The theory of the reentrant effect in susceptibility of cylindrical mesoscopic samples

G.A. Gogadze
B. Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine
47, Lenin Av., 61103 Kharkov Ukraine
e-mail: gogadze@ilt.kharkov.ua

Abstract
A theory has been developed to explain the anomalous behavior of the magnetic susceptibility of a normal metal-superconductor (NS) structure in weak magnetic fields at millikelvin temperatures. The effect was discovered experimentally by A.C. Mota et al [10]. In cylindrical superconducting samples covered with a thin normal pure metal layer, the susceptibility exhibited a reentrant effect: it started to increase unexpectedly when the temperature lowered below 100 mK. The effect was observed in mesoscopic NS structures when the N and S metals were in good electric contact. The theory proposed is essentially based on the properties of the Andreev levels in the normal metal. When the magnetic field (or temperature) changes, each of the Andreev levels coincides from time to time with the chemical potential of the metal. As a result, the state of the NS structure experiences strong degeneracy, and the quasiparticle density of states exhibits resonance spikes. This generates a large paramagnetic contribution to the susceptibility, which adds up to the diamagnetic contribution thus leading to the reentrant effect. The explanation proposed was obtained within the model of free electrons. The theory provides a good description for experimental results [10].

1 Introduction

Mesoscopic systems [1–3] can exhibit surprising properties at comparatively low temperatures. For pure normal metals there is a length scale \( \xi_N = \hbar V_F / k_B T \) (\( V_F \) is the Fermi velocity, \( T \) is the temperature, \( k_B \) is the Boltzmann constant) which has the meaning of a coherence length in a system with a disturbed long-range order. When this length is comparable with the characteristic dimensions of the system, the interference effects can come into play. Theoretically this was first demonstrated by Kulik [4] for a thin-wall normal pure-metal cylinder in the vector potential field. It appears that the magnetic moment of such a system is an oscillating function of the magnetic flux through the cross-section of the cylinder, the oscillation period being equal to
the flux quantum of the normal metal $hc/e$. The effect is generated by quantization of the electron motion and due to the sensitivity of the states of the system to the vector potential field (Aharonov–Bohm effect [5]). Bogachek and this author showed the existence of oscillating component with the period $hc/e$ in the magnetic moment of a singly connected normal cylinder in a weak magnetic field. Oscillations with this period are produced by the magnetic surface levels of the cylindrical sample in a weak magnetic field [6]. The effect of flux quantization in a normal singly connected cylindrical conductor was first detected experimentally in 1976 by Brandt et al. when they were investigating the longitudinal magnetoresistance in pure $Bi$ single crystals [7],[8]. This was actually the first observation of the interference effect of flux quantization in nonsuperconducting condensed matter.

Recent advanced technologies of preparation of pure samples have enabled investigation of the coherent properties of mesoscopic structures taking proper account of the proximity effect [9]. The samples were superconducting $Nb$ wires with a radius $R$ of tens of $\mu m$ coated with a thin layer $d$ of high-purity $Cu$ or $Ag$. The metals were in good contact and the electron mean free path exceeded the typical scale $\xi_N$. The magnetic susceptibilities of copper and silver were measured. The breakdown field $H_b$, the supercooled field $H_{sc}$ and the superheated field $H_{sh}$ were estimated as functions of temperature and normal metal thickness. While continuing their experiments on these samples, Mota and co-workers [10] detected a surprising behavior of the magnetic susceptibility of a cylindrical $NS$ structure ($N$ and $S$ are for the normal metal and the superconductor, respectively) at very low temperatures ($T < 100mK$) in the external magnetic field parallel to the $NS$ boundary.

Most intriguingly, a decrease in the sample temperature below a certain point $T_r$ (at a fixed field) produced a reentrant effect: the decreasing magnetic susceptibility of the structure unexpectedly started growing. A similar behavior was observed with the isothermal reentrant effect in a field decreasing to a certain value $H_r$ below which the susceptibility started to grow sharply. It is emphasized in Ref.[11] that the detected magnetic response of the $NS$ structure is similar to the properties of the persistent currents in mesoscopic normal rings. It is assumed [9] – [12] that the reentrant effect reflects the behavior of the total susceptibility $\chi$ of the $NS$ structure: the paramagnetic
contribution is superimposed on the Meissner effect-related diamagnetic contribution and nearly compensates it. Anomalous behavior of the susceptibility has also been observed in AgTa, CuNb and AuNb structures [11], [13].

The possibility of the paramagnetic contribution to the susceptibility of the NS structure needs further clarification. The NS structure in question is essentially a combination of two subsystems capable of electron exchange, which corresponds to the establishment of equilibrium in a large canonical ensemble (with fixed chemical potential). Assume that these systems are initially isolated with a thick dielectric layer. It is known that the superconductor response to the applied magnetic field generates superfluid screening current near the cylinder surface (Meissner effect). How does the normal mesoscopic layer respond to the weak magnetic field? Kulik [4] shows (see above) that in a weak magnetic field the magnetic moment of a thin-wall normal cylinder oscillates with the flux. The magnetic moment oscillations are equivalent to the existence of persistent current. Since the energies of the individual states and; hence, the total energy are dependent on the flux, the average current is nonzero. The current state corresponds to the minimum free energy, therefore the inclusion of weak dissipation would not lead to the decay of the current state. When the N and S metals are isolated, the quantum states of the quasiparticles in the N-metal are formed at the expense of specular reflection of the electrons from the dielectric boundaries. The amplitude of the magnetic moment oscillations in the N layer is small, which is determined by the smallness of the parameter $1/k_F R$ in the problem and by the paramagnetic character of the persistent current [4], [6] (when the magnetic field tends to zero, the magnetic susceptibility is positive). Thus, in the absence of the proximity effect, the total susceptibility of the NS structure is only governed by the diamagnetic contribution of the S-layer (the paramagnetic contribution is very small).

When the proximity effect is present in the NS structure, we assume that the probability of the electron transit from the superconductor to the N metal is close to unity. This significantly affects the properties of the NS structure. The diamagnetic response of the superconductor persists but new properties appear, that are brought about by the proximity effect. Now two kinds of electron reflection are observed in the normal film – a specular reflection from one boundary
and the Andreev reflection from other. Along with the trajectories closed around the cylinder circle, new trajectories appear in a weak field, which “screen” the normal metal. The new trajectories of ”particles” and ”holes” confine the quantization area of the triangle whose base is a part of the NS boundary between the points of at which the quasiparticle collides with this boundary. This area is maximum for the trajectories touching the superconductor. It is shown below that at certain values of the flux through the triangle area, the electron density of states experiences flux-dependent resonance spikes. Thus, in the presence of the proximity effect, the periodic flux-induced oscillations of the thermodynamic values typical of the normal layer in the NS structure give way to periodic resonance spikes with a period equal to a superconducting flux quantum $\frac{hc}{2e}$ [16]. The response of the normal mesoscopic layer to a weak magnetic field ($H \lesssim 100e$) is paramagnetic and the susceptibility amplitude is large. The picture, however, changes when the quantized magnetic flux through the triangle area increases and its value divided by $\frac{hc}{2e}$ starts to exceed the highest Andreev ”subband” number. A phase transition occurs in a certain field $H_r$. As a result, the $N$ layer is now screened only by the trajectories of those quasiparticles that do not collide with the superconducting boundary. Their amplitudes are rather small (see above) against the large diamagnetic response. We can thus conclude that the resonance contribution to the paramagnetic susceptibility of the NS structure can only appear in comparatively weak magnetic fields. At this condition the reentrant effect may be generated. The conclusion correlates well with the experimental observations [9] – [14].

The origin of paramagnetic currents in NS structure was discussed in several theoretical publications. Bruder and Imry [17] analyze the paramagnetic contribution to susceptibility made by quasiclassical (”glancing”) trajectories of quasiparticles that do not collide with the superconducting boundary. The authors [17] point to a large paramagnetic effect within their physical model. However, their ratio between the paramagnetic and diamagnetic contributions is rather low and cannot account for experimental results [9] – [14].

Fauchere, Belzig and Blatter [18] explain the large paramagnetic effect assuming a pure repulsive electron–electron interaction in noble metals. The proximity effect in the $N$ metal induces an order
parameter whose phase is shifted by $\pi$ from the order parameter $\Delta$ of the superconductor. This generates the paramagnetic instability of the Andreev states, and the density of states of the $NS$ structure exhibits a single peak near the zero energy. The theory in [18] essentially rests on the assumption of the repulsive electron interaction in the $N$ metal. Is the reentrant effect a result of specific properties of noble metals? or Does it display the behavior of any normal metal experiencing the proximity effect from the neighboring superconductor? Only experiment can provide answers to these questions. We just note that the theories of [17], [18] do not account for the temperature and field dependencies of the paramagnetic susceptibility and the nonlinear behavior $\chi$ of the $NS$ structure. The current theories cannot explain the origin of the anomalously large paramagnetic reentrant susceptibility in the region of very low temperatures and weak magnetic fields.

It is worth mentioning the assumption made by Maki and Haas [19] that below the transition temperature ($\sim 10mK$) some noble metals ($Cu$, $Ag$, $Au$) can exhibit $p$-wave superconducting ordering, which may be responsible for the reentrant effect. This theory does not explain the high paramagnetic reentrant effect either.

In this paper a theory of the reentrant effect is proposed which is essentially based on the properties of the quantized levels of the $NS$ structure. Levels with energies no more than $\Delta$ ($2\Delta$ is the gap of the superconductor) appear inside the normal metal bounded by the dielectric (vacuum) on one side and contacting the superconductor on the other side. The number of levels $n_0$ in the well is finite. Because of the Aharonov-Bohm effect [5], the spectrum of the $NS$ structure is a function of the magnetic flux in a weak field. The specific feature of the quantum levels of the structure is that in a varying field $H$ (or temperature $T$) each level in the well periodically comes into coincidence with the chemical potential $\zeta$ of the metal. As a result, the state of the system suffers strong degeneracy and the density of states of the $NS$ sample experiences resonance spikes.

It is shown that the phenomenon of resonance appears in a certain interval of weak magnetic fields at temperatures no higher than a hundred of millikelvins. Resonance is realizable only in pure mesoscopic $N$ layers under the condition of the Aharonov-Bohm effect. The resonance produces a large paramagnetic contribution $\chi^p$ to the
susceptibility of the $NS$ structure. When $\chi^p$ is added to the diamagnetic contribution $\chi^d$ produced by the Meissner effect, the total susceptibility displays the features of the reentrant effect \cite{20}.

2 Spectrum of quasiparticles of the $NS$ structure

Consider a superconducting cylinder with the radius $R$ which is covered with a thin layer $d$ of a pure normal metal. The structure is placed in a weak magnetic field $\vec{H}(0, 0, H)$ oriented along the symmetry axis of the structure. It is assumed that the field is weak to an extent that the effect of twisting of quasiparticle trajectories becomes negligible. It actually reduces to the Aharonov-Bohm effect \cite{5}, i.e. allows for the increment in the phase of the wave function of the quasiparticle moving along its trajectory in the vector potential field.

We proceed from a simplified model of $NS$ structure in which the order parameter magnitude changes stepwise at the $NS$ boundary. It is also assumed that the magnetic field does not penetrate into the superconductor. The coherent properties observed in the pure normal metal can be attributed to its large ”coherence” length $\xi_N$ at very low temperatures.

One can easily distinguish two classes of trajectories inside the normal metal. One of them includes the trajectories which collide in succession with the dielectric and $NS$ boundaries. The quasiparticles moving along these trajectories have energies $\varepsilon < \Delta$ and are localized inside the potential well bounded by a high dielectric barrier ($\simeq 1eV$) on one side and by the superconducting gap $\Delta$ on the other side. On its collisions, the quasiparticle is reflected specularly from the dielectric and experiences the Andreev scattering at the $NS$ boundary \cite{15}. We introduce an angle $\alpha$ at which the quasiparticle hits the dielectric boundary. The angle is counted off the positive direction of the normal to the boundary (Fig. 1). in this case the first class contains the trajectories with $\alpha$ varying within $0 \lesssim \alpha \leq \alpha_c$ ($\alpha_c$ is the angle at which the trajectory touches the $NS$ boundary). The other class includes the trajectories whose spectra are formed by collisions with the dielectric only, i.e. the trajectories with $\alpha > \alpha_c$.

The two groups of trajectories produce significantly different spectra of quasiparticles. The distinctions are particularly obvious in the
presence of the magnetic field. The trajectories with $\alpha \lesssim \alpha_c$ form a spectrum of Andreev levels which contains a supplement in the form of an integral of the vector potential field. The spectrum characterizes the magnetic flux through the area of the triangle between the quasiparticle trajectory and the part of the $NS$ boundary. It is also determines the magnitude of the screening current produced by ”particles” and ”holes” in the $N$ layer. These states are responsible for the reentrant effect. The trajectories with $\alpha > \alpha_c$ do not collide with the $NS$ boundary. The states induced by these trajectories are practically similar to the ”whispering gallery” type of states appearing in the cross-section of a solid normal cylinder in a weak magnetic field \cite{6, 21}. The size of the caustic of these trajectories is of the order of the cylinder radius, i.e. they correspond to high magnetic quantum numbers $m$. The spectrum thus formed carries no information about the parameters of the superconductor and it is impossible to meet the resonance condition in this case. These states make a paramagnetic contribution to the thermodynamics of the $NS$ structure but their amplitude is small ($\sim 1/k_F R$). It is therefore discarded from further consideration. Our interest will be concentrated on the trajectories with $\alpha \leq \alpha_c$.

The spectrum of quasiparticles of the NS structure can be obtained
easily using the multidimensional quasiclassical method generalized for the case of the Andreev scattering in the system \cite{16}, \cite{22}. After collision with the \textit{NS} boundary the ”particle” transforms into a ”hole”. The ”hole” travels practically along the path of the ”particle” but in the reverse direction. In the strict sense, however, the path of the ”hole” is somewhat longer because under the condition of Andreev elastic scattering the momentum of the ”particle” exceeds that of the reflected ”hole”. According to the law of conservation of the angular momentum, the angle $\alpha'$ at which the ”hole” comes up to the dielectric boundary and hence the distance covered by the ”hole” are larger. Eventually, the trajectory of the quasiparticle becomes closed due to its displacement along the perimeter of the \textit{N} layer. However, as the quasiparticle energy decreases and approaches the value of the chemical potential, the difference $\alpha - \alpha'$ starts tending to zero. Since our further interest is concerned with low-lying Andreev levels, we assume that the ”hole” trajectory is strictly reversible. The distance covered by the ”particle” (”hole”) between two boundaries is $L_0 \simeq 2d/\cos \alpha$.

According to the multidimensional quasiclassical method \cite{16}, \cite{22}, there are two congruences of ”particle” rays – towards the dielectric (\textit{I}) and in the opposite direction (\textit{II}). There are also two congruences of ”hole” rays – towards the \textit{NS} boundary (\textit{III}) and away from it (\textit{IV}). The covering space is constructed of four similar \textit{NS} structures whose edges are joined in accordance with the law of quasiparticle reflection from a dielectric and a \textit{NS} boundary. At the dielectric boundary the congruences \textit{I} and \textit{II} are joined. The congruences \textit{III} and \textit{IV} are joined independently. The covering space consists of the outer (”particles”) and inner (”holes”) toroidal surfaces. Each surface contains only a part of the single independent integration contour. The path of the ”particle” is $2d$. The ”hole” travels the same length whereupon the trajectory of the quasiparticle closes. The total length of the closed contour along the covering surface of the \textit{NS} structure is $4d$.

It is possible to choose two independent integration contours within a tours that do not contract into a point. One condition of quantization relates the caustic radius to the magnetic quantum number $m$. We replace it with an angle of incidence of the quasiparticle on the dielectric boundary. The other condition of quantization introduces the radial quantum number $n$. Thus, the complete set of quantum
numbers describing the motion of the quasiparticle includes \( n, \alpha, q \), where \( q \) is the quasimomentum component along the symmetry axis of the cylinder.

Assume that the condition \( d \ll R \) is obeyed for the \( NS \) structure. We can then neglect the curvature of the cylinder boundary and assume that it is flat. The condition of quasiclassical quantization can be written as

\[
\int_{\mathcal{L}_0} \left( \vec{p}_0 - \frac{|e|}{c} \vec{A} \right) d\vec{s} - \int_{\mathcal{L}_0} \left( \vec{p}_1 + \frac{|e|}{c} \vec{A} \right) d\vec{s} = 2\pi \hbar \left( n + 1 + \frac{1}{\pi} \arccos \frac{\varepsilon}{\Delta} \right),
\]

where \( p_0 \) \( (p_1) \) are the quasimomentum of the "particle" ("hole"), \( \varepsilon \) is the "quasiparticle" energy, \( \vec{A} \) is the vector potential \((0, 0, Hy)\), \( |\mathcal{L}_0| \) is the trajectory length covered by the "particle" ("hole"). The unity in the right-hand side of Eq. (1) appears when two collisions of the quasiparticle with the dielectric boundary are taken into account [22]. The term \( \frac{1}{\pi} \arccos \frac{\varepsilon}{\Delta} \) accounts for the phase delay of the wave function under the Andreev scattering of quasiparticles [16]. The quasimomentum \( p_0 \) and \( p_1 \) in Eq. (1) can be expanded in the parameter \( \varepsilon/\zeta \) retaining the first-order terms and replacing \( n + 1 \) by \( n \). As a result, Eq. (1) furnishes the sought for spectrum of the \( NS \) structure in a weak magnetic field (\( \mathcal{L} \) is the quasiparticle trajectory):

\[
\varepsilon_n(q, \alpha; \Phi) = \frac{\pi \hbar v_\mathcal{L}(q) \cos \alpha}{2d} \left( n + \frac{1}{\pi} \arccos \frac{\varepsilon}{\Delta} - \frac{\tan \alpha}{\pi} \Phi \right).
\]

Here \( v_\mathcal{L}(q) = \sqrt{p_F^2 - q^2/m^*} \), \( p_F \) is the Fermi momentum, \( q \) is the quasiparticle momentum component along the cylinder axis, \( m^* \) is the effective mass of the quasiparticle, \( \Phi_0 = hc/2e \) is the superconducting flux quantum. The positive \( \alpha \)-values refer to "particles" \((n > 0)\), while the negative ones are for "holes" \((n < 0)\).

The last term in Eq. (2) has the meaning of "phase"

\[
\Phi = \frac{2\pi}{\Phi_0} \int_0^d A(x) dx,
\]

which is dependent on the vector potential field and varies with the angle \( \alpha \) characterizing the trajectory of the quasiparticle.

The spectrum of Eq. (2) is similar to Kulik’s spectrum [23] for the current state of an \( SNS \) contact. However, Eq. (2) includes an
angle-dependent magnetic flux instead of the phase difference of the contacting superconductors.

The value of the "phase" (flux) controls the diamagnetic and paramagnetic currents in the $NS$ structure. To calculate it, we should know the distribution of the vector potential field inside the normal metal.

The problem of the Meissner effect in superconductor-normal metal (proximity) sandwiches was solved by Zaikin [24]. It was shown that the proximity effect caused the Meissner effect bringing an inhomogeneous distribution of the vector potential field over the $N$ layer of the structure: $A(x) = H x + \frac{4\pi}{c} j(a)x(d - \frac{x}{2})$. For convenience we introduce the notation $a = \int_0^d A(x)dx$. This expression can be obtained from the Maxwell equation $\text{rot} \overrightarrow{H} = \frac{4\pi}{c} \overrightarrow{j}$ with the boundary conditions $A(x = 0) = 0$ and $\partial_x A(x = d) = H$. The screening (diamagnetic) current $j$ is a function of $a$, $j(a) = -j_s \varphi(a/\Phi_0)$, where $j_s$ is the superfluid current and $\varphi(x)$ is function of flux. Thus, we can write down the self-consistent equation for $a$ [25] – [26]:

$$a = \frac{H d^2}{2} + \frac{4\pi}{3c} j(a)d^3.$$  \(4\)

The diamagnetic current $\overrightarrow{j}^d(a)$ was calculated in terms of the microscopic theory as a sum of currents of quasiparticles ("particles" and "holes") for all quasiclassical trajectories characterized by the angles $\theta$ and $\varphi$ [24], [26] (below the system of units $k_B = \hbar = c = 1$ is used):

$$j^d(\Phi, T) =$$

$$= -AT \sum_{\omega_n>0} \int_0^{\pi/2} d\theta \int_0^{\pi/2} d\varphi \frac{\sin^2 \theta \cos \varphi \sin[2\Phi \tan \theta \cos \varphi]}{\sqrt{\omega^2 + \Delta^2} \sin \alpha_n + \frac{\omega_n}{\Delta} \cos \alpha_n}$$

$$+ \cos^2(\Phi \tan \theta \cos \varphi),$$

(5)

where $A = 2ek_F^2/\pi^2$, $\omega_n = (2n + 1)\pi T$, $2\Delta$ is the superconductor gap, $\alpha_n = 2\omega_n d/v_F \cos \theta$, and $\Phi$ is given by Eq. (3). The function $j^d(\Phi)$ is noted for interesting features. In small magnetic fields ($\Phi \ll 1$) $j^d \approx -j_s \Phi$. Such low fields can lead to the effect of extrascreening of the external magnetic field (see [24]). When the field increases ($\Phi \simeq 1$), the current starts oscillating and for certain "phases" it
turns to zero at regular intervals ”phases” $\Phi$. With high values of the inequality ($\Phi \gg 1$), the current amplitude decreases.

3 Resonance spikes in the density of states of \textit{NS} structure in weak magnetic fields

In the region of weak magnetic fields, the density of states of the quasiparticles that are described by the spectrum of Eq. (2) exhibits sharp singularities. The spectrum of Eq. (2) is formed by the trajectories of the quasiparticles which collide with the dielectric and superconducting boundaries. It encloses a certain area penetrated by a magnetic flux. At any instant when the magnetic flux becomes a multiple of the superconducting flux quantum, the density of states experiences resonance spikes.

Let us consider the cross-section of a \textit{NS} structure. Assume that the superconducting cylinder radius $R$ and the normal layer thickness $d$ have a mesoscopic scale. The density of states $\nu(\varepsilon)$ can be calculated proceeding from the expression

$$\nu(\varepsilon) = \sum_{n, \alpha, \sigma} \int dq \delta[\varepsilon - \varepsilon_n(q, \alpha)].$$

The summation is taken over all quantum numbers $n$, $q$, $\alpha$ and spin $\sigma$. Since we are not interested in the contribution from the states formed by the trajectories of the quasiparticles with $\alpha > \alpha_c$, we can write down

$$\nu(\varepsilon) = \int_{-\alpha_c}^{\alpha_c} d\alpha \nu(\varepsilon; \alpha),$$

where $\nu(\varepsilon; \alpha)$ is the contribution to the density of states from the pre-assigned trajectory with a fixed $\alpha$. Eq. (2) for the low-lying Andreev levels ($\varepsilon \ll \Delta$) is taken as a spectrum. After integration with respect to $q$ and introduction of the notation $\beta = \pi h/2dm^*$, we can pass on to the dimensionless energy $\epsilon = \varepsilon/\beta p_F$. For $\nu(\varepsilon, \alpha)$ we have the expression

$$\nu(\epsilon, \alpha) = \frac{2p_F}{\pi^2 \beta d} \epsilon^2 \sum_n \frac{\sec^2 \alpha \theta[|n + \kappa| - \epsilon \sec \alpha]}{(n + \kappa)^2 \sqrt{(n + \kappa)^2 - \epsilon^2 \sec^2 \alpha}},$$

(8)
where \( \kappa = 1/2 - \Phi \tan \alpha / \pi \), and \( \theta(x) \) is the stepwise Heaviside function. Eq. (8) suggests two cases depending on the parameter \( n + \kappa \).

a) Non-resonance case. If \( n + \kappa \neq 0 \), the energy dependence under the radical sign in Eq. (8) can be neglected for small energies (\( \epsilon \to 0 \)). Then, the nonresonance contribution to the density of states is

\[
\nu^{(0)} \sim \frac{2pF}{\pi^2 \beta d} \epsilon^2 \int_0^{\alpha_c} d\alpha \sum_{n=\infty}^{+\infty} \frac{\sec^2\alpha}{(n + \kappa)^3}.
\]

The series in Eq.(9) is calculated readily by the formula in [27]:

\[
\sum_{k=-\infty}^{+\infty} \frac{1}{(k + \kappa)^n} = (-1)^{n-1} \frac{\pi}{(n-1)!} \frac{d^{n-1}}{d\kappa^{n-1}} \cot \pi \kappa.
\]

After calculation of the integral we obtain

\[
\nu^{(0)} \sim \frac{pF}{\beta d} \epsilon^2 \Phi_0 \cot \frac{2\pi a}{\Phi_0} \sqrt{\frac{2R}{d}},
\]

where \( \sqrt{\frac{2R}{d}} \approx \tan \alpha_c \).

b) Resonance case. Now we go back to Eq. (8). We find \( \nu^{\text{res}} \) as

\[
\nu^{\text{res}} \sim \epsilon^2 \int_0^{\alpha_c} d\alpha \sum_{n} \frac{\sec^2\alpha \theta(|a_n - btga| - \epsilon \sec a)}{|a_n - btga|^2 \sqrt{|a_n - btga|^2 - \epsilon^2 \sec^2 a}},
\]

where the notations \( a_n = n + 1/2 \), \( b = \frac{2a}{\Phi_0} \) are introduced. Eq. (11) shows that at certain values of the flux (b), the radicand in the denominator turns to zero.

Prior to calculation of \( \nu^{\text{res}} \), let us discuss the question of the contribution of different angles \( \alpha \) to the resonance amplitude. It is reasonable to assume that because of the factor \( \sec^2\alpha \) in the numerator of Eq. (11), the angles \( \alpha \sim \alpha_c \) are the main contributors to the integral. It is convenient to employ in the integral a new variable of integration \( x = \tan \alpha \). Then the neighborhood of the upper limit \( x_0 = \tan \alpha_c \) is the main contributor to the integral. Introducing the notation \( \tilde{a} = a_n - bx_0 \) and the small deviation \( \xi = x_0 - x \ll 1 \), we can write down the equation for the resonance condition as:

\[
(b^2 - \epsilon^2) \xi^2 + 2(\tilde{a}b + \epsilon^2 x_0) \xi + \tilde{a}^2 - \epsilon^2 (1 + x_0^2) = 0.
\]
The point of our interest is the asymptotics \( \nu(\epsilon) \) at low \( \epsilon \to 0 \). Eq. (12) is solved to the accuracy within first-order terms of \( |\epsilon| \):

\[
\xi_{1,2} \simeq \frac{\tilde{a}}{b} \pm \frac{|\epsilon|}{b} \sqrt{1 + x_0^2}.
\]  

(13)

The expression in front of the radical in the denominator of Eq. (11) has the second order smallness in \( |\epsilon| \), i.e. \( |\tilde{a}|^2 \lesssim |\epsilon|^2(1 + x_0^2) \), which leads to its cancellation with the similar small parameter in the numerator.

The remaining integral is estimated to be a constant of about unity. The resonance -induced spike of the density of states always appear when the Andreev level coincides with the Fermi energy at a certain flux in the \( N \) layer. In the vicinity of the chemical potential there is a strong degeneracy of the quasiparticle states with respect to the quantum number \( q \). As a result, a macroscopic number of \( q \) states contribute to the amplitude of the effect. Near the resonance, the ratio of the resonance and nonresonance amplitudes of the density of states is

\[
\frac{\nu^{\text{res}}}{\nu^{(0)}} \sim \frac{1}{|\epsilon|^2} \gg 1.
\]  

(14)

It is thus shown that a change in the magnetic flux leads to resonance spikes in the density of states of the \( NS \) structure. The flux interval between the spikes is equal to the superconducting flux quantum \( \Phi_0 \).

4 Calculation of susceptibility of \( NS \) contact

To explain the reentrant effect, we need to have an expression for the susceptibility of the \( NS \) structure. We assume that in a weak magnetic field the total susceptibility of the \( NS \) sample consists of two contributions. Firstly, the response of the superconductor to the applied magnetic field generates the Meissner effect. Note that the diamagnetic response is observed in all fields up to the critical one. The amplitude of the diamagnetic current increases monotonously with lowering temperature. On the other hand, the presence of a pure normal metal in the \( NS \) structure produces a paramagnetic contribution. In a weak magnetic field the contribution is due to
the Aharonov–Bohm effect and the quantization of the quasiparticle spectrum of the mesoscopic system. When the penetrability of the barrier between the metals is small, the electrons of the normal metal are reflected specularly from its boundaries. As compared to the diamagnetic contribution from the superconductor, the paramagnetic contribution produced by the $N$ layer has a small amplitude and can therefore be neglected. Thus, the paramagnetic and diamagnetic contributions cannot compete in the absence of the proximity effect in the $NS$ structure. However, if the penetrability of the barrier is close to unity, the mechanism of the Andreev reflection becomes active at the $NS$ boundary. The quasiparticle spectrum of the $N$ layer undergoes a significant transformation and resonance spikes appear in the amplitude of the density of states in a certain regions of magnetic fields and temperatures. Simultaneously, the distribution of the vector potential field in the normal layer becomes inhomogeneous. As shown below at certain values of the parameters of the problem, the paramagnetic contribution to the susceptibility of the $NS$ structure can become equal to the diamagnetic contribution. This is the reason why the reentrant effect appears in pure mesoscopic $NS$ structures.

Theoretically, the resulting susceptibility including the reentrant effect can be represented as a sum of the paramagnetic contribution $\chi^p$ of the $NS$ structure caused by the Andreev scattering and the diamagnetic susceptibility $\chi^d$ of the system in which there is no proximity effect between the $N$ and $S$ metals. The temperature-induced behavior of the diamagnetic current in such a system is well known. As the temperature decreases, the diamagnetic current amplitude increases and becomes saturated at temperatures about several millikelvins. At high temperatures $k_B T \gg \hbar V_F / d$, the diamagnetic current decreases rapidly following the law $j \sim T^{-1} \exp(-4\pi k_B T d / \hbar V_F)$. Note that in a $NS$ structure in which the electrons are reflected specularly at both boundaries of the normal metal, the susceptibility is negative (i.e. diamagnetic) in the whole interval of temperatures $0 < T < T_c$. However, we will not use this approach to estimate the resulting susceptibility. Below we calculate the screening current of the $NS$ structure. It naturally allows for the paramagnetic contribution at certain values of the magnetic field and temperature. We focus our attention on calculation of the paramagnetic contribution in structures with a pronounced proximity effect. This is important especially in the context of the re-
cient statement \[28\] that no paramagnetic reentrance can occur in \(NS\) proximity cylinders in the absence of electron-electron interaction in the \(N\) layer.

a) Paramagnetic susceptibility of \(NS\) contact

The contribution of the states in Eq(2) to the paramagnetic susceptibility of the normal layer in a \(NS\) contact can be calculated proceeding from the expression for the thermodynamic potential \((k_B = 1)\)

\[
\Omega = -T \sum_{\sigma, \alpha, n, q} \ln[1 + \exp(-\varepsilon_n(q, \alpha)/T)],
\]

where the summation is taken over the spin \((\sigma)\) and all the states related to the trajectories of the quasiparticles with \(a \lesssim a_c\). The expression for susceptibility (per unit volume \(V\) of the normal metal) is found using the formula

\[
\chi = -\frac{1}{V} \frac{\partial^2 \Omega}{\partial H^2}.
\]

After performing the summation over the spin and taking into account two signs of the angle \(\alpha\) and of the quasimomentum component \(q\), we arrive at the initial expression for paramagnetic susceptibility \((\zeta\) is the chemical potential of the metal):\

\[
\chi = \frac{d}{2T m^* F_0^2} \int_{-\zeta}^{\infty} d\varepsilon \frac{\exp(\varepsilon/T)}{[\exp(\varepsilon/T) + 1]^2} \times
\]

\[
\times \sum_n \int_0^{\alpha_c} d\alpha \cos \alpha \sin^2 \alpha \int_0^{p_F} dq (p_F^2 - q^2)^{3/2} \delta(\varepsilon - \varepsilon_n(q, \alpha)).
\]

In Ref.\[20\] we lost one of the radicals \((p_F^2 - q^2)^{1/2}\) in the similar initial expression for \(\chi\). As a result, the amplitude of the paramagnetic contribution appeared to be underestimated. This mistake is corrected in this work.

It is convenient to present the spectrum in terms of \(\beta = \frac{\pi h}{2m^* d}\) and \(\varkappa = \frac{1}{2} - \frac{ta_0}{\pi} \Phi\) as \(\varepsilon_n(q, \alpha) = \beta \cos \alpha (n + \varkappa) \sqrt{p_F^2 - q^2}\). Now we introduce the dimensionless energy \(\epsilon = \frac{\varepsilon}{\beta p_F} = \frac{\varepsilon}{\delta \varepsilon}\), \(\delta \varepsilon = \frac{\pi h V_F}{2d}\) is the distance between the Andreev levels in the \(SN\) structure. Since \(\zeta / \delta \varepsilon \gg 1\), the lower limit of the energy integral can be replaced with \(-\infty\). By introducing the variable \(x = \tan \alpha\) and the notation \(a_n = n + 1/2\),
\[ b = b(H, T) = 2a/\Phi_0, \quad a = \int_0^d A(x) dx, \quad x_0 = \tan\alpha_0 = \sqrt{2R/d} \]

and taking into account the parity of the integrand we obtain, instead of Eq. (16):

\[ \chi = C \int_0^\infty \frac{d\epsilon \epsilon^4}{\cosh^2(\eta\epsilon/2)} \sum_{n=0}^{n_0} \int_0^{x_0} \frac{dx x^2}{(a_n - bx)^4} \frac{\theta[a_n - bx - \epsilon \sqrt{1 + x^2}]}{\sqrt{(a_n - bx)^2 - \epsilon^2(1 + x^2)}}. \tag{17} \]

In Eq. (17) the summation is taken over the quantum numbers of the "particles". Here \( C = \frac{c^2 d}{\Phi_0^2}, \quad \eta = \frac{\delta \epsilon}{T}, \quad n_0 \) is the number of Andreev levels in the potential well and \( \theta \) is the Heaviside step function. It is seen in Eq. (17) that for the given "subzone" \( n \) the amplitude of the paramagnetic susceptibility increases sharply whenever the Andreev level coincides with the chemical potential of the metal. The resonant spike of susceptibility occurs when \( a_n - bx \) tends to zero on a change in the magnetic field (or temperature). Because of the finite number of Andreev levels, the existence region of the isothermal reentrant effect is within \( 0 < H \lesssim H_{\text{max}} \).

Let us calculate the integral over \( x \) in Eq. (17). It contains a singularity under the radical \( R(x) = \sqrt{Ax^2 + Bx + C} \) where \( A = b^2 - \epsilon^2, \quad B = -2a_n b, \quad C = a_n^2 - \epsilon^2 \). The singularity is determined by the roots of the quadratic equation \( x_{1,2} = \frac{a_n b}{b^2 - \epsilon^2} \pm \frac{|\epsilon|}{b^2 - \epsilon^2} \sqrt{b^2 + a_n^2 - \epsilon^2} \). On introducing the notation \( \alpha_0 = \frac{a_n}{b} \), the expression for the roots can be written with a linear accuracy with respect to \( \epsilon \) as

\[ x_{1,2} \approx \alpha_0 \pm \frac{|\epsilon|}{b} \sqrt{1 + \alpha_0^2}. \tag{18} \]

The main contribution to the integral over \( x \), Eq. (17), is made by the vicinity of the point \( \epsilon \to 0 \). If we exclude the singular points from the interval of integration, the indefinite integral over \( x \) can be calculated accurately (see the details in the Appendix). Because the \( \theta \)-function is present under the integral, the integration intervals \((0, x_1)\) and \((x_2, x_0)\) make a finite contribution to the integral. On substituting the limits of integration, the expressions obtained have different powers of the parameter \( |\epsilon|^{-1} \). We retain only the most important terms in order \( |\epsilon|^{-4} \) that determine amplitude of the effect. The discarded terms have higher orders of \( \epsilon \)-smallness. The intervals \((0, x_1)\) and \((x_2, x_0)\) make contributions of the same order of \( \epsilon \)-magnitude. The region \((x_1, x_2)\) does not contribute to the integral at all.
The estimate for the integral over $x$ is

$$\frac{4}{3 b(1 + \alpha_0^2)^2} \frac{1}{\epsilon^4}.$$  \hspace{1cm} (19)

On substituting Eq. (19) into Eq. (17), the parameter $\epsilon^4$ drops out of the energy integral and we can take it quite easily. Taking into account the energy limits $\theta(a_n - |\epsilon|)$ appearing in the process of calculation we can obtain the expression for the paramagnetic contribution to the susceptibility of the NS structure, which in dimensional units has the form

$$\chi_p \approx \frac{16 \zeta^2 d^2}{3 \pi \hbar V_F \Phi_0^2} \sum_{n=0}^{n_0} b(H, T) \text{th} \left[ \frac{\pi \hbar V_F}{4dk_B T}(n + 1/2) \right] \left[ 1 + \left( \frac{b(H, T)}{n+1/2} \right)^2 \right]^2.$$  \hspace{1cm} (20)

In Eq. (20) the summation over the quantum number $n$ is taken within finite limits, where $n_0$ has the meaning of the maximum number of the Andreev levels inside the potential well of the NS structure. Its order of magnitude in $n_0 \sim \Delta/\delta \varepsilon$, where $\delta \varepsilon$ is the distance between the Andreev levels, $\delta \varepsilon = \pi \hbar V_F/2d$, and $2\Delta$ is the energy gap. The flux $b(H, T) = 2a/\Phi_0$ depends on both the magnetic field and temperature. In the pre-assigned field its value is dictated by the screening current of the NS structure $j = -j_s \varphi(a/\Phi_0)$ (see Eq. (4)). The obtained expression for $\chi_p$ manifests a more rapid decrease susceptibility at the increasing parameter $b(H, T)$ than it was evidenced by Eq. (5) in Ref. [20].

We first discuss the isothermal case of a very low temperature and clear up the qualitative behavior of susceptibility in Eq. (20). We shall proceed from the region of very strong magnetic fields ($a/\Phi_0 \gg 1$) in which the second term in Eq. (14) is negligible. Then the dimensionless flux $b(H, T) \gg 1$ and the amplitude of the paramagnetic contribution in Eq. (20) decreases as $b(H, T)$ raised to power 3. In comparatively weak magnetic fields $a/\Phi_0 \sim 1), the function $\varphi(x)$ is actually an oscillating function of $H$ and here we can expect the reentrant effect. Indeed as the field decreases to a certain value and the parameter $b(H, T)/n_0$ becomes $\sim 1$ ($n_0$ is the number of the Andreev levels in the potential well), the amplitude of the paramagnetic susceptibility of the NS structure accepts for the first time an appreciable contribution from the highest Andreev ”subband” (level). On a further decrease in
this field, the contribution from the highest "subband" persists, but in a certain lower field an additional contribution appears from the neighboring lower-lying "subband" $n_0 - 1$. Finally, in a very weak field all the "subbands" of the $NS$ structure start to contribute and the paramagnetic susceptibility amplitude reaches its peak. However, at $H \rightarrow 0 \ (a/\Phi_0 \rightarrow 0)$, the paramagnetic contribution turns to zero, as follows from Eq. (20). The reason is that the resonance condition for the Andreev levels (Eq. (2)), cannot be realized in a zero field.

Now we change to the case when the temperature of the $NS$ structure varies but the field is kept constant. We assume the field to be weak ($H \sim 2 \cdot 10^{-1} Oe$). The second term in Eq. (4) for the flux is very important. It is highest at millikelvin temperatures. As a result, the parameter $b(H, T)$ has the lowest value. In this temperature region the hyperbolic tangent is close to unity and the paramagnetic contribution is dependent only on the parameter $b(H, T)$. Under this condition, all the "subbands" of the $NS$ structure contribute to the amplitude of the effect. As the temperature rises, the parameter $b(H, T)$ increases smoothly. Simultaneously, the argument of the hyperbolic tangent decreases. At a certain temperature, when the condition $k_B T > \pi \hbar V_F / 4d$ is met, the contribution from the lowest "sub-band" starts dying down and its amplitude is decreasing linearly with growing $T$. On a further rise of the temperature, the contributions from the higher "subbands" of the spectrum die down in succession. Finally, at a very high temperature the paramagnetic contribution tends to zero.

Let us estimate the amplitude of the paramagnetic contribution. The parameter $b(H, T)$ is dependent on the value of the flux $a = \int_0^d A(x)dx$, which at constant $T$ can be found by solving the self-consistent equation Eq. (4). In the region of millikelvin temperatures and magnetic fields $H \sim 2 \cdot 10^{-1} Oe$ the paramagnetic contribution has the largest amplitude. We obtain $b(H, T) \sim 10^{-4}$ in this region of $T$ and $H$. The coefficient before the sum in Eq. (20) can be found by substituting $\zeta_{Ag} \simeq 8.75 \cdot 10^{-12} erg$, $d = 3.3 \cdot 10^{-4} cm$ $V_F^{Ag} \sim 1.39 \cdot 10^8 cm/sec$ for the characteristic parameters of the normal $Ag$ layer. We thus obtain $16\zeta^2 d^2 / 3\pi \hbar V_F \Phi_0^2 \simeq 2.418 \cdot 10^3$. The product of this coefficient and the parameter $b(H, T)$ yields the order of magnitude of the paramagnetic contribution amplitude. It is seen
that the largest amplitude of the paramagnetic contribution exceeds that of the diamagnetic contribution in the vicinity of $T = 0$.

b) Full magnetic susceptibility of $NS$ structure in the presence of proximity effect

Let us consider a structure in which the electrons experience the Andreev scattering at the $NS$ boundary. In the presence of magnetic field, the screening current is induced in the normal layer due to the Meissner effect. We estimate the susceptibility generated by this current.

The total current $J$ is related to the magnetic moment $M$ as

$$M = \frac{1}{c} JS_0,$$  \hspace{1cm} (21)

where $S_0 \simeq \pi R^2$ is the cylinder cross-section ($d \ll R$). Let the average current density be $j$. The total current is then $J = S j$, where $S = dL$ ($L$ is the cylinder generatrix). The density of the screening current in $NS$ proximity sandwiches was calculated by Zaikin [24], [28]. We reproduce the formula for the current density (see Eq. (5)), which is valid at arbitrary values of temperature and magnetic field. At $T \ll \hbar V_F/d$ it is

$$j(\Phi) \simeq -\frac{4 e k_F^2 T}{\pi^2} \sum_{\omega_n > 0} \int_0^{\pi/2} d\theta \int_0^{\pi/2} d\varphi \sin^2 \theta \cos \varphi \sin[2 \arctan \cos \varphi \Phi] \frac{\sin[2 \arctan \cos \varphi \Phi]}{\sin^2 \alpha_n + \cos^2[\arctan \cos \varphi \Phi]}. \hspace{1cm} (22)$$

Here $\alpha_n = \frac{2 \omega_n d}{V_F \cos \theta}$, $\omega_n = (2n + 1)\pi T$ and the phase $\Phi$ follows from Eq. (3). Near $T = 0$ the summation of frequencies in Eq. (22) can be replaced with integration. For $\Phi \leq 1$ the response of the current is

$$j(\Phi) \simeq -\frac{e k_F^2 V_F}{\pi^2 d} \int_0^{\pi/2} d\theta \int_0^{\pi/2} d\varphi \sin^2 \theta \cos \varphi \sin[2 \arctan \cos \varphi \Phi]. \hspace{1cm} (23)$$

If the field is small enough to meet the condition $\Phi \ll 1$, Eq. (23) reduces to the result that was obtained for the first time in [24]:

$$j(\Phi) = -\frac{e k_F^2 V_F}{6 \pi^2 d} \Phi. \hspace{1cm} (24)$$

At ”phases” $\Phi \gg 1$, the screening current of Eq. (23) turns to zero.
The current-phase dependence at $T = 0$ is plotted in Fig.2. The dependence is nonlinear and its amplitude has a maximum at a certain value of $\Phi$. Knowing the current-phase dependence, we can determine the susceptibility of the $NS$ structure using the equation $\chi = dM/dH$. It is seen in Fig.2 that the susceptibility $\chi$ of the $NS$ structure (the derivative of current with respect to field) changes its sign at a certain low value of the magnetic field $H_r$. The susceptibility is "diamagnetic" in the region of high magnetic fields and "paramagnetic" at $H < H_r$. The "paramagnetic" portion of the curve is due to the proximity effect at the $NS$ boundary and to the Andreev levels in the $N$ layer.

Let us estimate $\chi$ in the linear-response regime near $T = 0$ when this dependence is described by Eq.(24). In such weak fields we obtain $\Phi \simeq \frac{3\pi H \lambda_N^2(T)}{\Phi_0}$, where the "penetration depth" $\lambda_N$ into the normal metal is dependent on temperature [26]:

$$\lambda_N^{-2}(0) = \frac{4\pi ne^2}{m^*c^2}; \quad (T \simeq 0)$$

$$\lambda_N^{-2} \sim \lambda_N^{-2}(0) \frac{6T}{T_A} \exp(-2T/T_A); \quad (T \gg T_A = \frac{\hbar V_F}{2\pi d}).$$

The estimate of susceptibility in the millikelvin region is $\chi \sim -\frac{R e\kappa^2_{VF}}{4\pi d} \frac{\lambda_N^2(0)}{\Phi_0}$. For the parameters of the problem $d = 3.3 \cdot 10^{-4}cm$,
\( R = 8.2 \cdot 10^{-4} \text{cm}, \ k_F^{Ag} \sim 1.2 \cdot 10^8 \text{cm}^{-1}, \ V_F^{Ag} \sim 1.39 \cdot 10^8 \text{cm/sec}, \lambda_N(0) \sim 2 \cdot 10^{-6} \text{cm} \) we obtain \( \chi = -0.06 \), which is close to \( \chi = -\frac{3}{4\pi} \).

Now we keep the magnetic field (assuming it weak) constant and plot the screening current versus temperature in a wide \( T \)-range. This dependence plotted using Eq. (5) is shown in Fig. 3. It is seen that the current amplitude has a maximum at a certain \( T_r \).

5 Discussion

In this study we have investigated the behavior of a superconducting cylinder covered with a thin layer of a pure normal metal. It is assumed that the normal metal and superconductor are in good contact. The system was placed in a magnetic field directed along the \( NS \) boundary. The \( NS \) structure has mesoscopic scale dimensions. It is assumed that the mean free path of the quasiparticles in the \( N \) layer exceeds the characteristic length \( \xi_N = \hbar V_F/k_B T \), which has the meaning of the coherence length for a system with disturbed long-range order. The goal of this study was to interpret the experiments in which A.C. Mota et al. [10] – [14] observed anomalous behavior of the magnetic susceptibility of a \( NS \) structure at varying temperature in a constant magnetic field or in a varying magnetic field at a constant temperature. This phenomenon was called a reentrant effect. Until recently it has not been explained adequately.

Earlier [20] the author clarified the nature of the reentrant effect. As was found, the origin of the paramagnetic contribution is closely connected with the properties of the quantized Andreev levels that are dependent on the magnetic flux varying with both temperature and magnetic field. Typically, the levels in the \( NS \) structure time from time (at certain values of the field \( H \) or temperatures) coincide with the chemical potential of the metal. As a result, the state of the system is highly degenerate and the density of states of the \( NS \) structure experiences resonance spikes. The response of the normal mesoscopic layer to a weak magnetic field is paramagnetic.

A theory of the reentrant effect has been developed in this study. We calculated the paramagnetic contribution separately and analyzed its behavior in a varying magnetic field and at varying temperature. In the course of this calculation we corrected the mistake made in [20].
which led to underestimation of the effect amplitude. The paramagnetic response is determined only by the trajectories of the quasiparticles that collide with the \(NS\) boundary. It is shown that the reentrant effect can occur in a certain range of weak magnetic fields at temperatures no higher than \(100\,mK\). We believe that paramagnetic reentrant effect is an intrinsic effect of mesoscopic \(NS\) proximity structures in the very low temperature limit.

Assume that the temperature of the \(NS\) structure is about \(10^{-3}\,K\) and the magnetic field is increasing. As soon as the field exceeds a certain value \(H_r\), the isothermal reentrant effect must vanish. In strong fields the Andreev levels cease to make a resonance contribution to the paramagnetic susceptibility. Now the paramagnetic contribution is made by the states formed by the trajectories of the quasiparticles that collide only with the dielectric boundary. However, their contribution to the resulting susceptibility of the structure is small because of the smallness of the quasiclassical parameter of the problem \(1/k_F R\). Under this condition the susceptibility exhibits diamagnetic behavior in all strong fields up to the critical one.

A self-consistent calculation of the screening current of the \(NS\) structure was performed taking into account the contribution from the Andreev levels. The analysis of the derived expression suggests the paramagnetic contribution to current. For example, Fig.2 illustrates the dependence of the current upon the phase (magnetic field). The values of the current \(j\) to the left of the extremum \(\Phi_r\) account for the contribution of the Andreev levels. The derivative of this curve with respect to the field is proportional to the magnetic susceptibility of the \(NS\) structure. It is positive ("paramagnetic") in the region of low magnetic fields and negative ("diamagnetic") in high fields.

Similar behavior is observed when the susceptibility of the \(NS\) structure is measured as a function of temperature in a pre-assigned weak magnetic field: it is "paramagnetic" in the region \(T < T_r\) and "diamagnetic" at \(T > T_R\) up to the critical temperature. Temperature dependence of magnetic susceptibility in the \(NS\) structure at fixed magnetic field will be investigated in detail in separate publication.

In the absence of the proximity effect in the \(NS\) structure, when the penetrability of the barrier between the \(S\) and \(N\) metals is small, the electrons of the normal metal are reflected specularly from its boundaries. In this case the \(SN\) structure is a total of two isolated
subsystems (normal metal and superconductor) placed into a magnetic field. Because of the Meissner effect, diamagnetic current develops near the superconductor surface. In normal metal, because of the Aharonov-Bohm effect, the quantized spectrum of quasiparticles is dependent on the magnetic flux through the cross-section of the cylinder. The flux generates a paramagnetic contribution to the susceptibility whose quasiclassical parameter of the problem $1/k_F R$ is small. Hence, in the absence of the proximity effect no competition is possible between the paramagnetic and diamagnetic contributions in the $NS$ structure, and the reentrant effect is unobservable in such $NS$ sample.

To conclude, it should be noted that the explanation proposed in this study for the reentrant effect was developed within a model which does not assume the electron-electron interaction in the $N$ layer of the $NS$ structure. In terms of the free-electron model, a large paramagnetic contribution to the susceptibility of the $NS$ structure appears in the region of very low temperatures in a weak magnetic field. If we increase the thickness $d$ of the pre-assigned normal metal, this would lead to a greater number of the Andreev levels $n_0$ in the potential well and affect the solutions of the self-consistent equation for $a$. As a result, the shape of the curve of the paramagnetic susceptibility would be slightly ”deformed” though its qualitative behavior would remain the same.
Appendix

Let us calculate the integral taken over \( x \) in Eq. (15):

\[
J = \int_0^{x_0} dxx^2\theta[a_n - bx - \epsilon\sqrt{1 + x^2}] / (a_n - bx)^4 \sqrt{(a_n - bx)^2 - \epsilon^2(1 + x^2)}.
\] (A1)

After introducing the notation \( \alpha_0 = a_n/b \), we can see that the function in front of the radical in the denominator has a singularity at the point \( x = \alpha_0 \). Besides, as was noted in the text, the integrand has singularities at the points \( x_1, x_2 \).

Integral (A1) can be written as a sum of four integrals

\[
J = \int_0^{x_0} dx \ldots = \lim_{\varepsilon \to 0} \left\{ \int_0^{x_1-\varepsilon} dx \ldots + \int_{x_1+\varepsilon}^{\alpha_0-\varepsilon} dx \ldots + \int_{\alpha_0+\varepsilon}^{x_2-\varepsilon} dx \ldots + \int_{x_2+\varepsilon}^{x_0} dx \ldots \right\}.
\]

It is obvious that the presence of the \( \theta \)-function makes the second and the third integrals equal to zero. We first calculate the integral \( J_1 \):

\[
J_1 = \frac{1}{b^4} \lim_{\varepsilon \to 0} \int_0^{x_1-\varepsilon} \frac{dxx^2}{(\alpha_0 - x)^4 \sqrt{Ax^2 + Bx + C}},
\] (A2)

where \( A = b^2 - \epsilon^2, B = -2a_n b, C = a_n^2 - \epsilon^2 \). On substituting the variable \( \alpha_0 - x = 1/t \), the indefinite integral becomes \( \int \frac{dt}{\sqrt{\alpha t^2 + \beta t + \gamma}} \),

where \( \alpha = -(1 + \alpha_0^2)\epsilon^2, \beta = 2\alpha_0\epsilon^2 \). It can be calculated by the method of undetermined coefficients:

\[
\int \frac{dt f(t)}{\sqrt{\alpha t^2 + \beta t + \gamma}} = (A_1 t^{n-1} + A_2 t^{n-2} + \ldots + A_n) \sqrt{\alpha t^2 + \beta t + \gamma} + A_{n+1} \int \frac{dt}{\sqrt{\alpha t^2 + \beta t + \gamma}}
\]

if \( f(t) \) is the polynonial to power \( n \). Although the calculation is tedious, it is actually simple. The coefficients \( A_1, A_2, A_3 \) and \( A_4 \) are readily found as:

\[
A_1 = -\frac{\alpha_0^2}{3\epsilon^2(1 + \alpha_0^2)}, \quad A_2 = \frac{\alpha_0(1 + \alpha_0^2/\epsilon)}{\epsilon^2(1 + \alpha_0^2)^2},
\]

\[
A_3 = -\frac{2a_n^2}{3\epsilon^4(1 + \alpha_0^2)^2} + \frac{-3 + 5\alpha_0^2 + \alpha_0^4/2}{3\epsilon^2(1 + \alpha_0^2)^3},
\]

\[
A_4 = \frac{2a_n^2}{3\epsilon^2(1 + \alpha_0^2)^3} - \frac{11\alpha_0^2 + 11\alpha_0^4/2}{3\epsilon^3(1 + \alpha_0^2)^3}.
\]
A_4 = \frac{a_n^2(\alpha_0^2/2 - 1)}{\epsilon^2\alpha_0(1 + \alpha_0^2)^2} + \frac{\alpha_0(2 - \alpha_0^2/2)}{(1 + \alpha_0^2)^3}.

It is seen that the coefficients have different orders of $\epsilon^{-1}$-magnitude: $A_1, A_2, A_4 \simeq \epsilon^{-2}$, $A_3 \simeq \epsilon^{-4}$. Finally, we have to calculate six integrals

\[ J_1 = \frac{1}{b^4} \lim_{\varepsilon \to 0} \int_{t_0}^{t_1 - \varepsilon} dt \left\{ 2A_1 t \sqrt{R(t)} + A_2 \sqrt{R(t)} + A_3 \alpha \tau^3 / \sqrt{R(t)} + (A_2 \alpha + A_1 \beta / 2) \right\}, \quad (A3) \]

where $R(t) = \alpha t^2 + \beta t + A$ and the designations $t_0 = \frac{1}{\alpha_0}$, $t_1 - \varepsilon = (\alpha_0 - x_1 + \varepsilon)^{-1}$ are introduced. All the six indefinite integrals in expression (A3) can be calculated accurately [29]. After substituting the limits of integration, integrals 1, 2, 3, 4 and 5 are bounded above on energy, which is due to the term $\sqrt{R(t)} \simeq \sqrt{a_n^2 - \epsilon^2/\alpha_0}$, i.e. $\theta(a_n - \epsilon)$. Taking into account the determined coefficients $A_i$ ($i = 1, 2, 3, 4$), we can obtain the final expression for $J_1$:

\[ J_1 \simeq \frac{1}{b^4} \left\{ \frac{b(\alpha_0^2 + 1/3)}{3\epsilon^2(1 + \alpha_0^2)^2} - \frac{b(1 + \alpha_0^2/6)}{\epsilon^2(1 + \alpha_0^2)^2} + \frac{2a_n^2b}{3\epsilon^4(1 + \alpha_0^2)^2} + \frac{b(1 - \frac{5}{3}\alpha_0^2 - \alpha_0^4/6)}{\epsilon^2(1 + \alpha_0^2)^3} + \frac{\alpha_0 b^2(\alpha_0^2/2 - 1)}{\epsilon^3(1 + \alpha_0^2)^5/2} + \frac{\alpha_0(2 - \alpha_0^2/2)}{\epsilon(1 + \alpha_0^2)^7/2} - \frac{b(\alpha_0^2/2 - 1)}{\epsilon^2(1 + \alpha_0^2)^2} - \frac{(2 - \alpha_0^2/2)}{(1 + \alpha_0^2)^3b} \right\}. \quad (A4) \]

Of all the terms in (A4), the most significant contribution is made by the third term because there is a factor $\epsilon^4$ in the numerator of the integral over the energy in Eq. (17). The contributions of the other terms are negligible. We thus obtain the estimate

\[ J_1 \simeq \frac{2\alpha_0^2}{3b(1 + \alpha_0^2)^2 \epsilon^4}. \quad (A5) \]

A similar calculation of the integral

\[ J_4 = \frac{1}{b^4} \lim_{\varepsilon \to 0} \int_{x_2 + \varepsilon}^{x_0} \frac{dx x^2}{(x - \alpha_0)^4 \sqrt{A x^2 + B x + C}} \]

25
gives a contribution, which is identical in the order of magnitude with (A5). As a result, we obtain the $J$ estimate present in the text, Eq. (19).

The author is sincerely grateful to A.N. Omelyanchouk for helpful discussions and support, to S.I. Shevchenko for valuable comments.

References

[1] Y. Imry, In: Direction in Condensed Matter Physics. B. Grinstein and G. Mazenko (eds), World Scientific, Singapure (1986), p.101.
[2] S. Washburn and R. A. Webb, Adv. Phys. 35, 375 (1986).
[3] A. G. Aronov and Yu. V. Sharvin, Rev. Mod. Phys. 59, 755 (1987).
[4] I. O. Kulik, JETP Lett. 11, 275 (1970).
[5] Y. Aharonov and D. Bohm, Phys. Rev. 115, 485 (1959).
[6] E. N. Bogachek and G. A. Gogadze, Zh. Eksp. Teor. Fiz, 63, 1839 (1972) [Sov. Phys. JETP, 36, 973 (1973)].
[7] N. B. Brandt, V. D. Gitsu, A. A. Nikolaeva, and Ya. G. Ponomarev, JETP Lett., 24, 272 (1976); Zh. Eksp. Teor. Fiz, 72, 2332 (1977) [Sov. Phys. JETP 45, 1226 (1977)].
[8] N. B. Brandt, E. N. Bogachek, D. V. Gitsu, G. A. Gogadze, I. O. Kulik, A. A. Nikolaeva, and Ya. G. Ponomarev, Fiz. Nizk. Temp. 8, 718 (1982) [Sov. J. Low Temp. Phys. 8, 358 (1982)].
[9] A. C. Mota, P. Visani, and A. Pollini. J. Low Temp. Phys. 76, 465 (1989).
[10] P. Visani, A. C. Mota and A. Pollini, Phys. Rev. Lett., 65, 1514 (1990).
[11] A. C. Mota, P. Visani, A. Pollini and K. Aupke, Physica B 197, 95 (1994).
[12] F. B. Muller-Allinger and A. C. Mota, Phys. Rev. Lett., 84, 3161 (2000).
[13] F. B. Muller-Allinger and A. C. Mota, cond-mat/0007331 (2000).
[14] R. Frassanito, P. Visani, M. Niderost, A. C. Mota, P. Smeibidl, K. Swieca, W. Wendler, and F. Pobell, Proceedings LT-21, Part S4-LT; Properties of Solids 1, Prague (1996), p.2317.
[15] A.F. Andreev, Zh. Eksp. Teor. Fiz. 46 (1964), 1823 [Sov. Phys. JETP 9 (1964), 1228]

[16] G.A. Gogadze, R.I. Shekhter and M. Jonson, Low Temp. Phys., 27 (2001), 913 [Fiz. Nizk. Temp. 27 (2001), 1237]

[17] C. Bruder and Y. Imry, Phys. Rev. Lett., 80 (1998), 5782.

[18] A.L. Fauchere, W. Belzig and G. Blatter, Phys. Rev. Lett., 82 (1999), 3336.

[19] K. Maki and S. Haas, cond-mat/0003413 (2000).

[20] G.A. Gogadze, J. Low Temp. Phys., 31 (2005), 94 [Fiz. Nizk. Temp., 31 (2005), 120.

[21] G.A. Gogadze, Fiz. Nizk. Temp., 9 (1983), 1051 [Sov. J. Low Temp. Phys., 9 (1983), 543]

[22] J.B. Keller and S.I. Rubinow, Ann. Phys. (N.Y), 9 (1960), 24.

[23] I.O. Kulik, Zh. Eksp. Teor. Fiz., 57 (1969), 1745 [Sov. Phys. JETP, 30 (1969), 944].

[24] A.D. Zaikin, Solid State Commun., 41 (1982), 533.

[25] W. Belzig, G. Bruder and G. Schon, Phys. Rev. B53 (1996), 5727.

[26] A.L. Faucher and G. Blatter, Phys. Rev. B56 (1997), 14102.

[27] A.P. Prudnikov, Yu. A. Brychkov, and O.I. Marichev, Integrals and Series (Nauka, Moscow, 1984) (in Russian).

[28] A.V. Galaktionov and A.D. Zaikin, Phys. Rev. B67 (2003), 184518.

[29] I.S. Gradshtein and I.M. Ryzhik. Tables of Integrals, Sums, Series, and Products, Nauka, Moscow (1971).