Finite-State Markov Modeling of Tunnel Channels in Communication-based Train Control (CBTC) Systems

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Abstract—Communication-based train control (CBTC) is gradually adopted in urban rail transit systems, as it can significantly enhance railway network efficiency, safety and capacity. Since CBTC systems are mostly deployed in underground tunnels and trains move in high speed, building a train-ground wireless communication system for CBTC is a challenging task. Modeling the tunnel channels is very important to design and evaluate the performance of CBTC systems. Most of existing works on channel modeling do not consider the unique characteristics in CBTC systems, such as high mobility speed, deterministic moving direction, and accurate train location information. In this paper, we develop a finite state Markov channel (FSMC) model for tunnel channels in CBTC systems. The proposed FSMC model is based on real field CBTC channel measurements obtained from a business operating subway line. Unlike most existing channel models, which are not related to specific locations, the proposed FSMC channel model takes train locations into account to have a more accurate channel model. The distance between the transmitter and the receiver is divided into intervals, and an FSMC model is applied in each interval. The accuracy of the proposed FSMC model is illustrated by the simulation results generated from the model and the real field measurement results.

I. INSTRUCTION

Urban rail transit systems are developing rapidly around the world. Due to the huge urban traffic pressure, improving the efficiency and capacity of urban rail transit systems is increasingly in demand. Being a key subsystem in rail transit systems, communication-based train control (CBTC) is an automated train control system using bidirectional train-ground communications to ensure the safe operation of rail vehicles [1]. It can enhance the level of safety and service offered to customers and improve the utilization of railway network infrastructure. CBTC is a modern successor of a traditional railway signaling system using interlockings, track circuits, and signals [2].

Building a train-ground wireless communication system for CBTC is a challenging task. As urban rail transit systems are mostly deployed in underground tunnels, there are a large amount of reflections, scattering and barriers that severely affect the propagation performance of wireless communications. Moreover, due to the available commercial-off-the-shelf equipments, wireless local area networks (WLANs) are often adopted as the main method of train ground communications for CBTC systems [3]. However, most of the current IEEE 802.11 WLAN standards are not originally designed for the high speed environment in tunnels [3], [4]. Furthermore, the fast movement of trains will cause frequent handoffs between WLAN access points (APs), which could affect CBTC performance severely.

Modeling the channels of urban rail transit systems in tunnels is very important to design and evaluate the performance of CBTC systems. There are some previous works on radio wave propagation in tunnels [5], [6]. A path loss model of tunnels is given in [6], which describes the characteristics of the large scale fading. Authors of [7] illustrate a measurement method of 2.4 GHz in a subway tunnel, and the research object is a distributed antenna system, which is not often applied in CBTC systems. Due to the good balance between accuracy and complexity, finite state Markov channel (FSMC) model has been successfully used in different channels, including Rayleigh fading [8], Ricean fading [9] and Nakagami fading [10].

Although some excellent works have been done on modeling channels, most of them do not consider the unique characteristics in CBTC systems, such as high mobility speed, deterministic moving direction, and accurate train location information. In this paper, we develop a finite state Markov channel model for tunnel channels in CBTC systems. Some distinct features of the proposed channel model are as follows.

- The proposed FSMC model is based on real field CBTC channel measurements obtained from business operating Beijing Subway Changping line.
- Unlike most existing channel models, which are not related to specific locations, the proposed FSMC channel model takes train locations into account to have a more accurate channel model.
- The distance between the transmitter and the receiver is divided into intervals, and an FSMC model is applied in each interval.
- Lloyd-Max technique [11] is used to determine the SNR level boundaries in the proposed FSMC model.
- The accuracy of the proposed FSMC model is illustrated by the simulation results generated from the model and the real field measurement results. The effects of different
parameters are also discussed.

The rest of this paper is organized as follows. Section II describes an overview of CBTC systems. In Section III the real field measurement configuration and scenario are described. In Section IV the FSMC model is introduced. Then, Section V presents the real field measurement results and discussions. Finally, the paper is concluded in Section VI with future work.

II. OVERVIEW OF COMMUNICATION-BASED TRAIN CONTROL

In CBTC systems, continuous bidirectional wireless communications between each mobile station (MS) on the train and the wayside access point (AP) are adopted instead of the traditional fixed-block track circuit. The railway line is usually divided into areas or regions. Each area is under the control of a zone controller (ZC) and has its own radio transmission system. Each train transmits its identity, location, direction and speed to the ZC. The radio link between each train and the ZC should be continuous so that the ZC knows the locations of all the trains in its area at all the time in order to guarantee train operation safety and efficiency.

Wireless channels in CBTC systems are different from those in other wireless systems, since most CBTC systems are deployed in underground tunnels, where there are a large amount of reflections, scattering and barriers that severely affect the propagation performance of wireless communications. In order to design and evaluate the performance of CBTC systems, modeling of tunnel channels in CBTC systems should be carefully studied.

III. REAL FIELD CBTC CHANNEL MEASUREMENTS

The objective of the real field CBTC channel measurements is to get the real field data of WLAN propagation in tunnels under real conditions of the subway line, which can be used to build an FSMC model.

A. Measurement Equipment

Two sets of Cisco 3200 are used, and one is set as AP while the other one is set as the mobile station (MS). Both of them are set to work at the frequency of 2.412GHz, which is also called channel 1. The output power of the AP is set as 30dBm. The AP is located on the wall of the tunnel, while the MS is located on the measurement vehicle. The transmitting antenna is a Yagi antenna connected with the AP, which is directional and vertically polarized. The half power beam width (HPBW) is 30° and the gain of Yagi antenna is 13.5dBi. In addition, the Shark-fin antenna is applied as the receiving antenna connected with the MS, which is also directional and vertically polarized. The HPBW is 40° and the gain of Shark-fin antenna is 10dBi. The measurement configuration settings are shown in Table I

| Frequency | 2.412GHz |
|-----------|----------|
| Transmitting Power | 30dBm |

| Transmitting Antenna | Yagi Antenna |
|----------------------|--------------|
| Type | Yagi Antenna |
| Polarization Direction | Vertical |
| Gain | 13.5dBi |
| HPBW | 30° |

| Receiving Antenna | Shark-fin Antenna |
|-------------------|-------------------|
| Type | Shark-fin Antenna |
| Polarization Direction | Vertical |
| Gain | 10dBi |
| HPBW | 40° |

The location of the train is obtained through a velocity sensor installed on the wheel of the measurement vehicle, which can detect the realtime velocity, and the resolution of position is millimeter per second. When the measurement vehicle is moving, the velocity sensor gets the speed transmitted to the singlechip computer immediately through a serial port. At the same time, the MS captures the signal strength and SNR at the current position, and the signal information together with the integrated displacement data by the singlechip computer can be stored in the laptop. Therefore, the signal strength and SNR mapping with the location of receiver can be obtained, which is useful to build an FSMC model depending on the distance between the transmitter and the receiver.

B. Measurement Scenario

The measurement was performed in the straight section of tunnels in Beijing Subway Changping Line, and the cross section of tunnel is rectangular. The height of the tunnel is 4.91m and the width is 4.4m. The transmitting antenna is located 0.15m below the tunnel roof, which is 4.76m. The receiving antenna is set on the top of an iron bar, which is 3.8m and also the height of the top of the train. As the threshold of the receiver is −90dBm, the coverage of one AP is about 0m-500m, which is also the experimental zone in our measurements. The tunnel where we performed the measurement is a section of straight tunnel, and Fig. I shows the cross section of tunnel in Changping Subway Line, the Shark-fin antenna, the Yagi antennas and the AP set on the wall.

IV. THE FINITE-STATE MARKOV CHAIN CHANNEL MODEL

To capture the characteristics of tunnel channels in CBTC systems, we define channel states according to the different received SNR levels, and use an FSMC to track the state variation. In this section, we first describe the FSMC model, followed by the determination of key model parameters, including SNR levels and SNR distribution.

A. The Finite State Markov Channel Model

Let $\Gamma$ denote the SNR of the received signal, whose range can be obtained from the experimental data. The range of SNR is partitioned into $N$ non-overlapping levels with thresholds $\{\Gamma_n, n = 1, 2, 3, ..., N + 1\}$. Let $S = \{s_1, s_2, ..., s_n\}$ denote the finite channel states, and the channel state is $s_n$ when the SNR of the received signal belongs to the range $(\Gamma_n, \Gamma_{n+1})$. Then $\{S_n\}$ is a Markov process and the transition probability

\begin{align}
\mathbb{P}(S_n = s_n | S_{n-1} = s_{n-1}) &= \begin{cases} 
\theta & \text{if } s_n = s_{n-1} \\
(1-\theta) & \text{if } s_n \neq s_{n-1}
\end{cases}
\end{align}

\[\theta = \frac{1}{N}, \quad (1-\theta) = \frac{N-1}{N}.\]
we can define a $k = 1, 2, 3, \ldots$ and $n, j \in \{1, 2, \ldots, N + 1\}$.

According to the property of first-order Markov chain, we assume that each state can only transit to the adjacent states, which means $p_{n,j} = 0$, if $| n - j | > 1$. With the definition, we can define a $K \times K$ state transition probability matrix $P$ with elements $p_{n,j}$.

Due to the effect of large scale fading, the amplitude of SNR depends on the distance between the transmitter and the receiver. It is obvious that the SNR is usually higher when the receiver is close to the transmitter; while it is lower when the receiver is far away from the transmitter. As a result, the transition probability from the high received SNR state to the low received SNR state is different when the receiver is near or far away from the transmitter, which means that the Markov state transition probability is related to the location of the receiver. Therefore, only one state transition probability matrix, which is independent of the location of the receiver, may not generate accurate enough models to describe the tunnel channels. Thus, we divide the tunnel into $L$ intervals and one state transition probability matrix is generated for each interval. Specifically, $P, l \in \{1, 2, \ldots, L\}$, is the state transition probability matrix corresponding to the $l$th interval, and the relationship between the transition probability and the location of the receiver can be built. Then, $p_{n,j}^l$ is the state transition probability from state $s_n$ to state $s_j$ in the $l$th interval.

Based on the measurement results, we need to determine the value of the state probability $p_{n}^l$ and the state transition probability $p_{n,j}^l$.

\[ p_{n,j} = P\{S_{k+1} = s_n \mid S_k = s_j\}, \quad (1) \]

where $k = 1, 2, 3, \ldots$ and $n, j \in \{1, 2, \ldots, N + 1\}$.

B. Determine the SNR Level Thresholds of the FSMC Model

As mentioned above, getting the thresholds of SNR levels is the key factor that affects the accuracy of the FSMC model. There are many methods to select the SNR level boundaries, and the equiprobable partition method is frequently used in previous works [8]–[10]. As nonuniform amplitude partitioning may be useful to obtain more accurate estimates of system performance measures [12], we choose the Lloyd-Max technique [11] instead of the equiprobable method to partition the amplitude of SNR in this paper. Lloyd-Max is an optimized quantizer, which can decrease the distortion of scalar quantization. Lloyd-Max can realize uniform scalar quantization and non-uniform scalar quantization, and the latter one is used in this paper to divide the amplitude range of SNR.

Firstly, a distortion function $D$ is defined as follows.

\[ D = \sum_{k=2}^{N+1} \int_{x_{k-1}}^{x_k} f(\tilde{x}_k - x)p(x)dx, \quad (2) \]

where $x_k$ is the threshold of the $k$th SNR level, $f(x)$ is the error criterion function, and $p(x)$ is the probability distribution function of SNR.

The error criterion function $f(x)$ is often taken as $x^2$ [13]. As a result, Then, the necessary conditions for minimum distortion are obtained by differentiating $D$ with respect to $x_k$ and $\tilde{x}_k$ as follows.

\[ x_k = \frac{\tilde{x}_k + \tilde{x}_{k+1}}{2}, \quad (3) \]

\[ \int_{x_{k-1}}^{x_k} (\tilde{x}_k - x)p(x)dx = 0. \quad (4) \]

Therefore, all elements of $\Gamma_n$ can be obtained according to (4). Combined with (3), the value of $\Gamma_n$ can be updated until the value of $D$ is the minimum, and the optimal thresholds of the SNR levels can be got. As $p(x)$ is still not determined, we should discuss the distribution of SNR according to the experimental sampling data, which is the last step to obtain the thresholds of SNR regions.

C. Determine the Distribution of SNR

Deriving the distribution of SNR is the crucial step of partitioning the levels of SNR. In fact, there are some classic models to describe the distribution of signal strength, such as Rice, Rayleigh and Nakagami, and then the corresponding models of SNR can also be obtained [14]. We firstly obtain the distribution of the signal strength in order to determine the model of SNR.

The Akaike information criterion (AIC) is adopted in this paper to get the approximate distribution model of the signal strength. The AIC is a measure of the relative goodness of fit of a statistical model. The general case of AIC is [15]

\[ AIC = -2 \ln L + 2\eta, \quad (5) \]

where $\eta$ is the number of parameters in the statistical model, and $L$ is the maximized value of the likelihood function for the estimated model. In fact, according to the relationship of $\eta$ and the number of samples $n$, AIC needs to be changed to
Akaike information criterion with a correction (AICc) when \( n/\eta < 40 \) [15].

\[
AICc = AIC + \frac{2\eta(\eta + 1)}{n - \eta - 1}. \tag{6}
\]

AICc is adopted to estimate the model of the signal strength distribution instead of the classic AIC in the paper. In practice, one can compute AICc for each of the candidate models and select the model with the smallest value of AICc. The candidate models include Rice, Rayleigh, and Nakagami in the paper.

Our model is related to the distance between the transmitter and receiver, and the tunnel should be divided into intervals. Thus, as mentioned above, we should divide the amplitude of SNR into several levels and firstly calculate the value of AICc for each model to determine the distribution function for each interval. Now we assume there are \( L\) intervals, and then we select the most appropriate model based on the frequency of the minimum AICc value of different candidate models. In order to obtain enough data for each interval, we set the length of each interval as 40 wavelengths of WLANs [16], and then there are 100 intervals. Fig. 2 shows the frequencies of AICc of different distributions. From Fig. 2 we can observe that the Nakagami distribution provides the best fit in a majority of the cases. As a result, we can define \( p(x) \) as the Nakagami distribution.

According to [14], we can obtain the distribution of SNR, after the distribution of the signal strength is obtained.

\[
p(x) = \frac{m^m x^{m-1}}{\bar{x}^m \Gamma(m)} e^{-\frac{m x}{\bar{x}}}, \tag{7}
\]

where \( x \) is the SNR data, \( \bar{x} \) is the mean of SNR, \( m \) is the fading factor of Nakagami distribution, and \( \Gamma(.) \) is the gamma function. In fact, \( m \) can be calculated when applying AICc through the maximum likelihood estimator for each interval.

Based on (3), (4) and (7), the thresholds \( \{\Gamma_n, n = 1, 2, ..., N + 1\} \) of SNR in each distance interval can be derived. Table II and Table III demonstrate the thresholds of SNR levels at the location of 100 m for different intervals, where we divide SNR into four and eight levels. As the distance intervals are different, the range of SNR is different and it brings different thresholds, which can provide one more accurate model.

### V. Real Field Measurement Results and Discussions

In this section, we compare our FSMC model with real field test results to illustrate the accuracy of the model. The effects of different parameters in the proposed model are discussed. The number of states in our model is first set as 4. We also use 8 states to study the effects of the number of states on the accuracy of the proposed model. In order to obtain the effects of distance intervals on the model, we choose the intervals as 5 m, 10 m, 20 m, 50 m, 100 m.

We perform the measurements in the tunnels of Beijing Subway Changping Line many times so that enough data can be captured. We verify the accuracy of the FSMC model through another set of measurement data. First of all, we get the statistical state transition probabilities. Table IV illustrates the state transition probabilities of the FSMC model and the measurement data at the same location (35 m - 40 m) when there are eight states, and the distance interval is 5 m. Fig. 3 shows the simulation results generated from our FSMC model and the experimental results from real field measurements. We can observe that there is greater agreement between them when the distance is 5 m than that with the 100 m distance interval. Next, we derive the Mean Square Error (MSE) to measure the degrees of approximation, shown in Fig. 4. With the distance interval increasing, the MSE does also increase, which means the accuracy decreases. Moreover, it is obvious that the MSE of the FSMC model with 4 states is larger than that with 8 states. The number of states in the FSMC model plays a key role.
TABLE IV
THE VALUES OF TRANSITION PROBABILITIES FOR THE FSMC MODEL WITH 8 STATES AND 5m INTERVAL

| k   | P_{k,k-1} | P_{k,k} | P_{k,k+1} | P_{k,k-1} | P_{k,k} | P_{k,k+1} |
|-----|-----------|---------|-----------|-----------|---------|-----------|
| 1   | -         | 0.75    | 0.25      | -         | 0.78    | 0.22      |
| 2   | 0.25      | 0.5     | 0.25      | 0.269     | 0.47    | 0.26      |
| 3   | 0.25      | 0.5     | 0.25      | 0.23      | 0.5     | 0.26      |
| 4   | 0.72      | 0.66    | 0.11      | 0.22      | 0.65    | 0.12      |
| 5   | 0.125     | 0.5     | 0.25      | 0.126     | 0.64    | 0.24      |
| 6   | 0.095     | 0.81    | 0.048     | 0.089     | 0.86    | 0.049     |
| 7   | 0.13      | 0.6     | 0.27      | 0.12      | 0.61    | 0.26      |
| 8   | 0.013     | 0.98    | -         | 0.013     | 0.98    | -         |

VI. CONCLUSIONS AND FUTURE WORK

Modeling the tunnel wireless channels of urban rail transit systems is important in designing and evaluating the performance of CBTC systems. In this paper, we have proposed an FSMC model for tunnel channels in CBTC systems. Since the train location is known in CBTC systems, the proposed FSMC channel model takes train locations into account to have a more accurate channel model. The distance between the transmitter and the receiver is divided into intervals, and an FSMC model is designed in each interval. The accuracy of the proposed model has been illustrated by the simulation results generated from the proposed model and the real field measurement. In addition, we have shown that the number of states and the distance interval have impacts on the accuracy of the proposed FSMC model. Future work is in progress to study the effects of wireless channels on the control performance of CBTC systems based on the proposed channel model.

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