Superconducting correlation in the one-dimensional $t$-$J$ model with anisotropic spin interaction and broken parity

J. Shiraishi, Y. Morita and M. Kohmoto

Institute for Solid State Physics, University of Tokyo 7-22-1 Roppongi Minato-ku,
Tokyo 106, Japan

Abstract

A variant of the one-dimensional $t$-$J$ model with anisotropic spin interaction and broken parity is studied by the nested algebraic Bethe-ansatz method. The gapless charge excitations and the gapful spin excitations are obtained. It is shown that the singlet-superconducting correlation dominates in the low-density region by applying the finite-size scaling analysis in the conformal field theory.
In these years, strongly-correlated electron systems have drawn much attention. It is partly caused by the discovery of the copper-oxide high-$T_c$ superconductors. The one-dimensional models of strong correlation play an important role since some of them can be solved exactly. The non-perturbative results thus obtained contain some of the essential properties of the strongly correlated systems. Close scrutiny, however, is required when one tries to discuss higher dimensions based on the one-dimensional results, since some of them are specific to one dimension.

Besides the Hubbard model [1–3], the $t$-$J$ model [4] is regarded as one of the most basic models which contains the essence of strong correlation. The Hamiltonian is

$$
H_{tJ} = \sum_{\langle i,j \rangle} \left[ -t \sum_{\sigma=\uparrow,\downarrow} \mathcal{P} (c_{i\sigma}^{\dagger} c_{j\sigma} + c_{j\sigma}^{\dagger} c_{i\sigma}) \mathcal{P} + J \left( S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z - \frac{1}{4} n_i n_j \right) \right],
$$

where $n$'s are the number operators given by $n_i = n_{i\uparrow} + n_{i\downarrow} = c_{i\uparrow}^{\dagger} c_{i\uparrow} + c_{i\downarrow}^{\dagger} c_{i\downarrow}$ and the spin operators are $S_i^k = \frac{1}{2} \sum_{\alpha,\beta} \tau_{i\alpha}^k \tau_{i\beta}^\alpha c_{i\beta}^{\dagger} c_{i\alpha}$ with the usual Pauli matrices $\sigma$'s. The Gutzwiller projector $\mathcal{P} = \prod_{j=1}^{L} (1 - n_{j\uparrow} n_{j\downarrow})$ restricts the Hilbert space by forbidding double occupancies hence represents strong correlation. In one dimension, the exact solution was obtained for the supersymmetric case ($2t = J$) [3–8] using the Bethe-ansatz method. The long-distance behavior of the correlation functions was also investigated by applying the finite-size scaling analysis in the conformal field theory to the excitation spectra obtained by the Bethe-ansatz method [9]. It was shown that the superconducting correlation does not exceed the others such as the spin density wave (SDW) and the charge density wave (CDW) correlations at any filling. However, the region where the superconducting correlation is dominant was found between the low-density supersymmetric region and the “phase-separated” region ($J \gg t$) by numerical diagonalization of finite clusters [10]. Since the numerical results are not totally reliable due to the finite-size effect, it is highly desirable to have exact results.

In this paper we study a one-parameter family of the one-dimensional correlated electron systems which includes the ordinary supersymmetric $t$-$J$ model. The Hamiltonian is

$$
\mathcal{H}_{tJ} = \sum_i \left[ - \sum_{\sigma} \mathcal{P} (c_{i\sigma}^{\dagger} c_{i+1\sigma} + c_{i+1\sigma}^{\dagger} c_{i\sigma}) \mathcal{P} + 2 \left( S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z - \frac{\Delta}{4} n_i n_{i+1} \right) \right] + \eta \left( S_i^z n_{i+1} - n_i S_{i+1}^z \right) + 2 \Delta n_i,
$$

(1)
where $\Delta^2 - \eta^2 = 1$ and we parameterize them as $\Delta = \cosh \gamma$ and $\eta = \sinh \gamma \ (\gamma \in \mathbb{R}_{\geq 0})$. When $\gamma = 0$, our Hamiltonian reduces to the ordinary supersymmetric $t$-$J$ model. When the number of electrons coincides with the number of lattice sites, our model becomes $s = \frac{1}{2}$ XXZ spin chain. The Hamiltonian (1) commutes with the transfer matrix of the solvable two-dimensional classical lattice model associated with the supersymmetric quantum affine superalgebra $U_q(\hat{sl}(2|1))$ ($q = e^\gamma$), which is a special case of the model given by Perk and Schultz [12,13]. Thus one could say that the Hamiltonian is “$q$-supersymmetric”. The ground state, excitations and the correlation functions are obtained by the nested algebraic Bethe-ansatz method, the finite-size scaling analysis in the conformal field theory and the numerical diagonalization for small clusters. We find spin gap and observe that the superconducting correlation dominates in the low-density region in contrast to the ordinary supersymmetric $t$-$J$ model.

The third term of the Hamiltonian (1) breaks the parity invariance. The role of this term was investigated by diagonalizing the system without it numerically. It turned out that the spin gap survives in some cases.

1. Bethe-Ansatz Equations (BAEs). The diagonalization of the Hamiltonian (1) with periodic boundary condition reduces to solving the coupled algebraic equations (BAEs) derived by the nested algebraic Bethe-ansatz technique. They are

\[
\left(\frac{\sin(p_j + \frac{i\gamma}{2})}{\sin(p_j - \frac{i\gamma}{2})}\right)^L = (-1)^N \prod_{\beta=1}^{M} \frac{\sin(p_j - \Lambda_{\beta} + \frac{i\gamma}{2})}{\sin(p_j - \Lambda_{\beta} - \frac{i\gamma}{2})} \quad j = 1, 2, \cdots, N, \tag{2}
\]

\[
\prod_{j=1}^{N} \frac{\sin(\Lambda_{\alpha} - p_j + \frac{i\gamma}{2})}{\sin(\Lambda_{\alpha} - p_j - \frac{i\gamma}{2})} = -\prod_{\beta=1}^{M} \frac{\sin(\Lambda_{\alpha} - \Lambda_{\beta} + i\gamma)}{\sin(\Lambda_{\alpha} - \Lambda_{\beta} - i\gamma)} \quad \alpha = 1, 2, \cdots, M, \tag{3}
\]

where $L$ is the number of lattice sites, $N$ is the number of electrons, $M$ is the number of down-electrons (magnons), $p$’s are the quasi-momenta of electrons and $\Lambda$’s are the magnon rapidities [14].

Hereafter we take the following ansatz for the ground state and the elementary excitations: $\Lambda$’s are one-strings $\{\Lambda_{\alpha} \in \mathbb{R} | \alpha = 1, \cdots, M\}$ and $p$’s consist of the one-strings
\{p_j = u_j \in \mathbb{R}| j = 1, \cdots, N - 2M\} \text{ and the two-strings } \{p_\alpha^\pm = \Lambda_\alpha \pm i\gamma/2 | \alpha = 1, \cdots, M\}. \text{ Note that the real parts of the two-strings coincide with the magnon rapidities and, if } M = N/2, \text{ there is no degrees of freedom for one strings of the quasi-momenta. This is essentially the same ansatz established in Refs. [3–5]. In our case, however, } \Lambda's \text{ and } u's \text{ are in the interval } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \text{ due to the periodicity of the BAEs (2), (3). By taking the logarithm of the BAEs, we have}

\begin{align*}
L \phi(u_j, \frac{\gamma}{2}) &= 2\pi i I_j + \sum_{\beta=1}^{M} \phi(u_j - \Lambda_\beta, \frac{\gamma}{2}) \\
L \phi(\Lambda_\alpha, \gamma) &= 2\pi i J_\alpha + \sum_{j=1}^{N-2M} \phi(\Lambda_\alpha - u_j, \frac{\gamma}{2}) + \sum_{\beta=1}^{M} \phi(\Lambda_\alpha - \Lambda_\beta, \gamma) \\
\alpha &= 1, \cdots, M, \tag{4}
\end{align*}

where \(\phi(z, \alpha) \equiv \log \frac{\sin(z+i\alpha)}{\sin(z-i\alpha)}\) (see [13]), and \(\{ I_j | j = 1, 2, \cdots, N - 2M\}\) is a set of integers (or half-odd integers) if \(M\) is even (or odd) and the set \(\{ J_\alpha | \alpha = 1, 2, \cdots, M\}\) is a set of integers (or half-odd integers) if \(N + M + 1\) is even (or odd). We order the quantum numbers \(I's\) and \(J's\) according to \(I_j > I_{j+1}\) and \(J_\alpha > J_{\alpha+1}\).

In the thermodynamic limit \(L, N \to \infty\), the distributions of \(\Lambda's\) can be described by the continuous density given by \(L \rho(\Lambda_\alpha) = \lim_{L,N \to \infty} 1/(\Lambda_{\alpha+1} - \Lambda_\alpha)\). The energy and momentum up to \(O(1)\) are

\begin{align*}
E &= i \sinh \gamma \sum_{j=1}^{N} \phi'(p_j, \frac{\gamma}{2}) = i \sinh \gamma \left( L \int d\Lambda' \rho(\Lambda') \phi'(\Lambda', \gamma) + \sum_{j=1}^{N-2M} \phi'(u_j, \frac{\gamma}{2}) \right), \\
\text{and}
\end{align*}

\begin{align*}
P &= i \sum_{j=1}^{N} \phi(p_j, \frac{\gamma}{2}) = i L \int d\Lambda' \rho(\Lambda') \phi(\Lambda', \gamma) + i \sum_{j=1}^{N-2M} \phi(u_j, \frac{\gamma}{2}). \tag{7}
\end{align*}

The integral intervals in (6) and (7) will be discussed in the following sections.

2. Ground State. We set \(N\) to be even for simplicity. Since the spin interaction is antiferromagnetic, the total \(S^z\) for the ground state is expected to be zero. This can be achieved by setting \(M = N/2\), i.e. for the sector without one-strings \(p_j = u_j\). We also require that the momentum \(P\) to be zero. We propose the following ansatz for \(J's\): the distributions
of \( J \)'s for the ground state is restricted as \( J_{\text{max}} \geq |J_\alpha| \geq J_{\text{min}} \), where \( J_{\text{max}} = \frac{L - M - 1}{2} \) and \( J_{\text{min}} = \frac{L - 2M + 1}{2} \).

In the thermodynamic limit, we assume that \( \Lambda \)'s are distributed only in the regions \([-\pi/2, -Q_g] \) and \([Q_g, \pi/2] \) in accordance with the distribution of the quantum numbers \( J \)'s. BAEs (4) and (5) are reduced to

\[
2\pi i \rho_g(\Lambda) = -\phi'(\Lambda, \gamma) + \left[ \int_{-\pi/2}^{-Q_g} + \int_{Q_g}^{\pi/2} \right] d\Lambda' \rho_g(\Lambda') \phi'(\Lambda - \Lambda', \gamma),
\]

where \( Q_g \) is determined by \( \left[ \int_{-\pi/2}^{-Q_g} + \int_{Q_g}^{\pi/2} \right] d\Lambda \rho_g(\Lambda) = \frac{N/2}{L} \equiv \frac{n}{2} \). The ground-state energy \( E_g \) is given by

\[
E_g = i L \sinh \gamma \left[ \int_{-\pi/2}^{-Q_g} + \int_{Q_g}^{\pi/2} \right] d\Lambda \rho_g(\Lambda) \phi'(\Lambda, \gamma).
\]

These equations can be solved numerically for arbitrary filling \( n \). The results are given in Fig. 1.

3. Charge Excitations. The charge excitations are those caused by the replacements of the \( \Lambda \)'s while keeping \( M = N/2 \) namely \( S^z \) remains to be zero. Thus the elementary excitations for the charge sector consists in making a jump (hole) at the point \( J_{\alpha h} \) and putting a quantum number \( J_{\alpha p} \) at a previously unoccupied region. In the thermodynamic limit, BAEs (4) and (5) are reduced to

\[
2\pi i \rho_c(\Lambda) = -\phi'(\Lambda, \gamma) - \frac{2\pi i}{L} \delta(\Lambda - \Lambda_h) + \frac{1}{L} \phi'(\Lambda - \Lambda_p, \gamma) + \left[ \int_{-\pi/2}^{-Q_c} + \int_{Q_c}^{\pi/2} \right] d\Lambda' \rho_c(\Lambda') \phi'(\Lambda - \Lambda', \gamma),
\]

retaining terms up to \( O(L^{-1}) \), where \( \Lambda_p \) and \( \Lambda_h \) denote the position of the hole and particle in the sea of two strings associated with the quantum number \( J_{\alpha h} \) and \( J_{\alpha p} \) respectively. \( Q_c \) is determined by \( \left[ \int_{-\pi/2}^{-Q_c} + \int_{Q_c}^{\pi/2} \right] d\Lambda \rho_c(\Lambda) = \frac{(N-2)/2}{L} \). For convenience, we decompose \( \rho_c(\Lambda) \) into the regular part and the singular part as \( \rho_c(\Lambda) = \rho_{c0}(\Lambda) - \frac{1}{L} \rho_{c1}(\Lambda) - \frac{1}{L} \delta(\Lambda - \Lambda_h) \), where \( \rho_{c0}(\Lambda) \) satisfies

\[
2\pi i \rho_{c0}(\Lambda) = -\phi'(\Lambda, \gamma) + \left[ \int_{-\pi/2}^{-Q_c} + \int_{Q_c}^{\pi/2} \right] d\Lambda' \rho_{c0}(\Lambda') \phi'(\Lambda - \Lambda', \gamma).
\]
For $\rho_{c1}(\Lambda)$, we have

$$2\pi i \rho_{c1}(\Lambda) = \phi'(\Lambda - \Lambda_h, \gamma) - \phi'(\Lambda - \Lambda_p, \gamma) + \left[ \int_{-\pi/2}^{-Q_c} + \int_{Q_c}^{\pi/2} \right] d\Lambda' \rho_{c1}(\Lambda') \phi'(\Lambda - \Lambda', \gamma)$$  \hspace{1cm} (12)

The excitation energy $\Delta E$ from the ground state and the momentum $P$ are given by

$$\Delta E = i \sinh \gamma \left( \phi'(\Lambda_p, \gamma) - \phi'(\Lambda_h, \gamma) - \left[ \int_{-\pi/2}^{-Q_c} + \int_{Q_c}^{\pi/2} \right] d\Lambda \rho_{c1}(\Lambda) \phi'(\Lambda, \gamma) \right),$$  \hspace{1cm} (13)

$$P = i \left( \phi(\Lambda_p, \gamma) - \phi(\Lambda_h, \gamma) - \left[ \int_{-\pi/2}^{-Q_c} + \int_{Q_c}^{\pi/2} \right] d\Lambda \rho_{c1}(\Lambda) \phi(\Lambda, \gamma) \right).$$  \hspace{1cm} (14)

Solving (12) numerically, the dispersion relation for the elementary charge excitations was obtained. The result for $\gamma = 2$ and $n = 0.45$ is shown in Fig. 2. The results for other parameters do not change in an essential manner, namely the charge excitation is always gapless.

4. Spin Excitations. The spin excitations can be considered as excitations coming from destroying the two-strings $p^{\pm}$'s and creating one-strings $u$'s. To study the elementary ones, let us consider the case of $M = N/2 - 1$ magnons. We assume that, in the sea of the quantum numbers $J$'s there are no jumps \[7,8\]. Then, in the thermodynamic limit, BAE (\[\]) becomes

$$2\pi i \rho_s(\Lambda) = -\phi'(\Lambda, \gamma) + \frac{1}{L} \phi'(\Lambda - u_1, \frac{\gamma}{2}) + \frac{1}{L} \phi'(\Lambda - u_2, \frac{\gamma}{2}) + \left[ \int_{-Q_s}^{-\pi/2} + \int_{Q_s}^{\pi/2} \right] d\Lambda' \rho_s(\Lambda') \phi'(\Lambda - \Lambda', \gamma),$$  \hspace{1cm} (15)

where $u_1$ and $u_2$ are one-string quasi-momenta, and $Q_s$ is determined by $\left[ \int_{-\pi/2}^{\pi/2} d\Lambda \rho_s(\Lambda) \right] = \frac{(N-2)/2}{L}$. It is convenient to decompose $\rho_s(\Lambda)$ into contributions of order $O(1)$ and $O(L^{-1})$ as $\rho_s(\Lambda) = \rho_{s0}(\Lambda) - \frac{1}{L} \rho_{s1}(\Lambda)$, where $\rho_{s0}(\Lambda)$ satisfies the same equation as (\[\]) obtained by replacing all the suffices $c$ to $s$. Then the integral equation for $\rho_{s1}(\Lambda)$ is obtained as

$$2\pi i \rho_{s1}(\Lambda) = -\phi'(\Lambda - u_1, \frac{\gamma}{2}) - \phi'(\Lambda - u_2, \frac{\gamma}{2}) + \left[ \int_{-\pi/2}^{-Q_s} + \int_{Q_s}^{\pi/2} \right] d\Lambda' \rho_{s1}(\Lambda') \phi'(\Lambda - \Lambda', \gamma),$$  \hspace{1cm} (16)
The excitation energy $\Delta E$ from the ground state and the momentum $P$ are

$$\Delta E = i \sinh \gamma \left( \phi'(u_1, \frac{\gamma}{2}) + \phi'(u_2, \frac{\gamma}{2}) - \left[ \int_{-Q_s}^{-\pi/2} + \int_{\pi/2}^{Q_s} \right] d\Lambda \rho_{s_1}(\Lambda) \phi'(\Lambda, \gamma) \right) \quad (17)$$

$$P = i \left( \phi(u_1, \frac{\gamma}{2}) + \phi(u_2, \frac{\gamma}{2}) - \left[ \int_{-Q_s}^{-\pi/2} + \int_{\pi/2}^{Q_s} \right] d\Lambda \rho_{s_1}(\Lambda) \phi(\Lambda, \gamma) \right) \quad (18)$$

Solving (16) numerically, the dispersion relation for the elementary spin excitations was obtained. The result for $\gamma = 2$ and $n = 0.45$ is shown in Fig. 3. The results for other parameters do not change in an essential manner, namely the spin excitation is always gapful. The spin gap as a function of $\gamma$ is also shown in Fig. 4. and one can see that the gap increases as holes are doped.

To study the effect of the parity violating term, we calculated the spin-spin correlations $\langle S_i^z S_j^z \rangle$ for the parity-unbroken Hamiltonian $H_{tJ} - \sum_i \eta(S_i^z n_{i+1} - n_i S_{i+1}^z)$ [17]. The results are shown in Fig. 5 and they indicate that the correlation decays exponentially. Hence there is still excitation gap.

5. Correlation Functions. Consider a field-theoretic description of the low-lying excitations. Since the dispersion for the low energy charge sector is approximately linear for $0 < n < 1$, and the gapful spin sector is irrelevant for the low-energy behavior, we can expect the system can be described by the conformal field theory [18].

Let us consider the excitations described by the density $\rho(\Lambda)$ satisfying

$$2\pi i \rho(\Lambda) = -\phi'(\Lambda, \gamma) + \left[ \int_{-\pi/2}^{Q_+} + \int_{Q_+}^{\pi/2} \right] d\Lambda' \rho(\Lambda') \phi'(\Lambda - \Lambda', \gamma), \quad (19)$$

and apply the general method of Kawakami-Yang [8] for the finite-size scaling method [19,20]. Using the Fourier-transform technique, we rewrite (19) as

$$\rho(\Lambda) = 2R_q(2\Lambda) + \int_{Q_-}^{Q_+} d\Lambda' 2R_q(2(\Lambda - \Lambda')) \rho(\Lambda'), \quad (20)$$

where we have introduced the deformed Shiba-function [22] defined by $R_q(v) = \frac{1}{2\pi} \sum_{m \in \mathbb{Z}} \frac{e^{imv}}{1+q^2|m|^2}$. The energy is given by
\[ E/L = 2 \cosh \gamma - 2\pi \sinh \gamma \left[ 2R_q(0) + \int_{Q_+}^{Q_-} d\Lambda \ 2R_q(-2\Lambda) \rho(\Lambda) \right]. \] (21)

Thanks to (20) and (21), we can immediately apply the general argument and the results are: i) the charge sector can be described by the \( c = 1 \) bosonic conformal field theory, i.e. it belongs to the universality class called the Tomonaga-Luttinger liquid, ii) the compactification radius \([21]\) is given by \( r = \xi(Q) \), where the dressed charge \( \xi(\Lambda) \) satisfies \( \xi(\Lambda) = 1 + \int_{Q_+}^Q d\Lambda' 2R_q(2(\Lambda-\Lambda')) \xi(\eta) \), and \( Q \) is determined by \( \left[ \int_{-\pi/2}^{-Q} + \int_{Q}^{\pi/2} \right] d\Lambda \rho(\Lambda) = \frac{N/2}{L} \).

As usual, we parameterize \( r \) by \( K_\rho = r^2/2 \). The equation for \( \xi \) was solved numerically and \( K_\rho \) as functions of \( n \) are shown in Fig. 6. The relations between \( K_\rho \) and the critical exponents are shown in Table I \([23,24]\). As long as \( \gamma \neq 0 \), the singlet-superconducting correlation is dominant when \( K_\rho > 1 \) in the low density region (high doping). However, for the usual supersymmetric case (\( \gamma = 0 \)), the superconducting correlation can not be dominant in any filling \([3]\) as seen in Fig. 6. This indicates that deformed \( t-J \) models including ours may be more appropriate to study superconducting mechanisms than the ordinary \( t-J \) model.

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\[ H_{XY} = \sum_{i} \left[ - \sum_{\sigma} \mathcal{P} (c_{i\sigma}^\dagger c_{i+1\sigma} + c_{i+1\sigma}^\dagger c_{i\sigma}) \mathcal{P} + 2 (S_{i}^x S_{i+1}^x + S_{i}^y S_{i+1}^y) - (e^{\gamma} n_{i \uparrow} n_{i+1 \downarrow} + e^{-\gamma} n_{i \downarrow} n_{i+1 \uparrow}) \right]. \]

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\[ \text{Im} \, \phi(x, \alpha) \text{ is a continuous monotonic decreasing function in } -\pi/2 < x < \pi/2. \]
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FIGURES

Fig. 1. Ground state energy per site as a function of electron density $n$. The solid lines are obtained by solving (8). We also performed direct numerical diagonalizations of the Hamiltonian for $L = 14$ with $\gamma = 2.0, 1.0$ and 0.5. The results are plotted by □, + and ◦ respectively.

Fig. 2. Dispersion of the elementary excitations in the charge sector for $\gamma = 2$ and $n = 0.45$. Sufficiently many points in the continuous spectrum are shown. The momentum $P$ is periodic with period $2\pi$. The gapless points are at $P = 0$, $2k_F$ and $2\pi - 2k_F$.

Fig. 3. Dispersion of the elementary excitations in the spin sector for $\gamma = 2$ and $n = 0.45$. Sufficiently many points in the continuous spectrum are shown. The momentum $P$ is periodic with period $2\pi$. There is no gapless point. Note that the momentum $P$ for the elementary spin excitation is restricted in $[-P_m, P_m]$, where $P_m$ depends on $\gamma$ and $n$.

Fig. 4. Spin gap as a function of $\gamma$. For $n = 1$, the spin gap of our model reduces to that of the XXZ spin chain (see Eq.(1)) [16].

Fig. 5. Spin-spin correlations on a logarithmic scale for the parity-unbroken Hamiltonian $H_{t,J} - \sum_i \eta (S_i^+ n_{i+1} - n_i S_{i+1}^z)$. The results are obtained by numerical diagonalization of $L = 12$ systems for (a) $N = 10$, $\gamma = 1.5$, (b) $N = 8$, $\gamma = 1.5$ and (c) $N = 2$, $\gamma = 2.0$.

Fig. 6 $K_\rho(n)$’s are shown for $\gamma = 0.0$, 0.5, 1.0 and 2.0. The broken line (for $\gamma = 0$ i.e. the ordinary supersymmetric case) denotes the data from Ref. 8. It can be shown analytically that $K_\rho(0) = 2$ and $K_\rho(1) = 1/2$ for any $\gamma > 0$. 
TABLES

TABLE I. Relation between $K_\rho$ and the critical exponents of the correlation functions.

| Correlations                        | exponents          |
|-------------------------------------|--------------------|
| $2k_F$ SDW (spin density wave)      | exponential decay  |
| $2k_F$ CDW (charge density wave)    | $K_\rho$           |
| SS (singlet superconductivity)      | $1/K_\rho$         |
| TS (triplet superconductivity)      | exponential decay  |
| $4k_F$ CDW (charge density wave)    | $4K_\rho$          |