Poisson-Lie T-duality and (1,1) supersymmetry

C. Klimčík
IHES, 91440 Bures-sur-Yvette, France

Abstract
A duality invariant action for (1,1) supersymmetric extension of Poisson-Lie dualizable $\sigma$-models is constructed.
1. The Poisson–Lie (PL) $T$-duality \cite{1} is a generalization of the traditional non-Abelian $T$-duality \cite{2}–\cite{5} and it is proved to enjoy \cite{1}, \cite{3}–\cite{13}, at least at the classical level, most of the structural features of the traditional Abelian $T$-duality \cite{14} and \cite{15}.

Perhaps the last remaining problem to be solved in order to complete the classical story consists in construction of the supersymmetric (SUSY) extension of the PL duality. The first step in this direction was undertaken by Sfetsos in \cite{16} where he noted that the (1,1) SUSY extensions (including the spectator fields) of the PL dual pair of $\sigma$-models have equivalent equations of motion. In other words, a solution of the field equations of one model can be explicitly mapped to a solution of the dual model. In \cite{16} and later in a series of papers by Parkhomenko \cite{17} also the case of extended supersymmetry was treated, however, the PL duality was always established only in the ”on shell” way described above. After experience with the purely bosonic case, it is plausible that an ”off shell” equivalence of the SUSY models can be also established which would amount to showing that the map sending a solution of the one model to the solution of the dual model is actually a symplectomorphism (or a canonical transformation) between the phase spaces of the models. In the purely bosonic case, the fact that the duality map is the symplectomorphism was established in \cite{1} by means of constructing the generating functional of the canonical transformation. Later in \cite{8}, we have given a manifest evidence of the off shell equivalence of the (purely bosonic) models: we parametrized the common phase space of the models in a duality invariant way (as a coadjoint orbit of the loop group of the underlying Drinfeld double) and found the first-order duality invariant Hamiltonian action reflecting simultaneously the dynamics of both models in the dual pair. In the SUSY case, such a duality invariant description of the phase space and a duality invariant action are missing; the goal of this paper is to fill this gap.

The main trouble in addressing the problem consists in the fact that in the first-order Hamiltonian description the world-sheet supersymmetry cannot be manifest. As the matter of the fact, already in the purely bosonic case one misses the explicit world-sheet Poincaré symmetry in the first-order action \cite{8}. However, we were able to establish (with P. Ševera) that the symplectic structure on the phase space is just the Kirillov-Kostant symplectic structure on the coadjoint orbit of the loop group and it was sufficient only to calculate the Hamiltonian which would give the dynamics of the PL dual pair of $\sigma$-models. The Poincaré symmetry of the theory then somehow miraculously
followed in the second-order $\sigma$-model description.

In what follows, we will work out the supersymmetric case in the same spirit. Namely, we will find the duality invariant parametrization of the configurations of the model and an explicit formula for the duality invariant action. We shall first do the (1,0) case (which is itself interesting as it is relevant for the description of the heterotic string) where we perform a detailed calculation that illustrates the method of obtaining the duality invariant action from the second order $\sigma$-model description. Then we shall use the method to solve the (1,1) case.

2. For the description of the PL $T$-duality, we need the crucial concept of the Drinfeld double, which is simply a Lie group $D$ such that its Lie algebra $D$ (viewed as a vector space) can be decomposed as the direct sum of two subalgebras, $G$ and $\tilde{G}$, maximally isotropic with respect to a non-degenerate invariant bilinear form on $D$. It is often convenient to identify the dual linear space to $G$ ($\tilde{G}$) with $\tilde{G}$ ($G$) via this bilinear form.

From the space-time point of view, we have identified the targets of the mutually dual $\sigma$-models with the group manifolds $G$ and $\tilde{G}$, corresponding to the Lie algebras $G$ and $\tilde{G}$.

Consider now the (1,0) SUSY extension of the PL dualizable $\sigma$-model on the target $G$ (cf. [1, 6, 8]):

$$S = \frac{1}{2} \int d\sigma d\tau d\theta \{ \bar{\Lambda}^a (R_{ab} + \Pi(G)_{ab}) \lambda^b + \bar{\Lambda}^a (\partial_{GG^{-1}})_a - (D + GG^{-1})_a \lambda^a \}. \quad (1)$$

Here the (cylindrical) world-sheet is parametrized by $\tau \in [-\infty, \infty], \sigma \in [0, 2\pi]$ and

$$\partial_{\pm} = \partial_{\tau} \pm \partial_{\sigma}. \quad (2)$$

The Grassmann variable $\theta$ is the superpartner of $\xi^+ = \frac{1}{2}(\tau + \sigma)$, hence

$$D_+ = \partial_{\theta} + \theta \partial_{\pm}. \quad (3)$$

The superfield $G$ is given in components as

$$G = g + \theta \gamma g, \quad g \in G, \quad \gamma \in \mathcal{G}; \quad (4)$$

$\gamma$ is the Majorana-Weyl fermion. The auxiliary superfields $\bar{\Lambda}^a$ and $\lambda^a$ can be written also as expansions in $\theta$ with generic coefficients; note, however, that
Λ is an odd superfield while λ is an even one and they both are not group valued. The components of the currents $dGG^{-1}$ are taken in a basis $T^a$ of $\mathcal{G}$. Of course, the Gaussian integration over the auxiliary superfields can be performed, yielding, perhaps, a more standard way of writing the 2d $\sigma$-model action on a group target.

Finishing the explanations of the symbols in (1), we note that $R_{ab}$ is an arbitrary non-degenerate constant matrix and $\Pi(G)_{ab}$ is the bivector field on the group manifold $G$ [18]. For our purposes, it is convenient to describe $\Pi(G)$ more explicitly; consider matrices $a(g), b(g)$ and $d(g)$ defined as

$$g^{-1}T^a = a(g)^a \tilde{T}^b, \quad g^{-1}\tilde{T}_a = b(g)_{ab}T^b + d(g)_a^{\ b}\tilde{T}_b,$$

where $T^a(\tilde{T}_a)$ are generators of the Lie algebra $\mathcal{G}(\tilde{\mathcal{G}}) \subset \mathcal{D}$. Now

$$\Pi(g) = b(g)a(g)^{-1}. \quad (6)$$

Note also a few useful properties of the matrices $a, b$ and $d$:

$$d'(g) = a(g^{-1}), \quad b'(g) = b(g^{-1}), \quad b(g)a(g^{-1}) + d(g)b(g^{-1}). \quad (7)$$

The field equations of the model (1) are given by

$$(R + \Pi(G))\lambda + \partial_- GG^{-1} = 0, \quad (8)$$

$$\bar{\Lambda}(R + \Pi(G)) - D_+ GG^{-1} = 0 \quad (9)$$

and

$$D_+ \chi - \partial_- \bar{\Xi} + [\bar{\Xi}, \chi] = 0. \quad (10)$$

Here $\chi$ and $\bar{\Xi}$ are understood to be elements of $\tilde{\mathcal{G}}$ and their coordinates in the basis $\tilde{T}_a$ are related to $\lambda$ and $\bar{\Lambda}$ as follows

$$\chi^a \equiv -d(G)_{ba}\lambda^b, \quad \bar{\Xi}^a \equiv -d(G)_{ba}\bar{\Lambda}^b. \quad (11)$$

Note, that the equation (10) can be interpreted as the $(1,0)$ SUSY zero-curvature condition. Indeed, it can be integrated to give

$$\chi = -\partial_- \tilde{H}\tilde{H}^{-1}, \quad \bar{\Xi} = -D_+ \tilde{H}\tilde{H}^{-1}, \quad (12)$$

where

$$\tilde{H} = \tilde{h} + \theta\tilde{\eta}\tilde{h}, \quad \tilde{h} \in \tilde{\mathcal{G}}, \quad \tilde{\eta} \in \tilde{\mathcal{G}}. \quad (13)$$

3
Thus we have obtained the $(1,0)$ version of the standard fact (cf. [1]) that every solution $G(\sigma, \tau, \theta)$ of the model (1) gives a string configuration $\tilde{H}(\sigma, \tau, \theta)$ propagating in the dual target $\tilde{G}$.

As in the bosonic case [1], we shall prove that the configuration $\tilde{G}(\tau, \sigma, \theta)$, defined by

$$G(\tau, \sigma, \theta) \tilde{H}(\tau, \sigma, \theta) = \tilde{G}(\tau, \sigma, \theta) H(\tau, \sigma, \theta), \quad \tilde{G}(\tau, \sigma, \theta) \in \tilde{G}, \quad H(\tau, \sigma, \theta) \in G,$$

(14)
is a solution of the dual $(1,0)$ $\sigma$-model. The action of the latter is the same as of (1), except all quantities bear tilde and $R_{ab}$ is replaced by its inverse matrix; explicitly:

$$\tilde{S} = \frac{1}{2} \int d\sigma d\tau d\theta \{ \tilde{\Lambda}_a ((R^{-1})^{ab} + \tilde{\Pi}(\tilde{G})^{ab}) \tilde{\lambda}_b + \tilde{\Lambda}_a (\partial_+ \tilde{G} \tilde{G}^{-1})^a - (D_+ \tilde{G} \tilde{G}^{-1})^a \tilde{\lambda}_a \}.$$  

(15)

To prove the statement, note that, using (11) and (12), the field equations (8) and (9) can be rewritten as

$$\langle D_+ LL^{-1}, R_a^- \rangle = 0, \quad R_a^- \equiv R_{ba} T^b - \tilde{T}_a,$$

(16)

$$\langle \partial_+ LL^{-1}, R_a^+ \rangle = 0, \quad R_a^+ \equiv R_{ab} T^b + \tilde{T}_a,$$

(17)

where

$$L(\tau, \sigma, \theta) = G(\tau, \sigma, \theta) \tilde{H}(\tau, \sigma, \theta).$$

(18)

In the same way, the field equations of the dual model are

$$\langle D_+ \tilde{L} \tilde{L}^{-1}, (R^{-1})^{ba} \tilde{T}_b - T^a \rangle = 0,$$

(19)

$$\langle \partial_+ \tilde{L} \tilde{L}^{-1}, (R^{-1})^{ab} \tilde{T}_b + T^a \rangle = 0,$$

(20)

where $\tilde{L}(\tau, \sigma, \theta) = \tilde{G}(\tau, \sigma, \theta) H(\tau, \sigma, \theta)$. Thus, under the identification $L = \tilde{L}$, the equations (16) and (17) are the same as (19) and (20). This finishes the proof

\footnote{It is, perhaps, worth adding a comment concerning the global topology of the world sheet. In general, $\tilde{G}(\tau, \sigma, \theta)$ may turn out not to be single-valued and, thus, not to correspond to a closed string solution. If we restrict the space of solutions of the model (1) in such a way that $\tilde{H}(\tau, \sigma, \theta)$ is single-valued (and the same thing we do with $H(\tau, \sigma, \theta)$ in the dual model (15)) then both $G(\tau, \sigma, \theta)$ and $\tilde{G}(\tau, \sigma, \theta)$ will be single valued. Actually, we must to impose this restriction in order to have the "off shell" duality, see page 6.}

1
3. The equations (16) and (17) are duality invariant \((L = \tilde{L})\). It is therefore natural to ask, if there is a duality invariant formulation of the dynamics of both models (1) and (15). The answer to this question is positive; we can construct this action starting directly from the expression (1). We shall work in components; first note

\[
D_+ GG^{-1} = \gamma + \theta(\partial_+ gg^{-1} + \gamma \gamma),
\]

\[
\partial_- GG^{-1} = \partial_- gg^{-1} + \theta(\partial_- \gamma + [\gamma, \partial_- gg^{-1}]),
\]

\[
\bar{\Xi} \equiv \bar{\kappa} + \theta \bar{x}, \quad \chi \equiv x + \theta \kappa.
\]

Of course, the components of \(\bar{\Xi}\) and \(\chi\) are also in \(\tilde{G}\); \(\bar{x}, x\) are real bosons and \(\bar{\kappa}, \kappa\) are Majorana-Weyl fermions. We perform the integration over \(\theta\) to arrive at

\[
S = \frac{1}{2} \int d\sigma d\tau \{\langle \bar{x}, g^{-1} T^a g \rangle \langle g^{-1} R^+_a g, x \rangle - \langle \bar{x}, g^{-1} \partial_- g \rangle + \langle g^{-1} \partial_+ g - g^{-1} \gamma \gamma g, x \rangle \\
- \langle \bar{\kappa}, g^{-1} T^a g \rangle \langle g^{-1} R^+_a g, \kappa \rangle + \langle \bar{\kappa}, g^{-1} \partial_- \gamma g \rangle - \langle g^{-1} \gamma g, \kappa \rangle \\
- \langle \bar{\kappa}, g^{-1} [T^a, \gamma] g \rangle \langle g^{-1} R^+_a g, x \rangle - \langle \bar{\kappa}, g^{-1} T^a g \rangle \langle g^{-1} [R^+_a, \gamma] g, x \rangle \}.
\]

(24)

Varying with respect to \(\kappa\), we obtain

\[
\gamma + \langle \bar{\kappa}, g^{-1} R^-_a g \rangle T^a = 0,
\]

(25)

or

\[
\langle \psi, R^-_a \rangle = 0, \quad \psi \equiv \gamma - g\bar{\kappa}g^{-1}.
\]

(26)

In the course of the derivation, the fermions \(\gamma, \bar{\kappa}\) and \(\psi\) will be understood to fulfil the constraints (25,26). Now redefining

\[
\bar{x} = \bar{x}' - \langle \{\bar{\kappa}, g^{-1} \gamma g\}, T^a \rangle \tilde{T}^a,
\]

(27)

we have

\[
S = \frac{1}{2} \int d\sigma d\tau \{\langle \bar{x}', g^{-1} T^a g \rangle \langle g^{-1} R^+_a g, x \rangle - \langle \bar{x}', g^{-1} \partial_- g \rangle \\
+ \langle g\bar{\kappa}g^{-1}, \partial_- \gamma + [\gamma, \partial_- gg^{-1}] \rangle + \langle g^{-1} \partial_+ g + g^{-1} \psi \psi g, x \rangle \}.
\]

(28)

Set

\[
x^a = \rho^a + \pi^a \equiv \rho^a + \langle \partial_\sigma U^{-1} + \frac{1}{2} \psi \psi, g T^a g^{-1} \rangle,
\]

(29)
\[ \bar{x}^a = \rho^a - \pi^a \equiv \rho^a - \langle \partial_a l^{-1} + \frac{1}{2} \psi \psi, g T^a g^{-1} \rangle, \]  
\[ l = g \tilde{h}, \quad \tilde{h} \in \tilde{G} \]  
(30)

and eliminate \( \rho \):

\[ S = \frac{1}{2} \int d\sigma d\tau \left\{ 2 \langle \partial_a \tilde{h} \tilde{h}^{-1}, g^{-1} \partial g \rangle + \langle \tilde{\kappa} \tilde{\kappa}, g^{-1} \partial g \rangle + \langle g \tilde{\kappa} g^{-1}, \partial \gamma \rangle \right. 
\[ \left. - \langle \partial_a l^{-1} + \frac{1}{2} \psi \psi, (A - \text{Id})(\partial_a l^{-1} + \frac{1}{2} \psi \psi) \rangle \right\}. \]  
(32)

Here \( A \) is a linear idempotent self-adjoint map from the Lie algebra \( D \) of the double into itself. It has two equally degenerate eigenvalues, +1 and -1, and the corresponding eigenspaces are linear envelopes of \( R_+^a \) and \( R_-^a \), respectively. \( \text{Id} \) is the identity map from \( D \) to \( D \).

The transformation (29,30) deserves some explanation. We have traded the field \( \pi \in \tilde{G} \) for a group-valued field \( \tilde{h} \in \tilde{G} \). Indeed, it is easy to see that

\[ \pi^a = (\partial_a \tilde{h} \tilde{h}^{-1})^a + \frac{1}{2} \langle \psi \psi, g T^a g^{-1} \rangle. \]  
(33)

In order to write the duality invariant action, we have to require that \( \tilde{h}(\tau, \sigma) \) is a single-valued function of \( \sigma \) (otherwise the string configuration \( \tilde{g}(\tau, \sigma) \) in the dual target would not be closed). This sets a constraint of unit monodromy of the quantity (cf. footnote 1)

\[ \pi - \frac{1}{2} \langle \psi \psi, g T^a g^{-1} \rangle \tilde{T}_a \in \tilde{G}. \]  
(34)

This is the analogue of what happened in the bosonic case \[1, 8\] and the resolution of the issue is the same: we have to constrain the model (1) by allowing only those of its solutions, which fulfil the constraint of the unit monodromy of the quantity (34). The same must be obviously true also in the dual case (15), where the quantity dual to (34) is constructed in the exactly corresponding way. Only after imposing both constraints, the PL T-duality between (1) and (15) takes place.

Consider the Polyakov-Wiegmann formula \[19\]

\[ I(g \tilde{h}) = I(g) + I(\tilde{h}) + \frac{1}{4\pi} \langle \partial_a \tilde{h} \tilde{h}^{-1}, g^{-1} \partial g \rangle, \]  
(35)
\[ I(l) \equiv \frac{1}{8\pi} \{ \int d\sigma d\tau \langle \partial_\sigma ll^{-1}, \partial_\tau ll^{-1} \rangle + \frac{1}{6} d^{-1} \langle dl l^{-1}, [dl l^{-1}, dl l^{-1}] \rangle \}. \]  

(36)

Using PW formula (35), Eq. (25) and the fact that both algebras \( \mathcal{G} \) and \( \tilde{\mathcal{G}} \) are isotropic, we arrive at

\[ S[l(\tau, \sigma), \psi(\tau, \sigma)] = \]

\[ = \frac{1}{8\pi} \int \left\{ \langle \partial_\sigma l l^{-1}, \partial_\tau l l^{-1} \rangle + \frac{1}{6} d^{-1} \langle dl l^{-1}, [dl l^{-1}, dl l^{-1}] \rangle - \frac{1}{2} \langle \psi, \partial_\psi \rangle 
- \langle \partial_\sigma ll^{-1} + \frac{1}{2} \psi \psi, (A - Id)(\partial_\sigma ll^{-1} + \frac{1}{2} \psi \psi) \rangle \right\}. \]  

(37)

Of course, \( \psi \in \mathcal{D} \) is thought to fulfil \( \langle \psi, R^- \rangle = 0 \).

The action (37) is the duality invariant first-order action, that we have been looking for. Since it was directly derived from the \( \sigma \)-model action (1), it is equivalent to it (with the constraint of the unit monodromy). In particular, it must give the same equations of motion. As a simple check of correctness of our calculation we can derive those equations directly from (37). In doing that one has to fix conveniently a small gauge symmetry \( l(\tau, \sigma) \rightarrow l(\tau, \sigma)l_0(\tau) \). The field equations then read

\[ \langle \psi, R^- \rangle = 0, \quad \langle \partial_+ ll^{-1} + \psi \psi, R^- \rangle = 0, \]  

(38)

\[ \langle \partial_- ll^{-1}, R^+ \rangle = 0, \quad \langle \partial_- \psi + [\psi, \partial_- ll^{-1}], R^+ \rangle = 0. \]  

(39)

If we combine the fields \( l \) and \( \psi \) in a single group-valued superfield

\[ L = l + \theta \psi l, \]  

(40)

we can rewrite the component equations (38) and (39) as

\[ \langle D_+ LL^{-1}, R^- \rangle = 0, \quad \langle \partial_- LL^{-1}, R^+ \rangle = 0. \]  

(41)

Those are precisely the field equations (16) and (17).
4. Consider now the (1,1) SUSY extension of the PL dualizable $\sigma$-model on the target $G$:

$$S = \frac{1}{2} \int d\sigma d\tau d\theta^+ d\theta^- \{ \Lambda_+^a (R_{ab} + \Pi(G)_{ab}) \Lambda^b_- + \Lambda_+^a (D_- GG^{-1})_a - (D_+ GG^{-1})_a \Lambda^a_+ \}. \quad (42)$$

Now we have two Grassmann world-sheet coordinates $\theta^\pm$, the superpartners of $\xi^\pm = \frac{1}{2} (\tau \pm \sigma)$, respectively. Hence

$$D_\pm = \partial_{\theta^\pm} + \theta^\pm \partial_\pm. \quad (43)$$

The superfield $G$ is given in components as

$$G = (1 + \theta^+ \gamma^+ + \theta^- \gamma^- + \theta^+ \theta^- (K + \frac{1}{2} [\gamma^- , \gamma^+])) g, \quad g \in G, \quad \gamma^\pm, K \in \mathcal{G}; \quad (44)$$

$\gamma^\pm$ are components of the Majorana fermion, $K$ is a real auxiliary bosonic field. The odd auxiliary superfields $\Lambda^a_\pm$ can be written in the standard way as expansions in $\theta$-s with generic coefficients.

The field equations of the model (42) are given by

$$(R + \Pi(G)) \Lambda_- + D_- GG^{-1} = 0, \quad (45)$$

$$\Lambda_+ (R + \Pi(G)) - D_+ GG^{-1} = 0 \quad (46)$$

and

$$D_+ \Xi_+ + D_- \Xi_- + \{ \Xi_+, \Xi_- \} = 0. \quad (47)$$

Here $\Xi_\pm$ are understood to be elements of $\tilde{\mathcal{G}}$ and their coordinates in the basis $\tilde{T}_a$ are related to $\Lambda_\pm$ as follows

$$\Xi^a_\pm \equiv -d(G)_{ba} \Lambda^b_\pm. \quad (48)$$

Note, that the equation (47) can be interpreted as the (1,1) SUSY zero-curvature condition. Indeed, it can be integrated to give

$$\Xi_\pm = -D_\pm \tilde{H} \tilde{H}^{-1}, \quad (49)$$

where

$$\tilde{H} = (1 + \theta^+ \tilde{\eta}^+ + \theta^- \tilde{\eta}^- + \theta^+ \theta^- (\tilde{M} + \frac{1}{2} [\tilde{\eta}^-, \tilde{\eta}^+])) \tilde{h}, \quad \tilde{h} \in \tilde{G}, \quad \tilde{\eta}^\pm, \tilde{M} \in \tilde{G}. \quad (50)$$
If we set
\[ L(\tau, \sigma, \theta^\pm) = G(\tau, \sigma, \theta^\pm) \tilde{H}(\tau, \sigma, \theta^\pm), \] (51)
then the field equations (45) and (46) can be written as (cf. (16),(17)):
\[ \langle D_\pm LL^{-1}, R^\tau_a \rangle = 0. \] (52)

The dual (1,1) \( \sigma \)-model (the interested reader can easily write its action by combining (1),(15) and (42)) has the identical equations of motions and we can look for the duality invariant formulation of the common dynamics of both models which would use only \( L \) as a configuration. We can proceed exactly as in the (1,0) case by eliminating (all but one component of the) auxiliary fields \( \Lambda^\pm \). The resulting duality invariant action is given by
\[
S[l(\tau, \sigma), \psi^\pm(\tau, \sigma)] = \frac{1}{8\pi} \int \left\{ \langle \partial_\sigma l^{-1}, \partial_\tau l^{-1} \rangle + \frac{1}{6} \langle dl^{-1}, [dl^{-1}, dl^{-1}] \rangle \\
- \frac{1}{2} \langle \psi^+, \partial_- \psi^+ \rangle + \frac{1}{2} \langle \psi^-, \partial_+ \psi^- \rangle + \frac{1}{4} \langle \psi^+ \psi^+, \psi^+ \psi^+ \rangle - \frac{1}{4} \langle \psi^- \psi^-, \psi^- \psi^- \rangle \\
+ \langle \partial_\sigma l^{-1}, \psi^+ \psi^+ + \psi^- \psi^- \rangle - \frac{1}{4} \langle \{ \psi^-, \psi^+ \}, A \{ \psi^-, \psi^- \} \rangle \\
- \langle \partial_\sigma l^{-1}, \frac{1}{2} \psi^+ \psi^+ - \frac{1}{2} \psi^- \psi^- - A(\partial_\sigma l^{-1} + \frac{1}{2} \psi^+ \psi^+ - \frac{1}{2} \psi^- \psi^-) \rangle \right\}. \] (53)

Here \( l \in D \) and \( \psi^\pm \) are components of the Majorana spinor with values in \( D \), such that \( \langle \psi^+, R^\tau_a \rangle = 0 \).

This action (53) is our final result. Note that it has the correct (1,0) limit (if we set \( \psi^- \) equal 0) and correct (0,0) limit (8) if we set both spinors \( \psi^\pm \) to zero. We may also find easily the field equations following from (53). They are
\[ \langle \psi^+, R^\tau_a \rangle = 0, \quad \langle \partial_\pm l^{-1} + \psi^\pm \psi^\pm, R^\tau_a \rangle = 0, \] (54)
\[ \langle \partial_\pm \psi^\mp, \partial_\pm l^{-1} + \frac{1}{2} \psi^\pm \psi^\pm \rangle \pm \frac{1}{2} [A \{ \psi^-, \psi^+ \}, \psi^\pm], R^\tau_a \rangle = 0. \] (55)

If we combine the fields \( l \) and \( \psi^\pm \) in a single group-valued superfield
\[ L = (1 + \theta^+ \psi^+ + \theta^- \psi^- + \theta^+ \theta^- (F + \frac{1}{2} [\psi^- \psi^+] ) ) l, \] (56)
\( F \in \mathcal{D} \) is an auxiliary field) we can rewrite the component equations (54) and (55) as
\[
\langle D_\pm LL^{-1}, R_\alpha^+ \rangle = 0. \tag{57}
\]
Those are precisely the field equations (52).

I thank P. Ševera and K. Sfetsos for discussions and correspondence.

References

[1] C. Klimčík and P. Ševera, Phys. Lett. B351 (1995) 455, hep-th/9502122; P. Ševera, Minimálne Plochy a Dualita, Diploma thesis, Praha University, May 1995 (in Slovak); C. Klimčík, Nucl. Phys. (Proc. Suppl.) 46 (1996) 116

[2] X. de la Ossa and F. Quevedo, Nucl. Phys. B403 (1993) 377

[3] B.E. Fridling and A. Jevicki, Phys. Lett. B134 (1984) 70

[4] E.S. Fradkin and A.A. Tseytlin, Ann. Phys. 162 (1985) 31

[5] A. Giveon and M. Roček, Nucl. Phys. B421 (1994) 173; E. Álvarez, L. Álvarez-Gaumé and Y. Lozano, Nucl. Phys. B424 (1994) 155; K. Sfetsos, Phys. Rev. D50 (1994) 2784; T. Curtright and C. Zachos, Phys. Rev. D49 (1994) 5408; O. Alvarez and C-H. Liu, Commun. Math. Phys. 179 (1996) 185; Y. Lozano, Phys. Lett. B355 (1995) 165

[6] A.Yu. Alekseev, C. Klimčík and A.A. Tseytlin, Nucl. Phys. B458 (1996) 430

[7] E. Tyurin and R. von Unge, Phys. Lett. B382 (1996) 233

[8] C. Klimčík and P. Ševera, Phys. Lett. B372 (1996) 65

[9] C. Klimčík and P. Ševera, Phys. Lett. B376 (1996) 82

[10] C. Klimčík and P. Ševera, Phys. Lett. B381 (1996) 56

[11] C. Klimčík and P. Ševera, Phys. Lett. B383 (1996) 281
[12] C. Klimčík and P. Ševera, Nucl. Phys. B488 (1997) 653

[13] C. Klimčík and P. Ševera, T-duality and the moment map, preprint IHES/P/96/70; hep-th/9610198

[14] K. Kikkawa and M. Yamasaki, Phys. Lett. B149 (1984) 357; N. Sakai and I. Senda, Prog. Theor. Phys. 75 (1986) 692

[15] T.H. Buscher, Phys. Lett. B194 (1987) 51 and B201 (1988) 466; K. Meissner and G. Veneziano, Phys. Lett. B267 (1991) 33; A.A. Tseytlin, Nucl. Phys. B350 (1991) 395 and Mod. Phys. Lett. A6 (1991) 1721; M. Roček and E. Verlinde, Nucl. Phys. B373 (1992) 630; A.S. Schwarz and A.A. Tseytlin, Nucl. Phys. B399 (1993) 691; A. Giveon and M. Roček, Nucl. Phys. B380 (1992) 128; E. Kiritsis, Mod. Phys. Lett. A6 (1991) 2871 and Nucl. Phys. B405 (1993) 109; E. Álvarez, L. Álvarez-Gaumé, J. Barbón and Y. Lozano, Nucl. Phys. B415 (1994) 71; A. Giveon, M. Porrati and E. Rabinovici, Phys. Rep. 244 (1994) 77; E. Álvarez, L. Álvarez-Gaumé and Y. Lozano, Nucl. Phys. Proc. Suppl. 41 (1995) 1 and Phys. Lett. B336 (1994) 183; C. Klimčík and P. Ševera, Mod. Phys. Lett. A10 (1995) 323

[16] K. Sfetsos, Poisson-Lie T-duality and supersymmetry, Utrecht preprint THU-96/38, hep-th/9611199

[17] S.E. Parkhomenko, Pisma Zh. Eksp. Teor. Fiz. 64 (1996) 823, hep-th/9612034; Poisson-Lie T-duality in N=2 superconformal WZNW models on compact groups, Landau Inst. preprint TMP/05/97, hep-th/9705233; Poisson-Lie T-duality and complex geometry in N=2 superconformal WZNW models, Landau Inst. preprint TMP/06/97, hep-th/9706199

[18] V.G. Drinfeld, Quantum Groups, in Proc. ICM, MSRI, Berkeley, 1986, p. 708; F. Falceto and K. Gawędzki, J. Geom. Phys. 11 (1993) 251; A.Yu. Alekseev and A.Z. Malkin, Commun. Math. Phys. 162 (1994) 147

[19] A. Polyakov and P. Wiegmann, Phys. Lett. B131 (1983) 121