Precision measurements of $W$ properties at LHC

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ABSTRACT: Exploiting the large number of collected events and the symmetries in the $W$ production and decay at LHC, it is possible to measure the $W$ double differential cross section of transverse momentum and rapidity and the angular coefficients. With the proposed method a determination of the integrated spectrum of transverse momentum of $W$ bosons is possible with an unprecedented granularity and increased precision with respect to the state-of-the-art generators. This enables the possibility to perform a theory-agnostic measurement of the $W$ mass with competitive uncertainty by constraining the $W$ production directly on data. This paper shows preliminary results derived on a subset of events collected by CMS during Run 2, for an integrated luminosity of 36 fb$^{-1}$. 

$^1$Paper prepared for a special issue of JHEP celebrating its 25th anniversary
1 Precision measurements as a tool for discovery

The Standard Model of Particle Physics has an outstanding success in correctly describing physics observed at colliders. However, it can not be the candidate for the ultimate theory: experimental evidence of Dark Matter and neutrino oscillations are examples of phenomena not finding their theoretical explanation within the Standard Model. Historically, electroweak fits have played a major role in predicting observables before their measurement, and even more after the discovery of the Higgs boson the program of delivering more and more precise experimental determination of key quantities has become imperative for testing the Standard Model.

The outstanding luminosity of the LHC has opened a new era of precision measurements in the electroweak sector with an unprecedented number of events of $W$ and $Z$ bosons having been produced. Although the clean environment of past $e^+e^-$ facilities such as LEP and SLC has been the most favourable for precision measurements, they have been eventually limited by statistics. On the other hand, with its many interesting events LHC is now reaching the level of precision set by $e^+e^-$ machines in the determination of key observables of the electroweak sector, like $\sin \theta_W$ [1].

After the discovery of the Higgs boson [2, 3] and the measurement of its mass [4] the electroweak fit predicts the mass of the $W$ boson to be 80350 MeV with the outstanding precision of 8 MeV [5]. The uncertainty of the world average of the experimental determinations has been 13 MeV, with a $2\sigma$ level-agreement with the predicted value, until March 2022, when the CDF Collaboration has announced a new measurement of the $W$ mass of 80433.5 ± 9.4 MeV [6]. Being 83 ± 12 MeV above the Standard Model prediction, this represents the first large upset to the Standard Model [7]. It is of paramount importance to confirm or rule out the observation by CDF with a new measurement of the $W$ mass with similar precision, using the unprecedented statistics collected by the LHC experiments.

Figure 1 shows the measurements of the mass of the $W$ boson. The $W$ mass has been measured by each LEP experiment with a precision of about 50 MeV in the hadronic or
semileptonic final state of the two $W$’s produced in pair in the $e^+e^-$ collision. This level of uncertainty has been possible given the precise knowledge of the initial state.

This technique can not be used at hadron colliders and only events with a $W$ decaying leptonically can be triggered and used for the measurement of its mass. This brings a major drawback: since an undetectable neutrino is present in the final state, it is not possible to reconstruct the invariant mass of each event. Then, a proxy carrying information about the $W$ mass has to be defined. The transverse momentum $p_T$ of the charged lepton in the final state shows a jacobian peak at the $W$ mass value. However, this observable is not a Lorentz invariant and its assessment depends on the underlying kinematics of the $W$ boson, which is not experimentally accessible and has to be inferred from external inputs. The limited knowledge of these inputs is the limiting factor in measuring the $W$ mass at hadron colliders and accounts for the largest contribution in the systematic uncertainty of the measurement.

![Figure 1: Measured value of $m_W$, from the ALEPH [8], DELPHI [9], L3 [10], OPAL [11], D0 [12], ATLAS [13], LHCb [14] and CDF [6] experiments. The current prediction of $m_W$ from the global electroweak fit is also included.](image)

2 An innovative method to determine $W$ properties at LHC

Exploring the symmetry of the production of $W$ at LHC and the asymmetry of their decay, it is possible to devise a new method of analysis to mitigate the large systematic uncertainties stemming from the dependence of the unknown $W$ production spectra [16].

When a $W$ is produced with small transverse momentum, its principal production mechanism involves the leading order parton subprocesses $u\bar{d} \to W^+$ and $\bar{u}d \to W^-$. Due to the valence quark content of the proton, it is likely that the $W$ will be moving in the direction of the initial-state quark, as opposed to antiquark. Since the electroweak charged current totally violates parity, the quark must be left-handed and the antiquark right-handed. A simple argument of angular momentum conservation [15] states that the
direction of the spin of the $W$ must preferentially be opposite to its direction of motion. Therefore, $W$ have preferentially negative helicity, with a small dilution occurring in case the antiquark is carrying a higher fraction of the proton momentum than the quark. This argument includes cases when a $W$ is produced by a sea quark-antiquark pair. In this case the entity of the dilution will be higher and more $W$ will be produced with the spin of the $W$ along its direction of motion.

Parity is totally violated in $W$ decays and $\mu^+$ from the $W^+$ with negative helicity are preferentially emitted backward with respect to the $W^+$ direction of motion, with a rapidity that is 0.5 smaller than the rapidity of the $W^+$. Conversely, $\mu^-$ from $W^-$ with negative helicity are preferentially emitted forward, with a rapidity that is 0.5 larger than the rapidity of the $W^-$. Therefore, these plots, made using observables that can be directly measured, can be used to discriminate the $W$ polarisation, which is a quantity not directly detectable.

Going forward, if we reproduce the plots of Figure 2 in a small bin of rapidity $y$ of
the $W$, which is feasible using a simulation, we can exploit this correlation to discriminate $y$ in addition to $W$ polarisation. A practical recipe for this measurement is thoroughly discussed in [17], where it is argued that after taking into account the systematic effects, the statistical precision of this analysis has a substantially smaller uncertainty than the prediction from Parton Distribution Functions of the rapidity spectrum and polarisation of $W$.

Following these prescriptions, CMS has delivered a measurement of the $W$ rapidity distribution for left-handed and right-handed helicity states[18]. It has been performed on CMS data collected during 2016, equivalent to an integrated luminosity of $35.9\,\text{fb}^{-1}$, in $W \rightarrow \mu\nu$ and $W \rightarrow e\nu$ channels. Figure 9 of [18] shows indeed that the final experimental uncertainty on the fitted distributions is smaller than the prediction from the simulation.

Moving on from the simplified case in which $W$’s are produced with small transverse momentum $q_T$, we discuss the general case of finite $W$ transverse momentum. In this situation, further polarisation states are possible for $W$’s decaying leptonically: all the nine elements of the full $W$ spin matrix. It is possible to define a parametrisation of the $W$ production cross section allowing to disentangle the production of the $W$ from its decay, in terms of the spherical harmonics [19]:

$$
\frac{d\sigma}{dq_T^2 dy d\cos\theta^* d\phi^*} = \frac{3}{16\pi} \frac{d\sigma^{U+L}}{dq_T^2 dy} \left[ (1 + \cos^2\theta^*) + \sum_{i=0}^{7} A_i(q_T, y) P_i(\cos\theta^*, \phi^*) \right],
$$

(2.1)

$\cos\theta^*$ and $\phi^*$ being the polar and azimuthal angles of the decay of the muon in a $W$ rest frame. The angular coefficients $A_i$ are functions of the $W$ transverse momentum and rapidity, $q_T$ and $y$, and together with the unpolarised cross section $\sigma^{U+L}$, encode the $W$ production. On the other hand, the functions $P_i$, spherical harmonics of degree 0, 1, 2 only depend on muon variables and therefore carry information on the kinematics of the detectable muon.

It comes naturally to apply the same idea discussed in [17] to the general case in which $W$’s are produced with longitudinal and transverse momentum, using the Collins-Soper [20] reference frame for the calculations, which is convenient and widely used in literature.

For fixed values of $q_T$ and $y$ and for an uniform distribution in the $\cos\theta^*$ and $\phi^*$ plane, the variables in the laboratory frame $p_t, \eta$ and $\Phi = \phi^\mu - \phi^W$ are correlated through the fixed value $m_W/2$ of the muon momentum in the $W$ rest frame. They describe the surface shown in Figure 3 for $y = 0$ and $q_T = 10\,\text{GeV}$. The figure shows also the $\eta, p_T$ correlation plot. The muon momentum $p_T$ depends on $q_T$ and on the decay angle $\Phi = \phi^\mu - \phi^W$, which is not experimentally detectable and that is integrated upon. The correlation plot $\eta, p_T$ encodes the values of $y$ and $q_T$. The population is maximal at the edges of the $\eta, p_T$ contour due to the jacobian of the integration in $\Phi$. Moreover two events with same value of $\cos\theta^*$ but opposite $\Phi$ will be mapped in the same point of the $\eta, p_T$ plane. This means that all the harmonics in Eq. 2.1 containing a function $\sin\phi$ will have a null density on the $\eta, p_T$ plane. This is the case for the harmonics related to $A_3, A_6, A_7$.

In Figure 4 we show the correlation plot for $q_T = 10\,\text{GeV}$ and a set of $y$ values: $(0, 1, 2)$. As expected the plot shifts from $\eta = 0$ as $y$ increases. More interesting is Figure 5 that
Figure 3: Illustration showing the effect of the projection of the angle $\Phi = \phi^\mu - \phi^W$ in the $\eta$ and $p_T$ plane. In this example $q_T = 10$ GeV and $y = 0$ shows what happens if we keep a constant $y = 0$ and plot a set of $q_T$ values ($1, 5, 10$) GeV. In this case we can observe how the width of the plot becomes thicker as $q_T$ grows.

Figure 4: Correlation plots in $\eta$ and $p_T$ plane. 4a: $y = 0$ and $q_T = 10$ GeV. 4b: $y = 1$ and $q_T = 10$ GeV. 4c: $y = 2$ and $q_T = 10$ GeV.

Figure 5: Correlation plots in $\eta$ and $p_T$ plane. 5a: $y = 0$ and $q_T = 1$ GeV. 5b: $y = 0$ and $q_T = 5$ GeV. 5c: $y = 0$ and $q_T = 10$ GeV.

If we go further in this analysis we can add a polarisation component to the $W$ using Eq. 2.1, turning on one $A_i$ at a time and setting it to 1 while keeping all the others at 0. This will change the distribution in the Collins-Soper plane and the population in the $\eta$, $p_T$ plot, but not its contour. In this way we can produce for each $y$ and $q_T$ a peculiar set
of plots reflecting the characteristics of the muon decay for each spherical harmonic. They are shown in Figure 6 for the coefficients $A_0$, $A_1$, $A_2$, $A_3$, $A_4$. If for example we pick $A_4$, in Figure 6e, this depends on the $\cos \theta^*$ variable alone and encodes the complete violation of parity in $W$ decays. This can be observed in the plot as an accumulation of events in the right part of $\eta, p_T$ contour. In the other plots similar features can be observed, which can allow discriminating $q_T, y$ and polarisation of the $W$ if a set of templates is produced and a fit similar to the one shown in [17] is performed.

To sum up, carrying out the complete calculation for the correlation in the $\eta, p_T$ plane as a function of the kinematics of a $W$ and the muon from its decay in the Collins-Soper frame, we have extended the method introduced in [17] to unfold not only the rapidity of the $W$ and its polarisation, but also its transverse momentum and the angular coefficients. This procedure has been applied to data collected by the CMS experiment to carry out a measurement of the distributions of $W$ rapidity and transverse momentum and the angular coefficients in [16].

![Figure 6](image-url)

**Figure 6**: Correlation plots in $\eta$ and $p_T$ plane, where a polarisation component to the $W$ using Eq. 2.1, turning on one $A_i$ at a time and setting it to 1 while keeping all the others at 0. 6a shows $A_0$, 6b shows $A_1$, 6c shows $A_2$, 6d shows $A_3$, 6e shows $A_4$. $y = 0, q_T = 10$ GeV.

### 3 A preliminary measurement of $W$ production at CMS

The analysis illustrated in the previous section is now applied to data collected by the CMS experiment in 2016 at $\sqrt{s} = 13$ TeV, for an integrated luminosity of 35.9 fb$^{-1}$. The aim of this measurement consists in the experimental assessment of the double differential cross section of $W q_T, y$ and the angular coefficients as a function of $q_T$ and $y$.

First, $W$ events are selected from data and are expected to be affected by a number of backgrounds whose entity can be reduced with the selection, but not completely removed,
since their kinematics partially overlaps with the signal spectra. The main reducible backgrounds in this analysis are muons from semileptonic decays of heavy flavour mesons and $Z$ decays to two muons, in case one muon escapes detection. Muons from $W$ boson decays to $\tau$ are also considered a source of background. Minor reducible backgrounds, contributing to less than the percent level to the overall yield include muons from $t\bar{t}$ and single top, and diboson events: $ZZ$, $WW$ and $WZ$. The backgrounds of electroweak origin are well modelled by the Monte Carlo simulations and can therefore be directly subtracted using this prediction. On the other hand, the contribution from non-prompt muons from QCD multijet events is not well modelled in the simulation and is estimated from data using the fake rate method.

$W$ events are selected in data and simulated samples requiring exactly one identified muon in the event, with $25 < p_T < 55$ GeV, $|\eta| < 2.4$ and with small impact parameters with respect to the beamspot $d_{xy} < 0.05$ cm. The latter is used to suppress the contributions from QCD and top, where muons decaying from $b$ quarks can have large impact parameters. In addition, the identified muon must have fired the trigger. In order to have fully populated distributions, the signal templates are limited to the region $q_T < 60$ GeV and $|y| < 2.4$, where the acceptance is higher than 20%. The events in the region of phase space $q_T > 60$ GeV or $|y| > 2.4$ are gathered in a single template that is considered a source of background. This process will be referred as low acceptance in the text.

Figure 7 shows a collection of the muon $p_T$ and $\eta$ templates for the aforementioned background sources.

| (a) | (b) | (c) |
|-----|-----|-----|
| (d) | (e) | (f) |

**Figure 7**: Templates for the background processes in the $W^+$ analysis. Figure 7a shows the template for the low acceptance events. Figure 7b, 7c, 7d, 7e, 7f show respectively the templates for $W$ to $\tau$, Drell-Yan, top, diboson and QCD processes. The z axis scale is different in each plot and shows the equivalent number of events.
The main step of this analysis is the construction of the signal templates, distributions of $p_T$ and $\eta$ of the muons used to unfold the cross sections of the $W$ boson production. This is done using a generator of $W$ events decaying to muons. Distributions of muon $p_T$ and $\eta$ are generated in bins of $W$ $q_T$ and $y$, for spherical harmonics according to Eq. 2.1. The angular coefficient decomposition holds in the hypothesis of the vectorial nature of a $W$ boson decaying to two fermions, and while constructing the templates it is crucial to restore this regime. This allows to decouple the $W$ production from its decay products so that the information on $W$ production is encoded in the value of the angular coefficients and the unpolarised cross section in each bin of $q_T$ and $y$. However, the physics parametrisation of Eq. 2.1 has the disadvantage of not being linear in the parameters of interest, since the angular coefficients show the unpolarised cross section in the denominator. For this reason, the decomposition in helicity cross sections presented in [19] is preferred to build the signal templates.

This procedure is independent on the details of the generator used in the simulation, therefore it is possible to unfold the double differential cross section of $q_T$ and $y$ and the helicity cross sections fitting the sum of signal and background templates to the distribution of muon $p_T$ and $\eta$ measured in data. The results are then presented in terms of the unfolded cross sections and angular coefficients. Since there is a one to one correspondence between helicity cross sections and angular coefficients, the latter are computed after the fit using the full covariance matrix. Figure 8 shows a collection of signal templates obtained with this procedure.

In this process, it is essential that the propagation of the generated muon inside the simulation of the CMS detector is accurate enough, so that theoretical and experimental effects can be decoupled using well calibrated templates. The fit is performed in the hypothesis that all differences between data and simulation is due to $W$ production, and this is ensured applying systematic uncertainty of experimental origin to signal and background templates. The dominant contribution to the analysis is due to the propagated uncertainty of the scale factors for the efficiencies. In addition, some systematic uncertainties of theoretical origin are applied to background templates to take into account the unknown $W$ spectra outside the fit range. Figure 9 shows the agreement of the distribution of $\eta$ and $p_T$ in data with the simulated samples for signal and backgrounds. Before the fit, the distributions of data and simulated samples agree at the level of few percent.

These figures show the unrolled measured $p_T, \eta$ distributions where each peak corresponds to the $p_T$ distribution in a given $\eta$ bin, $\eta$ ranging from -2.4 to +2.4. The signal simulation is shown in red. The light blue shows the low acceptance events, i.e. muons typically from $W$ at large $|y|$, which fall inside the fit acceptance. They are more copious for $W^+$ than for $W^-$, as discussed in section 1. The dark blue shows the QCD background.

The information synthetised in the signal and background templates is condensed in a global fit aimed at unfolding the double differential cross section of $q_T$ and $y$ and the angular coefficients from the observed distribution of muon $p_T$ and $\eta$. The fit is performed using an extended binned maximum likelihood fit, independently for each $W$ boson charge. The observed muon $p_T$ and $\eta$ distribution is fitted to the sum of the templates of the signal and background processes. The fit is allowed to modify the freely-floating signal strength
Figure 8: Representative examples of the signal templates for the six helicity cross sections (each row) in different bins of $y$ and $q_T$ (each column), in $W^+$. The $z$ axis scale is different in each plot and shows the equivalent number of events.
modifiers and to unfold the related cross section. Nuisance parameters associated to the systematic uncertainties contribute to the minimisation and can act on the freely-floating signal strength modifiers, on the fixed background strength modifiers, and to the unfolded cross sections. Details of the technical implementation of the fit are discussed thoroughly in [16].

The adopted procedure allows to obtain the first measurement of the double differential unpolarised cross section of the $W$ boson. This can be integrated in rapidity, in the region allowed in the fit range ($|y| < 2.4$) or in $q_T$, in the fiducial region $q_T < 60$ GeV. These integrated measurements are of notable interest since they can be performed with an uncertainty which is competitive with the state-of-the-art calculations. While the first has been recently published by CMS [18], the second is a new result which is important per se, and can also be used to improve the measurement of the $W$ mass.

This procedure also allows measuring for the first time the angular coefficients of the $W$ as a function of $q_T$ and $y$. In this case the precision is smaller and the purpose of including them in the fit is limited to the constraint in situ of the degrees of freedom needed for a theory-agnostic $W$ mass measurement. Among the angular coefficients, $A_4$ plays a major role since it is the only one manifesting at leading order and encodes the majority of the dependence of the $W$ production from the Parton Distribution Functions.

The fit procedure has been validated using a $W$ sample produced with a different $q_T$ and $y$ spectrum. In this way, the unbiasedness of the fit has been checked while factorising the experimental effects on the templates from the theoretical ones. The robustness of the fit has been tested also randomising this modified dataset with poissonian fluctuations on many toy experiments.

**Figure 9**: Agreement before the fit of the distribution of $\eta$ and $p_T$ in data with the simulated samples and the fake rate, for the unrolled distribution in $W^+$ and $W^-$, Figure 9a and Figure 9b, respectively.
Finally, the fit has been run on CMS data. For each fit configuration, $W^+$ and $W^-$, the results are presented in [16] in terms of unfolded cross sections and angular coefficients.

In Figure 10a the double differential unpolarised cross section is shown as a function of $W \, q_T$ and $y$, for $W^+$, unrolled in one dimension. The related integrated plot in $q_T$ as a function of $y$ is shown in 10c and the integrated plot in $y$ as a function of $q_T$ is shown in 10b. Figure 10d, 10e and 10f show the same results for $W^-$. We observe that the results for the unpolarised cross section integrated in $q_T$ as a function of $y$, shown in Figure 10c for $W^+$ and 10f for $W^-$ are fully consistent with the previously published results by CMS in [18], while the plots integrated in $y$ as a function of $q_T$, shown in Figure 10b for $W^+$ and 10e for $W^-$, represent the first measurements of the $W \, q_T$ at a small granularity. Both $W^+$ and $W^-$ show differences with respect to the prediction by POWHEG [21] MiNNLO [22, 23] of the order of 20%.

![Figure 10](image)

**Figure 10**: Fitted double differential cross section in $W^+$ and $W^-$ $q_T$ and $y$ (resp. Figure 10a and Figure 10d), integrated in $y$ (resp. Figure 10b and Figure 10e) and in $q_T$ (resp. Figure 10c and Figure 10f). The black dots represent the fit results on data, while the green bands are representative of the theoretical uncertainties of the simulation. The lower panel shows the ratio between the fitted value and the prediction encoded in the simulation.
4 A statistically dominated uncertainty on W mass

The experimental setup of this analysis enables for a simultaneous measurement of the W differential cross sections and mass. The fit is run on data to assess the uncertainty on $m_W$. The central value is not disclosed since it is blinded by the CMS Collaboration and moreover the muon momentum scale corrections are not yet applied to the samples used for this assessment.

The final uncertainty for $W^+$ is 11.2 MeV and for $W^-$ it is 16.1 MeV, where the large component is due to the statistical uncertainty. This method allows trading the systematic uncertainties originating from the theoretical model in the simulation for statistical uncertainties, which can be reduced increasing the data sample and the size of the simulation samples used to build the templates. The largest systematic component is the statistical error of the efficiency scale factors amounting to about 4.3 MeV in $W^+$ and 3.6 MeV in $W^-$. Combining the two measurements from $W^+$ and $W^-$ and assuming the statistical uncertainties to be fully uncorrelated, the uncertainty amounts to 9 MeV.

This assessment does not include the uncertainties on the muon momentum scale and on the muon final state radiation.

5 Conclusions

The analysis method described in [16] and summarized briefly in this paper allows measuring the double differential cross section and angular coefficients of the W production at LHC with sufficient precision for a theory-agnostic measurement of the W mass. A measurement of the W mass using this method is limited by statistics and by experimental systematics on efficiencies and momentum scale. Therefore, performing such a measurement on the full dataset collected by LHC during Run 2 would allow shedding light on the discrepancy between theory and measurements recently observed by CDF.

Acknowledgments

This work has been partially supported by MIUR, PRIN 2017, through the PRIN 2017F28R78 Project. The authors are grateful to the members of the CMS W mass analysis group for discussions and suggestions, especially Josh Bendavid, Valerio Bertacchi, Lorenzo Bianchini and Suvankar R. Chowdhury who have helped in different phases of the development of the analysis.

References

[1] A. M. Sirunyan et al. [CMS], “Measurement of the weak mixing angle using the forward-backward asymmetry of Drell-Yan events in pp collisions at 8 TeV,” Eur. Phys. J. C 78 (2018) no.9, 701 doi:10.1140/epjc/s10052-018-6148-7 [arXiv:1806.00863 [hep-ex]].

[2] G. Aad et al. [ATLAS], “Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC,” Phys. Lett. B 716 (2012), 1-29 doi:10.1016/j.physletb.2012.08.020 [arXiv:1207.7214 [hep-ex]].
[3] S. Chatrchyan et al. [CMS], “Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC,” Phys. Lett. B 716 (2012), 30-61 doi:10.1016/j.physletb.2012.08.021 [arXiv:1207.7235 [hep-ex]].

[4] G. Aad et al. [ATLAS and CMS], “Combined Measurement of the Higgs Boson Mass in pp Collisions at $\sqrt{s} = 7$ and 8 TeV with the ATLAS and CMS Experiments,” Phys. Rev. Lett. 114 (2015), 191803 doi:10.1103/PhysRevLett.114.191803 [arXiv:1503.07589 [hep-ex]].

[5] J. de Blas, M. Pierini, L. Reina and L. Silvestrini, “Impact of the recent measurements of the top-quark and W-boson masses on electroweak precision fits,” [arXiv:2204.04204 [hep-ph]].

[6] T. Aaltonen et al. [CDF], “High-precision measurement of the W boson mass with the CDF II detector,” Science 376 (2022) no.6589, 170-176 doi:10.1126/science.abk1781

[7] C. Campagnari and M. Mulders, “An upset to the Standard Model,” Science 376 (2022) no.6589, 136 doi:10.1126/science.abm0101

[8] S. Schael et al. [ALEPH], “Measurement of the W boson mass and width in e$^+$e$^-$ collisions at LEP,” Eur. Phys. J. C 47 (2006), 309-335 doi:10.1140/epjc/s2006-02576-8 [arXiv:hep-ex/0605011 [hep-ex]].

[9] J. Abdallah et al. [DELPHI], “Measurement of the Mass and Width of the W Boson in e$^+$e$^-$ Collisions at $\sqrt{s} = 161$-GeV - 209-GeV,” Eur. Phys. J. C 55 (2008), 1-38 doi:10.1140/epjc/s10052-008-0585-7 [arXiv:0803.2534 [hep-ex]].

[10] P. Achard et al. [L3], “Measurement of the mass and the width of the W boson at LEP,” Eur. Phys. J. C 45 (2006), 569-587 doi:10.1140/epjc/s2005-02459-6 [arXiv:hep-ex/0511049 [hep-ex]].

[11] G. Abbiendi et al. [OPAL], “Measurement of the mass and width of the W boson,” Eur. Phys. J. C 45 (2006), 307-335 doi:10.1140/epjc/s2005-02440-5 [arXiv:hep-ex/0508060 [hep-ex]].

[12] V. M. Abazov et al. [D0], “Measurement of the W Boson Mass with the D0 Detector,” Phys. Rev. Lett. 108 (2012), 151804 doi:10.1103/PhysRevLett.108.151804 [arXiv:1203.0293 [hep-ex]].

[13] M. Aaboud et al. [ATLAS], “Measurement of the W-boson mass in pp collisions at $\sqrt{s} = 7$ TeV with the ATLAS detector,” Eur. Phys. J. C 78 (2018) no.11, 898 [erratum: Eur. Phys. J. C 78 (2018) no.11, 898] doi:10.1140/epjc/s10052-017-5475-4 [arXiv:1701.07240 [hep-ex]].

[14] R. Aaij et al. [LHCb], “Measurement of the W boson mass,” JHEP 01 (2022), 036 doi:10.1007/JHEP01(2022)036 [arXiv:2109.01113 [hep-ex]].

[15] Z. Bern, G. Diana, L. J. Dixon, F. Febres Cordero, D. Forde, T. Gleisberg, S. Hoeche, H. Ita, D. A. Kosower and D. Maitre, et al. “Left-Handed W Bosons at the LHC,” Phys. Rev. D 84 (2011), 034008 doi:10.1103/PhysRevD.84.034008 [arXiv:1103.5445 [hep-ph]].

[16] E. Manca, “Precision measurements of W detected at CMS,” CERN-THESIS-2021-271.

[17] E. Manca, O. Cerri, N. Foppiani and G. Rolandi, “About the rapidity and helicity distributions of the W bosons produced at LHC,” JHEP 12 (2017), 130 doi:10.1007/JHEP12(2017)130 [arXiv:1707.09344 [hep-ex]].

[18] A. M. Sirunyan et al. [CMS], “Measurements of the W boson rapidity, helicity, double-differential cross sections, and charge asymmetry in pp collisions at $\sqrt{s} = 13$ TeV,” Phys. Rev. D 102 (2020) no.9, 092012 doi:10.1103/PhysRevD.102.092012 [arXiv:2008.04174 [hep-ex]].
[19] E. Mirkes, “Angular decay distribution of leptons from W bosons at NLO in hadronic collisions,” Nucl. Phys. B 387 (1992), 3-85 doi:10.1016/0550-3213(92)90046-E

[20] J. C. Collins and D. E. Soper, “Angular Distribution of Dileptons in High-Energy Hadron Collisions,” Phys. Rev. D 16 (1977), 2219 doi:10.1103/PhysRevD.16.2219

[21] S. Alioli, P. Nason, C. Oleari and E. Re, “NLO vector-boson production matched with shower in POWHEG,” JHEP 07 (2008), 060 doi:10.1088/1126-6708/2008/07/060 [arXiv:0805.4802 [hep-ph]].

[22] P. F. Monni, P. Nason, E. Re, M. Wiesemann and G. Zanderighi, “MiNNLO\textsubscript{PS}: a new method to match NNLO QCD to parton showers,” JHEP 05 (2020), 143 doi:10.1007/JHEP05(2020)143 [arXiv:1908.06987 [hep-ph]].

[23] P. F. Monni, E. Re and M. Wiesemann, “MiNNLO\textsubscript{PS}: optimizing 2 \rightarrow 1 hadronic processes,” Eur. Phys. J. C 80 (2020) no.11, 1075 doi:10.1140/epjc/s10052-020-08658-5 [arXiv:2006.04133 [hep-ph]].