Short Range Correlations and the EMC Effect in Effective Field Theory

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Introduction: Deep Inelastic Scattering (DIS) of leptons on hadrons can be precisely described as high-energy (perturbative) lepton-quark scattering weighted by the parton distribution functions (PDFs) that describe the probability of finding a quark or gluon inside the hadron. DIS has been used to map out the quark and gluon parton distributions for the proton and subsequently nuclei. In recent years, these experiments have revealed new and intriguing glimpses of nuclear structure that we seek to derive using effective field theory (EFT) methods.

In 1983, the European Muon Collaboration [1] measured the structure functions $F_2(x, Q^2)$ describing DIS for iron and deuterium targets, where Bjorken $x = Q^2/(2p \cdot q)$ and $Q^2 = -q^2$ are defined in terms of the target four-momentum $p$ and the momentum transfer from the lepton to the target, $q$. The results of these experiments could not be explained by nuclear structure (i.e., momentum distribution of nucleons inside the nucleus) without modifying the nucleon structure [1]. This “EMC effect” was unexpected since the typical binding energy per nucleon is much smaller (<1%) than the nucleon mass and the energy transfer involved in a DIS process. The EMC effect has now been mapped out for DIS on targets ranging from helium to lead (see Refs. [2–6] for reviews) and similar medium modifications of parton structure have been investigated in other reactions [5, 7]. The picture that has emerged is that the ratio

$$R_{EMC}(A, x) = \frac{F_2^A(x, Q^2)}{AF_2^d(x, Q^2)},$$

with $A$ the atomic number and $d$ the deuteron, can deviate from unity by up to 20% over the range $0.05 < x < 0.7$. The ratio has very little dependence on $Q^2$ and so we suppress it. Experimental data also suggest that for an isoscalar nucleus, the $x$ and $A$ dependence of $R_{EMC}$ is 1 factorizable. That is, the shape of the deviation of $R_{EMC}$ from unity is independent of $A$ while the magnitude of the deviation depends only on $A$ [8, 9].

$R_{EMC}$ forms a straight line in intermediate $x$, and one can express the magnitude of the EMC effect by the slope $dR_{EMC}(A, x)/dx$ for $0.35 \leq x \leq 0.7$. Since Bjorken $x$ is defined with respect to the parent nucleon of the struck parton, it is bounded in the range $0 \leq x \leq A$.

In recent experiments at Jefferson Lab, it was found that the ratio of quasi-elastic (QE) scattering cross sections,

$$a_2(A, x) \equiv \frac{2\sigma_A}{A\sigma_d} \Bigg|_{1.5 < x < 2},$$

forms an $x$-independent plateau with negligible $Q^2$ dependence for targets from $^3$He to $^{197}$Au [10–14]. This factor $a_2$ is referred to as the short range correlation (SRC) scaling factor. A remarkable empirical discovery is that the EMC slope and the SRC scaling factor $a_2$ are linearly related [15, 16].

In this Letter, we explain this linear relationship using EFT and compute $a_2$ in light nuclei. We first review the EFT description of the EMC effect of Ref. [17] which explained the factorization of $x$ and $A$ dependence of $R_{EMC} - 1$, and then show that the linear relation follows naturally from this. Factorization also shows that, up to higher order corrections, $a_2$ is scheme and scale independent even though it arises from scheme- and scale-dependent matrix elements in different nuclei. Finally, the values of $a_2$ for $^3$He and $^4$He are compared using the Green’s function Monte Carlo (GFMC) method with both chiral and Argonne-Urbana potentials to confirm the scheme and scale independence and are compared with data, showing close agreement. Results for $^9$Be and $^{12}$C extracted from variational Monte Carlo (VMC) calculations [18] are also discussed.

EFT Analysis: Chiral EFT is constructed based on the chiral symmetry of QCD. It has been successfully applied to many aspects of meson [19], single [20], and multi-nucleon systems [21]. In particular, chiral EFT has been applied to PDFs in the meson and single-nucleon
The structure functions describing lepton-nucleus DIS, \( F_2^A(x, Q^2) \), can be expressed in terms of nuclear PDFs \( q_i^A(x, Q) \) (for simplicity of presentation, we choose the DIS scheme where the renormalization and factorization scale are set equal to the hard scale of DIS, \( \mu = \mu_F = Q \), although the results below do not depend on the scheme) as \( F_2^A(x, Q^2) = \sum_i Q_i^2 x q_i^A(x, Q) \), where the sum is over quarks and anti-quarks of flavor \( i \) of charge \( \pm Q_i \) in a nucleus \( A \). In what follows, we focus on the isoscalar PDFs, \( q_A^A = q_\uparrow^A + q_\downarrow^A \); in the relevant experiments, nuclear PDFs are typically “corrected” for isospin asymmetry of the targets. The dominant (leading-twist) parton distributions are determined by target matrix elements of bilocal light-cone operators. Applying the operator product expansion, the Mellin moments of the parton distributions, \( \langle x^n \rangle_A(Q) = \int_{-A}^A x^n q_A(x, Q) dx \), \( n \geq 1 \), are determined by matrix elements of local operators,

\[
\langle A; p|O^{\mu_0 \cdots \mu_n}|A; p \rangle = \langle x^n \rangle_A(Q) p^{(\mu_0} \cdots p^{\mu_n)} \tag{4}
\]

with

\[
O^{\mu_0 \cdots \mu_n} = \gamma^{(\mu_0} iD^{\mu_1} \cdots iD^{\mu_n)} q, \tag{5}
\]

where \( (\ldots) \) indicates that enclosed indices have been symmetrized and made traceless and \( D^\mu = (\vec{D}^\mu - \vec{D}^\mu)/2 \) is the covariant derivative.

In nuclear matrix elements of these operators, there are other relevant momentum scales below \( Q: \Lambda \sim 0.5 \) GeV is the range of validity of the EFT, and \( P \sim m_\pi \) is a typical momentum inside the nucleus \( (m_\pi \sim \text{the pion mass}) \). These scales satisfy \( Q \gg \Lambda \gg P \) and the ratio \( \Lambda/Q \) is the small expansion parameter in the twist expansion while the ratio \( \epsilon = \Lambda/P \sim 0.2 - 0.3 \) is the small expansion parameter for the chiral expansion.

In EFT, each of the QCD operators is matched to hadronic operators at scale \( \Lambda \) [17]

\[
O^{\mu_0 \cdots \mu_n} \rightarrow (x^n)_{N} N^\mu_0 v^{(\mu_0} \cdots v^{\mu_n)} N^\dagger N \left[ 1 + \alpha_n N^\dagger N \right] + \langle x^n \rangle_{\pi} \pi^\alpha i\partial^{\mu_0} \cdots i\partial^{\mu_n} \pi^\alpha + \cdots, \tag{6}
\]

where the operators enclosed by \( : : \) are normal ordered (with respect to the vacuum state), \( N (\pi) \) is the nucleon (pion) field, \( v \) is the nucleon four velocity and \( \langle x^n \rangle_{\pi} \) is the \( n \)th moment of the isoscalar quark PDF in a free nucleon (pion). The \( \langle x^n \rangle_{\pi} \) terms are one-body operators acting on a single hadron only, while the \( \alpha_n \) terms are two-body operators. Here we have only kept the SU(4) (spin and isospin) singlet two-body operator \( \propto (N^\dagger N)^2 \) and neglected the SU(4) non-singlet operator \( \propto (N^\dagger \sigma N)^2 - (N^\dagger \tau N)^2 \) which changes sign when interchanging the spin \( (\sigma) \) and isospin \( (\tau) \) matrices [34]. The latter operator has an additional \( O(1/N_c^2) \sim 0.1 \) suppression in its prefactor [35] with \( N_c \), the number of colors. We also replace the nucleon velocity by the nucleus velocity and include the correction \( i\partial/M \) to higher orders.

The relative importance of the hadronic operators of Eq. (6) in a nuclear matrix element can be systematically estimated from the power counting of the EFT, which assigns a power of the small expansion parameter \( \epsilon \) to each Feynman diagram. In Weinberg’s power counting scheme [36], the nucleon one-body operator is \( O(\epsilon^{-3}) \), the nucleon two-body operator is \( O(\epsilon^0) \), while the pion one-body operator connecting two nucleons is \( O(\epsilon^{-1}) \). Since \( \langle x^n \rangle_\pi = 0 \) for even \( n \) due to charge conjugation symmetry, the \( n = 1 \) pion operator enters at \( O(\epsilon^1) \) but for higher \( n \) the contributions either vanish or are higher order compared with the other operators in Eq. (6).

The same order of importance for these operators is also found when using the alternate power counting of Refs. [37–39], but with a less suppressed two-body effect compared with the one-body nucleon operator. Other higher dimensional operators are omitted here because they are higher order in the power counting [17].

Using nucleon number conservation, \( \langle A| : N^\dagger N : |A \rangle = A \), the nuclear matrix element of Eq. (6) for \( n \neq 1 \) is

\[
\langle x^n \rangle_A(Q) = \langle x^n \rangle_N (Q) \left[ A + \alpha_n (\Lambda, Q) (A) : (N^\dagger N)^2 : |A \rangle \right], \tag{7}
\]

where \( \alpha_n \) is \( A \) independent but \( \Lambda \) dependent and is completely determined by the two-nucleon system. After an inverse Mellin transform, the isoscalar PDFs satisfy

\[
q_A(x, Q)/A \simeq q_N(x, Q) + g_2(A, \Lambda) \tilde{q}_2(x, Q, \Lambda), \tag{8}
\]

where

\[
g_2(A, \Lambda) = \frac{1}{2A} \langle A| : (N^\dagger N)^2 : |A \rangle, \tag{9}
\]

and \( \tilde{q}_2(x, Q, \Lambda) \) is an unknown function independent of \( A \).\(^1\)

This result also holds at the level of the structure function [17],

\[
F_2^A(x, Q^2)/A \simeq F_2^N(x, Q^2) + g_2(A, \Lambda) f_2(x, Q^2, \Lambda). \tag{10}
\]

The second term on the right-hand side of Eq. (10) is the nuclear modification of the nucleon structure function \( F_2^N \). The shape of distortion, i.e., the \( x \) dependence of \( f_2 \), which is due to physics above the scale \( \Lambda \), is \( A \) independent and hence universal among nuclei. The magnitude of distortion, \( g_2 \), which is due to physics below the scale \( \Lambda \), depends only on \( A \) and \( \Lambda \).

**Linear EMC-SRC relation in EFT:** At smaller \( Q^2 \), we can generalize the analysis in the previous section to

\(^1\) The exception of \( n = 1 \) in Eq. (7) results from the relevant contribution of the pionic operator in that case. This implies that factorization is violated only for \( x = 0 \) [40].
all higher twist terms in the operator product expansion. For a higher twist operator $O^{μ_0...μ_n}$, its indexes need not to be symmetric nor traceless, but the matching is still similar to Eq.(6). The only difference is that chiral symmetry dictates that the pion one-body operator has at least two derivatives in the chiral limit even if the operator has no index. For example, twist-three operators $G_α^2$ and $m_q π q$ are matched to $(∂^2)\,r$ and $m_q^2 \pi^2$ operators. Therefore the same power counting result holds to all orders in the twist expansion and we have

$$\sigma_{A}/A \simeq \sigma_N + g_2(A, \Lambda)\sigma_2(\Lambda), \quad (11)$$

where the $E$ (initial electron energy), $x$ and $Q^2$ dependence of $\sigma_N$ is suppressed.

With $\sigma_N$ vanishing for $x > 1$, Eqs. (2) and (11) imply

$$a_2(A, x > 1) \simeq \frac{g_2(A, \Lambda)}{g_2(2, \Lambda)}, \quad (12)$$

for both DIS and QE kinematics yielding a plateau in $a_2$ as observed experimentally at 1.5 < $x$ < 2. (Fermi motion, an $O(\epsilon)$ effect in the EFT, extends the contribution of the single nucleon PDF to $x$ slightly above 1 so the onset of the plateau is also pushed to larger $x$.) Since $a_2(A, x)$ is a ratio of physical quantities, it is independent of the EFT cutoff scale $\Lambda$. The EFT analysis also predicts that the scale dependence of $g_2(A, \Lambda)$ is independent of $A$ as suggested in [41].

From Eqs. (1) and (10), direct computation shows the that

$$\frac{dR_{EMC}(A, x)}{dx} \simeq C(x) [a_2(A) - 1] \quad (13)$$

has a linear relation with $a_2$, with $C(x) = g_2(2)[f_2^N f_2^N - f_2^N f_2^N]/[f_2^N + g_2(2)f_2^2]$ independent of $A$ and $\Lambda$ (here, $f' = df/dx$).

**SRC scaling factor:** Short-range correlations in light nuclei have been examined theoretically from several points of view [18, 42–47]. However, the focus of previous studies was on the one- or two-body distribution functions in coordinate or momentum space, which are scale and scheme dependent [48, 49].

Here we discuss their observable ratio, the SRC scaling factor, Eq. (12). We calculate $a_2$ using the GFMC method, which is one of the most accurate methods for solving the many-body Schrödinger equation for nuclei up to $A \leq 12$ [50]. The GFMC method projects out the lowest-energy state of a given Hamiltonian $H$ from a trial wave function $|Ψ_Γ\rangle$ via the many-body imaginary-time Green’s function

$$\lim_{τ \rightarrow \infty} e^{-Hτ} |Ψ_Γ\rangle \rightarrow |Ψ_0\rangle, \quad (14)$$

with $τ$ the imaginary time and $|Ψ_0\rangle$ the exact many-body ground state. A limitation of diffusion Monte Carlo methods is that they require local potentials in practice, while nuclear forces derived from chiral EFT are usually nonlocal. Recently local chiral EFT interactions have been derived up to next-to-next-leading order (N^2LO) in Weinberg power counting [51–55]. This enables us to use the GFMC method with chiral EFT as well as phenomenological interactions to study the scale and scheme independence of $a_2$.

The function $g_2(A, \Lambda)$ of Eq. (9) can be obtained from the isoscalar two-body distribution

$$\rho_{2,1}(A, r) = \frac{1}{4\pi\rho^2} \langle Ψ_0 | \sum_{i<j} δ(r - |r_i - r_j|) |Ψ_0\rangle, \quad (15)$$

as a matrix element of a local operator,

$$g_2(A, \Lambda) = \rho_{2,1}(A, r = 0)/A. \quad (16)$$

In Eq. (15), $r_i$ is the position of the $i$th nucleon and the sum runs over all pairs in the nucleus, so that the integral over $\rho_{2,1}(A, r)$ is normalized to $A(A - 1)/2$. We note that our EFT approach is not based on the experimentally observed np-pair dominance [57, 58], but instead on the fact that, of the two $S$-wave two-nucleon operators, the SU(4)-symmetric operator, $(N^4N)^2$ (counting all pairs) is dominant over the SU(4)-nonsymmetric operator (suppressed by a factor $O(1/N^2) \sim 0.1$). Thus, we include all pairs in Eq. (15). Nevertheless, at short internucleon separations, we find a predominance of np pairs over pp pairs by a factor $\sim 5–10$ for $^4$He and $^{12}$C. In our GFMC calculations, the two-body distribution function is obtained from a mixed estimate; for details see Ref. [59].

Figure 1 shows the scaled two-body distribution function $\rho_{2,1}(A, r)/A$ for $A = 2, 3, 4$ nuclei for chiral two- and

![Figure 1](image-url)
three-nucleon interactions at N²LO as well as for the phenomenological Argonne v₁₈ (AV18) two-nucleon [60] plus the UIX three-nucleon [61] potentials. The varying behavior of the two-body distributions at small separation \( r \) makes clear that \( g_2(A,A) \) depends both on the scheme and scale, where the latter is especially clear from the cutoff dependence (\( R_0 = 1.0 \) fm vs. \( R_0 = 1.2 \) fm). Analogous to PDFs, one- and two-body distribution functions depend on the renormalization scheme and scale and hence are not physical quantities [49]. However, the factorization derived in EFT shows the ratio \( a_2 \) should be scheme and scale independent.

Using Eqs. (12) and (16), \( a_2 \) is obtained from the ratio

\[
a_2 \simeq \lim_{r \to 0} \frac{2 \rho_{2,1}(A,r)}{A \rho_{2,1}(2,r)}, \tag{17}
\]

where we calculate the behavior at \( r = 0 \) by linearly extrapolating from the smallest two \( r \) values to zero separation. In EFT, locality only means a shorter distance than the resolution scale. Hence, we expect one can replace \( r \to 0 \) in Eq. (17) by smearing within \( r < R \) (a scale set by, but not necessarily equal to, \( R_0 \)), and still get the same \( a_2 \).

We see indeed this is the case in Fig. 2. The left two panels show \( a_2 \) for \(^3\)He and \(^4\)He calculated using the GFMC method with the chiral N²LO interactions and for the phenomenological AV18+UIX potentials. The right panel shows results extracted from VMC calculations [56] for the AV18+UX potentials for \(^9\)Be and \(^{12}\)C. The red and blue bands in the left panel represent a combined uncertainty estimate from the truncation of the chiral expansion [62] added in quadrature to the GFMC statistical uncertainties. The \( O(\epsilon^2) \sim 0.1 \) corrections to the operator are also contained within this conservative uncertainty estimate. We display the band obtained for the \( R_0 = 1.0 \) fm cutoff which encompasses the N²LO calculations with both cutoffs (\( R_0 = 1.0, 1.2 \) fm). For each panel, it is clear that a plateau in the ratio sets in at a value \( R \) depending on the scale and scheme. Moreover, we observe from Fig. 2 that the \( r = 0 \) value is a conservative estimate for \( a_2 \) given that the statistical uncertainties in the calculation of the two-body distributions grow as we approach zero separation. As is evident from Fig. 2, the GFMC values for \( a_2 \) are in very good agreement with experiment [16] while the preliminary VMC results are also encouraging. We summarize the extracted SRC scaling factors \( a_2 \) of the GFMC calculations and the comparison with experiment in Table I.

**Summary and outlook:** We have shown that the linear relation between the magnitude of the EMC effect at intermediate \( x \) and the SRC scaling factor \( a_2 \) is a natural consequence of scale separation and have derived this result using EFT. We have also computed \( a_2 \) for \(^3\)He

![Table I](image)

| \( A \) | \( a_2 \) (GFMC) | \( a_2 \) (VMC) | \( a_2 \) (Exp) |
|---|---|---|---|
| \(^3\)He | 2.1(2) - 2.3(3) | 2.0(4) | 2.13(4) |
| \(^4\)He | 3.8(7) - 4.2(8) | 3.4(3) | 3.60(10) |

**Fig. 2.** Ratio of the two-body distribution functions for \(^3\)He (blue) and \(^4\)He (red) to the two-body distribution function for the deuteron, \( 2\rho_{2,1}(A,r)/A\rho_{2,1}(2,r) \), as a function of relative separation \( r \). Results are shown for chiral interactions at N²LO with two different cutoffs (left panel) and for the AV18+UIX potentials (middle panel) calculated using the GFMC method. In the left panel, the darker (lighter) points are for \( R_0 = 1.0 \) fm (\( R_0 = 1.2 \) fm) and the bands represent a combined uncertainty estimate from the truncation of the chiral expansion added in quadrature to the GFMC statistical uncertainties. The right panel shows the ratio for \(^9\)Be (green) and \(^{12}\)C (black) for AV18+UX obtained from VMC results [56]. These ratios are compared to the experimental values for \( a_2 \) from Ref. [16], given by the horizontal lines.
and $^4$He using the GFMC method with both chiral and Argonne-Urbana potentials to confirm the scheme and scale independence.

GFMC calculations with chiral interactions for $^9$Be, $^{12}$C and other light nuclei will allow further tests of the EFT understanding of these phenomena. In the case of $^9$Be, it would be especially interesting to confirm whether $a_2$ is determined by local instead of global nuclear density [63]. It would also be very insightful to complete our theoretical understanding of the EMC-SRC relation by computing the $C(x)$ coefficient in Eq. (13) from lattice QCD calculations of $f_2(x)$ from the deuteron [64-66].

The EFT approach to the partonic structure of nuclei has broader applicability than to the isoscalar structure that we have discussed above. For the $F_3(x, Q^2)$ structure function that is accessible in weak-current DIS, EFT predicts a relation analogous to Eq. (10) with $F_2$ replaced by $F_1$, and $q_2$ replaced by an isospin-dependent nuclear matrix element. The resulting analogue of Eq. (13) is also expected to hold. The generalization to spin-dependent parton structure and to generalized parton distributions [67] is similarly straightforward. EFT could also shed light on whether a plateau of $\sigma_A/\sigma_{^3\text{He}}$ for $2 < x < 3$ exists, which is still inconclusive experimentally [10, 14, 68].

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SUPPLEMENTAL MATERIAL

A. DETAILS ON EFFECTIVE FIELD THEORY

In Weinberg’s power counting scheme, the typical nucleon momenta $|q|$ are counted as $O(\epsilon)$, where $\epsilon$ is the ratio of the soft to hard scale, while their energies $q^0$ are $O(\epsilon^2)$. Two-nucleon contact operators $(N^1N)^2$ are counted as $O(\epsilon^0)$, while the three-body contact operator $(N^1N)^3$ is counted as $O(\epsilon^3)$, both according to their mass dimension.

This counting can be applied to Eq. (6) of the Letter where the leading twist quark operator is matched to hadronic operators

\[
O^{\mu_0\ldots\mu_n} \rightarrow : (x^n)_N M^n v^{(\mu_0\ldots\mu_n)} (N^1N)^1 [1 + \alpha, N^1N] + \langle x^n \rangle \pi^A_i i\partial^{(\mu_0} \ldots i\partial^{\mu_n)} \pi^A + \ldots :
\]

We will focus on the tensor component with all $\mu_i = 0$. Since $\varphi^0 = 1$, the $v^{(\mu_0\ldots\mu_n)} (N^1N)^2$ is $O(\epsilon^0)$ while $v^{(\mu_0\ldots\mu_n)} (N^1N)^3$ is $O(\epsilon^3)$. The one derivative operator $\partial^{(\mu_0} v^{\mu_1} \ldots v^{\mu_n)} (N^1N)$ is $O(\epsilon^2)$, and its net effect is to replace $\varphi^0$ in Eq. (4) of the Letter from $AM$ to $M_A$.

The two derivative operator $\partial^{(\mu_0} \partial^{\mu_1} v^{\mu_2} \ldots v^{\mu_n)} (N^1N)$ is $O(\epsilon)$, and it can “spill” $q_{MN}(x)$ to $x > 1$. This is the Fermi-motion effect. Although it is higher order than the two-body operator, if $q_{\alpha}(x)$ is very small when $x$ is just above one, then the Fermi-motion effect could become larger and explain why the $d_2$ plateau only sets in at $x > 1.5$. It is important to note that in EFT off-shell effects that enter through Fermi motion can be absorbed into two-body operators through a field redefinition [69, 70]. Therefore the separation between “Fermi motion” and
“two-body effects” are meaningful only after the theory is clearly specified.

The pion one-body operator \( \pi^a i\partial_\mu \sigma^{\alpha \beta} \cdots i\partial_\nu \sigma^{\gamma \delta} \pi^a \) inserted in the one-pion-exchange diagram contributes at \( \mathcal{O}(\epsilon^{-1}) \). Since \( \langle x^n \rangle_\tau = 0 \) for even \( n \) due to charge conjugation symmetry, the \( n = 1 \) pion operator enters at \( \mathcal{O}(\epsilon^0) \), but for higher \( n \) the contributions either vanish or are higher order compared with the other operators in Eq. (6) of the Letter. This pion contribution is proportional to \( \delta(x)/x \) at \( \mathcal{O}(\epsilon^0) \). It breaks the factorization of the \( x \) and \( A \) dependence in nuclear PDFs but only at \( x = 0 \). So Eq. (8) of the Letter still holds for \( x \neq 0 \).

All the other operators in the matching are power counted and found to be higher order than \( \epsilon^0 \). In Ref. [43], a large pionic effect was found by computing the pion number in a nucleus. This result is not in contradiction to our discussion above, since the pion number operator \( \pi^a \pi^a \) is an \( \mathcal{O}(\epsilon^2) \) effect, but this operator does not appear in the matching of Eq. (18).

**B. MIXED ESTIMATES IN GFMC CALCULATIONS**

To calculate the two-body distributions \( \rho_{2,1}(r) \) defined in Eq. (15) of the Letter, we employ the GFMC method, starting from a variational Monte Carlo (VMC) calculation for the best possible trial wave function \( |\Psi_T\rangle \) that minimizes the expectation value \( \langle \Psi_T | H | \Psi_T \rangle \). The GFMC propagation, by acting with the imaginary-time propagator \( e^{-H\tau} \), evolves this trial state to the many-body ground state of the Hamiltonian \( |\Psi_T\rangle \rightarrow |\Psi_0\rangle \). Ideally, one would propagate both trial wave functions in a given expectation value to large imaginary time: \( \langle O(\tau) \rangle = \frac{\langle \Psi(\tau)|O|\Psi(\tau)\rangle}{\langle \Psi(\tau)|\Psi(\tau)\rangle} \), with \( |\Psi(\tau)\rangle = e^{-H\tau}|\Psi_T\rangle \), such that

\[
\lim_{\tau \rightarrow \infty} \langle O(\tau) \rangle \rightarrow \langle O \rangle = \frac{\langle \Psi_0|O|\Psi_0\rangle}{\langle \Psi_0|\Psi_0\rangle}.
\]

While such an expectation value is possible to compute in principle, it is difficult in practice for spin- and isospin-dependent operators and more so for momentum-dependent operators.

Instead, an approximation involving a so-called “mixed estimate” is employed:

\[
\langle O \rangle = \frac{\langle \Psi_0|O|\Psi_0\rangle}{\langle \Psi_0|\Psi_0\rangle} \approx 2\frac{\langle \Psi_0|O|\Psi_T\rangle}{\langle \Psi_0|\Psi_T\rangle} - \frac{\langle \Psi_T|O|\Psi_T\rangle}{\langle \Psi_T|\Psi_T\rangle},
\]

where the term \( \frac{\langle \Psi_0|O|\Psi_T\rangle}{\langle \Psi_0|\Psi_T\rangle} \) is the mixed estimate. Equation (20) is obtained by assuming \( |\Psi_T\rangle = |\Psi_0\rangle + \delta|\Psi\rangle \), with \( |\delta\Psi\rangle \) small, and keeping terms only to \( \mathcal{O}(\delta\Psi) \). This approximation has been tested thoroughly for many operators. For more details see Ref. [59].

So long as the extrapolation, (the difference between the mixed estimate and the variational estimate), is relatively small, then one obtains a robust value of the observable independent of the starting trial wave function.

We have checked in all cases in this Letter that the difference between the mixed estimate and the variational estimate is no more than 5% (15%) of the mixed estimate for the chiral EFT (AV18 + UIX) interactions.

**C. DETAILS ON TWO-BODY DISTRIBUTIONS**

In Fig. 2 of the Letter, we show that when \( r \) is smaller than the resolution scale \( R \), then the \( r \) dependence in the ratio drops out. This implies that the physics of \( a_2 \) is not governed by distance scales shorter than \( R \), which cancel in the ratio of two-body distributions. Instead, it is governed by physics at a scale larger than \( R \). Because the physics of the \( a_2 \) ratio is governed by a scale larger than \( R \), higher partial-wave contributions are naturally suppressed. In fig. 3, we show that the non S-wave two-body distributions \( \rho_{2}^{(ST)}(r) \) for the two-body spin/isospin (ST) = (00) (circles) and (11) (squares) channels in \(^4\)He are indeed much smaller than the leading SU(4)-symmetric S-wave combination \( \rho_{2,1}(r) \), suppressed by at least an order of magnitude. Note that we should not expect the ratios in fig. 3 to be constant in \( r \) at small \( r \) since those operators renormalize differently.
The ST distributions are defined in terms of $S$ and $T$ projectors for a pair of nucleons $|ij\rangle$:

$$P_{ij}^{(S=0)}(S=0) \equiv \frac{1}{4}(1 - \sigma_i \cdot \sigma_j), \quad (21a)$$

$$P_{ij}^{(S=1)}(S=1) \equiv \frac{1}{4}(3 + \sigma_i \cdot \sigma_j), \quad (21b)$$

and similarly for $P_{ij}^{(T)}$, with $\sigma \rightarrow \tau$. Then,

$$\rho_{2}^{(ST)}(r) \equiv \frac{1}{4\pi r^2} \left\langle \Psi_0 \left| \sum_{i<j} P_{ij}^{(S)} P_{ij}^{(T)} \delta(r - r_{ij}) \right| \Psi_0 \right\rangle, \quad (22)$$

with $r_{ij} \equiv |r_i - r_j|$. Even though our approach is based on $N_c$ counting instead of $np$-pair dominance, we also show the ratio of $np$ to $pp$ pairs in $^4$He and $^{12}$C in fig. 4. These results are extracted from VMC calculations with the AV18 + UX potentials [56]. The ratio ranges from 5–10 for $r < 1$ fm, consistent with Fig. 3 of Ref. [72], which shows that $\rho_{np}(q)/\rho_{pp}(q)$ ranges from 3–7 for $q > 1.5$ fm$^{-1}$, with $q$ the relative momentum between the two nucleons. Note that this result is with the center-of-mass momentum $Q$ of the two nucleons integrated. For $Q = 0$, the ratio is higher (see Fig. 2 of Ref. [72]).