Valley beam splitter based on strained graphene

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\textbf{Abstract.} We investigate theoretically the lateral displacements of valley-unpolarized electron beams in graphene after traversing a strained region. Valley double refraction occurs at the interface between the incident (unstrained) region and the strained region, in analogy with optical double refraction. It is shown that the exiting positions of $K$ and $K'$ transmitted beams, together with their distance $D$, can be tuned by the strain strength and the inclusion of an electrostatic potential. In addition, $D$ can be enhanced by the wave effect near the valley-dependent transmission resonances. The enhancement is remarkable for graphene n–p–n (or p–n–p) junctions. Thus the Goos–Hänchen effect of transmitted beams for a normal/strained/normal graphene junction can be utilized to design a valley beam splitter.

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1. Introduction

Electronic analogies of optical phenomena such as reflection, refraction, polarization, focusing and collimation have inspired many interesting proposals, which range from the spin field-effect transistor [1] to negative refraction and spin lenses [2–4] in p–n junctions made from graphene or topological insulators. The similarity between optics and electronics originates from the common wave nature of light and electrons. It is well known that a totally reflected beam of light is laterally shifted from the position predicted by geometrical optics. This wave effect, referred to as Goos–Hänchen (GH) shift [5, 6], has attracted much attention in the field of electronics in recent years. The GH shifts of transmitted beams have been utilized to design a spin beam splitter based on a two-dimensional (2D) electron gas with a local magnetic modulation [7]. In p–n–p junctions built on a carbon monolayer (graphene), the GH effect has been predicted to result in an $8e^2/h$ conductance plateau [8].

Graphene has many remarkable electronic properties [9] due to the gapless linear spectrum near corners of the hexagonal Brillouin zone. The orbital inequivalence of two opposite corners (the Dirac points $K$ and $K'$) provides a twofold valley degree of freedom, which has been exploited to design electronic devices with spintronic analogies [10–19]. The valley degeneracy in graphene is broken when the electron energy is far away from the Dirac point. In this situation, the group velocity of electrons becomes valley dependent. This observation leads to the proposal of a valley beam splitter [16] in analogy with optical double refraction. Near the Dirac point, it has been shown theoretically that [17, 18] valley-polarized current can be generated in bulk graphene by means of a valley-dependent transmission gap. The latter is caused by a combination of perpendicular magnetic fields and substrate strains. The elastic deformation shifts the $K$ and $K'$ valleys in the opposite direction [9, 19–23] and respects the time-reversal symmetry of graphene. It alone, however, cannot produce a valley polarization in two-terminal graphene devices [17] when valley-unpolarized currents incident from all directions are detected. This constraint may be circumvented by either collecting part of the output current or injecting a collimated electron beam, as the strategy in two kinds of proposed spin-filter devices [24, 25] utilizing only the spin–orbit interaction.

In this work, we examine the spatial separation of a valley-unpolarized beam of electrons with energy near the Dirac point. Note that the collimation of Dirac electrons has been reported in graphene p–n junctions [26] and the supercollimation has been predicted in graphene superlattices [27]. Our proposal relies on the valley double refraction at an interface between...
a strained and a normal region of a bulk graphene. For a graphene junction consisting of a strained region and two normal regions, we find that the GH shifts for the transmitted $K$ and $K'$ beams are different and are tunable with junction parameters. This provides an alternative way to generate a fully valley-polarized current without the need for external magnetic fields.

2. Model and formalism

The elastic deformation in the graphene plane (the $(x, y)$-plane) can be described by a gauge vector potential $[9]$, $A_S$. We assume that it changes the hopping integral only along a given $x$-direction $[17, 20]$, $A_S(x) = A_S(x)\hat{e}_y$. Such a strain can be induced by a tension along the $x$-direction applied to the substrate rather than the graphene itself, as in $[21]$. When a scalar potential $U(r) = U(x)$ is further considered, the low-energy Hamiltonian for Dirac electrons in the $\xi$ valley $(\xi = K/K'$ or $\pm 1)$ reads $[9]$

$$H_\xi = -i\hbar v_F(\sigma_x \partial_x + \sigma_y \partial_y) + \xi A_S(x)\sigma_y + U(x)\sigma_0,$$

where $v_F \approx 0.86 \times 10^6$ is the Fermi velocity, $\sigma_x$ and $\sigma_y$ are Pauli matrices and $\sigma_0$ is the $2 \times 2$ unit matrix. The translational invariance of equation (1) along the $y$-direction enables us to write its solution as $\Psi_\xi(r) = \Psi_\xi(x; k_\xi)\exp(ik_x y)$. Here $k_\xi$ is the conserved transverse wave vector, $\Psi_\xi$ satisfies $H_{\xi,k_\xi}\Psi_\xi = E\Psi_\xi$, and $H_{\xi,k_\xi}$ is an effective one-dimensional Hamiltonian obtained from equation (1) with the replacement $\partial_y \rightarrow ik_y$.

For a given region $j$ with a constant gauge potential $A_{Sj}$ and scalar potential $U_j$, the forward-propagating eigensolutions with energy $E$, if present, can be written as

$$\psi_{j\xi}(x; k_\xi) = \frac{\exp[ik_{x,j\xi}(x - x_j)]}{\sqrt{2}} \left(\exp(-i\frac{\theta_{j\xi}}{2}), \exp(i\frac{\theta_{j\xi}}{2})\right)^T,$$

$$k_{x,j\xi} = \text{sgn}(E_j)\sqrt{|E_j/(\hbar v_F)|^2 - k_{y,j\xi}^2},$$

where $E_j = E - U_j$ and $k_{x,j\xi} = k_\xi + \xi A_{Sj}/(\hbar v_F)$, $x_j$ is a reference point in region $j$ and $\theta_{j\xi}$ satisfies $\exp(i\theta_{j\xi}) = \hbar v_F(k_{x,j\xi} + ik_{y,j\xi})/E_j$. The velocity $v_{j\xi}$ of such a propagating mode has the $x$- and $y$-components

$$v_{x,j\xi} = v_F\psi_{j\xi}^+\sigma_x\psi_{j\xi} = v_F\cos\theta_{j\xi} > 0,$$

$$v_{y,j\xi} = v_F\psi_{j\xi}^+\sigma_y\psi_{j\xi} = v_F\sin\theta_{j\xi}.$$

Note that the amplitude of the velocity is constant ($v_{j\xi} \equiv v_F$), while the orientation of the velocity is characterized by the angle $\theta_{j\xi}$ with respect to the $x$-axis. Thus electrons from opposite valleys move along different directions in the strained region. In an unstrained region $j$, the valley index $\xi$ in $\theta_{j\xi}$ and $\psi_{j\xi}$ is omitted.

3. Kinematical analysis of interface scattering

We consider that a valley-unpolarized beam coming from an unstrained normal region ($j = N$) is scattered by an interface at $x = 0$, as illustrated in figure 1(a). The strained region ($j = S$) has
Figure 1. (a) Schematic representation of valley double refraction of Dirac electrons in graphene at an $N/S$ interface. The incident beam is valley unpolarized. (b) Kinematical construction of the refraction angles. The Fermi surface in the strained region consists of two circles centered at $(0, -q_S)$ and $(0, +q_S)$ (with $q_S = A_S/(\hbar v_F)$), which are for $K$ (red line) and $K'$ (blue line) electrons, respectively. (c) Refraction angle $\theta_{S\pm}$ plotted as a function of the incident angle $\theta_N$ for $E_N = E_S = 5A_S$. (d) The same as in (c) but for three other typical cases: (1) $E_N = 7A_S$, $E_S = 5A_S$ (solid lines); (2) $E_N = 2A_S$, $E_S = 5A_S$ (dashed lines); (3) $E_N = 2A_S$, $E_S = 0.5A_S$ (dotted lines). The characteristic angles are explained in the text.

a constant gauge potential $A_S > 0^5$. Total reflection happens for $\xi$ electrons when the incident angle $\theta_N$ satisfies

$$|E_N \sin \theta_N + \xi A_S| \geq |E_S|. \quad (5)$$

For this case, the dashed line in figure 1(b) representing the conserved transverse wave vector has no intersection with the isoenergy surface of the $\xi$ valley in the $S$ region. In the opposite case, the incident wave of $\xi$ electrons is transmitted (partly) into the $S$ region with refraction angle $\theta_{S\xi}$ and probability

$$T_{N/S}^{\xi} = \frac{\cos \theta_N \cos \theta_{S\xi}}{\cos^2[(\theta_N + \theta_{S\xi})/2]} \quad (6)$$

5 The refraction angles of $K$ and $K'$ electrons are swapped when $A_S$ switches its polarity and change signs as $E_S$ becomes $-E_S$. As for the change $E_N \to -E_N$, one has $\theta_{S\pm} \to -\theta_{S\pm}$. Therefore, for the valley double refraction it is sufficient to consider the situation $A_S, E_S, E_N > 0$. The transmission probability $T_{N/S}^{\xi}$, however, depends strongly on the relative signs of $A_S, E_S$ and $E_N$.

New Journal of Physics 13 (2011) 083029 (http://www.njp.org/)
When both $k_{x,S^+}$ and $k_{x,S^-}$ are real, the incident beam after being transmitted splits into two beams with opposite valley index that propagate at different angles ($\theta_{S^+} \neq \theta_{S^-}$). In this case, the interface acts as a valley beam splitter. Since $H_+$ and $H_-$ are related by the time-reversal symmetry, $\theta_{S^\pm}$ satisfies

$$\theta_{S^-}(\theta_N) = -\theta_{S^+}(\theta_N). \quad (7)$$

In figure 1(c), the variation of the refraction angle $\theta_{S\pm}$ with the incident angle $\theta_N$ is plotted for the simplest case $E_S = E_N$. For normal incidence the two refraction angles satisfy $\theta_{S^+} = -\theta_{S^-} \neq 0$, which is distinct from the optical double refraction in uniaxial crystals and the spin–orbit-induced beam splitting in 2D semiconductor heterostructures [25]. Note that the spatial separation of refraction beams for normal incidence has been demonstrated theoretically for a graphene $n$–$p^*$ junction [16]. The mechanism in [16] relies on the valley dependence of the group velocity caused by the trigonal warping of energy bands far away from the Dirac point. In contrast, in our case the low energy region is concerned and the transverse velocity is valley-dependent due to strain-induced gauge potential. The angular separation of the two refraction beams, $\Delta \theta_S = |\theta_{S^+} - \theta_{S^-}|$, increases with $|\theta_N|$ if the double refraction occurs. Obviously, $\Delta \theta_S$ can be controlled by the strain strength.

In analogy with the spin optics considered in [25], we define several characteristic angles related to valley filtering. The $K'$ electrons have a maximal outgoing angle $\theta_{Sc}$. If electrons are incident from all angles but are collected only in the interval $(\theta_{Nc}, \pi/2)$ (as in [24]), one will obtain a fully valley-polarized current. Total reflection occurs for $K$ electrons as the incident angle $\theta_N$ exceeds a critical angle $\theta_{Nc} > 0$ (equation (5)). The valley double refraction happens for $\theta_{Sm} = \theta_{S^+}(\theta_{Nc}) = \theta_{S^-}(\theta_{Nc})$. To achieve a valley beam splitter, one can restrict the incident angle to be within an interval $(\theta_{Nf}, \theta_{Nc})$ so that $K$ electrons are scattered into the angle interval $(\theta_{Sm}, \pi/2)$, while $K'$ electrons are scattered into the angle interval $(\theta_{Sf}, \theta_{Sm})$. Note that for $1 < E_S/A_S < 2$, the separation angle $\theta_{Sm}$ of two valley-polarized beams is negative.

The valley double refraction can be tuned by the electric potentials $U_N$ and $U_S$. In the case of $E_N > E_S + A_S$ and $E_S > A_S$, there exist both a lower and an upper critical angle, $\theta_{Nc}'$ of $\theta_{Nc}$, for the total reflection of $K$ electrons (see the solid lines in figure 1(d)). Note that $\theta_{Nc}' < 0 < \theta_{Nc} < |\theta_{Nc}'|$. The discussions on figure 1(c) can be applied in this case with the exception of $\theta_{S^\pm} = \pi/2$. When $0 < E_S - A_S < E_N \leq E_S + A_S$, the features of the valley double refraction are the same as those in figure 1(c). In this case, we set $\theta_{Nc}' = -\pi/2$. For the special case $E_S = A_S$, one of the two critical angles ($\theta_{Nc}'$ and $\theta_{Nc}$) becomes zero. In the case, of $0 < E_S < A_S$, double refraction disappears, as observed from the dotted lines in figure 1(d). In contrast, double refraction occurs for all incident angles if $0 < E_N \leq E_S - A_S$ (see the dashed lines in figure 1(d)). In this case, we can take $\theta_{Nc}' = \pi/2$ and $\theta_{Nc}' = -\pi/2$.

The kinematical analysis performed above is for a sharp $N/S$ interface. We now consider a smooth interface [25] where the potentials vary slowly on the scale of the electron wavelength $\lambda_F$ and the parameter $\eta = \max\{(dA_S/dx)/(A_S/\lambda_F), (dU/dx)/(U/\lambda_F)\} \sim \lambda_F/d \ll 1$. Here $d$ is the effective width of the interface. The wave function $\psi_\xi$ satisfies

$$\left\{ -\partial_x^2 + \left[ k_y + \frac{\xi}{\hbar v_F} A_S(x) \right]^2 - \left[ \frac{E - U(x)}{\hbar v_F} \right]^2 - i \frac{\partial_x U}{\hbar v_F} \sigma_x + \xi \sigma_z \frac{\partial_x A_S}{\hbar v_F} \right\} \psi_\xi = 0. \quad (8)$$
Figure 2. (a–c) Valley-dependent GH shift $\sigma_+$ (red solid lines) and $\sigma_-$ (blue dash-dotted lines) together with $\sigma^{\pm}_{cl}$ (black dashed lines) and $\sigma^{\pm}$ (black dotted lines) plotted as a function of the incident angle $\theta_N$. (d) Valley-resolved transmission probability versus incident angle. The value of $E_N$ is taken to be 2 in (a), 5 in (b) and 7 in (c) and (d). Other parameters are $E_S = 5$, $A_S = 1$ and $L = 3$. The inset in panel (a) illustrates schematically the lateral displacements $\sigma^\pm$ of transmitted electron beams $\psi^\pm_{\text{out}}(L, y)$. A detector placed in a proper position of the outgoing region can collect only the $K$ ($K'$) electrons.

For $\eta \ll 1$, the reflected wave can be neglected if refraction occurs and $\theta_N$ is not close to any critical angle. Further, the Wentzel–Kramers–Brillouin (WKB) ansatz based on equations (8) and (1) is of the form

$$
\psi_\xi \sim \left( \exp \left[ -\frac{i}{2} \theta_\xi(x) \right], \exp \left[ \frac{i}{2} \theta_\xi(x) \right] \right)^T,
$$

where $k_{x, \xi}(x)$ and $\theta_\xi(x)$ have similar expressions as $k_{x, j\xi}$ and $\theta_{j\xi}$. Thus, for such a smooth interface, the motion of electrons can be treated within ray optics. Then the kinematical separation of the $K$ and $K'$ electron trajectories may be utilized to construct a valley filter.

4. Goos–Hänchen (GH) shift of transmitted beams: analytical results

The valley double refraction at an $N/S$ interface serves a basis for creating valley-splitting beams in another unstrained region $O$, as depicted in the inset of figure 2(a). To this end, the incident (valley-unpolarized) electronic beam $\psi_{\text{in}}$ is assumed to be well collimated [26, 27].

New Journal of Physics 13 (2011) 083029 (http://www.njp.org/)
around an angle $\theta_N$, i.e.
\[
\psi_{in}(x, y) = \int e^{i k_y' y} \psi_N(x; k_y') f(k_y' - k_y) \, dk_y',
\]
where $k_y = (E_y/hv_F) \sin \theta_N$, and the profile of the beam is taken as a Gaussian with width $\Delta_q$, i.e. $f(q) = \exp[-q^2/(2\Delta_q^2)]$. The incident beam has a central position $y_{in}$ at the $N/S$ interface $x_N = 0$. The transmitted beam $\psi_{out}^\xi$ is formed by $\xi$ electrons traversing the middle $S$ region (with width $L$):
\[
\psi_{out}^\xi(x, y) = \int t_\xi(E, k_y') e^{i k_y' y} \psi_O(x; k_y') f(k_y' - k_y) \, dk_y'.
\]
Here $t_\xi(E, k_y')$ is the transmission amplitude for $\xi$ electrons with energy $E$ and transverse wave vector $k_y'$, which is normalized so that the corresponding transmission probability is given by $T_\xi = |t_\xi|^2$. The exiting position of this beam at the $S/O$ interface $x_O = L$ becomes $y_{out}^\xi = y_{in} + \sigma_\xi$. The exiting positions of $K$ and $K'$ transmitted beams have a distance
\[
D = |y_{out}^+ - y_{out}^-| = |\sigma_+ - \sigma_-|.
\]

For the rectangular profile of the potentials, the GH shift $\sigma_\xi$ is generally not equal to the classical result $\sigma_\xi = L \tan \theta_{S\pm}$ and thus the wave effect should be considered.

From the stationary-phase approximation [28] (which is valid when $\Delta_q \ll |E_N|/(h v_F)$), the two components of $\psi_{out}^\xi(L, y)$ are Gaussians centered at $\partial[-\text{Im} \ln t_\xi(E, k_y) \pm \theta_O/2]/\partial k_y$. Then we yield the general expression for the lateral displacement
\[
\sigma_\xi(E, \theta_N) = -\text{Im} \frac{\partial \ln t_\xi(E, k_y)}{\partial k_y}.
\]
From the fact that $\sigma_x H_{k_x} \sigma_x^{-1} = H_{-k_x}$, we obtain $t_-(E, -k_y) = t_+(E, k_y)$ and then
\[
\sigma_-(E, \theta_N) = -\sigma_+(E, -\theta_N).
\]
When $A_S$ changes to $-A_S$, the exiting positions of the two valley-polarized outgoing beams are swapped.

For the considered system with a simplified potential profile and $U_N = U_O$, a direct calculation gives
\[
\begin{align*}
t_{\xi}^{-1} &= \cos \varphi_\xi - i \sin \varphi_\xi F_0(\theta_N, \theta_S), \quad (14) \\
\sigma_\xi &= \frac{[F_1(\theta_N, \theta_S) + F_2(\theta_N, \theta_S) \sin(2\varphi_\xi)/(2\varphi_\xi)]L \cos \theta_N}{\cos^2 \theta_N \cos^2 \theta_S + (\sin \theta_N - \sin \theta_S)^2 \sin^2 \varphi_\xi}, \quad (15)
\end{align*}
\]
where $\varphi_\xi = k_x \theta_S L$, and
\[
\begin{align*}
F_0(\theta_N, \theta_S) &= \frac{1 - \sin \theta_N \sin \theta_S}{\cos \theta_N \cos \theta_S}, \quad (16) \\
F_1(\theta_N, \theta_S) &= \sin \theta_S (1 - \sin \theta_N \sin \theta_S), \quad (17) \\
F_2(\theta_N, \theta_S) &= (\sin \theta_N - \sin \theta_S) \left(1 - \frac{E_S \cos^2 \theta_S}{E_N \cos^2 \theta_N}\right). \quad (18)
\end{align*}
\]

6 The two components of $\psi_{in}(0, y)$ are centered at two different mean $y$-coordinates [8] $y^{u/d}_{in} = \pm 0.5 \theta_N / \partial k_y$. Their distance is $|\partial \theta_N / \partial k_y|$, which should be small for valley separation. The position $y_{in}$ is $(y^{u}_{in} + y^{d}_{in})/2 = 0.$
The lateral displacement $\sigma_\xi$ in equation (15) depends on the valley index through the propagating phase $\varphi_\xi$ and the refraction angle $\theta_{S\xi}$. The factor $F_0$ takes the value 1 only when $\theta_N = \theta_{S\xi}$, for which $\sigma_\xi = \sigma_\xi^{cl}$. Since $F_0 > 1$ for a real $\theta_{S\xi} \neq \theta_N$, the condition for perfect transmission ($T_\xi = 1$) is given by $\sin \varphi_\xi = 0$. The latter can be equivalently expressed in terms of the resonant refraction or incident angle,

$$\theta_{S\xi} = \theta_{S\xi}^{res} = \pm \arccos \left[ \frac{m \pi}{(L/E_S)} \right],$$

(19)

$$\theta_N = \theta_N^{res} = \arcsin \left[ \frac{E_S \sin \theta_{S\xi}^{res} - \xi A_S}{E_N} \right],$$

(20)

where the integer $m$ satisfies $1 \leq m \leq |E_S|L/\pi$ and $\xi A_S \leq \text{sgn}(E_S) \sqrt{E_S^2 - (m \pi/L)^2} < \xi A_S + E_N$. Note that resonant refraction angles $\theta_{S\xi}^{res}$ are determined only by $|E_S|$ and $L$, while resonant incident angles $\theta_N^{res}$ depend on $E_S$ and $E_N$ as well as $\theta_{S\xi}^{res}$, namely, $\theta_N^{res} = \theta_N^{res}(\theta_{S\xi}^{res}, E_S, E_N)$. At resonant incident angles, the lateral displacement $\sigma_\xi$ has the form

$$\sigma_\xi^{res} = F_0^{res} \xi L \tgg \theta_{S\xi}^{res},$$

(21)

whose amplitude is always larger than the corresponding classical value. The magnified factor $F_0^{res} = F_0(\theta_{N}^{res}, \theta_{S\xi}^{res})$ is approximately the peak-valley ratio of the transmission resonance because the minimum of transmission near $\theta_{S\xi}^{res}$ can be estimated as $1/F_0^{res}$.

5. GH shift of transmitted beams: numerical results

For brevity, hereafter all quantities are expressed in dimensionless units with a basic length $a$ and energy $E_0 = h v_F / a$ (which is 26 meV for $a = 20$ nm). The angular dependence of $\sigma_\xi$ is shown in figure 2 for a strained graphene n–n′–n junction with the simplified potential distribution. Due to the relation (13), it is sufficient to plot only the results for the case of $\theta_N \geq 0$. For a real $k_x, S\xi$, a remarkable deviation of $\sigma_\xi$ from $\sigma_\xi^{cl}$ may appear only when the incident angle $\theta_N$ has a large amplitude or is close to a critical angle. When the valley double refraction occurs for all angles at the $N/S$ interface (figure 2(a)), the valley spatial separation $D$ can greatly exceed $D^{cl} = |\sigma_+^{cl} - \sigma_-^{cl}|$ at large $|\theta_N|$. Near a critical angle $\theta_{\theta_k} \in \{\theta_{Nc}, -\theta_{Nc}'\}$ (figures 2(b) and (c)), $\sigma_\xi$ exhibits peaks and the difference between $D$ and $D^{cl}$ can be either positive or negative. It can be seen from figures 2(c) and (d) that the maximum of $|\sigma_\xi|$ appears at resonant incident angles. When $|\theta_N^{res} - \theta_{S\xi}^{res}|$ is small, the magnified factor $F_0^{res}$ is close to its minimum value 1 (see the first several resonances in figures 2(b) and (c)). Around the transmission peaks of $K$ electrons with a large $F_0^{res}$, $\sigma_\_\$ is almost the same as $\sigma_\xi^{cl}$. As a result, the valley spatial separation $D$ can be enhanced at the transmission resonance. For $\theta_N > \theta_{\theta_k}$, the value of $\cos \theta_S$ as well as $\varphi_\xi$ in equation (12) becomes imaginary. The corresponding GH shift $\sigma_\xi$ decays rapidly for $\theta_N$ close to $\theta_{\theta_k}$. As $\theta_N$ is far away from $\theta_{\theta_k}$, the transmission of $\xi$ electrons is suppressed drastically and the transmitted $\xi$ beam is difficult to detect.

When the parameter $E_S$ in figure 2 changes its sign by tuning gate voltages, the critical angles are unchanged, while $\sigma_\xi^{cl}$ switches its polarity. For such a n–p–n junction, the valley-dependent lateral displacement is plotted in figure 3 as a function of the incident angle. It can be seen that negative refraction happens for $K$ electrons when $\theta_N \in [0, \theta_{Nc})$. For $K'$ electrons, the transition from positive to negative refraction appears at the incident angle $\theta_{N0} = \arcsin(A_S/E_N)$. Accordingly, the GH shift $\sigma_\_\xi$ changes from positive to negative values with increasing $\theta_N$ (see figure 3(a)). Double negative refraction occurs as $\theta_{N0} < \theta_N < \theta_{Nc}$. For $E_N = 7$, the resonant incident angles in figure 3(d) coincide with those in figure 2(d).
Figure 3. The same as in figure 2 but for $E_S = -5$. Some values of the resonant GH shift for $K$ electrons are marked in (b) and (c).

because $\theta_{res}^{E_N}\left(-\theta_{res}^{E_S}, -E_S, E_N\right) = \theta_{res}^{E_N}\left(\theta_{res}^{E_S}, E_S, E_N\right)$. The corresponding peak-valley ratios in the two figures, however, have a large difference. Accordingly, the amplitude of resonant GH shifts for $E_S = -5$ is distinct from that for $E_S = 5$. At the resonant incident angle nearest and next nearest to the critical angle $\theta_{Nc}$, the value of $\sigma_+^-$ in figure 3(c) is $-125$ and $-26$, respectively, while in figure 2(c), it is $38$ and $10$, respectively. Such a contrast is more remarkable for $E_N = 5$ (see figures 2(b) and 3(b)). This indicates that for strained graphene n–p–n junctions, an enhanced valley spatial separation can be obtained for some incident angle away from the critical angle. For $E_N = 2$, a resonant GH shift for $K$ electrons appears in figure 3(c) and is difficult to be discerned in figure 2(a). These observations can be understood from the fact that $F_0(\theta_{Nc}^{E_N}, \theta_{S\pm}^{E_S}) < \sec \theta_{Nc}^{E_N} \sec \theta_{S\pm}^{E_S} < F_0(\theta_{Nc}^{E_N}, -\theta_{S\pm}^{E_S})$ when $\theta_{Nc}^{E_N} \theta_{S\pm}^{E_S} > 0$.

The GH shifts $\sigma_\pm$ should vary with the width $L$ of the strained region because the resonant refraction angle (19) together with $F_0^{res}$ depends on $L$. Such a variation is shown in figure 4 in the presence of double refraction. In figure 4(a), an n–p–n junction ($E_S = -5$) is concerned and the incident angle is chosen to be far away from the critical angle (see figure 3(b)). In figures 4(b) and (c), $E_S = 5$ and $\theta_N$ is close to $\theta_{Nc}$. In these figures the oscillation of $\sigma_+ - \sigma_-^{el}$ ($\sigma_- - \sigma_-^{el}$) with $L$ has a large (small) amplitude. In comparison with the results in figure 4(b), the small amplitude of the refraction angles in figure 4(a) results in a small $D^{el}$. However, the valley spatial separation $D$ in figure 4(a) can exceed its counterpart in figure 4(b), due to the wave effect. In figure 4(d), a large value $\theta_N = 80^\circ$ is taken. Now both $\sigma_+$ and $\sigma_-$ can be either positive or negative and show a rapid oscillation with $L$ due to a large $|k_{x, S\pm}|$. The two oscillations have nearly the same amplitudes but different periods ($k_{x, S+} \neq k_{x, S-}$). At some value of $L$, $\sigma_+$ is positive and is close to its maximum while $\sigma_-$ is negative and near its minimum (see the vertical lines in figure 4(d)). Thus, the distance $D$ can change from zero to a value much larger.
Figure 4. Valley-dependent GH shift $\sigma_+$ (red solid lines) and $\sigma_-$ (blue dash-dotted lines) together with $\sigma_{\pm}^{cl}$ (black dashed lines), and $\sigma_{\mp}^{cl}$ (black dotted lines) plotted as a function of the width $L$. In (a)–(d), we set $\theta_N$ to be $40^\circ$, $50^\circ$, $30^\circ$ and $80^\circ$ and take $E_N$ to be 5, 5, 7 and 2, respectively. The parameter $E_S$ is $-5$ in (a) and 5 in (b)–(d). $A_S = 1$. The vertical thin lines in (d) indicate some optimal values of $L$ for valley spatial separation under given parameters.

than $D^{\pm}$. The oscillations arise from the Fabry–Perot interference and are contributed by the term $\alpha \sin(2\varphi_0)$ in expression (15). Note that the average of $\sigma_\xi$ over $\varphi_0$ in a period $\pi$ is [28] approximately $\sigma_{\xi}^{cl}$.

The results presented so far were calculated for a rectangular profile of the uniaxial strain. One may wonder whether they are valid for a realistic elastic strain, which varies smoothly on the scale of the graphene lattice constant. To answer this problem, we consider a typical potential profile (see figure 5(a)) as $A_S(x) = A_S F(x)$ and $U(x) = U_S F(x)$ with

$$F(x) = \text{erf}(4x/L_b - 2) + \text{erf}[4(L - x)/L_b - 2]/2,$$

where $\text{erf}(x)$ is the error function and the width of the transition region is set at $L_b = 0.2L$. For electrons traversing the strained region with such a potential distribution, the valley-resolved transmission probability $T_\xi$ and lateral displacement $\sigma_\xi$ were calculated numerically and are shown in figures 5 and 6. From the angular dependence of $T_\xi$ and $\sigma_\xi$, it is seen that near a transmission resonance of $\xi$ electrons, $\sigma_\xi$ can be enhanced and the maximal amplitude of $\sigma_\xi$ together with the valley spatial separation $D$ depends strongly on the peak-valley ratio. For $E_N(E_N - U_S) < 0$, a remarkable valley spatial separation can be achieved for some incident angle away from the transmission forbidden region (see dotted lines in figures 5(b) and (d)). These features have been discussed for the simplified potential profile (figures 2 and 3). The oscillatory variation of $\sigma_\xi$ with the width $L$ is shown in figure 5(c), which is very similar to that in figures 4(a) and (b).
Figure 5. (a) A smooth distribution of strain-induced gauge potential with amplitude $A_S = 1$ (see text). (b) Valley-resolved transmission probability versus incident angle $\theta_N$. (c, d) Valley-dependent GH shift $\sigma_+$ (red lines) and $\sigma_-$ (blue lines) plotted as a function of the width $L$ and incident angle. We set $E_N = 5$, $A_S = 1$ and $U_S = 0$ or 10. In (c), the incident angle is taken as $50^\circ$ for $U_S = 0$ and $40^\circ$ for $U_S = 10$. In (b) and (d), the width $L$ is fixed at 3.

For a fixed width $L = 3$, incident energy $E_N = 5$ and incident angle $\theta_N = 40^\circ$, the tunability of the valley-dependent GH shift is demonstrated in figures 6(a) and (b). With increasing strain strength $A_S$ from 0, both $T_+$ and $T_-$ can come into resonances before the onset of transmission blockade for $K$ electrons (see figure 6(c)). In this range ($0 < A_S < 2$), the valley spatial separation $D$ can be adjusted greatly by $A_S$ (see figure 6(a)). In figure 6(d), the variation of $T_\pm$ with $U_S$ is plotted. There exist transmission gaps and the gap interval for $K$ electrons includes that for $K'$ electrons. For a given transmission gap, the corresponding GH shifts near its left and right terminals have opposite signs (see figure 6(c)). Thus, the amplitude and sign of $\sigma_\xi$ can be tuned by the strength of the scalar potential.

6. Conclusions and remarks

In summary, we have proposed an efficient way to create valley-splitting beams in graphene based on the strain effect. The strain-induced gauge potential acting on Dirac electrons leads to a valley dependence of their transverse velocity. Thus a valley-unpolarized beam after being transmitted through an $N/S$ interface may split into two valley-polarized beams. For an $N/S/N$ graphene junction, the lateral displacements of transmitted $K$ and $K'$ beams together with their difference can be enhanced by the wave effect and are adjustable with junction parameters such as the strain strength, the width of the $S$ region and the amplitude and polarity of an additional electric potential.
Figure 6. Valley-dependent GH shift $\sigma_{\pm}$ and transmission $T_{\pm}$ plotted as a function of $A_S$ and $U_S$. We set $L = 3$, $E_N = 5$ and $\theta_N = 40^\circ$. In (a) and (c) $U_S$ is set to 0 or 10. In (b) and (d) $A_S$ is fixed at 1.

For a realistic $N/S$ interface, the picture of valley double refraction under ray optics is valid when the electron wavelength $\lambda_F$ satisfies $d_c < \lambda_F \ll d$, where $d_c$ is the scale that the potentials have a large variation and $d$ is the effective width of the interface. As a wave effect, the valley-dependent GH shift, however, has no such limit. To obtain a valley separation from a $N/S/N$ graphene junction, the incident energy and incident angle should satisfy (see footnote 5) $|E_N \cos \theta_N| \gg 1$. Due to the large valley separation of outgoing beams, each of the generated valley-polarized beams can be collected alone by a detector (see the inset of figure 2(a)). If the detector consists of bulk graphene with broken inversion symmetry [13], the injection of such a valley-polarized current will lead to a measurable transverse voltage across the detector. Another possible detector is a bulk graphene sheet under the modulation of a substrate strain and a ferromagnetic stripe [17]. The longitudinal conductance of this detector depends on the valley polarization of the injected current.

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