Proposal of Empirical Homogenization of Masonry Wall Made of AAC Masonry Units

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Abstract. The article presents an own validation procedure of numerical FEM model of masonry wall made of autoclaved aerated concrete (AAC) masonry units. Empirical calibration of masonry’s mechanical parameters was carried out in the standard tests of the compressed and diagonally compressed wall. In the numerical calculations, an elastic-plastic material model with degradation was used. This model in the compression phase uses the Menetrey-William (M-W-3) plasticity surface and in a tensioning state, the Rankine criterion is used. In the first stage, axially compressed walls were analysed. Fracture energy was changed, leaving the remaining parameters unchanged. The relation between the load – vertical and horizontal strains was compared. In the second stage, the calibration involved changing the tensile strength of the wall in diagonally compression test. The relationship between shear stresses and form strain angle was compared. Satisfactory convergence in the range of maximum stresses and significant differences in terms of form strains were obtained. In the third stage, the modulus of elasticity was calibrated, the results of laboratory tests and numerical calculations were compared. The best convergence in terms of maximum tangential stress and scratch morphology was obtained in the model in which the tensile strength was reduced by 80%. The reliability of the proposed validation method was additionally verified on wall models of larger dimensions. The article is a continuation of the author's considerations regarding the author's validation method of the FEM masonry model.

1. Introduction
The correct selection of the material model of the wall and the method of homogenization and division into finite elements affect the accuracy of calculations for each element of the masonry structure (pillars, wall strips) including morphology of scratches, cracking and destructive (maximum) forces, deformation angles and stiffness of the wall. Obtained by validation (calibration) compliance of calculations of wall micro-models with test results entitles to analysis consisting in estimation the interaction between particular walls and slabs, and as a consequence to the correct prediction of internal forces, even on real scale buildings. In masonry structures characterized by anisotropic behaviour, resulting from the fact of joining at least two different materials – masonry units and mortar, there is always a complex state of stress, complicating the choice of material model and validation procedure [1-3]. The wall can be discreetly modelled [4, 5]. The mortar contact with masonry units or mortar layers can be replaced with contact elements with corresponding calculation parameters for both head and bed joints. Various strategies for such modelling are shown in Figure 1 [3, 6]. This approach although undoubtedly the most accurate is associated with a considerable degree of complexity in model construction and time-consuming calculations. Considering the attempt to create a FEM model [6-12] that reliably reflects the real work of the masonry structure, appropriate
simplifications can be made by introducing an equivalent homogeneous model with modified input parameters. However, it should be taken into account that the elimination of elements representing connections of masonry units generating significant local deformations may be associated with the occurring of difficult to eliminate errors, especially in the case of shearing and bending.

Figure 1. Various strategies of masonry modelling according to [3, 4] and authors proposal

The aim of the work is to develop own validation procedure for the FEM numerical model of a wall made of autoclaved aerated concrete (AAC) based on the concept of a macro-model, in which mechanical features were determined by experimental research - empirical homogenization. In order to reconstruct the behavior of the numerical wall model as accurately as possible, an approach was chosen to consist in the selection of parameters in axial and diagonally compression tests performed as part of the work [3, 13].

2. Validation of the homogeneous model

2.1. Research models

The subject of analyses were masonry walls made of autoclaved aerated concrete with dimensions 1,18 x 1,21 x 0,18 m (b x h x t) subjected to axial and diagonal compression [3, 13, 14] in accordance with PN-EN 1052-1: 2000 [15, 16]. Measuring bases are placed on the research samples to register deformations in a parallel and perpendicular direction to the load. In axially compressed wall models, the measuring bases were parallel and perpendicular to the plane of the bed joints. In diagonally compressed models (according to [17]) the position of the measuring bases was located at an angle of about 45 degrees in relation to the plane of the bed joints. In [3, 13], standard wall models were used to validate discrete FEM models using contact elements in the plane of bed and head joints. Using the author’s procedure, satisfactory convergence of test and calculation results was obtained, and the relative error was ± 2%. Comparison of obtained test results and calculations of models made as part of the work [3, 13] is shown in Figure 2.
2.2. Modelling strategy adopted

In the numerical calculations [3, 13], an elastic-plastic model with degradation was used. It is often used in the analysis of concrete structures. The model in the compression state uses the Menétréy-Willam boundary surface (M-W-3) [2, 18, 19, 20], while in the tensioning state the Rankine criterion is used. In addition, in the areas representing the contact of masonry units, contact elements with the Coulomb-Mohr criterion in the field of compressive stress and an elliptical cap in the range of tensile stress normal to the contact plane were used. Identical to the cited paper [3, 13], a plane model with finite elements with two degrees of freedom at each node was used. For the needs of own model, contact elements were omitted, treating the wall as a homogeneous, isotropic material. The initial input calculation parameters of the homogeneous model were the parameters summarized in Table 1, specified in [3, 13, 21]. Calibration of the calculation parameters of the model was carried out in three stages using the order shown in the block diagram - Figure 3.

Table 1. Initial calculation parameters for the elasto-plastic model with degradation

| No. | parameter                                      | value            |
|-----|-----------------------------------------------|------------------|
| 1   | initial modulus of elasticity \( E_0 [\text{N/mm}^2] \) | 2204             |
| 2   | poisson’s ratio \( \nu \)                     | 0.200            |
| 3   | tensile strength \( f_t [\text{N/mm}^2] \)     | 0.430            |
| 4   | fracture energy \( G_f [\text{MN/m}] \)       | \( 5,350 \times 10^{-5} \) |
| 5   | compression strength \( f_c [\text{N/mm}^2] \) | -2.970           |
| 6   | plastic deformation under compression \( \varepsilon_{cp} [-] \) | \(-4,180 \times 10^{-4}\) |
| 7   | ultimate displacement under compression [m]   | \(-5,000 \times 10^{-4}\) |
| 8   | reduction of compressive strength [-]         | 0.800            |
| 9   | shear stiffness reduction factor [-]           | 20               |
| 10  | average aggregate size [mm]                   | 20               |
In the first stage axially compressed walls were analysed. Only the fracture energy \( G_f \) of the masonry elements were changed, leaving the other parameters unchanged. To assess the correctness of the estimation, the load-vertical and horizontal deformation relationships were compared in each case. The mean square error value of the obtained relationships was calculated. Appropriate crack energy is the value at which the local minimum mean square error (MSE) was obtained. In the second stage, the calibration involved changing the tensile strength \( f_t \) of the wall in diagonally compression test. The procedure was identical except that a local minimum error for tensile strength was sought. The relationship between shear stress and deformation angle was compared. In the third stage, FEM calculations and test results of the compression wall, which was calibrated with mechanical parameters were compared. In case of unsatisfactory results, another correction was made.

3. Model validation results

3.1. Stage I. Calculations of axially compressed walls

Eight FEM models marked with symbols from 01 to 08 were used to calculate compressed walls. The original value of the fracture energy \( G_f = 5.21 \times 10^{-5} \) [MN/m] obtained in [3] was changed to \( G_f' \). In the models numbered from 02 to 04, the fracture energy was reduced by half in relation to the 01 model. In models 05 and 06 the value was increased twice and four times in relation to the model 01. In the model 07, the fracture energy was equal to the original \( G_f \) value, and in the model 08 \( G_f \) parameter was doubled – Table 2.

| No. | fracture energy \( G_f' \), MN/m | \( G_f'/G_f \) | MSE x10^8 for \( \epsilon_y \) |
|-----|----------------|---------|------------------------|
| 01  | 1.070 x 10^{-5} | 0.21    | 14.09                  |
| 02  | 5.350 x 10^{-6} | 0.10    | 12.38                  |
| 03  | 2.675 x 10^{-6} | 0.05    | 7.87                   |
| 04  | 1,388 x 10^{-4} | 0.03    | 5.22                   |
| 05  | 2,140 x 10^{-3} | 0.41    | 16.32                  |
| 06  | 4,280 x 10^{-3} | 0.82    | 12.13                  |
| 07  | 5,210 x 10^{-3} | 1.00    | 12.08                  |
| 08  | 1,042 x 10^{-4} | 2.00    | 11.94                  |

Based on the results of calculations, the following charts were made: normal stress - vertical (\( \epsilon_y \)) and horizontal deformation (\( \epsilon_x \)). The charts also indicate the average results of compression walls tests included in [3, 13] – Figure 4a. On this basis, diagrams for mean square error (MSE) – fracture energy \( G_f'/G_f \) were prepared – Figure 4b. Numerical calculations also allowed to compare the scratch mechanisms of compressed walls. Morphology of scratches obtained at the highest stresses and actual damages of the models are shown in Figure 5. Along with the reduction of the fracture energy value, the mean square error of vertical deformations also decreased. In the case of horizontal deformations,
this tendency was not so pronounced, but the lowest error value was obtained in the 06 model. The reduction in fracture energy also affected the way the walls were scratched - the smaller the parameter value, the scratches were closer to the vertical edge of the model. Higher values of fracture energy in models from 05 to 08 determined the diagonal course of scratches at maximum stresses. The optimal value of the parameter was considered to be the fracture energy adopted in the model designated as 04, which characterized the acceptable compliance of the scratch mechanism with the test results and the lowest mean square error value for vertical deformations.

Figure 4. Calculations results in the I stage: a) Stress – vertical and horizontal strain relationships for the analysed models 01÷08, b) mean squared error – Gf'/Gf relationships

Research sample [[6]]

Figure 5. Cracking patterns of FEM models at the maximum compressive stresses 01, 03, 04

3.2. Stage II. Calculations of diagonally compressed models
Eight FEM models marked with symbols from 09 to 16 were also used to calculate diagonally compressed walls. In all models, the fracture energy equal to Gf' = 1,388 x 10^-6 MN / m obtained in the model 04 was used for calculations, and the value of tensile strength f' changed in relation to the reference value f' = 0,61 N/mm² – Table 3. A comparison of the actual morphology of the wall crack with the results of numerical calculations is shown in Figure 6.

| No. | tensile strength f', N/mm² | f'/f₀ |
|-----|---------------------------|------|
| 09  | 1,270                     | 2,08 |
| 10  | 0,610                     | 1,00 |
| 11  | 0,305                     | 0,50 |
| 12  | 0,183                     | 0,30 |
| 13  | **0,128**                 | **0,21** |
| 14  | 0,061                     | 0,10 |
| 15  | 0,031                     | 0,05 |
| 16  | 0,013                     | 0,02 |

Table 3. Tensile strength of diagonally compressed walls
Diagrams of the relationship between the shear stress and the deformation angle for the analysed models were also made – Figure 7. In the models 09 and 10, in which the tensile strength was the highest, destruction in the form of chipping the corner of the model was observed, and the values of the maximum stress were overestimated. Along with decreasing the value of the changed parameter, the wall scratched vertically between opposite corners. The highest compliance of the maximum stresses compared to the test results was obtained in the model 13 (less than 1%). However, all models were characterized by greater stiffness. Model 13 best reflecting reality both in terms of the cracks mechanism and the level of maximum stress was characterized by almost twice lower values of deformation angles.

![Research sample](image)

**Figure 6.** Cracking patterns of FEM models at the time of maximum shear stresses 13, 15, 16

![Graph](image)

**Figure 7.** Calculations results in the II stage in the relationships of mean shear stress - deformation angle of 09-16 models

3.3. Stage III. Additional parameter calibration results

Due to the lack of satisfactory convergence of the solution in stage II, in stage III the modulus of elasticity of the wall and tensile strength were changed – **Table 4.**

| No. | $f'/f_t$ | $E'/E$ |
|-----|----------|--------|
| 17  | 0,21     | 0,75   |
| 18  | 0,21     | 0,60   |
| 19  | 0,21     | 0,80   |
| 20  | 0,21     | 0,55   |
| 21  | 0,23     | 0,50   |

Calibration was performed on diagonally compressed walls. The results obtained in the form of diagrams shear stress - deformation angle are shown in Figure 8a. The change in the modulus of elasticity caused a clear and rapid reduction of the model's rigidity, with an almost linear relationship with shear stresses, without changing the value of the maximum shear stress with a clear plastic
reduction – Figure 8a. The best convergence of results was obtained in the model 21 in which the tensile strength was corrected. Due to the change in the modulus of elasticity, another iteration was necessary consisting in comparing the obtained calculation results of the compressed numerical model with the corrected parameters - Figure 8b. In the compression model 22, the calibrated fracture energy from the model 04 and the changed tensile strength (model 21) were adopted. In the model 23, the modulus of elasticity was additionally changed (all modified parameters of the 21 model). As expected, the model 23 had lower stiffness, higher vertical strain values were obtained at the same stresses. The calibration did not change the maximum stress values and transverse strain.

3.4. Validation of calculations on larger wall models
The work [3, 21] presents the test results of walls made of AAC of larger dimensions with different proportions of height to length. The models were subjected to horizontal shear at different levels of initial compressive stress 0,1; 0,75 and 1,0 N/mm². As part of the verification of the proposed empirical homogenization method of the masonry wall - equivalent numerical models of the walls were made with unmodified input parameters and parameters calibrated according to the proposed method – Table 5. The charts: average shear stress – deformation angle (Figure 9) and values of maximum stresses and cracking stresses with corresponding deformation angles were compared with test results obtained in [3, 13].

### Table 5. List of analysed walls with a level of initial compression stress and calculation parameters

| No. | description         | geometry, m h x l x t | compression stress, N/mm² | G’/Gf | f’/f | E’/E |
|-----|---------------------|-----------------------|---------------------------|-------|-----|------|
| 24  | HOS-AAC-1/3-010/1_MES_H | 2,43 × 1,48 x 0,18   | 0,10                     | 1,00  | 1,00| 1,00 |
| 25  | HOS-AAC-1/3-010/1_MES_HK1 | 2,43 × 1,48 x 0,18   | 0,10                     | 0,03  | 0,21| 0,50 |
| 26  | HOS-AAC-1/3-075/1_MES_H | 2,43 × 1,48 x 0,18   | 0,75                     | 1,00  | 1,00| 1,00 |
| 27  | HOS-AAC-1/3-075/1_MES_HK1 | 2,43 × 1,48 x 0,18   | 0,75                     | 0,03  | 0,21| 0,50 |
| 28  | HOS-AAC-1/3-10/1_MES_H | 2,43 × 1,48 x 0,18   | 1,00                     | 1,00  | 1,00| 1,00 |
| 29  | HOS-AAC-1/3-10/1_MES_HK1 | 2,43 × 1,48 x 0,18   | 1,00                     | 0,03  | 0,21| 0,50 |
| 30  | HOS-AAC-0/5-010/1_MES_H | 2,43 × 2,36 x 0,18   | 0,10                     | 1,00  | 1,00| 1,00 |
| 31  | HOS-AAC-0/5-010/1_MES_HK1 | 2,43 × 2,36 x 0,18   | 0,10                     | 0,03  | 0,21| 0,50 |
| 32  | HOS-AAC-0/5-075/1_MES_H | 2,43 × 2,36 x 0,18   | 0,75                     | 1,00  | 1,00| 1,00 |
| 33  | HOS-AAC-0/5-075/1_MES_HK1 | 2,43 × 2,36 x 0,18   | 0,75                     | 0,03  | 0,21| 0,50 |
| 34  | HOS-AAC-0/5-10/1_MES_H | 2,43 × 2,36 x 0,18   | 1,00                     | 1,00  | 1,00| 1,00 |
| 35  | HOS-AAC-0/5-10/1_MES_HK1 | 2,43 × 2,36 x 0,18   | 1,00                     | 0,03  | 0,21| 0,50 |
| 36  | HOS-AAC-2/3-010/1_MES_H | 2,43 × 2,95 x 0,18   | 0,10                     | 1,00  | 1,00| 1,00 |
| 37  | HOS-AAC-2/3-010/1_MES_HK1 | 2,43 × 2,95 x 0,18   | 0,10                     | 0,03  | 0,21| 0,50 |
| 38  | HOS-AAC-2/3-075/1_MES_H | 2,43 × 2,95 x 0,18   | 0,75                     | 1,00  | 1,00| 1,00 |
| 39  | HOS-AAC-2/3-075/1_MES_HK1 | 2,43 × 2,95 x 0,18   | 0,75                     | 0,03  | 0,21| 0,50 |
Compliance of numerical calculations performed on models 29 – 47 in relation to real wall tests is insufficient. As homogeneous models without contact elements were analysed - the angles of form deformation were not compared. The best compliance of the cracking shear stress values was obtained in model 35 in relation to the research sample - 4%. However, the results obtained are characterized by a large spread of individual results. A similar situation occurs in the case of the value of maximum stresses. The highest compliance (1%) was obtained in the model 45 in relation to test results. It can be assumed that the more contact elements in a particular model - the greater the inconsistencies in relation to the true values obtained during the tests. This is indicated by the bed joint surface ratio (the ratio of the surface of the bed joint of each model in relation to the surface of bed joint in reference sample - diagonally compressed wall) compared with the average relations of cracking and maximum stresses (Table 6). Cracks in numerical homogeneous models with basic parameters occurred locally in opposite corners of the walls. The calibrated models had a different scratch morphology - the scratches were diagonal - in compliance with the cracking pattern of the research models - an example of such behavior is shown in Figure 10.

**Figure 9.** Average shear stress – deformation angle relationships obtained in FEM calculations and test results a) 24 – 29 models b) 42 – 47 models

| No. | description | \( \tau_{\text{cr,cal}} \) N/mm² | \( \tau_{\text{u,cal}} \) N/mm² | \( \frac{\tau_{\text{cr,cal}}}{\tau_{\text{cr}}} \) | \( \frac{\tau_{\text{u,cal}}}{\tau_{\text{u}}} \) | mean | mean | bed joint surface ratio |
|-----|-------------|-------------------------------|-------------------------------|---------------------|---------------------|------|------|-------------------------|
| 24  | HOS-AAC-1/3-010/1_MES_H | 0,236 | 0,267 | 1,82 | 1,86 | 1,00 | 1,31 | 2,81 |
| 25  | HOS-AAC-1/3-010/1_MES_HK1 | 0,018 | 0,021 | 0,14 | 1,40 | 1,00 | 1,31 | 2,81 |
| 26  | HOS-AAC-1/3-075/1_MES_H | 0,295 | 0,377 | 1,45 | 1,45 | 1,00 | 1,31 | 2,81 |
| 27  | HOS-AAC-1/3-075/1_MES_HK1 | 0,135 | 0,245 | 0,67 | 0,94 | 1,00 | 1,31 | 2,81 |
| 28  | HOS-AAC-1/3-010/1_MES_H | 0,272 | 0,370 | 1,16 | 1,28 | 1,00 | 1,31 | 2,81 |
| 29  | HOS-AAC-1/3-010/1_MES_HK1 | 0,173 | 0,261 | 0,74 | 0,90 | 1,00 | 1,31 | 2,81 |
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A three-step concept of calibrating the parameters of a numerical model of a wall made of AAC with thin bed joints and unfilled head joints was proposed. In the first stage, the fracture energy $G_f$ was changed which affected the morphology of the scratches and the course of the stress-strain relationship. The characteristics of the normal stress – vertical and horizontal strain were similar to the test results. The most similar course of the calculated vertical strain and deformations obtained in the tests was obtained in the model 04 (the smallest mean square error - MSE) with a significant reduction in the cracking energy compared to the original value. In the second stage, the tensile strength was calibrated by performing calculations on diagonally compressed models - changing the tensile strength $f_t$. The best convergence in terms of maximum tangential stresses and crack pattern was obtained in the model 13 in which the tensile strength was reduced by approximately 80%. However, the values of the deformation angle were more than twice smaller than empirically determined. Therefore, in the third stage of the calculations, the modulus of elasticity (the third parameter) was calibrated. In this case, the reduction of the modulus of elasticity resulted in matching the shear stress – deformation angle relationship satisfactorily. Unfortunately, this approach generated an increase in error in the calculation of compressed walls, especially in terms of vertical strain. Validation of calculations carried out on wall models of larger dimensions indicated the need for more accurate calibration of reference models (especially the diagonally compressed model). It can be assumed that the larger the contact surface (the surface of bed joints in a particular model) in relation to the model on which the parameters were calibrated, the greater the inaccuracies of the numerical calculations. By applying the proposed method of homogeneous model calibration in the case of larger-sized walls - a reliable reflection of the crack pattern was obtained in relation to the real-scale tests. However, approximations of maximum stresses and cracking stresses obtained in numerical calculations are insufficient. The highest agreement of results in relation to the tests was 4% and 1% respectively for cracking stresses.

| Model | $G_f$ | $f_t$ | $\gamma$ |
|-------|-------|-------|----------|
| HOS-AAC-2/3-010/1_MES_H | 0.109 | 0.236 | 0.52 |
| HOS-AAC-1/1-010/1_MES_HK1 | 0.032 | 0.203 | 0.15 |
| HOS-AAC-2/3-075/1_MES_H | 0.375 | 0.502 | 1.92 |
| HOS-AAC-2/3-075/1_MES_HK1 | 0.137 | 0.293 | 0.70 |
| HOS-AAC-0/5-10/1_MES_H | 0.345 | 0.494 | 1.98 |
| HOS-AAC-0/5-10/1_MES_HK1 | 0.181 | 0.294 | 1.04 |
| HOS-AAC-2/3-010/1_MES_H | 0.107 | 0.264 | 0.55 |
| HOS-AAC-2/3-010/1_MES_HK1 | 0.024 | 0.216 | 0.13 |
| HOS-AAC-2/3-075/1_MES_H | 0.373 | 0.548 | 1.85 |
| HOS-AAC-2/3-075/1_MES_HK1 | 0.067 | 0.336 | 0.33 |
| HOS-AAC-2/3-10/1_MES_H | 0.359 | 0.558 | 1.32 |
| HOS-AAC-2/3-10/1_MES_HK1 | 0.085 | 0.272 | 0.31 |

4. Conclusions

Figure 10. Comparison of the cracking pattern at the time of failure a) obtained in research for HOS-AAC-2/3-010/1 [21] b) HOS-AAC-2/3-010/1_MES_H c) HOS-AAC-2/3-010/1_MES_HK1

The characteristics of the normal stress – vertical and horizontal strain were similar to the test results. The most similar course of the calculated vertical strain and deformations obtained in the tests was obtained in the model 04 (the smallest mean square error - MSE) with a significant reduction in the cracking energy compared to the original value. In the second stage, the tensile strength was calibrated by performing calculations on diagonally compressed models - changing the tensile strength $f_t$. The best convergence in terms of maximum tangential stresses and crack pattern was obtained in the model 13 in which the tensile strength was reduced by approximately 80%. However, the values of the deformation angle were more than twice smaller than empirically determined. Therefore, in the third stage of the calculations, the modulus of elasticity (the third parameter) was calibrated. In this case, the reduction of the modulus of elasticity resulted in matching the shear stress – deformation angle relationship satisfactorily. Unfortunately, this approach generated an increase in error in the calculation of compressed walls, especially in terms of vertical strain. Validation of calculations carried out on wall models of larger dimensions indicated the need for more accurate calibration of reference models (especially the diagonally compressed model). It can be assumed that the larger the contact surface (the surface of bed joints in a particular model) in relation to the model on which the parameters were calibrated, the greater the inaccuracies of the numerical calculations. By applying the proposed method of homogeneous model calibration in the case of larger-sized walls - a reliable reflection of the crack pattern was obtained in relation to the real-scale tests. However, approximations of maximum stresses and cracking stresses obtained in numerical calculations are insufficient. The highest agreement of results in relation to the tests was 4% and 1% respectively for cracking stresses.
and maximum stresses (unfortunately, these are single cases). Further direction of work will be
detailing the parameters calibrating the reference model (diagonally compressed wall).

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