Structure and Evolution of Regular Galaxies

Jaan Einasto

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The Thesis was an attempt to combine data from three previously independent areas: the structure and kinematics of stellar populations of the Galaxy, photometric and dynamical models of galaxies, and models of the dynamical and physical evolution of galaxies. The Thesis was completed in November 1971 and defended in March 17, 1972 for the degree of Doctor of Science in astronomy and celestial mechanics. This synthesis was made with the goal to understand better the vast topic of the structure and evolution of galaxies.

The main results of the study can be divided into methodical and astronomical. The methodical results include: (i) extrapolation of the mass distribution function beyond the Sun’s distance, and the determination of the circular velocity at the Sun’s distance from the Galactic centre; (ii) conditions of physical correctness of models are developed, and the generalised exponential model is suggested; (iii) a method has been developed to construct spatial and hydrodynamical models of stellar systems, using a combination of observational data on populations, and data on physical evolution of populations.

The main astronomical results are: (i) spatial and hydrodynamical models of the Galaxy and the Andromeda galaxy M31 are elaborated in several approximations; (ii) the dynamical evolution of the Galaxy is reconstructed using kinematical characteristics of stellar populations of different ages; (iii) on the basis of stellar evolutionary tracks and star formation function, a theory of the evolution of galaxies is elaborated. The basic conclusion of the study is: it was impossible to reproduce the observed rotation curves of galaxies with known stellar populations.

When the work was finished it was clear that there are difficulties in the classical picture. Thus, immediately after the Thesis was completed, I started together with my Tartu collaborators searching for solutions to open problems. This search led to accepting the presence of non-stellar dark matter in galaxies. To understand the properties of dark matter in galaxies, we studied the environment of galaxies and the distribution of galaxies in space, which culminated with the discovery of the cosmic web. A short overview of the development of ideas directly connected with the topic of the Thesis is given in the Epilogue.

In some sense the Thesis is a time-capsule of the state of affairs just at the verge of the paradigm shift in cosmology. The Thesis was written in Russian and its most important parts were never published. Thus, it would be useful to make this study available for the astronomical community by translating the Thesis into English.

The Thesis consists of four parts and is divided to 23 chapters. Chapters 4, 7, 20, 21, 22, 23 were unpublished and the present translation is their first publication. These chapters were translated in full. Chapters 7, 11, 17, 19 were published in Tartu Observatory Publications or conference proceedings, but form the methodical and data basis to understand the main topic of the Thesis, thus these Chapters were also translated. The rest of Chapters, published in Tartu Observatory Publications, describe the general background of the topic, and are written in the English version as short summaries of respective papers in Russian.

Here I give a Summary of the Thesis and the Epilogue. The original Thesis in Russian can be accessed in Dspace link: http://hdl.handle.net/10062/6113, and the English translation in link: http://hdl.handle.net/10062/76090.
Pareno (1948) introduced the concept of practical stellar dynamics to denote the study of the structure of stellar systems based on observational data with the application of theoretical relations, derived in the dynamics of stellar systems. Currently theories of stellar evolution and nucleosynthesis, gas dynamics, relativistic astrophysics, etc. are also applied to the study of the structure of stellar systems. Retaining the convenient term of practical stellar dynamics, it is reasonable to accept for its goal the application of the results of the theory in the study of the structure and evolution of particular stellar systems.

The goal of practical stellar dynamics is the developing of models of stellar systems, as representative as possible, in which the synthesis of heterogeneous observational information is based on the results of the theory of stellar systems. The problem of evolution is considered primarily as an observational one, i.e. evolutionary conclusions are drawn on the basis of theoretical interpretation of suitable observational data. Due to our position inside the Galaxy, it is difficult to get a picture of its structure as a whole. To enrich our understanding of the global structure of the Galaxy, the study of other similar galaxies, among which Andromeda galaxy M31 is the most suitable, plays an essential role.

The body of works, which served as the basis for the present dissertation, is devoted to the development of methods of practical stellar dynamics and their application to investigate the structure and evolution of regular galaxies like our Galaxy and the Andromeda galaxy M31. We did not set as a goal the further development of theory and obtaining new observational data, since the already available theoretical and observational information is much more than could be processed and combined in one study. Thus, four cycles of works arose: (i) on the spatial and kinematical structure of the Galaxy, (ii) on the methods of building models of galaxies, (iii) on the study of the structure of the M31 galaxy, and (iv) on the evolution of galaxies, which form the content of four parts of the dissertation.

1.1 Spatial and kinematical structure of the Galaxy

1.1.1 Kinematics of stars

In the first part of the Thesis a method has been developed to use tangential velocities to determine kinematical parameters of star samples (Chapter 1 (Einasto, 1954)). Next a method has been developed to determine the mean velocity dispersion of samples of star using radial, tangential or spatial velocities, taking into account observational errors (Chapter 2 (Einasto, 1955)).

In Chapter 3 we analysed the asymmetric shift of stellar velocity centroids (Einasto, 1961). We calculated the mean velocity dispersion,

$$\sigma = \sqrt{\frac{1}{3}(\sigma_R^2 + \sigma_\theta^2 + \sigma_z^2)},$$

and the mean heliocentric centroid velocity in rotational direction, $V_\theta$, where $\sigma_R$, $\sigma_\theta$, and $\sigma_z$ are velocity dispersions in galactic cylindrical coordinates.

In Chapter 4 we made a new collection of mean velocity dispersions, centroid velocities and ages for Galactic populations, using data available in 1971. The Strömgren diagram for populations is given in Figure [ ]. Populations with metal deficit are represented by open circles, populations with normal metal content by points, the interstellar gas by a cross. The smooth curve shows the mean dependence between $\sigma$ and $V_\theta$ of populations of different ages; the latter is indicated in $10^9$ years, starting from the formation of oldest galactic populations known, using age estimates given in Chapter 22. The analyse of kinematical, metal content and age data of populations yields following results.

1. If we attribute all metal deficient populations to the halo, then it appears that the halo is rather heterogeneous in its kinematical properties; it contains all populations with velocity dispersion $\sigma \geq 50$ km/s with flattening $e \geq 0.10$ (Einasto, 1974b).
Figure 1.1: The Strömgren diagram for populations. In the horizontal axis, we show the heliocentric centroid velocity of the population in the direction of the Galactic rotation; in the vertical axis the mean velocity dispersion $\sigma$. Open circles are for metal-poor populations, dots for populations with normal metal abundance. The numbers give the birth-dates in billion years starting from the formation of the oldest populations, assuming for the age of the Galaxy $10^{10}$ years (Einasto, 1974b).

2. Direct age determinations of stellar populations are too inaccurate to estimate the duration of the initial galactic collapse. There exists evidence that the collapse proceeded in a short time scale compared with the age of the Galaxy, discussed in Chapter 21.

3. The populations of the galactic disc have mean velocity dispersions, $15 \leq \sigma \leq 50$ km/s, and respectively axial rations, $0.02 \leq \epsilon \leq 0.10$ (Einasto, 1970a).

Figure 1.2: Left: The extrapolation of the mass distribution function beyond the Sun’s distance $R > R_0$ (dashed lines) with different values for the circular velocity near the Sun, $V_0$. Limiting radii of models, $R_{lim}$, are indicated. Right: The dependence of limiting radii, $R_{lim}$, and apogalactic distances, $R_{apogal}$, on circular velocity near the Sun, $V_0$. Two cases of smooth extrapolation of the mass distribution function with different $R_{lim}$ are indicated. Apogalactic distances are given for two values of Oort’s limiting velocity, $\Delta v$.

1.1.2 System of Galactic parameters and extrapolation of the density

In Chapters 5 and 6 we elaborated the concept of the system of Galactic parameters, and derived a new system of parameters (Einasto, 1961; Einasto & Kutuzov, 1964a; Einasto, 1964; Einasto & Kutuzov, 1964b). The Galactic parameter system was presented in IAU XII General Assembly in Hamburg 1964. The system was improved several times to take into account new data; the latest version of the system was calculated including massive corona (dark matter halo) to the model by Einasto (1979), see Table 2.1.

As an essential ingredient of Galaxy modeling a method has been developed to extrapolate the mass distribution function beyond the Sun’s distance, and to determine the circular velocity at the
Table 1.1: Parameters of test populations of the Galaxy

| Popul. | $\epsilon$ | $a_0$ | $N$ | $x_0$ | $(\sigma_z)_0$ | $\sigma_z$ | $\sigma_R$ | $V_0$ | $t$ |
|--------|-----------|--------|-----|-------|----------------|-----------|-----------|-------|-----|
| Flat   | 0.02      | 8.0    | 0.5 | 0.0   | 8.8            | 8.8       | 16.4      | 250   | 0.9 |
| Disc 1 | 0.05      | 7.4    | 1.0 | 1.5   | 20.4           | 19.9      | 37.3      | 239   | 3.9 |
| Disc 2 | 0.10      | 6.4    | 1.5 | 3.0   | 34.7           | 34.4      | 64.5      | 216   | 7.6 |
| Halo 1 | 0.20      | 4.5    | 2.0 | 4.5   | 52.8           | 51.7      | 84.3      | 185   | 9.1 |
| Halo 2 | 0.40      | 1.9    | 3.0 | 7.5   | 34.7           | 34.4      | 64.5      | 216   | 7.6 |
| Halo 3 | 0.60      | 0.9    | 4.0 | 10.5  | 92.6           | 85.9      | 100.9     | 96    | 9.7 |
| Halo 4 | 0.80      | 0.6    | 5.0 | 13.5  | 108.3          | 98.5      | 109.8     | 46    | 10.0|

Sun’s distance from the Galactic centre (Chapter 7 [Einasto, 1965]). The method takes into account two conditions. Firstly, the radial gradient of the mass distribution function must be in accordance with the mean observed radial gradient of the spatial density, $G_R(\rho)_0$ (see Eq. (1.4)). Secondly, the limiting radius of the model, $R_{\text{lim}}$, must be equal to $R_{\text{apogal}}$, the apogalactic distance of stars moving near the Sun with the maximal galactocentric velocities, given by Oort’s limiting velocity, $\Delta v$. It is known that there exist no stars with heliocentric velocities exceeding $\Delta v = 65$ km/s in the direction of galactic rotation. The limiting velocity corresponds to the velocity required to reach the boundary of the Galaxy. Using the mass distribution model we can calculate the gravitational potential, and find apogalactic distances, $R_{\text{apogal}}$, of stars moving with the velocity $V_a = V_0 + \Delta v$ in the direction of the rotation of the Galaxy. We show in the left panel of Fig. 1.2 the mass distribution function for two values of the circular velocity, $V_0$. In both cases, the function of differential rotation is the same, and the extrapolation of mass function beyond the Sun’s distance, $R > R_0$, is smooth. In the right panel of Fig. 1.2 the dependence of $R_{\text{lim}}$ and $R_{\text{apogal}}$ on $V_0$ is shown.

In Fig. 1.2 the extrapolation of the mass distribution function was made graphically using the conditions of physical correctness, discussed in Chapter 8. A physically better motivated way for extrapolation is the use of suitable analytical expression, as the generalised exponential profile (1.24).

1.1.3 Models of the Galaxy

In Chapters 5 and 7 we analysed critically existing models of the Galaxy, and suggested in two approximations new models [Einasto, 1965, 1969a, 1970a; Einasto & Einasto, 1972]. After the Thesis was finished we made a third approximation [Einasto, 1979]. It has an improved system of Galactic parameters and includes a non-stellar dark corona (halo), see Tables 2.1, 2.2 and Figs. 2.5, 2.5 and 2.7 in the Epilogue.

For seven test populations of the Galaxy we calculated various descriptive functions. Data on test populations are given in Table 1.1. Here $(\sigma_z)_0$ is the vertical dispersion in Jeans approximation, $\sigma_z$ the vertical dispersion, taken into account the tilt of the velocity ellipsoid, $\sigma_R$ is the radial velocity dispersion, and $V_0$ the centroid velocity in the direction of Galactic rotation, all kinematical data are for the meridional plane $z = 0$ at $R = 10$ kpc. In these models we used for spatial density the modified exponential profile:

$$
\rho(a) = \rho_0 \exp[x_0 - (x_0^2 + \xi^2)^{1/(2N)}],
$$

where $N$ is the structural parameter, determining the shape of density distribution, and the parameter $x_0$ was applied to avoid a sharp density peak and the resulting minimum in velocity dispersion near the centre of the model.

Kinematical descriptive functions of test populations are shown in Figs. 1.3 to 1.9, which have three panels. In upper panels we show density $\rho(10, z)$ and vertical velocity dispersion $(\sigma_z)_0$ in Jeans approximation as functions of $z$ at $R = 10$ kpc, the adopted at this time distance of the Sun from Galactic centre. Solid lines show the density logarithm, $\ln \rho$, short dashed lines show the vertical density gradient, $l = -\partial \log \rho / \partial z$, and long dashed lines show the velocity dispersion $(\sigma_z)_0$. The vertical scale $z$ is shown at the top of the panel, the density scale is on the left border, and the velocity dispersion scale on the right border. Figures show that the vertical density gradient
increases from zero at \( z = 0 \) to maximum value on the periphery of the model, the shape depends on the density profile parameter \( N \) of the model. The velocity dispersion is almost constant.

Central panels shows with solid lines the logarithm of the density, \( \rho \), with dotted lines the radial density gradient, \( m = -\partial \log \rho / \partial R \), and with various dot-dashed lines velocity dispersions \( \sigma_R, \sigma_\theta, \sigma_z \). These data are given as functions of \( R \) on the plane of the Galaxy, \( z = 0 \). The radial distance scale \( R \) is shown at the lower border of the Figure, the density scale is on the left vertical border, and the velocity dispersion scale on the right border. Densities are given in units of the central density, gradients \( l \) and \( m \) in kpc\(^{-1}\), dispersions in km/s.

Vertical velocity dispersions \( \sigma_z \) were calculated from the second hydrodynamical equation (1.14), using for \( k_z \) Kuzmin equation (1.23) from the theory of irregular forces. Radial velocity dispersions \( \sigma_R \) were found from definition equation (1.17) from \( \sigma_z \) and \( k_z \), taking into account the Kuzmin equation (1.23). For the test populations with \( \epsilon \geq 0.10 \) radial velocity dispersions were also calculated from the first hydrodynamical equation (1.13) from circular velocity \( V_c \) and rotational velocity \( V_\theta \), this dispersion is marked as \( \sigma^*_R \). Figs. 1.5 – 1.9 show that in disc populations both versions of the radial velocity dispersion are very similar, but in halo populations \( \sigma_z \) is larger than \( \sigma^*_z \).

Disc population velocity dispersions have maxima at center and decrease with the distance from the Galactic center. In halo populations velocity dispersions near the center have a moderate minimum, a maximum not far from the center, and decrease at larger distance. The radial density gradient is zero at \( R = 0 \), and increases continuously with increasing distance for disc populations, \( \epsilon = 0.02, 0.05 \). For halo populations the gradient has a maximum at certain distance from the center, and slowly decreases at larger distance.
In bottom panel we show isolines, $\rho = \text{const}$ and $(\sigma_z)_0 = \text{const}$, in the plane of $R$, $z$-coordinates, shown at the bottom and the left border of the panel. Velocity dispersion $\sigma_z$ was calculated within ranges $0 \leq R \leq 30$ kpc and $0 \leq z \leq z_u$, where $z_u$ is the outer vertical limit of the population. We see that in disc populations lines $(\sigma_z)_0 = \text{const}$ are vertical, i.e. the velocity dispersion at given radial distance is constant. In halo populations lines $(\sigma_z)_0 = \text{const}$ are similar to lines of constant density. A similar picture is observed in the M31 galaxy, see Fig. 1.13.

1.2 Methods to calculate spatial and hydrodynamical models of regular stellar systems

1.2.1 Conditions of physical correctness

In Chapter 8 conditions of physical correctness of models of stellar systems are formulated. We use as the principal descriptive function in mass modelling of galaxies the spatial density $\rho(a)$ as function of the major semiaxis $a$ of the isodensity surface. The spatial density is the basic physical quantity determining the dynamical behaviour of a stellar system.

Conditions of physical correctness can be expressed in the following way (Einasto, 1969a):
(a) the spatial density $\rho(a)$ must be non-negative and finite,

$$0 \leq \rho(a) < \infty; \quad (1.3)$$
(b) the density should decrease with growing distance from the centre of the system:

\[ G(\rho(R)) = \frac{\partial \ln \rho}{\partial \ln R} \leq 0; \]  

(1.4)

(c) descriptive functions, calculated from the spatial density, should not have breaks;

(d) some moments of the mass-function should be finite:

\[ \mathcal{M}_i = \int_0^{\infty} \mu(a) a^i d(a) < \infty, \]  

(1.5)

where

\[ \mu(a) = 4\pi \epsilon \rho(a) a^2 \]

is the mass function – the mass of an ellipsoidal sheet of unit thickness and ratio of vertical to horizontal axes \( \epsilon \);

(e) the model should allow stable circular motions. In that case

\[ G(F^0_\rho(R)) = \frac{\partial \ln F^0_\rho}{\partial \ln R} > -1, \]  

(1.6)

where \( F^0_\rho(R) = F_\rho(R)/\mathcal{M} \) is the normalised acceleration function – the ratio of radial acceleration of the model to the radial acceleration of a mass-point model with the same mass \( \mathcal{M} \). For a mass point \( F^0_\rho(R) \equiv 1 \).

Real stellar systems have finite dimensions and finite densities. Therefore, all moments of the mass function, \( \mathcal{M}_i, i \geq -2 \), are finite. But the requirement of the finiteness of all moments is too
Figure 1.6: Descriptive functions for the test population of flattening $\epsilon = 0.20$.

strict. Therefore, we suppose that only moments of the order, $-2 \leq i \leq 2$, must be finite. Moments $\mathcal{M}_{-1}$ and $\mathcal{M}_0$ determine the central potential and the total mass of the model, respectively. Moment $\mathcal{M}_1$ defines the effective radius of the model.

### 1.2.2 Calculation of hydrodynamical models

In Chapters 9 to 11 methods to construct spatial and hydrodynamical models of stellar systems are described (Einasto, 1968c, 1969a, 1970b). We assume that the isodensity surfaces of galaxies are ellipsoids of revolution with constant axial ratio $\epsilon$. In this case we may use known formulae for the attraction of infinitely thin ellipsoidal layer (Kuzmin, 1952a). For the derivates of the gravitational potential $\Phi$ we get

$$K_R(R, z) = R G \int_0^1 \frac{\mu(a)u^2 du}{a^2 \sqrt{1 - e^2 u^2}}$$  \hspace{1cm} (1.7)

and

$$K_z(R, z) = z G \int_0^1 \frac{\mu(a)u^2 du}{a^2 (\sqrt{1 - e^2 u^2})^3}.$$  \hspace{1cm} (1.8)

Here $G$ is the gravitational constant, $\mu(a) = 4\pi a^2 \rho(a)$ is the mass function, $\rho(a)$ is the spatial mass density, $a$ is the major semiaxis of an isodensity ellipsoid, and $e = \sqrt{1 - e^2}$ is the eccentricity of the meridional section of an isodensity ellipsoid. For a point at $R, z$ outside the spheroid with major semiaxis $a$ the integrating variables $a$ and $u$ are connected by formula $a^2 = u^2[R^2 + z^2/(1 - (eu)^2)]$, for a point inside the spheroid $u = 1$. In the plane of symmetry $z = 0$ we get for the circular velocity $V_c$

$$V_c^2(R) = R K_R(R, 0).$$  \hspace{1cm} (1.9)
The gravitational attraction in radial and vertical directions is balanced by rotational velocity and random motions, expressed by hydrodynamical equations (Kuzmin, 1965; Einasto, 1969a):

\[
\frac{1}{R} \left( \sigma_R^2 - \sigma_\theta^2 \right) + \frac{1}{\rho} \frac{\partial}{\partial R} \left( \rho \sigma_R^2 \right) + \frac{1}{\rho} \frac{\partial}{\partial z} \left[ \rho \gamma \left( \sigma_R^2 - \sigma_z^2 \right) \right] - \frac{V_\theta^2}{R} = -K_R, \tag{1.10}
\]

\[
\frac{1}{R} \gamma \left( \sigma_R^2 - \sigma_z^2 \right) + \frac{1}{\rho} \frac{\partial}{\partial R} \left[ \rho \gamma \left( \sigma_R^2 - \sigma_z^2 \right) \right] + \frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho \sigma_z^2 \right) = -K_z. \tag{1.11}
\]

In these equations \( \sigma_R, \sigma_\theta, \sigma_z \) are velocity dispersions in cylindrical galactocentric coordinates \( R, \theta, z \); \( V_\theta \) is the centroid velocity; and

\[
\gamma = \frac{1}{2} \tan 2\alpha, \tag{1.12}
\]

where \( \alpha \) is the inclination angle of the major axis of the velocity ellipsoid with respect to the galactic symmetry plane. It is assumed that the major axis of the velocity ellipsoid lies in the meridional plane of the galaxy, and the remaining components of the centroid velocity are \( V_R = V_z = 0 \).

Calculating the necessary derivatives, Eqs. (1.10) and (1.11) can be written as

\[
V_\theta^2 - p \sigma_R^2 = RK_R = V_c^2, \tag{1.13}
\]

and

\[
\frac{1}{\rho} \frac{\partial (\rho \sigma_z^2)}{\partial z} + q \frac{\sigma_z^2}{R} = -K_z, \tag{1.14}
\]
Figure 1.8: Descriptive functions for the test population of flattening $\epsilon = 0.60$.

where

$$p = (1 - k_\theta) + G_R\{\rho\} + G_R\{\sigma_R^2\} + \frac{R}{z} \gamma (1 - k_z) [G_z\{\rho\} + G_z\{\gamma\} + G_z\{1 - k_z\}]$$, \hspace{1cm} (1.15)

and

$$q = \gamma \left( \frac{1}{k_z} - 1 \right) \left[ 1 + G_R\{\rho\} + G_R\{\gamma\} + G_R\{\sigma_R^2 - \sigma_z^2\} \right]$$.

In these equations

$$k_\theta = \frac{\sigma_\theta^2}{\sigma_R^2}, \quad k_z = \frac{\sigma_z^2}{\sigma_R^2}$$, \hspace{1cm} (1.17)

and $G\{ \}$ is the logarithmic derivative, e.g.

$$G_R\{\rho\} = G\{\rho(R)\} = \frac{\partial \ln \rho}{\partial \ln R}$$ \hspace{1cm} (1.18)

Equations (1.13) and (1.14) include five unknown kinematical functions: $\sigma_z$, $V_\theta$, $k_\theta$, $k_z$, $\gamma$. To calculate these functions, we have only two equations at present, thus the system of hydrodynamical equations is not closed. To solve the problem, one needs to have three additional independent relations between these unknown functions. It is convenient to give these additional relations for $k_\theta$, $k_z$, and $\gamma$, which determine the shape and the orientation of the velocity ellipsoid. In this case Eq. (1.14) allows to calculate the dispersion $\sigma_z$, giving the scale of the velocity dispersion, and
Figure 1.9: Descriptive functions for the test population of flattening $\epsilon = 0.8$.

Eq. (1.13) allows to calculate the centroid velocity $V_\theta$, giving the shift of the velocity ellipsoid with respect to the local standard of rest. The calculation of the hydrodynamical model of a galaxy reduces thus to the problem of finding equations for auxiliary kinematical functions $k_\theta$, $k_z$, and $\gamma$.

One additional relation is the Poisson equation, which can be written in the form (Kuzmin,[1952b]);

$$4\pi G \rho_t = C_c^2 - 2(A_c^2 - B_c^2).$$

(1.19)

Here $\rho_t$ is the total density of matter, and we used Oort-Kuzmin constants, $A_c$, $B_c$ and $C_c$, in dynamical sense:

$$(A_c - B_c)(3A_c - B_c) = \left(\frac{\partial^2 \Phi}{\partial R^2}\right)_{z=0},$$

(1.20)

and

$$C_c^2 = -\left(\frac{\partial^2 \Phi}{\partial z^2}\right)_{z=0}.$$  

(1.21)

Another additional equations are the Lindblad formula

$$k_\theta = -\frac{B}{A - B} = \frac{1}{2}[1 + G(V_\theta(R))],$$

(1.22)

and the formula

$$\frac{1}{\sigma_z^2} = \frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_R^2}.$$  

(1.23)
found from the theory of irregular forces for the case \( z \ll R \) (Kuzmin 1961). At larger distances from the galactic plane the inclination angle of the velocity ellipsoid must be taken into account. For this we may use a relation from the theory of the third integral of motions of stars (Kuzmin 1952c)

\[
\gamma = \frac{Rz}{(R^2 + z_0^2 - z^2)},
\]

where \( z_0 \) is a constant depending on the gravitational potential of the whole galaxy.

In Chapter 11 the calculation of these functions is discussed in detail. A novel method is suggested to calculate all basic kinematical functions of populations: rotational velocity \( V_\theta \), velocity dispersions \( \sigma_R^2, \sigma_\theta^2, \sigma_z^2 \), the angle of the velocity ellipsoid in respect to the plane of the system, \( \alpha \), as functions of spatial coordinates \( R, z \). In principle, the method allows to calculate consistently line-of-sight velocity dispersions and rotational velocities at all points of a galaxy. The method is applied to calculate hydrodynamical models of the Galaxy (Chapter 7, Figs. 1.3–1.9) and the Andromeda galaxy M31 (Chapters 18 and 20, Figs. 1.11 and 1.12).

1.2.3 Families of models of stellar systems

In Chapter 12 virial theorem and its application to the determination of masses of stellar systems is described. In Chapters 13 to 16 families of models of stellar systems are discussed. It is shown that most models of stellar systems are particular cases of two model families: polynomial and binomial models (Einasto 1968a,b,c). A special case of the binomial model is the generalised exponential model,

\[
\rho(a) = \rho_0 \exp \left[ -\left( \frac{a}{a_0 k} \right)^\nu \right],
\]

where \( \rho_0 \) is central density, \( a_0 \) is effective radius of the model, \( k \) is a dimensionless normalising parameter, and \( \nu = 1/N \) is a structural parameter of the model, determining the concentration of mass to the centre. The effective radius, defined through the first moment, characterises the real extent of a population rather well, being nearly independent of the parameter \( \nu \). It is shown that most models used so far contradict to some condition of physical correctness. The generalised exponential
function, Eq. (1.24), is the only function which agrees with all conditions of physical correctness. It allows to describe the structure of stellar systems and their components over the whole distance interval from zero to infinity, and in this way extrapolate the density to large distances from the centre.

Figure 1.11: Left: Vertical velocity dispersion $\sigma_z$ of the components of M31 in the $z$-axis of symmetry. Right: Vertical velocity dispersion $\sigma_z$ of the components of M31 in the plane of symmetry.

Figure 1.12: Left: Axial ratios of the velocity ellipsoid in the plane of symmetry and in the axis of M31. Right: Radial velocity dispersion $\sigma_R$ as function of $R$ for components of M31: 2 — bulge and halo; 3 — disc; 4 — flat; $\Sigma$ — galaxy as a whole. Horizontal scale is given in units $R^{1/3}$ and in $R$ on bottom and top, respectively.

1.3 Spatial and kinematical structure of the Andromeda galaxy

In the third part of the Thesis the kinematical and spatial structure of the Andromeda galaxy M31 is studied and its spatial and hydrodynamical models developed in two approximations (Chapters 17 – 20 (Einasto, 1969b; Einasto & Rümmel, 1970; Einasto, 1972b; Einasto & Rümmel, 1972)). The more advanced model consists of the following components: nucleus, core, bulge, halo, disc, flat. Structural and scale parameters of components were found from photometric data, see Fig. 1.10. These parameters are: effective radius $a_0$, flattening of equidensity ellipsoids $\epsilon$, parameter $\nu$ of the generalised exponential function, and the inclination angle $i$ of the symmetry axis of the galaxy to the line of sight.

The main difficulty in the determination of parameters of models of external galaxies is related to calculations of mass-to-light ratios of populations. These ratios are not free parameters to get the best fit of the observed rotation curve. In addition to the rotation curve also spectrophotometric data on population stellar content must be used, as well as velocity dispersions of small systems (open and globular clusters) with similar stellar content. Stellar populations get their photometric
properties during the evolution, thus data on the physical evolution of populations must be used, see Chapters 22 and 23.

Figure 1.13: Left: Isolines of velocity dispersion $\sigma_z(R,z)$ and density $\rho(R,z)$ in the meridional plane $R,z$ of M31 for three test populations of flattening $\epsilon = 0.02, 0.08, 0.30$. Right: Similar isolines for $\epsilon = 0.80$. For explanations see Chapter 7.

Using the combination of all data we calculated the rotation curve of M31, shown in the right panel of Fig. 1.10. Applying the hydrodynamical method, described in Chapter 11, we calculated all hydrodynamical functions for the Andromeda galaxy. In Fig. 1.11 we show vertical velocity dispersions $\sigma_z$ of the main components of M31 along the $z$-axis and in the plane of symmetry $z = 0$. In the left panel of Fig. 1.12 we show axial ratios of the velocity ellipsoid in the plane of symmetry, $k_\theta(R,0)$, $k_z(R,0)$, and in the axis, $k_z(0,z)$, of M31. In the right panel of Fig. 1.12 we give radial velocity dispersions $\sigma_R(R,0)$ of main components in the plane of symmetry $z = 0$. Velocity dispersions $\sigma_z(R,z)$ of four populations in the meridional plane are shown in Fig. 1.13.

1.4 Evolution of galaxies

The fourth part of the Thesis is devoted to the study of the evolution of galaxies. In Chapter 21 we find the evolutionary conclusions that can be made on the basis of kinematical and spatial data on populations of the Galaxy. In Chapter 22 we develop a model of physical evolution of stellar systems, in Chapter 23 we discuss star formation function and the evolution of galaxies and galactic populations.

1.4.1 Reconstruction of the dynamical evolution of the Galaxy

Data presented in Chapter 4 suggest that there exist three population groups in the Galaxy with different properties – flat, disc and halo. Kinematical data on flat populations, presented in the Strömgberg diagram in Fig. 1.11 indicate that the interstellar gas and very young stellar populations rotate in the Galaxy with a speed, about 14 km/s lower than the circular velocity. Young stars populate in velocity space an elliptical region around the point, which corresponds to circular velocity, and young stars just “fall” in the direction of the Galactic centre after their birth. A similar effect is observed in the vertical direction. Radio observation suggests that the shell of interstellar gas does not coincide exactly with the plane of the Galaxy and has a wave form. In the Solar neighbourhood, these waves have an amplitude of about 50 pc. After formation, stars are free from electromagnetic forces and fall towards the Galactic plane. Very young stars are still located close to their places of origin and oscillate both in radial and vertical direction. The frequency of vertical oscillations is measured by Kuzmin parameter $C$. Jouveet (1972, 1974) used these vertical oscillations to find the Kuzmin parameter $C = 70$ km/s/kpc.
Figure 1.14: Left: The time evolution of the distance from galactic centre $R$ of gas clouds of various angular momentum $h$. Right: Possible evolution of the flatness $\epsilon$ of stellar and gas populations with time. The evolution of stellar populations is shown by thin lines, that of the gas by bold line. Evolution is shown for two cases: the bold solid line is for the case when gas immediately arrives the final $\epsilon$, the bold dashed line is the case when for $\epsilon \leq 0.2$ gas gets the flatness as given by respective present stellar population flatness.

Eggen et al. (1962) used motions of old stars to give evidence that the Galaxy collapsed in its early evolution. Our collection of kinematical data, presented in Fig. 1.1, shows that stellar populations with metal deficite occupy in Strömberg diagram a large area with mean velocity dispersion $\sigma \geq 50$ km/s, and galactocentric rotation velocity $-V_\theta \geq 50$ km/s. Assuming the constancy of angular momentum of gas clouds during the evolution we calculated the evolutionary paths of clouds with various angular momentum $h$ in radial direction, shown in left panel of Fig. 1.14. During the evolution the flatness $\epsilon$ of gas population decreases, see right panel of Fig. 1.14. The Figure shows two possibilities, either the gas arrives immediately to a thin sheet and gets the final flatness, or the decrease of the flatness for $\epsilon \leq 0.2$ during the evolution is slower. In the first case the flatness of stellar disc populations must increase with time to obtain the presently observed thickness. In the second case stellar disc populations have constant flatness during the evolution.

1.4.2 Model of physical evolution of galaxies

On the basis of stellar evolutionary tracks and star formation function we calculated a model of the physical evolution of galaxies in Chapter 22. Following Salpeter (1955) we assume that the number of stars of mass $M$, $F(M)$, formed in unit time interval per cubic parsec, can be expressed by the following equation,

$$F(M) = a \times M^{-n},$$

(1.25)

where $a$ and $n = 2.35$ are constants. This equation can be used in the interval of stellar masses $M_0 \leq M \leq M_u$, where $M_0$ and $M_u$ are minimal and maximal masses of forming stars. We take as the upper limit of forming stars $M_u = 100 M_\odot$. For the minimal mass we cannot use a value $M_0 = 0$, since the integral of $F(M)$ is not converging in this case. For this reason, we take $M_0$ as the effective lower limit of mass of forming stars, and take $F(M) = 0$, if $M < M_0$. For stars with normal chemical composition, $Z = 0.02$, we use $M_0 = 0.03$ in solar units (Reddish, 1966).

Following Schmidt (1959) we assume that the star formation rate is proportional to the density of gas in power $S$:

$$R_I = \frac{d\rho_s}{dt} = \gamma \rho_g^S,$$

(1.26)

where $\rho_s$ and $\rho_g$ are star and gas densities, respectively, and $\gamma$ and $S$ are constants. We assume that the full matter density in a volume element, $\rho = \rho_s + \rho_g$, does not depend on time. In this case after
integration of Eq. (1.26) we get

$$\rho_g = \rho [1 + (S - 1)\tau]^{1/(S - 1)},$$

(1.27)

where

$$\tau = t/K$$

(1.28)

is the dimensionless time, and the characteristic time $K$ is expressed as

$$K = (\gamma \rho^{S-1})^{-1}.$$  

(1.29)

We calculated stellar population evolution models for all values, $S = 0, 1, 2$, using characteristic time $K$ from 0.3 to 20 billion years.

To calculate stellar population evolutionary models we used stellar evolutionary tracks by Iben, Demarque, Faulkner, Paczynski and others. Most series were calculated for chemical abundance: $X = 0.71, Y = 0.27$ and $Z = 0.02$. Using these models we tabulated evolutionary tracks for following stellar masses: 0.05, 0.1, 0.2, 0.4, 0.6, 0.8, 1.0, 1.25, 1.5, 2.25, 3, 5, 9, 15, 30, 60 $M_\odot$. In all tracks 19 points were given, each point corresponds to a certain stage in the evolution of stars. Evolutionary tracks are given in coordinates $\log T_e, \log L/L_\odot$, observational data are given as colours and absolute magnitudes, $B - V, M_V$. To find the relationship between these data as well as to calculate mass-to-light ratios of populations and the energy distribution in spectra of model galaxies, we found bolometric corrections, $BC = M_b - M_V$, and intrinsic colours, $(S-V)_0$, of stars of all types. Most calculations were done using following initial data for star formation rate: $n = 7/3, M_0 = 0.03 M_\odot, M_u = 100 M_\odot, S = 0 (K = 20), S = 1 (K = 0.5, 3), S = 2 (K = 0.3)$; here the characteristic time of galaxy formation $K$ is expressed in billion years. We calculated model galaxies for ages 0.01, 0.03, 0.3, 1, 2, 4, 6, 8, 9, 10, 15, and 20 billion years.

Figure 1.15: Left: Integral bolometric luminosities of galaxies of mass $M = 10^{11} M_\odot$ as function of the age $t$ in years for various parameters $S$ and $K$ of star formation function. Right: Mass-to-light ratio of galaxies, $f_B = M/L_B$, as function of the age $t$ for various parameters of the star formation function for extremal values of the metal content $Z$.

The evolutionary path of stellar populations and galaxies depend critically on parameters of star formation function. We show in left panel of Fig. 1.15 integral colorimetric luminosities of galaxies of mass $M = 10^{11} M_\odot$ as function of the age $t$ in years for various parameters $S$ and $K$ of star formation function. If star formation rate does not depend on gas density, case $S = 0$, the bolometric luminosity $E$ is much lower than in other cases and slightly increases with time. If star formation rate is proportional to the gas density, case $S = 1$, the bolometric luminosity increases initially, and after a maximum decreases. The initial luminosity and the location of the maximum depend on the characteristic time $K$. If star formation rate is proportional to density square, case $S = 2$, the initial luminosity is higher than in previous cases and increases slightly, has a maximum at $t \approx 3 \times 10^7$ years, and then decreases almost linearly in log-log plot.
Evolutionary changes in the mass-to-light ratios $f_B$ are shown in the right panel of Fig. 1.15 for three extremal values of the metal content $Z$. We see that after a minimum at epoch $t \approx 3 \times 10^6$ years mass-to-light ratios for all populations increase considerably up to thousand times, depending on the metal content.

The choice of parameters $M_0$, $M_u$ and $n$ is crucial in the modeling the physical evolution of galaxies. To explain differences in mass-to-light ratios of globular clusters ($f_V = 2L/L_V \approx 1$), dwarf galaxies ($f_V \approx 10$), and giant elliptical galaxies ($f_V \approx 100$), lower mass limits $M_0$ must depend on the fraction of heavy elements in the interstellar gas during the formation of stars of different populations. To get tracks for population rich in heavy elements we added to tracks, calculated for composition $Z = 0.02$, the following corrections:

$$
\Delta \log t = -0.12, \quad \Delta \log T_e = -0.10, \quad \Delta \log L = -0.20. 
$$

(1.30)

These corrections were based on tracks found by Iben (1967), Paczyński (1970) and Schlesinger (1969). We attribute these corrections to stars of heavy element content $Z = 0.08$. To get tracks for metal deficit stars we added to tracks for $Z = 0.02$ corrections:

$$
\Delta \log t = -0.22, \quad \Delta \log L = 0.25. 
$$

(1.31)

We attributed these corrections to stars of composition $Z = 0.001$.

Figure 1.16: The dependence of the luminosity function of galaxies in B system on the age of the galaxy $t$ in billion years. As argument we use the absolute magnitude in B system. Star formation function parameters are taken as follows: left: $S = 0$, $n = 2.33$, $M_0 = 0.03 M_\odot$, $R = 5 M_\odot$ per year; right $S = 2$, $K = 0.3 \times 10^9$ years. The mass of the galaxy is $M = 10^{11} M_\odot$.

We calculated the following functions: luminosity function, integrated luminosity in solar units and magnitudes, the contribution of stars of different luminosity to the summed luminosity, mass-to-light ratio. All functions were found in bolometric units and in photometric system UBVRIJKL. The luminosity functions of model galaxies with $S = 0$, $n = 2.33$, $M_0 = 0.03 M_\odot$, $R = 5 M_\odot$ per year (left panel) and $S = 2$ and $K = 0.3 \times 10^9$ years (right panel) are presented in Fig. 1.16. We see that already at a very early stage of the evolution, $t = 10^7$ years, the high luminosity tail of the function is rather strong, and at low luminosity, $M_B = 30$, the white dwarf region is visible. At later epochs the high luminosity tail gets weaker, and the low-luminosity tail increases. Note differences in high-luminosity regions between models with different parameter $S$.

1.4.3 Star formation function and evolution of galactic populations

In Chapter 23 we found numerical values of parameters of the function and estimated the dependence of this function on the chemical composition of the interstellar gas. Also, we discussed on
the basis of star formation function how stellar populations of various age and composition of the Galaxy could be formed.

An essential parameter of the Galaxy is its age. There exist three independent methods to derive ages of galaxies. The first method is based on cosmological considerations: it is clear that ages of galaxies are smaller than the age of the Universe. Most often it is assumed that the Hubble constant is \( H = 75 \text{ km s}^{-1} \text{ Mpc}^{-1} \), and the acceleration parameter \( q_0 = 0.5 \), which yields for the cosmological age \( T_U = 8.7 \pm 1.5 \times 10^9 \) years. Another method to estimate the age of our Galaxy is to use determinations of ages of its oldest halo populations. This method can be used for globular star clusters. Different determinations vary in reasonable limits. According to Sandage (1970) the probable age of globular clusters is \( T = 10 \pm 0.8 \times 10^9 \) years. The most accurate age estimates of the Galaxy come with the radiative isotope method. The mean value obtained with the isotope method is, \( T = 9.5 \pm 0.7 \times 10^9 \) years. When we use results of all three methods, we get for the age of the Galaxy \( T = 9.5 \pm 0.75 \times 10^9 \) years. Instead, we shall use in further calculations a round value \( T = 10 \times 10^9 \) years.

To apply the star formation model of galaxies, described by Eq. 1.26, we need values of basic parameters of the model. Following Schmidt (1959, 1963) (see also Einasto (1972d)), we accepted \( S = 2 \). To find the value of the other parameter of the model, \( \gamma \), we used data on the mass of gas and total mass of Andromeda galaxy M31 and Small Magellanic Cloud, and data on the spatial distribution of gas. Data on both galaxies yield with good mutual agreement a mean value \( \gamma = 4 \times M_\odot pc^{-3} (\text{Gyr})^{-1} \).

During the evolution the chemical composition of galactic populations changes due to the enrichment of interstellar medium with heavy elements by massive stars, which after their active life explode as supernovae. This process was most active in the early phase of the evolution of galaxies during the formation of the halo. Using data on the model of M31 we estimated the expected growth of metal content during the formation of the halo of M31. The growth model is in good agreement with actual data on the chemical composition of halo and disc.

An important factor in formation of galaxies of different morphological type and various galactic populations is the mean density of gas. Applying the model of physical evolution of galaxies we calculated the change of the fraction of gas in galaxies (and populations) of various mean density, see left panel of Fig. 1.17. Our model suggests that in high-density case the fraction of gas rapidly decreases. This explains the absence of gas in central regions of spiral galaxies, observed in galaxies like M31. In contrast, in low-density systems the fraction of gas decreases slowly. Similar changes are found in expected luminosities of galaxies of different mean density, see right panel of Fig. 1.17.

Figure 1.17:  \textit{Left:} The evolution of the fraction of gas mass in model galaxies of various mean density. \textit{Right:} The evolution of galactic total luminosity for galaxies of different mean density. In both panels solid lines are for low density, dashed lines for medium density, and dot-dashed lines for high-density galaxies or populations. Ages \( t \) are expressed in billion years.
CHAPTER 2
EPILOGUE

The defence of the Thesis on March 17, 1972 was successful. However, two related problems remained — it was impossible to reproduce the observed rotation curves of galaxies with known stellar populations, and data on mass-to-light ratios of populations were uncertain. For this reason, I started searches to find solutions to these open questions immediately after the defence. The story of events after the defence of the Thesis is described in detail in my book \cite{Einasto2014} and in the review paper \cite{Einasto2018}. Here I give a short overview of the development of ideas directly connected with the topic of the Thesis.

Both problems are connected with the possibility of the presence of dark matter in galaxies. I had serious reasons to believe that there is only a limited quantity of dark matter in galaxies like our own Galaxy. This problem has been studied by Tartu astronomers long ago. \cite{Opik1915} was one of the first to study the dynamics of the Galaxy with the goal to find the density of matter in Solar neighbourhood. He understood that due to the flat shape of the Galaxy, the dynamical density can be determined from the comparison of motions and spatial distributions of stars in the vertical direction. He found that the vertical attraction of known stars is sufficient to explain the observed distributions, and that there is no reason to add invisible matter (the term “dark matter” had not yet been suggested). \cite{Kuzmin1955, Eelsalu1958} repeated this study with new and better data and confirmed \cite{Opik1915} result. The problem was also discussed by \cite{Oort1960}, who found that the dynamical density near the Sun is larger than found by \cite{Kuzmin1952b, Kuzmin1955, Eelsalu1958}. In other words, there is a need for dark invisible matter. Since the matter density and possible presence of dark matter are of fundamental importance, my Tartu collaborator \cite{joeveer1968, joeveer1972} made a new analysis, using a completely different method, see Chapter 21. Ages of young stars are known, this allows to find parameters of vertical oscillations of young B stars and cepheids, which led to parameter $C = 70$ km/s/kpc and dynamical density $\rho_{\text{dyn}} = 0.09 M_\odot/pc^3$. On the basis of these studies, I supported the classical paradigm with no large amounts of dark matter in the Solar neighbourhood. More accurate recent data support this conclusion \cite{Gilmore1989}.

More data accumulated on rotation velocities of galaxies. New data suggested the presence of almost flat rotation curves of galaxies, thus, it was increasingly difficult to accept my previous solution of the discrepancy with large non-circular motions. I discussed the problem with my colleague Enn Saar in spring 1972, who suggested abandoning my earlier idea that galaxies have relatively sharp boundaries but may have extended envelopes.

The possible presence of dark matter in the Galaxy in Solar vicinity at least in some quantity was suggested by \cite{Oort1932, Oort1960}, and in clusters of galaxies by \cite{Zwicky1933, Karachentsev1966, Rood1972}. A numerical study of the stability of flat galaxies suggest the presence of massive halos of galaxies \cite{Ostriker1973}. Rotation velocity measurements of galaxies by \cite{Roberts1966, Roberts1967, Roberts1969} and \cite{Rubin1970} suggest that galaxies have indeed large and massive envelopes. My detailed analysis of properties of known stellar populations demonstrated that no known stellar population can be responsible for flat rotation curves of galaxies. As I discussed in \cite{Einasto2018}, the tacit assumption in earlier studies was that the stuff, responsible for this effect in clusters, galaxies in general and near the plane of the Galaxy, is the same everywhere.

After the discussion with Enn, I noticed that here lies a controversy. Dynamical data suggest that eventual dark matter in Solar vicinity is strongly concentrated toward the plane of the Galaxy, thus dissipation is needed for its formation. By contrast, if the rotation of galaxies in outer regions is influenced by a new hypothetical population, then this population should form a large, massive, and almost spherical population. In particular, for its formation, dissipation is not needed. Different size, shape, mass and dissipation properties suggest a different formation history and nature. Following these considerations, I concluded that there must exist two types of dark matter: the "local dark matter" near the Sun close to the plane of the Galaxy, and the "global dark matter", forming envelopes of galaxies and clusters of galaxies \cite{Einasto1972a, Einasto1974a}.

To have a better reproduction of observed rotation curves, it would be reasonable to look at which properties the population of global dark matter should have using available data on known stellar populations and galaxy rotation data. To avoid confusion with the known halo population,
consisting of old metal-poor stars, I called the new population “corona” (Einasto, 1972a, 1974a). To check this possibility, I used my programs to calculate dynamical models of galaxies. It was easy to find a new set of models with one addition component — dark corona. As the first approximation, I assumed that the total mass of the M31 corona is equal to the mass of the sum of known stellar populations (Einasto, 1972a, 1974a). I made two versions of models of galaxies in the Local Group and giant elliptical galaxy M87, variant A without corona and variant B with corona, see Fig. 2.1. This calculation showed that the adding of coronas improves model rotation curves, but not enough.

Figure 2.1: The distribution of mass-to-light ratio, \( f_B = M/L_B \), in galaxies of the Local Group and M87: models without (A) and with (B) dark corona (Einasto, 1974a).

I reported new results at the First European Astronomy Meeting in Athens on September 8, 1972 (Einasto, 1974a). It was clear that coronas cannot be made of stars because outer stellar populations consist of old halo-type stars with very low mass-to-light ratio, but the mass-to-light ratio of the corona is very high. The coronal matter cannot be in the form of neutral gas, since this gas would be observable. Initially I suspected that it could be ionised hot gas (Einasto, 1972a, 1974a). However, the total mass of coronas was not known yet, and the evidence for the presence of coronas was not strong.

So far, I had concentrated my efforts on the study of the structure of galaxies. It was now clear that the environment of galaxies was also important. The dark matter problem was discussed a long time ago in clusters of galaxies. Also masses of groups of galaxies, measured from the velocity dispersion of galaxies, were larger than summed masses of individual galaxies, see Holmberg (1937, 1969) and Karachentsev (1966). A similar discrepancy was found in the Local Group (the M31 - MW system) by Kahn & Woltjer (1959). Reading these papers on the mass discrepancy in clusters, groups and galaxies, I realised that the problem of dark matter in galaxies is the same as in clusters. This allows to find masses and radii of dark coronas of galaxies. I noticed that if galactic coronas are large enough, then companion galaxies should lie inside coronas of the main galaxies. Thus, companion galaxies can be considered as test particles to measure the gravitational attraction of the main galaxy.

I collected data for pairs of galaxies. The analysis was soon ready, see Fig. 2.2. Our analysis suggested that galactic coronas have masses about ten times larger than masses of their visible populations. In those years, Soviet astrophysicists had the tradition to organise Winter Schools. In 1974, the School was held in the Terskol winter resort. I presented my report on the masses of galaxies on January 29, 1974. I stressed in my talk arguments, suggesting that the presence of coronas around galaxies is a general phenomenon. Also, I suggested that galactic coronas probably have the same origin as dark matter in clusters and groups, and that coronas are probably not of stellar origin.
Prominent Soviet astrophysicists like Yakov Zeldovich, Iosif Shklovsky, and others participated in the Winter School. After my talk, the atmosphere was as if a bomb had exploded. For Zeldovich and his group, the presence of a completely new, massive non-stellar population was a surprise. Two questions dominated: What is the physical nature of the dark matter? and What is its role in the evolution of the Universe?

I had to hurry with the publication of our results, since massive halos were already discussed by Ostriker & Peebles (1973) to stabilise orbits of flat population stars. Following a suggestion by Yakov Zeldovich we sent the paper to “Nature” (Einasto et al., 1974a). Soon it was clear that it was just in time. Ostriker et al. (1974) got similar results using similar arguments; their paper was published several months after our “Nature” paper, and has a reference to our preprint. Both papers suggest that the total cosmological density of dark matter in galaxies is about 0.2 of the critical cosmological density.

In the “Nature” paper, we noted that dark matter in clusters cannot be explained by hot X-ray emitting gas, since its mass is insufficient to stabilise clusters. Ostriker et al. (1974) did not notice that dark matter forms a new population of unknown nature; authors write in the discussion that the very great extent of spiral galaxies can perhaps be understood as due to a giant halo of faint stars.

Soon the reaction to the results of both papers appeared: Burbidge (1975) formulated difficulties of the dark corona/halo concept. The main problem is in the statistical character of dynamical determinations of masses of multiple galaxies. If companion galaxies used in mass determination are not real physical companions but random interlopers, as suggested by Burbidge, then the mean velocity dispersion reflects random velocities of field galaxies, and no conclusions on the mass distribution around giant galaxies can be made.

Our “Nature” paper (Einasto et al., 1974a) together with a similar paper from the Princeton group by Ostriker et al. (1974) and the response by Burbidge (1975) started the “dark matter” boom. As noted by Kuhn (1970), a scientific revolution begins when leading scientists in the field start to discuss the problem and argue in favour of the new over the old paradigm.

Difficulties connected with the statistical character of our arguments were evident, thus we started a study of properties of companion galaxies to find evidence for some other regularity in the satellite system which surrounds giant galaxies. Soon we discovered that companion galaxies are segregated morphologically (Einasto et al., 1974b). Elliptical (non-gaseous) companions lie close to the primary galaxy whereas spiral and irregular (gaseous) companions of the same luminosity have larger distances from the primary galaxy. The distance of the segregation line from the primary galaxy depends on the luminosity of the satellite galaxy, see Fig. 2.3. This means that there exist physical interactions between companions and the coronal gas of the main galaxy — ram-pressure removal of gas from companion galaxies by the coronal gas of the main galaxy.
It was also clear that coronas form an extended population of the main central galaxy. But here lies a contradiction: inside a luminous galaxy with its non-luminous corona there exist companion galaxies, orbiting within the corona of the main galaxy. In other words, a dwarf galaxy inside the giant galaxy. To avoid confusion, we proposed with Arthur Chernin to call giant galaxies together with their coronas and satellites “hypergalaxies” (Chernin et al., 1976). We found that almost all dwarf galaxies are located near luminous galaxies. This led us to the conclusion that galaxies do not form in isolation, but as systems, and that hypergalaxies are the main sites of galaxy formation. However, the term “hypergalaxies” is not accepted by the astronomical community, instead the term “halo” is used.

The dark matter problem was discussed in a special session of the Third European Astronomy Meeting in Tbilisi, Georgia, in summer 1975. This was the first international discussion between the supporters of the classical paradigm with conventional mass estimates of galaxies, and of the new one with dark matter. Arguments favouring the classical paradigm were presented by Materne & Tammann (1976). Their most serious argument was: Big Bang nucleosynthesis suggests a low-density Universe with the density parameter $\Omega \approx 0.05$; the smoothness of the Hubble flow also favours a low-density Universe. If one excludes inconvenient data by Zwicky (1933) on the Coma cluster, by Kahn & Wolter (1959) data on the mass of the double system M31-Galaxy, and recent data on flat rotation curves of galaxies by Roberts (1966) and Rubin & Ford (1970), as written explicitly by Materne & Tammann (1976), then everything fits well into this classical cosmological paradigm. It was clear that the problem cannot be solved by dispute — new data were needed.

Soon new radio measurements of neutral hydrogen for a large number of galaxies were published by Bosma (1978). Another series of extended rotation curves of spiral galaxies was made by Roberts & Whitehurst (1975) using radio data, and by Rubin et al. (1978, 1979, 1980) using optical measurements. Observations confirmed the general trend that the mean rotation curves remain flat over the whole observed range of distances from the centre up to $\approx 40$ kpc for several galaxies. The internal mass within the radius $R$ increases over the whole distance interval. However, the nature of dark matter was still unknown.

The dark matter problem was discussed during the IAU General Assembly in Grenoble at the Commission 33 Meeting in August, 1976. In my talk I presented arguments for the non-stellar nature of dark corona (Einasto et al., 1976b). After the lecture, Ivan King came to me and asked to repeat the main arguments against the stellar origin of dark matter. The basic arguments are as follows.

Physical and kinematical properties of stellar populations depend almost continuously on the age of the population, see Fig. 2.4. The continuity of stellar populations of various age is reflected.

![Figure 2.3](image-url) Distribution of luminosity of companion galaxies of different morphology vs. distance from the central galaxy; spiral and irregular companions are marked with open circles, elliptical companions with filled circles (Einasto et al., 1974b).
Figure 2.4: Strömbärg diagram for galactic populations according to data presented in Chapter 4. Kinematical data for the corona are taken from the model by Einasto (1979). The Strömbärg fit was taken from original Russian version of Chapter 4, it does not take into account the non-stationary status of very young populations.

also in their kinematical characteristics, such as the velocity dispersion and the heliocentric centroid velocity, expressed in the Strömbärg diagram. The oldest halo populations have the lowest metallicity and $M/L$-ratio, the highest velocity dispersion, and the largest (negative) heliocentric velocity, see Fig. 2.4. There is no place to put the new population into this sequence. The dark population is almost spherical and non-rotating. It has a much larger radius than all known stellar populations. In order to be in equilibrium in the Galactic gravitational potential, these coronal stars must have a high velocity dispersion, about $\sigma \approx 200$ km/s, much more than all known stellar populations, up to 125 km/s, see Fig. 2.4. Jaaniste & Saar (1975) investigated the possible stellar nature of the corona. Authors found no fast moving stars, possible candidates for coronal objects.

The $M/L$ value, and the spatial and kinematical distribution of the dark population differ greatly from respective properties of all known stellar populations, and there are no intermediate populations. Thus, the corona must have been formed much earlier than all known populations to form the gap in relations between various physical, kinematical and spatial structure parameters. The total mass of the new population exceeds the masses of known populations by an order of magnitude, thus we have a problem: How to transform at an early stage of the evolution of the Universe most of the primordial matter into invisible stars? It is known that star formation is a very inefficient process: in a star-forming gaseous nebula only about 1% of matter transforms into stars.

As discussed above, neither neutral nor hot ionised gas is a suitable candidate for dark matter. Thus, the nature of coronas remained unclear. It was only much later that the non-baryonic nature of dark matter became evident, as discussed in a conference in Tallinn, April 7 – 10, 1981, and in Vatican Study Week, September 28 – October 2, 1981. Leading Soviet physicists and astronomers attended the Tallinn conference. Several talks were devoted to the formation of the structure of the Universe with neutrinos as dark matter (Yakov Zeldovich, Andrei Doroshkevich, Igor Novikov). In the Vatican Study Week neutrinos as dark matter candidates were discussed by Martin Rees, Joe Silk, Jim Gunn and Dennis Sciama. Difficulties of the neutrino-dominated dark matter were evident, and soon the Cold Dark Matter (CDM) was suggested by Bond et al. (1982), Pagels & Primack (1982), Peebles (1982), and Blumenthal et al. (1984).

The presence of massive dark matter coronas influences galactic models. Thus, I continued together with my collaborators Urmas Haud and Ants Kaasik to develop new models which included dark coronas. To develop the model of our Galaxy, a system of galactic parameters is needed. One of important parameters is the circular velocity near the Sun. Using the method described in Chapter 7 and shown in Fig. 1.2 we found for the circular velocity $V_0 = 220 \pm 7$ km/sec (Einasto et al., 1979). This value is lower than our previous estimate, discussed in Chapter 7, due to the addition of dark corona in the new model. The model of the Galaxy was described in the preliminary form by Einasto et al. (1976b) and in a more polished form by Einasto (1979). In this model, we found
an improved system of galactic parameters with \( R_0 = 8.5 \) kpc, presented in Table 2.1. Parameters of galactic populations according to this model are given in Table 2.2. For disc and flat populations parameters are given for positive mass components.

In the top left panel of Fig. 2.5 we show the circular velocity (solid line) and observed rotation velocity (symbols) of the model by Einasto (1979). In the right panel of the Figure we give the surface density of the Galaxy and its components of the same model. In the bottom left panel of Fig. 2.5 we show the velocity dispersion \( \sigma_z \) as the function of the distance from the galactic plane, and in the right panel the velocity dispersion \( \sigma_R \) as the function of the distance from galactic centre (Einasto 1979). Data are given for the main populations: flat, bulge, disc, halo and corona. Also we show the mean dispersion, and the critical dispersion by Toomre (1964). We see that the mean velocity dispersion is larger than the critical Toomre dispersion, thus the model is stable against small radial perturbations.

The critical point in model construction is the determination of mass-to-light ratios for individual populations. For the nucleus and core this ratio can be determined from observations by two methods, from spectrophotometric data and from virial theorem. For the halo we can use the value for open clusters, as found from velocity dispersion, from the rotation velocity at distance from the center where the disc or bulge dominate, and from calculations of the physical evolution of populations. We assume that the bulge and disc have the same chemical composition and mass-to-light ratios can be checked by independent velocity dispersion data in small regions of the Galaxy and its components.

The dependence of \( f_B \) of individual galactic populations on the total mass of galaxies is shown in Fig. 2.6, and on B-V and U-B colours in Fig. 2.7.

Mass-to-light ratios \( f = M/L_B \) of galactic populations are formed during the evolution of stars, and are incorporated in dynamical models of galaxies. \( M/L_B \)-ratios depend on the age and the chemical content of populations, and are fixed by the minimal mass of stars in the star-formation function, \( M_0 \). I accepted for stellar populations with a normal metal content a lower star formation limit \( M_0 = 0.03 \) \( M_\odot \), for metal-rich populations a limit \( M_0 = 0.001 \) \( M_\odot \), and for metal-poor populations a limit \( M_0 = 0.1 \) \( M_\odot \). Most limits are lower than the lowest masses needed to start hydrogen burning in stars, \( M^* = 0.08 \) \( M_\odot \). Using these lower mass limits, I got for old metal-poor populations \( M/L_B \leq 3 \), for old intermediate populations \( M/L_B \leq 10 \), and for old extremely metal-rich populations \( M/L_B \leq 100 \), see Fig. 2.6. The spatial distribution of mass in populations is well determined, and \( M/L_B \)-ratios can be checked by independent velocity dispersion data in small systems of different age and chemical content (open and globular clusters, nuclei of galaxies). As our model calculations showed, it is impossible to reproduce with known populations the observed

### Table 2.1: Galactic parameters

| Parameter | Unit     | Observed | Smoothed | Adopted | Reference |
|-----------|----------|----------|----------|---------|-----------|
| \( R_0 \) | kpc      | 8.8 ± 0.7| 8.5 ± 0.3| 8.5     | 1, 2      |
| \( V \)   | km/sec   | 220 ± 10 | 221 ± 5  | 225     | 3         |
| \( W \)   |          | 120 ± 15 | 133 ± 4  | 131.8   | 4         |
| \( A \)   | km/sec/kpc| 16 ± 1  | 15.7 ± 0.4| 15.5    | 5 - 7     |
| \( C \)   |          | 70 ± 5   | 74       | 14      |
| \( \omega \)|          | 26 ± 2   | 26.0 ± 0.7| 26.5    | 8 - 10    |
| \( k_2 \) |          | 0.282 ± 0.02 | 0.285 ± 0.008 | 0.293 | 11        |
| \( \rho_0 \)| \( M_\odot \)-pc\(^2\) | 0.1 ± 0.02 | 0.097 ± 0.02 | 0.007 | 12, 13    |

References: 1. Oort & Plaut (1975), 2. Harris (1976), 3. Einasto et al. (1979), 4. Haud (1984), 5. Crampton & Fernie (1969), 6. Balona & Feast (1974), 7. Crampton & Georgelin (1975), 8. Asteriadis (1977), 9. Fricke (1977), 10. Dieckvoss (1978), 11. Einasto (1972c), 12. Jõeveer (1974), 13. Woolley & Stewart (1967), 14. Jõeveer (1974).

### Table 2.2: Parameters of galactic components

| Quantity | Unit       | Nucleus | Bulge | Halo | Disc | Flat | Corona |
|----------|------------|---------|-------|------|------|------|--------|
| \( \epsilon \) |           | 0.6     | 0.6   | 0.3  | 0.10 | 0.02 | 1      |
| \( N \)   | 1          | 1       | 1     | 4    | 1    | 0.5  | 0.5    |
| \( a_0 \) | kpc        | 0.005   | 0.21  | 1.9  | 4.62 | 6.4  | 75     |
| \( \Omega \) | \( 10^{10} M_\odot \) | 0.009 | 0.442 | 1.2 | 7.68 | 1.0  | 110    |
flat rotation curves of spiral galaxies. In contrast, models based on rotation velocities (Schmidt (1957), Brandt & Scheer (1965), Roberts (1966), Rubin & Ford (1970)) have a very rapid increase of $M/L_B$-ratios on the periphery of M31. But these models contain no hint to understand how to explain this increase.

During one of 1976 IAU General Assembly meetings, Sandra Faber discussed her recent measurements of spectra of elliptical galaxies (Faber & Jackson, 1976). New data suggested that velocity dispersions of the nuclei of elliptical galaxies are much lower than accepted so far, which leads to a considerable decrease of mass-to-light ratios of elliptical galaxies. This suggests that corrections are needed to my previous galaxy evolution models. This can be done by changing the lower mass limit of forming stars, and using for all populations identical lower mass limits, $M_0 \approx 0.1 \, M_\odot$, which yields lower $M/L$ values for all populations. A very detailed review of masses and mass-to-light ratios of galaxies is given by Faber & Gallagher (1979). Their Table 1 gives $M/L_B$ values within Holmberg radius of galaxies with extended rotation curves. These measured mass-to-light values lie in the interval $0.6 \leq M/L_B \leq 12$, with a mean value about 4, which corresponds to the disc of galaxies.

In galactic models, the main task is the determination of parameters of populations. First, a crude preliminary model is calculated, model functions are compared with observed functions, and differences are found. In earlier models a simple trial-and-error procedure was applied to find proper values of population parameters. In late 1970s, Urmas Haud suggested applying an automatic procedure for model parameters search. As in previous model calculations, first preliminary values of model parameters are selected, model functions are calculated and compared with observed func-
Figure 2.6: Dependence of mass-to-light ratio $f_B$ of old galactic populations on the total mass of galaxies (Einasto, 1974a).

Figure 2.7: Dependence of mass-to-light ratio $f_B$ of old galactic populations on their B-V and U-B colours (Einasto, 1974a).

With this method, first a new model of the Galaxy was found by Haud & Einasto (1989), models of other galaxies were published by Tenjes et al. (1991, 1994, 1998).

Table 2.3: Parameters of components of M31

| Quantity | Unit | Nucleus | Core | Bulge | Halo | Disc | Flat | Corona |
|----------|------|---------|------|-------|------|------|------|--------|
| $\epsilon$ |       | 0.69    | 0.82 | 0.67  | 0.47 | 0.10 | 0.02 | 1      |
| $N$      |       | 1.2     | 1.5  | 2.4   | 4.9  | 1.3  | 0.3  |        |
| $a_0$    | kpc  | 0.0039  | 0.10 | 0.75  | 4.8  | 4.1  | 11.1 | 60     |
| $2R$     | $10^{18} M_\odot$ | 0.031  | 0.20 | 1.0   | 0.8  | 8.4  | 0.75 | 320    |
| $f_B$    | $M_\odot/ L_\odot$ | 32     | 13   | 2.6   | 2.0  | 15   | 1.1  |        |
| $U - B$  |       | 0.88    | 0.80 | 0.54  | 0.21 | 0.90 | -0.38|        |
| $B - V$  |       | 1.03    | 0.97 | 0.79  | 0.79 | 1.01 | 0.45 |        |

We show in Table 2.3 parameters of components of the model of M31 by Tenjes et al. (1994), and in Fig 2.8 rotation and mass-to-light curves according to the model. Parameters for the disc and flat subsystems are for positive mass components. The most essential change in comparison with earlier models of M31 is the decrease of masses and mass-to-light ratios of the nucleus, core and bulge, and the addition of a massive corona. In the case of the M31 model the decrease of masses and mass-to-light ratios reduces the height of the peak of the model circular velocity at
Figure 2.8: Left: The rotation curve of M31 according to the model by Tenjes et al. (1994). Open circles – observations, thick line – model, dashed lines – model curves of components. Right: Local mass-to-light ratios for visible populations and for the model with dark corona.

small distances from the center, and the addition of the corona improves the model circular velocity on large distance from the center.

In galactic models, we used the modified exponential model, Eq. (1.2). Our automatic model parameter search showed that for all stellar populations the optimal value of the parameter is $x_0 = 0$. In other words, there is no need for this modification of the exponential profile. Presently this profile in the form Eq. (1.24) is called “Einasto profile” and is used mainly to describe the spatial density distribution of dark matter halos (Merritt et al., 2005).

The study of the morphology of satellite galaxies was our first step in the investigation of the environment of galaxies. Following a suggestion by Iosif Shklovsky we studied the dynamics of the Magellanic Stream, discovered by Mathewson et al. (1974). The Magellanic Stream is a huge strip of gas through Magellanic Clouds. We noticed that most companions of the Galaxy, including Magellanic Clouds, the Magellanic Stream, and another stream of high-velocity hydrogen clouds lie close to a plane that is almost perpendicular to the Galactic plane (Einasto et al., 1976a). In this paper we used velocities of satellite galaxies and Magellanic Stream gas clouds to determine the mass of the Galaxy together with its satellites – our Local Hypergalaxy: $M_{\text{tot}} = 1.2 \pm 0.5 \times 10^{12} M_\odot$.

Inspired by this pioneering work Urmas Haud continued the study of the dynamics of high-velocity hydrogen clouds surrounding the Galaxy.

Figure 2.9: The spiral pattern of four galaxies according to Jaaniste & Saar (1976). Bold lines indicate the loci of new-born stars. The thickness of spiral arms (shaded areas) is determined by the lifetime of massive stars. Dotted curves are hydrogen spiral arms.

Another inspiration suggested by the Magellanic Stream phenomenon concerns the formation of spiral structure of galaxies. Jaaniste & Saar (1976) and Einasto et al. (1976b) noticed that in many giant galaxies dwarf satellite companions are located close to planes perpendicular to the main plane of the central giant galaxy. It is natural to assume that similar to the Magellanic Stream also other giant galaxies have gaseous streams surrounding the main galaxy near the plane of satellites. Gas in these streams falls to the central galaxy along the intersection of planes of the main galaxy and its satellites. These streams initiate perturbations in the gas of the central galaxy and give rise to
star formation. Due to the rotation of central galaxies star forming regions form a spiral pattern. Jaaniste & Saar (1976) suggested that the accretion of gas can be the main physical mechanism in the formation of spiral structure of galaxies. Using observed rotation curves of several galaxies authors calculated the expected form of spiral pattern. Results are close to actually observed spiral pattern of galaxies, see Fig. 2.9.

The study of the environment of galaxies was actually only an introduction to a much wider research area — the distribution of galaxies on large scales. Our involvement in these studies was emphasised by Yakov Zeldovich. After my report on dark matter in galaxies in the Terskol winter school he turned to me and asked to collaborate with him in the study of the Universe. He was developing a theory for the formation of galaxies (Zeldovich, 1970), alternative theories were suggested by Peebles & Yu (1970) and Ozernoi (1974), and he was interested to find some observational evidence that can be used to discriminate between these theories.

Initially, we did not know how we can contribute to the problem of galaxy formation. The expected consequences of the Zeldovich model were discussed by Doroshkevich et al. (1974) in the IAU Cosmology Symposium in Krakow 1973. According to this scenario, the first forming objects are superclusters of galaxies which fragment into galaxies. The Peebles & Yu (1970) scenario suggests that the first forming objects are small systems (galaxies or even star clusters), which by gravitational clustering form superclusters of galaxies. Ozernoi (1974) model did not predict any spatial distribution of galaxies. When discussing the problem with my Tartu collaborators, I remembered my previous experience in the study of galactic populations: kinematical and structural properties of galactic populations evolve only slowly, and thus remember their previous state. Large aggregates of galaxies remember their history better, since the crossing time in these systems is larger. Thus we had a leading idea for the search — we have to search for regularities in the large-scale distribution of galaxies.

In this way, we started to collect data on spatial distribution of galaxies in the nearby Universe. This resulted in the discovery of the cosmic web by Jõeveer et al. (1977), Jõeveer & Einasto (1978) and Jõeveer et al. (1978). The observed pattern of the distribution of galaxies has some similarity with the expected distribution, as found with numerical experiments by Doroshkevich & Shandarin (1977). Following this similarity, we called the observed distribution as “cellular”. Subsequently we used the term “supercluster-void network” (Einasto et al., 1980). Presently the structure is called “cosmic web”, following a suggestion by Bond et al. (1996).

The development of our understanding of the structure and evolution of the Universe is described in detail by Einasto (2014, 2018). This forms a natural extension to my earlier studies on the structure and evolution of galaxies.
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