Particle renormalizations in presence of dissipative environments

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We study the Aharonov-Bohm oscillations of a charged particle on a ring of radius $R$ coupled to a dirty metal environment. With Monte-Carlo methods we evaluate the curvature of these oscillations which has the form $1/M^* R^2$, where $M^*$ is an effective mass. We find that at low temperatures $T$ the curvature approaches at large $R > l$ an $R$ independent $M^* > M$, where $l$ is the mean free path in the metal. This behavior is also consistent with perturbation theory in the particle - metal coupling parameter. At finite temperature $T$ we identify dephasing lengths that scale as $T^{-1/4}$ at $R > l$ and as $T^{-1/2}$ at $R \ll l$.

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I. INTRODUCTION

The problem of interference in presence of a dissipative environment is fundamental for a variety of experimental systems. Interference has been monitored by Aharonov-Bohm (AB) oscillations in mesoscopic rings[1,2,3] or in quantum Hall edge states[4,5,6,7] in presence of noise from gates or other metal surfaces . Cold atoms trapped by an atom chip are sensitive to the noise produced by the chip[8,9,10]. In particular giant Rydberg atoms are studied[8,9,10] whose huge electric dipole is highly susceptible to such noise.

An efficient tool for monitoring the effect of the environment, as proposed by Guinea[9], is to find the AB oscillation amplitude as function of the radius $R$ of the ring. This amplitude is measured by the curvature of the oscillation amplitude as function of the radius $R$ of the ring. This amplitude is measured by the curvature

$$S^{(m)} = \frac{1}{2} M R^2 \int_0^{1/T} \left( \frac{\partial \theta}{\partial \tau} + 2 \pi m T \right)^2 d\tau + \alpha \int_0^{1/T} \int_0^{1/T} \frac{\pi T^2 K[\theta(\tau) - \theta(\tau')] + 2 \pi m T (\tau - \tau')^2}{\sin^2 \pi T (\tau - \tau')} d\tau d\tau'$$

(1)

where the effect of environments, in each of the 3 cases, is negligible.

$$K(z) = \begin{cases} \sin^2 \frac{z}{2}; & \alpha = \frac{3}{8k_F^2} \sqrt{\gamma} R^2 \quad (i) \\ 1 - \left[4r^2 \sin^2 \frac{z}{2} + 1\right]^{-1/2}; & \alpha = \frac{3}{8k_F^2} \sqrt{\gamma} \quad (ii) \\ 1 - \left[4r^2 \sin^2 \frac{z}{2} + 1\right]^{-3/2}; & \alpha = \frac{3}{8k_F^2} \sqrt{\gamma} \quad (iii) \end{cases}$$

(2)

Case (i) is the CL system where $\gamma$ is the coupling to a harmonic oscillator bath; case (ii) is a charge coupled to a dirty metal where $k_F$ is the Fermi wavevector, $l$ is the mean free path in the metal, and $r = R/l$; case (iii) is an electric dipole of strength $p$ coupled to a dirty metal.

We note that the forms (ii) and (iii) are based[13,14] on a wavevector and frequency dependent dielectric function for the metal of the form $\epsilon(q, \omega) = 1 + 4 \pi \sigma / (-i \omega + D q^2)$ valid at $q < 1/l$, where $\sigma$ is the conductivity and $D$ is the diffusion constant of the metal. The $q$ integrals are cutoff...
by \( q < 1/t \), hence the the forms (ii) and (iii) are valid at \( r \geq 1 \). We will use below these forms also at \( r < 1 \) since they represent qualitatively the decrease of \( K(z) \) with \( r \). Furthermore, at \( r \rightarrow 0 \) the form (ii) reduces to that of the CL model (i) with \( \alpha_{CL} = 2\nu^2 \).

We also note that in model (ii) \( \alpha < 1 \) for relevant metals. However, model (iii) allows for a large \( \alpha \) since the dipole parameter \( p \) can be large, as e.g. in the Rydberg atoms.

We are interested in the effect of the environment on the visibility of quantum interference as measured by the transform yield

\[
\omega \alpha = 0, \quad \text{for which}
\]

where

\[
\tau \equiv 0, \quad \text{by}
\]

in perturbation theory and to the need for either RG or variational treatments, or an equivalent variational scheme.

The cutoff \( \omega_c \) replaces a possibly higher environment cutoff, since significant renormalizations start only below \( \omega_c \) where the linear \( |\omega| \) dispersion leads to ln\( \omega \) terms in perturbation theory and to the need for either RG treatment, or an equivalent variational scheme. Note that \( K''(0) = \frac{1}{2}r^2/3r^2 \) in the 3 models above, hence \( \omega_c = \pi r/M \) in case (i), while \( \omega_c \sim \alpha/Mt^2 \) in cases (ii) and (iii).

### III. MONTE CARLO PROCEDURE

For the MC numerical method we need to discretize the time axis into a Trotter number \( N_T \) of segments, i.e. the time interval of each segment is \( \Delta \tau = 1/(TN_T) \). The discrete action is

\[
S^{(m)} = \frac{1}{2}\left[ MR^2 N_T + \alpha K''(0)\right] \sum_n (\theta_{n+1} - \theta_n + \frac{2\pi m}{N_T})^2
\]

\[+ \frac{\alpha \pi^2}{N_T^2} \sum_{n \neq n'} K(\theta_n - \theta_{n'}) + 2\pi m(n - n')/N_T \].

The \( \frac{1}{2} \alpha K''(0) \) term comes from the \( n = n' \) interaction term by expanding \( K(z) \) around \( z = 0 \). A key issue in our MC study is the choice of energy cutoff \( 1/\Delta \tau \) and the corresponding Trotter number \( N_T = 1/(T \Delta \tau) \). The correct choice is such that the free kinetic term dominates over the single \( n = n' \) interaction term, i.e. \( N_T \gg \omega_c/T \), with \( \omega_c \) from Eq. (6). Hence \( \Delta T \approx 1/\omega_c \) corresponds to the cutoff \( \omega_c \) as identified by RG or variational methods. A previous MC study on the charge problem has chosen \( N_T \) in the range \( 1/t \) to \( 4/t \), i.e. an energy cutoff of \( \approx 1/MR^2 \). For large \( r \) this cutoff is much smaller than \( \omega_c \) and is therefore insufficient.

Eqs. (11) identify \( 1/M^*(T)R^2 = 2\pi^2 T(m^2)|_{\phi_r=0} \) or that the MC evaluates the fluctuations in winding number \( \langle m^2 \rangle \) at external flux \( \phi_r = 0 \). The procedure is to start with some \( m \), update \( \theta_n \) at a time position \( n \) to \( \theta_n' \) and accept or reject the change according to the MC rule with probability \exp[S(m)|\{\theta_n\} - S(m')|\{\theta_n'\}] \). After the \( N_T \) points are successively updated, the winding number is shifted to \( m' = m + 1 \) and the shift is accepted or rejected with the probability \exp[S(m)|\{\theta_n\} - S(m')|\{\theta_n\}] \). An update of \( \theta_n \) is done randomly with a step size that produces an acceptance ratio of about 50%.

The inset in Fig. 1 shows the \( N_T \) dependence of \( M/M^* \) for the charge problem with \( r = 5, t = 0.2, \alpha = 0.019 \). A choice for \( N_T \) in the range \( 1/t \) to \( 4/t \) is clearly insufficient; saturation sets in around \( N_T \approx 100 \) which is of order of \( \omega_c/T = 30 \). In the following we choose our \( N_T \), in the charge problem, to be \( N_T = 40 \alpha r^2/t = 10 \omega_c/(\pi T) \), i.e. \( N_T = 95 \) for the inset parameters. For the dipole case, where \( \omega_c \) is 3 times higher we choose \( N_T = 120 \alpha r^2/t = 10 \omega_c/(\pi T) \). Fig. 1 shows that for \( r = 5, t = 0.2, \alpha = 0.02 \) (red squares) saturation indeed sets in near \( N_T = 300 \).

This high value of \( N_T \) restricts realistic MC studies. We have noticed, however, that this high \( N_T \) is necessary only in the vicinity of \( n = n' \) in the double sum of (7), where the summand is rapidly varying. Hence the double sum is taken over all points only in the vicinity of the singularity, i.e. for \( |n - n'| < 0.03N_T \). For points that are further separated we coarse grain the sum with fewer points, corresponding to an effective \( N_T = 1/t \).

The results of this procedure are shown by the green circles in Fig. 1, and are in agreement with the full calculation that includes all \( N_T \) points. The double sum has then \( \approx \frac{1}{2}10^{-3} N_T^2 + \frac{1}{2}t^{-2} \) terms, much less than the \( \frac{1}{2} N_T^2 \) terms of the full calculation. We also show data where the double sum is coarse grained at all points, including those near \( n = n' \), by blue triangles. Here the double sum has only \( \frac{1}{2}t^{-2} \) terms; this data has significant deviations from the full calculation.

We proceed to discuss our error estimates. At low temperatures we evaluate \( \langle m^2 \rangle \), and the average involves typically many values of \( m \). To estimate errors we evaluate the correlation function for a given run and deduce a correlation length \( \xi \). We discard the initial \( 10^4 \) MC iter-
and small $\alpha$ first order in $\theta$ (Appendix I). The perturbation is formally to be determined by the double sum Eq. (2) – red squares, (ii) For points $|n - n'| > 0.03N_T$, sum is coarse grained (see text) – green circles, (iii) the whole sum is coarse grained – blue triangles. Inset: The charge case with $r = 5, t = 0.2, \alpha = 0.019$ using all $N_T$ points in the sums.

At high temperatures $t > 1$, where $M/M^* \lesssim 10^{-3}$, the probability of $m \neq 0$ becomes extremely small so that just $m = \pm 1$ determine the outcome. Hence we evaluate $\langle m^0 \rangle = 2(e^{S_1 - S_0})_0$, averaging with $e^{-S_0}$. In this method we find a rather long correlation length of $\sim 10^5$, yet there is no need to vary $m$ and a 2% accuracy can be achieved after $\approx (1 - 2) \cdot 10^5$ iterations.

IV. MC RESULTS

We present here our data for the dirty metal, system (ii). In Fig. 2 we show our data for $\alpha = 0.019$ at low temperatures, $t < 0.3$; we note saturation at $t < 0.2$. In Fig. 3 we collect the limiting low $t$ values of our data for various alpha, typically achieved at $t \approx 0.1 - 0.01$. The data is limited to Trotter numbers $N_T = 40\alpha r^2/t < 9000$.

We compare in Fig. 3 the data with results of perturbation theory (Appendix I). The perturbation is formally first order in $\alpha$, however, it should be valid also for large $\alpha$ and small $r$ such that $x \lesssim 2$, where at $t = 0$ we define $x = M^*(t = 0)/M$. The perturbation curves are a good fit to the data for $r \lesssim 1$, while at $r > 1$ and small $\alpha$ the fit is qualitatively good, in the sense that saturation is achieved at large $r$. We have also attempted to fit these data by a scaling function of the form $x = 1 + r^{2-c}g(\alpha r^c)$, that is consistent with the $r \to 0$ form of the perturbation expansion. In particular, this form with $c = 2$ would scale onto the CL system at $r \to 0, \alpha \to \infty$. However, we could not find a satisfactory fit even for the small $r \lesssim 2$ regime.

Our data shows for the lowest $\alpha = 0.019$ and for $r \geq 3$
that $M/M^*$ reaches saturation with $M/M^* \approx 0.9$, almost independent of $r$. The data at $r = 20$ (shown in Fig. 2) is consistent with this saturation, though it is not shown in Fig. 3 to keep a convenient scale. In view of this saturation at $3 < r < 20$ we expect it to persist at higher $r$. In terms of $M^* \sim r^\mu$, our data shows that $\mu \lesssim 0.05$ and is consistent with $\mu = 0$. We note that with our revised values of $N_T$ we were not able to reach a saturation regime at larger $\alpha$, see Fig. 3.

Our result shows that the AB curvature $\sim 1/R^2$ is the same as for free particles, i.e. the ground state has no anomaly, at least for weak $\alpha = 0.019$. Furthermore, Fig. 2 shows that $M^*$ determines the finite temperature behavior, as long as $T \ll \omega_c$. Thus if we replace $M \to M^* = M/0.9$ in Eq. (4) we obtain the lower curve $0.9f(t/0.9)$ in Fig. 2 which is a good fit to the data. The thermal length is then $L_T \sim 1/\sqrt{M^*T}$.

In Fig. 4 we show our $r \geq 3$ data up to $t = 2$. The data falls in between two lines: $0.9f(t/0.9)$ and $f(t)$. The lower curve $0.9f(t/0.9)$ corresponds to the renormalized system and fits data with $T \ll \omega_c$, i.e. $t \ll 4\pi\alpha r^2$. For a fixed $t$ as $r$ decreases $T$ approaches $\omega_c$ and the data approaches the upper curve which is the unrenormalized free particle form $f(t)$.

We therefore parameterize our data by a function $x(r, t)$ such that $M/M^* = f(tx)/x$. In this way we avoid the obvious $t$ dependence associated with mass renormalization and focus on additional temperature effects. In Fig. 5 we show that for $r \geq 1$ the data for $x(t, r)$ scales with $t/r$. Since $t \sim T R^2$ the scaling parameter is $\sim T R$, identifying a length scale $\sim 1/T$. A dephasing length scale has been recently derived in a non-equilibrium study\cite{non_eq} which for $r \geq 1$ indeed scales with $1/T$. We propose therefore that the additional $T$ dependence embedded in our variable $x(t, r)$ is related to dephasing of the non-equilibrium situation.

We note that the perturbation expansion yields for $r \gg 1$,

$$\frac{M}{M^*} = 1 - 4\alpha + O\left(\frac{\alpha t}{r} \ln r\right) \quad r \gg 1. \quad (8)$$

While the dependence on $t/r$ is consistent with Fig. 5 (up to a $\ln r$ factor), we note that the $t/r$ form in the perturbation form (8) is valid only at $t \ll 1$ and $r \gtrsim 10$. Hence the observed scaling, Fig. 5, with $t/r$ up to $t \approx 1$ and at $3 < r < 20$ is an unexpected feature.

In Fig. 6 we show that for $r \ll 1$ the data scales as $t r^2$. At $t r^2 \lesssim 0.04$ both $x(t, r)$ and $x(0, r)$ are close to 1 and the errors in $1/x(t, r) - 1/x(0, r)$ are too large to draw a conclusion in this regime. The same difficulty is with all data of small $\alpha$, hence Fig. 6 shows only $\alpha = 0.2, 1$. At $t r^2 \gtrsim 0.04$ the data in Fig. 6 supports a $t r^2$ scaling. Since $t \sim T R^2$ this implies a length scale $\sim T^{-1/4}$. We note again that similar dependence for a dephasing length was found for $r \ll 1$ in the non-equilibrium study\cite{non_eq}.

For $r \ll 1$ we can use the perturbation result Eq. (A12)

$$\frac{M}{M^*} = 1 - 2\alpha \sum n a_n + 4\alpha t r^2 \quad r \ll 1. \quad (9)$$

This shows the $\alpha^2$ scaling at $t r^2 \ll 1$. It is remarkable that our data in Fig. 6 supports $\alpha r^2$ scaling up to rather high temperatures of $t \lesssim 1$.

As noted above, the $r$ dependence of $K(z)$ is reliable only at $r \gtrsim 1$ where the low $q, \omega$ form of $\epsilon(q, \omega)$ can be used, or at $r \ll 1$, which is the CL limit. In fact, for a general $\epsilon(q, \omega)$ one can expand the response in $R$ and obtain that the leading term is $K(z) \sim R^2$, i.e. the CL form. We conclude then that at both small and large $r$,
FIG. 6: Scaling of the variable $\frac{1}{x(1, r)} - \frac{1}{x(0, r)}$ for $r \ll 1$ cases, with $\alpha = 0.2$ and $\alpha = 1$.

where $K(z)$ is reliable, the $T$ dependent length scale of the equilibrium observable $M^*/M$ can be identified with a dephasing length.

V. DISCUSSION

The possible dependence of $M^*(r)$ at $T = 0$ has been of interest as a means of monitoring anomalies in the ground state\cite{13, 14} of metals. Previous studies proposed $M^* \sim r^\mu$ with either a small $\mu$ or a large $\mu$; we find it highly unlikely that an $r$ dependence will reappear at $r > 20$. We propose then $\mu = 0$ at $\alpha = 0.019$, implying $\mu = 0$ at all $\alpha$ (if larger $\alpha$ would show a $\mu \neq 0$ it would imply an unlikely singular line in the $\alpha, r$ plane). We propose then that $\mu = 0$ for all $\alpha$ at $r \gg 1$ and that the effect of the environment is a mass renormalization, in agreement with the variational study\cite{13}.

We have found temperature dependent length scales. For $r \gg 1$ we find $T^{-1}$, while for $r \ll 1$ we find $T^{-1/4}$. We note that the same $T$ dependence was found for dephasing lengths in a nonequilibrium study based on the purity of a reduced density matrix\cite{16} for the dirty metal situation. A dephasing length was deduced\cite{16} by comparing a dephasing rate with a mean level separation as a condition for coherence. It is remarkable that the agreement in these dephasing lengths is obtained in both regimes $r \gg 1$ and $r \ll 1$ where the form of Eq. (2ii) is valid for a dirty metal environment; the $r \ll 1$ form is also valid for other realizations of a CL environment. We have therefore the intriguing observation that equilibrium scales can identify non-equilibrium dephasing length scales.

APPENDIX A: PERTURBATION EXPANSION

Consider the action of a particle on a ring in presence of a dissipative environment and a flux $\phi_x$ through the ring Eq. (1) with the dirty metal environment:

$$K(z) = 1 - [4r^2 \sin^2 \frac{z}{2} + 1]^{-1/2} = \sum_{n=1}^\infty a_n \sin^2 \left(\frac{nz}{2}\right); \quad \alpha = \frac{3}{8k_B T^2} \quad (A1)$$

For a low $T$ expansion it is efficient to perform a duality transformation using the Poisson sum:

$$\sum_m g(m) = \int_{-\infty}^\infty d\phi \sum_p e^{2\pi i\phi p} g(\phi) \quad (A2)$$

where the sums $m, p$ run on all integers. Hence Eq. (1) becomes

$$Z = Z_1 \int_{-\infty}^\infty d\phi \sum_p e^{2\pi i\phi (p+\phi_x)} - \pi^2 \phi^2 \sum_n a_n \int_0^\beta d\tau \int_0^\beta d\tau' \frac{\pi^2 T^2}{2\sin^2 \pi T (\tau - \tau')} \left(1 - \cos(2\pi n T \phi (\tau - \tau')) \cos[\pi(n\theta(\tau) - \theta(\tau'))]\right) \quad (A3)$$
where \( t = 2MR^2T, \beta = 1/T, Z_1 = \int D\theta \exp(-S_1{\theta}) \) and the \( \langle ... \rangle_0 \) average is taken with respect to \( \exp(-S_1) \), where

\[
S_1{\theta} = \int_0^\beta d\tau \frac{1}{2} M R^2 \left( \frac{\partial \theta}{\partial \tau} \right)^2 \tag{A4}
\]

For a Gaussian average we have

\[
\langle \cos[n(\theta(\tau) - \theta(\tau'))] \rangle_0 = \exp\left[-\frac{n^2}{2} \left( \langle \theta(\tau) - \theta(\tau') \rangle^2 \right)_0 \right] = \exp\left[-\frac{n^2}{2} \sum_\omega \langle \mid \theta(\omega) \mid^2 \rangle_0 \left( 1 - \cos \omega(\tau - \tau') \right) \right] = \exp\left[-\frac{2n^2}{\beta^2} \sum_\omega \frac{1 - \cos \omega(\tau - \tau')}{\omega^2} \right] = e^{-n^2|\tau - \tau'|/\beta t} \tag{A5}
\]

where \( \theta(\tau) = \frac{1}{\beta} \sum_\omega e^{-i\omega t} \theta(\omega) \) and \( \omega \) are Matsubara frequencies \( \omega = 2\pi T \times \text{integer} \).

For periodic functions we can change integration variables to \( \tau_1 = \tau - \tau', \tau_2 = \frac{1}{\beta}(\tau + \tau') \) with \( d\tau_2 = \beta, \) and \( |\tau_1| \) in \( \langle A5 \rangle \) is chosen in the range \( (-\beta/2, \beta/2) \) to allow for periodicity and continuity at \( \tau_1 = 0; \) hence,

\[
Z = \int_{-\infty}^{\infty} d\phi \sum_p e^{2\pi i(p + \phi_c)\tau} \int_{-\beta/2}^{\beta/2} \frac{\pi^2 T^2}{2 \sin^2(\pi T \tau)} \left( 1 - \cos(2\pi n T \phi \tau) e^{-n^2|\tau'|/\beta t} \right) \tag{A6}
\]

Integrating \( \phi \) we obtain

\[
Z \sim \sum_p e^{-\frac{(p+\phi_c)^2 \tau}{2}} - \beta \alpha \sum_n a_n \int_{0}^{\beta/2} \frac{\pi^2 T^2}{\sin^2(\pi T \tau)} \left( e^{-\frac{(p+\phi_c)^2 \tau}{2}} - \frac{1}{\beta} e^{-\frac{(p+\phi_c-\alpha n T \tau)^2}{2} \tau} \right) - \frac{1}{\beta} \int e^{-\frac{(p+\alpha T \tau)^2}{2} \tau} \right) \tag{A7}
\]

where the correction to the free energy \( \delta F \) is

\[
\delta F = \alpha \sum_n a_n \int_{0}^{\beta/2} \frac{\pi^2 T^2}{\sin^2(\pi T \tau)} \left[ 1 - e^{-n^2 \frac{2}{T} \pi^2 t^2 - n^2 \frac{2}{T} \tau} \cos(2\pi n T \phi_c T/t) \right] + O(e^{-1/t \ln \omega_c T}) \tag{A9}
\]

The effective mass \( M^* \) is defined in terms of the curvature, so that the 1st order correction is

\[
\delta \frac{1}{M^* R^2} = \frac{\partial^2 \delta F}{\partial \phi^2_c} \bigg|_{0} = -\alpha \sum_n a_n \int_{0}^{\beta/2} \frac{\pi^2 T^2}{\sin^2(\pi T \tau)} (2n T \tau/t)^2 e^{-n^2 \frac{2}{T} \tau^2 - n^2 \frac{2}{T} \tau} \tag{A10}
\]

Note that there is no divergence at \( \tau = 0 \). The dominant integration range is \( \tau < t/Tn^2 \) so that the 1st term in the exponent can be expanded; keeping terms to order \( t^2 \) we obtain in terms of \( x = \tau n^2/2MR^2 \),

\[
\frac{\delta M}{M^*} = -2\alpha \sum_n a_n \int_{-\infty}^{\infty} \left( 1 + \frac{\pi^2 t^2}{3n^4} x^2 - \frac{t^4}{n^4} x^2 + \frac{t^2}{2n^4} x^4 + ... \right) e^{-x} dx = -2\alpha \sum_n a_n \left( 1 - \frac{2t}{n^2} + \left( \frac{2\pi^2}{3} + 12 \right) \frac{t^2}{n^4} + ... \right) \tag{A11}
\]
Hence to 1st order in $t$

$$
\frac{M}{M^*} = 1 - 2\alpha \sum_n a_n + 4\alpha \sum_n \frac{a_n}{n^2}
$$

(A12)

At $t = 0$ this result is consistent with Eq. 9 of Ref. 13.

The following sum rules are useful for evaluating these sums. Integrating Eq. (A1) we obtain:

$$
\sum_{n=1}^{\infty} a_n = 2 - 2 \frac{\int_0^{\pi} dz}{\pi \int_0^{\pi} \frac{dz}{\sqrt{4r^2 \sin^2 \frac{1}{2}z + 1}}}
$$

(A13)

Fourier transform of Eq. (A1)

$$
a_n = \frac{-4}{\pi} \int_0^{\pi} \left(1 - \frac{1}{\sqrt{4r^2 \sin^2 \frac{1}{2}z + 1}}\right) \cos nz \, dz
$$

(A14)

and performing the $n$ summation, we obtain

$$
\sum_{n=1}^{\infty} \frac{a_n}{n^2} = \frac{4}{\pi} \int_0^{\pi} \frac{1}{\sqrt{4r^2 \sin^2 \frac{1}{2}z + 1}} \left(\frac{\pi^2}{6} - \frac{\pi z}{2} + \frac{z^2}{4}\right) \, dz.
$$

(A15)

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