Study of Fragmentation from A Quark to Hadrons

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Abstract. Hadrons are known to be strongly interacting particles. A single quark is not detected in experiments but is always bound in a hadron because of color confinement. We observe hadrons in the final states as a result of quark reactions. We now consider a process of a quark to hadrons in $e^+e^-$ annihilation. An electron collides with a positron, and a quark-antiquark pair is generated. The quark and antiquark have opposite momenta. The quark radiates gluons and the gluons produce further quark-antiquark pairs. These quarks and gluons finally form hadrons. This process is called 'fragmentation'. Such a process is formulated by the fragmentation function. I will describe the process of the fragmentation from a quark to hadrons by using several models and also fragmentation function, based on lepton-nucleon deep inelastic scatterings and $e^+e^-$ reactions where I am involved in data analyses.

1. Introduction

Parton fragmentation refers to the process of converting high-energy, colored partons (quark and gluon) out of a hard scattering into hadron jets observed in detectors. Dramatic demonstrations of quark substructure are obtained in $e^+e^-$ annihilation to hadrons at very high energy (Figure 1.). The hadrons appear in the form of two 'jets' collimated around the $q\bar{q}$-axis. The first observation of jet was in 1975 at Stanford Linear Accelerator Center on $e^+e^-\rightarrow q + \bar{q}$ reaction [1]. After that, jet was observed on deep inelastic lepton scattering and proton-antiproton reaction. Now fragmentation function, which associate hadron with parton, is a powerful probe for the study of particle physics. For example, 'spin structure' of the proton can be investigated when the experimental information from deep inelastic scattering is combined with the fragmentation function from $e^+e^-$ collider. However, we need to think about some models to associate the jet with parton because nonperturbative treatment is necessary for this. We now mention two of the famous fragmentation function models.

![Figure 1. Fragmentation process in $e^+e^-$ annihilation](image)
2. Fragmentation Function

Fragmentation functions represent the probability for a parton to fragment into a particular hadron carrying a certain fraction of the parton’s energy. A parton fragmentation \( D_h^i(z) \) encodes the probability that the parton \( i \) fragments into a hadron \( h \) carrying an energy fraction \( z \) of the parton’s momentum. Energy fraction \( z \) is written as:

\[
z = \frac{p_h}{|p|}
\]

where \( p_h \) is the momentum of a hadron \( h \), \( p \) is momentum of parton \( i \). In an actual experiment, \( z \) is decided like this.

\[
z = \frac{p_h}{|p|} \approx \frac{2E_h}{E_{CM}}
\]

Figure 2. shows an energy fraction \( z \) of a pion generated from \( e^+e^- \) annihilation experiment of \( E_{CM} \) (center-mass-energy) = 10.58 GeV.

The following relation consists in \( D_h^i(z) \) and cross sections:

\[
\frac{1}{\sigma} \frac{d\sigma(e^+e^- \rightarrow hX)}{dz} = \frac{\sum_q e_q^2[D_h^i(z) + D_h^\bar{q}(z)]}{\sum_q e_q^2}
\]

where \( e_q \) is a charge of quarks. The right-hand value doesn’t depend on center-mass-energy \( E_{CM} \) and only depends on \( z \). And that is scaling consists. Figure 3. shows a distribution varying \( E_{CM} \) in \( e^+e^- \) annihilation experiment[2]. This data uses the fact \( \sigma \propto E_{CM}^{-1} \). The distribution shows it’s independence of \( E_{CM} \).

Fragmentation Function describes parton characteristics, thus it doesn’t depend on the method of making parton. At inclusive \( ep \) scattering \( (ep \rightarrow hX) \), cross section \( \sigma \) is represented like this:

\[
\frac{1}{\sigma} \frac{d\sigma(ep \rightarrow hX)}{dz} = \frac{\sum_q e_q^2 f_q(x) D_h^i(z)}{\sum_q e_q^2 f_q(x)}
\]

where \( f_q \) is parton distribution function of proton. Sums are extends to all quarks and anti-quarks which can become mother of hadron \( h \). A function \( f_q(x) \) represents momentum distribution of a parton in proton. Variable \( x \) is called Bjorken x and defined as \( x = \frac{Q^2}{2M \mu} \). \( Q^2 \) is momentum transfer from electron to proton and \( \mu \) is energy transfer. \( M \) is mass of the proton.

Figure 2. A energy fraction \( z \) of a pion generated from \( e^+e^- \) annihilation experiment of \( E_{CM} = 10.52 \) GeV. All bins are normalized.

Figure 3. \( E_{CM}^{-1}d\sigma/dz \) distributions with \( E_{CM} = 14, 22, 34 \) GeV in \( e^+e^- \) annihilation experiment[2]

Figure 4. Hadron Production from Deep inelastic lepton scattering: \( ep \rightarrow hX \)
3. Fragmentation Models
For comparing an experimental data to the theory, it is necessary to understand how a parton is converted into hadrons. The rigorous method doesn’t exist mathematically now but there are various approaches. Two of the very popular models are the independent fragmentation and the string fragmentation.

3.1. Independent fragmentation model
Independent fragmentation model is based on a rule that scaling consists about parton’s longitudinal momentum and distribution of transverse momentum doesn’t depend on it’s energy [3]. We assume that parton mass is zero and transverse momentum is ignored. Then parton $q_i$ with longitudinal momentum $q_\parallel$ makes meson $M(q_i\bar{q}_{i+1})$ with longitudinal momentum $p_{mz} = zp_{q_\parallel}$ choosing $\bar{q}_{i+1}$ from $q_{i+1}\bar{q}_{i+1}$ in vacuum at the probability of $f(z)dz$. The other parton $q_{i+1}$ is going through at the probability of $1 - f(z)dz$ (Figure 5).

When hadron mass is not ignored, we use light cone variable $(E + p_\parallel)$ instead of momentum. Then energy fraction $z$ is defined as

$$z = \frac{(E_h + p_{h\parallel})}{(E_q + p_{q\parallel})} \quad (5)$$

If parton $q_i$ and $q_j$ hadronizes independently at each process, we can use Monte Carlo calculations to this iteration.

The total probability $D(z)dz$ of first parton to hadronize consists of the probability of the parton to mesons $f(z)dz$ at first hadronization and that of other partons.

$$D(z) = f(z) + \int_z^1 f(1 - \eta)D(\frac{z}{\eta})\frac{d\eta}{\eta} \quad (6)$$

$$\int f(z)dz = 1 \quad (7)$$

Then rapidity $\eta$ is

$$\eta = \frac{1}{2} \ln \left( \frac{E_h + p_{h\parallel}}{E_q + p_{q\parallel}} \right) = \ln \left( \frac{E_h + p_{h\parallel}}{E_q + p_{q\parallel}} \right)m_T \sim \ln z + \text{const.} \quad (8)$$

In light quarks (u, d, s), $f(z)$ and $D(z)$ are

$$f(z) = (1 + \alpha)(1 - z)^\alpha \quad (9)$$

$$D(z) = (1 + \alpha)\left(\frac{1 - z}{z}\right)^\alpha \quad (10)$$

where $\alpha = 0.6$ is a free parameter to set calculations.
3.2. String fragmentation model
String fragmentation model innovates a string concept of QCD [4],[5],[6]. After $q\bar{q}$ arises at $t = x = 0$, they go through along a light-cone $t^2 - x^2 = 0$. Then they store a potential energy proportional to their distance. So if parton has a small energy compare to this, the parton loses it’s kinetic energy at the end point of x-axis. They move inverse direction and their trajectory gets to zigzag (Figure 6.(a) ). If parton has a large energy, they break up string and make another $q\bar{q}$ pairs (Figure 6.(b) ). The $q\bar{q}$ pairs are created according to the probability of a tunnelling process $\exp(-\pi \frac{m_{q,T}^2}{\kappa})$ which depends on the transverse mass squared $m_{q,T}^2 \equiv m_q^2 + p_{Tq}^2$ and the string tension $\kappa \sim 1$ GeV/fm. The transverse momentum $p_T$ is locally compensated between quark and antiquark. Due to the dependence on the parton mass $m_q$ and hadron mass $m_h$, the production of strange and heavy-quark hadrons is suppressed. The light-cone momentum fraction $z = \frac{(E + p_{\|})_h}{(E + p)_q}$, where $p_{\|}$ is the momentum of the formed hadron $h$ along the direction of the quark $q$, is given by the string-fragmentation function

$$f(z) \sim \frac{(1 - z)^a}{z} \exp(-b z \frac{m_{h,T}^2}{z})$$

(11)

where $a$ and $b$ free parameters. These parameters are obtained by measured data $a = 0.11$ and $b = 0.52$ GeV$^{-2}$ as determined in [6]

![Figure 6](image)

Figure 6. (a) The motion of $q$ and $\bar{q}$ in the CM frame. The hatched areas show where the field is nonvanishing.
(b) The final picture when $q$ and $\bar{q}$ move with large energies in opposite directions. The field has broken at many places by the production of $q\bar{q}$ pairs. Hatched areas indicate nonvanishing field.

4. Summary
Fragmentation function described the process from a quark to Hadrons . It gives detailed information about the parton with observed hadron jets. A parton fragmentation function $D^h$ depends on energy fraction $z$. To compare a experimental data to the theory, independent fragmentation model and string fragmentation models are effective.

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