ERIC HAROLD MANSFIELD
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Eric Mansfield was a structural engineer who worked at the Royal Aircraft Establishment Farnborough from 1943 to 1983 on a wide range of problems in the field of aircraft structures. His main working tool was the mathematical theory of elasticity, which he deployed with skill, ingenuity, perseverance and imagination to investigate many problems related to the design of aircraft structures during a period of development of higher flight performance and new materials. His studies included aircraft wings and fuselages, particularly in the presence of cut-outs and openings—for which he invented the ‘neutral hole’ concept—and problems involving large-deflection structural behaviour of plates, including thermal effects such as flexural and torsional buckling of fins and wings heated in supersonic flight. He also developed the novel and powerful tension field theory for understanding the post-buckling behaviour of thin webs in shear, and the analogous inextensional bending theory of thin wings. His mathematical models were confirmed by elegant experiments. Mansfield’s book *The bending and stretching of plates* contains clear descriptions of many of his research investigations; it has become a classic text. All of Mansfield’s writing is incisive and clear. He was widely regarded as a seminal figure in the field of applied structural mechanics in the second half of the twentieth century.

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Early years

Eric Harold Mansfield was born on 24 May 1923 in South Croydon. He was the second child of Harold Goldsmith Mansfield MC (b. 1879) and his wife Grace, née Pfundt (b. 1892). Eric’s sister, Daphne Grace, was two years older.

Eric’s father, after attending Whitgift School, embarked on a life of adventure. He started on a sheep farm in New Zealand, becoming an expert horseman and rifle shot, before serving with the New Zealand Mounted Rifles in the Boer war (1899–1902). Then he moved to Canada to join the Royal Canadian Mounted Police, where he helped in road-building through the wilds. In 1906 he was second in command of a surveying party that established the boundary between British Columbia and the Yukon, through mountains and wilderness; and in 1924 both a mountain and a creek there were named after him. During the First World War he served with the Essex Regiment, rising to the rank of captain; he fought in Egypt and Palestine, where he was wounded, and was awarded the Military Cross. In 1919 he published a book of 17 poems, *By Jaffa Way: the Judah 'ills*, about his war experiences.

Harold and Grace married in 1920, but Grace died when Eric was barely one year old. Eric and his sister were looked after for two years by their paternal grandmother, until she died, then by an Aunty May. Harold found the settled life in Croydon unbearable; and the small printing firm that he ran was not successful.

Aged 7 and 5 respectively, the children were sent to a co-educational boarding school in Somerset. Harold decided to earn his living by leading tourist parties through Europe during the summer months, and in the winter he gave illustrated talks across the United Kingdom about his various experiences. He let out their family home entirely, apart from a small attic, to pay the school fees. Thus, Eric and Daphne Grace embarked at an early age on a most unusual upbringing: living at school during term time, then farmed out together to various relatives during the holidays. There was no home for them to return to, and no continuity of care. But their father brightened their lives on the few occasions each year when he was able to join them: he created an enchanted make-believe world of Canadian ‘First Nation’ life—their names were Bearskin, Shining Cloud and Running Deer, and there was much tracking and stalking. As Daphne Grace expressed it later in an evocative poem: ‘While others walked sedate and upright, we on belly crept; face pressed against the leaf-mould, while one eye reconnoitred . . .’.

Eric knew nothing about his mother for 80 years, until his daughter researched the family history; no mention of her was ever made to the children. Grace was the youngest of the six children of Rudolph and Isobel Pfundt. Rudolph had come from Württemberg in 1875. He worked for Charles Barry, a tea-importer, and married his daughter.

School and university

At St Lawrence College, Ramsgate (1933–1941), Eric shone at mathematics, his best subject (‘all others being indifferent’). Towards the end of his time there, he helped with the teaching of mathematics. As he reached 18, Eric volunteered for the Royal Air Force (he had ambitions to be a rear-gunner) but was rejected on the grounds of poor eyesight.

He won a State Bursary to Trinity Hall, Cambridge, and began residence in October 1941. His desire was to read mathematics; but war-time arrangements in the university resulted in
his being attached instead to the Engineering Department, to read for the Mechanical Sciences Tripos. One of his supervisors there was W. H. Wittrick (FRS 1980). In the 1942 preliminary examination for the Tripos he was placed in class I; he obtained full marks in the mathematics paper, and was awarded an Exhibition by his college. In the Tripos examination, 1943, he was placed in class II.

With his life-long friend E. G. (Ted) Broadbent (FRS 1977), Eric was involved in regular rooftop fire-watching—and also in playing Bridge. He also had a gramophone and a small collection of classical records.

ROYAL AIRCRAFT ESTABLISHMENT FARNBOROUGH

Under war-time regulations, after a two-year course, science students were directed by a national committee, chaired by C. P. Snow, either into active service or to a research posting. Thus it was that Eric found himself posted to the Royal Aircraft Establishment (RAE) in late 1943. As it turned out, he remained at RAE until he retired in 1983. He rose spectacularly through the (Civil Service) ranks: senior principal scientific officer (individual merit) in 1959, and ultimately to chief scientific officer, a very lofty grade, by 1980.

During his time at RAE, Eric served on several Aeronautical Research Council (ARC) committees and sub-committees: Mechanics, Computation and Structures. He was also co-ordinator for Strength of Structures on a Western European Union Anglo-French collaboration on kinetic heating. Related external work included membership and vice-chairmanship of the
Royal Aeronautical Society’s Structures committee. He was the Royal Society’s representative on the British National Committee for Theoretical and Applied Mechanics, a member of the Mathematics Committee of the UK Science Research Council and a member of the reviewing panels of Mathematical Reviews and Applied Mechanics Reviews.

Eric chose to stay at RAE, rather than move on to an academic career. As he wrote later in an autobiographical note to the Royal Society:

my lack of a normal home life contributed, I think, to my becoming shy, reserved, and lacking in confidence. (This is only supposition, however, as my sister is quite the opposite in these respects.) I also developed a stammer at an early age, which I only finally conquered when I was about 30 years old; but this may have been a blessing in disguise, because I realised when I joined the RAE that, in order to get anywhere, I would have to write.

RESEARCH TOPICS

Eric’s topics of research are described briefly in the following sections.

Early work on wing structures

In his early years at RAE, Eric spent much of his time working on problems in the detailed design of aircraft wings. From the structural viewpoint, a wing is a tapered beam cantilevered out from the fuselage; and since the 1930s the bending moments in all-metal wings have been carried by spars with vertical cross-section, in conjunction with the sheet material that forms the aerofoil surfaces. In steady flight the upper surface carries compressive, and the lower surface tensile, longitudinal stress. Retracted undercarriages are usually stored inside wings, so that cut-outs are necessary; they are usually formed by omission of structural sheet material between the spars. Since the lower sheet material is in longitudinal tension, its omission poses problems of ‘stress concentration’ in both the sheet and its adjacent spar on account of the structural discontinuity.

Eric made computations of such stress concentrations, particularly near corners of cut-outs, and he expressed his results for concentration factors in the form of graphs and design charts, plotted in terms of the most relevant dimensionless groups \(1, 2\)*. Previous studies had provided only weak correlations with experimental flight data obtained by Eric and others by means of strain gauges; but Eric’s design charts gave better results.

The introduction of swept wings, as an aid to faster flight, exposed further problems. Thus, a conventional unswept wing curves without twist under pure bending moment and, reciprocally, twists without curving under pure torque. In a swept wing, on the other hand, where the ribs are skew to the spars, bending moments produce both curvature and twist, and torque produces both twist and curvature—facts that may in turn, through aerodynamic effects, adversely affect flight control.

Eric’s investigation of such phenomena involved a model of the essential structure of the wing as a shallow box of rectangular cross-section, with the longitudinal stiffening stringers idealized as stiff wires embedded in the skin sheet, while the skew ribs were likewise represented as stiffening wires \(3\). These two sets of non-orthogonal wires were then ‘smeared out’ into the sheets so as to make anisotropic elastic plates for completing the box. Eric was

* Numbers in this form refer to the bibliography at the end of the text.
then able to characterize the elastic response of the box to bending and twisting moments, and
to find axes of zero curvature and zero twist, about which applied moments produce pure twist or
pure flexure of the wing, respectively.

Later, Eric extended this work to show how a swept wing could be designed to be aero-
isoclinic (so that as the loading changes and the wing deforms, the ribs remain aligned to the
flight direction) by the provision of stiffening stringers swept back at an angle larger than the
sweepback angle of the wing (6).

Neutral holes

Following his detailed work on stress concentrations associated with cut-outs in wings, Eric
turned his attention to the idea of making and reinforcing a hole in a stress-bearing sheet in
such a way that the strength and stiffness of the sheet are not altered.

Reissner and Morshuchow (1949) had investigated how to reinforce a circular hole in a
stressed sheet in such a way that the stress in the sheet outside the hole is not affected by the
presence of the hole. The circular reinforcement, required to be elastically equivalent to the
removed sheet, could be very cumbersome for a sheet under unequal biaxial tension. Eric made
a radical attack on this problem (4, 7) by considering the use of purely tensile reinforcement
at the edge of the hole, and using Airy’s (1863) scalar stress function $\phi$ as an incisive tool.
This immediately showed that the shape of the neutral hole, as he called it, is determined by
the state of stress in the sheet.

In Cartesian co-ordinates, the stress components $\sigma_{xx}$, $\sigma_{yy}$, $\sigma_{xy}$ (or $\tau_{xy}$) are given by:

$$\frac{\partial^2 \phi}{\partial x^2} = \sigma_{yy}; \quad \frac{\partial^2 \phi}{\partial y^2} = \sigma_{xx}; \quad \frac{\partial^2 \phi}{\partial x \partial y} = -\sigma_{xy}$$

and they automatically satisfy the equations of static equilibrium. The boundary of the hole is
defined by $\phi = \text{constant}$, while the required tension in the edge reinforcement is given by the
slope-discontinuity between the plane $\phi = \text{constant}$ over the (stress-free) hole, and its values in
the stressed sheet. Use of Airy’s stress function in the solution of problems in plane elasticity
was well known (e.g. Timoshenko & Goodier 1951); but Eric’s use of it in solving a geometric
design problem was a brilliant stroke.

The cross-sectional area of the edge-reinforcing strip is determined by matching the
elastic strain along the strip and in the adjacent uncut sheet; that is a straightforward matter,
which produces moderately complicated formulas of a kind that Eric habitually handled with
elegance. When the sheet sustains a uniform state of stress, $\phi$ is a quadratic in $x$ and $y$.
Consequently, the boundary of the hole is a conic section: a parabola for uniaxial tension,
a circle for equal biaxial tension and an ellipse for unequal principal stresses of the same sign.
Eric provided a range of solutions for simple situations, including the ellipse with ratio of
axes $\sqrt{2}$ for a plane sheet with ratio 2 of the principal stresses—a case relevant to the design
of windows in a pressurized cylindrical fuselage. (See Eric’s article from 1963 (24) for the
design of a row of such windows).

Eric’s first paper on this topic included a study of the ‘efficiency’ of designs (4). In general,
the weight of edge reinforcement exceeded the weight of sheet removed; but Eric also found
some exceptional cases where the opposite was true.

Inextensional theory of large-deflection of thin plates

Together with a young colleague, Eric was contemplating a calculation of the fundamental
frequency of vibration of a cantilevered swept-back wing, here regarded as a plate. A
Biographical Memoirs

preliminary model, cut from thin card, immediately revealed that there was a concentration of bending at the corner where the trailing edge meets the fuselage, but also it focused attention on the fact that a thin flat sheet deflects inextensionally into a developable surface if that is kinematically possible—as it is, of course, for a thin cantilevered plate.

Truly inextensible continuous deformation of an initially plane surface can only occur by bending into locally cylindrical or conical ruled surfaces (see e.g. Hilbert & Cohn-Vossen 1952).

Figure 2a considers an elementary portion of the surface, bounded by two straight generators that intersect at point H, on or outside the boundary, that adopts a conical form with apex at H (8–10, 46). The local bending moments in the surface may be computed elastically once the out-of-plane curvature of the cone has been specified: they will be inversely proportional to distance from H; and the magnitude of the out-of-plane curvature must be such that the resulting bending moment on line KL is in equilibrium with the loading applied to the plate beyond that line. The geometry of the deformed surface is thus specified by \( \alpha \) and \( \eta \) as functions of \( x \) (see figure 2a) and the applied loading. Eric showed how to integrate the total strain energy \( U \) of the deformed plate, which may be expressed as a function of \( \alpha, x, \frac{dx}{d\alpha} \) and the applied loading.

The location of generators in a given case maximizes the total strain energy. This result may be obtained by pointing out that the total strain energy would not be changed if hypothetical thin, weightless rigid rods were to be glued onto the generators (46), and that the strain energy would be reduced by imposing the constraint of any straight rod not on a generator. Application of the calculus of variations (Jefferies & Jefferies 1950) then produces a non-linear second-order differential equation for \( x \); and this may be set up and solved when the form of the plate and its given loading have been prescribed.

Eric and F. W. Kleeman solved several cases involving triangular cantilever plates under uniform and tip loading (9, 10); and in the second edition of his book The bending and stretching of plates (46), Eric provides solutions to other problems. Figure 2b shows a typical result, for a triangular plate clamped along edge BC and loaded by a normal point force \( P \) at its tip A. The straight lines are typical generators of the deformed surface, while the curves give contours of dimensionless principal bending stress at the plate’s surface. The highest stresses are found on the lower edge AB near the support: stresses decrease towards the other edge on account of the conical nature of the deformed surface. The plot of figure 2b may also be read for all values less than 60° of the angles marked along edge AC, since each straight generator in turn may be taken as the clamped boundary of a plate.

With Kleeman, Eric performed some simple experiments on thin triangular cantilever steel plates coated with brittle lacquer and loaded at their tips; cracking of the lacquer confirmed the straight generators and their location. They also used an elegant photographic technique on thin elastic sheets with mirror finish, likewise to display the pattern of straight generators.

This scheme of analysis of an inextensional bending model of a plate is a subtle and direct approach to a difficult practical problem. It is characteristic of Eric’s general aim of focusing analysis on clear physical—here, geometrical—principles, in order to reveal directly the main features of a physical situation. This novel work was a clear advance on contemporary approaches to the same problem via finite-element methods.
Figure 2. (a) General scheme for the analysis of inextensional bending of a cantilevered plate, defining key variables. (b) Solution for a triangular plate, clamped along edge BC and carrying a point load $P$ at its tip $A$. Straight lines are generators (and alternative clamped edges); curves are contours of $(\sigma^2/3P)\tan \frac{1}{2} \phi$, where $\sigma$ is the bending stress at the plate’s surface, $t$ is the plate’s thickness and $\phi$ is the angle at the apex - here $60^\circ$. (Redrawn from (8.).)

**Plastic collapse of plates**

Having spent 10 years at RAE, Eric was released for the academic year 1953–1954 in order to attend a one-year postgraduate course in structures at the Cambridge Engineering Department. The course had been set up by Professor (later Lord) John F. Baker (FRS 1956), whose active field of research was the plastic design of structures: that is, the rational design of structures made from material that exhibits irreversible plastic deformation under rising stress. Although such an approach is not necessarily relevant, *prima facie*, to the design of aircraft structures, Eric evidently relished a return to his Alma Mater for an advanced course.

His subsequent paper, published in 1957, is an original, systematic study of ‘plastic collapse’ mechanisms for different plate geometries, boundary conditions and patterns of loading (11), and it contains a wide-ranging, detailed catalogue of 39 such mechanisms. A shorter version, published in 1960, was written in the context of the design of reinforced-concrete slabs (18).

It is clear from Eric’s first draft of the 1957 paper that he came to appreciate the importance of ‘large-deflection’ effects when the ‘collapse’ mechanism creates
a non-developable surface—a matter that was to exercise him greatly in later years.

Vibration

Vibration is an important recurring problem in aircraft structures, and from time to time Eric investigated natural frequencies and modes of vibration in relevant elastic structures. In 1951 he analysed a rectangular open-ended box-like tube, with ‘booms’ along its edges, vibrating in torsion (5), which he found to involve the use of two distinct dimensionless groups. Flexural vibration of a similar long box-tube as a classical beam provides simple results for frequencies in sinusoidal modes of arbitrary wavelength (14), but these must be modified to take account of shear strains in the walls, in ways which Eric elucidated in that paper.

Eric also studied the torsional vibration of wings, treated as solid plates with appropriate cross-sections. First, he considered a long strip of diamond section and found, by small-deflection plate theory, a family of simple modes with arbitrary longitudinal wavelength, whose frequencies were inversely proportional to wavelength (23). In the lower modes the centre-lines of cross-sections remained straight, while for higher modes the displacements were more concentrated near the edges. In other papers he considered torsional vibration of tapered, solid cantilevered wings (42, 43).

Thermal stress and the stiffness of thin wings

In 1957 Eric began a sequence of papers, during the course of which his thoughts about increasingly urgent topics of large-deflection, thermal stress and buckling of plates gradually developed and matured.

The starting point was a study of the torsion and bending of a thin solid wing or fin of a missile. Unusual phenomena arise from the stresses caused by aerodynamic, kinetic heating if the missile flies supersonically. These first appeared in the mid 1950s, when the wings of a model broke off in an American wind-tunnel test, causing considerable damage to the tunnel. NASA and various research centres in the USA discovered a phenomenon associated with the loss of torsional stiffness, that could explain the failure. They realized that, during the early stages of supersonic flight, kinetic heating of the wing would result in the heating up of the thinner parts, near the leading and trailing edges, sooner than the central region, so that the average temperature through the thickness of the wing would vary across the wing’s chord, as indicated schematically in figure 3a. That distribution of temperature produces longitudinal compressive (negative) stresses in the leading and trailing edges, balanced by tensile stresses in the centre, as shown.

Now when the wing is twisted by the application of a torque, these regions of compressive longitudinal stress introduce a component of torque in the same sense as the applied torque, thus providing a reduction of the torsional stiffness of the wing. The reduction from its value for an isothermal wing is proportional to the temperature difference $\Delta T$ (see figure 3b), and a reduction of, say, 50% might cause the wing to flutter and break up.

Eric investigated the possibility that these same thermal stresses might also reduce the flexural stiffness of the wing, with likewise potentially disastrous effects (12). His perceptive qualitative argument was that when an elastic strip is bent, its cross-section curves in the opposite sense for two reasons. First, there is a Poisson ratio effect, as seen clearly when a rubber eraser is bent. Second, by analogy with Brazier’s (Brazier 1927) effect whereby, for instance, a lightly bent drinking-straw ovalizes, the compressed and tensioned regions
Figure 3. (a) Schematic plot of average temperature through a wing’s thickness against position across its chord, together with corresponding regions of compressive (−) and tensile (+) longitudinal stress. (Redrawn from Mansfield’s unpublished book.) (b) Plot of torsional and flexural stiffness, relative to their values when $\Delta T = 0$, against the temperature strain parameter $\Sigma = \alpha \Delta T (a/t_0)^2$, for a solid (parabolic) lenticular cross-section, where $\alpha$ is the coefficient of linear thermal expansion of the material, $a$ is the chord-length and $t_0$ is the mid-wing thickness. (Redrawn from (12).)

move, respectively, away from and towards the overall centre of curvature. The net result is that the reduction in flexural stiffness of the wing depends on $(\Delta T)^2$. Figure 3b shows how dimensionless ‘relative stiffness factors’ for torque and bending moment, respectively, vary with a ‘temperature strain parameter’ $\Sigma$, which is proportional to $\Delta T$. In the example of figure 3b, the values of $\Delta T$ required for complete loss of torsional and flexural stiffness of the wing are equal; but that is not necessarily the case in general, as Eric showed by a mathematical tour de force (22).

All of this above work strictly applies to wings having a high aspect ratio, so that end-effects are negligible; and Eric devoted a long Appendix in his paper on flexural rigidity to the assessment of such effects (12).

Eric’s next paper made a comprehensive study of thin wings of lenticular cross-section under combined flexure and torsion in the presence of a parabolic distribution of temperature and with different values of $\Delta T$, either positive or negative (13). In all cases, the transverse chord-wise curvature across the wing’s central surface turns out to be uniform—a point that excited Eric’s curiosity. An unusual result is that, under zero bending moment and torque, a snap-through flexural buckling phenomenon can occur under negative $\Delta T$, ‘similar to that in a toy metal “clicker”’. 

Leading-edge buckling on account of aerodynamic heating

Eric extended his study of thermally induced structural effects in thin wings to an investigation of the circumstances in which leading-edge buckling can occur when the compressive stress is sufficiently localized there (16). After an extensive piece of detailed analysis, he reached the simple conclusion that local buckling may be initiated near the edge, in a mode similar to one that he had found in the torsional vibration of a wing with diamond cross-section—but here
with indeterminate longitudinal wavelength—when the longitudinal (compressive) stress $\sigma_y$ at the edge reaches the value $G\beta^2$, where $G$ is the elastic shear modulus and $\beta$ is the included angle of the sharp leading edge of the wing profile. He showed that the same formula also applies in other buckling problems (17).

**Large-deflection behaviour of a thin strip under load**

In a 1959 paper Eric moved away from heated wings and considered instead a long, initially straight elastic strip having a parabolic lenticular cross-section, deformed by the application of uniform bending moment and torque (15). This investigation was to develop into penetrating novel analyses of a range of structural phenomena.

I have noted already that, when bent, a thick strip such as a rubber eraser shows a uniform transverse curvature, of opposite sense from that of the longitudinal curvature on account of the material’s Poisson ratio. In the flexure of a wider strip, whose width is of the order of, say, 15 times its uniform thickness, it is found that such an effect is transitory and that, in general, the central portion of the strip deforms into an inextensional cylindrical surface that is flanked at the edges by boundary layers in which there is transverse curvature. (Such boundary layers were first discovered by H. Lamb (FRS), as a referee adjudicating a dispute between A. E. H. Love (FRS) and Lord Rayleigh (FRS) over the boundary conditions of thin shells: see Calladine 1984, 1988.) The width of the boundary layers is of the order of $\sqrt{(Rt)}$, where $R$ is the current radius of curvature of the strip and $t$ is the thickness. In other words, as the curvature $1/R$ increases, the transverse profile of the strip changes progressively, as the boundary layers steadily decrease in width. In contrast, as noted above, Eric discovered that if instead the strip has a (parabolic) lenticular cross-section, the transverse profile of its middle surface is always one of uniform curvature. In his paper, Eric makes a thorough investigation of the response of such a strip to combined bending moment and torque via an energy analysis (15). He investigates the behaviour of the strip both before and after any buckling; and he shows, for instance, that a strip loaded in pure torque can buckle into a helical form in which the middle surface of the strip approaches an inextensional cylindrical surface.

**Elastic plates of variable thickness**

As Eric remarked in an unfinished book (see below), the specially simple large-deflection mechanics of a strip with a lenticular cross-section—first noted in the context of thermal effects in thin wings—might be expected to carry over to other problems involving, for instance, a circular disc of lenticular cross-section. That clue was Eric’s motivation for undertaking a systematic derivation of the formal large-deflection equations for elastic plates of variable thickness, including thermally induced stresses.

Now the classical, small-deflection theory of bending for an elastic plate of uniform thickness involves a single governing equation $D\nabla^4 w = q$, together with appropriate boundary conditions; here $w$ is the transverse displacement, $q$ is the applied normal load per unit area, $D$ is the bending stiffness and $\nabla^4$ is the biharmonic operator.

Theodore von Kármán (1910) set out the system of two coupled non-linear differential equations that are sufficient for the investigation of large-deflection behaviour of elastic plates. His first equation describes the equilibrium of the plate: it includes the classical small-deflection theory plus the effect of membrane-like stress resultants acting through the now-curved middle surface. His second equation uses the Airy function to describe the in-plane stress resultants, and it ensures the geometrical compatibility of the surface strains, taking into account changes in Gauss curvature.
Eric’s special version of these two equations (20) includes several features not found in the original: variations in both thickness and temperature over the plate, plus a uniform temperature gradient through the thickness—expressed as the constant curvature $\kappa_T$ that would be imparted to every separate plate element by that gradient. As in von Kármán’s version, the principal variables are the transverse displacement $w$ and the Airy stress function $\phi$. These coupled ‘large-deflection’ equations assume nevertheless that all strains are small. They are an improvement on the classical small-deflection equations in that they allow properly for surface straining on account of Gauss curvature. (Eric’s earlier large-deflection treatment of inextensional bending ran along classical small-deflection lines; it is valid for deflections of at least the order of magnitude of the thickness of the plate, since there is no change in Gauss curvature, just as in the deflection of a simple cantilever beam, where the concept of Gauss curvature is not relevant.)

Eric solved his coupled equations for a range of relatively simple problems (20). His last example concerned the large-deflection bending of an unsupported circular plate of lenticular cross-section, subject to a uniform through-thickness temperature gradient producing a ‘thermal curvature’ $\kappa_T$. The plate’s thickness is given by $t = t_0(1 - (r/r_0)^2)$, where $t_0$ is the thickness at the centre and $r_0$ is the plate’s radius. The bending stiffness is given by $D = D_0(1 - (r/r_0)^2)^3$, where $D_0$ is the bending stiffness at the centre. In the small-deflection range, the plate is initially stress-free and the thermal strains produce a spherical curvature (of the middle surface) according to $w = -\frac{1}{2}\kappa_T r^2$. From previously spotted clues, Eric suspected that solution of the general equations would take the form

$$w = -\frac{1}{2}\kappa r^2,$$

where $\kappa$ would be a function of $\kappa_T$. By postulating that $\phi \propto D$ and substituting into the two equations, subject to the boundary conditions, Eric finds that, precisely,

$$\kappa_T - \kappa = ((1 - \nu)r_0^4\kappa^3)/(2(7 + \nu)t_0^2),$$

where $\nu$ is the Poisson ratio of the material.

This progressive reduction in $\kappa$ relative to $\kappa_T$ corresponds to the growing membrane stresses that are required by the increasing Gauss curvature on account of $\kappa$. This relation is plotted in figure 4 as the curve that departs from the small-deflection solution $\kappa = \kappa_T$.

In subsequent papers Eric extended the analysis to cover ‘the well-known fact’ that a thin unsupported plate tends to curl when subjected to a severe through-thickness temperature gradient (19, 21); that is, he considered the possibility of a non-spherical mode of deformation. For this purpose he changed to Cartesian co-ordinates, replacing [1] by $w = -\frac{1}{2}(\kappa_x x^2 + \kappa_y y^2)$.

Now for $\kappa_T = 5.15t_0/r_o^2$ (for $\nu = 0.3$)—i.e. when the out-of-plane displacement at the edge $= 1.7t_0$—a point of bifurcation is reached; and for larger values of $\kappa_T$ the plate buckles into an asymmetric mode, with two different principal curvatures, as shown in figure 4. After buckling, the Gauss curvature of the plate remains constant as $\kappa_T$ increases further, and the larger principal curvature becomes asymptotic to $(1 + \nu)\kappa_T r_o^2t_0$ as $\kappa_T$ continues to increase; the plate curls up into an approximately cylindrical surface—at least, if the prescribed temperature gradient can be maintained.
Eric also investigated in (21) a variant of the problem, in which the lenticular plate’s centre-surface was not initially plane, but spherical. In this case, the bifurcation and buckling phenomena were much richer: bifurcation buckling was now supplemented by snap-through buckling from a limit point (where a path changed from stable to unstable equilibrium) to another stable path. Then he investigated the problem in which an initially flat lenticular circular plate was subjected to a temperature distribution that was constant through the thickness, but now varied parabolically in the plane of the plate according to $T = T_1 (r/r_o)^2$ (19, 29). The plate buckled by bifurcation into a saucer- or a saddle-shape, depending on whether the temperature was highest at the centre or the circumference.

Eric separately investigated elliptical plates (26, 27). A particular result is that when an initially flat plate curls up under a through-thickness thermal gradient, the principal axis of curling is indeterminate. Experimental confirmation of this indeterminacy was obtained via simulation of a temperature gradient by cementing together two sheets of rubber, one of which was uniformly pre-stretched. The elliptical plate measured 12 in. × 6 in. and, ‘for ease of manufacture’, was of constant thickness. When free to deflect, it exhibited no preferred direction of curling.

Another paper, written with G. Z. Harris, is a comprehensive investigation of large-deflection vibrations of unsupported circular and elliptical plates of lenticular cross-section (28). Non-linearities arising from large deflections are of practical significance only for the lower modes; and attention was confined to three ‘fundamental’ modal components—two bending and one twisting. In general, the vibrations involve different time-wise variations of all three components, and recourse must be made to numerical integration of the equations;
but there are many special cases where the modal components vibrate in unison, and the integration may be done analytically. In a paper written 15 years later (41), Eric extended this earlier scope of (28) to include in-plane and through-thickness temperature variations; again, there were cases in which the modal components vibrated in unison, including the curling modes of certain elliptical plates with special temperature distributions.

Eric’s final paper in this series (44) investigated the normal small-deflection modes of lenticular elliptical plates, in the presence of in-plane quadratic temperature distributions of the kind experienced during supersonic flight. Eigenvalues and vectors of large matrices were computed on an industrial scale. Surprisingly, it turned out that the consequences of applied thermal stress could be studied simply by an equivalent change in the value of the Poisson ratio; and in practice only the lower modes were affected by the thermal effects.

‘Plates of lenticular cross-section’ was Eric’s mathematical device for producing surprisingly simple, analytical large-deflection solutions to a range of plate problems. By suppressing the formation of boundary layers at the plate edges, they allow large-deflection behaviour to be followed with relative clarity. Thus, they tend to refute a remark by Landau & Lifshitz (1959) to the effect that von Kármán’s equations are very complicated and cannot be solved exactly, even in the simplest cases.

**Tension field theory**

The behaviour of wrinkled membranes was first investigated by Wagner (1929), whose primary concern was to explain the stable behaviour of thin metal webs in wing spars when they carry a shear load well in excess of the initial buckling value. Such problems are non-linear, and their full solution presents formidable difficulties.

In the shearing experiment shown in figure 5a, the web is an aluminium-coated Melanex sheet 0.015 mm thick, gripped by the horizontal members of a rectangular frame with corner pivots (31). It suggests that, since the buckled wavelength is small, the material is acting essentially as a set of straight strings in simple diagonal tension—and indeed as if the sheet had been separated into discrete ‘tension rays’ by a family of many straight slits. Note, however, that the inclination of the ‘slits’ varies between 70° to the horizontal at the ends and ~50° in the centre; and that there are also ‘un-slit’, triangular regions at the ends, where the membrane is not attached to the frame’s verticals. Thus the behaviour is evidently not trivial.

Eric’s interest in problems of this sort during the late 1960s was stimulated by the potential application of stretched-membrane components in lightweight structures for use in space, and also of inflated membrane structures for use in portable lightweight bridges, etc.

In relation to the situation shown in figure 5a, Eric realized that the problem of finding the orientation of the straight tension rays in the membrane was similar to the problem of locating the straight generators in the theory of inextensional bending of plates, which he had solved a dozen years earlier. Indeed, he was able to point out a direct analogy between the hypothetical ‘slits’ of the tension field and the hypothetical ‘attached rigid rods’ of the inextensional bending theory; and indeed for the entire scheme of fixing them by maximizing the strain energy. This is an extension of a well-known analogy between the equations of bending and stretching of a flat plate (see e.g. Prager 1957). There are, however, some subtle differences in detail between the two theories (30, 32).

For example, figure 5b shows a jointed frame, in the form of a parallelogram, attached to a thin membrane on all sides. The two cases (i) and (ii) show the orientations of typical ‘slits’ when the acute angle of the frame is either closed by a small amount (i), or opened (ii); the
Figure 5. (a) Tension-ray lines in a thin rectangular membrane under shear, with some angles of inclination. The membrane is not attached to the short sides of the frame. (From (31), used with permission.) (b) Tension-ray lines in a thin membrane attached to a parallelogram-frame with corner joints, when the acute corners are (i) closed and (ii) opened. (Redrawn from (32).)

‘sfit’ lines bisect the acute and obtuse angles, respectively. The same two diagrams may also be read in terms of the families of straight generators when equal normal forces are applied at the corners of a plate in the form of the parallelogram, away from the reader at the acute corners and towards the reader at the obtuse corners in both (i) and (ii); in (i) the deformed plate is concave towards the reader, but in (ii) it is convex. Thus, in the plate interpretation of these diagrams there are two distinct possible solutions for a single loading condition; and case (ii) is the correct one (for a perfectly flat plate), because the strain energy of bending is larger. By contrast, in the tension field interpretation, (i) and (ii) represent two distinct loading conditions.

In the solution of inextensional plate-bending problems discussed earlier, for a corner-loaded cantilevered plate (figure 2b), any one of the straight generators may equally be
regarded as the cantilever-root of another triangular plate. Likewise, any tension-ray solution of a membrane under shear loading remains when particular regions are removed. For instance, Eric showed experimentally (31) that when material in the left corner of figure 5a, up to the 70° line, and in the right corner, beyond the 60° line, was removed, the pattern of the remaining rays was unchanged. A dozen years later (39), Eric extended his theory to the curved wrinkles that may form, for instance, when a heavy rectangular flexible membrane is suspended with slack from two adjacent corners.

Eric’s tension field theory is a good example of an asymptotic solution of a difficult general problem that can be drastically simplified—in this case by means of a great reduction in the thickness of the membrane. The buckling of the sheet involves negligible compressive stress, and there is no attempt to characterize the wavelength of the ripples. The solution shows that the post-buckling force/displacement relationship is asymptotically linear, which provides a useful guide for the design engineer.

Fibre-reinforced structures

The use of fibre-reinforced composite materials in aircraft structures increased steadily from the late 1960s. Eric made theoretical studies of the consequences of anomalous packing of fibres for the orthotropic elastic properties of unidirectional composites (33, 34, 35) and on diffusion lengths in elastically anisotropic composite bars and plates (36).

With A. J. Sobey he made a major study (37) of a helicopter blade, modelled as a tube of arbitrary cross-section with a multi-ply glassfibre-reinforced plastic (GFRP) composite wall. The general aim of the work was to provide designers with a tool for choosing asymmetric fibre lay-ups to facilitate aero-elastic tailoring, i.e. favourable patterns of deformation under load. The study with Sobey provided detailed, worked examples that showed, in graphical form, the consequences of various design decisions on the overall mechanical performance of the blade. This was the first paper to capture clear cross-coupling effects in composite structures, and it was awarded a prize in 1979 by the Royal Aeronautical Society. Later (40), Eric widened the scope of the work to include a two-cell model of the blade.

In 1979 Eric showed how a wing with fibre-composite surfaces may be analysed as an isotropic cantilever plate with variable rigidity, under bending, torsional, shearing load and an arbitrary chord-wise pressure distribution (38). This led to a novel engineering bending/torsion theory for application in the initial stages of design of wings intended to deform in a favourable manner under load in flight.

Floating bodies supported partly by surface tension

Eric’s last substantial paper (47) was on a different subject from all his other work. In his retirement he had a garden containing a pond. His curiosity was aroused by ‘water-skater’ (Gerris) insects that alighted on the water’s surface and were supported by surface tension.

On a sunny day Eric noticed six circular ‘shadows’, about 10 mm in diameter under each water-skater, on the sloping sides of the pond, about 150–250 mm below the surface; they evidently originated from where the insect legs rested on the surface of the water. He decided to study these phenomena, and recruited Edward Eastwood, of the University of Hertfordshire, to perform optical experiments with light objects floating on water and on such matters as the proportions of a fly’s weight supported on its three pairs of legs; and H. R. Sepanghi, a visitor from Iran at the University of Surrey, to help with the associated computation and graphics.
Figure 6. Cross-sections, to scale, of rafts of four long strips of width $b$, for three different values of dimensionless width $\beta = b \sqrt{\gamma g/S}$; see text for definitions. (From (47), used with permission.)

It is a thorough study of the detailed mechanics of an idealized problem in which four side-by-side flat plates are supported by a combination of buoyancy effects and surface tension, according to a simple equation, due to Young and Laplace early in the nineteenth century:

$$S(\kappa_1 + \kappa_2) - \gamma gz = 0,$$

where $S$ is the surface tension per unit length, $\kappa_1$ and $\kappa_2$ are the principal curvatures of the surface, $\gamma$ is the density of the liquid, $g$ the gravitational acceleration and $z$ the deflection of the surface. Eric produced solutions of this equation, in dimensionless terms, that involved deep mathematics and subtle arguments; and it produced results on mutual attraction or repulsion that he and his colleagues verified experimentally. Figure 6 displays theoretical results for rafts of four adjacent strips of different dimensional widths, which were confirmed by direct experiment.

**Eric’s books**

*The bending and stretching of plates* (25) was published in 1964 by Pergamon Press. The first six chapters present the classical ‘small-deflection’ theory of elastic plates, and the remaining three cover ‘large-deflection’ behaviour, reproducing much of his original work. The book was aimed at engineers rather than mathematicians, particularly structural engineers in civil, aeronautical and mechanical engineering, and structural research workers.
The enlarged second edition (46) was published by Cambridge University Press in 1989. It included additions to all nine chapters, and in particular the tension-field and inextensional bending theories, and the use of matrices to characterize the elastic response of multi-layer anisotropic fibre-reinforced plates. This book is the most widely cited of all his scientific works.

An unpublished book

In his retirement Eric drafted a second book, which was never published. The manuscript covers those research topics that particularly excited him, and it is presented in a ‘not so mathematical’ style, with a strong emphasis on physics; and in a generally ‘chatty’ way that draws out points of research method based on his own experience. It also explains the background history of some of his projects—matters hardly mentioned in his other writing—which has greatly aided the writing of this memoir.

Family

Eric’s first assignment at RAE was in the Aerodynamics Department, ‘working on doodlebugs’. George Purves Douglas was in charge of the department, and his daughter Mary Ola Purves (b. 1924) was employed there too, working on aerofoil design with Ted Broadbent. Eric and Ola married in 1947, and Ola relinquished her career in order to bring up their children: Peter Turquand (b. 1949), Daniel Ian (b. 1950) and Susan Carol (b. 1953) (figure 7).

Ola’s father died in 1954, shortly after his retirement. Eric and Ola decided to use the resulting inheritance to purchase a large house in Farnborough. Eric put up an enormous swing, climbing ropes and a rope bridge at high level through the woods at the bottom of the extensive garden. There were bluebells under the trees, an orchard and a fox’s den in the corner. Eric’s telescope often came out, and he and Ola introduced the children to the planets and to wider aspects of astronomy.

As well as their common interest in mathematics and science, Eric and Ola shared a love of outdoor life (figure 8). All holidays were taken under canvas, mostly near the sea; and some included fossil-hunting. Holidays always included an attempt to ‘live off the land’: the family snorkelled for spider-crabs and lit fires on the beach to cook their catch. Eric and Ola first took their family camping on the Continent in 1964. Nothing was booked in advance and most stops were made by the sea-side; but there were also visits to prehistoric cave paintings in France and northern Spain.

At home, Eric would entertain the children with card tricks, card games such as Pit and Peggity and with board games; including Buccaneer and one called Orbit, which he had invented and which involved getting fuel and supplies to the Moon. This game was tactical and exciting, and winnable by the youngest player; participants became familiar with cosmic dust, the Van Allen radiation belt and friendly space stations, ‘but Waddingtons turned it down’.

When family duties lessened, Ola involved herself in the local Girl Guide movement, running camps and serving as District Commissioner. Eric was not gregarious; he enjoyed Bridge at the local club, but generally did not enjoy social gatherings.

In 1973 Eric’s marriage to Ola was dissolved, and in 1974 he married Eunice Waller (née Shuttleworth Parker). They lived quietly in Church Crookham, and later moved to a house in Lower Bourne, near Farnham, with a large garden, a pond and many pine trees. Eunice was a keen gardener, and they both enjoyed walking the dog. In about 2005, when Eric was in his early 80s, he and Eunice moved to a modern house in Cheriton. It was here that he wrote
his unpublished book. Eric joined the local Bridge club, where his cheerfulness and ready wit were appreciated; but gradually he developed dementia and ceased playing Bridge. Eric’s final days were spent in hospital, following a broken hip.

Concluding remarks

Eric Mansfield was a private man, who flourished as a lone worker. He was one of the last to have fashioned a distinguished career by working in the well-trodden field of classical elasticity. Although much of his time was spent in solving relatively straightforward problems of the kind that abound in the design of aircraft structures (and presenting results in clear ways that enabled others to apply them successfully), he was frequently able to take dramatic innovative steps that involved sophisticated mathematics and provided fresh insight for the practising structural engineer.

In the Preface of his well-received book, The bending and stretching of plates (25), Eric nailed his colours to the mast: his steady objective was to devise analytical solutions rather than to make assays based on numerical methods such as Finite Elements, which have now become for many engineers the ‘default’ approach to structural mechanics; and which, he insisted, give little direct information about the key structural design parameters, such as
dimensionless groups. But, lest he be regarded by some as a kind of latter-day dinosaur, I would point out that dinosaurs actually evolved into birds that fly. Eric’s contributions of seminal ideas such as neutral holes and tension fields put him into the category of those who rise above the mundane and make a distinctive contribution to the advance of applied mechanics and useful engineering science. Furthermore, his resolution of an urgent problem over buckling of a wing through thermal effects in supersonic flight led him to the mathematical manoeuvre of studying plates of lenticular cross-section; and that led to clean solutions of von Kármán’s ‘large-deflection’ equations, thereby confounding the long-established conventional wisdom that such equations are practically impossible to solve analytically. Here Eric provided an example of inspired intuitive steps that open new vistas in the field of structural mechanics.

In fact, Eric frequently made use of numerical calculations—with generous acknowledgement to his helpers—in order to present clearly the solid consequences of his analyses; he was always careful to seek efficient ways of presenting his results, and it should not be overlooked that he conducted some neat experiments, in order to emphasize his points.

In presenting seminars to university groups and lectures at international meetings, Eric gave lucid, well-rehearsed talks that were punctuated by well-conceived and dramatic experiments.
Here he displayed the sense of fun and wit—doubtless inherited from his father—by which he also enriched the lives of his young children.

It would be remiss of me not to mention Eric’s prowess, as well as life-long interest, in the card-game Bridge; including his book, Bridge: the ultimate limits—a collection of demanding puzzles, or inferential problems, originally set as Christmas competitions (45). Here, both his intellectual power and wit are manifest.

**Honours and Awards**

1951  Winner (with I. T. Minhinnick), World Par Bridge Olympiad
1957  ScD degree, University of Cambridge
1960  Fellow, Royal Aeronautical Society
1964  Founding Fellow, Institute of Mathematics and its Applications
1967  Bronze Medal, Royal Aeronautical Society
1971  Fellow, the Royal Society of London
1973  Member, British National Committee for Theoretical and Applied Mechanics (to 1979)
1976  Founding Fellow, Fellowship of Engineering (from 1992, Royal Academy of Engineering)
1979  (With A. J. Sobey) George Taylor of Australia prize, Royal Aeronautical Society
1991  James Alfred Ewing Gold Medal, Institution of Civil Engineers
1994  Royal Medal, the Royal Society of London

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The frontispiece portrait is © Godfrey Argent Studio, 1986. The other photographs were kindly provided by the Mansfield family. Figures 2, 3 and 5b were created by the author for this memoir.

**Author Profile**

*Chris Calladine FRS FREng*

Chris Calladine FRS FREng is a retired professor of structural mechanics in the Engineering Department at the University of Cambridge, where he has spent most of his career. His research has been concerned with various aspects of structural engineering and design. After working in the nuclear power industry, he developed tools for understanding the behaviour of structures and components undergoing plastic deformation and creep at elevated temperature. Then he turned more generally to the mechanics of shell structures, and showed how Gaussian curvature can be used to advantage in analysis, as a scalar variable. He pioneered investigations into ‘tensegrity’ and related
structures, which paradoxically violate Maxwell’s rule for the construction of stiff frames. He has also explained some structural phenomena in the microbiological field. First, with Aaron Klug (FRS), he explained the construction of the helical flagellar filaments that are used in bacterial propulsion: the key to their helical polymorphism is a particular ‘mechanically’ bi-stable interface between neighbouring identical protein building-blocks in the tubular structure. With Horace Drew he interpreted the role of base-pair stacking phenomena in switching between the B- and A-forms of DNA; and also explained the mechanism for coding curvature into the molecule.

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