Dynamics of the return distribution in the Korean financial market

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Abstract

In this paper, we studied the dynamics of the log-return distribution of the Korean Composition Stock Price Index (KOSPI) from 1992 to 2004. Based on the microscopic spin model, we found that while the index during the late 1990s showed a power-law distribution, the distribution in the early 2000s was exponential. This change in distribution shape was caused by the duration and velocity, among other parameters, of the information that flowed into the market.

Key words: Econophysics, Emerging market, Log return, Power law distribution, Exponential distribution

PACS: 89.65.Gh, 89.75.Fb, 89.75.Hc

1 Introduction

Interdisciplinary research is now routinely carried out, with econophysics being one of the most active interdisciplinary fields [1,2,3,4,5]. Many research papers on mature markets have already been published. However, since emerging markets show different characteristics to those of mature markets, they represent an active field for econophysicists. The Korean market, one of the foremost emerging markets, has already been studied by physicists [4,5]. We concentrate on the particular properties of the Korean market through the return distribution.

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It is broadly assumed that the distribution of price changes takes the form of a Gaussian distribution, and that all information is applied to the market immediately by the efficient market hypothesis (EMH) [6]. Using the EMH, the trading profit with arbitrage cannot be obtained from the superiority of information. The price changes in an efficient market cannot be predicted and change randomly. This is suited to classical economics theory. However, experimental proofs reveal that Gaussian distributions of price changes do not exist in real markets [7,8].

Mandelbrot determined empirically that the tail part of the distribution is wider and the center of the distribution is sharper and higher than a Gaussian distribution by examining price changes of cotton; this distribution of price changes is termed the Lévy stable distribution [7]. Fama also found a Lévy stable distribution for the New York Stock Exchange (NYSE) [8]. After Mandelbrot’s study, the distribution of price changes was identified as non-Gaussian by many researchers [9,10,11,12]. It was reported that distributions in mature markets have a power-law tail, while those in emerging markets have an exponential tail [13,14].

Silva et al. [11] and Vicente et al. [12] reported that the distributions of price changes vary with time lag. Price changes have a power-law distribution for a short time lag, and an exponential distribution when the time lag is long. Moreover, for a very long time lag, the distribution becomes Gaussian. This transition problem was solved analytically by Heston using a stochastic model [15].

Financial markets are adaptive evolving systems, so the distributions of price changes are also different for various periods and countries. Especially, the Korean stock market has different properties compared with other countries, and the distribution of price changes is not stable and changes with time.

In this paper, we study the characteristics of the Korean stock market using probability distribution functions (PDFs) of the Korean Composition Stock Price Index (KOSPI) and investigate why phenomena different to other countries occur. We also carry out a simulation using the microscopic spin model to explain these differences.

2 Empirical data and analysis

We use the KOSPI data for the period from 1992 to 2004, and observe the PDFs of the KOSPI price changes for a time window of 1 year. We use only intra-day returns to exclude discontinuous jumps between the previous day’s close and the next day’s open price due to overnight effects. The price change
log return is defined by:

\[ S(t) \equiv \ln Y(t + \Delta t) - \ln Y(t), \]  

(1)

where \( Y(t) \) is the price at time \( t \) and \( \Delta t \) is the time lag.

Fig. 1a shows the log return distribution for the KOSPI. The distribution for an 1-min time lag represents a power-law distribution, that for 10 min is exponential, and that for 30 min is also exponential but close to Gaussian. These results are in accordance with previous reports [11,12].

In the Korean stock market, the log return distribution for an 1-min time lag shows some peculiar phenomena. Fig. 1b shows the PDFs of the KOSPI log return from 1998 to 2002, while Fig. 1c is a graph of the tail index, power law exponent of tail part of the log return distribution, as a function of time from 1993 to 2004. The shape of the distribution in 1998 (○) in Fig. 1b is close to a Lévy distribution and the tail part shows a power-law distribution. However, the tail index of the PDFs increases over the years and the shape of tail part changes to exponential with increasing time. This phenomenon can be confirmed in Fig. 1c. As well the tail index in the early 1990s is approximately 2.0, increases from the mid-1990s to the early 2000s, finally changes to an exponential tail. Although a discontinuity in the increasing trend occurred during the 1997 Asian financial crisis, the tail index continued to increase thereafter.

In Fig. 2a, the decay time for the autocorrelation function of log return is continuously decreasing, while the tail index abruptly varied around the time of the 1997 Asian financial crisis. This suggests that the decay time is related to the increasing trend of the tail index, regardless of the Asian financial crisis. Thus, we investigated the relation between the decay time for the autocorrelation function of log return and the tail index of the log return distribution to identify why this phenomenon is happened. Fig. 2b shows the relation between the tail index and the decay time. The decay time of the autocorrelation function is inversely proportional to the tail index by a factor of 4.

The decay time of the autocorrelation function decreases as the log return distribution of the KOSPI changes from a power-law to an exponential distribution (Fig. 2). This decrease in decay time means that the duration of information in the market is diminished compared with the past, that is, the market is more rapidly affected by the information flow and the influence of past information decreases more rapidly when the decay time is longer. Information and communication technology such as high-speed internet connections and electronic trading systems were not fully utilized in the past, so it took a longer time to deliver information to the market. Moreover, the information flow was small because social structures in the past were relatively
simple. For this reason, much more information flows into the market for a specific time interval now compared with the past. Therefore, the time scale of the past differs from the current scale. The amount of information and its velocity in reaching the market for 1 min now may be the same as that for 2 or 3 min or more in the past.

3 Model and results

Eguiluz and Zimmermann [16], Krawiecki et al. [17], Chowdhury and Stauffer [18], and Cont and Bouchaud [19] used microscopic models to simulate the financial market, and these models describe well the characteristics of the financial market. Krawiecki et al. described the market as a spin model. Agents and traders comprising the market are represented by spins, and interaction between agents and external information is represented by fields.

We modify the Krawiecki microscopic model of many interacting agents to simulate the variation of log return distribution for the Korean stock market. The number of agents is $N$, and we consider $i = 1, 2, \ldots, N$ agents with orientations $\sigma_i(t) = \pm 1$, corresponding to the decision to sell ($-1$) or buy ($+1$) stock at discrete time-steps $t$. The orientation of agent $i$ at the next step, $\sigma_i(t+1)$, depends on the local field:

$$I_i(t) = \frac{1}{N} \sum_j A_{ij}(t)\sigma_j(t) + h_i(t),$$

(2)

where $A_{ij}(t)$ represent the time-dependent interaction strength among agents, and $h_i(t)$ is an external field reflecting the effect of the environment. The time-dependent interaction strength among agents is:

$$A_{ij}(t) = A\xi(t) + a\eta_{ij}(t),$$

(3)

with $\xi(t)$ and $\eta_{ij}(t)$ determined randomly in every step. $A$ is an average interaction strength and $a$ is a deviation of the individual interaction strength. The external field reflecting the effect of the environment is:

$$h_i = h \sum_{k=0}^{\infty} \zeta_i(t-k)e^{-k/\tau},$$

(4)

where $h$ is an information diffusion factor, and $\zeta_i(t)$ is an event happening at time $t$ and influencing the $i$-th agent. $\tau$ is the duration time of the information, which represents how long the event at time $t$ retains influence on the opinion
of agents on market prices. At every step, agents are assumed to receive newly generated information. The later the event, the greater is the influence on agents and the market. The most recent event has a strong influence on agents, while old events have a weak influence. Influence on the market is considered to be reduced exponentially. Moreover, for larger $\tau$, an event at time $t$ retains a relatively strong influence on agent opinions and market price changes for a long time. On the other hand, for shorter $\tau$, the information from an event is rapidly delivered to agents and applied to market prices immediately, so the information quickly vanishes from the market (Fig 3). If we assume that the information flow is the same ($= 1$) whether $\tau$ is long or short, then $h$ is equal to $1/\tau$.

From the local field determined as above, agent opinions in the next step are determined by:

$$\sigma_i(t + 1) = \begin{cases} 
+1 \text{ with probability } p \\
-1 \text{ with probability } 1 - p
\end{cases}.$$  

(5)

where $p = 1/(1 + \exp\{-2I_i(t)\})$. In this model, price changes are:

$$x(t) = \frac{1}{N} \sum \sigma_i(t).$$  

(6)

We simulate a stock market with 1000 agents, and the values $\xi(t)$, $\eta(t)$, and $\zeta(t)$ are generated randomly within the range $[-1, 1]$.

Fig. 4a shows the relation between the diffusion factor, $h$, and the tail index of the PDFs. The tail index is directly proportional to $h$. This is similar to the trend line (solid line) in Fig. 1c. Fig. 4b and c show the PDFs of log return for various $h$ values. Similar to the experimental result, a power-law tail is evident for small $h$ (large $\tau$), and an exponential tail for large $h$ (small $\tau$). For very large $h$, the distribution is close to Gaussian (Fig. 4d)

The time lag, $\Delta t$, can be determined for 1 min, 10 min, 1 h, 1 day, etc., when price changes are calculated. However, time is not uniform, because the volume, the number of contracts, and the information flow into the market are not homogeneous. In other words, 1 min of today may not be same as 1 min of tomorrow. Thus, it is necessary to study the time uniformity.
4 Conclusions

The distribution of the KOSPI log return has recently taken the form of an exponential, while it showed a power-law tail in the 1990s. Moreover, the decay time of the autocorrelation function is continually decreasing. Thus, the duration of information received by agents is also decreasing as the amount of information increases. According to the EMH, the distribution of log return becomes Gaussian when the velocity of information flow is very fast, and all information received immediately affects the opinion of agents in the market. We could identify and confirm a relationship between the distribution of price changes, the velocity of information flow, and the duration of the influence of information for a time series of the Korean stock market. A similar phenomenon occurred in Japan around 1990, as identified from daily data [20]. However, in mature markets, including the NYSE, the tail index is not increasing or changing in shape. The reason for this is the robustness and maturity of the market. Mature markets are solid enough to endure external shocks, while emerging markets are susceptible to shocks and environmental changes. Modeling of the robustness and maturity of the market is planned as further work in the near future.

Acknowledgements

We wish to thank H. Jeong and O. Kwon for helpful discussions and supports.

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Fig. 1. (a) Probability distribution functions (PDFs) of log return for the KOSPI. Rectangle(■) represents 1-min time lag, circle(●) 10-min, and triangle(▲) 30-min, respectively. (b) PDFs of log return for the KOSPI from 1998 to 2002: ◇ 1998, ▽ 1999, △ 2000, ○ 2001, and ✷ 2002. (c) Evolution of the KOSPI tail index. The tail first increases and then changes to exponential. The dashed line represents the 1997 Asian financial crisis.
Fig. 2. (a) Evolution of the decay time for the autocorrelation function (ACF) and the tail index. (b) Relation between decay time and tail index for the ACF.

Fig. 3. Influence of information flow into the market. The latest information has a strong influence, and influence on the market diminishes as time passes. (a) If the information flow is small, the influence of the information slowly decreases. (b) If the information flow is large, the influence of the information rapidly decreases. The amount of information used by traders to determine whether to sell or buy is constant. Thus, the area of these functions has to be constant.
Fig. 4. (a) Relation between $h (= 1/\tau)$ and the tail index of PDFs. (b) Semi-log plot for the PDFs of price changes as a function of $h$. □, $h = 0$; ○, $h = 1$; △, $h = 2$; and ▽, $h = 3$. (c) Log-log plot for the PDFs. The straight line is guide line for power law distribution and the curved line is for exponential distribution. Symbols are as for (b). (d) PDF of price changes when $h$ is very large ($h = 10$).