Critical currents at the Bragg glass to vortex glass transition

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We present simulations of the transport properties of superconductors at the transition from the Bragg glass (BG) to the vortex glass (VG) phase. We study the frustrated anisotropic 3D XY model with point disorder, which has been shown to have a first order transition as a function of the intensity of disorder. We add an external current to the model and we obtain current-voltage curves as a function of disorder at a low temperature. We find that the in-plane critical current has a steep increase at the BG-VG transition, while the c-axis critical current has a discontinuous jump down, this later result in agreement with the first-order character of the transition.

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The study of the vortex phase diagram of anisotropic superconductors with point disorder has been of interest since the discovery of high $T_c$ superconductivity. It is now clear that at low temperatures and low magnetic fields there is a “Bragg glass” (BG) with an elastically distorted vortex lattice. This vortex lattice undergoes a first order melting transition to a vortex liquid (VL) when increasing the temperature $T$. At higher magnetic fields and low $T$ there is a disordered vortex state, the “vortex glass” (VG). A disorder driven transition from the BG to the VG has been proposed to occur when increasing the magnetic field $H$ or disorder. Experimental observations with increasing $H$ at low $T$, that have been attributed to the BG-VG transition, are: the destruction of the Bragg peaks in neutron scattering, a dip in the differential resistance, the onset of the “second magnetization peak”, or a jump in the Josephson plasma resonance. Numerical simulations in particle-like models with random pinning have found a transition from an ordered lattice to a disordered lattice when increasing particle density (i.e., the magnetic field). Recently, Monte Carlo simulations in the frustrated 3D XY model with disorder have established that the BG-VG transition is of first-order type.

Several of the experimental evidences of the BG-VG transition are directly or indirectly related to transport properties. For example, the most common determination of the BG-VG line is through the onset of a “second magnetization peak”, which is attributed to an steep increase of the in-plane critical currents. From the point of view of transport, the BG has zero linear resistivity ($\rho_{\text{lin}} = 0$). The VG phase was originally proposed to have zero resistivity. However, studies of the XY gauge glass model with finite screening ($\kappa < \infty$) have found that VG is unstable in $d = 3$, and the disordered phase is a frozen vortex liquid that has a very small finite resistivity. Only for the $\kappa \to \infty$ disordered XY and gauge glass models the VG phase is stable in $d = 3$ at low $T$, in which case $\rho_{\text{lin}} = 0$. Therefore, it is important to understand how the the non-linear current-voltage (IV) curves and their critical currents change across a disorder driven first-order transition. We present here numerical calculations of both the in-plane and the c-axis IV curves across the disordered driven BG-VG transition in the $\kappa = \infty$ frustrated XY model used in which is the only model where the first-order nature of the transition has been clearly determined so far. We find that the c-axis critical current has a discontinuous jump at the BG-VG transition, which could be tested experimentally.

The hamiltonian of the frustrated 3D XY model is:

$$H = \sum_r \sum_{\mu = \hat{x}, \hat{y}} J_{r\mu} V(\theta_\mu(r)) - \frac{\Gamma}{4} \cos(\theta_\hat{z}(r))$$

where $\theta_\mu(r) = \theta(r + \mu) - \theta(r) - A_\mu(r)$ and $A_\mu(r) = \frac{2\pi}{\Phi_0} \int_{a}^{(r + \mu)A} \mathbf{A} \cdot d\mathbf{l}$ and $\mathbf{r} = (n_x, n_y, n_z)$ defines a lattice site in a cubic grid. To minimize the pinning of the numerical grid, we have chosen $V(\theta_\mu(r)) = -R_0 - R_1 \cos(\theta_\mu(r)) - R_2 \cos(2\theta_\mu(r))$ and adjusted the coefficients $R_0$, $R_1$ and $R_2$ as in . We model uncorrelated random point pinning in the xy plane with $J_{r\mu} = J(1 + p_{r\mu})$, $\mu = \hat{x}, \hat{y}$, where $p_{r\mu}$ are independent random variables with $\langle p_{r\mu} \rangle = 0$ and $\langle p_{r\mu} \rangle^2 = 1$. In the presence of an external magnetic field $H$ along the $\hat{z}$ direction, we have $A_x(r) = A_x(r + y) + A_y(r + x) - A_y(r) = 2\pi f$, and we take $f = H\alpha^2 / \Phi_0 = \frac{1}{4}$. We model the current between two grid points with the RSJ model: $I_{\mu}(r, t) = S_\mu(r, t) + N_\mu(r, t) + \eta_\mu(r, t)$ with the superconducting current $S_\mu(r, t) = \frac{2\pi}{\Phi_0} \frac{\partial H}{\partial \theta_\mu(r)}$, the normal current $N_\mu(r, t) = \frac{h}{2eR_{\mu\mu}} \frac{\partial \eta_\mu(r)}{\partial t}$ with $R_{\mu\mu} = R_{\mu\mu}$ for $\mu = \hat{x}, \hat{y}$ and $R_{\mu\mu} = \Gamma R_{\mu\mu}$ for $\mu = \hat{z}$, and the thermal noise fluctuations have correlations $\langle \eta_\mu(r, t) \eta_{\mu'}(r', t') \rangle = (2k_B T / R_{\mu\mu}) \delta_{\mu, \mu'} \delta_{r, r'} \delta(t - t')$. We take periodic boundary conditions in all directions with a fluctuating twist $\alpha(t)$ such that $A_\mu(r, t) = A_\mu(0) - \alpha(t)$. This allows to obtain the voltage in each direction as $V_\mu = \frac{h}{2e} \frac{\partial \alpha(t)}{\partial t}$.
We consider current conservation in each node: \( \sum_{\mu} I_\mu(r) - I_\mu(r - \mu) = 0 \), and we fix the total current in each direction, consistently with the periodic boundary condition, by [17]:

\[
I^\text{ext}_\mu = \frac{\hbar}{2eR_\mu} \frac{d\alpha_\mu}{dt} + \frac{1}{L_xL_yL_z} \sum_r [S_\mu(r) + \eta_\mu(r,t)]
\]

We integrate with a second order Runge-Kutta stochastic algorithm with \( \Delta t = 0.1 \hbar^2/(4e^2R_{ab}J) \) and we average over an interval of \( 10^5 - 10^6 \Delta t \). We consider system sizes \( 48 \times 48 \times L_z \) with \( L_z = 12 - 32 \) and anisotropy \( \Gamma = 40 \). Most of the results shown are for \( L_z = 32 \) and for a single disorder realization. We have considered other realizations of disorder for some values of \( T \) and \( p \) and have found similar behavior.

Let us now discuss the in-plane transport at the transition. In Fig. 1(a) we plot the intensity of the structure factor, \( S(G) \), at one of the first reciprocal lattice vectors \( G \), showing the melting of the vortex lattice at \( T \approx 0.28 \). The resistivity is calculated by driving the system with a very small current and obtaining voltage. We show in Fig. 1(a) the \( c \)-axis resistivity \( \rho_c \) vs. \( T \) which drops sharply at \( T_c \); we find similar behavior in \( \rho_{ab} \) (not shown).

At a low temperature \( T = 0.1J/k_B \), we vary disorder and study the behavior of the lattice order parameter, \( S(G) \), which is shown in Fig. 1(b) for two system sizes. We find that \( S(G) \) drops at \( p_c \approx 0.11 \), indicating a transition from the BG to the VG. Typical structure factors \( S(k) \) for \( p < p_c \) and \( p > p_c \) are shown in the insets. Repeating this analysis for other \( T \), we have found that \( p_c(T) \) is approximately independent of \( T \) for low temperatures. Similar phase boundary \( p_c(T) \) for the BG phase were reported in [13, 14]. For very low temperatures, both [13] and [14] agree that there is a first-order transition from the BG phase to the VG phase in this model. (At higher \( T \) there is disagreement on the existence of a “vortex slush” phase).

From now on, we focus on the disorder driven transition at low \( T \), where [13, 14] agree on the phase diagram. We fix a low temperature \( T = 0.1J/k_B \) and we vary the disorder strength \( p \). In Fig. 2(a) we show the calculated in-plane IV curves for different \( p \). They are obtained by applying a uniform current along the \( x \) direction, \( I^\text{ext}_{\mu} = I_{ab}\delta_{\mu,x} \), and calculating the average voltage along \( x \), \( V_{ab} = \langle d\alpha_x/dt \rangle \). Before doing further analysis it is important to discuss the effect of a small driving current in the BG-VG transition. We find that currents much smaller than the critical currents have a negligible effect on \( p_c \). In Fig. 3 we show \( S(G) \) vs. \( p \) when the driving current in a given direction is of the order of the corresponding critical current. In the case of a \( y \)-direction current \( I_y = 0.010 \sim I^{ab}_c \), there is a very small effect with a critical \( p_c \) slightly smaller than the \( I = 0 \) case. In the case of an in-plane current \( I_{ab} = 0.002 \sim I^{ab}_c \), we find that there is a first drop of \( S(G) \) to a finite value at a \( p < p_c \). This corresponds to the situation when the applied current \( I_{ab} \) equals the critical current for that \( p \). Then a second drop of \( S(G) \) to the very small values of the VG phase occurs near the critical \( p_c \) but also at a slightly smaller value. In both cases, one can conclude that finite currents of the order of the critical current have a small effect on the value of \( p_c \), lowering it in about \( \Delta p_c \approx -0.005 \).

Let us now discuss the in-plane transport at the transition. In Fig. 2(a) we show with open symbols the IV curves for \( p < p_c \) and with filled symbols the IV curves for \( p > p_c \) (where \( p_c = 0.11 \) is the zero current critical disorder). We observe that in both phases the IV curves are strongly nonlinear and that they tend to zero resistivity.

![Figure 1](image1.png)

**FIG. 1**: (a) Intensity of the Bragg peak \( S(G) \) and \( c \)-axis resistivity \( \rho_c \) vs. \( T \) for \( p = 0.01 \) and size \( 48 \times 48 \times 32 \). \( T \) is normalized by \( J/k_B \). (b) \( S(G) \) vs. \( p \) for \( T = 0.1J/k_B \). \( \Delta: 48 \times 48 \times 12; \square: 48 \times 48 \times 32 \). Left inset: intensity plot of \( S(k) \) for \( p = 0.07 \). Right inset \( S(k) \) for \( p = 0.14 \).

We first study the model at equilibrium (in the absence of external currents). At low disorder, \( p = 0.01 \), we find a transition upon increasing \( T \) from a vortex lattice to a vortex liquid. We have calculated the structure factor \( S(k) = (1/Nf^2)(\sum_{r_{z},z} n_z(r_{z},z)e^{ikr_{z}}r_{z}^2) \), where \( n_z \) is the vorticity in the \( xy \) planes and \( N = L_xL_yL_z \). In
for low currents ($V/I \to 0$). In order to observe if there is a change when crossing the BG-VG line, we fix a value of the current $I_{ab}$, slightly above the critical currents, and we calculate the voltage $V_{ab}$ as a function of disorder strength $p$. This is shown in Fig. 4(a). We see that at $p_c$ there is a jump down in $V_{ab}$. This jump is observed for different values of $I_{ab}$. The location of the jump does not depend strongly on $I_{ab}$. The experiment of [7] finds a clear decrease of the differential resistance, measured for a current above the critical current, when increasing magnetic field at low temperatures in YBaCuO samples. This is in agreement with our result of Fig. 4(a) for the behavior of the in-plane voltage at the BG-VG transition. One can interpret this jump down in voltage as a consequence of a jump up in the critical current. It is difficult to define a critical current, since at $T > 0$ there is always a non-zero voltage for any current. Experimentally, the “critical current”, $I_c$, is obtained from the IV curves as $V(I_c) = V_{th}$, with the threshold voltage $V_{th}$ small enough. If $I_c$ is almost independent of $V_{th}$, then it corresponds to a crossover current separating two regimes in the IV curve. In our case, $I_c$ separates the ohmic flux flow regime from the low current nonlinear regime which has $\rho_{lin}(I \to 0) = 0$ for the two phases. (In the case of the VG with finite screening, $\kappa < \infty$, $I_c$ would separate two ohmic regimes). To obtain the $ab$ critical current, $I_{ab}^c$, we choose different values of $V_{th}$ slightly above our noise value of voltage. One of the $V_{th}$ used is shown in Fig. 2(a) with a dashed line. The obtained critical currents are plotted in Fig. 5(a) as a function of $p$ (we see that different $V_{th}$ give similar values of $I_{ab}^c$, within the error bars). We observe in the plot that there is a steep increase of
$I_{c}^{ab}$ at $p_c$. In the magnetization experiments in YBaCuO

$V_c$ for different values of $p$. For very low $I_z$, we find $V_c \approx 0$ for all $p$, consistent with both phases being superconducting. For intermediate values of $I_z$ we find that $V_c \approx 0$ for $p < p_c$ with a sharp jump to finite voltage $V_c$ at $p_c$. For values of $I_z$ above the critical currents, there is a finite voltage $V_c$ in both phases, with a clear discontinuous jump up at $p_c$. We obtain the critical currents $I_z^c$ with a voltage criterion $V_{th}$, which is shown in Fig.2(b) with a dashed line. It is obvious in this case that the values obtained for $I_z^c$ are insensitive to the choice of $V_{th}$. The c-axis critical currents are plotted as a function of disorder strength $p$ in Fig. 5(b). We observe in the plot that $I_z^c$ has a clear discontinuous jump down at $p_c$. This result is consistent with the Josephson plasma resonance measurements of [10], which are sensitive to the c-axis Josephson coupling, where a sharp decrease of the plasma resonance frequency was observed at the transition. Recently, the c-axis Josephson critical current has been measured experimentally in BiSrCaCuO samples varying magnetic field at the melting transition line, the BG-VG line [19]. A jump down of the critical current was found, although at a field slightly lower than the equilibrium case. It will be interesting if a similar experiment could be carried out across the BG-VG line, since our results imply that c-axis transport would give a better evidence (compared to the ab-plane transport) of the first-order nature of this transition.

The 3D XY model is very adequate to the description of the c-axis transport, since the Josephson coupling between planes is treated exactly. The c-axis current-voltage characteristics, $V_z$ vs. $I_z$, are shown in Fig.2(b) for different values of $p$ at $T = 0.1J/k_B$. The IV curves for $p < p_c$ are plotted with open symbols and the IV curves for $p > p_c$ are plotted with filled symbols. We observe that in all cases the IV curves show a well-defined critical current where the voltage drops steeply. The IV curves can be clearly separated in two different sets of curves, one set for $p < p_c$ with high critical currents and another set for $p > p_c$ with low critical currents. This can also be observed if for a fixed current $I_z$ we vary $p$ and we calculate $V_z$, as we show in Fig.4(b). For very low $I_z$ we find $V_z \approx 0$ for all $p$, consistent with both phases being superconducting. For intermediate values of $I_z$ we find that $V_z \approx 0$ for $p < p_c$ with a sharp jump to finite voltage $V_z$ at $p_c$. For values of $I_z$ above the critical currents, there is a finite voltage $V_z$ in both phases, with a clear discontinuous jump up at $p_c$. We obtain the critical currents $I_z^c$ with a voltage criterion $V_{th}$, which is shown in Fig.2(b) with a dashed line. It is obvious in this case that the values obtained for $I_z^c$ are insensitive to the choice of $V_{th}$. The c-axis critical currents are plotted as a function of disorder strength $p$ in Fig. 5(b). We observe in the plot that $I_z^c$ has a clear discontinuous jump down at $p_c$. This result is consistent with the Josephson plasma resonance measurements of [10], which are sensitive to the c-axis Josephson coupling, where a sharp decrease of the plasma resonance frequency was observed at the transition. Recently, the c-axis Josephson critical current has been measured experimentally in BiSrCaCuO samples varying magnetic field at the melting transition line, the BG-VG line [19]. A jump down of the critical current was found, although at a field slightly lower than the equilibrium case. It will be interesting if a similar experiment could be carried out across the BG-VG line, since our results imply that c-axis transport would give a better evidence (compared to the ab-plane transport) of the first-order nature of this transition.

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