The recent observations of the anomalous X-ray pulsars 4U 0142+61 and 1RXS J170849.0-400910 by the Imaging X-ray Polarimetry Explorer (IXPE) opened up a new avenue to study magnetars, neutron stars endowed with superstrong magnetic fields ($B \gtrsim 10^{14}$ G). The detected polarized X-rays from 4U 0142+61 exhibit a 90° linear polarization swing from low photon energies ($E \lesssim 4$ keV) to high energies ($E \gtrsim 5.5$ keV). We show that this swing can be explained by photon polarization mode conversion at the vacuum resonance in the magnetar atmosphere; the resonance arises from the combined effects of plasma-induced birefringence and Quantum electrodynamics (QED)-induced vacuum birefringence in strong magnetic fields. This explanation suggests that the atmosphere of 4U 0142 is composed of partially ionized heavy elements and that the surface magnetic field is comparable to or less than $10^{14}$ G, consistent with the dipole field inferred from the measured spindown. It also implies that the spin axis of 4U 0142+61 is aligned with its velocity direction. The polarized X-rays from 1RXS J170849.0-400910 do not show such 90° swing, consistent with magnetar atmospheric emission with $B \gtrsim 5 \times 10^{14}$ G.

Magnetars are neutron stars (NSs) whose energy outputs (even in quiescence) are dominated by magnetic field dissipations (1, 2). Recently, the NASA/ASI Imaging X-ray Polarimetry Explorer (IXPE) (3) reported the detection of linearly polarized X-ray emission from the anomalous X-ray pulsar (AXP) 4U 0142+61, a magnetar with an inferred dipole magnetic field (based on the spindown rate) of $\sim 10^{14}$ G (4). This is the first time that polarized X-rays have been detected from any astrophysical point sources. The overall phase-averaged linear polarization degree is $12 \pm 1\%$ throughout the IXPE band (2 to 8 keV). Interestingly, there is a substantial variation of the polarization signal with energy: the polarization degree is $14 \pm 1\%$ at 2 to 4 keV and $41 \pm 7\%$ at 5.5 to 8 keV, while it drops below the detector sensitivity around 4 to 5 keV, where the polarization angle swings by $\sim 90°$.

Taverna et al. (4) considered several possibilities to explain the observed polarization swing and suggested that the thermal X-rays from 4U 0142+61 are emitted from an extended region of the condensed neutron star (NS) surface. In this scenario, the 2 to 4 keV radiation is dominated by the O-mode (polarized in the plane spanned by the local magnetic field and the photon wave vector), while the 5.5 to 8 keV radiation by the X-mode (which is orthogonal to the O-mode) because of reprocession by resonant Compton scattering (RCS) in the magnetosphere.

While it is premature to draw any firm conclusion without detailed modeling, the “condensed surface + RCS” scenario may be problematic for several reasons: i) At the surface temperature of $T_s \simeq 5 \times 10^6$ K and $B_{14} \equiv B/(10^{14} \text{G}) \simeq 1$, as appropriate for 4U 0142, it is unlikely that the NS surface is in a condensed form, even if the surface composition is Fe (5, 6), simply because the cohesive energy of the Fe solid is not sufficiently large (7). ii) A stronger surface magnetic field is possible, but the emission from a condensed Fe surface (for typical magnetic field and photon emission directions at the surface) is dominated by the O-mode only for photon energies less than $E_\gamma \simeq 0.1 \eta Z^{2/5} B_{14}^{1/5}$ keV, where $\eta \lesssim 1$ (8, 9). An unrealistically strong field ($B_{14} \gtrsim 10^5$ for $Z = 26$) would be required to make $E_\gamma \gtrsim 4$ keV. iii) As acknowledged by Taverna et al. (4), the assumption that the phase-averaged low-energy photons are dominated by the O-mode would imply that the NS spin axis (projected in the sky plane) is orthogonal to the proper motion direction. This is in contradiction to the growing evidence of spin-kick alignment in pulsars (10–14).

In this paper, we show that the 90° linear polarization swing observed in 4U 0142 could be naturally explained by photon mode conversion associated with the “vacuum...
resonance" arising from QED and plasma birefringence in strong magnetic fields. The essential physics of this effect was already discussed in ref. 15, where it was shown that for neutron stars with H atmospheres, thermal photons with \( E \lesssim 1 \) keV are polarized orthogonal to photons with \( E \gtrsim 4 \) keV, provided that the NS surface magnetic field is somewhat less than \( 10^{14} \) G. The purpose of this paper is to reexamine the mode conversion effect under more general conditions (particularly the atmosphere composition) and to present semianalytic calculations of the polarization signals for parameters relevant to 4U 0142. Most recently, IXPE detected polarized X-rays from another AXP 1RXS J170849.0-400910 and found that the polarization angle \( \rho < \rho V \), however, the normal modes become circularly polarized as a result of the "cancellation" of the plasma and vacuum effects (Fig. 1). The half width of the vacuum resonance (defined by \( |\rho| < 1 \)) is

\[
\epsilon \equiv \frac{\Delta \rho}{\rho V} = \frac{2 \cos \theta_{KB}}{u_e^2 \sin^2 \theta_{KB}}.
\]

When a photon propagates in an inhomogeneous medium, its polarization state will evolve adiabatically (i.e., following the \( K_+ \) or \( K_- \) curve in Fig. 1) if the density variation is sufficiently gentle. Thus, an X-mode (O-mode) photon will be converted into the O-mode (X-mode) as it traverses the vacuum resonance, with its polarization ellipse rotated by 90° (Fig. 1). This resonant mode conversion is analogous to the Mikheyev–Smirnov–Wolfenstein neutrino oscillation that takes place in the Sun (30, 31) and similar level-crossing phenomena in other areas of sciences (e.g., Landau–Zener transition in atomic physics; EM wave propagation in inhomogeneous media and metamaterials; ref. 32). For this conversion to be effective, the adiabatic condition must be satisfied (25)

\[
E \gtrsim E_{\text{ad}} = 2.52 \left( f \tan \theta_{KB} \right)^{2/3} \left( \frac{1 \text{ cm}}{H_0} \right)^{1/3} \text{ keV},
\]

where \( H_0 = |d\ln \rho| \) is the density scale height (evaluated at \( \rho = \rho_V \)) along the ray. In general, the probability for nonadiabatic "jump" is given by

\[
P_j = \exp \left[ -\pi \left( \frac{E}{E_{\text{ad}}} \right)^3 \right].
\]

The mode conversion probability is \((1 - P_j)\).

### Calculation of Polarized Emission

To quantitatively compute the observed polarized X-ray emission from a magnetic NS, it is necessary to add up emissions from all surface patches of the star, taking account of beaming/anisotropy due to magnetic fields and light bending due to general relativity (15, 33–37). While this is conceptually straightforward, it necessarily involves many uncertainties related to the unknown distributions of surface temperature \( T_s \) and magnetic field \( B \). In addition, the atmosphere composition is unknown, the opacity data for heavy atoms/ions for general magnetic field strengths are not available, and atmosphere models for many surface patches (each with different \( T_s \) and \( B \)) are needed. Finally, to determine the phase-resolved lightcurve and polarization, the relative orientations of the line of sight, spin axis, and magnetic dipole axis are needed. Given all these complexities, we present a simplified, approximate calculation below. Our goal is to determine under what conditions (in terms of the magnetic field strength, surface composition, etc.) the polarization swing can be produced.

We consider an atmosphere plasma composed of a single ionic species (each with charge Ze and mass \( Am_p \)) and electrons. This is of course a simplification as, in reality, the atmosphere would consist of multiple ionic species with different ionizations. With the equation of state \( P = \rho kT/(\mu m_p) \), hydrostatic balance...
The density scale-height along a ray is

\[ \xi = \frac{K^2 \sin^2 \theta_{\text{KB}} + (1 + K^2 \cos^2 \theta_{\text{KB}}) u_e}{1 + K^2} \]

where the density \( \rho \) at density \( \rho \) is given by

\[ y = \frac{\rho k T}{\mu m_p g} = 0.41 \frac{\rho_1 T_6}{\mu g_2} \text{g/cm}^2, \tag{7} \]

where \( T = 10^6 T_6 \text{ K} \) is the temperature, \( \mu = A/(1 + Z) \) is the "molecular" weight, \( \rho_1 \) is the density in \( 1 \text{ g/cm}^3 \), and \( g_2 \) is the surface gravity \( g = (GM/R^2)(1 - 2GM/Rc^2)^{-1} \) in units of \( 2 \times 10^{14} \) (For \( M = 1.4M_\odot \), \( R = 12 \text{ km} \), we have \( g_2 \approx 1.00 \).) The density scale-height along a ray is

\[ H_\rho \approx \frac{k T}{\mu m_p g \cos \alpha} = 0.41 \frac{T_6}{\mu g_2 \cos \alpha} \text{cm}, \tag{8} \]

where \( \alpha \) is the angle between the ray and the NS surface normal.

To simplify our calculations, we shall neglect the electron scattering opacity and the bound–bound and bound–free opacities. The former is generally subdominant compared to the free–free opacity, while the latter are uncertain or unavailable. The free–free opacity of a photon mode (labeled by \( i \)) can be written as

\[ \kappa_i = \kappa_0 \xi_i, \tag{9} \]

where the \( B = 0 \) opacity is (setting the Gaunt factor to unity)

\[ \kappa_0 \approx 9.3 (Z^2/A^2) \rho_1 T_6^{-1/2} E_1^{-3} G, \tag{10} \]

with \( G = 1 - \exp(-E/kT) \). For the photon mode \( E \propto (iK, 1) \), the dimensionless factor \( \xi \) is given by (26, 38)

\[ \xi = \frac{K^2 \sin^2 \theta_{\text{KB}} + (1 + K^2 \cos^2 \theta_{\text{KB}}) u_e}{1 + K^2}; \tag{11} \]

implies that the column density \( y \) at density \( \rho \) is given by

Fig. 1. Polarization ellipticity of the photon mode as a function of density near the vacuum resonance. The two curves correspond to the (+) and (−) modes. In this example, the parameters are \( B = 10^{12} \text{G}, E = 5 \text{ keV}, \) and \( \theta_{\text{KB}} = 30^\circ \). The ellipticity of a mode is specified by the ratio \( K = -iE_y/E_x \), where \( E_x (E_y) \) is the photon’s electric field component along (perpendicular to) the \( k \)-\( B \) plane. The O-mode is characterized by \( |K| \gg 1 \), and the X-mode \( |K| \ll 1 \).

Fig. 2. The behavior of \( \xi_X, \xi_O, \xi_+, \) and \( \xi_- \) as a function of density. We see that for typical \( \theta_{\text{KB}} \) ’s, \( \xi_O \sim 1 \) and \( \xi_X \sim \nu_e^{-1} \ll 1 \) except near \( \rho = \rho_V \).

For a given photon energy \( E \) and wave vector \( k \) (which is inclined by angle \( \alpha \) with respect to the surface normal), the transfer equation for mode \( i \) (with \( i = X, O \) or \( i = +, − \)) reads

\[ \cos \alpha \frac{dI_i}{dy} = \kappa_i \left( I_i - \frac{1}{2} B_0 \right), \tag{12} \]

where \( B_0(T) \) is the Planck function and \( T = T(y) \) is the temperature profile (to be specified later). To calculate the emergent polarized radiation intensity from the atmosphere, taking account of partial mode conversion, we adopt the following procedure (33): i) We first integrate Eq. 12 for the X-mode and O-mode \( (i = X, O) \) from large \( y (= \infty) \) to \( y_V \), the column density at which \( \rho = \rho_V \). This gives \( I_{XV} \) and \( I_{OV} \), the X- and O-mode intensities just before resonance crossing. ii) We apply partial mode conversion

\[ I_{X}' = I_{XV} P_j + I_{OV}(1 - P_j), \]

\[ I_{O}' = I_{OV} P_j + I_{XV}(1 - P_j), \]

to obtain the mode intensities just after resonance crossing. This partial conversion treatment is valid since the resonance width is small \( (\Delta \rho/\rho_V \ll 1; \text{ Eq. 4}) \). iii) We then integrate Eq. 12 for the X-mode and O-mode again from \( y = y_V \) (with the "initial" values \( I_{XV}', I_{OV}' \)) to \( y \ll 1 \). This then gives the mode intensities emergent from the atmosphere, \( I_X, I_O, \).

An alternative procedure is to integrate Eq. 12 for \( i = X, O, +, − \) from \( y \gg 1 \) to \( y \ll 1 \) (without applying partial mode conversion), which gives the intensities \( I_X(0), I_O(0), I_+(0), \) and
\( I(0) \) at \( y \approx 0 \). Then, apply the partial conversion
\[
I_{XV} = I_X(0)P_J + I_+(0)(1 - P_J),
\]
where \( P_J \) is evaluated at \( y = y_V \).

It is straightforward to show that the above two procedures are equivalent. For example, after obtaining \( I_{XV} \), we can get the emergent X-mode intensity by
\[
I_{XV} = I_{XV}^\prime \exp \left( -\frac{\tau_{XV}}{\cos \alpha} \right) + \Delta I_{XV}. \tag{17}
\]
where \( \tau_{XV} = \int_0^{y_V} \kappa_X dy \) is the optical depth of X-mode (measured along the surface normal) at \( y = y_V \), and \( \Delta I_{XV} \) is the contribution to \( I_{XV} \) from the region \( 0 < y < y_V \):
\[
\Delta I_{XV} = \int_0^{\tau_{XV}/\cos \alpha} \exp \left( \frac{\tau_{XV}}{\cos \alpha} \right) \frac{1}{2} B_v(T) \frac{d\tau_{XV}}{\cos \alpha}. \tag{18}
\]
On the other hand, when integrating Eq. 12 for \( i = X \) from \( y \gg 1 \) to \( y \ll 1 \), we obtain
\[
I_X(0) = I_{XV}^0 \exp \left( -\frac{\tau_{XV}}{\cos \alpha} \right) + \Delta I_{XV}. \tag{19}
\]
Similarly,
\[
I_+(0) = I_{OV} \exp \left( -\frac{\tau_{XV}}{\cos \alpha} \right) + \Delta I_{XV}. \tag{20}
\]

It is easy to see that Eq. 17 together with Eq. 13 (the first procedure) and Eq. 15 with Eqs. 19–20 (the second procedure) yield the same emergent \( I_{XV} \).

**Photosphere Densities and Critical Field.** Before presenting our sample results, it is useful to estimate the photosphere densities for different modes and the condition for polarization swing.

When the vacuum polarization effect is neglected (\( \rho_V = 0 \)), \(|\beta| \gg 1 \) at all densities (for typical \( \theta_B \)'s not too close to 0), the \( \xi \)-factors for the O-mode (\(|K| \gg 1 \)) and for the X-mode (with \(|K| \ll 1 \)) are
\[
\xi_O \approx \sin^2 \theta_B, \quad \xi_X \approx \frac{1}{u_e \sin^2 \theta_B}. \tag{21}
\]

The photospheres of the O-mode and X-mode photons are determined by the condition
\[
\int_0^{y_{O,V}} \kappa_{O,V} \frac{dy}{\cos \alpha} = 2/3. \tag{22}
\]

The corresponding photosphere densities can be estimated as
\[
\rho_{O,V} = \frac{\xi_{O,V}}{B_{O,V}} \rho_0, \tag{23}
\]
\[
\rho_{O,V} = \frac{\xi_{O,V}}{B_{O,V}} \rho_0, \tag{24}
\]
where the “zero-field” photosphere density is
\[
\rho_0 \approx 0.59 \left( \frac{\mu g_2 \cos \alpha}{G} \right)^{1/2} \left( \frac{E_1}{Z} \right)^{3/2} \left( \frac{A}{T_{1.6}} \right) \text{g cm}^{-3}. \tag{25}
\]

The effect of the vacuum resonance on the radiative transfer depends qualitatively on the ratios \( \rho_V/\rho_O \) and \( \rho_V/\rho_X \), given by
\[
\frac{\rho_V}{\rho_O} = \left( \frac{B}{B_{O,V}} \right)^2, \quad \frac{\rho_V}{\rho_X} = \frac{B}{B_{X,V}}. \tag{26}
\]

where
\[
B_{O,V} = 7.8 \times 10^{13} \left( \frac{\mu g_2 \cos \alpha}{ZG E_1} \right)^{1/4} \left( \frac{f}{T_{1.6}} \right) \text{ G}, \tag{27}
\]
\[
B_{X,V} = 7.1 \times 10^{16} \left( \frac{\mu g_2 \cos \alpha}{ZG E_1} \right)^{1/2} \left( \frac{f^2 \sin \theta_B}{T_{1.6}^{7/4}} \right) \text{ G}. \tag{28}
\]

Clearly, the condition \( B \ll B_{O,V} \) or \( \rho_V \ll \rho_X \) is satisfied for almost all relevant NS parameters of interest, while \( B_{O,V} \) defines...
the critical field strength for the $90^\circ$ polarization swing (Figs. 3 and 4): If $B \lesssim B_{OV}$, the emergent radiation is dominated by the X-mode for $E \lesssim E_{ad}$ and by the O-mode for $E \gtrsim E_{ad}$; if $B \gtrsim B_{OV}$, the X-mode is dominant for all $E$'s.

We can quantify the role of $B_{OV}$ more precisely by estimating how vacuum resonance affects the photosphere densities. In the limit of no mode conversion (i.e., $E \ll E_{ad}$), it is appropriate to consider the transport of the X-mode and O-mode, with the mode opacities modified around the vacuum resonance (Fig. 2). The O-mode opacity has a dip near $\rho = \rho_V$ (where $\xi \approx \sin^2 \theta_{kB}/2$), and since the resonance width $\Delta \rho / \rho_V \ll 1$, the photosphere density $\rho_{O}'$ is almost unchanged from the no-
vacuum value, i.e., $\rho' \approx \rho_0$. On the other hand, the X-mode opacity has a large spike at $\rho = \rho_V$ (where $\xi = \sin^2 \theta_B/2$) compared to the off-resonance value ($\xi \sim u^{-1}$). The X-mode optical depth across the resonance (from $\rho_V - \Delta \rho$ to $\rho_V + \Delta \rho$) is of order $\Delta \tau_V \approx \epsilon (\rho_V/\rho_0)^2$, where $\epsilon$ is given by Eq. 4. Thus, the modified X-mode photosphere density is $\rho_x' \approx \rho_V$ for $\Delta \tau_V \gtrsim 1$ and

$$\rho_x' \approx \rho_x \left[1 - \epsilon (\rho_V/\rho_0)^2 \right]^{1/2}, \quad [29]$$

for $\Delta \tau_V \lesssim 1$.

In the limit of complete mode conversion (i.e., $E \gg E_{ad}$), it is appropriate to consider the transport of $(+)$-mode and $(\pm)$-mode, with the mode opacities exhibiting a discontinuity at $\rho = \rho_V$ (Fig. 2). The $(+)$-mode photosphere density $\rho_+^2$ is given by

$$\rho_+^2 \approx \rho_0^2 + \left(1 + \frac{\xi_X}{\xi_0} \right) \rho_V^2 \approx \rho_0^2 + \rho_V^2. \quad [30]$$

The $(-)$-mode photosphere density $\rho_-^2$ is affected by the vacuum resonance only if $\rho_0 > \rho_V$. Thus, $\rho_- = \rho_0$ for $\rho_0 < \rho_V$, [31] and

$$\rho_-^2 = \rho_0^2 + \frac{\xi_0}{\xi_X} (\rho_0^2 - \rho_V^2) \quad \text{for} \quad \rho_0 > \rho_V. \quad [32]$$

For general $E$’s with partial mode conversion, the emergent mode intensities are approximately given by

$$I_{O,e} \approx \frac{1}{2} P_J B_0 (\rho_0) + \frac{1}{2} (1 - P_J) B_0 (\rho_-), \quad [33]$$

$$I_{X,e} \approx \frac{1}{2} P_J B_0 (\rho_x') P_J + \frac{1}{2} (1 - P_J) B_0 (\rho_+). \quad [34]$$

where (for example) $B_0 (\rho_0)$ is the Planck function evaluated at $\rho = \rho_0$. These results are schematically depicted in Figs. 3 and 4.

**Results.** To compute the polarized radiation spectrum emergent from a NS atmosphere patch using the method presented above (Eq. 12 with Eqs. 13–14 or with Eqs. 15–16), we need to know the atmosphere temperature profile $T(y)$. This can be obtained only by self-consistent atmosphere modeling, which has been done for only a small number of cases (in terms of the local $T$, $B$ and composition). Here, to explore the effect of different $B$ and compositions, we consider two approximate models:

- **Model (i):** We use the profile $T_H(y)$ for the $T_s = 5 \times 10^6$ K H atmosphere model with a vertical field $B = 10^{14}$ G [Fig. 5 and equation 48 of (33); this model correctly treats the partial model conversion effect] and rescale it to take account of

![Fig. 5.](https://doi.org/10.1073/pnas.2216534120) Polarization degree $P_L$ (defined by Eq. 37) of the emergent radiation normal to the surface as a function of the photon energy $E$ for H and He atmospheres with different magnetic field strengths and directions ($\theta_B$, the angle between the surface B and the surface normal vector). All results are based on the temperature profile Model (ii).
the modification of the free–free opacity for different $Z$, $A$
(so that the rescaled profile yields the same effective surface
{
 temperature $T_s$):

$$T(y) = \left(2\mu Z^3/A^2\right)^{1/8.5} T_H(y).$$  \[35\]

- Model (ii): We use a smooth (monotonic) fit to the $T_H$ profile,
given by

$$\log_{10} T_H(y) = 0.11 + 0.147 \left[ \log_{10}(0.4y) + 3 \right], \quad [36]$$

and then apply Eq. 35 for rescaling.

Figs. 5 and 6 show a sample of our results for the polarization
degree of the emergent radiation, defined by

$$P_L \equiv \frac{I_{X,e} - I_{O,e}}{I_{X,e} + I_{O,e}}. \quad [37]$$

We see that for the H and He atmospheres (Fig. 5), $P_L$ transitions
from being positive at low $E$’s to negative at high $E$’s for $B_{14} \lesssim 0.5$, in agreement with the critical field estimate (Eq. 27). The
transition energy (where $P_L = 0$) is approximately given by $E_{ad}$
and has the scaling $E_{ad} \propto (\mu \tan^2 \theta_B)^{1/3}$, where $\theta_B$ is the angle
between the surface $B$ and the surface normal (Eq. 5). To obtain a
transition energy of 4 to 5 keV (as observed for 4U 0142) would
require most of the emission to come from the surface region
with $\theta_B \gtrsim 70^\circ$.

On the other hand, for a partially ionized heavy-element
atmosphere (such that $\mu/Z$ is much larger than unity), the critical
field $B_{OV}$ can be increased (Eq. 27). Fig. 6 shows that at $B_{14} = 1$,
a $Z = 2$, $A = 56$ atmosphere can have a sign change in $P_L$ around
$E \sim 3 - 5$ keV depending on the $\theta_B$ value.

To determine the observed polarization signal, we must
consider the propagation of polarized radiation in the NS
magnetosphere, whose dielectric property in the X-ray band is
dominated by vacuum polarization (39). As a photon propagates
from the NS surface through the magnetosphere, its polarization
state evolves following the varying magnetic field it experiences,
up to the “polarization-limiting radius” $r_{pl}$, beyond which the
polarization state is frozen. It is convenient to set up a fixed
coordinate system $XYZ$, where the $Z$-axis is along the line of
sight, and the $X$-axis lies in the plane spanned by the $Z$-axis and $\Omega$
(the NS spin angular velocity vector). The polarization-limiting
radius $r_{pl}$ is determined by the condition

$$1/k = 2|d\phi_B/ds|,$$

where $d\phi_B$ is the azimuthal angle of the magnetic field along the ray ($s$ measures the distance along the
For a NS with surface dipole field $B_d$ and spin frequency $\nu = \Omega/(2\pi)$, we have (33) $r_{pl}/R \sim 150 (E_1 B_{d14}^2 / \nu_1)^{1/6}$, where $B_{d14} = B_d/(10^{14} \text{G})$ and $\nu_1 = \nu/\text{Hz}$. Note that since $R \ll r_{pl} \ll r_1$ (with $r_1 = c/\Omega$ the light-cylinder radius), only the dipole field determines $r_{pl}$. Regardless of the surface magnetic field structure, the radiation emerging from most atmosphere patches with mode intensities $I_{X,e}$ and $I_{O,e}$ evolves adiabatically in the magnetosphere such that the radiation at $r > r_{pl}$ consists of approximately the same $I_{X,e}$ and $I_{O,e}$, with a small mixture of circular polarization generated around $r_{pl}$ (33). The exception occurs for those rays that encounter the quasi-tangential point (where the photon momentum is nearly aligned with the local magnetic field) during their travel from the surface to $r_{pl}$ (40). Since only a small fraction of the NS surface radiation is affected by the quasi-tangential propagation, we can neglect its effect if the observed radiation comes from a large area of the NS surface. Let the azimuthal angle of the $B$ field at $r_{pl}$ be $\Phi_B(r_{pl})$ in the XYZ coordinate system. The observed Stokes parameters (normalized to the total intensity $I$) are then given by

$$Q/I = -P_l \cos 2\Phi_B(r_{pl}), \quad [38]$$
$$U/I = -P_l \sin 2\Phi_B(r_{pl}). \quad [39]$$

Note that when $r_{pl} \ll r_1$, the transverse (XY) component of $B(r_{pl})$ is opposite to the transverse component of the magnetic dipole moment $\mu$; thus, $\Phi_B(r_{pl}) \simeq \pi + \Phi_\mu$, where $\Phi_\mu$ is the azimuthal angle of $\mu$.

To determine the emission from the whole NS surface, we need to add up contributions from different patches (area element $dS$), including the effect of general relativity (41, 42). For example, the observed spectral fluxes $F_L$, $F_Q$ (associated with the intensities $I$, $Q$) are

$$F_L = g^3 \int dS \cos \alpha \left( I_{X,e} + I_{O,e} \right), \quad [40]$$
$$F_Q = g^3 \int dS \cos \alpha \left( I_{O,e} - I_{X,e} \right) \cos 2\Phi_B(r_{pl}). \quad [41]$$

where $g \equiv (1 - 2GM/Rc^2)^{1/2}$ and $\alpha$ is the angle between the ray and the surface normal at the emission point). Clearly, to compute $F_L$, $F_Q$ and $F_U$ requires the knowledge of the distributions of NS surface temperature and magnetic field, as well as various angles (the relative orientations between the line of sight, the spin axis, and the dipole axis). This is beyond the scope of the paper. [Note that since $\Phi_B(r_{pl}) \simeq \pi + \Phi_\mu$, the phase variation of $F_Q$, $F_U$ follows the rotation of the magnetic dipole, as in the rotating vector model.] Nevertheless, our results depicted in Figs. 5 and 6 (with different values of local field strengths and orientations) show that the 90° linear polarization swing observed in AXP 4U 0142 can be explained by emission from a partially ionized heavy-element atmosphere with surface field strength about $10^{14}$ G or a H/He atmosphere with $B_{i14} \lesssim 0.5$ and a more restricted surface field geometry (i.e., most of the radiation comes from the regions with $\theta_B \gtrsim 70°$).

**Discussion**

We have shown that the observed X-ray polarization signal from AXP 4U 0142, particularly the 90° swing around 4 to 5 keV, can be naturally explained by the mode conversion effect–associated vacuum resonance in the NS atmosphere. In this scenario, the 2 to 4 keV emission is dominated by the X-mode, while the 5 to 8 keV emission by the O-mode as a result of the adiabatic mode conversion from the X-mode to the O-mode. This interpretation of the polarization swing would imply that the NS’s kick velocity is aligned with its spin axis, in agreement with the spin–kick correlation observed in other NS systems.

It is important to note that in our scenario, the existence of the X-ray polarization swing depends sensitively on the actual value of the magnetic field on the NS surface (Eq. 27). To explain the observation of AXP 4U 0142, the magnetic fields in most region of the NS surface must be less than about $10^{14}$ G, and a lower field strength would be preferred in terms of producing the polarization swing robustly (for a wide range of geometrical parameters). Using the pulsar spindown power derived from force-free electrodynamics simulations (43), $L_d = (\mu^2 B_4^4 / c^3)(1 + \sin^2 \theta_\mu)^{-1}/2$. Where $R_0$ is the NS radius in units of 10$^6$ cm, and $L_X$ is the moment of inertia in units of 10$^{45}$ g cm$^2$. Note that with $R_0 \simeq 1.3$ (44, 45), the above estimate is reduced by a factor of 2. In addition, if the magnetar possesses a relativistic wind with luminosity $L_w > L_d$, the wind can open up field lines at $r_{open} \sim (B_d^2 R_0^4 c/L_w)^{1/4} \lesssim c/\Omega$ and significantly enhance the spindown torque (46–48). This would imply that a smaller $B_d$ is needed to produce the observed $\dot{P}$ in 4U 0142. Thus, the observation of X-ray polarization swing is consistent with the indirectly “measured” dipole field and requires that the high-order multipole field components be not much stronger than the dipole field.

We reiterate some of the caveats of our work: We have not attempted to calculate the synthetic polarized radiation from the whole NS and to compare with the X-ray data from AXP 4U 0142 in detail; our treatment of partially ionized heavy-element magnetic NS atmospheres is also approximate in several aspects (e.g., bound–free opacities are neglected; the vertical temperature profiles are assumed based on limited H atmosphere models). At present, these caveats are unavoidable, given the uncertainties (and large parameter space needed to do a proper survey) in the surface temperature and magnetic field distributions on the NS and the fact that self-consistent heavy-element atmospheres models for general magnetic field strengths and orientations have not been constructed, especially in the regime where the vacuum resonance effects are important.

In this work, we have interpreted the 2 to 8 keV polarization signal from AXP 4U 0142 in terms of thermal emission from the NS surface. It is well established that for quiescent magnetars, the spectrum turns up above 10 keV, such that the bulk of their energy comes out as nonthermal hard X-rays (2, 49). The 0.5 to 10 keV spectrum can be parameterized either by an absorbed blackbody plus a power-law component (of a photon index between −4 and −2) or by the sum of several blackbodies. So it is possible that the 5 to 8 keV emission from AXP 4U 0142 has a significant nonthermal contribution. Thompson and Kostenko (50) have studied a model in which the hard X-ray emission of quiescent magnetars comes from the “annihilation bremsstrahlung” of an electron–positron magnetospheric plasma. Such emission is mainly polarized in the O-mode. If this emission dominates the 5 to 8 keV spectrum of AXP 4U 0142 while negligible at lower energies, it could provide an alternative explanation for the polarization swing for a wide range of surface
field strengths (recall that for $B \geq B_{\text{OV}}$, the atmospheric thermal emission is dominated by the X-mode for all $E$'s (Eq. (27)).

Very recently, while this paper was under review, IXPE reported the detection of polarized X-rays from another magnetar, 1RXS J170849.0−400910 (16): The phase-averaged polarization signal exhibits an increase with energy, from $\sim$20% at 2 to 3 keV to $\sim$80% at 6 to 8 keV, while the polarization angle is independent of the photon energy. This constant polarization angle is consistent with atmospheric emission dominated by the X-mode at all energies and is expected since the measured dipole field (based on spindown) for this AXP, $B_d \simeq 5 \times 10^{14}$ G, is significantly larger than $B_{\text{OV}}$ (Eq. (27)). The increase of linear polarization with $E$ can be a result of the magnetic field geometry (relative to the spin axis and line-of-sight) and the surface temperature distribution [ref. 33 for examples].

Overall, our work demonstrates the important role played by the vacuum resonance in producing the observed X-ray polarization signature from magnetars (and NSs with weaker magnetic fields). The observations of AXP 4U 0142 and 1RXS J170849.0−400910 by IXPE have now opened up a new window in studying the surface environment of NSs. Future X-ray polarization mission [such as eXTP; ref. 51] will provide more detailed observational data. Comprehensive theoretical modelings of magnetic NS surface radiation and magnetosphere emission will be needed to confront these observations.

Data, Materials, and Software Availability. All study data are included in the main text.

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