RELATIVISTIC JET DYNAMICS AND CALORIMETRY OF GAMMA-RAY BURSTS

N. WYGODA1,2, E. WAXMAN3, AND D. A. FRAIL3
1 Department of Particle Physics and Astrophysics, The Weizmann Institute of Science, Rehovot 76100, Israel
2 Department of Physics, NRCN, P.O. Box 9001, Beer-Sheva 84015, Israel
3 National Radio Astronomy Observatory, Array Operations Center, Socorro, NM 87801, USA

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ABSTRACT

We present numerical solutions of the two-dimensional relativistic hydrodynamics equations describing the deceleration and expansion of highly relativistic conical jets, of opening angles 0.05 ≤ θ0 ≤ 0.2, propagating into a medium of uniform density. Jet evolution is followed from a collimated relativistic outflow to the quasi-spherical non-relativistic phase. We show that relativistic sideways expansion becomes significant beyond the radius r0 at which the expansion Lorentz factor drops to θ0−1. This is consistent with simple analytic estimates, which predict faster sideways expansion than has been claimed based on earlier numerical modeling. For t > tθ = r0/c the emission of radiation from the jet blast wave is similar to that of a spherical blast wave carrying the same energy (significant deviations at t ≈ tθ occur only for well off-axis observers, θobs ≈ 1 ≫ θ0). Thus, the total (calorimetric) energy of gamma-ray burst blast waves may be estimated with only a small fractional error based on t > tθ observations.

Key words: gamma-ray burst: general – hydrodynamics – methods: numerical – radiation mechanisms: non-thermal – relativistic processes

1. INTRODUCTION

The dynamics of gamma-ray burst (GRB) jets and the spectral and temporal evolution of their afterglows remain an important problem (see Granot & Ramirez-Ruiz 2010 for a review). Much of what we know about GRB progenitors and the central engines that power them comes from multi-wavelength observations of their afterglows. From the analytic models which predict the evolution of these light curves it is possible to extract estimates of jet opening angles, the energetics of the outflows, and the properties of the circumburst medium (Panaitescu & Kumar 2002; Centeno et al. 2010; Yost et al. 2003). However, more recent numerical modeling has claimed that there are strong discrepancies between the analytic and numerical models of GRB jets.

A conical jet-like outflow expanding at a Lorentz factor Γ evolves as if it were a conical section of a spherical outflow as long as Γ ≥ θ0−1, since for Γ ≥ θ0−1 the (rest frame) transverse light crossing time of the jet is larger than the expansion/deceleration time. At this stage, the flow is described by the spherical Blandford–McKee (BM) blast wave solutions (Blandford & McKee 1976), with Γ2r03 = 17 Eiso/16πnmρc2. Here, r is the blast wave radius, n is the ambient medium number density, and Eiso is the isotropic equivalent energy, related to the (two-sided) jet energy by Ejet = 1/2θ2 θiso (note that θ0 is the angular radius). For Γ > θ0−1 a distant on-axis observer cannot distinguish a jet from a sphere since the emitted radiation is beamed into a 1/Γ cone.

The flow decelerates to Γ = θ0−1 at source frame time

\[ t_0 = r_0/c = 230 \left( \frac{E_{\text{iso,53}}/n_0}{10^{-3}} \right)^{1/3} \theta_{0,-1}^{2/3} \text{ days}, \]  

(1)
corresponding to an observer’s frame time (Waxman 1997b)

\[ t_{0,\oplus} = \frac{1}{4\sqrt{2}} t_0 = 0.6 \left( \frac{E_{\text{iso,53}}}{10^{53}} \right)^{1/3} \theta_{0,-1}^{8/3} \text{ days}, \]  

(2)

where θ0 = 10−1θ0−1, Eiso = 1053 Eiso,53 erg, and n = 1n0 cm−3 (for a burst located at redshift z, all observed times should be increased by a factor 1 + z; we do not explicitly show this correction in our equations). The sideways expansion is expected to be relativistic as long as the blast wave is relativistic and the post-shock energy density is relativistic (Rhoads 1999). If this is the case, at t > tθ the lateral expansion rapidly increases the jet opening angle and accelerates its deceleration (Rhoads 1999), reducing Γ to ∼ 1 with only a logarithmic increase of r (to ~ ln θ0−1 × r0). Thus, the observed timescale for the flow to become transrelativistic is (Livio & Waxman 2000)

\[ t_{\oplus} = t_0 + r_0/c \approx 230 \left( \frac{E_{\text{iso,53}}/n_0}{10^{53}} \right)^{1/3} \theta_{0,-1}^{2/3} \text{ days}. \]  

(3)

On a similar timescale, the flow is expected to become quasi-spherical, i.e., the jet is expected to expand to θ ∼ 1, and the outflow is subsequently expected to evolve into the spherical non-relativistic Sedov-von Neumann–Taylor (ST) flow.

This simple analytic description of jet expansion was challenged by a series of numerical calculations (Granot et al. 2001; Cannizzo et al. 2004; Zhang & MacFadyen 2009; Meliani & Keppens 2010). It was argued, based on the numerical results, that the sideways expansion of the jet is not relativistic, and that the jet retains its narrow original opening angle, θ0, as long as it is relativistic (Granot 2007; Zhang & MacFadyen 2009). This implies that the jet continues to evolve like a conical section of a spherical outflow with energy Eiso, with Lorentz factor following the BM solution, up to the radius rNR = cτNR at which it becomes sub-relativistic,

\[ r_{NR} = \left( \frac{17 E_{\text{iso}}}{16\pi n_0 \rho c^2} \right)^{1/3} = 1100 \left( \frac{E_{\text{iso,53}}}{n_0} \right)^{1/3} \text{ days}. \]  

(4)

The different descriptions of jet evolution inferred from analytic and numeric modeling lead to different predictions for the observed properties of GRB afterglows (e.g., van Eerten et al. 2010). For example, the suppression of the observed flux produced by the jet blast wave at t > tθ, compared to that produced by a spherical blast wave with the same Eiso, is smaller (i.e., the “jet break” is less pronounced) if the jet does not
expands significantly while it is relativistic. Furthermore, if the jet does not expand significantly and remains highly collimated and relativistic at $t > t_s$, then the accuracy of the late-time calorimetric estimates of the jet energy, which assume quasi-spherical emission at $t \sim t_s$ (Frail et al. 2000; Berger et al. 2004), is questionable.

The main goal of this Letter is to resolve the apparent discrepancy between the analytic and numeric description of jet expansion, and in particular to confirm the simple analytic models in light of the claims that they do not properly capture the essential jet dynamics. A more detailed analysis of the numerics will be given in N. Wygoda & E. Waxman (2011, in preparation). Our numerical calculations are described in Section 2, and their results regarding jet expansion are described in Section 3. In Section 4 we briefly discuss the implications for jet breaks and GRB calorimetry. Our conclusions are summarized in Section 5.

2. NUMERICAL CALCULATIONS

We use the RELDAFNA code (Klein 2010) to numerically solve the two-dimensional (2D) special relativistic hydrodynamics equations describing the flow of an ideal fluid with a constant polytropic index, $\gamma = 4/3$. RELDAFNA is a Godunov type Eulerian code, with second-order accuracy in time and space integration. It uses adaptive mesh refinement and is massively parallelized, allowing the use of effectively high resolution even in multiscale problems such as the current jet simulations. RELDAFNA was tested (Klein 2010; see also Section 3) by comparing its solutions for standard test problems to those of similar codes (Zhang & MacFadyen 2006; Meliani et al. 2007) and was shown to perform similarly.

The initial conditions chosen for our numerical calculations were a conical section of opening angle $\theta_0$ within which the flow fields are given by the BM solution for $E_{\text{iso}} = 10^{53}$ erg and initial density $n = 1 \, \text{cm}^{-3}$, surrounded by a static uniform cold gas of density $n = 1 \, \text{cm}^{-3}$ and pressure $p_0 = 10^{-10} \, \text{cm}^{-3} \, \text{g} \, \text{cm}^{-2}$. The radius of the conical section was chosen so that the Lorentz factor of the fluid behind the shock is $\Gamma = 20$. We present solutions for $\theta_0 = 0.2$, 0.1, and 0.05 (corresponding to $E_{\text{jet}} = 2 \times 10^{51}$, $5 \times 10^{50}$, and $1.25 \times 10^{50}$ erg). The $\theta_0 = 0.2$ simulation is similar to the simulation presented in Zhang & MacFadyen (2009, hereafter ZM09). The only difference is that we use $\gamma = 4/3$, instead of an equation of state for which $\gamma$ varies smoothly between $\gamma = 4/3$ for relativistic material and $\gamma = 5/3$ for non-relativistic material. Our choice is inaccurate for late times, when the flow becomes non-relativistic, but this inaccuracy is not expected to affect our results qualitatively. Moreover, if a significant fraction of the post-shock energy density is carried by magnetic fields and relativistic electrons, as required in order to account for afterglow observations, $\gamma$ remains close to its relativistic value further into the non-relativistic flow stage.

The size of the finest numerical cells in the simulation was initially taken as $5.6 \times 10^{13}$ cm, similar to ZM09. The results of our simulations were checked for convergence by increasing the grid resolution by a factor of four in each dimension. Increased resolution calculations were carried out both for initial conditions identical to those of the nominal calculations, and for initial conditions with a reduced radius of the conical section corresponding to a post-shock BM Lorentz factor of $\Gamma = 40$. These convergence tests indicated that while the Lorentz factor behind the shock, as well as the early time light curves that depend strongly on the high $\Gamma$ region, are not yet fully converged, the spreading of the jet is converged to a level of 10%. For example, the time it takes the jet angle to double its initial value decreased in the convergence test by $\sim 8\%$ for $\theta_0 = 0.2$ and by $\sim 25\%$ for $\theta_0 = 0.05$. The results presented in the next sections are the ones obtained with the higher numerical resolution. A more detailed analysis of the numerics will be given in N. Wygoda & E. Waxman (2011, in preparation).

3. JET EXPANSION

Figure 1 presents the evolution of the jet opening angle, $\theta(t)$, as a function of time, $\theta$ is defined as the cone opening angle within which 90% of the energy, excluding rest mass energy,
is included. We find that significant sideways expansion begins at $t \sim t_0 \ll t_{NR}$, in accordance with the analytic estimates described in Section 1. Narrower jets begin expanding earlier, $t_0 \propto \theta_0^{2/3}$, in accordance with the analytic estimates, and are therefore expected to also decelerate earlier.

The latter point is demonstrated in Figure 2, which shows the density distribution of the $\theta_0 = 0.2$ and 0.05 jet flows at identical time $t = 0.95t_{NR}$ (note that $t_{NR}$ is independent of $\theta_0$). The $\theta_0 = 0.2$ jet has tripled its opening angle and its tip is still close to its “isotropic equivalent location,” i.e., the location of a spherical blast wave with the same $E_{iso}$. The opening angle of the $\theta_0 = 0.05$ jet has increased by more than an order of magnitude, and its lateral spreading has significantly slowed down its radial propagation. The influence of jet expansion at $t_{NR}$ is much stronger for the narrower jet, in accordance with the analytic analysis described in Section 1: the ratio of $t_{NR}$ to $t_0$ is close to unity for the $\theta = 0.2$ jet and significantly larger for the $\theta = 0.05$ jet: at the source frame $t_0 \approx t_0 + r_0/c = 2t_0$ so that $t_{NR}/t_0 \approx 1/20^{2/3} = 1.5, 3.7$ for $\theta_0 = 0.2, 0.05$.

We thus find that the jet sideways expansion is relativistic and becomes significant at $t \sim t_0 \propto \theta_0^{2/3}$, and that this expansion leads to deceleration to sub-relativistic velocity on a timescale $t_1 \propto \theta_0^{2/3}$, which for $\theta_0 \ll 1$ is much smaller than $t_{NR} \propto \theta_0^5$. This behavior is consistent with the analytic analysis, and inconsistent with the claims based on earlier numerical modeling, that jet expansion is not significant up to $t \sim t_{NR}$.

In order to identify the origin of this apparent discrepancy, we compare our numerical results to those of ZM09 in Figure 3. The figure demonstrates that the jet expansion obtained in our calculation is similar to that obtained in ZM09. The conclusion that sideways expansion is not relativistic, and unimportant until $t \sim t_{NR}$, was reached in ZM09 based on noting that the growth of $\theta(t)$ is much slower than the exponential growth expected for relativistic sideways expansion (see Figure 3). This conclusion is, however, not valid, since exponential growth is expected only for $\theta^{-1} \gg \Gamma \gg 1$, and is not applicable for the evolution of the $\theta_0 = 0.2$ jet under consideration, for which expansion becomes significant only for $\Gamma < \theta_0^{-1} = 5$.

For this regime of $\Gamma$, one cannot use the exponential approximation, but should rather solve the differential equation

$$d\theta/dr = c_s/\Gamma c_r,$$

(5)

describing relativistic sideways expansion at the post-shock speed of sound $c_s$ in the jet frame ($c_s = c/\sqrt{3}$ for $\Gamma \gg 1$), along with mass and momentum conservation, that determine $\Gamma(r, \theta)$ (for more details see Rhoads 1999). Note that we replace Equation (3) of Rhoads (1999) with the more accurate Equation (5) (see also Piran 2000). This modification leads to a significant modification of $\theta(r)$ only for $\theta > 0.4$. For $\theta_0^{-1} > \Gamma \gg 1$, the solution of Equation (5) is indeed exponential, $\ln \theta/\theta_0 \propto r^{3/2}$. However, such a regime does not exist for $\theta_0 = 0.2$. As demonstrated in the figure, the solution of the simple model of Equation (5) provides a good description of the evolution of the jet opening angle, defined as the cone angle within which 90% of the energy is contained.

The $\theta_0^{2/3}$ scaling of $t_0$ and $t_1$, which is the essential prediction of the analytic model, is confirmed by the simulations independently of the details of the definition of the jet opening angle. The value of the opening angle as inferred from the simulations does, on the other hand, depend on the exact definition, e.g., on the percentage of the energy required to be contained within the...
cone’s opening angle. Thus, the accuracy of using the analytic models for light curve calculations cannot be simply determined from a comparison of the numerical and analytical opening angles. A direct comparison of light curves predicted by the simple analytic model for the mildly relativistic phase and those derived from the numerical simulation is given in Section 4.

Various authors have concluded, like ZM09, that lateral expansion is not relativistic, in contrast with what is shown here. We briefly explain the reasons that led to this misinterpretation of simulations’ results. In Granot (2007) the numerical results, which are similar to the ones obtained here, are compared to those of a model similar to that of Rhoads (1999) out of its domain of validity (Γ ≈ 1). Cannizzo et al. (2004) concluded that sideways expansion is negligible due to their low numerical resolution, which suppresses lateral expansion (Granot 2007; Meliani et al. 2007). Finally, Meliani & Keppens (2010) base their conclusion on a simulation with θ0 = 0.35 rad, for which ts ≈ tNR, in which case lateral expansion is not expected before the jet slows down. Note that Meliani et al. (2007) have concluded, using different initial conditions, that lateral expansion is indeed relativistic (0.4c – 0.7c) and causes the jet to deaccelerate differently than in the isotropic case.

4. LIGHT CURVES AND CALORIMETRY

We have calculated the synchrotron emission expected to be produced by shock accelerated electrons assuming that the magnetic field and the electrons hold a constant fraction $\epsilon_e = \epsilon_B = 0.1$ of the internal energy, and that the electron energy distribution is a power law with index $p = 2.4$. Electron cooling and synchrotron self-absorption are neglected. The effects of self-absorption, which may be significant at early time for low frequencies (the self-absorption frequency is independent of time before sideways expansion, $\nu_A \sim 1e^{-1/2}E^{1/2}_{iso}E^{1/2}_{jet}n_0^{3/2}$ GHz (Waxman 1997a) and decreases afterward), will be discussed in N. Wygoda & E. Waxman (2011, in preparation).

Figure 4 shows radio light curves ($\nu \approx 3$ GHz) predicted by the numerical model for the $\theta_0 = 0.2$ jet, for observers lying on the jet axis and at an angle $\theta_{obs} = \theta_0$. The numerical light curves of the jet are compared with those predicted for spherical (one-dimensional (1D)) fireballs with total energy $E_{iso}$ and $E_{jet}$, as well as with that predicted for a conical section with opening angle $\theta_0$ of a spherical $E_{iso}$ fireball, representing the light curve predicted for a non-expanding jet. Also shown is the radio flux of a spherical fireball with energy $E_{jet}$, assuming its evolution is described by the non-relativistic ST solution.

The jet emission is suppressed, compared to that of a spherical blast wave with energy $E_{iso}$, at $t > t_0$. The suppression is larger than would be predicted for a non-expanding jet (i.e., not due to the “missing flux” from the absent $\theta > \theta_0$ parts of the shell, but rather to the jet spreading, in accordance with van Eerten et al. 2011). Although at $t \sim t_s$, the jet has not yet reached full spherical symmetry and is still mildly relativistic and collimated (see Figure 1), the figure demonstrates that for observers lying both on-axis and at an angle $\theta = \theta_0$, the observed flux at $t > t_s$ is similar to that of a spherical fireball with energy $E_{jet}$.

In Figure 5 we compare jet light curves to those of spherical (1D) fireballs with the corresponding $E = E_{jet}$. Scaling $t_0$ with $t_{s,0}$ brings the (on-axis observer) light curves to a similar (universal) form, implying that significant jet spreading occurs at $t \sim t_0 \propto \theta_0^{2/3}$. Although, as noted above, at $t \approx t_s$ the jet does not yet reach full spherical symmetry, the (on-axis observer) light curves do not depart from those of the 1D $E = E_{jet}$ fireballs by more than $\sim 50\%$ for $t \gtrsim 0.3t_s$. Significant deviation from the 1D model predictions at $t \sim t_s$ occurs only for well off-axis observers, $\theta_{obs} \sim \pi/4 \gg \theta_0$, who do not observe the initial burst. A more detailed analysis of the jet light curves will be presented in N. Wygoda & E. Waxman (2011, in preparation).

5. DISCUSSION AND CONCLUSIONS

While there still remain many unsolved problems regarding the structure and dynamics of GRB jets that can only be
addressed by detailed hydrodynamic modeling, our work has demonstrated that analytic estimates provide a reasonable description of the behavior of the jet and the evolution of its afterglow.

We have shown that relativistic sideways expansion becomes significant at \( t > t_0 \propto \theta_0^{2/3} \) (Figure 1), in accordance with analytic estimates, and that the expansion is well described by the modified Rhoads model (Equation (5)). Our numerical results are consistent with those of ZM09, who calculated \( \theta_0 = 0.2 \) jet evolution. The apparent discrepancy between earlier numerical and analytic results arose because the simulations were not compared to the full solution of the Rhoads’ model (Figure 3, Section 3), and because of the large \( \theta_0 \) chosen, it is difficult to test the relativistic expansion assumption since \( t_{\text{NR}} \) and \( t_s \) are similar, \( t_{\text{NR}}/t_s \approx 1/20^{2/3} = 1.5 \) (see Section 3).

Jet expansion has a significant effect on its observed properties. The suppression of the flux at \( t > t_0 \) is stronger than in the absence of spreading (Figure 4), and at \( t > t_s \) the emission of radiation from the jet blast wave is similar to that of a spherical blast wave carrying the same energy (Figures 4 and 5). Moreover, although at \( t \sim t_s \) the jet has not yet reached full spherical symmetry and is still mildly collimated (see Figure 1), at \( t > t_s \) the flux is well approximated by that of a blast wave following the non-relativistic ST evolution.
(Figures 4 and 5; significant deviations at $t \sim t_s$ occur only for well off-axis observers, $\theta_{\text{obs}} \sim 1 \gg \theta_0$, who do not observe the initial burst). Thus, the total (calorimetric) energy of GRB blast waves may be estimated with only a small fractional error based on $t > t_s$ observations. We expect to see this technique and its variants (Frail et al. 2000; van der Horst et al. 2008; Shivvers & Berger 2011) applied to increasing numbers of GRB afterglows when the new generation of facilities (EVLA, LOFAR) starts full operation.

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REFERENCES

Berger, E., Kulkarni, S. R., & Frail, D. A. 2004, ApJ, 612, 966
Blandford, R. D., & McKee, C. F. 1976, Phys. Fluids, 19, 1130
Cannizzo, J. K., Gehrels, N., & Vishniac, E. T. 2004, ApJ, 601, 380

Cenko, S. B., Frail, D. A., Harrison, F. A., et al. 2010, ApJ, 711, 641
Frail, D. A., Waxman, E., & Kulkarni, S. R. 2000, ApJ, 537, 191
Granot, J. 2007, RevMexAA Conf. Ser., 27, 140
Granot, J., Miller, M., Piran, T., Suen, W. M., & Hughes, P. A. 2001, in Gamma-ray Bursts in the Afterglow Era, ed. E. Costa, F. Frontera, & J. Hjorth (Berlin: Springer-Verlag), 312
Granot, J., & Ramirez-Ruiz, E. 2010, arXiv:1012.5101
Klein, Y. Y. 2010, Masters thesis, Hebrew Univ, Jerusalem, Israel
Livio, M., & Waxman, E. 2000, ApJ, 538, 187
Meliani, Z., & Keppens, R. 2010, A&A, 520, L3
Meliani, Z., Keppens, R., Casse, F., & Giannios, D. 2007, MNRAS, 376, 1189
Panaitescu, A., & Kumar, P. 2002, ApJ, 571, 779
Piran, T. 2000, Phys. Rep., 333, 529
Rhoads, J. E. 1999, ApJ, 525, 737
Shivvers, I., & Berger, E. 2011, ApJ, 734, 58
van der Horst, A. J., Kamble, A., Resmi, L., et al. 2008, A&A, 480, 35
van Eerten, H. J., Leventis, K., Meliani, Z., Wijers, R. A. M. J., & Keppens, R. 2010, MNRAS, 403, 300
van Eerten, H. J., Meliani, Z., Wijers, R. A. M. J., & Keppens, R. 2011, MNRAS, 410, 2016
Waxman, E. 1997a, ApJ, 489, L33
Waxman, E. 1997b, ApJ, 491, L19
Yost, S. A., Harrison, F. A., Sari, R., & Frail, D. A. 2003, ApJ, 597, 459
Zhang, W., & MacFadyen, A. 2009, ApJ, 698, 1261
Zhang, W., & MacFadyen, A. I. 2006, ApJS, 164, 255