Soret Effect on Chemically Radiating MHD Oscillatory Flow with Heat Source through Porous Medium in Asymmetric Wavy Channel

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Abstract. The oscillatory flow of an incompressible viscous and electrically conducting thin fluid in an asymmetric wavy channel filled with porous medium under the influence of externally applied magnetic field and heat source is investigated. Assuming Boussinesq approximation, the governing equations of the flow i.e momentum equation, energy equation and concentration equations are formulated. Under the combined influence of heat source and thermo-diffusion (Soret) effects, closed form solutions of the governing equations are obtained for the velocity, the temperature, and the concentration profiles. The effects of the various parameters entering into the problem on dimensionless velocity, temperature, concentration distributions are presented graphically for various values of Soret number \(Sr\), Buoyancy ratio \(N\), Grashof number \(Gr\), \(Ge\), radiation parameter, heat source parameter and magnetic field effect \(M\).

1. Introduction

The study of flows through porous media has become of main interest due to its applications in many scientific and engineering problems associated with petroleum engineering to study the movements of natural gas, oil and water through the oil reservoirs; in chemical engineering for the filtration and water purification processes. Further, to study the underground water resources and seepage of water in river beds one need the knowledge of the fluid flow through porous medium. Muthuraj [1] investigated heat transfer effect on MHD oscillatory flow through asymmetric wavy channel and Satya Narayana [2] presented heat and mass transfer effects on MHD oscillatory flow through irregular channel.

The porous medium is in fact a non-homogeneous medium but it may be possible to replace it with a homogeneous fluid having dynamical properties equivalent to those of non-homogeneous continuum. Hence one can study the flow of hypothetical fluid and the complicated problem of the flow through a porous medium as reduced flow problem of homogeneous fluid. Free convection define , the change in temperature cause density variation leading to buoyancy forces acting on the fluid elements.

Hossain [3] and K.D. Singh [4] have presented some problems of mixed convection and heat transfer in the vertical rotating channel.

In processes such as drying, evaporation at the surface of water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occurs simultaneously.
Heat transfer for an electrically conducting fluid flow under the influence of magnetic fields are considered significant due to its applications in many engineering problems such as nuclear reactors and those dealing with liquid metals. Eckert and Drake [5] have pointed out that when a convective flow of the mass is caused by temperature difference one cannot neglect the thermal diffusion effect which is commonly known as Soret effect due to its practical applications in engineering and science.

Makinde [6] have presented heat transfer to MHD oscillatory flow in a channel filled with porous medium and Falade [7] studied the combined effect of suction/injection with chemical reaction on MHD oscillatory flow.

Many practical diffusive operations involve the molecular diffusion of a species in the presence of a chemical reaction within or at the boundary. There are two types of reactions, one is the homogeneous reaction, which occurs uniformly throughout a given phase. The species generation in a homogeneous reaction is analogous to the internal source of the heat generation. The second one is the heterogeneous reaction, which takes place in a restricted region or within the boundary of a phase. It can also be treated as a boundary condition similar to the constant heat flux condition in heat transfer.

Misra [8] have presented slip velocity in blood flow through stenosed arteries. In addition Ramachandra Rao and Ogulu [9, 10] reported MHD oscillatory flow of blood through channels of variable cross section and heat transfer on oscillatory blood flow in indentured porous artery. All industrial chemical processes are designed to transform cheaper raw materials to high value products (usually via chemical reactions). A reactor, in which such chemical transformations take place, has to carry out several functions like bringing reactants into intimate contacts, providing an appropriate environment (temperature and concentration fields) at adequate time, and allowing for the removal of products. Fluid dynamics plays a vital role in establishing the relationship between the reactor hardware and the reactor performance.

Sasikumar [11, 12] have analyzed free convective MHD oscillatory flow past parallel plates in a porous medium with heat source, chemical reaction and slip flow effects in asymmetric channel. Sasikumar [11] have investigated effects of heat and mass transfer on MHD oscillatory flow with chemical reaction and slip conditions in asymmetric wavy channel. Adesanya [13] studied MHD oscillatory slip flow and heat transfer filled with porous media.

Many researchers Sachin Ahmed [17] have investigated suction effects on MHD oscillatory flow through porous channel. Acharya [18] have presented magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and heat flux. Devika and Misra [19, 20] have analyzed MHD oscillatory flow of a visco-elastic fluid in a porous channel with chemical reaction and MHD oscillatory channel flow heat and mass transfer in a physiological fluid in presence of chemical reaction respectively. When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of a more intricate nature. It has been observed that an energy flux can be generated not only by temperature gradients but also by concentration gradients. The energy flux caused by a concentration gradient is termed the diffusion-thermo (Dufour) effect. On the other hand, mass fluxes can also be created by temperature gradients and this embodies the thermal-diffusion (Soret) effect.

M.C.Raju, A.J Chamka [21] studied the soret effect combined with radiation and chemical reaction on MHD Mixed convective flow past semi vertical plate. [22] investigated the effects of heat and mass transfer on two-dimensional unsteady MHD free convection flow past a vertical
porous plate in a porous medium in the presence of thermal radiation under the influence of Dufour and Soret effects. Soret-Dufour and radiation effect on MHD flows arise in many areas of engineering and applied physics. The study of such flow has application in MHD generators, chemical engineering, nuclear reactors, geothermal energy, reservoir, engineering and astrophysical studies.

[23] analysed Soret and Dufour effects on the magnetohydrodynamic (MHD) peristaltic flow of variable viscosity fluid in a symmetric channel. [24] examined the combined effect of spatially stationary surface waves and the presence of fluid inertia on the free convection along a heated vertical wavy surface embedded in an electrically conducting fluid saturated porous medium, subject to the diffusion-thermo (Dufour), thermo-diffusion (Soret) and magnetic field effects. Diffusion-thermo implies that the heat transfer is induced by concentration gradient, and thermo-diffusion implies that the mass diffusion is induced by thermal gradient. [25] A numerical study of heat and mass transfer in two-dimensional stagnation-point flow of an incompressible viscous fluid over a stretching vertical sheet in the present of thermal-diffusion (Soret) and diffusion-thermo (Dufour) numbers is investigated.

[26] The heat and mass transfer characteristics of natural convection about a vertical surface embedded in a saturated porous medium subjected to a magnetic field is numerically studied, by taking into account the diffusion-thermo (Dufour) and thermal-diffusion (Soret) effects. In most of the studies related to heat and mass transfer process, Soret and Dufour effects are neglected on the basis that they are of a smaller order of magnitude than the effects described by Fouriers and Ficks laws. But these effects are considered as second order phenomena and may become significant in areas such as hydrology, petrology, geosciences, etc. The Soret effect, for instance, has been utilized for isotope separation and in mixture between gases with very light molecular weight (H2, He) and of medium molecular weight (N2, air). The Dufour effect was found to be of order of considerable magnitude so that it cannot be neglected (Eckert and Drake [5]).

[26] The heat and mass transfer characteristics of natural convection about a vertical surface embedded in a saturated porous medium subjected to a magnetic field is numerically studied, by taking into account the diffusion-thermo (Dufour) and thermal-diffusion (Soret) effects.

2. Mathematical Formulation

Consider the flow of viscous, incompressible, electrically conducting and chemically radiating optically thin fluid in an asymmetric wavy channel in presence of heat source. The channel wavy walls are given by

\[
\begin{align*}
H_1 &= d_1 + a_1 \cos \left( \frac{2\pi x}{\lambda} \right) \\
H_2 &= -d_2 - b_1 \cos \left( \frac{2\pi x}{\lambda} + \phi \right)
\end{align*}
\]  

(1)

where \(a_1, b_1, d_1, d_2, \phi\) are given by \(a_1^2 + b_1^2 + 2a_1b_1 \cos \phi \leq (d_1 + d_2)\)

Channel wall temperatures are maintained at \(T_1, T_2\) respectively in order to induce radiative heat transfer. Electrical conductivity of the fluid and electro magnetic force are assumed to be very small compared to the applied external magnetic field normal to the channel walls. The basic flow in the medium is entirely due to buoyancy force caused by temperature difference between the wall and the medium. All fluid properties are assumed to be constant except the density variation with temperature. Assuming pressure gradient to be oscillatory across the ends of the channel, the resulting flow is unsteady oscillatory.

Under the usual Boussinesq approximation the equations governing the flow are as follows: The governing equations of the flow i.e momentum, energy and species concentration equations
are formulated as follows:

\[
\begin{align*}
\frac{\partial u}{\partial t} &= -\frac{1}{\rho} \frac{\partial \rho}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} + g\beta_T (T - T_2) + g\beta_C (C - C_2) - \frac{\nu}{K^*} u \\
\frac{\partial T}{\partial t} &= \frac{k}{\rho C_P} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_P} \frac{\partial q}{\partial y} + \frac{Q^*}{\rho C_P} (T - T_2) \\
\frac{\partial C}{\partial t} &= D \frac{\partial^2 C}{\partial y^2} - K^*_r (C - C_2) + \frac{DK_T}{T_m} \left( \frac{\partial^2 T}{\partial y^2} \right)
\end{align*}
\]

With boundary conditions

\[
\begin{align*}
u &= 0, T = T_1, C = C_1 \text{ on } y = H_1 \\
u &= 0, T = T_2, C = C_2 \text{ on } y = H_2
\end{align*}
\]

The heat flux due to radiation is given by Plank’s approximation as

\[
\frac{\partial q}{\partial y} = 4\alpha^2 (T_2 - T)
\]

We introduce the following non-dimensional quantities

\[
\begin{align*}
\xi &= \frac{x}{d}; \eta = \frac{y}{d}; \tau = \frac{t}{\nu}; \xi_t = \frac{d}{d_t}; H_1 = \frac{h_1}{d_t} \\
H_2 &= \frac{h_2}{d_t}; d = \frac{d_2}{d_t}, a = \frac{a_1}{d_t}, b = \frac{b_1}{d_t}, Da = \frac{K^*}{d_t} \\
Re &= \frac{U d_t}{\nu}; Q = \frac{\rho C_P}{K} \frac{d_2^2}{d_t}; \Gamma = \frac{1}{Da}; Sc = \frac{D}{\nu d}; \\
K^*_r &= \frac{K_T}{d_t}; \tau = \frac{\rho d_t^2}{\nu d_t}; P_c = \frac{U d_t C_P}{K} \\
M &= \frac{\rho d_t^2 d_2^2}{\nu d_t}; G_T = \frac{\rho d_2 (T_1 - T_2) d_2^2}{\nu U}; G_C = \frac{\rho d_2 (C_1 - C_2) d_2^2}{\nu U} \\
\theta &= \frac{T - T_2}{T_1 - T_2}; \phi = \frac{C - C_2}{C_1 - C_2}; s^2 = \frac{1}{Da}; Sr = \frac{DK_T}{T_m d_t} \left( \frac{T_1 - T_2}{C_1 - C_2} \right)
\end{align*}
\]

The boundary conditions in non-dimensional form

\[
\begin{align*}
h_1 &= 1 + \frac{a \cos 2\pi x}{d} \\
h_2 &= -d - \frac{b \cos (2\pi x + \phi)}{d} \text{ where } a, b, d \text{ and } \phi \text{ satisfy } a^2 + b^2 + 2ab \cos \phi \leq (1 + d)^2.
\end{align*}
\]

The governing equation in non-dimensional form

\[
Re \frac{\partial u}{\partial t} = -\frac{\partial \rho}{\partial x} + \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gc\phi - (s^2 + M^2) u
\]
\[ P \epsilon \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + (N^2 + Q) \theta \tag{10} \]
\[ \frac{\partial \phi}{\partial t} = Sc \frac{\partial^2 \phi}{\partial y^2} - Kr \phi + Sr \frac{\partial^2 \theta}{\partial y^2} - kc \phi \tag{11} \]

with boundary conditions in non-dimensional form
\[
\begin{align*}
    u = 0, \theta = 1, \phi = 1, & \text{ on } y = h_1 \\
    u = 0, \theta = 0, \phi = 0, & \text{ on } y = h_2
\end{align*}
\tag{12}
\]

3. Method of Solution

Assuming pressure gradient for purely oscillatory flow as
\[
\begin{align*}
    \frac{\partial p}{\partial x} &= \lambda e^{i\omega t} \\
    u(y, t) &= u_0(y) e^{i\omega t} \\
    \theta(y, t) &= \theta_0(y) e^{i\omega t} \\
    \phi(y, t) &= \phi_0(y) e^{i\omega t}
\end{align*}
\tag{13}
\]

Substituting (13) in (9), (10), (11), we obtain the following set of ordinary differential equations:
\[
\begin{align*}
    \left[ D^2 - (s^2 + M^2 + i\omega Re) \right] u_0 &= -\lambda - Gr \theta_0(y) - Gc \phi_0(y) \tag{14} \\
    \left[ D^2 + (N^2 + Q - i\omega Pe) \right] \theta_0 &= 0 \tag{15} \\
    \left( D^2 + L^2 \right) \phi_0 &= 0 \tag{16}
\end{align*}
\]

The corresponding boundary conditions are given by
\[
\begin{align*}
    u_0 &= 0, \theta_0 = 1, \phi = 1, & \text{ on } y = h_1 \\
    u_0 &= 0, \theta_0 = 0, \phi = 0, & \text{ on } y = h_2
\end{align*}
\tag{17}
\]

Solving ordinary differential equations (14), (15), (16) subject to the boundary conditions (17), the exact solutions for velocity, temperature and concentration are obtained as follows:
\[
\begin{align*}
    u_0 &= (A_2 e^{Ty} + B_2 e^{-Ty}) + \left( \frac{\lambda}{T^2} \right) + \left( \frac{Gr}{L^2 + T^2} \right) \left\{ \frac{\sin[L(y - h_2)]}{L} \right\} \\
    &\quad - Gc \left[ A_1 e^{Py} + B_1 e^{-Py} \right] + \left( \frac{L^2}{L^2 + T^2} \right) \left( \frac{Sr}{Sc} \right) \left( \frac{1}{L^2 + T^2} \right) \left\{ \frac{\sin[L(y - h_2)]}{L} \right\} \\
    \theta_0 &= \frac{\sin[L(y - h_2)]}{\sin[L(h_1 - h_2)]} \tag{18} \\
    \phi_0 &= \left\{ A_1 e^{Py} + B_1 e^{-Py} - \left( \frac{L^2}{L^2 + P^2} \right) \left( \frac{Sr}{Sc} \right) \frac{1}{\sin[L(h_1 - h_2)]} \right\} \tag{19}
\end{align*}
\]
The exact solution for fluid velocity, temperature and concentration satisfying boundary conditions (17) are given by

\[
\begin{align*}
    u &= \left[ (A_2 e^{Ty} + B_2 e^{-Ty}) + \left( \frac{\lambda}{T^2} \right) \right] + \left( \frac{Gr \theta_0}{T^2} \right) - Gr \left[ \frac{A_1 e^{Py} + B_1 e^{-Py}}{P^2 - T^2} \right] \\
    &\quad + \left( \frac{L^2}{L^2 + T^2} \right) \left( \frac{Sr}{Sc} \right) \left( \frac{1}{L^2 + T^2} \sin L(h_1 - h_2) \right) e^{i\omega t} \\
    \theta &= \left[ \frac{\sin[L(y - h_2)]}{\sin L(h_1 - h_2)} \right] e^{i\omega t} \\
    \phi &= \left[ A_1 e^{Py} + B_1 e^{-Py} - \left( \frac{L^2}{L^2 + P^2} \right) \left( \frac{Sr}{Sc} \right) \left( \frac{1}{L^2 + T^2} \sin L(h_1 - h_2) \right) \right] e^{i\omega t}
\end{align*}
\]

The skin friction co-efficient across the channels wall is given by

\[
\tau = \mu \left[ \frac{\partial u}{\partial y} \right]_{y=h_1, h_2} = \mu(A_2 e^{Ty} + B_2 e^{-Ty}) + \left( \frac{Gr \cdot L}{L^2 + T^2} \right) \cos[L(y - h_2)] \\
- Gr \left[ \frac{A_1 e^{Py} - B_1 e^{-Py}}{P^2 - T^2} \right] + \left( \frac{L^2}{L^2 + T^2} \right) \left( \frac{Sr}{Sc} \right) \left( \frac{1}{L^2 + T^2} \sin L(h_1 - h_2) \right) e^{i\omega t}
\]

The rate of heat transfer across the channels wall is given by

\[
Nu = - \left( \frac{\partial \theta}{\partial y} \right)_{y=h_1, h_2} = L \frac{\cos[L(y - h_2)]}{\sin L(h_1 - h_2)} e^{i\omega t}
\]

All the above constants of equations (21), (22) (23) are given in Appendix A.

4. Results and Discussions

In this analysis of MHD oscillatory flow with radiation and chemical reaction in an asymmetric channel, the effect of various flow parameters including soret number on the distributions of velocity, temperature and concentration have been studied through graphs drawn using MATLAB.

Figure 2 to 8 represent variations on velocity distribution at many sample time periods for different values of Grashoff, modified Grashoff numbers (Gr, Gc), magnetic parameter (M), radiation parameter (N), frequency of oscillation (\(\omega\)), heat source parameter (Q) and soret number (Sr). Figure 2 and 3 indicate that fluid velocity increases as Gc increases and velocity decreases for the increase in parameter Gr values for time \(t = 0.1\), \(Q = 1\), \(Pe = 1\), \(Sr = 1\), \(Sc = 1\), \(N = 1\), \(K_r = 1\), \(M = 1\), \(\omega = 1\), \(\lambda = 1\).

Figure 4 and 6 reveal the reducing effect on velocity due to increase in magnetic field strength M and oscillations \(\omega\) at time \(t = 0.1\), \(Q = 1\), \(Pe = 1\), \(Sr = 1\), \(Sc = 0.51\), \(Gr = Gc = 1\), \(K_r = 1\), \(N = 1\), \(\lambda = 1\).

Figure 5 and 7 shows that velocity increases as radiation parameter N and heat source parameter Q increases with \(t = 0.1\), \(Pe = 1\), \(Sr = 1\), \(Sc = 0.51\), \(Gr = Gc = 1\), \(K_r = 1\), \(M = 1\), \(\omega = 1\), \(\lambda = 1\).
Figure 2: Effect of $G_c$ on velocity

Figure 3: Effect of $Gr$ on velocity

Figure 4: Effect of $M$ on velocity

Figure 5: Effect of radiation on velocity

Figure 6: Effect of $\omega$ on velocity

Figure 7: Effect of $Q$ on velocity

$\omega = 1$.

Figure 8 reflects soret effect on velocity for $t = 0.1, Q = 1, Pe = 1, N = 1, Sc = 0.51, Gr = Gc = Kr = 1, M = 1, \lambda = 1$, that velocity increases as soret number increases.

Figure 9, 10 and 11 exhibit variations in temperature profile under the influence of radiation parameter, Peclet number and heat source parameter.
Figure 8: Effect of soret on velocity

Figure 9: Temperature profiles

Figure 9 indicates temperature profiles oscillatory with variations in radiation parameter $N$ at $t = 0.2$ with $P_e = 0.3, Q = 2, \omega = 1$.

Figure 10 and 11 indicates that temperature profiles are decreasing with increasing values of Peclet number and temperature increases with decreasing values in heat source parameter $Q$.

Figure 12 - 15 present variations in concentration profiles influenced by different parameters like chemical reaction parameter, heat source parameter, Schmidt number and soret number.

Figure 12 and 14 shows the effect of chemical reaction parameter (Kr) and Schmidt number (Sc) on concentration profile, resulting increasing in concentration profile.

Figure 13 indicate that concentration decreases with the increase in heat source parameter $Q$.

Figure 15 exhibits soret effect on concentration for $Q = 1$, as $Sr$ increases concentration profiles are decreasing and for $Q = 2$, concentration profiles increase. Hence concentration profiles are oscillating with the combined effect of $Sr$ and $Q$. 
5. Conclusion
In this paper, the investigations of the Soret effects on MHD Oscillatory flow with heat source in presence chemical reaction in an asymmetric wavy channel filled with porous medium is carried out. The linear coupled governing partial differential equations are converted into ODE and then exact solutions are obtained. Velocity, temperature, concentration profiles are analysed graphically for various flow parameters using MATLAB.

The following are the findings of present investigation:

- Velocity of fluid for various parameters are parabolic with maximum value near centre of the channel. Magnitude of velocity decreases with the increase in frequency of oscillations and magnetic field strength. It is observed that velocity is increasing when flow parameters like soret number, radiation parameter, Grashoff number and heat source parameters increase.

- Temperature profiles are oscillating with increasing pattern as the radiation parameter increases. It is observed that temperature profiles are decreasing with the increase in Peclet number and the pattern is reverse for the increase in heat source parameter.

- Concentration profiles are oscillating as combined effect of increase in soret number and heat source parameter and concentration profile decreases as heat source parameter and chemical reaction parameter increase. Concentration profile increases with the increase in Schmidt number.
6. Nomenclature

| Symbol | Description |
|--------|-------------|
| \(a_1, b_1\) | amplitudes of the wavy walls |
| \(a, b\) | amplitude ratios |
| \(B_0\) | electromagnetic induction |
| \(c_p\) | specific heat at constant pressure |
| \(d\) | Width of channel |
| \(D\) | - Axial velocity |
| \(A\) | - Gravitational force |
| \(G_r\) | - Hartmann number |
| \(N_u\) | Nusselt number at the wall \(y = h_1\) |
| \(Nu_2\) | Nusselt number at the wall \(y = h_2\) |
| \(Pe\) | Peclet number |
| \(p\) | - Pressure |
| \(q\) | Radiative heat flux |
| \(\theta\) | Fluid temperature |
| \(\beta_r\) | coefficient of thermal expansion |
| \(\beta_G\) | coefficient of mass expansion |
| \(k\) | magnetic permeability |
| \(\sigma\) | Conductivity of the fluid |
| \(\rho\) | Fluid density |
| \(\nu\) | - kinematics viscosity coefficient |
| \(\lambda\) | Wave length |
| \(\omega\) | Frequency of the oscillation |
| \(\alpha\) | mean radiation absorption coefficient |
| \(\tau\) | skin friction at the wall |
| \(Sr\) | Soret number |

7. Appendix

\[L^2 = N^2 + Q - i\omega Pe\]
\[P^2 = (i\omega + Kr)/Sc\]
\[T^2 = s^2 + M^2 + i\omega Re\]
\[A_1 = \left\{ \frac{2 \sinh[P(h_1 - h_2)]}{e^{-P h_2}} \right\} \left\{ 1 + \frac{Sr}{Sc} \left( \frac{L^2}{L^2 + T^2} \right) \right\} \]
\[B_1 = \left\{ \frac{2 \sinh[P(h_1 - h_2)]}{e^{-P h_2}} \right\} \left\{ 1 + \frac{Sr}{Sc} \left( \frac{L^2}{L^2 + T^2} \right) \right\} \]
\[A_2 = e^{-Th_2} \left( \frac{-\lambda}{T^2} - B_2 e^{-Th_2} \right)\]
\[B_2 = \frac{1}{2 \sinh T(h_1 - h_2)} \left[ \frac{\lambda}{T^2} \left( e^{Th_2} - e^{Th_1} \right) + e^{Th_2} \left( \frac{Gr}{L^2 + T^2} - GcA_3 \right) \right]\]
\[A_3 = \frac{1}{P^2 - T^2} + \left\{ \frac{1}{P^2 - T^2} + \frac{1}{L^2 + T^2} \right\} \left( \frac{L^2}{L^2 + T^2} \right) \left( \frac{Sr}{Sc} \right)\]

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