Measurement of the $B^0$-$\bar{B}^0$ Mixing Parameter $\Delta m_d$ using Semileptonic $B^0$ Decays

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Abstract

We present a measurement of the $B^0$-$\bar{B}^0$ mixing parameter $\Delta m_d$ using neutral $B$ meson pairs in a 29.1 fb$^{-1}$ data sample collected at the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB asymmetric-energy $e^+e^-$ collider. We exclusively reconstruct one neutral $B$ meson in the semileptonic $B^0 \rightarrow D^{*-}\ell^+\nu$ decay mode and identify the flavor of the accompanying $B$ meson from its decay products. From the distribution of the time intervals between the two flavor-tagged $B$ meson decay points, we obtain $\Delta m_d = (0.494 \pm 0.012 \pm 0.015) \text{ ps}^{-1}$, where the first error is statistical and the second error is systematic.

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$B^0$-$\bar{B}^0$ mixing plays a unique role in the determination of basic parameters in the standard model (SM) of elementary particles. It is characterized by the oscillation frequency $\Delta m_d$, which is the difference between the two mass eigenvalues of neutral $B$ meson states. In the SM, the mixing is due to second-order weak interactions known as box diagrams \[1\] whose amplitudes involve $V_{td}$, an element of the quark mixing matrix governing transitions between the top and down quarks \[2\]. The mixing also induces large time-dependent CP violation in neutral $B$ meson decays, which has been observed recently \[3\]. For such CP violation measurements, precise $\Delta m_d$ measurements are important.

In this Letter, we report a $\Delta m_d$ measurement with 31.3 million $B^0$-$\bar{B}^0$ pairs, collected with the Belle detector at the KEKB asymmetric-energy $e^+e^-$ (3.5 on 8 GeV) collider \[4\] operating at the $\Upsilon(4S)$ resonance. The time evolution is described as $e^{-|\Delta t|/\tau_{B^0}/(4\tau_{B^0})}\{1 \pm \cos(\Delta m_d\Delta t)\}$, where the plus (minus) sign is taken when the flavor of one $B$ meson is opposite to (the same as) the other, $\tau_{B^0}$ is the lifetime of the neutral $B$ meson and $\Delta t$ is the proper time difference between the two $B$ meson decays. At KEKB, the $\Upsilon(4S)$ is produced with a Lorentz boost of $\beta\gamma = 0.425$ nearly along the electron beamline ($z$). Since the $B$ mesons are approximately at rest in the $\Upsilon(4S)$ center-of-mass system (c.m.s), $\Delta t$ can be determined from the displacement in $z$ between the two $B$ decay vertices: $\Delta t \approx (z_{rec} - z_{tag})/\beta\gamma c \equiv \Delta z/\beta\gamma c$

The Belle detector \[1\] is a large-solid-angle spectrometer that consists of a silicon vertex detector (SVD), a central drift chamber (CDC), an array of aerogel threshold Čerenkov counters (ACC), time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter comprised of CsI(Tl) crystals (ECL) located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux-return located outside of the coil is instrumented to detect $K_L^0$ mesons and to identify muons (KLM).

We use the decay chain $B^0 \rightarrow D^{*-}\ell^+\nu$, $D^{*-} \rightarrow D^0\pi^-$, and $D^0 \rightarrow K^{+}\pi^-$, $K^{+}\pi^-\pi^0$ or $K^{+}\pi^-\pi^+\pi^- \[7\]$. The large branching fractions and distinctive final states of the semileptonic decay allow for the efficient isolation of a high-purity $B^0$ sample. The event selection criteria are almost the same as those for our previous CP violation measurement \[3\]. Charged particles are selected from tracks with associated SVD hits. Track momenta for $D^0 \rightarrow K^{+}\pi^-\pi^+\pi^-$ decays are required to be larger than 0.2 GeV/c. Candidate $\pi^0 \rightarrow \gamma\gamma$ decays are pairs of photons with energies greater than 0.08 GeV that have an invariant mass within 0.011 GeV/c$^2$ of $m_{\pi^0}$ and a total momentum greater than 0.2 GeV/c. We require the invariant mass within 0.013 GeV/c$^2$ of $m_{D^0}$ for $D^0 \rightarrow K^{+}\pi^-$ or $K^{+}\pi^-\pi^+\pi^-$, and $-0.037 < M_{K^{+}\pi^-\pi^0} - m_{D^0} < +0.023$ GeV/c$^2$ for $D^0 \rightarrow K^{+}\pi^+\pi^0$. The mass difference between the $D^{*-}$ and $D^0$ candidates, $M_{\text{diff}}$, should be within 1 MeV/c$^2$ of the nominal value. The c.m.s angle between the $D^{*-}$ candidate and a lepton (an electron or a muon that has a c.m.s momentum within 1.4 < $p_{\ell}^{\text{c.m.s}}$ < 2.4 GeV/c) is required to be greater than 90 degrees. The energies and momenta of the $B$ meson and the $D^{*}\ell$ system in the c.m.s should satisfy $M_{\nu}^2 = (E_{B}^{\text{c.m.s}} - E_{D^{*}\ell}^{\text{c.m.s}})^2 - |\vec{p}_{B}^{\text{c.m.s}}|^2 - |\vec{p}_{D^{*}\ell}^{\text{c.m.s}}|^2 + 2|\vec{p}_{B}^{\text{c.m.s}}||\vec{p}_{D^{*}\ell}^{\text{c.m.s}}| \cos \theta_{B,D^{*}\ell}$, where $\nu$ is the neutrino mass and $\theta_{B,D^{*}\ell}$ is the angle between $\vec{p}_{B}^{\text{c.m.s}}$ and $\vec{p}_{D^{*}\ell}^{\text{c.m.s}}$. We calculate $\cos \theta_{B,D^{*}\ell}$ setting $M_{\nu} = 0$. Figure \[3\] shows the $\cos \theta_{B,D^{*}\ell}$ distribution. The signal region is defined as $|\cos \theta_{B,D^{*}\ell}| < 1.1$.

We identify the flavor of the accompanying $B$ meson from the properties of the decay products \[3\]. Several categories of well measured tracks that have a charge correlated with the $b$ flavor are selected: high momentum leptons from $b \rightarrow \ell^-\nu$, lower momentum leptons from $c \rightarrow s\ell^+\nu$, charged kaons and $\Lambda$ baryons from $b \rightarrow c \rightarrow s$, high momentum pions originating from decays of the type $B^0 \rightarrow D^{(*)}X$ (where $X = \pi^+, \rho^+, a_1^+, \text{etc}$), and slow pions from $D^{*-} \rightarrow D^0\pi^-$. Information extracted from each track is combined for the $b$-flavor
determination, taking into account correlations in case tracks in more than one category are present. For each flavor decision, we assign a MC-determined flavor-tagging dilution factor $r$, which ranges from $r = 0$ for no flavor discrimination to $r = 1$ for unambiguous flavor assignment. It is used only to sort data into six intervals of $r$, according to estimated flavor purity. More than 99.5% of the events are assigned a non-zero value of $r$.

We reconstruct the $B^0 \to D^*-\ell^+\nu$ decay vertex using the $\bar{D}^0$ trajectory, the lepton track and the interaction-point profile (IP) convolved with the finite $B$ flight length in the plane perpendicular to the $z$ axis (21 $\mu$m). The reduced $\chi^2$ of the vertex is required to be less than 15. The method of reconstructing the tagging side $B$ vertex is described elsewhere [3].

We find 16397 candidates after flavor tagging and vertex reconstruction. The signal fraction is estimated to be 80.4%. The backgrounds consist of fake $D^*$ mesons (7.8%), $B \to D^{**}\ell\nu$ events (7.4%), random combinations of $D^*$ mesons with leptons with no angular correlation (2.6%; called “uncorrelated background”) and continuum events (1.8%). Here $D^{**}$ consists of non-resonant $D^*\pi$ components and charmed mesons heavier than $D^*$. The background due to a combination of a fake lepton and a true $D^*$ from the same $B$ meson is estimated with MC to be negligible. We estimate the fake $D^*$ background fraction from the $\bar{D}^0$ mass sideband events and from fake $D^*$ events reconstructed with wrong-charge slow pions. The uncorrelated background fraction is evaluated by counting candidates where we invert the lepton momentum vector artificially. We estimate the continuum background fraction by scaling the off-resonance data (2.3 fb$^{-1}$) with the integrated luminosity. We fit the $\cos\theta_{B,D^*\ell}$ distribution in a range $-10 < \cos\theta_{B,D^*\ell} < 1.1$ to estimate the $B \to D^{**}\ell\nu$ background fraction; the $\cos\theta_{B,D^*\ell}$ shapes for the signal and $B \to D^{**}\ell\nu$ are modelled using MC and all the other background fractions and distributions are fixed from the aforementioned special background samples.

The $\Delta t$ resolution function for the signal, $R_{\text{sig}}(\Delta t)$, is expressed as

$$R_{\text{sig}}(\Delta t) = g_1 G(\Delta t; \mu_1, \sigma_1) + (1 - g_1) G(\Delta t; \mu_2, \sigma_2),$$

$$\sigma_{1(2)} = S_{1(2)} \sqrt{\sigma_{\text{rec}}^2 + \sigma_{\text{tag}}^2},$$
where $G(x; \mu_{1(2)}, \sigma_{1(2)})$ is the main (tail) Gaussian component with $\mu_{1(2)}$ and $\sigma_{1(2)}$ as the mean and standard deviation, respectively, $g_\ell$ is the fraction of the main component, $S_{1(2)}$ is a scale factor that corrects our imperfect error estimation, and $\sigma_{\text{rec}(tag)}$ is the $\Delta t$ error calculated from the vertex error for the reconstructed (tagged) $B$ meson determined for each event. We extract the above parameters from the $\Delta t$ distribution of the candidate $B^0 \to D^{*+}\ell^+\nu$ events without distinguishing between different flavor assignments. The signal probability density function (PDF) is given by $F_{\text{sig}}(\Delta t) = \int_{-\infty}^{\infty} \Lambda(\Delta t'; \tau_{B^0}) R_{\text{sig}}(\Delta t - \Delta t')d\Delta t'$, where $\Lambda(\Delta t; \tau_{B^0}) = \exp(-|\Delta t|/\tau_{B^0})/(2\tau_{B^0})$. We define the likelihood value for each event as $L_i = (1 - f_{bg}) F_{\text{sig}}(\Delta t_i) + f_{bg} F_{bg}(\Delta t_i)$, where $f_{bg}$ is the overall background fraction of 0.196 and $F_{bg}(\Delta t_i)$ is the background PDF given by $\sum_k f_k F_k(\Delta t_i)$. Here $F_k$ and $f_k$ are the PDF and the fraction, respectively, for each of the four background components. We use the signal PDF for the $B \to D^{**}\ell\nu$ component. For the other background components, we use $F_k(\Delta t) = \int_{-\infty}^{\infty} [(1 - f_{\delta k}) \Lambda(\Delta t'; \tau_k) + f_{\delta k} \delta(\Delta t')] G(\Delta t - \Delta t'; \mu_k, \sigma_k) d\Delta t'$, and $\sigma_k = S_k \sqrt{\sigma_{\text{rec}}^2 + \sigma_{\text{tag}}^2}$. Here $\delta(\Delta t')$ is the Dirac’s delta function that accounts for components with small or zero lifetimes. The parameters in $F_k(\Delta t)$ are obtained from the upper $M_{\text{diff}}$ sideband ($0.155 < M_{\text{diff}} < 0.165$ GeV/$c^2$) for fake $D^*$, the lepton-momentum-inverted events for uncorrelated background, and off-resonance data for continuum background, respectively.

We perform a likelihood fit to determine $R_{\text{sig}}(\Delta t)$ with $\tau_{B^0} = 1.548$ ps \cite{8} and with the background parameters fixed to the obtained values. We find that the fraction of the main component is large ($g_\ell = 0.87^{+0.06}_{-0.09}$) and the $\Delta t$ error estimation is correct ($S_1 = 0.99^{+0.09}_{-0.10}$).

The typical rms resolution on $\Delta t$ is 1.43 ps.

We determine $\Delta m_d$ by an unbinned maximum-likelihood fit to the $\Delta t$ distributions. We define the likelihood value for each event as follows:

$$L_i^{\text{OF(SF)}} = (1 - f_{bg}) \{ (1 - f_{D^{**}\ell\nu}) F_{\text{sig}}^{\text{OF(SF)}}(\Delta t_i) 
+ f_{D^{**}\ell\nu} F_{D^{**}\ell\nu}^{\text{OF(SF)}}(\Delta t_i) \}
+ f_{bg} \sum_k f_k^{\text{OF(SF)}} F_k^{\text{OF(SF)}}(\Delta t_i),$$

where OF (SF) denotes $B^{0}\overline{B}^{0}$ ($B^{0}\overline{B}^{0}$ or $\overline{B}^{0}\overline{B}^{0}$), i.e. a state with the opposite (same) flavor, $F_{\text{sig}}^{\text{OF(SF)}}$, $F_{D^{**}\ell\nu}^{\text{OF(SF)}}$ and $F_k^{\text{OF(SF)}}$ are PDFs for the OF (SF) signal events, $D^{**}\ell\nu$ decays and other backgrounds, respectively, $f_{bg}^{(l = 1, 6)}$ is an overall background fraction excluding $B \to D^{**}\ell\nu$ in each $r$ region and $(1 - f_{bg}^{(l = 1, 6)}) f_{D^{**}\ell\nu}$ corresponds to a fraction of $D^{**}\ell\nu$ decays.

Other background fractions $f_k^{\text{OF(SF)}}$, where the relation $f_k^{\text{OF}} + f_k^{\text{SF}} = 1$ holds, are obtained from the control sample for the uncorrelated and fake $D^*$ backgrounds, and from MC events for the continuum.

The PDFs for the OF and SF signal events are given by

$$F_{\text{sig}}^{\text{OF(SF)}}(\Delta t) = \int_{-\infty}^{\infty} P_{\text{mix}}^{\text{OF(SF)}}(\Delta t') R_{\text{sig}}(\Delta t - \Delta t') d\Delta t',$$

where

$$P_{\text{mix}}^{\text{OF(SF)}}(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} \{ 1 \pm (1 - 2w_1) \cos(\Delta m_d \Delta t) \}.$$
of the vertex resolution, we measure $\Delta m$ variation on the $\Delta m$ unity in the MC (with all others set to zero), and repeat the analysis; we take the largest difference from the main result, which is obtained without the fake $D^*$ background. We take the largest difference from the main result, which is obtained without the fake $D^*$ background. We perform the fit with 10 free parameters (listed in Table I) to the $\Delta t$ distributions of SF and OF events in the signal region and the $B \rightarrow D^{**}\ell\nu$ dominant region defined as $-10 < \cos\theta_{B,D^*\ell} < -1.1$. In this way, the background parameters $f_{B^+}$, $f_\delta$ and $\tau'_{B^+}$ are determined simultaneously. Additional correction factors, $r_\delta$ and $r_{B^+}$, are introduced only in the signal region to account for the difference between the two regions. We use MC to determine $r_\delta = 0.62^{+0.14}_{-0.12}$ and $r_{B^+} = 1.04 \pm 0.02$.

The fit result is summarized in Table I. Figure 3 shows the observed $\Delta t$ distributions for the OF and SF events. Figure 4 shows the corresponding flavor asymmetry, $A(\Delta t) = [N_{OF}(\Delta t) - N_{SF}(\Delta t)]/[N_{OF}(\Delta t) + N_{SF}(\Delta t)]$, where $N_{OF(SF)}$ denotes the number of OF (SF) events.

Table I: Summary of mixing fit. Errors are statistical only. For each wrong tag fraction, the $r$ interval and the number of candidate events are also shown.

| parameter | result |
|-----------|--------|
| $\Delta m_d$ | $0.494 \pm 0.012$ ps$^{-1}$ |
| $w_1$ ($0 < r \leq 0.25$, 6360 events) | $0.467 \pm 0.010$ |
| $w_2$ ($0.25 < r \leq 0.5$, 2364 events) | $0.360 \pm 0.016$ |
| $w_3$ ($0.5 < r \leq 0.625$, 1453 events) | $0.254 \pm 0.020$ |
| $w_4$ ($0.625 < r \leq 0.75$, 1702 events) | $0.182 \pm 0.017$ |
| $w_5$ ($0.75 < r \leq 0.875$, 1958 events) | $0.103 \pm 0.014$ |
| $w_6$ ($0.875 < r \leq 1$, 2560 events) | $0.032 \pm 0.010$ |
| $f_{B^+}$ | $0.70^{+0.12}_{-0.13}$ |
| $f_\delta$ | $0.28^{+0.10}_{-0.09}$ |
| $\tau'_{B^+}$ | $1.87^{+0.24}_{-0.18}$ ps |

The systematic errors are summarized in Table II. $D^{**}$ branching fractions used in this analysis are based on theoretical assumptions [3]. We set each such branching fraction to unity in the MC (with all others set to zero), and repeat the analysis; we take the largest variation on the $\Delta m_d$ result as the systematic error. To account for uncertainties in the tails of the vertex resolution, we measure $\Delta m_d$ by setting the upper limit on $|\Delta t|$ that ranges from 5 to 55 ps. We take the largest difference from the main result, which is obtained without the $|\Delta t|$ upper limit, as a systematic error. Systematic errors from the background PDFs are obtained by varying each shape parameter individually, repeating the fit procedure, and adding each contribution in quadrature. We also perform a MC study where we obtain background PDFs by two methods: one from the background control samples, and the other directly from the signal region. The difference between two $\Delta m_d$ fit results is included in the systematic error. A fit with the $B \rightarrow D^{*}\ell\nu$ and $B \rightarrow D^{**}\ell\nu$ MC events yields the $\Delta m_d$ value that is consistent with the input value within 1.7$\sigma$. We conservatively take the difference,
FIG. 2: The $\Delta t$ distributions for (a) the OF events and (b) the SF events. The solid lines are the result of the unbinned maximum likelihood fit. The dashed lines show the background distribution.

FIG. 3: The observed time-dependent flavor asymmetry. The curve is the result of the unbinned maximum likelihood fit.

which we attribute to MC statistics, as a systematic uncertainty. The systematic error due to the IP constraint is estimated by varying ($\pm 10\mu m$) the smearing used to account for the $B$ flight length. Other sources of systematic errors are obtained by changing each parameter by $1\sigma$, repeating the fit procedure and adding each contribution in quadrature. We also perform a $\Delta m_d$ fit in each $r$ region and find no systematic trend.

In summary, we have measured the $B^0$-$\bar{B}^0$ mixing parameter $\Delta m_d$ using $B^0 \rightarrow D^{*-}\ell^+\nu$ decays in a 29.1 fb$^{-1}$ data sample collected with the Belle detector at the KEKB $e^+e^-$ collider operating at the $\Upsilon(4S)$ resonance. From an unbinned maximum likelihood fit to the
The result is one of the most precise measurements performed so far, and is consistent with the world average value of $\Delta m_d = 0.472 \pm 0.017$ (ps$^{-1}$) \cite{8} as well as other recent measurements \cite{10}.

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\[ \Delta m_d = 0.494 \pm 0.012 \text{(stat)} \pm 0.015 \text{(syst)} \text{ ps}^{-1}. \]

### TABLE II: Summary of the systematic errors (ps$^{-1}$) on the $\Delta m_d$ measurement.

| source                                      | error (ps$^{-1}$) |
|---------------------------------------------|-------------------|
| $D^{**}$ branching fractions                | 0.007             |
| $|\Delta t|$ range                           | 0.007             |
| Background shape                            | 0.006             |
| Resolution function                         | 0.006             |
| $B^0$ lifetime                              | 0.005             |
| Fit bias                                    | 0.004             |
| Background fraction                         | 0.003             |
| $B \rightarrow D^{**} \ell \nu$ fraction   | 0.002             |
| IP constraint                               | 0.002             |
| $B^\pm$ wrong tag fraction                  | < 0.001           |
| $B^\pm$ shape parameter                     | < 0.001           |
| total                                       | 0.015             |
Throughout this Letter, the inclusion of the charge conjugate mode decay is implied.