What have we learnt about the Sun from the measurement of the $^8$B neutrino flux?

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Abstract

By combining the results of SNO and Super-Kamiokande one can derive - in the absence of sterile neutrinos - the total neutrino flux produced from $^8$B decay in the Sun. We use this information to check the accuracy of several input parameters of solar model calculations. Opacity and p-p fusion cross section are constrained by the $^8$B flux measurement to the level of few percent. The central solar temperature is determined to the one-percent level. We also find an upper limit for the flux on Earth of sterile neutrinos. We discuss the role of nuclear physics uncertainties on these determinations.

I. INTRODUCTION

Electron neutrinos from $^8$B decay in the Sun have been detected at the Sudbury Neutrino Observatory (SNO) by means of the charged current (CC) reaction on deuterium [1]. The result, $\Phi_e = (1.75\pm0.14) \cdot 10^6$ cm$^{-2}$ s$^{-1}$ (here and in the following one sigma statistical errors are combined in quadrature with systematical errors), is a factor three smaller than the SSM prediction of Bahcall et al. [2]:

$$\Phi^{SSM} = 5.05 \cdot (1^{+0.20}_{-0.16}) \cdot 10^6 \text{ cm}^{-2}\text{ s}^{-1}. \quad (1)$$

The CC reaction is sensitive exclusively to $\nu_e$, while Electron Scattering (ES) also has a small sensitivity to $\nu_\mu$ and $\nu_\tau$. Comparison of $\Phi_e$ to the Super-Kamiokande (SK) precision result on ES yields a $3.3\sigma$ difference, providing evidence that there is a non-electron flavor active neutrino component in the solar flux.

Extraction of this flux, $\Phi_{\mu+\tau}$, can be done in a model independent way by exploiting the similarities of the response functions of SNO and of SK, see [3,4]. In this way, one determines from the two experiments the total active neutrino flux, $\Phi^{EXP} = \Phi_e + \Phi_{\mu+\tau}$, produced by $^8$B decay in the sun [3]:

$$\Phi^{EXP} = 5.20 \cdot (1^{+0.20}_{-0.16}) \cdot 10^6 \text{ cm}^{-2}\text{ s}^{-1}. \quad (2)$$
The close agreement with the theoretical prediction (1) is an important confirmation of the robustness of SSM calculations, see Table I.

Actually, one can go somehow in more detail and use this experimental information to provide an independent estimate of the accuracy of several input parameters that are used in solar model calculations. Among others, we shall consider the solar metal abundance, opacity and p-p fusion cross section.

We shall also consider the central solar temperature. As well known, this is not an independent parameter, rather it is the result of solar model calculations, mainly dependent on the assumed values for metals, opacity and luminosity. The boron flux measurement essentially provides a measurement of the central solar temperature.

So far we neglected the possibility of oscillations of $\nu_e$ into sterile neutrinos. In fact one can use the experimental result on active neutrinos $\Phi^{\text{EXP}}$ and the theoretical prediction on the total flux, $\Phi^{\text{SSM}}$ to provide an upper bound on the flux of sterile neutrinos on earth.

When connecting the $^8\text{B}$ flux with quantities characterizing the solar interior, an important link is provided by nuclear physics, through the nuclear cross sections of the processes which lead to $^8\text{B}$ production. We shall discuss the relevance of nuclear physics uncertainties in this respect.

II. POWER LAWS

The neutrino flux from $^8\text{B}$ decay in the Sun depends on several inputs $Q_i$, which can be grouped as nuclear and astrophysical parameters, see Table II.

The first group contains the zero energy astrophysical S-factors for the reactions which are involved in the production of $^8\text{B}$ nuclei. $S_{11}$ refers to $p+p\rightarrow d+e^++\nu$, $S_{33}$ and $S_{34}$ refer respectively to $^3\text{He}+^3\text{He}\rightarrow^4\text{He}+2p$ and to $^3\text{He}+^4\text{He}\rightarrow^7\text{Be}+\gamma$. Once $^7\text{Be}$ nuclei have been formed, the probability of forming a $^8\text{B}$ nucleus is essentially given by the ratio of proton to electron capture on $^7\text{Be}$, i.e on $S_{17}/S_{e7}$.

The $^8\text{B}$ neutrino flux depends on several astrophysical inputs. Concerning the dependence on the solar luminosity $L_\odot$ and age $t_\odot$, a more luminous Sun requires a higher internal temperature, which implies a higher Boron flux. An older Sun is also a more luminous sun.

The metal content $Z/X$ determines opacity. If $Z/X$ increases, opacity increases and the radiative transfer of the solar luminosity requires higher temperature gradients, which in turn implies higher temperature and a larger $^8\text{B}$ flux.

Opacity calculation are complex, and also for fixed metal content different codes yield (slightly) different opacities. For this reason we introduce, by means of a scaling parameter $\kappa$, the possibility of a uniform scaling of opacity in the solar interior.

Another scaling factor, $D$, is allowed for varying the calculated diffusion coefficients. If these inputs are changed with respect to the values used in the SSM calculations, $Q_i^{\text{SSM}}$, the $^8\text{B}$-neutrino flux $\Phi$ changes according to a power law:

$$\Phi = \Phi^{\text{SSM}}(Q_i/Q_i^{\text{SSM}})^{\alpha_i} .$$ (3)

We have calculated these coefficients by using solar models which include helium and heavy elements diffusion, see Table II. They are in agreement with the values calculated (neglecting diffusion) in [4] (see Table II) and in [10].
Most of these values can be understood by using simple, semi-quantitative arguments, see e.g. \[10\]. As an example, one can easily show that at equilibrium the production of $^7\text{Be}$ nuclei scales as $S_{34}/\sqrt{S_{33}}$ \[12,10\] and that the production of $^8\text{B}$ nuclei, which is a minor perturbation of the pp-II chain, scales as $S_{17}/S_{e7}$.

Each input is affected by uncertainties. The values shown in Table II, 4-th row, correspond to one sigma percentage error. They have been obtained according to the following criteria:

i) For the nuclear cross sections, we generally consider the values and uncertainties recommended in ref. \[13\] which are quite similar to those quoted in ref. \[14\]. Concerning $S_{17}$, we quote an uncertainty of 9%, taking into account recent measurements \[15\].

ii) Concerning solar luminosity, age and metals, we adopt the same uncertainties quoted in \[16\], which are derived by recent measurements of the solar constant, by a detailed comparison of meteorite radioactive datings and by analysis of meteoric and photospheric chemical compositions \[17\].

iii) Regarding the opacity and diffusion uncertainties, there is no experimental guidance and one has to resort to the comparison among different theoretical calculations. A factor 2.5% is the typical difference between the calculated values of the Livermore and Los Alamos opacities, see \[18\]. Concerning diffusion, Bahcall and Loeb \[19\] quote and uncertainty of about 15% by comparing results of different theoretical calculations. (We have found that agreement with helioseismic information fixes diffusion to the level of 10% \[20\].)

We also show in Table II, fifth line, the contributed fractional error, $\alpha \Delta Q/Q$, i.e. the contribution of each parameter to the error on the neutrino flux. Summing up in quadrature, the nuclear uncertainties yield an error on the flux $(\Delta \Phi/\Phi)_{\text{nuc}}=13.3\%$, which matches closely the error resulting from astrophysical uncertainties calculated according to the same prescription, $(\Delta \Phi/\Phi)_{\text{ast}}=12.2\%$. All in all, the total calculated uncertainty, $(\Delta \Phi/\Phi) = \sqrt{(\Delta \Phi/\Phi)_{\text{nuc}}^2 + (\Delta \Phi/\Phi)_{\text{ast}}^2}$ is about 18%, in agreement with ref. \[2\].

### III. RESULTS

The experimental result, Eq. (2), is in excellent agreement with the theoretical prediction, Eq. (1). The two determinations are affected by a similar error and, as we have just seen, the theoretical error gets comparable contributions from nuclear and astrophysical uncertainties.

One can expect that in the future, with increasing statistics and with better understanding of the systematical uncertainties, the experimental error will be reduced, possibly by a factor of two. In view of this, one should improve the theoretical calculation by a comparable factor, which requires a better understanding of the nuclear physics and astrophysics which are involved, see again Table II.

We remind that experiments are only sensitive to active neutrinos. The comparison with the SSM prediction can be used to derive an upper bound on the presence of sterile neutrinos, by requiring that $\Phi_s = \Phi^{\text{SSM}} - \Phi^{\text{EXP}}$. This gives:

$$\Phi_s < 2.5 \times 10^6 \text{ cm}^{-2} \text{s}^{-1} \quad \text{(at 2\sigma)} \quad .$$

\[4\]
Improvement of this bound will require that both theoretical and experimental determinations become more accurate.

Neglecting the possibility of sterile neutrinos, one can use the $^8$B-flux measurement as an independent way of estimating the accuracy of several nuclear and astrophysical parameters.

In fact, the measurement of $\Phi$ can be interpreted as a way of determining each of the parameters listed in Table I, by means of:

$$Q_i = \left(\frac{\Phi^{\text{EXP}}}{\Phi^{\text{SSM}}}\right)^{1/\alpha_i} \Pi_{j \neq i} \left(\frac{Q_j}{Q_j^{\text{SSM}}}\right)^{-\alpha_j \alpha_i}$$  \hspace{1cm} (5)

In this way the measurement of $\Phi$ together with the independent information on $Q_j$ ($j \neq i$) can be translated into a measurement of $Q_i$.

For each parameter in Table I, we have estimated the corresponding accuracy by taking into account the $^8$B result and the uncertainty on all other parameters, in turn. The resulting values,

$$(\Delta Q_i/Q_i)_B = \frac{1}{\alpha_i} \sqrt{\left(\frac{\Delta \Phi}{\Phi}\right)^2 + \Sigma_{j \neq i} \left(\frac{\alpha_j}{\alpha_i} \frac{\Delta Q_j}{Q_j}\right)^2},$$  \hspace{1cm} (6)

are listed in Table I, last row. We remark the following points:

- The opacity scaling factor $\kappa$ is determined to be unity with an accuracy of 9.3%. This is somehow worse than the estimated input accuracy of 2.6%. We remind however that this uncertainty was essentially estimated from the comparison between two different theoretical calculations.

- The metal content $Z/X$ is confirmed within 18%. Again this is worse than the estimated input uncertainty (6.1%) from studies of solar photosphere, which however might not be representative of the metal content of the solar interior if it has been enriched in metals after the sun formation.

- The astrophysical $S$-factor for the pp fusion reaction, $S_{11}$, is determined with accuracy of 9%. This quantity is not measured and it is the result of theoretical errors. The estimated uncertainty of these calculations (“input error”) is 1.7% Helioseismology allows a few per cent accuracy [7].

- The errors on $S_{33}$, $S_{34}$, $S_{e7}$, $S_{17}$, $L_\odot$ and $t_\odot$ are not competitive with more direct measurements in the literature.

IV. THE CENTRAL SOLAR TEMPERATURE

A final remark concerns the accuracy of the central solar temperature $T_c$ (see Table I for theoretical predictions of $T_c$).

Solar model builders have been claiming for decades that $T_c$ is known with an accuracy of one per cent or better. Helioseismology has provided strong support to this finding. Although helioseismology measures sound speed and not temperature, consistency with
Helioseismic data has been found only for solar models which produce $T_c$ within one percent of the SSM prediction [8,9]. The measurement of $^8$B provides an important test in this respect.

In fact, the temperature is not an independent variable. Its precise value is determined by some of the parameters which we have been discussing: the cross section for the pp-reaction, the metal content of the Sun, the adopted values for the radiative opacity, the solar age and luminosity and also the diffusion coefficients.

On the other hand, the central temperature is not affected by the other nuclear parameters listed in Table [II]. In full generality, the relationship between the boron flux and $T_c$ is (see [10]):

$$\Phi = \Phi^{SSM} \left( \frac{T_c}{T^{SSM}_c} \right)^\beta \frac{S_{nuc}}{S^{SSM}_{nuc}},$$

(7)

Where $S_{nuc}$ includes the dependence on the nuclear cross sections

$$S_{nuc} = S^{0.84}_{34} S^{-0.43}_{33} S^{17}_{17}/S^{e7},$$

(8)

As discussed above, $S_{nuc}$ is determined with an accuracy of 13.3%.

Concerning the exponent $\beta$ in Eq. (7), it is weakly dependent on which parameter is being varied for obtaining a changement of $T_c$. We have calculated the exponent by using solar models which include helium and heavy elements diffusion, see Table [II] and Fig. [I]. On these grounds we will take $\beta = 20$. We remark that if one neglects diffusion, slightly higher values of $\beta$ are obtained, see Table [III] last column. In fact in ref. [11] by studying solar models without diffusion a somehow higher value was found, $\beta = 24$. Anyhow the choice of a slightly different value does not induce a significant difference in the conclusions.

The agreement between theory and experiment on $\Phi$ thus implies that the central temperature of the Sun agrees with the SSM prediction to within one percent:

$$T_c = 15.7(1 \pm 1\%)10^6 K,$$

(9)

where the error gets comparable contributions from the uncertainty on $\Phi$ and from nuclear physics.

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| TABLE I. Predictions of some SSM calculations: |
|-----------------------------------------------|
| BP2000  | FRANEC97 | RCVD96 | JCD96 | GARSOM2 |
| $T_c$ [10^6K] | 15.696 | 15.69 | 15.67 | 15.668 | 15.7 |
| $\Phi_B$ [10^6cm^{-2}s^{-1}] | 5.05 | 5.16 | 6.33 | 5.87 | 5.30 |

| TABLE II. Nuclear and Astrophysical parameters related to the determination of $^8$B flux |
|-----------------------------------------------|
| $Q$ | Nuclear | Astrophysical |
| $S_{11}$ | $S_{33}$ | $S_{34}$ | $S_{e7}$ | $S_{17}$ | $L_{\odot}$ | $t_{\odot}$ | $Z/X$ | $\kappa$ | D |
| $\alpha$ | -2.7 | -0.43 | 0.84 | -1 | 1 | 7.2 | 1.4 | 1.4 | 2.6 | 0.34 |
| $\alpha$ [6] | -2.59 | -0.40 | 0.81 | -1 | 1 | 6.76 | 1.28 | 1.26 | - | - |
| Input error (%) | 1.7 | 6.1 | 9.4 | 2 | 9 | 0.4 | 0.4 | 6.1 | 2.5 | 15 |
| Contributed error to $\Phi$ (%) | 4.6 | 2.6 | 7.9 | 2 | 9 | 2.9 | 0.56 | 8.5 | 6.5 | 5 |
| Uncertainty derived from $^8$B measurement (%) | 9 | 58 | 28 | 25 | 23 | 3.5 | 18 | 18 | 9.3 | 73 |

| TABLE III. Dependence of the boron flux from central temperature, see Eq. (7) |
|-----------------------------------------------|
| $Q_i$ | $\beta$ (with diffusion) | $\beta$ (without diffusion) |
| $S_{11}$ | 19.5 | 21 |
| $L_{\odot}$ | 21.1 | |
| $t_{\odot}$ | 17.0 | 18 |
| $Z/X$ | 17.4 | 21 |
| $\kappa$ | 18.8 | 19 |
| D | 21.2 | |
FIG. 1. The behaviour of the $^8$B neutrino flux as a function of the central temperature, when varying different solar model inputs of ±10%. The straight line corresponds to the power law $T_c^{20}$. 