Color-flavor locked strange quark matter in a strong magnetic field

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The quark quasiparticle model is extended to study the properties of color-flavor locked strange quark matter at finite chemical potential and in a strong magnetic field. We present a self-consistent thermodynamic treatment by employing a chemical potential dependent bag function. It is found that the magnetized color-flavor-locked (MCFL) matter is more stable than other phases within a proper magnitude of magnetic field. The stability window is graphically shown for the MCFL matter compared with ordinate magnetized matter. The anisotropic structure of MCFL matter is dominated by the magnetic field and almost independent of the energy gaps. A critical maximum magnetic field of about $1.56 \times 10^{18}$ G is found, under which MCFL matter is absolutely stable with respect to nuclear matter.

I. INTRODUCTION

Properties of strange quark matter (SQM) at low temperature and high density regime have received a lot of theoretical attentions. The stability of SQM in the interior of a compact star has been investigated by many works [1–3]. The most symmetric pairing state, color flavor locked (CFL) state, can only be realized if the mass split between light quarks and strange quarks is small and/or the chemical potential $\mu$ satisfies $\mu > m_s^2/2\Delta$ [4] with $m_s$ being the strange quark mass and $\Delta$ the pairing gap. CFL state is predicted to be more stable than ordinary strange quark matter. So a hybrid star is suggested to be constructed with color superconducting quark matter in the interior and nuclear matter in the surface [5]. Recently, the special properties of neutron star matter [6] and dense quark matter in the presence of a strong magnetic field have aroused a lot of interest [7, 8]. One may naturally wonder how the external magnetic field affects the stability condition and the equations of state of CFL matter.

In the past years, the magnetized strange quark matter (MSQM) and CFL matter have been studied in the bag model [9] and improved in the Nambu-Jona Lasinio (NJL) model [10–12]. The CFL state has a wide range of model parameters characterized by the so-called stability window [13]. In an external strong magnetic field, the magnetized color flavor locked matter (MCFL) is expected to have interesting properties [14–16], where the magnetic field contribution is considered as the vacuum term in the thermodynamical potential [17]. Due to the effect of the strong magnetic field, the anisotropic pressure changes the mechanical stability condition of strange quark matter, which is especially important for the finite size spherical drop of SQM [2, 18]. In literature, there are two kinds of treatments with the chemical potential relation between various components of the quark matter. One is the weak equilibrium for bulk matter, such as the general treatment of u, d, and s quarks in quark stars. The other is non-$\beta$ equilibrium proposed in a first order deconfinement transition of hadron matter to color superconductivity. When the phase transition is unavoidable, the non-$\beta$-stable intermediate state is assumed to have a short life time [19].

In literature, the quark quasiparticle model has been developed by many authors in studying the dense quark matter at finite temperature and density [20, 22]. The importance of the quasiparticle model is the introduction of the medium-dependent quark mass scale in describing QCD nonperturbative properties. A widely accepted quark mass scale is derived at the zero-momentum limit of the dispersion relations from an effective quark propagator by resuming one-loop self-energy diagrams in the hard dense loop approximation [2]. In the scale, the quark mass is a function of its chemical potential and the temperature. So the pressure and energy functions of SQM have corrections by an additional bag function. As is known, the system pressure and various component chemical potentials are very important quantities for the equations of state of SQM. To balance the electrostatic repulsion of electrically charged particles, Poincare assumed a pressure to exist inside the particles [23, 24]. The pressure is exerted by the physical space outside the bag in the bag model. It has also been shown through the dynamical quark mass in the NJL model [25]. In the quark quasiparticle model, it is presented by a chemical-potential dependent bag function, which is derived by the self-consistency condition of thermodynamics. Recently, we have applied this model to study the strangelets [26] and magnetized SQM with interesting results [27]. The aim of this paper is to investigate the special properties of the CFL state in an external strong magnetic field. We derive a chemical potential and field-strength dependent bag function. Then we show the self consistency of the thermodynamic treatment. Our results show that the pairing effect together with an external magnetic field in a proper magnitude can generally enlarge the stability window of MCFL matter.

This paper is organized as follow. In Sec. III we derive the self-consistent thermodynamic formulas of CFL and MCFL matter in the quasiparticle model with and without the magnetic field. Based on the running coupling constant between quarks, we...
investigate the stability of different phases of SQM. Specially, the stability windows of MSQM and MCFL matter are shown. In Sec. III, we show the numerical results and have a discussion on the stability and the anisotropic structure of SQM in strong magnetic field. The last section is a short summary.

II. CFL MATTER AND MCFL PHASE IN THE QUASIPARTICLE WITH A NEW EFFECTIVE BAG FUNCTION

A. Thermodynamic treatment of CFL matter in the quasiparticle model

We begin with the thermodynamical potential density of unpaired quarks. At zero temperature, the thermodynamic potential density $\Omega_{\text{free}}$ is $\Omega_{\text{free}} = \sum_{i=u,d,s} \frac{d_i}{48\pi^2} \left\{ p_f \left[ 3(2p_f^2 + m_i^2) \sqrt{p_f^2 + m_i^2} - 8\mu_i p_f \right] - 3m_i^4 \ln \frac{p_f^2 + \sqrt{p_f^2 + m_i^2}}{m_i} \right\}$, (1)

where $d_i$ is 6 for quarks and $d_i = 2$ for electrons. All the thermodynamic quantities can be derived from the characteristic function by obeying the self-consistent thermodynamic relation [26].

In order to write down the expression for the thermodynamic potential density of three-flavor quark matter in the color flavor locked phase, as usually done, we assume that all nine (three flavors times three colors) quarks have a common gap $\Delta$. To order $\Delta^2$, the pairing contribution to the thermodynamic potential density of color-flavor locked SQM can be written as $\Omega_{\text{CFL}} = \Omega_{\text{free}} + \Omega_{\text{pair}}$, (2)

where the pairing term from one quasiparticle with gap $\Delta_1$ and eight quasiparticles with gap $\Delta_2$ is $\Omega_{\text{pair}} \simeq -\left( \frac{\mu_i}{2\pi} \right)^2 (\Delta_1^2 + 8\Delta_2^2)$

$\simeq -3\Delta^2 \bar{\mu}^2 / \pi^2$. (3)

where the approximate relation $\Delta_1 = 2\Delta_2 = 2\Delta$ is used between the singlet and octet gaps. The mean chemical potential is $\bar{\mu} = (\mu_u + \mu_d + \mu_s) / 3$. The common Fermi momentum $p_f$ is obtained by solving

$\sum_i \sqrt{p_f^2 + m_i^2} = 3\bar{\mu}$. (5)

In this paper, the effective quark mass in the framework of the quasiparticle model is taken to be $m_i(\mu_i) = m_{i0} \sqrt{\frac{2}{4 + \frac{g^2\mu_i^2}{6\pi^2}}}$, (6)

where $m_{i0}$, $\mu_i$, and $g$ are, respectively, the quark current mass, quark chemical potential, and the strong interaction coupling constant. In principle, the strong coupling constant is running [33, 34], and one can use a phenomenological expression such as [35],

$g^2(T = 0, \mu_i) = \frac{48\pi^2}{29} \left[ \ln \left( \frac{0.8\mu_i^2}{\Lambda^2} \right) \right]^{-1}$. (7)

However, to make the expressions simple and understandable, one can adopt a constant $g$ with value in the range of (0, 5), as was done in the previous literature [2]. Through the following sections and figures, we choose the running coupling value in the calculations. The current mass can be neglected for up and down quarks. The strange quark current mass can be adopted as 120 MeV. If the vanishing current mass is assumed for up and down quarks, Eq. (6) can be reduced to the simple form

$m_i = \frac{g\mu_i}{\sqrt{6\pi}}$. (8)

The pressure and the energy density for the CFL matter are, respectively, given by

$P = -\Omega_{\text{CFL}} - B^*$, (9)

$E = \Omega_{\text{CFL}} + \sum_i \mu_i n_i + B^*$. (10)
where the effective bag function $B'$ is introduced as a residual interaction energy density. To meet the thermodynamic consistency condition, the $B'$ function can be divided into two parts: $\mu_i$-dependent part and the definite integral constant, i.e., $B' = \sum B_i(\mu_i) + B_0$ ($i = u, d, s$) where $B_0$ is similar to the conventional bag constant. The concrete expression of the chemical potential dependence of the effective bag function $B'$ is to be self-consistently derived in the next paragraph. The advantage of this particular implementation is to show the automatic confinement characteristic in the model.

The quark number density of the component $i$ is given as

$$n_i = \frac{p_i^3}{\pi^2} + \frac{2\Delta^2 \bar{\mu}}{\pi^2}. \quad (11)$$

The total baryon number density $n_b$ is the sum of $n_i$ divided by 3, i.e., $n_b = (n_u + n_d + n_s)/3$. Eq. (5) explicitly means that all the three flavors of quarks have the same number density, i.e. $n_u = n_d = n_s$. Therefore, the CFL phase is naturally neutralized, and the number density of electrons is thus zero. Accordingly, the weak equilibrium condition $\mu_u + \mu_d = \mu_s$ becomes $\mu_u = \mu_d = \mu_s$ due to the zero chemical potential of electrons.

In order to satisfy the fundamental thermodynamic equation

$$E = -P + \sum_i \mu_i \dfrac{\partial P}{\partial \mu_i}, \quad (12)$$

the following requirement must be satisfied [2, 36],

$$\left(\dfrac{\partial P}{\partial m_i}\right)_{\mu_i} = 0. \quad (13)$$

Considering Eq. (13), we have the vacuum energy density function $B_i(\mu_i)$ through the following differential equation,

$$\dfrac{dB_i(\mu_i)}{d\mu_i} \dfrac{d\mu_i}{dm_i} = -\dfrac{d\Omega_{CFL}}{dm_i}. \quad (14)$$

If the current mass of light quarks is neglected, one can integrate Eq. (31) under the condition $B_i(\mu_i = 0) = 0$ and have

$$\dfrac{dB_{u,d}^\ast}{dm_i} = -\dfrac{d_i}{4\pi^2} \left[ m_i p_f \sqrt{m_i^2 + p_f^2} - m_i^3 \ln \left( \dfrac{p_f + \sqrt{p_f^2 + m_i^2}}{m_i} \right) \right]. \quad (15)$$

The effective bag constant including the effect of pairing was investigated in the CFL matter by Ref. [37]. Here we derive the effective bag function by self-consistent requirement from Eq. (31). The pairing contributions is included in the thermodynamic potential density. If we apply the running coupling $g(\mu)$, the expression of the effective bag function is changed into

$$B_i(\mu_i) = -\int_{\mu_i}^{\mu_i^*} \dfrac{d\Omega}{dm_i} \bigg|_{T = \mu_i} \dfrac{dm_i}{d\mu_i} d\mu_i$$

$$= -\dfrac{d_i}{4\pi^2} \int_{\mu_i}^{\mu_i^*} \left[ m_i p_f \sqrt{m_i^2 + p_f^2} + m_i^3 \ln \left( \dfrac{p_f + \sqrt{p_f^2 + m_i^2}}{m_i} \right) \right] \dfrac{dm_i}{d\mu_i} d\mu_i, \quad (16)$$

where the lower limit $\mu_i^*$ of the integration over $\mu_i$ should satisfy $B_i(\mu_i^*) = 0$. This requirement is equivalent to including the integration constant into the $B_0$.

For the pairing energy gap, $\Delta = 80$ MeV (dashed line) and $\Delta = 100$ MeV (solid line), the energy per baryon of CFL matter versus the baryon number density is plotted in Fig. 1. On the top of the panel, the $\Delta = 0$ curve marked by a dotted line represents the unpaired SQM. For larger pairing gap, the CFL matter would have a smaller free energy per baryon. The minimum values of the curves are located at the stable zero-pressure points marked by open circles. This is the result of the thermodynamic self-consistency requirement.

### B. MCFL phase in the present of the strong magnetic field

To definitely describe the magnetic field of a compact star, we assume a constant magnetic field ($H_{mz} = H$) along the $z$ axis. Due to the quantization of orbital motion of charged particles in the presence of a strong magnetic field, known as Landau diamagnetism, the single particle energy spectrum is [38]

$$\epsilon_i = \sqrt{p_i^2 + m_i^2} + e_i H(2n - \eta + 1), \quad (17)$$
where \( p_e \) is the component of the particle momentum along the direction of the magnetic field \( H \), \( e_i \) is the absolute value of the electronic charge (e.g., \( e_i = 2/3 \) for the up quark and \( 1/3 \) for the down and strange quarks), \( n = 0, 1, 2, ... \), is the principal quantum number for the allowed Landau levels, and \( \eta = \pm 1 \) refers to quark spin-up and -down state, respectively. For the sake of convenience, one usually sets \( 2\nu = 2n - \eta + 1 \), where \( \nu = 0, 1, 2, ... \). The single particle energy then becomes

\[
\epsilon_i = \sqrt{p_e^2 + \tilde{m}_{i,\nu}^2},
\]

where \( \tilde{m}_{i,\nu} = m_i^2 + 2\nu e_i B \) denotes the square of quark effective mass in the presence of a magnetic field.

In literature, the MCFL matter was successfully investigated in the NJL framework [39–43]. Due to the mixture of the photon field \( A_\mu \) and the eighth component \( G_\mu^8 \) of the gluon field, the original \((u, d, s)\) quark representation is divided into neutral, positively, and negatively charged spinors with the quark rotated charges \( \hat{Q} \) [44]. The \( \hat{Q} \) of different quarks are in units of \( \hat{e} = e \cos \theta \) [43], where \( \cos \theta = g/\sqrt{\nu^2/3 + g^2} \) [45, 46]. By considering the relation between the magnitude of the applied external magnetic field \( H_{\text{ext}} \) outside and the rotated field \( \hat{H} \) inside the color superconducting matter, one has \( \hat{H} = H_{\text{ext}} \cos \theta \). This can be simplified as \( \hat{H} = H_{\text{ext}} \) with \( g \gg e \). In the following section, we will use "magnetic field" in short when referring to the rotated magnetic field \( \hat{H} \). The original symmetry pattern is \( \text{SU}(3)_{C-L+R} \) has been broken to the residual subgroup \( \text{SU}(2)_{C-L+R} \) [40, 42]. The gap structure is indirectly affected by the external field through the coupled gap equations [40]. The original gap of CFL matter is split into two different gaps, \( \phi \) and \( \Delta_H \), in the implementation of the MCFL matter. In principle, the gaps are determined by the solution of the gap equation [39, 40]. The gap \( \phi \) is formed only by the pairs of neutral quarks, while \( \Delta_H \) function has contributions from quarks with opposite charges and neutral quarks [42]. By an analogy with the pairing contribution in Eq. (3) for CFL matter, the total thermodynamic potential density \( \Omega_{\text{MCFL}} \) is the sum of pairing term and the thermodynamic potential \( \Omega_{\text{MSQM}} \) to lowest nonzero order in \( \Delta_H/\mu_q \) at the quark chemical potential \( \mu_q = 400 \sim 500 \text{ MeV} \),

\[
\Omega_{\text{MCFL}} = \Omega_{\text{MSQM}} - \frac{\mu^2}{\pi^2} (\phi^2 + 2\Delta_H^2). \tag{19}
\]

where \( \Omega_{\text{MSQM}} = \sum \Omega_\nu(m_i, \mu_i) \). This approximation is obviously available to lowest nonzero order in \( \phi/\mu \) because \( \phi \) could be smaller than \( \Delta_H \) [39, 42]. The dependence of the gap \( \Delta_H \) on the magnetic field can be understood through the gap equation in NJL model [39, 40]. As is known to us in the bag model, the quark interaction is simply included into a single bag constant, which represents the energy density difference. Therefore in order to obtain the gap value as the function of the magnetic field, we can only resort to the approximation solution from the gap equation in NJL model. For MCFL matter, the gap parameters can be solved numerically through the gap equation. In the strong-field limit, all the Landau levels have been drop down in addition to the lowest Landau level and the analytical solution can be found in the equation,

\[
1 \approx \frac{g^2}{3\Lambda^2} \int_\Lambda \frac{1}{\sqrt{(q - \mu^2 + 2(\Delta_H)^2)}} + \frac{g^2\hat{e}H}{6\Lambda^2} \int_\Lambda \frac{dq}{(2\pi)^2} \frac{1}{\sqrt{(q - \mu)^2 + (\Delta_H)^2}}. \tag{20}
\]

For more detailed introduction, one can see Ref. [39, 40]. Because the solution of Eq. (20) is associated with Landau energy level, the gap parameters show oscillation behavior, known as the de Haas-van Alphen effect, as long as \( 0.2 < \hat{e}H/\mu_q^2 < 0.5 \). The
oscillation ceases when only the lowest Landau level contributes to the gap equations, namely, \( \tilde{e}H / \mu_i^2 > 1 \) [43]. In this case, the analytical function was obtained for the dependence of the gap on the magnetic field in Refs. [39]. In the region \( \tilde{e}H / \mu_i^2 \approx 0.1 \) we are interested in, the parameters \( \phi \) and \( \Delta H \) show a narrow range of the oscillation. The gap parameters with very weakly oscillation behavior can be briefly supposed as constants in our investigation. The thermodynamically self-consistence with this approach will be shown in following section that zero pressure is obtained simultaneously with the minimum of free energy. In the special case of \( \tilde{H} \approx 5 \times 10^{17} \text{G} \), the gap parameters can be approximately invariant.

Because the ground state \( (\nu = 0) \) is single degenerate while all \( \nu \neq 0 \) states are doubly degenerate, we assign the spin degeneracy factor \( (2 - \delta_{00}) \) to the index \( \nu \) Landau level. We can do the calculations with the redefined charges of the quarks in the 9-dimensional flavor-color representation [39],

\[
\begin{align*}
s_b & \quad s_g & \quad s_r & \quad d_b & \quad d_g & \quad d_r & \quad u_b & \quad u_g & \quad u_r \\
0 & \quad 0 & \quad -1 & \quad 0 & \quad -1 & \quad +1 & \quad +1 & \quad 0
\end{align*}
\]  

(21)

The thermodynamic potential density for charged quarks at zero temperature is simplified to give

\[
\Omega_i(m_i, \mu_i) = -\frac{d_i \tilde{e}_i H}{2 \pi^2} \sum_{\nu=0}^{\nu_{\text{max}}} (2 - \delta_{00}) \int_0^{p_z} (\mu_i - \epsilon_i) dp_z'
\]

\[
= -\frac{d_i \tilde{e}_i H}{4 \pi^2} \sum_{\nu=0}^{\nu_{\text{max}}} (2 - \delta_{00}) \left\{ \mu_i \sqrt{\mu_i^2 - m_{i,v}^2} - m_{i,v} \ln \left( \frac{\mu_i + \sqrt{\mu_i^2 - m_{i,v}^2}}{m_{i,v}} \right) \right\}.
\]

(22)

where the \( d_i = 2 \) only refers the spin degeneracy factor. The upper limit \( \nu_{\text{max}} \) of the summation index \( \nu \) can be understood from the positive value required by the logarithm and square-root function in Eq. (22). So we have

\[
\nu \leq \nu_{\text{max}} \equiv \text{int} \left[ \frac{\mu_i^2 - m_{i,v}^2}{2 \tilde{e}_i H} \right],
\]

(23)

where “int” means the number before the decimal point.

The quark number density is

\[
n_i = \frac{d_i \tilde{e}_i H}{2 \pi^2} \sum_{\nu=0}^{\nu_{\text{max}}} (2 - \delta_{00}) \sqrt{\mu_i^2 - m_{i,v}^2} + \frac{2 \mu_i}{3 \pi^2} \left( \phi^2 + 2 \Delta_{H}^2 \right).
\]

(24)

In the preceding paragraphs, we only take into account the influence of the magnetic field on the Landau energy levels of the particles. In fact, the magnetic field pressure should be considered in phenomenological presentations as in the previous work [47], which should be distinguished with a hydrodynamical pressure [24]. Due to the anisotropic structure, the pressure is different in the directions parallel and perpendicular to the field [48]. By including the matter contribution and the field contribution, the total parallel pressure is given by

\[
P_{\parallel} = -\Omega_{\text{MCFL}} - \frac{\tilde{H}^2}{2} - B^*.
\]

(25)

and the transverse pressure is

\[
P_{\perp} = -\Omega_{\text{MCFL}} - M_f \tilde{H} + \frac{\tilde{H}^2}{2} - B^*.
\]

(26)

where the system magnetization is \( M_f = -\left( \partial \Omega_{\text{MCFL}} / \partial H \right) = \sum M_i \). The contribution from the \( i \)th quark species reads

\[
M_i = \frac{\partial \Omega}{\partial H} = \frac{\tilde{e}_i d_i}{2 \pi^2} \sum_{\nu=0}^{\nu_{\text{max}}} (2 - \delta_{00}) \int_0^{p_z} \left( \mu_i \sqrt{\mu_i^2 - m_{i,v}^2} - m_{i,v} \ln \left( \frac{\mu_i + \sqrt{\mu_i^2 - m_{i,v}^2}}{m_{i,v}} \right) \right) dp_z.
\]

(27)

Substituting Eq. (27) into Eq. (26), one has the analytical expression,

\[
P_{\perp} = \frac{d_i |\tilde{e}_i|^2 \tilde{H}^2}{2 \pi^2} \sum_{i=1}^{\nu_{\text{max}}} \nu \left[ \mu_i + \sqrt{\mu_i^2 - m_{i,v}^2} \right] \ln \left( \frac{\mu_i + \sqrt{\mu_i^2 - m_{i,v}^2}}{m_{i,v}} \right) + \frac{\tilde{H}^2}{2} - B^*.
\]

(28)

The anisotropy of pressures in a strong magnetic field was recently investigated by the bag model [49] and the NJL model [50]. In our quasiparticle model, the quark effective mass takes the place of the current mass in bag model to reflect the medium effect in the high density environment. The pressure splitting is

\[
P_{\perp} - P_{\parallel} = -M_f \tilde{H} + \tilde{H}^2.
\]

(29)
To guarantee the vanishing of both the parallel and transverse pressure, one should require that $P_\parallel = 0$ and $\partial P_\parallel / \partial \tilde{H} = 0$. It is obvious that the bag constant should depend on the magnetic field [18, 47].

Similar to the approach ruled by the fundamental thermodynamic relation in CFL matter, we have the following requirement,

$$
\left( \frac{\partial P_\parallel}{\partial m_i} \right)_{\mu_i, \tilde{H}} = 0.
$$

(30)

Consequently, we have the following differential equation,

$$
\frac{dB_i(\mu_i)}{d\mu_i} \frac{d\mu_i}{dm_i} = - \frac{\partial \Omega_{\text{MCFL}}}{\partial m_i}.
$$

(31)

We can express the effective bag function of MCFL matter as

$$
B^* = \sum_i \sum_{\nu} B_{i,\nu}(\mu_i) + B_0,
$$

(32)

where the contribution of the component $i$ at the $\nu$ level is given as

$$
B_{i,\nu}(\mu_i) = - \int_{\mu_i}^{\mu_i'} \frac{\partial \Omega_{\text{MCFL}}}{\partial m_i} \bigg|_{T=0, \mu_i} \frac{dm_i}{d\mu_i} d\mu_i
$$

$$
= - \frac{d_\nu \tilde{\epsilon}}{2\pi^2} \int_{\mu_i}^{\mu_i'} m_i \ln \left( \frac{\mu_i + \sqrt{\mu_i^2 - m_i^2}}{m_i, \nu} \right) \frac{dm_i}{d\mu_i} d\mu_i.
$$

(33)

For uncharged particles in the rotated representation, we can not obtain discrete Landau levels and the integration of $\Omega_i$ can be carried out to give [26]

$$
\Omega_i = - \frac{d_i}{48\pi^2} \left[ |\mu_i| \sqrt{\mu_i^2 - m_i^2} \left( 2\mu_i^2 - 5m_i^2 \right) + 3m_i^4 \ln \left( \frac{|\mu_i| + \sqrt{\mu_i^2 - m_i^2}}{m_i} \right) \right].
$$

(34)

Correspondingly, the number density [24] and bag function [33] for uncharged quarks should be changed [26].

III. NUMERICAL RESULTS: THE STABILITY AND ANISOTROPIC STRUCTURE OF MCFL

FIG. 2: The comparison of free energy per baryon versus the baryon number density for MCFL, MSOM, CFL, and SQM respectively. The magnetic field strength and energy gaps are adopted as $\tilde{H} = 10^{17}$ G, $\phi = 15$ MeV, and $\Delta_H = 20$ MeV.
It is known that different constitutions of three flavor quarks and external environment will have different effects on the stability of SQM. The stability for different phases was recently investigated by an inequalities expression [13]. In the framework of the preceding quasiparticle model, we have done the numerical calculations with the quark current mass values \( m_q = 5 \text{ MeV}, m_d = 10 \text{ MeV}, \) and \( m_s = 120 \text{ MeV}. \) The constant term \( B_0 \) is \((145 \text{ MeV})^4. \) In Fig. 2, we give the energy per baryon for the SQM, MSQM, CFL, and MCFL phases respectively. From the top to down in the panel, there are obviously two features. Firstly, the quark pairing effect greatly improves the stability of SQM. Secondly, the external magnetic field in a proper magnitude lowers the free energy per baryon through the rearrangement of Landau energy level of magnetized quark matter. In a proper range of magnetic field strength, we can get the general inequality relation of the energy per baryon in the four kinds of quark matter phases as

\[
\frac{E}{A}_{\text{MCFL}} < \frac{E}{A}_{\text{MSQM}} < \frac{E}{A}_{\text{CFL}} < \frac{E}{A}_{\text{SQM}}.
\]  

(35)

Here we should emphasize that the validity of the comparison result in Eq. (35) will depend on the gap value of CFL matter. If given a larger gap value \( \sim 3 \Delta_H, \) the CFL phase will be more stable than MSQM phase.

Now we investigate the range of the magnetic field strength required by the stability condition of the MSQM and MCFL matter. As is known that the effect of the magnetic field on the equation of state of the magnetized SQM is remarkable [27]. Undoubtedly, the influence of the magnetic field on the stabilities will depend on its magnitude \( \mathcal{H}. \) One may wonder how high is the field magnitude is allowed in SQM according the stability hypothesis. The possible strongest magnetic field in nature is generated either in the interior of a rotating protoneutron star, or in the in ultrarelativistic heavy ion collisions for a brief timescales of order fm/c. The physical upper limit can be estimated as \( \mathcal{H}_{\text{max}} \sim 10^{20} \text{ G} \) by assuming equivalent uniform field and mass density under energy-conservation arguments by applying the equipartition theorem on the magnetic energy and the energy from quark matter [51]. In this paragraph, we will investigate the allowed field range by considering the stability condition criterion. By comparing the energy per baryon with that of the \(^{56}\text{Fe}\) nucleus (roughly 930 MeV), the stability window \((E/n_b < 930 \text{ MeV})\) is obtained at zero pressure condition for MSQM (dashed line) and MCFL (solid line) in Fig. 3. For MCFL matter, the two energy gaps are chosen as \( \phi = 15 \) and \( \Delta_H = 20 \text{ MeV}. \) It is clearly seen that the pairing energy can enlarge the stability window in the parameter space. The maximum value of the magnetic field is allowed up to \( \mathcal{H}_m = 1.56 \times 10^{18} \text{ G} \) in the absolute stability region, which is smaller than that showed in Fig.5 of Ref. [47]. However, the special values of the allowed maximum field are associated with the energy gaps and the QCD scale parameter \( \Lambda. \)

![Fig. 3: Stability windows for MCFL matter and MSQM are denoted by solid curve and dashed curve respectively. The parameters are the same as Fig.2. The two curves corresponds to the borderline values 930 MeV of the energy per baryon.](image)

The anisotropic structure was investigated recently in dense neutron matter under the presence of a strong magnetic field [51]. The difference of \( P_L \) and \( P_T \) reflects the breaking of the rotational symmetry by the magnetic field. The effect of anisotropic pressure will become very important especially in calculating the properties of strangelets, the spherical drop of SQM. In contrast to the previous work about MCFL matter [13], our equation of state in the quasiparticle model implies \( P_L < P_T \) by including the field Maxwell contribution. To satisfy the equilibrium condition of the self bound matter, it is required that the parallel pressure and the transverse pressure should vanish simultaneously \( P_L = P_T = 0 [47]. \) A natural and realizable method is that the bag parameter is field-dependent [16]. Consequently, the residual part of the bag constant in the present version of the quasiparticle model should reflect its dependence on \( \mathcal{H} \) under the equilibrium condition. Just as pointed out in Ref. [47], one can consider a
FIG. 4: The transverse pressure and parallel pressure of magnetized CFL matter and unpaired matter as functions of the magnetic field strength. The case-1, case-2, and case-3 of energy gaps ($\phi$, $\Delta H$) are chosen as (30, 40), (15, 20), and (0, 0) in unit of MeV, which are marked by solid, dashed, and dotted lines respectively. The case-3 is the unpaired magnetized strange quark matter.

FIG. 5: The effective bag function $B^*$ versus the baryon number density for different parameter sets of energy gaps. The field strength is $\tilde{H} = 10^{17}$ G. The other parameters are the same as Fig. 4.

field-dependent bag constant for gravitationally bound stars, because the internal pressure produced by the magnetic field can be compensated by the gravitational pressure. In Fig. 4 we show the parallel $P_\parallel$ and the transverse $P_\perp$ pressures as functions of the magnetic field strength. We call to the reader’s attention that our numerical result of Fig. 4 is automatically obtained by zero pressure condition instead of fixing a baryon number density. The solid and dashed lines represent the parameter set ($\phi$, $\Delta H$) of the MCFL matter with (30, 40) and (15, 20) in unit of MeV, respectively. The dotted line with $\phi = \Delta H = 0$ denotes the recovery of the unpaired MSQM phase. The pressure lines feel a weak effect of the pairing gaps. It is found that the pressure splitting is dominated by the magnetic field and independent of the pairing energy gap. The magnetic field strength has a common threshold value $\tilde{H}_{th} \sim 3 \times 10^{17}$ G for different gap values. When the magnetic field strength is larger than the threshold value, the pressure anisotropy starts to become noticeable: the transverse $P_\perp$ (or the parallel $P_\parallel$) will increase (or decrease) rapidly far beyond the constant values. Because the contribution of Maxwell term to the pressure becomes dominant with the high value of the field strength, the ascending branches as well as the descending branches will become to overlap at high magnitude of the magnetic field. In Fig. 5 we compare the effective bag function at the fixed field strength $\tilde{H} = 10^{17}$ G. It is obvious that the residual interaction energy density $B^*$ will be decreasing with the decreased distance between quarks, which is required by the behavior of confinement potential energy. However at the same baryon number density, the $B^*$ increases with the gap value of MCFL.
matter. For ultrastrong field, i.e., $\tilde{e}H > \mu_i^2$, the similar comparison can be out of the scope of the present work.

IV. SUMMARY

We have extended the quark quasiparticle model to study the properties of both CFL matter and MCFL matter in an external strong magnetic field. The self-consistent thermodynamic treatment is obtained through an additional bag function, which is dependent on the chemical potential and/or the magnetic field. It is used to describe the residual interaction of quarks with the quark effective masses in the quasiparticle model. The stability properties of CFL and MCFL matter are calculated and compared with ordinary SQM and MSQM. For a proper magnitude of the magnetic field, about $\tilde{e}H < 0.2\mu_i^2$, the MCFL with smaller energy per baryon is more stable than other phases of SQM. The magnetic field makes both the paired and unpaired quark matter more stable by decreasing the energy per baryon. Moreover, we find that the MCFL matter has an enlarged stability window by comparison with MSQM due to the quarks’ pairing effect. Because of the formation of pairing quarks and the participation without electrons in MCFL matter, there is not a drop behavior for the strange quarks even in the strongest magnetic field. Therefore the MCFL is stable against the transition of the SQM to non strange quark matter in the neutron stars [52].

The anisotropic structure of MCFL matter is also investigated in our quasiparticle model. The isotropic symmetry in the ordinary CFL matter is broken by the magnetic field. The anisotropic pressure splitting is dominated by the magnetic field strength. Importantly, it is found that the anisotropic structure is almost independent on the pairing gap, and the threshold value of the magnetic field is the same for the different phases, i.e., MCFL and MSQM. The result can be understood for that the contribution of the Maxwell term of the magnetic field is much higher than that of the pairing effect.

Since the MCFL matter has a larger stability window, one can expect extensive research of MCFL matter in astrophysics. For example, we can hope that the CFL and/or MCFL matter can be energetically favored in neutron stars or pulsars even if ordinary SQM could not be allowed in the core of neutron star [53]. In particular, MCFL matter has a larger probability of existing in stars due to the two mechanisms, namely, cooper pairing effects and the Landau magnetic quantum levels. The study of magnetic field in the inner regions of compact stars is helpful to distinguish the different quark matter phases. The anisotropic equations in ultrastrong magnetic field are still worth investigating for non-spherical strangelets and compact stars in future work.

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[1] E. Witten, Phys. Rev. D 30, 272 (1984); E. Farhi and R. L. Jaffe, Phys. Rev. D 30, 2379 (1984).
[2] K. Schertler, C. Greiner and M. H. Thoma, Nucl. Phys. A 616, 659 (1997).
[3] M. Buballa and M. Oertel, Phys. Lett. B 457, 261 (1999).
[4] M. Alford, C. Kouvaris, and K. Rajagopal, Phys. Rev. Lett. 92, 222001 (2004).
[5] M. Alford, K. Rajagopal, S. Reddy, and F. Wilczek, Phys. Rev. D 64 074017 (2001).
[6] R. C. Duncan, C. Thompson, Astrophys. J. 392, L9 (1992).
[7] S. Chakraborty, Phys. Rev. D 54, 1306 (1996).
[8] D. Ebert, K. G. Klimenko, Nucl. Phys. A 728, 203 (2003).
[9] R. G. Felipe, H. J. Mosquera Cuesta, A. P. Martínez, and H. P. Rojas, Chin. J. Astron. Astrophys. 5, 399 (2005).
[10] A. J. Mizher, and M. N. Chernodub, and E. S. Fraga, Phys. Rev. D 39, 0451 (1984); E. Farhi and R. L. Jaffe, Phys. Rev. D 39, 261 (1989).
[11] S. Fayazbakhsh, and N. Sadooghi, Phys. Rev. D 39, 261 (1989).
[12] D. P. Menezes, M. Benghi Pinto, S. S. Avancini, and C. Providência, Phys. Rev. C 80, 065801 (2009); S. S. Avancini, D. P. Menezes, and C. Providência, Phys. Rev. C 83, 065805 (2011).
[13] L. Paulucci, J. I. Horvath, Phys. Rev. C 78, 064907 (2008).
[14] C. Manuel, Nucl. Phys. A 785, 114c (2007).
[15] R. González Felipe, D. Manreza Paret, and A. Pérez Martínez, Eur. Phys. J. A 47, 1 (2011).
[16] A. P. Martínez, R. González Felipe, and D. M. Paret, Int. J. Mod. Phys. E 20, 84 (2011).
[17] J. S. Schwinger, Phys. Rev. 82, 664 (1951).
[18] R. González Felipe and A. Pérez martínez, J. Phys. G 36, 075202 (2009); R. González Felipe, E. López Fune, D. Manreza Paret, and A. Pérez Martínez, J. Phys. G 39, 045006 (2012).
[19] G. Lugones and I. Bombaci, Phys. Rev. D 72, 065021 (2005).
[20] K, Schertler, C. Greiner, P. K. Sahu, and M. H. Thoma, Nucl. Phys. A 637, 451 (1998).
[21] P. Romatschke, arXiv:hep-ph/0312152.
[22] X. P. Zheng, M. Kang, X. W. Liu, S. H. Yang, Phys. Rev. C 72, 025809 (2005).
[23] A. Einstein, The Meaning of Relativity, Princeton Univ. Press, p.106 (1922).
[24] T. D. Lee, Nucl. Phys. A 750, 1 (2005).
[25] M. Alford, K. Rajagopal and F. Wilczed, Phys. Lett. B 422, 247 (1998); P. Rapp, T. Schafer, E. V. Shuryak and M. Velkovsky, Phys. Rev. Lett. 81, 53 (1998); M. Huang, P. F. Zhuang and W. Q. Chao, Phys. Rev. D 65, 076012 (2002); M. Buballa, Phys. Rept. 407, 205 (2005).
[26] X. J. Wen, Z. Q. Feng, N. Li, and G. X. Peng, J. Phys. G 36, 025011 (2009).
[27] X. J. Wen, S. Z. Su, D. H. Yang, and G. X. Peng, Phys. Rev. D 86, 034006 (2012).
[28] G. Lugones, and J. E. Horvath, Phys. Rev. D 66, 074017 (2002).
[29] G. X. Peng, X. J. Wen, Y. D. Chen, Phys. Lett. B 633, 314 (2006).
[30] I. A. Shovkovy, Found. Phys. 35, 1309 (2005).
[31] R. D. Pisarski, Nucl. Phys. A 498, 423c (1989).
[32] K. Schertler, C. Greiner and M. H. Thoma, J. Phys. G 23, 2051 (1997).
[33] D. V. Shirkov and I. L. Solovtsov, Phys. Rev. Lett. 79, 1209 (1997).
[34] X. J. Wen, J. Y. Li, J. Q. Liang, and G. X. Peng, Phys. Rev. C 82, 025809 (2010).
[35] B. K. Patra and C. P. Singh, Phys. Rev. D 54, 3551 (1996).
[36] M. I. Gorenstein and S. N. Yang, Phys. Rev. D 52, 5206 (1995); N. Prasad and C. P. Singh, Phys. Lett. B 501, 92 (2001).
[37] G. Lugones and J. E. Horvath, Phys. Rev. D 69, 063509 (2004).
[38] L. D. Landau and E. M. Lifshitz, Quantum Mechanics (Pergamon Press, New York, 1965).
[39] E. J. Ferrer, V. de la Incera and C. Manuel, Phys. Rev. Lett. 95, 152002 (2005).
[40] E. J. Ferrer, V. de la Incera, C. Manuel, Nucl. Phys. B 747, 88 (2006); E. J. Ferrer, V. de la Incera, C. Manuel, PoS JHW 2005, 022 (2006); E. J. Ferrer, V. de la Incera, and C. Manuel, J. Phys. A 39, 6349 (2006).
[41] B. Feng, E. J. Ferrer, V. de la Incera, Nucl. Phys. B 853, 213 (2011).
[42] J. L. Noronha, and I. A. Shovkovy, Phys. Rev. D 76, 105030 (2007).
[43] K. Fukushima and H. J. Warringa, Phys. Rev. Lett. 100, 032007 (2008).
[44] M. G. Alford, J. Berges and K. Rajagopal, Nucl. Phys. B 571, 269 (2000); R Casalbuoni and R Gatto, Phys. Lett. B 464, 111 (1999).
[45] K. Rajagopal and F. Wilczek, in At the Frontier of Particle Physics/Handbook of QCD, edited by M. Shifman (World Scientific, Singapore, 2000), Vol. 3, p. 2061.
[46] S. B. Rüster, I. A. Shovkovy, and D. H. Rischke, Nucl. Phys. A 743, 127 (2004); K. Fukushima, C. Kouvaris, and K. Rajagopal, Phys. Rev. D 71, 034002 (2005).
[47] L. Paulucci, E. J. Ferrer, Vivian de la Incera, and J. E. Horvath, Phys. Rev. D 83, 043009 (2011).
[48] M. Strickland, V. Dexheimer, and D. P. Menezes, Phys. Rev. D 86, 125032 (2012).
[49] R. González Felipe, A. Pérez Martínez, H. Pérez Rojas, and M. G. Orsaria, Phys. Rev. C 77, 015807 (2008).
[50] E. J. Ferrer, V. de la Incera, J. P. Keith, I. Portillo, and P. L. Springsteen, Phys. Rev. C 82, 065802 (2010).
[51] A. A. Isayev, J. Yang, Phys. Lett. B 707, 163 (2012).
[52] A. A. Isayev and J. Yang, J. Phys. G 40, 035105 (2013).
[53] M. Alford and S. Reddy, Phys. Rev. D 67, 074024 (2003).