Tool diameter optimization in S-shaped test piece machining

Fu Meng¹, Guan Liwen¹, Wang Liping¹, Mo Jiao¹ and Zhao Xiao²

Abstract
S-shaped test piece is used to assess the performance of five-axis numerical control machine tools. Compared to traditional test method and test pieces, it performs better and was agreed to be included in the draft international standard at the 79th ISO/TC39SC2 meeting. However, the lack of theoretical research is the barrier to apply for standard and to be widely used. Therefore, this paper aims to study one of the theoretical research that how to choose the suitable tool diameter. We proposed a new research method based on software AdvantEdge, Vector Method for error calculation, Microelement Model of cutting force and deformation model of cantilever beam. The simulation results show that the 20 ≤ diameter < 30 is the proper choices and the 20 is the best choice, which is consistent with the empirical consequence. This method not only provides the theoretical support for the S-shaped test piece, but also can be applied to other parts with complex curved surfaces.

Keywords
Five axis machining, S-shaped test piece, theoretical error, deformation error, tool diameter optimization

Date received: 7 January 2020; accepted: 4 December 2020
Handling Editor: James Baldwin

Introduction
S-shaped test piece,¹,² proposed by Chengdu Aircraft Industry Group, is a standard test piece and can be used for comprehensive testing of five axis machine tools. This standard specimen aims at solving the problem that machining parts with complex surfaces (such as the aerospace thin-walled components) may be unqualified when the machine tool has passed the traditional test method. Five-axis machine tool is different from three-axis one, so the previous detection methods (e.g. R-test³) or test pieces (e.g. NAS specimen⁴) cannot reflect the processing performance of five-axis machine accurately. Even though in these methods or sample testing qualified basis, Out-of-roundness-tolerance parts will still exist when processing. Through the newly proposed S-shaped test piece, the performance of the qualified machine tools can meet the processing requirements. Therefore, this test piece was included in the draft standard (DIS) at the 79th ISO/TC39SC2 meeting in May 2016.⁵

However, the shortcoming of S-shaped test piece is that it was proposed based on practical experience. Therefore, despite of repeated modification and research, its theoretical support is still lacking, which is a serious problem to apply for international standards.⁶,⁷ In addition, due to the lack of theoretical

¹Department of Mechanical Engineering, Institute of Manufacturing Engineering, Tsinghua University, Beijing, PR China
²Department of Mechatronic Engineering, Institute of Manufacturing Engineering, University of Electronic Science and Technology, Chengdu, PR China

Corresponding author:
Guan Liwen, Institute of Manufacturing Engineering, Department of Mechanical Engineering, Tsinghua University, Haidian District, Beijing 100084, PR China.
Email: guanlw@mail.tsinghua.edu.cn

Creative Commons CC BY: This article is distributed under the terms of the Creative Commons Attribution 4.0 License (https://creativecommons.org/licenses/by/4.0/) which permits any use, reproduction and distribution of the work without further permission provided the original work is attributed as specified on the SAGE and Open Access pages (https://us.sagepub.com/en-us/nam/open-access-at-sage).
support, S-shaped test piece cannot find where the issues exist if unqualified, as the previous methods do. Therefore, the theoretical research on S-shaped test piece and the study of error tracing based on the theoretical research are the necessary ways to improve and popularize the S-shaped test piece.

One of the theory research is the selection of the tool diameter when machining S-shaped test piece. Chengdu Aircraft Industry Group have found the 20 mm of diameter is most appropriate for machining S-shaped test piece, which cause deformation error and the theoretical error in the optimum state. However, the company failed to give the theoretical explanation of this choice. This paper studies on this problem. The importance of this issue is that the S-shaped test piece is proposed to test the performance of the machine tool. Because the deformation error and the theoretical error caused by unreasonable tool diameter cannot reflect the performance of the machine, so they must be optimized as much as possible.

However, because of the mass production, the engineering experience and multiple processing early trials are enough to choose suitable tool diameter. Therefore, the related theoretical research is little. This paper puts forward the research method of the selection of the optimal tool diameter when machining S-shaped test piece. During the pre-treatment, we use the Vector Method for error calculation in order to simplify the calculation. Then, the milling simulation software AdvantEdge is used to obtain the cutting parameters. Next, the cutting force is calculated according to the Microelement Model. Finally, discrete points of tool axis can be obtained according to the Deflection Formula and the total error can be calculated. The simulation results show that the ideal tool diameter range is 20–30 mm, and the optimal tool diameter is 20 mm. This study not only provides theoretical support for cutting tools with diameter of 20 mm when cutting S-shaped test piece, but also can provide theoretical reference to the selection of the tool diameter when machining any parts with complex surface, which has important significance in theory and practice.

**Theoretical error and deformation error**

Figure 1 shows the model of the S-shaped test piece, which is consist of two S surfaces A and B, and the surfaces are composed of two S type B spline curves, top curve \( C_1(u) \) and bottom curve \( C_2(u) \).

The projection of top curve \( C_1(u) \) and bottom curve \( C_2(u) \) is not consistent, so the existence of twist angle \( \gamma \) cause the theoretical error of the S-shaped test piece, as shown in Figure 2. Because the theoretical error cannot be eliminated due to twist angle, the researchers have proposed all kinds of method to position tool. This paper adopt the traditional Single Point Offset (SPO) method with top curve to conduct the simulation, as it is the most popular method.

When the method to position the tool is the same, the smaller tool diameter would lead to little theoretical error. In the extreme case where tool diameter equals zero, the theoretical error would disappear. However, the smaller the tool diameter is, the worse the stiffness of tool is. Therefore, the tool is more prone to large deformation in the machining process. Since the deformation error decreases with the decrease of tool diameter and the theoretical error decreases with the increasing of tool diameter, the selection of tool diameter is crucial. The processing performance should not contain the theoretical error and deformation error, so giving the appropriate recommendation value of tool diameter is necessary.

**Optimization of tool diameter based on simulation**

**Optimization process**

The block diagram of the principle of simulation in this paper is shown in Figure 3. Firstly, according to the calculation principle of SPO, we compile MATLAB program to generate G code files, and import G code data to milling processing simulation software AdvantEdge According to the calculation principle of SPO, we
compile MATLAB program to generate G code files, import G code data to milling processing simulation software AdvantEdge, and simulate milling of S-shaped test piece. The AdvantEdge can give the corresponding cutting parameters for each row G code. Secondly, the cutting parameter is introduced into the Microelement Model to calculate the cutting force. Then the flexural cantilever model and the Deflection Formula obtain the deformation value and deformation direction of each point on the tool axis. Finally, the actual coordinate of discrete points of the tool axis can be calculated, so the minimum distance between the discrete points and test points is considered as the error value, which includes theoretical error and deformation error.

However, for the sake of accuracy, the G code data imported to the AdvantEdge need to be especially dense, which makes G code data lengthy (up to hundreds of thousands of lines). In addition, when the discrete points are finally obtained, the data becomes larger and larger and makes the calculation slow. Therefore, using the Vector Method for error calculation and finding the main G code data would decrease the data size. In this process, the objective tool axis appears the minimum distance to the test point is considered. Then, one G code corresponds to one tool axis, so extract G code data near the objective G code can decrease data size a lot. For example, assuming that there are 30 test points and we consider ±10 lines away from the objective G code, only 6300 lines of G code will be used for subsequent calculation. Compared to the hundreds of thousands of lines at the beginning, the optimized method (shown in Figure 4) significantly decrease the computing time.

### The Vector Method for error calculation

The Vector Method is shown in Figure 5. The test point M is the point on the ruled surface. O is the point of tool tip. T is the unit vector of tool axis from the tool tip to the tool handle. P is the unit vector from the tool tip to the point M. d need solved, is the distance between the point M and the tool axis. In calculation, the sign of \( P \cdot T \) need to judge. \( P \cdot T < 0 \) means the angle
between \( P \) and \( T \) is more than 90 degrees, so the minimum distance from the point \( M \) to the tool axis is \( ||MO|| \). If the angle between \( P \) and \( T \) is less than 90 degrees, \( d = (||P||^2 - (P \cdot T)^2)^{1/2} \) is what we desire.

The Microelement Model

Figure 6 shows the milling force of Microelement Model. In the process of cutting the workpiece, the corresponding cutting center angle range is \([\theta_{\text{min}}, \theta_{\text{max}}]\). The range is divided into \( N \) equal proportion \( \theta_1, \theta_2, \ldots, \theta_{n-1}, \theta_n \), where \( \theta_i \) represents a range of \( d\theta \) (For example, \( \theta_i \) represent \([\theta_{\text{min}}, \theta_{\text{min}} + d\theta]\), and so on). This method is equivalent to discrete a spiral-milling cutter to many tiny discs along the tool axis at equal intervals \( dz \). (With the spiral angle \( \beta \) and the tool radius \( R \), we obtain \( dl = \tan \beta \cdot dz = R \cdot d\theta \), so \( dz = R \cdot d\theta / \tan \beta \).

Every tiny disc can be seen as a thin spiral-milling cutter with axial width \( dz \), so according to the empirical formula, it can be obtained:

\[
\begin{align*}
F_t(\theta_i) &= K_t \cdot h(\theta_i)^{m_t} \cdot dz \\
F_r(\theta_i) &= K_r \cdot h(\theta_i)^{m_r} \cdot dz \\
F_a(\theta_i) &= K_a \cdot h(\theta_i)^{m_a} \cdot dz 
\end{align*}
\]

where \( F_t(\theta_i), F_r(\theta_i), \) and \( F_a(\theta_i) \) are tangential force, radial force and axial force, respectively. \( K_t, K_r, \) and \( K_a \) are coefficients of tangential force, radial force and axial force, respectively. \( m_t, m_r, \) and \( m_a \) are the index of the effect that cutting thickness put on tangential force, radial force and axial force, respectively. The above parameters can be obtained by referencing to the calibration methods. In addition, \( h(\theta_i) \) represents the instantaneous cutting thickness.

In the above parameters, \( dz \) depends on \( d\theta \), which can be given as needed. However, \( \theta_{\text{min}}, \theta_{\text{max}} \) and \( h(\theta_i) \) are still unknown. As seen in Figure 7, \( B_0B_2 \) is the trajectory of current cutting edge and \( A_1A_2 \) is the trajectory of preceding cutting edge. Besides, \( A_2B_0B_1B_2 \) is the part of the workpiece to be processed and \( v_f \) is the feed speed. Therefore, the maximum cutting center angle \( \theta_{\text{max}} \) equals to \( \arccos(R - a_c/R) \), where \( a_c \) is maximum cutting thickness. Moreover, the minimum cutting center angle \( \theta_{\text{min}} \) is caused by the processing remnants. Because this article focuses on qualitative analysis, the requirements of results are not very high. It is beneficial to speed up the calculation that supposing \( \theta_{\text{min}} = 0 \). \( h \), used for \( h(\theta_i) \), is the minimum cutting center angle when the cutting thickness equals to \( a_c \). From the Figure 7, we obtain \( \theta_h = \arctan \left( \sqrt{R^2 - (R^2 - a_c^2)}/R - a_c \right) \), where \( f_z \) is the feed engagement.

The expression of calculation of instantaneous cutting thickness is as follows:

\[
\begin{align*}
h &= \begin{cases} 
R - \frac{R - a_c}{\cos \theta} & , \theta \in (\theta_{\text{min}}, \theta_{\text{max}}) \\
R - \left( \frac{R^2 + \left( \frac{f_z}{m_a} \right)^2}{2} - \left( \frac{1}{2} + \frac{f_z \cdot \cos \theta}{2\pi R + Z_a \cdot \cos \theta} \right)^2 \right)^{1/2} & , \theta \in (\theta_{\text{min}}, \theta_{\text{max}}) 
\end{cases}
\end{align*}
\]

Deformation Computation

The schematic diagram of cutting force of a microelement is shown in Figure 8. Assuming that the cutting center angle of the tiny disc of the tool is \( \theta_{\text{c}} \), the tangential force \( F_t(\theta_c) \) produces rotational torque, which makes the tool have the trend of torsion. However, the radial force \( F_r(\theta_c) \) points to the tool axis, which causes the deformation of the tool along the direction of the radial force. When the radial concentrated load is acting on the cutter in different directions, the tool produces a corresponding deflection deformation along the direction of each load.

The actual deformation is particular complex and there are still many deformation directions under the
Microelement Model. It is necessary to ignore secondary factors in order to simplify the calculation and analysis, so only the deflection deformation along the direction of vector $N$ are obtained where the $N$ is the normal vector of the S surface. Besides, deflection model of cantilever beam is applied due to its simple and intuitionistic.

As shown in Figure 9, the point O is the fixed point and the point K is the loading point. Therefore, the deflection $\omega(z)$ is calculated according the Deflection Formula and shown in the below:

$$\omega(z) = \frac{FZ_k}{6EI} (3Z_k - z), z \in [0, Z_k]$$
$$\omega(z) = \frac{FZ_k^3}{6EI} + \sin\left(\frac{FZ_k^3}{6EI}\right) \cdot (z - Z_k), z \in [Z_k, L]$$

where $F$ is the applied load, the $E$ is the elastic modulus of the equivalent cantilever beam, and the $I$ is the polar moment of inertia of the equivalent cantilever beam.

According to the principle of superposition method to calculate the deflection, the sum of the deflection of the tiny disk under each $F_r$ can be calculated. In addition, the Microelement Model in Section 3.3 can obtain all $F_r$, and the force location of $F_r$ is the position of the tiny disk.

Therefore, according to the Microelement Model in Section 3.3, the process diagram to calculate the actual coordinates of the discrete points of tool axis after the deformation is shown in Figure 10. Firstly, for each discrete point $T_i$ of tool axis, we calculate $\omega_i$ (the sum of deflection caused by all $F_r$) along the direction of $N$. Then, the actual coordinates $T_i'$ of the discrete points can be obtained as $T_i' = T_i + N \cdot \omega_i$.

**Simulation results**

The simulation results are shown in Figure 11 with the $d = 18, 20, 22,$ and $30$. The selection of tool diameter sequence is referenced by the recommended value of GBT 6117.1-1996.15 The following conclusions can be drawn from the simulation results:

1. From the numerical point of view, the deformation error may make up for the theoretical error
The simulation results of theoretical error and deformation error:

- (a) $d = 18 \text{ mm}$ and $H = 7 \text{ mm}$
- (b) $d = 20 \text{ mm}$ and $H = 7 \text{ mm}$
- (c) $d = 18 \text{ mm}$ and $H = 15 \text{ mm}$
- (d) $d = 20 \text{ mm}$ and $H = 15 \text{ mm}$
- (e) $d = 18 \text{ mm}$ and $H = 23 \text{ mm}$
- (f) $d = 20 \text{ mm}$ and $H = 23 \text{ mm}$
- (g) $d = 22 \text{ mm}$ and $H = 7 \text{ mm}$
- (h) $d = 30 \text{ mm}$ and $H = 7 \text{ mm}$
- (i) $d = 22 \text{ mm}$ and $H = 15 \text{ mm}$
- (j) $d = 30 \text{ mm}$ and $H = 15 \text{ mm}$
- (k) $d = 22 \text{ mm}$ and $H = 23 \text{ mm}$
- (l) $d = 30 \text{ mm}$ and $H = 23 \text{ mm}$

### Conclusion

The proposed S-shaped test piece is of great significance for the accuracy inspection and performance evaluation of five axis machine tools. However, the theoretical support is rather deficient, which limits the popularization and application of the S-shaped test piece. This paper analyzes the reason of the theoretical error and deformation error, and explains the trend of the error changes with tool diameter changes. The core is to put forward the reasonable selection method of tool diameter. This method combines the Vector Method, the Microelement Model and the Deflection Formula. The whole calculation process is clear, simple and direct, and finally give the appropriate selection of the tool diameter when machining S-shaped test piece, which is consistent with the company’s experimental result. This method can not only be applied to the selection of tool diameter for S-shaped test piece, but also provide theoretical reference for the selection of tool diameter when machining arbitrary parts with complex surfaces.
Figure 12. The deformation error when $d = 18$ and $d = 20$. 
Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This research was financially supported by The National Natural Science Foundation of China (Grant No. 51675301).

ORCID iDs

Guan Liwen https://orcid.org/0000-0001-9052-4235
Mo Jiao https://orcid.org/0000-0003-0346-8066

References

1. Su Z and Wang L. Latest development of a new standard for the testing of five-axis machine tools using an s-shaped test piece. Proc Inst Mech Eng B J Eng Manuf 2015; 229: 1221–1228.
2. Wang W, Jiang Z, Li Q, et al. A new test part to identify performance of five-axis machine tool-part ii validation of s part. Int J Adv Manuf Technol 2015; 79: 739–756.
3. Li J, Xie F, Liu XJ, et al. A geometric error identification method for the swiveling axes of five-axis machine tools by static R-test. Int J Adv Manuf Technol 2016; 89: 3393–3405.
4. AIA/NAS National Aerospace Standard *979 69, Uniform Cutting Tests - NAS Series Metal Cutting Equipment Specifications, Aerospace Industries Association of America, Inc. 1969, pp.34-37.
5. Mou WP. Test conditions for machining centers, part 7: accuracy of a finished test piece-M5. In: The 74th meeting of ISO/TC39/SC2, Hangzhou, China, 24–28 September 2012, pp.24-28.
6. Guan L, Mo J, Fu M, et al. An improved positioning method for flank milling of S-shaped test piece. Int J Adv Manuf Technol 2017; 92: 1349–1364.
7. Guan L, Mo J, Fu M, et al. Theoretical error compensation when measuring an S-shaped test piece. Int J Adv Manuf Technol 2017; 93: 2975–2984.
8. Sui XL, Yang R, Wang Q, et al. Analysis on milling force based on AdvantEdge FEM. Appl Mech Mater 2013; 300–301: 253–260.
9. Song G, Li J and Sun J. Approach for modeling accurate undeformed chip thickness in milling operation. Int J Adv Manuf Technol 2013; 68: 1429–1439.
10. Rubio DW, Lagarrigue P, Dessein G, et al. Calculation of tool paths for a torus mill on free-form surfaces on five-axis machines with detection and elimination of interference. Int J Adv Manuf Technol 1998; 14: 13–20.
11. Redonnet JM, Rubio W and Dessein G. Side milling of ruled surfaces: optimum positioning of the milling cutter and calculation of interference. Int J Adv Manuf Technol 1998; 14: 459–465.
12. Bedi S, Mann S and Menzel C. Flank milling with flat end milling cutters. Comput Aided Des 2003; 35: 293–300.
13. Ni QM, Li CX, Wu GL, et al. Modeling and simulation of milling forces for flexible ball-end milling cutters. Chin J Mech Eng 2002; 38(3): 108–112.
14. Wang L, Si H, Guan L, et al. Comparison of different polynomial functions for predicting cutting coefficients in the milling process. Int J Adv Manuf Technol 2018; 94: 2961–2972.
15. GB/T 6117.1-1996. Type and size of milling cutter.