Three-dimensional cavity-assisted spontaneous emission as a single-photon source: Two cavity modes and Rabi resonance

M. Khanbekyan
Institut für Theoretische Physik, Universität Magdeburg, Postfach 4120, D-39016 Magdeburg, Germany
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Within the framework of exact quantum electrodynamics in dispersing and absorbing media, we have studied the emission from an initially in the upper state prepared emitter in a high quality cavity in the case, when there are two cavity modes that resonantly interact with the emitter. In the case, when one of the modes is in resonance with the emitter transition and the other mode is tuned to the frequency equal to the Rabi splitting of the first mode, the effect of Rabi resonance is observed. In particular, the second mode exhibits a substantial increase of the one-photon Fock state efficiency and enhancement of the emission spectrum in comparison to the exact resonant case.

I. INTRODUCTION

The interaction of a single emitter with the electromagnetic field inside a high quality (Q) cavity has promising implications towards realization of various schemes in quantum optics [11] and related fields such as quantum information science [2]. For scalable and integrable quantum information processing it is furthermore vital to obtain high-efficiency quantum entanglement. Various implementations in recent years include polarization entanglement of photons by means of parametric downconversion in nonlinear optical crystals [3], by using the biexciton (XX)–exciton (X) cascade of single semiconductor quantum dots (QDs) [4], or via time-bin entanglement [5]. Beyond that, the usage of the strong coupling of the electromagnetic field with an emitter placed in a high-finesse cavity promises the most substantial improvement for both the efficiency of entanglement distribution schemes and demonstration of violation of a Bell inequality at larger scales (see, e.g., Ref. [6]). Very widespread schemes of a strong coupling regime between a single emitter and a single cavity mode in the domains of atomic [7, 8] and semiconductor [9] systems have been realized. Recently, increasing attention has been given to a new perspective of quantum entanglement, namely, generation of multipartite quantum W states, that is, states exhibiting a uniform distribution of a single photon across multiple electromagnetic field modes [10, 11]. Genuine N-partite entanglement states emerge as a resource for exploring fundamental aspects of quantum theory (see, e.g., Refs. [11, 12]) and hold great promise for a number of applications as quantum information protocols [13] or robust quantum network schemes (see, e.g., Refs. [14, 15]). In this context, various attempts have been made to realize an experiment that can unambiguously demonstrate the nonlocality of a single particle [16–18].

The recent progress in semiconductor nanotechnology has substantially facilitated the fabrication of various optical resonator structures characterized by a large confinement of light in well-defined spatial and spectral mode profiles, enabling low mode-volume, high quality factor emitter–cavity systems [19]. In particular, several experimental groups realized cavity QED (cQED) systems with large vacuum field amplitude at a QD placed in a high-Q low volume microcavity [20]. In the past decade the strong coupling regime of coupling of single QDs with optical cavities has been realized in a number of high-quality semiconductor micro- and nanostructures, as in photonic crystals [21], micropillars [22], and microdisks [23].

The interesting feature of micropillar cavities is the existence of two spectrally separated orthogonal, linearly polarized high-Q components of the fundamental optical mode. Here, the frequency splitting arises as the result of a slightly elliptical cross-section of the micropillar, which lifts the degeneracy of the resonator fundamental mode. It has been shown that these two modes can couple with a single QD in the strong coupling regime [24], where temperature tuning has been used for variation of the transition mode frequency. Moreover, the two polarization modes can couple with the common QD gain medium in micropillar lasers [25]. In particular, in the latter case the collective interaction of QDs with the micropillar modes gives rise to an unconventional normal-mode coupling between the modes [26]. Then, spontaneous emission of a two-level atom resonantly interacting with two modes of the cavity at the same time theoretically has been discussed in Ref. [27] within a simple generalized Jaynes-Cummings approach. The model, however, does not allow the study of the outgoing field spectra corresponding to the individual cavity modes.

In earlier works, we have studied within the frame of exact quantum electrodynamics [28] the single-photon emission of a single emitter in a high-Q cavity. In particular, we have shown the generation of single-photon wave packets with time-symmetric spatio-temporal shapes [29], and the generation of single-photons with the wave packets shorter than the cavity decay time [28]. More recently, we have presented the feasibility of generation of one-photon Fock state wave packets of desired shape with high efficiency by adjusting the shape of the pump pulse applied to the three-level

*E-mail address: khanbekyan@gmail.com
A-type emitter [30].

In the present article we extend the theory to the case of strong coupling of a single two-level emitter with two orthogonal high-Q polarization modes at the same time. The practical realization of simultaneous interaction of a single emitter with two cavity modes can be realized, e.g., in semiconductor systems [31, 32]. In the case of the micropillar cavities [32], two orthogonal, linearly polarized high-Q components of the fundamental optical mode can be spectrally separated due to slight asymmetry of the micropillar cross-section. For a single QD interaction the QD exciton transition can be tuned by means of variation of the temperature. Here, we study, in particular, the quantum state of the excited outgoing mode is studied. A summary and some concluding remarks are given in Sec. IV.

The paper is organized as follows. The basic equations for the resonant interaction of an emitter with a cavity-assisted electromagnetic field are given in Sec. II. The theory is then presented for the case of the resonant interaction with two polarization modes of high-Q cavity. In Sec. III the Wigner function of the quantum state of the two polarization outgoing modes, which allows us to study various coupling regimes of emitter–bimodal cavity interaction.

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II. BASIC EQUATIONS

A. Quantization scheme

Let us consider a single emitter (position \( r_A \)) that interacts with the electromagnetic field in the presence of a dispersing and absorbing dielectric medium with a spatially varying and frequency-dependent complex permittivity

\[
\varepsilon(r, \omega) = \varepsilon'(r, \omega) + i\varepsilon''(r, \omega),
\]

with the real and imaginary parts \( \varepsilon'(r, \omega) \) and \( \varepsilon''(r, \omega) \), respectively. Applying the multipolar-coupling scheme in electric dipole approximation, we may write the Hamiltonian that governs the temporal evolution of the overall system, which consists of the electromagnetic field, the dielectric medium (including the dissipative degrees of freedom), and the emitter coupled to the field, in the form of [33, 64]

\[
\hat{H} = \hat{H}_{\text{field}} + \hat{H}_A + \hat{H}_{\text{int}},
\]

where

\[
\hat{H}_{\text{field}} = \int d^3r \int_0^\infty d\omega \hbar \omega \hat{f}(r, \omega) \cdot \hat{f}(r, \omega)
\]

is the Hamiltonian of the field–medium system, where the fundamental bosonic fields \( \hat{f}(r, \omega) \) and \( \hat{f}(r, \omega) \),

\[
[\hat{f}(r, \omega), \hat{f}(r', \omega')] = \delta(\omega - \omega')\delta(r - r')
\]

and

\[
[\hat{f}(r, \omega), \hat{f}(r', \omega')] = 0,
\]

play the role of the canonically conjugate system variables. Further,

\[
\hat{H}_A = \sum_m \hbar \omega_m \hat{S}_{mm}
\]

is the Hamiltonian of the emitter, where \( \hat{S}_{mm} \) are the flip operators,

\[
\hat{S}_{nn} = |m\rangle\langle n|,
\]

with \(|m\rangle\) being the energy eigenstates of the emitter. Finally,

\[
\hat{H}_{\text{int}} = -\hat{d} \cdot \hat{E}(r_A)
\]

is the emitter-field coupling energy, where

\[
\hat{d} = \sum_{mn} d_{mn} \hat{S}_{mn}
\]

is the electric dipole-moment operator of the emitter \( \langle d_{mn} = \langle m|d|n\rangle \rangle \), and the operator of the medium-assisted electric field \( \hat{E}(r) \) can be expressed in terms of the variables \( \hat{f}(r, \omega) \) and \( \hat{f}(r, \omega) \) as follows:

\[
\hat{E}(r) = \hat{E}^{(+)}(r) + \hat{E}^{(-)}(r),
\]

\[
\hat{E}^{(+)}(r) = \int_0^\infty d\omega \hat{E}(r, \omega), \quad \hat{E}^{(-)}(r) = [\hat{E}^{(+)}(r)]^\dagger,
\]

\[
\hat{E}(r, \omega) = \frac{i\sqrt{\hbar}}{\varepsilon_0 c} \int d^3r' \sqrt{\varepsilon''(r', \omega)} G(r, r', \omega) \cdot \hat{f}(r', \omega).
\]

In the above, the classical (retarded) Green tensor \( G(r, r', \omega) \) is the solution to the equation

\[
\nabla \times \nabla \times G(r, r', \omega) - \frac{\omega^2}{c^2} \varepsilon(r, \omega) G(r, r', \omega) = \delta^{(3)}(r - r')
\]

together with the boundary condition at infinity, \( G(r, r', \omega) \to 0 \) if \( |r - r'| \to \infty \), and defines the structure of the electromagnetic field formed by the presence of dielectric bodies.

B. Two-level emitter in a two-mode cavity

Let us focus on a single atom-like emitter placed in a resonator cavity and assume that only a single transition \( |1\rangle \leftrightarrow |2\rangle \), frequency \( \omega_{21} \) is quasi resonantly coupled
to a narrow-band cavity-assisted electromagnetic field, Fig. 1. In this case, the interaction Hamiltonian $\hat{H}_{\text{int}}$ in the rotating-wave approximation reads

$$
\hat{H}_{\text{int}} = -i\sqrt{\frac{\hbar}{\varepsilon_0}} \int_0^\infty \omega^2 \frac{\varepsilon^\prime(r, \omega)}{c^2} \int d^3r \sqrt{\varepsilon^\prime(r, \omega)}
$$

$$
\times \mathbf{d}_{21} \cdot \mathbf{G}(r_A, r, \omega) \cdot \hat{f}(r, \omega) \hat{S}_{21} + \text{H.c.}
$$

(14)

In what follows we assume that the emitter is initially (at time $t = 0$) prepared in the upper state $|0\rangle$ and the rest of the system, i.e., the part of the system that consists of the electromagnetic field and the dielectric medium (i.e., the cavity) is prepared in the ground state $|\{0\}\rangle$, defined by $\hat{f}(r, \omega)|\{0\}\rangle = 0$. Since in the case under consideration we may approximately span the Hilbert space of the whole system by the single-excitation states, we expand the state vector of the overall system at later times $t$ ($t \geq 0$) as

$$
|\psi(t)\rangle = C_2(t)e^{-i\omega_{21}t}|\{0\}\rangle|2\rangle +
$$

$$
+ \int d^3r \int_0^\infty \omega e^{-i\omega t} C_1(r, \omega, t) \cdot \hat{f}(r, \omega)|\{0\}\rangle|1\rangle,
$$

(15)

where $\hat{f}(r, \omega)|\{0\}\rangle$ is an excited single-quantum state of the combined field–cavity system.

It is not difficult to prove that the Schrödinger equation for $|\psi(t)\rangle$ leads to the following system of differential equations for the probability amplitudes $C_1(r, \omega, t)$ and $C_2(t)$:

$$
\dot{C}_2 = -\frac{1}{\sqrt{\pi\hbar\varepsilon_0}} \int_0^\infty \omega^2 \frac{\varepsilon^\prime(r, \omega)}{c^2} \int d^3r \sqrt{\varepsilon^\prime(r, \omega)}
$$

$$
\times \mathbf{d}_{21} \cdot \mathbf{G}(r_A, r, \omega) \cdot C_1(r, \omega, t)e^{-i(\omega - \omega_{21})t},
$$

(16)

$$
\hat{C}_1(r, \omega, t) = \frac{1}{\sqrt{\pi\hbar\varepsilon_0}} \int_0^\infty \omega^2 \frac{\varepsilon^\prime(r, \omega)}{c^2} \int d^3r \sqrt{\varepsilon^\prime(r, \omega)}
$$

$$
\times \mathbf{d}_{21}^* \cdot \mathbf{G}^*(r_A, r, \omega) C_2(t)e^{i(\omega - \omega_{21})t}.
$$

(17)

Further, substituting the formal solution to Eq. (17) [with the initial condition $C_2(0) = 1$ and $C_1(r, \omega, 0) = 0$] into Eq. (16) and using the integral relation

$$
\frac{\omega^2}{c^2} \int d^3r \varepsilon''(r', \omega) G(r, r', \omega) G^*(r'', \omega) = \text{Im} G(r, r'', \omega)
$$

we can derive the integro-differential equation

$$
\dot{C}_2 = \int_0^t dt' K(t - t')C_2(t'),
$$

(19)

where the kernel function $K(t)$ reads

$$
K(t) = -\frac{1}{\pi\hbar\varepsilon_0} \int_0^\infty \omega^2 \frac{\varepsilon^\prime(r, \omega)}{c^2} e^{-i(\omega - \omega_{21})t}
$$

$$
\times \mathbf{d}_{21} \cdot \text{Im} \mathbf{G}(r_A, r_A, \omega) \cdot \mathbf{d}_{21}^*.
$$

(20)

Having solved Eq. (19) for $C_2(t)$ we may eventually calculate $C_1(r, \omega, t)$ according to Eq. (17)

$$
C_1(r, \omega, t) = \frac{1}{\sqrt{\pi\hbar\varepsilon_0}} \int_0^\infty \omega^2 \mathbf{d}_{21}^* \cdot \mathbf{G}^*(r_A, r, \omega)
$$

$$
\times \int_0^t dt' \sqrt{\varepsilon^\prime(r, \omega)} C_2(t')e^{i(\omega - \omega_{21})t'}.
$$

(21)

Without loss of generality, we may assume that the cavity is formed as a stratified system, i.e., whose material properties do not change throughout each plane orthogonal to a fixed direction (z axis) defined as the direction of the cavity outgoing field, $\varepsilon(r, \omega) \rightarrow \varepsilon(z, \omega)$. A stratified system admits separation into an angular spectrum representation of $s$- and $p$-polarized fields which do not mix, and the (nonlocal part of the) Green tensor can be expressed as a two-dimensional Fourier integral

$$
G(r, r', \omega) = \frac{1}{(2\pi)^2} \int d^2k e^{i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}')} G(z, z', \mathbf{k}, \omega),
$$

(22)

with $r = (z, \mathbf{r})$, and

$$
G(z, z', \mathbf{k}, \omega)
$$

$$
= \sum_{\sigma = s, p} \left[ e_\sigma^+ (k) g_\sigma^+ (z, z', \mathbf{k}, \omega) + e_\sigma^- (k) g_\sigma^- (z, z', \mathbf{k}, \omega) \right],
$$

(23)

where $\sigma$ indicates two polarization directions ($s$ and $p$) with unit vectors $\mathbf{e}_\sigma^+$ and $\mathbf{e}_\sigma^-$ for outgoing and incoming waves, correspondingly, with the orthogonality relations

$$
e_\sigma^\pm (k) \cdot e_{\sigma'}^\pm (k) = \delta_{\sigma\sigma'},
$$

(24)

$$
g_\sigma^\pm (k) \cdot g_{\sigma'}^\pm (k) = 0, \quad \sigma \neq \sigma'.
$$

(25)

The Green tensor $G(r, r', \omega)$ determines the spectral response of the resonator cavity. For a sufficiently high $Q$, the excitation spectrum effectively turns into quasi-discrete sets of lines of midfrequencies $\omega_{k,l}$ and
with widths $\Gamma_{k,l}$ in $s$- and $p$-polarization directions, correspondingly, according to the poles of the Green tensor at the complex frequencies

$$\Omega_{k,l} = \omega_{k,l} - \frac{1}{2}i\Gamma_{k,l}. \quad (26)$$

In the following we assume that two modes of the cavity-assisted field, namely, the $k$th mode of the $s$-polarization direction and the $l$th mode of the $p$-polarization direction, are quasi resonantly coupled with the emitter transition $|1\rangle \leftrightarrow |2\rangle$, and

$$\Gamma_k, \Gamma_l, \frac{1}{2}|\omega_k - \omega_l| < \frac{1}{2}|\omega_m - \omega_{k,l}|, \quad m \neq k, l. \quad (27)$$

To calculate the kernel function $K(t)$, Eq. (20), we may assume that the cavity-assisted field that resonantly interacts with the emitter can be approximated by Lorentzian functions corresponding to the $k$th and $l$th modes

$$4\omega \sqrt{\frac{\rho_0}{\pi \hbar}} d_{21} \cdot \text{Im} G(r_A, r_A, \omega) \cdot d_{21}^* = \sum_{j=k,l} \alpha_j(\omega_j) \frac{1}{\pi} \frac{1}{|\omega - \Omega_j|^2}. \quad (28)$$

Then, we note that, within the approximation scheme used, the frequency integration can be extended to $\pm \infty$. Thus, using Eq. (28) we derive

$$K(t) = -\frac{1}{4} \sum_{j=k,l} \alpha_j(\omega_j) \Omega_j e^{-i(\Omega_j - \omega_{21})(t-t')} \quad (29)$$

From Eq. (19) together with Eq. (29) we can conclude that $R_j \equiv \sqrt{\alpha_j(\omega_j)\omega_j}$ can be regarded as vacuum Rabi frequencies of emitter–cavity mode interactions for the $k$th and $l$th modes, correspondingly.

### III. OUTGOING TWO-MODE FIELD

#### A. Field operators

For the sake of transparency, let us restrict our attention to the case when the cavity is embedded in free space. Then, inserting Eq. (22) together with Eq. (23) into Eq. (12) we find

$$\hat{E}(r, \omega) = \frac{1}{(2\pi)^2} \sum_{\sigma=s,p} \int d^2 k e^{ik \cdot \rho}$$

$$\times \left[ \hat{e}^+_{\sigma}(k) \hat{E}^+_{\sigma}(z, k, \omega) + \hat{e}^-_{\sigma}(k) \hat{E}^-_{\sigma}(z, k, \omega) \right], \quad (30)$$

with

$$\hat{E}^\pm_{\sigma}(z, k, \omega) = 2 \omega \left[ \hat{e}^\pm_{\sigma}(k) \hat{E}^\pm_{\sigma}(z, k, \omega) \times \int d'z' \sqrt{\varepsilon^\prime(z', \omega)} g^\pm_{\sigma}(z, z', k, \omega) \cdot \hat{f}(z', k, \omega). \quad (31)$$

where

$$\hat{f}(z, k, \omega) = \int d^2 \rho e^{-i k \cdot \rho} \hat{f}(z, \rho, \omega) \quad (32)$$

$[\hat{f}(z, \rho, \omega) \equiv \hat{f}(r, \omega)]$. In the following, we are interested in the two polarization directions of the outgoing field $\hat{E}^\pm_{\sigma}(0^+, k, \omega)$ at the point $z = 0^+$ on the axis of the cavity outgoing field propagation right outside the cavity. Using Eq. (31) together with Eq. (4) we find

$$\left[ \hat{E}^\pm_{\sigma}(0^+, k, \omega), \hat{E}^\mp_{\sigma'}(0^+, k', \omega') \right] = \delta_{\sigma\sigma'} \Omega_{\sigma\sigma'} \delta(\omega - \omega') \delta(k - k'), \quad (33)$$

where

$$\hat{c}^\pm_{\sigma}(k, \omega) = \frac{4\pi \hbar \omega^2}{c^2} \int d'z' \varepsilon'(z', \omega) \Omega_{\sigma\sigma'} \hat{g}^\pm_{\sigma}(0^+, z', k, \omega)^2 \quad (34)$$

From Eq. (33) we can see that bosonic outgoing field operators can be introduced according to

$$\hat{E}^\pm_{\sigma}(0^+, k, \omega) = \sqrt{\hat{c}^\pm_{\sigma}(k, \omega)} \hat{b}_{\sigma}(k, \omega), \quad (35)$$

with

$$[\hat{b}_{\sigma}(k, \omega), \hat{b}^\dagger_{\sigma'}(k', \omega')] = \delta_{\sigma\sigma'} \delta(\omega - \omega') \delta(k - k'). \quad (36)$$

Notice, at this point no assumption is made about the details of stratification of the resonator media, the geometry and dielectric properties of which define the functions $g^\pm_{\sigma}(0^+, z', k, \omega)$.

#### B. Quantum state of the outgoing field

To calculate the quantum state of the outgoing field we start from the multimode characteristic functional

$$C_{\text{out}}[\beta_{\sigma}(k, \omega), t] = \langle \psi(t) \rangle \times \exp \left[ \sum_{\sigma} \int_0^\infty d\omega \int d^2 k \beta_{\sigma}(k, \omega) \hat{b}_{\sigma}(k, \omega) - \text{H.c.} \right] \langle \psi(t) \rangle. \quad (37)$$

Applying the Baker–Campbell–Hausdorff formula and recalling the commutation relation (36), we may rewrite $C_{\text{out}}[\beta_{\sigma}(k, \omega), t]$ as

$$C_{\text{out}}[\beta_{\sigma}(k, \omega), t] = \exp \left[ -\frac{1}{2} \sum_{\sigma} \int_0^\infty d\omega \int d^2 k |\beta_{\sigma}(k, \omega)|^2 \right]$$

$$\times \langle \psi(t) \rangle \exp \left[ \sum_{\sigma} \int_0^\infty d\omega \int d^2 k \beta_{\sigma}(k, \omega) \hat{b}_{\sigma\dagger}(k, \omega) \right]$$

$$\times \exp \left[ -\sum_{\sigma} \int_0^\infty d\omega \int d^2 k |\hat{\beta}_{\sigma\dagger}(k, \omega)|^2 \hat{b}_{\sigma}(k, \omega) \right] \langle \psi(t) \rangle. \quad (38)$$
To evaluate $\mathrm{C}_{\text{out}}[\beta_{\sigma}(k,\omega),t]$ for the state $|\psi(t)\rangle$ given by Eq. (15) we first note that from the commutation relation $[\phi(k,\omega),\phi^{\dagger}(k,\omega)]$ together with the relation $\hat{f}(z,k,\omega)|\{0\}\rangle=0$ it follows that

$$\hat{f}(z,k,\omega)|\psi(t)\rangle = C_{1}(z,k,\omega,t)e^{-\alpha_{k}\rho}C_{1}(r,\omega,t).$$

Hence, on recalling Eqs. (31) and (35), it can be shown that

$$\hat{b}_{\sigma}(k,\omega)|\psi(t)\rangle = F_{\sigma}^{*}(k,\omega,t)|\{0\}\rangle,$$

where

$$F_{\sigma}(k,\omega,t) = -i \frac{1}{\sqrt{c_{r}(k,\omega)}} \sqrt{\frac{\hbar}{\varepsilon_{r}(k,\omega)}} e^{i\omega t} \times \int\! dz \sqrt{\varepsilon''(z,\omega)} g_{\sigma}^{++}(0^{+},z,k,\omega) \cdot C_{1}(z,k,\omega,t).$$

with $C_{1}(z,k,\omega,t)$ being determined by Eq. (40) together with Eq. (21). Then, combining Eqs. (38) and (41) it is not difficult to obtain

$$\mathrm{C}_{\text{out}}[\beta_{\sigma}(k,\omega),t] = \exp \left[-\frac{1}{2} \sum_{\sigma} \int_{0}^{\infty} d\omega \int d^{2}k |\beta_{\sigma}(k,\omega)|^{2} \right] \times \left[1 - |\beta_{\sigma}(t) + \beta_{\rho}(t)|^{2}\right],$$

where

$$\beta_{\sigma}(t) = \int_{0}^{\infty} d\omega \int d^{2}k \beta_{\sigma}(k,\omega) F_{\sigma}(k,\omega,t).$$

As we can see from Eq. (43), the characteristic function of the outgoing field does not factorize into a product of characteristic functions in $s$- and $p$-polarization directions. The polarization modes are quantum correlated in a nontrivial fashion, which shows the entangled nature of the quantum state under consideration.

C. Mode functions and Rabi Resonance

The characteristic functions for the $s$- ($p$)-polarization directions can be found from $\mathrm{C}_{\text{out}}[\beta_{\sigma}(k,\omega),t]$ by simply setting, correspondingly, $\beta_{p}(k,\omega)=0$ or $\beta_{s}(k,\omega)=0$ in Eq. (43). Then, following Ref. [29], we Fourier transform the characteristic functions for every polarization direction and by choosing appropriate sets of nonmonochromatic modes, we obtain the multimode Wigner functions of the quantum state of the outgoing field in $\sigma$-polarization direction to be

$$W_{\sigma_{\text{out}}}(\gamma_{\sigma_{1}},t) = W_{\sigma_{1}}(\gamma_{\sigma_{1}},t) \prod_{i \neq 1} W_{\sigma_{i}}^{(0)}(\gamma_{\sigma_{i}},t),$$

where

$$W_{\sigma_{1}}(\gamma_{\sigma_{1}},t) = [1 - \eta_{\sigma}(t)] W_{\sigma_{1}}^{(0)}(\gamma_{\sigma_{1}}) + \eta_{\sigma}(t) W_{\sigma_{1}}^{(1)}(\gamma_{\sigma_{1}}),$$

with $W_{\sigma_{1}}^{(0)}(\gamma_{\sigma_{1}})$ and $W_{\sigma_{1}}^{(1)}(\gamma_{\sigma_{1}})$ being the Wigner functions of the vacuum state and the one-photon Fock state, respectively, for the $i$th nonmonochromatic mode of $\sigma$-polarization. As we can see from Eq. (45), the nonmonochromatic modes with $i \neq 1$ are in the vacuum state. The modes labeled $i = 1$ with the nonmonochromatic mode functions

$$F_{\sigma_{1}}(k,\omega,t) = \frac{F(k,\omega,t)}{\sqrt{\eta_{\sigma}(t)}}$$

are in the mixed state described by the Wigner functions $W_{\sigma_{1}}(\gamma_{\sigma_{1}},t)$. Moreover, Eq. (46) reveals that $\eta_{\sigma}(t)$ defined by

$$\eta_{\sigma}(t) = \int_{0}^{\infty} d\omega \int d^{2}k |F_{\sigma_{1}}(k,\omega,t)|^{2}$$

can be regarded as the efficiency to prepare the excited outgoing $\sigma$-polarization wave packet in a one-photon Fock state.

To calculate the nonmonochromatic mode functions $F_{\sigma_{1}}(k,\omega,t)$ we combine Eqs. (21) and (42) to derive

$$F_{\sigma_{1}}(k,\omega,t) = \frac{1}{\sqrt{c_{r}(k,\omega)}} \times \int\! dz \sqrt{\varepsilon''(z,\omega)} d_{21} \cdot G(z_{A},z,k,\omega) \cdot g_{\sigma}^{++}(0^{+},z,k,\omega) \times \int_{0}^{t} dt' C_{2}(t') e^{i\omega_{2}(t'-t)} e^{i\omega_{2}t'}. \quad (49)$$

As we have mentioned above, the Green tensor $G(r,r',\omega)$, or in particular, $g_{2}^{++}(0^{+},z_{A},k,\omega)$ and $G(z_{A},z,k,\omega)$ in Eq. (49), determine the spectral resonances of the high-$Q$ cavity. In particular, the Green tensor terms define the $k$th and $l$th resonance frequencies of the $s$- and $p$-polarization directions of the cavity, respectively [complex frequencies $\Omega_{k,l}$, Eq. (26)]. Moreover, the first scalar product in Eq. (49), $d_{21} \cdot G(z_{A},z,k,\omega)$, can be identified as the overlap of the emitter electric dipole moment and the cavity field. The second scalar product $G(z_{A},z,k,\omega) \cdot g_{\sigma}^{++}(0^{+},z,k,\omega)$ describes the wanted radiative losses of the cavity modes $\gamma_{k,l}$ due to the transmission of the radiation through the fractionally transparent mirror. It can be shown [30] that the efficiencies $\eta_{\sigma}(t)$ are proportional to the ratio $\gamma_{k,l}/\Gamma_{k,l}$, where the damping parameters $\Gamma_{k,l}$ [see, e.g., Eq. (26)] can be regarded as being the sum $\Gamma_{k,l} = \gamma_{k,l} + \gamma_{k,l}'$, where $\gamma_{k,l}'$ describe the unwanted losses due to scattering and absorption, defined by the complex permittivity $\varepsilon(z,\omega)$, Eq. (1). Note, unwanted losses as low as 30% have been reported in modern high-$Q$ semiconductor cavities [36, 37], which corresponds to $\gamma_{k}/\Gamma_{k} = \gamma_{k}/\Gamma_{k} = 0.7$. 


Thus, to obtain the outgoing nonmonochromatic mode functions $F_{s1}(k, \omega, t)$ for the $s$- and $p$-polarization directions, we first numerically solve the integro-differential Eq. (19) together with Eq. (29) and insert the solution for $C_2(t)$ into Eq. (40). To further evaluate Eq. (40), we model the cavity as a planar dielectric multilayer system, where the cavity body is confined by layers of perfectly reflecting and fractionally transparent mirrors (see Ref. [38] for details and Appendix A for the expression of the corresponding Green tensor). The spectral mode functions $F_{s1}(k, \omega, t)$ for the $s$- and $p$-polarization directions are plotted in Fig. 2 when the $k$th mode ($s$-polarization) is in exact resonance with the emitter transition for the two following cases. In the first case (Fig. 2(a)), mode $Q$ factors are 8300 and 6640 for the $k$th and $l$th modes, correspondingly, which is in accordance with the experimentally observed values [32], the $l$th mode ($p$-polarization) is also in resonance with the emitter transition.

Then, both mode spectra exhibit two peaks around the resonance frequency featuring vacuum Rabi splitting of the emitter-field interaction. Obviously, since the Rabi frequency $R_k = 20\Gamma_k$ is higher than $R_l = 6\Gamma_k$, the one-photon Fock state efficiency of the outgoing wave packet in $s$-polarization is higher than the one of the $p$-polarization. Further, in the second case, when the frequency of the $l$th mode coincides with the one of the peaks of the Rabi splitting of the $k$th mode–emitter interaction [Fig. 2(b)], mode $Q$ factors are 8300 and 6632 for the $k$th and $l$th modes, correspondingly, both modes spectra exhibit three-peak structures. Interestingly, by comparison of Figs. 2(a) and 2(b) we notice, that in the latter case the enhancement of the $p$-polarization emission spectrum and diminution of the $s$-polarization emission spectrum is observed in comparison to the exact resonance case. Moreover, one-photon Fock state efficiency of the $p$-polarization direction is more than 3 times larger in comparison to the exact resonance case.

IV. SUMMARY AND CONCLUDING REMARKS

Based on macroscopic QED in dispersing and absorbing media, we have presented an exact description of the resonant interaction of a two-level emitter in a high-$Q$ cavity with the cavity-assisted electromagnetic field. In particular, we have studied the case when two orthogonal closely-spaced polarization modes of the cavity at the same time resonantly interact with the emitter. We have applied the theory to the determination of the Wigner function of the quantum state of the excited outgoing field.

Assuming that the Hilbert space of the total system is spanned by a single-quantum excitation, we have shown that the polarization directions do not factorize in the Wigner function of the outgoing field. Thus, we may conclude that the quantum state of the outgoing field represents a nonseparable coherent superposition of the two polarization directions.

To further exploit the nature of the resonant bimodal interaction, then we have studied quasiprobability distributions of the individual polarization directions. In the case when both polarization modes are in exact resonance with the emitter transition, the spectral mode functions of the two polarization directions exhibit two-peak structures around the resonance frequency. The single-photon Fock state efficiencies for every of the polarization directions are determined by the corresponding Rabi frequencies of the emitter-field coupling. More interesting is the case when one of the modes is in resonance with the emitter transition, and the frequency of the second polarization mode is out of resonance but coincides with the one of the peaks of the Rabi splitting of the first mode. In particular, in this case both spectra of the resonant and the off-resonant modes exhibit three-peak structures. This is an example of the well-known Rabi resonance scenario [39, 40], which shows that mode interaction in the bimodal cQED systems may drastically change the features of the nonclassical radiation field emission. In particular, as an important effect of
the assisted Rabi resonance we observe at the Rabi resonance frequency region drastic enhancement of the emission spectrum of the off-resonant mode. At the same time diminution of the spectrum of the resonant mode is observed. Moreover, the one-photon Fock state efficiency of the off-resonant mode is substantially larger in the case of the Rabi-resonance case in comparison to the exact resonance case.

In summary, spectral mode functions of the outgoing field of two cavity modes simultaneously interacting with a single emitter in the exact resonant case exhibit similar two-peak structures featuring Rabi splitting of the strong coupling.

In the latter case, when the first mode is in resonance and the second mode is tuned to the Rabi frequency of the coupling of the first mode, the spectral mode functions of both cavity modes are split into triplets. Therefore, to observe the effect of Rabi resonance in a practical situation, another degree of freedom is necessary to distinguish spectra of the individual modes. To observe the effect of Rabi resonance in a practical situation, another degree of freedom such as polarization is needed for the mode-resolved spectroscopy of the outgoing field. Another possible realization of the simultaneous strong coupling of two modes with a single emitter may be provided by the interaction of transverse modes with a two-level atom, where mode-resolved spectroscopy is also possible, see, e.g., Ref. [41]. In Ref. [11], however, only the case when both modes are in exact resonance with the emitter is considered.

Finally, as an outlook, the system featuring tunable interaction of a single emitter with two closely-spaced cavity modes can be used to generate genuine multipartite polarization entangled states of light. Quantification of the entanglement of the field modes in the exact resonance and Rabi resonance scenarios is the subject of our future investigations.

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Appendix A: Green tensor for three-dimensional dielectric multilayer structure

The nonlocal part of the Green tensor for a three-dimensional dielectric multilayer structure reads

\[
G(z, z', k, \omega) = \frac{i}{2} \sum_{\sigma = s, p} \sum_{n = 0}^{\infty} \left[ \epsilon_{\sigma}^{(j)}(z, k, \omega) \epsilon_{\sigma}^{(j')}(z', -k, \omega) \Theta(z - z') + \epsilon_{\sigma}^{(j)}(z, k, \omega) \epsilon_{\sigma}^{(j)}(z', -k, \omega) \Theta(z' - z) \right],
\]

where \( z (z') \) belongs to the layer \( j \) (\( j' \)), and \( \xi_p = 1, \xi_s = -1 \). In the above, the functions \( \epsilon_{\sigma}^{(j)}(k, \omega, z) \) and \( \epsilon_{\sigma}^{(j)}(k, \omega, z') \) denote waves of unit strength, traveling, respectively, rightward and leftward in the \( j \)th layer, and being reflected at the boundary,

\[
\epsilon_{\sigma}^{(j)}(z, k, \omega) = e_{q+j}^{(j)}(k) e^{i\beta_j(z - d_j)} + r_{q/j}^{(j)} e_{q-j}^{(j)}(k) e^{-i\beta_j(z - d_j)},
\]

\[
\epsilon_{\sigma}^{(j)}(z, k, \omega) = e_{q-j}^{(j)}(k) e^{-i\beta_j z} + r_{j/q}^{(j)} e_{q+j}^{(j)}(k) e^{i\beta_j z},
\]

and

\[
\Xi_{q}^{j'} = \frac{1}{\beta_{n}^{q/j} n_{0/n}} \frac{t_{0/j}^{q} e^{i\beta_{j} d_{0}}}{D_{n j}} \frac{t_{n/j}^{q} e^{i\beta_{j} d_{n}}}{D_{n j'}}
\]

where

\[
D_{n j} = 1 - r_{j/0}^{q} r_{0/j}^{q} e^{2i\beta_{j} d_{j}}
\]

\((d_{0} = d_{n} = 0)\).

Here,

\[
\beta_{j} = \sqrt{k_{j}^{2} - k^{2}} = \beta_{j}^{*} + i\beta_{j}'' \quad (\beta_{j}', \beta_{j}'', \geq 0)
\]

\((k = |k|)\),

\[
k_{j} = \sqrt{\varepsilon_{j}(\omega)} \frac{\omega}{c} = k_{j}^{*} + ik_{j}'' \quad (k_{j}', k_{j}'', \geq 0),
\]

and \( t_{j/j'} \) and \( r_{j/j'} \) are, respectively, the transmission and reflection coefficients between the layers \( j' \) and \( j \). Finally, the unit vectors \( e_{p \pm}^{(j)}(k) \) in Eqs. (A2) and (A3) are the polarization unit vectors for transverse electric (\( q = s \)) and transverse magnetic (\( q = p \)) waves,

\[
e_{s \pm}^{(j)}(k) = \frac{k}{k_{j}} \times e_{z},
\]

\[
e_{p \pm}^{(j)}(k) = \frac{1}{k_{j}} \left( \mp \beta_{j} \frac{k}{k} + k e_{z} \right).
\]

In the limiting case where the space outside the structure may be regarded as being vacuum the coefficients \( c_{\sigma}^{+}(k, \omega) \), Eq. (54) read

\[
c_{\sigma}^{+}(k, \omega) = \frac{\pi \hbar \omega^{2}}{c^{2}} \frac{1}{\varepsilon_{0} \beta_{0}}.
\]
