An explicit scheme for ohmic dissipation with smoothed particle magnetohydrodynamics

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ABSTRACT

In this paper, we present an explicit scheme for Ohmic dissipation with smoothed particle magnetohydrodynamics (SPMHD). We propose an SPH discretization of Ohmic dissipation and solve Ohmic dissipation part of induction equation with the super-time-stepping method (STS) which allows us to take a longer time step than Courant–Friedrich–Levy stability condition. Our scheme is second-order accurate in space and first-order accurate in time. Our numerical experiments show that optimal choice of the parameters of STS for Ohmic dissipation of SPMHD is \( t_{\text{sts}} \sim 0.01 \) and \( N_{\text{sts}} \sim 5 \).

Key words: magnetic fields – MHD – methods: numerical – stars: formation – stars: protostars.

1 INTRODUCTION

Magnetic field plays an important role in various astrophysical problems. In star formation processes, magnetic field changes the formation and evolution of protostars, discs and jets (e.g. Machida, Tomisaka & Matsumoto 2004; Matsumoto & Tomisaka 2004; Matsumoto 2007; Inutsuka, Machida & Matsumoto 2010; Machida, Inutsuka & Matsumoto 2011). Until recently, these interesting phenomena in collapsing magnetized cloud core have been investigated with nested-grid code or adaptive-mesh-refinement code.

Smoothed particle hydrodynamics (SPH) is a suitable numerical scheme for the protostellar collapse simulations because of its adaptive nature at high-density region and several authors have investigated the formation and evolution of protostar and disc in molecular cloud core (e.g. Bate 1998; Stamatellos, Whitworth & Hubber 2011; Tsukamoto & Machida 2011, 2013). In spite of the importance of magnetic field, however, most of the simulations with SPH do not include a magnetic field because, until recently, robust magnetohydrodynamic (MHD) schemes for SPH had not been developed.

Recently, several authors proposed robust smoothed particle magnetohydrodynamics (SPMHD) schemes. Tricco & Price (2012) proposed an SPMHD scheme with the hyperbolic divergence cleaning method (Dedner et al. 2002), which is originally proposed by Price & Monaghan (2005). They improved the original method of Price & Monaghan (2005) by changing discretization forms for \( \nabla \cdot B \) and \( \nabla \phi \). With this method, they successfully simulated protostellar collapse and formation of jets (see, also Price, Tricco & Bate 2012).

Iwasaki & Inutsuka (2011) proposed an SPMHD scheme based on Godunov SPH (GSPH) proposed by Inutsuka (2002). We refer to their method as Godunov SPMHD (GSPMHD). Instead of the artificial dissipation terms which are used in Price & Monaghan (2005), they use a solution of a non-linear Riemann problem with magnetic pressure and the method of characteristics to calculate the interactions between SPH particles. This method significantly reduces the numerical diffusion (compare fig. 2 of Iwasaki & Inutsuka 2011 and fig. 6 of Price & Monaghan 2005). They also have developed hyperbolic divergence cleaning method for GSPMHD (Iwasaki & Inutsuka, in preparation) and successfully simulated formation of jets (Iwasaki, in preparation).

In the previous studies about star formation processes with SPMHD, ideal MHD was assumed. But the assumption of ideality is generally not correct for the star formation processes because interstellar gas is partially ionized and several magnetic diffusion processes (e.g. Ohmic dissipation, Hall effect and ambipolar diffusion) play roles. Especially, Ohmic dissipation is effective at high-density region \( (\rho \gtrsim 10^{-12} \text{ g cm}^{-3}) \) and formation and evolution of circumstellar discs, protostars and jets are significantly affected by Ohmic dissipation (see, e.g. Machida, Inutsuka & Matsumoto 2006).

To investigate the magnetic field in the intracluster medium of galaxy clusters, SPMHD simulations with Ohmic dissipation were performed by Bonafede et al. (2011). But they only considered the spatially constant magnetic resistivity. This assumption is generally not verified for protostellar collapse simulations because the resistivity has large spatial variation according to the gas density and temperature. Therefore, an Ohmic dissipation scheme for SPMHD which includes the effect of the spatially varying resistivity is desired to investigate formation and evolution of protostars, discs and jets.

In this paper, we propose a new explicit scheme for Ohmic dissipation with SPMHD. In Section 2, we describe our SPH discretization for Ohmic dissipation and time-stepping method. We present the results of several numerical tests in Section 3. Finally, we summarize our results in Section 4.
2 EXPLICIT SCHEME

2.1 Discretization

The induction equation with Ohmic dissipation is

$$\frac{d(B/\rho)}{dt} = \frac{B}{\rho} \cdot \nabla v - \frac{1}{\rho} \nabla \times (\eta \nabla \times B),$$

(1)

where $\rho$, $B$, $v$, $\eta$ denote the density, magnetic field, velocity and resistivity, respectively. Equation (1) is solved by an operator splitting approach and we focus on the solution of the second term on the right-hand side. The equation of Ohmic dissipation is given as

$$\frac{d(B/\rho)}{dt} = - \frac{1}{\rho} \nabla \times (\eta \nabla \times B).$$

(2)

Equation (2) is written as

$$\frac{d(B/\rho)}{dt} = - \frac{1}{\rho} \left\{ \partial_i (\eta \partial_i B_v - \eta \partial_i B_m) \right\} \equiv \frac{1}{\rho} \partial_i F_{\mu v}.$$  

(3)

Here, we used Greek letter, $\mu, \nu$ to denote the components of vector and we used Einstein summation convention. There are several choices for the discretization of the $(\nabla \cdot F)/\rho$. In this study, we adopted the following discretization. Discretization form of $(\nabla \cdot F)/\rho$ of $i$th particle is

$$\left( \frac{1}{\rho} \partial_i F_{\mu v} \right) = - \int \frac{\partial_i (\eta \partial_i B_v - \eta \partial_i B_m)}{\rho} W_i d^3r$$

$$= \int \eta (\partial_i B_v - \partial_i B_m) \partial_v \left( \frac{W_i}{\rho} \right) d^3r$$

$$\sim \sum_j m_j \left\{ \eta (\partial_i B_v - \partial_i B_m) \right\} \partial_i W_{ij}.$$  

(4)

Here, we used Latin letter, $i, j$ to denote the particle number and $W_i = W[x_i, h(x)]$ and $W_j = W[x_j, h_j]$, where we adopted the mean smoothing length as $h_{ij} = (h_i + h_j)/2$. We also investigated the following formula for $(\nabla \cdot F)/\rho$:

$$\left( \frac{1}{\rho} \partial_i F_{\mu v} \right) =$$

$$\sum_j m_j \left[ \frac{4}{\rho \rho_j} \eta \eta_j \partial_j (\partial_i B_v - \partial_i B_m) \right] (x_{ij} - x_{ij}) \partial_i W_{ij}$$

$$+ \left\{ \frac{\eta (\partial_i B_v - \partial_i B_m)}{\rho_{ij}} \right\} \partial_i W_{ij}.$$  

(5)

The discretization of the first term on the right-hand side is suggested by Cleary & Monaghan (1999). The spatial resolution of this formula is slightly better than that of equation (4) but this introduces larger divergence error because the discretizations of the derivative of the magnetic field and the volume factor are inconsistent between the first and the second term. Therefore, we adopted equation (4).

There are also several choices for the gradient tensor of magnetic field. In the following test calculations, we adopted

$$(\nabla B)_j = \frac{1}{\rho_i} \sum_j m_j (B_j - B_i) \nabla W[x_i - x_j, h_{ij}].$$

(6)

We use the cubic spline kernel of Monaghan & Lattanzio (1985),

$$W(r, h) = C_i \left\{ \begin{array}{ll} 1 - \frac{3}{2} q^2 + \frac{3}{2} q^3, & 0 \leq q < 1 \\ \frac{1}{2} (2 - q)^3, & 1 \leq q < 2 \\ 0 & 2 < q \end{array} \right. + \nu_{st},$$

(7)

where $q = r/h$ and $C_i = \frac{1}{\sqrt{\pi}}, \sqrt{\frac{10}{\pi}}$ for three and two dimensions, respectively. The smoothing length of $i$th particle is determined iteratively by the relation

$$h_i = C_h \left( \frac{m_i}{\rho_i} \right)^{1/d}.$$  

(8)

where $d$ is the dimension of the problem. $C_h$ is a parameter and set to be 1.2.

Although, we do not solve the energy equation in the following test calculations, it would be useful to derive the SPH discretization of Ohmic dissipation term in the energy equation. The energy equation of Ohmic dissipation is given as

$$\frac{D E}{D t} \big|_{\text{Ohm}} = \frac{1}{\rho} \nabla \cdot (\eta \nabla \times (\nabla B \times \eta \nabla \times B))$$

$$= \frac{1}{\rho} \nabla \cdot \left\{ \eta \nabla \left( \frac{B^2}{2} \right) - \eta \nabla \cdot B \nabla B \right\} \equiv \frac{1}{\rho} \nabla \cdot S,$$  

(9)

where $\epsilon = \frac{1}{2} \nabla \cdot (u + \frac{\rho^2}{2\rho})$ is the specific total energy and $u = P/[(\gamma - 1)\rho]$ is the specific internal energy. The discretization form of $(\nabla \cdot S)/\rho$ of $i$th particle is

$$\left( \frac{1}{\rho} \partial_i S_{ij} \right) = \int \partial_i S_{ij} W_i d^3r = - \int S_{ij} \partial_i \left( \frac{W_i}{\rho} \right) d^3r$$

$$\sim \sum_j m_j \left\{ \frac{(S_{ij})}{\rho_j} + \frac{(S_{ij})}{\rho_j} \right\} \partial_i W_{ij},$$  

(10)

where $S_{ij}$ is calculated as

$$S_{ij} = \eta \left\{ \partial_i \left( \frac{B^2}{2} \right) - B_j \partial_i B_i \right\} = \eta \left\{ B_i (\partial_i B_j) - B_j (\partial_i B_i) \right\},$$

(11)

and equation (6). The equation (10) is antisymmetric under particle exchange and it is obvious that the error of the total energy is within machine epsilon by this discretization.

2.2 Time-stepping

During the protostellar collapse, high-density region $\rho \gtrsim 10^{-10}$ g cm$^{-3}$ appears. In the high-density region, the time-scale of Ohmic dissipation is shorter than the dynamical time-scale of the gas and the computational cost for Ohmic dissipation becomes large. To reduce the computational cost, we adopt the super-time-stepping method (STS) proposed by Alexiades, Amiez & Gremaud (1996). This method was used for Ohmic dissipation in Tomida et al. (2013) and ambipolar diffusion in Choi, Kim & Wiita (2009). In STS, Courant–Friedrichs–Levy (CFL) stability condition is relaxed by requiring the stability not at the end of each time step but at the end of a cycle of $N_{\text{sts}}$ steps. Following Alexiades et al. (1996), we define a super time step,

$$\Delta T_{\text{sts}} = \sum_{j=1}^{N_{\text{sts}}} \tau_j,$$

where $\tau_j$ is the sub-step and is given as

$$\tau_j = \Delta t_{\text{exp}} \left[ (1 - v_{\text{sts}}) \cos \left( \frac{2j - 1}{N_{\text{sts}}} \pi \right) + 1 + v_{\text{sts}} \right]^{-1}.$$  

(12)
Thus, the super time step is
\[ \Delta T_{\text{sts}} = \sum_{j=1}^{N_{\text{sts}}} \tau_j = \Delta t_{\exp} \frac{N_{\text{sts}}}{2\nu_{\text{sts}}} \left( 1 + \nu_{\text{sts}}^{1/2} \right)^{2N_{\text{exp}}} \frac{1}{\left( 1 + \nu_{\text{sts}}^{1/2} \right)^{2N_{\text{exp}}} + \left( 1 - \nu_{\text{sts}}^{1/2} \right)^{2N_{\text{exp}}} }, \]

(13)

where \( \Delta t_{\exp} \) is the explicit time step for Ohmic dissipation and we use \( \Delta t_{\exp} = C_{\text{CFL}} h^2/2 \). Here, \( C_{\text{CFL}} \) is CFL number and \( h \) is the smoothing length. \( \nu_{\text{sts}} \) is a parameter that controls the stability and the acceleration of the scheme. With smaller \( \nu_{\text{sts}} \), the scheme becomes faster but unstable. Optimal choice of \( \nu_{\text{sts}} \) depends on the problem and we investigate the optimal choice for \( \nu_{\text{sts}} \) in Section 3.

With STS, magnetic field is updated as
\[ \left( \frac{B}{\rho} \right)_{t+\Delta T_{\text{exp}}} = \left( \frac{B}{\rho} \right)_{t} + \sum_{j=1}^{N_{\text{sts}}} \tau_j \frac{d(B/\rho)}{dt} \bigg|_{t+\sum_{k=1}^{j-1} \tau_k}. \]

(14)

For comparison, we also performed simulations with a simple Euler method such as
\[ \left( \frac{B}{\rho} \right)_{t+\Delta t_{\exp}} = \left( \frac{B}{\rho} \right)_{t} + \Delta t_{\exp} \frac{d(B/\rho)}{dt} \bigg|_{t}. \]

(15)

3 NUMERICAL TESTS

3.1 Sinusoidal diffusion problem

At first, we consider a simple problem in which sinusoidal magnetic field diffuses with a constant resistivity. The initial magnetic field is
\[ B_x = 0, \ B_y = 0, \ B_z(x) = \sin(2\pi x). \]

(16)

The resistivity is set to be \( \eta = 1 \). The computational domain is two dimensions and \( x, y \in [-0.5, 0.5] \). We imposed periodic boundary conditions for each direction. We performed convergence tests by changing the time step \( \Delta t_{\exp} \) and the smoothing length \( h \). As a measure of the error, we calculated \( L_1 \) norm of \( \dot{B}/\rho \), defined as
\[ L_1 = \frac{1}{N_{\text{tot}}} \sum_{i=1}^{N_{\text{tot}}} \left| \dot{B}_{\text{ref},i}(r_i) - \dot{B}_i(r_i) \right|. \]

(17)

As reference solutions, \( \dot{B}_{\text{ref},i} \), we adopted the results with \( N_{\text{tot}} = 128^2 \), \( \Delta t_{\exp} = 6 \times 10^{-8} \) for the convergence test of the time step and \( N_{\text{tot}} = 256^2 \), \( \Delta t_{\exp} = 1.5 \times 10^{-6} \) for the convergence test of the smoothing length. For the calculations of both solutions, we used the Euler method. For STS, we adopted the value of \( \nu_{\text{sts}} = 0.01 \), \( N_{\text{sts}} = 5 \).

Fig. 1 shows the \( L_1 \) norms as a function of time step. We show both results of Euler method (solid) and STS (dashed). The horizontal axis is \( \Delta t_{\exp} \) for the Euler method and \( \bar{\tau} \) for STS. Here, \( \bar{\tau} \) is defined as \( \bar{\tau} = \sum_{j=1}^{N_{\text{sts}}} \tau_j / N_{\text{sts}} \). The figure shows that both schemes scale linearly and are of first order in time. This figure also shows that the error of STS is slightly larger than the Euler method at the same \( \Delta t \). This means that, with the same computational cost, the error of STS is slightly larger than the Euler method. This is simply because the error of STS is proportional not to \( \tau \), but to \( \Delta T_{\text{sts}} \).

Fig. 2 shows the \( L_1 \) norms as a function of smoothing length. The time step is fixed to be \( \Delta t_{\exp} = 1.5 \times 10^{-6} \) and the Euler method is used. The figure shows that the error is proportional to \( h^2 \) and it is confirmed that our discretization is of second order in space.

3.2 Gaussian diffusion problem

Next, we consider magnetic diffusion of \( B_z \) in Gaussian profile. The initial profile of magnetic field is given as
\[ B_x = 0, \ B_y = 0, \ B_z(x, y) = \frac{1}{4\eta t_0} \exp \left[ -\frac{x^2}{4\eta t_0} - \frac{y^2}{4\eta t_0} \right]. \]

(18)

where \( t_0 \) is the initial time and set to be unity. We set magnetic resistivity as \( \eta = 1 \). The computational domain is two dimensions and \( x, y \in [-16, 16] \). We impose periodic boundary conditions for each direction. The particle number for each direction is fixed to be 128.

Fig. 3 shows the \( L_1 \) norm as a function of time step at \( t = 8 \). Again, we considered both the Euler method (solid) and STS (dashed). The reference solution is the result with \( N_{\text{tot}} = 128^2 \), \( \Delta t_{\exp} = 1.4 \times 10^{-4} \) and the Euler method. The figure shows the same tendency of the results of sinusoidal diffusion problem, i.e. both schemes are of first order and the error of STS is slightly larger than the Euler method at the same time step.

To understand how the efficiency of STS depends on the parameters, we defined acceleration efficiency, \( F = \Delta T_{\text{sts}} / (N_{\text{sts}} \Delta t_{\exp}) \) and plotted it in Fig. 4 with \( \nu_{\text{sts}} = 10^{-2}, 10^{-3}, 10^{-4} \). The figure
shows that the maximum of acceleration efficiency is determined by given $\nu_{\text{sts}}$ and the maximum value is $F_{\text{max}} = 1/\sqrt{\nu_{\text{sts}}}$. Therefore, small $\nu_{\text{sts}}$ is preferable for the acceleration. But the small $\nu_{\text{sts}}$ makes the scheme unstable. This figure also shows the efficiency saturates around $N_{\text{sts}} \sim 1/\sqrt{\nu_{\text{sts}}}$. Therefore, optimal choice of $N_{\text{sts}}$ is $\sim 1/\sqrt{\nu_{\text{sts}}}$.

To seek the optimal choices of $\nu_{\text{sts}}$ and $N_{\text{sts}}$ for Ohmic dissipation, we investigated the behaviour of the solutions at $t = 8$ by changing the parameters. We choose the parameter sets as $\nu_{\text{sts}} = (10^{-2}, 10^{-3}, 10^{-4})$ and $N_{\text{sts}} = 5, 10$. The CFL number is set to be $C_{\text{CFL}} = 0.3$ for all calculations. The results at $y = 0$ are shown in Fig. 5. In this figure, only 64 particles are plotted to make the results more visible. The exact solution,

$$B_z(x, y) = \frac{1}{4\pi \eta(t_0 + t)} \exp \left( \frac{-x^2 + y^2}{4\eta(t_0 + t)} \right),$$

(19)

is also plotted. The results with $N_{\text{sts}} = 5$ (left-hand panel) and ($\nu_{\text{sts}}$, $N_{\text{sts}}$) = (10$^{-2}$, 10) (circles in the right-hand panel) agree well with the exact solution. In the case of $N_{\text{sts}} = 10$, as $\nu_{\text{sts}}$ becomes small, the solution becomes distorted and the result with $\nu_{\text{sts}} = 10^{-4}$ shows the significant overshoot. This results shows that $\nu_{\text{sts}} \sim 0.01$ is preferable for the stability. From Fig. 4, we can see that $F$ already saturates at $N_{\text{sts}} \sim 5$ for $\nu_{\text{sts}} = 0.01$. Therefore, we recommend $\nu_{\text{sts}} \sim 0.01$ and $N_{\text{sts}} \sim 5$ as the optimal values of the parameters.

### 3.3 A test with spatially varying resistivity

In this subsection, we consider the diffusion of $B_z$ in Gaussian profile with the spatially varying resistivity. The resistivity distribution is given as

$$\eta(r) = \exp \left[ -(x^2 + y^2 + z^2) \right],$$

(20)

and the initial magnetic field is

$$B_x = 0, B_y = 0, B_z(r) = \exp \left[ -(x^2 + y^2) \right].$$

(21)

The computational domain is three dimensions and $x, y, z \in [-3, 3]$. We impose periodic boundary conditions for each direction.

Fig. 6 shows the contour maps of $B_x$ and $B_z$ obtained at $t = 1$ with the Euler method with $\Delta_{\text{exp}} = 10^{-3}$ and STS with $C_{\text{CFL}} = 0.3$, $\nu_{\text{sts}} = 0.01$, $N_{\text{sts}} = 5$. The particle number of each
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Figure 6. Magnetic field distributions of the 3D diffusion problem with spatially varying resistivity at $t = 1$, $y = 0$ planes are shown. The magnetic field is solved with the Euler method and $\Delta t_{\text{exp}} = 10^{-3}$ for top panels and with STS and $C_{\text{FL}} = 0.3$, $v_{\text{sts}} = 0.01$, $N_{\text{sts}} = 5$ for bottom panels. Left-hand panels show $B_x$ and contour levels are $B_x = -0.09, -0.08, \ldots, 0.09$. Right-hand panels show $B_z$ and contour levels are $B_z = 0.05, 0.1, \ldots, 0.95$.

direction is 48. The results are consistent with each other and also consistent with the results calculated with the grid-code (see, Matsumoto 2011). But the $B_z$ around the centre is slightly overestimated with STS.

To confirm that our discretization is of second order in space with the spatially varying resistivity, we show the $L_1$ norm of $B_z$ at $t = 1$ as a function of smoothing length in Fig. 7. The solutions are obtained with the Euler method and $\Delta t_{\text{exp}} = 10^{-3}$. The reference solution is the result with $N_{\text{ext}} = 96^3$ and $\Delta t_{\text{exp}} = 10^{-3}$.

This figure shows that the error is proportional to $h^2$ and it is confirmed that our discretization is of second order in space.

3.4 Gravitational collapse of magnetized cloud core

Finally, we consider the gravitational collapse of magnetized cloud core. The initial condition is as follows. The initial molecular cloud core has a mass of $1 M_\odot$ and radius $R_c = 2.7 \times 10^4$ au. The free-fall time of the core is $2.4 \times 10^4$ yr. The core is rigidly rotating with the angular velocity of $\Omega = 1.8 \times 10^{-13}$ s$^{-1}$. For the

Figure 7. $L_1$ norm of error as a function of smoothing length for the 3D diffusion problem with spatially varying resistivity at $t = 1$. Solid line denotes the results with the Euler method and $\Delta t_{\text{exp}} = 10^{-3}$. The dash-dotted lines are in proportion to $h$ and $h^2$, respectively.
boundary condition, we fix the particles whose radius is larger than $2.6 \times 10^7$ au.

We adopt a barotropic equation of state

$$P = \begin{cases} 
\rho^2 (1 + (\rho/\rho_c)^{2/5}), & \rho < \rho_c \\
\rho^{7/5} (\rho/\rho_0)^{2/5}, & \rho_0 \leq \rho < \rho_c, \\
\rho^{11/5} (\rho/\rho_0)^{1/5}, & \rho_c \leq \rho
\end{cases}$$

where $c_s = 190$ m s$^{-1}$, $\rho_c = 4 \times 10^{-14}$ g cm$^{-3}$, $\rho_0 = 4 \times 10^{-9}$ g cm$^{-3}$ and $\rho_c = 4 \times 10^{-13}$ g cm$^{-3}$. The initial magnetic field is parallel to the z-axis with the magnitude of $B_z = 189$ $\mu$G and the initial plasma beta is $\beta = 2.5$. The cloud core is modelled with $5 \times 10^5$ particles.

We use the GSPMHD scheme of Iwasaki & Inutsuka (2011) with hyperbolic divergence cleaning method (Iwasaki & Inutsuka, in preparation) to solve ideal MHD part and Barnes–Hut tree algorithm with opening angle $\theta = 0.5$ for gravity part. Ohmic dissipation is solved with present method. We adopted the resistivity $\eta$ as

$$\eta = \frac{7.4 \times 10^5}{X_e} \sqrt{\frac{T}{10 \, \text{K}}} \left[ 1 - \tanh \left( \frac{n}{10^{12} \, \text{cm}^{-3}} \right) \right] \, \text{cm}^2 \, \text{s}^{-1},$$

where $T$ and $n$ are the gas temperature and number density, and $X_e$ is the ionization degree of the gas and

$$X_e = 5.7 \times 10^{-4} \left( \frac{n}{\text{cm}^{-3}} \right)^{-1}.$$ (24)

This model has the similar form to the model adopted in Machida, Inutsuka & Matsumoto (2007) but is artificially shifted to lower density to emphasize the effect of Ohmic dissipation in the first core. With our model, Ohmic dissipation is effective at $10^{-13}$ g cm$^{-3} \lesssim \rho \lesssim 10^{-10}$ g cm$^{-3}$.

In Fig. 8, the magnetic energy of the central part (the region of $\rho > 0.1 \rho_c$, where $\rho_c$ is the central or maximum density of the cloud core) normalized by the thermal energy as a function of central density is shown. The solid line and crosses show the results with the STS and Euler methods, respectively. The result of ideal MHD is also shown with the dashed line for comparison. The parameters for STS are $v_{\text{in}} = 0.01, N_{\text{ini}} = 5$.

When the central density is small ($10^{-16} < \rho_c < 10^{-14}$ g cm$^{-3}$), Ohmic dissipation is ineffective and there is no difference between resistive and ideal MHD models. The magnetic energy of the resistive MHD models begins to decrease at $\rho_c \sim 10^{-15}$ g cm$^{-3}$ and becomes more than three orders of magnitude smaller than the ideal MHD model at $\rho_c = 10^{-10}$ g cm$^{-3}$. This figure also shows that the result with STS agree very well with that of the Euler method. Therefore, STS is proved to be beneficial for the realistic star formation problems.

In Fig. 9, the density distributions at the centre of the cloud when $\rho_c \sim 5 \times 10^{-3}$ g cm$^{-3}$ are shown. The velocity field is shown with red arrows.

In the ideal MHD model (left), the black thick line denotes the velocity contour of $|v_z| = 0$. This line clearly shows that the outflow forms at the centre of the cloud. On the other hand, in the resistive MHD model (right), the outflow does not form because of the large resistivity in the first core. The structure of the first core is also very different from the ideal MHD model because the magnetic braking is ineffective. The detailed simulations and analysis of the
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formation and evolution of the outflow with ideal GSPMHD can be found in Iwasaki (in preparation).

4 SUMMARY AND PERSPECTIVE

In this paper, we presented an explicit scheme for Ohmic dissipation with SPMHD. We proposed a SPH discretization of Ohmic dissipation term in the induction equation. Ohmic dissipation part is solved with STS which relaxes the CFL stability condition requiring the numerical stability not at the end of each time step but at the end of a cycle of \( N_{\text{sts}} \) steps. Our scheme is second-order accurate in space and first-order accurate in time. The scheme successfully solves 2D and 3D tests. Our scheme is simple and can be easily implemented to any SPMHD codes.

We showed that STS introduces slightly larger error compared to the Euler method if we fix the computational costs. This comes from the fact that the error of STS is proportional not to \( \tau \) but to \( \Delta T_{\text{sts}} \).

We found that optimal choice of the parameters of STS for Ohmic dissipation of SPMHD is \( v_{\text{sts}} \sim 0.01 \) and \( N_{\text{sts}} \sim 5 \) and these values are consistent with the values suggested by Tomida et al. (2013).

Our present scheme is only first-order accurate in time. Recently, Meyer, Balsara & Aslam (2012) suggest a method which extends STS to second-order accurate in time. They applied this method to solve thermal conductivity. It is possible to solve Ohmic dissipation or other magnetic diffusion with their method. Note that, however, the efficiency of acceleration of their method is not so good as first-order STS at small \( N_{\text{nts}} \) and large \( N_{\text{nts}} \) is required to achieve better acceleration. We plan to improve accuracy in time of our scheme in future works.

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