d-Wave Superconductivity Induced by Chern-Simons Term in High-$T_c$ Cuprates

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We show that a Chern-Simons term for a gauge field describing a fluctuation of spins is induced by integrating out hole fields in the presence of spin-orbit coupling which originates from a buckling of the CuO$_2$ plane. Through the Chern-Simons term, holes behave like skyrmion excitations in a spin system and become a superconducting state with $d_{x^2-y^2}$ symmetry after the antiferromagnetic long-range order is destroyed.

KEYWORDS: high-$T_c$ superconductivity, mechanism of d-wave superconductivity, Chern-Simons term, skyrmion excitation, buckling of CuO$_2$ plane, spin-orbit coupling

Since the discovery of high-$T_c$ superconductivity in cuprates, much experimental and theoretical effort has been invested to clarify its mechanism of superconductivity. Results of experimental studies indicate that the following properties are essential features. First, the CuO$_2$ layered structure is intrinsic to superconductivity and both the undoped and carrier-doped CuO$_2$ planes are characterized as two-dimensional systems on the basis of their magnetic, transport, and optical properties.

Second, superconductivity occurs in a disordered spin background. For the undoped case, the system is a Mott insulator and spins at Cu sites show antiferromagnetic long-range order below the Néel temperature $T_N$. Upon doping, $T_N$ decreases to zero and spin-glass behavior is observed. Superconductivity emerges upon further doping. Apparently, disorder in the spin system is introduced by doped holes. Third, the Cooper pair is spin-singlet and has $d_{x^2-y^2}$ symmetry.

In addition to these properties, the occurrence of superconductivity appears to be closely related to the structural phase transition point. Moreover, the transition temperature $T_c$ is closely related to the buckling of the CuO$_2$ plane. Among the effects on the conduction electron system accompanied by a buckling of the CuO$_2$ plane, there is spin-orbit coupling. Spin-orbit coupling can have an important effect on conduction electrons through the Berry phase induced by the background spin configuration.

In this Letter, we propose a mechanism of d-wave superconductivity in a disordered spin background based on a two-dimensional model with spin-orbit coupling. We assume for the spin-orbit coupling term that it is induced by the buckling of the CuO$_2$ plane, as shown in Fig. 1. We show that a Chern-Simons term for a gauge field, which describes the fluctuation of spins at Cu sites, is induced by integrating out the hole fields. Through this Chern-Simons term, holes behave like skyrmion excitations for the spin system. When the antiferromagnetic long-range order is destroyed by these skyrmion excitations, the Chern-Simons term leads to Cooper pairing of holes. We show that the pairing state is spin-singlet with $d_{x^2-y^2}$ symmetry using a transformation to the previously considered model.

Our model is described by the following Hamiltonian:

$$H = -t_0 \sum_{\langle i,j \rangle} \left( c_{i\sigma}^\dagger c_{j\sigma} + H.c. \right) + J_K \sum_j s_j \cdot S_j + H_{so} + H_{spin}, \tag{1}$$

where $c_{i\sigma}^\dagger = \left( c_{i\uparrow}^\dagger \ c_{i\downarrow}^\dagger \right)$ and $c_i = \left( c_{i\uparrow} \ c_{i\downarrow} \right)^T$ denote a creation and an annihilation operator for holes in a spinor representation. In the first term, the summation is taken over the nearest-neighbor sites. The second term denotes Kondo coupling between the spin of holes: $s_j = \frac{1}{2} c_{i\sigma}^\dagger \sigma c_{i\sigma}$, and the spin at Cu sites: $S_j = \frac{1}{2} \sum_i c_{i\sigma}^\dagger \sigma c_{i\sigma}$. The Hamiltonian (1) is based on a model which distinguishes electrons at the Cu site and holes at the O site. We implicitly exclude the double occupancy of holes at O sites because there is a strong on-site Coulomb repulsion. A similar model without $H_{so}$ is proposed in refs. 21, 22 and 23. Note that if we take the limit $J_K \to \infty$, eq. (1) without $H_{so}$ corresponds to the so-called t-J model. For spin-orbit coupling, we as-

![Fig. 1. Buckling of the CuO$_2$ plane. O atoms are displaced from the CuO$_2$ plane. Arrows represent the spins at Cu sites. The hole at the O site interacts with the spin via Kondo coupling $J_K$.](image-url)
sume the following form:

\[ H_{so} = i \sum_{j} \sum_{\alpha=x,y} c_{j}^{\dagger} \lambda^{(\alpha)} \cdot \sigma c_{j+a\hat{a}_{\alpha}} + H.c., \]

where \( \lambda^{(\alpha)} = (\lambda_{x}^{(\alpha)}, \lambda_{y}^{(\alpha)}) \), \( \hat{a}_{\alpha} \) is a unit vector along the \( \alpha \)-axis, and \( a \) is the lattice constant. Spin-orbit coupling is produced by a buckling of the CuO_{2} plane. An example is shown in Fig. 1. In the presence of spin-orbit coupling, there is a generally a Dyalsloshinskii-Moriya-type interaction for the spin system. However, we ignore this term because it does not play an important role in the mechanism of the system. In order to describe the spin system we introduce the Schwinger bosons.

The use of this transformation is based on the assumption that the constraint \( \sqrt{t_{0}^{2} + \lambda_{z}^{2}} \eta \tau_{i}(i\sigma_{y}) \)

is the lattice constant. Spin-orbit coupling parameter in the Hamiltonian and the antiferromagnetic are added:

\[ h_{ij}(\lambda^{(\alpha)}) = -\sqrt{t_{0}^{2} + \lambda_{z}^{2}} \lambda^{(\alpha)} \cdot \sigma - i\lambda_{x}A_{ji}. \]

After this transformation is performed, an additional sign change occurs for both the Kondo coupling term and the component of the gauge field. However, as we will discuss later, transformation changes the symmetry of the pairing matrix. Therefore, we must perform the inverse transformation to obtain the physical state of the original system. In the following, we call the system obtained after transformation the “F-system” and the original system the “AF-system.” In the F-system, the action for the holes is given by

\[ S_{h} = \int dt \sum_{j} (\tau_{j}(t))G^{-1}(\{\hat{k}_{\mu} - A_{\mu}\})c_{j}(t), \]

where \( \hat{k}_{\mu} \) is defined by \( c_{j a\hat{a}_{\mu}} = c_{j+a\hat{a}_{\mu}}, \) for \( \mu = x, y, \) and \( \hat{k}_{i} = -i\partial_{i}. \) The inverse of the Green’s function is given by

\[ G^{-1}(\{k_{\mu}\}) = (k_{0} + 2t_{0}\eta \sum_{\alpha=x,y} \cos k_{\alpha} - g(k) \cdot \sigma, \]

where

\[ g(k) = \left(2\eta \sum_{\alpha=x,y} \lambda_{z}^{(\alpha)} \sin k_{\alpha}, 2\eta \sum_{\alpha=x,y} \lambda_{y}^{(\alpha)} \sin k_{\alpha}, -J_{K}/4\right). \]

Note that we cannot take the limit \( J_{K} \to \infty \) in the presence of spin-orbit coupling. If we take the limit \( J_{K} \to \infty \), then the spin of holes is projected in the direction antiparallel to the spin at Cu sites. However, the hopping process always involves the opposite spin of holes as long as \( \lambda_{\alpha} \neq 0 \).

In order to calculate the Chern-Simons term, we take a continuum limit. (The condition of taking this limit will be discussed later.) The induced Chern-Simons term is given by:

\[ S_{CS} = -\frac{\theta}{2\pi} \int dt \int d^{2}\mathbf{r} A_{x}^{\tau}(\partial_{x}A_{y}^{\mu} - \partial_{y}A_{x}^{\mu}). \]

Note that only the Abelian Chern-Simons term appears in eq. (10) because the SU(2) gauge field \( A_{\mu} \) is reduced to the Abelian Chern-Simons term upon using the curl-free condition. We retain \( A_{x}^{\tau} \) because it describes the
staggered spin fluctuation. From eq. (12), we obtain
\[ \theta = \frac{1}{2} \times \text{sgn}(J_K \Lambda), \]
where \( \Lambda \equiv \lambda_\tau^{(x)} \lambda_y^{(y)} - \lambda_y^{(x)} \lambda_x^{(y)} \), and we have used the continuum form of (1): \( g(k) = (2\eta \sum_{s} \lambda_x^{(a)} k_x + \sum_{s} \lambda_y^{(a)} k_y, -J_K / 4) \). In contrast to the anyon system, the value of \( \theta = \pm 1/2 \) does not alter the statistics of particles.

We can extend the above calculation to a finite temperature. However, if we concentrate on the region \( k_B T < J_K \), we can neglect finite temperature corrections. Since spin-orbit coupling term (8) involves a process of hopping between different d-orbitals at the same site, the external electromagnetic gauge field \( A_{\mu}^\tau \) does not couple to it. Therefore, there is no Chern-Simons term for \( A_{\mu}^\tau \). In deriving eq. (10), we have taken the continuum limit for the gauge field \( A_{\mu}^{\alpha} \). Since the length scale of the gauge field is given by \( v/\Delta_{sw} \), where \( \Delta_{sw} \) and \( v \) denote the gap and the velocity of the spin wave mode respectively, the condition of taking the continuum limit is \( \Delta_{sw} / (v/a) < \lambda_\tau / t_0 \), which can be seen from eq. (5). For \( \lambda_\tau \), a rough estimation gives \( \lambda_\tau \sim 2 \text{meV} \). We assume that this condition is satisfied in the underdoped region because there \( \Delta_{sw} \) may be very small and \( v/a \) is close to the value of the undoped case \( \sim 200 \text{meV} \).

Although the presence of spin-orbit coupling is essential for the derivation of the Chern-Simons term, it has no importance for other physical processes. Therefore, we can neglect it in the following discussion.

Now we discuss the effect of the Chern-Simons term. We take
\[ S = S_h + S_{CS} + S_{\text{spin}}, \]
for the effective action. The action for the spin system is given by the Cooper pair model (13): \[ S_{\text{spin}} = (2/g) \int d^3x \sum_\sigma (|\partial_\mu - i A_\mu^\sigma| \sigma^2 + (\Delta_{sw}^\sigma / v^2) |\sigma|^2). \]
By integrating out the \( \alpha \)-field, we obtain the equation between the spin density of the hole and the “magnetic” field \( B(r, t) = \partial_\tau A_\tau^\sigma(r, t) - \partial_x A_x^\sigma(r, t) \). If we take the x-axis as the quantization axis for the spin, we obtain \( \sum_\sigma s_\sigma \rho_\sigma(r, t) = \frac{\theta}{2\pi} B(r, t) \), where \( s_\tau = 1 \) for \( \tau = \uparrow \) and \( s_\tau = -1 \) for \( \tau = \downarrow \). In the AF-system, this relation involves the isospin index, that is,
\[ \sum_\sigma s_\tau s_\sigma \rho_\sigma(r, t) = \frac{\theta}{2\pi} B(r, t). \]
Here \( s_\tau = 1(-1) \) for \( \tau \) belongs to the A(B)-sublattice. Therefore, \( \uparrow(\downarrow) \)-spin at the A-sublattice induces a (anti-)skyrmion excitation in the localized spin system and \( \downarrow(\uparrow) \)-spin at the B-sublattice induces a (anti-)skyrmion excitation in the localized spin system. Since the skyrmion and anti-skyrmion excitations introduce disorder into the localized spin system and the number of them is the same as that of holes, disorder in the spin system increases upon doping. If the hole density is sufficiently small that the magnetic long-range order is preserved, then the Meissner effect occurs for the gauge field \( A_{\mu}^\tau \) and holes are pinned because skyrmions break the translational invariance, that is, the system is an insulator.

After the magnetic long-range order is destroyed by the skyrmion excitations, the Chern-Simons term becomes dominant in the long wavelength and low-energy physics. For the holes, it leads to a pairing state. Coupling between the hole current and the gauge field \( A_{\mu}^\sigma \) in the F-system is given by
\[ S_{j-A} = \int dt \int d^2r \sum_{\sigma = \uparrow, \downarrow} j_\sigma(r, t) \cdot \mathbf{A}^\sigma(r, t), \]
where \( j_\sigma(r, t) \) is the hole current for \( \sigma \)-spin. Since eq. (14) describes minimal coupling between the hole current and the gauge field \( \mathbf{A}^\sigma \), it gives rise to a Lorentz force. Such a Lorentz force is induced between holes passing each other. Therefore, it leads to a chiral pairing state. The chirality is determined by the sign of \( \theta \). From its pairing mechanism, the possibility of the s-wave pairing state is excluded.

Now we investigate the pairing state of the AF-system through the F-system. Before doing that, we must know the relationship of the pairing matrix between them. We assume that \( i \) and \( j \) are nearest neighbor sites. If we take \( \Delta_{ij}^\sigma = (c_i \gamma c_j^\dagger - c_j \gamma c_i^\dagger) \) for the spin-singlet pairing order parameter for the AF-system, then after performing transformation (16) we obtain \( \Delta_{ij}^\sigma \rightarrow (c_i \gamma c_j^\dagger + c_j \gamma c_i^\dagger) \). Therefore, the spin-singlet pairing state is transformed into the spin-triplet pairing state and vice versa. In k-space, holes at the A-sublattice are described by the fields \( \alpha_{k\sigma} = \frac{1}{\sqrt{2}} \left( c_{k\sigma} + c_{k+Q\sigma} \right) \) and holes at the B-sublattice are described by \( \beta_{k\sigma} = \frac{1}{\sqrt{2}} \left( c_{k\sigma} - c_{k+Q\sigma} \right) \), where \( Q = (\pi/a, \pi/a) \). Here, we assume that the A-sublattice is the set of (even, even) and (odd, odd) and the B-sublattice is the set of (even, odd) and (odd, even). The pairing matrix may be given by
\[ \Delta_{\sigma_1\sigma_2}^k = \sum_{k'} V_{AF}^{\sigma_1\sigma_2}(k') \langle \beta_{-k'\sigma_2} \alpha_{k\sigma_1} \rangle_{AF}, \]
where \( \sum_{k'} (f_{k'} + f_{k'+Q}) = \sum_{k} f_{k} \). Here, we do not need the explicit form of \( V_{AF} \) because we solve the gap equation through that of the F-system. Transformation (16) in k-space is given by \( \beta_{k\sigma} \rightarrow i \sigma_y \beta_{k\sigma} \). By performing this transformation, the gap equation (15) is transformed into
\[ \Delta_{\sigma_1\sigma_2}^k = \sum_{k'} V_{F}^{\sigma_1\sigma_2} \langle i \sigma_y \beta_{-k'\sigma_2} \alpha_{k\sigma_1} \rangle_{F}. \]
For the singlet pairing case \( \Delta_{\sigma_1\sigma_2}^k \), eq. (16) is reduced to
\[ \Delta_{\sigma_1\sigma_2}^k = \left( \Delta_{\sigma_1\sigma_2}^{(1)k} + \Delta_{\sigma_1\sigma_2}^{(2)k} \right) / 2, \]
where
\[ \Delta_{\sigma_1\sigma_2}^{(1)k} = \sum_{k'} V_{F}^{\sigma_1\sigma_2} \langle -c_{-k'\sigma_2, c_{k'\sigma_1}} \rangle_{F}, \]
\[ \Delta_{\sigma_1\sigma_2}^{(2)k} = -\sum_{k'} V_{F}^{\sigma_1\sigma_2} \langle c_{-k'\sigma_2} c_{k'\sigma_1} \rangle_{F}. \]
The minus sign in eq. (20) originates from the sign change in the kinetic term in \( G^{-1}(k_0, k + Q) \). Although the vector \( g(k) \) changes as \( g(k + Q) =...
diag(-1, -1, 1)g(k), the sign of θ does not change. In the continuum approximation, eqs. (19) and (20) are reduced to
\[ \Delta_k = \pm \sum_{k'} \frac{4\pi i}{\theta} \frac{k \times k'}{|k-k'|^2} \mathcal{E}_k, \]
(21)
where \( E_k = \sqrt{\Delta_k^2 + |\Delta_k|^2} \), at \( T = 0 \). Following the analysis of ref. [23], we can solve eq. (21). The solution is given by \( \Delta_1^{\uparrow \uparrow} = \Delta_k \exp \left( \pm 2i\ell \theta_k \right) \), and \( \Delta_2^{\uparrow \downarrow} = \Delta_k \exp \left( \mp 2i\ell \theta_k \right) \), where \( \Delta_k \) is a function of \( k = |k| \) and \( \theta_k = \arctan k_y/k_x \). Here, \( 2\ell \) is the relative angular momentum of the Cooper pair and is not equal to zero. The gap of superconductivity \( \Delta \mu \) is of the order of Fermi energy \( \epsilon_F \) and the coherence length of superconductivity \( \xi_{SC} \) is given by \( \xi_{SC}/a \sim 4\epsilon_F/\Delta \mu \). Here, \( x \) is the hole concentration. Since \( \Delta_k \) is of the order of \( \epsilon_F \), this value may be smaller than the average distance between holes: \( \sim a/\sqrt{x} \). From eq. (18), we obtain
\[ \Delta_1^{\uparrow \downarrow} = -\Delta_1^{\downarrow \uparrow} = \Delta_k \cos \left( 2\ell \theta_k \right). \]
(22)
Since the smallest \( \ell \) is realized in the ground state, we set \( \ell = 1 \). In this case, eq. (22) describes the \( d_{x^2-y^2} \) pairing state because \( \cos \left( 2\theta_k \right) = (k_x^2 - k_y^2)/k^2 \). For the triplet pairing case, we find that \( \Delta_1^{\uparrow \downarrow} = \Delta_1^{\downarrow \uparrow} = \left( \Delta_2^{\uparrow \downarrow} + \Delta_2^{\downarrow \uparrow} \right)/2 \), and \( \Delta_1^{\uparrow \downarrow} = \Delta_1^{\downarrow \uparrow} = 0 \). However, such a pairing state is not stable in the bulk of the system because the d-vector satisfies \( d_k \parallel \hat{e}_y \), that is, the spins of Cooper pairs lie in the plane perpendicular to the y-axis. As a result, the pairing state has spin-singlet and \( d_{x^2-y^2} \) symmetry.

There is also some contribution to the pairing mechanism from other spin fluctuations, which may be characterized by the Maxwell term: \( \sim -\frac{1}{2} \left( \partial_\mu A_\mu - \partial_\alpha A_\alpha \right)^2 \) in the gauge field description because coupling to the spin system is mediated by the gauge field \( A_\mu \). Meanwhile, our spin fluctuation is characterized by the Chern-Simons term. Since there is an extra derivative for the former compared with the latter, our mechanism may be more dominant in the long wavelength and low-energy limit than other spin fluctuation mechanisms. Moreover, the Chern-Simons term only exists in the 2+1 dimension. Therefore, our spin fluctuation is unique to the 2+1 dimension. In contrast, the Maxwell term exists in any dimension.

For the application to the orthorhombic phase of La2−xSrxCuO4, [11] we require one more transformation after transformation \( \left( \mathbf{1} \right) \), that is, \( c_{(odd,even)} \rightarrow i\sigma_z c_{(odd,even)} \), and \( c_{(even,odd)} \rightarrow -i\sigma_z c_{(even,odd)} \). Also in this case, the value of \( \theta \) is given by eq. (12) and the pairing state has spin-singlet and \( d_{x^2-y^2} \) symmetry.

In summary, we have studied a model of the CuO2 plane with buckling and have shown that the Chern-Simons term for the gauge field, which describes the fluctuation of the spin system, is induced. Through this Chern-Simons term, the doped hole behaves like a skyrmion or anti-skyrmion excitation depending on its spin or isospin, that is, whether it resides on the A-sublattice or B-sublattice. After the antiferromagnetic long-range order is destroyed by the skyrmion excitations, the Chern-Simons term becomes dominant for long wavelength and low-energy physics and leads to the spin-singlet \( d_{x^2-y^2} \) superconducting state.

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