Disoriented chiral condensate formation from tubes of hot quark plasma

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We investigate the time evolution of a system of quarks interacting with $\sigma$ and pion fields starting from an initial configuration consisting of a tube of hot quark plasma undergoing a boost-invariant longitudinal expansion. We work within the framework of the linear sigma model using classical transport equations for the quarks coupled to the mean-field equations for the meson fields. In certain cases we find strong amplifications of any initial pion fields. For large-radius tubes, starting from quark densities that are very close to critical, we find that a disoriented chiral condensate can form in the centre of the tube. Eventually the collapse of the tube drives this state back to the true vacuum. This process converts the disoriented condensate, dominated by long-wavelength pion modes, into a coherent excitation of the pion field that includes significant components with transverse momenta of around 400 MeV. In contrast, for narrow tubes or larger initial temperatures, amplification occurs only via the pion-laser-like mechanism found previously for spherical systems. In addition, we find that explicit chiral symmetry breaking significantly suppresses the formation of disoriented condensates.

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I. INTRODUCTION

Relativistic heavy-ion collisions provide a way to form regions of hot, dense hadronic matter. If the temperature is high enough, this matter is expected to be in a phase where chiral symmetry is restored and quarks are unconfined. Recently there has been much interest in the possibility that, as such a system cools, it could lead to regions in which the quark condensate is misaligned with respect to the physical vacuum. These regions can also be thought of as coherent excitations of the pion fields along particular directions in isospin space. Such a state is known as a disoriented chiral condensate (DCC) and a signal for its

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formation would be anomalously large event-by-event fluctuations in the ratio of charged to neutral pions.

Many of the studies of DCC's found in the literature use idealised geometries in order to simplify the calculations. These include uniform matter in a finite box \cite{6}, infinite matter undergoing a boost-invariant expansion in one direction \cite{12} and isotropically expanding infinite matter \cite{13,14,12}. An important exception is the work of Asakawa et al. \cite{15}, who considered the classical evolution of the chiral fields for cylindrical systems undergoing boost-invariant longitudinal expansion.

In a previous paper \cite{16} (hereafter referred to as I), we examined the effects of finite size on the evolution of the chiral fields in the case of spherical systems. The framework used was the linear sigma model \cite{17} with explicit quark degrees of freedom. This model has also been studied by Csernai and Mishustin \cite{9}, for the case of infinite matter expanding in one direction. Here we extend our studies to the case of cylindrical systems undergoing a boost-invariant longitudinal expansion. This geometry should be more relevant to ultra-relativistic heavy-ion collisions, and in particular to the central regions of such collisions where particle production is expected to be independent of rapidity \cite{21}.

As in Ref. \cite{9}, we assume a rapid quench that leaves the quarks out of thermal equilibrium with the chiral fields. The subsequent evolution of the system is then described by the classical Euler-Lagrange equations for the fields coupled to a relativistic transport equation for the quarks \cite{22}. We work in the classical limit where the transport equation reduces to a relativistic Vlasov equation for the distribution of the quarks in phase space. Further details of the model and method of solution can be found in I.

For spherical droplets of hot quark matter, our studies in I showed that the quarks stream rapidly away leaving the chiral fields in an unstable configuration. During their subsequent evolution these fields always “roll” towards the physical vacuum. This behaviour can be thought of as the inward collapse of the surface of the droplet as the quarks escape from it. Although such systems show no tendency to form DCC’s, we found that coherent amplification of any initial pion fluctuation could occur through a pion-laser-like mechanism. This is a consequence of the strong oscillations of the σ field that can pump energy into oscillations of the pion field. Similar behaviour has also been seen in Refs. \cite{23,24}. We have shown in I that this mechanism is a robust one: for example, it does not require a chiral phase transition in order to produce enhanced pion fields.

For cylindrical systems studied here, we find a competition between two mechanisms. The first consists of the transverse flow of the quarks and resulting collapse of the surface of the tube. This leads to behaviour that is similar to that seen in the spherical case. The second is the dilution of the quark density inside the tube as a result of its longitudinal expansion. The latter can lead to DCC formation, as one might expect from the results of Ref. \cite{6}. However, the extent to which this mechanism operates is very sensitive to details of the initial configuration. The formation time for any DCC relative to the time for the tube to collapse determines whether a DCC forms, and how long it can exist. The collapse

\footnote{For reviews of DCC’s, as well as further references, see: \cite{1,3}.}

\footnote{Other recent work using this model can be found in Refs. \cite{18,20}.}
also changes the momentum distribution of the pions from that of the initial DCC, by coherently exciting pion modes with higher momenta. In addition we find that inclusion of explicit chiral symmetry breaking, with the strength needed to give the observed pion mass, significantly suppresses DCC formation.

The paper is organized as follows. In Sec. II we present the linear sigma model that we use together with the classical transport and field equations. Since the basic approach has already been described in I, we concentrate on those features that are specific to the longitudinally expanding cylinder. Our results are described in Sec. III and we discuss their implications in Sec. IV.

II. MODEL

We work here with the linear sigma model \[17\], which provides a simple model for the physics associated with chiral symmetry. The model describes quarks interacting with a chiral four-vector of meson fields \(\sigma, \pi\). Since we study here configurations in which only one component of the pion field is nonvanishing, we keep only one pion field and, as in I, we further simplify the model by neglecting the isospin dependence of the quark-pion coupling.

The Lagrangian we use is thus

\[
\mathcal{L} = \bar{\psi} \left( i \partial - g(\sigma + i\pi \gamma_5) \right) \psi + \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi \partial^\mu \pi \right) - U(\sigma, \pi),
\]

in which the meson fields of an O(2) linear sigma model are coupled to two flavours and three colours of quark.

The interactions among the meson fields are described by the potential \(U\), which we take to be of the form

\[
U(\sigma, \pi) = \frac{\lambda^2}{4} (\sigma^2 + \pi^2 - \nu^2)^2 - f_\pi m_\pi^2 \sigma,
\]

where \(f_\pi = 93\) MeV is the pion decay constant. The “Mexican-hat” form of this potential leads to spontaneous breaking of the chiral symmetry through the nonzero vacuum expectation value of \(\sigma\), which corresponds to the quark condensate of the QCD vacuum. The parameter \(\lambda\) is chosen to give the \(\sigma\) meson a mass in the range 600–1000 MeV. In the physical vacuum the quarks develop a mass \(M_q = g f_\pi\). We consider values of \(g\) that correspond to quark masses in the range 300-500 MeV.

For the situation considered here, it is convenient to work in terms of polar coordinates \(\rho\) and \(\phi\) for the transverse position, proper time \(\tau = \sqrt{t^2 - z^2}\), and (space-time) rapidity \(\eta = \frac{1}{2} \ln[(t+z)/(t-z)]\). For a cylindrically symmetric system undergoing a boost-invariant expansion, the fields depend only on \(\rho\) and \(\tau\). In this case the equations for the meson fields take the form

\[
\frac{1}{\tau} \frac{\partial}{\partial \tau} \left( \tau \frac{\partial \sigma}{\partial \tau} \right) = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial \sigma}{\partial \rho} \right) - \left[ \lambda^2 \left( \sigma^2(\tau, \rho) + \pi^2(\tau, \rho) - \nu^2 \right) + g^2 S_q(\tau, \rho) \right] \sigma(\tau, \rho) + f_\pi m_\pi^2,
\]

\[
\frac{1}{\tau} \frac{\partial}{\partial \tau} \left( \tau \frac{\partial \pi}{\partial \tau} \right) = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial \pi}{\partial \rho} \right) - \left[ \lambda^2 \left( \sigma^2(\tau, \rho) + \pi^2(\tau, \rho) - \nu^2 \right) + g^2 S_q(\tau, \rho) \right] \pi(\tau, \rho).
\]
The source density in these equations, \( S_q(\tau, \rho) \), is proportional to the scalar density of quarks. Its detailed form is given below.

The Vlasov equation \([22]\) is conveniently expressed in terms of momentum variables defined in the local rest frame of the matter. In this frame, which is specified by the unit four-vector \( u = (t, 0, z) / \tau \) \([23]\), the longitudinal momentum \( p_\parallel \) corresponding to a classical particle with three-momentum \( p \) moving in the presence of mean scalar and pseudoscalar fields is

\[
p_\parallel = \frac{p_z t - E z}{\tau},
\]

where

\[
E(x, p) = \sqrt{p^2 + M^2(x)},
\]

and

\[
M(x) = g \sqrt{\sigma^2(x) + \pi^2(x)}.
\]

Similarly, the energy of such a particle in this frame is

\[
\epsilon = p \cdot u = \frac{Et - p_z z}{\tau}.
\]

The transverse components of the momentum, denoted \( p_\perp \), are of course unchanged by the boost to the local rest frame. In terms of these variables, the Vlasov equation for the phase-space distribution of quarks \( f(\tau, \eta, r_\perp, p_\parallel, p_\perp) \) can be written

\[
\left[ \frac{\partial}{\partial \tau} + \frac{p_\parallel}{\tau \epsilon} \frac{\partial}{\partial \eta} + \frac{1}{\epsilon} p_\perp \cdot \nabla r_\perp - \frac{p_\parallel}{\tau} \frac{\partial}{\partial p_\parallel} - \left( \nabla r_\perp \epsilon(\tau, r_\perp, p_\parallel, p_\perp) \right) \cdot \nabla p_\perp \right] f(\tau, \eta, r_\perp, p_\parallel, p_\perp) = 0,
\]

where we have assumed boost invariance, \( i.e. \) that \( M(x) \) is independent of \( \eta \). The antiquark distribution, denoted \( \tilde{f}(\tau, \eta, r_\perp, p_\parallel, p_\perp) \), satisfies an equation of similar form.

The Vlasov equation \([3]\) describes freely streaming classical quarks and antiquarks. These particles have energies that are related to their three-momenta by Eq. \([3]\) and obey relativistic single-particle equations of motion, which, in terms of the coordinates and momenta defined above, take the form:

\[
\dot{r}_\perp(\tau) = \frac{p_\perp(\tau)}{\epsilon},
\]

\[
\dot{p}_\perp(\tau) = -\nabla r_\perp \epsilon,
\]

\[
\dot{\eta}(\tau) = \frac{p_\parallel(\tau)}{\tau \epsilon},
\]

\[
\dot{p}_\parallel(\tau) = -\frac{p_\parallel(\tau)}{\tau},
\]
where the overdots denote derivatives with respect to \( \tau \) and the particle’s energy in the local rest frame is

\[
\epsilon(\tau, r_\perp(\tau), p_\parallel(\tau), p_\perp(\tau)) = \sqrt{M^2(\tau, r_\perp(\tau)) + p_\parallel^2(\tau) + p_\perp^2(\tau)}.
\]  

(14)

The source term in the field equations, \( S_q(x) \), is given by the integral of \( [f(x, p) + \tilde{f}(x, p)]/E(x, p) \) over all three-momenta \([23,10]\). Changing variables to \( p_\parallel \) and \( p_\perp \), we can write it in the form

\[
S_q(x) = \int dp_\parallel d^2p_\perp \frac{f(x, p_\parallel, p_\perp) + \tilde{f}(x, p_\parallel, p_\perp)}{\epsilon(x, p_\parallel, p_\perp)}.
\]  

(15)

Rather than solving Eq. (1) directly as a partial differential equation in seven dimensions, we use the test-particle method \([25,26]\). In this approach, the smooth distributions \( f \) and \( \tilde{f} \) are approximated by a set of classical particles obeying the equations of motion (10)–(13).

For the numerical results presented here, we used 40,000 test quarks and antiquarks.

The numerical techniques for solving the equations of motion (3), (4) and (9) are very similar to those applied to a soliton bag model in Ref. [27]. Details of the method for the case of the linear sigma model can be found in I. In the present case, the boost invariance of the system leads to a number of simplifications. The quark distributions are independent of \( \eta \) and hence we do not need to consider the longitudinal motion of the particles, Eq. (12).

In addition, the longitudinal momenta satisfy Eq. (13) and so they simply scale like \( 1/\tau \) \([9]\).

Because of the longitudinal expansion, there is no finite, conserved energy for systems with this geometry. Nonetheless it is convenient to define an energy per unit rapidity in the local rest frame. This takes the form

\[
E = \tau \int d^2r_\perp \left\{ \frac{1}{2} \dot{\sigma}^2 + \frac{1}{2} \dot{\pi}^2 + \frac{1}{2} (\nabla_{r_\perp} \sigma)^2 + \frac{1}{2} (\nabla_{r_\perp} \pi)^2 + U(\sigma, \pi)
\right\}
\]

\[+ \int dp_\parallel d^2p_\perp \epsilon \left[ f(\tau, \eta, r_\perp, p_\parallel, p_\perp) + \tilde{f}(\tau, \eta, r_\perp, p_\parallel, p_\perp) \right], \]  

(16)

where a constant has been added to \( U(\sigma, \pi) \) so that it vanishes in the vacuum. By making use of the equations of motion, one finds that the rate of change of \( E \) is given by

\[
\frac{dE}{d\tau} = -\tau \int d^2r_\perp \left\{ \frac{1}{2} \dot{\sigma}^2 + \frac{1}{2} \dot{\pi}^2 - \frac{1}{2} (\nabla_{r_\perp} \sigma)^2 - \frac{1}{2} (\nabla_{r_\perp} \pi)^2 - U(\sigma, \pi)
\right\}
\]

\[+ \int dp_\parallel d^2p_\perp \frac{p_\parallel^2}{\epsilon} \left[ f(\tau, \eta, r_\perp, p_\parallel, p_\perp) + \tilde{f}(\tau, \eta, r_\perp, p_\parallel, p_\perp) \right]. \]  

(17)

We have used this equation as a check on our numerical integration of the equations of motion.

### III. RESULTS

In this section, we present the results of our simulations corresponding to various initial conditions of the system. As in I, the results shown are for our “standard” choice of parameters, \( m_\sigma = 1000 \text{ MeV} \), \( M_q = 300 \text{ MeV} \) and, if chiral-symmetry breaking is included,
m_\pi = 139 \text{ MeV}. For these parameters, the temperature below which the phase with \( \sigma \neq 0 \)
becomes the ground state is \( T_0 \approx 235 \text{ MeV} \).

The initial conditions are specified by three parameters: the radius \( r_0 \) of the tube, the
temperature \( T \) of the quark plasma and the proper time \( \tau_0 \) at which the evolution according
to Eqs. (3), (4) and (9) starts. A chemical potential \( \mu \) can also be introduced to allow for
a nonzero initial baryon number, but this does not qualitatively change the behaviour of
the system and so we present here only cases with \( \mu = 0 \). We assume, as before, that the
plasma inside the tube is initially uniform.\(^3\)

In order to study whether initial pionic fields can be amplified during the evolution, we
add a small pionic perturbation to the initial configuration. As in our previous work, we
have considered only uniform initial fluctuations of the pion field since our aim is to study
whether DCC formation is possible in these systems. A more realistic approach would be
to take initial fluctuations from a thermal distribution using a method similar to that in
\cite{28}. We would then expect to find the formation of regions of differently oriented DCC’s,
as seen in the work of Asakawa \textit{et al.} \cite{15}. Such studies will require integration of the full
three-dimensional equations of motion and so will be much more computationally intensive.

For many choices of initial conditions, and in particular for cases with relatively small
initial tubes \( (r_0 < 5 \text{ fm}) \), we find similar behaviour to that seen for the spherical droplets of
quark plasma studied in I. In these cases, the rapid outward streaming of the quarks leaves
the chiral fields in an unstable configuration. The surface of the tube collapses inwards, with
the chiral fields “rolling” towards the physical vacuum. The \( \sigma \) field then executes strong
oscillations about its vacuum value. If a nonzero initial pion field is present, then this can
be amplified by the laser-like mechanism also seen in I.

In contrast, for large enough initial tubes, we find a rather different behaviour. This is
particularly clear in the chiral limit. An example of this type is shown in Figs. 1–5. These
show the behaviour of a tube of initial radius \( r_0 = 6 \text{ fm} \) and temperature \( T = 250 \text{ MeV} \)
at \( \tau_0 = 1 \text{ fm}/c \). This initial proper time is the same as that used in Refs. \cite{8,10,11,14,15},
but is significantly smaller than the estimates of the freeze-out time in the work of Csernai,
Mishustin and coworkers \cite{38,20}. Note that the two examples for which we display results
here are not necessarily the most realistic ones; rather they have been chosen since they
most clearly illustrate the types of behaviour that are possible, and the conditions under
which these can occur.

Figs. 1 and 2 show the evolution of the chiral fields for this example of large tube with an
initial temperature just above \( T_0 \). The behaviour of the central region of the tube is domi-
nated, at least initially, by the longitudinal expansion. In particular the quark density there
drops below its critical value before the quarks have a chance to start streaming outwards.
At this point the chirally restored phase becomes unstable and any small fluctuation from
it can start to grow exponentially. In the present case, this happens at \( \tau \approx 2.5 \text{ fm}/c \), when
the pion field at small radii can be seen to rise rapidly and then oscillate about a value close
to \( f_\pi \).

This pion field is uniform across the central region of the tube. It is an example of a
DCC: a region of misaligned vacuum. The collapse of the surface of the tube means that

\(^3\) For further details of the implementation of the initial conditions in this model, see Sec. IV of I.
the DCC does not persist in this form for proper times longer than $r_0/c$. In Figs. 1 and 2, one can see nonzero $\sigma$ fields appearing at successively smaller radii and the corresponding pion fields ceasing to oscillate in phase with that at the centre of the tube. From $\tau \sim 7 \text{ fm}/c$ onwards, the behaviour of the system resembles that of the ones studied in I, the sigma field oscillating violently until $\tau \sim 12 \text{ fm}/c$ and then settling down to its vacuum value.

By Fourier analysing the pion field at successive times, we have also investigated the spectrum of the pion modes that are excited. In terms of the transverse Fourier transform of the field,

$$\tilde{\pi}(\tau, k_\perp) = \int d^2 r_\perp \exp(i k_\perp \cdot r_\perp) \pi(\tau, \rho),$$

and its proper-time derivative $\dot{\tilde{\pi}}(\tau, k_\perp)$ we define the corresponding intensity in momentum space (per unit rapidity):

$$E_\pi(t, k_\perp) = \frac{1}{2} \left( |\tilde{\pi}(t, k_\perp)|^2 + \omega_k^2 |\dot{\tilde{\pi}}(t, k_\perp)|^2 \right),$$

where $\omega_k = \sqrt{k_\perp^2 + m_\pi^2}$. At large times, when the pion fields are sufficiently weak that they are well described by a linearised equation of motion, this is just the energy density of the pion field in momentum space. A convenient measure of the total strength of the pion field is provided by

$$N_\pi(\tau) = \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{1}{\omega_k} E_\pi(t, k_\perp),$$

which, for weak fields, is equal to the total number of pions per unit rapidity.

In Fig. 3 we show the behaviour of $N_\pi(\tau)$. Initially this rises very rapidly and then shows strong oscillations. This is similar to what was seen in the studies of Rajagopal and Wilczek [6,1]. After $\tau \sim 7 \text{ fm}/c$, these oscillations disappear and are replaced by a more gradual rise in $N_\pi(\tau)$. From the spectrum shown in Fig. 4, we see that the initial oscillations are due to the strong excitation of modes with the lowest frequencies and so are characteristic of a DCC. As the tube collapses, the nearly uniform DCC is destroyed and strength is removed from the lowest modes. Nonetheless significant coherent pion fields continue to be present although, unlike the original DCC, these oscillate in space and time.

In Fig. 5 we plot the same spectrum as a function of pion momentum at various proper times. At $\tau = 4 \text{ fm}/c$ one sees the very large strength concentrated in modes with transverse momenta of less than 150 MeV. This pattern is characteristic of the initial, nearly uniform DCC. At later times, this strength falls off and modes with higher momenta become excited. In particular, significant strength builds up in modes with transverse momenta in the range 300–500 MeV. There is also a smaller peak for momenta of 700–900 MeV Although the final pion fields are no longer dominated by the lowest momentum components, their amplitude still reflects the fact that a DCC was formed. For example, $N_\pi$ is enhanced by a factor of about 2000. This is far larger than the effects seen in the spherical systems studied in I, where the maximum enhancement factors were less than 100.

When explicit chiral symmetry breaking is present, in the form of the final term in the potential (2), there is no phase transition but only a crossover between vacua with small and large values of the $\sigma$ field. For the physical value of $m_\pi$, the crossover is fairly rapid and
so some of the features of the phase transition do survive. Nonetheless explicit symmetry
breaking does lead to some qualitative differences in the evolution of the chiral fields after
a quench. An example is shown in Figs. 6–8. In this case the radius of the initial tube is
again \( r_0 = 6 \) fm and \( \tau_0 = 1 \) fm. The initial temperature was taken to be \( T = 225 \) MeV,
which lies within the crossover region for our standard parameter set.

The explicit symmetry-breaking term in (2) tilts the Mexican hat potential in the direc-
tion of the true vacuum. Hence the system never evolves towards a maximally misaligned
vacuum of the sort seen in the previous example. Despite this, significant amplification of
any initial pion field can occur and so a DCC can still form. This is shown in Fig. 4 by
the appearance of a more-or-less uniform pion field of \( \sim 20 \) MeV in the centre of the tube.
This field starts to oscillate around the true vacuum, \( \pi = 0 \). In an infinite system, like those
studied in Refs. [8,20], these oscillations would continue indefinitely. In the present case,
they are terminated at \( \tau \simeq 7 \) fm/c by the collapse of the tube.

The fact that the fields inside the tube tend to roll towards the true vacuum, as a
result of the tilted potential, also means that much less energy is released when the tube
finally collapses. Indeed, in the example shown, the tube never completely collapses and a
cylindrical region of cold quark matter is formed. Formation of such a region of quark matter
was predicted by Csernai and Mishustin [1] and was also seen in I for parameter sets with
strong quark-meson couplings. This matter is created in a highly excited configuration, as
can be seen in Fig. 6 from the strong oscillations of the \( \sigma \) field from \( \tau \sim 10 \) fm/c onwards. As
this matter settles down, it radiates a significant fraction of its energy in the form of pions,
which is the reason for the continued growth of \( N_\pi \) with proper time seen in Fig. 8. However,
as discussed in I, we believe that the formation of such matter represents an artifact of the
model and so this behaviour should not be taken too seriously. By \( \tau \sim 10 \) fm/c, before
this behaviour sets in, \( N_\pi \) has been enhanced by a factor of about 200. Although this
enhancement is significantly less than that in the previous example, it is still much larger
than that found in the spherical systems studied in I.

IV. DISCUSSION

We have studied the evolution of systems of quarks coupled to chiral fields, starting from
an initial tube of hot quark plasma undergoing a boost-invariant longitudinal expansion.
Such a geometry is expected to be relevant to the central regions of ultra-relativistic heavy-
ion collisions. We assume that a rapid quench occurs at some proper time \( \tau_0 \), leaving the
quarks out of thermal equilibrium with the chiral fields. The subsequent evolution of the
system is treated in the classical approximation, the quarks being described by a relativistic
Vlasov equation in the presence of \( \sigma \) and pion fields which satisfy the mean-field equations
of the linear sigma model.

Unlike the spherical droplets studied previously in I, these cylindrical configurations can
form DCC’s of the sort seen in infinite systems following a quench [3]. Similar behaviour
is also seen in infinite systems undergoing longitudinal expansion [1,9,11,13,20]. However we
find that the transverse flow of the quarks out of the tube tends to act against the formation
of a DCC. This effect does not occur for infinite systems. It leads to an inward collapse of
the surface of the tube, which is similar to what happens for spherical droplets. Although
this collapse does not generate DCC’s, it can enhance any initial pion fluctuation through
a pion-laser-like mechanism whereby strong oscillations of the $\sigma$ field pump energy into the pion fields $^{[23][24]}$.

In the cylindrical case, the presence of two competing mechanisms leads to considerable sensitivity to the initial conditions. We find that significant DCC formation occurs only in large tubes ($r_0 > 5$ fm) where the longitudinal expansion rapidly reduces the quark density below the critical value needed to keep the system in the chirally symmetric phase. For a DCC to be produced, the initial density of quarks and antiquarks should be close to the critical value (i.e. the temperature should be close to $T_0$) and the proper time at freeze-out should be small compared to the radius of the tube.

If the initial proper time $\tau_0$ is taken to be $\sim 1$ fm/$c$, as in Refs. $[8,10,11,14,15]$ and the illustrative examples shown here, then tubes with radii of $\sim 5$ fm or more have time to form significant DCC’s. On the other hand, if $\tau_0$ is $\sim 5-10$ fm/$c$, as suggested by Csernai and Mishustin $[9,18,20]$, then it seems unlikely that DCC’s can form in tubes of realistic radii. Clearly it is crucial to have better models for the early stages of ultra-relativistic heavy-ion collisions in order to determine the temperature and proper time at freeze-out.

We also find that the subsequent evolution of such systems can significantly affect the transverse-momentum distribution of the emitted pions. Even if a system does form an initial DCC involving only very low-momentum modes of the pion field, the energy released by the oscillations of the $\sigma$ field as the tube collapses can lead to coherent excitation of pionic modes with higher transverse momenta.

A further feature of our results is that explicit breaking of chiral symmetry has a significant effect on the evolution of these systems. In particular, by tilting the Mexican hat in the direction of the true vacuum, it reduces the degree of misalignment of the vacuum that can appear at the centre of the tube. A similar effect is also seen in infinite systems $[20]$. As a result, the enhancement of the number of pions, as given by Eq. (20), is typically much smaller than that seen in the chiral limit. Nonetheless, even in the broken-symmetry example shown above, the pionic enhancement is much larger than that seen in the spherical systems studied in I, where no DCC was formed.

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FIG. 1. Time evolution of the $\sigma$ field at various radii. The parameters of the model are $m_\sigma = 1000$ MeV and $M_q = 300$ MeV, and the system is chirally symmetric ($m_\pi = 0$). The tube has initial radius $r_0 = 6$ fm and temperature $T = 250$ MeV at $\tau_0 = 1$ fm/c.
FIG. 2. Time evolution of the pion field at various radii. The model parameters and initial conditions are the same as in Fig. 1.
FIG. 3. Time evolution of $N_\pi$. The model parameters and initial conditions are the same as in Fig. 1.
FIG. 4. Time evolution of the pion energy density in momentum space for the lowest ten modes of the field. The model parameters and initial conditions are the same as in Fig. 1.
FIG. 5. Momentum dependence of the pion energy density at various proper times. The model parameters and initial conditions are the same as in Fig. 1.
FIG. 6. Time evolution of the $\sigma$ field at various radii. The parameters of the model are $m_\sigma = 1000$ MeV, $m_\pi = 139$ MeV and $M_q = 300$ MeV. The tube has initial radius $r_0 = 6$ fm and temperature $T = 225$ MeV at $\tau_0 = 1$ fm/$c$. 
FIG. 7. Time evolution of the pion field at various radii. The model parameters and initial conditions are the same as in Fig. 4.
FIG. 8. Time evolution of $N_\pi$. The model parameters and initial conditions are the same as in Fig. 4.