Estimation of Bourgoyne and Young Model Coefficients to Predict Optimum Drilling Rates and Bit Weights using Genetic Algorithms – a case study of the Faihaa Oil Field in Iraq

Abdulkareem A Khaleel¹, Mohammed S Adnan², Salem J Alhamd³

¹,²,³ College of Engineering – University of Karbala

¹Corresponding author email: abdulkareemalrubaiey@gmail.com

Abstract. Drilling optimization requires maximizing rate of penetration (ROP) by controlling parameters such as bit weight (WOB) and rotary speed (N) to reduce drilling time and cost and to mitigate hole problems. The Bourgoyne and Young model was selected to study the effects of all parameters concerned with oil well drilling (depth, pore pressure, equivalent circulating density, bit weight, rotary speed, bit tooth dullness and jet impact force) based on actual bit records for a well in Faihaa oil field. A multiple regression analysis technique and genetic algorithm procedure were employed to analyze the field data, with the results used to develop a general equation relating rate of penetration to all variables. All the constants of the model were thus determined, and the optimized bit weight calculated for different depths of well to achieve the optimum penetration rate.

Keywords: Bourgoyne and Young model; penetration rate; genetic algorithm; optimum bit weight

1. Introduction

Optimizing a drilling operation is the paramount objective for operators and drilling contractors seeking to minimize well costs and maximize drilling performance. One of the most effective solutions for reducing well cost is to maximize the rate of penetration based upon optimum selection of applied drilling parameters and tools, and various mechanical specific energy (MSE) and statistical approaches have been implemented to enhance the rate of penetration (ROP) to drill new wells more efficiently and cost-effectively [1]. In the oil industry, the cost of drilling operations is very high, especially the cost of renting a drilling rig; one of the main aims of petroleum engineering is thus to reduce the costs by increasing the speed of drilling or ROP. Increasing drilling performance and minimizing costs in any drilling operation begins with an accurate understanding of the lithological characteristics in the wellbore and, perhaps more importantly, the formation rock strength or “drillability”. An accurate estimation of these parameters is vital to reducing costs by improving the overall drilling performance of any well [3].

One of the first attempts at drilling optimization was presented by Graham and Muench in 1959 [4]. They analytically evaluated various weights on the bit and rotary speed combinations to derive empirical mathematical expressions for bit life expectancy and drilling rates as a function of depth, rotary speed, and bit weight [5]. In 1963, Galle and Woods [5] produced graphs and procedures for field applications to determine the best combination of drilling parameters. One of the most important drilling optimization studies was performed in 1974 by Bourgoyne and Young [6], however. They proposed the use of a linear drilling penetration rate model and performed multiple regression analysis to select optimal drilling parameters using a minimum cost formula, showing that maximum rate of penetration generally coincided with a minimum cost approach if technical limitations were ignored. In the 1990s, various different drilling planning approaches were developed, along with new techniques to identify the best possible well construction performance. Later on, Drilling the Limit optimization techniques were also introduced, and towards the end of the previous millennium, real-time monitoring techniques were emplaced, allowing
drilling parameters started to be monitored remotely [7]. A few years later, real-time operations support centers began to be constructed, and some operators proposed advanced techniques for monitoring drilling parameters at the rig site [8].

1.1. Case study data:
The necessary analyses for this research study were performed using data for a vertically drilled well (Faihaa-1) in block 9, located in southern Iraq, 25 kilometers north of Basra, on the border of Iran, with the Majnoon and Sindbad fields to the left and right, respectively. The Faihaa oil field was discovered in 2014.

1.2. Objective of study:
1-To determine the exponents for the Bourgoyne and Young model using multiple regression analysis.
2-To predict the modelled rate of penetration and to compare this with actual field data.
3-To determine the optimum bit weight based on the rate of penetration equation and flounder points.

2. Theory
Several drilling factors influence drilling operations, some of which are controllable, and others of which are not [7], as shown in figure 1. Some of these factors may thus be tightly controlled in order to obtain the required speed during drilling to break the rock formation, in addition to avoiding problems that may arise during the drilling process [7]. Controllable operational factors that may affect the rate of penetration include the weight on bit (WOB), rotations per minute (RPM), the type of bit used, jet impact force, and bit hydraulics. A parametric sensitivity analysis was performed, as discussed later, to investigate which controllable operational parameter most significantly affect the developed ROP model.
The permeability and the strength of the formation affect the rate of penetration, as do various drilling fluid properties such as fluid density, rheology, viscosity, chemical composition, solid content, and filtration characteristics [3]. ROP tends to decrease on increases of fluid viscosity, fluid density, and solid and lubricant content, and increase on increases in filtration rate. Other factors such as torque, cuttings transport and the equivalent circulating density (ECD) also influence the rate of penetration; for example, ROP tends to increase as ECD decreases [7].

In order to maximize rates of penetration and minimize drilling costs, both quantitative and qualitative assessments are required to enhance drilling process efficiency [7]. The classic drilling curve, which is divided into three regions as shown in figure 3, is used for this purpose.

Many mathematical models have been proposed in an effort to describe the relationship of several drilling variables with the penetration rate. According to previous studies [13,14,15], such models analyse the parameters, describe their relationship to the rate of penetration, and find solutions that offer control over them. Several named models are thus now used in the field [4]:

a) MSE (Mechanical Specific Energy)

b) Burgoyne and Young Model

c) Warren Model

d) Modified Warren Model

Figure 2. Factors Affecting ROP (7)

Figure 3. The classic drilling curve (ROP vs. WOB) [1].
e) Real-Time Bit Wear Model  
f) Hareland and Rampersad Model  
g) Maurer Model.

3. Methodology and model  
Burgoyne and Young’s model may be the most important among the models named earlier, as it is based on statistical analysis of previous drilling parameters [6]. A linear penetration model is then introduced and multiple regression analysis over rate of penetration is conducted.

The model proposed by Burgoyne and Young has thus been adopted for this project in order to derive equations to perform ROP estimation using the available input data. This model was selected as the most complete mathematical drilling model in use in the industry for roller-cone type bits. Burgoyne and Young proposed the following equation to model the drilling process when using roller cone bits:

\[ \frac{df}{dt} = e^{[a_1 + \sum_{j=2}^{n} a_j x_j]} \]  
where 
\[ \frac{df}{dt} \] - rate of penetration 
\( a_1 - a_8 \) - constants 
\( x_1 - x_8 \)-drilling parameters or functions

3.1. Model equations  
The model can therefore be expressed as [6]

\[ ROP = f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 \]  
(2)

The first term \( (f_1) \) expresses the effect of rock drillability

\[ f_1 = e^{2.303 a_1} \]  
(3)

\( a_1 \) – Formation type parameter

The second term \( (f_2) \) models the compaction effect and is given by

\[ f_2 = e^{2.303 a_2 (10000 - D)} \]  
(4)

\( a_2 \) – Normal compaction parameter

\[ x_2 = (10000 - D) \]  
(5)

\[ f_2 = e^{2.303 a_2 x_2} \]  
(6)

The third term models \( (f_3) \) under-compaction due to differential pressure as

\[ f_3 = e^{2.303 a_3 D 0.69 (g_p - 9.0)} \]  
(7)

\( a_3 \)-Under compaction parameter

\[ x_3 = D 0.69 (g_p - 9) \]  
(8)

\[ f_3 = e^{2.303 a_3 x_3} \]  
(9)

where \( g_p \) is the pore pressure gradient in pounds per gallon equivalent.

The fourth term \( (f_4) \) is the effect of differential pressure

\[ f_4 = e^{2.303 a_4 D (g_p - p_c)} \]  
(10)

\( a_4 \) – Pressure differential parameter

\[ x_4 = D (g_p - p_c) \]  
(11)

\[ f_4 = e^{2.303 a_4 x_4} \]  
(12)

where \( (p_c) \) is the mud weight in pound per gallon.
The fifth term \((f_5)\) models the effect on ROP caused by changing the WOB

\[
f_5 = \left[ \frac{(WOB)}{d_b} - \frac{(WOB)}{d_b} \right]^{a_5} \\
\frac{4.0 - (WOB)}{d_b} \cdot t
\]

\(a_5 \quad \text{Weight on bit (WOB) parameter}
\)

\[
x_5 = \ln \left[ \frac{(WOB)}{d_b} - \frac{(WOB)}{d_b} \right]^{1} \\
\frac{4.0 - (WOB)}{d_b} \cdot t
\]

\(f_5 = e^{2.303a_5x_5}
\)

The sixth term \((f_6)\) models the effect of rotary speed (RPM) on the ROP and is given by

\[
f_6 = \left( \frac{N}{60} \right)^{a_6}
\]

\(a_6 \quad \text{RPM parameter}
\)

\[
x_6 = \ln \left( \frac{N}{60} \right)
\]

\(f_6 = e^{2.303a_6x_6}
\)

The seventh term \((f_7)\) models the effect of bit wear on the ROP; this depends on bit type and formation type and is given by

\[
f_7 = e^{-a_7h}
\]

\(a_7 \quad \text{Tooth wear parameter.}
\)

\(f_7 = e^{a_7x_7}
\)

The last term \((f_8)\) is the effect of bit hydraulics on the ROP, given as

\[
f_8 = \left( \frac{F_j}{1000} \right)^{a_8}
\]

\(a_8 \quad \text{Hydraulics parameter}
\)

\[
F_j = \frac{350 \mu d_n}{pq}
\]

\(x_8 = \left[ \frac{350 \mu d_n}{pq} \right]
\]

\(f_8 = e^{a_8x_8}
\)

where \((F_j)\) is the hydraulic jet impact force beneath the bit and

- \(D\) - True vertical depth (ft)
- \(d_b\) - Bit diameter (in)
- \(F_j\) - Jet impact force (lbf)
- \(g_p\) - Pore pressure gradient (lbf/gal)
- \(h\) - Fractional bit tooth wear
- \(\rho_e\) - Equivalent mud density (lbf/gal)
- \(N\) - Rotary speed (rpm)
- \(WOB\) - Weight on bit (1000 lbf)
- \((WOB/d_b)\) - Threshold bit weight per inch

Thus, the general equation is
\[
\frac{dF}{dt} = \left[ e^{a_1} e^{a_2 x_2} e^{a_3 x_3} e^{a_4 x_4} e^{a_5 x_5} e^{a_6 x_6} e^{a_7 x_7} e^{a_8 x_8} \right] \tag{26}
\]

as shown in figure 4.

3.2. Genetic algorithm
Burgoyne and Young recommended a multiple regression method to determine unknown coefficients. However, applying multiple regressions leads to physically meaningless values in some situations, and while new mathematical model methods have recently been developed to reach meaningful results, in order to develop a more accurate prediction and physically meaningful coefficients, a genetic algorithm was used to determine the coefficients in the current work [11].

3.3. Optimum weight on bit
Optimum weight on bit can be calculated as [12]:

\[
\left( \frac{WOB}{d_b} \right)_{\text{opt}} = \frac{a_5 H_1 \left( \frac{WOB}{d_b} \right)_{\text{max}} + a_6 \left( \frac{WOB}{d_b} \right)_{L}}{a_5 H_1 + a_6} \tag{27}
\]

where

\( (WOB/d_b) \) opt. - optimized weight on bit to bit diameter (1000Ib/in.)
\( H_1 \) - Constant that depend on bit type
\( (WOB/d_b) \) max. - Maximum weight on bit to bit diameter (1000Ib/in.)

Figure 4. Burgoyne and Young Model parameters (9)
4. Results and discussion

Field data were taken from Faihaa well 1 as shown in table 1. The primary drilling variables required for the regression analysis are depth, penetration rate, bit weight per inch of bit diameter, rotary speed, fractional tooth wear, jet impact force, mud density, and pore pressure gradient.

Table 1. Field data for Faiha 1 well

| Data entry | Depth (ft) | RPM | Pore Gradient (ppg) | ECD (ppg) | W/d (1000lb/in) | H tooth dull. | Impact force (lb) |
|------------|------------|-----|---------------------|-----------|----------------|---------------|------------------|
| 1          | 1977.84    | 100 | 9.33893             | 9.3296    | 0.461538       | 0.5           | 1.86             |
| 2          | 3512.88    | 110 | 9.33893             | 9.3296    | 0.461538       | 0.25          | 1.82             |
| 3          | 4165.60    | 110 | 9.75584             | 9.7461    | 0.461538       | 0.25          | 2.35             |
| 4          | 4555.92    | 100 | 10.00596            | 9.9960    | 0.384515       | 0.25          | 1.77             |
| 5          | 7002.80    | 100 | 10.08937            | 10.0793   | 0.857143       | 0.375         | 1.85             |
| 6          | 8019.6     | 100 | 10.25614            | 10.2459   | 0.857143       | 0.25          | 2.16             |
| 7          | 8583.76    | 100 | 10.33952            | 10.3292   | 0.857143       | 0.25          | 2.0              |
| 8          | 8918.32    | 100 | 10.7564             | 10.7457   | 0.85714        | 0.125         | 1.96             |
| 9          | 9184.00    | 110 | 11.1733             | 11.1622   | 1.46938        | 0.125         | 2.22             |
| 10         | 9971.20    | 100 | 11.1733             | 11.1622   | 1.22449        | 0.25          | 2.24             |
| 11         | 10850.24   | 100 | 11.2567             | 11.2455   | 1.22449        | 0.125         | 1.88             |
| 12         | 11076.56   | 100 | 11.2567             | 11.2455   | 1.22449        | 0.125         | 2.18             |
| 13         | 11319.28   | 100 | 11.2567             | 11.2455   | 1.63265        | 0.25          | 1.69             |
| 14         | 13198.72   | 110 | 13.5081             | 13.4946   | 1.63265        | 0.125         | 2.15             |
| 15         | 13448.00   | 50  | 14.1752             | 14.1610   | 0.59701        | 0.125         | 2.37             |
| 16         | 13612.00   | 90  | 14.5087             | 14.4942   | 1.79104        | 0.126         | 2.40             |
| 17         | 14491.04   | 90  | 14.5087             | 14.4942   | 1.19403        | 0.127         | 2.16             |

The parameters \(x_1\) through \(x_8\) were calculated using Equations 5 through 23 for each data entry shown in table 2 and uniform formation imposed. To calculate the best values of the regression constants \(a_1\) through \(a_8\)using the data shown in the table 2, eight equations with the eight unknowns, \(a_1\) through \(a_8\), were obtained from \(x_1\) through \(x_8\). Using the values of \(x_2 - x_8\) shown in table 2 for the relevant data points in table 1 in the general equation for rate of penetration gives:

\[
185.9634 = a_1 + 1057.562*a_2 + 1447.931*a_3 + 106.6964*a_4 + \ldots \\
\ldots + 5.555*a_5 + 0.480083*a_6 - 0.21341*a_7 - 6.18955*a_8
\]

(28)

Table 2. Values of Parameters \(x_2 - x_8\)

| \(x_2\)   | \(x_3\)   | \(x_4\) | \(x_5\)   | \(x_6\)   | \(x_7\)   | \(x_8\)   |
|----------|----------|---------|----------|----------|----------|----------|
| 8064.80  | 62.79865 | 18.05464| 4.880719 | 0.510826 | -0.5     | -6.28718|
| 6529.76  | 93.96315 | 32.37595| 4.880719 | 0.616138 | -0.25    | -6.30892|
| 5877.04  | 236.0085 | 40.18278| 4.880719 | 0.606136 | -0.25    | -6.05334|
| 5468.72  | 334.3458 | 45.11475| 4.69818  | 0.510826 | -0.25    | -6.33678|
| 3039.84  | 488.1872 | 70.15354| 5.500258 | 0.510826 | -0.375   | -6.29257|
| 2023.04  | 618.4547 | 81.73113| 5.500258 | 0.510826 | -0.25    | -6.13765|
| 1458.88  | 691.3491 | 88.22294| 5.500258 | 0.510826 | -0.25    | -6.21461|
The eight equations were solved for the eight unknowns and the constants, $a_1$ through $a_8$, thus obtained. A rate of penetration model was then constructed for the field. The prediction of rate of penetration using the constructed model and the actual rate of penetration taken from the actual drilling report are shown in figure 5 and the values of Actual and Predicted ROP are shown in table 4.

![Figure 5. Actual vs. Predicted ROP for Faihaa-1 well](image)

**Table 3.** Values of constants $a_1$ to $a_8$ from regression analysis

| Parameters                  | Constant | Value  |
|-----------------------------|----------|--------|
| Formation strength          | $a_1$    | 29.6151|
| Normal Compaction           | $a_2$    | -0.00284|
| Under compaction            | $a_3$    | 0.006345|
| Pressure differential       | $a_4$    | -0.32809|
| Weight on bit               | $a_5$    | 1.239237|
| Rotary speed                | $a_6$    | -4.80025|
| Tooth wear                  | $a_7$    | 5.463498|
| Jet impact force            | $a_8$    | -0.06485|
Table 4. Values of actual and predicted ROP

| Dataset numbers | Actual ROP | Predicted ROP |
|-----------------|------------|---------------|
| 1               | 9.085446   | 11.68409      |
| 2               | 29.53396   | 25.19321      |
| 3               | 31.83753   | 30.07397      |
| 4               | 45.8201    | 43.27922      |
| 5               | 42.70037   | 44.08499      |
| 6               | 55.39556   | 79.42219      |
| 7               | 74.59493   | 74.78371      |
| 8               | 164.3644   | 167.6999      |
| 9               | 212.5898   | 231.7731      |
| 10              | 189.1154   | 119.5773      |
| 11              | 157.7752   | 191.9705      |
| 12              | 191.8943   | 177.8083      |
| 13              | 151.2346   | 124.4362      |
| 14              | 66.18127   | 108.5359      |
| 15              | 120.4356   | 131.6141      |
| 16              | 339.5423   | 281.8025      |
| 17              | 171.3342   | 133.1121      |

While a multiple regression method may result in negative or zero values, such negative or zero values for coefficients are physically meaningless. Applying a genetic algorithm using the trust-region method allowed the computed coefficients gained to all be physically meaningful and in recommended bounds, as shown in table 5.

Table 5. Values of constants (using a genetic algorithm)

| Parameter                   | Constant | Value  |
|-----------------------------|----------|--------|
| Formation strength          | a1       | 30.7   |
| Normal compaction           | a2       | 0.00025|
| Under compaction            | a3       | 0.00059|
| Pressure differential       | a4       | 0.0157 |
| Weight on bit               | a5       | 1.1877 |
| Rotary speed                | a6       | 4.023  |
| Tooth wear                  | a7       | 8.2355 |
| Jet impact force            | a8       | 5.0548 |

In terms of optimization of the WOB, the prediction results for data numbers 7, 9, 13, and 15, as shown in table 1, can be optimized by applying equation 25; the results are shown in table 6.
Table 6. Optimized Value of Weight on Bit

| Data number | Depth-ft  | Rotary speed-rpm | Actual WOB-lb | Optimized WOB-lb |
|-------------|-----------|------------------|---------------|------------------|
| 7           | 8541.12   | 100              | 15000         | 2304.711         |
| 9           | 9141.36   | 110              | 18000         | 16000            |
| 13          | 11191.36  | 100              | 20000         | 16000            |
| 15          | 13247.92  | 50               | 5000          | 10000            |

Figure 6 illustrates the simulation of weight on bit versus rate of penetration for a depth of 8541.12 ft. The increase of weight on bit varies linearly with rate of penetration, and thus cannot be predicted for an optimized WOB. The optimized weight on bit value for this depth is thus 23047.11 lb, as calculated using equation 27; however, this value occurs after the flounder region and thus cannot be applied to datasets 1 to 8.

Figure 7 illustrates the relationship of weight on bit with rate of penetration at a depth of 13,247.92 ft where the actual weight on bit equals 5,000 lb and the optimized weight on bit equals 10,000 lb; this value occurs before the flounder region, and can be used to eliminate the flounder region.
As shown in figure 8, another two points for actual weight on bit 18,000 and 20,000 lb, at 9,141.36 ft and 11,191.36 ft, respectively were selected. The optimized weight on bit was 16,000 lb, which can thus be identified as the optimum value to enhance performance after the flounder region.

5. Conclusion

Determination of optimum weight on bit is very important in drilling operations, as this parameter can be changed during drilling operation to improve efficiency.
1. The optimization of weight on bit can help optimize the whole drilling operation, increasing the rate of penetration and thus reducing the time needed for drilling, which reduces the overall cost of drilling operations.

2. The constants $a_1$ to $a_9$, which represent the effects of different drilling parameters, can be estimated using Multiple Regression analysis; this was done for the first time for the region used in this study.

3. The Bourgoyne and Young Model produces a reliable Rate of Penetration (ROP) model. On datasets 9, 10, 12, 15, 16, and 17, it predicted accurate ROP as compared with the actual ROP obtained from the field.

4. Optimization of Weight on Bit (WOB) showed that, for a depth of 8,541.12 ft, the optimal WOB was 23,047.11 lb, compared to 15,000 lb at 9,141.36 ft; similarly, the optimized value of WOB was 16,000 lb at 13,247.92 ft.

5. The results of this research provide guidance for further drilling operations close to the observed well in Faihaa Field, as these optimal values can be used as reference to obtain optimum drilling performance and reduce drilling cost.

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