High resolution particle spectroscopy in $^{208}\text{Pb}$ with the Q3D magnetic spectrograph of the Maier-Leibnitz-Laboratorium at München

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Abstract. By using the Q3D magnetic spectrograph of the Maier-Leibnitz-Laboratorium at München, particle-hole states in $^{208}\text{Pb}$ are investigated. The reaction $^{208}\text{Pb}(p, p')$ via seven known isobaric analog resonances in $^{209}\text{Bi}$ and the reaction $^{207}\text{Pb}(d, p)$ were studied. An instrumental resolution of about 3 keV for protons without energy loss in the target is achieved; depending on the effective target thickness the peak shape becomes asymmetric with a tail of 3-8 keV width. A dozen doublets of states at less than 2 keV or even vanishing distance are resolved by the selective excitation in different analog resonances. For excitation energies $3.9 < E_x < 6.1\text{ MeV}$ in $^{208}\text{Pb}$, 80 states with negative parity and 30 states with positive parity are predicted by the shell model. Most states are identified and work is going on to determine their structure.

In the doubly magic nucleus $^{208}\text{Pb}$ the lowest states are expected to be well described by one-particle one-hole configurations with neutron holes for $82 \leq N \leq 126$, neutron particles for $126 \leq N \leq 184$, and proton holes for $50 \leq Z \leq 82$, proton particles for $82 \leq Z \leq 126$. In each group of particles or holes one intruder exists, hence about one third of the states are expected to have positive parity.

The level density is rather high. For the region $4.8 < E_x < 6.1\text{ MeV}$, the shell model predicts a mean distance of 15 keV between two states with any spin or parity. Gamma-spectroscopy with a resolution of 2 keV may resolve most states. In particle spectroscopy, the resolution has been improved only slowly, from 15 keV in 1969 to 4 keV ten years ago (see Table and [5]).

Experiments of $^{208}\text{Pb}(p, p')$ and $^{207}\text{Pb}(d, p)$ were performed with the Q3D magnetic spectrograph of the Maier-Leibnitz-Laboratorium at München [3, 4, 6, 7, 8] (Fig. 1). In the reaction $^{208}\text{Pb}(p, p')$, proton energies $E_p = 14.8 - 18.2\text{ MeV}$ were chosen to cover the known isobaric analog resonances (IAR) [1]. Targets of $100 - 300\mu\text{g/cm}^2$ thickness were used. The protons were detected by the Q3D magnetic spectrograph at scattering angles $\Theta = 20^\circ - 140^\circ$. The multiwire detector gathers protons in a solid angle of $3^\circ \times 3^\circ$. A spectrum of 1 MeV length (highly linear in the momentum space) is gathered in typically half an hour with sufficient statistics. The reaction $^{207}\text{Pb}(d, p)$ was performed with $E_d = 22.0\text{ MeV}$, too [3, 4].

The line shape is asymmetric (Fig. 2). With thin targets, an instrumental resolution for protons of 3 keV (corresponding to a relative energy resolution of $3 \cdot 10^{-4}$) is achieved. (The target must be correctly placed to avoid the protons crossing the carbon backing.) Because of the energy loss by straggling in the target, an exponential tail appears, linear on the logarithmic scale. The width of the tail depends on the target thickness, the scattering angle, the target
In the following, the inelastic proton scattering on the doubly magic nucleus $^{208}\text{Pb}$ via isobaric analog resonances in $^{209}\text{Bi}$ [1, 3, 7] is shortly explained (Fig. 3). In the doubly magic nucleus $^{208}\text{Pb}$ there are 44 excess neutrons. By adding one neutron a particle state with orbital angular momentum $L$ and spin $J$ in $^{209}\text{Pb}$ is created. The isobaric analog of such a state consists of 45
Figure 2. Extract of a $^{208}$Pb($p, p'$) spectrum of 1 MeV length taken on the $j_{15/2}$ and $d_{5/2}$ IAR. The 4861 $8^+$, 4868 $7^+$ states with dominant structure $j_{15/2}$/$p_{1/2}$ are resolved. On the logarithmic scale the line shape is asymmetric. (lower panel, magenta left arrow.) The instrumental resolution is 3 keV as determined for protons not suffering an energy loss by straggling in the target. Satellites produced by the knockout of $L_{I,II,III}$-electrons are seen near $E_x = 4.88$ MeV. (upper panel, magenta right arrow.) The width of the exponential tail depends on the effective target thickness.

In each component one excess neutron is converted into a proton,

$$ |\Psi_{LJ}^{IAR}(^{209}\text{Bi})\rangle = \frac{1}{\sqrt{2T_0 + 1}} T_-( |LJ, \nu\rangle \otimes |^{208}\text{Pb}(0^+ \text{g.s.})\rangle) $$ \hspace{1cm} (1)$$

In the proton decay of such an analog resonance (right part of Fig. 3) either the proton escapes with the unchanged energy (this corresponds to the elastic proton scattering), or $^{208}$Pb is left in an excited state and a superposition of up to 30 neutron particle-hole configurations is created. The proton energy can be precisely measured; relative distances between two states can thus be determined with an uncertainty of 100 eV up to excitation energies of 8 MeV.

Inelastic proton scattering via IAR has many advantages.

(i) By adjusting the proton energy to a certain analog resonance, the neutron particle of a particle-hole configuration can be chosen. Doublets may be resolved by selective excitation in different IAR. Examples are the doublets $E_x = 4.71$ and 5.40 [3], 4.93, 5.19 and 5.99 [7], 5.65 [5, 6], 5.69 [6], 6.42 MeV [8].
(ii) The particle of a particle-hole configuration is determined from excitation functions (Fig. 4). Only the $j_{15/2}$ intruder IAR excites positive parity states. Among more than 150 identified states, the parity of 30 states is determined to be positive [7].

(iii) Amplitudes of particle-hole configurations are deduced from angular distributions. The relevant single particle widths can be determined in a self-consistent manner, since on each of the seven IAR, they must be the same for the holes $p_{1/2}$, $p_{3/2}$, \ldots.

(iv) If mainly a single configuration contributes to the cross section of some state, the shape of the angular distribution is given by pure geometry and the spin can be determined [3, 6].

Almost all states below $E_x < 6.1$ MeV and a few states up to $E_x = 7.0$ MeV are identified. For many states spin and parity is determined [3, 6, 7, 8]. Fig. 6 compares the excitation energies of positive parity states below $E_x = 6.0$ MeV to predictions of the shell model without residual interaction [3, 7]; their structure is determined [7]. Similarly, work is going on to determine the structure of negative parity states.

The goal of the work is to determine the residual interaction from experiment [2, 4]. Within the shell model without residual interaction [3, 7], we consider a set of configurations $|n, I\pi\rangle$ and a set of physical states $|n, I\pi\rangle$ with spin $I$ and parity $\pi$ and consecutive number $n = 1, 2, \ldots$. In the shell model the states consist of particle-hole configurations $|k, I\pi\rangle$ and inversely particle-hole configurations consist of states,

$$|n, I\pi\rangle = \sum_k t_{nk}(I\pi)|k, I\pi\rangle, \quad |k, I\pi\rangle = \sum_n t_{nk}^\dagger(I\pi)|n, I\pi\rangle. \quad (3)$$

Here $t_{nk}(I\pi)$ denotes the unitary transformation matrix of the Hilbert space of physical states $|n, I\pi\rangle$ into the configuration space $|k, I\pi\rangle$. The transformation matrix is real because of the time reversal invariance of the Hamiltonian. The amplitudes of the transformation matrix $t_{nk}$ can be determined from angular distributions for $^{208}$Pb$(p, p')$ via IAR, $^{208}$Pb$(d, p)$, and $^{209}$Bi$(d, ^3$He).

The Hamiltonian acting on the the states $|n, I\pi\rangle$ and the Hamiltonian acting on the configurations $|k, I\pi\rangle$ have the eigenvalues

$$H_{nm}(I\pi) = \delta_{nm}E_n(I\pi), \quad h_{kl}(I\pi) = \delta_{kl}\epsilon_k(I\pi). \quad (4)$$

The energies $E$ of the physical states are deduced from experiment. By assuming the lowest states in the nuclei $^{209}$Bi, $^{208}$Pb to be single particle states and the lowest states in the nuclei $^{207}$Tl, $^{207}$Pb to be single hole states and from the mass differences of the four neighbor nuclei to $^{208}$Pb with $A = 208 \pm 1$, the configuration energies $\epsilon$ are derived [3, 7].

Figure 3. Schematic description of the inelastic proton scattering via isobaric analog resonances in $^{209}$Bi. (left panel) The creation of an IAR and (right panel) the proton decay of an IAR exciting a coherent superposition of up to 30 neutron particle-hole configurations in states of $^{208}$Pb.
Figure 4. Excitation functions of $^{208}\text{Pb}(p, p')$ via IAR in $^{209}\text{Bi}$ (from [7]). All seven known IAR between $E_p = 14.9 - 17.5$ MeV are covered (black arrows). The width of the $j_{15/2}$ IAR (red arrow) is 210 keV, the distance to the $d_{5/2}$ IAR (green arrow) is 100 keV. For the four states in $^{208}\text{Pb}$ with the given excitation energies in keV, the particle of the particle-hole configuration is determined to be $j_{15/2}$, hence the parity is positive.

Figure 5. Determination of matrix elements of the residual interaction from experiment [Eq. (5)]. The two $0^-$ states in $^{208}\text{Pb}$ are the only known bound $0^-$ states in the whole nuclear chart. They offer an excellent example of a two-level scheme since the next configurations are predicted at a distance of ten times the off-diagonal matrix element determined as $v_{12} = v_{21} = 110 \pm 10$ (expt. $\pm 15$ (syst.) keV [4]. The systematic uncertainty of 15 keV derives from the estimate of the deviation of the matrix $t_{nk}$ [Eq. (3)] from unitarity by comparing admixtures of $s_{1/2}p_{1/2}$ in the two $0^-$ states to nine $1^-$ states.

The two Hamiltonians $H, h$ differ by the residual interaction $v$,

$$v_{kl}(I^\pi) = \sum_{nm} t_{kn}(I^\pi) H_{nm}(I^\pi) t_{ml}^\dagger(I^\pi) - h_{kl}(I^\pi).$$

(5)
Similarly as done for the two $0^-$ states [4] (Fig. 5), for all states below $E_x \lesssim 6.1$ MeV with spins $1^- - 8^-$ and $5^+ - 10^+$, the amplitudes of the transformation matrix [Eq. (3)] are being derived from experimental data. Most states (see Fig. 6 for positive parity states) are identified and their dominant configurations are already determined. Some doublets have still to be discussed and some spin assignments are not yet settled.

**Figure 6.** For spins $5^+, 6^+, 7^+, 8^+, 9^+, 10^+$ and $E_x < 6.0$ MeV, the excitation energies $E_x$ of states in $^{208}$Pb predicted by the shell model without residual interaction [3, 7] are shown (marked blue). Almost all predicted states are identified [7] (marked magenta). The configuration numbers correspond to the shell model predictions [Eq. (3)]. The configurations $j_{15/2}p_{1/2}$, $j_{15/2}f_{5/2}$, $j_{15/2}p_{3/2}$ are marked by colors.

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