The axion shield

A. Andrianov

V.A. Fock Institute of Physics, Sankt-Petersburg State University,
Ulianovskaya 1, 198504 St. Petersburg, Russia,

D. Espriu

CERN, 1211 Geneva, Switzerland,

AND

F. Mescia and A. Renau

Departament d’Estructura i Constituents de la Matèria and
Institut de Ciències del Cosmos (ICCUB), Universitat de Barcelona,
Martí i Franqués 1, 08028 Barcelona, Spain

Abstract

We investigate the propagation of a charged particle in a spatially constant, but time dependent, pseudoscalar background. Physically this pseudoscalar background could be provided by a relic axion density. The background leads to an explicit breaking of Lorentz invariance; as a consequence the process $p \rightarrow p\gamma$ is possible and the background acts as a shield against extremely energetic cosmic rays, an effect somewhat similar to the GZK cut-off effect. The effect is model independent and can be computed exactly. The hypothetical detection of the photons radiated via this mechanism would provide an indirect way of verifying the cosmological relevance of axions.

*On leave of absence from DECM and ICCUB, Universitat de Barcelona
1 Introduction

Cold relic axions resulting from vacuum misalignment\cite{1, 2} in the early universe is a popular and so far viable candidate to dark matter. If we assume that cold axions are the only contributors to the matter density of the universe apart from ordinary baryonic matter its density must be\cite{3}

\[
\rho \simeq 10^{-30}\, \text{g cm}^{-3} \simeq 10^{-46}\, \text{GeV}^4. \tag{1}
\]

Of course dark matter is not uniformly distributed, its distribution traces that of visible matter (or rather the other way round). The galactic halo of dark matter (assumed to consist of axions) would correspond to a typical value for the density\cite{4}

\[
\rho_a \simeq 10^{-24}\, \text{g cm}^{-3} \simeq 10^{-40}\, \text{GeV}^4 \tag{2}
\]

extending over a distance of 30 to 100 kpc in a galaxy such as the Milky Way. Precise details of the density profile are not so important at this point. Even in this case this is a rather low density; the photon background provides a very diffuse concentration of pseudoscalar particles, but then the photon density is also low and yet it has an impact on ultra-high energy cosmic rays imposing the GZK cutoff\cite{5}. What are the consequences of this diffuse axion background on high-energy cosmic ray propagation? This is the question we answer in this work.

The fact that the axion is a pseudoscalar, being the pseudo Goldstone boson of the broken Peccei-Quinn symmetry\cite{6}, is quite relevant. Its coupling to photons will take place through the anomaly term; hence the coefficient is easily calculable once the axion model is known

\[
\Delta L = g_{a\gamma\gamma} \frac{\alpha}{2\pi} \frac{a}{f_a} \tilde{F} F. \tag{3}
\]

Two popular (and so far viable) axion models are the DFSZ\cite{8} and the KSVZ\cite{7} ones. In both models \( g_{a\gamma\gamma} \simeq 1 \). Here \( a \) is the axion field while \( f_a \), the axion decay constant, is bound to be of \( \mathcal{O}(10^{11}) \) GeV or more. As to the axion mass, \( m_a \) it should lie in the range \( 10^{-3}\text{eV} > m_a > 10^{-6} \) eV.

The axion field is originally misaligned and in the process of relaxing to the equilibrium configuration coherent oscillations with \( q = 0 \) are produced, provided that the reheating temperature after inflation is below the Peccei-Quinn transition scale. In late times the axion field evolves according to

\[
a(t) = a_0 \cos m_a t. \tag{4}
\]

Then, integrating by parts in the previous equation we get

\[
\Delta L = -g_{a\gamma\gamma} \frac{\alpha a_0}{\pi f_a} \sin(m_a t) e^{ijk} A_i F_{jk}. \tag{5}
\]
A particle travelling at the speed of light will see coherent regions with quasi-constant values of the axion background of a size ranging from the millimeter to the meter, depending on the axion mass. Small as this size is, it is very many orders of magnitude bigger than the wavelength of a particle travelling with a momentum \( p \) characteristic of a very high energy cosmic ray. For processes with a high momentum transfer we can treat this slowly varying term as a constant and thus replace (5) by

\[
\Delta L = \frac{1}{4} \eta \varepsilon^{ijk} A_i F_{jk},
\]

where the “constant” \( \eta \) will change sign with a period \( 1/m_a \). \( \eta \) is obviously related to the axion density. Numerically we expect

\[
|\eta| \simeq g_{a\gamma\gamma} \frac{2\alpha \sqrt{p}}{f_a} \approx 10^{-24} - 10^{-25} \text{eV},
\]

for \( f_a \) in the range \( 10^{11} - 10^{12} \) GeV and where we have included and additional factor of \( \frac{1}{2} \) to roughly account for the variability of the \( \sin m_a t \) term. Equation (6) can also be written as

\[
\Delta L = \frac{1}{2} \eta_\alpha A_\beta \tilde{F}^{\alpha\beta},
\]

with \( \eta_\alpha = (\eta, 0, 0, 0) \). The expression uses a covariant language but it is of course Lorentz invariance violating as \( \eta_\alpha \) is a constant axial vector.

## 2 Solving QED in a cold axion background

Let us study the effects of explicit breaking of Lorentz invariance by means of a time-like constant axial vector. Consider electromagnetism in such a background

\[
\mathcal{L} = \mathcal{L}_{\text{INV}} + \mathcal{L}_{\text{LIV}}
\]

\[
\mathcal{L}_{\text{INV}} = -\frac{1}{4} F^{\alpha\beta} F_{\alpha\beta} + \bar{\psi} (i \gamma^\mu \not{\partial} - e \gamma^\mu A_\mu - m_\psi) \psi \quad \mathcal{L}_{\text{LIV}} = \frac{1}{2} m_\gamma^2 A_\mu A^\mu + \frac{1}{2} \eta_\alpha A_\beta \tilde{F}^{\alpha\beta}
\]

It will be useful for us to keep \( m_\gamma > 0 \). It need not be a fundamental mass; but an effective or induced in-medium mass. Otherwise gauge invariance is manifest.

The equations of motion are

\[
\left\{ g^{\lambda\nu} (k^2 - m_\gamma^2) + i \varepsilon^{\lambda\nu\alpha\beta} \eta_\alpha k_\beta \right\} \dot{A}_\lambda(k) = 0.
\]

We can build two complex and space-like chiral polarization vectors \( \varepsilon_\pm^\mu(k) \) which satisfy the orthonormality relations

\[
- g_{\mu\nu} \varepsilon_+^{\mu\ast}(k) \varepsilon_+^\nu(k) = 1 \quad g_{\mu\nu} \varepsilon_-^{\mu\ast}(k) \varepsilon_-^\nu(k) = 0
\]
In addition we have
\[ \varepsilon_T^\mu(k) \sim k^\mu, \quad \varepsilon_L^\mu(k) \sim k^2 \eta^\mu - k^\mu \eta \cdot k \] (13)

They fulfill
\[ g_{\mu\nu} \varepsilon_A'^\mu(k) \varepsilon_B'(k) = g_{AB}, \quad g^{AB} \varepsilon_A'^\mu(k) \varepsilon_B'(k) = g^{\mu\nu}. \] (14)

These polarizations correspond only approximately to the usual ones of QED. As a consequence light propagation in an axion background may be subject to modifications. However, for visible light and even less for radio waves, there is no marked separation of scales with respect to the axion mass and therefore the time variation of the background cannot be treated adiabatically. The net effect is expected to be zero or extremely small [11]. We refer the reader to [9, 10] for additional details.

The polarization vectors of positive and negative chirality are solutions of the vector field equations if and only if
\[ k_{\pm}^\mu = (\omega_{k_{\pm}}, k), \quad \omega_{k_{\pm}} = \sqrt{k^2 + m^2_{\pm} \eta |k|}, \quad \varepsilon_{\pm}^\mu(k, \eta) = \varepsilon_{\pm}^\mu(k_{\pm}) \quad \left( k_{\pm}^0 = \omega_{k_{\pm}} \right). \] (15)

In order to avoid problems with causality we want \( k_{\pm}^2 \geq 0 \). For photons of a given chirality (negative if \( \eta > 0 \), positive if \( \eta < 0 \)) this can be if and only if
\[ |k| < \frac{m^2_{\gamma}}{\eta} \equiv \Lambda_{\gamma}. \] (16)

In fact for \( m_{\gamma} = 0 \) these photons cannot exist as physical asymptotic states. If they are produced they will eventually decay (to three photons of like chirality) in a cascade process that leads to a red-shift.

As is known to everyone the processes \( e^- \rightarrow e^- \gamma \) or \( \gamma \rightarrow e^+ e^- \) cannot occur in vacuum. However in the present situation, because of the preferred reference frame, physics is different in different frames and for the latter process
\[ \omega_{k_{\pm}} = \sqrt{k^2 + m^2_{\gamma} \pm \eta |k|} = \sqrt{p^2 + m^2_{\epsilon}} + \sqrt{(p - k)^2 + m^2_{\epsilon}}, \] (17)

as discussed in [9, 10] it is possible for photons of the opposite chirality (positive if \( \eta > 0 \), negative if \( \eta < 0 \)) if
\[ |k| \geq \frac{4m^2_{\tau}}{\eta} \equiv k_{\text{th}} \quad (m_{\gamma} = 0) \] (18)

Here we shall be concerned with the related process \( e \rightarrow e \gamma \) whose kinematics will be discussed in detail in the following sections. In fact since we shall be concerned about the possible influence of the pseudoscalar background on cosmic ray propagation, which consist mostly of protons, we shall discuss the process \( p \rightarrow p \gamma \). In this case there is a large hierarchy between the proton momentum
and the time variation of the background. We shall see below that the process \( p \rightarrow p \gamma \) is perfectly possible kinematically. This is a prompt process that is not affected by a slight (in relative terms) time variation of the background. The net effect shall not average to zero.

As for the value of \( m_{\gamma} \) to use, the density of electrons in the universe is expected to be at most of the order of \( n_e = 10^{-7} \, \text{cm}^{-3} \). Photons pick up a mass of the order of

\[
4\pi\alpha \frac{n_e}{m_e}.
\]

This number is of the order of \( 10^{-15} \, \text{eV} \), but we shall assume the more conservative limit \( 10^{-18} \, \text{eV} \), compatible with some astrophysical bounds in the \( 10^{-16} \) to \( 10^{-17} \, \text{eV} \) region \[12\]. It is a very small number, but it is non-zero for sure (it actually depends on the local density of electrons, of course).

### 3 Kinematic constraints

Having found out the different polarizations and dispersion relations in the axion background let us now turn to kinematical considerations. Let us consider the process \( p(p) \rightarrow p(p-k)\gamma(k) \) with \( p^\mu = (E,p), p = |p| \) and \( k^\mu = (w_k,k), k = |k| \). Using \[15\] energy conservation leads to

\[
\sqrt{E^2 + k^2 - 2pk \cos \theta} + \sqrt{k^2 + \eta k + m_{\gamma}^2} - E = 0.
\]

For simplicity we take the positive sign for the polarization, in the understanding that changing the sign of \( \eta \) amounts to exchanging positive and negative chiral polarizations for the photon.

Let us first consider the case \( m_{\gamma} = 0 \). Then the above energy conservation equation reduces to

\[
k^2(4E^2 - 4p^2 \cos^2 \theta - \eta^2 - 4p\eta \cos \theta) + k(4p^2\eta + 4m_e^2\eta) = 0.
\]

This equation has the trivial solution \( k = 0 \), where nothing happens, and

\[
k = \frac{4p^2\eta + 4m_e^2\eta}{-4E^2 + 4p^2 \cos^2 \theta + \eta^2 + 4p\eta \cos \theta}.
\]

To find out the kinematical restrictions on \( k \) we search for the extrema of the denominator. There is only one such extremum which is \( \cos \theta = -\eta/2p \). Plugging this into the expression for \( k \) one gets \( k = -\eta \). Since \( k \) has to be positive this process is kinematically impossible unless \( \eta < 0 \). Of course for \( \eta = 0 \) it is also impossible as this is the familiar vacuum case. So the process \( p(p) \rightarrow p(p-k)\gamma(k) \) is actually possible, for positive values of \( \eta \) if the emitted photon is of negative chirality. On the other hand, if \( \eta \) is negative, the process becomes possible only for positive chirality photons. Here we shall take \( \eta \) to be always negative, but it is trivial to recover the results for positive \( \eta \) by exchanging chiralities.
This extremal value (for negative $\eta$) is clearly providing the minimum value for $k$, $k_{\text{min}} = |\eta|$. There is no other extremal point, so the maximum value for $k$ has to lie at one of the two values $\cos \theta = \pm 1$. It is not difficult to convince oneself (and quite intuitive) that the maximum is found for $\cos \theta = 1$ and it corresponds to

$$k_{\text{max}} = \frac{E^2}{p - \frac{m_p^2}{\eta}}$$

so $k_{\text{max}} \simeq E$ for $E \gg m_p^2/|\eta|$ and $k_{\text{max}} \simeq |\eta|E^2/m_p^2$ for $E \ll m_p^2/|\eta|$.

Let us now consider the case $m_\gamma > 0$. Conservation of energy leads to the following two roots for $k$

$$k_{\pm} = \frac{2E^2\eta - 2pm_\gamma^2 \cos \theta - \eta m_\gamma^2 \mp 2E \sqrt{E^2\eta^2 - 4E^2m_\gamma^2 + m_\gamma^4 + 4p^2m_\gamma^2 \cos^2 \theta + 2pm_\gamma^2 \cos \theta}}{-4E^2 + 4p^2 \cos^2 \theta + 4pm \cos \theta + \eta^2}. \quad (24)$$

The equations above are meaningful only for those values $\theta$ and $p$ providing a positive discriminant $\Delta$. From the condition that $\Delta \geq 0$ we get

$$\sin^2 \theta \leq \frac{\eta^2}{4p^2m_\gamma^2} \frac{1}{1 + \frac{1}{4p}(p - p_-)(p - p_+)}, \quad (25)$$

where

$$p_{\pm} = \frac{m_\gamma^2}{-\eta} \pm \frac{2m_\gamma m_\gamma - \eta \sqrt{1 - \frac{\eta^2}{4m_\gamma^2}}}{-\eta}. \quad (26)$$

Let us examine these two values $p_{\pm}$. Since $\eta$ is taken negative, it is clear that $p_+ > 0$ and $p_- < 0$. Therefore for $\sin^2 \theta$ to be a positive quantity we need

$$p > p_+ = p_{\text{th}} \simeq \frac{2m_\gamma m_p}{-\eta}. \quad (27)$$

This is the threshold value below which the process cannot take place kinematically. For $p >> p_{\text{th}}$

$$\sin^2 \theta_{\text{max}} \to \frac{\eta^2}{4m_\gamma^2}. \quad (28)$$

$\theta_{\text{max}}$ is the value of $\theta$ where the bound in $\sin^2 \theta$ of eq. (25) is saturated. We see that photons are emitted in a rather narrow cone. This justifies a posteriori the approximation $\cos \theta \simeq 1 - \frac{1}{2} \sin^2 \theta$ that has been used.

At $\theta_{\text{max}}$ the square root in (24) vanishes and $k_+ = k_- = k(\theta_{\text{max}})$ in (24). Then, keeping only the leading terms we get

$$k(\theta_{\text{max}}) \simeq \frac{2m_\gamma^2}{-\eta} (1 + 3 \frac{pm_\gamma^2}{E^2\eta} p >> p_{\text{th}}) \frac{2m_\gamma^2}{-\eta} \frac{2m_\gamma^2}{-\eta}. \quad (29)$$

From eq. (24) we work out the value for $\theta = 0$, which is the minimum value of $\theta$ from the bound in eq. (25).

$$k_+(0) \simeq \frac{E^2\eta - pm_\gamma^2 - E \sqrt{E^2\eta^2 - 4m_p^2m_\gamma^2 + 2pm_\gamma^2 \cos \theta}}{2pm - 2m_\gamma^2} \frac{E^2}{p - \frac{m_p^2}{\eta}}. \quad (30)$$
which is the same result obtained before, and

$$k_-(0) \simeq \frac{E^2\eta - pm^2 + E\sqrt{E^2\eta^2 - 4m_p^2 m^2 + 2pm^2}}{2p\eta - 2m_p^2} \approx \frac{m^2}{\eta}$$  \hspace{1cm} (31)$$

Now we notice that \(k_-(0) < k(\theta_{max}) < k_+(0)\). To show that \(k_{\text{min}} = k_-(0)\) and \(k_{\text{max}} = k_+(0)\) we have to study the derivative of \(\theta\) versus \(k\), namely we should have \(d\cos\theta/dk < 0\) for \(k < k(\theta_{max})\) and \(d\cos\theta/dk > 0\) for \(k > k(\theta_{max})\).

We isolate \(\cos\theta\) from the energy conservation relation (20)

$$\cos\theta = \frac{m^2 + \eta k - 2E\sqrt{m^2 + k^2 + \eta k}}{-2pk}.$$  \hspace{1cm} (32)$$

and

$$\frac{d\cos\theta}{dk} = \frac{m^2}{2k^2p} - \frac{E}{2k^2p} \frac{\eta k + 2m^2}{\sqrt{\eta k + k^2 + m^2}} \approx \frac{\eta k + 2m^2}{2k^2\sqrt{\eta k + k^2 + m^2}}.$$  \hspace{1cm} (33)$$

For \(k \to k(\theta_{max})\) \(d\cos\theta/dk \to 0\) and is the only zero which of course means that this value of \(k\) corresponds to a minimum of \(\cos\theta\) (i.e. to a maximum of \(\sin\theta\)). On the other hand, for \(k < k(\theta_{max})\), \(d\cos\theta/dk < 0\) and for \(k > k(\theta_{max})\), \(d\cos\theta/dk > 0\). Then \(k_{\text{min}} = k_-(0)\) and \(k_{\text{max}} = k_+(0)\).

We have said before that the actual cone of photon emission is quite narrow for the values of \(m^2\) and \(\eta\) relevant in the present discussion, but for the sake of completeness let us discuss the case where \(m^2\) is small. We use again the expression (32) and

$$\frac{dk}{d\cos\theta} = \frac{2pk^2\sqrt{m^2 + k^2 + k\eta}}{m^2\sqrt{m^2 + k^2 + k\eta} - 2Em^2}\hspace{1cm}.$$  \hspace{1cm} (34)$$

The extremum conditions leads to \(k = 0\) or \(m^2 + k^2 + k\eta = 0\). The first possibility is unphysical since it is incompatible with (32). The second one gives

$$k = -\eta \pm \eta\sqrt{1 - \frac{4m^2}{\eta^2}},$$  \hspace{1cm} (35)$$

which is real only as long as \(\eta^2 > 4m^2\). These expressions have a smooth \(m^2 \to 0\) limit which has already been discussed at the beginning of this section.

4 Differential decay probability

Here we shall compute the relevant matrix element and the differential decay probability. The calculation is standard but in view of the rather peculiar properties of Chern-Simons QED it is probably useful to go into some details. Since the process is possible only for one chirality of the photon (depending on the sign of \(\eta\)) we shall not sum over the final polarization of the radiated photon.
Using the standard Feynman rules we get

$$|M|^2 = e^2 \varepsilon^\mu_\nu \varepsilon^\nu_\mu \text{tr} \left[ u(q) \bar{u}(q) \gamma^\mu u(p) \bar{u}(p) \gamma^\nu \right]$$  \hspace{1cm} (36)$$

Summing and averaging over the final and initial proton helicities, respectively, and performing the trace we get

$$|M|^2 = 2e^2 \left\{ -p \cdot k + \left[ \varepsilon^\mu_\mu(k) \varepsilon^\nu_\nu(k) + \varepsilon^\mu_\mu(k) \varepsilon^\nu_\nu(k) \right] p^\mu p^\nu \right\} = 2e^2 \left\{ -p \cdot k + 2|\varepsilon^\mu_\mu(k)p^\mu| \right\}, \hspace{1cm} (37)$$

where \(\varepsilon^\mu_\mu(k)\) is the polarization vector defined in section 2. Recall that \([9]\)

$$\varepsilon^\mu_\mu(k) \varepsilon^\nu_\nu(k) + \varepsilon^\mu_\mu(k) \varepsilon^\nu_\nu(k) = -S^{\mu\nu}/\eta^2|k|^2 \hspace{1cm} (38)$$

where

$$S^{\mu\nu} = \left( \eta \cdot k \right)^2 - \eta^2 k^2 g^{\mu\nu} - \left( \eta \cdot k \right) \left( \eta^\mu k^\nu + \eta^\nu k^\mu \right) + k^\mu \eta^\nu + \eta^2 k^\mu k^\nu. \hspace{1cm} (39)$$

Then

$$|M|^2 = 2e^2 \left( -p \cdot k + |p|^2 \sin^2 \theta \right). \hspace{1cm} (40)$$

It is not difficult to see that in the kinematical conditions where the process is possible \(p \cdot k < 0\), thus guaranteeing positivity.

The differential decay width will be

$$d\Gamma(Q) = (2\pi)^4 \delta^{(4)}(q + k - p) \frac{1}{2E|\bar{M}|^2} dQ, \hspace{1cm} (41)$$

where \(dQ\) refers to the final state phase space. The final result is

$$d\Gamma(Q) = \frac{1}{16\pi} \frac{|k|}{|p|} \frac{|M|^2}{E\omega_k} d|k| = \frac{\alpha}{2} \frac{|k|}{|p|} \frac{1}{E\omega_k} (-p \cdot k + |p|^2 \sin^2 \theta) d|k| \hspace{1cm} (42)$$

after using the energy-momentum conservation delta and performing the integral over all the angles. In the previous expression \(\sin \theta\) corresponds to

$$\cos \theta = \frac{m_\gamma^2 + \eta |k| - 2E\omega_k}{-2|p||k|}. \hspace{1cm} (43)$$

5 Energy loss and radiation spectrum

Since in this section no confusion with four-vectors is possible, we revert to the lighter notation \(p = |p|, k = |k|\) and \(w = w_k\). The relevant quantity to compute is the following

$$\frac{dE}{dx} = -\frac{1}{v} \int d\Gamma(Q) w(Q). \hspace{1cm} (44)$$

Then, using the previous results,

$$\frac{dE}{dx} = -\frac{\alpha}{2} \frac{1}{p^2} \int kdk [-\frac{1}{2} \left( m_\gamma^2 + \eta k \right) + p^2 (1 - \cos^2 \theta)]. \hspace{1cm} (45)$$
In the previous expression \( \cos \theta \) is given by (43). Then the l.h.s of this identity equals

\[
- \frac{\alpha}{8p^2} \int \frac{dk}{k} \left[ -2\eta k^3 - 2m_p^2 k^2 + 4p^2 k^2 - \eta^2 k^2 - m_p^4 - 4E^2 w^2 - 2m_p^2 \eta k + 4Em_p^2 \omega + 4\eta E \omega k \right].
\]  

(46)

Expanding \( \omega = k \sqrt{1 + \frac{\eta}{k} + m_p^2 \gamma k^2} \approx k + \frac{\eta}{2} + \frac{m_p^2}{2k} \), recalling the integration limits \( \frac{m_p^2}{\eta} \) and \( \frac{\eta E^2}{m_p^2 - m_p^2} \) and neglecting at this point the effective photon mass we get

\[
\frac{dE}{dx} = -\frac{\alpha}{8p^2} \frac{\eta E^2}{(p\eta - m_p^2)^3} \left[ -\frac{2}{3} \eta^3 E^4 + 2\eta E^2 (p\eta - m_p^2) (m_p^2 + \eta E - 2\eta p - \frac{\eta^2}{4} + \eta^2 \frac{D}{E}) \right].
\]  

(47)

There are two relevant limits

\[
E \ll \frac{m_p^2}{|\eta|} \quad \Rightarrow \quad \frac{dE}{dx} = -\frac{\alpha \eta^2 E^2}{4m_p^2}.
\]  

(48)

\[
E \gg \frac{m_p^2}{|\eta|} \quad \Rightarrow \quad \frac{dE}{dx} = -\frac{\alpha |\eta|}{3} E.
\]  

(49)

At this point it becomes obvious why we have bothered to keep the proton mass \( m_p \) all along the calculation.

There are two key scales in this problem. One, discussed in sections 2 and 3, is the threshold energy where the process \( p \rightarrow p\gamma \) becomes kinematically possible in the presence of a pseudoscalar background represented by \( \eta \neq 0 \), namely \( E_{th} \simeq 2m_{\gamma}m_p/|\eta| \). The other relevant scale is \( m_p^2/|\eta| \) where there is a crossover: the energy loss per unit length well below this energy is effectively proportional to \( \eta^2 \), while well above that scale is proportional to \( \eta \). Thus even if we are talking about very energetic particles the mass is a relevant parameter when Lorentz violating interactions are present.

If \( E \gg m_p^2/|\eta| \) the energy loss is given by

\[
E(x) = \exp \left( -\frac{\alpha |\eta|}{3} x \right).
\]  

(50)

For \( \eta \) in the range of values described in section 1, this would give a mean free path in the range \( \mathcal{O}(1) \) to \( \mathcal{O}(10) \) kpc. This would imply that cold axions act as a powerful shield against very energetic cosmic rays. Since \( E_{th} \simeq 10^{15} \) eV the detection of cosmic rays above that energy would in fact impose a rather stringent bound on the combination \( \sqrt{\rho_a/f_a} \).

However, this is not so because even for the most energetic cosmics, just below the GZK cut-off of \( 10^{20} \) eV, we are well below the cross-over scale \( m_p^2/|\eta| \). In this regime the expression for \( E(x) \) is

\[
E(x) = \frac{E(0)}{1 + \frac{\alpha |\eta|}{4m_p^2} E(0) x}.
\]  

(51)

It is peculiar to see that for extremely large distances \( E(x) \sim \frac{1}{x^2} \), independently of the energy of the primary particle. However the distances where this behaviour could be seen are unphysically large because \( \frac{\alpha |\eta|}{4m_p^2} \) is very small. 

Thus the effect of the pseudoscalar background on the reach of cosmic rays is totally negligible for reasonable energies. Does this mean that the effect is completely invisible? Not quite.

Let us assume for the sake of definiteness that $E(0) \simeq 10^{20}$ eV. Then, the photon emission due to the axion background goes all the way up to $w_{\text{max}} \sim 10^{-2}$ eV (for $\sqrt{\rho_a/f_a}$ just in the currently allowed upper limit, and getting smaller as $\eta$ decreases). These energies corresponds to radio emission in the submillimetric range ($10^{-5}$ m or larger).

For electrons or positrons the numbers are different. The threshold energy for the radiation loss due to the interaction with the axion background can be as low as 1 TeV (for $f_a \sim 10^{11}$ GeV). The radiation emitted in this case would be with wavelengths characteristic of X-ray emission, or larger.

It is not easy to say whether these photons could be detected at all, because even over cosmological distances the rate of radiation is very low for protons (higher for electrons), but the emission of cosmic rays is sometimes copious and this could offset the low probability of radiation. The shape of the spectrum is completely calculable by the formulae we provide here, and the radiation is emitted in a very narrow cone around the charged particle. These two characteristics may be instrumental to detect it or, alternatively, to exclude its presence. Excluding it with some confidence level would place an interesting bound on $\sqrt{\rho_a/f_a}$. It should be mentioned that, being low energy, this radiation once is produced is not affected by the pseudoscalar background as discussed in section 2.

6 Conclusions and outlook

In this work we have considered the effect on charged particles of a mildly (compared to the particle momentum) time dependent pseudoscalar background. A physical situation worth exploring is the influence of a diffuse relic cold axion background on cosmic ray propagation. The effect is model independent, universal and can be computed unambiguously.

The effect is completely calculable in great detail because particle propagation is governed by a modification of QED (Maxwell-Chern-Simons Electrodynamics) that is exactly solvable. We have determined the kinematical constraints, the characteristics of the emitted radiation and the rate of energy loss of charged particles moving in such a medium. Some rather non-intuitive features appear and the results, we believe, are interesting per se. We find that for protons the phenomenon may appear (depending on the value of $\sqrt{\rho_a/f_a}$) at energies around $10^3$ TeV, but for those primaries with energies that survive the GZK cut-off the suppression brought about by the interaction with this 'axion shield' is quite negligible, being only relevant for extremely energetic particles. However, the 'bremmstrahlung' radiation may be measurable. The characteristic energies and extremely strong
colimation associated to given primaries may be crucial for that.

We have also investigated the relevance of this phenomena for electrons and positrons. In this case the threshold energy is really around the corner (around 1 TeV, maybe even less as it depends on the photon effective mass and on the value of $\sqrt{\mu_a/f_a}$) and the radiation could be as energetic as X-rays.

An analogous phenomenon may present itself in a completely different context, namely heavy ion collisions at high density and relatively low temperature. As emphasized recently a phase where a parity breaking condensate is present may well appear even at moderate densities. Once formed this condensate would be time dependent due to the expansion of the fireball produced in the collision. Such a background would have rather non trivial effects on the propagation of different particles. It is known to potentially influence the behaviour of scalars and pseudoscalars, dramatically changing the spectrum, but it would also influence, via a mechanism similar to the one discussed here, photons and leptons. This effect is being investigated.

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