Rates of reconnection in pulsar winds

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ABSTRACT

Using the formulation of relativistic reconnection by Lyutikov & Uzdensky (2002) we estimate the upper possible rates of reconnection in pulsar winds using Bohm prescription for resistivity. We find that the velocity of plasma inflow into the reconnection layer may be relativistic, of the order of the speed of light in the plasma rest-frame. This in principle may allow efficient dissipation of the magnetic field energy in the wind and/or destruction of the toroidal magnetic flux. The efficiency of reconnection realized in pulsar winds remains an open question: it should depend both on the microphysical properties of plasma and on the three-dimensional structure of the reconnection flow.

Subject headings: stars: pulsars - plasmas - magnetic fields

1. Introduction

Magnetic reconnection is a very important phenomenon in many laboratory and astrophysical plasmas (Biskamp 2000, Priest & Forbes 2000). Most reconnection models are based on two classical schemes suggested by Sweet & Parker and Petschek. Recognition that magnetic reconnection processes are also of great importance in high energy astrophysics, where dynamic behavior is often dominated by super-strong magnetic fields, with energy density $B^2/(8\pi)$ larger than the rest energy of the matter $\rho c^2$, has led Lyutikov & Uzdensky (2002) to formulate the relativistic Sweet-Parker reconnection. In this Letter we apply the model of relativistic reconnection to the problem of energy conversion in pulsar winds.

The $\sigma$ paradox ($\sigma$ is conventionally defined as the ratio of Poynting to particle fluxes or, equivalently, as a ratio of the magnetic to plasma energy density) is a long-standing problem in pulsar physics (Rees & Gunn 1974, Kennel & Coroniti 1984). Models of the pulsar magnetosphere (Goldreich & Julian 1969, Arons & Scharlemann 1979, Ruderman & Sutherland 1975) predict that near the light cylinder most of the spin-down luminosity of...
a pulsar should be in a form of Poynting flux, $\sigma \gg 1$. On the other hand, modeling of the dynamics of the Crab nebula (and other PWNs like W44, Shelton et al. 1999, Vela, Pavlov et al. 2001, PWN around PSR B1509-58, Gaensler et al. 2002) gives a low value of $\sigma$ at what is commonly believed to be a reverse shock - strongly magnetized flows cannot match the boundary conditions (Rees & Gunn 1974, Kennel & Coroniti 1984).

Most promising resolution of the $\sigma$ paradox invokes internal dissipation of the magnetic fields in the equatorial flow due to break down of MHD approximation (Coroniti 1990, Michel 1994, Melatos & Melrose 1996). In the equatorial plane of an oblique rotator, the MHD wind forms striped structures, in which alternating $B_\phi$ regions are separated by current sheets. As plasma flows out from the pulsar, the plasma density decreases in proportion to $r^{-2}$ reaching a critical radius, where it becomes less than the critical charge density $n_{\text{crit}} \sim r^{-1}$ required to carry the current. Beyond this limiting radius MHD approximation breaks down. Coroniti (1990) has argued that such a breakdown of MHD would lead to effective dissipation of the field. This process was called reconnection since it is supposed to destroy magnetic field and transfer energy to particles, similar to the effects of reconnection in a classical (e.g. Solar physics) sense. Later Michel (1994) extended the Coroniti’s model, arguing that if reconnection is efficient, the structure of the wind nebula may be very different from the ideal MHD one envisioned by the Kennel & Coroniti (1984). In particular, the flow may not have a reverse shock, but may simply decelerate smoothly to match nebular boundary conditions. This model has been criticized by Lyubarsky & Kirk (2001) (LK afterward) who argued that acceleration of the flow, resulting from extra pressure released during reconnection, may not leave enough time for destruction of the field.

The physical model used by all the above authors is, in fact, not a reconnection model in the classical sense. The geometry and dynamics of the flow in the above models is completely different from (and in some sense contradictory to) the reconnection models. In the original Coroniti model, the thickness of the dissipation layer, which is assumed to be equal to the Larmor radius based on the external magnetic field and internal thermal particle velocity, plays a passive role. As the magnetic field of the wind decreases with radius, the thickness of the dissipation layer increases: the reconnection layer “eats out” the magnetic field. At the same time the inflowing plasma always remains at rest with respect to reconnection layer. This type of a dynamical behavior is quite contrary to reconnection, where magnetic field actually flows into the reconnection region. The flow dynamics proposed by LK is even more different from the classical reconnection picture. LK have argued that the expansion of the reconnection layer will push away the magnetic field - a situation reverse to that of reconnection, where magnetic field is “sucked” into the reconnection layer.
In this paper we reconsider the problem of reconnection in pulsar winds, applying the Lyutikov & Uzdensky (2002) model of relativistic reconnection. Lyutikov & Uzdensky (2002) presented a relativistic generalization of the simplest model of magnetic reconnection — the Sweet–Parker model — to strongly magnetized plasmas. In the spirit of the Sweet–Parker model, the reconnection layer is assumed to have a rectangular shape with a width $L$ and thickness $\delta \ll L$. The width $L$ of the reconnection layer is determined by the global system size; also prescribed are the ratio $\sigma$ of the magnetic field energy density to the plasma energy density in the ideal-MHD inflow region, and the plasma resistivity $\eta$. In contrast, the thickness $\delta$ of the reconnection region, and the plasma inflow and outflow velocity are calculated as a part of the analysis. As the plasma enters the reconnection layer, it slows down, coming to a halt at a stagnation point. At the same time, magnetic energy is dissipated and converted into internal energy of the pair-rich plasma. In the out-flowing region, the plasma is accelerated by the pressure gradients, reaching some terminal relativistic velocity $\gamma_{\text{out}}$.

Lyutikov & Uzdensky (2002) have found that the structure of the reconnection layer (its thickness, the inflow and outflow velocities) depend on the ratio of two large dimensionless parameters of the problem - magnetization parameter $\sigma \gg 1$ and the Lundquist number

$$S = \frac{Lc}{\eta} \gg 1. \quad (1)$$

In the sub-alfvenic regime, $\sigma \ll S^2$, the flow is determined by the set of equations

$$\beta_{\text{in}} \gamma_{\text{in}} \sim \frac{L}{\delta} \frac{1}{S};$$
$$\gamma_{\text{in}}(1 + \sigma) \sim \gamma_{\text{out}};$$
$$\beta_{\text{in}} \sim (1 + \sigma) \frac{\delta}{L}. \quad (2)$$

The inflow velocity is non-relativistic, $\beta_{\text{in}} \ll 1$, for $\sigma \ll S$, strongly relativistic sub-alfvenic, $1 \ll \gamma_{\text{in}} \ll \gamma_A$ for $S \ll \sigma \ll S^2$ ($\gamma_A = \sqrt{2\sigma}$ is the Lorentz factor of the Alfvén wave velocity in the incoming region), and near alfvenic, $\gamma_{\text{in}} \sim \gamma_A$ for $\sigma \geq S^2$. The outflowing plasma is moving always relativistically, $\gamma_{\text{out}} \gg 1$ if $\sigma \gg 1$.

2. Reconnection in pulsar winds

Beyond the light cylinder the pulsar wind is quasi-radial, moving with strongly relativistic velocity (typical Lorentz factor $\gamma_0 = \sqrt{\sigma} \sim 100$), and is strongly magnetized

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¹These relations are valid for both the non-relativistic reconnection, $\sigma \ll 1$, and for relativistic, $\sigma \gg 1$. 
\( \sigma \sim 10^4 \), dominated by the toroidal magnetic field \( B \sim 1/r \). In the equatorial plane of an oblique rotator the alternating polarity of the magnetic field create conditions favorable for reconnection (Michel 1994, Coroniti 1990, LK).

We are interested in the maximum possible reconnection rate in the pulsar wind (conventionally, the term “reconnection rate” refers to the inflow velocity of plasma in terms of Alfvén velocity). The maximum reconnection rate corresponds to the maximum value for the resistivity and thus the minimal Lundquist number, which may be estimated using Bohm’s arguments that the maximum diffusion coefficient in the magnetized plasma cannot be much larger than \( r_L v \) where \( r_L \) is the Larmor radius and \( v \) is the typical velocity of the electrons (of the order of the speed of light in our case). Thus, in the limit \( \sigma \gg 1 \)

\[
\eta \sim \frac{c^2}{\omega_B} \quad \text{(3)}
\]

which gives

\[
S \sim \frac{L}{r_L} \quad \text{(4)}
\]

\[
\delta \sim \frac{r_L}{\beta_{in} \gamma_{in}} \quad \text{(5)}
\]

\[
\beta_{in}^2 \gamma_{in} \sim \frac{\sigma}{L} \quad \text{(6)}
\]

The cyclotron frequency \( \omega_B \) in the wind frame

\[
\omega_B \sim \frac{\omega_{B,LC} r_{LC}}{\gamma_0 r} \quad \text{(7)}
\]

where \( r_{LC} \sim c/\Omega \) is the pulsar light cylinder, and \( \omega_{B,LC} \) is the cyclotron frequency at the light cylinder.

The maximum width of the reconnection layer cannot be larger than the hydrodynamically causally connected sector of the wind \( L \sim r/\gamma_0 \). Using eqns. (4-7) we can then estimate the Lundquist number in the wind

\[
S \sim \frac{\omega_{B,LC}}{\gamma_0 \Omega} \quad \text{(8)}
\]

Note, that the Lundquist number is independent of radius.

For a “typical” pulsar with the surface field \( B_{NS} \sim 10^{12} \) G and \( \Omega \sim 10 \) rad/sec, \( \omega_{B,LC} \sim 10^9 \) rad/sec the Lundquist number is

\[
S \sim 10^4 \quad \text{(9)}
\]

and thus for \( \sigma \leq 10^4 \) we expect a weakly relativistic inflow velocity with

\[
\beta_{in}^2 \gamma_{in} \sim \frac{\sigma}{S} \sim 1 \quad \text{(10)}
\]
3. Discussion

Using the formulation of relativistic reconnection by Lyutikov & Uzdensky (2002) we have found that the dynamical and kinematic constraints set on the rates of reconnection in pulsar winds may still allow very efficient reconnection given by eq. (10). If the inflow velocity in the plasma frame indeed reaches $\sim c$, then magnetic field of the wind would annihilate after propagating $\sim \gamma_0^2 r_{LC} \sim 10^4 r_{LC}$. We do not believe that such high rates are indeed realized (see, though, Kirk et al. 2002, who have argued that efficient dissipation within several light cylinders may be responsible for pulsar high energy emission). By choosing the Bohm prescription for resistivity and by neglecting the pressure of the ambient plasma we have estimated the upper limits on the rates of reconnection. More realistic calculations should be based on the microphysics of the reconnection layer, e.g. tearing mode in relativistic regimes (Zelenyi & Krasnoselskikh 1979) and computer simulations of relativistic reconnection layers (Zenitani & Hoshino 2001, Larrabee et al. 2002). This should provide the estimates for the Lundquist number in the layer. In addition, heating of the plasma by dissipating magnetic fields should also be taken into account. The corresponding model should be at least two-dimensional, allowing for the extra pressure to be relieved in the $\theta$ direction.

In spite of these limitations, this simple estimate shows that reconnection can be very efficient in relativistic plasmas. In the case of the Crab nebula, the magnetic flux has to be destroyed only by the time the wind reaches the wisps located at $r_w \sim 10^{17}$ cm, or $\sim 10^8$ light cylinder radii away. Assuming that initially $\gamma_0 \sim 100$, the inflowing velocity necessary to destroy the magnetic field needs to be only $10^{-4}$ of the speed of light. This is 4 orders of magnitude smaller than the maximum reconnection velocity (10). To provide such an inflow velocity the Lundquist number may be 8 orders of magnitude larger. Given such large range of allowed parameters, it is possible that the reconnection indeed is able to destroy effectively the magnetic flux in pulsar winds.

Perhaps the main argument against reconnection in the wind comes from the absence of observed high energy emission from the central part of the Crab nebula (Weisskopf et al. 2000). Reconnection is usually accompanied by efficient particle heating and acceleration which should result in radiative losses. In fact, in the case of pulsar winds, reconnection need not to destroy the magnetic field - it need only destroy the magnetic flux (e.g., by changing the topology of the field lines) to allow the flow to match the non-relativistically moving boundary of the nebular. Changing of topology may require very little dissipation. Alternatively, the pulsar wisps, conventionally associated with the reverse shock, may indeed be the signs and the sites of reconnection (Michel 1994) happening in an initially cold wind which has not crossed the fast sonic point (cold winds have asymptotic fast Mach
number at most unity, Kennel et al. 1983; thus, in order for reconnection to be efficient in slowing down the pulsar wind the flow must be subsonic).

The results of this work may be contrasted with those of Coroniti (1990) and LK. On a microscopical level there are some similarities. For example, comparing the thickness of reconnection sheet assumed by Coroniti and LK with our result for the Bohm-type diffusion, eq. \((\ref{5})\), shows that the thickness of the reconnection layer does become of the order of Larmor radius, \(\delta \sim r_L \propto r\), but only when the inflowing velocity becomes weakly relativistic \(\beta_{in} \gamma_{in} \sim 1\).

The key difference is that the “reconnection models” of Coroniti (1990) and LK are one-dimensional. If the flow is forced to be one-dimensional then the dissipated energy has to stay inside the reconnection layer in a thermalized form, leading to the unusual dynamics derived by LK. The assumption of one-dimensionality is in a sharp contrast to classical reconnection which is at least a two-dimensional (and most likely three-dimensional) process. Thus, if reconnection happens in pulsar winds the flow structure will be at least two-dimensional, so that the particle pressure created by the dissipation of the magnetic field will be relieved in the direction orthogonal to the initial inflow direction (radial) and not along it as argued by LK.

In the two-dimensional Sweet-Parker model the outflowing velocity is directed along the external magnetic field lines. In the case of the equatorial flow in pulsar winds, this corresponds to the azimuthal direction. In a perfectly axisymmetric picture such outflow is obviously impossible. Contrary to LK we suggest that instead of preserving the axially symmetric form at all costs, the flow would become non-axially symmetric and thus two-dimensional (and most likely three dimensional). As soon as reconnection starts in some localized region, a localized deposition of energy will distort the flow and break the azimuthal symmetry, setting up some complicated velocity pattern, with plasma moving in and out of the reconnection regions (see Fig. 1). Mutual interaction of different layers then becomes an important issue. Redeposition of energy back into the flow from reconnection regions would affect the rates of reconnection only after a considerable fraction of the magnetic field has been dissipated. Under certain circumstances reconnection in one localized region may push plasma into other reconnection sites, speeding up reconnection, as is illustrated in Fig. 1. These issues are beyond the scope of this Letter.

Based on these arguments we conclude that the one-dimension approach to the problem of reconnection in pulsar winds and the ensuing result that reconnection is not important is likely to be incorrect - one has to consider at least two-dimensional (and possible full three-dimensional) problem. Our simple estimates show that in pulsar winds the plasma may flow into the reconnection region with mildly relativistic velocities insuring a much
more efficient reconnection than argued by LK. Since we have calculated only the upper limits on the reconnection rate, we cannot make a conclusive statement if reconnection indeed occurs efficiently. This requires an understanding of the microphysical processes (e.g., evolution of tearing mode) in strongly relativistic magnetized plasmas and of the three-dimensional structure of the flow.

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Fig. 1.— A sketch of possible two-dimensional reconnection in a pulsar wind. Solid lines represent magnetic field, dashed lines - field separatrix, thick boxes - reconnection regions; arrows indicate the direction of the flow. The initially azimuthally symmetric striped magnetic field breaks into separate reconnection regions. In the vicinity of each reconnection region the structure of the velocity field resembles a conventional two-dimensional Sweet-Parker flow. This cartoon serves only to show, that giving up azimuthal symmetry, it is possible to have locally a classical two-dimensional reconnection picture.