Determination of shower central position in laterally segmented lead-fluoride electromagnetic calorimeters

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ABSTRACT: The spatial resolution of laterally segmented electromagnetic calorimeters, built of lead fluoride material, is studied on the basis of Monte-Carlo simulations. Parametrization of the relative resolution on the shower position is proposed and optimized in terms of the energy of incoming particles and the elementary size of the calorimeter blocks. A new fit algorithm method is proposed that improves spatial resolution at high energies (> 5 GeV), and provides guidance for the design optimization of electromagnetic calorimeters.

KEYWORDS: Calorimeter methods; Calorimeters; Data processing methods; Detector modelling and simulations I (interaction of radiation with matter, interaction of photons with matter, interaction of hadrons with matter, etc)

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1 Introduction

Electromagnetic calorimeters are fundamental elements of numerous experiments ranging from nuclear to hadronic and high-energy physics. Their main goal is to measure precisely the energy of the reconstructed electrons and photons. The depth of the calorimeters is generally taken to be large enough to minimize energy leaks and to allow a full development of the electromagnetic shower created by the incident particles. In addition to the energy measurement, the impact position of particles, corresponding to the shower central position in case of normal incidence, is usually required in order to provide refined information as particle identification. Indeed, the knowledge of the photon or the electron energy and both the impact and interaction vertex positions allow the determination of the particle four-vector. This is of direct relevance for instance in the experimental determination of the origin of two photon events from the decay of $\pi^0$- or $\eta$-mesons where the two photon invariant mass allows identifying the meson nature. It has been shown in some experiments, that the spatial resolution is of equal importance than the energy resolution for particle identification [1].

The knowledge of the particle impact position requires a laterally segmented calorimeter where the energy is released in a cluster of adjacent blocks. The impact position can then be determined using the energy deposited in each block. Laterally segmented calorimeters such as lead-fluoride (PbF$_2$) are frequently employed to determine the energy and the position of the showering particle [2–4]. This high-density material has a short radiation length ($X_0 = 0.93$ cm) and a small Molière radius ($r_M = 2.12$ cm) leading to compact detector geometries [5, 6].

Many efforts are still developed for optimizing the performance of such lead-fluoride calorimeters, in particular their energy and spatial resolution [7]. We investigate in this work, on the basis of full shower simulations with GEANT4 [8, 9], the spatial resolution of a PbF$_2$ calorimeter for several block sizes and different particle energies. Two approaches are successively considered,
discussed, and optimized to obtain a parametrization of the spatial resolution in terms of the block size and the particle energy. The effect of the electromagnetic calorimeter energy resolution on the spatial resolution is addressed, and the two approaches are finally compared.

2 Simulation

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{calorimeter.png}
\caption{Schematic of the geometry of the simulated calorimeter.}
\end{figure}

The electromagnetic calorimeter is simulated within the GEANT4 framework following a $15 \times 15 \text{PbF}_2$ block matrix. The depth of the calorimeter is chosen to be $50X_0$ to have no shower longitudinal leakage and each block has a transverse square shape with dimensions $d \times d \text{ cm}^2$. Different configurations corresponding to 13 different transverse size varying from $0.3r_M$ to $5r_M$ are studied. The minimum block size ($0.3r_M$) is consistent with a large enough minimal calorimeter ($4.5r_M \times 4.5r_M$) to contain the electromagnetic shower. For each configuration, photons of energy $E$ are sent parallel to the $z$-axis and impinge normally on the central region of the calorimeter (figure 1). The known impact position $(x, y)$ of incident particles is distributed uniformly over the area of the central block $(-d/2, -d/2) < (x, y) < (d/2, d/2)$. Eleven photon energies varying from 500 MeV to 20 GeV are generated, leading to a total of 143 configurations. In each case, the response on the calorimeter is studied for $10^5$ generated initial photons: the event-by-event energy deposit in each individual block is recorded and used to reconstruct the shower central position $(x_c, y_c)$ as described in the following sections. In this paper, focus is put on the $x_c$ coordinate since the $y_c$ coordinate can be deduced following the same methods.

Two approaches can be considered for reconstructing the shower central position $x_c$. The first method is based on a centre-of-gravity formula depending on the $x_i$-coordinate of the $i^{th}$ block centre and the energy deposit $E_i$ in this block. The second method developed in the present work relies on fitting the calorimeter block response with a known profile function where $x_c$ becomes the free parameter of the fit.
3 Centre-of-gravity based method

Various formulas have been proposed to determine the shower central position [10–14]. The centre of gravity method is one of the most common formula

\[ x_c = \frac{\sum_i w_i x_i}{\sum_i w_i}, \tag{3.1} \]

where the sum runs over the number of blocks in the shower cluster and \( w_i \) is a weight factor depending of the energy \( E_i \). In the simplest case where \( w_i = E_i \), it has been shown that the obtained \( x_c \) depends on \( x \) in a non-linear way. A term, depending of the size \( d \) of the blocks and the exponential radial falloff of the shower, has to be added to eq. (3.1) to correct from the correlation between \( x_c - x \) and \( x \) [10], the so-called S-curve. Many other formulas and algorithms have been discussed and compared in reference [12]. Lately, a simple method based on eq. (3.1) has been proposed and is still largely employed nowadays in many experimental analysis. It has been shown that it gives results similar or superior in quality to all those discussed previously and does not need position correction [14]. In this method the energy weight \( w_i \) is given by

\[ w_i = \max\left\{ 0; W_0 + \ln\left( \frac{E_i}{E} \right) \right\}, \tag{3.2} \]

where \( W_0 \) is a free dimensionless parameter. Consequently, only the blocks having an energy deposition larger than \( E e^{-W_0} \) are taken into account in the calculation of \( x_c \). In addition, \( W_0 \) allows to set the relative weight of the blocks, with a small energy deposition, used in the sum of eq. (3.1). Indeed, high \( W_0 \) values attribute almost an equal weight to the blocks entering the sum, while small \( W_0 \) values favor the highest energy blocks. It exists an optimal value \( W_0 \approx 4 \) giving the best position reconstruction [14] and corresponding to an energy deposition threshold in the cluster blocks equal to 1.8% \( E \).

![Figure 2](image.png)

**Figure 2.** Relative position resolution \( \sigma/d \), for \( d = r_M \) and \( E = 10 \text{GeV} \), as a function of \( W_0 \) (statistical uncertainties are smaller than the point size).

Actually the optimal \( W_0 \) value, called \( W_{0}^{CG} \), depends on the size \( d \) of the blocks as well as the energy \( E \) of the incident particle. Figure 2 shows a typical behavior of the position resolution \( \sigma \),
defined by the root mean square (RMS) of the \((x_c - x)\) distribution, as a function of \(W_0\). The minimal value of \(\sigma\), called \(\sigma^{cg}\), defines \(W^{cg}_0\) which is determined with an uncertainty of 0.1. Both \(W^{cg}_0\) and \(\sigma^{cg}\) are determined for each geometry and energy configuration previously described (section 2). Figure 3 shows the energy dependence of \(W^{cg}_0\) for different block sizes. At a given size, the reconstructed energy deposit in the blocks surrounding the central block becomes small and very sensitive to the sampling fluctuations of the shower when the energy \(E\) decreases. Including these blocks in the calculation of \(x_c\) could then degrade the resolution. These blocks are removed by small \(W^{cg}_0\) values as shown in figure 3. At given energy \(E\), \(W^{cg}_0\) increases as the block size \(d\) becomes larger. Indeed, for large blocks the energy of the shower is essentially deposited in the central block. Having a high threshold with small \(W_0\) excludes the remaining blocks and the \(x_c\) position becomes the central block coordinate with a resolution relative to the block size. An empirical parametrization of \(W^{cg}_0\) as function of \(d\) and \(E\) is proposed here following the expression

\[
W^{cg}_0 = \ln \left( \frac{100 \ E (\text{GeV})}{2.02 e^{-d/m} + [4.98 e^{-d/m} + 0.30] E (\text{GeV})} \right), \tag{3.3}
\]

where the three numerical constants are determined from a global fit of the 143 configurations. The relative uncertainty on the fitted parameters is less than 0.1%. Figure 3 shows the result of this parametrization for some particular values of \(d\).

The obtained resolution \(\sigma^{cg}\) is represented in figure 4 as function of \(E\) for different block sizes. The spatial resolution is sensitive to the number of calorimeter blocks used for the shower position reconstruction: it decreases with the block size \([15]\). In addition, \(\sigma^{cg}\) is expected to scale with \(1/\sqrt{E}\) at high energies \([16]\). The \(E\)-dependent \(\sigma^{cg}\) graphs for each block size are then fitted with the expression

\[
\frac{\sigma^{cg}}{d} = \frac{\alpha}{\sqrt{E}} + \beta, \tag{3.4}
\]
Figure 4. The relative resolution $\sigma_{\text{cg}} / d$ as a function of the shower energy $E$. The block size $d$ relative to the Molière radius is reported in the legend for each configuration. Dashed lines represent the parametrization of eq. (3.6).

Figure 5. The fitted coefficients $\alpha$ (left) and $\beta$ (right) as a function of $d$ (in $r_M$ units). Dashed lines represent the parametrization of eq. (3.5).

where $\alpha$ and $\beta$ are the free parameters of the fit. The resulting coefficients $\alpha$ and $\beta$ are represented on figure 5 as function of the block size. The $d$-dependence of $\alpha$ and $\beta$ can be parametrized as

$$\alpha = a_{\text{cg}} \left( \frac{d}{r_M} \right)^{b_{\text{cg}}}$$
$$\beta = c_{\text{cg}} \left( \frac{d}{r_M} \right) + f_{\text{cg}},$$

(3.5)

where $a_{\text{cg}}, b_{\text{cg}}, c_{\text{cg}}$ and $f_{\text{cg}}$ are four constants relative to the centre-of-gravity method. The global fit of the $\sigma_{\text{cg}}$ resolution for each configuration leads to the semi-empirical parametrization of the relative resolution

$$\frac{\sigma_{\text{cg}}}{d} = a_{\text{cg}} \left( \frac{d}{r_M} \right)^{b_{\text{cg}}} \sqrt{E (\text{GeV})} + c_{\text{cg}} \left( \frac{d}{r_M} \right) + f_{\text{cg}},$$

(3.6)

where $a_{\text{cg}} = 0.110, b_{\text{cg}} = -0.334, c_{\text{cg}} = 3.55 \times 10^{-3}$ and $f_{\text{cg}} = 5.02 \times 10^{-3}$. The relative uncertainty on these fitted coefficients is less than 1%. The results of this expression are shown in figure 4 for some particular block sizes. The resolution $\sigma_{\text{cg}}$ decreases at high energies and for small block sizes. However, the $d$-dependence of the relative resolution $\sigma_{\text{cg}} / d$ at fixed energy is more involved.
It is worth noting that eq. (3.6) suggests the existence of an optimum block size corresponding to an optimum relative resolution at a given $E$, allowing optimizing the calorimeter block size for a given energy measurement range.

4 Fit based method

![Figure 6. Average transverse shower profile function for $E=1$ GeV photons.](image)

A second approach for reconstructing the shower central position relies on the knowledge of the lateral profile of the electromagnetic shower. Figure 6 shows the profile function $F(x)$ obtained from the simulated response of an infinite lead fluoride calorimeter to $N_{\text{sim}}=10^5$ photons of 1 GeV. This response is represented as function of the distance from the shower centre along $x$-axis. The energy deposit is determined per generated photon, as the transverse dimension integral normalized by the initial photon energy

$$F(x) = \frac{1}{N_{\text{sim}}} \frac{1}{E} \int_{-\infty}^{+\infty} \frac{d^2E(x,y)}{dxdy} dy = \frac{1}{N_{\text{sim}}} \frac{1}{E} \frac{dE(x)}{dx}, \quad (4.1)$$

where $dE(x)$ is the energy released by the shower in a vertical column calorimeter centred at $x$, having a width $dx=1$ mm and an infinitely large height. This profile function turns out to be the same for photons and electrons and no energy dependence is expected according to the definition of the Molière radius. A bin-to-bin linear interpolation of the figure 6 histogram allows to define the profile function $F(x)$ which will be used to fit the calorimeter response and deduce the shower central position.

In the calorimeter described in section 2, the expected energy deposit for a given simulated event in a column $i$ identified by its central coordinate $x_i$ writes

$$E_i^{\text{exp}} = E \int_{x_i-d/2}^{x_i+d/2} F(x-x_c) \, dx, \quad (4.2)$$

where $x_c$ is the unknown shower central position, and $E$ is the incident particle energy corresponding to the total energy deposit in the calorimeter if we neglect the energy resolution effect. The clear
energy deposit in that column for the same event expresses

\[ E_i = \sum_j E_{ij} \Theta(E_{ij} - E e^{-W'_0}) , \]  

(4.3)

where \( W'_0 \) is a dimensionless parameter related to the energy threshold applied on the blocks, and \( \Theta(x)=1 \) when \( x > 0 \) and 0 otherwise is the Heaviside function. In the previous equation, \( E_{ij} \) represents the energy deposit in the block belonging to column \( i \) and row \( j \). Fitting for each event, with the MINUIT package \cite{17}, the \( E_i \) distribution (eq. (4.3)) with the expected distribution \( E_i^{\text{exp}} \) (eq. (4.2)) allows to extract the single free parameter of the fit \( x_c \). Following eq. (4.3), this algorithm rejects the blocks with energy deposit smaller than the threshold \( E e^{-W'_0} \). Figure 7 shows the influence of \( W'_0 \) on the obtained resolution \( \sigma \) given by the RMS of the \( (x_c - x) \) distribution for a particular value of \( E \) and \( d \). As expected, higher \( W'_0 \) values allow a larger number of blocks contributing to eq. (4.3) and then lead to a better relative resolution. However, the resolution remains constant after a certain \( W'_0 \) since very small energy deposit blocks do not contribute significantly to \( x_c \) calculation and consequently not change the fit result. The optimum \( W'_0 \), denoted \( W^\text{fit}_0 \) here-after, is defined as the value above which the resolution does not improve by more than 1%.

A logarithmic scaling of \( W^\text{fit}_0 \) with the block size is visible in figure 8, following the parametrization

\[ W^\text{fit}_0 = 1.61 \ln \left( \frac{d}{r_M} \right) + 5.55 . \]  

(4.4)

A small energy dependence of \( W^\text{fit}_0 \) is also observed in figure 8 but could be neglected. For large block size, the shower energy is mainly deposited in one single block so the energy threshold must be reduced to include the surrounding blocks in order to improve the spatial resolution.

The relative resolution of the fit method is shown in figure 9 as function of the shower energy \( E \) for different block sizes. As for the centre-of-gravity method, the resolution can be parametrized following the expression

\[ \frac{\sigma}{d'} = \frac{a^\text{fit} \left( \frac{d}{r_M} \right)^{b^\text{fit}}}{\sqrt{E(\text{GeV})}} + c^\text{fit} \left( \frac{d}{r_M} \right) + f^\text{fit} , \]  

(4.5)
Figure 8. Block size dependence of $W_0^{\text{fit}}$ for two different photon energies; the dashed line corresponds to the parametrization of eq. (4.4).

Figure 9. The relative resolution $\sigma_{\text{fit}}/d$ as a function of the shower energy $E$. The block size $d$ is indicated in the legend for each configuration. Dashed lines represent the parametrization of eq. (4.5).

where $a^{\text{fit}} = 0.121$, $b^{\text{fit}} = -0.349$, $c^{\text{fit}} = 1.78 \times 10^{-3}$ and $f^{\text{fit}} = 1.89 \times 10^{-3}$. These four constants are deduced from a global fit of the studied configurations.

In ref. [12], the shower profile was approximated by a single exponent term and a least-squares fit algorithm was developed for extracting the shower position taking into account all of the responding blocks of the calorimeter. To compare our results to the performance of such exponential approximation, a fit of the calorimeter response with the profile function $F(x) = \frac{1}{2b} e^{-|x|/b}$ was performed for the 143 configurations. The two free parameters of the fit are $x_c$ and $b$ related to the shower transverse width. The corresponding resolution $\sigma_{\text{fit}}^{\text{expo}}$ is compared in figure 10 to the resolution $\sigma_{\text{fit}}^{\text{real}}$ obtained with the more realistic profile function deduced from figure 6. The exponential profile appears systematically less performant than the realistic profile. At high energies, when the reconstructed energy deposit in the blocks surrounding the central block becomes sufficiently high and then accurate, a better knowledge of the profile function leads to a better position resolution.
Figure 10. Energy dependence of the relative difference between the position resolution obtained with a realistic shower profile and an exponential profile. The block size $d$ is indicated in the legend for each configuration.

The fit algorithm described in this section is obviously more computing time consuming at the data analysis level than the centre-of-gravity based method. Nevertheless, modern computers can easily and efficiently handle this problem: for instance, more than $10^4$ events per second can be analyzed with a modest 2 GHz processor.

5 Effect of the energy resolution

The resolution of the energy measurement per block originates solely from the shower sampling fluctuations in the current simulation approach. Experimentally, lead-fluoride calorimeters are based on the detection of Čerenkov light and the fluctuations in the number of collected photo-electrons dominates the energy resolution [16]. The generation and tracking of Čerenkov photons is not performed here because of unrealistic computing times and strong sensitivity to exact optical properties of crystal and wrapping surfaces which are known to differ from an experimental device to another. This effect can globally be parametrized by an additional smearing of the energy deposition $E_i$ in each block $i$. This is done by adding to $E_i$ a random number following a Gaussian distribution centred at zero and having the following width

\[ \sigma^s = s \sqrt{E_i}, \]

where $s$ is a constant related to the global relative energy resolution of the calorimeter. Several $s$-values in the range 1%-10% are used in the following to study the effect of the energy resolution on the position resolution. The degradation of the position resolution $\sigma$ is defined as

\[ \sigma^{\text{deg}} = \sqrt{\sigma^2(s) - \sigma^2(s = 0)}, \]

where $\sigma(s)$ is the position resolution at a given $s$-value, and $\sigma(s=0)$ is the position resolution in absence of energy smearing, both determined for any of the two methods. This study shows that $\sigma^{\text{deg}}$ is proportional to $s$. 

\[ - 9 - \]
Figure 11. \( \sigma^{\text{deg}}/(sd) \) as a function of \( 1/\sqrt{E} \) for the \( d = r_M \) configuration. The same behavior is observed for other configurations which are omitted for sake of clarity. The dashed line represents the parametrization of eq. (5.3).

Figure 12. The fitted parameter \( a^{\text{deg}} \) (eq. (5.3)) as a function of \( d/r_M \). The dashed line corresponds to the relation \( a^{\text{deg}} = 0.112 \left( \frac{d}{r_M} \right) + 1.12 \) and represents the first order polynomial fit of the distribution.

Figure 11 shows the typical energy dependence of the relative average resolution \( \sigma^{\text{deg}}/(sd) \) for a particular block size. As expected from eq. (5.1), the degradation of the position resolution can be parametrized at high energies \( (E > 5 \text{ GeV}) \) as

\[
\frac{1}{s} \frac{\sigma^{\text{deg}}}{d} = a^{\text{deg}} \sqrt{E},
\]

(5.3)

where \( a^{\text{deg}} \) is a parameter depending linearly on the block size \( d \) as shown in figure 12. Finally the obtained position resolution \( \sigma \) taking into account energy resolution effects can be expressed as

\[
\frac{\sigma}{d} = \frac{\sigma^{\text{cg,fit}}}{d} \oplus s \frac{0.112 \left( \frac{d}{r_M} \right) + 1.12}{\sqrt{E(\text{GeV})}},
\]

(5.4)

where \( \sigma^{\text{cg,fit}} \) is either given by eq. (3.6) or eq. (4.4) according to the chosen method of the position determination.
Figure 13. The relative contribution of the energy resolution to the final position resolution determined with the fit method for $s = 3\%$ (left) and $s = 8\%$ (right). Similar results are obtained with the center of gravity method. The block size $d$ is indicated in the legend for each configuration.

The energy resolution of lead-fluoride calorimeters is frequently better than $s=8\%$ [3, 4, 7, 18]. For example, the reported energy resolution in ref. [3] corresponds to $s=2.4\%$ for a calorimeter block size $d=1.4\ r_M$. The relative contribution of the second term of eq. (5.4) to the final position resolution $\sigma/d$ is shown in figure 13 for two particular $s$ values. For $s=3\%$ the contribution of this term is less than 10\% for any of the studied configurations. At higher $s$ values the effect of the energy resolution on the spatial resolution may become significant for large block sizes. Experimentally, other effects such as the physics and electronics background, energy calibration, radiation damage... can contribute to the energy resolution and then degrades the position resolution. However, in a well designed experiment these effects are not dominating the position resolution. Finally, the systematic uncertainty coming from the experimental knowledge of the coordinates $x_i$ of the block centres has to be added quadratically to eq. (5.4) to obtain the final position resolution but this contribution can generally be neglected, as connected to the accuracy of the mechanical design and mounting. In ref. [3], the obtained experimental resolution on the reconstructed position using the centre of gravity method is 2 mm, in good agreement with the 1.9 mm predicted by eq. (5.4).

6 Discussion

The centre-of-gravity and fit methods are compared in figure 14 for a typical distribution of the shower central position obtained at a given configuration with $d=r_M$ and $E=10\ \text{GeV}$, disregarding energy resolution effects discussed previously. In opposition to the fit determination, the centre-of-gravity determination of $x_c$ does not exhibit the constant behavior expected from the uniform generation of the shower $x$ origin. This effect can be understood with figure 15 which represents the difference between the reconstructed shower position $x_c$ and its true position $x$ as a function of $x$ for both studied methods. As reported in ref. [14], the centre-of-gravity method allows a significant improvement of the spatial resolution by decreasing the $S$-curve size relatively to the case $w_i=E_i$ in eq. (3.1). However, there is still some correlations between the reconstructed and the true position of the shower leading to figure 14 behaviour. This appears as a consequence of the number of blocks taken into account in eq. (3.1). It was found in this study that the average number of blocks entering eq. (3.1) is approximately 4 but could slightly differ depending on the configuration type.
Figure 14. Reconstructed shower position $x_c$ (in block size units) using the centre-of-gravity method (solid line) and the fit method (dashed line) for $d = r_M$ and $E = 10\text{ GeV}$.

Figure 15. Difference between the reconstructed shower position $x_c$ and the true position $x$ (in block size units) as a function of $x$ for $d = r_M$ and $E = 10\text{ GeV}$. $x_c$ is determined with the centre-of-gravity method (left) and the fit method (right). The dashed line represents the average points position when $w_i$ is equal to $E_i$ in eq. (3.1).

(for instance, 4.2 in the configuration shown in figure 14). Thus on average, two different block coordinates only are contributing to the determination of $x_c$ in eq. (3.1). The measured position is then biased towards the centre of the block containing the largest energy deposit. This restriction does not show-up in the fit method where the energy threshold for each block is always lower than the one used in the centre-of-gravity method. Indeed, $W^\text{fit}_0$ is always larger than $W^\text{cg}_0$ and allows a larger number of blocks to contribute to the determination of the shower central position. No correlations between the reconstructed and the true position of the shower are then expected for this method as shown in figure 15.

Figure 16 shows the relative difference between the spatial resolution obtained with the two previously described methods. For small block sizes and particle energies the shower fluctuations become important in the blocks surrounding the central block. The fit method includes a larger number of such blocks relatively to the centre-of-gravity method leading to a slightly degraded spatial resolution. However, when the block size comes closer or higher than the Molière radius, the fit method can provide as high as 40% better resolution at high energies.
Figure 16. Energy dependence of the relative difference between the spatial resolution of the centre-of-gravity and fit methods without energy resolution effects. The block size $d$ is indicated in the legend for each configuration.

Figure 17. Energy dependence of the relative difference between the spatial resolution of the centre-of-gravity and fit methods for calorimeter energy resolution $s=3\%$ (left) and $s=8\%$ (right). The block size $d$ is indicated in the legend for each configuration.

Figure 17 shows similar results when the energy resolution effect is included as described in section 5. For small and reasonable experimental $s$ values ($\sim 3\%$) the effect of the energy resolution on the spatial resolution is negligible and the gain brought by the fit method is preserved. For large $s$ values ($\sim 8\%$) this gain decreases but remains significant (5-30\%) at high energies ($E > 5$ GeV) and when $d \geq r_M$.

Within the present work, particles with normal incidence only are considered whereas experimentally one can have different incident angles. It has been shown that the position resolution obtained with the centre-of-gravity method is rather insensitive to this angle for moderate values [14]. For large incidence angles, a simulated optimizable geometrical correction depending on the shower depth must be taken into account since the impact position of the particle is shifted relatively to the shower central position [19, 20]. Similarly, different lateral shower profile functions depending on the incidence angle can be simulated and exported within the fit method to obtain the shower position and the corrected impact position.
7 Conclusion

The present work discussed the determination of the shower central position in a laterally segmented electromagnetic calorimeter following the widely used centre of gravity method and a new fit method here developed. A semi-empirical parametrization of the relative position resolution as a function of the incident particle energy and the calorimeter block size is proposed and optimized for each case. Energy resolution effects on the position resolution are also discussed and quantified. The fit method is shown to improve by 5-30% the position resolution as compared to the centre-of-gravity method for incident particle energies higher than 5 GeV and calorimeter block sizes equal to or larger than the Molière radius.

Despite the present application to lead-fluoride calorimeters, the results presented here can be exported to several electromagnetic calorimeter materials through their parametrization in terms of the Molière radius. In addition, as far as simulations are concerned, the profile function at the heart of the fit method has energy and material universality features. This global study not only provides useful parametrization for the design optimization of electromagnetic calorimeters in future experiments looking for a good spatial resolution, but procures also a new algorithm to improve position resolution at the data analysis level.

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