PRIMORDIAL GRAVITATIONAL WAVES:
A PROBE OF THE VERY EARLY UNIVERSE

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Summary

We discuss the potential cosmological role of gravitational wave astronomy as a probe of the very early universe. The next generation of detectors—now in production—may be able to observe a stochastic background of gravitational waves produced by violent processes during the earliest moments after the creation of the universe. Viable theoretical scenarios within detector sensitivity include strongly first-order phase transitions, possibly at the end of inflation, and networks of cosmic strings. At this stage, other primordial backgrounds from slow-roll inflation, global topological defects and the standard electroweak phase transition appear to be out of range. The discovery of any of these possible cosmological sources will have enormous implications for our understanding of the very early universe and for fundamental physics at the highest energies.

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The discovery of the cosmic microwave background radiation was a watershed in modern cosmology. The more recent observations of fluctuations in this background provide us with a detailed snapshot of the universe at about 400,000 years after the Hot Big Bang, just as the universe became transparent to electromagnetic radiation. The scientific impact of the discovery of the CMBR is difficult to overestimate, yet it serves merely as a useful foil in the context of this essay. Here, we wish to discuss the prospect of gravitational wave astronomy providing snapshots, not from a few hundred thousand years, but from the very first fractions of a second after the creation of the universe. Gravitational radiation easily penetrates the electromagnetic surface of last scattering and propagates freely to the present day from its time of emission, which can be as early as the Planck epoch at $10^{-43}$ seconds (as illustrated in fig. 1).

The remarkable transparency of the universe to gravitons is due to their very weak coupling with ordinary matter which makes them extremely difficult to observe. In recent years, pioneering experiments have been proposed to directly detect gravitational waves using terrestrial and space-based laser interferometers\(^1\). These experiments have been primarily designed to search for bursts of radiation from astrophysical sources, such as black hole–black hole mergers, but they also have the potential to detect a stochastic background of gravitational waves produced by violent processes in the very early universe. Such background noise might at first be an annoyance for experimentalists, as it was for Penzias and Wilson, but its implications would be far-reaching because the processes responsible for its creation would undoubtedly involve physics beyond the standard model. Currently, a number of viable theoretical models produce detectable gravitational wave backgrounds, including first-order phase transitions, topological defects, and inflation (refer to fig. 2), but we must not preclude the strong possibility of the emergence of some fundamentally new physical insight; after all, serendipity and observational cosmology seem to be habitual associates. Nevertheless, we are not suggesting that this exciting, but somewhat speculative, cosmological scenario should override the solid astrophysical case which has already been made for the next generation of interferometers. Rather our aim is to emphasise, with some specific examples, the potential rewards of such a unique probe of the very early universe.

Stochastic backgrounds of gravitational waves are normally quantified by their relative spectral density $\Omega_g(f)$ given at a frequency $f$, that is, the energy density in gravitational radiation in an octave frequency bin centred on $f$ relative to the critical density of the universe.
This is directly related to the dimensionless wave amplitude \( h_c \propto \sqrt{\Omega_g/f} \) which is measured experimentally, except that we must allow for cosmological uncertainties by quoting sensitivities in terms of \( \Omega_g h^2 \), where ‘little \( h \)’ is the rescaled Hubble parameter \( 0.4 < h < 0.9 \). At present, there are four main frequency bands for studying gravitational radiation (each shown in fig. 2).

First, there is the tensor contribution to the microwave background anisotropies detected by the COBE-DMR experiment; this implies an upper bound of \( \Omega_g < 7 \times 10^{-11} \) at frequencies around \( 3 \times 10^{-17} h \) Hz. A second upper limit comes from pulsar observations, since a stochastic background would lead to timing noise in the incoming periodic signal; this imposes the tighter constraint, \( \Omega_g h^2 < 6 \times 10^{-8} \) at \( 4 \times 10^{-9} \)Hz. Thirdly, proposed ground-based interferometers

* It should be noted that the statistical veracity of this result has been questioned by recent re-analyses of the same data suggesting both stronger and weaker bounds. For the purposes of this essay we shall use the original
will study frequencies around 100Hz with a maximum sensitivity of about $\Omega_g h^2 \approx 10^{-7}$ for the first LIGO detector\(^6\) and $\Omega_g h^2 \approx 10^{-10}$ for the advanced LIGO and VIRGO\(^7\) detectors. Finally, the ambitious space-based interferometer LISA\(^8\) will have a sensitivity of $\Omega_g h^2 \approx 10^{-10}$ at about $10^{-3}$Hz.

Gravitational radiation produced by classical, causal processes in the early universe will have a characteristic frequency related to its time of emission since the causal horizon provides an upper limit on the wavelength. Figure 1 illustrates the implications of this for the LIGO and VIRGO detectors which remarkably probe times as early as $10^{-25}$ seconds, while LISA potentially looks back to $10^{-15}$ seconds.

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Figure 2: Summary of the potential cosmological sources of a stochastic gravitational radiation background, including inflationary models, first-order phase transitions and cosmic strings\(^9,10\), as well as a primordial 0.9K blackbody graviton spectrum (the analogue of the blackbody photon radiation). Also plotted are the relevant constraints from the COBE measurements, pulsar timings, and the sensitivities of the proposed interferometers. Notice that local cosmic strings and strongly first-order phase transitions may produce detectable backgrounds, in contrast to standard slow-roll inflation models.
First, however, we focus attention on the quantum creation of gravitational waves during an inflationary epoch. Inflation is a period of rapid expansion which occurs if the potential energy of a scalar field dominates the energy density of the early universe. Inflationary scenarios are popular because they resolve the well-known horizon, flatness and monopole problems of the standard cosmology. However, probably their most significant testable prediction is the quantum mechanical production of a nearly scale-invariant spectrum of adiabatic density perturbations. An important by-product of this process is the parametric amplification of tensor modes resulting in a background of gravitational waves.

The standard picture for inflation is that the scalar field rolls slowly down its potential as shown in fig. 3(a), finally ending in the re-heating of the universe when the remaining vacuum energy is converted into radiation. During this slow-roll regime the amplitude of the spectrum of gravitational waves produced is proportional to $H^2$, where $H^{-1}$ is the Hubble radius. After creation at about $10^{-35}$ seconds, a particular mode is driven outside the Hubble radius (or ‘horizon’) by the rapid expansion and it effectively ‘freezes’ until it returns inside the horizon during the subsequent radiation or matter dominated eras. This process is illustrated by the dashed ‘super-horizon’ evolution in fig. 1.

The important dynamical fact is that $H$ must decrease monotonically and therefore the largest contribution to the gravitational wave spectrum is due to modes that were driven outside the horizon early in inflation and have just come back inside the horizon at the present day. Fig. 1 shows that these are the frequencies probed by CMBR experiments and the observed anisotropies can be used to normalize $H$ early in inflation. Once this fact is known, the entire spectrum can be calculated using the dynamical equations for the scalar field. Assuming that
the entire COBE signal is due to tensor modes, a weak upper bound on the inflationary signal can be deduced, that is, \( \Omega_g h^2 < 10^{-14} \) for frequencies which came inside the horizon before equal matter-radiation. This is well below the quoted sensitivities of proposed detectors.

In fact, for any realistic slow-roll inflation model the situation is considerably worse and, paradoxically, the larger the CMBR tensor contribution, the lower the contribution to the frequencies relevant for direct detection; \( H \) must decrease more rapidly in models which have a large gravitational wave contribution in COBE frequencies\(^{11}\). The actual spectrum for two simple inflationary models is illustrated in fig. 2. A simple chaotic model has a negligible COBE signal but an almost flat spectrum with amplitude \( \Omega_g h^2 \approx 10^{-16} \) at a few Hz. In contrast, for a power law inflation model, almost all the COBE signal is due to gravitational waves but the spectrum falls very rapidly to an amplitude of only \( \Omega_g h^2 \approx 10^{-24} \) by the LISA frequency band.

While the prospects for detecting gravitational waves from standard slow-roll inflation seem poor, there are a number of alternative scenarios which can produce a detectable signal. These include speculative superstring-inspired models\(^{12}\) and also those which do not end in the standard re-heating scenario. Extended inflation\(^{13}\) and hybrid inflation\(^{14}\) exit through a phase transition which provides an extra, potentially more powerful source of gravitational waves if this final transition is strongly first-order\(^{15}\).

A simple potential for a first-order phase transition is illustrated in fig. 3(b); the field becomes trapped in a metastable local minimum of the potential—the false vacuum—before the transition to the true vacuum takes place by bubble nucleation and growth (as shown in fig. 4). When these bubbles collide copious amounts of gravitational radiation can be emitted, particularly if the relative velocity of the bubble walls is highly relativistic as in a strongly first-order phase transition. For an extended inflation model, the contribution to the gravitational wave background from such a transition could be as high as \( \Omega_g h^2 \approx 10^{-8} – 10^{-9} \) and it would lie in the LIGO/VIRGO sensitivity range if the re-heat temperature of the universe was close to \( 10^8 \)GeV or \( 10^9 \)GeV. We have included a crude sample spectrum for such a model in fig. 2.

First- and second-order phase transitions are a generic phenomena in cosmology since spontaneously broken symmetries are now an integral part of modern particle physics. If the standard electroweak phase transition were strongly first-order, then there would be a contribution of about \( \Omega_g h^2 \approx 10^{-9} \) around a frequency of \( 10^{-3} \)Hz, inside the LISA band with a detectable amplitude\(^{16}\) (see fig. 2). Unfortunately, the minimal standard model is currently
A strongly first-order phase transition can produce a stochastic background of gravitational waves. The nucleated bubble walls expand at highly relativistic velocities and complete the transition to the true vacuum through violent collisions.

believed to have only a weakly first-order transition with the bubble walls reaching velocities well below the speed of light; estimates in this case,\(^{17}\) yield only \(\Omega_g h^2 \sim 10^{-22}\). Nevertheless, there are a number of well-motivated extensions to the standard model (like supersymmetry) which could entail symmetry-breaking just above the electroweak scale; if strongly first-order, such phase transitions would create a distinctive and detectable LISA signal. Fortuitously from this point of view, LISA has a very interesting frequency response range.

Symmetry breaking phase transitions in the early universe will also inevitably produce topological defects of one form or another (refer to the recent review\(^ {18}\)). Of particular interest in our context are line-like defects, known as cosmic strings, formed at a grand-unification scale (\(10^{-35}\) seconds). Due to their large mass per unit length (typically \(\mu \sim 10^{22}\) g cm\(^{-1}\)), such strings provide a viable mechanism for seeding the formation of large-scale structure\(^ {19,20}\). A string network has been shown in numerical simulations\(^ {21,22}\) to evolve towards a self-similar scaling regime in which the number of strings per horizon volume remains fixed (see fig. 5). The network maintains this constant relative density by creating loops which oscillate relativistically and decay radiatively. For local cosmic strings, this loop decay channel is gravitational waves, thus creating a stochastic background over a vast range of frequencies from \(10^{-13}\) Hz to \(10^{10}\) Hz.

Under mild assumptions about the loop emission spectrum and the particle content of the universe, the radiation background due to loops created in the radiation era has a flat spectrum with amplitude \(\Omega_g h^2 \approx 10^{-8}\) for frequencies between \(10^{-9}\) Hz and \(10^{10}\) Hz. In contrast, the background produced in the matter era is sensitive to the loop spectrum. If this spectrum is not cut-off by radiation backreaction\(^ {23}\), then loop radiation can feed into higher (\(\sim 10^{-8}\) Hz) frequencies, because it does not feel the full impact of the redshifting after the time of equal
Figure 5: The scale-invariant evolution of a string network is maintained by small loop creation; the oscillating loops then decay into a stochastic background of gravitational waves. One horizon volume of a radiation era simulation is shown containing about 40 long strings. Interestingly, it is precisely these frequencies that are relevant for the pulsar timing experiments$^{3,4,5}$. Using an appropriately truncated loop spectrum and the dimensionless string parameter normalised to COBE$^{24}$, $G\mu/c^2 \approx 10^{-6}$, the predicted radiation background can be calculated numerically$^{10}$ as shown in fig. 2. Alternatively, if one assumes that $G\mu/c^2$ is arbitrary, then the pulsar timing experiment$^{3}$ can be used to deduce the bound $G\mu/c^2 < 3.5(\pm 0.8) \times 10^{-6}$.

The contribution from cosmic strings to the frequency ranges relevant for direct detection is likely to be slightly lower than that for pulsar timing. This is due to particle mass thresholds, where the number of relativistic degrees of freedom $N$ decrease, effectively diluting the relative contribution of any pre-existing decoupled radiation. The cosmic string spectrum shown in fig. 2 illustrates this effect with a gradual rise at frequencies around $10^{-4}$Hz. This is caused by particle annihilation near the QCD and electroweak phase transitions. If the particle physics model has more degrees of freedom at higher energies there could be other steps associated with other phase transitions. However, the dependence on $N$ is reasonably weak and therefore we can conservatively estimate that $\Omega_gh^2 > 5.0 \times 10^{-9}$ at $10^{-3}$Hz and $\Omega_gh^2 > 1.0 \times 10^{-9}$ at $100$Hz for $G\mu/c^2 \approx 1.0 \times 10^{-6}$. We note also that very precise determinations of the stochastic background from cosmic strings at different frequencies would measure $N$ in different cosmological epochs, providing fascinating insight into the particle content of the early universe (at times much...
earlier than the electroweak phase transition using LIGO or VIRGO).

Finally, we should note that there are other types of topological defects which do not produce such a large gravitational wave background, notably those formed when global symmetries are broken. For global cosmic strings, which primarily radiate Goldstone bosons, a simple calculation suggests that gravitational radiation will be suppressed by approximately four orders of magnitude relative to local strings, as illustrated in fig. 2. It is likely that a background produced by other global defects, such as global textures, will be likewise suppressed.

We have summarized the potential cosmological sources of gravitational waves and concluded that, of the candidates already proposed, strongly first-order phase transitions, possibly at the end of inflation, and local cosmic string networks provide tangible hope of direct detection. On the other hand, slow-roll inflation, global topological defects and the standard electroweak phase transition create signals which appear to be too weak. However, it is important to note that the study of gravitational wave emission in the early universe is a relatively unexplored domain. In light of the dramatic technological advances being made by experimentalists, the time is clearly ripe for more detailed quantitative studies of the characteristic signals produced by cosmological sources. Moreover, such theoretical foresight could positively influence observational strategies.

To conclude, then, it is clear that gravitational waves can potentially provide a unique and unparallelled probe of the very early universe. The proposed interferometers, both terrestrial and space-based, and improved pulsar timings could rule out or severely constrain a number of viable theoretical models. On the other hand, the detection of a primordial background of gravitational waves would have a profound impact on our understanding of high energy physics and cosmology, providing an unprecedented view of the earliest moments of our Universe.

Bibliography

1. Thorne K. [1996], in Particle and nuclear astrophysics and cosmology in the next millennium, Kolb E.W & Peccei R., eds. (World Scientific, Singapore).
2. Davis R.L., Hodges H.M., Smoot G.F., Steinhardt P.J. & Turner M.S. [1992], Phys. Rev. Lett. 69, 1856.
3. Kaspi V.M., Taylor J.H. & Ryba M.F. [1994], Ap. J. 428, 713.
4. Thorsett S.E. & Dewey R.J. [1996], to appear Phys. Rev. D.
5. McHugh M.P., Zalamansky G., Vernotte F. & Lantz E. [1996], submitted to Phys. Rev. D.
6. Abramovici A. et al [1992], Science 256, 325.
7. Bradachia el al [1990], Nucl. Instrum. & Methods A289, 518.
8. Jafry Y.R., Cornelisse J. & Reinhard R. [1994], ESA Journal 18, 219.
9. Caldwell R.R. & Allen B. [1992], Phys. Rev. D45, 3447.
10. Battye R.A., Caldwell R.R. & Shellard E.P.S [1996], DAMTP preprint.
11. Liddle A. [1994], Phys. Rev. D49, 3805. Erratum: Phys. Rev. D51, 4603.
12. Brustein R., Gasperini M., Giovannini M. & Veneziano G. [1995], Phys. Lett. 361B, 45.
13. La D. & Steinhardt P.J. [1989], Phys. Rev. Lett. 62, 376.
14. Linde A.D. [1994], Phys. Rev. 49, 748.
15. Turner M.S. & Wilczek F. [1990], Phys. Rev. Lett. 65, 3080.
16. Kowosky A., Turner M.S. & Watkins R. [1992], Phys. Rev. Lett. 69, 2026.
17. Kamionkowski M., Kowosky A. & Turner M.S. [1994], Phys. Rev. D49, 2837.
18. Vilenkin A. & Shellard E.P.S. [1994], Cosmic strings and other topological defects (Cambridge University Press).
19. Zel’dovich Ya.B. [1980], M. N. R. A. S. 192, 663.
20. Vilenkin A. [1981], Phys. Rev. Lett. 46, 1169. Erratum: Phys. Rev. Lett. 46, 1496.
21. Bennett D.P. & Bouchet F.R. [1990], Phys. Rev. D41, 2408.
22. Allen B. & Shellard E.P.S. [1990], Phys. Rev. Lett. 64, 119.
23. Battye R.A. & Shellard E.P.S [1996], Phys. Rev. D53, 1811.
24. Allen B., Caldwell R.R., Shellard E.P.S., Stebbins A. & Veeraraghavan S. [1995], work in progress.