Intelligent Bayesian regularization networks for bio-convective nanofluid flow model involving gyro-tactic organisms with viscous dissipation, stratification and heat immersion

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ABSTRACT

In the current study, a novel intelligent numerical computing paradigm based upon the foundation of the artificial neural networks legacy involving the Bayesian regularization (ANN-BR) approach has been implemented for the investigation of the non-uniform heat preoccupation process with the bio-convective flow dynamics of nanomaterial involving gyro-tactic microorganisms. The designed bio-convective stratified nanofluid flow (BCSNF) model initially represented by a system of PDEs is transformed into nonlinear ODEs by exploring appropriate transformations. The reference dataset for the BCSNF model was generated by the Adams numerical method for six scenarios by variation of the magnetic number, Brownian motion parameter, Prandtl number, bio-convection Lewis number, thermophoretic parameter, and bio-convection Peclet number. The approximate solutions were determined with 5–7 decimal places of accuracy and interpreted for the BCSNF model by the testing, training, and validation processes of the designed ANN-BR scheme. To check the efficiency of the introduced ANN-BR method, absolute error analysis, histogram studies, regression indices, and mean squared error (MSE) based figures of merit were used exhaustively to solve the variants of the BCSNF model involving gyro-tactic microorganisms with viscous dissipation, stratification, and heat immersion to study the influence of prominent parameters on the velocity, temperature, concentration, and motile density profiles.

Nomenclature

| Symbol | Description |
|--------|-------------|
| V      | Velocity profile (ms⁻¹) |
| u      | Velocity component in the x-direction (m s⁻¹) |
| v      | Velocity component in the y-direction (m s⁻¹) |
| w      | Velocity component in the z-direction (m s⁻¹) |
| (x, y, z) | Cartesian coordinates |
| Ec     | Eckert number |
| T, C  | Temperature (K) and concentration (M) |
| Tₐ, Cₐ | Ambient temperature (K) and ambient concentration (M) |
| ρf    | Nanofluid density (kg m⁻³) |
| ρm    | Microorganism density (kg m⁻³) |
| g     | Gravitational angle |
| q      | Non uniform heat generation/absorption |
| Dm    | Microorganism motile density |
| DB    | Brownian diffusion coefficient (m² s⁻¹) |
| DT    | Thermophoresis diffusion coefficient (m² s⁻¹) |
| f     | Dimensionless velocity component |
| C_f   | Skin friction coefficient |
| Pr    | Prandtl number |
| Nb    | Dimensionless Brownian motion parameter |
| Nt    | Dimensionless thermophoresis parameter |
| Lb    | Bio-convection Lewis number |
| M     | Hartmann number |
| Nr    | Buoyancy ratio parameter |
| D     | Diffusion coefficient (m² s⁻¹) |
| Tw    | Wall temperature (K) |
| Cw    | Wall concentration (M) |
| ρ     | Density (kg m⁻³) |
| v     | Kinematic viscosity (m² s⁻¹) |
| θ     | Dimensionless parameter |
| φ     | Dimensionless concentration |
| α     | Inclination angle |
| α₁    | Thermal diffusivity (m² s⁻¹) |
| η     | Dimensionless parameter |
| ξ     | Motile density |
| Rb    | Bio-convection Rayleigh number |
| S     | Thermal stratification |

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Mixed convection

Schmidt number

Bio-convection Peclet number

Microorganism concentration

Motile density

1. Introduction

Commercial products including bio-fertilizers, biofuel, etc. are prepared industrially by utilizing microorganisms, e.g. algae, cyanobacteria, and eukaryotic microalgae, that have metabolic characteristics and are useful in biofuel manufacturing (Bruzaite et al., 2020; Majid et al., 2018; Qin et al., 2020; Sartaj et al., 2020). Bio-nanoconvection dynamics involving microorganisms have tremendous medical applications and are of significant importance as a tactic in removing/reducing/controlling microorganism activities in human health (Basha & Sivaraj, 2021; Bhatti, Marin, et al., 2020; Waqas et al., 2019). Moreover, motile microorganisms have been extensively utilized by industry to prepare physical items such as biofuel, ethanol, hydrogen gas, biodiesel, etc. (Khan et al., 2017; Khan, Raja, et al., 2020). In view of promising applications and demand for biofluids, researchers and scientists are engaged in analyzing different problems involving microorganisms with bio-convective effects due to gyro-tactic microorganisms. They investigated flow over a convective wall. Their analysis found that self-propelled microorganisms present resistance to nanomaterial accumulation. Convective boundary layer fluid dynamics over a horizontal flat wall entrenched in a porous medium occupied by nanomaterial involving gyro-tactic microorganisms has been analyzed by Rao et al. (2021). The authors’ analysis found that bio-convexion constraints considerably affect the promulgation rate of motile microorganisms. Hydromagnetic flow of nanofluid involving gyro-tactic microorganisms in a boundary layer region past a vertical wall with slippage effects have been investigated by Kumaraswamy Naidu et al. (2021). Rashad & Nabwey (2019) analyzed hydromagnetic nanomaterial rheology with bio-convective effects due to gyro-tactic microorganisms. They investigated flow over a convective wall. Zhang et al. (2020) investigated the bio-convexion phenomenon involving oxytactic microorganisms in the rheology of nanomaterial. The authors analyzed a Riga plate incorporating Darcy–Brinkman–Forchheimer medium properties. A study of entropy generation in hydromagnetic radiative nanomaterial rheology involving gyro-tactic microparticles has been presented by Sohail et al. (2020). They considered the effects of chemical reaction and nonlinear thermal radiation. Heat and mass transport properties in bio-convexion nanomaterial rheology have been studied by Elanchezhiyan et al. (2020). They incorporated the effects of convective mass flux phenomena. Rehman et al. (2018) analyzed motile bio-convexion characteristics in hydromagnetic stratified nanomaterial. Awais, Awan, Raja, Parveen, et al. (2021) explored variable transport properties in the analysis of simultaneous heat and mass transfer in hydromagnetic bio-convective nanomaterial rheology. They discussed the properties of gyro-tactic microorganisms and performed computational numerical analysis. The properties of bio-convexion analysis of nanomaterial rheology with heat immersion effects have been presented by Awais, Awan, Raja, and Shoaib (2021). They evaluated stratification and viscous dissipation phenomena. Recent investigations related to gyro-tactic organism characteristics in bio-convective nanofluid flow with stratification can be found in Alhussain et al. (2021), Arafa et al. (2021), Chu et al. (2021), Haq et al. (2020), Khan, Nadeem, et al. (2020), Majeed et al. (2020) and Rana et al. (2020) and the references cited therein. All these techniques introduced for the analysis of gyro-tactic organism phenomena in bio-convective nanofluid flow models involve deterministic analytical and numerical solvers, while artificial intelligence (AI) methodologies based on stochastic numerical solvers look promising for the investigation of such systems.

The introduction of AI methods for the stochastic numerical analysis and precise detection of indent arrangements of these complex problems has been reported exhaustively (Khan et al., 2020; Peng et al., 2021). Recently, AI methods involving ‘learning’ and ‘generalization’ through artificial neural networks (ANNs), i.e. mathematical models inspired by human genetic processes, have been presented. In the past few years, researchers and scientists have utilized ANNs as modern AI techniques for analyzing several industrial and technical problems in the fields of weather forecasting (Andelković and Bajatović, 2020), nonlinear transport models (Çolak, 2021), nanofluid heat transfer management (Awais, Bibi, et al., 2021; Shoaib et al., 2021; Uddin et al., 2021), and medicine (Liu et al., 2021; Yang & Yu, 2021), as well as heuristic computational analysis with optimized cubic splines for nonlinear Thomas–Fermi systems (Ahmad et al., 2020). Mehmood et al. (2019) investigated integrated computing methodology for heat transfer analysis in the rheology of micropolar fluids. A new stochastic methodology for nonlinear Painlevé II equations arising in some applications of random matrix theory was presented by Raja et al. (2018). Neuro-evolutionary computing methodology for Painlevé II
equations with applications in nonlinear optics was analyzed by Ahmad et al. (2018). A fractional order cuckoo search algorithm for hyper-chaotic financial systems was presented by Yousri and Mirjalili (2020). Kang (2020) presented a recurrent neural network to predict fault diagnosis in chemical processes. Parveen et al. (2020) investigated the pressure rise phenomenon and heat transfer for hybrid nanomaterial rheology via ANNs. All these valuable contributions have inspired or motivated authors to investigate an AI based computing paradigm for solving fluid dynamics problems of paramount interest.

In the current investigation, our aim is to venture further into the regime of ANN applications. The contributions and innovative insights of the present study are summarized as follows.

- A novel AI based numerical computing application premised on artificial neural networks supported by a Bayesian regularization (ANN-BR) approach is presented to investigate the non-uniform heat absorption process and bio-convective flow dynamics of nanomaterial involving gyro-tactic microorganisms.
- The reference dataset for the BCSNF model is generated by exploiting the Adams numerical technique in Mathematica software for six scenarios by varying the magnetic number, Brownian motion parameter, Prandtl number, bio-convection Lewis number, thermophoretic parameter, and bio-convection Peclet number.
- Approximate solutions are calculated for the designed BCSN system by the ANN-BR testing, training, and validation process, and results are consistently found to be in good agreement with standard solutions.
- To check the efficiency of the designed ANN-BR, regression analyses, histogram studies, and mean squared error indices are used to analyze the solution of the BCSNF model effectively.
- Furthermore, the influence of prominent system model parameters on the velocity, temperature, concentration, and motile density profile is also examined and exhaustively interpreted for different physical scenarios.
- Ease in implementation, simplicity of the concept, provision of continuous solutions, reliability, stability, expendability, and convergence are noticeable characteristics of the proposed ANN-BR computing paradigm.

2. System model

Consider the flow dynamics of gyro-tactic microbes in a stratified bio-convective nanofluid as presented in

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  
(1)

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_f}[(\rho_p - \rho_f)g(C - C_{\infty})] - \frac{1}{\rho_f}[(n - n_{\infty})g\gamma(\rho_m - \rho_f)] + \frac{\mu}{\rho_f} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho_f} B_0^2 \sin^2(\alpha) u + (1 - C_{\infty}) \beta g(T - T_{\infty}) \]  
(2)

\[ u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{\mu}{\rho_c \rho_f} (\frac{\partial u}{\partial y})^2 + \frac{\sigma}{\rho_c} B_0^2 \sin^2(\alpha) u^2 + \frac{\bar{q}}{\rho_c \rho_f} \]  
(3)

\[ u \frac{\partial n}{\partial x} + v \frac{\partial n}{\partial y} + \frac{b W_c}{C_w - C_0} \left[ \frac{\partial}{\partial y} \left( n \frac{\partial C}{\partial y} \right) \right] = D_m \frac{\partial^2 n}{\partial y^2} \]  
(4)

Note that $u$ and $v$ are the velocities along the $x$- and $y$-directions. Moreover $\rho_f$, $\rho_m$, $T$, $\alpha$, and $\alpha$ are the nanofluid density, microorganism mass per unit volume ratio, temperature, thermal diffusivity, and inclination angle of the wall, respectively. Furthermore, $D_B$, $D_T$, $D_m$, and $\bar{q}$ represent the Brownian motion, thermophoresis, microorganisms motile density, non-uniform heat generation ($\bar{q} > 0$) or absorption ($\bar{q} < 0$), respectively. The
mathematical relation for $\tilde{q}$ is

$$\tilde{q} = \frac{k u_x}{x y} \left[ A (T_s - T_\infty) f' + B (T - T_\infty) \right]$$

The wall properties in the presence of stratification are expressed by as (Awais et al. (2021a))

$$u = \begin{cases} 
U_s = \alpha x, & v = 0, \\
T = T_s = T_0 + b_1 x, \\
C = C_0 + d_1 x, & \text{at } y = 0 \\
n = n_s = n_0 + \epsilon_1 x \\
0, & T \to T_\infty, \\
C \to C_\infty, & n = n_\infty \quad \text{as } y \to \infty
\end{cases}$$

Invoking the following variables

$$\eta = \sqrt{\frac{a}{v y}}, \quad \psi = \sqrt{\nu x f} (\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_s - T_0}$$

$$\phi(\eta) = \frac{C - C_\infty}{C_s - C_0}, \quad \xi(\eta) = \frac{n - n_\infty}{n_s - n_0}$$

Equations (2)–(7) become (Awais et al. (2021a))

$$\frac{d^3 f}{d \eta^3} + f \frac{d^2 f}{d \eta^2} - M^3 \sin^2(\alpha) \frac{df}{d \eta} - \left( \frac{df}{d \eta} \right)^2 + \lambda (\theta - N_\tau \phi - Rb \xi) = 0$$

$$\frac{d^2 \theta}{d \eta^2} + N_b \frac{d \phi}{d \eta} \frac{d \theta}{d \eta} + N_t \left( \frac{d \theta}{d \eta} \right)^2$$

$$+ M^2 \sin^2(\alpha) \left( \frac{df}{d \eta} \right)^2 + A_1 \frac{df}{d \eta}$$

$$+ A_2 \theta + Pr \left( f \frac{d \theta}{d \eta} + Ec \left( \frac{d^2 f}{d \eta^2} \right)^2 - S \frac{df}{d \eta} - \frac{df}{d \eta} \right)$$

$$= \frac{df}{d \eta}$$

$$\frac{d^2 \varphi}{d \eta^2} + Sc \left( f \frac{d \varphi}{d \eta} - \frac{df}{d \eta} \varphi - Q \frac{df}{d \eta} \right) + \frac{N_t}{N_b} \frac{d^2 \theta}{d \eta^2} = 0$$

$$\frac{d^2 \xi}{d \eta^2} + Lb \left( f \frac{d \xi}{d \eta} - \frac{df}{d \eta} \xi - B \frac{df}{d \eta} \right)$$

$$- Pe \left[ \frac{d^2 \varphi}{d \eta^2} (\xi + \Omega) + \frac{df}{d \eta} \frac{d \xi}{d \eta} \right] = 0$$

along with the wall properties

$$f(0) = 0, \quad \theta(0) = 1 - S, \quad \frac{df}{d \eta}(0) = 1, \quad \varphi(0) = 1 - Q,$$

$$\xi(0) = 1 - B \frac{df}{d \eta}(\infty) = \theta(\infty) = \varphi(\infty) = \xi(\infty) = 0$$

### 3. Solution methodology

The necessary description of the solution methodology adopted for the BCSNF model as represented with Equations (9)–(13) is presented here based on ANNs backpropagated with Bayesian regularization. The overall work flow of the solution methodology is presented in Figure 2, the layer structures are portrayed in Figure 3, and a thorough development of the single neuron model is shown in Figure 4. The neural networks environment in MATLAB software is exploited via ‘nftool’ for the execution of the developed scheme based on ANNs backpropagated with Bayesian regularization, i.e. ANN-BR. The solution procedure comprises a significant dataset description and an execution process for executing the proposed ANN-BR.

The mathematical expressions for bio-convective stratified nanofluid flow (BCSNF) model (9–12) using numerical values for scenario 5 of case 3 can be expressed as follows:

$$\frac{d^3 f}{d \eta^3} + f \frac{d^2 f}{d \eta^2} - 0.75 \frac{df}{d \eta} - \left( \frac{df}{d \eta} \right)^2 + 0.5 \theta$$

$$- 0.25 \varphi - 0.05 \xi = 0$$

$$\frac{d^2 \theta}{d \eta^2} + 0.5 \frac{d \varphi}{d \eta} \frac{d \theta}{d \eta} + 1.5 \left( \frac{d \theta}{d \eta} \right)^2 + 3 \left( \frac{df}{d \eta} \right)^2 + 0.4 \frac{df}{d \eta}$$

$$+ 0.5 \theta + f \frac{d \theta}{d \eta} + 0.1 \left( \frac{d^2 f}{d \eta^2} \right)^2 - 0.5 \frac{df}{d \eta} - \frac{df}{d \eta} = 0$$

$$\frac{d^2 \varphi}{d \eta^2} + 0.5 \left( f \frac{d \varphi}{d \eta} - \frac{df}{d \eta} \varphi - 0.5 \frac{df}{d \eta} \right) + 3.0 \frac{d^2 \theta}{d \eta^2} = 0$$

$$\frac{d^2 \xi}{d \eta^2} + 0.1 \left( f \frac{d \xi}{d \eta} - \frac{df}{d \eta} \xi - 0.5 \frac{df}{d \eta} \right) - 0.1 \left[ \frac{d^2 \varphi}{d \eta^2} (\xi + 0.5) + \frac{df}{d \eta} \frac{d \xi}{d \eta} \right] = 0$$

along with the boundary conditions

$$f(0) = 0, \quad \theta(0) = 0, \quad \varphi(0) = \xi(0) = 0.5, \quad \frac{df}{d \eta}(0) = 1,$$

$$\frac{df}{d \eta}(\infty) = \theta(\infty) = \varphi(\infty) = \xi(\infty) = 0$$

In above equations, $M$, $N_b$, $N_t$, $Sc$, $\lambda$, $\alpha$, $N_r$, $Rb$, and $Pr$ represent the Hartman number, the Brownian motion index, the thermophoresis parameter, the Schmidt number, the mixed convective parameter, the inclination angle, the buoyancy ratio parameter, the bio-convection parameter, and the Prandtl number, respectively. Note
**Figure 2.** Workflow diagram of the proposed ANN-BR algorithm for solving the BCSNF model.

**Figure 3.** A mathematical equivalent representation of a neuron model in neural network methodology for the BCSNF model.
that $\lambda > 0$ represents assisting flow, $\lambda < 0$ indicates opposing flow and $\lambda = 0$ implies the forced convection effect. Moreover $S = b_2/b_1$, $Ec$, $Lb$, $Pe$, $\Omega = d_2/d_1$, $B = e_2/e_1$, respectively, represent the thermal stratification, the Eckert number, the bio-convection Lewis number, the bio-convection Peclet number, the concentration difference of microorganisms, the mass stratification number, and the motile density. Furthermore, $A$ and $B$ represent the heat generation and absorption phenomena. For non-uniform heat generation, $A > 0$ and $B > 0$, while for internal heat absorption, $A < 0$ and $B < 0$. These dimensionless quantities are expressed mathematically as follows:

$$M^2 = \frac{\sigma B_0^2}{\rho c}, \quad Sc = \frac{\nu}{D_B}, \quad N_b = \frac{\tau D_B (C_s - C_\infty)}{\nu}, \quad \lambda = \frac{Gr_x}{Re_x^2},$$

$$Gr_x = \frac{g_0 \beta_T (T_s - T_\infty) x^3}{\nu^2}, \quad Re_x = \frac{U_j x}{\nu},$$

$$N_i = \frac{\tau D_T (T_s - T_\infty)}{v T_\infty},$$

$$Pr = \frac{\nu}{a_m}, \quad Ec = \frac{U_i^2}{c_p (T_s - T_2)}, \quad Lb = \frac{\nu}{D_m}, \quad Pe = \frac{b W_c}{D_m}.$$

Similarly, mathematical expressions for all scenarios of all cases for the BCSNF model can be expressed in the same manner. The reference dataset of the proposed ANN-BR is generated through the Ademical numerical approach (Awais, Raja, et al., 2021; Awan et al., 2020, Awan, Awais, et al., 2021, Awan, Raja, et al., 2021; Qureshi et al., 2021; Ullah, Ali, et al., 2021, Ullah, Hayat, Alsaedi, et al., 2021) for inputs within the range of 0–3 having a time interval 0.05. For the generation of a reference dataset, the Mathematica software package was utilized through the built-in routine ‘NDSolve’ by varying the magnetic number, the Brownian motion parameter, the Prandtl number, the bio-convection Lewis number, the thermophoretic parameter, and the bio-convection Peclet number as represented in Table 1. The values of the physical parameters of interest in the system model, i.e. Equations (14)–(17) as tabulated in Table 1, are set with extreme care, detailed literature review, convergence, and stability analysis of the model for an inclusive and exhaustive description of the findings.

The reference datasets for the velocity profile, i.e. $f(\eta)$, $f'(\eta)$, and $f''(\eta)$, the temperature profile, i.e. $\theta(\eta)$ and $\theta'(\eta)$, the concentration profile $\varphi(\eta)$ and $\varphi'(\eta)$, and the motile density profile $\xi(\eta)$ and $\xi'(\eta)$ are generated for 61 inputs in which 75% of the data are utilized for training, 20% for testing, and 5% for validation of the proposed ANN-BR using a neural network, as shown in Figure 4.

4. Analysis and discussion of the results

The outcomes of the BCSNF model presented in Equations (9)–(13) through artificial numerical computation for the developed artificial neural networks backpropagated with Bayesian regularization are analyzed. The six different scenarios of the BCSNF model by varying the magnetic number, the Brownian motion parameter, the

| Scenario | Case | $M$ | $N_b$ | Pr | Lb | $N_i$ | Pe |
|----------|------|----|------|----|----|-------|----|
| 1        | 1    | 0.5| 0.5  | 1.0| 0.1| 0.5   | 0.1|
| 2        | 1    | 1.0| 0.5  | 1.0| 0.1| 0.5   | 0.1|
| 3        | 1.5  | 0.5| 1.0  | 0.1| 0.5| 0.1   | 0.1|
| 4        | 2.0  | 0.5| 1.0  | 0.1| 0.5| 0.1   | 0.1|
| 5        | 1    | 1.0| 0.1  | 1.0| 0.1| 0.5   | 0.1|
| 6        | 1    | 1.0| 0.1  | 1.0| 0.1| 0.5   | 0.1|
| 3        | 1    | 1.0| 0.5  | 0.5| 0.1| 0.5   | 0.1|
| 2        | 1.0  | 0.5| 1.0  | 0.1| 0.5| 0.1   | 0.1|
| 3        | 1.0  | 0.5| 1.5  | 0.1| 0.5| 0.1   | 0.1|
| 4        | 1.0  | 0.5| 2.0  | 0.1| 0.5| 0.1   | 0.1|
| 1        | 1.0  | 0.5| 1.0  | 0.1| 0.5| 0.1   | 0.1|
| 2        | 1.0  | 0.5| 1.0  | 0.4| 0.5| 0.1   | 0.1|
| 3        | 1.0  | 0.5| 1.0  | 0.7| 0.5| 0.1   | 0.1|
| 4        | 1.0  | 0.5| 1.0  | 1.0| 0.5| 0.1   | 0.1|
| 5        | 1    | 1.0| 0.5  | 1.0| 0.1| 0.5   | 0.1|
| 6        | 1    | 1.0| 0.5  | 1.0| 0.1| 0.5   | 0.1|
| 3        | 1.0  | 0.5| 1.0  | 0.1| 0.5| 0.1   | 0.1|
| 4        | 1.0  | 0.5| 1.0  | 0.1| 0.5| 0.1   | 0.1|
| 1        | 1.0  | 0.5| 1.0  | 0.1| 0.5| 0.1   | 0.1|
| 2        | 1.0  | 0.5| 1.0  | 0.1| 0.5| 0.1   | 0.1|
| 3        | 1.0  | 0.5| 1.5  | 0.1| 0.5| 0.1   | 0.1|
| 4        | 1.0  | 0.5| 2.0  | 0.1| 0.5| 0.1   | 0.1|

Figure 4. Neural network architecture for solving the BCSNF model.
Prandtl number, the bio-convection Lewis number, the thermophoretic parameter, and the bio-convection Peclet number are formulated for four different cases for the velocity, temperature, concentration, and motile density profile of the BCSNF model as listed in Table 1.

The reference dataset for the velocity profile, i.e. \( f(\eta) \), \( f'(\eta) \), and \( f''(\eta) \), the temperature profile, i.e. \( \theta(\eta) \) and \( \theta'(\eta) \), the concentration profile \( \phi(\eta) \) and \( \phi'(\eta) \), and the motile density profile \( \xi(\eta) \) and \( \xi'(\eta) \) of the developed ANN-BR network as shown in Figure 1. The obtained dataset in terms of the velocity profile, i.e. \( f(\eta) \), \( f'(\eta) \), and \( f''(\eta) \), temperature profile, i.e. \( \theta(\eta) \) and \( \theta'(\eta) \), concentration profile \( \phi(\eta) \) and \( \phi'(\eta) \), and motile density profile \( \xi(\eta) \) and \( \xi'(\eta) \) are used later on as the reference outcome in the current study.

The ANN-BR executes the solution of the bio-convection nanofluid flow model by using the ‘nftool’ built-in command in MATLAB neural networks toolbox. The reference/standard datasets for velocity profile, i.e. \( f(\eta) \), \( f'(\eta) \), and \( f''(\eta) \), the temperature profile, i.e. \( \theta(\eta) \) and \( \theta'(\eta) \), the concentration profile \( \phi(\eta) \) and \( \phi'(\eta) \), and the motile density profile \( \xi(\eta) \) and \( \xi'(\eta) \) of the developed ANN-BR is generated by utilizing the strengths of the Adams numerical approach in the range 0–3, having a step size of 0.05 for all four cases of each scenario of the BCSNF model. The obtained dataset in terms of the velocity profile, i.e. \( f(\eta) \), \( f'(\eta) \), and \( f''(\eta) \), temperature profile, i.e. \( \theta(\eta) \) and \( \theta'(\eta) \), concentration profile \( \phi(\eta) \) and \( \phi'(\eta) \), and motile density profile \( \xi(\eta) \) and \( \xi'(\eta) \) are created for 61 inputs in which 75% of the data samples are utilized for training, 20% for testing and 5% for validation of the ANN-BR networks as shown in Figure 1.

The solutions of ANN-BR for all six scenarios of various cases in terms of performance index, state transition, histogram plots, regression, and fitness function are illustrated in Figures 5–10. Furthermore, convergence via MSE learning curves for testing and training samples, best performance index, gradient, Mu, epochs, sum of squares parameters, effective parameters, and time taken are presented in Tables 2–7 for each scenario of all cases.

Figure 5 illustrates the outputs of ANN-BR for case 3 of scenario 1 of the BCSNF model. Figure 5(a) shows the convergence or learning curves on MSE for both training and testing, with the best MSE training performance being 5.53303E-12, which is achieved at 132 epochs. Figure 5(b) shows the state transition results and it is observed that the gradient and Mu parameter of the Bayesian regularization are 1.1302E-08 and 5000, respectively, at 132 epochs, whereas the \( \text{Nu}_m \) parameter and sum of squares parameter values are 100.4185 and 112.2329, respectively. The error histogram plot is shown in Figure 5(c), while correlation studies are also performed to investigate the regression analysis as illustrated in Figure 5(d). The efficient outcome of ANN-BR is achieved by matching outcomes of the Adams numerical solver for scenario 2 of case 3 as shown in Figure 5(e). One may observe that the correlation value \( R \) being close to unity specifies perfect modeling in terms of training and testing, which certifies the correctness of the proposed ANN-BR for the BCSNF model. Table 2 is constructed for scenario 1 of all four cases and it is noticed that MSE is around E-10 to E-13, while the gradient is E-05 to E-08. Moreover, the Mu, epoch, effective parameter, sum of squares parameter and time taken for the case of scenario 1 are listed in Table 2.

Figure 6 illustrates the results/outcomes of ANN-RB for case 4 of scenario 2 of the BCSNF model. Figure 6(a) shows the learning curves on MSE for both training and testing samples with the best MSE training performance being 5.53303E-12, which is achieved at 132 epochs. Figure 6(b) shows the state transition results and it is observed that the gradient and Mu parameter of the Bayesian regularization are 1.1302E-08 and 5000, respectively, at 132 epochs, whereas the \( \text{Nu}_m \) parameter and sum of squares parameter values are 100.4185 and 112.2329, respectively. The error histogram plot is shown in Figure 6(c), while correlation studies are also performed to investigate the regression analysis as illustrated in Figure 6(d). The efficient outcome of ANN-BR is achieved by matching outcomes of the Adams numerical solver for scenario 2 of case 4 as shown in Figure 6(e). One may observe that a correlation value \( R \) close to unity specifies perfect modeling in terms of training and testing, which certifies the correctness of the proposed ANN-BR for the BCSNF model. Table 3 is constructed for scenario 2 of all four cases and it is noticed that the MSE is around E-09 to E-13, while the gradient is E-06 to E-08. Moreover, Mu, epoch, effective parameter, sum of squares parameter and time taken for the case of scenario 2 is listed in Table 3.

Figure 7 represents the outcomes of ANN-RB for scenario 3 of case 3 of the BCSNF model. Figure 7(a) shows the convergence of MSE for both training and testing with the best MSE training performance being 1.681E-12, which is achieved at 134 epochs. Figure 7(b) shows the state transition results and it is observed that the gradient and Mu parameter of Bayesian regularization are 8.929E-08 and 500, respectively, at 134 epochs, whereas the \( \text{Nu}_m \) parameter and sum of squares parameter values are 110.1462 and 160.6779, respectively. The error histogram plot is shown in Figure 7(c), while \( C_0 \) relation studies are also performed to investigate the regression analysis as illustrated in Figure 7(d). The efficient outcome of ANN-BR is achieved by matching outcomes of the Adams numerical solver for scenario 3 of case 3 as presented in Figure 7(e). One may observe that the correlation value \( R \) being close to unity specifies perfect modeling in terms of training and testing, which certifies the correctness of the proposed ANN-BR for the BCSNF model. Table 4 is constructed for scenario 3 of all four
Figure 5. Outcomes of ANN-BR for scenario 1 of case 3 for solving the BCSNF model.
Figure 6. Outcomes of ANN-RB for scenario 2 for solving the BCSNF model.
Figure 7. Outcomes of ANN-RB for scenario 3 of case 3 for solving the BCSNF model.
Figure 8. Outcomes of ANN-RB for scenario 4 of case 4 for solving the BCSNF model.
Figure 9. Outcomes of ANN-RB for scenario 5 for solving the BCSNF model.
Figure 10. Outcomes of ANN-RB for scenario 6 for solving the BCSNF model.
Table 2. Summary of results for ANN-BR in the case of scenario 1 of the BCSNF model.

| Case | Training MSE | Testing MSE | Performance index | Gradient | Mu | Epoch | Sum of squares parameter | Effective parameter | Time |
|------|--------------|-------------|-------------------|----------|----|-------|--------------------------|---------------------|------|
| 1    | 5.85E-12     | 6.92E-13    | 5.86E-12          | 9.86E-08 | 500| 97    | 79.7                     | 99.8                | 0.00 |
| 2    | 3.49E-11     | 1.22E-11    | 3.50E-11          | 2.75E-08 | 500| 75    | 93.9                     | 96.4                | 0.00 |
| 3    | 1.30E-10     | 2.86E-10    | 1.30E-10          | 1.73E-07 | 500| 1000  | 6.09E+03                 | 98.6                | 0.00 |
| 4    | 3.12E-10     | 3.64E-10    | 3.13E-10          | 3.24E-05 | 500| 1000  | 6.28E+03                 | 106                 | 0.00 |

Table 3. Summary of results for ANN-BR in the case of scenario 2 of the BCSNF model.

| Cases | Training MSE | Testing MSE | Performance index | Gradient | Mu | Epoch | Sum of squares parameter | Effective parameter | Time |
|-------|--------------|-------------|-------------------|----------|----|-------|--------------------------|---------------------|------|
| 1     | 2.022E-09    | 3.15E-09    | 2.02E-09          | 8.24E-06 | 500| 1000  | 6.30E+03                 | 105                 | 0.00 |
| 2     | 1.478E-12    | 3.849E-13   | 1.48E-12          | 9.91E-08 | 500| 254   | 157                      | 105                 | 0.00 |
| 3     | 1.649E-11    | 1.171E-11   | 1.65E-11          | 1.83E-08 | 500| 113   | 116                      | 101                 | 0.00 |
| 4     | 5.33E-12     | 1.01E-10    | 5.53E-12          | 1.13E-08 | 500| 132   | 112                      | 100                 | 0.00 |

cases and it is noticed that the MSE is E-11 to E-13, while the gradient is E-08. Moreover, the Mu, epoch, effective parameter, sum of squares parameter and time taken for the case of scenario 3 are listed in Table 4.

Figures 8–10 represent the outcomes of ANN-RB for scenario 4 of case 4, scenario 5 of case 4 and scenario 6 of case 3 of the BCSNF model. Figures 8(a), 9(a), and 10(a) show the convergence of the MSE for both training and testing with the best MSE training performances being 4.407E-12, 5.792E-13, and 1.935E-12 for scenarios 4–6, respectively, which are achieved at 136, 321, and 191 epochs for scenarios 4–6, respectively. Figures 8(b), 9(b), and 10(b) show the state transition results and it is observed that the gradient and Mu parameter of Bayesian regularization are 1.291E-08, 3.5569E-09, and 7.456E-09 and Mu 5000 at epochs 136, 321, and 191 for scenarios 4–6, whereas the Nu_m parameters and sum of squares parameter values are 105.5395, 110.7292, and 107.681, and 135.4491, 159.7407, and 231.5897, for scenarios 4–6, respectively. Error histogram plots are shown in Figures 8(c), 9(c), and 10(c), while correlation studies are also performed to investigate the regression analysis as illustrated in Figures 8(d), 9(d), and 10(d) for scenarios 4–6, respectively. The efficient outcome of ANN-BR is achieved by matching the outcomes of the Adams numerical solver for scenario 4 of case 4, scenario 5 of case 1, and scenario 6 of case 2 as illustrated in Figures 8(e), 9(e), and 10(e) for scenarios 4–6, respectively. One may witness that correlation values R close to unity specify perfect modeling in terms of training and testing, which certifies the correctness of the proposed ANN-BR for the BCSNF model. Tables 5–7 are constructed for scenarios 4–6 of all four cases and it is noticed that the MSEs are around E-11 to E-12, E-11 to E-13, and E-11 to E-13, respectively, while the gradients are E-08 to E-09 for each scenario. Moreover, the Mu, epoch, effective parameter, sum of squares parameter values are 105.5395, 110.7292, and 107.681, and 135.4491, 159.7407, and 231.5897, for scenarios 4–6, respectively.

Table 4. Summary of results for ANN-BR in case of scenario 3 of the BCSNF model.

| Cases | Training MSE | Testing MSE | Performance index | Gradient | Mu | Epoch | Sum of squares parameter | Effective parameter | Time |
|-------|--------------|-------------|-------------------|----------|----|-------|--------------------------|---------------------|------|
| 1     | 1.83E-12     | 4.11E-13    | 1.83E-12          | 9.45E-08 | 500| 187   | 158                      | 103                 | 0.01 |
| 2     | 9.31E-13     | 1.49E-13    | 9.31E-13          | 9.97E-08 | 500| 352   | 201                      | 106                 | 0.01 |
| 3     | 1.68E-12     | 9.45E-12    | 1.68E-12          | 8.93E-08 | 500| 134   | 161                      | 110                 | 0.00 |
| 4     | 1.17E-11     | 2.04E-11    | 1.18E-11          | 1.53E-08 | 500| 84    | 106                      | 105                 | 0.00 |

Table 5. Summary of results for ANN-BR in case of scenario 4 of the BCSNF model.

| Cases | Training MSE | Testing MSE | Performance index | Gradient | Mu | Epoch | Sum of squares parameter | Effective parameter | Time |
|-------|--------------|-------------|-------------------|----------|----|-------|--------------------------|---------------------|------|
| 1     | 3.90E-11     | 1.33E-11    | 3.90E-11          | 3.21E-08 | 500| 84    | 107                      | 90.5                | 0.00 |
| 2     | 6.27E-12     | 3.07E-12    | 6.28E-12          | 1.13E-08 | 500| 135   | 142                      | 100                 | 0.00 |
| 3     | 1.75E-12     | 2.04E-11    | 1.76E-12          | 4.37E-09 | 500| 184   | 157                      | 101                 | 0.00 |
| 4     | 4.40E-12     | 1.84E-11    | 4.41E-12          | 1.29E-08 | 500| 136   | 135                      | 106                 | 0.01 |
Table 6. Summary of results for ANN-BR in case of scenario 5 of the BCSNF model.

| Cases | Training | Testing | Performance index | Gradient | Mu   | Epoch | Sum of squares parameter | Effective parameter | Time  |
|-------|----------|---------|-------------------|----------|------|-------|-------------------------|---------------------|-------|
| 1     | 5.70E-11 | 5.70E-11| 5.70E-11          | 7.12E-08 | 5000 | 64    | 105                     | 101                 | 0.00.00|
| 2     | 1.79E-12 | 2.68E-13| 1.79E-12          | 9.99E-08 | 500  | 315   | 155                     | 106                 | 0.00.01|
| 3     | 2.55E-11 | 8.13E-11| 2.55E-11          | 3.26E-08 | 5000 | 111   | 111                     | 101                 | 0.00.00|
| 4     | 5.79E-13 | 2.51E-10| 5.79E-13          | 3.36E-09 | 5000 | 321   | 321                     | 0.00.00             | 0.00.01|

Table 7. Summary of results for ANN-BR in case of scenario 6 of the BCSNF model.

| Cases | Training | Testing | Performance index | Gradient | Mu   | Epoch | Sum of squares parameter | Effective parameter | Time  |
|-------|----------|---------|-------------------|----------|------|-------|-------------------------|---------------------|-------|
| 1     | 3.26E-11 | 5.31E-11| 3.26E-11          | 4.22E-08 | 5000 | 75    | 118                     | 102                 | 0.00.00|
| 2     | 7.31E-13 | 8.03E-13| 7.32E-13          | 3.93E-09 | 5000 | 281   | 217                     | 110                 | 0.00.01|
| 3     | 1.93E-12 | 4.00E-12| 1.94E-12          | 7.46E-09 | 5000 | 191   | 232                     | 108                 | 0.00.01|
| 4     | 5.06E-13 | 2.98E-13| 5.07E-13          | 2.87E-09 | 5000 | 437   | 283                     | 106                 | 0.00.02|

The reliable, precise and stable performance of the proposed design paradigm ANN-BR is verified and validated by the numerical data in Tables 2–7.

4.1. Effects of prominent parameters on velocity, temperature, concentration and motile density profile

Figures 11–21 are plotted in order to show a comparison between ANN-BR along with the Adams numerical method for velocity, temperature, concentration, and motile density profile. The outcomes of ANN-BR were examined via MATLAB software for investigating the influence of the variation of the magnetic number on velocity, temperature, and concentration profile with an absolute error as presented in Figures 11–21.

From Figure 11(a), it is noticed that the velocity profile decreases with an enhancement in the magnetic number values, whereas the absolute error (AE) is found to be in the range $10^{-04} \rightarrow 10^{-08}$, which proves the validity of the proposed algorithm, as shown in Figure 11(b).

From the point of view of physical and technical analysis, it can be stated that, for the situation in which any liquid is exposed to a magnetic field, then its seeming viscosity surges to critical values up to the point of viscoelastic solid. In this regard, it is noted that the yield stress of liquids can be controlled with the assistance of magnetic field variations. These technical properties give rise to several control-based applications in engineering processes including hydro-magnetic power generation, electromagnetic casting of metals, ion propulsion, generators, etc. It can be observed from

![Figure 11](image-url)

(a) Variation of $M$ with $f'(\eta)$

(b) Analysis on AE

Figure 11. Comparison of ANN-RB outcomes from numerical solutions for $M$ with $f'(\eta)$.
Figure 12. Comparison of ANN-RB outcomes from numerical solutions for \( M \) with \( \eta \).

Figure 13. Comparison of ANN-RB outcomes from numerical solutions for \( M \) with \( \eta \).

Figure 12(a) that temperature enhances with increments in the values of \( M \), while the opposite trend is noticed for the concentration profile with the variation of \( M \), as presented in Figure 13(a). The AE for both \( \theta(\eta) \) and \( \phi(\eta) \) for various values of \( M \) is found to be in the negligible range, i.e. \( 10^{-04} \rightarrow 10^{-08} \), which validates the proposed algorithm. Increase in the magnetic field retards the liquid velocity, which results in less molecular movement. The vibrations of molecules are the main cause of temperature control. For the case when molecular vibration is low, temperature rise within the system results in increased temperature profile, whereas the opposite behavior is observed for the concentration of species. Figures 14 and 15 illustrated the variation of the Brownian motion parameter with both temperature and concentration profile, respectively. It is found from Figure 14(a) that \( \eta \) increases with an enhancement in the values of \( N_b \), whereas the opposite trend is noticed for the concentration profile as presented in Figure 15(a). The AE for both \( \theta(\eta) \) and \( \phi(\eta) \) for different values of \( N_b \) found in the negligible limit is illustrated in Figures 14(b) and 15(b). A rise in the Brownian motion parameter enhances the temperature profile owing to the fact that the Brownian motion parameter is directly proportional to temperature and inversely proportional to the kinematic viscosity of the material. Larger values of the Brownian motion parameters increase the temperature profile and decreases in the concentration profile. The influence of the Prandtl number \( Pr \) on \( \theta(\eta) \) and \( \phi(\eta) \) is presented in Figures 16 and 17, respectively. It is noticed from Figure 16 that \( \eta \) decreases for higher values of \( Pr \), while \( \eta \) enhances for higher values of \( Pr \) as
Figure 14. Comparison of ANN-RB outcomes from numerical solutions for $N_b$ with $\eta$.

Figure 15. Comparison of ANN-RB outcomes from numerical solutions for $N_b$ with $\eta$.

Figure 16. Comparison of ANN-RB outcomes from numerical solutions for $Pr$ with $\eta$. 
Figure 17. Comparison of ANN-RB outcomes from numerical solutions for Pr with $\eta$. The AE observed from Figures 16(b) and 17(b) lies within the limits $10^{-05}$ to $10^{-09}$, hence it can easily be neglected and verifies the correctness of the proposed ANN-BR algorithm. The Prandtl number is the ratio of momentum diffusivity to thermal diffusivity. For larger values of the Prandtl number, diffusive momentum dominates thermal diffusivity results, decreasing the temperature profile significantly.

Figure 18. Comparison of ANN-RB outcomes from numerical solutions for Lb with $\eta$. The effect of varying the bio-convection Lewis number on the motile density profile is presented in Figure 18. It is quite clear from Figure 18(a) that an enhancement in value of Lb will tend to decrease the motile density profile, which means that motile density has lower values for higher values of Lb. From a physical point of view, the bio-convective Lewis number Lb has an inverse relationship with microorganism motile density, and therefore larger values of bio-convective Lewis number Lb result in a decrease in microorganism motile density. The AE exists in the negligible range, i.e. $10^{-05}$ to $10^{-09}$, as illustrated in Figure 18(b).
Figure 19. Comparison of ANN-RB outcomes from numerical solutions for $N_t$ with $\eta$.

Figure 20. Comparison of ANN-RB outcomes from numerical solutions for $N_t$ with $\eta$.

Figure 21. Comparison of ANN-RB outcomes from numerical solutions for $Pe$ with $\eta$. 
in Figure 20(a). The validity of the proposed algorithm can be verified from the graphs of AE, which is found to be within the limits of $10^{-04} → 10^{-09}$ as presented in Figures 19(b) and 20(b). The variation of the bio-convection Peclet number with the motile density profile is depicted in Figure 21(a) mean, while it is noticed that the motile density profile decreases for higher values of Pe owing to the fact that the bio-convection Peclet number is inversely proportional to the microorganism motile density. It is also observed from Figure 21(b) that AE is found to be within negligible limits, i.e. $10^{-04} → 10^{-08}$ hence the correctness of our proposed algorithm is verified.

5. Conclusions

A novel application of an intelligent numerical computing paradigm via artificial neural networks optimized with a Bayesian regularization approach has been presented for the investigation of the non-uniform heat absorption process with the bio-convective flow dynamics of nanomaterial involving gyro-tactic microorganisms. The PDEs governing the gyro-tactic microorganism characteristics in a bio-convective nanofluid flow model with a stratification process were transformed into nonlinear ODEs through similarity variables. The state-of-the-art Adams numerical method was applied for the generation of a dataset of transformed ODEs to model the BCSNF to measure the effects of velocity, temperature, concentration, and motile density profiles on various physical parameters such as the magnetic number, the Brownian motion parameter, the Prandtl number, the Lewis number, the thermophoretic parameter, and the Peclet number. In the ANN-BR method, 75% arbitrarily selected data were used for training, 20% for testing and 5% for validation to find approximate solutions with the BCSNF model for each scenario. The proposed and reference outcomes show the authenticity of the model with a precision of the order of $10^{-11}$ to $10^{-04}$ consistently for each case. The reliability, stability, and convergence of the proposed ANN-BR algorithm were further certified by a mean squares errors based fitness metric, histogram illustrations, and regression analysis for each variant of the BCSNF model.

In the future, new variants of artificial intelligence based integrated intelligent networks (Ahmadi, Sadeghzadeh, et al., 2019; Alotaibi et al., 2020; Duan et al., 2020; Park et al., 2020; Sabir et al., 2020) will be developed by the interested researcher for solving stiff/nonlinear/singular systems representing the fluid mechanics models (Ahmadi, 2021; Ahmadi, Mohseni-Gharyehsafa, et al., 2019; Awais, Raja, et al., 2021; Siddiqua et al., 2018; Ullah, Hayat, et al., 2021; Ullah, Ullah, et al., 2021), circuit theory dynamics (Mehmood, Zameer, Aslam, et al., 2020, Mehmood, Zameer, Ling, et al., 2020), mathematical biological systems (Umar et al., 2020; Wang et al., 2020), information security models (Liu et al., 2020), and astro/plasma/nuclear/atomic physics models (Bukhari et al., 2020; Ilyas et al., 2021; Jadoon et al., 2021; Zameer et al., 2020).

Disclosure statement

No potential conflict of interest was reported by the authors.

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