Physical properties of three short period close binaries: KIC 2715417, KIC 6050116 and KIC 6287172 *

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Abstract We present the physical parameters of three short period close binaries using data observed from the Kepler Space Telescope. All of these observations were taken in a single bandpass (which approximates the Johnson V-band). Our three systems are KIC 2715417, KIC 6050116 and KIC 6287172. The first system, KIC 2715417, is considered a semi-detached system with the secondary component filling its Roche lobe. The second system, KIC 6050116, is an overcontact system, while the third system, KIC 6287172, belongs to ellipsoidal variables as deduced from the Roche lobe geometry. For photometric analysis, we used the PHOEBE software package, which is based on the Wilson−Devinney code. Due to lack of spectroscopic data, the photometric mass ratios are determined from the analyses of light curves using the q-search method. The absolute parameters are determined using three different methods (Harmanec, Maceroni & Van’t Veer and Gazeas & Niarchos).

Key words: stars: binaries: eclipsing — stars: fundamental parameters — stars: luminosity function, mass function — stars: individual (KIC 2715417, KIC 6050116 and KIC 6287172)

1 INTRODUCTION

We have many types of close binary systems as a result of close binary evolution. These types can be classified as follows:

- The type called semi-detached systems is found to have the surfaces of the low mass components in contact with the inner Lagrangian surfaces (the critical potential surfaces) and the surfaces of the more massive components are bounded within a separate equipotential surface. In Beta Lyrae systems, where one of the two components has become a giant or supergiant in the course of its evolution, matter can escape from the critical potential surface at which the gravitation at their surface is so weak that material can be transferred to the other component. W UMa type or late type contact systems have both of their components filling the inner Lagrangian surfaces and share a common envelope. Ellipsoidal variables (ELVs) can be described as very close binaries with the two components having non-spherical shapes (ellipsoidal shapes) due to their mutual gravitation. These binary systems can be considered as close binaries for the following reasons:

- Close binaries have short periods. The distance separating the two components is comparable to their size. The two components are close enough that their shapes are distorted by mutual gravitational forces to non-spherical shapes such as in the type known as ELVs. The surfaces of both components overflow their critical Lagrangian surfaces and share a common envelope such as in the case of an overcontact type. When one of their two components becomes giant or supergiant in the final stages of its evolution, matter may freely flow from one component to the other as in the case of the Beta Lyrae type.

1.1 Observations

Kepler is a space telescope launched by NASA on 2009 March 7 to discover extrasolar planets orbiting around other stars in the field of the constellation Cygnus. Kepler has a primary mirror 1.4 meters in diameter. The field of view of the Kepler spacecraft is 105 deg². The photometer...
is composed of an array of 42 CCDs and each CCD has $2200 \times 1024$ pixels$^1$. The *Kepler* catalog ID, coordinates, $V$ magnitude, color excess and interstellar extinction of the target objects are listed in Table 1.

From the *Kepler* data archive$^2$, we have collected observations of the light curves for the above three systems. Observations of the KIC 2715417 system started at 2454964.51259 JD and ended at 2456390.95883 JD. The light curve of these data is shown in Figure 1.

In the case of the KIC 6050116 system, observations started at 2454964.51229 JD and ended at 2456390.95881 JD, and the corresponding light curve is shown in Figure 2. For the system KIC 6287172, the observations started at 2454953.53905 JD and ended at 2456390.95893 JD. The corresponding light curve of this system is shown in Figure 3.

In the case of the eclipsing binary system KIC 2715417 and from the *Kepler Input Catalog*, it is found that the effective temperature of the primary component is 5189 K, the *Kepler* magnitude is 14.070 mag, metallicity $= -0.458$, log$_{10} g$ (log$_{10}$ surface gravity) = 4.830 and color excess reddening $E(B - V) = 0.063$. It is considered a semi-detached binary with the secondary component filling its Roche lobe. The associated ephemeris of the system can be written as

$$HJD (\text{Min I}) = 2454964.666158 + 0.2364399 \times E,$$  

where HJD (Min I) represents the minima epoch times in Heliocentric Julian Date and $E$ is the integer number of cycles.

For the eclipsing binary system KIC 6050116 and from the *Kepler Input Catalog*, it is found that the effective temperature of the primary component is 4569 K, the *Kepler* magnitude equals 14.25 mag, metallicity (solar = 0.0134) = $-0.849$, log$_{10} g$ (log$_{10}$ surface gravity) = 4.524 and color excess reddening $E(B - V) = 0.059$, indicating it is an overcontact system. The associated ephemeris of the system can be written as

$$HJD (\text{Min I}) = 2454964.708006 + 0.2399081 \times E.$$  

Using the same *Kepler Input Catalog* for the eclipsing binary system KIC 6287172, we also found that the effective temperature of the primary component is 6646 K, the *Kepler* magnitude is equal to 12.714 mag, metallicity (solar = 0.0134) = $-0.470$, log$_{10} g$ = 4.286 and color excess reddening $E(B - V) = 0.114$. It can be considered an ELV. The associated ephemeris of the system can be written as

$$HJD (\text{Min I}) = 2454953.651911 + 0.2038732 \times E.$$  

### 2 LIGHT CURVE ANALYSES

For the three systems (KIC 2715417, KIC 6050116 and KIC 6287172), it appears that their temperatures are below 7200 K, which indicates that they are convective, hence the gravity darkening for convective stars $g_1 = g_2 = 0.32$ (Lucy 1967). The bolometric albedo $A_1 = A_2 = 0.5$ (Ruciski 1969) and the limb darkening coefficients were adopted from VanHamme (1993) based on the linear cosine law model.

Due to the lack of any spectroscopic data for these three systems, it is difficult to determine the mass ratios ac-

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1. [http://www.nasa.gov/missionpages/kepler/spacecraft/index.html](http://www.nasa.gov/missionpages/kepler/spacecraft/index.html)
2. [http://keplerreb.s.villanova.edu/](http://keplerreb.s.villanova.edu/)
3. [http://archive.stsci.edu/kepler/keplerfov/search.php](http://archive.stsci.edu/kepler/keplerfov/search.php)
accurately. Thus, we used the \( q \)-search method as discussed in Deb et al. (2010) to determine the approximate value of the photometric mass ratios for these systems. We have adjusted the PHOEBE software model of the unconstrained binary system to estimate a set of parameters that represent the observed light curve. In order to determine the photometric mass ratio, we used a series of mass ratios with values ranging from 0.1 to 1.0 in steps of 0.1. Subsequently, we took the three values of mass ratio corresponding to the minimum values of the residuals. Then, we used the same technique but with steps of 0.01. For each value of mass ratio, we obtained the sum of squared deviations (residuals) from the fitting solutions of the light curve modeling.

Figure 4 shows that the minimum occurs at value \( q \approx 0.29 \) for the system KIC 2715417. Figure 5 indicates that the minimum occurs at value \( q \approx 0.57 \) for the system KIC 6050116. Finally Figure 6 demonstrates that the minimum occurs at value \( q \approx 0.61 \) for the system KIC 6287172, which should not be trusted due to the very small inclination of ELV type. Rather, the models offered here are the best ones that can be applied with the available data.

### 2.1 Light Curve Analysis of the System KIC 2715417

The light curve of the system KIC 2715417 in the \( V \)-band as shown in Figure 1 has been analyzed using the PHOEBE package, version 0.31a (Prsa & Zwitter 2005) which is based on the code of Wilson & Devinney (1971). From the Kepler Input Catalog, it is found that the corresponding temperature \( T_1 = 5189 \text{ K} \). First, from the PHOEBE software, we have used the model of the unconstrained binary system to find approximate values of the parameters that represent the observed light curve. Data obtained from the PHOEBE model of the unconstrained binary system were analyzed to find the fill-out factor \( (f) \) using the software Binary Maker 3 (BM3). This software is a modification of the parameter defined by Lucy & Wilson (1979) to specify equipotentials for contact, overcontact and detached systems after specifying the mass ratio. In case of the detached type, the fill-out is given by formula (A). For overcontact, the fill-out is given by formula (B)

\[
f = \frac{\Omega_{\text{inner}} - \Omega_{\text{outer}}}{\Omega_{\text{inner}} - \Omega_{\text{outer}}}, \quad \text{for } \Omega_{\text{inner}} > \Omega_{\text{outer}} \quad (\text{Overcontact}) \quad (B).
\]

Thus, the fill-out factor \( (f) \) for detached stars will lie between \((-1 < f < 0)\). The fill-out factor \( (f) \) for overcontact systems will lie between \( (0 < f < 1) \). In the case of a contact system, the fill-out factors of two stars equal zero \( (f = 0) \). When \( (f) \) is near zero, the system can be described as a near contact system (e.g. the smaller star has a fill-out factor \( (f_2) = 0.00 \) and the larger star has \( (f_1) = -0.02 \)).

In our case, we found the fill-out factor of the primary component \( f_1 = -0.1150 \) and the secondary component has fill-out factor \( f_2 = 0.0013 \). According to the above mentioned rules, the best model describing the binary system KIC 2715417 is a near contact system. The best photometric fitting has been reached after several runs, which shows that the primary component is more massive and hotter than the secondary one, with a temperature difference of about 478 K.

The orbital and physical parameters of the system KIC 2715417 are listed in Table 2. Figure 7 displays the observed light curve for the interval 2454964.51259 JD – 2454972.99293 JD, together with the synthetic curve in the \( V \)-band while Figure 8 displays the light curve residual error for different phases. According to the effective temperature of both the primary and secondary components of the system KIC 2715417 and from the calibration of Morgan-Keenan (MK) spectral types for main sequence stars (Drilling & Landolt 2002), the spectral types are nearest to K0 and K2 respectively.

Using the orbital and physical parameters listed in Table 2 with the BM3 software, we present the shape of the system KIC 2715417 at phases 0.0, 0.25, 0.5 and 0.75 in Figure 9. Also we present the Roche lobe geometry of the system in Figure 10.

Because of the O’Connell effect (O’Connell 1951), we included cool spots on the surface of the star to achieve the best fit. We have one cool spot on both the primary and secondary components for the system KIC 2715417 as shown in Figure 9. Table 2 gives the spot parameters where the spot of the secondary component has a larger radius and cooler temperature than the primary component.

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### Table 1 Catalog ID, Coordinates and \( V \) Magnitude

| Kepler Catalog ID  | \( \alpha \)2000 (hh: mm: ss.ss) | \( \delta \)2000 (± dd: mm: ss.ss) | \( V \)mag | \( E(B-V) \) | \( A_v \) |
|--------------------|--------------------------------|-------------------------------|-----------|-------------|---------|
| KIC 2715417        | 19:27:52.565                  | +37:55:39.97                  | 14.070    | 0.063       | 0.427   |
| KIC 6050116        | 19:36:33.113                  | +41:20:22.56                  | 14.258    | 0.059       | 0.369   |
| KIC 6287172        | 19:29:59.69                   | +41:37:45.0                   | 12.714    | 0.114       | 0.339   |

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4 http://www.binarymaker.com/
Fig. 4 The $q$-search diagram for the system KIC 2715417.

Fig. 5 The $q$-search diagram for the system KIC 6050116.

Fig. 6 The $q$-search diagram for the system KIC 6287172.

Fig. 7 Light curve in $V$ filter showing the coherence between observed and synthetic curves.

Table 2 Orbital and Physical Parameters of the System KIC 2715417

| Parameter                  | Value          | Parameter                  | Value          |
|----------------------------|----------------|----------------------------|----------------|
| Epoch                      | 2454964.666158 | $X_1$                      | 0.655          |
| Period (d)                 | 0.2364399      | $X_2$                      | 0.711          |
| Inclination ($i$)          | 56.56$^\circ$ ± 0.04$^\circ$ | $g_1$  | 0.32          |
| Mass ratio ($q$)           | 0.289 ± 0.002  | $g_2$                      | 0.32           |
| TAVH ($T_1$)               | 5189K (Fixed)  | $A_1$                      | 0.50           |
| TAVC ($T_2$)               | 4711 ± 14K     | $A_2$                      | 0.50           |
| PHSV ($\Omega_1$)         | 2.7635 ± 0.0061 | $\frac{L_1}{L_1+L_2}$ | 0.7909 ± 0.0016 |
| PHSV ($\Omega_2$)         | 2.4416 ± 0.0032 | $\frac{L_2}{L_1+L_2}$ | 0.2091 ± 0.0030 |
| Phase Shift                | 0.00402 ± 0.00082 | Residual $\sum (O - C)^2$ | 0.0401         |
| Fill-out factor ($f_1$)    | -0.1150        | ($f_2$)                    | 0.0013         |
| $r_1$ (back)               | 0.4716 ± 0.0005 | $r_2$ (back)               | 0.3109 ± 0.0024 |
| $r_1$ (side)               | 0.4625 ± 0.0004 | $r_2$ (side)               | 0.2932 ± 0.0017 |
| $r_1$ (pole)               | 0.4505 ± 0.0003 | $r_2$ (pole)               | 0.2764 ± 0.0015 |
| $r_1$ (point)              | 0.4859 ± 0.0008 | $r_2$ (point)              | 0.3256 ± 0.0029 |
| Mean fractional radii ($r_1$) | 0.4616 ± 0.0004 | $r_2$ (mean)               | 0.2935 ± 0.0019 |

Spot Parameters

| Parameter          | Spot 1 (primary) | Spot 2 (secondary) |
|--------------------|------------------|--------------------|
| Temp Factor        | 0.86 ± 0.04      | 0.80 ± 0.05        |
| Spot Radius        | $7^\circ$ ± $2^\circ$ | $14^\circ$ ± $3^\circ$ |
| Longitude ($\lambda$) | $74^\circ$ ± $12^\circ$ | $87^\circ$ ± $11^\circ$ |
| Co-Latitude ($\phi$) | $122^\circ$ ± $8^\circ$ | $46^\circ$ ± $9^\circ$ |
Fig. 8 Light curve residual error for different phases of the system KIC 2715417.

Fig. 9 Shape of the system KIC 2715417 at phases 0.0, 0.25, 0.5 and 0.75.

Fig. 10 Roche lobe geometry of the system KIC 2715417.

Fig. 11 Light curve in V filter showing the coherence between observed and synthetic curves.
The temperature of the primary component is 5189 K while the temperature factor of the cool spot is 0.86 ± 0.04. This means that the primary cool spot has a temperature of 4462 ± 208 K. On the other hand, the temperature of the secondary component is 4711 ± 14 K and the temperature factor of the cool spot is 0.80 ± 0.05. Thus, the temperature of the cool spot on the secondary component is 3769 ± 236 K.

2.2 Light Curve Analysis of the System KIC 6050116

The light curve of the system KIC 6050116 in the V-band as shown in Figure 2 has been analyzed using the PHOEBE package. It is found that the corresponding temperature $T_1 = 4569$ K from the Kepler Input Catalog. With the PHOEBE software we have used the model of an unconstrained binary system to estimate a set of parameters that represent the observed light curve. The best photometric fitting has been reached after several runs.

From the analyses of the BM3 software, the type of binary system is considered to be overcontact, since the fill-out factor of the first component is $f_1 = 0.0704$ and that of the secondary component is $f_2 = 0.0704$. The fill-out factor is equal for the two components in contact and overcontact binaries as the two stars are in contact or overcontact with each other. They must have the same gravitational equipotential (Ω), otherwise gas will literally leak away from the system until it reaches equilibrium.

The primary component is more massive and hotter than the secondary one, with a temperature difference of
about 84 K. The orbital and physical parameters of the system KIC 6050116 are listed in Table 3.

Figure 11 displays the observed light curve for the interval 2454964.51229 JD – 2454974.48433 JD, together with the synthetic curve in the V band while Figure 12 depicts the light curve residual error for different phases.

According to the effective temperatures of both the primary and secondary components of the system KIC 6050116 and from the calibration of MK spectral types for main sequence stars, the spectral types are nearest to K5 for both components. Using the orbital and physical parameters listed in Table 3 with the BM3 program, we present the shape of the system KIC 6050116 at phases 0.0, 0.25, 0.5 and 0.75 in Figure 13. We also present the Roche lobe geometry of the system in Figure 14.

We also have one cool spot on the secondary component for the system KIC 6050116 as shown in Figure 13. Table 3 gives the temperature of the secondary component as $485 \pm 17$K and the temperature factor of the cool spot is $0.76 \pm 0.04$. Thus, the temperature of the cool spot on secondary component is $3409 \pm 179$K.

2.3 Light Curve Analysis of the System KIC 6287172

The light curve of the system KIC 6287172 in the V-band as shown in Figure 3 has been analyzed using the PHOEBE package. We found that the corresponding temperature $T_1 = 6646$K from the Kepler Input Catalog. With the PHOEBE software we have used the model of an unconstrained binary to determine approximate values of the physical and geometrical parameters that represent the observed light curve. After the best photometric fitting has been reached, we have used the data obtained from the PHOEBE software input into the other software BM3 to find the type of binary system from the fill-out factors of the two components.

We found that the fill-out parameter of the primary star is $f_1 = 0.432657$ and that for the secondary star is $f_2 = 0.432657$, so this system is considered a non-eclipsing overcontact type or ELV. The primary component is more massive and hotter than the secondary one, with a temperature difference of about 22 K. The orbital and physical parameters of the system KIC 6287172 are listed in Table 4.

Figure 15 displays the observed light curve for the interval 2454953.53905 JD – 2454967.31191 JD, together with the synthetic curve in the V-band, while Figure 16 presents the light curve residual error for different phases.

According to the effective temperature of both the primary and secondary components of the system KIC 6287172 and from the calibration of MK spectral types for main sequence stars, the spectral types are nearest to F5 for both components. Using the orbital and physical parameters listed in Table 4 with the BM3 program, we present the shape of the system KIC 6287172 at phases 0.0, 0.25, 0.5 and 0.75 in Figure 17. We display the Roche geometry of the system in Figure 18.

3 ABSOLUTE PARAMETERS OF THE SYSTEMS

The physical parameters of effective temperature $T_{\text{eff}}$, absolute magnitude $M_V$, relative radius $R/R_\odot$, relative luminosity $L/L_\odot$, and surface gravity $g$ were calculated using the equations of stellar structure. The adopted constants which we used are $T_{\text{eff}} \odot = 5772$K, $\log g \odot = 4.44$ and $M_{\text{bol}} \odot = 4.74$.

We have three methods to determine only the primary masses for each type of binary. These three methods are from Harmanec (1998), Maceroni & Van’t Veer (1996) and Gazeas & Niarchos (2006).

3.1 Harmanec’s Method

We compute the physical parameters according to the empirical relation derived by Harmanec (1998) from his work about stellar masses and radii based on modern binary data. These relations are obtained by a least-squares fit via Chebyshev polynomials for the data introduced by Popper.
(1980). Harmanec’s method is the mass—temperature relation used to determine the masses of the primary and secondary components, \( M_1 \) and \( M_2 \) respectively, for detached binaries. However, in the case of overcontact binaries and any other type of binaries, it is only applicable for determination of the primary component \( M_1 \), which is slightly affected by thermal contact with the secondary low mass stars. If we assume that the effect of temperature of the secondary component on the primary is negligible because the primary has higher temperature, the mass of the primary can be deduced from the previous relations with good approximation. The mass of the secondary component cannot accurately be calculated using this method because the secondary component is greatly affected by thermal con-

### Table 3 Orbital and Physical Parameters of the System KIC 6050116

| Parameter       | Value       | Parameter       | Value       |
|-----------------|-------------|-----------------|-------------|
| Epoch           | 2454964.708006 | \( X_1 \)       | 0.787       |
| Period (d)      | 0.2399081   | \( X_2 \)       | 0.795       |
| Inclination (i) | 69.2° ± 0.7°| \( g_1 \)       | 0.32        |
| Mass ratio (q)  | 0.573 ± 0.012| \( g_2 \)       | 0.32        |
| TAVH (\( T_1 \))| 4569 K (Fixed) | \( A_1 \)       | 0.50        |
| TAVC (\( T_2 \))| 4485 ± 14 K | \( A_2 \)       | 0.50        |
| PHSV (\( \Omega_1 \)) | 2.9899 ± 0.0016 | \( \frac{L_1}{L_1+L_2} \) | 0.6504 ± 0.0020 |
| PHSV (\( \Omega_2 \)) | 2.9899 ± 0.0016 | \( \frac{L_1}{L_1+L_2} \) | 0.3496 ± 0.0037 |
| Phase Shift     | 0.00857 ± 0.00035 | \( \sum (O - C)^2 \) | 0.107112   |
| \( f_1 \)       | 0.0704      | \( f_2 \)       | 0.0704      |
| \( r_1 \) (back)| 0.4618 ± 0.0040 | \( r_2 \) (back)| 0.3637 ± 0.0084 |
| \( r_1 \) (side)| 0.4315 ± 0.0025 | \( r_2 \) (side)| 0.3291 ± 0.0062 |
| \( r_1 \) (pole)| 0.4067 ± 0.0018 | \( r_2 \) (pole)| 0.3144 ± 0.0053 |
| \( r_1 \) (point)| 0.5570 ± 0.0021 | \( r_2 \) (point)| 0.4430 ± 0.0021 |
| \( r_1 \) (mean)| 0.4245 ± 0.0028 | \( r_2 \) (mean)| 0.3337 ± 0.0045 |

**Spot Parameters**

| Parameter       | Value       |
|-----------------|-------------|
| Temp Factor     | 0.76 ± 0.04 |
| Spot Radius     | \( 10^5 \) ± 3° |
| Longitude (\( \lambda \)) | 62° ± 7° |
| Co-Latitude (\( \phi \)) | 117° ± 9° |

### Table 4 Orbital and Physical Parameters of the System KIC 6287172

| Parameter       | Value       | Parameter       | Value       |
|-----------------|-------------|-----------------|-------------|
| Epoch           | 2454953.651911 | \( X_1 \)       | 0.507       |
| Period (d)      | 0.2038732   | \( X_2 \)       | 0.509       |
| Inclination (i) | 7.1° ± 0.2° | \( g_1 \)       | 0.32        |
| Mass ratio (q)  | 0.606 ± 0.001 | \( g_2 \)       | 0.32        |
| TAVH (\( T_1 \))| 66.46 K (Fixed) | \( A_1 \)       | 0.50        |
| TAVC (\( T_2 \))| 60.24 ± 11 K | \( A_2 \)       | 0.50        |
| PHSV (\( \Omega_1 \)) | 2.9213 ± 0.0037 | \( \frac{L_1}{L_1+L_2} \) | 0.6137 ± 0.0062 |
| PHSV (\( \Omega_2 \)) | 2.9213 ± 0.0037 | \( \frac{L_1}{L_1+L_2} \) | 0.3863 ± 0.0062 |
| Phase Shift     | 0.02983 ± 0.00031 | \( \sum (O - C)^2 \) | 0.475962   |
| \( f_1 \)       | 0.4327      | \( f_2 \)       | 0.4327      |
| \( r_1 \) (back)| 0.4954 ± 0.0004 | \( r_2 \) (back)| 0.4133 ± 0.0008 |
| \( r_1 \) (side)| 0.4537 ± 0.0002 | \( r_2 \) (side)| 0.3609 ± 0.0005 |
| \( r_1 \) (pole)| 0.4232 ± 0.0002 | \( r_2 \) (pole)| 0.3409 ± 0.0004 |
| \( r_1 \) (point)| 0.5513 ± 0.0002 | \( r_2 \) (point)| 0.4487 ± 0.0002 |
| \( r_1 \) (mean)| 0.4245 ± 0.0028 | \( r_2 \) (mean)| 0.3337 ± 0.0045 |
tact with the primary hotter star. The masses of primary stars according to Harmanec were calculated from the following equation

$$
\log \frac{M_1}{M_\odot} = (1.771141 X - 21.46965) X \\
+ 88.05700 X - 121.6782,
$$

where $X = \log(T_{\text{eff}1})$. Equation (4) is applicable for $4.62 \geq \log(T_{\text{eff}}) \geq 3.71$. The masses of secondary components are determined from the photometric mass ratio, $(q = \frac{M_2}{M_1})$.

### 3.2 Maceroni & Van’t Veer’s Method

The physical parameters can be determined from the relation between total mass ($M_T$) and total luminosity ($L_T$) in the binary systems from Maceroni & Van’t Veer (1996), under the assumption that interaction between the two components of the binary system does not affect the total luminosity of the system. Therefore the common envelope radiates the luminosity given by the sum of the internal luminosities. In other words, the total luminosity is the same for the binary system. Due to the previous assumption, this
method can be applicable for any type of binary system such as detached, overcontact or ellipsoidal configurations to determine the total mass of the binary system. Using the value of mass ratio for each binary, we can get the individual masses $M_1$ and $M_2$ for the binary system as shown in the following equations

$$
\log(L_T) = \frac{2}{3} \log(M_T) + c_1 ,
$$

where

$$
c_1 = \log \left( \frac{cEP^2(r_1^2T_1^4 + r_2^2T_2^4)}{G} \right).
$$

In the same paper (Maceroni & Van’tVeer 1996) in equation (2), the constant $c$ was written as

$$
c = \left( \frac{4\pi}{G} \right)^{\frac{2}{3}} G^2 \sigma.
$$

However, we note that there is a mistake in the formula for $c$ and it should be written as

$$
c = \left( \frac{4\pi}{G} \right)^{\frac{2}{3}} G^2 \sigma.
$$

Using the evolutionary tracks for a non-rotating model which have been computed by Mowlavi et al. (2012) for zero age main sequence (ZAMS) stars with metallicity $Z = 0.014$, we correlate the total luminosity and total mass from fitting data of the binary system of stars located on the ZAMS as follows

$$
\log(L_T) = 4.3 \log(M_T) - 0.539 .
$$

From the intersections between the straight lines of binary systems represented by Equations (5) and (6) as shown in Figures 19, 23 and 27, we can deduce the total mass $(M_T)$ of the binary system. From knowledge of the mass ratio $(q)$ and total mass $(M_T)$ of the binary system, the individual masses $(M_1$ and $M_2$) can readily be calculated from Equation (7)

$$
M_1 = \frac{M_T}{q + 1}, \quad M_2 = M_1 \times q = \frac{M_T \times q}{(q + 1)}.
$$

### 3.3 Gazeas & Niarchos’ Method

This is the period—mass relation used to determine the mass of the primary component. It is only applicable for short period eclipsing binary systems such as overcontact or near contact types with orbital period $\log P < -0.25$ and does not include ELVs (non-eclipsing binaries) according to Gazeas & Niarchos (2006) and Gazeas & Stepien (2008). The mass of the primary star $(M_1)$ can be obtained from the following expression

$$
\log(M_1) = (0.755 \pm 0.059) \log(P) + (0.416 \pm 0.024) ,
$$

where $P$ is the orbital period of a W UMa type binary system. All of these three methods are used to calculate only the mass of the primary component in the binary system. The mass of the secondary component in the binary system is found from the photometric mass ratio $(q = \frac{M_2}{M_1})$ and the primary mass. The photometric mass ratio of the contact type is more precise than undercontact type such as the detached binary. All of these three methods are used to calculate only the individual masses of the binary systems. From Kepler’s third law, the semi-major axis of the orbit for the binary system can be determined using the following equation

$$
a = \sqrt{\frac{G \times (M_1 + M_2) \times P^2}{4\pi^2}} ,
$$

where $G$ is the gravitational constant ($G = 6.67428 \times 10^{-11}$ kg$^{-1}$ m$^3$ s$^{-2}$). Knowing the semi-major axis $(a)$, thus we can calculate the radius of primary and secondary stars according to the relation $R_j = r_j a$, where $j$ can take values 1 for the primary component or 2 for the secondary component and $r$ is the mean fractional radii of the stars. The luminosities of the primary and secondary stars were calculated using the direct equation

$$
\frac{L_j}{L_\odot} = \left( \frac{R_j}{R_\odot} \right)^2 \times \left( \frac{T_j}{T_\odot} \right)^4.
$$

The bolometric magnitudes of the primary and secondary components of binary systems can be calculated using the following equation

$$
M_{bol j} = M_{bol\odot} - 2.5 \log_{10} \frac{L_j}{L_\odot} .
$$

The absolute magnitude, $M_V$, is related to the bolometric magnitude, $M_{bol}$, via

$$
M_V = M_{bol} - BC ,
$$
where $BC$ is the bolometric correction given by Reed (1998)

$$
BC = - 8.499 \log (T) - 4 + 13.421 \log (T) - 4 - 8.131 \log (T) - 4^2 - 3.901 \log (T) - 4 - 0.438.
$$

We used the relation in Equation (12) to calculate the absolute magnitude for all three systems. Using the interstellar extinction value ($A_v$) corresponding to the equatorial coordinates J2000 obtained from Schlafly & Finkbeiner (2011) for the $V$-band at effective wavelength $\lambda = 5517$ Å, we finally found that, for system KIC 2715417 ($A_v = 0.427$), for system 6050116 ($A_v = 0.369$) and for system 6287172 ($A_v = 0.339$). We can apply the distance modulus relation to calculate the distance of the three systems as follows

$$
D = 10^{0.2 (m - M + 5 - A_v)},
$$

where the distance is in parsec (pc), $m$ represents the apparent magnitude and $M_V$ is the absolute magnitude. All the absolute parameters (mass, semi-major axis, radius, luminosity, bolometric magnitude and distance) of the three systems which have been calculated are listed in Tables 5, 6 and 7.

4 EVOLUTIONARY STATE OF THE SYSTEMS

In order to study the evolutionary state of the three systems (KIC 2715417, KIC 6050116 and KIC 6287172), we have plotted the physical parameters listed in Tables 5, 6 and 7 for the components of the three systems on the $H - R$, $M - R$ and $M - L$ diagrams in Figures 19 – 30. We used evolutionary tracks for a non-rotating model which have been computed by Mowlavi et al. (2012) for both ZAMS stars and terminal age main sequence (TAMS) stars with metallicity $Z = 0.014$ (solar metallicity). As is clear from Figures 19, 23 and 27, the straight line (Binary System) represents the relation between the total luminosity and total mass of the binary system as written in Equation (5). We can determine the total mass from the intersection between the two lines, Binary System and ZAMS.

In Figures 20, 24 and 28, the primary components of the three systems are located on ZAMS, however the secondary components of the three systems are located above ZAMS. This can be attributed to the rise in temperature of the secondary component due to thermal contact between the two components of the binary systems and consequently the value of luminosity will rise without any change in mass.

In Figures 21, 25 and 29, the primary components of the three systems are located on ZAMS, but the secondary components of the three systems are located below ZAMS. That may be due to thermal contact.

In Figures 22 and 26, the primary components of the two systems (KIC 2715417, KIC 6050116) are located on ZAMS, but the secondary components of these two systems are located far away from ZAMS. In Figure 30 we note that the primary component of the system KIC 6287172 is located above ZAMS with additional accumulated mass due to matter transfer from the secondary component. This system belongs to the type ELV, which can be described as a very close non-eclipsing binary whose components have a non-spherical shape due to their mutual gravitation. The associated light variation can change as seen from the Earth as a result of rotation of the two components because their surfaces facing the observers are changing.
We presented the physical properties of the systems KIC 2715417, KIC 6050116 and KIC 6287172 derived from the analyses of *Kepler* data according to the above results. We arrived at the following conclusions: The system KIC 2715417 is considered a near contact system. The system KIC 6050116 is an overcontact system, while the system KIC 6287172 belongs to ELV or non-eclipsing overcontact type. The effective temperatures of the primary star for all the three systems are slightly higher than those of the secondary component. We found that the primary components of our systems are located on the ZAMS line while the secondary components are located away from ZAMS. This can be due to heating of the secondary components in the close binaries by thermal contact.

We used the three mentioned methods to determine the masses of the primary components of the binary systems and calculated the masses of the secondary components from the photometric mass ratios obtained from the *q*-search method which gives good values compared with the spectroscopic mass ratio, particularly for contact type. This enabled us to obtain the required comparison of the systems under consideration. It is found that the method of

5 CONCLUSIONS AND DISCUSSION

Table 5 The Absolute Parameters of the System KIC 2715417

| Parameter | Harmanec  | Maceroni  | Gazeas  |
|-----------|-----------|-----------|---------|
| $M_1 (M_\odot)$ | $0.902 \pm 0.076$ | $0.888 \pm 0.032$ | $0.877 \pm 0.132$ |
| $M_2 (M_\odot)$ | $0.261 \pm 0.022$ | $0.257 \pm 0.009$ | $0.254 \pm 0.038$ |
| $R_1 (R_\odot)$ | $1.692 \pm 0.049$ | $1.683 \pm 0.020$ | $1.677 \pm 0.080$ |
| $R_2 (R_\odot)$ | $0.781 \pm 0.023$ | $0.778 \pm 0.009$ | $0.774 \pm 0.037$ |
| $L_1 (L_\odot)$ | $0.497 \pm 0.014$ | $0.497 \pm 0.014$ | $0.492 \pm 0.024$ |
| $L_2 (L_\odot)$ | $0.399 \pm 0.023$ | $0.394 \pm 0.009$ | $0.391 \pm 0.038$ |
| $M_1 \text{bol}1$ | $5.739 \pm 0.064$ | $5.750 \pm 0.026$ | $5.759 \pm 0.102$ |
| $M_2 \text{bol}2$ | $7.002 \pm 0.064$ | $7.014 \pm 0.026$ | $7.023 \pm 0.102$ |
| $D$ (pc) | $324 \pm 9$ | $322 \pm 4$ | $320 \pm 16$ |

Notes: The system KIC 6287172 belongs to ELV type. The Gazeas & Niarchos (2006) method is not included in Table 7 because it is only applicable to W UMa type.

Table 6 The Absolute Parameters of the System KIC 6050116

| Parameter | Harmanec  | Maceroni  | Gazeas  |
|-----------|-----------|-----------|---------|
| $M_1 (M_\odot)$ | $0.702 \pm 0.059$ | $0.788 \pm 0.063$ | $0.887 \pm 0.133$ |
| $M_2 (M_\odot)$ | $0.402 \pm 0.034$ | $0.452 \pm 0.036$ | $0.508 \pm 0.076$ |
| $R_1 (R_\odot)$ | $1.680 \pm 0.049$ | $1.746 \pm 0.048$ | $1.816 \pm 0.086$ |
| $R_2 (R_\odot)$ | $0.757 \pm 0.022$ | $0.741 \pm 0.020$ | $0.771 \pm 0.037$ |
| $L_1 (L_\odot)$ | $0.606 \pm 0.018$ | $0.582 \pm 0.016$ | $0.605 \pm 0.029$ |
| $L_2 (L_\odot)$ | $0.225 \pm 0.013$ | $0.216 \pm 0.012$ | $0.233 \pm 0.023$ |
| $M_1 \text{bol}1$ | $6.360 \pm 0.064$ | $6.406 \pm 0.060$ | $6.320 \pm 0.101$ |
| $M_2 \text{bol}2$ | $6.945 \pm 0.064$ | $7.032 \pm 0.060$ | $6.946 \pm 0.101$ |
| $D$ (pc) | $233 \pm 7$ | $228 \pm 6$ | $237 \pm 11$ |

Table 7 The Absolute Parameters of the System KIC 6287172

| Parameter | Harmanec  | Maceroni  |
|-----------|-----------|-----------|
| $M_1 (M_\odot)$ | $1.376 \pm 0.12$ | $1.239 \pm 0.054$ |
| $M_2 (M_\odot)$ | $0.883 \pm 0.070$ | $0.751 \pm 0.033$ |
| $R_1 (R_\odot)$ | $1.899 \pm 0.055$ | $1.834 \pm 0.026$ |
| $R_2 (R_\odot)$ | $0.934 \pm 0.027$ | $0.902 \pm 0.013$ |
| $L_1 (L_\odot)$ | $0.779 \pm 0.023$ | $0.752 \pm 0.011$ |
| $L_2 (L_\odot)$ | $1.532 \pm 0.087$ | $1.429 \pm 0.041$ |
| $M_1 \text{bol}1$ | $4.277 \pm 0.064$ | $4.352 \pm 0.031$ |
| $M_2 \text{bol}2$ | $4.688 \pm 0.064$ | $4.764 \pm 0.031$ |
| $D$ (pc) | $401 \pm 12$ | $387 \pm 6$ |
Maceroni & Van’t Veer (1996) gives the best results for our close binary systems which are in a state of thermal contact.

For the system KIC 2715417, the primary component has a mass of $0.888 \pm 0.032 M_\odot$ while the secondary component has a mass of $0.257 \pm 0.009 M_\odot$. The distance equals $322 \pm 4$ pc. Each component has a cool spot while the spectral types are K0 and K2 for the primary and secondary components respectively. The system KIC 6050116 consists of two components with masses of $0.788 \pm 0.063 M_\odot$ and $0.452 \pm 0.036 M_\odot$ for the primary and secondary components respectively. This system is located at a distance of $228 \pm 6$ pc. Only the primary component has a cool spot, while the spectral type is K5 for both components of the system.
The intersection between the two lines of binary system and ZAMS, applied to get the total mass and luminosity of the system KIC 6287172.

The third binary system KIC 6287172 has larger masses of 1.239 ± 0.054 $M_\odot$ and 0.751 ± 0.033 $M_\odot$ for the primary and secondary components respectively. The system is further away than the previous two systems at a distance of 387 ± 6 pc, while the spectral type is F5 for both the primary and secondary components without any spot for both components. The difference between two maxima in any light curve indicates the presence of a spot on the surface of the component which is called the O’Connell effect (O’Connell 1951). The presence of cool spots on the primary and secondary components of the system KIC 2715417 and the primary component of the system KIC 6050116 signifies the existence of a magnetic field inside the spots which inhibits convection, similar to the case of sunspots.

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