Slepton NLG (Non-Linear Gauge) in GRACE/SUSY-loop

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Abstract

We have been developing a program package called GRACE/SUSY-loop which is for the automatic calculations of the MSSM amplitudes in one-loop order. The non-linear gauge (NLG) fixing conditions play the crucial role in the calculations in one-loop order which contain a large number of Feynman diagrams. We present the recent progress in GRACE/SUSY-loop which is obtained by extending the non-linear gauge formalism to the slepton sector.

1 Introduction

Supersymmetry (SUSY) between bosons and fermions at the unification-energy scale is one of the most promising hypotheses in the theory beyond the standard model (BSM), which is expected to resolve the remaining problems in the standard model (SM). The minimal supersymmetric extension of the SM (MSSM) is consistent with all the known high-precision experiments at a level comparable to the SM.

Since it is a broken symmetry at the electroweak-energy scale, the relic of SUSY is expected to remain as a rich spectrum of heavy SUSY particles, i.e. partners of usual matter fermions (leptons and quarks), gauge bosons and Higgs bosons, which are named sfermions (sleptons and squarks), gauginos and higgsinos, respectively. The quest of these new particles is one of the most important aims of the high-energy physics at present and future colliders of sub-TeV-energy or TeV-energy region.

In particular, experiments at the ILC offer high-precision determination of SUSY parameters via $e^-e^+$-annihilation processes. Since the theoretical predictions with the high accuracy comparable to that of experiments is required to extract important physical results from the data, we have to include at least one-loop contributions in perturbative calculations of amplitudes.

Recently, we have calculated the radiative corrections to production processes and decay processes of SUSY particles in the framework of the MSSM using GRACE/SUSY-loop \([1, 2, 3]\) which is a program package for automated computations of the MSSM in one-loop order. For the test of numerical results, we have used the non-linear gauge (NLG) formalism \([4, 5, 6, 7, 8, 9]\) applied to GRACE/SUSY-loop \([10]\).

In this paper, we show the recent progress in GRACE/SUSY-loop which is obtained by extending the non-linear gauge formalism to the slepton sector.
2 Features of GRACE/SUSY-loop

For many-body final states, each production process or decay process is described by a large number of Feynman diagrams even in tree-level order. There are still more Feynman diagrams in one-loop order even for two-body final states. For this reason, we have developed the GRACE system \[11\], which enables us to calculate amplitudes automatically.

A program package called GRACE/SUSY-loop is the version of the GRACE system for the calculation of the MSSM amplitudes in one-loop order, which includes the model files of the MSSM and can produce corresponding 2-point functions and counter terms. For the automatic calculation of the MSSM amplitudes in one-loop order, there exist other program packages independently developed by other groups, SloopS \[12\] and FeynArt/Calc \[13\].

As explained in \[1\], the renormalization scheme adopted for the electroweak (EW) interactions in GRACE/SUSY-loop is a variation of the on-mass-shell scheme \[14\] \[15\] \[16\] \[17\] \[18\], which is an MSSM extension of the scheme in the SM used in GRACE-loop \[11\]. There are some degrees of freedom in the renormalization conditions of the sfermion sector. We can choose different sets of residue conditions, decoupling conditions on the transition terms between the lighter and the heavier sfermions.

In GRACE/SUSY-loop, we use the technique of the NLG formalism \[4\] \[5\] \[6\] \[7\] \[8\] \[9\] in order to confirm the validity of calculations by imposing the NLG invariance on physical results. The NLG formalism is an extension of the linear \(R\) gauge. The gauge fixing lagrangian for the EW interactions in the NLG \[10\] \[12\] is given as follows:

\[
\mathcal{L}_{gf} = - \frac{1}{\xi_W} |F_W|^2 - \frac{1}{2\xi_Z} (F_Z)^2 - \frac{1}{2\xi_\gamma} (F_\gamma)^2, \\
F_W = (\partial_\mu \mp i\alpha A_\mu \pm ig_{W} \tilde{\beta} Z_\mu) W^{\mp \mu} \pm i\xi_W \frac{g}{2} (v + \tilde{\delta} H^0 + \tilde{\epsilon} h^0 \pm i\kappa G^0) G^\pm, \\
F_Z = \partial_\mu Z^\mu + \xi_Z \frac{g Z}{2} (v + \epsilon_H H^0 + \epsilon_h h^0) G^0, \\
F_\gamma = \partial_\mu A^\mu, 
\]

where \(v = \sqrt{v_1^2 + v_2^2}\), \(M_W = \frac{m_T}{2}\), \(M_Z = \frac{m_T}{2}\), \(h^0\) and \(H^0\) stands for the lighter and heavier CP even Higgs boson, respectively, \(G^\pm\) and \(G^0\) stands for the Goldstone boson which corresponds to gauge boson \(W^\pm\) and \(Z\), respectively. The gauge fixing lagrangian \[1\] contains seven independent NLG-parameters, \(\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}_H, \tilde{\delta}_h, \kappa, \epsilon_H, \epsilon_h\). The numerical tests are performed by varying these parameters.

3 Renormalization conditions in the slepton sector

The bare mass term in the slepton sector of MSSM lagrangian is given by

\[
\mathcal{L}_0^{mass} = - \begin{pmatrix} \tilde{\ell}_L \cr \tilde{\ell}_R \end{pmatrix}_0 \begin{pmatrix} m_{\tilde{\ell}_L}^2 & m_{\tilde{\ell}_{LR}}^2 \\ m_{\tilde{\ell}_{LR}}^2 & m_{\tilde{\ell}_R}^2 \end{pmatrix}_0 \begin{pmatrix} \tilde{\ell}_L \\ \tilde{\ell}_R \end{pmatrix}_0, \quad \ell = e, \mu, \tau, 
\]

where

\[
m_{\tilde{\ell}_L}^2 = \tilde{m}_{\ell_L}^2 + m_{\tilde{\ell}_L}^2 + M_{\tilde{\ell}_L}^2 \cos 2\beta (T_{3\ell} - Q_\ell \sin^2 \theta_W), \\
m_{\tilde{\ell}_R}^2 = \tilde{m}_{\ell_R}^2 + m_{\tilde{\ell}_R}^2 + M_{\tilde{\ell}_R}^2 \cos 2\beta Q_\ell \sin^2 \theta_W, \\
m_{\tilde{\ell}_{LR}}^2 = -m_\ell (A_\ell + \mu \tan \beta), 
\]
It contains three parameters $\tilde{m}_{\ell_L}, \tilde{m}_{\ell_R}$ and $A_\ell$. Diagonalizing the mass matrix, we determine the mixing angle $\theta_\ell$ and the mass eigenvalues $m_{\tilde{\ell}_1}, m_{\tilde{\ell}_2}$.

\[
\begin{pmatrix}
\cos \theta_\ell & \sin \theta_\ell \\
-\sin \theta_\ell & \cos \theta_\ell
\end{pmatrix}
\begin{pmatrix}
m_{\tilde{\ell}_L}^2 \\
m_{\tilde{\ell}_R}^2
\end{pmatrix}
= \begin{pmatrix}
m_{\tilde{\ell}_1}^2 \\
m_{\tilde{\ell}_2}^2
\end{pmatrix},
\]

while the mass of the $\tilde{\nu}$ is given by $(m_{\tilde{\nu}}^2)_0 = (\tilde{m}_{\tilde{\nu}_L}^2 + \frac{1}{2}M_0^2 \cos 2\beta)_0$. We assume the SU(2)$_L$ conditions on their left-handed soft SUSY-breaking mass terms, $\tilde{m}_{\tilde{\ell}_L}^2 = \tilde{m}_{\tilde{\nu}_L}^2, \tilde{m}_{\tilde{\nu}_L}^2 = \tilde{m}_{\tilde{\ell}_L}^2$ and $\tilde{m}_{\tilde{\tau}_L}^2 = \tilde{m}_{\tilde{\nu}_L}^2$, which lead to the following condition,

\[
m_{\tilde{\nu}}^2 = \cos^2 \theta_\ell m_{\tilde{\nu}_1}^2 + \sin^2 \theta_\ell m_{\tilde{\nu}_2}^2 - m_\ell^2 + M_0^2 \cos 2\beta.
\]

This relation is valid among the bare quantities as well as among the renormalized quantities.

In the slepton sector, there are three mass renormalization constants, five wavefunction renormalization constants, and one mixing angle renormalization constants for each generation. They are $\delta m_{\tilde{\ell}_1}, \delta m_{\tilde{\ell}_2}, \delta m_{\tilde{\nu}_i}, \delta Z_{\tilde{\ell}_i\tilde{\ell}_j}(i,j = 1,2), \delta Z_{\tilde{\nu}_i}, \delta \theta_\ell$. We introduce the wavefunction renormalization constants in the unmixed fields $\tilde{\ell}_L$ and $\tilde{\ell}_R$ for each charged slepton.

\[
\begin{pmatrix}
\tilde{\ell}_L \\
\tilde{\ell}_R
\end{pmatrix}
= \begin{pmatrix} Z_{\tilde{\ell}_1/2} & 0 \\
0 & Z_{\tilde{\ell}_R/2}
\end{pmatrix}
\begin{pmatrix}
\tilde{\ell}_L \\
\tilde{\ell}_R
\end{pmatrix}
= \begin{pmatrix}
\cos \theta_\ell & -\sin \theta_\ell \\
\sin \theta_\ell & \cos \theta_\ell
\end{pmatrix}
\begin{pmatrix}
\tilde{\ell}_1 \\
\tilde{\ell}_2
\end{pmatrix}.
\]

The first equation of (10) means, in particular,

\[
\begin{pmatrix}
\tilde{\ell}_L \\
\tilde{\ell}_R
\end{pmatrix}
= \begin{pmatrix}
\cos \theta_\ell & -\sin \theta_\ell \\
\sin \theta_\ell & \cos \theta_\ell
\end{pmatrix}
\begin{pmatrix}
\tilde{\ell}_1 \\
\tilde{\ell}_2
\end{pmatrix}.
\]

and inserting this expression in (9), we find

\[
\begin{pmatrix}
\tilde{\ell}_L \\
\tilde{\ell}_R
\end{pmatrix}
= \begin{pmatrix} Z_{\tilde{\ell}_1/2} & 0 \\
0 & Z_{\tilde{\ell}_R/2}
\end{pmatrix}
\begin{pmatrix}
\cos \theta_\ell & -\sin \theta_\ell \\
\sin \theta_\ell & \cos \theta_\ell
\end{pmatrix}
\begin{pmatrix}
\tilde{\ell}_1 \\
\tilde{\ell}_2
\end{pmatrix}.
\]

Therefore, four wavefunction renormalization constants, $Z_{\tilde{\ell}_1/2}, Z_{\tilde{\ell}_2/2}, Z_{\tilde{\nu}_1/2}, Z_{\tilde{\nu}_2/2}$ can be expressed in terms of three independent renormalization constants, $Z_{\tilde{\ell}_1/2}, Z_{\tilde{\ell}_R/2}$ and $\delta \theta_\ell$, for each charged slepton.

We have adopted the following renormalization conditions in our paper on the chargino pair-production and decays [11].

- the on mass-shell conditions for all the three sleptons in each generation
- the residue conditions for all the three sleptons in each generation
- the decoupling conditions for the on-shell $\tilde{\ell}_i$ with $\tilde{\ell}_j, (j \neq i, \tilde{\ell} = \tilde{\ell}, \tilde{\ell}^\prime, \tilde{\tau})$
- SU(2) relation for $\delta \theta_\ell$

\[^1\text{We use the convention, } m_{\tilde{\ell}_1} < m_{\tilde{\ell}_2}\]
which lead to the following expressions for the renormalization constants.

\[
\begin{align*}
\delta m^2_{\tilde{\ell}} &= -\text{Re} \Sigma^R(m^2_{\tilde{\ell}}), \quad \tilde{\ell} = \tilde{e}_1, \tilde{e}_2, \tilde{\nu}_e, \tilde{\mu}_1, \tilde{\mu}_2, \tilde{\nu}_\mu, \tilde{\tau}_1, \tilde{\tau}_2, \tilde{\nu}_\tau, \\
\delta Z_{\tilde{\ell}\tilde{\ell}} &= \Sigma'(m^2_{\tilde{\ell}}), \quad \tilde{\ell} = \tilde{e}_1, \tilde{e}_2, \tilde{\mu}_1, \tilde{\mu}_2, \tilde{\tau}_1, \tilde{\tau}_2, \\
\delta Z_{\tilde{\nu}_\ell} &= \Sigma'(m^2_{\tilde{\nu}_\ell}), \quad \ell = e, \mu, \tau, \\
\frac{1}{2} \delta Z_{\tilde{\ell}_i \tilde{\ell}_j} &= -\frac{\Sigma_{\tilde{\ell}_i \tilde{\ell}_j}(m^2_{\tilde{\ell}_i})}{m^2_{\tilde{\ell}_i} - m^2_{\tilde{\ell}_j}}, \quad i \neq j, \quad \tilde{\ell} = \tilde{\ell}, \tilde{\mu}, \tilde{\tau}, \\
\delta \theta_\ell &= \frac{\delta m_{\tilde{\nu}_\ell} - \delta (M^2 - m^2_{\tilde{\ell}}) - \cos^2 \theta_\ell \delta m^2_{\tilde{\ell}_i} - \sin^2 \theta_\ell \delta m^2_{\tilde{\ell}_j}}{\sin 2 \theta_\ell (m^2_{\tilde{\ell}_j} - m^2_{\tilde{\ell}_i})}, \quad \ell = e, \mu, \tau.
\end{align*}
\]

4 Extension of non-linear gauge formalism

We can extend the NLG functions \([2]\) and \([3]\) by including bilinear forms of sleptons with new NLG parameters \(\tilde{c}\) as follows:

\[
\begin{align*}
F_{W^+} &= (\partial_\mu + i e \tilde{A}_\mu + ig c \tilde{W} Z_\mu) W^{+\mu} + i \xi_W \frac{g}{2} (v + \tilde{\delta} H^0 + \tilde{\delta} h^0 + i \tilde{\kappa} G^0) G^+ \\
&+ i \xi_W g \left[ \sum_{i=1,2} \left\{ \tilde{c}^e_i (\tilde{e}_i^* \tilde{\nu}_e) + \tilde{c}^\mu_i (\tilde{\mu}_i^* \tilde{\nu}_\mu) + \tilde{c}^\tau_i (\tilde{\tau}_i^* \tilde{\nu}_\tau) \right\} \right], \\
F_Z &= \partial_\mu Z^\mu + \xi_Z \frac{g Z}{2} (v + \tilde{\delta} H^0 + \tilde{\delta} h^0) G^0 \\
&+ \xi_Z g Z \left[ \tilde{c}^{e\nu} (\tilde{e}_e^* \tilde{\nu}_e) + \tilde{c}^{\mu\nu} (\tilde{\mu}_\mu^* \tilde{\nu}_\mu) + \tilde{c}^{\nu\tau} (\tilde{\nu}_\mu^* \tilde{\tau}_\mu) \\
&+ \sum_{i,j=1,2} \left\{ \tilde{c}^{ij} (\tilde{e}_i^* \tilde{e}_j) + \tilde{c}^{i\mu} (\tilde{\mu}_i^* \tilde{\mu}_j) + \tilde{c}^{ij} (\tilde{\tau}_i^* \tilde{\tau}_j) \right\} \right]
\end{align*}
\]

while \(F_{W^+}\) is hermitian conjugate to \(F_{W^-}\).

In this paper, we focus on the NLG parameter \(\tilde{c}^1\) in the NLG function \(F_{W^\pm}\), and set \(\xi_W = 1\) in order to avoid the instability in the one-loop calculations. The Feynman rules of vertices in the linear gauge are modified by introducing the NLG parameter \(\tilde{c}^1\) as shown in Figure 1.

The gauge invariance of NLG in the one-loop calculations is guaranteed by the BRST transformation, which leads to the introduction of the Faddeev-Popov ghosts, \(\tilde{\omega}_\pm, \tilde{\omega}_Z\) and \(\tilde{\omega}_\gamma\), and anti-ghosts, \(\tilde{\bar{\omega}}_\pm, \tilde{\bar{\omega}}_Z\) and \(\tilde{\bar{\omega}}_\gamma\). The corresponding ghost lagrangian to the NLG parameter \(\tilde{c}^1\) is given as follows:

\[
\begin{align*}
\mathcal{L}_{\text{ghost}} &= -\xi_W g \tilde{c}^{1 \tau} \tilde{\omega} + \left( \frac{i}{2} \right) \left[ \sqrt{2} g \cos \theta_\tau \tilde{\omega} - \cos \theta_\tau g Z \omega_Z (\cos \theta_\tau \tilde{\tau}_1 - \sin \theta_\tau \tilde{\tau}_2) \\
&- 2 \epsilon_\omega_\gamma (\cos \theta_\tau \tilde{\tau}_1 - \sin \theta_\tau \tilde{\tau}_2) \right] \tilde{\nu}_\tau \\
&- i \xi_W g \tilde{c}^{1 \tilde{\tau}} \tilde{\omega} + \left( \frac{i}{2} \right) (2 g Z \sin \theta_\tau \omega_Z - 2 \epsilon_\omega_\gamma) (\tilde{\tau}_1) \tilde{\nu}_\tau \\
&- i \xi_W g \tilde{c}^{1 \tau} \tilde{\omega} + \left( \frac{i}{2} \right) \left[ \sqrt{2} g \omega_\gamma (\cos \theta_\tau \tilde{\tau}_1 - \sin \theta_\tau \tilde{\tau}_2) + g_\omega Z \tilde{\nu}_\gamma \right] + (h.c.).
\end{align*}
\]

5 Numerical tests

We have calculated the slepton decay widths in one-loop order including the NLG gauge fixing functions \([13]\). Here we present the results of the process, \(\tilde{\tau}_2 \rightarrow \tilde{\tau}_1 + h^0\). Typical
\[ W^+ - \tilde{\nu}_\tau - \tilde{\tau}_1 \]
\[ G^+ - \tilde{\nu}_\tau - \tilde{\tau}_1 \]
\[ \tilde{\nu}_\tau - \tilde{\tau}_1 - \tilde{\nu}_\tau - \tilde{\tau}_1 \]

Figure 1: Feynman rules including the NLG couplings

Feynman diagrams of this process concerned in the NLG coupling \( \tilde{c}_1 \) is shown in Figure 2. We have investigated coefficients of the zeroth power to fourth power of \( \tilde{c}_1 \) in the UV part and UV finite part. Table 1 shows numerical results, in which we have used SUSY parameters as \( M_2 = 400 \text{ GeV}, \mu = -100 \text{ GeV}, \tan \beta = 30, m_{\tilde{\tau}_1} = 495.84 \text{ GeV}, m_{\tilde{\tau}_2} = 608.23 \text{ GeV} \) and \( \theta_\tau = 0.74\pi \). Then we have confirmed the NLG invariance of vertices for the two-body decays in one-loop order.

We have also calculated cross sections of the scattering processes systematically to test the NLG invariance of up to four-point vertices in one-loop order. Here we present the results of the process, \( \tilde{\tau}_1 + \tilde{\tau}_1^* \to \tilde{\tau}_1 + \tilde{\tau}_1^* \). Typical Feynman diagrams of this process concerned in the NLG coupling \( \tilde{c}_1 \) is shown in Figure 3. Table 2 shows numerical results.

6 Summary

We have developed the program package GRACE/SUSY-loop for the MSSM amplitudes in one-loop order, and extended the non-linear gauge formalism applied to GRACE/SUSY-loop by introducing the gauge fixing terms of bilinear forms of sleptons. Then we have confirmed the NLG invariance of the MSSM amplitudes in one-loop order for decay processes and scattering processes using GRACE/SUSY-loop.

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Figure 2: Typical Feynman diagrams of $\tilde{\tau}_2 \rightarrow \tilde{\tau}_1 + h^0$

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Figure 3: Typical Feynman diagrams of \( \tilde{\tau}_1 + \tilde{\tau}_1^* \rightarrow \tilde{\tau}_1 + \tilde{\tau}_1^* \)
Table 1: Test for NLG invariance of $\tilde{\tau}_2 \rightarrow \tilde{\tau}_1 + h^0$

| graph | UV part | Finite part |
|-------|---------|-------------|
| $(\tilde{c}_1^0)$ | $(\tilde{c}_1^1)$ | $(\tilde{c}_1^2)$ | $(\tilde{c}_1^3)$ | $(\tilde{c}_1^4)$ |
| Virtual | | | | |
| 1 | $-2.864048E+02$ | $8.107528E+02$ | $-5.728097E+02$ | $-6.902655E+00$ | $6.31073E+00$ |
| 2 | $2.640148E+02$ | $-3.733722E+02$ | $0.000000E+00$ | $4.340137E+00$ | $0.000000E+00$ |
| 3 | $1.28103E+03$ | $-2.015626E+03$ | $2.864048E+02$ | $2.219785E+01$ | $-3.032222E+00$ |
| 4 | $-4.259420E+02$ | $3.478145E+02$ | $0.000000E+00$ | $-4.16543E+00$ | $0.000000E+00$ |
| 6 | $0.000000E+00$ | $0.000000E+00$ | $0.000000E+00$ | $2.651421E-01$ | $0.000000E+00$ |
| 7 | $-1.470710E+03$ | $1.877379E+03$ | $2.864048E+02$ | $-2.268075E+00$ | $-3.192840E+00$ |
| 8 | $0.000000E+00$ | $0.000000E+00$ | $0.000000E+00$ | $-3.626202E-01$ | $0.000000E+00$ |
| 9 | $0.000000E+00$ | $0.000000E+00$ | $0.000000E+00$ | $-7.996177E-02$ | $-8.572352E-02$ |
| 10 | $1.839419E+02$ | $-5.202662E+02$ | $3.67837E+02$ | $5.671871E+00$ | $-4.659912E+00$ |
| 50 | $0.000000E+00$ | $0.000000E+00$ | $0.000000E+00$ | $1.638727E+00$ | $1.756807E+02$ |
| 82 | $1.467649E+03$ | $2.020973E+02$ | $0.000000E+00$ | $-2.152933E+00$ | $0.000000E+00$ |
| 94 | $-9.034220E+01$ | $0.000000E+00$ | $-3.67837E+02$ | $0.000000E+00$ | $4.642344E+00$ |
| 134 | $0.000000E+00$ | $-2.025188E+02$ | $0.000000E+00$ | $1.726551E+00$ | $0.000000E+00$ |
| 135 | $0.000000E+00$ | $-2.025188E+02$ | $0.000000E+00$ | $1.726551E+00$ | $0.000000E+00$ |
| 138 | $-1.279135E+03$ | $2.020973E+02$ | $0.000000E+00$ | $-2.266272E+00$ | $0.000000E+00$ |
| Counter Term | | | | |
| 145 | $5.33066E+02$ | $-1.25523E+02$ | $0.000000E+00$ | $2.666161E+00$ | $0.000000E+00$ |
| Total | $-6.03204E-20$ | $5.714153E-27$ | $1.019217E-29$ | $-3.995728E-23$ | $-2.514610E-31$ |

Table 2: Test for NLG invariance of $\tilde{\tau}_1 + \tilde{\tau}_1^* \rightarrow \tilde{\tau}_1 + \tilde{\tau}_1^*$