Super-Dense Matter at Super-Strong Magnetic Fields

Efrain J. Ferrer and Vivian de la Incera

Department of Physics, Western Illinois University, Macomb, IL 61455, USA

Abstract. Our Universe is full of regions where extreme physical conditions are realized. A most intriguing case is the super-dense core of neutron stars, some of which also have super-strong magnetic fields, hence called magnetars. In this paper we review the current understanding of the physical properties of the different phases of quark matter at very high densities in the presence of large magnetic fields. We also discuss how the Meissner instability produced at moderate densities by a pairing stress due to the medium neutrality and \( \beta \)-equilibrium constraints can lead to the spontaneous generation of a magnetic field.

Keywords: Color Superconductivity, Magnetic Field Generation.
PACS: 12.38.Aw, 12.38.-t, 24.85.+p

INTRODUCTION

Neutron stars, the leftover core of an exploding supergiant star, are so compact that gravity binds nucleons there 10 times more strongly than the strong-nuclear force binds nucleons in nuclei. Their average density can reach up to \( 10^{18} \text{kg/m}^3 \). At those tremendous densities the superdense matter at the cores of those compact objects can realize a quark deconfined phase that produces a color superconducting state [1, 2].

A common characteristic of neutron stars is their strong magnetization. It is known that, since the surface area of a supergiant star shrinks by a factor of about \( 10^{10} \) after the supernova collapse, the resultant field strength is increased by the same factor. Surface magnetic fields of these compact objects range from \( B = 1.7 \times 10^8 G \) (PSR B1957+20) up to \( 2.1 \times 10^{13} G \) (PSR B0154+61), with a typical value of \( 10^{12} G \) [3]. The existence of stellar objects - known as magnetars- with even stronger surface magnetic fields of order \( B \sim 10^{14} - 10^{15} G \) [4] is already an observational fact. In the core of these compact objects, the field may be considerably larger due to flux conservation during the core collapse. By applying the scalar virial theorem it can be shown that the interior field can reach values of order \( B \sim 10^{18} G \) [5].

From the above arguments, it becomes imperative to investigate the interplay between color superconductivity and strong magnetic fields in order to understand the real state of matter in the inner region of compact astronomical objects. Although a color superconductor (CS) is in principle an electric superconductor because the diquark condensate carries nonzero electric charge, in spin-zero phases like the color-flavor-locked (CFL) and the 2SC phases [6], there is no Meissner effect for a new in-medium electromagnetic field \( A_\mu \). This in-medium or "rotated" electromagnetic field is a combination of the regular electromagnetic field and the \( 8^{th} \) gluon [6]. As the quark pairs are all neutral with respect to the "rotated" electromagnetic charge \( \tilde{Q} \), the "rotated" electromagnetic field \( \tilde{A}_\mu \)
remains long-range within the superconductor.

It has been recently found [7]-[11], that the magnetic field can modify the ground state of the CS. An external magnetic field can lead to the splitting of the gap parameters [7] and produce Haas-van Alphen oscillations of the gap [9] and the magnetization [10]. Moreover, it has been shown that the appearance of chromomagnetic instabilities triggered either by a strong magnetic field [11] or, in the absence of the field, by pairing stress occurring at moderate densities, leads to the formation of inhomogeneous gluon condensates that can either boost the existing field [11] or spontaneously generate one [12].

These effects, all of which connect magnetism with CS, create a new scenario for magnetized high-dense matter where different phases can be realized. In that scenario, the boundaries between different magnetic phases are determined by the various scales that characterize the CS [8]. Ignoring the quark masses, the main scales of this medium are the baryon chemical potential $\mu$, the dynamically generated gluon mass $m_M \sim g\mu$ and the gap parameter $\Delta \sim \frac{\mu}{g} e^{-\alpha/g}$, with $\alpha$ a constant that is dominated by magnetic gluon exchanges [13]. We can assume that at sufficiently large $\mu$, the running strong coupling $g$ becomes weak enough for the hierarchy of the scales to be $\Delta \ll m_g \ll \mu$.

In a CS with three flavors a phase transmutation from CFL to the so-called magnetic CFL phase (MCFL) takes place when $\tilde{B} \simeq \Delta_{CFL}^2$ [8]. During the phase transmutation no symmetry breaking occurs, since in principle once a magnetic field is present the symmetry is theoretically that of the MCFL phase. For even larger fields ($\tilde{B} \geq \tilde{B}_{PCFL} = m_M^2$, with $m_M$ being the magnetic mass of the charged gluons) a second phase transition occurs from MCFL to the so-called paramagnetic CFL phase (PCFL) [8, 11]. This one is a real phase transition because the PCFL phase breaks the translational symmetry and the remaining rotational symmetry in the plane perpendicular to the applied magnetic field. In the following sections we will present the main characteristics of various CS phases with either external or spontaneously generated magnetic fields.

**EFFECT OF STRONG MAGNETIC FIELDS ON SUPER-DENSE MATTER**

The symmetry breaking patterns of the MCFL and CFL phases are different. In the CFL phase the symmetry breaking is given by

$$\mathcal{G} = SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_B \times U(1)_{e.m.} \rightarrow SU(3)_{C+L+R} \times \tilde{U}(1)_{e.m.}.$$  (1)

This symmetry reduction leaves nine Goldstone bosons: a singlet associated to the breaking of the baryon symmetry $U(1)_B$, and an octet associated to the axial $SU(3)_A$ group. Once a magnetic field is switched on, the difference between the electric charge of the $u$ quark and that of the $d$ and $s$ quarks reduces the original flavor symmetry of the theory and consequently also the symmetry group remaining after the diquark condensate is formed. Then, the breaking pattern for the MCFL-phase [7] becomes

$$\mathcal{G}_{\delta B} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_A^{(1)} \times U(1)_B \times U(1)_{e.m.}.$$
The group $U(1)_{A}^{(1)}$ (not to be confused with the usual anomaly $U(1)_{A}$) is related to the current which is an anomaly-free linear combination of $s, d, u$ axial currents [14]. In this case only five Goldstone bosons remain. Three of them correspond to the breaking of $SU(2)_{A}$, one to the breaking of $U(1)_{A}^{(1)}$, and one to the breaking of $U(1)_{B}$. Thus, an applied magnetic field reduces the number of Goldstone bosons in the superconducting phase, from nine to five.

The MCFL phase is not just characterized by a smaller number of Goldstone fields, but also by the fact that all these bosons are neutral with respect to the rotated electric charge. Hence, no charged low-energy excitation can be produced in the MCFL phase. This effect can be relevant for the low energy physics of a color superconducting star’s core and hence for its transport properties. In particular, the cooling of a compact star is determined by the particles with the lowest energy; so a star with a core of quark matter and sufficiently large magnetic field can have a distinctive cooling process.

Despite the difference between MCFL and CFL, the two phases are hardly distinguishable at weak magnetic fields. The symmetry of the CFL phase can be considered as an approximated symmetry in the presence of an external magnetic field as long as the field-induced mass of the charged Goldstone bosons is smaller than twice the gap, so these mesons cannot decay into a quasiparticle-quasihole pair [8]. This explains why at a threshold field $\sim \Delta_{CFL}^{2}$ a symmetry transmutation between the CFL and MCFL phases takes place. Since, strictly speaking, the exact symmetry in the presence of the magnetic field (ignoring quark masses) is that of MCFL, the transition from the "approximated" CFL to MCFL at some threshold field is not a phase transition, but a crossover or symmetry transmutation. Only at fields comparable to $\Delta_{CFL}^{2}$ the main features of MCFL emerge through the low-energy behavior of the system. At the threshold magnetic field, only five of the original nine Goldstone bosons that characterize the low-energy behavior of the CFL phase remain. These are precisely the five neutral Goldstone bosons determining the new low-energy behavior of the genuinely realized MCFL phase.

As discussed in Ref. [8] there is a close analogy between the CFL-MCFL transmutation and what can be called a "field-induced" Mott transition [15]. Mott transitions have been discussed in condensed matter [15] and in QCD [16] to describe delocalization of bound states into their constituents at a temperature defined as the Mott temperature. By definition, the Mott temperature $T_{M}$ is the temperature at which the mass of the bound state equals the mass of its constituents, so the bound state becomes a resonance at $T > T_{M}$. In the CFL-MCFL transmutation, the usual role of the Mott temperature is played by the threshold magnetic field.

Now, if we keep increasing the magnetic field until it reaches the next energy scale $m_{M} \sim g\mu$, that is for $B > B_{PCFL} = m_{M}^{2}$ [11], one of the modes of the charged gauge field becomes tachyonic. This phenomenon is a consequence of the well known "zero-mode problem" for spin-1 charged fields in the presence of a magnetic field found for Yang-Mills fields [17], for the $W_{\mu}^{\pm}$ bosons in the electroweak theory [18], and even for higher-spin fields in the context of string theory [19]. This effect is due to the interaction of the applied magnetic field with the charged gluon anomalous magnetic moment $\langle ie\tilde{f}_{\mu\nu}G_{\mu}^{+}G_{\nu}^{-}\rangle$. 

\[ \rightarrow SU(2)_{C+L+R} \times \tilde{U}(1)_{\text{e.m.}}. \]
Similarly to other spin-1 theories with magnetic instabilities [17, 18], the solution of the zero-mode problem leads to the restructuring of the ground state through the formation of an inhomogeneous condensate \( \overline{G} \) of the modulus of the charged gluons, as well as the boost of the existing magnetic field \( \tilde{B} = \nabla \times \tilde{A} \) due to the backreaction of the \( \overline{G} \) condensate on the rotated magnetic field. It can be corroborated [11] that the minimum equation for the \( \overline{G} \) field is equivalent to the Ginzburg-Landau equation appearing in the Abrikosov’s approach [20] to type II metal superconductivity for the limit situation when the applied field is near the critical value \( H_{c2} \). As in the Abrikosov’s case, the order parameter, \( \overline{G} \), continuously increases from zero with the applied field, thus signaling a second-order phase transition where a gluon crystalline vortex state with the corresponding magnetic flux tubes condenses, thus breaking the translational symmetry and the remaining rotational symmetry in the plane perpendicular to the applied magnetic field.

A peculiarity of the present situation [11] is that contrary to what occurs in conventional type-II superconductors, where the applied field only penetrates through the flux tubes and with a smaller strength, the gluon vortex state exhibits a paramagnetic behavior; that is, outside the flux tube the applied field \( \tilde{B} \) totally penetrates the sample, while inside the tubes the magnetic field becomes larger than \( \tilde{B} \). Hence, since the \( \tilde{Q} \) photons remain massless in the presence of the condensate \( \overline{G} \), the \( U(1)_{em} \) symmetry remains unbroken. At asymptotically large densities, because \( \Delta_{CFL} \ll m_M \), we have \( B_{MCFL} \ll B_{PCFL} \) for each \( \mu \) value.

Under the assumption that the contribution of the sextet (symmetric) gaps can be neglected at all values of the magnetic field, a new two-flavor phase, the 2SCds, was recently found [9] at fields much larger than \( \mu^2 \).

**MAGNETIC FIELD GENERATION IN SUPER-DENSE MATTER**

Matter inside compact stars should be neutral and remain in \( \beta \) equilibrium. When these conditions along with the mass of the s-quark, \( M_s \), are taken into account in the dynamics of the gluons, some gluon modes become tachyonic [21, 22], indicating that the system ground state should be restructured. In Ref. [12] we addressed this problem for the Meissner unstable region of the so-called gapped 2SC phase.

As known, once the gluons are taken into consideration, the gapped 2SC becomes unstable at certain values of the baryon chemical potential [21]. As it turns out, the tachyonic modes in this unstable phase are associated with the rotated charged gluon fields. Note that even though the gluons are neutral with respect to the regular electric charge, in the 2SC phase some gluons have nonzero rotated charge. Since the charged gluons are the first becoming tachyonic as the baryon chemical potential is decreased toward moderate density values, it is natural to expect that the stable ground state should incorporate the condensation of them. In Ref. [12] we allowed for an inhomogeneous condensate \( \overline{G} \) of such gluons. Taking into account that this kind of solution may generate rotated electromagnetic currents, the rotated magnetic field \( \tilde{B} \) was also included in the general framework of the condensation phenomenon, but now as an induced field to be found self-consistently from the minimum equations of the system free energy. The
solution of the minimum equations implied that the instability is actually removed by the formation of a gluon condensate $\bar{G}$ that induces a magnetic field $\tilde{B}$. Since what condensates is the modulus of the charged gluons, the condensate is neutral and thus preserves the rotated electromagnetic gauge invariance $\tilde{U}_{em}(1)$ of the CS.

We underline that contrary to what occurs in the PCFL phase where a strong applied magnetic field is needed to produce the inhomogeneous gluon condensation, the condensation phenomenon here is connected to a Meissner instability triggered by pairing stress. The pairing stress develops at moderate densities due to the neutrality and $\beta$-equilibrium of the medium. The spontaneous induction of a rotated magnetic field in CS systems that have pairings with mismatched Fermi surfaces is a new kind of phenomenon that can serve to generate magnetic fields in stellar compact objects as magnetars.

When the absolute value of the magnetic mass becomes of order $m_g$, the gluon condensate could produce a magnetic field of order $10^{16} - 10^{17}$ G. The possibility of generating a magnetic field of such a large magnitude in the core of a compact star without relying on a magneto-hydrodynamic effect can be an interesting alternative to address the main criticism [23] of the observational conundrum of the standard magnetar paradigm [4]. Then, we conclude that if color superconductivity is realized in the core of compact stellar objects at such expected densities that a Meissner unstable phase is attained, the theory of the origin of stellar magnetization should consider the mechanism addressed in [12]. On the other hand, to have a mechanism that associates the existence of high magnetic fields to CS at moderate densities can serve to single out the magnetars as the most probable astronomical objects for the realization of this high-dense state of matter.

We stress that for even lower densities the contribution of $\bar{G}$ and $\tilde{B}$ in the quark quasiparticle propagators cannot be neglected, thereby affecting the gap equation. As a consequence, the inhomogeneity of the gluon condensate will be naturally transferred to the diquark condensate. Hence, a LOFF-type phase may appear as a back reaction of the inhomogeneous gluon condensate on the gap solution. It is natural to expect that the additional reduction in free-energy due to the $\bar{G} - \tilde{B}$ condensation will make this phase energetically favored over a pure LOFF one [24].

We anticipate that a $\bar{G} - B$ condensate will likely remove the chromomagnetic instability in the three-flavor system too. In that case there are four gluons with tachyonic masses. Following the results of the last paper in Ref. [22], the four tachyonic gluons are $A_1$, $A_2$ and two combinations of $A_3$, $A_8$ and $A_γ$. A third combination of $A_3$, $A_8$ and $A_γ$ is massless, hence it represents the rotated electromagnetic field in that phase, and the gluons $A_1$ and $A_2$ acquire rotated charge since they couple to the new rotated electromagnetic field through its $A_3$ component. This implies that $A_1$ and $A_2$ are analogous to the charged gluons that become tachyonic in the 2SC case. If $A_1$ and $A_2$ condense in an inhomogeneous condensate following the same mechanism we found in Ref. [12], this condensate could induce a rotated magnetic field and also give real masses to the two combinations of $A_3$, $A_8$ and $A_γ$ that were tachyonic. Exploring this idea will require a clever approach that will permit us to circumvent the difficulty of dealing with the big jump in the Meissner masses at the onset of the chromomagnetic instability.

In addition to our mechanism [12] for spontaneous generation of a magnetic field in a
CS, a field may be also spontaneously generated by an induced Goldstone boson current, as reported in [25].

ACKNOWLEDGMENTS

We acknowledge the support of DOE Nuclear Theory grant DE-FG02-07ER41458.

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