The NP right-chiral CC coupling constant estimation in neutrino oscillation experiments

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Abstract

The error probability of the discrimination of the Standard Model (SM) with massive neutrinos and its new physics (NP) model extension in experiments of the muon neutrino oscillation, following the pion decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$, is calculated. The stability of the estimation of the NP charged current coupling constant $\varepsilon_R$ is analysed and the robustness of this estimation is checked.

Keywords: Neutrino oscillation, Density matrix, Relative entropy, Statistical information, Quantum measurements

1. The muon neutrino density matrix

The well known modelling of the chiral right-handed currents is connected with left-right symmetric extensions of the Standard Model (SM) [1–3]. There are also effective-Lagrangian SM extensions which can be used to inspect the existence of the chiral right-handed interactions [4, 5, 6]. This paper follows this path. Let the muon neutrino $\nu_\mu$ be produced in the decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$ of pion to muon and the muon Dirac neutrino $\bar{\nu}_\mu$. The neutrino $\nu_\mu$ produced in this process is the relativistic one. The muon flavour neutrino state $|\nu_\mu\rangle$ is a superposition of the stationary states $|\nu_\mu\rangle_\nu \equiv |p, \lambda, \iota\rangle$ of definite masses $m_\nu$, $i = 1, 2, 3$, helicities $\lambda = -1$ or $+1$ and four-momentum $p$ [8]. By including new physics (NP) interactions [9, 10], e.g., the chiral right-handed interactions [6], this superposition composes the mixed state $|\nu_\mu\rangle_{1;2;3}$ [11, 12]. The other reason of the departure from the pure state can be connected, e.g., with the existence of scalar interactions [8]. From the $\pi^+ \rightarrow \mu^+ + \nu_\mu$ decay experiments we know that the fraction of the right-handed $N_{\nu_{1;2}}$ to the left-handed $N_{\nu_{1;2}}$ neutrinos fulfils the constraint $N_{\nu_{1;2}}/N_{\nu_{1;2}} < 0.002$ [11, 12]. Let us assume that the pion decays effectively both in the left ($L$) and right ($R$) chiral charged current (CC) interactions [10] via the exchange of the SM W-boson only. Then, at the W-boson energy scale, the $R$ and $L$ chiral pion decay constants [13, 14, 15] are equal and the (pseudo)scalar correction can be neglected due to its smallness [16, 17]. The invariant amplitudes $A^L_{i;1,\lambda}$ [10] in the decay $\pi^+ \rightarrow \mu^+ + \nu_\lambda$, where $\lambda = -1$ or $+1$ is the muon helicity, are related as follows:

$$|A^L_{1;1,+1}(p)|^2 = |A^L_{1;1,-1}(p)|^2 \frac{|e_R|^2|U^{R\ast}_{ii}|^2}{|e_L|^2|U^{L\ast}_{ii}|^2},$$

where $U^L_{ii}$ and $U^R_{ii}$ are the $L$ and $R$ chiral neutrino mixing matrices, which enter into the CC Lagrangian in the products with the coupling constants $e_L$ and $e_R$, respectively [10]. $U^L_{ii}$ is the Maki-Nakagawa-Sakata-Pontecorvo neutrino mixing matrix [18, 19]. For relativistic neutrinos, the dependance of the production process on the neutrino masses can be neglected [7]. Then, in the production (P) process, in the center of mass (CM) frame and in $|\nu_\mu\rangle_\nu$ basis, the elements of the general form of the 3×3-dimensional nonzero muon neutrino reduced mass-helicity density matrix (obtained from the full density matrix by tracing out the other degrees of freedom) are as follows [11, 12, 13]:

$$\varrho_{-1;1,-1}^{P;1,\iota} = \frac{A^L_{1,1}}{N_\nu} |e_L|^2 |U^{L\ast}_{ii}|^2 U^{R\ast}_{ii}, \quad \varrho_{+1;1,+1}^{P;1,\iota} = \frac{A^R_{1,1}}{N_\nu} |e_R|^2 |U^{R\ast}_{ii}|^2 U^{L\ast}_{ii},$$

(2)

where $N_\nu = A_{e_L}^2 |e_L|^2 + A_{e_R}^2 |e_R|^2$ is the normalization constant and $A_{e_L} = A_{e_R}$. The functions $A_{e_L}$ and $A_{e_R}$ are the amplitudes for the CC vector-axial processes, i.e., V-A and V+A, respectively. They depend on the energies and momenta of the particles in the production process of the neutrino. Thus, the density matrix elements are as follows:

$$\varrho_{-1;1,-1}^{P;1,\iota} = \frac{|e_L|^2 |U^{L\ast}_{ii}|^2 |L_{\mu \mu}|^2}{|e_R|^2 |U^{R\ast}_{ii}|^2 + |e_L|^2 |U^{L\ast}_{ii}|^2}, \quad \varrho_{+1;1,+1}^{P;1,\iota} = \frac{|e_R|^2 |U^{R\ast}_{ii}|^2 |L_{\mu \mu}|^2}{|e_R|^2 |U^{R\ast}_{ii}|^2 + |e_L|^2 |U^{L\ast}_{ii}|^2}. \quad (3)$$

They constitute the muon neutrino 6×6-dimensional block diagonal density matrix $\rho^{P;1,\iota} = \varrho_{1;1;}^{P;1,\iota}$ (where $\varrho_{1;1;}^{P;1,\iota}$ is diagonal).

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We choose $U_{ai}^R = U_{ai}^L$, otherwise there is not only the neutrino helicity mixing but also the mass mixing \[8\]. Since the density matrix elements, Eq. (3), depend on the norms of $\varepsilon_L$ and $\varepsilon_R$, and not on their phases, we assume in the analysis that these coupling constants are real. The NP values of $\varepsilon_L$ and $\varepsilon_R$ can deviate slightly from the SM values 1 and 0, respectively. Using the constraint $N_{e,i}/N_{e,i} < 0.002$ from Eq. (1) the bound on the ratio $R = |\varepsilon_R/\varepsilon_L| < 0.0447 \approx 0.045$ results. Since the Fermi constant constraint $\varepsilon_L^2 + \varepsilon_R^2 \equiv 1$ should also hold, we obtain (due to the 4th power) that $|\varepsilon_R| < 0.0447 \approx 0.045$ constraining the density matrix $(\rho_{\mu,i+1}^{\mu})$ of the initial neutrino.

The muon neutrino $\nu_\mu$ produced in the process $\pi^+ \to \mu^+ + \nu_\mu$ is the relativistic one (the neutrino energy $> 100$ MeV in the Laboratory (L) frame). Thus, the effect of the helicity Wigner rotation is negligible \[3\] resulting in $\rho_{\mu,i}^{\mu}(\mathbf{p}_L) = \rho_{\mu,i}^{\mu}(\mathbf{p})$ for the density matrix in the L frame, where $\mathbf{p}_L$ and $\mathbf{p}$ are the neutrino momenta in the L and CM frames, respectively. Only the neutrino produced in the L frame in the forward direction along the $z$-axis reaches the detector and this axis is chosen as the quantization one \[7\]. After production the neutrino $\nu_\mu$ propagates in matter and we assume that this is the non-dissipative \[20\] homogeneous medium. By virtue of quantum mechanical unitarity of the muon-environment time evolution, the interactions of the entangled muons with their environment cannot affect, in any experiment, the probability of neutrino oscillation that follows the pion decay \[21\]. Thus, in the relativistic case, when the distance $z$ and the propagation time $t$ approach the relation $z = t$ \[7\] (see Appendix), the evolution rule for the neutrino density matrix is as follows:

\[
\rho^{\mu}(t = 0) \to \rho^{\mu}(t) = e^{-i\mathcal{H}_i t} \rho^{\mu}(t = 0) e^{i\mathcal{H}_i t},
\]

(4)

where $\rho^{\mu}$ is the initial density matrix \[3\] and $\mathcal{H}$ is the effective Hamiltonian. Although the coherence properties of the neutrino beam resulting from pion decay are influenced \[8\] by the initial pion state, for standard neutrinos no coherence loss is expected on terrestrial scales \[21\]. Under above assumptions, the oscillation probability from $\mu$ to $\beta$ flavour at the detection (D) point at $z = L$ is equal to $P_{\mu-\beta}(L) = \text{Tr}[\rho^{\mu}(L) \hat{P}^{\beta}]$. Here $\hat{P}^{\beta} \equiv (\hat{P}_{\beta,j}^{\beta}) = \text{diag}((U_{\mu,L}^R)^2, (U_{\mu,R}^L)^2)$ is a $6 \times 6$-dimensional block diagonal projection operator to the $\beta$ flavour direction in the neutrino flavour space \[22\].

With three massive and two helicity neutrino states, $\mathcal{H}$ has the $6 \times 6$-dimensional representation (see, e.g., \[6, 10, 23\]):

\[
\mathcal{H} = M + \mathcal{H}_{\text{int}},
\]

(5)

where the $6 \times 6$-dimensional diagonal matrix $M$ is the mass term \[6, 7\]. Here $\mathcal{H}_{\text{int}}$ is the $6 \times 6$ matrix representation of the interaction Hamiltonian \[10, 23\] for the coherent neutrino scattering inside the non-dissipative homogeneous medium \[6, 10, 23, 24\]. We will see that under the above conditions, data obtained in all earth’s oscillation experiments in which the muon neutrinos are produced in the process $\pi^+ \to \mu^+ + \nu_\mu$ fall into one category of results that together enable the discrimination of the SM from the NP model (expressed by Eq. (3)).

Note. Usually, the precise knowledge of the evolution of the neutrino density matrix during oscillation experiments (Appendix), ruled by the particular form of the Hamiltonian $\mathcal{H}$, is necessary. It is the case, for example, in the consistency analysis \[23\] of the values of parameters of $U_{\mu,i}^{\mu}$ with the predictions of (the type of) the Aharonov-Anandan neutrino geometric phase considerations \[25\].

2. The SM and NP model discrimination

The value of the departure of the purity of the quantum state $\text{Tr}[(\rho^\mu)^2]$ from 1, \[26\], is a second order effect in the NP parameter $\varepsilon_R$ \[8\]. In order to find the distance in the statistical space of distributions, it is convenient to represent the density operator in the spectral-decomposition form, i.e.:

\[
\rho^\mu(z) = \sum_{j=1}^{N} p^j(z) |w^j(z)\rangle \langle w^j(z)|,
\]

(6)

where $p^j(z) \geq 0$ and $|w^j(z)\rangle$ are the eigenvalues and (normalized) eigenvectors of $\rho^\mu(z)$, respectively, while $N$ is the rank of $\rho^\mu$ and $\sum_{j=1}^{N} p^j(z) = 1$. The NP and SM neutrino quantum states are given by the density matrices $\rho^\mu_{NP}$ and $\rho^\mu_{SM}$, respectively. The spectral decomposition of the density matrix is unique in the sense that for it the von Neumann entropy $S(\rho^\mu) := -\text{Tr}(\rho^\mu \ln \rho^\mu)$ is equal to the Shannon entropy $S(p) = -\sum_{j=1}^{N} p^j \ln p^j$ of the probability distribution $p \equiv \{p^j\}_{j=1}^{N}$ \[26\].
The advantage of the spectral-decomposition form is that via 
Fisher-Rao metric (which is related to Shannon entropy [26]), 
the eigenvalues \( p^j \) enter into the calculation of the classical 
lower bound for the variance of the unbiased estimator of a para-
meter (see Section 2.2.1).

The NP effects change the neutrino state in the course of the 
ocillation in a different way than the SM [28]. Yet, due to the 
unitarity of the evolution given by Eq. (4) the eigenvalues of the 
density matrices \( \rho_{SM}^j \) and \( \rho_{NP}^j \), which we denote as \( p_{SM}^j \) and 
\( p_{NP}^j \), respectively, do not vary with \( z \). This happens only if 
the neutrino evolution remains unitary, as it is, e.g., in the case of 
the (constant density) slab approximation [7]. Thus 
\[
\begin{align*}
p_{SM}^j &= p_{SM}^j(z = 0) = p_{SM}^j(z = L) \\
p_{NP}^j &= p_{NP}^j(z = 0) = p_{NP}^j(z = L), \quad j = 1, 2, \ldots, 6
\end{align*}
\] (7)

and the NP v.s. SM discrimination in the probability 
boundaries \( p_{NP} \equiv \{p_{NP}^j\} \) and \( p_{SM} \equiv \{p_{SM}^j\} \) does not change 
during the course of the neutrino propagation from the moment 
of its production at \( z = 0 \) up to the point of its detection at 
\( z = L \). Therefore, \( p_{NP}^j \) and \( p_{SM}^j \) are the invariants of the neutrino 
oscillation phenomenon. This is the reason why for neutrinos \( \nu_\mu \) 
produced in \( \pi^- \to \mu^- + \nu_\mu \), all \( \nu_\mu \to \nu_\mu \) survival experiments are 
integral with each other and form one general experiment from 
which all data can be taken simultaneously. The limits on the 
unitary evolution can appear when, e.g., the sterile neutrinos 
[27], heavy neutrinos [28], decoherence and dissipation [20] 
or other phenomena [27] are included, thus violating the result 
given by Eq. (7). In the SM the produced muon neutrino is in the 
pure state with helicity \( \lambda = -1 \), i.e., only one eigenvalue of \( \rho_{SM}^j \) 
is nonzero, say \( p_{SM}^1 = 1 \), and \( p_{SM}^j = 0 \) for \( j = 2, \ldots, 6 \). In the NP 
case the muon neutrino is in the mixture of two helicity states 
\( \lambda = -1 \) and \( +1 \), i.e., two eigenvalues of \( \rho_{NP}^j \) are nonzero, say 
\( p_{NP}^j \) and \( p_{NP}^j \) and the others are \( p_{NP}^j = 0 \), \( j = 2, 3, 5, 6 \). From the 
spectral-decomposition of \( \rho_{NP}^j \), Eq. (3), we obtain numerically 
(with the accuracy of the expansion coefficients up to the forth 
decimal place) the truncated series expansion in \( \varepsilon_R \) parameter:
\[
\begin{align*}
p_{NP}^j &= \varepsilon_R |p_{NP}^j| \text{ } |R| + 1.4911 1 |p_{NP}^j| 6 \quad \text{ for } \varepsilon_R < 0.045 \\
p_{NP}^j &= 1 - p_{NP}^j \quad \text{ and } \quad p_{NP}^j = 0, \quad j = 2, 3, 5, 6
\end{align*}
\] (8)

and \( p_{NP} \to p_{SM} \) for \( \varepsilon_R \to 0 \). Through the work, the symbol 
of approximate equality will appear as a consequence of the 
approximation occurring in Eq. (5).

### 2.1. The sensitivity problem

We are unaware whether we are sampling from the SM or NP 
model distribution. The sensitivity problem for two probability 
model discrimination is connected with the erroneous identifi-
cation of the probability distribution in the \( N \)-dimensional 
sampling. The upper minimal bound on the probability \( P_E \) of this 
error for distributions generated by two density matrices, here 
\( \rho_{NP}^j \) and \( \rho_{SM}^j \), was found by Huai and Petz [29]:
\[
P_E(\rho_{NP}^j, \rho_{SM}^j) = e^{-NS(\rho_{SM}^j|\rho_{NP}^j)}, \quad (9)
\]

where the number \( N \) of quantum copies of the system is very 
large (in principle infinite) and
\[
S(\rho_{SM}^j|\rho_{NP}^j) := \text{Tr}(\rho_{SM}^j \ln \rho_{SM}^j - \rho_{NP}^j) \quad (10)
\]
is the always nonnegative Umegaki quantum relative entropy 
[30, 31]. \( S(\rho_{SM}^j|\rho_{NP}^j) \) is the measure of how far from each other 
are the NP and SM neutrino quantum states. By the monotonic-
ty of the relative entropy it was proven that [32]
\[
S(\rho_{SM}^j|\rho_{NP}^j) \leq S(\rho_{NP}^j|\rho_{SM}^j) \leq S(\rho_{NP}^j|\rho_{NP}^j) \quad (11)
\]

Here
\[
S(\rho_{SM}^j|\rho_{NP}^j) := \sum_{j=1}^{6} \rho_{SM}^j \text{ ln } \rho_{SM}^j - \rho_{NP}^j) \quad (12)
\]
is the classical Kullback-Leibler relative entropy and 
\( S(\rho_{SM}^j|\rho_{NP}^j) \) is the quantum relative entropy that takes 
the supremum over all possible Positive Operator Valued Mea-
sures [26]. We have to stress that from Eq. (7) it follows that 
the relative entropy \( S(\rho_{SM}^j|\rho_{NP}^j) \) does not vary with \( z \), having 
therefore the same value at the points of the neutrino production 
and detection. By using the relative entropy \( S(\rho_{SM}^j|\rho_{NP}^j) \) in 
Eq. (9) instead of \( S(\rho_{SM}^j|\rho_{NP}^j) \), the significance of the difference 
between the NP and SM states is underestimated. However it 
gives the operationally easier (classical) bound \( P_E(\rho_{SM}, \rho_{NP}) \) 
for the calculation of the error probability [26]:
\[
P_E(\rho_{SM}^j, \rho_{NP}^j) \leq P_E(\rho_{SM}, \rho_{NP}) := e^{-NS(\rho_{SM}|\rho_{NP})} \quad (13)
\]
The Fisher-Rao metric consists of one component only:

\[ s_{\xi\xi} = \left( \frac{p_{S_M}}{p_{SM}} \right)^2 \frac{\partial \ln p_{SM}}{\partial \xi} \frac{\partial \ln p_{SM}}{\partial \xi} , \]  

which, for the distribution \( p_{NP} \) given by Eq. (8), is equal to:

\[ s_{\xi\xi}(\xi_R) \approx 4 \left( 1 - 2|\xi_R|^2 + 5.4556|\xi_R|^4 \right) \quad \text{for} \quad \mathcal{R} < 0.045. \] (15)

The Fisher information on \( \xi_R \) in the \( N \)-dimensional sample is equal to \( I_F(\xi_R) = N s_{\xi\xi}^{\xi\xi} \) [33] and from the scalar Cramér-Rao inequality [33] we obtain in the classical approach the lower bound \( \sigma^2(\tilde{\xi}_R) \) on the variance of any unbiased estimator \( \tilde{\xi}_R \) of \( \xi_R \):

\[ \sigma^2(\tilde{\xi}_R) \geq \frac{1}{N g_{\xi\xi}} \geq \frac{1}{4N} \quad \text{for} \quad \mathcal{R} < 0.045 \] (16)

and thus the standard error \( \sigma(\tilde{\xi}_R) = \sqrt{\sigma^2(\tilde{\xi}_R)} \geq \sigma(\tilde{\xi}_R) = \sqrt{\sigma^2(\tilde{\xi}_R)} \approx (1 + |\xi_R|^2 - 1.2278 |\xi_R|^4)/(2 \sqrt{N}) \). The values of the lower bound \( \sigma(\tilde{\xi}_R) \) for the standard error \( \sigma(\tilde{\xi}_R) \) as the function of \( \xi_R \) for some \( N \) are shown on Figure 2. Finally, in the classical approach the Rao distance between the distributions \( p_{NP} \) and \( p_{SM} \) in \( S \) is after applying Eq. (15) equal to

\[ D_{Rao}(p_{SM}, p_{NP}) = \int_0^{\xi_R} \sqrt{g_{\xi\xi}^{\xi\xi}}(\xi_R) d\xi_R \]

(17)

\[ \approx 2 |\xi_R| - \frac{2}{3} |\xi_R|^3 + 0.8911 |\xi_R|^5 \quad \text{for} \quad \mathcal{R} < 0.045. \]

2.2. The stability of the \( \xi_R \) estimation

To learn about the stability of the estimation of \( \xi_R \) the lower bound on the variance of its estimator \( \tilde{\xi}_R \) has to be found. The relationship between two lower bounds, classical and quantum, will be determined. It will be shown that the classical lower bound is not smaller than quantum, therefore, from an experimental point of view, the classical bound (which needs the bigger sample) is more restrictive than the quantum one.

2.2.1. The classical lower bound

In the classical (c) approach it is the Fisher information on \( \xi_R \) parameter that has to be calculated. In general, a probability distribution \( p_\xi \) is parameterized by a \( n \)-dimensional parameter \( \xi = (\xi^a)_{a=1}^n \in \Xi \), where \( \Xi \) is a subset of \( \mathbb{R}^n \). The Riemannian metric \( g^{\xi\xi} \) of the statistical model \( S = (p_\xi|\xi = (\xi^a)_{a=1}^n \in \Xi) \) is called the Fisher-Rao metric [33]. In this paper \( \xi \) is reduced to the scalar NP parameter \( \xi_R \) and the \( n = 1 \)-dimensional manifold \( S = (p_{SM}(\xi_R)|\xi_R \in (0, 1)) \) is coordinatized by the parameter \( \xi_R \). Then, the Fisher-Rao metric consists of one component only:

\[ s_{\xi\xi}^{\xi\xi} = \left( \frac{p_{SM}}{p_{SM}} \right)^2 \sum_{j=1}^n \left( \frac{\partial \ln p_{SM}}{\partial \xi_R} \right) \left( \frac{\partial \ln p_{SM}}{\partial \xi_R} \right) , \]

(14)
From Eq.\( \ref{eq:20} \) we see that as the classical lower bound \( \sigma^2(\hat{\varepsilon}_R) \) on \( \sigma^2(\tilde{\varepsilon}_R) \) is bigger than the quantum lower bound \( \sigma(\tilde{\varepsilon}_R) \) hence the quantum estimation is more effective. Yet, since \( \sigma^2(\tilde{\varepsilon}_R) \) is calculated from Eq.\( \ref{eq:14} \) with the eigenvalues \( p^i_{NP} \), Eq.\( \ref{eq:8} \), thus, unlike \( \sigma^2(\tilde{\varepsilon}_R) \), the classical bound neither depends on the relativistic neutrino energy nor on the baseline of the experiment. Finally, let us note that, from a practical point of view, it appears that if the classical lower bound is experimentally satisfactory (see Conclusions) then the quantum one, though not designated, is even more powerful.

3. Conclusions

Two model characteristics were evaluated in this paper: (i) the model selection one for the sensitivity of the NP-SM discrimination with the change of the NP right-chiral CC coupling constant \( \varepsilon_R \) based on the upper bound \( P_E(p_{SM}, p_{NP}) \) of the erroneous identification of the model probability distribution \( P_E(q_{SM}, q_{NP}) \) and (ii) the one of the stability of the NP model estimator \( \tilde{\varepsilon}_R \) with the change of \( \varepsilon_R \) based on classical Fisher-Rao metric. The decay \( \pi^+ \rightarrow \mu^+ + \nu_\mu \) of the high energy pion followed by the relativistic neutrino unitary propagation constituted the background for the considerations.

Due to the unitarity of the evolution of the density matrix \( q_{NP} \) its eigenvalues \( p^i_{NP} \), Eq.\( \ref{eq:8} \), do not depend on the relativistic neutrino energy nor on the baseline of the experiment. From this point of view all \( \nu_\mu \rightarrow \nu_\mu \) survival experiments form one general class. With these \( p^i_{NP}(\varepsilon_R) \) the classical lower bound \( \sigma^2(\tilde{\varepsilon}_R) \), Eq.\( \ref{eq:16} \), on the variance of \( \hat{\varepsilon}_R \), which is bigger than the quantum lower bound \( \sigma^2(\tilde{\varepsilon}_R) \), Eq.\( \ref{eq:20} \), was calculated. Thus, with Eq.\( \ref{eq:16} \) the analysis of the robustness of the estimation of \( \varepsilon_R \) (see text below Eq.\( \ref{eq:20} \)) can be performed globally, i.e., for all production-oscillation (PO) experiments taken jointly. To summarize, \( N \) in Eq.\( \ref{eq:16} \) can be taken as the total size of all samples obtained in all survival experiments.

There exists the upper (not of the oscillation experiments origin) bound on the ratio \( R \equiv |\varepsilon_R/\varepsilon_L| < 0.045 \) (Section 1). It follows from the analysis of \( \pi^+ \rightarrow \mu^+ + \nu_\mu \) decay experiments, in which the polarization of the emitted muon was measured \([11, 12]\). From the PO experiments, Eq.\( \ref{eq:16} \) says that for \( |\varepsilon_R| \approx 0.045 \) to diminish the standard error \( \sigma(\tilde{\varepsilon}_R) \geq \sigma(\tilde{\varepsilon}_R) \) below |\varepsilon_R| value, i.e., for the robust \( \varepsilon_R \) estimation, \( N \gtrsim 125 \) is required. Then also, for \( N = 125 \), \( P_E(p_{SM}, p_{NP}) \approx 0.78 \) and the probability of the erroneous identification of the NP model with SM would be high. Yet, at the end of 2017 the number \( N \) of survival \( \nu_\mu \rightarrow \nu_\mu \) events in all \( \pi^+ \rightarrow \mu^+ + \nu_\mu \) experiments (which form a combination of T2K, NOvA and mainly MINOS observations) was already about 6500 \([36-39]\). For \( N = 6500 \) we obtain \( \sigma(\tilde{\varepsilon}_R) \approx 0.006 \) and simultaneously the probability \( P_E(p_{SM}, p_{NP}) \approx 2.32 \times 10^{-6} \) is significantly small, leading to good NP-SM discrimination.

In conclusion, if \( R \) is only slightly smaller than 0.045, both the significant result for the NP-SM discrimination and robust estimation of the right chiral CC interaction parameter \( \varepsilon_R \) in neutrino PO experiments have been already reached. It is anticipated that 2026 will be the first year of the beam operations in the DUNE experiment \([40]\), which is to result in the observation of more than 7900 \( \nu_\mu \) survival events over 3.5 years. Therefore, in ten years we will obtain \( N = 14600 \) survival events, and even for \( R = 0.02 \) the conventional value \( P_E(p_{SM}, p_{NP}) < 0.003 \) will be reached, suggesting (if not yet observed) the nonexistence of the right chiral CC neutrino interactions.

Appendix: The density matrix at the detection point

The effective Hamiltonian \( \mathcal{H} \), Eq.\( \ref{eq:5} \), and neutrino density matrix \( \rho^{\nu_\mu} \), Eq.\( \ref{eq:3} \), have the 6 \times 6 matrix representations. The diagonalisation of \( \mathcal{H} \) \([11, 28, 36]\) gives \( \mathcal{H} = \frac{1}{2 E} W \text{diag}(\vec{\mu}^2) W^\dagger \).
where \( W \) is the diagonalising unitary matrix defined by the eigenvectors of \( 2E_i\mathcal{H} \), and the corresponding real eigenvalues \( \hat{m}_i^2 \), \( i = 1, \ldots, 6 \), are the neutrino effective squared masses. \( E_i \) is the neutrino energy, neglecting the mass contribution. \( W \equiv (W_{i,k,l}) = (\psi_{i} | \mathcal{H} | \psi_{l}) \) defines the transformation from the helicity-mass basis \( | \psi_{i} >, | p, \lambda, i > \) to the eigenvector basis \( | l > \) of \( \mathcal{H} \). For the relativistic neutrino \( \nu_{\mu} \) and in the non-dissipative homogeneous medium, from Eq.\((21)\) it follows that in \( | \psi_{\mu} > \) basis the density matrix at the point \( z = T \) of the density matrices in the \( L \) frame and CM frame is assumed
\[ \rho_{\mu \mu} = \mathcal{T}(\rho_{\mu \mu}(t = T)) = \sum_{i,l} \sum_{i',l'} W_{i,i'} W_{l,l'}^{*} e^{-i\Delta E_{il'} \cdot T} W_{i,i'}^{*} \rho_{i' l'} W_{l,l'} , \]
where \( T \) is the time between neutrino production and detection, \( \Delta E_{il'} = E_{i} - E_{l'} = \frac{\hat{m}_{i}^{2} - \hat{m}_{l'}^{2}}{2E_{i}} \). The equality \( \rho_{\mu \mu}(\hat{p}_{L}) = \rho_{\mu \mu}(\hat{p}) \) of the density matrices in the \( L \) frame and CM frame is assumed \( \mathcal{L} \) (Section\(1)\). Because of the \( W \) matrix unitarity, the \( L \) frame neutrino density matrix at the detection point is normalized, i.e., \( \text{Tr}(\rho_{\mu \mu}(t = T)) = 1 \). Eq.\((21)\) is valid in the so-called light-ray approximation \( T = L \) \( \mathcal{O} \). The deviation of \( T \) from the relation \( T = L \) is experimentally significant if some corrections \( \epsilon_{\mu \mu} \) \( \mathcal{O} \) to the oscillation phases \( \Delta \phi_{\mu \mu} = | \Delta E_{\mu \mu} | T \) are also significant. As \( \epsilon_{\mu \mu} \) are functions of \( \Delta \phi_{\mu \mu} \), this would require \( \Delta \phi_{\mu \mu} \gg 1 \) \( \mathcal{O} \). However, for the oscillations to be measurable at all, it is necessary that \( \Delta \phi_{\mu \mu} \sim 1 \), in which case the corrections \( \epsilon_{\mu \mu} \) to \( \Delta \phi_{\mu \mu} \) can be neglected \( \mathcal{O} \), validating the light-ray approximation.

Finally, using \( \rho_{\mu \mu}(t = T) \), Eq.\((21)\), one can also calculate, e.g., the geometric phase of the \( \mu \) flavour neutrino state \( \mathcal{O} \) or the cross section \( \sigma_{\mu \mu} \mathcal{O} \) for the detection of the \( \beta \) flavour neutrino in the \( L \) frame \( \mathcal{O} \).

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