Confinement effects from a PNJL model at zero temperature regime

O A Mattos, O Lourenço and T Frederico
Departamento de Física, Instituto Tecnológico de Aeronáutica, DCTA, 12228-900, São José dos Campos, SP, Brazil
E-mail: odilonam@ita.br, odilon@ita.br, tobias@ita.br

Abstract. The Polyakov-Nambu-Jona-Lasinio (PNJL) model is a model that incorporates confinement effects in the Nambu-Jona-Lasinio (NJL) model through the addition of the Polyakov loop ($\Phi$). These effects are studied at finite temperature regime. However, at zero temperature its modified Fermi-Dirac distributions become step functions and $\Phi$ disappears from the equations of state (EOS), as well as the Polyakov potential, leading the model to the conventional form of the NJL model. In this work we propose a variation of the PNJL model where all the couplings depend on $\Phi$ with the constraint that the interactions vanish at the deconfinement phase where $\Phi$ reaches its maximum value and the quarks behave as free particles. In this approach, coupling constants of original PNJL model become now dependent on $\Phi$. As a consequence, all equations of state present a $\Phi$ dependence even at zero temperature regime. The thermodynamics of this new model is discussed.

1. Introduction

The NJL model is an effective quark model widely used to describe the mechanism of mass generation of quarks [1, 2]. An disadvantage of this model is its lack of the deconfinement effects. The PNJL model is an improved version of the NJL one in the sense that the confinement/deconfinement phenomenon is taken into account through the inclusion of the Polyakov loop ($\Phi$) at finite temperature regime [3]. However, at zero temperature regime, the structure of the PNJL model completely loses information on $\Phi$ (Wilson loop at finite temperature [4]), since its equations become the same as the NJL model. We propose in this work a modification in the scalar and vector coupling strengths of the two flavor NJL model by making them dependent on $\Phi$. As a constraint, we require that all interactions disappear in the deconfined phase (free quarks regime, $\Phi \sim 1$).

2. NJL model

The NJL model was initially proposed in Refs. [6, 7] basically as a model for elementary nucleons (point particles) interacting to each other through a contact interaction. By that time Quantum Chromodynamics (QCD) did not yet exist, but after it was developed, physicists realized that NJL model shares some conceptually important features with QCD such as chiral symmetry breaking and dynamical mass generation. It has been used as an effective model for QCD ever since. Since then, it is also used as a phenomenologial model for quarks. It can be used to study the SU(2) system where only two quarks (up and down) are considered in quark matter. In this
context, the Lagrangian density of the model reads

$$\mathcal{L}_0 = \bar{\psi}(i\partial - m_0)\psi + G_s \left[ (\bar{\psi}\gamma^5\psi)^2 - (\bar{\psi}\gamma^5\tau\psi)^2 \right] - G_V \left[ (\bar{\psi}\gamma_\mu\gamma^5\psi)^2 + (\bar{\psi}\gamma_\mu\gamma^5\tau\psi)^2 \right],$$

(1)

with \(m_0 = m_u = m_d\) being the current quark mass. \(G_s\) and \(G_v\) regulate the strength of the scalar and vector interactions, respectively. The \(T_{00}\) component of the energy-momentum tensor gives the energy density of the system (symmetric in our case), which in the mean-field approach is written as,

$$\mathcal{E} = G_s\rho_s^2 + G_V\rho^2 + \frac{\gamma}{2\pi^2} \int_0^{k_F} dk k^2 (k^2 + M^2)^{1/2} - \frac{\gamma}{2\pi^2} \int_0^\Lambda dk k^2 (k^2 + M^2)^{1/2} - \mathcal{E}_{\text{vac}}.$$  

(2)

The chemical potential is given by \(\mu = (k_F^2 + M^2)^{1/2} + 2G_V\rho\) and the pressure can then be defined as \(P = \mu\rho - \mathcal{E}\). Its form is

$$P = G_V\rho^2 - G_s\rho_s^2 + \frac{\gamma}{6\pi^2} \int_0^{k_F} dk k^4 \left( \frac{k^2 + M^2}{k^2 + M^2} \right)^{1/2} + \frac{\gamma}{2\pi^2} \int_0^\Lambda dk k^2 (k^2 + M^2)^{1/2} - P_{\text{vac}},$$

(3)

where \(\lambda\) is the cutoff parameter, and \(k_F\) is Fermi momentum related to the quark density \(\rho\) through \(\rho = (\gamma/6\pi^2)k_F^3\). \(\gamma = N_s \times N_f \times N_c = 12\) is the degeneracy factor due to the spin, flavor, and color numbers \((N_s = 2, N_f = 2\) and \(N_c = 3\)). The vacuum terms \((\mathcal{E}_{\text{vac}}\) and \(P_{\text{vac}}\) are obtained when \(k_F = 0\) and are inserted in the model in order to impose \(\mathcal{E} = P = 0\) at \(\rho = 0\). The constituent quark mass is

$$M = m_0 + \frac{G_s\gamma}{\pi^2} \int_{k_F}^{\Lambda} \frac{M}{(k^2 + M^2)^{1/2}},$$

(4)

with the quark condensate is given by \(\rho_s = \langle \bar{\psi}\gamma^5\psi \rangle = (m_0 - M)/2G_s\).

In Fig. 1 we show the plots for the pressure and quark condensate (in units of its value in vacuum \(\rho_{s(\text{vac})}\)) as a function of \(\mu\).

![Graph](image)

**Figure 1.** For NJL model, (a) \(P \times \mu\) and (b) \(\rho_s/\rho_{s(\text{vac})} \times \mu\) for \(G_V = 0\) with parametrization of table 2.2 of Ref. [5], where \(\Lambda = 587.9\) Mev, \(m_0 = 5.6\) Mev and \(G_s\Lambda^2 = 2.44\).

Notice from the figure a clear signature of a first order phase transition, similar to that presenting in relativistic hadronic mean-field models [8, 9] at moderate temperatures [10].
3. Our results

The PNJL model [11, 12, 13, 14] is an extended version of the NJL in which the so called Polyakov loop Φ is is included in the Lagrangian density of the NJL model through a Polyakov potential constructed in order to adjust QCD lattice results of the pure gauge system. It is a phenomenological measure of the quark confinement, with Φ = 0 (Φ = 1) representing confinement (deconfinement). All the EOS of the NJL model are modified by the inclusion of Φ in the thermodynamical quantities. However, at $T = 0$, the Polyakov loop completely disappear of these EOS and the PNJL model is reduced to the conventional NJL model again.

In this work we propose the modification of the coupling constants of the NJL model by making them dependent on Φ, namely, $G_s \rightarrow G_s (1 - Φ^2)$ and $G_V \rightarrow G_V (1 - Φ^2)$ with the condition that the interactions regulated by them vanish at the deconfinement phase (Φ = 1). The similar proposition for the constants can be seen in the references [15, 16, 17, 18]. These replacements give rise to the following energy density and pressure, respectively,

$$\mathcal{E} = G_s \rho_s^2 + G_V \rho^2 + \frac{γ}{2\pi^2} \int_0^{k_F} dk k^2 (k^2 + M^2)^{1/2} - \frac{γ}{2\pi^2} \int_0^{k_F} dk k^2 (k^2 + M^2)^{1/2} \cdot U(Φ, ρ_s, ρ) - \mathcal{E}_{vac}, \tag{5}$$

$$P = G_V \rho^2 - G_s \rho_s^2 + \frac{γ}{6\pi^2} \int_0^{k_F} dk \frac{k^4}{(k^2 + M_f^2)^{1/2}} + \frac{γ}{2\pi^2} \int_0^{k_F} dk (k^2 + M_f^2)^{1/2} - 2G_V \Phi^2 \rho^2 - U(Φ, ρ_s, ρ) - P_{vac}, \tag{6}$$

where it is possible to define a Polyakov loop potential as

$$U(Φ, ρ_s, ρ) = -G_s Φ^2 \rho_s^2 - G_V Φ^2 ρ^2 + a_3 T_0^4 \log \left[1 - 6Φ^2 + 8Φ^3 - 3Φ^4\right], \tag{7}$$

with an explicit back reaction of the quark system into the gluonic one, since $U$ is also a function of the quark quantities $ρ_s$ and $ρ$ besides $Φ$ as well. In Eq. (7), we have included the logarithmic term in order to ensure $Φ$ limited to $Φ = 1$, see Ref [19]. Here, $T_0 = 190$ MeV and $a_3$ is taken as a free parameter.

This specific modification in the NJL model turn the model in a type of PNJL in which it is possible to investigate the deconfinement effects at zero temperature. As an example we show in Fig. 2 the $Φ$ dependence of $\mathcal{E}$ for $ρ$ corresponding to a baryonic density of $ρ_B = 3ρ_0$ ($ρ_0 = 0.15$ fm$^{-3}$).

![Figure 2](image-url)  

Figure 2. Energy density of Eq. (5) as a function of $Φ$ for different $a_3$ values. $G_V$ is fixed to $G_V = 1.25G_s$. The parameters is the same of the figure 1.
Notice from the Fig. 2 that there is a set of $a_3$ values that produce a minimum at $\Phi > 0$ for the energy density as a function of $\Phi$. This feature is not present in the traditional PNJL model ($G_s$ and $G_V$ fixed) at zero temperature since in that case, energy density and pressure have no dependence on $\Phi$, i.e., $\Phi$ in Eqs. (5) and (6).

4. Conclusions and perspectives

We could verify that our modification in the NJL model at zero temperature, namely, the replacements $G_s \rightarrow G_s(1 - \Phi^2)$ and $G_V \rightarrow G_V(1 - \Phi^2)$, ensures a nonvanishing contribution of $\Phi$ to the EOS of the model allowing the study the confinement/deconfinement phase transition, what is not possible in the traditional PNJL model. We intend to investigate in more details the thermodynamical quantities of this modified model in order to completely define the range of parameters for $a_3$ and $G_V$ that leads to $\Phi \neq 0$ solutions.

5. Acknowledgments

This work was partially supported by CNPq under grants 310242/2017-7 (O.L.) and 308486/2015-3 (T.F.), by FAPESP under the thematic projects 2013/26258-4 (O.L.) and 2017/05660-0 (T.F.), and by INCT-FNA project 464898/2014-5. This study was also financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001.

[1] Vogl U, Weise W 1991 Prog. Part. Nucl. Phys 27 195-272
[2] Klevansky S P 1992 Rev. Mod. Phys. 64 649-708
[3] Fukushima K, Skokov V 2017 Prog. Part. Nucl. Phys. 96 154-199
[4] Wilson K G 1974 Phys. Rev. D 10 2445-2459
[5] Buballa M 2005 Phys. Rept. 407 205-376
[6] Nambu Y, Jona-Lasinio G 1961 Phys. Rev. 122 345
[7] Nambu Y, Jona-Lasinio G 1961 Phys. Rev. 124 246
[8] Lourenço O, Dutra M, Delfino A, Amaral R L P G 2007 Int. J. Mod. Phys. E 16 3037; Lourenço O, Santos B M, Dutra M, Delfino A 2016 Phys. Rev. C 94 045207; and Lourenço O, Dutra M, Menezes D P 2017 Phys. Rev. C 95 065212
[9] Santos B M, Dutra M, Lourenço O, Delfino A 2015 Phys. Rev. C 92 015210; 2014 Phys. Rev. C 90 035203
[10] Silva J B, Lourenço, A Delfino, Sá Martins J S, Dutra M 2008 Phys. Lett. B 664 246-252
[11] Fukushima K 2004 Phys. Lett. B 591 277-284
[12] Lourenço O, Dutra M, Delfino A, Malheiro M 2011 Phys. Rev. D 84 125034
[13] Lourenço O, Dutra M, Frederico T, Delfino A, Malheiro M 2012 Phys. Rev. D 85 097504
[14] Dutra M, Lourenço O, Delfino A, Frederico T, Malheiro M 2013 Phys. Rev. D 88 114013
[15] Ferreira M, Costa P, Lourenço O, Frederico T, Providência C 2014 Phys. Rev. D 89 116011
[16] Farias R L S, Gomes K P, Krein G, Pinto M B 2014 Phys. Rev. C 90 025203; and Farias R L S, Timóteo, V S, Avancini S S, Pinto M B, Krein G 2017 Eur. Phys. J. A 53 101
[17] Timóteo V S, Farias R L S, Avancini S S, Pinto M B, Tavares W R 2018 EPJ Web Conf. 171 20001
[18] Avancini S S, Farias R L S, Pinto M B, Tavares W R, Timóteo V S 2017 Phys. Lett. B 767 247-252
[19] Dexheimer V A, Schramm S 2010 Phys. Rev. C 81 045201