Detecting the Most Unusual Part of Two and Three-dimensional Digital Images

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Abstract

The purpose of this paper is to introduce an algorithm that can detect the most unusual part of a digital image in probabilistic setting. The most unusual part of a given shape is defined as a part of the image that has the maximal distance to all non intersecting shapes with the same form. The method is tested on two and three-dimensional images and has shown very good results without any predefined model. A version of the method independent of the contrast of the image is considered and is found to be useful for finding the most unusual part (and the most similar part) of the image conditioned on given image.

The results can be used to scan large image databases, as for example medical databases.

Keywords: image processing, image statistics, image recognition

1 Introduction

In this paper we are trying to find the most unusual part with predefined shape of a given image. If we consider an one-dimensional quasi-periodical image, as for example electrocardiogram (ECG), the most unusual parts with length about one second will be the parts that correspond to rhythm abnormalities \textsuperscript{6}. Therefore they are of some interest. Considering two and three dimensional images, we can suppose that the most unusual part of the image can correspond to something interesting of the image.

Recently we have presented an algorithm that can detect the most unusual part of a digital image, referring to two-dimensional images \textsuperscript{8}. In fact the
algorithm can be used in more than two dimensions. The present paper is an extension of this method in the case of three-dimensional images.

Of course, if we have a clear mathematical model of what the interesting part of the image can be, it would be probably better to build a mathematical model that detects those unusual characteristics of the image part that are interesting. However, as in the case of ECG, the part that we are looking for, can not be defined by a clear mathematical model, or just the model can not be available. In such cases the most unusual part can be an interesting instrument for screening images.

To state the problem, we need first of all a definition of the term “most unusual part”. Let us chose some shape $S$ within the image $A$, that could contain that part and let us denote the cut of the figure $A$ with shape $S$ and origin $\vec{r}$ by $A_S(\vec{r}, \vec{r})$, e.g.

$$A_S(\vec{r}, \vec{r}) \equiv S(\vec{r})A(\vec{r} + \vec{r})$$

where $\vec{r}$ is the in-shape coordinate vector, $\vec{r}$ is the origin of the cut $A_S$ and we used the characteristic function $S(.)$ of the shape $S$. Further in this paper we will omit the arguments of $A_S$. We can suppose that the most unusual part is the one that has the largest distance with the rest of the cuts with the same shape.

Speaking mathematically, we can suppose that the most unusual part is located at the point $\vec{r}$, defined by:

$$\vec{r} = \arg \max_{\vec{r}} \min_{\vec{r}':|\vec{r}' - \vec{r}| > \text{diam}(S)} ||A_S(\vec{r}') - A_S(\vec{r})||.$$

(1)

Here we assume that the shifts do not cross the border of the image. The norm $||.|.$$ is assumed to be $L_2$ norm.

As the parts of an image that intersect significantly are similar, we do not allow the shapes located at $\vec{r}'$ and $\vec{r}$ to intersect, avoiding this by the restriction on $\vec{r}' : |\vec{r}' - \vec{r}| > \text{diam}(S)$.

If we are looking for the part of the image to be unusual in a context of an image database, we can assume that further restrictions on $\vec{r}'$ can be added, for example restricting the search to avoid intersection with several images.

The definition above can be interesting as a mathematical construction, but if we are looking for practical applications, it is too strict and does not correspond exactly to the intuitive notion of the interesting part as there can be several of them. Therefore the correct definition will be to find the outliers of the distribution of the distances $||.|.$ between the blocks.

In $d$-dimensional space the figure with linear size $N$ has $N^d$ points and if $||S|| \ll ||A||$, in order to find deterministically the most unusual part, we need $N^d$ operations. This is unacceptable for large two dimensional images, and it is even worse in the case of 3D image databases. Therefore we are looking for

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1Similar results are achieved with $L_1$ norm. The algorithm was not tested with $L_{max}$ norm due to its extreme noise sensitivity. We use $L_2$ because of its relation with PSNR criteria that closely resembles the human subjective perception.
an algorithm that provides an approximate solution of the problem and solves it within some probability limit in acceptable execution time.

As is defined above in Eq. (1), the problem is very similar to the problem of location of the nearest neighbor between the blocks. This problem has been studied in the literature, concerning Code Book and Fractal Compression [1]. However, the problem of finding $\vec{r}$ in the above equation, without specifying $\vec{r}'$, as we show in the present paper, can be solved by using probabilistic methods avoiding slow calculations.

2 The Method

2.1 Projections

The problem of estimating the minima of Eq. (1) is complicated because the blocks are multidimensional. Therefore we can try to simplify the problem by projecting the block $B \equiv A_S(\vec{r})$ in one dimension using some projection operator $X$. For this aim, we consider the following quantity:

$$b = |X.B_1 - X.B| = |X.(B_1 - B)|, \quad |X| = 1.$$ (2)

The dot product in the above equation is the sum over all $\rho$-s:

$$X.B \equiv \sum_{\rho} X(\rho)B(\rho; \vec{r}).$$ (3)

If $X$ is random, and uniformly distributed on the sphere of corresponding dimension, then the mean value of $b$ is proportional to $|B_1 - B|$; $\langle b \rangle = c|B_1 - B|$ and the coefficient $c$ depends only on the number of points of the block, that can be treated as its dimensionality, considering the projection operator. However, when the size of the block, e.g. its dimensionality increases, the two random vectors ($B_1 - B$ and $X$) are close to orthogonal and the typical projection is small. But if some block is far away from all the other blocks, then with some probability, the projection will be large. The method resembles that of Ref. [5] for finding nearest neighbor.

As mentioned above we must look for outliers in the distribution. This would be difficult in the case of many dimensions, but easier in the case of one dimensional projection.

We will regard only projections orthogonal to the vector with components proportional to $X_0(\rho) = 1, \forall \rho$. The projection on the direction of $X_0$ is proportional to the mean brightness of the area and thus can be considered as not so important characteristics of the image. An alternative interpretation of the above statement is by considering all blocks that differ only by their brightness to be equivalent.

Mathematically the projections orthogonal to $X_0$ have the property:

$$\sum_{\rho} X(\rho) = 0.$$ (4)
The distribution of the values of the projections satisfying the property Eq. (4) is well known and universal [12] for the 2D natural images. The same distribution seems to be valid for a vast majority of the images. The distribution of the projections derived for the X-ray image, shown in Fig. 1, is shown in Fig. 2.

In the case of a three-dimensional image, Fig. 3, the corresponding histogram, obtained by using the above method is shown in Fig. 4. One observes a higher asymmetry of the distribution of the projections in this case, compared to the same distribution of two-dimensional images, but qualitatively it is of the same type.

Roughly speaking, if the blocks are small enough, the distribution satisfies a power law distribution with exponential drop at the extremes. When the blocks are big enough, the exponential part is predominant.

If $A_r$ and $A'_r$ have similar projections, then they will belong to one and the same or to adjacent bins. Therefore we can look for blocks that have a minimal number of similar and large projections. But these, due to the universality of the distribution, are exactly the blocks with large projection values.

As a first approximation, we can just consider the projections and score the points according to the bin they belong to. The distribution can be described by only one parameter that, for convenience, can be chosen to be the standard deviation $\sigma_X$ of the distribution of $X.B$.

The notion of ”large value of the projection” will be different for different projections but will be always proportional to the standard deviation $\sigma_X$. Therefore we can define a parameter $a$ and score the blocks with $|X.B| > a\sigma_X$.

\[2\] In general, the standard deviation will be larger for projections with larger low-frequency components. That is why we choose the criterion proportional to $\sigma_X$ and not as an absolute value for all projections $X$. 

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Figure 1: The original test image. X-ray image of a person with ingested coin.

Figure 2: The distribution of the projection value for square shape with a size 48x48 pixels.
2.2 Algorithm

Resuming, in order to find the most unusual blocks of shape $S$ in an image $A$, we propose the following

**Algorithm:**

0. Initialize: Construct a figure $B$ with the same shape as $A$ and with all pixels equal to zero. The result of the algorithm will be saved in $B$.
1. Generate a random projection operator $X$, with carrier with shape $S$, zero mean and norm one.
2. Project all blocks (convolute the figure). We denote the resulting figure as $C$.
3. Calculate the standard derivation $\sigma_X$ of the result of the convolution.
4. For all points of $C$ with absolute values greater than $a\sigma_X$, increment the corresponding pixel in $B$.
   Repeat steps 1-4 for $M$ number of times.
5. Select the maximal values of $B$ as the most singular part of the image.

The acceptable values of $a$ are discussed in the next section. The number of iterations $M$ can be fixed empirically or until the changes in $B$, normalized by that number, become insignificant. Following the algorithm, one can see that the time to perform it is proportional to $MN^d \log N$. The speed per image of size $1024 \times 2048$ on one and the same computer, with $S$, a square of size $56 \times 56$ points, is about 3 seconds compared to about an hour, using the direct search by implementing Eq. (1). We use a laptop with 2GHz Intel Celeron CPU and 1GB of memory.\(^3\)

\(^3\)If the block is small enough, the convolution can be performed even faster in the space domain and it is possible to improve the execution time.
Figure 5: Score values for different size of the shape (24x24, 32x32, 40x40, 56x56). The value of the parameter $a$ in all the cases is 12.

3 Empirical Assessment

Applying the algorithm above, we are looking for the most unusual part of the image in different settings. We also generalize the method in order to improve it and to amplify the range of applications.

3.1 Two dimensional images

Some results are presented in Figs. 5-6, where we used square shapes with different size, 30 random projection operators and different values of $a$.

Because the distribution of the projections (Fig. 2) is universal, it is not surprising that the algorithm is operational for different images. We have tested it with some 100 medical Xray images and the results of the visual inspections were good.

It can be noted that the number of projection operators is not critical and can be kept relatively low and independent of the size of the block. Note that with significantly large blocks, the results can not be regarded as an edge detector. This empirical observation is not a trivial result at all, indicating that the degrees of freedom are relatively few, even with large enough blocks, something that depends on the statistics of the images and can not be stated in general. With more than 20 projections we achieve satisfactory results, even for areas with more than 3000 pixels (some $10^5$ in 3D). The increment of the number of the projections improves the quality, but with more than 30 projection practically no improvement can be observed.

A phenomenological argument can be given, observing that in the case of 30 projections, the pixels with maximal values are larger than 5. In order to
Figure 6: Score values for different parameter $a$ ($a = 8, 10, 12, 16$). The size of the shape is 24x24.

distinguish a binary criteria (unusual/usual) this value is satisfactory large.

It is possible to look at that algorithm in a different way, namely, if we are trying to reconstruct the figure by using some projection operators $X_C$ (for example DCT as in JPEG), then the length of the code, one uses to code a component with distribution like Fig. 2, will be proportional to the logarithm of the probability of some value of the projection $X_C.A$. Therefore, what we are scoring is the block that has some component of the code larger than some length in bits (here we ignore the psychometric aspects of the coding). Effectively we score the blocks with longer coding, e.g. the ones that have lower probability of occurrence.

Using a smoothed version of the above algorithm in step 4, without adding only one or zero, but for example, penalizing the point with the square of the projection difference with respect to the current block divided by $\sigma$, and having in mind the universal distribution of the projection, one can compute the penalty function as a function of the value of the projection $x$, that results to be just $1/2 + x^2/2\sigma^2$. Summing over all projections, we can obtain that the probability of finding the best block is approximated given by $1/2[1+\text{erfc}(M(1/2+x^2/2\sigma^2))]$ as a consequence of the Central Limit Theorem. The above estimation gives an idea why one needs few projections to find the most unusual block, in sense of the global distribution of the blocks, almost independently of the size of the block. The only dependence of the size of the blocks is given by $\sigma^2$ factor, that is proportional to its size. Further, the probability of error will drop better than exponentially with the increment of $M$.

The non-smoothed version performs somewhat better that the above estimation in the computer experiments.
3.2 Three dimensional images

Further we investigate how the algorithm works in 3D. As noticed in the previous section, the distribution of the projection is similar, but more irregular and asymmetric.

We have noticed that in 3D the method works better by using spherical shape, instead of cubic one. Most probably this is due to the fact that in the cube the most distant boundary voxel is $\sqrt{3}$ times further that the closest boundary voxel. This is significantly more that $\sqrt{2}$ as it is in the case of 2D pixels, although some ”squaring” effect can be noted also in 2D (See Fig.9, $\gamma = 0.5$).

Comparing the quality of the method on two and three-dimensional images, (Fig. 7), one can say that when the structure is clearly three dimensional, the algorithm working in 3D separates this structure much better than working in 2D section. The irrelevance of the dimension for the algorithm is probably the main advantage with respect to other algorithms, as for example the Hough transform [4]. The maximum execution time scales with $N$ as the number of pixels $MN^d \log_2 N^d$. Once again in 3D, as in the 2D case, $M$ can be chosen very modest, about 30. The memory cost is four times the memory needed to save a single image, using naive FFT implementation of the convolution.

3.3 Contrast

However, there is an evident objection against the proposed algorithm. Namely, if some area of the image with high contrast is selected, then the projection is proportional to the contrast of that area. This will actually select the most contrast areas as the most unusual ones. Mathematically this is in fact so, the boundaries are the most unusual parts of the images, but for practical reasons
the dependence of the contrast should be eliminated or at least attenuated with similar argumentation as the one we have used for the brightness.

Namely, two images that differ only by their contrast could be considered as equivalent. To eliminate the influence of the contrast, the best is to normalize the projections using the contrast of the block. Let us regard as a contrast the standard deviation $\sigma_B$ of the block $B$ in question. Then the change in the algorithm is just evident: substitute each of the projections Eq. (3) with its normalized value $X'_B \equiv X_B/\sigma_B$. However, the distribution $X'_B$ is no longer similar to that shown in Fig. 4. The distribution is just normal \[7\]. To illustrate this, we represent the distribution and its quartile normal-plot in Fig. 8.

Using this normalization procedure makes the algorithm sensible to the noise, converting the flat noisy areas to the most unusual ones because of the randomness of the noise. Also the contrast, as an important characteristics, is better to be partially preserved in the normalized projection. Therefore it is much better not to eliminate the dependence of the contrast, but just to attenuate it. We found that using

$$X_B(\gamma) \equiv X_B/\sigma_B^\gamma$$

with different $\gamma$-s serves well in order to give an appropriate weight of the contrast. When $\gamma = 0$ we have the case of uncorrected projections, while when $\gamma = 1$, the effect of the contrast is totally eliminated. Also $\gamma$ can be assumed to be the tradeoff between the texture and the shape of the area. In Fig. 9 we represent the results for different $\gamma$-s ($\gamma = 0, 0.5, 1, 1.5, 2$). We can see that different structures are highlighted dependent on the values of $\gamma$ - the texture.

\[4\] The distribution ought to be tested with caution because the low-pass filtering will flat the top of the distribution. Also the precision of the pixels ought to be at least 2 bytes in order to avoid rounding errors.
\[ \gamma = 0 \]
\[ \gamma = 0.5 \]
\[ \gamma = 1 \]
\[ \gamma = 2 \]

Figure 9: Different normalizations corresponding to the values of \( \gamma = 0, 0.5, 1, 2 \).
The influence of the borders diminishes and the influence of the texture increases.

### 3.4 Network

The pitfall of the consideration in the previous subsection is that the detected blocks are unusual in absolute sense, e.g. with respect to all figures that satisfy the power law or similar distribution of the projections. Actually this is not desirable. If for example, in X-ray image, several spinal segments appear, although these can be unusual in the context of all existing images, they are not unusual in the context of thorax or chest X-ray images.

Therefore the parts of the images with many similar projections must “cancel” each other. This gives us the idea to build a network, where its components with similar projection are connected by a negative feedback corresponding to the blocks with similar projections.

As we have seen in the previous section, the small projection values are much more probable and therefore less informative. Using this empirical argument, we can suggest that the connections between the blocks with large projections are more significant.

The network is symmetrical by its nature, because of the reflexivity of the
distances. We can try to build it in a way similar to the Hebb network [2] and define Lyapunov o energy function of the network. Thus the network can be described in terms of artificial recursive neural network. Connecting only the elements of the image that produce large projections, the network can be build extremely sparse [13], which makes it feasible in real cases.

Let us try to formalize the above considerations. For each point we define a neuron. The neurons corresponding to some point \( \vec{r} \) and having projection \( x \) receive a positive input flux, which is proportional to \(-\log p(x)\), where \( p \) is the probability of having projection with value \( x \). The same element, if its projection is large, also receives a negative flux from the points \( \vec{r}' \) with nearest projections that satisfy the condition \(|\vec{r} - \vec{r}'| > \text{diam}(S)| \). The flux in general is a function of \( p(x) \) and \( x' - x \).

As a first approximation we assume that the flux is constant with \( p(x) \) and the dependence on \( x' - x \) is trivial: the weight is 1 if \(|x' - x| < \delta\) and zero otherwise, where \( \delta \) is some parameter of the model.

In other words, we reformulate our problem in terms of a Hebb-like neural network with external field

\[
h = -h_0 \sum_{i=1}^{M} \log p(x_i)
\] (5)

and weights

\[
w_{rr'} = -\sum_{i=1}^{M} \sum_{|x_i| > a\sigma_i, |x'_i| > a\sigma_i, |x'_i - x_i| < \delta, x'_ix_i > 0} 1.
\] (6)

The extra parameter \( h_0 \) balances between the global and the local effects. It can be chosen in a way that the mean fluxes of positive and negative currents are equal in the whole network. The parameter \( \delta \), as a proof of concept value, can be assumed to be equal to infinity. So the only parameter, as in the previous case, is \( a \).

The dynamics of the network over time \( t \) is given by the following equation [3]:

\[
s_{\vec{r}}(t + 1) = g(\beta[h_{\vec{r}} + \sum_{\vec{r}'} w_{\vec{r}\vec{r}'} s_{\vec{r}'}(t) - T]),
\]

where \( g(.) \) is a sigmoid function, \( s_{\vec{r}}(t) \) is the state of the neuron \( s \) at position \( \vec{r} \) and time \( t \), \( \beta \) is the inverse temperature and \( T \) is the threshold of the system. The result must be insensitive to the particular chose of \( g(.) \).

Once the network is constructed, we need to choose its initial state. If the a priori probabilities for all points to be the origin of the most unusual block are equal, one can choose \( s_{\vec{r}}(0) = 1, \forall \vec{r} \). Due to the non-linearity, the analysis of the results is not straightforward. The existence of the attractor is guaranteed by the symmetrical nature of the weights \( w \), which is a necessary condition for the existence of an energy function.
We can further refine the results of the previous section by fixing the global threshold $T$ in a way to have only some fraction of the excited neurons. Thus we obtain a bump activity of the network, previously considered in [9, 10, 11]. A sample result is shown in Fig. 10.

Regarding the time analysis of the procedure, one can see that the execution times are proportional to the number of the weights $w$. Having in mind that the connectivity is between the blocks, and that we can use a fraction of blocks less than $1/N^d$, the execution time can drop to order inferior to the $N^{2d}$ limit. Thus, the number of steps to achieve the attractor is of order $d \log N$. It does not grow faster with the dimension than in a linear manner.

### 3.5 Conditional distribution

A case of special practical interest is to find the most unusual part of the image with respect to some database of images or with respect to a single image. Therefore, we are looking for the conditional probability of the occurrence of the blocks with respect to that database/image. We can impose that conditional distribution by using the network constructed in the previous section. However, if we look for the dependence and the conditional distribution of only one test image $A_p$, we can do it in an easier way, namely we can take from the image $A_p$ the patterns, $\{X\}$ with a shape $S$ in a random manner. That means to change step 1 of the algorithm to the following:

1c. Select at random a point of $A_p$ as an origin of the shape $S$. Use this area as a projection operator (normalizing and subtracting the mean brightness).

Then we can answer the following two questions at the same time - What is the part of the image $A$ with shape $S$ that is most similar to the image $A_p$ and what is the part of the image $A$ most dissimilar to the parts of the image $A_p$? The first answer is related to pattern recognition problem. As an example, if
Figure 11: Finding the most similar part using the algorithm. The test image has size of $54 \times 145$. The size of the rectangle block $S$ is shown for each image.

we look for the colon in the CT image shown in Fig.3, the test image can look like Fig.12.

If the shape $S$ is small, then the statistics would be more or less universal and we cannot expect that the result would be very specific to the image $A_p$. If we increase the size of the shape $S$, the result will be more and more specific. If the size of the $S$ is similar to the size of $A_p$, we can expect highly specific response. We have found empirically that in order to achieve satisfactory result we must use $\gamma \approx 1$, e.g. to eliminate the dependence on the contrast.

The results of the conditioning are shown in Fig.11. We condition one image of the colon in Fig. 3 to the one in Fig.12. We find that the recognition is very good. It does not depend on the dimensionality of the image. The first two panels of Fig.11 with smallest $S$, $16 \times 16$ and $32 \times 32$, have strong mixture between negative and positive large projections (not shown in the figure). It accentuates more or less the borders, but with mixed sign of the projections.
The last image has only positive correlations with the test image in the upper left corner. The position of the colon (See Fig 3) is detected correctly.

Because the methods of using Hebb-like network, described in the previous section is independent of the one, described in this section, we can combine them by simply applying them together. Actually we need to connect only the points corresponding to the $M$ blocks of $A_p$, that we have selected in step 1c, to compute the projections. Note that we used only 30 samplings in the previous experiments. The effect of the network application is pruning of the spurious part of the images, especially with big size of $S$.

4 Discussion and Future Directions

In this paper we present a method to find the most unusual part in two and higher dimensional images, when its shape is fixed, but in general arbitrary. The method is almost independent on the size of the shape in terms of the execution speed and time. It gives good results on experimental images without predefined model of the interesting event.

The method works equally well for 2D and 3D images. It is also fast enough using 3D images.

One necessary future development of the algorithm is to achieve practical and computable criteria of the ”rareness” of the block and comparing the results on large enough database in order to have qualitative measure of the results. The criterion must be different from Eq. (1), because its direct computing tends to be very slow and unstable.

It is possible to speed up the process by using multi-resolution approach. For example, we can use the downsized images and after that we can search in the original images only in the points detected in the smaller images. However, this approach needs future exploration, because in its naive version it can be
used only when the convolution in space domain is faster than the convolution in frequency domain.

Among the future applications of the present method, one could mention the achievement of experiments on different type of images and large image databases and experiments on acceleration of the network due to the special equivalence class construction.

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References

[1] Y. Fisher, Fractal Image Compression, ISBN 0387942114, Springer Verlag (1995).

[2] D. Hebb, The Organization of Behavior: A Neurophysiological Theory, Wiley, New York (1949).

[3] J. Hopfield: Neural networks and physical systems with emergent collective computational properties, Proc. Natl. Acad. Sci. USA, 79 (1982) 2554.

[4] P. Hough et al.: Method and Means for Recognizing Complex Patterns, U.S. Patent 3,069,654 (1962).

[5] P. Indyk: Uncertainty Principles, Extractors, and Explicit Embedding of L2 into L1, in the Proceedings of 39th ACM Symposium on Theory of Computing, (2007).

[6] E. Keogh, J. Lin and A. Fu: HOT SAX: efficiently finding the most unusual time series subsequence, in the Proceedings of 39th ACM Symposium on Theory of Computing, (2007).

[7] K. Koroutchev, J.R. Dorronsoro: Factorization and structure of natural 4 x 4 patch densities, Optical Engineering, 45 (2006) 127003.

[8] K. Koroutchev and E. Korutcheva, Detecting the most unusual part of a digital image, Combinational Image Analysis, LNCS 4958 (2008) 286-294.

[9] K. Koroutchev and E. Korutcheva: Bump formations in binary attractor neural network, Phys. Rev. E 73 (2006) 026107.
[10] K. Koroutchev and E. Korutcheva: Bump formations in attractor neural network and their application to image reconstruction, in the Proceedings of the 9th Granada Seminar on Computational and Statistical Physics, AIP, ISBN 978-0-7354-0390-1 (2006) 242.

[11] Y. Roudi and A. Treves: An associate network with spatially organized connectivity, JSTAT, 1 (2004) P07010.

[12] D. Ruderman: The statistics of natural images, Network : Computation in Neural Systems, 5 (1994) 598.

[13] M. Tsodyks and M. Feigel’man: The enhanced storage capacity in neural networks with low activity level, Europhys.Lett., 6 (1988) 101.