Static scalar field condensation in regular asymptotically AdS reflecting star backgrounds

Yan Peng

1 School of Mathematical Sciences, Qufu Normal University, Qufu, Shandong 273165, China

Abstract

We study condensation behaviors of static scalar fields in the regular asymptotically AdS reflecting star spacetime. With analytical methods, we provide upper bounds for the radii of the scalar hairy reflecting stars. Above the bound, there is no scalar hair theorem for the star. Below the bound, we numerically obtain charged scalar hairy reflecting star solutions and in particular, the radii of the hairy stars are discrete, which is similar to known results in other reflecting object backgrounds. For every set of parameters, we search for the largest AdS hairy star radius, study effects of parameters on the largest hairy star radius and also find difference between properties in this AdS reflecting star background and those in the flat reflecting star spacetime. Moreover, we show that scalar fields cannot condense around regular AdS reflecting stars when the star charge is small.

PACS numbers: 11.25.Tq, 04.70.Bw, 74.20.-z

yanpengphy@163.com
I. INTRODUCTION

The famous no-scalar-hair theorem announces that the static massive scalar fields cannot exist in asymptotically flat black holes and it was believed that this property is mainly due to the fact that a classical black hole horizon could irreversibly absorb matter and radiation fields, for references see [1]-[15] and a review see [16]. In fact, this no scalar hair behavior is not restricted to spacetime with a horizon. Hod firstly found that regular asymptotically flat neutral compact reflecting stars cannot support the static massive scalar fields [17]. This no hair theorem for regular reflecting stars leads to works searching for no scalar hair behavior in the horizonless spacetime.

As a further step, it was found that massless scalar fields nonminimally coupled to the gravity cannot exist around the asymptotically flat neutral compact reflecting stars [18]. In addition, Bhattacharjee and Sudipta extended the discussion of no hair behaviors to reflecting stars in the asymptotically dS gravity background [19]. It seems that no hair theorem may widely hold in regular reflecting objects gravities. Since there is no hair theorem for asymptotically flat charged black holes [20, 21], it is interesting to further examine whether scalar fields can exist in the asymptotically flat charged reflecting objects background. Very different from cases in black holes, it was lately found that charged scalar fields can condense around a charged reflecting shell and it was found that the radius of the hairy shell is discrete in a range, where the spacetime outside the shell is supposed to be absolutely flat [22, 23]. In fact, the existence of composed scalar fields and reflecting objects configurations doesn’t depend on the choice of flat spacetime limit. In curved backgrounds of asymptotically flat reflecting stars, it was found that the reflecting star can support massive scalar fields and the scalar hairy reflecting star radius is also discrete [24, 25].

A simple way to invade the no hair theorem of static scalar fields is enclosing the gravity system in a confined box [26-30]. And it is well known that the AdS boundary could also serve as an infinity potential to confine the scalar field and dynamical formations of scalar hair due to the confinement was studied in [31]. At present, AdS scalar hairy black holes have been widely studied with the interest of the application of holographic theories [32]-[45]. On the other side, the scalar field configurations were constructed in the regular AdS reflecting shell background with analytical methods [46]. For a AdS shell, we mean that the shell charge and mass is very small compared to the shell radius or the spacetime outside the shell is pure AdS. Along this line, it is interesting to extend the discussion to the more general AdS reflecting star background.
by including the nonzero star charge and mass to examine whether there are still no scalar hair behaviors and also try to construct scalar field configurations supported by AdS charged reflecting stars.

This paper is organized as follows. In section II, we introduce the gravity model constructed with a scalar field coupled to asymptotically AdS charged reflecting stars. In part A of section III, we obtain an upper bound for the scalar hairy reflecting star radius. And in part B of section III, we show that the scalar field could condense around a AdS reflecting star and study properties of the largest hairy star radius. The last section is a summary of our main results.

II. EQUATIONS OF MOTION AND BOUNDARY CONDITIONS

In this paper, we study the system of a scalar field coupled to a charged reflecting star in the four dimensional asymptotically AdS spacetime. And the corresponding Lagrange density reads

\[ L = -\frac{1}{4} F^{MN} F_{MN} - |\nabla_\mu \psi - q A_\mu \psi|^2 - m^2 \psi^2, \]

(1)

As usual we define \( A_\mu \) as the ordinary Maxwell field with only nonzero \( tt \) component as \( A_\mu = -\frac{Q}{r} dt \) and \( \psi = \psi(r) \) is the scalar field with only radial dependence. Here, \( m \) is the scalar field mass and \( q \) corresponds to the scalar field charge.

We set the ansatz of the AdS spherically symmetric star geometry as

\[ ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \]

(2)

where \( f(r) = \frac{r^2}{L^2} - \frac{2M}{r} + \frac{Q^2}{r^2} \) with \( M \) as the star mass, \( Q \) as the star charge and \( L \) corresponding to the AdS radius. We also define the radial coordinate \( r = r_s \) as the radius of the reflecting star. In this work, we study the regular reflecting star without a horizon or there is \( g(r) > 0 \) for all \( r \geq r_s \).

The scalar field equation can be obtained as

\[ \psi'' + \left( \frac{2}{r} + \frac{g'}{g} \right) \psi' + \left( \frac{q^2 Q^2}{r^2 g^2} - \frac{m^2}{g} \right) \psi = 0, \]

(3)

where \( g = \frac{r^2}{L^2} - \frac{2M}{r} + \frac{Q^2}{r^2} \).

We can set \( L = 1 \) in the calculation with the symmetry

\[ L \to \alpha L, \quad r \to \alpha r, \quad q \to q/\alpha, \quad t \to \alpha t, \quad M \to \alpha M, \quad Q \to \alpha Q, \quad m \to m/\alpha. \]

(4)

Near the AdS boundary, asymptotic behaviors of the scalar fields are \( \psi = \frac{\psi_+}{r^{\lambda_+}} + \frac{\psi_0}{r^{\lambda_-}} + \cdots \) with \( \lambda_\pm = (3 \pm \sqrt{9 + 4m^2L^2})/2 \). For \( m^2L^2 > 0 \), only the second scalar operator \( \psi_+ \) is normalizable. So we fix \( \psi_- = 0 \)
and the scalar fields around the infinity behave as

\[ \psi = \frac{\psi_+}{r^{\lambda_+}} + \cdots. \]

(5)

We also impose reflecting boundary conditions at the surface of the star as

\[ \psi(r_s) = 0. \]

(6)

### III. SCALAR FIELD CONDENSATION BEHAVIORS AROUND ADS CHARGED REFLECTING STARS

#### A. Upper bounds for the radius of the scalar hairy AdS reflecting star

With a new radial function \( \tilde{\psi} = \sqrt{r}\psi \), the equation of the scalar field can be expressed as

\[ r^2 \tilde{\psi}'' + (r + \frac{rg'}{g}) \tilde{\psi}' + (-\frac{1}{4} - \frac{rg'}{2g} + \frac{Q^2}{g^2} - \frac{m^2 r^2}{g}) \tilde{\psi} = 0, \]

(7)

with \( g = \frac{r^2}{L^2} - \frac{2M}{r} + \frac{Q^2}{r^2} \).

From the boundary conditions (5) and (6), we have

\[ \tilde{\psi}(r_s) = 0, \quad \tilde{\psi}(\infty) = 0. \]

(8)

The function \( \tilde{\psi} \) must have (at least) one extremum point \( r = r_{\text{peak}} \) between the surface \( r_s \) of the reflecting star and the AdS boundary \( r_b = \infty \). At this extremum point, the scalar field is characterized by

\[ \{ \tilde{\psi}' = 0 \text{ and } \tilde{\psi}\tilde{\psi}'' \leq 0 \} \text{ for } r = r_{\text{peak}}. \]

(9)

According to the relations (7) and (9), we arrive at the inequality

\[ -\frac{1}{4} - \frac{rg'}{2g} + \frac{Q^2}{g^2} - \frac{m^2 r^2}{g} \geq 0 \text{ for } r = r_{\text{peak}}. \]

(10)

Then we have

\[ m^2 r^2 g \leq q^2 Q^2 - \frac{rg'}{2} - \frac{1}{4}g^2 \text{ for } r = r_{\text{peak}}. \]

(11)

We divide radii of hairy reflecting stars into two types: \( r_s < \max\{\sqrt{4ML^2}, \sqrt{QL}\} \) and \( r_s \geq \max\{\sqrt{4ML^2}, \sqrt{QL}\} \). For radii satisfying \( r_s \leq \max\{\sqrt{QL}, \sqrt{4ML^2}\} \), we naturally have an upper bound \( mr_s \leq \max\{m\sqrt{QL}, m\sqrt{4ML^2}\} \). For other radii satisfying \( r_s \geq \max\{\sqrt{4ML^2}, \sqrt{QL}\} \), we obtain an upper bound in the following.

For radii satisfying \( r_s \geq \max\{\sqrt{4ML^2}, \sqrt{QL}\} \), there is

\[ g = \frac{r^2}{L^2} - \frac{2M}{r} + \frac{Q^2}{r^2} = \frac{r^2}{2L^2} - \frac{2M}{r} + \frac{Q^2}{r^2} = \frac{r^2}{2L^2} + \frac{1}{2r^2}(r^3 - 4ML^2) + \frac{Q^2}{r^2} \geq \frac{r^2}{2L^2} \]

(12)
and
\[ g'(r) = \left(\frac{r^2}{L^2} - \frac{2M}{r} + \frac{Q^2}{r^2}\right)' = \frac{2r}{L^2} + \frac{2M}{r^3} - \frac{2Q^2}{r^3} = \frac{2}{r^3L^2}(r^4 - Q^2L^2) + \frac{2M}{r^2} \geq 0. \] (13)

From the regular condition \( g(r) > 0 \) for all \( r > r_s \) and relations (11)-(13), we arrive at
\[ m^2 r_s^2 \frac{r_s^2}{L^2} \leq m^2 r^2 \frac{r^2}{L^2} \leq m^2 r^2 \cdot 2g(r) \leq 2(q^2Q^2 - \frac{r^2g'}{2} - \frac{1}{4}g^2) \leq 2q^2Q^2 \quad \text{for} \quad r = r_{\text{peak}}. \] (14)

According to (14), there is
\[ m^2 r_s^2 \frac{r_s^2}{L^2} \leq 2q^2Q^2. \] (15)

It also can be expressed with dimensionless quantities with the symmetry (4) as
\[ mr_s \leq \sqrt{2} \sqrt{qQmL}. \] (16)

Then we have two types of hairy star radii

- case 1: \( r_s \leq \max\{\sqrt{QL}, \sqrt{4ML^2}\} \) or \( mr_s \leq \max\{m\sqrt{QL}, m\sqrt{4ML^2}\} \);
- case 2: \( r_s \geq \max\{\sqrt{QL}, \sqrt{4ML^2}\} \) with \( mr_s \leq \sqrt{2} \sqrt{qQmL} \).

So we obtain an upper bound for the regular AdS hairy reflecting star radii as
\[ mr_s \leq \max\{m\sqrt{QL}, m\sqrt{4ML^2}, \sqrt{2} \sqrt{qQmL}\}. \] (17)

B. Scalar field configurations supported by a regular AdS charged reflecting star

In this part, we construct the regular AdS scalar hairy charged reflecting star solutions and also study properties of the hairy reflecting star radii. Since we impose reflecting boundary conditions for the scalar field at the star surface, the scalar field can be putted in the form \( \psi = \psi_0(r - r_s) + \cdots \) around the star radius. As the scalar field equation is linear with respect to \( \psi \), we can fix \( \psi_0 = 1 \) in the calculation. We will numerically integrate the equation from various reflecting boundary to the infinity to search for hairy star radii \( r_s \) satisfying the boundary conditions (5).

We show the case of \( qL = 2, m^2L^2 = \frac{1}{4}, \frac{Q}{L} = 2 \) and \( \frac{M}{L} = 7.5 \) in Fig. 1. In the left panel, when we impose a reflecting boundary at \( \frac{r}{L} = 1.900 \), the scalar field is zero around \( \frac{r}{L} \approx 3.061 \) and it decreases to be more negative far away. And with a little larger reflecting boundary at \( \frac{r}{L} = 1.909 \) in the middle panel, the scalar field decreases to be zero at a larger coordinate \( \frac{r}{L} \approx 5.297 \) and becomes more negative far away from the reflecting boundary. For a reflecting boundary at \( \frac{r}{L} = 1.918 \) in the right panel, there is no additional
zero point of the scalar field and the scalar field increases to be more positive far from the star. In front cases, it is clearly that the reflecting boundaries cannot be fixed as the star radius since the scalar field should asymptotically approach zero far from the star surface according to (5). However, there may exist a critical reflecting boundary between $r_L = 1.909$ and $r_L = 1.918$ and for this critical reflecting boundary, the zero points of the scalar field is at the infinity or the scalar field asymptotically approaches zero according to (5). This possible critical reflecting boundary can be fixed as the radius of the AdS scalar hairy reflecting star.

![Graphs](image)

**FIG. 1:** (Color online) We plot the scalar field $\psi$ as a function of the radial coordinate $r$ with $qL = 2$, $m^2L^2 = \frac{1}{2}$, $Q_L = 4$, $M_L = 7.5$ and various reflecting boundaries. The left panel corresponds to the case of the reflecting boundary at $r_L = 1.900$, the middle panel represents the case of the reflecting boundary at $r_L = 1.909$ and the right panel shows the case of the reflecting boundary at $r_L = 1.918$.

With more detailed calculations in cases of $qL = 2$, $m^2L^2 = \frac{1}{2}$, $Q_L = 4$ and $M_L = 7.5$, we arrive at the critical discrete reflecting star radius $r_L ≈ 1.91077$ with the scalar field asymptotically approaches zero at the infinity. We have further checked that when the reflecting boundary $r_L \to 1.91077$, the corresponding scalar field converges to the nonzero limit with $r_L = 1.91077$. On the other side, the general solutions of equation (3) behave in the form $\psi = A \cdot \frac{1}{r^{\lambda_-}} + B \cdot \frac{1}{r^{\lambda_+}} + \cdots$ with $\lambda_\pm = (3 \pm \sqrt{11})/2$ as $r \to \infty$. We have numerically found $A < 0$ for reflecting boundary $r_L$ below 1.91077 and there is $A > 0$ for reflecting boundary $r_L$ above 1.91077 in accordance with results in Fig. 1. So a critical radius $r_L \approx 1.91077$ with $A = 0$ should exist and the corresponding scalar field behaves as $\psi \propto \frac{1}{r^{(3+\sqrt{11})/2}}$ at the infinity.

We plot behaviors of the scalar field with the surface at $r_L \approx 1.91077$ in the case of $qL = 2$, $m^2L^2 = \frac{1}{2}$, $Q_L = 4$ and $M_L = 7.5$ in the left panel of Fig. 2. In fact, $r_L \approx 1.91077$ is the largest radius of the hairy star since there is no larger zero points of the scalar field below the upper bound $r_L \leq \max\{2, \sqrt{30}, 4\} = 4$ due to (17). Integrating the equation from $r_L = 1.91077$ to smaller radial coordinates in the right panel of Fig. 2, we can obtain various discrete radii of AdS hairy reflecting stars around $mr_s \approx 1.35114$, 1.31465, 1.31105, 1.31062, · · · below the upper bound $mr_s \leq \max\{\sqrt{2}, \frac{\sqrt{30}}{\sqrt{2}}, 2\sqrt{2}\} = 2\sqrt{2} \approx 2.82843$ according to (17). Here, we find that radii of AdS scalar hairy reflecting stars are discrete qualitatively the same to analytical results in other reflecting object background.
FIG. 2: (Color online) We show the scalar field \( \psi \) as a function of the radial coordinate \( r_L \) with \( q_L = 2, m^2 L^2 = \frac{1}{2}, \frac{Q}{M} = 4 \) and \( \frac{M}{L} = 7.5 \). The left panel is with the largest radius putted at \( \frac{r_s}{L} = 1.91077 \) and the right panel shows various discrete hairy star radii.

We show the largest hairy star radius \( mR_s \) as a function of star mass and charge with dimensionless quantities according to the symmetry (4) in Fig. 3. With \( qL = 2, m^2 L^2 = \frac{1}{2} \) and \( \frac{Q}{M} = 4 \) in the left panel, we see that the larger star mass \( qM \) corresponds to a larger \( mR_s \) similar to known results in the asymptotically flat reflecting star background \[25\]. In contrast, we can see that larger \( qQ \) leads to a smaller \( mR_s \) with \( qL = 2, m^2 L^2 = \frac{1}{2} \) and \( \frac{M}{L} = 7.5 \) in the right panel, which is very different from cases of reflecting stars in the asymptotically flat background \[25\].

FIG. 3: (Color online) We plot the largest hairy star radius with respect to various star charge and mass. We show effects of the star mass \( qM \) on \( mR_s \) with \( qL = 2, m^2 L^2 = \frac{1}{2} \) and \( \frac{Q}{M} = 4 \) in the left panel and the right panel corresponds to behaviors of \( mR_s \) as a function of \( qQ \) with \( qL = 2, m^2 L^2 = \frac{1}{2} \) and \( \frac{M}{L} = 7.5 \).

It is interesting to extend the discussion in the right panel of Fig. 3 to smaller star charge and also examine whether there are neutral AdS hairy reflecting stars. We show the largest star radius \( mR_s \) and the largest black hole horizon \( mR_h \) as a function of star charge with \( qL = 2, m^2 L^2 = \frac{1}{2} \) and \( \frac{M}{L} = 7.5 \) in Fig. 4. Since the reflecting star is assumed to be regular, the star surface coordinate should be larger than the largest horizon of a black hole with the same charge and mass. We find that the largest radius is above the largest horizon in the range \( qQ \geq 4 \) and they both increase as we decrease the star charge. When we choose smaller star charge, the largest star radius becomes nearer to the largest horizon and for small star charge, they almost coincide with each other.
FIG. 4: (Color online) We show the largest hairy star radius \(mR_s\) and largest horizon \(mR_h\) as a function of the star charge \(qQ\) with \(qL = 2\), \(m^2L^2 = \frac{1}{2}\) and \(\frac{M}{L} = 7.5\). The top blue line shows behaviors of \(mR_s\) and the bottom red line is with values of \(mR_h\).

We also show values of the largest star radius \(mR_s\) and the largest horizon \(mR_h\) in Table I with \(qL = 2\), \(m^2L^2 = \frac{1}{2}\), \(\frac{M}{L} = 7.5\) and various \(qQ\). The data again shows that the star surface becomes nearer to the largest horizon as we decrease the star charge. And when the star charge is \(qQ \approx 4\), the star radius is almost equal to the horizon and our further numerical results suggest that there is no reflecting hairy star with the reflecting surface outside the horizon for \(qQ < 4\). So we conclude that there is no regular asymptotically AdS hairy reflecting star solutions or no scalar hair theorem holds for small star charge.

| \(qQ\) | 4    | 5    | 6    | 7    | 8    |
|-------|------|------|------|------|------|
| \(mR_s\) | 1.67585 | 1.63194 | 1.57063 | 1.48316 | 1.35112 |
| \(mR_h\) | 1.67585 | 1.63189 | 1.57004 | 1.47850 | 1.31063 |

### IV. CONCLUSIONS

We studied static scalar field configurations supported by a reflecting star in the AdS spacetime. With analytical methods, we provided upper bounds for the radius of the scalar hairy star as \(mr_s \leq \max\{m\sqrt{qL}, m\sqrt{4ML^2}, \sqrt{2}\sqrt{qQmL}\}\). Above the bound, the AdS reflecting star cannot support the scalar field or there is no scalar hair theorem. Below the bound, we numerically obtained the charged scalar hairy reflecting star solutions and the radius of the hairy star is discrete, which is similar to cases in other reflecting object backgrounds. For each fixed set of parameters, we searched for the largest radii \(mR_s\) and also examined effects of parameters on the largest AdS hairy star radii. We found that larger star mass \(qM\) corresponds to a larger \(mR_s\) similar to cases in the asymptotically flat reflecting star background. However, we saw that larger star charge \(qQ\) leads to a smaller \(mR_s\), which is very different from results in asymptotically flat reflecting
star spacetime. And our further detailed calculation showed that scalar fields cannot condense around regular AdS reflecting stars or no scalar hair theorem also exists when the star charge is small.

Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grant No. 11305097; Shandong Provincial Natural Science Foundation of China under Grant No. ZR2018QA008.

[1] J. D. Bekenstein, Phys. Rev. Lett. 28, 452 (1972).
[2] J. E. Chase, Commun. Math. Phys. 19, 276 (1970); C. Teitelboim, Lett. Nuovo Cimento 3, 326 (1972); J. D. Bekenstein, Phys. Today 33, 24 (1980).
[3] R. Ruffini and J. A. Wheeler, Phys. Today 24, 30 (1971).
[4] S. Hod, Phys. Rev. D 86, 104026 (2012) [arXiv:1211.3202].
[5] S. Hod, The Euro. Phys. Journal C 73, 2378 (2013) [arXiv:1311.5298].
[6] S. Hod, Phys. Rev. D 90, 024051 (2014) [arXiv:1406.1179].
[7] S. Hod, Class. and Quant. Grav. 32, 134002 (2015) [arXiv:1607.00003].
[8] S. Hod, Phys. Lett. B 758, 181 (2016) [arXiv:1606.02306].
[9] C. A. R. Herdeiro and E. Radu, Phys. Rev. Lett. 112, 221101 (2014).
[10] C. L. Benone, L. C. B. Crispino, C. Herdeiro, and E. Radu, Phys. Rev. D 90, 104024 (2014).
[11] C. Herdeiro, E. Radu, and H. Runarsson, Phys. Lett. B 739, 302 (2014).
[12] C. Herdeiro and E. Radu, Class. Quantum Grav. 32 144001 (2015).
[13] J. C. Degollado and C. A. R. Herdeiro, Gen. Rel. Grav. 45, 2483 (2013).
[14] P. V. P. Cunha, C. A. R. Herdeiro, E. Radu, and H. F. Runarsson, Phys. Rev. Lett. 115, 211102 (2015).
[15] Y. Brihaye, C. Herdeiro, and E. Radu, Phys. Lett. B 760, 279 (2016).
[16] J. D. Bekenstein, arXiv:gr-qc/9605059.
[17] S. Hod, No-scalar-hair theorem for spherically symmetric reflecting stars, Physical Review D 94, 104073 (2016).
[18] S. Hod, No nonminimally coupled massless scalar hair for spherically symmetric neutral reflecting stars, Physical Review D 96, 024019 (2017).
[19] Srijit Bhattacharjee, Sudipta Sarkar, No-hair theorems for a static and stationary reflecting star, Phys.Rev.95.084027.
[20] S. Hod, Stability of the extremal Reissner-Nordström black hole to charged scalar perturbations, Phys.Lett. B713, 505 (2012).
[21] S. Hod, No-bomb theorem for charged Reissner-Nordström black holes, Physics Letters B 718, 1489 (2013).
[22] S. Hod, Charged massive scalar field configurations supported by a spherically symmetric charged reflecting shell, Physics Letters B 763, 275 (2016).
[23] S. Hod, Marginally bound resonances of charged massive scalar fields in the background of a charged reflecting shell, Physics Letters B768(2017)97C102.
[24] Shahar Hod, Charged reflecting stars supporting charged massive scalar field configurations, European Physical Journal C 78, 173 (2017).
[25] YanPeng, Scalar field configurations supported by charged compact reflecting stars in a curved spacetime, Physics Letters B780(2018)144C148.
[26] Sam R Dolan, Supakchaispongirtsakul, Elizabeth Winstanley, Stability of black holes in Einstein-charged scalar field theory in a cavity Phys. Rev. D 92, 124047 (2015).
[27] Pallab Basu, Chethan Krishnan, P.N.BalaSubramanian, Hairy black holes in a box JHEP 11(2016)041.
[28] Yan Peng, Studies of a general flat space/boson star transition model in a box through a language similar to holographic superconductors, JHEP 07(2017)042.
[29] Yan Peng, Bin Wang, Yunqi Liu, On the thermodynamics of the black hole and hairy black hole transitions in the asymptotically flat spacetime with a box, Eur Phys J C (2018)78:176.
[30] J. X. Lu, Shibaji Roy, Zhiguang Xiao, Phase transitions and critical behavior of black branes in canonical ensemble, JHEP 01(2011)133.
[31] Pablo Bosch, Stephen R. Green, and Luis Lehner, Nonlinear Evolution and Final Fate of Charged AntiCde Sitter Black Hole Superradiant Instability, Phys. Rev. Lett. 116, 141102 (2016).
[32] J.M. Maldacena, The large-N limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys. 2(1998)231 [Int. J. Theor. Phys. 38(1999)1113].
[33] S.S. Gubser, I.R. Klebanov, and A.M. Polyakov, Gauge theory correlators from non-critical string theory, Phys. Lett. B 428(1998)105.
[34] E. Witten, Anti-de Sitter space and holography, Adv. Theor. Math. Phys. 2(1998)253.
[35] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, Building an AdS/CFT superconductor, Phys. Rev. Lett. 101(2008)031601.
[36] Rong-Gen Cai, Huai-Fan Li, Hai-Qing Zhang, Analytical Studies on Holographic Insulator/Superconductor Phase Transitions, Phys. Rev. D 83(2011)126007.
[37] Yunqi Liu, Yungui Gong, Bin Wang, Non-equilibrium condensation process in holographic superconductor with nonlinear electrodynamics, JHEP 02(2016)116.
[38] F. Aprile and J.G. Russo, Models of holographic superconductivity, Phys. Rev. D 81(2010)026009.
[39] J. Jing, Q. Pan, and S. Chen, Holographic Superconductors with Power-Maxwell field, JHEP 11(2011)045.
[40] Y. Brihaye and B. Hartmann, Holographic superconductors in $3+1$ dimensions away from the probe limit, Phys. Rev. D 81(2010)126008.
[41] P. Basu, J. He, A. Mukherjee, M. Rozali and H. H. Shieh, Competing Holographic Orders, JHEP 10(2010)092.
[42] Davood Momeni, Hossein Gholizade, Muhammad Raza, Ratbay Myrzakulov, Holographic Entanglement Entropy in 2D Holographic Superconductor via $AdS_2/CFT_2$, Phys. Lett. B 747(2015)417.
[43] Yan Peng and Yunqi Liu, A general holographic metal/superconductor phase transition model, JHEP 02(2015)082.
[44] Q. Pan, B. Wang, E. Papantonopoulos, J. Oliveira and A.B. Pavan, Holographic Superconductors with various condensates in Einstein-Gauss-Bonnet gravity, Phys. Rev. D 81(2010)106007.
[45] Yi Ling, Peng Liu, Jian-Pin Wu, Note on the butterfly effect in holographic superconductor models, Phys. Lett. B 768(2017)288.
[46] Yan Peng, Studies of scalar field configurations supported by reflecting shells in the AdS spacetime, arXiv:1803.09148.
[47] S. Chandrasekhar, The Mathematical Theory of Black Holes, (Oxford University Press, New York, 1983).