Stability characteristics of porous functionally graded plate in thermal environment

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Abstract: This paper deals with the investigation of stability characteristics of functionally gradient plates with microstructural defect (porosity). The mathematical formulations have been done using recently developed non-polynomial deformation theory by the authors'. The characteristic equations are derived using a variational principle. Finite element formulation is done using $C^0$ continuous nine-noded elements with seventy-two degrees of freedom per element. Porosity effect has been incorporated in the material properties using therecently developed mathematical model by the authors'. Convergence and validation studies have been shown to establish the authenticity and reliability of the solutions. The effect of porosity volume fraction, temperature distribution and boundary conditions on the stability of the gradient plate is presented in diagrammatically.

1. Introduction: Functionally graded structures (FGS) have proven their worthiness over thetraditional composite structure. These types of thestructure show their incredible attribute to withstand in extreme thermo-mechanical loading conditions. The material properties of such materials generally vary even and imperceptibly along the thickness direction, thus, FGS demonstrate its superiority over conventional laminated structures by evading delaminationrelated problems. In open reports, several studies are evident in this subject. Reddy [1]presented the analytical solution for graded plate using TSDT. Javaheri and Eslami[2] used CPT to assess the critical buckling loads ofthinf graded plate. Talha and Singh [3,4]explored the dynamic response of graded plate using HSDT with 13 DOF per node. This work has been extended by Gupta et al. [5,6] to examined the dynamic characteristics of the graded plate with unconventional boundary conditions and elastic foundation. Gupta and Talha[7]reviewed the recent development in the field of graded structures. In addition, Wattanasakulpong et al. [8] emphasized on the accumulation of porosity during the fabrication process of FGM. The large amplitude vibration response of graded beam with porosity inclusion is given by Wattanasakulpong and Ungbhakorn[9].Atmane et al. [10]investigated the mechanical response of porous FGM beam using nonpolynomialHSDT. Literature related to the FGM plate with porosity is still scarce and there is a need for further investigations. This paper investigates the influence of porosities on the stability of graded plate in the thermal environment using nonpolynomialHSNDT [11,12]. The equation of motion is obtained using variational principle. Finite element formulation is done using $C^0$ continuous nine-noded elements with seventy-two degrees of freedom per element. The validation study is done to substantiate the efficacy of the present model. The response of graded structure due to the presence of microstructural defect (porosity) has been explored extensively.
2. Displacement field

Consider a graded plate with \((axb)\) cross-section having an constant thickness \(\text{‘}h\text{’}\). According to the recently developed NP-HSNQTDT\[11,13,14\], the displacement field is given as

\[
\begin{align*}
\vec{X}_1 &= x_1 - x_1 \left[ Y_x + \left( \frac{AB}{h} \right) H_x \right] + \text{Bsinh}^{-1} \left( \frac{\Delta x_1}{h} \right) H_x \\
\vec{X}_2 &= x_2 - x_2 \left[ Y_y + \left( \frac{AB}{h} \right) H_y \right] + \text{Bsinh}^{-1} \left( \frac{\Delta x_2}{h} \right) H_y \\
\vec{X}_3 &= x_3 + \text{Acosh}^{-1} \left( \frac{\Delta x_3}{h} \right) H_z
\end{align*}
\]

(1)

The field variables can be denoted as \(\{ \Phi \} = \{ \mu_0, \nu_0, \gamma_x, \gamma_y, H_x, H_y, H_z \}^T\). The parameter \(\text{‘}A\text{’}\) is ascertained as 3.4 in the post-processing phase.

3. Strain-displacement terms

The various strains developed from the displacement kinematics in Eq. (1) is as follows:

\[
\begin{bmatrix}
\varepsilon_{xx}^0 \\
\varepsilon_{yy}^0 \\
\varepsilon_{zz}^0 \\
\gamma_{xy}^0 \\
\gamma_{yz}^0 \\
\gamma_{zx}^0
\end{bmatrix} = \begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz} \\
\gamma_{xy} \\
\gamma_{yz} \\
\gamma_{zx}
\end{bmatrix} + f(x_1) + \phi(x_3) + g(x_3)
\]

(2)

Where, \(f(z) = \psi \left[ \text{sinh}^{-1} \left( \frac{\Lambda x_3}{h} \right) - \frac{\Lambda x_3}{h} \right] \), \(g(z) = \Lambda \text{cosh}^{-2} \left( \frac{\Lambda x_3}{h} \right) \), \(\phi(x_3) = g'(x_3)\), \(\varepsilon_{xx}^0 = \frac{\partial x_1}{\partial x_1} \), \(\varepsilon_{yy}^0 = \frac{\partial x_2}{\partial y_2} \), \(\gamma_{xy}^0 = \frac{\partial x_1}{\partial y_2} \).

4. Constitutive equations

The material properties of the graded structure with microstructural defect (porosity) are calculated using the modified power law as given in Eq. 3:\[12\]:

\[
E(x_1) = [E_x - E_n] \left[ \frac{2x_1 + h}{2x_1} \right]^\alpha - \log(1 + \lambda/2) [E_x + E_n] \left[ 1 - \frac{2x_1}{h} \right] + E_n
\]

(3)

\[
\rho(x_1) = [\rho_x - \rho_n] \left[ \frac{2x_1 + h}{2x_1} \right]^\alpha - \log(1 + \lambda/2) [\rho_x + \rho_n] \left[ 1 - \frac{2x_1}{h} \right] + \rho_n
\]

\(\lambda\) is defined as porosity volume fraction \((\lambda \leq 1)\). \(\lambda = 0\) denote the graded plate without any defects. Typical temperature dependent property \(\text{‘}P\text{’}\) can be conveyed as a function of temperature:
\[ P = P_0 \left( P_4 T^{-1} + 1 + P_2 T + P_3 T^2 + P_4 T^3 \right) \]

Where \( P_0, P_1, P_2 \) and \( P_3 \) are the coefficients of temperature \( T(\kappa) \). The effective temperature field along the transverse direction is given as,

\[
T(x_j) = \left( \frac{T_e + T_m}{2} - \frac{T_e - T_m}{2} \cos \left( \frac{\pi x_j}{h} \right) \right), \quad \text{with} \quad T(-h/2) = T_m, T(h/2) = T_e
\]

(4)

The constitutive relation used in the present study is as follows:

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\tau_{xy} \\
\tau_{yz} \\
\tau_{xz}
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\
Q_{21} & Q_{22} & Q_{23} & 0 & 0 & 0 \\
Q_{31} & Q_{32} & Q_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & Q_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & Q_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & Q_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz} \\
\gamma_{xy} \\
\gamma_{yz} \\
\gamma_{zx}
\end{bmatrix} =
\begin{bmatrix}
1 \\
1 \\
1 \\
0 \\
0 \\
0
\end{bmatrix}
\alpha \Delta T
\]

\[ Q_{11} = Q_{22} = Q_{33} \frac{E(x_j)(1-\nu^2)}{1-2\nu^2-2\nu^4} \]

\[ Q_{12} = Q_{21} = Q_{31} = Q_{13} = Q_{23} = Q_{32} \frac{E(x_j)\nu(1+\nu)}{1-2\nu^2-2\nu^4} \]

\[ Q_{44} = Q_{55} = Q_{66} = \frac{E(x_j)}{2(1+\nu)} \]

5. Governing Equations

The strain energy equation for \( i^{th} \) element of the plate is written as:

\[
\Pi = \frac{1}{2} \int_A \{\varepsilon\}^T \{\sigma\} dV = \frac{1}{2} \int_A \{\varepsilon\}^T \{Q\} \{\varepsilon\} dV
\]

\[
= \frac{1}{2} \int_A \{\bar{\varepsilon}\}^T \{M\} \{\bar{\varepsilon}\} dV = \frac{1}{2} \int_A \{\bar{\varepsilon}\}^T \{\bar{M}\} \{\bar{\varepsilon}\} dV = \frac{1}{2} \int_A \{\bar{\varepsilon}\}^T \{B\}^T \{D\} \{B\} \{\bar{\varepsilon}\} dA
\]

\[
= \frac{1}{2} \int_A \{\bar{s}\}^T \{K\} \{\bar{s}\} dA
\]

(5)

The kinetic energy of the plate is given as:

\[
\Delta = \frac{1}{2} \int_A \rho \{\bar{s}\}^T \{\bar{s}\} dV
\]

and \( \{\bar{s}\} = [\bar{N}] \{\bar{\varepsilon}\} \)

(6)

(7)

Putting Eq. (7) into Eq. (6), the kinetic energy of an element is obtained as:

\[
\Delta^e = \frac{1}{2} \int_A \rho \{\bar{s}\}^T \{\bar{N}\}^T \{N\} \{\bar{s}\} dA
\]

\[
= \frac{1}{2} \int_A \{\bar{s}\}^T \{m\} \{\bar{s}\} dA
\]

Where \( \{m\}^e \) is the mass matrix of the element.

The required governing equations for stability analysis is given as:

\[
\lambda_{cr} \left[ K_e \right] + \left[ K \right] \{\bar{s}\} = 0
\]

(9)

Where \( \{K\}, \{Kg\} \) and \( \lambda_{cr} \), are global linear and geometric stiffness matrix, critical load of buckling, global.
6. Result and discussion

6.1 Convergence and validation

Example 1: This problem is considered to confirm the convergence study of a present model for the stability analysis of Al/Al₂O₃ graded plate. The buckling load parameter is assumed \( \lambda_{\text{cr}} = \frac{\lambda_{\text{cr}}}{E_c h^3} \). The various material properties of the material are provided in Table 1. The buckling load is computed with the gradual mesh refinement for various values of 'n' as shown in Fig 1a. It is found from the convergence study that the present solution shows the fast convergence with mesh size.

Example 2. In the second example, the nondimensional buckling load of (Al/Al₂O₃) graded plate is obtained and the outcomes are compared with results given by Thai and Kim [15] which are based on TSDT. The mass density and Young’s modulus are: \( \rho = 3800 \text{ kg/m}^3 \), \( E_c = 380 \text{ GPa} \) for Al₂O₃; \( \rho = 2702 \text{ kg/m}^3 \), \( E_m = 70 \text{ GPa} \) for Al. The Buckling load parameter is plotted against volume fraction exponent for uniaxial and biaxial compression of the graded plate as shown in Fig 1b. It is visible that both the results are matches well.

![Figure 1.](image)

Figure 1. Convergence and Comparison study of the present solution with different mesh sizes.

6.2 Benchmark results

This section addresses the calculation of buckling load parameters of (Ti-6Al-4V/ZrO₂) graded plate with amicrostructural defect (porosity). The material properties are shown in Table 1. The non-dimensionalization is done using \( \lambda_{\text{cr}} = \frac{\lambda_{\text{cr}} h^3}{D_c} \) where \( D_c = E_c h^3 / 12(1-\nu^2) \). The effect of different boundary conditions, porosity volume fraction and thermal environment on the stability of graded plate is investigated. In Fig. 2a-b, the change of buckling load parameter of (SSSS) graded plate is plotted against volume fraction exponent for various value of \( \lambda_{\text{cr}} \). The temperature distribution is governed by the Eq. (4) whereas the material properties of the porous graded plate is estimated using Eq.(3). It is clear from the results that by increasing ‘n’, the buckling load decreases. Because larger value of ‘n’ means less ceramic contents, hence stiffness will reduce. It is also manifested from the results that the buckling load parameter decreases as the porosity volume fraction increases. It is observed that the buckling load decreased as the temperature difference between the top and bottom surface increased from 0K to 1000K.
Table 1: Temperature-dependent material coefficients for metal and ceramics

| Material      | Properties | $P_1$ | $P_0$ | $P_1$ | $P_2$ | $P_3$ | $T$ (T = 300K) |
|---------------|------------|-------|-------|-------|-------|-------|----------------|
| ZrO$_2$      | $E$ (Pa)   | 2.4427e+9 | -1.371e-3 | 1.214e-6 | -3.681e-10 | 168.063e+9 |
| ZrO$_2$      | $\alpha$ (1/K) | 12.766e-6 | -1.491e-3 | 1.006e-5 | -6.788e-11 | 18.591e-6 |
| ZrO$_2$      | $\rho$ (kg/m$^3$) | 3000 | 0 | 0 | 0 | 3000 |
| Ti-6Al-4V    | $E$ (Pa)   | 122.56e+9 | -4.586e-4 | 0 | 0 | 105.698e+9 |
| Ti-6Al-4V    | $\alpha$ (1/K) | 7.75788e-6 | 6.638e-4 | -3.147e-6 | 0 | 6.941e-6 |
| Ti-6Al-4V    | $\rho$ (kg/m$^3$) | 4429 | 0 | 0 | 0 | 4429 |

Fig. 2c-d, reflect the variation of nondimensional buckling load of fully clamped graded plate with \( \lambda \) for various values of \( \lambda \). It depicts the undeviating response as discussed for (SSSS) graded plate in Fig. 2a-b. It is worth noting that the percentage change in buckling load is approximately 2.4-7% when temperature difference increased from 0K to 100K for SSSS boundary condition, whereas this value is nearly 0.2-2% for CCCC boundary condition. Furthermore, the change in buckling load varies from 7-12% when \( \lambda \) increases from 0 to 0.4 in SSSS boundary condition, whereas this value, is approximately 13-21% for CCCC boundary conditions.

Fig 3a-b present the change in first five mode of buckling of (SSSS and CCCC) graded plate due to porosity when the temperature difference between the top and the bottom surface of FGM plate is 100K. It is found that the higher modes are influenced more due to porosity in comparison to the first mode of buckling.

![Figure 2](image)

**Figure 2.** Buckling load parameter Vs volume fraction index under thermal environment (a-b: SSSS; c-d: CCCC)
Figure 3. Initial five modes of buckling of the graded plate with porosity. (a) SSSS (b) CCCC

7. Conclusion: In this paper, the stability of graded plate under thermal environment is investigated using recently developed HSNDT. The various numerical results are computed to examine the consequence of porosity and thermal environment on buckling response of FGM plate. It is concluded that the porosity contributes to decrease the buckling load parameter. It is also observed that porosity has a significant influence on the stability of fully clamped graded plate. It is also concluded that the buckling load decrease as the temperature difference between the two surface increases.

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