Importance of Mixing for Exotic Baryons

MICHAL PRASZALOWICZ

M. Smoluchowski Institute of Physics, Jagellonian University, Reymonta 4, 30-059 Krakow, Poland

Exotic antidecuplet baryons are predicted to be not only surprisingly light but also very narrow. First we explain how small decay width arises in the quark soliton model. Next, we study possible mixing of exotic antidecuplet with Roper octet and discuss its phenomenological consequences.

PACS numbers: 11.30.Rd, 12.39.Dc, 13.30.Eg, 14.20.–c

1. Introduction

Despite recent scepticism concerning early announcements of the discovery of exotic strange baryon Θ+(1540) two collaborations DIANA and LEPS confirmed their original results [1, 2, 3]. We refer the reader to recent experimental reviews [4, 5, 6]. In this article following [7] we assume that Θ+ exists with a mass equal 1540 MeV and total width Γ < 1 MeV. An immediate consequence is the existence of the whole exotic SU(3) multiplet: 10. Apart from truly exotic states antidecuplet contains cryptoexotic nucleon-(N10) and Σ-like states (Σ10). The interpretation of these states is not well understood: one may try to associate them with some known resonances, or one may postulate the existence of new resonances with nucleon or Σ quantum numbers. This is the approach which we adopt here [7]. Following Refs. [8]–[12] we assume that there exists new, narrow nucleon resonance N(1685) which we will interpret as a member of 10. If so, N10 decays have to satisfy the following constraints [7] originally discussed in [8].

\[ \Gamma_{N_{10} \to \pi N} < 0.5 \text{ MeV}, \quad \text{Br}_{N_{10} \to \eta N} > 0.2, \quad 5 \text{ MeV} < \Gamma_{\text{tot}}^{N_{10}} < 25 \text{ MeV}. \quad (1) \]

Unfortunately the small partial width of N(1685) to \( \pi N \) contradicts SU(3) symmetry relations between the decay constants of \( N_{10} \).

In this short note we would like to emphasize the importance of mixing both for decays and mass spectra of the putative exotic antidecuplet.
baryons. The exotic states have been already anticipated by the founders of the quark model although they did not elaborate on them. Later the arguments have been raised that they should be heavy and wide. In contrast, chiral soliton models predicted that the pentaquark masses were generically small (i.e. in the range of 1.5–1.6 GeV) [13, 14, 15]. It was much more difficult to accommodate the small decay width of Θ+ [16]. In Sect. 2 we explain how small decay width arises naturally in the Chiral Soliton Quark Model (χQSM). Next, in Sect. 3 we argue that the decay coupling $g_{ΘNK}$ is further reduced due to Gell-Mann–Okubo (GMO) mixing caused by the nonzero $m_s$. In Sect. 4 we show how additional mixing of $\bar{10}$ with Roper octet can change decay patterns of $N_{\bar{10}}$. We estimate allowed range of mixing angles and present predictions for remaining members of $\bar{10}$: Σ$\bar{10}$ and Ξ$\bar{10}$. Conclusions are presented in Sect. 5.

2. Decay widths in Chiral Soliton Quark Model

In Ref. [15] the following nonrelativistic formula for the decay width has been used:

$$\Gamma_{B_1 \rightarrow B_2 \varphi} = \frac{g_{B_1 B_2 \varphi}^2}{2\pi(M_1 + M_2)^2} p_\varphi^3.$$  (2)

It follows from the Goldberger-Treimann relation between axial and strong decay constants. Here $M_1$ is the mass of the decaying baryon, $M_2$ the mass of the decay product and $p_\varphi$ is the outgoing meson momentum. Generically $\Gamma_{Θ+ \rightarrow NK}$ given by (2) would be still in the range of a few hundreds of MeV [16] if not for the terms non-leading in $1/N_c$ expansion. Indeed, even without mixing the decay constant $g_{B_1 B_2 \varphi}$, which stands for the matrix element of the tensor decay operator $O^{(8)}_\varphi$ between the physical baryon wave functions $|B_{\text{phys}}\rangle$:

$$g_{B_1 B_2 \varphi} = \left< B_2^{\text{phys}} \right| O^{(8)}_\varphi \left| B_1^{\text{phys}} \right>$$  (3)

is the sum of three different contributions that are formally of different order in $1/N_c$. However, they are multiplied by the SU(3) Clebsch-Gordan coefficients that also depend on $N_c$ [17]. For example:

$$g_{ΘNK}^2 = \frac{9(N_c + 1)}{(N_c + 3)(N_c + 7)} G_{\Theta \bar{10}}^2 \quad \text{with} \quad G_{\Theta \bar{10}} = G_0 - \frac{N_c + 1}{4} G_1 - \frac{1}{2} G_2$$  (4)

where $G_0 \sim N_c^{3/2}$, $G_{1,2} \sim N_c^{1/2}$. Similarly

$$g_{\Delta N\pi}^2 = \frac{3(N_c - 1)(N_c + 5)}{2(N_c + 1)(N_c + 7)} G_{10}^2 \quad \text{with} \quad G_{10} = G_0 + \frac{1}{2} G_2.$$  (5)
Chiral soliton models provide us with specific predictions for constants $G_{0,1,2}$ [18]. Had we neglected $G_1$ and $G_2$ in Eqs. (4,5) (which would be inconsistent for $g_{\Theta NK}$ because of $N_c$ enhancement of $G_1$) we would have obtained for $N_c = 3$:

$$g_\Delta N = g_{\Theta NK} \sim 17.6$$

estimating $G_0$ from the experimental value of $\Delta$ decay width, and consequently

$$\Gamma_{\Theta NK} \sim 150 \text{ MeV.} \quad (6)$$

We see that small decay width of $\Theta^+$ results from the cancelation in (4).

Indeed, the authors of Ref. [15] have shown that in the nonrelativistic limit of $\chi QSM$ one obtains that

$$G_0 = -(N_c + 2)G, \quad G_1 = -4G, \quad G_2 = -2G,$$

with $G \sim N_c^{1/2}$ and consequently $\Gamma_{\Theta NK} = 0$. In the same limit $\chi QSM$ predicts that $g_A = 3/5$ and $\mu_p/\mu_n = -3/2$. It follows that antidecuplet decay constants are small.

### 3. Gell-Mann Okubo Mixing

Treating $m_s$ corrections as perturbation introduces mixing [7] for $m_s$

$$\left| B_{\text{phys}}^{10} \right> = \left| 8_{1/2}, B \right> + c_{10} \left| 10_{1/2}, B \right> + c_{27} \left| 27_{1/2}, B \right>,$$

$$\left| B_{\text{phys}}^{35} \right> = \left| 35_{1/2}, B \right> + d_{35} \left| 27_{1/2}, B \right>, \quad (7)$$

where subscripts refer to spin. For some specific states mixing constants $c_{10}, d_{35} \sim m_s$ may be equal zero due to the isospin. For example $\Theta^+$ mixes only with $35$, but this component of the wave function does not contribute to the decays to octet. Therefore only mixing of the final nucleon with $10$ and $27$ modifies the decay constant:

$$g_{\Theta NK}^2 = \frac{3}{5} \left[ G_{10} + \frac{5}{4} G_{10} H_{10} - \frac{7}{4} G_{27} H'_{27} \right]^2. \quad (8)$$

Since $G_{10}$ is small the admixtures proportional to the reduced matrix elements

$$H_{10} \sim \left< 10_{1/2}, B' \right| \hat{O}_r^{(8)} \left| 10_{1/2}, B \right>, \quad H'_{27} \sim \left< 27_{1/2}, B' \right| \hat{O}_r^{(8)} \left| 10_{1/2}, B \right> \quad (9)$$

are important even if mixing parameters $c_{10}$ and $c_{27}$ are not large (for definitions see Ref. [19]). Neither $H_{10}$ nor $H'_{27}$ vanish in the nonrelativistic limit. Therefore in this limit $\Theta^+$ decay occurs entirely due to the mixing. For realistic model parameters (when $G_{10} > 0$) there is a cancelation (note that $H_{10} < 0$ and $H'_{27} > 0$) between different terms in (8) and $g_{\Theta NK}$ is
further suppressed. Mixing affects decay patterns of pentaquarks violating SU(3) relations between the decay constants $g_{B_1 B_2 \phi}$ [19].

On somewhat more phenomenological ground let us consider only $8 \leftrightarrow \bar{10}$ mixing [7] defining

$$g_{\theta NK} = \cos \alpha \, g_{\bar{10}} + \sin \alpha \, h_{\bar{10}}$$

(10)

which can be directly extracted from (2) if $\Gamma_{\Theta NK}$ is known. Throughout this paper we assume that $\Gamma_{\Theta NK} \approx 1$ MeV, hence $g_{\theta NK} \approx 1.4$. It is then possible to express decay constants of other members of antidecuplet in terms of measurable physical parameters such as $g_{\pi NN} \approx 13.2$, $\varepsilon = F/D \approx 0.56$, mixing angle $\alpha$ and one a priori unknown parameter $h_{\bar{10}}$ which can be estimated from $\chi$QSM calculations. Here we take $h_{\bar{10}} = -7$ [7]. Then we have for example:

$$g_{\pi NN} = \frac{1}{2} \cos \alpha \, g_{\theta NK} - \tan \alpha \sqrt{3} g_{\pi NN},$$
$$g_{\eta NN} = \frac{1}{2} \cos \alpha \, g_{\theta NK} - \frac{1}{2} \sin 2\alpha \, h_{\bar{10}} + \tan \alpha \frac{3\varepsilon - 1 \, g_{\pi NN}}{1 + \varepsilon} \sqrt{3}.$$  

(11)

If we want to interpret $N(1685)$ as $N_{\bar{10}}$ we need to satisfy bounds [1]. This is quite difficult within one angle scenario. It is possible to nullify $g_{\pi NN}$ by a suitable choice of mixing angle $\alpha \approx 0.03$, but then the mean octet mass (i.e. the nucleon mass before mixing) which is experimentally 1151 MeV comes out wrong [7].

$$M_8 = M_{N_{\bar{10}}}^\text{phys} \cos^2 \alpha + M_{N_{\bar{10}}}^2 \sin^2 \alpha \approx M_{N_{\bar{10}}}^\text{phys}.$$  

(12)

The mixing angle that satisfies (11) is order of magnitude too small to account for baryon masses. For realistic mixing angles $g_{\pi NN}$ is dominated by $g_{\pi NN}$ and $\Gamma_{\pi NN} > \Gamma_{\eta NN}$ in contradiction with experimental data for $N(1685)$.

4. Mixing with Roper

In order to satisfy conditions (11) with more realistic mixing angles we have considered in Ref. [7] scenario in which exotic antidecuplet can mix with Roper resonance octet (mixing angle $\phi$). For Roper octet GMO mass formulae work with much worse accuracy than for the ground state octet [20, 21], so there is a need for additional mixing. Since $\varepsilon_{\text{Roper}} \sim 1/3$ [21] Roper decay to $N\eta$ is negligible. However Roper admixture contributes to $N\pi$ and other decay modes:

$$g_{\pi NN} = \frac{1}{2} \cos \phi \, \cos \alpha \, g_{\theta NK} + \cos \phi \tan \alpha \sqrt{3} g_{\pi NN} - \tan \phi \, g_{\pi NN},$$
$$g_{\eta NN} = \frac{1}{2} \cos \phi \, \cos \alpha \, g_{\theta NK} - \frac{1}{2} \sin 2\alpha \, h_{\bar{10}} + \cos \phi \, \tan \alpha \frac{3\varepsilon - 1 \, g_{\pi NN}}{1 + \varepsilon} \sqrt{3}.$$  

(13)
importance-of-mixing\textsuperscript{v3} printed on December 23, 2012

Fig. 1. Total (solid black line) decay width (divided by 25) and partial decay width of $N_{10}^{-}$ to $\pi N$ (long dashed red line) in MeV together with branching ratio (short dashed blue line) of $N_{10}^{-} \rightarrow \eta N$ as functions of mixing angle $\alpha$ along the line (14). Thin vertical lines correspond to the limits on the mixing angle $\alpha$. The plot is made for $h_{10}^{-} = -7$.

Since $g_{RN\pi} \sim 12$ is comparable with $g_{\pi NN}$ one may suppress $g_{N_{10}^{-}N\pi}$ without changing much $g_{N_{10}^{-}N\eta}$. In Ref. [7] we have found that conditions (1) are satisfied in the vicinity of the line

$$\phi(\alpha) = 0.0508 - 2.207\alpha, \quad 0.079 < \alpha < 0.159.$$  \hspace{1cm} (14)

We see that mixing angles are reasonable. The decay widths and branching ratio to $N\eta$ along (14) are plotted in Fig. 1.

In Fig. 2 we plot $g_{N_{10}^{-}N\pi}$ and $g_{N_{10}^{-}N\eta}$ together with their different components along the line (14). We see that indeed $g_{N_{10}^{-}N\pi}$ is small due to the cancellation between $g_{\pi NN}$ and $g_{RN\pi}$, while $g_{N_{10}^{-}N\eta}$ rises moderately when mixing increases.

Having established the range of mixing angles we can predict masses of the remaining antidecuplet members [7]:

$$1795 \text{ MeV} < M_{N_{10}^{-}} < 1830 \text{ MeV},$$  \hspace{1cm} (15)

$$1900 \text{ MeV} < M_{\Xi_{10}^{-}} < 1970 \text{ MeV}.$$  \hspace{1cm} (16)

Note that bound (16) contradicts the result of NA49 $M_{\Xi_{10}^{-}} \sim 1860 \text{ MeV}$ [22]. From our analysis it follows that total decay width of $\Xi_{10}^{-}$ to $K\Sigma$ and $\pi\Xi$
is of the order of 10 MeV. Total width of $\Sigma_{10}$ does not exceed 30 MeV but is also constrained from below to be larger that 10 MeV. Most prominent decay channels are $KN$ and $\pi \Lambda$ with branching ratios approximately 60% and 20% respectively. Due to the mixing SU(3) forbidden decays to decuplet are possible, but small, at the level of 5 to 9%.

5. Summary and Conclusions

Mixing induced by $m_s$ was first studied within the $\chi$QSM already in Ref. [15] but only in the leading order in $N_c$. It was extended to non-leading terms in Ref. [8] and [19]. Mixing appears also in other approaches to pentaquarks. For example in a diquark model [23] antidecuplet mixes with an accompanying cryptoexotic octet. Diagonalization of strangeness induces ideal (large) mixing between these two representations. The resulting physical states of nucleon quantum numbers have been interpreted as Roper and $N^*(1710)$. However due to the ideal mixing these two states should have comparable widths [24], while experimentally they are differ substantially. The discussion of masses and decay widths of the $N^*$ states under assumption that they correspond to the Roper and $N^*(1710)$ done in Ref. [25] still indicates that it is impossible to match the mass splittings with the observed branching ratios for these two resonances even for arbitrary mixing. Whether any different assignment of the diquark $N^*$ states would be compatible with the decay patterns deserves a separate study.

In this short note we have argued that due to the smallness of the reduced matrix elements of $10 \rightarrow 8$, which is natural in chiral soliton models, mixing with other SU(3) representations has to be taken into account. Unfortunately, for the time being we can only speculate which mixing scenario is phenomenologically impossible, allowed or desired. Here we have examined a possibility that antidecuplet mixes with the Roper octet. Mixing of Roper and the ground state octets is presumably very small. Indeed the first order GMO mass formulae work very well for the ground state octet so there is almost no space for additional mixing.

Acknowledgements This paper is based on a common work with Maxim Polyakov and Klaus Goeke. I would like to thank the organizers of the workshop "Excited QCD" for stimulative and creative atmosphere.

REFERENCES

[1] T. Nakano et al. [LEPS Collaboration], Phys. Rev. C 79, 025210 (2009) [arXiv:0812.1035 [nucl-ex]].

[2] T. Hotta [LEPS Collaboration], Acta Phys. Polon. B 36, 2173 (2005).
[3] V. V. Barmin et al. [DIANA Collaboration], Phys. Atom. Nucl. 70, 35 (2007) [arXiv:hep-ex/0603017] and [arXiv:0909.4183 [hep-ex]].

[4] T. Nakano, Nucl. Phys. A 755, 3 (2005).

[5] V. D. Burkert, Int. J. Mod. Phys. A 21, 1764 (2006) [arXiv:hep-ph/0510309].

[6] M. V. Danilov and R. V. Mizuk, Phys. Atom. Nucl. 71, 605 (2008).

[7] K. Goethe, M. V. Polyakov and M. Praszalowicz, [arXiv:0912.0469 [hep-ph]].

[8] R. A. Arndt, Y. I. Azimov, M. V. Polyakov, I. I. Strakovsky and R. L. Workman, Phys. Rev. C 69 (2004) 035208 [arXiv:nucl-th/0312126].

[9] V. Kuznetsov [GRAAL Collaboration], [arXiv:hep-ex/0409032].

[10] V. Kuznetsov et al., Phys. Lett. B 647 (2007) 23. [hep-ex/0606065]

[11] V. Kuznetsov et al., Acta Phys. Polon. B 39, 1949 (2008) [arXiv:0807.2316 [hep-ex]].

[12] V. Kuznetsov et al., [arXiv:1003.4585 [hep-ex]].

[13] L.C. Biedenharn and Y. Dothan, Monopolar Harmonics in SU(3)F as eigenstates of the Skyrme-Witten model for baryons, E. Gotsman and G. Tauber (eds.), From SU(3) to gravity, p. 15-34; L.C. Biedenharn, Y. Dothan and A. Stern, Phys. Lett. B 146, 289 (1984).

[14] M. Praszalowicz, talk at Workshop on Skyrmions and Anomalies, M. Ježabek and M. Praszalowicz eds., World Scientific 1987, page 112 and Phys. Lett. B 575 (2003) 234 [hep-ph/0308114].

[15] D. Diakonov, V. Petrov and M. V. Polyakov, Z. Phys. A 359 (1997) 305 [arXiv:hep-ph/9703373].

[16] H. Weigel, Eur. Phys. J. A 2 (1998) 391 [arXiv:hep-ph/9804260] and AIP Conf. Proc. 549 (2002) 271 [arXiv:hep-ph/0006191].

[17] M. Praszalowicz, Phys. Lett. B 583 (2004) 96 [arXiv:hep-ph/0311230].

[18] J. R. Ellis, M. Karliner and M. Praszalowicz, JHEP 0405 (2004) 002 [arXiv:hep-ph/0401127].

[19] M. Praszalowicz, Acta Phys. Polon. B 35 (2004) 1625 [arXiv:hep-ph/0402038].

[20] D. Diakonov and V. Petrov, Phys. Rev. D 69, 094011 (2004) [arXiv:hep-ph/0310212].

[21] V. Guzey and M. V. Polyakov, [arXiv:hep-ph/0501010] and Annalen Phys. 13, 673 (2004) and [arXiv:hep-ph/0512355].

[22] C. Alt et al. [NA49 Collaboration], Phys. Rev. Lett. 92, 042003 (2004) [arXiv:hep-ex/0310014].

[23] R. L. Jaffe and F. Wilczek, Phys. Rev. Lett. 91 (2003) 232003 [arXiv:hep-ph/0307341].

[24] T. D. Cohen, Phys. Rev. D 70 (2004) 074023 [arXiv:hep-ph/0402056].

[25] S. Pakvasa and M. Suzuki, Phys. Rev. D 70 (2004) 036002 [arXiv:hep-ph/0402079].