Probing the $H\gamma\gamma$ coupling via Higgs exclusive decay at the LHC

Hongxin Dong,1,2,* Peng Sun,2,† and Bin Yan3,‡

1Department of Physics and Institute of Theoretical Physics, Nanjing Normal University, Nanjing, Jiangsu 210023, China
2Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou, Gansu Province 730000, China
3Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

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It is well known that the precise measurement of $H \to \gamma\gamma$ can generate two degenerate parameter spaces of $H\gamma\gamma$ anomalous coupling. We propose to utilize the exclusive Higgs rare decays $H \to \Upsilon(ns) + \gamma$ to break above mentioned degeneracy and further to constrain the $H\gamma\gamma$ anomalous coupling at the HL-LHC. We demonstrate that the branching ratios of $H \to \Upsilon(ns) + \gamma$ can be significantly enhanced in the non-SM-like parameter space from $H \to \gamma\gamma$ measurement, due to the destructive interference between the direct and indirect production of $\Upsilon(ns)$ in the SM. By applying the NRQCD factorization formalism, we calculate the partial decay widths of $H \to \Upsilon(ns) + \gamma$ at the NLO accuracy of $\alpha_s$. We show that it is hopeful to break such degenerate parameter space at the HL-LHC if we can further highly suppress the background and enhance the signal efficiency compared to the ATLAS preliminary simulation.

I. INTRODUCTION

After the discovery of the Higgs boson at the Large Hadron Collider (LHC) [1, 2], precise measuring the Higgs properties at the LHC and future colliders has become one of the major tasks of particle physics. The rare decay process, $H \to \gamma\gamma$, as one of the golden channel to discover the Higgs boson has received much attention in the high energy physics community due to the excellent performance of photon reconstruction and identification at the LHC. It is useful to indirectly search the potential new physics (NP) effects through measuring the $H\gamma\gamma$ anomalous coupling if the NP contributes to this rare decay by quantum loops. From the recent global analysis of the ATLAS collaboration, the signal strength of $\gamma\gamma$ mode is consistent with the SM prediction with a high accuracy, therefore, the $H\gamma\gamma$ anomalous coupling has been severely constrained by the LHC data [3].

However, all the knowledge of the $H\gamma\gamma$ anomalous coupling is inferred from the Higgs decay branching ratio measurements, as a result, it would be a challenge to probe the NP effects in the so called “faked-no-new-physics” (FNNP) scenario that the NP contribution is to probe the FNNP scenario at the HL-LHC through the exclusive Higgs boson decay process $H \to \Upsilon(ns) + \gamma$, with $n = 1, 2, 3$. There are two separate production mechanisms for the quarkonium state: the direct production (see Fig. 1(a),(b)) and the indirect production (see Fig. 1(c)). The indirect production of the $\Upsilon(ns)$ in this process can be induced by the top quark and $W$-boson loop in the SM. It has been shown that this process can be used to constrain the light quark Yukawa coupling and its CP violation effects via the interference between the direct and indirect process [15–22]. In this paper, we demonstrate that this exclusive Higgs decay can also be used to probe the FNNP scenario at the HL-LHC.

Owing to the destructive interference between the direct and indirect production of the process $H \to \Upsilon(ns) + \gamma$ in the SM, the branching ratio of this rare decay is around $O(10^{-9}) \sim O(10^{-8})$ [15, 16, 18]. Thus, it would be a challenge to observe the SM signal with the current or foreseeable dataset. But, the branching ratio could

* 211002010@njnu.edu.cn
† pengsun@impcas.ac.cn
‡ yanbin@ihep.ac.cn (corresponding author)
be enhanced by one to two orders of magnitude for the FNNP scenario due to the change of the sign of the interference term between the direct and indirect production processes. As a result, it would be hopeful to exclude or test the FNNP scenario through this Higgs rare decay at the HL-LHC. In this paper, we will consider the next-to-leading order (NLO) QCD correction for the decay width of \( H \to \Upsilon(ns) + \gamma \) with the \( H \gamma \gamma \) anomalous coupling under the non-relativistic QCD (NRQCD) framework, and demonstrate below that the FNNP parameter space originated from \( H \to \gamma \gamma \) could be excluded or tested by the rare decay \( H \to \Upsilon(ns) + \gamma \) at the HL-LHC.

II. THEORETICAL ANALYSIS

In this work we adapt the effective Lagrangian approach to parameterize the possible NP effects and consider the following effective couplings which are related to the Higgs exclusive rare decay,

\[
\mathcal{L} = \frac{\alpha_{em}}{4\pi v} (\kappa_{\gamma\gamma} H A_{\mu\nu} A^{\mu\nu} + \kappa_{Z\gamma} H Z_{\mu\nu} A^{\mu\nu}) - \frac{y_b}{\sqrt{2}} \kappa_b H \bar{b} b,
\]

where \( v = 246 \text{ GeV} \) is the vacuum expectation value and \( \alpha_{em} \) is the electromagnetic coupling. \( Z_{\mu\nu} \) and \( A_{\mu\nu} \) are the field strength tensor of the \( Z \) boson and photon, respectively. \( y_b \) is the bottom quark Yukawa coupling and \( \kappa_b \) is introduced to parametrize possible NP effect in the bottom quark Yukawa sector and \( \kappa_b = 1 \) in the SM. It has been demonstrated that the contribution from \( H Z \gamma \) anomalous coupling to the rare decay \( H \to \Upsilon(ns) + \gamma \) will be suppressed by \( \mathcal{O}(m_b^4/m_Z^4) \) [18], therefore, it is reasonable to ignore its contribution in this work. The \( H \gamma \gamma \) anomalous coupling can be well constrained by the \( H \to \gamma \gamma \) measurements at the LHC. The partial decay width of \( H \to \gamma \gamma \) after including the anomalous coupling \( \kappa_{\gamma\gamma} \) is given by,

\[
\Gamma(H \to \gamma\gamma) = \frac{m_H^3}{16\pi} \left( \frac{\alpha_{em}}{2\pi v} \right)^2 (\kappa_{\gamma\gamma}^{SM} + \kappa_{\gamma\gamma})^2,
\]

where \( \kappa_{\gamma\gamma}^{SM} \) is the contribution from the \( W \)-boson and top quark loops in the SM [23],

\[
\kappa_{\gamma\gamma}^{SM} = \frac{1}{2} \left[ 3Q_t^2 A_t(\tau_t) + A_W(\tau_W) \right].
\]

Here the functions \( A_t \) and \( A_W \) are the contribution from top quark and \( W \)-boson respectively, and their definitions can be found in Ref. [23] with \( \tau_t = m_H^2/4m_t^2 \); \( Q_t \) is the electric charge of top quark. The bottom quark loop contribution is suppressed by the Yukawa coupling \( y_b \) and is ignored. The contribution from \( W \)-boson will dominates over the one from the top quark loop owing to a large beta function coefficient of \( W \) loop, which could be understood from the Higgs low energy theorem in the limit of \( \tau_t \to 0 \) [24–26], i.e. \( A_W(\tau_W) = -8.324 \) and \( A_t(\tau_t) = 1.376 \).

The properties of the Higgs boson have been studied extensively in the diphoton final state by the ATLAS and CMS experiments [3, 27–31]. The most updated limit is from the ATLAS collaboration at the 13 TeV with 139 fb\(^{-1}\) and the signal strength is \( \mu = 1.04 \pm 0.1 \) [3]. Therefore, we can get the bound for the \( H\gamma\gamma \) anomalous coupling at 1\( \sigma \) level as following,

\[
0.94 \leq \frac{\Gamma(H \to \gamma\gamma)}{\Gamma_{SM}(H \to \gamma\gamma)} \leq 1.14, \quad (4)
\]

where the subscript “SM” indicates the prediction from SM, i.e. \( \kappa_{\gamma\gamma} = 0 \). It yields a bound on \( \kappa_{\gamma\gamma} \) as,

\[
-0.22 \leq \kappa_{\gamma\gamma} \leq 0.10, \quad 6.39 \leq \kappa_{\gamma\gamma} \leq 6.71. \quad (5)
\]

It clearly shows that there are two-fold solutions for \( \kappa_{\gamma\gamma} \) when we only consider the branching ratio of \( H \to \gamma\gamma \) measurements. Breaking the degenerate solutions of \( H\gamma\gamma \) anomalous coupling plays a crucial role to understand the nature of the Higgs boson.

Next, we consider the rare decay \( H \to \Upsilon(ns) + \gamma \) to probe the FNNP scenario at the HL-LHC. From the factorization of the NRQCD [32], the partial decay width could be written as,

\[
\Gamma(H \to \Upsilon(ns) + \gamma) = \hat{\Gamma}(H \to (b\bar{b}) + \gamma)(\mathcal{O}^{\Upsilon(ns)}(\beta S_1)), \quad (6)
\]

where \( \hat{\Gamma}(H \to (b\bar{b}) + \gamma) \) is the short distance coefficient and can be calculated from the matching between the perturbative QCD and NRQCD, while the long-distance matrix element \( \mathcal{O}^{\Upsilon(ns)}(\beta S_1) \) is a non-perturbative parameter which can be determined either from the lattice QCD calculation or the branching ratio measurements of \( \Upsilon(ns) \to \ell^+\ell^- \), with \( \ell^\pm = e^\pm, \mu^\pm, \tau^\pm \). Recently, the complete three-loop QCD corrections to the leptonic decay of \( \Upsilon(ns) \) has been finished in Ref. [33]. We also note that the relativistic correction effects will be suppressed by the relative velocity of the bottom quarks in the meson rest frame and its numerical effects is very small and can be ignored as well in this analysis [16, 22]. Below, we calculate the partial decay width of \( H \to \Upsilon(ns) + \gamma \) under the NRQCD factorization formalism at the leading order (LO) and NLO accuracy of strong coupling \( \alpha_s \).

A. LO partial decay width

As shown in Fig. 1, there are two different production mechanisms for this exclusive decay \( H(p_H) \to \Upsilon(ns)(2p_b) + \gamma(p_\gamma) \). We use the covariant projection operator to calculate the scattering amplitudes, which is defined as,

\[
\Pi = \frac{\Psi_{\Upsilon(ns)}(0)}{2\sqrt{m_T}} f_T^\mu(p_T) \left( p_T + m_T \right) \otimes \frac{1}{\sqrt{N_c}}, \quad (7)
\]

where \( f_T^\mu(p_T) \) is the polarization vector of the \( \Upsilon \) with the momentum \( p_T \) and \( \Psi_{\Upsilon(ns)}(0) \) is the Schrödinger wave
Illustrative Feynman diagrams at one-loop level can be
exclusive Higgs decay process
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significantly enhanced within the FNNP parameter space (e.g.
destructive, but the partial decay width can be signifi-
cantly enhanced within the FNNP parameter space).

We checked that our result agrees with that in Refs. [15,
18]. It clearly shows that the interference effect between
the direct and indirect production of
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at the LO is,

\[ M_{\text{dir}} = \frac{2 \sqrt{2} e_b \kappa_b \psi_{\Upsilon(n\bar{s})}(0)}{3 \sqrt{m_b^2 m_H^2}} \left[ m_H^2 e_Y^* \cdot e_Y^* - 4 p_b \cdot e_Y^* p_Y \cdot e_Y^* \right], \]

where \( e \) is the electron charge and \( e_Y \) is the polarization vector of the photon. The indirect-production amplitude
from Fig. 1(c) is given by,

\[ M_{\text{indir}} = \frac{\alpha_{\text{em}}}{4 \pi} \sqrt{2} e_b \psi_{\Upsilon(n\bar{s})}(0) \left[ \frac{\kappa_{\gamma\gamma}^\text{SM}}{\sqrt{m_b^2 m_H^2}} m_H^2 \right] \]

Combing the amplitudes from the direct and indirect pro-
duction processes, we obtain the partial decay width of
\( H \rightarrow \Upsilon(n\bar{s}) + \gamma \) at the LO,

\[ \Gamma_0 = \frac{e^2 \psi_{\Upsilon(n\bar{s})}(0)}{12 \pi m_H m_b} \left[ \frac{\alpha_{\text{em}} m_H^2}{4 \pi^2 m_b} \left( \kappa_{\gamma\gamma}^\text{SM} + \kappa_{\gamma\gamma} \right) + \sqrt{2} e_b \kappa_b \right]^2. \]

We checked that our result agrees with that in Refs. [15,
18]. It clearly shows that the interference effect between
the direct and indirect production of
H \rightarrow \Upsilon(n\bar{s}) + \gamma
in the SM (\( \kappa_{\gamma\gamma}^\text{SM} = -3.2445 \), \( \kappa_{\gamma\gamma} = 0 \) and \( \kappa_b = 1 \)) is
destructive, but the partial decay width can be signifi-
cantly enhanced within the FNNP parameter space (e.g.
K \gamma \gamma \sim -2 \kappa_{\gamma\gamma}^\text{SM}).

B. NLO QCD correction

Now we consider the NLO QCD correction to the ex-
clusive Higgs decay process
H \rightarrow \Upsilon(n\bar{s}) + \gamma. The illus-
trative Feynman diagrams at one-loop level can be
found in Fig. 2. To regularize the ultraviolet (UV) and
infrared (IR) divergences at the loop level, we adopt
the dimensional regularization scheme in our calculate-
tion and define the dimensionality of space-time \( d = 4 - 2 \epsilon \).

We should note that the NLO QCD correction contains
the Coulomb singularity from box diagrams of
H \rightarrow \Upsilon(n\bar{s}) + \gamma.

The LO is,

\[ \Gamma_{\text{NLO}} = \Gamma_0 \left( 1 + \frac{\alpha_s}{\pi} C_F \frac{\pi^2}{v_b} + \frac{\alpha_s}{\pi} F + \mathcal{O}(\alpha_s^2) \right) \]

Here the factor \( F \) is the finite part of the NLO QCD
coefficient. The Coulomb singularity from box diagrams
of direct production and triangle diagram of indirect
production has been factorized out and can be absorbed
into the definition of the long-distance matrix element
\( \langle O^\Upsilon(n\bar{s})(3S_1) | \rangle \).
where $\Gamma_{\text{dir}}, \Gamma_{\text{indir}}^0$, and $\Gamma_{\text{int}}^0$ are corresponding to the LO partial decay width from the direct, indirect production and the interference between them, respectively; see the details in Eq. (11). Note that the Coulomb singularity from box and indirect triangle diagrams has been matched into the long-distance matrix element. Combining all the parts, we obtain the partial decay width of $H \rightarrow \Upsilon(ns) + \gamma$ at the NLO level,

$$\Gamma_{\text{NLO}} = \Gamma_0 - \frac{\alpha_s(\mu)}{\pi} C_F \left[ \frac{7}{2} + \frac{2\pi^2}{3} - 2\ln^2 2 \right. \\
- 4\ln 2 \left( 1 + \ln \frac{m_b^2}{m_H^2} \right) \Gamma_{\text{dir}}^0 \\
- \frac{\alpha_s(\mu)}{2\pi} C_F \left[ \frac{15}{2} + \frac{\pi^2}{3} - \ln^2 2 \right. \\
- 2\ln 2 \left( 1 + \ln \frac{m_b^2}{m_H^2} \right) \Gamma_{\text{int}}^0 - 4\frac{\alpha_s(\mu)}{\pi} C_F \Gamma_{\text{dir}}^0 \Gamma_{\text{indir}}^0.
$$

(18)

The NLO QCD correction to the direct production process agrees well with the result in Refs. [35, 36] in the limit of $m_b \to 0$. We have numerically checked that the $m_b$ correction to the partial decay width $\Gamma_{\text{NLO}}$ is very small and will be ignored in the following numerical analysis.

III. THE $H \gamma\gamma$ ANOMALOUS COUPLING

Below, we utilize the rare decay of $H \rightarrow \Upsilon(ns) + \gamma$ to constrain the $H \gamma\gamma$ anomalous coupling. The parameters of the SM are choose as follows [37],

$$m_W = 80.385 \text{ GeV}, \quad m_Z = 91.1876 \text{ GeV},$$

$$m_H = 125 \text{ GeV}, \quad \Gamma_H = 4.07 \text{ MeV},$$

$$G_\mu = 1.1663785 \times 10^{-5} \text{ GeV}^{-2}.$$

(19)

The weak mixing angle is fixed under the $G_\mu$ scheme [38], which is $c_W = m_W / m_Z$ and the electromagnetic coupling $\alpha_{\text{em}} = \sqrt{2} G_F m_W^2 G_\mu / \pi$. We define the running bottom quark Yukawa coupling $y_b(m_H)$ of the direct production by the running mass at next-to-next-to-leading order in the $\overline{\text{MS}}$ scheme, i.e. $m_b(m_H) = 2.79 \text{ GeV}$ [37]. The long-distance matrix elements ($\mathcal{O}(\Upsilon(ns)^3 S_1)$) can be obtained from the branching ratio measurements of $\Upsilon(ns) \rightarrow \ell^+\ell^-$ and can be found in Table 1 of Ref. [12].

Table I shows the predicted branching ratios of $H \rightarrow \Upsilon(ns) + \gamma$ at the LO and NLO with renormalization scale $\mu = m_H$ for the SM and FNNP case ($\kappa_{\gamma\gamma} = -2\kappa_{\gamma\gamma}^{\text{SM}}$ and $\kappa_b = 1$). Note that the accuracy of the long-distance matrix elements should be consistent with the short-distance coefficient when we calculate the branching ratios. It clearly shows that the branching ratios of $H \rightarrow \Upsilon(ns) + \gamma$ under the FNNP case could be enhanced about one to two orders of magnitude compared to the SM predictions.

The rare decay of $H \rightarrow \Upsilon(ns) + \gamma$ has been measured by the ATLAS collaboration at the 13 TeV LHC with an integrated luminosity of 13.6 fb$^{-1}$ [39]. A 95% confidence level (CL) upper limits on the branching ratios of Higgs decays to $\Upsilon(ns)\gamma$ have been obtained, i.e. (4.9, 5.9, 5.7) $\times 10^{-4}$. With a much larger integrated luminosity is collected at the HL-LHC, we could expect that the upper limits of these branching ratios could be improved by two to three orders of magnitude compared to Ref. [39]. In fact, an estimated projection for the Higgs decays to $J/\Psi\gamma$ has been made by the ATLAS collaboration at the HL-LHC and it shows that the expected 95% CL upper limit for the branching ratio of $H \rightarrow J/\Psi + \gamma$ with $J/\Psi \rightarrow \mu^+\mu^-$ is around $\mathcal{O}(10^{-5})$ [40]. From the analysis of the ATLAS experiment [39], we note that both the signal and background events will be reduced about a factor 2 from $H \rightarrow \Upsilon(ns) + \gamma$ to $H \rightarrow J/\Psi + \gamma$ due to the different quarkonium mass window and cut efficiencies, while the signal efficiency will be enhanced about 20%. To estimate the event numbers of the signal $\Upsilon(ns)\gamma$ and background at the HL-LHC, we assume this relation is still hold and rescale the event numbers from the analysis of Ref. [40], i.e. the event number for $\Upsilon(ns)$ can be obtained by

$$n_\Upsilon \sim 0.6 n_{J/\Psi} \frac{\text{BR}(H \rightarrow \Upsilon(ns) + \gamma) \text{BR}(\Upsilon(ns) \rightarrow \ell^+\ell^-)}{\text{BR}(H \rightarrow J/\Psi + \gamma) \text{BR}(J/\Psi \rightarrow \ell^+\ell^-)}.$$

(20)

where $n_{J/\Psi}$ is the event number for the measurement $H \rightarrow J/\Psi + \gamma$ at the HL-LHC [40]. Similar to Ref. [12], we will also combine the measurements from $\Upsilon(ns) \rightarrow e^+e^-, \mu^+\mu^-$ and $\tau^+\tau^-$ and assume the same detection efficiency for all three decay channels.

To estimate the sensitivity for testing the hypothesis with parameters ($\kappa_{\gamma\gamma}, \kappa_b$) against the hypothesis with SM, we define the likelihood function as [41],

$$L(\kappa_{\gamma\gamma}, \kappa_b) = \prod_i \frac{(s_i(\kappa_{\gamma\gamma}, \kappa_b) + b_i)^{n_i} e^{-s_i(\kappa_{\gamma\gamma}, \kappa_b)}}{n_i!},$$

(21)

where $b_i$ and $n_i$ are the event numbers for the background and observed events in the $i$-th process ($H \rightarrow \Upsilon(ns)+\gamma$ at the ATLAS and CMS collaborations, with $i = 1, 2, 3$ and $\Upsilon(ns) \rightarrow \ell^+\ell^-$), respectively. The parameter $s_i(\kappa_{\gamma\gamma}, \kappa_b)$ is the event number of the signal for the parameters ($\kappa_{\gamma\gamma}, \kappa_b$) of the $i$-th data sample, which could be obtained from the rescaling of the $J/\Psi\gamma$ simulation [40]; see Eq. (20). Here the label $i$ denotes the experiments from $n = 1, 2, 3, \ell = e, \mu, \tau$, and ATLAS, CMS experi-

| BR($H \rightarrow \Upsilon(ns) + \gamma$) | $\Upsilon(1s)$ | $\Upsilon(2s)$ | $\Upsilon(3s)$ |
|--------------------------------------|-------------|-------------|-------------|
| LO ($\kappa_{\gamma\gamma} = 0$)     | 0.51        | 0.24        | 0.18        |
| NLO ($\kappa_{\gamma\gamma} = 0$)   | 3.03        | 1.44        | 1.05        |
| LO ($\kappa_{\gamma\gamma} = -2\kappa_{\gamma\gamma}^{\text{SM}}$) | 90.3        | 42.9        | 31.1        |
| NLO ($\kappa_{\gamma\gamma} = -2\kappa_{\gamma\gamma}^{\text{SM}}$) | 83.6        | 39.7        | 28.8        |
ments. The observed events is a combination of the SM signal and background, i.e. \( n_s = s_i(\kappa_{\gamma \gamma} = 0, \kappa_b = 1) + b_i \). The test statistic \( q \) is defined as the ratio of the likelihood function,

\[
q^2 = -2 \ln \frac{L(\kappa_{\gamma \gamma} \neq 0, \kappa_b \neq 1)}{L(\kappa_{\gamma \gamma} = 0, \kappa_b = 1)}.
\]

Based on the even numbers of signals and backgrounds, we obtain

\[
q^2 = 2 \left[ \sum_i n_i \ln \frac{n_i}{n_i'} + n_i' - n_i \right].
\]

Here \( n_i' = s_i(\kappa_{\gamma \gamma}, \kappa_b) + b_i \) and the 1-\( \sigma \) (i.e. 68% CL) upper limit for the parameter space is setting \( q = 1 \).

Figure 3(a) shows the expected 68% CL constraints on the parameters \((\kappa_{\gamma \gamma}, \kappa_b)\) obtained from measuring the rare decay \( H \rightarrow \Upsilon(n_s) + \gamma \) at the HL-LHC (orange band). In the same figure, we also show the constrains from the current branching ratio measurements of \( H \rightarrow \gamma \gamma \) and \( H \rightarrow bb \) (black band) and \( H \rightarrow bb \) [42] (blue band). Though the branching ratios of \( H \rightarrow \Upsilon(n_s) + \gamma \) can be significantly enhanced under the FNNP scenario, it is still very difficult to probe this signal at the HL-LHC based on the event numbers of Ref. [40]. However, we note that the background analysis from the simulation is evaluated by scaling the observed background at the 8 TeV LHC data and the systematic uncertainty of the background shape is assumed by a constant [40]. As a result, the background estimation from the current analysis should have a large uncertainty and could be overestimated. With expected advances in the experimental measurement and analysis, it is quite possible that both the signal \((\epsilon_s)\) and background \((\epsilon_b)\) efficiencies could be improved at the time of HL-LHC runs. We introduce the parameters \( \lambda_{b,s} \) to denote the possible improvement of the total efficiencies for the signal and background, respectively, i.e. \( \epsilon_{s,b} = \lambda_{s,b} \epsilon_{b,s} \), where \( \epsilon_{s,b} \) denote the efficiencies from the current simulation. We show the required improvement in the detection efficiencies in order to exclude or test the FNHP scenario in Fig. 3(b) with \( \kappa_b = 1 \) and \( \kappa_{\gamma \gamma} = 6.39 \). It shows that such goal could be achieved if the background can be suppressed further about two orders of magnitude and the signal efficiency is enhanced about 5 times compared to the ATLAS preliminary simulation.

IV. CONCLUSIONS

In this paper, we propose to probe the \( H\gamma\gamma \) anomalous coupling via Higgs exclusive decay \( H \rightarrow \Upsilon(n_s) + \gamma \) at the HL-LHC. Owing to the destructive interference between the direct and indirect production mechanisms of the quarkonium, we notice that the branching ratios of \( H \rightarrow \Upsilon(n_s) + \gamma \) could be significantly enhanced under the FNHP scenario. As a result, it is hopeful to break the degeneracy of the \( H\gamma\gamma \) anomalous coupling by this rare decay at the HL-LHC, as implied by the branching ratio of \( H \rightarrow \gamma \gamma \) measurements at the LHC. Based on the NRQCD factorization formalism, we calculate the branching ratios of \( H \rightarrow \Upsilon(n_s) + \gamma \) to the NLO accuracy of \( \alpha_s \). To explore the potential of probing the \( H\gamma\gamma \) anomalous coupling at HL-LHC, we rescale the background and signal event numbers from the ATLAS simulation for the process \( H \rightarrow J/\Psi + \gamma \). It shows that the measurement of Higgs decays to \( \Upsilon(n_s)\gamma \) could break the degeneracy of the \( H\gamma\gamma \) anomalous coupling, if the background can be further suppressed about two orders of magnitude and the signal efficiency can be improved by a factor of 5, as compared to the current simulation of the ATLAS experiment.

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