A hierarchical process for optimizing bus stop distribution

Zhengdong Huang*

School of Urban Design, Wuhan University, Wuhan, China

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Transit stop spacing is an important indicator in deploying public transit services. This paper presents a hierarchical process for optimizing distribution of bus stops in the context of multi-modal transit development in large cities. Firstly, connection stops are generated manually to connect with other transit facilities. Secondly, key stops are identified based on a coverage model that minimizing total distance, using node centrality value as weight. Thirdly, ordinary bus stops are optimized with coverage model that maximizing potential transit demand. The analytical process is based on a combination of raster and vector data models in Geographical information system, which allows for flexible computing and effective evaluation.

Keywords: bus stop; optimization; hierarchical; coverage model; Flowmap

Introduction

Public transit has been advocated for facilitating mobility and mitigating environmental impacts of transport in large cities. Transit stop spacing is an important indicator in deploying public transit services. Large cities typically have a more complicated land use structure and development density, and in many cases they provide multi-modal transit service, such as for rail, bus rapid transit (BRT), and bus. In fast growing cities, public transit system needs to be structured dynamically to serve changing transit demand. On the one hand, in newly developed areas, transit service has to be planned to connect with the current system. On the other hand, economic growth generates new and increasing travel demand within cities, which requires the provision of more efficient and integrated modal transit systems.

However, irrational distribution of bus stops leads to a low public bus service quality. For example, redundant distribution of bus stops within short distances in most central areas increase unnecessary bus stopping and passenger waiting time (Murray, 2001). Furthermore, bus stop inadequacy often exists in urban outskirts. Such dispersed pattern results in a low walking accessibility (Murray, 2003), satisfying less public transport demands and causing social inequity as well. The performance of a transit system can be significantly improved if the spacing of bus stops is optimized.

Typically, coverage models and their variants have been applied to optimize the distribution of transit stops. However, these methods put most emphasis on spatial distribution, while less attention is given to network structure and actual transit demand in prioritizing distribution of transit stops. More importantly, the study of stop optimization in a multi-modal transit environment is seriously inadequate.
This paper presents a hierarchical process for optimizing the distribution of bus stops in the context of multi-modal transit development in large cities. Firstly, reviews are made on stop spacing and location optimization, as well as measurement of node centrality in a road network. Secondly, a hierarchical process for bus stop optimization is introduced. Then, the process is evaluated with a case study of Wuhan city in China. Last, factors related to the optimizing process are discussed, and conclusions are given.

**Literature review**

**Coverage model for optimization stop locations**

Stop spacing is important for single bus routes. As each stopping consumes time, for a bus route with a given length, more stops implicate a longer stopping time. On the other hand, fewer stops along a route implies longer walking distance for passengers, which might lead to lower bus patronage. In order to keep the operational efficiency, there should be a balance between bus speed and stop spacing. In addition, spacing of bus stops has close linkage with transit trip demand distribution. Through mathematical models, stop spacing along a given bus route may be optimized based on varying transit demand along the route (Wirasinghe & Ghoneim, 1981).

In addition to spacing stops along single routes, stop coverage calculations allow for assessing the whole bus system in a city. The proportion of area or population covered by bus service is evaluated by stop coverage, i.e. buffer areas around bus stops. For multimodal transit system, transit stops may have different service extent, e.g. travellers might be willing to walk longer distance to take a subway or a shuttle bus. Redundancy exists in stop coverage when two stops are too close with each other. The standard of service distance in central area might be different from other areas of a city. The number of bus stops might be optimized with computer simulation method (Alterkawi, 2006).

Murray, Tong, and Kim (2010) have summarized three basic types of deterministic coverage location models: (1) the model of location set covering problem (LSCP) that requires completely coverage of all demand with the minimum number of facilities; (2) the model of maximal covering location problem (MCLP) that allows covering as much demand as possible using a limited number of facilities; and (3) those that increase the likelihood of facility availability through the provision of backup coverage by lower-level facilities. In addition, there is multi-level location set covering problem (ML-LSCP), in which facilities need to cover demand points a number of times while demand is also changing (Church & Gerrard, 2003).

LSCP provides an integer mathematical model for optimizing locations of facilities, such as emergency service facilities (Toregas, Swain, ReVelle, & Bergman, 1971). Owing to its ability of measuring coverage efficiency, LSCP is typically adapted to optimizing location of bus stops (Murray, 2001). A hybrid set cover problem has been proposed for supporting the analysis of both transit access and accessibility in existing and expanded areas (Matisziw, Murray, & Kim, 2006; Murray, 2003).

**Node centrality in road network**

In road network, nodes serve as candidate stops and demand points in coverage model for stop optimization. The importance of each node may be measured by its topological centrality in the network. Different types of centrality have been discussed, such as degree centrality, betweenness centrality, closeness centrality, and eigenvector centrality.
Among these measures, the betweenness of a node indicates the number of times that a node appears in the shortest path of any pair of other nodes (Freeman, 1978). The centrality can also be measured with node status. The status value of a node is the total distance from the node to all other nodes in the graph, in which all distances between nodes are computed with shortest path (Buckley & Harary, 1990).

The integration value in space syntax can be applied to indicate the centrality of key points in a street network (Jiang & Claramunt, 2002). The graph of street network is composed of selected points, in which each point can be seen from its neighbor points. This approach is adapted from the traditional approach of axial line and axial map in space syntax (Hillier & Hanson, 1984). The formed network is a non-value graph in which each connectivity of adjacent points is assigned a value of 1.

Another approach to identifying the centrality of network nodes is based on dynamic simulation, such as random walk (RW). A RW is a mathematical formalization of a trajectory that consists of taking successive random steps, which was first introduced by Pearson (1905). The movement might be on a line, a graph, a regular lattice, or a 3-D space. Factors in a RW include the choice of the starting point, the length of a step, the distance of the walk, the time of departure, the total walking time, and the choice of direction in the next step. All these factors are inherently stochastic. A measure of betweenness centrality based on RWs, which expands the shortest-path based measure and maximum-flow measure, has been described (Newman, 2005). The random-walk betweenness has been tested on social network and proven to be effective.

In urban area, random walking algorithm is applied to classify the positional value of each road node by simulating citizens’ daily travelling (Li, Zhao, & Yuki, 2009). The simulation process takes into account the total distance travelled, which makes it different from the space syntax approach. The road nodes and segments through which the citizens are walking with a large number of times are considered as places with high connectivity and accessibility in road networks.

Methodology

General framework

In this study, bus stops are categorized into three levels. The first level refers to stops that connect major transit facilities, including subway stations, BRT stations, and passenger stations for inter-city transport. The second level consists of key stops at bus transfer hubs. The third level is the ordinary stops that mainly serve as passenger collectors.

A hierarchical process is presented for optimizing bus stop distribution in the context of multi-modal transit in large cities (Figure 1). Firstly, connection stops are created with reference to existing important transport facilities. Secondly, key bus stops are generated using a coverage model for minimizing average weighted distance. Network node centrality is utilized as weight for this purpose. Finally, ordinary bus stops are optimized with a coverage model for maximizing custom coverage. The custom coverage in this case refers to transit demand that is represented by number of people.

The framework is based on technology of Geographical information system (GIS). A GIS platform, based on ArcGIS from ESRI, is set up for integrating various types of data, generating data for the coverage models, evaluating modelling outcomes, as well as presenting final results. In particular, network node centrality and population coverage are computed with ArcGIS, and served as weights for coverage models. Coverage models are implemented with Flowmap, a standalone package tool for geographical
analyses. Flowmap is specialized in interaction analysis, network analysis and service location modeling (Breukelman, Brink, & Jong, 2009). The relocation model in Flowmap meets the requirement of this study. Flowmap provides interfaces for exchanging GIS data, which facilitates evaluation of modeling results in GIS.

Two-dimensional spatial features may be represented with either vector or raster data models in GIS. Vector data model records geographical coordinates, and is suitable for representing point, line and polygon features. Raster data model records regularly partitioned grids, and is suitable for representing continuous spatial phenomena. The initial GIS database in this framework is composed of three types of feature layers, including road network, transit facilities, and population. The road network is based on an arc-node structure, in which nodes are potential candidate bus stops. Transit facilities include subway stations, inter-city passenger transport stations, and other transport hubs. Population data are stored in raster data-set, in which the value of raster cell represents number of people.

Two types of coverage models are designed, respectively for optimizing key stops and ordinary stops. The modeling parameters are defined as follows:

\[
\begin{align*}
  i & \quad \text{network node} \\
  j,k & \quad \text{candidate stop, all network nodes are candidates} \\
  r_i & \quad \text{node } i's \text{ weight measured with node random-walk centrality} \\
  g_i & \quad \text{node } i's \text{ weight measured with trip demand} \\
  d_{ik} & \quad \text{distance between node } i \text{ and stop } k \\
  x_{ik} & \quad \text{equals to 1 if node } i \text{ is exclusively covered by stop } k, \text{ equals to 0 otherwise} \\
  X_k & \quad \text{equals to 1 if candidate stop } k \text{ is chosen, equals to 0 otherwise} \\
  D & \quad \text{maximal distance covered by stop} \\
  m & \quad \text{total number of connection stops} \\
  n & \quad \text{total number of key stops} \\
  p & \quad \text{total number of ordinary stops}
\end{align*}
\]

**Stage 1: connection stop generation**

Large cities may build subway, light rail, BRT, or important transport facilities, bus routes have to be designed to connect these facilities. Therefore, connection bus stops must be included in any optimized stop set. When the locations of these facilities are
available, closest nodes will be identified as connection bus stops manually. During the optimization process, these connection stops serve as restraints to other stops.

**Stage 2: key stop optimization**

Key stops are identified in areas that are not covered by connection stops within a specified distance. These two types of stops form a spatial skeleton structure that shapes the urban activity areas. Therefore, in optimizing key stops, connection stops are included in the model as must-select predefined stops. The coverage model for key stop optimization is a MCLP type model, with an objective of minimizing total weighted distance. As the purpose at this stage is to search for strategic locations of key stops, the strength of network node is utilized for the weight. In a topological road network, the centrality of node fits this purpose. The objective function and constraints of the model is described in Equation (1).

\[
\begin{align*}
\min & \sum_k \sum_i r_i d_{ik} x_{ik} X_k \\
\text{Subject to:} & \sum_k X_k = m + n \\
& \sum_k x_{ik} \leq 1, \forall i \\
& X_k = \begin{cases} 1, & \text{chosen} \\ 0, & \text{otherwise} \end{cases} \\
& x_{ik} = \begin{cases} 1, & \text{if } d_{ik} \leq D \text{ and } \sum_{j \neq k} x_{ij} = 0 \\ 0, & \text{otherwise} \end{cases}
\end{align*}
\]

The model allocates nodes to candidate stops. If a node \(i\) is within distance \(D\) of a candidate stop \(k\), and it has not been allocated to other stops, then the node is allocated \(k\). As the total number of stops is pre-defined, some nodes might fall outside of any stop and could not be allocated. On the other hand, if we set a larger distance \(D\) and a larger number of key stops, then there is a possibility that all nodes are allocated with fewer stops than \(m + n\).

A random-walk centrality (RWC) measure is applied to indicate node centrality in road network. A single random walking task starts from a network node, and randomly traverses a pre-defined distance. For each node traversed, its RWC value is increased by 1. The process stops when all nodes have been assigned as starting point for RW. To fit into the coverage model, the RWC values of all nodes are normalized by reversely scaling between 0 and 1, i.e. node \(i\)'s normalized value \(r_i = (\text{RWC}_{\text{max}} - \text{RWC}_i) / (\text{RWC}_{\text{max}} - \text{RWC}_{\text{min}})\).

At each step of a random walking session, one road segment connecting the current node has to be chosen as the next step of walking. Three rules are set for one walking session. First, the walking itinerary should not circulate itself, i.e. a walk should not go back to any road segment that it has traversed. Second, the probability of choosing next road segment \(S_i\) is defined by \(P_i = W_i / \sum W_j\), where \(W_j\) is the width of road segment \(S_j\) that is connected to the current node and has not been traversed. Third, to simulate citizen’s transit travel, random walking distance values (in meters) are generated in accordance with normal distribution of \(N(\mu, \delta^2)\).

**Stage 3: ordinary stop optimization**

With the availability of connection stops and key stops from the previous stages, this stage generates the ordinary bus stops. The objective of the model is for maximizing
the customer coverage within serving distance of stops, using trip demand at each network node. The model requires setting a maximal serving distance for the candidate stop. In general, the maximal distance that transit passengers are willing to walk is around 400 m. As the central area of a city usually has higher density, the service distance of stop can be smaller than that for outside area.

The coverage model for ordinary bus stop is defined as follows,

$$\max \sum_i g_i x_{ik} X_k$$
Subject to:
$$\sum_k X_k = m + n + p$$
$$\sum_k x_{ik} \leq 1, \forall i$$
$$X_k = \begin{cases} 1, & \text{chosen} \\ 0, & \text{otherwise} \end{cases}$$
$$x_{ik} = \begin{cases} 1, & \text{if} \; d_{ik} \leq D \text{ and } \sum_j x_{ij} = 0 \\ 0, & \text{otherwise} \end{cases}$$

(2)

This is a MCLP type coverage model for maximizing total custom demand. In the coverage model, network nodes serve as both candidate stops and demand points. A demand point is allocated to a stop if it is within a distance D of the stop and not allocated to any other stop. Some demand points might not be allocated. In principle, if all demand points are allocated to stops, then a maximal number of stops are derived. Therefore, the total number of stops $m + n + p$ to be optimized should not exceed this maximal number.

To evaluate customer coverage, potential transit demand $g_i$ is assigned to each node. Transit trip demand estimation requires detailed survey of travelers’ socioeconomic status, so that specific demand model may be calibrated. Due to limitation on data, this study makes use of population data directly as the potential demand of transit trips. Population data are usually available from census. In developed counties, the census systems provide spatial units at hierarchical levels. For cities of these countries, the centroids of small statistical parcels may be utilized directly as demand points. In other countries such as China, such spatial units are not available at lower spatial levels, and there is a need to disaggregate population data from large spatial units to smaller spatial units. The process requires land use data that are classified into various categories based on types of human activities. In China, urban land is classified into eight categories, including residential, administrative, commercial, industrial, logistics, transport, utilities, and green (MOHURD, 2011). For example, if a statistical district has 10,000 residents, and land use map of the district is available, then the population may be disaggregated into small areas (such as raster cells) based on residential land use distribution. Details of the disaggregating method have been described in Huang, Ottens, and Masser (2007). Population are disaggregated into raster cells, and then allocated to each node based on Thiessen polygon analysis in ArcGIS. The Thiessen polygon of each node defines an exclusive area of influence, so that any location inside the polygon is closer to that node than to any other nodes. By overlaying population raster layer with Thiessen polygon layer, population covered by each polygon is derived, and is linked to the corresponding node.

**Case study of Wuhan, China**

**The city of Wuhan**

Wuhan is a typical metropolitan city with 4.65 million population and a built-up area of approximately 450 km². As a historical city, Wuhan has been the most important
transport hub in central China since 400 years ago. High population density, fast economic growth, and rapid increase in number of vehicles contribute to a high transport demand. Limited by its natural terrain, Wuhan is geographically partitioned into three parts by the Yangtze River and the Han River. Although the two rivers endow the city with a distinctive morphology, they also block movements between the towns. Till 2013, five bridges and one tunnel crossing the Yangtze River have been built. However, only two bridges serve as the major corridors of bus transport. More than 50 bus routes are concentrated on each of the two bridges.

The public transit system in Wuhan comprises bus, taxi, ferry, and rail transport. Bus transit is the major transit mode, for example, buses covered 73% of all transit trips and over 4 million passenger trips per day in 2011 (WHTPI, 2012). However, the bus dominated transit structure will change, as the city has put great emphasis on rail transport system since 2010. According to the plan, from 2012 to 2017, the city will open one rail line for each year. This raises the problem of restructuring bus transit system, including the optimization of bus stops.

The process and results of optimization

The original data-sets for the case of Wuhan city include road network, population, land use, candidate stop, railway station, planned subway station, ferry port, and so on. Candidate stops are road network nodes. There are 1335 nodes in total.

In the first stage, connection stops are identified with reference to transport facilities, i.e. railway station, planned subway station, and ferry port. Nodes that are closest to these facilities are designated manually as connection stops. In total, there are 160 connection stops created.

In the second stage, the node centrality value is computed firstly with random walking procedure developed in ArcGIS. The walking distance has an average value of 5000 m, and a standard deviation of 1000. The node centrality value is utilized as weight in the coverage model of Flowmap. Based on the scale in question, the total number of key stops and connection stops is set to 200, about 30% of total bus stops to be created. Therefore, 40 key stops need to be optimized in this stage. The maximal distance allowed for stop coverage is set to 1500 m.

In the third stage, the ordinary stops are searched using the coverage model that maximizing potential custom coverage. Based on the size of the study area and small variation of coverage distance, the total number of stops is roughly estimated to fall between 580 and 750. Although it is possible to test scenarios of different total stop number, this study only takes 616 stops for optimizing. Therefore, apart from the 200 connection and key stops, 416 ordinary stops need to be searched. In this model, network nodes are candidate stops as well as demand points. Population data are available from larger statistical zones, and are disaggregated into raster cells based on residential land use distribution. The raster cell size is 30 m by 30 m. By aggregating data from raster cells into the Thiessen polygon zones of network nodes, each node may be assigned a demand value of population.

The model requires setting maximal distance $D$ between demand points and stops. In order to reflect the geographical difference in land development density, the study area is divided into two zones, i.e. the central zone and the outer zone. The two zones utilize different maximal coverage distance. The central zone is more densely developed, so the coverage distance is set to 350 m. In the outer zone, a 450-m coverage is applied.
The optimized stop locations, road network, and central zone are depicted in Figure 2. Three types of stops are marked with different symbols. The large white areas within which no roads appear imply the Yangtze River and several large lakes in the city. The boundaries of these areas are not indicated.

Analysis and discussion

The three-stage optimizing process has generated satisfactory result. Compared to the existing 733 stops, the modeling process generates 616 stops, which indicates a drop of 117 bus stops through optimization (Table 1). After optimization, there are noticeable changes of number of stops in the central and outer zones. The number of stops in the central zone is reduced from 415 to 260, while in the outer zone the number increases from 318 to 356. This result indicates a heavy redundant of stops in central area, as well as an insufficient stop coverage in outer area. There are totally 314 stops appear in both the existing and the optimized stop sets, which explains 42.8% of the existing stop set and 67.0% of the optimized stop set.

While the total number of stops is reduced, the total covered area within 500 m is enlarged from the existing 283.69 to 334.58 km². This implies a good improvement on level of service in terms of spatial coverage. The area covered by 300 m also shows a small increase.

The story is not so straightforward, however, concerning population served within 300 and 500 m (Table 2). Here, the population data in raster cells, rather than that in network nodes, are utilized for statistics. The percentage value indicates the covered population over total population. The total population is 3.416 million in the whole

![Figure 2. Optimized bus stop distribution.](attachment:image.png)
study area, 1.991 million in the central zone, and 1.425 million in the outer zone. Compared with the existing stop set, while there is a slight drop of served population in the central zone, there is a big increase in the outer zone.

The population coverage can only present a partial evaluation on transit service coverage. For a transit trip, there is an origin stop and a destination stop. Considering work trip in the morning, the origin stops are those in or close to residential neighborhoods, while the destination stops are attached to the employment places. Therefore, the statistics may be further improved by bringing in employment distribution. In this perspective, there is a great need to incorporate employment demand into the coverage model, rather than only using population data.

Spatial structure is a key factor for shaping urban transit system. In addition to the rivers, Wuhan city is also renowned for its big lakes. The rivers and lakes endow the city with unique urban morphology. In order to meet the needs of social, cultural, and economic activities, bridges and tunnels have to be built to connect the built-up areas. The bridges and tunnels also become transport bottlenecks due to their limited capacities. This situation undoubtedly has impact on the deployment of bus routes and stops, which may be explored in future studies of integrated route-stop optimization. The stop optimization model presented in this research does not reflect the special morphological need.

Road network follows the general link-node structure that has been utilized in many network related applications. A specific feature of this study is that nodes are used as both candidate stops and demand points. Due to the irregularity of road network, optimized stops are usually unevenly spaced. For generating satisfactory stop distribution, the spacing between nodes needs to be kept small. With the link-node structure, a long link (i.e. road segment) indicates a large spacing between its two nodes. These long links may be divided into smaller ones to maintain shorter node spacing. However, if the spacing is set as too small, the dividing process may create large amounts of nodes, which has impact on computing efficiency.

Node centrality value and node demand value are, respectively utilized as weights in coverage models for generating key stops and ordinary stops. These two types of weight values are derived with relatively simple assumption, and may be further investigated.

| Buffer distance (m) | Central zone | Outer zone | Sum |
|---------------------|--------------|------------|-----|
|                     | x10,000 | %          | x10,000 | %          | x10,000 | %          |
| Existing            | 300     | 160.85 | 80.78   | 55.89 | 39.22 | 216.74 | 63.44 |
|                     | 500     | 193.46 | 97.16   | 104.27 | 73.16 | 297.73 | 87.15 |
| Optimized           | 300     | 116.91 | 58.71   | 55.28 | 38.79 | 172.19 | 50.40 |
|                     | 500     | 189.58 | 95.21   | 109.44 | 76.79 | 299.03 | 87.53 |

Table 1. Number of stops and spatial areal coverage.

| | Number of stops in two zones | Covered area (km²) |
|------------------|-------------------------|------------------|
|                  | Total | Central | Outer | 300 m buffer | 500 m buffer |
| Existing         | 733   | 415     | 318   | 156.78       | 283.69       |
| Optimized        | 616   | 260     | 356   | 159.96       | 334.58       |

Table 2. Population coverage.
for the optimizing process. On the one hand, the node centrality value is derived with random walking algorithm, taking road width into consideration. In central urban area, where road width is similar and road distribution is in regular grid pattern, the nodes have similar centrality value. On the other hand, concerning the potential transit demand at network nodes, simply using population or employment data at network nodes may not correctly indicate transit travel demand. Therefore, it is necessary to build demand models for transit travel by utilizing more detailed socioeconomic data and modeling techniques.

A potential drawback comes from the key stop generation. Key stops and connection stops serve as anchor points in bus network, and should be put at strategic locations. In this study, while the connection stops are identified with predefined rules, key stops are optimized with coverage model. The coverage model generates stops with mutually exclusive and relatively equal area of service, which implies key stops are evenly distributed. Therefore, the coverage model does not respond to variable development density in different urban areas. While it is possible to optimize on an area-by-area basis, as in the stage of ordinary stop optimization, a self-adaptive method sensitive to urban structure is more effective. While stop coverage reflects the walking accessibility to bus system, bus routes network indicates the effectiveness of moving travelers among the stops. Therefore, it is more important to jointly optimize the routes and stops. Optimizing bus route-stop system requires a broader process, in which stops and routes are optimized in an integrated way. The GIS-based framework of this study may be expanded to incorporate bus route optimization.

Conclusions
This study has presented a framework for hierarchically optimizing distribution of bus stops in a multi-modal transit system. Three levels of bus stops are defined, and two levels of stops are optimized with coverage models. The result has shown a good improvement on service coverage with fewer number of stops. Particularly, redundant stops in central area of the city are reduced, and more stops are deployed in the outer area to improve bus accessibility. Future research concerns improving key stop location model, enhancing weight factor of the coverage models, as well as optimizing the whole route-stop system.

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