Effects of Long-Wavelength Track Irregularities Due to Thermal Deformations of Railway Bridge on Dynamic Response of Running Train

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Abstract: In a fixed-end arch railway bridge restraining the displacement and rotation at the support to constrain the longitudinal deformation of the superstructure, vertical deformation occurs according to temperature change. Due to such deformation, periodic change in long-wavelength track irregularity occurs, which, by increasing the vertical train body acceleration, degrades ride comfort. In the present study, the vertical deformation of a fixed-end arch railway bridge and the accompanying track irregularity changes were measured during the summer and winter, respectively. Based on the measured data, the relationships among the ambient temperature, the temperature of the bridge members, the deformation of the bridge, and the track irregularity were investigated. Additionally, the correlation between the train body acceleration and the long-wavelength track irregularity was examined, and a method of controlling long-wavelength track irregularity considering seasonal temperature change was discussed.

Keywords: railway bridge; temperature; deformation; long-wavelength track irregularity; train body acceleration

1. Introduction

Since the dynamic behavior of a train traveling on a railway bridge is sensitive to changes in track geometry due to bridge deformation, there are strict restrictions on deformation amplitude [1,2]. In addition to vertical deformation, longitudinal expansion or contraction of a bridge increases the axial force on the rail. To control this, it is necessary to restrain excessive longitudinal deformation and/or to install rail expansion joints or special fastening devices [2,3]. For this reason, the fixed-end arch design is often chosen for long-span railway bridges, as this type effectively controls longitudinal deformation of the superstructure by restraining deformation and rotation at the supports. However, the restraints at the supports cause vertical displacement of the arch due to temperature change, causing track-geometrical errors (which is often referred to as “track irregularities”), increasing train vibration, and degrading ride comfort thereby. Bridge deformation due to temperature change and resulting track irregularities does not, as general track irregularities caused by long-term bridge deflection or subgrade settlement do, show gradual growth in a certain direction; rather, it shows annual fluctuation corresponding to seasonal temperature variation; therefore, permanent maintenance is difficult. Unlike ballasted track, the geometry of which can be periodically adjusted through ballast tamping and track lifting, in the case of slab track, the rail is fixed to slabs that are integrated with the bridge superstructure, which design renders periodic track-geometrical changes very difficult to deal with.
To date, a number of analytical studies on the dynamic behavior of bridges during train travel have been carried out [4–15]. On the other hand, there are relatively few studies on the correlation between bridge-deformation-caused long-wavelength track irregularities and train response. Choi et al. [16] analyzed the effects of the shape, wavelength and amplitude of track irregularity on the safety and riding comfort of trains, and found that vertical track irregularity greatly affects vertical train acceleration. However, they did not consider the interaction between train and bridge. Yang and Hwang [17] proposed a dynamic analysis model that takes into account train-track-bridge interaction by the direct stiffness and mode-superposition methods. Using this model, they assessed the effect of steel-arch-bridge vertical deformation due to temperature change on the running safety and ride comfort of high-speed trains. Based on their results, they showed that a temperature-change-induced large vertical deformation has a great effect on the safety and ride comfort of high-speed trains, but that the effect of this deformation on bridge structural safety is not significant. Hung and Hsu [18] used three-dimensional finite element analysis to show that long-wave track irregularity caused by long-term bridge deflection due to concrete creep causes low-frequency vibration close to the natural frequency of the train suspension system. They validated their model by comparing the theoretical results with the experimental results, and proposed a track-geometrical adjustment pattern through shim insertion for avoidance of the resonance of the train suspension system.

In sum, the previous studies show that long-wavelength track irregularity caused by bridge deformation have a great influence on train vibration, which depends on the bridge span length, the characteristics of the vehicle suspension system, and the train speed. The present study aimed to evaluate the correlations among the vertical deformation of fixed-end arch bridge, long-wavelength vertical track irregularity and vehicle vibrations, and to propose a management strategy for track irregularity change in an appropriate range. For these purposes, temperature changes, vertical deformation and track-geometrical changes of fixed-end arch bridges were measured during the summer and winter seasons, and the relationships among ambient temperature, main arch temperature, vertical bridge deformations and track irregularity were assessed. Also, the correlation between train acceleration and long-wavelength track irregularity was analyzed by the train running test, based on which a management strategy for consideration of seasonal temperature change is herein proposed.

2. On-Site Measurements

2.1. Test Bridge

Long-span bridge designs are increasingly utilized in the construction of railway bridges, but they are inevitably disadvantageous in terms of dynamic stability and track-bridge interaction, which ultimately limits their application. Alternatively, fixed-end arch bridge designs have advantages in terms of stiffness and serviceability (specifically required for railway use), and its application is gradually increasing. Two of the most typical examples are the Contreras Bridge [19] of 261-m central span length, and the Sobre El Tajo Bridge [20] of 324-m central span length, both in Spain. These bridges, however, support ballasted track, which, compared with slab track, is easier to adjust geometrically for deformation due to thermal expansion or contraction.

The target bridge in the present study is a three-span continuous arch bridge built for Gyungbu High Speed Railways in Korea. This comprises two side spans of 55 m and one center span of 70 m (see Figure 1). The main arch rib, of the encased composite girder type with concrete-filled steel box, is rigidly supported on the piers and foundations. This is a double-arch structure in which reinforced concrete sub-arch ribs are placed on the upper part of the main arch rib at intervals of 11 m. The wall and bridge deck are integrated with the sub arch ribs. The main arch ribs are fixed to the bridge piers and foundations in order to minimize longitudinal deformation and, thus also, axial stresses of the rail due to the track-bridge interaction. However, with this fixed-end type design, vertical deformations of the main arch rib according to temperature change are inevitable. The concrete slab tracks were laid, and the turnouts were installed on the bridge.
Figure 1. Dimensions of test bridge: (a) side view; (b) typical cross section.
2.2. Measurements

2.2.1. Temperature

To determine the seasonal temperature change of the test bridge, the main arch ribs and concrete deck slabs were measured using 24 and 6 thermocouples that had been installed on the main arch ribs and concrete deck slab, respectively (see Figure 2). Additionally, one thermocouple had been installed in the shade on the upper part of the bridge pier in order to measure the ambient temperature. The temperature measurements were carried out from 10 December 2015 to 24 October 2016 (about 10 months).

![Figure 2. Locations of temperature sensors installed on test bridge.](image)

2.2.2. Track Geometry

In the high-speed railway under commercial operation, the track geometry is periodically checked (e.g., once per month) by a track inspection car. However, the car measures track irregularity using a chord versine method that measures the maximum deviation on a 10 to 30-m straight line, it is impossible to directly measure the absolute extent of long-wavelength track irregularity. Therefore, the change of the track geometry caused by a vertical deformation of the main arch element of the bridge during the winter and summer seasons was measured, in the present study, by applying a 3D precision survey technique. The absolute positions of the top of the rail were measured, as a baseline, against fixed points on the bridge using the GRP system FX [21]. The survey was conducted for a total length of 360 m, and measurements were taken at one point for every three sleepers (approximately 1.8 to 2 m). The number of measuring points was minimized, considering that the measurements had to be done within the night repair time (about 3 h and 30 min). Table 1 shows the ambient and arch-member temperatures at the track-geometrical measurement points.
2.3. Accelerations of Running Trains and Bridge

In order to investigate the correlations between temperature-change-caused deformation of the arch bridge and the responses of the bridge and train, the accelerations of the bridge and train as measured during the winter (February 2016) and summer (July and August 2016) seasons by Lee et al. [22] were assessed.

The method of measuring the acceleration of a train is based on the simplified method described in UIC 518 OR [23]. Acceleration sensors and train speed sensors were installed in the train body and bogie as shown in Figure 3. Train acceleration was filtered by the ride-comfort filter shown in Figure 4 [22] according to UIC 518 OR.

**Table 1. Temperatures at track-geometrical measurement points.**

| Season | Date and Time | Track     | Ambient Temperature (°C) | Average Temperature of Main Arch Ribs (°C) |
|--------|--------------|-----------|---------------------------|-------------------------------------------|
| Winter | February 02 2016 01:00 ~ 04:30 | North bound | −2 | −2 |
|        | February 03 2016 01:00 ~ 04:30 | South bound | −5 | −1 |
| Summer | August 10 2016 01:00 ~ 04:30 | North bound | +23 | +28 |
|         | August 11 2016 01:00 ~ 04:30 | South bound | +25 | +30 |

**Figure 3.** Locations of accelerometers installed in test train (from Lee et al. [22]).

**Figure 4.** Ride-comfort filter for train acceleration (UIC 518 OR).
Bridge accelerations were measured through six accelerometers, one for each crown of the main arch. Table 2 summarizes the ambient temperatures and the train passing speeds at the measurement dates [22].

| Season  | Date           | Track       | Ambient Temperature (°C) | Average Temperature of Main Arch Ribs (°C) | Train Speed (km/h) |
|---------|----------------|-------------|--------------------------|--------------------------------------------|-------------------|
| Winter  | February 19 2016 | South bound | 12                       | 12                                         | 300               |
|         |                 | North bound | 10                       | 6                                          | 300               |
| Summer  | July 22 2016    | South bound | 37                       | 32                                         | 270               |
|         | (1st)           | North bound | 26                       | 28                                         | 270               |
|         | August 12 2016  | South bound | 38                       | 37                                         | 230               |
|         | (2nd)           | North bound | 34                       | 32                                         | 300               |

3. Results

3.1. Temperature

Typical plots of the temporal fluctuation of the center arch temperature and ambient temperature are shown in Figure 5. It can be seen that the annual variation patterns of the ambient temperature and member temperature are close to each other, but that the member temperature is slightly higher than the ambient temperature. This was due to the fact that the members are directly exposed to the solar radiation heat and that their temperatures drop more slowly than the ambient temperature.

Figure 5. Temporal fluctuation of measured temperature: (a) temporal fluctuation of temperature; (b) daily temperature variation in winter; (c) Daily temperature variation in summer.
The relationship between the ambient temperature and the temperature of each arch member is shown more clearly in the Figure 6 plot of the daily maximum and minimum temperatures. As can be seen, the relationship is linear and the maximum temperature is higher than the ambient temperature on both the south-bound line (Track T1) and the north-bound line (Track T2). Since the south- and north-bound lines are separate on the test bridge, there is a difference in the time of receiving direct sunlight, and accordingly, there is also a difference in the temperature change tendencies of the bridge members. The difference between the arch temperature and the ambient temperature is higher in the north-bound line, which the sunlight reaches in the afternoon, and the scatter is also greater. The daily minimum temperature of the arch rib is higher than the ambient temperature, but the difference and scatter are smaller than the daily maximum temperature. This tendency is almost the same in the south- and north-bound lines.

![Figure 6](image-url)

**Figure 6.** Relationship between ambient and arch-rib temperatures: (a) daily maximum temperature on south-bound line; (b) on north-bound line; (c) daily minimum temperature on south-bound line; (d) on north-bound line.
3.2. Track Geometry

Figure 7 shows the track-geometrical measurement results, which are expressed as the rail level relative to the level of the base point. As can be seen in the figure, in the mid-span of the three arches, large downward deflections in winter and upward deflections in summer occurred. The differences of the vertical rail levels in winter and summer are as large as 25 mm in the center arch and 15 to 18 mm in the side arches. This phenomenon was almost the same in both the south- and north-bound lines. On the other hand, the seasonal differences of vertical rail levels were relatively small in the supports. The deformed shape was almost symmetrical in winter and summer, and exactly matched the shape of the main arch ribs. Also, it could be seen, in the plot of the peak temperature of the arch ribs and the maximum rail levels shown in Figure 8, that the vertical deformation of the bridge increased proportionally to the temperature of the member, and that the vertical deformation of the center arch with the longer span was larger than that of the side arch with the shorter span. Therefore, it could be confirmed that the main cause of the change of rail levels is the contraction and expansion of the main arch ribs due to temperature change.
By contrast, the changes in the lateral alignment of the rails from winter to summer were not large in either the south- or north-bound line. The effect of deformation due to main-arch temperature change on the rails’ lateral alignment, then, was very limited. The singular point of the lateral alignment of the rails at the position of approximately 100 m in Figure 7b,d was due to FACOP, which is a kind of kinematical gauge optimization system for reducing wear on the tongue rail in the turnouts [24]; it was not related to track-geometrical errors.
3.3. Accelerations

Tables 3 and 4 show the results of acceleration measurements of the train body and bridge. Acceleration of the train body is directly related to ride comfort. UIC 518 OR specifies that it should not exceed 3.0 m/s\(^2\). As can be seen from Table 3, the peak value of the train vertical acceleration in summer was 25 to 50\% larger than that in winter. On the other hand, the bridge acceleration differences between summer and winter were relatively small, and did not show any specific trends. This suggests that long-wavelength track irregularities caused by bridge deformation have a great effect on train acceleration but little effect on bridge acceleration. This result agrees with previous work by Yang and Hwang [17].

**Table 3. Peak values of vertical acceleration of train body.**

| Season      | Train Speed (km/h) | Location     | Peak Acceleration of Train (m/s\(^2\)) | Criteria \(^1\) (m/s\(^2\)) |
|-------------|--------------------|--------------|----------------------------------------|----------------------------|
| Winter      | 300 South bound    | 0.26         |                                        |                            |
|             | 300 North bound    | 0.27         |                                        |                            |
| Summer 1st  | 270 South bound    | 0.40         |                                        |                            |
| Summer 2nd  | 270 North bound    | 0.33         |                                        |                            |
|             | 230 South bound    | 0.39         |                                        |                            |
|             | 300 North bound    | 0.35         |                                        |                            |

\(^1\) According to UIC 518 OR [23] and KS R 9216 [25].

**Table 4. Peak values of vertical acceleration of bridge deck.**

| Season      | Train Speed (km/h) | Location | Peak Acceleration of Bridge Deck (g) | Criteria \(^1\) (g) |
|-------------|--------------------|----------|-------------------------------------|--------------------|
|             |                    |          | Side Arch (south) | Center Arch | Side Arch (north) |          |          |
| Winter      | 300 South bound    | 0.08     | 0.12       | 0.07       |                  |          |          |
|             | 300 North bound    | 0.06     | 0.09       | 0.08       |                  |          |          |
| Summer 1st  | 270 South bound    | 0.09     | 0.10       | 0.07       |                  |          |          |
|             | 270 North bound    | 0.08     | 0.09       | 0.09       |                  |          |          |
| Summer 2nd  | 230 South bound    | 0.08     | 0.08       | 0.06       |                  |          |          |
|             | 300 North bound    | 0.07     | 0.10       | 0.09       |                  |          |          |

\(^1\) According to UIC 518 OR [23] and KS R 9216 [25], 1 g \(\approx\) 9.81 m/s\(^2\).
4. Discussion

4.1. Validation of Causes of Vertical Deformation of Bridge Superstructure

To validate the cause of bridge vertical deformation, the deformation along with the variation of the track geometry due to temperature change were calculated using the commercial finite element analysis program Abaqus. The structural analysis model used in the calculation is shown in Figure 9. The rail and arch members were modeled with frame elements, the wall with plate elements, and spring elements were applied to the support of each arch rib.

![Finite element mesh of test bridge: (a) south-bound; (b) north-bound.](image)

The input temperature for each measuring position of the bridge deck slab and main arch was selected as the average of temperatures measured from 0 to 6 o’clock (see Table 5).

| Track            | Locations                        | Input Temperatures (°C) |
|------------------|----------------------------------|-------------------------|
|                  |                                  | Summer | Winter |
| North bound      | Mid-arch                         | +28.5  | −1.8   |
|                  | Side arch                        | +27.8  | −2.0   |
|                  | Deck slab (on mid-arch)          | +29.9  | −3.6   |
|                  | Deck slab (on side-arch)         | +25.8  | −3.1   |
| South bound      | Mid-arch                         | +29.8  | +2.4   |
|                  | Side arch                        | +29.3  | +2.2   |
|                  | Deck slab (on mid-arch)          | +26.9  | −0.2   |
|                  | Deck slab (on side-arch)         | +31.0  | +0.1   |

1 Selected as the average of temperatures measured from 0 to 6 o’clock.

Figure 10 compares the rail-level differences between summer and winter from the structural analysis and measurements. Since the absolute vertical displacement of the rail depends on its initial level, the differences between the rail levels of summer and winter were examined for the comparison. Although there were considerable discrepancies between the calculated and measured rail-level differences, the change patterns are exactly the same as the measured patterns. Therefore, it
could be confirmed that the cause of the seasonal change of vertical deformation was the temperature fluctuation of the arch members. The difference between the analysis and measurement results seems to be attributed to the difference of boundary conditions such as spring stiffness as well as the difference between the actual temperatures and the input temperatures chosen as the average temperatures of the members.

![Graph showing differences in rail levels between winter and summer](image-url)

**Figure 10.** Vertical track irregularities caused by thermal deformation of bridge superstructure: (a) south-bound; (b) north-bound.

### 4.2. Effect of Vertical Track Irregularities on Train Body Accelerations

In Korean high-speed railways, as noted above, the track geometry is measured based on the chord versine method, not the absolute geometry. Ten (10)-m and 30-m chords are used in Korea. The 10-m-chord track irregularity underestimates the long-wavelength components, which are closely related to train vibration and passenger comfort. On the other hand, the 30-m-chord track irregularity better reflects the long-wavelength components of track irregularity [25]. Thus, it is necessary to investigate 30-m-chord track irregularity in order to control the temperature deformation of the bridge in terms of train vibration and passenger comfort.
Figure 11 shows 30-m vertical track irregularities converted from the measured track geometry. The blue line represents the upper and lower limits beyond which more “attention” should be paid [26].

Meanwhile, to examine the influence of the amplitude of track irregularities for train and bridge accelerations, the track irregularity waveform should be measured at the time of train passing. However, in this study, the track geometry was measured during the night repair time, and thus the track geometry in the test run could only be estimated from the measured data. If we assume that the track geometry is linearly proportional to the temperature of main arch rib, the track-geometrical change \( \frac{dy_r}{dT} \) (mm/°C) can be estimated by the relationship.
where $T_s$, $T_w$ are the temperatures (°C) of the main arch rib in summer and winter, respectively, and $y_{r,s}$, $y_{r,w}$ are the rail levels relative to the reference point (mm) at the measuring point in summer and winter, respectively.

Using Equation (1), the track-geometrical data at a certain temperature other than the temperature of measurements can be obtained, and the chord-length-based track irregularity can also be estimated. Figures 12a and 13a show the rail levels, and Figures 12b and 13b show 30-m-chord track irregularities obtained by this method in the test run.

\[ \frac{dy_r}{dT} = \frac{y_{r,s} - y_{r,w}}{T_s - T_w} \]  

Figure 12. Vertical accelerations of train body and track vertical irregularities in test run (south-bound line, summer): (a) vertical track irregularities from absolute geometry at peak temperature; (b) 30-m-chord vertical track irregularities at peak temperature; (c) vertical accelerations of train body.
In comparing the 30-m-chord track irregularities in Figures 12b and 13b with those in Figure 11, one can find that the difference between those for nighttime and those for daytime temperatures is not large in summer, and so the waveforms look similar. On the other hand, since the temperature of the daytime is higher relative to nighttime in winter, the waveform is not clear. This is why the train body accelerations measured in summer are higher than those in winter. However, this also implies that in winter, the train body accelerations in the early morning and nighttime are larger than those measured in the daytime.
It can be found from the comparison of Figure 12b,c and Figure 13b,c that the waveforms of the train body vertical accelerations in the time domain coincide very well with those of the 30-m-chord track irregularities. The values of the train body accelerations and the 30-m-chord track irregularities are also well correlated, as shown in Figure 14.

**Figure 14.** Relationship between peak vertical train body acceleration and peak 30-m-chord vertical track irregularities.

This high correlation between the 30-m vertical track irregularity and the vertical acceleration of the train body is due to the relationship between the amplification factor of the chord versine method and the transfer characteristics of vehicle acceleration [27]. The relationship between track irregularities by the chord versine method and absolute track-geometrical errors can be expressed as

\[ \delta_{30m}(\lambda) = H_{c}(\lambda) \delta_{abs}(\lambda) \]  

(2)

where \( \delta_{30m}(\lambda) \) represents the amplitude of the 30-m-chord track irregularities in the wavelength domain and \( \delta_{abs} \) stands for the absolute track-geometrical errors. Also, the relationship between the train body acceleration and the absolute track-geometrical errors can be defined by the transfer function \( H_{a}(\lambda) \) as

\[ A(\lambda) = H_{a}(\lambda) \delta_{abs}(\lambda) \]  

(3)

where \( A(\lambda) \) represents the amplitude of the train body acceleration in the wavelength domain. Here, if the waveforms of the above two functions—\( H_{c}(\lambda) \) and \( H_{a}(\lambda) \)—are similar in the major wavelength bands, the correlation between the 30-m vertical track irregularity and the vertical acceleration of the train body will be high.

Figure 15 shows the amplification factor of the chord versine method; i.e., \( H_{c}(\lambda) \), according to chord lengths when the waveform of the track irregularity is represented by the sine function [27].
Figure 15. Transfer function of track irregularities (Ono et al. 2012 [27]).

The transfer ratio of the acceleration by track-geometrical error is greatly increased at a critical wavelength. The critical wavelengths are divided into those determined by the natural frequencies of the vehicle suspension system and those determined by the distance between the vehicle bogies regardless of the train suspension system.

The critical wavelengths determined by the natural frequency of the vehicle suspension system are proportional to the train speed as follows:

\[ \lambda_n = \frac{V}{f_n} \quad (n = 1, 2, 3, \ldots) \tag{4} \]

where \( V \) is the train speed and \( f_n \) is the natural frequency of the vehicle suspension system.

The critical wavelengths determined by the bogie distance can be obtained by [18,28] as follows.

- Critical wavelengths causing car body bouncing:

\[ \lambda_b = \frac{B}{m} \quad (m = 1, 2, 3, \ldots) \tag{5} \]

- Critical wavelengths causing car body pitching:

\[ \lambda_p = \frac{B}{(l - 1/2)} \quad (l = 1, 2, 3, \ldots) \tag{6} \]

where \( B \) is the center-to-center distance of the bogies. Thus, the first critical wavelengths corresponding to the bogie distance of a Korean KTX high-speed train (18.7 m) are 18.7 m and 37.4 m for bouncing and for pitching, respectively. As can be seen in Figure 15, in the case of application of the 30-m chord at these critical wavelengths, the amplification ratio is close to 1 or more than 1. This is why the 30-m-chord track irregularity and train body acceleration are highly correlated.

On the other hand, the critical wavelength related to the natural frequency of the train suspension system depends on the train speed, and so the correlation with the 30-m-chord track irregularity and train body acceleration will vary with the train speed. When applying the 30-m chord, the amplification ratio is larger than 0.8 up to approximately 70 m, and good correlation can be expected. The natural frequencies of the suspension system of the Korean KTX high-speed trains and corresponding wavelengths are shown in Table 6. The natural frequencies of the suspension system of the trains were calculated based on the 2-D mass-spring-damper vehicle model used in the train-track analysis by Yang and Hwang [17]. At the train speed of 230 to 300 km/h, the critical wavelengths corresponding to the pitching mode of the
train body are between 63.9 and 83.3 m, and the critical wavelengths corresponding to the bouncing mode of the train body are between 91.3 and 119.0 m.

| Mode         | Natural Frequencies (Hz) | Critical Wavelength (m) Corresponding to Natural Frequencies at Train Speed of V (km/h) |
|--------------|--------------------------|--------------------------------------------------------------------------------------|
|              | Train Body Bogie         | V = 230   V = 250   V = 270   V = 300   V = 230   V = 250   V = 270   V = 300 |
| Bouncing     | 0.7                      | 6.9       | 91.3       | 99.2       | 107.1      | 119.0      | 9.3        | 10.1       | 10.9       | 12.1       |
| Pitching     | 1.0                      | 9.3       | 63.9       | 69.4       | 75.0       | 83.3       | 6.9        | 7.5        | 8.1        | 9.0        |

In sum, it is appropriate to manage the track geometry with a 30-m chord for absolute track-geometrical errors of wavelengths of up to approximately 70 m. However, if the wavelength is longer (i.e., in cases of longer bridge spans), the longer chord length should be considered, because the dominant wavelength of the track-geometrical errors might, depending on the train speed, come close to the critical wavelength bands related to the natural frequency of the vehicle suspension.

Figure 16 plots the FFT results of vertical track irregularities converted according to temperature in the test run. As could easily be predicted, the highest amplitude was observed at 70 m, which corresponds to the center arch span of the bridge, 45 and 60 m, which correspond to spans slightly smaller or larger than the span of the side arch (55 m), and 180 m, which corresponds to the entire length of the bridge. In summer, the peaks appeared at 70 m and 180 m. In winter, however, the amplitude, at around 70 m, was decreased, but was greatly increased beyond 180 m. This was due to the fact that, in winter, the temperature in the daytime increases relative to nighttime and the amplitude of the track-geometrical errors near 70 m decreases. This is one of the reasons why the acceleration of the train body was smaller in winter than in summer.

Figure 17 plots the FFT results for the vehicle vertical accelerations in the wavelength domain. As can be seen, the peak vertical accelerations of the train body were observed in the vicinity of 40 to 50 and 60 to 70 m, which coincided with the dominant wavelengths of the track-geometrical errors in both summer and winter. As mentioned above, because the most dominant wavelengths of the track-geometrical errors are not superposed on the critical wavelengths for the train body vibration, the train body accelerations were not significant, and did not exceed the limits for ride comfort. However, if the train speed decreases, the critical wavelengths corresponding to the natural frequencies of the
vehicle suspension come close to 70 m, the dominant wavelength of the geometrical errors. According to Table 5, the critical wavelength for the pitching-mode resonance of the train body is 69.4 m at the train speed of 250 km/h. Therefore, at 250 km/h, the train body acceleration will be the largest.

Figure 17. Train body accelerations in wavelength domain.

Figure 18 plots the train acceleration versus the unit track irregularity for the 30-m chord at different train speeds. There is no measurement data for 250 km/h, but Figure 18 shows that the train acceleration per unit track irregularity when the train travels at speeds of 230 and 270 km/h is slightly larger than that at 300 km/h. This suggests that reducing train speed to control train vibration is not always effective, especially in summer when track-geometrical error increases; rather, it is desirable to adjust the train speed so that the critical wavelength corresponding to the vehicle natural frequency will not be close to the dominant wavelength of the track irregularity.

Figure 18. Peak train body acceleration per unit 30-m-chord track irregularity versus train speed.
In Figure 18, it is also noteworthy that the train acceleration is somewhat higher in winter than in summer when the same size of track-geometrical error occurs. This may be attributed to the fact that the spring constants of the train suspension system consisting of air springs increases with decreasing temperature.

4.3. Effect of Vertical Track Irregularities on Acceleration of Bridge

As shown in Table 4, the acceleration of the bridge, unlike train acceleration, does not have a significant correlation with vertical track irregularity. The acceleration of a bridge increases when the excitation frequency of the train approaches the natural frequency of the bridge [29]. As shown in Table 7, the natural frequencies corresponding to the vertical vibration modes of the bridge are 4.81 Hz from the numerical model and 4.4 Hz from the measured vertical acceleration of bridge deck from the train test run. The excitation frequencies corresponding to the dominant wavelengths of the track-geometrical errors (45, 60, 70 m) are between 0.91 and 1.85 Hz at train speeds of 230 to 300 km/h. Even if we consider the effect of the mass of the train on the natural frequencies, they are far from each other, and therefore, it is reasonable that long-wavelength track irregularities do not affect the acceleration of the bridge. Rather, what does affect bridge acceleration is the excitation frequency determined by the bogie distance and axle distance of the train [7]. However, this dynamic behavior of the bridge due to the excitation frequency of the train should be validated at the design stage [2].

| Source | Major Mode Direction | Natural Frequencies (Hz) |
|--------|----------------------|-------------------------|
| Numerical model | Transverse | 1.24, 1.42, 2.06, 3.05, 4.07 |
| | Longitudinal | 3.46, 11.84 |
| | Vertical | 4.81, 6.71, 7.24, 8.33, 9.20 |
| Measured deck acceleration from test run | Vertical | 4.4, 8.9 |

4.4. Suggestion for Design and Maintenance

In the design of fixed-end arch bridges, it is necessary to determine vertical deformation due to temperature change and, thus also, to ensure that the bridge span lengths will not coincide with the critical wavelengths for vehicle vibration. However, it is not easy to predict track-geometrical errors due to temperature change, because the rail levels at the construction stage cannot be precisely known, and, moreover, the support conditions and input temperatures in the numerical models are not exactly the same as those in the real structure.

Therefore, at the maintenance stage or commissioning stage, it is important to readjust the rail levels within an appropriate range. To do this, first, the rail profiles should be measured for a certain range of temperature, and then the peak track irregularities at the maximum temperature (during the daytime in summer) and at the minimum temperature (at nighttime in winter) can easily be predicted, as was done in the present study. This information will facilitate determination of the extent of rail-height adjustment. The rail profiles after readjustment are given as

$$y_r(x, T) = y_{ri}(x, T) + \Delta y_r(x)$$  \hspace{1cm} (7)

where $$y_{ri}(x, T)$$ is the reference rail level at temperature $$T$$, $$y_r(x, T)$$ is the rail level after adjustment, and $$\Delta y_r(x)$$ is the extent of the rail height readjustment (i.e., shim thickness) at $$x$$. From the measurement data, the reference rail level $$y_{ri}(x, T)$$ at a specific temperature can be obtained, and the readjusted rail level should be determined to meet the following requirements:

$$a_{peak,s} = v_{30m,s} \left( \frac{a_{peak}}{v_{30m}} \right) \leq a_{all}$$  \hspace{1cm} (8)
and

\[ a_{\text{peak},w} = v_{30m,w} \left( \frac{a_{\text{peak}}}{v_{30m}} \right) \leq a_{\text{all}} \]  

(9)

where \( a_{\text{peak},s} \) and \( a_{\text{peak},w} \) are the peak accelerations of the train body in summer and winter, respectively, \( v_{30m,s} \) and \( v_{30m,w} \) are the peak 30-m-chord track irregularities obtained based on the readjusted rail profiles \( y_r(x, T) \) in summer and winter, respectively, \( a_{\text{peak}} / v_{30m} \) is the peak train body acceleration per unit 30-m-chord track irregularity, and \( a_{\text{all}} \) is the allowable limit of train body acceleration. To meet the requirements in both summer and winter, the extent of the rail height readjustment \( \Delta y_r(x) \) should be determined so that the waveform of the 30-m-chord track irregularities in summer and winter are almost symmetrical with respect to the rail profiles at neutral temperature; i.e., \( v_{30m,s} \approx v_{30m,w} \). The relationship between track irregularity and train body acceleration, as shown in Figures 13 and 17, can be obtained from the running test at different speeds, or alternatively, it can be predicted by an appropriate numerical dynamic analysis model, which should first be validated based on the test data.

5. Conclusions

The present study investigated the correlations among the vertical deformation of fixed-end arch railway bridges, long-wavelength track irregularities, and train vibration. Based on the results, the present study suggests how to evaluate the suitability of track geometry and how to prepare the maintenance plan. The conclusions drawn from this study are as follows.

- Field measurements and a numerical analysis were used to clarify the phenomenon by which large vertical long-wavelength track irregularity is periodically generated by the thermal expansion of the bridge arch rib fixed at both ends due to seasonal temperature changes in fixed-end arch bridges. As a result, the dominant wavelength of the track irregularity is determined by the span length of the main arch.
- Long-wavelength track irregularity increases the vertical acceleration of the train body, especially in the daytime on summer days when the track irregularity is the largest. On the other hand, in winter, the temperature increases during the daytime relative to nighttime, and track irregularity becomes smaller than at nighttime, the result being that the measured train acceleration is lower than that in summer, but is expected to increase in the early morning and at night.
- When track irregularity is represented by the chord versine method with a 30-m chord, the waveform with the train body acceleration is very similar to that of the track irregularity, and the peak acceleration magnitude is also highly correlated with the peak value of the 30-m-chord track irregularity. It is considered that this is due to the fact that amplification factors of the 30-m chord in the critical wavelength band for the vertical train body acceleration are close to or larger than 1; it is also considered that it is appropriate to manage long-wavelength track irregularity with the 30-m chord when the bridge span length is less than 70 m. If the bridge span is longer than 70 m, the dominant wavelength of the track irregularity might, depending on the train speed, be close to the critical wavelength band corresponding to the natural frequency of the train suspension, and so it is necessary to further extend the chord length.
- Unlike train vibration, excitation frequency due to long-wavelength track irregularity to the bridge structure is very small compared with the natural frequency of the bridge, so that the acceleration of the bridge is hardly affected by track irregularity.
- To control long-wavelength track irregularity due to seasonal temperature changes and to prevent the degradation of passenger comfort, it is necessary during the design stage to predict vertical deformation of the bridge due to temperature change and the corresponding long-wavelength track irregularity. Also, it is preferable to determine the main span length in such a way that it does not coincide with the critical wavelength of the train.
- At the maintenance stage or before the start of train operation, readjustment of the rail height can be carried out to reduce long-wavelength track irregularity. To do so, it is recommended to
measure track-geometrical errors at different temperatures and to obtain the correlation between long-wavelength track irregularity and train body acceleration from a test run.

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References
1. UIC 776-3 R: Deformation of Bridges, 1st ed.; International Union of Railways: Paris, France, 1989.
2. EN 1990, Eurocode—Basis of Structural Design; European Committee for Standards: Brussels, Belgium, 2002.
3. UIC 774-3 R, Track/bridge Interaction. Recommendations for Calculations, 2nd ed.; International Union of Railways: Paris, France, 2001.
4. Fryba, L. Vibration of Solids and Structures under Moving Loads; Noordhoff International: Groningen, The Netherlands, 1972.
5. Matsuura, A. Study of dynamic behaviours of bridge girders for high-speed railway. J. JSCE 1976, 256, 35–47.
6. Yang, Y.-B.; Lin, C.W. Vehicle–bridge interaction analysis by dynamic condensation method. J. Struct. Eng. 1995, 121, 1636–1643. [CrossRef]
7. Fryba, L. Dynamics of Railway Bridges; Thomas Telford: London, UK, 1996.
8. Wu, Y.S.; Yang, Y.-B. Steady-state response and riding comfort of trains moving over a series of simply supported bridges. Eng. Struct. 2003, 25, 251–265. [CrossRef]
9. Ju, S.H.; Lin, H.T. Resonance characteristics of high-speed trains passing simply supported bridges. J. Sound Vib. 2003, 267, 1127–1141. [CrossRef]
10. Yang, Y.-B.; Yau, J.D.; Wu, Y.S. Vehicle–Bridge Interaction Dynamics: With Application to High-Speed Railways; World Scientific: Singapore, 2004.
11. Yang, Y.-B.; Lin, C.W. Vehicle–bridge interaction dynamics and potential applications. J. Sound Vib. 2005, 284, 205–226. [CrossRef]
12. Xia, H.; Zhang, N.; Gao, R. Experimental analysis of railway bridge under high-speed trains. J. Sound Vib. 2005, 282, 517–528. [CrossRef]
13. Xia, H.; Zhang, N.; Guo, W.W. Analysis of resonance mechanism and conditions of train–bridge system. J. Sound Vib. 2006, 297, 810–822. [CrossRef]
14. Liu, K.; Reynders, E.; De Roeck, G. Experimental and numerical analysis of a composite bridge for high-speed trains. J. Sound Vib. 2009, 320, 201–220. [CrossRef]
15. Yang, Y.-B.; Yau, J.D. Vertical and pitching resonance of train cars moving over a series of simple beams. J. Sound Vib. 2015, 337, 135–149. [CrossRef]
16. Choi, I.Y.; Um, J.-H.; Lee, J.S.; Choi, H.-H. The influence of track irregularities on the running behavior of high-speed trains. Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit 2013, 227, 94–102. [CrossRef]
17. Yang, S.C.; Hwang, S.H. Train-track-bridge interaction by coupling direct stiffness method and mode superposition method. J. Bridge Eng. 2016, 21, 04016058. [CrossRef]
18. Hung, C.F.; Hsu, W.L. Influence of long-wavelength track irregularities on the motion of a high-speed train. Veh. Syst. Dyn. 2017, 56, 95–112. [CrossRef]
19. Manterola, J.; Martinez, A.; Martin, B.; Navarro, J.A.; Gil, M.A.; Fuente, S.; Blanco, L. Long railway viaducts with special spans: Part I. Arch construction by balanced cantilever with auxiliary cable. In Proceedings of the International Conference on Multi-Span Large Bridges, Porto, Portugal, 1–3 July 2015; pp. 381–390.
20. Manterola, J.; Martinez, A.; Bartin, B.; Gil, M.A.; Fuente, S.; Blanco, L. Railway arch Bridge over the Tajo River in the Alcantara Reservoir; IABSE Symposium Report; International Association for Bridge and Structural Engineering: Madrid, Spain, 2014; pp. 1228–1235.

21. Amberg Technologies. Available online: http://www.ambergtechnologies.ch/en/products/rail-surveying/grp-system-fx/ (accessed on 2 September 2018).

22. Lee, K.-C.; Kim, S.; Hur, H.-M.; Jeon, S.H. Evaluation of traffic safety and passenger comfort of KTX considering vertical deformation of steel-composite arch bridge due to temperature change. J. Korean Soc. Railway 2018, 21, 47–54. [CrossRef]

23. UIC 518 OR: Testing and Approval of Railway Vehicles from The Point of View of Their Dynamic Behavior, 4th ed.; International Union of Railways: Paris, France, 2009.

24. Voestalpine BWG GmBH. Available online: http://www.voestalpine.com/bwg/en/products/Kinematic-Gauge-Optimization-FAKOP/ (accessed on 8 August 2018).

25. KS R 9216, Railway Rolling Stock—Test and Evaluation Method for Passenger Comfort; Korean Agency for Technology and Standards: Eumseong, Korea, 2015.

26. KR. Guideline for Track Maintenance; Korea Railroad Network Authority: Daejeon, Korea, 2016.

27. Ono, S.; Ukai, T. Development of a track management method for Shinkansen speed increases. JR East Tech. Rev. 2012, 12, 70–75.

28. Iwnicki, S. Handbook of Railway Vehicle Dynamics; CRC Press: Boca Raton, FL, USA, 2006.

29. Mao, L.; Lu, Y. Critical speed and resonance criteria of railway bridge response to moving trains. ASCE J. Bridge Eng. 2013, 18, 131–141. [CrossRef]

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