Comparison of the effect of 1st order and 2nd order fluctuating temperature field on convective heat transfer coefficient in an unsteady laminar-boundary layer

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Abstract. An unsteady thermal boundary layer analysis over a heat source (flat plate) has revealed that first order temperature change in source and core flow temperatures affects the time average or RMS value of h'S. It also emphasize that 1st order changes are important. Analysis also indicates that a phase difference between pressure and temperature exists at the source in order to increase the RMS value of h'S. Value of h'S is directly proportional to the tan/sine of the above phase difference, This implies that entropy oscillations must exist at the heat source. The purpose of this paper is to compare the result of 2nd order analysis previously done, with the 1st order study carried out in this paper. In second order analysis time average of h”S depends upon 2nd order change due to 1st order effects. Finally, existence of phase difference between pressure and temperature, thus having entropy oscillations are evident, comparison implies although there is similarity between 1st order and 2nd order results but first order effects are more important than 2nd order effects.

1. Introduction
Engineers and scientists always have desire to get maximum amount of heat to be transferred from a heated surface carried by the flowing fluid over it. So that the surface area can be minimized which will be cost as well as space effective. The space required is very critical in many applications such as boiler and nuclear reactor. Conventionally, steady state heat transfer from a surface (straight wall or circular pipe with diameter very large compare to boundary layer thickness) is purely due to conduction through stagnation layer of the fluid in the vicinity of the surface. Heat conducted is carried away by convection in the fluid, convection heat transfer coefficient is not the property of the fluid but it depends on flow characteristics and surface characteristics. It has been observed in [1] that oscillatory mode results in higher heat transfer rates through the wall. It suggests that it will improve convective heat transfer as well.

2. Literature review
When a plate is heated or cooled uniformly over the entire surface at a temperature T, from the leading edge of the plate. Temperature gradients are set up in the fluid due to difference in temperature of wall and free stream temperature away from the wall. The temperature of the fluid immediately vicinity of the wall is T, and free stream temperature is T∞. In the region, adjacent to the wall temperature varies from T, to T∞, is called the thermal boundary layer (Figure 1). This is analogous to the hydrodynamic boundary layer. Above thermal boundary layer temperature is everywhere T∞ [2].
Figure 1. Physical model of thermal and hydrodynamic boundary layer.

Thermal system made by human beings the fluctuating regimes are commonly established in addition to permanent regimes. In [3], attempt has been made to determine effect of fluctuating temperature regimes on the thermal boundary layer growth and thickness as well as convective heat transfer coefficient, the techniques used for analysis are mainly mathematical extensions of differential and to the integral methods of Karma Pathausen in boundary layer flow. Mr. S.P Mahulikar and C.P Tso have worked on a new classification of fluid flow with infusion of thermal energy in a circular tube under laminar forced convection [4]. Keith Condle and Donald M.Mc. Eligot have worked on convective heat transfer for pulsating flow in the take down pipe of a V-6 Engine [5]. They found that the convective heat transfer parameters for pulsating flow are enhanced. Heat transfer is measured based on values of parameters determined experimentally.

Eduardo Ramos, Brain D Storey et al, have analyzed the oscillatory boundary layer flow generated by sinusoidal temperature field in the presence of wall. They observed that increase in amplitude of temperature is nonlinear, the temperature oscillation remains periodic but the time average heat transfer from the boundary to fluid for long duration is zero [6]. Hongweili, school of Mechanical Engineering Purdue University and Mr. M. Razi Nalim, Department of Mechanical Engg., Indiana University- Purdue University have studied the varying temperature fluid flow in heat exchangers, Nuclear reactors, non-steady flow devices and combustion engines. They obtained transient heat transfer correlation which shows that quasi steady models prediction commonly used, can be significantly different in both magnitude and direction of heat transfer [7]. G. Polidori, M. Lachi, J. Padet have studied the transient heat exchange between a steady laminar boundary layer and a flat sample with no thickness. They observed the time and space evolution of both the surface temperature and heat exchange coefficient [8].

Ernst R G Eckert, University of Minnesota has published a report on heat transfer in boundary layer flow at high velocities and high temperature [9]. Mr. M.F Blair had conducted experiments to study the influence of free stream turbulence on zero pressure gradient in a fully turbulent boundary layer flow. Convective heat transfer coefficients and and the mean velocity and temperature profile inside the boundary layer were obtained for constant free stream turbulence ranging from 1/4 to 7% for fully turbulent boundary layer flow. He found that both skin friction and heat transfer were substantially increased (up to 20%) [10].

3. Scope and problem formulation
In aforesaid literature review, it is observed that many researchers and scientists have used different systems to study heat transfer and related physics of the systems. They were enthusiastic to steady, unsteady heat transfer using different approaches. It is observed that the approach used in the present
work is very simple and will be easily understood by any reader. This will have far reaching implications in future heat transfer systems.

To study the effect of fluctuating temperature on laminar boundary layer heat transfer and thus to obtain a fluctuating temperature profile at the heat source (a flat wall or circular cylinder having dia. very large compare to boundary layer thickness), a differential analysis of thermal boundary has been carried out for unsteady case along the lines indicated in [11]. Boundary condition obtained from the analysis is used to determine the constant in temperature profile [12]. Temperature profile is obtained for 1st order as well as second order studies. First order and 2nd order temperature profile are used separately to determine 1st and 2nd order convective heat transfer coefficients.

4. Procedure
Consider a fluid flowing over a flat plate as shown in Figure 2. If the plate surface (heat source) temperature is $T_s$ and if the free stream temperature is $T_\infty$ (A typical temperature profile of the fluid temp near the solid boundary is also shown in Figure 3). Near the wall fluid particles are stationary therefore the rate of heat transfer from the surface is given by [11].

![Figure 2. Control volume for differential analysis of thermal boundary layer.](image)

![Figure 3. Temperature and velocity profile.](image)
\[ Q_{HT} = -kS \left( \frac{\partial T}{\partial y} \right)_{y=0} \] (1)

Where \( k \) = the thermal conductivity of the fluid.

\( \left( \frac{\partial T}{\partial y} \right)_{y=0} \) the value of the temperature gradients at \( y=0 \). The co-ordinate \( y \) is measured along the normal to the surface.

The rate of heat transfer \( Q_{HT} \) from a solid surface (heat source) to surrounding fluid is described in [11] and is given below.

\[ Q_{HT} = H_{HT} (T_s - T_\infty) \] (2)

Equating equation (1) and (2) for steady state

\[ H_{HT} = \frac{-kS}{(T_s - T_\infty)} \left( \frac{\partial T}{\partial y} \right)_{y=0} \] (3)

Thus for determination of convective heat transfer coefficient, temperature gradient has to be determined at the solid surface.

### 4.1. Determination of First order convective heat transfer coefficient \( h' \)

To obtain the actual expression for \( h s' \), we should know the temperature profile at the heated wall.

The temperature profile used is given by [12].

\[ T = T_s - \frac{3}{2} \left( T_s - T_\infty \right) (y/\delta) - \frac{1}{2} \left( T_s - T_\infty \right) (y/\delta)^3 \]

\[ + \in [b_0 + b_1 (y/\delta)+b_2(y/\delta)+b_3(y/\delta)^3] \cos (\omega t + \phi_T) \] (4)

The constants in above profile are obtained by using following boundary conditions.

1. At \( y=0 \), \( T_s = a_sT + b_s\cos(\omega t + \phi_T) \)

2. At \( y=0 \) \[ \rho \left( \frac{\partial T}{\partial y} \right)_{y=0} = \left[ k \left( \frac{\partial^2 T}{\partial y^2} \right)_{y=0} + K \left( \frac{\partial^2 T}{\partial y^2} \right)_{y=0} \right] \]

Above boundary condition is obtained from the differential analysis of thermal boundary layer about a heat source for unsteady case [13].

Where \( s \) is specific internal energy of the fluid and \( h \) is the enthalpy of the fluid. Rho (\( \rho \)) is given by

\[ \rho = \frac{P}{RT} \]

\[ P = P + \epsilon b_p \cos (\omega t + \phi) \]

Pressure \( P \) is assumed constant across the boundary layer

3. At \( y=\infty \), \( T_s = a_{sT} + \epsilon b_{sT} \cos (\omega T + \phi_T) \)

   where \( \infty \) signifies core flow constants.

4. At \( y=\infty \), \( \frac{\partial T}{\partial y} = 0 \)

We get the following Temperature profile,

\[ T = T_s - \frac{3}{2} \left( T_s - T_\infty \right) (y/\delta) - \frac{1}{2} \left( T_s - T_\infty \right) (y/\delta)^3 \]

\[ + \in [b_0 + b_1 (y/\delta)+b_2(y/\delta)+b_3(y/\delta)^3] \cos (\omega t + \phi_T) \]

\[ - \frac{\partial T}{\partial y} \left( \frac{\partial T}{\partial y} \right)_{y=0} \] (5)

\[ \left( \frac{\partial T}{\partial y} \right)_{y=0} = \left[ k \left( \frac{\partial^2 T}{\partial y^2} \right)_{y=0} + K \left( \frac{\partial^2 T}{\partial y^2} \right)_{y=0} \right] \] (6)

\[ \rho = \frac{P}{RT} \]

\[ P = P + \epsilon b_p \cos (\omega t + \phi) \]

Pressure \( P \) is assumed constant across the boundary layer

3. At \( y=\infty \), \( T_s = a_{sT} + \epsilon b_{sT} \cos (\omega T + \phi_T) \)

   where \( \infty \) signifies core flow constants.

4. At \( y=\infty \), \( \frac{\partial T}{\partial y} = 0 \)

We get the following Temperature profile,
Where, 
\[ b_2 = \frac{\delta^2}{16 \pi a_T \sin (\phi_p - \phi_T)} \]  
(10)

Now, \( h_s' \) is given by 
\[ h_s' = \frac{-k}{T_s - T_\infty} \left( \frac{dT}{dy} \right)_{y=0} \]  
(11)

Which gives 
\[ h_s' = \frac{3}{2} \frac{k}{T_s - T_\infty} \frac{(b_sT - b_\infty T)}{\delta} + \frac{1}{2} \frac{\delta \rho \omega b_{st} (c_p + c_v) \sin (\phi_p - \phi_T) \cos (\omega t + \phi_T)}{(T_s - T_\infty) 2 R a_{st} \cos (\phi_p - \phi_T)} \]  
(12)

Therefore
\[ < h_s' > = \frac{3}{2} \frac{k}{T_s - T_\infty} \frac{(b_sT - b_\infty T)}{\delta} + \text{RMS of 2}^{\text{nd}} \text{term} \]

4.2. Determination of 2\text{nd} order connective heat transfer coefficient \( h_s \)

Introducing 2\text{nd} order term in Temperature profile as given below 
\[ \epsilon^2 \left[ (e_0 + e_1 (y/\delta) + e_2 (y/\delta)^2 + e_3 (y/\delta)^3) + (d_0 + d_1 (y/\delta)) \right. \]
\[ + \left. (b_sT - b_\infty T) \right] \cos (\omega t + \phi_T) \]  
(13)

And following the procedure as given for 1\text{st} order analysis, following temperature profile is obtained.

\[ T = T_s - \frac{3}{2} \frac{b_sT - b_\infty T}{(T_s - T_\infty)} \frac{y}{\delta} - \frac{1}{2} \left( \frac{T_s - T_\infty}{\delta} \right) \left( \frac{y}{\delta} \right)^3 \]
\[ + \epsilon^2 (e_0 + e_1 (y/\delta) + e_2 (y/\delta)^2 + e_3 (y/\delta)^3) \]
\[ + b_2 \left( \frac{1}{2} (y/\delta) + (y/\delta)^2 - \frac{1}{2} (y/\delta)^3 \right) \cos (\omega t + \phi_T) \]
\[ + \epsilon^2 (e_0 + e_1 (y/\delta) + e_2 (y/\delta)^2 + e_3 (y/\delta)^3) + d_2 (d_0 + d_1 (y/\delta)) \]
\[ + \frac{1}{2} (d_sT - d_\infty T) (y/\delta)^2 \]  
(14)

Where \( b_2 = \frac{\delta^2}{16 \pi a_T \cos \phi_T} \)  
(15a)

\[ e_2 = \frac{\delta^2}{16 \pi a_T} \left[ b_p b_{st} (2 c_p + c_v) \cos (\phi_p - \phi_T) \right. \]
\[ - \left. \frac{b_{st}}{2 T_s \cos \phi_T} (c_p T_s b_p \sin \phi_p - (c_p + c_v) P b_{st} \sin \phi_T) \right] \]  
(15b)
\[ d_2 = \frac{\delta^2}{2kRT_c \cos \phi_T} \left[ -2p_d T \left( C_p + C_v \right) \omega \sin \phi_T + 2c_p T \omega \sin \phi_T \cdot \frac{\beta_p b_{ST} (C_p + C_v) \omega}{2} \sin \left( \phi_T + \phi_T \right) - \frac{b_{ST}}{2} \left( c_p T + b_p \omega \sin \phi_T - \left( 2c_p T \omega \sin \phi_T \cos \left( 2 \omega t + \phi_T \right) \right) \right] \]  

Second order heat transfer from the source is given by

\[ h''_s = \left( -k \right) \frac{\partial T'}{\partial y} \]  

Where \( T'' \) is the coefficient of \( c^2 \) in the expression of temperature profile given in equation (14). Value of \( T'' \) is substituted in equation (16) and simplified to get expression for \( h''_s \),

\[ h''_s = \left( \frac{k}{T_T - T_\infty} \right) \frac{\partial T'}{\partial y} \]  

From which time average of \( h''_s \) is obtained as

\[ \frac{2k}{25} \left( c_{ST} - c_{ST} \right) + \frac{\delta b_{ST}}{8RT_c (T_T - T_\infty)} \sin ( \phi_T - \phi_T ) \]  

5. Results and discussion

The first term of the time average of \( h_s \), relates to the first order change in the source and core flow temperatures. It is proportional to \( k \) and inversely proportional to \( \delta \). The second term is first order term arising from product of first order and steady state term, and it has many practical implications. It indicates that a phase difference between pressure and temperature must exist at the heat source. Heat convection is directly proportional to the tan of the phase difference. In isentropic case, no phase difference between pressure and temperature is possible. This implies that entropy oscillations must exist at the heat source. The second term also implies that \( h'_s \) is proportional to

1. The frequency of oscillations
2. The ratio of the first order source temperature oscillation magnitude to steady state source temperature.
3. The magnitude of the steady state pressure.
4. Thermal boundary layer thickness.

The first term of time average of \( h'_s \), relates to the second order changes in the source and core flow temperatures. The second term in a second term arising from 1\text{st} order effects. As in the first order results it also show the phase difference between pressure and temperature is critical and thus entropy oscillations must exist at the heat source. First term of both \( h'_s \), and \( h''_s \), are proportional to \( k/ \delta \) and the difference of coefficients of 1\text{st} order and 2\text{nd} order respectively. So, Profile of \( h'_s \) and \( h''_s \), will be same only magnitude will be different depending upon A & B and keeping K constant (Figure 4).

\[ A = \frac{2}{3} (b_{ST} - b_{ST}) \]
\[ B = \frac{2}{3} (c_{ST} - c_{ST}) \]

Thus as \( \delta \) increases \( h'_s \) and \( h''_s \) decreases and at a particular pt 1\text{st} order \( h'_s \) is greater than 2\text{nd} order \( h''_s \).
2nd term also implies 1, 2 & 4 as written above and 3rd in magnitude of the first order pressure oscillation in place of steady slate values. It looks that relative magnitude of h', and h'', depends on the magnitude of 1st and 2nd order oscillations and if a mechanism can be found to control (Øp - ØT) heat transfer may be controlled. 2nd term of 1st order h', is proportional to δ tan(Øp - ØT) with some constant terms and 2nd term of 2nd order h'' is proportional to δ sin(Øp - ØT) with some constant terms. Constant term of 1st order is greater than constant of 2nd order. The Graphical representation of keeping δ constant and (Øp – Øt) is varying as shown in Figure 5. With increase in δ, variation in h', and h'', is similar and increases (Figure 6).

Figure 4. Variation of h', and h'', with respect to δ.

Figure 5. Change in Values of h’, and h’’, with change in values of (Øp – Øt).
Figure 6. Change in values of \( h' \) and \( h'' \), with respect to \((\phi_p - \phi_t)\) with variation in \( \delta \).

From above analysis it can be seen that steady state convective heat transfer is effective for small values of \( \delta \) and 1\textsuperscript{st} & 2\textsuperscript{nd} order analysis reveals that higher values of convective heat transfer is obtained for higher values of \( \delta \).

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