Abstract. Characteristic trends in physics of nuclear clustering are discussed. Emphasis is put on the cluster structure physics because we have recently remarkable developments for this subject. As representative examples three points are discussed: (1) novel types of cluster structure in neutron-rich nuclei such as those formed by molecular orbits and dineutron correlation, (2) novel types of cluster structure in stable nuclei such as cluster gas states, (3) ab initio calculations of cluster structure. I also discuss some of new developments in other fields than nuclear structure. Various new phenomena and systems of clustering imply the basic importance of clustering dynamics in nuclear many body systems which give us rich prospects in future studies of nuclear clustering.

1. Characteristic trends in physics of nuclear clustering
The subjects of this conference which started by the introductory talk of Brink cover almost all important subjects of nuclear cluster physics which were discussed also in recent cluster conferences as important subjects. A characteristic feature of this conference is that we heard many talks for the subject of "cluster structure of stable and unstable nuclei". It is rather remarkable compared with former cluster conferences. This is just the reflection of the characteristic recent developments of the cluster structure physics. I think the following three points represent the recent new developments of the cluster structure physics: (1) "Novel types of cluster structure in neutron-rich nuclei". This point was discussed by Kanada-En'yo A. Milin, Itagaki, Ito, Descouvemont, Marques, Saito, Takashina, Nakamura, Orr, Wolski, Kimura, Timofeyuk, Haigh, Bohlen, Kikuchi, W. Horiuchi, Madurga, Dorsch, and others. (2) "Novel types of cluster structure in stable nuclei". This point was discussed by Fynbo, Itoh, Courtin, Yamada, Tohsaki, Buck, Funaki, Kawabata, Ohkubo, Brenner, Alvarez-Rodrigues, and others. (3) "Ab initio calculations of cluster structure". This point was discussed by Neff, Forssen, Myo, Togashi, Sugimoto, and others.

1.1. Novel types of cluster structure in neutron-rich nuclei
Neutron-rich nuclei are new style of nuclear matter for which even our basic knowledge of nuclear physics is forced to change such as saturation (density, energy), magic number, proportional density and shape of neutrons and protons, and so on. Before 1990’s it was uncertain whether cluster structure exists or not in neutron-rich nuclei. We now know that the answer is yes. But the cluster structures in neutron-rich nuclei are novel kinds of cluster structure, for which even Ikeda’s threshold rule of cluster structure for stable nuclei is not obeyed. It is because dynamics of excess neutrons play important roles for the clustering dynamics.
Molecular orbits

Ground states and low-lying levels of Be isotopes are well described by molecular orbits of excess neutrons around \( \alpha + \alpha \) core. We have a long history of molecular orbit model for nuclei such as Seya-Kohno-Nagata, von Oertzen, Okabe-Abe, Greiner, Itagaki. AMD by Kanada-En’yo has verified this model. In Be isotopes we now know the important roles of \( \sigma \) orbit (intruder \( sd \) orbit): for example, \( \sigma \)-orbit sustains or enhances clustering, \( \sigma \)-orbit causes breaking of \( N=8 \) magicity. What is remarkable is that we now know the existence of states of many kinds of cluster structure with \( \sigma^0, \sigma^1, \) and \( \sigma^2 \) configurations.

An important question is the universality of cluster structure with molecular orbits. Kimura discussed, by using AMD, eventual neutron molecular orbits around \( \alpha + ^{16}O \) core and those around \( \alpha + ^{15}N \) core. He predicted the existence of molecular orbital states with \( \sigma^1 \), and \( \sigma^2 \) configurations in low excitation energy region around 2 MeV in \( ^{27}F \) and \( ^{29}F \).

A few talks discussed the atomic orbital configuration of neutrons at somewhat higher excitation energy region. In this configuration, neutrons move not over the whole system but around one cluster core and therefore the resultant structure is just the molecular structure composed of neutron-rich nuclei. The molecular band in \( ^{12}Be \) found by Freer et al. was discussed to have mainly such structure. Ito discussed a unified theoretical framework to treat both molecular and atomic orbital configurations.

Dineutron

P.G. Hansen explained the neutron halo of \( ^{11}Li \) by the di-neutron cluster model: \( ^{11}Li = ^9Li + \) di-neutron[1]. This dineutron model was one of the earliest ideas which imply the importance of clustering in neutron-rich nuclei. G. Bertsch and H. Esbensen[2] solved 3-body model of \( ^9Li + n + n \) and found the formation of di-neutron in the surface region. However, \( B(E1) \) strength distributions obtained by Coulomb breakup reactions performed at RIKEN, GSI, and MSU were not in good agreements with Bertsch-Esbensen results.

Recently T. Nakamura and his group remeasured carefully the breakup reaction of \( ^{11}Li \) and obtained new results which disagree with former experimental data. As was shown in Nakamura’s talk, their new data are found to be now in good agreement with Bertsch-Esbensen results. Other recent theoretical calculations which reproduce Nakamura’s new data also show the formation of a dineutron. For example the talk by Myo in this conference which succeeds in reproducing the halo nature, the breaking of \( N=8 \) magicity, and Nakamura’s new data, show the dineutron formation.

It is known that the pairing interaction is much stronger for dilute density than for ordinary density. Therefore it is very much likely that we have formation of dineutrons in dilute surface region of neutron-rich nuclei. Recently Matsuo and his group made Hartree-Fock-Bogoliubov calculations and argued that the dineutron correlations are strong in medium-mass region [3].

Extremely neutron-rich nuclei

We had several reports about neutron correlations in light nuclei. Marques treated 2n correlation in \( ^{6}He, ^{11}Li, ^{14}Be, \) and 4n correlation in \( ^{8}He, ^{14}Be \). Arickx studied extremely neutron-rich light systems like 4n, 6n, \( ^{3}H, ^{7}H \). Orr, Wolski, and Povoroznyk discussed light unbound neutron-rich nuclei like \( ^{7}He, ^{9}He, \) and \( ^{10}Li \).

1.2. Novel types of cluster structure in stable nuclei

Gas state of clusters

Gas state of nucleons has been an important subject of nuclear physics for a long time. Such a state has a very high excitation energy and therefore is a subject of nuclear matter and nuclear
reaction rather than nuclear structure. On the other hand, the gas state of clusters, if it exists, is not so highly excited, and can be a discrete state accessible spectroscopically.

Gas state of clusters is a new concept of nuclear structure and this concept was first proposed for the Hoyle state of $^{12}$C in 1970’s[4]. However, the discussion at that time was confined only for the Hoyle state, the $0^+$ state at $E_x = 7.66$ MeV near the $3\alpha$ breakup threshold at 7.27 MeV. Now in 2000’s, gas state of clusters is regarded as being universal and is studied in many nuclei both theoretically and experimentally. Present trend has started since the idea of $3\alpha$ BEC ( Bose Einstein condensation ) was proposed for the Hoyle state[5].

In the case of $^{12}$C, the discussion is not confined to only one state, the Hoyle state which is regarded as being a $3\alpha$ BEC state where every $\alpha$ cluster occupies an identical $0S$ cluster-orbit. Funaki argued that the state formed by the excitation of one $\alpha$ cluster from the $0S$ cluster-orbit to the $0D$ cluster-orbit is realized as the observed $2^+$ state at $E_x = 9.9$ MeV. Furthermore, it is argued by Kato and his group that the observed $0^+$ state at $E_x = 10.3$ MeV may be the state formed by the excitation of one $\alpha$ cluster from the $0S$ cluster-orbit to the $1S$ cluster-orbit.

The experimental confirmation of the existence of a $2^+$ state near $E_x = 10$ MeV is important for clarifying the $3\alpha$ cluster structure in $^{12}$C. We had important talks by experimentalists, Fynbo and Itoh, about this problem.

Many talks were related to this subject of gas state of clusters: In $^{12}$C, we had talks by Kanada-En’yo, Itagaki, Neff, Yamada, Alvarez-Rodrigues, Kawabata, Ohkubo, in addition to Funaki, Fynbo, and Itoh. For $^{11}$B, Kawabata and Kanada-En’yo discussed that the third $3/2^-$ state is a good candidate of $2\alpha + t$ gas-like state. For $^{14}$C, Kawabata, Kanada-En’yo, Yamada, Tohsaki discussed $3\alpha + n$ gas-like states. For $^{16}$O, Funaki discussed $4\alpha$ BEC state by comparing experimental data by Wakasa with $4\alpha$ OCM (orthogonality condition model) results which give $4\alpha$ BEC state around $4\alpha$ breakup threshold in addition to the reproduction of well-known states with $^{12}$C + $\alpha$ structure.

Other related talks are by Brenner on ALAS and liquid-gas, by Roepke on low density nuclear matter, by Zinner on ultracold atomic gasses, and by Ashwood on resonant scattering.

1.3. Ab initio calculations of cluster structure
One of the most characteristic features of the present-day nuclear theory is the development of ab initio calculations of nuclear structure. And one of the most remarkable results of ab initio structure calculations is the result for $^8$Be by Wiringa, Pieper, Carlson, and Pandharipande[6]. This result verified undoubtedly the $\alpha - \alpha$ structure of $^8$Be which has spatial localization of $\alpha$ clusters with inter-$\alpha$ distance 4 fm. It invalidates the argument that the cluster state is nothing but a special shell model state with a specific Elliott’s SU(3) symmetry.

Clustering and realistic nuclear force
The ab initio calculation of $^8$Be showed that the very large contribution of tensor force in $^4$He is inherited in $^8$Be with almost the same percentage of the contribution. This fact implies that we are now able to study the relation between clustering and realistic nuclear force especially tensor force.

A good example which shows important relation between clustering and tensor force was given by Myo’s talk on $^{11}$Li in this conference. Myo treats explicitly the second order effect of tensor force. He argues that the two-particle two-hole jump of $^9$Li core due to tensor force is Pauli-blocked when valence neutrons have $(0p)^2$ configuration while it is not blocked when valence neutrons have $(0s)^2$ configuration. This Pauli-blocking effect is largely responsible for energy-lowering of $(0s)^2$ configuration which is the breaking of $N=8$ magicity and for the formation of dineutron cluster which gives the halo structure of two neutrons. As already mentioned, Myo’s result reproduces Nakamura’s new data of Coulomb breakup cross sections.
Coexistence of mean-field-like structure and cluster structure

Coexistence of mean-field-like structure of the ground state and cluster structure of excited states is desired to be reproduced by ab initio calculations. FMD+UCOM by Neff and Feldmeier is an almost ab initio approach which can treat many excited states even with the same $J$ and $\pi$ and can describe clustering excited states. It is remarkable that the excited states with $3\alpha$ structure in $^{12}\text{C}$ are obtained together with the shell-model-like ground state by FMD+UCOM. It is very impressive and important to see that the density distributions of the $^{12}\text{C}$ states by FMD+UCOM are very similar to those by AMD.

1.4. New developments in other fields than cluster structure

Other subjects of cluster structure

Needless to say there are many other subjects of cluster structure than the subjects discussed in the above subsections. Here I quote such subjects which were reported in this conference. Proton-rich nuclei were discussed by Curtis. Algebraic models for clustering were discussed by Fortunato and Levai who treat supersymmetric model, and also by Tchuvil’sky and Cseh who treat Elliott’s SU(3) model and quasidynamical model, respectively. Few body approaches for cluster structure were discussed by Theeten and also by Vasilevsky, Arickx, and Fujiwara by using Faddeev formalism. Akimune reported cluster structure with excited alpha cluster. Yamazaki reported cluster structure composed of three-nucleon cluster.

Nuclear reaction

There have been much progress in nuclear reaction theories for nuclear reaction processes with unstable nuclear beam. Due to the weak binding nature of the system elaborate description of resonance states has been developed.

We heard many talks on reactions for neutron-rich nuclei; Sakuragi discussed coupled channel approach, Tengborn treated transfer reaction, Kikuchi discussed breakup process. Talks by Chuvilekaya, Marques, Orr, Wolski, and Chouma are also on reactions with unstable nuclear beam.

Masui discussed resonance and continuum in terms of Gamow state. Jensen gave careful analysis of sequential and direct decays. Decay process was also discussed by Fu, Grigorenko, and Raciti. Last two talkers studied 2p or $^2\text{He}$ decay. Quasi-free process was discussed by Balashov and breakup fusion was discussed by Singh. There were talks on scattering problems by Ashwood, Norbby, Lonnroth, R.Suzuki, Sakuta, Kamouni, and others. Ashwood was on resonant scattering, Lonnroth discussed level density, R.Suzuki used complex scaling method, Sakuta discussed ALAS, and Kamouni discussed microscopic approach.

Molecular states

Study of molecular resonance states has now become more interesting since recently the experimental knowledge of superdeformed and hyperdeformed states has been much accumulated in sd and pf shell region[7]. We heard several talks on molecular states in this conference and they are more or less concerned with relation between molecular states and superdeformed states. Courtin and Salsac discussed molecular states in $^{28}\text{Si}$ and $^{48}\text{Cr}$, respectively. Lepine-Szily discussed $^{12}\text{C} + ^{24}\text{Mg}$ system. Taniguchi argues possible relation of hyperdeformed states of $^{40}\text{Ca}$ with $^{12}\text{C} + ^{28}\text{Si}$ molecular states. Cseh’s talk is also concerned with molecular states in $^{40}\text{Ca}$, $^{40}\text{Ca}$, and Ni. Although in somewhat different context, von Oetzen discussed hyperdeformed compound states in $^{56}\text{Ni}$ and $^{60}\text{Zn}$.

A few years ago Kimura and myself studied the relation between $^{16}\text{O} + ^{16}\text{O}$ molecular states and superdeformed states of $^{32}\text{S}$. Several HF calculations had predicted the bandhead $0^+$ state of superdeformed band of $^{32}\text{S}$ around $Ex = 9 \text{ MeV}$[9], while Ohkubo and others had pre-
dicted the lowest $0^+$ state of $^{16}\text{O} + ^{16}\text{O}$ molecular structure also around $E_x = 9$ MeV[10]. The prediction of the lowest molecular state was made by using the uniquely determined optical potential (real part) of $^{16}\text{O} - ^{16}\text{O}$ system. We made AMD calculations and obtained energy curve as a function quadrupole deformation which is almost the same as those of the HF results. We found that the superdeformed band obtained by the GCM calculation along the energy curve contains $^{16}\text{O} + ^{16}\text{O}$ molecular component by about 44%. The energy of the superdeformed minimum was about 10 MeV lower than the minimum energy obtained by using $^{16}\text{O} + ^{16}\text{O}$ Brink wave function, which is mainly due to the energy gain from the spin-orbit force in the AMD calculation. We diagonalized the microscopic Hamiltonian by using as the basis states the AMD states along the energy curve and also Brink wave functions with various inter-cluster distances. The results show that the superdeformed band is not changed by this diagonalization while the third band contains $^{16}\text{O} + ^{16}\text{O}$ molecular component by about 100%, which corresponds to the band of observed molecular resonances.

**Heavy and superheavy nuclei**
Various subjects were discussed for heavy and superheavy nuclei. Greiner gave a theoretical review of physics of superheavy nuclei. Morita explained experimental approaches to the synthesis of superheavy elements and reported his successful synthesis of Z=113 element at RIKEN by using cold fusion method developed at GSI. Decay of heavy nuclei was discussed by Ren, Gugliemetti, and Royer. Ren’s talk was on alpha decay and Gugliemetti talked on C decay, while Royer discussed quasi molecular shape path. Fission was discussed by Skalski. Rotational bands of heavy nuclei were discussed by Merchant and Buck. Buck’s talk was on the discovery of an interesting phenomenological formula of rotational energies and he argued that it can be derived by assuming cluster structure.

**Clustering in nuclear matter and nuclear collisions**
Cluster formation in dilute nuclear matter was discussed by Roepke who also reported about cluster condensates paying attention to alpha condensate problem in finite nuclei. Clustering features in nuclear collisions were discussed by several talkers. Papa discussed isospin degree of freedom, Carevic and Dzelalija discussed strange particle, Abdel-Asiz discussed particle correlation at GeV/A, and Ricciardi discussed even-odd cluster production.

**Clustering in nuclear astrophysics**
In the talk of Wiescher important roles of cluster states in astrophysical reactions were discussed by using the Ikeda diagram. Processes like triple alpha burning, He burning, and C burning were explained in detail. S. Kubono also sometimes used Ikeda diagram in similar discussions by calling it "Cluster Nucleosynthesis Diagram (CND)"[11]. Neutron star was discussed by Khan by studying clusters in neutron star crust, and also by Sonoda who studied nuclear pasta in collapsing core. Tumino reported about nuclear reactions for nuclear astrophysics, Trojan horse method in particular.

**Clustering in other field physics**
Quark cluster approaches were discussed by Fujiwara and Musulmanbekov. Fujiwara reported their unified study of B-B force, and Musulmanbekov discussed quark correlation. Atomic cluster physics was discussed by Poenaru.

2. Developing trends and future perspectives
If we regard nuclear clustering as being the physics of dynamical assembling and disassembling of nucleons, clustering is a basic nuclear dynamics and appears abundantly in many problems of nuclear structure and nuclear dynamics. It is because nucleons are easy to assemble and
disassemble because of the saturation property of binding energy and density and also because of rather strong correlations of nucleons. We can say that the formation of clusters is a fundamental aspect of nuclear many body dynamics together with the formation of mean field. Here we recall the summary talk by Prof. Arima at the Chester cluster conference in England in 1984 in which it is said that coexistence of the clustering aspect and mean field aspect (single particle aspect) is a unique feature of nuclear many body system.

Various novel types of cluster structure both in unstable and stable nuclei which we discussed in this conference are the manifestation of the omnipresence of the cluster dynamics in nuclei. In particular, the following novel types of cluster structure which I stressed are remarkable subjects as general nuclear structure issues. In order to confirm or establish these structures, further intensive studies both experimental and theoretical are required: (1) Molecular orbit structure in neutron-rich nuclei in sd-shell and heavier region. (2) Dineutron cluster structure and also dineutron correlation in wide region of neutron-rich nuclei. (3) Gas-like states of clusters in wide region of nuclear chart.

Another new trend of nuclear cluster physics which I stressed is ab initio calculations of cluster structure and it belongs to the general trend of present-day nuclear theory. Except very light nuclei cluster structures show up in excited states, which imply that the ab initio calculation of cluster structure is nothing but the ab initio calculation of the coexistence of the cluster structure of excited state and the mean-field-like structure of the ground state. Actually ab-initio-like calculations with FMD+UCOM have shown us such coexistence of cluster structure and mean-field-like structure. Ab initio calculation including the use of the proper effective nuclear force applicable to cluster model space constitutes an important part of future perspectives of theoretical studies of nuclear cluster physics. We expect that roles of tensor force which is vital for the binding of alpha cluster are disclosed giving us deep understanding of the nature of alpha clustering.

Also for other subjects than cluster structure, much progress has been made and also is expected in the future. We can quote many attractive key words as below: For "nuclear reactions", key words may be 'loosely bound states', 'resonance', 'decay', 'direct reactions'. For "molecular states", they may be 'reactions', 'spectroscopy', and 'superdeformation'. For "heavy and superheavy nuclei", they may be 'synthesis of SHE', 'exotic decay', and 'surface clustering'. For "nuclear collisions and nuclear matter", they may be 'clustering toward and in dilute matter' and 'isospin degree of freedom'. For "nuclear astrophysics" we expect variety of clustering aspects in more processes.

I think that nuclear cluster physics is nothing but the manifestation of the richness of nuclear many body systems namely subatomic systems.

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