Scalable and Passive Wireless Network Clock Synchronization

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Abstract—Clock synchronization is ubiquitous in wireless systems for communication, sensing and control. In this paper we design a scalable system in which an indefinite number of passively receiving wireless units can synchronize to a single master clock at the level of discrete clock ticks. Accurate synchronization requires an estimate of the node positions. If such information is available the framework developed here takes position uncertainties into account. In the absence of such information we propose a mechanism which enables simultaneous synchronization and positioning. Furthermore we derive the Cramer-Rao bounds for the system which show that it enables synchronization accuracy at sub-nanosecond levels. Finally, we develop and evaluate an online estimation method which is statistically efficient.

I. INTRODUCTION

Time synchronization plays a key role in wireless communication, sensing and control. Indeed, many wireless applications require upkeep of timing in accomplishing their objectives.

In wireless cellular communications, accurate time information is traditionally needed for signal acquisition, demodulation, multiple access coordination, etc [1], [2]. Accurate timing and synchronization are also requirements in real-time wireless channel characterization and in several concepts in wireless communications, including beamforming and interference alignment [3]–[6]. Such requirements are also mentioned in [8] as the main challenge for distributed beamforming to work in the next generation wireless communication systems. Similarly, in [4]–[6], accurate time synchronization is shown to be a requirement for interference alignment work. Emerging concepts like femto-cells pose more challenging synchronization requirements in terms of scalability and accuracy as discussed in [7]. The sub-nanosecond time and phase synchronization is also needed in distributed radar applications [8]. Wireless ranging and positioning require time synchronization in time-difference-of-arrival (TDOA) based schemes, where anchor nodes are synchronized in time [2]. Wireless control networks are also critically dependent on synchronized sensors and actuators [10].

Variants of the Network Time Protocol (NTP) [11] and the Precision Time Protocol (PTP) [12] constitute the most popular methods for time reference and synchronization in wired networks [13]. The emergence of a variety of wireless networks during the past decade has led to the development of wireless time-synchronization protocols and localization schemes. The Reference Broadcast Synchronization (RBS) and the Time synchronization Protocol for Sensor Networks (TPSN) emerged as popular wireless time synchronization protocols around the same time [14], [15]. In RBS, the nodes in a wireless network synchronize through a broadcast by a master node and inter-node exchanges to remove any sender uncertainty. TPSN works by creating a hierarchical tree-based structure where every leaf node synchronizes to its parent node through message exchanges. Neither RBS nor TPSN accounts for propagation delays nor do they enable passive synchronization. For the aimed accuracies of these protocols, the signal time-of-flight over a wireless channel is assumed to be negligible. The protocol developed in [16] enables higher accuracy by using separate channels for communication and measurements required for synchronization.

Global Positioning System (GPS) signals are also used for synchronizing time in wireless communication systems [6]–[8], [17], [18]. GPS-based timing solutions enable an indefinite number of nodes to perform simultaneous self-localization and time synchronization. This joint feature is important in deployed wireless sensor networks where both position and time need to be resolved at each sensing node. In GPS-based solutions, a signal known as pulse per second (PPS) is extracted from pseudorange measurements and satellite ephemeris data at the GPS receivers. The PPS signal is then used as a reference in frequency synthesizers to generate high frequency signals [5], [8], [19]. However, GPS signals cannot be accessed indoors and the timing accuracy obtained does not reach nanosecond levels.

In this paper we develop a scalable system in which passive, receiver-only nodes can synchronize to a single master clock at the level of discrete clock ticks. We show that the synchronization performance of the system can reach sub-nanosecond levels. When position information is lacking, we propose a mechanism which enables simultaneous synchronization and...
A. Prior art and our contributions

Time synchronization schemes are evolving to provide nanosecond-level synchronization, which requires accounting for signal time-of-flight between nodes. A scalable multihop scheme to synchronize the nodes to nanosecond accuracy was proposed in [20].

Several works have developed system proposals as well as presented theoretical analyses of time synchronization, cf. [21]–[23]. Fundamental limits on time synchronization in sensor networks were given in [21]. The authors of [22] suggested using factor-graph methods for network clock estimation. In [23], clock synchronization is achieved using eavesdropping measurements. The synchronizing unit is a receiver-only node and hence the method is claimed to be energy efficient. In [24], a joint localization method for source nodes was proposed using TDOA which implicitly synchronizes an arbitrary number of anchor nodes.

Our proposed method for clock synchronization in wireless networks enables system performance beyond the state-of-the-art. Specifically, we highlight the following attributes of our proposal.

• **Accuracy:** We focus on enabling nanosecond accuracy. NTP provides millisecond accuracy over IP networks and has been overtaken by PTP over wired networks. PTP provides accuracy levels of a few nanoseconds using specialized hardware. For wireless solutions, such as RBS and TPSN, the accuracy for sensor network synchronization methods is on the order of microseconds. These methods do not need to take time of flight into account as their requirements are less stringent. GPS-based synchronization methods can typically synchronize to a 100 nanosecond-level. For nanosecond levels, time of flight needs to be estimated accurately as in [20].

• **Scalability:** Another feature of the proposed synchronization method is its scalability. Scalability has been addressed previously in a few papers, albeit only implicitly. RBS, TPSN and the scheme proposed in [20] are scalable by virtue of providing synchronization to nodes through adhoc multihop connections. In these systems, nodes synchronize through mutual exchanges of signals among them. The signal exchange could be two-way round-trip time measurements or timestamps recording time-of-arrival information. By contrast, systems like GPS and the one proposed in [23] are receiver-only systems and hence they allow any number of nodes to synchronize with the reference clock. Our proposed method is similar to the latter class of scalable solutions. Indeed, we develop a method that is scalable as each synchronizing node requires only a receiver to synchronize to a reference, as in GPS. The lack of transmission requirement for the synchronizing nodes makes the solution energy efficient.

• **Positioning for synchronization:** Our proposed solution is similar to GPS with respect to scalability but enables nanosecond accuracy using existing hardware technologies. In addition, it can be used in indoor scenarios.

We will propose a local positioning system along with the synchronization mechanism to enable time-of-flight estimation. The proposed positioning system for synchronization builds upon our previous works [25]–[28].

We consider a general scenario as illustrated in Figure 1. The observed clock time in a wireless network is traditionally modeled as a continuous function of clock skew \( \alpha \) and the phase offset \( \beta \) [23], [29], [30].

\[
C_m(t) = t \quad \text{and} \quad C_u(t) = \alpha t + \beta, \quad (1)
\]

where \( C_m(t) \) denotes a reference master clock and \( C_u(t) \) denotes the local clock of a node \( u \) in the network. In this model the local clock time can be resolved into that of the master clock by identifying the clock parameters. Network synchronization is achieved by resolving the observed time at each node to a common clock.

In digital clocks, however, time is recorded by counting the number of periods of a repeating clock signal. At each rising clock edge of the periodic signal, an integer time counter will have a relative offset \( \alpha \). The lack of transmission requirement for the synchronizing nodes makes the solution energy efficient.

### B. Problem formulation

The state of each digital clock is the integer number of cycles that have elapsed since some initialization event. Suppose the master clock operates with a period \( T_m \). Then its clock state \( n_m \in \{0, 1, 2, \ldots \} \) corresponds to times

\[
C_m \in \{0, T_m, 2T_m, \ldots \}.
\]

The master clock initializes the counters by transmitting a signal across the wireless network. The clock at node \( u \), which operates with period \( T_u \), will have a relative offset \( \phi_u \) due to
the propagation delay and nonsynchronicity as illustrated in Fig. 2. Its clock state \( n_u \in \{0, 1, 2, \ldots \} \) corresponds to times

\[
C_u = \{ \phi_u, T_u + \phi_u, 2T_u + \phi_u, \ldots \}
\]

Therefore the current clock state \( n_u \) of the node can be resolved into a common time if the clock parameters \( \phi_u \) and \( T_u \) are identified. In addition, identification of \( T_m \) enables also coordination with respect to the master periodic signal across the wireless network.

Based on the previous discussion we may write

\[
C_m = T_m n_m \quad \text{and} \quad \begin{cases} 
C_1 = T_1 n_1 + \phi_1 \\
C_2 = T_2 n_2 + \phi_2 \\
\vdots \\
C_U = T_U n_U + \phi_U 
\end{cases}
\] (2)

By identifying the clock parameters at each node \( u = 1, \ldots, U \), synchronization is achieved since a common time frame is shared across the entire network. This enables coordination relative to the master clock among all nodes.

Note that nominal values of the clock frequencies, and therefore of the periods \( T_u \) and \( T_m \), are typically available given. However, usually, these values are not sufficiently precise. To obtain more accurate estimates of \( T_u \) it is possible to use a device that measures the intervals between ticks. Similarly, as the signal from the master clock is repeated periodically after \( M \) cycles, \( T_m \) can also be estimated accurately. The primary challenge, however, is to estimate the relative offset \( \phi_u \).

In this paper, we design a system in which passively receiving nodes are synchronized by estimating their respective clock parameters. The system is scalable to an indefinite number of nodes, i.e. \( U \gg 1 \). Furthermore, we study the resolution limits of the system using the Cramér-Rao bounds. Using existing hardware performance figures, we show that the proposed system enables sub-nanosecond accuracy. While the estimation of \( T_m \) and \( T_u \) can be performed separately from \( \phi_u \), we derive a joint online estimator that takes into account the uncertainties of all estimates. The proposed estimator is subsequently evaluated in several numerical experiments.

*Remark:* An implementation of the estimator along with numerical simulation examples is available at the webpage of KTH Dept. Signal Processing under ‘Reproducible research’.

## II. System model

To achieve the objectives stated above, we propose a system with the following features:

1) All passive units can measure time-intervals \( \Delta = t - t' \) between events at times \( t \) and \( t' \), using a time-measurement device. This enables observations at a higher resolution than that of the digital clock and is grounded in the emerging TDC and ADC technologies.

2) The master periodically transmits a time-resolvable signal after \( M \) clock cycles. Among others, this ensures the identifiability of \( T_m \). The transmission event from the master defines the starting point of a system-wide clock with period \( T_m \). We call the period of \( M \) clock cycles an *epoch*.

3) The master node \( m \) is located at a known position \( x_m \). The position of an arbitrary synchronizing node \( u \), denoted \( x \), is unknown. Together with the assumption that an epoch is longer than the clock period of any synchronizing node, i.e., \( M T_m > T_u \), that fact that \( x_m \) is known enables the identifiability of \( \phi_u \) as we will show below.

We will model the unknown position as \( x \sim \mathcal{N}(\bar{x}, \Lambda_x^{-1}) \) when we have access to a prior estimate \( \bar{x} \) with a dispersion matrix \( \Lambda_x^{-1} \). When such prior position information is lacking, i.e. when \( \Lambda_x = 0 \), then \( \phi_u \) cannot be identified. To ensure identifiability in such a case, under the assumption that the positions are expressed in three-dimensional coordinates, we consider a system with the following additional features:

4) There exist three transceiving nodes, deployed at known positions \( \{x_1, x_2, x_3\} \), cf. Fig. 3. The transceivers transmit sequentially in the order \( \{m, 1, 2, 3\} \), and repeatedly.

5) When receiving a signal from the preceding transmitter in the above order, the subsequent transceiver transmits after a fixed delay \( \Delta_0 \), which can be generated independently of the local clock [8]. This is to avoid interfering signals from the master and transceivers during an epoch. Specifically, we assume

\[
M T_m \gg \Delta_0 > \text{max. distance to transmitter}/c,
\]

where \( c \) is the propagation velocity. Then each transmitted signal can reach all nodes before the subsequent signal is transmitted.

Making use of these features together, we will show that it is possible to synchronize any number of passively receiving nodes. That is, each synchronizing node \( u \) can resolve the unknown clock parameters \( \phi_u, T_u \) and \( T_m \) in (2). The **scalable wireless network synchronization system** is abbreviated **SWINS**.

![Fig. 2. Space-time diagram of nodes m and u, with one vertical spatial dimension and a horizontal time dimension. The digital clock states correspond to discrete events or ticks along the time-axes (dots). The master node m and passive node u have clock periods \( T_m \) and \( T_u \), respectively. The transmission event from the master defines the initial tick of the system clock (white). Upon receiving the signal, the corresponding initial tick on the local clock (gray) will be subject to an unknown offset \( \phi_u \).](image)
The time of flight equals velocity and \( \rho \) interval between the received signal and the subsequent clock tick at \( u \). The time of flight equals \( \frac{1}{c} \rho_{m,u} \).

A. Data model

First, consider the initial signal received by a passive node \( u \) from \( m \), as depicted in Fig. 4. Node \( u \) can only record time intervals, and we define \( \Delta_1 \) as the time between the received signal and the next clock tick at \( u \). Given that the time of flight of the signal is \( \frac{1}{c} \rho_{m,u} \), where \( c \) is the signal propagation velocity and \( \rho_{m,u} = \|x_m - x\|_2 \) is the range between \( m \) and \( u \), the following relation

\[
\Delta_1 = \phi_u - \frac{1}{c} \rho_{m,u} \quad (3)
\]

applies to the first epoch.

At node \( u \), the number of clock cycles till the subsequent epoch begins, denoted \( N \), is recorded and corresponds to a constant time interval \( NT_u \geq MT_m \). Observing each \( N \)th clock tick we can derive a relation between the observed intervals as follows, see Fig. 5 which illustrates the basic principle. Let \( \Delta_k \) denote the time between receiving a signal and the \( k \)th clock tick for \( k > 1 \). Then it follows that \( \Delta_{k-1} + NT_u = MT_m + \Delta_k \). This relation for \( k \) intervals together with (3) can be written as the following recursion

\[
\Delta_k = \Delta_{k-1} + NT_u - MT_m
\]

\[
\Delta_2 = \Delta_1 + NT_u - MT_m
\]

\[
\Delta_1 = \phi_u - \frac{1}{c} \rho_{m,u}
\]

which comprises the unknown clock and position parameters. Using this recursion, we can write the observed interval \( \Delta_k \) at the \( k \)th epoch as

\[
y_{\phi,k} = \phi_u - \frac{1}{c} \rho_{m,u} + (k - 1)(NT_u - MT_m) + w_{\phi,k}, \quad (4)
\]

where \( w_{\phi,k} \) is a zero-mean noise. From the above equation we see that \( \phi_u \) cannot be identified without determining also the range \( \rho_{m,u} \), which is a function of the unknown position \( x \).

Next, we show that it is possible to resolve \( x \) using scheduled transmissions from the three transceivers during an epoch. The basic principle is illustrated in Fig. 6. When the master signal reaches transceiver node 1, it transmits after a known delay \( \Delta_0 \). The subsequent transceiving nodes do the same according to the given transmission order \( \{m, 1, 2, 3\} \). For the \( k \)th epoch, the time-intervals between each received signals at node \( u \) can be written as

\[
y_{1,k} = \frac{1}{c} \rho_{m,1} + \Delta_0 + \frac{1}{c} \rho_{1,u} - \frac{1}{c} \rho_{m,u} + w_{1,k},
\]

\[
y_{2,k} = \frac{1}{c} \rho_{1,2} + \Delta_0 + \frac{1}{c} \rho_{2,u} - \frac{1}{c} \rho_{1,u} + w_{2,k}, \quad (5)
\]

\[
y_{3,k} = \frac{1}{c} \rho_{2,3} + \Delta_0 + \frac{1}{c} \rho_{3,u} - \frac{1}{c} \rho_{2,u} + w_{3,k},
\]

where \( \rho_{i,j} = \|x_i - x_j\|_2 \). Each time-interval measurement produces a hyperbolic constraint on \( x \), cf. the principles of TDOA approach [28]. Thus three constraints are sufficient for identifying \( x \) in the three-dimensional space, and therefore also for resolving \( \phi_u \).

At the end of the \( k \)th epoch, its duration is recorded, resulting in

\[
y_{m,k} = MT_m + w_{m,k}, \quad (6)
\]

where \( M \) is known and therefore we can resolve \( T_m \) from (5). Similarly, for each epoch at \( u \), \( N \) ticks are recorded at the
local clock, and the observed time interval is
\[ y_{u,k} = NT_u + w_{u,k}. \]  

In sum, using the above observations, made at a passive node \( u \), ensures that \( \phi_u, T_u \), and \( T_m \) are identifiable parameters. This enables wireless synchronization to the master clock \( m \). The additional transceivers also render \( x \) identifiable and therefore enable self-localization at each node \( u \).

B. Noise model

Each observed time-interval above is subject to two sources of error arising from its start and stop events, respectively. In (5) and (6), the start and stop events are triggered by uncorrelated RF signals. A nominal value of error variance \( \sigma_k^2 \) from such events can be assigned but in practice varying RF conditions produce outliers so that we assume a varying \( \sigma_k^2 \). The total noise variance for these measured intervals is \( \text{E}[w_{i,k}^2] = 2\sigma_k^2 \) for \( i = m, 1, 2, 3 \) since they are based on a pair of RF measurements. In (6), one RF measurement is shared with (4) so that the errors of the observed intervals are correlated: \( \text{E}[w_{i,k}w_{m,k}] = \sigma_k^2 \). Furthermore, because two consecutive measurements share one RF measurement in (6) and (5), we can write:
\[ \text{E}[w_{i,k}w_{j,k}] = \begin{cases} 2\sigma_k^2, & i = j, \\ \sigma_k^2, & j \text{ follows } i, \text{ or vice versa,} \\ 0, & \text{otherwise.} \end{cases} \]

We assume that the RF noise yields the dominant part of \( \sigma_k \) and that the noise contribution of the timing device itself is only a small fraction \( 0 < \alpha < 1 \) of \( \sigma_k \), which depends on the performance figures of the device. In practice \( \alpha = 0.1 \) is a reasonable value for existing hardware [33], and this is the value we will assume in what follows. Then since the start and stop events of the interval in (7) are triggered solely by two clock ticks, we have \( \text{E}[w_{u,k}^2] = 2\alpha^2\sigma_k^2 \). Finally, because (4) is based on one RF and one clock tick we have \( \text{E}[w_{\phi,k}^2] = (1 + \alpha^2)\sigma_k^2 \). We model the noise sources as jointly Gaussian and omit the correlation between consecutive epochs.

III. CRAMÉR-RAO BOUNDS

To study some basic properties of \( S_{\text{WINS}} \), we begin by collecting the observed time intervals from epoch \( k \) in a vector
\[ y_k \triangleq S_k \begin{bmatrix} y_{\phi,k} & y_{u,k} & y_{m,k} & y_{1,k} & y_{2,k} & y_{3,k} \end{bmatrix}^T \in \mathbb{R}^{n_k}, \]  

where
\[ S_k = \begin{cases} I_6 & \text{if transceiving nodes present in epoch } k, \\ [I_3 \ 0_{3 \times 3}] & \text{otherwise} \end{cases} \]  

is a selection matrix and \( n_k \) is the number of measured intervals in epoch \( k \). Combining (4), (7), (6), and (5), we can write (8) as
\[ y_k = \mu_k + H_k c + \frac{1}{c} G_k \rho(x) + w_k \in \mathbb{R}^{n_k}, \]  

where \( c \triangleq [\phi_u \ T_u \ T_m]^T \) contains the parameters of interest. The mean vector
\[ \mu_k = S_k \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{c} \|x_m - x_1\| + \Delta_0 \\ \frac{1}{c} \|x_1 - x_2\| + \Delta_0 \\ \frac{1}{c} \|x_2 - x_3\| + \Delta_0 \end{bmatrix} \in \mathbb{R}^{n_k} \]

is known and the vector of ranges
\[ \rho(x) = \begin{bmatrix} \|x - x_m\|_2 \\ \|x - x_1\|_2 \\ \|x - x_2\|_2 \\ \|x - x_3\|_2 \end{bmatrix} \in \mathbb{R}^4, \]

is a function of the unknown position \( x \). The known system matrices in (10) can be written as
\[ H_k = S_k \begin{bmatrix} 1 & (k-1)N & -(k-1)M \\ 0 & N & 0 \\ 0 & 0 & M \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \]
\[ G_k = S_k \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \]  

Based on the noise model introduced in Section II-B the measurement noise vector \( w_k \) has a covariance matrix...
\[ \sigma^2_k Q_k \triangleq E[w_k w_k^\top], \] given by:
\[
Q_k = S_k \begin{bmatrix}
(1 + \alpha^2) & 0 & 1 & 0 & 0 & 0 \\
0 & 2\alpha^2 & 0 & 0 & 0 & 0 \\
1 & 0 & 2 & 1 & 0 & 0 \\
0 & 0 & 1 & 2 & 1 & 0 \\
0 & 0 & 0 & 1 & 2 & 1 \\
0 & 0 & 0 & 0 & 1 & 2 \\
\end{bmatrix}
\]

As the noise \( w_k \) is modeled as Gaussian, we have
\[
y_k | c, x, \sigma^2_k \sim \mathcal{N} (\mu_k + H_k c + \frac{1}{c} G_k \rho(x), \sigma^2_k Q_k). \tag{11}\]

This data model enables an analysis of how accurately the clock parameters can be estimated in SWINS.

### A. Cramér-Rao bound

Define the vector
\[
\theta \triangleq \begin{bmatrix} c \\ x \end{bmatrix} \in \mathbb{R}^{3+d},
\]
where \( d = 2 \) or 3 is the spatial dimension. The Fisher information matrix of \( \theta \) for the \( k \)th epoch data model in \( [11] \) is given by \([34] \) ch. 3 \([35] \) App. B.3:
\[
J_k(x, \sigma^2_k) = \frac{1}{\sigma^2_k} \begin{bmatrix} H_k \frac{1}{c} G_k \Gamma(x) \end{bmatrix}^\top Q_k^{-1} \begin{bmatrix} H_k \frac{1}{c} G_k \Gamma(x) \end{bmatrix}, \tag{12}\]

where the Jacobian of the range function \( \rho(x) \) is
\[
\Gamma(x) \triangleq \partial_x \rho(x) = \begin{bmatrix} (x-x_m) \quad (x-x_1) \quad (x-x_2) \quad \cdots \quad (x-x_d) \end{bmatrix} \in \mathbb{R}^{4 \times d}.
\]

In the above model, the data from each epoch are mutually uncorrelated. Therefore the information from each epoch is additive and the total information matrix after \( k \) epochs equals
\[
\Lambda_k = \Lambda_{k-1} + J_k, \tag{13}\]
where \( \Lambda_0 = 0 \). Then the mean-square error (MSE) matrix of any unbiased estimator \( \hat{\theta} \) is bounded via the Cramér-Rao inequality:
\[
E_y[(\theta - \hat{\theta})(\theta - \hat{\theta})^\top] \succeq \Lambda_k^{-1},
\]
and specifically for \( \hat{c} \) we have
\[
E_y[(c - \hat{c})(c - \hat{c})^\top] \succeq (\Lambda_{c,k} - \Lambda_{xc,k}^\top \Lambda_{xc,k}^{-1} \Lambda_{xc,k})^{-1}, \tag{14}\]
where the right-hand side is obtained by partitioning the information matrix as
\[
\Lambda_k = \begin{bmatrix} \Lambda_{c,k} & \Lambda_{xc,k}^\top \\ \Lambda_{xc,k} & \Lambda_{x,k} \end{bmatrix}.
\]

### B. Hybrid Cramér-Rao bound

When an informative prior for \( x \) exists, the unknown position can be modeled as a random variable \( x \sim \mathcal{N}(\bar{x}, \Lambda_x^{-1}) \). Then the MSE matrix of any unbiased estimator \( \hat{c} \), when averaged over all possible values of \( x \), is bounded via the Hybrid Cramér-Rao inequality \([36]\):\[
E_y, x [(c - \hat{c})(c - \hat{c})^\top] \succeq (\Lambda_{c,k} - \Lambda_{xc,k}^\top \Lambda_{xc,k}^{-1} \Lambda_{xc,k})^{-1}, \tag{16}\]
where the right-hand side is obtained from the expected information matrix
\[
\bar{\Lambda}_k = E_x[\Lambda_k] + \begin{bmatrix} 0 & 0 \\ 0 & \Lambda_x \end{bmatrix} = \begin{bmatrix} \bar{\Lambda}_{c,k} & \bar{\Lambda}_{xc,k}^\top \\ \bar{\Lambda}_{xc,k} & \bar{\Lambda}_{x,k} \end{bmatrix}.
\]

The expectation is approximated numerically using Monte Carlo simulations.

To illustrate the spatial variation of \( [16] \) for \( d = 2 \), we plot the transceiving nodes and plot in Fig. 8 the Hybrid Cramér-Rao Bound (HCRB) of \( \phi_n \) as a function of the prior mean \( \bar{x} \).

The master position and the precision matrix of the prior of \( x \) are given by:
\[
x_m = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \Lambda_x = \begin{bmatrix} 0.1^2 & 0 \\ 0 & 0.01^2 \end{bmatrix}^{-1}.
\]
This corresponds to an position error ellipse whose axis correspond to standard deviations of 0.1 and 0.01 meters, respectively. There is greater uncertainty along the $\bar{x}_2$-axis than the $\bar{x}_1$-axis. Consequently the bound on the clock error, which depends on the range to the master, is greater when $\bar{x}$ is on positions along one axis than the other. This variation in accuracy.

\[ x \approx \text{arg max}_x \text{ln} p(x) \]

\[ J(x) = \text{diag}(\sigma_x^2) \]

\[ \hat{\sigma}^2(x) = \frac{1}{n} \| y - \bar{x} \|_Q^{-1} \]

\[ \Pi_{H} = \text{diag}(\sigma_x^2) \]

\[ \hat{\sigma}^2(x) = \frac{1}{n} \| y - \hat{x} \|_Q^{-1} \]

\[ \hat{x} = \text{arg min}_x \text{ln} V_0(x) + V_1(x) \]

\[ \hat{\theta}_0 = \begin{bmatrix} \bar{x} \\ 0 \end{bmatrix} \text{ and } \hat{J}_0 = \begin{bmatrix} 0 & 0 \\ 0 & \Lambda_x \end{bmatrix} \]

denote the prior estimate and the corresponding information matrix, respectively. Then we can compute the following linear combination recursively:

\[ \hat{\theta}_k = \left( \sum_{i=0}^{k} \hat{J}_i \right)^{-1} \left( \sum_{i=0}^{k} \hat{J}_i \hat{\theta}_i \right) = \hat{\Lambda}_k^{-1} \hat{s}_k \]

\[ \hat{\Lambda}_k = \hat{\Lambda}_{k-1} + \hat{J}_k \]

\[ \hat{s}_k = \hat{s}_{k-1} + \hat{J}_k \hat{\theta}_k \]

\[ \hat{\theta}_0 = \hat{J}_0 \]

\[ \hat{s}_0 = \hat{J}_0 \hat{\theta}_0. \]

Remark: If a constant noise level $\sigma_0^2$ is used in (12), i.e., $\hat{J}_k = J_k(\bar{x}_k, \sigma_0^2)$, then one can verify that (19) is invariant to the nominal value $\sigma_0^2 > 0$. To make (19) robust with respect to noise outliers the corresponding estimate of $J_k > 0$ should decrease when there are outlying observations in epoch $k$. This can be achieved using the estimated noise variance from (17) for each epoch, which we denote $\hat{\sigma}_k^2$. More concretely, we use $\hat{\sigma}_k^2 = \max(\sigma_k^2, \sigma_0^2)$. In this way, the estimator adapts to noise outliers that exceed a nominal $\sigma_0^2$ and at the same time occasional overestimation of the information matrix is prevented when $\hat{\sigma}_k^2$ is small.

B. Minimization method

We propose a computationally efficient gradient-based method to solve (17). First we note that the negative log-likelihood can be expressed as

\[ -\ln p(y_k, \bar{x}, \sigma_k^2) = \frac{\sigma_k^2}{2} \| y_k - \mu_k - H_k e \|_Q^{-1} + \frac{1}{2} \ln \sigma_k^2 + \frac{1}{2} \| \bar{x} - \bar{x} \|_Q^{-1} + K, \]

\[ \Pi_{H} = \text{diag}(\sigma_x^2) \]

is a projector matrix. After inserting (21) into (20), the maximum likelihood estimate of $\bar{x}$ can be obtained by solving

\[ \bar{x} = \text{arg min}_x \text{ln} V_0(x) + V_1(x) \]

\[ \Delta V(\bar{x}) \]

where

\[ V_0(x) = \hat{\sigma}^2(x) \text{ and } V_1(x) = \frac{1}{n} \| x - \bar{x} \|_Q^{-1} \Lambda_x. \]
The gradient of $V(x)$ can be written as
\[ \partial V(x) = \frac{1}{V_0(x)} \partial V_0(x) + \partial V_1(x), \]
where compact expressions of the gradients $\partial V_0(x)$ and $\partial V_1(x)$ are given in Appendix A. Starting from an initial point $x^0$, we formulate a gradient descent method
\[ \hat{x}^{i+1} = \hat{x}^i + \alpha_i p_i, \]  
(24)
where
\[ p_i = \frac{\partial V_0(x) + V_0(x) \partial V_1(x)}{\|\partial V_0(x) + V_0(x) \partial V_1(x)\|} \times -\partial V(x) \]  
(25)
and the step size $\alpha_i$ is chosen by a line search
\[ \min_{\alpha_i \in I} V(\hat{x}^i + \alpha_i p_i) \]  
(26)
in the interval $I = [0, \eta \|\hat{x}^i - \hat{x}^{i-1}\|]$ where $\eta$ is a user parameter which determines the upper limit on the step size. When prior information is available the initial point can be taken as $\hat{x}^0 = \hat{x}$. If it is unavailable the centroid of the known transmitting node coordinates, i.e., $\hat{x}^0 = \frac{1}{N} \sum_i x_i$, or the estimate from a previous epoch can be used.

In summary, for each epoch, (17) is solved by iterating (24) until convergence, followed by insertion of the position $I$ in the interval
\[ \vec{\phi} = \frac{\theta}{\phi}, \]
and the step size $\alpha_i$ is set to 1.2. We set the tolerance $\epsilon$ to $10^{-7}$.

\textbf{Algorithm 1} Online estimator at a generic epoch

1: Input: $y$, $s$ and $\hat{\Lambda}$
2: Initialize $i = 0$ and $\hat{x}^i$
3: repeat
4: Compute $p_i$ via (25)
5: Set $\alpha_i$ using (26)
6: $\hat{x}^{i+1} = \hat{x}^i + \alpha_i p_i$
7: $i := i + 1$
8: until $\alpha_i < \epsilon$
9: Compute $\hat{\sigma}$ and $\hat{\sigma}$ via (21)
10: Compute $\hat{J}$ via (12)
11: $\hat{\Lambda} := \hat{\Lambda} + \hat{J}$
12: $s := s + \frac{3}{2} \hat{\theta}$
13: $\hat{\theta} = \frac{\hat{\Lambda}}{s}$
14: Output: $\hat{\theta}$, $s$ and $\hat{\Lambda}$

\section{V. NUMERICAL EXPERIMENTS}

We perform a numerical evaluation of SWINS, comparing the accuracy of the online estimator with the Cramér-Rao bounds. The root mean-square error (RMSE) of the parameter estimates was computed using $10^3$ Monte Carlo simulations.

In the following examples we set the unknown clock parameters to $T_m = 50 \times 10^{-9}$ and $T_u = 50 \times 10^{-9}$ [s]. The unknown $\phi_u$ contains the time of flight and the offset $\Delta_1$ that we set to $5 \times 10^{-9}$ [s]. Note however that the bounds are invariant to these parameter values. The numbers of clock cycles were set to $M = 100$ and to $N = 101$. In all examples the master is located at the following coordinates
\[ x_m = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \]

In the first scenario we consider a situation in which we have prior information about the position, modeled by the distribution $N(\bar{x}, \Lambda^{-1})$, and no additional transceivers are present. In the second scenario, we consider no prior information (i.e., $\Lambda_x = 0$) but add transceivers located at
\[ x_1 = \begin{bmatrix} 11 \\ 11 \end{bmatrix}, x_2 = \begin{bmatrix} 11 \\ 11 \end{bmatrix} \text{ and } x_3 = \begin{bmatrix} 11 \\ 11 \end{bmatrix}, \]
cf. the configuration in Fig. 7.

For the online estimator we set the nominal $\sigma_0$ to 10 [ns] and let the estimator adapt to noise outliers that exceed $\sigma_0^2$. The upper limit on the relative step size, $\eta$, is set to 1.2. We set the tolerance $\epsilon$ to $10^{-7}$.

\textbf{A. Master node and no transceivers}

In the first scenario, the prior information is given by
\[ \hat{x} = \begin{bmatrix} 9 \\ 8 \end{bmatrix} \text{ and } \Lambda_x = \sigma_x^{-2}I_2, \]
where $\sigma_x$ parameterizes the precision of $\hat{x}$ in meters. The unknown position of the node is randomized as $x \sim N(\bar{x}, \Lambda_x^{-1})$.

The resolution limits of SWINS, given by the HCRB (16), are shown in Fig. 9. When $\sigma_x$ is 20 [cm] and the measurement noise level $\sigma_k$ is 2 [ns], we note that the HCRB of $\phi_u$ reaches sub-nanosecond levels as the number of epochs $k$ increases. The bound of $T_m$ eventually collapses to that of $T_u$, whose accuracy is fundamentally limited by the errors of the timing device, cf. (7). In this scenario the online estimator achieves the HCRB for all parameters.

Fig. 9 illustrates also how the accuracy of the initial position estimate $\hat{x}$, namely $\sigma_x$, limits the accuracy of $\phi_u$. For 500 epochs, a position accuracy about $\pm 50$ cm ($\sigma_x = 0.25$) results in sub-nanosecond resolution limit for $\phi_u$. The bounds for $T_m$ and $T_u$ are left virtually unaffected by $\sigma_x$.

\textbf{B. Master node with three transceivers}

The unknown position of the node is fixed at $x = [9 8]^T$. The resolution limits of SWINS, given by the CRB in (14), are shown in Fig. 10. For a noise level of $\sigma_k = 2$ [ns], the CRB of $\phi_u$ reaches sub-nanosecond levels already at 10 epochs. Similar to the previous scenario the online estimator attains the bounds, which now decrease steadily with the number of epochs.

Fig. 10 illustrates also how the measurement noise level limits the accuracy of $\phi_u$. The estimation errors decrease as the unknown noise decreases $\sigma_k \to 0$. A small gap to the CRB for $\phi_u$ is visible when the noise level increases to 5 [ns].
We have designed a scalable system, denoted SWINS, in which an indefinite number of receiving wireless units can synchronize to a single master clock. The synchronization is performed at the level of discrete clock ticks and the mechanism can be implemented with passive receivers, thereby obviating the need for two-way communication and timestamp exchanges.

By deriving Cramer-Rao bounds for the data model we can conclude that SWINS advances the limits wireless synchronization towards sub-nanoseconds levels based on state-of-the-art hardware components. An online estimator based on the maximum likelihood approach was also developed that can operate with prior position information or, when such information is absent, with the proposed positioning infrastructure. The numerical experiments show that the estimator is statistically efficient.

In future work we will consider applications which can benefit from precise timing information and, furthermore, study the impact on performance of the geometric configuration of the transmitting nodes.

**Appendix A**

Derivation of Gradient

The gradient of $V_1$ is readily obtained as

$$\partial V_1 = \frac{2}{n} A_x (x - \bar{x}). \quad (27)$$

Due to the logarithm $\ln V_0$, we can equivalently redefine $V_0$ as $V_0 = n \sigma^2$. Then to obtain the gradient of $V_0(x)$ we first re-write the function as

$$V_0 = \rho^T W \rho - 2 \sigma^T \rho + (y - \mu)^T Q^{-1} \Pi_H^\perp (y - \mu). \quad (28)$$
where
\[ W = e^{-2G^T Q^{-1} \Pi_H G}, \]
\[ w = e^{-1G^T Q^{-1} \Pi_H G(y - \mu)}. \]

Because (28) equals
\[ V_0 = \sum_i \sum_j |W|_{ij} \rho_i \rho_j - 2 \sum_i w_i \rho_i + K, \]
where \( K \) is a constant, the gradient can be expressed as
\[ \partial V_0 = \sum_i \sum_j |W|_{ij} (\gamma_i \rho_j + \rho_i \gamma_j) - 2 \sum_i w_i \gamma_i, \quad (29) \]
where
\[ \gamma_i = \frac{\partial_x \rho_i}{\partial_x (||x - x_i||^2)^{1/2}} = \frac{x - x_i}{||x - x_i||}. \]

The gradients in (29) and (27) are used in (25).

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