LOW-FREQUENCY GRAVITATIONAL WAVES FROM BLACK HOLE MACHO BINARIES

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ABSTRACT

Nakamura et al. have recently estimated the initial distribution of binary MACHOs in galactic halos, assuming that the MACHOs are primordial black holes of mass $\sim 0.5 M_\odot$ and considered their coalescence as a possible source for ground-based interferometer gravitational wave detectors such as LIGO. Evolving their binary distribution forward in time to the present, the low-frequency ($10^{-5} < f < 10^{-1}$ Hz) spectrum of gravitational waves associated with such a population of compact binaries is calculated. The resulting gravitational waves would form a strong stochastic background in proposed space interferometers such as LISA and the OMEGA. Low-frequency gravitational waves are likely to become a key tool for determining the properties of binaries within the dark MACHO population.

Subject headings: black hole physics — dark matter — Galaxy: halo — gravitation — gravitational lensing

1. INTRODUCTION

The MACHO project has thus far detected 15 candidate microlensing events in the direction of the Large Magellanic Cloud (LMC) (Cook et al. 1998) and two in the direction of the Small Magellanic Cloud (SMC) (Alcock et al. 1998). One of the LMC events (Bennett et al. 1996) and one of the SMC events (Alcock et al. 1998) appear to have been caused by binary MACHO lenses. The LMC observations suggest that perhaps half of the halo consists of MACHOs of average mass $\sim 0.5 M_\odot$. Observational constraints make it unlikely that these MACHOs are main-sequence red dwarfs (Bahcall et al. 1994; Graff & Freese 1996a, 1996b) or white dwarfs (Charlot & Silk 1995; Adams & Laughlin 1996). Given these difficulties, more speculative forms of MACHOs have been seriously considered, such as primordial black holes or boson stars.

Recently, Nakamura et al. (1997; hereafter, NSTT) considered the possibility that all of the halo MACHOs consist of primordial black holes (BH-MACHOs) (Nakamura et al. 1997). They derived an estimate of the initial binary distribution function and determined the rate at which BH-MACHOs in binaries would coalesce due to emission of gravitational radiation. They found extragalactic halo BH-MACHO coalescences to be promising, and proposed gravitational waves (GWs) for ground-based interferometer detectors such as LIGO.

In this Letter, BH-MACHOs are again examined as possible sources of gravitational radiation but now in the low-frequency ($10^{-5} - 10^{-1}$ Hz) band in which space-based interferometers, such as the proposed Laser Interferometer Space Antenna (LISA) (Bender et al. 1998) and Orbiting Medium Explorer for Gravitational Astrophysics (OMEGA) (Hellings et al. 1998) instruments, will be most sensitive. The NSTT binary distribution function is assumed and evolved forward from the time of black hole formation to the present to determine the distribution of BH-MACHO binaries in semimajor axis and eccentricity today. The black hole binaries with orbital frequencies higher than about $10^{-5}$ Hz are in highly eccentric orbits, which greatly increases their efficiency as sources of gravitational radiation and spreads that radiation over a large number of harmonics of the orbital frequency (Peters & Mathews 1963). Binary BH-MACHOs are shown to create a strong stochastic background in the low-frequency GW band, possibly dominating over all other galactic background sources.

2. EVOLUTION OF THE NSTT BINARY DISTRIBUTION

The distribution of black hole MACHO binaries obtained by NSTT defines the probability distribution of values for the semimajor axis $a$ and eccentricity $e$ of the binary’s orbit. Utilizing a simple model of primordial black hole formation, they found the probability distribution to be

$$F(a, e) \, da \, de = (3/2) a^{1/2} \tilde{x}^{-3/2} (1 - e^2)^{-3/2} \, da \, de,$$

where $\tilde{x} = 1.1 \times 10^{16} (M_{\text{BH}}/M_\odot)^{1/3} (\Omega h^2)^{-4/3}$ cm, and, as usual, $\Omega$ is the cosmological density parameter and $h$ is the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$. Allowable values for the semimajor axis are bounded above by $\tilde{x}$, and the eccentricity for each value of $a$ is also bounded above, by $e_{\text{max}} = [1 - (a/\tilde{x})^{3/2}]^{1/2}$.

The spatial distribution of black hole MACHO binaries is assumed to follow the standard spherical flat rotation halo model

$$\rho = \hat{\rho} R^2 + a^2 r^2 + a^2, \quad (2)$$

where $\hat{\rho} = 0.0079 M_\odot$ pc$^{-3}$ is the local density of dark matter, $r$ is the Galactocentric radius, $R = 8.5$ kpc is the Galactocentric radius of the Sun, and $a = 5.0$ kpc is the halo core radius. The distribution of equation (2) is valid from $r = 0$ out to some halo radius $R_\odot$; typical values are $R_\odot = 50$ kpc for a halo extending to the LMC or $R_\odot = 300$ kpc for a halo extending half-way to M31.

The NSTT distribution given by equation (1) is very strongly peaked toward binaries with large eccentricities due to the $(1 - e^2)^{-3/2}$ factor in $F$. For example, for binaries with initial orbital frequencies of $\sim 10^{-10}$ Hz (which will evolve into the low-frequency band today), 90% of the binaries are within $\Delta e = 0.00198$ of the maximum eccentricity, $e_{\text{max}} = 0.9999775$. In view of this, a simplifying assumption will be made: that all black hole MACHO binaries initially have $e = e_{\text{max}}$. The distribution of equation (1) may then be
rewritten in simplified form, after an integration over \( e \), as

\[
F(a)da = \frac{3}{2} \left[ \left( \frac{a}{x} \right)^{3/4} - \left( \frac{a}{x} \right)^{3/2} \right] \frac{da}{a}.
\]

The approximated NSTT distribution (with all binaries at their maximum value of \( e \)) describes the binary black hole MACHO population at the time of its formation, in the early universe. In order to calculate the spectrum of gravitational radiation produced by the binary black holes, it is necessary to know the distribution of binaries today. The population will have evolved due to a variety of effects, such as accretion, gravitational interactions with other MACHOs and with ordinary matter, and due to the emission of gravitational radiation. Only this last effect will be considered here.

In order to determine the probability distribution of black hole MACHO binary orbital parameters today, the initial distribution given in equation (3) must be evolved forward in time from the early universe to the present epoch. The evolution of a binary system of point masses due to gravitational radiation reaction in the weak field limit is well understood (Peters & Mathews 1963; Peters 1964). The distribution was evolved by numerically integration using a fourth-order adaptive step-size Runge-Kutta routine. The results obtained are characterized by their maximum value of \( a_i(\alpha_0) \), the present semimajor axis as a function of the initial semimajor axis (a subscript 0 indicates the present-day value of a quantity), and \( e_i(\alpha_0) \), the present eccentricity as a function of the initial semimajor axis.

The form of the distribution today is illustrated in Figure 1, in which the present eccentricity \( e_i \) is plotted against the present orbital frequency \( f_0 \). The circulating effect of gravitational radiation reaction is clearly visible; those binaries with orbital frequencies today of \( 10^{-5} \) Hz or below are still highly eccentric, while those which have evolved to higher frequencies such as \( 10^{-2} \) Hz are basically circular today. Integration of a 50 kpc halo with \( \Omega h^2 = 0.1 \) gives 2.6 \times 10^7 binaries in the low-frequency band today, comparable to the expected number of close white dwarf binaries in the Galaxy (Hils, Bender, & Webbink 1990). In contrast, a 300 kpc halo with \( \Omega h^2 = 1.0 \) will have 1.6 \times 10^9 black hole binaries in the low-frequency band today.

3. STOCHASTIC GRAVITATIONAL WAVE BACKGROUND

The calculation of the gravitational radiation spectrum emitted by the halo population of BHMACHO binaries differs substantially from the more familiar calculations for Galactic white dwarf binaries. This is because the BHMACHO binaries will generally have non-negligible eccentricity. A binary system with an eccentric orbit will emit significantly more power in GWs than a circular system with the same semimajor axis. Eccentric binaries will also emit GWs on a multitude of harmonics of the orbital frequency, whereas circular binaries emit purely in the \( n = 2 \) mode. Peters & Mathews (1963) found that the orbit-averaged power emitted in gravitational radiation by a binary system in the \( n \)th harmonic of the orbital frequency is

\[
L(f, n, e) = \frac{32}{5} \frac{m_1 m_2 (m_1 + m_2)}{a^5} g(n, e),
\]

where \( f \) is the orbital frequency associated with semimajor axis \( a \) (not the frequency of the harmonic radiation), and \( g(n, e) \) is given by

\[
g(n, e) = \frac{n^4}{32} \left[ J_{n+2}^2(ne) + 2eJ_{n+1}(ne) \right.
\]

\[
+ \frac{2}{n} J_n(ne) + 2eJ_{n+1}(ne) - J_{n+2}(ne) \right] \left[ 1 - e^2 \right]^{1/2}
\]

\[
\left. + \frac{4}{5n^2} J_n(ne)^2 \right].
\]

The total GW signal at frequency \( f \) will incorporate contributions from a large number of binaries, and hence the signal will be stochastic in nature. The characteristic spectral amplitude at frequency \( f \) may be written as a sum over the uncorrelated contributions from the binaries for which \( f \) is a harmonic:

\[
h_i(f) = \frac{2}{\pi f} \left( \frac{1}{d^2} \right)^{1/2} \sum_{j=2}^{\infty} L(f, j, e_j) \frac{dN}{df},
\]

where \( \langle d^{-2} \rangle \) is the inverse distance to the source squared, averaged over the source distribution, and the frequency \( f_j \) is the orbital frequency of a binary system for which the \( j \)th harmonic is at frequency \( f_j = jf \). Similarly, \( e_j \) is the eccentricity of the NSTT evolved binary for which the \( j \)th harmonic is at frequency \( f_j \). The upper limit on the summation, \( f_{\text{max}} \), is the value of \( f \) for which the semimajor axis of the binary is at the maximum value \( a \sim x \). For each value of \( j \), \( dN/df \) gives the number of sources per unit frequency interval at frequency \( f_j \), and \( L(f, j) \) is the power emitted at frequency \( f_j \) by a binary with orbital frequency \( f_j \).

The average squared inverse distance between halo sources and the Sun may be found by integrating over the source distribution given in equation (2). For a 50 kpc halo, this gives \( \langle d^{-2} \rangle^{-1/2} = 16.8 \) kpc, while for a 300 kpc halo, \( \langle d^{-2} \rangle^{-1/2} = 39.8 \) kpc.

The procedure followed to determine the low-frequency GW spectrum of the NSTT distribution of black hole MACHO bi-
naries is as follows. First, a frequency \( f \) is chosen. This fixes the frequencies (and hence semimajor axes) of the binaries whose \( j \)th harmonic will contribute to the sum in equation (6). For each harmonic index \( j \) in equation (6), the luminosity \( L(f, j) \) and the number of binaries per unit frequency interval \( dN/df \) must then be determined.

In order to determine the luminosity from equation (4), the harmonic index \( j \) must be known along with the semimajor axis \( a \) and the eccentricity \( e \). The semimajor axis is trivially obtained from the orbital frequency \( f_0 \), while the eccentricity is also a function of \( f_0 \), determined by the numerical evolution of the approximate NSTT, as illustrated in Figure 1. Once the eccentricity is known, the function \( g(j, e) \) may be evaluated.

Direct numerical computation of Bessel function values was used for the lowest 10 harmonics \((j \leq 10)\). Accurate evaluation of the Bessel functions becomes more difficult for larger values of \( j \). An asymptotic expansion was used to fix the value of \( g(n, 1) \) [which depends only on \( J_1(n) \)], and the shape of \( g(n, e) \) for arbitrary \( e \) was then modeled analytically using a combination of exponential and trigonometric functions. Trial variation of parameters and functional forms within the analytic fit indicate that the resulting luminosity is robustly estimated to about 1% accuracy.

The evaluation of the number of binaries present today per unit frequency interval \( dN/df \) for an arbitrary binary frequency \( f \) may be split into three pieces:

\[
\frac{dN}{df} = N_{\text{halo}} \frac{dF}{da} \frac{da}{da_0} \frac{da_0}{df} \tag{7}
\]

Here \( dF/da_0 \) is the initial fraction of all black hole MACHO binaries per unit semimajor axis interval, \( da/da_0 \) is the amount by which an infinitesimal interval in semimajor axis has expanded or contracted in evolving to the present day, and \( da_0/df \) is the final conversion from semimajor axis to present orbital frequency via Kepler’s third law. Given an orbital frequency \( f_0 \) today, the initial value of the semimajor axis \( a_0 \) may be determined from the results of the numerical evolution of the approximate NSTT distribution. Then, \( dF/da_0 \) may be directly evaluated from equation (3). The evolution of \( da/da_0 \) is also determined from the numerically evolved NSTT distribution. Multiplication of the factors in equation (7) gives the number of binaries per unit frequency interval as a function of frequency, today. The results confirm the notion that the total GW signal from the black hole MACHO binaries will be stochastic in nature. Assuming an integration time of 4 months (typical of LISA or OMEGA), every \( 10^{-2} \) Hz bin will include modes from multiple binaries for all frequencies \( f \leq 10^{-2} \) Hz. Above that frequency, individual bins may be occupied by individual binaries evolving toward coalescence or may be empty, opening windows in which weaker sources (e.g., extragalactic) could be observed.

The total gravitational wave spectral amplitude was then evaluated for a set of frequencies \( f \) incorporating contributions from harmonics up to a maximum value \( j_{\text{max}} \). Setting \( j_{\text{max}} \) equal to 1000 was found to provide an accuracy over the entire frequency range \( 10^{-5}-10^{-1} \) Hz of better than 1%; specifically, the inclusion of \( 10^4 \) harmonics rather than \( 10^3 \) changes the spectral amplitude by less than 1% over this range.

The resulting spectrum of gravitational radiation is shown in Figure 2. Both curves illustrated represent small halos \((R_h = 50 \text{ kpc})\) with \( \Omega h^2 = 0.1 \) for the lower curve and \( \Omega h^2 = 1.0 \) for the upper curve. The spectrum is only weakly dependent on the value of the halo radius; a larger halo simply adds additional sources at large distance. Curves for a 300 kpc halo are of the same shape as those illustrated but with spectral amplitude increased by a constant factor of \( \sim 1.106 \) over the full frequency range (on the logarithmic plot of Fig. 2, this corresponds to moving the curves upward by 0.0436).

The peak in the spectrum at around \( f = 10^{-3} \) Hz is due to a combination of circumstances. Binaries whose \( n = 2 \) mode is at that frequency have eccentricities of around 0.4 (see Fig. 1), so that about 50% of their power is still being emitted in the \( n = 2 \) mode (Peters & Mathews 1963). At the same time, lower frequency binaries within an order of magnitude in frequency are significantly eccentric, so that they contribute substantial power at \( 10^{-3} \) Hz through their higher harmonics. At higher frequencies, the lower frequency binaries are more nearly circular and their higher harmonics do not contribute so much power. At lower frequencies, the fundamental mode’s contribution (and, later, the higher modes’ contributions) drops off sharply due to the \( a^{-3} \) dependence in equation (4).

At high frequencies, the spectrum is seen to approach a power-law behavior, \( h_{\text{f}} \sim f^{-7/6} \). Since binaries in the higher frequency range are nearly circular, essentially all of the signal is generated by the \( n = 2 \) harmonic. The observed power law is then explained by noting that \( L \sim f^{10/3} \), while the factors of \( dN/df \) in equation (7) have the following asymptotic form: \( dF/da_0 \sim \text{constant}, \text{ da}/da_0 \sim f^{-3}, \text{ da}_0/df \sim f^{-5/3} \). Using these asymptotic forms in equation (6) and only including the \( j = 2 \) mode yields \( h_f \sim f^{-7/6} \).

The contribution due to BH MACHO binaries in the halos of other galaxies may be estimated in the same general manner as has been done for white dwarf binaries (Hils et al. 1990; Kosenko & Postnov 1998). The effect of including the contribution of extragalactic BH MACHOs is to raise the predicted spectrum by an amount varying between 17% (0.068 on the logarithmic plot of Fig. 2) for 50 kpc halos in a universe with \( \Omega h^2 = 1.0 \) and 50% (0.176 on the logarithmic plot) for 300 kpc halos in a universe with \( \Omega h^2 = 0.1 \). The shape of the predicted spectrum is generally the same, with the peak at \( 10^{-3} \) Hz somewhat broadened toward lower frequencies.

Also shown in Figure 2 are the sensitivity curves for the
proposed LISA (Bender et al. 1998) and OMEGA (Hellings et al. 1998) space interferometer GW detectors and a recent estimate of the background due to Galactic close white dwarf binaries (CWDBs) (D. L. Hills & P. Bender 1998, private communication). Except at low frequencies, the BHMAChO binary background would be notably stronger than the CWDB background. The halo black hole binaries would form a strong “confusion noise” stochastic background, significantly larger than the instrument noise in either proposed interferometer over a large frequency range $10^{-6}$ Hz $< f < 10^{-7}$ Hz. Of course, while the stochastic signal is in one sense a noise, it also provides direct information on the nature and distribution of MACHO binaries in the Galactic halo. It is conceivable that the Galactic and extragalactic components could be separated by the anisotropy of the Galactic component, although it would be more difficult than the corresponding separation for Galactic disk white dwarf binaries (Giampieri & Polnarev 1997).

Whether the halo MACHOs are primordial black holes, white dwarfs, boson stars, or some other type of compact object, some fraction of them will be in binary systems. Given the huge number of MACHOs in the Galaxy’s halo, the low-frequency gravitational wave signal from these binary systems will quite likely be detectable by the space-based interferometers. These interferometers will be excellent instruments for determining the nature and population of the halo MACHO binaries.

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