Title - Newton's 2nd Law

Objective

In this experiment we will attempt to confirm the validity of Newton’s 2nd Law by analyzing the motion of two objects (glider and hanging mass) on a horizontal air-track. First, we will calculate the theoretical acceleration by applying Newton’s 2nd Law ($F_{\text{net}} = M\ddot{A}$), neglecting friction, to the glider and hanging mass. Next, we will calculate the experimental acceleration of the glider by applying the kinematic equations of motion as it moves between two markers (photogates) on the track. We will then compare the experimental acceleration to the theoretical acceleration.

Theory

a) Acceleration using Newton’s 2nd Law

Apparatus Setup

Free-Body Diagram

Apply Newton’s 2nd Law to mass $M_1$ and $M_2$.

Mass ‘$M_1$’

$\sum F_x = T = M_1\ddot{a}$

Mass ‘$M_2$’
\[ \Sigma F_y = M_2g - T = M_2a \]

Adding both equations gives:

\[ M_2g = M_1a + M_2a \]

\[ a_{theo} = \frac{M_2g}{(M_1 + M_2)} \]

b) Acceleration using Kinematic Equations

Using the kinematic equation

\[ V_2^2 = V_1^2 + 2a(x - x_0) \]

we will calculate the experimental acceleration of the glider as it moves between the two photogates. We will take the origin of our coordinate system at the first photogate.

\[ d = \text{distance between photogates} \]
\[ V_1 = \frac{s}{t_1} \text{ velocity of the glider through photogate 1} \]
\[ V_2 = \frac{s}{t_2} \text{ velocity of the glider through photogate 2} \]
\[ s = \text{diameter of small flag on glider} \]
\[ t_1 = \text{time for small flag to go through photogate 1} \]
\[ t_2 = \text{time for small flag to go through photogate 2} \]

\[ a_{exp} = \frac{V_2^2 - V_1^2}{2d} \]

Apparatus

Refer to theory section for apparatus setup

One air track(#21), blower(#2), blower hose and power supply
One digital photogate(#2C) and one accessory photogate(#2A)
One glider(#1B)
One flat accessory box(#22A)
String
Electronic pan balance(#1)
Vernier Calipers (#12c)

Procedure

1. Measure the mass of the glider and hanging mass.
2. Setup the air track and blower as indicated by instructor.
3. Measure the distance between photogates.
4. Measure the diameter of the small flag on glider with vernier calipers.
5. Release glider 10 cm away from photogate 1 and record times through both photogates.
6. Repeat step (5) four more times.
Data

M₁ = 4750 g
M₂ = 50.00 g
g = 9.80 m/s²
d = 60.65 cm
s = 1.01 cm

| Run # | t₁    | t₂    | V₁ (cm/s) | V₂ (cm/s) | d (cm) | a exp (cm/s²) |
|-------|-------|-------|-----------|-----------|--------|---------------|
| 1     | 0.039 | 0.023 | 25.5      | 43.0      | 60.65  | 9.91          |
| 2     | 0.043 | 0.024 | 23.0      | 41.5      | 60.65  | 9.86          |
| 3     | 0.044 | 0.023 | 22.5      | 42.5      | 60.65  | 10.7          |
| 4     | 0.041 | 0.023 | 24.5      | 42.5      | 60.65  | 9.97          |
| 5     | 0.038 | 0.032 | 26.0      | 43.5      | 60.65  | 10.1          |

Calculations

Theoretical Acceleration:

\[ a_{\text{theo}} = \frac{M_2 g}{M_1 + M_2} = \frac{50.00 \text{ g} \times 980 \text{ cm/s}^2}{4750 \text{ g} + 50.00 \text{ g}} \]

\[ a_{\text{theo}} = 10.2 \text{ cm/s}^2 \]

Experimental Acceleration:

\[ a_{\exp} = \frac{V_2^2 - V_1^2}{2d} = \frac{(43.5 \text{ cm/s})^2 - (26.0 \text{ cm/s})^2}{2 \times 60.65 \text{ cm}} \] (sample calculation Run #5)

\[ a_{\exp} = (9.91 + 9.86 + 10.7 + 9.97 + 10.1)/5 = 10.1 \text{ cm/s}^2 \] (average experimental acceleration)

\[ \% \text{ error} = \left| \frac{\text{exp} - \text{theo}}{\text{theo}} \right| \times 100 \]

\[ \% \text{ error} = \left| \frac{10.1 - 10.2}{10.2} \right| \times 100 = 0.98 \% \]
Conclusion

1. The theoretical acceleration using Newton’s 2nd Law was 10.21 cm/s² and the average experiment acceleration using the kinematic equations was 10.10 cm/s². The percent error between experiment and theory was only 1%. Although the percent error was small, there were still systematic and random errors present.

2. Based on the relative small % error of 0.98% we can conclude that the objective of confirming Newton’s 2nd Law was accomplished.

3. In measuring the velocity of the gliders through the photogates we used the average velocity instead of the instantaneous velocity. This resulted in the average velocity always being smaller than the instantaneous velocity. This will then cause \( a_{exp} = \frac{V_f^2 - V_i^2}{2d} \) to be consistently smaller than \( a_{theo} \) which resulted in a systematic error. A second systematic error was that in applying Newton’s 2nd Law to derive \( a_{theo} \) of the glider we neglected the frictional force. The resulting equation should have been \( a_{theo} = \frac{(M_2 g - f k) / (M_1 + M_2)} \). Neglecting friction on the \( a_{theo} \) equation should result in \( a_{theo} \) being consistently larger than \( a_{exp} \). The data shows this to be true with the exception of one data point.

4. In addition to the random errors involved due to the uncertainty of the measuring devices, other random errors involved in the experiment include:

   a) Not releasing the glider from same initial point every run.
   b) Trying to balance the air track.
   c) Having the hanging mass \( M_2 \) swinging when releasing \( M_1 \) from rest.

All these random errors contributed to the uncertainty in the final results for the accelerations. These random errors also contributed to the 0.98% error in the final results.