Generation of asymmetric incommensurable torque signals

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Abstract. Asymmetric incommensurable torque signals can be achieved by exploiting magnetic nonpolar repulsion techniques, see also \cite{1}, aiming at designing kinetic energy harvester (KEH) systems. This paper confirms by using three different simulation methods that an asymmetric incommensurable torque signal is feasible to be reached, exploiting non-conservative magnet fields. All shown calculations are in full agreement with the classical Electromagnetic Theory. The first method (I) is based on time consuming full 3D FE generated torque and force signals. The second method (II) is using also FE generated torque and force signals, but they are calculated only in orthogonal directions, approximated with Fourier Series and normalized to provide 2D stiffness signals. The third approach (method III) focuses on filamentary current rings to calculate the magnetic field of permanent magnets (PM) to generate in a purely analytically way the same force and torque signals as in the previous methods. However, only speculation about the energy source as required to maintain the forecasted persistent torque can be given.

1. Introduction
Energy Harvesting is a technology for capturing non-electrical energy from ambient energy sources, converting it into electrical energy and storing it to power wireless electronic devices \cite{2, 3, 4}. The process of capturing mechanical energy such as shocks and vibrations is a subject area of energy harvesting requiring specific types of devices, so called kinetic energy harvesters (KEH) and there are many types of KEH’s \cite{4}. For the devices described in this paper we employ still the naming convention KEH, as effectively rotor mass and rotor inertia are used to start an everlasting oscillation in theory. Microscopically, there might be yet unknown sources why such a persistent motion is possible to obtain \cite{5}. In chapter 2 asymmetric incommensurable torque signals are modeled using the three different approaches. In chapter 3 a proposition for a KEH prototype buildup is discussed. In the conclusions, chapter 4 we summarize the findings.

2. Design of asymmetric incommensurable torque signals
The force or torque of two permanent magnets (PM) can be calculated using Lorentz force law

\[ F_{\text{mag}} = q \left( E + (v \times B) \right) \equiv q(v \times B) \]  

where \( F_{\text{mag}} \) is the resulting force, \( E \) the electric field-, \( B \) the magnetic field- and \( v \) the point charge \( q \) velocity trajectory-vector. As in these KEH systems, velocities are extremely low in respect to the speed of light, \( E \) can be disregarded (which is always assumed in calculations for electrical machines, see for instance \cite{6}). Generally, magnetic forces will do no work on isolated electric charges \cite{7}, since

\[ W_{\text{mag}} = \int F_{\text{mag}} \cdot dl = \int \int q(v \times B) \cdot v_0 dt = 0 \] 

The result in (2) follows when \( v \) and the created path velocity vector \( v_0 \) are equal, which is always the case using a point charge. However, for the PM-based KEH systems proposed in this paper, we show
that the calculated total magnetic work \( W_{\text{mag}} \) is unequal zero by applying method I and II using (a) well-deliberated closed trajectories \( \Gamma \), and (b) instead of a point charge, a disk PM or a current loop (both having geometrical extensions). So, in contrast to (2),

\[
W_{\text{mag}} = \oint_{\Gamma} \mathbf{F}_{\text{mag}} \cdot d\mathbf{l} \neq 0
\]  

(3)

This result must be validated in an experiment as both methods I and II rely on Maxwell Stress tensor approach in which the calculated torque-signal is difficult to evaluate, see also [8]. However, the purely analytical method III will result in a total magnetic work \( W_{\text{mag}} = 0 \) as expected from (2). The simplest possible configuration of a KEH system that can exert asymmetric incommensurable torque is shown in Figure 1a. Figure 1b, c show the normalized 2D stiffness for the torque- and force-signal assuming an airgap of 1mm between stator-rotor PM set. The generation of such amplitude diagrams can for instance be approximated with Fourier Series. It is sufficient to consider only terms up to the forth order, see also [1]. These normalized signals for torque and force components, given by

\[
\tau_{\text{rad}}(\phi, z) = C_{\tau} f_{\text{rad}}(\phi, z)
\]

\[
F_{ax}(\phi, z) = k_{\tau} f_{\tau}(\phi, z)
\]

(4)

(5)

respectively, are stiffness functions, where the torque component \( C_{\tau} (Nm/rad) \) and its counterpart force component \( k_{\tau} (N/m) \) can be easily measured as the maximal break-torque respectively break-force. Introducing a harmonic axial cam movement of the rotor as

\[
z_{R}(\phi) = A_{\text{co}} \sin(\omega_{\text{co}} \phi + \xi_{\text{co}})
\]

(6)

where the lateral movement \( z_{R} \) is constrained to the rotating DoF \( \phi \) with a given cam amplitude \( A_{\text{co}} \), an initial offset \( \xi_{\text{co}} \) and an angular velocity \( \omega_{\text{co}} \).

![Figure 1](image1.png)

**Figure 1.** Concept view (a) of rotary-translatory nonlinear PM spring system, with a rotary (\( \phi \)), an axial (\( z \)) and a radial (r) DoF. Normalized 2D torque spring and force spring amplitude for \( \phi \) and \( z \) direction shown in (b) resp. (c). Shown data represents a disk PM with diameter 10mm, height 5mm, residual mag. N52 (\( B \approx 1.4457 \)).

In the following simulations, the values in (6) are kept as \( \omega_{\text{co}} = 3 \) and \( \xi_{\text{co}} = 0 \), to make the case simple. The setup is illustrated in Figure 2, showing also a cam function with a realistically small cam amplitude of \( A_{\text{co}} = 1mm \) and an asymmetric rotor to stator displacement of 9mm.

![Figure 2](image2.png)

**Figure 2.** Concept buildup of a rotating and axial wobbling 3PM rotor set and an asymmetrically placed 3PM stator set in a single line distribution (axR3S3sl) with developed view (a) and front view (b).
Figure 3a shows the optimization process for creating an unbalanced torque signal by sweeping the stator-rotor asymmetry from $d_{\text{sym}} = 0 \ldots 20\text{mm}$. Keeping the stator and rotor PM in the same z-axis with an enabled cam movement at 0mm, no asymmetric torque nor force signal in one revolution occurs. As soon as an asymmetry (z offset) between rotor and stator PM is introduced, the (normalized) torque signal will increase, and at half the PM-diameter it reaches a maximum, followed by a decline. When offset reaches 15mm, no torque signal at all is generated, due to the lacking mutual PM magnet influence. The torque signal (blue) is depicted in %, showing 50% as a balanced torque signal. No energy surplus is occurring, e.g. the created force signal needs accordingly to be damped to generate an energy surplus (see also chapter 3). Figure 3c shows generated torque signals with FE simulation (red) and Fourier Series approximation (blue) plus analytical model using Lorentz approach [9], [10], [11]. In all three cases, a net torque of all summed up rotor disk PMs is clearly positive (61%) over one full revolution; peak torque is reached at 10°, 130° and 250°.

This behavior is intriguing, even though the analytical calculations using method III [10], [11] no energy surplus is forecasted. However, the damping in the axial direction (constraining force) of the proposed system buildup can be damped. An energy source must therefore be present somehow, and some conjecture about such a source is given in [1].

3. Proposed system for buildup
A system concept buildup is proposed in Figure 4, with focus of keeping the cam amplitude low (with $A_{\text{co}} \leq 3\text{mm}$) and allowing an angular cam velocity coupling of $\omega_{\text{co}} = 1$ only.

Using the PM configuration and the sketched cam movement in Figure 4, the motion profile (torque DE) with constraining force created from lateral movement (6) follows (using Lagrange approach):

$$J\phi'' + \frac{1}{2}A_{\text{co}}^2m\omega_{\text{co}}^2\phi'' + \frac{1}{2}A_{\text{co}}^2m\omega_{\text{co}}^2\cos(2\omega_{\text{co}}\phi)\phi'' - \frac{1}{2}A_{\text{co}}^2m\omega_{\text{co}}^2\sin(2\omega_{\text{co}}\phi)\phi'' + \tau_\phi(\phi, \dot{\phi}) + \tau_S(\phi) = 0 \quad (7)$$

In case the torque signals from the constraining force are neglected from (7), e.g. $A_{\text{co}} = 0$, the torque DE can be written much simpler as

$$J\phi'' + \tau_\phi(\phi, \dot{\phi}) + \tau_S(\phi) = 0 \quad (8)$$
with $J$ denoting the inertia, $\tau_0$ the torque due to friction and $\tau_\Sigma$ the asymmetric incommensurable torque.

The simulation model confirms that, with a realistic rotor mass of $m = 200 g$ ($I = 45 \mu g m^2$), a persistent motion is generated, as shown in Figure 5d, having applied a nonlinear friction model (compare Figure 5c, used also in [1]), and an approximated SDoF dynamic model for which the acceleration and deceleration due to the lateral cam movement is taken into account (solid lines in Figure 5b). Also, the nonlinear torque signal $\tau_0$ should be considered as

$$
\tau_0(\phi', \phi) = \tau_{\text{Rad}}(\phi') + \tau_{\text{ax}}(\phi', \phi) = (D_{\text{Rad}}(\phi') + D_{\text{ax}}(\phi') \big| f_f(\phi, z_\phi(\phi))) \big| \phi' \big)
$$

with torque friction component $D_{\text{Rad}}(\phi')$ and $D_{\text{ax}}(\phi')$. Both components are described with a nonlinear friction model, depicted in Figure 5c. In (6), $D_{\text{Rad}}(\phi')$ is the torque friction for revolving the rotor (which could have been also modeled with a linear function, as the rotor is revolving at steady state well above $\phi' > 10 \text{rad/s}$), and $D_{\text{ax}}(\phi') \big| f_f(\phi, z_\phi(\phi)) \big| \phi'$ is the additional torque friction introduced when rotor PMs are in proximity of the stator PMs. The normalized force $f_f(\phi, z_\phi(\phi))$ has been introduced earlier (see Figure 1c, as we deal with friction, the absolute value of $f_f$ must be taken, as the rotor exerts on either cam well a positive damping force resp. damping torque; resulting constraining force $F_{\text{ax}}$ depicted in Figure 5a). This constraining force cannot be eliminated (but it could be minimized with an elaborate stator-rotor PM arrangement), as generated forces in the rotor PM rendez-vous changing sign (compare also Figure 5a and Figure 4, PM rendez-vous at 1a and 2a are doubling negative $z$-forces and after 180°, at 1b and 2b, these forces changing sign).

For creating an everlasting motion it is important to note that the friction torque signal over one full revolution must be smaller than the asymmetric incommensurable torque signal $\tau_\Sigma$ created by the stator-rotor PM pair arrangement (shown in yellow, Figure 5a). The magnetic energy $W_{\text{mag}}$ of this dynamic system in steady-state for one revolution must fulfill

$$
W_{\text{mag}} = \int_0^{2\pi} \tau_\Sigma(\phi)d\phi = \int_0^{2\pi} \tau_0(\phi', \phi)d\phi' \big)
$$

The complexity of this proposed configuration is modest and expenditures for prototyping is small. Even though the lateral constraining $z$-forces are symmetrical, this lateral energy component is not canceled out over one revolution and a low friction cam system must be used to guarantee that the energy component from the asymmetric incommensurable torque signal is larger.
Figure 5. Diagram (a) depicts the resulting normalized torque and force signals from proposed setup in Figure 4, using $A_{co} = 3 \text{ mm}$ and $\omega_{co} = 1$ ($\xi_{co} = 0$) applying the 3 different calculation methods; (b) shows motion functions of approximated cam signal with and without ($A_{co} = 0$) constraining force (and keeping $\tau_{\Sigma}$ zero); (c) friction model from [12] for radial and axial friction (in set rotor position without additional lateral load); (d) resulting motion signals.

The torque asymmetry can be further increased using different PM shapes (for instance, using block magnets, an asymmetry of nearly 60% could be achieved and larger force signals will result). In addition, such a system can be easily stacked in z-direction, or the coupling increased to $\omega_{co} = 2$ (making it necessary to double the PM arrangement frequency on the surface for stator and rotor, compare Figure 4) to basically unfold an energy surplus as desired.

4. Conclusions

It is stated that, in theory, by combining trajectories of a rotor with accordingly stator-rotor PM distributions, an asymmetric incommensurable torque signal would be achievable based on calculations with three different methods. The presented trajectories have been limited only in rotational- ($\phi$) and translational- ($z$) direction, excluding movements that involve also radial direction (changing rotor diameter distance $r$, see also Figure 1a). While keeping a cam harmonic, the generated force signal must be damped in all cases – by using a low friction cam setup, whether using a PM setup from Figure 2 or from Figure 4 for creating a persistent motion. Also important in this study is the foresight, that magnet field distributions could exert energy and are not per se conservative fields. According to the presented results, the magnetic field itself is neither conservative nor non-conservative; it all depends on the closed path trajectories followed through the field. Non-conservative trajectories can be generated which allow netto work while rotating a rotor over one full revolution. The fundamentals for the forecasted energy source, which is necessary to deliver this netto work, remain a research question, and further opensource experiments are indispensable to verify the presented theoretical claims.

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