Adaptive iterative error recovery detection algorithm for uplink massive multiple-input multiple-output systems

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Abstract
In this paper, an error recovery (ER) detection based on the lattice reduction (LR) algorithm with adaptive iteration is presented for uplink massive multiple-input multiple-output (MIMO) systems to obtain an improved bit error rate (BER) performance, especially when the number of active users increases. In the proposed algorithm, as the result of LR preprocessing, a larger diagonally dominant index (DDI) is used to measure the diagonal dominance of gram matrix for the initialisation of the ER detection. In addition, the larger DDI could be further used to realise adaptive iteration, which helps to strike a trade-off between the BER performance and the computational complexity. Moreover, two properties of the DDI are derived, and the simulation results show that the larger DDI leads to better detection performance, and it could be widely applied to linear elimination and MIMO detection algorithms.

1 | INTRODUCTION

In February 2019, the 12th issue of Cisco Mobile Visual Networking Index Forecast (2017–2022) was released. According to Cisco’s prediction, the overall mobile data traffic is expected to grow to 77 exabytes per month by 2022, which should be a seven-fold increase over that of 2017 [1]. To sustain speed exceeding 1 Gbps and ultra-low latency communications, massive multiple-input multiple-output (MIMO) transmission technology was applied in the 5th generation wireless systems, where hundreds of antennas at the base station (BS) were implemented to serve tens of single-antenna users [2, 3].

However, the large number of antennas will pose challenges for the receiver to perform signal detection efficiently, and various detectors have been proposed to tackle this problem [4–13]. Inspired by machine learning and artificial intelligence, Chockalingam et al. proposed a series of metaheuristic detectors such as likelihood ascent search detector [4, 5], reactive taboo search detector [6], and belief propagation detector [7, 8]. Those detectors in [4–8] are appealing for massive MIMO systems, which have a similarly large number of transmit and receive antennas. However, they are computationally uncompetitive when the number of active users is much smaller than the number of BS antennas. Alternatively, some others focused on iterative approaches for solving a system of linear equations derived from the minimum mean-squared error (MMSE) criterion in uplink massive MIMO systems, including error recovery (ER) algorithm [9], Newton iteration (NI) [10, 11], Richardson iteration (RI) [12], and Jacobi iteration [13] and so forth. Unfortunately, the algorithms in [9–13] all suffer from significant performance loss as the number of active users increases, and they are normally set as invariable iterations.

In this paper, a modified ER method is proposed to improve the bit error rate (BER) performance so that the detector performs well even the number of active users increases. In the modified method, the diagonally dominant index (DDI) is first introduced to evaluate the diagonal dominance of a non-zero square matrix, which helps to measure the orthogonality of matrices. Then, the properties of the DDI are mathematically derived for the gram matrix and triangular matrix, which could be widely used in linear elimination.

On the other hand, researchers have proposed several methods to improve the BER performance [14, 15]. Usually, these methods are based on obtaining a better initial solution or utilising interference cancellation. In [14], the authors utilised the faster decision and better initial solution for iteration reduction.
and performance enhancement in Gauss–Seidel Iteration detection algorithm. Reference [15] improved the interferences from different users by iterative sequential detection to obtain the performance enhancement and iteration reduction. Though these two algorithms used different ways to approach the performance enhancement, their performance also decreases rapidly with the number of users increases. Notably, the lattice reduction (LR) preprocessing [16–20] is a feasible method to improve the orthogonality of the channel matrix, which can be transferred to the diagonal dominance of the gram matrix based on channel state information (CSI). With a more orthogonal CSI, a better initial solution of the iterative detector can be obtained in the massive MIMO system. Thus, the LR preprocessing is capable of drawing an initial solution with fewer deviations. Mathematicians have proposed many algorithms to do LR, including Gauss reduction algorithm and its complex counterpart, Lenstra–Lenstra–Lovász (LLL) [16] reduction algorithm and its complex LLL (CLLL) counterpart [19], Kannan reduction algorithm [20], and so on. It should be noted that Gauss reduction aims at two-dimensional lattices, and Kannan reduction is not an efficient practical solution. By contrast, the CLLL reduction can produce a reduced basis from any given lattice basis in polynomial time.

Unlike [21], in this paper, the LR preprocessing is focused on the utilisation to increase the DDI value of the gram matrix so that an improved initial solution of the ER detector is obtained. Moreover, the setting of iterations could also be done according to the DDI of the gram matrix for the sake of a good trade-off between the BER performance and the computational complexity. In the rest of this paper, the properties of DDI and adaptive iterative algorithm with complexity analysis are presented in details. Simulation results are finally provided to attest the effectiveness of the proposed techniques.

# System Model and Traditional ER Detector

In this section, one mathematical model of the massive MIMO system is described, and an overview of the ER detector [9] is presented.

## System model

We consider an uplink massive MIMO system with $M$ single-antenna users at the transmitter and $N$ antennas at the BS receiver as shown in Figure 1. Note that $M$ and $N$ should be large, and $N$ is much larger than $M$. The transmitted symbols are denoted as $s = [s_1 \ s_2 \ \cdots \ s_M]^T$, in which $s_k \ (k = 1, 2, \cdots, M)$ is uncoded information symbol transmitted by the $k$th user. As a further note, each entry in $s$ is drawn independently from a constellation set, such as phase-shift keying or quadrature amplitude modulation (QAM), with average symbol energy $E_s$. The flat fading CSI $H$ is described by an $N \times M$ complex matrix whose entries are independently and identically distributed (i.i.d.) random variables with zero mean and unit variance, that is, $CN(0, 1)$. Thus, the received symbol vector $y$ is given by

$$y = Hs + w$$

(1)

where $w$ is the $N \times 1$ additive noise vector whose entries are also i.i.d. variables following $CN(0, \sigma^2)$ with $\sigma^2$ being the average noise power. The average received signal-to-noise ratio is given by $E_s / \sigma^2$.

## 2.2 Traditional ER detector

At the receiver of massive MIMO systems, the MMSE detector is an appealing candidate to recover the transmitted symbols with near-optimal performance and relatively low complexity when $N$ is much larger than $M$ [22]. Assuming that the CSI is perfectly known at the BS, the estimated vector based on the MMSE criterion can be mathematically described as

$$\hat{s} = (H^H H + \frac{\sigma^2}{E_s} I_M)^{-1} H^H y$$

(2)

in which $(\cdot)^H$ is the conjugate transpose operation, and $I_M$ is an $M \times M$ identity matrix. For simplicity, Equation (2) can be written as

$$\hat{s} = A^{-1} b$$

(3)

by setting $H^H H + (\sigma^2 / E_s) I_M = A$ and $H^H y = b$.

The traditional ER detector is an iterative approach derived from the MMSE solution. It is based on the condition that $N \gg M$, which leads to the near-orthogonality of channel matrix $H$. Therefore, the gram matrix $G = H^H H$ becomes diagonally dominant, and as a beneficial result, the initial solution can be approximately calculated as

$$s^{(0)} = D^{-1} b$$

(4)

where $D$ is the diagonal matrix of $A$. The residual $s_r$ between $\hat{s}$ and $s^{(0)}$ can be modelled as

$$e = A s_r = b - A s^{(0)}.$$
Thus, the error elements can be recovered in an ordered fashion. First, a quality metric is computed according to each element of $e$ as $q_j = |e_j|/\|h_j\|^2$, where $e_j$ is the $j$th element of $e$, and $h_j$ is the $j$th column of $H$. Then, a sequence of subscripts can be obtained by sorting the quality metrics in descending order, that is, $\Gamma = \{ f_1, f_2, \ldots, f_M \}$, where $f_j$ is the subscript of the maximal $q_j$, $j = 1, 2, \ldots, M$ etc. Next, the $4$th element $s_4(f_4)$ of the error vector $s$ can be detected by the following formula:

$$
\tilde{s}_4(f_4) = D^{-1} \left( f_4, f_4 \right) \left( e_{f_4} - \sum_{k = f_1}^{f_4-1} A \left( f_4, k \right) \tilde{s}_k(k) \right) \quad (6)
$$

where $1 < i \leq M$. Finally, the output solution is updated as

$$
\tilde{s}^{(1)} = \tilde{s}^{(0)} + \tilde{s}_4 
$$

Moreover, in order to improve the performance of the ER detector, multiple iterations are necessary.

In general, the better initial solution of ER detector, as shown in Equation (4), could be brought from the better diagonal dominance of the gram matrix. The better diagonal dominance of the gram matrix could also lead to fewer iterations. Therefore, in this paper, the DDI would be presented as follows, which could be used to measure the diagonal dominance of a gram matrix.

## 3 | DIAGONALLY DOMINANT INDEX

In this section, we derive the mathematical properties for the DDI of the gram matrix and triangular matrix. At first, we introduce the DDI to evaluate the degree of diagonal dominance for a non-zero square matrix.

For a non-zero matrix $B$ with $n$ rows and $n$ columns, we define the DDI of $B$ as

$$
DDI \ (B) = \frac{1}{2n} \sum_{i=1}^{n} \left( \frac{|b_{i,i}|}{\|B(i,:)\|_1} + \frac{|b_{i,i}|}{\|B(:,i)\|_1} \right) \quad (8)
$$

in which $B(i,:)$ is the $i$th row of $B$, and $B(:,i)$ is the $i$th column of $B$. It is not surprising that the range of DDI is $[0,1]$. 

### 3.1 | DDI of the gram matrix

**Lemma 1.** Given a set of vectors $X = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n]$ with their gram matrix $G_x$, we have $DDI (G_x) = 1$ if the vectors are pairwise orthogonal and vice versa.

**Proof.** According to the definition of the gram matrix, we have

$$
G_x = \begin{bmatrix}
(\mathbf{x}_1, \mathbf{x}_1) & (\mathbf{x}_1, \mathbf{x}_2) & \cdots & (\mathbf{x}_1, \mathbf{x}_n) \\
(\mathbf{x}_2, \mathbf{x}_1) & (\mathbf{x}_2, \mathbf{x}_2) & \cdots & (\mathbf{x}_2, \mathbf{x}_n) \\
\vdots & \vdots & \ddots & \vdots \\
(\mathbf{x}_n, \mathbf{x}_1) & (\mathbf{x}_n, \mathbf{x}_2) & \cdots & (\mathbf{x}_n, \mathbf{x}_n)
\end{bmatrix}
$$

Since the vectors are pairwise orthogonal, we have $(\mathbf{x}_i, \mathbf{x}_j) = 0$ for $j \neq i$, and $(\mathbf{x}_i, \mathbf{x}_i) = \| \mathbf{x}_i \|^2$, $i, j = 1, 2, \ldots, n$. Therefore, $G_x$ takes on the form of a diagonal matrix, that is,

$$
G_x = \begin{bmatrix}
\| \mathbf{x}_1 \|^2 & 0 & \cdots & 0 \\
0 & \| \mathbf{x}_2 \|^2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \| \mathbf{x}_n \|^2
\end{bmatrix}
$$

Naturally, the DDI of $G_x$ can be computed by Equation (8), which equals 1. Hence, we have proved that the DDI of the gram matrix of the given vectors is 1 if the vectors are pairwise orthogonal.

Conversely, the vectors are pairwise orthogonal if the DDI of the gram matrix equals 1. According to Equation (8), $DDI (G_x) = 1$ is equivalent to

$$
\sum_{i=1}^{n} \left( \frac{|x_{i,i}|}{\|X(:,i)\|_1} + \frac{|x_{i,i}|}{\|X(:,i)\|_1} \right) = 2n.
$$

Note that $0 \leq |x_{i,i}|/\|X(:,i)\|_1 + |x_{i,i}|/\|X(:,i)\|_1 \leq 2n$, if and only if $|x_{i,i}|/\|X(:,i)\|_1 = |x_{i,i}|/\|X(:,i)\|_1 = 1$ for each $i = 1, 2, \ldots, n$, the equal sign holds. Thus, we have

$$
|x_{i,i}| = \|X(:,i)\|_1 = \|X(:,i)\|_1.
$$

That is to say, $(\mathbf{x}_i, \mathbf{x}_i) = 0$ holds for all $j \neq i$. Geometrically speaking, $\mathbf{x}_i$ is orthogonal to $\mathbf{x}_j$ for each $j \neq i$. 

In the uplink massive MIMO systems, $N \gg M$ is a condition often satisfied [23]. As a beneficial result, the channel vectors associated with different users may become asymptotically orthogonal, which means the DDI of $G$ is close to 1. Based on the above condition, the ER detector could obtain an initial solution with few errors. However, if the number of active users increases in the uplink massive MIMO system, the condition $N \gg M$ will not hold any more, which leads to the decreased value of $DDI (G)$. Thus, there will be more errors in the initial solution of the ER detector. Therefore, in this paper, the DDI is used to measure the orthogonality of the channel matrix through the diagonal dominance of the gram matrix based on perfect CSI. In a word, there is a negative correlation between the DDI of the gram matrix and the performance of ER detector.

### 3.2 | DDI of the triangular matrix

In an upper triangular matrix, all the entries below the main diagonal are zero, which indicates the probability that the DDI of the triangular matrix may have a lower bound.

**Lemma 2.** For a given upper triangular matrix $R$ with $n$ rows and $n$ columns, the DDI of $R$ is always larger than $1/n$.

**Proof.** According to the definition,
\[
R = \begin{bmatrix}
r_{1,1} & r_{1,2} & \cdots & r_{1,n} \\
0 & r_{2,2} & \cdots & r_{2,n} \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & r_{n,n}
\end{bmatrix}
\]

For the first row,

\[
\frac{|r_{1,1}|}{\|R(1,:)\|_1} + \frac{|r_{1,1}|}{\|R(:,1)\|_1} = \frac{|r_{1,1}|}{\sum_{j=1}^{n} |r_{1,j}|} + 1,
\]

and it is not difficult to derive that

\[
1 < \frac{|r_{1,1}|}{\sum_{j=1}^{n} |r_{1,j}|} + 1 < 2.
\]

For the \(n\)th row,

\[
\frac{|r_{n,n}|}{\|R(n,:)\|_1} + \frac{|r_{n,n}|}{\|R(:,n)\|_1} = 1 + \frac{|r_{n,n}|}{\sum_{k=1}^{n} |r_{k,n}|};
\]

similarly, we have

\[
1 < 1 + \frac{|r_{n,n}|}{\sum_{k=1}^{n} |r_{k,n}|} < 2.
\]

For the second to \((n-1)\)th rows, that is, \(i = 2, 3, \ldots, n-1,\)

\[
0 < \frac{|r_{i,i}|}{\|R(i,:)\|_1} + \frac{|r_{i,i}|}{\|R(:,i)\|_1} < 2
\]

If we sum them, we will have

\[
2 < \sum_{j=1}^{n} \left( \frac{|r_{j,j}|}{\|R(j,:)\|_1} + \frac{|r_{j,j}|}{\|R(:,j)\|_1} \right) < 2n.
\]

According to the definition of DDI,

\[
\text{DDI}(R) = \frac{1}{2n} \sum_{j=1}^{n} \left( \frac{|r_{j,j}|}{\|R(j,:)\|_1} + \frac{|r_{j,j}|}{\|R(:,j)\|_1} \right).
\]

Hence, \(1/n < \text{DDI}(R) < 1.\)

It should be noted that the gram matrix and the triangular matrix are widely used in detection algorithms based on linear elimination. Thus, the properties of their DDI can be exploited.

4 | PROPOSED TECHNIQUES

In this section, we proposed two methods to further improve the performance of the ER detector. One improves the initial solution of the traditional ER detector based on increasing the DDI of the gram matrix in the MMSE equaliser. The other searches for an adaptive iteration scheme in the detector.

4.1 | Employing LR

A better initial solution of the ER detector, as shown in Equation (4), could be brought from a better diagonal dominance of the gram matrix based on CSI. As presented in [16–20], LR preprocessing is an effective method to improve the orthogonality of CSI for better initialisation. That is to say, LR is an effective method to improve the diagonal dominance of the gram matrix based on CSI. Inspired by this, we apply LR preprocessing before the ER detector to the CSI \(H\) as \(\hat{H}, T) = LR(H).\) It should be noted that,

\[
\hat{H} = HT, \quad (9)
\]

where \(\hat{H}\) is the equivalent channel matrix whose columns are more orthogonal, \(T\) is a unimodular matrix, that is, the real and imaginary parts of all entries in \(T\) are integers and the determinant of \(T\) is \(\pm 1\). Then, the received signal vector \(y\) can be written as

\[
y = HTT^{-1}s + n = \hat{H}c + n. \quad (10)
\]

in which \(c = T^{-1}s\). Hence, the problem of estimating \(s\) in the original system turns into the estimation of \(c\) in the new system [19]. It is worth pointing out that each entry in \(s\) is chosen from a constellation set, but the entry of \(c\) is not. According to Equation (9), we know that the unimodular matrix \(T\) varies with \(H\). Therefore, the set of possible values for each element in \(c\) will vary as \(H\) changes, which brings a challenge to the quantification of \(c\). To solve this problem, we transform the estimate of \(c\) to the estimate of \(s\) by

\[
\hat{s} = T\hat{c}. \quad (11)
\]

Thereby, the quantification of \(c\) is avoided and the decision of \(s\) is improved.

To facilitate a better understanding of the transformation benefit, the gram matrix after LR is written as

\[
G = \hat{H}^H \hat{H} = T^H H^H HT. \quad (12)
\]

As we mentioned in Section 3, the DDI of \(G\) is closer to 1 than \(G\), since the orthogonality of \(H\) is improved. Thus, we could obtain a better approximated initial solution of \(c\) as [21]

\[
c^{(0)} = \left( \text{diag}\left( \frac{\sigma^2}{E_s} I_M \right) \right)^{-1} \hat{H}^H y, \quad (13)
\]

where \(\text{diag}(\cdot)\) creates a square diagonal matrix with the main diagonal elements.
FIGURE 2  Comparison of diagonally dominant index between the traditional gram matrix and the lattice reduction preprocessing (LR-aided) one.

For example, assuming that there are 100 antennas at the BS, the uplink massive MIMO systems with 4-QAM modulation at each user, the channel matrix $H$ is generated from the Rayleigh function and it is perfectly known at the BS. Based on such a channel matrix, the DDI values of the gram matrix could be shown in Figure 2. The DDI of the gram matrix for the ER detector with LR preprocessing (we call it LR-aided) is larger than that of the gram matrix for the traditional ER detector. To be more specific, the DDI of the gram matrix can be improved by about 0.1 when $M/N$ is larger than 0.5, which helps to improve BER performance under the same situation as the traditional ER detector.

4.2  Setting iterations adaptively

The block diagram of the proposed system is shown in Figure 3. It is observed that the ER process will repeat several times, which is controlled by the DDI of the gram matrix after LR pre-processing; a greater number of iterations are required if the number of active users increases in the system.

Assuming that there are 100 antennas at the BS in the system model described above, we give the value of DDI($\hat{G}$) with different $M$ (user number) in Table 1, which reveals that the closer $M$ gets to $N$, the closer DDI($\hat{G}$) gets to 0 and the more iterations the detector requires.

It is clear that a larger DDI value could lead to fewer iterations with lower computational complexity. Then, in order to strike a good trade-off between the performance and complexity for the detector even if the number of active users increases, we propose to set the number of iterations (denoted by $k$) based on the value of DDI($\hat{G}$). As shown in Table 2, only one iteration is required for the detector if the value of DDI($\hat{G}$) is quite close to 1, and seven iterations are at most when the value of DDI($\hat{G}$) approaches to 0.

5  NUMERICAL RESULTS

5.1  Performance analysis

Simulation results are given to illustrate the effectiveness of the proposed techniques for improving BER performance in the

![FIGURE 3  Block diagram of the proposed system](image)

| $M$ | $\text{DDI}(\hat{G})$ | $M$ | $\text{DDI}(\hat{G})$ |
|-----|----------------------|-----|----------------------|
| 10  | 0.775                | 60  | 0.250                |
| 20  | 0.610                | 70  | 0.228                |
| 30  | 0.471                | 80  | 0.198                |
| 40  | 0.376                | 90  | 0.192                |
| 50  | 0.309                | 100 | 0.188                |

Note: DDI is diagonally dominant index.
TABLE 2 Setting iterations according to DDI(\(\bar{\gamma}\))

| \(k\) | DDI(\(\bar{\gamma}\)) |
|------|----------------------|
| 1    | 0.75 < DDI \leq 1   |
| 2    | 0.45 < DDI \leq 0.75|
| 3    | 0.35 < DDI \leq 0.45|
| 4    | 0.3 < DDI \leq 0.35 |
| 5    | 0.25 < DDI \leq 0.3  |
| 6    | 0.2 < DDI \leq 0.25  |
| 7    | 0 < DDI \leq 0.2     |

FIGURE 4 Bit error rate (BER) performance of the proposed detector and the traditional error recovery (ER) detector when \(M = 30\) \((k\) is iterations) considered circumstance. In the simulation, we considered 100 \(\times\) 30 and 100 \(\times\) 60 massive MIMO systems with 4-QAM modulation at each user, and the CSI is assumed to be Rayleigh-faded and known perfectly at the receiver. During each channel, all the users transmit their uncoded information symbols simultaneously. All the results are calculated over random channel and noise conditions as shown in Equation (1).

Comparisons of the proposed detector (LR-aided ER) and the traditional ER detector [9] under different iterations are given in Figures 4 and 5. It is worth pointing out that ER algorithm [9] is an approximated iterative method based on the MMSE criterion, which involves matrix inversion. However, in the massive MIMO scenario, the computational complexity of matrix inversion (i.e. \(O(N^3)\)) is unacceptable. Hence, we give out the BER performance curve of LR-aided MMSE (MMSE detection with LR pre-processing) as optimal BER curve for comparison. It is not surprising that the LR-sided ER detector performs better than the traditional ER detector by about 3 dB under various iterations.

Next, we will show how the number of iterations affects the performance and complexity of the detector. The traditional ER detector sets invariable iterations, which will bring extra complexity if it iterates too many times or too many detection errors if iterates very little times as shown in Figures 4 and 5, respectively. It is observed that there is no significant change in the BER performance by using multiple iterations if the number of active users in the system is only 30. Hence, two iterations are enough in this context, and redundant iterations bring nothing but unnecessary computational complexity. However, the near-optimal (optimal means the LR-sided MMSE) BER performance is achieved under six iterations or above if there are 60 users; two iterations are far from enough to provide acceptable performance. Therefore, adaptive iterations are appealing to strike a good trade-off between the BER performance and the computational complexity.

Considering that there are 100 antennas at the BS, BER performance under different iterations is shown in Figure 6. The relation curve of the proposed number of iterations and the number of users is depicted in Figure 7. It should be noted

![FIGURE 4 BER performance of the proposed detector and the traditional ER detector when \(M = 30\) \((k\) is iterations)](image1)

![FIGURE 5 BER performance of the proposed detector and the traditional ER detector when \(M = 60\) \((k\) is iterations)](image2)

![FIGURE 6 BER performance and iterations with the proposed LR-aided ER detector)](image3)
that the proposed iterations are set according to the DDI as shown in Table 2, which is related to the CSI. It can be observed that the proposed number of iterations is able to ensure that the BER is below $10^{-4}$. Figure 7 is just an example when there are 100 antennas at the BS, which should not be interpreted that the number of proposed iterations relies only on the number of active users. As shown in Figures 8 and 9, the proposed adaptive LR-aided ER detector is able to achieve the near-optimal BER performance even the number of active users increases.

5.2 Complexity analysis

The average computational complexity of the proposed detector is measured and compared with the traditional ER detector in terms of the number of real-valued operations. Since the LR algorithm is carried out in the preprocessing phase, its computational complexity is shared by symbols within the coherence time of the channel. In the proposed detector, the CLLL method is adopted to carry out LR preprocessing. The incremental complexity inherent in the use of CLLL is determined by the number of iterations (say $k_c$) required [24], the average complexity of which is

$$O\left( M^2 \log_{1.33} \frac{M}{N - M + 1} \right).$$

(14)

As a further note, $k_c$ mainly depends on the ‘quality’ of the channel matrix, which is usually well-conditioned for the uplink massive MIMO. Therefore, the average per-symbol complexity increase is tiny.

It should be noted that the computational complexity analysis in [9] is based on a real-valued CSI matrix, but the LR preprocessing algorithm is capable of dealing with a complex-valued CSI matrix. In order to demonstrate the superiority of the adaptive iterative ER detector in computational complexity, we recalibrate the real-valued operations of the iterative part of the traditional ER detector in the complex-valued system, which is shown in Table 3.

Considering that there are some massive MIMO systems with 100 antennas at the BS, and 30 to 60 users at the transmitting side. The traditional ER detector iterates six times no matter how many users activated in the system. It can be seen from

FIGURE 7 The proposed iterations and the number of users in a multiple-input multiple-output system with 100 antennas at the base station in the proposed detector

FIGURE 8 BER performance of the proposed adaptive LR-aided ER detector with different number of users

FIGURE 9 Comparison of BER performance when signal-to-noise ratio $= 8$ dB

TABLE 3 Real-valued operations of traditional error recovery (ER)

| Operation | Times          |
|-----------|----------------|
| Multi     | $k(6M^2 + (4N + 2)M)$ |
| Add       | $k(6M^2 + (4N - 2)M)$ |
possible to give further research on the methods using DDI to help improve the spatial and diversity gain for multi-user massive MIMO systems.

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Table 4 The real-valued operations of the proposed detector and traditional ER detector

| M   | Operation | ER        | Proposed |
|-----|-----------|-----------|----------|
| 30  | Add       | 104,040   | 34,680   |
|     | Multi     | 104,760   | 34,920   |
| 40  | Add       | 153,120   | 76,560   |
|     | Multi     | 154,080   | 77,040   |
| 50  | Add       | 209,400   | 139,600  |
|     | Multi     | 210,600   | 140,400  |
| 60  | Add       | 272,880   | 272,880  |
|     | Multi     | 274,320   | 274,320  |

Figure 10 that even the traditional ER detector iterates six times, its BER performance is still worse than the proposed detector. The real-valued operations of the proposed detector and traditional ER detector is shown in Table 4. The results show that the proposed adaptive iteration criterion could reduce about 30% to 60% computational complexity with the active user number $M < 60$.

6 | CONCLUSION

In this context, we introduced two improved techniques, which are LR preprocessing and adaptive iterations, for ER detector based on the DDI measurement. As we pointed out in Section 1, the drawbacks of ER detector are shared by iterative approaches derived from the MMSE criterion, such as diagonal band Newton iteration (DBNI) and RI. Therefore, the two enhancement methods are also effective for DBNI and RI. The DDI could be a useful tool to measure the diagonal dominance of the gram matrix. Moreover, the number of antennas in the MIMO system is also related to spatial division and diversity [25], so it is
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