Numerical simulation of helical flow in a cylindrical channel

A Vasiliev, A Sukhanovskii and R Stepanov
Institute of Continuous Media Mechanics UB RAS, 1, Acad. Korolev St., Perm, 614013, Russia
E-mail: vasiliev.a@icmm.ru

Abstract. Numerical simulation of the helical flow in a cylindrical channel with diverter was carried out using open-source software OpenFOAM Extend 4.0. The velocity, vorticity and helicity density distributions were analyzed. It was shown that azimuthal contribution of helicity is negative near the wall and positive in the center. In opposite axial helicity contribution is negative in the center and positive near the wall. Analysis of helicity of non-axisymmetric part of the flow showed that it has substantial values near the diverter but than rapidly decreases with \( y \) (axial coordinate) and further downstream it can be neglected. Dependencies of integrated values of azimuthal \( H\phi \) and axial \( H_y \) contributions of helicity density on \( y \) show a remarkable quantitative similarity. It was found that integral values of \( H\phi \) and \( H_y \) are negative for all \( y \). Magnitudes of \( H\phi \) and \( H_y \) decrease after the diverter up to \( y \approx 70 \) mm and after that monotonically increase. The flow behind the diverter is characterized by substantial amount of helicity and can be used as a helicity generator.

1. Introduction
The volume integral of hydrodynamic helicity density \( H \) is an inviscid invariant of the Euler equation and it is related to the linkage of the vortex lines of the flow [1]. Helicity density \( H \) is defined as a scalar product of the local velocity vector \( \mathbf{U} \) and the local vorticity vector \( \mathbf{W} \)

\[
H = \mathbf{U} \cdot \mathbf{W}.
\]

Helicity density is a pseudoscalar quantity which is a signature of broken mirror symmetry of a flow. It is assumed that helicity can lead to the generation of large-scale magnetic fields (MHD – dynamo [2]) and atmospheric vortices (hydrodynamic alpha effect [3, 4]). The inhomogeneity of turbulent helicity may have an effect on turbulent momentum transport of screw pipe flow [5] and contributes to the sustainment of large-scale vorticity field in a three-dimensional mean flow [6].

The measurement of helicity is extremely difficult as the simultaneous measurements of both the velocity and vorticity vectors are required. Modern optical techniques such as dual or tomographic PIV (Particle Image Velocimetry) make possible reconstruction of helicity density distribution in a plane or even in a volume. Spatial distribution of the helicity density in screw jets was experimentally investigated in [7, 8] where helical flows was generated by screw nozzle. In [7] dual-plane dye laser PIV was used for measurements of all components of vorticity. From the analysis of the results, it was found that regions with high helicity were correlated with regions of high turbulent kinetic energy dissipation.
Tomographic PIV, which is currently under intensive development, makes possible to measure the instantaneous 3D velocity fields in the selected measuring volume of the flow. The helical modes have been studied in low-and high-swirl turbulent jets by using the high-speed tomographic PIV technique [8].

One of the main problems of laboratory studies of helicity is to produce the flow with substantial value of helicity. This problem is far from trivial [9]. One of the promising configuration for generation of such flow is an axial flow in the channel with a diverter [10, 11]. Qualitative observations showed that the flow in a torus with a diverter is very complex and consist of helical motions of different scales. There was no direct measurements of helicity or even estimation of its value. The main goal of the present study is to give a clear answer on the possibility of using such a system as a helicity generator. Special attention is paid to the azimuthal inhomogeneity and anisotropy of flow.

2. Numerical simulation

Numerical simulation of the forced air screw flow in a cylindrical channel was carried out using open-source software OpenFOAM Extend 4.0. For injection of screw flow eight blade diverter was used. Similar diverters were used in studies of screw flows in torus [10, 11]. Inclination angle of the blades was 30°. Figure 1(a) presents the computational domain, where a diverter placed at distance of 20 mm downstream from the inlet boundary. Computational domain is a cylindrical channel of diameter $d = 60$ mm and length $L = 400$ mm. The origin of a Cartesian coordinate system $(x, y, z)$ is located at the centre of the diverter. The axis of a cylinder $y$ coincides with the direction of the flow.

The discretization of three-dimensional Navier-Stokes equations for an incompressible flow was carried out by the method of control volumes. The PIMPLE (merged PISO-SIMPLE) algorithm was used for the pressure-velocity coupling. We employed implicit Euler method second order accuracy temporal discretization. For the discretization of the divergence term total variation diminishing (TVD) scheme of second order accuracy was applied. The second order accuracy central difference scheme was applied for discretization of diffusion term. The iterative solvers PCG and PBiCG – the preconditioned conjugate gradient and preconditioned biconjugate gradient methods were used for solving the obtained system of linear algebraic equations. The following boundary conditions are used in calculations: at the inlet, velocity assumed to be uniform $U = (0, 0.1, 0)$ m/s and normal gradient of pressure is zero; at the outlet, pressure assumed to be uniform $p = 0$ and normal gradient of velocity is zero. For the diverter and cylinder surface no-slip condition was used. Numerical simulation was done using unstructured mesh. Computation mesh had about 4.8 millions elements. All the results have been obtained using initial conditions equal to zero for velocity and pressure in the whole computational domain.

3. Results

Numerical simulation was done for the fixed value of Reynolds number $Re = u_0 d/ \nu = 370$, where $\nu$ is kinematic viscosity, $d$ is a diameter of cylinder and $u_0$ is velocity at the inlet. Streamlines shows the three-dimensional structure of the flow in figure 1(a). After the diverter an intensive screw flow is produced.

For analysis of velocity and vorticity fields in a cylindrical channel it is more convenient to use cylindrical $(r, \phi, y)$ coordinate system instead of Cartesian. Figures 2 and 3 show the spatial distributions of the time-averaged azimuthal and the axial components of the velocity and vorticity in three different cross-sections perpendicular to the $y$ axis. In the present study we did not analysed the radial components of velocity and vorticity, because they (and their gradients) are approximately an order of magnitude smaller than other components. Near the exit from diverter the flow is characterized by eight distinct jets produced by blades of
Figure 1. a) – Streamlines of the mean velocity field, b) – the diverter.

Figure 2. Upper row: time-averaged azimuthal component of the velocity. Lower row – time-averaged axial component of the velocity. Column correspond to cross-sections at $y = 0.05$ m, $y = 0.15$ m and $y = 0.20$ m (left-right).

divertor. Stagnation zone in a central part of the channel is a result of chosen configuration of the diverter (see figure 1(b)). Further from the divertor at $y = 0.15$ m we see that diffusion of
momentum by viscosity leads to the rapid axisymmetrization of the flow and instead of eight helical jets the flow took the shape of octagon. Due to non-slip condition on the walls there is a velocity shear near the walls of the channel which has strong influence on the flow evolution. Downstream at $y = 0.2$ m the shape of the flow became almost circular. Helicity describes the correlation between velocity and vorticity vectors so analysis of vorticity distribution is also very important. Blades of the diverter produced mainly two components of vorticity - azimuthal and axial. Similar to the velocity evolution the vorticity distribution became more homogeneous further from the diverter due to diffusion of vorticity. There is evident spatial separation of azimuthal vorticity by sign. Negative azimuthal vorticity is concentrated in the central part of the flow and positive azimuthal vorticity has maximum values near the walls. It is interesting that having similar distributions just after the diverter azimuthal and axial vorticity evolve differently. Negative axial vorticity also concentrated in the center but in a very narrow region in comparison with azimuthal vorticity. Positive axial vorticity is located in a wide band near the walls.

After analysis of velocity and vorticity fields we came to the most interesting and intriguing point - helicity density distribution (see figure 4)

$$H = U \cdot W = U_r W_r + U_\phi W_\phi + U_y W_y = H_r + H_\phi + H_y$$

$$= U_r (\partial U_y / \partial \phi - r \partial U_\phi / \partial y) / r + U_\phi (\partial U_r / \partial y - \partial U_y / \partial r) + U_y (\partial r U_\phi / \partial r - \partial U_r / \partial \phi) / r. \ (2)$$

Figure 3. Upper row: time-averaged azimuthal component of the vorticity. Lower row – time-averaged axial component of the vorticity. Column correspond to planes $y = 0.05$ m, $y = 0.15$ m and $y = 0.20$ m (left-right).
We consider two main contributions of helicity - azimuthal $H_\phi$ and axial $H_y$. Radial contribution $H_r$ has non-zero value near the diverter exit but than rapidly decays. At small values of $y$ distributions of $H_\phi$ and $H_y$ have eight maximums of helicity density (positive and negative) produced by spiral jets from the diverter blades. Despite similarity of the spatial structure of $H_\phi$ and $H_y$ distributions of their positive and negative parts are completely different. Azimuthal helicity has positive values in a central part and negative near the walls which is opposite to axial helicity. Helicity density distribution behind the diverter is concentrated in the circular band closer to the walls and characterized by periodic variation of sign. Like evolution of velocity and vorticity helicity density distribution became more axisymmetric but the axisymmetrization of $H_y$ is much faster than of $H_\phi$.

At the next step we consider azimuthal modes and its radial profiles. The contributions of different velocity and vorticity components and helicity density is decomposed into the complex Fourier amplitudes:

$$\hat{f}(r, y) = \frac{1}{2\pi} \int_0^{2\pi} f(r, \phi, y)e^{-im\phi} d\phi,$$

where $m$ is the azimuthal wavenumber. Figure 5 shows radial profiles of different velocity components for the axisymmetric mode $m = 0$ at different $y$. As we mentioned before radial component of velocity is relatively small and can be neglected. Profiles of azimuthal and axial velocity components showed that they evolve along channel in a different way. The viscous stress on the wall supresses azimuthal velocity. The evolution of profile of axial velocity is more complex. Since the total flowrate is fixed the mean (in the transverse cross-section) value of axial velocity is also fixed so the change of radial profile of axial velocity is a result of redistribution without decay. We see that there is a strong change of radial profile of $U_y$ up to $y = 0.15$ m when maximum of $U_y$ is shifted from the wall with increasing of $U_y$ in the central part. After $y = 0.15$ m the profiles became similar with a slight growth in the center.

Radial profiles of azimuthal and axial vorticity components for the axisymmetric mode $m = 0$ at different $y$ are presented in figure 6. Let's consider evolution of the azimuthal vorticity profile for the axisymmetric mode $m = 0$. Behind the diverter ($y = 0.05$ m) near the wall there almost linear dependence of positive values of $W_\phi$ than close to the center the sign of $W_\phi$ is reversed and we see the band of weakly changing negative values of $W_\phi$. At $y = 0.15$ m there is remarkable change of the azimuthal vorticity profile. The slope of linear part near the wall strongly decreases and there is almost uniform distribution of negative values of $W_\phi$ in the central part. The axial vorticity profile has two quasilinear parts with different slopes behind the diverter, negative in the center and positive near the wall. Downstream this difference in slopes is vanished and axial vorticity profile became almost linear at all radii.

Radial profiles of helicity for mode $m = 0$ at different $y$ is a result of both velocity and vorticity evolution. Downstream the profiles of azimuthal and axial contributions of helicity density for mode $m = 0$ became more smooth and stable (see figure 7). Azimuthal contribution of helicity is negative near the wall and positive in the center. In opposite axial helicity contribution is negative in the center and positive near the wall. Total helicity has almost linear profile for $r < 0.2$ m and vanishes at boundary. The screw flow in a described system can be treated as a sum of axisymmetric mode $m = 0$ and non-axisymmetric mode $m = 8$. Analysis of helicity of non-axisymmetric mode of the flow showed that it rapidly decreases with $y$ and further downstream it can be neglected (see figure 8). It means that the main role in generation of helicity in the considered system plays axisymmetric part of the flow.

Finally we present dependences of integrated over $\phi$ and $r$ values of $H_\phi$ and $H_y$ on the coordinate $y$ (see figure 8, right). Please note that at $y > 300$ mm the boundary condition at the exit influences on the flow structure and we exclude this region from our analysis. Radial contribution decays very fast because the walls of the channel effectively suppress all
Figure 4. Upper row: time-averaged azimuthal helicity density $H_\phi$. Middle row – time-averaged axial helicity density $H_y$. Lower row – time-averaged helicity density $H$. Column correspond to planes $y = 0.05$ m, $y = 0.15$ m and $y = 0.20$ m (left-right).
radial motions. Considering azimuthal and axial contribution there is a remarkable quantitative similarity. Despite completely different spatial distribution their integral values belong to the one curve. The integral values of $H_\phi$ and $H_y$ are negative for all $y$. Magnitudes of $H_\phi$ and $H_y$ decrease after the diverter up to $y \approx 70$ mm and after monotonically increase. Evidently this result requires more attention and further study.
4. Conclusions
Numerical simulation of the helical flow in a cylindrical channel with diverter was carried out using open-source software OpenFOAM Extend 4.0. There are several key points about helicity generation in the pipe flow past through fixed diverter: amount of total helicity in outlet flow, scale distribution, axial inhomogeneity and anisotropy. Therefore 3D distributions of velocity, vorticity and helicity density were analyzed in details. Helicity density distribution is characterized by different sign in the central and near wall regions. Azimuthal contribution of helicity is negative near the wall and positive in the center. In opposite axial helicity contribution is negative in the center and positive near the wall. The spiral flow in a described system can be treated as a sum of axisymmetric mode $m = 0$ and non-axisymmetric mode $m = 8$. Analysis of helicity of non-axisymmetric mode of the flow showed that it has substantial values near the diverter but rapidly decreases with $y$ and further downstream it can be neglected. It means that the main role in generation of helicity in the considered system plays axisymmetric part of the flow. Radial contribution of helicity decays very fast downstream because the walls of the channel effectively suppress all radial motions. Despite different spatial distribution variation of mean values of $H_\phi$ and $H_y$ along the channel has remarkable quantitative similarity. It was shown that integral values of $H_\phi$ and $H_y$ are negative for all $y$. Magnitudes of $H_\phi$ and $H_y$ decrease after the diverter up to $y \approx 70 \text{ mm}$ and after that monotonically increase. Evidently this result requires more attention and further study.

We can conclude that for the first time we demonstrated that the flow behind the diverter is characterized by substantial amount of helicity concentrated in largest scale and can be used as a helical flow injector. In particular the structure of flow develops fast to axisymmetric distribution with well balanced a azimuthal $H_\phi$ and axial $H_y$ contributions. Axisymmetrization of the flow and rapid decay of radial velocity makes possible to use only radial dependence of azimuthal and axial velocity components to estimate helicity as $H \approx -U_\phi (\partial U_y/\partial r) + U_y (\partial r U_\phi/\partial r)/r$. This result has important outcome for laboratory studies because in case of forced screw flows in channels StereoPIV (measurements three components of velocity in a plane) can be used for direct measurements of the helicity.

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