Three-Loop Corrections to the Higgs Boson Mass and Implications for Supersymmetry at the LHC

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In supersymmetric models with minimal particle content and without left-right squark mixing, the conventional wisdom is that the 125.6 GeV Higgs boson mass implies top squark masses of $\mathcal{O}(10)$ TeV, far beyond the reach of colliders. This conclusion is subject to significant theoretical uncertainties, however, and we provide evidence that it may be far too pessimistic. We evaluate the Higgs boson mass, including the dominant three-loop terms at $\mathcal{O}(\alpha^3 \alpha_s^2)$, in currently viable models. For multi-TeV stops, the three-loop corrections can increase the Higgs boson mass by as much as 3 GeV and lower the required stop mass to 3 to 4 TeV, greatly improving prospects for supersymmetry discovery at the upcoming run of the LHC and its high-luminosity upgrade.

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Introduction. The Higgs boson, recently discovered at the LHC by the ATLAS and CMS Collaborations [1, 2], is now the subject of impressive precision studies. In particular, combining the results of all channels, the currently available data, consisting of 25 fb$^{-1}$ collected at $\sqrt{s} = 7$ and 8 TeV, constrain the Higgs boson mass to be

\begin{align}
\text{ATLAS (combined)} & \quad 125.5 \pm 0.2^{+0.5}_{-0.6} \text{ GeV} \quad \text{(1)} \\
\text{CMS (combined)} & \quad 125.7 \pm 0.3 \pm 0.3 \text{ GeV} \quad \text{(2)}
\end{align}

where the first uncertainties are statistical and the second systematic. Because the Higgs boson has been seen in purely leptonic and photonic channels without missing $E_T$, its mass is already known with a fractional uncertainty smaller than any of the quarks, providing a potentially stringent bound on ideas for new physics.

The Higgs mass measurement is especially important for supersymmetry. In supersymmetry, the Higgs quartic coupling is determined, at tree level, by the gauge couplings, removing this a priori free Standard Model parameter. The Higgs mass $m_h$ also receives large radiative corrections, which are functions of superpartner masses. As a result, $m_h$ provides useful guidance as to the mass scale of the superpartners, with implications for direct discovery prospects for supersymmetry at colliders. Unfortunately, this potential is currently clouded by theoretical uncertainties in the Higgs boson mass calculation, which are arguably much larger than the experimental uncertainties. In this study, we extend previous work by including the dominant 3-loop contributions to $m_h$ derived in Refs. [3–5], and we explore implications for supersymmetry discovery prospects at the LHC.

The Higgs Mass at 3-Loops. In supersymmetric models with minimal field content, the tree-level Higgs boson mass cannot exceed $m_Z \simeq 91$ GeV. The 1-loop contributions were explored long ago [7,9], and many studies now incorporate 2-loop contributions, available with public codes such as FEYNHIGGS [10–13], SOFTSUSY [13], SUSPECT [15], and SPHENO [16, 17].

The radiative corrections to the Higgs boson mass are most sensitive to the top squark sector. At tree-level, the top squark mass matrix is

\begin{equation}
(m^2_{L} + m^2_{R} + \Delta_{L}) \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix} = \begin{pmatrix} m_{t} X_t & m_{t} \Delta \beta \\ m_{t} \Delta \beta & m_{t} \Delta \beta \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix},
\end{equation}

where $X_t \equiv A_t - \mu \cot \beta$, $\Delta_{L} \equiv (\frac{2}{3} - \frac{2}{3} \sin^2 \theta_{W}) m_Z^2 \cos 2\beta$, and $\Delta_{R} \equiv \frac{2}{3} \sin^2 \theta_{W} m_Z^2 \cos 2\beta$. Diagonalizing this matrix gives the physical masses of the lighter stop $\tilde{t}_1$ and heavier stop $\tilde{t}_2$. The radiative contributions are maximized for heavy stops and large left-right mixing with $X_t/M_S \approx \sqrt{6}$, where $M_S = \sqrt{m_{tL} m_{tR}}$. This “maximal mixing” relation is valid at 1-loop; it is modified by higher-order corrections, but remains within $\sim 20\%$ of the 1-loop value. For $X_t \ll M_S$, however, conventional 2-loop analyses imply that the measured Higgs mass requires stops with masses $\sim 5 - 10$ TeV. If this is the characteristic mass scale of all squarks, they will be far beyond the reach of the LHC or any near-future collider.

To improve the accuracy of current estimates of $m_h$, we use here the program H3M [5]. Building on the 1- and 2-loop terms provided by FEYNHIGGS [10–13], H3M includes the roughly 16,000 diagrams that are the leading 3-loop corrections at $\mathcal{O}(\alpha^3 \alpha_s^2)$ [5, 6].

When evaluating $m_h$, special care has to be taken to use accurate numbers for the values of the input parameters entering the calculation, most notably the top quark mass $m_t$ and the strong coupling constant $\alpha_s$ in SUSY-QCD, renormalized in the $\overline{MS}$ scheme (i.e., using dimensional reduction and modified minimal subtraction), at a specific renormalization scale $\mu$. These must be calculated from the experimentally accessible values of the top quark pole mass and $\alpha_s(m_Z)$ in five-flavor QCD.
In the original version of H3M, the transition of $m_t$ from the on-shell to the \( \overline{\text{DR}} \) scheme could suffer from large logarithms if superpartners masses or renormalization scales $\mu$ are much larger than $m_t$. Since null results from the LHC increasingly favor this possibility, the program has been improved in the following way. First, we calculate $m_t(\mu)$ in five-flavor QCD in the \( \overline{\text{MS}} \) scheme using 4-loop running as implemented in the numerical package RunDec \cite{18}. This value is transferred to the \( \overline{\text{DR}} \) scheme via a finite renormalization at 3-loop order \cite{19} \cite{20}. Finally, the transition from five-flavor QCD to SUSY-QCD is performed using the 2-loop decoupling coefficient of $m_t$ \cite{21} \cite{22}. This procedure is faster, more robust, and more accurate than the old code. The new version of H3M is publicly available at http://www.ttp.kit.edu/Progdata/ttp10/ttp10-23.

Results as a Function of Weak-Scale Parameters. We now present results for the Higgs boson mass, including the 3-loop corrections described above, as functions of weak-scale supersymmetry parameters. We set $\tan \beta = 20$ so that the tree-level Higgs boson mass is within 1 GeV of its maximal value, and we consider nearly degenerate, unmixed stops, with $m_{\tilde{t}_L} = m_{\tilde{t}_R}$ and $X_t = 0$. The dependence on other parameters is relatively mild; we set $\mu = 200$ GeV, assume gaugino mass unification with $m_{\tilde{g}} = 1.5$ TeV, and set all other soft mass parameters equal to $m_{\tilde{t}_{L,R}} + 1$ TeV. For multi-TeV values of the sfermion masses, these models have scalar masses far heavier than gaugino and Higgsino masses.

The results are shown in Fig. 1. For $m_{\tilde{t}_i}$ in the range 1–10 TeV, 1-loop corrections raise the Higgs mass by 18 to 31 GeV, and 2-loop corrections raise the mass further by another 4 to 7 GeV. The experimental value of $m_h$ is apparently obtained for $m_{\tilde{t}_i} \sim 5$ TeV. However, the 3-loop effects raise the Higgs mass by another 0.5 to 3 GeV. The magnitude of the corrections decreases with increasing loop order, indicating a well-behaved, if slowly converging, perturbative expansion, and the size of the 3-loop corrections is consistent, within uncertainties, with the NLL analysis of Ref. \cite{23}. Clearly, however, the 3-loop corrections are still sizable, and they reduce the required top squark mass to 3 to 4 TeV, a reduction with potentially great significance for supersymmetry discovery, as we discuss below.

Ref. \cite{23} observes partial cancellations between leading logarithm terms of $\mathcal{O}(\alpha_t\alpha_s^2)$ and $\mathcal{O}(\alpha_s^2\alpha_s)$ in a particular scenario. We advocate a full calculation at $\mathcal{O}(\alpha_s^2\alpha_s)$ to investigate whether this behaviour is universal.

In Fig. 1 the width of the bands is determined by the parametric uncertainty induced by the uncertainty in the top quark mass and $\alpha_s$. It is dominated by the uncertainty in the top mass. The top mass has been constrained by kinematic fits in combined analyses of Tevatron \cite{24} and LHC \cite{25} data, and may also be stringently constrained in the future by cross section measurements (see, e.g., Ref. \cite{26}). For now, we consider the range $m_{t}^{\text{pole}} = 173.3 \pm 1.8$ GeV. The resulting parametric uncertainty is 0.5 to 2 GeV; it exceeds the experimental uncertainty and is comparable to that expected from 4- and higher-loop effects in the theoretical prediction.

In Fig. 2 we compare our results to those of 2-loop codes. The 2-loop results differ significantly from each other, with differences of up to 4 GeV for stop masses in the 1 to 10 TeV range shown. The 3-loop results are within this range for $\sim$ TeV stop masses, as found in Refs. \cite{23} \cite{26}. However, for multi-TeV stop masses, the 3-loop contributions may significantly enhance $m_h$.

Some of the differences between the 2-loop results can be explained by different default choices for the renormalization scale. They also differ in how the running top mass is extracted from its pole mass. This difference is formally of higher order \cite{27}. The different treatment of parameters also explains the difference between H3M’s 2-loop results and \textsc{FeynHiggs}. For example, \textsc{FeynHiggs} uses 1-loop running for $\alpha_s$ and $m_t$, which is formally correct since the 2-loop results are leading order in $\alpha_s$.

Results for mSUGRA and Implications for Supersymmetry at the LHC. To determine the implications of the 3-loop corrections for the LHC, we consider here the well-known framework of minimal supergravity (mSUGRA), defined in terms of GUT-scale parameters, for which detailed collider studies have been carried out.

![Graph showing Higgs boson mass as a function of $m_t$](image_url)
In Fig. 2 we show contours of $m_h$ with 3-loop corrections in two well-studied $(m_0, M_{1/2})$ planes of mSUGRA. To highlight the regions of parameter space preferred by $m_h$, at each point in parameter space, we define a theoretical uncertainty $\Delta_{th} \equiv \sqrt{(\Delta_{pert})^2 + (\Delta_{para})^2}$, where

$$\Delta_{pert} \equiv \frac{1}{2} \left| m_h^{(3\text{-loop})} - m_h^{(2\text{-loop})} \right|,$$

$$\Delta_{para} \equiv \left| m_h^{(\alpha_s=0.1177)} - m_h^{(\alpha_s=0.1184)} \right|. \quad (4)$$

The quantity $\Delta_{pert}$ is the estimated uncertainty from neglecting higher-order terms in the perturbation series. It is motivated by observing that the scale variation of the two-loop prediction underestimates the 3-loop corrections, and is typically in the 0.5 to 1.5 GeV range. The parametric uncertainty $\Delta_{para}$ arises dominantly from the uncertainty in the top quark mass. In the figure, we shade regions where the calculated $m_h$ is within $\Delta_{th}$ and $2\Delta_{th}$ of the experimental central value 125.6 GeV.

The positive 3-loop terms significantly impact the preferred range of superpartner masses and the prospects for supersymmetry discovery at the LHC. In Fig. 3 top panel, $A_0 = 0$ and stop mixing is negligible throughout the plane. Requiring that the theoretical prediction be within $2\Delta_{th}$ of the experimental central value, and imposing the further requirement that thermal relic neutralinos make up all the dark matter (the focus point region [22, 32]), scalar mass parameters as low as $m_0 \sim 4 - 5$ TeV, corresponding to stop masses as low as 3 to 4 TeV, and ghino masses as low as $m_3 \approx 2.8M_{1/2} \approx 2$ TeV are consistent with the measured Higgs mass. These are far lighter than the squark masses required if only 1- and 2-loop corrections to $m_h$ are included. Current bounds do not challenge this parameter space [28], but the 14 TeV LHC with 100 fb$^{-1}$ will already start probing the favored parameter space, and a high-luminosity upgrade to 3 ab$^{-1}$ may probe most of it [29]. The LHC reach was extrapolated from a study that used $\tan^2 \beta = 45$ [29] by...
transferring the \((m_{\tilde{q}}, m_{\tilde{g}})\) values on the reach contours to the space with \(\tan \beta = 10\). The sensitivities are determined by searches for multiple jets and missing energy along with a variable number of leptons and are expected to be approximately independent of \(\tan \beta\). Of course, lighter squark masses and brighter discovery prospects are possible if one relaxes the cosmological requirement.

If there is significant stop mixing, the implications may be even more dramatic. This is illustrated in Fig. 3 bottom panel, where \(A_0 = -2m_0\). With the 3-loop corrections included, the preferred region moves to \(m_{\tilde{q}}\) as low as 1 TeV, and the 2\(\sigma\) region even overlaps the region with the correct thermal relic density of neutralinos (the stau co-annihilation region). Current bounds exclude some of the favored region, but the 14 TeV LHC will probe most of it, and it will be explored fully by the LHC high-luminosity upgrade.

**Conclusions.** 3-loop contributions to the Higgs boson mass may be as large as 3 GeV in supersymmetric theories with multi-TeV superpartners. Given the extreme sensitivity of the stop mass to such changes, this lowers the preferred stop mass to as low as 3 to 4 TeV, with striking implications for supersymmetry discovery at the LHC. In models with a characteristic squark mass scale, these results imply that even without significant mixing or additional particles, 1st and 2nd generation squarks may be within reach of the 14 TeV LHC with 100 fb\(^{-1}\), with much more promising prospects for a high-luminosity upgrade. Given the rapidly diminishing experimental uncertainty on \(m_h\), these results highlight the importance of improved theoretical calculations of \(m_h\), incorporating improved determinations of the top quark mass, to refine the implications of the Higgs boson discovery for supersymmetry.

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