Measuring time delay in path MTU discovery in transmitting a packet in IPv4 and IPv6 network

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Abstract Path MTU Discovery (PMTUD) was initially designed for Internet protocol version 4 (IPv4) to prevent the communication loss due to smaller path MTU. This protocol is then further developed for Internet protocol version 6 (IPv6) with new set of constraints. In IPv4 network, the PMTUD activates when the packets DF bit is set, while as in IPv6, PMTUD is always running for every packet. In this paper, we analyzed the time consumed to transmit a single packet from source to destination in IPv6 and IPv4 network in the presence of PMTUD. Based on our analysis we concluded that the communication time increases with the varying MTU of the intermediate nodes. Moreover, we formulated the mathematical model to determine the communication delay in a network. Our model shows that the asymptotic lower bound for time taken is $\Omega(n)$ and the asymptotic upper bound is $\Theta(n^2)$, using path MTU discovery.

Introduction

The Path MTU Discovery (PMTUD) protocol was designed for Internet protocol version 4 (IPv4) with the aim to discover the minimum path MTU of the path from source to destination. The goal is to reduce the packet drop frequency in networks. The protocol works for the packets where the don’t fragment (DF) bit is set or in scenarios where the intermediate nodes are not allowed to fragment the incoming packets. The protocol make use of ICMP messages to inform the source about the minimum MTU of the path from the source to the destination RFC792.

The ICMP message encapsulates the size of the last node’s forwarding MTU value along the cause of the packet drop with message as "fragmentation needed and DF bit set". On reaching this message to the source node it further fragments the packet and re-transmits it into small chunks of size equal to or lower then the said MTU value informed by the ICMPv4 message, It keeps on repeating the process until it reaches to destination RFC1911.
In case of IPv6 network, the intermediate nodes are not allowed to fragment the packets RFC8200 [2017]. This decision was carried out mainly for the reason as the fragmentation is considered harmful Kent and Mogul [1995] and has many effects on security and network performance of wired, wireless Pope and Simon [2013a], Internet of Things (IOT) Pope and Simon [2013b] and 6LoWPAN Mesrinejad et al. [2011] networks. As a result, PMTUD protocol is always active in such networks. The Path MTU discovery v6 uses ICMPv6 message protocol with (Type 2, Code 0) error message as "Packet too Big" RFC4443 [2006]. On receiving this ICMPv6 packet RFC4443 [2006] it contains the next Path MTU value of the problem occurred node and the source node regenerates the same packet of size equal to the informed MTU value in ICMPv6 message and re-transmits it and this process keeps on repeating until the packet is reached to destination RFC8201 [2017].

In both the networks, IPv4 and IPv6, Path MTU Discovery results in increase in time delay if the process of re-transmission keeps on repeating and hence adversely affects the network throughput as it largely depends on the time delay. There are many factors for the re-transmission of a single packet through a network path but most of the common factors are:

- Random and decreasing MTU value of the nodes in the path.
- ICMP message unreachable due to firewall restrictions Luckie, Cho and Owens [2005], MTU mismatch Chuachan, Djemame and Puangpronpitag [2020], routers are configured to not send ICMP destination unreachable messages, software bugs responsible for PMTUD failures Luckie and Stasiewicz [2010] and PMTUD Black holes BB12 [2012] cases the source to continuously send packets without knowing the path MTU after every timeout.
- The change in path MTU value over time due to the change in routing topology Piltzecker and Posey [2008]; Teixeira et al. [2004] after every next re-transmission making increased use of PMTUD algorithm results in time consumption in sending a packet RFC1911 [1990].

The aim of this paper is to analyze time consumption due to re-transmission of packet in the network concentric only to the first and the last factor and excluding the middle factor i.e ICMP unreachable, using Path MTU discovery for both IPv4 and IPv6 networks. We analyzed the best to the worst situation that could happen using PMTUD in IPv6. The calculations and analysis made for IPv6 can easily be extended to PMTUDv4 in IPv4. The mathematical model proposed can easily be integrated in any state-of-the-art network simulator to calculate the effect of PMTUD on various network parameters.

In this paper the time delay due to PMTUD protocol is expressed by a term "Time Wastage" and is symbolised as $P_{TW}$. Thus, "Time Wastage" is defined as the extra time that has been taken by PMTUD protocol to decrease the MTU to the minimum MTU of the path.

**Contributions:** Let us summarize the contributions of this paper:

- We prove that the asymptotic upper bound and lower bound of time wastage in
using Path MTU discovery is $\Theta(n^2)$ and $\Omega(n)$ respectively, in IPv6 and IPv4 network with DF=1.

- We calculated the maximum time wastage in sending a single packet with DF bit set in PMTUD, is of two degree polynomial equation and the minimum time wastage is one degree polynomial equation.
- We designed a mathematical model to find out the total time wastage in sending a single packet for different scenarios using PMTUD protocol in IPv6 and IPv4 network with DF=1.

The rest of the treatise is as follows. Section discusses the methods of calculating total time delay in PMTUD followed by Section where we analyse the time wastage in PMTUD. The paper concludes in Section .

**Method**

**Measurement on Time Delay in PMTUD**

To implement Path MTU Discovery algorithm is really simple, but the use of PTB messages in form of ICMP error messages made it unreliable in IPv4 and fail more often in IPv6 network. Many studies has been carried out regarding the PMTUD failure and its effects on IPv4 protocol but none of these have proposed or discussed any mathematical methods of calculating or measuring the delay associated with Path MTU Discovery in a more generalised way. We came across modeling a new start-of-art protocol and find a real need of such models in calculating the robustness and performance of our new designed algorithm to that of the Path MTU discovery, please note that in this paper we are only discussing the mathematical modeling and analysis of time delay in Path MTU Discovery, which is the most basic parameter in Internet system to measure the performance and Quality of the network, network parameters, network equipment and protocols.

Before going into the analysis we have begin with a case study where we designed a theoretical network configuration as shown in Figure 1. This network configuration can be used for both IPv4 and IPv6 packets analysis as in this study we are analysing the effect of PMTUD in time delay, so both of them using PMTUD protocol and no separate network configuration and analysis is needed Hussain and Bashir [2020]. In Figure 1 shows a network configuration between a source and destination, where the source tries to sends a packet of size 1800 octet to destination and follows different steps to complete the transmission with a path of varying MTUs. We will first measure and analyse the time delays using Path MTU Discovery. The outputs from this case study will be used later in the paper, to further analyse time consumption.

**Effect on Total Time Delay Using PMTUD**

In Figure 1 the source begins to transmit the first transmission with initial packet of size 1800 bytes, the packet travels up-to node 2, at node 2 the next-interface MTU
is lower than the incoming packet size so the node truncates the packet and sends an ICMPv6 type 2 message to source “Packet too Big”. The time taken by a packet to reach at node 2 from source is $2T_{D1}$, where $T_{D1}$ is End to End delay (E2ED) due to first transmission and the factor 2 is because it travels two links e1 and e2. Similarly, time delay for the ICMPv6 message is $2T'_{D1}$ where $T'_{D1}$ is the End to End delay between hops for ICMPv6 message for first transmission. The source on receiving the ICMPv6 message it fragments the packet and initiate the $2_{nd}$ re-transmission. The total time taken for the fragmentation process is $T_f$. Since, the waste of time up-to $2_{nd}$ re-transmission is:

$$T_W = 2(T_{D1} + T'_{D1}) + T_f$$

(1)

Where $P T_W$ is the time wastage in first transmission.

In $2_{nd}$ re-transmission of packet the size is now lower than the previous transmission and the packet travels up-to node 3 and is again been truncated by $3_{rd}$ node and send the same ICMPv6 type 2 message to source “Packet too Big”. The E2ED from source to the node 3 is $3T_{D2}$ where the factor 3 is because the packet traveled 3 links and the subscript 2 identifies the $2_{nd}$ re-transmission. The E2ED for the ICMPv6 message from node 3 to source is $3T'_{D2}$ such that, $T'_{D2} < T_{D2}$. At source the packet is again fragmented and initiate the third transmission. The fragmentation at source takes some time to fragment the packet which is $T_F$. So the E2ED for $2_{nd}$ re-transmission from source to initiation of $3_{rd}$ re-transmission is calculated as:

$$T_W = 3(T_{D2} + T'_{D2}) + T_F$$

(2)

FIGURE 1. End to End Delay using PMTUD.
Similarly, in the $3_{rd}$ re-transmission the transmission is again failed at node 5 which sends back the ICMPv6 (Type 2) message “Packet too Big” to source. Therefore the E2ED for the $5_{th}$ transmission is the sum of time delay for packet to reach node 5 from source, the time delay of ICMPv6 message from node 5 to source and the fragmentation time at source. i.e
\[
T_{W3} = 5(T_{D3} + T'_{D3}) + T_F
\]  

In $4_{th}$ re-transmission the packet size is least compared to the packet size of all previous transmissions and is equal to the lowest MTU of the node in the path. This transmission gets successful and packet reaches to destination and sends back an acknowledgement (ACK) packet to source. Since the E2ED in sending the packet from source to destination in $4_{th}$ re-transmission is given by:
\[
T_4 = p_TD(n + 1)  
\]
\[
T_4 = p_TD(5 + 1)  
\]
\[
T_4 = 6(p_TD)  
\]
where $T_4$ is E2ED of $4_{th}$ transmission which is also a successful transmission and $p_TD$ is the average time delay between consecutive node and n is number of nodes in the path.

Therefore, the total loss of time or time wastage ($p_TW$) on sending the packet from source to destination from $1_{st}$ transmission to last transmission is:
\[
p_TW = T_{W1} + T_{W2} + T_{W3}  
\]
\[
p_TW = ((2T_{D1} + T'_{D1} + T_F) + (3T_{D2} + T'_{D2} + T_F) + (5T_{D3} + T'_{D3}) + T_F)  
\]

Since there is no time loss in $4_{th}$ re-transmission as its successful transmission.

The E2ED between nodes in transmission 1, 2 , 3 are not similar that’s why we add subscript to them. It’s because of the reason that, as packet size increases the E2ED between nodes increases due to increase in over all overhead on node from processing, queueing and transmission which cause the total increase in the E2ED between nodes. The increasing order of the E2ED between nodes of the transmissions is
\[
T_{D1} > T_{D2} > T_{D3} > T_{D4}  
\]

Lets take the worst case scenario that the E2ED of the failed transmission will be equal to E2ED of last successful transmission for the sack of easy calculation i.e;
\[
T_{D1} = T_{D2} = T_{D3} = T_{D4}  
\]

Our this assumption may lightly decrease the total time wastage in PMTUD. The actual time wastage will be slightly higher then the time wastage shown in the Equation 10, which is because from Equation 9 we have:
\[
T_{Di} - T_{D4} < 0  
\]
where, \( i = (1, 2, 3) \)

Therefore, Equation 8 becomes:

\[
P_T = 10(T_{D4} + T_{D4}') + 3T_F
\]  

(10)

Since, the 4th transmission is also the successful transmission.

Therefore,

\[
T_{D4} = P_T D
\]

Hence,

\[
P_T W = 10(P_T D + T_D') + 3T_F
\]  

(11)

Therefore, The total time for transmitting the packet in PMTUD is:

\[
Total time = T_4 + P_T W
\]

\[
P_T = 6(P_T D) + 10(P_T D + P_T D') + 3T_F
\]  

(12)

So, we would neglect the changing of E2ED between nodes due to different packet size in each transmission and will take E2ED between nodes of all transmission equal to the E2ED between nodes of successful transmission. So from now own in our calculation we would neglect the E2ED between nodes of various packet size and apply the E2ED between nodes of successful transmission.

We have calculated the time wastage in IPv4 and IPv6 network due to the packet drop by intermediate nodes which we also called as extra time delay or exceed time delay using all these name reflect same meaning. The time wastage or extra time delay can be understand as data exchange between Bob and Alice, that when Bob transmits a packet to Alice it expected total time delay of \( T_D \) without any node dropping that packet, but if any node drops the packet then the Bob has to re-transmit a new packet with decreased packet size to Alice and the packet arrives to Alice in the expected time \( T_D \), since the time consumed in the previous failed transmission which comprises the E2ED of packet and the E2ED of ICMPv6 which is named as the time wastage or extra time delay. Now the total time delay in sending the packet will be :

\[
Total time delay(T) = (T_D) + (T_W)
\]

where \( T_D \) is expected time without packet loss and \( T_W \) is time wastage due to failed transmission and \( T \) is actual observed time. In other words the sum of total time delay of failed transmissions due to successive single packet loss by intermediate nodes is called time wastage or extra time delay. In the following theorems we will show different time wastage depending upon the order by which nodes drop packet and number of nodes involved using PMTUD algorithm. We will now draw some further conclusions on time wastage using PMTUD in more general way.
Analysis of Time Wastage in Path MTU Discovery

**Theorem 1.** The maximum or the worst-case total time wastage ($P_{TW}$) for the packet drop using PMTUD algorithm is:

$$P_{TW} = S_n[P_{TD} + P_{TD}^'] + nT_F,$$

where $S_n$ is sum of $n$-terms and $n$ is defined as the number of nodes between source and destination.

**Proof.** When the first node drop packet, then it has to send ICMP message to source and then source send a new packet which reaches node 1 doing this the time wastage will be the sum of E2ED from source to node 1 and the ICMP message packet delay from node 1 to source i.e $P_{TD} + P_{TD}'$. Similarly, if the packet is drop by second node only, then the time wastage will be $2(P_{Tp} + P_{TD}')$ as the node has to reach the source for Ack packet and send the packet again up-to 2nd node it will take $2(P_{Tp} + P_{TD}')$ extra time to do that. Since, if the packet is dropped by any number of nodes in the path between source and destination, then the time wastage is the effecting node multiply to the sum of E2ED of packet and ICMPv6 packet and it forms an arithmetic progression of difference of $(P_{Tp} + P_{TD}')$, so by A.P the $an$th term is given by

$$a_n = a + (n - 1)d, \forall n \in \text{(nodes)} \rightarrow \mathbb{Z}^+$$

Since, $a = P_{TD} + P_{TD}' + T_F$, $d = P_{TD} + P_{TD}'$

Then the time wastage for incremental nodes dropping packets can be derived and is given by:

$$P_{TD} + P_{TD}' + T_F, \ 2P_{TD} + 2P_{TD}' + T_F, \ 3P_{TD} + 3P_{TD}' + T_F, \ 4P_{TD} + 4P_{TD}' + T_F, \ 5P_{TD} + 5P_{TD}' + T_F, \ 6P_{TD} + 6P_{TD}' + T_F, \ \ldots, \ nP_{TD} + nP_{TD}' + T_F$$

Then the total time wastage is given by adding time wastage due to each node 1 upto $n^{th}$:

$$P_{TW} = [P_{TD} + P_{TD}'] + [2P_{TD} + 2P_{TD}'] + [3P_{TD} + 3P_{TD}'] + [4P_{TD} + 4P_{TD}'] + \ldots + [nP_{TD} + nP_{TD}'] + nT_F$$

$$= S_n[P_{TD} + P_{TD}'](1 + 2 + 3 + 4 + \ldots + n) + nT_F$$

$$= S_n[P_{TD} + P_{TD}'] + nT_F, \ \forall n \in \mathbb{Z}^+$$

**Corollary 1.1.** The asymptotic upper bound for the worst-case time wastage using Path MTU discovery is of $\Theta(n)$. 
Measuring Time Delays

Proof. Since, from the Theorem 1, the equation 13 which can be written as function of \( n \): 

\[
\begin{align*}
\frac{P_T D + P_T'}{2} + n \left( \frac{P_T D + P_T'_{D}}{2} + T_f \right) 
\end{align*}
\]

Which is in the form of two-degree polynomial equation of form \( an^2 + bn + c \) where \( a = \left[ \frac{P_T D + P_T'}{2} \right]/2, b = \left[ \frac{P_T D + P_T'_{D}}{2} + T_f \right], c = 0 \) and \( a > 0 \), therefore we can present this equation in asymptotic notation which has the positive constant of \( c_1, c_2 \) and \( n_0 \) therefore the constants can be found as:

\[
\begin{align*}
& c_1 n^2 \leq f(n) \leq c_2 n^2 \\
& c_1 n^2 \leq n^2 \left( \frac{P_T D + P_T'}{2} \right) + n \left( \frac{P_T D + P_T'_{D}}{2} + T_f \right) \leq c_2 n^2 \\
& c_1 \leq \left( \frac{P_T D + P_T'}{2} \right) + \left( \frac{P_T D + P_T'_{D} + 2T_f}{2n} \right) \leq c_2 
\end{align*}
\]

The right hand inequality hold for any value of \( n \geq 1 \) by choosing any kind of constant \( c_1 \leq P_T D + P_T' \). Similarly , we make the left side inequality hold true for any value of \( n \geq 1 \) by selecting any constant \( c_2 \geq P_T D + P_T' + T_f \). Since by Choosing \( c_1, c_2, \) and \( n \) at these given values we can say that:

\[
\begin{align*}
f(x) = \Theta(n^2)
\end{align*}
\]

Since, the asymptotic upper bound of worst-case time wastage in PMTUD algorithm is \( \Theta(n^2) \), where \( n \) is the number of nodes in the path between source and the destination which the packet has traversed.

Since \( f(x) \) has resulted in a asymptotic tight bound of \( \Theta(n^2) \) which can yield both upper bound and lower bound of time wastage in PMTUD. The upper bound can also be expressed as \( O(n^2) \) Cormen et al. [2001]. However, the lowest bound at worst-case time wastage of PMTUD algorithm is \( \sigma(n^2) \) and can’t be the overall lower bound of time wastage in PMTUD as in normal packet drop in PMTUD the lower bound of time delay is \( \sigma(n) \) as detailed in Corollary 2.1.

Theorem 2. When a packet has been truncate by any single intermediate node using PMTUD algorithm, then the time wastage \( ^{P}T_{W} \) for transmitting the single packet is given by:

\[
^{P}T_{W} = n_1 [P_T D + ^{P}T_{D'}] + T_f, \\
\forall n_1 \subset n,
\]

where \( n \in \text{ (nodes) } \rightarrow \mathbb{Z}^+, \) such that \( n_1 \in n_i \) represents the node which dropped packet first.
Proof. Let the time delay between hops be $PT_D$ and the time delay for ICMPv6 message be $PT'_D$ as the ICMPv6 message can’t be greater then 1280 octets, therefore ICMPv6 time delay be less then then transmitting packet delay. On sending back the new packet the source need to do fragmentation and it will take a small time called fragmentation time $T_F$.

When $1^{st}$ node drop packet the time wastage would be the time until it route the packet to $2^{nd}$ node that will be the delay between the source to $1^{st}$ hop and the ICMP message delay by hop to the source and time for the fragmentation by source.

\[ PT_D + PT'_D + T_F \]

Similarly, for other nodes the time wastage is,

- $1^{st}$, \[ PT_W = PT_D + PT'_D + T_F \text{ or } 1[PT_D + PT'_D] + T_F \]
- $2^{nd}$, \[ PT_W = 2PT_D + 2PT'_D + T_F \text{ or } 2[PT_D + PT'_D] + T_F \]
- $3^{rd}$, \[ PT_W = 3PT_D + 3PT'_D + T_F \text{ or } 3[PT_D + PT'_D] + T_F \]
- $4^{th}$, \[ PT_W = 4PT_D + 4PT'_D + T_F \text{ or } 4[PT_D + PT'_D] + T_F \]
- $5^{th}$, \[ PT_W = 5PT_D + 5PT'_D + T_F \text{ or } 5[PT_D + PT'_D] + T_F \]
- $6^{th}$, \[ PT_W = 6PT_D + 6PT'_D + T_F \text{ or } 6[PT_D + PT'_D] + T_F \]

Since, from Equation 15 to 20 we see that the $PT_W$ for each node depends on the position number of the node factor with $[PT_D + PT'_D]$, having constant $T_F$ throughout. Therefore, the general equation for the time wastage for $n_1$ node drop packet is:

\[ PT_W = n_1[PT_D + PT'_D] + T_F, \]

\[ \forall n_1 \subset n, \text{ where } n \in (\text{nodes}) \rightarrow \mathbb{Z}^+. \]

Corollary 2.1. The asymptotic lower bound for the time wastage using Path MTU discovery is of $\Omega(n)$

Proof. From theorem 2 the equation 14 form a linear 1 degree polynomial equation of form $an + b$ where $a = [PT_D + PT'_D]$ and $b = T_F$ which shows that:

\[ f(n) = an + b \]
\[ f(n) = \Omega(n) \]

where \[ cn \leq \frac{[PT_D + PT'_D]}{n} + T_F \]
dividing both the sides of inequality by \( n \) we get:

\[
c \leq \left[ P \frac{T_D}{n} + P' \frac{T_D'}{n} \right] + \frac{T_f}{n}
\]

The left hand inequality can be hold for any value \( n \geq 0 \) choosing any constant \( c \leq \left[ P \frac{T_D}{n} + P' \frac{T_D'}{n} \right] \).

Therefore, the asymptotic lower bound is \( \Omega(n) \). which implies that this is the best case scenario of the time wastage in Path MTU discovery.

\[\text{Theorem 3.} \quad \text{The minimum total time wastage for nodes dropping packet at least in one position in the path between source and destination in PMTUD is given by,}\]

\[
P_T = \sum_{i=1}^{a} \left[ n_i \left( P \frac{T_D}{n} + P' \frac{T_D'}{n} \right) \right] + aT_F
\]  

(21)

Where \( n_i \) is the position of the node in the path which drops packet at \( i \) times, also \( T_F \) is the time of fragmentation by source node and \( a \) is number of times packet dropped in the path.

\[\text{Proof.} \quad \text{In Theorem} 2\text{we have} P_T = n_1 \left[ P \frac{T_D}{n} + P' \frac{T_D'}{n} \right] + T_F \text{ which is time wastage for a node dropping packet in the path between the source and destination. The time wastage for two nodes dropping packets in the path is given by} \ [n_1 + n_2] \left[ P \frac{T_D}{n} + P' \frac{T_D'}{n} \right] + 2T_F,
\]

\[\text{where,} \quad n_1 \in n \text{ and} \quad n_2 \in n - (1, 2, 3, ..., n_1) , \quad n_1 \neq n_2 , \quad n_1 < n_2, \text{ such that} \quad n \in (\text{nodes}) \to \mathbb{Z}^+ \text{are first and second nodes respectively which dropped packets in the path. Similarly, as we go on increasing the number of nodes which dropping packets to} \ i \text{ their is always a term} \ n_i \left[ P \frac{T_D}{n} + P' \frac{T_D'}{n} \right] + T_F \text{ is incremented to the preceding time wastage, so in general for all of nodes which drop packet in the path in different order can be given by :}\]

\[
P_T = \left( \left( P \frac{T_D}{n} + P' \frac{T_D'}{n} \right) \sum_{i=1}^{a} [n_i] \right) + aT_F,
\]

(22)

\[\text{where} \quad a \text{ is the number of nodes which dropped packet. Now the Equation} 22\text{is a general equation for the total time wastage for all cases for nodes which drop packet from} \ i = 1 \text{ to} \ i = n, \text{ where} \quad n \in (\text{nodes}) \to \mathbb{Z}^+. \]

\[\text{Theorem 4.} \quad \text{The minimum time wastage equals to the maximum total time wastage using PMTUD algorithm at} \ a = n , \text{ which is given by,}\]

\[
P_T = S_n \left[ P \frac{T_D}{n} + P' \frac{T_D'}{n} \right] + nT_f.
\]

(23)
Proof. If all the nodes in between the path of source and destination drops the packet consecutively then the theorem for the minimum total time wastage is given by:

\[ P_{TW} = (P_{TD} + P_{TD}') \sum_{i=1}^{n} [n_i] + nT_F \]  

(24)

where \( a = n \), as all nodes are dropping packet

Expanding Equation [24] by putting values of \( i \) which runs from 1 to \( n \) and is given by:

\[ P_{TW} = [n_1(P_{TD} + P_{TD}') + T_F] + [n_2(P_{TD} + P_{TD}') + T_F] + [n_3(P_{TD} + P_{TD}') + T_F] + \ldots + [n_n(P_{TD} + P_{TD}') + T_F] 
\]

\[ = [P_{TD} + P_{TD}'][n_1 + n_2 + n_3 + \ldots + n_n] + [1 + 1 + 1 + \ldots + n - times]T_F 
\]

\[ = [P_{TD} + P_{TD}'] [1 + 2 + 3 + 4 + \ldots + n] + nT_F 
\]

\[ = [P_{TD} + P_{TD}'] S_n + nT_F 
\]

\[ P_{TW} = S_n [P_{TD} + P_{TD}'] + nT_F \]  

(25)

Since the Equation [25] which is derived from General formula of minimum time wastage is equal to the maximum total time wastage at \( a = n \).

\[ P_{TW} = S_n [P_{TD} + P_{TD}'] + nT_F \]

\[ \quad \text{since, the minimum lower bound } P_{TW} \text{ is } n_1 [P_{TD} + P_{TD}'] + T_F \text{ when one node drop packet at } n_1^{th} \text{ position in the path. As we goes on increasing value of } a \text{ the } P_{TW} \text{ also goes on increasing up-to a certain point where } a = n \text{ then } P_{TW} = S_n [P_{TD} + P_{TD}'] + nT_f \text{, which is the upper bound total time wastage} \]

\[ \therefore n_1 [P_{TD} + P_{TD}'] + T_F \leq P_{TW} \leq S_n [P_{TD} + P_{TD}'] + nT_f \]  

(26)

\[ \text{Since, the minimum lower bound } P_{TW} \text{ is } n_1 [P_{TD} + P_{TD}'] + T_F \text{ when one node drop packet at } n_1^{th} \text{ position in the path. As we goes on increasing value of } a \text{ the } P_{TW} \text{ also goes on increasing up-to a certain point where } a = n \text{ then } P_{TW} = S_n [P_{TD} + P_{TD}'] + nT_f \text{, which is the upper bound total time wastage} \]

\[ \therefore n_1 [P_{TD} + P_{TD}'] + T_F \leq P_{TW} \leq S_n [P_{TD} + P_{TD}'] + nT_f \]  

(27)

(28)

(29)

Therefore, the value of \( P_{TW} \) can’t be lower then \( n_1 [P_{TD} + P_{TD}'] + T_F \) and can’t be higher then \( S_n [P_{TD} + P_{TD}'] + nT_f \).

\[ \square \]
Conclusions & Future Work

In this paper, we analyzed the effect of Path MTU Discovery in IPv6 and IPv4 networks. We concluded that the time wastage varies with the increase in the number of re-transmissions for a single packet. We adopted new theorems and corollaries to clearly calculate the time wastage resulted due to continuous use of Path MTU Discovery in IPv6 and IPv4 networks. The comprehensive analysis carried-out in the paper are one of the first stages of research in time delays encountered in Path MTU Discovery in IPv4 and IPv6 networks. The conclusion that is drawn from these analysis is that the time wastage due to Path MTU discovery has an asymptotic lower bound of $\Omega(n)$ and upper bound of $\Theta(n^2)$ with the domain depending only on the number of nodes the packet traverses the path.

The analysis can be used by the research community to fine tune some of the parameters in order to reduce the delays associated with PMTUD protocol. The analysis can also been used as the way to find the optimistic and robustness of a new protocol design than the present one.

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Key Words

Packet Drop; Time Delay; Ipv6 Protocol; Time Wastage; Path MTU Discovery; Path MTU.