The Local Space-Time Lorentz Transformation: a New Formulation of Special Relativity Compatible with Translational Invariance

J.H. Field

Département de Physique Nucléaire et Corpusculaire Université de Genève . 24, quai Ernest-Ansermet CH-1211 Genève 4.

E-mail: john.field@cern.ch

Abstract

The apparent times and positions of moving clocks as predicted by both ‘non-local’ and ‘local’ Lorentz Transformations are considered. Only local transformations respect translational invariance. Such transformations change temporal but not spatial intervals, so breaking space-time exchange symmetry and forbidding the conventional relativity of simultaneity and length contraction effects of special relativity. Two satellite-borne experiments to test these predictions are proposed.

PACS 03.30.+p
1 Introduction

The relativistic Length Contraction (LC)\(^1\) and Time Dilatation (TD) effects were pointed out as consequences of the space-time Lorentz Transformation (LT) in Einstein’s original paper [1] on Special Relativity (SR). The LC effect was clearly stated to be an ‘apparent’ one in Reference [1]; even so, in many text books on SR it is stated to be not only an, in principle, experimentally observable effect but also a ‘real’ one implying that there is actually some dynamical contraction of the body, as discussed for example, in References [2, 3].

It was only on making a more careful analysis of the physics of the observation process that it was realised, some five decades after Einstein’s original paper, that the length contracted sphere that he considered would appear to be, to a distant observer, not flattened into an ellipsoid, but simply rotated [4, 5]. This is because ‘observation’ actually means the detection of photons emitted by, or scattered from, the observed object. Not only the LC effect but also the the effects of light propagation time delays of the observed photons and optical aberration (that is, the change in the direction of motion of a photon due to the LT of its momentum) must also be properly accounted for. In the present paper it is assumed throughout that all observations are corrected for the last two effects, so that the apparent positions are predicted by the LT only.

In a recent paper by the present author [6] it was noticed that the LC effect itself is closely akin to the effect of light propagation time delays. In the latter, in order to arrive at the observer at the same time (this is the definition of the observation that gives LC) the photons coming from parts of the viewed object at different distances along the line of sight must be emitted from the object at different times. Similarly, because of the ‘relativity of simultaneity’ proposed by Einstein, the photons recorded by the stationary observer at a fixed time must be emitted, in the rest frame of the moving object, at different times from different positions along the direction of motion of the object. Thus LC is a strict consequence of the relativity of simultaneity. In the same paper [6] two other apparent distortions of space-time due to the LT: Space Dilatation (SD) in which the moving object appears longer (not shorter as in LC) and Time Contraction (TC) in which which the moving equivalent clock viewed at a fixed position in the observer’s frame appears to be running faster (not slower as in TD) than an identical stationary clock, were pointed out. The four effects LC, TD, SD and TC are all related to the projective geometry of the LT equations, and correspond to the projections with \(\Delta t = 0\), \(\Delta x' = 0\), \(\Delta t' = 0\) and \(\Delta x = 0\) respectively \(^2\). The ‘apparent’ nature of all the effects is then manifest since a moving object cannot be ‘really’ contracted in, for example, the LC effect, if, with a different observation procedure (SD) it appears to be elongated. A similar consideration applies to the (opposite) TD and TC effects.

In an even more recent paper [7] the relation between the ‘real’ positions of moving

---

\(^1\)In References [6, 7], by the present author, the acronym ‘LFC’ for ‘Lorentz-Fitzgerald Contraction’ was used. But as this name is more properly assigned to a conjectured dynamical effect in a pre-relativistic theory (see Section 8 below), ‘LC’ for Length Contraction or Lorentz Contraction (a consequence of the Lorentz Transformation) is used throughout the present paper.

\(^2\)The space and time coordinates: \(x, t; x', t'\) are measured in two inertial frames, \(S; S'\) in relative motion along their common \(x\)-axis.
objects (i.e. those defined, for different objects, in a common reference frame) to the apparent positions predicted by the LT was considered in detail. In particular, the well known ‘Rockets-and-String’ [8] and ‘Pole-and-Barn’ [9] paradoxes were re-discussed and a procedure for measuring the real positions of moving objects in a single reference frame was proposed.

The present paper pursues further the relation between the real and apparent positions of moving objects as well as analysing the times registered by two, spatially separated, synchronised, moving clocks viewed by a stationary observer. It is noted that, due to the ambiguity in the choice of the origin of spatial coordinates in the rest frame of the moving objects or clocks, no definite predictions are obtained for either the times recorded by, or the apparent positions of, the moving clocks, but that, in every case, there is a violation of translational invariance. Only if a local LT is used, in which the origin of spatial coordinates in the rest frame of the moving object coincides with the spatial coordinate of the transformed event, is translational invariance found to be respected. In this case there is no ‘relativity of simultaneity’ and the related LC, TC and SD effects do not occur. However, TD, that results from a local LT, is unaffected by the restriction to this type of transformation. It is shown in Section 8 below that, in fact, TD is the only relativistic space-time effect that is confirmed experimentally. Only time intervals, not space intervals, are modified by a local LT, thus breaking, in this case, the space-time exchange symmetry recently proposed as a mathematical basis for SR [10].

The plan of this paper is as follows: In the following section the discussion of the distinction between the ‘real’ and ‘apparent’ positions of moving objects first given in [7] is repeated, as this concept is important for the arguments presented later in the paper; indeed, for a local LT there is no distinction between ‘real’ and ‘apparent’ positions. In Section 3 translational invariance is discussed in a general way in connection with the observation of two identical and similarly accelerated clocks. In Section 4 the observation of the times of clocks subjected to a similar, constant, acceleration in their proper frames is discussed for local and non-local LTs. In Section 5 the real and apparent positions of the moving clocks are discussed in relation to the LC effect. In Section 6, the principle of ‘Source Signal Contiguity’ is proposed and some causal paradoxes of SR are resolved by use of the local LT. In Section 7 the Minkowski space-time plot is considered and relativistic kinematics is discussed. Section 8 contains an analysis of the Michelson-Morley experiment considered as a ‘photon clock’. A concise review of the experimental tests of SR performed to date is presented in Section 9. In Section 10 two different satellite-borne experiments are proposed. The first, an extension of a previously performed cesium clock experiment, is a proposal to observe TC. This is an $O(\beta^2)$ effect. The second experiment proposes a ‘photon clock’ similar to the longitudinal arm of a Michelson interferometer, constituted by two satellites following the same orbit close to the surface of the Earth, to measure directly the ‘relativity of simultaneity’ (RS) effect. As the relativistic effects are here of $O(\beta)$ less precise time measurements are sufficient to test the predictions in this case.

3 $\beta \equiv v/c$ where $v$ is the relative velocity of the satellite and ground-based observer at the time of the experiment.
The Real Positions of Moving Objects

Figure 1: Space-time trajectories in the frame $S$ of the real positions of $O1$ and $O2$ when subjected to constant and identical proper accelerations. Units are chosen with $c = 1$. Also $L = a = 1$, $t_{acc} = \sqrt{3}$.

The ‘real’ positions of one or more moving objects are defined here as those specified, or measured, in a single frame of reference. The latter may be either inertial, or with an arbitrary accelerated motion. By introducing the concept of a co-moving inertial frame, at any instant of the accelerated motion, the distance between the objects can always be defined as the proper distance between them in a certain inertial frame. If this distance in the different co-moving inertial frames is constant the ‘real’ distance is said to be constant. No distinction is made between the real distance between the points on a rigid extended object and that between discrete, physically separated, objects coincident in space-time with these points. This is because the LT, that relates only space-time events in different inertial frames, treats, in an identical manner, points on extended or discrete physical objects.

The utility of the science of the ‘real’ positions of moving objects (astronomy, railways, military ballistics, air traffic control, space travel, GPS satellites...) is evident and the validity of the basic physical concepts introduced by Galileo (distance, time, velocity and acceleration) are not affected, in any way, by SR. The ‘real’ positions of objects in a given reference frame are those which must be known to, for example, avoid collisions between moving objects in the case of railway networks, or, on the contrary, to assure them in the case of military ballistics or space-travel. From now on, in this paper, the words ‘real’ and ‘apparent’ will be written without quotation marks.
For clarity, a definite measuring procedure to establish the real distance between two moving objects in a given frame of observation is introduced. Suppose that the objects considered move along the positive x-axis of an inertial coordinate system S. The co-moving inertial frame of the objects is denoted by S'. It is imagined that two parallel light beams cross the x-axis in S, at right angles, at a distance \( \ell \) apart. Each light beam is viewed by a photo-cell and the moving objects are equipped with small opaque screens that block the light beams during the passage of the objects. Two objects, O1 and O2, with \( x(O2) > x(O1) \), moving with the same uniform velocity, \( v \), will then interrupt, in turn, each of the light beams. Suppose that the photo-cells in the forward (F) and backward (B) beams are equipped with clocks that measure the times of extinction of the beams to be (in an obvious notation) \( t(F1) \), \( t(F2) \), \( t(B1) \) and \( t(B2) \). Consideration of the motion of the objects past the beams gives the following equations:

\[
\begin{align*}
  t(B1) - t(F1) &= \frac{\ell}{v} \\
  t(B2) - t(F2) &= \frac{\ell}{v} \\
  t(B2) - t(F1) &= \frac{\ell - L}{v}
\end{align*}
\]

where \( L \) is the real distance between the moving objects. Eqns(2.1) and (2.2) give the times of passage of the objects O1 and O2, respectively, between the light beams, whereas Eqn(2.3) is obtained by noting that, if \( \ell > L \), each object moves the distance \( \ell - L \) during the interval \( t(B2) - t(F1) \). If \( \ell < L \) each object moves a distance \( L - \ell \) during the interval \( t(F1) - t(B2) \) and the same equation is obtained. Taking the ratios of Eqn(2.3) to either Eqn(2.1) or Eqn(2.2) yields, after some simple algebra, the relations:

\[
\begin{align*}
  L &= \ell \frac{t(B1) - t(B2)}{t(B1) - t(F1)} \quad (2.4) \\
  L &= \ell \frac{t(F1) - t(F2)}{t(B2) - t(F2)} \quad (2.5)
\end{align*}
\]

Subtracting Eqn(2.3) from Eqs(2.1) or (2.2), respectively, gives:

\[
\begin{align*}
  t(B1) - t(B2) &= \frac{L}{v} \quad (2.6) \\
  t(F1) - t(F2) &= \frac{L}{v} \quad (2.7)
\end{align*}
\]

Taking the ratios of Eqn(2.6) to (2.2) and Eqn(2.7) to (2.1) then yields two further equations, similar to (2.4) and (2.5) above:

\[
\begin{align*}
  L &= \ell \frac{t(B1) - t(B2)}{t(B2) - t(F2)} \quad (2.8) \\
  L &= \ell \frac{t(F1) - t(F2)}{t(B1) - t(F1)} \quad (2.9)
\end{align*}
\]

Equations (2.4),(2.5),(2.8) and (2.9) show that any three of the four time measurements are sufficient to determine the real separation, \( L \), between the two moving objects. In

\textsuperscript{4}forward’ and ‘backward’ are defined from the viewpoint of the moving objects. Thus the forward beam lies nearest to the origin of the x-axis
these equations, the times: $t(F2)$, $t(B1)$, $t(F1)$ and $t(B2)$, respectively, are not used to determine $L$. In order to combine all four time measurements to obtain the best, unbiased, determinations of $v$ and $L$, Eqns(2.1) and (2.2) may be added to obtain:

$$t(B1) + t(B2) - t(F1) - t(F2) = \frac{2\ell}{v}$$  (2.10)

while subtracting two times Eqn(2.3) from (2.10) gives:

$$t(B1) - t(B2) + t(F1) - t(F2) = \frac{2L}{v}$$  (2.11)

The velocity, $v$, is obtained by transposing Eqn(2.10):

$$v = \frac{2\ell}{t(B1) + t(B2) - t(F1) - t(F2)}$$  (2.12)

while the ratio of Eqn(2.11) to (2.10) gives:

$$L = \ell \frac{t(B1) - t(B2) + t(F1) - t(F2)}{t(B1) + t(B2) - t(F1) - t(F2)}$$  (2.13)

It is interesting to note that the real spatial positions, as well as the instantaneous velocity and acceleration, at any time, of two objects subjected to a symmetric, uniform, acceleration in the frame $S$, can also be determined from the four time measurements just considered. In this case there is no redundancy; the time measurements determine four equations which may be solved for the four quantities just mentioned. In the case of uniform motion, the constant velocity hypothesis may be checked by comparing the independent determinations of $v$ provided by Eqns(2.1) and (2.2). Furthermore, as already mentioned, any three of the four time measurements is sufficient to determine $L$. Evidently, if $t(F1) = t(B2)$, then $L = \ell$ in the case of an arbitrary accelerated motion of the two objects. Thus, by varying $\ell$, the real distance between the co-moving objects can be determined even if the acceleration program of their co-moving frame is not known.

The two objects, moving with equal and constant velocities along the $x$-axis in $S$, discussed above, are now considered to be set in motion by applying identical acceleration programs to two objects initially at rest and lying along the $x$-axis in $S$. The two objects considered are then, by definition, subjected to the same acceleration program in their common rest frame, or, what is the same thing, their common rest frame, (with respect to which the two objects are, at all times, at rest) is accelerated. Under these circumstances the distance between the objects remains constant in the instantaneous co-moving inertial frame of the objects. At the start of the acceleration procedure, the instantaneous co-moving inertial frame is $S$, at the end of the acceleration procedure it is $S'$. Therefore the separation of the objects in $S$ at the start of the acceleration procedure is the same as that in $S'$ at the end of it. Note that there is no distinction between the real and apparent separations for objects at rest in the same inertial frame. Also ‘relativity of simultaneity’ can play no role, since the proper time of both objects is always referred to the same co-moving inertial frame. Since the acceleration program of both objects starts at the same time in $S$, and both objects execute identical space time trajectories, the real separation of the objects must also remain constant. This necessarily follows from space-time geometry. Similarly, since the acceleration program stops at the same time in $S'$ for
both objects the real separation of the objects remains constant in this frame and equal to the original separation of the objects in S. This behaviour occurs for any symmetric acceleration program, and is shown, for the special case of a constant acceleration in the rest frame of the objects (to be calculated in detail below), in Fig 1. Since, in the above discussion, both objects are always referred to the same inertial frame there is no way that SR can enter into the discussion and change any of the above conclusions. Indeed, SR is necessary to derive the correct form of the separate space-time trajectories in S, but the symmetry properties that guarantee the equalities of the real separations of the objects cannot be affected, in any way, by SR effects.

Two objects, O1 and O2, originally lying at rest along the x-axis in S and separated by a distance $L$ are now simultaneously accelerated, during a fixed time period, $t_{acc}$, in S, starting at $t = 0$, with constant acceleration, $a$, in their common proper frame, up to a relativistic velocity $v/c \equiv \beta = \sqrt{3}/2$ corresponding to $\gamma = 1/\sqrt{1-\beta^2} = 2$ and $t_{acc} = c\sqrt{3}/a$. The equations giving the velocity $v$ and the position $x$ in a fixed inertial frame, using such an acceleration program, were derived by Marder [11] and more recently by Nikolic [12] and Rindler [13]. The positions and velocities of the objects in the frame S are:

for $t \leq 0$

\begin{align*}
  v_1 &= v_2 = 0 \\
  x_1 &= -\frac{L}{2} \\
  x_2 &= \frac{L}{2}
\end{align*}

for $0 < t < t_{acc}$

\begin{align*}
  v_1(t) &= v_2(t) = v(t) = \frac{act}{\sqrt{c^2 + a^2t^2}} \\
  x_1(t) &= c \left[ \frac{\sqrt{c^2 + a^2t^2} - c}{a} \right] - \frac{L}{2} \\
  x_2(t) &= c \left[ \frac{\sqrt{c^2 + a^2t^2} - c}{a} \right] + \frac{L}{2}
\end{align*}

and for $t \geq t_{acc}$

\begin{align*}
  v_1(t) &= v_2(t) = v(t_{acc}) \\
  x_1(t) &= v(t_{acc})(t - t_{acc}) + x_1(t_{acc}) \\
  x_2(t) &= v(t_{acc})(t - t_{acc}) + x_2(t_{acc})
\end{align*}

The origins of S and S’ have been chosen to coincide at $t = t' = 0$. The real positions of the objects in S are shown, as a function of $t$ for $a = 1$ and $t_{acc} = \sqrt{3}$, in Fig.1. The velocities of the two objects are equal at all times, as is also the real separation of the objects $x_2 - x_1 = L$. SR is used only to derive Eqn(2.17). The time-varying velocity is then integrated according to the usual rules of classical dynamics in order to obtain Eqns(2.18),(2.19) for the positions of the objects during acceleration. These
are, by definition, the real positions of the objects O1 and O2 in S. The equalities of the velocities and the constant real separations are a direct consequence of the assumed initial conditions and the similarity of the proper frame accelerations of the objects. These are the sets of equations that must be used to specify the distance between O1 and O2 and any other objects whose real positions are specified in S, in order to predict collisions or other space-time interactions of the objects.

3 Moving Clocks and Translational Invariance

![Diagram of moving clocks](image_url)

Figure 2: a) clocks A, B and C at \( t = t' = 0 \). They are at rest in S. b) After an identical acceleration program, during time \( t_{acc} \) in S, the clocks A and B (at rest in S') move in S with velocity \( v_f \) parallel to the x-axis and are viewed from S. Due to relativistic effects, the apparent times \( t'_A, t'_B \) observed in S are less than \( t \), but, from translational invariance, \( t'_A = t'_B \).

In the following, in order to investigate the properties of the space-time LT, it will be found convenient to consider two identical clocks, A and B, which perform identical motion parallel to the x-axis of a ‘stationary’ inertial frame S. The common co-moving frame, at any instant, of A and B, is denoted by S’. Initially, A and B, which have each been synchronised with a reference clock, C, at rest in the frame S, are at rest in S, separated by a distance, \( L \) (see Fig.2a). At time \( t = 0 \) each clock is subjected to an identical acceleration program during the time \( t_{acc} \) in S. Because the velocities of A and B are the same at any instant then, as discussed in the previous section, a common co-moving inertial frame always exists for the two clocks. As also discussed in the previous
section, their separation, as measured in this co-moving frame, or in the frame S, remains $L$ at all times. Because of relativistic effects, that will be calculated in the following section for a specific acceleration program, the observed time in S, $t'$, registered by the moving clocks A and B will differ from that shown by the reference clock C that is at rest in S. However, the times indicated by A and B must be identical. This is a consequence of translational invariance. The relativistic effects of the acceleration program must be the same whether they are applied to a clock initially at rest at position $x$ or one initially at rest at position $x + L$. After the time $t_{acc}$, the acceleration of each clock ceases and they continue to move with the same constant relativistic velocity $\beta_f$, separated by the distance $L$, as shown in Fig2b.

4 Time Dilatation of Moving Clocks using ‘Local’ and ‘Non-Local’ Lorentz Transformations

Figure 3: Times indicated at time $t = t_{acc}$ in S by the clocks A, B and C, according to the space-time LT, for different choices of the origin, $O'$, of $S'$: a) local LT for both A and B ($x'_A = x'_B = 0$), b) $O'$ at A, c) $O'$ at B, d) $O'$ midway between A and B. Only a) gives a prediction consistent with translational invariance. Units with $a = c = 1$ are used and $v_f = \sqrt{3}/2$, $\gamma_f = 2$, $t_{acc} = \sqrt{3}$.

It is now assumed that the clocks A and B, introduced above, initially at rest in S, are subjected to the same uniform acceleration, $a$, in their own rest frames, at the same instant in S, in the direction of the positive x-axis, for a time, in S, of duration $t_{acc}$. This
is exactly the same acceleration program as that discussed for the objects O1 and O2 in Section 2 above. The formulae giving the relativistic velocity, \( \beta(t) = \frac{v(t)}{c} \), and position, \( x(t) \), in the frame S as a function of the elapsed time, \( t \), in this frame are, as in (2.17) and (2.18) above:

\[
\beta(t) = \frac{at}{\sqrt{c^2 + a^2t^2}} \quad (4.1)
\]

\[
x(t) = \frac{c}{a} \left[ \sqrt{c^2 + a^2t^2} - c \right] \quad (4.2)
\]

where, in this case, the clock is situated at the origin of S at \( t = 0 \). The observed time, \( t' \), indicated by the clocks in S, as a function of \( t \) is calculated by integrating the differential form of the LT of time in accordance with the time variation of \( \beta \) given by Eqn(4.1). In the first case a local LT is used for each clock. This corresponds to the choice \( x' = 0 \) in the general space-time LT:

\[
x' = \gamma(x - vt) \quad (4.3)
\]

\[
t' = \gamma(t - \frac{\beta x}{c}) \quad (4.4)
\]

\[
x = \gamma(x' + vt') \quad (4.5)
\]

\[
t = \gamma(t' + \frac{\beta x'}{c}) \quad (4.6)
\]

That is, the origin of coordinates in \( S' \) is chosen to be at the position of the transformed space point (the position of the clock A or B). A non-local LT is one in which the origin of coordinates in \( S' \) is not at the same position as the transformed space point. There are clearly an infinite number of such LT equations for any transformed space point, corresponding to an arbitrary choice of origin in \( S' \). Situating clock A at \( x' = 0 \), the differential form of (4.6) corresponding to Eqn(4.1) is:

\[
dt' = \frac{dt}{\gamma(t)} \quad (4.7)
\]

where:

\[
\gamma(t) = \frac{1}{\sqrt{1 - \beta(t)^2}}
\]

or, using Eqn(4.1),

\[
dt' = \frac{dt}{\sqrt{1 + \frac{a^2t^2}{c^2}}} \quad (4.8)
\]

Performing the integral over \( t \) gives:

\[
t'_A(t, x'_A = 0) = \frac{c}{a} \ln \left( \frac{at}{c} + \sqrt{1 + \frac{a^2t^2}{c^2}} \right) \quad (4.9)
\]

local LT, \( x' = 0, \ 0 < t < t_{acc} \)

For \( t \geq t_{acc} \) Eqn(4.7) simplifies to:

\[
dt' = \frac{dt}{\gamma(t_{acc})} \quad (4.10)
\]
so that for \( t \geq t_{\text{acc}} \):

\[
t'_A = t'_A(t_{\text{acc}}, x'_A = 0) + \frac{t - t_{\text{acc}}}{\gamma(t_{\text{acc}})}
\]

(4.11)

where \( t' = t'(t_{\text{acc}}, x' = 0) \) is given by Eqn(4.9). An identical result is, of course, given by a local LT for clock B. Thus, in this case, translational invariance, as depicted in Fig.1a, is evidently respected.

A non-local LT is now used to evaluate \( t' \) for the clock B. The origin of the LT in \( S' \) is chosen at the position of the clock A. The value of \( t' \) for A, \( t'_A \), is then given by Eqns(4.9),(4.11). Now the position of B corresponds to a non-local LT with \( x'_B = L \). In this case, the differential form of (4.6) gives, instead of (4.8), the expression:

\[
dt' = \frac{dt}{\sqrt{1 + \frac{a^2 t^2}{c^2}}} - \frac{L}{c}d\beta
\]

(4.12)

and, on integrating over \( t \):

\[
t'_B(t, x'_B = L) = \frac{c}{a} \ln \left( \frac{at}{c} + \sqrt{1 + \frac{a^2 t^2}{c^2}} \right) - \frac{Lat}{c^2 \sqrt{1 + \frac{a^2 t^2}{c^2}}}
\]

(4.13)


\[
\text{non-local LT, } x'_B = L, \quad 0 < t < t_{\text{acc}}
\]

That is:

\[
t'_B(t, x'_B = L) = t'_B(t, x'_B = 0) - \frac{L\beta(t)}{c}
\]

(4.14)

Since, for the local LT with \( x' = 0 \) for both A and B:

\[
t'_A(t, x'_A = 0) = t'_B(t, x'_B = 0)
\]

(4.15)

it follows that:

\[
t'_B(t, x'_B = L) = t'_A(t, x'_A = 0) - \frac{L\beta(t)}{c}
\]

(4.16)

Evidently \( t'_A \) and \( t'_B \) are different, and so translational invariance is not respected if \( L \neq 0 \), i.e. if the LT applied to the clock B is non-local.

Alternatively, choosing the origin in \( S' \) of the LT for clock A to coincide with the position of B \( (x'_A = -L) \) or to lie midway between the two clocks \( (x'_A = -L/2, x'_B = L/2) \) gives the results, valid in the interval \( 0 \leq t \leq t_{\text{acc}} \):

\[
t'_A(t, x'_A = -L) = t'_A(t, x'_A = 0) + \frac{L\beta(t)}{c}
\]

(4.17)

\[
t'_A(t, x'_A = -L/2) = t'_A(t, x'_A = 0) + \frac{L\beta(t)}{2c}
\]

(4.18)

\[
t'_B(t, x'_B = L/2) = t'_A(t, x'_A = 0) - \frac{L\beta(t)}{2c}
\]

(4.19)

The times registered by the clocks A and B, as viewed from S, at time \( t = t_{\text{acc}} \), are shown, for the different choices of the origin in \( S' \) of the LT considered above, in Fig.3.
Units are chosen such that $c = a = 1$. Also $L = 1$ and $\beta_f = \beta(t_{\text{acc}}) = \sqrt{3}/2$, so that $\gamma_f = 1/\sqrt{1 - \beta_f^2} = 2$ and $t_{\text{acc}} = \sqrt{3}$. The time interval $L\beta_f/c = \sqrt{3}/2$ corresponds to a $90^\circ$ rotation of the hands of the clocks A and B, while $t_{\text{acc}} = \sqrt{3}$ corresponds to a $180^\circ$ rotation of the hand of C. The time shift between clocks A and B depends only on the final velocity and the spatial separation of the clocks—it is independent of the acceleration $a$. The same shift therefore occurs for instantaneous acceleration into the frame $S'$ from $S$, as often considered in idealised space-time thought experiments.

Since the procedure described for accelerating the clocks is physically well defined and unique, the times indicated by the clocks A and B must also be unique, so at most one of the four cases shown in Fig.3 can be physically correct. Since it has been argued in Section 3 above that translational invariance requires that the clocks A and B indicate always the same time, the only possibility is that shown in Fig.3a, corresponding to a local LT for each clock. The non-local LT used in Figs.4b,c,d must then be unphysical. In each of these cases where events contiguous in space-time with one clock,(Fig. 3b and 3c) or both clocks,(Fig. 3d) are subjected to a non-local LT, it can be seen that clock A is apparently in advance of clock B by the fixed time interval $L\beta_f/c$. This is just the apparent effect of ‘relativity of simultaneity’ resulting from the different spatial positions of the clocks introduced in Einstein’s first paper on SR [1]. It results from the term $\beta x'/c$ in Eqn(4.6). The argument just presented shows that the physical existence of such an effect is in contradiction with translational invariance. It must then be concluded that, if translational invariance is respected, a non-local space-time LT is unphysical and that ‘relativity of simultaneity’ does not exist. Since LC is a direct consequence of the relativity of simultaneity [7] it also cannot exist if translational invariance is respected. The LC effect is further discussed in the following Section.

It may be noted that in all cases shown in Fig.3, the formula (4.11) is valid, on substituting the appropriate value of $t'(t_{\text{acc}})$. If $t_1 > t_2 > t_{\text{acc}}$ it then follows, independently of the choice of a local or non-local LT, or of the position of the origin of the latter in $S'$, that:

$$\Delta t' = t'_1 - t'_2 = \frac{t_1 - t_2}{\gamma(t_{\text{acc}})} = \frac{\Delta t}{\gamma(t_{\text{acc}})}$$

(4.20)

Thus the rate, as observed in $S$, of both clocks A and B, after acceleration, is slowed down in accordance with the well known Time Dilatation (TD) effect, although the actual times recorded by the clocks do depend on the choice of the $S'$ origin for the non-local LT. Indeed, as shown in Fig.3c, for the choice $x'_A = -L$, the clock A is apparently in advance of the stationary clock C at the end of the acceleration period. The TD effect for uniformly moving clocks is given by the $\Delta x' = 0$ projection of the LT [6] and, as shown in Section 5 below, is defined as the result of successive local LTs performed on the moving clock. Thus the experimentally confirmed TD effect is perfectly compatible with translational invariance.

It is important for the calculation of $t'(t)$ that the clocks A, B, C be properly synchronised at time $t = 0$ when they are at rest (see Fig.2 ). As discussed at length in Reference [1] the problem of synchronisation of clocks, situated at different spatial positions, is a non-trivial one. In fact in Reference [1] the ‘relativity of simultaneity’ was derived by comparing the synchronisation of clocks in a stationary and a moving frame. However, it is not necessary to discuss clock synchronisation to understand that the phys-
ical situation depicted in Fig3a is the only possible one. For this it is convenient to introduce ‘radioactive clocks’ which need no synchronisation. If the clocks A, B and C consist of similar samples of radioactive nuclei, equipped with a detector, then the proper time $t'$ is determined by the number of radioactive decays, $N_{obs}$, recorded. In the limit that $N_{obs} \to \infty$:

$$t' = \tau N \ln \left( \frac{N_0}{N_0 - N_{obs}/\epsilon} \right)$$

(4.21)

where $N_0$ is the number of unstable nuclei at time $t = 0$, $\tau N$ is the mean life of the radioactive nucleus and $\epsilon$ is the efficiency of detection. Allowing for the statistical accuracy in the determination of $t'$ due to the finite value of $N_{obs}$, it is clear that two such clocks, initially situated at different spatial positions and then subjected to identical acceleration programs, must record equal values of $t'$ at any time $t$ in S. Thus Figs.3b,c,d clearly correspond to physically impossible situations.

5 Local and Non-Local Lorentz Transformations and the Relativistic Length Contraction

Figure 4: a) real positions of clocks A and B in either S at $t = 0$ or S’. b)-f) show the apparent positions in S at $t = 0$ of the clocks A and B for different choices of the origin, $O'$, in S’: b) $O'$ at A, c) $O'$ at B, d) $O'$ midway between A and B. A, e) $x_A' = -3L/2$, f) local LT for both A and B ($x_A' = x_B' = 0$). In all cases clocks are synchronised so that $t = t' = 0$ when the origins of S and S’ coincide. Units and parameters as in Fig.3.
In order to discuss the LC effect it will be convenient to consider the positions of the clocks A and B, introduced above, for \( t > t_{\text{acc}} \). Both clocks then move with their final constant relativistic velocity \( \beta_f = \beta(t_{\text{acc}}) \). The real separation of the clocks, in both S and S', is \( L \), as a consequence of the identical acceleration program to which they were subjected. As in the case of the calculation of \( t'(t) \) discussed in the previous Section, different LT, both local and non-local, will be used to calculate the apparent positions of the clocks in S and the clock synchronisation between S and S' will be performed during the phase of uniform motion following acceleration.

The first case considered is same as an early discussion of the LC effect by Einstein \[14\], and is similar to the presentation of LC given in most text books on SR. The origin of the coordinate system in S is chosen to coincide, at some instant when the clocks are synchronised so that \( t = t' = 0 \), with that of S', which is located at the position of clock A. Applying the LT Eqn(4.3) then gives \( x_{A}^{\text{app}} = 0 \) at this time, i.e. the apparent position, according to the local LT at the clock A is the same as the real position of clock A at \( t = 0 \). The LT (4.3), with the above choice of clock synchronisation, is now applied to the clock B. This non-local LT with \( x_{B}^{\prime} = L \) gives, for the apparent position of B, \( x_{B}^{\text{app}} = L/\gamma_f \). Thus, the apparent position of B, according to the LT, is shifted from the actual position, \( x_{B} = L \) of B at \( t = 0 \). The distance between the apparent positions of A and B is:

\[
x_{B}^{\text{app}} - x_{A}^{\text{app}} = L/\gamma_f - 0 = L/\gamma_f,
\]

the well-known LC effect. The results of repeating this type of calculation, with different choices of the origin in S', but always synchronising the clocks so that \( t = t' = 0 \) when the origins of S and S' coincide, are presented in Table 1 and shown in Fig.4. Fig.4a shows the real positions of the clocks in either S at \( t = 0 \) or S'. In Fig.4b-f the apparent positions of the clocks according to the LT, as observed in S at \( t = 0 \), are shown for different choices of the origin O' of the LT. In Fig.4b, corresponding to Einstein’s calculation of Reference \[14\] the LT is local for A but not for B, In Fig.4c it is local for B but non-local for A. In Figs.4d,e it is non-local for both clocks. Finally, in Fig.4f it is local for both clocks. Since, as shown in the previous Section, only the case shown in Fig.4f is consistent with translational invariance of \( t'(t) \) this is the only physically possible solution. In can be seen that, in this case, the real and apparent positions of the clocks are the same and there is no LC effect. In fact, it is assumed that the local LT are performed for all points of the spatially extended clocks A and B. Thus the apparent sizes of the clocks are the same in S and S' (see Fig.4f). If a fixed origin is chosen in S' for the LT of events contiguous in S' with the clocks, as in Figs.4b-e, the clocks are apparently contracted, parallel to their direction of motion, by the factor \( 1/\gamma_f \).

Indeed it is clear by an even more basic requirement of a physical theory, that it gives some well defined prediction, that the LC cannot be even an apparent physical phenomenon. There are an infinite number of different predictions for the apparent positions of the moving clocks, in all of which they are separated by the ‘Lorentz-contracted’ distance \( L/\gamma_f \), differing only by the arbitrary choice of the position of the origin, O', of the non-local LT. In fact the apparent position of, say, \( x_{A} \), can be anywhere on the x-axis with a suitable choice of O’. As can be seen from the fourth row of Table 1, \( x_{A} = X_{A} \) and \( x_{B} = X_{A} + L/\gamma_f \), for any \( X_{A} \), by choosing \( x'_{A} = -\gamma_f X_{A}/(\gamma_f - 1) \). The problem here can be formulated in terms of a general principle that should be obeyed by any acceptable physical theory. It might be called CIPP for ‘Coordinate Independence of Physical Predictions’ and is equivalent to translational invariance:
Table 1: Apparent positions and times of clocks A and B at \( t = 0 \) for different choices of origin in \( S' \). In all cases the clocks in \( S \) and \( S' \) are synchronised at \( t = t' = 0 \) when the origins of \( S \) and \( S' \) are coincident.

| \( x'_A \) | \( x'^{\text{app}}_A - x_A \) | \( x'^{\text{app}}_B - x_A \) | \( t_A \) | \( t_B \) | \( t'_A \) | \( t'_B \) |
|---|---|---|---|---|---|---|
| 0 | 0 | \( \frac{L}{\gamma_f} \) | 0 | 0 | 0 | \( -\frac{\beta_f L}{c} \) |
| \(-L\) | \( \frac{(\gamma_f-1)L}{\gamma_f} \) | \( L \) | 0 | 0 | \( \frac{\beta_f L}{c} \) | 0 |
| \(-\frac{L}{2}\) | \( \frac{(\gamma_f-1)L}{2\gamma_f} \) | \( \frac{(\gamma_f+1)L}{2\gamma_f} \) | 0 | 0 | \( \frac{\beta_f L}{2c} \) | \( -\frac{\beta_f L}{2c} \) |
| \(-L + D\) | \( \frac{(\gamma_f-1)(L+D)}{\gamma_f} \) | \( L + \frac{(\gamma_f-1)D}{\gamma_f} \) | 0 | 0 | \( \frac{\beta_f (L+D)}{c} \) | \( \frac{\beta_f D}{c} \) |

The predictions of physical phenomena by any theory must be independent of the choice of spatial coordinate system.

It is shown clearly in Fig.4 that this principle is not respected by a non-local LT. Since, for a local LT the arbitrariness in the choice of the origin of coordinates is, by definition, absent, CIPP is not applicable and unique predictions for the observed space-time positions of events in \( S \) are always obtained.

As will be discussed in Section 9 below, and unlike for TD, there is, at the present time, no experimental evidence for the relativity of simultaneity or the LC. The same is true of the two other effects of SR [6], Space Dilatation (SD), the \( \Delta t' = 0 \) projection of the LT, and Time Contraction (TC) the \( \Delta x = 0 \) projection of the LT, both of which are also direct consequences of the relativity of simultaneity. On the other hand, the well verified TD effect is, by definition, the prediction of a local LT. Typically, the moving clock is assumed to be situated at \( x' = 0 \) so that the LT of Eqns(4.3) and (4.4) becomes:

\[
x' = 0 \\
t' = \frac{t}{\gamma} \\
x = \gamma vt' \\
t = \gamma t'
\]

Local LT of an event at \( x' = 0 \)

Eqns(5.2) and (5.4) are identical and, by use of Eqn(5.2), Eqn(5.3) reduces to:

\[
x = vt
\]

which is just the trajectory of the origin of \( S' \) in \( S \). Thus, unlike in the case of a non-local LT, there is no distinction between the apparent position of the observed event in \( S \) and the actual position, in this frame, of the corresponding point in \( S' \). Eqn(5.4) is just the TD effect.
The inverse local LT is:

\[ x = 0 \]  \hspace{1cm} (5.6) 
\[ t = \frac{t'}{\gamma} \]  \hspace{1cm} (5.7) 
\[ x' = -\gamma vt \]  \hspace{1cm} (5.8) 
\[ t' = \gamma t \]  \hspace{1cm} (5.9) 

Local LT of an event at \( x = 0 \)

Again, Eqns(5.7) and (5.9) are identical, and Eqns(5.7) and (5.8) may be combined to give:

\[ x' = -vt' \]  \hspace{1cm} (5.10) 

the trajectory of the origin of S in S’. Eqn(5.9) is the TD effect for a clock at rest in S when viewed from S’.

Since the spatial position of the transformed event can always be chosen at the origin of S’, it is proposed here that Eqns(5.1)-(5.10) embody the essential physical content of the space-time LT. Thus, only time and not space is modified in the passage from Galilean to Special Relativity. The local LT differs from the general LT of Eqns(4.4) and (4.5) only in the specific choice of origin for the transformed event. As seen above, SR based on the local LT (5.1)-(5.4) gives, unlike the non-local LT, a unique prediction that respects translational invariance. In such a theory there is no relativity of simultaneity or the associated apparent distortions of space and time: LC, SD, TC. The apparent lengths of physical objects, or their spatial separations, are the same in all inertial frames, and correspond to the real positions of the objects, described by the usual Galilean kinematical laws, in each frame. Only the apparent time, described by the TD formula, changes from one inertial frame to another.

In fact, setting \( \gamma = 1 \) in (5.2)- (5.4) and (5.7)- (5.9) yields a local Galilean transformation and its inverse. The difference between such a transformation and a LT is then only an \( O(\beta^2) \) effect. In contrast non-local Galilean and Lorentz transformations differ also by the \( O(\beta) \) terms giving the spatial dependence of the transformation of time in (4.4) and (4.6). Just these terms are responsible for relativity of simultaneity and the associated LC, SD and TC effects, although only the relativity of simultaneity effect itself is an \( O(\beta) \) effect. As discussed in Section 9 below, at the time of writing there is no experimental evidence for the existence of such \( O(\beta) \) terms. The second of the satellite experiments to be proposed in Section 10 below can easily confirm of exclude the importance of such terms.

In the following Sections the differences in space-time geometry of the local and non-local LT are explored in more detail, as well as relativistic kinematics, which is shown to be unchanged by the restriction to a local LT.
6 ‘Source Signal Contiguity’ and the Resolution of Some Causal Paradoxes of Special Relativity

In Section 4 above, the breakdown of translational invariance by a non-local LT was demonstrated by considering the apparent times of similarly accelerated clocks. This breakdown is however directly evident, in space, in Figs 4a-4e. It is sufficient to consider Fig 4b, corresponding to Einstein’s demonstration [14] of the LC. The apparent position of A is the same as the real position of the clock, whereas for the identical clock B, only displaced by the distance $L$ along the x-axis, the apparent position differs by $L/2$ from its real one. Since the clocks are identical apart from their position along the x-axis, the relationship between the real and apparent positions must be the same for both clocks. Since the non-local LT predicts that this is not the case it clearly violates translational invariance.

Consider now single photons emitted (or reflected) from A and B, which, when detected in S, constitute an observation of the apparent positions of the clocks in this frame. In S’, the proper frame of the clocks, it is always assumed that the space-time events corresponding to the photon emission process and that describing the position of the source, are contiguous. The same is true, in the example of Fig 4b, for the space-time event corresponding to the observation in S of the emission of a photon from A, and that corresponding to the real position of A at the time of emission. This is not the case for a photon emitted by B. Not only is it apparently spacially displaced from the real position of B at the time of observation, but also, as shown in the first row of Table 1, it is emitted from B at the earlier time, $t' = -\beta f L/c$, when the real position of B is displaced in S by the distance $-\gamma f \beta_f^2 L = -3/2$, as compared to that shown in Fig 4a, where the clocks A and B are separated by one spatial unit. Thus the photon appears to be ‘hanging in time’ in S for the period $\gamma f \beta_f L/c = \sqrt{3}$ between its times of emission and observation.

A similar consideration of the example shown in Fig. 4d, presented elsewhere [7], shows that the photon emitted from A is apparently observed in S before A reaches the position at which the photon is emitted in S’, thus apparently violating causality. None of these paradoxes occur when local LT at A and B are used to calculate the apparent positions in S of photon emission as shown in Fig 4f. The observed positions of photon emission are then contiguous in space-time with the real positions of A and B.

The causal paradox of the ‘backward running clocks’ pointed out in a recent paper [15] is also resolved by use of the local LT. In Fig. 1c of Reference [15], the different apparent times registered by the clocks A’, B’ and C’ are due to the term $\beta x'/c$ in Eqn (4.6) that leads to ‘relativity of simultaneity’. This figure shows a similar effect to that of Fig 2b,c,d of the present paper. In order that the three clocks show the same time when they are brought to rest, as shown in Fig 1d of Reference [15] (thus, tacitly, imposing the condition of translational invariance) it is necessary that the clock A’ apparently runs ‘backward in time’ and C’ ‘forward in time’ during the deceleration. If local LT are used to calculate the apparent times of A’ and C’ (as is already done, in the example, for B’) all the moving

---

5 The observer in S will in fact observe the photon at a later time due to propagation delays and at a different position due to optical aberration. As mentioned in the Introduction, it is assumed, in the following discussion, that corrections for these effects have been applied.
clocks in Fig.1c of Reference [15] indicate the same apparent time and the paradox is resolved.

Since essentially all our knowledge of objects in the external world is derived from our observation (direct or indirect) of photons emitted by, or scattered from, them, it is only possible to obtain knowledge of the real positions or sizes of such physical objects by making an assumption, that may be called ‘Source Signal Contiguity’ (SSC). The latter may be stated as the following physical principle:

Space-time events corresponding to photon emission processes and the space-time positions of their sources are contiguous in all frames of observation.

This is the tacit assumption made, for example, in all astronomical measurements where, say, the diameter of the Sun or Moon is deduced from a pattern of photons detected by a telescope. As shown above, the SSC principle is not respected by the non-local LT. To give another concrete example of this in Astrophysics, consider a star moving perpendicular to the line of sight with velocity \( \beta c \) relative to the Earth, that explodes, emitting a light pulse of very short duration in its proper frame. As viewed from the Earth, with coarse time resolution, the image of the explosion will be, according to a calculation using a non-local LT, elongated parallel to the velocity direction by the factor \( \gamma \) due to the SD effect [6]. In S, the observed photons from the star would not respect the SSC principle. A calculation using a local LT for each emitted photon, after applying appropriate corrections for light propagation time delays and optical aberration, does respect SSC and, as in Fig.4f, predicts an observed image that faithfully reflects the real spatial distribution of the different photon sources in the frame of observation.

7 The Minkowski space-time plot and Relativistic Kinematics

Consider two clocks C1 and C2 situated on the \( x' \) axis at \( x_1 = 0 \) and \( x_2 = L' \) respectively. The times registered by these clocks are denoted by \( t_1' \) and \( t_2' \) respectively. If \( t \) is the time recorded by a fixed clock at an arbitrary position in S, then, with a particular choice of spatial coordinate system in S, the equation of motion of C1 in S may be written: \( x_1(t) = vt \). The LT connecting the spatial coordinates in S and S' and the times \( t_1' \) and \( t \) may then be written:

\[
\begin{align*}
    x_1' &= \gamma [x_1 - vt] = 0 \\
    t_1' &= \gamma [t - \frac{v x_1}{c^2}]
\end{align*}
\] (7.1) (7.2)

The space transformation equation is a necessary consequence of the choice of spatial position of C1 in S', and the choice of coordinate system in S – both of which do not depend on the relative velocity, \( v \), of S’ relative to S. When \( t = 0 \) it follows from (7.1) and (7.2) that \( t_1' = 0 \), so that C1 and the clock in S are synchronised at this instant. Using the same coordinate system in S, the equation of motion of C2 is written as \( x_2(t) = vt + L \), where \( L \equiv x_2(0) \) is a constant, independent of \( v \). The LT connecting the spatial coordinates of
Figure 5: Minkowski plots for the clocks C1 and C2 showing the absence of the ‘length contraction’ and ‘relativity of simultaneity’ effects. See text for discussion.
C2 in S’ to those in S and the time \( t' \) to \( t \) is:

\[
x'_2 - L' = \gamma [x_2 - L - vt] = 0 \tag{7.3}
\]

\[
t' = \gamma \left[ t - \frac{v(x_2 - L)}{c^2} \right] \tag{7.4}
\]

When \( t = 0 \), (7.3) and (7.4) require that \( t'_2 = 0 \), so that, at this instant, C1, C2 and the clock in S are all synchronised. As \( v \to 0 \), \( x \to x' \) and \( \gamma \to 1 \). For \( v = 0 \) (7.3) is then written:

\[
x'_2 - L' = x'_2 - L = 0 \tag{7.5}
\]

so that \( L' = L \) –as discussed Section 2 above there is no ‘length contraction’ effect. Eliminating \( x_1 \) from (7.2) and \( x_2 \) from (7.4) by use of (7.1) and (7.3) respectively, gives the TD relations

\[
t'_1 = \frac{t}{\gamma} = t'_2 \tag{7.6}
\]

so that C1 and C2 remain synchronised at all times –there is no ‘relativity of simultaneity’ effect. Introducing the local coordinate systems defined by

\[
\tilde{x}_1(0) \equiv x_1, \quad \tilde{x}'_1(0) \equiv x'_1; \quad \tilde{x}_2(L) \equiv x_2 - L, \quad \tilde{x}'_2(0) \equiv x'_2 - L
\]

(7.3) and (7.4) are written as a local LT similar to (7.1) and (7.2):

\[
\tilde{x}'_2(L) = \gamma [\tilde{x}_2(L) - vt] = 0 \tag{7.8}
\]

\[
t'_2 = \gamma \left[ t - \frac{v\tilde{x}_2}{c^2} \right] \tag{7.9}
\]

(7.1) and (7.2) are recovered by setting \( L = 0 \) in (7.8) and (7.9). With the aid of the TD relations (7.6), (7.8) and (7.9) may be written as:

\[
\tilde{x}_2(L) = vt = \gamma \beta t'_2 \tag{7.10}
\]

\[
t = \gamma t'_2 \tag{7.11}
\]

These equations show that events: \( \tilde{x}_1(0), t_1 \) on the world line of C1 and \( \tilde{x}_2(L), t_2 \) on the world line of C2 lie, as a consequence the identity: \( \gamma^2 - \beta^2 \gamma^2 \equiv 1 \), on the similar hyperbolae:

\[
c^2 t'^2 - \tilde{x}'^2(0)^2 = c^2(t_1')^2 \tag{7.12}
\]

\[
c^2 t'^2 - \tilde{x}'^2(L)^2 = c^2(t'_2)^2 \tag{7.13}
\]

In Fig. 5, these hyperbolae at the instant \( t'_1 = t'_2 = t' \) are plotted, as well as the world lines in S of C1 and C2 for \( \beta = 0 \) and \( \beta = 1/3 \). The absence of ‘length contraction’ and ‘relativity of simultaneity’ effects is evident from inspection of this figure.

Events on the world-line of a massive physical object, \( O(m) \), of Newtonian mass \( m \), are time-like separated and so lie on a hyperbola such as (7.12) or (7.13). The TD relation (7.11) gives:

\[
\Delta t = t_1 - t_2 = \gamma(t'_1 - t'_2) = \gamma \Delta \tau \tag{7.14}
\]

where \( \tau \equiv t' \) is the proper time of the object. The energy momentum and velocity 4-vectors, \( P \) and \( V \) respectively of \( O(m) \) are defined as [17]:

\[
P \equiv m \frac{dx}{d\tau} = (P_0, P_x) = mV \tag{7.15}
\]

\[
V \equiv (c\gamma, c\beta \gamma) \tag{7.16}
\]
where 

\[ x \equiv (ct, x) = (x_0, x) \]

and Eqn(7.14) has been used to relate \( d\tau \) and \( dt \). Consider now a LT from S into another inertial frame S” moving with velocity \( u \) along the x-axis relative to S. The corresponding spatial LT analogous to (4.3) is:

\[ x'' = \gamma_u (x - \beta_u x_0) \] (7.17)

where

\[ \beta_u = \frac{u}{c}, \quad \gamma_u = \frac{1}{\sqrt{1 - \beta_u^2}} \]

Multiplying by \( m \) and taking the derivative with respect to the proper time gives:

\[ m\frac{dx''}{d\tau} = P''_x = \gamma_u (P_x - \beta_u P_0) \] (7.18)

It follows from Eqn(7.15) and (7.16) and the analogous four-vector definitions in S” that:

\[ P'^2_0 - P'^2_x = (P''_0)^2 - (P''_x)^2 = m^2 c^2 \] (7.19)

Squaring Eqn(7.18), adding \( m^2 c^2 \) to the LHS, using Eqn(7.19) and performing some algebra allows the derivation of the LT of the relativistic energy of \( O(m) \):

\[ P''_0 = \gamma_u (P_0 - \beta_u P_x) \] (7.20)

Taking the ratio of Eqn(7.18) to Eqn(7.20):

\[ \frac{P''_x}{P''_0} = \beta'' = \frac{P'_x/P'_0}{1 - \beta_u P'_x/P'_0} = \frac{\beta - \beta_u}{1 - \beta_u \beta} \] (7.21)

This equation may be rearranged as:

\[ \beta = \frac{\beta'' + \beta_u}{1 + \beta'' \beta_u} \] (7.22)

which is the usual relativistic velocity addition formula. It has been derived here from the differential TD relation (7.14) and the derivative with respect to the proper time of the spatial LT between S and S”. Thus, the result is not dependent on the use of the LT of time Eqn(4.4), containing the term \(-\beta \gamma x/c\) that is responsible, in the case of a non-local space-time LT, for the unphysical ‘relativity of simultaneity’. Alternatively Eqn(7.22) can be more directly obtained by taking the derivative, with respect to the proper time, of the non-local temporal LT between S and S”:

\[ x''_0 = \gamma_u (x_0 - \beta_u x) \] (7.23)

Because of the derivative, the choice of coordinate origins is again of no importance.

In summary, the usual formulae of relativistic kinematics can be derived either by using only the spatial LT, and a differential TD relation, or by taking derivatives with respect to proper time in both the spatial and non-local temporal LT . No distinction
between ‘local’ and ‘non-local’ LT is then required in the derivation of kinematic formulae in SR.

8 The ‘Lorentz Fitzgerald Contraction’ and the Michelson-Morley Experiment as a ‘Photon Clock’

As discussed in most text-books on SR, the Lorentz Fitzgerald Contraction hypothesis was introduced by Fitzgerald [18] and further developed by Lorentz [19] in an attempt to reconcile the null result of the Michelson-Morley (MM) experiment [20] with the propagation of light as a wave motion in a luminiferous aether, through which the Michelson-Morley interferometer was conjectured to move with velocity \( v_{ae} \). The null result of the experiment is explained if, due to a dynamical interaction with the aether, the length of all moving bodies is contracted by the factor \( \sqrt{1 - (v_{ae}/c)^2} \).

In Einstein’s SR there is no aether, and the LC effect of the same size as that conjectured by Fitzgerald and Lorentz is found to be a geometrical consequence (\( \Delta t = 0 \) projection) of a non-local space-time LT. The usual text-book discussion of the MM experiment, in terms of SR, claims to explain the null result by invoking a geometrical contraction of the arm of the interferometer parallel to the direction of motion by the Lorentz-Fitzgerald factor. Since this is often interpreted as experimental evidence for the LC, which it has been argued in the previous sections of the present paper, cannot be a real physical effect, it is mandatory to now re-examine carefully the description of the MM experiment in SR.

Consider a Michelson interferometer with arms of equal length, \( L \), at rest in the frame \( S' \). The half-silvered plate is denoted by P and the mirrors by M1 and M2. The arm P-M2 is parallel to the direction of uniform motion of the interferometer, with velocity \( v \), in the frame \( S \) (see Fig.6a). At time \( t' = 0 \), a pulse of photons moving parallel to the \( x' \)-axis is split by P into two sub-pulses moving along the two arms of the interferometer. Typical photons moving in the arms P-M1 and P-M2 are denoted as \( \gamma_1 \) and \( \gamma_2 \) respectively. Each arm of the interferometer can now be considered as an independent ‘photon clock’ which measures the proper time interval in \( S' \) corresponding to the time for the photons to make the round trip from P to the mirrors and back again to P: \( \Delta \tau = 2L/c \). First, the conventional argument [21] for the existence of a LC effect for the arm P-M2 will be examined, before analysing in detail the sequence of events, as predicted by the LT, seen by an observer in \( S \) during the passage of the photons through the interferometer.

The relativistic velocity addition formula (7.23) predicts that photons (or, in general, any massless particles) have the same speed, \( c \), in any inertial frame. Under the apparently plausible assumption (to be discussed further below) that the space-time events A (reflection of \( \gamma_1 \) from M1, Fig.6b), B (reflection of \( \gamma_2 \) from M2, Fig.6c) and C (arrival of both reflected photons back at P, fig.6d) can be calculated by following photon paths in the frame \( S \), the following relations are derived:

\[
\begin{align*}
ct_{OA} &= \sqrt{L^2 + v^2 \ell^2} = \ct_{AC} = \sqrt{L^2 + v^2 \ell^2} \\
ct_{OB} &= \ell + vt_{OB}, \quad \ct_{BC} = \ell - vt_{BC}
\end{align*}
\]
Here \( t_{OA} \) and \( t_{AC} \) refer to time intervals along the path in S of \( \gamma_1 \), while \( t_{OB} \) and \( t_{BC} \) are similarly defined for \( \gamma_2 \). The ‘observed length’ of the arm P-M2 is denoted by \( \ell \). The total transit time in S of \( \gamma_1 \) is:

\[
t_{OAC} = t_{OA} + t_{AC} = \frac{2L}{c\sqrt{1 - (\frac{v}{c})^2}}
\]  

(8.3)

while that of \( \gamma_2 \) is

\[
t_{OBC} = t_{OB} + t_{BC} = \frac{2\ell}{c(1 - (\frac{v}{c})^2)}
\]  

(8.4)

Assuming now that \( \gamma_1 \) and \( \gamma_2 \) are observed in space-time coincidence when they arrive back at C: \( t_{OAC} = t_{OBC} \), it follows from Eqns(8.3) and (8.4) that:

\[
\ell = L\sqrt{1 - (\frac{v}{c})^2}
\]

(8.5)

The ‘observed’ length of the arm P-M2 is reduced, compared to its length in S’, by the same factor as in the LC. However, the space-time observations performed here are quite different to the \( \Delta t = 0 \) projection that defines the LC. As will become clear when the pattern of space-time events seen by an observer in S is calculated in detail, at no point is a simultaneous observation made of both ends of the arm P-M2. Thus the SR ‘effect’ embodied in Eqn(8.5) is quite distinct from the LC, even though the length contraction factor is the same.

The sequence of events observed in S during the passage of the photons along the two arms of the interferometer is now considered. It is imagined that each mirror, as well as the plate P, are equipped with small photon detectors, with a time resolution much smaller than \( L/c \), that are used to trigger luminous signals in the close neighbourhoods of P, M1 and M2 so that the observer in S observes space-time events corresponding to the passage of the photon pulses through P, their reflections from M1 and M2 and their return to P. It is also assumed that the observer in S records the actual (real) position of the interferometer for comparison with the apparent positions of the signals generated by the passage of the photons.

The first case considered is a LT with origin in S’ coincident with P. The LT is thus local for events situated at P or M1 but non-local for events situated at M2. The positions and times in S’ 6, the apparent positions and times in S, as well as the corresponding real positions of P of the four events: (i) initial passage of photons through P, (ii) reflection of \( \gamma_1 \) from M1, (iii) reflection of \( \gamma_2 \) from M2 and (iv) return passage of \( \gamma_1 \) and \( \gamma_2 \) through P, are presented in Table 2 and shown in Fig.6. In Figs.6, 7 and 8 the real positions in S of M1 and M2 are denoted by cross-hatched rectangles and their apparent positions by open rectangles, and also directly (e.g. M2a in Fig.6c). The indicated positions of the plate P correspond to real positions in S unless otherwise indicated (as Pa). The apparent positions in S of the photons at the indicated times are those of the tip of the corresponding arrowhead. The distances and times shown in Figs.6, 7 and 8 correspond, as previously, to the choice of parameters: \( L = c = 1, \beta = \sqrt{3}/2 \). As shown in Fig.6a,b and d the events (i), (ii) and (iv) are observed in spatial coincidence with the positions of P, M1 and, again, P, respectively. However, the events (ii) and (iii), that are simultaneous

---

6In S’ there is evidently no distinction between the real and apparent positions of events
Figure 6: The sequence of events observed in S during the passage of photon pulses through a Michelson interferometer moving with constant velocity $\beta = \sqrt{3}/2$. a) initial pulse arrives at the half-silvered plate, P, and divides into sub-pulses moving along the arms P-M1 ($\gamma_1$) and P-M2 ($\gamma_2$); b) $\gamma_1$ reflects at M1 (event A); c) $\gamma_2$ reflects at M2 (event B); c) $\gamma_1$ and $\gamma_2$ return to P (event C). The origin of S' is at P. Real positions of the moving mirrors are indicated by cross-hatched rectangles, apparent ones by open rectangles, or directly (M2a).

| Event          | $x'$ | $t'$ | $x_{app}'$ | $t_{app}$ | $x_{app}'$ | $x_P$ |
|---------------|------|------|------------|-----------|------------|-------|
| $\gamma_1, \gamma_2$ pass P | 0    | 0    | 0          | 0         | 0          | 0     |
| $\gamma_1$ at M1      | 0    | $\frac{L}{c}$ | $\gamma\beta L$ | $\frac{2L}{c}$ | $\gamma\beta L$ | $\gamma\beta L$ |
| $\gamma_2$ at M2      | $L$  | $\frac{L}{c}$ | $\gamma(1 + \beta)L$ | $\frac{(1+\beta)L}{c}$ | $\gamma\beta(1 + \beta)L$ | $\gamma\beta L$ |
| $\gamma_1, \gamma_2$ back at P | 0    | $2\frac{L}{c}$ | $2\gamma\beta L$ | $\frac{2\gamma L}{c}$ | $2\gamma\beta L$ | $2\gamma\beta L$ |

Table 2: Coordinates of space time events during photon transit of a Michelson interferometer. Origin in S’ at P. The origin of S is at P at $t = 0$. 


Figure 7: As Fig.6 except that the origin of $S'$ is at $M2$

Figure 8: As Fig.6 except that local LT are used for all three events A, B and C
Table 3: Coordinates of space time events during photon transit of a Michelson interferometer. Origin in S' at M2. The origin of S is at distance $\gamma \beta L$ from M2 at $t = -\gamma \beta L/c$.

| Event                  | $x'$ | $t'$ | $x^{app}$ | $t^{app}$ | $x_P^{app}$ | $x_P$          |
|------------------------|------|------|-----------|-----------|-------------|----------------|
| $\gamma_1, \gamma_2$ pass P | $-L$ | 0    | $-\gamma L$ | $-\gamma \frac{L}{c}$ | $-\gamma L$ | $-(1 + \gamma \beta^2)L$ |
| $\gamma_1$ at M1       | $-L$ | $\frac{L}{c}$ | $-\gamma (1 - \beta) L$ | $\gamma (1-\beta) \frac{L}{c}$ | $-\gamma (1 - \beta) L$ | $-\gamma (1 - \beta) L$ |
| $\gamma_2$ at M2       | 0    | $\frac{L}{c}$ | $\gamma \beta L$ | $\frac{2L}{c}$ | $-\gamma (1 - \beta) L$ | $-\gamma (1 - \beta) L$ |
| $\gamma_1, \gamma_2$ back at P | $-L$ | $2\frac{L}{c}$ | $-\gamma (1 - 2 \beta) L$ | $\frac{2 \gamma (2-\beta) L}{c}$ | $-\gamma (1 - 2 \beta) L$ | $(2\gamma \beta - 1 - \gamma \beta^2)L$ |

Table 4: Coordinates of space time events during photon transit of a Michelson interferometer. Local origins in S' at P ($O_1'$) and M2 ($O_2'$). The origin, $O_1$, of S is at P at $t = 0$. $x^{app}$ and $x_P$ are relative to $O_1$. In this case there is no distinction, in the reference frame S, between the apparent positions of events and the real positions of their sources.

| Event                  | $x'$ | $t'$ | $x^{app} = x_{source}$ | $t^{app}$ | $x_P$ |
|------------------------|------|------|------------------------|-----------|-------|
| $\gamma_1, \gamma_2$ pass P | 0    | 0    | 0                      | 0         | 0     |
| $\gamma_1$ at M1       | 0    | $\frac{L}{c}$ | $\gamma \beta L$ | $\frac{2L}{c}$ | $\gamma \beta L$ |
| $\gamma_2$ at M2       | 0    | $\frac{L}{c}$ | $L (1 + \gamma \beta)$ | $\frac{2L}{c}$ | $\gamma \beta L$ |
| $\gamma_1, \gamma_2$ back at P | 0    | $2\frac{L}{c}$ | $2 \gamma \beta L$ | $\frac{2 \gamma \beta L}{c}$ | $2 \gamma \beta L$ |
in $S'$, are not so in $S$. The reflection of $\gamma_2$ on $M_2$ is observed at time $\gamma\beta Lc$ later than the reflection of $\gamma_1$ from $M_1$. Also the observed reflection from $M_2$ is shifted from the actual position of $M_2$ in $S$ at the time of observation (Fig.6c).

The sequence of events presented in Table 3 and shown in Fig.7 results from choosing the origin of $S'$ to be at $M_2$. Now it is only event (iii) that is observed in spatial coincidence with the actual position of the relevant element of the interferometer ($M_2$). Events (i), (ii) and (iv) are all observed at apparent positions separated from the actual positions of $P$, $M_1$ and, again $P$, respectively at the times of observation. In addition the whole sequence of events is shifted in time by $-\gamma\beta L/c$ as compared to those shown in Table 2 and Fig.6. This is the result of different clock synchronisation between $S$ and $S'$ in the two cases. For Table 2, the synchronised clock in $S$ is situated at the position of $P$ at $t = 0$ (Fig.6a), whereas in Table 3 it is at the position of $M_2$ at $t = 0$.

The same critical remarks must be made concerning the different sequences of events predicted by the non-local LT in Figs.6 and 7, as previously made concerning the different apparent times and positions of the moving clocks in Figs.4b,c and d and Figs.5b,c,d and e, respectively. They correspond to physically impossible situations where different sequences of events are observed (with apparent positions of the events differently shifted from the actual positions of the mirrors and plate) depending only on an arbitrary choice of the origin in $S'$ of a non-local LT. As in the case of the discussion of the LC in Section 5, the number of different possibilities is infinite. In the general case where the LT is non-local for all the events (i)-(iv), all of them will be observed at apparent positions different from the actual positions of $P$, $M_1$ and $M_2$ at the observation times in $S$. The sequences of events shown in Figs.6 and 7 clearly violate the ‘CIPP’ principle introduced in Section 5, as well as the ‘SSC’ principle of Section 6. For example, in the case of Table 2 and Fig.6, the photon reflection event and the mirror $M_2$ are contiguous in space-time in $S'$, but not, (as shown in Fig.6c) in $S$. Other examples of the breakdown of SSC in the frame $S$ are shown in Figs.7a, b and d.

The results of using a local LT with origin in $S'$ at $P$ for events (i), (ii) and (iv), and at $M_2$ for event (iii) are presented in Table 4 and shown in Fig.8. In this case the CIPP principle does not apply since there is no arbitrariness in the choice of coordinate origins, and in all cases SSC is respected. Transmission events and the position of $P$, and reflection events and the positions of the mirrors, are contiguous in space-time in both $S'$ and $S$. In this case, there is no apparent contraction of the arm $P-M_2$ as the local LT leaves invariant spatial separations. It is conjectured here that if the gedankenexperiment just discussed were to be actually performed, the observations would be as in Table 4 and Fig.8. Indeed, in the case of a non-local LT there is no definite prediction to be tested, since what are shown, in Tables 2 and 3, and Figs.6 and 7, are just two of an infinite number of predictions in each of which apparent events are observed at different positions, relative to the actual positions of elements the interferometer in $S$.

In contrast to the predictions for the apparent positions of the events (i)-(iv) the apparent time differences between (i) and (ii): $\gamma L/c$ and (i) and (iii): $\gamma(1 + \beta)L/c$ are the same for any choice of the origin of the non-local LT. These may be compared with the corresponding intervals: both equal to $\gamma L/c$ for a local LT. This difference is the basis of the proposed experimental test of RS in the second satellite-borne experiment described.

26
in Section 10 below. The time difference between events (i) and (iv) (one cycle time of either the longitudinal or the transverse photon clocks) is predicted to be \(2\gamma L/c\) by both local and non-local LT. This is in accordance with the universal TD effect for uniformly moving clocks of any nature.

It should be remarked that the simple calculation, based on the hypothesis of an apparent light speed in S of \(c\), recapitulated above, that is used to infer length contraction of the arm P-M2, is consistent with the sequence of events shown if Figs.6 and 7, and is indeed so for any choice of origin in S’ of the non-local LT. The apparent contraction of the length of the arm P-M2 by the factor \(1/\gamma\) is evident in Fig.6c and Fig.7b and d. The situation is similar to that shown in Fig.4b,c,d and e where, although the non-local LT predicts different apparent positions of the clocks, the apparent distance between the clocks is constant and shows the LC effect. As in the case of LC given by the \(\Delta t = 0\) projection of the LT, the LC effect appears in Figs. 6 and 7 is also correlated with the ‘relativity of simultaneity’ effect, none of the entries in Tables 2 or 3 corresponds to a \(\Delta t = 0\) projection.

It will now be found instructive to consider in more detail the two independent ‘photon clocks’ constituted by the two arms of the moving interferometer. That corresponding to the arm P-M1 may be called a ‘transverse’ \(\gamma\)-clock, that to P-M2 as a ‘longitudinal’ one. By comparing the events listed in Tables 2, 3 and 4, it can be seen that the transverse \(\gamma\)-clock gives a similar temporal sequence of events for a non-local or local LT. The transit times from O to A and from A to B (see Fig.6) are equal and given by Eqn (8.1). Direct calculation using the coordinates presented in Tables 2, 3 and 4 shows that the apparent distances in S between events (i) and (ii) and between (iii) and (iv), are, in all cases, \(\gamma L\), while the corresponding elapsed time is \(\gamma L/c\), thus yielding an apparent light speed of \(c\), consistent with the assumption used to derive Eqn (8.1). The behaviour of the transverse \(\gamma\)-clock therefore does not depend on the choice of non-local or local LT to analyse its behaviour. The situation is different for the longitudinal \(\gamma\)-clock based on the arm P-M2. As can be seen from Tables 2 and 3 (the same is true for any other origin in S’ of the non-local LT) and already mentioned above, the event (iii) is observed to occur in S at time \(\gamma\beta L/c\) later than (ii). Also the apparent speed in S of the photon \(\gamma_2\) in both the path OB and BC is always \(c\), and so is consistent with Eqn (8.2) above. When a local LT is used to transform every event from S’ to S, as in Table 4, different apparent speeds are found for the paths OB and BC:

\[
s^{app}_{OB} = c\left(\frac{1}{\gamma} + \beta\right) \quad (8.6)
\]
\[
s^{app}_{BC} = c\left(\frac{1}{\gamma} - \beta\right) \quad (8.7)
\]

Since the total apparent path length according to Table 4 is \(2L\) and the total elapsed time in S is \(2\gamma L/c\) the average apparent speed in the arm P-M2, \(\langle s^{app}_{long}\rangle\) is:

\[
\langle s^{app}_{long}\rangle = \frac{c}{\gamma} \quad (8.8)
\]

Since a local LT modifies time intervals but not spatial separations when transforming events between two inertial frames, and as the apparent speed is just the ratio of the spatial separation to the time interval, Eqn (8.8) is a natural consequence of the TD effect.
It can be seen from Eqns(8.6), (8.7) and (8.8) that $s_{OB}^{app} > c$, $s_{BC}^{app} < c$ and $\langle s_{long}^{app} \rangle < c$. It may seem, at first sight, that Eqns(8.6) and (8.7) are in contradiction with the relativistic velocity addition formula (7.23). However the meanings of these equations are completely different. Eqn(7.23) describes the relation between velocities of some physical object as measured in three different inertial frames, whereas Eqn(8.6) and (8.7) describe apparent photon speeds that correspond to the observation, from an inertial frame, of a clock, in another inertial frame, that uses the assumed constancy of the speed of light propagating over a known fixed distance to define a time interval. These different apparent speeds are a simple consequence of TD of this time interval, specified by the photon clock, and the Lorentz-invariance of spatial separations.

Finally, in this Section, it may be remarked that Eqn(8.8) has a simple physical interpretation in the case that all space-time events are transformed locally between S’ and S. When a local LT is used for a moving macroscopic clock (of any type) the movement of all space points of the clock is apparently slowed down by the universal factor $1/\gamma$. Compare the longitudinal $\gamma$-clock of this type where the photon makes a round trip from P to M2 and back to P, with a conventional analogue clock, where the hands make a single rotation, thus also returning to the original configuration. Just as the rate of rotation of the hand of the clock is apparently slowed down by the fraction $1/\gamma$, due to TD, so too is the apparent speed of the photon in the $\gamma$-clock slowed down by the same factor over one complete period of operation.

9 A Summary of Experimental Tests of Special Relativity

In this Section a brief review is made of the experimental tests of SR that have been performed to date. They can be grouped under four broad categories: (i) tests of Einstein’s second postulate (universality, isotropy and source motion independence of the velocity of light) (ii) measurements of the Transverse Doppler Effect, (iii) tests of relativistic kinematics and (iv) tests of Time Dilatation. Some more general (but model dependent) tests based on electron and muon $g - 2$ experiments are also briefly mentioned. Finally, possible indirect evidence for LC derived from models of particle production in nucleon-nucleon collisions is discussed.

An attempt has been made to be exhaustive concerning the different types of test that have been performed, but not as regards the detailed literature for each type. As far as possible, the most recent and precise limits are quoted. However the review is limited to experiments that test directly predictions of SR rather than searching for new physics effects beyond SR (for example breakdown of Lorentz Invariance) which has been, in recent years, an important new research topic [22].

Isotropy and Source Independence of the Speed of Light

The first experiment of this type, already discussed in the previous Section, is that of MM [20]. The limit obtained for the maximum possible velocity, $v_{ae}$, of the Earth relative to the aether of about 5 km/sec is much less than either the Earth’s orbital speed around
the Sun (30 km/s) or the Earth’s velocity relative to the rest frame of the microwave background radiation (380 km/s). However, because the MM interferometer had arms of equal length, the ‘aether drag’ Lorentz Fitzgerald Contraction hypothesis could not be excluded. This was done by the Kennedy-Thorndike (KT) [23] experiment which was a repetition of the MM experiment with unequal length arms. In more recent years the use of laser beams with frequencies servo-locked to standing-wave modes of a Fabry-Perot interferometer of precisely controlled length [24, 25] allowed a 300 fold improvement in the limit on a possible anisotropy of the speed of light from \( \Delta c/c \leq 3 \times 10^{-8} \) in the original KT experiment to \( \Delta c/c \leq 10^{-10} \) [25]. An even more recent experiment of the same type using two orthogonal cryogenic optical resonators [26] has further improved this limit to: 

\[
\Delta c/c = (2.6 \pm 1.7) \times 10^{-15}.
\]

One of the earliest suggestions to test the independence of the velocity of light from the motion of its source was that of De Sitter [27] who suggested observation of the periodic variation of the Doppler effect in the light from binary star systems. If the velocity of light depends on that of the source according to the classical velocity addition formula, the half-periods observed using the Doppler effect of each member of the binary system are predicted to be unequal [28]. It was pointed out by Fox [29] that if the light is scattered from interstellar matter before arriving at the Earth, then due to the optical ‘Extinction Theorem’, the source of the observed light is, effectively, the scattering atoms, not the moving stars, thus invalidating the test. This objection is not applicable to photons in the X-ray region due to their much smaller scattering cross-section. An analysis [30] of pulsating X-ray sources set a limit of \( k < 2 \times 10^{-9} \) in the modified classical velocity addition formula: 

\[
c' = c + kv.
\]

An accelerator experiment, in which the speed of photons originating in the decay of \( \pi^0 \) mesons of energy about 6 GeV was measured by time-of-flight, set the limit \( k = (-3 \pm 13) \times 10^{-5} \) [31]. The latter two experiments are, then, essentially tests of the relativistic velocity addition formula (7.23).

A review of different experiments, up until 1991, testing the isotropy of the one-way speed of light, and their interpretation in the Mansouri and Sexl ‘test theory’ for parametrising breakdown of Lorentz Invariance [32] can be found in [33].

**Measurements of the Transverse Doppler Effect**

Following the pioneering experiment of Ives and Stilwell [34] using a beam of atomic hydrogen, the Transverse Doppler Effect (essentially the prediction corresponding to the first term on the RHS of Eqn(7.21)) has been measured using many different experimental techniques: temperature dependence of the Mössbauer effect [35], variation of the absorption of photons from a rotating Mössbauer source [36, 37], double photon excitation, by a single laser, of beams of neon [38, 39] or collinear saturation spectoscopy using a \(^7\text{Li}^+\) beam with \( \beta = 0.064 \) and two independent laser beams [40] and laser excitation of an 800 MeV (\( \beta = 0.84 \)) atomic hydrogen beam [41]. The two photon excitation experiments measure the pure Transverse Doppler Effect. In Reference [38] this term was measured with an accuracy of \( 4 \times 10^{-5} \) for \( \beta = 4 \times 10^{-3} \). In a later experiment of the same type [39], the precision was improved to \( 2.3 \times 10^{-6} \). The experiment of Reference [40] reached a similar precision of \( 1.1 \times 10^{-6} \). The experiment of Reference [41] is interesting both because of the highly relativistic nature of the atomic Hydrogen beam (\( \gamma = 1.84 \)).
in comparison with the atomic beams used in References [38, 39, 40] and that, not only
the Transverse Doppler Effect, but, in fact, the whole relativistic energy transformation
equation Eqn(7.21) is tested with a relative precision of $2.7 \times 10^{-4}$.

Tests of Relativistic Kinematics

As just discussed, relativistic kinematics is already tested, using photons, in the Transverse Doppler Effect experiments. The relativistic kinematics of the electron embodied
in the relation $p = \gamma mv$ provided provided one of the first pieces of evidence [42] for the
correctness of the kinematical part of Einstein’s theory of SR. More recent tests of relativis-
tic kinematics using electrons have included measurements of the relation between kinetic
energy and velocity [43] or kinetic energy and momentum [44, 45]. The first of these
experiments used a Van de Graaff accelerator to produce an electron beam with kinetic
energies from 0.5-15 MeV, whose velocity was subsequently measured by time-of-flight.
The latter two experiments used radioactive $\beta$-decay sources, a magnetic spectrometer
to measure the momentum and total absorption detectors to measure the kinetic energy.
An experiment performed at SLAC measured, by time-of-flight, the velocity of 15-20 GeV
photons and electrons over a distance of $\approx 1$ km. The fractional velocity difference was
found to be zero within $2 \times 10^{-7}$ [46]. This is essentially a test of the relation between
energy momentum and mass (Eqn(7.20) as well as that between velocity, momentum and
energy: $\beta = pc/E$.

Tests of Time Dilatation

Precise test of TD have been performed either by using decaying subatomic particles
as ‘radioactive clocks’, as described in Section 4 above, or by comparing an atomic clock,
at rest on the surface of the Earth, with a similar clock in movement (or that has been
in movement) in an aircraft, a ballistic rocket or a satellite moving around the earth.

In the experiment of Ayres et al. [47] the measured lifetimes of $\approx 300$ MeV ($\gamma = 2.55$)
$\pi^{\pm}$ produced at a cyclotron were compared with the known lifetime of pions at rest. The
velocity of the pions, measured by time-of-flight, was used to calculate $\gamma$. The time dilated
lifetime was found to agree with the prediction of SR to within $4 \times 10^{-3}$.

Bailey et al. [48] measured the lifetimes of 3.1 GeV ($\gamma = 29.3$) $\mu^{\pm}$ decaying in the
storage ring of the last CERN g-2 experiment. The ratio of the muon lifetimes in flight
$\tau^{\pm}$ to the known values at rest $\tau_{0}^{\pm}$ were compared with the quantity $\tau$ derived from the
measured cyclotron frequency of the stored muons. For $\mu^{+}$, where the lifetime at rest
is known with the best accuracy, the ratio $\tau^{+}/\tau_{0}^{+}$ was found to agree with $\tau$ to within
$(2 \pm 9) \times 10^{-4}$. The physical meaning of this test is discussed in more detail below.
Another important feature of this experiment is the demonstration that the TD effect
depends only on the absolute value of the velocity. The measured effect is the same as in
an inertial frame even though the transverse acceleration of the circulating muons is $10^{19}$
m s$^{-2}$.

In the experiment of Hafele and Keating [49] four cesium clocks were flown around
the world in commercial airliners, once from west to east (WE) and once from east to
west (EW). They were then compared with similar reference clocks at the U.S. Naval
Observatory, thus realising the ‘twins paradox’ of SR in an actual experiment. The
effects of Special Relativity (TD) and of General Relativity (the gravitational red-shift) were predicted to be of comparable size, but to add on the EW and subtract on the WE trips. The observed time losses of the flown clocks relative to the reference clocks: \(59 \pm 10\) ns (WE) and \(273 \pm 7\) ns (EW) were found to be in good agreement with the relativistic predictions of \(40 \pm 23\) ns and \(275 \pm 21\) ns respectively.

A combined test of SR and General Relativity (GR) was also made in the experiment of Vessot et al. [50] in which a hydrogen maser was flown in a rocket executing a ballistic trajectory with a maximum altitude of \(10^4\) km. Again the SR and GR corrections affecting the observed rate of the maser were of comparable magnitude and opposite sign. They were seen to cancel exactly at a certain time during both the ascent and descent of the rocket. The quoted accuracy of the combined SR and GR test was quoted as \((2.5 \pm 70) \times 10^{-6}\).

In the Spacelab experiment NAVEX [51], a cesium clock in a Space Shuttle was compared with a similar, ground based, clock. The difference in rate \(R\) of the two clocks is predicted to be:

\[
R = \frac{1}{c^2} (\Delta \phi - \frac{\Delta v^2}{2})
\]  

where \(\Delta \phi\) is the difference in gravitational potential and \(\Delta v^2\) the difference in the squared velocities between the moving and reference clocks. The measured value of \(R\): \(-295.02 \pm 0.29\) ps/s was found to be in good agreement with the relativistic prediction: \(-295.06 \pm 0.10\) ps/s. The GR part of the prediction for \(R\): \(R_{GR} = 35.0 \pm 0.06\) ps/s is thus tested with a relative precision of about 1%, and the SR part (TD) with a relative precision of 0.1%. In this experiment the Shuttle was orbiting at \(\approx 330\) km above the Earth at a velocity of \(7712\) m/s (\(\beta = 2.5 \times 10^{-5}\)). As discussed in the following section, a straightforward extension of this type of experiment could test the relativity of simultaneity of SR. Such a test has never been performed.

**Relativity Tests in Lepton g-2 Experiments**

The rate of precession, \(\omega_a\), of the spin vector of a charged lepton, relative to its momentum vector, in a uniform magnetic field, is independent of the energy of the lepton. Closer examination shows that this fact results from a subtle cancellation of several SR effects in the three angular frequencies that make up \(\omega_a\): the Larmor precession frequency, observed in the laboratory frame, \(\omega_L\), the Thomas precession frequency, \(\omega_T\), and the cyclotron frequency, \(\omega_c\). In fact [52]:

\[
\omega_a = \omega_L + \omega_T - \omega_c = \frac{eB}{mc} \left[ \frac{g \gamma_E}{2 \gamma_t} + \frac{1 - \gamma_T}{\gamma_M} - \frac{1}{\gamma_M} \right]
\]  

where the 3 terms in the large square bracket correspond to \(\omega_L\), \(\omega_T\) and \(\omega_c\) respectively. The four different \(\gamma\)-factors \(\gamma_E\), \(\gamma_t\), \(\gamma_T\) and \(\gamma_M\) correspond to the Lorentz transformations of the electromagnetic field and time, to the kinematical Thomas precession: \((1 - \gamma_T)\omega_c\) and the relativistic mass-energy relation: \((\gamma_M = E/(mc^2))\), respectively. The quantity \(\gamma_t\) referred to above, as determined from the cyclotron frequency of the CERN g-2 experiment [48] is, in fact, \(\gamma_M\), while the ratio of the muon lifetimes at rest and in flight is \(\gamma_t\). Thus, in this experiment, the relation \(\gamma_t = \gamma_M\) is checked. In the pion decay
experiment of Ayres et al. [47] the relation tested is \( \gamma_t = \gamma_v \) where \( \gamma_v \equiv \sqrt{1 - (v/c)^2} \), since the velocity of the deaying pions is directly measured. To make further tests, based directly on the measurement of \( \omega_a \), additional assumptions are necessary. For example, Newman et al. [53] assume that \( \gamma_t = \gamma_E = \gamma_T = \gamma_k \) since they are all ‘kinematical’ quantities, but that \( \gamma_M \) might be different. By comparing measurements of \( \omega_a \) for the electron at velocities \( \beta \simeq 0.5 \) and \( 5 \times 10^{-5} \) it is concluded. [53] that \( \gamma_k \) and \( \gamma_M \) are equal to within a fractional error of \( (5.3 \pm 3.5) \times 10^{-9} \), yielding one of the most precise experimental confirmations of SR. Combley et al. [52] test SR using Eqn.(9.2) but with considerably weaker theoretical assumptions than Newman et al. By considering the transverse acceleration of a charged particle in a magnetic field in both the laboratory and particle rest frames, the consistency condition:

\[ \gamma_M \gamma_E = \gamma_t^2 \]  

may be simply derived. Combining Eqns(9.2) and (9.3) with the direct measurements of \( \gamma_t \) and \( \gamma_M \) from the CERN muon g-2 experiment both \( \gamma_E \) and \( \gamma_T \) were derived. All four \( \gamma \)-factors are found to be consistent, within their experimental errors, the largest of which is \( 2 \times 10^{-3} \) [52].

Test of Length Contraction with Particle Production Models

In the statistical model of particle production in high energy nucleon-nucleon collisions proposed by Fermi [54, 55] and the related hydrodynamical model of Belenkij and Landau [56], the initial state is supposed to consist of a length-contracted ‘pancake’ of high density nuclear matter (see, for example, Fig.1 of Reference [55]) which then develops into the observed multiparticle final state. The multiplicity of produced particles is then expected to be different in such models according to whether the volume, \( V \), of the initial state takes into account, or not, LC. Fermi remarked [55] that the agreement of the model with experimental measurements ‘seems to indicate that the assumption that the volume, \( V \), should be Lorentz contracted is not greatly in error.’ However, the following note of caution was added: ‘One should keep in mind, however, that the number of particles emitted in a collision of this type depends only on the fourth root of the volume \( V \). A change of a factor of 2 or 3 would produce only a relatively minor variation in the expected number of particles’. In a later review article Feinberg [57] re-examined the question as to whether the comparison of such models with experiment could be interpreted as providing positive evidence for LC, and concluded that this was not the case. With our present knowledge of the quark substructure of nucleons and of QCD the physical basis of the models of Fermi and Landau may, of course, be questioned. The present writer concludes, finally, on this question, that statistical and hydrodynamical models of particle production do not provide any convincing experimental evidence for the existence, or not, of LC as a physical effect.

10 Proposals for Two Satellite-borne Experiments to Test the Relativity of Simultaneity

The review of experimental tests of SR presented in the previous Section shows that, at the time of writing, although there is ample experimental confirmation of Einstein’s
Table 5: Coordinates of space time events in $S$ and $S'$. The origin of $S'$ is at clock B, so that the LT is non-local for clock A. The origin of $S$ is at $C$ at $t_C = 0$.

| Event       | $x_C$ | $t_C$ | $x_A^{\text{app}}$ | $t_A^{\text{app}}$ | $x_B^{\text{app}}$ | $t_B^{\text{app}}$ |
|-------------|-------|-------|---------------------|---------------------|---------------------|---------------------|
| B passes C  | 0     | 0     | $-L/\gamma$          | $\beta L/c$       | 0                   | 0                   |
| A passes C  | 0     | 0     | $-L/\gamma\beta c$   | $L/\gamma$         | $L/\gamma\beta c$  |                     |

second postulate, relativistic kinematics and time dilatation, this is not the case for length contraction and the other apparent space-time effects: time contraction and space dilatation [6]. This disparity in the experimental verification of SR is not at all reflected in textbook discussions where no distinction is made between the well tested TD effect, and the experimentally untested LC effect. To the writer’s best knowledge, no experiment has even attempted to observe LC or the relativity of simultaneity, much less the recently noted TC and SD effects that are also direct consequences of the relativity of simultaneity. In this Section tests of the relativity of simultaneity using similar techniques to the previously performed Spacelab experiment NAVEX (SEN) [51], are proposed. The first experiment is essentially a method to observe the previously proposed [6] TC effect. The second is a ‘photon clock’ similar to that constituted by the longitudinal arm of the Michelson interferometer discussed in Section 7 above.

The principle of the first experiment is illustrated by considering observation of the two clocks A and B, introduced in Section 3 above, from the fixed position, in the stationary frame $S$, of the clock C. The clocks A and B are separated by the fixed distance $L$ in their common rest-frame, $S'$, and are synchronised (see Figs.2a, 3a, 9a and 10a). The frame $S'$ moves with uniform velocity $v$, relative to $S$, along their common x-axis. Clock C is synchronised with B at time $t = t' = 0$ when B has the same x coordinate in $S$ as C. The results of the conventional SR calculation\(^7\), choosing the origin in $S'$ at the position of clock B, and so using a non-local LT for the clock A, are shown in Table 5 and Fig.9. The results of using a local LT for both clocks A and B are shown in Table 6 and Fig.10. In the case of the non-local LT for A the apparent x coordinate of this clock coincides with the x coordinate of C at time $t_c = T_{NL} = L/\gamma v$ when the time, $t_A^{\text{app}}$, indicated by A when viewed from $S$, is given by $t_A^{\text{app}} = L/v$. Thus $t_A^{\text{app}} = \gamma T_{NL}$. The moving clock A thus appears to be in advance of the stationary clock C by $\gamma - 1$ times the time interval in $S$, $T_{NC}$, between the passages of the clocks B and A past C. This is just the TC effect ($\Delta x = 0$ projection of the LT) pointed out in Reference [6]. Because clock C is synchronised with B at $t = 0$ it is easy to see that, unlike the apparent positions of the clocks discussed in Section 5, the TC effect is the same for any choice of the origin of the non-local LT in S’. This follows (see Figs.3b,c,d and Table 1) because the relativity of simultaneity gives always gives the same apparent time difference $t_A^{\text{app}} - t_B^{\text{app}} = \beta L/c$ between the times indicated by A and B in $S$. Thus, unlike in the case of the prediction of the apparent positions of the clocks, the TC effect is unambiguously predicted for any choice
Figure 9: Clocks A and B, at rest in S’, move with velocity $v = \frac{\sqrt{5}}{2}c$ along the x-axis in S. When B passes the fixed clock in S, these clocks are synchronised ($t = t' = 0$), as are A and B in S’. The times of A and B as observed in S’ at $t' = 0$ and in S at $t = 0$ are shown in a) and b) respectively. c) shows the times indicated by the clocks when the apparent x-position of A coincides with the x-position of C. The origin of S’ is at B, i.e. the LT is non-local for clock A.
of the origin of the non-local LT, and so is experimentally testable.

Use of local LT for both A and B gives the results (respecting translational invariance): \( t'_A = t'_B \) (for any \( t_C \)) and \( t^{upp}_A = T_L/\gamma \). Thus clock A appears to be delayed relative to the stationary clock, by \((\gamma - 1)/\gamma\) times the time interval in S, \( T_L \), between the passages of the clocks B and A past C. Also, since there is no LC effect in this case, \( T_L = \gamma T_{NL} = L/v \).

In SEN a cesium clock (that may be identified with the clock B above) in a Space Shuttle, executing an almost circular orbit around the Earth, was compared with a similar ground-based clock (corresponding to clock C above) at the culmination times of the orbit, i.e. the times at which the distance between the orbiting and ground based clocks was minimum. In order to realise the experiment described above, it is sufficient to add a third clock (corresponding to A in the above example) following the same orbit as B but separated from it by a distance \( L \). Comparison of A and C at culmination, having previously synchronised B and C at culmination, then essentially realises the ideal experiment discussed above and so enables a test of the relativity of simultaneity of SR.

To estimate the order of magnitude of the expected effects and the corresponding measurement uncertainties, the movement of the ground based clock due to the rotation of the earth is, at first, neglected. It is also assumed that the clocks A and B are in the same inertial frame. For this discussion, the orbit parameters of SEN [51] will be assumed. The Space Shuttle was in circular orbit 328 km above the surface of the Earth, moving with a velocity of 7712 m/s (\( \beta = 2.5 \times 10^{-5} \)). A convenient separation of A and B along their common orbit is then 200km. In this case synchronisation signals can be

---

7 This is similar to Einstein’s calculation of LC in Ref [14]
Table 6: Coordinates of space time events in S and S’. Local LT are used for both clocks A and B. The origin of S is at C at $t_C = 0$.

| Event   | $x_C$ | $t_C$ | $x_A$ | $t_A^{app}$ | $x_B$ | $t_B^{app}$ |
|---------|-------|-------|-------|-------------|-------|-------------|
| B passes C | 0     | 0     | $-L$  | 0           | 0     | 0           |
| A passes C | 0     | $\frac{L}{\beta c}$ | 0     | $\frac{L}{\gamma \beta c}$ | $L$   | $\frac{L}{\gamma \beta c}$ |

The gravitational red shift, due to the Earth’s field, has the effect of increasing the advance, (or decreasing the delay) of A, relative to C, by about 10% for the case of a non-local, (or local) LT applied to A.

The LC effect for the non-local LT of A results in a smaller value of $T$: $T_{NL} = T_L / \gamma = T_L (1 - \beta^2 / 2 + O(\beta^4))$. In principle, this difference is measurable if the absolute distance, $L$, between A and B in S’ is precisely known, as well as the time difference between the culminations of A and B and the velocity of the orbiting clocks. Although the first condition can perhaps be met by using interferometry, (the LC effect corresponds to a difference of length of $\sim 60 \, \mu m$ over 200 km) the rotation of the Earth would seem to preclude any possibility to measure the latter quantities with the required uncertainty in time of about 1 ns in 30 s, and knowledge of the orbit velocity with a similar precision. The reason for this is that the movement of C on the surface of the Earth between the culminations of A and B, is expected to modify $T$ by about 4%; while A is moving the distance of 200 km so as to occupy the position of B at culmination, the clock C moves about 8.5 km due to the Earth’s rotation. This is not serious for the tests of TC and TD since the expected time shifts are, in first approximation, simply scaled according to the actual value of $T$. However, the spatial separation between the culminations of A and B is clearly quite different to the ideal case (i.e., neglecting the rotation of the Earth). The separation must be known with a relative precision of $\sim 3 \times 10^{-10}$ in order to test directly the LC effect. This hardly seems possible.

The second proposed experiment also uses two satellites, A and B, following the same
Figure 11: Scheme of an experimental realisation of Einstein’s clock synchronisation procedure using two satellites in low Earth orbit. ‘Relativity of Simultaneity’ is directly tested in the experiment by observation at the ground station C of the times of arrival of the ‘photon clock’ signals $S^{(1)}_A$ and $S^{(2)}_A$ from the satellite A [a) and c)] and $S_B$ from the satellite B [b)]. C is viewed from the co-moving frame of A and B. Coordinate systems and geometrical and temporal parameters used in the analysis are defined. See the text for more the details of the experiment.
orbit close to the surface of the Earth. The experiment is shown schematically in Fig.11. The satellites are separated by a distance $L$ and follow an orbit that passes at altitude $H$ over a ground station, $C$, equipped for the detection and measurement of the arrival times of microwave signals sent from the satellites. Cartesian coordinate systems $S_1'$ and $S_3'$ with origins $O_1'$ and $O_3'$, $x$-axes parallel to the orbit and $y$-axes perpendicular to the surface of the Earth, are defined. Satellite A is at $O_1'$ and satellite B at $O_3'$. The origin $O_2'$ of a third system $S_2'$, with parallel axes, lies midway between those of $S_1'$ and $S_3'$. It is assumed that the frames $S_1'$, $S_2'$ and $S_3'$ may be considered, to a sufficiently good approximation, to be defined in a common instantaneous co-moving inertial reference system for A and B. Cartesian reference systems with $x$-axes in the plane of the satellite’s orbit, and directions parallel to that of $S_2'$ at the instant of closest approach of $S_2'$ to C, are also defined. The relative velocity, at closest approach, of $S_2'$ and $S_2$ is denoted by $v$. At the instant of closest approach, $O_1$, $O_2$ and $O_3$ are aligned with $O_1'$, $O_2'$ and $O_3'$ respectively.

A microwave signal is sent from B towards A at such a time that the reflected signal from A arrives back at B at the instant of closest approach of $O_2'$ and $O_2$. At this instant the $x$-coordinates of $O_2'$ and $O_2$ coincide in both $S_2'$ and $S_2$ (Fig.11b), and clocks in $S_2$, $S_1'$, $S_2'$ and $S_3'$ are synchronised so that $t_2 = t_2' = t_3' = 0$. This implies that the initial signal from B arrives at A at the time $t_1' = t_2' = t_3' = -L/c$. The reception of this signal by A triggers the emission, after a delay time $t_D(A)$, of the signal $S_A^{(1)}$ that is sent to C (Fig.11a). Reception of the reflected signal back at B triggers the emission, after a delay time $t_D(B)$ of the signal $S_B$ that is also sent to C (Fig.11b). Finally, after reflection at B, the initial signal arrives for a second time at A, and after delay $t_D(A)$ a second signal $S_A^{(2)}$ is sent to C by A (Fig.11c). The proposed experiment is to simply compare the time interval $\delta t_{BA}$ between the arrival times of $S_A^{(1)}$ and $S_B$ at C with $\delta t_{AB}$ which is the difference between the arrival times at C of $S_B$ and $S_A^{(2)}$.

The calculation of these time intervals using the conventional non-local LT of SR is done using the frames $S_2'$ and $S_2$. The space-time coordinates of the emission events of the signals $S_A^{(1)}$, $S_B$ and $S_A^{(2)}$ as calculated using (4.3)-(4.6) are presented in Table 7. Taking into account the propagation times of the signals from the satellites to the ground station, the following values are obtained for the arrival times of the three signals at C:

$$Non - local LT$$

| Event         | $x_2'$ | $t_2'$ | $x_2^{app}$ | $t_2^{app}$ |
|---------------|--------|--------|-------------|-------------|
| $S_A^{(1)}$ emitted | $-\frac{L}{2}$ | $-\frac{L}{c} + t_D(A)$ | $-\gamma L\left(\frac{1}{2} + \beta - \frac{vt_D(A)}{L}\right)$ | $-\frac{2L}{c}(1 + \frac{\beta}{2} - \frac{vt_D(A)}{L})$ |
| $S_B$ emitted     | $\frac{L}{2}$   | $t_D(B)$            | $\gamma L\left(\frac{1}{2} + \frac{vt_D(B)}{L}\right)$ | $\frac{\gamma L}{c}\left(\frac{\beta}{2} + \frac{vt_D(B)}{L}\right)$ |
| $S_A^{(2)}$ emitted | $-\frac{L}{2}$ | $\frac{L}{c} + t_D(A)$ | $-\gamma L\left(\frac{1}{2} - \beta - \frac{vt_D(A)}{L}\right)$ | $\frac{\gamma L}{c}(1 - \frac{\beta}{2} + \frac{vt_D(A)}{L})$ |

Table 7: Coordinates of space time events in $S_2'$ and $S_2$. The origin of $S_2'$ is midway between the satellites A and B giving a non-local LT. The origin of $S_2$ is at C.
\[
\begin{align*}
t(S_A^{(1)}) &= \frac{R - L}{c} + \frac{Ld_A}{c} \left( \frac{1}{\beta} - \frac{L}{2R} \right) + \frac{\beta L}{2c} \left( \frac{L}{R} - 1 \right) \\
t(S_B) &= \frac{R}{c} + \frac{Ld_B}{c} \left( \frac{1}{\beta} + \frac{L}{2R} \right) + \frac{\beta L}{2c} \\
t(S_A^{(1)}) &= \frac{R + L}{c} + \frac{Ld_A}{c} \left( \frac{1}{\beta} - \frac{L}{2R} \right) - \frac{\beta L}{2c} \left( \frac{L}{R} + 1 \right)
\end{align*}
\]  

(10.1) 

(10.2) 

(10.3) 

where \( \beta \equiv v/c \), \( R \equiv \sqrt{H^2 + L^2/4} \), \( d_{A,B} \equiv vt_{D}(A, B)/L \) and only terms of \( O(\beta) \) have been retained. (10.1)-(10.3) give the following time intervals:

\[
\delta t_{BA} \equiv t(S_B) - t(S_A^{(1)}) = \frac{L}{c} + \frac{L^2}{2c\beta} (d_B - d_A) + \frac{L^2}{2Rc} (d_B + d_A) + \frac{\beta L}{c} - \frac{\beta L^2}{2cR} 
\]

(10.4) 

\[
\delta t_{AB} \equiv t(S_A^{(2)}) - t(S_B) = \frac{L}{c} - \frac{L^2}{c\beta} (d_B - d_A) - \frac{L^2}{2Rc} (d_B + d_A) - \frac{\beta L}{c} - \frac{\beta L^2}{2cR} 
\]

(10.5) 

So that

\[
\Delta t_{NL} \equiv \delta t_{BA} - \delta t_{AB} = \frac{2\beta L}{c} + \frac{2L^2}{c\beta} (d_B - d_A) + \frac{L^2}{cR} (d_B + d_A) 
\]

(10.6) 

The time interval \( \Delta t_L = \delta t_{BA} - \delta t_{AB} \) is now calculated using local LT at the satellites A and B, i.e. using the frame \( S_1' \) for A and \( S_3' \) for B. The x-coordinates in \( S_1', S_2' \) and \( S_3' \) are connected by the relations (see Fig.11a):

\[
x_1' = x_2' + \frac{L}{2}, \quad x_3' = x_2' - \frac{L}{2}
\]

(10.7) 

At \( t_1' = t_2' = t_3' = t_2 = 0 \) when \( O_2' \) and \( O_2 \) coincide in x, local LT at A and B relate \( S_1' \) and \( S_3' \) to the frames \( S_1 \) and \( S_3 \) co-moving with \( S_2 \). At this instant The origins \( O_1' \) and \( O_1 \) coincide in x as viewed from both \( S_1' \) and \( S_1 \), and similarly the origins \( O_3' \) and \( O_3 \) coincide in x as viewed from both \( S_3' \) and \( S_3 \). The symmetry of these relations and the special relativity principle\(^8\) then require, in view of (10.7) that (see Fig.11b and compare with Fig.4):

\[
x_1 = x_2 + \frac{L}{2}, \quad x_3 = x_2 - \frac{L}{2}
\]

(10.8) 

Using the local LT (5.3),(5.4) to transform the signal emission events from the co-moving inertial frame of the satellites A and B to that of the ground station C, as well as (10.8) to express all coordinates in the frame \( S_2 \), gives the signal emission space-time events presented in Table 8. Taking into account signal emission delays and the propagation times to the ground station and neglecting terms of \( O(\beta^2) \) and higher, the following arrival times at C are found for the signals:

Local LT

\(^8\)What is invoked here is the restricted kinematical form of the SR Principle that states the reciprocal nature of space-time measurements by any two inertial observers. See [10].
Table 8: Coordinates of space time events in $S_2^\prime$ and $S_2$. Local LT are used for the satellites A and B. The origin of $S_2$ is at C.

| Event      | $x_2'$ | $t_2'$ | $x_2$  | $t_2^{app}$ |
|------------|--------|--------|--------|-------------|
| $S_A^{(1)}$ emitted | $-\frac{L}{2}$ | $-\frac{L}{c} + t_D(A)$ | $-\frac{L}{2} - \gamma \beta (L - ct_D(A))$ | $-\frac{2}{c}(L - ct_D(A))$ |
| $S_B$ emitted       | $\frac{L}{2}$  | $t_D(B)$     | $\frac{L}{2} + \gamma \beta ct_D(B)$ | $\gamma t_D(B)$  |
| $S_A^{(2)}$ emitted | $-\frac{L}{2}$ | $\frac{L}{c} + t_D(A)$ | $-\frac{L}{2} + \gamma \beta (L + ct_D(A))$ | $\frac{2}{c}(L + ct_D(A))$ |

yielding the time intervals:

\[
\delta t_{BA} = \frac{L}{c} \left\{ \left( 1 + \frac{\beta L}{2R} \right) + \frac{d_B}{\beta} \left( 1 + \frac{\beta L}{2R} \right) \left( 1 - \frac{d_A}{\beta} \right) \right\} 
\] (10.12)

\[
\delta t_{AB} = \frac{L}{c} \left\{ \left( 1 - \frac{\beta L}{2R} \right) \left( 1 + \frac{d_A}{\beta} \right) \right\} \left( 1 + \frac{\beta L}{2R} \right) \left( 1 + \frac{d_B}{\beta} \right) 
\] (10.13)

which give:

Local LT

\[
\Delta t_L = \delta t_{BA} - \delta t_{AB} = \frac{2L}{c^2\beta} (d_B - d_A) + \frac{L^2}{cR} (d_B + d_A) 
\] (10.14)

The delay-dependent terms are identical, at $O(\beta)$, to those of the calculation using non-local LT in (10.6). In the absence of these terms it is found that $\Delta t_L = 0$ whereas from (10.6) $\Delta t_{NL} = 2\beta L/c$. Substituting $L = 200\text{km}$ and $\beta = 2.5 \times 10^{-5}$ as for the NAVEX spacialab experiment gives $\Delta t_{NL} = 33\text{ns}$, a time difference easily measurable in a single experiment. Thus a clear and unambiguous discrimination between the predictions of local and non-local LT can be made in this way.

The delay-time dependent terms in (10.6) and (10.14) are:

\[
\Delta t_D = 2(t_D(B) - t_D(A)) + \frac{\beta L}{R}(t_D(B) + t_D(A)) 
\] (10.15)

The contribution of the second term is quite negligible. For $L/R = 1$ and $\beta = 2.5 \times 10^{-5}$, even for delays as long as 1µs (300 light-metres), it amounts only to 0.05ns. Thus knowledge of $t_D(B) - t_D(A)$ to within a few ns is adequate to establish the expected difference in the predictions of local and non-local LT, which may be compared with the time resolution for caesium clock signals of a few ps obtained in SEN.
The second term in (10.15) also enables the systematic uncertainty due to imprecise knowledge of the time of culmination to be estimated. For SEN the standard deviation of twelve culmination times \[51\] is 26ns. The corresponding uncertainty in \(\Delta t\) given by (10.15) is then \(1.3 \times 10^{-3}\)ns for \(L/R = 1\) and \(\beta = 2.56 \times 10^{-5}\), as compared to a predicted value of \(\Delta t_{NL} = 33\)ns.

The parameters of the experiment have been chosen to simplify the analysis as much as possible. However only small modifications are needed to describe more general experimental configurations. For example, the ground station C was located at: \((x_2, y_2, z_3) = (0, 0, 0)\). If it is located instead at \((0, 0, D)\) all the above equations are valid with the replacement \(R \rightarrow \sqrt{H^2 + D^2 + L^2}/4\). Earlier or later times for the signal sequence \(S_{A}^{(1)}, S_B\) and \(S_{A}^{(2)}\) are taken into account by a suitable choice of the parameters \(t_D(A)\) and \(t_D(B)\). In fact during the time \(L/c\) of passage of the light signals between the satellites, the latter move, in the frame \(S_2\), by only a distance \(\beta L \simeq 5\)m with the above parameter choice. The propagation distance of the three signals from the satellites to C is thus essentially constant for positions near to culmination of \(O_2\).

The experiment just proposed is an actual physical realisation of the clock synchronisation procedure proposed in Einstein’s first paper on special relativity \[1\]. However, unlike in the case of the test of TC in the first experiment discussed above, no actual clock synchronisation is necessary. The observer at the ground station simply measures the arrival times of \(S_{A}^{(1)}, S_B\) and \(S_{A}^{(2)}\), constructs \(\delta t_{BA}\) and \(\delta t_{AB}\) and compares them. However, if synchronised clocks are available at A and B the ‘photon clock’ part of the experiment is unnecessary. A sends signals to C at \(t' = -L/c\) and \(L/c\), B a signal at \(t' = 0\). The predictions for the arrival times of these signals at C are the same as those for the photon clock experiment just described, with all time delays set to zero. Indeed if the clocks have been previously synchronised using Einstein’s light signal procedure the relativistic interpretation of the two experiments is identical. In one the synchronisation is performed in ‘real time’, in the other not.

The availability of synchronised clocks at A and B enables an even simpler test of the relativity of simultaneity to be made. If signals \(S_A\) and \(S_B\) are sent at the same time \(t' = 0\) by the two satellites, the non-local LT (4.5) and (4.6) predict space-time coordinates in \(S_2\) of: \(x_2 = -\gamma L/2, t_2 = -\gamma \beta L/2c\) for the emission of \(S_A\) and \(x_2 = \gamma L/2, t_2 = \gamma \beta L/2c\) for the emission of \(S_B\). Hence it is predicted that \(S_B\) will arrive at C with time delay of \(\gamma \beta L/c\) relative \(S_A\). Use of local LT at A and B give instead the predictions \(x_2 = -L/2, \ t_2 = 0\) and \(x_2 = L/2, t_2 = 0\) for the emission of \(S_A\) and \(S_B\) respectively, so that the two signals arrive simultaneously at C. With the same values of \(L\) and \(\beta\) as assumed above, a difference of 16.5ns in the arrival times of the signals is predicted by the non-local LT. In fact this last experiment is an actual physical realisation, for a non-local LT, of the ‘Space Dilatation’ (SD) effect proposed in \[6\]. The distance in \(S_2\) between the signal emission events predicted by a non-local LT is \(\gamma L\), as compared to \(L\) for local LTs. The difference is only an \(O(\beta^2)\) effect that, as discussed above for the LC effect of similar relative magnitude, is very difficult to measure. However, the associated time difference is an \(O(\beta)\) effect that is large enough to to be easily measurable with modern techniques.

It may be at first sight surprising that special relativity can be tested by the observation of \(O(\beta)\) effects. Both the experimentally confirmed TD and the well-known but untested
LC effects are of $O(\beta^2)$. It is important to notice, in this connection, that, as already mentioned in Section 5 above, the non-local LT of time (4.4) and (4.6) do not differ from the corresponding Galilean transformations only by terms of $O(\beta^2)$. It is the second, space-dependent, $O(\beta)$ terms in these equations that are responsible for the different predicted values of $\Delta t_{NL}$ and $\Delta t_L$ in the experiment described above. It is just these terms, which are non-vanishing only for a non-local LT, that are responsible for ‘relativity of simultaneity’ and the associated $O(\beta^2)$ LC, SD and TC effects. Conversely, the TD effect is, by definition, given by a local LT. Although, as discussed above, direct experimental confirmation, or the contrary, of the $O(\beta^2)$ LC and SD effects, seems difficult, both the TC effect, of similar magnitude, but opposite sign to the experimentally confirmed $O(\beta^2)$ TD effect, and direct $O(\beta)$ ‘relativity of simultaneity’ can indeed both be easily tested by the experiments proposed above.

A variation of the second experiment in which the satellites A and B are replaced by GPS satellites, and the ground station C by a GPS signal detector on a satellite in low Earth orbit, is described in Reference [58]. In this case, a very large value of $\Delta t_{NL}$ of 3.2$\mu$sec is predicted.

Acknowledgements

I thank Y.Bernard for valuable help in the preparation of the figures. I would also like to especially thank Y.Keilman for pointing out a calculational error (now corrected) in an earlier version of this paper and D.Utterback for interesting related questions and encouragement, as well for suggestions leading to an improvement of the presentation in several places.
References

[1] A.Einstein, Annalen der Physik 17, 891 (1905). English translation by W.Perrett and G.B.Jeffery in ‘The Principle of Relativity’ (Dover, New York, 1952) P37, or in ‘Einstein's Miraculous Year’ (Princeton University Press, Princeton, New Jersey, 1998) P123.

[2] R.A.Sorensen, Am. J. Phys. 63 413 (1995).

[3] J.S.Bell, ‘How to teach special relativity’, in ‘Speakable and Unspeakable in Quantum Mechanics’, Cambridge University Press, (1987), pp67-80.

[4] J.Terrell, Phys. Rev. 116 1041 (1959).

[5] R.Penrose, Proc. Cambridge Phil. Soc. 55 137 (1959).

[6] J.H.Field, Am. J. Phys. 68, 367 (2000).

[7] J.H.Field, ‘On the Real and Apparent Positions of Moving Objects in Special Relativity: The Rockets-and-String and Pole-and-Barn Paradoxes Revisited and a New Paradox’, arXiv pre-print: physics/0403094.

[8] E.Dewan and M.Beran, Am. J. Phys. 27 517 (1959).

[9] E.Dewan, Am. J. Phys. 31 383 (1963).

[10] J.H.Field, Am. J. Phys. 69, 569 (2001).

[11] L.Marder, ‘Time and the Space Traveller’, Allen and Unwin, London, 1971, Ch 3.

[12] H.Nikolic, Am. J. Phys. 67, 1007 (1999).

[13] W.Rindler, ‘Introduction to Special Relativity’, 2nd Edition (O.U.P. Oxford, 1991) Section 14 P33.

[14] A.Einstein, ‘Relativity the Special and General Theory’, Routledge, London, 1964, Ch 12.

[15] V.S.Soni, Eur. J. Phys. 23, 225 (2002).

[16] H.Minkowski, ‘Space and Time’, in ‘The Principle of Relativity’, Dover, New York, 1952, p75

[17] A.Einstein, ‘The Meaning of Relativity’, Princeton University Press, Princeton, New Jersey, 1955, p46.

[18] O.Lodge, Nature 46, 165 (1892).

[19] H.A.Lorentz, Verst. Kon. Akad v Wet Amsterdam 1, 74 (1892).

[20] A.A.Michelson and E.W. Morley, Amer. Jour. of Sci. 34, 1333 (1887).

[21] G.Stephenson and C.W.Kilmister, ‘Special Relativity for Physicists’, Longmans Green and Co, London, 1958, Ch 2.
[22] M.Pospelov and M.Romalis, ‘Lorentz Invariance on Trial’, Physics Today, July 2004 and References therein.

[23] R.J.Kennedy and E.M.Thorndike, Phys. Rev. 42, 400 (1932).

[24] A.Brillet and J.L.Hall, Phys. Rev. Lett. 42, 549 (1979).

[25] D.Hils and J.L.Hall, Phys. Rev. Lett. 64, 1697 (1990).

[26] H.Müller et al., Phys. Rev. Lett. 91, 020401 (2003).

[27] W. de Sitter, Phys. Z. 14, 429,1267 (1913).

[28] J.Aharoni, ‘The Special Theory of Relativity’, O.U.P, Oxford, 1965, p313.

[29] J.G.Fox, Am. J. Phys. 30, 297 (1962).

[30] K.Brecher, Phys. Rev. Lett. 39, 1051 (1977).

[31] T.Alväger, et al., Phys. Lett. 12, 260 (1964).

[32] R.Mansouri and R.U.Sexl, Gen. Rel. Grav. 8, 497, 515, 809 (1977).

[33] C.M.Will, Phys. Rev. D45, 403 (1992).

[34] H.E.Ives and G.R.Stilwell, J. Opt. Soc. Am. 28, 215 (1938).

[35] R.V.Pound and G.A.Rebka Jr, Phys. Rev. Lett. 4, 274 (1960).

[36] J.J.Hay et al., Phys. Rev. Lett. 4, 165 (1960).

[37] W.Kündig, Phys. Rev. 129, 2371 (1963).

[38] M.Kaivola et al., Phys. Rev. Lett. 54, 255 (1985).

[39] R.W.McGowan et al., Phys. Rev. Lett. 70, 251 (1993).

[40] R.Grieser et al., Appl. Phys. B59, 127 (1994).

[41] D.W.MacArthur et al., Phys. Rev. Lett. 56, 282 (1986).

[42] A.H.Bucherer, Phys. Z. 9, 755 (1908).

[43] W.Bertozzi, Am. J. Phys. 32, 551 (1964).

[44] S.Parker, Am. J. Phys. 40, 241 (1972).

[45] K.N.Geller and R.Kollantis, Am. J. Phys. 40, 1125 (1972).

[46] Z.G.T.Guiragossián et al., Phys. Rev. Lett. 34, 335 (1975).

[47] D.S.Ayres et al., Phys. Rev. D3, 1051 (1971).

[48] J.Bailey et al., Nature 268, 301 (1979).

[49] J.C.Hafele and R.E.Keating, Science 177, 166,177 (1972).
[50] R.F.C.Vessot et al., Phys. Rev. Lett. 45, 2081 (1980).
[51] E.Sappl, Naturwissenschaften 77, 325 (1990).
[52] F.Combley et al., Phys. Rev. Lett. 42, 1383 (1979).
[53] D.Newman et al., Phys. Rev. Lett. 40, 1355 (1978).
[54] E.Fermi, Prog. Theor. Phys. 5, 570 (1950).
[55] E.Fermi, Phys. Rev. 81, 683 (1951).
[56] S.Z.Belenkij and L.D.Landau, Nuovo Cimento Supp. 3, 15 (1956).
[57] E.L.Feinberg, Sov. Phys. Usp. 18, 624 (1976).
[58] J.H.Field, ‘Proposals for Two Satellite-Borne Experiments to Test Relativity of Simultaneity in Special Relativity’, physics/0509213.