The structure around \( p\bar{p} \) threshold of \( J/\psi \rightarrow \gamma\eta'\pi^+\pi^- \): A revised version

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In this paper, we update the study of the structure around \( p\bar{p} \) threshold discovered in \( \eta'\pi^+\pi^- \) invariant mass spectra of the process of \( J/\psi \rightarrow \gamma\eta'\pi^+\pi^- \). The \( NN \) re-scattering is taken into account, and the distorted-wave Born approximation is applied to get the decaying amplitude through \( J/\psi \rightarrow \gamma NN \rightarrow \gamma\eta'\pi^+\pi^- \). The \( NN \) scattering amplitudes are obtained by solving the Lippmann-Schwinger equation with the potentials given by chiral effective field theory. To fix the unknown couplings, we fit the amplitudes to the data sets of the latest measurements on the invariant mass spectra of \( J/\psi \rightarrow \gamma\eta'\pi^+\pi^- \), \( J/\psi \rightarrow \gamma pp \), as well as the phase shifts and inelasticities given by partial wave analysis. We vary the cut-offs (\( R=0.9, 1.0, \) and \( 1.1 \) fm) and find stable solutions. The structure around \( p\bar{p} \) threshold of \( J/\psi \rightarrow \gamma\eta'\pi^+\pi^- \) is caused by threshold behaviour of \( NN \) intermediate states.

I. INTRODUCTION

Physicists have long been interested in 6-quark dibaryons and 3-quark-3-anti-quark baryon states. They can be one new kind of inner structure of matters and give clues to experimentalists to find new resonances. One possible candidate of the baryonium, \( X(1835) \) first discovered in the invariant mass spectrum of \( M_{\eta'\pi^+\pi^-} \) near the \( p\bar{p} \) threshold by BES collaboration [1], attracts both theoretical and experimental attention as the proton and neutron are the basic components of nucleus. Its mass and width are given as \( M=1833.7\pm6.1 \) MeV/c\(^2\) and \( \Gamma=67.7\pm20.3 \) MeV/c\(^2\) [1]. Subsequently, this resonance is also observed in decays of \( J/\psi \rightarrow \gamma 3(\pi^+\pi^-) \) [2], \( J/\psi \rightarrow \gamma K_S^0K_S^0\eta \) [3] and \( J/\psi \rightarrow \gamma \phi \) [4], while it is faintly supported by a few other experiments [5, 6]. A few years ago, BESIII increased the statistics of their measurements on the \( J/\psi \rightarrow \gamma\eta'\pi^+\pi^- \) and observed a clear structure around \( p\bar{p} \) threshold, but there is no peak around 1835 MeV. Therefore, one would wonder the inner links between the structure around \( p\bar{p} \) threshold in \( M_{\eta'\pi^+\pi^-} \), the \( X(1835) \), and \( NN \) re-scattering. Indeed, the anomalous behaviour around \( p\bar{p} \) threshold has been observed in quite a bit other processes such as \( B^+ \rightarrow K^+p\bar{p} \) and \( B^0 \rightarrow D^{*0}p\bar{p} \) by Belle collaboration [7, 8], \( J/\psi \rightarrow \gamma p\bar{p} \) by BES collaboration [9], \( e^+e^- \rightarrow p\bar{p}/n\bar{n} \) by Babar [10] and SND [11] collaborations, and \( e^+e^- \rightarrow 2(\pi^+\pi^-\pi^0), 3(\pi^+\pi^-) \) by Babar collaboration [12]. Meanwhile, there are many relative theoretical research such as \( e^+e^- \rightarrow p\bar{p}/n\bar{n} \) [13, 14], \( J/\psi \rightarrow \gamma p\bar{p} \) [15, 16], and \( e^+e^- \rightarrow 2(\pi^+\pi^-\pi^0), 3(\pi^+\pi^-) \) [17].

In the previous paper [16], the \( NN \) final state interactions (FSI)\(^1\) have been included. The strategy is establishing a two-step process, \( J/\psi \rightarrow \gamma NN \rightarrow \gamma\eta'\pi^+\pi^- \), where the distorted-wave Born approximation (DWBA) is used to implement FSI of \( NN \). The underlying physics behind the first process \( J/\psi \rightarrow \gamma NN \) is following the experimental measurements of anomalous energy-dependent behaviour near the \( p\bar{p} \) threshold in \( J/\psi \rightarrow \gamma NN \). See, e.g., Ref. [9], where there is an apparent threshold enhancement near the \( p\bar{p} \) threshold in the decay of \( J/\psi \rightarrow \gamma p\bar{p} \) as observed by BES collaboration. Echoing these measurements, some theoretical models do find that the structure around \( p\bar{p} \) threshold in \( M_{\eta'\pi^+\pi^-} \) invariant mass spectrum of \( J/\psi \rightarrow \gamma\eta'\pi^+\pi^- \) is caused by the intermediate \( NN \) re-scattering [15, 16], while no resonance pole is found in the \( ^1S_0 \) partial wave.

Recently, BESIII collaboration performed the latest measurements on \( J/\psi \rightarrow \gamma\eta'\pi^+\pi^- \) again, and the uncertainty of the \( \eta'\pi^+\pi^- \) invariant mass spectrum has been reduced near the \( p\bar{p} \) threshold [21]. Thus, it would be necessary to include these new data sets and update the research of Ref. [16]. Also, in the previous work, only one cut-off (\( R=0.9 \) fm) was considered, and it is essential to check whether the structure is stable with different cut-offs.

This paper is organized as follows. In Sec. II, the formulas to calculate the reactions of \( J/\psi \rightarrow \gamma\eta'\pi^+\pi^- \), \( J/\psi \rightarrow \gamma p\bar{p} \) and \( p\bar{p} \rightarrow \eta'\pi^+\pi^- \) are given. Through solving a set of coupled channel equations, e.g., Lippmann-Schwinger equation (LSE) and DWBA, one can get the scattering and decaying amplitudes. Then the fit results and discussions are given in Sec. III. The \( J/\psi \rightarrow \gamma p\bar{p} \) amplitude is fixed by fitting to the \( p\bar{p} \) invariant mass spectra and the phase shifts of \( ^1S_0 \) partial waves of \( NN \) scattering. The \( J/\psi \rightarrow \gamma\eta'\pi^+\pi^- \) amplitude is then fixed by fitting to the invariant mass spectra. The effects of varying cut-offs are also discussed. Finally, a summary is given in Sec. IV.

II. FORMALISM

As mentioned above, the \( J/\psi \rightarrow \gamma\eta'\pi^+\pi^- \) decaying amplitude is obtained through a two-step process of \( J/\psi \rightarrow \gamma NN \rightarrow \gamma\eta'\pi^+\pi^- \). The \( NN \) off-shell scattering amplitude is the kernel to be input into DWBA equa-

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\(^1\)For different approaches implementing FSI, see e.g. Refs.[18–20].
mass of the system
\[ Q \]

Notice that the meaning of invariant mass is parameterized as
\[ V = \frac{Q^2 - t_1 + m_{\eta'}^2}{2Q}, \]
\[ E_{\pi^+} = \frac{Q^2 - t_2 + m_{\pi'}^2}{2Q}, \]
\[ E_{\pi^-} = \frac{t_1 + t_2 - m_{\pi}^2 - m_{\pi'}^2}{2Q}. \]

The upper and lower limits of the integrals of Eq.(4) are
\[ t_1^+ = 4m_{\pi}, \quad t_1^- = (Q - m_{\eta'}^2), \]
\[ t_2^+ = \lambda_{1/2}(t_1^2, m_{\eta'}^2), \quad t_2^- = \lambda_{1/2}(t_2^2, m_{\eta'}^2), \]
\[ \lambda_{1/2}(t_1^2, m_{\eta'}^2) = \frac{1}{4t_1^2} \left[ (Q^2 - m_{\eta'}^2)^2 - (t_1^2, m_{\eta'}^2) \right], \]
\[ \lambda_{1/2}(t_2^2, m_{\eta'}^2) = \frac{1}{4t_2^2} \left[ (Q^2 - m_{\eta'}^2)^2 - (t_2^2, m_{\eta'}^2) \right]. \]

with the Källén function defined as \( \lambda(x, y, z) = (x - y - z)^2 - 4yz \). The explicit equations to solve the scattering and decaying amplitudes are as follows

\[ T_{N\bar{N} \rightarrow N\bar{N}}(p', p; E_p) = V_{N\bar{N} \rightarrow N\bar{N}}(p', p) + \int_0^{\infty} \frac{dkk^2}{(2\pi)^3}, \]
\[ V_{N\bar{N} \rightarrow N\bar{N}}(p', k) = \frac{1}{2E_p - 2E_k + i\epsilon} T_{N\bar{N} \rightarrow N\bar{N}}(k, p; E_p), \]
\[ F_{J/\psi \rightarrow N\bar{N}}(Q) = A_{J/\psi \rightarrow N\bar{N}}(p) + \int_0^{\infty} \frac{dkk^2}{(2\pi)^3}, \]
\[ A_{J/\psi \rightarrow N\bar{N}}(k) = \frac{1}{2E_p - 2E_k + i\epsilon} T_{N\bar{N} \rightarrow N\bar{N}}(k, p; E_p), \]
\[ F_{N\bar{N} \rightarrow \eta'\pi^\pm}(Q) = V_{N\bar{N} \rightarrow \eta'\pi^\pm}(p) + \int_0^{\infty} \frac{dkk^2}{(2\pi)^3}, \]
\[ T_{N\bar{N} \rightarrow N\bar{N}}(p, k; E_k) = \frac{1}{2E_k - Q + i\epsilon} V_{N\bar{N} \rightarrow \eta'\pi^\pm}(k), \]
\[ F_{J/\psi \rightarrow \eta'\pi^\pm}(Q) = A_{J/\psi \rightarrow \eta'\pi^\pm}(Q) + \int_0^{\infty} \frac{dkk^2}{(2\pi)^3}, \]
\[ F_{J/\psi \rightarrow \eta'\pi^\pm}(E_k) = \frac{1}{Q - 2E_k + i\epsilon} V_{N\bar{N} \rightarrow \eta'\pi^\pm}(k), \]

where \( E_p, E_k \) are the energy of the nucleon or anti-nucleon in the center of mass frame, with \( E_p = Q/2 \) for the processes of \( J/\psi \rightarrow \gamma N\bar{N} \) and \( N\bar{N} \rightarrow \eta'\pi^\pm \).

III. RESULTS AND DISCUSSIONS

This section will discuss the fit to the experimental data and the physics underlying the structure around
$p\bar{p}$ threshold. Since the decaying amplitude of $J/\psi \to \gamma\eta'\pi^+\pi^-$ is constructed based on a two-step process, it would be convenient to divide this section into two parts: one is about the physics of the process of $J/\psi \to \gamma p\bar{p}$; and the other is of $J/\psi \to \eta'\pi^+\pi^-$.  

A. Analysis of $J/\psi \to \gamma p\bar{p}$

Firstly we focus on the process of $J/\psi \to \gamma p\bar{p}$. As discussed above, the $N\bar{N}$ final state interactions should be considered. It is reasonable to assume that the relative angular momentum between $\gamma$ and $p\bar{p}$ should be the lowest one, i.e., $S$-wave. Then the relevant partial wave of $N\bar{N}$ should be $1S_0$, with isospin to be either one or zero according to the unfixed isospin of the photon. Here we follow the previous work [16] and set the ratio between different isospin components to be $T_{N\bar{N}\to p\bar{p}} = 0.4T^0 + 0.6T^1$. Indeed, this ratio gives the best $\chi^2_{d.o.f.}$ on fitting to the data. In the previous work [16], the process of $J/\psi \to \gamma p\bar{p}$ has been analyzed at $R = 0.9$ fm. To study the stability of the solutions with different cut-offs, we analyze the process again with a set of cut-offs, e.g., $R = 0.9, 1.0, 1.1$ fm.

The $N\bar{N}$ scattering amplitudes are solved by LSE. Similar to Ref. [16], the phase shifts of $1S_0$ $N\bar{N}$ partial wave with isospin $I=1$ are refit to reproduce the invariant mass spectra of $J/\psi \to \gamma p\bar{p}$. The values of the low-energy constants (LECs) of the chiral $N\bar{N}$ scattering potential of $I = 1$ $1S_0$ partial wave are listed in Table I. The fitting uncertainties, respectively.

![FIG. 1. Fit to the PWA’s. The top graphs are the $I = 1$ phase shifts of $1S_0$ partial wave of N$^3$LO with cut-offs $R = 0.9, 1.0$ and $1.1$ fm. The red points represent the results of PWA [23]. The bottom graphs are the results for N$^2$LO and N$^3$LO, with cut-off $R = 1.0$ fm. The pink and sky blue bands are their apparent difference between ours and PWA’s. The reason is that the phase shifts are the same in the low energy region for the processes of $J/\psi \gamma \to NN$ and $N\bar{N} \to NN$, but they do not need to be the same in the high energy region. Also, the $p\bar{p}$ invariant mass spectra of $J/\psi \gamma \to p\bar{p}$ are smooth and decrease slowly in the high energy region, which requires the phase shifts to have similar behavior.

With the $N\bar{N}$ scattering amplitude, one can obtain the $J/\psi \gamma \to NN$ decaying amplitude according to Eq. (2). Here we set $C_{J/\psi \gamma \to NN} = 0$ as a constant transition amplitude $A_{J/\psi \gamma \to NN}^\text{LO}$ is enough to describe the data well. Also we fix $C_{J/\psi \gamma \to p\bar{p}} = 1$ due to the overall normalization factor for the events data. Notice that the phase shifts and inelasticity of $I = 1$ $1S_0$ partial wave and the invariant mass spectra from BESIII [24], BES [9] and CLEO [25] are fit simultaneously. The fitting results of the decay rate of the process of $J/\psi \to p\bar{p}$ are shown in Fig 2. The top graphs of Fig 2 are those of N$^3$LO with different cut-offs: The purple dashed, black solid, and brown dotted curves are for cut-offs $R = 0.9, 1.0$, and $1.1$ fm, respectively. All curves with different cut-offs are consistent with the experimental data. The results with $R = 0.9$ and $1.0$ fm are somewhat similar to each other, and they have only a slight difference with the result of $R = 1.1$ fm. The comparison between the decay rates of N$^2$LO and N$^3$LO are given in the bottom graphs of Fig 2, shown as the dashed blue and solid black curves. The error bands of the results with $R = 1.0$ fm are given by the pink and sky blue bands for N$^2$LO and N$^3$LO, respectively.

The estimation is based on a Bayesian method [22, 26]. Though the fit of N$^3$LO is not so good as that of N$^2$LO, especially in the energy region around the threshold, both

| LECs | N$^2$LO | N$^3$LO |
|------|-------|-------|
| $R$ (fm) | 1.0 | 0.9 | 1.0 | 1.1 |
| $\bar{C}_{31S_0}$ (GeV$^{-2}$) | 0.0480 | 0.3355 | 0.2674 | 0.0664 |
| $C_{31S_0}$ (GeV$^{-4}$) | -1.8397 | -3.7566 | -5.1411 | -5.3367 |
| $D_{31S_0}^1$ (GeV$^{-6}$) | -7.9227 | -15.0857 | -25.0000 |
| $D_{31S_0}^2$ (GeV$^{-6}$) | 11.1314 | 20.0000 | 25.0000 |
| $\bar{C}_{33S_0}$ (GeV$^{-1}$) | -0.0102 | -0.0120 | -0.0907 | -0.1154 |
| $C_{33S_0}$ (GeV$^{-3}$) | -4.5259 | -3.0480 | -3.8174 | -4.6828 |

TABLE I. The values of low energy constants of the $N\bar{N}$ scattering potentials of $I = 1$ $1S_0$ partial wave. All parameters are multiplied with a factor $10^3$. The results of $N\bar{N}$ scattering phase shifts and inelasticity at N$^3$LO for different cut-offs are at the top of Fig 1. The results of N$^2$LO and N$^3$LO with cut-off $R = 1$ fm are at the bottom of Fig 1. Both are consistent with the phase shifts and inelasticity of partial wave analysis (PWA) [23] in the low-energy region ($T_{lab}$ below 100 MeV). This is guaranteed by the fact that we fit the first four points to constrain the low energy behavior of the off-shell $N\bar{N}$ scattering amplitudes $T_{N\bar{N}\to NN}$, which will be input in the DWBA. While in the high energy region, there is an
are compatible with the experimental measurements.

As observed by the experiments, there is a clear threshold enhancement near the \( p\bar{p} \) threshold. This inspires people to believe that there is a baryonium state related to the \( NN \), e.g., the \( X(1835) \). In our analysis, such a state is not found in the origin \( NN \) scattering amplitudes, neither in \( I = 0 \) nor in \( I = 1 \). Nonetheless, with the modest modified \( I = 1 \) partial wave, such a bound state can be found below the \( p\bar{p} \) threshold. For instance, the pole location for \( R = 1.0 \) fm is \( E_B = -49.8 - i49.2 \) MeV of \( N^3LO \), and \( E_B = -1.0 - i100.2 \) MeV of \( N^2LO \). Which are compatible with those found in Refs. [15, 16]. Nevertheless, the width (\( \Gamma = 2 \text{Im} E_B \)) of the state is large, and thus it is not easy to conclude that the anomalous structure around \( NN \) threshold of other processes are purely caused by the \( X(1835) \). It should be stressed that the pole found here may not be suitable to be treated as evidence of confirming the baryonium origin of the \( X(1835) \), as the data above the \( NN \) or \( BB \) threshold could determine the amplitudes below threshold unreliably.

### B. Analysis of \( J/\psi \to \gamma\eta'\pi^+\pi^- \)

As mentioned above, to determine the \( J/\psi \to \gamma\eta'\pi^+\pi^- \) decay amplitude, one needs to know the amplitudes of two processes, \( J/\psi \to \gamma NN \) and \( NN \to \eta'\pi^+\pi^- \). The former decaying amplitude has been given in the previous sub-section, and the latter is calculated out by the last equation of Eq. (2). Notice that only isoscalar \( S_0 \) wave is needed. The unknown couplings \( \tilde{C}_{NN \to \eta' \pi^+ \pi^-} \) (\( C_{NN \to \eta' \pi^+ \pi^-} \) is set to be zero) is fixed by fitting to the scattering cross section, \( \sigma(pp \to \eta' \pi^+ \pi^-) = 2.23 \text{ mb} \), which is obtained by multiplying the measured branching ratio \( BR(pp \to \eta' \pi^+ \pi^-) = 0.626\% \) [29] and the total annihilation cross section of \( pp \) [30]. With these two amplitudes, we can now use DWBA to get the decaying amplitude of \( J/\psi \to \gamma\eta'\pi^+\pi^- \). Note that the parameters \( \tilde{C}_{J/\psi \to \gamma NN} \), \( \tilde{C}_{NN \to \eta' \pi^+ \pi^-} \) and normalization factors for the \( M_{\eta' \pi^+ \pi^-} \) invariant mass spectra are multiplied together. Therefore, we can set \( \tilde{C}_{J/\psi \to \gamma NN} = 1 \) again. Finally one only needs to determine 3 parameters by fitting to the \( J/\psi \to \gamma\eta'\pi^+\pi^- \) invariant mass spectra, i.e., \( \tilde{C}_{J/\psi \to \gamma\eta'\pi^+\pi^-} \), \( C_{J/\psi \to \gamma\eta'\pi^+\pi^-} \) and a normalization factor. Notice that \( \tilde{C}_{J/\psi \to \gamma\eta'\pi^+\pi^-} \) and \( C_{J/\psi \to \gamma\eta'\pi^+\pi^-} \) can be complex numbers, and hence there are four degrees of freedom for them. In the previous work [16], these two parameters are taken as real numbers. However, in this analysis, for cut-offs \( R=1.0 \) and 1.1 fm, the complex parameters would give a better quality of the fits. The parameters of our solutions are shown in Table II.

| \( R \) (fm) | \( N^2LO \) | \( N^3LO \) |
|-------------|------------|------------|
| \( \tilde{C}_{pp \to \eta' \pi^+ \pi^-} \) (GeV\(^{-2}\)) | 0.0075 | 0.0069 | 0.0072 | 0.0075 |
| \( C_{J/\psi \to \gamma\eta'\pi^+\pi^-} \) (GeV\(^{-2}\)) | -0.2498 | -0.1072 | -0.1561 | -0.2283 |
| \( C_{J/\psi \to \gamma\eta'\pi^+\pi^-} \) (GeV\(^{-3}\)) | +0.0105 | -0.0844 | -0.0342 | +0.0177 |
| \( C_{J/\psi \to \gamma\eta'\pi^+\pi^-} \) (GeV\(^{-3}\)) | 0.0881 | 0.0360 | 0.0538 | 0.0830 |
| \( C_{J/\psi \to \gamma\eta'\pi^+\pi^-} \) (GeV\(^{-3}\)) | +0.0101 | +0.0462 | +0.0249 | +0.0025 |

TABLE II. The values of parameter \( \tilde{C}_{pp \to \eta' \pi^+ \pi^-} \), \( \tilde{C}_{J/\psi \to \gamma\eta'\pi^+\pi^-} \) and \( C_{J/\psi \to \gamma\eta'\pi^+\pi^-} \) in Eq. 3. All parameters are in units of \( 10^3 \).

Recently, BESIII collaboration performed a new measurement on the invariant mass spectra of the decay rate

\(^2\)Indeed, in some other processes, there have been observed (baryon-anti-baryon) threshold enhancements. However, it is not necessary to have a baryonium state there [27, 28].
of $J/\psi \rightarrow \gamma \eta' \pi^+ \pi^-$ [21]. In this new measurement, two ways to collect the events of the final state $\eta'$ are considered: one is from $\eta' \rightarrow \gamma \pi^+ \pi^-$, and the other is from $\eta' \rightarrow \pi^+ \pi^- \eta$. Correspondingly, there are two different data sets, and we label them as BESIII 2022 (I) and (II) respectively. These new data sets have smaller errors than that of the previous measurement [31], and thus it is necessary to include them in the present analysis. The fitting results are shown in Fig. 3. The fitting results of $N_{2\text{LO}}$, $N_{3\text{LO}}$, and $N_{\text{I}}$ are shown in Fig. 3. The fitted threshold pole in the $N\eta' \pi^+ \pi^-$ invariant mass spectra of $J/\psi \rightarrow \gamma \eta' \pi^+ \pi^-$, $C_{J/\psi \rightarrow \gamma \eta' \eta}$ and $C_{J/\psi \rightarrow \gamma \eta' \pi}$ be complex and thus the degrees of freedom of the fit have been increased. Also, notice that the results of $J/\psi \rightarrow \gamma p\bar{p}$ at $N^2\text{LO}$ are worse than that at $N^3\text{LO}$. Nevertheless, the mechanism of how the structure around $p\bar{p}$ threshold is generated would be similar to that of the previous paper [16], see discussions below.

To study the structure around $p\bar{p}$ threshold in the $M_{N\eta' \pi^+ \pi^-}$ invariant mass spectra of $J/\psi \rightarrow \gamma \eta' \pi^+ \pi^-$, we separate two contributions to the $M_{N\eta' \pi^+ \pi^-}$ spectra: one is from $NN$ re-scattering ($J/\psi \rightarrow \gamma NN \rightarrow \gamma \eta' \pi^+ \pi^-$), and the other is the background, i.e., the contribution from the transition amplitude $A^0_{J/\psi \rightarrow \gamma \eta' \pi}$. The individual contributions of these two parts are calculated by Eq. (2), with one part switched on while the other switched off. The results are shown in Fig. 4.

![Fig. 3](https://example.com/fig3.png)

**FIG. 3.** Fit to the $J/\psi \rightarrow \gamma \eta' \pi^+ \pi^-$ decay rate. The top graphs are of $N^3\text{LO}$ with cut-offs $R=0.9$, $1.0$, and $1.1$ fm. The bottom graphs are for $N^2\text{LO}$ and $N^3\text{LO}$, with cut-off $R=1.0$ fm. The data sets labeled by "BESIII 2022 (I)" and "BESIII 2022 (II)" are from Ref. [21] and the one labeled by "BESIII 2016" is from Ref. [31]. The pink and sky blue bands are the error estimation of the results for $N^2\text{LO}$ and $N^3\text{LO}$, respectively, with $R=1.0$ fm. The vertical line is the $p\bar{p}$ threshold.

$N^3\text{LO}$ with cut-offs $R=0.9$, $1.0$ and $1.1$ fm are shown in the top graphs of Fig. 3, denoted by dashed purple, solid black, and dotted brown curves, respectively. It can be found that all our solutions are consistent with the data sets. The difference between the results of different cut-offs is relatively tiny, revealing that the solutions are somewhat cut-off independent. The error bands of $R=1.0$ fm, both for $N^2\text{LO}$ and $N^3\text{LO}$, are shown by the pink and sky blue bands as plotted in Fig. 3. The error estimation method is again from the Bayesian method as Ref. [22, 26]. Interestingly, the solution of $N^3\text{LO}$ has almost the same quality as that of $N^3\text{LO}$. This is because we have let the parameters of the transition amplitude $A^0_{J/\psi \rightarrow \gamma \eta' \pi^+ \pi^-}$, $C_{J/\psi \rightarrow \gamma \eta' \eta}$ and $C_{J/\psi \rightarrow \gamma \eta' \pi}$ be complex and thus the degrees of freedom of the fit have been increased.

As can be seen, the backgrounds are pretty smooth and more prominent, while the $NN$ re-scattering term contributes little except for a cusp-like structure around the $p\bar{p}$ threshold. This confirms our conclusion, i.e., the structure around the $p\bar{p}$ threshold in the $M_{N\eta' \pi^+ \pi^-}$ invariant mass spectra of $J/\psi \rightarrow \gamma \eta' \pi^+ \pi^-$ should be generated by the interference between the background and the $NN$ re-scattering. On the other hand, there is no pole ($NN$ bound state or resonance) found in the $I=0^+ S_0$ partial wave of $NN$ scattering amplitude for any of the cut-offs, neither in the $N^2\text{LO}$ case nor in the $N^3\text{LO}$ case.

Hence, our analysis indicates that there is no need for a $NN$ bound state to quantitatively describe the struc-

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3We are aware that in some other models [32–34], they find a near-threshold pole in the $NN$ $I=0^+ S_0$ partial wave.
ture around the $p\bar{p}$ threshold in $M_{\pi^+\pi^-}$ invariant mass spectra of $J/\psi \to \gamma\eta'\pi^+\pi^-$. 

IV. CONCLUSION

We analyzed the decaying processes of $J/\psi \to \gamma\eta'\pi^+\pi^-$ and $J/\psi \to \gamma p\bar{p}$ with chiral effective field theory and distorted wave Born approximation. The latest measurements on the $M_{\pi^+\pi^-}$ invariant mass spectra of $J/\psi \to \gamma\eta'\pi^+\pi^-$ by BESIII collaboration are included. Our results show that the structure around the $p\bar{p}$ threshold found in the invariant mass spectra is caused by the $NN$ threshold effect, while no bound state or resonance is found in the relevant $I = 0^+S_0$ partial wave. To check the reliability, we test the effects of different cut-offs and find that our conclusion is not sensitive to the cut-offs.

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[1] M. Ablikim et al. Observation of a resonance $X(1835)$ in $J/\psi \to \gamma\eta'\pi^+\pi^-$. Phys. Rev. Lett., 95:262001, 2005.
[2] M. Ablikim et al. Observation of a structure at 1.84 GeV/c$^2$ in the $3(\pi^+\pi^-)$ mass spectrum in $J/\psi \to 3(\pi^+\pi^-)$ decays. Phys. Rev. D, 88(9):091502, 2013.
[3] M. Ablikim et al. Observation and Spin-Parity Determination of the $X(1835)$ in $J/\psi \to \gamma K_S^0 K_S^0\eta$. Phys. Rev. Lett., 115(9):091803, 2015.
[4] M. Ablikim et al. Study of $\eta(1475)$ and $X(1835)$ in radiative $J/\psi$ decays to $\gamma\phi$. Phys. Rev. D, 97(5):051101, 2018.
[5] X. H. He et al. Search for the process $e^+e^- \to J/\psi X(1835)$ at $\sqrt{s} \approx 10.6$ GeV. Phys. Rev. D, 89(3):032003, 2014.
[6] C. C. Zhang et al. First study of $\eta_c(1760)$ and $X(1835)$ production via $\eta'\pi^+\pi^-$ final states in two-photon collisions. Phys. Rev. D, 86:052002, 2012.
[7] Kazuo Abe et al. Observation of $B^{\pm} \to p\bar{p}K^{\pm}$. Phys. Rev. Lett., 88:181803, 2002.
[8] Kazuo Abe et al. Observation of $B^0 \to D^{\ast 0} p\bar{p}$. Phys. Rev. Lett., 89:151802, 2002.
[9] J. Z. Bai et al. Observation of a near threshold enhancement in the $p\bar{p}$ mass spectrum from radiative $J/\psi \to \gamma p\bar{p}$ decays. Phys. Rev. Lett., 91:022001, 2003.
[10] J. P. Lees et al. Study of $e^+e^- \to p\bar{p}$ via initial-state radiation at Babar. Phys. Rev. D, 87(9):092005, 2013.
[11] M. N. Achasov et al. Experimental study of the $e^+e^- \to n\bar{n}$ process at the VEPP-2000 $e^+e^-$ collider with the SND detector. Eur. Phys. J. C, 82(8):761, 2022.
[12] Bernard Aubert et al. The $e^+e^- \to 3(\pi^+\pi^-), 2(\pi^+\pi^-\pi^0)$ and $K^0\bar{K}^0(2\pi^+\pi^-)$ cross sections at center-of-mass energies from production threshold to 4.5-GeV measured with initial-state radiation. Phys. Rev. D, 73:052003, 2006.
[13] J. Haidenbauer, X. W. Kang, and U. G. Meißen. The electromagnetic form factors of the proton in the timelike region. Nucl. Phys. A, 929:102–118, 2014.
[14] Qin-He Yang, Ling-Yun Dai, Di Guo, Johann Haidenbauer, Xian-Wei Kang, and Ulf-G. Meißen. New insights into the oscillation of the nucleon electromagnetic form factors. 6 2022.
[15] Xian-Wei Kang, J. Haidenbauer, and Ulf-G. Meißen. Near-threshold $p\bar{p}$ invariant mass spectrum measured in $J/\psi$ and $\psi'$ decays. Phys. Rev. D, 91(7):074003, 2015.
[16] Ling-Yun Dai, Johann Haidenbauer, and Ulf-G. Meißen. $J/\psi \to \gamma\eta'\pi^+\pi^-$ and the structure observed around the $p\bar{p}$ threshold. Phys. Rev. D, 98(1):014005, 2018.
[17] Johann Haidenbauer, Christoph Hanhart, Xian-Wei Kang, and Ulf-G. Meißen. Origin of the structures observed in $e^+e^-$ annihilation into multipion states around the $p\bar{p}$ threshold. Phys. Rev. D, 92(5):054032, 2015.
[18] L. Y. Dai, Meng Shi, Guang-Yi Tang, and H. Q. Zheng. Nature of $X(2600)$. Phys. Rev. D, 92(1):014020, 2015.
[19] Ling-Yun Dai and Michael R. Pennington. Comprehensive amplitude analysis of $\gamma \to \pi^+\pi^-\pi^0\pi^0$ and $KK$ below 1.5 GeV. Phys. Rev. D, 90(3):036004, 2014.
[20] De-Liang Yao, Ling-Yun Dai, Han-Qing Zheng, and Zhi-Yong Zhou. A review on partial-wave dynamics with chiral effective field theory and dispersion relation. Rept. Prog. Phys., 84(7):076201, 2021.
[21] M. Ablikim et al. Observation of a State $X(2600)$ in the $\pi^+\pi^-\eta'$ System in the Process $J/\psi \to \gamma\pi^+\pi^-\eta'$. Phys. Rev. Lett., 129(4):042001, 2022.
[22] Ling-Yun Dai, Johann Haidenbauer, and Ulf-G Meißen. Antinucleon-nucleon interaction at next-to-next-to-next-to-leading order in chiral effective field theory. JHEP, 07:078, 2017.
[23] Daren Zhou and Rob G. E. Timmermans. Energy-dependent partial-wave analysis of all antiproton-proton scattering data below 925 MeV/c. Phys. Rev. C, 86:044003, 2012.
[24] M. Ablikim et al. Spin-Parity Analysis of $pp$ Mass Threshold Structure in $J/\psi$ and $\psi'$ Radiative Decays. Phys. Rev. Lett., 108:112003, 2012.
[25] J. P. Alexander et al. Study of $\psi(2S)$ Decays to $\gamma p\bar{p}$, $\pi^0 p\bar{p}$ and $\eta p\bar{p}$ and Search for $p\bar{p}$ Threshold Enhancements. Phys. Rev. D, 82:092002, 2010.
[26] E. Epelbaum, H. Krebs, and U.G. Meißen. Improved chiral nucleon-nucleon potential up to next-to-next-to-next-to-leading order. Eur. Phys. J. A, 51(5):53, 2015.
[27] Ling-Yun Dai, Johann Haidenbauer, and Ulf G. Meißen. Re-examining the $X(4630)$ resonance in the reaction $e^+e^- \to \Lambda^+_c\Lambda^-$. Phys. Rev. D, 96(11):116001, 2017.
[28] Johann Haidenbauer, Ulf-G. Meißen, and Ling-Yun Dai. Hyperon electromagnetic form factors in the timelike region. Phys. Rev. D, 103(1):014028, 2021.
[29] Claude Amsler et al. Production and decay of $\eta'(958)$ and $\eta(1440)$ in $\bar{p}p$ annihilation at rest. *Eur. Phys. J. C*, 33:23–30, 2004.

[30] A Bertin et al. $\bar{p}p$ annihilation cross-section at very low-energy. *Phys. Lett. B*, 369:77–85, 1996.

[31] Medina Ablikim et al. Observation of an anomalous line shape of the $\eta'\pi^+\pi^-$ mass spectrum near the $p\bar{p}$ mass threshold in $J/\psi \rightarrow \gamma\eta'\pi^+\pi^-$. *Phys. Rev. Lett.*, 117(4):042002, 2016.

[32] J. P. Dedonder, B. Loiseau, B. El-Bennich, and S. Wycech. On the structure of the X(1835) baryonium. *Phys. Rev. C*, 80:045207, 2009.

[33] A. I. Milstein and S. G. Salnikov. Interaction of real and virtual $p\bar{p}$ pairs in $J/\psi \rightarrow p\bar{p}\gamma(\rho, \omega)$ decays. *Nucl. Phys. A*, 966:54–63, 2017.

[34] J. P. Dedonder, B. Loiseau, and S. Wycech. Photon or meson formation in $J/\psi$ decays into $p\bar{p}$. *Phys. Rev. C*, 97(6):065206, 2018.