**The Role of Center Vortices in QCD**

C. Alexandrou\(^a\), Ph. de Forcrand\(^b\) and M. D’Elia\(^c\)

\(^a\)PSI, CH-5232 Villigen, Switzerland and University of Cyprus, CY-1678 Nicosia, Cyprus

\(^b\)ETH, CH-8092 Zürich, Switzerland

\(^c\)University of Pisa and INFN, I-56127 Pisa, Italy

Center vortices are unambiguously identified after Laplacian Center Gauge fixing and their influence on confinement and chiral symmetry breaking is investigated on a sample of SU(2) configurations at zero and finite temperature.

1. Introduction

Within the vortex theory of confinement, put forward about twenty years ago [1], the QCD vacuum is considered as a condensate of colour magnetic vortices with a flux quantized in terms of the center of the group \(Z_N\). A center vortex is associated with a singular gauge transformation with a discontinuity given by a gauge group center element. It has the effect of multiplying the Wilson loop linked to this vortex by an element of \(Z_N\), i.e. \(W(C) \rightarrow e^{2\pi i n/N}W(C), n = 1, ..., N - 1\). Assuming that center vortices condense in the QCD vacuum, the area law behaviour of large Wilson loops follows from fluctuations in the number of vortices linking the loops.

A procedure to identify these vortices on the lattice via gauge fixing was shown recently to yield results in accord with vortex condensation theory [2]. The main idea consists of choosing a gauge that makes the link variables \(U\) as close as possible to the center of the gauge group. The Direct Maximal Center (DMC) gauge proposed by Del Debbio et al. [3] determines a gauge transformation \(g \in SU(N)\) that maximises the quantity

\[
R[U] = \sum_{x,\mu} |\text{Tr} \ U_{\mu}^{GF}(x)|^2
\]

where \(U_{\mu}^{GF}(x) = g(x)U_{\mu}(x)g^\dagger(x + \hat{\mu})\). Then the gauge fixed links are projected onto \(Z_N\), i.e. one replaces each \(U^{GF}\) by its closest center element \(Z\) in the evaluation of the observables. For SU(2) the center projection is defined by

\[
Z_\mu(x) = \text{sign} \left[\text{Tr} \ U_{\mu}^{GF}(x)\right] I
\]

and from now on, for simplicity, we will be discussing only SU(2). A plaquette in the \(Z_2\) projected theory with value \(-1\) represents a defect called a P-vortex. The idea of center dominance is that center vortices, identified as P-vortices in the DMC gauge, are

---

*Talk presented by C. Alexandrou*
the relevant nonperturbative degrees of freedom. It is supported by numerical results: the $Z_2$-theory shows a string tension similar to that of the full-theory $[3]$; the deconfinement phase transition $[4]$ is well described in the center vortex picture as a percolation transition.

The center-dominance scenario received further support by the following test carried out in Ref. $[5]$: The P-vortices were removed by considering a modified ensemble with links $U'_\mu(x) \equiv Z_\mu(x)U_\mu(x)$ which projects onto the trivial $Z_2$ vacuum. It was shown $[5]$ that in this modified ensemble, both confinement is lost and chiral symmetry is restored.

We have investigated the chiral content of the $Z_2$-projected theory further, by looking at the quark condensate $\langle \bar{\psi}\psi \rangle$ in the $Z_2$ sector as a function of the quark mass $m_q$, at zero and finite temperature. At zero temperature it extrapolates linearly to a non-zero value as $m_q \to 0$ as shown in Fig. 1. For very small quark masses it diverges as $1/m_q$, revealing the presence of a few extremely small eigenvalues, which may be caused by the non-trivial topology of the original $SU(2)$ gauge field. This behaviour is strikingly similar to that observed with domain-wall fermions $[6]$. It is well described by the functional form

$$\langle \bar{\psi}\psi \rangle_{Z_2} = a + b/m_q + c m_q.$$  \hspace{1cm} (3)

This ansatz also works well at finite temperature. The comparison of data on $16^3 \times N_t$ volumes at $\beta = 2.4$, with $N_t = 16, 8$ and 4 (the latter in the deconfined phase) reveals that $a, b$ and $c$ do not vary much with temperature. $c$ remains close to its free-field value. $a$ tends to decrease somewhat in the deconfined, chirally symmetric phase, but remains surprisingly large. $b$ may also show some decrease, but not in the same proportion as the fluctuations of the $SU(2)$ topological charge. Therefore, one has to be cautious in relating the $Z_2$ condensate to the chiral properties of $SU(2)$.

![Figure 1](image-url) Quark condensate versus quark mass, in the original (+), the center-vortex free (x), and the center-projected (squares) theories.

2. Laplacian Center Gauge

A local iterative maximization of the DMC gauge-fixing condition Eq. $[1]$ selects any one of the many possible maxima. Each of these Gribov copies will have its own set of P-vortices, which may show dramatically different properties $[7]$. In order to find a center
gauge fixing procedure that is free of Gribov copies, we first note that DMC is equivalent to maximizing $\sum_{x,\mu} Tr_{adj} U_{\mu}(x)$, since

$$|Tr U|^2 = Tr U^2 + 2 = Tr_{adj} U + 1 \quad \text{taking} \quad Tr \sigma_a \sigma_b = 2\delta_{ab}.$$ \hspace{1cm} (4)

The idea is thus to smooth the center-blind, adjoint component of the gauge field as much as possible, then to read the center component off the fundamental gauge field. Therefore, Maximal Center Gauge is just another name for adjoint Landau gauge.

The problem of Gribov copies in the fundamental Landau gauge was solved in [8]: If one relaxes the requirement that $g \in SU(2)$, the maximization of the gauge-fixing functional is achieved by taking for $g^\dagger$ the eigenvector $\vec{v}$ associated with the smallest eigenvalue of the covariant Laplacian $\Delta_{xy} = 2d \delta_{xy} - \sum_{\pm \mu} U_{x \pm \mu}(x) \delta_{x \pm \mu, y}$. At each site, $v(x)$ has 2 complex color components. The Laplacian gauge condition consists of taking for $g^\dagger$ the $SU(2)$ projection of $v$, thus rotating $v(x)^\dagger$ along direction $(1,1)$ at all sites.

We follow this construction for the adjoint representation. The covariant Laplacian is now constructed from adjoint links $U_{ab}^{\dagger} = \frac{1}{2} Tr [U \sigma^a U^\dagger \sigma^b]$, $a, b = 1,2,3$. It is a real symmetric matrix. The lowest-lying eigenvector $\vec{v}$ has 3 real components $v_i$, $i = 1,2,3$ at each site $x$. One can apply a local gauge transformation $g(x)$ to rotate it along some fixed direction. Note, however, that this does not specify the gauge completely: Abelian rotations around this reference direction are still possible. What we have achieved at this stage is a variation of Maximal Abelian Gauge which is free of Gribov ambiguities. This Laplacian Abelian Gauge has been proposed in [9] and, as it was shown there, monopoles are directly identifiable by the condition $|v(x)| = 0$ for smooth fields. Abelian monopole worldlines appear naturally as the locus of ambiguities in the gauge-fixing procedure: the rotation to apply to $v(x)$ cannot be specified when $|v(x)| = 0$.

To fix to center gauge, we must go beyond Laplacian Abelian Gauge and specify the Abelian rotation. This is done most naturally by considering the second-lowest eigenvector $\vec{v}^{\prime}$ of the adjoint covariant Laplacian, and requiring that the plane $(v(x), v'(x))$ be parallel to, for instance, $(\sigma_3, \sigma_1)$ at every site $x$. This fixes the gauge completely, except where $v(x)$ and $v'(x)$ are collinear. Collinearity occurs when $\frac{v_2}{v_3} = \frac{v'_{2}}{v'_{3}} = \frac{v_{1}}{v_{3}}$, i.e. 2 constraints must be satisfied. Thus, gauge-fixing ambiguities have codimension 2: in 4$d$, they are 2$d$ surfaces. They can be considered as the center-vortex cores [11].

3. Results

We have applied Laplacian Center Gauge fixing and center projection to an ensemble of $SU(2)$ configurations. The main difference with DMC gauge is an increase in the $P$-vortex density ($\sim 11\%$ vs $\sim 5.5\%$ on a 16$^4$ lattice at $\beta = 2.4$), similar to the increase in the monopole density for Laplacian Abelian Gauge [11]. As in [8], the string tension, the quark condensate and the topological charge all vanish upon removal of the $P$-vortices. Fig. 2 displays the Creutz ratios $\chi(R, R) = -\ln [(W(R,R)(W(R-1,R-1))/(W(R,R-1))^2]$ constructed from averages $\langle W(R,T) \rangle$ of $R$ by $T$ Wilson loops. For large $R$, $\chi(R,R)$ approaches the string tension $\sigma$. On the modified configuration the string tension goes to zero whereas in the $Z_2$-projected theory it reproduces the $SU(2)$ value of Ref. [10]. The quark condensates behave as in Fig. 1.

If one applies Laplacian Center Gauge fixing to a cooled one-instanton configuration, one finds a very small number ($\lesssim 100$) of P-vortices, regardless of the original instanton size. These P-vortices are all near the instanton center, as illustrated in Fig. 3.
Figure 2. Creutz ratios for the original, the modified and the $Z_2$ projected ensembles. The dashed line is the string tension result of Ref. 10.

4. Conclusions

Any Gribov ambiguities that cast doubt on the physical relevance of P-vortices (pointed out e.g. by [7]) are removed by the Laplacian Center Gauge fixing. This gauge appears naturally as an extension of Laplacian Abelian Gauge. It allows the direct identification of center vortices (and monopoles) by inspection of the two lowest eigenmodes of the covariant adjoint Laplacian, without gauge-fixing. As in DMC gauge, center dominance seems to hold: (i) At zero temperature the string tension is well reproduced in the $Z_2$-projected theory, and even the $Z_2$ quark condensate is non-zero; (ii) In the deconfined phase, the $Z_2$ string tension vanishes: however the $Z_2$ quark condensate does not.

Although here we have only discussed SU(2), our gauge fixing procedure readily generalizes to SU($N$): complete gauge-fixing is achieved by rotating the first ($N^2-2$) eigenvectors of the adjoint Laplacian along some reference directions. Ambiguities arise whenever these ($N^2-2$) eigenvectors [each with ($N^2-1$) real components] become linearly dependent. This again defines codimension-2 center-vortex cores.

REFERENCES

1. G. t’Hooft, Nucl. Phys. B153 (1979) 141; N. K. Nielsen and P. Olesen, Nucl. Phys. B144 (1978); G. Mack, Cargèse lectures, (1979); J. Ambjørn and P. Olesen, Nucl. Phys. B170 (1980).
2. L. Del Debbio, M. Faber, J. Greensite and Š. Olejník, Phys. Rev. D55 (1997) 2298.
3. L. Del Debbio et al., Phys. Rev. D58 (1998) 094501.
4. M. Engelhardt, K. Langfeld, H. Reinhardt and O. Tennert, hep-lat/9904004.
5. Ph. de Forcrand and M. D’Elia, Phys. Rev. Lett. 82 (1999) 4582.
6. P. Chen et al., Nucl. Phys. (Proc.Suppl.) B73 (1999) 207.
7. T. G. Kovacs and E. T. Tomboulis, hep-lat/9905029.
8. J. C. Vink and U. Wiese, Phys. Lett. B289 (1992) 122.
9. A.J. van der Sijs, Nucl. Phys. (Proc.Suppl.) B53 (1997) 535; hep-lat/9803001; hep-lat/9809126.
10. C. Michael and M. Teper, Phys. Lett. B199 (1987) 95.
11. C. Alexandrou, M. D’Elia and Ph. de Forcrand, hep-lat/9907028.