The stability of planets in the Alpha Centauri system

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ABSTRACT

This paper investigates the long-term orbital stability of small bodies near the central binary of the Alpha Centauri system. Test particles on circular orbits are integrated in the field of this binary for 32 000 binary periods or approximately 2.5 Myr. In the region exterior to the binary, particles with semi-major axes less than roughly three times the binary’s semi-major axis $a_b$ are unstable. Inside the binary, particles are unstable if further than 0.2 $a_b$ from the primary, with stability closer in a strong function of inclination: orbits inclined near 90° are unstable in as close as 0.01 $a_b$ from either star.

1. Introduction

Though the formation of multiple star systems is possibly quite different from that of single stars like our Sun, it is plausible that multiple stars also host planetary systems. If this is the case, then the high frequency of binary and multiple systems implies that such planetary systems have been created in large numbers in our Galaxy. However, the question of whether planets might persist for long periods within such a system remains unanswered. Alpha Centauri, a triple system with two of the stars forming a close binary (semi-major axis 23 AU) and a third orbiting this pair at a much greater distance (12 000 AU), is extraordinary only in its proximity to the Sun (1.3 pc). For this reason, it is a prime place to prospect for planets, and a logical starting point for our theoretical investigations of the stability of planetary orbits in multiple systems.

Stability considerations can constrain the locations where planets are likely to exist. As direct imaging and astrometric techniques are most suited to detecting planets on large orbits, while spectroscopic methods are better at detecting small orbits, an understanding of long-term stability in binary systems can increase the efficiency of searches for extra-solar planets. We seek regions of phase space where test particles (planets) could remain for times on the order of the ages of the stars. More precisely, we will determine those regions in which planets cannot be stable on such time scales. Our integrations follow test particles for only a few million years, and thus cannot assure stability over the $\alpha$ Cen system’s probable 5 billion year age (Noels et al. 1991). However, even such relatively short integrations are sufficient to identify large regions in which single planets are unstable, and thus cannot exist today.
2. Method and Models

We adopt a simple, empirical, observationally motivated criterion for stability. The term “stable” will be applied to test particles whose time-averaged semi-major axis does not vary from its initial value by more than 5% over the whole integration, the remainder being termed “unstable”. Thus our definition of stability excludes planets which remain bound to the binary, but migrate to larger or smaller orbits, encompassing only such planets as remain near their initial orbits. We also compute Lyapunov exponents, which measure the rate of exponential divergence of nearby orbits, and are correlated with stability lifetimes.

2. Method and Models

The numerical integrations in this paper used the symplectic mapping for the N-body problem described by Wisdom and Holman (1991). This technique is typically an order of magnitude faster than conventional integration methods and has the additional advantage of showing no spurious dissipation other than that introduced by roundoff error. Lyapunov times were computed by evolving a tangent vector associated with each test particle during the orbit calculations (Mikkola and Innanen 1994). This procedure has two advantages over the common approach of measuring the Lyapunov time by evolving two nearby trajectories. First, using the tangent vector avoids the saturation and renormalisation problems that accompany the two-trajectory technique. Second, the variational method is faster because the most expensive calculations required (the distances between the test particle and planets) do not need to be computed twice.

We approach the problem with a simple model which captures the overall dynamics. We ignore the distant third star, α Cen C (Proxima), as it appears likely that it is not bound to the central binary (Anosova et al. 1994), and because the perturbations it could inflict were it bound are extremely small. The orbit of the central pair is thus taken to be a fixed Kepler ellipse. The semimajor axis of the central binary $a_b$ is 23.4 AU, and its eccentricity is 0.52 and the inclination of its orbit to the plane of the sky is 79° (Heintz 1978). The primary, α Cen A, has a mass of 1.1 M$_\odot$; the secondary, α Cen B, has a mass of 0.91 M$_\odot$ (Kamper and Wesselink 1978). Their physical properties are outlined in Table 1. In the field of this binary we integrate a battery of massless test particles representing low-mass planets. As these particles do not interact with one another, this paper does not address the stability of multiple planet systems.

| Star     | Mass (M$_\odot$) | MK class | $V$ | $M_V$ | $L$ (L$_\odot$) |
|----------|------------------|----------|-----|-------|-----------------|
| α Cen A  | 1.1              | G2V      | −0.01 | 4.37  | 1.6             |
| α Cen B  | 0.91             | K1V      | 1.33 | 5.71  | 0.45            |

Table 1: Physical characteristics of α Cen A and B, including their mass, spectral type, apparent and absolute visual magnitude and luminosity (Kamper and Wesselink 1978; Hoffleit and Jaschek 1982; Lang 1992).
3. Initial Conditions

Test particles are initially placed in circular orbits in two separate regions. The interior region is centred on the primary, and extends from 0.01 to 0.5 times the binary semi-major axis $a_b$ (0.23 to 11.7 AU). The exterior region is centred on the barycentre, and spans $1.5a_b$ to $5a_b$ (35 to 117 AU). Note that the mass fraction in the secondary (0.45) exceeds the maximum value ($\sim 0.005$ for a binary eccentricity of 0.52, Danby 1964) at which the $L_4$ and $L_5$ Lagrange points are stable, so no particles are expected to survive there.

No separate study of the dynamics of orbits centred on $\alpha$ Cen B was performed due to the similarity of the masses of the primary and secondary. Such a study is expected to produce results qualitatively very similar to those obtained for orbits centred on $\alpha$ Cen A.

As the central binary has minimum and maximum separations of $0.48a_b$ and $1.52a_b$, particles on circular orbits with semi-major axes in this range suffer close encounters with the secondary, and are unlikely to be stable. Thus, no test particles are started in the semi-major axis range $0.5a_b$ to $1.5a_b$.

The integration is started with the perturber at apastron, and on the opposite side of the primary from the particles. The particles are initially in the plane of the binary, but have a range of inclinations.

Thirteen different inclination values were examined, ranging from $0^\circ$ to $180^\circ$ in $15^\circ$ increments. All particles were started on circular orbits, relative to the primary in the inner shell, and relative to the barycentre in the outer one. Particles were distributed evenly in initial semi-major axis $a$, 50 particles for each value of the inclination in the inner region ($0.01a_b$ particle separation), and 36 particles for each value of the inclination ($0.1a_b$ particle separation) in the outer one, for a total of 1118 particles in both regions. The integration proceeded for 32 000 binary periods, approximately 2.5 Myr of simulated time. The time step used was $3 \times 10^{-3}$ of the binary period in the outer region; in the inner region, the step size was $10^{-4}$ from 0.11 to $0.5a_b$, and $3 \times 10^{-5}$ for the particles with semi-major axes less than and including $0.1a_b$. These step sizes translate into 33 steps per particle orbit at $0.01a_b$, 360 per orbit at $0.11a_b$ and 610 per orbit at $1.5a_b$.

When integrating particles in the inner region, the mass of the secondary was grown adiabatically over 500 binary periods in order to eliminate transients in the particles’ motions which would not be present in a mature planetary system. The time span of 500 binary periods (roughly 40 000 years) is comparable to the precession period of test particle orbits with semi-major axes as small as $0.05a_b$. The adiabatic growth procedure was found to have little effect on the final results, and was omitted in the calculations of the exterior region. In the inner region, 498 of the 650 particles became unstable during the adiabatic growth phase, with those with the largest orbits typically being lost first.
4. Simulations

The inner region proves to be largely unstable over the integration time scale (Figure 1). Each cell of Figure 1 represents one of the test particles’ initial conditions. A white cell indicates a particle that was ejected or had a close encounter (defined to be a passage within 0.25 \( a_b \)) with the secondary. Two other colours indicate those particles that survived for the entire simulation: those whose time-averaged semi-major axis deviated from its initial value by less than 5% are indicated in black, those which deviated by more than 5% are shown in grey.

The region where particles are longest-lived is close to the primary as might be expected, but with a wide gap at inclinations between 60° and 120°. This gap presumably closes at smaller distances from the central star. Orbits in the plane of the binary are stable out to larger distances than those with significant inclinations. Retrograde orbits survive out to larger radii than prograde ones, as might be expected from the shorter encounter times suffered by retrograde orbits and from studies of distant outer planet satellites (Hénon 1970).

There are only three grey particles in Figure 1. The two grey ones at \( a = 0.23 a_b \) survive but migrate out onto larger orbits outside the binary (\( a \sim 5 a_b \)). The grey one at \( i = 0°, a = 0.13 a_b \) moves inward slightly to roughly 0.1\( a_b \). Overall, the \( a-i \) plane is divided fairly cleanly into two parts, one stable and one unstable on million year time scales.

A plot showing Lyapunov times for the inner region appears in Figure 2. The plot shows the same general stable/unstable division as Figure 1. There are, however, regions in Figure 2 that show Lyapunov times below the maximum detectable level (which is about 1200 binary orbital periods, or 0.1 Myr in the inner region), but which have not moved significantly from their initial positions. As the time scale for large qualitative changes in test orbits has been observed to be much longer than the Lyapunov times in some cases (Lecar et al. 1992), such particles may move away from their initial positions over longer time scales.

Plots analogous to those in Figures 1 and 2 but for the outer region are shown in Figures 3 and 4. In Figure 3, we again see a division of the region into stable and unstable regions. Particles within roughly 3\( a_b \) are for the most part unstable, with the stable region reaching further inwards for retrograde orbits, while orbits outside 4\( a_b \) survive for the length of the integration.

There are many grey cells in Figure 3, indicating particles which have survived the integration but whose semi-major axes wander significantly from their initial values. Almost all of these particles move to larger orbits well outside the binary. Only one moves to a more tightly bound orbit, and a few along the stability edge remain within 50% of their initial semi-major axis, but will presumably move away on longer time scales.

The behaviour of the Lyapunov times in the outer region, displayed in Figure 4, is generally consistent with Figure 2. Particles which show only small changes in semi-major axis generally have longer Lyapunov times, except for a few isolated particles. It is unclear if these exceptions
Fig. 1.— The change in semi-major axis of test particles in the inner region of the α Cen binary, on a grid of inclination $i$ and semi-major axis $a$. A white cell indicates a particle that was ejected or had a close encounter with the secondary. Particles which survived the whole integration time, but whose average semi-major axis differs from its initial value by more than 5% are indicated by a grey cell, while a deviation of less than 5% is indicated by a black cell.
Fig. 2.— The Lyapunov time of test particles in the inner region of the $\alpha$ Cen binary, on a grid of inclination $i$ and semi-major axis $a$. Black indicates the longest detectable Lyapunov times (approximately 1200 binary periods), shading to white, the lowest, at or near zero. Particles which were ejected or which moved by more than 50% of their initial semi-major axis are grouped with the lowest Lyapunov times (white).
4. SIMULATIONS

Fig. 3.— The change in semi-major axis of test particles in the outer region of the $\alpha$ Cen binary, on a grid of inclination $i$ and semi-major axis $a$. The shadings are the same as in Figure [1].
Fig. 4.— The Lyapunov time of test particles in the outer region of the $\alpha$ Cen binary. The shadings are the same as in Figure 2, except that the maximum Lyapunov time is now roughly 2800 binary periods.
5. **QUESTIONS AND COMMENTS**

Indicate isolated pockets of chaotic behaviour associated with narrow resonances, or possibly chaos on time scales of order or slightly larger than the maximum detectable Lyapunov time, and which are just at the edge of detectability in these simulations. The maximum detectable Lyapunov time is slightly longer outside than inside, at roughly 2800 orbits or 0.2 Myr.

In order to further investigate the stability of planets over even longer time scales, all particles between 0.05 \( a_b \) and 0.15 \( a_b \) and at zero inclination were run 50 times longer (1.6 million binary periods or 130 million years). Particles inside and including that at 0.1 \( a_b \) were stable, with no signs of chaos on time scales less than the maximum detectable Lyapunov time (roughly 50 000 binary periods or 4 million years) while the particles outside 0.1 \( a_b \) were all ejected or suffered close encounters. Comparison of this result with those in Figures 1 and 2 indicates that the inner edge of what the “stable” region may be eroded somewhat as integration times are extended, though the time scale for this effect and whether or not it will reach arbitrarily far inwards is unclear.

5. **Questions and Comments**

Although these integrations cannot assure the stability of planets on time scales greater than about a million years, they do identify important unstable regions. The zone in which planets cannot have survived since the formation of the \( \alpha \) Cen system extends from at least 0.15\( a_b \) (3.5 AU) to 3\( a_b \) (70 AU) for orbits which lie in the binary’s orbital plane. Retrograde orbits may be stable as far as 0.2\( a_b \) (4.7 AU) from the primary, and in as far as 2.8\( a_b \) (66 AU). Orbits lying perpendicular to the plane are unstable in as close as 0.01\( a_b \) (0.23 AU) from the primary, though smaller stable orbits are not excluded by our studies.

The stable regions, as seen when projected onto the plane of the sky, are presented in Figures 3 and 4. The density of plotted points is proportional to the projected density of planets in the \( \alpha \) Cen system if the phase space corresponding to the black cells in Figures 1 and 3 is uniformly populated with circular orbits.

The habitable zone for planets, as defined by Hart (1979), lies about 1.2–1.3 AU (1") from \( \alpha \) Cen A. A similar zone may exist 0.73–0.74 AU (0.6") from \( \alpha \) Cen B. From our investigations, it appears that planets in this habitable zone would be stable in the sense used here, at least for certain inclinations.

Our results are in qualitative agreement with Harrington’s (1968; 1972) studies of hierarchical triple star systems. He found that the inner binary tended to be unstable when its orbital plane was perpendicular to the orbital plane of the most distant member.

Benest (1988) also investigated the stability of planets in the \( \alpha \) Cen system. He only explored the case of zero planetary inclination but did allow for non-zero planetary eccentricities. Benest found that eccentric retrograde orbits were stable over a greater range of initial distances from the
Fig. 5.— The projected density of stable interior planets around the $\alpha$ Cen binary. The orbit of the secondary is based on Worley and Heintz (1983), and the secondary’s position is indicated for the epoch 2000.0. The axes are in arcseconds.
Fig. 6.— The projected density of stable exterior planets around α Cen with semi-major axes less than 5 $a_b$. The axes are in arcseconds.
primary than prograde ones, but found the opposite to be the case for circular orbits. This result is different from what we observe here, possibly due to the short duration of his simulations, which lasted only 100 binary periods, while ours run over three hundred times longer.

The fact that planets would seem to be more stable when in the plane of the binary’s orbit may increase the likelihood of planets existing in the α Cen system. If one assumes that the planets form and then remain roughly in the primary’s equatorial plane, as they have in our Solar System, the coincidence of α Cen’s equatorial and orbital planes (Doyle and Lorre 1984; Hale 1994) indicates that, should planetary formation have proceeded in a manner similar to that in which it did here, it is plausible that planets might remain in the system to this day.

6. Conclusions

Our studies reveal that much of the region around the central α Cen binary is unstable. However, there are zones in which planets on circular orbits could be stable in the α Cen system on million year time scales. These zones are located both far from \((a \gtrsim 70 \text{ AU})\) and near to \((a \lesssim 3 \text{ AU})\) the primary. Stability is a strong function of the inclination for interior orbits, less so exterior orbits. The inner stable region encompasses Hart’s (1979) habitable zone, however a planet orbiting in the more distant stable region would be inhospitable to life.

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