On Simple GP – Injective Modules

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ABSTRACT

In this paper, we study rings whose simple right R-module are GP-injective. We prove that ring whose simple right R-module is GP-injective it will be right s\pi-weakly regular ring. Also, proved that if R is N duo ring or R is NCI ring whose simple right R-module is GP-injective is S-weakly regular ring.

Keywords: Modules, weak regular rings, N duo ring, NCI ring.

1- Introduction

Throughout in this paper, R is associative ring with identity and all modules are unitary. For a subset X of R, the left(right) annihilator of X in R is denoted by l(X)(r(X)). If X={a}, we usually abbreviate it to l(a)(r(a)). We write J(R),N(R), N(R)=\sum_{i=1}^{n} N_i, P(R)=\cap_{i=1}^{n} P_i, for the Jacobson radical, the set of nilpotent elements, the nil radical (that means the sum of all nil ideals), prime radical (that means the intersection of all prime ideals) respectively.

A ring R is called NI if N(R)=0 [1]. A ring R is 2-primal if N(R) \subseteq P(R) [2]. A ring R is said to be semiprimitive if J(R)=0 [1]. An element a in the ring R said to be right (left) weakly regular if aRaR=0 (aRaR \neq 0) [12].

A right R-module M is called Generalized Principally injective (briefly, GP-injective) if for any a \in R, there exists a positive integer n such that a^n \neq 0 and any right R-homomorphism of a^n R into M extends to one of R into M [8]. Right GP-injective modules are called right YJ-injective modules by several authors [16].
2. Some Properties of Rings whose Simple Right \( R \)-module are GP-injective.

We give a different prove that proved by Kim, et. al. in [8].

**Theorem 2.1**

Let \( R \) be a ring whose every simple right \( R \)-module is GP-injective. Then \( R \) is semiprime.

**Proof:**

We shall show that is no nilpotent ideal in \( R \), if not, suppose there exists \( 0 \neq a \in R \) with \((aR)^2 = 0\), \( aRaR = 0 \), that means \( RaR \subseteq r(a) \), there exists a maximal right ideal \( M \) of \( R \) containing \( r(a) \), \( R/M \) is GP-injective. Hence, there exists an appositive integer \( n=1 \) such that \( a \neq 0 \) and any \( R \)-homomorphism of \( aR \) into \( R/M \) extends to one of \( R \) into \( R/M \), we define \( f: aR \rightarrow R/M \) such that \( f(ar) = r + M \) where \( r \in R \). We have to show that if \( f \) is well defined \( R \)-homomorphism, let \( x, y \in R \), \( a(x-y) = 0 \), \( (x-y) \in r(a) \subseteq M \), \( x+M=y+M \), \( f(ax)=x+M=y+M=f(ay) \), \( f(ax)=f(ay) \), so \( f \) is well defined right \( R \)-homomorphism, since \( R/M \) is GP-injective, there exists \( b + M \in R/M \) such that \( 1+M=f(a)=(b+M)(a+M)=ba+M \), \( 1+M=ba+M \), \( 1-ba \in M \), since \( ba \in RaR \subseteq M \), we get that \( 1 \in M \), which is a contradiction. Therefore, \( a=0 \). This shows that \( R \) is semiprime.

We give a different prove that is proved by Xue in [16].

**Proposition 2.2**

Let \( R \) be a ring whose simple right \( R \)-module is GP-injective. Then, \( R \) is semiprimitive.

**Proof:**

We shall show that if \( f(R) = 0 \), if not, there exists \( 0 \neq a \in J(R) \), then either \( Ra^n R + r(a^n) = R \), or not, if not, there exists a maximal right ideal \( M \) of \( R \) containing \( r(a^n) \), \( R/M \) is GP-injective, there exists a positive integer \( n \) and \( a^n \neq 0 \). Any \( R \)-homomorphism of \( a^n R \) into \( R/M \) extends to one of \( R \) into \( R/M \), Let \( f: a^n R \rightarrow R/M \) such that \( f(a^n r) = r + M \), where \( r \in R \), we have to show that if \( f \) is well defined, let \( a^n x = a^n y \) where \( x, y \in R \), \( a^n(x-y) = 0 \), \( (x-y) \in r(a^n) \subseteq M \), then \((x-y)+M=M \), \( x+M=y+M \), \( f(a^n x)=x+M=y+M=f(a^n y) \), \( f(a^n x)=f(a^n y) \), so \( f \) is well defined right \( R \)-homomorphism, since \( R/M \) is GP-injective, there exists \( 1+M=f(a^n)=(b+M)(a^n+M)=ba+M \), \( 1+M=ba+M \), \( 1-ba \in M \), since \( ba \in Ra^n R \subseteq M \), we get that \( 1 \in M \), which is a contradiction. That means \( Ra^n R + r(a^n) = R \), in particular there exists \( y, z \in R \) and \( v \in r(a^n) \) such that \( y a^n z + v = 1 \), \( a^n y a^n z + a^n v = a^n , a^n y a^n z = a^n, a^n(1 - ya^n z) = 0 \), since \( a \in J(R) \), so \( ya^n z \in J(R) \), \( 1 - ya^n z \) is invertible, there exists \( u \in R \) such that \((1 - ya^n z)u = 1, a^n = a^n(1 - ya^n z)u = 0 \), must \( a^n = 0 \) which is a contradiction with \( a^n \neq 0 \). Therefore, \( a=0 \), \( J(R) = 0 \). This shows that \( R \) is semiprimitive.

**Corollary 2.3**

Let \( R \) be a ring whose simple right \( R \)-module is GP-injective. Then, \( N^n(R) = 0 \).

**Proof:**
We shall show that $N^*(R) = 0$, since $N^*(R)$ is the large nil ideal of $R$. It is clearly that $J(R)$ containing every nil ideal, so $N^*(R) \subseteq J(R)$ but $J(R) = 0$ by Proposition 2.3. This shows that $N^*(R) = 0$.

**Theorem 2.4**

Let $R$ be a ring whose simple right $R$-module is GP-injective. Then, the set $N_2(R)$ is right weakly regular.

**Proof:**

We shall show that $RbR + r(b) = R$, for all $b \in N_2(R)$, if not, suppose there exists $0 \neq a \in N_2(R)$, such that $RaR + r(a) \neq R$, then there exists a maximal right ideal $M$ of $R$ containing $RaR + r(a)$, $R/M$ is GP-injective, there exists an invertible integer $n=1$ such that $a \neq 0$ and any $R$-homomorphism of $aR$ into $R/M$ extends to one of $R$ into $R/M$. Now, let $f: aR \to R/M$ such that $f(ar) = r + M$ where $r \in R$. Note that $f$ is well defined right $R$-homomorphism, since $R/M$ is GP-injective there exists $b + M \in R/M$ such that $1+M=f(aM)=(a+M)(a+M)=ba+M, 1=M=ba+M, 1-ba \in M$ since $ba \in RaR \subseteq M$, we get that $1 \in M$, which is a contradiction.

Therefore, $RaR + r(a) = R$. In particular, there exists $y, z \in R$ and $v \in r(a)$ such that $yaz + v = 1, ayaz + av = a, ayaz = a$, that is for all $a \in N_2(R)$. This shows that the set $N_2(R)$ is right weakly regular.

**Theorem 2.5**

Let $R$ be a ring without zero divisors whose simple right $R$-module is GP-injective. Then, $R$ is a simple ring.

**Proof:**

We shall show that there is no two sided ideal of $R$, if not there exists a two sided ideal of $R$, $RaR$ is a two sided ideal for some $0 \neq a \in R$, since $0 \neq a, RaR \neq 0$, if $RaR \neq R$, there exists a maximal right ideal $M$ of $R$ containing $RaR$, $R/M$ is GP-injective, there exists a positive integer $n$ and $a^n \neq 0$ such that any $R$-homomorphism of $a^nR$ into $R/M$ extends to one of $R$ into $R/M$, we define $f: a^nR \to R/M$ such that $f(a^n) = r + M$, where $r \in R$, let $r_1, r_2 \in R$ such that $a^n r_1 = a^n r_2, a^n (r_2 - r_2) = 0, r_1 - r_2 \in r(a^n) = 0$, since $a$ is a non-zero divisor, so must $r_1 = r_2$. $f(a^n r_1) = r_1 + M = r_2 + M = f(a^n r_2)$, so $f$ is well defined $R$-homomorphism, since $R/M$ is GP-injective, there exists $b + M \in R/M$ such that $1+M=f(a^n)\Rightarrow (b+M)(a^n+M)=ba^n+M, 1+M=ba^n+M$, but $ba^n \in RaR \subseteq M, 1-ba^n \in M, 1 \in M$, which is a contradiction. Therefore, $RaR = R$, for all $a \in R$, that means $R$ not containing any two sided ideal of $R$. This shows that $R$ a simple ring.

**3. Rings Whose Simple Right R-module are GP-injective and it relation with other Rings.**

In this section, we give different conditions to the ring whose simple right $R$-module is GP-injective to get the reduced, S-weakly regular, regular, strongly regular ring.

A ring $R$ is said to be N duo if aR=Ra, for all $a \in N(R)[15]$. 

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Theorem 3.1
Let $R$ be a $N$ duo ring whose every simple right $R$-module is GP-injective. Then, $R$ is a reduced ring.

Proof:
We shall show that $N(R) = 0$, if not, there exists $0 \neq a \in N(R)$ with $a^2 = 0$, if $aR + r(a) \neq R$, there exists a maximal right ideal $M$ of $R$ containing $aR + r(a), R/M$ is GP-injective, there exists an apposite integer $n=1$ such that $a \neq 0$ and any $R$-homomorphism of $aR$ into $R/M$ that extends to one of $R$ into $R/M$. Let $f : aR \rightarrow R/M$ such that $f(ar) = r + M$ where $r \in R$. Note that $f$ is well defined right $R$-homomorphism, since $R/M$ is GP-injective there exists $b + M \in R/M$ such that $1 + M = f(a) = (b + M)(a + M) = ba + M$, $1 + M = b + M$, $1 - ba \in M$, since $R$ is a $N$ duo ring and $a \in N(R)$, we get $aR = Ra$, $ba \in Ra = aR \subseteq M$. so $1 \in M$, which is a contradiction. Therefore, $aR + r(a) = R$. In particular, there exists $z \in R$ and $v \in r(a)$ such that $az + v = 1, a^2 z + av = 0 = a$, for all $a \in N(R)$. This shows that $R$ is a reduced ring.

Call a ring $R$ NCI if $N(R)$ is containing a non-zero ideal of $R$ whenever $N(R) \neq 0$. Clearly, NI ring is NCI [5].

Theorem 3.2
Let $R$ be an NCI ring whose simple right $R$-module is GP-injective. then $R$ is a reduced ring.

Proof:
We shall show that $N(R) = 0$, if not, $0 \neq N(R)$, since $R$ is an NCI ring, so $N(R)$ is containing a non-zero ideal $I$, but $I$ is nil ideal. It is clearly that $J(R)$ containing every nil ideal, so $I \subseteq J(R) = 0$, form proposition 2.4, $I = 0$, that is mean $N(R)$ must be an ideal, similarly $N(R) \subseteq J(R) = 0$, $N(R) = 0$. This is shows that $R$ is reduced ring.

Theorem 3.3
Let $R$ be aring whose simple right $R$-module is GP-injective. Then, the following conditions are equivalent:
1- $R$ is reduced ring.
2- $R$ is N duo ring.
3- $R$ is 2-priml ring.
4- $R$ is NI ring.
5- $R$ is NCI ring.

Proof:
1 $\rightarrow$ 2, is clear and 2 $\rightarrow$ 1, by Theorem 3.2
1 $\rightarrow$ 3 $\rightarrow$ 4 $\rightarrow$ 5, is clear and 5 $\rightarrow$ 1, by Theorem 3.1

Call a ring $R$ S-weakly regular ring if $a \in aRa^2 R$, for all $a \in R$ [14].

Theorem 3.4
Let $R$ be aring whose simple right $R$-module is GP-injective. Then, $R$ is S-weakly regular ring. If satisfies one of the following conditions.
1- $R$ is a reduced ring.
2- $R$ is N duo ring.
3- $R$ is 2-priml ring.
4. \( R \) is NI ring.
5. \( R \) is NCI ring.

**Proof:**

We shall prove that \( R \) is S-weakly regular when \( R \) is reduced, and the proof of the other condition that is clearly form Theorem 3.3.

We shall show that \( \text{Rd}^2 \text{R} + r(d) = \text{R} \), for all \( d \in \text{R} \) if not, there exists \( 0 \neq a \in \text{R} \) such that \( \text{Ra}^2 \text{R} + r(a) \neq \text{R} \) there exists a maximal right ideal \( M \) of \( R \) containing \( \text{Ra}^2 \text{R} + r(a) \), \( \text{R/M} \) is GP injective, there exists an apposite integer \( n \) such that \( a^n \neq 0 \) and any \( R \)-homomorphism of \( a^n \text{R} \) into \( \text{R/M} \) extends to one of \( \text{R} \) into \( \text{R/M} \).

Let \( f: a^n \text{R} \to \text{R/M} \) such that \( f(a^n r) = r + M \), where \( r \in \text{R} \). let \( a^n x = a^n y \) where \( x, y \in \text{R}, a^n(x - y) = 0, (x - y) \in r(a^n) = r(a) \subseteq M \), since \( R \) is a reduced ring, then \( (x-y)+M=M, x+M=y+M, f(a^n x)=x+M=y+M=f(a^n y), f(a^n x)=f(a^n y), \) so \( f \) is well defined right \( R \)-homomorphism, since \( \text{R/M} \) is GP-injective there exists \( b + M \in \text{R/M} \) such that \( 1+M=f(a^n)=f(b+M)(a^n+M)=ba^n+M, 1+M=b+M \). Now \( ba^n \in Ra^2R \subseteq M \), it is true when \( n \geq 2 \), \( 1 - ba^n \in M, 1 \in M \), which is a contradiction. Therefore, \( Ra^2R + r(a^n) = \text{R} \), for \( n \geq 2 \). Now, when \( n=1 \), \( f: aR \to R/M, f(ar) = r + M, 1+M=f(a)=(b+M)(a+M)=ba+M, 1+M=ba+M \), by multiply \( a+M \) in the left side and \( b+M \) in the right side, we have \( ba+b^2a^2+M \), since \( b^2a^2 \in Ra^2R \subseteq M, \) then \( ba - b^2a^2 \in M \), we get that \( ba \in M \), since \( 1+M=ba+M, 1-ba \in M, 1 \in M \), which is a contradiction. Therefore, \( Ra^2R + r(a) = \text{R} \), for all \( a \in R \). In particular, there exists \( y, z \in \text{R} \) and \( v \in r(a) \) such that \( ya^2z + v = 1, ay^2z + av = ay^2z = a \). This shows that \( R \) is an S-weakly regular ring.

An element \( a \) of ring \( R \) is said to be a right regular element if the right annihilator ideal is zero \( (r(a)=0) \) [6]. A ring \( R \) is said to be MERT if and only if every maximal essential right ideal of \( R \) is an ideal [14]. A ring \( R \) is called Kasch if every simple right \( R \)-module embeds in \( R \), equivalently, for every maximal right ideal \( M \) of \( R \) is a right annihilator of \( R \) [3]. Call a ring \( R \) a right SF-ring if each simple right \( R \)-module is flat [13]. A ring \( R \) is said to be regular if \( a \in aRa \), for all \( a \in R \) [11].

**Lemma 3.5** [6]

Let \( R \) be a semiprime ring with maximum condition on left and right annihilators. Then, every essential right ideal contains a regular element.

**Lemma 3.6** [13]

\( R/I \) is right flat \( R \)-module if and only if for each \( x \) in \( R \) there is some \( y \) in \( R \) such that \( x=xy \).

**Lemma 3.7** [7]

Let \( R \) be a MERT ring, then the following conditions are equivalent:
1. \( R \) is regular.
2. \( R \) is right SF.

**Theorem 3.8**

Let \( R \) be a MERT ring whose every simple right module is GP-injective and satisfies maximum condition on left and right annihilators. Then \( R \) is Kasch ring and right SF-ring, hence \( R \) is regular ring.

**Proof:**

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We shall prove that every maximal right ideal is direct summand, if not, suppose that M a maximal right ideal of R which is not a direct summand of R, then M is a maximal essential right ideal of R, by Theorem 2.1 and Lemma 3.5, we have M containing a non-zero divisor a, R/M is GP-injective, there exists a positive integer n and \( a^n \neq 0 \) such that any R-homomorphism of \( a^nR \) into R/M extends to one of R into R/M. Let \( f: a^nR \to R/M \) such that \( f(a^n) = r + M \), where \( r \in R \), since a is a non-zero divisor f is well defined right R-homomorphism, since R/M is GP-injective, there exists \( b + M \in R/M \) such that \( 1 + M = f(a^n) = ba^n + M \). By Theorem 2.1 and Lemma 3.5, we have M containing a non-zero divisor a, R/M is GP-injective, so \( ba^n \in M \). Therefore, M is essential right ideal and R is MERT ring, M is an ideal, so \( ba^n \in M \), we get that \( 1 \in M \), which is a contradiction. Therefore, M a direct summand. This shows that every maximal right ideal of R is a direct summand.

There exists J right ideal for any maximal right ideal M such that \( M \subseteq J \), in particular there exists \( m \in M \) and \( j \in J \) such that \( m+j=1 \), so for all \( d \in M \), \( jd=0 \), then \( M \subseteq r(j) \), but M is a maximal right ideal, we have \( M=r(j) \), for every maximal right ideal. This shows that R is a right kasch ring.

Also, md=d for all \( d \in M \), from Lemma 3.6, we get that R/M is flat right R-module and that is for all M maximal right ideal of R. This shows that R is a right SF-ring.

Since, R is MERT and right SF-ring, by using Lemma 3.7, we get that R is a regular ring.

**Theorem 3.9**

Let R be N duo ring and MERT whose simple right R-module is GP-injective. Then, R is a strongly regular ring.

**Proof:**

We shall show that \( dR + r(d) = R \), for all \( d \in R \). If not then there exists \( aR + r(a) \neq R \), for some \( a \in R \), there exists a maximal right ideal M of R containing \( aR + r(a) \). M is either essential or direct summand, if M is not essential, then \( M=r(e) \) for some \( 0 \neq e = e^2 \in R \), by Theorem 3.1, R is a reduced ring, \( a \in r(e) = l(e), ae = 0 \). e \in r(a) \subseteq r(e), c \in r(e), \( e^2 = 0 \), but \( e = e^2 \), hence e=0, and \( M=r(e)=r(0)=M \), R=M, which is a contradiction. Therefore, M is an essential right ideal of R. Thus, R/M is GP-injective, there exists a positive integer n such that \( a^n \neq 0 \) and any R-homomorphism of \( a^nR \) into R/M extends to one of R into R/M, Let \( f: a^nR \to R/M \) be defined by \( f(a^n) = r + M \), where \( r \in R \). Note that f is well defined right R-homomorphism, because R is a reduced ring, since R/M is GP-injective, there exists \( b + M \in R/M \) such that \( 1 + M = f(a^n) = ba^n + M \). Since \( a \in M \), M is an essential right ideal and R is MERT ring, M is an ideal, so \( ba^n \in M \), we get that \( 1 \in M \), which is also contradiction. Therefore, \( aR + r(a) = R \), for all \( a \in R \). This shows that R is a strongly regular ring.

Finally, we give the following important result.

In [16] Xue proved, if every simple left R-module is GP-injective, then for any nonzero \( a \in R \), there exists a positive integer \( n=n(a) \) such that \( a^n \neq 0 \) and \( RaR + l(a^n) = R \) [Proposition 2].

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In the above proof, we have $RaR + l(a^n) = R$, and $Ra^n R + l(a^n) = R$, it is clear that proof $Ra^n R + l(a^n) = R$ leads us to $RaR + l(a^n) = R$ because $Ra^n R \subseteq RaR$, we give a new proof that strengthens the above $Ra^{2n} R + r(a^n) = R$, hence $Ra^{2n} R \subseteq Ra^n R \subseteq RaR$.

A ring $R$ is said to be right (left) $\pi$-weakly regular ring if, for every $a \in R$, there exists a positive integer $n$, depending on $a$ such that $\pi^n \in a^n Ra^{2n} R \ (a^n \in Ra^{2n}Ra^n)$ [10].

**Theorem 3.10**

Let $R$ be a ring whose simple right $R$-module is GP-injective. Then $R$ is right $\pi$-weakly regular ring.

**Proof:**

Let $a \in R$ and $a$ is not a nilpotent element, if $Ra^{2n} R + r(a^n) \neq R$, then there exists a maximal right ideal $M$ of $R$ containing $Ra^{2n} R + r(a^n)$, by hypothesis $R/M$ is GP-injective, we define $f: a^n R \rightarrow R/M$ such that $f(a^n r) = r + M$, where $r \in R$, we show that $f$ is well defined, let $a^n x = a^n y$ where $x, y \in R$, $a^n (x - y) = 0$, $(x - y) \in r(a^n) \in M$, then $(x - y) + M = M$, $x + M = y + M$, $f(a^n x) = x + M = y + M = f(a^n y)$, $f(a^n x) = f(a^n y)$. So $f$ is well defined right $R$-homomorphism, since $R/M$ is GP-injective, there exists $b + M \in R/M$ and $a^n \neq 0$ such that $1 + M = f(a^n) = (b + M)(a^n + M) = ba^n + M$, $1 + M = ba^n + M$, $1 - ba^n \in M$, since $1 + M = ba^n + M$, multiply by $a^n + M$ from left and $b + M$ from right, we get $ba^n + M = b^2 a^{2n} + M$, $ba^n - b^2 a^{2n} \in M$. But $b^2 a^{2n} \in Ra^{2n} R \subseteq M$, so $ba^n \in M$, but $1 - ba^n \in M$, we get that $1 \in M$, which is a contradiction. Therefore, $Ra^{2n} R + r(a^n) = R$. In particular, there exists $y, z \in R$ and $v \in r(a)$ such that $ya^{2n} z + v = 1, a^n y a^{2n} z + a^n v = a^n, a^n y a^{2n} z = a^n$, and that is for all $a \in R$ which is not nilpotent elements. When $a$ is nilpotent element, there exists a positive integer $m$ such that $a^m = 0$, so $a^m = a^m ra^{2m}$. for any $r, s \in R$. This shows that $R$ is a right $\pi$-weakly regular ring.
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