Analysis of influence of shield tunneling on overlying underground pipelines based on HSS model

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Abstract. Small strain stiffness properties of soil play an important role in correctly predicting the surrounding deformation of an underground excavation. Based on a case of analysis for overlying pipelines adjacent to the metro tunnel of Changzhou North Railway Station, a three-dimensional finite element model to analyze this typical problem is proposed. Small-strain of soil is considered to investigate the influence of shield tunneling on overlying underground pipelines. Based on Plaxis 3D, the hardening small-strain (HSS) constitutive model of soil is used in simulation analysis of the ground surface settlement and pipelines deformation. By comparing simulation results with the measured value, it shows that the actual deformation of engineering can be predicted more reasonably when the small strain stiffness of soil is considered in numerical analysis. After studying the influences on deformation characteristics of pipelines due to different burial depths, materials and the relative positions to the tunnel, it is found that the closer the pipeline is adjacent to the tunnel, the greater the influence is from the shield tunneling; and the increase of the stiffness ratio between the pipeline and surrounding soil can effectively reduce the settlement of the pipeline, but it increases the internal force of the pipeline.

1. Introduction
City subway is generally developed in densely populated urban areas with a high concentration of buildings (structures). Construction of subway tunnels inevitably leads to displacement and stress relaxation of the surrounding soil, which causes additional settlement and bending strain in the adjacent pipelines. The influence of tunnel excavation on overlying buried pipelines and on the ability of the pipelines to function normally and safely is one of the focuses of urban subway tunnel engineering.

Considering the highly nonlinear characteristics of soil, researchers often use numerical analysis methods to analyze the influence of tunnel excavation on overlying pipelines. Klar [1], Marshall [2]...
used numerical analysis methods to perform parametric analysis of the influence of tunnel excavation on continuous pipelines, and they compared their results with the results of continuous elastic theory and centrifugal model tests. Using the finite-element method based on the Winkler foundation model theory, Wang et al. [3] and Shi et al. [4] performed parametric analysis of the influence of tunnel excavation on continuous pipelines and presented a design diagram to estimate the maximum bending moment of the pipelines, the results of which were compared with the results of centrifugal model tests. Wang et al. [5] analyzed the influence of tunnel construction on existing rigid pipelines using the three-dimensional finite-element method (FEM) in engineering practices, with the results indicating that the relative positions of the pipeline and tunnel have a relatively larger effect on the settlement and deformation of the pipeline, as well as the differential settlement of the pipeline and soil. They recommended pipeline strain as the pipeline deformation control standard. Through FEM analysis, Wei et al. [6] pointed out that the maximum horizontal displacement and vertical displacement of the pipeline were linearly related to the burial depth of the pipeline and decreased with increasing pipe–soil relative stiffness ratio. At the same time, based on the analytical methods and numerical analysis, pipeline safety evaluation methods and control techniques during tunnel excavation have been proposed [7-9].

Many engineering studies have shown that with the disturbance caused by tunnel construction, except for a small part of the soil layers that will undergo plastic deformation, the vast majority are in a state of small strain within 0.01%–0.1% [10-13]. Under the small strains, soil exhibits deformation characteristics including high stiffness, obvious nonlinearity, anisotropy, yield continuity, and structuredness [14]. At present, limited by experimental conditions, the engineering industry pays less attention to the importance of small-strain stiffness characteristics of soil in predicting soil deformation and often uses conventional tests to determine the stiffness of soils. This method of reflecting soil deformation using destructed soil indicators often seriously underestimates the mechanical properties of the soil [15]. Consideration of the small-strain stiffness characteristics of the soil is important for correctly predicting its deformation around underground structures, especially for tunnels or foundation pits that are dominated by deformation control and located in downtown areas.

Benz et al. [16-17] proposed a small-strain overlay model by theoretical derivation. Compared with the conventional elastic-plastic model, it only needs to add two material parameters to describe the small-strain characteristics of soil. The proposed small soil strain model has now been adopted by commercial software packages, such as Plaxis, which can be used to perform FEM analysis considering the small-strain characteristics of soil in practice [18]. The hardening small-strain (HSS) constitutive model is usually adopted to reflect the small-strain characteristics of soil to predict soil deformation more realistically. The HSS constitutive model is derived from the HS (hardening soil) constitutive model and combines the small-strain characteristics of soil [19]. Therefore, it possesses the characteristics of the HS model, such as its consideration of the hardening characteristics of soft clay. It can distinguish between loading and unloading, and the soil stiffness depends on the characteristics of stress history and stress path. In addition, it can also consider the behavior of the shear modulus attenuating with strain in a small strain range.

In this study, we proposed as analysis case of a shield tunnel section of Changzhou Metro Line 1 adjacent to the Changzhou North Station. Considering the small-strain stiffness characteristics of soil and the HSS model parameters of typical soil layers, the Plaxis FEM software is used to simulate and analyze the disturbance effect of tunnel excavation on overlying underground pipelines, and influences of the burial depth of pipelines, pipeline materials, the pipeline–tunnel relative position, and other factors on pipeline deformation are discussed.

2. Project overview and geological conditions
The section of the Changzhou North Station-Xinqiao Station in this study is the middle section in the first phase of the Changzhou Rail Transit Line 1 project. It is in an area in the Xinbei District to be reserved for development, Changzhou City, and most of the site is open space. As the section of the line departs from the Changzhou North Station, the upper and lower lines deflect to the northwest with
a small radii of 340 m and 360 m, respectively. In the process, the tunnel passes underneath sewers and storm drains under the Beiyi Road. The minimum burial depth of the tunnel section is 9.2 m, and the maximum is 14.2 m. The tunnel lining is assembled using universal wedge-shaped rings with staggered joints. The lining segments have an outer diameter of 6200 mm, an inner diameter of 5500 mm, a segment thickness of 350 mm, and a ring width of 1200 mm.

Changzhou North Station is a two-story underground island station, and the underground building area is 14,518 m². The pipelines around the station are distributed closely and complexly. The affected pipelines are listed in Table 1.

Table 1. Surrounding pipelines of Changzhou North Railway Station.

| No. | The type of pipelines               | The status                                         |
|-----|------------------------------------|---------------------------------------------------|
| 1   | DN300/DN400 sewage pipe            | Polyvinyl chloride (PVC) material, buried depth of 2m |
| 2   | D300/D400 drain pipe               | Polyvinyl chloride (PVC) material, buried depth of 3m |
| 3   | D400/D800 drain pipe               | Concrete material, buried depth of 3.5m            |
| 4   | DN400 service pipe                 | Plastic material, buried depth of 2m               |
| 5   | 10kV/35kV power piping             | Plastic material, buried depth of 3.7m             |
| 6   | Telecom pipe                       | Plastic material, buried depth of 3.7m             |

According to the geological survey reports and engineering experience in the Changzhou area, the soil of the shield tunnel section is representative of the Changzhou area, with the typical characteristics of the soil in this area. From top to bottom, the stratum of the tunnel section is successively ① miscellaneous fill, ①2 plain fill, ③1 clay, ③2 silty clay, ⑤1-1 silty sand mixed with silty clay, ⑤1-2 silty sand, ⑤1-3 silty sand, ⑤2 silty sand, ⑦2 silty clay, ⑧1 silt, and ⑧2 silt. The location and soil layers of the shield tunneling are shown in Figure 1.

![Figure 1. The location and soil layers of the shield tunneling](image)
3. Numerical simulation for the shield tunnel

3.1. Calculation model
In this study, we intend to use the FEM software for large-scale geotechnical engineering, Plaxis3D, to simulate the tunnel excavation process. To simplify the problem, the mutual influence of the excavation within a double-hole tunnel is not be considered. We only calculate and analyze the effect of the single-hole tunnel excavation on the overlying pipelines. In consideration of the symmetry of tunnel excavation, half of the tunnel using a symmetrical model for this calculation is analyzed. In the calculation model, the soil layer is taken at a depth of 35 m, the pipeline is located 2 m underground, and similar types of soil layers are combined: \( \text{③}_1 \) and \( \text{③}_2 \) are combined into the upper silty clay layer; \( \text{⑤}_1, \text{⑤}_2, \text{⑤}_3, \) and \( \text{⑤}_2 \) are combined into silty sand layers; and \( \text{⑦}_2 \) and \( \text{⑧}_1 \) are combined into silt layers. These are the three main soil layers in the overall calculation model. The sand layer is where the shield tunnel is mainly located and also where the shield machine mainly works. The simulation model is shown in Figure 2.

![Figure 2. Simulation model of shield tunneling by Plaxis 3D](image)

The linear elastic model is adopted for the shield machine, concrete segmental lining, and pipelines, with the parameters given in Table 2. The shield machine is simulated with the plate element, and the tunnel after lining is simulated with the concrete material. Under the principle of equal stiffness, the beam element is used to simulate pipelines, and the surface load is used to simulate the earth pressure on the shield head, grouting pressure on the shield tail, and jack thrust. The excavation parameters of the shield machine are given in Table 3.

| Table 2. Basic parameters of materials. |
|----------------------------------------|
| Name | E (kN/m\(^2\)) |
| Shield tunneling machine | 34.5\( \times 10^6 \) |
| Concrete segment | 200\( \times 10^6 \) |
| PVC pipeline | 3.53\( \times 10^6 \) |

| Table 3. Excavation parameters of shield tunneling. |
|----------------------------------------|
| Parameters | Range |
| Earth pressure in the chamber | 0.21\( \sim \)0.25MPa |
| Cylinder thrust force | 12000\( \sim \)16000kN |
| Rotating speed of cutter | 1\( \sim \)1.2rpm |
| Grouting pressure | 0.36\( \sim \)0.45MPa |
3.2. Soil layer parameters.

In the calculation model, an isotropic HSS model is used as the constitutive model of the soil. The HSS model contains many parameters, including not only eleven HS model parameters but also two small strain parameters. Among them, the static earth pressure coefficient K0 can be determined according to the recommendations of Mayne & Kulhawy [20]; the loading and unloading Poisson’s ratio υur can be the value suggested by Brinkgreve and Broere [21], generally taken as 0.2; and the dilatancy angle ψ can be obtained from the study by Bolton [22]. For sand soil, it can be taken as \( \phi' - 30^\circ \). When the internal friction angle \( \phi' \) is less than \( 30^\circ \), ψ can be 0. For clay soil, ψ is generally taken as 0. The remaining parameters can be measured jointly using a consolidation apparatus, a stress path triaxial apparatus, and a resonant column, which include the following: the effective strength indices \( c', \phi' \), the modulus values \( E_{50}^{ref}, E_{ur}^{ref}, E_{oed}^{ref} \) under the reference stress \( p^{ref} \) (100 kPa), the failure ratio \( R_f \), the modulus stress level correlation index \( m \), reference initial shear modulus \( G_0^{ref} \), and reference strain \( \gamma_0 \).

We collect samples from top to bottom in the three typical soil layers in the depth range of the subway foundation, and the basic soil parameters are obtained, as given in Table 4.

| Soil layer | Soil type   | Water content (%) | Natural density (kN/m³) | Void ratio | Sample depth (m) |
|------------|-------------|-------------------|-------------------------|------------|-----------------|
| ③2        | Silty clay  | 22                | 19.5                    | 0.677      | 2.6~6.4         |
| ⑤2        | Silty sand  | 19.8              | 20.4                    | 0.577      | 6.6~9.4         |
| ⑧1        | Silt        | 25                | 18.9                    | 0.779      | 10.1~12.9       |

Four tests are carried out: the triaxial consolidation drainage shear test, the triaxial consolidation drainage loading and unloading test, the standard consolidation test, and the resonance column test to obtain small-strain model parameters of the soil layer: \( c', \phi', p^{ref}, E_{50}^{ref}, E_{ur}^{ref}, E_{oed}^{ref}, R_f, G_0^{ref}, \) and \( \gamma_{0.7} \), as given in Table 5.

| Soil layer | \( c' \)/kPa | \( \phi' \)/\( {}^\circ \) | \( p^{ref} \)/kPa | \( E_{50}^{ref} \)/MPa | \( E_{ur}^{ref} \)/MPa | \( E_{oed}^{ref} \)/MPa | \( R_f \) | \( \gamma_{0.7} \)/\( \times 10^{4} \) |
|------------|--------------|-----------------|-----------------|-----------------|-----------------|-----------------|---------|-----------------|
| ③2        | 6.7          | 29.1            | 100             | 5.5             | 32.0            | 4.8             | 28.8    | 0.72            | 4.0       |
| ⑤2        | 11.7         | 39.4            | 100             | 8.7             | 48.0            | 7.1             | 66.9    | 0.56            | 1.9        |
| ⑧1        | 12.5         | 41.4            | 100             | 14.2            | 88.9            | 12.9            | 68.9    | 0.62            | 2.2        |

4. Calculation result and analysis

This study numerically simulates the tunnel excavation process by defining 25 m completely built tunnels in advance and defines concrete material to simulate the built tunnel segment lining structure. The shield machine head is simulated as a 7.4 m plate element. Linear shrinkage method is used to simulate the over-excavation ahead of the shield machine and the ground loss during excavation. The ground loss rate is set at 0.5%, according to the domestic subway construction experience. The linear incremental surface loads are used to simulate grouting pressure and earth pressure at the shield machine head and tail in different depths. When the shield machine head drives forward, the shield machine tail completes grouting, and then the segment lining is applied, thereby simulating the excavation process. The tunnel is deemed to have passed underneath the pipeline, after the shield
machine tail has passed underneath the pipeline and the segmental lining is applied. At that time, the deformation of the pipeline basically represents that after the tunnel passes through the pipeline.

4.1. Vertical deformation of soil
After the shield tunnel passes underneath the pipeline, the overlying soil on the tunnel will settle owing to the loss of soil during excavation. The deeper the soil is, the greater the vertical deformation of the soil is when it is closer to the tunnel. The monitoring data and calculation data of ground surface settlement vertically above the pipeline are shown in Figure 3. As can be seen from it, the actual monitoring value is in line with the calculated value of the surface settlement, which basically conforms to the theoretical Gaussian settlement curve; the maximum surface settlement value in the calculation data is slightly larger than the monitoring, and the simulated settlement influence range is slightly less than the measured. It can be concluded that the numerical simulation results based on the HSS model can well reflect the actual situation.

![Figure 3. Comparison between the calculation data and the monitoring data of the ground surface deformation.](image)

![Figure 4. Comparison between the calculated value and the measured of the pipeline vertical settlement.](image)

4.2. Vertical deformation of pipelines
The subsequent vertical deformation of the pipeline compared with the monitoring data is shown in Figure 4. As shown in Figure 4, when the tunnel passes through the pipeline perpendicularly, the pipeline appears to be settling. The maximum settlement occurs directly above the tunnel, and the settlement decreases as the distance between the tunnel and the pipeline increases. Comparing the calculation data with the monitoring data, the vertical settlement curve of the pipeline conforms to a consistent law of deformation; the calculated maximum settlement value is slightly larger than the measured, but within one order of magnitude. Therefore, the numerical simulation based on the HSS model can reflect the actual situation of the engineering.
Figure 5. Relationship curves between pipeline maximal settlement and the distance to cutter.

When the shield machine excavates, the deformation of the pipeline has a certain relationship with the horizontal distance between the pipeline and the cutter of shield machine head. Figure 5 shows the relationship between the settlement value at the maximum settlement point of the pipeline and the horizontal distance from the maximum settlement point to the shield head, as well as the comparison between the measured data and the simulated. As seen in Figure 5, the simulated data are consistent with the measured, which reflects the deformation law of pipelines directly above the tunnel during the excavation. When shield machine does not reach the pipeline position, a slight uplift occurs in the pipeline. When the cutter head reaches directly below the pipeline, the settlement speed of the pipeline increases. Finally, when the grouting is completed in the shield machine tail and the lining is completed, the pipeline settlement lessens slightly and then stabilizes.

4.3. Influencing factors analysis

In actual engineering practices, the area influenced by shield tunnel excavation is often encountered different types of pipelines with different buried depths at different working conditions. In this section, FEM simulation analysis is conducted on the effect of tunnel excavation on the deformation of overlying pipelines under different influencing factors, such as the depth of pipelines, the materials of pipelines, and the relative positions of the pipelines and the tunnel.

4.3.1. Influence of pipeline depth on pipeline deformation. Generally speaking, the depth of pipelines buried underground from 1 m to 6 m. To study the effect of pipeline depth on pipeline deformation, we simulate the situations with pipelines at different buried depths from 0.5 m to 6.5 m on a vertical line with intervals of 0.5 m. The variation of pipeline additional internal force with the buried depth of pipeline is shown in Fig. 6. Figure 6 shows that under the influence of shield tunnel construction, the
internal force of the pipeline increases with the increase in pipeline depth. When the buried depth of the pipeline is 0.5 m, the maximum additional axial force, the maximum additional shearing force, and the maximum additional bending moment of the pipeline are 7.55 kN, 0.8 kN, and 0.36 kN·m, respectively. When the buried depth of the pipeline reaches 6.5 m, the maximum additional axial force, the maximum additional shearing force, and the maximum additional bending moment are 164.1 kN, 8.52 kN, and 2.91 kN·m, respectively, and the additional internal force has increased significantly. Among them, the maximum additional axial force increases 21.7 times, the maximum additional shear force increases 10.7 times, and the maximum additional bending moment increases 8.1 times. The reason is that, the deeper the pipeline is buried, the closer to the tunnel, the greater the deformation and the additional internal force are. It indicates that the buried depth of pipeline has an important impact on the pipeline settlement. Therefore, in practice, special attentions should be paid to monitoring and protecting pipelines that are buried deeper and closer to the tunnel.

4.3.2. Influence of different pipeline materials on pipeline deformation. Pipelines consisting of different materials have different deformation stiffnesses. We conduct numerical simulation of several pipeline material types that are more commonly used in actual projects to analyze the effect of tunnel excavation on the deformation of overlying pipelines. The material rigidity of different pipelines is shown in Table 6. With other factors unchanged, numerical simulations are performed on pipelines with different materials at a depth of 2.0 m. The relationships between the additional internal force of the pipeline and the material rigidity of pipeline after the tunnel passes underneath the pipeline are shown in Figure 7. As seen from Figure 7, the additional internal force of the pipeline increases with the material rigidity. During the excavation of shield tunnel, the loss of stratum will affect the pipelines buried in the soil. Under the interaction between the pipe and soil, the pipeline will also have a certain deformation. Because of the difference in material rigidity between the pipelines and soil, the
(a) Relationship between the maximal additional axial force and the rigidity of pipeline.  

(b) Relationship between the maximal additional shearing force and the rigidity of pipeline.  

(c) Relationship between maximal additional bending moment and the rigidity of pipeline.  

Figure 7. Relationships between maximal additional bending moment and the material rigidity of pipeline.

deformation of pipelines and soil cannot be coordinated; thus, additional internal force will be generated inside pipelines. The greater the pipeline rigidity and the higher the rigidity ratio of pipe to soil, the worse the deformation coordination between pipe and soil, and the greater the additional internal force generated in the pipe body.

We replace the pipe rigidity coordinate with the pipe–soil rigidity ratio and obtain the relationship between the maximum settlement of the pipeline and the pipe–soil ratio, as shown in Figure 8. The increase in pipeline rigidity can reduce the maximum settlement value, and the larger the pipe–soil rigidity ratio, the smaller the pipeline settlement. At the same time, for a flexible pipe with less rigidity, the effect of increasing the rigidity for reducing the settlement value is significantly higher than that with large rigidity. According to the numerical simulation data, when the rigidity of a flexible pipeline is increased three times, the settlement value is reduced by 1.86%. When the rigidity of a rigid pipe is increased 5.86 times, the settlement value is only reduced by 0.08%.
Figure 8. Relationship curve between the maximal settlement and the rigidity ratio.

4.3.3. Influence of pipeline–tunnel relative position on pipeline deformation. The deformation of the pipeline during tunnel excavation is simulated when the pipeline and tunnel are parallel. The most unfavorable condition is considered, namely, when the pipeline is located directly above the centerline of the tunnel and is parallel to the tunnel. Comparing the simulation results in this situation with the calculation results when the pipeline and the tunnel are at mutually perpendicular positions, the following are found. (1) The maximum settlements are almost same for the two situations, and they are all near the tunnel centreline; (2) Under the influence of tunnel construction, the additional axial force of the pipeline parallel to the shield tunneling is significantly greater than that of the pipeline perpendicular to the shield tunneling, but the additional shearing force and additional bending moment are both relatively close.

5. Conclusions
Based on the tunneling analysis model abstracted from the shield section of Changzhou North Station-Xinqiao Station, the HSS model is used to analyze the deformation characteristics of overlying underground pipelines under the influence of tunnel excavation, and the following conclusions are drawn.

1) Analysis of the tunnel excavation based on the HSS model considering the small strain characteristics of the soil can obtain more reasonable calculation of the soil deformation, and it can lay a good foundation for the safety evaluation of overlying underground pipelines.

2) The buried depth of pipeline has a more important impact on the settlement and deformation of the pipeline. The closer it is to the tunnel, the more dangerous the pipeline is. Increasing the pipe–soil rigidity ratio is conducive to reducing the deformation of the pipeline, but it could increase the internal force of the pipeline. When the pipe–soil rigidity ratio is greater than 200, the effect of reducing the pipeline settlement is not significant. Under the influence of tunnel construction, the additional axial force generated by the underground pipeline parallel to the shield tunnel is significantly larger than that of the pipeline perpendicular to the tunnel, whilst the additional shearing force and additional bending moment are almost same.

3) After obtaining the basic profile of a subway project, computational simulation and analysis of the influence of tunnel excavation on overlying pipelines can be conducted in advance to study the degree of danger caused by tunnel excavation to overlying underground pipelines in service, so that prevention and protection measures can be planned.

4) The underground pipelines are divided into continuous and sectional pipelines. This study only analyzes the continuous pipelines, and the influence on sectional pipelines needs further evaluation.
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References
[1] Klar A, Vorster T E, Soga K, and Mair R J 2007 Elastoplastic solution for soil-pipe-tunnel interaction J. Geotech. Geoenviron. 133 782-92
[2] Marshall A M, Klar A and Mair R J 2010 Tunneling beneath buried pipes: View of soil strain and its effect on pipeline behavior J. Geotech. Geoenviron. 136 1664-72
[3] Wang Y, Shi J and Ng C W 2011 Numerical modeling of tunneling effect on buried pipelines Can. Geotech. J. 48 1125-37
[4] Shi J, Wang Y and Ng C W 2013 Buried pipeline responses to ground displacements induced by adjacent static pipe bursting Can. Geotech. J. 50 481-92
[5] Wang T, Luo F R, Liu W N, and Li X G 2011 Influence of metro station construction by drift-pile-beam-arch method on soil and rigid-joint pipeline Rock Soil Mech. 8 2533-38
[6] Wei G, Wei X J, Qiu X G, and et al 2009 3D numerical simulation of effect of underground urban street-passage tunnel construction on adjacent pipeline Chin. J. Rock Mech. Eng. pp 2853-59
[7] Li X G, and Wang T 2008 Simple method for evaluating safety of flexible pipeline Rock Soil Mech. 7 1861-64
[8] Jia R H, Yang J S, Ma T, and Liu S Y 2009 Field monitoring and numerical analysis of shield tunneling considering existing tunnel Chin. J. Rock Mech. Eng. 3 425-30.
[9] Gao B L 2014 Influence of bored subway tunnel construction on the deformation of existing pipelines and control techniques Modern Tunnelling Technology 4 96-101.
[10] Zhong X C, Zhu W and Qing J S 2003 The comparison of shield tunnel with segment seams in sequence and in stagger Chin. J. Rock Mech. Eng. 25 109-12
[11] Ye Y D, Zhu H H, and Wang R L 2007 Analysis on the current status of metro operating tunnel dam age in soft ground and its cause Chin. J. Under. Sp. Eng. 1 157-61
[12] Wei G and Xi R Q 2005 Prediction of longitudinal ground deformation due to tunnel construction with shield in soft soil Chin. J. Rock Mech. Eng. 27 1077-81
[13] Burland J B 1989 Small is beautiful-The stiffness of soils at small strains Can. Geotech. J. 26 499-516
[14] Mair R J 1993 Unwin Memorial Lecture 1992: Developments in geotechnical engineering research: application to tunnels and deep excavations Proc. ICE-Civil Engineering 97 27-41
[15] Cao Q, Shi J Y, Chai S X, Wang P and Zhang J X 2009 Non-linear analysis of stiffness of soils under small strain Chin. J. Geotech. Eng. 31 699-703
[16] Benz T, Vermeer P A and Schwab R 2009 A small-strain overlay model Int. J. Numer. Anal. Met. 33 25-44
[17] Benz T 2006 Small-strain stiffness of soils and its numerical consequences (Stuttgart: Stuttgart University)
[18] Brinkgreve R B J and Broere W 2007 Plaxis 2D manual (Delft: Delft University of Technology Press) p 102
[19] Schanz T, Vermeer A and Bonnier P 1999 The hardening soil model: formulation and verification Beyond 2000 Comput. Geotech. (Amsterdam: Balkema) p 281
[20] Mayne P H and Kulhawy F H 1982 K0-OCR relationships in soils J. Geotech. Eng. 108 851-72
[21] Brinkgreve R B J and Broere W 2006 Plaxis material models manual (Delft: Delft University of Technology Press) p 72
[22] Bolton M D 1986 The strength and dilatancy of sands Géotechnique 36 65-78