EOS OF NONEQUILIBRIUM PARTON PLASMA IN AN EXPANDING FLUX TUBE

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Abstract. We investigate a new version of the flux-tube model based on joint action of two different processes responsible for vacuum creation of the parton plasma: the dynamical Schwinger mechanism and the geometrical mechanism linked to the dynamical Casimir effect. These two mechanisms act at different time scales. The Casimir mechanism provides the abundant generation of partons from the sea at the very early stage after the fusion of colliding nuclei and release of color degrees of freedom. Then the created plasma initiates the back-reaction mechanism via the standard Schwinger scenario. This process apparently leads to fast thermalization of secondary partons. The model allows for the self-consistent description of pre-equilibrium evolution of the plasma, fully determined by the total energy and the total number of valence quarks of colliding nuclei. The Equation of State of the created parton plasma is discussed.

1. Introduction

The present work is an attempt to combine kinetics of vacuum particle creation in strong fields, which is similar to the flux-tube model (FTM) widely employed for the description of ultra-relativistic heavy-ion collisions, with the non-stationary Casimir effect, applied to the pattern of longitudinally expanding quark-gluon plasma inside the flux tube. The main assumptions of the model are as follows. The system of two identical colliding ions is considered in their center-of-mass frame. Before the collision both ions are represented by two Lorentz-contracted disks with the thickness (for simplicity, central collision is assumed)

\[ d_0 = 2r_0 A^{1/3} \frac{m_N}{\sqrt{S_{NN}}}, \tag{1} \]

where \( r_0 = 1.2 \text{ fm} \) is the nucleon radius and \( \sqrt{S_{NN}} \) is the energy per nucleon, \( A \) is the mass number of a nucleus. During the collision the nuclear matter is heated up, e.g., by the shock waves, the significant amount of energy is released within the overlapped volume, and the total deconfinement takes place. The color degrees of freedom brought by the colliding nuclei come into play. This leads to formation of the original quark-gluon plasma (QGP), containing \( 6A \) valent quarks in the volume \( V_0 = \pi A^{2/3} r_0^2 d_0 \) with density

\[ \rho_0 = \frac{\sqrt{S_{NN}}}{\pi r_0^3 m_N}. \tag{2} \]
The subsequent stage is characterized by strong longitudinal expansion of the QGP. It is assumed that the color cylinder has plane external boundaries deprived of diffuse layers. The initial velocities of these boundaries are defined by the initial average velocities of colliding nuclear matter,

\[ v_0 = \sqrt{1 - m_N^2 / S_{NN}}. \]  

The relations (1)-(3) represent the initial conditions of the system. The strong quasi-classical gluon fields, emerged between the flying away valent quarks, generate the secondary quarks and gluons from the sea by means of Schwinger mechanism (SM) of vacuum particle creation [1]. In the FTM the main problem is the adequate description of non-stationary SM, where the kinetic approach is appropriate. The expanding hot and dense partonic matter reaches the (quasi)equilibrium state, and, finally, experiences the dynamical phase transition to hadronic matter. This picture corresponds to a standard scenario of the FTM [2].

The understanding and mathematical realization of the kinetic model of the QGP pre-equilibrium evolution, which takes into account parton vacuum creation, has long history [3]. The problem is not fully solved yet. One of the most developed kinetic models is based on the quantum electrodynamics (QED) approximation [4]. Here the kinetic equation (KE), describing the vacuum creation of electron-positron pairs under action of the time dependent quasi-classical electromagnetic fields, was derived and investigated. This KE is a direct consequence of the QED equation of motion in non-dissipative approximation. Therefore, it is applicable for the description of a QED system, e.g., electron-positron plasma in a strong laser fields both the X-ray [5] and optical range [6]. Some foundation for applicability of the QED equation to study formation and pre-equilibrium evolution of the QGP is provided by the Abel projection approximation [7]. The obtained KE was employed also in the FTM [8], and results expected from the qualitative considerations were regained. Below we will limited by the frame of this approximation and will call the colorless fermions (spin 1/2) and bosons (spin 0) as ”quarks” and ”gluons”, respectively.

Meantime, certain progress was achieved also in the quantum chromodynamics (QCD) based kinetic theory [9], where the KE for quark and gluon subsystems in a strong quasi-classical gluonic field was obtained. Unfortunately, the corresponding system of partial differential equations is very complicated and cannot be solved numerically at present. Therefore, further investigation of the kinetics of vacuum particle production in the framework of QED approach is an important task for the development of the FTM. It brings to our attention a whole sequence of problems important for the kinetic theory, such as scattering process of particles at the presence of strong fields, its influence on non-equilibrium dynamics of the system, the role of finite volume of the system, etc.

The last problem is directly related to the FTM, where the size of expanding QGP does not exceed few nucleonic radii. That leads to necessity of taking into account the vacuum polarization stipulated by finite size of the system or, in other words, the Casimir effect [10]. Since the flux tube is expanding, the dynamical Casimir effect first considered by Schwinger [11] (see also [12, 13]) is relevant. The necessary conditions for this effect in the ideal case are infinite surface conductivity of the plasma and the impenetrable walls (for quasi-classical field) of the flux tube [10]. These conditions are certainly fulfilled for the QGP (see, e.g., [14]). It is worth noting that the same conditions are used independently for justification of the bag model [15]. In a similar way, the Casimir forces were suggested as one of the possible mechanisms responsible for the confinement of quarks [13]. Thus, the expanding flux tube can be interpreted as a dynamical Casimir bag.

The aim of the present work is to develop a kinetic theory of vacuum creation of parton plasma, which should include both action of a quasi-classical time-dependent chromoelectric field and a non-stationary Casimir mechanism (CM) between two flying away parallel plates.
Thus, the picture of expanding in the longitudinal direction cylinder is quite appropriate (we neglect here the transverse expansion). The corresponding closed system of equations is obtained in Sec.2 within the framework of non-perturbative dynamics. This system consists of KE for the description of vacuum pair creation under the joint action of SM and CM, and Maxwell equation for the description of the back-reaction problem. In general case, this system of equations is rather complicated and, therefore, some additional simplifications are needed. For example, in our previous study [17] the expansion velocity of color tube was set constant and back reaction was not taken into account, although the initial conditions for partonic distribution functions were formulated in accord with general Eqs. (1)-(3). The equation of state (EOS) of the model, which is necessary for self-consistent definition of the expansion velocity of the flux tube, is also derived here. Results of the numerical analysis are presented in Sec.3. It is shown that the CM provides the abundant generation of partons from the sea at the very early stage after the fusion of colliding nuclei and release of color degrees of freedom. This bunch of plasma initiates the back-reaction mechanism on the SM basis, which apparently leads to fast thermalization of secondary partons. The pre-equilibrium evolution of the parton plasma is described self-consistently (without invoking initiating short pulse of the strong quasi-classical field) and is determined solely by the initial conditions and parameters of colliding system. Finally, conclusions are drawn in Sec.4.

2. Basic equations
2.1. Kinetic equations. The KE for the description of vacuum particle creation was obtained first in [4] within the QED on the basis of time-dependent Bogoliubov transformation [10] for the case of unbounded space. This KE was investigated in [7]. Below we study the modified KE obtained by direct transition from the continuous momentum space to the nonstationary discrete one with the instantaneous cell size \( L(t) \). Such transition corresponds to implementation of the dynamical CM (for details see [16, 17]).

We use the metrics \( g^{\mu\nu} = diag(1, -1, -1, -1) \) and the natural units \( \hbar = c = k = 1 \), where \( k \) is Boltzmann constant. Denote the initial distance between the plates as \( d_0 \). It is identical to the disk thickness (1) at the initial time \( t_0 \) of overlapping of two ions. The plasma conductivity (particularly, the surface conductivity) is assumed to be very large: in the QGP case, it is provided by the smallness of effective coupling constant [14]. That guarantees the applicability of the standard theory of Casimir effect [10, 13]. As shown below, the CM generates the significant amount of parton sea at the first short period of the system evolution. The corresponding strong currents initiate a strong inner longitudinal time-dependent chromoelectric field, which is characteristic for the flux-tube back-reaction dynamics. Thus, it is necessary to take into account this quasi-classical field. The whole system is assumed to be homogeneous. In the traditional FTM, the action of the back-reaction mechanism is provided by the power pulse of an external field. Note that this model artefact is absent in the presented model. A new element of the model is time dependence of the boundary conditions, \( x_3 \in [-L(t)/2, L(t)/2] \), with \( L(0) = d_0 \). Chromoelectric field is directed along the same axis (in the Hamilton gauge \( A_0 = 0 \)).

\[
\mathbf{E}(t) = (0, 0, E(t)), \quad E(t) = -\dot{A}^3(t). \tag{4}
\]

The relevant KE has the general form

\[
\dot{f}_\pm(p, t) = S_\pm(p, t), \tag{5}
\]

where \( f_\pm(p, t) \) are the distribution functions and \( S_\pm(p, t) \) are the non-Markovian source terms for bosons(+) and fermions(−), respectively,

\[
S_\pm(p, t) = 2w_\pm(p, t) \int_0^t dt'w_\pm(p, t')[1 \pm (f_\pm(p, t') + \dot{f}_\pm(p, t'))] \cos \vartheta(p, t, t'), \tag{6}
\]
with \( \tilde{f}_{\pm}(p, t') \) being the distribution functions of antiparticles. The coefficients \( w_{\pm} \) determining the momentum spectrum of the quasi-particles are

\[
\begin{align*}
    w_{+}(p, t) &= \frac{\omega(p, t)}{2\omega(p, t)}, \\
    w_{-}(p, t) &= \frac{\varepsilon_{\pm} \dot{P}}{2\omega_{\pm}^{2}(p, t)}, \\
    \varepsilon_{\pm}^2 &= m^2 + p_{\perp}^2, \\
    \omega^2(p, t) &= \varepsilon_{\pm}^2 + P^2, 
\end{align*}
\]

(7)

\( p_{\perp} \) is the transverse momentum, and \( P = p^3 - eA^3(t) \). Finally,

\[
\vartheta(p, t, t') = 2 \int_{\nu}^{t} d\tau \omega(p, \tau). 
\]

(9)

The back-reaction problem and initial conditions for KE (5) were discussed briefly in [16].

The KE (5) can be reduced to system of ordinary differential equations convenient for further numerical analysis

\[
\dot{f}_{\pm} = \frac{1}{2} w_{\pm} u_{\pm}, \quad \dot{u}_{\pm} = w_{\pm} \left[ 1 \pm (f_{\pm} + \tilde{f}_{\pm}) \right] - 2\omega v_{\pm}, \quad \dot{v}_{\pm} = 2\omega u_{\pm}. 
\]

(11)

The KE for antiparticles has similar form (5) but different initial conditions (Sect.1).

The back-reaction problem and initial conditions for KE (5) were discussed briefly in [16]. Here a selfconsistent description of the recession velocity, namely the function \( \dot{L}(t) \), is considered. Generally, the corresponding EOS should be derived for this purpose. The simplest formulation of the problem implies zero initial conditions \( f_{\pm}(t_0) = 0 \) for (5), which is written in the collisionless approximation (to take collisions in the account see, e.g., [18]).

2.2. Expansion rate and equation of state.

Let us introduce the total energy density of parton plasma

\[
\varepsilon(t) = \varepsilon_{+}(t) + \varepsilon_{-}(t), \\
\varepsilon_{\pm}(t) = g_{\pm} \int [dp] \omega(p, t) [f_{\pm}(p, t) + \tilde{f}_{\pm}(p, t)],
\]

where \( g_{\pm} \) are the deneracy factors for gluons (\( g_{+} \)) and quarks (\( g_{-} \)), and

\[
\int [dp] = \frac{1}{(2\pi)^2 L(t)} \sum_{n} \int d^2 p_{\perp}. 
\]

(14)

The transverse and longitudinal parts of the pressure are

\[
\begin{align*}
    P_{\perp}(t) &= g_{\pm} \int_{\perp} \frac{[dp] t}{2\omega(p, t)} \left\{ p_{\perp}^2 [f_{\pm}(p, t) + \tilde{f}_{\pm}(p, t)] + u_{\pm}(p, t) \beta_{\perp}^{2} \right\}, \\
    P_{\parallel}(t) &= g_{\pm} \int_{\parallel} \frac{[dp] t}{\omega(p, t)} \left\{ p_{\parallel}^2 [f_{\pm}(p, t) + \tilde{f}_{\pm}(p, t)] + u_{\pm}(p, t) \beta_{\parallel}^{2} \right\}, 
\end{align*}
\]

(15)
where the functions $u_{\pm}(p, t)$ are defined by Eqs. (11) and $\beta_{\pm}^1 = (p_\perp^2 - 2\omega^2/3)$, $\beta_{\pm}^3 = p_\parallel^3/\varepsilon_\perp$.

To define the expansion rate it is convenient to employ the energy conservation law. Neglecting the transversal expansion of the flux tube, one gets

$$L(t) = \frac{A^{1/3}}{\pi r_0^2 \varepsilon(t)}. \quad (16)$$

The flux-tube expansion is accompanied by cooling of the plasma. Introducing the effective temperature one can define the duration of the pre-equilibrium evolution of the system, when the deconfinement temperature $T_c$ associated with the critical length $L_c$ of the flux tube is attained. For $L \geq L_c$ (i.e. $T \leq T_c$) the plasma hadronization takes place, however, this process lies out of scope of the present study. The energy density of the non-equilibrium state is

$$\varepsilon_{ef}(t) = \varepsilon_{ef}^+(t) + \varepsilon_{ef}^-(t) = \varepsilon(t), \quad (17)$$

$$\varepsilon_{ef}^{\pm}(t) = g_{\pm} \int [dp]\omega(p, t)[f_{\pm}^0(p, t) + \tilde{f}_{\pm}^0(p, t)], \quad (18)$$

where

$$f_{\pm}^0(p, t) = \left\{ \exp \left( \beta_{\pm}^+(t)\omega(p, t) - \mu_{\pm}^+(t) \right) \right\}^{-1}, \quad (19)$$

and $\beta_{\pm}^{\pm}$ is the inverse temperature. The chemical potentials of particles and antiparticles are related simply as $\tilde{\mu}_\pm = -\mu_\pm$.

3. Creation of particles in an expanding Casimir bag

Now the formalism formulated above can be applied to study the vacuum creation of bosons and fermions in an expanding color flux tube. For the sake of simplicity, only light $u$ and $d$ quarks are considered. Their masses and the maximum value of the chromoelectric field within the tube are chosen as follows: $m_{u,d} = 230$ MeV and $eE = 0.9$ GeV/fm, respectively. The time dependence of the field is modeled as a half-period of harmonic oscillation. In line with
the Bjorken model, longitudinal expansion of the tube is assumed to proceed linearly with the velocity $0.9c$ and initial width $L_0 = 0.1$ fm.

It is worth mentioning that the two key parameters which govern the particle production in the two processes, namely, $E(t)$ and $L(t)$, are varying at different time scales. This circumstance leads to qualitatively and quantitatively different momentum distribution of secondary particles. Note, that according to previous studies [7] the SM predicts a characteristic two-peaked spectrum for bosons and a single-peaked distribution for fermions. In contrast, the momentum distribution of fermions produced due to the CM, presented in Fig. (1), has a two-hump structure. In this sense, the geometrical mechanism does not distinguish between the particle statistics (bosons or fermions). As can be seen in Fig. (2), the CM dominates over the SM at the very early stage of the expansion (the Casimir effect acts on the limited short distances), when the mean gluon field is yet small and the rate of the SM is small, while already at times $t \geq 0.3$ fm/$c$ the production of new particles is governed by the Schwinger-like mechanism.

Spectra of particles produced via these two different processes are also different: the CM favors the production of ultrarelativistic particles with high transverse momentum $p_t/m \gg 1$ [16]. Further expansion of the QGP and creation of new particles via the flux-tube mechanism leads to quasi-equilibrium momentum distribution and significant softening of the particle momentum distribution. Here the mean momentum of created particles is of order of the particle mass. To check these theoretical estimates experimentally one has to perform, for instance, a combined analysis of longitudinal momentum distributions of fermions and bosons in the $p_t$-intervals.

Fig. (3) shows the behavior of energy density and pressure of parton plasma. These pictures confirm the existence of two different regions corresponds to CM and SM. The appearance of the negative pressure is stipulated by the forced character of the flux tube expansion with a constant velocity and it will be eliminated obviously by the transition to the selfconsistent description based on the Eq. (16). The strong oscillations of the observable variables are typical for the nonequilibrium stage of the vacuum tunneling process, when the balance between the creation and annihilation processes is not reached yet. The strong oscillations of the observable variables are typical for the nonequilibrium stage of the vacuum tunneling process, when the balance between the creation and annihilation processes is not reached yet. Results of the numerical calculations are stable with respect to change of initial conditions.

**Figure 3.** The time dependence of energy density (left panel) and of transversal and longitudinal pressure of created fermions.
4. Conclusions
In summary, the particle production in a flux tube created between two heavy ions, passing each other at ultrarelativistic energies, is considered. The necessity to take into account the dynamic finite size effects was shown in [19]. In addition to standard Schwinger-like mechanism of particle creation in strong chromoelectric fields the model incorporates the so-called geometrical mechanism based on the dynamical Casimir effect. At the very beginning of the reaction, when the gluonic fields are weak, the production of particles is determined merely by the CM, while later on the Schwinger-like mechanism dominates. Longitudinal and transverse momentum distribution of fermions and bosons, produced either via the CM or the FTM, are found to be different: the $p_t$-spectra of CM-particles are much harder, and longitudinal momentum distributions of both fermions and bosons have similar two-peaked structure. In contrast, the $p_t$-spectra of FTM-particles are softer, and longitudinal momentum spectra demonstrate two peaks for bosons and only one peak for fermions. Although the numerical results are very sensitive to the choice of initial conditions and the expansion rate of the flux tube, the aforementioned qualitative differences represent the main trend of particle production via the two competing mechanisms, and can be studied experimentally. The proposed model allows for the thermodynamical description of the QGP expansion and cooling up to the deconfinement temperature. We plan to consider this aspect in the forthcoming publications.

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