Investigation of Penetrative Convection in Stratified Fluids Through 3D PTV

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Abstract The adequacy of existing theories of flow and transport in stably stratified fluids when “Penetrative Convection” occurs requires experimental methods able to provide fully three-dimensional descriptions of both the Eulerian velocity and Lagrangian particle trajectories within the system. Here we report on boundary layer experiments that rely upon a three camera 3D-PTV photogrammetric technique. A combination of image and object space based information was employed to establish the spatio-temporal correspondences between particle position at consecutive time steps. The system calibration features have been examined in details. An appropriate calibration procedure for the stereoscopic system is necessary. It provides the intrinsic and extrinsic parameters of the stereoscopic system in order to be used for determining the correspondence of points in the object or world reference frame and in the image reference frame. The photogrammetric principles used by 3D Particle Tracking Velocimetry are described. First the fundamental mathematical model of the collinearity condition and its extensions are explained. Then the epipolar line intersection method built upon multicamera correspondences (structure from stereo) is discussed as well as the procedure of reconstruction of particle trajectories in a single step by conjunctly establishing the spatial and temporal correspondences between particle images. The laboratory model employed to simulate the evolution of the mixing layer in a stably stratified fluid body consists of a tank filled with water and subjected to heating from below. Pollen particles were employed as the tracer. Thermocouples were used to measure temperature. Tests on simulated data were performed as well to ensure the method’s operability and robustness. The great variety of data sets that were processed during the development of the spatio-temporal matching algorithm show its general applicability for a wide range of different 3D-PTV measurement tasks.

1. Introduction

A penetrative convective flow is observed in the atmosphere when solar heating creates an unstable boundary layer at the earth surface. In many lakes an analogous phenomenon occurs starting from the upper free surface and penetrating downward with time. In the ocean, under calm conditions, the upper 20 or 30 m usually exhibit a continuous, moderately stable density distribution. When wind begins to blow over the surface, turbulence in the water is generated both by the mean shear and by sporadically breaking waves. With time, entrainment or erosion by the turbulence of underlying denser water causes the turbulent convective layer to deepen. Relatively rapid mixing creates an approximately uniform density, temperature and velocity distribution in the upper layer, and entrainment takes place across the interface between the turbulent and stable fluids. The flow field when penetrative convection occurs is organized into domes or spatial regions with significant vertical motion. In the atmosphere the domes spring up from the heated surface below and stress the stable layer above creating internal waves above the mixing layer. In lakes and oceans, domes start at the cooler surface and stress the stable layer below. Convection is initially organized in persistent coherent structures, but later the flow becomes turbulent and the structures form and break randomly in space. Within the unstable layer, the fluid temperature, density and velocity change rapidly with time.

The dynamics of penetrative convection in nature influences the transport and mixing features of stratified fluids, in fact the flux through the interface between the mixing layer and the stable layer plays a fundamental role in characterizing and forecasting the distribution of chemical species with implication for:
- air or water quality (dispersion of pollutants, released inside the mixing layer, is confined inside it)
- absorption of UV radiation (in case of ozone, a natural filter for UV in the upper atmosphere, but a harmful contaminant in the lower troposphere)
- climate change (in case of greenhouse gases)
- water turnover, ecosystems, algal blooms and eutrophization (in case of oxygen and nutrients in oceans and lakes)

When 2D techniques are employed to detect the velocity field, the flow is illuminated with a thin light sheet and only the velocity components within this sheet can be evaluated. Although a few methods exist for analyzing 3D velocities in a point (3D laser Doppler anemometry; Hinsch and Hinrichs, 1996) or plane (3D stereo-PIV; Stuer et al., 1999), only a fully 3D technique based on the illumination of a flow volume rather than a flow sheet will give the information needed to construct the instantaneous 3D velocity fields. There exist a number of imaging-based measurement techniques for determining 3D velocity fields in an observation volume. Among these are:

- scanning techniques (Guezzennec et al., 1994; Moroni and Cushman, 2001);
- holographic techniques (Hinsch and Hinrichs, 1996; Zhang et al, 1997);
- defocusing techniques (Willert and Gharib, 1992);
- photogrammetric techniques (3D-PTV; Maas, 1992; Kasagi and Nishino, 1990).

We will focus on 3D-PTV which is a 3D extension of the 2D particle-tracking method. Instead of tracking particles in a thin light sheet, the particles are now tracked in an illuminated volume. The obtained 3D particle trajectories can be used to calculate the 3D velocity field. Because the actual path of the particles is analyzed, particle-tracking techniques are generally more accurate than PIV-based techniques (Cowen and Monismith, 1997; Moroni and Cenedese, 2005). 3D-PTV is based on reconstructing 3D trajectories of reflecting tracer particles through a photogrammetric recording of image sequences.

Suzuki et al (2000), Kief et al (2002), Doh (2004), Willneff and Grueg (2002), present the application of three-dimensional particle tracking velocimetry in different fluid mechanics experiments, demonstrating the increasing interest 3D-PTV is gaining and the progresses in the algorithms developed.

Here we present the application of 3D-PTV to the study of penetrative convection in stratified fluid heated from below. The 3D-PTV system has been designed to be able to: image a volume far away the boundary walls, lengthen the trajectories, and improve the accuracy of the procedure through a careful test on synthetically generated data. A physically based photogrammetric calibration of the stereoscopic arrangement was employed and its accuracy tested. The effects of multimedia geometry on calibration parameters is taken into account. The combination of image and object space based information is employed to establish the correspondences between particle positions (structure from stereo reconstruction). A particle tracking algorithm was then employed to reconstructed 3D trajectories.

The paper is organized as follows. In Section 2 the experimental set-up and procedures are described. Section 3 presents a thorough discussion of the photogrammetric particle tracking algorithm showing the details of the calibration procedure, the structure from stereo analysis. The performance of the matching algorithm applied to synthetic data as well as the results of the fluid flow measurements are discussed in Section 4. The article ends with a brief discussion and some conclusions.
2. Experimental set-up

The laboratory model consists of a convection chamber containing an initially stable, density stratified fluid, which is heated from below to cause destabilization.

![Sketch of the experimental set up](image)

The bottom of the parallelepiped test section is horizontal and hence the fluid is homogeneous laterally. Heating the fluid from below creates penetrative convection. Distilled water is used for the fluid phase and pollen particles of about 80 µm mean diameter are used for the passive tracer to reconstruct particle trajectories. The test section is a tank with a square base (41×41 cm²) and height 40 cm (Figure 1). Its lateral sides are insulated by 3 cm thick removable polystyrene sheets. When images are acquired, the insulation on the sides facing the cameras is removed. A diffuser, which also acts to insulate the upper surface, floats on the surface of the water as it fills the tank. A warm tank of water drains by gravity into a continually stirred colder tank and that tank in turn drains to the diffuser. While the diffuser floats upwards, it fills the test section creating a linear stratification of the fluid, cold to hot from bottom to top. Sixteen thermocouples are spaced vertically in the tank to record changes in temperature.

| Experiment | \( T_{b0} \) (K) | \( T_{bc} \) (K) | \( 1/\alpha = (\partial \bar{T} / \partial z) \) (K/m) |
|------------|-----------------|-----------------|-----------------------------------|
| 1          | 283.15          | 308.15          | 120.38                            |
| 2          | 293.15          | 308.15          | 55.29                             |

Table 1 Details of initial condition for exp #1 and #2. \( T_{b0} \) is the temperature of the bottom before heating starts, \( T_{bc} \) is the final heating temperature and \( 1/\alpha \) is the temperature gradient.

Following initial stratification of the tank, a hot cryostatically controlled water bath is attached to a
metal base plate and experiment begins. A 15×15×40 cm$^3$ light volume produced by a high power lamp (1000 W) was employed to illuminate the central region of the test section. Images of the pollen particles were recorded using three synchronized CCD cameras with a time resolution of 25 frames per second and a space resolution of 764×576 pixels. Synchronization of the cameras is obtained by using the sink signal from one of the cameras as triggering signal for the other ones. Each camera is connected to a different color input of a RGB frame grabber and the full resolution image are stored as single channel bmp files. Two experiments are presented here. Details are reported in Table 1.

Figure 2A displays the stratification profiles before heating starts. The profiles measured are overlapped to linear interpolation that is very close to the experimental data. When heating from below it started, the temperature profile changes with time as far as the phenomenon evolves (Figure 3). Vertical temperature profiles allow measuring of the growth of the mixing layer with time ($z_i(t)$).

Figure 2. Stratification profiles before heating starts for exp #1 and #2 (A) and temporal evolution of the mixing layer height ($z_i(t)$) for the same experiments (B).

Figure 3. Temperature vertical profiles evolution during heating for experiment #1 and #2 (the stratification profile is the one at $t=0$ s)

Three portions characterize each profile. The portion of the profile close to the boundary presents a negative gradient related to the existence of the thermal boundary layer. Then, the profile has a uniform temperature, $\overline{T}(t)$, where the mixing layer is located. Finally, above the mixing region, the temperature profile practically collapses onto the straight line of the initial stratification. The
temperature profile in the stable layer is not noticeably affected by the growing mixing layer. Each temperature profile is associated to a time given in the legend even though it was obtained through averaging temperature data acquired for 20 seconds at each thermocouple location. \( \bar{T}(t) \) is related to \( z_i(t) \) through the following relation:

\[
z_i(t) = \alpha(\bar{T}(t) - T_{i0})
\]

Knowing the mean temperature within the mixing layer, the height can therefore be calculated (figure 2 B). As we can clearly see from Figure 2, the vertical temperature gradient plays a fundamental role for the mixing layer dynamics. Starting from the same heating forcing, a greater temperature gradient, see exp #1, opposes a greater resistance to increasing the convective region, resulting in a lower mixing layer height at each time step.

3. Photogrammetric Particle Tracking

The geometrical features of the experimental apparatus have determined the camera arrangement; three CCD cameras were mounted equidistant from the test section with each optical axis perpendicular to the corresponding side of the tank it faces. Subsequently, with a square based tank, the optical axes of successive cameras are at 90° angles.

The starting points for the photogrammetric reconstruction of a 3D object in real space from 2D image projections are: the pinhole model for cameras and the mathematical relations of photogrammetry based on the collinearity assumption. According to the pinhole principle a camera is modeled by its projective center \( C \) and the image plane \( R \). A 3-D point \( P \) is projected into an image point \( P' \) given by the intersection of \( R \) with the line containing \( C \) and \( P \). The line containing \( C \) and orthogonal to \( R \) is called the optical axis and its intersection with \( R \) is the principal point (PP). The distance between \( C \) and \( R \) is the focal distance \( c \) (Figure 4A).

Introduce the following reference frames as sketched in Figure 4A:

- the world reference frame \( OXYZ \) is an arbitrary 3D reference frame where the position of 3D points in the scene are expressed and can be measured directly;
- the image reference frame \( \xi\eta\zeta \) is the coordinate system where the position of pixels in the image are expressed (\( \zeta = 0 \) for all image point and \( \zeta = c \) for the projective center);
- the camera standard reference frame \( Cxyz \) is a 3D frame solidal to the camera, centered in \( C \), with the \( z \) axis coincident with the optical axis, \( x \) parallel to \( \xi \) and \( y \) parallel to \( \eta \).

According to those reference frames the fundamental relations of photogrammetry allow one to determine the image coordinates \( \xi \) and \( \eta \) of a generic image point \( P' \) from the \( X, Y, Z \) coordinates of the corresponding object point \( P \) in the world reference frame, knowing the camera space orientation and its intrinsic parameters.

Starting from the collinearity condition, which states that the object point, camera projective center, and image point must lie on a straight line (Mass, 1992), the relations are analytically described by Eqs. 1 and 2:

\[
\xi = \xi_0 - \frac{r_{11}(X - X_0) + r_{21}(Y - Y_0) + r_{31}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)} = \xi_0 - \frac{Z_\zeta}{N} \\
\eta = \eta_0 - \frac{r_{12}(X - X_0) + r_{22}(Y - Y_0) + r_{32}(Z - Z_0)}{r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)} = \eta_0 - \frac{Z_\eta}{N}
\]

\((X, Y, Z)\): object point coordinates in the world reference frame
\((X_0, Y_0, Z_0)\): camera projective center coordinates in the world reference frame
\(R = r_{ij}\): elements of \(3 \times 3\) rotation matrix with angles \(\omega, \phi, \kappa\)
\((\xi_0, \eta_0)\): image principal point
\(c\): image focal distance
The spatial rotation matrix $R$ indicates the spatial arrangement of the frame related to the coordinate system $OXYZ$. The elements $r_{ik}$ can be expressed as a function of the rotations $\omega$, $\phi$ and $\kappa$ (3):

$$
R_{oxyz} =
\begin{pmatrix}
\cos \phi \cos \kappa & -\cos \phi \sin \kappa & \sin \phi \\
\cos \omega \sin \kappa + \sin \omega \sin \phi \cos \kappa & \cos \omega \cos \kappa - \sin \omega \sin \phi \sin \kappa & -\sin \omega \cos \phi \\
\sin \omega \sin \kappa - \cos \omega \sin \phi \cos \kappa & \sin \omega \cos \kappa + \cos \omega \sin \phi \sin \kappa & \cos \omega \cos \phi
\end{pmatrix}
$$

where $\omega$ is the rotation around the X axis, $\phi$ is the rotation around the Y axis and $\kappa$ is the rotation around the Z axis (see figure 4A). The order the rotations have to be applied is $\omega$, $\phi$ and $\kappa$. The positive values correspond to anticlockwise rotations.

Figure 4. A. Collinearity condition with camera inverted for drawing purposes; $\xi_0=\eta_0=0$ (from Maas, 1993). The reference system $OX'Y'Z'$, parallel to the image system $\xi\eta\zeta$ and rotated with respect as OXYZ, is useful to derive eqs (1) and (2); B. Sketch of calibration procedure.

Eqs 1 and 2 show that every point in the world space have a correspondent point in the image plane, but if we try to find the X,Y,Z coordinates of a point from one view, knowing $\xi$ and $\eta$, we can notice that infinite points in the object space correspond to a single point in the image plane. For this reason, reconstructing the 3D geometry from one view is impossible, and at least a second view of the same object is needed. Parameters to be determined to employ equations (1) and (2) are the intrinsic parameters ($\xi_0$, $\eta_0$, $c$) and the extrinsic parameters ($\omega$, $\phi$, $\kappa$, $X_0$, $Y_0$ and $Z_0$). The camera
constructor insures $\xi_0$ and $\eta_0$ are zero. System calibration is needed to deduce the remaining 7 unknown parameters. Additional parameters for systematic image deformation and the multimedia geometry are not included in the calibration procedure, but calibration was conducted both inside water and air to directly check the effects of a multimedia geometry and include those effects onto the calibration parameters.

We have chosen a world coordinate reference system (OXYZ) solidal to a lower corner of the test section. Considering the point of view of camera 2, the axes are parallel to the three edges of the test section with the X axis oriented from left to right, the Z axis from the bottom to the top and the Y axis moving deeper inside the tank (Figure 4B).

### 3.1 Calibration

The aim of the calibration procedure for a stereoscopic system is to determine the intrinsic and extrinsic parameters of cameras in order to solve the photogrammetric problem. The intrinsic parameters describe the geometrical and optical characteristics of the camera-optics-lens system: focal distance, coordinates of the principal point in the image reference frame, the pixel shape coefficient (0 corresponds to rectangular pixels), the radial and tangential distortion coefficients due to the camera optics.

The extrinsic parameters map the camera reference coordinates xyz to the world reference coordinates XYZ. They describe the position in the object space of cameras in terms of the translation vector $(X_0,Y_0,Z_0)$ and rotation angles $(\omega, \varphi, \kappa)$.

In many cases, the overall performance of the structure from stereo reconstruction step strongly depends on the accuracy of the camera calibration, which is a very sensitive procedure to be conducted with extreme carefulness. Several methods for geometric camera calibration are presented in the literature divided into explicit, based on the above physical parameters, and implicit methods, where the physical parameters are replaced by a set of non-physical implicit parameters that are used to interpolate between some known target points (Heikkila and Silven, 1997).

We developed an explicit physically based calibration procedure using Eqs (1) and (2) in their inverse form.

Pre-calibration is performed to ensure each camera’s optical axis is orthogonal to the corresponding test section face; it provides the starting values of the rotation angles for the calibration procedure (see table 2). The final number of parameters to be determined for each camera is seven: $c$, $X_0$, $Y_0$, $Z_0$, $\omega$, $\varphi$, $\kappa$.

The procedure adopted for calibration is summarized here: a number of 3D positions, whose $(X_i, Y_i, Z_i)$ coordinates are known, are sequentially imaged by each camera. To check the quality of the calibration procedure the expected position of points in image space given their 3D position and interior and exterior calibration parameters is computed. Statistics (mean and variance) of the differences between expected and measured positions provide an estimator for the reliability of the entire procedure (table 3).

The following steps allow calibrating the acquisition system (figure 4B):

1) place a 1 cm equally spaced grid target inside the test section (the grid presents alternatively distributed black and white squares).
2) mount the target on a precision motor positioner, which can produce motor controlled and accurate movements along two perpendicular directions;
3) for each camera acquire images of the target for seven different positions, each parallel to the others and to the corresponding face of the test section, along the optical axis direction of the considered camera. The coordinates of each target point in the world reference frame $(X_i,Y,Z_i)$ are measured (step 1 in fig. 4B);
4) reconstruct the target point coordinates in the image reference frame $(\xi_0,\eta_0)$, through a corner detection code (step 2 in fig. 4B);
5) convert the image coordinates from pixels to centimeters using the camera sensible area dimensions (for our CCD 0.64906x0.48306 cm$^2$ corresponds to 782x582 pixels);

6) identify the camera intrinsic and extrinsic orientation parameters in the absolute reference system using the photogrammetric approach (step 3 in fig. 4B).

7) compute the estimated coordinates $(\zeta_i, \eta_i)$ using the photogrammetric equations, together with the calibration parameters found in the previous step, and perform the error analysis (step 4 in fig. 4B).

Let us accurately describe the point 6. We use the classical approach of minimizing a nonlinear error function through the least square estimation.

The latter is well known when the relation among “observations” and “unknowns” is linear. In fact, we have a system of linear equations:

$$ I_m = a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mu}x_u = a_{mk}x_k $$

where

- $x_k$ are unknowns;
- $I_m$ are the observations;
- $a_{mk}$ are coefficients.

If the number of observations is equal to $u$, then we have a system of $u$ linear equations to determine $u$ unknowns $x_k$. The solution of the system (in matrices) $I = Ax$ is $x = A^{-1}I$. If the number of observations is $n$, where $n > u$, as it should be to check the observations and to increase the accurateness of the unknowns determined, the problem of compensation arises. It can be solved by forcing the sum of the square of the corrections $v$ to be minimum. Let us rewrite the system $I = Ax$ in the form of “observation equations”

$$ v = Ax - I $$

The unknown $x_k$ can be obtained forcing the minimum condition:

$$ v^T v = \text{min} = (Ax - I)^T (Ax - I) = x^T A^T A x - 2 I^T A x + I^T I $$

The solution is:

$$ \frac{\partial(v^T v)}{\partial x} = 2x^T A^T A - 2I^T A = 0 $$

$$ \hat{x} = (A^T A)^{-1} A^T I $$

The matrix $(A^T A)$ is called matrix of “normal equations” (or normal matrix); the matrix $A$ is called matrix of the “observation equations” (or draw matrix).

It stands to reason that equations of photogrammetry describe a non-linear relation between coordinates in image plane (observations) and calibration parameters (unknowns); thus, in order to solve the calibration problem we need to extend the least square compensation method described above to a more general non-linear system as:

$$ \tilde{I}_m = f_m(x_1, x_2, \ldots, x_u) $$

Since the compensation algorithm is supposed to be made of linear equation, a Taylor expansion of (6) is required, with starting values $x_1^0, x_2^0, \ldots, x_u^0$:

$$ \tilde{I}_m = f_m(x_1^0, x_2^0, \ldots, x_u^0) + \left(\frac{\partial f_m}{\partial x_1}\right)^0 dx_1 + \left(\frac{\partial f_m}{\partial x_2}\right)^0 dx_2 + \ldots + \left(\frac{\partial f_m}{\partial x_u}\right)^0 dx_u $$

where the partial derivatives are computed at the approximate values $x_1^0, x_2^0, \ldots, x_u^0$.

Comparing (7) and (4), the following correspondences can be highlighted:
\[ a_{mk} \to \left( \frac{\partial f_m}{\partial x_k} \right)^0 \]

\[ l_m \to \bar{l}_m - f_m(x_1^0, x_2^0, \ldots, x_u^0) \]

\[ x_k \to dx_k \]

From the above it follows that the indirect observation compensation method coupled with the least squares method requires equations 1 and 2 to be partially derived:

\[ \frac{\partial \xi}{\partial X_0} = -\frac{c}{N^2} (r_{iz}Z_i - r_{iz}N) = a_2 \]

\[ \frac{\partial \eta}{\partial Y_0} = -\frac{c}{N^2} (r_{iz}Z_i - r_{iz}N) = a_3 \]

\[ \frac{\partial \xi}{\partial Z_0} = -\frac{c}{N^2} (r_{iz}Z_i - r_{iz}N) = a_4 \]

\[ \frac{\partial \xi}{\partial \omega} = -\frac{c}{N} \left( (Y - Y_0)r_{3z} - (Z - Z_0)r_{2z} \right) \]

\[ \frac{\partial \eta}{\partial \omega} = -\frac{c}{N} \left( (Y - Y_0)r_{3z} - (Z - Z_0)r_{2z} \right) \]

\[ \frac{\partial \xi}{\partial \phi} = \frac{c}{N} \left( Z_i \cos \kappa - Z_i \sin \kappa \right) \frac{Z_i}{N} + N \cos \kappa \]

\[ \frac{\partial \eta}{\partial \phi} = \frac{c}{N} \left( Z_i \cos \kappa - Z_i \sin \kappa \right) \frac{Z_i}{N} + N \sin \kappa \]

Each point \( P_i \) of unknown or known coordinates \((X_i, Y_i, Z_i)\) and known image coordinates \((\xi_i, \eta_i)\), provides an equation

\[ \begin{align*}

v_{\xi_i} &= \left( \frac{\partial \xi}{\partial X_0} \right)^0 dX_0 + \left( \frac{\partial \xi}{\partial Y_0} \right)^0 dY_0 + \left( \frac{\partial \xi}{\partial Z_0} \right)^0 dZ_0 + \left( \frac{\partial \xi}{\partial c} \right)^0 dc + \left( \frac{\partial \xi}{\partial \omega} \right)^0 d\omega + \\

&\quad \left( \frac{\partial \xi}{\partial \phi} \right)^0 d\phi + \left( \frac{\partial \xi}{\partial \kappa} \right)^0 d\kappa - (\xi_i^0 - \xi_i^0) \\

v_{\eta_i} &= \ldots \quad \text{(analogous)}
\end{align*} \]

Equations (22) corresponds to (7). The partial derivatives \( (\cdot)^0 \) are computed for approximate values of the unknown. \( \xi_i^0 \) and \( \eta_i^0 \) are the approximate coordinates computed by (1) and (2).

3.1.1. Calibration results

The three-camera set-up of figure 4B involves the following rotation matrices computed from the starting values of \( \omega, \phi \) and \( \kappa \) reported in Table 2 (respectively for camera #1, camera #2 and camera #3):

\[ R_1 = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \quad R_3 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \]

If we consider the pre-calibration unreliable, then the rotation matrix angles are unknowns. For each camera, Table 2 displays the number of calibration points.
Table 2. Data related to the calibration procedure

| N points | ω_st | φ_st | κ_st | N points | ω_st | φ_st | κ_st |
|----------|------|------|------|----------|------|------|------|
| Camera #1 | 2225 | 90   | 270  | 0        | 2375 | 90   | 270  |
| Camera #2 | 2246 | 90   | 0    | 0        | 2451 | 90   | 0    |
| Camera #3 | 2230 | 90   | 90   | 0        | 2429 | 90   | 90   |

The final calibration parameters are obtained by applying the least squares method several times for different starting values. At each iteration, the starting values are set close, but not equal to the outputs of the previous iteration. To make sure the minimum reached is not local, various starting values are employed to insure the results do not vary.

Figure 5 presents the comparison between the expected positions of points in image space, computed by employing their 3D position and interior and exterior calibration parameters, and measured points for the camera labelled camera #2.

Table 3 shows results of the calibration procedure both for calibration conducted in water and in air.  
Mean and standard deviation of the difference between the quantity $\sqrt{xi^2 + η^2}$ computed for expected and measured coordinates was evaluated.
Table 3. Results of the calibration procedure for calibration inside water and air (we use 25 mm optics for each camera).

### 3.2 Structure-from-stereo

The purpose of structure-from-stereo is the reconstruction of the 3D geometry in a scene from two or more views taken with pinhole cameras. Two processes are involved: correspondence or matching and reconstruction. Correspondence estimates which points in the images are projections of the same scene point. Two cameras are enough to reconstruct a 3D image, but additional views increase confidence in the results. The coordinates of corresponding points are related by the epipolar geometry. Assuming the orientation parameters of the cameras are known from the calibration procedure, given a point in one image, its conjugate must belong to a line in the other image which is called epipolar line (fig 6).

![Figure 6. Sketch of correspondence and reconstruction procedure, used to lengthen 3D trajectories](image)

Figure 6 describes the procedure to establish the stereoscopic correspondences. Starting from a point \( P_1 \) in the first image, all epipolar lines \( EP_{12} \) in the second image and \( EP_{13}, EP_{23}, EP'_{23}, EP''_{23} \) in the third are derived and which respectively candidates \( P_2, P_2, P'_2, \) and \( P_3, P_3, P'_3 \) are found.
The particle on the intersection between the epipolar lines \( P_3 \) will be the one we were looking for.

Errors can be taken into account by introducing a tolerance \( \varepsilon \) and the search area for the corresponding particle image becomes a narrow two-dimensional band-shaped window in image space.

The 2D displacement vector between corresponding points within couples of images (called conjugate pairs) is called disparity. Reconstruction recovers the full 3D coordinates of points using the estimated disparity and intrinsic and extrinsic calibration parameters. At the end a PTV algorithm can be applied to lengthen 3D trajectories.

4. Results

4.1. Simulated curling trajectories

The algorithm was tested on a synthetically generated data set simulating curling trajectories. The starting location was randomly distributed in the world reference system, the number of spots variable from trajectory to trajectory and mean velocity constant along each trajectory. Particles are allowed to move along the positive direction of the Z axis.

Image sequences were created by assuming a typical 3D-PTV set-up with three cameras. The three-dimensional trajectories have been projected on the image plane of three cameras (C1, C2 and C3) replicating the exterior and interior calibration parameters of the acquisition system employed. The synthetic trajectory lengths (in SI units) are similar to the ones expected from the experiments.

| Trajectory number | % of matching (projection using water calibration) | Error in calibration parameters | air calibration parameters |
|-------------------|---------------------------------------------------|---------------------------------|---------------------------|
|                   |                                                   | err0 | err1 | err2 |                               |
| 100               | 100                                               | 100  | 94   | 52   | 8                             |
| 200               | 100                                               | 100  | 89   | 51   | 4                             |
| 500               | 100                                               | 100  | 75   | 27   | 3                             |
| 1000              | 100                                               | 100  | 44   | 8    | 0                             |
| 2000              | 100                                               | 100  | 14   | 1    | 0                             |

Table 4. Results of the matching procedure on the synthetically generated data set (tolerance= 3 pixel)

According to the methodology previously described the matching procedure employs the epipolar geometry to determine corresponding baricenters. The 3D position of matched baricenters is compared to the synthetically generated data set and the percentage of correspondences determined. The effect of an increasing number of particles seeding the measurement volume on the matching...
procedure was tested (Table 4). The effect of a wrong evaluation of the calibration parameters was investigated as well. In the latter case, the image sequences were created by employing a fixed set of calibration parameters, while the reconstruction was carried out by using a modified set of calibration parameters, either introducing an arbitrary error (err1 and err2 in table 4) and using the calibration set found not considering the effect of water refraction. In table 4 the results labelled “err1” were obtained using a modified set of parameters producing a minimum error of 1 pixel in the estimated coordinates of the calibration target points \((\xi_i, \eta_i)\), while “err2” refers to a modified set of parameters producing a minimum error of 2 pixels of the same points. Table 4 shows as the matching procedure is extremely sensitive to errors in the calibration parameters. The percentage of matching, in fact, drastically drops introducing a very little calibration error. Using the calibration results for air with experiments conducted in water also produce a sensible loss in matching performance. The same behaviour can be observed increasing the number of trajectories.

### 4.2. Application on real images

The stereoscopic system, properly calibrated as described in section 3.3.1, and the matching-reconstruction algorithms, tested with synthetic data as described in the previous section, has been applied to real images to trace dome structures in penetrative convection experiments (for experiments details see section 2).

![Figure 8 Projections to XZ plane (A) and YZ plane (B) of matched centroids after 220 s. 20 s consecutive times are overlaid, dark colour refers to later times (exp#1 of table 1).](image)

Figure 8 shows a 20 s overlap of matched centroids and projected to XZ plane (A) and YZ plane (B) for clarity of display. The figure refers to exp #1 of table 1 after 220 s from the beginning of heating from below. Darker colours have been linked with later times, while lighter colours with earlier times. From both the two projections it can be clearly seen that the bottom turbulent region, feeding the mixing layer, is moving upward against the almost quiet stable layer above. The 3D-PTV procedure is suitable for reconstructing the displacement field (i.e. particle trajectories) in both the mixing layer and the stable one.
4. Conclusions

3D-PTV allows a more likely description of the velocity field, which occurs during evolution of the convective mixing layer, than more traditional 2D techniques. Further photogrammetric 3D-PTV, rather than "scanning" 3D-PTV (Moroni and Cushman, 2001) is more accurate when the tracer particles density is high, because particles may be tracked directly in the 3D space rather than through matching of 2D projections.

Sensitivity tests conducted on matching algorithm prove that calibration accuracy is fundamental to obtain a correct matching and particle tracking; small error in calibration parameters, or not considering water refraction effects, drastically reduces matching performance. On the other hand we can reach an accuracy less than 1 pixel (about 0.3 mm) in our calibration procedure.

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