Minimal universal extra dimensions in CalcHEP/CompHEP

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New Journal of Physics 12 (2010) 075017 (28pp)
Received 26 February 2010
Published 16 July 2010
Online at http://www.njp.org/
doi:10.1088/1367-2630/12/7/075017

Abstract. We present an implementation of the model of minimal universal extra dimensions (MUED) in CalcHEP/CompHEP. We include all level-1 and level-2 Kaluza–Klein (KK) particles outside the Higgs sector. The mass spectrum is automatically calculated at one loop in terms of the two input parameters in MUED: the inverse radius \( R^{-1} \) of the extra dimension and the cut-off scale of the model \( \Lambda \). We implement both the KK number conserving and the KK number violating interactions of the KK particles. We also account for the proper running of the gauge coupling constants above the electroweak scale. The implementation has been extensively cross-checked against known analytical results in the literature and numerical results from other programs. Our files are publicly available and can be used to perform various automated calculations within the MUED model.

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1. Introduction

The standard model (SM) of particle physics has been successfully verified by experiment at low energies. Nevertheless, even if the Higgs boson is discovered, the SM will still be considered to be an incomplete theory, as it fails to provide the long-sought missing link between Einstein’s General Relativity and Quantum Mechanics. The leading candidate for a quantum theory of gravity, string theory, typically posits the existence of several new ingredients, which are absent in the SM: new spatial dimensions, a symmetry between bosons and fermions (supersymmetry), as well as new gauge interactions. All of these new ingredients are manifestly present at the Planck scale, but it is not at all clear which of them survive down to low energies. Traditionally, supersymmetry and extra gauge interactions have attracted the most attention, and their consequences for collider phenomenology have been extensively studied \[1, 2\]. Within the last 10 years or so, there has been a resurgence of interest in models with extra spatial dimensions, whose presence might be revealed in high-energy collider experiments, such as the Tevatron at Fermilab, the Large Hadron Collider (LHC) at CERN, or the proposed International Linear Collider (ILC). By now a whole plethora of extra-dimensional models have been described and studied to various extents in the literature. Roughly speaking, they can all be classified according to the following two criteria.

- How many and which of the SM particles can access the extra dimensions (the bulk)? The two extremes here are provided by the ‘large’ extra dimension models (also known as ADD, after the initials of their original proponents) \[3\], in which only gravity can enter into
the bulk, and the universal extra dimensions (UED) models [4], in which all SM particles are allowed to propagate in the bulk.

- What is the metric of the bulk? It can be flat (e.g. in UED) or warped [5].

Extra dimensions accessible to standard model fields are of interest for various reasons. They could allow gauge coupling unification [6], and provide new mechanisms for supersymmetry breaking [7] and the generation of fermion mass hierarchies [8]. More complicated versions have also been proposed, motivated by ideas about electroweak symmetry breaking [9], neutrino masses [10, 11], proton stability [12] or the number of generations [13].

In this paper, we shall concentrate on the simplest case of a single flat extra dimension, which is accessible to the full SM particle content [4] (see [14–19] for the case of two UEDs). This particular scenario has recently been studied in relation to collider phenomenology [20–37], indirect low-energy constraints [38–51], dark matter [52–74] and cosmology [75–81]. It is therefore of great interest to have an implementation of the minimal UED (MUED) model (reviewed below in section 2) in the most popular general-purpose computer programs for collider and astroparticle phenomenology. The main goal of this paper is to present one such implementation, suitable for either CalcHEP [82] or CompHEP [83]. There are several advantages in choosing CalcHEP and CompHEP for this purpose:

- CalcHEP and CompHEP can be used for parton-level event generation, preserving the full spin correlations in both production and decay.
- CalcHEP and CompHEP can easily be interfaced [84] to a general-purpose event generator, such as PYTHIA [85], for the simulation of fragmentation, hadronization and showering.
- CalcHEP and CompHEP can easily be interfaced with a dark matter program, such as micrOMEGAs [86], for the calculation of the relic density and detection rates of a generic dark matter candidate.
- The implementation of new models is very straightforward and user-friendly, as we shall demonstrate below with the example of MUED.

The paper is organized as follows. In section 2, we review the MUED model, introducing the relevant new particles, couplings and interactions. In section 3, we explain how these were incorporated into CalcHEP and CompHEP. Throughout the paper, we assume that the readers are already familiar with these programs, so that we only need to explain the additional *.mdl model files related to our UED implementation. In section 4 we discuss how the implementation can be used to study the collider phenomenology of MUED and show some illustrative results. In the appendices, we list some more technical results that may be useful to some readers. For example, appendix A contains the five-dimensional (5D) UED Lagrangian and appendix C contains the resulting Feynman rules for the level 1 Kaluza-Klein (KK) particles after compactification.

2. The minimal universal extra dimensions (MUED) model

2.1. Kaluza–Klein (KK) decomposition

The 5D UED model [4] is simply the SM placed in an extra dimension compactified on an $S_1/Z_2$ orbifold, as shown in figure 1. Let us label the usual 3 + 1 space–time dimensions with $S_1/Z_2$ orbifold, as shown in figure 1. Let us label the usual 3 + 1 space–time dimensions with.
Figure 1. The $S_1/Z_2$ compactification of a single extra dimension on a circle with opposite points identified, as indicated by the gray arrows. The blue dots represent the fixed (boundary) points and $y$ is the coordinate along the extra dimension.

$x^\mu$, $\mu = 0, 1, 2, 3$, reserving the coordinate $y$ for the extra dimension. In order to end up with chiral fermions in four dimensions and to project out unwanted gauge degrees of freedom, one typically imposes an additional symmetry, thus creating a manifold with boundaries. For example, in the case of the $S_1/Z_2$ orbifold shown in figure 1, one identifies the opposite points on the circle, which creates two fixed points, denoted by the blue dots. Any 5D field can now be assigned a definite parity with respect to the orbifold projection $P_5: y \to -y$. For example, consider a generic scalar field $\phi(x, y)$. An even scalar field $\phi^+(x, y)$ is expanded in KK modes as

$$\phi^+(x, y) = \frac{1}{\sqrt{\pi R}} \phi_0^+(x) + \frac{2}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \phi_n^+(x) \cos \frac{ny}{R},$$

and obeys Neumann boundary conditions at the two fixed points:

$$\left( \frac{\partial \phi^+(x, y)}{\partial y} \right)_{y=0} = \left( \frac{\partial \phi^+(x, y)}{\partial y} \right)_{y=\pi R} = 0.$$

Here $x$ is the usual 4D space–time coordinate $x^\mu$, $R$ is the size of the extra dimension and $n$ labels the KK level. The SM modes correspond to $n = 0$. In contrast, the KK decomposition of an odd scalar field

$$\phi^-(x, y) = \frac{2}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \phi_n^-(x) \sin \frac{ny}{R},$$

is missing a zero mode ($n = 0$) and obeys Dirichlet boundary conditions

$$\phi^-(x, 0) = \phi^-(x, \pi R) = 0.$$

One can similarly assign a definite $P_5$ parity to each component of a gauge field $A_M(x, y)$, $M = 0, 1, 2, 3, 5$. The usual $3+1$ components $A_\mu$, $\mu = 0, 1, 2, 3$, are chosen to be even, which ensures the presence of the SM gauge fields $A^0_\mu(x)$ at the $n = 0$ level, while the extra-dimensional component $A_5$ is taken to be odd. The corresponding KK expansions of the 5D gauge fields are given by

$$A_\mu(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ A^0_\mu(x) + \sqrt{2} \sum_{n=1}^{\infty} A^n_\mu(x) \cos \frac{ny}{R} \right\},$$

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At the two fixed points \( y = 0 \) and \( y = \pi R \), the components \( A_\mu(x, y) (A_3(x, y)) \) obey Neumann (Dirichlet) boundary conditions analogous to equation (2) (equation (4)).

The KK decomposition of a fermion is rather interesting. Since there is no chirality in five dimensions, the KK modes of the SM fermions come in vector-like pairs, i.e. there is a left-handed and a right-handed KK mode for each SM chiral fermion. For example, the SU(2)\(_W\)-singlet chiral fermions \( \psi^0_R(x) \) of the SM (which happen to be all right-handed) are obtained from the following decomposition:

\[
\psi^+_R(x, y) = \frac{1}{\sqrt{2\pi R}} \psi^0_R(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \psi^n_R(x) \cos \frac{ny}{R},
\]

\[
\psi^-_R(x, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \psi^n_R(x) \sin \frac{ny}{R},
\]

where upon compactification, the two KK fermions \( \psi^+_R(x) \) and \( \psi^-_R(x) \) at any given KK level \( n \) pair up to give a Dirac fermion of mass \( n/R \). Similarly, the SU(2)\(_W\)-doublet SM fermions \( \psi^0_L(x) \) (which happen to be left-handed) arise from

\[
\Psi^+_L(x, y) = \frac{1}{\sqrt{2\pi R}} \Psi^0_L(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \Psi^n_L(x) \cos \frac{ny}{R},
\]

\[
\Psi^-_L(x, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \Psi^n_L(x) \sin \frac{ny}{R},
\]

where the massive Dirac fermion at each \( n \) is now formed from \( \Psi^+_L(x) \) and \( \Psi^-_L(x) \).

From equations (7)–(10), we see that there exist left-handed KK modes \( \psi^+_L(x) \) that are associated with the right-handed SM fermions \( \psi^0_R(x) \). Similarly, there are right-handed KK modes \( \psi^-_R(x) \) that are associated with the left-handed SM fermions \( \psi^0_L(x) \). This often leads to some confusion in the literature when it comes to the labeling of fermion KK partners. It should be understood that the chiral index (L or R) of a KK mode fermion refers to the chirality of its SM partner. Here we shall also utilize an alternative convention, introduced in [90], where the KK fermions are identified by their SU(2)\(_W\) quantum numbers instead: SU(2)\(_W\)-doublets (SU(2)\(_W\)-singlets) are denoted with uppercase (lowercase) letters. This convention was already employed in equations (7)–(10). With those conventions, the fermion content of the MUED model is listed in table 1.

Finally, notice that the geometry in figure 1 is still invariant under the interchange of the two fixed points. The corresponding \( Z_2 \) symmetry is the celebrated KK parity and will be a symmetry of the Lagrangian as long as one continues to treat the two boundary points in a symmetric fashion.

### 2.2. The KK mass spectrum

At tree level, the mass \( m_n \) of any KK mode at the \( n \)th KK level is given by

\[
m^2_n = \frac{n^2}{R^2} + m_0^2,
\]

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Table 1. Fermion content of the MUED model. SU(2)_L-doubles (SU(2)_L-singlets) are denoted with uppercase (lowercase) letters. KK modes carry a KK index \( n \), and for simplicity we omit the index ‘0’ for the SM zero modes.

| SU(2)_L representations | SM mode | KK modes |
|-------------------------|---------|----------|
| Quark doublet           | \( q_L(x) = (U_L(x) \ D_L(x)) \) | \( Q^b_L = \begin{pmatrix} U_B(x) \\ D_B(x) \end{pmatrix}, Q^b_R = \begin{pmatrix} U_B(x) \\ D_B(x) \end{pmatrix} \) |
| Lepton doublet          | \( L_L(x) = (\nu_L(x) \ E_L(x)) \) | \( L^e_L(x) = \begin{pmatrix} \nu^{e}_{L}(x) \\ E_{L}(x) \end{pmatrix}, L^e_R(x) = \begin{pmatrix} \nu^{e}_{R}(x) \\ E_{R}(x) \end{pmatrix} \) |
| Quark singlet           | \( u_R(x) \) | \( u^b_R(x), u^b_L(x) \) |
| Quark singlet           | \( d_R(x) \) | \( d^b_R(x), d^b_L(x) \) |
| Lepton singlet          | \( e_R(x) \) | \( e^b_R(x), e^b_L(x) \) |

Figure 2. The spectrum of the first KK level at (a) tree level and (b) one loop, for \( R^{-1} = 500 \text{ GeV}, \Lambda R = 20, m_h = 120 \text{ GeV} \), and assuming vanishing boundary terms at the cut-off scale \( \Lambda \). (From [90].)

where \( R \) is the radius of the extra dimension as illustrated in figure 1, and \( m_0 \) is the mass of the corresponding SM particle (zero mode). The resulting mass spectrum for the first KK level is shown in figure 2(a) for \( R^{-1} = 500 \text{ GeV} \), and it can be seen to be highly degenerate. In fact, several of the lightest \( n = 1 \) KK modes have no allowed decays and are absolutely stable.

However, this drastic conclusion is completely reversed once radiative corrections are taken into account [90]. First, the mass spectrum gets renormalized by bulk interactions, which are uniquely fixed in terms of the SM gauge and Yukawa couplings and thus contain no new parameters beyond those already appearing in the SM. At the same time, the KK masses also receive contributions from terms localized on the boundary points (the two blue dots in figure 1). The coefficients of the boundary terms are, in principle, new free parameters of the theory. The MUED model makes the ansatz that all boundary terms simultaneously vanish at some high scale \( \Lambda > R^{-1} \). The boundary terms are then regenerated at lower scales through Renormalization Group Equation (RGE) running and lead to additional corrections to the KK mass spectrum [90]. The resulting one-loop corrected mass spectrum is shown in figure 2(b). The mass splittings among the different \( n = 1 \) KK modes are now sufficiently large to allow prompt cascade decays to the lightest KK particle (LKP). For the parameter values shown in the
The mass eigenstates of the KK photon $\gamma_n$ and the KK $Z$-boson $Z_n$ are mixtures of the corresponding interaction eigenstates: the KK mode $B_n$ of the hypercharge gauge boson and the KK mode $W^3_n$ of the neutral SU(2)$_W$ gauge boson. The mixing angle $\theta_n$ is obtained by diagonalizing the mass matrix in the $(B_n, W^3_n)$ basis

$$\begin{pmatrix}
\frac{n^2}{2\Lambda^2} + \frac{1}{4} g_1^2 v^2 + \delta m^2_{B_n} & \frac{1}{4} g_1 g_2 v^2 \\
\frac{1}{4} g_1 g_2 v^2 & \frac{n^2}{2\Lambda^2} + \frac{1}{4} g_2^2 v^2 + \delta m^2_{W^3_n}
\end{pmatrix},$$

(12)

where $g_1$ ($g_2$) is the hypercharge (weak) gauge coupling, $v = 246$ GeV is the vacuum expectation value (VEV) of the SM Higgs boson, and $\delta$ represents the total one-loop correction, including both bulk ($\bar{\delta}$) and boundary ($\delta$) contributions [90]:

$$\hat{\delta} m^2_{V_n} \equiv \delta m^2_{V_n} + \bar{\delta} m^2_{V_n}.$$  

(13)

Note that for $n \geq 1$ the KK mixing angle $\theta_n$ is, in general, different from the zero-mode (Weinberg) angle $\theta_0 \equiv \theta_W$ in the SM. For typical values of $R^{-1}$ and $\Lambda$, $\theta_n \ll \theta_W$, and the neutral gauge boson KK mass eigenstates become approximately aligned with the corresponding interaction eigenstates: $\gamma_n \approx B_n$ and $Z_n \approx W^3_n$ for $n \geq 1$. This approximation will be used in our MUED implementation described below in section 3.

2.3. KK interactions

The bulk interactions of the KK modes are already fixed by the SM. The 5D MUED Lagrangian is a straightforward generalization of the SM Lagrangian to five dimensions, as discussed in appendix A. Upon compactification, integrating over the extra-dimensional coordinate $y$, one recovers the bulk interactions among the various KK modes and their SM counterparts (see appendix C). Since translational invariance holds in the bulk, all these bulk interactions conserve both KK number and KK parity.

However, as already alluded to in the previous subsection, there may also exist ‘boundary’ interactions localized on the fixed points in figure 1. They do not respect translational invariance and therefore break KK number by even units. Such interactions may already appear at the scale $\Lambda$, being generated by the new physics which is the ultraviolet completion of UED. In the MUED version, one makes the assumption that no such terms are present at the scale $\Lambda$. Even so, upon renormalization to lower energy scales, boundary terms are radiatively generated from bulk interactions. This is illustrated in figure 3, where we show how an effective coupling between a level-2 KK gauge boson $V_2$ and two SM fermions is generated at one loop from a diagram with level-1 KK particles running in the loop. This effective coupling

$$-i \frac{g}{\sqrt{2}} \left( \frac{\delta m^2_{A_2}}{m^2_2} - 2 \frac{\delta m_{f_2}}{m^2_f} \right) \bar{\psi}_0 \gamma^\mu T^a P_+ \psi_0 A_{2\mu},$$

can be expressed in terms of the boundary contributions $\delta m_n$ (see equation (13)) to the one-loop mass corrections [90]. The explicit form of this effective coupling is summarized in table 2 for each type of level-2 KK gauge boson and for the various possible SM fermion pairs.

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6 Strictly speaking, the true LKP in figure 2(b) is the KK graviton $G_1$ (not shown). However, due to its extremely weak couplings, $G_1$ is irrelevant for collider phenomenology. For its astrophysical implications, see [91].
Having reviewed the MUED model, we are now in a position to describe its implementation in CalcHEP and CompHEP. Each one of these programs gives its users an opportunity to incorporate new physics in the already existing framework of the SM, MSSM, etc. To this end, one must simply supply an updated version of the four model files defining a given physics scenario in CalcHEP and CompHEP: prtclsN.mdl, varsN.mdl, funcN.mdl and
Table 3. KK gauge bosons.

| Name  | A   | A+   | 2*spin | Mass | Width | Color |
|-------|-----|------|--------|------|-------|-------|
| $G_{\mu}^1$ | KG  | KG   | 2      | MKG  | wKG   | 8     |
| $B_{\mu}^1$ | B1  | B1   | 2      | MB1  | 0     | 1     |
| $Z_{\mu}^1$ | Z1  | Z1   | 2      | MZ1  | wZ1   | 1     |
| $W_{\mu}^1$ | $\sim$W+ | $\sim$W− | 2 | MW1  | wW1   | 1     |
| $G_{\mu}^2$ | $\sim$G2 | $\sim$G2 | 2      | MKG2 | wKG2  | 8     |
| $B_{\mu}^2$ | B2  | B2   | 2      | MB2  | wB2   | 1     |
| $Z_{\mu}^2$ | Z2  | Z2   | 2      | MZ2  | wZ2   | 1     |
| $W_{\mu}^2$ | $\sim$W2 | $\sim$w2 | 2 | MW2  | wW2   | 1     |

Table 4. KK leptons.

| Name  | A   | A+   | 2*spin | Mass | Width | Color |
|-------|-----|------|--------|------|-------|-------|
| $e_L^1$ | $\sim$eL | $\sim$EL | 1 | DMe  | wDe1  | 1     |
| $\mu_L^1$ | $\sim$mL | $\sim$ML | 1 | DMm  | wDe2  | 1     |
| $\tau_L^1$ | $\sim$tL | $\sim$TL | 1 | D Mt  | wDe3  | 1     |
| $e_R^1$ | $\sim$eR | $\sim$ER | 1 | SM e  | wSe1  | 1     |
| $\mu_R^1$ | $\sim$mR | $\sim$MR | 1 | SMm  | wSe2  | 1     |
| $\tau_R^1$ | $\sim$tR | $\sim$TR | 1 | S M t  | wSe3  | 1     |
| $\nu_e^1$ | $\sim$n1 | $\sim$N1 | 1 | DMen | wDn1  | 1     |
| $\nu_\mu^1$ | $\sim$n2 | $\sim$N2 | 1 | DMm n | wDn2  | 1     |
| $\nu_\tau^1$ | $\sim$n3 | $\sim$N3 | 1 | DM t n | wDn3  | 1     |
| $e_L^2$ | $\sim$le | $\sim$lE | 1 | DMe2 | wDe12 | 1     |
| $\mu_L^2$ | $\sim$lm | $\sim$lM | 1 | DMm2 | wDe22 | 1     |
| $\tau_L^2$ | $\sim$lT | $\sim$tL | 1 | D Mt2 | wDe32 | 1     |
| $e_R^2$ | $\sim$re | $\sim$rE | 1 | SM e2 | wSe12 | 1     |
| $\mu_R^2$ | $\sim$rm | $\sim$rM | 1 | SMm2 | wSe22 | 1     |
| $\tau_R^2$ | $\sim$rt | $\sim$rT | 1 | S M t2 | wSe32 | 1     |
| $\nu_e^2$ | $\sim$en | $\sim$eN | 1 | DMen2 | wDn12 | 1     |
| $\nu_\mu^2$ | $\sim$mn | $\sim$mN | 1 | DMm n2 | wDn22 | 1     |
| $\nu_\tau^2$ | $\sim$tn | $\sim$tN | 1 | DM t n2 | wDn32 | 1     |

lgrngN.mdl, where N stands for the numerical label of the physics scenario in the model menu of CalcHEP and CompHEP. We shall now discuss each one of those files, which are available from http://home.fnal.gov/~kckong/mued/.

3.1. Particles

New particles are defined in the prtclsN.mdl model file. We incorporate the $n=1$ and $n=2$ KK modes of the gauge bosons (see table 3), leptons (see table 4) and quarks (see table 5). In tables 3–5, the KK number is represented by a superscript $n=1$ or $n=2$, while the subscript is either the Lorentz index ($\mu$) of the vector particles in table 3 or the chirality index of the fermion particles in tables 4 and 5. We remind the reader that all KK fermions are
Table 5. KK quarks.

| Name | A   | A+  | 2*spin | Mass  | Width | Color |
|------|-----|-----|--------|-------|-------|-------|
| $u_1^L$ | Du  | DU  | 1      | DMu   | wDu   | 3     |
| $d_1^L$ | Dd  | DD  | 1      | DMd   | wDd   | 3     |
| $c_1^L$ | Dc  | DC  | 1      | DMc   | wDc   | 3     |
| $s_1^L$ | Ds  | DS  | 1      | DMs   | wDs   | 3     |
| $t_1^L$ | Dt  | DT  | 1      | DMtop | wDt   | 3     |
| $b_1^L$ | Db  | DB  | 1      | DMb   | wDb   | 3     |
| $u_2^L$ | ~Du | ~DU | 1      | DMu2  | wDu2  | 3     |
| $d_2^L$ | ~Dd | ~DD | 1      | DMd2  | wDd2  | 3     |
| $c_2^L$ | ~Dc | ~DC | 1      | DMc2  | wDc2  | 3     |
| $s_2^L$ | ~Ds | ~DS | 1      | DMs2  | wDs2  | 3     |
| $t_2^L$ | ~Dt | ~DT | 1      | DMtop2| wDt2  | 3     |
| $b_2^L$ | ~Db | ~DB | 1      | DMb2  | wDb2  | 3     |
| $u_2^R$ | ~Su | ~SU | 1      | SMu   | wSu   | 3     |
| $d_2^R$ | ~Sd | ~SD | 1      | SMd   | wSd   | 3     |
| $c_2^R$ | ~Sc | ~SC | 1      | SMc   | wSc   | 3     |
| $s_2^R$ | ~Ss | ~SS | 1      | SMs   | wSs   | 3     |
| $t_2^R$ | ~St | ~ST | 1      | SMtop | wSt   | 3     |
| $b_2^R$ | ~Sb | ~SB | 1      | SMb   | wSb   | 3     |

vector-like and the chirality index refers to the chirality of their SM counterparts. The corresponding masses and widths of the KK fermions in tables 4 and 5 carry ‘D’ or ‘S’ to indicate their nature: SU(2)$_W$-doublet or SU(2)$_W$-singlet, respectively. The new particles listed in tables 3–5 are in addition to the usual SM particles, which are not shown here.

3.2. Variables

The input parameters for any given physics scenario are defined in the varSN.mdl model file. In principle, MUED has only two additional input parameters beyond the SM: the radius $R$ of the extra dimension and the cut-off scale $\Lambda$. For convenience, we use the inverse radius $R^{-1}$ and the number of KK levels $\Lambda R$, which can fit below the scale $\Lambda$. $R^{-1}$ has dimensions of GeV, while $\Lambda R$ is dimensionless. Our additions to the varSN.mdl model file are listed in table 6. As seen from the table, we also include several other variables of interest. RG is used to turn on and off the running of coupling constants, while scaleN is the renormalization scale $\mu$ at which the couplings are evaluated. The remaining parameters in table 6 are some useful numerical constants related to the RGE running of the gauge couplings (see section 3.5).
Table 6. Parameters added to the \texttt{varsN.mdl} model file.

| Parameters | Default values | Symbols | Comments |
|------------|----------------|---------|----------|
| Rinv       | 500            | $R^{-1}$| Inverse radius of the extra dimension |
| LR         | 20             | $\Lambda R$| The number of KK levels below $\Lambda$ |
| RG         | 1              | $\Lambda R$| 1 turn on the running of the coupling constants |
|            |                |         | 0 turn off the running of the coupling constants |
|            |                |         | Renormalization scale, $\mu = n/R$ |
| scaleN     | 2              | $n$     | $n = 2$ can be used for KK level 1 pair production |
|            |                |         | or level 2 single production |
|            |                |         | $n = 4$ can be used for KK level 2 pair production |
| cb1        | 6.8333         | $b_1$   | The coefficient of the SM $\beta$-function for $U(1)_Y$ |
| cb2        | -3.16667       | $b_2$   | $-\frac{19}{7}$, The coefficient of the SM $\beta$-function for $SU(2)_W$ |
| cb3        | -7             | $b_3$   | $-7$, The coefficient of the SM $\beta$-function for $SU(3)_c$ |
| cb1t       | 6.8333         | $\tilde{b}_1$ | The coefficient of the KK $\beta$-function for $U(1)_Y$ |
| cb2t       | -2.83333       | $\tilde{b}_2$ | $-\frac{17}{7}$, The coefficient of the KK $\beta$-function for $SU(2)_W$ |
| cb3t       | -6.5           | $\tilde{b}_3$ | $-\frac{13}{2}$, The coefficient of the KK $\beta$-function for $SU(3)_c$ |
| c1MZ       | 98.4151        | $\alpha_1^{-1}$ | $\alpha_1^{-1}(\mu = M_Z)$ |
| c2MZ       | 29.5846        | $\alpha_2^{-1}$ | $\alpha_2^{-1}(\mu = M_Z)$ |
| c3MZ       | 8.53244        | $\alpha_3^{-1}$ | $\alpha_3^{-1}(\mu = M_Z)$ |

3.3. Constraints

The \texttt{funcN.mdl} model file is reserved for variables that are not numerical inputs but are instead computed in terms of the parameters already defined in the \texttt{varsN.mdl} model file. In our case, we use \texttt{funcN.mdl} to supply the masses and two-body decay widths of the KK particles introduced in section 3.1. Therefore they are automatically computed by CalcHEP/CompHEP at the beginning of each numerical session. The masses for all KK particles are evaluated based on the one-loop formulae of [90], and we have also made numerical cross-checks with the results from the private code used in [90]. Our formulae for the widths have been derived analytically and cross-checked with CalcHEP/CompHEP (see section 4). A partial list of two-body decay widths can be found in [22, 23, 28] and our formulae agree with their expressions. In the older versions of CalcHEP/CompHEP, defining the widths as constraints was very convenient in our implementation, since one did not have to launch a separate numerical session for their calculation and then enter their numerical values as input parameters. However, the more recent versions of CalcHEP and CompHEP allow for the automatic calculation of the particle widths on the fly, using the interactions defined in the \texttt{lgrngN.mdl} model file. Our implementation thus allows for backward compatibility with older versions of CalcHEP/CompHEP.

3.4. Interactions

The new interactions of the KK particles of section 3.1 are added to the \texttt{lgrngN.mdl} model file. We include the usual bulk interactions, as well as the KK number violating boundary interactions listed in table 2 [90]. Since the Weinberg angle $\theta_n$ for any $n \geq 1$ is small [90],

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we ignore the mixing among the neutral KK gauge bosons. Thus the KK-photon $\gamma_n$ is identical to the hypercharge gauge boson $B_n$ and the KK Z-boson $Z_n$ is identical to the neutral SU(2)$_W$ gauge boson $W^3_n$. We also ignore the mixing between SU(2)$_W$-doublet and SU(2)$_W$-singlet KK fermions.

Our lgrngN.mdl model file includes all interactions of level-1 and level-2 KK particles, except for the KK Higgs bosons. The phenomenology of the KK Higgs bosons is very model dependent, depending on the value of the SM Higgs mass $m_h$ and the bulk Higgs mass term (see [90] for details). Therefore, we omit any interactions involving KK Higgs bosons.

The collider phenomenology of the KK Higgs bosons has been discussed in [90] for details. Therefore, we omit any interactions involving KK Higgs bosons.

The UED Lagrangian can easily be derived as shown in appendix C. Here we only point out how to deal with four-point interactions involving KK gluons, since this case requires special treatment when implemented in CalcHEP/CompHEP.

The Lagrangian for the quartic interactions with KK gluons is the following:

$$\mathcal{L}_4 = -\frac{1}{4} g_3^2 f^{abc}_a f^{ade}_b G^{0,b}_\mu G^{0,\mu}_\nu G^{0,\nu}_a G^{0,\nu}_e - \frac{g_3^2}{2} f^{abc}_a f^{ade}_b G^{0,d}_\mu G^{0,e}_\nu G^{1,b}_\mu G^{1,e}_\nu - \frac{g_3^2}{4} (f^{abc}_a (G^{0,b}_\mu G^{1,c}_\nu + G^{0,c}_\nu G^{1,b}_\mu))^2 - \frac{1}{4} \frac{3}{2} g_3^2 f^{abc}_a f^{ade}_b G^{1,b}_\mu G^{1,c}_\nu G^{1,d}_\mu G^{1,e}_\nu. \quad (14)$$

The color structure of these four-point interactions cannot be directly written down in CalcHEP/CompHEP format. Hence, to implement this vertex in CalcHEP/CompHEP, we use the following trick. We introduce three auxiliary tensor fields $t^{a}_{\mu \nu}, s^{a}_{\mu \nu}$ and $u^{a}_{\mu \nu}$ in the same way as the original CalcHEP/CompHEP approach for SM gluons. Then one can rewrite the Lagrangian as

$$\mathcal{L} = -\frac{1}{2} t^{a}_{\mu \nu} t^{a}_{\mu \nu} + i \frac{g_3}{\sqrt{2}} t^{a}_{\mu \nu} f^{abc}_a f^{ade}_b G^{0,b}_\mu G^{0,e}_\nu G^{0,\mu}_a G^{0,\nu}_e + i \frac{g_3}{\sqrt{2}} t^{a}_{\mu \nu} f^{abc}_a f^{ade}_b G^{1,b}_\mu G^{1,e}_\nu G^{1,\mu}_a G^{1,\nu}_e

- \frac{1}{2} s^{a}_{\mu \nu} s^{a}_{\mu \nu} + i \frac{g_3}{2} s^{a}_{\mu \nu} f^{abc}_a f^{ade}_b G^{1,b}_\mu G^{1,e}_\nu G^{0,\nu}_a G^{0,\nu}_e

- \frac{1}{2} u^{a}_{\mu \nu} u^{a}_{\mu \nu} + i \frac{g_3}{\sqrt{2}} u^{a}_{\mu \nu} f^{abc}_a f^{ade}_b (G^{0,b}_\mu G^{1,e}_\nu + G^{1,b}_\mu G^{0,e}_\nu)

= -\frac{1}{2} (t^{a}_{\mu \nu} - ig_3 \frac{1}{\sqrt{2}} f^{abc}_a f^{ade}_b G^{0,b}_\mu G^{0,c}_\nu G^{0,\mu}_a G^{0,\nu}_e - ig_3 \frac{1}{\sqrt{2}} f^{abc}_a f^{ade}_b G^{1,b}_\mu G^{1,c}_\nu G^{1,\mu}_a G^{1,\nu}_e)^2 \quad (15)

- \frac{1}{4} g_3^2 f^{abc}_a f^{ade}_b (G^{0,b}_\mu G^{0,c}_\nu G^{1,b}_\mu G^{1,c}_\nu + G^{0,b}_\mu G^{1,b}_\mu G^{0,c}_\nu G^{1,c}_\nu + G^{0,b}_\mu G^{0,c}_\nu G^{1,b}_\nu G^{1,c}_\mu + G^{0,b}_\mu G^{1,b}_\mu G^{0,c}_\nu G^{1,c}_\nu + G^{0,b}_\mu G^{1,b}_\mu G^{0,c}_\nu G^{1,c}_\nu + G^{0,b}_\mu G^{1,b}_\mu G^{1,c}_\nu G^{1,c}_\nu)

- \frac{1}{2} (s^{a}_{\mu \nu} - ig_3 \frac{1}{2} f^{abc}_a f^{ade}_b G^{0,b}_\mu G^{1,c}_\nu G^{0,\mu}_a G^{1,\nu}_e - ig_3 \frac{1}{2} f^{abc}_a f^{ade}_b G^{1,b}_\mu G^{1,c}_\nu G^{0,\nu}_a G^{1,\nu}_e)^2

- \frac{1}{2} (u^{a}_{\mu \nu} - ig_3 \frac{1}{\sqrt{2}} f^{abc}_a f^{ade}_b (G^{0,b}_\mu G^{1,c}_\nu + G^{1,b}_\mu G^{0,c}_\nu))^2

- \frac{1}{4} g_3^2 (f^{abc}_a (G^{0,b}_\mu G^{1,c}_\nu + G^{1,b}_\mu G^{0,c}_\nu))^2.

It is easy to show that the functional integration over the three auxiliary tensor fields reproduces the four-gluon interactions (14).

---

7 The collider phenomenology of the KK Higgs bosons has been discussed in [31, 32, 37].
3.5. Running of the coupling constants

Due to the additional contributions from the KK modes to the beta functions, the gauge

couplings run faster in theories with extra dimensions. The RGE for $\alpha_i \equiv g_i^2/4\pi$ is given by [6]

$$\frac{d\alpha_i^{-1}}{dt} = -\frac{b_i - \tilde{b}_i}{2\pi} - \frac{\tilde{b}_i X_\delta}{2\pi} \left( \frac{\mu}{\mu_0} \right)^\delta,$$

where $\delta$ is the number of extra dimensions, $\mu_0$ is some reference energy scale, $X_\delta = 2\pi^{\delta/2}/\delta \Gamma(\delta/2)$,

$$(b_1, b_2, b_3) = \left( \frac{41}{6}, -\frac{19}{6}, -7 \right)$$

are the SM beta function coefficients, while

$$(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = \left( \frac{41}{6}, -\frac{17}{6}, -\frac{13}{2} \right)$$

correspond to the contributions of the KK states at each massive KK excitation level [92, 93].

The solution to (16) becomes

$$\alpha_i^{-1} = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln \frac{\mu}{M_Z} + \frac{\tilde{b}_i}{2\pi} \ln \frac{\mu}{\mu_0} - \frac{\tilde{b}_i X_\delta}{2\pi \delta} \left[ \left( \frac{\mu}{\mu_0} \right)^\delta - 1 \right].$$

The effect of the RGE running (19) can be accounted for by setting the RG parameter in table 6 to 1 and choosing the appropriate renormalization scale via $\text{scaleN}$.

4. Discussion

4.1. Code validation

In general, the availability of CaCalHEP/CompHEP model files opens the door to a number of applications related to collider phenomenology and dark matter searches. Each such individual study contributes to the validation of the code. Further consistency checks are provided by comparing to existing analytical and/or numerical results in the literature.

- For starters, we have compared the KK mass spectrum calculated with our implementation to the results shown in figure 2, which were obtained independently in [90]. Using identical inputs and neglecting the running of the gauge couplings (as was done in [90]), we found perfect agreement.
- The interaction vertices of appendix C can be independently derived with the automated tool LanHEP [94]. We checked some of the more technically challenging cases (especially the self-interactions of gauge bosons) and also found agreement.
- To minimize the possibility of typing mistakes, we computed analytically the cross-sections for a selected number of simple scattering processes and compared them to the analytical expressions derived by CaCalHEP/CompHEP.
- We have similarly checked that the KK particle widths calculated from our analytical expressions agree with those computed with CaCalHEP/CompHEP by means of our MUED implementation.
Figure 4. Strong production of $n = 1$ KK particles at the LHC for $\sqrt{s} = 7$ TeV: (a) KK–quark pair production; (b) KK–quark/KK–gluon associated production and KK–gluon pair production. The cross-sections have been summed over all quark flavors and also include charge-conjugated contributions, such as $Q_1 \bar{q}_1$ and $\bar{Q}_1 q_1$, $g_1 \bar{Q}_1$. We use CTEQ6L parton distributions [98] and choose the scale of the strong coupling constant $\alpha_s$ to be equal to the parton-level center-of-mass energy.

- Our analytic formulae for decay widths agree with the expressions given in [22, 23, 28].
- Our implementation was used for the analytic calculation of all (co)annihilation cross-sections of level 1 KK particles [67] and the results were in complete agreement with [52, 66].
- Our model files have already been used for various collider studies [25, 27–30, 34, 67, 95–97]. One example is given in figure 4, which shows the strong production cross-section of level 1 KK particles at the imminent LHC energy of 7 TeV.
- We have compared the results for various production cross-sections in MUED to those in published papers [20, 21] and found agreement.
- Our model files were also cross-checked against the known analytical expressions for various invariant mass distributions [26, 99, 100].
- Our model files have also been tested by other groups, for example in creating Pythia\textsubscript{UED} [87, 101, 102], which implemented the matrix elements for certain processes in PYTHIA [85]. Another extensive comparison to an independent MUED implementation via FeynRules was done by Christensen \textit{et al} in [89].

4.2. Outlook

Moving forward, it is important to be mindful of the limitations of our implementation. First of all, it is still MUED, and the spectrum is quite constrained, given in terms of only two parameters: $R^{-1}$ and $\Lambda$. If a signal consistent with UED is discovered at the LHC or the Tevatron, one would like to start testing the data with a more general UED framework, which
allows for the presence of arbitrary boundary terms at the scale $\Lambda$. Work along these lines has already started and a beta version of the corresponding UED model files is available from the authors upon request.

Acknowledgments

We are grateful to Priscila de Aquino, Neil Christensen and Claude Duhr for independent extensive testing of our model files against the results from FeynRules, in the process of which a typo in the original version of our MUED model files was uncovered. AD is partially supported by funding available from the Department of Atomic Energy, Government of India, for the Regional Centre for Accelerator-based Particle Physics, Harish-Chandra Research Institute. KK is partially supported by the US Department of Energy (DOE) under contract number DE-AC02-76SF00515. KM is partially supported by the US DOE under grant number DE-FG02-97ER41029.

Appendix A. The UED Lagrangian in five dimensions

The Lagrangian for the 5D UED model is written as

$$L_{\text{Gauge}} = \int_0^{\pi R} dy \left\{ -\frac{1}{4} B_{MN} B^{MN} - \frac{1}{4} W^a_{MN} W^{aMN} - \frac{1}{4} G^A_{MN} G^{AMN} \right\},$$  \hspace{1cm} (A.1)

$$L_{\text{GF}} = \int_0^{\pi R} dy \left\{ -\frac{1}{2\xi} (\partial^\mu B_\mu - \xi \partial_5 B_5)^2 - \frac{1}{2\xi} (\partial^\mu W^a_\mu - \xi \partial_5 W^a_5)^2 - \frac{1}{2\xi} (\partial^\mu G^A_\mu - \xi \partial_5 G^G_5)^2 \right\},$$  \hspace{1cm} (A.2)

$$L_{\text{Leptons}} = \int_0^{\pi R} dy \left\{ i \bar{L}(x, y) \Gamma^M D_M L(x, y) + i \bar{e}(x, y) \Gamma^M D_M e(x, y) \right\},$$  \hspace{1cm} (A.3)

$$L_{\text{Quarks}} = \int_0^{\pi R} dy \left\{ i \bar{Q}(x, y) \Gamma^M D_M Q(x, y) + i \bar{u}(x, y) \Gamma^M D_M u(x, y) + i \bar{d}(x, y) \Gamma^M D_M d(x, y) \right\},$$  \hspace{1cm} (A.4)

$$L_{\text{Yukawa}} = \int_0^{\pi R} dy \left\{ \lambda_u \bar{Q}(x, y) u(x, y) i \tau^2 H^*(x, y) + \lambda_d \bar{Q}(x, y) d(x, y) H(x, y) + \lambda_e \bar{L}(x, y) e(x, y) H(x, y) \right\},$$  \hspace{1cm} (A.5)

$$L_{\text{Higgs}} = \int_0^{\pi R} dy \left[ (D_M H(x, y))^\dagger (D^M H(x, y)) + \mu^2 H^\dagger(x, y) H(x, y) - \lambda \left( H^\dagger(x, y) H(x, y) \right)^2 \right],$$  \hspace{1cm} (A.6)
in terms of 5D fields decomposed as discussed in section 2.1:

\[
H(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ H(x) + \sqrt{2} \sum_{n=1}^{\infty} H_n(x) \cos \left( \frac{n\gamma}{R} \right) \right\},
\]

\[
B_{\mu}(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ B_{\mu}^0(x) + \sqrt{2} \sum_{n=1}^{\infty} B_{\mu}^n(x) \cos \left( \frac{n\gamma}{R} \right) \right\},
\]

\[
B_5(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} B_5^n(x) \sin \left( \frac{n\gamma}{R} \right),
\]

\[
W_{\mu}(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ W_{\mu}^0(x) + \sqrt{2} \sum_{n=1}^{\infty} W_{\mu}^n(x) \cos \left( \frac{n\gamma}{R} \right) \right\},
\]

\[
W_5(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} W_5^n(x) \sin \left( \frac{n\gamma}{R} \right),
\]

\[
G_{\mu}(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ G_{\mu}^0(x) + \sqrt{2} \sum_{n=1}^{\infty} G_{\mu}^n(x) \cos \left( \frac{n\gamma}{R} \right) \right\},
\]

\[
G_5(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} G_5^n(x) \sin \left( \frac{n\gamma}{R} \right),
\]

\[
Q(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ q_L(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[ P_L Q_L^n(x) \cos \left( \frac{n\gamma}{R} \right) + P_R Q_R^n(x) \sin \left( \frac{n\gamma}{R} \right) \right] \right\},
\]

\[
u(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ u_R(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[ P_R u_R^n(x) \cos \left( \frac{n\gamma}{R} \right) + P_L u_L^n(x) \sin \left( \frac{n\gamma}{R} \right) \right] \right\},
\]

\[
d(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ d_R(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[ P_R d_R^n(x) \cos \left( \frac{n\gamma}{R} \right) + P_L d_L^n(x) \sin \left( \frac{n\gamma}{R} \right) \right] \right\},
\]

\[
L(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ L_L(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[ P_L L_L^n(x) \cos \left( \frac{n\gamma}{R} \right) + P_R L_R^n(x) \sin \left( \frac{n\gamma}{R} \right) \right] \right\},
\]

\[
e(x, y) = \frac{1}{\sqrt{\pi R}} \left\{ e_R(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[ P_R e_R^n(x) \cos \left( \frac{n\gamma}{R} \right) + P_L e_L^n(x) \sin \left( \frac{n\gamma}{R} \right) \right] \right\}.
\]

Here \(H(x, y)\) is the 5D Higgs scalar field and \((B_{\mu}(x, y), B_5(x, y)), (W_{\mu}(x, y), W_5(x, y))\) and \((G_{\mu}(x, y), G_5(x, y))\) are the 5D gauge fields \(B_M, W_M\) and \(G_M\) for \(U(1)_Y, \text{SU}(2)_W\) and \(\text{SU}(3)_c\), respectively. The 5D index \(M\) runs over \(M = \mu, 5\), where \(\mu = 0, 1, 2\) and 3. The \(\text{SU}(2)_W\) and \(\text{SU}(3)_c\) gauge fields are

\[
W_M \equiv W_M^a \frac{\tau^a}{2},
\]
\[ G_M = G_M^A \frac{\lambda^A}{2}, \]

where \( \tau^a, a = 1, 2 \) and 3 are the usual Pauli matrices and \( \lambda^A, A = 1, 2, \ldots, 8 \), are the usual Gell–Mann matrices. The 5D field strength tensors for \( U(1)_Y, \) \( SU(2)_W \) and \( SU(3)_c \) are defined as follows:

\[
B_{MN} = \partial_M B_N - \partial_N B_M, \\
W_{MN}^a = \partial_M W_N^a - \partial_N W_M^a + g_2^{(S)} \epsilon^{abc} W_M^b W_N^c, \\
G_{MN}^A = \partial_M G_N^A - \partial_N G_M^A + g_3^{(S)} f^{ABC} G_M^B G_N^C, \tag{A.8}
\]

where \( \epsilon^{abc} \) and \( f^{ABC} \) are the 5D metric.

Finally, \( Q(x, y) \) and \( L(x, y) \) are the SU(2)\(_w\)-doublet fermions from table 1, while \( u(x, y) \), \( d(x, y) \) and \( e(x, y) \) are the corresponding SU(2)\(_w\)-singlet fermions from table 1. \( P_{LR} = \frac{1+\gamma^5}{2} \) are the 4D chiral projectors in terms of the usual \( \gamma_5 \) matrix. The gamma matrices in 5D

\[
\Gamma^M = (\gamma^\mu, i\gamma^5) \tag{A.10}
\]

satisfy the Dirac–Clifford algebra

\[
\{\Gamma^M, \Gamma^N\} = 2 g^{MN}, \tag{A.11}
\]

where \( g^{MN} \) is the 5D metric

\[
g_{MN} = \begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & -1 \end{pmatrix}, \tag{A.12}
\]

and \( g^{\mu\nu} = (+ - - -) \) is the usual 4D metric.

The covariant derivatives act on 5D fields as follows:

\[
D_M Q(x, y) = \left( \partial_M + ig_3^{(S)} G_M + i \frac{Y_Q}{2} g_1^{(S)} B_M \right) Q(x, y), \\
D_M u(x, y) = \left( \partial_M + ig_3^{(S)} G_M + i \frac{Y_u}{2} g_1^{(S)} B_M \right) u(x, y), \\
D_M d(x, y) = \left( \partial_M + ig_3^{(S)} G_M + i \frac{Y_d}{2} g_1^{(S)} B_M \right) d(x, y), \tag{A.13}
\]

\[
D_M L(x, y) = \left( \partial_M + ig_2^{(S)} W_M + i \frac{Y_L}{2} g_1^{(S)} B_M \right) L(x, y), \\
D_M e(x, y) = \left( \partial_M + i \frac{Y_e}{2} g_1^{(S)} B_M \right) e(x, y),
\]

where the fermion hypercharges are \( Y_Q = \frac{1}{3}, Y_u = \frac{4}{3}, Y_d = -\frac{2}{3}, Y_L = -1 \) and \( Y_e = -2 \).
It is now a rather straightforward but tedious exercise to substitute the expansions (A.7) into the 5D Lagrangians (A.1)–(A.6) and perform the integration over \( y \) with the help of the orthonormality relations listed in appendix B. The resulting Feynman rules in terms of 4D fields are listed in appendix C.

**Appendix B. Orthonormality relations**

The following orthonormality relations can be used in the process of compactifying the 5D Lagrangian listed in appendix A.

\[
\int_0^{\pi R} dy \cos \left(\frac{my}{R}\right) \cos \left(\frac{ny}{R}\right) = \frac{\pi R}{2} \delta_{m,n},
\]

\[
\int_0^{\pi R} dy \sin \left(\frac{my}{R}\right) \sin \left(\frac{ny}{R}\right) = \frac{\pi R}{2} \delta_{m,n},
\]

\[
\int_0^{\pi R} dy \cos \left(\frac{my}{R}\right) \cos \left(\frac{ly}{R}\right) = \frac{\pi R}{4} \Delta^1_{mnl},
\]

\[
\int_0^{\pi R} dy \cos \left(\frac{my}{R}\right) \cos \left(\frac{ly}{R}\right) \cos \left(\frac{ky}{R}\right) = \frac{\pi R}{8} \Delta^2_{mnl},
\]

\[
\int_0^{\pi R} dy \sin \left(\frac{my}{R}\right) \sin \left(\frac{ny}{R}\right) \sin \left(\frac{ly}{R}\right) \sin \left(\frac{ky}{R}\right) = \frac{\pi R}{8} \Delta^3_{mnl},
\]

\[
\int_0^{\pi R} dy \sin \left(\frac{my}{R}\right) \sin \left(\frac{ny}{R}\right) \cos \left(\frac{ly}{R}\right) \cos \left(\frac{ky}{R}\right) = \frac{\pi R}{8} \Delta^4_{mnl},
\]

\[
\int_0^{\pi R} dy \cos \left(\frac{my}{R}\right) \sin \left(\frac{ny}{R}\right) = 0,
\]

\[
\int_0^{\pi R} dy \sin \left(\frac{my}{R}\right) \sin \left(\frac{ny}{R}\right) \sin \left(\frac{ly}{R}\right) = 0,
\]

\[
\int_0^{\pi R} dy \sin \left(\frac{my}{R}\right) \cos \left(\frac{ny}{R}\right) \cos \left(\frac{ly}{R}\right) = 0,
\]

\[
\int_0^{\pi R} dy \sin \left(\frac{my}{R}\right) \cos \left(\frac{ny}{R}\right) \cos \left(\frac{ly}{R}\right) \cos \left(\frac{ky}{R}\right) = 0,
\]

\[
\int_0^{\pi R} dy \sin \left(\frac{my}{R}\right) \sin \left(\frac{ny}{R}\right) \sin \left(\frac{ly}{R}\right) \cos \left(\frac{ky}{R}\right) = 0,
\]

where the \( \Delta \) symbols are defined as

\[
\Delta^1_{mnl} = \delta_{l,m+n} + \delta_{n,l+m} + \delta_{m,l+n},
\]

\[
\Delta^2_{mnl} = \delta_{k,l+m+n} + \delta_{l,m+n+k} + \delta_{m,n+k+l} + \delta_{n,k+l+m} + \delta_{k+m,l+n} + \delta_{k+l,m+n} + \delta_{k+n,l+m},
\]

\[
\Delta^3_{mnl} = -\delta_{k,l+m+n} - \delta_{l,m+n+k} - \delta_{m,n+k+l} - \delta_{n,k+l+m} - \delta_{k+m,l+n} - \delta_{k+l,m+n} - \delta_{k+n,l+m},
\]
\[\Delta_{mnl}^4 = -\delta_{l,m+n} + \delta_{n,l+m} + \delta_{m,l+n}, \quad (B.5)\]
\[\Delta_{mnl}^5 = -\delta_{k,l+m+n} + \delta_{n,k+l+m} + \delta_{l+k+m+n} + \delta_{k+l+m+n} + \delta_{k+n,l+m}. \quad (B.6)\]

**Appendix C. Feynman rules**

Here, we list some of the KK number conserving vertices with subscripts ‘\(n\)’ standing for the KK level, which are obtained after compactifying the 5D Lagrangian of appendix A with the help of the orthonormality relations of appendix B. \(I_3, Y\) and \(Q_f\) denote the isospin, hypercharge and charge of the corresponding fermion, respectively (\(Q_f = I_3 + Y/2\)). The CKM matrix element between two flavors \(f\) and \(f'\) is denoted as \(V_{ff'}\). For KK-number violating (but still KK-parity conserving) vertices, refer to figure 3 and table 2.

\[G_{n}^{c} = -ig_{3}\gamma^{\mu}T_{ba}^{c}, \quad (G_{n}^{c})^{\dagger} = -i g_{3}\sqrt{2} \gamma^{\mu}T_{ba}^{c}, \quad (G_{n}^{c})^{\dagger} = -ig_{3}\gamma^{\mu}T_{ba}^{c}, \quad (G_{n}^{c})^{\dagger} = -ig_{3}\gamma^{\mu}T_{ba}^{c}. \]

\[G_{n}^{c} = -ig_{3}\gamma^{\mu}T_{ba}^{c}, \quad (G_{n}^{c})^{\dagger} = -i g_{3}\sqrt{2} \gamma^{\mu}T_{ba}^{c}, \quad (G_{n}^{c})^{\dagger} = -ig_{3}\gamma^{\mu}T_{ba}^{c}, \quad (G_{n}^{c})^{\dagger} = -ig_{3}\gamma^{\mu}T_{ba}^{c}. \]

\[G_{n}^{\nu b} = i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \quad (G_{n}^{\nu b})^{\dagger} = -i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \]

\[G_{n}^{\nu b} = i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \quad (G_{n}^{\nu b})^{\dagger} = -i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \]

\[G_{n}^{\nu b} = i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \quad (G_{n}^{\nu b})^{\dagger} = -i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \]

\[G_{n}^{\nu b} = i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \quad (G_{n}^{\nu b})^{\dagger} = -i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \]

\[G_{n}^{\nu b} = i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \quad (G_{n}^{\nu b})^{\dagger} = -i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \]

\[G_{n}^{\nu b} = i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \quad (G_{n}^{\nu b})^{\dagger} = -i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \]

\[G_{n}^{\nu b} = i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \quad (G_{n}^{\nu b})^{\dagger} = -i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \]

\[G_{n}^{\nu b} = i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \quad (G_{n}^{\nu b})^{\dagger} = -i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \]

\[G_{n}^{\nu b} = i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \quad (G_{n}^{\nu b})^{\dagger} = -i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \]

\[G_{n}^{\nu b} = i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \quad (G_{n}^{\nu b})^{\dagger} = -i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \]

\[G_{n}^{\nu b} = i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \quad (G_{n}^{\nu b})^{\dagger} = -i g_{3} f^{abc} [(p - q)_{\lambda} g_{\mu\nu} + (q - r)_{\mu} g_{\lambda\nu} + (r - p)_{\nu} g_{\lambda\mu}], \]
\[
G^\nu_{\lambda} = \frac{-i}{2} g^2 \left[ f^{a b e} f^{c d e} (g_{\lambda \nu} g_{\mu \rho} - g_{\lambda \rho} g_{\nu \mu}) + f^{a c d} f^{b d e} (g_{\lambda \mu} g_{\nu \rho} - g_{\lambda \rho} g_{\mu \nu}) + f^{a d e} f^{b c e} (g_{\lambda \mu} g_{\nu \rho} - g_{\lambda \rho} g_{\mu \nu}) \right],
\]

\[
\frac{\gamma}{\gamma} = -i Q \gamma^\mu, \quad \quad Z = -i \frac{g_2}{\cos \theta_W} (2 I_3 + \sin^2 \theta_W Q_f) \gamma^\mu, \quad \quad W^\pm = -i \frac{g_2}{\sqrt{2}} \gamma^\mu V_{f'}, \quad \quad V_{f'} = \frac{g_2}{\sqrt{2}} \gamma^\mu P_L, \quad \quad B_n = -i \frac{Y}{2} g_1 \gamma^\mu P_L, \quad \quad B_n = -i \frac{Y}{2} g_1 \gamma^\mu P_R, \quad \quad Z_n = -i I_3 g_2 \gamma^\mu P_L, \quad \quad W^\pm_n = -i \frac{g_2}{\sqrt{2}} \gamma^\mu P_L V_{f'}, \quad \quad \bar{f}_0 = \frac{g_2}{\sqrt{2}} \gamma^\mu P_L.
\[ B_2 \sim f^P_1 = i \frac{Y}{2} \sqrt{2} \gamma^\mu \gamma^5, \]
\[ \bar{f}^D_1 \]
\[ Z_2 \sim f^D_1 = i I_3 \frac{g_2}{2} \gamma^\mu \gamma^5, \]
\[ \bar{f}^D_1 \]
\[ W^\pm_2 \sim f^S_1 = -i \frac{g_2}{2} \gamma^\mu P_L V_{ff'}, \]
\[ \bar{f}^S_1 \]

\[ A_\mu \sim \begin{array}{c} \bar{k}_3 \\ \bar{W}^\mu_- \end{array} \]
\[ k_1 \quad \begin{array}{c} l_k \end{array} \]
\[ k_2 \]
\[ k_3 = -i e \left[ (k_1 - k_2) g_{\mu\nu} + (k_2 - k_3) g_{\nu\lambda} + (k_3 - k_1) g_{\lambda\mu} \right], \]
\[ W^\mu_+ \]

\[ Z^\mu_\lambda \sim \begin{array}{c} \bar{k}_3 \\ \bar{W}^\mu_- \end{array} \]
\[ k_1 \quad \begin{array}{c} l_k \end{array} \]
\[ k_2 \]
\[ k_3 = -i g_2 \left[ (k_1 - k_2) g_{\mu\nu} + (k_2 - k_3) g_{\nu\lambda} + (k_3 - k_1) g_{\lambda\mu} \right], \]
\[ W^\mu_+ \]

\[ Z_\mu \sim \begin{array}{c} \bar{k}_3 \\ \bar{W}^\mu_- \end{array} \]
\[ k_1 \quad \begin{array}{c} l_k \end{array} \]
\[ k_2 \]
\[ k_3 = -i g_2 \cos \theta_W \left[ (k_1 - k_2) g_{\mu\nu} + (k_2 - k_3) g_{\nu\lambda} + (k_3 - k_1) g_{\lambda\mu} \right], \]
\[ W^\mu_+ \]
\[ W_{1+}^{1+} \]

\[ Z_{\nu}^{2} = -i \frac{e^2}{\sqrt{2}} \cos \theta_W [(k_1 - k_2)g_{\mu\nu} + (k_2 - k_3)g_{\nu\lambda} + (k_3 - k_1)g_{\lambda\mu}], \]

\[ W_{1-}^{1-} \]

\[ A_{\mu} \]

\[ W_{\rho}^{n+} = -ie^2 (2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}), \]

\[ A_{\nu} \]

\[ W_{\sigma}^{n-} = -i \frac{e^2}{\sin \theta_W} (2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}), \]

\[ A_{\mu} \]

\[ W_{\rho}^{n-} = -i \frac{e^2}{\sin \theta_W} (2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}), \]

\[ Z_{\nu}^{n} = -i \frac{1}{\sqrt{2}} \frac{e^2}{\sin \theta_W} (2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}), \]

\[ W_{\sigma}^{n+} \]

\[ W_{\rho}^{1+} = -i \frac{1}{\sqrt{2}} \frac{e^2}{\sin \theta_W} (2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}), \]

\[ Z_{\nu}^{2} = -i \frac{1}{\sqrt{2}} \frac{e^2}{\sin \theta_W} (2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}), \]
\[ W_{\mu}^n + W_{\nu}^\sigma = i g_2^2 (2g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}), \]

\[ W_{\nu}^+ W_{\sigma}^- = -i g_2^2 (2g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}), \]

\[ W_{\mu}^+ Z_{\rho}^\sigma = -i \cos \theta_W g_2^2 (2g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}), \]

\[ W_{\nu}^- Z_{\sigma}^n = -i \frac{1}{\sqrt{2} \sin \theta_W} g_2^2 (2g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}), \]

\[ W_{\mu}^{1-} Z_{\rho}^2 = i \frac{3}{2} g_2^2 (2g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}), \]
\[ W^{n+}_{\mu} \rightarrow Z_\rho \]
\[ W^{n-}_{\nu} \rightarrow Z_\sigma \]
\[ = -i \cos^2 \theta_W g^2_W (2g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}), \]

\[ W^{1+}_{\mu} \rightarrow Z_\rho \]
\[ W^{1-}_{\nu} \rightarrow Z_2^2 \]
\[ = -i \frac{1}{\sqrt{2}} \cos \theta_W g^2_W (2g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}), \]

\[ W^{n+}_{\mu} \rightarrow Z_\rho \]
\[ W^{n-}_{\nu} \rightarrow Z^n_\sigma \]
\[ = -i \frac{3}{2} g^2_W (2g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}), \]

\[ W^{1+}_{\mu} \rightarrow Z_\rho \]
\[ W^{1-}_{\nu} \rightarrow Z_2^2 \]
\[ = -i \frac{1}{2} g^2_W (2g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}), \]

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