A Note on Obesity as Epidemic in Korea

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Abstract
Objective: To analyze the incidence of obesity in adults aged 19–59 years in Korea and predict its trend in the future.
Methods: We considered a two-compartmental deterministic mathematical model Susceptible-Infected-Susceptible (SIS), a system of difference equations, to predict the evolution of obesity in the population and to propose strategies to reduce its incidence.
Results: The prevention strategy on normal-weight individuals produced a greater improvement than that produced by treatment strategies.
Conclusions: Mathematical model sensitivity analysis suggests that obesity prevention strategies are more effective than obesity treatment strategies in controlling the increase of adult obesity in Korea.

1. Introduction

Obesity is an unbalanced condition in which accumulated dietary calories exceed the body’s energy consumption. Currently, obesity is one of the most serious health problems all over the world, including in Korea. It is an ongoing problem for the whole of society, as well as for individuals. Obesity is associated with illnesses such as cardiovascular diseases, diabetes, musculoskeletal disorders such as osteoarthritis, and some cancers, and significantly devalues the quality of life [1]. From a societal perspective, the annual expenditure on controlling diseases related to obesity is astronomical [2].

Recently, obesity has been hypothetically regarded as a social epidemic in the sense that it can spread from one person to another [3,4]. There is consistent evidence that obesity is a health concern that spreads by social peer pressure or social contact [5]. Mathematical modeling of the progression of most infectious diseases is useful to discover the likely outcome of an epidemic or to help in their management. Indeed, mathematical studies indicate that obesity is transmitted socially [5–8].

In this paper, we applied a new difference equation model to study obesity in Korean adults aged 19–59 years and also projected the findings into the future. The main purpose of this study was to predict the future prevalence of obesity in 19–59-year-old Koreans and to propose some strategies to reduce obesity in Korea. Moreover, we identified important predictive factors of obesity in the Korean adult population.
2. Materials and Methods

2.1. Model description

We introduce a two-compartmental deterministic mathematical model, a system of difference equations, to predict the evolution of obesity in the Korean population and to propose strategies to reduce its incidence. Basically, the idea of modeling the obesity epidemic comes from the disease model Susceptible-Infected-Susceptible (SIS) [9].

As in [8], we divided the 19–59-year-old population in Korea into two subpopulations based on their body mass index (BMI; weight/height² (kg/m²)). The classes or subpopulations were individuals defined as normal weight $S_t$ (BMI, <25) and obese individuals $O_t$ (BMI, ≥25) according to the definition of WHO [10].

We regarded obesity as an infectious disease caused by social peer pressure or social contact that influences the probability of transmission of sedentary lifestyle and unhealthy nutritional habits. From this position, let us propose an epidemiological-type model to study the epidemic evolution of obesity. We adopted the following assumptions [8]:

1. Once an adult starts an unhealthy lifestyle, he/she would continue it and develop obesity $O_t$ because of this lifestyle. In class $O_t$, individuals who are able to stop his/her unhealthy lifestyle can move to class $S_t$.
2. Populations of humans are homogeneously mixed, which means the rate of interaction between two different subpopulations is proportional to the product of the numbers in each of the subsets concerned [11,12]. The transit is modeled using the term $\beta S_t O_t$ [13].
3. The subpopulation’s sizes and behaviors with time will decide the dynamic evolution of adulthood obesity.

The model can be described as shown in Figure 1.

Without loss of generality and for the sake of clarity, the 19–59-year-old adult population is normalized to unity; for all time $t$, $S_t + O_t = 1$. Under the above assumptions, we have the following nonlinear system of difference equations:

$$
\begin{align*}
S_{t+1} &= S_t - \mu S_t + \mu S^0 - \beta S_t O_t + \rho O_t \\
O_{t+1} &= O_t - \mu O_t + \mu O^0 + \beta S_t O_t - \rho O_t
\end{align*}
$$

(2.1)

![Figure 1. Schematic diagram of the mathematical model.](image)

The state variables of the model are shown in Table 1 and parameters used in the model in Table 2.

Model (2.1) is analyzed qualitatively to investigate the existence and stability of equilibria. First, we identify the equilibrium of nonlinear system of difference equations (2.1). We denoted by $(\bar{S}, \bar{O})$ the equilibrium points of system (2.1). Computing the steady state, we obtain two equilibrium points; the disease-free equilibrium (DFE) for (2.1) is:

$$(\bar{S}, \bar{O}) = (1, 0)$$

(2.2)

and the endemic equilibrium (EE) for (2.1) is:

$$(\overline{S}, \overline{O}) = \left( \frac{\omega - \sqrt{\omega^2 + 4\beta \mu O^0}}{2\beta} + 1, \frac{-\omega + \sqrt{\omega^2 + 4\beta \mu O^0}}{2\beta} \right)$$

(2.3)

DFE $(1,0)$ is locally asymptotically stable if $0 < \rho + \mu - \beta < 2$ and the endemic equilibrium $(\overline{S}, \overline{O})$ (2.3) is locally asymptotically stable if $0 < \rho + \mu - \beta(\overline{S} - \overline{O}) < 2$. On the one hand, model

![Figure 2. Parameter estimation for $\beta$ and $\rho$ using least square method.](image)

| Table 1. State variables for population dynamics |
|-----------------------------------------------|
| $S_t$ | The proportion of normal weight individuals at time $t$ |
| $O_t$ | The proportion of obese individuals at time $t$ |

| Table 2. Time-invariant parameters for population dynamics |
|------------------------------------------------------------|
| $1/\mu$ | Average time of stay in the system of 19–59-year-old adults |
| $\rho$ | Rate at which an obese individual moves to the normal weight subpopulation |
| $\beta$ | Transmission rate because of social pressure to adopt an unhealthy lifestyle |
| $S^0, O^0$ | Proportions of normal weight and obese populations in the 18-year-old age group |
(2.1) can be simplified to the following one difference equation using the assumption $S_t + O_t = 1$.

\[ O_{t+1} = -\beta (O_t)^2 + (1 + \beta - \rho - \mu)O_t + \mu O^0 \]  

(2.4)

\[ \overline{O} = -\rho + \mu - \beta + \sqrt{(\rho + \mu - \beta)^2 + 4\beta \mu O^0}/(2\beta) \] is a locally asymptotically stable equilibrium of (2.4) if

\[ 0 < (\rho - \mu - \beta)^2 + 4\beta \mu O^0 < 4. \]

Additionally, DFE (1,0) is locally asymptotically stable of (2.1) if and only if \( \overline{O} \) is a locally asymptotically stable equilibrium of (2.4) and the endemic equilibrium \((\overline{S}, \overline{O})\) (2.3) is locally asymptotically stable of (2.1) if and only if \( \overline{O} \) is a locally asymptotically stable equilibrium of (2.4). See Appendix for details.

### 2.2. Numerical computation

Parameter estimation was performed to produce the model of the current Korean situation.

First, we need to know the parameters $m$, $S^0$, and $O^0$ of the model. $\mu$ is inversely proportional to the mean time spent by an adult in the system. The total 40 years from age 19 to 59 is 2080 weeks (accepting 1 year = 52 weeks) so $\mu$ should be 1/2080. $S^0$ and $O^0$ are the proportions of the normal weight and obese populations, respectively, in the 18 years age group. According to the statistical data of the Korea Institute of Sport Science in 2007 [14], the obesity rate of 18-year-old Koreans was 16.4%.

Parameters $\beta = 0.0009038$ and $\rho = 0.0000336$ were estimated by least-square method using obesity data (from 1998 to 2008) of the fourth Korea National Health and Nutrition 2008 (Figure 2) [15].

Recall $O_{t+1} = -\beta (O_t)^2 + (1 + \beta - \rho - \mu)O_t + \mu O^0$ (2.4). To know the trend of the obese population of 19—59-year-old Koreans, using the parameter in Table 3, we simulate our model (2.4) (Figure 3).

From Figure 3, we see that the trend of obese population of persons aged 19 to 59 years in Korea increases.

Now, we compare prevention strategy versus treatment by present simulations of the mathematical model and varying some of the parameters. The aim of varying the parameters is to observe how the final prediction can be affected by these changes. This perturbation allows us to propose obesity prevention strategies.

Figures 4 and 5 suppose an increase of physical activity in the obese class $O_t$. Consequently, we tried to increase gradually the parameter $\rho$ from 0% to 2000%.

### Table 3. Estimated parameters

| Parameter | Value |
|-----------|-------|
| $\mu$     | 1/2080|
| $\beta$   | 0.0009038|
| $\rho$    | 0.0000336|
| $S^0$     | 0.8365|
| $O^0$     | 0.1635|

**Figure 3.** Numerical simulation of the mathematical model.

**Figure 4.** Numerical simulation of the mathematical model when parameter $\rho = 0.0000336$ is increased from 0% to 2000%.
initially given $\rho = 0.0000336$. In this case, this treatment strategy on obese individuals produces a small improvement. Figures 6 and 7 suppose a decrease in the social transmission parameter $\beta$. In this case, we tried to reduce gradually the parameter $\beta$ from 0% to 100% initially given $\beta = 0.000938$. The prevention strategy on the normal-weight individuals produced a bigger improvement than that produced by the previously mentioned treatment strategies.

3. Results

Data of the fourth Korea National Health and Nutrition 2008 show that there has been a noticeable increase in the rate of obesity in Korea [15]. Performing the simulations with the above model showed the same increases in coming years. In other words, the numerical simulations carried out with our proposed mathematical model and with the estimated parameters indicate an

![Figure 5. Dynamics of obesity depending on perturbation of $\rho$.](image)

![Figure 6. Numerical simulation of the mathematical model when parameter $\beta = 0.0009038$ is reduced by 0–100%](image)
increasing trend in obesity among the 19—59-year-old population in future.

From the results of the numerical simulations (Figures 4—7), we conclude that the prevention strategy on normal-weight individuals to adopt physical activity and healthy nutritional habits is better than the treatment strategy that motivates physical activity and healthy nutritional habits in the obese populations due to the finding that the reduction observed in the incidence of obesity is greater in the former case.

4. Discussion

To reduce the incidence of adulthood obesity among the 19—59-year-old population in Korea in future, from Figures 4—7 we could see that it would be more beneficial to focus on the promotion of physical activity and healthy nutritional habits in normal-weight individuals than on implementing similar campaigns for obese individuals. To achieve lower obesity prevalence, these technical suggestions would produce positive effects on reducing the incidence of obesity based on the above numerical simulations.

5. Conclusions

Mathematical model sensitivity analysis suggests that obesity prevention strategies are more effective than obesity treatment strategies in controlling the increase of adulthood obesity in Korea. Consequently, prevention strategy is recommended for tackling the obesity social epidemic.

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Appendix.

To identify the equilibrium of nonlinear system of difference equations (2.1), we set

\[ \bar{S} = S - \mu S + \mu S^0 - \beta S \bar{O} + \rho \bar{O}, \]  
\[ \bar{O} = O - \mu O + \mu O^0 + \beta S \bar{O} - \rho \bar{O}. \]  

(A1)  

(A2)

Since \( \bar{S} + \bar{O} = 1 \), (A2) can be expressed by

\[ \beta (\bar{O})^2 + (\rho + \mu - \beta) \bar{O} - \mu O^0 = 0. \]  

(A3)

By putting \( \omega = \rho + \mu - \beta \) we can express (A3) as

\[ \beta (\bar{O})^2 + \omega \bar{O} - \mu O^0 = 0. \]  

(A4)

If \( \bar{O} = 0 \), then \( O^0 = 0 \). Therefore, the disease-free equilibrium (DFE) for (2.1) is

\[ (\bar{S}, \bar{O}) = (0, 1) \]  

(A5)

Now, suppose that \( \bar{O} \neq 0 \). Then, the quadratic equation (A4) has roots \( \bar{O} = -\omega \pm \sqrt{\omega^2 + 4\beta \mu O^0} / (2\beta) \). For looking up the positive equilibrium, we need for \( \bar{O} > 0 \), then we get \( \bar{O} = -\omega + \sqrt{\omega^2 + 4\beta \mu O^0} / (2\beta) \). Here, clearly \( \bar{O} < 1 \), for \( -\omega + \sqrt{\omega^2 + 4\beta \mu O^0} / (2\beta) < 1 \) if and only if \( \frac{\omega^2}{\beta} < \frac{\omega^2}{\beta} + 4\beta \mu O^0 \). Since \( \bar{S} + \bar{O} = 1 \), we have \( \bar{S} = -\omega - \sqrt{\omega^2 + 4\beta \mu O^0} / (2\beta) + 1 \). Therefore, the endemic equilibrium (EE) for (2.1) is

\[ (\bar{S}, \bar{O}) = \left( \frac{-\omega + \sqrt{\omega^2 + 4\beta \mu O^0}}{2\beta} + 1, \frac{-\omega - \sqrt{\omega^2 + 4\beta \mu O^0}}{2\beta} \right) \]  

(A6)

Next, we check the stability of the equilibrium points \( (\bar{S}, \bar{O}) \).

**Theorem 1**

DFE \( (\bar{S}, \bar{O}) \) in (A5) and the endemic equilibrium \( (\bar{S}, \bar{O}) \) in (A6) are locally asymptotically stable if \( 0 < \rho + \mu - \beta < 2 \) or if \( 0 < \rho + \mu - \beta (\bar{S} - \bar{O}) < 2 \), respectively.

**Proof**

The Jacobian matrix evaluated at equilibrium \( (\bar{S}, \bar{O}) \) is of the form [16]:

\[ J(\bar{S}, \bar{O}) = \begin{pmatrix} 1 - \mu - \beta \bar{O} & -\beta \bar{S} + \rho \\ \beta \bar{O} & 1 - \mu + \beta \bar{S} - \rho \end{pmatrix} \]

By Jury Test for \( n = 2 \) [17, 18], if \( |\text{Tr}(J)| < 1 + \text{det}(J) < 2 \) holds then the equilibrium point \( (\bar{S}, \bar{O}) \) is locally asymptotically stable. Otherwise, the equilibrium \( (\bar{S}, \bar{O}) \) is unstable.

It is easy to check the condition \( |\text{Tr}(J)| < 1 + \text{det}(J) < 2 \) is equivalent to the conditions in Theorem for each case.

**Remark**

Since \( S_t + O_t = 1 \), the system of difference equations (2.1) can be simplified to one difference equation:

\[ \begin{align*} 
S_{t+1} &= S_t + O_t + \beta (1 - O_t) O_t - \rho O_t - \mu O_t \\
&= -\beta (O_t)^2 + (1 + \beta - \rho - \mu) O_t + \mu O^0 \\
&= f(O_t) 
\end{align*} \]  

(A7)

Now, we may check the stability of the equilibrium point \( \bar{O} \) in (A7) as follows:

**Theorem 2**

\( \bar{O} = -\omega + \sqrt{\omega^2 + 4\beta \mu O^0} / (2\beta) \) is a locally asymptotically stable equilibrium of (A7) if \( 0 < \omega^2 + 4\beta \mu O^0 < 4 \), where \( \omega = \rho + \mu - \beta \).

**Proof**

Put \( f(x) = -\beta x^2 + (1 + \beta - \rho - \mu) x + \mu O^0 \). We need to find the condition \( |f'(\bar{O})| < 1 \) [19, 20]. Since \( \omega = \rho + \mu - \beta \), we see that \( f'(\bar{O}) = -2\beta \bar{O} + (1 + \beta - \rho - \mu) = -2\beta \left( -\omega + \sqrt{\omega^2 + 4\beta \mu O^0} / (2\beta) \right) + (1 + \beta - \rho - \mu) = -\omega - \sqrt{\omega^2 + 4\beta \mu O^0} + 1 - \omega = -\sqrt{\omega^2 + 4\beta \mu O^0} \), whence we have that \( |f'(\bar{O})| < 1 \) if \( \frac{-\sqrt{\omega^2 + 4\beta \mu O^0}}{2\beta} < 1 \) or \( \frac{-\sqrt{\omega^2 + 4\beta \mu O^0}}{2\beta} > 1 \). Squaring both sides, we get \( 0 < (\rho + \mu - \beta)^2 + 4\beta \mu O^0 < 4 \).

**Remark**

By simplifying the condition \( \rho + \mu - \beta (S - O) \), one may see that the DFE \( (\bar{S}, \bar{O}) \) in (A5) and the endemic equilibrium \( (\bar{S}, \bar{O}) \) in (A6) are locally asymptotically stable if and only if \( \bar{O} \) in (A7) is a locally asymptotically stable equilibrium.