Spontaneous emission spectrum of the non-lasing supermodes in semiconductor laser arrays

Holger F. Hofmann and Ortwin Hess
Institute of Technical Physics, DLR
Pfaffenwaldring 38-40, D-70569 Stuttgart, Germany

July 18, 2018

Abstract

It is shown that the interference between spontaneous emission into the non-lasing supermode and the laser field of a semiconductor laser array causes spatial hole-burning which couples the dynamics of the spontaneous emission with the laser field. In particular, phase locking between the spontaneous emission and the lasing mode leads to the formation of a spectral triplet composed of in-phase relaxation oscillation sidebands and an out-of-phase line at the lasing frequency.

The investigation of spontaneous emission into the non-lasing modes of a semiconductor laser operating in cw-mode can provide valuable insights into the carrier dynamics of the laser device. This is especially useful in the study of phase-locked laser arrays, such as two-dimensional arrays of vertical cavity surface emitting lasers (VCSEL arrays). Such coherently coupled VCSEL arrays are fabricated e.g. to obtain maximal coherent output power. They usually exhibit stable anti-phase locking between adjacent lasers resulting in the lasing of only the supermode with highest frequency.

However, there is still spontaneous emission into the other non-lasing modes. Generally, the interference terms between the laser light and the non-lasing modes determine the spatial intensity distribution inside the laser cavity. The significance of these terms depends on the phase relation between the spontaneous emission and the laser light. It is particularly strong if the spontaneous emission is in phase with the laser light. Consequently one can expect that the in-phase spontaneous emission will cause spatial hole-burning and spatial relaxation oscillations, while the out-of-phase spontaneous emission will not couple to the carrier dynamics.

In this letter, we describe the dynamics of the lasing antisymmetric supermode and the non-lasing symmetric supermode of a two laser array using a rate equation model similar to the one introduced by Winful and Wang. In this model, the spatial distribution of the carrier density is approximated by assigning separate carrier densities to the individual lasers of the array. For a two laser array, the dynamical equations of the field and the carrier densities may be written in terms of the anti-symmetric supermode $E_-$ and the symmetric supermode $E_+$. The validity of this discretization of the electromagnetic field and the limitation to two modes is established in a more detailed model based on partial differential equations. The total carrier density $N$, which is the sum of both carrier densities in the lasers, interacts equally strong with both modes. The spatial carrier density difference between the two lasers, $\Delta$, is a measure of spatial hole-burning effects on the length scale of the array. The dynamical equations then read

$$\frac{d}{dt} E_+ = \frac{w}{2} N (1 - i\alpha) E_+ - (\kappa_+ + i\omega_+) E_+$$
$$+ \frac{w}{2} \Delta (1 - i\alpha) E_-$$

$$\frac{d}{dt} E_- = \frac{w}{2} N (1 - i\alpha) E_- - (\kappa_- + i\omega_-) E_-$$
$$+ \frac{w}{2} \Delta (1 - i\alpha) E_+$$

$$\frac{d}{dt} N = \mu - \gamma N - w (E_+^* E_+ + E_-^* E_-) N$$
$$- w (E_+^* E_- + E_-^* E_+) \Delta$$

$$\frac{d}{dt} \Delta = - (\gamma + 2\Gamma) \Delta - w (E_+^* E_+ + E_-^* E_-) \Delta$$
$$- w (E_+^* E_- + E_-^* E_+) N.$$
period equal to the distance $r$ between the lasers in the array. The diffusion rate is thus

$$\Gamma = \frac{4\pi^2}{r^2} D_{\text{diff}}. \quad (2)$$

Note that such a treatment of diffusion represents an extension of the original model of Winful and Wang as discussed elsewhere.

The stable solution of the array dynamics is given by $\Delta = 0$, $N = 2\kappa_-/w$, $E_+ = 0$ and $E_- = \sqrt{I_0} \exp[-i(\omega_- + \alpha \kappa_-)t]$, where the laser intensity $I_0$ is a linear function of the injection rate $\mu$. The linearized dynamics of small fluctuations in the non-lasing mode $E_+$ and the spatial hole-burning parameter $\Delta$ read

$$\frac{d}{dt} E_+ = - (\kappa_+ - \kappa_- + i \omega_+ + i\alpha \kappa_-) E_+ + \frac{w}{2} \Delta (1-i\alpha) \sqrt{I_0} e^{-i(\omega_- + \alpha \kappa_-)t}$$

$$\frac{d}{dt} \Delta = - (\gamma + 2\Gamma + wI_0) \Delta - 2\kappa_- \sqrt{I_0} (e^{-i(\omega_- + \alpha \kappa_-)t} E_+^* + e^{+i(\omega_- + \alpha \kappa_-)t} E_+). \quad (3a)$$

Note that fluctuations in the lasing mode $E_-$ or the total density $N$ do not appear in the linearized equations for $E_+$ and $\Delta$. The fluctuations in the non-lasing mode are therefore not correlated with the fluctuations in the lasing mode.

The laser field itself does affect the dynamics of fluctuations, however, as expressed by the final terms of the linearized equations which are sensitive to both the amplitude $\sqrt{I_0}$ and the phase $-(\omega_- + \alpha \kappa_-)t$ of the laser field. In order to describe the phase relation between the fluctuations of the non-lasing mode $E_+$ and the laser field, $E_+$ can be expressed in terms of the component $f_\parallel$ in phase with the laser field and the component $f_\perp$ which is $\pi/2$ out of phase with the laser field,

$$E_+ = (f_\parallel - if_\perp) e^{-i(\omega_- + \alpha \kappa_-)t}. \quad (4)$$

Using this parameterization the Langevin equation describing the spontaneous emission from the non-lasing mode formulated as a matrix equation reads

$$\frac{d}{dt} \begin{pmatrix} f_\parallel \\ f_\perp \\ \Delta \end{pmatrix} =$$

$$= \begin{pmatrix} -s + \Omega & \frac{\sqrt{I_0}}{w} & \frac{w}{2} \Delta \\\n-\Omega & -s - \frac{\alpha \sqrt{I_0}}{w} & 0 \\
-4\kappa_- \sqrt{I_0} & 0 & -\gamma - 2\Gamma - wI_0 \end{pmatrix} \begin{pmatrix} f_\parallel \\ f_\perp \\ \Delta \end{pmatrix} + \begin{pmatrix} Q_\parallel \\ Q_\perp \\ 0 \end{pmatrix}, \quad (5)$$

where $\Omega = \omega_- - \omega_+$ and $s = \kappa_+ - \kappa_-$. are the quantum noise terms acting on the cavity mode from the quantum vacuum outside the cavity and from the dipole fluctuations in the gain medium. Pump noise in the carrier density has been neglected since its effect is much smaller than that of the quantum fluctuations in the case of strong relaxation oscillations. The minimal quantum fluctuations for complete inversion in the gain medium are given by

$$\langle Q_\parallel(t)Q_\parallel(t+\tau) \rangle = \langle Q_\perp(t)Q_\perp(t+\tau) \rangle = \kappa \delta(\tau). \quad (6)$$

The fluctuation dynamics depends on four timescales, the damping coefficients of the field $s$ and of the carrier density difference $\gamma + 2\Gamma + wI_0$, the frequency difference between the supermodes $\Omega$ and the coupling frequency between the field and the carrier density difference $\sqrt{2\kappa_- wI_0}$. Note that this coupling frequency is identical with the frequency of relaxation oscillations in the total intensity. Physically, it represents the spatial variations in the relaxation oscillations associated with fluctuating spatial hole-burning. If the spatial hole-burning effect represented by $\Delta$ is negligible, the fluctuations in $f_\parallel$ and $f_\perp$ are rotations with frequency $\Omega$ damped at a rate of $s$. This corresponds to a single Lorentzian spontaneous emission line at $\omega_+$ with a width of $2s$.

We will now examine the opposite case of strong spatial hole-burning effects, however, which applies if the frequency difference $\Omega$ is much smaller than the relaxation oscillation frequency $\sqrt{2\kappa_- wI_0}$. In this limit, the light field is phase locked to the laser field by the spatial hole-burning associated with the in-phase field fluctuations $f_\parallel$. The field dynamics separates into two uncorrelated solutions. The out-of-phase fluctuations are described by an exponential relaxation of fluctuations in $f_\perp$,

$$\Delta \approx f_\perp \approx 0 \quad f_\perp \approx f_0 e^{-(s+\alpha \Omega)t}. \quad (7)$$

The in-phase fluctuations are actually phase shifted with respect to the laser light by the linewidth enhancement factor $\alpha$. They are given by relaxation oscillations with

$$f_\parallel \approx f_0 \cos(\sqrt{2\kappa_- wI_0} t) e^{-(\gamma + 2\Gamma + wI_0 + s - \alpha \Omega)t/2} \quad f_\parallel \approx \alpha f_0 \cos(\sqrt{2\kappa_- wI_0} t) e^{-(\gamma + 2\Gamma + wI_0 + s - \alpha \Omega)t/2}$$

$$\Delta \approx -2\sqrt{2\kappa_-} f_0 \sin(\sqrt{2\kappa_- wI_0} t) e^{-(\gamma + 2\Gamma + wI_0 + s - \alpha \Omega)t/2}. \quad (8)$$

The complete solution of the Langevin equation and the resulting two-time correlation functions are analogous to the ones derived for the polarization fluctuations in VCSELs.
The spectrum of the spontaneous emission into the non-lasing mode is a triplet composed of a central line at the laser frequency representing the out-of-phase emission processes and two sidebands representing the relaxation oscillations caused by in-phase spontaneous emission. The total spectrum reads

\[ I(\delta \omega) = \frac{\kappa}{2\pi} \frac{(1 + \alpha^2)}{(s + \alpha \Omega)^2 + \delta \omega^2} \]

\[ + \frac{\kappa}{2\pi} \frac{(1 + \alpha^2)}{(\gamma + 2\Gamma + w I_0 + s - \alpha \Omega)^2 + 4(\delta \omega - \sqrt{2\kappa - w I_0})^2} \]

\[ + \frac{\kappa}{2\pi} \frac{(1 + \alpha^2)}{(\gamma + 2\Gamma + w I_0 + s - \alpha \Omega)^2 + 4(\delta \omega + \sqrt{2\kappa - w I_0})^2} \]  

(9)

The Intensity is given in terms of the average photon number inside the optical cavity. The emission rate is found by multiplying with \(2\kappa+\). \(\delta \omega\) is the frequency difference to the position of the laser line at \(\omega_+ + \alpha \omega_-\). Figure 1 shows the spontaneous emission triplet of the non-lasing supermode for a typical choice of parameters, \(\gamma = 10 \text{ GHz}, \quad \Gamma = 20 \text{ GHz}, \quad \kappa_- = 2000 \text{ GHz}, \quad \alpha = 3, \quad \Omega = 5 \text{ GHz} \) and \(s = 15 \text{ GHz}\). The laser intensity is given in units of \(\gamma/w\), which typically corresponds to about 10000 photons in the cavity.

The central line does not change as the laser intensity increases. The reason lies in the fact that the interference term of the out-of-phase fluctuations and the laser light vanishes. The spatial intensity distribution is not modified by out-of-phase spontaneous emission and no spatial hole-burning results. Therefore, this component of the spontaneous emission is the unchanged emission from the carrier density \(N\) pinned at \(N = 2\kappa_-/w\).

The sidebands at \(\delta \omega = \pm \sqrt{2\kappa - w I_0}\) represent that part of the spontaneous emission which has a component in phase with the laser light. The non-zero interference term between the non-lasing mode and the laser light causes spatial hole-burning as represented by the parameter \(\Delta\). Consequently, there are spatial relaxation oscillations at the relaxation oscillation frequency \(\sqrt{2\kappa - w I_0}\). Since the interference term between the non-lasing mode and the laser light represents the intensity difference between the two lasers in the array, the oscillation in the in-phase component of the non-lasing field \(f_\parallel\) corresponds to a spatial intensity oscillation. The phase of the sideband emission processes is not the same as the phase of the laser field, however. As can be seen in equation (9), the out-of-phase component \(f_\perp\) is \(\alpha\) times as large as the in-phase component \(f_\parallel\). This is a consequence of the spatial change in the refractive index. While the gain-guiding properties associated with a non-zero carrier density difference \(\Delta\) convert the field of the lasing supermode into the non-lasing supermode at the same phase, the index guiding properties change the phase by \(\pi/2\). Since the ratio of the index guiding and the gain guiding induced by the changes in carrier density is given by \(\alpha\), the phase difference between the laser field and the relaxation oscillation sidebands of the non-lasing mode is equal to \(\arctan(\alpha)\).

Since the relaxation oscillation sidebands represent the spatial effects induced by the in-phase component of spontaneous emission into the non-lasing mode, the linewidth, the frequency, and the total intensity are a function of the carrier dynamics as well as of the laser field. The dependence on laser intensity is shown in figure 3, in which the out-of-phase contribution has been removed. The total intensity in the two sidebands \(I_{sb}\) compared to the total intensity of the central line \(I_{cl}\) is

\[ \frac{I_{sb}}{I_{cl}} = \frac{s + \alpha \Omega}{\gamma + 2\Gamma + w I_0 + s - \alpha \Omega} \]  

(10)

The diffusion \(\Gamma\) and the total rate of induced and spontaneous transitions \(\gamma + w I_0\) both suppress spontaneous emission processes into the non-lasing supermode by damping the carrier density difference \(\Delta\). In this manner, the total spontaneous emission into the non-lasing supermode decreases as laser intensity increases, even though the average carrier density remains pinned at \(N = 2\kappa_-/w\).

The spontaneous emission triplet of the non-lasing supermode should be observable in the spectrum emitted from the symmetric non-lasing supermode of the laser array cavity. Since the symmetric supermode has its far-field intensity maximum in the center of the far-field, where the stable anti-symmetric laser mode should have zero intensity, it seems likely, that this spectrum can be observed by measurements of the weak intensities in the center of the far field. The assumptions of the model applied in this letter may then be tested by measurements of the linewidths and frequencies. An interesting point would be the comparison of sideband intensities. If the coherent coupling \(\Omega\) is such that \(\sqrt{2\kappa - w I_0} \gg \Omega\) does not hold anymore, then the lower frequency sideband intensity should be stronger than the high frequency sideband intensity – up to the point where \(\Omega \gg \sqrt{2\kappa - w I_0}\) and there is a single spontaneous emission line left at \(\omega_+\). This case would imply extremely strong coupling between the individual lasers of the array, however, implying that the array acts as a single laser with a modified cavity.

In conclusion, we have shown that the carrier dynamics of a two laser array modifies the spontaneous emission in the non-lasing supermode by phase locking it to
the laser field and by modulating the in-phase component through relaxation oscillations of the spatial hole-burning.

1. H. F. Hofmann and O. Hess, “Quantum Noise and Polarization Fluctuations in Vertical Cavity Surface Emitting Lasers,” Phys. Rev. A 56, 868–876 (1997).
2. A. K. J. van Doorn, M. P. van Exter, A. M. van der Lee, and J. P. Woerdman, “Coupled-mode description for the polarization state of a vertical-cavity semiconductor laser,” Phys. Rev. A 55, 1473 (1997).
3. J. M. Catchmark, L. E. Rogers, R. A. Morgan, M. T. Asom, G. D. Guth, and D. N. Christodoulides, “Optical Characteristics of Multitransverse-Mode Two-Dimensional Vertical-Cavity Top Surface-Emitting Laser Arrays,” IEEE J. Quant. Electr. 32, 986–995 (1996).
4. R. A. Morgan and K. Kojima, “Optical characteristics of two-dimensional coherently coupled vertical-cavity surface-emitting laser arrays,” Opt. Lett. 18, 352 (1993).
5. R. A. Morgan, K. Kojima, T. Mullally, G. D. Guth, M. W. Focht, R. E. Leibenguth, and M. Asom, “High-power coherently coupled 8×8 vertical cavity surface emitting laser array,” Appl. Phys. Lett. 61, 1160–1162 (1992).
6. M. Orenstein, E. Kapon, J. P. Harbison, L. T. Florez, and N. G. Stoffel, “Large two-dimensional arrays of phase-locked vertical cavity surface emitting lasers,” Appl. Phys. Lett. 60, 1535 (1992).
7. S. S. Wang and H. G. Winful, “Dynamics of phase-locked semiconductor laser arrays,” Appl. Phys. Lett. 52, 1774–1776 (1988).
8. H. G. Winful and S. S. Wang, “Stability of phase locking in coupled semiconductor laser arrays,” Appl. Phys. Lett. 53, 1894–1896 (1988).
9. M. Münk, F. Kaiser, and O. Hess, “Stabilization of spatiotemporally chaotic semiconductor laser arrays by means of delayed optical feedback,” Phys. Rev. E. 56 (1997).
10. H. F. Hofmann and O. Hess, “The Split Density Model: A Unified Description of Polarization and Array Dynamics for Vertical Cavity Surface Emitting Lasers,” Quant. Semiclass. Opt. (1997).

Fig. 1. Spontaneous emission triplet for $\gamma = 10$ GHz, $\Gamma = 20$ GHz, $\kappa_\perp = 2000$ GHz, $\alpha = 3$, $\Omega = 5$ GHz and $s = 15$ GHz. (a) shows the dependence of the frequencies, the intensities and the linewidths on the laser intensity as a three dimensional plot and (b) shows the triplet at Intensities of $wI_0 = 2.5$ GHz, $I_0 = 5$ GHz, $I_0 = 7.5$ GHz and $I_0 = 10$ GHz.

Fig. 2. Sidebands without the out-of-phase line at $\delta \omega = 0$. As the intensity increases, the in-phase contribution of spontaneous emission is suppressed by the stimulated emission term $wI_0$. 

4
