Matrix Model for Dirichlet Open String

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Abstract

We discuss the open string ending on D $p$-branes in IKKT framework. First we determine the boundary conditions of Green-Schwarz superstring which are consistent with supersymmetry and $\kappa$-symmetry. We point out some subtleties arising from taking the Schild gauge and show that in this gauge the system incorporates the limited dimensional D $p$-branes ($p = 3, 7$). The matrix regularization for the Dirichlet open string is given by gauge group $SO(N)$. When $p = 3$, the matrix model becomes the dimensional reduction of a 6 dimensional $\mathcal{N} = 1$ super Yang-Mills theory.
1 Introduction

During the recent developments in the superstring theory, the matrix theories give a new framework to understand the non-perturbative aspects of the string theory[1].

Among some variants of such theories, we focus on IKKT matrix model [2] which is the large-$N$ reduced model of ten dimensional $U(N)$ super Yang-Mills theory. It is supposed to give a constructive definition of the type IIB superstring theory in the Schild gauge. To understand the non-perturbative properties, it is indispensable to have an appropriate description of $p$-branes. Originally, such objects were studied from classical configuration with $[X^i, X^j] \propto 1$ [1] [2] [3].

In this paper, we propose another picture of $p$-branes by defining the open string ending on D $p$-branes, so-called Dirichlet open string, in IKKT framework. It is an extension of our previous work [4] where open supermembrane ending on $M$ five-brane was investigated by using the regularization technique similar to that in [5].

In sec.2 we determine the boundary conditions for the type IIB Green-Schwarz open superstring which are consistent with space-time supersymmetry and $\kappa$-symmetry. We describe the boundary conditions for fermions in a linear form and discuss subtleties in the analytic continuation of fermionic variables which is mandatory for IKKT model.

In sec.3 we account for implications coming from the Schild gauge. One of the main consequences is that the system is consistently accompanied only by the limited dimensional D $p$-branes ($p = 3, 7$). We then identify an unbroken part of supersymmetry. Finally we show that such an open string is regularized by $SO(N)$ matrix model. When $p = 3$, in particular, it becomes the reduced model of a 6 dimensional $\mathcal{N} = 1$ super Yang-Mills theory.

In appendix we summarize the convention of $SO(1, 9)$ gamma matrices that we use in this paper.

2 Boundary Conditions for the Type IIB Superstring

In this section, we fix the boundary conditions of the type IIB Green-Schwarz (GS) superstring ending on D $p$-branes. We consider the case in which the purely bosonic gauge field $A_{\mu}(X)$ on the D $p$-brane world volume couples to the boundary $\partial\Sigma$ of the string world sheet $\Sigma$. Our investigation is made in the rigid superspace for simplicity.

2.1 Variations of GS superstring action with boundary

The action of the type IIB GS superstring [4] is

$$S_{GS} = -\int_{\Sigma} d^2\sigma \left[ \sqrt{-g} + \mathcal{L}_{WZ} \right] - \int_{\partial \Sigma} dX^\mu A_\mu(X) ,$$

$$\mathcal{L}_{WZ} = i\epsilon^{ab} \partial_a X^\mu (\bar{\theta}^1 \Gamma_{\mu} \partial_b \theta^1 - \bar{\theta}^2 \Gamma_{\mu} \partial_b \theta^2) - \epsilon^{ab} (\bar{\theta}^1 \Gamma^\mu \partial_a \theta^1)(\bar{\theta}^2 \Gamma_{\mu} \partial_b \theta^2) .$$ (1)

where $\sigma^a$ ($a = 0, 1$) are world sheet coordinates, and $(X^\mu(\sigma), \theta^a(\sigma))$ ($\mu = 0, 1, \ldots 9; \alpha = 1, \ldots, 32; A = 1, 2$) is the embedding map from $\Sigma$ into the $1+9$ dimensional superspace.
Here $\theta^A (A = 1, 2)$ are Majorana-Weyl spinors in 1+9 dimensional space-time with the same chiralities. $g$ is the determinant of the induced metric $g_{ab}$ on $\Sigma$:

$$g_{ab} = \Pi_a^\mu \Pi_b^\nu \eta_{\mu\nu},$$
$$\Pi_a^\mu = \partial_a X^\mu - i\bar{\theta}^1 \Gamma^\mu \partial_a \theta^1 - i\bar{\theta}^2 \Gamma^\mu \partial_a \theta^2.$$

(2)

When the world sheet $\Sigma$ has no boundary, the action (1) is invariant under $\mathcal{N} = 2$ space-time supersymmetry transformations,

$$\delta_{\text{SUSY}} \theta^A = \epsilon^A,$$
$$\delta_{\text{SUSY}} X^\mu = -i\bar{\theta}^1 \Gamma^\mu \delta_{\text{SUSY}} \theta^1 - i\bar{\theta}^2 \Gamma^\mu \delta_{\text{SUSY}} \theta^2,$$

(3) where $\epsilon^A$ are constant Majorana-Weyl spinors. It is also invariant under local fermionic transformations (so-called $\kappa$-symmetry transformations),

$$\delta_{\kappa} \theta^A = \alpha^A,$$
$$\delta_{\kappa} X^\mu = i\bar{\theta}^1 \Gamma^\mu \delta_{\kappa} \theta^1 + i\bar{\theta}^2 \Gamma^\mu \delta_{\kappa} \theta^2,$$

(4) where

$$\alpha^1 = (1 + \bar{\Gamma}) \kappa^1,$$
$$\alpha^2 = (1 - \bar{\Gamma}) \kappa^2,$$
$$\bar{\Gamma} = \frac{1}{2\sqrt{-g}} \Sigma_{\mu\nu} \Gamma^\mu^\nu,$$
$$\Sigma^\mu^\nu = \epsilon^{ab} \Pi^\mu_a \Pi^\nu_b.$$

(5)

In the above, $\kappa^A$ are Majorana-Weyl spinors depending on the world sheet coordinates $\sigma^a$ and $\bar{\Gamma}$ is subject to $\bar{\Gamma}^2 = 1$ and $\text{Tr}(\bar{\Gamma}) = 0$.

The lagrangian of the action (1) is invariant under these fermionic transformations modulo total derivative terms. In the presence of the world sheet boundary, the variations of the action (1) under these fermionic transformations leave the boundary terms. The situation is parallel to that in the type IIA theory [4]. The explicit form of $\delta_{\text{SUSY}} S_{\text{GS}}$ and $\delta_{\kappa} S_{\text{GS}}$ is obtained from eqs.(6) and (8) in ref. [4] by the replacement,

$$\frac{1 + \Gamma_{11}}{2} \theta^1 \mapsto \bar{\theta}^1,$$
$$\frac{1 - \Gamma_{11}}{2} \theta^2 \mapsto \bar{\theta}^2.$$

(6)

In what follows we will determine the boundary conditions along the line of the analysis in the type IIA theory [4]. Because we consider the situation in which the open superstring ends on the D $p$-branes, we have

$$\delta X^\mu = 0,$$
$$\bar{\theta}^1 \Gamma^\mu \delta \theta^1 + \bar{\theta}^2 \Gamma^\mu \delta \theta^2 = 0,$$

on $\partial \Sigma$.

(7)

Here $\mu(= 0, 1, \ldots, p)$ and $\nu(= p + 1, \ldots, 9)$ denote, respectively, the directions which are parallel and perpendicular to the D $p$-brane. We find that we are able to respect supersymmetry, $\kappa$-symmetry and variational principle by imposing, besides eq.(7), the boundary conditions,

$$F_{\mu\nu}(\bar{\theta}^1 \Gamma^\mu \delta \theta^1 + \bar{\theta}^2 \Gamma^\mu \delta \theta^2) = 0,$$
$$\sqrt{-g} n^a \Pi^a_{\mu} - F_{\mu\nu} n_a \epsilon^{ab} \Pi^b = 0,$$

on $\partial \Sigma$.

(8)

(9)

where $n^a$ denotes a unit vector normal to $\partial \Sigma$, and $F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$.
2.2 Linear boundary conditions for fermions

We rewrite the boundary conditions in a linear form w.r.t. fermions $\theta^A$ in order to preserve a fraction of supersymmetry. We look for such boundary conditions first in the case that $F_{\mu\nu} = 0$. We begin by focusing on eqs.(7) and (8). We set the following ansatz:

$$\theta^A = (\Gamma_{(p)}^\alpha \otimes M^{AB}) \theta^B , \quad (\alpha, \beta = 1, \ldots, 32; A, B = 1, 2) , \quad \text{on } \partial \Sigma , \quad (10)$$

where $\Gamma_{(p)} = \Gamma_0 \Gamma_1 \cdots \Gamma_p$, and $M$ is a $2 \times 2$ matrix. In order that this ansatz is self-consistent, $(\Gamma_{(p)} \otimes M) \theta$ must be a Majorana-Weyl spinor with the same chiralities as $\theta$ and the relation $(\Gamma_{(p)} \otimes M)^2 = 1$ should hold. These consistency conditions require that $M^* = M$ and that $p$ should be odd. Substituting eq.(10) into eqs.(7) and (8) with $F_{\mu\nu} = 0$, we find that

$$M = \begin{cases} \pm \sigma^1 & p = 4m + 1 \\ \pm i \sigma^2 & p = 4m + 3 \end{cases} . \quad (11)$$

It yields that, for any odd integer $p$,

$$\theta^1 = \pm \Gamma_{(p)}^2 \theta^2 , \quad \text{on } \partial \Sigma . \quad (12)$$

By substituting eqs.(10) and (11) into eq.(9), we eventually find the desired boundary conditions:

$$\delta X^\mu = 0 , \quad \frac{1}{2} \left( 1 - \Gamma_{(p)} \otimes M \right) \theta = 0 , \quad (13)$$

Let us now turn on the field strength $F_{\mu\nu}$ on the $D$-brane world volume. We restrict ourselves to the constant $F_{\mu\nu}$. By an analysis similar to that in the type IIA theory, we obtain the linear boundary conditions that reproduce the conditions (7) and (8),

$$\theta = e^{-\frac{1}{2} Y_{\mu\nu} \Gamma_{(p)} \otimes \sigma^3} (\Gamma_{(p)} \otimes M) \theta , \quad \text{on } \partial \Sigma , \quad (14)$$

where $Y_{\mu\nu}$ is defined such that $F_{\mu\nu} = (\tanh Y)_{\mu\nu}$. In general, however, it is difficult to rewrite the condition (14) in a simple form, since it is fairly non-linear.

We point out that the result (14) is consistent with that obtained in the light-cone gauge and yields the same supersymmetry breaking pattern as is given in ref.[8].

2.3 Analytic continuation of $\theta^2$

When IKKT showed the correspondence between their matrix model and the type IIB theory in the Schildd gauge, they started from the action in which the signs of the $\theta^2$-bilinear terms are reversed:

$$S_{GS}^{(IKKT)} = - \int d^2 \sigma \left[ \sqrt{-\frac{1}{2} \Sigma^2} + ie^{ab} \partial_a X^\mu (\bar{\theta}^1 \Gamma_{(p)} \partial_b \theta^1 + \bar{\theta}^2 \Gamma_{(p)} \partial_b \theta^2) + e^{ab} (\bar{\theta}^1 \Gamma_{(p)} \partial_a \theta^1) (\bar{\theta}^2 \Gamma_{(p)} \partial_b \theta^2) \right] , \quad (15)$$
where $\Sigma^{\mu\nu}$ is defined in eq. (15) and we note that $\frac{1}{2} \Sigma^2 = g$. In the present case, however, $\Pi^\mu_a$ is defined as

$$\Pi^\mu_a = \partial_a X^\mu - i \bar{\theta}^1 \Gamma^\mu \partial_a \theta^1 + i \bar{\theta}^2 \Gamma^\mu \partial_a \theta^2.$$  \hspace{1cm} (16)

In order to obtain the action (15) from the conventional one, we need to perform an analytic continuation, $\theta^2 \rightarrow i \theta^2$ [2]. Here we should note that the Dirac conjugation $\bar{\psi}$ of a spinor $\psi$ is now supposed to be defined by eq. (33): $\bar{\psi} = -\psi^T C^{-1}$. Hereafter we use this redefinition for all the spinors regardless of whether they are Majorana or not.

The $\mathcal{N} = 2$ space-time supersymmetry and the $\kappa$-symmetry transformations are given by reversing the signs of $\theta^2$-bilinear terms in eqs. (3) and (4).

In what follows, we consider the boundary conditions for a type IIB Dirichlet open string defined by the action (13). We now concentrate on the case in which the field strength $F_{\mu\nu}$ on the D $p$-branes is switched off. The boundary conditions are obtained by reversing the sign of the $\theta^2$-bilinear terms in eqs. (7), (8) and (9) with $F_{\mu\nu} = 0$. We should rewrite them into a linear form w.r.t. fermionic coordinates $\theta^A$. Applying to eq. (12) the analytic continuation, $\theta^2 \rightarrow i \theta^2$, we obtain the linear boundary conditions,

$$\theta^1 = \pm i \Gamma(p) \theta^2 \text{ , on } \partial \Sigma .$$  \hspace{1cm} (17)

It implies that $M$ in eq. (13) is replaced by

$$M = \begin{cases} \pm \sigma^2 & p = 4m + 1 \\ \pm i \sigma^1 & p = 4m + 3 \end{cases} .$$  \hspace{1cm} (18)

We remark on the Majorana condition for $\theta$. From the above we find that $M^* \neq M$. It means that these linear boundary conditions do not satisfy the Majorana condition. In fact there is not such a matrix $M$ as fulfill the boundary conditions and the Majorana condition simultaneously. In the present case, however, there are no strong reasons to impose the Majorana condition on $\theta$. This is because we have performed the analytic continuation and the Majorana condition become subtle. We will henceforth ignore the Majorana condition.

### 3 Matrix Regularization of an Open Superstring

In this section, we regularize a type IIB Dirichlet open superstring by a matrix model, by using prescriptions similar to those in ref. [2]. We restrict ourselves to the case that the open string ends on two parallel D $p$-branes or on a single D $p$-brane. In this situation, only the DD and NN sectors emerge and we do not have to consider DN or ND sector.

#### 3.1 Schild gauge formulation

In this section we consider an open superstring in the Schild gauge,

$$\psi \equiv \theta^1 = \theta^2 .$$  \hspace{1cm} (19)
As is done by IKKT, we introduce an auxiliary field $\sqrt{h}$, which is a positive definite scalar density on the world sheet. The action is rewritten \([2]\) into

$$S_{\text{Schild}} = \int_\Sigma d^2 \sigma \left[ \sqrt{h} \left( \frac{1}{4} \{X^\mu, X^\nu\}^2 - \frac{i}{2} \bar{\psi} \Gamma^\mu \{X_\mu, \psi\} \right) + \beta \sqrt{h} \right], \quad (20)$$

where $\{\ast, \ast\}$ is the Lie bracket defined as $\{X, Y\} = \frac{\epsilon^{ab}}{\sqrt{h}} \partial_a X \partial_b Y$, for arbitrary functions $X(\sigma)$ and $Y(\sigma)$ on the world sheet. Combining with the $\kappa$-symmetry transformations, we can define the Schild gauge-preserving supersymmetry transformations \([2]\) as

$$\begin{align*}
\delta^{(1)} \psi &= -\frac{1}{2\sqrt{h}} \sigma_{\mu \nu} \Gamma^{\mu \nu} \eta, \\
\delta^{(1)} X^\mu &= i \eta \Gamma^\mu \psi, \\
\delta^{(1)} \sqrt{h} &= 0, \\
\delta^{(2)} \psi &= \xi, \\
\delta^{(2)} X^\mu &= 0, \\
\delta^{(2)} \sqrt{h} &= 0.
\end{align*} \quad (21)$$

The boundary conditions are modified to

$$\begin{align*}
\delta X = 0, \\
\psi &= \pm i \Gamma(p) \psi, \\
n^a \partial_a X = 0, \\
n^a \partial_a \psi &= \mp i \Gamma(p) n^a \psi, \quad \text{on } \partial \Sigma.
\end{align*} \quad (22)$$

In order that the above conditions for the fermionic sector should be self-consistent, $\Gamma(p)$ have to satisfy $(i \Gamma(p))^2 = 1$, i.e. $\Gamma(p)^2 = -1$. Combined with eq.$(32)$, this implies that

$$p = 4m + 3 = 3, 7. \quad (23)$$

We find that the Schild-type gauge choice restricts the possible dimensions of D branes. Such a restriction must be a gauge artifact. The situation seems to be similar to the difficulties in describing transverse five-branes in the BFSS matrix theory. In the following we restrict ourselves to the case that $p = 4m + 3$.

We make some comments on the gauge symmetry on the open world sheet. When the world sheet $\Sigma$ is a closed surface, the Schild-type action \((20)\) has gauge symmetry whose gauge group is area-preserving diffeomorphisms (APD) on $\Sigma$:

$$\delta_{\text{gauge}} X^\mu = -\{\zeta(\sigma), X^\mu(\sigma)\}, \quad \delta_{\text{gauge}} \psi(\sigma) = -\{\zeta(\sigma), \psi(\sigma)\}, \quad (24)$$

where $\zeta(\sigma)$ is an infinitesimal arbitrary function globally well-defined on $\Sigma$. In order that the boundary contributions of $\delta_{\text{gauge}} S_{\text{Schild}}$ should vanish, the transformation parameter $\zeta(\sigma)$ has to obey the Dirichlet boundary condition. The situation is the same as the open supermembrane in the light-cone gauge \([4]\).

### 3.2 Unbroken supersymmetry algebra

Let us now consider the commutator algebra of the unbroken supersymmetry transformations. Because they have to preserve the boundary conditions \((22)\), the parameters in eq.$(21)$ are subject to the chirality projection on the D $p$-brane world volume: $\eta = \mp i \Gamma(p) \eta$ and
\[ \xi = \pm i \Gamma_{(p)} \xi. \] The resulting commutator algebra turns out to be almost the same as that in ref.\[3\]. The only difference resides in the variation of the bosonic coordinates,

\[ [\delta_{\eta}^{(1)}, \delta_{\xi}^{(2)}] X_{\underline{\mu}} = -i \eta \Gamma_{\underline{\mu}} \xi, \quad [\delta_{\eta}^{(1)}, \delta_{\xi}^{(2)}] X_{\bar{\mu}} = 0. \] (25)

This implies that the super-translation algebra reduces to that on the D \( p \)-brane world volume.

### 3.3 Matrix regularization of open superstring

In this section we investigate the matrix model for a Dirichlet open superstring in the Schild gauge. We consider only the case in which the topology of the world sheet \( \Sigma \) is a cylinder. We identify the world sheet coordinate such that \( \tau (= \sigma_{0}) \) parametrizes the \( S^1 \)-direction (i.e. \( \tau \sim \tau + 1 \)) and \( \sigma (= \sigma_{1}) \in [0, \frac{1}{2}] \) parametrizes the \( I \)-direction of the cylinder \( S^1 \times I \).

We introduce the notations,

\[ \psi = \psi^{(D)} + \psi^{(N)}, \]

where

\[ \psi^{(D)} \equiv \frac{1}{2} (1 \mp i \Gamma_{(p)}) \psi, \quad \psi^{(N)} \equiv \frac{1}{2} (1 \pm i \Gamma_{(p)}) \psi. \] (26)

From eq.(22) we find that \( \psi^{(D)} \) and \( \psi^{(N)} \) obey the Dirichlet and the Neumann boundary conditions respectively.

We approximate the real fields on the world sheet by \( N \times N \) hermitian matrices. By using the same reasoning that is made in ref.\[4\], we obtain the correspondence rules:

- **DD sector**: \( X^{\underline{\mu}}, \psi^{(D)} \xrightarrow{N \to \infty} N \times N \) antisymmetric matrices,
- **NN sector**: \( X^{\underline{\mu}}, \psi^{(N)} \xrightarrow{N \to \infty} N \times N \) symmetric matrices. (27)

As is mentioned in sec.3.1, the transformation parameters of APD must belong to the Dirichlet sector. It follows that

parameters of APD \( \xrightarrow{N \to \infty} N \times N \) antisymmetric matrices. (28)

We can now write down the matrix model to regularize an open superstring. Following the standard procedure, we replace real fields on the world sheet, \( \int_{\Sigma} d^{2} \sigma \sqrt{h} \) and \( i \{*, *\} \) with \( N \times N \) hermitian matrices, \( \text{Tr} \) and \( \{*, *\} \) respectively. Consequently, we obtain the matrix regularization of the action (20):

\[ S = \alpha \left[ \frac{1}{4} \text{Tr} \left( [X^{\underline{\mu}}, X^{\underline{\nu}}]^{2} + 2[X^{\underline{\mu}}, X^{\underline{\nu}}]^{2} + [X^{\underline{\mu}}, X^{\underline{\nu}}]^{2} \right) \right. 
- \frac{1}{2} \text{Tr} \left( + \bar{\psi}^{(D)} \Gamma^{\underline{\mu}} \{X_{\underline{\mu}}, \psi^{(D)}\} + \bar{\psi}^{(N)} \Gamma^{\underline{\mu}} \{X_{\underline{\mu}}, \psi^{(N)}\} \ight. 
\left. + \bar{\psi}^{(D)} \Gamma^{\underline{\mu}} \{X_{\underline{\mu}}, \psi^{(N)}\} + \bar{\psi}^{(N)} \Gamma^{\underline{\mu}} \{X_{\underline{\mu}}, \psi^{(D)}\} \right) \right] + \beta \text{Tr} 1. \] (29)

The APD gauge transformation (24) becomes

\[ \delta_{\text{gauge}} X^{\underline{\mu}} = i \{ \zeta , X^{\underline{\mu}} \}, \quad \delta_{\text{gauge}} \psi = i \zeta \psi, \] (30)
where $\zeta$ is an antisymmetric matrix as is found in eq. (28). This gauge transformation can be identified with $SO(N)$ gauge transformation. $X^\mu$ and $\psi^{(D)}$ belong to the adjoint representation and $X^a$ and $\psi^{(N)}$ belong to the 2nd rank symmetric representation of $SO(N)$ respectively. The situation is very similar to the case of a light-cone open supermembrane \[4\] [9].

We point out here that in the $p = 3$ case the bosonic and the fermionic physical degrees of freedom match. The matter contents in this case are given as follows,

\[
\begin{align*}
\text{Neumann sector :} & \quad \begin{cases} 
\text{bosonic} & X^0, X^1, X^2, X^3 \\
\text{fermionic} & \psi^{(N)} \equiv \frac{1 + \Gamma(p)}{2} \psi
\end{cases} \\
\text{Dirichlet sector :} & \quad \begin{cases} 
\text{bosonic} & X^4, X^5, X^6, X^7, X^8, X^9 \\
\text{fermionic} & \psi^{(D)} \equiv \frac{1 + \Gamma(p)}{2} \psi
\end{cases}
\end{align*}
\]

(31)

It is obvious that, in the Neumann sector, the physical degrees of freedom of bosons and fermions are both four. In the Dirichlet sector, while we have four fermionic degrees of freedom, there appear to be six bosonic ones. Owing to the gauge symmetry, however, the number of the latter reduces by two. Thus the bosonic and fermionic physical degrees of freedom match in each sector.

We note that the above matter contents can be interpreted as the zero volume limit of the six dimensional $\mathcal{N} = 1$ $SO(N)$ super Yang-Mills theory which couples to a hypermultiplet in the second rank symmetric representation of the gauge group. We hope that this model will play a role in understanding D-3 branes, which are sometimes difficult to analyze because of their self-duality.

Finally we mention the relationship between the recently proposed large-$k$ $USp(2k)$ matrix model \[10\] and our $SO(N)$ model. The authors of \[10\] find that, in order to have unbroken supersymmetry, there are two ways of projecting the ten bosonic matrix coordinates into $n_-$ components in the adjoint representation of $USp(2k)$ and $n_+$ ones in the antisymmetric representation. One is $(n_-, n_+) = (6, 4)$ and the other is $(n_-, n_+) = (2, 8)$. They correspond to $(3+1)$- and $(7+1)$-dimensional orientifold fixed planes respectively. This result should be naturally derived by extending our analysis to an unoriented superstring. What we have learned are that $SO(N)$ matrix models and $USp(2k)$ ones incorporate D branes and orientifold planes respectively, and that the possible dimensions of these objects undergo the identical restriction. It might be interesting to further investigate these relationship especially in the type I theory.

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\section*{Appendix}

1+9 dimensional gamma matrices $\Gamma^\mu$ ($\mu = 0, 1, \ldots, 9$) satisfy the $SO(1,9)$ Clifford algebra $\{\Gamma^\mu, \Gamma^\nu\} = -2\eta^{\mu\nu}$, where we use the almost plus sign convention as the ten dimensional Minkowski metric $\eta_{\mu\nu} = \text{diag}(-, +, \ldots, +)$. In this convention $\Gamma^0$ is defined to be hermitian.
and $\Gamma^i$ ($i = 1, \ldots, 9$) are to be anti-hermitian. Thus we find $\Gamma^\dagger_\mu = \Gamma^0 \Gamma_\mu \Gamma^0$. The charge conjugation matrix $C$ satisfies $\Gamma^T_\mu = -C^{-1} \Gamma_\mu C$, and $C^T = -C$. It follows that

$$\Gamma^2_{(p)} = (-)^{\frac{1}{2}(p+3)} I_{32}, \quad \Gamma_{(p)} T = (-)^{\frac{1}{2}(p+1)(p+2)} C^{-1} \Gamma_{(p)} C,$$

(32)

where $\Gamma_{(p)} \equiv \Gamma_{01}^{(p)}$. The charge conjugation of a spinor $\theta$ is defined as $\theta^c = \bar{\theta}^T$, where $\bar{\theta} \equiv \theta^T \Gamma^0$ is the Dirac conjugate of $\theta$. It follows that the Majorana condition $\psi = \psi^c$ means that

$$\overline{\psi} = -\psi^T C^{-1} \quad \text{for Majorana spinor } \forall \psi.$$

(33)

References

[1] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, Phys. Rev. D55 (1997) 5112.

[2] N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, Nucl. Phys. B498 (1997) 467. M. Fukuma, H. Kawai, Y. Kitazawa and A. Tsuchiya, “String Field Theory from IIB Matrix Model”, [hep-th/9705128].
H. Aoki, S. Iso, H. Kawai, Y. Kitazawa and T. Tada, “Space-Time Structures from IIB Matrix Model”, [hep-th/9802085].

[3] T. Banks, N. Seiberg and S. Shenker, Nucl. Phys. B490 (1997) 91.
I. Chepelev, Y. Makeenko and K. Zarembo, Phys. Lett. B400 (1997) 43.
A. Fayyazuddin and D.J. Smith, Mod. Phys. Lett. A12 (1997) 1447.

[4] K. Ezawa, Y. Matsuo and K. Murakami, “Matrix Regularization of Open Supermembrane –towards M-theory five-brane via open supermembrane”, [hep-th/9707200].

[5] B. de Wit, J. Hoppe and H. Nicolai, Nucl. Phys. B305 (1988) 545.
B. de Wit, K. Peeters and J. Plefka, “Open and Closed Supermembranes with Winding”, [hep-th/9710215].

[6] M.B. Green and M. Gutperle, Nucl. Phys. B476 (1996) 484.
see also C. Chu, P. Howe and E. Sezgin, “Strings and D-Branes with Boundaries”, [hep-th/9801202].

[7] M.B. Green and J.H. Schwarz, Phys. Lett. B136 (1984) 367; Nucl. Phys. B143 (1984) 285.

[8] E. Bergshoeff, R. Kallosh, T. Ortín and G. Papadopoulos, Nucl. Phys. B502 (1997) 149.

[9] N. Kim and S.-J. Rey, Nucl. Phys. B504 (1997) 187.

[10] H. Itoyama and A. Tokura, “USp(2k) Matrix Model : Nonperturbative Approach to Orientifolds”, [hep-th/9801084].