**Numerical Computation of Electromagnetic Fields in Metals Using A Modified Finite-difference Time Domain Method**

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The Gandhi's scaled-frequency FDTD (finite-difference time domain) method for electromagnetic fields of low frequency was modified, and so can be adaptable to the numerical computation of electromagnetic phenomena in metals. The computational results by a simplified model agreed well with the analytical solutions. A new treatment for the interface of metal and dielectric was also presented, further how and why it is was explained. The low-frequency electromagnetic fields were computed successfully due to this treatment. The computational results are in excellent agreement with the experimental ones.

KEY WORDS: scaled frequency FDTD method; electromagnetic fields; numerical simulation; electromagnetic continuous casting.

1. Introduction

With the development of metallurgical industry, electromagnetic theory has been introduced and applied to improve the quality of products. In the continuous casting process, the EMS (electromagnetic stirring), soft contact EMC (electromagnetic continuous casting), and electromagnetic casting have been wildly studied recently.1–4)

Exact or analytical solutions to the time-dependent Maxwell’s equations that describe the electromagnetic fields used in metallurgical industry process are almost impossible except for very few special cases with rectangular geometrical systems and/or uniform electromagnetic properties.5) Therefore, some numerical methods emerged for electromagnetic fields (EMFs). The FDTD method is one typical example and is a direct approximation of the derivative of Maxwell’s time-dependent rotation equations originally for high frequency.5,6) It has been noticed widely because it is easier in form and faster than other methods.5–7)

In metallurgy process, the frequency of interest is usually within the range of 50–20 000 Hz, 1–4) so the application of FDTD method is very limited because of two challenges. First, during the whole processes aforementioned, the temperature of metal is more than 760°C, the magnetic permeability μ and the permittivity of the metal are equal to the ones μ0 and ε0 of vacuum respectively. To ensure stability of the solution, the time step is taken to be δ/2c. Assuming that δ has the value of 5 mm, the iterations will be great up to 108 times. Then the computer cost is very heavy and the accuracy of the computational results will be viewed with quite skepticism because of the accumulation of iterative errors of computation. A scaled-frequency FDTD method has been presented by Gandhi to reduce the iterations.7)

This method was developed originally for human model and not suitable for metal without modification. Second, the metal concerned in previous numerical simulation is taken to be perfectly conducting scatter as employed in Refs. 8), 9) when the frequency is up to 107 Hz. But it’s not the fact while the frequency in present numerical simulation is less than 20 000 Hz and great errors may be conducted.

In this study, the scaled-frequency FDTD method has been modified to simulate the EMFs in metal at low frequency. A simplified physical model with analytical solutions is used to validate the modified method. For another problem of metal and dielectric interface treatment at low frequency existing in FDTD method, a new improvement has been proposed and experiments have been made to verify the method.

2. Scaled-frequency FDTD Method

The scaled-frequency method was presented by Gandhi to reduce the number of iterations in the simulation of low-frequency EMFs in1992 and an equation suited for human model was also presented.7)

\[ E_{\text{in}}(f) = \frac{f\sigma'}{f'\sigma} E_{\text{in}}(f') \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cd - 683–688

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obtained from the following equation when the frequency is 60 Hz and electric conductivity is $1 \times 10^6$ S/m.

$$\delta x = \frac{2}{\omega \mu \sigma} \quad \ldots \ldots \ldots \ldots (2)$$

Obviously Gandhi’s scaled-frequency FDTD method cannot be used in metal without modification.

3. Modification to the Scaled-frequency FDTD method

The model for the modification is shown in Fig. 3, assuming that $a \ll b$, $b \ll l$ and the space between coil and the metal can be neglected. The electric conductivity of metal $\sigma \gg \omega \varepsilon$, so the induced current $\vec{J} = \sigma \vec{E}$. Using the MKS system of units, Maxwell’s time-dependent curl equations are

$$\nabla \times \vec{H} = \sigma \vec{E} \quad \ldots \ldots \ldots \ldots (3a)$$

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \ldots \ldots \ldots \ldots (3b)$$

In metal, Eq. (3) can be rewritten as

$$\left( \nabla^2 - \mu \sigma \frac{\partial}{\partial t} \right) \left[ \frac{\vec{H}}{\vec{E}} \right] = 0 \quad \ldots \ldots \ldots \ldots (4)$$

From Eq. (4), we can write by using $\vec{H} = \vec{H}_{e^{i\omega t}}$ and $\vec{E} = \vec{E}_{e^{i\omega t}}$

$$\left( \nabla^2 - j \omega \mu \sigma \right) \left[ \frac{\vec{H}}{\vec{E}} \right] = 0 \quad \ldots \ldots \ldots \ldots (5)$$

To solve the differential equation, following steps are listed:

(1) Equation Simplification. Assuming that $a \ll b$,

$b \ll l$, following simplification can be employed; Only $z$-directed magnetic flux density exist and then only $y$-directed component of electric field exist.

Then

$$\vec{B} = k \sqrt{2} B_z(x) \sin(\omega t + \theta_y), \quad \vec{E} = j k E_y(x) \quad \ldots \ldots (6)$$

Now, the following equation can be deduced from Eq. (5)

$$\frac{d^2}{dx^2} \frac{\vec{H}_z}{\vec{E}} - \gamma^2 \frac{\vec{H}_z}{\vec{E}} = 0 \quad \ldots \ldots \ldots \ldots (7)$$

(2) Boundary Condition: The boundary conditions can be described as

$$\frac{\vec{H}_z}{\vec{E}} \left( \frac{a}{2} \right) = \frac{Ni}{l} \quad \ldots \ldots \ldots \ldots (8)$$

So the magnetic intensity and the induced current in the metal can be obtained

$$\vec{H}_z \left( \frac{a}{2} \right) = \frac{Ni}{l} \left( e^{-\frac{\mu}{2} \pm e^{\frac{\mu}{2}}} \right) (e^{-\frac{\mu}{2} \pm e^{\frac{\mu}{2}}} \ldots \ldots (9)$$
\[ \mathbf{J} = \sigma \mathbf{E} = (\nabla \times \mathbf{H}) \times \mathbf{e}_x = -\frac{\partial \mathbf{H}}{\partial x} \quad \ldots \ldots \ldots (10) \]

The following equation can be deduced from Eqs. (9) and (10)
\[ \mathbf{J} = -\frac{NI}{l} \mathbf{h} \left( \Gamma \frac{a}{2} \right) \quad \ldots \ldots \ldots (11) \]

Equations (9) and (11) implied that a higher quasi-static frequency \( f' \) may be used with the same \( \Gamma \) and in the same boundary conditions. So Eq. (1) can be used in metal, but the electric conductivity at higher irradiation frequency \( \sigma' \neq \sigma \). A proper value of the electric conductivity at higher irradiation frequency should be determined with the same value of \( \Gamma \).

4. Comparison of Computational Solutions and Analytical Solutions

To check the validity of the modified scaled-frequency FDTD method, the model shown in Fig. 3 was adopted. The metal has the dimension of 2000×200×20 mm. Considering the symmetry of the system, only a quarter of the space was computed. 800×40×4 rectangular grids were used. The computational results were obtained using an irradiation frequency \( f' \) of 5×10^7 Hz followed by scaling to 200 Hz, 400 Hz, and 600 Hz with the same boundary condition. From Figs. 4–6, it can be seen that the computational results agree with a maximum deviation of less than 3% from analytical results. The modified method was also adopted in the simulation of EMFs of soft contact EMC mold, and the result shown in Fig. 7 is almost the same as that presented in ref. (3).

5. Treatment for Interface of Metal and Dielectric

In metallurgy process, interfaces of different media such as metal and air, metal and mold flux are frequently encountered in the numerical simulation of EMFs. The main difference of all those electromagnetic properties of the materials on both sides of the interface is electric conductivity. In this study, only the difference of electric conductivity was considered.

The tangential components of electric field intensity and normal components of magnetic field intensity are continuous on the interface. Assuming that \( x \)-direction is the normal, Maxwell’s equation on the interface can be expressed as
\[ \frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\varepsilon} (\nabla \times \mathbf{H} - \sigma \mathbf{E}) \quad \ldots \ldots \ldots (12) \]
where \( i = 1, 2 \) means two kinds of materials on both sides of the interface. \( E_{i} \) and \( E_{z} \) are two tangential components of electric field intensity respectively. In a Cartesian coordi-
nate system, scalar equation of \( E_y \) can be written as

\[
\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon} \left( \frac{\partial H_z}{\partial z} - \sigma_y E_y \right) \tag{13}
\]

Considering the continuity of \( E_y \), \( H_x \) and \( \partial H_x/\partial z \) on the interface, the following equation can be deduced

\[
\left( \frac{\partial H_z}{\partial x} \right)_1 + \sigma_1 E_y = \left( \frac{\partial H_z}{\partial x} \right)_2 + \sigma_2 E_y \tag{14}
\]

For both sides of the interface, Eq. (13) can be expressed as follows

\[
\begin{align*}
\varepsilon \frac{\partial E_y}{\partial t} &= \frac{\partial H_z}{\partial z} - \sigma_1 E_y \\
\varepsilon \frac{\partial E_y}{\partial t} &= \frac{\partial H_z}{\partial z} - \sigma_2 E_y
\end{align*} \tag{15}
\]

Finite-difference discretizations of \( \partial H_z/\partial x \) and \( \partial H_z/\partial x \) are

\[
\begin{align*}
\left( \frac{\partial H_z}{\partial x} \right)_1 &= H_z(m) - H_z(m - 1/2) \\
\left( \frac{\partial H_z}{\partial x} \right)_2 &= H_z(m + 1/2) - H_z(m
\end{align*} \tag{16}
\]

where \( m \) is the space step number of the interface, \( m + 1/2 \) is half a step away from the interface on the side of material 2, \( m - 1/2 \) and is half a step away from the interface on the side of material 1.

Then from Eqs. (14) and (16), the magnetic field intensity on the interface is

\[
H_z(m) = \frac{1}{2} \left( H_z(m - 1/2) + H_z(m + 1/2) + (\sigma_2 - \sigma_1) E_z \frac{\Delta x}{2} \right) 
\]

\[\tag{17}\]

From Eqs. (16) and (17), we can rewrite Eq. (15)

\[
\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon} \left( \frac{\partial H_z}{\partial z} - \frac{H_z(m + 1/2) - H_z(m - 1/2)}{\Delta x} - \frac{1}{2} (\sigma_1 + \sigma_2) E_y \right) 
\]

\[\tag{18}\]

Equation (18) can also be expressed as follows

\[
\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon} \left( \frac{\Delta H_z}{\Delta x} - \frac{\partial H_z}{\partial y} = \frac{1}{2} (\sigma_1 + \sigma_2) E_y \right) \tag{19}
\]

Same as the deducing of Eq. (19), scalar equation of \( E_z \) on the interface can be obtained

\[
\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon} \left( \frac{\Delta H_y}{\Delta x} - \frac{\partial H_y}{\partial y} = \frac{1}{2} (\sigma_1 + \sigma_2) E_z \right) \tag{20}
\]

Equations (19) and (20) implied that the electric conductivity of the interface is \((\sigma_1 + \sigma_2)/2\). The yielding of Eqs. (19) and (20) implied that this expression can only be adopted when the electric conductivity is the main difference of the two materials and the difference of permittivity and the magnetic permeability can be ignored.

6. Experiments

To check the validity of the new treatment for the interface, experiments were made. Figure 1 shows the schematic diagram, and the electric conductivity and the dimensions of the metal are listed in Table 1. In order to eliminate the
effects of other electric properties, the aluminum was chosen to be the experimental medium. The Gauss meter was used to measure the magnetic flux density outside the metal with the frequency of 50 Hz, and the locations of measured points are shown in Fig. 8.

7. Comparison of Computational Results and Experimental Results

From Figs. 9–11, it can be seen that the computational results agree with a maximum deviation of less than 6% from the experimental results. But the facts should be noted that, the experiments contains the error of 2%, and the discrepancies between the numerical and measured values may be partly due to errors in the experimentally determined data. Finally, the assumption of symmetry of system (in fact the experimental system is not exactly systematical) can also introduce errors to experiments.

8. Conclusions

(1) Assuming that the value of the electric conductivity at higher irradiation frequency be determined with the same value of $I$, the Gandhi’s scaled-frequency FDTD method was modified to be suited for the simulation of EMFs in metals in this study.

(2) The computational solutions using the modified method showed excellent agreement with analytical solutions.

(3) In the computation of the tangential components of electric field intensity on the interface of metal and dielectric in metallurgy processing, the electric conductivity can be regarded as the mathematical average of the materials on both sides of the interface. This method can only be adopted when the difference of permittivity and the magnetic permeability of the materials can be ignored.

(4) The comparison of numerical results and experimental ones indicated that, the new treatment of interface in the FDTD method for metal could provide acceptable accuracy for metallurgical industry process. And the results shown an attractive numerical method for electromagnetic fields in metal with high accuracy.

Nomenclature

$E$: Electric field intensity (V/m)

$H$: Magnetic field intensity (A/m)

$m$: Space grid number index of the interface in $x$-direction
\( x, y, z \): Coordinate axes in Cartesian geometry
\( \varepsilon \): Permittivity (F/m)
\( \mu \): Magnetic permeability (H/m)
\( \delta \): Cell size (m)
\( c \): Maximum velocity of EM wave (m/s)
\( \Delta t \): Time step (s)
\( \omega \): Angular frequency (rad/s)
\( j_c \): Electric current density (A/m\(^2\))
\( f \): Frequency of low-frequency electromagnetic fields (Hz)
\( f' \): Frequency of high-frequency electromagnetic fields (Hz)
\( \sigma_1 \): Electric conductivity of material 1 (S/m)
\( \sigma_2 \): Electric conductivity of material 2 (S/m)
\( \sigma \): Electric conductivity at low frequency (S/m)
\( \sigma' \): Electric conductivity at high frequency (S/m)
\( \delta_s \): Skin depth (m)

Subscripts
\( x, y, z \): Coordinate axes
1: Medium 1
2: Medium 2

Superscripts
\( \rightarrow \): Vector
\( \cdot \): Complex number

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