Spontaneous Physical Compactification of the Extra Dimensions

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Abstract

We conjecture that the extra dimensions are physical non-compact at high energy scale or high temperature; after the symmetry breaking or cosmological phase transition, the bulk cosmological constant may become negative, and then, the extra dimensions may become physical compact at low energy scale. We show this in a five-dimensional toy brane model with three parallel 3-branes and a real bulk scalar whose potential is temperature dependent. We also point out that after the global or gauge symmetry breaking, or the supersymmetry breaking in supergravity theory, the spontaneous physical compactification of the extra dimensions might be realized.

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1 Introduction

Gauge hierarchy problem is one of the deep problems in the Standard Model. About three years ago, it was suggested that the large compactified extra dimensions may also be the solution to the gauge hierarchy problem [1], because a low \((4 + n)\)-dimensional Planck scale \((M_X)\) may result in the large 4-dimensional Planck scale \((M_{pl})\) due to the large physical volume \((V^n_p)\) of extra dimensions: \(M_{pl}^2 = M_X^{2+n} V^n_p\).

In addition, about two years ago, Randall and Sundrum [2] proposed another scenario that the extra dimension is an orbifold, and the size of extra dimension is not large but the 4-dimensional mass scale in the Standard Model is suppressed by an exponential factor from 5-dimensional mass scale because of the exponential warp factor. Furthermore, they suggested that the fifth dimension might be coordinate non-compact [3], and there may exist only one brane with positive tension at origin, but, there exists the gauge hierarchy problem and the physical size of the fifth dimension is indeed compact. The remarkable aspect of the second scenario is that it gives rise to a localized graviton field. Recently, a lot of 5-dimensional models with 3-branes were built [4-5]. We constructed the general models with parallel 3-branes on the five-dimensional space-time, and obtained that the 5-dimensional GUT scale on each brane can be identified as the 5-dimensional Planck scale, but the 4-dimensional Planck scale is generated from the low 4-dimensional GUT scale exponentially in our world. Furthermore, the models with codimension-1 brane(s) were constructed on the six-dimensional and higher dimensional space-time [6-9].

On the other hand, it is well known that five kinds of superstring theories live in the 10-dimensional space-time manifold and M-theory lives in the 11-dimensional space-time manifold due to the anomaly cancellation. In short, it seems that the low energy phenomenology and the current fundamental theories favour the existence of extra dimensions. Therefore, we might want to ask the question: why are the extra dimensions physical compact at low energy scale? From our philosophy, in the early Universe or when the energy scale or temperature is very high, all the space-time dimensions might be physical non-compact, furthermore, all the space dimensions might be equivalent, i.e., there may be no difference between the 3-dimensional space we observe and the extra space dimensions. If this philosophy was right, then, we should understand how the extra dimensions become physical compact at low energy scale.

We conjecture that at high energy scale or high temperature, the extra dimensions are physical non-compact. After the cosmological phase transition, or the global or gauge symmetry breaking, or the supersymmetry breaking in supergravity theory, the bulk cosmological constant becomes negative, and then, the extra dimensions become physical compact.

In this paper, in order to support our conjecture, we give a 5-dimensional toy brane model with three parallel 3-branes and one real bulk scalar which has temperature dependent potential. We assume that at high energy scale or temperature, the bulk cosmological constant is zero, because in string theory, it is natural to take the bulk cosmological constant to be zero since the tree-level vacuum energy in the
generic critical closed string compactifications (supersymmetric or not) vanishes, and the zero bulk cosmological constant is natural in the scenario in which the bulk is supersymmetric (though the brane need not be), or the quantum corrections to the bulk are small enough to be neglected in a controlled expansion. For simplicity, we also assume that at high energy scale, the metric is flat and diagonal. When the energy scale or temperature goes down, there is a second order cosmological phase transition. After the cosmological phase transition, the bulk cosmological constant becomes negative, and then, the metric has warp factor, which makes the fifth dimension physical compact. In fact, at low energy scale, this model is the same model discussed previously in [3]. Of course, this idea can be generalized to all the previous brane models and brane networks with constant bulk cosmological constant [2-9].

The key point to have the spontaneous physical compactification of the extra dimensions is that, after the cosmological phase transition or symmetry breaking, the bulk cosmological constant becomes negative, and then, the extra dimensions become physical compact due to the warp factor. Therefore, after the global or gauge symmetry breaking, or the supersymmetry breaking in supergravity theory, the spontaneous physical compactification of extra dimensions might be happened for the cosmological constant might become negative. And it is interesting to discuss the spontaneous physical compactification of the extra dimensions in the model buildings, and in the string theory or M-theory compactifications.

2 One Toy Model

In this section, we would like to present a toy brane model with spontaneous physical compactification of the extra dimension.

We consider the space-time is five-dimensional, and the fifth dimension is $R^1$. In addition, in order to obtain the cosmological phase transition, we introduce a real scalar $\phi$ whose bulk potential is temperature dependent. Assuming that we have three parallel 3-branes along the fifth dimension, and their fifth coordinates are: $-\infty < y_1 < y_2 < y_3 < +\infty$. We obtain the metric in each brane from the five-dimensional metric $g_{AB}$ where $A, B = \mu, y$ by restriction

$$g^{(i)}_{\mu\nu}(x^\mu) \equiv g_{\mu\nu}(x^\mu, y = y_i).$$

In this paper, for simplicity, we assume that the metric is diagonal, and at high energy scale or temperature, the metric is $\text{diag}(-1, 1, 1, 1, 1)$.

The classical action with a bulk scalar is given by

$$S = S_{\text{Bulk}} + S_{\text{Brane}},$$

$$S_{\text{Bulk}} = \int d^4x \, dy \, \sqrt{-g} \left\{ \frac{1}{2} M^3 X R - \frac{1}{2} \partial_A \phi \partial^A \phi - \Lambda(\phi, T) \right\},$$

$2$
\[ S_{\text{Brane}} = \sum_{i=1}^{3} \int d^4x \; dy \; \sqrt{-g^{(i)}} \{ \mathcal{L}_i - V_i(\phi, T) \} \delta(y - y_i) , \] (4)

where \( M_X \) is the 5-dimensional Planck scale, \( T \) is the temperature, \( \Lambda(\phi, T) \) is the temperature dependent bulk potential for \( \phi \) \([10]\), and \( V_i(\phi, T) \) where \( i = 1, 2, 3 \) is the brane tension which is dependent on \( \phi \) and \( T \). Our ansatz for \( \Lambda(\phi, T) \) and \( V_i(\phi, T) \) is\(^2\)

\[ \Lambda(\phi, T) = D \left( T^2 - T_o^2 \right) \phi^2 + \frac{\lambda(T)}{4M_X} \phi^4 , \] (5)

\[ V_i(\phi, T) = C_i(T) \; M_X \; \phi^2 , \] (6)

where \( D \) and \( T_o \) are constant terms, \( \lambda(T) \) is a slowly varying function of \( T \). The \( C_i(T) \) is also a slowly varying function and will be determined later. The temperature dependent potential for \( \phi \) has \( Z_2 \) symmetry, which is invariant under \( \phi \leftrightarrow -\phi \).

The solutions to the minimum or maximum of the potential \( \Lambda(\phi, T) \) are given by

\[ < \phi > |_{T=0} , \] (7)

\[ < \phi > |_{T} = \left( \frac{2D(T_o^2 - T^2)M_X}{\lambda(T)} \right) \] (8)

Therefore, the critical temperature is given by \( T_o \). At \( T > T_o \), the origin \( < \phi > = 0 \) is the minimum. At the same time, only the solution \( < \phi > = 0 \) does exist. In this case, the bulk cosmological constant is zero, and all the brane tensions are zero or one can think that the brane tensions are very small. At \( T = T_o \), both solutions collapse at \( < \phi > = 0 \). At \( T < T_o \), the origin \( < \phi > = 0 \) is the maximum, and the solution \( < \phi > = \sqrt{\frac{2D(T_o^2 - T^2)M_X}{\lambda(T)}} \) gives the minimum, so, the \( Z_2 \) symmetry is broken.

This cosmological phase transition is called of second order because there is no barrier during the cosmological phase transition.

When \( T < T_o \), we can expand the \( \phi \) around its minimum, and write \( \phi \) as

\[ \phi = \phi' + \sqrt{\frac{2D(T_o^2 - T^2)M_X}{\lambda(T)}} , \] (9)

so, we can express the \( \Lambda(\phi, T) \) and \( V_i(\phi, T) \) as \( \Lambda(\phi', T) \) and \( V_i(\phi', T) \), respectively

\[ \Lambda(\phi', T) = \frac{\lambda(T)}{4M_X} \left( \phi'^2 + 2\phi' \sqrt{\frac{2D(T_o^2 - T^2)M_X}{\lambda(T)}} \right)^2 \]

\(^2\)There might exist the terms proportional to \( T^5 \) and \( T^4 \) in the \( \Lambda(\phi, T) \) and \( V_i(\phi, T) \), respectively, which will vanish at low temperature. Those additional terms might not change the compactness of the space, but, will make the discussions of the spontaneous physical compactification of extra dimension a little bit complicated. So, as a toy model, we do not consider those terms and do not consider how to derive the ansatz Eqs. (5) and (6) here.
\[- \frac{D^2}{\lambda(T)} M_X (T_o^2 - T^2)^2, \]  

\[V_i(\phi', T) = C_i(T) M_X \left( \phi'^2 + 2\phi' \sqrt{\frac{2D(T_o^2 - T^2) M_X}{\lambda(T)}} + \frac{2D(T_o^2 - T^2) M_X}{\lambda(T)} \right). \]  

Neglecting the field \( \phi' \) for \( \langle \phi' \rangle = 0 \), during the cosmological phase transition, we obtain the dominant classical action

\[S_{\text{Bulk}} = \int d^4 x \ dy \sqrt{-g} \left( \frac{1}{2} M_X^3 R + \frac{D^2}{\lambda(T)} (T_o^2 - T^2)^2 \right), \]  

\[S_{\text{Brane}} = \sum_{i=1}^{3} \int d^4 x \ dy \sqrt{-g^{(i)}} \left( L_i - \frac{2 C_i(T) D (T_o^2 - T^2) M_X}{\lambda(T)} \right) \delta(y - y_i). \]  

Because the calculations are similar to those in Ref. [5], we just give the result here. Assuming that there exists a solution that respects 4-dimensional Poincare invariance in the \( x^\mu \)-directions, one obtains the 5-dimensional metric:

\[ds^2 = e^{-2\sigma(y,T)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2. \]  

With this metric, the Einstein equations reduce to

\[\left( \frac{d\sigma}{dy} \right)^2 = \frac{D^2}{6 M_X^2 \lambda(T)} (T_o^2 - T^2)^2, \]  

\[\frac{d^2 \sigma}{dy^2} = \sum_{i=1}^{3} \frac{2 C_i(T) D (T_o^2 - T^2)}{3 M_X \lambda(T)} \delta(y - y_i). \]  

The general solution to above differential equations is

\[\sigma(y, T) = \sum_{i=1}^{3} (-1)^{i+1} k(T) |y - y_i| + f[T_o^2 - T^2], \]  

where \( f[T_o^2 - T^2] \) is the polynomial with variable \( T_o^2 - T^2 \) and without constant term, and \( f[T_o^2 - T^2] \) is similar to the constant \( c \) in [5].

From Eqs. (15-17), we obtain

\[k(T) = \sqrt{\frac{1}{6\lambda(T)}} \frac{D(T_o^2 - T^2)}{M_X}, \]  

\[C_i(T) = (-1)^{i+1} \sqrt{\frac{3\lambda(T)}{2}}. \]
In addition, for simplicity, we define that \( \sigma(y) \equiv \sigma(y, T = 0) \), \( \lambda_o \equiv \lambda(T = 0) \), \( C^o_i \equiv C_i(T = 0) \), and \( k_o \equiv k(T = 0) \). After the cosmological phase transition, i.e., at very low temperature or energy scale, we obtain the metric

\[
\sigma(y) = \sum_{i=1}^{3} (-1)^{i+1} k_o |y - y_i| + f[T^2_o],
\]

(20)

where

\[
k_o = \sqrt{\frac{1}{6 \lambda_o}} \frac{DT^2_o}{M_X}.
\]

(21)

And we have

\[
C^o_i = (-1)^{i+1} \sqrt{\frac{3 \lambda_o}{2}}.
\]

(22)

From above metric, we obtain that, after the cosmological phase transition, the fifth dimension becomes physical compact because of the warp factor. And the gauge hierarchy problem can be solved in this model, as discussed in Ref. [2, 5].

One might notice that in \( \Lambda(\phi', T) \), the lowest order \( \phi' \) term is order of \( \phi'^2 \), but in \( V_i(\phi', T) \), the lowest order \( \phi' \) term is order of \( \phi' \). In fact, the lowest order \( \phi' \) term in \( V_i(\phi', T) \) can be order of \( \phi'^2 \), for example, if we defined the original \( V_i(\phi, T) \) as

\[
V_i(\phi, T) = C_i(T) M_X \left( \phi^2 - 2 \theta(T_o - T) \sqrt{\frac{2D(T^2_o - T^2)M_X}{\lambda(T)}} \phi \right),
\]

(23)

or

\[
V_i(\phi, T) = C_i(T) M_X \left( \phi^2 - 2 \sqrt{\frac{2D|T^2_o - T^2|M_X}{\lambda(T)}} \phi \right),
\]

(24)

where \( \theta(x) \) is the step function. The definition in Eq. (23) preserves the \( \mathbb{Z}_2 \) symmetry before the cosmological phase transition. With definition of \( V_i(\phi, T) \) in Eq. (23) or Eq. (24), the discussions of the spontaneous physical compactification of extra dimension are similar to the above discussions if one made the following transformation: \( C_i(T) \rightarrow -C_i(T) \), and \( C^o_i \rightarrow -C^o_i \), so, we will not repeat it here.

One can also discuss the first order cosmological phase transition if one introduced the term \( -E(T) \sqrt{T} \phi^3 \) in the bulk potential, however, in this case, there exists barrier during the cosmological phase transition [10].

In addition, this idea can be generalized to all the five-dimensional brane models with parallel 3-branes and the brane networks discussed in [2-9].

Furthermore, if one considered a complex scalar in the bulk, the global symmetry will be \( U(1) \). Global symmetries are argued to be broken by quantum gravity effects [11]. So, one might need to consider it as a gauge \( U(1) \) symmetry and put additional particles in the bulk. Similarly, the spontaneous physical compactification of extra dimension might also be happened in this scenario.
3 Discussion and Conclusion

The key point in above model to have the spontaneous physical compactification of extra dimension is that, after the cosmological phase transition or symmetry breaking, the bulk cosmological constant becomes negative, and then, the extra dimension becomes physical compact due to the warp factor. Therefore, in order to search other scenarios where the spontaneous physical compactification of extra dimensions may be realized, we need to explore other possibilities to have negative vacuum energy after the symmetry breaking.

Obviously, the global or gauge symmetry breaking (Higgs mechanism) is one candidate, because after the symmetry breaking, the cosmological constant will be negative if the cosmological constant is zero in original theory. For example, we can put the gauge group and some particles in the bulk, the gauge symmetry might be broken by radiative corrections at low energy scale (similar to the electroweak symmetry breaking in the mSUGRA model), then, the bulk cosmological constant might become negative and the spontaneous physical compactification of extra dimensions might be happened.

The other possibility is the supersymmetry breaking. As we know, in the supergravity theory, the gravitino mass term is negative in the potential. And if the supersymmetry was broken, the gravitino mass will give a negative contribution to the cosmological constant. So, the cosmological constant might be negative after the supersymmetry breaking, and then, we might have the spontaneous physical compactification of extra dimensions. Of course, at high energy scale, the supersymmetry is preserved, so, the cosmological constant is zero and the extra dimensions are physical non-compact.

How to realize the spontaneous physical compactification of extra dimensions due to the global or gauge symmetry breaking, or the supersymmetry breaking in the model buildings and in the string theory or M-theory compactifications is an interesting subject and deserves further study.

In short, we conjecture that at high energy scale or high temperature, the extra dimensions are physical non-compact. After the cosmological phase transition, or the global or gauge symmetry breaking, or the supersymmetry breaking in supergravity theory, the bulk cosmological constant becomes negative, and then, the extra dimensions become physical compact. We show this in a five-dimensional toy brane model with three parallel 3-branes and a real scalar whose bulk potential is temperature dependent. In the mean time, the gauge hierarchy problem can be solved in the toy model.

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