Abstract

After a short review of the production mechanisms of the light scalars which reveal their nature and indicate their quark structure, we suggest to study the mixing of the isovector $a_0^0(980)$ with the isoscalar $f_0(980)$ in spin effects.

1 Introduction.

The scalar channels in the region up to 1 GeV became a stumbling block of QCD. The point is that both perturbation theory and sum rules do not work in these channels because there are not solitary resonances in this region.

As Experiment suggests, in chiral limit confinement forms colourless observable hadronic fields and spontaneous breaking of chiral symmetry with massless pseudoscalar fields. There are two possible scenarios for QCD realization at low energy: 1. $U_L(3) \times U_R(3)$ non-linear $\sigma$ model, 2. $U_L(3) \times U_R(3)$ linear $\sigma$ model. The experimental nonet of the light scalar mesons suggests $U_L(3) \times U_R(3)$ linear $\sigma$ model.

2 $SU_L(2) \times SU_R(2)$ Linear $\sigma$ Model, Chiral Shielding in $\pi\pi \rightarrow \pi\pi$ [1]

Hunting the light $\sigma$ and $\kappa$ mesons had begun in the sixties. But the fact that both $\pi\pi$ and $\pi K$ scattering phase shifts do not pass over $90^\circ$ at putative resonance masses prevented to prove their existence in a conclusive way.

Situation changes when we showed that in the linear $\sigma$ model there is a negative background phase which hides the $\sigma$ meson [1]. It has been made clear that shielding wide lightest scalar mesons in chiral dynamics is very natural. This idea was picked up and triggered new wave of theoretical and experimental searches for the $\sigma$ and $\kappa$ mesons.

Our approximation is as follows (see Fig. 1): $T_0^{(0\text{tree})} = \frac{m^2 - m^2_f}{2s} [5 - 3 \frac{m^2 - m^2_f}{m^2 - s} - 2 \frac{s - 4m^2_f}{s - 4m^2}] \times \ln \left(1 + \frac{s - 4m^2_f}{m^2_s} \right)$, $T_0^{0\text{tree}} = \frac{2\text{Im}T_0^{0\text{tree}}}{M_{\rho_{\pi\pi}^{\text{res}}}/\rho_{\pi\pi}^{\text{res}}} = \frac{e^{2i\delta_{\rho_{\pi\pi}^{\text{res}}}} - 1}{2\rho_{\pi\pi}^{\text{res}}}$, $T_{bg} = \frac{e^{2i\delta_{\rho_{\pi\pi}^{\text{res}}}} - 1}{2\rho_{\pi\pi}^{\text{res}}} = \frac{\lambda(s)}{1 - i\rho_{\pi\pi}^{\text{res}}\lambda(s)}$, $\lambda(s) = \frac{m^2 - m^2_f}{32\pi f^2}$ $[5 - 2 \frac{s - 4m^2_f}{s - 4m^2}] \times \ln \left(1 + \frac{s - 4m^2_f}{m^2_s} \right)$, $\text{Re} \Pi_{\pi\pi}^{\text{res}}(s) = -\frac{g_{\pi\pi}^{\text{res}}(s)}{16\pi} \lambda(s) \rho_{\pi\pi}^{2\text{tree}}$, $\text{Im} \Pi_{\pi\pi}^{\text{res}}(s) = \sqrt{s} \Gamma_{\pi\pi}^{\text{res}}(s) = \frac{g_{\pi\pi}^{\text{res}}(s)\rho_{\pi\pi}^{2\text{tree}}}{16\pi}$, $g_{\pi\pi}(s) = \frac{g_{\pi\pi}^{\text{res}}(s)}{1 + i\rho_{\pi\pi}^{\text{res}}\lambda(\pi)}$, $M^2_{\pi\pi} = m^2_{\pi\pi} - \text{Re} \Pi_{\pi\pi}^{\text{res}}(M^2_{\pi\pi})$, $\rho_{\pi\pi} = \sqrt{1 - \frac{4m^2_f}{s}}$, $g_{\pi\pi}^{\text{res}}(s) = \sqrt{\frac{3}{2}} g_{\pi\pi}^{\text{res}}(s)\rho_{\pi\pi}^{2\text{tree}}$, $T_{0\text{tree}} = \frac{\text{Im}T_0^{0\text{tree}}}{\rho_{\pi\pi}^{\text{res}}} = \frac{e^{2i\delta_{\rho_{\pi\pi}^{\text{res}}}} - 1}{2\rho_{\pi\pi}^{\text{res}}}.

The results in our approximation are: $M_{\pi\pi} = 0.43$ GeV, $\Gamma_{\pi\pi}^{\text{res}}(M^2_{\pi\pi}) = 0.67$ GeV, $m^2_{\pi\pi} = 0.93$ GeV, $\Gamma^{\text{renorm}}(M^2_{\pi\pi}) = \frac{\Gamma_{\pi\pi}^{\text{res}}(M^2_{\pi\pi})}{1 + d\text{Re} \Pi_{\pi\pi}^{\text{res}}(s)/ds = A^2_{\pi\pi}^2}$ $= 0.53$ GeV, $g_{\pi\pi}^{\text{res}}(s) = 0.43$, $a_0 = 0.18 m^2_{\pi\pi}$, $\delta_{\pi\pi} = -0.04 m^2_{\pi\pi}$, the Adler zeros $(s_A)_{0}^{0} = -0.45 m^2_{\pi\pi}$ and $(s_A)_{0}^{0} = -2.02 m^2_{\pi\pi}$. The chiral shielding of the $\sigma(600)$ meson in $\pi\pi \rightarrow \pi\pi$ is illustrated in Fig. 2 with the help of the $\pi\pi$ phase shifts $\delta_{\pi\pi}$, $\delta_{\pi\pi}^0$, $\delta_{\pi\pi}^0(a)$, and with the help of the corresponding cross sections (b).
Figure 1: The graphical representation of the $S$ wave $I = 0$ $\pi\pi$ scattering amplitude $T_0^0$.

![Diagram](image)

Figure 2: The $\sigma$ model. Our approximation. $\delta_0^0 = \delta_{\text{res}} + \delta_{\text{bg}}$. $(\sigma_0^0, \sigma_{\text{res}}, \sigma_{\text{bg}}) = \frac{12\pi}{a}(|T_0^0|^2, |T_{\text{res}}|^2, |T_{\text{bg}}|^2)$.

3 The $\sigma$ Propagator

$1/D_\sigma(s) = 1/[M_{\text{res}}^2 - s + \text{Re}\Pi_{\text{res}}(M_{\text{res}}^2) - \Pi_{\text{res}}(s)]$. The $\sigma$ meson self-energy $\Pi_{\text{res}}(s)$ is caused by the intermediate $\pi\pi$ states, that is, by the four-quark intermediate states. This contribution shifts the Breit-Wigner (BW) mass greatly $m_\sigma - M_{\text{res}} \approx 0.50$ GeV. So, half the BW mass is determined by the four-quark contribution at least. The imaginary part dominates the propagator modulus in the region $0.3$ GeV $< \sqrt{s} < 0.6$ GeV. So, the $\sigma$ field is described by its four-quark component at least in this energy (virtuality) region.

4 Four-quark Model

The nontrivial nature of the well-established light scalar resonances $f_0(980)$ and $a_0(980)$ is no longer denied practically anybody. As for the nonet as a whole, even a cursory look at PDG Review gives an idea of the four-quark structure of the light scalar meson nonet, $\sigma(600)$, $\kappa(700 - 900)$, $f_0(980)$, and $a_0(980)$, inverted in comparison with the classical $P$ wave $q\bar{q}$ tensor meson nonet $f_2(1270)$, $a_2(1320)$, $K_2^*(1420)$, $\phi_2^*(1525)$. Really, while the scalar nonet cannot be treated as the $P$ wave $q\bar{q}$ nonet in the naive quark model, it can be easy understood as the $q^2\bar{q}^2$ nonet, where $\sigma$ has no strange quarks, $\kappa$ has the $s$ quark, $f_0$ and $a_0$ have the $s\bar{s}$ pair. Similar states were found by Jaffe in 1977 in the MIT bag.

5 Radiative Decays of the $\phi$ Meson and the $K^+K^-$ Loop Model

Ten years later we showed that $\phi \rightarrow \gamma a_0 \rightarrow \gamma \pi \eta$ and $\phi \rightarrow \gamma f_0 \rightarrow \gamma \pi \pi$ can shed light on the problem of the $a_0(980)$ and $f_0(980)$ mesons. Now these decays are studied not only theoretically but also experimentally. When basing the experimental investigations, we suggested one-loop model $\phi \rightarrow K^+K^- \rightarrow \gamma a_0/f_0$, see Fig. 3. This model is used in the data treatment and is ratified by experiment, see Fig. 4. Gauge invariance gives the conclusive arguments in favor of the $K^+K^-$ loop transition as the principal mechanism.
of the $a_0(980)$ and $f_0(980)$ meson production in the $\phi$ radiative decays.

6 The $K^+K^-$ Loop Mechanism is Four-Quark Transition \[3\]

In truth this means that the $a_0(980)$ and the $f_0(980)$ are seen in the $\phi$ meson radiative decays owing to the $K^+K^-$ intermediate state. So, the mechanism of the $a_0(980)$ and $f_0(980)$ production in the $\phi$ meson radiative decays is established at a physical level of proof. We are dealing with the four-quark transition. A radiative four-quark transition between two $q\bar{q}$ states requires creation and annihilation of an additional $q\bar{q}$ pair, i.e., such a transition is forbidden by the OZI rule, while a radiative four-quark transition between $q\bar{q}$ and $q^2\bar{q}^2$ states requires only creation of an additional $q\bar{q}$ pair, i.e., such a transition is allowed by the OZI rule. The large $N_C$ expansion supports this conclusion.

7 Scalar Nature and Production Mechanisms in $\gamma\gamma$ collisions \[4\]

Twenty seven years ago we predicted the suppression of $a_0(980) \rightarrow \gamma\gamma$ and $f_0(980) \rightarrow \gamma\gamma$ in the $q^2\bar{q}^2$ MIT model, $\Gamma_{a_0 \rightarrow \gamma\gamma} \sim \Gamma_{f_0 \rightarrow \gamma\gamma} \sim 0.27$ keV. Experiment supported this prediction.

Recently the experimental investigations have made great qualitative advance. The Belle Collaboration published data on $\gamma\gamma \rightarrow \pi^+\pi^-$, $\gamma\gamma \rightarrow \pi^0\pi^0$, and $\gamma\gamma \rightarrow \pi^0\eta$, whose statistics are huge \[5\], see Fig. 5. They not only proved the theoretical expectations based on the four-quark nature of the light scalar mesons, but also have allowed to elucidate the principal mechanisms of these processes. Specifically, the direct coupling constants of the $\sigma(600)$, $f_0(980)$, and $a_0(980)$ resonances with the system are small with the result that their decays into $\gamma\gamma$ are the four-quark transitions caused by the rescatterings $\sigma(600) \rightarrow \pi^+\pi^- \rightarrow \gamma\gamma$, $f_0(980) \rightarrow K^+K^- \rightarrow \gamma\gamma$ and $a_0(980) \rightarrow K^+K^- \rightarrow \gamma\gamma$ in contrast to the $\gamma\gamma$ decays of the classic $P$ wave tensor $q\bar{q}$ mesons $a_2(1320)$, $f_2(1270)$ and $f_2(1525)$, which are caused by the direct two-quark transitions $q\bar{q} \rightarrow \gamma\gamma$ in the main. As a result the practically model-independent prediction of the $q\bar{q}$ model $g_{f_2\gamma\gamma}^2: g_{a_2\gamma\gamma}^2 = 25:9$ agrees with experiment rather well. The two-photon light scalar widths averaged over resonance mass
Systematic error

The presence of a strong ($|\langle f_0 \rangle| \sim 0.19$ keV, $|\langle a_0 \rangle| \sim 0.3$ keV and $|\langle \sigma \rangle| \sim 0.45$ keV. As to the ideal $q\bar{q}$ model prediction $g^2_{f_0\gamma\gamma} : g^2_{a_0\gamma\gamma} = 25 : 9$, it is excluded by experiment.

8 Summary of the Above [1, 3, 4]

(i) The mass spectrum of the light scalars, $\sigma(600)$, $\kappa(800)$, $f_0(980)$, $a_0(980)$, gives an idea of their $q^2\bar{q}^2$ structure. (ii) Both intensity and mechanism of the $a_0(980)/f_0(980)$ production in the $\phi(1020)$ radiative decays, the $q^2\bar{q}^2$ transitions $\phi \rightarrow K^+K^- \rightarrow \gamma|a_0(980)/f_0(980)|$, indicate their $q^2\bar{q}^2$ nature. (iii) Both intensity and mechanism of the scalar meson decays into $\gamma\gamma$, the $q^2\bar{q}^2$ transitions $\sigma(600) \rightarrow \pi^+\pi^- \rightarrow \gamma\gamma$ and $[f_0(980)/a_0(980)] \rightarrow K^+K^- \rightarrow \gamma\gamma$, indicate their $q^2\bar{q}^2$ nature also.

9 The $a_0^0(980) - f_0(980)$ Mixing in Polarization Phenomena [6]

The $a_0^0(980) - f_0(980)$ mixing as a threshold phenomenon was discovered theoretically in 1979 in our work [6]. Now it is timely to study this phenomenon experimentally[1].

The main contribution originates from the $a_0^0(980) \rightarrow (K^+K^- + K^0\bar{K}^0) \rightarrow f_0(980)$ transition, see Fig. 6. Between the $K\bar{K}$ thresholds

$$|\Pi_{a_0f_0}(m)| \approx \frac{|g_{a_0K^+K^-}g_{f_0K^+K^-}|}{16\pi} \sqrt{\frac{2(m_{K^0} - m_{K^+})}{m_{K^0}}} \approx 0.127 \frac{|g_{a_0K^+K^-}g_{f_0K^+K^-}|}{16\pi} \approx 0.03 \text{GeV}^2.$$ 

It dominates for two reasons. i) It has the $\sqrt{m_d - m_u} \sim \sqrt{\alpha}$ order. ii) The strong coupling of the $a_0^0(980)$ and $f_0(980)$ to the $K\bar{K}$ channels, $|g_{a_0K^+K^-}g_{f_0K^+K^-}|/4\pi \simeq 1 \text{GeV}^2$.

We noted in 2004 [6] that the phase jump, see Fig. 6(b), suggest the idea to study the $a_0^0(980) - f_0(980)$ mixing in polarization phenomena. If a process amplitude with a suitable spin configuration is dominated by the $a_0^0(980) - f_0(980)$ mixing then a spin asymmetry of a cross section jumps near the $K\bar{K}$ thresholds. An example is the reaction $\pi^- p_1 \rightarrow (a_0^0(980) + f_0(980)) n \rightarrow a_0^0(980)n \rightarrow \eta\pi^0 n$, see Fig. 7. Performing the polarized target experiments on the reaction $\pi^- p \rightarrow \eta\pi^0 n$ at high energy could unambiguously and very easily establish the existence of the $a_0^0(980) - f_0(980)$ mixing phenomenon through the presence of a strong ($\sim 1$) jump in the normalized azimuthal spin asymmetry of the $S$ wave $\eta\pi^0$ production cross section near the $K\bar{K}$ thresholds. In turn it could give an exclusive information on the $a_0^0(980)$ and $f_0(980)$ coupling constants with the $K\bar{K}$ channels, $|g_{a_0K^+K^-}g_{f_0K^+K^-}|/4\pi$.

1In Ref. [7] the search program of the $a_0^0(980) - f_0(980)$ mixing at the $C/\tau$ factory has been proposed. Recently the VES Collaboration published the data on the first effect of the $a_0^0(980) - f_0(980)$ mixing, $f_1(1420) \rightarrow \pi^0a_0^0(980) \rightarrow \pi^0f_0(980) \rightarrow 3\pi$ [8], in agreement with our calculation 1981 [6].
Figure 6: The “resonancelike” behavior of the modulus (a) and phase (b) of the $a_0^0(980) - f_0(980)$
mixing amplitude $\Pi_{a_0f_0}(m)$.

Figure 7: The spin asymmetry in $\pi^-p_1 \rightarrow (a_0^0(980) + f_0(980))n \rightarrow \eta\pi^0n$.

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