Deterministic fatigue crack-growth simulations for a railway axle by Dual Boundary Element Method

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Abstract The maintenance of railway axles requires an optimization of their inspection intervals in order to reduce costs. Such components can endure more than 30 years of service life and are subjected to scheduled Non Destructive Testing (NDT) inspections. It is therefore important to assess the influence of the severe in-service loads on the propagation of cracks in the axles. This manuscript reports deterministic fracture calculations, for the simulation of cracks propagating through a railway axle, by the Dual Boundary Element Method. Once Stress Intensity Factors along the crack front are obtained, the residual fatigue life estimates were calculated through a user made spreadsheet, taking into account the complex loading conditions of the axles. The presented results support the application of the crack propagation simulation for the determination of axle NDT intervals under different loading conditions.

1. Introduction
Fatigue of railway axles has been the topic that drove early fatigue studies by Wöhler in the nineteenth century. Nowadays, the problem of the fatigue strength assessment of such components is still of great importance for the railway industry, as their failures may cause serious accidents [1]. During their lifetime, railway axles are regularly inspected by means of Non Destructive Testing (NDT) inspections, generally based on visual observation, Ultrasonic Testing (UT) or Magnetic Particle Inspection (MPI). The interval between these inspections can be defined as the mileage that can be safely run between two consecutive NDT controls. This interval can be mainly considered as a function of the minimum detectable defect size and of the crack-growth rates for the material of interest. The present document is concentrated on the numerical simulation of the fracture phenomenon in a railway axle. The main objective is to predict the fatigue life of the axle in presence of an edge crack nucleated on its outer surface. These predictions are essential for these kind of mechanical components, also to support the determination of their inspection intervals.

Nowadays, several methodologies have been demonstrated as capable to simulate fracture phenomena, among them the Finite Element Method (FEM) [2-4], the Dual Boundary Element Method (DBEM) [5-9] and a combined use of the two approaches [10-13]. The current work provides a numerical framework by means of which deterministic life predictions of the axle in presence of defects were obtained. The DBEM code BEASY [14] was here used to tackle the fracture phenomenon numerically, and some of the related results were compared with analytical evaluations for validation.
The fatigue lives were predicted with an in-house made spreadsheet which, fed with the DBEM outcomes, allowed to take into account of the complex load spectra commonly considered for these railway applications. Even though no dedicated experimental data were available for the specific train under investigation, three different loading spectra were arranged by using literature data. Particularly, the load spectra were derived from experimental data as measured on board for similar trains and slightly modified to be compliant with the current investigation.

2. DBEM simulations

Numerical analyses were performed in a DBEM environment by considering a simplified modelling of the axle under investigation. The models consisted of a straight bore axle having diameter equals to 180 mm and length equals to 720 mm, undergoing a pure bending condition with maximum axial stress $\sigma_{max}$. Kinematic coupling conditions were considered for the two ending cross-sections in order to comply with the Euler-Bernoulli beam theory assumptions. An alternating bending condition with stress ratio $R = \sigma_{min}/\sigma_{max} = -1$ was simulated, as also made in similar works [15, 16]. The only loading condition providing the maximum bending stress was explicitly computed, thus getting the corresponding $K_{max}$ values, whereas, as per a linear elastic analysis, $K_{min} = R K_{max} = -K_{max}$. The material of the axle was the widely used high strength steel 30NiCrMoV12, according to UNI 6787 [17], with Young’s modulus equals to 200 GPa and Poisson’ ratio equal to 0.3.

![Figure 1. DBEM model for the initial crack (red dot) with highlight of the local remeshing surface in blue.](image)

The uncracked DBEM model is shown in figure 1. It comprised nearly 840 4-noded quadrilateral surface elements with 3 translational dof’s for the uncracked model, and this number increased along with the advancing of the crack, reaching nearly 1800 elements for the last propagation steps. A step-by-step local remeshing procedure was preferred to reduce the computational burden and the remeshing surface (in blue in figure 1) increased in size with the crack extension. 8-noded quadrilateral and 6-noded triangular quadratic surface elements with 3 translational dof’s were used to fill the remeshing surface, while a “structured” line of elements was guaranteed along the crack front. Discontinuous elements were used to mesh both crack faces, where dual equations of displacements and tractions were alternatively used for collocation [18]. Rings of internal points were properly positioned along crack fronts to calculate the $J$-integral and get in turn the $K$ values [19-20].
The fracture simulation was setup as an incremental approach that, starting from a crack having a given user defined size and shape, allowed to simulate the crack-growth by consecutive discrete steps of propagation until a critical crack size was reached. A semi-circular crack with radius equal to 2 mm was assumed as initially existing in the middle transversal plane of the cylinder (see figure 1 for highlight of the initiation point). Such initial crack advanced step-wise through the cross-section of the axle in nearly 65 discrete incremental steps. The average crack advance per step was equal to 1 mm for the first 10 steps and equal to 2 mm for the remaining steps up to failure. Each i-th step of crack-growth consisted in the following sub steps:

a) crack insertion - the i-th crack is inserted in the model, involving a local rearrangement of the surrounding mesh in order to accommodate the i-th crack geometry. Such localized mesh rearrangement was adopted to keep the coarse original mesh far from the crack, thus reducing runtimes.

b) model solving - the i-th cracked model is solved and $K$ values along i-th crack front are calculated by using the J-integral technique.

c) crack extension - the (i+1)-th crack front is predicted by considering the i-th $K$ values and the model geometry. Kinking of the crack was calculated by the Minimum Strain Energy Density (MSED) criterion [21-22], even if a pure Mode I crack-growth was expected.

3. Fatigue life prediction procedure

Even if the DBEM code was capable of handling different load spectra, it was preferred to compute the life predictions of the axle with an in-house made spreadsheet. $K$ values and the corresponding crack depths $a$ were then imported in the spreadsheet, in which the following contributes were implemented: 1) a NASGRO crack-growth formulation, 2) three different load spectra, 3) a procedure to estimate the residual mileage of the train. All the details of this procedure are provided in the followings.

3.1. NASGRO equation calibration

A NASGRO formulation was selected since it allowed to take into account of: 1) the three distinct parts of the crack-growth diagram (near-threshold, Paris’ range and critical zone); 2) the crack closure effects according to Newman definition [23]; 3) the stress ratio influence on crack-growth rates.

Different features of the NASGRO formulations were initially documented in [24] and continuously developed along the years. The main equation to calculate crack-growth rates is given by:

$$\frac{da}{dN} = C \left[ \left( \frac{1 - f}{1 - R} \right) \Delta K \right]^n \left( \frac{1 - \Delta K_{th}/\Delta K}{1 - K_{max}/K_c} \right)^q,$$

(1)

where $N$ is the number of fatigue cycles, $a$ is the crack depth and $da/dN$ represents the crack-growth rate. $\Delta K$ is the stress intensity factor range equals to $K_{max} - K_{min}$ and can be considered as the key parameter in Linear Elastic Fracture Mechanics (LEFM).

$C, n, p$ and $q$ are empirical constants required by NASGRO formulation to fit the crack-growth rate experimental data. $K_c$ is the critical $K$ value for which final rupture of component is expected. These data were available from full-scale axles tests data in [16].

The crack opening function, $f$, allows to consider that the $\Delta K$ contributes only partially to the crack-growth. The phenomenon related to crack closure was first discovered in [25-26] leading to a widely accepted definition [23] as:

$$f = \frac{K_{op}}{K_{max}} = \begin{cases} \max \left( R, A_0 + A_1 R + A_2 R^2 + A_3 R^3 \right) & , \ R \geq 0 \\ A_0 + A_1 R & , \ -2 \leq R \leq 0 \end{cases},$$

(2)

$$a = \frac{-\Delta K}{2 \pi}$$

$$\Delta K = \left( \frac{K_{op}}{K_{max}} \right) \left( \frac{a}{R} \right)^{1/2}$$

$$\frac{1 - f}{1 - R} \Delta K$$

$$\left( \frac{1 - \Delta K_{th}/\Delta K}{1 - K_{max}/K_c} \right)^q$$
where the coefficients \( A_i \) were given by:

\[
\begin{align*}
A_0 &= \left(0.825 - 0.34\alpha + 0.05\alpha^2\right)\left[\cos\left(\frac{\pi}{2}\frac{S_{\text{max}}}{\sigma_0}\right)\right]^{1/\alpha} \\
A_1 &= (0.415 - 0.071\alpha)\frac{S_{\text{max}}}{\sigma_0} \\
A_2 &= (1 - A_0 - A_1 - A_3) \\
A_3 &= 2A_0 + A_1 - 1
\end{align*}
\]

(3)

\( \alpha \) is a plane stress/strain constraint factor that, for materials such as the high-strength steel here considered, presents relatively high values (2.5 or higher); the considered value here was equal to 2.5. \( \frac{S_{\text{max}}}{\sigma_0} \) is the ratio of the maximum applied stress \( S_{\text{max}} \) to the flow stress \( \sigma_0 \), the latter generally defined as an average between yield stress \( \sigma_y \) and ultimate tensile stress \( \sigma_{UTS} \). Even though such ratio varies through the propagation, especially when considering load spectrum effects, for sake of simplicity it was assumed to be as constant for all the made calculations.

The threshold stress intensity factor range as a function of \( R, \Delta K_{th} \), was given by:

\[
\Delta K_{th} = \Delta K_{th,0} \frac{\sqrt{\alpha/\left(\alpha + a_0\right)}}{\left(1 - f\right)\left(1 - R\right)}
\]

(4)

where \( \Delta K_{th,0} \) is the threshold \( \Delta K \) for \( R = 0 \), \( C_{th} \) is a fitting parameter with different values for positive or negative \( R \) values, whereas \( a_0 \) is the El-Haddad’ parameter according to [27]. \( \Delta K_{th,0} \) represents the value of \( \Delta K \) below which no crack-growth would occur for \( R = 0 \). These values were available from [16]. Finally, the El-Haddad’s parameter \( a_0 \) allows to take into account of the crack-growth for short cracks; this is generally always assumed to be equals to 0.0381 mm. For all the load cases considered in the followings, \( \Delta K_{th} \) was exceeded already for the initial crack depth of 2 mm.

The data adopted for the NASGRO formulation are listed in table 1.

| Table 1. NASGRO formulation parameters. |
|------------------------------------------|
| \( \sigma_{UTS} \) [MPa] | \( \sigma_y \) [MPa] | \( K_C \) [MPa/m] | \( C \) [MPa/m\(^n/2\)] | \( n \) [-] | \( p \) [-] |
| 980 | 880 | 120 | 1.15e-10 | 2.41 | 0.65 |
| q [-] | \( a_0 \) [m] | \( C_{th} \) [-] | \( C_{th} \) [-] | \( \Delta K_{th,0} \) [MPa/m] | \( \alpha \) [-] |
| 0.001 | 3.81e-5 | 1.4 | -0.07 | 6.13 | 2.5 |

3.2. NDT inspection methods

Both size (2 mm radius) and shape (semi-circular) of the initial crack were arbitrarily assumed. However, larger cracks with more realistic shapes, as numerically predicted starting from the aforementioned initial crack, were considered as effective starting cracks to be used for life predictions, namely the first part of crack propagation is only needed to generate a realistic crack shape from which to trigger the cycle counting. Particularly, such starting cracks were selected by considering the NDT inspection method adopted to detect them. Figure 2 reports the Probability Of Detection (POD) curve available in [28] for different NDT techniques. “Far-End Scan” UT POD was considered in the current investigation since presented the lowest POD among the curves, thus was chosen in a conservative perspective with respect to “Near-End Scan” and “Magnetic Particle Inspections” (MPI). The two realistic crack sizes here considered were equals to 4.8 mm and 13.9 mm, selected in such a way to consider a 50% and 90% of probability to be detected respectively.
3.3. Load spectra

No dedicated experimental data were available in literature for the specific train under investigation. Nevertheless, on the basis of load spectra obtained from on-board experimental measurements on different trains, available in [15, 16], three simple block loading cycles were arranged (figure 3, table 2) for the current investigation. In particular, all the load spectra were proportionally scaled to correspond to a total mileage of 2000 km on railway lines, considering an average railway wheel diameter equal to 880 mm. Load spectra in figure 3 consisted of a series of stress amplitude $\sigma_j$, each of them associated with a given number of cycles $n_j$, the latter defining the amount of repetitions that the axle undergoes at each given $\sigma_j$ along its operation. The three load cases considered three different levels of the lowest flexural stress, 50, 42.5 and 35 MPa, with identical stress ratios. Based on the Linear Elastic Fracture Mechanics (LEFM) assumption for which $K \propto \sigma \sqrt{\pi a}$, $K$ values related to each distinct stress value can be calculated without any further simulation once that reference $K$ are computed for a given stress level.
Figure 3. Different stress spectra for the railway axle.

Table 2. Number of cycles and related stress levels for the three loading spectra.

| Number of cycles $n_j$ [-] | 30  | 100 | 1300 | 6000 | 26000 | 140000 | 550000 |
|----------------------------|-----|-----|------|------|-------|--------|--------|
| Stress level $\sigma_j$ for LS1 [MPa] | 159 | 150 | 130 | 105 | 85 | 62 | 50 |
| Stress level $\sigma_j$ for LS2 [MPa] | 135.15 | 127.5 | 110.5 | 89.25 | 72.25 | 52.7 | 42.5 |
| Stress level $\sigma_j$ for LS3 [MPa] | 111.3 | 105 | 91 | 73.5 | 59.5 | 43.4 | 35 |

3.4. Life prediction procedure

The load spectra were inserted in a user made spreadsheet in which an algorithm to perform fatigue life predictions was implemented. For each $(n_j, \sigma_j)$, the calibrated NASGRO equation (equation 1) was rearranged in the following form to calculate the corresponding $\Delta a$:

$$\Delta a = \sum_j n_j \cdot C \left[ \left( \frac{1-f}{1-R} \right) \Delta K_j \right]^\alpha \left( \frac{1-\Delta K_{th}/\Delta K_j}{1-K_{max,j}/K_c} \right)^\beta.$$  (5)

It is worth noting that $\Delta K_j$ were calculated based on the abovementioned LEFM assumption for each distinct $\sigma_j$ value of each $j$-th load block of the considered spectrum. $\Delta a$ represents the crack advance produced after a single load spectrum application, thus corresponding to a mileage of 2000 km. Such $\Delta a$ was used to calculate the amount of repetitions of the load spectrum that can be run between two numerical crack advances $a_i$ and $a_{i+1}$. Namely, equation 6 allowed to calculate how many load spectra are applied for each given numerical crack advance $\Delta a_{i,i+1} = a_{i+1} - a_i$:

$$\Delta N_{i,i+1} = \frac{\Delta a_{i,i+1}}{\Delta a} \cdot N_{LS}.$$  (6)
In final, summing up the $\Delta N_{i,j+1}$ from the considered initial step and up to failure, the total $\Delta N$ representative of the predicted fatigue life was obtained, in turn obtaining the corresponding residual mileage.

4. Results

The crack-growth simulation started assuming an initial semi-circular crack with radius equals to 2 mm positioned in the middle transversal cross section of the axle (figure 1). The mode I SIFs ($K_I$) were the only drivers of crack-growth since $K_{II}$ and $K_{III}$ were null. This was expected in correspondence of a pure bending load, with a perfectly in-plane crack-growth (figure 4). In figure 5, the $K_I$ values, calculated by DBEM simulations and analytical formulation (as available in [22]), in correspondence of the mid-side node of crack front (deepest point), were plotted vs. crack depth; the results showed a very good agreement.

![Figure 4](image)

**Figure 4.** Cross sections of DBEM models at various steps along the crack-growth simulation.

![Figure 5](image)

**Figure 5.** $K_I/\sigma_{nom}$ ratio for mid-side point along crack front (crack depth point) calculated by DBEM and by an analytical formulation.

Fatigue life predictions (in terms of residual mileages) were computed using SIFs corresponding to the mid-side nodes of crack fronts by means of the NASGRO formulation above described (figure 6). Equivalent results would have been obtained if considering different nodes along crack front. The residual mileage of the axle was predicted by means of the $K$ values calculated numerically (figure 5),
by considering the two abovementioned initial crack depths (i.e., 4.8 mm and 13.9 mm) and also for each of the three load spectra (figure 3). For the initial part of crack-growth, and especially for the smaller initial crack, $\Delta K$ were higher than $\Delta K_{th}$ only for a part of the load spectrum. Namely, the lowest stress amplitudes corresponding to $\Delta K$ lower than $\Delta K_{th}$ did not give a contribute to crack-growth for small crack depths. This aspect was taken into account in the calculations.

Crack-growth rates (expressed in residual mileage) considering the three load spectra and two different initial cracks are shown in figure 7, whereas the total residual mileages at rupture were reported in table 3.

![Figure 6](image1.png)

**Figure 6.** Fitting of NASGROv3 equation and corresponding full-scale axle 30NiCrMoV12 propagation data.

![Figure 7](image2.png)

**Figure 7.** Comparison of DBEM results in terms of residual mileage for the three load spectra and an initial crack depth of 4.8 mm and 13.9 mm (load spectra definition in figure 3; table 2).
Table 3. DBEM residual mileages for the three load cases.

| Load spectrum [-] | LS1     | LS2     | LS3     |
|-------------------|---------|---------|---------|
| Residual mileage [km] for initial crack of 4.75 mm | 91.097  | 23.693  | 256.345 |
| Residual mileage [km] for initial crack of 13.9 mm  | 16.495  | 8.209   | 35.463  |

5. Conclusions

An algorithm to estimate fatigue life of railway axles in presence of a defect was built up. To this aim, a set of fatigue crack propagation material data was available in literature for full-scale axles made of 30NiCrMoV12 steel. Such data were used to calibrate a NASGRO formulation to predict the residual mileage of the axle considering different sizes of defects. A simplified DBEM model of the axle was used for such a purpose and the results compared with analytical data showing a very good agreement. The residual life predictions were performed for three realistic load spectra in order to acquire a sensibility on results given by the loading conditions of the axle.

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