MPM dynamic simulation of a seismically induced sliding mass

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Abstract. In some geotechnical applications, material can undergo large displacement combined with excessive deformation; e.g. the sliding mass problem. Owing to the limitations of classical Lagrangian and Eulerian finite element methods to model these problems, the Material Point Method (MPM) has been developed about two decades ago to cope with the large deformation. In MPM, the continuum field is represented by Lagrangian material points (particles), which can move through a fixed background of a computational mesh. Therefore, it can be seen as a mesh-based method formulated in arbitrary Lagrangian–Eulerian description. Although MPM represents the continuum by material points, solution is performed on the computational mesh. Thus, imposing boundary conditions is not aligned with the material representation. In this paper, a non–zero kinematic condition is introduced where an additional set of particles is incorporated, which tracks the moving boundary by carrying the time–dependent boundary evolution. Furthermore, the material point method has been adopted to simulate the progressive failure of a sliding granular slope triggered by a seismic excitation. In order to represent the topographical bottom of the sliding mass, on which the seismic motion is applied, a rigid boundary is implemented by introducing an additional set of particles. A frictional contact algorithm is defined between the boundary and the descending mass, which allows sliding and rolling with friction. The traction due to contact is incorporated into the discretised momentum equation as an external force where the solution of this equation is performed separately for each body in contact. Defining the local coordinate system accurately in this algorithm is essential to avoid interpenetration. Thus, a two-dimensional triangular discretisation is utilised within the three-dimensional tetrahedral elements to track the surface progression of each body in contact. Complying with other continuum models findings performed on granular materials, the present model overpredicts the lateral deformations. Therefore, a local damping proportional to the out-of-balance nodal forces is included. In spite of the simple Mohr- Coulomb failure criteria being used, the results of the present numerical model are comparative to another continuum based model.

1. Introduction
The stability of slopes is one of the prominent problems in practical engineering. The major causes responsible for triggering a landslide includes heavy rainfall, imposed loads, strength degradation due to weathering and seismic excitation. The earthquake–induced landslides are among the most destructive slope movements, where the excessive failure might take place [11]. In some of the cases, it becomes inevitable to study the behaviour of the slopes even after the failure as the large scale movement over broad areas can result in serious damages and casualties. In these cases, the failures

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cannot be prevented, but it becomes a primary interest of research to know the extent and probability of an eventual slide to reduce the damage. Numerical methods based on mesh deformation, e.g. Finite Element Method (FEM), have difficulties in modelling large deformations due to problems of mesh distortion and entanglement. In order to such type of large deformation problems, various methods have been proposed by coupling the two frame of references in a unified approach. Coupled Eulerian-Lagrangian (CEL) method is one of these methods, in which an updated Lagrangian finite element method (FEM) is coupled with the Eulerian description via interface models. The FE-software Abaqus utilised this feature and therefore it has been used in this paper. The Material Point Method (MPM), which is an innovative mesh-free particle method, performs the coupling procedure in an arbitrary form where the natural movement of the material is traced. MPM has been applied for many geotechnical applications involving large deformation of collapsing slopes and landslides (see for example [1,6]). In this paper, a strain-smoothening techniques based on nodal mixed discretisation has been implemented in MPM to relax the mesh locking problem, whereas the frictional contact algorithm is extended so that a prescribed velocity can be assigned directly to one of the two bodies in contact. Moreover in this paper, the two methods (MPM and CEL) have been applied to a landslide progression failure occurred during the 1999 Chi-Chi earthquake of Taiwan. To show the potential of applying the two continuum based models, some results are presented.

2. Coupled Eulerian-Lagrangian (CEL) method
Coupled Eulerian-Lagrangian (CEL) method is numerical framework to combine Lagrangian (has relatively small deformation) with Eulerian material (fluid behaviour) combined together via the interface of the Lagrangian object. By using this method, the Eulerian material is tracked as it flows through the mesh by computing its Eulerian Volume Fraction (EVF). Each Eulerian material is designated a percentage, which represents the portion of that element filled with a material. The coupling procedure between the two material descriptions inside CEL allows simulating the rock material as a flow running over the rigid base. The Eulerian mesh, where the sliding mass flows with large deformations, has no problems regarding mesh and element distortion as the Eulerian mesh remains fixed spatially.

2.1. Time integration
The CEL method is implemented in Abaqus/Explicit using explicit time integration [4]. The discretised equation of motion can be written in the form

\[ \ddot{a} = M^{-1} \cdot (F^{\text{ext}} - F^{\text{int}}) \]

where, \( M \) is the mass matrix, \( \ddot{a} \) is the nodal acceleration vector with the superposed dot is a material derivative of the nodal displacement \( a \) with respect to time, \( F^{\text{ext}} \) and \( F^{\text{int}} \) are the external and internal nodal force vectors, respectively. Equation (1) is integrated explicitly using central difference integration as follows

\[ \ddot{a}^{(i+\frac{1}{2})} = \ddot{a}^{(i-\frac{1}{2})} + \frac{\Delta t^{(i+1)} + \Delta t^{(i)}}{2} \cdot \dot{a}^{(i)} \]

\[ a^{(i+1)} = a^{(i)} + \Delta t^{(i+1)} \cdot \ddot{a}^{(i+\frac{1}{2})} \]

where \( \ddot{a} \) is velocity and \( \Delta t \) is the time increment. The superscript \( i \) refers to the increment number while \( (i-1/2) \) and \( (i+1/2) \) points to mid increment values. The central difference integration operator is explicit in that the kinematic state can be advanced using known values from previous increment. Explicit integration is conditionally stable, and is bounded with the following limits

\[ \Delta t \leq \frac{2}{\omega_{\text{max}}} \]

in which \( \omega_{\text{max}} \) is the maximum frequency of the system. Abaqus/Explicit uses an adaptive algorithm to determine conservative bounds for the highest element frequency. An estimate of the highest
eigenvalue in the system can be obtained by determining the maximum element dilatational mode of
the mesh. The stability limit based upon this highest element frequency is conservative in that it will
give a smaller stable time increment than the true stability limit that is based upon the maximum
frequency of the entire model. Abaqus/Explicit contains a global estimation algorithm, which
determines the maximum frequency of the entire model. At the beginning of the analysis, the program
evaluates the time step size based on element by element estimation. As the step proceeds, the stability
limit will be determined from the global estimator once the algorithm determines that the accuracy of
the global estimation is acceptable.

3. Material Point Method (MPM)
In MPM, the continuum body is discretised by Lagrangian material points. The momentum equation is
solved on the background Eulerian mesh which provides a convenient means of calculating discrete
derivatives.

3.1. Time integration
Euler-forward time integration scheme is adopted in MPM to solve Equation (1), which yields
\[
\dot{a}^{t+\Delta t} = \dot{a}^t + \Delta t \ddot{a}^t, \quad \ddot{a}^t = \left[ M_p^t \right]^{-1} F^t
\]  
(6)
where \( t \) is the current time increment, \( \dot{a}^t \) and \( \dot{a}^{t+\Delta t} \) are the nodal velocities at time \( t \) and \( t+\Delta t \),
respectively. The incremental nodal displacement is obtained by integrating the nodal velocity by the
Euler-backward rule (e.g. Sulsky et al. [12], Wieckowski [14]).
\[
\Delta a^{t+\Delta t} = \Delta t \ \dot{a}^{t+\Delta t}
\]  
(7)
The positions of the particles are subsequently updated from
\[
x_p^{t+\Delta t} = x_p^t + N_p \Delta a^{t+\Delta t}
\]  
(8)
in which \( x_p^t \) and \( x_p^{t+\Delta t} \) are the particle positions at time \( t \) and \( t+\Delta t \) respectively. After getting the grid
node velocities, the strain increment of a material point \( p \) is calculated and based on the calculated
strain increment, the stress increment at each material point is updated by applying the appropriate
constitutive model. The constitutive models are applied at material points and this allows easy
evaluation and tracking of history-dependent variables. At the end of time \( t+\Delta t \) all the material point
variables are updated and a new cycle is begun.

3.2. Contact algorithm
The contact defined between the soil and rigid walls is a frictional contact. The contact algorithm
proposed by Bardenhagen et al. [2] is used in this paper, which can be seen as a predictor-corrector
scheme formulated in explicit manner, in which the velocity is predicted from the solution of each
body separately and then corrected using the velocity of the coupled bodies following Coulomb
friction. This solution scheme uses the concept of comparing the single and combined body velocities
for a contact node and defining its behaviour accordingly. The algorithm is able to detect whether two
bodies are approaching or separating from one another. In the case of approaching, the contact
behaves according to the coulomb friction law. If the two bodies are separating, the contact algorithm
allows free separation where each one moves according to its own equation of motion. Following this
procedure, the bodies in contact would feel each other as soon as they contribute to the same
computational grid node.
For the considered problem of the sliding mass, the bottom is defined as a rigid object being in contact
with the sliding deformable mass. The calculation of the interface orientation is performed be tracking
the configuration of the rigid body, then the deformable body is updated consequently so that the
interpenetration of the two objects is prevented. As mentioned earlier, there is no need to specify the
contact surface a priori as the algorithm detect it automatically, which is valid for explicit integration
schemes where the time step size is rather small. For the bottom object, a prescribed velocity is
assigned to the material points; therefore, Equation (1) is carried out only for the deformable object.
3.3. Prescribed velocity
In traditional dynamic FEM, the prescribed velocity is defined over nodes. These nodes always define part of the Lagrangian body boundary. On the other hand, in MPM the continuum is defined by Lagrangian particles which might change position from one element to another. Hence, there is no interface surface where prescribed velocity can be applied. Within the framework of MPM, prescribed velocity can be applied directly on the material points of a rigid body representing the moving boundary for simple one-dimensional problem. The active boundary nodes must be tracked. For applications with axial movement, part of the mesh can be displaced as a moving mesh having a prescribed value while the rest is stretched uniformly [9]. This becomes very complicated and inconvenient when applied to applications like imposing seismic motion for a slope problem. As an alternative, an additional set of particles is introduced which tracks the moving boundary by carrying the time-dependent boundary evolution [5]. At the beginning of a time step, a prescribed velocity is assigned to the boundary particle. Next, the prescribed velocity is mapped via the shape functions from the prescribed particles to the computational nodes, where the discrete equations are solved. Following the same methodology in this paper, prescribed velocities are applied as a boundary condition for the rigid walls and a contact is defined between the rigid walls and soil.

4. Earthquake induced landslide debris flow
The dynamic process of landslides induced by earthquakes is very complex in its nature. Various numerical methods are used to simulate different parts of the whole process of evolution of a landslide starting from the development of a critical sliding surface to initiation and triggering of failure to disintegration of the sliding mass to debris flow and finally deposition. Here, CEL and MPM are used to simulate the last part of the process i.e. progression of the sliding mass and deposition. For this, the Chiu-fen-erh-shan landslide of 1999 is used as a case study.

4.1. The Chiu-fen-erh-shan landslide
The Chiu-fen-erh-shan landslide is one of the major landslides caused by the disastrous Mw 7.6 Chi-Chi earthquake of 1999 in Taiwan. The landslide has been characterised as a translational rock-block slide on dip slope and known to be disintegrated into fragmented rock avalanche travelling long distance at high velocity [7]. The landslide debris travelled a distance of more than 1 km and covered an area of 1.95 km$^2$. Many studies have been conducted since then to understand the mechanism and movement of the landslide [8, 7, 15]. These studies concluded that the slope had undergone gravitational creep and was unstable even prior to the earthquake. A slip surface had been developed as a result of flexural folding. Observations by Wang et al [13] suggested the presence of clay seams between the alternating beds of shale and sandstone that composed the slope. The existence of the clay seam provides a very smooth surface for the slide to occur. The slope was retained in its position because of a sandstone bed that formed resistant ridges at the foot of the slope. The sandstone bed was probably damaged seriously during the earthquake giving way to the massive landslide. The topography and horizontal component of the velocity are depicted in Figure 1 [3].

![Figure 1](chi-chi-landslide-topography-and-velocity.png)

**Figure 1.** Chi-Chi landslide: (left) topography and (right) time history of the horizontal velocity
4.2. Numerical analyses

In this section, both CEL and MPM analyses are presented considering restrictions in each model. In CEL, the base block is assumed as a Lagrangian rigid body while the sliding mass is considered as an Eulerian material. The sliding mass, which is comprised of a highly jointed rock mass is modelled as a continuum with a density of 2550 kg/m³, Young’s Modulus of 500 MPa and a Poisson’s ratio of 0.19. The Mohr-Coulomb failure criterion is used in the analysis. The strength behavior of the sliding mass is mainly ruled by the strength parameters of the rock joints, hence, the strength parameters of joints are directly used to define the shear strength of the continuum. The values adopted for cohesion and friction are 20 kPa and 24° respectively [3]. The simulation is performed in two steps. In the first step, the quasi-static solution for the gravity stresses is obtained, while the seismic excitation is given in the second step to the base block. The numerical model of the slope is meshed with 3-dimensional, linear eight noded elements with reduced integration. The contact between the Lagrangian and Eulerian domain of the model is implemented by the general contact algorithm, which enforces the use of penalty contact method.

The same problem above has been repeated in MPM, where a three-dimensional program using tetrahedral elements has been used. Therefore, the plane-strain condition of the present two-dimensional problem is imposed using one element in depth while the out-of-plane boundary condition is restricted. To relax the numerical problem of the mesh locking associated with the low-order element, strain-smoothening technique based on nodal mixed discretisation has been integrated. Prescribed velocity is imposed to the particles of the sliding surface as a rigid block. Numerical damping has been added to the sliding material to imitate the material damping. The progression of the flow resulting from the landslide simulated by the two methods is depicted in Figure 2.

For the MPM model, an artificial damping has been added to approach the measurements (Figure 1). Therefore, the MPM deformation in Figure (2) shows the final steady-state whereas the sliding object in the Abaqus model flows further beyond the edge of the slope.

![Figure 2. Progression of landslide: (left) CEL (right) MPM](image)

5. Conclusion

The potential of simulating an important geotechnical application has been presented in this paper using CEL and MPM. Owing to the fact that both methods are based on continuum modelling, they overestimate the lateral movement of the sliding material when a simple constitutive model is used, (as the Mohr-Coulomb one implemented in this paper). Therefore, implementing more advanced
constitutive model is essential to get better prediction or to add a numerical damping as an alternative. However, the amount of the required damping is difficult to evaluate. For the MPM model, the artificial damping has been shown to be efficient to predict the final configuration beyond failure, whereas the Rayleigh damping integrated in Abaqus was not sufficient to hold the sliding mass (see Figure (2)). Remembering that CEL treats the sliding material in Eulerian manner, therefore, tracking the local displacement is not possible. On the other hand, the Lagrangian nature of MPM allows obtaining the incremental displacement in more natural fashion (see the layering representation in Figure (2)). In order to proceed further with the continuum modelling of rock slopes, more advanced constitutive models are required, as well as updating the contact evolution basing on the mobilised resistance rather than the constant assumption in the current models.

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