The effect of self interacting isoscalar-vector meson on finite nuclei and infinite nuclear matter.

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A detailed study is made for the nucleon-nucleon interaction based on relativistic mean field theory in which the potential is explicitly expressed in terms of mass and the coupling constant of the meson fields. A unified treatment for self-coupling of isoscalar-scalar $\sigma$-, isoscalar-vector $\omega$-mesons and their coupling constant are given with a complete analytic form. The present investigation is focused on the effect of self-interacting higher order $\sigma$ and $\omega$ field on nuclear properties. An attempt is made to explain the collapsing stage of nucleon by higher order $\omega$-field. Both infinite nuclear matter and the finite nuclear properties are included in the present study to observe the behaviour or sensitivity of this self interacting terms.

Keywords: Relativistic mean field, Nucleon-nucleon Potential, Energy density, Pressure density, Binding energy, Excitation energy

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1. Introduction

The Nucleon-Nucleon (NN) interaction problem was started from last half century. Probably this is a long standing question in history of nuclear physics. In fact, describing the nuclear properties in terms of the interactions between the nucleons pairs is indeed the main goal for nuclear physicists. The nucleon-nucleon NN-interaction in terms of mediated mesons is put forwarded by Yukawa in 1935. Although the meson theory is not fundamental in the view of QCD, still the approach has improved our understanding of the nuclear forces as well as highlight some good quantitative results. The modern theory of NN potential through particle exchanges is made possible by the development of quantum field theory. However, at low-energy, one can assume that the interactions is instantaneous and therefore the concept of interaction potential becomes useful. The derivation of a potential through particle exchange, is important to understand the nuclear force as well as structural properties.

Nowadays, there are large development in the nuclear theory by introducing quark and gluon in connection with the NN-potential. These models give the
fundamental understanding of NN interaction at present. Here, we are not addressing all these rich and long standing subject about NN-potential, but some basic facts and important issues of the NN-interactions arising from relativistic mean field (RMF) Lagrangian. The behaviour of this potential gives an idea about the breaking of nucleon and the formation of quark – gluon – plasma medium at very high energy i.e. the collapsing state of nucleons.

This paper is organized as follows. In Section II, we briefly discuss the theoretical formalism of NN-interaction from relativistic mean field theory. The general forms of the NN potentials are expressed in the coordinate space (r-space) in terms of mass and coupling constant of the force parameters. In Section III, we review the effect of modified term in the Lagrangian and their effect on the finite nucleus and infinite nuclear matter observables. In Section IV, we address few comments about the current form of the NN-interaction to the collapsing stage of the nucleons.

2. The theoretical frameworks

The nuclear interaction in relativistic mean field is possible via various mesons interaction with nucleons. The linear relativistic mean field (RMF) Lagrangian density for a nucleon-meson many-body system is given as:

\[
\mathcal{L} = \bar{\psi}_i \left[ i \gamma^\mu \partial_\mu - M \right] \psi_i + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - g_\sigma \bar{\psi}_i \psi_i \sigma \\
- \frac{1}{4} \Omega^{\mu \nu} \Omega_{\mu \nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - g_\omega \bar{\psi}_i \gamma^\mu \psi_i \omega_\mu - \frac{1}{4} \tilde{B}^{\mu \nu} \tilde{B}_{\mu \nu} \\
+ \frac{1}{2} m_\rho^2 \tilde{R}^\mu \tilde{R}_\mu - g_\rho \bar{\psi}_i \gamma^\mu \tau_3 \psi_i \tilde{R}_\mu - \frac{1}{4} F^{\mu \nu} F_{\mu \nu} - e \bar{\psi}_i \gamma^\mu \frac{1}{2} \left( 1 - \tau_3 \right) \psi_i A_\mu, \tag{1}
\]

where, the field for \( \sigma \) meson is denoted by \( \sigma \), for \( \omega \) by \( V_\mu \), and for the iso-vector \( \rho \) mesons by \( \tilde{R}_\mu \), respectively. The \( \psi_i \), \( \tau_3 \) and \( \tau_3 \) are the Dirac spinors for nucleons, the iso-spin and the 3\(^{rd}\) component of the iso-spin, respectively. Here \( g_\sigma \), \( g_\omega \), \( g_\rho \) and \( g_\delta \) are the coupling constants for \( \sigma \), \( \omega \), and \( \rho \) mesons and their masses are denoted by \( m_\sigma \), \( m_\omega \) and \( m_\rho \), respectively. The field tensors for \( V_\mu \) and \( \tilde{R}_\mu \) are given by \( \Omega^{\mu \nu} \) and \( \tilde{B}_{\mu \nu} \), respectively. If, we neglect the \( \rho \)-meson, it corresponds to the Walecka model in its original form. From the above relativistic Lagrangian, we obtain the field equations for the nucleons and mesons as,

\[
\left( -i \alpha \cdot \nabla + \beta \left( M + g_\sigma \sigma \right) + g_\omega \omega + g_\rho \tau_3 \rho_3 + g_\delta \delta \tau \right) \psi_i = \epsilon_i \psi_i, \tag{2}
\]

\[
\left( - \nabla^2 + m_\sigma^2 \right) \sigma(r) = - g_\sigma \rho_\sigma(r), \tag{3}
\]

\[
\left( - \nabla^2 + m_\omega^2 \right) \omega(r) = g_\omega \rho(r), \tag{4}
\]

\[
\left( - \nabla^2 + m_\rho^2 \right) \rho_\rho(r) = g_\rho \rho_3(r). \tag{5}
\]
for Dirac nucleons and corresponding mesons in the Lagrangian. In the limit of one-meson exchange and mean-field (the fields are replaced by their number), for a heavy and static baryonic medium, the solution of single nucleon-nucleon potential for scalar ($\sigma$) and vector ($\omega, \rho$) fields are given by $\ref{15,16}$

$$V_{\sigma}(r) = -\frac{g_\sigma^2}{4\pi} e^{-m_\sigma r},$$

and

$$V_{\omega}(r) = \frac{g_\omega^2}{4\pi} e^{-m_\omega r}, \quad V_\rho(r) = \frac{g_\rho^2}{4\pi} e^{-m_\rho r}.$$  \hspace{1cm} (6)

The total effective nucleon-nucleon potential is obtained from the scalar and vector parts of the meson fields. This can be expressed as $\ref{8}$

$$v_{\text{eff}}(r) = V_{\omega} + V_\rho + V_{\sigma} = \frac{g_\omega^2}{4\pi} e^{-m_\omega r} + \frac{g_\rho^2}{4\pi} e^{-m_\rho r} - \frac{g_\sigma^2}{4\pi} e^{-m_\sigma r}. \hspace{1cm} (8)$$

### 2.1. Non-linear case

The Lagrangian density in the above Eqn. (1) contains only linear coupling terms, which is able to give a qualitative description of the nuclei $\ref{15,16}$. The essential nuclear matter properties like incompressibility and the surface properties of the finite nuclei cannot be reproduced quantitatively within this linear model. Again the interaction between a pair of nucleons when they are embedded in a heavy nucleus is less than the force in empty space. This suppression of the two-body interactions within a nucleus in favour of the interaction of each nucleon with the average nucleon density, means that the non-linearity acts as a smoothing mechanism and hence leads in the direction of the one-body potential and shell structure $\ref{17,18,19,20}$. The replacement of mass term $\frac{1}{2}m_\sigma^2 \sigma^2$ of $\sigma$ field by $U(\sigma)$ and $\frac{1}{2}m_\omega^2 V^\mu V_\mu$ of $\omega$ field by $U(\omega)$. This can be expressed as

$$U(\sigma) = \frac{1}{2}m_\sigma^2 \sigma^2 + \frac{1}{3}g_2 \sigma^3 + \frac{1}{4}g_3 \sigma^4,$$  \hspace{1cm} (9)

$$U(\omega) = \frac{1}{2}m_\omega^2 V^\mu V_\mu + \frac{1}{4}c_3 (V^\mu V_\mu)^2.$$  \hspace{1cm} (10)

The terms on the right side of Eqns (9-10), except the first term other are from the non-linear self coupling amongst the $\sigma$ and $\omega$ mesons, respectively $\ref{17,18}$. Here, the non-linear parameter $g_2$ and $g_3$ due to $\sigma-$ fields are adjusted to the surface properties of finite nuclei $\ref{21,22}$. The most successful fits yield, the $+ve$ and $-ve$ signs for $g_2$ and $g_3$, respectively. The negative value of $g_3$ is a serious problem in quantum field theory. As, we are dealing within the mean field level and with normal nuclear matter density, the corresponding $\sigma$ field is very small and the $-ve$ value of $g_3$ is still allowed $\ref{21,22}$. With the addition of the non-linear terms of the Eqns (9-10) to the Lagrangian, the field equation for $\sigma$ and $\omega-$ fields (in Eqn. (6-7)) are
modified as:

\[-\nabla^2 + m_\sigma^2 \sigma(r) = -g_\sigma \rho_\sigma(r) - g_2 \sigma^2(r) - g_3 \sigma^3(r),\]

\[-\nabla^2 + m_\omega^2 \omega(r) = g_\omega \rho_\omega(r) - c_3 W^3(r).\]  

(11)

Here, \(W(r) = g_\omega V_0(r)\) and \(c_3\) is the non-linear coupling constant for self-interacting \(\omega\)-mesons. Because of the great difficulty in solving the above nonlinear differential equations, it is essential to have a variation principle available for the estimation of the energies associated with various source distributions. In the static case, the negative sign of the third term in the Lagrangian is computed with the correct source function and an arbitrary trial wave function. The limit on the energy has a stationary value equal to the correct energy when the trial function is in the infinitesimal neighborhood of the correct wave function. Now, the solution for the modified \(\sigma\) and \(\omega\) fields are given as [21]

\[V_\sigma = -\frac{g^2_o}{4\pi} e^{-m_\sigma r} + \frac{g^2_2}{4\pi} e^{-2m_\sigma r} + \frac{g^2_3}{4\pi} e^{-3m_\sigma r},\]

\[V_\omega = -\frac{g^2_o}{4\pi} e^{-m_\omega r} + \frac{c^2_3}{4\pi} e^{-3m_\omega r}.\]  

(12)

The new NN-interaction analogous to \(M3Y\) form and is able to improve the incompressibility and deformation of the finite nuclei results [23]. In addition to this, the non-linear self coupling of the \(\sigma\) and \(\omega\)-mesons help to generate the repulsive and attractive part of the NN potential at long as well as at short distance respectively to satisfy the saturation properties (Coester-band problem) [22]. Thus also, generate the most discussed \(3\)-body interaction. The modified effective nucleon-nucleon interaction is defined as [8]

\[v_{\text{eff}}(r) = V_\omega + V_\rho + V_\sigma\]

\[= g^2_o \frac{e^{-m_\omega r}}{4\pi r} + g^2_2 \frac{e^{-m_\omega r}}{4\pi r} + g^2_3 \frac{e^{-m_\omega r}}{4\pi r} + \frac{g^2_2}{4\pi} e^{-2m_\omega r} + \frac{g^2_3}{4\pi} e^{-3m_\omega r} + \frac{c^2_3}{4\pi} e^{-3m_\omega r}.\]  

(13)

### 3. Results and Discussion

The above expression in Eqn. (13) shows that the effective nucleon-nucleon potential is presented eloquently in terms of the well known inbuilt RMF theory parameters of \(\sigma\), \(\omega\) and \(\rho\) meson fields. Here, we have used RMF (NL3) force parameter along
with varying $c_3$ for $\omega$-self interactions to determine the nuclear properties. The values of these ardent parameter for NL3-force are listed in Table 1. Although, the $\omega^4$ term is already there in the FSU-Gold parameter set, here we are interested to see the effect of non-linear self coupling of $\omega$ meson. Thus, we have added the self-interaction of $\omega$ with coupling constant $c_3$ on the top of NL3 sets and observing the possible effects.

First of all, we have calculated the NN-potential for linear and non-linear cases using Eqs (8) and (13), respectively. The obtained results for each cases are shown in Fig. 1. From the figure, it is clear that without taking the non-linear coupling for RMF (NL3), one cannot reproduce a better NN-potential. In other word, the depth of the potential for linear and non-linear are $\sim 150$ MeV and $50$ MeV, respectively. Thus, the magnitude of the depth for linear case is not reasonable to fit the NN-data. Again, considering the values of $c_3$, there is no significant change in the total nucleon-nucleon potential. For example, the NN-potential does not change at all for $c_3 \simeq \pm 0.6$, which can be seen from Fig. 1.

Further, we have calculated the individual contribution of meson fields to the NN-potential in particular case of $\sigma$ and $\omega$-mesons. In case of $\sigma$-field, we have calculated the linear and non-linear contribution separately, and combined to get the total $\sigma$-potential, which is shown in Fig. 2. From the figure, one can find the non-linear self-interacting terms in the $\sigma$-field play an important role in the repulsive
Fig. 2. The contribution of $\sigma$-potential from linear, non-linear and total as a function of distance $r$ for NL3 parameter set.

Fig. 3. The contribution of $\omega$-potential from linear, non-linear and total as a function of distance $r$ for NL3 parameter set.
core of the total NN-potential.\textsuperscript{21} The linear and non-linear contribution of the $\omega$-field at various $c_3$ are shown in Fig. 3. The important feature in this figure is that the linear term give an infinity large repulsive barrier at $\sim 0.4$ fm, at which range, the influence of the non-linear term of the $\omega-$meson is till zero. However, this non-linear terms is extremely active at very short distance ($\sim 0.2$ fm), which can be seen from the figure. Further, for both $\pm$-values of $c_3$, the contribution having same magnitude but in opposite directions. The strongly attractive potential of the non-linear $\omega-$meson part towards the central region make some evidence to explain the collapsing stage of the nucleons during the formation of quark–gluon–plasma QGP at high energy heavy-ion-collision. That means, mostly the (i) $\sigma-$ meson is responsible for the attractive part of the nuclear force (nuclear binding energy) (ii) the non-linear terms are responsible for the repulsive part of the nuclear force at long distance, which simulate the 3-body interaction of the nuclear force\textsuperscript{21} (help to explain the Coester band problem) (iii) similarly, the $\omega-$ meson is restraint for the repulsive part of the nuclear force (known as hard core) and (iv) the non-linear self-coupling of the $\omega-$ meson ($\frac{1}{4}c_3 V^\mu V^\mu$ is responsible for the attractive part in the very shortest ($\sim 0.2 fm$) region of the NN-potential. This short range strong attractive component of the $\omega-$ meson is important for the collapsing stage of the nucleons, when they come close to each other and overcome the barrier during the heavy-ion-collision (HIC) experiment and makes the QGP state of matter. The shifting of the barrier towards the central region make some evidence to explaining the collapsing state of the nucleon. It is worthy to mention that the values of these constants are different for different forces of RMF theory. Hence, the NN-potential somewhat change a little bit in magnitude by taking different forces, but the nature of the potential remains unchanged.

\subsection*{3.1. Energy density and Pressure density}

In the present work, we study the effect of the additional term on top of the NL3 force parameter to the Lagrangian, which comes from the self-interaction of the vector fields with $c_3$ as done in the Refs.\textsuperscript{24,25} The inclusion of this term is not new, it is already taken into account for different forces of RMF and effective field theory motivated relativistic mean field theory (E-RMF). Here, our aims to see the effect of $c_3$ to the nuclear system and the contribution to the attractive part of the hard core of NN-potential. We have solved the mean field equations self-consistently and estimated the energy and pressure density as a function of baryon density. The NL3 parameter set\textsuperscript{26,27} along with the additional $c_3$ is used in the calculations. The obtained results for different values of $c_3$ are shown in Figs. 4 and 5, respectively. From the figure, it is clearly identify that the $-ve$ value of $c_3$ gives the stiff equation of state (EOS), meanwhile the $+ve$ value shows the soft EOS. It is to be noted that mass and radius of the neutron star depends on the softness and stiffness of EOS. Here, in our investigation, we observed that the softening of the EOS depends on the non-linear coupling of the $\omega-$ meson.\textsuperscript{25} The recent
measurement of Demorest et al. \cite{28} put a new direction that the NL3 force needs slightly softer EOS. However, when we deals with G2 (E-RMF) model, the results of the Ref. \cite{29} demands a slightly stiffer EOS. This implies that, the value of $c_3$ should be fixed according to solve the above discussed problem.

3.2. Binding energy, Excitation energy and Compressibility

To see the sensitivity of $c_3$ on the finite nuclei, we calculated the binding energy (BE), giant monopole excitation energy ($E_x$) for $^{40}$Ca and $^{208}$Pb nuclei as representative cases as a function of $c_3$. The obtained results are shown in upper and lower panel of the Fig. 6, respectively. From the figure, one can observed a systematic variation of binding energy by employing the isoscalar-vector self-coupling parameter $c_3$. For example, the binding energy is monotonically changes for all $\pm e$ values of $c_3$.

Further, we analyzed the variation of compressibility modulus of $^{40}$Ca and $^{208}$Pb in the upper panel of the Fig. 7 and excitation energy of these nuclei in the lower panel. The value of the coupling constant($c_3$) varies from $-2$ to $+2$, where the result have reasonable values. We included both positive and negative value of $c_3$ to know the the discrepancy between the sign of $c_3$. Excitation energy and compressibility modulus in finite nuclei are calculated by using scaling method and the extended Thomas-Fermi approaches to relativistic mean field theory (RETF) \cite{30}. From the

Fig. 4. The energy per particle of symmetric nuclear matter as a function of baryon density for various values of $c_3$. 
calculated results, we find that the compressibility modulus as well as the monopole excitation energy of finite nuclei do not change with the increase of $c_3$ to some optimum value. It is interesting to notice that although we get a stiff equation of state with negative value of $c_3$ for infinite nuclear matter system, this behaviour does not result in finite nuclei, i.e. the $K_A$ and $E_x$ do not change for negative $c_3$. May be the density with which we deal in the finite nucleus is responsible for this discrepancy. However, $K_A$ and $E_x$ increase substantial after certain value of $c_3$, i.e. the finite nucleus becomes too much softer at about $c_3 \sim 1.0$. As a result, the incompressibility becomes large.

4. Summary and Conclusions

In this paper, we tested the effect of the non-linear self-coupling of the $\omega -$ vector meson. At extremely short distance, it gives a strongly attraction for $-ve$ value of $c_3$, which is mostly responsible for the asymptotic properties of the quarks. This short range distance is about $0.2 fm$, below which the vector—meson itself shows a strong attraction due to its self-interaction. This short-range strong attraction makes the nucleon collapse and form quark—gluon—plasma when a highly energetic projectile nucleon approaches another target nucleon of a distance of $\sim 0.2$ fm.
Fig. 6. The binding energy of $^{40}$Ca and $^{208}$Pb in their ground state for different values of $c_3$.

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Fig. 7. (a) The excitation energy as a function of $c_3$ for $^{40}$Ca and $^{208}$Pb. (b) The incompressibility of $^{40}$Ca and $^{208}$Pb as a function of $c_3$.

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