Validation of computational code UST3D by the example of experimental aerodynamic data

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Abstract. Numerical simulation of the aerodynamic characteristics of the hypersonic vehicles X-33 and X-34 as well as spherically blunted cone is performed using the unstructured meshes.
It is demonstrated that the numerical predictions obtained with the computational code UST3D are in acceptable agreement with the experimental data for approximate parameters of the geometry of the hypersonic vehicles and in excellent agreement with data for blunted cone.

1. Introduction
The author’s computational code UST3D (Un-Structure Tetrahedral 3-Dimensional) is intended for calculation of aerodynamic characteristics of hypersonic vehicles of complex geometry on the basis of the Navier-Stokes equations.

This computational code was created with the use of the explicit method of the splitting on physical processes. The simplest variant of the method oriented on gas dynamic problem for structured orthogonal grids has its origin in [1,2]. Recently such a method was applied for unstructured tetrahedral grids [3].

High efficiency of this method was demonstrated by numerical study of aerodynamic characteristics of hypersonic vehicles (HV) which are similar to X–43 and X–51 [3,4]. The mathematical modeling of aerodynamic characteristics was performed with the use of multi-nuclear working stations of the middle performance. The method of parallelization on the base of compiler 'Intel® Parallel Studio XE 2015’ was used.

Development of the splitting method by improvement approach for calculation of a decay of a discontinuity on edges of tetrahedral elements had resulted in decreasing the numerical dissipation [5]. However, unfortunately, the calculation time was increased about in three times. Nevertheless, it was concluded that there are reasons to use both these methods depending on the purpose of a numerical simulation.

It is well known that to achieve high degree of reliability of numerical predictions it is necessary to use validated computer codes. General goal of the present paper is the validation of the computational code UST3D by the examples of rebuilding aerodynamic coefficients of some hypersonic vehicles that were measured in experimental research on wind tunnels.

Experimental data for models of two hypersonic vehicles, X–43 [6] and X–51 [7], and for spherically blunted cone [8] are analyzed and numerically reconstructed in the given paper.
It should be noted that numerical technology of the validation of a computer code includes several significant stages. These are:

- Creation of electronic surface of investigated vehicles,
- Creation of computational grids for surface and volume,
- Numerical rebuilding of aerodynamic characteristics of aircrafts known from experimental measurements.

The electronic images of surfaces HV, investigated in present work, as well as the surface and volume computational meshes used in present work were created in [9], where numerical format of * . neu was used for the grids. Numerical approaches to creation of the computational meshes were based on works [10 – 12].

The computational code UST3D consists of three parts. The first part is the preprocessor, in which all necessary data for each elementary computational domain are collected. Each elementary computational volume of the tetrahedral is characterized by the following data:

- the unique number of each vertex of each side,
- the direction cosines of each side;
- area of the each side,
- volume,
- numbers of elementary volumes bounded with each of fourth edges,
- characteristics of the volumes bordering with the each surface (there are several kinds of the volumes: the volume inside the computational domain, the fictive volume outside the free surface of the computational domain, the fictive volume inside solid body).

The second part is the main computational solver, which realizes numerical simulation of the Navier-Stokes equations. The third part is the postprocessor, which is intended for graphical representation of obtained results of numerical simulation.

2. Aerodynamic characteristics of the X–33

Experimental data that are taken as the base for the validation are given in [7]. It was proposed that the semi-scale prototype of a rocket-based vehicle X–34 will demonstrate key design and operational aspects of a single-stage-to-orbit (SSTO), reusable launch vehicle (RLV). The X–33 concept had a lifting body shape with two integrated linear aerospike rocket engines and flies a suborbital trajectory to simulate important aero-thermodynamic aspects of ascent and reentry environments for a full-scale RLV.

The X–33 vehicle is a lifting-body delta platform with twin vertical tails, canted fins, and body flaps. The canted fins have a dihedral of 20 deg and a –8.58 deg incidence angle. Reference dimensions and the geometry details are given in [7], where two configurations of the X–33 vehicle were investigated. These are: the F–Loft and Rev–F configurations. The Rev–G configuration has minor modifications to the aft, upper surface of the vehicle.

We used experimental data obtained at two facilities of NASA Langley Research Center (LaRC) Aerothermodynamics Facilities Complex (AFC).

Nominal flow conditions realized in the 20–Inch Mach 6 and the 31–Inch Mach 10 Air Tunnels are presented in table 1.

| Table 1. Nominal flow conditions |
|----------------------------------|
| $M$ | 6 | 10 |
| $p_{\infty},$ erg/cm$^3$ | 11180. | 2397. |
| $T_{\infty},$ K | 62.1 | 48.9 |
| $\gamma$ | 1.4 | 1.4 |
| $T_{surf},$ K | 300. | 300. |
Table 2. Reference dimensions for the X–33 model

| Dimension | 0.007-scale model |
|-----------|-------------------|
| $S_{\text{ref}}$, cm$^2$ | 73.2 |
| $S_{\text{mid}}$, cm$^2$ | 28.2 |
| c.g.$S_{\text{ref}}$, cm | 8.91 |
| X$_{\text{nose}}$, cm | 5.64 |

The results of numerical simulation for the aerodynamic coefficients $C_L$ and $C_D$ and their comparison with available experimental data [7] are shown in figure 1. The coefficients of the lift-body force $C_L$ and of the drag-force $C_D$ data flow coordinate system are calculated as follows:

$$ C_L = C_x \cos \alpha - C_y \sin \alpha, \quad C_D = C_x \cos \alpha + C_y \sin \alpha, $$(1)

where $C_x = C_A$ and $C_y = C_N$ are axial force coefficient and normal force coefficient; $\alpha$ is the angle of attack.

Aerodynamic coefficients $C_x$ and $C_y$ are calculated as following:

$$ C_x = F_x / q_w S_{\text{mid}}, \quad C_y = F_y / q_w S_{\text{mid}}, $$

where $q_w = 0.5 \rho_w V_w^2$,

$$ F_x = \iint \left[ (p - p_w) \omega_x + \tau \omega_x \right] ds, $$

$$ F_y = \iint \left[ (p - p_w) \omega_y + \tau \omega_y \right] ds, $$

where $p$ and $p_w$ are the pressure near surface and in input flow; $\tau$ is the frictional force; that is tangential to elementary surface $ds$ with unit vector $\tau$; and $\omega_x, \omega_y, \omega_x', \omega_y'$ are the directional cosines:

$$ \omega_x = \cos (\mathbf{i} \cdot \mathbf{n}), \quad \omega_y = \cos (\mathbf{j} \cdot \mathbf{n}), $$

$$ \omega_x' = \cos (\mathbf{i} \cdot \mathbf{\tau}), \quad \omega_y' = \cos (\mathbf{j} \cdot \mathbf{\tau}), $$

where $\mathbf{i}, \mathbf{j}$ are the unit vectors of the Cartesian coordinate system, $\mathbf{n}, \mathbf{\tau}$ are the normal and tangent to the elementary surface.

For recalculation of the coefficients for dimensions of experimental models it is used the following formula

$$ C_{L,D}^* = C_{L,D} S_{\text{mid}} / S_{\text{ref}}. $$

All calculations were performed for the following set of parameters of the computational mesh: $N_{\text{el}} = 3275358$ is the total number of elemental volumes, $N_{\text{points}} = 569087$ is the total number of points, $N_{\text{surf,el}} = 72079$ is the number of tetrahedral elements on the streamlined surface, and $N_{\text{fict,el}} = 93160$ is the number of fictive elements.

The calculations were performed for half-space bounded by the plane $z=0$. Boundary conditions of the symmetry were used on this surface. The surface and volume meshes are shown in figure 2. Distribution of temperature near the surface is also shown in figure 1a.
Figure 1. Aerodynamic coefficients of drag (a), lifting (b), and lifting-to-drag relation (c) for the model X–33.

Figure 2 (a) Surface and (b) volume computational grid for the model X–33.
Distributions of Mach number ($a$), pressure ($b$), and temperature (in K; $c$) along the surface and in the plane $z=0$ at Mach numbers $M = 10$ and $M = 6$ and the angle of attack of 50 deg are shown in figures 3 and 4. Some disagreement between the experimental and calculation data is explained by the differences between real experimental geometry and the electronic geometry that was created on the basis of some images from scientific papers.

Differences between the real and the electronic geometries does not impedes to investigate the aerodynamic characteristics of the hypersonic vehicles, similar to the X–33, because created surface and volume grids allow to perform an analysis of gas dynamic functions similar to those shown in figures 3 and 4, and to perform cross-verification of various computational codes.

![Figure 3](image1.png)

**Figure 3.** Distributions of (a) Mach number, (b) pressure (in erg/cm$^3$), and (c) temperature (in K) along surface and in the plane $z=0$ at $M=10$ and angle of attack of 50 deg.
3. Aerodynamic characteristics of the X−34

Aerodynamic experimental data for the X-34 are presented in [6]. The model X−34 is a model of an unmanned suborbital vehicle, designed to be used as a flying test bed to demonstrate key vehicle and operational technologies applicable to future reusable launch vehicles (RLVs). It was presumed that the X−34 will be air-launched from a L−1011 carrier aircraft at approximately Mach number $M = 0.7$ and altitude of ~12 km. An onboard engine will be used for acceleration the vehicle to speeds exceeding $M = 7$ and altitudes ~76 km. Then an unpowered entry will follow including autonomous landing.

One can note that the X−34 platform is similar to the Space Shuttle Orbiter. Real construction (length) of the X−34 is approximately 16.5 m with the wingspan of 8.54 m. A double-delta wing with sweep angles with 80 and 45 deg is employed. The X−34 has been designed and build by the Orbital Sciences Corporation (OSC). The parameters of trajectory of the X−34 are presented in [6].
The most of the data in [6] were obtained with models representing the latest so called Outer-Mold-Line geometry. The models were built in two scales, namely, 0.018 scale (there were two similar models) and 0.033 scale.

The X−34 reference dimensions, which were used in present paper, are presented in table 3.

| Dimension       | 0.018-scale model |
|-----------------|-------------------|
| $S_{\text{ref}}$, cm$^2$ | 107.68            |
| $S_{\text{mid}}$, cm$^2$ | 24.0              |
| c.g., cm         | 19.2              |
| X, cm            | 24.83             |

The aerodynamic data of this paper were obtained with the use of three NASA Langley Research Centre facilities: the Low-Turbulence Pressure Tunnel – LTPT; the Unitary Plan Wind Tunnel – UPWT, and the 20–Inch Mach 6 Air Tunnel. Brief information about these facilities is in [6].

The nominal flow conditions borrowed from [6] and used in this paper are shown in table 4.

| Facility    | M   | $p_\infty$, erg/cm$^3$ | $T_\infty$, K |
|-------------|-----|------------------------|---------------|
| LTPT        | 0.4 | 850 000                | 315           |
|             | 0.8 | 623 000                | 288           |
|             | 0.9 | 561 000                | 280           |
|             | 1.05| 215 000                | 266           |
| UPWT        | 2   | 144 000                | 181           |
|             | 6   | 2 244                  | 60            |
| 20–Inch Mach 6 | 6   | 4 937                  | 61.7          |
|             | 6   | 10 322                 | 61.7          |
|             | 6   | 15 259                 | 63.3          |

All calculations in this paper were carried out for the X−34 0.018-scale model.

All calculations were performed for the following set of parameters of the computational mesh: $N_{\text{el}} = 4916505$ is the total number of elemental volumes, $N_{\text{points}} = 875947$ is the total number of points, $N_{\text{surf,el}} = 160292$ is the number of tetrahedral elements on the streamlined surface, and $N_{\text{fict,el}} = 221872$ is the number of fictive elements.

The calculations were performed for half-space bounded by the plane $z = 0$. Boundary conditions of the symmetry were used on this surface. The volume mesh is shown in figure 5.

![Figure 5](image-url)
The results of numerical simulation for the aerodynamic coefficients of lifting $C_L$ and drag $C_D$ for the model of X–34 are shown in figure 6. Experimental data obtained in [6] are also shown in this figure.

![Figure 6](image)

**Figure 6.** Aerodynamic coefficients of (a) drag and (b) lifting for the model X–34.

Distribution of pressure and Mach number along the surface and on the plane $z=0$ are shown in figure 7.

![Figure 7](image)

**Figure 7.** Distributions of (a) pressure (in erg/cm$^3$) and (b) Mach number for the model X–34 at angle-of-attack of $\alpha=16$ deg.

4. Aerodynamic characteristics of the spherically blunted cone

In the present paper the numerical simulation of the aerodynamic coefficients of the spherically blunted cone has been performed to validate the computational code UST3D. It is significant that the geometry of the blunted cone is known precisely.

Detailed experimental investigation of the aerodynamic coefficients of the spherically blunted cone has been performed in [8]. Force and moment tests were conducted in this work for sharp and spherical blunted cone configuration having half the cone angle of $90^\circ$ in the Langley 11-inch hypersonic tunnel at Mach number of 6.77, the Reynolds number per meter of $5.315 \times 10^6$, and the angle-of-attack range from $0^\circ$ to $180^\circ$. 
Calculations of the aerodynamic coefficients are performed with the use of equations (1) – (5). Schematic of the calculational task and a definition of the sign of aerodynamic moment are shown in figure 8.

A pitch moment for elementary surface element is defined as follows:

$$\Delta M_{i,z} = x_i \Delta F_{i,y} - y_i \Delta F_{i,x},$$

where $x_i$, $y_i$ are the coordinates of the $l$-th surface element, which are counted out from the origin of the laboratory frame of reference; $\Delta F_{i,x}$, $\Delta F_{i,y}$ are the projections of the surface force effecting on the $l$-th surface element.

The elementary moment of the pitch relative to the center of gravity is calculated as follows:

$$\Delta M_{i,z,c,g} = \Delta M_{i,z} - (x_{cg} \Delta F_{i,y} - y_{cg} \Delta F_{i,x}) = \Delta F_{i,y} (x_i - x_{cg}) - \Delta F_{i,x} (y_i - y_{cg}),$$

where $x_{cg}$, $y_{cg}$ are the coordinates of the center of gravity in the laboratory frame of reference. It should be noted that the longitudinal coordinate $x_{cg,ref}$ is counted from the critical point of the HV, therefore

$$x_{cg} = x_{cure} + x_{cg,ref},$$

where $x_{cure}$ is the longitudinal coordinate of the critical point. It was supposed that

$$y_{cg} = z_{cg,ref} = z_{cg} = z_{cg,ref} = 0.$$ 

In accordance with (6) the positive moment is counted in counter clock-wise direction. The coefficient of the pitch moment is calculated by the following formula:

$$m_z = \frac{\sum_{i=1}^{N_{ele}} \Delta M_{i,z,c,g}}{L_{ref} \cdot q_s S_{ref}},$$

here $L_{ref}$ is the reference length.

Initial conditions used in the calculations are presented in table 5.

| Table 5. Nominal flow conditions for blunted cone |
|-----------------------------------------------|
| $p_{\infty}$, erg/cm$^3$                      | 188 000 |
| $T_{\infty}$, K                               | 56.9    |
| $\gamma$                                     | 1.4     |
| $M_{\infty}$, K                              | 6.77    |

Reference dimensions for this model are shown in table 6.
The results of numerical simulation for the aerodynamic coefficients of lifting $C_L$, drag $C_D$, and pitch moment for the spherically blunted cone are shown in figures 9. Experimental data obtained in [8] are also shown in this figure.

**Table 6. Reference dimensions for blunted cone**

| Dimension | Unit | Value |
|-----------|------|-------|
| $R_N$     | cm   | 0.926 |
| $X_A$     | cm   | 5.000 |
| $Y_A = R_N \cos \alpha$ | cm | 0.914 |
| $X_B$     | cm   | 17.12 |
| $Y_B$     | cm   | 2.858 |
| $x_{nose}$ | cm | 4.219 |
| $x_{cg}$  | cm   | 11.298 |
| $x_{cg,ref}$ | cm | 7.079 |
| $L_{ref}$ | cm   | 5.715 |
| $S_{ref}$ | cm²  | 25.65 |

**Figure 9.** Aerodynamic coefficients of (a) drag, (b) lifting, and (c) pitch moment for the blunted cone at $M = 6.77$. 
All calculations were performed for the following parameters of the computational mesh: \( N_{el} = 686,145 \) is the total number of elemental volumes, \( N_{points} = 140,889 \) is the total number of points, \( N_{surf,el} = 37,116 \) is the number of tetrahedral elements on the streamlined surface, and \( N_{fict,el} = 102,556 \) is the number of fictive elements.

Distribution of Mach, pressure and temperature along the surface and on the plane \( z = 0 \) are shown in figure 10.

![Distributions of Mach number, pressure (in erg/cm³), and temperature (in K) along surface and on the plane \( z = 0 \) at \( M = 6.77 \) and angle of attack of 27 deg.](image)

**Figure 10.** Distributions of (a) Mach number, (b) pressure (in erg/cm³), and (c) temperature (in K) along surface and on the plane \( z = 0 \) at \( M = 6.77 \) and angle of attack of 27 deg.

**Conclusion**

Numerical simulations of the aerodynamic coefficients of the hypersonic vehicles X–33 and X–34 as well as for spherically blunted cone have been performed with the aim of the validation of the author’s computational code UST3D.

Experimental aerodynamic data used for the validation were obtained in wind tunnels of NASA Langley Research Center.
It is demonstrated that the numerical predictions obtained with the computational code UST3D are in acceptable agreement with the experimental data for approximate parameters of the geometry of the hypersonic vehicles and in good agreement with precise data for blunted cone.

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