General constraints on light resonances in a strongly coupled symmetry breaking sector

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Abstract
In this paper we consider the information that can be obtained about a strongly-interacting symmetry breaking sector from precision measurements of four-fermion processes at LEP2 or a (500 GeV) NLC. Using a “Z-peak” subtracted approach to describe four-fermion processes, we show that measurements of the cross section for muon production, the related forward-backward asymmetry, and the total cross section for the production of hadrons (except $t$’s) can place constraints on (or measure the effects of) the lightest vector or axial resonances present in a strong symmetry breaking sector. We estimate that such effects will be visible at LEP2 for resonances of masses up to approximately 350 GeV, and at a 500 GeV NLC for resonances of masses up to approximately 800 GeV. Multiscale models, for example, predict the presence of light vector and axial mesons in this mass range and their effects could be probed by these measurements.

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1 Introduction.

Precision measurements \[1\] of electroweak quantities at LEP and SLC, as well as in neutral current and atomic parity violation experiments, place constraints on models of dynamical electroweak symmetry breaking. The most severe of these constraints arise from measurements of the “oblique” parameters S and T \[2, 3\], which measure the extra isospin conserving and violating contributions to the gauge boson self-energies, and from a measurement of the ratio of widths \(R_b = \frac{\Gamma_{Zb}}{\Gamma_{Z\text{had}}}\) \[4, 5\].

The extra contributions to T and to \(R_b\) in these models come largely from the physics of top-quark mass generation. The absence of large deviations in these quantities from their standard model predictions strongly suggests that, if electroweak symmetry breaking occurs dynamically, top-quark mass generation must take place in a separate sector which does not give rise to the bulk of electroweak symmetry breaking \[6\]. Examples of this kind include top-condensate models \[7\], such as top-color \[8\], and top-color assisted technicolor \[9\]. Contributions to \(R_b\) and T can be largely eliminated, or at least greatly reduced, in models of this sort. A natural implication in these cases is the existence of extra light scalars, such as “top-pions”, and possibly light vector resonances as well.

The constraints coming from S are potentially more problematic: even if the bulk of electroweak symmetry breaking is due to an isospin-conserving strongly-interacting sector, there will necessarily always be extra contributions to the gauge-boson self-energies. Following Peskin and Takeuchi, an estimate of the oblique effect S can be made as follows. We model the contributions of the symmetry breaking sector to the custodial triplet vector and axial spectral functions each in terms of a single narrow resonance:

\[
R_{VV,AA}(s) = 12\pi^2 F^2_{V,A} \delta(s - M^2_{V,A})
\]

The extra contribution from the symmetry breaking sector to S can then be written:

\[
S = 4\pi \left[ \frac{F^2_V}{M^2_V} - \frac{F^2_A}{M^2_A} \right].
\]

If we further assume that the electroweak symmetry breaking sector is QCD-like, in that it obeys the analogs of the first- and second-Weinberg sum rules \[10\], we find that this expression may be written

\[
S = 4\pi \frac{F^2_\pi}{M^2_V} \left[ 1 + \frac{M^2_V}{M^2_A} \right].
\]

where, to the extent that this sector is responsible for the bulk of electroweak symmetry breaking, \(F_\pi \approx 246\text{GeV}/\sqrt{N_D}\) is the analog of the pion decay constant in QCD. Note that this expression is always positive and, scaling from QCD, Peskin and Takeuchi \[2\] estimate it’s value to be

\[
S \approx 0.25 \frac{N_D N_{TC}}{3}
\]

where \(N_D\) and \(N_{TC}\) are the number of technidoublets and technicolors respectively. This should be compared with the most recent experimental value \[11\]

\[
S = -0.26 \pm 0.16
\]
some 3.5 standard deviations away from the estimate above.

As emphasized by Lane \cite{12}, however, it is necessary that a phenomenologically viable theory of dynamical electroweak symmetry breaking be unlike QCD. As noted twenty years ago, in the absence of a GIM mechanism any extension of a QCD-like symmetry breaking sector that can account for the observed \( s \) and \( c \)-quark masses is likely to produce unacceptably large flavor-changing neutral-currents \cite{13}. The most elegant mechanism proposed to solve this problem is “walking technicolor” \cite{14}. In this case, unlike QCD, the technicolor coupling does not fall quickly just above the chiral symmetry breaking scale: i.e. the coupling “walks” to zero instead of running to zero quickly. This slowly falling coupling enhances the masses of ordinary fermions and may allow one to construct theories without dangerous flavor-changing neutral currents.

Part and parcel of this solution, however, is that the asymptotic behavior of the coupling is not like QCD \cite{12}. This fact casts doubt on the use of the first and second Weinberg sum rules in evaluating \( S \). Furthermore, some methods used to produce a walking coupling invoke fermions in different representations of the technicolor gauge group \cite{15}. In these “multiscale” models, TeV-scale resonances are expected from the bound-states of the technifermions in the highest representation of the gauge group, but lighter resonances occur as well. Indeed, the lightest resonances may be as light as 300 – 400 GeV \cite{15}.

For these reasons, we will consider the implications of precision electroweak data that can be gathered at future colliders for more general strongly-interacting symmetry breaking sectors. Instead of the simple, single-resonance model used in \cite{2}, we consider the case when the spectral function can be approximated as a sum of narrow resonances, all beyond the production threshold:

\[
R_{VV}(s) = 12\pi^2 \sum_i F_{V_i}^2 \delta(s - m_{V_i}^2) 
\]

and

\[
R_{AA}(s) = 12\pi^2 \sum_j F_{A_j}^2 \delta(s - m_{A_j}^2) 
\]

The dispersive evaluation of \( S \) then gives:

\[
S = \left[ \sum_i \frac{F_{V_i}^2}{M_{V_i}^2} - \sum_j \frac{F_{A_j}^2}{M_{A_j}^2} \right].
\]

In the absence of further information, very little can be determined about the parameters in the expression above (note that, a priori, no reason exists why the ratio \( F_{V,A,n}^2/M_{V,A,n}^2 \) should decrease quickly with increasing \( n \), even if the resonances’ masses do actually increase with \( n \)).

The aim of this paper is to show that precision measurements at high energy \( e^+e^- \) colliders can conversely yield information about the lightest resonances in the vector and axial vector channel, at least in a situation of “reasonable” increase of the masses in the families. In the next section, we illustrate the method, in section 3, we perform a quantitative analysis, and a short final discussion is given in section 4.
2 Section 2

2.1 Review of the approach

In a model that predicts the existence of one or several vector resonances strongly coupled to the weak gauge bosons and to the photon, one naturally expects that sizeable contributions to the gauge bosons self-energies will arise from the transition gauge boson-resonance-gauge boson. In the specific case of $e^+e^-$ annihilation, neutral resonances would contribute, at the one loop level, the real part of the three transverse self-energies $A_\gamma(q^2)$, $A_\gamma Z(q^2)$, $A_Z(q^2)$ where $q^2$ denotes the c.m. squared total energy of the process and the definition of $A(q^2)$ is:

$$A_{i,j}(q^2) = A_{i,j}(0) + q^2 F_{i,j}(q^2)$$

($i,j = \gamma, Z; A_{i,i} \equiv A_i$).

In fact, the situation is slightly more subtle at the one loop level. In order to better understand this statement, it is useful to consider the specific case of production of a final muon-antimuon couple, and in particular to concentrate one’s attention on the contribution to the invariant scattering amplitude coming from the $Z$ exchange that, at the tree level, reads

$$T^{(0)}_{e\mu}(q^2, \theta) = i\sqrt{2}G_{\mu,0}M^2_{Z,0}\bar{v}_e \gamma^\rho(g_{V\mu} - g_{A\mu} \gamma^5)u_\mu \gamma^\rho(g_{V\mu} - g_{A\mu} \gamma^5)v_\mu$$

where $G_{\mu,0}, M_{Z,0}$ are the “bare” Fermi coupling and $Z$ mass and we used the identity

$$\sqrt{2}G_{\mu,0}M^2_{Z,0} = \frac{g_0^2}{4c_0^2}$$

($s_0^2 = 1 - c_0^2 = \frac{4}{g_0^2}$).

When one moves to one loop, it is straightforward to verify that the term into the square bracket of eq.(10) is replaced by

$$\frac{\sqrt{2}G_{\mu,0}M^2_{Z,0}}{q^2 - M^2_{Z,0}} \to \frac{\sqrt{2}G_{\mu}M^2_Z}{q^2 - M^2_Z + iM_Z\Gamma_Z}[1 + \frac{\delta G_{\mu}}{G_{\mu}} + \frac{Re\bar{A}_{\mu}(0, \theta)}{M^2_Z} - I^{\mu}_{Z}(q^2, \theta)]$$

where $G_{\mu}, M_Z, \Gamma_Z$ are now the physical (conventionally defined) Fermi coupling, the $Z$ mass and the $Z$ width; $\bar{A}_{Z}(q^2, \theta)$ is a certain gauge-invariant combination of self-energies, vertices and boxes whose self-energy component is $\bar{A}_{Z}(q^2)$ defined by eq.(1) and

$$\bar{I}^{e,\mu}_{Z}(q^2, \theta) = \frac{g_0^2}{q^2 - M^2_Z}Re[\bar{F}_{Z}^{(e,\mu)}(q^2, \theta) - F_{Z}^{(e,\mu)}(M^2_Z, \theta)]$$

where the self-energy component of $\bar{F}_{Z}^{e,\mu}$ is $F_{Z}(q^2)$, defined again by eq.(9). The full expression of $\bar{A}_{Z}(q^2, \theta)$ that includes vertices and boxes has been given in previous references.
It has been obtained following closely the Degrassi-Sirlin conventions and philosophy, and it is essentially based on the simple property that quantities contributing different Lorentz structures of the invariant scattering amplitude must be separately gauge-independent. Since the discussion of refs. 16 is rather detailed, we shall not repeat it here, particularly because in the special case that we are considering only the TC contributions to the self-energies will be relevant. In this very definite sense, we can from now on concentrate our attention on the transverse self-energy content of the various gauge-independent quantities. The only possible exception to this rule, provided by the process where a final $b\bar{b}$ couple is produced, will be separately discussed at the end of this section.

Eq. (12) shows the transformation at one loop of the term into the square bracket of eq. (10). To fully take into account the electroweak one loop structure it is sufficient to replace the bare quantity $g^0_{Vl}$ in eq. (10) by the corresponding finite and gauge-independent term

$$g^0_{Vl} \rightarrow g^1_{Vl} \equiv \frac{1}{2} [1 - 4s^2(q^2, \theta)]$$

where

$$s^2_l(q^2, \theta) = s^2_l[1 + \tilde{\Delta} \kappa(q^2, \theta)]$$

$$\tilde{\Delta} \kappa(q^2, \theta) = \frac{c_1}{s_1} [\tilde{F}_{\gamma Z}(q^2)] + \frac{c_2^2}{c_1^2 - s_1^2} \left( \frac{\delta \alpha}{G_\mu} - \frac{\delta G_\mu}{\delta M^2_Z} \right)$$

Here $\tilde{F}_{\gamma Z}(q^2)$ is a gauge-invariant combination of self-energies, vertices and boxes whose self-energy part is $F_{\gamma Z}(q^2)$ defined by eq. (10) and $s^2_l c_1^2 \equiv s^2_l(1 - s^2_l) = \frac{\pi \alpha}{\sqrt{2g_\mu M^2_Z}}$. Note that one has at $q^2 = M^2_Z$

$$s^2_l(M^2_Z, \theta) \equiv s^2_l(M^2_Z)$$

(boxes do not contribute at this $q^2$ value) where $s^2_l(M^2_Z)$ is by definition the effective weak “leptonic” angle measured on $Z$ resonance. Analogously, one finds

$$\left( \frac{\delta G_\mu}{G_\mu} + \frac{Re \tilde{A}^{(\epsilon, \mu)}_Z(0, \theta)}{M^2_Z} \right) - \tilde{I}^\mu_Z(M^2_Z, \theta) \equiv \epsilon_1$$

where $\epsilon_1$ is the first of the three Altarelli-Barbieri parameters, whose combination with the third parameter $\epsilon_3$ produces $s^2_l(M^2_Z)$

$$s^2_l(M^2_Z) = s^2_l + \frac{s^2_l}{c_1^2 - s_1^2} [\epsilon_3 - c_1^2 \epsilon_1 + c_1^2 (\Delta \alpha(M^2_Z) + \text{"vertices")}]$$

where $\Delta \alpha(M^2_Z) = F_\gamma(0) - F_\gamma(M^2_Z)$ takes into account the running of $\alpha_{QED}$.

For the purpose of this paper, it is convenient to introduce now the leptonic $Z$ width $\Gamma_l$, defined in terms of $\epsilon_{1,3}$ as
\[
\frac{\Gamma_l}{M_Z} = \frac{1}{24\pi\sqrt{2}}(1 + \delta^{\text{QED}})G_\mu M_Z^2[1 + \epsilon_1][1 + v_l^2(M_Z^2)]
\]  

(20)

where \(v_l \equiv 1 - 4s_\ell^2\) and \(\delta^{\text{QED}}\) is a known QED “correction”.

The previous expressions, that we wrote in order to achieve a fully self-contained discussion, are sufficient to proceed now in a quicker way towards our specific goal. The next step in this direction is that of writing the expression for the “Z-Z component” of the differential \(e^-\mu^+\) cross section, that will read at one loop

\[
\left(\frac{d\sigma^{(1)}(Z)}{d\Omega}\right)_{e\mu}(q^2, \theta) \approx \left\{ \frac{\sqrt{2}G_\mu M_Z^2}{q^2 - M_Z^2}[1 + \frac{\delta G_\mu}{G_\mu} Re \tilde{A}_Z^{(e-\mu)}(0, \theta) - \tilde{I}_Z^{e,\mu}(q^2, \theta)] \frac{1}{4}[1 + (1 - 4s_\ell^2(q^2, \theta))^2]\right\}^2
\]  

(21)

The previous formula can be rewritten by formally “adding and subtracting” \(\tilde{I}_Z^{e,\mu}(q^2 = M_Z^2, \theta)\) and \(s_\ell^2(q^2 = M_Z^2, \theta)\). Throwing away higher order terms and using the definitions eqs. (17-21) one concludes that at the one loop level, the alternative “Z-peak subtracted” representation can be used:

\[
\left(\frac{d\sigma^{(1)}(Z)}{d\Omega}\right)_{e\mu}(q^2, \theta) \approx \left\{ \frac{\Gamma_l}{M_Z}[1 - R^{(l,\mu)}(q^2, \theta) - \frac{8s_1c_1v_l(M_Z^2)}{1 + v_l^2(M_Z^2)} V^{(l,\mu)}(q^2, \theta)] \right\}^2
\]  

(22)

where the two “form-factors”

\[R^{(e,\mu)}(q^2, \theta) \equiv \tilde{I}_Z^{e,\mu}(q^2, \theta) - \tilde{I}_Z^{e,\mu}(M_Z^2, \theta)\]  

(23)

and

\[V^{(e,\mu)}(q^2, \theta) \equiv \tilde{F}^{e,\mu}_Z(q^2, \theta) - \tilde{F}^{e,\mu}_Z(M_Z^2, \theta)\]  

(24)

are now subtracted at \(q^2 = M_Z^2\) and, in the remaining theoretical expression, the Fermi coupling \(G_\mu\) has disappeared, having been replaced by quantities measured on Z peak \((\Gamma_l, s_\ell^2(M_Z^2))\). Clearly, this will cause a loss of precision in the theoretical prediction. This has been fully discussed in Refs. [16], showing that the accuracy of the experimental measurements on Z resonance is sufficient to guarantee that no sensible theoretical error will be induced at LEP2, LC given their (optimal) expected experimental accuracies. For this totally pragmatic reason, we can conclude that eqs. (21) and (22) provide two practically equivalent theoretical expressions for what concerns realistic future \(e^+e^-\) colliders measurements in the two final muon channel.

In a totally analogous way, all the theoretical expressions for all possible “Z-Z” components of observables in the final two fermion processes can be rewritten in such a way that the new expressions contain Z-peak subtracted “form-factors” and Z-peak measured quantities (in general, Z widths and asymmetries). The full discussion is given in refs. [16] and we do not insist on it here. Also, the method can be easily extended to the various “\(\gamma - Z\)” type components leading to similar Z-peak subtracted expressions, while for the “\(\gamma - \gamma\)” components the existing theoretical scheme does not need any extra change: it will
contain $\alpha_{QED}$ and “$\gamma\gamma$” form factors subtracted at $q^2 = 0$, of the general non universal form

$$\tilde{\Delta}(e,f)(q^2, \theta) = \Delta(0) + "l - f vertices, boxes"$$

(25)

where $\tilde{\Delta}(e,f)(q^2 = 0, \theta) = 0$, and the universal self-energy component $\Delta(0)$ is defined after eq.(19).

The next question that arises immediately at this point is that of under which circumstances it will be convenient to abandon the conventional representation where $G$ is retained and to adopt the Z-peak subtracted one. Although a general prescription is not simple to be obtained, we can provide here an example of two specific situations where, in our opinion, the use of the subtracted expression will be rather useful. The first case is one in which one wants to write down a simple, analytic theoretical expression, that may provide an approximate “satisfactory” estimate of the complete numerical value of various relevant observables in the Standard Model at one loop. As we shall show in the next Section, this will indeed be the case if a suitably defined “Z-peak subtracted” Born approximation, built in the spirit of our approach, will be systematically utilized. The second case is one in which one wants to compute the virtual effects at one loop of a model of new physics that contains a number of parameters, whose $q^2$-dependence cannot be systematically neglected, so that their effects in the subtracted form-factors will not be generally negligible. Such would be the case of models of technicolour type, to which we now devote the second part of this section for a fully illustrative discussion.

2.2 Effects of models of TC type

We shall start by considering the relatively simple case of a technicolour-type model where two families of resonances exist, one of vector and one of axial type, that can only contribute self-energies through the chain gauge-boson-resonance-gauge-boson.

Other possible effects will be ignored. the only exception being represented by the contribution to the $Zb\bar{b}$ vertex, to be discussed later on. Therefore, assuming a number $N_V$ and a number $N_A$ of vector and axial-vector resonances, under the reasonable assumption that the various widths can be in first approximation neglected with respect to the masses, the model can be parametrized by giving $2(N_V + N_A)$ parameters, conventionally called “strengths” and masses, to be denoted e.g. as $F_{Vi}, M_{Vi}, F_{Ai}, M_{Ai}$. Using as a first approach a delta-type parametrization, we shall write following conventional normalizations:

$$ImA_V(q^2) = \sum_{i=1}^{N_V} \pi q^2 F_{Vi}^2 \delta(q^2 - M_{Vi}^2)$$

(26)

$$ImA_A(q^2) = \sum_{j=1}^{N_A} \pi q^2 F_{Aj}^2 \delta(q^2 - M_{Aj}^2)$$

(27)
where $A_{V,A}$ are the vector and axial-vector components of the three neutral self-energies $A_{i,j}(q^2)$ eq.(8), and we shall follow the usual request that parity and strong isospin are conserved in the model, writing

$$A_{\gamma\gamma}(q^2) = e^2 A_V(q^2)$$  \hspace{1cm} (28)

$$A_{ZZ}(q^2) = \frac{e^2}{4s_1^2c_1}[(1 - 2s_1^2)^2 A_V(q^2) + A_A(q^2)]$$  \hspace{1cm} (29)

$$A_{\gamma Z}(q^2) = \frac{e^2}{2s_1c_1}(1 - 2s_1^2) A_V(q^2)$$  \hspace{1cm} (30)

(we are neglecting for the moment possible $\omega$-type resonances).

The simplified delta function approximation of eqs.(24-27) is very useful provided that the use of unsubtracted dispersion relation is allowed to compute the resonant contribution to the relevant self-energies. This provides a very general and to some extent model independent way of proceeding. Whenever such an approach is not feasible, other more specifically model dependent approaches must be used, that introduce in general extra parameters and assumptions. Although this is in principle a tolerable attitude, the loss of simplicity and generality thus introduced is certainly not an advantage.

On this very specific point, the replacement of the conventional representation by the Z-peak subtracted one plays the essential role. This can be seen by concentrating again on the special example provided by the equivalent eqs.(24) and (26). As one easily sees, the self-energy content of eqs.(26) is represented by three separate quantities, i.e. $(A_{Z}(0) - A_{W}(0) M_Z^2)$, (the $W$ self-energy $A_{W}(0)$ is coming from $\frac{\delta G_{\mu}}{G_{\mu}}$), $I_Z(q^2)$ defined after eq.(5) and $\Delta \kappa(q^2)$ the self-energy component of eq.(8). Quite generally, neither $(A_{Z}(0) - A_{W}(0) M_Z^2)$ nor $\Delta \kappa(q^2)$ satisfy unsubtracted dispersion relations, since in the limit of large $q^2$ all $F_i(q^2)$ approach a non vanishing constant (in particular, $A_i(q^2)$ diverges linearly). This difficulty is completely removed in eq.(26), where only subtracted quantities $\simeq [F_i(q^2) - F_i(M_Z^2)]$ survive, since for such differences it is always possible to write the (formally) unsubtracted dispersion relation:

$$[F_i(q^2) - F_i(M_Z^2)] = \left(\frac{q^2 - M_Z^2}{\pi}\right) P \int_0^{\text{inf}} ds \text{Im} F_i(s) \left(\frac{s - q^2}{s - M_Z^2}\right)$$  \hspace{1cm} (31)

As a consequence of this fact, the complete TC resonant contribution to self-energies in LEP2, LC two fermion production in the approximation of eqs.(27),(28), will be expressed in terms of the resonances’ strengths and masses, and more precisely it will affect the three universal form-factors in the following way:

$$R^{(TC)}(q^2) = \frac{\pi \alpha(q^2 - M_Z^2)}{s_1^2c_1^2} \left(1 - 2s_1^2\right)^2 \sum_i \frac{F_{V_i}^2}{M_{V_i}^2} \left(\frac{1}{1 - \frac{M_Z^2}{M_{V_i}^2}}\right)^2 \left(\frac{1}{M_{V_i}^2 - q^2}\right) +$$
\[ \sum_{i} \frac{F_{Ai}^2}{M_{Ai}^2} \left( \frac{1}{1 - \frac{M_{Ai}^2}{M_{Ai}^2 - q^2}} \right)^2 \left( \frac{1}{M_{Ai}^2 - q^2} \right) \]  

(32)

\[ V^{(TC)}(q^2) = \frac{2\pi\alpha(q^2 - M_2^2)}{s_1 c_1} \left\{ (1 - 2s_1^2) \sum_{i} \frac{F_{Vi}^2}{M_{Vi}^2} \left( \frac{1}{1 - \frac{M_{Vi}^2}{M_{Vi}^2 - q^2}} \right) \right\} \]  

(33)

\[ \Delta^{(TC)}(q^2) = -4\pi\alpha \sum_{i} \frac{F_{Vi}^2}{M_{Vi}^2} \left( \frac{1}{M_{Vi}^2 - q^2} \right) \]  

(34)

Eqs. (32-34) can be easily derived starting from the definitions of \( R, V, \Delta \alpha \) eqs. (24), (25), (20) and from eqs. (27-30). Note that the delta-function approximation is not essential in our derivation. More realistic parametrizations \( a-la \) Breit-Wigner could be used, but for not unreasonably large resonance widths the results would not be essentially modified, as one could easily verify.

Starting from eqs. (32-34) it is now straightforward to compute the TC contribution to all the observables of the process \( e^+e^- \rightarrow f\bar{f} \) at variable \( q^2 \). This will be discussed in detail in the next final Section 3. But before doing that we want to stress another consequence of the use of our subtracted approach that is, we believe, rather useful. Looking at eqs. (32, 33) one sees that in the sums over the two family indices each separate \( F_i^2/M_i^2 \) term is multiplied by a factor \( \simeq 1/(M_i^2 - q^2) \). This has the consequence that a factor \( \simeq 1/[1 + (M_n^2 - M_1^2)/(M_1^2 - q^2)] \) will weight the \( n \)-th coefficient \( F_n^2/M_n^2 \) in the sum, thus strongly depressing the \( n > 1 \) contributions when \( q^2 \) increases (and approaches values not dramatically smaller than \( M_1^2 \)) and \( M_n, n > 1, \) is “reasonably” larger than \( M_1 \). This mechanism favours therefore the possibility of isolating, or at least privileging, the contributions coming from the lightest vector and axial-vector resonances, independently of the existence and the features of heavier components in the family.

This situation should be compared with that obtainable from measurements on \( Z \) resonance, that was exhaustively discussed in refs. [4]. In particular, the effect of a TC model like the one that we are considering here on \( \epsilon_3 \) (which is equivalent to the effect on the self-energy part of \( \epsilon_3 \), practically on the Peskin-Takeuchi parameter \( S \)) would be of the following kind:

\[ \epsilon_3^{(TC)} \simeq \frac{\alpha}{4s_1^2} S_{TC} \simeq \frac{\alpha}{s_1^2} \left\{ \sum_{i} \frac{F_{Vi}^2}{M_{Vi}^2} - \sum_{j} \frac{F_{Aj}^2}{M_{Aj}^2} \right\} \]  

(35)

(we did not impose in eq. [10] the validity of the two Weinberg sum rules).

A glance at eq. (35) shows that, indeed, the type of information derivable at LEP1 from \( \epsilon_3 \) (\( S \)) on a model with families is quite different and therefore independent of that derivable at larger \( q^2 \) values at LEP2, LC. In fact, no depression factor exists in eq. (35), and one measures in fact the sum of all \( F_{i,j}^2/M_{i,j}^2 \) terms. Only for the special case of one resonance per family, the informations at LEP1, LEP2, LC are of the same type.

Keeping this fact in mind, in the remaining part of this paper we shall examine a case where only the 2 lightest resonances (a “techni-\( \rho \)” and a “techni-\( A \)” will contribute...
In other words, we shall assume that the next terms in the two sums are sufficiently depressed with respect to the lightest ones. The quantitative analysis of this particularly simple case will be shown in the forthcoming Section 3.

3 TC contribution to realistic observables

3.1 Formal expressions

From the expressions of the TC contribution to the three universal form factors \( R, V, \Delta \alpha \) given by eqs. (32-34) it is simple to derive the corresponding effect on any observable quantity of the process \( e^+e^- \rightarrow 2 \) fermions. In practice, though, this procedure is unlikely to lead to interesting consequences unless the predicted effect is, on a certain suitable scale, relatively “large”. In particular, a rather essential condition is that the experimental accuracy with which a certain observable can be measured is “suitable”, since the size of “visible” effects should be in any case larger (by an amount dictated by the aimed confidence level of the research) than that of the experimental error.

At LEP2, at the optimal expected experimental conditions illustrated in previous studies [19], the best experimental precision, of a relative size of less than one percent, will be achieved in the measurements of three observables i.e. \( \sigma_\mu \) (the cross section for muon production), \( A_{FB,\mu} \) (the related forward-backward asymmetry) and \( \sigma_5 \) (the cross section for production of hadrons). The same situation will probably occur at a next linear \( e^+e^- \) collider of 500 GeV (NLC) [20] at least in a first stage where initial state longitudinal polarization will not be achievable. For this reason, we have decided in this paper to concentrate our attention on the three previously considered observables, leaving a more complete analysis of extra quantities (like specific heavy quark production cross sections and asymmetries and/or various longitudinal polarization asymmetries) to a future investigation.

As a first step towards our goal, we will write three approximate expressions valid at one loop, where only the self-energy component of the various form factors has been retained. In order to avoid unnecessary complications, we shall moreover retain only, for what concerns the SM component of the involved self-energies, the pure fermionic contribution (the latter is automatically gauge-invariant). Also, terms that are numerically irrelevant (at the percent level fixed by the experimental accuracy) will be systematically neglected, following the discussion given in refs. [19]. As a result of this procedure, we shall be led to the following approximate expressions, valid in the “Z-peak subtracted” approach (and, strictly, at the one loop level):

\[
\sigma_\mu(q^2) \simeq \frac{4}{3} \frac{\pi q^2}{\alpha}(\frac{\alpha^2}{q^4}(1 - \Delta \alpha(q^2)))^2 + \left[ \frac{3 \Gamma_i}{M_Z(q^2 - M_Z^2)} \right]^2 \left[ 1 - R(q^2) - 8 s_1 c_1 v_{l(M_Z^2)} V(q^2) \right]^2 \\
+ \left[ \frac{2 \alpha}{q^2} \frac{3 \Gamma_i}{M_Z(q^2 - M_Z^2)} v_{l(M_Z^2)} \right]^2
\]  

(36)
A few comments on eq. (36) are appropriate at this point. The contribution from the “Z-Z” term can be reconstructed directly from eq. (23). For sufficiently large \( q^2 \) values, \( q^2 > M_Z^2 \), it is much less important than the pure photon contribution. For the latter we have retained a factor \( 1/(1 - \Delta \alpha) \) that, for what concerns its SM fermionic component, resums over all the leading logarithmic contributions. For contributions from other sources, i.e. of TC origin, we shall identify (at our one-loop level) \( 1/(1 - \Delta \alpha^{(TC)}) \) with \( [1 + \Delta \alpha^{(TC)}] \).

The third contribution in eq. (28) is due to \( \gamma - Z \) interference. This is, in the case of \( \sigma_\mu \), definitely smaller than the other two terms (almost two orders of magnitude) at lowest order. We have not used in this case the extra \( O(\alpha) \) correction factor, that would contain \( \Delta \alpha \), \( R \) and \( V \), since the practical effect of this correction would be invisible, at the given experimental conditions.

The derivation of eq. (36) is particularly simple. In a perfectly equivalent way one can derive two approximate expressions for the remaining considered observables, as it has been shown in refs. [16]. More precisely, we shall define:

\[
A_{FB,\mu}(q^2) = \left[ \frac{\sigma_{FB,\mu}(q^2)}{\sigma_\mu(q^2)} \right]
\]

where \( \sigma_\mu \) will be approximated by eq. (36) and

\[
\sigma_{FB,\mu}(q^2) \simeq \pi q^2 \left\{ \frac{2\alpha}{q^2} \left[ \left( \frac{3\Gamma_l}{M_Z} \right) \left( \frac{1}{q^2 - M_Z^2} \right) \left( \frac{1}{1 + \nu l^2(M_Z^2)} \right) \left[ \frac{1}{1 - \Delta \alpha(q^2)} \right] + \frac{3\Gamma_l}{M_Z} \frac{4\nu l^2(M_Z^2)}{(q^2 - M_Z^2)^2} \right] \right\}
\]

Note that, away from the Z peak, the pure Z contribution (second term in eq. (38)) is negligible with respect to the \( \gamma - Z \) interference (first term). For this reason, like in the previous case, we did not consider the extra correction factor containing \( R \) and \( V \) that would multiply it.

Quite similar considerations, that lead to several welcome simplifications, finally allow to write an approximate expression for \( \sigma_5(q^2) \) (\( \equiv \sigma_u(q^2) + \sigma_d(q^2) + \sigma_s(q^2) + \sigma_c(q^2) + \sigma_b(q^2) \)) that is relatively simple, and reads:

\[
\sigma_5(q^2) \simeq N_q^{QCD}(q^2)^4 \frac{4}{3} \pi q^2 \left\{ \left[ \frac{11\alpha^2}{9q^4} \right] \left[ \frac{1}{1 - \Delta \alpha(q^2)} \right]^2 + \left[ \frac{3\Gamma_l}{M_Z} \right] \left[ \frac{3\Gamma_{had}}{M_Z N_q^{QCD}(M_Z^2)} \right] \left( \frac{1}{q^2 - M_Z^2} \right) \left[ \left[ 1 - 2R(q^2) - s_1 c_1 V(q^2) \right] \frac{32}{10} \frac{\Gamma_c}{\Gamma_{had}} + \frac{48}{13} \frac{\Gamma_b}{\Gamma_{had}} \right] \right\}
\]

where \( \Gamma_c, \Gamma_b, \Gamma_{had} \) are the Z widths into \( c\bar{c}, b\bar{b} \) and hadrons (measured on top of Z resonance) and, to obtain some of the numerical factors, the appropriate expression \( s_1^2 = 1/4 \) has been used (only in the terms that are already negligibly small or of \( O(\alpha) \). Note
that, in the case of \( \sigma_5 \), the pure photon and the pure Z terms are almost equivalent, and much more important than the \( \gamma - Z \) interference contribution (last term in eqs. (39)), for which consequently several approximations (like that of considering \( \Gamma_u = \Gamma_c \) and \( \Gamma_d = \Gamma_s = \Gamma_b \)) have been used, whose effect on the overall expressions is well beyond the experimental accuracy, as fully discussed in ref. [16].

From eqs. (36-39) it is straightforward to derive the TC vector resonances (VR) virtual contribution (at one loop) to the considered observables. More precisely, we shall have in our approach the following TC “shifts”:

\[
\delta \sigma^{(TC)VR}_\mu (q^2) = \frac{4}{3} \pi q^2 \left\{ \frac{\alpha^2}{\alpha_0^2} \right\} 2 \Delta \alpha^{(TC)}(q^2) \left[ \left( \frac{3 \Gamma_I}{M_Z} \right)^2 \frac{1}{(q^2 - M_Z^2)^2} \right] \left[ 2 R^{(TC)}(q^2) + 16 s_1 c_1 V^2(q^2) \right] \]  

\[
\delta \sigma^{(TC)VR}_{FB,\mu} (q^2) = \frac{N QCD}{3} \pi q^2 \left\{ \frac{11 \alpha^2}{\alpha_0^2} \right\} 2 \Delta \alpha^{(TC)}(q^2) \left[ \left( \frac{3 \Gamma_I}{M_Z} \right)^2 \frac{1}{(q^2 - M_Z^2)^2} \right] \left[ \Delta \alpha^{(TC)}(q^2) - R^{(TC)}(q^2) \right] \]  

\[
\delta \sigma^{(TC)VR}_5 (q^2) = \frac{4}{3} \pi q^2 \left\{ \frac{\alpha^2}{\alpha_0^2} \right\} 2 \Delta \alpha^{(TC)}(q^2) \left[ \left( \frac{3 \Gamma_I}{M_Z} \right)^2 \frac{1}{(q^2 - M_Z^2)^2} \right] \left[ 2 R^{(TC)}(q^2) + s_1 c_1 V^2(q^2) \right] \left\{ \frac{32 \Gamma_c}{10 \Gamma_{had}} + \frac{48 \Gamma_b}{13 \Gamma_{had}} \right\} \]  

Starting from eqs. (10-12) and taking into account the definition eq. (37) one can easily obtain the formal expressions of the various effects by simply inserting eqs. (32-34). At this point, everything is ready for the derivation of exclusion limits on the parameters from suitable conventional fits, at a given confidence level. This will be done in the forthcoming Section 3.2. But before doing that we want to devote the last part of this half-Section to a quantitative discussion, related to the actual accuracy of the approximated formulae eqs. (36-39) that we have used for what concerns the conventional Standard Model prediction.

On the basis of the “Z-peak subtracted” philosophy, we would actually expect that eqs. (36-39) should give a reasonable approximation, within the SM scheme, of all those contributions that have been already “partially” reabsorbed by quantities measured either at \( q^2 = M^2 \) (Z widths, asymmetries) or at \( q^2 = 0 \) (\( \alpha \)). The genuinely electroweak terms that cannot be reabsorbed in this way (QED boxes are supposedly known and separately calculable) are those coming from weak boxes, since these are kinematically vanishing at these points. This suggests that a more ambitious approximation to one-loop observables should contain the effect from weak box diagrams, in particular at higher \( q^2 \) values where such terms might become more relevant (e.g. “pure Z” boxes will be multiplied in our
representation by \((q^2 - M_Z^2)\) factors). In fact, the complete existing programs \cite{21} confirm this feeling in details.

In order to achieve this additional accuracy, we have proceeded as follows. First of all, we have decided from now on to work for what concerns the SM component in the t’Hooft \(\xi = 1\) gauge. In this gauge, we have then simply computed the SM WW and ZZ boxes following the prescriptions given by Consoli and Hollik \cite{22}. These extra terms have then been added to the SM component of eqs.\((36-39)\). The resulting expressions are those that we expect to provide a satisfactory approximation to the SM predictions for \(q^2 > M_Z^2\).

They contain a certain amount of vertices already reabsorbed in the new theoretical inputs; the SM self-energy content should be practically negligible (it is systematically subtracted) with the exception of the fermionic contribution to \(\Delta \alpha(q^2)\), that is in fact the only quantity that we have retained; the weak boxes content is exactly added. We have defined these approximate expressions with an “S” index, that means “subtracted”, and written formally

\[
\sigma^{(SM,S)}(q^2) \equiv [\sigma^{eq.\,(28)}]\]_{SM} + \sigma^{W,Zboxes}(q^2) \tag{43}
\]

\[
A^{(SM,S)}_{FB,\mu}(q^2) \equiv [A^{eq.\,(29-30)}]\]_{SM} + A^{W,Zboxes}_{FB,\mu}(q^2) \tag{44}
\]

\[
\sigma^{(SM,S)}_5(q^2) \equiv [\sigma^{eq.\,(31)}]\]_{SM} + \sigma^{W,Zboxes}_5(q^2) \tag{45}
\]

To verify whether our feelings were correct, we have then carried on a systematic comparison with the rigorous predictions obtainable e.g. by using the available program TOPAZ0 \cite{21}. We show the results of this comparison in the following Table I at several \(q^2\) values, chosen for simplicity in the LEP2 range i.e. for \(\sqrt{q^2} \lesssim 200\) GeV. Note that the purely electroweak contribution to \(\sigma_5\) has been computed. In order to obtain the complete expression that includes the strong interaction effects, a conventional rescaling factor \((1 + \alpha_s(q^2)/\pi)\) should be applied. This is straightforward and should not require any special comment.

As one sees from inspection of Table I, the numerical agreement between our approximate “subtracted” (S) expressions eqs.\((43)-\tag{45}\) and the full rigorous TOPAZ0 (T) calculation is indeed impressive, since the numerical differences are systematically well below the optimal expected \((\sim 1\%)\) experimental accuracy, with one possible exception around the critical value \(\sqrt{q^2} = 2M_W\), where the “resonant” effects of the W vertex, that cannot be taken into account by our method, are not negligible. This means that, for \(\sqrt{q^2}\) not very close to \(2M_W\), we shall be entitled to use our expressions as an adequate description of the SM predictions as long as searches of visible effects of New Physics will be performed. This is exactly what we shall do in the next forthcoming Section 3.2.

3.2 Numerical analysis of the TC effects

From the previous long discussion we have concluded that there might be an effect in the three observables \(\sigma_\mu\), \(A_{FB,\mu}\) and \(\sigma_5\), at \(q^2 > M_Z^2\), essentially produced by the two
lightest members of two families of vector and axial vector resonances, when the next resonances are sufficiently heavier than the lightest ones, that are in turn supposed to be not too far away from the available total c.m. $e^+e^-$ energy. In this case, there would be four parameters to be fitted at a given $q^2$, i.e. $F^2_V/M^2_V$, $M^2_V$, $F^2_A/M^2_A$, and $M^2_A$ (from now on, $V$, $A$ will denote the lightest resonances).

We can introduce at this point another simplification that is somehow motivated by technical considerations. More precisely, we shall accept that both $M^2_V$, $M^2_A$ are definitely larger than $M^2_Z$:

$$\frac{M^2_Z}{M^2_V} << 1 \quad ; \quad \frac{M^2_Z}{M^2_A} << 1 \quad (46)$$

The practical consequence of this assumption is that the number of parameters that appear in the three form-factors $\Delta \alpha$, $R$, $V$ eqs.(32-34) is now reduced to two, i.e.

$$X(q^2) = \left(\frac{F^2_V}{M^2_V}\right)(\frac{1}{M^2_V - q^2}) \quad (47)$$
$$Y(q^2) = \left(\frac{F^2_A}{M^2_A}\right)(\frac{1}{M^2_A - q^2}) \quad (48)$$

and the numerical program devised to calculate bounds at a given confidence level becomes rather simple. We have decided to proceed in this way to get a first set of results valid in the approximation of eq.(46), encouraged by the fact that our ansatz is actually only affecting the two form factors $R$ and $V$ but not $\Delta \alpha$ that, in the three considered observables, gives a large part of the effect (in the particular case of $\sigma_\mu$, $\Delta \alpha$ is in practice the only relevant correction). Therefore our approximation will only affect a fraction of the calculation, mostly for what concerns the contribution of the axial vector resonance (and only if $M^2_A$ is not sufficiently larger than $M^2_Z$).

In the first part of our analysis we have derived model-independent bounds for the two quantities $X(q^2)$, $Y(q^2)$ at two different values of $\sqrt{q^2} = 192$ $GeV$ and $500$ $GeV$ that correspond to the highest energy situations at LEP2 and at a future NLC $e^+e^-$ linear collider. With this aim, we have parametrized the relative TC shifts of the considered observables from their SM values as follows

$$\frac{\delta \sigma^{(TC)}_\mu(q^2)}{\sigma_\mu} = \frac{\sigma^{(TC)}_\mu - \sigma^{(SM)}_\mu}{\sigma_\mu} = a_1(q^2)X(q^2) + b_1(q^2)Y(q^2) \quad (49)$$
$$\frac{\delta A^{(TC)}_{FB,\mu}(q^2)}{A^{(SM)}_{FB,\mu}} = \frac{A^{(TC)}_{FB,\mu} - A^{(SM)}_{FB,\mu}}{A^{(SM)}_{FB,\mu}} = a_2(q^2)X(q^2) + b_2(q^2)Y(q^2) \quad (50)$$
$$\frac{\delta \sigma^{(TC)}_5(q^2)}{\sigma_5} = \frac{\sigma^{(TC)}_5 - \sigma^{(SM)}_5}{\sigma_5} = a_3(q^2)X(q^2) + b_3(q^2)Y(q^2) \quad (51)$$
where the quantities $a_j, b_j$ are certain functions of $q^2$ whose numerical values can be easily derived from eqs.([10],[12]) and from the given expressions of the SM formulae used in our program.

In the practical derivation of bounds we have assumed that the experimental accuracies for the three observables are, both at LEP2 [19] and at NLC [20], respectively

\[
\frac{\delta \sigma^{(\text{exp})}_\mu}{\sigma_\mu} = \frac{\delta A_{FB,\mu}^{(\text{exp})}}{A_{FB,\mu}} = \pm 0.009; \quad \frac{\delta \sigma_5^{(\text{exp})}}{\sigma_5} = \pm 0.007
\] (52)

Under the assumption of non observability of any deviation from the SM predictions, we have then derived the 95% CL exclusion curves in $\text{TeV}^{-2}$ for $X, Y$ depicted in Fig.(1) (LEP2) and Fig.(2) (NLC) respectively (since both $X$ and $Y$ are by definition positive quantities, we have only shown the relevant quadrant in the $(X, Y)$ plane). As one sees, the dependence of $X$ and $Y$ is rather different in the observables: in particular, $\sigma_\mu$ gives, as expected, essentially information on $X$, while $\sigma_5$ and the asymmetry are also sensitive to $Y$. As a consequence of the different sensitivities, the combined 95% CL exclusion ellipse can be reasonably approximated in the allowed $(X, Y > 0)$ region, by a line of equation

\[
Y = a + bX
\] (53)

with $(a, b) = (2.8, -1.7)$ at LEP2 and $(0.38, -1.8)$ at NLC. In practice, the limiting values would be roughly of $O(1)$ at LEP2 and ten times smaller at NLC.

Figs.(1) and (2) show a result that is, to a certain extent, model independent in the sense that no specific assumption e.g. on the strengths of the couplings has been used. From the curves given in these Figures one can, as a next step, derive bounds for a subset of the four original parameters ($F_i^2/M_i^2, M_i^2$) if certain conditions on the remaining subset are imposed. To give an example of such a procedure we have first parametrized the two effective strengths in the following way:

\[
\frac{F_V}{M_V} = \alpha \frac{f_\rho}{m_\rho}; \quad \frac{F_A}{M_A} = \beta \frac{F_V}{M_V}
\] (54)

where $f_\rho$ and $m_\rho$ are the $\rho$ parameters and $\frac{f_\rho}{m_\rho} \simeq 1/(2\sqrt{2\pi})$. The choice $\alpha = 1$ would be dictated by QCD analogy. From our previous introductory considerations, we are led to discard values of $\alpha$ too close to 1. On the other hand, smaller $\alpha$ values would lead to unduely pessimistic boundaries values for the resonant masses. To proceed with a mildly optimistic compromise, we chose therefore the value $\alpha = 2$. For a first investigation we also fixed the values $\beta = 1$ and redid our analysis having now as residual free parameters the two masses $M_V$ and $M_A$.

The results of our investigation for this particular case are shown in Figs.(3) (LEP2) and (4) (NLC). One sees that the 95% CL exclusion limits would now lie in a range of approximately $M_V \simeq 500 \text{ GeV}, M_A \simeq 350 \text{ GeV}$ for LEP2(192) and $M_V = 1.5 \text{ TeV}, M_A = 0.9 \text{ TeV}$ for NLC(500). Although these values are given for a special choice of the strengths, they provide a qualitative measure of what could be a realistic reach for
searches of two lightest technivectors in models of the considered type, with reasonable
and self-consistent choices of the two effective strengths $F_V^2/M_V^2$ and $F_A^2/M_A^2$. Clearly,
from the general result given in Figs.(1) and (2), one would easily derive other types of
bounds for different input choices dictated by the specific model.

Having established, at least qualitatively, the type of negative bounds that will be
derivable at LEP2, NLC, we tried to examine the more optimistic possibility of indentification
of a TC signal.

Let us first notice that from eq.(49-51) a TC signal should satisfy the general constraint
(independent of the precise values of the TC parameters):

$$\frac{\delta a_{FB,\mu}^{(TC)}}{A_{FB,\mu}} \simeq c_1 \frac{\delta \sigma_5^{(TC)}}{\sigma_5} + c_2 \frac{\delta \sigma_\mu^{(TC)}}{\sigma_\mu}$$

with $(c_1 \simeq 0.9, \ c_2 \simeq -1.2)$ at LEP2 and $(c_1 \simeq 1.3, \ c_2 \simeq -1.6)$ at NLC.

From eq.(55), it would not be difficult to draw the plane that, in the space of the three
shifts, would correspond to the considered TC model. For simplicity, at this qualitative
stage, rather than one three-dimensional plot, we considered three two-dimensional ones
in the planes $(\sigma_\mu, \ A_{FB,\mu}), (\sigma_\mu, \ \sigma_5)$ and $(A_{FB,\mu}, \ \sigma_5)$, respectively. The SM values were
computed using our “subtracted” program, and the TC contributions was added following
our previous equations, without computing in a first stage the QED radiation effects.

In Figs.(5),(6) we show a typical situation occuring in the $(\sigma_\mu, \ \sigma_5)$ plane. This is in
fact the most interesting situation, since these two variables are more sensitive than the
asymmetry to the TC effects. Moreover, there is in this case a rather typical feature rep-resented by the fact that the TC shifts on $\sigma_\mu$ and $\sigma_5$ are essentially negative. This is true
whenever the parametrization eq.(32-34) for $R^{(TC)}, \Delta\alpha^{(TC)}$ can be used, independently
of the number of resonances, and would still be valid if a less simplified parametrization
a la Breit-Wigner were used. To conclude that a signal of possible TC origin was seen,
one should first verify whether the shift lies in the “suggested” region in the $(\sigma_\mu, \ \sigma_5)$
plane, and consider this as a (quasi) necessary condition. In order to make more defi-nite claims, though, a derivation of $\simeq 5\sigma$ should be found. This would require values of
the masses, in our representative example, of order $(M_V \simeq 310 \ GeV, \ M_A \simeq 310 \ GeV)$
or $(M_V \simeq 470 \ GeV, \ M_A \simeq 260 \ GeV)$ at LEP2. In the more ambitious situation of
NLC, one would be able to see an effect for $(M_V \simeq 0.8 \ TeV, \ M_A \simeq 0.8 \ TeV)$ or
$(M_V \simeq 1.4 \ TeV, \ M_A \simeq 0.67 \ TeV)$.

The previous conclusions were drawn without taking into account the QED radiation
effects. The reason why we proceeded in this way is that such an analysis was already
performed in a previous paper [23] for a situation that is extremely similar to that met
in this paper. In ref [23],in fact, the QED convoluted effect was computed using the so-called
structure-functions approach [24] for a case in which the SM prediction, computed
essentially in the same “subtracted” philosophy, was implemented by corrections due
to assumed anomalous triple gauge boson couplings. These were parametrized by simple
analytic expressions that were certain functions of $q^2$ only (and not of the scattering angle
$\theta$). The conclusion of ref [23] was that, provided that a suitable cut on the hard photon
energy was enforced, the simple “not QED convoluted” effects were totally unaltered. Since our TC model affects the SM predictions by a simple $q^2$ dependent function, the conclusions of ref. [23] will remain the same, as we checked in a few simple representative cases. We do not insist on this point here, since it has been rather exhaustively discussed in the aforementioned ref. [23] to which we defer the interested reader for more details. We simply say that Fig.(5) can be considered as a realistic one, that is not altered by the QED radiation effects which, for $\sigma_{\mu}$, $A_{FB,\mu}$ and $\sigma_5$, can be eliminated by a cut of “standard” type.

To conclude this Section, we would like to add a final comment about possible TC effects in the $Zb\bar{b}$ vertex that we have not considered in our analysis. These would not affect $\sigma_{\mu}$ and $A_{FB,\mu}$ and would enter $\sigma_5$ since they could affect the $b\bar{b}$ component $\sigma_b$.

Two comments are useful at this point. The first one is that, in any case, only the “pure Z” component of $\sigma_b$ in $\sigma_5$ would be modified via $Zb\bar{b}$ vertex effects. Numerically, at Born level, this amounts to, approximately, one tenth of the total observable. Therefore, in order to produce a visible effect, the modification of the $Zb\bar{b}$ vertex should be, at the considered $q^2$ values, exceptionally large. The second comment is that only the $Zb\bar{b}$ vertex that does vary with $q^2$ would be effective, since the value of the vertex at $q^2 = M_Z^2$ is automatically subtracted in our procedure, and reabsorbed by the new theoretical input provided by the $Zb\bar{b}$ partial width $\Gamma_b$ (whose value is now in essential agreement with the SM, so that no sources of theoretical troubles are introduced by its use in our approach). In order to make an effect in $\sigma_5$ at the considered $q^2$ values, a TC model should therefore generate a “huge” effect in the $Zb\bar{b}$ vertex that varies “dramatically” with $q^2$ when one moves from $q^2 = M_Z^2$ to higher values, which seems to us, at least for the preliminary LEP2 case, rather peculiar. We believe that this possibility might be examined, in particular considering the specific observable $\sigma_b(q^2)$, whose future realistic experimentally accuracy remains to be fully understood, as a special case whenever a theoretical model that meets the previous requested would (will) be proposed.

4 Concluding remarks

In this paper we have considered the information that can be obtained about a strongly-interacting symmetry breaking sector from precision measurements of four-fermion processes at LEP2 or a (500 GeV) NLC. Using a “Z-peak” subtracted approach to describe four-fermion processes, we have shown that measurements of the cross section for muon production, the related forward-backward asymmetry, and the total cross section for the production of hadrons (except $t$’s) can place constraints on (or measure the effects of) the lightest vector or axial resonances present in a strong symmetry breaking sector. We estimate that such effects will be visible at LEP2 for resonances of masses up to approximately 350 GeV, and at a 500 GeV NLC for resonances of masses up to approximately 800 GeV. Multiscale models, in particular, predict the presence of light vector and axial mesons in this mass range and their effects could be probed by these measurements.
The four-fermion processes measured here provide a complementary probe to the $e^+e^- \to W^+W^-$ measurement suggested by Barklow [25]. The four-fermion processes are sensitive only to effects in the gauge-boson self energies and, because of the “Z-peak” subtraction, are mostly sensitive to the lightest scale objects in the symmetry breaking sector. Two-gauge boson production, on the other hand, is sensitive largely to gauge-boson rescattering effects which are likely to be dominated by whatever sector of the model is responsible for the bulk of electroweak symmetry breaking.

A final remark is that we did not consider, in the discussion of NLC possibilities, that represented by the availability of longitudinally polarized lepton beams. That would produce an enrichment of experimental measurements, fully discussed in a previous paper [26], and would lead to a definite improvement of bounds and of effects. The reason why we did not include the discussion here is that, to our knowledge, a fully rigorous discussion of the QED radiation effects for these longitudinal polarization observables at NLC is still missing [27]. The problem is being considered at the moment, and work along that direction is already in progress.

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Table A1: Standard Model values for the observables $\sigma_\mu$, $A_{FB,\mu}$, and $\sigma_5$
for $\sqrt{q^2}$ between 150 and 190 GeV
For each energy, the upper row shows the predictions of the “Z-subtracted” approach
$(S)$ and the lower row the predictions of the complete calculation as encoded in the
program TOPAZ0 $(T)$, ref.[21].

| $\sqrt{q^2}$ (GeV) | $\sigma_\mu^{(S)}(T)$ | $A_{FB,\mu}^{(S)}(T)$ | $\sigma_5^{(S)}(T)$ |
|---------------------|-----------------------|-----------------------|---------------------|
| 150                 | 5.842                 | 0.635                 | 45.176              |
|                     | 5.807                 | 0.635                 | 44.901              |
| 161                 | 4.953                 | 0.607                 | 36.384              |
|                     | 4.876                 | 0.605                 | 35.547              |
| 165                 | 4.661                 | 0.600                 | 33.579              |
|                     | 4.599                 | 0.598                 | 32.945              |
| 175                 | 4.046                 | 0.583                 | 27.968              |
|                     | 4.013                 | 0.583                 | 27.710              |
| 180                 | 3.788                 | 0.576                 | 25.729              |
|                     | 3.767                 | 0.576                 | 25.602              |
| 190                 | 3.346                 | 0.563                 | 22.056              |
|                     | 3.340                 | 0.564                 | 22.115              |
References

[1] For a review, see the latest results from the “Electroweak Working Group”, http://www.cern.ch/LEPEWWG.

[2] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. 65 (1990) 964 and Phys. Rev. D46 (1992) 381.

[3] M. Golden and L. Randall, Nucl. Phys. B361 (3) 1991; B. Holdom and J. Terning, Phys. Lett. B247 (88) 1990; A. Dobado, D. Espriu, and M. Herrero, Phys. Lett. B253 (161) 1991.

[4] R. S. Chivukula, S. B. Selipsky, and E. H. Simmons, Phys. Rev. Lett. 69, 575 (1992); R. S. Chivukula, E. Gates, J. Terning, and E. H. Simmons Phys. Lett. B311 (1993) 157; R. S. Chivukula, J. Terning, and E. H. Simmons, Phys. Lett. B331 (1984) 383.

[5] See also, for example, G.-H. Wu Phys. Rev. Lett. 74 (1995) 4137.

[6] For a review, see R. S. Chivukula, talk presented at the SLAC Topical Workshop, Stanford, July 19-21, 1995, hep-ph/9509384.

[7] Y. Nambu, Enrico Fermi Institute Preprint EFI 88-39; V. A. Miransky, M. Tanabashi, and K. Yamawaki, Phys. Lett. B221 (1989) 177 and Mod. Phys. Lett. A4 (1989) 1043 (1989); W. A. Bardeen, C. T. Hill, and M. Lindner, Phys. Rev. D41 (1647) 1990.

[8] C. T. Hill, Phys. Lett. B266 (1991) 419; S. Martin, Phys. Rev. D45 (1992) 4283 and D46 (1992) 2197; N. Evans, S. King, and D. Ross, Z. Phys. C60 (1993) 509.

[9] C.T. Hill, Phys. Lett. B345 (1995) 483.

[10] S. Weinberg, Phys. Rev. Lett. 18 (1967) 507.

[11] P. Langacker and J. Erler, hep-ph/9703428.

[12] K. Lane, lectures given at Theoretical Advanced Study Institute (TASI 93) in Elementary Particle Physics, Boulder, CO, 6 Jun - 2 Jul 1993, hep-ph/9401324.

[13] S. Dimopoulos and L. Susskind, Nucl. Phys. B155 (1979) 237; E. Eichten and K. Lane, Phys. Lett. B90 (1980) 125.

[14] B. Holdom, Phys. Rev. D24 (1981) 1441; B. Holdom, Phys. Lett. B150 (1985) 301; K. Yamawaki, M. Bando, and K. Matumoto, Phys. Rev. Lett. 56 (1986) 1335; T. Appelquist, D. Karabali, and L.C.R. Wijewardhana, Phys. Rev. Lett. 57 (1986)
T. Appelquist and L.C.R. Wijewardhana, Phys. Rev. **D35** (1987) 774;
T. Appelquist and L.C.R. Wijewardhana, Phys. Rev. **D36** (1987) 568.

[15] K. Lane and E. Eichten, Phys.Lett. **B222** (1989) 274.

[16] F.M. Renard and C. Verzegnassi, Phys. Rev. **D52** (1995) 1369. Phys. Rev. **D53** (1996) 1290.

[17] G. Degrassi and A. Sirlin, Nucl. Phys. **B383** (1992) 73 and Phys. Rev. **D46** (1992) 3104.

[18] G. Altarelli, R. Barbieri and F. Caravaglios, Phys. Lett. **B314** (1993) 357.

[19] Physics at LEP2, Proceedings of the Workshop-Geneva, Switzerland (1996), CERN 96-01, G. Altarelli, T. Sjostrand and F. Zwirner eds.

[20] $e^+e^-$ Collisions at 500 GeV: The Physics Potential, Proceedings of the Workshop-Munich, Annecy, Hamburg, 92-123B (1992) 491, P.M. Zerwas ed.

[21] See e.g.: “TOPAZ0” G. Montagna, O. Nicrosini, G. Passarino and F. Piccinini, “TOPAZ0 2.0 - A program for computing deconvoluted and realistic observables around the $Z^0$ peak”, CERN-TH.7463/94, in press on Comput. Phys. Commun.; G. Montagna, O. Nicrosini, G. Passarino, F. Piccinini and R. Pittau, Comput. Phys. Commun. 76 (1993) 328.

[22] See, e.g.: M. Consoli and W. Hollik, in “Z Physics at LEP1”, Vol. 1, p. 7, G. Altarelli, R. Kleiss, C. Verzegnassi eds.

[23] A. Blondel, F.M. Renard, L. Trentadue and C. Verzegnassi, Phys. Rev. **D54** (1996) 5567.

[24] O. Nicrosini, ibid., p. 73. O. Nicrosini and L. Trentadue, Phys. Lett. **B196** (1987) 551, Z. f. Phys. **C39** (1988) 479. For a review see also: O. Nicrosini and L. Trentadue, in Radiative Corrections for $e^+e^-$ Collisions, J. H. Kühn, ed. (Springer, Berlin, 1989), p. 25; in QED Structure Functions, G. Bonvicini, ed., AIP Conf. Proc. No. 201 (AIP, New York, 1990), p. 12.

[25] T. Barklow, in Physics and Experiments with Linear Colliders, A. Miyamoto, et. al., eds. (World Scientific, Singapore, 1996).

[26] F.M. Renard and C. Verzegnassi Phys. Rev. **D55** (1997) 4370.

[27] See discussion in ref.(26).
Figure captions

**Fig.1** Limits (at 2σ) on light TC parameters X,Y at LEP2(192 GeV), from σµ, AFB,µ, σ₅, and their quadratic combinations (the observability domain lies above the lines).

**Fig.2** Limits (at 2σ) on light TC parameters X,Y at NLC(500 GeV), same captions as in Fig.1.

**Fig.3** Limits (at 2σ) on light TC masses Mᵥ, Mₐ, in the α = 2, β = 1 model, at LEP2(192 GeV), same captions as in Fig.1 but the observability domain region lies below the lines.

**Fig.4** Limits (at 2σ) on light TC masses Mᵥ, Mₐ, in the α = 2, β = 1 model, at NLC(500 GeV), same captions as in Fig.3.

**Fig.5** Allowed domain for light TC effects in the (σ₅, σµ) plane, at LEP2(192 GeV). The three curves correspond to 1,2,5 standard deviations, the observability domain lies inside the triangle.

**Fig.6** Allowed domain for light TC effects in the (σ₅, σµ) plane, at NLC(500 GeV), same captions as in Fig.5.
Fig. 1

\[ Y = \frac{T eV}{BnZr^2} \]

192 GeV

\[ \sigma_{\mu} \]
\[ A_{FB,\mu} \]
\[ \sigma_{5} \]
\[ \text{total} \]
$M_A \ (TeV)$

$M_V \ (TeV)$

$\sigma_\mu$

$A_{FB,\mu}$

$\sigma_5$

$\text{total}$

192 GeV

Fig. 3
Fig. 4
Fig. 5
Fig. 6