Expressibility of Norms in Temporal Logic

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Abstract

In this short note we address the issue of expressing norms (such as obligations and prohibitions) in temporal logic. In particular, we address the argument from [1] that norms cannot be expressed in Linear Time Temporal Logic (LTL).

Recently, doubts were raised in [1] regarding suitability of LTL and other temporal logics for expressing ‘real life’ norms. We claim that the argument in [1] does not show that LTL is unsuitable for expressing norms. Instead it shows that a translation of deontic notions such as obligations and permissions into temporal logic which interprets ‘obligatory’ as ‘always true’ and ‘permitted’ as ‘eventually true’ does not work, as could be expected. However, [1] is now often cited as an argument against using temporal logic for specifying norms in general. We would like to revisit the example which is considered paradoxical when specified in LTL in [1], and show that it is possible to exactly specify the set of conditions on runs which satisfy the norms from the example using standard LTL.

We assume that the reader is familiar with temporal logic, in particular with the syntax and semantics of LTL. We use $U$ for the Until operator, $G$ for always in the future, $F$ for some time in the future.

The example is as follows (we compress it slightly without changing the meaning, and use the same variable names for propositions):

1. collection of personal information ($A$) is forbidden unless authorised by the court ($C$)

2. The destruction of personal information collected illegally before accessing it ($B$) excuses the illegal collection

3. collection of medical information ($D$) is forbidden unless collection of personal information is permitted

As pointed out in [1], this classifies possible situations as compliant and non-compliant as follows:
• situations satisfying $C$ are compliant
• situations not satisfying $C$, where $A$ happens but $B$ happens as well, are weakly compliant
• situations where $C$ is false, where $A$ happens and $B$ does not, are violations
• situations not satisfying $C$ where $D$ happens are violations
• situations not satisfying $C$ but also not satisfying $A$ and $D$ are compliant

The classification above is not very precise, since $A$, $B$, $C$, and $D$ are treated as state properties which are true or false at the same time. Later in [1] a temporal relation between $A$ and $B$ is introduced: if $C$ is false and $A$ happens, then $B$ should happen some time after that to compensate for the violation of $A$.

Hence it is very easy to classify runs into compliant or violating in LTL:

• Fully compliant runs:
  $$G(C \lor (\neg C \land \neg A \land \neg D))$$
  (everywhere, either there is a court authorisation, or there is no collection of personal or medical information)

• Weakly compliant runs:
  $$F(\neg C \land A) \land G(\neg C \land A \rightarrow FB) \land G(\neg C \rightarrow \neg D)$$
  (there is at least one violation of prohibition on collection of personal information, but each such violation is compensated by $B$ in the future; there are no violations of prohibition on collecting medical information)

Finally, violations are specified as follows:

• Violating runs:
  $$F(\neg C \land (D \lor (A \land \neg FB)))$$

Note that the three formulas above define a partition of all possible runs. Clearly, there is nothing paradoxical in this specification of the set of norms.

For the sake of completeness we reproduce here the formalisation of the same set of norms in [1] and analyse where the paradoxical results come from. The set of norms is formalised in [1] as follows:

N1 $\neg C \rightarrow (\neg A \otimes B)$
N2 $C \rightarrow FA$
N3 $G\neg A \rightarrow G\neg D$

Footnote: It would have perhaps been better not to treat $B$ as a state property, but as a property of a run, ‘data not accessed until destroyed’, which is expressible as $\neg Read\otimes Destroyed$, but we will stick with the formalisation in [1] to make comparison easier. Another issue is that instead of requiring $B$ to happen ‘eventually’, in real life there would be some time limit on when it should happen (such as in the next state).
N4 $\mathcal{F}A \rightarrow \mathcal{F}D$

N1 is intended to say that $B$ compensates for a violation $\neg C \land A$. It uses a connective $\otimes$ which was introduced for expressing contrary to duty obligations. The truth definition for $\otimes$ as given in [1] is

$TS, \sigma \models \phi \otimes \psi$ iff $\forall i \geq 0, TS, \sigma_i \models \phi$ or $\exists j, k : 0 \leq j \leq k, TS, \sigma_j \models \neg \phi$ and $TS, \sigma_k \models \psi$

where $TS$ is a transition system, and $\sigma$ a run in $TS$. This makes $\otimes$ equivalent to $\mathcal{G}\phi \lor \mathcal{F}(\neg \phi \land \mathcal{F}\psi)$

This condition is similar to our characterisation of weakly compliant runs, although it is stated as a property which should be true for all runs. The condition N2 is one of the really problematic ones. It aims to say that if $C$ holds, then $A$ is permitted; ‘permitted’ is identified with ‘will eventually happen’. It is quite clear that permission of $A$ cannot be expressed as ‘$A$ will eventually happen’; the two have completely different meanings. This does not mean that LTL cannot be used for specifying norms, it just means that this particular way of specifying norms in LTL is inappropriate. N4 is problematic in the same way: instead of saying that an occurrence of $D$ is not a violation under the same conditions as when an occurrence of $A$ is not a violation, it says that if $A$ is going to happen then $D$ is going to happen – which is again a completely different meaning. N3 attempts to say that if $A$ is prohibited then $D$ is prohibited. However, instead it implies that if $A$ happens (the antecedent $\mathcal{G}\neg A$ is false) then it does not matter whether $D$ happens (the implication is still true). Given this formalisation, which is inappropriate in multiple ways, [1] produces an example run where N1–N4 are true and the prohibition on collecting medical information is violated. The first two states on this run are $t_1, t_2$:

$t_1 \models \neg C \land A \land D$

$t_2 \models B$

which makes it a weakly compliant run as far as violating prohibition of $A$ is concerned, but a non-compliant run as far as violating the prohibition of $D$ is concerned. With our LTL specification of the set of norms it is classified as a violating run since $\mathcal{F}(\neg C \land D)$ holds on it. It does satisfy N1–N4, but clearly the problem is with N1–N4 rather than with the intrinsic difficulty of expressing norm violating patterns in temporal logic.

References

[1] Guido Governatori. Thou shalt is not you will. In Ted Sichelman and Katie Atkinson, editors, Proceedings of the 15th International Conference on Artificial Intelligence and Law, ICAIL 2015, pages 63–68. ACM, 2015.