Abstract

The hadronic decay $B \to J/\psi K^*$ is analyzed within the framework of QCD factorization. The spin amplitudes $A_0$, $A_\parallel$ and $A_\perp$ in the transversity basis and their relative phases are studied using various different form-factor models for $B - K^*$ transition. The effective parameters $a^h_2$ for helicity $h = 0, +, -$ states receive different nonfactorizable contributions and hence they are helicity dependent, contrary to naive factorization where $a^h_2$ are universal and polarization independent. QCD factorization breaks down even at the twist-2 level for transverse hard spectator interactions. Although a nontrivial strong phase for the $A_\parallel$ amplitude can be achieved by adjusting the phase of an infrared divergent contribution, the present QCD factorization calculation cannot say anything definite about the phase $\phi_\parallel$. Unlike $B \to J/\psi K$ decays, the longitudinal parameter $a^0_2$ for $B \to J/\psi K^*$ does not receive twist-3 corrections and is not large enough to account for the observed branching ratio and the fraction of longitudinal polarization. Possible enhancement mechanisms for $a^0_2$ are discussed.
I. INTRODUCTION

It has been well known that the factorization approach (naive or generalized) fails to explain the production ratio \( R = \mathcal{B}(B \to J/\psi K^*)/\mathcal{B}(B \to J/\psi K) \) and the fraction of longitudinal polarization \( \Gamma_L/\Gamma \) in \( B \to J/\psi K^* \) decay. We consider two representative form-factor models for \( B - K(K^*) \) transitions, the Ball-Braun (BB) model based on the light-cone sum rule (LCSR) analysis \( ^{[1]} \) and the Melikhov-Stech (MS) model \( ^{[2]} \) based on the constituent quark picture. Both are consistent with the lattice calculation at large \( q^2 \), the constraint from heavy-quark symmetry on the \( q^2 \) dependence of heavy-light transition (see Sec. IV for more details). We see from Table I that in general the predicted longitudinal polarization is too small, whereas the production ratio is too large.

### TABLE I. The ratio of vector meson to pseudoscalar production \( R \) and the longitudinal polarization fraction \( \Gamma_L/\Gamma \) in \( B \to J/\psi K^{(*)} \) decays calculated in two representative form-factor models using the factorization hypothesis.

|       | BB   | MS   | CDF  | CLEO | BaBar | Belle |
|-------|------|------|------|------|-------|-------|
| \( R \) | 3.40 | 3.11 | 1.53 ± 0.32 | 1.45 ± 0.26 | 1.38 ± 0.11 | 1.43 ± 0.13 |
| \( \Gamma_L/\Gamma \) | 0.47 | 0.46 | 0.61 ± 0.14 | 0.52 ± 0.08 | 0.60 ± 0.04 | 0.60 ± 0.05 |

This is understandable because the parameter \( a_2 \), which governs \( B \to J/\psi K(K^*) \) decays, is assumed to be universal according to the factorization hypothesis, namely \( a_2^B(J/\psi K^*) = a_2(J/\psi K) \) where \( h = 0, +, - \) refer to the helicity states 00, ++ and -- respectively. In the above-mentioned form-factor models, one has \( h_0 = 5.98, h_+ = 6.23 \) and \( h_- = 0.43 \) (in units of \( \text{GeV}^3 \)) in the BB model and \( h_0 = 5.47, h_+ = 5.92 \) and \( h_- = 0.73 \) in the MS model, where \( h_i \) are the helicity amplitudes given by

\[
\begin{align*}
    h_0 &= \frac{f_{J/\psi}}{2m_K^*} \left[ (m_B^2 - m_{J/\psi}^2 - m_{K^*}^2)(m_B + m_{K^*}) A_{1}^{BK^*}(m_{J/\psi}^2) - \frac{4m_B^2 p_c^2}{m_B + m_{K^*}} A_{2}^{BK^*}(m_{J/\psi}^2) \right], \\
    h_\pm &= m_{J/\psi} f_{J/\psi} \left[ (m_B + m_{K^*}) A_{1}^{BK^*}(m_{J/\psi}^2) \pm \frac{2m_B p_c}{m_B + m_{K^*}} V_{BK^*}(m_{J/\psi}^2) \right].
\end{align*}
\]

It is obvious that \( h_+ > h_0 \gg h_- \). Therefore, under naive factorization \( \Gamma_L/\Gamma \approx (a_2^0 h_0)^2/[(a_2^0 h_0)^2 + (a_2^+ h_+)^2] = h_0^2/(h_0^2 + h_+^2) \ll 1/2 \) and \( R \) is expected to be greater than unity due to three polarization states for \( J/\psi K^* \). These two problems will be circumvented if nonfactorized terms contribute differently to each helicity amplitude and to different decay modes so that \( a_2^0(J/\psi K^*) > a_2^+(J/\psi K^*) \neq a_2^-(J/\psi K^*) \) and \( a_2(J/\psi K) > a_2^0(J/\psi K^*) \). In other words, the present data imply that the effective parameter \( a_2^0 \) should be non-universal and polarization dependent. Recently two of us have analyzed charmless \( B \to VV \) decays.
within the framework of QCD factorization \cite{footnote1}. We show that, contrary to phenomenological
generalized factorization, nonfactorizable corrections to each partial-wave or helicity amplitude
are not the same; the effective parameters $a_i$ vary for different helicity amplitudes. The
purpose of the present paper is to study the nonfactorizable effects in $B \to J/\psi K^*$ decay
within the same framework of QCD factorization.

The decays $B \to J/\psi K(K^*)$ are of great interest as experimentally only a few color
suppressed modes in hadronic $B$ decays have been measured so far. The recent measurement
by BaBar \cite{Footnote2} has confirmed the earlier CDF observation \cite{Footnote3} that there is a nontrivial strong
phase difference between polarized amplitudes, indicating final-state interactions. However,
no such evidence is seen by CLEO \cite{Footnote4} and more recently by Belle \cite{Footnote5}. It is interesting to
check if the current approach for $B$ hadronic decays predicts a departure from factorization.
Therefore, the measurements of various helicity amplitudes in $B \to J/\psi K^*$ decays will
provide a nice ground for testing factorization and differentiating various theory approaches
in which the calculated nonfactorizable terms have real and imaginary parts.

It is known that in the QCD factorization approach the coefficient $a_2$ is severely sup-
pessed in the absence of hard spectator interactions. It has been shown in \cite{Footnote6} that $|a_2|$ in
$B \to J/\psi K$ is of order 0.11 to the leading twist order, to be compared with the experimental
value of order 0.25. The twist-3 effect in hard spectator interactions will enhance $a_2$ to the
value of $0.19^{+0.14}_{-0.12}$. We shall see later that, contrary to the $J/\psi K$ case, $a_2^0$ in $B \to J/\psi K^*$ does
not receive twist-3 contributions and it is dominated by twist-2 hard spectator interactions.

The layout of the present paper is as follows. In Sec. II we first outline the necessary
ingredients of the QCD factorization approach for describing $B \to J/\psi K^*$ and then we pro-
ceed to compute vertex and hard spectator interactions. The ambiguity of the experimental
determination of spin amplitude phases is addressed in Sec. III. Numerical calculations and
results are presented in Sec. IV. Discussions and conclusions are shown in Sec. V.

\section*{II. $B \to J/\psi K^*$ IN QCD FACTORIZATION} \label{sec:II}

\subsection*{A. Factorization formula} \label{sec:IIA}

The general $B \to J/\psi K^*$ amplitude consists of three independent Lorentz scalars:

\begin{equation}
A[B(p) \to J/\psi(\epsilon_{J/\psi}, p_{J/\psi})K^*(\epsilon_{K^*}, p_{K^*})] \propto \epsilon_{J/\psi}^{\mu} \epsilon_{K^*}^{\nu} (ag_{\mu\nu} + bp_{\mu}p_{\nu} + ic\epsilon_{\mu\nu\alpha\beta}p_{\alpha}^{\mu}p_{\beta}^{\nu}), \tag{2.1}
\end{equation}

where $\epsilon^{0123} = +1$ in our convention, the coefficient $c$ corresponds to the $P$-wave amplitude,
and $a$, $b$ to the mixture of $S$- and $D$-wave amplitudes. Three helicity amplitudes can be
constructed as \cite{Footnote7}

\*For $\overline{B} \to J/\psi K^*$ decay the transverse amplitudes are given by $H_{\pm} = -a \pm m_{BP}c$.
\[ H_0 = -\frac{1}{2m_{j/\psi}m_K^*} \left[ (m_B^2 - m_{j/\psi}^2 - m_{K^*}^2)a + 2m_B^2 p_e^2 c \right], \]

\[ H_\pm = a \pm m_B p_e c, \quad (2.2) \]

where \( p_e \) is the c.m. momentum of the vector meson in the \( B \) rest frame. If the final-state two vector mesons are both light as in charmless \( B \to V_1 V_2 \) decays with \( V_1 \) being a recoiled meson and \( V_2 \) an ejected one, it is expected that \(|H_0|^2 > |H_+|^2 > |H_-|^2\) owing to the argument that the amplitude \( H_+ \) is suppressed by a factor of \( \sqrt{2}m_2/m_B \) as one of the quark helicities in \( V_2 \) has to be flipped, while the \( H_- \) amplitude is subject to a further chirality suppression of order \( m_1/m_B \) \([9]\). However, for \( B \to J/\psi K^* \) decay, \( \sqrt{2}m_{j/\psi}/m_B \) is of order unity and hence in practice \( H_+ \) and \( H_0 \) can be comparable.

Note that the polarized decay amplitudes can be expressed in several different but equivalent bases. For example, the helicity amplitudes can be related to the spin amplitudes in the transversity basis \( (A_0, A_\parallel, A_\perp) \) defined in terms of the linear polarization of the vector mesons, or to the partial-wave amplitudes \( (S, P, D) \) via:

\[ A_0 = H_0 = -\frac{1}{\sqrt{3}} S + \frac{\sqrt{2}}{3} D, \]
\[ A_\parallel = \frac{1}{\sqrt{2}} (H_+ + H_-) = \frac{\sqrt{2}}{3} S + \frac{1}{\sqrt{3}} D, \]
\[ A_\perp = \frac{1}{\sqrt{2}} (H_+ - H_-) = P; \quad (2.3) \]

where we have followed the sign convention of \([10]\). The decay rate reads

\[ \Gamma(B \to J/\psi K^*) = \frac{p_e}{8\pi m_B^2} \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* |(H_0)|^2 + |H_+|^2 + |H_-|^2, \]
\[ = \frac{p_e}{8\pi m_B^2} \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* |A_0|^2 + |A_\parallel|^2 + |A_\perp|^2, \]
\[ = \frac{p_e}{8\pi m_B^2} \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* (|S|^2 + |P|^2 + |D|^2). \quad (2.4) \]

The effective Hamiltonian relevant for \( B \to J/\psi K^* \) has the form

\[ \mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{cb} V_{cs}^* \left[ c_1(\mu) O_1(\mu) + c_2(\mu) O_2(\mu) \right] - V_{tb} V_{ts}^* \sum_{i=3}^{10} c_i(\mu) O_i(\mu) \right\} + \text{h.c.}, \quad (2.5) \]

where

\[ O_1 = (\bar{c}b)_{V-A}(\bar{s}c)_{V-A}, \quad O_2 = (\bar{s}b)_{V-A}(\bar{c}c)_{V-A}, \]
\[ O_{3,5} = (\bar{s}b)_{V-A} \sum_{q'} (\bar{q}' q')_{V-A(V+A)}, \quad O_{4,6} = (\bar{s}b)_{V-A} \sum_{q'} (\bar{q}' q')_{V-A(V+A)}; \quad (2.6) \]
\[ O_{7,9} = \frac{3}{2} (\bar{s}b)_{V-A} \sum_{q} e_{q'}(\bar{q}' q')_{V+A(V-A)}; \quad O_{8,10} = \frac{3}{2} (\bar{s}b)_{V-A} \sum_{q'} e_{q'}(\bar{q}' q')_{V+A(V-A)}; \]
with $O_3 - O_6$ being the QCD penguin operators, $O_7 - O_{10}$ the electroweak penguin operators, and $(q_1q_2)_{\gamma\alpha}(1 \pm \gamma_5)q_2$. Under factorization, the decay amplitude of $B \to J/\psi K^*$ reads

$$A(B \to J/\psi K^*) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* (a_2 + a_3 + a_5 + a_7 + a_9) X^{(BK^*,J/\psi)},$$

(2.7)

where

$$X^{(BK^*,J/\psi)} \equiv \langle J/\psi | (\bar{c}c)_{V-A} | 0 \rangle \langle K^* | (\bar{b}s)_{V-A} | B \rangle$$

$$= -if_{J/\psi} m_{J/\psi} \left( (\varepsilon_{K^*} \cdot \varepsilon_{J/\psi})(m_B + m_{K^*}) A_{1}^{BK^*}(m_{J/\psi}^2) \right)$$

$$- (\varepsilon_{K^*} \cdot p_B) (\varepsilon_{J/\psi} \cdot p_B) 2 A_2^{BK^*}(m_{J/\psi}^2) - i \varepsilon_{\mu\nu\alpha\beta} \varepsilon_{J/\psi} \varepsilon_{K^*} p_\mu p_\nu p_\alpha p_\beta \frac{2V^{BK^*}(m_{J/\psi}^2)}{m_B + m_{K^*}} \right].$$

(2.8)

Note that for $\overline{B} \to J/\psi K^*$ decay, the factorizable amplitude $X^{(BK^*,J/\psi)} \equiv \langle J/\psi | (\bar{c}c)_{V-A} | 0 \rangle \langle K^* | (\bar{s}b)_{V-A} | \overline{B} \rangle$ is the same as (2.8) except that the last term proportional to $i \varepsilon_{\mu\nu\alpha\beta}$ has a positive sign. Comparing (2.8) with (2.2) leads to the helicity amplitudes

$$H_0 = -\tilde{a}(J/\psi K^*) h_0, \quad H_\pm = \tilde{a}(J/\psi K^*) h_\pm,$$

(2.9)

where $\tilde{a}(J/\psi K^*) = a_2 + a_3 + a_5 + a_7 + a_9$. Note that the helicity amplitudes $H_\pm$ in $\overline{B} \to J/\psi K^*$ are precisely the ones $H_\mp$ in $B \to J/\psi K^*$ decays. Hence, in the factorization approach one has $|H_-| > |H_+|$ for the former and $|H_+| > |H_-|$ for the latter. This is consistent with the picture that the $s$ quark produced in the weak process $b \to c \bar{c}s$ in $\overline{B} \to J/\psi K^*$ has helicity $-1/2$ in the zero quark mass limit. Therefore, the helicity of $K^*$ in $\overline{B} \to J/\psi K^*$ cannot be $+1$ and the corresponding helicity amplitude $H_+$ vanishes in the chiral limit [11].

**B. QCD factorization**

Under naive factorization, the coefficients $a_i$ are given by $a_{2i} = c_{2i} + \frac{1}{N_c} c_{2i-1}$, $a_{2i-1} = c_{2i-1} + \frac{1}{N_c} c_{2i}$. Hence, $a_{2h}^b(J/\psi K^*) = a_2(J/\psi K)$ for $h = 0, +, -$. In the present paper, we will compute nonfactorizable corrections to $a_{2h}^b(J/\psi K^*)$. The effective parameters $a_{2h}^b$ entering into the helicity amplitudes $H_0$ and $H_\pm$ are not the same.

The QCD-improved factorization approach advocated recently in [12] allows us to compute the nonfactorizable corrections in the heavy quark limit since only hard interactions between the $(BV_1)$ system and $V_2$ survive in the $m_b \to \infty$ limit. Naive factorization is recovered in the heavy quark limit and to the zeroth order of QCD corrections. In this approach, the light-cone distribution amplitudes (LCDAs) play an essential role. The LCDAs of the vector meson are given by [13] [14].

5
\begin{align}
\langle V(P,\varepsilon)|\bar{q}(x)\gamma_\mu q'(0)|0\rangle &= f_{V}\mu \int_{0}^{1} d\xi \, e^{i\xi P \cdot x} \left[ \varepsilon^x \cdot P \phi_{\parallel}(\xi) + (\varepsilon^x \cdot P) \phi_{\perp}(\xi) \right], \\
\langle V(P,\varepsilon)|\bar{q}(x)\gamma_\mu\gamma_5 q'(0)|0\rangle &= \frac{1}{4} m_{V} \left( f_{V} - f_{V}^T \frac{m_q + m_{q'}}{m_{V}} \right) \epsilon_{\mu\nu\alpha\beta} \epsilon^{\nu\sigma} P_{\sigma} x_{\beta} \int_{0}^{1} d\xi \, e^{i\xi P \cdot x} \phi_{\parallel}(\xi), \\
\langle V(P,\varepsilon)|\bar{q}(x)\sigma_{\mu\nu} q'(0)|0\rangle &= -if_{V}^{T}(\varepsilon^x P_{\nu} - \varepsilon^{x}_{\nu} P_{\mu}) \int_{0}^{1} d\xi \, e^{i\xi P \cdot x} \phi_{\perp}(\xi) \\
\langle V(P,\varepsilon)|\bar{q}(x)\gamma_\mu q'(0)|0\rangle &= \left( f_{V} - f_{V}^T \frac{m_q + m_{q'}}{m_{V}} \right) \left( \varepsilon^x \cdot x \right) m_{V}^2 \int_{0}^{1} d\xi \, e^{i\xi P \cdot x} h_{\parallel}(\xi), \\
\langle V(P,\varepsilon)|\bar{q}(x)q'(0)|0\rangle &= i \frac{1}{2} \left( f_{V} - f_{V}^T \frac{m_q + m_{q'}}{m_{V}} \right) \langle P \cdot \varepsilon \rangle m_{V}^2 \int_{0}^{1} d\xi \, e^{i\xi P \cdot x} h_{\perp}(\xi),
\end{align}

where \( x^2 = 0 \), \( \xi \) is the light-cone momentum fraction of the quark \( q \) in the vector meson, \( f_{V} \) and \( f_{V}^{T} \) are vector and tensor decay constants, respectively, but the latter is scale dependent.

In Eq. (2.10), \( \phi_{\parallel}(\xi) \) and \( \phi_{\perp}(\xi) \) are twist-2 DAs, while \( h_{\parallel}(\xi) \), \( g_{\parallel}(\xi) \) and \( g_{\perp}(\xi) \) are twist-3 ones.

Since
\begin{equation}
\frac{\varepsilon \cdot x}{P \cdot x} P^\mu = \varepsilon^\parallel + \frac{\varepsilon \cdot x}{P \cdot x} \frac{m_{V}^2}{2P \cdot x} \varepsilon^\parallel,
\end{equation}

it is clear that to order \( O(m_{V}^2/m_B^2) \) the approximated relation \( \frac{\varepsilon \cdot x}{P \cdot x} P^\mu = \varepsilon^\parallel \) holds for a light vector meson, where \( \varepsilon^\parallel \) (\( \varepsilon^\parallel \)) is the polarization vector of a longitudinally (transversely) polarized vector meson. Also, to a good approximation one has \( \varepsilon^\parallel = P_{V}^\mu/m_{V} \) for a light vector meson like \( K^* \). Hence, \( P \cdot \varepsilon_{\perp} = 0 \) and Eq. (2.10) can be simplified for \( K^* \) as
\begin{align}
\langle K^*(P,\varepsilon)|\bar{q}(x)\gamma_\mu s(0)|0\rangle &= f_{K^*} m_{K^*} \int_{0}^{1} d\xi \, e^{i\xi P \cdot x} \left[ \varepsilon^z \cdot \phi_{\parallel}^{K^*}(\xi) + \varepsilon^{z\perp} \phi_{\perp}^{K^*}(\xi) \right], \\
\langle K^*(P,\varepsilon)|\bar{q}(x)\gamma_\mu\gamma_5 s(0)|0\rangle &= \frac{1}{4} m_{K^*} f_{K^*} \epsilon_{\mu\nu\alpha\beta} \epsilon^{\nu\sigma} P_{\sigma} x_{\beta} \int_{0}^{1} d\xi \, e^{i\xi P \cdot x} \phi_{\perp}^{K^*}(\xi), \\
\langle K^*(P,\varepsilon)|\bar{q}(x)\sigma_{\mu\nu} s(0)|0\rangle &= -if_{K^*}^{T}(\varepsilon^{z\parallel} P_{\nu} - \varepsilon^{z\perp} P_{\mu}) \int_{0}^{1} d\xi \, e^{i\xi P \cdot x} \phi_{\perp}^{K^*}(\xi), \\
\langle K^*(P,\varepsilon)|\bar{q}(x)s(0)|0\rangle &= -\frac{1}{2} f_{K^*} m_{K^*} \int_{0}^{1} d\xi \, e^{i\xi P \cdot x} h_{\perp}^{(s)}(\xi),
\end{align}

where \( h_{\parallel}(\xi) = dh(\xi)/d\xi \) and we have neglected light quark masses and applied the relation
\begin{equation}
(P_{\mu} x_{\nu} - P_{\nu} x_{\mu}) \frac{\varepsilon \cdot x}{(P \cdot x)^2} m_{V}^2 = \frac{\varepsilon \cdot x}{P \cdot x} (P_{\mu} P_{\nu} - P_{\nu} P_{\mu}) + (\varepsilon_{\mu} P_{\nu} - \varepsilon_{\nu} P_{\mu}),
\end{equation}

which vanishes for a light vector meson. From Eq. (2.12) we see that the twist-3 DA \( h_{\parallel}^{(t)} \) of \( K^* \) does not make a contribution.

In the heavy quark limit, the \( B \) meson wave function is given by
\begin{equation}
\langle 0|\bar{b}_{\alpha}(x)q_{\beta}(0)|B(p)\rangle |_{x_{+} = x_{-} = 0} = -\frac{if_{B}}{4} [(\bar{p} + m_{B}) \gamma_{5}]_{\beta\gamma} \int_{0}^{1} d\rho \, e^{-i\rho p \cdot x} [\Phi_{B}^{\rho}(\rho) + \Phi_{2}^{\rho}(\rho)]_{\gamma\alpha}.
\end{equation}

with \( n_{-} = (1, 0, 0, -1) \) and the normalization conditions
\[
\int_0^1 d\bar{\rho} \Phi_1^B(\bar{\rho}) = 1, \quad \int_0^1 d\bar{\rho} \Phi_2^B(\bar{\rho}) = 0. \tag{2.15}
\]

Likewise, to the leading order in \(1/m_c\), the \(J/\psi\) wave function has a similar expression

\[
\langle J/\psi(p, \varepsilon) | \bar{c}(x) c_\beta(0) | 0 \rangle |_{x_s = x_\perp = 0} = \frac{f_{J/\psi}}{4} \left[ i^x (\bar{\rho} + m_{J/\psi}) \right]_\beta \gamma \times \int_0^1 d\xi e^{-i\xi p + x - [\Phi_{J/\psi}^1(\xi) + \Phi_{J/\psi}^2(\xi)]} \gamma_\alpha. \tag{2.16}
\]

Since the \(J/\psi\) meson is heavy, the use of the light-cone wave function for \(J/\psi\) is problematic. The effects of higher twist wave functions have to be included and may not converge fast enough. Because the charmed quark in \(J/\psi\) carries a momentum fraction of order \(\sim m_c/m_{J/\psi}\), the distribution amplitudes of \(J/\psi\) vanish in the end point region. In the following study we adopt \(\Phi||\) as the DA of the non-local vector current of \(J/\psi\) rather than \(g(\varepsilon)\) as the DA of the \(\epsilon_\perp\) component since the latter does not vanish at the end point. Hence, we will treat the \(J/\psi\) wave function on the same footing as the \(B\) meson. Comparing Eq. (2.16) with Eq. (2.10) we see that at the leading order in \(1/m_c\) one has

\[
\Phi_{J/\psi}^1(\xi) = \Phi_{J/\psi}^2(\xi) = \Phi_{J/\psi}^\perp(\xi), \quad f^{T_{J/\psi}} = f_{J/\psi}. \tag{2.17}
\]

The inclusion of vertex-type corrections and hard spectator interaction in QCD factorization leads to

\[
\begin{align*}
a_2^h &= c_2 + \frac{c_1}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_1 F^h, \\
a_3^h &= c_3 + \frac{c_1}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_4 F^h, \\
a_5^h &= c_5 + \frac{c_6}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_6 (-F^h - 12), \\
a_7^h &= c_7 + \frac{c_8}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_8 (-F^h - 12), \\
a_9^h &= c_9 + \frac{c_{10}}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_{10} F^h, \tag{2.18}
\end{align*}
\]

where \(C_F = (N_c^2 - 1)/(2N_c)\) and the superscript \(h\) denotes the polarization of the vector mesons: \(h = 0\) for helicity 0 state, and \(h = \pm\) for helicity \(\pm\) ones. In the naive dimensional regularization (NDR) scheme for \(\gamma_5\), \(F^h\) in Eq. (2.18) has the form

\[
F^h = -12 \ln \frac{\mu}{m_b} - 18 + f_1^h + f_{II}^h, \tag{2.19}
\]

where the hard scattering function \(f_1^h\) arises from vertex corrections [see Figs. 1(a)-1(d)] and \(f_{II}^h\) from the hard spectator interactions with a hard gluon exchange between the emitted vector meson and the spectator quark of the \(B\) meson, as depicted in Figs. 1(e)-1(f).
FIG. 1. Vertex and spectator corrections to $B \to J/\psi K^*$.

C. Vertex corrections

The calculation of vertex corrections in Fig. 1 is very similar to that in $B \to J/\psi K$ decay and the detail can be found in [8]. In terms of the two hard kernels $f_I$ and $g_I$ given by

$$f_I = \int_0^1 d\xi \Phi_{J/\psi}^{\parallel}(\xi) \left\{ \frac{2z\xi}{1 - z(1 - \xi)} + (3 - 2z) \frac{\ln \xi}{1 - \xi} \right.$$  

$$+ \left( -3\frac{1}{1 - z\xi} + 1 \frac{1}{1 - z(1 - \xi)} - \frac{2z\xi}{[(1 - z(1 - \xi))^2]} \right) z\xi \ln z\xi$$  

$$+ \left( 3(1 - z) + 2z\xi + \frac{2z^2\xi^2}{1 - z(1 - \xi)} \right) \ln(1 - z) - i\pi \right\}$$  

$$+ \int_0^1 d\xi \Phi_{J/\psi}^{\perp}(\xi) \left\{ -4r \frac{\ln \xi}{1 - \xi} + \frac{4zr \ln z\xi}{1 - z(1 - \xi)} - 4zr \frac{\ln(1 - z) - i\pi}{1 - z(1 - \xi)} \right\}, \quad (2.20)$$

and

$$g_I = \int_0^1 d\xi \Phi_{J/\psi}^{\parallel}(\xi) \left\{ \frac{-4z\xi}{(1 - z)(1 - \xi)} \ln \xi + \frac{z\xi}{[1 - z(1 - \xi)]^2} \ln(1 - z) + \left( \frac{1}{(1 - z\xi)^2} - \frac{1}{[1 - z(1 - \xi)]^2} \right) \ln(1 - z) \right.$$  

$$+ \frac{2(1 + z - 2z\xi)}{(1 - z)(1 - z\xi)^2} z\xi \ln z\xi - i\pi \frac{z\xi}{[1 - z(1 - \xi)]^2} \right\}.$$
\[ + \int_0^1 d\xi \Phi_{J/\psi}^{I/\psi}(\xi) \left\{ \frac{4r}{(1-z)(1-\xi)} \ln \xi - \frac{4rz}{(1-z)(1-z\xi)} \ln z\xi \right\}, \]  

(2.21)

where \( r = f_{J/\psi}^T m_c/(f_{J/\psi} m_{J/\psi}) \), \( z \equiv m_{J/\psi}^2/m_B^2 \), the first scattering function \( f_l^0 \) induced from vertex corrections has the form

\[
f_l^0 = f_I + g_I (1-z) \frac{A_0^{BK^*}(m_{J/\psi}^2)}{A_3^{BK^*}(m_{J/\psi}^2)} ,
\]

\[
f_l^\pm = f_I ,
\]

(2.22)

where

\[
\tilde{A}_3(q^2) = \frac{m_B + m_K*}{2m_K*} A_1(q^2) - \frac{m_B^2 - m_{J/\psi}^2 + m_K^*}{2m_K*(m_B + m_K*)} A_2(q^2).
\]

(2.23)

In writing Eqs. (2.20) and (2.21) we have distinguished the contributions from \( \Phi_{J/\psi}^{I/\psi} \) and \( \Phi_{J/\psi}^{I/\psi} \) for reader’s convenience, though later we will apply Eq. (2.17). Also notice that we have applied the relation \( r \equiv f_{J/\psi}^T m_c/(f_{J/\psi} m_{J/\psi}) = 2 \left( \frac{m_c}{m_{J/\psi}} \right)^2 = 2\xi^2 \).

(2.24)

Three remarks are in order. (i) As shown in \[8\], the transverse DA \( \Phi_{J/\psi}^{I/\psi} \) contributes not only to the transverse amplitudes \( H_\pm \) but also to the longitudinal amplitude \( H_0 \), and vice versa for the longitudinal DA \( \Phi_{J/\psi}^{I/\psi} \). This occurs because \( J/\psi \) is heavy: the coefficient in front of \( \Phi_{J/\psi}^{I/\psi} \) in Eq. (2.11) consists of not only the longitudinal polarization but also the transverse one. (ii) It is easily seen that in the zero \( J/\psi \) mass limit,

\[
f_l^0 \rightarrow \int_0^1 d\xi \phi^{J/\psi}(\xi) \left( 3 \frac{1 - 2\xi}{1-\xi} \ln \xi - 3i\pi \right),
\]

(2.25)

in agreement with \[12\] for \( B \rightarrow \pi\pi \), as it should be. (iii) The expression of \( A_0/\tilde{A}_3 \) in Eq. (2.22) can be further simplified by applying equations of motion. Neglecting the mass of

\[It is known from HQET that below the \( m_c \) scale, where \( m_c \) is the pole mass of the charmed quark, the vector and tensor currents receive the same anomalous dimensions; that is, \( f_{J/\psi}^T \) and \( f_{J/\psi} m_c \) scale as the same power. Up to the \( m_b \) scale, \( f_{J/\psi}^T \) rescales with a factor \( [\alpha_s(m_b)/\alpha_s(m_c)]^{4/3b} \), \( m_c \) with \( [\alpha_s(m_b)/\alpha_s(m_c)]^{4/6b} \), and the ratio of \( f_{J/\psi}^T/f_{J/\psi} \) becomes \( [\alpha_s(m_c)/\alpha_s(m_b)]^{8/3b} \). Since \( m_c(m_b) = (1.1 - 1.2) \times 2m_c(m_b)/m_{J/\psi} \), where \( b = (11N_c - 2n_f)/3 \) and \( m_c(m_b) \) is the running charmed quark mass at the \( m_b \) scale. However, the scale factor \( [\alpha_s(m_c)/\alpha_s(m_b)]^{8/3b} \sim (1.1 - 1.2) \) is relatively small and can be neglected for our purposes.
light quarks, applying the equation \( \bar{s}p_K^*(1 - \gamma_5)b = 0 \) and sandwiching it between the \( K^* \) and \( B \) states leads to the result:

\[
- \frac{m_{J/\psi}^2}{2m_B m_{K^*}} A_2(q^2) = A_3(q^2) - A_0(q^2)
\]  (2.26)

and hence \( A_0^{B K^*}(m_{J/\psi}^2)/A_3^{B K^*}(m_{J/\psi}^2) = 1 \). Consequently, \( f_I^0 = f_I + g_I(1 - z) \).

D. Hard spectator interactions

For hard spectator interactions, we write

\[
f_{II} = f_{II(2)} + f_{II(3)},
\]  (2.27)

where the subscript \( (...) \) denotes the twist dimension of the LCDA. To the leading-twist order, we obtain

\[
f_{II(2)}^0 = 4\pi^2 \frac{\alpha_s(\mu_h)}{\alpha_s(\mu)} \frac{f_B f_{J/\psi} f_{K^*}}{h_0} (1 - z) \int_0^1 d\xi d\bar{\rho} d\bar{\eta} \frac{\Phi_1^B(\bar{\rho})\Phi_{J/\psi}(\xi)\Phi_{K^*}(\bar{\eta})}{\bar{\rho}(\bar{\rho} - \bar{\eta} + \bar{\eta}z + 4\xi^2z)}
\]

\[
\times \frac{\bar{\rho} - \bar{\eta} + (\bar{\rho} - 2\xi + \bar{\eta})z + 4\xi^2z}{\bar{\rho}(\bar{\rho} - \bar{\eta} + \bar{\eta}z)[(\bar{\rho} - \xi)(\bar{\rho} - \eta) + (\bar{\eta} - \bar{\rho} + \bar{\eta} - \bar{\rho})z]}.
\]  (2.28)

This can be further simplified by noting that \( \bar{\rho} \sim O(\Lambda_{QCD}/m_b) \rightarrow 0 \) in the \( m_b \rightarrow \infty \) limit.

\[
f_{II(2)}^0 = 4\pi^2 \frac{\alpha_s(\mu_h)}{\alpha_s(\mu)} \frac{f_B f_{J/\psi} f_{K^*}}{h_0} \int_0^1 d\xi d\bar{\rho} \int_0^1 d\bar{\eta} \frac{\Phi_1^B(\bar{\rho})\Phi_{J/\psi}(\xi)\Phi_{K^*}(\bar{\eta})}{\bar{\rho}(\bar{\rho} - \bar{\eta} + \bar{\eta}z)[(\bar{\rho} - \xi)(\bar{\rho} - \eta) + (\bar{\eta} - \bar{\rho} + \bar{\eta} - \bar{\rho})z]},
\]  (2.29)

where the \( z \) terms in the numerator cancel after the integration over \( \xi \) via Eq. (2.24).

Likewise, for transversely polarization states, we find

\[
f_{II(3)}^0 = \frac{4\pi^2}{N_c} \frac{\alpha_s(\mu_h)}{\alpha_s(\mu)} \frac{2f_B f_{J/\psi} f_{K^*}}{m_B h_{\perp}} (1 \pm 1)
\]

\[
\times \int_0^1 d\bar{\rho} d\xi d\bar{\eta} \frac{\Phi_1^B(\bar{\rho})\Phi_{J/\psi}(\xi)\Phi_{K^*}(\bar{\eta})}{\bar{\rho}(\bar{\rho} - \bar{\eta} + \bar{\eta}z)[(\bar{\rho} - \xi)(\bar{\rho} - \eta) + (\bar{\eta} - \bar{\rho} + \bar{\eta} - \bar{\rho})z]}\frac{1 - 2\xi}{\bar{\rho}\bar{\eta}(1 - z)}.
\]  (2.30)

Note that the hard gluon exchange in the spectator diagrams is not as hard as in the vertex diagrams. Since the virtual gluon’s momentum squared there is \( k^2 = (-\bar{\rho}p_B + \bar{\eta}p_{K^*})^2 \approx -\bar{\rho}m_B^2 \sim -\Lambda_h m_b \), where \( \Lambda_h \) is the hadronic scale \( \sim 500 \text{ MeV} \), we will set \( \alpha_s \approx \alpha_s(\sqrt{\Lambda_h m_b}) \) in the spectator diagrams. The corresponding Wilson coefficients in the spectator diagrams are also evaluated at the \( \mu_h = \sqrt{\Lambda_h m_b} \) scale. As for twist-3 contributions to hard spectator interactions, we find

\[
f_{II(3)}^0 = 0,
\]  (2.31)
\[ f_{II(3)}^\pm = \frac{4\pi^2}{N_c} \frac{\alpha_s(\mu_h)}{\alpha_s(\mu)} \frac{2f_B f_{J/\psi} f_{K^*-J/\psi} m_{K^*}}{m_B^2 h_\pm} \times \int_0^1 d\bar{\rho} \frac{\Phi_1^B(\bar{\rho})}{\bar{\rho}} \int_0^1 d\xi \frac{\Phi_{J/\psi}(\xi)}{\xi} \int_0^1 d\bar{\eta} \left( g_{\perp}^{K^*(v)}(\bar{\eta}) + g_{\perp}^{K^*(a)}(\bar{\eta}) \right). \] (2.32)

Since asymptotically \( \Phi^{K^*}(\bar{\eta}) = 6\bar{\eta}(1 - \bar{\eta}) \), the logarithmic divergence of the \( \bar{\eta} \) integral in Eq. (2.23) implies that the spectator interaction is dominated by soft gluon exchanges between the spectator quark and the charmed or anti-charmed quark of \( J/\psi \). Hence, QCD factorization breaks down even at the twist-2 level for \( f_{II(2)}^+ \). Thus we will treat the divergent integral as an unknown “model” parameter and write

\[ Y \equiv \int_0^1 \frac{d\bar{\eta}}{\bar{\eta}} = \ln \left( \frac{m_B}{\mu_h} \right)(1 + \rho_H), \] (2.33)

with \( \rho_H \) being a complex number whose phase may be caused by soft rescattering [12]. Note that linear divergences are cancelled owing to the relation (2.24). Needless to say, how to treat the unknown parameter \( \rho_H \) is a major theoretical uncertainty in the QCD factorization approach.

**E. Distribution amplitudes**

If we apply the asymptotic form for the vector meson’s LCDAs [13]

\[ \Phi_{J/\psi}(x) = 6x(1 - x), \]
\[ g_{\perp}^{(a)}(x) = \frac{3}{4} \left[ 1 + (2x - 1)^2 \right], \] (2.34)

it is easy to check that \( f_{II(3)}^\pm = 0 \). Since the scale relevant to hard spectator interactions is of order \( \mu_h = \sqrt{\Lambda_h m_h} \approx 1.5 \text{ GeV} \), it is important to take into account the evolution of LCDAs from \( \mu = \infty \) down to the lower scale. The leading-twist LCDA \( \Phi_M \) can be expanded in terms of Gegenbauer polynomials \( C_n^{3/2} \) [13]:

\[ \Phi_M(x, \mu) = 6x(1 - x) \left( 1 + \sum_{n=1}^{\infty} a_{2n}^M(\mu) C_n^{3/2}(2x - 1) \right), \] (2.35)

where the Gegenbauer moments \( a_n^M \) are multiplicatively renormalized. To \( n = 2 \) we have

\[ \Phi_{J/\psi}(x, \mu) = 6x(1 - x) \left[ 1 + 3a_1^\perp \xi + \frac{3}{2} a_2^\perp (5\xi^2 - 1) \right], \]
\[ \Phi_{J/\psi}(x, \mu) = 6x(1 - x) \left[ 1 + 3a_1^\parallel \xi + \frac{3}{2} a_2^\parallel (5\xi^2 - 1) \right], \] (2.36)
where $\xi = 2x - 1$. For twist-3 DAs we follow [15] to use

$$
g_\perp^{(v)}(x, \mu) = 6x(1-x)\left[1 + a_2^\parallel \xi + \left\{\frac{1}{4}a_2^\parallel + \frac{5}{3} \zeta_3 \left(1 - \frac{3}{16}\omega_3^A + \frac{9}{16}\omega_3^V\right)\right\}(5\xi^2 - 1)\right]
+ 6\delta_+ [3x(1-x) + (1-x) \ln(1-x) + x \ln x],
+ 6\delta_- [(1-x) \ln(1-x) - x \ln x],
$$
g_\perp^{(v)}(x, \mu) = \frac{3}{4} (1 + \xi^2) + \frac{3}{2} a_1^\parallel \xi^3 + \left\{\frac{3}{7}a_2^\parallel + 5\zeta_3\right\}(3\xi^2 - 1)
+ \left[\frac{9}{112}a_2^\parallel + \frac{15}{64} \zeta_3 (3\omega_3^V - \omega_3^A)\right](3 - 30\xi^2 + 35\xi^4)
+ \frac{3}{2}\delta_+ [2 + \ln x + \ln(1-x)] + \frac{3}{2}\delta_- [2\xi + \ln(1-x) - \ln x],
\tag{2.37}
$$

where the Gegenbauer moments and couplings $\eta_3, \omega_3^{V,A}, \delta_{+,-}$ for $K^*$ at the scale $\mu^2 = 1$ GeV$^2$ and $\mu^2 = 5$ GeV$^2$ can be found in [15]. It turns out that the end-point behavior of $g_\perp^{(v)}$ for $K^*$ is substantially modified and is very different from that of the asymptotic form (see Fig. 3 of [14]).

### III. EXPERIMENTS

The angular analysis of $B^+ \to J/\psi K^{*+}$ and $B^0 \to J/\psi K^{*0}$ has been carried out by CDF [3], CLEO [4] and most recently by the $B$ factories BaBar [5] and Belle [6]. The three polarized amplitudes are measured in the transversity basis with results summarized in Table IV. Experimental results are conventionally expressed in terms of spin amplitudes $\hat{A}_{0,\perp,\parallel}$ normalized to unity, $|\hat{A}_0|^2 + |\hat{A}_\perp|^2 + |\hat{A}_\parallel|^2 = 1$. Since the measurement of interference terms in the angular distribution is limited to $\text{Re}(A_\parallel A_0^\ast)$, $\text{Im}(A_\perp A_0^\ast)$ and $\text{Im}(A_\parallel A_0^\ast)$, there exists a phase ambiguity:

$$
\phi_\parallel \to -\phi_\parallel, \\
\phi_\perp \to \pm \pi - \phi_\perp, \\
\phi_\perp - \phi_\parallel \to \pm \pi - (\phi_\perp - \phi_\parallel).
\tag{3.1}
$$

Take the BaBar measurement [5] as an example:

$$
\phi_\perp = -0.17 \pm 0.17, \quad \phi_\parallel = 2.50 \pm 0.22, \quad \Rightarrow \quad |H_+| < |H_-|, \tag{3.2}
$$

where the phases are measured in radians. The other allowed solution is

$$
\phi_\perp = -2.97 \pm 0.17, \quad \phi_\parallel = -2.50 \pm 0.22, \quad \Rightarrow \quad |H_+| > |H_-|. \tag{3.3}
$$

\footnote{Note that there is a slight difference for the expressions of $g_\perp^{(v,a)}$ in [13] and [14].}
As pointed out in [11], the solution (3.2) indicates that $A_\parallel$ has a sign opposite to that of $A_\perp$ and hence $|H_+| < |H_-|$, in contradiction to what expected from factorization. Therefore, we will compare solution (3.3) with the factorization approach. Obviously there is a 3-σ effect that $\phi_\parallel$ is different from $\pi$ and this agrees with the CDF measurement. However, such an effect is not observed by Belle and CLEO (see Table IV). In Table IV we will only list those amplitude phases from solution (3.3).

The measured branching ratios are

$$
B(B^+ \to J/\psi K^{*+}) = \begin{cases} (13.7 \pm 0.9 \pm 1.1) \times 10^{-4} & \text{BaBar} [5] \\ (12.9 \pm 0.8 \pm 1.2) \times 10^{-4} & \text{Belle} [6] \\ (14.1 \pm 2.3 \pm 2.4) \times 10^{-4} & \text{CLEO} [4] \end{cases}
$$

and

$$
B(B^0 \to J/\psi K^{*0}) = \begin{cases} (12.4 \pm 0.5 \pm 0.9) \times 10^{-4} & \text{BaBar} [5] \\ (12.5 \pm 0.6 \pm 0.8) \times 10^{-4} & \text{Belle} [4] \\ (13.2 \pm 1.7 \pm 1.7) \times 10^{-4} & \text{CLEO} [4] \end{cases}
$$

**IV. NUMERICAL RESULTS**

To proceed we use the next-to-leading Wilson coefficients in the NDR scheme [16]

\[ c_1 = 1.082, \quad c_2 = -0.185, \quad c_3 = 0.014, \quad c_4 = -0.035, \quad c_5 = 0.009, \quad c_6 = -0.041, \]

\[ c_7/\alpha = -0.002, \quad c_8/\alpha = 0.054, \quad c_9/\alpha = -1.292, \quad c_{10}/\alpha = 0.263, \quad c_g = -0.143, \]

at $\mu = \overline{m}_b(m_b) = 4.40$ GeV for $\Lambda_{\overline{\text{MS}}}^{(5)} = 225$ MeV taken from Table XXII of [16] with $\alpha$ being an electromagnetic fine-structure coupling constant. For the decay constants, we use

\[ f_{K^*} = 221 \text{ MeV}, \quad f_{J/\psi} = 405 \text{ MeV}, \quad f_B = 190 \text{ MeV}, \]

and we will assume $f_V^T = f_V$ for the tensor decay constant. For LCDAs we use those in Sec. II.E, and the $B$ meson wave function

$$
\Phi_B^B(\bar{\rho}) = N_B \bar{\rho}^2 (1 - \bar{\rho})^2 \exp \left[ -\frac{1}{2} \left( \frac{\bar{\rho}m_B}{\omega_B} \right)^2 \right],
$$

with $\omega_B = 0.25$ GeV and $N_B$ being a normalization constant.

In the following study, we will consider eight distinct form-factor models: the Bauer-Stech-Wirbel (BSWI) model [17,18], the modified BSW model (referred to as the BSWII model) [19], the relativistic light-front (LF) quark model [20], the Neubert-Stech (NS) model [21], the QCD sum rule calculation by Yang [22], the Ball-Braun (BB) model based on the light-cone sum rule analysis [1], the Melikhov-Stech (MS) model based on the constituent quark picture [23] and the Isgur-Wise scaling laws based on the SU(2) heavy quark symmetry.
TABLE II. Form factors $A_{1}^{BK^*}$, $A_{2}^{BK^*}$ and $V^{BK^*}$ at $q^2 = 0$ and $q^2 = m_{J/\psi}^2$ in various form-factor models.

|       | BSWI | BSWII | LF  | NS  | Yang | BB  | MS  | YYK |
|-------|------|-------|-----|-----|------|-----|-----|-----|
| $A_{1}^{BK^*}(0)$ | 0.33 | 0.33  | 0.26 | 0.30 | 0.18 | 0.34 | 0.36 | 0.49 |
| $A_{1}^{BK^*}(m_{J/\psi}^2)$ | 0.45 | 0.45  | 0.37 | 0.39 | 0.24 | 0.43 | 0.43 | 0.49 |
| $A_{2}^{BK^*}(0)$ | 0.33 | 0.33  | 0.24 | 0.30 | 0.17 | 0.28 | 0.32 | 0.30 |
| $A_{2}^{BK^*}(m_{J/\psi}^2)$ | 0.46 | 0.63  | 0.43 | 0.48 | 0.31 | 0.45 | 0.50 | 0.42 |
| $V^{BK^*}(0)$ | 0.37 | 0.37  | 0.35 | 0.30 | 0.21 | 0.46 | 0.44 | 0.39 |
| $V^{BK^*}(m_{J/\psi}^2)$ | 0.55 | 0.82  | 0.42 | 0.51 | 0.40 | 0.86 | 0.77 | 0.87 |

(YYK) so that the form factor $A_1$ is mostly flat, $A_2$ is a monopole-type form factor and $V$ is a dipole-type one [23]. The values of the form factors $A_{1}^{BK^*}$, $A_{2}^{BK^*}$ and $V^{BK^*}$ at $q^2 = 0$ and $q^2 = m_{J/\psi}^2$ in various form-factor models are shown in Table II.

Among the eight form-factor models, only a few of them are consistent with the lattice calculations at large $q^2$, constraint from $B \rightarrow \phi K^*$ at low $q^2$ and the constraint from heavy quark symmetry for the form-factor $q^2$ dependence. The BSWI model assumes a monopole behavior (i.e. $n = 1$) for all the form factors. However, this is not consistent with heavy quark symmetry for heavy-to-heavy transition. The BSWII model takes the BSW model results for the form factors at zero momentum transfer but makes a different ansatz for their $q^2$ dependence, namely a dipole behavior (i.e. $n = 2$) is assumed for the form factors $F_1$, $A_0$, $A_2$, $V$, motivated by heavy quark symmetry, and a monopole dependence for $F_0$, $A_1$. However, the equality of the form factors $A_{1}^{BK^*}$ and $A_{2}^{BK^*}$ at $q^2 = 0$ is ruled out by recent measurements of $B \rightarrow \phi K^*$ decays [7]. Lattice calculations of $V^{BK^*}$, $A_{0}^{BK^*}$ and $A_{1}^{BK^*}$ at large $q^2$ [24] in conjunction with reasonable extrapolation to $q^2 = m_{J/\psi}^2$ indicate that $V^{BK^*}(m_{J/\psi}^2)$ is of order 0.70-0.80.

The parameters $\tilde{a}^h(J/\psi K^*)$ defined by

$$\tilde{a}^h(J/\psi K^*) = a_2^h + a_3^h + a_5^h + a_7^h + a_9^h$$  \hfill (4.4)

are calculated using Eq. (2.18) and their results are shown in Table III. Since the penguin parameters $a_{5,7,9}^h$ are small, in practice we have $\tilde{a}^h \approx a_2^h$. Note that $a_0^h$ and $a_2^h$ are independent of the parameter $\rho_H$ introduced in Eq. (2.33); that is, they are infrared safe. Since $h_-$ is quite small due to the compensation between the $A_1^{BK^*}$ and $V_1^{BK^*}$ terms and $f_+^{\pm (3)}$ is inversely proportional to $h_-$, $\tilde{a}^-$ becomes more sensitive than $\tilde{a}^+$ to the form-factor model chosen.

From the experimental measurement of spin amplitudes, it is ready to extract the parameters $\tilde{a}^h$ in various form-factor models. We use the averaged decay rate $\Gamma(B \rightarrow J/\psi K^*) =$
TABLE III. The calculated parameters $\tilde{a}^h(J/\psi K^*)$ ($h = 0, +, -$) for $B \rightarrow J/\psi K^*$ decay in QCD factorization using various form-factor models for $B - K^*$ transition. The experimental results for $\tilde{a}^h(J/\psi K^*)$ are obtained using the averaged branching ratio of $B \rightarrow J/\psi K^*$ measured by BaBar, Belle and CLEO in conjunction with the central values of the BaBar measurement for the spin amplitudes $|\hat{A}_{0,\perp}|^2$. Only the central values of $\tilde{a}^h_{\text{expt}}$ are shown here.

| Model | $\tilde{a}^0$ | $|\tilde{a}^0|_{\text{expt}}$ | $\tilde{a}^+$ | $|\tilde{a}^+|_{\text{expt}}$ | $\tilde{a}^-$ | $|\tilde{a}^-|_{\text{expt}}$ |
|-------|---------------|-----------------|-----------|-----------------|-----------|-----------------|
| BSWI  | 0.11 - i0.06  | 0.19            | 0.16 - i0.05 | 0.18               | -0.01 + i0.05 | 0.06             |
| BSWII | 0.15 - i0.06  | 0.25            | 0.14 - i0.05 | 0.15               | -0.07 + i0.05 | 0.14             |
| LF    | 0.14 - i0.06  | 0.25            | 0.19 - i0.05 | 0.23               | -0.02 + i0.05 | 0.07             |
| NS    | 0.14 - i0.06  | 0.25            | 0.18 - i0.05 | 0.20               | -0.03 + i0.05 | 0.08             |
| Yang  | 0.23 - i0.06  | 0.43            | 0.25 - i0.05 | 0.30               | -0.16 + i0.05 | 0.20             |
| BB    | 0.12 - i0.06  | 0.20            | 0.14 - i0.05 | 0.16               | -0.15 + i0.05 | 0.23             |
| MS    | 0.13 - i0.06  | 0.22            | 0.14 - i0.05 | 0.16               | -0.07 + i0.05 | 0.14             |
| YYK   | 0.09 - i0.06  | 0.16            | 0.13 - i0.05 | 0.15               | -0.06 + i0.05 | 0.12             |

$(5.34 \pm 0.23) \times 10^{-16}$ GeV obtained from Eqs. (3.4) and (3.5) and the central values of the spin amplitudes measured by BaBar as an illustration:

$$\hat{A}_0| = 0.597 \pm 0.028 \pm 0.024, \quad |\hat{A}_\perp|^2 = 0.160 \pm 0.032 \pm 0.014,$$

$$|\hat{A}_\parallel|^2 = 0.243 \pm 0.034 \pm 0.017.$$  \hspace{1cm} (4.5)

Then $\tilde{a}^0$ can be determined from $\Gamma_L(B \rightarrow J/\psi K^*) = \Gamma(B \rightarrow J/\psi K^*) \times |\hat{A}_0|^2$ and likewise for $\tilde{a}^\pm$. The results are shown in Table III. It is evident the “experimental” values of $\tilde{a}^h$ are polarization dependent: $|\tilde{a}^0| > |\tilde{a}^+| > |\tilde{a}^-|$, whereas the present QCD factorization calculation yields $|\tilde{a}^+| > |\tilde{a}^0| > |\tilde{a}^-|$.

Normalized spin amplitudes and their phases in $B \rightarrow J/\psi K^*$ decays calculated in various form-factor models using QCD factorization are exhibited in Table IV, where the unknown parameter $\rho_H$ in Eq. (2.33) is taken to be real and unity. For comparison, we also carry out the analysis in the partial wave basis as the phases of $S$, $P$ and $D$ partial wave amplitudes are the ones directly related to the long-range final state interactions. We see from the Tables that the predicted $|\hat{A}_0|^2$, $|\hat{D}|^2$, and branching ratios are too small, whereas $|\hat{A}_\perp|^2 = |\hat{D}|^2$ is too large. It is also clear that a non-trivial phase $\phi_\parallel$ deviated from $-\pi$ is seen in some form-factor models, but it is still too small compared to the BaBar measurement. Nevertheless, a large phase $\phi_\parallel$ as implied by BaBar can be achieved by adjusting the phase of the complex parameter $\rho_H$, but admittedly it is rather arbitrary. In other words, the present QCD factorization calculation cannot say something definite for the phase $\phi_\parallel$. The partial wave decompositions $S, P$ and $D$ corresponding to the relative orbital angular momentum
$L = 0, 1, 2$ between $J/\psi$ and $K^*$ uniquely determine the spin angular momentum. Our results are difficult to account for the observation $|S|^2 : |D|^2 : |P|^2 \simeq 3.5 : 1 : 1$ from recent Babar and Belle measurements.

**TABLE IV.** Normalized spin amplitudes and their phases (in radians) in $B \rightarrow J/\psi K^*$ decays calculated in various form-factor models using QCD factorization. The branching ratios given in the Table are for $B^+ \rightarrow J/\psi K^{*+}$. For comparison, experimental results form CDF, CLEO, BaBar and Belle are also exhibited.

|     | $|\hat{A}_0|^2$ | $|\hat{A}_\perp|^2$ | $|\hat{A}_\parallel|^2$ | $\phi_\perp$ | $\phi_\parallel$ | $B(10^{-3})$ |
|-----|----------------|---------------|----------------|--------------|----------------|------------|
| BSWI| 0.43           | 0.33          | 0.24           | -3.05        | -2.89          | 0.76       |
| BSWII| 0.38         | 0.36          | 0.26           | 3.13         | -3.12          | 0.73       |
| LF | 0.41           | 0.34          | 0.25           | -3.09        | -2.95          | 0.69       |
| NS | 0.40           | 0.34          | 0.25           | -3.10        | -2.99          | 0.70       |
| Yang | 0.38         | 0.36          | 0.25           | -3.12        | -3.11          | 0.64       |
| BB | 0.41           | 0.34          | 0.25           | -3.04        | -3.05          | 0.77       |
| MS | 0.40           | 0.35          | 0.25           | -3.08        | -3.05          | 0.75       |
| YYK | 0.44           | 0.32          | 0.23           | -2.99        | -2.95          | 0.84       |
| CLEO [4] | 0.52 ± 0.08 | 0.16 ± 0.09 | 0.32 ± 0.12 | -3.03 ± 0.46 | -3.00 ± 0.37 | 1.41 ± 0.31 |
| CDF [3] | 0.59 ± 0.06 | 0.13 ± 0.13 | 0.28 ± 0.12 | -2.58 ± 0.54 | -2.20 ± 0.47 |           |
| BaBar [3] | 0.60 ± 0.04 | 0.16 ± 0.03 | 0.24 ± 0.04 | -2.97 ± 0.17 | -2.50 ± 0.22 | 1.37 ± 0.14 |
| Belle [3] | 0.60 ± 0.05 | 0.19 ± 0.06 | 0.21 ± 0.08 | -3.15 ± 0.21 | -2.86 ± 0.25 | 1.29 ± 0.14 |

There are several major theoretical uncertainties in the calculation: $B - K^*$ form factors, the twist-3 LCDAs of $K^*$ at the scale $\mu_h$ and the infrared divergences occurred in twist-2 and twist-3 contributions. It has been advocated that Sudakov form factor suppression may alleviate the soft divergence [23]. Hence, we have studied Sudakov effects explicitly and the detailed results will be presented in a future publication. When partons in the meson carry the transverse momentum through the exchange of gluons, the Sudakov suppression effect will be naturally generated due to large double logarithms $\exp[-\alpha_s C_F \ln^2 (Q^2 / k^2_\perp)]$, which will suppress the long-distance contributions in the small $k_\perp$ region and give a sizable average $\langle k^2_\perp \rangle \sim \bar{\Lambda} m_B$, where $\bar{\Lambda} = m_B - m_b$. This can resolve the singularity problem occurring at the end point. Basically, there is no Sudakov suppression in the vertex correction since the end-point singularity in the hard kernel is cancelled in the convolution. However, for the hard spectator interaction, we can have large Sudakov suppression effects at the end point since there are sizable $\langle k^2_\perp \rangle$ contributions in the propagators. Especially, the end-point singularities without $k_\perp$ do not compensate in the twist-3 contributions. We find that $\tilde{a}_2^0$ is suppressed whereas $\tilde{a}_2^2$ is enhanced by the Sudakov effect and conclude that Sudakov suppression cannot help to solve the discrepancy between theory and experiment.
TABLE V. Normalized partial wave amplitudes and their phases (in radians) in $B \rightarrow J/\psi K^*$ decays calculated in various form-factor models using QCD factorization and fitted from the data, where $\phi_P = \arg (PS^*)$, $\phi_D = \arg (DS^*)$ and there exists a phase ambiguity: $\phi_D \rightarrow -\phi_D$ and $\phi_P \rightarrow \pm \pi - \phi_P$.

| Model | $|S|^2$ | $|P|^2$ | $|D|^2$ | $\phi_P$ | $\phi_D$ |
|-------|--------|--------|--------|----------|----------|
| BSWI  | 0.60   | 0.33   | 0.07   | -0.04    | 2.75     |
| BSWII | 0.60   | 0.36   | 0.04   | 0.02     | 3.10     |
| LF    | 0.60   | 0.34   | 0.06   | -0.05    | 2.80     |
| NS    | 0.60   | 0.34   | 0.06   | -0.05    | 2.86     |
| Yang  | 0.59   | 0.36   | 0.05   | 0.002    | 3.07     |
| BB    | 0.60   | 0.34   | 0.06   | 0.05     | 2.99     |
| MS    | 0.60   | 0.35   | 0.05   | 0.01     | 2.97     |
| YYK   | 0.60   | 0.32   | 0.08   | 0.05     | 2.85     |
| CLEO  | 0.77 ± 0.19 | 0.16 ± 0.09 | 0.07 ± 0.03 | 0.04 ± 0.59 | 2.9 ± 0.59 |
| CDF   | 0.61 ± 0.34 | 0.13^{+0.13}_{-0.11} | 0.26 ± 0.20 | 0.10 ± 0.34 | 2.17 ± 0.34 |
| BaBar | 0.65 ± 0.13 | 0.16 ± 0.03 | 0.19 ± 0.10 | -0.13 ± 0.21 | 2.44 ± 0.21 |
| Belle | 0.66 ± 0.14 | 0.19 ± 0.06 | 0.15 ± 0.03 | -0.14 ± 0.29 | 2.80 ± 0.29 |

V. DISCUSSIONS AND CONCLUSIONS

The hadronic decay $B \rightarrow J/\psi K^*$ is analyzed within the framework of QCD factorization. The spin amplitudes $A_0$, $A_\parallel$ and $A_\perp$ in the transversity basis and their relative phases are studied using various different form-factor models for $B \rightarrow K^*$ transition. The effective parameters $a_{h}^2$ for helicity $h = 0, +, −$ states receive different nonfactorizable contributions and hence they are helicity dependent, contrary to naive factorization where $a_{h}^2$ are universal and polarization independent. QCD factorization breaks down even at the twist-2 level for transverse hard spectator interactions. Although a nontrivial strong phase for the $A_\parallel$ amplitude can be achieved by adjusting the phase of an infrared divergent contribution, the present QCD factorization calculation cannot say anything definite about the phase $\phi_\parallel$. In QCD factorization we found that $a_2^0$ and $a_2^−$ are infrared safe.

Unfortunately, our conclusion is somewhat negative. the longitudinal parameter $a_2^0$ calculated by QCD factorization which is of order 0.15 in magnitude is not large enough to account for the observed decay rates and the fraction of longitudinal polarization. In QCD factorization, the ratio $R$ of vector meson to pseudoscalar production is close to unity with large uncertainties arising from the chirally enhanced and infrared sensitive contributions to $B \rightarrow J/\psi K$. (In the naive factorization approach, $R$ ranges from 1.3 to 4.2, but it is difficult to account for $R$, $\Gamma_L/\Gamma$, and $|P|^2$ simultaneously.) This is mainly ascribed to
the smallness of \( a_0^2 \). It is instructive to compare \( a_0^2(J/\psi K^*) \) in \( B \to J/\psi K^* \) decay with \( a_2(J/\psi K) \) in \( B \to J/\psi K \). It is found in [8] that \( a_2(J/\psi K) = 0.19^{+0.14}_{-0.12} \) for \( |\rho_H| \leq 1 \) and that twist-2 as well as twist-3 hard spectator interactions are equally important. As for \( a_0^2(J/\psi K^*) \), it is dominated by twist-2 hard spectator interactions. We have studied Sudakov form factor suppression on end-point singularities and found that it does not help to solve the discrepancy between theory and experiment.

Since the predicted \( a_0^2 \) in QCD factorization is too small compared to experiment, one may explore other effects that have not been studied. One possibility is that soft final-state interactions (FSIs) may enhance \( a_0^2 \) substantially [27]. A recent observation of \( B^0 \to D^{0(*)}\pi^0 \) decay by Belle [28] and CLEO [29] indicates \( a_2(D^{(*)}\pi) \sim 0.40 - 0.55 \) much larger than the naive value of order 0.25. It is thus conceivable that some sort of inelastic FSIs could make substantial nonperturbative contributions to \( a_0^2 \). The other possibility arises from the gluon component in the \( K^* \) wave function. Consider the diagram in which one of outgoing charmed quarks emits a hard gluon before they form the \( J/\psi \) meson and the gluon fragments into a parton of the \( K^* \) meson. Neglecting the charmed quark mass, because the charmed quark’s helicity is conserved in the strong interaction, this gluon has zero helicity, i.e., it is longitudinally polarized. Following the same argument right after Eq. (2.2), the hybrid \( K^* \) will make a contribution to \( H_0 \) and \( H_\perp \). Although this amplitude is suppressed by order of \( \Lambda_{QCD}/m_b \) owing to the presence of an additional propagator compared to the leading diagram, it is enhanced by the large Wilson coefficient \( c_1 \) and hence cannot be ignored. A similar mechanism can also give a contribution to the \( B \to J/\psi K \) mode but it is difficult to make a quantitative estimate since the chirally enhanced twist-3 contribution is still quite uncertain. Good candidates to search for evidence of this effect are \( B \to \rho_0 \rho_0, \rho_0 \omega, \omega \omega \). Without taking into account the hard gluon emission, the branching ratios of these decays which are color suppressed and dominated by \( b \to d \) penguin contributions are of order \( 10^{-7} \) [30,31,7]. Nevertheless, they can receive large contributions, proportional to \( c_1 \) at the amplitude level, from the hard gluon emission mechanism so that the branching ratios become \( 10^{-6} \sim 10^{-5} \).

**Note added:** We learned the paper by X.S. Nguyen and X.Y. Pham (NP) (hep-ph/0110284, v2) in which a similar analysis in QCD factorization was carried out. However, their results differ from ours in some aspects: (i) There are some discrepancies between Eqs. (2.20-2.22) in the present paper and Eqs. (36) and (37) of NP. Also the expression of \( F_{II}^z \) given by Eq. (39) of NP originally derived in [7] is valid only for two light vector mesons in the final state. It will get some modifications for heavy \( J/\psi \). It should be stressed that Eq. (28) adopted by NP for describing LCDAs works only for a light vector meson, but not for a heavy meson like \( J/\psi \). (ii) For hard spectator interactions, we have considered contributions from leading wave functions of \( B \) and \( J/\psi \) and twist-3 DAs of \( K^* \) [see Eq. (2.32)], which are absent
in NP. Also we have taken into account the relevant scale $\mu_h = \sqrt{\Lambda_h m_b}$ for hard spectator interactions. (iii) Unlike NP we did not consider the higher twist expansion for the $J/\psi$ wave function. The twist expansion of LCDAs is applicable for light mesons but it is problematic for heavy mesons like $J/\psi$. Note that although twist-3 $J/\psi$ contributions to hard spectator interactions were considered by NP, they did not consistently compute the twist-3 effects of $J/\psi$ in vertex corrections.

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