Cauchy Horizons, Thermodynamics and Closed Time-like Curves in Planar Supersymmetric Space-times

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ABSTRACT

We study geodesically complete, singularity free space-times induced by supersymmetric planar domain walls interpolating between Minkowski and anti-de Sitter (AdS$_4$) vacua. A geodesically complete space-time without closed time-like curves includes an infinite number of semi-infinite Minkowski space-times, separated from each other by a region of AdS$_4$ space-time. These space-times are closely related to the extreme Reissner Nordström (RN) black hole, exhibiting Cauchy horizons with zero Hawking temperature, but in contrast to the RN black hole there is no entropy. Another geodesically complete extension with closed time-like curves involves space-times connecting a finite number of semi-infinite Minkowski space-times.

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In this Letter we analyze geodesically complete space-times induced by supersymmetric domain walls interpolating between $3 + 1$ Minkowski and Anti-deSitter ($AdS_4$) vacua.\cite{1,2} These are planar and causally non-trivial exact solutions to Einstein’s equations without space-time singularities. In addition, they provide a new class of space-times, asymptotically Minkowski in a direction away from the wall, in which the thermodynamic properties of Cauchy horizons and the physics of closed time-like curves can be studied, and with the attractive feature that the curvature is everywhere finite.

The ADM mass of supersymmetric (Minkowski or $AdS_4$) vacua in $N = 1, d = 4$ supergravity vanishes\cite{3}, thus ensuring the degeneracy of supergravity vacua. A complementary way of establishing stability of such vacua exploits\cite{4} the existence of a Bogomol’nyi bound for ADM mass/area stored in the wall of the bubble tunnelling between two supersymmetric vacua. Namely, the minimal ADM mass/area of such a bubble wall is compatible with the Coleman-DeLuccia bound\cite{5} only in the limit of infinite bubble radius, i.e., in the limit when the tunnelling is absolutely suppressed. As a consequence of vacuum degeneracy one expects domain wall solutions interpolating between isolated supersymmetric vacua not only in the global case\cite{6,7}, but also when gravity is turned on.

Domain walls interpolating between non-degenerate supersymmetric minima of a supergravity matter potential were found in Ref.\cite{1} and further studied in Ref.\cite{2}. A field theoretical realization of such walls exists within $N = 1, d = 4$ supergravity coupled to chiral superfields. We consider the simplest case of one chiral supermultiplet, $T$, with a nonzero superpotential, $W(T)$. The bosonic part of the Lagrangian density (consisting of the metric $g_{\mu\nu}$ and the complex scalar field component $T$) is

$$L = -\frac{1}{2\kappa} R + K_{T\bar{T}} g^{\mu\nu} \partial_\mu T \partial_\nu \bar{T} - e^{\kappa K} (K^{T\bar{T}} |D_T W|^2 - 3\kappa |W|^2)$$  \hspace{1cm} (1)

where $K(T, \bar{T}) = \text{Kähler potential}$, $D_T W = e^{-\kappa K}(\partial T e^{\kappa K} W)$ and $\kappa = 8\pi G$. Supersymmetric vacua satisfy $D_T W = 0$; therefore, supersymmetric vacua with $W = 0$.
correspond to Minkowski space-times while those with $W \neq 0$ correspond to $AdS_4$ space-times with cosmological constant $\Lambda = -3|\kappa We^{\frac{2\kappa}{T}}|^2 = -3\alpha^2$.

One of the minimal energy domain wall solutions corresponds to static, planar domain walls$^{[1,2]}$ interpolating between an isolated supersymmetric Minkowski vacuum as $z \to \infty$ and an isolated supersymmetric $AdS_4$ vacuum as $z \to -\infty$. The matter and the metric coefficients satisfy the first order differential equations corresponding to the supersymmetric bosonic backgrounds. One finds a static solution both for the scalar matter field $T(z)$ and the conformally flat metric:

$$ds^2 = A(z)(dt^2 - dx^2 - dy^2 - dz^2).$$  \hspace{1cm} (2)$$

The conformal factor $A(z)$ has the asymptotic behaviour:

$$A(z) \rightarrow \begin{cases} 
1, & z \to +\infty \\
(\alpha z)^{-2}, & z \to -\infty
\end{cases} \hspace{1cm} (3)$$

while in the wall region ($z \sim 0$) it smoothly interpolates between the two regions. Here, $\Lambda = -3\alpha^2$ is the cosmological constant of the $AdS_4$ vacuum. The wall’s ADM mass/area $\sigma = (2/\sqrt{3})\kappa^{-1}|\Lambda|^{1/2} \equiv 2\alpha$ saturates the Bogomol’nyi bound.

The possibility of a static juxtaposition of $AdS_4$ and Minkowski space-times can be understood from a general relativistic perspective without referring to the underlying matter field configuration. For this purpose we approximate the wall as infinitely thin with the conformal factor $A(z)$ changing from the $AdS_4$ form, $A(z) = (\alpha z)^{-2}$, to the Minkowski form, $A(z) = 1$, at a chosen value of $z = z_0 \equiv -\alpha^{-1}$. Because Einstein’s tensor is of second order in derivatives of the metric, it has a $\delta$-function singularity at $z_0$, and we therefore use Israel’s$^{[8]}$ method of singular hypersurfaces. The wall is found to satisfy the equation of state of a domain wall.

By adapting Tolman’s$^{[9]}$ mass formula to give a gravitational surface mass density, the effective gravitational mass/area of the wall is calculated to be $-2\alpha$ whereas that of the semi-infinite $AdS_4$ space-time is $2\alpha$. The exact cancellation of the
effective masses allows for a semi-infinite Minkowski space-time of zero effective gravitational mass to exist adjacent to the wall with the semi-infinite AdS$_4$ space-time on the other side. The above result follows from Einstein’s equations and the assumed form of the metric. Supergravity provides a field theoretic realization of such domain walls with finite thickness and an ADM mass/area precisely cancelling the effective gravity of the AdS$_4$ vacuum.

We now discuss the space-time induced by the metric (2), (3). We first study the motion of test particles which define the geodesics\textsuperscript{[2]}. The nulls are trivial since the metric is conformally flat. In addition, due to the boost invariance of the metric in the $x, y$ directions, we can without loss of generality move to a frame in which a test particle moves only transverse to the wall. Thus, it is sufficient to study time-like geodesics in the $1 + 1$ system $ds^2 = A(z)(dt^2 - dz^2)$. As the metric is static, there is a conserved energy parameter $\epsilon = A(z)dt/d\tau$ where $\tau$ is the proper time. For time-like world-lines on the AdS$_4$ side (where $A(z) = (\alpha z)^{-2}$) the equation of motion yields\textsuperscript{[2]}

\[ z^2 - t^2 = (\alpha \epsilon)^{-2}. \] (4)

This is the same world-line as a particle undergoing a constant proper acceleration of $(\alpha \epsilon)$ moving in a Minkowski space-time, \textit{i.e.}, a Rindler particle\textsuperscript{[11]}. The constant scalar curvature of AdS yields, through the equivalence principle, a freely falling Rindler particle\textsuperscript{[10]}.

The proper distance of a constant time slice, $d(z) = \int^z \sqrt{A(z')}dz'$ is logarithmically divergent on the AdS$_4$ side. However, the time-like geodesics leaving the wall at $\alpha z = -1$ and moving into the AdS$_4$ side reach $z = -\infty$ with $t = \infty$ in the finite proper time $\tau = -\int_{-1/(\alpha \epsilon)}^{-\infty} A(z)dz/\sqrt{\epsilon^2 - A(z)} \approx \pi/2\alpha$, where $A(z) \approx (\alpha z)^{-2}$ was used. Therefore, the coordinates $z, t$, which completely cover the semi-infinite Minkowski side, are not geodesically complete on the AdS$_4$ side\textsuperscript{[12]}. To make the space-time geodesically complete we must extend the AdS$_4$ side beyond the horizon at $z = -\infty$ onto a new patch. On this new patch, we define a new coordinate
and identify \( z' = +\infty \) with the old coordinate \( z \) at \( z = -\infty \). In addition, \( t = \infty \) becomes \( t' = -\infty \) upon crossing the horizon, which continues the forward flow in the time-like direction. By introducing the new semi-infinite \( AdS_4 \) region, we have added \( 2\alpha \) to the effective gravitational mass/area of the system, and to remain in gravitational equilibrium we place an identical domain wall centered at \( z'_0 = +\alpha^{-1} \). The smooth extension of the scalar field creating the new wall is \( T(z') = T(-z) \). This new wall interpolates between the new \( AdS_4 \) region and another semi-infinite Minkowski space-time.

At this point we have two domain walls separated by a region of \( AdS_4 \) and outer regions of semi-infinite Minkowski space-times. Clearly, the time-like geodesics can leave this system and thus another extension must be specified\(^{14}\). Depending on the choice of identifications there are different possibilities, some of which we enumerate here and are depicted in Fig. 1 by their conformal diagram.

(A) One can choose the covering space of the domain wall system, which makes no identifications and thus contains no closed time-like curves. The system is an infinite lattice of semi-infinite Minkowski universes separated by a continuous \( AdS_4 \) core. This space-time is similar to that of the extreme Reissner-Nordström (RN) black hole\(^{14}\). However, in the extreme RN space-time there is a time-like curvature singularity, while here the singularity is replaced by a domain wall and the curvature is everywhere finite.

(B) One can identify semi-infinite Minkowski regions living vertically adjacent to one another. This identification yields closed time-like curves. In this case we see that the closed time-like curves of pure \( AdS_4 \) become a time-machine for the semi-infinite Minkowski universes: a particle can pass through the wall into the \( AdS_4 \) region, cross the Cauchy horizon, and reemerge from the wall at an earlier Minkowski time.

(C) One can make an identification between adjacent Minkowski half-spaces at finite distance from the wall. In this case the \( AdS_4 \) space acts as a wormhole in connecting two regions of Minkowski space-time\(^{15}\). The Minkowski times at the
wormhole mouths (domain walls) need not be the same, which leads, as in (B), to the existence of a time-machine for the Minkowski universes.

**Figure 1** Conformal diagram of the extended domain wall system. Coordinates $x, y$ are suppressed; therefore, each point represents an infinite plane with distances in the plane conformally compressed by $A(z)$. The compact null coordinates of 1 + 1 Minkowski space-time define the axes: $u', v' = 2 \tan^{-1}[\alpha(t \mp z)]$. These coordinates can be smoothly extended across the nulls separating the diamonds$^{[16]}$. Past and future null infinity for the semi-infinite Minkowski regions are $I^-$ and $I^+$, respectively. The domain walls are the double time-like lines splitting the diamonds. Time-like geodesics on the Minkowski universes are arcs beginning at past time-like infinity and ending at future time-like infinity. A time-like geodesic on the $AdS_4$ side is the periodic line passing from diamond to diamond. Cauchy horizons for data placed on the constant time slices in one diamond are the dashed nulls separating the $AdS_4$ patches. For possibility (B) one makes the identifications $\beta = \delta$. For possibility (C) one makes the identifications $\beta = \gamma = \delta$. Possibility (A) involves no identifications.

One of the most important aspects of these space-times is that they have Cauchy horizons, where predictability breaks down at the classical level. The nulls defining the Cauchy horizons are boundaries beyond which information placed on an infinite constant time slice in one of the diamonds is insufficient to specify the evolution of the data. These nulls are the boundaries between the diamonds defined by one domain wall, i.e., for the first diamond (see Fig. 1) the Cauchy horizon is at $u' = \pi, -\pi < v' < \pi$. Additionally, near the horizons, space-time is the maximally supersymmetric $AdS_4$ vacuum. It is interesting to note for motion in the transverse direction, the 1 + 1 line element near the Cauchy horizon can be thought of as the two-dimensional truncation of the maximally supersymmetric Robinson-Bertotti (RB) metric $ds^2_{RB} = (\rho/GM)^2 dt^2 - (GM/\rho)^2 d\rho^2, \rho \to 0$, where $z = -\alpha^{-2} \rho^{-1}$ and $\alpha = (GM)^{-1}$. Recall near the horizon ($r \to GM$) of the two-dimensional extremal RN black hole, the metric $ds^2_{RN} = (1 - GM/r)^2 dt^2 - (1 - GM/r)^{-2} dr^2$ is that of RB$^{[17]}$ where $\rho = r - GM$. Thus, as one passes through the RN horizon at $r = GM, t = \infty$, the radial motion can be described by the metric of 1 + 1 $AdS_2$. Outside the RN horizon, one has $z^{-1} = -t^{-1} = 0^-$ and inside the RN horizon one uses the next patch $z'^{-1} = -t'^{-1} = 0^+$. In this way the domain
wall induces a singularity free gravitational field with a Cauchy horizon similar to
the horizon of the extremal RN black hole.

We may examine the thermodynamics of the domain wall system, in particu-
lar, such properties associated with the covering space-time, case (A), which has
no closed time-like curves\cite{18}. Due to the Cauchy horizon, information will be lost
to the next diamond. However, there is no Hawking radiation from this horizon
and therefore the Hawking temperature of the system is zero. This zero tem-
perature result is apparent for the following reasons: (i) The wall is realized as
a supersymmetric bosonic configuration which is associated with zero tempera-
ture\cite{19}. Recall also that the extreme RN black hole is a supersymmetric bosonic
configuration\cite{20} and is known to have zero temperature. (ii) The Euclidean sec-
tion of the space-time does not exhibit a Euclidean time with finite period. (iii)
The gravitational mass behind the horizon is zero, which is consistent with its sur-
faced gravity, $\kappa^i = -g^{1/2} r\hat{r}_0\hat{0}$, being zero at the horizon. Here hats refer to a local
orthonormal frame.

The entropy of a black hole can be associated with the number of states ac-
cessible to collapsing matter forming the hole\cite{21}. In the case of the domain wall,
the system has zero temperature and thus only the degeneracy of the ground state
contributes to the entropy. In this case one has only one state (the solitonic config-
uration of the domain wall) and thus the entropy vanishes. In other words, there
is only one field configuration minimizing the action and interpolating between the
supersymmetric vacua which produce the horizon and only one smooth space-time
extension across the horizon. Alternatively, we can obtain this result by evaluating
the “surface term” at the horizon\cite{22}. Using Einstein’s equations with the domain
wall Ansatz for the Lagrangian $L$ from Eq. (1) yields the on-shell action

$$ I = \int L\sqrt{-g}d^4x = -\frac{1}{2\kappa} \int \frac{d}{dz} dA(z) dz \int dt dx dy $$

which is a pure surface term. It vanishes for this domain wall system. A lack of
entropy associated with a horizon is not un-precedented; however, the only known
examples are the minimal energy dilatonic “electro-magnetic” black-holes\textsuperscript{[23]} which have space-time singularities.

We complete our analysis by examining the behaviour of classical objects and quantum fields on this space-time, especially near the Cauchy horizons. Recall for the extreme RN case one has finite tidal forces and finite quantum field energies at the horizon; we will see the same results here. Away from these horizons there is no problem. Any tidal force or vacuum polarization is due solely to the domain wall and the constant curvature of the $AdS_4$ space-time. At the horizon, the space-time is $AdS_4$ which implies that if a classical object can withstand a local tidal force of magnitude $\alpha^2$ in all directions, passage through the Cauchy horizon into the next diamond is possible.\textsuperscript{[24]}

In case (A) one can evaluate the vacuum polarization which the gravitational field induces on a massless conformally coupled quantum field by relating it to the local curvature. A conformally coupled scalar field in a geometry conformally related to the Minkowski space-time has a vacuum stress-energy tensor equal to\textsuperscript{[25]} $\langle T_{\mu \nu} \rangle = -\frac{\hbar}{256\pi^3} S_{\mu \nu}$ where $S_{\mu \nu}$ is a second order curvature term\textsuperscript{[26]} and we expose factors of $\hbar$ to emphasize the quantum nature of this energy. This stress-energy tensor is regular everywhere, including the region near the Cauchy horizon and it vanishes on the Minkowski side. Note that this is yet another way of seeing that the Hawking temperature of the system is zero. In addition, a major contribution to the stress-energy tensor is due to the vacuum polarization near the wall where the gradient energy is concentrated.

The classical solution had an exact cancellation of the gravitational mass of the negative vacuum energy on the $AdS_4$ side and the gravitational mass of the wall. By breaking supersymmetry and letting a scalar field live on this background one expects the total gravitational mass to change, and in a semiclassical theory of gravitation this would lead to gravitational forces in the classically Minkowski region. To investigate this point, we calculate the Tolman\textsuperscript{[9]} mass/area from the
quantum field:

\[
\Sigma(z)_v = \frac{\hbar}{2880\pi^2} \left\{ \left[ \frac{1}{6} H^3 - H'' - HH' \right]_z - \int_{-\infty}^{z} \left[ 5H'^2 + (H' + H)^2 \right] dz' \right\},
\]

where \( H(z) \equiv A'(z)/A(z) \). On the AdS\(_4\) side, the quantum field induces a small change in the cosmological constant which means the geometry is virtually unchanged. Because the boundary term vanishes both at the horizon and in the Minkowski region far from the wall, the gravitational mass of the quantum vacuum is positive definite. Hence, as seen from the Minkowski side, there is a domain wall with a positive effective gravitational mass near \( z = 0 \). The gravitational force on a test particle is thus attractive, in contrast to the conventional case of \( \lambda \phi^4 \) non-supersymmetric domain wall which has repulsive gravitational interactions. It is well known\(^{[27]}\) that there exists no static vacuum solution of Einstein’s field equations describing this space-time. Therefore, the quantum corrections induce time-dependence in the metric. This example demonstrates the essential role played by supersymmetry in the existence of static Minkowski-AdS\(_4\) domain wall space-times.

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9. R. C. Tolman, Phys. Rev. 35, 875 (1930).

10. The spherical coordinates for pure AdS4 : \( ds^2 = (\alpha \cos \psi)^{-2}(dt_c^2 - d\psi^2 - \sin^2 \psi d\Omega_2^2) \) where \(-\pi \leq t_c \leq \pi, 0 \leq \psi < \pi/2\), are related to the planar coordinates through \( \alpha t = -\frac{\cos t_c}{\sin t_c - \cos \theta \sin \psi} \), \( \alpha z = \frac{\cos \psi}{\sin t_c - \cos \theta \sin \psi} \), \( \alpha x = \frac{\sin \psi \sin \theta \cos \phi}{\sin t_c - \cos \theta \sin \psi} \), \( \alpha y = \frac{\sin \psi \sin \theta \sin \phi}{\sin t_c - \cos \theta \sin \psi} \) yielding \( ds^2 = (\alpha z)^{-2}(dt^2 - dx^2 - dy^2 - dz^2) \). The transformation is defined on a particular null diamond, for example, \( 0 \leq t_c \pm \psi \leq \pi \), where time-like geodesics live. Under this transformation, the transverse time-like geodesics \( z^2 - t^2 = (\alpha \epsilon)^{-2} \) become the periodic radial geodesics \( \sin^2 \psi = [(1 - (\alpha \epsilon)^{-2})/(1 + (\alpha \epsilon)^{-2})]^2 \sin^2 t_c \). The planar coordinates with \( z < 0 \) combined with \( z > 0 \) completely cover all of \( AdS_4 \).

11. See for example W. Rindler, Essential Relativity, Springer-Verlag 1979.

12. In Refs. \[1\] and \[2\] the geodesic incompleteness of the \( AdS_4 \) space-time induced by the domain walls was not recognized.
13. There are analogous geodesically complete space-times for domain walls interpolating between two supersymmetric $AdS_4$ vacua, classified as type II and type III domain walls in Ref. [2]. For type II walls ($AdS_4 - AdS_4$ walls with superpotential $W$ passing through $W = 0$) the Penrose conformal diagram (for a smooth extension of the scalar field across the horizon) fills out the plane. For type III walls ($AdS_4 - AdS_4$ walls where $W$ does not pass through $W = 0$) the space-time essentially corresponds to the $AdS_4$ with a $z$ dependent cosmological constant. In this case the geodesically complete Penrose diagram (again for a smooth extension of the scalar field) is a strip, i.e., the middle of the $AdS_4$-Minkowski Penrose diagram.

14. S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time*. Cambridge 1973.

15. J. Friedmann, M. S. Morris, I. D. Novikov, F. Echeverria, G. Klinkhammer, K. S. Thorne, and U. Yurtsever, Phys. Rev. D42, 1915 (1990).

16. As Fig. 1 exhibits, we can extend the coordinates $u', v'$ across the Cauchy horizon. Explicitly this is seen by writing the $1 + 1$ line element near the horizon as $ds^2 = (\alpha z)^{-2}(dt^2 - dz^2) = [\alpha \sin(1/2(u' - v'))]^{-2}du'dv'$ which has a smooth extension across the null $u' = \pi, -\pi < v' < \pi$ as well as all the other Cauchy horizons. The full $3 + 1$ metric has coordinate singularities in the $x, y$ directions crossing the null.

17. Near the horizon the extremal $3 + 1$ RN black hole takes on the Robinson-Bertotti metric $AdS_2 \times S^2$. See for example: G. W. Gibbons in: *Supersymmetry, Supergravity and Related Topics*, (F. del Aguila et al. eds.) World Scientific, Singapore 1985, p. 147; D. Brill, Phys. Rev. D46, 1560 (1992); R. Kallosh and A. Peet, *Dilaton Black Holes near the Horizon*, Stanford preprint SU-ITP-92-27, hep-th/9209110.

18. The possibilities (B) and (C) with their closed time-like curves are problematic due to the ambiguity in formulating the Cauchy problem [15]. One may speculate with Hawking that the infinities discussed in his Chronol-
ogy Protection Conjecture [S. W. Hawking, Phys. Rev. D46, 603 (1992)]
could be cancelled due to the usual fermionic and bosonic cancellations in
supersymmetric theories.

19. G. W. Gibbons in: Supersymmetry, Supergravity and Related Topics, (F. del
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20. G. W. Gibbons and C. M. Hull, Phys. Lett. 109, 190 (1982).

21. For a review see P. C. W. Davies, Rep. Prog. Phys. 41, 1313 (1978).

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66, 2281 (1991); F. Wilczek, lecture notes at Princeton University, 1992,
unpublished.

23. R. E. Kallosh, A. D. Linde, T. M. Ortín, A. W. Peet, and A. van Proeyen,
Supersymmetry as a Cosmic Censor, Stanford preprint SU-ITP-92-13, hep-
th/9205027, May 1992.

24. Space-time singularities form when a homogeneous cloud of pressure-free
dust is introduced into pure $AdS_4$ due to the focusing effect of time-like
geodesics every $1/2$ the $AdS_4$ period: $\pi/2\alpha$: see F. J. Tipler, C. J. S. Clarke
and G. F. R. Ellis in General Relativity and Gravitation, vol. 2, edited by A.
Held, 1980 Plenum Press, New York. The same would happen in the domain
wall system if one allowed for an infinite plane of dust to fall through the
wall. However, for realistic finite perturbations the geometry is stable. In
addition, there are no uncontrollable tidal forces in the background geometry
and thus for physical particles with repulsive forces (i.e. non-zero pressures),
able to withstand the finite tidal forces produced by themselves and by the
background, no collapse occurs.

25. N. D. Birrell and P. C. W. Davies, Quantum Fields in Curved Space , chapter
6, Cambridge University Press 1982.

26. For completeness, we present the calculation of the renormalized stress-
energy tensor for a massless conformally coupled field on a conformally
flat space-time. Note we use the results for a space-time conformal to flat Minkowski space-time which is appropriate for one domain wall diamond. The formulae used can be found in Ref. [25]. The second order curvature terms are defined by

\[ S_{\mu \nu} = \frac{1}{6}(1)H_{\mu \nu} - (3)H_{\mu \nu} \]

where (1) \( H_{\mu \nu} = -2R_{\mu ;\nu} + 2R_{\rho ;\nu} \delta^\mu_{\nu} - \frac{1}{2}R^2 \delta_{\mu \nu} + 2RR_{\mu \nu} \) and (3) \( H_{\mu \nu} = R_{\mu \rho} R^\rho_{\nu} - \frac{2}{3}RR_{\mu \nu} - \frac{1}{2}R_{\alpha \beta} R^{\alpha \beta} \delta_{\mu \nu} + \frac{1}{4}R^2 \delta_{\mu \nu} \).

Defining \( H(z) \equiv A'(z)/A(z) \) we find

\[ A^2 S^t_\tau = -H''' + \frac{1}{2}HH'' - \frac{1}{4}(H')^2 + H^2H' - \frac{1}{8}H^4 \]

and

\[ A^2 S^z_z = -\frac{3}{2}HH'' + \frac{3}{4}(H')^2 + \frac{3}{8}H^4 \] .

By boost invariance along the wall it follows that \( S^t_\tau = S^x_x = S^y_y \).

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