Numerical analysis of terahertz surface plasmon polaritons propagating in a parallel plate configuration

Sukhdeep Kaur¹,*, Sarabjeet Kaur¹, Manpreet Kaur¹ and Gagan Kumar²

¹Department of Physics, Guru Nanak Dev University, Amritsar, Punjab-143005, India
²Department of Physics, Indian Institute of Technology Guwahati, Assam, India

Email: *¹sukhdeep.iitd@gmail.com

Abstract: In this paper, we present numerical analysis of terahertz (THz) surface plasmon polaritons propagating in a parallel plate configuration. A planar metal surface has the ability to support loosely bound surface waves, called Zenneck waves at terahertz frequencies. Two parallel metal plates separated by a narrow vacuum region can lead to the highly confined terahertz surface plasmon modes at specific frequencies. The frequency of the terahertz mode can be changed with a change in the gap between the plates. Further, we observed that the carrier concentration of plates strongly effect the frequency of the terahertz mode as well as its properties. We have examined in detail the dispersion properties of the terahertz surface plasmons supported by the parallel plate configuration with a varying gap width as well as carrier density of the plates.

1. Introduction
Terahertz radiations cover a narrow range of frequencies between optics and microwave region in the electromagnetic spectrum. In recent years, terahertz related research and technology development has acquired a significant interest of the scientific community all around the globe. This is because of the various applications that terahertz radiations can advance [1-5]. This include high speed communication [6], sub wavelength imaging [7], slow light devices [8], bio sensing [9], spectroscopy etc. The development of the waveguides at terahertz frequencies has been one of the primary focus of research in this fast growing area of science and technology. Although, several waveguide geometries have been examined both experimentally and numerically in last more than a decade, however parallel metal configuration has been one of the most promising and widely explored techniques as it has proven its potential in applications such as imaging, sensors and strong field confinement. In this direction, Zhan et al [10] have experimentally demonstrated two dimensional confinement of THz waves in tapered waveguide. It is accomplished by coupling of plasmonic edge modes across the air gap. By optimizing the width and separation, THz waves have been confined down to a size of 10 µm (=λ/260). Mueckstein and Mitrofanov [11] have studied THz surface plasmon waves excited on a gold surface by a focused beam. Jeon and Grischkowsky [12] have experimentally studied the propagation of THz Zenneck surface wave on an aluminium sheet, which are basically THz surface plasmons. They obtained weak guided THz surface plasmon propagation with high attenuation rate due to metal surface than the one predicted by Zenneck theory. Grischkowsky and co-workers have also developed a comprehensive...
understanding of the terahertz modes supported by the parallel metal configuration and experimentally demonstrated the ability of this configuration in sensing with greater sensitivity.

The excitation of surface plasmons and their propagation in metals at terahertz frequencies is very challenging as metal exhibit very high conductivity. Zenneck had reported the loosely bound terahertz surface waves in metal, calling them Zenneck waves which are confirmed by Grischkowsky and his co-workers as mentioned above. By placing another sheet of metal separated by a thin air gap can reduce the radiative losses. In this regard parallel metal configurations have been developed and examined for various applications as discussed above. However, an analytical description and a comprehensive understanding in this direction is not well established. Furthermore, a change in the metals with other materials such as semiconductors can greatly influence the terahertz modes propagating in the gap. Therefore, a comprehensive understanding of the terahertz modes and their propagation properties depending upon the carrier concentration of the side plates is greatly required.

In this paper, we examine the propagation properties of the terahertz (THz) surface plasmon polaritons in a parallel plate configuration. We examine the dispersion relations of the terahertz surface modes with different widths of the air film surrounded by the two parallel plates. We developed a theoretical model to obtain dispersion relations of terahertz modes. Next, we have examined the dispersion relations with a change in the carrier concentration of the plates. The dispersion relations are plotted for different carrier concentrations which are chosen keeping in mind silicon substrates under different carrier dopant level.

2. Dispersion relation of terahertz surface plasmon wave in a parallel plate configuration

We consider two parallel metal half spaces \( z < -b/2 \) and \( z > b/2 \) separated by a thin vacuum region \((-b/2 < z < b/2)\) (Figure 1). Surface plasmon waves are coherent delocalized electron waves that exist at the interface between two materials where the real part of the dielectric function changes sign across the interface (e.g. a metal-dielectric interface such as metal sheet in air).

The effective permittivity of metal at frequency \( \omega \) is,

\[
\epsilon_m = \epsilon_r - \frac{\omega_p^2}{\omega^2(1+i\nu/\omega)}
\]  

where \( \epsilon_r \) is the lattice dielectric constant of the metal, \( \omega_p = (n_0 e^2/m\epsilon_0)^{1/2} \), \( n_0 \), \( -e \) and \( m \) are the density, charge and effective mass of the free electron, \( \epsilon_0 \) is free space permittivity, \( c \) is the speed of light in vacuum and \( \nu \) are the collisions. Further, we may write it in terms of real and imaginary parts i.e.

\[
\epsilon_m \approx \epsilon_r - \frac{\omega_p^2}{\omega^2} + i \frac{\nu}{\omega} \frac{\omega_p^2}{\omega^2}
\]

The field distribution for the surface plasmon mode for the said configuration with \( t, x \) variation is given as \( \exp[-i (\omega t-k_z x)] \). The wave equation governing electric field in three different media is given by,
\[ \frac{\partial^2 E_x}{\partial z^2} - \left( k_x^2 - \frac{\omega^2}{c^2} \epsilon_m \right) E_x = 0, \]  

(2)

where \( k_x = \frac{\omega}{c} \left( \frac{\epsilon_m}{1+\epsilon_m} \right)^{1/2} \), using equation (1) we may get real and imaginary parts as \( k_x = k_x r + i k_x i \) and \( k_x i \) will govern the decay loss. Within parallel plate configuration, propagation of wave is in x-direction and mode of propagation of surface plasmon polariton is basically a TM mode. In zx plane, component of electric field obtained from the waveguide equations are given by

\[ E_z = \frac{i}{(\frac{\omega^2}{c^2} \epsilon_m - k^2)} \left( k \frac{\partial E_x}{\partial y} + \omega \frac{\partial B_z}{\partial z} \right), \]  

(3)

\[ E_y = \frac{i}{(\frac{\omega^2}{c^2} \epsilon_m - k^2)} \left( k \frac{\partial E_x}{\partial y} - \omega \frac{\partial B_z}{\partial z} \right), \]  

(4)

Using Gauss law \( \vec{\nabla}. \vec{E} = 0 \), we obtain the wave equation governing \( E_x \) in the three media, where \( \epsilon_m = 1 \) for \(-b/2 < z < b/2\) and \( \epsilon_m \) for \( z > b/2 \) and \( z < -b/2 \).

In region I, \( z > b/2 \) the wave equation is given as,

\[ \frac{\partial^2 E_x}{\partial z^2} - \left( k_x^2 - \frac{\omega^2}{c^2} \epsilon_m \right) E_x = 0, \]  

(5)

The solution of equation (5) will be,

\[ E_x = B_3 e^{\beta_1 z} + B_0 e^{-\beta_1 z} \]  

(6)

For \( z > b/2 \), at \( z \to \infty \), \( B_3 = 0 \), one may write \( E_x = B_0 e^{-\beta_1 z} \) and \( E_z = \frac{i k_x}{\beta_1} E_x \)

where \( \beta_1 = \left( k_x^2 - \frac{\omega^2}{c^2} \epsilon_m \right)^{1/2} \). The total electric field in region \( z > b/2 \) will be, \( \vec{E} = E_x \hat{x} + E_z \hat{z} \), i.e.,

\[ \vec{E}_I = \left[ \hat{x} + \frac{i k_x}{\beta_1} \hat{z} \right] B_0 e^{-\beta_1 z} \]  

(7)

In region II, \( -b/2 < z < b/2 \), \( \epsilon = 1 \) (vacuum region), the component of electric field is,

\[ \frac{\partial^2 E_x}{\partial z^2} - \left( k_x^2 - \frac{\omega^2}{c^2} \right) E_x = 0, \]  

(8)

Following Liu and Tripathi [13], one may obtain solution in component form from above equation, \( E_x = B_2 e^{-\beta_2 z} + B_1 e^{\beta_2 z} \) and \( E_z = \frac{i k_x}{\beta_2} E_x \), where \( \beta_2 = \left( k_x^2 - \frac{\omega^2}{c^2} \right)^{1/2} \). The total electric field in this region is given by, \( \vec{E} = E_x \hat{x} + E_z \hat{z} \), gives

\[ \vec{E}_{II} = \left[ \hat{x} - \frac{\hat{z} i k_x}{\beta_2} \right] B_1 e^{\beta_2 z} + \left[ \hat{x} + \frac{\hat{z} i k_x}{\beta_2} \right] B_2 e^{-\beta_2 z} \]  

(9)

In region III, \( z < -b/2 \), the wave equation will be,

\[ \frac{\partial^2 E_x}{\partial z^2} - \left( k_x^2 - \frac{\omega^2}{c^2} \epsilon_m \right) E_x = 0 \]  

(10)
Solution of equation (10) will be given by, \( E_x = B_3 e^{\beta_1 z} + B_0 e^{-\beta_2 z} \). For \( z < -b/2 \), at \( z \to -\infty \), \( B_0 = 0 \), therefore, \( E_x = B_3 e^{\beta_1 z} \) and \( E_z = \frac{ik_x}{\beta_1} E_x \) will give the complete solution as \( \vec{E} = E_x \hat{x} + E_z \hat{z} \), i.e.,

\[
\vec{E}_{III} = \left[ \hat{x} - \frac{ik_x}{\beta_1} \right] B_3 e^{\beta_1 z} \tag{11}
\]

Thus, in each region we have,

\[
\vec{E}_I = \left[ \hat{x} + \frac{ik_x}{\beta_1} \hat{z} \right] B_0 e^{-\beta_1 z}, \quad z > b/2 \tag{12}
\]

\[
\vec{E}_{II} = \left[ \hat{x} - \frac{ik_x}{\beta_2} \right] B_1 e^{\beta_2 z} + \left[ \hat{x} + \frac{ik_x}{\beta_2} \right] B_2 e^{-\beta_2 z}, \quad -b/2 < z < b/2 \tag{13}
\]

\[
\vec{E}_{III} = \left[ \hat{x} - \frac{ik_x}{\beta_1} \right] B_3 e^{\beta_1 z}, \quad z < -b/2 \tag{14}
\]

Applying boundary conditions, continuity of \( E_x \) and \( \varepsilon_m E_x \) at \( z = b/2 \) and \( z = -b/2 \), i.e. we obtain the following equations,

\[
B_1 e^{\beta_2 b/2} + B_2 e^{-\beta_2 b/2} = B_0 e^{-\beta_1 b/2} \tag{15}
\]

\[
B_1 e^{\beta_2 b/2} - B_2 e^{-\beta_2 b/2} = - \left( \varepsilon_m \frac{\beta_2}{\beta_1} \right) B_0 e^{-\beta_2 b/2}, \tag{16}
\]

\[
B_1 e^{-\beta_2 b/2} + B_2 e^{\beta_2 b/2} = B_3 e^{-\beta_1 b/2}, \tag{17}
\]

\[
B_1 e^{-\beta_2 b/2} - B_2 e^{\beta_2 b/2} = \left( \varepsilon_m \frac{\beta_2}{\beta_1} \right) B_3 e^{-\beta_1 b/2}. \tag{18}
\]

Solving these equations, we obtain the dispersion relation,

\[
\frac{\beta_2^2}{\beta_1^2} = \left| \frac{1 - e^{\beta_2 b}}{1 + e^{\beta_2 b}} \right|^2 \frac{1}{\varepsilon_m^2}. \tag{19}
\]

Simplifying further we obtain,

\[
(4k_x c \omega^2 - 2k_x c \omega_p^2 + \omega^2 \omega_p) \exp \left( \frac{-\omega^2 b}{2k_x c^2} \right) = (2k_x c \omega_e^2 + \omega^2 \omega_p) \exp (-k_x b), \tag{20}
\]

In order to estimate the decay loss, we can write the dispersion relation as following,

\[
F = \frac{\beta_2^2}{\beta_1^2} = \left| \frac{1 - e^{\beta_2 b}}{1 + e^{\beta_2 b}} \right|^2 \frac{1}{\varepsilon_m^2}.
\]

One may separate it in real and imaginary parts as \( F(k_{xr} + ik_{xi}, \epsilon_{mr} + i\epsilon_{mi}) = 0 \). Upon simplification, the one may obtain the absorption coefficient, \( k_{xi}^{-1} \) of the field in the vacuum region given by,

\[
k_{xi} = -\epsilon_{mi} \frac{\partial F/\partial \epsilon_m}{\partial F/\partial k_x} \cong k_{xr} \frac{\epsilon_{mi}}{2} \left( 1 - \frac{1}{1+\epsilon_{mr}} \right). \tag{21}
\]

One may use equation (21) to calculate the propagation length of the plasmonic mode in vacuum region sandwiched by metal regions. Since metal behaves like conductors at terahertz frequency, field
penetration inside the metal for terahertz is negligible e.g. skin depth inside Au metal is ~ 80 nm at 1 THz. Therefore, one can neglect the loss of terahertz field inside the metal regions.

We have solved the dispersion relations given by equation (20) for different values of air gap and carrier densities of the plates. Dispersion relation is examined for two different values of air gap viz. $b = 1 \text{mm}$ and $500 \mu\text{m}$ for the carrier densities $n = 10^{18}/\text{cm}^3$, and $n = 10^{20}/\text{cm}^3$ of the plates. Figure 2 shows the dispersion relations for parallel plate configuration when $b = 500 \mu\text{m}$ for plate carrier densities $n = 10^{18}/\text{cm}^3$, and $n = 10^{20}/\text{cm}^3$ respectively. In both the plots, one may note that as the wave number increases, initially frequency monotonically increases and then saturates at higher wave number. For higher value of wave number $k_x$, the dispersion curves saturate. The plasma frequency gets changed with density of the plates. In case of $n = 10^{18}/\text{cm}^3$, it is $\omega_p = 56.4 \times 10^{12} \text{rad/s}$ and for $n = 10^{20}/\text{cm}^3$, it is $\omega_p = 56.4 \times 10^{13} \text{rad/s}$. It may be noted that as the carrier density of the plates in increased, the saturation value of the dispersion relation decreases signifying a lower cut-off of the terahertz modes supported by the parallel plate configuration. We also examine the dispersion relation of surface plasmon mode while changing the width of the air gap for two different carrier concentrations of the metal plates. Figure 3 shows dispersion relations of the terahertz surface plasmon polaritons propagating inside the parallel plate configuration with gap width $b = 1 \text{ mm}$ for two different values of carrier densities of the plates, (a) $n = 10^{18}/\text{cm}^3$ and (b) $n = 10^{20}/\text{cm}^3$. It is important to highlight that the terahertz field in the vacuum region sandwiched by

![Figure 2](image1.png)

(a)

![Figure 3](image2.png)

(b)
two metal plates could be significant for variety of application such as sensing, acceleration etc. Therefore it is important to analyse the field profile of the plasmon mode as it propagates through the gap. In order to have the perceptivity of fields in different regions of the parallel plate configuration, Figure 4. THz field profile in parallel plate configuration comprising two metallic layers. The field is maximum at the metal air interface, however decays exponentially in both the media with different decay constants. The field profile is calculated for the parameters: $a = 500 \, \mu m$, $\omega_p = 30.4 \, THz$, $B_z = 2 \, kV / cm$, $\omega = 9.64 \, THz$.

In Figure 4, we have plotted terahertz field profile as it travels in the waveguide. Equations (7)-(11) are used to calculate the fields of THz in different regions for parameters: $a = 500 \, \mu m$, $\omega_p = 30.4 \, THz$, $B_z = 2 \, kV / cm$, $\omega = 9.64 \, THz$, $k_z = 0.19 \times 10^5 \, m^{-1}$. It may be noted that the field is maximum at the surface of the metals and minimum in the middle. Further, the THz field penetration inside the plates is very low because of their high conductivity. We calculated $1/e$ penetration depth $\sim 3 \, \mu m$ for the above given parameters. The field in $E_y$ direction is negligible for the surface plasmon mode.

In summary, we carried out analysis for the propagation of terahertz surface plasmon polaritons supported by the parallel plate configuration. The dispersion properties of the modes are examined for different widths of the air gaps as well as carrier densities of the plates. The dispersion relation saturates at higher frequency when carrier density of the plate is increased for all the values of air gaps. The
results indicate that one may design the waveguide with different parameters such as air gap and parallel plate properties to excite modes at specific frequencies. The above analysis could be significant in designing parallel plate THz plasmonic waveguides with materials such as semiconductors replacing conventional wave guiding techniques comprising metals.

Acknowledgements

The authors would like to acknowledge the funding from the project (Project No. PHY/P/GK/02) for the financial assistance.

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