Coulomb Drag Between Parallel
Ballistic Quantum Wires

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The Coulomb drag between parallel, ballistic quantum wires is studied theoretically in the presence of a perpendicular magnetic field $B$. The transresistance $R_D$ shows peaks as a function of the Fermi level and splitting energy between the 1D subbands of the wires. The sharpest peaks appear when the Fermi level crosses the subband extrema so that the Fermi momenta are small. Two other kinds of peaks appear when either \textit{intra}- or \textit{inter}-subband transitions of electrons have maximum probability; the \textit{intra}-subband transitions correspond to a small splitting energy. $R_D$ depends on the field $B$ in a nonmonotonic fashion: it decreases with $B$, as a result of the suppression of backscattering, and increases sharply when the Fermi level approaches the subband bottoms and the suppression is outbalanced by the increase of the Coulomb matrix elements and of the density of states.

PACS: 73.20 Dx

(March 24, 2022)
I. INTRODUCTION

Experimentally \[1\] and theoretically \[2\] momentum transfer between spatially separated electron layers or Coulomb drag has been studied mostly between two-dimensional (2D) layers. Theoretically, this drag has also been studied between very long one-dimensional (1D) layers (quantum wires) in which the wire length \(L\) is much longer than the mean free path \(l_i\) (diffusive regime \[3\]) and recently between 1D layers of length \(L \ll l_i\), in which the electron motion along the layer, at low temperatures, is mostly ballistic \[4\] \[5\] (ballistic regime). Even when most of the electrons pass along the wires without collisions, a few of them experience backscattering due to the interaction with the electrons of the other wire and this modifies the time-averaged distribution functions in such a way that the drag effect occurs. In both regimes the drag response is found to be maximal when the subbands in the two wires line up precisely. It is important that in the ballistic regime the transresistance is determined only by the Coulomb interaction between the electrons and such basic properties of the layers as the number of occupied subbands, and does not include the relaxation characteristics of the system such as scattering times. Therefore, the ballistic regime provides a possibility to obtain more direct information about the Coulomb interaction in 1D electron systems.

Motivated by the results of Refs. \[4\] and \[5\] we undertook an extended theoretical study of the drag in the ballistic regime, without tunneling between the wires, but in the presence of a perpendicular magnetic field \(B\). In Secs. II and III, we generalize the theory of Ref. 4 to include the effects of intersubband transitions in electron-electron collisions and account for the influence of a magnetic field on the Coulomb drag; a limited account of this influence, valid when only the lowest subbands in the two wires are occupied, appeared in Ref. \[5\]. Concluding remarks follow in Sec. IV.
II. GENERAL FORMALISM

We use a model of a four-terminal double-quantum-wire system, as shown schematically in Fig. 1, similar to the systems investigated in the "directional coupler" problem [1]. Two closely spaced quantum wires, numbered 1 and 2 and centered at \(y_1\) and \(y_2\), are contacted independently to four leads at \(x = 0\) and \(x = L\), where \(L\) is the length of the wires. The leads have chemical potentials \(\mu_{1,2}(0) = \mu_{1,2}^+\) and \(\mu_{1,2}(L) = \mu_{1,2}^-\). Applying the bias \(V = (\mu_2^+ - \mu_2^-)/e\) to the leads of wire 2 (drive wire) we obtain the current \(I\) flowing through it. This current induces a voltage \(V_D = (\mu_1^+ - \mu_1^-)/e\) in wire 1 (drag wire). This is the typical setup for drag measurements [1]. We assume that the barrier between the wires is high enough to allow the neglect of tunneling.

Below we assume that the electrons in each wire are parabolically confined by the potentials \(U_j = \varepsilon_j^0 + m^*\Omega_j^2(y - y_j)^2/2\), \(j = 1, 2\). In the presence of a perpendicular magnetic field \(B\), introduced through the vector potential \(A = (-By, 0, 0)\), the normalized wave functions are \(\Psi_{jnk}(x, y) = e^{ikx}\chi_{jnk}(y), \chi_{jnk}(y) = (\pi^{1/2}\ell_j^{2n}n!)^{-1/2} H_n((y - Y_j)/\ell_j)\exp(-(y - Y_j)^2/2\ell_j^2)\), where \(n\) is the 1D subband number, \(k\) the wave vector of electrons, and \(H_n(x)\) the Hermite polynomials. Neglecting spin splitting the corresponding energy spectrum \(\varepsilon_{jn}(k)\) reads

\[
\varepsilon_{jn} = \varepsilon_j^0 + \hbar\omega_j(n + 1/2) + (\hbar^2/2m_j)(k - y_j/\ell_j^2)^2. \tag{1}
\]

Here \(\omega_j^2 = \omega_c^2 + \Omega_j^2\), \(\omega_c = eB/m^*\) is the cyclotron frequency, \(m_j = m^*\Omega_j^2/\Omega_j^2\), \(\ell_c = (\hbar/m^*\omega_c)^{1/2}\) is the magnetic length, \(\ell_j^2 = \hbar/m^*\omega_j\), and \(Y_j = [\Omega_j^2y_j + \hbar\omega_ck/m^*]/\omega_j^2\) are the \(k\)-dependent centers of the oscillators. The expressions for the kinetic energies \((\hbar^2/2m_j)(k - y_j/\ell_j^2)^2\) of the electrons can be simplified by a gauge invariant transformation resulting in a shift of \(k\) by an arbitrary constant. Since we neglect tunneling, we do not consider electron transitions between the wires and can make such shifts independently for each wire; this does not affect the kinetic equations written below. Explicitly, we will shift the wave vectors in the manner \(k - y_1/\ell_c^2 \rightarrow k\) for wire 1 and \(k - y_2/\ell_c^2 \rightarrow k\) for wire 2. Then the kinetic energies in Eq. (1) will read \(\hbar^2k^2/2m_j\) and the oscillator centers \(Y_j = y_j + (\hbar\omega_c/m^*\omega_j^2)k\).
If the distribution functions \( f_{jk}(x) \equiv f_{jk} \) change over distances much longer than both the electronic wavelength \( \pi/k \) and the characteristic radius of the interaction potential, we can write the Boltzman kinetic equations as

\[
\frac{\hbar k}{m_j} \frac{\partial f_{jn_k}(x)}{\partial x} = -\frac{4\pi}{\hbar} \sum_{j'k'q} \sum_{n'n''} \left| M^{j'j}_{n'n''}(k, k', q) \right|^2 \delta(\varepsilon_{jn_k} + \varepsilon_{j'n'k'} - \varepsilon_{jn_1,k-q} - \varepsilon_{j'n'_1,k'+q})
\times [f_{jn_k}(1 - f_{jn_1,k-q})f_{j'n'k'}(1 - f_{j'n'_1,k'+q}) - f_{jn_1,k-q}(1 - f_{jn_k})f_{j'n'_1,k'+q}(1 - f_{j'n'k'})],
\]

where the collision integral accounts only for electron-electron scattering. The Coulomb matrix elements \( M^{j'j}_{n'n''}(k, k', q) \) are given by

\[
M^{j'j}_{n'n''}(k, k', q) = \frac{2e^2}{\kappa} \int dy \int dy' K_0(|q||y - y'|) \chi_{jn_k}(y)\chi_{j'n'k'}(y')\chi_{j'n'_1,k'+q}(y')\chi_{jn_1,k-q}(y),
\]

where \( \kappa \) is the dielectric constant and \( K_0 \) the modified Bessel function.

It is convenient to write separately the distribution functions for the forward- and backward-moving electrons as \( f_{jk} = f_{jk}^+ |_{k>0} \) and \( f_{jk} = f_{jk}^- |_{k<0} \), respectively. For these functions the boundary conditions are given in the Landauer-Buttiker approach by

\[
f_{jn_k}(0) = f(\varepsilon_{jn_k} - \delta \mu_j^-) \quad \text{and} \quad f_{jn_k}(L) = f(\varepsilon_{jn_k} - \delta \mu_j^+),
\]

where \( \delta \mu_j^+ = \mu_j^+ - \mu \), \( \mu \) is the equilibrium chemical potential, \( f(\varepsilon) = [e^{(\varepsilon-\mu)/k_BT} + 1]^{-1} \), and \( T \) the temperature. For \( j = 1 \) and \( j = 2 \), Eq. (2) gives two coupled kinetic equations whose solution allows us to express the unknown potentials \( \mu_1^- \) and \( \mu_1^+ \) through the fixed \( \mu_2^- \) and \( \mu_2^+ \) values and thereby calculate the transresistance.

**III. RESULTS**

If most of the electrons move through the wires ballistically, Eq. (2) can be solved by simple iterations. The zero-order approximation gives \( f_{jn_k}^+(x) = f(\varepsilon_{jn_k} - \delta \mu_j^+) \) and \( f_{jn_k}^-(x) = f(\varepsilon_{jn_k} - \delta \mu_j^-) \). Substitution of these functions in the collision integral gives a non-zero contribution for backscattering collisions between the electrons of different wires. This is
the main contribution which will be considered in detail in the following. If more than a single subband in a wire is occupied, the intersubband transitions within one wire also contribute to the collision integral of Eq. (2) (if only the lowest subband is occupied, the intralayer part of the collision integral completely vanishes because of the relation \( q = k - k' \) following from the momentum and energy conservation rules). However, within the iterative approach used here, we can neglect the influence of the intralayer collisions on the distribution functions of the drive layer \((j = 2)\), since the transport regime is nearly ballistic. Further, the intralayer collisions do not modify considerably the distribution functions of the drag layer \((j = 1)\) because \( \mu_1^+ - \mu_1^- \) is assumed to be much smaller than \( \mu_2^+ - \mu_2^- \), and the main effect on \( f_{\pmnk}(x) \) results from the interlayer Coulomb interaction. Considering only contributions linear in \( V \), we substitute the equilibrium Fermi-Dirac functions \( f_{\pmnk}(x) = f(\varepsilon_{\pmnk}) \) in the collision integral and obtain

\[
\begin{align*}
    f_{+nk}(x) &= f(\varepsilon_{nk} - \delta \mu_1^+) - eV(m_1/\hbar k)\lambda_n(k)x, \\
f_{-nk}(x) &= f(\varepsilon_{nk} - \delta \mu_1^-) + eV(m_1/\hbar k)\lambda_n(-k)(x - L),
\end{align*}
\]

where the factor

\[
\lambda_n(k) = \frac{4\pi}{\hbar k_B T} \sum_{n'_{1},n'_{1'}k'q} \left\{ \left| M_{n_{1}n_{1}'n'_{1}n'_{1'}}^{1212}(k, k', q) \right|^2 \delta(\varepsilon_{nk} + \varepsilon_{2n'k'} - \varepsilon_{n_{1}k_{1}q} - \varepsilon_{n'_{1}k'_{1}q}) \\
    \times f(\varepsilon_{nk}) \left[ 1 - f(\varepsilon_{nk+q}) \right] f(\varepsilon_{n'k'}) \left[ 1 - f(\varepsilon_{n'k'+q}) \right] \right\}_{k > 0, k' > 0, q > 0}
\]

is determined by the Coulomb matrix elements and the equilibrium distribution functions only. The current flowing in the drag wire is given by

\[
I_D = \frac{e}{\pi} \sum_n \int_0^\infty dk \left( \hbar k/m_1 \right) \left[ f_{+nk}(x) - f_{-nk}(x) \right].
\]

\( I_D \) does not depend on \( x \) due to the property \( \sum_n \int_\infty^\infty \lambda_n(k)dk = 0 \), which follows from detailed balance. Substituting Eqs. (4) and (5) into Eq. (7), using the requirement \( I_D = 0 \), and defining the transresistance \( R_D \) as \( R_D = -V_D/I \) through the ballistic current \( I = V/R_N \), where \( R_N = h/2e^2N \) is the ballistic resistance of wire 2 and \( N \) the number of occupied subbands, we finally obtain
\[ R_D = \frac{\hbar L}{NN_D e^2 k_B T} \sum_{n,n_1=0}^{N_D-1} \sum_{n',n_1'=0}^{N-1} \int_{-\infty}^{0} dk' \int_{-k'}^{\infty} dq \int_0^q dk \left| M^{1221}_{n_1' n' n}(k, k', q) \right|^2 \]

\[ \times \delta(\varepsilon_{1nk} + \varepsilon_{2n'k'} - \varepsilon_{1n,k-q} - \varepsilon_{2n',k'+q}) f(\varepsilon_{1nk})[1 - f(\varepsilon_{1nk,k-q})] f(\varepsilon_{2n',k'})[1 - f(\varepsilon_{2n',k'+q})] \]  

(8)

Here \( N_D \) is the number of occupied subbands in the drag wire (wire 1). Note that the introduction of both \( N \) and \( N_D \) assumes that the theory is valid when the Fermi energy \( \mu - \varepsilon_j^0 - \hbar \omega_j(n + 1/2) \) with respect to the highest occupied level is larger than \( k_B T \). This, of course, implies that the 1D subband separations \( \hbar \omega_1 \) and \( \hbar \omega_2 \) are much larger than \( k_B T \) and is true at \( T \sim 1 \) K for electrostatically defined electron channels.

Below we consider the case of identical wires \( \Omega_1 = \Omega_2 = \Omega \), which entails \( \omega_1 = \omega_2 = \omega \), \( \ell_1 = \ell_2 = \ell \), and \( m_1 = m_2 \). To further evaluate expression (8), it is convenient to detach the contribution \( R^{(1)}_D \) from \( R_D \) that expresses the equality \( n + n' = n_1 + n_1' \) for which the energy conservation law gives \( q = k - k' \). Then we have \( R_D = R^{(1)}_D + R^{(2)}_D \) with

\[ R^{(1)}_D = \frac{m^* L}{NN_D e^2 k_B T} \sum_{n,n_1=0}^{N_D-1} \sum_{n',n_1'=0}^{N-1} \frac{\delta_{n+n',n_1+n_1'}}{k_n k_n'(k_n + k_n')} \frac{\Delta^2_{n,n_1'}}{\sinh^2 \Delta_{n,n_1'}} \]

\[ \times \left| M^{1221}_{n_1' n' n}(k_n, k_n', k_n + k_n') \right|^2. \]  

(9)

Here \( \Delta_{n,n_1'} = [\Delta + \hbar \omega(n - n_1')]/2k_B T \) and \( \Delta = \varepsilon_1^0 - \varepsilon_2^0 \) is the interwire splitting energy between the lowest subbands. Further, \( k_n = (\omega/\Omega)[2m^*(\mu - \varepsilon_1^0 - \hbar \omega(n + 1/2))/\hbar^2]^{1/2} \) and \( k_n' = (\omega/\Omega)[2m^*(\mu - \varepsilon_2^0 - \hbar \omega(n' + 1/2))/\hbar^2]^{1/2} \) are the Fermi wave numbers for the states 1, \( n \) and 2, \( n' \), respectively. The part \( R^{(2)}_D \), corresponding to \( n + n' \neq n_1 + n_1' \), is obtained as

\[ R^{(2)}_D = \frac{m^* L^2}{4\hbar NN_D e^2 k_B T Q^2} \sum_{n,n_1=0}^{N_D-1} \sum_{n',n_1'=0}^{N-1} \int_{-\infty}^{0} dk \int_0^q dk' (1 - \delta_{n+n',n_1+n_1'})/p(k, k') \]

\[ \times \Theta(k k' + \omega^2(n + n' - n_1 - n_1')/\Omega^2 \ell^2) \left| M^{1221}_{n_1' n' n}(k, -k', q) \right|^2 f(\varepsilon_{1nk}) f(\varepsilon_{2n'k'}) \]
\[ 
\times [1 - f((\varepsilon_{1nk} + \varepsilon_{2n'k'})/2 + \Delta(k, k')/2)] [1 - f((\varepsilon_{1nk} + \varepsilon_{2n'k'})/2 - \Delta(k, k')/2)] \quad (10) 
\]

where \( p(k, k') = [(k + k')^2/4 + \omega^2(n + n' - n_1 - n'_1)/\Omega^2\ell^2]^{1/2} \), \( q = (k + k')/2 + p(k, k') \), and \( \Delta(k, k') = \Delta + \hbar\omega(n_1 - n'_1) - (\Omega/\omega)^2\hbar^2 p(k, k')(k - k')/m^* \). The statistical factor in Eq. (10) is small unless \(|\varepsilon_{1nk} - \mu|, |\varepsilon_{2n'k'} - \mu|, \) and \(|\Delta(k, k')|\) are small enough and comparable to \( k_B T \). This allows the integrals over \( k \) and \( k' \) are to be carried out in narrow regions around \( k_n \) and \( k_{n'} \), respectively. We used the same property to reduce the contribution \( R_D^{(1)} \) to expression (9). Although the requirement \(|\Delta(k, k')| \sim k_B T \) imposes certain restrictions on the values of \( \mu, \Delta, \) and \( \omega \), the processes with \( n + n' \neq n_1 + n'_1 \) can give a considerable contribution to \( R_D \), especially for \(|\Delta + \hbar\omega(n - n'_1)| \gg k_B T \) and \( R_D^{(1)} \) small. We stress that the previous theoretical work [4] on the Coulomb drag in the ballistic regime took into account only the processes with \( n = n_1 \) and \( n' = n'_1 \), thus neglecting other processes completely from the beginning. The numerical calculations given below demonstrate that this limitation is considerable in many cases.

If only the lowest subbands are occupied in each wire, i.e., for \( n = n_1 = n'_1 = 0 \), the calculation of the transresistance is considerably simplified. Only \( R_D^{(1)} \) contributes to \( R_D \) and Eq. (9) can be rewritten as

\[
R_D = \frac{2e^2m^*^3\omega^6Lk_B T}{\pi\hbar^8k^2\Omega^6k_1k_V(k_1 + k_V)} \left( \frac{(\Delta/2k_B T)^2}{\sinh^2(\Delta/2k_B T)} \right) \times e^{-(\omega_c/\omega)^2\ell^2(k_1+k_V)^2} \left[ \int_{-\infty}^{\infty} du e^{-u^2/2} K_0 ((k_1 + k_V)d + \ell u) \right]^2. \quad (11)
\]

Expression (11) is convenient for assessing the magnetic-field dependence of the transresistance \( R_D \). It directly demonstrates a significant reduction [5] of the drag effect by the magnetic field \( B \), mostly due to the exponential factor. The decrease of \( R_D \) starts as \( R_D(B) - R_D(0) \sim -B^2 \) and becomes exponential with increasing \( B \). The physical reason for this decrease is the suppression of backscattering in electron-electron collisions as the oscillator centers for forward- and backward-moving electrons are pulled apart by the magnetic field. The characteristic field \( B_0 \) for this suppression depends on the position of the
Fermi level $\varepsilon_F$ and is estimated as $B_0 \sim (m^*/e)(m^*\Omega^3/h)^{1/2}/(k_1 + k_{1'})$. If $\varepsilon_F$ is not far from the subband bottoms, $B_0$ is big and the suppression is weak. If $\varepsilon_F$ is well in between the 1D subbands, $B_0$ is estimated as 1 tesla for typical wire parameters. However, when $\varepsilon_F$, with the increase of $B$, approaches the subband bottom, the opposite effect takes place: the transresistance increases because the wave vectors $k_1$, $k_{1'}$, and $q$ become progressively smaller and the suppression of backscattering becomes less important than the increase of the Coulomb matrix element and of the density of states.

Below we present numerical results for the transresistance $R_D$, expressed in units of the fundamental resistance $R_0 = h/2e^2$, at $T = 1.3$ K, $L = 0.4$ μm, $d = |y_1 - y_2| = 50$ nm, $\hbar \Omega = 4$ meV, $m^* = 0.067 m_0$, and $\kappa = 13$. Figure 2 shows the dependence of $R_D$ on the Fermi energy $\varepsilon_F$, defined as $\varepsilon_F = \mu - (\varepsilon_0^1 + \varepsilon_0^2)/2$, calculated at $B = 0$ and $B = 1$ tesla. The calculations were done assuming that up to two subbands can be populated in each wire. As seen in part (a), for $\Delta = 0$ there are pronounced sharp peaks of $R_D$ when $\varepsilon_F$ crosses the bottoms of the first and second subbands. The sharpness of the peaks is explained by a strong enhancement of the Coulomb collision probability when $k_n$, $k_{n'}$, and $q$ are small, cf. Eqs. (9) and (10). In Fig. 2 (b), for $\Delta = 1$ meV, one can see three peaks; the middle one appears after $\varepsilon_F$ crosses the bottom of the second subband in wire 2. This peak exists due to the processes with $n + n' \neq n_1 + n'_1$. The third, most prominent peak in Fig. 2 (b), appears after $\varepsilon_F$ crosses the bottom of the second subband in wire 1, so that two subbands in both wires are populated. The processes with $n + n' \neq n_1 + n'_1$ give the main contribution to this peak as well.

The application of the magnetic field shifts the peaks to higher Fermi energies, due to the increased confinement energy, and sharpens them due to the suppression of backscattering in the regions far from the subband edges. The resulting decrease of the transresistance due to this suppression is illustrated in Fig. 3 for one ($\varepsilon_F = 3.5$ meV) and two ($\varepsilon_F = 8$ meV) subbands populated. These dependences are non-monotonic: when, with the increase of $B$, $\varepsilon_F$ approaches the first or second subband bottom, $R_D$ starts to grow sharply. This
dependence is basically the same for both $\Delta = 0$ and $\Delta = 1$ meV.

Figure 4 shows the dependence of $R_D$ on the level splitting $\Delta$ at several constant values of $\mu - \varepsilon_1^0 = \varepsilon_F - \Delta/2$. This means that the subband positions of wire 1 remain constant with respect to the Fermi level but those of wire 2 do not; this can be experimentally achieved, for example, by changing the voltage of the gate adjacent to wire 2 while keeping that adjacent to wire 1 at a constant voltage. The curves are plotted for one (a) or two (b) subbands populated in wire 1 but for different $\mu$, far from (solid) and close to (dashed) the upper populated subband edge. Both curves of Fig. 4 (a) show two peaks: the sharp ones appear when the second subband of wire 2 becomes populated while the broad ones appear when the second subband of wire 2 is aligned with the first one of wire 1, at $\Delta = \hbar \omega \simeq 4.36$ meV. A similar behavior is seen in Fig. 4 (b). At large negative $\Delta$ only one subband is populated in wire 2 while at $\Delta \simeq -1.5$ (solid) and $-0.5$ meV (dashed) the second subband of wire 2 becomes populated as well. This transition is reflected by strong and sharp peaks in $R_D$. Other strong peaks appear at $\Delta = 0$, when the subbands are aligned; note that on the dashed curve such a peak merges with that at $\Delta \simeq -0.5$ meV and is not resolved.

The minor peaks in the regions of negative and positive $\Delta$ exist due to the inter-subband transitions with $(n, n_1, n', n'_1) = (0, 1, 0, 0)$ and $(1, 0, 0, 0)$ and $(n, n_1, n', n'_1) = (0, 1, 1, 1)$ and $(1, 0, 1, 1)$, respectively. The maxima of these peaks occur when $\Delta(k, k') \approx \Delta(k_n, k_{n'})$, cf. Eq. (10), goes to zero. Thus, the level-splitting dependence of $R_D$ shows a rich structure of peaks indicating that both the intra- and inter-subband transitions of electrons contribute to $R_D$.

All calculations described in this section were repeated for different values of the interwire separation $d$. An increase of $d$ considerably decreases the transresistance: $R_D$ drops by more than one order of magnitude as $d$ varies from 40 to 60 nm, mainly due to the dependence of the Bessel function on its argument. However, all qualitative features presented above are preserved.
IV. REMARKS AND CONCLUSIONS

The treatment of the drag effect in the ballistic transport regime demonstrates the salient properties of electron-electron collisions in double-layer quasi-1D electron systems. The reduced dimensionality dramatically decreases the scattering probabilities at low temperatures due to the restrictions imposed by the momentum and energy conservation laws. As a result, the transresistance shows peaks as a function of either the Fermi level position or the interlayer level splitting energy. The peaks always appear when the Fermi level crosses the bottom of a subband, so that a new subband $n$ is involved in the scattering process; the Fermi wave number $k_n$ for this subband is small, the density of states is high, and this results in a higher scattering probability. When subband $n$ is aligned to another one, the conservation rules allow electron transitions inside the subband $n$, the corresponding momentum transfer $\hbar q \simeq 2\hbar k_n$ is small, and the Coulomb matrix element is large thus giving rise to an additional increase of the peak. Next, the peaks appear when two subbands from different layers are aligned; this favors transitions which conserve the sum of the subband numbers, $n+n' = n_1+n'_1$, especially the transitions between the electrons inside the aligned subbands, cf. Eq. (9). Finally, peaks occur under special conditions, for $\Delta(k_n, k_{n'}) \simeq 0$, cf. Eq. (10); this implies a maximum probability for intersubband transitions with $n+n' \neq n_1+n'_1$. Although the peaks associated with these transitions are usually weaker than those under subband alignment, they give a considerable contribution which cannot be neglected. The described rich structure of the peaks is best seen in the level-splitting dependence of the transresistance shown in Fig. 4.

A magnetic field $B$ applied perpendicular to the wire plane reduces the overlap between the wave functions for forward- and backward-moving electrons and thereby tends to suppress electron-electron scattering. This results in a decrease of the transresistance. In addition, the application of $B$ modifies the quantization energies and leads to a shift of the subbands with respect to the Fermi level. Since the scattering probability increases when a subband edge comes close to the Fermi level, the transresistance $R_D$ may increase with the
increase of $B$. Therefore, the dependence of $R_D$ on $B$ is basically non-monotonic as shown in Fig. 3.

The results obtained here are valid when the 1D electron gas in either wire is described as a normal Fermi liquid. We used this model because the wires are short, the transport is nearly ballistic, and the properties of the 1D electrons are determined by those of the 2D reservoirs they are injected from. The case of the Coulomb drag between 1D electron systems described as Luttinger liquids has been studied in Ref. 7.

Concerning experimental results we are aware only of those of Ref. [8] where the transresistance $R_D$ was measured as a function of side gate voltages controlling the confining potentials of the parallel, submicron-long quantum wires, thus allowing to change the positions of the 1D subbands with respect to the Fermi level, the interlayer subband splitting $\Delta$, the wire widths $W_j$, and the interwire distance $d$. It was found that $R_D$ shows sharp peaks when the Fermi level crosses the bottom of a 1D subband. When the gate adjacent to the drag wire was kept at a constant voltage, corresponding to one populated subband in it, the transresistance, as a function of the voltage of the gate adjacent to the drive wire, showed two peaks. The shape and position of these peaks permit us to identify them with those of Fig. 4 (a), since the situation described by Fig. 4 (a) corresponds roughly to this type of measurements. These experimental results provide qualitative support for our theoretical predictions. However, as no formal connection is made in our model between the gate voltages and the parameters $\varepsilon_F$, $\Delta$, $d$, and $W_j$, we cannot attempt a more detailed comparison. Such a connection requires a detailed knowledge of the gate-induced modification of the double-wire confining potential which could be obtained only by a self-consistent solution of the electrostatic problem for the three-gate structure investigated in Ref. 8. We expect though that further experimental and theoretical work will test sufficiently the drag in the ballistic regime and our results.
ACKNOWLEDGMENTS

The work of PV was supported by the Canadian NSERC Grant No. OGP0121756.
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FIGURES

FIG. 1. Schematic diagram of a coupled quantum-wire device.

FIG. 2. Dependence of the transresistance $R_D$ on the position of the Fermi level for (a) aligned, $\Delta = 0$, and (b) shifted, $\Delta = 1$ meV, levels in quantum wires. The dashed and solid curves correspond to $B = 0$ and $B = 1$ tesla, respectively. The other parameters are listed in the text.

FIG. 3. Dependence of $R_D$ on the magnetic field $B$, with one ($\varepsilon_F = 3.5$ meV) and two ($\varepsilon_F = 8$ meV) populated 1D subbands, at $\Delta = 0$ (a) and $\Delta = 1$ meV (b).

FIG. 4. Dependence of $R_D$ on the level splitting energy $\Delta$ at $B = 1$ tesla when one (a) or two (b) populated subbands of wire 1 remain constant with respect to the Fermi level, $\varepsilon_F - \Delta/2 = \text{const}$. (a): $\varepsilon_F - \Delta/2 = 3.5$ meV (dashed) and 4 meV (solid). (b): $\varepsilon_F - \Delta/2 = 7$ meV (dashed) and 8 meV (solid).
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