Committee Machines—A Universal Method to Deal with Non-Idealities in RRAM-Based Neural Networks

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Artificial neural networks (ANNs) are notoriously power- and time-consuming when implemented on conventional von Neumann computing systems. Recent years have seen an emergence of research in hardware that strives to break the bottleneck of von Neumann architecture and optimise the data flow; namely to bring memory and computing closer together. One of the most often suggested solutions is the physical implementation of ANNs in which their synaptic weights are realised with analogue resistive devices, such as resistive random-access memory (RRAM). However, various device- and system-level non-idealities usually prevent these physical implementations from achieving high inference accuracy. We suggest applying a well-known concept in computer science—committee machine (CM)—in the context of RRAM-based neural networks. Using simulations and experimental data from three different types of RRAM devices, we show that CMs employing ensemble averaging can successfully increase inference accuracy in physically implemented neural networks that suffer from faulty devices, programming non-linearities, random telegraph noise, cycle-to-cycle variability and line resistance. Importantly, we show that the accuracy can be improved even without increasing the number of devices.

I. INTRODUCTION

Artificial neural networks (ANNs), with all of their variants, are now the main tools in machine learning tasks, such as classification. The vast amounts of data being constantly produced have enabled successful training and operation of these ANNs. However, to achieve high inference accuracies, it is usually necessary for ANNs to have a large number of parameters. This results in both training [1] and inference [2] stages being time- and power-consuming. This is largely caused by the need to transfer data from memory to computing units—physical separation of memory and computing is the essence of any von Neumann system.

One of the most promising solutions to these problems is the paradigm of non-von Neumann computing and, specifically, analogue implementations of synapses (weights) in physical ANNs. Because there are many more synapses than there are neurons in ANNs, the matrix-vector multiplications, in which the synaptic weight values are used, are the costliest operations in these networks, both in terms of power and time. Thus, the most popular approach is using analogue memory devices as proxies for synaptic weights of ANNs (both fully connected and their variants [3, 4]). A common technique is to arrange such devices in a structure, called crossbar array, in which every device (or a pair of devices) is used to represent a single synaptic weight. Devices, such as phase-change memories (PCMs) [5, 6] or resistive random-access memories (RRAMs) [7, 8], have been considered as candidates for such tasks. Although we here focus on ex-situ training, such systems have been successfully utilised for in-situ training too [9, 10].

In the case of RRAM implementations, the main concern is that various non-idealities associated with these devices can prevent them from achieving high inference accuracies [11, 12]. Examples of such non-idealities include, but are not limited to, devices not being able to electroform, devices stuck in one of the resistance states after electroforming, limited endurance, non-linear resistance modulation when applying identical voltage pulses, random telegraph noise (RTN) and device-to-device variability. To mitigate the effects of these device non-idealities, it is often necessary to modify device structure [8], to use more advanced programming schemes [13] or to use additional circuitry in conjunction with RRAM elements [14]. On the system-level, there is an issue of line resistance which affects the readings of the currents and thus decreases inference accuracy. These line resistance effects can be partially compensated for algorithmically [15] or partially mitigated by using multiple smaller crossbar arrays [16]. Examples of past efforts at dealing with these and other non-idealities of memristive devices and systems are listed in Table I; most of these non-idealities are still the main focus of the research in neuromorphic RRAM community.

In this work, we propose a simple way to mitigate the effects of all types of non-idealities. We suggest combining several non-ideal physically implemented neural networks into committees to achieve better accuracy. The committee machine (CM) method we propose significantly increases the inference accuracy and does not increase the computation time because ANNs in such committees work in parallel. Although one could argue that

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| First author (year) | Non-ideality | Device type | Proposed solution |
|---------------------|--------------|-------------|-------------------|
| C. Sung (2018) [17] | Current/voltage non-linearity | TaOₓ RRAM | Hot-forming step is adopted |
| C. Li (2018) [14]  | Current/voltage non-linearity | HfO₂ RRAM | 1T1R architecture is adopted |
| Y. Fang (2018) [18] | Device-to-device variability | HfOₓ RRAM | Ultra-thin ALD-TiN buffer layer is introduced |
| B. Govoreanu (2013) [19] | Device-to-device variability | Al₂O₃/TiO₂ (VMCO) RRAM | Non-filamentary RRAM is adopted |
| A. J. Kenyon (2019) [20] | Device-to-device variability | SiOₓ RRAM | The roughness of bottom electrodes is increased |
| L. Xia (2017) [13]  | Faulty devices | - | A modified mapping algorithm and redundancy schemes are used |
| S. Ambrogio (2018) [6] | Limited dynamic range | PCM | Two pairs of conductance of varying significance for every synaptic weight are used |
| M. Hu (2016) [15]   | Line resistance | - | Advanced mapping algorithms are used to compensate for line resistance effects |
| W. Wu (2018) [21]   | Programming non-linearity | HfOₓ RRAM | Electro-thermal modulation layer is deposited on the switching layer |
| J. Woo (2016) [8]    | Programming non-linearity | HfO₂ RRAM | Bilayer structure is adopted |
| S. Ambrogio (2018) [6] | Programming non-linearity | PCM | PCM devices are used together with CMOS transistors |
| Z. Chai (2018) [22] | Random telegraph noise | TiO₂/a-Si (aVMCO) RRAM | Non-filamentary RRAM is adopted |

Table I: Examples of past efforts at dealing with non-idealities of memristive devices and their systems.

This technique requires more area and power (because of additional networks used), we find that instead of using one large ANN, one can use a committee of several smaller ANNs—when the committee of small ANNs uses the same number of devices as the large ANN, the committee still achieves higher accuracy. Besides, the introduced modularity increases robustness and flexibility—if any constituent ANN fails beyond repair, it could be ignored and thus not affect the performance of the committee.

II. RESULTS AND DISCUSSION

After fully connected ANNs were trained in software to recognise handwritten digits (using MNIST data base [23]), their physical implementations were simulated to reflect various device- and system-level non-idealities. After this, the networks were combined into CMs that employed ensemble averaging. The principle of ensemble averaging is shown in Figure 1A—several networks are combined in parallel and then their outputs are averaged. After that, the prediction is made using the averaged vector—the prediction is the label corresponding to the largest entry in the vector.

Ensemble averaging is used even with conventional ANNs and often produces better accuracy than that of the best individual network [24]. Although there are other types of CMs besides ensemble averaging, they often rely on training additional gating networks or boosting networks during the training stage. Using a gating network in this scenario would produce additional problems—to avoid it acting as a performance bottleneck, it too would have to be implemented on crossbar arrays. Various non-idealities would decrease the effectiveness of this gating network which is responsible for making the decisions about the whole committee of ANNs. Likewise, we speculate that boosting of networks would not be feasible in ex-situ training because it requires information about where individual ANNs perform poorly—this cannot be known precisely until they are implemented physically on crossbar arrays and the non-idealities manifest themselves. To authors’ best knowledge, the application of boosting in the context of memristive neural networks seems to have been explored only once before [25]; as expected, it requires
training each physical implementation differently because non-idealities manifest themselves differently in different crossbar arrays. Because of these reasons, we use only ensemble averaging in this work but are open to the idea that other CM methods could be utilised successfully for ex-situ training.

In ensemble averaging, we find that even when the physical ANNs, which go into a committee, all use the same continuous weights that are mapped onto crossbar arrays (see Figure 1B), committee can still increase the accuracy of these physical implementations. Although all networks have the same continuous weights before mapping, their physical implementations (which we call ‘disturbances’ in Figures 1B, C because they can usually be represented by the modification of individual weights) will be different. For example, in one crossbar array, a certain set of devices will not be able to electroform, while in the other crossbar array, it will be a different
set, etc. This will result in different physical implementations having slightly different learned representations of the data set, or, to paraphrase, different networks will be affected differently due to the non-idealities. This means that these committees will be able to combine different representations, and thus achieve higher accuracy. However, by definition, such approach would never yield a committee accuracy that is higher than the accuracy of a single continuous network.

A better approach is to use different continuous networks for different physical implementations that go into a committee (see Figure 1C). This approach much more resembles the conventional application of ensemble averaging in computer science. In the context of RRAM crossbar arrays, it would not only help to mitigate the effects of the non-idealities (as in the case of Figure 1B), but would also allow to combine the representations of continuous networks that were different even before the mapping stage. We used this method in this analysis. In this work, any given committee used only one network architecture but each network was initialised differently before training, thus trained networks had different sets of weights. Although it was not explored in this work, combining different network architectures in a committee of disturbed networks might be advantageous. Most importantly, having different continuous networks, that are implemented physically, allows to achieve higher accuracies which are sometimes even higher than that of individual continuous networks.

A. Experiments

To test how technology-agnostic CM method is, we quantified non-idealities of three different types of RRAM devices—filamentary silicon oxide-based (SiO$_x$), filamentary tantalum oxide-based (Ta$_2$O$_5$), and non-filamentary vacancy modulated conductive oxide-based (aVMCO) devices. Types of non-idealities explored are very common in virtually all RRAM technologies.

1. Non-linear programming

The most energy-efficient and most straightforward procedure to modulate the resistance of RRAM devices is by application of identical voltage pulses. This approach assumes that the modulation is always linear with respect to the number of pulses applied. However, for most RRAM technologies, this is not true, and the pulsing curves are to different degrees non-linear. Although this is less relevant for inference, one might end up with different resistance states than desired if no resistance verification scheme is used to confirm the resistance values. Figure 2A shows very non-linear resistance changes when identical voltage pulses are applied to SiO$_x$ RRAM devices. These data were used to investigate what happens when one assumes linear behaviour without verify-

![Figure 2: Experimental data. A) Non-linear programming with voltage pulses. Gradual resistance changes, as well as linear fit, are shown for a SiO$_x$ device. B) Cumulative probability plots of RTN-induced relative current deviations for all 8 resistance states. Lognormal fits are shown for each resistance state of a Ta$_2$O$_5$ device. C) Cumulative probability plots of relative C2C current deviations for 10 out of 100 resistance states (every eleventh state is included). Weibull fits are shown for these resistance states of a Ta$_2$O$_5$ device.](image-url)
ing actual resistance values, i.e. when linear fit instead of actual resistance values in Figure 2A are used.

2. Random telegraph noise

RRAM devices often suffer from RTN resulting in a different accuracy at any given instant in time. Ta$_2$O$_5$ and aVMCO devices were characterised by measuring the current of their resistance states multiple times. Figure 2B shows the cumulative probability plots for 8 resistance states of Ta$_2$O$_5$ devices, together with lognormal fits. These fits were used to disturb the weights of ANNs in order to investigate the effect of RTN on inference accuracy. Ta$_2$O$_5$ simulation results are presented in the main text, while those of aVMCO can be found in the supplementary information, although both are summarised in Figure 5A.

3. Cycle-to-cycle variability

Although for inference RRAM devices have to be switched and set to correct state only once, there is often high variability between devices. When different devices are electroformed, each of their conductive filaments is slightly different. Therefore, the way they respond to various inputs might be different too which might result in resistance states that are difficult to control in a reliable way across multiple devices. This device-to-device variability is to some extent comparable to cycle-to-cycle (C2C) variability—when devices are switched multiple times, their conductive filaments change too, producing variation similar to that found from device to device. Figure 2C shows current deviation data for C2C variability of 10 resistance states of Ta$_2$O$_5$ devices, together with Weibull distribution fits in these cumulative probability plots. These were used to disturb the ANNs to reflect C2C variability which itself is representative of device-to-device variability. aVMCO device data were used too and the resulting simulation results are provided in the supplementary information, although the simulation data for both types of devices is summarised in Figure 5B. In the case of ex-situ training, a poorly set resistance state could be reprogrammed again, thus this analysis represents the worst-case scenario when no adjustments are made.

B. Simulations

7 different types of non-idealities have been considered: devices not being able to electroform, electroformed devices stuck at high resistance state (HRS), electroformed devices stuck at low resistance state (LRS), non-linear programming, RTN, C2C variability and line resistance. For the first six non-idealities, the simulations have been performed for two network architectures: 784(+1):100(+1):100(+1):10 (large network) and 784(+1):30(+1):10 (small network). In this subsection, box plots for these six non-idealities are shown for the large network architecture only, but the results for the effectiveness of committees are summarised for both network architectures in Figure 5A and all remaining box plots can be found in the supplementary information. Line resistance simulations used only small network architecture for practicality—it is computationally expensive to simulate currents in large crossbar arrays. All box plots contain accuracies for continuous and discretised networks before disturbance, accuracy of individual networks after disturbance, as well as accuracies for committees of disturbed networks (ranging from size 2 to size 5) employing ensemble averaging. In all simulations, we observe that using more networks in a committee results in higher median accuracy.

1. Faulty devices

Left-side panels of Figure 3 show the effectiveness of ensemble averaging when dealing with faulty devices—different subfigures explore different ways in which devices can fail, and in each subfigure 10% of these devices are simulated as not working properly. The results for 20% of devices being faulty are included in Figure 5A and are presented in full in the supplementary information.

Figure 3A demonstrates the committees in action when 10% of the devices are not able to electroform. The median accuracy of individual non-ideal networks, in this case, is $\sim$94.0%. When they are combined into committees of size 5, the median accuracy improves to $\sim$97.5%. Figure 3B shows the effect of 10% of the devices getting stuck at HRS. In this case, the median accuracy of individual non-ideal networks is much higher—96.4%. This results in the median accuracy of a committee being higher too—98.0%—for a committee of size 5. This accuracy is even higher than that of individual networks with continuous weights. Finally, in Figure 3C, the effect of 10% of the devices getting stuck at LRS is shown. In this case, the effect is even more detrimental than that of devices not being able to electroform—median accuracy of individual non-ideal networks is $\sim$93.4%. However, with the help of the committees of size 5, the accuracy improves to $\sim$97.3%.

2. Non-linear programming

In the analysis of the effect of non-linear programming, we observe that the drops in accuracy due to deviations from linear programming model are comparable to the drops in accuracy due to discretisation, as seen in Figure 3D. This is a result of the ratio between highest and lowest resistance states (HRS/LRS ratio) of 3.55 being not particularly large. However, the committees im-
proved the accuracy in a very similar way as with other non-idealities. The median accuracy went from $\sim 96.7\%$ in individual non-ideal networks to $\sim 98.1\%$ in committees of size 5.

3. Random telegraph noise

Although a high HRS/LRS ratio of 8 was used in the simulations involving RTN, a significant drop in accuracy of ANNs is observed when they are discretised, as seen in Figure 3E. That is because only 8 resistance states were used. When high HRS/LRS ratios are used and re-
### Figure 4: Analysis of line resistance.

**A)** The average decreases in output currents for a crossbar of size $785 \times 30$ for different values of interconnect resistance when no reordering of inputs or outputs is used. Decreases in current were computed by taking into account the absolute values of currents from two crossbars (of positive and negative weights).

**B)** Effectiveness of committees when dealing with interconnect resistance of $1 \Omega$. **C)** Effectiveness of committees when dealing with interconnect resistance of $2.5 \Omega$. Both **B** and **C** used pixel-intensity-aware input reordering, as well as random reordering of the inputs and outputs of second synaptic layer. In box plots in **B** and **C**, the maximum whisker length is set to $1.5 \times IQR$.

Resistance states are equally spaced (as is the case here), more states are required for the accuracy to saturate [26]. In the specific case of Figure 3E, the median accuracy with discretised weights using 8 equally spaced resistance states is $\sim 97.1\%$. Individual non-ideal networks achieve $\sim 96.8\%$ median accuracy, but when they are in committees of size 5, it increases to $\sim 98.1\%$.

4. **Cycle-to-cycle variability**

The effect of C2C variability is much more detrimental to accuracy than that of RTN, as shown in Figure 3F. However, using committees improved the accuracy significantly, just like with other non-idealities. The median accuracy increased from $\sim 83.9\%$ in individual non-ideal networks to $\sim 95.4\%$ in committees of size 5.

5. **Line resistance**

The effect of line resistance can be extremely detrimental in many crossbar-based physical implementations of ANNs. That is especially the case if the synaptic layers are large in size. Because in a neural network many of the inputs are nonzero at any given time, a lot of current accumulates in the vertical bit lines which results in significant voltage drops across the interconnects, and thus the current distribution across the crossbar is affected in a major way. Figure 4A shows typical drops in output currents for a single synaptic layer of one neural network—simulation used crossbar of size $785 \times 30$ with 8 equally spaced resistance states for which $LRS = 25 \, k\Omega$ and $HRS = 200 \, k\Omega$. Current decreases for different values of interconnect resistance, $R_L$, are shown. Even with such relatively high RRAM resistance states, we experience significant drops in current—when $R_L = 2.5 \, \Omega$, the average current decrease is more than $50\%$. Naturally, this results in significant drop in accuracy. To reduce current decrease, we use pixel-intensity-aware input reordering. Additionally, to increase the variability between different physical ANNs, we recommend random reordering of inputs and outputs of crossbar arrays. Both methods improve inference accuracy; they are explained in more detail in the supplementary information.

When employing these methods, we find that for $R_L = 1 \, \Omega$, the accuracy of individual non-ideal networks is $\sim 93.3\%$, as shown in Figure 4B. When they are combined into committees of size 5, the accuracy increases to $\sim 96.8\%$. Likewise, for $R_L = 2.5 \, \Omega$, the accuracy...
goes from \( \sim 90.2\% \) in individual non-ideal networks to \( \sim 94.9\% \) in committees of size 5, as shown in Figure 4C. We note that this non-ideality seems to be the most difficult to deal with by employing committees. That might be because of the nature of this non-ideality mentioned earlier—the disturbances are position-dependent which makes it difficult to reduce their effect by combining several neural networks which were all affected in a similar way. Nevertheless, the observed accuracy improvement is very significant.

6. The Big Picture

The results for accuracy improvements when using committees of size 5 are summarised in Figure 5A. This graph reveals two important points. Firstly, it shows that committees employing ensemble averaging can successfully mitigate the effects of all types of non-idealities. This is significant because, for most RRAM technologies, one optimisation method might mitigate the effects of one non-ideality but amplify the effect of others. Secondly, the graph suggests that it is still important to aim for high accuracies of individual networks, and therefore the optimisation of individual devices, that are affected by these non-idealities—the higher the median accuracy of individual networks, the higher the median accuracy of committees of those same networks. We stress that this might be the most important factor determining the accuracy of a committee—one can infer the median accuracy of a committee mostly from the median accuracy of individual networks, even without knowing the non-ideality or the network architecture. Only the left brown data point representing large line resistance \((R_L = 2.5 \Omega)\) deviates from the overall trend more than others. Although more difficult to deal with, we still see a large improvement in accuracy even with this non-ideality.

It is also important to consider whether using larger networks, instead of committees of smaller networks, would yield the same results if the same number of synapses (or RRAM devices) was used in the large network as in the committee of smaller networks. In our previous work we found that the accuracy of networks before disturbance (which we call starting accuracy) has a huge effect on the robustness—the larger the starting accuracy, the more robust the networks become [26]. One way to achieve higher starting accuracy is to have larger networks, e.g. if we have a network with one hidden layer, we might increase the number of neurons in that hidden layer, which would likely result in higher accuracy after training and thus higher robustness. This means that when such networks are disturbed, their accuracy does not drop as much; this is illustrated by the red curve of Figure 5B. In this figure, each of the curves shows median accuracy for different number of hidden neurons—30, 60, 120 and 240—in individual disturbed networks with one hidden layer, as well as in committees of those networks. We notice that after applying C2C disturbance to a committee of 2 networks with 30 hidden neurons in each (left-most data point of the orange curve), it achieves only slightly higher accuracy \((\sim 0.2\%)\) than a single network with 60 hidden neurons (second data point from the left of the red curve) which requires almost the same number of synapses. However, a committee of two networks that each have 120 hidden neurons (third data point from the left of the orange curve) achieves considerably higher
accuracy (∼4.0%) than a network with 240 hidden neurons (rightmost data point of the red curve), despite both requiring almost the same number of synapses. This difference in accuracy improvement stems from the fact that a continuous network with 60 hidden neurons can be trained to achieve considerably higher accuracy than a network with 30 hidden neurons. However, a continuous network with 240 hidden neurons achieves only slightly higher accuracy than a network with 120 hidden neurons after training, i.e., the accuracy starts to saturate as we add more neurons (see Figure S7 in the supplementary information). Because of that, a network with 240 hidden neurons is only slightly more robust than a network with 120 neurons. We also see that as we increase the number of networks in committees, their advantage becomes even more apparent, e.g., a committee of four networks that each have 60 hidden neurons achieve much higher accuracy (∼5.4%) than a single network with 240 neurons, despite both requiring almost the same number of synapses. Thus, the benefits of CMs of RRAM-based neural networks are obvious, unless the networks are especially tiny. When line resistance is taken into account, the advantages of committees of smaller networks become even more evident.

III. CONCLUSIONS

Using experimental data from three different types of RRAM devices, we simulated physical ANNs implemented using these devices and showed that CMs employing ensemble averaging can be used to mitigate the effects of device- and system-level non-idealities. Ensemble averaging allows to achieve higher inference accuracy in physically implemented neural networks that suffer from faulty devices, programming non-linearities, random telegraph noise, cycle-to-cycle variability, and even line resistance. This method is a universal way to deal with the most common non-idealities and is straightforward to implement during the fabrication stage. Although some level of non-idealities in RRAM devices is unavoidable, CM method allows us to deal with these on the system level and is agnostic to a particular technology or, to some degree, type of the non-ideality. Increased modularity of these physically implemented neural network systems will increase not only their inference accuracy, but also their robustness and flexibility, even without the need to sacrifice area.

Methods

A. Experiments

All of RRAM devices used in this paper show bipolar switching characteristics. The filamentary silicon oxide RRAM consists of 280 nm of Mo (bottom electrode), 35 nm of sputtered switching SiO₂, and the 115 nm layer of Au with 3 nm Ti layer adhesion layer in-between. A shadow mask was used to define the electrode areas with individual electrode size of 200 µm × 200 µm. The filamentary based Ta₂O₅ device consists of a TiN/4nm stoichiometric Ta₂O₅/20 nm nonstoichiometric TaOₓ/10 nm TaN/TiN stack with a cross-sectional area of 75 nm × 75 nm, while the non-filamentary based amorphous Vacancy Modulated Conductive Oxide (aVMCO) has a cross-sectional area of 135 nm × 135 nm and is composed of a TiN/8 nm amorphous-Si/8 nm anatase TiO₂/TiN stack. The detailed fabrication process parameters can be found in references [27–29] for Ta₂O₅, aVMCO and SiO₂ RRAMs respectively. Ta₂O₅ and aVMCO devices were fabricated by imec. All electrical tests for Ta₂O₅ and aVMCO devices were done with a Keysight B1500A, and with a Keithley 4200 for SiO₂ devices. The RTN data is extracted by switching the device into 8 uniformly distributed resistance levels between 25 kΩ and 200 kΩ, and 8 nearly uniformly distributed resistance levels between 1 MΩ and 7.5 MΩ with incremental RESET DC sweeps [30] for Ta₂O₅ and aVMCO respectively. RTN measurement is then carried out at each resistance level at a 0.1 V and 3 V read-out for Ta₂O₅ and aVMCO respectively, with a sampling time of 2 ms/point and 10,000 sampling point per resistance level for a RTN measurement period of 20 s. For C2C variability extraction, the Ta₂O₅ device was cycled 100 times—within each cycle, 100 AC pulses were applied for RESET (100 µs @ −1.7 V), and one DC Sweep for SET (0 V → 1.5 V → 0 V; \( I_{CC} = 100 \mu A \)). For the aVMCO device, SET and RESET are both performed using 500 AC pulses (SET: 20 µs @ −2.5 V; RESET: 100 µs @ 5.3 V) in each cycle. In both devices, the mean value of measured conductance during cycling after each pulse is calculated and used as the reference. The total range of the reference conductance, i.e., on/off window, is divided into 100 equally spaced states for variability evaluation. For each state, the pulse that produces the reference conductance nearest to this specific state is chosen. The variability parameters of this state can be extracted from the Weibull distribution of \( \Delta I_{C2C}/I_{ref} \) generated by this pulse. The depression curve of SiO₂ devices in Figure 2A is obtained by applying identical voltages pulses of width 500 ns @ 0.9 V.

B. Simulations

Multiple ANNs were trained using conventional software methods and their physical implementations employing RRAM crossbar arrays were simulated, thus mimicking ex-situ training. Various RRAM non-idealities were considered, some of which have been quantified using experimental data. ANNs were arranged into committees to mitigate the effects of those non-idealities. In this work, feed-forward ANNs with fully connected layers and continuous weights were trained to recognise handwritten digits using the MNIST data base. All 60,000 MNIST training images were used during the
training stage; training set consisted of 45,000 images and verification set consisted of 15,000 images. All 10,000 test images were used to evaluate the inference accuracy of ANNs. Networks used 784 input neurons representing pixel intensities of MNIST images of $28 \times 28$ pixel size, as well as one bias neuron. 10 output neurons were used; they represented the ANNs’ predictions of 10 handwritten digits. Hidden layers used sigmoid activation functions, while the output layer used softmax activation function. Weights were adjusted by employing backpropagation algorithm and cross-entropy error function. A higher than required number of ANNs was trained and only 30 best performing ones (for each architecture) were selected for the simulations. Mainly two network architectures were investigated: 2 hidden layers with 100 neurons in each (in addition to a bias neuron) and 1 hidden layer with 30 neurons (in addition to a bias neuron). To produce Figure 5B, additional networks with 1 hidden layer were trained—with 60, 120 and 240 hidden neurons (in addition to a bias neuron). The code was implemented in Python.

Continuous weights were mapped onto RRAM conductances using proportional mapping scheme—synaptic weights were made proportional to conductances. The zero weight was interpreted as given—in practise, it would be implemented by not electroforming the device, thus resulting in its negligible conductance. Although aVMCO devices do not have electroforming stage, for consistency we assumed that additional insulating circuit elements could be used to implement the zero weight. Negative weights would be implemented by employing a second crossbar array. Thus, with $N_G$ conductance states of electroformed devices, one would achieve $2N_G + 1$ discrete weight levels. Maximum discrete weights were optimised separately for each set of network architecture and conductance levels; in each case this was done by excluding a certain proportion, $p_L$, of weights with largest absolute values. What $p_L$ values were used for each experiment is summarised in Table SI in the supplementary information. More details on discretisation and mapping procedures can be found in our past work [26].

All non-idealities, except for line resistance, were simulated by disturbing the weights of the discretised neural networks. To investigate line resistance, currents in every branch of every crossbar had to be simulated. By setting up simultaneous linear equations using Kirchhoff’s current and voltage laws, those were solved by employing LU decomposition and sparse matrix representations in Python’s library scipy.

After simulating RRAM non-idealities, committees of different ANNs were composed. Committees used ensemble averaging, i.e. the outputs of individual networks in a committee were averaged to produce a single output. In ensemble averaging, the output vectors of individual networks can simply be added together (if the weightings of different networks are the same, as we propose here); the label corresponding to the entry with the highest value would be the prediction of the committee. This addition can be performed either in software, or, if the activation function of the last neuronal layer can be implemented physically, it can be performed by adding corresponding currents produced by the circuitry of this activation function.

In the simulations, neural networks that go into a committee were chosen randomly. This was done to reflect the most convenient strategy when manufacturing such systems—because one does not need to selectively choose the networks, manufactured crossbars can be easily programmed without the need to replace them if they perform poorly when working individually (unless their effect is so detrimental that they have to be ignored which can be made possible with this technique). Besides, devices might change over time, thus these simulations, which show what happens when one does not selectively choose the networks, are valuable to investigate conditions where it is not possible to replace the networks. In the simulations, 30 base networks were used (each having different set of weights) for each of the architectures. Then they were all discretised using HRS/LRS values extracted from experiments. Finally, to reflect each of the non-idealities, all networks were disturbed multiple times (except for non-linear programming and line resistance simulations because these disturbances were deterministic). In each disturbance iteration, multiple combinations of networks were chosen and their performance as a committee of certain size was evaluated. In total, for each network architecture and for most non-idealities, 10,000 data points were recorded for a committee of every size—these data captured the variations of base networks, their combinations and different disturbance iterations. Because non-linear programming and line resistance simulations were deterministic, more combinations were tried for each committee size, but the total number of data points was still limited to 10,000.

**Author Contributions**

A.M. and D.J. conceived the idea and designed the study. A.M., P.F. and Z.C. performed the experimental measurements. D.J. performed the simulations and analysed the experimental and simulation results. W.H.N. fabricated SiO$_x$ devices. A.M., W.D.Z. and A.J.K. supervised the research. D.J. wrote the initial manuscript. All authors contributed to the discussions of the results and improved the text.

**Competing Interests Statement**

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.
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Supplementary Information for the Paper
"Committee Machines—A Universal Method to Deal with Non-Idealities in RRAM-Based Neural Networks"

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Pixel-Intensity-Aware Input Reordering

To reduce current decreases in crossbar arrays, we suggest using pixel-intensity-aware input reordering. That is, average pixel values are recorded over the training set and then they are used to reorder the crossbar inputs so that the ones that on average have the highest values would be at the bottom of the bit lines, while those with lowest values—at the top. This means that high current would concentrate mostly at the bottom of bit lines, thus the voltage drops across interconnects at the top of bit lines would be negligible—the crossbar currents would be disturbed less in this way. This method indeed increases the accuracy of individual non-ideal networks when tested on previously unseen MNIST test set. However, it should only be used when the training set is really representative of the test set because simple averaging does not take into account the possibility that there might be significantly more examples from one class than another.

Random Reordering of Inputs and Outputs

When combining non-ideal networks into committees, we also suggest to randomly reorder the inputs to every synaptic layer (except the first one, where pixel-intensity-aware input reordering would be used) as well as the outputs of the last synaptic layer. The disturbances to the network in crossbar simulations, unlike with other non-idealities, are position-dependent. Thus to increase the variability between different networks that go into a committee we found it useful to apply such random reordering—it makes ANN committees achieve higher accuracy in this way.
Figure S1: Effectiveness of committees when dealing with faulty RRAM devices in 784(+1):100(+1):100(+1):10 networks. A) 20% of devices not able to electroform. B) 20% of devices stuck at HRS. C) 20% of devices stuck at LRS. In all box plots, the maximum whisker length is set to $1.5 \times \text{IQR}$. 
Figure S2: Effectiveness of committees when dealing with faulty RRAM devices in 784(+1):30(+1):10 networks. A) 10% of devices not able to electroform. B) 10% of devices stuck at HRS. C) 10% of devices stuck at LRS. In all box plots, the maximum whisker length is set to 1.5 × IQR.
Figure S3: Effectiveness of committees when dealing with faulty RRAM devices in 784(+1):30(+1):10 networks. 

A) 20% of devices not able to electroform. 

B) 20% of devices stuck at HRS. 

C) 20% of devices stuck at LRS. In all box plots, the maximum whisker length is set to 1.5 × IQR.
**Figure S4:** Effectiveness of committees when dealing with non-idealities of aVMCO RRAM devices in 784(+1):100(+1):100(+1):10 networks. **A)** RTN. **B)** C2C variability. In both box plots, the maximum whisker length is set to $1.5 \times \text{IQR}$.
Figure S5: Effectiveness of committees when dealing with non-idealities of RRAM devices in 784(+1):30(+1):10 networks. A) Non-linear programming with identical voltage pulses (SiO\textsubscript{x} devices). B) RTN (Ta\textsubscript{2}O\textsubscript{5} devices). C) C2C variability (Ta\textsubscript{2}O\textsubscript{5} devices). In all box plots, the maximum whisker length is set to 1.5 × IQR.
Figure S6: Effectiveness of committees when dealing with non-idealities of aVMCO RRAM devices in 784(+1):30(+1):10 networks. A) RTN. B) C2C variability. In both box plots, the maximum whisker length is set to $1.5 \times \text{IQR}$. 
Figure S7: Median accuracy of continuous networks with one hidden layer for different number of hidden neurons.
| Figure | Non-ideality | Device type | HRS/LRS Conductance states | Number of conductance states | Spacing of states | Network architecture | \( p_L \) (%) |
|--------|-------------|-------------|-----------------------------|-------------------------------|-------------------|----------------------|------------|
| 3A     | Devices not able to electroform | SiO\(_2\) | 3.550 | 10 | Equally spaced conductance states | 784(+1):100(+1):100(+1):10 | 0.8 |
| 3B     | Devices stuck at HRS | SiO\(_2\) | 3.550 | 10 | Equally spaced conductance states | 784(+1):100(+1):100(+1):10 | 0.8 |
| 3C     | Devices stuck at LRS | SiO\(_2\) | 3.550 | 10 | Equally spaced conductance states | 784(+1):100(+1):100(+1):10 | 0.8 |
| 3D     | Programming non-linearity | SiO\(_2\) | 3.550 | 10 | Equally spaced conductance states | 784(+1):100(+1):100(+1):10 | 0.8 |
| 3E     | Random telegraph noise | Ta\(_2\)O\(_5\) | 8.000 | 8 | Equally spaced resistance states | 784(+1):100(+1):100(+1):10 | 0.0 |
| 3F     | Cycle-to-cycle variability | Ta\(_2\)O\(_5\) | 4.421 | 100 | Equally spaced conductance states | 784(+1):100(+1):100(+1):10 | 0.5 |
| 4B     | Line resistance | Ta\(_2\)O\(_5\) | 8.000 | 8 | Equally spaced resistance states | 784(+1):30(+1):10 | 0.1 |
| 4C     | Line resistance | Ta\(_2\)O\(_5\) | 8.000 | 8 | Equally spaced resistance states | 784(+1):30(+1):10 | 0.1 |
| 5B     | Cycle-to-cycle variability | Ta\(_2\)O\(_5\) | 4.421 | 100 | Equally spaced conductance states | 784(+1):30(+1):10 | 0.2 |
| 5D     | Cycle-to-cycle variability | Ta\(_2\)O\(_5\) | 4.421 | 100 | Equally spaced conductance states | 784(+1):60(+1):10 | 0.1 |
| 5E     | Cycle-to-cycle variability | Ta\(_2\)O\(_5\) | 4.421 | 100 | Equally spaced conductance states | 784(+1):120(+1):10 | 0.1 |
| 5F     | Cycle-to-cycle variability | Ta\(_2\)O\(_5\) | 4.421 | 100 | Equally spaced conductance states | 784(+1):240(+1):10 | 0.1 |
| S1A    | Devices not able to electroform | SiO\(_2\) | 3.550 | 10 | Equally spaced conductance states | 784(+1):100(+1):100(+1):10 | 0.8 |
| S1B    | Devices stuck at HRS | SiO\(_2\) | 3.550 | 10 | Equally spaced conductance states | 784(+1):100(+1):100(+1):10 | 0.8 |
| S1C    | Devices stuck at LRS | SiO\(_2\) | 3.550 | 10 | Equally spaced conductance states | 784(+1):100(+1):100(+1):10 | 0.8 |
| S2A    | Devices not able to electroform | SiO\(_2\) | 3.550 | 10 | Equally spaced conductance states | 784(+1):30(+1):10 | 0.2 |
| S2B    | Devices stuck at HRS | SiO\(_2\) | 3.550 | 10 | Equally spaced conductance states | 784(+1):30(+1):10 | 0.2 |
| S2C    | Devices stuck at LRS | SiO\(_2\) | 3.550 | 10 | Equally spaced conductance states | 784(+1):30(+1):10 | 0.2 |
| S3A    | Devices not able to electroform | SiO\(_2\) | 3.550 | 10 | Equally spaced conductance states | 784(+1):30(+1):10 | 0.2 |
| S3B    | Devices stuck at HRS | SiO\(_2\) | 3.550 | 10 | Equally spaced conductance states | 784(+1):30(+1):10 | 0.2 |
| S3C    | Devices stuck at LRS | SiO\(_2\) | 3.550 | 10 | Equally spaced conductance states | 784(+1):30(+1):10 | 0.2 |
| S4A    | Random telegraph noise | aVMCO | 7.500 | 8 | | 784(+1):100(+1):100(+1):10 | 0.0 |
| S4B    | Cycle-to-cycle variability | aVMCO | 6.692 | 100 | Equally spaced conductance states | 784(+1):100(+1):100(+1):10 | 0.2 |
| S5A    | Programming non-linearity | SiO\(_2\) | 3.550 | 10 | Equally spaced conductance states | 784(+1):30(+1):10 | 0.2 |
| S5B    | Random telegraph noise | Ta\(_2\)O\(_5\) | 8.000 | 8 | Equally spaced resistance states | 784(+1):30(+1):10 | 0.1 |
| S5C    | Cycle-to-cycle variability | Ta\(_2\)O\(_5\) | 4.421 | 100 | Equally spaced conductance states | 784(+1):30(+1):10 | 0.2 |
| S6A    | Random telegraph noise | aVMCO | 7.500 | 8 | | 784(+1):100(+1):100(+1):10 | 0.1 |
| S6B    | Cycle-to-cycle variability | aVMCO | 6.692 | 100 | Equally spaced conductance states | 784(+1):30(+1):10 | 0.1 |

Table SI: Summary of parameters for each simulation in the main text and supplementary information. The nature of non-idealities did not depend on the device type in the simulations of faulty devices (i.e., devices not able to electroform or stuck at LRS or HRS) and line resistance (excluding the fact that devices with higher resistance were affected less by line resistance), but some parameters were still necessary to represent the weights using conductances. Thus, to reduce the number of arbitrary parameters, SiO\(_2\) distribution of states was used for the simulations of faulty devices and Ta\(_2\)O\(_5\) distribution of states (from RTN experiment) was used for the simulations of line resistance.