Is it possible to measure the Lense-Thirring effect on the orbits of the planets in the gravitational field of the Sun?

L. Iorio

Dipartimento di Fisica dell’Università di Bari, Via Amendola 173, 70126, Bari, Italy
e-mail: lorenzo.iorio@libero.it

Received , 2004; accepted , 2004

Abstract. In this paper we explore a novel approach in order to try to measure the post-Newtonian $1/c^2$ Lense-Thirring secular effect induced by the gravitomagnetic field of the Sun on the planetary orbital motion. Due to the relative smallness of the solar angular momentum $J$ and the large values of the planetary semimajor axes $a$, the gravitomagnetic precessions, which affect the nodes $\Omega$ and the perihelia $\omega$ and are proportional to $J/a^3$, are of the order of $10^{-3}$ arcseconds per century only for, e.g., Mercury. This value lies just at the edge of the present-day observational sensitivity in reconstructing the planetary orbits, although the future hermean mission BepiColombo should allow to increase it. The major problems come from the main sources of systematic errors. They are the aliasing classical precessions induced by the multipolar expansion of the Sun’s gravitational potential and the classical secular $N$–body precessions which are of the same order of magnitude or much larger than the Lense-Thirring precessions of interest. This definitely rules out the possibility of analyzing only one orbital element of, e.g., Mercury. In order to circumvent these problems, we propose a suitable linear combination of the orbital residuals of the nodes of Mercury, Venus and Mars which is, by construction, independent of such classical secular precessions. A 1-sigma reasonable estimate of the obtainable accuracy yields a 36% error. Since the major role in the proposed combination is played by the Mercury’s node, it could happen that the new, more accurate ephemerides available in future thanks to the BepiColombo mission will offer an opportunity to improve the present unfavorable situation.

Key words. Relativity – Gravitation– Celestial Mechanics– Sun: fundamental parameters – Planets and satellites: general– Methods: miscellaneous

1. Introduction

1.1. The Lense-Thirring effect

According to the linearized weak-field and slow-motion approximation of the General Theory of Relativity (GTR), valid throughout the Solar System, the secular gravitomagnetic Lense-Thirring precessions on the longitude of the ascending node $\Omega$ and the argument of pericentre $\omega$ of the orbit of a test particle freely orbiting around a central mass $M$ with proper angular momentum $J$ are (Lense & Thirring 1918)

$$\dot{\Omega}_{\text{LT}} = \frac{2GJ}{c^2a^3(1-e^2)^{5/2}}, \quad \dot{\omega}_{\text{LT}} = \frac{-6GJ \cos i}{c^2a^3(1-e^2)^{5/2}}.$$ (1)

In eq. (1) $G$ is the Newtonian constant of gravitation, $c$ is the speed of light in vacuum, $a$, $e$ and $i$ are the semimajor axis, the eccentricity and the inclination, respectively, of the orbit to the reference $\{x,y\}$ plane which coincides with the equatorial plane of the central mass. Its spin $J$ is assumed to be directed along the $z$ axis.

1.2. Attempts to measure the Lense-Thirring effect in the Solar System

Up to now, there is not yet any direct observational check of this prediction of GTR which can be considered reliable and undisputable.
Indeed, the only performed attempts to detect the Lense-Thirring precessions of eq. (1) in the Solar System arena are due to I. Ciufolini and coworkers (Ciufolini et al. 1998). They analyzed the orbital data of the existing laser-ranged geodetic LAGEOS and LAGEOS II satellites in the gravitational field of the Earth over an observational time span of a few years. The claimed total accuracy would be of the order of 20-30%, but, according to other scientists, such estimates would be largely optimistic (Ries et al. 2003).

In April 2004 the extraordinarily sophisticated GP-B mission (Everitt et al. 2001) has been launched. Its goal is to measure another gravitomagnetic effect in the terrestrial gravitational field, i.e. the precession of the spins of four superconductor gyroscopes (Schiff 1960) carried onboard. The claimed accuracy would be of the order of 1% or better. The experiment should last one year.

Almost twenty years ago it was proposed to launch a third LAGEOS-like satellite—the LAGEOS III/LARES—and to analyze the time series of the sum of the residuals of the nodes of LAGEOS and LARES (Ciufolini 1986) or some other combinations of residuals of the nodes and the perigees of LARES and both the existing LAGEOS satellites (Iorio et al. 2002). The obtainable accuracy would probably be of the order of 1%. Mainly funding problems have prevented, up to now, from implementing such relatively easy and cheap mission. Recently, the possibility of measuring the Lense-Thirring precessions of eq. (1) by means of the Relativity-dedicated OPTIS spacecraft, which could be launched in the same orbital configuration of LARES, has been considered (Iorio et al., 2004).

The recently proposed LATOR (Turyshhev et al. 2004) and ASTROD (Ni et al. 2004) missions would be sensitive to the gravitomagnetic part of the bending of light rays and time delay in the gravitational field of the Sun.

Finally, it must be noted that, according to K. Nordvedt Jr. (Nordvedt 2003), the multidecadal analysis of the Moon’s motion with the Lunar Laser Ranging (LLR) technique strongly supports the existence of the gravitomagnetic force\(^1\) as predicted by GTR, although in an indirect way. “It often has been claimed that the presence of gravitomagnetism within the total gravitational interaction has not been experimentally confirmed and measured. Indeed, different experiments have been under development to explicitly observe the effects of this historically interesting prediction of general relativity. But this gravitomagnetic acceleration already plays a large role in producing the fine shape of the lunar orbit, albeit in conjunction with the rest of the total equation of motion; the precision fit of the LLR data indicates that gravitomagnetism’s presence and specific strength in the equation of motion can hardly be in doubt. [...] It would be impossible to understand this fit of the LLR data without the participation of the gravitomagnetic interaction in the underlying model, and with strength very close to that provided by general relativity\(^2\), \(\gamma = 1^\circ\)."

In regard of the possibility of measuring explicitly the Lense-Thirring precessions of eq. (1) from the analysis of the orbital motion of proof masses in the gravitational field of a real rotating astronomical mass like the Earth, it must be pointed out that the main problems come from the aliasing effects induced by a host of classical orbital perturbations, of gravitational and non-gravitational origin, which unavoidably affect the motion of the probes along with GTR. In particular, the even zonal harmonics \(J_2\) of the multipolar expansion of the gravitational potential of the central mass induce secular classical precessions which, in many cases, are larger than the gravitomagnetic ones of interest. Moreover, also the non-gravitational perturbations, to which the perigees of the LAGEOS–like satellites are particularly sensitive, are another important source of bias. As we will see, the approach proposed by Ciufolini (1996) and Iorio (Iorio 2002; Iorio & Morea 2004) in the performed or proposed tests with LAGEOS and LAGEOS II consists of suitably designing linear combinations \(\sum c_i \delta \Omega_{i,\text{obs}}^j + \sum k_j \delta \omega_{j,\text{obs}}^j\) of orbital residuals which are able to reduce the impact both of the even zonal harmonics of the gravitational field of the central mass and of the non–gravitational perturbations. In general, the coefficients \(c_i\) and \(k_j\) which weigh the various orbital elements in the combinations are a compromise between these two distinct needs.

### 1.3. Aim of the paper

In this paper we wish to investigate the possibility of extending the Ciufolini-Iorio approach in order to try to measure the Lense-Thirring effect of eq. (1) in the gravitational field of the Sun from interplanetary ranging measurements to some of the inner planets of the Solar System. The relevant parameters are listed in Table II.

At this point it is important to clarify what is the current approach in testing post-Newtonian gravity from planetary data analysis followed by, e.g., the Jet Propulsion Laboratory (JPL). In the interplay between the real data and the equations of motions, which include also the post-Newtonian accelerations expressed in terms of the various PPN parameters (Will 1993), a set of astrodynamical parameters, among which there are also \(\gamma\) and \(\beta\), are simultaneously and straightforwardly fitted and adjusted and a correlation matrix is also released. This means that the post-Newtonian equations of motion are globally tested as a whole in terms of, among other parameters, \(\gamma\) and \(\beta\); no attention is paid

---

1. According to Nordvedt (2003), the Earth–Moon range is affected by long–periodic harmonic perturbations of gravitomagnetic origin whose amplitudes are of the order of 5 m and the periods are monthly and semi-monthly. The amplitudes of the lunar motion at both these periods are determined to better than half a centimeter precision in the total orbital fit to the LLR data.

2. Here the Eddington-Robertson-Schiff PPN \(\gamma\) parameter (Will 1993) is quoted.
Table 1. Relevant astronomical and astrophysical parameters used in the text. The value for the Sun’s angular momentum $J$ has been obtained from (Pijpers 2003). The planetary data can be retrieved at http://nssdc.gsfc.nasa.gov/planetary/factsheet/.

| Symbol | Description | Value | Units |
|--------|-------------|-------|-------|
| $G$    | Newtonian gravitational constant | $6.67259 \cdot 10^{-11}$ | m$^3$ kg$^{-1}$ s$^{-2}$ |
| $c$    | speed of light in vacuum | $2.99792458 \cdot 10^8$ | m s$^{-1}$ |
| $GM$   | Sun’s GM | $1.32712440018 \cdot 10^{20}$ | m$^3$ s$^{-2}$ |
| $R$    | Sun’s equatorial radius | $6.9599 \cdot 10^8$ | m |
| $J$    | Sun’s proper angular momentum | $1.9 \cdot 10^{41}$ | kg m$^2$ s$^{-1}$ |
| A.U.   | astronomical unit | $1.49597870691 \cdot 10^{11}$ | m |
| $\alpha_{\text{Mer}}$ | Mercury’s semimajor axis | $0.38709893$ | A.U. |
| $\alpha_{\text{Ven}}$ | Venus’s semimajor axis | $0.72333199$ | A.U. |
| $\alpha_{\text{Earth}}$ | Earth’s semimajor axis | $1.0000011$ | A.U. |
| $\alpha_{\text{Mar}}$ | Mars’s semimajor axis | $1.52366231$ | A.U. |
| $e_{\text{Mer}}$ | Mercury’s eccentricity | $0.20563069$ | - |
| $e_{\text{Ven}}$ | Venus’s eccentricity | $0.00677323$ | - |
| $e_{\text{Earth}}$ | Earth’s eccentricity | $0.01671022$ | - |
| $e_{\text{Mar}}$ | Mars’s eccentricity | $0.09341233$ | - |
| $i_{\text{Mer}}$ | Mercury’s inclination to the ecliptic | $7.00487$ | deg |
| $i_{\text{Ven}}$ | Venus’s inclination to the ecliptic | $3.39471$ | deg |
| $i_{\text{Earth}}$ | Earth’s inclination to the ecliptic | $0.00005$ | deg |
| $i_{\text{Mar}}$ | Mars’s inclination to the ecliptic | $1.85061$ | deg |

Table 2. Gravitomagnetic and classical nodal precession coefficients, in $\prime\prime$ cy$^{-1}$. The coefficients $\dot{\Omega}_\ell$ are $\partial \dot{\Omega}_\ell / \partial J_\ell$ and refer to the classical precessions induced by the oblateness of the central mass. The numerical values of Table 1 have been used in eq. (1) and eq. (5) (see below). $\dot{\Omega}_{\text{class}}$ are the nominal centennial rates released at http://ssd.jpl.nasa.gov/eleml_planets.html. They mainly include the $N$–body secular precessions.

| Precessions | Mercury | Venus | Earth | Mars |
|-------------|---------|-------|-------|------|
| $\dot{\Omega}_{LT}$ | $1.008 \cdot 10^{-3}$ | $1.44 \cdot 10^{-4}$ | $5.4 \cdot 10^{-7}$ | $1.5 \cdot 10^{-9}$ |
| $\dot{\Omega}_2$ | $-1.26878626476 \cdot 10^5$ | $-1.3068273031 \cdot 10^4$ | $-4.210107706 \cdot 10^3$ | $-9.80609460 \cdot 10^2$ |
| $\dot{\Omega}_4$ | $5.2774935 \cdot 10^1$ | $1.349709$ | $2.28040 \cdot 10^{-1}$ | $2.3554 \cdot 10^{-2}$ |
| $\dot{\Omega}_{\text{class}}$ | $-4.463 \cdot 10^2$ | $-9.9689 \cdot 10^2$ | $-1.822825 \cdot 10^4$ | $-1.02019 \cdot 10^3$ |

To this or that particular feature of the post-Newtonian accelerations. This is similar to the LLR approach outlined before. On the contrary, our aim is just to try to single out one particular piece of the post-Newtonian equations of motion, i.e. the gravitomagnetic acceleration.

2. The gravitomagnetic field of the Sun

In the case of the Sun and the planets, the Lense-Thirring effect is quite small: indeed, for, e.g., the node it is $\leq 10^{-3}$ arcseconds per century ($\prime\prime$ cy$^{-1}$), as it can be inferred from Table 1 and Table 2. The point is that the angular momentum of the Sun is relatively small and the Lense-Thirring precessions fall off with the inverse of the third power of the planet’s semimajor axis. It is important to note that, if we want to consider the detection of eq. (1) as a genuine test of GTR, it is necessary that the Sun’s angular momentum $J$ is known with a high accuracy from measurements which are independent from GTR itself. This is just the case: indeed, the helioseismic data from the Global Oscillations Network Group (GONG) and also from the Solar and Heliospheric Observatory (SoHO) satellite yield measurements of $J$ which are accurate to a few percent (Pijpers 2003).

2.1. Sensitivity analysis

According to the results of Table 3 (E.M. Standish, private communication, 2004), it should be possible to extract the gravitomagnetic signature from a multi-year analysis of the planetary nodal evolution. Let us explain how the results of Table 3 have been obtained. E.M. Standish averaged, among other things, the nodal evolution of some planets over two centuries by using the DE405 ephemerides (Standish 1998) with and without the post-Newtonian accelerations. Standish included in the force models also the solar oblateness with $J_2 = 2 \cdot 10^{-7}$, so that the so obtained numerical residuals accounted for the post-Newtonian effects only; the uncertainty in the determined shift for, e.g., Mercury, was $1.82 \cdot 10^{-4}$ $\prime\prime$ cy$^{-1}$. Note that the quoted uncertainty of Table 3 do not come from direct
observational errors. They depend on the fact that in the force models used in the numerical propagation many astrodynamical parameters occur (masses of planets, asteroids, etc.); their numerical values come from multiparameter fits of real data and, consequently, are affected by observational errors. Such numerical tests say nothing about if GTR is correct or not; they just give an idea of what would be the obtainable accuracy set up by our knowledge of the Solar System arena if the Einstein theory of gravitation would be true. It must be noted that our knowledge of the orbital motion of Mercury will improve thanks to the future hermean missions Messenger (see on the WEB [http://messenger.jhuapl.edu/] and [http://discovery.nasa.gov/messenger.html]), which has been launched in the summer 2004 and whose encounter with Mercury is scheduled for 2011, and, especially, BepiColombo (see on the WEB [http://sci.esa.int/science-e/www/area/index.cfm?fareaid=30], which is scheduled to fly in 2010-2012. A complete error analysis for the range and range-rate measurements can be found in (Iess & Boscagli 2001). According to them, a two orders of magnitude improvement in the Earth-Mercury range should be possible. According to a more conservative evaluation by E.M Standish (Standish, private communication 2004), improvements in the Mercury’s orbital parameters might amount to one order of magnitude; Indeed, the current accuracy in radar ranging is hundreds of meters; The new data could be reach the tens of meters level. These figures allow to get another evaluation of the present and future accuracy in measuring the Lense-Thirring precession on the hermean node. The shift in the position of Mercury’s node due to the solar gravitomagnetic field over a time span $T$ is $\Delta r = a\Omega \Delta T T$; from the data of Table it can be retrieved that, over one century, it amounts to 282 m. By assuming $\sigma_{\Delta r_{\text{Lageos}}} \sim 100$ m we have a 35% error which would reduce to 3% for $\sigma_{\Delta r_{\text{Merc}}} \sim 10$ m.

On the other hand, there would be severe limitations to the possibility of detecting the Lense-Thirring effect by analyzing the secular evolution of only one orbital element of a given planet due to certain aliasing systematic errors.$^4$ Indeed, as in the case of the Earth-LAGEOS-LAGEOS II system, we should cope with the multipolar expansion of the central mass, i.e. the Sun in this case. Indeed, it turns out that the aliasing secular precessions induced by its quadrupole mass moment $J_2$ on the planetary nodes and the perihelia would be almost one order of magnitude larger than the Lense-Thirring precessions if we assume $J_2 = 2 \cdot 10^{-7} \pm 4 \cdot 10^{-8}$ (Pireaux & Rozelot 2003). There are still many uncertainties about the Sun’s oblateness, both from a theoretically modelling point of view and from an observational point of view (Rozelot et al. 2004). Moreover, the perihelia are also affected by another relevant post-Newtonian secular effect, i.e. the gravitoelectric Einstein pericentre advance (Einstein 1915)

$$\omega_{\text{GE}} = \frac{3nGM}{c^2a(1-e^2)}.$$  

In it $n = \sqrt{GM/a^3}$ is the Keplerian mean motion of the orbiting particle. This effect has been measured for the first time in the Solar System with the interplanetary ranging technique at a $10^{-3}$ level of relative accuracy (Shapiro et al. 1972; 1976; Shapiro 1990). The Einstein precession is almost four orders of magnitude larger than the Lense-Thirring effect, so that its mismodelled part would still be one order of magnitude larger than the Lense-Thirring effect of interest. Finally, also the classical $N$–body secular precessions are to be considered because it turns out that they are quite large (see Table 2).

### 3. The linear combination approach

A possible way out could be to extend the Ciufolini-Iorio linear combinations approach to the Sun-planets scenario in order to built up some combinations with the nodes of the inner planets which cancel out the impact of the Sun’s oblateness and of the $N$– body precessions.

---

$^3$ While the spacecraft trajectory will be determined from the range-rate data, the planet’s orbit will be retrieved from the range data (Milani et al. 2002). In particular, the determination of the planetary centre of mass is important to this goal which can be better reached by a not too elliptical spacecraft’s orbit. The relatively moderate ellipticity of the planned 400× 1500 km polar orbit of BepiColombo, contrary to the much more elliptical path of Messenger, is, then, well adequate.

$^4$ Fortunately, contrary to the Earth-LAGEOS-LAGEOS II system, in this case the non-gravitational perturbations, which are proportional to the area-to-mass ratio $S/M$ of the orbiting probe, can be safely neglected. Indeed, $S/M \propto 1/r$ where $r$ is the radius of the probe assumed spherical.
3.1. A $J_2 - (N - \text{body})$ free combination

Let us write down the expressions of the residuals of the nodes of Mercury, Venus and Mars explicitly in terms of the mismodelled secular precessions induced by the quadrupolar mass moment of the Sun, the secular $N$–body precessions and of the Lense-Thirring secular precessions, assumed as a totally unmodelled feature. It is accounted for by a scaling parameter $\mu_{LT}$ which is zero in Newtonian mechanics and 1 in GTR

$$
\begin{align*}
\delta\hat{\Omega}_{\text{obs}}^{\text{Mercury}} &= \hat{\Omega}_{2}^{\text{obs}} \delta J_2 + \hat{\Omega}_{\text{class}}^{\text{Mercury}} + \hat{\Omega}_{LT}^{\text{Mercury}} \mu_{LT} + \Delta^{\text{Mercury}}, \\
\delta\hat{\Omega}_{\text{obs}}^{\text{Venus}} &= \hat{\Omega}_{2}^{\text{obs}} \delta J_2 + \hat{\Omega}_{\text{class}}^{\text{Venus}} + \hat{\Omega}_{LT}^{\text{Venus}} \mu_{LT} + \Delta^{\text{Venus}}, \\
\delta\hat{\Omega}_{\text{obs}}^{\text{Mars}} &= \hat{\Omega}_{2}^{\text{obs}} \delta J_2 + \hat{\Omega}_{\text{class}}^{\text{Mars}} + \hat{\Omega}_{LT}^{\text{Mars}} \mu_{LT} + \Delta^{\text{Mars}}.
\end{align*}
$$

The coefficients $\hat{\Omega}_{\ell}$ are defined as

$$
\hat{\Omega}_{\ell} = \frac{\partial \hat{\Omega}_{\text{class}}^{\ell}}{\partial J_\ell},
$$

where $\hat{\Omega}_{\text{class}}^{\ell}$ represent the classical secular precessions induced by the oblatteness of the central mass. The coefficients $\hat{\Omega}_{\ell}$ have been explicitly worked out from $\ell = 2$ to $\ell = 20$ (Iorio 2003); it turns out that they are functions of the semimajor axis $a$, the inclination $i$ and the eccentricity $e$ of the considered planet: $\hat{\Omega}_{\ell} = \hat{\Omega}_{\ell}(a, e, i; GM)$. For the first two even zonal harmonics we have

$$
\begin{align*}
\hat{\Omega}_2 &= -\frac{3}{2} \mu \left( \frac{R}{a} \right)^2 \cos i \left( \frac{1 + 2i^2}{(1 - e^2)} \right), \\
\hat{\Omega}_4 &= \hat{\Omega}_2 \left[ \frac{3}{8} \left( \frac{R}{a} \right)^2 \left( \frac{1 + 2i^2}{(1 - e^2)} \right)^2 \left( 7 \sin^2 i - 4 \right) \right].
\end{align*}
$$

$R$ is the Sun’s equatorial radius. The quantities $\Delta$ in eq. (3) refer to the other unmodelled or mismodelled effect which affect the temporal evolution of the nodes of the considered planets. In the present case we would mainly be represented by the precessions induced by the octupolar mass moment of the Sun (Rozelot et al., 2004). From the results of Table 2 and from the evaluations of (Rozelot et al. 2004) according to which the possible magnitude of $J_4$ would span the range $10^{-7} - 10^{-9}$, the secular precession induced by the octupolar mass moment of the Sun is negligible with respect the Lense-Thirring rates. These facts lead us to design a three-node combination which cancels out just the effects of $J_2$ and of the classical $N$–body precessions while is affected by the residual effect of $J_4$.

Indeed, if we solve eq. (3) for the Lense-Thirring parameter $\mu_{LT}$ it is possible to obtain

$$
\begin{align*}
\delta\hat{\Omega}_{\text{obs}}^{\text{Mercury}} + k_1 \delta\hat{\Omega}_{\text{obs}}^{\text{Venus}} + k_2 \delta\hat{\Omega}_{\text{obs}}^{\text{Mars}} &\sim X_{LT} \mu_{LT}, \\
k_1 &= \frac{\Omega_{\text{obs}}^{\text{Mars}} \Omega_{\text{obs}}^{\text{Mercury}} - \Omega_{\text{obs}}^{\text{Mercury}} \Omega_{\text{obs}}^{\text{Mars}}}{\Omega_{2}^{\text{Mercury}} \Omega_{\text{class}}^{\text{Mercury}} - \Omega_{2}^{\text{Mars}} \Omega_{\text{class}}^{\text{Mars}}} = -1.0441702 \cdot 10^3, \\
k_2 &= \frac{\Omega_{\text{obs}}^{\text{Mercury}} \Omega_{\text{obs}}^{\text{Venus}} - \Omega_{\text{obs}}^{\text{Venus}} \Omega_{\text{obs}}^{\text{Mercury}}}{\Omega_{2}^{\text{Mercury}} \Omega_{\text{class}}^{\text{Mercury}} - \Omega_{2}^{\text{Venus}} \Omega_{\text{class}}^{\text{Venus}}} = 9.765758, \\
X_{LT} &= \hat{\Omega}_{LT}^{\text{Mercury}} + k_1 \hat{\Omega}_{LT}^{\text{Venus}} + k_2 \hat{\Omega}_{LT}^{\text{Mars}} = -3.51 \cdot 10^{-4} \, \text{"} \, \text{cy}^{-1},
\end{align*}
$$

where the numerical values of the coefficients $k_1$ and $k_2$ and the slope $X_{LT}$ of the gravitomagnetic trend come from the values of Table 2. The meaning of eq. (6) is the following. Let us construct the time series of the residuals of the nodes of Mercury, Venus and Mars by using real observational data and the full dynamical models in which GTR is purposely set equal to zero, e.g. by using a very large value of $c$. We expect that, over a multidecadal observational time span, the so combined residuals will fully show the GTR signature and partly the mismodelled $N$–body effect in terms of a linear trend. The measured slope, divided by $X_{LT}$, yields $\mu_{LT}$ which should be equal to one if GTR was correct and if the bias from the residual $N$–body effect was sufficiently small. The systematic error affecting eq. (6) is totally negligible because it would be due only to the higher degree multipole mass moments of the Sun. The obtainable 1-sigma observational error, according to the results of Table 3 would amount to 36%. Note that this evaluation agrees with that presented in Section 2.4.5

---

5 It can be shown that it can be expressed in terms of the PPN parameter $\gamma$.

6 Possible aliasing time-dependent $N$–body residual effects with the periodicities of the outer planets, mainly Jupiter, should average out over a sufficiently long multidecadal time span. However, it would be possible to fit and remove them from the time series.
4. Conclusions

In this paper we have explored the possibility of measuring the post-Newtonian Lense-Thirring effect induced by the solar gravitomagnetic field on the motion of some of the Solar System planets. The magnitude of the gravitomagnetic precessions is very small amounting to $10^{-3} r_c y^{-1}$ for Mercury. The main systematic errors which would mask the relativistic effect of interest would be the quite larger secular precessions induced by the post-Newtonian gravitoelectric part of the Sun’s gravitational field, by the Sun’s oblateness and by the $N-$body interactions. By using a suitably designed linear combination of the orbital residuals of the nodes of Mercury, Venus and Mars it would be possible to cancel out the corrupting impact of the first solar even zonal harmonic plus the $N-$body classical secular precessions. Moreover, the proposed measurement would be aliased neither by the post-Newtonian gravitoelectric field because it affects only the perihelia and the mean anomalies. The obtainable observable accuracy should be 36% (1-sigma) for the proposed $J_2 - (N-$body) free combination. It would be a somewhat modest result for a reliable test of GTR. However, we note that should new, more accurate ephemerides for Mercury be available as a by-product from the Messenger and, especially, BepiColombo missions, the error’s evaluation presented here could become more favorable.

Acknowledgements. L. Iorio is grateful to L. Guerriero for his support while at Bari. Special thanks to E. M. Standish (JPL) for his help and useful discussions and clarifications, and to W.-T. Ni for the updated reference about ASTROD.

References

Ciufolini, I. 1986, Phys. Rev. Lett. 56, 278
Ciufolini, I. 1996, Il Nuovo Cimento A, 109, 1709
Ciufolini, I., Pavlis, E. C., Chieppa, F., Fernandes-Vieira, E., & Pérez-Mercader, J. 1998, Science, 279, 2100
Einstein, A. 1915, Preuss. Akad. Wiss. Berlin Sitzber., 47, 831
Everitt, C. F. W., Buchman, S., DeBra, D. B., and other members of the Gravity Probe B Team 2001, Gravity Probe B: Countdown to Launch. In Gyros, Clocks, and Interferometers: Testing Relativistic Gravity in Space, ed. C. Lämmerzahl, C. W. F. Everitt, & F.W. Hehl, (Springer–Verlag, Berlin), 52
Iess, L. & Boscagli, G. 2001, Plan. Space Sci., 49, 1597
Iorio, L. 2002, Class. Quantum Grav. 19, 5473
Iorio, L., Lucchesi, D.M., & Ciufolini, I. 2002, Class. Quantum Grav. 19, 4311
Iorio, L. 2003, Celest. Mech & Dyn. Astron. 86, 277
Iorio, L., & Morea, A. 2004, Gen. Rel. Gravit. 36, 1321
Iorio, L., Ciufolini, I., Pavlis, E.C., Schiller, S., Dittus, H., & Lämmerzahl, C. 2004, Class. Quantum Grav. 21, 2139
Lense, J., & Thirring, H. 1918, Phys. Z. 19, 156 translated by Mashhoon, B., Hehl, F. W., & Theiss, D. S. 1984, Gen. Rel. Grav., 16, 711
Milani, A., Vokrouhlický, D., Villani, D., Bonanno, C. & Rossi, A. 2002, Phys. Rev. D., 66, 082001
Ni, W.-T., Shiomi, S., & Liao, A.-C. 2004, Class. Quantum Grav. 21, S641
Nordvedt, K. 2003, Some considerations on the varieties of frame dragging. In Nonlinear Gravitodynamics. The Lense–Thirring Effect, ed. R. Ruffini, & C. Sigismondi, (World Scientific, Singapore), 35
Pijpers, F. P. 2003, A&A. 402, 683
Pireaux, S., & Rozelot, J.-P. 2003, Ap&SS, 284, 1159
Ries, J. C., Eanes, R. J., & Tapley, B. D. 2003, Lense-Thirring Precession Determination from Laser Ranging to Artificial Satellites. In Nonlinear Gravitodynamics. The Lense–Thirring Effect, ed. R. Ruffini, & C. Sigismondi, (World Scientific, Singapore), 201
Rozelot, J.-P., Pireaux, S., Lefebvre, S., & Corbard, T. 2004, preprint astro-ph/0403382
Schiff, L.I. 1960 Am. J. Phys., 28, 340
Shapiro, I., Pettengill, G. H., Ash, M. E., Ingalls, R. P., Campbell, D. B., & Dyce, R. B. 1972, Phys Rev. Lett., 28, 1594
Shapiro, I., Counselman, C., & King, R. 1976, Phys Rev. Lett., 36, 555
Shapiro, I. 1990, Solar system tests of general relativity: recent results and present plans. In Proceedings of the 12th International Conference on General Relativity and Gravitation, ed. N. Ashby, D. Bartlett, & W. Wyss, (Cambridge University Press, Cambridge), 313
Standish, E. M. 1998, JPL planetary and lunar ephemerides DE405/LE405, Interoffice Memorandum 312.F-98-048, 1
Turyshhev, S. G., Shao, M., & Nordvedt, K. 2004, Class. Quantum Grav. 21, 2773
Will, C. M. 1993, Theory and Experiment in Gravitational Physics, 2nd edition (Cambridge University Press, Cambridge)