Research on load simulation control system based on optimal \( H_\infty \) method

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Abstract: In order to optimize the accuracy of the vehicle load simulation system, the control framework of the system is established based on the speed tracking method. Based on \( H_\infty \) mixed sensitivity method, a disturbance observer is established. In order to solve the problem that the order of disturbance observer is too high to restrain disturbance directly, a design constraint framework of disturbance observer is proposed, and the optimization of disturbance observer is realized in the constraint framework. The simulation and experimental results show that the optimized disturbance observer has stronger disturbance rejection ability.

1. Introduction

Bench test technology plays a key role in design and development of vehicle. In order to ensure the authenticity and accuracy of the bench load simulation, it is necessary to compensate and load the test pieces running on the test bench. The core of this technology is called load simulation technology\(^{[1]} \). During the simulation process, the load simulation system may have some internal and external disturbances. These interference factors act on the test bench in the form of force or speed, which makes the test bench deviate from the ideal curve at a certain moment of the test. These disturbances can be divided into: model nonlinear factors, equipment parameter perturbation, high frequency unmodeled dynamics of the system, and signal noise of sensors\(^{[2-4]} \). They may affect the accuracy of the system, or make the control system unstable. Therefore, the control system should have the ability to suppress disturbance and improve the control performance of the system.

For general control systems, disturbance observer is a widely used disturbance compensation method in electromechanical systems\(^{[5]} \). Chen\(^{[6]} \) proposed a two-stage design method to separate the design of disturbance observer and controller. Doyle\(^{[7]} \) used \( H_\infty \) method to proposes a frequency-domain loop shaping method, which makes many robust performance indexes of the control system can be used as the system transfer function \( H_\infty \). Zhang\(^{[8]} \) used general \( H_\infty \) in control system. Then, by reducing the order of the controller model, a low order disturbance observer is obtained. Using \( H_\infty \), in general, the transfer function of the system and the weighted function representing the sensitivity and robustness of the system are obtained to solve the problem. The structure of the weighting function directly affects the structure of the final controller. However, there is no unified method for the selection of weighting.
function, and the general method depends on the experience of designers or test requirements. The controller obtained by this way is blind to some extent, and it is difficult to fully express the requirements of the controlled system.

In order to solve the problem, a load simulation control system based on speed tracking method is proposed in the text, and the disturbance observer is designed by $H_\infty$ method, and the optimization method for disturbance observer is studied.

2. Principle of load simulation
The structure of load simulation test bench is shown in Fig.1. The test vehicle is fixed in the middle. The driving wheels on both sides are connected with the load simulation equipment to cope with the output of the vehicle's bilateral half shafts. The load simulation equipment on each side is composed of motor and transmission mechanism.

![Fig. 1 Structure of load simulation system](image)

In order to deal with a variety of different tested vehicles on a test bench, the control loops of the load system and tested vehicle are independent of each other. Therefore, the the driving torque $T_e$ of the tested vehicle on the load model system is unknown and uncontrollable to the control system, which belongs to a kind of interference torque acted on the load simulation system. The real driving situation of load simulation system is that, the combined torque of motor output torque and driving torque $T_e$ drives the whole system to rotate, making it simulate the speed of vehicle. According to Fig.2, during the experiment, the system is affected by the model uncertainty of the system itself and the driving torque of the vehicle. Therefore, it is necessary to design the control method to make the accurate load simulation realized under the joint action of $T_e$ and load torque $T_m$.

3. Control strategy of load simulation system

3.1 Control loop based on velocity tracking method
At present, the commonly used control architectures for load simulation include inverse dynamics method, energy method and speed tracking method[9]. The energy method is suitable for the test under constant conditions, while the inverse dynamics method can simulate a small range of loads. The speed tracking method calculates the torque required by the load motor by using the feedback controller and feedforward compensation for the experimental platform. Its principle is shown in Fig. 3a.
In the figure, $T_e - T_l$ is the driving force output after the driving wheel output torque and driving resistance moment offset each other. $T_e - T_l$ deduces $\omega_{em}$ from the actual system model $G_{em}(s)$, and then calculates the required load motor torque $T_m$ by using the speed closed loop through the system controller $C(s)$. It can be seen that the action mechanism of the control system is: according to the actual speed and torque collected at the moment $n$, the target speed $\omega$ at the $n+1$ moment can be obtained by converting the vehicle dynamics model, and the actual speed $\omega_{em}$ at the $n+1$ moment can be obtained through the action of the control system and make $\omega_{em}$ continuously approximates to $\omega$, so as to realize accurate vehicle load simulation.

Set $T_e - T_l = T_{el}$ in Fig.3a, replace $G(s)$ with $G_n(s)(1 + \Delta(s))$ and consider the affection of $T_e$ at the same time. That is to say, the controller controls the system under the action of disturbance torque, and the motor and mechanical structure model have model uncertainty. Thus, the equivalent change of figure 3a is shown in Fig.3b.

Where $Q(s) = \frac{C(s)G_n(s)}{1 + C(s)G_n(s)}$. In this case, if controller $C(s)$ is the result of the optimal design of $G(s)$, then the system with disturbance observer obtained according to the above equivalent changes can also meet the above indexes, and has the optimality under the actual index conditions.

### 3.1.1 Disturbance observer design based on $H_\infty$ mixed sensitivity method

According to the design criteria of the disturbance observer, if the controller $C(s)$ is the result of the optimal design of the system to meet the design index, then the system obtained by the above equivalent change can also meet the above index and has the optimality under the design index condition. The controller $C(s)$ and disturbance observer filter $Q(s)$ are designed under $H_\infty$ mixed sensitivity method. Fig. 3b is a single-input single-output feedback system. Set $e, u, d$ as the tracking error, system input and disturbance respectively, $W_1(s)$ and $W_2(s)$ represent for weighted function of $e$ and $\omega$. According to $H_\infty$ mixed sensitivity method, the sensitivity and complementary sensitivity functions of the system are obtained as $S(s) = \frac{1}{1 + C(s)G_n(s)}$, $T(s) = \frac{C(s)G_n(s)}{1 + C(s)G_n(s)}$, which satisfy:

$$\min \left\| \frac{W_2 S}{W_3 T} \right\|_\infty = \gamma_0 \quad (1)$$

Where $\gamma_0$ is a positive constant. Since $S(s) + T(s) = 1$, these two functions cannot be designed arbitrarily at the same time, so a compromise is needed. In the control system, because the interference is often located in the middle low frequency band and the system noise is often in the middle high frequency band, so the amplitude of $S(s)$ in the low frequency band should be minimized, and the amplitude of $T(s)$ in the high frequency band should be minimized. Therefore, the main function of $Q(s)$ is to minimize the weight functions $W_1(s)$ and $W_3(s)$ in the given frequency band under the premise of robust stability of the control system. It is expressed in the form of infinite norm that:

$$\|W_1(s)S(s)\|_\infty < 1, \|W_3(s)T(s)\|_\infty < 1 \}

S(s), T(s) \in RH_\infty \quad (2)$$

In this formula $RH_\infty$ is a set of transfer functions with stability and authenticity. It can be seen that
the stability of the of the inner loop in system must be guaranteed when the filter is matched with the control system. Because the weight function to calculate the filter needs to fully characterize the system disturbance characteristics, so as to prevent the load simulation system from instability. Therefore, it is necessary to solve the filter from the transfer function of the system. The multiplicative uncertainty of load simulation system is expressed as \( G(s) = G_n(s)(1 + \Delta(s)) \). Therefore, the transfer function considering only the system uncertainty can be expressed as follows:

\[
G_u(s) = \frac{G(s)G_n(s)}{G_n(s) + G(s) - G_n(s)Q(s)} = \frac{(1 + \Delta(s))G(s)}{1 + T(s)\Delta(s)} \tag{3}
\]

In the low frequency band, when \( |T(s)| \to 1 \), equation (3) can be expressed as:

\[
G_d(s) = \frac{(1 + \Delta(s))G(s)}{1 + \Delta(s)} = G(s) \tag{4}
\]

Therefore, the control loop of the load simulation system considering input, output, and model uncertainties is directly affected by \( T(s) \). The transfer function \( W_3(s) \) represents the upper bound of the system disturbance, so \( W_3(s) \) is always greater than the upper limit of the disturbance amplitude frequency. According to formula (2), if \( \|W_1S\|_\infty < 1 \), the system meets the performance requirements and \( \|W_3T\|_\infty < 1 \), the system meets the requirement of robust stability. Therefore, the necessary and sufficient conditions for the system to satisfy robust performance are as follows:

\[
\|W_1S\| + \|W_3T\|_\infty < 1 \tag{5}
\]

### 3.1.2 System simulation

To verify the effectiveness of the disturbance observer, the platform shown in Fig 1 is taken as an example. Where the motor transfer function and the inertia part model of load simulation system is

\[
P(s) = \frac{4798}{0.000743s^2 + 0.00726s + 1} \quad G(s) = \frac{1}{512s + 24}
\]

respectively, then the open-loop transfer function is

\[
G_n(s) = \frac{4798}{(0.000743s^2 + 0.00726s + 1)(512s + 24)} \tag{6}
\]

Selection of \( W_3(s) \) is analyzed according to the system disturbance. The amplitude frequency characteristics of the system are obtained in Fig.4a. Due to the requirement that \( W_3(s) \) should cover the upper bound of all uncertainties, thus

\[
W_3(s) = \frac{654.3599(s + 50.84)(s^2 + 0.0516s + 0.00479)}{(s + 389.2)(s + 209.6)(s + 0.04688)}
\]

**Fig. 4 Amplitude frequency characteristic diagram of load simulation system**

| a. | Amplitude frequency characteristics of system | b. | Spectrum diagram of the system |

Selection of weight function \( W_1(s) \) is based on the spectrum curve analysis of the tested vehicle, and the spectrum diagram is shown in Fig.4b. It can be seen from the figure that the main frequency components of the system are all within 1rad/s. According to the purpose of the test design, generally, the loading error in the main frequency component of the system is less than 1\%, and according to the design experience, \( W_1(s) \) is selected as \( W_1(s) = \frac{s + 100}{s + 1} \). Thus, the transfer function of the disturbance observer is:
\[
Q(s) = \frac{16.6444(s+389.3)(s+209.5)}{(s+50.84)(s+2.759)(s^2+3.172s+7.306)(s^2+9.732s+1326)}
\] (7)

A 5000Nm step torque signal is used to excite the system. The velocity response deviation of the system is shown in Fig.5. It can be seen that compared with the traditional single loop with PI controller, the speed response deviation of the double loop control based on disturbance observer reduces from 2.2rad/s to 0.4rad/s.

![Fig.5 Speed response of disturbance observer based control system with step signal](image)

In this case, it can be found that the order of \( Q(s) \) is unduly high. While \( Q(s) = \) can be obtained by forced reduction. At this time, \( Q(s) \) can not completely offset the order of \( G_n^{-1} \). In addition, since the selection of weighting function does not directly reflect the suppression of system disturbance, the filter is not the optimal filter of the system.

The reason for the above problem is the weight function \( W_1(s) \) and \( W_2(s) \) in the control loop are obtained by repeated trial and error, lacking systematic solution. At the same time, the use of different weight functions has a great impact on the calculation results of the observer and the stability of the system. To summarise, the design method based on \( H_\infty \) mixed sensitivity optimization needs complete disturbance observer evaluation criteria. Therefore, in order to obtain a disturbance observer with stronger disturbance rejection ability, it is necessary to design the constraint structure of the disturbance observer which can be directly used for the establishment of systematic optimization calculation program and the disturbance suppression of the reflected controller.

### 3.1.3 Optimization of constraint structure

1) Order condition constraint according to the controlled object

The internal control loop model based on speed tracking method is similar to an internal model control system model. The control idea is to use the known mathematical model with Q-filter to form a disturbance observer to overcome the uncertainty of the system \( \Delta \) and the unknown disturbance input. Therefore, in the control loop, the Q-filter should not only truncates the noise energy, but also solve the problem that the inverse \( G_n^{-1}(s) \) of the nominal model cannot be realized. Suppose the transfer function \( G(s) \) and its nominal model \( G_n(s) \) is physically realizable, \( G_n^{-1}(s) \) must have redundant differential terms and cannot be realized alone. But as long as \( Q(s)G_n^{-1}(s) \) can be realized, this problem can be solved. Suppose the relative order of \( G_n(s) \) is \( k_g \). The relative order of \( Q(s) \) is \( k_Q \). The relative order of \( Q(s) \) and \( G_n(s) \) should be \( k_Q \geq k_g \).

2) Structural constraints based on disturbance

![Fig.6 Disturbance torque feedforward compensation](image)

a. Disturbance torque feedforward compensation

b. Equivalent structure diagram
Due to the disturbance torque of the test-bed $T_e$, that is to say, the output torque of the tested vehicle can be measured by the torque sensor, so it can be considered from $T_e$, as shown in Fig. 6a, according to the principle of structure invariance\textsuperscript{[11]} $K(s) = \frac{1}{G(s)}$ is the compensation controller. Via equivalent transformation, the control loop can be obtained as shown in Fig. 6b. It shows that disturbance $T_e$ can be suppressed by designing a disturbance observer. The equivalent control loop as shown in Fig. 7 is obtained by equivalent transformation of the structure block diagram of the disturbance observer. After the transformation, a controller $\frac{1}{1-Q(s)}$ is obtained. For disturbance observer, the highest order of disturbance that can be suppressed should be one of its main performances.

For the equivalent system loop described in Fig. 7, it is assumed that the disturbance is composed of a polynomial of order $q$. In order to suppress the disturbance completely, according to the internal model principle\textsuperscript{[10]}, the order $n_D$ of controller $D(s) = \frac{1}{1-Q(s)}$ must satisfy $n_D > q$, that is, its order should be at least $q + 1$. In this way, order $q$ reflects the main order characteristics of Q-filter.

\begin{equation}
|W_3(j\omega)| > \max\{|W_\zeta(j\omega)|, |W_d(j\omega)|\}
\end{equation}

In order to satisfy the condition of equation (8), the order of $W_3(s)$ is very high. Therefore, it is necessary to reduce the order of $W_3(s)$. Assuming that the upper bound of the system disturbance $W_3(s)$ is known, 2 functions $W_{3a}(s)$ and $W_{3b}(s)$ which are tangent to $W_3(s)$ can be selected at the low and high frequency positions respectively, so that there is $|W_{3a}(j\omega)| > |W_3(j\omega)|$ in the low frequency position and $|W_{3b}(j\omega)| > |W_3(j\omega)|$ at high frequency, as shown in Fig. 8. In this case, we assume that there is a new weighting function $W_{3c}(s)$ makes:

\begin{equation}
|W_{3c}(j\omega)| = \max\{|W_{3a}(j\omega)|, |W_{3b}(j\omega)|\}
\end{equation}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig7.png}
\caption{Structure diagram of disturbance observer with equivalent change}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig8.png}
\caption{Selection method of $W_3(s)$}
\end{figure}
It is not hard to get:

\[ \|W_3e(s)Q(s)\|_\infty < 1 \]  (10)

It is shown that \( W_3e(s) \) can be used as the weighting function of noise elimination and robust stability in the evaluation function.

Suppose that the expression of external interference is \( d(t) = c_q t^q + c_{q-1} t^{q-1} + \cdots + c_0 \), \( D(s) \) need to contain \( \frac{1}{s^{q+1}} \). Suppose \( Q(s) = \sum_{j=0}^{n} b_j s^j / \sum_{m=0}^{n} a_m s^m \), then we can get:

\[ D(s) = \frac{1}{1-Q(s)} = \frac{a_m s^m+\cdots+a_0}{s^{q+1} + \sum_{m=n+1}^{\infty} a_i s^i - q + 1} \]  (11)

If \( a_i = b_1 \), and \( q \) is a positive integer not greater than \( n \), then \( D(s) \) contains at least \( q+1 \) order integral links, thus,

\[ S(s) = 1 - Q(s) = \frac{s^{q+1} + \sum_{m=n+1}^{\infty} a_i s^i - q + 1} {s^{q+1} + \sum_{m=n+1}^{\infty} a_i s^i - q + 1} \]  (12)

In order to suppress the disturbance of the system, \( S(s) \) has \( q+1 \) zeros, so \( W_4(s) \) must have \( q+1 \) poles. Thus, in the case of \( W_4(s) \),

\[ W_4(s) = \frac{\delta}{(s+\lambda)^{q+1}} \]  (13)

Where \( \lambda \) is an infinitesimal constant, which can avoid the pole of \( W_4(s) \) appearing on the imaginary axis, and \( \delta \) is an arbitrarily selected constant.

Therefore, according to the summary above, the selection requirement of \( W_4(s) \) is obtained: since the order \( n \) of Q-filter depends on the order \( n_1 + k_q \) of the generalized controlled object, where \( n_1 \) is the order of \( W_4(s) \). At the same time, \( W_4(s) \) must have \( q+1 \) poles to suppress the \( q \)-order disturbance. In order to balance the order of the system and the order model of the disturbance, take \( q=1 \), then \( W_4(s) \) is the second-order weighting function. According to the design index of the Q-filter, the performance evaluation function of the disturbance observer system is defined as follows:

\[ \min_{Q(s)\in \Psi} \left\| \frac{W_4(s)Q(s)}{W_4(s)(1-Q(s))} \right\|_\infty = \gamma \]  (14)

Among them, for the set of transfer functions of stability and authenticity \( \Psi \), there are:

\[ \Psi = \begin{pmatrix} Q(s) = \frac{\sum_{j=0}^{n} b_j s^j / \sum_{m=0}^{n} a_m s^m}{1 - Q(s)} = \frac{\sum_{j=0}^{n} b_j s^j / \sum_{m=0}^{n} a_m s^m}{s^{q+1} + \sum_{m=n+1}^{\infty} a_i s^i - q + 1} \end{pmatrix} \]  (15)

Where \( a_i \) and \( b_j \) are rational real numbers. Equation (15) expresses the amplitude characteristics of the disturbance observer system in the whole frequency band, and constrains the order \( k \) of the Q-filter itself. Through the minimization of \( \gamma \), the optimal state of disturbance observer performance such as disturbance suppression performance, noise elimination performance and robust stability can be achieved.

According to the state space solution method of Ricatti equation\(^7\), the order of \( H_\infty \) is the sum of the order of the weighted function and the relative order of the controlled plant. Therefore, by reducing the order and constraining the weighting function, order \( n \) of the target filter \( Q_{opt}(s) \) can be achieved. When the order of the controlled system is known to be 3, then the relative order of Q-filter is \( k \geq 3 \). If \( q=1 \), then \( n_1 = 2 \). Therefore, the order of the designed filter is 5.
3.2 Simulation and result comparison

Investigate the load simulation system described in section 3.1.2. According to equation (15), set 
\[ W_1(s) = \frac{1}{(s+0.001)^2}, \quad W_3(s) = \frac{s^2}{(0.001s+1)^2}. \]
By introducing the above parameters into the generalized state space, when \( \gamma = 0.2823 \), the optimal controller is obtained as:
\[
Q_{opt}(s) = \frac{3407.9349(s+10^4)}{(s+4233)(s^2+3.825s+6.071)(s^2+9.735s+1326)}
\]  
(16)

The comparison between the two groups of filters \( Q(s) \) and \( Q_{opt}(s) \) are shown in Fig.9.

![Comparison of amplitude frequency characteristics between Q(s) and Q_{opt}(s)](image)

a. comparison of a-complement sensitivity function
b. comparison sensitivity function

Fig 9 Comparison of amplitude frequency characteristics between \( Q(s) \) and \( Q_{opt}(s) \)

In Fig.9 According to the calculation, at the frequency of about 10Hz, the noise suppression ability of \( Q_{opt}(s) \) is 3.3 times that of \( Q(s) \). For the suppression ability of sensitivity function at low frequency, \( |1 - Q_{opt}(j\omega)| \) is always 2.04dB smaller than \( |1 - Q(j\omega)| \). According to the calculation, the low frequency disturbance rejection ability of \( Q_{opt}(s) \) at 1Hz is 1.27 times that of \( Q(s) \).

4. Test and conclusion

In order to verify the disturbance suppression ability of the above control system, the load simulation device shown in Fig.10 is used to test the load capacity of a tracked vehicle under a load of 30t, and the test results are shown in Fig.11.

![Mechanical structure of load simulator](image)

Fig.10 Mechanical structure of load simulator

In order to simulate the sudden change of driving torque, the vehicle was tested repeatedly in upshift and downshift. According to figure 11a, figure 11b and chart 1, it can be seen that when the torque changes suddenly, the speed fluctuation of the control system designed based on the optimal \( H_{\infty} \) method is about 6 times smaller than general \( H_{\infty} \) method. The result proved that:

1. The control system based on speed tracking method and disturbance observer has fast response speed and strong disturbance suppression ability, which can realize accurate vehicle load simulation.
2. Compared with ordinary disturbance detectives, the optimazed \( H_{\infty} \) controller has a lower order and a stronger disturbance suppression ability.
Theoretical speed

\[ H_\infty \]

Speed under general control

Speed under optimized control

\[ H_\infty \]

Torque

Referencen

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