Dynamical Casimir-Polder force between an atom and a conducting wall

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The time-dependent Casimir-Polder force arising during the time evolution of an initially bare two-level atom, interacting with the radiation field and placed near a perfectly conducting wall, is considered. Initially the electromagnetic field is supposed to be in the vacuum state and the atom in its ground state. The analytical expression of the force as a function of time and atom-wall distance, is evaluated from the the time-dependent atom-field interaction energy. Physical features and limits of validity of the results are discussed in detail.

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I. INTRODUCTION

The presence of vacuum fluctuations is a remarkable property of quantum field theory. In quantum electrodynamics these fluctuations are responsible of many observable effects such as Casimir and Casimir-Polder forces [1, 2, 3, 4]. They are long-range electromagnetic interactions between neutral objects such as atoms/molecules or macroscopic bodies. These interactions are related to the properties of the vacuum state of the electromagnetic field, in particular to the fact that vacuum field fluctuations may polarize the atoms or exert a pressure on macroscopic bodies, and thus induce electromagnetic interactions. Recently, Casimir forces have been measured with remarkable precision for different topologies [5, 6]. The atom-wall Casimir-Polder force has also been measured with precision [7, 8], and a good agreement with theoretical predictions has been obtained.

In this paper we consider the Casimir-Polder force between an atom and a perfectly conducting plate in a dynamical (i.e. time-dependent) situation. Recent papers have given special attention on the problem of time-dependent Casimir-Polder forces between two or more atoms or dynamical Casimir forces between macroscopic objects [9, 10, 11, 12]. Current experiments are trying to detect the real photons emitted in the dynamical Casimir effect [13]. In this paper we consider the dynamical Casimir-Polder force arising between a perfectly conducting plate and an initially bare ground-state atom during its dynamical self-dressing. We obtain an analytical expression of the time-dependent Casimir-Polder force, which is characterized by a timescale corresponding to the time taken by the field emitted by the atom to go back to the atomic position, after reflection on the conducting plate. This is physically sound, because after this time the interaction of the atom with the (reflected) reaction field starts. We also find that the dynamical Casimir-Polder force can be attractive or repulsive according to time and atom-wall distance (on the contrary, the stationary Casimir-Polder force for a ground-state atom is attractive for any atom-wall separation [2]).

This paper is organized as follows. In Section II we introduce the Hamiltonian describing our system in the multipolar coupling scheme and solve the relevant Heisenberg equations for atomic and field operators. In Section III we discuss a method for obtaining the stationary atom-wall Casimir-Polder energy and force, which is then generalized to the time-dependent case that is the main subject of the paper. In Section IV we also discuss the results obtained and their physical interpretation, as well as possible future developments.

II. THE HAMILTONIAN AND THE HEISENBERG EQUATIONS

We consider a two-level atom in front of an infinitely extended and perfectly conducting wall placed at \( z = 0 \). In the multipolar coupling scheme and within dipole approximation, our system is described by the following Hamiltonian [3]

\[
H = H_0 + H_I
\]

\[
H_0 = \hbar \omega_0 S_z + \sum_{kj} \hbar \omega_j a_{kj} \dagger a_{kj}
\]

(1)

\[H_I = -i\sqrt{\frac{2\pi\hbar c}{V}} \sum_{kj} \sqrt{k} [\mu \cdot f(kj, r)] \times (S_+ a_{kj} + S_- a_{kj} - S_+ a_{kj}^\dagger - S_- a_{kj}^\dagger)\]

(2)

where \( \omega_0 = c k_0 \) is the transition frequency between the atomic levels and \( f(kj, r) \) are the field mode functions evaluated at the atomic position \( r \), that take into account the presence of the wall. \( S_+, S_- \) are the pseudospin operators of the two-level atom, and \( \mu \) is its electric dipole moment operator.

In the presence of an infinite perfectly conducting wall placed at \( z = 0 \), the mode functions \( f(kj, r) \) are [3, 13]...
\[ (f_{k,j})_x = \sqrt{s(\hat{e}_{k,j})_x} \cos \left[ k_x (x + \frac{L}{2}) \right] \sin \left[ k_y (y + \frac{L}{2}) \right] \sin (k_z z) \]
\[ (f_{k,j})_y = \sqrt{s(\hat{e}_{k,j})_y} \sin \left[ k_x (x + \frac{L}{2}) \right] \cos \left[ k_y (y + \frac{L}{2}) \right] \sin (k_z z) \]
\[ (f_{k,j})_z = \sqrt{s(\hat{e}_{k,j})_z} \sin \left[ k_x (x + \frac{L}{2}) \right] \sin \left[ k_y (y + \frac{L}{2}) \right] \cos (k_z z) \]

These expressions of the field modes should be considered in the limit \( L \to \infty \).

In order to evaluate in the next Section the dynamical Casimir-Polder energy, we need expressions of field and atomic operators in the Heisenberg representation. Thus we can write the Heisenberg equations for atomic and field operators, and solve them iteratively at the lowest significant order. After straightforward algebra, we obtain the following expressions for the Heisenberg operators relevant for the calculations in Section III.

\[ e_{k,j}^{(0)} = e^{-i\omega_k t} a_{k,j}^{(0)} \]
\[ S_{+}^{(0)} = e^{i\omega_0 t} S_{+}^{(0)} \]
\[ e_{k,j}^{(1)}(t) = e^{-i\omega_k t} \sqrt{\frac{2\pi \omega_k}{\hbar V}} (\mu \cdot f(k, r)) \left[ S_{+}^{(0)} F(\omega_0 + \omega_k, t) + S_{-}^{(0)} F(\omega_k - \omega_0, t) \right] \]
\[ S_{+}^{(1)}(t) = -2S_{z}^{(0)} e^{i\omega_0 t} \sum_{k,j} \sqrt{\frac{2\pi \omega_k}{\hbar V}} (\mu \cdot f(k, r)) \left[ a_{k,j}^{(0)} F(\omega_0 + \omega_k, t) - a_{k,j}^{\dagger}(0) F(\omega_k - \omega_0, t) \right] \]

(\text{the superscripts indicate the perturbative order), where we have introduced the auxiliary function}

\[ F(x, t) = \int_{0}^{t} e^{ixt} dt = \frac{e^{ixt} - 1}{ix} \]

\[ F(d, k_{0}) = -\frac{\mu^{2}}{12\pi d^{4}} \left( 8k_{d}d - 6(2k_{0}^{2}d^{2} - 1)f(2k_{0}d) \right) \]

where \( f(z) \) and \( g(z) \) are the auxiliary functions of the sine and cosine integral functions [10]. This force is negative for any atom-wall distance \( d \), yielding an attractive force, and behaves as \( d^{-4} \) for \( d \ll k_{0}^{-1} \) (near zone) and as \( d^{-5} \) for \( d \gg k_{0}^{-1} \) (far zone) [2].

It is easy to show, independently from the explicit form of \( H_{f} \), that

\[ \Delta E^{(2)} = \frac{D \langle 0, \downarrow | H_{f} | 0, \downarrow \rangle_{D}}{2} \]

where \( | 0, \downarrow \rangle_{D} \) is the first-order dressed ground state of the system

\[ | 0, \downarrow \rangle_{D} = | 0, \downarrow \rangle - \sum_{| \psi \rangle \neq | 0, \downarrow \rangle} \frac{\langle \psi | H_{f} | 0, \downarrow \rangle}{E_{\psi} - E_{0}} | \psi \rangle \]

This relation shows that the second-order energy shift can be also obtained from the average value of the interaction Hamiltonian on the dressed ground state of the system [17].

We now consider the time-dependent situation, when the atom is initially in its bare ground-state. This state
is not an eigenstate of the total Hamiltonian and thus it evolves in time (dynamical self-dressing) \[18, 19\], finally yielding a time-dependent Casimir-Polder force between the atom and the wall. In order to obtain the dynamical (i.e. time-dependent) Casimir-Polder energy-shift, and then the dynamical force, we can use a generalization of the second order in the atom-field interaction we obtain

\[ H^{(2)}_f(t) = -\frac{2\pi ic}{V} \sum_{k,j} k |\mu \cdot f(kj, r)|^2 \left( S_+(0) e^{i\omega_0 t} + h.c. \right) \left[ S_+(0) \left( e^{-i\omega_k t} F(\omega_0 + \omega_k, t) - e^{i\omega_k t} F^*(\omega_k - \omega_0, t) \right) - h.c. \right] \\
+ \frac{4\pi ic}{V} S_+(0) \sum_{kk',jj'} \sqrt{kk'} (\mu \cdot f(kj, r)) (\mu \cdot f(k'j', r)) \left[ a_{k'j'}(0) \left( e^{i\omega_k t} F^*(\omega_0 + \omega_k, t) - e^{-i\omega_k t} F^*(\omega_k - \omega_0, t) \right) + h.c. \right] \\
\times (a_{kj}(0)e^{-i\omega_k t} - h.c.) \]

(10)

In analogy with the stationary case outlined above, we can evaluate the time-dependent Casimir-Polder atom-wall energy shift as

\[ \Delta E^{(2)}(d, t) = \langle 0, \downarrow | H^{(2)}_f(t) | 0, \downarrow \rangle \]

\[ = -\frac{i\pi c}{V} \sum_{kj} (\mu \cdot f(kj, r))^2 \left( F^*(\omega_0 + \omega_k, t)e^{i(\omega_0 + \omega_k)t} - F(\omega_0 + \omega_k, t)e^{-i(\omega_0 + \omega_k)t} \right) \]

(11)

where we have defined the differential operator \( D_m \) as

\[ D_m = -\frac{\mu^2}{4\pi d^3} \left[ 2 - 2 \frac{\partial}{\partial m} + \frac{\partial^2}{\partial m^2} \right] \]

(14)

A similar approach for obtaining a time-dependent Casimir-Polder energy has already been used to calculate the dynamical Casimir-Polder force between a ground-state and an initially bare excited atom \[3\] as well as dynamical three-body Casimir-Polder forces \[10\]. The expression \[11\] can be explicitly evaluated at any time \( t > 0 \) compatible with our second-order perturbative expansion, and for any atom-wall distance \( d \).

In the continuum limit, after some algebraic manipulation, eq. \[11\] becomes

\[ \Delta E^{(2)}(d, t) = -\frac{\mu^2}{4\pi d^3} \int_0^\infty -2x \cos(x) + (2 - x^2) \sin(x) \]

\[ \times \left( 1 - \cos[a(x + x_0)] \right) dx \]

(12)

where \( x_0 = 2k_0d \) and \( a = \frac{c}{2d} \). This expression can be written in the more compact form

\[ \Delta E^{(2)}(d, t) = \lim_{m \to 1} \left[ D_m \int_0^\infty \frac{\sin(mx)}{x + x_0} \left( 1 - \cos[a(x + x_0)] \right) dx \right] \]

(13)

\[ \mbox{In fact, we shall evaluate } H_f(t)/2 \mbox{ in the Heisenberg representation and its average value on the initial state, that is the bare ground state of the atom-wall system. Using the expressions } 14 \mbox{ for the Heisenberg operators into the expression } 2 \mbox{ of the interaction Hamiltonian, at the second order in the atom-field interaction we obtain} \]
\[ F(d, t, k_0) = \frac{\partial (\Delta E^{(2)})(d, t)}{\partial d} = -\frac{\mu^2}{12\pi d^4} \left[ 8k_0d + \frac{16d^3k_0(c^2t^2 - 8d^2)\cos(ck_0t)}{(c^2t^2 - 4d^2)^2} \right] + \]
\[ -\frac{4d^3t^2 \sin(k_0ct)}{(4d^2 - c^2t^2)^2} \left[ 16d^4 \left( 2k_0^2d^2 - 9 \right) - 16c^2t^2d^2(k_0^2d^2 - 2) + c^4t^4 \left( 2k_0^2d^2 - 3 \right) \right] + \]
\[ + \left( Ci[k_0(2d + ct)] - 2Ci(2k_0d) + C[i[k_0(2d - ct)] \right] \left[ 2k_0d \cos(2k_0d) \left( 3 - 2k_0^2d^2 \right) - 3(1 - 2k_0^2d^2) \sin(2k_0d) \right] + \]
\[ + \left( Si[k_0(2d + ct)] - 2Si(2k_0d) + S[i[k_0(2d - ct)] \right] + \left( 3(1 - 2k_0^2d^2) \cos(2k_0d) + 2k_0d(3 - 2k_0^2d^2) \sin(2k_0d) \right] \]

where \( Si(x) \) and \( Ci(x) \) are respectively the sine- and cosine integral functions \[15\]. The parameter \( l \) in \[15\] is 0 for \( a < 1 \) and 1 for \( a > 1 \). This expression clearly shows the divergence at \( t = 2d/c \) discussed above.

Let now discuss some features of physical relevance of the expression obtained for the dynamical Casimir-Polder force. At \( t = 0 \) the force vanishes, due to the initial condition of a bare state. For successive times the forces increases with an oscillatory behaviour both in time and space, with scales given by \( k_0 \). Depending on the time \( t \) and the atom-wall distance \( d \), the force can be positive (attractive) or negative (repulsive). This is a new feature compared to the stationary case for the ground-state atom, where the force is always attractive. Figure \[11\] shows a plot of the Casimir-Polder force for a fixed value of \( d \) as a function of \( t \) for \( a < 1 \), that is \( ct < 2d \). The mentioned oscillations of the value of the force are evident. An oscillatory behaviour of the force occurs also for \( a > 1 \), that is for \( ct > 2d \), as shown in Figure \[8\]. This plot shows also that for large \( t \) the force settles to a negative value, asymptotically yielding a static attractive force. From \[13\] or \[15\] it is easy to show that the asymptotic value of the force coincides with that obtained with a time-independent approach. This indicates that, at large times, the atom is fully dressed and thus its interaction with the wall is the same as in the static case. It is also worth mentioning that the fact that the force is not zero for \( t < d/c \), i.e. before a light signal from the atom can reach the wall, should not surprise. In fact, for \( t > 0 \) the atom immediately “knows” of the existence of the wall, because it interacts with the field modes \[3\] which incorporate the presence of the conducting wall.

IV. CONCLUSION

In this paper we have considered the dynamical (time-dependent) Casimir-Polder force between an initially bare ground-state atom and a perfectly conducting wall. An analytical expression of the time-dependent force has been obtained, which shows that, contrarily to the well-known stationary case of an attractive force for any atom-wall distance, the force oscillates in time from attractive to repulsive values. The scale of these oscillations is related to the atomic transition frequency. A characteristic timescale of the dynamical Casimir-Polder force is shown to be twice the time taken by a light signal to cover the atom-wall distance. This is physically understandable, because this is the time taken by the field emitted by the atom to go back to the atomic position, after being reflected on the wall, and interact with its source. Work in progress concerns the dynamical atom-wall Casimir-Polder force in the case of an initially excited atom.

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FIG. 1: The dynamical Casimir-Polder force for $t < 2d/c$, that is before the back-reaction time. Units are such that $c = 1$ and $k_0 = 1$. The atom-wall distance is $d = 10$, thus the back-reaction time is $t = 20$. 
FIG. 2: The dynamical Casimir-Polder force for $t > 2d/c$, that is after the back-reaction time. Same units and parameters of Fig. 1 have been used.