Analyzing travel time belief reliability in road network under uncertain random environment

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Abstract
The reliable operation of urban road traffic is the vision of urban development. The reliability measurement method with travel time as a performance index is the starting point of this study. Based on the travel time data, we have studied the shortcomings of existing travel time reliability indicators. Most of them simplify or even ignore the information of traffic performance thresholds. According to the characteristics of the actual urban road network, we proposed measurement of travel time reliability under uncertain random environment, that is, the travel time belief reliability. By extracting the information of traffic service subject and object, this method takes into account the influence of cognitive and random uncertainty on reliability. Next, we established the belief reliability model of travel task under the uncertain random road environment. The model considers the route selection, departure status and road conditions and gives the route selection algorithm under a time-varying road network. In addition, the impact of road objective factors and driving state factors on the travel time threshold is discussed by using the uncertainty regression analysis method. Finally, we took the actual travel task in Beijing as an example to verify the feasibility and practicability of the model and algorithm.

Keywords Uncertainty quantification · Travel time reliability · Belief reliability · Uncertain random system

1 Introduction

In recent years, urban road traffic has become more and more complex. Travelers not only have connectivity and short-term requirements for road traffic services, but also pay more and more attention to travel reliability. Sometimes, they even sacrifice travel time to ensure high travel reliability. Travel time reliability is an indicator that measures the stability of travel time from the perspective of time, which reflects the resilience of the road network under random and fluctuating traffic conditions. The reliability of travel time is very important to travelers, especially commercial transporters. It can help them manage their time effectively and even increase their income.

Based on different definitions of travel time reliability, many different quantifiable measurement standards have been proposed. According to the calculation method, they can be divided into two categories: statistical measures and probability measures.

Statistics refers to the function of samples without unknown parameters. Statistical measurement is the general term for a series of indicators obtained by statistical analysis of travel time data. Many researchers put forward the reliability indicators of travel time which are statistical measures. Herman and Lam (1974), Sterman and Schofer (1976), Bell and Iida (1997) and Bates et al. (2001) take the variability of travel time as a reliability index, that is, using divergence to measure the stability of travel time. Standard deviation (Hollander 2006, 2010), coefficient of variation (Sun et al. 2016) and the inverse of standard deviation (Sterman and Schofer 1976) are representative indexes. The buffer time is a waste of time caused by passengers who want to arrive on time and leave early. The buffer time is usually calculated by subtracting the average value from 95% or 90% of the travel time (Sun 2018; Xiong 2006; Zegeer et al. 2014). Tardy trips measure
threshold (Xiaofei et al. 2014). In the previous literature, the threshold was often defined as the quantile of travel time or is to determine the acceptable time. In previous studies, the time. It is worth noting that the key point of these measures provides more intuitive reliability indexes at the same time. For example, a standard deviation value of 5 may confuse the traveler. When the buffer time is 30 min, it is suggested that starting 30 min earlier is more reliable but it does not directly reflect the reliability of the trip completion.

The probabilistic measurement is based on probability theory. They define travel time reliability as the probability that the travel can be completed within the acceptable time under a certain service level of road network service (Asakura and Kashiwadani 1991; Zhihua et al. 2004).

Compared with statistical measures, probabilistic measures retain the measure of travel time volatility, while providing more intuitive reliability indexes at the same time. It is worth noting that the key point of these measures is to determine the acceptable time. In previous studies, the threshold was often defined as the quantile of travel time or standard. However, this is a helpless method since it lacks information on the service level of the road network in its definition, and also ignores information on traveler acceptability, which will affect the acceptable time threshold (Xiaofei et al. 2014). In the previous literature, these travel time reliability measures were generally analyzed through travel time data. They simplified or ignored the road facility factors and the traveler’s travel status factors. However, one of them is the main body of the urban road traffic system, and the other is the object of service. Both of them are indispensable in the reliability measurement of the traffic network system.

Then, we further analyze the service characteristics of the urban road traffic network. On the one hand, with the random fluctuation of travel demand, there is inherent randomness in the system. The randomness is represented by the fact that the travel time will be different even under the same travel conditions. On the one hand, due to the random fluctuations in travel demand, there is inherent randomness within the system, which is intuitively manifested in the fact that travel time will be different even under the same travel conditions. On the other hand, the main service target of the transportation network is travelers, whose cognition of the transportation network and travel status will affect the reliability measurement. For example, people with different travel purposes will have different evaluations on the reliability of the same road. The objective factors of the road and the subjective factors of travelers make the reliability measurement epistemically uncertain. In the uncertain random environment, we need a new reliability measure that takes into account epistemic uncertainty and random uncertainty.

Therefore, this paper uses a mathematical theory called uncertainty theory to measure the travel time reliability. The uncertainty theory was founded in 2007 by Liu (2010) and then improved by many studies. The general purpose of the theory is to rationally deal with belief degrees evaluated by experts or related personnel in the field. Belief degree is a way to describe an indefinite quantity, and its counterpoint is frequency generated by samples related to the probability theory.

This new travel time reliability measurement will consider some factors in the urban road network, including road infrastructure, fuzzy perception of traveler information, etc. Through analyzing these factors, we try to make up for the deficiency of the existing reliability index information extraction. In addition, the evaluation index using the measurement of traveler’s trust can reflect the happiness of travelers to a certain extent and provide a statistical reference for urban traffic management.

The uncertainty theory has been viewed as an appropriate mathematical system to model epistemic uncertainty and applied in various fields, including statistical analysis, risk analysis, differential equations, optimization problems, etc. In 2010, Liu first described reliability as an uncertain measure mathematically and provided some formulas about the reliability of Boolean systems. Later, Zeng et al. (2013) named it the belief reliability and proposed an analysis method for unit level and product level. Then, the chance theory was proposed by Liu (2013b). It is a combination of uncertainty theory and probability theory, which can be used to analyze systems with random uncertainty and cognitive uncertainty. In 2019, Kang (2020) finally proposed a theoretical framework for belief reliability.

This paper assumes that the urban road network is an uncertain random network, that is, only study the epistemic uncertainty and random uncertainty of travel time performance.

This paper then introduces the basic definitions and theorems of uncertainty theory and chance theory. In Sect. 3, we give the definition and calculation formula of travel time belief reliability. In Sect. 4, we establish a travel time model for travel tasks in the urban road network. Then, the uncertainty regression model of the travel time threshold is established in Sect. 5. Finally, we give an example to verify the travel time belief reliability measure.
2 Preliminaries

In this section, we will introduce some basic definitions and theorems of uncertainty theory and chance theory, which are the mathematical basis of the travel time reliability. In addition, the uncertain regression models and parameter solving methods are introduced which guides the establishment of the following model.

2.1 Uncertainty theory and chance theory

The uncertainty measure represents the belief degree of the occurrence of an event, and the measure satisfies normality, duality, subadditivity axioms and product axioms.

Definition 2.1 (Uncertain variable) (Liu 2007) An uncertain variable is a function $\xi$ from an uncertainty space $(\Gamma, L, M)$ to the set of real numbers such that $\{\xi \in B\}$ is an event for any Borel set $B$ of real numbers.

Definition 2.2 (Uncertainty distribution) (Liu 2007) The uncertainty distribution $U$ of an uncertain variable is defined by

$$
\Phi(x) = M\{\xi \leq x\}
$$

for any real number $x$.

To deal with the random uncertainty and epistemic uncertainty in complex systems, this paper will use the chance theory which was proposed by Liu (2013b). The chance measure is defined based on the chance space as follows:

Definition 2.3 (Chance space) (Liu 2010) Let $(\Gamma, L, M)$ be an uncertainty space and let $(\Omega, A, Pr)$ be a probability space. Then, the product $(\Gamma, L, M) \times (\Omega, A, Pr)$ is called a chance space.

Definition 2.4 (Chance measure) (Liu 2010) Let $(\Gamma, L, M) \times (\Omega, A, Pr)$ be a chance space and let $A$ be a event and let $\Theta \in L \times A$ be an event. Then, the chance measure of $\Theta$ is defined as

$$
Ch(\Theta) = \int_0^1 Pr\{\omega \in \Omega \mid M\{\gamma \in \Gamma \mid (\gamma, \omega) \in \Theta\} \geq x\}dx.
$$

Definition 2.5 (Uncertain random variable) (Liu 2010) An uncertain random variable is a function $\xi$ from a chance space $(\Gamma, L, M) \times (\Omega, A, Pr)$ to the set of real numbers such that it is an event in $L \times A$ for any Borel set $B$.

Definition 2.6 (Chance distribution) (Liu 2010) Let $\xi$ be an uncertain random variable. Then, its chance distribution is defined by

$$
\Phi(x) = Ch\{\xi \leq x\}
$$

for any $x \in R$.

To obtain the chance distribution of the uncertain random variable, some operational laws will be used:

Theorem 2.1 (Liu 2010) Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. If the function $f(x_1, x_2, \ldots, x_n)$ is strictly increasing with respect to $x_1, x_2, \ldots, x_m$ and strictly decreasing with respect to $x_{m+1}, \ldots, x_n$, then the uncertain variable

$$
\xi = f(\xi_1, \xi_2, \ldots, \xi_n)
$$

has an inverse uncertainty distribution.

$$
\Psi^{-1}(x) = f(\Phi_1^{-1}(x), \ldots, \Phi_n^{-1}(x), \Phi_{m+1}^{-1}(1-x), \ldots, \Phi_n^{-1}(1-x)).
$$

Theorem 2.2 (Liu 2013a) Let $\eta_1, \eta_2, \ldots, \eta_m$ be independent random variables with probability distributions $\Psi_1, \Psi_2, \ldots, \Psi_m$, respectively, and let $\tau_1, \tau_2, \ldots, \tau_m$ be uncertain variables (not necessarily independent). Then, the uncertain random variable

$$
\xi = f(\eta_1, \eta_2, \ldots, \eta_m, \tau_1, \tau_2, \ldots, \tau_m)
$$

has a chance distribution

$$
\Phi(x) = \int_{\Omega^n} M\{f(\gamma_1, \gamma_2, \ldots, \gamma_m, \tau_1, \tau_2, \ldots, \tau_m) \leq x\}d\Psi_1(\gamma_1) \ldots d\Psi_m(\gamma_m)
$$

for any number $x$.

2.2 Uncertain regression analysis

Uncertain regression analysis is a statistical analysis method for uncertain variables, and it is mainly used to analyze the relationship between dependent variables and independent variables.

Let $(x_1, x_2, \ldots, x_p)$ be a vector of explanatory variables, and let $y$ be a response variable. Suppose the relationship between $(x_1, x_2, \ldots, x_p)$ and $y$ can be expressed by a function $f(\cdot)$. That is, the regression model is given as

$$
y = f(x_1, x_2, \ldots, x_p | \beta) + \varepsilon
$$

where $\beta$ is a vector of unknown parameters and $\varepsilon$ is a disturbance term.

Theorem 2.3 (Lio and Liu 2020) Let $(x_{i1}, x_{i2}, \ldots, x_{ir}, y_i), i = 1, 2, \ldots, n$, be a set of observed values, and let the regression model be

$$
y = f(x_1, x_2, \ldots, x_p | \beta) + \varepsilon
$$

where $\varepsilon$ is a normal uncertain variable $N(\varepsilon, \sigma)$ with unknown parameters $\varepsilon$ and $\sigma$. Then, the maximum
likelihood estimator \( (\beta^*, e^*, \sigma^*) \) solves the maximization problem,

\[
\max_{\beta, e, \sigma} L(\beta, e, \sigma|z_1, z_2, \ldots, z_n)
\]

where

\[
L(\beta, e, \sigma|z_1, z_2, \ldots, z_n) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left( \frac{n}{\sqrt{2\pi}\sigma} \sum_{j=1}^{n} (\frac{e - z_i}{\sigma})^2 \right)
\]

and \( z_i = y_i - f(x_{i1}, x_{i2}, \ldots, x_{ip} | \beta), i = 1, 2, \ldots, n. \)

Furthermore, \((\beta^*, e^*)\) solves the minimization problem,

\[
\min_{\beta, e, \sigma} \frac{n}{\sqrt{2\pi}\sigma} \sum_{j=1}^{n} (\frac{e - y_i + f(x_{i1}, x_{i2}, \ldots, x_{ip} | \beta)}{\sigma})^2
\]

and \(\sigma^*\) solves the maximization problem,

\[
\max_{\sigma > 0} \frac{n}{\sqrt{2\pi}\sigma} \sum_{j=1}^{n} (\frac{e - y_i + f(x_{i1}, x_{i2}, \ldots, x_{ip} | \beta^*)}{\sigma})^2
\]

3 Travel time belief reliability measure

In this section, we will introduce a new measure of travel time reliability under an uncertain random environment called travel time belief reliability. We will give calculation formulas and numerical solution algorithms for the new indicators.

3.1 Performance analysis

Travel time is one of the most common and intuitive indexes in the transportation system. At present, the traffic administrative department has many advanced technologies and detection instruments, so it is not difficult to obtain travel time data. Using big data mining methods to process these data is also a research hot spot.

We assume that travel time is a random variable. With such a large sample size, it can be considered that the frequency histogram of travel time is very close to the real density curve, which is in line with the premise of applying probability theory.

For travelers, reliable service is that the travel time is within their acceptable range. It is worth noting that this range may not necessarily have a lower limit, because the shorter the travel time, the better the travelers. Even if there is a lower limit, people are more willing to accept the loss of travel time below the lower limit. In this paper, the upper limit of the acceptable time range is called the travel time threshold. The travel time threshold is a variable affected by many external factors, such as traveler’s perception of the road, the choice of departure time and the objective state of the road for travel purpose. This also makes the travel time threshold complicated, and our knowledge of its true value is insufficient. In addition, human participation makes the information we collect about the threshold is subjective. These lead to epistemic uncertainty in the model. Therefore, we assume that the travel time threshold is an uncertain variable.

We define \( t \) as the travel time and notate the accepted travel time threshold as \( T \). The travel time is a smaller-the-better (STB) parameter (Zhang et al. 2018), which means the trip will fail when travel time is higher than the threshold. Then, the travel time reliable criterion is as follows:

\[
t_a < T_a.
\]

Namely, the trip is reliable for travelers when the criterion is met, and the failure criterion is the opposite, which is the same as the reliability of the engineering industry.

3.2 Travel time belief reliability (TTBR)

According to the above performance analysis, the definition of travel time belief reliability is given as follows:

**Definition 3.1** Travel time belief reliability (TTBR) Travel time belief reliability is a chance measure that travelers can reach their destination within their acceptable time threshold, the mathematical expression of travel time belief reliability as follows:

\[
R_B = Ch\{t < T\}
\]

where \( t \) is the travel time and obeys the probability distribution \( F(t) \). \( T \) is the travel time threshold and obeys the uncertainty distribution \( \Phi(t) \).

Then, the travel time belief reliability can be calculated as the formula:

\[
R_B = Ch\{t \leq T\} = Ch\{t - T \leq 0\} = \Psi(0).
\]

According to the chance theory and Theorem 2.2 mentioned in parts 2.1 and 2.2, we have the chance distribution:

\[
\Psi(x) = Ch(t - T \leq x) = \int M\{t - T \leq x\}dF(t)
\]
where $t - T$ is the performance margin of travel time.

When analyzing the urban route network, a travel task may involve multiple road sections or multiple alternative routes. At this time, the theoretical calculation may be difficult. We also give a numerical calculation method based on the structure–function as follows:

### Algorithm: Uncertain Random Simulation

Input: $t_i : F_i(t), T_s : \Phi_s(\tau)$

Output: $R : R = C_h\{f(t_i) - T_s \leq 0\}$

for $j \leq N_{\text{simulation}}$ do

Generate randomly samples $t_1^{(j)}, t_2^{(j)}, \ldots, t_n^{(j)}$;

Calculate the uncertain measure $M_j = M\{T_s - f(t_i) \geq 0\}$;

return $R = \sum_{j=1}^{N_{\text{simulation}}} M_j$

4 Distribution of travel time in the urban road network

In this section, we consider the real travel situation and analyze the travel time of a certain travel task, that is, from an origin to a destination. In the following, we will call it an OD pair. Then, we use a more sensitive goodness-of-fit test to determine the probability distribution function of the travel time.

4.1 Improved $K$ shortest path choice algorithm of the urban road network

In practical travel, travelers have to face the traffic information of the whole region. The route to the destination may be different because the travelers’ choices are not fixed. So route choice is important to work. Once the path is determined, the network problem can be reduced to an OD pair problem by reducing the dimension. However, the traffic network is a classical time-dependent network; that is to say, its state changes over time. We establish the $k$ shortest path model for this time-dependent random network.

We assume a traffic network with roads as edges and intersections as nodes. Travelers have an exact starting point and destination, which can be represented by network nodes. Moreover, travelers are provided with at least $k$ routes to their destination before departure, and they are sensible decision makers whose goals are to keep the trip as short as possible.

Some symbols are expressed as follows:

$(V, E \times T, G)$: the traffic network;

$V$ : the collection of $n$ nodes;

$E$ : the collection of $N$ links;

$T = [t_0, t_n]$ : the valid span on which we concerned;

$G = (g_{ij})$ : the matrix of the side weight function;

$g_{ij}(t)$ : travel time from node $i$ to $j$ at departure time $t$;

$f_{i}(t)$ : the travel time from node $v_i$ to end node $v_N$ departed at $t$;

$A_{ij}^{-1}$ : a collection of start nodes $v_i$ of the link $e_{ij}$;

$\text{Succ}(i) = j$: the successor node $v_i$ is $v_j$.

Dividing $T$ into $M$ parts by tiny interval time $\Delta$, let $S = \{t_0, t_0 + \Delta, t_0 + 2\Delta, \ldots, t_0 + (M - 1)\Delta\}$. When $\Delta$ is small enough, the expected value of each interval can be used to replace the travel time, which can also reflect the time dependence of the traffic network.

A $k$ shortest paths (KSP) algorithm based on improved Dijkstra (Fen et al. 2008) is proposed, which can get the $k$ shortest travel time paths in the sense of expected value.
Algorithm: Improved KSP Algorithm

Part I: Dijkstra algorithm of time-dependent stochastic networks

Input: \( f_{i,j} = \begin{cases} \infty & i \neq N \\ 0 & i = N \end{cases}, \ \text{List} = \{v_N\} \)

Output: \( \text{Succ} \)

While \( \text{List} \neq \emptyset \) do

Choose \( v_j = \text{List}(1) \)

For \( \forall v_i \in A_{ij} / v_j \) do

For \( \forall t \in S = \{\tau_0, \tau_0 + \Delta, \tau_0 + 2\Delta, \ldots, \tau_0 + (M-1)\Delta\} \) do

If \( f_i(t) > g_j(t) + f_j(t + g_j(t)) \) do

\( f_i(t) = g_j(t) + f_j(t + g_j(t)) \)

\( \text{succ}(i) = j \)

\( \text{List} = \{\text{List}, v_j\} \)

return \( \text{Succ} \)

Part II: Find K-shortest paths

Input: \( \text{Graph}(V, E \times T, G) \), origin node and destination node

Output: \( K \)-shortest travel time paths

Get the shortest path \( P_k(k=1) \) based on the improved Dijkstra algorithm

For \( k = 1, 2, \ldots, K \) do

For \( \forall v_i \in P_k / v_n \)

Let \( g_{i,0} = \infty \)

Get the shortest path \( P_i' \) from \( v_i \) to \( v_n \)

\[ P_k = \min \left\{ P_i' \right\} \]

return \( P_k(k=1, 2, \ldots, K) \)
Table 1 Administrative division of road grades

| Road grade | Road level |
|------------|------------|
| 1          | Intercity highway |
| 2          | Urban expressway |
| 3          | National highway |
| 4          | Provincial road |
| 5          | County road |
| 6          | Township road / other |

Through the above algorithm, we can get $K$ shortest travel time paths in the sense of expected value.

Systematically, the $K$ paths are in parallel, and the margin of stroke time is the difference between the minimum stroke time and the threshold. Therefore, the travel time of the travel network is

$$t_i(\tau) = \min[t_i(\tau)](i = 1, 2 \ldots, k)$$  \hspace{1cm} (18)

where $i(i = 1, 2 \ldots, k)$ means the traveler chooses the $i$th path and $\tau$ is the departure time.

4.2 Cumulative effect of travel time

In real situations, the entire transportation network changes dynamically over time. Travel is a time-dependent behavior. Every time a traveler finishes a link of the path and reaches the next link, the time will be cumulated, and the state of the two links will change.

We assume that the $i$th path is composed of some independent links. The travel time of each link is known, but it cannot be added directly. Assume that the traveler’s departure time is $\tau_0$. Then, the travel time of the $i$th path can be calculated by

$$t_i(\tau_0) = t_{i1}(\tau_0) + t_{i2}(t_{i1}) + \tau_0 + \cdots + t_{in}(\sum_{j=1}^{n-1} E(t_{ij}) + \tau_0)$$  \hspace{1cm} (19)

where $t_{ij}(i = 1, 2 \ldots, k; j = 1, 2 \ldots n_i)$ is the travel time of link $j$ in $i$th path, which obeys the random distributions $F_{ij}(t)$, and $t_{ij}(\tau)$ represents the travel time through link $j$ at time $\tau$.

4.3 Goodness-of-fit test of travel time distribution

To determine the travel time probability distribution, this paper uses the nonparameter test method to find the probability distribution function of travel time. First, we divided the 24 h of a day into n parts by 1440/n minutes and collected the travel time of vehicles leaving within 1440/i(i = 1, 2 \ldots, n) minutes. The observed samples $t_{i1}, t_{i2}, \ldots, t_{iN}$ are travel time of a road in the same $i$th minutes of $N$ days. Through the statistical analysis of these samples, we can get the daily travel time distribution pattern at the time 1440/i.

There are various distributions to model travel time in different situations (Aron et al. 2014). The lognormal distribution, the gamma one, the Weibull one and the burr one are used (Al-Deek and Emam 2006; Lu and Dong 2018; Pu 2011; Taylor 2012). We use the za.test function in the “DistributionTest” package in $R$ to test travel time probability distribution function, which is more sensitive and accurate to the drift of normal distribution and logarithmic normal distribution (Zhang 2002). If the test $P$-value passes the test, we will get the travel time distribution of one road.

Finally, in the dynamic urban road network, the travel time distribution of a particular travel task is as follows:

$$F_i(t) = 1 - \prod_{i=1}^{K} (1 - F_i(t)).$$  \hspace{1cm} (20)

And according to the convolution formula, the $F_i(t)$ is as follows:

$$F_i(t) = F_{i1}(t) * F_{i2}(t) * \ldots * F_{ij}(t) * \ldots * F_{in}(t).$$  \hspace{1cm} (21)

5 Uncertain regression model of travel time threshold

This section attempts to establish an uncertain regression model based on travel time thresholds of different departure times.

In this uncertain regression model, the independent variables are the objective state and travel states of the road, and the dependent variable is the travel time threshold evaluation given by the traveler. After a series of inquiries on historical travelers’ tolerance for travel time on this road, many empirical data were obtained. However, these people cannot give a very accurate description, and even different people may have different reliability. What they said contained real information about the actual threshold.

We choose the eight factors: road slope, traffic speed, length, bus stop, traffic lights, intersections, nearby commercial buildings and whether to travel during peak hours. Then, we collect some sample data through research. It is worth noting that the traffic speed data we collect may be inaccurate, but this will not lead to mathematical modeling problems because uncertain regression can be analyzed based on inaccurate data (Yao and Liu 2018).

Some symbols are used in the model, and they are described as follows:
• $x_1$: road grade, which is determined by the road design department and based on the administrative distinction, see Table 1 for specific grades;
• $x_2$: the road length;
• $x_3$: the number of bus stations on the road;
• $x_4$: the number of traffic lights on the road;
• $x_5$: the number of commercial buildings near this road;
• $x_6$: the number of intersections;
• $x_7$: the average speed of vehicles on the road;
• $x_8$:

$$x_8 = \begin{cases} 
1 & \text{Peak travel} \\
0 & \text{Off-peak travel} 
\end{cases}$$

Assume that the travel time threshold $T$ follows a normal uncertainty distribution. The sensitivity of travelers to the departure time, and the peak and off-peak trips are considered separately.

An uncertain linear regression model with seven factors and the travel time threshold is established:

$$T = \hat{\beta}_0 + \sum_{j=1}^{7} \hat{\beta}_j x_j + \epsilon$$  \hspace{1cm} (22)

where $\epsilon$ obeys the normal uncertain variable $\mathcal{N}(0, \sigma)$, $\hat{\beta}$ solves the minimization problem,

$$\min_{\beta, \epsilon} \left\{ \sum_{i=1}^{n} \left| T_i - \beta_0 - \sum_{j=1}^{7} \beta_j x_j \right| \right\}$$  \hspace{1cm} (23)

$\hat{\epsilon} = 0$ and $\hat{\sigma}$ solves the maximization problem,

$$\max_{\sigma > 0} \frac{-n}{\sqrt{3}\sigma} \exp \left( \frac{-n}{\sqrt{3}\sigma} \sum_{i=1}^{n} \left| T_i - \beta_0 - \sum_{j=1}^{7} \beta_j x_j \right| \right).$$  \hspace{1cm} (24)

According to Theorem 2.3, the maximum likelihood estimation of the parameters can be obtained, and the uncertainty regression equation of the travel time threshold and its uncertainty distribution can be obtained:

$$T = \hat{\beta}_0 + \sum_{j=1}^{7} \hat{\beta}_j x_j + \epsilon$$  \hspace{1cm} (25)

where $\epsilon \sim \mathcal{N}(0, \hat{\sigma})$ and $\hat{\sigma}$ satisfies formula (24).

Finally, we perform residual analysis to obtain the uncertainty distribution function of the travel time threshold:

$$T \sim \begin{cases} 
\mathcal{N}_1 \left( \hat{\beta}_0 + \sum_{j=1}^{7} \hat{\beta}_j x_j, \hat{\sigma} \right) & x_8 = 1 \\
\mathcal{N}_0 \left( \hat{\beta}_0 + \sum_{j=1}^{7} \hat{\beta}_j x_j, \hat{\sigma}' \right) & x_8 = 0 
\end{cases}$$  \hspace{1cm} (26)

### 6 Case study

In this section, we try to analyze the reliability of a specific travel task from the starting point to a destination. The actual map of the area we studied is shown in Fig. 1, and the blue dots are the traffic detectors. Furthermore, we collected travel speed data from July 27 to July 31 through these detectors.

Firstly, we established the traffic network through the longitude and latitude data of the detectors and road geographic information. The simplified network is shown in Fig. 2. The purpose of this study is to acquire the travel time belief reliability from the origin (116.3328,39.9929) to the destination (116.3533,39.9765) in a working day. The distance between these two locations is about 4 kms.

We collected the speed data between many monitoring points and preprocessed these data statistically to calculate the road travel time data. Furthermore, we studied the travel time of 288 periods by dividing 24 h into 5-minute intervals. Then, we use the za.test function in the “DistributionTest” package in R to test the travel time probability distribution function. Finally, the travel time distribution function of each route and each time period was obtained.

Next, we analyzed the travel time of all roads in the road network and used algorithm 1 to determine the real-time optimal route choices from the origin point to the destination in the sense of expected travel time. The travel demands of travelers in a whole day will change with time, which will lead to the change of the traffic capacity of the road network. In actual travel, the travelers will also choose different routes according to the departure time (Table 2).

In this real case study, we can collect road information through field trips and maps. We collect data on the 11 shortest paths and important roads of the region in different periods. The main factors include road grade, road length, number of bus stops on the road, the number of signal lights, number of surrounding entertainment buildings, number of intersections and average traffic speed. In particular, a path may consist of roads of multiple grades, and here, we use the average grade of the road as the level of the entire path. For the travel time threshold, we surveyed 20 people who had travel experience in this area through a questionnaire and collected their acceptable time data for this travel at different departure times. Our observations are...
inaccurate, and the above indicators are all uncertain variables. Based on the data above, for the study area, we established an uncertain regression model of travel time thresholds for different departure times (Table 3).
Based on the state data of important roads in the area, we fitted the uncertainty regression equation of the travel time threshold near Beihang University in the Haidian District. The residual analysis of the above two uncertain regression models can obtain the uncertainty distribution of the residual term:
\[ e_1 \sim \mathcal{N}(0, 1.50) \]
\[ e_0 \sim \mathcal{N}(0, 0.65). \]

Then, the forecast uncertain variable of \( T \) concerning \( \bar{x}_1, \bar{x}_2, \ldots, \bar{x}_7 \) is determined by

\[
T = \begin{cases} 
-0.3786x_1 + 0.7059x_2 - 0.0388x_3 + 0.1281x_4 + 0.8865x_5 - 0.0163x_6 + 0.0826x_7 + 0.4782 \text{ peak} \\
-0.3783x_1 + 0.5451x_2 + 0.1112x_3 + 0.0550x_4 + 0.7823x_5 + 0.0993x_6 + 0.0549x_7 + 0.1413 \text{ off-peak}.
\end{cases}
\]

The residual analysis of the above two uncertain regression models can obtain the uncertainty distribution of the residual term:
\[ e_1 \sim \mathcal{N}(0, 1.50) \]
\[ e_0 \sim \mathcal{N}(0, 0.65). \]
The travel time threshold $T$ will be estimated by the above, and it is natural to define the uncertain expect:

$$
\mu_1 = \hat{\beta}_0 + \sum_{i=1}^{7} \hat{\beta}_i E[\tilde{x}_i] \\
\mu_0 = \hat{\beta}_0 + \sum_{i=1}^{7} \hat{\beta}_i E[\tilde{x}_i].
$$

The uncertainty distribution of the travel time thresholds for these two different departure times is obtained:

$$T \sim \begin{cases} 
\mathcal{N}(6.2359, 1.50) & \text{peak} \\
\mathcal{N}(6.5476, 0.65) & \text{off} - \text{peak}. 
\end{cases}
$$

We use hypothesis testing and uncertainty regression analysis to obtain the random distribution of travel time and the uncertainty distribution of travel time threshold. Furthermore, according to formulas 32 and 33, we can obtain an analytical solution for travel time reliability. Since calculating the travel time probability distribution in this example involves multiple convolutions, the calculation process is complicated, so we use an uncertain stochastic simulation algorithm to obtain a numerical solution. The simulation scale is 10,000. Finally, the change of the OD’s travel time belief reliability with departure time is shown in Fig. 3.

Through this example, we analyzed the road traffic conditions in a certain area of Haidian, Beijing. There are two periods of low reliability in a day: morning peak and evening peak, and there will be a sharp decrease in the confidence reliability of the evening peak, which is obviously in line with common sense, but under this evaluation standard, the fluctuation curve is sawtooth, and the stability of the reliability in this area is worth improving. In addition, according to the empirical equation of travel time threshold, it can be seen that road length and surrounding entertainment buildings are important factors that affect travelers’ expected travel time.

7 Conclusion

This study analyzed the existing travel time reliability of urban traffic and proposed a new measurement—travel time belief reliability. In the previous literature, the travel time threshold is either a fixed window width or defined as the quantile of the probability travel time reliability index. We consider epistemic factors such as the uncertainty of the traveler’s knowledge of road condition information. The travel time threshold is assumed to be a variable. We have also established an uncertain regression model of the travel time threshold, which can be used to analyze the impact of people’s perception of objective road factors and operating conditions on the evaluation of travel time threshold. The regression equation can be used as the empirical equation of the travel time threshold on working days near the study area. Based on the reliability definition under the definition of performance margin, we give a new
definition of travel time reliability and give the calculation formulas for the unit level and task level. Especially for more complex travel task structures, we also give numerical calculation methods for uncertain stochastic simulation.

The method based on uncertainty theory and chance theory has a more credible basis in the establishment and analysis of the reliability model. It considers the impact of travelers’ understanding of the road and the objective state of the road on reliability and tries to solve the regret of insufficient information extracted by the existing reliability indicators by analyzing the uncertain random environment. On the one hand, the travel time reliability measurement based on belief data can reflect the well-being degrees of travelers. On the other hand, it provides pedestrians and urban traffic management departments with an intuitive index that is more consistent with the actual traffic environment which is beneficial for travel planning and urban road congestion management.

When using travel time to ensure reliability, we need to pay attention to the problem of data collection: The use of fixed detectors to obtain road section travel time data is one of the common methods, and this is how the data in the case are obtained. To quantify the travel time threshold, in addition to obtaining the data of the transportation infrastructure in the area, it is very important to collect the experience data of the travelers. Travelers’ perception of regional road conditions and the real-time changes of the road conditions are the contents of uncertain quantification in this paper.

Therefore, in future research, we will further analyze the objective factors of the road, extract more relevant features and optimize the empirical function of the travel time threshold. In addition, we can also evaluate the overall belief reliability of the urban road network and provide parameter support for solving the bottleneck problem of traffic congestion.

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