3D massive Dirac fermions with chemical potential in external magnetic field: Current-current correlation function

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Abstract

The response of fermionic system to external gauge fields in presence of non-quantized magnetic field is determined by current-current correlation function $\Pi_{\mu\nu}(B)$. We study $2D$ dimensional Dirac electron system and calculate current-current correlation function in a presence of magnetic field $B$, chemical potential $\eta$ and gap $m$.  

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1 Introduction

The experimental and theoretical study of graphene, two-dimensional graphite, is an extremely rapidly growing field of today’s condensed matter research. The reasons for enormous scientific interest are manifold. Graphene is a zero gap semiconductor because its conduction and valence bands meet at the Dirac point [1]. Electronic properties of graphene is sensitive to environmental conditions therefore there will be changed in presence of other layers. Graphene has peculiar band structure as a result electrons at Fermi energy are described an effective Lorentz invariant theory. Electrons propagating through graphene’s honeycomb lattice effectively lose their mass, in result producing quasi-particles that are described 2D analogue of Dirac equation [2]. The theoretical and experimental studies of the influence of the external fields on the graphene transport features are held recently [3, 4]. The constant magnetic field acts as a strong catalyst of dynamical symmetry breaking leading to the generation of fermion masses in 2 + 1 dimension. There is a striking similarity between the role of magnetic field in 2 + 1 dimensional models and the role of Fermi surface in the Bardeen-Cooper-Schrieffer (BCS) theory of superconductivity [5, 6]. The magnetic field influences on the high frequency conductivity and on the electromagnetic waves absorption of the graphene are investigated in. Magnetic field applied to graphene gives rise to discrete Landau levels which are essentially important in the explanation of a anomalous quantum Hall effect in graphene [7, 8, 9]. By analyzing properties of quantum Hall effect in a weak field was obtained that low energy excitations in graphene are Dirac quasiparticles. The dependence of Hall conductivity on the magnetic field intensity was investigated. The graphene conductivity have the oscillations when the magnetic field intensity changes [10]. However, current response functions are not fully studied in presence of magnetic field. In the paper [11] the transversal part of the polarization operator $\Pi^\mu\nu$ was calculated. The goal in this work is the calculation of current density correlation function $\Pi^{\mu 0}$ when we have third order Feynman’s diagram in presence of gap $m, \eta$ chemical potential and $B$ magnetic field. Exact expression of
polarization operator without magnetic field but for finite chemical potential \( \eta \) and gap \( m \) was calculated in \[12\].

## 2 Current-current correlation function \( \Pi^{\mu\nu}(B) \)

The action which describes the graphene in the Effective Field Theory (EFT) framework via \( N_f \) four-component massive Dirac fermions with instantaneous three-dimensional Coulomb interactions is the following (in Euclidean space time) \[13, 14\]

\[
S_g = -\sum_{i=1}^{N_f} \int d^2 x dt \bar{\psi}_i \left( \gamma^0 \partial_0 + v \gamma^k \partial_k + i A_0 \gamma^0 + m \right) \psi_i + \frac{1}{2g^2} \int d^2 x dt (\partial_\mu A_\mu)^2. \tag{1}
\]

Here \( v \) is the velocity, which can be taken as 1 in the calculations and then restore in the resulting formulas. In real graphene \( N_f = 2 \), \( \gamma \)-matrices satisfy to Euclidean Clifford algebra and can be chosen as

\[
\gamma^0 = \sigma^3 \otimes \sigma^3, \quad \gamma^i = \sigma^i \otimes 1, \quad \{ \gamma^\mu, \gamma^\nu \} = 2 \delta^{\mu\nu}. \tag{2}
\]

The four-component fermionic structure is conditioned by the existence of the quasiparticle excitations in two sublattices in the graphene around two Dirac points.

Since each Dirac point contributes to response function additively, below, for simplicity, we will be concentrated on calculation of current-current correlation function only for single Dirac point. Therefore we start from free Dirac action in three dimensional space-time with chemical potential \( \eta \), gap \( m \) and \( B \) magnetic field, which after Wick rotation to complex time/energy acquires the form

\[
S = \int \frac{d k d \omega}{(2 \pi)^3} \bar{\psi}_{k,\omega} \left[ \sigma k + \sigma_3 m - \left( \omega - i \eta \right) \right] \psi_{k,\omega}, \tag{3}
\]

where the Fourier transformation is done \( (k = \{k_1, k_2\}) \) and in the role of \( \gamma \) functions Pauli matrices are taken. Here we intend to calculate the current-current correlation function for the three-dimensional theory with the kinetic part for the fermions presented above and the interaction term with \( U(1) \) gauge field \( A_\mu \) in the third order approximation.
The magnetic field dependence of the current-current correlation function is defined by third order Feynman diagrams in Fig.1, where vector potential $A_{\rho}$ couples to vertex $\rho$. After some transformations diagram a) reads

$$
\Pi_{\mu 0} = Ng \int_{-\infty}^{+\infty} \frac{d^3k}{(2\pi)^3} \text{Tr}[\sigma_{\mu} G(\hat{k}^+) A_{\rho} \sigma_{\rho} G(\hat{k}^+ + \hat{p}) \sigma_3 G(\hat{k}^-)]
$$

where $G(\hat{k}) = \frac{\hat{k} - m}{\hat{k}^2 + m^2}$ is the Green function of the fermion and we have used the notation $k^\pm = (k^0 \pm \frac{q^0}{2}, \Omega \pm \frac{q^1}{2})$. By using identity $A_{\rho} \sigma_{\rho} \hat{p} = \tilde{A} \hat{p} + i\epsilon_{\nu\rho} A_{\nu} p_{\rho} \sigma_3 = iB \sigma_3$ in second row of the expression (4), where we have dropped $\tilde{A} \hat{p}$ term since it gives zero, we come to following Trace in the nominator

$$
B \text{Tr}[\sigma_{\mu}(\hat{k}^+ - m)\sigma_3\sigma_3(\hat{k}^- - m)] = 2B(\epsilon_{\mu\nu\sigma} k^+_{\nu} k^-_{\sigma} - m(k^+ + k^-)_{\mu})
$$

$$
= 2B[\epsilon_{\mu\nu}(q_{\nu} \Omega - k_{\nu} \omega) - 2mk_{\mu}]
$$

In the same way one can find corresponding expression for Trace for diagram of Fig.1(b), which coincides with (5).

We see, that in three dimensional space the third order Feynman’s diagrams are not vanish, therefore, summarizing Trace results we obtain $4B[\epsilon_{\mu\nu}(q_{\nu} \Omega - k_{\nu} \omega) - 2mk_{\mu}]$.
3 Calculation of $\Pi_{\mu 0}$

Using the trace result $\Pi_{\mu 0}$ is acquire following form

$$\Pi_{\mu 0}(B) = N g^2 \int_{-\infty}^{+\infty} \frac{d^3k}{(2\pi)^3} \left( \frac{4B[\epsilon_{\mu\nu}(q_\nu \Omega - k_\nu \omega) - 2mk_\mu]}{(k^2 + m^2)(k^2 + m^2)^2} + \frac{4B[\epsilon_{\mu\nu}(q_\nu \Omega - k_\nu \omega) - 2mk_\mu]}{(k^2 + m^2)(k^2 + m^2)^2} \right) \quad (6)$$

where $k^2 = \vec{k}^2 + (\Omega + \Gamma + i\eta)$. Generally $\Pi_{\mu\nu}$ must satisfy the condition of conservation of charge $\partial_\mu \Pi_{\mu\nu} = 0$. The evaluation of such integrals performs with the method of Feynman parametrization. These method gives opportunity to squeeze the three denominator factors into single quadratic as polynomial of $k$. After we shift $k$ by a constant. It is easy to begin with trivial case when in denominator we have two factors

$$1_{AB}^2 = 2 \int_0^1 dx_1 dx_2 \frac{\delta(x_1 + x_2 - 1)}{(x_1 A + x_2 B)^2} \quad (7)$$

When we have three factors then

$$1_{ABC}^3 = 2 \int_0^1 dx_1 dx_2 dx_3 \frac{\delta(x_1 + x_2 + x_3 - 1)}{(x_1 A + x_2 B + x_3 C)^3} \quad (8)$$

Our integral (6) has three factors in the denominator, therefore by using (8) we obtain

$$1_{AB}^2 = \frac{\Gamma(1 + 2)}{\Gamma(1)\Gamma(2)} \int_0^1 du_1 du_2 \frac{\delta(u_1 + u_2 - 1)u_2}{(u_1 A + u_2 B)^3} = 2! \int_0^1 du \frac{1 - u}{(uA + (1 - u)B)^3} \quad (9)$$

where $A = k^2 + m^2$, $B = k^2 + m^2$. Easy to find out, that making shift $k^\pm = k^\prime \pm (1/2 - u)q$ we come to very simple expressions

$$\frac{1}{AB} = 2 \int_0^1 du_1 dx_2 \frac{\delta(x_1 + x_2 - 1)}{[x_1 A + x_2 B]^2} \quad (10)$$

Then the polarization operator $\Pi_{\mu 0}$ defined by (6) acquires the form

$$\Pi_{\mu 0}(B) = 8B \int_0^1 du \frac{d^3k}{(2\pi)^3} \left[ \epsilon_{\mu\nu}(\Omega' + (\frac{1}{2} - u)\omega) - (k' + (\frac{1}{2} - u)q)_\nu \omega - 2m(k' + (\frac{1}{2} - u)q)_\nu \right] \frac{1 - u}{(k'^2 + m^2 + u(1 - u)q^2)^3} \quad (10)$$

5
\[
\begin{align*}
\Pi_{\mu\nu}(B) &= 8iB \int_0^1 du \frac{d^3k}{(2\pi)^3} \frac{\epsilon_{\mu\nu}q_{\nu}(\Gamma + i\eta) - m(1 - 2u)q_{\mu}}{(k^2 + m^2 + u(1 - u)q^2)^3} \\
&= \frac{3iB}{2} \int_0^1 du \frac{d\vec{k}}{(2\pi)^2} \frac{\epsilon_{\mu\nu}q_{\nu}(\Gamma + i\eta) - m(1 - 2u)q_{\mu}}{(\vec{k} + m^2 + u(1 - u)q^2)^{5/2}}
\end{align*}
\]

Now, by performing integration over \( \vec{k} \) using standard formula of dimensional regularization

\[
\int \frac{d^2k}{(2\pi)^2} \frac{1}{(k^2 + \Delta)^n} = \frac{1}{(4\pi)^{\frac{n}{2}}} \frac{\Gamma(n - \frac{d}{2})}{\Gamma(n)} \frac{1}{\Delta^{n - \frac{d}{2}}}
\]

and dividing the range of integration \([0, 1]\) in three part we obtain

\[
\begin{align*}
\Pi_{\mu0}(B) &= iB \left\{ \int_{u_1}^{u_2} du \frac{\epsilon_{\mu\nu}q_{\nu}(\Gamma + i\eta) - m(1 - 2u)q_{\mu}}{(m^2 + u(1 - u)q^2)^{3/2}} - \int_0^{u_1} du \frac{\epsilon_{\mu\nu}q_{\nu}(\Gamma + i\eta) - m(1 - 2u)q_{\mu}}{\eta^3} \right\} \\
&= iB \left\{ -\frac{1}{4\pi} \left[ -2 \frac{1 - 2u}{(m^2 + u(1 - u)q^2)^{1/2}} \right]_{u_1}^{u_2} + \frac{1}{\eta^3} \epsilon_{\mu\nu}q_{\nu}(\Gamma + i\eta)(1 + u_1 - u_2) + \frac{m}{\eta^3} \epsilon_{\mu\nu}q_{\nu}(u_1 - u_2)(u_1 + u_2 - 1) \right\} \\
&= -\frac{Bi \epsilon_{\mu\nu}q_{\nu}(\Gamma + i\eta)}{\pi} \sqrt{4(\eta^2 - m^2)/q^2} - \frac{Bi}{4|\eta|} \frac{\epsilon_{\mu\nu}q_{\nu}(\Gamma + i\eta)}{\eta^3} \left[ 1 - \sqrt{1 - 4(\eta^2 - m^2)/q^2} \right] \left[ 1 - \sqrt{1 - 4(\eta^2 - m^2)/q^2} \right]
\end{align*}
\]
where expressions $u_1 = \frac{1}{2} \left(1 - \sqrt{1 - \frac{4(\eta^2 - m^2)}{q^2}}\right)$, $u_2 = \frac{1}{2} \left(1 + \sqrt{1 - \frac{4(\eta^2 - m^2)}{q^2}}\right)$ are obtained from the equation $m^2 + u(1 - u)q^2 = \eta^2$.

4 Results

Finally, in case of $\frac{q^2}{4} \geq (\eta^2 - m^2) \geq 0$, when the square root in the expression of $u_{1,2}$ is real, the integral over $u$ gives

$$\Pi_{\mu0}(B) = -\frac{iB}{4\pi|\eta|}\epsilon_{\mu\nu}q_\nu(\Gamma + i\eta) \left( \frac{1}{m^2 + \frac{q^2}{4}}\sqrt{1 - \frac{4(\eta^2 - m^2)}{q^2}} + \frac{1}{\eta^2} \left(1 - \sqrt{1 - \frac{4(\eta^2 - m^2)}{q^2}}\right) \right)$$

(15)

Denote that for polarization operator take place the condition of conservation of charge.

When $\eta^2 - m^2 \geq \frac{q^2}{4}$, then $u_1 = u_2 = \frac{1}{2}$ and for $\Pi_{\mu3}(B)$ we obtain

$$\Pi_{\mu0}(B) = -\frac{iB}{4\pi|\eta|^3}\epsilon_{\mu\nu}q_\nu(\Gamma + i\eta)$$

(16)

For $\eta^2 - m^2 \leq 0$ then $u_1 = 0, u_2 = 1$ and in a result we have following expression

$$\Pi_{\mu\nu}(B) = -\frac{iB}{\pi m}\epsilon_{\mu\nu}q_\nu(\Gamma + i\eta) \frac{1}{4m^2 + q^2}$$

(17)

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