Electroweak Symmetry Breaking
and Extra Dimensions

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Abstract. The electroweak symmetry may be broken by a composite Higgs which arises naturally as a bound state of the top quark if the standard model gauge fields and fermions propagate in extra dimensions. The top quark mass and the Higgs mass can be predicted from the infrared fixed points of the renormalization group equations. The top quark mass is in good agreement with the experimental value, and the Higgs boson mass is predicted to be $\sim 200$ GeV. The bounds on the compactification scale can be quite low if all standard model fields propagate in the same extra dimensions due to the momentum conservation in extra dimensions. The current lower limits are about 300 GeV for one extra dimension and 400-800 GeV for two extra dimensions. The future collider experiments may either discover the Kaluza-Klein (KK) states of the standard model fields or raise their mass limits significantly. There may also be some other light bound states which could be observed at upcoming collider experiments.

In the standard model (SM), the electroweak symmetry is broken by a nonzero vacuum expectation value of a fundamental scalar Higgs field. However, the squared mass of a scalar field receives quadratically divergent radiative corrections, therefore suffers from the hierarchy problem if the cutoff is much higher than the weak scale. Large extra dimensions accessible to the gravitons can remove the large hierarchy between the weak scale and the Planck scale [1,2], but provide no understanding why the electroweak symmetry is broken, i.e., why there is a scalar field with the particular quantum numbers and a negative squared mass. We will show that if the SM fields also propagate in some extra compact dimensions of the size $\sim \text{TeV}^{-1}$, a composite Higgs field can arise naturally as a bound state of the SM fermions. This is because gauge interactions are non-renormalizable in more than four dimensions, therefore rapidly become strong at energies not far above the compactification scale. The strong SM gauge interactions then can form bound states from SM fermions. In particular, there is a Higgs bound state composed of the top quark and its Kaluza-Klein (KK) excitations. This provides the explanation of the Higgs quantum numbers [3,4].

Consider a one generation model in which the SM gauge fields and the third generation fermions live in six dimensions, with two of the six dimensions compactified at a scale $M_c \sim \text{TeV}^{-1}$. The theory is non-renormalizable hence needs a physical
cutoff $M_q$. A possible candidate is the scale of quantum gravity, which is determined by the sizes of the extra dimensions accessible to the gravitons [1,2]. In six dimensions, there exist four-component chiral fermions. We assign $SU(2)_W$ doublets with positive chirality, $\mathcal{Q}_+, \mathcal{L}_+$, and $SU(2)_W$ singlets with negative chirality, $\mathcal{U}_-, \mathcal{D}_-, \mathcal{E}_-$. Each fermion contains both left- and right-handed two-component spinors when reduced to four dimensions. We impose an orbifold projection such that the right-handed components of $\mathcal{Q}_+, \mathcal{L}_+$, and left-handed components of $\mathcal{U}_-, \mathcal{D}_-, \mathcal{E}_-$, are odd under the orbifold $\mathbb{Z}_2$ symmetry and therefore the corresponding zero modes are projected out. As a result, the zero-mode fermions are two-component four-dimensional quarks and leptons: $\mathcal{Q}_+^{(0)} \equiv (t, b)_L$, $\mathcal{U}_-^{(0)} \equiv t_R$, $\mathcal{D}_-^{(0)} \equiv b_R$, $\mathcal{L}_+^{(0)} \equiv (\nu_\tau, \tau)_L$, $\mathcal{E}_-^{(0)} \equiv \tau_R$.

At the cutoff scale, the SM gauge interactions are strong and will produce bound states. The squared-mass of a scalar bound state has quadratic dependence on the cutoff, and can become much smaller than the cutoff scale or even negative if the coupling is sufficiently strong. Using the one-gauge-boson-exchange approximation, one finds in general among possible scalar bound states, that $H_U = \mathcal{Q}_+, \mathcal{U}_-$, which has the correct quantum number to be the Higgs field, is the most attractive channel. Therefore it is most likely to acquire a negative squared-mass to break the electroweak symmetry. The composite Higgs is expected to have a large coupling to its constituents, so the theory not only predicts the correct Higgs quantum numbers, but also a heavy up-type quark (top quark). The $H_D = \mathcal{Q}_+ \mathcal{U}_-$ channel is also quite strongly bound while the other channels are not sufficiently strong to produce light bound states. The low-energy theory below $M_c$ is expected to be a two-Higgs-doublet model. In more general models, there may be other light bound states which may be accessible at the upcoming collider experiments in addition to the Higgs bosons. The possible light bound states depend on the model. For example, in the eight-dimensional model, there is a strongly bound state $\mathcal{Q}_Q^c$ transforming like the right-handed bottom quark under the SM gauge group [4].

Compared with the usual four-dimensional dynamical electroweak symmetry breaking (EWSB) models, the higher-dimensional model has the advantage that the binding force can be the SM gauge interactions themselves, without the need of introducing new strong interactions. In addition, it also gives a prediction of the top quark mass naturally in the right range. In the minimal four-dimensional top quark condensate model, the top quark is too heavy, $\sim 600$ GeV, if the compositeness scale is in the TeV range [5]. With extra dimensions, the KK excitations of the top quark also participate in the EWSB, so the top quark mass can be smaller. Another way of understanding of the top Yukawa coupling being $\sim 1$ instead of the strong coupling value $\sim 4\pi$ is that (the zero mode of) the top quark coupling receives a volume dilution factor because it propagates in extra dimensions. In fact, the top quark mass can be predicted quite insensitively to the cutoff because of the infrared fixed point behavior of the renormalization group (RG) evolution. The infrared fixed point is rapidly approached due to the power-law running in extra-dimensional theories [6], even though the cutoff scale is not much higher than the
weak scale. Similarly, the Higgs self-coupling also receives the extra-dimensional volume suppression. As a result, the physical Higgs boson is relatively light, $\sim 200$ GeV, in contrast with the usual strongly coupled four-dimensional models. It is also governed by the infrared fixed point of the RG equations. The numerical predictions of the top quark mass and the Higgs boson mass are shown in Fig. 1.

![Graph of top quark and Higgs masses](image)

**FIGURE 1.** The top quark mass (left) and the Higgs boson mass (right) as functions of the number of KK excitations, $N_{KK}$, and the compactification scale $M_c$ in the six-dimensional theory. The shaded area in the Higgs boson mass prediction corresponds to the top quark mass lying within 3$\sigma$ of the experimental value, $174.3 \pm 1.1$ GeV.

So far we have not discussed the first two generation fermions. If they are localized on some four-dimensional subspaces, they can also form some four-dimensional bound states, though probably not as strongly bound as the higher dimensional Higgs bound state, because they do not receive contributions from the extra components of the gauge fields. Due to the tree-level KK gauge boson exchange between the first two generation fermions, there are strong bounds on the compactification scale, $M_c \gtrsim 2$–5 TeV from the precision electroweak data [7].

Another interesting possibility is that all SM fields live in the same extra dimensions (universal extra dimensions). In that case, we need to introduce explicit flavor-breaking interactions from the cutoff scale to distinguish the three generations. The flavor-breaking interactions should enhance the third generation channels relative to the first two generation channels so that only the composite Higgs field from the third generation top quark have a negative squared-mass and is responsible for the EWSB. Some flavor-breaking interactions can also give masses to the other light fermions.

Because of the momentum conservation in extra dimensions, there is no tree-level contribution to the electroweak observables from the KK modes in the case of universal extra dimensions. There are loop corrections because KK modes can appear in the loop. The experimental bounds on the size of the universal dimensions are much weaker. The main constraints come from weak-isospin violation effects. The
lower limits for the compactification scales are $\sim 300$ GeV for one extra dimension and 400-800 GeV for two extra dimensions [8]. With such a loose bound, there is a hope that the upcoming collider experiments may discover the KK states.

At the colliders, the KK states have to be produced in pairs or more because of the KK number conservation. If there is no additional KK number violating interactions or the KK number violating interactions are sufficiently weak, some of the KK modes will be stable or long-lived. After being produced, they will hadronize into integer-charged states. The signatures will be highly ionizing tracks. By extrapolating the current lower mass limits on heavy stable quarks from the Run I of Tevatron [9], we estimate that the direct lower bound on the compactification scale is 300–350 GeV. If there are KK number violating interactions which allow the KK modes to decay inside the detector, the signatures and the limits depend on the KK number violating effects. The direct bounds are somewhat weaker, in the $\sim 200$ GeV range [8]. It is interesting that the direct limit is comparable to the indirect bounds from the electroweak data, which means that the future collider experiments may discover the KK modes or raise the lower bound significantly.

In summary, we have shown that if the standard model fields propagate in extra dimensions of TeV$^{-1}$ size, the standard model gauge interactions become strong at high energies and naturally produce a Higgs bound state from the top quark to break the electroweak symmetry. The top quark mass and the Higgs boson mass can be predicted from the RG infrared fixed points. The top quark mass is in good agreement with the experimental value and the Higgs mass is around 200 GeV. There may be other light bound states which could be observed in the future experiments. In the case all standard model fields propagate in the same extra dimensions, the bounds on the compactification scale are sufficiently loose that the KK modes of the standard model fields may be discovered at the upcoming experiments.

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