Model independent study of the Dirac structure of the nucleon-nucleon interaction

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Relativistic and non-relativistic modern nucleon-nucleon potentials are mapped on a relativistic operator basis using projection techniques. This allows to compare the various potentials at the level of covariant amplitudes were a remarkable agreement is found. In nuclear matter large scalar and vector mean fields of several hundred MeV magnitude are generated at tree level. This is found to be a model independent feature of the nucleon-nucleon interaction.

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A fundamental question in nuclear physics is the role which relativity plays in nuclear systems. The ratio of the Fermi momentum over the nucleon mass is about $k_F/M \approx 0.25$ and nucleons move thus with maximally about 1/4 of the velocity of light which implies only moderate corrections from relativistic kinematics. However, there exists a fundamental difference between relativistic and non-relativistic dynamics: a genuine feature of relativistic nuclear dynamics is the appearance of large scalar and vector mean fields, each of the magnitude of several hundred MeV. The scalar field $\Sigma_S$ is attractive and the vector field $\Sigma_V$ is repulsive. In relativistic mean field (RMF) theory, both, sign and size of the fields are enforced by the nuclear saturation mechanism [1].

At nuclear saturation density $\rho_0 \approx 0.16$ fm$^{-3}$ the empirical fields deduced from RMF fits to finite nuclei are of the order of $\Sigma_o \approx -350$ MeV and $\Sigma_V \approx +300$ MeV [2] ($\Sigma_o$ is the time-like component of $\Sigma_V$). The single particle potential in which the nucleons move originates from the cancellation of the two contributions $U_{s.p.} \approx \Sigma_o + \Sigma_S \approx -50$ MeV which makes it difficult to observe relativistic effects in nuclear systems. There exist, however, several features in nuclear structure which can naturally be explained within Dirac phenomenology while models based on non-relativistic dynamics have difficulties. Best established is the large spin-orbit splitting in finite nuclei. Also the so-called pseudo-spin symmetry, observed more than thirty years ago in single-particle levels of spherical nuclei, can naturally be understood within RMF theory as a consequence of the coupling to the lower components of the Dirac equation [3].

A connection to Quantum-Chromo-Dynamics (QCD) as the fundamental theory of strong interactions is established by QCD sum rules [4, 5]. The change of the chiral condensates $\langle \bar{q}q \rangle, \langle q'q' \rangle$ in matter leads to attractive scalar and repulsive vector self-energies which are astonishingly close to the empirical values derived from RMF fits to the nuclear chart. Also relativistic many-body calculations [4, 5, 8] yield scalar/vector fields of the same sign and magnitude as obtained from RMF theory or, alternatively, from QCD sum rules. Moreover, Dirac-Brueckner-Hartree-Fock (DBHF) calculations [5] agree even quantitatively surprisingly well with the QCD motivated approach of Ref. [3] where chiral fluctuations from pion-nucleon dynamics were considered on top of the chiral condensates.

These facts suggest that preconditions for the existence of large fields in matter or, alternatively, the density dependence of the QCD condensates, must already be inherent in the vacuum nucleon-nucleon (NN) interaction.

Relativistic one-boson-exchange potentials (OBEP), e.g. Bonn (A,B,C), [10] provide fits with fair precision to NN scattering data. High precision fits (with $\chi^2$/datum $\approx 1.01$) are provided by OBE type potentials such as the relativistic CD-Bonn [11] or the Nijmegen potentials [12] where fits are performed separately in each partial wave. In contrast to e.g. the spectator model of [13] the 'standard' OBE potentials are based on the no-sea approximation which excludes explicit excitations of anti-nucleons but absorbs such contributions into the model parameters, in particular into a large $\omega$ coupling. The present investigations are restricted to the standard-type models.

The long-range part of the interaction is generally mediated by the one-pion-exchange (OPE) while the scalar intermediate range attraction is mainly due to correlated two-pion-exchange. The short-range part, i.e. the hard core, is dominated by the heavy vector meson exchange, i.e. the isoscalar $\omega$ and the isovector $\rho$ meson. Widely used in nuclear physics are, however, also high precision non-relativistic empirical potentials such as the Argonne potential AV18 [14]. A systematic connection to QCD is established by chiral effective field theory (EFT). Up to now the two-nucleon system has been considered at next-to-next-to-next-to-leading order (N$^3$LO) in chiral perturbation theory [15, 16]. In such approaches the NN potential consists of one-, two- and three-pion exchanges and regularizing contact interactions which account for short range correlations. The advantage of EFT compared to the more phenomenological OBE potentials is the systematic expansion in terms of chiral power counting.

A better understanding of the common features of the various approaches is essential in order to arrive at a
more model independent understanding of the nucleon-nucleon interaction, in particular since all well established interactions fit NN scattering data with approximately the same precision. Substantial progress was recently achieved by the construction of a universal low energy NN potential \( V_{\text{low } k} \) based on renormalization group techniques \[15\]. However, like the EFT potentials \( V_{\text{low } k} \) is not covariantly formulated and has therefore not been used in relativistic nuclear structure calculations.

The present work applies projection techniques to map the various potentials on the operator basis of relativistic field theory which is given by the Clifford algebra in Dirac space. This allows to identify the different Lorentz components of the interaction and to calculate the relativistic self-energy operator in matter. The philosophy behind this approach is based on the fact, that any NN interaction, whether relativistic or not, is based on a common spin-isospin operator structure which invokes certain scales: long-range spin-isospin dependent forces, especially given by the one-pion exchange, short- and intermediate-range spin-independent interactions, short-range isoscalar spin-orbit interactions and quadratic isovector spin-orbit interactions.

In a covariant formulation the spin-isospin structure of the interaction is constrained by the symmetries of the Lorentz group. E.g. the Born scattering matrix of covariant OBE potentials is given by the sum over the corresponding scalar, pseudoscalar and vector mesons \( \alpha \)

\[
\tilde{V}(q',q) = \sum_{\alpha = s,p,s,v} F_\alpha^0(q',q) \Gamma_\alpha^{(2)} D_\alpha(q' - q) \Gamma_\alpha^{(1)},
\]

where \( q_\mu \) and \( q'_\mu \) are the c.m. momenta of the incoming and outgoing nucleons. The meson propagator is denoted by \( D_\alpha \), \( F_\alpha \) are form factors applied to the vertices \( \Gamma_\alpha \). Isospin factors \( \tau_1, \tau_2 \) are suppressed in \( \Gamma_\alpha \).

The Dirac structure of the potential is contained in the meson-nucleon vertices

\[
\Gamma_s = g_s \mathbf{1}, \quad \Gamma_p = g_p \frac{q' - q}{2M} \gamma_5, \quad \Gamma_v = g_v \gamma_\mu + \frac{f_\mu}{2M} \gamma_5.
\]

For the pseudoscalar mesons \( \pi \) and \( \eta \) a pseudovector coupling is used and the \( \omega \) meson has no tensor coupling, i.e. \( f_\mu^{(\omega)} = 0 \). The potential \( V(q',q) \), i.e. the OBE Feynman amplitudes are obtained by sandwiching \( \tilde{V} \) between the incoming and outgoing Dirac spinors.

From a low energy expansion of the OBE amplitudes one obtains the representation of non-covariant potentials

\[
V(q',q) = \sum_{\alpha} [V_\alpha + V_\alpha' \tau_1 \cdot \tau_2] O_\alpha
\]

where the most important operators (assuming identical particles) are

\[
O_1 = 1, \quad O_2 = \sigma_1 \cdot \sigma_2,
O_3 = \frac{1}{3} (\sigma_1 \cdot k) (\sigma_1 \cdot k) - \frac{1}{3} k^2 (\sigma_1 \cdot \sigma_2),
O_4 = \frac{i}{2} (\sigma_1 + \sigma_2) \cdot \mathbf{n}, \quad O_5 = (\sigma_1 \cdot \mathbf{n}) (\sigma_2 \cdot \mathbf{n})
\]

with \( k = q' - q, \quad P = \frac{i}{2}(q' + q) \) and \( n = P \times k \).

The modern non-covariant Nijm I and II potentials are constructed from approximate OBE amplitudes and thus based on the operator structure \( \Gamma \). However, since they are separately fitted in each partial wave they are often referred to as phenomenological models. The Argeome potential \( AV_{18} \) \[14\] is purely phenomenological in the sense that the OBE picture is released. Based on the symmetries of \( \Gamma \), the intermediate and short range part is parameterised by phenomenological functions \( V_\alpha \) and only one-pion-exchange is contained explicitly. The EFT potentials, containing one- and multi-pion exchange explicitly, are even more rigorous since most part of the nuclear repulsion is carried by regularizing counter terms. Such an approach is justified by the fact that heavy meson \((\rho, \omega)\) exchange cannot be resolved up to the momentum scale of about 400 MeV which is constrained by NN scattering data. We apply the EFT Idaho potential \[15\] which fits NN scattering data with similar quality as \( AV_{18} \) or the Nijmegen potentials. The same philosophy is behind \( V_{\text{low } k} \) which can be viewed as the condensation of the various formulations to a model independent result.

Although based on the same operator structure as OBEPs, it is not straightforward to fix the Lorentz character of the various pieces of the non-covariant interactions \( AV_{18}, \text{EFT (Idaho)} \) and \( V_{\text{low } k} \).

However, any two-body amplitude can be represented covariantly by Dirac operators and Lorentz invariant amplitudes \[18\]. A relativistic treatment invokes automatically the excitation of anti-nucleons. However, NN scattering, in both, relativistic and non-relativistic approaches is restricted to the positive energy sector and neglects the coupling to anti-nucleons. As a consequence one has to work in a subspace of the full Dirac space where on-shell two-body matrix elements can be expanded into five Lorentz invariants. A possible choice of a set of five linearly independent covariant operators are the scalar, vector, tensor, axial-vector and pseudo-scalar Fermi covariants \( \Gamma_m = \{S, V, T, P, A\} \) with \( S = 1 \otimes 1, \quad V = \gamma^\mu \otimes \gamma_\mu, \quad T = \sigma^{\mu\nu} \otimes \sigma_{\mu\nu}, \quad P = \gamma_5 \otimes \gamma_5, \quad A = \gamma_5 \gamma^\mu \otimes \gamma_5 \gamma_\mu \).

Working with physical, i.e. antisymmetrized matrix elements \( V^A \), one has to keep in mind that direct and exchange covariants \( \Gamma_m \) (where Dirac indices of particles 1 and 2 are interchanged) are coupled by a Fierz transformation. Since the low momentum part of the NN interaction is totally dominated by the pseudovector one-pion exchange (OPE) it is preferable to choose an operator basis where the pseudovector exchange is completely separated from the remaining operator structure. This can be achieved by the following set of covariants originally proposed by Tjon and Wallace \[15\]

\[
\Gamma_m = \{S, -\tilde{S}, (A - \tilde{A}), PV, -\tilde{P}V\}.
\]

PV and \( \tilde{P}V \) are the direct and exchange pseudovector covariants, analogous to the pseudoscalar covariant P,
however, with $\gamma_5$ replaced by $(\not{q} - \not{q}/2M\gamma_5$. Thus the on-shell ($|q| = |q'|$) scattering matrix is given by

$$V^{I}(q', q) = \sum_{m} g_{m}(|q|, \theta) \Gamma_{m},$$

where $\theta$ is the c.m. scattering angle and $I = 0, 1$ the isospin channel. For the Hartree-Fock self-energy it is sufficient to consider $\theta = 0$ when antisymmetrized matrix elements are used since $\theta = \pi$ contains only redundant information. The transformation of the Born amplitudes from an angular-momentum basis onto the covariant basis (6) is now standard and runs over the following steps: $|L,S,J\rangle \rightarrow \text{partial wave helicity states} \rightarrow \text{plane wave helicity states} \rightarrow \text{covariant basis}$. The first two transformation can e.g. be found in Refs. 19. The last step has to be performed numerically by matrix inversion.

With the covariant amplitudes at hand, one is now able to determine the relativistic mean field in nuclear matter by calculating the relativistic self-energy $\Sigma$ in Hartree-Fock approximation at tree level. We are thereby not aiming for a realistic description of nuclear saturation properties which would require a self-consistent scheme. Moreover, short-range correlations require to base such calculations on the in-medium T-matrix rather than the bare potential $V$. This leads to the relativistic Dirac-Brueckner-Hartree-Fock scheme which has been proven to describe nuclear saturation with quantitatively satisfying accuracy.

The self-energy for the nucleon $k$ follows from the interaction matrix $V$ by integrating over the occupied states $q$ in the Fermi sea.

$$\Sigma_{\alpha\beta}(k, k_F) = -i \int \frac{d^4q}{(2\pi)^4} G^D(q) \Sigma^{\alpha\beta}(k, q)_{\alpha\beta}. \tag{7}$$

The Dirac propagator $G^D(q) = [\not{q} + M]2\pi\delta(q^2 - M^2)\Theta(q_0)\Theta(k_F - |q|)$ describes the on-shell propagation of a nucleon with momentum $q$ and energy $E_q = \sqrt{q^2 + M^2}$ inside the Fermi sea. In isospin saturated nuclear matter the self-energy consists of a scalar $\Sigma_s$, a time-like vector $\Sigma_v$ and a spatial vector part $\Sigma_o$.

$$\Sigma(k, k_F) = \Sigma_s(k, k_F) - \gamma_0 \Sigma_o(k, k_F) + \gamma \cdot k \Sigma_v(k, k_F), \tag{8}$$

which are given by

$$\Sigma_s = \frac{1}{4} \int \frac{k_F^3 e^3 q}{(2\pi)^3} \frac{M}{E_q} \left[ 4g_\Sigma - g_\Sigma + 4g_\Lambda - \frac{(k^\mu - q^\mu)^2}{4M^2} g_{\Psi V} \right],$$

$$\Sigma_o = \frac{1}{4} \int \frac{k_F^3 e^3 q}{(2\pi)^3} \left[ g_s - 2g_\Lambda + \frac{E_k}{E_q} \frac{(k^\mu - q^\mu)^2}{4M^2} g_{\Psi V} \right],$$

$$\Sigma_v = \frac{1}{4} \int \frac{k_F^3 e^3 q}{(2\pi)^3} \frac{k \cdot q}{|k|^2 E_q} \left[ g_s - 2g_\Lambda + \frac{k_z (k^\mu - q^\mu)^2}{4M^2} g_{\Psi V} \right]. \tag{9}$$

Fig. 1 shows the resulting isospin-averaged amplitudes $g_{m}(|q|, 0)$ for the different NN potentials after projection on the Dirac operator structure. The pseudovector representation of the relativistic operator basis is used.

The potentials a coincidence at the level of covariant amplitudes is not completely unexpected. However, it is remarkable that this agreement transfers to the EFT Idaho potential and also to $V_{\text{low } k}$ which are of completely different character and theoretical background.

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$$\Sigma_v = \frac{1}{4} \int \frac{k_F^3 e^3 q}{(2\pi)^3} \frac{k \cdot q}{|k|^2 E_q} \left[ g_s - 2g_\Lambda + \frac{k_z (k^\mu - q^\mu)^2}{4M^2} g_{\Psi V} \right]. \tag{9}$$

Fig. 2 shows the tree level self-energy components obtained with the various potentials in nuclear matter at saturation density with a Fermi momentum $k_F = 1.35$ fm$^{-1}$. All potentials yield scalar/vector fields $\Sigma_s$ and $\Sigma_o$ of comparable strength: a large and attractive scalar field $\Sigma_s \simeq -(450 \pm 400)$ MeV and a repulsive vector field of $\Sigma_o \simeq +(350 \pm 400)$ MeV. The spatial components $k\Sigma_v$ agree as well. As known from self-consistent DBHF calculations 6, 7, the spatial vector self-energy is a moderate correction to the large scalar and time-like vector components $\Sigma_s$ and $\Sigma_o$. This is the case at tree level. The right part of Fig. 2 shows the single particle potential $U_{\text{esp}}(k, k_F) = M\Sigma_s/E_k - \Sigma_o + \Sigma_v k^2/E_k$, defined as the expectation value of the self-energy. It
Fig. 2: Tree level results for nuclear matter at $k_F = 1.35$ fm$^{-1}$ for the various potentials: scalar and vector self-energy components (left), spatial vector self-energy component (middle) and single particle potential $U_{s.p.}$ (right). Stars in the right panel denote a non-relativistic calculation of $U_{s.p.}$.

reflects the well known fact that various two-body potentials are rather different although they are phase-shift equivalent, i.e. they describe NN scattering data with about the same accuracy when iterated in the Lippmann-Schwinger equation [21]. The differences at tree level are mainly due to differences in the short-range part of the interaction [17]. Fig. 2 includes also the results from a ‘non-relativistic’ calculation of $U_{s.p.}$ where the partial wave amplitudes are directly summed up. The non-relativistic and the relativistic calculations show an excellent agreement which demonstrates the accuracy of the applied projection techniques.

We presented a model independent study of the Dirac structure of the nucleon-nucleon interaction. The potentials were projected on a relativistic operator basis in Dirac space using standard projection techniques which transform from a partial wave basis, where the potentials are originally given, to the basis of covariant amplitudes. The different approaches can now be compared at the level of these covariant amplitudes and, moreover, this allows to calculate the relativistic self-energy operator in nuclear matter. For both, the covariant amplitudes and the tree-level Hartree-Fock self-energy, we observe a remarkable agreement between the OBEPs (Bonn, CD-Bonn, Nijmegen), the phenomenological AV$_{18}$ potential and the EFT based Idaho and $V_{\text{low } k}$ potentials. The structure of the interaction enforces the existence of large scalar and vector fields as a model independent fact. The scale of these fields is set at the tree level. Although essential for nuclear binding and saturation, higher order correlations, i.e. Brueckner ladder correlations, change the size of the fields by less than 20% [12]. The magnitude of the fields is similar to that predicted by relativistic mean field phenomenology, relativistic many body correlations and also by QCD sum rules. We conclude that the appearance of large scalar and vector fields in matter is a general and model independent consequence of the vacuum structure of the NN interaction. Relativistic dynamics is therefore essential for nuclear systems.

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