We show how light spin-$\frac{1}{2}$ or spin-0 particles may be acceptable Dark Matter candidates, provided they annihilate sufficiently strongly through new interactions, such as those induced by a new light neutral spin-1 boson $U$. The corresponding interaction is stronger than weak interactions at lower energies, but weaker at higher energies.

Annihilation cross sections of (axially coupled) spin-$\frac{1}{2}$ Dark Matter particles, induced by a $U$ vectorially coupled to matter, are the same as for spin-0 particles. In both cases, the cross sections $(\sigma_{\text{ann}} v_{\text{rel}}/c)$ into $e^+e^-$ automatically include a $v_{\text{dm}}^2$ suppression factor, as desirable to avoid an excessive production of $\gamma$ rays from residual Dark Matter annihilations. We also relate Dark Matter annihilations with production cross sections in $e^+e^-$ reactions. As for spin-0, light spin-$\frac{1}{2}$ Dark Matter particles annihilating into $e^+e^-$ could be responsible for the bright 511 keV $\gamma$ ray line observed by INTEGRAL from the galactic bulge.

1 Introduction

What is (non-baryonic) Dark Matter made of? The familiar, although still unobserved, neutralinos of supersymmetric theories? What could be the other possibilities? Could Dark Matter particles be light? If so, how could they annihilate? May be through some new interactions, such as those induced by a new light spin-1 boson $U$, that would be sufficiently strong for this purpose? But if so, how could they have remained unnoticed? And how could such Light Dark Matter particle annihilations manifest? Could they be responsible for the bright 511 keV $\gamma$ ray line recently observed by the INTEGRAL satellite from the galactic bulge?

We adopt here the usual point of view, according to which Dark Matter is made of new neutral massive particles, not directly observed. Such particles have to annihilate sufficiently, otherwise their relic density would be too large. Therefore weakly-interacting massive neutral particles, taken as possible Dark Matter candidates, should not be too light.
Supersymmetric extensions of the Standard Model\(^1\) naturally provide weakly-interacting neutral particles\(^2\), stable as a result of \(R\)-parity conservation. Spin-\(\frac{1}{2}\) photinos, or more generally neutralinos, taken as possible Dark Matter candidates, should then be sufficiently heavy\(^3\). In any case, given the still unsuccessful hunt for superpartners, most notably at LEP, the lightest neutralino (LSP) of Supersymmetric extensions of the Standard Model is generally believed to be heavier than about \(\sim 30\) GeV.

Still, their may be room for other, non-conventional possibilities, such as those classified as in\(^4\) according to the mass and interaction strength of Dark Matter particles, which determine whether or not they are still relativistic at the time of their decoupling. More specifically, Light Dark Matter candidates having sufficiently “strong” interactions could remain coupled until they become non-relativistic and annihilate (cf. the left part of “Region II” in\(^4\)), with annihilation reactions freezing out at \(T_F = m_{dm}/x_F\), provided the interactions responsible for these annihilations be strong enough. But, can this really make sense?

Indeed, how could a light (annihilating) Dark Matter particle possibly exist, without leading to a too large relic density? At first, in any case, it should have no significant direct coupling to the \(Z\) boson, otherwise it would have been produced in \(Z\) decays at LEP. Despite that, it would have to annihilate sufficiently – and in fact, much more strongly than through ordinary weak interactions – otherwise its relic energy density would be too high! Can this happen at all, and what could then be the new interactions responsible for Light Dark Matter annihilations?

We have explored in\(^5\) under which conditions a light spin-0 particle could be a viable Dark Matter candidate. In the most interesting case the new interactions responsible for its annihilations are mediated by a new neutral spin-1 gauge boson \(U\), light and very weakly coupled, as introduced long ago\(^6\). The spin-1 induced Dark Matter annihilation cross section into \(e^+e^-\) \((\sigma_{ann} v_{rel}/c)\) also includes, naturally, a \(v_{dm}^2\) low-energy suppression factor, as desired to avoid an excessive production of \(\gamma\) rays originating from the residual annihilations of Dark Matter particles (if lighter than \(\sim 100\) MeV)\(^7\).

This applies as well to (Majorana or Dirac) spin-\(\frac{1}{2}\) particles\(^8\). Their annihilation cross sections into \(ff\) fermion pairs through the exchanges of a \(U\) boson have, also in this case, the desired \(v_{dm}^2\) suppression factor at threshold, provided the \(U\) boson is axially coupled to Dark Matter particles, and vectorially coupled to matter fermions. This last feature is, in any case, also necessary to avoid a problematic axionlike behavior of its longitudinal polarization state\(^6\). The annihilation, at threshold, of a \(C = +\) state (with \(J = S = 0\)) into a \(ff\) state with \(C' = (-)^{(J'+S')} = +\), through a \(C\)-violating interaction, is forbidden by charge conjugation. This ensures that \(\sigma_{ann} v_{rel}\) has the appropriate \(\sim v_{dm}^2\) behavior, automatically suppressing (by a factor \(\approx 10^{-5}\)) the late annihilations of non-relativistic relic Dark Matter particles.

Furthermore, the annihilation cross sections of spin-\(\frac{1}{2}\) and spin-0 Dark Matter particles are, in this case, given by exactly the same expressions. (Majorana or Dirac) spin-\(\frac{1}{2}\) particles then turn out to be acceptable Light Dark Matter (LDM) candidates, as well as spin-0 particles.

One crucial feature is that the new interactions mediated by the \(U\) boson should actually be “not-so-weak” (at lower energies and relatively to weak interactions) – i.e. \(<\sigma_{ann} v_{rel}/c> \approx 4–5\) picobarns for a Majorana particle, or 8–10 pb for a Dirac one or a complex scalar, so as to ensure for sufficient annihilations of light Dark Matter particles, whatever their spin\(^8\). More precisely, the new \(U\)-mediated Dark-Matter/Matter interactions will be stronger than ordinary weak interactions but only at lower energies, when weak interactions are really very weak. But weaker at higher energies, at which they are damped by \(U\) propagator effects (for \(s > m_U^2\)), when weak-interaction cross sections, still growing with energy like \(s\), become important. The smallness of the \(U\) couplings to ordinary matter, as compared to \(e\), by several orders of magnitude, and the resulting smallness of \(U\) amplitudes compared to electromagnetic ones, then accounts for the fact that these particles have not been observed yet.
We have indicated in May 2003 that “a gamma ray signature from the galactic centre at low energy could be due to the existence of a light new gauge boson”, inducing annihilations of Light Dark Matter particles into $e^+e^-$\(^5\). The observation, a few months later, by the satellite INTEGRAL of a bright 511 keV $\gamma$ ray line from the galactic bulge\(^9\), requiring the presence in this region of a rather large number of annihilating positrons, may then be viewed as originating from such Light Dark Matter particle annihilations\(^10\). Indeed spin-0, but just as well spin-$\frac{1}{2}$ particles, could then be responsible for this bright 511 keV line, which does not seem to have an easy interpretation in terms of known astrophysical processes involving conventional physics\(^8\,11\).

2 Dark Matter decoupling and relic density

Let us evaluate the (relatively large) cross sections needed to obtain a suitable relic abundance of Light Dark Matter particles, of mass $m_{\text{dm}}$\(^8\). Before that, we note that for spin-$\frac{1}{2}$ particles $\chi$ the interactions responsible for the annihilations may be written, in the local limit approximation, as effective four-fermion interactions $L \approx G \bar{\chi}\chi f f$. The corresponding cross sections, proportional to $G^2$, scale like $m_{\text{dm}}^2$. Such fermionic particles that would annihilate through exchanges of (charged) heavy bosons of masses $>\sim m_W$ cannot be light, since their annihilation cross sections would be too small.

- For $10 \text{ MeV} \lesssim m_{\text{dm}} \lesssim 1 \text{ GeV}$, the freeze-out occurs at $T_F$ (between $\approx 6$ and 50 MeV) after most muons have annihilated, but not electrons yet. The surviving particles get diluted by the expansion of the Universe, proportionally to $T^3$, with an extra factor $4/11$ corresponding to the subsequent annihilation of $e^+e^-$ pairs into photons, so that the relic density of Dark Matter particles may now be expressed as

$$n_{\odot} \approx \frac{4}{11} \frac{T_{\odot}^3}{T_F^3} n_{\text{dm}} \cdot$$

$T_{\odot} \approx 2.725 \text{ K}$ being the present photon temperature. We denote by $N_{\odot,\text{dm}} = (2) n_{\odot,\text{dm}}$ the total number density of Dark Matter (particles + antiparticles), with the factor 2 present only for non-self-conjugate particles. The freeze-out equation $\Gamma \approx H$, 

$$n_{\odot,\text{dm}} \frac{4}{11} \frac{T_F^3}{T_{\odot,\gamma}^3} <\sigma_{\text{ann}} v_{\text{rel}}> \approx 1.66 \sqrt{g_*=4\frac{3}{4}} \frac{T_F^2}{m_{\text{Pl}}} \cdot$$

fixes the relic energy density

$$\rho_{\text{dm}} \approx (2) n_{\odot,\text{dm}} m_{\text{dm}} \approx (2) \frac{4}{11} 1.66 \sqrt{g_*=4\frac{3}{4}} \frac{T_{\odot,\gamma}^3}{m_{\text{Pl}}} \frac{1}{<\sigma_{\text{ann}} v_{\text{rel}}>},$$

$$\simeq (2) \frac{x_F}{20} \frac{4.2 \times 10^{-56} \text{ GeV}^2}{<\sigma_{\text{ann}} v_{\text{rel}}>}. \quad (3)$$

Dividing by the critical density $\rho_c/h^2 \approx 1.054 \times 10^{-5} \text{ GeV/cm}^3$, we get the density ratio $\Omega_{\text{dm}} h^2$

$$\Omega_{\text{dm}} h^2 \simeq (2) \frac{x_F}{20} \frac{2 \times 10^{-36} \text{ cm}^2}{<\sigma_{\text{ann}} v_{\text{rel}}/c>}. \quad (4)$$

There is also an expected increase of the required averaged cross section by a factor $\approx 2$, for $<\sigma_{\text{ann}} v_{\text{rel}}>$ behaving at threshold like $v_{\text{dm}}^2$. (Indeed the later annihilations that would still
occur below the temperature \( T_F \) given by eq. (2) are further inhibited by this \( v_{dm}^2 \) factor, preventing the Dark Matter density from reaching the equilibrium value corresponding to this \( T_F \). Altogether, obtaining the right amount of Dark Matter (\( \Omega_{dm} h^2 \approx 0.1 \)) requires typically

\[
<\sigma_{ann} v_{rel}/c> \simeq (2) 4 \text{ pb ,}
\]

when \( <\sigma_{ann} v_{rel}> \) behaves like \( v_{dm}^2 \), the factor 2 being associated with non self-conjugate Dark Matter particles.

- For lighter particles, the evaluation of the relic abundance gets modified, without leading, however, to drastic effects in the cross sections. For particles lighter than about 2 to 3 MeV, that would decouple (at \( T_F \lesssim 0.15 \text{ MeV} \)) after most electrons have annihilated, the dilution factor 4/11 is no longer present in eqs. (1-3). The \( g_\nu \) at Dark Matter freeze-out (no longer 43/4) may be expressed in terms of the neutrino temperature as \( g_\nu \simeq 2 + \frac{7}{8} (2 \times 3) \frac{T_{\nu}^4}{T_{\gamma}^4} \). This would be \( \simeq 3.36 \) in the standard model. The neutrino contribution to \( g_\nu \), however, is no longer the same. Dark Matter particles annihilating after neutrinos decouple (normally at \( T \approx 3.5 \) to 2 MeV) would also heat up the photon gas as compared to neutrinos, so that the resulting \( T_{\nu} \) would be less than the usual \( (4/11)^{1/3} T_{\gamma} \), resulting in a lower contribution of neutrinos to the \( g_\nu \) at \( T_F \). If a significant fraction of Dark Matter annihilations were to occur after neutrino decoupling but before the \( n/p \) ratio freezes out, this could allow for less primordial helium than in the standard model! While light masses \( m_{dm} \lesssim 2 \) MeV, however, are found to be disfavored as they would severely disturb the BBN concordance, slightly larger masses could in fact improve it. In any case, for \( m_{dm} \lesssim 2 \) MeV, the required cross section gets slightly increased (from the absence of the 4/11 factor but with lower values of \( g_\nu \) and \( x_f \)) by \( \simeq 20\% \), up to about (2) times 5 pb – with no spectacular difference expected when \( m_{dm} \) grows from 2 to 10 MeV or more.

- Altogether for cross sections behaving like \( v_{dm}^2 \), the required \( <\sigma_{ann} v_{rel}/c> \)'s at freeze-out are of the order of 4 to 5 picobarns for a self-conjugate (Majorana) Dark Matter particle, or 8 to 10 pb for a complex scalar, or Dirac particle, as summarized below. We do not consider real self-conjugate spin-0 particles, as Bose statistics does not allow for the desired \( P \)-wave annihilation.

| Spin-\( \frac{1}{2} \) Majorana (\( \chi \)) | Spin-\( \frac{1}{2} \) Dirac (\( \psi \)) | Spin-0 (\( \varphi \) complex) |
|---------------------------------|---------------------------------|-------------------------------|
| 4 – 5 pb                        | 8 – 10 pb                       | 8 – 10 pb                     |

Since \( G_F^2 (1 \text{ GeV})^2 /2 \pi \simeq 10^{-38} \text{ cm}^2 \), or \( 10^{-2} \text{ pb} \), cross sections (at freeze-out) of weak interaction order are, for light masses \( m_{dm} \ll \text{ GeV} \), far too small for a correct relic abundance. Significantly larger annihilation cross sections are needed, requiring new types of interactions, if Dark Matter is to be made of such light particles.

This corresponds roughly, for present annihilations of residual Dark Matter particles having a velocity \( v_{dm} \approx 3 \times 10^{-3} \) the velocity at freeze-out in the primordial Universe (\( \simeq 0.4c \)), and cross sections \( <\sigma_{ann} v_{rel}> \propto v_{dm}^2 \), to

\[
<\sigma_{ann} v_{rel}/c> \simeq (4 \text{ to } 10) \times 10^{-5} \text{ pb}.
\]
This is the right order of magnitude for light Dark Matter particle (in the $\simeq$ MeV range) annihilations to be at the origin of the 511 keV $\gamma$ ray signal observed by INTEGRAL from the galactic bulge $^{9,10,8,11}$.

Given INTEGRAL results, we tend to favor rather light Dark Matter masses, so as to maximize the total number of $e^+$ produced in their annihilations, and therefore the observable signal expected from a given Dark Matter energy density. And, also, to avoid these $e^+$ and $e^-$ from Dark Matter annihilations, initially produced with an energy close to $m_{dm}$, having too much energy dissipated in $\gamma$ rays, before $e^+$ can form positronium and annihilate, possibly leading to the bright 511 keV $\gamma$ ray line.

3 Spin-0 annihilations through heavy fermion exchanges

Spin-0 Dark Matter particles ($\varphi$) having Yukawa interactions coupling ordinary quarks and leptons $f$ to heavy fermions $F$ such as mirror fermions (as inspired by $N = 2$ extended supersymmetric and/or higher-dimensional theories$^{13}$) may have relatively large annihilation cross sections, behaving as the inverse of the squared masses of the exchanged mirror fermions. The low-energy effective Lagrangian density responsible for their pair-annihilations into $f\bar{f}$ may indeed be written as an effective dimension-5 interaction,

$$L \approx \frac{C_l C_r}{m_F^2} \varphi^* \varphi \bar{f}_R f_L + \text{h.c.},$$

(7)

$C_l$ and $C_r$ denoting the Yukawa couplings to the left-handed and right-handed fermion fields, respectively. The resulting annihilation cross section at threshold, of the type

$$\sigma_{\text{ann}} v_{\text{rel}} \approx \frac{C_l^2 C_r^2}{\pi m_F^4},$$

(8)

(for non-chiral couplings, i.e. $C_l C_r \neq 0$), is largely independent of the Dark Matter mass; it can be quite significant and take the appropriate values, even for light spin-0 Dark Matter particles$^5$.

However, in the absence of a $P$-wave suppression factor $\propto v_{dm}^2$ one runs the risk, at least for lighter Dark Matter particles ($\lesssim 100$ MeV/$c^2$), of too much $\gamma$ ray production due to residual annihilations of Dark Matter particles$^7$ (unless there is an asymmetry between Dark Matter particles and antiparticles).

4 Exchanges of a new light spin-1 gauge boson $U$

It is thus preferable to consider annihilations induced through the virtual production of a new light neutral spin-1 gauge boson $U$ $^{5,6,8}$.

4.1 Effective Lagrangian density for spin-0 annihilations

Spin-0 Dark Matter annihilations may be described, in the local limit approximation, by an effective Lagrangian density involving the product of the Dark Matter ($\varphi$) and quark and lepton ($f$) contributions to the $U$ current, i.e.

$$L = \frac{C_U}{m_U^2} \varphi^* i \partial_\mu \varphi \left( f_V \bar{f} \gamma^\mu f + f_A \bar{f} \gamma^\mu \gamma_5 f \right),$$

(9)

$C_U$ and $f_V$ and/or $f_A$ denoting the couplings of the new gauge boson $U$ to the spin-0 Dark Matter field $\varphi$ and the matter fermion $f$ considered, respectively. The $f_V$ coupling (i.e. in fact the product $C_U f_V$) corresponds to an interaction invariant under Charge Conjugation, while for the $f_A$ coupling (i.e. $C_U f_A$) this interaction would be $C$-violating.
4.2 Restrictions on axial couplings of the \( U \) to ordinary matter.

These axial couplings \( f_A \) of the new light gauge boson \( U \) to matter fermions \( f \), however, are likely to be absent – or at least should be sufficiently small – otherwise they would lead to a generally-unwanted axionlike behavior of the light spin-1 \( U \) boson. Let us concentrate for a while on what would happen in the presence of such axial couplings \( (f_A) \) of the \( U \) boson.

The longitudinal polarisation state of a light (ultrarelativistic) \( U \) boson, with polarisation vector \( \epsilon^\mu_L \simeq k_U^\mu/m_U \), and coupled proportionately to \( f_{V,A} \epsilon^\mu_L \simeq (f_{V,A}/m_U) k_U^\mu \), would then behave very much as a spin-0 axionlike particle, having pseudoscalar couplings to quarks and leptons

\[
 f_{P,q,l} = 2 f_A \frac{m_{q,l}}{m_U}, \tag{10}
\]

as soon as there is a (non-conserved) axial part in the quark-and-lepton contribution to the \( U \) current. Indeed \( k_U^\mu \) acting on an axil current \( f \gamma^\mu \gamma_5 f \) resurrects an effective pseudoscalar coupling to \( f \gamma_5 f \) with an extra factor \( 2m_f \) in its coefficient, which leads to the effective pseudoscalar coupling \( (10) \), as shown in the second paper of Ref. 6 (while \( k_U^\mu \) acting on a vector matter current \( f \gamma^\mu f \) gives no such contribution).

There, the axial couplings \( f_A \) were expressed in terms of the extra \( U(1) \) gauge coupling \( g' \) as \( \tilde{g}'_T \), in the simplest case of a universal axial coupling of the new gauge boson \( U \) to quarks ans leptons (obtained for equal v.e.v.’s for the two Higgs doublets\(^b \), or \( v_1 = v_2 \), i.e. \( x = 1 \)). One also has (with \( r = 1 \), i.e. no extra Higgs singlet introduced, and the extra \( U(1) \) symmetry spontaneously broken at the electroweak scale), \( 2 f_A = \tilde{g}'_T = 2^{1/2} G_F^{1/2} m_U \simeq 4 \times 10^{-6} m_U(\text{MeV}) \). Expression (10) then reconstructs exactly the usual pseudoscalar coupling \( 2^{1/2} G_F^{1/2} m_{q,l} \) of a spin-0 axionlike particle (the one “eaten away” by the light spin-1 \( U \) boson) to quarks and leptons. This is phenomenologically unacceptable, as no such axionlike particle has been observed.

More generally, however, we can introduce as in 6 an extra Higgs singlet (with, possibly, a large v.e.v. so that the extra \( U(1) \) symmetry would then be broken “at a large scale” \( F \)), and the two Higgs doublet v.e.v.’s are not necessarily equal (their ratio being denoted by \( x \)). The effective pseudoscalar coupling (10) would then read:

\[
 f_{P,q,l} = 2 f_A \frac{m_{q,l}}{m_U} = 2^{1/2} G_F^{1/2} m_{q,l} \times \begin{cases} \frac{r}{x} & \text{for } u, c, t \text{ quarks}, \\ \frac{r}{x} & \text{for } d, s, b \text{ quarks and } e, \mu, \tau \text{ leptons.} \end{cases} \tag{11}
\]

This may now be acceptable if the ratios \( f_A/m_U \) (inversely proportional to the extra \( U(1) \) symmetry-breaking scale \( F \)) are sufficiently small (as it would happen for \( F \) at least slightly above or well above the electroweak scale), as in the “invisible \( U \) boson” or analogous “invisible axion” mechanisms 6. The values of \( f_{V,A}/m_U \) that we may like to consider (cf. Sec. 8) to generate sufficient annihilations of Light Dark Matter particles into \( f \bar{f} \) pairs (in practice \( e^+e^- \), or \( \nu\bar{\nu} \)), however, could be larger than what would be acceptable for the ratios \( f_A/m_U \), unless of course the Dark Matter coupling \( C_U \) is taken to be sufficiently large. (Note, in addition, that no direct effective \( U \gamma \gamma \) coupling is to be expected here, and that the \( U \) boson would not decay into \( \gamma \gamma \), in contrast to an axion.)

In particular, from the results of \( \psi \) and \( \Upsilon \) decay experiments, sensitive to the decays \( \psi \to \gamma + U \), \( \Upsilon \to \gamma + U \) (with the \( U \) having invisible decay modes), into Dark Matter particle or \( \nu\bar{\nu} \) \(^b\) Separately responsible for the down-quark and charged-lepton masses (through \( < \varphi_d > = v_1/\sqrt{2} \)), or up-quark masses (through \( < \varphi_u > = v_2/\sqrt{2} \)), as in supersymmetric extensions of the standard model\(^1 \), from which these theories originate.
pairs), we get limits on the axial couplings of the $U$ to the $c$ and $b$ quarks. These axial couplings may be expressed, using the language of Refs. 6,14, as $f_A \simeq 2 \times 10^{-6} m_U (\text{MeV}) \times (r x \text{ or } r/x)$ for the $c$ and $b$ quarks, respectively (cf. eq. (11)). $r \ll 1$ corresponds to an extra $U(1)$ symmetry broken “at a very high scale” $F$ much larger than the electroweak scale, so that the axionlike effects of the new gauge boson $U$ would then get quasi “invisible”. We then get the limits $r x < 0.75$ and $r/x < 0.4$ from the $\psi \rightarrow \gamma + U$ and $\Upsilon \rightarrow \gamma + U$ decays, respectively, which may be turned into the following restrictions on the axial couplings of the $c$ and $b$ quarks,

\[ f_{Ac} < 1.5 \times 10^{-6} m_U (\text{MeV}) \quad , \quad f_{Ab} < 0.8 \times 10^{-6} m_U (\text{MeV}) \ . \]  

(12)

This may be remembered approximately, at least in the latter case, as

\[ \frac{f_{Aq}^2}{m_U^2} < \frac{1}{10} G_F \ . \]  

(13)

Further, and potentially stronger, restrictions on the coupling $f_{Aq}$ may also be obtained from a careful analysis of searches for $K^+ \rightarrow \pi^+ + \text{unobserved } U$ boson, provided one is sufficiently confident about the possibility of estimating reliably the decay rate in terms of $f_{Aq}$ and $m_U$. As an illustration if we consider that the branching ratio $B (K^+ \rightarrow \pi^+ U)$ ought to be $\gtrsim 10^{-8} \frac{r^2}{x^2}$, and combine this with the experimental limit $B (K^+ \rightarrow \pi^+ + \text{unobserved } U) \lesssim 0.6 \times 10^{-10}$ (resp. $10^{-10}$) for $m_U \lesssim 70$ (resp. 100) MeV, we get $r/x \lesssim 1$, corresponding to

\[ f_{As} \lesssim 2 \times 10^{-7} m_U (\text{MeV}) \ , \]  

(14)

which may be remembered as $f_{As}^2/m_U^2 \lesssim (1/300) G_F$. A similar analysis may be done for the $U$ contribution to the anomalous magnetic moments of the charged leptons, with appropriate care due to the possibility of cancellations between (positive) vector and (negative) axial contributions, of opposite signs. In the limit in which $m_U$ is small compared to $m_\mu$, one gets from from $f_{V\mu}$ and $f_{A\mu}$ the two contributions

\[ \delta a_\mu \simeq f_{V\mu}^2 \frac{2}{8 \pi^2} - f_{A\mu}^2 \frac{m_\mu^2}{4 \pi^2 m_U^2} \ . \]  

(15)

If, to keep things simple, we were to disregard the $f_{V\mu}$ contribution to $\delta a_\mu$, we would get

\[ \delta a_\mu \simeq - \frac{f_{A\mu}^2}{4 \pi^2} \frac{m_\mu^2}{m_U^2} \simeq - \frac{G_F m_\mu^2}{8 \pi^2 \sqrt{2}} \frac{r^2}{x^2} \simeq 1.17 \times 10^{-9} \frac{r^2}{x^2} \]  

(16)

(as for the exchange of a pseudoscalar axionlike particle), which implies $r/x < 1.5$, corresponding to

\[ f_{A\mu} < 3 \times 10^{-6} m_U (\text{MeV}) \ , \]  

(17)

if we want the extra (negative) $f_{A\mu}$ contribution to $a_\mu$ to be, by itself, $\lesssim (2 \text{ to } 3) \times 10^{-9}$ in magnitude. This bound, approximately expressed as

\[ \frac{f_{A\mu}^2}{m_U^2} < G_F \ , \]  

(18)

\[ \text{We assume here that the } U \text{ decays preferentially into unobserved neutrals, as it would happen for a } U \text{ coupling to Dark Matter } C_U \text{ significantly larger than couplings to ordinary matter fermions } f. \text{ Otherwise the limits of (12) should be slightly weakened.} \]
may of course be relaxed in the presence of a vector coupling $f_{V\mu}$ sufficiently large compared to $f_{A\mu}$, as shown by eq. (15)\textsuperscript{d}.

Furthermore, atomic physics parity-violation experiments also provide strong constraints on the coupling constant product $f_{A\mu}f_{V\mu}$, such that $f_{A\mu}f_{V\mu}/m_{U}^{2}$ should be less than a small fraction of $G_{F}$\textsuperscript{17}. This may also be interpreted as pointing towards a purely vectorial coupling of the new gauge boson $U$ to ordinary matter – unless of course its couplings to matter are taken to be sufficiently small (which again would require the $U$ coupling to Dark Matter $C_{U}$ to be large enough).

 Altogether, we shall generally stick, at least for simplicity, to purely vector couplings $f_{V}$ of the $U$ boson to ordinary matter. This is also motivated by the fact that this occurs naturally in a number of $SU(3) \times SU(2) \times U(1) \times$ extra-$U(1)$, or $SU(5) \times$ extra-$U(1)$, gauge extensions of the Standard Model (cf. the third paper in Ref. 6), in which the matter couplings of the $U$ boson turn out to be given by a (conserved) linear combination of the $B$, $L$ (or $B - L$, in grand-unified theories) and electromagnetic currents.

5 Spin-0 annihilations through $U$ exchanges

The threshold behavior of the annihilation cross section $\sigma_{ann}(\varphi\bar{\varphi} \to f\bar{f})$ may be understood from simple arguments based on Charge Conjugation. The initial $\varphi\bar{\varphi}$ state has $C = +$ in an $S$ wave ($L = 0$). The final $f\bar{f}$ state then also has $C' = (-)^{(L'+S')} = +$, since angular momentum conservation requires $J' = J = 0$. In the case of an axial coupling ($f_{A}$) to the fermion field $f$, the relevant terms in the Lagrangian density (9), being $C$-violating, cannot induce the decay $\varphi\bar{\varphi} \to f\bar{f}$. For a vector coupling ($f_{V}$), the relevant terms are $C$-conserving, but the $f\bar{f}$ final state, being vectorially produced through the virtual production of a $U$ boson (as if it were through a one-photon exchange), must have $C' = -$ (instead of $C' = +$ from $J$ conservation). In both cases there can be no $S$-wave term in the annihilation cross section. The dominant ($P$-wave) terms in $\sigma_{ann} v_{rel}$ are proportional to the square of the Dark Matter particle velocity in the initial state,

$$\sigma_{ann} \propto v_{dm}^{2}$$

at threshold\textsuperscript{e}.

For a vector coupling to fermions ($f_{V}$), the annihilation cross section (which may also be obtained directly from the corresponding production cross section in $e^{+}e^{-}$ annihilations, cf. Sec. 7) is given by

$$\sigma_{ann} v_{rel}(\varphi\bar{\varphi} \to f\bar{f}) = \frac{2}{3\pi} v_{dm}^{2} \frac{C_{i}^{2} f_{V}^{2}}{(m_{U}^{2} - 4E^{2})^{2}} \sqrt{1 - \frac{m_{f}^{2}}{E^{2}}} \left( E^{2} + \frac{m_{f}^{2}}{2} \right) ,$$  

\textsuperscript{d}Furthermore if $m_{U}$ is large compared to $m_{e}$, we also have:

$$\delta a_{e} \simeq \frac{1}{12 \pi^{2}} \frac{m_{e}^{2}}{m_{U}^{2}} \left( f_{V}^{2} - 5 f_{A}^{2} \right) ,$$

which generally turns out to be less constraining than the corresponding limits from the muon $g - 2$; cf. eqs. (35,36) in Sec. 8. This may also be reexpressed, using eq. (11), as\textsuperscript{16}

$$\delta a_{e} \simeq \frac{f_{A}^{2} m_{e}^{2}}{4 \pi^{2} m_{U}^{2}} \left( \frac{5}{3} + \frac{1}{3} \frac{f_{V}^{2}}{f_{A}^{2}} \right) \simeq G_{F} m_{e}^{2} \frac{5}{8 \pi^{2} \sqrt{2}} \left( \frac{5}{3} + \frac{1}{3} \frac{f_{V}^{2}}{f_{A}^{2}} \right) \frac{v^{2}}{m_{f}^{2}} .$$

\textsuperscript{e}In other terms, the $U$ charge-density and current of a $\varphi\bar{\varphi}$ pair vanish at threshold; the annihilation amplitudes vanish proportionally to the rest-frame momenta $p_{dm}$ of the initial particles, or to $v_{dm}$, as a result of the derivative nature of the $U$ coupling to scalar particles.
where $\beta_f = v_f / (c) = (1 - m_f^2 / E^2)^{1/2}$, and
\[
\frac{3}{2} \beta_f - \frac{1}{2} \beta_f^3 = \sqrt{1 - \frac{m_f^2}{E^2}} \left( E^2 + \frac{m_f^2}{2} \right),
\]
is the usual kinematic factor relative to the vectorial pair production of spin-$\frac{1}{2}$ Dirac fermions.

For an axial coupling ($f_A$), replacing $f_V$ by $f_A$, and (21) by the factor $\beta_f^3$ appropriate to the axial production of a pair $ff$ of spin-$\frac{1}{2}$ particles, we get
\[
\sigma_{\text{ann}} v_{\text{rel}} (\varphi \bar{\varphi} \rightarrow ff) = \frac{2}{3\pi} v_{\text{dm}}^2 \frac{C_U^2 f_A^2}{(m_U^2 - 4E^2)^2} E^2 \left(1 - \frac{m_f^2}{E^2}\right)^{3/2}.
\]
If the $U$ coupling to the fermion field $f$ includes both vector and axial contributions, the annihilation cross section is the sum of the above contributions (20) and (22). In all cases, if behaves at threshold like $v_{\text{dm}}^2$.

6 Spin-$\frac{1}{2}$ annihilation cross sections through $U$ exchanges

In this case also, there will be no $S$-wave annihilations, for a $U$ boson having axial couplings to Dark Matter, with vector ones to ordinary matter.$^7$

Let us consider spin-$\frac{1}{2}$ Majorana fermions $\chi$, which can only have a purely axial coupling to the $U$ boson. The analysis, in fact, applies as well to Dirac fermions ($\psi$), provided they are also axially coupled to the $U$ boson (i.e. excluding the presence of a vector coupling of the $U$ to the Dark Matter Dirac fermion $\psi$).$^7$

The effective Lagrangian density may now be written as
\[
\mathcal{L} = \frac{C_U}{2 m_U^2} \bar{\chi} \gamma_\mu \gamma_5 \chi \left( f_V \bar{f} \gamma^\mu f + f_A \bar{f} \gamma^\mu \gamma_5 f \right).
\]
The coupling $f_V$ (i.e. in fact $C_U f_V$) is now $C$-violating while $f_A$ (still generally presumed to be absent as it would be related with an unwanted axionlike behavior of the $U$ boson, as we have discussed in subsection 4.2) would be $C$-conserving.

At threshold the antisymmetry of a 2-Majorana $\chi \chi$ state imposes that the total spin be $J = S = 0$, so that the production (indifferently through vector and/or axial couplings to $f$) of massless fermion pairs $ff$ with total angular momentum $\lambda = \pm 1$ is forbidden. But the ($S$-wave) annihilation cross section $\sigma_{\text{ann}} v_{\text{rel}}$ could in principle still include, at threshold (potentially dangerous) non-vanishing contributions proportional to $m_f^2$ – depending on whether the $U$ coupling to $f$ is taken to be vector or axial, as we shall see.

The initial $\chi \chi$ state has $C = +$. If it is in an $S$ wave (so that $J = S = 0$), the final $ff$ state must have $C^\prime = (-)^{(L^\prime + S^\prime)} = +$ (with $L^\prime = S^\prime$ from $J = 0$). The ($C$-violating) coupling product $C_U f_V$ cannot contribute to the $S$-wave $\chi \chi \rightarrow ff$ annihilation amplitude. (Also, a $U$ with a vector coupling to $f$ can only produce a $ff$ pair with $C^\prime = -$, while $C^\prime = +$ from angular momentum conservation.) The $f_V$ contribution to the $S$-wave annihilation cross section must vanish, so that
\[
\sigma_{\text{ann}} v_{\text{rel}} (\chi \chi \rightarrow ff) \propto v_{\text{dm}}^2 \quad \text{at threshold}.
\]

$^7$To relate the annihilation cross sections of Dirac and Majorana particles we can write the decomposition $\psi = (\chi - i \chi^\prime)/\sqrt{2}$ of the Dirac field $\psi$, so that $\bar{\psi} \gamma_\mu \gamma_5 \psi = \frac{1}{2} \bar{\chi} \gamma_\mu \gamma_5 \chi + \frac{i}{2} \bar{\chi}^\prime \gamma_\mu \gamma_5 \chi^\prime$, and consider an initial state in which each of the two annihilating particles is either a $\psi$ or a $\bar{\psi}$ (or, equivalently, either a $\chi$ or a $\chi^\prime$). The pair annihilation cross section of Dirac particles ($\psi$, with axial coupling $C_U$) is the same as for Majorana particles ($\chi$, with axial coupling $C_U/2$), so that $\sigma_{\text{ann}} v_{\text{rel}} (\bar{\psi} \psi \rightarrow e^+ e^-) = \sigma_{\text{ann}} v_{\text{rel}} (\chi \chi \rightarrow e^+ e^-)$. 

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This would be different for an axial coupling $f_A$ (corresponding to $C = +$). The $f_A$ contribution to $S$-wave annihilation has now no reason to vanish, as soon as $m_f \neq 0$. Constant terms proportional to $m_f^2$ (potentially undesirable for $m_{dm} \lesssim 100 \text{ MeV}$) then appear in $\sigma_{ann} v_{rel}$, as shown later in eq. (27).

It is remarkable that the possible constraint of vector couplings of the light $U$ to the matter fermions $f$, obtained here from the requirement of spin-$\frac{1}{2}$ annihilation cross sections strictly behaving like $v_{dm}^2$, corresponds to the one following from the fact that a light $U$ boson would have an unacceptable axionlike behavior if it had sizeable axial couplings $f_A$ to the matter fermions $f^6$, as discussed in subsection 4.2.

As for spin-0 particles, the annihilation cross sections of spin-$\frac{1}{2}$ Dark Matter particle pairs ($\chi \chi$ or $\bar{\psi} \psi$) into $e^+e^-$ may be related to their production cross sections in $e^+e^-$ annihilations (cf. Sec. 7). From the relation between the pair production of (axially-coupled) spin-$\frac{1}{2}$ particles, and spin-0 particles, in $e^+e^-$ annihilations, we find that their annihilation cross sections are the same, namely, when the electron mass is neglected,

$$\sigma_{ann} v_{rel} (\chi \chi \rightarrow e^+e^-) = \frac{2}{3\pi} v_{dm}^2 \frac{C_U^2 f_V^2}{(m_U^2 - 4E^2)^2} \sqrt{1 - \frac{m_e^2}{E^2}} \left( E^2 + \frac{m_f^2}{2} \right),$$

(25)

Since no $S$-wave annihilation cross section may be induced from a non-vanishing $m_e$ (in the case of $f_V$), the only expected effect of a non-vanishing $m_e$ (or more generally of fermion masses $m_f$) is a multiplication by the usual kinematic factor for the vectorial production of a $f \bar{f}$ pair, $\frac{1}{4} \beta_f - \frac{1}{2} \beta_f^2$:

$$\sigma_{ann} v_{rel} (\chi \chi \rightarrow f \bar{f}) = \frac{2}{3\pi} v_{dm}^2 \frac{C_U^2 f_V^2}{(m_U^2 - 4E^2)^2} \sqrt{1 - \frac{m_e^2}{E^2}} \left( E^2 + \frac{m_f^2}{2} \right),$$

(26)

exactly as in (20) for the pair annihilation of spin-0 particles. The same formula also applies for the annihilation cross section of Dirac particles, $\sigma_{ann} v_{rel} (\bar{\psi} \psi \rightarrow f \bar{f})$, as already mentioned.

Remarkably enough, this cross section (26) for the annihilation of spin-$\frac{1}{2}$ particles (axially coupled to the $U$ boson, this one vectorially coupled to the matter fermion $f$) is identical to (20) for the annihilation of spin-0 candidates! We get in both cases the same $v_{dm}^2$ suppression factor, as desirable to avoid an excessive production of gamma rays from residual light Dark Matter annihilations. The (collisional and free-streaming) damping effects$^{4,5,18}$ associated with such particles are also, in both cases, sufficiently small.

For comparison, the spin-$\frac{1}{2}$ annihilation cross section in the case of an axial matter fermion coupling $f_A$ (still with an axial coupling of the Dark Matter fermion $\chi$ or $\bar{\psi}$ to the $U$, and assuming for simplicity the $U$ heavy compared to $m_{dm}$ and $m_f$) is:

$$\sigma_{ann} v_{rel} (\chi \chi \rightarrow f \bar{f}) = \frac{1}{2\pi} \frac{C_U^2 f_A^2}{(m_U^2 - 4E^2)^2} \sqrt{1 - \frac{m_e^2}{E^2}} \left[ \frac{4}{3} \left( E^2 - m_f^2 \right) v_{dm}^2 + \frac{m_{dm}^2}{E^2} m_f^2 \right].$$

(27)

It coincides with (26) (replacing $f_A$ by $f_V$) in the limit of vanishing $m_f$. But the overall $v_{dm}^2$ suppression factor no longer subsists for $m_f \neq 0$, for which one recovers a non-vanishing $S$-wave term in the annihilation cross section, proportional to $m_f^2$.

Such a behaviour, however, could still be tolerated, from the point of view of the annihilation cross section, if $m_{dm}$ is sufficiently large compared to $m_e$, so that the “unwanted” term not proportional to $v_{dm}^2$, damped by a factor $\propto m_e^2/m_{dm}^2$, be sufficiently small. This provides a way
to ease out the requirement of a cross section $\sigma_{\text{ann}} v_{\text{rel}}$ proportional to $v_{\text{dm}}^2$. As an example for $m_{\text{dm}} \gtrsim 10$ MeV, the coefficient of the constant term in (27), although non-vanishing, is smaller than the coefficient of the $v_{\text{dm}}^2$ term by a factor of about $3 m_e^2/(64 m_{\text{dm}}^2) \lesssim 1.2 \times 10^{-4}$. The constant term in the annihilation cross section $<\sigma_{\text{ann}} v_{\text{rel}}> - the one potentially dangerous when considering residual annihilations of relic low-velocity Dark Matter particles – then represents less than $10^{-3}$ times the $<\sigma_{\text{ann}} v_{\text{rel}}> at freeze out (corresponding to $v_{\text{dm}} \simeq 10^{-5}c$), determined by the relic abundance estimate of Sec. 2. Remembering also that a certain fraction of Dark Matter annihilations, possibly around 60% or even more depending on $U$ couplings, could be into unobserved $\nu\bar{\nu}$ pairs rather than $e^+e^-$, this may well be sufficient to keep the gamma ray production from residual Dark Matter annihilations at a sufficiently small level, also in this case of an axial-axial interaction of a ($\gtrsim 10$ MeV) Dark Matter particle with ordinary matter.

7 Relating production and annihilation cross sections

The production (in $e^+e^-$ scatterings) and annihilation cross sections of Dark Matter particles may be related using CPT (or simply $T$) invariance. One gets for example, in the case of spin-0 particles:

$$\sigma_{\text{prod}} v_e (e^+e^- \to \varphi \bar{\varphi}) / v_{\text{dm}} \equiv \frac{1}{4} \sigma_{\text{ann}} v_{\text{dm}} (\varphi \bar{\varphi} \to e^+e^-) / v_e ,$$

which may also be used to derive the annihilation cross section from the production one. Indeed, from the electromagnetic pair production cross section of charged spin-0 particles in $e^+e^-$ annihilations (neglecting $m_e$), $\sigma_{\text{prod}}^{(e^+) \varphi} = \frac{4}{15 \pi} m_{\text{dm}}^3$, we get the production cross section, through $U$ exchanges, of neutral spin-0 Dark Matter particles,

$$\sigma_{\text{prod}} (e^+e^- \to \varphi \bar{\varphi}) = \frac{1}{12 \pi} \frac{C_U^2 f_V^2}{(4 E^2 - m_U^2)^2} E^2 m_{\text{dm}}^3 .$$

Multiplying it by $v_{\text{rel}} \simeq 2$, by the spin factor 4 and the velocity ratio $(\beta_e \simeq 1)/(\beta_{\text{dm}} = v_{\text{dm}})$, we get the corresponding annihilation cross section,

$$\sigma_{\text{ann}} v_{\text{rel}} (\varphi \bar{\varphi} \to e^+e^-) = \frac{2}{3 \pi} v_{\text{dm}}^2 \frac{C_U^2 f_V^2}{(m_U^2 - 4 E^2)^2} E^2 ,$$

which, once the kinematic factor $\frac{3}{2} \beta f - \frac{1}{2} \beta_f^3$ is reintroduced, coincides with (20). (Replacing $f_V$ by $f_A$, and reintroducing the kinematic factor $\beta_f^3$, we recover expression (22) of the annihilation cross section through an axial coupling to $f$.)

The $v_{\text{dm}}^2$ suppression factor in the annihilation cross section $\sigma_{\text{ann}} v_{\text{rel}}$ of spin-0 particles $\varphi$ appears simply as a reflection by CPT of the well-known $\beta_3$ factor for their pair production in $e^+e^-$ annihilations (with, in both cases, a $P$ wave for the $\varphi \bar{\varphi}$ state). This also applies to spin-$\frac{1}{2}$ particles, whether Majorana or Dirac, axially coupled to the $U$ boson. Their pair production through an axial coupling to the $U$ also involves a $\beta_{\text{dm}}^3$ factor, which reflects (by CPT) in a $v_{\text{dm}}^2$ suppression factor for the annihilation cross section at threshold.

Because the production cross sections of spin-0 and spin-$\frac{1}{2}$ particles in $e^+e^-$ annihilations are given by similar formulas the corresponding annihilation cross sections into $ff$ pairs are given, also, by similar formulas. For spin-$\frac{1}{2}$ Dirac particles, eq. (28) is replaced by

$$\sigma_{\text{prod}} v_e (e^+e^- \to \psi \bar{\psi}) / v_{\text{dm}} \equiv \sigma_{\text{ann}} v_{\text{dm}} (\psi \bar{\psi} \to e^+e^-) / v_e ,$$

with, for an axially coupled $U$ boson (with axial coupling $C_U$) and neglecting $m_e$,

$$\sigma_{\text{prod}} (e^+e^- \to \varphi \bar{\varphi}) = \frac{1}{4} \sigma_{\text{prod}} (e^+e^- \to \psi \bar{\psi}) ,$$
both cross sections being proportional to $\beta_{dm}^3$, so that
\[
\sigma_{ann}(\psi \bar{\psi} \rightarrow e^+ e^-) = \sigma_{ann}(\varphi \bar{\varphi} \rightarrow e^+ e^-),
\]
as given by (30), the same formula applying to a Majorana fermion $\chi$ (with axial coupling $C_U$).

This leads to eqs. (20), (22) and (26), when the appropriate phase space factors are reestablished.

8 Constraints on the $U$ couplings

To get an idea of the size of the couplings required to get appropriate values of the annihilation cross sections at freeze-out ($\approx 8-10$ picobarns for a complex spin-0 particle, or 4–5 for a Majorana one, cf. Sec. 2), we can write the above expressions (20,22,26) as
\[
\sigma_{ann} v_{\text{rel}} \simeq \frac{v_{dm}^2}{16} \left(\frac{C_U f_{V,A}}{10^{-6}}\right)^2 \left(\frac{m_{dm} \times 3.6 \text{ MeV}}{m_U^2 - 4 m_{dm}^2}\right)^2 \text{ pb},
\]

where $f_{V,A}$ are the vector and axial coupling constants, respectively, related to $C_U$.

This expression can be further simplified as
\[
\delta a_\mu \simeq \frac{f_{V,\mu}^2}{8 \pi^2} \simeq (2 \pm 2) \times 10^{-9},
\]
\[
\delta a_e \simeq \frac{f_{V,e}^2}{12 \pi^2} \frac{m_e^2}{m_U^2} \simeq (4 \pm 3) \times 10^{-11},
\]
so that
\[
f_{V,\mu} \lesssim 6 \times 10^{-4}, \quad f_{V,e} \lesssim 2 \times 10^{-4} m_U(\text{MeV}).
\]

One should also have, for a $U$ mass larger than a few MeV's,
\[
|f_{V,e} f_{V,\mu}| \lesssim G_F m_U^2 \lesssim 10^{-11} (m_U(\text{MeV}))^2
\]
so that $U$ exchanges do not modify excessively $\nu - e$ low-energy elastic scattering cross sections. This requires that the $U$ couplings to neutrinos, at least, be sufficiently small. Conversely, the $U$ coupling $C_U$ of Dark Matter should not be too small, so as to get, from eq. (34), large enough values of the annihilation cross section $<\sigma_{ann} v_{rel}>$.

If, in addition, all $f_V$'s turn out to be of the same order, they should then be smaller than about $3 \times 10^{-6} m_U(\text{MeV})$, or in other terms, approximately,
\[
\frac{f_V^2}{m_U^2} < G_F
\]
(to be contrasted with eqs. (12,13,14) in subsection 4.2, imposing a stricter bound for an axial coupling to quarks). Expression (34) of $<\sigma_{ann} v_{rel}>$ then implies that the $U$ coupling to Dark
Matter $C_U$ should be larger than about $0.2 \left( m_U^2 - 4 m_{dm}^2 \right)/m_U m_{dm}$ (for $<\sigma_{\text{ann}}v_{\text{rel}}>$ to be larger than 4 or 5 pb), in which case the self-interactions of Dark Matter would be quite significant, much stronger than ordinary weak interactions, by several orders of magnitude (depending also on the energy considered).

When the $U$ and $\varphi$ particles are light (and neglecting $m_e$), eq. (29) may be written as

$$
\sigma_{\text{prod}}(e^+e^- \rightarrow \varphi \bar{\varphi}) \simeq \frac{C_U^2 f_V^2}{48 \pi s} \simeq C_U^2 f_V^2 \frac{2.58 \mu b}{s(\text{GeV}^2)} \simeq \left( \frac{C_U f_V}{10^{-6}} \right)^2 \frac{2.6 \times 10^{-42} \text{cm}^2}{(\sqrt{s}(\text{GeV}))^2},
$$

which may also be applied in the case of an axial coupling by changing $f_V$ into $f_A$. This should still be multiplied by 4, or 2, for the production of a spin-$\frac{1}{2}$ $\psi \psi$ or $\chi \chi$ pair, respectively. It is easy to get from there the corresponding cross sections for $e^+e^- \rightarrow \gamma + \text{a pair of unobserved Dark Matter particles}$, and to compare them (as in $^2$) to the cross sections for $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ or $e^+e^- \rightarrow \gamma\bar{\gamma}\bar{\gamma}$. For the relevant values of $C_U f_V$ considered, these production cross sections get very small at high energies, much below neutrino production cross sections, so that the direct production of such Dark Matter particles (and $U$ bosons as well) is in general not expected to lead to easily observable signals in $e^+e^-$ annihilations, although this would deserve further studies. Note that, depending on the relative values of $C_U$ and $f_V$ (and to a lesser extent $f_{V_e}$), especially if we take $C_U$ relatively large compared to $f_V$, as indicated earlier, the $U$ boson may in general be expected to have preferred invisible decay modes into Dark Matter particles, dominating over visible decays $U \rightarrow e^+e^-$. 

9 Final remarks

Altogether spin-$\frac{1}{2}$ Dark Matter particles axially coupled to the $U$ boson have the required characteristics for Light Dark Matter (LDM) particles annihilating into $e^+e^-$, as well as spin-0 particles. In both cases, $U$-induced Dark-Matter/electron interactions should be significantly stronger than ordinary weak interactions at low energy (but weaker at high energies), which requires the $U$ to be more strongly coupled to Dark Matter than to ordinary matter – also resulting in significant $U$-induced Dark Matter self-interactions. Finally, light spin-$\frac{1}{2}$ Dark Matter particles appear more attractive than spin-0 ones, as the smallness of their mass is easier to understand, and provide valuable alternative scenarios to be discussed and confronted with the standard ones.

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