Optimal Positioning of PMUs for Fault Detection and Localization in Active Distribution Networks

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Abstract—This paper considers the problem of fault detection and localization in active distribution networks using Phasor Measurement Units (PMUs). The proposed algorithm consists in computing a set of weighted least squares state estimates whose results are used to detect, characterize and localize the occurrence of a fault. Moreover, a criteria to minimize the number of PMUs required to correctly perform the proposed algorithm is defined. Such a criteria, based on system observability conditions, allows the design of an optimization problem to set the positions of PMUs along the grid, in order to get the desired fault localization resolution. The performances of the strategy are tested via simulations on a benchmark distribution system.

Index Terms—Fault Detection and Localization, State Estimation, Phasor Measurement Units, Active Distribution Networks.

I. INTRODUCTION

Nowadays, Distribution Networks (DNs) are operated in a very different way with respect to their original design [1], [2]. The massive use of Distributed Energy Resources (DERs) causes non-unidirectional power flows with a dramatic impact on the protection scheme behaviour. The main issues can be classified as [3]: binding of protection, false tripping or sympathetic tripping, islanding problems, loss of coordination, auto-recloser problems. Literature proposes several protection schemes to address these problems. They are generally based on: limitation of DER capacity [4], use of distance protections and reconfiguration strategies [5], in line fault current limiters [6], adaptive protection schemes. Specifically, the latter assumes that relays are able to communicate each others.

Recent studies investigate the possibility of merging protection devices, distributed measurements and communication systems, in order to asses in real-time the system state and take decisions based on a complete view of the grid operating conditions. Such an approach opens to a plethora of applications both in operation and planning e.g. state estimation [7]–[9], system and load modelling, event detection and localization, optimal positioning of measurement devices and switches [10].

The focus of this paper is on fault detection and localization (FDL) in Active Distribution Networks (ADNs) using the measurements provided by PMUs. Recent literature proposes several interesting solutions. In [11], FDL is realized for ADNs by placing PMUs at each node of grid, allowing the localization of the faulted line. In [12], FDL is realized for radial passive DNs, allowing the localization of a fault between a couple of PMUs. This method requires pseudo-measurements that introduce uncertainty in the provided results. The technique developed in [13] allows FDL in radial passive DNs. In this case, PMUs are placed at each node of the grid. In [14], FDL is realized for ADNs by placing PMUs according to a suitable positioning criteria. With this method, localization results are approximated. In [15] FDL is carried out for ADNs by placing PMUs only to nodes with DERs. Here pseudo-measurements are used and localization regards the identification of the faulted portion of the grid. Reference [16] proposes a method for FDL for ADNs by placing PMUs at the point of connection with the main grid, at all nodes with DERs, and at a set of nodes defined by optimal positioning. In this case, localization results are approximated, but estimation errors are characterized by a probabilistic model.

The present paper proposes a method of FDL in radial ADN. The approach is similar to the one of [11], where a set of Weighted Least Squares (WLS) State Estimators (SEs) are performed considering all possible network topologies for each fault location (each line). Differently from [11], the objective is to perform FDL without placing PMUs at each node, but looking for the minimum necessary number of such devices. With this aim, a PMUs Optimal Positioning Algorithm (OPA) is developed based on observability conditions, suitably defined according to the properties of the adopted estimation procedure. The result of the OPA is the partition of the grid into connected lines clusters, where the proposed Fault Detection and Localization Algorithm (FDLA) is able to localize the fault. The advantage of the method is that the OPA can be customized to obtain a desired localization resolution, in terms of number and composition of the lines clusters.

The paper is organized as follows: Section III reports the general problem formulation; Section IV introduces the basic state-estimation based method; Section V defines observability and fault localizability conditions; Section VI provides the FDLA and the PMUs-OPA; Section VII describes the case study; Section VIII presents simulation results; Section IX...
reports the conclusions of the paper.

Notation. $\hat{U}^{abc}$ is a triplet vector $[\hat{U}^a \ \hat{U}^b \ \hat{U}^c]$. Real and imaginary components of a generic phasorial vector $\hat{U}$ are indicated as $U_{re}$ and $U_{im}$. $b_i$ indicates the $i$-th node of the grid; $\rho(b_i)$ is the degree of $b_i$, defined as the number of nodes connected with $b_i$.

II. PROBLEM FORMULATION

The purpose of this work is to detect and locate a short-circuit event whenever it occurs in a Medium Voltage (MV) distribution grid. The following assumption are made: 1) distribution network is radial and composed by $n$ nodes and $m$ lines; 2) the network admittance matrix $Y$ is known; 3) $d < n$ nodes are monitored via PMUs, and $n - d$ nodes are not; $\mathcal{I}_d$ is the set of the indices of the $d$ monitored nodes; 4) at each monitored node $b_i$ ($i \in \mathcal{I}_d$), the measurement system acquires the measures of three phase-to-ground voltages $V_{i}^{abc}$ and three phase node injected currents $I_{i}^{abc}$.

Following these assumptions, state vector $x$ is defined as

$$x = [V_{1, re}^{abc} \ ... \ V_{n, re}^{abc} \ V_{1, im}^{abc} \ ... \ V_{n, im}^{abc}]^T \in \mathbb{R}^N,$$

with $N = 6n$, and measurements vector $z$ is defined as

$$z = [z_V, z_I]^T \in \mathbb{R}^D, \ \ D = 12d,$$

$$z_V = [\ ... \ V_{re,i}^{abc} \ ... \ V_{im,i}^{abc} \ ... ], \ i \in \mathcal{I}_m^d,$$

$$z_I = [\ ... \ I_{re,i}^{abc} \ ... \ I_{im,i}^{abc} \ ... ], \ i \in \mathcal{I}_m.$$

PMUs introduce noise in the measurements, supposed to be additive, zero-mean and with a known covariance matrix $R$. Refer to [7] to analyze the validity of assuming that PMUs provide voltage and current in Cartesian coordinates.

The relation among $x$ and $z$ has the form:

$$z = H x + v,$$

where $H$ is a $D \times N$ matrix and $v$ is the measurement noise.

Matrix $H$ has the form:

$$H = \begin{bmatrix} H_V \\
H_I \end{bmatrix},$$

where $H_V \in \mathbb{R}^{6d \times N}$ relates the state vector $x$ with the component $z_V$ of $z$, and $H_I \in \mathbb{R}^{6d \times N}$ relates $x$ with the component $z_I$ of $z$. Therefore, $H_V$ is composed by zeros and ones to select the voltages directly measured by PMUs, $H_I$ is computed using the network admittance matrix $Y$, by removing from the following matrix

$$H_I = \begin{bmatrix} Re\{Y\} & -Im\{Y\} \\
Im\{Y\} & Re\{Y\} \end{bmatrix},$$

the rows corresponding to non-monitored nodes.

Under the mentioned assumptions, the objectives of the paper are: 1) to develop a Fault Detection and Localization Algorithm (FDLA) that, using the available PMUs measurements, is able to: 1.a) detect the occurrence of a fault; 1.b) characterize the fault, i.e. state in which phases it has occurred; 1.c) localize the fault; 2) to define the minimum number $d$ and the optimal positions of PMUs along the grid, that allow the FDLA to work correctly and obtain a desired fault localization resolution.

III. BASIC ALGORITHM

In this section, we provide the core algorithm of the general FDLA, which will be illustrated in Section [V]. The adopted approach is similar to the one proposed in [11]. The idea is that a fault on a line results in a sudden addition of one virtual node in the network, that is between two real nodes and absorbs the fault current. Therefore, $m + 1$ parallel WLS SEs [7] are realized, each returning, at any measurement time step, the estimate $\hat{x}^k$, $k = 0, 1, \ldots, m$. Estimate $\hat{x}^0$ is obtained by applying WLS without adding virtual nodes, as follows:

$$\hat{x}^0 = \left( H^T R^{-1} H \right)^{-1} H^T R^{-1} z.$$

Estimates $\hat{x}^k$, with $k > 0$, are computed by applying the WLS equation to the network extended with a virtual node placed in the middle of line $k$. Specifically, for each $k = 1, 2, \ldots, m$: a) the state vector $x$ is extended with the voltage of the virtual node $b_{n+1}$:

$$x_k = [V_{1, re}^{abc} \ ... \ V_{n+1, re}^{abc} \ V_{1, im}^{abc} \ ... \ V_{n+1, im}^{abc}]^T;$$

b) the admittance matrix of the extended grid $Y^k$ is computed and used to obtain the corresponding measurement matrix $H^k$;

$$z_k = \left( z - H^k \hat{x}^k \right)^T R^{-1} \left( z - H^k \hat{x}^k \right),$$

each SE is associated to the Weighted Measurement Residual (WMR),

$$w_k = (z - H^k \hat{x}^k)^T R^{-1} (z - H^k \hat{x}^k),$$

to evaluate the estimate accuracy.

As in [11] the idea is that, in a no-fault scenario, this process will return $m + 1$ estimates with comparable WMRs, since all SEs use a correct model of the grid topology, in which the virtual node will be suitably estimated to do not absorb any current. Differently, if a fault occurs for example on line $\alpha$, only the $\alpha$-th SEs will be based on a topology close to real one. This implies that all WMRs, excepting for the $\alpha$-th, will sudden increase. In particular, the $0$-th WMR will always increase after a fault, since the grid topology model used by the corresponding SE does not include any virtual node.

Therefore, the Basic Algorithm (BA) consists in the following steps: 1) compute the $m + 1$ WLS estimates; 2) detect the fault by registering an anomalous variation of $w^0$; 3) localize the fault by selecting the minimum WMR 4) characterize the fault by computing the estimated currents injected into the virtual node from the selected best estimate $\hat{x}^\alpha$ (using the corresponding admittance matrix: $Y^\alpha \hat{x}^\alpha$) and registering which of the three phase currents are different from zero.

Unfortunately, the BA does not generally works in the hypothesis that not all nodes are monitored by PMUs (i.e. $d < n$).
First, WLS estimates (8) and (10) cannot be implemented independently of the number and the position of PMUs, since this is possible only if the network observability is kept [17].

Second, even if estimates (8)−(10) are computable, localization cannot correctly carried out just selecting the minimum WMR.

This fact is clarified in the following section, where, moreover, the conditions to keep the network observability with a reduced number of PMUs are provided.

IV. GRID OBSERVABILITY AND FAULT LOCALIZABILITY

Following the definitions in [17], an electrical network is observable if, in the no noise ideal case, it is possible to calculate the voltage at all nodes starting from the available measurements. With the formulation used in this paper, we have observability if and only if matrix $H$ is full row-rank. It is well known that, if this does not hold true, matrix $H^T R^{-1} H$ in (8) is not invertible and, thus, WLS cannot be used.

Obviously, if all nodes are equipped with a PMU, all nodal voltages are directly known ($H_{V}$ is the identity matrix in $\mathbb{R}^{6n}$), and the network is observable. However, our objective is to reduce as much as possible the number of PMUs.

By applying simple electrotechnical computations it is easy to prove the following lemma.

Lemma 1: A radial grid with $n$ nodes and $m$ lines is observable if, for all $i = 1, 2, \ldots, n$:

a. if $\rho(b_i) > 1$, $b_i$ has at most one adjacent non-monitored node;

b. if $\rho(b_i) = 1$ and $b_i$ is non-monitored, then $b_i$ is adjacent to a monitored node.

Lemma 1 gives us the conditions to allow (8) to be implemented. However, we also need to implement estimates (10). Therefore, the observability of the $m$ networks extended with a virtual node placed in the middle of each line is required. Indeed, this will imply that matrices $H^{k}$ are full row-rank and, thus, $H^{k} R^{-1} H^{k}$ are invertible. The following theorem provides the conditions to satisfy such a requirement.

Theorem 1: Given a radial grid with $n$ nodes and $m$ lines, the extended grid with $n + 1$ nodes and $m + 1$ lines, obtained by adding the virtual node $b_{n+1}$ in the middle of one of the $m$ lines is observable if and only if, for all $i = 1, 2, \ldots, n$:

a. whatever given a couple of adjacent nodes, at least one of the two is monitored;

b. if $\rho(b_i) = 1$, $b_i$ is monitored.

Proof. By definition, $\rho(b_{n+1}) = 2$ and it is non-monitored. Thus, condition a. of Lemma 1 implies that, by positioning the virtual node in the middle of any line of the grid, it has to be adjacent to a monitored node. This is verified if and only if condition a. of Theorem 1 holds true. Condition b. of Lemma 1 is a necessary and sufficient for condition b. of Theorem 1.

Indeed, if the virtual node is located at a line incident to a node $b_i$ with degree $\rho(b_i) = 1$, we have a non-monitored node (the virtual one) with a node with degree $\rho = 1$. Thus, condition b. of Theorem 1 is verified only if condition b. of Lemma 1 holds true, and viceversa. The sufficient condition of Theorem 1 is, therefore, proved. The necessary condition can be proved by contradiction showing that there exist counterexamples when one of the two conditions is not satisfied.

Roughly speaking, Theorem 1 states that the terminal nodes of each feeder must be equipped with a PMU and there cannot be two adjacent nodes without PMU.

With Theorem 1 we have the conditions to make estimates (8)−(10) implementable and correctly working. Next Theorem 2 provides a key property of such estimates that makes a fault unlocalizable within specific portions of the grid by using the WLS estimates. Such grid portions are defined as follows.

Definition 1: An Unlocalizable Fault Cluster (UFC) is a connected portion of the grid, defined as a subset of lines $C \in \{1, 2, \ldots, m\}$, such that:

- if $C$ is composed by one single line, then this line connects two monitored nodes (single-line UFC);
- if $C$ is composed by more than one line, then: a) it is connected to the rest of the grid through a non-monitored node; and b) whatever given one line in $C$, it connects a monitored node to a non-monitored one.

Theorem 2: Consider a radial grid with $n$ nodes and $m$ lines and suppose that conditions a. and b. of Theorem 1 are satisfied. Whatever given an UFC $C$ composed by more than one line, and two estimates $\hat{z}^i$ and $\hat{z}^f$, computed by (10), associated to virtual nodes placed on two lines belonging to $C$, then, for all $z \in \mathbb{R}^{6}$, $w^i = w^f$.

We cannot provide a formal proof of Theorem 2 in this paper for space lacking.

Theorem 2 implies that, using the BA introduced in Section III we will obtain a set of grid clusters associated to WLS estimates with identical WMRs. Therefore, an idea could be to select the minimum WMR to discover in which UFC the fault has occurred. Unfortunately, differently on what happens when all nodes are monitored [17], in the present case, putting a virtual node in the middle of one line within the correct UFC does not necessarily result in the best representation of the faulted grid, excepting for the unlikely case of occurrence of the fault at the exact half of the line.

In particular, this happens when the fault occurs within a single-line UFC. We verified that, in these cases, if the fault occurs sufficiently far from the middle of the line, the WMR of any adjacent UFC could result to be smaller than the one associated to the right one. This can be explained by the fact that, in this scenario, both the WLS SEs associated to the right and the wrong adjacent UFCs use an approximated model of the grid topology. However, differently from the right single-line UFC, the wrong adjacent UFCs could include non-monitored nodes close to the real fault location, which gives to the estimation process a degree of freedom that allows to compensate the modelling error.

To fix this problem, the SEs associated to single-line UFCs should be put in condition to work with the same degrees of freedom of those associated to the adjacent UFCs. The solution is to add fake nodes in the middle of all single-line UFCs, which, by Definition 1, are all lines between two monitored nodes, and apply the approach to the so obtained extended
That realizes the minimum variation with respect to the average. Therefore, using also B. PMUs-OPA, which consists in the solution of the following non-monitored as much as possible. Finally, function characterizeFault getsCluster the faulted UFC* is determined by selecting the WMR if fork nodes (nodes with degree \( w \)) are not adjacent to another fork node, to apply (15)

\[
\begin{align*}
\text{Algorithm 1: FDLA} \\
[r, \{C^l\}] & \leftarrow \text{getUFC(Grid Topology, } T_m^d) \; ; \\
\text{for all} \; t & \; \text{measurement time step} \; t; \\
& \text{collect PMUs measurements } z(t); \;
& \text{compute } \hat{x}^t(t) \; \text{and } w^d(t), \; 0, 1, 2, \ldots, r; \;
& \text{if } |w^0(t) - w^0(t - 1)| > th_w \; \text{then} \; \\
& \quad \rightarrow \text{Fault Detected}; \\
& \quad \alpha = \min \{w^0(t) - \mu^w\}; \\
& \quad C^* = \text{getUFC}(\alpha) \rightarrow \text{Faulted Cluster}; \\
& \quad \text{characterizeFault}(\hat{x}^t) \rightarrow \text{Fault Type}; \\
\end{align*}
\]

mixed-integer optimization problem:

\[
\begin{align*}
\min_{\gamma} c^T \gamma \\
\text{s.t.} \quad & A \gamma \geq f \\
& \gamma_i = 1 \quad \text{if } \rho(b_i) = 1 \quad \forall i = 1, 2, \ldots, n
\end{align*}
\]

where: \( \gamma = [\gamma_1 \gamma_2 \cdots \gamma_n]^T \) is a vector of binary variables representing if the \( i \)-th node is monitored \( \gamma_i = 1 \) or not \( \gamma_i = 0 \); matrix \( A \in \mathbb{R}^{n \times n} \) describes the grid topology:

\[
A_{i,k} = \begin{cases} 
\rho(b_i) & \text{if } i = k \\
1 & \text{if } (i \neq k) \land (b_i \text{ and } b_k \text{ connected}) \\
0 & \text{otherwise} 
\end{cases}
\]

\( f \in \mathbb{R}^n \) with \( f_i = \rho(b_i) \); and \( c \in \mathbb{R}^n \) collects cost function weights \( c_i \).

Constraints in (13) imposes conditions of Theorem 1. Cost function weights \( c_i \) can be all equal to one if the unique objective is to minimize the number \( d \) of required PMUs, or defined as follows, if the objective is also to maximize the number \( r \) of UFC*’s:

\[
c_i = \begin{cases} 
1 & \text{if } \rho(b_i) \leq 2 \\
n \cdot \rho(b_i) & \text{if } \rho(b_i) > 2
\end{cases}
\]

In this way, optimization will strongly penalize the positioning of PMUs on fork nodes. Finally, if the user desires to distinguish two specific portions of the grid, for example to perform protections, we can state that they should be divided by a fork node \( b_c \). If this holds true, and \( b_c \) is not adjacent to another fork node, to apply (15) is enough to obtain the desired grid portioning. Differently, if \( b_c \) is adjacent to other fork nodes, we need to modify (15) by setting equal to one the weights \( c_i \) corresponding to all the fork nodes adjacent to \( b_c \).

B. PMUs-OPA

The localization resolution of the FDLA depends on the number \( r \) of UFC*’s. According to Definition 2, \( r \) is augmented if fork nodes (nodes with degree \( > 2 \)) are forced to be non-monitored as much as possible. Therefore, using also Theorem 1 to assure grid observability, we can define the PMUs-OPA, which consists in the solution of the following mixed-integer optimization problem:

\[
\begin{align*}
\min c^T \gamma \\
\text{s.t.} \quad & A \gamma \geq f \\
& \gamma_i = 1 \quad \text{if } \rho(b_i) = 1 \quad \forall i = 1, 2, \ldots, n
\end{align*}
\]

where: \( \gamma = [\gamma_1 \gamma_2 \cdots \gamma_n]^T \) is a vector of binary variables representing if the \( i \)-th node is monitored \( \gamma_i = 1 \) or not \( \gamma_i = 0 \); matrix \( A \in \mathbb{R}^{n \times n} \) describes the grid topology:

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0 & \text{otherwise} 
\end{cases}
\]

\( f \in \mathbb{R}^n \) with \( f_i = \rho(b_i) \); and \( c \in \mathbb{R}^n \) collects cost function weights \( c_i \).

Constraints in (13) imposes conditions of Theorem 1. Cost function weights \( c_i \) can be all equal to one if the unique objective is to minimize the number \( d \) of required PMUs, or defined as follows, if the objective is also to maximize the number \( r \) of UFC*’s:

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n \cdot \rho(b_i) & \text{if } \rho(b_i) > 2
\end{cases}
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In this way, optimization will strongly penalize the positioning of PMUs on fork nodes. Finally, if the user desires to distinguish two specific portions of the grid, for example to perform protections, we can state that they should be divided by a fork node \( b_c \). If this holds true, and \( b_c \) is not adjacent to another fork node, to apply (15) is enough to obtain the desired grid portioning. Differently, if \( b_c \) is adjacent to other fork nodes, we need to modify (15) by setting equal to one the weights \( c_i \) corresponding to all the fork nodes adjacent to \( b_c \).

VI. CASE STUDY

The performances of the proposed solution are evaluated by simulations on a modified version of the Cigré benchmark MV distribution network [13], shown in Fig. 1. There are 17 nodes, divided into two feeders starting from nodes 1 and 12, from which the neutral is earthed through grounding transformers. The grid has been implemented with the option of connecting
the neutral directly to the ground or by Petersen coil. Loads are delta-connected and represent secondary substations. Finally, three 400 kVA Diesel generators have been placed at nodes 5, 10 and 13.

Figure 1 reports the positions of PMUs, defined using the OPA. Figure 2 compares the results of two versions of the optimization. In Case A, cost function weights are all equal to 1. Thus, the unique objective is to minimize the number of PMUs, that results to be \( d = 11 \). With this solution, we have only 3 UFC*s, as shown in Figure 2(left). Therefore, the resulting fault localization resolution is not satisfactory. In particular, the entire Feeder 2 is included in the same UFC*, as shown in Figure 2(left). Therefore, the localization resolution is not satisfactory. In Case B, optimization cost function weights \([15]\) are adopted to pursue the dual objective of minimizing the number \( d \) of PMUs and maximizing the number \( r \) of UFC*s. In this case, we obtain \( d = 12 \), just one more with respect to Case A, whereas \( r = 7 \). Therefore, localization resolution, that we can measure by the number of UFC*s, is strongly augmented. In the simulation tests we adopt the results of Case B.

Simulations are realized on the Matlab platform. Measurements are simulated by adding noise to magnitude and phase of voltages and currents. Noises are assumed to be white and Gaussian, with the following standard deviations: \( 1.6 \cdot 10^{-3} \)\% and \( 4 \cdot 10^{-1} \)\% for voltage and current magnitudes, respectively; \( 5.1 \cdot 10^{-5} \text{rad} \) and \( 5.8 \cdot 10^{-3} \text{rad} \) for voltage and current phases, respectively. According to the capability of real PMUs, measures are sampled with a time step of 20 ms. Since measurements are in polar coordinates, a transformation to rectangular coordinates is applied \([7]\). All these assumptions and noise standard deviations are the same of \([11]\).

VII. RESULTS

The FDLA has been tested on a set of 8 different short-circuit events, with fault resistances set to 100 \( \Omega \). For single-phase faults, both the cases with neutral connected to the ground directly and by Petersen coil. The latter will be indicated as 1-phase-p, the former with 1-phase-g. In the FDLA, detection threshold \( t_{h_{w}} \) is equal to the 120\% of the maximum variation of \( w_{b} \) registered in pre-fault conditions.

The results of a Monte Carlo analysis, with 100 noise realizations for each fault, are reported in Table I. The table shows the number of occurrences of the following cases: (D-L) fault correctly detected and located; (D-nL) fault detected, but not correctly located; (n-D-L) fault not detected, but correctly located; (n-D-nL) fault not detected and not correctly located.

It results that the performances of the proposed method are more than satisfactory. Indeed, all detection and localization percentages are close to 100\%. Worst results are registered for single-phase faults, especially in the case of neutral connection by Petersen coil. This is not surprising since, in these scenarios, fault currents are low and measurement noise may cause difficulties both in detection and in localization. We remark that, in all the cases, characterization has resulted to be correct. Moreover, perfect performances has been verified with lower levels of the measurement noises.

Figures 3–5 show the detailed results of three specific fault occurrences, one 3-phase, one 2-phase and one 1-phase. In particular, the figures report the WMRs of seven SE, one (the 0-th) associated to the grid without virtual nodes, used for detection, and six to the UFCs. The same colors reported in Figures 3–5 show the detailed results of three specific fault occurrences, one 3-phase, one 2-phase and one 1-phase. In particular, the figures report the WMRs of seven SE, one (the 0-th) associated to the grid without virtual nodes, used for detection, and six to the UFCs. The same colors reported in Figures 3–5 and 7 are adopted to identify the different UFCs. Notice that the gray UFC is constituted by the two primary transformers, as thus it is not considered. Moreover, the figures report the estimated current injections on the virtual node used to
characterize the fault.

![WMR and Current Injection Graphs](image)

Fig. 3. WMRs (top) and estimated current injections at the virtual node (bottom) for a 3-phase fault at 50% of Line 4-5 (Cluster 2).

![WMR and Current Injection Graphs](image)

Fig. 4. WMRs (top) and estimated current injections at the virtual node (bottom) for a 2-phase fault at 75% of Line 9-10 (Cluster 3).

![WMR and Current Injection Graphs](image)

Fig. 5. WMRs (top) and estimated current injections at the virtual node (bottom) for a 1-phase-g fault at 25% of Line 7-8 (Cluster 3).

In all the cases, we can observe that at 0.5 s, when the faults occur, the 0-th WMR suddenly increases allowing the detection. In the same time, the estimated current injections on the virtual node, corresponding to the faulted phases (phase A and B in the 2-phase case, and phase A in the 1-phase case), becomes different from zero, allowing the fault characterization. Finally, we can observe that the WMR associated to the right cluster result to be the minimum immediately after the fault starting time, allowing a correct localization.

VIII. CONCLUSIONS

The objective of this paper was to determine the minimum number of PMUs required to realize a state estimation-based algorithm able to detect and localize faults in radial Active Distribution Networks (ADNs). We discovered that, based on the positions of PMUs along the grid, faults are localizable within portions of the grid defined as clusters of lines. As such clusters are smaller, as higher is the fault localization resolution that we can obtain. As a consequence, an Optimal Positioning Algorithm (OPA) has been developed, by which it is possible to simultaneously minimize the number of required PMUs and maximize the number of lines clusters. The approach has been successfully applied to a benchmark MV distribution network.

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