Social Influences in the Voter Model: the Role of Conformity

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Abstract

We introduce a model to study the effects of social influences in opinion dynamics. In particular, we analyze the voter model, from a socio psychological perspective, by considering the role of conformity. Conformity is a central issue in social psychology as it represents one of people’s behaviors that emerge as result of their interactions. We introduce a voter model where agents, linked in a network, change their opinion according to those of their neighbors and to their degree of conformity. In particular, agents can behave as conformists or non-conformists. In the former case, agents change opinion according to that of the majority of their social circle (i.e., the majority of their neighbors); in the latter case, they do the opposite, i.e., they assume the opposite opinion. We perform a computational study of the proposed model, with the aim to analyze the role of conformity in the voter model. Moreover, we want to investigate whether it is possible to achieve some kind of equilibrium or of order in the system. The two more interesting results are that the amount of non-conformist agents in the population plays a central role in these dynamics, and the topology of the agent network has not a prominent role in the proposed model. Finally, we study the outcomes of the model by considering that agents can change also their degree of conformity over time, i.e., they can turn their behavior from conformist to non-conformist and vice-versa.

Keywords:

toer model, opinion dynamics, agent-based models

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1. Introduction

During last years opinion dynamics have been investigated by several authors. Just to cite a few, Krapivsky and Redner proposed a spin model for studying the dynamics of majority rules, Galam introduced several sociophysics models, Holme and Newman analyzed the opinion spreading in terms of a nonequilibrium phase transition, and Bianconi and collaborators proposed a model to consider the role of social networks in these dynamics. The voter model represents one of the most famous models of opinion dynamics. Usually, this model considers a set of interacting agents, provided with a state that represents their opinion. In so doing, it is possible to perform computational studies to analyze the evolution toward consensus in the presence of different opinions. Agent-based models allow to study interesting phenomena in opinion dynamics (see for instance) or, more in general, in social dynamics. We put our attention on the dynamics of the voter model from a socio-psychological perspective. Social psychology provides fundamental theories to study interactions among individuals in social contexts (for instance see). In this work, we analyze the role of conformity, an important behavior of individuals that emerges as result of their interactions, by a simple implementation of the voter model. In particular, we consider a system with only two opinions (i.e., two possible states), and we provide agents with a degree of conformity, i.e., they can be conformists or non-conformists. Conformist agents assume the opinion of the majority of their neighbors, whereas the non-conformist agents do the opposite. We perform a computational study of the proposed model by considering different conditions, as the topology of the agent network and the amount of non-conformist agents in the population. Moreover, we analyze the system by considering agents that can change their degree of conformity over time. In particular, at each time step, agents decide whether to be conformists or non-conformists, depending on a score that they gain during the evolution of the system. The main result is that conformity is an important behavior in these dynamics as it strongly affects the outcomes of the proposed model. Moreover, it is worth to observe that we found very small differences between results achieved varying the topology of the agent network. The remainder of the paper is organized as follows: Section introduces the model for studying the role of conformity. Section shows results of numerical simulations.
Eventually, Section 4 ends the paper.

2. Conformists versus Non-Conformists

We introduce a simple voter model, considering two possible opinions, where agents interact over a network. Agents have an opinion, mapped to a state $s = \pm 1$ and, in addition, they are provided with a degree of conformity that represents their individual behavior. In particular, an agent can be conformist or non-conformist. Conformists modify their state (i.e., opinion) according to the majority of their neighbors, whereas non-conformists do the opposite, i.e., they follow the minority. Therefore, conformist agents change state over time as follows:

$$s_i(t+1) = \begin{cases} +1 & \text{if } \sum_{j=1}^{n_i} s_j(t) > 0 \\ -1 & \text{if } \sum_{j=1}^{n_i} s_j(t) < 0 \\ s_i(t) & \text{if } \sum_{j=1}^{n_i} s_j(t) = 0 \end{cases}$$

with $s_i(t)$ state of the $i$th agent at time $t$, $n_i$ number of neighbors of the $i$th agent and $s_j(t)$ state of the $j$th agent, linked with the $i$th agent. On the other hand, non-conformist agents follow an opposite rule for changing their state:

$$s_i(t+1) = \begin{cases} +1 & \text{if } \sum_{j=1}^{n_i} s_j(t) < 0 \\ -1 & \text{if } \sum_{j=1}^{n_i} s_j(t) > 0 \\ s_i(t) & \text{if } \sum_{j=1}^{n_i} s_j(t) = 0 \end{cases}$$

In so doing, at each time step, agents compute their state depending on their degree of conformity and the opinions of their social circle. Figure 1 shows an example of the proposed model. Furthermore, agents have a score $P$ computed by comparing their state $s$ with that of the whole system, defined as $S = \sum_{i=1}^{N} s_i$ (i.e., the total sum of states). In particular, agents increases their score of +1 every time their state is in accordance with the value of $S$ (i.e. both positive or both negative). For instance, in the event the $j$th agent has the state $s_j = -1$ and the system state is $S = -120$, its $P_j$ increases; otherwise it decreases or, in the event $S = 0$, its score does not change. The individual score $P$ allows to evaluate whether it is better to behave as a conformist or as a non-conformist voter. Finally, we introduce a small variation of the proposed model in order to let agents to change also their degree of conformity over time. In particular, a conformist agent can become
non-conformist, and vice-versa, if its state is not in accordance with $S$ (i.e., the total sum of states). For example, let us consider an agent $j$, in the state $s_j = +1$, that plays as a conformist at time $t$. In the event $S = -5$ at time $t$, then it becomes a non-conformist at $t + 1$. It is worthy to highlight that, in this case, agents change degree of conformity just to increase their individual score, and not because of social influences generated by their neighbors. This variation of the model allows to observe if agents consider more convenient to behave as conformists or non-conformists, during the evolution of the system.

3. Simulations

We perform numerical simulations of the proposed model with two different topologies of the agent network, i.e., scale-free networks and small-world networks. Scale-free networks have been achieved by the Barabasi-Albert model\cite{17}, whereas small-world networks have been achieved by the Watts-Strogatz model\cite{18}. In particular, to generate small-world networks, we start from a 2-dimensional regular lattice with 6 neighbors per node, then we rewire with probability $\beta = 0.1$ each edge at random. Eventually, both networks (i.e., scale-free and small-world) have $N = 10^4$ agents, with an average degree $\langle k \rangle = 6$. As shown in Figure\cite{2}, the evolution of the system, in terms of the summation of states $S(t)$, is similar in both agent networks. In gen-

![Figure 1: A small network of conformist agents (i.e., green nodes) and non-conformist agents (i.e., yellow nodes). The inner numbers represent the state of agents at time $t$, whereas the red numbers represent the state of agents at time $t + 1$.](image)
eral, $S(t)$ fluctuates around an average value that is positive in the event there are a few non-conformist agents, whereas it fluctuates around zero, or around lightly negative values, as the amount of non-conformist agents increases. In scale-free networks, it is interesting the behavior of $S(t)$ during first time steps by using a few non-conformist agents. In particular, $S(t)$ has a rapid increase indicating that a great fraction of agents has a state $+1$, followed by a decrease of $S(t)$ up to a small positive average value. In order to compare the two different behaviors of agents, we use the score $P$ defined above. Therefore, varying the amount of non-conformist agents, and considering both agent networks, we analyze the value of $\langle P \rangle$ over time –see Figure 3. In general, results show that it is more convenient to play the voter model as a conformist agent, with the exception of the case where there are many non-conformist agents in the system (we recall that this information is not known by agents).

3.1. The Degree of Conformity as a Degree of Freedom

Now, we analyze the proposed model considering the degree of conformity as a degree of freedom, i.e., agents can change their behavior from conformists to non-conformists over time. In particular, agents define their degree of conformity in order to increase their individual score. Therefore, we define the following simple rule: at each time step, agents compare their status $s$ with the total summation $S$; in the event $s$ and $S$ have a different sign.
Figure 3: Value of the average scores $\langle P \rangle$, over time, achieved by conformist agents ($M$) and non-conformist agents ($m$) varying $f_m$, i.e., the fraction of non-conformist agents. On the left, results achieved in scale-free networks. On the right results achieved in small-world networks. Results are averaged over 20 different realizations.

(i.e., an agent has an opinion different from that of the majority of the whole population) they change degree of conformity. Figure 4 shows the variation of $S(t)$ in a population where agents can change also their degree of conformity. As before, it is interesting to note that, although we used two different network structures, the related outcomes are still very similar.

Figure 4: Summation of states $S(t)$, in the whole population of agents, over time. Each curve refers to a different population containing, at $t = 0$, a fraction $f_m$ of non-conformist agents, as indicated in the legend. On the left, results achieved in scale-free networks. On the right results achieved in small-world networks. Results are averaged over 20 different realizations.
In particular, the value of $S(t)$ is positive only for a low amount of non-conformist agents at $t = 0$. In order to evaluate the density of agents that follow the two behaviors, i.e., conformist and non-conformist, we analyzed their density over time –see Figure 5. We can observe, in both kinds of the agent network, the density of non-conformist agents falls to zero in the event their density is smaller than 0.6 at time $t = 0$. In general, results of this analysis illustrate that agents find more convenient to behave as conformists, in order to increase their individual score $P$, even if there are a lot of non-conformist agents at $t = 0$.

3.2. Discussion

We study a simple voter model with the aim to analyze the role of social influences in opinion dynamics. In particular, we consider the conformity, a behavior considered relevant in these dynamics by social psychologists. In the proposed model, agents have a state that represents their opinion, and they are also endowed with a degree of conformity that represents their behavior, i.e., conformist or non-conformist. We consider a system with two opinions, therefore agents have a state equal to $\pm 1$. At each time step, agents compute their state according to those of their neighbors and to their degree of conformity. Moreover, agents have a score $P$ that is computed by comparing their state $s$ with summation of states in the population $S$. 

Figure 5: Density of conformist agents ($M$) and of non-conformist agents ($m$) over time, varying the fraction $f_m$ of non-conformist agents at $t = 0$. On the left, results achieved in scale-free networks. On the right, results achieved in small-world networks. Results are averaged over 20 different realizations.
In particular, in the event their state is equal to that of the majority of the population, their $P$ increases of +1, otherwise it decreases (of −1). This score allows to evaluate if it is more convenient to behave as a conformist or as a non-conformist voter. In general, numerical simulations performed on scale-free and small-world networks show similar results. For instance, the value of $S(t)$, i.e., the summation of states in the population, reaches a steady-state characterized by small fluctuations around a small average value (see figure 2). This latter is usually positive, with the exception of systems having a high fraction of non-conformist agents in the population. As shown in figure 3 agents achieve a higher score when behave as conformists, also in this case, with the exception of populations characterized by the presence of many non-conformist agents. Moreover, as for the value of $S(t)$, there very small differences among results achieved in scale-free networks and those achieved in small-world networks. Finally, we analyzed the system allowing agents to change both their opinion and their degree of conformity. In particular, at each time step each agent decides whether to behave as conformist or as a non-conformist. Agents modify their degree of conformity by comparing their state $s(t)$ with the summation of states $S(t)$, i.e., they change behavior when the their state has a different sign (±) from that of $S(t)$, as they aim to increase their individual score $P$ over time. It is interesting to note that, in this case, the value of $S(t)$ is positive if the fraction of non-conformist agents (introduced at time $t = 0$) is lower that 0.5, whereas is null if this fraction is equal to 0.5 and, eventually, it is negative for fractions higher than 0.5 – see figure 4. In figure 5 we observe the density of conformist agents versus that of non-conformist ones, varying the fraction of non-conformists (introduced at $t = 0$). In general, in both considered network structures, the majority of agents behaves as a conformist. In the light of these results, we can state that conformity strongly affects the outcomes of a simple voter model, as it happens also in real scenarios, according to social psychology theories. Moreover, it is worthy to note that using different structures of the agent network the related results are similar.

4. Conclusions

In this work, we study the role of social influences in the voter model. In particular, we consider the conformity, an important social behavior analyzed by social psychologists. We propose an agent-based model, where interacting agents change their opinion according to those of their neighbors and to their
degree of conformity. In particular, agents can behave as conformists or non-conformists. In the former case, agents set their opinion according to that of the majority of their neighbors; instead, in the latter case, agents set their opinion by considering the minority of their neighbors. Results of numerical simulations, of the proposed model, clearly show that the conformity is an important character that influences these dynamics (as it happens in real scenarios). Moreover, we found small differences in the results achieved by using scale-free networks and small-world networks to connect the agents. In general, it seems more convenient to play the voter model as a conformist voter. Furthermore, in the event agents change also their behavior over time, from conformist to non-conformist (and vice-versa), they prefer to become as conformists. Finally, we deem useful to represent social influences or behaviors to study models of opinion dynamics because, as shown in this work by numerical simulations, they strongly affect these dynamics.

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