Realistic Hadronic Matrix Element Approach to Color Transparency

B.K. Jennings
TRIUMF
Vancouver, B.C. V6T 2A3

G.A. Miller
Physics Department, FM-15
University of Washington, Seattle, Washington 98195

Abstract

Color transparency occurs if a small-sized wave packet, formed in a high momentum transfer process, escapes the nucleus before expanding. The time required for the expansion depends on the masses of the baryonic components of the wave packet. Measured proton diffractive dissociation and electron deep inelastic scattering cross sections are used to examine and severely constrain the relevant masses. These constraints allow significant color transparency effects to occur at experimentally accessible momentum transfers.

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Color Transparency (CT) is the postulated [1,2] absence of final (or initial) state interactions caused by the cancellation of color fields of a system of quarks and gluons with small spatial separation. For example, suppose an electron impinges on a nucleus knocking out a proton at high momentum transfer. The consequence of color transparency is that there is no exponential loss of flux as the ejected particle propagates through the nucleus. Thus, the usually “black” nucleus becomes transparent. We restrict our attention to processes for which the fundamental reaction is elastic, or at least a two-body reaction. This requires that the nuclear excitation energy be known well enough to ensure that no extra pions are created.

The existence of color transparency depends on: (1) forming a small-sized wave packet in a high momentum transfer reaction. (2) the interaction between such a small object and nucleons being suppressed (color neutrality or screening) and (3) the wave packet escaping the nucleus while still small. That color neutrality (screening) causes the cross section of small-sized color singlet configurations with hadrons to be small was found in Ref. 3, and is well-reviewed in Refs. 4,5 and 6. So we take item (2) as given. The others require more discussion.

The formation of a small-sized wave packet (1) at feasible energies is an open question even though asymptotic perturbative QCD predicts that the size of the ejected wave packet is of order of the inverse of the momentum transfer Q. Including the effects of gluon radiation (Sudakov suppression) further increases the importance of small separations between quarks [7] and, as a consequence leads to a faster decrease with Q [8]. But, the minimum value of Q required for the wave packet to be small is not known.

It is also true that at experimentally available energies, the small object does expand as it moves through the nucleus. Thus the final state interactions are suppressed but not zero. The importance of this expansion was found by Farrar et al. [9], and by Jennings and Miller [10]. See also Ref. [11].
Tantalizing but non-definitive evidence has been obtained in a pioneering \((p,pp)\) experiment at Brookhaven National Laboratory (BNL) \[12\]. Color transparency is the object of current searches using electron and proton beams \[13,14\]. The existence of color transparency has not yet been demonstrated, and it would be useful to improve the reliability of CT predictions. Here we use apparently unrelated diffractive dissociation (DD) and deep inelastic scattering (DIS) data to probe the existence of the small-sized wave packet and to constrain the expansion process.

To be specific, consider the high \(Q^2\) quasielastic \((e,e'p)\) reaction. A wave packet is formed when a bound proton absorbs the virtual photon. This wave packet is dubbed \[4\] a point like configuration \((PLC)\), in an optimistic notation. Thus

\[
|PLC\rangle = T_H(Q^2)|N\rangle,
\]

where the hard photon absorption operator is denoted as \(T_H(Q^2)\). The \(|N\rangle\) represents a nucleon at rest, and \(|N(q)\rangle\) represents one of momentum \(q\). The form factor is \(F(Q^2) = \langle N(q)|T_H(Q^2)|N\rangle\).

We assume that the PLC has no soft interaction \(U\) with surrounding nucleons. Then \[6\]

\[
0 = U T_H(Q^2)|N\rangle.
\]

In the optical approximation \(U = -4\pi i Im\hat{f}\rho\), in which \(\hat{f}\) represents the PLC-nucleon interaction as a sum of quark-nucleon scattering operators and \(\rho\) is the density of target nucleons. Only the dominant imaginary part of \(\hat{f}\) is kept, and the nucleonic matrix element \(\langle N|4\pi Im\hat{f}|N\rangle = \sigma_p\), the proton-nucleon total cross section. Thus we may abbreviate: \(4\pi Im\hat{f} \equiv \hat{\sigma}\). Taking the nucleon matrix element of Eq. (2) and using completeness yields

\[
0 = \sigma_p + \sum_{\alpha} \int \frac{dM_X^2}{(M+m_p)^2} \langle N(q)|\hat{\sigma}_p|\alpha, M_X^2\rangle \frac{\langle \alpha, M_X^2|T_H(Q^2)|N\rangle}{F(Q^2)}.
\]
in which an intermediate state of mass $M_X^2$ has a set of quantum numbers (including multiplicity) $\alpha$. It is useful to define the integral term of Eq. (3a) as $I(Q^2)$. Then

$$\sigma_p = - I(Q^2).$$

(3b)

As the PLC propagates through a length $\ell$, each component acquires a phase factor $e^{ipX\ell}$ with $p_X^2 = p^2 + M_N^2 - M_X^2$. Then one may define [15] an effective PLC-nucleon cross section, $\sigma_{eI}(\ell)$:

$$\sigma_{eI}(\ell) \equiv \sigma_p + \int_\alpha \int dM_X^2 \langle N(\bar{q})|\hat{\sigma}|\alpha, M_X^2 \rangle e^{i(pX-p)\ell} \frac{\langle \alpha M_X^2 | T_H(Q^2) | N \rangle}{F(Q^2)}$$

(4)

The reader may wonder how Eq. (3a) can ever be valid. This occurs in a model obtained by Jennings and Miller (JM)[10]. They represent the states $(\alpha M_X^2)$ by two-body harmonic oscillator eigenfunctions $|N_m\rangle$ in two spatial dimensions ($|N_0\rangle \equiv |N\rangle$). Then $T_H(Q^2)|N\rangle \propto |\bar{b} = 0\rangle$ and $\langle N_m|T_H(Q^2)|N\rangle = \langle N|T_H(Q^2)|N\rangle=F(Q^2)$. Further JM take $\hat{\sigma} = \sigma_p b^2/\langle N|b^2|N\rangle$. Then $b^2|N\rangle = [||N\rangle - |N_1\rangle]/\langle N|b^2|N\rangle$ where $|N_1\rangle$ is the symmetric $2h\omega$ state. This means that $M_X^2 = M_1^2$. Using these relations in Eq. (4) gives $\sigma_{eI}^{JM}$

$$\sigma_{eI}^{JM}(\ell) = \sigma_p \left(1 - e^{i(p_1-p)\ell} \right).$$

(5)

We examine Eq. (5) to understand the results to be presented. The quantity $(p-p_1) \approx (M_1^2 - M_N^2)/2p$, which means that $\frac{1}{(M_1-M_N)} \equiv \tau_0$ plays the role of a time scale for the expansion of the PLC. If $\tau_0 << \ell$, the two terms of eq(5) cancel and transparency occurs; otherwise, final state interactions do occur. (The relevant value of $\ell$ is about a nuclear radius.)

The previous two-state model has some desirable features, but it is not realistic because a continuum of proton states is excited in $pp \rightarrow pX$ reactions. We therefore use experimental observations of the matrix elements appearing in Eqs. (3) and (4). First we notice an apparent difficulty. Those matrix elements are off energy-shell extensions
of scattering amplitudes. The violation of conservation of energy is approximately 
\[(M_X^2 - M^2)/2\rho,\] so that if the integrals converge for low values of \(M_X^2\) (the virtuality 
is small) and off shell effects can be neglected. Then

\[
\left| \langle N(q) | \hat{\sigma} | \alpha, M_X^2 \rangle \right| = \left[ \frac{d^2\sigma^{DD}(\alpha)}{dt dM_X^2} \right]^{1/2}
\]

\[
\left| \langle \alpha, M_X^2 | T_H(Q^2) | N \rangle \right| = \left[ \frac{1}{\sigma_M} \frac{d^2\sigma^{DIS}(\alpha)}{d\Omega dE} \right]^{1/2}
\]

where \(DD\) and \(DIS\) stand for diffractive dissociation and deep inelastic scattering. In \(DD\) a fast proton breaks into the state \(\alpha, M_X^2\) without exciting the bound target nucleon. These are cross sections in which the final state is denoted by the quantum numbers \(\alpha\). Define probabilities \(P^{DD,DIS}(\alpha)\) so that

\[
d\sigma^{DD,DIS}(\alpha) = P^{DD,DIS}(\alpha, M_X^2) d\sigma^{DD,DIS},
\]

where \(\sum_\alpha P^{DD,DIS}(\alpha, M_X^2) = 1\). This is an often used reasonable approximation. Measurements of multiplicities [16,17] show that \(P^{DD,DIS}(\alpha)\) is a peaked but broad function of multiplicities.

One can see if existing data rule out Eq. (3) by noting that the integral term has a lower (negative) limit. This can be obtained by taking each product of matrix elements to be negative. Then the quantity \(-I(Q^2)\) of Eq. (3b) can be written as

\[
-I(Q^2) \leq \int_{(M+M_n)^2}^{\infty} dM_X^2 \left[ \frac{d^2\sigma^{DD}}{dt dM_X^2} W_2(x, Q^2) \right]^{1/2} \sum_\alpha \frac{\left( P^{DD}(\alpha, M_X^2) P^{DIS}(\alpha, M_X^2) \right)^{1/2}}{F(Q^2)}
\]

\[\equiv I_{\text{max}}.\]

If \(I_{\text{max}} < \sigma_p\), the data would rule out Eq. (3).

We next evaluate \(I_{\text{max}}\) to see if a \(PLC\) can be formed. We use Atwood’s [18] parameterization of \(W_2(x, Q^2)\) and Goulianos’s [19] tabulation of the \(s\) dependence of
\[ \frac{d^2 \sigma_{DD}}{ddM_X^2} \text{ at } t = -0.042 \text{ GeV}^2 \text{ since much data are taken at that low value.} \]

The interpretation of the \( pp \) data is somewhat problematical, since the measurements represent the diffractive dissociation process only if \( \frac{M_X^2 - M^2}{s} \lesssim \frac{m_\pi}{M} \)[19], so that a maximum value of \( M_X^2 \) is given by \( M_X^2(\text{max}) \approx \frac{m_\pi s}{M} + M^2 \). The probability functions \( P^{DD}, P^{DIS} \) are taken from Ref. [16] for DIS and Ref. [17] for diffractive dissociation. The sum over \( \alpha \) is then approximately 0.6, approximately independent of \( M_X^2 \). \( I_{max} \) is evaluated by performing the integral over \( M_X^2 \) up to a maximum value \( M_c^2 \). If \( M_c^2 \) exceeds \( M_X^2(\text{max}) \) by a large amount, we would say \( I_{max} < \sigma_p \) and color transparency would be ruled out.

The quantity \( \sigma_p \) is as tabulated in Ref. [20].

The use of the stated inputs shows that \( I_{max} \) is greater than or equal to \( \sigma_p \) for values of \( M_c^2 \) between 2.4 and 2.6 GeV\(^2 \), depending slightly on \( s \). These values of \( M_c^2 \) have small virtuality and do not exceed the bound required for diffractive dissociation to occur. Thus existing \( DD \) and \( DIS \) data allow the existence of color transparency. This is our strongest conclusion.

A further step is to use the above treatment of the integrand to evaluate \( \sigma_{eff} \) of Eq. (4). But this could be unrealistic: not all of the products of matrix elements are negative and a sharp cutoff of the \( DD \) cross section is not expected. In general, we should replace the factor \( \sum_{\alpha} [P^{DD}(\alpha, M_X^2) P^{DIS}(\alpha, M_X^2)]^{1/2} \) by the function \( g(M_X^2) \):

\[ g(M_X^2) = \sum_{\alpha} [P^{DD}(\alpha, M_X^2) P^{DIS}(\alpha, M_X^2)]^{1/2} \text{ Sign}(\alpha) \quad (7) \]

where \( \text{sign}(\alpha) \) is \( \pm 1 \) depending on the phases. Measuring the relative phases of \( DD \) and \( DIS \) amplitudes is difficult. Thus although \( g(M_X^2) \) is a measurable function, it is not known.

It is reasonable to try a form \( g(M_X^2) = \left( \frac{M}{M_X} \right)^\beta \) (power-law) instead of the previously used \( g(M_X^2) = \theta(M_c^2 - M_X^2)0.6 \) (sharp cutoff). Values of \( \beta \) ranging from 2.4 to 4.0 allow the sum-rule relation (3) to be satisfied at each value of \( Q^2 \). The use of the
power-law fall-off allows high mass \( M_X^2 \) states \( (M_X^2 \approx Q^2) \) to participate in the integral without emphasizing the importance of states of large virtuality.

We now turn to predicting nuclear color transparency. The function \( \sigma_{\text{eff}} \) is obtained by using the stated inputs products of matrix elements. The results are shown in fig 1, for \( s = 13GeV^2 \). (For electron scattering \( s = Q^2 + 4M^2 \).) If \( g(M_X^2) \) is given by the power fall-off, \( \sigma_{\text{eff}}(\ell) \sim \ell \) for small values of \( \ell \). This similar to the model of Ref [9]. If the sharp cut-off is used, one obtains \( \sigma_{\text{eff}}(\ell) \sim \ell^2 \) for small values of \( \ell \). \( \sigma_{\text{eff}} \) is generally smaller with the sharp cut-off because with \( M_c^2 \approx 2.2GeV^2 \) large values of \( M_X \) do not appear, \( p_X - p \) is prevented from becoming large, and the cancellation between the two terms of Eq.(4) is not disturbed much by the phase factor \( (p_X - p)\ell \).

Next we present predictions for the quasieelastic \((e,e',p)\) measurements being carried out at SLAC [13]. The ratios of cross sections \( \sigma/\sigma^{\text{BORN}} \) are shown in Fig. 2. The quantities \( \sigma \) are \((e,e'p)\) differential cross sections integrated over the scattering angles of the outgoing proton. (See Ref. 10 for details.) Full color transparency corresponds to a ratio of unity. We are concerned with the energies for which \( \sigma/\sigma^{\text{BORN}} \) approaches unity and for which it is substantially greater than that obtained with the standard Glauber treatment. Both choices of \( g(M_X^2) \) show that observable increases are obtained for values of \( \vec{q} \) as low as 5 GeV/c, or \( Q^2 = 9 \text{ GeV/c}^2 \). The results of using the sharp cutoff are very similar to those of using the model of Ref. 10, with \( M_1 = 1.44\text{GeV} \). This follows from the small value of \( M_c \) and is also a consequence of the results shown in Fig. 1.

The single published experiment aimed at observing the effects of color transparency is the BNL \((p,pp)\) work [12] at beam momenta \( p_L \) ranging from 6 to 12 GeV/c. The kinematics of the BNL experiment are such that the basic \( pp \) elastic scattering occurs at a center of mass angle of 90° if the target proton is at rest. Fig. 3 shows that the experimentally determined transmission \( \sigma/\sigma^{\text{BORN}} \) (ratio of nuclear to
hydrogen cross section per nucleon after removing the effects of nucleon motion) has unexpected oscillations with energy. Also shown is the expectation based on standard Glauber theory. This standard survived a rigorous examination in Ref. [21] and the independence on energy was confirmed in a detailed calculation that simulated the experimental conditions [22].

One possibility, suggested by Ralston and Pire [23], is that the energy dependence is caused by an interference between a hard amplitude which produces a small object, and a soft one (the Landshoff process) which does not. Zakharov and Kopeliovich [11] and Jennings and Miller [24] have pursued this by including the effects of the expansion of the PLC. The technique is to use Eq. (4) for the initial or final state nuclear interactions of the small object and to use the ordinary cross section \( \sigma_p \) for those of the large object. Another well-motivated mechanism is that of Brodsky and de Teramond [25] in which the two-baryon system couples to charmed quarks (there is a small (6q) and a large (6q, c\( \bar{c} \)) object) is also examined in Ref. [24]. None of these treatments reproduce the data.

Here we employ \( \sigma_{eff} \) of Eq. (4) as evaluated in Fig. 1. To approximate \( T_H(Q^2) \) by \( W_2^{1/2} \) is to assume that the proton-proton high \( Q^2 \) data vary in a manner similar to \( W_2 \). This is reasonable, because in each case the reaction starts with a quark absorbing high momentum. The Ralston-Pire mechanism is evaluated using a recent more accurate fit of the hard pp scattering data by Carlson et al [26]. Both the usual quark-counting and Landshoff amplitudes are included in their description of \( A_{nn} \) and the differential cross-sections. The results for the mechanisms of Refs. [23] and [25] are shown in Fig. 3. Both the power-law and sharp cutoff versions of \( g(M_X^2) \) are used. We take these as representing lower and upper limits to the predictions, and obtain a range of variation by shading the area between these curves. The enhancement at about 4 GeV is a new consequence of the amplitude of Ref. [26]. The Brodsky-deTeramond model along with
the sharp cutoff $g(M_X^2)$ seems closest to the data. But no calculation achieves good agreement with the data. One can say that the general trend is reproduced. The strong dependence on $g(M_X^2)$ shows that at least one measurement of color transparency is needed to determine this function. The new experiment [14] designed for higher energies and greater accuracy will certainly help.

Our results are that measured diffractive dissociation and deep inelastic scattering data lend support to the idea that color transparency occurs. The formation of a PLC is allowed, and its expansion is not too rapid. We eagerly await the new experimental results [13,14].

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Figure Captions

1. The real part of $\sigma_{eff}(\ell)/\sigma$. Dashed: sharp cut off $g(M_X^2)$, dotted: eq. (5) with $M_1 = 1.44 GeV$, dash-dot: power law $g(M_X^2)$.

2. Ratios of cross sections for the $(e, e'p)$ reaction. The solid line represents the standard Glauber calculation ($\sigma_{eff} = \sigma_p$). The other curves are defined in Fig. 1.

3 Energy dependence of $\sigma/\sigma^{BORN}$. Data points-Carroll et al.[12]. The area shaded vertically is obtained from the mechanism of Ref. [23] and amplitude of Ref. [26]. The area shaded horizontally is obtained from the mechanism of Ref. [25]. Upper bound: sharp cut off $g(M_X^2)$ Lower bound: power law $g(M_X^2)$. The solid curve assumes no color transparency.