A PHYSICAL MODEL FOR THE COEVOLUTION OF QSOs AND THEIR SPHEROIDAL HOSTS

GIAN LUIGI GRANATO,1,2 GIANFRANCO DE ZOTTI,1,2 LAURA SILVA,3 ALESSANDRO BRESSAN,1,2 AND LUIGI DANESI2

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ABSTRACT

We present a physically motivated model for the early coevolution of massive spheroidal galaxies and active nuclei at their centers. Within dark matter halos, forming at the rate predicted by the canonical hierarchical clustering scenario, the gas evolution is controlled by gravity, radiative cooling, and heating by feedback from supernovae and from the growing active nucleus. Supernova heating is increasingly effective with decreasing binding energy in slowing down the star formation and in driving gas outflows. The more massive protogalaxies virializing at earlier times are thus the sites of the faster star formation. The correspondingly higher radiation drag fastens the angular momentum loss by the gas, resulting in a larger accretion rate onto the central black hole. In turn, the kinetic energy carried by outflows driven by active nuclei can unbind the residual gas, thus halting both the star formation and the black hole growth, in a time again shorter for larger halos. For the most massive galaxies the gas unbinding time is short enough for the bulk of the star formation to be completed before Type Ia supernovae can substantially increase the Fe abundance of the interstellar medium, thus accounting for the α-enhancement seen in the largest galaxies. The feedback from supernovae and from the active nucleus also determines the relationship between the black hole mass and the mass, or the velocity dispersion, of the host galaxy, as well as the black hole mass function. In both cases the model predictions are in excellent agreement with the observational data. Coupling the model with GRASIL (Silva et al. 1998), the code computing in a self-consistent way the chemical and spectrophotometric evolution of galaxies over a very wide wavelength interval, we have obtained predictions in excellent agreement with observations for a number of observables that proved to be extremely challenging for all the current semianalytic models, including the submillimeter counts and the corresponding redshift distributions, and the epoch-dependent K-band luminosity function of spheroidal galaxies.

Subject headings: galaxies: elliptical and lenticular, cD — galaxies: evolution — galaxies: formation — quasars: general

1. INTRODUCTION

Although the traditional approach to galaxy formation and evolution regards nuclear activity as an incidental diversion, it is becoming clear, beyond any reasonable doubt, that the formation of supermassive black holes (BHs) powering nuclear activity is intimately linked to the formation of its host galaxy and plays a key role in shaping its evolution. Evidences supporting this view include the following: the discovery that massive dark objects (MDOs), with masses in the range \(10^8 - 10^9 M_\odot\), and a mass function matching that of the galaxy luminosity function of quasars, and the mass function of dark halos at the same redshift (Haehnelt, Natarajan, & Rees 1998; Monaco, Salucci, & Danese 2000); and the similarity between the evolutionary histories of the luminosity densities of galaxies and quasars (e.g., Cavaliere & Vittorini 1998). Recently, Shields et al. (2003) have found that the correlation between \(M_\text{BH}\) and the stellar velocity dispersion is already present at redshift up to \(z \approx 3\).

As discussed by Granato et al. (2001), the mutual feedback between galaxies and quasars during their early evolutionary stages may be the key to overcome some of the crises of the currently standard scenario for galaxy evolution. For example, predictions of semianalytic models (Devriendt & Guiderdoni 2000; Cole et al. 2000; Somerville, Primack, & Faber 2001; Menci et al. 2002) are persistently unable to account for the surface density of massive galaxies at substantial redshift detected by (sub)millimeter surveys with SCUBA and MAMBO (Blain et al. 2002; Scott et al. 2002) and by deep K-band surveys (Cimatti et al. 2002b; Kashikawa et al. 2003), unless ad hoc adjustments are introduced. The difficulty stems from the fact that the standard cold dark matter (CDM) scenario tends to imply that most of the star formation occurs in relatively small galaxies that later merge to make bigger and bigger objects. On the contrary, the data indicate that galaxies detected by (sub)millimeter surveys are mostly very massive, with very high star formation rates (SFRs; \(\sim 10^3 M_\odot \text{yr}^{-1}\)), at \(z > 2\) (Dunlop 2001; Ivison et al. 2002; Aretxaga et al. 2003; Chapman et al. 2003). All these data are more consistent with the traditional “monolithic” scenario, according to which elliptical galaxies formed most of their stars in a single burst, at relatively high redshifts, and underwent essentially passive evolution thereafter. On the other hand, the “monolithic”
scheme is inadequate to the extent that it cannot be fitted in a consistent scenario for structure formation from primordial density perturbations.

Clues on the timing of evolution of both galaxies and quasars are provided by chemical abundances (e.g., Friaca & Terlevich 1998). Spectroscopic observations demonstrate, even for the highest redshift quasars, a fast metal enrichment of the circumnuclear gas (e.g., Hamann & Ferland 1999; Fan et al. 2000, 2001; Freudling, Corbin, & Korista 2003). Statistical studies of local E/S0 galaxies, hosting supermassive BHs, show that the most massive galaxies are also the most metal-rich, the reddest, and, perhaps, the oldest (Forbes & Ponman 1999; Trager et al. 2000a, 2000b). Such galaxies show an excess $\alpha$-element/Fe ratio, compared to solar ($\alpha$-enhancement; Trager et al. 2000a, 2000b; Thomas, Maraston, & Bender 2002), suggestive of very intense but short star formation activity. In addition, observations of hosts of high-redshift quasars show that they are as massive as expected from the local $L_{\text{QSO}}-L_{\text{host}}$ relation and that most of their stars are relatively old, although star-forming regions are present (Kukula et al. 2001; Hutchings et al. 2002; Ridgway et al. 2002; Stockton & Ridgway 2001).

The work carried out by our group in the last several years, aimed at constructing physically grounded models for the joint formation and evolution of quasars and spheroidal galaxies in the framework of the standard hierarchical clustering scenario (Granato et al. 2001), has shown that, to account for the surface density of massive galaxies at substantial redshifts detected by submillimeter SCUBA surveys (which turns out to be far in excess of predictions of standard semianalytic models), allowing for the observed relationships between quasars and galaxies, it is necessary to assume that the formation of stars and of the central BH took place on shorter timescales within more massive DM halos. In other words, the canonical hierarchical CDM scheme—small clumps collapse first—is reversed for baryon collapse and the formation of luminous objects (antihierarchical baryon collapse scenario). This behavior, attributed to the feedback from supernova explosions and, for the most massive galaxies, from nuclear activity, may account simultaneously for evolutionary properties of quasars and of massive spheroidal galaxies (Monaco et al. 2000; Granato et al. 2001), for clustering properties of SCUBA galaxies (Magliocchetti et al. 2001; Perrotta et al. 2003), and for the metal abundances in spheroidal galaxies and in bulges of later Hubble types hosting supermassive BHs (Romano et al. 2002).

Recently, Cattaneo & Bernardi (2003), using the early-type galaxy sample in the Sloan Digital Sky Survey (Bernardi et al. 2003a), investigated the hypothesis that quasars formed together with the stellar population of elliptical galaxies, finding a consistency with the observed luminosity function of optical quasars.

However, there is not, as yet, a clear understanding of the physical mechanisms governing the interactions among the active nucleus and the host galaxy. Silk & Rees (1998) and Fabian (1999) proposed that the relationship between the BH mass and the velocity dispersion (or mass) of the host stellar spheroid may be the effect of the quasar feedback. However, in its present form, the proposed mechanism does not imply shorter star formation times for more massive galaxies and therefore cannot easily explain the $\alpha$-enhancement of more massive objects. Wang & Biermann (1998) suggested that the formation of both elliptical galaxies and supermassive BHs at their centers is related to the merging of two protodisks. Kauffmann & Haehnelt (2000) and Haehnelt & Kauffmann (2000) have analyzed the evolution of active nuclei and of host galaxies in the framework of the hierarchical clustering scenario, using a semianalytic approach. In their scheme, the merging process determines both the evolution of galaxies and the growth of the BHs at their centers. This model predicts a rapid evolution of galaxies with redshift. Volonteri, Haardt, & Madau (2002, 2003) presented a model in which most of the mass in BHs is assembled in accretion episodes triggered by merging. They found that the galaxy merging would leave about 10% of massive BHs distributed in galactic halos and a similar fraction of binary supermassive BHs in galactic centers. Di Matteo et al. (2003) considered the gas content in galaxies, as predicted by cosmological hydrodynamical simulations including subgrid prescriptions for gas cooling and star formation (but not for BH growth). They pointed out that the observed $M_{\text{BH}}-\sigma$ correlation is well reproduced, provided that a linear relationship between the gas and BH masses (at $z > 1$) is assumed.

In this paper we carry out an investigation of the physical processes driving the growth of the central BH and the effect of the energy released by the active nucleus on the surrounding protogalactic gas. The corresponding prescriptions for matter outflow, gas cooling and collapse, star formation, etc., are implemented in a model for the formation and evolution of galaxies, interfaced with our code computing their spectral energy distribution as a function of their age, taking into account the evolution of stellar populations, of metal abundances, and of the dust content and its distribution (GRASIL; Silva et al. 1998; Silva 1999; Granato et al. 2000). Our aim is to put on a more solid physical basis the approach by Granato et al. (2001), which partly relies on empirical recipes.

The model has been tested against the observed correlation of BH masses with the mass and velocity dispersion of the host galaxies, the observed $K$-band and submillimeter counts, and the associated redshift distributions, along with the chemical and photometric properties of the local E/S0 galaxies. In subsequent papers the model will be used to predict the time-dependent luminosity function of quasars up to $z > 6$ and of passively evolving elliptical galaxies, as well as their clustering properties, with reference to the observational capabilities of space missions such as SIRTF, already launched, and JWST, of ground millimeter telescopes such as ALMA, and of the forthcoming deep near-IR surveys from the ground.

In § 2 we describe the basic physical processes governing the evolution of the SFR in spheroidal galaxies, the growth of central BHs, and their feedback. In § 3 we present our main results, which are discussed in § 4. In § 5 we summarize our main conclusions.

We adopt a CDM cosmology with cosmological constant, consistent with the Wilkinson Microwave Anisotropy Probe (WMAP) data (Bennett et al. 2003), as well as with information from large-scale structure (Spergel et al. 2003): $\Omega_m = 0.29$, $\Omega_b = 0.047$, $\Omega_{\Lambda} = 0.71$, $H_0 = 72$ km s$^{-1}$, $\sigma_8 = 0.8$, and an index $n = 1.0$ for the power spectrum of primordial density fluctuations.

2. BASIC INGREDIENTS

2.1. Dark Matter Halos

Following Navarro, Frenk, & White (1997) and Bullock et al. (2001b), we identify as a virialized halo at redshift $z$ a volume of the universe of radius $r_{\text{vir}}$ enclosing an overdensity $\Delta_{\text{vir}}(z)$, which, for a flat cosmology, can be approximated by

$$\Delta_{\text{vir}} \approx \frac{18\pi^2}{2} \frac{\rho_{c}}{\rho(z)} ,$$

(1)
where \( x = \Omega(z) - 1 \) and \( \Omega(z) \) is the ratio of the mean matter density to the critical density at redshift \( z \) (Bryan & Norman 1998). The halo mass is then given by

\[
M_{\text{vir}} = \frac{4\pi}{3} \Delta_{\text{vir}}(z) \rho_{\text{c}}(z) r_{\text{vir}}^3,
\]

\( \rho_{\text{c}}(z) \) being the mean universal matter density. Useful quantities are the rotational velocity of the DM halo at its virial radius, \( v_{\text{vir}} = GM_{\text{vir}}/r_{\text{vir}} \), and the equilibrium gas temperature in the DM potential well,

\[
kT = \frac{1}{2} \mu m_p v_{\text{vir}}^2,
\]

where \( m_p \) is the proton mass and \( \mu m_p \) is the mean molecular weight of the gas.

The virialized halos exhibit a universal density profile (Navarro et al. 1997; Bullock et al. 2001b) well described by

\[
\rho(r) = \frac{\rho_s}{c \left( 1 + cx \right)^2},
\]

where \( x = r/r_{\text{vir}} \) and \( c \) is the concentration parameter. Analyzing the profiles of a large sample of virialized halos obtained through high-resolution N-body simulations, Bullock et al. (2001b) found that, at any redshift, \( c = \text{a function of } M_{\text{vir}}^{-0.13} \), while, at fixed mass, \( c \propto (1 + z)^{-1} \).

Numerical simulations and theoretical arguments show that the DM assembly in halos proceeds through a first phase of fast accretion, followed by a slow phase (Wechsler et al. 2002; van den Bosch 2002). Zhao et al. (2003) showed that the potential well of the halos is built up during the fast accretion. The subsequent slow accretion does not change the “identity” of the halo, although significantly increasing its mass. Guided by these results, in the following we assume that, for the mass and redshift ranges we are interested in, the virialization epoch of DM halos coincides with the end of the fast accretion phase and with the beginning of the vigorous star formation of the protospheroidal galaxies and that these galaxies keep their identity until the present time. Our approach implies, in the mass and redshift range considered, a one-to-one correspondence between halos and galaxies, consistent with the available data on clustering of Lyman break (Bullock, Wechsler, & Somerville 2002) and SCUBA galaxies (Magliocchetti et al. 2001; Perrotta et al. 2003).

The formation rate of massive halos (\( M_{\text{vir}} \gtrsim 2.5 \times 10^{11} M_\odot \)) at \( z \gtrsim 1.5 \) is approximated by the positive part of the time derivative of the halo mass function \( n(M_{\text{vir}}, z) \) (Haehnelt & Rees 1993; Sasaki 1994; Peacock 1999). The negative part is small for this mass and redshift range, consistent with our assumption that massive halos survive until the present time. Following Press & Schechter (1974), we write

\[
n(M_{\text{vir}}, z) = \frac{\rho}{M_{\text{vir}}^2} \nu f(\nu) \frac{d\ln \nu}{d\ln M_{\text{vir}}},
\]

where \( \rho \) is the average comoving density of the universe and \( \nu = \left[ \delta_c(z)/\sigma(M_{\text{vir}}) \right]^2 \), \( \sigma(M_{\text{vir}}) \) being the rms initial density fluctuations smoothed on a scale containing a mass \( M_{\text{vir}} \), and \( \delta_c \) the critical overdensity for spherical collapse. The latter quantity is given by \( \delta_c(z) = \delta_0/b(z) \), \( \delta_0 \) being the present-day critical overdensity (1.686 for an Einstein-de Sitter cosmology, with only negligible dependencies on \( \Omega_m \) and \( \Omega_L \)) and \( b(z) \) the linear growth factor, for which we adopt the approximation proposed by Li & Ostriker (2002). For the function \( \nu f(\nu) \) we adopt the expression given by Sheth & Tormen (2002), which also takes into account the effect of ellipsoidal collapse:

\[
\nu f(\nu) = A [1 + (a\nu)^p] \left( \frac{a\nu}{2} \right)^{1/2} e^{a\nu/2} \pi^{1/2},
\]

where \( A = 0.322, p = 0.3, \) and \( a = 0.707 \).

### 2.2. Star Formation Rate

Recent studies on the angular momentum of DM halos suggest that a significant fraction of the mass has a low specific angular momentum (Bullock et al. 2001a). In addition, the results of high-resolution simulations including gas suggest that a significant fraction of the angular momentum of the baryons is redistributed to the DM by dynamical friction (see, e.g., Navarro & Steinmetz 2000), although the heating of the gas in subgalactic halos may increase the tidal stripping and the angular momentum of the gas (Maller & Dekel 2002). As far as the effect of angular momentum can be neglected, as is probably the case for the formation of spheroidal galaxies, the collapse time of baryons within the host DM halo, \( t_{\text{coll}} \), is the maximum between the free-fall time,

\[
t_{\text{dyn}}(r) = \left[ \frac{3\pi}{32G\rho(r)} \right]^{1/2},
\]

and the cooling time,

\[
t_{\text{cool}}(r) = \frac{3}{2} \frac{\rho_{\text{gas}}(r)}{\mu m_p} \frac{kT}{C n_e^2(r) \Lambda(T)},
\]

both computed at the virial time \( t_{\text{vir}} \). In the above equations \( \rho \) is the total matter density, \( \rho_{\text{gas}} \) is the gas density, \( n_e \) is the electron density, \( \Lambda(T) \) is the cooling function, and \( C = \left[ n_e^2(r) / \langle n_e(r) \rangle \right]^2 \) is the clumping factor, assumed to be constant. In the following we adopt the cooling function given by Sutherland & Dopita (1999), which includes the dependence on metal abundance.

We consider a single-zone galaxy with three gas phases: diffuse gas in the outer regions, with mass \( M_{\text{diff}}(t) \), infalling on a dynamical timescale; cold gas with mass \( M_{\text{cold}}(t) \), available to form stars; and hot gas with mass \( M_{\text{hot}}(t) \), eventually outflowing. At the virialization we assume that \( M_{\text{diff}}(t_{\text{vir}}) = f_b M_{\text{vir}} \), with \( f_b = 0.16 \), the universal ratio of baryons to DM (Bennett et al. 2003; Spergel et al. 2003). The infalling gas, initially at the equilibrium temperature in the DM potential well (eq. [3]), is transferred to the cool star-forming phase at a rate

\[
\dot{M}_{\text{cold}}(t) = \frac{M_{\text{diff}}(t_{\text{vir}})}{\max \{ t_{\text{cool}}(r_{\text{vir}}), t_{\text{dyn}}(r_{\text{vir}}) \}}.
\]

The SFR is given by

\[
\psi(t) = \int_0^{t_{\text{vir}}} \frac{1}{\max \{ t_{\text{cool}}(r), t_{\text{dyn}}(r) \}} \frac{dM_{\text{cold}}(r, t)}{dr} dr,
\]

where we assume that the cold gas distribution still follows the DM distribution. This assumption is admittedly quite unrealistic since cold cloudlets should rather fall toward the center.
But again, a realistic modeling of the cold gas distribution would be too ambitious at the present stage.

As mentioned above, according to recent studies (Wechsler et al. 2002; van den Bosch 2002; Zhao et al. 2003), while the buildup of the potential well is rather fast, the mass of the halo goes on increasing over a timescale of the order of the Hubble time. The baryons associated with this slow accretion occupy a large volume so that they have a relatively low density and feel also the heating effect of the supernova and quasar feedback. It is therefore reasonable to assume that their cooling time is comparable to, or longer than, the Hubble time and does not participate in the star formation process.

Strictly speaking, in equation (9) the increase of the cooling time due to the dilution of the hot gas as the cold gas drops out should be taken into account. The effect is, however, minor because the fraction of gas in the cold phase is always less than 50% and can therefore be ignored given the exploratory nature of the present model.

Although the initial mass function (IMF) may depend on the physical properties of the gas, such as density and metallicity (see Eisenhauer 2001 and references therein), it is often assumed to be independent of time and galaxy mass. We consider in the following the IMF preferred by Romano et al. (2002) on the basis of chemical abundances in local elliptical galaxies, i.e., $\Phi(M) \propto M^{-0.4}$ for $M \leq 10 M_\odot$ and $\Phi(M) \propto M^{-1.25}$ for $10 M_\odot < M \leq 100 M_\odot$. A detailed discussion of the effects of varying the IMF on the properties of the present-day elliptical galaxies will be presented in a forthcoming paper.

The feedback due to supernova (SN) explosions moves the gas from the cold to the hot phase at a rate

$$M_{\text{cold}}^{\text{SN}} = \frac{2}{3} \psi(t) \epsilon_{\text{SN}} \frac{\eta_{\text{SN}} E_{\text{SN}}}{\sigma^2},$$

(11)

where $\eta_{\text{SN}}$ is the number of Type II SNe expected per solar mass of formed stars (determined by the IMF, adopting a minimum progenitor mass of $8 M_\odot$), $E_{\text{SN}}$ is the kinetic energy of the ejecta from each supernova ($10^{51}$ ergs; e.g., Woosley & Weaver 1986), and $\epsilon_{\text{SN}}$ is the fraction of this energy that is used to reheat the cold gas. Analyses show that about 90% of the SN kinetic energy may be lost by radiative cooling (Thronson et al. 1998; Heckman et al. 2000); we adopt $\epsilon_{\text{SN}} = 0.05$ as our reference value, consistent with the results by Mac Low & Ferrara (1999) and Wada & Venkatesan (2003).

We relate $V_{\text{vir}}$ to the line-of-sight velocity dispersion $\sigma$ using the relationship $\sigma/V_{\text{vir}} \approx 0.65$, derived by Ferrarese (2002) for a sample of local galaxies.

The chemical evolution of the gas is followed by using classical equations and stellar nucleosynthesis prescriptions, as, for instance, reported in Romano et al. (2002).

We can elucidate the dependence of the SFR on halo mass by means of a simple order-of-magnitude argument. Since the mean density within the virial radius is $\approx 200\rho_{\text{crit}}$, the mean value of $t_{\text{dyn}}$ is about a factor of 10 shorter than the expansion timescale, at all redshifts, independently of the halo mass. If the gas is in virial equilibrium in the DM potential well, the effective cooling time of a pure hydrogen plasma is, assuming uniform density,

$$t_{\text{cool, eff}} \approx 1.6 \times 10^{11} \left(1 + z_{\text{vir}}\right)^{-5/2} h_{100}^{1/3} \times \left(\frac{M_{\text{vir}}}{10^{12} M_\odot} \right)^{1/3} \left(\frac{M_{\text{vir}}/M_{\text{gas}}}{1/0.16}\right)^{1/2} C^{-1} \text{ yr.}$$

(12)

As discussed by Romano et al. (2002; see their Fig. 10), for spheroidal galaxies with $M_{\text{vir}} < 10^{11} M_\odot$, the ratio between $M_{\text{vir}}$ and the mass in stars at $z = 0$ (including remnants) $M_{\text{sph}}$ decreases with increasing $M_{\text{sph}}$ roughly as

$$\frac{M_{\text{vir}}}{M_{\text{sph}}} \propto M_{\text{sph}}^{-1/3},$$

(13)

as a result of the larger effect of stellar feedback in shallower potential wells. Since the mass of cooling gas is approximately equal to $M_{\text{sph}}/t_{\text{cool, eff}}$, the timescale for the gas to cool is set by the value of $t_{\text{cool, eff}}$. If $t_{\text{cool, eff}} < 10^{11} M_\odot$, $M_{\text{in}}/t_{\text{cool}} \propto M_{\text{sph}}^{-2/3}$. Thus, the SFR, $\psi(t) \approx M_{\text{sph}}/t_{\text{cool}}$, is approximately $\propto M_{\text{sph}}^{1/3}$, since both timescales are effectively independent of, or very weakly dependent on, $M_{\text{vir}}$. This means that stars form faster within larger DM halos, as in the hierarchical baryon collapse scenario by Granato et al. (2001). If, as argued above, the SFR is controlled by $t_{\text{dyn}}$, we have $\psi(t) \approx 32(M_{\text{gas}}/10^{11} M_\odot) (1 + z)^{3/2} M_\odot \text{ yr}^{-1}$.

2.3. Black Hole Growth

As discussed by Haiman, Ciotti, & Ostriker (2003), the available information on the evolution of both the global SFR and the quasar emissivity is broadly consistent with the hypothesis that star formation in spheroids and BH fueling are proportional to one another.

One mechanism yielding such proportionality has been discussed by Umemura (2001), Kawakatu & Umemura (2002), and Kawakatu, Umemura, & Mori (2003). In the central regions of protogalaxies the drag due to stellar radiation may result in a loss of angular momentum of the gas at a rate that in a clumpy medium is well approximated by

$$\frac{d \ln J}{dt} \approx \frac{L_{\text{sph}}}{c^2 M_{\text{gas}}} (1 - e^{-\tau}),$$

(14)

where $L_{\text{sph}}$ is the global stellar luminosity and $\tau$ is the effective optical depth of the spheroid. The latter quantity is given by $\tau = \bar{\tau} N_{\text{inf}}$, where $\bar{\tau}$ is the average optical depth of single clouds and $N_{\text{inf}}$ is the average number of clouds intersected by a light ray over a typical galactic path.

The gas can then flow toward the center, feeding a mass reservoir around the BH at a rate (Kawakatu et al. 2003)

$$M_{\text{inflow}} \approx -M_{\text{gas}} \frac{d \ln J}{dt} \approx \left(\frac{L_{\text{sph}}}{c^2}\right) (1 - e^{-\tau}).$$

(15)

During the early evolutionary stages the luminosity is dominated by massive main-sequence stars, $M \geq 5 M_\odot$, and is thus proportional to the SFR $\psi(t)$. For the adopted IMF we have

$$M_{\text{inflow}} \approx 1.2 \times 10^{-11} \psi(t) (1 - e^{-\tau}) M_\odot \text{ yr}^{-1}.$$  

(16)

While this expression is useful for the analytical estimates presented in § 3.1, in the full calculations we have adopted the
values $L_{\text{ph}}$ computed using our spectrophotometric code GRASIL. We parameterize the optical depth $\tau$ as

$$\tau = \tau_0 \left( \frac{Z}{Z_\odot} \right) \left( \frac{M_{\text{gas}}}{10^{12} M_\odot} \right)^{1/3},$$

(17)

and for $\tau_0$ we explore the range from 1 to 10 (see § 3).

An order-of-magnitude estimate of the relevant timescales and luminosities can be derived assuming that $M_{\text{inflow}}$ is simply the accretion rate on the central BH. Then, the timescale for it to grow to a mass $M_{\text{BH}}$ is, using the order-of-magnitude estimate of $\psi(t)$ given in the previous section,

$$t_{\text{BH}} \simeq M_{\text{BH}} / M_{\text{inflow}} \simeq 2.3 \times 10^9 \frac{M_{\text{BH}}}{10^8 M_\odot} \left( \frac{M_{\text{gas}}}{10^{11} M_\odot} \right)^{-1},$$

$$\times (1 + z)^{-3/2} \left( 1 - e^{-\tau} \right)^{-1} \text{yr.}$$

(18)

Thus, if the accretion is not limited, e.g., by radiation pressure or angular momentum, BHs can grow to very large masses from small (e.g., stellar mass) seeds in a time shorter than the age of the universe at all relevant redshifts. If $M_{\text{BH}} \propto M_{\text{gas}}$, the growth time is independent of $M_{\text{BH}}$.

For accretion with radiative efficiency $\eta$, the bolometric luminosity is $L_{\text{bol}} \simeq \eta M_{\text{inflow}} c^2$. Following Elvis, Risaliti, & Zamorani (2002), we adopt, as a reference value, $\eta = 0.15$. In units of the Eddington luminosity

$$L_{\text{Edd}} \simeq 1.26 \times 10^{46} \frac{M_{\text{BH}}(t)}{10^8 M_\odot} \text{ergs} \text{s}^{-1},$$

(19)

and setting $M_{\text{BH}}(t) \simeq \dot{M}_{\text{inflow}} t$, we have

$$\frac{L_{\text{bol}}}{L_{\text{Edd}}} \simeq \frac{3}{0.15} \left( \frac{t}{10^8 \text{yr}} \right)^{-1},$$

(20)

showing that the fast accretion phase and the buildup of supermassive BHs can be quite fast, thus accounting for QSOs at high $z$.

Obviously, super-Eddington accretion is not the only mechanism for rapid BH growth. At the other extreme we may have accretion with low radiative efficiency such as the BH mergers at high redshifts advocated by Haiman et al. (2003), which may be testable by gravitational wave experiments like LISA (Menou, Haiman, & Narayan 2001). Since these early evolutionary phases are expected to be heavily dust obscured, a powerful tool to discriminate among the various possibilities is hard X-ray emission. Far-IR/millimeter observations are also useful but less capable of distinguishing the effect of the AGN from that of a starburst.

While mechanisms for super-Eddington accretion have been proposed (Begelman 2001, 2002), fully unperturbed free fall is unrealistic. Correspondingly, $L_{\text{bol}} / L_{\text{Edd}}$ will not reach the extreme values indicated by equation (20) and the duration of the $L_{\text{bol}} / L_{\text{Edd}} \gtrsim 1$ phase can be longer. In our model, we let the material infalling at the rate given by equation (15) first accumulate in a circumnuclear mass reservoir and then flow toward the BH on a timescale depending on the viscous drag $\tau_{\text{visc}} \sim r^2 / \nu$. The reservoir accumulates mass at a net rate $M_{\text{res}} = M_{\text{inflow}} - M_{\text{BH}}$, where $M_{\text{BH}}$ is computed as follows. Following Duschl et al. (2000) and Burkert & Silk (2001), we adopt a viscosity $\nu = \Re_{\text{crit}}^2 r_\nu$, where $\Re_{\text{crit}} = 100 - 1000$ is the critical Reynolds number for the onset of turbulence. With these assumptions the viscous time can be expressed as $\tau_{\text{visc}} = \tau_0 \left( \frac{Z}{Z_\odot} \right) \left( \frac{M_{\text{gas}}}{10^{12} M_\odot} \right)^{1/3}$.

The dynamical time $\tau_{\text{dyn}} = (3\pi / 32G\rho_s)^{1/2}$ is referred to the system “BH plus reservoir.”

The accretion radius of the BH is given by $r_A = GM_{\text{BH}} / V_{\text{vir}}^2$. Defining the reservoir dimension $R_{\text{res}} = \alpha r_A$, we estimate $\rho_s$ as the mean density within a sphere of radius $R_{\text{res}}$ with mass $M_{\text{BH}} + M_{\text{res}}$, to get

$$\tau_{\text{dyn}} = \frac{\pi}{2} \frac{G}{V_{\text{vir}}^2} \left( \frac{M_{\text{BH}} + M_{\text{res}}}{M_{\text{BH}}} \right)^{1/2}.$$

(21)

The viscous accretion rate onto the BH can be defined as

$$M_{\text{BH}} = \frac{M_{\text{res}}}{\tau_{\text{visc}}} = k_{\text{accr}} \frac{\rho_s^3}{G} \left( \frac{M_{\text{res}}}{M_{\text{BH}}} \right)^{3/2} \left( 1 + \frac{M_{\text{BH}}}{M_{\text{res}}} \right)^{1/2}.$$

(22)

The constant $k_{\text{accr}} = [\pi(\alpha / 2)^{3/2} (V_{\text{vir}} / \sigma)^3 \Re_{\text{crit}}^{-1}]^{-1}$ has a rather wide range of possible values. For $\Re_{\text{crit}} = 100$ and $\alpha = 10$ we have $k_{\text{accr}} \sim 10^{-4}$, which we adopt as the reference value in the following. The actual accretion rate is then given by

$$M_{\text{BH}} = \min \left( M_{\text{BH}}^{\text{visc}}, A M_{\text{Edd}} \right),$$

(23)

where $M_{\text{Edd}} = L_{\text{Edd}} / (\gamma c^2)$ and $A = (L / L_{\text{Edd}})_{\text{max}}$ is the maximum allowed Eddington ratio. In our computations, we allow at most mildly super-Eddington accretion, i.e., $A \lesssim \text{a few}$.

2.4. QSO Feedback

The QSO activity affects the interstellar medium (ISM) of the host galaxy and also the surrounding intergalactic medium (IGM) through both the radiative output and the injection of kinetic energy producing powerful gas outflows. Quasar-driven outflows have been invoked to produce an intergalactic magnetic field (Furlanetto & Loeb 2001) and to preheat the intracluster medium (Valageas & Silk 1999; Kravtsov & Yepes 2000; Wu, Fabian, & Nulsen 2000; Bower et al. 2001; Cavaliere, Lapin, & Menci 2002; Platania et al. 2002). In the case of radio-loud QSOs there is evidence that up to about half of the total power is in the jets (Rawlings & Saunders 1991; Celotti, Padovani, & Ghisellini 1997; Tavecchio et al. 2000). In broad absorption line (BAL) QSOs, the kinetic power of the outflowing gas can be a significant fraction of the bolometric luminosity (Begelman 2003). X-ray observations of BAL QSOs revealed significant absorption ($N_H \gtrsim 10^{23} \text{cm}^{-2}$), implying large outflows ($M_{\text{out}} \sim 5 M_\odot \text{yr}^{-1}$) and large kinetic luminosities $L_K$ (Brandt, Gallagher, & Kaspi 2001; Brandt & Gallagher 2000).

Theoretical studies on the mechanisms responsible for AGN-driven outflows show that efficient acceleration could be due to radiation pressure through scattering and absorption by dust (see, e.g., Voit, Weymann, & Korista 1993) and scattering in resonance lines (see, e.g., Arav, Li, & Begelman 1994). Murray et al. (1995) presented a dynamical model for a wind produced just over the disk by a combination of radiation and gas pressure. In a similar way Proga, Stone, & Kallman (2000) showed that a wind can be launched from a disk around a supermassive BH with velocities up to 0.1$c$ and a mass-loss rate of $0.5 M_\odot \text{yr}^{-1}$. Following Murray et al. (1995), an approximate solution for the wind velocity produced by line acceleration as a function of the radius is

$$v = v_\infty \left( 1 - \frac{r_f}{r} \right)^{2.35},$$

(24)

where $r_f$ is the radius at which the wind is launched.
The asymptotic speed is
\[ v_\infty \sim \left( \frac{GM_{BH}}{r_f} \right) \frac{1}{2}, \tag{25} \]
where \( \gamma \) is related to the force multiplier (see, e.g., Laor & Brandt 2002). Adopting the reference values of the parameters of Murray et al. (1995), we have \( \gamma \approx 3.5 \). By replacing the BH mass with the corresponding Eddington luminosity \( L_{\text{Edd},46} \) in units of \( 10^{46} \) ergs s\(^{-1}\), we get
\[ \frac{v_\infty}{c} \sim 6.2 \times 10^{-2} \left( \frac{r_f}{10^{16} \text{ cm}} \right)^{-1/2} L_{\text{Edd},46}^{1/2}. \tag{26} \]
Detection of outflows with velocities ranging between 0.1c and 0.4c in the BAL quasars APM 08279+0522 (Chartas et al. 2002) and PG 1115+080 (Chartas, Brandt, & Gallagher 2003) has been reported, based on observations of X-ray BALs performed with the Chandra and XMM-Newton X-ray observatories. In the case of APM 08279+0522 an intrinsic bolometric luminosity of \( L_{\text{bol}} \approx 2.4 \times 10^{47} \) ergs s\(^{-1}\) has been estimated by Egami et al. (2000). Under the assumption of \( L_{\text{Edd}} \approx L_{\text{bol}} - 3L_{\text{bol}} \), the maximum observed velocity \( v \approx 0.4c \) is obtained with a launching radius \( r_f \approx (0.5-2) \times 10^{16} \) cm. Such small values of \( r_f \) are confirmed by the observed variability of the absorption-line energies and widths in APM 08279+0522 over a proper timescale of 1.8 weeks (Chartas et al. 2003).

The asymptotic speed is reached at \( r \gtrsim 40r_f \). If \( f_c \) is the covering factor of the outflow and using \( M_\text{w} = 4\pi r^2 \rho(r) v_r \sim 4\pi f_c \, m_H \, N_{22} \, 40r_f \, v_\infty \), we get
\[ M_\text{w} = 2.6f_cN_{22} \, L_{\text{Edd},46}^{1/2} \left( \frac{r_f}{10^{16} \text{ cm}} \right)^{1/2} \frac{M_\odot}{\text{yr}^{-1}}, \tag{27} \]
where \( N_{22} = N_{4} / 10^{22} \) cm\(^{-2}\). Adopting \( r_f = 1.5 \times 10^{16} \) cm as a reference value, the kinetic power in the outflow is
\[ L_K = \frac{1}{2} M_\text{w} v_\infty^2 \approx 3.6 \times 10^{44} f_c N_{22} L_{\text{Edd},46}^{3/2} \text{ ergs s}^{-1}, \tag{28} \]
and may thus amount to several percent of the accretion luminosity for highly luminous QSOs. Interestingly, this corresponds to the power required to account for the preheating of the intracluster medium (Bower et al. 2001; Platania et al. 2002; Lapi, Cavaliere, & De Zotti 2003). It is also interesting to notice that \( M_\text{w} / M_{\text{acc}} \approx 2.5(\epsilon / 0.1)f_c N_{22} L_{\text{Edd},46}^{1/2} \), implying that for highly luminous QSOs \( M_\infty \approx M_{\text{acc},i} \), if \( f_c N_{22} \approx 1 \). High-luminosity QSOs emitting at the Eddington limit are able to generate winds involving relatively small amounts of gas, but with very high velocities and significant kinetic energies. When the luminosity decreases below the Eddington limit, we replace the Eddington luminosity with the bolometric luminosity in equations (28) and (30).

Estimating the fraction of the kinetic luminosity \( L_K \) transferred to the ISM is a rather complex problem. One possible effect of outflows and jets is to transport the ambient gas to larger radii. This effect has been recently estimated through numerical simulations by Brüggen et al. (2002). By investigating the effects of radio cocoons on the intracluster medium, they concluded that frequent low-power activity cycles are rather efficient in stirring the environment gas, particularly in regions close to the injection point of the high-speed gas. Ensslin & Kaiser (2000) evaluated the energy accumulated in the cocoons around radio galaxies. Inoue & Sasaki (2001) argued that for nonrelativistic cocoon plasma, the fraction of the jet energy deposited in the intracluster medium can reach 40%, neglecting the radiative cooling. Similar conclusions have been reached by Bicknell, Dopita, & O’Dea (1997). Nath & Roychowdhury (2002), taking into account also the radiative losses, found that a large fraction (\( \gtrsim 0.5 \)) of the kinetic energy of BAL and radio-loud QSOs is transferred to an ambient gas with number density \( n \approx 0.1-1 \) cm\(^{-3}\) and temperature \( T_{\text{vir}} \approx 10^6 \) K. These values of density and temperature are quite similar to those of the gas in the outer regions of massive galactic halos.

We assume that the QSO feedback heats up the ISM at a rate \( L_h = f_h L_K \) and removes it from the cold phase at a rate
\[ \dot{M}_{\text{QSO}} = -\frac{2}{3} \frac{L_h}{\sigma^2} M_{\text{gas},i}, \tag{29} \]
where \( M_{\text{gas}} = M_{\text{cold}} + M_{\text{inf}} \) is the mass of the gas in the cold phase plus that of the gas that has not yet fallen in the star-forming region. Setting \( \epsilon_{\text{QSO}} = (f_h / 0.5)(\epsilon / 0.1)(N_{22} / 10) \), we get
\[ \dot{M}_{\text{QSO}} \approx -2 \times 10^3 \frac{\epsilon_{\text{QSO}} L_{\text{Edd},46}^{3/2}}{\sigma (300 \text{ km s}^{-1})^2} \frac{M_{\text{cold}}}{M_\odot} \text{ yr}^{-1}. \tag{30} \]
We have explored the range \( 1 \leq \epsilon_{\text{QSO}} \leq 10 \), adopting \( \epsilon_{\text{QSO}} = 6 \) as our reference value. The fraction \( M_{\text{cold}} / M_{\text{gas}} \) on the right-hand side of equation (29) has been introduced in order to share the QSO feedback between the cold and the infalling gas. Thus, the QSO feedback also removes the infalling gas at a rate
\[ \dot{M}_{\text{inf}} = -\frac{2}{3} \frac{L_h}{\sigma^2} M_{\text{inf}}. \tag{31} \]
The quasar feedback can easily heat the interstellar gas to temperatures \( \sim 1 \) keV (Valageas & Silk 1999; Bower et al. 2001; Nath & Roychowdhury 2002; Platania et al. 2002; Lapi et al. 2003), thus unbinding it and making it flow into the IGM. The corresponding energy injection in the IGM may account for the steepening of the X-ray luminosity–temperature correlation observed in groups of galaxies (O’Sullivan, Ponman, & Collins 2003). Given the low IGM gas density, only a small fraction of it will eventually cool down and fall back again to form stars.

3. RESULTS

The results presented here refer to halos with mass in the range \( 2.5 \times 10^{11} M_\odot \lesssim M_{\text{vir}} \lesssim 1.6 \times 10^{13} M_\odot \), formed at \( z_{\text{vir}} \approx 1.5 \). This lower limit to the virialization redshift allows us to crudely filter out halos hosting disks and irregular galaxies. In our view, late-virialized objects had more time to acquire a substantial angular momentum and are less likely to take on an early-type morphology. This is in keeping with the common notion that stellar populations in disks are on average substantially younger than those in spheroids.

The model, whose main baryonic components and corresponding mass transfer are sketched in Figure 1, allows us to follow the star formation and chemical enrichment histories of spheroidal galaxies, as well as the growth of their central BHs, once the virialization time and the halo mass are given. Interfacing it with the code GRASIL (see Silva et al. 1998 for details), which computes the spectrophotometric evolution from radio to X-ray wavelengths including the dust effects, we get the spectral properties of the spheroidal galaxies as a function of cosmic time. In the application of GRASIL, we
have made the standard assumption that the starburst is highly obscured throughout its duration, with a 1 μm optical depth of molecular clouds, where new stars are born, of $\tau_1 = 30$ for solar metallicity, and scaling linearly with $Z$. As for the dust emissivity index, we have adopted the canonical value of $2$. The results presented in this paper are only weakly sensitive to variations of $\tau_1$ by a factor of several.

The relationship between $\tau_1$ and the optical depth given by equation (17) is not straightforward. The starlight is processed by dust within molecular clouds and reemitted in the far-IR, with a peak at 60–100 μm. At these wavelengths, the optical depth is lower than that at 1 μm by a factor of $\sim 10^{-2}$. The effective optical depth to the intercloud radiation, responsible for the drag exerted on gas clouds as discussed in § 2.3, is therefore $\tau \sim 10^{-2} \tau_1 N_{\text{int}}$, where $N_{\text{int}}$ is the mean number of clouds intersected by a photon (Kawakatu & Umemura 2002). The latter number depends, among other things, on the radial distribution of the clouds within the galaxy. For instance, a population of giant molecular clouds with a typical size of 10 pc, masses of the order of $10^5 M_\odot$, and uniformly distributed in the central $\sim 10$ kpc of the galaxy give $N_{\text{int}} \sim 10$ for $M_{\text{gas}} \approx 10^{12} M_\odot$. As anticipated, for $\tau_0$ (eq. [17]) we explored the range 1–10.

In this section we present some of the most important results, obtained using the reference values of the parameters already discussed and summarized in Table 1. The discussion of the effects of varying the parameters within the allowed ranges is presented in the next section.

Examples of the time evolution of the various components for different virialization redshifts and halo masses are shown in Figures 2 and 3.

### 3.1. Time Delay between Star Formation and QSO Activity

A key result of the model is the prediction of the time delay between the onset of vigorous star formation, at the virialization epoch, and the peak of the QSO activity. The combined action of stellar and nuclear feedbacks, eventually sweeping out the ISM, determines the duration of the star formation.

As shown in Figure 4, the duration of the most active star formation phase decreases with increasing $M_{\text{vir}}$ and $z_{\text{vir}}$. The timescale $T_{100}$ is defined as the galactic age at which the gas mass still available for star formation or accretion, i.e., $M_{\text{cool}} + M_{\text{inf}}$, is reduced to 1% of the initial value. This corresponds approximately to the duration of the star formation burst empirically estimated by Granato et al. (2001) and given by their equation (8). In the redshift range $3 \leq z_{\text{vir}} \leq 6$, $T_{100} \approx 0.5$–1 Gyr for $M_{\text{vir}} \gtrsim 10^{12} M_\odot$, corresponding (see Fig. 5) to $M_{\text{sph}} \approx 2.5 \times 10^{10} M_\odot$. For smaller masses the duration of the actively star-forming phase increases to 1.5–3 Gyr for
\[ M_{\text{sph}} \approx 3 \times 10^9 M_\odot, \text{if virialized at } z_{\text{vir}} \approx 3, \text{ and is even longer if virialization occurs at lower } z. \]

A feeling of the role of the main ingredients of the model in shaping this result can be obtained ignoring the reservoir, so that the inflow onto the central BH is governed by equation (15). The inflow is definitively halted at a time \( t_{\text{out}} \) when the energy injected by the QSO in the ISM equals the binding energy of the gas (e.g., Cavaliere et al. 2002):

\[
\int_0^{t_{\text{out}}} f_h L_K \, dt = M_{\text{vir}} V_{\text{vir}}^2,
\]

where \( V_{\text{vir}} \) is related to the mass inside the virial radius, \( M_{\text{vir}} \), by (Bullock et al. 2001b)

\[
V_{\text{vir}} = 75(1 + z_{\text{vir}})^{1/2} \left( \frac{M_{\text{vir}}}{10^{11} h^{-1} M_\odot} \right)^{1/3} \text{ km s}^{-1}.
\]

Using equations (28) and (18) with \( \exp(-\tau) \ll 1 \) and assuming that the BH growth occurs in a time shorter than the expansion timescale, we obtain

\[
\begin{align*}
\int_0^{t_{\text{out}}} f_h L_K \, dt & = M_{\text{vir}} V_{\text{vir}}^2, \\
& = 7.9 \times 10^9 \frac{h^{4/15}}{f_h} \gamma^{-2/5} (1 + z_{\text{vir}})^{-1/2} \\
& \times \left( \frac{M_{\text{vir}}}{10^{12} M_\odot} \right)^{2/3} \left( \frac{M_{\text{gas}}}{10^{11} M_\odot} \right)^{-3/5} \\
& \approx 1.2 \times 10^9 h^{4/15} f_h^{-2/5} (1 + z_{\text{vir}})^{-1/2} \left( \frac{M_{\text{vir}}}{10^{12} M_\odot} \right)^{-7/30} \text{ yr,}
\end{align*}
\]

where the last step follows from equation (13), normalized to \( M_{\text{vir}}/M_{\text{sph}} \approx 20 \) for \( M_{\text{sph}} = 5 \times 10^{10} M_\odot \), assuming \( M_{\text{gas}} \approx M_{\text{sph}} \).

The final BH mass (except for the effect of later reactivation phases) can be roughly estimated to be \( \sim M_{\text{BH}}(t_{\text{out}}) \):

\[
\begin{align*}
M_{\text{BH}}(t_{\text{out}}) & \approx 4.3 \times 10^7 h^{4/15} f_h^{-2/5} \\
& \times \left( \frac{M_{\text{vir}}}{10^{12} M_\odot} \right)^{2/3} \left( \frac{M_{\text{gas}}}{10^{11} M_\odot} \right)^{2/5} (1 + z_{\text{vir}}) \, M_\odot \\
& \approx 3.2 \times 10^7 h^{4/15} f_h^{-2/5} \left( \frac{M_{\text{vir}}}{10^{12} M_\odot} \right)^{19/15} (1 + z_{\text{vir}}) \, M_\odot \\
& \approx 2.5 \times 10^8 \left( \frac{h}{0.7} \right)^{-1} \left( \frac{f_h}{0.3} \right)^{-2/5} \left( \frac{\sigma}{200 \text{ km s}^{-1}} \right)^{19/5} (1 + z_{\text{vir}})^{-9/10} M_\odot,
\end{align*}
\]

where we have used equations (13) and (33) and have adopted the relationship \( \sigma \approx 0.65 V_{\text{vir}} \) (Ferrarese 2002). This result is in remarkably good agreement with the determination by Gebhardt et al. (2000) of the \( M_{\text{BH}}-\sigma \) relationship. A full comparison of our model predictions with observational data is shown in Figure 6, where the expected spread due to the distribution of virialization redshifts is also illustrated. Note the predicted falloff of \( M_{\text{BH}} \) at low values of \( \sigma \) due to the combined effect of SN feedback, which is increasingly efficient with decreasing halo mass in slowing down the gas infall onto the central BH, and of the decreased radiation drag due to a decrease of \( \tau \) (eq. [17]), which, for these objects, \( \ll 1 \). This steepening of the lower part of the \( M_{\text{BH}}-\sigma \) relationship translates into a flattening of the lower part of the \( M_{\text{vir}}-M_{\text{BH}} \) correlation, in agreement with the results reported in Figure 5 of Ferrarese (2002).

In Figure 7 the predicted BH mass function at \( z = 0 \) is shown against the local BH mass function recently derived by F. Shankar et al. (2004, in preparation) following Salucci et al. (1999). The total local BH mass density is estimated to be \( 5.2 \times 10^5 M_\odot \, \text{Mpc}^{-3} \).

The physics that rules the fast growth of the central BH in massive halos is the key point: its growth is paralleled by an increasing feedback from the nuclear activity. In the most massive halos the quasar feedback eventually removes most of the gas (and dust), leaving the nucleus shining as an optical quasar until the reservoir mass is exhausted on a timescale \( \sim 10^7 \, \text{yr} \) (see Fig. 3). Since then the host galaxy evolves passively and the BH becomes dormant (apart from possible reactivations). The process slows down with decreasing halo mass, as the supernova feedback becomes increasingly important.

3.2. Photometric Properties and Metal Abundances of Spheroidal Galaxies

In small halos (\( M_{\text{vir}} \approx 10^{11} M_\odot \)) the SFR is kept low by stellar feedback, which heats up most of the gas and moves it to lower density outskirts, where the cooling time is very long. Only when the virial mass exceeds a few times \( 10^{11} M_\odot \) are the potential wells deep enough to allow a more effective star formation. Figure 5 shows the dependence on \( M_{\text{vir}} \) of the ratio \( M_{\text{vir}}/M_{\text{sph}} \) for three values of \( z_{\text{vir}} \). Bright galaxies virializing at \( z \lesssim 4 \) are predicted to have \( M_{\text{vir}}/M_{\text{sph}} \gtrsim 40 \). For comparison, McKay et al. (2002) report, for bright galaxies, \( M/L \) values in the SDSS g band of 171 \pm 40 based on the dynamics of satellites and of 270 \pm 35 based on weak-lensing measurements; adopting for the stellar component of early-type galaxies \( M/L \approx 4 - 5.9 \) (Fukugita, Hogan, & Peebles 1998), this translates (for \( L_v \approx L_g \)) into \( M_{\text{vir}}/M_{\text{sph}} \approx 30 - 70 \).

Once the star formation law is specified, the chemical evolution of the galaxy is followed by using classical equations and stellar nucleosynthesis prescriptions (Granato et al. 2001). The predicted mean metallicity (\( Z_v \)) is shown, as a function of the mass in stars, in Figure 8. The average metallicity increases from about half-solar to supersolar with increasing galaxy mass. In particular, for our reference model, the metal content is supersolar for \( M_{\text{vir}} \gtrsim 5 \times 10^{10} M_\odot \), or \( M_{\text{sph}} \gtrsim 10^{10} M_\odot \). It should be stressed that this result refers to the average metallicity of the galaxies, since our model is one-zone. In the central regions, star formation, and consequently metal enrichment, is faster because of the higher densities entailing shorter dynamical times. Adopting densities appropriate for the innermost 10\% of the galaxy mass, the predicted metallicity of the gas is between 3 and 5 times solar, with a general trend at
increasing with $M_{\text{vir}}$. This result compares fairly well with several recent direct chemical measurements of circumnuclear gas in high-$z$ QSOs (e.g., Dietrich et al. 2003).

Another evident trend is the increase with the galaxy mass of the ratio between the abundance of the so-called $\alpha$-elements and that of Fe. Figure 9 shows the mean $[\text{Mg/Fe}]$ ratio, in stars, as a function of the galaxy mass for three different values of $M_{\text{vir}}$ and $z_{\text{vir}} = 4$. Note the sharp break corresponding to the sweeping out of the ISM by the quasar feedback, for large $M_{\text{vir}}$.

objects the star formation activity is essentially ended when Type Ia SNe start to pollute the ISM with Fe.

Both the trend of the global metallicity and the ratio of the $\alpha$-elements to Fe abundance with the galaxy mass are in fair agreement with what is inferred from the spectrophotometric observations of local early-type galaxies. The narrowness and the inclination of the color-magnitude relation of elliptical galaxies hint at a rapid formation process and at a range in metallicity from about half-solar to twice solar (e.g., Bressan, Chiosi, & Fagotto 1994). Furthermore, narrowband observations not only provide evidence for a similar spread in
metallicity (Bernardi et al. 2003b) but also demand a significant enhancement of the \([\text{Mg}]/\text{Fe}\) elements in the more massive elliptical galaxies (Worthey et al. 1994). The observed run of Mg and Fe narrowband indices with the luminosity of the galaxy or with the \([\text{H}/\beta]\) index cannot be reproduced by stellar population models based solely on solar partition of heavy elements (Worthey et al. 1994; Trager et al. 2000a, 2000b; Bernardi et al. 2003b).

It is worth noticing at this level that while the model reproduces, for a relatively broad range of parameter choices, the main trends of the metal abundances inferred from observations of local elliptical galaxies, our predictions cannot yet be directly compared with observations. For example, narrowband indices show a complicated dependence on global metallicity, partition of heavy elements, and age of the stellar populations that renders the adoption of common scaling relations (e.g., Matteucci, Ponzone, & Gibson 1998) quite uncertain. We should thus generate mock catalogs of local early-type galaxies and investigate their spectrophotometric properties by means of adequate spectral synthesis tools. Work in this direction is in progress and will be reported in a subsequent paper.

### 3.3. Luminosity Function of Early-Type Galaxies

In these galaxies the \([\text{K}]\)-band luminosity is a quite good indicator of the mass in stars. In Figure 10 we compare the modeled \([\text{K}]\)-band local luminosity function with the recent determinations by Kochanek et al. (2001) and Huang et al. (2004).
(2003). As already remarked, the solid line includes the contribution of all spheroids with mass in the range $2.5 \times 10^{11} M_\odot \leq M_{\text{vir}} \leq 1.6 \times 10^{13} M_\odot$, formed at $z_{\text{vir}} \geq 1.5$. According to our model, spheroidal galaxies born in halos with $M_{\text{vir}} \leq 2.5 \times 10^{11} M_\odot$ have masses in stars $M_{\text{sph}} \leq 10^9 M_\odot$ and $K$-band magnitudes $\geq -21.3$ (see Fig. 5).

For a more direct comparison with the observational data, the dot-dashed line in Figure 10 shows the predicted local luminosity function for early-type galaxies only. It was obtained subtracting from the results shown by the solid line, the contributions of bulges of Sa and Sb galaxies estimated using the local $B$-band luminosity functions and the bulge-to-total luminosity ratios for galaxies of these morphological types derived by Salucci et al. (1999) and adopting a color $B-K = 4.1$. It should also be noted that the mass in stars for small halos is strongly dependent on the stellar feedback, namely, on the IMF and on the SN efficiency. An increase of the efficiency from 0.1 to 0.3 results in a decrease of mass in stars by a factor of $\approx 3$. This dependence weakens with increasing $M_{\text{vir}}$.

Figure 11 compares the predicted rest-frame $K$-band luminosity function at $z = 1.5$ with the observational determination by Pozzetti et al. (2003), which, at the highest luminosities, is substantially above predictions of the models by Kauffmann et al. (1999), Cole et al. (2000), and Menci et al. (2002).

### 3.4. SCUBA Galaxies and EROs

In large galactic halos the SFR turns out to be very high at high redshift, yielding a quick increase of the metallicity and of the dust mass. The latter is computed by GRASIL as proportional to the product of the gas mass by its metallicity, with a coefficient determined by the condition of a gas-to-dust ratio of 110 for solar metallicity (Silva et al. 1998). Thus, most of the star formation occurs in a dusty environment, so that these galaxies are powerful far-IR/submillimeter sources, highly obscured in the visual and near-IR bands. In Figure 12 we have plotted the model predictions at 850 $\mu$m against the SCUBA counts. The predicted redshift distributions are similar to those shown by Granato et al. (2001). In particular, for a flux density limit of 5 mJy, the model gives a median redshift of 2.2 and an interquartile range of 1.6–3.3, to be compared with $z_{\text{median}} = 2.4$ and the interquartile range of 1.9–2.8 found for the sample of Chapman et al. (2003).

After the ISM has been swept out, galaxies evolve passively. The combination of redshift and aging soon makes them extremely red. We computed the expected contribution to the extragalactic $K$-band counts of spheroidal galaxies in this phase. A comparison (Fig. 13) of the predicted with the observed redshift distribution of galaxies with $K \leq 20$ (Cimatti et al. 2002b) shows that they fully saturate the high-redshift tail of the distribution.

Cimatti et al. (2002a) selected a complete sample of extremely red objects (EROs) $[(R-K_s) \geq 5]$ with $K < 19.2$. More than 60% of the objects have redshift, mostly spectroscopic. On the basis of the spectra, the sample has been subdivided into dusty and nondusty EROs. Their data suggest that there is a significant number of old dust-free elliptical galaxies in place at $z \geq 1$. This result is confirmed by the subsequent analysis by Pozzetti et al. (2003), who found that
the bright end of the $K$-band luminosity function at $z \geq 1$ is dominated by red/early-type galaxies. Our model is consistent with these results (see Fig. 11).

Our expectation of the existence of a significant population of luminous red galaxies at substantial redshifts is also borne out by the results of the very deep near-infrared photometry of the Hubble Deep Field–South with ISAAC on the VLT (Franx et al. 2003) and by follow-up Keck spectroscopy (van Dokkum et al. 2003). Model predictions closely match the surface density ($\mathcal{N} \approx 8$ arcmin$^{-2}$) and the median redshift (2.6) of galaxies with $K_s < 22$ and $(J - K_s) > 2.3$ (Franx et al. 2003).

4. DISCUSSION

To highlight the dependence of galaxy properties on their halo masses, we make reference to three characteristic values, namely, $M_{\text{vir}} \approx 10^{11.4}, 10^{12.4}$, and $10^{13.4}$, referred to as low, intermediate, and high masses, respectively.

The clumping factor $C$ affects only the evolution of high and intermediate masses, where a decrease of $C$ from 20 to 1 strongly inhibits the capability of gas to quickly cool down, form stars, and feed the BH. For large galactic masses, the star formation activity and the growth of the BH, rather than being confined to the first ~1 Gyr after virialization, continue for a time comparable to the Hubble time. However, the final ratio $M_{\text{BH}}/M_{\text{sph}}$ is almost the same, both masses ultimately depending on the SFR. Conversely, at lower masses the collapse becomes increasingly limited by the dynamical time even when $C = 1$.

The supernova efficiency $\epsilon_{\text{SN}}$ has some effect on the evolution at all masses: $M_{\text{sph}}$, $\langle Z_s \rangle$, and $M_{\text{BH}}/M_{\text{sph}}$ decrease with increasing $\epsilon_{\text{SN}}$, while $T_{100}$ increases. Differences are modest at high masses but become dramatic at low masses. At $M_{\text{vir}} \approx 2 \times 10^{11} M_\odot$ and $\epsilon_{\text{SN}} \approx 0.2$, the evolution of the SFR can show damped oscillations at early epochs ($T \lesssim 1$ Gyr).

Conversely, the QSO efficiency, $\epsilon_{\text{QSO}}$, influences only intermediate and large masses, where an increase of this parameter determines a shorter active star-forming phase and lower values of $\langle Z_s \rangle$, $M_{\text{sph}}$, and $M_{\text{QSO}}$, while the ratio $M_{\text{BH}}/M_{\text{sph}}$ is little affected. An increase of the radiative efficiency $\epsilon$ has a very similar effect.

When the accretion is Eddington-limited (or, at most, mildly super-Eddington), $T_{100}$, $\langle Z_s \rangle$, and $M_{\text{sph}}$ decrease with increasing seed BH mass: less e-folding times are necessary to produce substantial effects on the environment. Typically, $M_{\text{sph}}$ decreases by 30% when the seed mass is increased from $10^3$ to $10^4 M_\odot$, but the ratio $M_{\text{BH}}/M_{\text{sph}}$ decreases by less than 10%. As already remarked, the development of low masses is instead weakly affected by the BH feedback. Conversely, if substantially super-Eddington accretion is allowed, the model is quite insensitive to the precise choice of the seed BH mass.

The ratio between QSO and SN feedback is an increasing function of the virial mass of the galaxy (see Fig. 14). For low-mass galaxies the integrated effect of the QSO is almost negligible compared to that of SNe but takes over (typically by a factor of a few) for intermediate masses ($M_{\text{vir}} \sim 10^{12.4} M_\odot$).
and dominates (by a factor of \(\gtrsim 10\)) at high masses \((M_{\text{vir}} \sim 10^{13.4} M_\odot)\). Note that the QSO effect usually increases exponentially with time, while that of SNe increases more slowly. Thus, the instantaneous QSO effect becomes dominant, if ever, only a few e-folding times before the maximum of QSO activity.

As mentioned at the beginning of §3, our definition of a galactic halo associated with a spheroidal galaxy is rather crude. However, the successful comparison with the observed population properties of spheroidal galaxies (Figs. 7, 10, 11, 12, 13, and 15) gives us some confidence about the meaningfulness of the criterion we have adopted.

Although we have not addressed the details of the formation of disk (and irregular) galaxies, we envisage them as associated primarily with halos virializing at \(z_{\text{vir}} \lesssim 1.5\), which have incorporated, through merging processes, a large fraction of halos less massive than \(2 \times 10^{11} M_\odot\) virializing at earlier times. These low-mass halos virialized at early times may become the bulges of late-type galaxies.

5. SUMMARY AND CONCLUSIONS

We have presented a detailed, physically grounded model for the early coevolution of spheroidal galaxies and of active nuclei at their centers. The model is based on very simple recipes that can be easily implemented. In summary, we start from the diffuse gas within the DM halo falling down into the star-forming regions at a rate ruled by the dynamic and cooling times. Part of this gas condenses into stars, at a rate again controlled by the local dynamic and cooling times. However, the gas also feels the feedback from supernovae and from active nuclei, heating it and possibly expelling it from the potential well. In addition, the radiation drag on the cold gas decreases its angular momentum, causing an inflow into a reservoir around the central BH. Viscous drag then causes the gas to flow from the reservoir into the BH, increasing its mass and powering the nuclear activity.

In the shallower potential wells (corresponding to lower halo masses and, for given mass, lower virialization redshifts), the supernova heating is increasingly effective in slowing down the star formation and driving gas outflows, resulting in an increase of star/DM ratio with increasing halo mass. As a consequence, the star formation is faster within the most massive halos, and the more so if they virialize at substantial redshifts. Thus, in keeping with the proposition by Granato et al. (2001), physical processes acting on baryons effectively reverse the order of formation of galaxies compared to that of DM halos.

A higher SFR also implies a higher radiation drag, resulting in a faster loss of angular momentum of the gas (Umemura 2001; Kawakatu & Umemura 2002; Kawakatu et al. 2003) and, consequently, a faster inflow toward the central BH. In turn, the kinetic energy carried by outflows driven by active nuclei through line acceleration is proportional to \(M_{\text{BH}}\)\(^2\) (Murray et al. 1995), and this mechanism can inject in the ISM a sufficient amount of energy to unbind it. The time required to sweep out the ISM, thus halting both the star formation and the BH growth, is again shorter for larger halos. For the most massive galaxies \((M_{\text{vir}} \gtrsim 10^{12} M_\odot)\) virializing at \(3 \lesssim z_{\text{vir}} \lesssim 6\), this time is less than 1 Gyr, so that the bulk of the star formation may be completed before Type Ia supernovae can significantly increase the Fe abundance of the ISM; this process can then account for the \(\alpha\)-enhancement seen in the largest galaxies.

The interplay between star formation and nuclear activity determines the relationship between the BH mass and the mass, or velocity dispersion, of the host galaxy, as well as the BH mass function. As illustrated by Figures 6 and 7, the model predictions are in excellent agreement with the observational data. A specific prediction of the model is a substantial steepening of the \(M_{\text{BH}}-\sigma\) relation for \(\sigma \lesssim 150\) km s\(^{-1}\): the mass of the BH associated with less massive halos is lower than expected from an extrapolation from higher masses because of the combined effect of SN heating, which is increasingly effective with decreasing galaxy mass in hindering the gas inflow toward the central BH, and decreased radiation drag (see eq. [15], with \(\tau \ll 1\)).

Coupling the model with GRASIL (Silva et al. 1998), the code computing in a self-consistent way the chemical and spectrophotometric evolution of galaxies over a very wide wavelength interval, we have obtained predictions for the

![Graph showing predicted star formation in spheroids and BH mass accretion rates per unit volume as a function of redshift.](image-url)
submillimeter counts and the corresponding redshift distributions, as well as for the redshift distributions of sources detected by deep K-band surveys, which proved to be extremely challenging for all the current semianalytic models. The results, shown by Figures 12 and 13, are again very encouraging.

A discussion of the evolutionary properties of AGNs predicted by the present model is deferred to a future paper, where we will address, among other things, the complex issue of how the bolometric luminosity produced by accretion processes shows up in different electromagnetic bands. We note, however, that the analysis by Granato et al. (2001), who approached the problem the other way round, i.e., inferred the formation history of spheroidal galaxies and of galactic bulges from the observed epoch-dependent luminosity function of quasars, got results in detailed agreement with those presented here. Thus, an at least qualitative agreement of the present model with the data on AGNs seems to be ensured. This is confirmed by the successful comparison of our model predictions for the relationship among the BH mass and the velocity dispersion of the host galaxy (Fig. 6) and for the local BH mass function (Fig. 7). Furthermore, the predicted history of global accretion rate onto the central BHs (Fig. 15), which, in our scheme, is directly proportional to the history of the bolometric luminosity density produced by AGNs, with its peak in the redshift range 2–3, is nicely consistent with the results of optical surveys (see, e.g., Fig. 8 of Fan et al. 2003).

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