Research on optimization of the number of acoustic arrays

Yan Wu, Junjun Zong*, Hui Yang, Chuanxu Liu, Guangxing Wang and Caofang Lv
PLA Army Academy of Artillery and Air Defense, Hefei, China

*Corresponding author: zjj_2008@163.com

Abstract. The reasonable number of base stations in multiple acoustic array azimuth cross location is studied. In this paper, the maximum likelihood estimation (MLE) algorithm is used to discuss the problem of increasing the number of arrays to improve the positioning accuracy, and the positioning error model is given. The experimental results show that with the increase of the number of base stations, the positioning accuracy will gradually improve, but if there are too many base stations, the improvement of positioning accuracy will gradually slow down.

Keywords: acoustic array, direction finding(DF), cross location, maximum likelihood estimation(MLE), positioning accuracy, pseudo-linear estimation(PLE)

1. Introduction
In order to improve the positioning accuracy of signal source, it is usually necessary to improve the direction finding accuracy of acoustic array and optimize the positioning algorithm. This paper is based on the maximum likelihood algorithm, and gives the positioning accuracy with the number of acoustic arrays. The structure of the paper is organized as follows. In the second section, we first review the principle of maximum likelihood estimation and the bearing-crossing location. Then introduce how to choose the initial coordinate and depict the accuracy error. The third section gives the simulation results and conclusions. The fourth section summarizes the whole paper.

2. The principle of multi-station DF cross location based on the MLE

2.1. The principle of DF cross location
The method of DF cross location is also known as triangulation method, it measure the emitter by high-precision DF equipment at two or more stations. The equipment can calculate the source coordinate with the measured angle data and the station coordinate.

The station position is represented by \( S_k = (x_k, y_k, z_k)^T \) \((k = 1, 2, \ldots, n)\), where \( T \) denotes matrix transpose. The true location of the source station is expressed by \( X_p = (x_p, y_p, z_p)^T \). We can measure azimuth \( (\theta_k) \) and elevation \( (\phi_k) \) angle at each station. The measurement unit is radian. At the same time, it is assumed that azimuth \( (\theta_k) \) and elevation \( (\phi_k) \) are independent of each other, and satisfy with zero mean Gaussian noise. The mean-square-deviation are denoted as \( \sigma_{\theta_k} \) and \( \sigma_{\phi_k} \). Location mode was shown in Figure 1.
2.2. MLE algorithm based on acoustic location

According to the position relationship, we can get the equation (1) and (2)

\[ \theta_k = \tan^{-1}\left( \frac{y - y_k}{x - x_k} \right) \]  
(1)

\[ \phi_k = \tan^{-1}\left( \frac{z_p - z_k}{\sqrt{(x_p - x_k)^2 + (y_p - y_k)^2}} \right) \]  
(2)

According to the principle of MLE, we can construct the cost function

\[ J_{ML}(p) = e^T(p)W^{-1}e(p) \]  
(3)

Where \( W \) is covariance matrix of the measurement noise and \( e(p) \) is the error vector, which can be expressed as follows.

\[ W = \text{diag}\left\{ \sigma_{\theta_1}^2, \sigma_{\phi_1}^2, \cdots, \sigma_{\theta_n}^2, \sigma_{\phi_n}^2, \cdots, \sigma_{\theta_n}^2 \right\} \]  
(4)

\[ e(p) = \left[ \tilde{\theta}_1 - \theta_1(p), \cdots, \tilde{\theta}_n - \theta_n(p), \tilde{\phi}_1 - \phi_1(p), \cdots, \tilde{\phi}_n - \phi_n(p) \right] \]  
(5)

The estimation value of the signal source can be expressed as

\[ \hat{p}_{ML} = \arg \min_p J_{ML}(p) \]  
(6)

The MLE is a nonlinear estimation, so it does not have a closed-form solution and requires a numerical search algorithm. The Gauss-Newton algorithm, which is a batch iterative minimization technique, can be employed to calculate the MLE. The GN algorithm consists of
\[ X_{i+1} = X_i - \left( J_i' W_i J_i \right)^{-1} J_i' W_i e(P_i) \]  \hspace{1cm} (7)

Where \( J_i \) is the Jacobian matrix of \( e(p) \)

\[
J_i = \begin{bmatrix}
\begin{vmatrix}
sin \hat{\phi}_i(i) & -\cos \hat{\phi}_i(i) \\
\ddot{d}_i & \\
\vdots & \\
\sin \hat{\phi}_n(i) & -\cos \hat{\phi}_n(i) \\
\ddot{d}_n & \\
\end{vmatrix}

\begin{vmatrix}
sin \hat{\theta}_i(i) \cos \hat{\phi}_i(i) & sin \hat{\theta}_i(i) \sin \hat{\phi}_i(i) & -\cos \hat{\theta}_i(i) \\
\frac{\|X_i - S_i\|}{d_i} & \frac{\|X_i - S_i\|}{d_i} & \frac{\mu_i}{d_i} \\
\vdots & \vdots & \vdots \\
\sin \hat{\theta}_n(i) \cos \hat{\phi}_n(i) & sin \hat{\theta}_n(i) \sin \hat{\phi}_n(i) & -\cos \hat{\theta}_n(i) \\
\frac{\|X_i - S_n\|}{d_n} & \frac{\|X_i - S_n\|}{d_n} & \frac{\mu_n}{d_n} \\
\end{vmatrix}
\end{bmatrix}
\hspace{1cm} (8)

Where \( \ddot{d}_k = \|X_i(1:2) - S_k(1:2)\| \) is denoted as the distance between the \( i \)th signal source and the \( k \)th station. \( X_i(1:2), S_k(1:2) \) is a vector which contain the first two entries of \( X_i \). The initial coordinate of the signal source can be calculated by PLE.

2.3. Position error
The covariance matrix of measurement error can be expressed as

\[
P_x = E\left[ \delta X \delta X^T \right] = \begin{bmatrix}
\sigma_x^2 & \rho_{xx} \sigma_y \sigma_z & \rho_{xz} \sigma_y \\
\rho_{yx} \sigma_x \sigma_y & \sigma_y^2 & \rho_{yz} \sigma_z \\
\rho_{zx} \sigma_x \sigma_z & \rho_{yz} \sigma_z & \sigma_z^2
\end{bmatrix}
\hspace{1cm} (9)
\]

Where the position of DF station and measurement error of azimuth \((\hat{\theta}_k)\) and elevation \((\hat{\phi}_k)\) are independent of each other, so the correlation in equation (9) can be expressed as \( \rho_{xy} = \rho_{xz} = 0 \), the model can be written as the following.

\[
P_x = \begin{bmatrix}
\sigma_x^2 & 0 \\
0 & \sigma_y^2 \\
0 & \sigma_z^2
\end{bmatrix}
\hspace{1cm} (10)
\]

Where: \( \sigma_{xy}^2 = E\left[ \Delta x_{ij}^2 \right] \), \( \sigma_{xy}^2 = E\left[ \Delta y_{ij}^2 \right] \), \( \sigma_{xy}^2 = E\left[ \Delta z_{ij}^2 \right] \), and \( \Delta x_{ij} = x_{pi} - x_{pj} \), \( \Delta y_{ij} = y_{pi} - y_{pj} \), \( \Delta z_{ij} = z_{pi} - z_{pj} \).

The distance error of the \( j \)th signal of the \( i \)th simulation can be written as

\[
\Delta X_{ij} = \sqrt{\sigma_{xy}^2 + \sigma_{xz}^2 + \sigma_{yz}^2} \quad (i = 1, \cdots, N, \ j = 1, \cdots, M)
\hspace{1cm} (11)
\]
The average and variance of $M$ target of $N$ simulation can be written as the following.

$$ E_{NM} = \frac{1}{N} \frac{1}{M} \sum_{i=1}^{N} \sum_{j=1}^{M} \Delta X_{ij} $$ (12)

$$ F_{NM} = \frac{1}{N-1} \sum_{i=1}^{N} \sqrt{(E_{iM} - E_{NM})^2} $$ (13)

3. Simulation result analysis

3.1. Simulation conditions

In order to find the best station numbers, we use simulation experiment technology to analysis. The experimental parameters are as follows: possible station number is 2 ~ 20, expanding width is 6km and depth is ± 0.2km, high and low is ± 0.2km, showed a random distribution. We assume azimuth ($\theta_k$) and elevation ($\phi_k$) has the standard errors, which are separated into two groups, one is 0.1°, 0.3°, 0.5°, the other is 0.5°, 1.0°, 1.5°. The area of target is assumed as: axis x is ±3km, axis y is 1km~7km, axis is ±0.4km.

3.2. Analysis of the simulation results

Figure 1 and table 1 are simulation results, by running 200 Monte-Carlo runs at 500 random targets. Results are compared for three different $\sigma$ which is 0.1°, 0.3°, 0.5°.

| Station number | Measuring error | $\sigma = 0.1°$ | $\sigma = 0.3°$ | $\sigma = 0.5°$ |
|----------------|----------------|----------------|----------------|----------------|
|                | Average        | variance       | Average        | variance       | Average        | variance       |
| 2              | 15.21          | 9.70           | 41.00          | 21.28          | 56.13          | 22.54          |
| 3              | 12.65          | 9.45           | 34.50          | 20.04          | 46.84          | 23.24          |
| 4              | 11.69          | 8.80           | 31.50          | 20.89          | 43.31          | 23.05          |
| 5              | 10.47          | 8.38           | 28.70          | 20.17          | 39.77          | 23.16          |
| 6              | 9.99           | 7.79           | 26.55          | 19.66          | 38.14          | 22.60          |
| 7              | 9.22           | 7.36           | 25.65          | 18.98          | 37.29          | 22.74          |
| 8              | 8.58           | 6.96           | 25.51          | 18.64          | 35.86          | 22.26          |
| 9              | 8.18           | 6.61           | 24.02          | 18.08          | 35.09          | 22.21          |
| 10             | 7.97           | 6.37           | 22.88          | 17.45          | 33.74          | 22.03          |
| 11             | 7.49           | 6.08           | 22.02          | 16.84          | 31.70          | 21.79          |
| 12             | 7.25           | 5.83           | 20.66          | 16.67          | 31.31          | 21.36          |
| 13             | 6.96           | 5.67           | 20.24          | 16.19          | 29.87          | 21.33          |
| 14             | 6.76           | 5.49           | 20.56          | 15.77          | 29.80          | 20.91          |
| 15             | 6.62           | 5.33           | 18.93          | 15.57          | 29.36          | 20.97          |
| 16             | 6.30           | 5.18           | 18.68          | 14.97          | 28.43          | 20.56          |
| 17             | 6.15           | 5.13           | 18.19          | 14.71          | 27.86          | 20.45          |
| 18             | 5.87           | 4.88           | 17.39          | 14.57          | 27.45          | 19.79          |
| 19             | 5.90           | 4.80           | 17.23          | 14.20          | 27.16          | 19.91          |
| 20             | 5.63           | 4.72           | 17.54          | 14.01          | 26.18          | 19.66          |
Figure 2. The changing curves of average and variance

We can conclude the conclusions from figure 2 and table 1:
Location accuracy can be improved with the stations number increasing. Curve’s changing is very rapid at the beginning, until the number of station are increased at the certain. The result show that if the DF station’s number achieve a reasonable number, there is no use to improve the station numbers.

Figure 2 and table 2 are simulation results of generalized precision, by running 200 Monte-Carlo runs at 500 random target, Results are compared for three different $\sigma$ which are $0.5^0$, $1.0^0$, $1.5^0$.

Table 2. Numerical changing of average value and variance

| Station number | Measuring error | $\sigma$ = $0.5^0$ | $\sigma$ = $1.0^0$ | $\sigma$ = $1.5^0$ |
|---------------|----------------|--------------------|--------------------|--------------------|
|               | Average | variance | Average | variance | Average | variance | Average | variance |
| 2             | 55.07   | 22.57    | 67.39   | 21.22    | 71.36   | 20.28    |
| 3             | 46.38   | 23.20    | 60.69   | 22.51    | 66.96   | 21.24    |
| 4             | 43.14   | 23.23    | 57.01   | 22.98    | 64.23   | 21.88    |
| 5             | 40.26   | 22.97    | 55.03   | 23.29    | 61.95   | 22.50    |
| 6             | 38.61   | 22.74    | 53.46   | 23.52    | 60.40   | 22.37    |
| 7             | 37.24   | 22.80    | 51.51   | 23.67    | 59.08   | 22.72    |
| 8             | 35.53   | 22.31    | 50.31   | 23.67    | 57.97   | 23.29    |
| 9             | 34.40   | 21.99    | 49.40   | 23.81    | 56.51   | 23.34    |
| 10            | 32.99   | 21.79    | 48.05   | 23.87    | 56.11   | 23.29    |
| 11            | 32.33   | 21.84    | 47.18   | 23.56    | 55.04   | 23.55    |
| 12            | 31.29   | 21.41    | 46.16   | 23.94    | 54.33   | 23.43    |
| 13            | 30.58   | 21.26    | 45.15   | 23.88    | 53.07   | 23.63    |
| 14            | 29.76   | 20.87    | 44.32   | 23.59    | 52.43   | 23.42    |
| 15            | 29.58   | 20.85    | 43.76   | 23.72    | 52.04   | 23.78    |
| 16            | 28.87   | 20.68    | 43.18   | 23.57    | 51.22   | 23.79    |
| 17            | 28.18   | 20.22    | 42.56   | 23.66    | 50.75   | 23.65    |
| 18            | 27.19   | 19.91    | 41.97   | 23.58    | 50.36   | 23.82    |
| 19            | 27.00   | 19.99    | 41.61   | 23.67    | 49.71   | 23.92    |
| 20            | 26.67   | 19.77    | 40.92   | 23.33    | 48.89   | 23.95    |
Figure 3. The changing curves of average and variance

We can conclude the conclusion from figure 3 and table 2:

Comparing the curve of the average on figure 2, we can find that with the station number increasing, the station’s DF is more precise and the convergence speed is faster, on the contrary, the station’s DF is slightly lower and the convergence speed is slower. The result is the opposite of the figure 1.

Comparing curves of the variance on figure 2, we can find that with the station number increasing, the curve is convergence if the station’s DF is more precise, however, if the station’s DF is lower than a reasonable limit, the curve is divergence.

Table 2 and Figure 2 for the general direction finding more conditions, using MLE for Monte-Carlo 200 experiments, the 500 random simulation of target distance error mean and variance of data and the corresponding curve.

Acknowledgements
This work was supported by the fund of national natural science, the item number is 61170252.

References
[1] Zong Junjun, Cui Xunxue. Algorithm and accuracy of weighted maximum likelihood estimation in multi-station DF crossing localization. 2015 4th international conference on computer, mechatronics, control and electronic engineering. September 28, 2015.
[2] K. Dogancay and G. Ibal. Instrumental variable estimator for 3D bearing-only emitter location. Intelligent Sensors, Sensor Networks and Information Processing Conference, ISSNIP 2005. December 2005.
[3] Zong Junjun, Cui Xunxue, et al. Accuracy analysis of sensor array DF crossing for the sound location of burst point [J]. the 4th international conference on computing, control and industrial engineering, October 2013.