Gaugino Condensation in the Early Universe

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Abstract

We examine the process of formation of the gaugino condensation within a Nambu-Jona-Lasinio type approach. We construct an effective Lagrangian description for the gaugino condensation which include a Weyl compensator superfield whose vacuum expectation value is related to the gaugino condensation.

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1 Introduction

There has recently been considerable attention focused on the study of supersymmetric models of elementary particle interactions. This is especially true in the context of grand unification theories, where remarkable studies have been done in the hope of solving the gauge hierarchy problem or unifying the gravitational interaction in the superstring formalism. Supersymmetric extension of the gravity(supergravity) seems necessary in introducing the soft breaking terms and making the cosmological constant vanish simultaneously. In supergravity models, the spontaneous breaking of local supersymmetry or super-Higgs mechanism may generate soft supersymmetry breaking terms that allow to fulfill such phenomenological requirements. However, the super-Higgs mechanism implies the existence of a supergravity breaking scale, intermediate between Planck scale($M_p$) and weak scale($M_W$). The intermediate scale is expected to be of $O(10^{13}\text{Gev})$. Here we expect that this intermediate scale is implemented by the mechanism of gaugino condensation in the hidden sector which couples to the visible sector by gravitational interaction. The effective action for the gaugino condensation is well studied by many authors[1], but we cannot solve the problem of the formation of the gaugino condensation in the early universe in the context of these effective theories. Usually there arises a potential barrier and the formation of the condensation is largely suppressed[3].

In this paper, we examine the formation of the gaugino condensation in the weak coupling domain and show that there can be a natural phase transition of gaugino condensation in the early universe.

2 Gaugino condensation in the early universe

In the standard superfield formalism of the locally supersymmetric action, we have:

$$ S = \frac{-3}{\kappa^2} \int d^8 z E \exp \left( -\frac{1}{3} \kappa^2 K_0 \right) + \int d^8 z \mathcal{E} \left[ W_0 + \frac{1}{4} f_0 W W \right] + h.c. $$

where we set $\kappa^2 = 8\pi/M_p^2$. In the usual formalism of minimal supergravity, the Weyl rescaling is done in terms of component fields. However, in order to understand the
anomalous quantum corrections to the classical action, we need a manifest supersymmetric formalism, in which the Weyl rescaling is also supersymmetric. It is easy to see that the classical action (2.1) itself is not super-Weyl invariant. However, the lack of the super-Weyl invariance can be remedied with the help of a chiral superfield \( \varphi \) (Weyl compensator).

For the classical action (2.1), the Kähler function \( K_0 \), the superpotential \( W_0 \) and the gauge coupling \( f_0 \) are modified:

\[
K_0 \rightarrow K = K_0 - 6\kappa^{-2} \text{Re} \log \varphi
\]
\[
W_0 \rightarrow W = \varphi^3 W_0
\]
\[
f_0 \rightarrow f = f_0 + \xi \log \varphi
\]

(2.2)

\( \xi \) is the constant that is decided by the super-Weyl anomaly.

Let us examine the simplest case. We do not include any chiral matter fields and moduli fields, and we fix the value of \( W_0 \) and \( f_0 \) as:

\[
W_0 = \mu^3
\]
\[
f_0 = \frac{1}{g^2}
\]

(2.3)

and rescale the field \( \varphi \) as:

\[
\tilde{\varphi} = \Lambda \varphi
\]

(2.4)

Finally we have:

\[
K = K_0 - 6\kappa^{-2} \text{Re} \log \left( \frac{\tilde{\varphi}}{\Lambda} \right)
\]
\[
W = \lambda \varphi^3
\]
\[
f = \frac{1}{g^2} + \xi \log \left( \frac{\tilde{\varphi}}{\Lambda} \right)
\]

(2.5)

where we set \( \lambda \equiv \mu^3/\Lambda^3 \). (We can set \( \mu = \Lambda = M_p \))

From the equation of motion for the auxiliary field of the super-Weyl compensator, we have the relation:

\[
\lambda \varphi^3 = \frac{1}{6} e^{-\kappa^2 \frac{k}{K}} \xi \lambda^\alpha \lambda_\alpha
\]

(2.6)

(Here we rescale the Weyl compensator and factor out \( e^{-\kappa^2 \frac{k}{K}} \) so that the equations looks like a familiar form.) The scalar potential is:

\[
V_0 = -3e^{\kappa^2 K} K^2 |W|^2
\]
\[
= -3e^{\kappa^2 K} K^2 \lambda^2 |\tilde{\varphi}^3|^2
\]

(2.7)
Equation of motion for the auxiliary field (2.6) suggests that the eq. (2.7) can be interpreted as a four-fermion interaction of the gaugino:

\[-\frac{1}{12} \kappa^2 |\text{Re} f|^2 \xi^2 |\lambda^\alpha \lambda_\alpha|^2\]  

(2.8)

where the factor of $\text{Re} f$ appears because we have rescaled the gaugino fields to have canonical kinetic terms. This four-fermion interaction becomes strong as $\text{Re} f$ reaches 0.

The strong coupling point is:

\[\tilde{\varphi}_s = \Lambda e^{-\frac{1}{g^2}}\]  

(2.9)

If the universe starts at very small gaugino condensation, $\tilde{\varphi}$ rolls down the hill (2.7). Near the origin, the shape of the scalar potential is not changed by 1-loop effect because small value of $\tilde{\varphi}$ suggests small interaction. As $\tilde{\varphi}$ gets large, 1-loop effect starts to dominate and lifts the effective potential. Finally, the rolling stops near the strong coupling point $\tilde{\varphi}_s$. This suggests that the gaugino condensation is naturally formed and the scale is expected to be $\sim \Lambda e^{-\frac{1}{g^2}}$.

For a second example, we include a dilaton superfield $S$. Now $f_0$ is not a constant and depends on the field $S$:

\[f_0 = S\]  

(2.10)

And the Kähler potential for the dilaton superfield is:

\[K_0 = -\log(S + \bar{S})\]  

(2.11)

Here we should include the effect of the existence of the dilaton field in the scalar potential. The tree level scalar potential is:

\[V_0 = h_S (G^{-1})^s \hat{S} h^S - 3e^{2G}\]  

(2.12)

here the auxiliary field of $S$ is:

\[h_S = e^{\hat{S}} G_S + \frac{1}{4} f_S \lambda^\alpha \lambda^\alpha\]

\[= \frac{1}{4} e^{\hat{S}} W 1 + \frac{6S_R \xi^{-1}}{S_R}\]  

(2.13)

where we set $G = K + ln(\frac{1}{4}|W|^2)$ and $S_R = S + \bar{S}$. The tree level potential can be given in a simple form:

\[V_0 = e^{2K} \frac{A}{16} |\tilde{\varphi}|^2\]  

(2.14)
where

\[ A = \lambda^2 \kappa^2 \left[ \left( 1 + \frac{6S_R}{\xi} \right)^2 - 3 \right] \]  

(2.15)

If \( S \) is small and \( A \) is negative at the early stage of the universe, we can expect that the tree level potential has the same characteristics as the simplest model. The field \( \tilde{\varphi} \) rolls down the hill, and finally reaches at the strong coupling point. In this case, there is no problematic potential barrier separating the weak and strong domain.

If the initial value of \( S \) is not so small and \( A \) is not negative, the situation changes. There appears a problematic potential barrier which suppresses the transition. We should also note that the global minimum is not stable for \( A > 0 \). However, we can be optimistic to expect that the instability of the true vacuum is not a problem because at the strong coupling point, an effective Lagrangian constructed in terms of the confined picture should be much more reliable. In ref.\[4\], cut-off parameter \( \Lambda_c \) has introduced to avoid this instability. Ref.\[4\] corresponds to a special case of our model.

Finally we will comment on the difference between ref.\[2\] and Nambu-Jona-Lasinio like approach. As is shown in ref.\[4\], our tree level Lagrangian can be cast to the same form as the effective superpotential in ref.\[2\], so one may wonder why the difference discussed above arises. The crucial difference is that \( \varphi \) in eq.(2.2) is an auxiliary field so the tree level scalar potential related to \( F_\varphi \) does not exist in our model. One may also wonder which is wrong. We cannot find a definite answer to this question, but we can say that the effective Lagrangian obtained from the confined picture may not be applied near the origin because such a limit should correspond to the deconfinement and may be singular.

3 Conclusion

We examined the formation of a gaugino condensation in the hidden sector of the supergravity models within a Nambu-Jona-Lasinio type approach.

First we considered a simplest model that contains only one gauge field and with no dependence on moduli fields. In this simplest model, we have shown that the phase transition can naturally occur and 1-loop effect can stabilize the vacuum.

We have also examined a model with a dilaton dependent coupling. If the initial value of \( S \) is small and remains small during the phase transition, we can expect that the phase
transition can take place. If the Hubble constant during inflation lifts the potential for $S$, small initial value of the dilaton field can naturally be realized and the phase transition can occur. The main difficulty is the stability of the global vacuum when $A > 0$. We should induce a cut-off scale by hand or merely expect that the non-perturbative effects like confinement would stabilize the vacuum.

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