Nucleosynthesis Bounds in Gauge-Mediated Supersymmetry Breaking Theories

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Abstract

In gauge-mediated supersymmetry breaking theories the next-to-lightest supersymmetric particle can decay during or after the nucleosynthesis epoch. The decay products such as photons and hadrons can destroy the light element abundances. Restricting the damage that these decays can do leads to constraints on the abundance and lifetime of the NLSP. We compute the freezeout abundance of the NLSP by including all coannihilation thresholds which are particularly important in the case in which the NLSP is the lightest stau. We find that the upper bound on the messenger scale can be as stringent as $10^{12}$ GeV when the NLSP is the lightest neutralino and $10^{13}$ GeV when the NLSP is the lightest stau. Our findings disfavour models of gauge mediation where the messenger scale is close to the GUT scale or results from balancing renormalisable interactions with non-renormalisable operators at the Planck scale. When combined with the requirement of no gravitino overabundance, our bound implies that the reheating temperature after inflation must be less than $10^7$ GeV.

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1. Gauge-mediated supersymmetry breaking theories provide an interesting alternative to the usual gravity-mediated scenarios in transmitting supersymmetry breaking effects to the low energy world \[1\] (for a review, see Ref. \[2\]). Among the most attractive features of these theories is the natural suppression of flavour-violating interactions which is guaranteed by the gauge symmetry. In addition low-energy phenomenology is now governed by the fact that the gravitino is the lightest supersymmetric particle and this may lead to interesting collider signals.

The next-to-lightest supersymmetric particle (NLSP) also plays an important role in low-energy phenomenology. Since the soft mass of a supersymmetric particle is determined by its gauge quantum numbers, the NLSP will be either a neutralino \(\tilde{N}_1\) or the lightest, mainly right-handed stau \(\tilde{\tau}_1\), depending on the choice of parameters (the possibility of a sneutrino NLSP is now marginal). The relevant parameters which define the gauge-mediated model are the messenger index \(N\) (twice the sum of the Dynkin indices of the messenger gauge representations), and the supersymmetric and supersymmetry-breaking messenger mass parameters \(M\) and \(F\). We also distinguish the supersymmetry breaking scale \(F\) felt by the messenger fields and the fundamental scale of supersymmetry breaking \(F_0\) which determines the gravitino mass and couplings by defining \(k \equiv F/F_0 \leq 1\). In our analysis, we will trade \(F\) for the NLSP mass which is a parameter of more direct physical meaning. The Higgs mass parameters \(\mu\) and \(B_\mu\) are new independent inputs, which can be determined by imposing electroweak symmetry breaking, and therefore be fixed in terms of \(\tan \beta\) and the algebraic sign of \(\mu\). As a result, the lightest neutralino turns out to be mainly \(B\)-ino. More details on the definition of the parameters can be found in Ref. \[2\].

The NLSP is not stable and eventually will decay into the gravitino. If these decays occur during the nucleosynthesis epoch the light element abundances can be drastically altered \[3\]–\[9\]. For example, if the NLSP is a neutralino the dominant decay mode produces a photon which affects the primordial \(^4\text{He}\) abundance. On the other hand hadronic decays can also prove dangerous for the nucleosynthesis products. In the case of the neutralino NLSP, the photon can hadronise or, in the case of a stau NLSP, the semileptonic decay of a tau lepton can produce hadronic showers which leads to energetic nucleons. Even though nucleosynthesis may be over, these energetic nucleons destroy \(^4\text{He}\) or synthesise \(^3\text{He}\) and tritium leading to the overproduction of the light elements. Alternatively if the NLSP actually decays hadronically during nucleosynthesis these nucleons will instead establish thermal equilibrium with the surrounding plasma by colliding with the ambient protons and \(^4\text{He}\) leading to the eventual increase of the \(n/p\) ratio. This results in a greater abundance of \(^4\text{He}\), \(^3\text{He}\) and deuterium \(D\).

In order to avoid the destructive effects on the nucleosynthesis products the lifetime of the NLSP must be restricted so as to decay sufficiently well before it can interfere with the nucleosynthesis products or if it decays during nucleosynthesis that the enhanced light element abundances are consistent with astrophysical observations. In addition the abundance of the NLSP at the time of decay will also be important. Since the abundance and lifetime are related to the messenger scale \(M\), an upper bound can be placed on the messenger scale (for a typical set of the other gauge-mediated parameters), depending on whether the neutralino or stau is the NLSP. These bounds will be shown to be fairly generic for most of the parameter space of gauge-mediated theories.

2. The damaging effects of the NLSP decay products during the nucleosynthesis epoch
constrains the abundance and lifetime of the decaying particle. In order to obtain a bound on the NLSP lifetime a detailed calculation of the NLSP abundance at the time of decay must be performed. This amounts to calculating the NLSP abundance at the time of freeze out when the NLSP is no longer in chemical equilibrium.

We will consider the two separate cases of a neutralino and stau NLSP with mass $m_{\text{NLSP}}$. For moderate values of $\tan \beta$, $\tilde{\tau}_1$ is lighter than the neutralino whenever

$$N > \frac{66}{5(13 \xi - 2)}, \quad \xi \equiv \frac{\alpha_2^2(m_{\text{NLSP}})}{\alpha_1^2(M)} = \left[ 1 + \frac{22}{4\pi} \alpha_1(m_{\text{NLSP}}) \ln \frac{m_{\text{NLSP}}}{M} \right]^2. \quad (1)$$

For large $\tan \beta$ this region becomes slightly larger because of the stau left-right mixing. In order to determine the NLSP abundance we consider all relevant channels which change the number of NLSP's. When the NLSP is a neutralino the most relevant annihilation channels are those consisting of fermions and gauge bosons as depicted in Table I. The annihilation channels into Higgs bosons are suppressed because the lightest neutralino is mainly $B$-ino. For the same reason, the neutralino NLSP generically is not degenerate in mass with other particles, so it will not be important to consider coannihilations for this case. We can also neglect annihilation channels which involve gravitino vertices since these lead to scattering amplitudes which are suppressed by a factor $m_{\text{NLSP}}^2/F_0$. Since we are interested in NLSP lifetimes of the order of the nucleosynthesis timescale, this suppression makes all gravitino-emission processes negligible.

On the other hand when the NLSP is the stau, the lightest smuon and selectron have a small mass difference with the NLSP:

$$\frac{m_{\tilde{\mu}_1, \tilde{e}_1} - m_{\tilde{\tau}_1}}{m_{\tilde{\tau}_1}} \approx \frac{m_\tau^2}{2m_{\tilde{\ell}_R}^2} \left[ \frac{(\mu \tan \beta - A_\tau)^2}{m_{\tilde{\mu}_L}^2 - m_{\tilde{\ell}_R}^2} - 1 \right]. \quad (2)$$

Here $A_\tau$ is the supersymmetry-breaking trilinear term and $m_{\tilde{\ell}_L, R}$ are the flavour-independent contributions to the left and right slepton masses (including the $D$-term contribution). Since in gauge-mediated theories $\mu^2 > m_{\tilde{\ell}_L}^2$, the mass difference in Eq. (2) is always positive. Because of the approximate mass degeneracy among the sleptons, the calculation of the stau relic abundance must include all coannihilation processes listed in Table I. We also need to compute the smuon and selectron density at the decoupling time, since these “co-NLSP’s” are also responsible for producing damaging decay products.

The NLSP abundance is determined by considering the evolution of the number density $n_i$ of particle $i$ which is governed by the Boltzmann equation. In the presence of coannihilations the Boltzmann equation can be written as

$$\frac{dn_i}{dt} = -3H n_i - \langle \sigma_{\text{eff}} v \rangle (n_i^2 - n_i^\text{eq}) \quad (3)$$

where $H$ is the Hubble expansion parameter, $n = \sum_i n_i$ and the thermal average of the effective cross section is defined as

$$\langle \sigma_{\text{eff}} v \rangle = \sum_{ij} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^\text{eq} n_j^\text{eq}}{n_i n_j}. \quad (4)$$
The individual cross sections $\sigma_{ij}$ include the processes listed in Table I. $v_{ij}$ is the relative velocity and $n^{\text{eq}}$ is the equilibrium number density.

The cross sections for all annihilation channels are numerically evaluated using the CompHEP software package [11]. After performing the thermal average and including all relevant coannihilation thresholds, the ratio $Y^{\text{eq}}$ of the equilibrium number density $n^{\text{eq}}$ to the entropy density $s$ is given by

$$Y^{\text{eq}}(T) \equiv \frac{n^{\text{eq}}}{s} = \frac{45x^2}{4\pi^4 g_{\text{eff}}(T)} \sum_i g_i \frac{m_i^2}{m_{\text{NLSP}}^2} K_2 \left( x \frac{m_i}{m_{\text{NLSP}}} \right), \quad (5)$$

where $x = m_{\text{NLSP}} / T$, $g_i$ is the number of internal degrees of freedom and $g_{\text{eff}} = 81$. The NLSP abundance $Y_{\text{NLSP}}$ is the sum of all the “co-NLSP” abundances. The effect of including the coannihilating channels changes the pure stau abundance by up to $50\%$. This can be seen in Fig. 1 where the stau abundance with and without coannihilations is shown. Notice the important reduction of the relic abundance for values of $m_{\tilde{\tau}_1}$ close to half the Higgs mass, caused by the resonant annihilation channel. It should be noted, however that given the present LEP bound on stable $\tilde{\tau}_1$, this possibility is no longer realistic for the stau.

3. Let us now consider the effect of the NLSP decaying during the nucleosynthesis epoch. Suppose first that the NLSP is a neutralino. The dominant decay mode of the neutralino is into a photon and gravitino with a decay rate given by [2,12,13]

$$\Gamma(N_1 \rightarrow \gamma \tilde{G}) = \frac{k^2 \kappa_\gamma m_{N_1}^5}{16\pi F^2} \quad (6)$$

where $\kappa_\gamma = |N_{11} \cos \theta_W + N_{12} \sin \theta_W|^2$ and $N_{11,12}$ are the NLSP gaugino components in standard notation. There are also other decay modes which involve the $Z$ or a Higgs boson, but these modes are suppressed by a $\beta^8$ phase-space factor. The photon of the dominant decay mode has a dramatic effect on the light element abundance\cite{14}. In the radiation dominated

\footnotetext{1}{Notice that the gravitinos released in the NLSP decays do not thermalise and their present contribution to the energy density of the Universe is negligible.}
FIG. 1. *Comparison of the stau abundance with and without coannihilation (lower and upper curves, respectively).* The gauge-mediated parameters are $M = 10^{13}$ GeV, $N = 12$, $\tan \beta = 1.1$ and $\text{sgn} \mu = 1$.

thermal background high energy photons initiate electromagnetic cascades and create many low-energy photons capable of photodissociating the light elements [3]. These photodissociation effects caused by the photon of the decaying neutralino can be parametrised by a “damage” factor

$$d_\gamma \equiv m_N Y_N \frac{1}{\eta}$$

where $\eta$ is the baryon-to-photon number density ratio. This quantity $d_\gamma$ is constrained by astrophysical observations of light elements. Recently new observations of the $D$ and $^4\text{He}$ primordial abundance have led to a reanalysis of the constraints on the abundance and lifetime of long-lived particles [4]. In particular, controversy in the measurement of primordial deuterium have led to two different values for the deuterium abundance $X_D$ (normalised to hydrogen). In Ref. [14] a high value of the deuterium abundance $X_D = (1.9 \pm 0.5) \times 10^{-4}$ is quoted while in Ref. [15] a low value $X_D = (3.39 \pm 0.25) \times 10^{-5}$ is obtained.

The neutralino abundance $Y_{N_1}$ depends on the details of the sparticle spectrum and can be calculated for a particular choice of the parameters $N, M, F, \tan \beta$ and $\text{sgn} \mu$. Using the
constraints in Ref. [9] we show in Fig. 2 the bound on the messenger scale for a representative neutralino NLSP scenario in which $N = 2$, $\tan \beta = 2$ and $\text{sgn} \mu = 1$, for both high and low values of the measured $X_D$. We have also assumed that $k = 1$ and the limiting value of $M$ grows approximately linearly with $k$. Notice that the high deuterium value gives a slightly more stringent bound than the low deuterium value, because for lifetimes less than $10^6$ seconds deuterium is effectively photodissociated.

![FIG. 2. Upper bound on the messenger scale $M$ as a function of the neutralino NLSP mass $m_{N_1}$ from photodissociations where $k = 1$, $N = 2$, $\tan \beta = 2$ and $\text{sgn} \mu = 1$. Both bounds use a $^4\text{He}$ abundance $Y_p(^4\text{He}) = 0.234$ but the dashed line uses a low value of the deuterium abundance while the solid line uses the high value of the deuterium abundance (see text).](image-url)

The bound will change slightly for different values of the gauge-mediated supersymmetry breaking parameters, but typically for the neutralino NLSP scenario lifetimes greater than $10^5$ seconds are ruled out by photodissociations.

Note that photodissociations are only important after nucleosynthesis is over (which corresponds to times later than $10^4$ seconds). This is because during earlier times when thermal photons are more energetic and numerous, photon-photon interactions are much more probable than photon-nucleus interactions. Consequently stronger constraints can only be obtained if we consider processes that affect nucleosynthesis at times earlier than $10^4$ seconds.
However before discussing the relevant processes for times earlier than \(10^4\) seconds it is possible to obtain stronger constraints than those from photodissociations by considering hadronic decays of the NLSP \([8]\). If the NLSP decays hadronically during times \(\gtrsim 10^4\) seconds then the photodissociation bound needs to be reconsidered since while D is destroyed by photodissociation it can be produced by the hadronic showers. In fact even if the NLSP decays exclusively into photons it is possible that hadronic showers will be generated. The main effect of hadronic showers is not only to increase the abundance of D but also the other light nuclei \(^3\)He, \(^6\)Li and \(^7\)Li. In particular D and \(^3\)He arise mainly from “hadrodestruction” processes, which occur when an energetic nucleon breaks up the ambient \(^4\)He nucleus into D and \(^3\)He. The other light nuclei, \(^6\)Li and \(^7\)Li arise from “hadrosynthesis” of \(^3\)He, T or \(^4\)He. More complete details can be found in Ref. \([5]\). Again the “damage” of the hadronic decays on the primordial nuclear abundances can be parametrised as

\[
d_B \equiv d_\gamma B^*_h
\]

where \(B^*_h = (\nu_B/5)B_hF\) is an effective baryonic branching ratio which depends on the true baryonic branching ratio \(B_h\), the baryonic multiplicity \(\nu_B\) and a factor \(F\) representing the dependence of the yields on the energy of the primary shower baryons.

One of the contributions to the neutralino hadronic branching ratio comes from the decay \(N_1 \to Z\tilde{G}\)

\[
B_h \simeq 0.7 \tan^2 \theta_W \left(1 - \frac{m_Z^2}{m_{N_1}^2}\right)^4
\]

where the hadronic branching ratio of the Z-boson is 0.7, and we have assumed a pure \(B\)-ino. Although this contribution, and the analogous one from Higgs decay, can be forbidden by phase space, there is always at least a contribution to \(B_h\) of order \(\alpha\) from photon conversion into a \(q\bar{q}\) pair. The explicit expression for \(B_h\) can be obtained from Eq. (13) of Ref. \([16]\). Using the corresponding value of \(d_B\), one finds that the overproduction of \(^7\)Li constrains the lifetime of the neutralino to be shorter than \(10^4\) seconds. This is also true if the NLSP is the stau. The stau decays predominantly into a tau and gravitino and since the tau has a large hadronic branching ratio, the corresponding value of \(d_B\) is much larger than that for the neutralino. Consequently the lifetime of the stau must also be less than \(10^4\) seconds if the overproduction of \(^7\)Li is to be avoided \([8]\).

It is clear that in order to obtain precise constraints on the abundance and lifetime of the NLSP we need to consider processes occurring at times earlier than \(10^4\) seconds. At these times the main decay products that interfere with nucleosynthesis are hadrons. Hadronic showers induce interconversions between the ambient protons and neutrons thus changing the equilibrium \(n/p\) ratio \([1]\). In particular during the lifetime interval \(\tau_{\text{NLSP}} \sim 1 - 100\) seconds the overall effect of the hadronic decays is to convert protons into neutrons. The additional neutrons that are produced are all synthesised into \(^4\)He and thus hadronic decays during this time interval are constrained by the observational upper bound on the primordial helium abundance \(Y_p(\text{^4He})\).

Eventually the neutron fraction falls to zero because all neutrons are contained in the \(^4\)He nuclei and the remaining neutrons created by the NLSP decay increase the deuterium D abundance. Furthermore for \(\tau_{\text{NLSP}} \sim 100 - 1000\) seconds the \(^3\)He abundance is also increased.
by D-D burning. After $\tau_{\text{NLSP}} \gtrsim 10^4$ seconds all the neutrons arising from NLSP decay will themselves decay before forming D. Thus in the interval $10^2 - 10^4$ seconds the appropriate constraint on hadronic decays arises from the observational bounds on $(D + ^3\text{He})/\text{H}$.

The overall effect of these hadronic decays has been considered in Ref. [4], where the constraints on the abundance and lifetime of the decaying particle are parametrised by

$$f = \frac{N_{\text{jet}}B_h}{2} \frac{\langle n(E_{\text{jet}}) \rangle}{\langle n(33 \text{ GeV}) \rangle}$$

(10)

where $N_{\text{jet}}$ is the number of jets, $B_h$ is the hadronic branching ratio and $\langle n(E_{\text{jet}}) \rangle$ is the average charge multiplicity for a jet with energy $E_{\text{jet}}$ and is given by

$$\langle n(E_{\text{jet}}) \rangle = 1 + 0.0135 \exp \left[ 1.9 \sqrt{2 \ln \left( \frac{E_{\text{jet}}}{0.15 \text{ GeV}} \right)} \right].$$

(11)

Since at the parton level the neutralino NLSP decays into three particles we will assume that $E_{\text{jet}} = m_{\text{NLSP}}/3$ and $N_{\text{jet}}$ is the number of quarks at the parton level. In the case of the stau in which the tau decays semi-leptonically we assume $E_{\text{jet}} = m_{\text{NLSP}}/6$.

The most stringent constraints on the NLSP abundance and lifetime in the interval $10^2 - 10^4$ seconds come from the primordial deuterium abundance. Previous constraints on $Y_{\text{NLSP}}f$ and the lifetime were obtained for $X_D < 10^{-4}$ and $\eta = 3 \times 10^{-10}, 10^{-9}$ [4]. In order to use the more recent measurements, we rescale the previous constraints for the new values $X_D = (1.9 \pm 0.5) \times 10^{-4}$ or $X_D = (3.39 \pm 0.25) \times 10^{-5}$. This rescaling can be done analytically by using the change in the deuterium abundance

$$\Delta X_D = \frac{\Delta Y_{\text{NLSP}}}{Y_H} \epsilon_D a_n,$$

(12)

where $Y_H$ is the hydrogen density, $\epsilon_D$ is the fraction of injected neutrons that end up in deuterium and $a_n$ is the number of $n\bar{n}$ pairs per NLSP decay. This equation is valid as long as $X_D \ll \epsilon_D$, which conveniently holds whenever the deuterium constraint is important. Since the limit comes from the overproduction of deuterium and the standard $X_D$ prediction decreases with $\eta$, we make the most conservative choice of $\eta = 6 \times 10^{-10}$, which is the largest value compatible with $^4\text{He}$ and $^6\text{Li}$ abundances.

When the high value of the deuterium abundance is used, one obtains no constraints in the region $10^2 - 10^4$ seconds. All values of the NLSP abundance and lifetime are consistent with high $X_D$ value and consequently there is no improvement on the lifetime upper bound of $10^4$ seconds obtained from the overabundance of $^7\text{Li}$.

On the other hand stringent constraints in the interval $10^2 - 10^4$ seconds are obtained when the low value of the deuterium abundance is used. Let us consider the two NLSP scenarios separately. First, when the NLSP is the neutralino the scaling parameter $f$ is determined using $N_{\text{jet}} = 2$ and $B_h = 10^{-2}$. Rescaling the solid curve of Fig. 4 in Ref. [4] enables one to determine the constraint arising from the low value of $X_D$ and $\eta = 6 \times 10^{-10}$. Thus calculating the value for the neutralino number density for a generic set of gauge-mediated supersymmetry breaking parameters leads to constraints on the abundance and lifetime of the neutralino which can be expressed as a bound on the messenger scale $M$, see
Fig. 3. Upper bound on the messenger scale $M$ as a function of the neutralino NLSP mass $m_{N_1}$ from hadronic decays where $k = 1$, $N = 2$, $\tan \beta = 2$ and $\text{sgn} \mu = 1$. The solid (dashed) line corresponds to the bound assuming a low (high) deuteriu m measurement. More specifically the dashed line represents the $10^4$ second lifetime contour which arises from the $^7\text{Li}$ overabundance.

The bound on $M$ does not significantly depend on the value of $\tan \beta$, but grows approximately linearly with $N$.

Let us now discuss the case of the stau NLSP. At the decoupling time, the abundances of smuons and selectrons are comparable to the stau abundance. The cosmological fate of the frozen-out smuons and selectrons depends on the mass difference in Eq. (2). If $\tan \beta$ is large, then $m_{\tilde{\ell}_1} - m_{\tilde{\tau}_1} > m_\tau - m_{\ell}$ ($\ell = e, \mu$) and the decay $\tilde{\ell}_1 \to \tilde{\tau}_1 \tau \ell$ is kinematically open. The value of $\tan \beta$ for which this transition occurs depends on the parameter choice, but it is typically of order 10, as it can be estimated from Eq. (2). If this mode is accessible, it dominates over the two-body decay into gravitino. This can be simply understood from the decay rate expression [17] in the limit of vanishing lepton masses and small slepton mass difference,

$$
\Gamma(\tilde{\ell}_1^- \to \tilde{\tau}_1^- \tau^+ \ell^-) \simeq \frac{4 G_F^2}{15 \pi^3} \tan^4 \theta_W \frac{M_W^4}{m_{N_1}^4} (m_{\tilde{\ell}_1} - m_{\tilde{\tau}_1})^5,
$$

(13)
\[
\Gamma(\ell^-_1 \to \tilde{\ell}^+_1 \tau^- \ell^-) \simeq \frac{4G_F^2}{15\pi^3} \tan^4 \theta_W \frac{M_W^4}{m_{\chi^0_1}^2 m_{\tilde{\ell}_1}^2} (m_{\tilde{\ell}_1} - m_{\chi^0_1})^5. \tag{14}
\]

Here \( m_{\chi^0_1} \) is the mass of the B-ino which mediates the decay, and is assumed to be much larger than \( m_{\tilde{\ell}_1} \). In this case, smuons and selectrons decay soon after decoupling, with each one producing a stau in the final state. Therefore, the relevant stau abundance at the nucleosynthesis epoch is determined by the sum over the three slepton abundances at the decoupling time. The stau hadronic branching fraction comes from the semileptonic tau decay, and it is given by \( B_h \simeq 0.65 \).

For smaller values of \( \tan \beta \) the neutralino-mediated three-body process is forbidden, and the rates of the two competing slepton decay modes are

\[
\Gamma(\tilde{\ell}^-_1 \to \ell \tilde{G}) = \frac{k^2 m_{\tilde{\ell}}^5}{16\pi F^2} \tag{15}
\]

\[
\Gamma(\tilde{\ell}^-_1 \to \tilde{\tau}^-_1 \nu_\ell \nu_\tau) \simeq \frac{G_F^2}{15\pi^3} \frac{M_W^4}{m_{\chi^0_1}^2 m_{\tilde{\ell}_1}^2} (m_{\tilde{\ell}_1} - m_{\chi^0_1})^5 \sin^2 \theta_\tau \sin^2 \theta_\ell. \tag{16}
\]

Here \( m_{\chi^0_1} \) is the chargino (gaugino) mass which mediates the decay, and \( \theta_\tau, \theta_\ell \) are the left-right mixing angles in the slepton system. The process in Eq. (16) is suppressed by the large value of \( F \) required in our study of nucleosynthesis and the process in Eq. (15) is suppressed by the small mixing angles proportional to the corresponding lepton masses. It turns out that in the small \( \tan \beta \) regime and for \( m_{\tilde{\ell}_1} \sim 100 \) GeV, the two rates are comparable when \( \sqrt{F} \sim 10^9 \) GeV, which is just the value of \( \sqrt{F} \) necessary to have a stau decay during nucleosynthesis. Therefore, depending on the particular choice of the gauge mediation parameters, we can encounter different situations. The first possibility is that \( \Gamma(\tilde{\ell}^-_1 \to \ell \tilde{G}) > \Gamma(\tilde{\ell}^-_1 \to \tilde{\tau}^-_1 \nu_\ell \nu_\tau) \), in which all light sleptons decay directly into gravitinos around the same time. In this case smuons and selectrons do not significantly contribute to the effective hadronic branching fraction, because real electrons and muons cannot decay into hadrons. As \( \tan \beta \) is increased and \( m_{\tilde{\ell}_1} \) is decreased, we go first to a regime in which \( \Gamma(\mu^- \to \tilde{\tau}^-_1 \nu_\mu \bar{\nu}_\tau) > \Gamma(\tilde{\ell}^-_1 \to \ell \tilde{G}) \) and then in a regime in which the three-body decays dominate for both \( \tilde{\mu}_1 \) and \( \tilde{e}_1 \). The two regimes are possible because the decay process in Eq. (16) depends on the lepton mass through the left-right mixing angle. In the first regime, only the smuon contributes to the effective hadronic branching fraction, while in the second one both the smuon and selectron contribute.

With respect to a neutralino NLSP of equal mass, a stau NLSP has a larger hadronic branching ratio \( B_h \), but a smaller relic abundance because of the additional annihilation channels. The two effects roughly compensate each other, although the bound on the NLSP lifetime is slightly weaker in the \( \tilde{\tau}^-_1 \) case. When expressed in terms of \( M \), the bound appears even weaker because we have to choose a very large value of \( N \) in order to satisfy the requirement of Eq. (1) for a neutralino NLSP. The bound on the messenger mass for \( N = 12 \) and \( \tan \beta = 1.1 \) is shown in Fig. 4. For this choice of parameters, the selectron and the smuon dominantly decay directly into gravitinos. In the figure we only show the bound arising from the low deuterium value. The bound for the high deuterium value corresponds to a lifetime which cannot be achieved for this choice of \( N \).
FIG. 4. Upper bound on the messenger scale $M$ as a function of the stau NLSP mass $m_{\tilde{\tau}}$ from hadronic decays where $k = 1$, $N = 12$, $\tan \beta = 1.1$ and $\text{sgn} \mu = 1$.

4. Although the mass spectrum of supersymmetric theories with gauge mediation can be predicted in terms of a few parameters, the experimental and cosmological features of the theory can drastically change as the messenger mass $M$ is varied in the allowed range between about 100 TeV and $10^{15}$ GeV. It is thus very important to study possible constraints on the value of $M$. The requirement that the NLSP decay products do not upset the successful predictions of standard big-bang nucleosynthesis provides such a constraint. Here we have studied in detail the effects of the decaying NLSP on the light-element primordial abundances and shown that the most dangerous damages come from the hadronic decay modes. After computing the NLSP relic abundance, including coannihilation effects, we conclude that the injection of hadronic jets in the Universe at times later than $10^4$ seconds grossly overproduces $^7$Li. This is true for both neutralino and stau NLSP. At earlier times, the most relevant bounds come from deuterium overproduction. At present, there is a disagreement on the extracted observational value of the primordial deuterium. Using $X_D = (3.39 \pm 0.25) \times 10^{-5}$ [3], no further limit on the NLSP is derived, while with $X_D = (1.9 \pm 0.5) \times 10^{-4}$ [4] the limit from $^7$Li can be improved. The corresponding limits on the messenger mass $M$ are shown in Figs. 3 and 4 for a representative choice of gauge mediation parameters, in the case of neutralino and stau NLSP respectively.
It should be noticed that the bounds on the messenger mass presented here can be evaded in the presence of other interactions which can lead to a fast NLSP decay, e.g. $R$-parity violation. Nevertheless these bounds have significant implications for model building. In particular they disfavour models in which $M$ is close to the GUT scale or models in which the messenger scale results from balancing renormalisable interactions with non-renormalisable operators at the Planck scale [18,19].

It is also interesting to notice that the nucleosynthesis bound discussed here is complementary to the bound obtained from gravitino overabundance. Indeed when $m_G$ is larger than about a keV, gravitinos which were in thermal equilibrium at early times, give a contribution to the present energy density larger than the critical value. It is then necessary to assume that gravitinos have been diluted by some mechanism. Let $T_{\text{max}}$ be the temperature at which the ordinary radiation-dominated Universe begins. This corresponds to either the reheating temperature after an inflationary epoch or to the temperature associated with significant entropy production. The requirement that gravitinos do not overclose the Universe gives the following constraints on $T_{\text{max}}$ [20,21]

\begin{equation}
T_{\text{max}} < 100 \text{ GeV} - 1 \text{ TeV} \quad \text{for} \quad 2h^2 \text{keV} \lesssim m_G \lesssim 100 \text{ keV}, \tag{17}
\end{equation}

\begin{equation}
T_{\text{max}} < 10 \text{ TeV} h^2 \left( \frac{m_G}{100 \text{ keV}} \right) \left( \frac{\text{TeV}}{m_\tilde{g}} \right)^2 \quad \text{for} \quad m_G \gtrsim 100 \text{ keV}, \tag{18}
\end{equation}

where $h$ is the Hubble constant in units of 100 km sec$^{-1}$ Mpc$^{-1}$ and $m_\tilde{g}$ is the gluino mass. To compare this bound with the nucleosynthesis result, it is convenient to express it in terms of the messenger mass $M$, the messenger index $N$, and the pure $B$-ino mass $m_{N_1}$. We find

\begin{equation}
T_{\text{max}} \approx 100 \text{ GeV} - 1 \text{ TeV} \quad \text{for} \quad 10^8 h^2 \text{ GeV} \lesssim \frac{M}{kN} \left( \frac{m_{N_1}}{100 \text{ GeV}} \right) \lesssim 5 \times 10^9 \text{ GeV}, \tag{19}
\end{equation}

\begin{equation}
T_{\text{max}} \approx 5 \times 10^{-6} h^2 \frac{M}{kN} \left( \frac{100 \text{ GeV}}{m_{N_1}} \right) \quad \text{for} \quad \frac{M}{kN} \left( \frac{m_{N_1}}{100 \text{ GeV}} \right) \lesssim 5 \times 10^9 \text{ GeV}. \tag{20}
\end{equation}

This shows that there is no gravitino problem as long as $M \lesssim 10^8 h^2 \text{ GeV} kN(100 \text{ GeV}/m_{N_1})$. For larger values of $M$, there is a very stringent constraint on $T_{\text{max}}$ which requires inflation at particularly low temperatures. As $M$ grows this limit becomes weaker, but eventually the nucleosynthesis bound on the messenger mass sets in. When the two bounds are combined, we find that the reheating temperature after inflation should be less than about $10^7 h^2 \text{ GeV}$ which is stronger than the bound $\sim 10^{10}$ GeV usually obtained in gravity-mediated scenarios [18,20]. Our limit is valid for gauge-mediated theories in which the LSP gravitino is heavier than a keV and in which there are no new interactions uncorrelated with the supersymmetry-breaking scale mediating the NLSP decay (like $R$-parity violating interactions).

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