A New Interval Type-2 Fuzzy Aggregation Approach for Combining Multiple Neural Networks in Clustering and Prediction of Time Series

Martha Ramírez1 · Patricia Melin1

Received: 20 April 2022 / Revised: 17 October 2022 / Accepted: 20 October 2022 / Published online: 17 November 2022
© The Author(s) under exclusive licence to Taiwan Fuzzy Systems Association 2022

Abstract Inspired by how some cognitive abilities affect the human decision-making process, the proposed approach combines neural networks with type-2 fuzzy systems. The proposal consists of combining computational models of artificial neural networks and fuzzy systems to perform clustering and prediction of time series corresponding to the population, urban population, particulate matter (PM2.5), carbon dioxide (CO2), registered cases and deaths from COVID-19 for certain countries. The objective is to associate these variables by country based on the identification of similarities in the historical information for each variable. The hybrid approach consists of computationally simulating the behavior of cognitive functions in the human brain in the decision-making process by using different types of neural models and interval type-2 fuzzy logic for combining their outputs. Simulation results show the advantages of the proposed approach, because starting from an input data set, the artificial neural networks are responsible for clustering and predicting values of multiple time series, and later a set of fuzzy inference systems perform the integration of these results, which the user can then utilize as a support tool for decision-making with uncertainty.

Keywords Neural networks · Time series · Prediction · Clustering · Type-2 fuzzy system

1 Introduction

During the last decades, many technological changes have contributed to the use of Artificial Neural Networks (ANNs) to solve complex problems, this is due to the ability of ANNs to learn from non-linear relationships [1, 2]. In the case of supervised ANNs, they can be trained to produce desired outputs in response to sample inputs, demonstrating their effectiveness for time series prediction [3–5]. On the other hand, unsupervised ANNs can be trained with input data, but the outputs associated with them are unknown [6, 7].

The most important modeling tool based on fuzzy set theory is the fuzzy system, which consists of fuzzy rules and fuzzy reasoning for performing logical inference. Fuzzy reasoning is an inference procedure that derives conclusions from a set of fuzzy if–then rules and known facts [8].

One of the latest advances in the field of medicine is the study of how the cognitive abilities affect the human decision-making process. In the case of the cognitive flexibility, it has been described as the ability to produce alternative solutions and to switch thoughts, by choosing and using appropriate information, therefore, understanding the situation and making decisions [9]. There are different degrees of cognitive flexibility, individuals that possess lower cognitive flexibility shows rigid cognitions and perseverative behaviors often referred to as cognitive inflexibility or ‘rigidity’ [10]. On the other hand, individuals with higher cognitive flexibility can modify mental scripts and behavioral routines to change task demands by categorizing ideas and concepts in multiple ways and establishing non-obvious relationships between them rather than simply reproducing these in ways they were originally learned [11].
Therefore, these advances have influenced the creation and application of bioinspired computational models. Such is the case with artificial neural networks [12–14] and fuzzy inference systems [15–17], which, although they emerged in the previous century, in accordance with the theory of cognitive flexibility have proven to be a robust support for decision-making.

Since the beginning of 2020, the world has been affected by the arrival of the coronavirus disease 2019 (COVID-19) pandemic, caused by infection with severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) [8]. At that time the adverse effects on infected people were unknown, what was certain was its high transmissibility, so the governments of the entire world were seen in the need to establish short-term strategies like face mask-wearing and social distancing among others, to maintain the flow of economic activities and, above all, protect people’s health. Being a new threat, difficult times were lived and in a fight against time, several researchers from different branches of knowledge, in an act of solidarity, joined ranks to find a way to combat this new virus, managing to create in a couple of months different vaccine proposals to face the global pandemic [18, 19].

With the passing of the months and the advancement of the vaccination plan in different countries, the world faced new threats upon discovering several genetic variations of the virus, linked to increased transmission, immune invasion, or severe disease. By characterizing new variations in Variant of concern (VOC), Variant of interest (VOI), or Variant of high consequence (VOHQ) [20, 21]. Although the whole world maintains the sum of efforts so that the COVID-19 pandemic comes to an end, there is still interest in the scientific community in discovering the impact and trend of the COVID-19 virus in the population [22]. Therefore, countless computational models have been developed for the prediction of time series of positive cases and cases of deaths from said disease [23–25].

It should be noted that another constant global challenge is to evaluate the increase in population and minimize the level of pollutant emissions in the environment [26], for which different computer models have been developed to forecast the trend of these variables, for example: population, urban population [27, 28], particulate matter (PM$_{2.5}$), carbon dioxide (CO$_2$) [29, 30], among others.

Traditionally, statistical models are used to predict time series, such as: Autoregressive Integrated Moving Average model (ARIMA) and Seasonal Autoregressive Integrated Moving Average (SARIMA). However, due to their ability to learn from non-linear relationships, in most cases the predictions of ANNs have shown better results than conventional statistical models [31–33].

The motivation to design a model inspired by the functioning of the cognitive functions used during decision-making is that every day people face new challenges to integrate multiple sources of information and we need to consider multiple variables simultaneously, so it would be of great help for them to have a computational
tool that integrates multiple results of the clustering and prediction of time series, considering different aspects to those used by traditional statistical models.

Thus, the main contribution of this paper consists of combining models of artificial neural networks and fuzzy systems, which partially simulate the behavior of cognitive functions in the human brain when performing decision-making based on the results of clustering and prediction of time series. Thus, a set of Self-Organizing Map Neural Networks (SOMs) are used to generate data clusters of multiple time series by identifying similarities in historical data trends between countries, simultaneously a set of Nonlinear Autoregressive Exogenous (NARX) model and Nonlinear Autoregressive (NAR) neural networks are used to make predictions of time series values. Finally, a set of type-1 and type-2 fuzzy inference systems perform the integration of these results, which the user can use as a support tool for decision-making.

This approach differs from most existing intelligent computational methods [22, 34, 35], by combining both supervised and unsupervised neural networks training algorithms, and fuzzy systems, for the prediction of time series, since most computational models in the literature...
Fig. 5 Illustration of monthly COVID-19 confirmed and death cases by country
Fig. 5 continued
only use supervised training algorithms to perform the prediction of time series. Thus, one of the great advantages is that our proposal in addition of clustering and prediction of time series, it contemplates the management of uncertainty for decision making and integration of results using type-2 fuzzy inference systems.

This paper is organized as follows. In Sect. 2, the theoretical aspects are shown. The problem to be solved is described in Sect. 3. The methodology is explained in Sect. 4. The experiments and discussion of results are shown in Sects. 5 and 6, respectively. Finally, in Sect. 7, the general conclusions are presented.

Fig. 5 continued

**Table 2** Attributes of the annual total population time series

| No | Attributes              |
|----|-------------------------|
| 1  | Country name            |
| 2  | Country code            |
| 3  | Indicator name          |
| 4  | Indicator code          |
| 5  | Year                    |
| 6  | Total population        |

**Table 3** Attributes of the annual total urban population time series

| No | Attributes              |
|----|-------------------------|
| 1  | Country name            |
| 2  | Country code            |
| 3  | Indicator name          |
| 4  | Indicator code          |
| 5  | Year                    |
| 6  | Total urban population  |

Fig. 6 Illustration of monthly total annual population by country
2 Basic Concepts

In this section we present a general summary of the theoretical aspects considered to carry out the development of our proposal, which mainly contemplate artificial neural networks and fuzzy systems that have been used as bioinspired methods.

2.1 Self-organizing Map (SOM) Neural Network

SOM is an unsupervised network for classification, as it accepts an input vector \( x \) and the input weight vector \( m \) and produces a vector having \( s \) elements (Fig. 1). The elements are the negative of the distances between the input vector and the weight matrix vectors formed from the \( r \) rows of the input weight matrix. The competitive transfer function accepts a net input vector for a layer and returns neuron outputs of 0 for all neurons except for the winner. The winner's output is 1. If all biases are 0, then the neuron whose weight vector is closest to the input vector has the least negative net input and, therefore, wins the competition with an output of 1, where if we define the best match to be at unit with index \( c \) it can be determinate by Eqs. (1) and (2) that represent the similarity matching:

\[
\|x - m_c\| = \min_i\|x - m_i\|, \quad (1)
\]

\[
\|x - (t_k) - m_c(t_k)\| = \min_i\{\|x - (t_k) - m_i(t_k)\|\}. \quad (2)
\]

The weights of the winning neuron are adjusted, which allows a neuron to learn an input vector, thus, the neuron whose weight vector was closest to the input vector is updated to be even closer, and can be written as Eq. (3):

\[
m_i(t_{k+1}) = m_i(t_k) + a(t_k)[x(t_k) - m_i(t_k)] \quad \text{for } i \in N_c
\]

\[
m_i(t_{k+1}) = m_i(t_k) \quad \text{otherwise},
\]

\[
(3)
\]

where \( m \) is the input weight vector, \( i \) is an index position, \( t_k \) is a vector time index, \( z \) is the learning rate and \( x \) is an input vector [36].

![Fig. 7 Illustration of monthly total annual urban population by country](image)

![Table 4 Attributes of the time series of PM2.5 air pollution](table)

| No | Attributes                        |
|----|----------------------------------|
| 1  | Country name                     |
| 2  | Country code                     |
| 3  | Indicator name                   |
| 4  | Indicator code                   |
| 5  | Year                             |
| 6  | PM2.5 air pollution              |

\[123\]
2.2 Nonlinear Autoregressive with Exogenous (NARX)

In the NARX neural network (Fig. 2), the future values of a time series $y(t)$ are predicted from past values of that series and past values of a second time series $x(t)$, and can be written as Eq. (4) [37]:

$$y(t) = f(y(t-1), y(t-2), \ldots, y(t-d), x(t-1), x(t-2), \ldots, x(t-d))$$

(4)

where $y(t)$ represents a value of the time series $y$ at time $t$, $x(t)$ represents a value of a second time series $x$ at time $t$, $d$ is a time delay parameter and $f$ is an activation function.

2.3 Nonlinear Autoregressive (NAR)

In the NAR neural network (Fig. 3), the future values of a time series $y(t)$ are predicted only from past values of that series and can be written as Eq. (5) [38]:

$$y(t) = f(y(t-1), y(t-2), \ldots, y(t-d))$$

(5)

where $y(t)$ represents a value of the time series $y$ at time $t$, $d$ is a time delay parameter, and $f$ is an activation function.

2.4 Type-2 Fuzzy Systems

The main idea of type-1 fuzzy logic is modeling the vagueness in linguistic concepts, while in Interval type-2 fuzzy logic the main goal is modeling uncertainty, which affects decision-making and appears in number different of forms. It is well known that uncertainty is an attribute of information.

The basic structure of a fuzzy inference system consists of three conceptual components: a rule base, which contains a selection of fuzzy rules, a database, which defines the membership functions used in the fuzzy rules; and a reasoning mechanism, which perform the inference procedure upon the rules and given facts to derive a reasonable output or conclusion, which is almost always fuzzy sets.
Therefore, we need a method of defuzzification to extract a crisp value that best represents a fuzzy set [8].

The basics of fuzzy logic do not change from type-1 to type-2 fuzzy sets, basically, type-2 fuzzy systems consist of fuzzy if–then rules, in which antecedent or consequent membership functions are type-2 fuzzy sets. Type-2 is a generalization of conventional fuzzy logic (type-1) in the sense that uncertainty is not only limited to the linguistic variables, but also is present in the definition of the membership functions. A type-reduced set of the type-2 fuzzy logic system (FLS) can be thought of as representing the uncertainty in the crisp output due to the perturbation. This is analogous to using intervals in a stochastic-uncertainty situation. We defuzzify the type-reduced set to get a crisp output from the type-2 FLS. The most natural way to do this seems to be finding the centroid of the type-reduced set. Finding the centroid is equivalent to finding the weighted average of the outputs of all the type-1 FLSs that are embedded in the type-2 FLS, where the weights correspond to the memberships in the type-reduced set [39]. The main difference is in the concept of membership degree, so in type-1 fuzzy logic the membership degree is a crisp value between 0 and 1, however, in type-2 fuzzy logic the membership degree is an interval with two boundaries between 0 and 1. So, the amount of uncertainty in a system can be reduced by using type-2 fuzzy logic because it offers better capabilities to handle linguistic uncertainties by modeling vagueness and unreliability of information [40].

The mathematical representation of an Interval Type-2 Fuzzy Set can be expressed as Eq. (6):

\[ J_x = \left\{ (x, u) \mid u \in \left[ \mu_A(x), \bar{\mu}_A(x) \right] \right\}, \]

where \( \mu_A(x) \) and \( \bar{\mu}_A(x) \) correspond to the boundaries of the fuzzy set, usually known as lower and upper membership functions, respectively.

The mathematical expression of the Footprint of Uncertainty (FOU) is presented as Eq. (7):

\[ \text{FOU} \in \left[ \mu_A(x), \bar{\mu}_A(x) \right], \]

where the \( \mu_A(x) \) and \( \bar{\mu}_A(x) \) are the lower and upper membership functions, respectively [41].

For the rule base and fuzzy inference engine, in an Interval Type-2 Mamdani FIS that performs the same process a Type-1, but for upper and lower rules firing forces respectively. The inference is computed based on the Modus ponens inference (fuzzy logic version), as shown in Eq. (8):
IF $x_i \in F_{l}$ and ... and $x_p \in F_{l}$, THEN $y \in G^l$, where $l = 1, ..., M$ [40].

3 Problem Description

We aim at identifying associations between different time series: population, urban population, particulate matter (PM$_{2.5}$), carbon dioxide (CO$_2$), and COVID-19 using an intelligent hybrid computational model.

For this work we selected five datasets for each of the following 13 countries (Fig. 4): Belize (BLZ), Brazil (BRA), Canada (CAN), China (CHN), Spain (ESP), France (FRA), Guatemala (GTM), India (IND), Italy (ITA), Mexico (MEX), Poland (POL), Russian Federation (RUS), United States (USA).

Below we show the description of each selected dataset, and it should be noted that no preprocessing was carried out prior to its use.

The first dataset consists of six attributes (Table 1) for 325 instances corresponding to the daily number of...
COVID-19 new confirmed cases and new deaths, from January 23, 2020 to January 23, 2022 [42].

Also, we obtained the monthly average value for each one of the 13 countries, 25 data points per variable (Fig. 5).

The second dataset consists of six attributes (Table 2) for 61 instances corresponding to the total annual population, from 1960 to 2020 (Fig. 6) [43].

The third dataset consists of six attributes (Table 3) for 61 instances corresponding to the total annual urban population, from 1960 to 2020 (Fig. 7) [44].

The fourth dataset consists of six attributes (Table 4) for 8 instances corresponding to the PM2.5 air pollution, mean annual exposure (micrograms per cubic meter) from 2010 to 2017 (Fig. 8) [45].
The fifth dataset consists of six attributes (Table 5) for 59 instances corresponding total annual CO₂ emissions, from 1960 to 2018 (Fig. 9) [46].

Also, below we can find the correlation matrices (Figs. 10, 11, 12, and 13) for the time series population, urban population, particulate matter (PM₂.₅), carbon dioxide (CO₂). For the case of the COVID-19 time series, and we selected the Brazil country to illustrate the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots (Figs. 14 and 15).

4 Proposed Method

In this work, we propose a computational model that consists in three phases: the first one is to use a Self-Organizing Map (SOM) Neural Network for clustering tasks with respect to their performance to find similarities and label each element of the dataset with the class that belongs to it, that means, it learns to classify inputs vectors into a given number of classes.
To solve the problem, it is a priority to find similarities between the attributes of the data set and because our interest is to classify the information based on the value of each variable, we have configured the SOM neural network to group the information into four classes: Cluster 1 (C1), Cluster 2 (C2), Cluster 3 (C3), and Cluster 4 (C4), respectively.

For the prediction tasks (in the second phase), because of the multiple time series considered in this study case, we used both a Nonlinear Autoregressive with Exogenous (NARX) neural network and a Nonlinear Autoregressive (NAR) neural network. Finally, in the third phase, a set of type-1 and type-2 fuzzy inference systems is used to classify the countries by integrating the results of the neural networks.

The structure of the type-1 fuzzy system used to integrate the results of the SOM neural networks consists of four inputs and one output (Table 6).

The fuzzy system is of Mamdani type, four inputs and one output, 8 rules and centroid defuzzification method, where the membership function parameters and fuzzy rules were exhaustively tested, until this selection of parameters values was obtained (Fig. 16).
Fig. 14 Illustration of Autocorrelation Function (ACF) for COVID-19 BRA time series

Fig. 15 Illustration of Partial Autocorrelation Function (PACF) for COVID-19 BRA time series
The structure of the type-1 fuzzy systems used to integrate the results of the NAR and NARX neural networks consists of two inputs and one output (Table 7).

It is of Mamdani type, two inputs and one output, 9 rules and centroid defuzzification method, where the membership function parameters and fuzzy rules were exhaustively tested, until this selection of parameters was obtained (Fig. 17).

The structure of the type-2 fuzzy system used to integrate the results of the SOM, NAR, and NARX neural networks consists of three inputs and one output (Table 8).

It is Mamdani type, three inputs and one output, 27 rules and centroid defuzzification method, where the type-2 membership function parameters and fuzzy rules were exhaustively tested, until this selection of parameter values was achieved (Fig. 18).

We can encompass our proposal as described below: For the first phase, once the information belonging to each cluster has been identified using SOM neural networks, then the prediction results are obtained using NAR and NARX neural networks, for the second phase, also a set of type-1 fuzzy inference systems is used to associate the results of the neural networks. So that in the third and last phase, with these results the inputs of type-2 fuzzy inference system are generated, which operates as an integrator of the results to achieve a final global result (Fig. 19).

In Eq. (9) we show how the final output of type-2 fuzzy system is calculated (after type-reduction), which is basically the weighted average of the outputs of the fuzzy rules, where the membership functions of the rules are the weights:

\[
y(t) = \frac{\mu_1 y_1(t) + \mu_2 y_2(t) + \ldots + \mu_{27} y_{27}(t)}{\mu_1 + \mu_2 + \ldots + \mu_{27}},
\]

where \(y_i(t)\) are the outputs of the rules, \(i = 1, \ldots, 27\), \(\mu_i\) are the membership function values at the outputs of the rules, \(i = 1, \ldots, 27\), and \(y(t)\) is the total output.

5 Experimental Results

By using SOM networks, we classify the variables into four classes based on the average total annual records (Tables 9 and 10). For the case of population (Cluster 1 includes 8 countries, Cluster 2 and Cluster 3 include 2 countries, respectively, and Cluster 4 includes one country), urban population (Cluster 1 includes 7 countries, Cluster 2
includes 3 countries, Cluster 3 includes one country, and Cluster 4 includes two countries), PM25 (Cluster 1 and Cluster 2 include 4 countries, respectively, Cluster 3 includes 3 countries and Cluster 4 includes two countries), CO₂ (Cluster 1 includes 9 countries, Cluster 2 includes two countries, Cluster 3 and Cluster 4 include one country, Cluster 3 includes one country, and Cluster 4 includes two countries).
respectively), Covid-19 deaths (Cluster 1 includes 9 countries, Cluster 2 includes two countries, Cluster 3 and Cluster 4 include one country, respectively), and finally, Covid-19 cases (Cluster 1 includes 10 countries, Cluster 2, Cluster 3, and Cluster 4 include one country, respectively).

For the prediction of four variables, we used a NARX neural network with 10 neurons in the hidden layer. For each experiment 30 executions were performed, and the complete data set was considered, 70% was used for training and 30% for testing (Table 11 shows the relative root mean squared error (RMSE)).

In Fig. 20, we show the response of outputs for the prediction of four variables population, urban population, CO2, Covid-19 cases, based on simulation results of

![Diagram](image-url)

**Fig. 17** Illustration of second type-1 fuzzy system: triangular membership functions and fuzzy rules

**Table 8** Type-2 fuzzy system variables (type-1 fuzzy systems results)

| Variables     | Membership function type | Membership function |
|---------------|--------------------------|---------------------|
| Input1 INDEX1F | Triangular               | Low                 |
|               | Triangular               | Medium              |
|               | Triangular               | High                |
| Input2 INDEX2F | Triangular               | Low                 |
|               | Triangular               | Medium              |
|               | Triangular               | High                |
| Input3 INDEX3F | Triangular               | Low                 |
|               | Triangular               | Medium              |
|               | Triangular               | High                |
| Output INDEX1 | Triangular               | Low                 |
|               | Triangular               | Medium              |
|               | Triangular               | High                |
Table 11. In Fig. 21, the autocorrelation error for multiple variables is illustrated. Also, we used a NAR neural network with 10 neurons in the hidden layer to make the prediction of the four variables: population, urban population, CO2, and Covid-19 cases. For each experiment, the complete data set was considered, 70% was used for training and 30% for testing, and 30 executions were performed (Table 12 shows the relative root mean squared error (RMSE)).

In Fig. 22, we show the response of outputs for the prediction of four variables: population, urban population, CO2, Covid-19 cases, based on simulation results of Table 12. In Fig. 23, the autocorrelation error for individual variables is shown.

Based on the prediction results obtained using NAR and NARX neural networks, we can find in Table 13 that in most cases the NARX network obtained better results, except for the CO2 prediction in which the NAR network produced a better result.

Subsequently, we classified using a Mamdani type-1 fuzzy Inference System with four inputs and one output, based on the variables: population, urban population, CO2, and Covid-19 cases, the level reached for each of the countries according to the clusters made by the neural system.

Table 12. In Fig. 21, the autocorrelation error for multiple variables is illustrated. Also, we used a NAR neural network with 10 neurons in the hidden layer to make the prediction of the four variables: population, urban population, CO2, and Covid-19 cases. For each experiment, the complete data set was considered, 70% was used for training and 30% for testing, and 30 executions were performed (Table 12 shows the relative root mean squared error (RMSE)).

In Fig. 22, we show the response of outputs for the prediction of four variables: population, urban population, CO2, Covid-19 cases, based on simulation results of Table 12. In Fig. 23, the autocorrelation error for individual variables is shown.

Based on the prediction results obtained using NAR and NARX neural networks, we can find in Table 13 that in most cases the NARX network obtained better results, except for the CO2 prediction in which the NAR network produced a better result.

Subsequently, we classified using a Mamdani type-1 fuzzy Inference System with four inputs and one output, based on the variables: population, urban population, CO2, and Covid-19 cases, the level reached for each of the countries according to the clusters made by the neural system.
Table 9 Clusters of countries by individual variables

| Variables         | Cluster 1 (C1) | Cluster 2 (C2) | Cluster 3 (C3) | Cluster 4 (C4) |
|-------------------|---------------|---------------|---------------|---------------|
| Population        | 8             | 2             | 2             | 1             |
| Urban population  | 7             | 3             | 1             | 2             |
| PM25              | 4             | 4             | 3             | 2             |
| CO2               | 9             | 2             | 1             | 1             |
| Covi19 deaths     | 9             | 2             | 1             | 1             |
| Covid19 cases     | 10            | 1             | 1             | 1             |

Table 10 Clusters of individual variables by country

| Countries | Population | Urban population | PM25 | CO2 | Covi19 deaths | Covid19 cases |
|-----------|------------|-----------------|------|-----|--------------|---------------|
| BLZ       | C1         | C1              | C1   | C1  | C1           | C1            |
| BRA       | C2         | C2              | C2   | C2  | C2           | C1            |
| CAN       | C1         | C1              | C3   | C1  | C1           | C1            |
| CHN       | C3         | C3              | C4   | C1  | C1           | C1            |
| ESP       | C1         | C1              | C3   | C1  | C1           | C1            |
| FRA       | C1         | C1              | C2   | C1  | C1           | C2            |
| GTM       | C1         | C1              | C1   | C1  | C1           | C1            |
| IND       | C3         | C4              | C4   | C2  | C2           | C3            |
| ITA       | C1         | C1              | C2   | C1  | C1           | C1            |
| MEX       | C1         | C2              | C1   | C1  | C1           | C1            |
| POL       | C1         | C1              | C1   | C1  | C1           | C1            |
| RUS       | C2         | C2              | C2   | C3  | C3           | C1            |
| USA       | C4         | C4              | C3   | C4  | C4           | C4            |
networks SOM (very few, few, many, too many) records. The membership function parameters and fuzzy rules were exhaustively tested, until this selection of parameters was obtained, seeking to integrate the results obtained by each of the SOM neural networks, with respect to the cluster assigned to each country, according to the corresponding variable (Tables 14 and 15).

The level of increase of a variable was calculated by subtracting from the final value (prediction made by the neural network) the initial value (original data) of the time series. The result obtained was then divided by the starting value (original data). Once the level of increase was obtained for each of the four variables: population, urban population, CO\textsubscript{2}, and Covid-19 cases, we used a second Mamdani type-1 fuzzy inference system with two inputs and one output, to classify the level of increase of population and urban population variables. The membership function parameters and fuzzy rules were exhaustively

Table 11 NARX Prediction of values for multiple variables

| Primary variable | Secondary variable | Average %RMSE | Best %RMSE | Worst %RMSE |
|------------------|--------------------|---------------|------------|-------------|
| Population       | Urban population   | 0.000096798   | 0.000004321| 0.001512516 |
| Urban population  | Population         | 0.000137810   | 0.000007525| 0.002548414 |
| CO\textsubscript{2} | Population        | 0.031689279   | 0.005474208| 0.078434301 |
| Covid19 cases    | Covid19 deaths     | 0.000076041   | 0.000018428| 0.000166901 |

Fig. 20 Illustration of response outputs time series prediction of multiple variables
tested, until this selection of parameter values was obtained (Tables 16 and 17).

Subsequently, a third Mamdani type-1 fuzzy inference system with two inputs and one output was used to classify the level of increase of the variables CO₂ and Covid-19 cases, where the membership function parameters and fuzzy rules were exhaustively tested, until this selection of parameter values was obtained (Tables 18 and 19).

Finally, we use a type-2 fuzzy inference system to classify the level of general increase of the variables population, urban population, CO₂, and Covid-19 cases, based on the outputs of the type-1 fuzzy inference systems (Tables 20 and 21).

### 6 Discussion of Results

In this work, we are looking for segregating the tasks obtained by each of the supervised and unsupervised neural networks, and then a decision-making process can be carried out by using a set of fuzzy inference systems. In this area, the results have shown that it is possible to use neural networks to find the clusters based on the similarity of the data and to be able to identify countries with similar statistics, with which it is possible to obtain a general view about the behavior of multiple variables in several places simultaneously.

On the other hand, the simulations carried out showed that both the NARX and NAR neural networks used to make predictions of the variables: population, urban

---

**Table 12 NAR Prediction of values for individual variables**

| Variable         | Average %RMSE | Best %RMSE | Worst %RMSE |
|------------------|---------------|------------|-------------|
| Population       | 0.000390816   | 0.000039335| 0.002322634 |
| Urban population | 0.001407223   | 0.000071357| 0.009816944 |
| CO2              | 0.001486522   | 0.000649318| 0.003356711 |
| Covid19 cases    | 0.010909516   | 0.005269325| 0.036478969 |

---

![Fig. 21 Illustration of autocorrelation of error for multiple variables](image-url)
population, CO\(_2\), and Covid-19 cases, are a good option to estimate time series variables. Type-1 and type-2 fuzzy inference systems proved to be useful for integrating the different results obtained using supervised and unsupervised neural networks, seeking to establish similarities in the historical information of multiple variables among a sample of countries.

### 7 Conclusion

We have presented in this work a model for the clustering and prediction of time series using a Self-Organizing Map Neural Network as an unsupervised method, and as a supervised method, the Nonlinear Autoregressive with Exogenous (NARX) and Nonlinear Autoregressive (NAR) neural networks were used.

The simulation results have shown that it is possible to use unsupervised SOM neural networks to find the clusters based on the similarity of the historical data trends between countries. Also, it is possible to use supervised NARX and NAR neural networks to make the prediction of time series: population, urban population, particulate matter (PM\(_{2.5}\)), carbon dioxide (CO\(_2\)), and Covid-19 cases. Finally, by using type-1 and type-2 fuzzy inference systems to classify the countries by integrating the results of neural networks, which partially simulate the behavior of cognitive functions in the human brain when deciding and so helping the end-user during the decision-making process.
Through the experiments that were carried out, we identified the advantages of proposed method, by combining models of artificial neural networks and fuzzy systems to perform clustering and prediction of time series, which by having information segments grouped by similar attributes allows us to obtain specific results for a particular country. Thus, one of the great advantages is that our proposal, in addition of clustering and prediction of time series, is that it contemplates the management of uncertainty for decision making and integration of results using type-2 fuzzy inference systems.

As future work, we consider conducting tests with new data sets, focusing on the fuzzy integration method, looking to optimize the learning rules and the parameters of the membership functions of the different type-1 and type-2 fuzzy systems, this is because it is also intended to improve the prediction results of each region by using hybrid classification and prediction techniques.

**Table 13** NAR vs NARX Comparison of prediction results

| Variable        | NAR Average %RMSE | NAR Best %RMSE  | NAR Worst %RMSE | NARX Average %RMSE | NARX Best %RMSE  | NARX Worst %RMSE |
|-----------------|-------------------|-----------------|-----------------|---------------------|------------------|-----------------|
| Population      | 0.000390816       | 0.000039335     | 0.002322634     | 0.000096798         | 0.00004321       | 0.001512516     |
| Urban population| 0.001407223       | 0.000071357     | 0.009816944     | 0.000137810         | 0.00007525       | 0.002548414     |
| CO2             | **0.001486522**   | **0.000649318** | **0.003356711** | 0.031689279         | 0.005474208      | 0.078434301     |
| Covid-19 cases  | 0.010909516       | 0.005269325     | 0.036478969     | **0.00076041**      | **0.00018428**   | **0.000166901** |

Bold means best result values in the table.
Table 14  SOM results as parameters of Type-1 FIS membership function

| Variables            | Membership function type | Membership functions | Parameter a | Parameter b | Parameter c |
|----------------------|--------------------------|----------------------|-------------|-------------|-------------|
| Input 1 Population   | Triangular               | Very few             | 0.122       | 0.422       | 1.298       |
|                      | Triangular               | Few                  | 0.894       | 1.544       | 2.166       |
|                      | Triangular               | Many                 | 1.894       | 2.460       | 3.200       |
|                      | Triangular               | Too many             | 2.900       | 3.520       | 3.920       |
| Input 2 Urban population | Triangular          | Very few             | 0.122       | 0.422       | 1.298       |
|                      | Triangular               | Few                  | 0.894       | 1.544       | 2.166       |
|                      | Triangular               | Many                 | 1.894       | 2.460       | 3.200       |
|                      | Triangular               | Too many             | 2.900       | 3.520       | 3.920       |
| Input 3 CO2           | Triangular               | Very few             | 0.122       | 0.422       | 1.298       |
|                      | Triangular               | Few                  | 0.894       | 1.544       | 2.166       |
|                      | Triangular               | Many                 | 1.894       | 2.460       | 3.200       |
|                      | Triangular               | Too many             | 2.900       | 3.520       | 3.920       |
| Input 4 Covid19 cases| Triangular               | Very few             | 0.122       | 0.422       | 1.298       |
|                      | Triangular               | Few                  | 0.894       | 1.544       | 2.166       |
|                      | Triangular               | Many                 | 1.894       | 2.460       | 3.200       |
|                      | Triangular               | Too many             | 2.900       | 3.520       | 3.920       |
| Output 1 Country level| Triangular              | Very few             | 0.122       | 0.422       | 1.298       |
|                      | Triangular               | Few                  | 0.894       | 1.544       | 2.166       |
|                      | Triangular               | Many                 | 1.894       | 2.460       | 3.200       |
|                      | Triangular               | Too many             | 2.900       | 3.520       | 3.920       |

Table 15  Output first Type-1 FIS country level

| Countries | BLZ | BRA | CAN | CHN | ESP | FRA | GTM | IND | ITA | MEX | POL | RUS | USA |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Level     | Few | Few | Few | Few | Few | Few | Few | Many| Few | Few | Few | Few | Too many |

Table 16  NARX results as parameters of Type-1 FIS membership function

| Variables           | Membership function type | Membership functions | Parameter a | Parameter b | Parameter c |
|---------------------|--------------------------|----------------------|-------------|-------------|-------------|
| Input 1 CO2         | Triangular               | Low                  | 0.000       | 0.600       | 1.200       |
|                     | Triangular               | Medium               | 0.900       | 1.600       | 2.200       |
|                     | Triangular               | High                 | 2.000       | 2.500       | 3.000       |
| Input 2 Covid19 cases | Triangular             | Low                  | 0.000       | 0.600       | 1.200       |
|                     | Triangular               | Medium               | 0.900       | 1.600       | 2.200       |
|                     | Triangular               | High                 | 2.000       | 2.500       | 3.000       |
| Output 1 Increase level | Triangular            | Low                  | 0.000       | 0.600       | 1.200       |
|                     | Triangular               | Medium               | 0.900       | 1.600       | 2.200       |
|                     | Triangular               | High                 | 2.000       | 2.500       | 3.000       |
### Table 17 Output second Type-1 FIS variables increase level

| Countries | BLZ | BRA | CAN | CHN | ESP | FRA | GTM | IND | ITA | MEX | POL | RUS | USA |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Increase Level | Medium | Medium | Medium | Low | Medium | Low | Medium | Low | Low | Medium | Low | Medium | Low |

### Table 18 NAR results as parameters of Type-1 FIS membership function

| Variables | Membership function type | Membership functions | Parameter a | Parameter b | Parameter c |
|-----------|--------------------------|----------------------|-------------|-------------|-------------|
| Input1 CO2 | Triangular | Low | 0.000 | 0.600 | 1.200 |
|           | Triangular | Medium | 0.900 | 1.600 | 2.200 |
|           | Triangular | High | 2.000 | 2.500 | 3.000 |
| Input 2 Covid19 cases | Triangular | Low | 0.000 | 0.600 | 1.200 |
|           | Triangular | Medium | 0.900 | 1.600 | 2.200 |
|           | Triangular | High | 2.000 | 2.500 | 3.000 |
| Output 1 Increase level | Triangular | Low | 0.000 | 0.600 | 1.200 |
|           | Triangular | Medium | 0.900 | 1.600 | 2.200 |
|           | Triangular | High | 2.000 | 2.500 | 3.000 |

### Table 19 Output third Type-1 FIS variables increase level

| Countries | BLZ | BRA | CAN | CHN | ESP | FRA | GTM | IND | ITA | MEX | POL | RUS | USA |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Increase Level | Medium | Medium | Medium | Low | Medium | Low | Medium | Low | Medium | Low | Medium | Low |

### Table 20 ANN and Type-1 FIS results as parameters of Type-2 FIS membership function

| Variables | Membership function type | Membership function a | b | c | Lower Scale | Lower Lag |
|-----------|--------------------------|----------------------|----|----|-------------|-----------|
| Input1 INDICATOR1F | Triangular | 0.1223 | 0.6223 | 1.2980 | 0.1000 | 0.2000 | 0.2000 |
|           | Triangular | 0.9000 | 1.4000 | 1.9000 | 1.1000 | 0.2000 | 0.2000 |
|           | Triangular | 1.9000 | 2.4000 | 2.9000 | 2.1000 | 0.2000 | 0.2000 |
| Input2 INDICATOR2F | Triangular | 0.1223 | 0.6223 | 1.2980 | 0.1000 | 0.2000 | 0.2000 |
|           | Triangular | 0.9000 | 1.4000 | 1.9000 | 1.1000 | 0.2000 | 0.2000 |
|           | Triangular | 1.9000 | 2.4000 | 2.9000 | 2.1000 | 0.2000 | 0.2000 |
| Input3 INDICATOR3F | Triangular | 0.1223 | 0.6223 | 1.2980 | 0.1000 | 0.2000 | 0.2000 |
|           | Triangular | 0.9000 | 1.4000 | 1.9000 | 1.1000 | 0.2000 | 0.2000 |
|           | Triangular | 1.9000 | 2.4000 | 2.9000 | 2.1000 | 0.2000 | 0.2000 |
| Output 1 KEY INDICATOR | Triangular | 0.0000 | 0.5000 | 1.0000 | 0.1000 | 0.2000 | 0.2000 |
|           | Triangular | 1.0000 | 1.5000 | 2.0000 | 1.1000 | 0.2000 | 0.2000 |
|           | Triangular | 2.0000 | 2.5000 | 3.0000 | 2.1000 | 0.2000 | 0.2000 |

### Table 21 Output Type-2 FIS variables increase level

| Countries | BLZ | BRA | CAN | CHN | ESP | FRA | GTM | IND | ITA | MEX | POL | RUS | USA |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Increase Level | Medium | Medium | Medium | Low | Medium | Low | Medium | Low | Medium | Low | Medium | Medium |
Intelligence and Computing and Cyber Science and Technology Congress (DASC/PIcom/DataCom/CyberSciTech), 2018, pp. 1074–1079, doi: https://doi.org/10.1109/DASC/PIcom/DataCom/CyberSciTech.2018.00178.

30. Austin, E., Coull, B., Zanobetti, A., Koutrakis, P.: A framework to spatially cluster air pollution monitoring sites in US based on the PM2.5 composition. Environ. Int. 59, 244–254 (2013). https://doi.org/10.1016/j.envint.2013.06.003

31. Arun-Kumar, K.E., et al.: Comparative analysis of Gated Recurrent Units (GRU), long Short-Term memory (LSTM) cells, autoregressive Integrated moving average (ARIMA), seasonal autoregressive Integrated moving average (SARIMA) for forecasting COVID-19 trends. Alexandria Eng. J. 61(10), 7585–7603 (2022). https://doi.org/10.1016/j.aej.2022.01.011

32. Parasyris, A., Alexandrakis, G., Kozyrakis, G.V., Spanoudaki, K., Kampanis, N.A.: Predicting meteorological variables on local level with SARIMA, LSTM and hybrid techniques. Atmosphere 13(6), 878 (2022). https://doi.org/10.3390/atmos13060878

33. Wasesa, M., et al.: SARIMA and artificial neural network models for forecasting electricity consumption of a microgrid based educational building. IEEE Int. Conf. Ind. Eng. Eng. Manag. (IEEM) 2020, 210–214 (2020). https://doi.org/10.1109/IEEM45057.2020.9309943

34. Ding, X., Hao, K., Cai, X., Tang, S., Chen, L., Zhang, H.: A novel similarity measurement and clustering framework for time series based on convolution neural networks. IEEE Access 8, 173158–173168 (2020). https://doi.org/10.1109/ACCESS.2020.3025048

35. Barbounis, T.G., Theocaris, J.B.: Locally recurrent neural networks for wind speed prediction using spatial correlation. Inform. Sci. 177(24), 5775–5797 (2007). https://doi.org/10.1016/j.ins.2007.05.024

36. Kohonen T.: Self-Organizing Feature Maps. In: Self-Organiz-ation and Associative Memory. Springer Series in Information Sciences, vol 8. Springer, Berlin, Heidelberg, 1988, pp. 127–136, https://doi.org/10.1007/978-3-662-00784-6_5

37. Xu, M., Song, R., Zhao, Y., Song, B., Tang, J.: Application of NARX dynamic neural network in blood glucose prediction model, 2020 IEEE 9th Data Driven Control and Learning Systems Conference (DDCLS), 2020, pp. 178–183, doi: https://doi.org/10.1109/DDCLS49620.2020.9275222.

38. Mónica, J.C., Melin, P., Sánchez, D.: Genetic Optimization of Ensemble Neural Network Architectures for Prediction of COVID-19 Confirmed and Death Cases. In: Castillo O., Melin P. (eds) Fuzzy Logic Hybrid Extensions of Neural and Optimization Algorithms: Theory and Applications. Studies in Computational Intelligence, vol 940. Springer, Cham., 2021, pp. 85–98, https://doi.org/10.1007/978-3-030-68776-2_5

39. Castillo, O., Melin, P.: 3 type-2 fuzzy logic. In: Type-2 Fuzzy Logic: Theory and Applications. Studies in Fuzziness and Soft Computing, vol 223. Springer, Berlin 2007, pp. 29–43

40. Melin, P., Ontiveros-Robles, E., Castillo, O.: Background and Theory. In: New Medical Diagnosis Models Based on Generalized Type-2 Fuzzy Logic. SpringerBriefs in Applied Sciences and Technology. Springer, Cham, 2021, pp. 5–28.

41. Ontiveros-Robles, E., Castillo, O., Melin, P.: Towards asymmetric uncertainty modeling in designing General Type-2 Fuzzy classifiers for medical diagnosis. Expert Syst. Appl. 183, 115370 (2021). https://doi.org/10.1016/j.eswa.2021.115370

42. Ritchie, H. et al.: Coronavirus Pandemic (COVID-19). Published online at OurWorldInData.org. Retrieved from: https://ourworldindata.org/coronavirus

43. The World Bank Data: Population, total, 2022. Retrieved from: https://data.worldbank.org/indicator/SP.POP.TOTL

44. The World Bank Data: CO2 emissions (kt), 2022. Retrieved from: https://data.worldbank.org/indicator/EN.ATM.CO2E.KT

45. The World Bank Data: PM2.5 air pollution, mean annual exposure (micrograms per cubic meter), 2022. Retrieved from: https://data.worldbank.org/indicator/EN.ATM.PM25.MC.M3

46. The World Bank Data: PM2.5 air pollution, mean annual exposure (micrograms per cubic meter), 2022. Retrieved from: https://data.worldbank.org/indicator/EN.ATM.PM25.MC.M3

Springer Nature or its licensor (e.g. a society or other partner) holds exclusive rights to this article under a publishing agreement with the author(s) or other rightsholder(s); author self-archiving of the accepted manuscript version of this article is solely governed by the terms of such publishing agreement and applicable law.

Martha Ramírez holds the B.S. in Computer Science, M.S. in Computer Science, and currently she is enrolled in the Ph.D. in Computer Science in Tijuana Institute of Technology. Her current research interests include hybrid intelligent systems, neural networks and fuzzy systems. She has published more 10 papers in time series prediction using neural network models.

Patricia Melin Prof. holds the Doctor in Science degree (Doctor Habilitatus D.Sc.) in Computer Science from the Polish Academy of Sciences. She is a Professor of Computer Science in the Graduate Division, Tijuana Institute of Technology, Tijuana, Mexico, since 1998. In addition, she is serving as Director of Graduate Studies in Computer Science and head of the research group on Hybrid Neural Intelligent Systems (2000-present). She is past President of NAFIPS (North American Fuzzy Information Processing Society) 2019–2020. Prof. Melin is the founding Chair of the Mexican Chapter of the IEEE Computational Intelligence Society. She is member of the IEEE Neural Network Technical Committee (2007 to present), the IEEE Fuzzy System Technical Committee (2014 to present) and is Chair of the Task Force on Hybrid Intelligent Systems (2007 to present) and she is currently Associate Editor of the Journal of Information Sciences and IEEE Transactions on Fuzzy Systems. She is member of NAFIPS, IFSA, and IEEE. She belongs to the Mexican Research System with level III. Her research interests are in Modular Neural Networks, Type-2 Fuzzy Logic, Pattern Recognition, Fuzzy Control, Neuro-Fuzzy and Genetic-Fuzzy hybrid approaches. She has published over 220 journal papers, 10 authored books, 30 edited books, more than 200 chapters in books and more than 300 papers in conference proceedings with h-index of 69 in Scopus. She has served as Guest Editor of several Special Issues in the past, in journals like: Applied Soft Computing, Intelligent Systems, Information Sciences, Non-Linear Studies, JAMRIS, Fuzzy Sets and Systems. Prof. Melin is Associate Editor of the IEEE Transactions of Fuzzy Systems, Journal of Information Sciences, Journal of
Engineering Applications of Artificial Intelligence and Journal of Complex and Intelligent Systems. She has been recognized as Highly Cited Researcher in 2017 and 2018 by Clarivate Analytics because of having multiple highly cited papers in Web of Science.