Optimal sizing of a networked microgrid using Nash equilibrium for mount magnet

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Abstract

In this paper, a technique of game theory is proposed based on a multi-objective imperialistic competition algorithm (ICA) for system optimization in order to design a networked microgrid in grid-connected mode. The selected networked microgrid, which consists of two different grid-connected microgrids with common load and grid, might have different combinations of generation resources including wind turbine, photovoltaic panels and batteries. To perform the effective sizing of networked microgrid, a Nash equilibrium based game theory is developed in which the rating of the generation systems is considered as players and annual profit as payoff. Moreover, in order to meet the equilibrium point and find the optimum sizes of generation resources in different coalitions between players, ICA, which is being frequently used in optimization applications, is implemented using MATLAB software. Finally, in order to validate the results, the sensitivity analysis is studied to examine the impact of electricity price and discount rates.

Keywords: Game theory, Nash equilibrium, photovoltaic panel, storage battery, wind turbine.

1. Introduction

In recent years, to control the increasing requirements of electricity price and demand, different renewable energy based generations are the hot topic. Novel researches showed that generation through renewable energy is the modern way and, the environmental concerns is another reason to increase its rapid use [1]-[2]. But, the generation from renewable resources like wind and sunlight depend on the condition of weather, and consequently very hard to achieve higher precision or to get most reliable generation. The solution to control the intermittent type of generation is the addition of different kind of energy storage systems [3]-[4].

In fact, generation through renewable resources is not an only way for reliable, safe and economical way, but it is a most encouraging way to develop modern form of power generation. In order to get most optimum results from the microgrid, the performance need to be improved by right planning and minimize the expenses within the system limitations [5]. In past, there are number of studies about the sizing and optimization of microgrid and different approaches are used to achieve desired outcomes like fuzzification mechanism is used in [6], loss of power supply probability method is adopted in [7], and the trade-off method is used in [8]-[9].

Networked microgrid is an architecture, where different connected microgrids are controlled in certain range of space. The concept of networked microgrid has been proposed in [10], where interconnected microgrids can supports each other to meet load requirements and also in the situation of emergencies. Microgrid networking has many other benefits over single microgrid like economic benefits and also can minimize power outage problems [11]. In a microgrid, generation resources like wind turbines, photovoltaic panels and batteries may belongs to different owners, but for the optimum sizing of each...
component, it is preferred to maximize their profit. In this regard, when various components are involved in microgrid to maximize the profit, the game theory is an advance type of multi-objective optimization to solve the decision making problem [12]. It is also evident in [13], in comparing non-cooperative game models, cooperative game models give more optimum results and maximum profit. In the game theory models, different algorithms are used to perform the optimization, like in [5] particle swarm optimization algorithm is used for designing of grid connected system, and in [13] an approach for hybrid power system planning is proposed, respectively. In [14] a colonial competition algorithm is used for maintaining the frequency stability in microgrid, and in [15] multi-objective imperialistic competition algorithm ICA is used for the problems of microgrid optimization. It has to be noticed that, in most of research papers ICA is used for optimization of single microgrid, however, in this paper it is applied for networked microgrid.

This study focuses to design the capacity allocation of generation resources of networked microgrid in planning stage. In the networked microgrid, each of microgrid consists of different combinations of generation resources. The game theoretical technique called Nash equilibrium is used for the optimization through iterative search procedure. To meet the load requirements, generation resources like wind turbine, photovoltaic panels and batteries are considered as players, and Nash equilibrium will be achieved among the players to get maximum profit. To keep the selected networked microgrid simple, two different microgrids are considered for this study. An imperialistic competition algorithm is used to design the model for networked microgrid in MATLAB to get most suitable sizing of generation resources and maximum annual profit. Moreover, to make sure the effectiveness of networked microgrid, sensitivity analysis will be performed.

2. Design of Networked Microgrid

The design of the networked microgrid is based upon some input variables like weather forecast, load and other information. In this paper, the feasibility of the proposed game model will be checked for Mount Magnet, a remote town of Western Australia that is 560 km northeast of Perth. To make the analysis for networked microgrid, the annual data for wind speed, solar radiation and electrical load are considered for period from June 2015 to May 2016 [16]. Fig. 1(a) illustrates that the peak electricity demand for Mount Magnet is approximately 1390 kW in the month of summer and the minimum load is

![Fig. 1. Hourly profiles: (a) electrical load (b) wind speed and (c) solar radiations.](image)
about 312 kW in the month of winter. The wind speed data is shown in Fig. 1(b) for hourly base, and the annual average and maximum speed is approximately 4.16 m/s and 11.22 m/s, respectively. The selected town has very good profile for solar radiations, it can be seen from Fig. 1(c) that the value is maximum about 1058 Watt/m² in the month of summer, however, trends down in the winter.

The networked microgrid can be a combination of m number of microgrid, however, the selected system consists of two microgrids with different combinations of generation resources. The block diagram of proposed networked microgrid is shown in Fig. 2 for a remote area, which consists of generation resources, load and the main grid. Wind turbine, photovoltaic panels and storage batteries are considered as the sources of power generation depending upon the weather forecast. For the proposed system, both microgrid are connected with main grid, therefore, if they fail to meet the load requirements, they have option to purchase power from main grid and in case of large generation, it can sell the excessive power to grid. The goal of this research is to find the optimum sizes for power generation resources and battery to meet the load requirements, and achieve maximum annual profit for networked microgrid.

2.1. Elements of game model

Game theory is a scientific field dealing with the study and analysis strategic, rational decision process of individuals and their interactions in environment. In other words, it is a decision making process that can resolve different kinds of conflicts between the decision makers and find the maximum payoff [17]. The main element of any game are the decision variables known as the players and each of the players must have more than one choice to make strategies to get the required payoff. In the networked architecture, if \( m, n, i, \) and \( P_i \) represents the number of microgrid, number of decision variable, generation resource, and decision variable, respectively. The maximum and minimum power values of the players are the constraints in a selected game model and represented as strategic space \( SS_i = [P_i^{min}, P_i^{max}] \).

The total annual profit for \( m \) microgrid can be find as:

\[
I_{MG, total} = \sum_{i=1}^{m} I_{MG,m} 
\]  

\[
I_{MG,m} = \sum_{i=1}^{n} I_i 
\]

2.2. Payoff of microgrid-1

In the microgrid-1, three generation resources \( i \) wind turbine, solar panel and battery are considered, and their decision variables or players are represented by \( P_{WT}, P_{SP}, \) and \( P_{BT} \), respectively. Similarly, the maximum payoff or profit for the \( WT, SP \) and \( BT \) are \( I_{WT}, I_{SP}, \) and \( I_{BT} \), respectively. The total annual profit for the microgrid-1 is:
In grid-connected mode, to get the maximum annual profit for the generation resource \( i \) different parameters are considered, like power selling income \( I_{SE,i} \), salvage value \( I_{SV,i} \), income from ancillary service \( I_{AS,i} \), initial investment cost \( C_{IN,i} \), compensation cost from energy cannot supplied \( C_{ES,i} \), purchasing power from grid \( C_{PR,i} \), and operation and maintenance cost \( C_{OM,i} \) etc. The annual profit can be find using the bellow equation:

\[
I_i = I_{SE,i} + I_{SV,i} + I_{AS,i} - C_{IN,i} - C_{OM,i} - C_{ES,i} - C_{PR,i}
\]

The benefit of \( I_{AS,i} \) is considered for battery, however, for wind turbine and solar panel is taken as zero. However, when the storage batteries are out of service their unbalance power in microgrid, load demand, and thermal overloading, in hour \( t \) where

\[
C_{AS} = \sum_{i=1}^{n=3} C_i
\]

\[
C_{ES} = \sum_{t=1}^{B7B4} 1.5 \times R(t) \times \{DP(t) - P_{GR}(t)\}
\]

\[
DP(t) = P_L(t) - P_{WT}(t) - P_{SP}(t) - (P_{BT}(t) - P_{BT\text{,min}})
\]

\[
P_{GR}(t) = \begin{cases} 
0 & DP(t) \leq 0 \\
DP(t) & 0 < DP(t) \leq P_{max}^{TL} \\
P_{max}^{TL} & DP(t) > P_{max}^{TL}
\end{cases}
\]

where \( C_{ES} \), \( R(t) \), \( DP(t) \), \( P_L(t) \) and \( P_{max}^{TL} \) are total annual energy not supplied cost, electricity price, unbalance power in microgrid, load demand, and thermal overloading, in hour \( t \).

The output power of wind turbine \( P_{WT}(t) \) and storage battery \( P_{BT}(t) \) can be find as:

\[
P_{WT}(t) = \begin{cases} 
0 & V(t) < V_{cl} \text{ or } V(t) \geq V_{co} \\
\frac{P_{WT} \times (V(t) - V_{cl})}{V_r - V_{cl}} & V_{cl} \leq V(t) < V_r \\
P_{WT} & V_r \leq V(t) < V_{co}
\end{cases}
\]

\[
P_{BT}(t) = \begin{cases} 
(P_{BT}(t) - 1) + \Delta(t - 1) & \Delta(t - 1) \geq 0 \\
(P_{BT}(t) - 1) + \Delta(t - 1) & \Delta(t - 1) < 0
\end{cases}
\]
\[ \Delta(t-1) = p_{WT}(t-1) + p_{SP}(t-1) - p_i(t-1) \]  

(14)

where \( V_{cl}, V_{co}, V_r, \) and \( E_c \), are cut-in wind speed, cut-out wind speed, rated wind speed, and battery charging efficiency, respectively.

The annual income from power selling \( I_{SE} \), can be calculated from the following:

\[ I_{SE}(t) = \sum_{t=1}^{8760} (1 + \alpha_s) * R(t) * P_{SE}(t) \]  

(15)

\[ P_{SE}(t) = \begin{cases} p_i(t) & P_{SU}(t) \leq 0 \\ \frac{p_i(t) * p_{mx}(t)}{(\sum_{i=1}^{n} p_i)} & P_{SU}(t) > 0 \end{cases} \]  

(16)

\[ P_{mx}(t) = P_e(t) + P_{max}^{TL} + (P_{BT} - p_{BT}(t)) \]  

(17)

\[ P_{SU}(t) = p_{WT}(t) + p_{SP}(t) - P_{mx}(t) \]  

(18)

where \( \alpha_s, P_{SE}(t), P_{SU}(t), \) and \( P_{mx}(t) \) are the subsidy coefficient, power selling, surplus power, and maximum power that can consumed, respectively.

The annual selling power \( P_{BT,SE}(t) \) and ancillary income for storage battery \( I_{BT,AS} \) can be find as:

\[ P_{BT,SE}(t) = \begin{cases} Dp_{BT}(t) & Dp_{BT}(t) > 0 \\ 0 & Dp_{BT}(t) \leq 0 \end{cases} \]  

(19)

\[ Dp_{BT}(t) = p_{BT}(t) - p_{BT}(t + 1) \]  

(20)

\[ I_{BT,AS} = I_{pu,RP} * \sum_{t=1}^{8760} (p_{BT}(t) - P_{BT,SE}(t) - P_{B,Min}) \]  

(21)

where \( Dp_{BT}(t) \) and \( I_{pu,RP} \) represent change in battery capacity in hour \( t \) and per unit income from reserve power, respectively.

2.3. Payoff of microgrid-2

Two generation resources \( i \) wind turbine \( WT \) and \( BT \) are considered to design a microgrid-2, and their decision variables are \( P_{WT} \) and \( P_{BT} \), respectively. The profit for both players are \( I_{WT} \) and \( I_{BT} \). The total annual profit for the microgrid-2 is:

\[ I_{MG,2} = \sum_{i=1}^{n=2} I_i \quad i \in \{WT, BT\} \]  

(22)

To get the maximum annual profit for microgrid-2, the technical parameters are considered, and the equation. 4 will be used for the \( WT \) and \( BT \). Lastly, the total annual profit of the networked architecture for microgrid-1 and microgrid-2 will be calculated by using the equation.1.

3. Nash Equilibrium and Algorithm

Game theory is an advance type of multi-objective optimization that has been applied for many years to solve different decision making problems. To design cooperative and non-cooperative game models, various kind of solution concept or techniques are used like Nash equilibrium, Pareto optimality etc. In the theory of games, Nash equilibrium is a fundamental concept and the most widely used technique to find the outcome of decision variables [18]. In this research, cooperative game models are designed using Nash equilibrium for optimum values of decision variables and to find the maximum profit. The proposed networked architectures consist of three different combination of generation resources \( i \), therefore, cooperative game model can have four different coalitions for the planning problem of three player’s game. The optimum values of decision variables is find through Nash equilibrium using iterative
procedure in the following steps:
1. Input the parameters like wind speed, solar radiation, electricity price, and discount rate etc.
2. For the selected microgrid, randomly choose initial values of decision variables \( (P_{WT}^0, P_{SP}^0, P_{BT}^0) \) from strategic space.
3. In case of generation resources through WT and SP are cooperating with each other and BT is self-sufficient. To explain it, consider a \( j \)th iteration \((P_{WT}^j, P_{SP}^j, P_{BT}^j)\) which depend upon previous iteration \((P_{WT}^{j-1}, P_{SP}^{j-1}, P_{BT}^{j-1})\), as:
   \[
   (l_{WT}^{j-1}, l_{SP}^{j-1}) = \arg \max_{P_{WT}, P_{SP}} l_{WT, SP} (P_{WT}, P_{SP}, P_{BT}^{j-1})
   \]
   \[
   p_{BT}^{j-1} = \arg \max_{P_{BT}} (h_{WT}^{j-1}, l_{SP}^{j-1}, P_{BT})
   \]
4. In this step, share with every player in coalition about strategic values of third step.
5. Check the condition of Nash equilibrium, if none of the player change its value during whole round of iteration, means \((P_{WT}, P_{SP})=(P_{WT}^j, P_{SP}^j)\), and \( P_{BT} = P_{BT}^j \), the Nash equilibrium is found. In case, results are not achieved, move towards step-3.

In order to design and simulate the proposed networked architecture in MATLAB, an imperialistic competition algorithm is used. The main operators of the algorithm are assimilation, revolution and imperialistic competition. It is a modern population based algorithm and used in various research areas to solve many optimization problems [19]. In this research work, to find most feasible value of decision variables 50 number of population or countries, 5 number of imperials, and maximum decades of 50, are considered.

4. Results and Analysis

The sizing of networked microgrid will be carried out with the help of game theory technique Nash equilibrium, and the simulation will be performed in MATLAB using ICA. In order to optimize the objective function and to find suitable sizes of decision variables, input parameters as listed in Table-1 [13] will be used for each of the generation resource.

In case of microgrid-1 three generation resources WT, PS and BT are considered, therefore, for the cooperative game model four different kinds of coalitions are possible. To find the cooperative game model with maximum profit and most suitable generation capacities, all possible game models are simulated. The optimum values of decision variables and annual profit for each of game model are listed in Table 2 for microgrid-1. It is evident from the result that the capacity of WT is higher that SP, however, SP capacity is higher than BT in all of the cases except case-2. The results of microgrid-1 illustrates that profit is maximum when all of the generation resources are in coalition with each other in case-1. Therefore, case-1 for microgrid-1 is giving the solution for Nash equilibrium with maximum value of profit and most feasible sizes for generation resources to meet the load requirements. The cooperative game models also show that if bigger sizes of generation resources are considered in a microgrid, the value of annual profit goes high. As the microgrid gets opportunity to sale additional power to the main grid, and it is easier for networked architecture to meet load requirements in any emergency situation.

Table 1. Input parameters for networked microgrid

| Parameters                  | Values   | Parameters                  | Values   |
|-----------------------------|----------|-----------------------------|----------|
| Electricity price \((R)\)   | 0.12 $/kWh | WT salvage value \((SV_{WT})\) | 77 $/kW  |
| Discount rate \((Dr)\)     | 12 %     | PV panel price \((U_{SP})\)  | 1,890 $/kW |
| Cut-in wind speed \((v_{ci})\) | 3 m/s | Life span of PV \((L_{SP})\)  | 20 Years |
| Cut-out wind speed \((v_{co})\) | 20 m/s | OM cost of PV \((OM_{SP})\)  | 20 $/(kW.year) |
| Rated wind speed \((v_{r})\) | 12 m/s | PV panels salvage vale \((SV_{SP})\) | 189 $/kW |
| Life span of WT \((L_{WT})\) | 20 Years | Life span of batteries \((L_{BT})\) | 10 Years |
| WT price \((U_{WT})\)      | 770 $/kW | Battery price \((U_{BT})\)  | 100 $/kW  |
| OM cost of WT \((OM_{WT})\) | 20 $/(kW.year) | OM cost of battery \((OM_{BT})\) | 1 $/(kW.year) |
Table 2. Nash equilibrium results for microgrid-1

| Case# | Coalition       | Decision Variables (kW) | Annual profit ($/year) |
|-------|----------------|--------------------------|------------------------|
|       |                | $P_{WT}$ | $P_{SP}$ | $P_{BT}$ | $I_{MG-1}$ |
| 1     | {WT, SP, BT}  | 44,876   | 8,000    | 6,233    | 2.4841E+7  |
| 2     | {WT, SP}, {BT} | 44,979   | 8,541    | 9,999    | 2.4685E+7  |
| 3     | {WT, BT}, {SP} | 44,952   | 15,020   | 9,304    | 2.2942E+7  |
| 4     | {WT}, {SP, BT} | 25,000   | 19,879   | 9,875    | 1.4871E+7  |

Table 3. Nash equilibrium results for microgrid-2

| Case# | Coalition | Decision Variables (kW) | Annual profit ($/year) |
|-------|-----------|--------------------------|------------------------|
| 1     | {WT, BT}  | $P_{WT}$ | $P_{BT}$ | $I_{MG-2}$ |
|       |           | 44,903   | 8,752    | 2.5037E+7  |

In case of microgrid-2, two generation resources $WT$ and $BT$ are considered, therefore, for the cooperative game model only one type of coalition is possible. Table-3 shows the optimum sizes of both decision variables and maximum annual profit at Nash equilibrium.

In the end, sensitivity analysis is performed to analyse the impact of changing the values of electricity price $R$ and discount rate $Dr$ for proposed networked architecture. These parameters are considered to observe the variations in the value of annual profit from microgrid-1 and 2. The influence to change the electricity price and discount rate are shown in Fig. 3(a) and 3(b), respectively. The results for both microgrid show that, as the electricity price decreases, the total annual profit decreases to. However, in case of high electricity prices, generation resources are earning more profit by selling power to the load and main grid. On the other side, as the value of discount rate increases, the value of profit is increasing sequentially, and vice versa. Moreover, it is also evident from both parameters that, the influence of electricity price is more sensitive than discount rate. Therefore, small increase in electricity price brings quick increase in profit value.

5. Conclusion

A game theory technique Nash equilibrium is used in this paper to model a networked microgrid, and optimization is performed in MATLAB using ICA. The main achievement of this research is make a suitable capacity allocation of generation resources for each microgrid, and find the maximum profit for proposed networked architecture. In this analysis, it is considered that the decision variables are cooperative with each other, and the results for all possible coalitions are calculated. It is clear from the results that the cooperation between the all players maximize the annual profit and the optimum values of generation resources can be achieved through Nash equilibrium. The sensitivity analysis validated the results of networked microgrid and checked the influence on decision variables of generation resources for both microgrids.

![Fig. 3. Sensitivity analysis: (a) electricity price and (b) discount rate.](image-url)
Conflict of Interest

The authors declare no conflict of interest.

Authors Contributions

The paper was a collaborative effort between the authors. Liaqat Ali, S.M Muyeen, Hamed Bizhani, and Arindam Ghosh contributed collectively to the theoretical analysis, modeling, simulation, and manuscript preparation.

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