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A discussion of the associated flow rule based on the FAY model and Nakajima tests

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Abstract. Constitutive models based on a non-associated flow rule (non-AFR) have received increased attention due to their flexibility in capturing laboratory experiments. On the other hand, the recently proposed Fourier Asymmetric Yield (FAY) model enables the definition of convex yield functions with arbitrary complexity, which can be used in conjunction to an associated flow rule and nevertheless accurately match experimental data. The present contribution aims comparing AFR and non-AFR based approaches with respect to their ability in capturing measured strain fields in controlled Nakajima experiments. It is shown that a sufficiently flexible yield function is able of delivering accurate results without having to renounce to restrictions such as flow rule association or convexity. The analysis is carried out using two common deep drawing materials, namely an AA6016 and a steel grade DC05.

1. Introduction
The accuracy and efficiency of forming simulations is greatly dependent on the chosen constitutive relationships. Material modelling is therefore a field of study which receives increased attention of the scientific community, especially in case of sheet metal forming, where pronounced anisotropy is expected. Many authors have recently reported significant advantages in accuracy and computational efficiency by the use of a non-associated flow rule (see e.g. [1]). The improvements are mostly argued based on more accurate earing prediction on cup drawing tests. A different approach has been provided by [2], who demonstrated that a non-AFR based approach can ease restrictions in the yld2000-2d model [3] to better capture stress states between plane strain and equi-biaxial. Although plenty of literature exists reporting the mentioned improvements as well as providing theoretical proofs of flow stability (see e.g. [4]) non-associated flow rule based models still do not penetrate the industrial simulation practice. This is primarily due to the well tested and validated robustness of AFR, which is an indirect expression of the fact that plastic deformation is primarily governed by Schmid’s law. It can be argued that this is a too restrictive assumption and that modern materials exhibit more complex deformation mechanisms potentially leading to non-AFR. The experimental evidence presently available is, however, very limited (see e.g. [5]). Furthermore the implications of non-AFR under complex stress and deformation conditions still remains largely unexplored. The present contribution aims revisiting [2] in order to provide further insight on the topic. The principal strain distributions of two different Nakajima test configurations are compared with the well-established yld2000-2d model both based on AFR and non-AFR. Furthermore the recently proposed FAY model is employed to obtain a viable alternative which accurately captures both configurations without having to renounce on AFR.
2. Fourier Asymmetric Yield Model

The used FAY (Fourier Asymmetric Yield) model is shortly reviewed here. Further details, especially about the identification procedure can be found in [6] and [7].

The approach adopted in the development of FAY is what may be called a bottom-up procedure. The only assumption about the yield function is, in fact, that it exists and that is periodic in the chosen parametrization. As it is known that every periodic function can be represented as a Fourier series, it can be concluded that instead of a-priori defining a functional description for the yield function, its Fourier representation can be directly identified by imposing the physical and material dependent restrictions at hand. These can be e.g. as symmetry, orthotropy and convexity as well as available experimental data.

The model is designed by re-parametrizing the space of plane stresses by a particular spherical system of coordinates

\[ r = \sqrt{\frac{\sigma_{xx}}{2} + \frac{\sigma_{yy}}{2} + \tau_{xy}^2}, \quad \varphi = \arccos \left( -\frac{\sigma_{xx} + \sigma_{yy}}{2r} \right) , \quad \psi = \frac{1}{2} \arctan \left( 2\tau_{xy}, (\sigma_{xx} - \sigma_{yy}) \right). \]  

(1)

The parametrization is chosen such that, for a uniaxial tensile or compressive stress state, the polar angle \( \varphi \) is a constant, the radius \( r \) is linearly correlated to the magnitude of the acting stress, and the azimuthal angle \( \psi \) corresponds to the angle between loading direction and rolling direction. A formally compact flow function is defined in terms of the new coordinates

\[ \bar{\sigma}^q = r^q f (\varphi, \psi) \]  

(2)

where \( f \) is a two-dimensional Fourier series of the defined angles

\[ f (\varphi, \psi) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} a_{m,n} \cos (m\varphi) \cos (n\psi) + \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} b_{m,n} \cos (m\varphi) \sin (n\psi) \]

\[ + \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} c_{m,n} \sin (m\varphi) \cos (n\psi) + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} d_{m,n} \sin (m\varphi) \sin (n\psi). \]  

(3)

By adjustment of the Fourier coefficients, virtually any shape of the flow surface can be realized. Specifically, it is possible to introduce anisotropy as well as tension-compression-asymmetry, if needed. Further, by adding more and more non-trivial coefficients, any number of experimental data (flow stresses under different conditions and loading directions, \( R \)-values) could be captured. However, care must be taken to not violate physical constraints such as orthotropy and convexity.

In addition to the 8 experiments used to calibrate yld2000-2d, 3 further variables were considered to calibrate FAY:

- The curvature of the surface at the equi-biaxial point was controlled via the closely related second derivative of the Fourier series with respect to \( \varphi \).
- The position of the plane strain points was adjusted via the radius \( r_{PS} \) and the polar angle \( \varphi_{PS} \). The flow function was calibrated such that the flow surface passes through these points and such that the outward normal lies parallel to the direction of the first principle stress. In order to properly account for anisotropy in plane strain, the radius \( r_{PS} \) has been scaled in the different directions proportionally to the stress ratios in uniaxial tension.

3. Results and Discussion

Two materials, AA6016 and DC05, have been used in this study, which were charcterized by tensile tests in three directions and a bulge test. The Nakajima test configurations with 100mm
width (B100) and 200mm width (B200, full specimen) have been chosen as good representatives of respectively the plane strain and equi-biaxial stress states. The yld2000-2d function has been identified for AFR and non-AFR cases. In the latter case a separate yld2000-2d function has been fitted for the yield locus and the yield potential with respectively the exponents $M_\sigma$ and $M_p$. More details about this procedure can be found in [2]. The simulations have been carried out using LS-Dyna using fully integrated quadrilateral shell elements with 5 integration points through the thickness. The material models have been implemented as a user subroutine (umat) following the explicit return mapping algorithm in [8]. The coefficient of friction has been chosen as 0.0 as special care has been taken to lubrication and the specimens failed centrally. The experimental strain distributions have been measured using the GOM ARAMIS optical measurement system.

3.1. Aluminium Alloy AA6016

The mechanical properties of the considered aluminium alloy can be found in Table 1. The Hockett-Sherby flow curve approximation has been used for the hardening behaviour with the parameters given in Table 2. The exponents of the AFR and non-AFR based yld2000-2d models have been adjusted until reaching a minimal error measure as defined in [2] between the measured and simulated major strain distributions at different stages and summed up for the different configurations (see Eq. 4).

$$\text{error} = 0.5 \sum_{i=1}^{k} \left[ \frac{1}{l} \sum_{j=1}^{l} (\varepsilon_{1,exp}^{(j)} - \varepsilon_{1,sim}^{(j)})^2 \right]_B + 0.5 \sum_{i=1}^{k} \left[ \frac{1}{l} \sum_{j=1}^{l} (\varepsilon_{1,exp}^{(j)} - \varepsilon_{1,sim}^{(j)})^2 \right]_B$$

(4)

Similarly the support points of the FAY models have been adjusted until an optimal configuration was found. Figure 1 gives a detailed overview of the resulting curves. It is seen that the optimally calibrated FAY model delivers equivalent if not slightly better results than the best fit non-AFR model. As also discussed in [2] this sensitivity in the region between equi-biaxial and plane strain stress states is primarily an issue with FCC metals such as aluminium which are characterized by a relatively flat plane strain region. Satisfying this requirement using the yld2000-2d model usually leads to a sharp corner in the biaxial region which may lead to unrealistic results, especially in case of high exponents. The identified models can be seen in Figure 2, whereas Figure 3 shows a detail in the equibiaxial region. It can be noted that the FAY model provides both a flatter region in equi-biaxial stress and is able to reproduce the flat plane strain region.

Table 1: Mechanical properties of AA6016

| $\sigma_0/\sigma_y$ | $\sigma_{45}/\sigma_y$ | $\sigma_{90}/\sigma_y$ | $\sigma_b/\sigma_y$ | $R_0$ | $R_{45}$ | $R_{90}$ | $R_b$ |
|---------------------|-----------------------|----------------------|-------------------|-------|---------|---------|-------|
| 1.000               | 0.965                 | 0.972                | 0.986             | 0.686 | 0.500   | 0.666   | 1.000 |

Table 2: Hockett-Sherby hardening curve parameters as in: $\sigma_y = B - (B - A) \exp(-m\varepsilon^n)$ for AA6016

| $A$    | $B$    | $m$  | $n$  |
|--------|--------|------|------|
| 123.7 MPa | 352.35 MPa | 5.62 | 0.865 |
AFR, $M = 4.5$

AFR, $M = 8$

non-AFR

FAY

exp.

Figure 1: Measured and predicted strain distributions during Nakajima test of AA6016 at strokes of 5 mm, 10 mm, 15 mm, 20 mm and 25 mm; for B200 additionally at 30 mm.

Figure 2: Predicted yield locus, uni-axial flow stresses and $R$-values for AA6016

3.2. DC05

the same approach as in AA6016 has been followed for DC05, with the sole difference that the punch friction coefficient has been chosen as 0.1 for the B200 configuration, in order to match the non-central localization of strains. The mechanical parameters and the Hockett-Sherby hardening parameters can be respectively found in Tables 3 and 4. In this case, it is observed that the yld2000-2d model with $M = 6$ and AFR delivers already results close to optimum. Also the non-AFR optimization delivered the best results for $(M_p = 6, M_p = 6)$ (AFR)). The identification procedure of FAY delivered practically the same yield locus as yld2000-2d (see Figure 4). As far as strain distributions are concerned FAY even delivers slightly worse results. This is however primarily due to the fact that the final considered strain states are very close to localization. This is especially the case for the B100 configuration (see Figure 5). These state is very sensitive to slight changes in the yield surface. It can be therefore concluded for DC05 that yld2000-2d with AFR delivers results which do not improve with additional flexibility.
Figure 3: Detail AA6016

Table 3: Mechanical properties of DC05

| Angle to rolling direction [°] | \( \sigma_{yy}/\sigma_y \) | \( \sigma_{45}/\sigma_y \) | \( \sigma_{90}/\sigma_y \) | \( \sigma_b/\sigma_y \) | \( R_0 \) | \( R_{45} \) | \( R_{90} \) | \( R_b \) |
|-----------------------------|----------------|----------------|----------------|----------------|--------|--------|--------|--------|
| 0                          | 1.02           | 1.04           | 1.06           | 2.00           | 1.47   | 2.52   | 0.85   |

Table 4: Hockett-Sherby hardening curve parameters as in: \( \sigma_y = B - (B - A) \exp(-m \varepsilon^n) \) for DC05

| \( A \) | \( B \) | \( m \) | \( n \) |
|--------|--------|--------|--------|
| 155.9 MPa | 453.6 MPa | 2.60   | 0.680  |

Figure 4: Predicted yield locus, uni-axial flow stresses and \( R \)-values for DC05
4. Summary and Conclusions

Material grades exhibiting complex anisotropy behaviour have brought by the need for ever increasing flexibility in the definition yield functions. A response to this need which recently received significant attention, is the use of non-AFR based models. It has been shown in the literature that the use of this approach can be beneficial where AFR based models reach the limits of their predictive ability. The present work aimed contributing to this discussion by showing that the advantages of non-AFR can be easily reproduced using AFR if the yield surface is appropriately designed. The recently proposed FAY model has been used for this purpose giving its capability to easily adjust its flexibility without significant increases in computational cost. The results in this work, as well as in [7] suggest that for the cases considered a convex yield locus can be found, which deliver an excellent approximation of the experiments, without having to renounce the well-established AFR.

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