Development of a comprehensive mathematical model of the dependence of the performance of front loader attachments on technical condition of hydraulic drive elements

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Abstract. The article presents a comprehensive mathematical model that describes the dependence of the performance of front loader attachments on technical condition of hydraulic drive elements. The mathematical model is implemented as an algorithm for performing calculations of a hydraulic system conditionally consisting of two elements. The main criterion for ensuring high performance is the prime cost of work performed by a front loader to move loads, such as loading loose hay or bales, wood chips, silage, etc. According to the developed method, calculations were performed to implement a deterministic model in the Mathcad system and the acceptable level of performance was determined at lowest prime cost of work performed. Additionally, simulation of the dependence of the prime cost on the nominal level of performance was carried out. The developed comprehensive mathematical model allows us to determine the optimal frequency of replacement or restoration of the first and the second elements of a hydraulic drive according to the criterion of ensuring minimum costs, and, as a consequence, minimizing the prime cost.

1. Introduction

To ensure the cost-effective operation of front-end loaders used for various loading and unloading operations, it is necessary to maintain high performance of attachments, performing a significant part of the main work on loading and unloading various materials in the agro-industrial complex. The technical performance of front loader attachments depends on the volume and weight of the goods being moved, such as loading loose hay or bales, wood chips, silage, etc. The operation time of attachments during the period of loads movement depends on correct operation of the elements of the hydraulic system, such as hydraulic cylinders, hydraulic motors, hydraulic distributors, hydraulic pumps. The correct operation of both one and all elements of the hydraulic system can vary widely, while significantly increasing the operating time of attachments. To determine the acceptable values of the increase in the operating time of attachments, it is necessary to introduce the criterion of the limit state at which a parametric failure occurs [1].

Parametric failure means the limit state of operation of hydraulic system elements, at which, as a result of wear, the performance of the entire system decreases, and, as a result, the prime cost of production increases [2]. An increase in the cost of diagnostics, maintenance and repair of both machines in general and individual systems leads to an increase in the prime cost of production, especially during seasonal work [3,4]. Timeliness of works on restoration or replacement of faulty elements allows reducing these costs and ensuring the cost-effective operation of front loader attachments.
machine elements during diagnostics and maintenance is of great importance for ensuring high profitability of enterprises of the agro-industrial complex [5,6,7].

The purpose of the article is to develop a comprehensive mathematical model of the dependence of attachments performance on the technical condition of hydraulic drive elements of a front loader using the example of a hydraulic system, conventionally consisting of two elements.

The tasks are: formalizing a deterministic mathematical model with subsequent calculations of the dependence of the prime cost on the nominal level of productivity, as well as simulating for a model with stochastic parameters.

2. Formalizing mathematical model

If one element with a low wear rate $\alpha_1$ operates in the system, then the dependence of its performance $y_1$ can be expressed as a function:

$$y_1 = (t - h_1) \cdot \alpha_1 + D_{\text{max}}$$

where $t$ is the time during which the work is performed by the object;

$h_1$ - the 1st period of correct operation at the $D_{\text{max}}$ level after replacement or restoration of the first element (in our task we assume that $h_1 = h_2 = \ldots = h_k$) without reducing performance.

$\alpha_1$ - the wear rate of the first element, set by the angular coefficient of the decrease in productivity;

$D_{\text{max}}$ - the maximum possible productivity value.

From expression (1), the value of the system operation period $a_{k,j}$ is determined, limited by the decrease in performance of the first element to the acceptable level $j$, equal to the value $y_1$:

$$a_{k,j} = \frac{-j + D_{\text{max}}}{\alpha_1}$$

where $j$ is the acceptable level of performance (set conditionally in the figure) upon reaching which the second element is replaced and when the first element reaches it, the replacement is also performed. In our example we accept $0 < j \leq D_{\text{max}}$.

The ordinate axis value shows the performance of the element or the system as a whole, which, in our case, is expressed in arbitrary units.

This dependence is shown graphically in Figure 1.

![Graphical representation of the model](image)

**Figure 1.** Dependence of changes in performance of the first element $y_1$ on time $t$.

The profit received from the operation of the first element for the $k$ period at the $j$ level will be determined by the expression:

$$P_{k,j} = \int_{h_k}^{h_k + a_{k,j}} ((t - h_k) \cdot \alpha_1 + D) \, dt + h_k \cdot D_{\text{max}}$$
The maximum possible profit from the operation of the entire system in the $k$ period at the $j$ level will be determined by the expression:

$$M_{k,j} = (a_{k,j} + h_j) \cdot D_{\text{max}}.$$  \hfill (4)

The maximum possible profit from the operation of the entire system, taking into account all $k$ periods of operation of the first element at the $j$ level, will be determined by the expression:

$$Y_j = \sum_k M_{k,j}.$$  \hfill (5)

The total costs, including losses from reduced performance and costs associated with downtime in the $k$ period at the $j$ level of the first element, will be determined by the expression:

$$C_{k,j} = (M_{k,j} - P_{k,j}) + r_k,$$

where $r_k$ are the costs associated with downtime to replace or restore the first element.

The total costs, including losses from reduced performance and costs associated with downtime for all $k$ periods of operation at the $j$ level of the first element, will be determined by the expression:

$$L_j = \sum_k C_{k,j}.$$  \hfill (7)

If another element with a higher wear rate $\alpha_2 > \alpha_1$ operates in the system, then the dependence of its performance $y_2$ is expressed as a function:

$$y_2 = (t - z_1) \cdot \alpha_2 + D_{\text{max}},$$

where $z$ is the period of operation of the system during which the second element works properly (due to its replacement or restoration or from the beginning of its operation);

$\alpha_2$ - the wear rate of the second element, set by the angular coefficient of the decrease in performance;

In the problem, it is assumed that $z_i = const$. In this case, the first element continues to work with the previously set performance $y_1$.

Let us graphically represent in Figure 2 how the total performance $y_5$ decreases if the first element with performance $y_1$ is not replaced for a long time, while the second element with performance $y_2$ is replaced several times.

**Figure 2.** Changes in the performance of a system consisting of two elements, depending on time $t$
$y_{S_i}$ – the dependence of the change in the performance of the second element for the $i$ period of operation ($b_{i,j}$). The dependence $y_{S}$ is taken equal to $y_{S1}$.

The value of the system operation period $b_{i,j}$ will be determined by the expression:

$$b_{i,j} = \left( -\frac{j + D_{\text{max}}}{\alpha_1 - \alpha_2} \right) \cdot \left( \frac{-\alpha_1}{-\alpha_1 - \alpha_2} \right) \cdot \left( \sum_{i=1}^{j} b_{i-1,j} + \sum_{i=1}^{j} z_i - z_i \right).$$  \hspace{1cm} \text{(9)}$$

The number of replacements or restorations of the second element is limited by the decrease in performance and the replacement or restoration of the first element, and has a finite value of replacements or restorations, depending on the value of the acceptable level $j$ according to the expression:

$$B_j = \sum_{i} (b_{i,j} \geq 0).$$  \hspace{1cm} \text{(10)}$$

The number of replacements or restorations can be unlimited in time or quantity. By replacing or restoring an element, we mean the restoration of performance to the level of the maximum value $D_{\text{max}}$ (axis values $y$).

According to the task, as shown in Figure 2, in case of failures of the second element, it is necessary to determine the optimal value of the acceptable level $j$, where the profit, subject to the replacements of the second element before the replacement of the first one, will be the maximum. In the task, when replacing or restoring the second element, we consider the costs $p_j$, which conventionally include:

- loss of profit due to system downtime associated with the performance of work to restore or replace an element;
- the cost of purchasing a new item or the cost of purchasing materials to restore an old one;
- the costs of performing the work themselves associated with replacing an element with a new one (dismantling an old one, installing a new one, checking it in operation) or restoring an old one (dismantling an old one, disassembling, repairing, assembling, adjusting, checking in operation).

The total costs of downtime at the $j$ level due to replacement or restoration for each $k$ period (in our example, we assume that $k$ varies from 0 to $N$, $N = 10$), in which the second element is replaced, we write it as an expression:

$$p_j = \sum_{i} (q \cdot b_{i,j}).$$  \hspace{1cm} \text{(11)}$$

where $q$ is the cost of replacing or restoring the second element.

In addition, the costs will include the loss of performance during the operation of the second element. In Figure 2, this area is bounded by points II, V, VI during the first period of operation.

Losses associated with a decrease in the performance of the second element at the $j$ level will be determined by the expression:

$$R_{i,j} = \int_{z_i}^{h_{i,j} + z_i} (t - z_1) \cdot \alpha_1 + D_{\text{max}} dt - \int_{z_i}^{h_{i,j} + z_i} (t - z_1) \cdot (\alpha_1 + \alpha_2) + D_{\text{max}} dt,$$  \hspace{1cm} \text{(12)}$$

$$R_j = \int_{\sum_{k=0}^{h_{i,j} + z_i}}^{\sum_{k=0}^{h_{i,j} + z_i}} (t - z_1) \cdot \alpha_1 + D_{\text{max}} dt - \int_{\sum_{k=0}^{h_{i,j} + z_i}}^{\sum_{k=0}^{h_{i,j} + z_i}} (t - \sum_{k=0}^{h_{i,j} + z_i}) \cdot (\alpha_1 + \alpha_2) + D_{\text{max}} dt.$$  \hspace{1cm} \text{(13)}$$

Total losses associated with a decrease in the performance of the second element at the $j$ level until the moment of replacement of the first element is reached and provided that the first element is replaced $k$ number of times will be determined by the expression:
\[ V_j = \sum_{i=1}^{B} \sum_{k} R_{i,j} \]. \quad (14)

Total costs during the operation of the second element of the system at the \( j \) level, due to performance losses and replacements or restorations, will be determined by the expression:
\[ E_j = V_j + p_j. \quad (15) \]

The profit earned from the operation of two elements in the first period includes:
- the area bounded by points I, II, III, IV, characterized by the period of correct operation of the first and second elements;
- the area bounded by points II, VI, VII, III, taking into account the decrease in system performance to the \( j \) level;
- the period of correct operation of the second element \( h_k \).

As a criterion for parametric failure, we express the value of the prime cost \( W_j \) depending on the acceptable level \( j \) from the operation of two elements in the form of the expression:
\[ W_j = \frac{L_j + E_j}{Y_j} \rightarrow \min. \quad (16) \]

3. Algorithm for implementing a deterministic model

According to the developed methodology, an algorithm was developed that implements calculations according to the above-mentioned model in the Mathcad system:
\[ D_{\text{max}} = 100; \]
\[ k_{\text{max}} = 0.995, \quad k_{\text{min}} = 0.895 \] – Setting the maximum and minimum performance levels.
\[ J = 20, \quad \Delta k = \frac{k_{\text{max}} - k_{\text{min}}}{J} \] – Setting the number of steps and the amount of change in the performance level.
\[ j = 1..J + 1 \] – Setting the performance level index.
\[ k_j = k_{\text{min}} + \Delta k \cdot (j - 1) \] – Setting a discrete change in performance level.
\[ t_s = 0, \quad t_f = 1000 \] – Setting the start and end times of the experiment.
\[ T = 10000, \quad \Delta t = \frac{t_f - t_s}{T} \] – Setting the number of steps and the amount of time change.
\[ t = 1..T \] – Setting the time index.
\[ t_i = t_s + \Delta t \cdot t \] – setting a discrete time change;
\[ h = 50; \quad z = 20; \quad r = 400; \quad p = 2; \quad \alpha_1 = -0.03; \quad \alpha_2 = -0.1. \]

We will perform calculations depending on the specified parameters
\[ W_j = \text{def} \_ S \left( D_{\text{max}}, k_j, \alpha_1, t_s, t_f, h, T, \alpha_2, z, r, p \right), \quad (17) \]

where \( \text{def} \_ S \) is the name of the procedure for calculating prime cost values and other related parameters developed for Mathcad.

We perform prime cost calculations based on fixed set values and graphically determine the minimum prime cost value \( W_j \) in Figure 3.
Based on the results of calculations, we obtain the minimum value of the prime cost $W_j$ at $k_j = 0.97$ equals to $W_j = 0.042$.

4. Simulation of the dependence of the prime cost on the nominal level of performance

In the second version of the task for all six parameters $\alpha_{1m_j}, h_{m_j}, \alpha_{2m_j}, z_{m_j}, r_{m_j}, p_{m_j}$ a uniform distribution law is set with changing values within conventional limits:

$$\alpha_{1m} = \text{runif}(J + 1, 0.02, 0.1), \quad \alpha_{2m} = \text{runif}(J + 1, 0.1, 0.5), \quad h_m = \text{round}((\text{runif}(J + 1, 0.6)), \quad z_m = \text{round}((\text{runif}(J + 1, 0.3)), \quad r_m = \text{round}((\text{runif}(J + 1, 0.8)), \quad p_m = \text{round}((\text{runif}(J + 1, 1.1)))$$

The calculations used the previous values of the source data. The experiment was carried out for $J = 10$ the values of nominal performance $k_j$ in the same range from $k_{\text{min}} = 0.895$ to $k_{\text{max}} = 0.995$ with a step of 0.01.

When performing simulation, for each of the 10 points, the values of six parameters were generated 100 times. For randomly set values of these parameters, the prime cost $W_j$ values were calculated using the algorithm (18)

$$W_j = \text{def } - S(D_{\text{max}}, k_{m_j}, \alpha_{1m}, t_j, h_{m_j}, T, \alpha_{2m}, z_{m}, r_{m_j}, p_{m_j}).$$

Then, for each of the values $k_j$, the minimum $W_{\text{min}, j}$, maximum $W_{\text{max}, j}$ and average $W_{\text{av}, j}$ values were determined, which are shown in Figure 4.
Figure 4. Dependence of the prime cost on the level of performance (stochastic case)

Figure 4 shows that the form of dependence $W_j$ on the value of the performance level $k_j$ remains the same as in Figure 3.

It can be concluded that there is a stable pattern - the nominal value $k_j$ is best chosen in the range from 0.95 to 0.98. This choice will ensure the minimum prime cost values $W_j$.

5. Conclusions

1. The development of a comprehensive mathematical model of the dependence of the performance of front loader attachments on the technical condition of hydraulic drive elements was carried out using the example of a hydraulic system, conventionally consisting of two elements.

2. Calculations based on deterministic and stochastic models of the dependence of the prime cost on the nominal level of performance showed within what limits the value of the level of performance $k_j$ can change to ensure the minimum value of the prime cost $W_j$.

3. The developed comprehensive mathematical model allows us to determine the optimal frequency of replacement or restoration of the first and the second elements according to the criterion of ensuring minimum costs, and, as a consequence, minimizing the prime cost $W_j$.

4. The results of the calculations can be in demand in the conditions of competitive market relations and used to increase the economic efficiency of the operation of both separate attachments and front-end loaders in general.

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