Flat directions, String Compactification and 3 Generation Models.

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Abstract

We show how identification of absolutely flat directions allows the construction of a new class of compactified string theories with reduced gauge symmetry that may or may not be continuously connected to the original theory. We use this technique to construct a class of 3 generation models with just the Standard Model gauge group after compactification. We discuss the low-energy symmetries necessary for a phenomenologically viable low-energy model and construct an example in which these symmetries are identified with string symmetries which remain unbroken down to the supersymmetry breaking scale. Remarkably the same symmetry responsible for stabilising the nucleon is also responsible for ensuring one and only one pair of Higgs doublets is kept light. We show how the string symmetries also lead to textures in the quark and lepton mass matrices which can explain the hierarchy of fermion masses and mixing angles.

1 Introduction

The construction of the effective low-energy theory from the string has undergone several iterations since the discovery of the heterotic string opened the way to building realistic 4-D theories with the Standard Model gauge group. The first attempts compactified the 10-D string theory using Calabi-Yau manifolds [1] or orbifolds [2] and examples of three generation models were identified [3]. Subsequently it was realised that one could directly construct 4-D models, many of which did not have a ten-dimensional interpretation, and so there was no absolute need to interpret our low-energy world as resulting from a higher dimensional theory. Recently, in order to reconcile the string prediction for the gauge coupling unification scale with the “experimental” value, it has been suggested [4] that the relevant underlying string theory is actually the strongly interacting heterotic string, or M-theory [5]. Many of the 4-D string theory constructions cannot be embedded in M-theory and as a result the discussion has come full circle and there is renewed interest in the construction compactified string theories. Although the starting point is eleven dimensional, much of the 10-D work remains
relevant because the strongly coupled heterotic string theory corresponds, in leading order in the string coupling, to M-theory compactified on the direct product of a 6-D Calabi-Yau manifold with the line interval, i.e. $CY_3 \otimes S_1/Z_2$.

In order to construct the effective low-energy theory descending from M-theory which directly realises the prediction of the string for the string unification scale, $M_X$, one must construct the compactified theory on a specific $CY_3 \otimes S_1/Z_2$ manifold such that the gauge group after compactification should be just $SU(3) \otimes SU(2) \otimes U(1)$, otherwise the prediction of the string relating gauge couplings will not refer to just those couplings measured in the laboratory but rather those of the Grand Unified group. This means there should be no GUT below the compactification scale! In this case one may ask why then does the Standard model have GUT properties? This need not be a difficult question to answer because the string does have an underlying GUT structure. In this case GUT breaking on compactification must leave some relic of an underlying string gauge symmetry in the low energy spectrum. It is the purpose of this paper to investigate in detail the structure to be expected in such cases by constructing explicit three generation string models compactified on Calabi-Yau manifolds.

To date, the three generation Calabi-Yau manifolds constructed have all had a gauge group larger than that of the Standard Model after compactification. The reason is straightforward. The models are based on the heterotic string with an underlying $E_8 \otimes E_6$ gauge symmetry. In order to break this group at the compactification scale it is necessary to implement Wilson line breaking on a non-simply-connected manifold. The construction of the three generation case involves the modding out of the simply connected manifold by a discrete symmetry group and it is this process that introduces the non-simply-connectedness of the manifold. The associated Wilson lines form a representation of some or all of the discrete symmetry group. However, since the discrete groups involved are $Z_3$ groups, the Wilson line breaking is limited and, at best, can only reduce the gauge group to $SU(3)^3$. However it is known that these compactifications, which have a $(2,2)$ world sheet supersymmetry, are related along flat directions to theories with $(2,0)$ supersymmetry. The latter have a reduced symmetry $E_8 \otimes SO(10)$ or $E_8 \otimes SU(5)$ and thus, after Wilson line breaking, may provide a basis for constructing a three generation model with just the Standard model gauge group. However not much is known about these $(2,0)$ models for they have proved difficult to construct. Some examples are known, built using the direct product of superconformal theories constrained to have the correct ghost charge, but none with just three generations. In this paper we will develop a method which does allow us to construct three generation examples with $(2,0)$ symmetry and we will show how they allow us to generate just the Standard Model gauge group. The method involves deforming the known three generation Calabi-Yau model along flat directions. In order to identify these directions it is necessary to start from a point in moduli space where the symmetries are larger than for the generic Calabi-Yau model because it is only by finding these symmetries that one may identify the relevant flat directions. Luckily the superconformal construction provides a way of constructing a 4-d string theory whose large radius limit is just the 3 generation Calabi-Yau theory. Moreover the symmetry of the superconformal theory is enlarged in just such a way as to allow us to identify absolutely flat directions in field space. Allowing vevs in these directions still leaves a viable string theory with three generations but reduces the
gauge symmetry. Such deformed theories need not be continuously connected to the original theory and provide a new class of four-dimensional string theories. Furthermore, as there are a large number of flat directions, there is a rich diversity of effective low-energy theories related to the original Calabi-Yau theory. While this may seem disappointing, adding further to the already vast number of possible string vacua, it does offer some hope for finding a phenomenologically viable theory. Using this flexibility we show how it is possible to construct a consistent version of the MSSM in which the residual low-energy symmetry of the theory guarantees there is just one pair of light Higgs doublets.

2 Symmetries and flat directions.

2.1 D-flat directions

We turn now to the central ingredient of the construction of new string theories, namely the identification of absolutely flat directions in the scalar potential from the symmetry properties of the theory. For a direction to be absolutely flat it is necessary for both the F and the D terms to vanish identically. Let us consider the D terms first. The general form is

\[ L_{\text{Scalar}}^{D} = \frac{1}{2} \sum_a |g_a \sum_i A_i^a T^a A_i|^2 \]  

where \( A_i \) is the scalar component of the chiral superfield \( \phi_i \), \( g_a \) is the gauge group coupling constant and \( T^a \) is the associated generator. A field direction will be D-flat if the fields acquiring vacuum expectation values (vevs) along a F-flat direction are singlets under the gauge group or if there is a cancellation between terms carrying opposite D-charge. To discuss the latter possibility consider first the simple case of a single \( U(1) \) factor. It is sufficient for D-flatness that there are at least two fields carrying \( U(1) \) charges with opposite signs provided that the relative magnitude of their vevs are not fixed by the F-terms of the theory. In this case the ratio of vevs of the two fields is determined by the condition of D-flatness. This result generalises immediately to more than one \( U(1) \). For the non-Abelian case the same conditions apply to the diagonal generators. Thus one ends up with \( N \) constraints amongst the vevs fields for the case they are charged under \( N_D \) Abelian gauge group factors.

2.2 F-flat directions

Let us consider now the constraints for F-flatness. From the above discussion on D-flatness we know that at least one combination of fields acquiring vevs along D-flat directions will be gauge invariant and therefore be allowed by the gauge symmetry to appear in the superpotential. For example, with a single \( U(1) \), the condition of D-flatness means there must be at least two fields, \( \phi_A, \phi_B \) carrying opposite sign charges \( Q_A \) and \( -Q_B \), where \( Q_A \) and \( Q_B \) are positive. Then the combination \( \phi_A^{Q_B} \phi_B^{Q_A} \) is gauge invariant. In string theory compactifications the charges \( Q_A \) and \( Q_B \) are rational and so the gauge invariant term \( \Phi = (\phi_A^{Q_B} \phi_B^{Q_A})^p \) for some integer \( p \) will involve only integer powers of the fields and thus be a possible superpotential term. Such a term is not F-flat by itself because both \( F_{\phi_A} \) and \( F_{\phi_B} \) are non-zero.
and there is no possibility for other terms to cancel them if only $\phi_A$ and $\phi_B$ have vevs. Of course if $\Phi$ has dimension $> 3$ it will be suppressed by some inverse power of mass (in general the string scale $\approx M_{Planck}$) but such a term will spoil the absolute F-flatness we are seeking here. In the case that additional fields carry vevs there is the possibility of a cancellation between different contributions to an F-term but this does not solve the problem because such additional terms necessarily give rise to new F-terms that are non-vanishing and hence does not lead to F-flatness. For example, if $\phi_C$ has non-vanishing vev and $Q_C$ is positive there is an additional term of the form $(\phi_C^Q B^Q C)^q$. This contributes to $F_{\phi_B}$ and can cancel the contribution from the first term. However $F_{\phi_C}$ is now non-zero and so one has not generated an F-flat potential.

As we have just seen continuous gauge symmetries do not lead to F-flat directions by themselves. Global symmetries can evade this conclusion and ensure F-flattness because there is no D-term associated with a global symmetry and the absence of the D-flatness constraint means that the fields acquiring vevs can all have the same sign of global charge. In this case no combination of charged fields will be neutral and hence invariant under the global symmetry and thus there is no term in the superpotential involving these fields. However in string compactification there are no continuous global symmetries and hence this possibility for generating flat directions is not realised. The only other symmetries available to us are discrete symmetries. Non-R symmetries do not lead to flat directions because we can form an invariant under such a symmetry from any gauge invariant combination of fields simply by raising it to a suitable power. For example if the gauge invariant combination of fields, $\Phi$, has a non-trivial charge under a $Z_M$ symmetry, $\Phi^M$ will be invariant and a candidate F-term. Thus we are led to the conclusion that the only symmetry capable of generating a flat direction in the scalar potential is a discrete R-symmetry \[10\]. The reason R-symmetries are so powerful in this respect is easy to see because an R-symmetry requires that the superpotential coordinates, $\theta$, transform non-trivially, $\theta \rightarrow e^{i\beta} \theta$. Thus the superpotential, $P$, must also transform non-trivially, $P \rightarrow e^{2i\beta} P$. If the gauge invariant combination of fields $\Phi$ is invariant under the R-symmetry then so too is any power of $\Phi$, and hence is not a possible F-term. Of course to ensure F-flatness it is also necessary to avoid any gauge invariant terms of the form $\phi_X \Phi_i$, where $\Phi_i$ is any combination of the chiral superfields making up $\Phi$ and $\phi_X$ is any of the chiral superfield which do not acquire vevs. However these latter terms may be absent due to the gauge and/or discrete symmetries.

### 2.3 Moduli

There is an exception to the conclusion discussed in the last section which applies to string moduli fields, $m$. These are fields which, due to the string symmetries, have absolutely flat directions when non-moduli fields have zero vevs. The reason is because the string symmetries ensure that the moduli do not appear in the superpotential on their own but only in non-renormalisable terms in conjunction with non-moduli fields. Because these fields have absolutely flat potentials their vevs are undetermined in the absence of supersymmetry breaking and they serve to determine the couplings of the theory. In Calabi Yau compactification, the moduli fields can acquire vevs through interactions with other fields, and these vevs can determine the shape and size of the compactification space. This is a crucial feature of compactification theories as it allows for the generation ofPlanck scale mass terms that are consistent with the observed low energy scale. The moduli fields also play a role in the generation of scalar potentials and the structure of the effective theory at low energies. The study of moduli fields and their interactions is a key aspect of string compactification models and is an active area of research in theoretical physics.
ification there are $h_{2,1}$ complex structure moduli fields whose vevs determine the Yukawa couplings of the theory. In addition there are $h_{1,1}$ fields whose vevs determine the shape of the compactification manifold and determine the Kahler structure of the theory. Since the complex structure moduli fields appear in the superpotential they may play a role in setting a F-term to zero and thus should be considered when looking for absolutely flat directions involving non-moduli fields.

2.4 Absolutely flat directions

From these considerations it is straightforward to quantify the conditions for absolutely flat directions. We start with $N$ fields acquiring vevs. The condition of D-flatness imposes $N_D$ constraints among the vevs. The condition of F-flatness imposes a further $N_F$ constraints corresponding to $N_F$ F-terms which involve (independent) combinations of the $N$ fields only. Clearly the condition for an absolutely flat direction requires

$$N \geq N_F + N_D$$

Equality only occurs if all vevs are determined (strictly this is not a flat direction but a point in field space) and this necessarily implies one or more of the non-moduli fields acquires Planck scale vevs. To see this consider the simplest example of a theory with no R-symmetry with single chiral superfield, $\phi$, which transforms trivially under any of the symmetries of the theory. The superpotential has the form

$$P = \lambda \phi^3 + \bar{\lambda} \frac{\phi^4}{M} + ...$$

In this case $N = N_F = 1$ and the vev of $\phi$ is determined. Clearly the condition that $F_\phi = 0$ is satisfied only if $\phi$ develops a vev of $O(M)$. In this case there is no residual flat direction and the non-trivial minimum is not connected to the trivial minimum with $\phi = 0$. In this case identification of a point at which the vacuum energy vanishes allows us to build a new string theory not continuously connected to the original. This is to be contrasted with the case in which the inequality is satisfied in eq(2) may be continuously connected to the original theory.

We wish to explore the possibility that a specific string theory possesses absolutely flat directions allowing us to construct new string compactifications\(^2\). The first point to notice is that an almost necessary condition is that there should be no terms allowed in the superpotential of the form $\Phi^n$ where $\Phi$ is the gauge invariant combination of fields introduced above. The reason for this is because any term of this form generates $N_\Phi$ F-terms involving the fields contributing to $\Phi$, all of which have non-zero vevs. Thus for this subset $N_\Phi$ of the $N$ fields there are $N_\Phi$ F-terms so by eq(2) all the vevs are determined. Thus for this to be an absolutely flat direction there should be no terms in the superpotential of the form $\phi_X \Phi^n$ for any of the fields $\phi_X$ of the theory. This is a very strong condition and is not realised in a typical string theory. On the other hand if the term $\Phi^n$ is forbidden then the condition for

\(^2\)For other work constructing new theories by deforming along flat directions, see [11, 12].
a flat direction becomes much simpler, namely the number $N_X$ of terms of the form $\phi_X \Phi^n$ should, by eq(2), be bounded by the constraint

$$N_\phi \leq N_X + N_D$$

(4)

In Section 5 we will construct explicit examples of a three generation string theory satisfying these conditions but first we present a brief review of the three generation string construction that provides the starting point of our discussion.

3 The Three generation Calabi Yau Model.

We start with a brief review of the superconformal version of the 3 generation Calabi Yau theory which provides the starting point for our new (2,0) theories with reduced gauge symmetry. The construction introduced by Gepner starts with the tensor product of $N=1$ superconformal theories in 2D. The resulting theory has an enhanced gauge group $G = E_8 \otimes E_6 \otimes U(1)^3$. In addition it has a much richer class of discrete symmetries including discrete R-symmetries. The latter is important because, alone amongst discrete symmetries, it can lead to absolutely flat directions in field space.

The trace anomaly for a level $k$ superconformal theory ($k$ is the principal quantum number labeling the superconformal theory) is given by

$$c = \frac{3k}{k+2}$$

(5)

By choosing an appropriate set of the minimal $N=2$ superconformal theories, we are able to provide the $c = 9$ central charge. Together with $c = 3$ contribution from the free space-time fields, which describe the flat four dimensional part of the model, the total central charge is 12, which is required for anomaly cancellation in the heterotic string. In analogy with heterotic compactifications, we also introduce two sectors (corresponding respectively to left and right movers) namely the gauge sector and the SUSY sector. The procedure leading to space-time supersymmetry and the required gauge group consists of imposing modular invariant constraints on the quantum numbers of the fields in the theory. This construction has been described in great details by Gepner so here we will only summarise the results of the analysis.

The primary fields in every minimal superconformal models are labeled by integers $l$ and $q$. The first one is the principal quantum number, taking its values in the range $0, ..., k$ while the second one labels the $U(1)^3$ charge of the field and is defined modulo $2(k+2)$. In addition, fields can belong to either Ramond or Neveu-Schwarz sector. We take this into account by introducing the third label $s$ (0 for Neveu-Schwarz and $\pm 1$ for Ramond sector). The most important thing in our construction of new models will be the $U(1)$ charges of the

\[\text{Deformations of Calabi-Yau theories along absolutely flat directions ensured by R-symmetries have previously been considered by Greene. This approach did not start with the superconformal construction used here.}

\[\text{The } U(1) \text{ symmetry is a part of the superconformal algebra associated with each tensor factor.}\]
fields. These are given by

\[ Q = -\frac{q}{k + 2} + s \frac{s}{2} \]  

(6)

The model we consider here is the three generation model, constructed by Gepner [13]. This case has been discussed in many papers [14] and it has been shown that it corresponds to the heterotic string theory compactified on a specific Calabi-Yau manifold (leading eventually to the three generations of fermions). The results we quote here are based on the extensive work of Scheich and Schmidt [15].

The three generation model is constructed by representing the internal degrees of freedom by a tensor product of four superconformal field theories, one with level \( k = 1 \) and three with \( k = 16 \). The same product is used in both supersymmetry and gauge sectors and a modular invariant theory is formed using the \( A [9] \) invariant for the \( k = 1 \) factor and \( E_7 \) affine modular invariants for \( k = 16 \) factors. At this stage the model contains 35 generations (27 of \( E_6 \)), 8 anti-generations (27 of \( E_6 \)) and 197 massless \( E_6 \) singlets. To reduce the net number of generation to three we mod out the tensor product of the superconformal theories by two discrete symmetries present in the model \( S \), which are the cyclic permutations of the three \( k = 16 \) sub-theories and a \( Z_3 \) symmetry generated by (0,3,6,0) element (the four numbers correspond respectively to \( Z_3 \) charge in the first subtheory and the three \( Z_{18} \) charges in the remaining ones) [13].

The fields are labeled by the quantum numbers of the superconformal factors in the form

\[ \left( \begin{array}{lll} l & q & s \\ \bar{l} & \bar{q} & \bar{s} \end{array} \right) \]  

(7)

The notation used here refers to the \( SO(10) \) \( 10 \) component in the decomposition of the full 27-plet of \( E_6 \) into the subgroup \( SO(10) \times U(1) \). To get the full 27-plet, one has to take three different fields corresponding respectively to the \( 1, 10 \) and \( 16 \) of \( SO(10) \). These three different representations can be generated one from each other by acting with an operator which results in shifting \( q \rightarrow q + 1 \) and \( s \rightarrow s + 1 \). Applying this successively generates the set \( 10 \rightarrow 16 \rightarrow 1 \rightarrow 10 \).

An analogous procedure applies to the supersymmetry sector. A given field, described by \( (l, \bar{q}, \bar{s}) \) corresponds to only one (for example scalar) component of the chiral superfield. To get the full supermultiplet, one has to sum three different Lorentz states. Again, we obtain them using a specific operator (in this case it is the supersymmetry charge), which shifts the quantum number in the SUSY sector: \( q \rightarrow q + 1 \), \( s \rightarrow s + 1 \). The notation of eq(7) refers to the scalar component. From it we generate the one corresponding to fermions (in the \( -\frac{1}{2} \) picture) by applying the supersymmetry charge operator once. Applying the supersymmetry charge twice yields the vertex operator for the auxiliary F-field.

For the scalar components of the chiral supermultiplets which transform under \( SO(10) \) as the \( 10 \) component of an \( E_6 \) 27 supermultiplet the massless fields are given in Table [14].

It is important to remember that in all these fields (and the ones which are listed further below), the three \( k=16 \) factors can be interchanged. This may give one, three or six different fields depending on whether the the quantum numbers in all the factors are the same, two are the same or all are different respectively. Upon modding out by the cyclic permutations, the surviving fields are the symmetric combinations. If Wilson lines are associated with the
$l_1: \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 12 & 12 & 0 \\ 12 & 12 & 0 \end{pmatrix}$

$l_2: \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix}$

$l_3: \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix}$

$l_4: \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix} \begin{pmatrix} 12 & 12 & 0 \\ 12 & 12 & 0 \end{pmatrix}$

$l_5: \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 12 & 12 & 0 \\ 12 & 12 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix}$

$l_6: \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 10 & 10 & 0 \\ 10 & 10 & 0 \end{pmatrix}$

$l_7: \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix}$

$l_8: \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2 & 2 & 0 \\ 8 & 8 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 8 & 8 & 0 \end{pmatrix} \begin{pmatrix} 2 & 2 & 0 \\ 8 & 8 & 0 \end{pmatrix}$

$l_9: \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2 & 2 & 0 \\ 8 & 8 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 2 & 2 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 8 & 8 & 0 \end{pmatrix}$

Table 1: The massless fields of the three generation Gepner construction transforming as the 27 representation of $E_6$. The four factors refer to the $1.16^3$ superconformal factors of the construction. The quantum numbers given refer to the scalar component transforming as the 10 of the SO(10) subgroup of $E_6$. 

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Table 2: The massless fields of the three generation Gepner construction transforming as the $27$ representations of $E_6$. The four factors refer to the $1.16^3$ superconformal factors of the construction. The quantum numbers given refer to the scalar component transforming as the 10 of the $SO(10)$ subgroup of $E_6$. Cyclic permutations then the states left light will correspond to different combinations such that the product of the discrete gauge group factor and the permutation are singlet.

The $27$ mirror generations are given in Table 2. Finally the gauge singlets are given in Table 3.

In addition there are 9 complex structure moduli fields which determine the Yukawa couplings of the theory and 6 Kahler structure moduli which determine the metric.

In the construction of phenomenologically relevant models, one has to break the gauge group from $E_6$ to a smaller group leading, in the end, to the Standard Model gauge group. In the case of Calabi-Yau compactifications, the best way to do it \cite{16} is to use Wilson lines. In the Gepner model there is an exact analog of this mechanism \cite{17} which, as in the Calabi-Yau case, requires the introduction of additional fields with non-trivial transformation properties under the action of the discrete group which we embed in the gauge group. In the case this is the $Z_3$ cyclic permutation group, the new fields carry the quantum numbers of the left-handed quarks, $q$, and the left-handed antiquarks, $Q$. They are given by the combinations of the fields given above which transform as $\alpha$ and $\alpha^2$ ($\alpha = e^{i2\pi/3}$) respectively under the cyclic permutation group.

For completeness we list in Table 4 the fields appearing in the case that the discrete group associated with the Wilson lines is the phase twist generated by (0,3,6,0) element \cite{13}. 

\begin{table}[h]
\centering
\begin{tabular}{c|c|c}
| $T_1$ | $0 0 0$ | $2 2 0$ |
|-------|---------|--------|
| $T_2$ | $1 1 0$ | $4 4 0$ |
|-------|---------|--------|
| $T_3$ | $1 3 2$ | $2 2 0$ |
|-------|---------|--------|
| $T_4$ | $0 0 0$ | $6 6 0$ |
|-------|---------|--------|
| $T_5$ | $0 0 0$ | $0 0 0$ |
|-------|---------|--------|
| $T_6$ | $0 0 0$ | $12 12 0$ |
\end{tabular}
\end{table}
\begingroup
\begin{align*}
\phi_1 : & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 6 & 6 & 0 \\ 4 & 0 & 0 \end{pmatrix} \\
\phi_2 : & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 12 & 12 & 0 \\ 6 & 6 & 0 \end{pmatrix} \\
\phi_3 : & \begin{pmatrix} 0 & 0 & 0 \\ 8 & 8 & 0 \\ 8 & 8 & 0 \\ 2 & 2 & 0 \end{pmatrix} \\
\phi_4 : & \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \\ 10 & 10 & 0 \\ 6 & 4 & 0 \end{pmatrix} \\
\phi_5 : & \begin{pmatrix} 6 & 6 & 0 \\ 6 & 6 & 0 \\ 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix} \\
\phi_6 : & \begin{pmatrix} 6 & 6 & 0 \\ 6 & 6 & 0 \\ 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix} \\
\phi_7 : & \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \\ 10 & 10 & 0 \\ 6 & 6 & 0 \end{pmatrix} \\
\phi_8 : & \begin{pmatrix} 4 & 4 & 0 \\ 12 & 12 & 0 \\ 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix} \\
\phi_9 : & \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \\ 10 & 10 & 0 \\ 6 & 6 & 0 \end{pmatrix} \\
\phi_{10} : & \begin{pmatrix} 4 & 4 & 0 \\ 12 & 12 & 0 \\ 10 & 10 & 0 \\ 6 & 6 & 0 \end{pmatrix} \\
\phi_{11} : & \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \\ 10 & 10 & 0 \\ 6 & 6 & 0 \end{pmatrix} \\
\phi_{12} : & \begin{pmatrix} 2 & 2 & 0 \\ 8 & 8 & 0 \\ 8 & 8 & 0 \\ 2 & 2 & 0 \end{pmatrix} \\
\phi_{13} : & \begin{pmatrix} 4 & 4 & 0 \\ 12 & 12 & 0 \\ 10 & 10 & 0 \\ 6 & 6 & 0 \end{pmatrix} \\
\phi_{14} : & \begin{pmatrix} 4 & 4 & 0 \\ 12 & 12 & 0 \\ 10 & 10 & 0 \\ 6 & 6 & 0 \end{pmatrix} \\
\phi_{15} : & \begin{pmatrix} 1 & 1 & 0 \\ 1 & 3 & 2 \\ 4 & 4 & 0 \\ 4 & 0 & 0 \end{pmatrix} \\
\phi_{16} : & \begin{pmatrix} 1 & 1 & 0 \\ 1 & 3 & 2 \\ 4 & 4 & 0 \\ 4 & 0 & 0 \end{pmatrix} \\
\phi_{17} : & \begin{pmatrix} 1 & 1 & 0 \\ 1 & 3 & 2 \\ 4 & 4 & 0 \\ 4 & 0 & 0 \end{pmatrix} \\
\phi_{18} : & \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix}
\end{align*}
\endgroup
\[ \phi_{19} : \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & -6 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & 0 & 0 \end{pmatrix} \]

\[ \phi_{20} : \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & -2 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & -2 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & -2 & 0 \end{pmatrix} \]

\[ \phi_{21} : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4 & 2 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 2 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 2 & 0 \end{pmatrix} \begin{pmatrix} 10 & 10 & 0 \\ 10 & -10 & 0 \end{pmatrix} \]

\[ \phi_{22} : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4 & 2 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & -4 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & -4 & 0 \end{pmatrix} \begin{pmatrix} 10 & 10 & 0 \\ 6 & 2 & 0 \end{pmatrix} \]

\[ \phi_{23} : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 10 & 10 & 0 \end{pmatrix} \begin{pmatrix} 10 & 10 & 0 \\ 10 & -8 & 0 \end{pmatrix} \]

\[ \phi_{24} : \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 2 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 2 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \]

\[ \phi_{25} : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 6 & -2 & 0 \\ 4 & -2 & 0 \end{pmatrix} \begin{pmatrix} 10 & 10 & 0 \\ 10 & 10 & 0 \end{pmatrix} \]

\[ \phi_{26} : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 6 & -2 & 0 \\ 6 & -2 & 0 \end{pmatrix} \begin{pmatrix} 10 & 10 & 0 \\ 10 & 10 & 0 \end{pmatrix} \]

| φ_{19} | φ_{20} | φ_{21} | φ_{22} | φ_{23} | φ_{24} | φ_{25} | φ_{26} |
|---|---|---|---|---|---|---|---|
| \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & -6 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & 0 & 0 \end{pmatrix} | \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & -2 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & -2 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & -2 & 0 \end{pmatrix} | \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4 & 2 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 2 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 2 & 0 \end{pmatrix} \begin{pmatrix} 10 & 10 & 0 \\ 10 & -10 & 0 \end{pmatrix} | \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4 & 2 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & -4 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & -4 & 0 \end{pmatrix} \begin{pmatrix} 10 & 10 & 0 \\ 6 & 2 & 0 \end{pmatrix} | \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 10 & 10 & 0 \end{pmatrix} \begin{pmatrix} 10 & 10 & 0 \\ 10 & -8 & 0 \end{pmatrix} | \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 2 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 2 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} | \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 6 & -2 & 0 \\ 4 & -2 & 0 \end{pmatrix} \begin{pmatrix} 10 & 10 & 0 \\ 10 & 10 & 0 \end{pmatrix} | \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 6 & -2 & 0 \\ 6 & -2 & 0 \end{pmatrix} \begin{pmatrix} 10 & 10 & 0 \\ 10 & 10 & 0 \end{pmatrix} |

Table 3: The scalar components of the massless $E_6$ gauge singlet fields.

\[ q_1 : \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 8 & 8 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

\[ q_2 : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 8 & 8 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 10 & 10 & 0 \\ 10 & 10 & 0 \end{pmatrix} \]

\[ q_3 : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 8 & 8 & 0 \end{pmatrix} \]

\[ Q_1 : \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 8 & 8 & 0 \end{pmatrix} \]

\[ Q_2 : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 10 & 10 & 0 \\ 10 & 10 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 8 & 8 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

\[ Q_3 : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 8 & 8 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \]

Table 4: Quark and lepton fields for the case of Wilson line breaking with the twist generated by the (0,3,6,0) element.
3.1 Couplings in the Gepner model.

The crucial element in our determination of flat directions in the Gepner model is the identification of the discrete symmetries of the model. Thus we wish to determine the correlation functions of the form

\[ < V_{-1} V_{-1/2} V_{-1/2} \cdots V_0 >, \]  

(8)

where \( V_{-1(0)} \) is a vertex operator for a space-time scalar in the \((-1)((0))\) picture and \( V_{-1/2} \) is a vertex operator for a space-time fermion in the \((-1/2)\) picture. The choice of vertex operators in the \((0)\) picture for \( n > 3 \) is dictated by the need to have the correct ghost charge \((-2)\) for the non-vanishing correlator computed at the string tree level \([18]\). All the quantum numbers listed above corresponded to vertex operators for the space-time scalars in the \((-1)\) picture

\[ V_{-1} \sim 4 \bigotimes_{i=1}^4 \left( \frac{l_i}{\mathcal{I}_i} \frac{q_i}{\mathcal{Q}_i} \frac{s_i}{\mathcal{S}_i} \right). \]

(9)

As mentioned above, to obtain vertex operators for fermions in the \((-\frac{1}{2})\) picture we have to act with the supersymmetry charge which shifts the quantum numbers giving

\[ V_{-1/2} \sim 4 \bigotimes_{i=1}^4 \left( \frac{l_i}{\mathcal{I}_i} \frac{q_i + 1}{\mathcal{Q}_i} \frac{s_i + 1}{\mathcal{S}_i} \right). \]

(10)

The transformation from the \((-1)\) picture to the \((0)\) picture is carried out by one of the superpartners of the stress tensor \([19]\) and results in the fields with the following quantum numbers

\[ V_0 \sim \sum_{j=1}^4 \sum_{i=1}^{j-1} \left( \frac{l_i}{\mathcal{I}_i} \frac{q_i}{\mathcal{Q}_i} \frac{s_i}{\mathcal{S}_i} \right) \otimes \left( \frac{l_j}{\mathcal{I}_j} \frac{q_j}{\mathcal{Q}_j} \frac{s_j + 2}{\mathcal{S}_j} \right) \otimes \bigotimes_{i=j+1}^4 \left( \frac{l_i}{\mathcal{I}_i} \frac{q_i}{\mathcal{Q}_i} \frac{s_i}{\mathcal{S}_i} \right). \]

(11)

As discussed in detail in \([20]\), the allowed correlation functions of eq(8) are entirely specified by the symmetries present in the model. Of particular importance are the \( U(1) \) factors associated with each \( N = 2 \) superconformal factor. In order to discuss these conditions it is convenient to define the \( U(1) \) charges for all the subtheories

\[ \alpha_i^J = -\frac{q_i^J}{k_i + 2} + \frac{s_i^J}{2} \]

(12)

\[ \alpha_i^J = -\frac{q_i^J}{k_i + 2} + \frac{s_i^J}{2} \]

(13)

The index \( i \) labels the subtheories, while \( J \) refers to the different fields entering the coupling.

In the case of the SUSY sector, the condition is the same for all types of couplings and reflects the picture changing needed to provide the total ghost charge equal \(-2\). Taking into account the rules of picture changing mentioned earlier in this section, we obtain the following conservation law

\[ \sum_{j=1}^n \left( \alpha_i^J - \frac{2}{k_i + 2} + 1 \right) + \sum_{j=4}^n d_i^J = 0 \quad (\forall i). \]

(14)
In the last term, \( d_i^J \) can be equal 0 or 1 with the condition

\[
\sum_{i=1}^{4} \sum_{J=1}^{n} d_i^J = n - 3. \tag{15}
\]

Obviously, this condition exists only for \( n > 3 \) (it is only when we have more than three fields in the coupling that we have to change the picture to \( (0) \)).

In the case of the gauge sector, the conservation rule depends on the type of coupling we are discussing. As we mentioned above, the quantum numbers of the fields listed in the Tables correspond to the 10 representation of the \( SO(10) \) factor of \( E_6 \). This means that for example in the case of the coupling \( 27 \times 27 \times 27 \), in order to make a gauge invariant coupling we have to change two of the three fields from 10 to 16. This gives us

\[
\sum_{J=1}^{n} \left( \alpha_i^J - \frac{2}{k_i + 2} + 1 \right) = 0 \quad (\forall i). \tag{16}
\]

In the case of couplings of the type \( 1 \times 27 \times 27 \) there is no need to change any representations because the quantum numbers for the fields listed in the Tables generate the term \( 1 \times 10 \times 10 \) which is already gauge invariant. This leads to the simple conservation law

\[
\sum_{J=1}^{n} \alpha_i^J = 0 \quad (\forall i). \tag{17}
\]

The same is obviously true for the \( 1 \times 1 \times 1 \) couplings.

As we shall now discuss the constraints of eqs (14, 15, 16, 17) are quite restrictive and lead to absolutely flat directions of the type discussed in Section 2.

### 3.2 Flat directions

We start this section with a simple example of a flat direction involving \( E_6 \) non-singlet fields. In particular we consider the case that the \( N \) component of the fields \( l_7 \) and \( \bar{I}_4 \) acquires a non-zero vev.

We first establish that there are no terms allowed in the superpotential of the form \((l_7 \bar{I}_4)^m\). From Table 1 and 2 we see that these fields both have vanishing \( \alpha_1 \) charge. From eq (14) we see that for a non-vanishing correlator the sum of the \( \alpha_1 \) charges should be \( \frac{1}{3} \mod 3 \), i.e. the symmetry is an R-symmetry. Thus we see immediately that no terms are allowed of the form \((l_7 \bar{I}_4)^m\). This establishes \( l_7, \bar{I}_4 \) as a good candidate for an absolutely flat direction. However to complete the proof that this is the case we need to check that there are no non-vanishing F-terms of the type \((l_7 \bar{I}_X, l_X \bar{I}_4, \phi_X)(l_7 \bar{I}_4)^m\). This is in principle straightforward to do using the rules of eqs (14) to (17). In practice it is necessary to use algebraic computer programme to do the search and the results quoted below have been obtained using MAPLE. We find there is only one coupling of the dangerous type namely

\[
l_7 \bar{I}_4 \phi_{15}, \tag{18}\]

which apparently spoils the flatness of the \((l_7)_N, (\bar{I}_4)_N\) direction. However there are three ways this conclusion may be avoided.

13
• The first possibility applies if there are couplings of higher order of the type

\[ \frac{1}{M^{2(n-1)}} (l_7\bar{l}_4)^n \phi_{15} \]  

(19)

One can easily check that the symmetries allow terms with \( n = 3p + 1 \) giving the superpotential

\[ \phi_{15}(l_7\bar{l}_4) + \frac{1}{M^6}(l_7\bar{l}_4)^4 + \frac{1}{M^{12}}(l_7\bar{l}_4)^7 + ... \]  

(20)

We expect such terms to arise in the effective theory descending from the string with \( M \) given by the string scale. This means that although there is no absolute flat direction for \( l_7 \) and \( \bar{l}_4 \), there still exist a discrete zero of potential for non-zero value of \( <l_7\bar{l}_4> = O(M^2) \). This is interesting because it corresponds to a vacuum solution not continuously connected to the original Calabi Yau vacuum.

• The second way the \((l_7)_N, (\bar{l}_4)_N\) direction may be flat is through a cancellation of its contribution to \( F_{\phi_{15}} \) through other fields acquiring vevs. In particular one finds the following terms

\[ \phi_3\phi_4\phi_{15}, \]  

(21)

\[ \phi_1\phi_2\phi_{15}, \]  

(22)

\[ \phi_6\phi_5\phi_{15}, \]  

(23)

which we can use to give a non-zero VEV to \( F_{\phi_{15}} \). This can happen by adjusting VEVs of some of these singlet fields in such a way that the whole term multiplied by \( \phi_{15} \) in

\[ \phi_{15}(l_7\bar{l}_4 + \phi_3\phi_4 + \phi_1\phi_2 + \phi_6\phi_5) \]  

(24)

vanishes. We cannot use \( \phi_3 \) and \( \phi_4 \) because it would introduce a further non-zero F term through the allowed coupling \( l_7\bar{l}_3\phi_3 \). There is no such coupling for the fields \( \phi_1 \) and \( \phi_2 \) so we consider whether there are new non-zero F terms introduced in higher order if we allow them to acquire vevs. The dangerous couplings are of the type

\[ (l_7\bar{l}_4)^n \phi_1^m \phi_2^k, \]  

(25)

\[ (l_7\bar{l}_4)^n \phi_1^m \phi_2 \phi_x, \]  

(26)

\[ (l_7\bar{l}_4)^n \phi_1^m \phi_2^k l_7\bar{l}_x, \]  

(27)

\[ (l_7\bar{l}_4)^n \phi_1^m \phi_2^k l_x\bar{l}_4. \]  

(28)

Using MAPLE one can check that in fact the only allowed couplings belonging to this set are \((l_7\bar{l}_4)^n \phi_1^m \phi_2 \phi_{15}\), for some specific \( n \), \( m \) and \( k \). Including them the most general superpotential involving \( l_7, \bar{l}_4, \phi_1 \) and \( \phi_2 \) are

\[ \phi_{15}[l_7\bar{l}_4 + (l_7\bar{l}_4)^4 + (l_7\bar{l}_4)^7 + \]  

+ (higher order terms involving only \( l_7 \) and \( \bar{l}_4 \)) +

+ \( \phi_1\phi_2 \)

+ (higher order terms involving only \( \phi_1 \) and \( \phi_2 \)) +

+ \( l_7\bar{l}_4\phi_1\phi_2 + (l_7\bar{l}_4)^2\phi_1\phi_2 + (l_7\bar{l}_4)^3\phi_1^2\phi_2^2 + \)

+ (higher order terms involving only \( l_7, \bar{l}_4, \phi_1 \) and \( \phi_2 \)).
Clearly the vanishing of $F_{\phi_{15}}$ implies a relation between $<l_7\bar{t}_4>$ and $<\phi_1\phi_2>$, but leaves a flat direction corresponding to the magnitude of $<l_7\bar{t}_4>$. 

- Finally, the third option involves the moduli fields. The moduli, $m$, determine the strength of the coupling in eq(18) so we have the coupling 

$$\phi_{15}(\lambda(m)l_7\bar{t}_4 + \frac{\lambda'(m)}{M^6}(l_7\bar{t}_4)^4 + \frac{\lambda''(m)}{M^{12}}(l_7\bar{t}_4)^7 + ...)$$

Now $F_{\phi_{15}}$ can be made zero through the moduli fields acquiring vevs to make the term in brackets vanish. Note that because this coupling involves the field $\phi_{15}$ with vanishing vev this is the only condition that needs be satisfied to ensure $F$-flatness. It is clear that if the term had involved fields all of which acquire vevs this would not have been true and all the moduli would have been determined by the condition $F_{m_i} = 0$. We shall discuss this case later when we consider directions which are only approximately flat.

This example has conveniently illustrated all the possibilities for achieving flat directions. What we have shown is that there are equivalent solutions to the compactified string theory in which the symmetry is broken to $SO(10)$ through the $l_7$ and $\bar{t}_4$ vevs along the $N$ direction. Since these flat directions may have Planck scale vevs they are of a different character to those hitherto investigated with vevs at intermediate scales triggered by soft supersymmetry breaking mass terms. These new solutions are on an equal footing to the original compactifications with $E_6$ symmetry. Indeed what we have done is to demonstrate that there is a richer moduli space describing the string vacuum than just the string moduli discussed above. The disappointment the reader may feel at yet another contribution to the degeneracy of string vacua should be tempered by the fact that these string vacua may provide a more promising starting point for a phenomenologically viable string theory. We shall demonstrate this in the remainder of this paper investigating the prospects for using these new solutions to build a realistic three generation string theory.

4 Low Energy string symmetries

In this section we wish to explore the question whether we can use the techniques discussed above to find a viable low energy three generation theory with just the Standard Model gauge group at the compactification. The starting point is the identification of the low-energy symmetries that must be left unbroken until the electroweak breaking scale or the supersymmetry breaking scale. In addition to the $SU(3) \times SU(2) \times U(1)$ gauge symmetry there must be a symmetry capable of preventing rapid nucleon decay. The latter may be the $Z_2$ R-parity of the MSSM or it may be a larger discrete group. In addition there must be a further symmetry which prevents one, and only one\footnote{This condition is necessary if the success of the gauge unification predictions is to be maintained.}, pair of Higgs doublets from acquiring a large invariant mass. In the MSSM this symmetry is absent and an arbitrarily large $\mu$...
term is allowed, so technically the MSSM is unnatural. In our opinion such an unnatural tuning of a parameter is not acceptable in a string theory because any term allowed by string symmetries typically occur unsuppressed. Thus we require that the origin of the light Higgs be guaranteed by the string symmetries. Moreover, as there are many more than a single pair of Higgs doublets in the compactified string model, we have the additional problem of ensuring that only one pair remains light.

4.1 R-parity in the Calabi Yau 3 generation model.

As we noted above the starting point for our string construction is the three generation superconformal theory that is equivalent, in the large radius limit, to a three generation Calabi-Yau theory. The gauge group of this model can at most be broken to $SU(3)^3$ by Wilson lines. To break this group further $E_6$ non-singlet fields must acquire vevs. It is convenient to label the states of a 27 by its $SU(3)^3$ transformation properties. We have

$$27 = (1, 3, \bar{3}) \oplus (3, \bar{3}, 1) \oplus (\bar{3}, 1, 3)$$

where we take the first $SU(3)$ factor to be the colour group. Then the leptons and Higgs states belong to the $(1, 3, \bar{3})$ representation. As discussed below the “normal” assignment is given by

$$\begin{pmatrix} H_1 \\ E^+ \\ l \\ \nu_R \\ N^0 \end{pmatrix}$$

where $H_{1,2}$ are Higgs doublets, $l$ is a lepton doublet, $E^+$ is a charged lepton, $\nu_R$ is a right-handed neutrino component and $N$ is an $SO(10)$ singlet field (for the $SO(10) \otimes U(1)$ subgroup of $E_6$). If we are to construct a deformation of the model with just the Standard Model gauge group it is necessary for us to find flat directions which involve fields transforming $\nu_R$ and $N$ in eq(31). Giving these vevs breaks the gauge group to $SU(3) \times SU(2) \times U(1)$ as required. Our task is to find flat directions which generate this breaking but leave unbroken symmetries which prevent rapid nucleon decay and which keep just one Higgs field light. It turns out that these requirements largely determine the structure of the low-energy theory.

Consider the case that there is a $Z_2$ R-parity. The simplest possibility is to identify it with a discrete symmetry of the string compactification. However this turns out not to be consistent with the requirement of a single Higgs pair of fields and three quark and lepton generations. To see this we note that Higgs doublets are R-parity even and must come from R-parity even $(1, 3, \bar{3})$ and $(1, \bar{3}, 3)$ multiplets in the underlying $SU(3)^3$ theory. Our assumption is that the R-parity commutes with the gauge symmetry and thus all three doublets of an R-parity even multiplet of eq(31) must be Higgs doublets (not as shown in eq(31)). Thus we see that a $(1, 3, \bar{3})$ representation contains two Higgs doublets with weak hypercharge +1 and one Higgs doublet with weak hypercharge -1. Upon $(1, 3, \bar{3})$ and $(1, \bar{3}, 3)$ breaking some of the Higgs fields may acquire mass through the coupling of positive and negative hypercharge states. It is clear that to have the possibility that only one pair of Higgs fields with opposite hypercharge are left light we must have

$$N_{R^+} - N_{\bar{R}^+} = 0$$
where $N_{R^+, R^+}$ are the number of positive R-parity states in the $(1, 3, \bar{3})$ and $(1, \bar{3}, 3)$ representations respectively. Quark and leptons belong to R-parity odd states and in order for there to be just three generations we must have

$$N_{R^-} - N_{\bar{R}^-} = 3$$  \hspace{1cm} (33)

Note that the “three generation” condition of the underlying Calabi-Yau manifold, $N_{R^+} + N_{R^-} - N_{R^+} - N_{\bar{R}^-} = 3$, does not guarantee three quark and lepton generations; this requires a definition of quark and lepton which R-parity supplies. The only $Z_2$ symmetry in the model of Section 3 is the anticyclic permutation of the three $k=16$ factors of the underlying superconformal model. In this case only the fields $l_4 - l_5$, $l_8 - l_9$, and $T_5 - T_6$ are odd under the $Z_2$, giving $N_{R^+} = 7$, $N_{R^-} = 2$, $N_{\bar{R}^+} = 5$, and $N_{\bar{R}^-} = 1$. Clearly this does not satisfy eqs (32) and (33) so we cannot identify this $Z_2$ symmetry with R-parity.

Since there are several $Z_3$ symmetries it may be possible to use one to generate a baryon parity [21]. We will investigate this possibility elsewhere but here we wish to point out a novel possibility which utilises an approximate $Z_2$ symmetry to implement an approximate R-parity. For example in Table 5 one sees that $l_8$ and $l_9$ couple only in pairs meaning that these couplings are invariant under an effective $Z_2$ symmetry under which $l_8, l_9$ are odd. The point is that the $Z_3$ symmetries give rise to an effective $Z_2$ symmetry in the trilinear couplings which may be used to define the R-parity.

With this motivation we consider whether any of these approximate symmetries are capable of generating such an R symmetry. Since the discrete gauge group factor remains the same the constraints of eqs (32) and (33) still apply. It is straightforward to enumerate

\begin{verbatim}
Table 5: Tri-linear couplings for the matter fields.
\end{verbatim}
all possible $Z_2$ symmetries consistent with the terms appearing in $\mathfrak{g}$. For example $l_3$ is necessarily even while $l_{8,9}$ may both be odd or both be even etc. However in all cases the number of $Z_2$ odd states is even in both the $(1,3,\overline{3})$ and $(1,\overline{3},3)$ sectors. Thus eq(33) cannot be satisfied and even the approximate symmetries cannot be identified with R-parity.

Luckily there is another way to implement this approximate R-parity. The problem we have encountered follows because the R-symmetry has been assumed to commute with $SU(3)^3$ leading immediately to eqs(32) and (33). If, however, we identify R-parity with $CU$ where $C$ is a $Z_2$ discrete symmetry and $U$ is the $Z_2$ discrete gauge group factor given by the $SU(2)_L \otimes SU(2)_R$ diagonal group elements $(-1, -1, 1)_L \otimes (-1, -1, 1)_R$ the counting changes. In this case, cf eq(31), a $C^+$ $(1,3,\overline{3})$ state contains two R-parity even doublets which are to be identified with Higgs doublets carrying opposite weak hypercharge together with a R-parity odd doublet which is to be identified with a lepton doublet. The identifications are reversed for a $C^-$ state. With similar identifications for the $(1,\overline{3},3)$ states we find the conditions of eqs(32) and (33) are replaced by

$$N_{C^+} - N_{\overline{C}^+} = 3$$
$$N_{C^-} - N_{\overline{C}^-} = 0$$

Consider first the possibility we identify $C$ with the $Z_2$ given by the anticyclic permutation of the three $k=1$ factors of the underlying superconformal model. This corresponds to $N_{C^+} = 7, N_{\overline{C}^+} = 5, N_{C^-} = 2, N_{\overline{C}^-} = 1$. In this case the constraints of eqs(32) and (33) are not satisfied. Of course one may go to higher dimension discrete symmetries and this indeed is a possibility given that there are several $Z_3$ symmetries of the theory. We shall investigate this possibility in another paper but here we wish to explore the novel possibility discussed above which utilises an approximate $Z_2$ symmetry to implement an approximate R-parity. In this case it is possible to satisfy these equations through the identification of $C$ with the approximate $Z_2$ symmetry under which $l_{8,9}$ and $l_{1,3}$ are odd while all other states
are even. For it \( N_{C+} = 7 \), \( N_{C-} = 4 \), \( N_{\bar{C}+} = 2 \), \( N_{\bar{C}-} = 2 \), satisfying eqs(34). The symmetry clearly leaves the terms of Table \( \text{3} \) invariant. Provided the flat direction has non-zero vevs for R-parity even states only R-parity will be preserved. This happens if \( \langle N >, <\bar{N} > \) belong to \( C \) even states and \( \langle \nu_R >, <\bar{\nu}_R > \) belong to \( C \) odd states only. However this, by itself, is not sufficient to guarantee that R-symmetry is preserved because higher dimension terms may become important along these flat directions and these may not respect the approximate \( Z_2 \) symmetries. Thus it is important to identify the origin of this approximate symmetry so that one may be sure that it is not broken when fields develop vevs along flat directions. From Tables \( \text{1} \) and \( \text{2} \) we see that the states \( l_{8,9} \) and \( \bar{l}_{1,3} \) are distinguished by being the only states with charge \( q_i \), \( \bar{q}_i = 2 \) \( \bar{q}_i \) \( \text{Mod} \) 6 , \( i = 2, 3, 4 \). The approximate \( Z_2 \) is due to the underlying \( Z_3 \) symmetries associated with these charges. The flat direction we explore in detail here preserves all \( SU(3)^3 \) singlet combinations carrying zero \( \bar{q}_i \) charge \( \text{Mod} \) 6, so it is this \( Z_3 \) (called \( Z_{3}^\text{G} \) henceforth) that maintains the approximate \( Z_2 \). The trilinear couplings \( l^3 \) allowed by this symmetry have \( \bar{q}_i \) charge 0, 0.4, 0.2, and 4.4.2. This implies a potential conflict with the need to obtain a \( Z_2 \) symmetry because the assignment assumed above assigned states carrying charge 0 and 4 positive \( Z_2 \) charge while states carrying charge 2 had negative \( Z_2 \) charge. This assignment is inconsistent with a coupling 4.4.2. In Table \( \text{3} \) this troublesome coupling is absent but one must check whether it is generated at an unacceptable level when large vevs develop along flat directions. For a coupling to be allowed

\[
\sum_j \bar{q}_i^j = 16, i = 2, 3, 4 \tag{35}
\]

where \( \bar{q}_i^j \) is the \( \bar{q}_i \) charge for the jth field. From Table \( \text{1} \) we see that the trilinear term \( l_3 l_6 l_8 \) can satisfy this condition. It does not appear in Table \( \text{5} \) because the other charge constraints corresponding to other symmetries are not satisfied. However if all other discrete symmetries are broken at some stage this term will be allowed. Does it lead to rapid nucleon decay? In fact it does not because the superfield \( l_3 \) is a permutation singlet and hence does not contain quark components. As a result the R-parity violating coupling is only allowed in the lepton sector generating lepton number violation but not baryon number violation.

The only 4.4.2 coupling which does involve quarks is \( l_6 l_6 l_{8,9} \). However this coupling is forbidden by the constraint of eq(35). The simplest way to see this is to note that \( \sum_{i,j} \bar{q}_i^j = 54 \) instead of 48 required by eq(35). We must also check that this coupling does not appear together with the \( SU(3)^3 \) singlet combinations of fields acquiring large vevs so that it does not appear when the theory is deformed along a flat direction. This will be the case for the phenomenologically interesting direction discussed below, but due to a combination of the constraints of eq(35).

In conclusion we have investigated the possibility of preventing rapid nucleon decay via discrete symmetries in the three generation Gepner model. We have found that it is possible to ensure stability to the required order through a new, approximate, \( Z_2 \) symmetry involving the quark sector. This symmetry allows dimension 4 lepton number violating terms and is thus different from the normal R-parity of the MSSM. In fact it is a version of “Baryon” parity [21]. Of course suppression of the dimension 4 terms leading to nucleon decay may not be sufficient because dimension 5 terms can still occur at an unacceptable rate. These terms require a detailed model and we will discuss them once we have filled in some of these
4.2 Light Higgs doublets

While our discussion of R-parity allowed for the possibility that there should be left light just one pair of Higgs doublets with opposite weak hypercharge, it did not guarantee that this should be the case. However it is important that a symmetry should ensure the lightness of the Higgs states because the Higgs doublets can acquire an $SU(3) \otimes SU(2) \otimes U(1)$ invariant mass and the symmetry is needed to prevent this happening, without fine tuning of parameters, when the underlying $SU(3)^3$ symmetry is broken. To address this question we will try to identify an effective $Z_2$ symmetry involving the chiral Higgs supermultiplets which will generate a chirality ensuring that one Higgs pair can not acquire a mass. We denote by $N_{H_{1,2}^\pm}$, and $N_{\overline{H}_{1,2}^\pm}$ the number of Higgs fields with $Z_2$ “chirality” $\pm 1$ coming from the $(1,3,\overline{3})$ sector and the $(1,\overline{3},3)$ sector respectively. The definition of “chirality” here requires that masses involve a coupling of a positive to a negative chirality state. The requirement that there should be just one pair of Higgs doublets forbidden by this chirality from acquiring a mass is then given by eq(34) together with

$$N_{H_{1,2}^+} - N_{H_{1,2}^-} + N_{\overline{H}_{1,2}^+} - N_{\overline{H}_{1,2}^-} = 1 \quad (36)$$

It is now straightforward to check whether there is a symmetry in the compactified string model capable of satisfying this constraint. Consider the allowed mass terms consistent with the $Z_3^G$ symmetry which ensures nucleon stability. A Higgs mass term can be generated by $l^3$ or $\overline{t}$ couplings or by a $l\overline{t}$ coupling. As discussed above The $l^3$ couplings allowed by the $Z_3^G$ symmetry have $\overline{t}$, charge 0.0.4, 0.2.2, and 4.4.2. The discussion is made more complicated because, as mentioned above, the 4.4.2 term violates lepton number and so mixes Higgs and leptons. Let us first ignore this term, in which case there is a well defined separation of Higgs and leptons as discussed above eq(34). Consider first the mass terms generated by the $<N>$ vev. Such a $l^3$ term will generate a mass term for an $H_{1,2}$ pair coupling $\overline{t}_i$ charge 0 to 4 states and $\overline{q}_i$ charge 2 to 2 states. Thus a consistent assignment of the $Z_2^G$ “chirality” is to make all the 0 states positive and all the 4 states negative. The 0.2 mass terms generated by a $<\nu_R>$ vev in the 2 direction is consistent with this $Z_2^G$ “chirality” if the Higgs state in the 2 multiplet is odd. The mass term arising from the 0.2 coupling with a $<N>$ vev in the 0 multiplet involves lepton states only and plays a role in the determination of the lepton parities as discussed below. We turn now to the mass terms generated by $\phi \overline{t}$ couplings. Note that the mass terms generated by a $\phi$ vev involve 0.0, 2.(-2) and 4.(-4) $\overline{q}_i$ charges. Thus in the $\overline{t}$ sector we must assign all 0 Higgs states negative and all the -4 and -2 states positive $Z_2^G$ parity. Finally consideration of the $\overline{t}^l$ couplings shows that they are consistent with these assignments. The resulting assignments for the “Higgs” states are shown in Table 8.

For the leptons the assignment of parity follows using the same general argument and the results are also shown in Table 8. At this point we may include the coupling between the “Higgs” states and the “lepton” states arising from the 2.4.4 with a $<\nu_R>$ vev in the 2 direction. The associated mass terms are consistent with the assignments of 8 (in fact the
relative sign between the leptons and quarks is determined by this coupling to be that given in $\mathbb{Z}_2^G$. It is now straightforward to determine the number of states which must be left light due to chirality. We have $N_{H_1+,+} = N_0 + N_7 + N_{\overline{T}} = 8$, $N_{H_2,-} = N_{\overline{T}} + N_4 + N_2 = 7$ and so there are $N_{H_1+,+} - N_{H_2,-} = 1$ Higgs doublets $H_1$ left light. Similarly we find $N_{H_2,+} - N_{H_1,-} = 1$ Higgs doublets $H_2$ left light. Similarly we find $N_{L_+} = N_0 + N_4 + N_2 + N_{\overline{T}} = 11$, $N_{\overline{T},-} = N_{\overline{T}} + N_7 + N_2 = 8$ so the number of lepton doublets left light is $N_{L_+} - N_{\overline{T},-} = 3$ as desired. One may check that the $4.4.2$ couplings which mix Higgs and leptons do not disturb this counting for using the chiralities of Table $8$ which were determined including this coupling we find $N_{H_2,+} + N_{L_+} - N_{H_1,-} - N_{\overline{T},-} = 4$ and $N_{H_1,+} - N_{H_2,-} = 1$ corresponding to three “lepton” and one “Higgs” doublet which mix and one Higgs doublet of the opposite hypercharge. Remarkably this demonstrates that the same symmetry that is responsible for stabilising the nucleon also ensures three generations of leptons and one light pair of Higgs states remains light.

5 A Three generation $SU(3)\otimes SU(2)\otimes U(1)$ Compactified String Theory

Having identified the string symmetries we need to remain unbroken to the supersymmetry breaking scale we are now in a position to determine whether we can obtain a compactified string theory with these symmetries by deforming the three generation Gepner model along absolutely flat directions. The breaking along the $N$ direction should be $C$ even to implement the approximate R-parity discussed in Section $4.1$. They should also be in the even $\mathbb{Z}_2^G$ parity sector to maintain a light Higgs state as discussed in Section $4.2$. Further, as discussed in Section $2$, absolutely flat directions require a discrete R-symmetry. Consider the discrete symmetry associated with the charge $\alpha_1$ as defined in eq(12). Terms allowed by this symmetry in the superpotential must have charge $+1$ and so there are no terms involving charge 0 fields on their own. As a result (cf. Section $2$), these are likely to be absolutely flat directions. We thus consider the $C$ even, $\mathbb{Z}_2^G$ parity even, $\alpha_1$ neutral states $l_{4,5,7}$ and $\overline{t}_{4,5,6}$ as candidate states to acquire non-vanishing vevs along the $N$ direction. We concentrate here on the possibility only $l_7$ and $\overline{t}_4$ acquire vevs in the $N$ direction and postpone a discussion of the other possibilities to a future publication. Turning to the breaking along the $\nu_R$ direction we

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
$O$ : & $H_1^{+} H_1^{+} l_1^{+} E_{+}^+ \overline{\nu}_{R_+} N_1^{+}$ \\
$H_2^{-} l_1^{-} H_2^{-} E_{-}^- \overline{\nu}_{R_-} N_2^{-}$ \\
$H_1^{0} H_2^{0} E_{0}^0$ \\
\hline
$\overline{O}$ : & $H_-^{+} H_-^{+} l_1^{-} E_{-}^+ \overline{\nu}_{R_-} N_1^{-}$ \\
$H_2^{-} l_1^{-} H_2^{-} E_{-}^- \overline{\nu}_{R_-} N_2^{-}$ \\
$H_1^{0} H_2^{0} E_{0}^0$ \\
\hline
\end{tabular}
\caption{$\mathbb{Z}_2^G$ chiralities of the Higgs and lepton states. Here $H, H$ have the quantum numbers of the Higgs doublets $H_{1,2}$ with opposite weak hypercharge. The Higgs states $\overline{H}, H$ can mix with the lepton states $\overline{t}, l$.}
\end{table}
follow the logic just discussed and conclude this should be along a $C$ odd, $Z_2^G$ parity odd, $\alpha_1$ neutral direction. In this case there is no choice and it must in the $l_8$, $\bar{l}_1$ direction. However this direction is not D-flat due to the charge, $\pi_1$, carried by these fields. To obtain a flat direction we allow $l_2$ to acquire a vev along the $N$ direction. Since it has the opposite sign of charge under the $\overline{U}_4$ its vev will adjust to cancel this D-term.

It is straightforward to check, following the same discussion as in Section 3.2, that the combination of fields $l_8|_\nu_R$, $\tilde{t}_1|_\nu_R$, $l_7|_N$, $l_2|_N$ and $\bar{t}_4|_N$ is indeed flat to all orders. The only gauge invariant combination of fields has the form $(l_2|_\nu_R t_8|_\overline{T})^a (l_7|_T)^{m}$. This has the charge structure $(1, 16, 4, 10)^n(0, 12, 12, 12)^m$ where we have denoted the charges $(3\alpha_1, 18\alpha_1, i = 2, 3, 4)$ and we must allow symmetric permutations in the last three charges. Clearly eq(14) requires $n = 3p + 1$ for some integer $p$. Inserting this and applying eq(14) again gives

$$
16a + 4b + 10c + 12m = 18r + 16
$$

$$
16b + 4c + 10a + 12m = 18s + 16
$$

$$
16c + 4a + 10b + 12m = 18t + 16
$$

$$
a + b + c = 3p + 1
$$

where $a, b, c, m, r, s, t, p \in Z$. The appearance of the separate terms involving $a, b, c$ follows from the possibility of making cyclic permutations of charges. It is straightforward to check that these equations have no solution to the term $(l_2|_\nu_R t_8|_\overline{T})^a (l_7|_T)^{m}$ is not allowed. In this case the R-symmetry guaranteeing its absence arises as a combination of the discrete symmetries and not just the $\alpha_1$ charge as before. This provides an example of a flat direction breaking $E_6$ to $SU(5)$ just as is desired. Allowing for Wilson line breaking the gauge group can be further broken to just the $SU(3) \otimes SU(2) \otimes U(1)$ of the Standard Model.

5.1 The spectrum at the compactification scale

It is straightforward now to determine the spectrum in the model allowing for the string scale vevs along the flat direction just discussed and the Wilson line breaking that follows if they are associated with the freely acting group of cyclic permutations that is modded out when forming the 3 generation model. After Wilson line breaking the leptons belong to the permutation symmetric states, the left-handed quark states belong to the states transforming as $e^{i2\pi/3}$, and the right-handed quark states belong to the states transforming as $e^{i4\pi/3}$ under cyclic permutations. In order to avoid a confusing profusion of indices we will not make explicit the permutation symmetry in the discussion below.

The Higgs doublets left light are the $H_{1,2}$ pairs of doublets in the $C$-even states $l_{1,4,5}$ the $H_3$ doublets in the $C$-odd states $l_{8,9}$ and the $\overline{H}_3$ doublets in the $C$-odd states $\overline{l}_{1,3}$. The remaining Higgs doublets acquire mass by their coupling to the $N, \overline{N}$ vevs via $27^3$ or $\overline{27}^3$ or $27.\overline{27}.1$ couplings.

The lepton doublets left light are in the $C-$even states $l_{1,2,3,4,5,6}$ and $\overline{l}_{2,5,6}$.

Finally there are $C$-even $SU(2)_L$ and $SU(2)_R$ quark doublets left light in the multiplets $l_{1,2,4,5,6}$ and $\overline{l}_{5,6}$. In addition there are left light $C-$odd $SU(2)_L$ and $SU(2)_R$ quark doublets in $l_9$ and $\overline{l}_3$. There are also $SU(2)_L$ and $SU(2)_R$ quark singlets left light in the $C$-even multiplets $l_{1,4,5}$. 

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It is possible to give vevs to further fields without disturbing the flat direction. For example the field $\phi_{16}$ may acquire a vev giving a mass to the quark and lepton doublets in the multiplets $l_5, \overline{7}_6$. It is clear that a complete discussion of the various possibilities require a careful analysis of the many possible flat directions and this is beyond the scope of the present paper. Here we wish to demonstrate that the methods developed above offer a rich building kit for generating phenomenologically interesting models. To complete this programme we take the model thus developed, including the $\phi_{16}$ vev, and show that symmetry breaking below the compactification scale can lead to a viable model.

5.2 Intermediate scale breaking

As we have seen the breaking at the compactification scale leaves a model with the required $SU(3) \otimes SU(2) \otimes U(1)$ gauge group but with many additional states beyond the MSSM spectrum lying in vectorlike representations with respect to $SU(3) \otimes SU(2) \otimes U(1)$. It turns out it is not possible to give all these additional states a mass through vevs developing along absolutely flat directions. However it is known that radiative corrections can (and in fact often do) drive scalar masses negative triggering further breaking. In general this will not occur along absolutely flat directions so the breaking typically occurs at scales beneath the compactification scale but far above the electroweak scale. Such breaking can readily give mass to the additional states as we now demonstrate. Let us consider whether the flat direction discussed above can be extended to include the scalar fields $\phi_8, \phi_{11}, \phi_{16}$. In this case one may find a superpotential term allowed by all the symmetries of the form

$$\lambda(m)(l_2|^T|\phi_8\phi_{16})^2 l_7|^T|/M^\nu.$$ As discussed in Section 4 the appearance of this term means the theory is not F-flat. However large vevs may still develop along the $\phi_8, \phi_{16}$ directions if the sum of the soft supersymmetry breaking masses squared for these fields, $m_{\phi_8}^2 + m_{\phi_{16}}^2$ is negative. Moreover radiative corrections involving the Yukawa couplings of these fields is very likely to drive these mass squared negative so breaking along these field directions is quite reasonable. Assuming this does indeed happen it is straightforward to determine the scale of this breaking. Given string scale vevs for the fields $l_8|^R|, \overline{1}_7|^R|, l_7|^N|, l_2|^N$ and $\overline{\nu}_1|^N|$ develop along the absolutely flat direction as discussed in Section 5 the largest F-term are the ones which reduce the power of $\phi_8, \phi_{16}$ since these acquire vevs below the Planck scale, i.e. $F_{\phi_8, \phi_{16}}$. However these terms may be made much smaller by adjusting the moduli field vevs, $< m >$ (it is at the stage of intermediate scale breaking that we expect the string moduli vevs to be fixed). As a result the F-terms may all be made of the same order as $F_m$ or $F_\ell, \ell, \ell$. These are of order $(\phi_8\phi_{16})^2/M^2$ leading to the potential

$$V = m_{\phi_{16}}^2 \phi_{16}^2 + m_{\phi_8}^2 \phi_8^2 + O((\phi_8\phi_{16})^4/M^4)$$

Minimisation of this potential for $m_{\phi_{16}}^2 + m_{\phi_8}^2$ negative leads to the vevs $\phi_{16} \approx \phi_8 = O((m_{\phi_{16}}^2 + m_{\phi_8}^2)M^4)^{1/6} \approx 10^{13} \text{GeV}$ where we have used a supersymmetry breaking scale of $O(1 \text{TeV})$. In a similar manner we may deduce that $\phi_{11}$ may acquire a vev of the same order. Allowing for such breaking below the compactification scale we find that all the masses allowed by the residual symmetries of the model are generated. In particular all vectorlike states with respect to these residual symmetries acquire intermediate scale masses. From Table 3 we see that the fields $\phi_8, \phi_{11}, \phi_{16}$ all have zero $\overline{\nu}_{2,3,4}$ charge Mod 6. This means they may acquire vevs
without breaking the $Z_3^G$ symmetry of Section 4.2 and 4.1. As a result we are guaranteed to have three generations of leptons and one pair of Higgs doublets left light after intermediate scale breaking.

5.3 Nucleon decay

After intermediate scale breaking we arrive at a low energy supersymmetric theory with just the Standard Model gauge group and with the MSSM minimal matter content. However the theory is not identical to that of the MSSM because, as discussed above, there is a Baryon parity stabilising the nucleon which allows lepton-number-violating dimension 4 terms in the Lagrangian. At this stage we are in a position to discuss the dimension 5 contributions to nucleon decay. There are two dimension 5 operators, namely $QQQl|_F$ and $Q^c_1Q^c_1Q^c_1E|_F$. The potential problem with dimension 5 operators follows because, although suppressed by the inverse of a large mass, the suppression of them is typically inadequate by itself to increase the nucleon lifetime to be consistent with experimental bounds. However the second operator turns out to have a very strong suppression due to mixing angles and the suppression factor that occurs when dressing the operator to change the sfermions to fermions. Thus we concentrate here on the former operator. The first point to note is that these operators are absent before vevs develop along flat directions due to the underlying gauge symmetry because there is no $27.27.27.27$ coupling. However once we allow large vevs for fields in the $27$ representations these terms can arise through tree level graphs giving the higher dimensional couplings $\frac{1}{M^2}27.27.27.27. <27| <27^\dagger> |_F$. The magnitude of these terms depends on the effective mass scale, $M_{eff}^{-1} = \frac{27}{M^2} <27| <27^\dagger> |_F$. The necessary suppression factor is much smaller. It is straightforward to check that our Baryon parity does not by itself forbid such terms. For example for the case in which we mod out by a phase twist the couplings of Table 5 generate the term $Q_1Q_2Q_3l_2 <l_{2N}^\dagger>$ through the exchange of the third component of $Q_2$ which acquires a mass through $<l_{2N}>$. Thus in this case $M^2 \approx | <l_{2N}^\dagger> |^2$ so the dimension 5 operator $Q_1Q_2Q_3l_2$ is suppressed by $1/ <l_{2N}>$. Allowing for the (s)quark mixing angles going from current to light quark mass eigenstates gives a factor of approximately $(\sin \theta_c)^2$ (assuming mixing angles in the squark sector are comparable to the quark sector) leaving a further suppression of $10^{-4}$ still needed. Here we consider two effects that may be responsible for the remaining suppression.

5.3.1 Quark and lepton mixing angle suppression

One possible source of suppression is that there is a small mixing angle associated with the lepton doublet $l_2$. As we have discussed the lepton sector is very rich so the determination
of the mixing angles is quite involved. In particular, as discussed in more detail in the next Section, the lepton doublets may be mixtures of the lepton doublets in the fields $l_{1,2,3,4,5,6,7,8,9}$ and $l_{1,3}$ and also of the “Higgs” doublet in $\tilde{7}_2$. In the case of the operator $Q_1Q_2Q_3l_2$ there is a reason why we expect the mixing angle to be very small, much less than the minimum required. This follows because of the coupling (cf Table 5) $l_2\nu_R$. The lepton doublet in $l_2$ acquires a large mass from the term $l_2l_9 < l_8|\nu_R >$. As a result the component of $l_2$ in the light lepton doublets may be extremely small, the mixing angle being suppressed by the factor $1/ < l_8|\nu_R >$. This can readily suppress the contribution to nucleon decay of the dimension 5 operator $Q_1Q_2Q_3l_2$ below the experimental limit even for values of $< l_{2N} >$ much less than the Planck scale. Of course one must look at all dimension 5 operators. In the example just considered one may see from Table 5 that there are potentially large contributions involving the fields $l_{1,2,3,4,5,6}$. Thus we obtain suppression of dimension five operators only if the light leptons are dominantly in the $l_{7,8,9}$ and $l_{1,3}$ and $l_{2,4}H$ directions. Whether this happens depends sensitively on the relative magnitude of the large vevs and on the intermediate scale breaking and we will not consider this further here.

5.3.2 R-parity suppression

Our discussion of the magnitude of the coefficient of the operator $Q_1Q_2Q_3l_2$ assumed that we could ignore the effect of Kaluza Klein and string excitations. This is not true when we have large vevs developing along flat directions because the states light in the absence of such vevs may acquire mass by mixing with the massive excitations. For example, there may be a coupling $Q_2X_q\nu_R$ where $X_q$ is a Kaluza Klein or string state with the gauge quantum numbers of $q$. Since $X_q$ must have a term in the effective Lagrangian $M_XX_q\tilde{X}_Q$ the net effect of the large vev, $< l_8|\nu_R >$, is to generate a mass for the component proportional to $(< l_8|\nu_R > Q_2|3 + M_XX_Q|2)$. If $< l_8|\nu_R >$ is larger than $M_X$ this will be principally along the $Q_2$ direction showing this mixing term is important when determining the mass of the “light” states. One may now see that in the limit of large $< l_8|\nu_R >$ there is a suppression of dimension five contributions to nucleon decay due to the effective $R$-parity discussed above. This follows because the quark and lepton states are $C-$even while $l_8$ is $C-$odd. Thus one sees that the term $\frac{1}{M^2}27.27.27.27. < 27^\dagger > |_F$ responsible for nucleon decay involves only $C-$even states in the numerator. For the particular example discussed above the $< 27^\dagger >$ vev is the $C-$even vev $< l_2^\dagger >$. However, as just discussed, the mass, $M$, associated with this term is that of the third component of $Q_2$ and is given by $M \approx < l_8|\nu_R >$. As a result the net contribution has the effective mass scale given by $M_{e^f}^{-1} = \frac{< 27^\dagger >}{M^2} \approx \left(\frac{< l_2^\dagger >}{< l_8|\nu_R >}\right)^2 \frac{1}{< l_{2N}^\dagger >}$. We see that relative to our previous estimate there is a suppression factor $\left(\frac{< l_2^\dagger >}{< l_8|\nu_R >}\right)^2$ which, for large $< l_8|\nu_R >$, can explain the needed suppression. Of course one must consider too the contribution associated with the “light” component proportional to $(M_XQ_2|3 - < l_8|\nu_R > X_Q|2)$ which is orthogonal to the heavy state proportional to $(< l_8|\nu_R > Q_2|3 + M_XX_Q|2)$. It is straightforward to see that this acquires mass of $O(< l_{2N} >)$ and could apparently give a larger $M_{e^f}^{-1}$. However, for $M_X \approx < l_2^\dagger >$ it gives a contribution of the same order as the one just discussed because the coupling to the quark and lepton states is suppressed by the square of a mixing angle of $O(\frac{M_X}{< l_8|\nu_R >})$ in the large $< l_8|\nu_R >$ limit. Thus we see that the
residual approximate $R$–parity of the theory offers the possibility of an elegant explanation of the suppression of dimension five contributions to nucleon decay. We will return to a more complete discussion of this possibility elsewhere.

6 Quark and Lepton masses

We conclude with a discussion of the form of the quark and lepton masses that result in the theory just discussed with a residual baryon parity. We start with a discussion of the lepton mass matrix. The structure of the mass matrix is constrained by the residual $Z_3^G$ symmetry. This means the left-handed charge conjugate leptons ($SU(2)$ singlets) are mixtures of the positive chirality states with $Z_3^G$ charges 0, 4. In fact there is a separate conservation of chirality for the two charges so from Table 1 we see there are left light two states with charge 0 and one state with charge 4. The left-handed lepton doublets have a similar structure but in this case the terms 0, 2, 0, 2, 0, 4 and 4, 2 induce mixing of the “lepton” states 4, 0, 4 and the “Higgs” states $\mathbf{4}_H$ and a mixing of the lepton state 4, 0 with the lepton state in 2, 4, 0.

The light Higgs are the positive chirality states of Table 1, a mixture of the states 0, 0, 4, and 4, 4. The resulting mass matrix has the form

$$
egin{pmatrix}
0 & 0 & <0_H>
\end{pmatrix}
\begin{pmatrix}
0 & 0 & <0_H>
<0_H> & 0 & <0_H>
<0_H> & <0_H> & 0
\end{pmatrix}
\begin{pmatrix}
0_l + 2_l + \mathbf{4}_l
0_l + 2_l + \mathbf{4}_l
4_l + \mathbf{4}_H
\end{pmatrix}
$$

where we have suppressed the Yukawa couplings and the mixing factors determining the light states composition. Note that the L-R symmetry is broken by the mixing in the $l$ doublet sector. This mass matrix has two non-zero eigenvalues. Their values depend on the precise nature of the intermediate scale breaking and on the Yukawa couplings in both the low-energy sector and the sector involving compactification scale masses. We will explore this for a specific intermediate breaking scheme elsewhere. Here we consider the more pressing problem whether the low-energy symmetries allow for an electron mass. The masslessness of this state at tree level is ensured by the $Z_3^G$ symmetry. At radiative order mass is generated by the term $0, 4, 0|F$ but SUSY protection means that this is generated by SUSY breaking through the $D$–term $0, 4, 0|F = 0, 4, 0|F$ leading to the suppression factor of $O(h^2 m_4^2 M^2)$ where $h$ is the appropriate combination of the various Yukawa couplings involved. This is too small to be identified with the ratio $m_e/m_\tau$ and so this radiative term is not able to generate an acceptable electron mass. However the $Z_3^G$ symmetry may be broken to give an electron mass without losing the $Z_3^G$ protection for the light Higgs states or nucleon stability. This is because we may induce mixing in the light lepton states via the couplings $0_{E,l}, \phi_{E,l}, <\phi >$ and $4_{E,l}, \phi_{E,l}, <\phi >$ where we have allowed a Standard Model singlet field, $\phi$, with $Z_3^G$ charge $-4$ to acquire a vev. In this case the mass matrix becomes

$$
\begin{pmatrix}
0_E + 4_E & 0_E + 4_E & 4_E + 0_E
\end{pmatrix}
\begin{pmatrix}
<0_H> & <0_H> & <0_H>
<0_H> & <0_H> & <0_H>
<0_H> & <0_H> & <0_H>
\end{pmatrix}
$$
\[
0_1 + 2_1 + \frac{3}{2}_1 + 4_1 + \frac{5}{2}_H \\
0_1 - 2_1 + \frac{3}{2}_1 + 4_1 + \frac{5}{2}_H \\
4_1 - \frac{1}{2}_H + 0_1 + 2_1 + \frac{3}{2}_1
\]  
(40)

where

\[\eta = \frac{< \phi_{-4} > < \phi_0 >}{< \phi_{-4} >^2 + < \phi_0 >^2} \]  
(41)

Clearly the electron now acquires a mass and its smallness may be explained if \(\frac{< \phi_{-4} >}{< \phi_0 >}\) is small. This provides an illustration how the string symmetries can generate an hierarchical structure in the lepton masses. In this case the same symmetry responsible for light Higgs states and for nucleon stability is responsible for the electron mass hierarchy! One sees that a detailed prediction for the lepton masses requires determination of a complicated mixing pattern amongst the leptons and the Higgs, but this is characteristic of all attempts to explain the fermion mass structure through Froggatt Nielsen mixing [23].

Of course it is necessary to check that the vev \(< \phi_{-4} >\) does not lead to large masses for the light Higgs doublets. To see that this is indeed the case note that the constraints of holomorphicity of the superpotential require that the only mass terms in the Higgs sector generated by \(< \phi_{-4} >\) come from the terms \(0_4 H, < \phi_{-4} >\) and \(0_0, < \phi_{-4} >\). This only involves odd \(Z_2^G\) chirality fields, all of which acquire large compactification scale masses by coupling to even \(Z_2^G\) chirality fields. Adding a “Majorana” type mass to the odd chirality states only does not affect the light even chirality sector because there is no mass mixing of these states with the the odd chirality fields. Of course at radiative order the light even chirality sector will mix with the heavy sector but, just as in the electron mass case discussed above, the resultant mass generated is negligibly small. Similarly it is straightforward to verify that the new terms do not spoil the approximate \(R\)–parity needed to stabilise the nucleon. The reason is the same; the new couplings involve the fields descending from the \(27\) and these are all odd parity quark fields. The light quarks have even parity and do not couple to these new couplings.

We postpone to another publication a detailed discussion of the form of the neutrino mass structure. The Dirac mass matrix has a similar form to the charged lepton mass matrix given above. The Majorana mass matrix for the right handed fields comes from couplings of the form

\[4\tau_R.0\tau_R.(-2)\tau^o.(-2)\tau^o. \]  
(42)

In addition there are mass terms coming from the mixing of the right handed neutrinos with singlet fields from the couplings of the form given in Table 1. For example we have

\[< l_8|\nu_R > \bar{l}_1|\tau_R \phi_{18} \]  
(43)

together with another seven such couplings. This gives a complicated mass matrix the analysis of which lies beyond the scope of this paper.

We turn now to the quark masses. Consider the case that the Wilson line breaking is associated with the cyclic permutation group. The vectorlike pairs of up quarks acquire mass due to the large-scale breaking only via \(1.27.27\) related couplings. As a result the light quark states involve mixing only between \(0_{q,Q}\) and \(4_{q,Q}\) respectively giving the mass matrix of the form

\[27\]
\[
\begin{pmatrix}
0_U + 4_U & 0_U + 4_U & 4_U + 0_U \\
< 0_H > \eta & < 0_H > \eta & < 0_H > \\
< 0_H > \eta & < 0_H > \eta & < 0_H > \\
< 0_H > & < 0_H > & < 0_H > \eta
\end{pmatrix}
\cdot
\begin{pmatrix}
0_q + 4_q \\
0_q + 4_q \\
4_q + 0_q
\end{pmatrix}
\]  

(44)

Note that this structure preserves the underlying $L - R$ symmetry because there is no mixing with $\overline{7}$ in the quark sector. For this reason the case with \( \frac{<\phi_{-4}>}{<\phi_0>} << 1 \) is untenable because it leads to two massive degenerate quark states and one light state. As we note below this case may be rescued if we consider the case the Wilson line breaking is associated with the phase twist generated by the $(0,3,6,0)$ element. First however we note that it is possible to live with this form of the up quark matrix in the limit \( \frac{<\phi_{-4}>}{<\phi_0>} >> 1 \). As we noted above a large \( <\phi_{-4}> \) does not spoil the light Higgs protection or destabilise the nucleon so this limit is acceptable. Depending on the details of the intermediate scale breaking and the Yukawa couplings the diagonal entries in the mass matrix may be large so the symmetric structure of the matrix may be acceptable. We will discuss this possibility in detail elsewhere. In eq(40) one may see the lepton mass matrix is also acceptable in this limit although the smallness of the electron mass no longer follows simply from the magnitude of \( \frac{<\phi_{-4}>}{<\phi_0>} \).

Finally we consider the down quark masses. Its structure is similar to the leptons due to the mixing in the down quark sector generated by the $27^3$ and $\overline{27}^3$ terms. We find the mass matrix has the form

\[
\begin{pmatrix}
0_D + 4_D + \overline{2}_D & 0_D + 4_D + \overline{2}_D & 4_D + 0_D + \overline{2}_D \\
< 0_H > \eta & < 0_H > \eta & < 0_H > \\
< 0_H > \eta & < 0_H > \eta & < 0_H > \\
< 0_H > & < 0_H > & < 0_H > \eta
\end{pmatrix}
\cdot
\begin{pmatrix}
0_d + 4_l \\
0_l + 4_l \\
4_l + 0_l
\end{pmatrix}
\]  

(45)

Again this has an acceptable structure although the hierarchy of masses is not explained in the limit \( \eta >> 1 \). Note that the down quark and lepton masses may still be related by the underlying $E_6$ symmetry despite the fact that the gauge group after compactification is $SU(3) \otimes SU(2) \otimes U(1)$. The reason is that the quark and lepton fields are made up of the same $E_6$ multiplets but with different phases associated with the components. Thus the lepton multiplets, being permutation singlets, have the form $A + B + C$ where $A \rightarrow B \rightarrow C \rightarrow A$ under the permutation symmetry. The quark doublets have the form $A + \alpha^2 B + \alpha C$ where $\alpha = e^{2\pi i/3}$. As a result the couplings in the compactified model are related to the same underlying $E_6$ couplings, albeit modified by an overall phase. We will discuss in another paper whether this phase can be responsible for the observed CP violation in the Standard Model.

To complete this section we consider the structure of quark masses for the case the Wilson line breaking is associated with the phase twist generated by the $(0,3,6,0)$ element. This illustrates some of the diversity associated with different string constructions and also gives an example in which the case $\eta << 1$ is tenable and some of the fermion mass hierarchy is
determined by the same symmetry responsible for light Higgs states and for nucleon stability.
In this case there are only three generations of quarks on compactification and no vectorlike representations. As a result the structure of the quark mass matrix is simplified as there is no mixing to consider when determining the light spectrum. From Table 4 and Table 5 we find the following form for the quark mass matrices

\[
\begin{pmatrix}
Q_1 & Q_2 & Q_3
\end{pmatrix} \cdot
\begin{pmatrix}
0 & l_5 & l_{4,6} \\
l_4 & l_2 & l_1 \\
l_{5,6} & l_1 & l_{2,3}
\end{pmatrix} \cdot
\begin{pmatrix}
q_1 \\
q_2 \\
q_3
\end{pmatrix}
\]  

(46)

From the discussion above we know that the light Higgs are mixtures of the $Z^G_3$ charge 0,2,4 states. Only the 0 states are relevant here, namely $l_{1,2,4,5,7}$. From eq(46) we see that a Higgs with these components this will generate an acceptable mass matrix although the hierarchy of the second and third generations will require a cancellation between the terms involving $l_{1,2}$. We shall discuss ways in which this may happen naturally in a separate paper.

As for the leptons the relative magnitude of the first generation masses is set by the $Z^G_3$ symmetry breaking structure via the ratio $\frac{<l_{4,5}>}{<l_{1,2}>}$. A particularly interesting feature is the appearance of the texture zero in the (1,1) position. Such texture zeros are known to be consistent with the observed pattern of fermion masses and if, in addition, the (1,2) entry equals the (2,1) entry one obtains a prediction for the Cabibbo angle which is known to be in excellent agreement with experiment [24]. It is of interest to note that the symmetries of the string model lead readily to such a symmetric structure. This follows if $Z_2$ associated with anticyclic permutations of the three level 16 factors is unbroken. In this case the Higgs can only involve the $Z_2$ even combination $l_{4+} = l_4 + l_5$ leading to the mass matrix

\[
\begin{pmatrix}
Q_1 & Q_2 & Q_3
\end{pmatrix} \cdot
\begin{pmatrix}
0 & l_{4+} & l_{4+} \\
l_{4+} & l_2 & l_1 \\
l_{4+} & l_1 & l_{2,3}
\end{pmatrix} \cdot
\begin{pmatrix}
q_1 \\
q_2 \\
q_3
\end{pmatrix}
\]  

(47)

This structure reproduces the good texture zero prediction for the CKM matrix given by [24]

\[
|V_{us}| = \sqrt{\frac{M_D}{M_s}} - \sqrt{\frac{M_u}{M_c}} e^{i\sigma}
\]

\[
c.f.(0.218 - 0.224) = |(0.16 - 0.33) - (0.047 - 0.07)e^{i\sigma}|
\]

As a bonus the matrix has relatively small entries in the (3,1) and (1,3) positions because, with the Higgs having the appropriate $l_{4+}$ magnitude to give the correct (1,2) element, one finds that the (3,1) and (1,3) elements do not affect the masses to leading order. They can affect the mixing angles but one may check that they give the relations

\[
|V_{ub}| \approx \sqrt{\frac{M_u}{M_c}} |V_{cb}|
\]

\[
|V_{tu}| \approx \sqrt{\frac{M_d}{M_s}} \sqrt{\frac{M_c}{M_u}} |V_{ub}|
\]
where the $\simeq$ means equality up to coefficients of $O(1)$. These relations are also in excellent agreement with experiment. Thus we see that texture zeros and a hierarchical structure can follow very naturally from the underlying string symmetries of the three generation Calabi-Yau model.

7 Summary and Conclusions

In this paper we have explored a new class of string compactifications obtained by allowing vevs to develop along absolutely flat directions. In practice these directions may only be recognised starting with a four dimensional string theory at a point in moduli space with enhanced symmetry. In this discrete symmetries play a crucial role as only they are capable of ensuring flatness to all non-renormalisable orders in the superpotential. It turns out that the Gepner construction which builds a four dimensional theory via products of $N = 2$ superconformal theories has such R symmetries together with additional gauge and discrete symmetries. It is known that these compactifications are equivalent in the large radius limit to Calabi Yau compactification and thus are of relevance to M-theory compactification.

As an example of the method we studied the flat directions in a three generation Calabi Yau model built using the Gepner construction. The original theory has an underlying $E_6$ gauge group which may be broken by Wilson lines to $SU(3)^3$ but not to the Standard Model. The hope is that the perturbation along flat directions will allow for a reduction of the original symmetry to $SU(5)$ so that after Wilson line breaking the group can be reduced to $SU(3) \otimes SU(2) \otimes U(1)$. Amongst other things this relates the compactification scale determined in M-theory with the gauge unification scale found by continuing the couplings of the MSSM and offers the prospect of a quantitative string theory prediction for this scale. We found that the theory does indeed have flat directions which lead to a reduction of the string gauge symmetry to $SU(5)$.

Construction of a phenomenological string theory requires that there be low energy symmetries which stabilise the nucleon and also leave just one pair of Higgs doublets light. We showed that the three generation Calabi Yau model has just such a symmetry following from the original discrete symmetries of the string. Furthermore we showed that the flat direction leading to the reduced gauge symmetry leaves this symmetry unbroken so that is possible to build a viable low energy theory which has just the Standard Model $SU(3) \otimes SU(2) \otimes U(1)$ gauge symmetry below the compactification scale. The stability of the nucleon follows from a “Baryon parity” which forbids rapid baryon number violating processes but allows lepton number processes to occur through the exchange of particles with supersymmetry breaking scale masses. The stability of just one pair of Higgs doublets follows through a new “Higgs chirality” which has an excess of one positive chirality Higgs field of each weak hypercharge. Remarkably the stability of the nucleon and the lightness of just one pair of Higgs doublets is ensured by the same $Z_3^G$ discrete symmetry. A feature of the theory at the compactification scale is that there are several additional states left light in vectorlike representations with respect to $SU(3) \otimes SU(2) \otimes U(1)$. We show that these states are all likely to acquire intermediate scale masses somewhere between the compactification and the electroweak scale. This is generated by intermediate scale vevs triggered by $SU(3) \otimes SU(2) \otimes U(1)$ scalar fields
acquiring negative masses squared due to radiative corrections. We studied the possibility for such intermediate scale breaking and showed that it is likely to occur at a very high scale due to the constrained form of the underlying string symmetry. We demonstrated that this can happen along directions in field space which leave the $Z_3^G$ symmetry intact.

Given knowledge of the string symmetries it is straightforward to determine the pattern of quark and lepton masses. In the three generation Calabi Yau theory the residual low-energy symmetries impose constraints on the allowed form of these matrices. We showed that an acceptable pattern of lepton masses is possible in which the lightness of the electron may be determined by the ratio of $Z_3^G$ symmetry breaking vevs. We studied the quark masses in two variants of the theory in which the Wilson line breaking is associated with different discrete groups which are modded out in the construction of the three generation theory. In both cases an acceptable pattern of masses for the up and down quarks is consistent with the low-energy symmetries.

In the case the Wilson line breaking is associated with the cyclic permutation group one loses the association of the electron mass hierarchy with the ratio of $Z_3^G$ symmetry breaking vevs. However one finds in this case that the quark and lepton Yukawa couplings responsible for generating masses have a residual Grand Unified structure associated with the underlying $E_6$ which may preserve the successful $SO(10)$ relations between the charged lepton and down quark masses.

In the case the Wilson line breaking is associated with the phase twist generated by the $(0,3,6,0)$ element the quark mass structure is consistent with the electron mass hierarchy being related to the $Z_3^G$ symmetry breaking and in addition the lightness of the first generation of quarks is also determined by the ratio of $Z_3^G$ symmetry breaking vevs. Moreover we found that a symmetric mass matrix and texture zeros in the (1,1) and (1,3) positions follow quite readily from the symmetries of the theory. This structure leads to excellent predictions for the quark masses.

To go further it is necessary to study a specific pattern of intermediate scale breaking. A given pattern will have further structure in the quark and lepton masses following from the underlying rich structure of string symmetries. We will report elsewhere on a study of a particularly promising choice for this intermediate scale breaking.

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