Effect of Coriolis force on the self-gravitational instability of dusty plasma in the presence of magnetic field

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Abstract. In view of the importance of Coriolis force in an astrophysical context, the problem of self-gravitational instability of dusty plasma in the presence of magnetic field is investigated. Equations of the problem are stated and the dispersion relation has been derived with the help of linearized perturbation equations. We find that the Jeans criterion of instability remains valid but the expression of the critical Jeans wave-number is modified. Mathematical calculations have been performed and some figures are plotted between the growth rate of instability and wave numbers. From the curves, it is found that dust sonic speed has a stabilizing effect.

1. Introduction

The importance of dusty plasma in understanding the problem of star formation has increased the attraction and attention of scientists in the recent past. The experimental observations from satellite and space studies have revealed the presence of charged dust grains in terrestrial atmosphere and also in some laboratory devices. The dust grains also exist in comets, planetary rings, asteroids and the magnetosphere, tokamaks MHD generators and plasma etching techniques are the examples of laboratory plasma in which dust particles present. The probable reason of anomalous scattering of radio waves in the ionosphere may be due to the dust grains in that region. The size of dust particles may range between .05µm to 10µm. When the grain average separation is greater in comparison to plasma Debye length, then we call it “dust in plasma”, otherwise “dusty plasma”. The role of dust grains is important for the process of formation of stars and planets. The collapsing of dusty plasma into the small clouds of mass less than 1 MΘ has been discussed by Horedt [1], Alfvén [2] and Carlqvist [3]. The analyses of dust related problems with the three-fluid model were studied by D’Angelo et al. [4] and D’Angeloet al. [5]. Many researchers [6-11] have analyzed in this field. In all the above studies the gravitational instability of the plasma system with rotating charged dust particles has not been investigated. Therefore, in the present work, we shall discuss the effect of charged dust grains on the gravitational instability of plasma system.
2. Basic equations and dispersion relation
To study the present problem, we consider a three component fluid model of dusty plasma. This model consists of electrons, ions and charged dust particles subjected to an external field \( \vec{B}_0 \). As a simplified model, we assume that dust grains have uniform mass, behave like a point charge. The self-gravitation for both ions and dust particles is considered by taking Poisson’s equation for each. The momentum equation for each of dust, ion and electrons are

\[
m n D v = Z n e E + Z n e (v \times B) - \gamma T n \n + 2 m n (v \times \Omega)
\]

(1)

\[
m n D v = n_0 E + n_0 (v \times B) - \gamma T n \n + m n \n \n U_i
\]

(2)

\[
0 = -n_0 E - \frac{n_0}{c} (v \times B) - \gamma T n \n
\]

(3)

where \( n_j, v_j, m_j, T_j \) and \( \gamma_j \) are, respectively, the total number density, velocity, mass, temperature and ratio of specific heat of the particle species \( j (j = e \) for electrons and \( i \) for ions). The gravitational potential for ion and dust are denoted by \( U_i \) and \( U \), respectively. The terms \( n, v, m, T, \) and \( \gamma \) are the number density, velocity, mass, temperature and ratio of specific heat for dust grains, respectively. The Coriolis force acting on the dust fluid is \( 2 m n (v \times \Omega) \). The value of \( Z \) is taken positive or negative according to the positively or negatively charged dust particles. The numerical value of \( \gamma, \gamma_e, \) and \( \gamma_i \) are to be appropriately chosen. For dust particles and ions, the continuity equations and Poisson equations are

\[
\sum n_j + [\nabla \cdot (n_j v_j)] = 0 (4), \quad \sum \frac{\partial n_j}{\partial t} + [\nabla \cdot (n_j v_j)] = 0 (5), \quad \nabla \times U = -4 \pi G m n \n \]

(6)

\[
\nabla \times U_i = -4 \pi G m n_i \n
\]

(7)

The electromagnetic fields \( E \) and \( B \), are given by Maxwell equations coupled to the equations (1)-(3). By assuming quasi-neutrality the displacement current for low frequencies the simplified form of these equations can be written as

\[
\nabla \times B = \frac{4 \pi e}{c} (n_0 v_e + Z n v - n_e v_e) \quad (8), \quad \nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t} \quad (9), \quad n_e = n_i + Z n
\]

(10)

The value of \( \nu_e \) from equation (8) is substituted in equation (3) to obtain an expression for \( E \) i.e.

\[
E = -\frac{n_i}{c n_e} v_i \times B - \frac{Z n}{c n_e} v \times B + \frac{1}{4 \pi n_e} (\nabla \times B) \times B - \frac{\gamma T_e \nabla n_e}{n_e} \nabla n_e
\]

(11)

By eliminating \( E \) from equation (11) and using equation (10), equation (1) and (2) can be rewritten as

\[
m n D v = \frac{Z n}{4 \pi n_e} (\nabla \times B) \times B + \frac{Z n}{c n_e} (v - v_i) \times B - Z \gamma T e \nabla n_e - \gamma T n \n + mn \n \n U + 2 m n (v \times \Omega)
\]

(12)

\[
m n D v = \frac{n_i}{4 \pi n_e} (\nabla \times B) \times B - \frac{Z e}{Z n} n_0 n_i (v - v_i) \times B - \frac{\gamma T_e \nabla n_e}{n_e} \nabla n_e - \gamma T \n \n \n + m n \n \n U_i
\]

(13)

After eliminating \( E \) between equation (1) and (9), we have

\[
\frac{\partial B}{\partial t} = \nabla \times (v \times B) + \frac{2 m c}{Ze} \nabla \times (v \times \Omega)
\]

(14)

where we have assumed that for low-frequency wave \( \omega << Ze B_0 \) so the term \( (mc/Ze) \nabla \times (Dv) \) can be neglected in comparison to the first term. The set of equations (4), (7), (10) and (12)-(14) represents completely a self-gravitating system of magnetized rotating dusty plasma. In real, the dust particle is much heavier than ions so the ion inertia can also be neglected. With this approximation, we get by adding equations (12) and (13) and using equation (10).

\[
m n D v = \frac{1}{4 \pi} (\nabla \times B) \times B - (\gamma T e + \gamma T) \n \n + mn \n \n U + 2 m n (v \times \Omega) + m n \n \n U_i
\]

(15)

Equation (13) yields with the above approximation

\[
\sum n_j + [\nabla \cdot (n_j v_j)] = 0 (4), \quad \sum \frac{\partial n_j}{\partial t} + [\nabla \cdot (n_j v_j)] = 0 (5), \quad \nabla \times U = -4 \pi G m n \n
\]

(6)

\[
\nabla \times U_i = -4 \pi G m n_i \n
\]
\[ \nu_L = \frac{1}{B^2} B \times (\nabla \times B) - \frac{1}{n_e 4\pi n B^2} B \times [(\nabla \times B) \times B] - \frac{c n_e m_i}{Z n_e B^2} B \times \nabla U_i \frac{c n_e y_i T_i}{Z n_e B^2} B \times \nabla n_e. \quad (16) \]

In equation (16) the last two terms appear due to the thermal effects. Moreover, the first term in the equation corresponds to \( E \times B \) drift and is a dominant one; therefore, we neglect the thermal terms. We substitute the value of \( \delta n \) in equation (5) to get
\[
\frac{\partial n}{\partial t} + \nabla \cdot \left[ \frac{n_e}{B^2} B (\nabla \times B) - \frac{c}{Z e} \frac{n_i}{4\pi n B^2} B \times [(\nabla \times B) \times B] - \frac{c n_e}{Z n_e B^2} B \times \nabla U_i \right]. \quad (17)
\]

### 3. Dispersion Relation

Equations (4), (14), (15) and (17) can be used as the basic equations for the considered self-gravitating dusty plasma system. These equations are linearized by assuming that perturbed quantity varies as \((k \cdot x - \omega t)\). The perturbation state is given by

\[ n = n_0 + \delta n, \quad n_i = n_{i0} + \delta n_i, \quad n_e = n_{e0} + \delta n_e, \quad B = B_0 + B, \quad \nu = \nu_0 + \nu_1, \quad U = U_0 + \delta U, \quad D = \frac{\partial}{\partial t} + \nabla \cdot \nu \]

The subscripts “0” and “1” denotes the equilibrium and the perturbed quantities, respectively. On solving these equations the general dispersion relation is obtained as

\[ \omega^2 = (V_s^2 + C_s^2 - 4\pi G \frac{m^2 n_i^2 + m^2 n_e^2}{mn_0}) + \Omega B_0 - 2\Omega V_A^2 \quad (18), \quad V_A = \frac{B_0}{(4\pi G n_0)^{1/2}} \left[ \frac{\gamma n_e y_i n_i T_i + \gamma n_e y_e T_e}{mn_0} \right]^2 \]

represents the Alfvén velocity in terms of dust particles mass density and \( C_s \) represents the sound speed as the ratio of total particle pressure to the dust mass density. The dispersion relation (18) shows the combined influence of temperature, sound speed, Alfvén speed in terms of the dust particle mass density and rotation parameter on the self-gravitational instability of a dusty plasma. The present results are also similar to that of Rao [10] in the absence of self-gravitational and rotation in that case. Thus with these correlations, we find that the dispersion relation (18) is modified due to the combined effects of rotation, sound speed, Alfvén velocity in terms of the dust particle mass, rotation and self-gravitational of the system.

The system is unstable for \( k < k_{ji} \), where \( k_{ji} \) is modified Jeans critical wave number and it is given as

\[ k_{ji} = \left[ \frac{4\pi G}{(V_s^2 + C_s^2 - \Omega B_0 + 2\Omega V_A^2)} \left( \frac{m^2 n_i^2 + m^2 n_e^2}{mn_0} \right) \right]^{1/2} \quad (19) \]

Thus, we note that the Jeans criterion is modified due to the inclusion of rotation and dust particles in the system and the critical Jeans wave number depends on the rotation and dust mass density of dust along with the density of ions and electrons. In the absence of rotation, the relation (18) reduces to that of critical wave number already obtained by Chhajliani and Parihar [11]. Thus, we find the expression of the critical Jeans wave number which is modified due to the presence of rotation, sound speed and Alfvén velocity in terms of the dust particle mass density. If we ignore the self-gravitation of the system then we obtain a fast Alfvén wave of phase velocity \( V_{A}^2 + C_s^2 \). If fluid pressure dominates the magnetic pressure then wave turned into acoustic wave moving with velocity \( C_s \). If the magnetic pressure is much greater in comparison to the fluid pressure then this turns as Alfvén wave with velocity \( V_A \). The Alfvén velocity depends on the dust mass density. We write the dispersion relation (19) in non-dimensional form in terms of self-gravitational as

\[ \omega^* = V_{A}^2 + V_{s}^2 + k^2 (1 + \delta) - \Omega V_A^* \quad (20) \]

where the various non-dimensional parameters are defined as

\[ \omega^* = \frac{\omega}{(4\pi G \rho_0)^{1/2}}, \quad V_{A}^* = \frac{B_0^2 (4\pi G \rho_0)^{1/2} k^2}{4\pi G \rho_0} - \frac{\rho_i^2}{\rho_0}, \quad V_{s}^* = \frac{\gamma T/m}{4\pi G \rho_0} k^2, \quad \omega^* = \frac{\Omega}{(4\pi G \rho_0)^{1/2}} k^2 \]

\[ \Omega^* = \frac{(\rho_i \gamma T_i/m_i) + (\rho_e \gamma T_e/m_e)}{(4\pi G \rho_0)} k^2 \]
In figures 1 and 2, we have depicted the non-dimensional growth versus nondimensional wave number for various arbitrary values of the dust sonic speed (V_{sd}^*), rotational (\Omega^*) and the ratio of the density of dust particles (\delta). Figure 1 is plotted for the growth rate of an unstable mode (positive real root of (\omega^*)) against the wavenumber (k^*) with variation in the sound speed of dust (V_{sd}^*) parameter with other values as V_A^*=5, \delta=10 and \Omega^*=1. Hence the sonic speed of dust has a destabilizing influence on the growth rate of the instability.

Figure 1

Figure 2 curve plotted for the growth rate of unstable mode against the wave number taking \omega^* as y and k^* as x with variation in the \Omega^* other values as V_{sd}^*=4, \delta=10 and V_A^*=2. We find that the growth rate of the instability decreases with increase in \Omega^*. Hence the rotation parameter \Omega^* has a stabilizing influence on the growth rate of instability of the system.

5. Conclusions
We have investigated the self-gravitational instability of magnetized dusty plasma under the effect of rotation. The general dispersion relation is obtained, which is modified due to the presence of these parameters. This dispersion relation is reduced for propagation perpendicular to the z-axis and propagation parallel to the z-axis. We find that the Jeans criterion of instability remains valid but the expression of the critical Jeans wave number is modified. We obtain a self-gravitating Alfvén mode modified by the presence of rotation. We find that the condition of instability is affected by the presence of magnetic field and rotation. From the curves, it is found that rotation have a stabilizing effect but the sonic speed of dust has a destabilizing influence on the system.

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