Two degrees of freedom parallel linkageto track solarthermal platforms installed on ships

I Visa1, A Cotorcea1, 2 M Moldovan1 and M Neagoe1
1Renewable Energy Systems and Recycling Research Centre, Transilvania University of Brasov, Romania
2Department of Naval and Port Engineering and Management, Mirceacel Batran Naval Academy, Romania

E-mail: visaion@unitbv.ro

Abstract. Transportation is responsible at global level for one third of the total energy consumption. Solutions to reduce conventional fuel consumption are under research, to improve the systems’ efficiency and to replace the current fossil fuels. There already are several applications, usually on small maritime vehicles, using photovoltaic systems to cover the electric energy demand on-board and to support the owners’ commitment towards sustainability. In most cases, these systems are fixed, parallelly aligned with the deck; thus, the amount of solar energy received is heavily reduced (down to 50%) as compared to the available irradiance. Large scale, feasible applications require to maximize the energy output of the solar convertors implemented on ships; using solar tracking systems is an obvious path, allowing a gain up to 35...40% in the output energy, as compared to fixed systems. Spatial limitations, continuous movement of the ship and harsh navigation condition are the main barriers in implementation. This paper proposes a solar tracking system with two degrees of freedom, for a solar thermal platform, based on a parallel linkage with spherical joints, considered as Multibody System. The analytical model for mobile platform position, pressure angles and a numerical example are given in the paper.

1. Introduction

Solar tracking systems functionality consists of changing the platform’s position (supporting solar thermal collectors or photovoltaic modules) to follow the apparent path of the Sun on the sky and thus to increase the amount of input solar radiation and, consequently, the output energy up to 35% comparing to fixed, optimally tilted, systems [1, 2]. The Sun-Earth geometry is described through a pair of angles: solar elevation angle (α) and solar azimuth angle (ψ) as in figure 1.

Figure 1. Solar elevation angle (α) and solar azimuth angle (ψ) describing Sun-Earth geometry.
For ground mounted solar tracking systems, these values range between 0° and 90° for the elevation angle and usually between 0° and 240° for the azimuth angle depending on the day and implementation location [3]. Usually, these motions can be done using serial and parallel mechanisms [1, 4].

According to figure 1, these angles are described in the OXYZ system corresponding to the local horizontal plane, where the OX axis has the East-West direction and positive sense toward the East, the OY axis has the South-North direction and positive sense toward North, and OZ axis perpendicular to the local horizontal plane with upward positive sense for right orthogonal OXYZ system.

The tracking mechanisms for the platforms installed in a concrete and fixed location have the mobility M=2, usually by two successive rotations around two well defined axes, from which one is a fixed axis and the second one is a mobile axis. The diversity of these tracking mechanisms and the required rotation angles of the platform were previously described [1] for all types of tracking mechanisms: azimuthal, pseudo-azimuthal, pseudo-equatorial and equatorial. For all these systems, the rotation angle around the fixed axis is an absolute angle and the rotation angle around the mobile axis is a relative angle. In this case, for each independent rotation a specific mechanism is used (linkage type, gears type, cam type etc.), using as independent driving motions translation (linear actuator) or rotation (rotary actuator). Very often linear actuators are used. A complex solution of this type, based on a four bar mechanism is described in [5] and is sketched in figure 2a.

The tracking mechanisms based only on absolute angles (directly related to the basis) are of parallel linkage type. In this case, both drivers are connected between the basis and the tracked platform. Literature mentions these types of tracking parallel linkages having mobility M=2, as in figures 2b [6], 2c [7] and 2d [8].

![Figure 2. Tracking mechanisms of parallel linkages type with M=2.](image)

For a fixed location, all the previously described mechanisms allow the necessary motions by an optimal design. Solar tracking systems installed on the ships, i.e. with a changing location, must meet additionally requirements defined by the ship motion on different directions and latitudes, and by roll and pitch motions. In this case, the tilt angle and diurnal tracking motion cover a much broader range, and as a result new tracking mechanisms must be considered.

This paper investigates a new type of parallel linkage, with mobility M=2, based on spherical joints and driving linear actuators, to be used as tracking mechanism feasible for solar tracking systems on the ships.

![Figure 3. Proposed parallel tracking mechanism with M=2: CAD model (a) and structural scheme (b).](image)
2. MBS structure of the parallel tracking linkage
The general form of the mechanism (in figure 3) consists of: the basis (1), the mobile platform (2), four sphere-sphere (SS) connections between the two bodies, and two linear actuators connected between the bodies, according to structural synthesis described in [6] by using Multibody System Method (MBS) [9]. The characteristics of the kinematical constraints are presented in table 1.

| Location | Type | Bodies | Kinematic constraints $c_{ij}$ |
|----------|------|--------|-------------------------------|
| AA$_2$   | SS   | 1-2    | 1                             |
| BB$_2$   | SS   | 1-2    | 1                             |
| CC$_2$   | SS   | 1-2    | 1                             |
| DD$_2$   | SS   | 1-2    | 1                             |

Sum of kinematic constraints $\Sigma c_{ij} = 4$

The mechanism mobility ($M$) is calculated using equation (1), for a number of two bodies ($n_b = 2$) in the general space ($S = 6$), having the sum of kinematic constraints $\Sigma c_{ij} = 4$:

$$M = S \cdot (n_b - 1) - \Sigma c_{ij} = 6 \cdot (2 - 1) - 4 = 2$$

(1)

The mechanism mobility is equal with the number of the driving constraints represented by the two linear actuators PP$_2$ and QQ$_2$, in figure 3b.

3. The parallel tracking linkage geometry
To define the geometrical model, two reference systems are used: the system OXYZ of the local horizontal plane of the body 1 and the system O$_2$X$_2$Y$_2$Z$_2$ of the mobile platform 2.

Table 1. The characteristics of the kinematic constraints between bodies.

The indirect connections AA$_2$, BB$_2$, CC$_2$ and DD$_2$ are geometrically defined through the constant lengths between the respective joints $L_{AA2}$, $L_{BB2}$, $L_{CC2}$ and $L_{DD2}$.
4. The position and orientation of the mobile platform
The position and orientation of the platform 2 is described by 6 generalized coordinates: the coordinates of the origin O₂ of the mobile system O₂X₂Y₂Z₂ (of the mobile platform 2) relative to the fixed system OXYZ and the Bryant angles ϕ₁, ϕ₂, and ϕ₃ described in figure 4.

![Figure 4](image)

Figure 4. Bryant angles ϕ₁, ϕ₂, and ϕ₃ describing the orientation of the mobile system O₂X₂Y₂Z₂ of the mobile platform 2 relative to the fixed system OXYZ.

The corresponding rotation matrix are:

\[
[M_{ϕ₁}] = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos ϕ₁ & \sin ϕ₁ \\
0 & \sin ϕ₁ & \cos ϕ₁
\end{bmatrix}
\]

\[
[M_{ϕ₂}] = \begin{bmatrix}
\cos ϕ₂ & 0 & \sin ϕ₂ \\
0 & 1 & 0 \\
-\sin ϕ₂ & 0 & \cos ϕ₂
\end{bmatrix}
\]

\[
[M_{ϕ₃}] = \begin{bmatrix}
\cos ϕ₃ & -\sin ϕ₃ & 0 \\
\sin ϕ₃ & \cos ϕ₃ & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

and the general rotation matrix is:

\[
[M_2] = [M_{ϕ₁}][M_{ϕ₂}][M_{ϕ₃}]
\]

\[
=M_{ϕ₁}M_{ϕ₂}M_{ϕ₃}
\]

\[
= \begin{bmatrix}
\cos ϕ₂ \cos ϕ₃ & -\cos ϕ₂ \sin ϕ₃ & \sin ϕ₂ \\
\cos ϕ₁ \cos ϕ₃ + \sin ϕ₁ \sin ϕ₂ \cos ϕ₃ & \cos ϕ₁ \sin ϕ₃ - \sin ϕ₁ \sin ϕ₂ \sin ϕ₃ & -\sin ϕ₁ \cos ϕ₂ \\
\sin ϕ₁ \sin ϕ₃ - \cos ϕ₁ \sin ϕ₂ \cos ϕ₃ & \sin ϕ₁ \cos ϕ₃ + \cos ϕ₁ \sin ϕ₂ \sin ϕ₃ & \cos ϕ₁ \cos ϕ₂
\end{bmatrix}
\]

The elevation αₙ and azimuthal ψₙ angles (figure 5) are calculated with equation (4).

\[
αₙ = 90° - θₙ = 90° - \cos^{-1}(n_z; \ Ψₙ = \tan^{-1}\frac{n_y}{n_x})
\]

wherenₓ, nᵧ, nₜ are the direction cosines of O₂Z₂ axis in OXYZ system (the last column in matrix 3).

![Figure 5](image)

Figure 5. Elevation and azimuthal angles of the mobile platform.
5. The position functions

The system of the position functions (equation 5) consists of four equations describing the geometrical constraints (equations F1 to F4) and two driving constraints equations (equations F5 and F6). Thus, the system has 6 equations and 6 unknowns: \(X_{O2}, Y_{O2}, Z_{O2}, \phi_1, \phi_2, \phi_3\).

The general form of the equations is:

\[
F_1: (X_A - X_{A2})^2 + (Y_A - Y_{A2})^2 + (Z_A - Z_{A2})^2 - L_{AA2}^2 = 0
\]

\[
F_2: (X_B - X_{B2})^2 + (Y_B - Y_{B2})^2 + (Z_B - Z_{B2})^2 - L_{BB2}^2 = 0
\]

\[
F_3: (X_C - X_{C2})^2 + (Y_C - Y_{C2})^2 + (Z_C - Z_{C2})^2 - L_{CC2}^2 = 0
\]

\[
F_4: (X_D - X_{D2})^2 + (Y_D - Y_{D2})^2 + (Z_D - Z_{D2})^2 - L_{DD2}^2 = 0
\]

\[
F_5: (X_P - X_{P2})^2 + (Y_P - Y_{P2})^2 + (Z_P - Z_{P2})^2 - L_{PP2}^2 = 0
\]

\[
F_6: (X_Q - X_{Q2})^2 + (Y_Q - Y_{Q2})^2 + (Z_Q - Z_{Q2})^2 - L_{QQ2}^2 = 0
\]

The system described by (5) is nonlinear and can be solved by numerical methods, usually by Newton-Raphson algorithm. The mechanism configuration for which the mobile platform 2 is parallel with the fixed platform 1 is considered as initial solution.

In this case,

\[
\begin{align*}
(X_{O2})_0 &= 0, (Y_{O2})_0 = 0, (Z_{O2})_0 = OO_2, (\phi_1)_0 = 0, (\phi_2)_0 = 0, (\phi_3)_0 = 0, \\
(L_{PP2})_0 &= \sqrt{(X_P - X_{P2})^2 + (Y_P - Y_{P2})^2 + (Z_P - Z_{P2})^2} \\
(L_{QQ2})_0 &= \sqrt{(X_Q - X_{Q2})^2 + (Y_Q - Y_{Q2})^2 + (Z_Q - Z_{Q2})^2} \\
X_{P2} &= X^{(2)}_{P2}, Y_{P2} = Y^{(2)}_{P2}, Z_{P2} = OO_2 + Z^{(2)}_{P2}, X_{Q2} = X^{(2)}_{Q2}, Y_{Q2} = Y^{(2)}_{Q2}, Z_{Q2} = OO_2 + Z^{(2)}_{Q2}
\end{align*}
\]

The coordinates of the points \(A_2, B_2, C_2\) and \(D_2\) in the system (5) are given by:

\[
\begin{align*}
X_{A2} &= X_{O2} + [M_2] X^{(2)}_{A2}, & Y_{A2} &= Y_{O2} + [M_2] Y^{(2)}_{A2}, & Z_{A2} &= Z_{O2} + [M_2] Z^{(2)}_{A2} \\
X_{B2} &= X_{O2} + [M_2] X^{(2)}_{B2}, & Y_{B2} &= Y_{O2} + [M_2] Y^{(2)}_{B2}, & Z_{B2} &= Z_{O2} + [M_2] Z^{(2)}_{B2} \\
X_{C2} &= X_{O2} + [M_2] X^{(2)}_{C2}, & Y_{C2} &= Y_{O2} + [M_2] Y^{(2)}_{C2}, & Z_{C2} &= Z_{O2} + [M_2] Z^{(2)}_{C2} \\
X_{D2} &= X_{O2} + [M_2] X^{(2)}_{D2}, & Y_{D2} &= Y_{O2} + [M_2] Y^{(2)}_{D2}, & Z_{D2} &= Z_{O2} + [M_2] Z^{(2)}_{D2}
\end{align*}
\]

Thus, the system (5) has the general form

\[
[F_1(X_{O2}, Y_{O2}, Z_{O2}, \phi_1, \phi_2, \phi_3, t)] = 0
\]

where: time \(t\) is under explicit form in the equations corresponding to the driving constraints:

\[
L_{PP2} = L_{PP2\min} + a_1 \cdot t_1, \quad L_{QQ2} = L_{QQ2\min} + a_2 \cdot t_2
\]

and \(a_1, a_2\) are the linear actuators velocities in mm/s.

For the initial configuration:

\[
(t_1)_0 = \frac{(t_{PP2})_0 - L_{PP2\min}}{a_1}, \quad (t_2)_0 = \frac{(t_{QQ2})_0 - L_{QQ2\min}}{a_2}
\]
For the next configurations:
\[ t_1 = (t_1)_0 \pm \Delta t_1, \quad t_2 = (t_2)_0 \pm \Delta t_2 \] (14)

considering "+" for increasing \( L_{PP_2}, L_{QQ_2} \) and "-" for decreasing \( L_{PP_2}, L_{QQ_2} \).

The incident angle defined as the angle between the sunray direction (figure 1) and the normal to the mobile platform (direction \( O_2Z_2 \)) is required to assess the tracking efficiency. This angle is given by:
\[ \nu = \cos^{-1}\left(\cos \alpha \cdot \cos \alpha_n \cdot \cos(\psi - \psi_n) + \sin \alpha \cdot \sin \alpha_n\right) \] (15)

where \( \alpha_n \) and \( \psi_n \) are the elevation and diurnal angles of the mobile platform 2.

6. The velocity functions
To evaluate the pressure angle, the velocities of the points \( P_2 \) and \( Q_2 \) are required. These are given by:
\[
\begin{align*}
\dot{X}_{P_2} &= [\dot{X}_{O_2}, \dot{Y}_{O_2}, \dot{Z}_{O_2}] + [M_2] \dot{X}_{P_2}^{(2)}, \\
\dot{Y}_{P_2} &= [\dot{X}_{O_2}, \dot{Y}_{O_2}, \dot{Z}_{O_2}] + [M_2] \dot{Y}_{P_2}^{(2)}, \\
\dot{Z}_{P_2} &= [\dot{X}_{O_2}, \dot{Y}_{O_2}, \dot{Z}_{O_2}] + [M_2] \dot{Z}_{P_2}^{(2)}
\end{align*}
\] (16)

where the generalized velocities:
\[ \dot{q} = [\dot{X}_{O_2}, \dot{Y}_{O_2}, \dot{Z}_{O_2}, \dot{\phi_1}, \dot{\phi_2}, \dot{\phi_3}]^T \] (17)
can be obtained from the first derivative of the equations in system (4) relative to time, which leads to the general form:
\[ [J][\dot{q}] = -\left[\frac{\partial F}{\partial t}\right] \] (18)

where \([J]\) is the Jacobian of the system (4), and \[\frac{\partial F}{\partial t}\] has the form as given by (19):
\[\left[\frac{\partial F}{\partial t}\right] = [0 \ 0 \ 0 \ -2a_1L_{PP_2} \ -2a_2L_{QQ_2}]^T \] (19)

In (16), \([M_2]\) is given by:
\[ [M_2] = \frac{\partial [M_1]}{\partial \phi_1} \phi_1 + \frac{\partial [M_1]}{\partial \phi_2} \phi_2 + \frac{\partial [M_1]}{\partial \phi_3} \phi_3 \] (20)

7. The pressure angles
The pressure angles (\( \beta_p \) and \( \beta_q \)) can be obtained from the scalar product of the actuator direction and absolute velocity vectors of the points of interest \( P_2 \) and \( Q_2 \):
\[ \beta_p = \cos^{-1}\left(\frac{PP_2 \cdot \bar{v}_{P_2}}{|PP_2| |\bar{v}_{P_2}|}\right) \] (21)

where:
\[ PP_2 = (X_P - X_{P_2})i + (Y_P - Y_{P_2})j + (Z_P - Z_{P_2})k \] (22)
\[ \bar{v}_{P_2} = \dot{X}_{P_2}i + \dot{Y}_{P_2}j + \dot{Z}_{P_2}k \] (23)
\[ |PP_2| = \sqrt{(X_P - X_{P_2})^2 + (Y_P - Y_{P_2})^2 + (Z_P - Z_{P_2})^2} \] (24)
8. Numerical example
This parallel tracking mechanism was virtually prototyped in SolidWorks, and two motion studies are further presented, for the mechanism positioning the mobile platform from the initial configuration (figure 6a – horizontal position) towards East (figure 6b) and, respectively towards West (figure 6c).

![Figure 6. Parallel tracking mechanism positions: initial (a), East (b) and West (c).](image)

The geometrical model of the proposed parallel tracking mechanism is presented in table 3.

| Body | Cartesian coordinates |
|------|-----------------------|
| Body 1 | \[
\begin{bmatrix}
X_A \\
Y_A \\
Z_A \\
\end{bmatrix} = \begin{bmatrix}
-800 \\
-800 \\
150 \\
\end{bmatrix},
\begin{bmatrix}
X_B \\
Y_B \\
Z_B \\
\end{bmatrix} = \begin{bmatrix}
-800 \\
800 \\
150 \\
\end{bmatrix},
\begin{bmatrix}
X_C \\
Y_C \\
Z_C \\
\end{bmatrix} = \begin{bmatrix}
800 \\
800 \\
150 \\
\end{bmatrix},
\begin{bmatrix}
X_D \\
Y_D \\
Z_D \\
\end{bmatrix} = \begin{bmatrix}
800 \\
-800 \\
150 \\
\end{bmatrix},
\begin{bmatrix}
X_P \\
Y_P \\
Z_P \\
\end{bmatrix} = \begin{bmatrix}
0 \\
-800 \\
150 \\
\end{bmatrix},
\begin{bmatrix}
X_Q \\
Y_Q \\
Z_Q \\
\end{bmatrix} = \begin{bmatrix}
800 \\
0 \\
150 \\
\end{bmatrix}
\end{bmatrix},
\end{align}
\]

Body 2
\[
\begin{align}
[A^{(2)}_2] & = \begin{bmatrix}
X_{A_2}^{(2)} \\
Y_{A_2}^{(2)} \\
Z_{A_2}^{(2)} \\
\end{bmatrix} = \begin{bmatrix}
-550 \\
-550 \\
-150 \\
\end{bmatrix},
[B^{(2)}_2] = \begin{bmatrix}
X_{B_2}^{(2)} \\
Y_{B_2}^{(2)} \\
Z_{B_2}^{(2)} \\
\end{bmatrix} = \begin{bmatrix}
-550 \\
550 \\
-150 \\
\end{bmatrix},
[C^{(2)}_2] = \begin{bmatrix}
X_{C_2}^{(2)} \\
Y_{C_2}^{(2)} \\
Z_{C_2}^{(2)} \\
\end{bmatrix} = \begin{bmatrix}
550 \\
550 \\
-150 \\
\end{bmatrix},
[D^{(2)}_2] = \begin{bmatrix}
X_{D_2}^{(2)} \\
Y_{D_2}^{(2)} \\
Z_{D_2}^{(2)} \\
\end{bmatrix} = \begin{bmatrix}
550 \\
-550 \\
-150 \\
\end{bmatrix},
[P^{(2)}_2] = \begin{bmatrix}
X_{P_2}^{(2)} \\
Y_{P_2}^{(2)} \\
Z_{P_2}^{(2)} \\
\end{bmatrix} = \begin{bmatrix}
-335 \\
-550 \\
-150 \\
\end{bmatrix},
[Q^{(2)}_2] = \begin{bmatrix}
X_{Q_2}^{(2)} \\
Y_{Q_2}^{(2)} \\
Z_{Q_2}^{(2)} \\
\end{bmatrix} = \begin{bmatrix}
550 \\
335 \\
-150 \\
\end{bmatrix}
\end{align}
\]

The position of the mobile platform 2 during the motion from horizontal towards the East and West positions are defined through the coordinates of its reference system origin $O_2$ and Bryant angles presented in table 4 and 5, respectively along with its elevation and azimuth. As driving motions are considered the linear actuator velocities $a_1=75\text{mm/s}$ and $a_2=15\text{mm/s}$.

\[|v_{P_2}| = \sqrt{X_{P_2}^2 + Y_{P_2}^2 + Z_{P_2}^2}\] (25)
Table 4. Positions of the mobile platform during its motion from horizontal to East position.

| Time \( t \) [s] | Platform origin coordinates | Bryant angles \( \phi_1 \) [°] | \( \phi_2 \) [°] | \( \phi_3 \) [°] | Elevation \( \alpha_n \) [°] | Azimuth \( \psi_n \) [°] |
|-----------------|----------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0               | X \(_{O2}\) [mm] 0.00 | Y \(_{O2}\) [mm] 0.00 | Z \(_{O2}\) [mm] 1375.74 | 0.00 | 0.00 | 0.00 | 90.00 | 90.00 |
| 1               | 54.48          | 20.72          | 1357.25          | 0.69 | 11.77 | 1.80 | 88.22 | 83.92 |
| 2               | 28.14          | 28.82          | 1293.97          | 0.98 | 25.76 | 2.12 | 88.05 | 78.38 |
| 3               | -4.11          | -20.49         | 1195.79          | 0.25 | 37.96 | -0.72 | 89.40 | 71.81 |
| 4               | -26.11         | -113.35        | 1060.28          | -4.47 | 48.03 | -11.13 | 81.87 | 65.95 |
| 5               | -31.26         | -235.13        | 869.58           | -18.17 | 52.32 | -37.01 | 65.37 | 62.00 |

Table 5. Positions of the mobile platform during its motion from horizontal to West position.

| Time \( t \) [s] | Platform origin coordinates | Bryant angles \( \phi_1 \) [°] | \( \phi_2 \) [°] | \( \phi_3 \) [°] | Elevation \( \alpha_n \) [°] | Azimuth \( \psi_n \) [°] |
|-----------------|----------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0               | X \(_{O2}\) [mm] 0.00 | Y \(_{O2}\) [mm] 0.00 | Z \(_{O2}\) [mm] 1375.74 | 0.00 | 0.00 | 0.00 | 90.00 | -90.00 |
| 1               | 200.28         | 49.94          | 1347.77          | 0.79 | 6.14 | 5.49 | 84.53 | -86.54 |
| 2               | 171.33         | 164.81         | 1295.53          | 2.04 | 19.29 | 9.06 | 81.33 | -80.49 |
| 3               | 135.05         | 226.77         | 1214.55          | 4.39 | 29.64 | 14.62 | 76.87 | -75.08 |
| 4               | 102.42         | 257.43         | 1119.34          | 7.94 | 37.75 | 21.54 | 72.07 | -70.60 |
| 5               | 73.90          | 269.93         | 1013.83          | 12.81 | 44.00 | 29.90 | 67.05 | -66.85 |

Based on the coordinates of the linear actuators joints \( P_2 \) and \( Q_2 \), the lengths of the linear actuators during the motion from horizontal to East position are presented in table 6.

Table 6. Linear actuators joints coordinates on the mobile platform, and linear actuators lengths variation during the motion from horizontal to the East position.

| Time \( t \) [s] | Linear actuators joints coordinates \([mm]\) | Linear actuators lengths \([mm]\) |
|-----------------|---------------------------------|---------------------------------|
|                 | \( X_{P2} \) | \( Y_{P2} \) | \( Z_{P2} \) | \( X_{Q2} \) | \( Y_{Q2} \) | \( Z_{Q2} \) | \( L_{PP2} \) | \( L_{QQ2} \) |
| 0               | -313.6 | -512.2 | 1185.8 | 521.8 | 345.1 | 1180.9 | 1019.4 | 1056.8 |
| 1               | -158.5 | -596.8 | 1155.7 | 496.0 | 403.3 | 1177.9 | 944.4 | 1081.8 |
| 2               | -52.2  | -639.8 | 1093.1 | 356.5 | 477.6 | 1119.5 | 869.4 | 1106.8 |
| 3               | 30.1   | -601.2 | 1005.3 | 208.0 | 568.9 | 1012.6 | 794.4 | 1131.8 |
| 4               | 95.5   | -508.3 | 888.4  | 69.1  | 668.8 | 851.4  | 719.4 | 1156.8 |
| 5               | 137.5  | -391.6 | 728.7  | -56.0 | 761.5 | 618.1  | 644.4 | 1181.8 |

Based on the position and velocity vectors of the points of interest \( P_2 \) and \( Q_2 \), the pressure angles in these joints are evaluated and the values are presented in table 7.
Table 7. Velocity and pressure angles of the joints P2 and Q2 during the motion from horizontal to the East position.

| Time t [s] | Velocities of the joints P2 and Q2 [mm/s] | Pressure angles [°] |
|------------|-----------------------------------------|-------------------|
|            | \( \dot{x}_{P2} \) | \( \dot{y}_{P2} \) | \( \dot{z}_{P2} \) | \( v_{P2} \) | \( \dot{x}_{Q2} \) | \( \dot{y}_{Q2} \) | \( \dot{z}_{Q2} \) | \( v_{Q2} \) | \( \beta_{P2} \) | \( \beta_{Q2} \) |
| 0          | 185.0 | -41.7 | -21.7 | 190.8 | 80.4 | 80.4 | 26.2 | 116.7 | 66.9 | 74.1 |
| 1          | 125.4 | -88.1 | -45.7 | 159.9 | -112.7 | -112.7 | -32.3 | 162.6 | 62.1 | 74.7 |
| 2          | 91.9 | 2.2 | -76.2 | 119.4 | -150.2 | -150.2 | -82.8 | 228.0 | 51.1 | 70.0 |
| 3          | 73.7 | 70.1 | -100.3 | 142.9 | -144.7 | -144.7 | -132.0 | 243.5 | 58.3 | 66.5 |
| 4          | 56.1 | 111.2 | -136.4 | 184.7 | -132.6 | -132.6 | -193.7 | 269.5 | 66.0 | 66.1 |
| 5          | 22.9 | 112.4 | -180.7 | 214.1 | -117.1 | -117.1 | -275.3 | 321.3 | 69.5 | 72.0 |

The results for the elevation angle (\( \alpha_n \)), azimuthal angle (\( \psi_n \)) and pressure angle (\( \beta_{P2}, \beta_{Q2} \)) depend on the mechanism geometry, driving actuators position and velocity. Numerical results validate the linkage analytical model. An optimum design based on these parameters is required in the next step to allow the necessary positions of the mobile platform with acceptable pressure angles.

9. Conclusions
A new mechanism, of parallel linkage type, is proposed to track solar thermal platforms installed on ships. The mechanism has four connections SS type between the mobile platform and the basis and two driving linear actuators connected to the basis and mobile platform (two absolute motions). This mechanism allows to get the necessary position of the mobile platform according to the specific motions of the ships on different latitudes. The paper describes the analytical model and validates it through a numerical example. As a next step, the mechanism synthesis is necessary to get the optimal collection efficiency of the available solar radiation (defined by the incident angle between the Sunray and the normal to the mobile platform) and the optimum mechanism functionality (pressure angles and actuator strokes, relative motions in the spherical joints, loads).

10. References
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Acknowledgments
We hereby acknowledge the structural funds project PRO-DD (POS-CCE, O.2.2.1., ID 123, SMIS 2637, contract no. 11/2009) for providing the infrastructure used in this work and the project PNII Cooperation Project, EST in URBA no. 28/2012 financially supported by the Romanian National Research Council.