSP becomes challenging, mirroring the complexity of classical spectrum calculations. By including vortex rotations and vortex-CNOTs in the ansatz, we prepare approximate ground states of \( N = 8 \) and \( N = 12 \) Kitaev lattices with \( 10^{-3} \) infidelity and comparable energy distance (see SM, sec. D). Performing GSP for increasing \( h^z \) and both ferromagnetic (FM, \( J = -1 \)) and antiferromagnetic (AFM, \( J = 1 \)) Kitaev couplings, we show relevant system observables in Fig. 3. Specifically, in Fig. 3(a,b) we show the normalized total magnetization \( M^z/N = \sum_{i=1}^{N} \langle \psi_{\theta_{opt}} | Z_i | \psi_{\theta_{opt}} \rangle / N \) (dots), closely following true GS magnetization shown by solid curves. In Fig. 3(c,d) we plot the average vortex number deviation \( \eta = |2W_{\text{tot}}/n - 1| \), showing the transition between QSL physics to trivial physics (see SM, sec. D). Intriguingly, already at small system sizes we observe qualitative changes that resemble transitions in Ref. [19].

Results: dynamics.—Finally, we generalize our considerations to study dynamical effects. Quantum simulators excel in evolving quantum systems, where a propagator \( \hat{U}(t) = \exp(-iHt) \) is approximated by a sequence of gates [24]. Then, preparing the Kitaev GS at zero field, simulation of dynamics for \( h \neq 0 \) opens the route to quench studies of strongly correlated spins [18]. Let us define bond-bond correlations functions [108, 109] using time-dependent expectations of two-body operators \( m_{kk'}(t) = \langle \psi_{\theta} | U(0) Z_k Z_{k'} U(t) | \psi_{\theta} \rangle \) and four-body operators \( c_{kk'jj'}(t) = \langle \psi_{\theta} | U(0) Z_k Z_{k'} U(t) Z_j Z_{j'} | \psi_{\theta} \rangle \). We define the static correlator as \( C_{ii'jj'} = c_{ii'jj'}(0) - m_{ii'}(0)m_{jj'}(0) \), and the dynamic correlator as \( S_{ii'jj'}(t) = c_{ii'jj'}(t) - m_{ii'}(t)m_{jj'}(0) \). Such correlations can be accessed by dynamical QS with a proven advantage over classical computation [109].

In Fig. 4 we show dynamical bond-bond correlations for the variationally prepared GS of \( H_0 \) that is evolved under the biased Kitaev Hamiltonian with \( h^z/|J| = 0.5 \), simulating the quench. We propagate the system using the Trotterization approach with 10 steps. The result is shown by blue and magenta dots for horizontal and diagonal bond orientations (SM, sec. E). We confirm that \( S_{ii'jj'} \) can be successfully reproduced with the proposed workflow, following the solid curves obtained for the true GS and idealized QS. The results can be further improved if analog QS is used [110, 111]. Additionally, we studied static bond correlations at \( h \neq 0 \) (SM, sec. D), and note that our results can be extended to track long-range spin-spin correlations [112, 113]. Also, similar protocol can be used to study QSL quasiparticles in non-zero fields [114].

Discussion and conclusion.—We observe that for limited circuit depth we can prepare high-quality QSL ground states in the noiseless limit. This is the first step towards scalable GSP and QS for Kitaev models. For this scaling to persist when implemented on a real hardware, we require a noise level that does not impede...
optimization. Given large gradients, we expect the procedure to work at relatively low shot numbers. We assume a gate set based on rotations plus CNOTs, and assign a single-qubit and CNOT errors as \( \varepsilon_1 \) and \( \varepsilon_2 \). Using the simplest noise model, which was shown to describe large scale quantum experiments \([116]\), we estimate fidelity as \( F = (1-\varepsilon_1)^{n_R} (1-\varepsilon_2)^{n_{\text{CNOT}}} \), where \( n_R \) and \( n_{\text{CNOT}} \) is the number of rotations and CNOTs, respectively (see details in SM, sec. F). For \( N = 18 \), \( \varepsilon_1 = 10^{-4} \) and \( \varepsilon_2 = 10^{-3} \) we can achieve GS with \( F = 82\% \) fidelity, and for \( \varepsilon_1 = 10^{-5} \) and \( \varepsilon_2 = 10^{-4} \) this can improve to \( F = 98\% \). Furthermore, we note that the coupling to environment may make QS more rich, and lead to exceptional spin liquids \([115]\). In SM, sec. F, we also consider the question of connectivity and boundary conditions, presenting possible mappings between the hexagonal lattice and hardware-native square \([116]\) and heavy-hex \([117]\) lattices. As for the initialization, we stress that studies of Kitaev’s QSL physics perfectly aligns with efforts towards quantum error correction \([118–122]\), where the stabilization procedure was demonstrated. Given that for zero magnetic field the model is classically tractable and can be verified at increasing \( N \), can it become a benchmarking problem on the road to quantum advantage?

Recent state-of-the-art experiments include realizing topologically ordered toric code ground states on a quantum processor \([123]\), working at the scale of 31 qubits. Here, we suggest Kitaev spin liquid as a material science challenge that may be tackled by modern quantum devices. In fine, we proposed the efficient variational workflow for the ground state preparation utilizing symmetries of the Kitaev model, initialization, and vortex-based operations. When paired with dynamical quantum simulation, our toolbox allows studying the physics of strongly-correlated spins that is classically inaccessible.

Acknowledgements.— We acknowledge valuable discussions with Laura Baez in the beginning of the project. We also thank Annie Paine for useful suggestions on the lattice mappings and reading the manuscript. T. A. B. thanks the University of Exeter for hosting her visit as a part of the MSc project work.

Note added.— While completing this work several independent variational protocols of kagome Heisenberg models and square-octagon-lattice Kitaev model appeared in \([124]\), \([125]\), and \([126]\). While having the same spirit and pushing NISQ boundaries, our work highlights the importance of stabilization and adds dynamical aspects.

\footnote{Electronic address: o.kyriienko@exeter.ac.uk}

\begin{thebibliography}{99}
\bibitem{Witczak-Krempa2014} W. Witczak-Krempa, Gang Chen, Yong Baek Kim, and L. Balents, \textit{Correlated quantum phenomena in the strong spin-orbit regime}, Annu. Rev. Condens. Matter Phys., \textbf{5}, 57 (2014).
\bibitem{Loh2019} E. Y. Loh, Jr., J. E. Gubernatis, R. T. Scalettar, S. R. White, D. J. Scalapino, and R. L. Sugar, \textit{Sign problem in the numerical simulation of many-electron systems}, Phys. Rev. B \textbf{41}, 9301 (1990).
\bibitem{Savary2017} L. Savary and L. Balents, \textit{Quantum spin liquids: a review}, Rep. Prog. Phys. \textbf{80}, 016502 (2017).
\bibitem{Balents2010} L. Balents, \textit{Spin liquids in frustrated magnets}, Nature \textbf{464}, 199 (2010).
\bibitem{Tomasello2019} B. Tomasellos, C. Castelnovo, R. Moessner, and J. Quintanilla, \textit{Correlated Quantum Tunneling of Monopoles in Spin Ice}, Phys. Rev. Lett. \textbf{123}, 067204 (2019).
\bibitem{Takagi2019} H. Takagi, T. Takayama, G. Jackeli, G. Khaliullin, and S. E. Nagler, \textit{Kitaev quantum spin liquid - concept and materialization}, Nat. Rev. Phys. \textbf{1}, 264 (2019).
\bibitem{Anderson1987} P. W. Anderson, \textit{The resonating valence bond state in La\textsubscript{2}CuO\textsubscript{4} and superconductivity}, Science \textbf{235}, 1196 (1987).
\bibitem{Lee2007} P. A. Lee, \textit{From high temperature superconductivity to quantum spin liquid: progress in strong correlation physics}, Rep. Prog. Phys. \textbf{71}, 012501 (2007).
\bibitem{Jackeli2009} G. Jackeli and G. Khaliullin, \textit{Mott Insulators in the Strong Spin-Orbit Coupling Limit: From Heisenberg to a Quantum Compass and Kitaev Models}, Phys. Rev. Lett. \textbf{102}, 017205 (2009).
\bibitem{Chaloupka2010} J. Chaloupka, G. Jackeli, and G. Khaliullin, \textit{Kitaev-Heisenberg Model on a Honeycomb Lattice: Possible Exotic Phases in Iridium Oxides \textit{A}_{2}\textit{IrO}_{3}}, Phys. Rev. Lett. \textbf{105}, 027204 (2010).
\bibitem{Rau2015} J. G. Rau, E. K.-H. Lee, and H.-Y. Kee, \textit{Spin-orbit physics giving rise to novel phases in correlated systems: iridates and related materials}, Annu. Rev. Condens. Matter Phys. \textbf{7}, 195 (2016).
\bibitem{Kubota2015} Yumi Kubota, Hidekazu Tanaka, Toshio Ono, Yasuo Narumi, and Koichi Kindo, \textit{Successive magnetic phase transitions in \alpha-RuCl\textsubscript{3}: XY-like frustrated magnet on the honeycomb lattice}, Phys. Rev. B \textbf{91}, 094422 (2015).
\bibitem{Janssen2017} L. Janssen, E. C. Andrade, and M. Vojta, \textit{Magnetization processes of zigzag states on the honeycomb lattice: Identifying spin models for \alpha-RuCl\textsubscript{3} and Na\textsubscript{2}\textit{IrO}_{3}}, Phys. Rev. B \textbf{96}, 064430 (2017).
\bibitem{Gordon2019} J. S. Gordon, A. Catuneanu, E. S. Sørensen, and Hae-Young Kee, \textit{Theory of the field-revealed Kitaev spin liquid}, Nature Commun. \textbf{10}, 2470 (2019).
\bibitem{Norman2016} M. R. Norman, \textit{Colloquium: Herbertsmithite and the search for the quantum spin liquid}, Rev. Mod. Phys. \textbf{88}, 041102 (2016).
\bibitem{Kitaev2006} A. Kitaev, \textit{Anyons in an exactly solved model and beyond}, Ann. Phys. \textbf{321}, 2 (2006).
\bibitem{Trebst2017} S. Trebst, \textit{Kitaev Materials}, arXiv:1701.07056 [cond-mat.str-el] (2017).
\bibitem{Hermanns2018} M. Hermanns, I. Kimchi, and J. Knolle, \textit{Physics of the Kitaev model: fractionalization, dynamic correlations, and material connections}, Annu. Rev. Condens. Matter Phys. \textbf{9}, 17 (2018).
\bibitem{Hickey2010} C. Hickey and S. Trebst, \textit{Emergence of a field-driven \textit{U}(1) spin liquid in the Kitaev honeycomb model}, Nature Commun. \textbf{10}, 530 (2019).
\bibitem{Lau2011} A. M. Läuchli, J. Sudan, E. S. Sørensen, \textit{Ground-state energy and spin gap of spin-1/2 Kagome-Heisenberg antiferromagnetic clusters: Large-scale exact diagonalization results}, Phys. Rev. B \textbf{83}, 212401 (2011).
\bibitem{Wietek2018} A. Wietek and A. M. Läuchli, \textit{Sublattice coding algorithm and distributed memory parallelization for large-scale exact diagonalizations of quantum many-body systems}, Phys. Rev. E \textbf{98}, 033309 (2018).
\end{thebibliography}
[22] Simeng Yan, D. A. Huse, and S. R. White, *Spin-Liquid Ground State of the S = 1/2 Kagome Heisenberg Antiferromagnet*, Science **332**, 1173 (2011).

[23] Hong-Chen Jiang, Hong Yao, L. Balents, *Spin liquid ground state of the spin-1/2 square J1-J2 Heisenberg model*, Phys. Rev. B **86**, 024424 (2012).

[24] I. M. Georgescu, S. Ashhab, and F. Nori, *Quantum simulation*, Rev. Mod. Phys. **86**, 153 (2014).

[25] A. A. Houck, H. E. Tureci, and J. Koch, *On-chip quantum simulation with superconducting circuits*, Nat. Phys. **8**, 292 (2012).

[26] A. A. Houck, H. E. Tureci, and J. Koch, *On-chip quantum simulation with superconducting circuits*, Nat. Phys. **8**, 292 (2012).

[27] M. Dobsicek, G. Johansson, V. Shumeiko, and G. Babbush, D. W. Berry, J. R. McClean, and H. Neven, *Quantum virtual cooling*, Phys. Rev. X **9**, 031013 (2019).

[28] T. A. Bessalova and O. Kyriienko, *Hamiltonian Operator Approximation for Energy Measurement and Ground-State Preparation*, PRX Quantum **2**, 030318 (2021).

[29] F. Arute et al., *Quantum Simulators: Architectures and Opportunities*, PRX Quantum **2**, 017003 (2021).

[30] S. Ebadi, T. T. Wang, H. Levine, A. Keesling, G. Semeghini, A. Omran, D. Bluvstein, R. Samajdar, H. Pichler, Wen Wei Ho, Soonwon Choi, S. Sachdev, M. Greiner, V. Vuletic, and M. D. Lukin, *Quantum phases of matter on a 356-atom programmable quantum simulator*, Nature **595**, 227 (2021).

[31] G. Semeghini, H. Levine, A. Keesling, S. Ebadi, T. T. Wang, D. Bluvstein, R. Verresen, H. Pichler, M. Kallinowski, R. Samajdar, A. Omran, S. Sachdev, A. Vishwanath, M. Greiner, V. Vuletic, and M. D. Lukin, *Probing Topological Spin Liquids on a Programmable Quantum Simulator*, arXiv:2104.04119 [quant-ph] (2021).

[32] S. McArdle, S. Endo, A. Aspuru-Guzik, S. C. Benjamin, and Xiao Yuan, *Quantum computational chemistry*, Rev. Mod. Phys. **92**, 15003 (2020).

[33] Y. Cao, J. Romero, J. P. Olson, M. Degroote, P. D. Johnson, M. Kieferová, I. D. Kivlichan, T. Menke, B. Peropadre, N. P. D. Sawaya, Sukin Sim, L. Veis, and A. Aspuru-Guzik, *Quantum Chemistry in the Age of Quantum Computing*, Chem. Rev. **119**, 10856 (2019).

[34] V. E. Elfving, B. W. Broer, M. Webber, J. Gavartin, M. D. Halls, K. P. Lorton, and A. Bochevarov, *How will quantum computers provide an industrially relevant computational advantage in quantum chemistry?*, arXiv:2009.12472 [quant-ph] (2020).

[35] A. Aspuru-Guzik, A. D. Dutoi, P. J. Love, and M. Head-Gordon, *Simulated Quantum Computation of Molecular Energies*, Science **309**, 1704 (2005).

[36] M. Dobsicek, G. Johansson, V. Shumeiko, and G. Wendin *Arbitrary accuracy iterative quantum phase estimation algorithm using a single ancillary qubit: A two-qubit benchmark*, Phys. Rev. A **76**, 030306(R) (2007).

[37] T. E. O’Brien, B. Senjean, R. Sagastizabal, X. Bonet-Monroig, A. Dukiewicz, F. Buda, L. DiCarlo, and L. Visscher, *Calculating energy derivatives for quantum chemistry on a quantum computer*, npj Quantum Inf. **5**, 113 (2019).

[38] D. W. Berry, M. Kieferova, A. Scherer, Y. R. Sanders, Guang Hao Low, N. Wiebe, C. Gidney, and R. Babbush, *Improved techniques for preparing eigenstates of fermionic Hamiltonians*, npj Quantum Information **4**, 22 (2018).

[39] R. Babbush, D. W. Berry, J. R. McClean, and H. Neven, *Quantum Simulation of Chemistry with Sublinear Scaling in Basis Size*, npj Quantum Information **5**, 92 (2019).

[40] R. M. Parrish and P. L. McMahon, *Quantum Filter Diagonalization: Quantum Eigendecomposition without Full Quantum Phase Estimation*, arXiv:1909.08925 (2019).

[41] O. Kyriienko, *Quantum inverse iteration algorithm for programmable quantum simulators*, npj Quantum Information **6**, 7 (2020).

[42] N. H. Stair, R. Huang, and F. A. Evangelista, *A Multitarget Quantum Krylov Algorithm for Strongly Correlated Electrons*, J. Chem. Theory Comput. **16**, 2236 (2020).

[43] J. Cotler, Soonwon Choi, A. Lukin, H. Gharibyan, T. Grover, M. E. Tai, M. Rispoli, R. Schittko, P. M. Preiss, A. M. Kaufman, M. Greiner, H. Pichler, and P. Hayden, *Quantum virtual cooling*, Phys. Rev. X **9**, 031013 (2019).

[44] M. Ganzhorn, D. J. Egger, P. Barkoutsos, P. Ollitrault, K. Bharti, A. Cervera-Lierta, Thi Ha Kyaw, T. Haug, P. J. J. O’Malley, R. Babbush, I. D. Kivlichan, J. Romero, J. R. McClean, R. Barends et al., *Scalable Quantum Simulation of Molecular Energies*, Phys. Rev. X **6**, 031007 (2016).

[45] A. Kandala, A. Mezzacapo, K. Temme, M. Takita, M. Brink, J. M. Chow, and J. M. Gambetta, *Hardware-efficient variational quantum eigensolver for small molecules and quantum magnets*, Nature (London) **549**, 242 (2017).

[46] F. Arute et al., *Hartree-Fock on a superconducting qubit quantum computer*, Science **369**, 1084 (2020).

[47] M. Dall’Agnol, A. Arrasmith, R. Babbush, S. C. Benjamin, S. Endo, K. Fujii, J. R. McClean, K. Mitarai, Xiao Yuan, L. Cincio, and P. J. Coles, *Variational Quantum Algorithms*, Nat. Rev. Phys. **3**, 625 (2021).

[48] R. Schittko, P. M. Preiss, M. E. Tai, M. Rispoli, M. Degroote, H. Heimonen, J. S. Kottmann, T. Menke, Wai-Keong Mok, and A. Aspuru-Guzik, *Noisy intermediate-scale quantum (NISQ) algorithms*, arXiv:2101.08448 [quant-ph] (2020).

[49] R. M. Parrish, E. G. Hohenstein, P. L. McMahon, and T. J. Martinez, *Quantum Computation of Electronic Transitions Using a Variational Quantum Eigensolver*, Phys. Rev. Lett. **122**, 230401 (2019).

[50] V. E. Elfving, J. A. Gámez, and C. Gogolin, *Simulating quantum chemistry in the restricted Hartree-Fock space on a qubit-based quantum computing device*, Phys. Rev. A **103**, 032605 (2021).

[51] S. Yalouz, B. Senjean, J. Günther, F. Buda, T. E. O’Brien, and L. Visscher, *A state-averaged orbital-optimized hybrid quantum-classical algorithm for a democratic description of ground and excited states*, Quantum Sci. Technol. **6**, 024004 (2021).

[52] Bo Peng and K. Kowalski, *Variational quantum solver employing the PDS energy functional*, Quantum **5**, 473 (2021).

[53] M. Ganzhorn, D. J. Egger, P. Barkoutsos, P. Ollitrault, G. Salis, N. Moll, M. Roth, A. Fuhrer, P. Mueller, S. Woerner, I. Tavernelli, and S. Filip, *Gate-Efficient Simulation of Molecular Eigenstates on a Quantum Computer*, Phys. Rev. Applied **11**, 044092 (2019).

[54] C. Kokail, C. Maier, R. van Bijnen, T. Brydges, M. K.
Y. Nam, J.-S. Chen, N. C. Pisenti, K. Wright, C. Delaney, D. Maslov, K. R. Brown, S. Allen, J. M. Amini, J. Apisdorf, K. M. Beck, A. Blinov, V. Chaplin, M. Chmielowski, C. Collins, S. Deb Nath, A. M. Ducore, K. M. Hudek, M. Keesan, S. M. Kreikemeier, J. Mizrahi, P. Solomon, M. Williams, J. D. Wong-Campos, C. Monroë, and J. Kim, *Ground-state energy estimation of the water molecule on a trapped ion quantum computer*, npj Quantum Inf. 6, 33 (2020).

A. J. C. Woitzik, P. Kl. Barkoutsos, F. Wudarski, A. Buchleitner, and I. Tavernelli, *Entanglement production and convergence properties of the variational quantum eigensolver*, Phys. Rev. A 102, 042402 (2020).

T. L. Patti, K. Najafi, Xun Gao, and S. F. Yelin, *Entanglement devised barren plateau mitigation*, Phys. Rev. Research 3, 033090 (2021).

L. Funcke, T. Hartung, K. Jansen, S. Kühn, and P. Stornati, *Dimensional Expressivity Analysis of Parametric Quantum Circuits*, Quantum 5, 422 (2021).

K. Yeter-Aydeniz, B. T. Gard, J. Jakowski, Swarnadeep Majumder, G. S. Barron, G. Siopsis, T. Humble, and R. C. Posser, *Benchmarking Quantum Chemistry Computations with Variational, Imaginary Time Evolution, and Krylov Space Solver Algorithms*, arXiv:2102.05511 [quant-ph] (2021).

J. R. McClean, S. Boixo, Vadim N. Smelyanskiy, Ryan Babbush, and Hartmut Neven, *Barren plateaus in quantum neural network training landscapes*, Nature Commun. 9, 4812 (2018).

M. Cerezo, A. Sone, T. Volkoff, L. Cincio, and P. J. Coles, *Cost-Function-Dependent Barren Plateaus in Shallow Quantum Neural Networks*, Nature Commun. 12, 1791 (2021).

M. Metcalf, N. P. Bauman, K. Kowalski, and W. A. de Jong, *Resource Efficient Chemistry on Quantum Computers with the Variational Quantum Eigensolver and The Double Unitary Coupled-Cluster approach*, J. Chem. Theory Comput. 16, 10, 6165 (2020).

N. P. Bauman, J. Chládek, L. Veis, J. Pittner, and K. Kowalski, *Variational Quantum Eigensolver for Approximate Diagonalization of Downfolded Hamiltonians using Generalized Unitary Coupled Cluster Ansatz*, Quantum Sci. Technol. 6, 034008 (2021).

M. D. Sapova and A. K. Fedorov, *Variational quantum eigensolver techniques for simulating carbon monoxide oxidation*, arXiv:2108.11167 [physics.chem-ph] (2021).

A. W. Harrow and R. A. Low, *Random Quantum Circuits are Approximate 2-designs*, Comm. Math. Phys. 291, 257 (2009).

H. R. Grimley, S. E. Economou, E. Barnes, and N. J. Mayhall, *An adaptive variational algorithm for exact molecular simulations on a quantum computer*, Nature Commun. 10, 3007 (2019).

H. L. Tang, V. O. Shkolnikov, G. S. Barron, H. R. Grimley, N. J. Mayhall, E. Barnes, and S. E. Economou, *Qubit-ADAPT-VQE: An Adaptive Algorithm for Constructing Hardware-Efficient Ansätze on a Quantum Processor*, PRX Quantum 2, 020310 (2021).

D. Claudino, J. Wright, A. J. McCaskey, and T. S. Humble, *Benchmarking adaptive variational quantum eigensolvers*, Front. Chem. 8, 606863 (2020).

P. Suchsland, P. Kl. Barkoutsos, I. Tavernelli, M. H. Fischer, and T. Neupert, *Simulating a ring-like Hubbard system with a quantum computer*, arXiv:2104.06428 [quant-ph] (2021).

D. Chivilikhin, A. Samarín, V. Ulyantsev, I. Torsh, A. R. Oganov, O. Kyriienko, *MoG-VQE: Multiobjective genetic variational quantum eigensolver*, arXiv:2007.04424 (2020).

Shi-Xin Zhang, Chang-Yu Hsieh, Shengyu Zhang, Hong Yao, *Differentiable Quantum Architecture Search*, arXiv:2010.08561 [quant-ph] (2021).

Yuxuan Du, Tao Huang, Shan You, Min-Hsiu Hsieh, Dacheng Tao, *Quantum circuit architecture search: error mitigation and trainability enhancement for variational quantum solvers*, arXiv:2010.10217 [quant-ph] (2020).

Sukin Sim, J. Romero, J. F. Gonthier, and A. A. Kunitsa, *Adaptive pruning-based optimization of parameterized quantum circuits*, Quantum Sci. Technol. 6, 025019 (2021).

M. Blikis, M. Cerezo, Guillaume Verdon, Patrick J. Coles, Lukasz Cincio, *A semi-agnostic ansatz with variable structure for quantum machine learning*, arXiv:2103.06712 [quant-ph] (2021).

Chuangtao Chen, Zhimin He, Lvzhou Li, Shenggen Zheng, and Haozhou Situ, *Quantum Architecture Search with Meta-learning*, arXiv:2106.06248 [quant-ph] (2021).

M. Ostaszewski, L. M. Trenkwalder, W. Masarz, E. Scerri, and V. Dunjko, *Reinforcement learning for optimization of variational quantum circuit architectures*, arXiv:2103.16089 [quant-ph] (2021).

B. T. Gard, L. Zhu, G. S. Barron, N. J. Mayhall, S. E. Economou, and E. Barnes, *Efficient symmetry-preserving state preparation circuits for the variational quantum eigensolver algorithm*, npj Quantum Inf. 6, 10 (2020).

G. S. Barron, B. T. Gard, O. J. Altman, N. J. Mayhall, E. Barnes, and S. E. Economou, *Preserving Symmetries for Variational Quantum Eigensolvers in the Presence of Noise*, Phys. Rev. Applied 16, 034003 (2021).

Y. Herasymenko and T. E. O’Brien, *A diagrammatic approach to variational quantum ansatz construction*, arXiv:1907.08157 [quant-ph] (2019).

V. O. Shkolnikov, Nicholas J. Mayhall, Sophia E. Economou, Edwin Barnes, *Avoiding symmetry roadblocks and minimizing the measurement overhead of adaptive variational quantum eigensolvers*, arXiv:2109.05340 [quant-ph] (2021).

Hans Hon Sang Chan, N. Fitzpatrick, J. Segarra-Martí, M. J. Bearpark, and D. P. Tew, *Molecular Excited State Calculations with Adaptive Wavefunctions on a Quantum Eigensolver Emulation: Reducing Circuit Depth and Separating Spin States*, arXiv:2105.10275 [quant-ph] (2021).

E. Grant, L. Wossnig, M. Ostaszewski, M. Benedetti, *An initialization strategy for addressing barren plateaus in parametrized quantum circuits*, Quantum 3, 214 (2019).

Wen Wei Ho and T. H. Hsieh, *Efficient variational simulation of non-trivial quantum states*, SciPost Phys. 6, 029 (2019).

A. Kaviani, A. Pervishko, J. Biamonte, and D. Yudin, *Numerical hardware-efficient variational quantum simulators*, Quantum 10, 606863 (2020).
ulation of a soliton solution, Phys. Rev. A 104, L020402 (2021).

[85] U. L. Heras, A. Mezzacapo, L. Lamata, S. Filipp, A. Wallraff, and E. Solano, Digital Quantum Simulation of Spin Systems in Superconducting Circuits, Phys. Rev. Lett. 112, 200501 (2014).

[86] O. Kyriienko and A. S. Sørensen, Floquet Quantum Simulation with Superconducting Qubits, Phys. Rev. Applied 9, 064029 (2018).

[87] R. Wiersema, Cunlu Zhou, Y. de Sereville, J. F. Carrasquilla, Yong Baek Kim, and Henry Yuen, Exploring Entanglement and Optimization within the Hamiltonian Variational Ansatz, PRX Quantum 1, 020319 (2020).

[88] D. Wierichs, C. Gogolin, and M. Kastoryano, Avoiding local minima in variational quantum eigensolvers with the natural gradient optimizer, Phys. Rev. Research 2, 043246 (2020).

[89] T. Gonzalez-Rayya, R. Asensio-Perea, Ana Martin, L. C. Celeri, M. Sanz, P. Lougovski, and E. F. Dumitrescu, Digital-Analog Quantum Simulations Using the Cross-Resonance Effect, PRX Quantum 2, 020326 (2021).

[90] Jin-Guo Liu, Yi-Hong Zhang, Yuan Wan, and Lei Wang, Variational quantum eigensolver with fewer qubits, Phys. Rev. Research 1, 023025 (2019).

[91] L. Slattery and B. K. Clark, Quantum Circuits For Two-Dimensional Isometric Tensor Networks, arXiv:2108.02792 [quant-ph] (2021).

[92] Saptarshi Mandal and A. M. Jayannavar, An introduction to Kitaev model-I, arXiv:2006.11549 [cond-mat.str-el] (2020).

[93] Han-Dong Chen and Zohar Nussinov, Exact results of the Kitaev model on a hexagonal lattice: spin states, string and brane correlators, and anyonic excitations, J. Phys. A: Math. Theor. 41, 075001 (2008).

[94] Xiao Xiao, J. K. Freericks, A. F. Kemper, Determining quantum phase diagrams of topological Kitaev-inspired models on NISQ quantum hardware, arXiv:2006.05524 [quant-ph] (2021).

[95] V. Lahtinen, G. Kells, A. Carollo, T. Stitt, J. Vala, and J. Pachos, Spectrum of the non-abelian phase in Kitaev’s honeycomb lattice model, Annals of Physics 323, 2286 (2008).

[96] G. Kells, A. T. Bolukbasi, V. Lahtinen, J. K. Slingerland, J. K. Pachos, and J. Vala, Topological Degeneracy and Vortex Manipulation in Kitaev’s Honeycomb Model, Phys. Rev. Lett. 101, 240404 (2008).

[97] V. Lahtinen, Interacting non-Abelian anyons as Majorana fermions in the honeycomb lattice model, New J. Phys. 13, 075009 (2011).

[98] M. Suchara, S. Bravyi, and B. Terhal, Constructions and noise threshold of topological subsystem codes, J. Phys. A: Math. Theor. 44, 155301 (2011).

[99] Yi-Chan Lee, G. C. Brell, and S. T. Flammia, Topological quantum error correction in the Kitaev honeycomb model, J. Stat. Mech. 083106 (2017); DOI:10.1088/1742-5468/aa7ee2.

[100] H. Bombin, Topological subsystem codes, Phys. Rev. A 81, 032301 (2010).

[101] A. G. Fowler, M. Mariantoni, J. M. Martinis, and A. N. Cleland, Surface codes: Towards practical large-scale quantum computation, Phys. Rev. A 86, 032324 (2012).

[102] V. Lahtinen and J. K. Pachos, Topological phase transitions driven by gauge fields in an exactly solvable model, Phys. Rev. B 81, 245132 (2010).

[103] V. Lahtinen, A. W. W. Ludwig, J. K. Pachos, and S. Trebst, Topological liquid nucleation induced by vortex-vortex interactions in Kitaev’s honeycomb model, Phys. Rev. B 86, 075115 (2012).

[104] Notably, the iterative Krylov type diagonalization for low energies failed to find the correct ground state in certain cases, struggling to conserve the number of vortices unless explicitly encoded. At the same time, the imaginary time propagation preserves symmetries by construction.

[105] Xiu-Zhe Luo, Jin-Guo Liu, Pan Zhang, Lei Wang, Yao.jl: Extensible, Efficient Framework for Quantum Algorithm Design, Quantum 4, 341 (2020).

[106] Jinfeng Zeng, Yufeng Wu, Jin-Guo Liu, Lei Wang, and Jianping Hu, Learning and inference on generative adversarial quantum circuits, Phys. Rev. A 99, 052306 (2019).

[107] O. Kyriienko, A. E. Paine, and V. E. Elfring, Solving nonlinear differential equations with differentiable quantum circuits, Phys. Rev. A 103, 052416 (2021).

[108] Shuo Yang, Shi-Jian Gu, Chu-Pu Sun, and Hai-Qing Lin, Fidelity susceptibility and long-range correlation in the Kitaev honeycomb model, Phys. Rev. A 78, 012304 (2008).

[109] M. E. Baez, M. Goihl, J. Haferkamp, J. Bermejo-Vega, M. Gluza, and J. Eisert, Dynamical structure factors of dynamical quantum simulators, PNAS 117, 26123 (2020).

[110] R. Schmied, J. A. Wiesenfeld, and D. Leibfried, Quantum simulation of the hexagonal Kitaev model with trapped ions, New J. Phys. 13, 115011 (2011).

[111] H. Weimer, M. Muller, H. P. Buchler, and I. Lesanovsky, Digital quantum simulation with Rydberg atoms, Quantum Inf. Process. 10, 855 (2011).

[112] G. Baskaran, Saptarshi Mandal, and R. Shankar, Exact Results for Spin Dynamics and Fractionalization in the Kitaev Model, Phys. Rev. Lett. 98, 247201 (2007).

[113] Saptarshi Mandal, Subhro Bhattacharjee, K. Sen, Gupta, R. Shankar, and G. Baskaran, Confinement-deconfinement transition and spin correlations in a generalized Kitaev model, Phys. Rev. B 84, 155121 (2011).

[114] J. Knolle, D. L. Kovrizhin, J. T. Chalker, and R. Moessner, Dynamics of a Two-Dimensional Quantum Spin Liquid: Signatures of Emergent Majorana Fermions and Fluxes, Phys. Rev. Lett. 112, 207203 (2014).

[115] Kang Yang, Siddharth C. Morampudi, and Emil J. Bergholtz, Exceptional Spin Liquids from Couplings to the Environment, Phys. Rev. Lett. 126, 077201 (2021).

[116] F. Arute et al., Quantum supremacy using a programmable superconducting processor, Nature 574, 505 (2019).

[117] C. Chamberland, G. Zhu, T. J. Yoder, J. B. Hertzberg and A. W. Cross, Topological and Subsystem Codes on Low-Degree Graphs with Flag Qubits, Phys. Rev. X 10, 011022 (2020). See also the press release by IBM.

[118] J. Kelly, R. Barends, A. G. Fowler, A. Megrant, E. Jeffrey, T. C. White, D. Sank, J. Y. Mutus, B. Campbell, Yu Chen, Z. Chen, B. Chiaro, A. Dunsworth, I.-C. Ho, C. Neill, P. J. J. O’Malley, C. Quintana, P. Roushan, A. Vainsencher, J. Wenner, A. N. Cleland, and J. M. Martinis, State preservation by repetitive error detection in a superconducting quantum circuit, Nature 519, 66 (2015).

[119] A. D. Corcoles, E. Magesan, S. J. Srinivasan, A. W.
Cross, M. Steffen, J. M. Gambetta and J. M. Chow, Demonstration of a quantum error detection code using a square lattice of four superconducting qubits, Nat. Commun. 6, 6979 (2015).

[120] C. C. Bultink, T. E. O’Brien, R. Vollmer, N. Muthusubramanian, M. W. Beekman, M. A. Rol, X. Fu, B. Tarasinski, V. Ostroukh, B. Varbanov, A. Bruno, L. DiCarlo, Protecting quantum entanglement from leakage and qubit errors via repetitive parity measurements, Science Advances 6, (2020); doi:10.1126/sciadv.aay3050

[121] Google Quantum AI, Exponential suppression of bit or phase errors with cyclic error correction, Nature 595, 383 (2021).

[122] Laird Egan, Dripto M. Debroy, Crystal Noel, Andrew Risinger, Daiwei Zhu, Debopriyo Biswas, Michael Newman, Muyuan Li, Kenneth R. Brown, Marko Cetina, Christopher Monroe, Fault-Tolerant Operation of a Quantum Error-Correction Code, arXiv:2009.11482 [quant-ph] (2020).

[123] K. J. Satzinger, ..., and P. Roushan, Realizing topologically ordered states on a quantum processor, arXiv:2104.01180 [quant-ph] (2021).

[124] J. Kattmölle and J. van Wezel, Variational quantum eigensolver for the Heisenberg antiferromagnet on the kagome lattice, arXiv:2108.02175 [quant-ph] (2021).

[125] J. L. Bosse and A. Montanaro, Probing ground state properties of the kagome antiferromagnetic Heisenberg model using the Variational Quantum Eigensolver, arXiv:2108.08086 [quant-ph] (2021).

[126] A. C. Y. Li, M. S. Alam, T. Iadecola, A. Jahin, D. Murat Kurcuoglu, Richard Li, P. P. Orth, A. B. Ozguler, G. N. Perdue, and N. M. Tubman, Benchmarking variational quantum eigensolvers for the square-octagon-lattice Kitaev model, arXiv:2108.13375 [quant-ph] (2021).