Modeling and analysis of a reactive low-pressure hydraulic turbine

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Abstract. This work is devoted to the development of a low-pressure reactive micro-hydroelectric power plant that operates independently from the general power supply. Impellers of a reactive hydro turbine proposed for a micro-hydroelectric power station includes a curved confusor nozzle. In this design, the cavitation effect practically does not affect the impeller system. In the turbine, the above-mentioned drawbacks of the existing turbines are minimized. The results of theoretical calculations and tables, graphical dependencies of an experimental reactive micro-hydroelectric power station are presented.

1. Introduction

It is known that the main part of existing rivers, channels, and sources of hydroelectric power have low heads. Today almost all over the world the use of this hydroelectric potential for the purpose of generating electricity is of particular interest and an urgent task in the field of hydropower, the efficient use of environmentally friendly sources of hydroelectric power is also one of the priority tasks of economic development [1, 2].

In the work performed the main part of a micro-hydroelectric power plant using water flow low–pressure turbines consists of parts with a bucket, propeller, vane impeller, and the improvement they aim to change the angle of water impact on the blades, changing the curvature of the surface of the blades and to determine the optimal values of their sizes [3 – 6].

The existing designs of jet hydro turbines (radial-axial, propeller, rotary-blade, two-pin) are characterized by the fact that they work effectively at heads greater than 4 m [7 – 10].

The analysis of the source shows that propeller turbines have a significant disadvantage, which is that their efficiency changes sharply with changes in load, and the zone of high efficiency values is observed only in a narrow range of power changes. This disadvantage significantly reduces the efficiency of propeller turbines when used in systems with an energy deficit [11]. At low heads, the efficiency of these turbines decreases sharply, which gives an unsatisfactory result in a low-pressure watercourse.

As follows from the above, the problem of creating new hydro turbines that provide electrical energy with high efficiency in low-pressure water flows has not been solved. Therefore, the importance of developing new designs of micro-hydroelectric power plants that work effectively in low-pressure water flow has been identified.
2. Methods
At the suggestion of the authors of this work, the solution to this goal is achieved by the fact that in a reactive low-pressure hydraulic turbine, containing an impeller with channels for water outflow and a stator with reflectors to increase efficiency by improving the reactive recoil and simplifying the design the impeller is made in the form of a cylinder, blades, and channels with an outlet nozzle for the outflow of water, located on the same horizontal plane of the bottom of the working cylinder. The channels for water outflow represent a concave conical in cross-section of the pipe, located perpendicular to the inner radius of the impeller, which has an outlet nozzle, allowing the water flow coming out of the nozzle to be directed perpendicular to the tangent plane, drawn to the point of the center of the arc of a concave and vertically mounted circular – cylindrical reflector [11].

The water flow inside the cylinder moving from the center along the radius of the impeller acts with active forces on the inner blades of the nozzle and during further movement is reflected from the inner wall of the nozzle and directed to the outlet giving a reactive force (Fig. 1).

When water moves in the transition from the Central cylinder to the nozzle the water flow branches into N the number of the nozzle, since the area is a member of the water nozzle greater than the area of the Central cylinder so there is a reduction in the average water flow rate and pressure loss due to the rotation of water at 90°. Taking this into account, solving the Bernoulli equation for the velocity \( V_3 \) of water entering the nozzle, we obtain [12]:

\[
V_3 = \sqrt{V^2 \left[ 1 + \frac{2S_2}{aNS_3} \left( 1 - \frac{S_2}{NS_3} \right) - \frac{1}{a} \left( \frac{S_2}{NS_3} \right)^2 \left( 1 - \frac{S_2}{NS_3} \right)^2 \right] + \frac{2gH_2}{\alpha}} \tag{1}
\]

When solving the Bernoulli equation for the average water velocity at the outlet of the nozzle, we get the following result:

\[
V_c = V_3 \frac{S_3}{S_c} - \sqrt{\left( \frac{S_3}{S_c} - 1 \right)^2 - \xi_{90°}} \tag{2}
\]

For the water pressure \( H_0=2 \text{ m} \), and the speed at the entrance of the water flow to the turbine \( V_c=5.95 \text{ m/s} \), \( d=0.273 \text{ m} \), at a temperature of 15°C, the coefficient of dynamic viscosity will be considered equal to \( n=0.0114 \), we get the Reynolds number [13]: \( Re=140921 \) \( (10^5 \leq Re \leq 10^8) \).

Since the water flow inside the turbine has a vortex character, to calculate the coefficient of hydraulic friction, you can use the formula proposed by Nikuradze:

\[
\lambda = 0.0032 + 0.22Re^{-0.237} \tag{3}
\]

When \( Re=140921 \lambda=0.0165 \), then the pressure loss will be determined as follows:

\[
h = \frac{\lambda L}{d} \cdot \frac{V_c^2}{2g} = \frac{0.0165 \cdot 1.1}{0.3} \cdot \frac{V_c^2}{2g} = 0.0605 \cdot \frac{V_c^2}{2g} \tag{4}
\]

here \( L \) is the distance to which water flows.

From (6) it can be seen that at low pressure, due to friction during the turbulent movement of water inside the turbine, a maximum of about 6% -7% of the kinetic energy of the liquid flow is lost.

To calculate the pressure loss when branching water, the geometric shapes, and sizes of all the nozzles, their water consumption is considered the same, then to calculate the pressure loss, we get the following expression:

\[
h_{b-v} = \frac{Q^2}{N^2} \cdot \frac{\rho}{2g} \cdot \left( 1 - \frac{S_2}{NS_3} \right)^2 \tag{5}
\]
Solving equations (3)-(6) together, we find the total coefficient of energy loss when the water flow enters the nozzle:

\[ \xi_{\text{coef}} = 1 + \frac{2S_2}{aNS_3} \left( 1 - \frac{S_2}{NS_3} \right) - \frac{1}{a} \left( \frac{S_2}{NS_3} \right)^2 (1 - \frac{S_2}{NS_3})^2 \]

(6)

All the components of the impeller and the water inside it spin together. Their moment of inertia relative to the rotating shaft is calculated by integration. Table 1 shows the results of the theoretical calculation for the prepared prototype:

| №  | \( I_1 \)  | \( I_2 \)  | \( I_3 \)  | \( I_4 \)  | \( I_5 \)  | \( I_6 \)  | \( I_t \) \( (\text{kg}^*\text{m}^2) \) |
|----|-----------|-----------|-----------|-----------|-----------|-----------|----------------|
| 1  | 2.371787  | 2.507981  | 1.899842  | 7.412789  | 0.24357   | 0.30396   | 18.18973      |

In the construction under consideration (Figure 1) the movement of the impeller occurs under the action of a reactive force \( F \), then the moment of rotational force is obtained from the General theorem about the change in the kinematic moment of a solid [13]:

\[ M_z = -N\pi\rho R^3_1 v_c (v_c - \omega_z R_c) = -N\pi\rho R^3_1 v_c^2 \left( 1 - \cos \beta \right). \]

(7)

The reaction force \( F \) acting on point A in Fig.1, will be equal to the geometric difference between the pulses of the amount of water entering the nozzle and the amount of water leaving it:

\[ F = K_c - K_3; \quad K_3 = \rho S_3 \theta^2_3; \quad K_c = \rho S_c \theta^2_c; \quad K_3 = K_{rt} = P_{s_3} v^2_{rt} \]

(8)

\[ F = K_c - (-K_3 \cos \alpha) = K_c + K_3 \cos \alpha \]

(9)
\[ \omega_x = \frac{\vartheta_c (R_c^2 - r_c^2)}{R_c^2 R_c \vartheta_c + R_c^2 \vartheta_c} \]  

(10)

here, \( R_c \) is the distance from the axis of rotation to the center of the nozzle, where the water comes out; \( r_c \) is the radius of the place where the water exits the nozzle; \( \vartheta_b, \vartheta_t \) are the speed of the water at the entrance to the nozzle and at the exit from the nozzle, respectively.

For example, for local resistance to water to be minimal, the following condition must be met:

\[ \frac{S_3}{S_c} \geq \sqrt{\xi_{90} - \xi_2} \]  

(11)

here, \( S_c \) is the output cross-section of the nozzle; \( \xi_{90} \) is the coefficient of resistance when turning the water by 90°, and \( \xi_2 \) is the coefficient of resistance at the confusor section of the nozzle.

3. Results and Discussions

Theoretical analyses were performed for the proposed design, and the dependence of the torque of the forces and the efficiency of the hydro turbine on the water pressure was obtained. For rice.2 shows their graphical relationships.

With increasing \( H_0 \) due to the resistance at the turning points in the hydro turbine, the loss of energy increases in proportion to the square of the speed. To ensure that these energy losses are minimal, condition (11) must be met. Table 2 shows the corresponding energy values of the hydro turbine obtained by theoretical calculation.

Figure 2. a) Graph of the dependence of the efficiency of the micro-hydroelectric turbine on the water head; b) the dependence of the efficiency of the torque hydro turbine on the water head.

Changes in the energy characteristics of a hydro turbine in relation to changes in its size can be analyzed based on the data in table 2. Table 2 shows that when the water pressure increases, the speed of the outflow of water also increases, and as a result, a lot of water is consumed. This leads to the fact that in places where the volume of water consumed is insufficient, the hydro turbine operates with low energy indicators. Therefore, with a constant amount of water, changing the size of the components of the hydro turbine, according to the water pressure, leads to an increase in its efficiency.

Figure 3 shows the dependence of the efficiency of the hydraulic turbine on the output radius of the nozzle. It is known that, if the size of the hydraulic turbine is unchanged, an increase in the head will lead to an increase in the amount of water flow due to an increase in the speed of water output. As a result, the hydro turbine operates with low efficiency. The graph shows that the high efficiency of the hydraulic turbine is observed only at a critical value of the radius of the nozzle outlet. Such water turbines should be used in places where it is possible to increase water consumption. In the constant output radius and number of the impeller nozzle, when the water pressure increases after 5 m, there is
a slow increase in the efficiency and power of micro-hydroelectric power plants, accordingly, the flow rate of the liquid increases.

Table 2. Changes in the energy parameters of micro-hydroelectric power plants correspond to changes in the geometric dimensions of the hydro turbine

| $H_0$ | $V_0$ | $V_f$ | $V_j$ | $V_C$ | $\omega_0$ | $P$ | $M$ | $FIK$ | $n_r$ | $Q$ |
|-------|-------|-------|-------|-------|-----------|-----|-----|------|------|-----|
| M     | m/s   | m/s   | m/s   | m/s   | rad/s     | Vm  | m   | %    | rpm  | l/s |
|-------|-------|-------|-------|-------|-----------|-----|-----|------|------|-----|
| 1     | 3.26  | 2.91  | 3.42  | 6.33  | 17.92     | 988.91 | 55.19 | 59.85 | 166.15 | 168.43 |
| 2     | 5.32  | 4.58  | 5.14  | 9.51  | 32.01     | 2371.26 | 74.07 | 67.54 | 296.85 | 178.93 |
| 3     | 6.79  | 5.78  | 6.42  | 11.87 | 44.21     | 3762.53 | 85.11 | 70.12 | 409.96 | 182.32 |
| 4     | 7.98  | 6.77  | 7.48  | 13.83 | 55.36     | 5156.11 | 93.14 | 71.42 | 513.35 | 183.99 |
| 5     | 9.03  | 7.64  | 8.41  | 15.54 | 65.80     | 6550.67 | 99.55 | 72.19 | 610.17 | 184.99 |
| 6     | 9.96  | 8.42  | 9.24  | 17.09 | 75.71     | 7945.73 | 104.95 | 72.71 | 702.09 | 185.65 |
| 7     | 10.81 | 9.13  | 10.01 | 18.50 | 85.21     | 9341.09 | 109.63 | 73.08 | 790.15 | 186.13 |
| 8     | 11.60 | 9.79  | 10.72 | 19.82 | 94.36     | 10736.64 | 113.78 | 73.36 | 875.04 | 186.48 |
| 9     | 12.34 | 10.40 | 11.39 | 21.05 | 103.23    | 12132.33 | 117.53 | 73.58 | 957.27 | 186.76 |
| 10    | 13.04 | 10.99 | 12.02 | 22.22 | 111.85    | 13528.11 | 120.95 | 73.75 | 1037.20 | 186.98 |

Figure 3. Dependence of the efficiency of a hydro turbine on the output radius of the nozzle

Figure 4 shows a diagram of the relative location of the output channel 4, nozzle 5, and reflectors 7. From the geometric arrangement, it follows that the values of the radius of the impeller $R_{pk}$, the radius of the stator $R_{st}$, the radial height of the nozzle $h_{cn}$, the radial height of the reflector $h_{cm}$ and the radial distance between the nozzle point far from the center of the impeller and the point nearest to the center of the reflector $b$ are interrelated by the equation:

$$R_{cm} - R_{pk} = h_{cn} + h_{cm} + b$$

(12)

The number of reflectors is determined by the formula:
The equation for the coefficient $k$ is given by:

$$k = \frac{\pi (R_c + h_{om})}{d_c},$$

where $R_c + h_{om}$ is the internal radius of the stator.

To achieve the maximum torque of the reaction force, it is necessary to choose the optimal relationship between the values of the radial height of the reflector $h_{om}$ and the radial width of the rectangular nozzle $d_c$, the stepwise circular distance between adjacent stator reflectors $l$ and the angle between the tangent drawn around the impeller circle at the nozzle center point and the direction of the flow coming out of the nozzle $\beta$. At a distance $b$ equal to $0.7 d_c$ the maximum speed of the exited flow from the nozzle is preserved, in other words, the expression takes place:

$$b = 0.7 d_c$$

Under the condition $b > 0.7 d_c$ the water flow rate begins to decrease due to scattering and, consequently, the reactive force decreases; under the condition $b < 0.7 d_c$, hydraulic resistance appears due to the narrow space between the nozzle and the reflectors, which also leads to a loss of reactive recoil of the water flow.

For maximum reactive impact water flow is necessary to choose the height of the reflector and a circular step-by-step distance between adjacent reflectors of the stator $l$, the magnitude of which also depends on the size of the nozzle and defined by the expressions:

$$h_{om} = 2 d_c \cos \beta; \quad l = 2d_c$$

Perpendicular to the stress emerging from the nozzle water flow is necessary to choose the value of the angle between a tangent circle of the impeller in the nozzle and the direction emerging from the nozzle flow $\beta$ in the range $20^\circ \div 30^\circ$, what follows from the calculation of the maximum return of the reaction force and the experiment conducted on the layout of a hydraulic turbine.

A specific example of a jet hydraulic turbine design has the following dimensions: - impeller diameter 600 mm, - the height of the impeller 100 mm – number of drainage channels 12 pieces, the outer diameter of the stator 700 mm, the number of reflectors on the inner wall of the stator 36 pieces the diameter of a vertically installed shaft 40 mm, shaft height 1300 mm. The water flow through pipes with a diameter of 274 mm is fed to the hydraulic turbine, the shaft of which is equipped with a pulley. The rotation of the shaft at a speed of 180÷200 turnover/minutes is transmitted through the
pulley and the connecting belt with an acceleration coefficient of ≈ 5.2 to the shaft of the electric generator. The created "micro-HPP" with a jet hydraulic turbine has the following technical characteristics:
- water pressure ≈ 230 mm,
- power 4 kW, voltage 220÷230 V, current frequency 50 Hz, dimensions 700×700×1300 mm, - weight ≈120 kg (Fig.4). The micro-hydro power plant was tested in a natural environment with a technical capacity of 3924 W, the water pressure of 2 m, and a water flow rate of 200 l/s.

Table 3. Obtained energy parameters from the micro-HPP test

| Experience № | Water consumption (l/s) | Voltage (V) | Amperage (A) | Power (kW) | efficiency factor η (%) | The rotational speed of turbine n (rpm) |
|--------------|------------------------|-------------|--------------|------------|------------------------|---------------------------------------|
| 1            | 195-200                | 215         | 10.37        | 2.23       | 57                     | 144                                   |
| 2            | 195-200                | 210         | 10           | 2.10       | 54                     | 141                                   |
| 3            | 195-200                | 225         | 10.31        | 2.32       | 59                     | 145                                   |
| 4            | 195-200                | 218         | 9.85         | 2.15       | 54.5                   | 144                                   |
| 5            | 195-200                | 220         | 9.54         | 2.10       | 54                     | 145                                   |
| 6            | 195-200                | 216         | 10.18        | 2.20       | 56                     | 143                                   |
| 7            | 195-200                | 222         | 9.81         | 2.18       | 55.6                   | 146                                   |
| 8            | 195-200                | 218         | 10.6         | 2.33       | 59.5                   | 144                                   |
| 9            | 195-200                | 224         | 10.7         | 2.4        | 61.2                   | 146                                   |
| 10           | 195-200                | 216         | 10.2         | 2.2        | 56                     | 142                                   |

The test results are shown in Table 3.

As can be seen from Table 3, the average efficiency of the micro-HPP installation was 57%. If the efficiency of the generator ηg=0.95, the efficiency of additional installations ηd=0.95, then calculate the efficiency of the hydro turbine:

$$\eta_T = \frac{\eta_{\text{MHPP}}}{\eta_g \cdot \eta_d} = \frac{0.56}{0.95 \cdot 0.95} = 100\% = 62.05\%$$

The efficiency of the hydro turbine was 62.05%. The difference between the speed of rotation of a hydro turbine and theoretical calculations:

$$\Delta n_s = n_{sH} - n_s = 155.8 - 144 = 11.8 \text{ rpm}$$

The difference in the frequency of the hydro turbine was 11.8 rpm less than the theoretical calculations. The result of the experiment differs from the indicator of theoretical calculations by 7.57%. The results of the micro-HPP experiment with reactive hydroturbine are shown in Table 3.
In contrast to the analog [7, 8], when implementing these features, the proposed design of a reactive hydraulic turbine has the following advantages (Fig. 5):
- the impeller is a cylinder and allowed to RUB against the body only on one upper circle, on the lower side of the impeller in the form of a cylinder is mounted on a support, therefore reducing the loss of energy to mechanical friction;
- channel to drain the water from the cylinder of the impeller is rectangular in cross-section shaped tube with the tip of a nozzle, and provided mutual perpendicularity of the radius of curvature of the impeller and a channel to drain the water, as well as mutual perpendicular directions emerging from the nozzle the water flow and the tangential plane passing the center point of an arc mounted vertically concave and circular-cylindrical reflector for maximum efficiency torque reaction force;
- contains an effective shape base that allows vertical drop and rapid runoff of the maximum kinetic and potential energy of water; - high speed of water flow in the discharge channels, while the speed of rotation of the wheel is equal to the speed of water flow in the channel, which means that the maximum reactive return is achieved; - improving the energy and economic performance of the hydro turbine by simplifying the design and reducing material consumption.

4. Conclusions
For the design of micro-hydro obtaining the following conclusions:
- the conditions of maximum values of internal active forces by ensuring the impact of water flow on the surface of the nozzle vanes at an angle of 25°-30° with non-participating in the rotation of the impeller jet turbine guide blades efficiency of using the torque of the reaction force;
- it was found that by effective use of centrifugal forces and Coriolis forces in the impeller of a reactive hydro turbine, as well as by reducing internal friction and local resistance by using internal guide blades, it is possible to reduce energy losses in the hydro turbine by 20-40%;
- it was found that the use of low-frequency asynchronous generators in micro-hydroelectric power plants with a capacity of up to 50 kW excludes the use of additional elements in the form of power loss reducers and multistage pulleys. - developed a new design of micro-hydro, working on reactive principle and defines the relationship of technical and economic parameters of its component parts, jet turbine, and electric generator;
- it was found that changing the size of the hydraulic turbine impeller based on the water pressure leads to an increase in efficiency, the efficiency, in this case, varies within 59% -75%; for non-variable
turbine sizes, in the critical radius of the water outlet nozzle, the efficiency of the hydraulic turbine changes accordingly to the water pressure within 56% - 67%.

Thus, the proposed design of a reactive hydraulic turbine is operable, easy to implement, and can be used as the basis for creating new high-efficiency vertical hydro turbines for micro and mini hydroelectric power plants, as well as for upgrading existing ones.

The electricity produced in the proposed installation can be actively used far from the electrical networks, in remote villages that have a low-pressure hydro potential.

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