THE PERFORMANCE OF COUNT PANEL DATA ESTIMATORS: A SIMULATION STUDY AND APPLICATION TO PATENTS IN ARAB COUNTRIES

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Abstract: This paper provides four estimators of count panel data (CPD) models; fixed effects Poisson (FEP), random effects Poisson (REP), fixed effects negative binomial (FENB), and random effects negative binomial (RENB). In FEP and FENB models, we used conditional maximum likelihood (CML) estimation method. While for REP and RENB models, we used maximum likelihood (ML) estimation method. We conducted a Monte Carlo simulation study to compare the behavior of these estimators in the four models. The results of simulation show that the best estimator is FENB compared to other estimators (FEP, REP, and RENB), because it has minimum values for Akaike information criterion (AIC) and Bayesian information criterion (BIC), especially when the model or the data has an overdispersion problem. Moreover, a real dataset has been used to study the effect of some economic variables on the number of patents for seven Arab countries over the period from 2000 to 2016. Application results indicate that the RENB is the suitable model for this data, and the important (statistically significant) variables that effect on the number of patents is the gross domestic product per capita.

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1. INTRODUCTION

Recently, panel data or longitudinal data sets have become one of the most exciting fields in econometrics literature due to new sources of data which observes the cross-sections of individuals over time. This allows constructing and testing more realistic behavioral models that could not be identified using a single cross-section or a single time-series data set. Therefore, panel data analysis is a core field in modern econometrics and multivariate statistics. Thus, panel data sets have become widely available, where there are many of the contributions and recent studies which have analyzed panel data, e.g. Baltagi [1] stated that the panel data refers to the pooling of observations on a cross-section of households, countries, firms, etc., over several time periods.

According to Vijayamohan [2], the panel data refers to a data set containing observations on multiple phenomena over multiple time periods, where it has two dimensions; the spatial dimension (cross-sectional) and temporal dimension (time series). Greene [3] pointed out that the analysis of panel data is one of the important topics and common in economics, because it allows great flexibility in modeling differences in behavior across individuals and provide rich sources of information and rich environment for the development of estimation techniques. Furthermore, the researchers are uses time-series cross-sectional data to examine issues that could not be studied in either cross-sectional or time-series alone. Also, the analysis of panel data allows the model builder to learn about economic processes considering both heterogeneity across individuals, firms, countries, etc., and dynamic effects that are not visible in cross sections.

Abonazel [4] explained that pooling cross-sectional and time series data (panel data) achieves a deep analysis for the data and gives a richer source of variation, which allows for more efficient estimation of the parameters and more effective in identifying and estimating effects that are simply not detectable in cross-sectional or time series data. Also, panel data sets are more effective
in studying complex issues of dynamic behavior.

Panel data models have become increasingly popular among applied researchers due to their heightened capacity for capturing the complexity of human behavior as compared to cross-sectional or time-series data models. Therefore, we will discuss the most popular models in panel data modeling, which is the fixed effects and random effects models.

In general, the fixed effects model has different intercepts, where the intercept is differing from unit to unit and fixed over time. The general form of the fixed effects model is [5, 6, 7]:

$$y_{it} = \alpha_i + x_{it}'\beta + u_{it}, \quad i = 1, 2, \ldots, N; \quad t = 1, 2, \ldots, T,$$

where $y_{it}$ is the response variable for individual $i$ at time $t$, $x_{it}$ is the vector of explanatory variables, $\alpha_i$ is a scalar constant (the intercept) include the unobserved effect for special variables to the $i^{th}$ individual over time, $\beta$ is the vector of the regression coefficients, and $u_{it}$ is the error term of the model.

In fixed effects model, the individual effects $\alpha_i$ are treated as fixed constants over time where individual effects are parts of the intercept, however in random effects model puts the individual effects into the error term and treat the individual effects, like $u_{it}$ as random variables. The random effects model assumes that the unit’s error term is not correlated with the predictors and the variation across entities is assumed to be random, in addition to the random effects model assumes that there is one constant term ($\alpha$) for all across unites, and the differences of the intercept term can be captured in the error term, hence the error term become have new assumptions [6, 8]. The random effects model is given by:

$$y_{it} = \alpha + x_{it}'\beta + \varepsilon_{it}, \quad i = 1, 2, \ldots, N; \quad t = 1, 2, \ldots, T,$$

where $\varepsilon_{it} = v_i + u_{it}$; this means that the error term of the model consists two components, $v_i$ and $u_{it}$, where $v_i$ denotes the unobservable individual effects, which are unobservable factors affecting $y$ and which do not vary over time or the unit’s unobserved ability that is not included in the regression, such as managerial skills, level of intelligence, and the unobservable entrepreneurial of unit. While $u_{it}$ denotes the disturbances, which varies with units and time and can be thought of as the usual disturbance in the regression or represents the other variables
influencing $y$ but which vary both over time and units.

The unobservable individual effects ($\nu_i$) and the disturbances ($u_{it}$) are assumed to be independently distributed across units, where $\nu_i$ is uncorrelated with each independent variable included in the model [1].

On the other side, if the dependent variable of the panel data model takes non-negative integer value such as (0, 1, 2, …), in this case the model is called the count panel data (CPD) model. Actually, the CPD analysis is a data type used with increasing frequency in empirical research in economics, social sciences, and medicine, etc., for example, the number of patents in some countries of the world over several years, the number of deaths from covid-19 in the countries of the world in multiple time periods, and the number of accidents in several areas over several years.

In econometrics literature, commonly used models that fit this data are Poisson and negative binomial models, where there are many economic studies that discussed these models in panel data modeling, e.g. [9, 10, 11, 12, 13].

Count regression models are varied depending on the types of data, where the count data is treated as dependent variable, so linear estimation methods, such as least squares that are designed to deal with continuous variable, are not appropriate for count data. Since the linear regression model assumes that the dependent variable follows the normal distribution, then it is not suitable for the count data. In addition to, the linear regression model may produce negative estimates for the response variable which is incorrect for the count data. So, the Poisson and negative binomial distributions are the basis of count data analysis.

The rest of the paper is organized as follows: section 2 provides Poisson panel models. In section 3 presents negative binomial panel models. Section 4 will be devoted to determining the settings of the simulation through design of Monte Carlo experiment and how the data is generated, where presents the main steps for making the Monte Carlo simulation study. Section 5 offers the results of the simulation study. In section 6, the empirical study on patents for seven Arab countries is presented. Finally, section 7 contains concluding remarks.
2. **POISSON PANEL MODELS**

The most common probability models for modelling CPD is Poisson panel model. In the Poisson distribution is the mean and the variance are the same, the higher the value of the mean of the distribution, the greater the variance or variability in the data [14]. The Poisson panel model assumes that the dependent variable \( y_{it} \) has a Poisson distribution. The probability mass function of \( y_{it} \) with parameter \( \lambda_{it} \) can be expressed as:

\[
f(y_{it}; \lambda_{it}) = \frac{\exp(-\lambda_{it})(\lambda_{it})^{y_{it}}}{y_{it}!}, \quad i = 1, 2, ..., N; \quad t = 1, 2, ..., T,
\]

where \( y_{it} \) represents a variable consisting of count values and \( \lambda_{it} > 0 \). \( \lambda_{it} \) is the expected or predicted mean of the count variable \( y_{it} \), and the subscripts \( (i) \) and \( (t) \) indicates that the model describes each observation in the data. In the model (3), the mean and the variance of \( y_{it} \) must be equal, i.e. \( E(y_{it}) = var(y_{it}) = \lambda_{it} \).

The Poisson panel model has one parameter \( (\lambda_{it}) \) which it must be positive. It is convenient to specify \( \lambda_{it} \) as an exponential function of the independent variables. The exponential form ensures that \( \lambda_{it} \) remains positive for all possible combinations of parameters and independent variables.

2.1 **Fixed Effects Poisson Model**

In the FEP model, all characteristics that are not time-varying are captured by the individual effects \( (\alpha_i) \). The intercept (constant term) is merged into \( \alpha_i \), hence the explanatory variables \( (x_{it}) \) do not contain an intercept [15]. The conditional probability function of the FEP model as:

\[
f(y_{it} | x_{it}, \alpha_i, \beta) = \frac{\exp(-\alpha_i \lambda_{it})(\alpha_i \lambda_{it})^{y_{it}}}{y_{it}!}, \quad i = 1, 2, ..., N; \quad t = 1, 2, ..., T,
\]

where \( \lambda_{it} = \exp(x_{it}' \beta) \). The last equality specifies an exponential functional form. To estimate the parameters of the model (4), it can use the CML estimation method that developed by Hausman et al. [16]. Since \( y_{it} \) and \( \sum_{t=1}^{T} y_{it} \) are follow the Poisson distribution, then the conditional joint density function (CJDF) for the \( t^{th} \) observation is:

\[
f(y_{1t}, ..., y_{Tt} | \sum_{t=1}^{T} y_{it}) = \frac{(\sum_{t=1}^{T} y_{it})!}{(\sum_{t=1}^{T} \lambda_{it})^{\sum_{t=1}^{T} y_{it}}} \times \prod_{t=1}^{T} \frac{\lambda_{it}^{y_{it}}}{y_{it}!}.
\]

when taking the logarithm of CJDF and summing over all individuals, the conditional loglikelihood is:
\[
\ln L = \sum_{i=1}^{N} \left( \ln \left( \sum_{t=1}^{T} y_{it} \right) + \sum_{t=1}^{T} \ln y_{it}! + \sum_{t=1}^{T} \left[ y_{it} x_{it}' \beta - y_{it} \ln \left( \sum_{t=1}^{T} \exp(x_{it}'\beta) \right) \right] \right),
\]

it can obtain the estimated parameters for the FEP model by solving:

\[
\sum_{i=1}^{N} \sum_{t=1}^{T} x_{it}' \left( y_{it} - \frac{\sum_{t=1}^{T} y_{it}}{\sum_{i=1}^{N} \lambda_{it}} \lambda_{it} \right) = 0.
\]

2.2 Random Effects Poisson Model

In the REP model, the individual effects (unobserved heterogeneity) are expressed as \( \nu_i \) instead of \( \alpha_i \), while the intercept is included and merged into \( x_{it} \). The individual effect \( \nu_i \) must follow a specified distribution in order to estimate the parameters of the REP model. Therefore, many researchers assumed that the individual effect in the REP model has a gamma distribution with parameters \( (\gamma, \gamma) \), see e.g. [5, 14, 16, 17, 18].

The REP model assumes that the response variable \( (y_{it}) \) has a Poisson distribution and the individual effect has a gamma distribution, then ML estimation method should be used to estimate the parameters of the REP model. The ML function for the \( it^{th} \) observation is:

\[
f(y_{it} | \nu_i, x_{it}) = \prod_{t=1}^{T} \left( \frac{\lambda_{it}^{y_{it}} y_{it}!}{\gamma+t} \right)^y \left[ \Gamma(\gamma+y) \right] [\gamma + \sum_{t=1}^{T} \lambda_{it}]^{-\gamma} \sum_{t=1}^{T} y_{it},
\]

and the log-maximum likelihood function is:

\[
\ln L = \sum_{i=1}^{N} \left( \sum_{t=1}^{T} (y_{it} x_{it}' \beta - \ln y_{it}!) + \gamma \ln y - \gamma \ln[\gamma + \sum_{t=1}^{T} \exp(x_{it}'\beta)] + \ln[\Gamma(\gamma+y)] - \sum_{t=1}^{T} y_{it} \ln [\gamma + \sum_{t=1}^{T} \exp(x_{it}'\beta)] \right),
\]

thus, it can obtain the estimated parameters of this model by solving:

\[
\sum_{i=1}^{N} \sum_{t=1}^{T} x_{it}' \left[ y_{it} - \lambda_{it} \left( \frac{y_{it}^{y_{it}+y}}{\lambda_{it}^{y_{it}+y}} \right) \right] = 0.
\]

3. Negative Binomial Panel Models

The negative binomial model is one of the basic models for count data analysis. This model has found a widespread use in the fields of health, social, economic, and physical sciences when the response variable comes in the form of non-negative integers or counts [19].

In general, the negative binomial panel model introduced as a generalized version of Poisson model that allows the variance of the dependent variable to differ from its mean. The negative binomial panel model is a two-parameter model; with mean \( \lambda_{it} \) and dispersion parameters \( \phi_i \).
The mean of the negative binomial panel model is understood in the same manner as the Poisson mean, but the variance of the negative binomial has a much wider scope than is allowed by the Poisson model. When the variance of count data exceeds the mean, i.e. if \( \text{var} (y_{it}) > E(y_{it}) \), then we speak about overdispersion. But if \( \text{var} (y_{it}) < E(y_{it}) \), then this is called underdispersion. The Poisson model does not allow for overdispersion or underdispersion. Hence, we used the negative binomial model instead of the Poisson model [19].

3.1 Fixed Effects Negative Binomial Model

The FENB model assumes that for a given unit \( i \), the response variable \( y_{it} \) is independent over time and \( \sum_{t=1}^{T} y_{it} \) has a negative binomial distribution with parameters \( \theta_i \) and \( \sum_{t=1}^{T} \lambda_{it} \). These assumptions imply that:

\[
\sum_{t=1}^{T} y_{it} \sim NB(\theta_i \sum_{t=1}^{T} \lambda_{it} , (\theta_i \sum_{t=1}^{T} \lambda_{it}) (1 + \theta_i))
\]

where \( \theta_i = \alpha_i / \phi_i \). Hausman et al. [16] showed that the CJDF of the FENB model for the \( i^{th} \) observation is:

\[
f(y_{i1}, ..., y_{iT} | \sum_{t=1}^{T} y_{it}) = \frac{\Gamma(\sum_{t=1}^{T} \lambda_{it}) \Gamma(\sum_{t=1}^{T} y_{it} + 1)}{\Gamma(\sum_{t=1}^{T} \lambda_{it} + \sum_{t=1}^{T} y_{it})} \times \prod_{t=1}^{T} \frac{\Gamma(\lambda_{it} + y_{it})}{\Gamma(\lambda_{it}) \Gamma(y_{it} + 1)},
\]

where \( \Gamma(\cdot) \) is the gamma function. In order to estimate the parameters of this model, Hausman et al. [16] used the CML estimation method. Thus, it can obtain the CML estimation of this model by maximizing the following log-conditional maximum likelihood function:

\[
\ln L = \sum_{i=1}^{N} \left[ \ln \Gamma(\sum_{t=1}^{T} \lambda_{it}) + \ln \Gamma(\sum_{t=1}^{T} y_{it} + 1) - \ln \Gamma(\sum_{t=1}^{T} \lambda_{it} + \sum_{t=1}^{T} y_{it}) + \sum_{t=1}^{T} [\ln \Gamma(\lambda_{it} + y_{it}) - \ln \Gamma(\lambda_{it}) - \ln \Gamma(y_{it} + 1)] \right].
\]

3.2 Random Effects Negative Binomial Model

For the RENB model, Hausman et al. [16] assumed that the dependent variable \( y_{it} \) specified to be independent and identically distributed negative binomial, and \( 1/(1 + \delta_i) \) is distributed as beta with parameters \( (a, b) \), where \( \delta_i = \nu_i / \phi_i \), i.e. \( 1/(1 + \delta_i) \sim Beta(a, b) \). The mean and the variance of the response variable \( y_{it} \) are \( \lambda_{it} \delta_i \) and \( \lambda_{it} \delta_i (1 + \delta_i) \), respectively.

To estimate the parameters of RENB model, it can use the ML estimation method. Then the joint density function for the \( i^{th} \) observation is:
\[ f(y_{it}|x_{it}) = \frac{\Gamma(a+b)\Gamma(a+\sum_{t=1}^{T} \lambda_{it}) \Gamma(b+\sum_{t=1}^{T} y_{it})}{\Gamma(a)\Gamma(b)\Gamma(a+b+\sum_{t=1}^{T} \lambda_{it}+\sum_{t=1}^{T} y_{it})} \times \prod_{t=1}^{T} \left[ \frac{\Gamma(\lambda_{it}+y_{it})}{\Gamma(\lambda_{it})\Gamma(y_{it}+1)} \right]. \]

The ML estimation of the RENB model can be obtained by maximizing the following log-maximum likelihood function:

\[
\ln L = \sum_{i=1}^{N} \{ \ln \Gamma(a + b) + \ln \Gamma(\alpha + \sum_{t=1}^{T} \lambda_{it}) + \ln \Gamma(b + \sum_{t=1}^{T} y_{it}) - \ln \Gamma(\alpha) - \ln \Gamma(b) - \ln \Gamma(\alpha + \sum_{t=1}^{T} \lambda_{it} + \sum_{t=1}^{T} y_{it}) + \sum_{i=1}^{T} [\ln \Gamma(\lambda_{it} + y_{it}) - \ln \Gamma(\lambda_{it}) - \ln \Gamma(y_{it} + 1)] \}. 
\]

4. Simulation Design

We will use the Monte Carlo simulation for making a comparison between the behavior of FEP, REP, FENB, and RENB estimators of the four CPD models above. We used R language to conduct our Monte Carlo simulation [20, 21]. Several studies have been relied upon when conducting a Monte Carlo simulation study such as [4, 22, 23, 24, 25, 26].

4.1 In Case of Moderate and Large Samples

The simulation study was carried out in the moderate and large samples based on the following:

1. The values of \( N \) were chosen to be 30, 50, 100, 200, 300, and 500 to represent moderate and large samples for the number of individuals.
2. The values of \( T \) were chosen to be 10, 15, 40, 50, 100, and 200 to represent different size for the time period.
3. The values of \( \beta_1, \beta_2, \) and \( \beta_3 \) were chosen to be 1.
4. The response variable \( (y_{it}) \) is generated from the negative binomial distribution with different values of the dispersion; where \( \phi \) were chosen to be 0.5, 1, and 5.
5. The individual effects \( (\alpha_i) \) were generate as independent normally distribution with mean -1 and standard deviation 0.5, where \( \alpha_i \) is differing from unit to unit and fixed over time.
6. We generate the explanatory variables using random numbers following the uniform distribution from -1 to 1.
7. For all experiments we ran 1000 replications and all the results for all separate experiments are obtained by precisely the same series of random numbers.

We can note that the generated model in our simulation is FENB model with three cases of the
dispersion parameter ($\phi$). In the first case the dispersion parameter $\phi < 1$ (i.e., $\phi = 0.5$), therefore we speak about underdispersion. While in the second case the dispersion parameter $\phi = 1$, this is called equidispersion. In the third case the dispersion parameter $\phi > 1$ (i.e., $\phi = 5$), thus we speak about overdispersion.

4.2 In Case of Small Samples

In this section, we will study the behavior of the four estimators in case of small samples. The data were generated by the same method in the case of moderate and large samples with the difference in cross section size to be 5, 10, 15, and 20 and time series to be 15 and 20, and the dispersion parameter is one.

The Monte Carlo experiment has been designed to compare the small, moderate, and large samples performances of ML estimators of REP and RENB models and CML estimators of FEP and FENB models based on AIC [27] and BIC [28].

5. Simulation Results

The results of the Monte Carlo simulation study for the moderate and large samples have been provided in tables from 1 to 6, while figures from 1 to 4 displays the small samples results. Each table represents AIC and BIC values (rounded to integer) for different values of $T$ and $\phi$. Tables from 1 to 6 present the estimation results (AIC and BIC) of FEP, REP, FENB, and RENB estimators for different values of $N$.

In tables from 1 to 3, when the dispersion parameter equal 0.5 or 1, we find that the AIC and BIC values of FENB estimator have smallest values than the FEP, REP, and RENB estimators. For example, in table 1 when $\phi = 1$ and $T = 10$, the AIC value of FENB is 388, but the AIC values of FEP, REP, and RENB are 413, 579, and 556, respectively. While the BIC value of FENB is 403, but the BIC value of FEP, REP, and RENB are 424, 598, and 575, respectively. And when the dispersion parameter is increasing to 5, then AIC and BIC values of FENB estimator are decreasing dramatically and still AIC and BIC values of FENB estimator is the smallest. For example, in table 1 when $\phi = 5$ and $T = 10$, the AIC value of FENB is 364, but the AIC values
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of FEP, REP, and RENB are 596, 779, and 615, respectively. While the BIC value of FENB is 378, but the BIC values of FEP, REP, and RENB are 607, 797, and 633, respectively. So, the results of tables from 1 to 3 showed that the FENB estimator is better than FEP, REP, and RENB estimators in case of moderate samples \((N = 30, 50, 100)\).

For the results of large samples \((N = 200, 300, 500)\), in tables 4 to 6, showed that when the time periods \((T)\) is increasing from 10 to 15 or 200 and the dispersion parameter \((\phi)\) is increasing from 0.5 to 1 or 5, the AIC and BIC values of FENB estimator is still smaller than the AIC and BIC values of other estimators. For example, in table 5 when \(N = 300, \phi = 5, \text{ and } T = 10\), the AIC value of FENB is 3511, but the AIC values of FEP, REP, and RENB are 5883, 7684, and 6026, respectively. While the BIC value of FENB is 3535, but the BIC values of FEP, REP, and RENB are 5901, 7714, and 6057, respectively. Whereas when \(N = 300, \phi = 5, \text{ and } T = 50\), the AIC value of FENB is 23092, but the AIC values of FEP, REP, and RENB are 38116, 40751, and 30809, respectively. While the BIC value of FENB is 23122, but the BIC values of FEP, REP, and RENB are 38139, 40789, and 30847, respectively. So, the results of tables from 4 to 6 showed that the FENB estimator is better than FEP, REP, and RENB estimators in case of large samples.

Figures from 1 to 4 display the AIC and BIC values of different estimators for small samples in \(N\) and \(T\). These figures showed that in case of increasing the number of units \((N)\), the AIC and BIC values of all estimators are increased. But still the FENB estimator is better than other estimators in case of small samples, even if the dispersion parameter equal one.
### Table 1: AIC and BIC values of different estimators when $N = 30$

| Criterion | Estimator | $T = 10$ | $T = 15$ | $T = 40$ | $T = 50$ | $T = 100$ | $T = 200$ |
|-----------|-----------|----------|----------|----------|----------|-----------|-----------|
| AIC       | FEP       | 442      | 722      | 2075     | 2420     | 6090      | 10811     |
|           | REP       | 619      | 912      | 2319     | 2658     | 6392      | 11147     |
|           | FENB      | 429      | 705      | 2016     | 2369     | 5883      | 10512     |
|           | RENB      | 605      | 894      | 2260     | 2606     | 6183      | 10848     |
| BIC       | FEP       | 454      | 735      | 2090     | 2435     | 6108      | 10831     |
|           | REP       | 637      | 933      | 2344     | 2685     | 6422      | 11181     |
|           | FENB      | 444      | 722      | 2037     | 2390     | 5907      | 10539     |
|           | RENB      | 623      | 915      | 2285     | 2633     | 6213      | 10881     |

- **$\phi = 0.5$**

| Criterion | Estimator | $T = 10$ | $T = 15$ | $T = 40$ | $T = 50$ | $T = 100$ | $T = 200$ |
|-----------|-----------|----------|----------|----------|----------|-----------|-----------|
| AIC       | FEP       | 413      | 664      | 2122     | 3118     | 5687      | 12793     |
|           | REP       | 579      | 847      | 2359     | 3378     | 5978      | 13139     |
|           | FENB      | 388      | 619      | 1970     | 2847     | 5262      | 11553     |
|           | RENB      | 556      | 806      | 2225     | 3134     | 5600      | 12034     |
| BIC       | FEP       | 424      | 676      | 2138     | 3133     | 5705      | 12813     |
|           | REP       | 598      | 868      | 2384     | 3405     | 6008      | 13173     |
|           | FENB      | 403      | 635      | 1990     | 2869     | 5286      | 11580     |
|           | RENB      | 575      | 826      | 2251     | 3161     | 5630      | 12068     |

- **$\phi = 1$**

| Criterion | Estimator | $T = 10$ | $T = 15$ | $T = 40$ | $T = 50$ | $T = 100$ | $T = 200$ |
|-----------|-----------|----------|----------|----------|----------|-----------|-----------|
| AIC       | FEP       | 596      | 1037     | 2571     | 3869     | 6440      | 15335     |
|           | REP       | 779      | 1247     | 2810     | 4135     | 6723      | 15673     |
|           | FENB      | 364      | 621      | 1692     | 2371     | 4260      | 9504      |
|           | RENB      | 615      | 968      | 2261     | 3159     | 5389      | 11869     |
| BIC       | FEP       | 607      | 1049     | 2586     | 3885     | 6458      | 15355     |
|           | REP       | 797      | 1267     | 2836     | 4161     | 6753      | 15707     |
|           | FENB      | 378      | 637      | 1712     | 2393     | 4284      | 9530      |
|           | RENB      | 633      | 989      | 2287     | 3185     | 5419      | 11902     |

- **$\phi = 5$**
Table 2: AIC and BIC values of different estimators when $N = 50$

| Criterion | Estimator | $T = 10$ | $T = 15$ | $T = 40$ | $T = 50$ | $T = 100$ | $T = 200$ |
|-----------|-----------|----------|----------|----------|----------|-----------|-----------|
| AIC       | FEP       | 786      | 1055     | 3255     | 4199     | 8974      | 18073     |
|           | REP       | 1086     | 1364     | 3638     | 4609     | 9462      | 18615     |
|           | FENB      | 760      | 1030     | 3178     | 4098     | 8716      | 17604     |
|           | RENB      | 1059     | 1338     | 3560     | 4508     | 9202      | 18143     |
| BIC       | FEP       | 798      | 1069     | 3272     | 4216     | 8994      | 18095     |
|           | REP       | 1107     | 1387     | 3666     | 4638     | 9495      | 18652     |
|           | FENB      | 776      | 1049     | 3200     | 4122     | 8742      | 17633     |
|           | RENB      | 1080     | 1361     | 3588     | 4537     | 9234      | 18179     |
|           |           |          |          |          |          |           |           |
| AIC       | FEP       | 721      | 1133     | 3363     | 4707     | 9710      | 18455     |
|           | REP       | 996      | 1449     | 3745     | 5138     | 10209     | 18994     |
|           | FENB      | 671      | 1052     | 3141     | 4321     | 8898      | 17171     |
|           | RENB      | 951      | 1373     | 3551     | 4795     | 9477      | 17868     |
| BIC       | FEP       | 734      | 1147     | 3379     | 4724     | 9729      | 18476     |
|           | REP       | 1017     | 1473     | 3773     | 5167     | 10242     | 19030     |
|           | FENB      | 688      | 1071     | 3163     | 4345     | 8924      | 17199     |
|           | RENB      | 972      | 1396     | 3579     | 4824     | 9510      | 17904     |
|           |           |          |          |          |          |           |           |
| AIC       | FEP       | 973      | 1493     | 5297     | 5876     | 13395     | 23832     |
|           | REP       | 1272     | 1824     | 5733     | 6307     | 13886     | 24376     |
|           | FENB      | 574      | 939      | 3114     | 3714     | 8208      | 15316     |
|           | RENB      | 998      | 1448     | 4238     | 4903     | 10538     | 19055     |
| BIC       | FEP       | 986      | 1507     | 5314     | 5893     | 13414     | 23853     |
|           | REP       | 1293     | 1848     | 5761     | 6336     | 13918     | 24412     |
|           | FENB      | 591      | 957      | 3137     | 3737     | 8234      | 15345     |
|           | RENB      | 1019     | 1472     | 4266     | 4932     | 10570     | 19091     |
### Table 3: AIC and BIC values of different estimators when \( N = 100 \)

| Criterion | Estimator | \( T =10 \) | \( T =15 \) | \( T =40 \) | \( T =50 \) | \( T =100 \) | \( T =200 \) |
|-----------|-----------|-------------|-------------|-------------|-------------|-------------|-------------|
| AIC \( \phi = 0.5 \) | FEP       | 1456        | 2173        | 7069        | 8833        | 17302       | 37893       |
|           | REP       | 2016        | 2791        | 7868        | 9696        | 18261       | 39011       |
|           | FENB      | 1417        | 2116        | 6863        | 8570        | 16848       | 36765       |
|           | RENB      | 1976        | 2735        | 7661        | 9432        | 17803       | 37880       |
| BIC \( \phi = 0.5 \) | FEP       | 1470        | 2188        | 7088        | 8852        | 17324       | 37916       |
|           | REP       | 2040        | 2818        | 7900        | 9729        | 18297       | 39050       |
|           | FENB      | 1436        | 2138        | 6888        | 8596        | 16877       | 36797       |
|           | RENB      | 2001        | 2761        | 7693        | 9464        | 17839       | 37920       |
| AIC \( \phi = 1 \) | FEP       | 1541        | 2440        | 7428        | 9623        | 19331       | 38778       |
|           | REP       | 2109        | 3073        | 8234        | 10472       | 20300       | 39869       |
|           | FENB      | 1414        | 2244        | 6836        | 8836        | 17832       | 35839       |
|           | RENB      | 1994        | 2899        | 7708        | 9773        | 18980       | 37298       |
| BIC \( \phi = 1 \) | FEP       | 1555        | 2456        | 7447        | 9642        | 19352       | 38802       |
|           | REP       | 2133        | 3099        | 8265        | 10504       | 20336       | 39909       |
|           | FENB      | 1433        | 2266        | 6862        | 8863        | 17861       | 35870       |
|           | RENB      | 2018        | 2926        | 7740        | 9806        | 19016       | 37337       |
| AIC \( \phi = 5 \) | FEP       | 2035        | 2900        | 9162        | 11617       | 26275       | 53460       |
|           | REP       | 2643        | 3553        | 9984        | 12476       | 27288       | 54587       |
|           | FENB      | 1213        | 1821        | 5739        | 7338        | 15927       | 32642       |
|           | RENB      | 2070        | 2828        | 7734        | 9695        | 20405       | 40924       |
| BIC \( \phi = 5 \) | FEP       | 2049        | 2916        | 9181        | 11637       | 26297       | 53484       |
|           | REP       | 2667        | 3580        | 10015       | 12508       | 27324       | 54627       |
|           | FENB      | 1233        | 1843        | 5764        | 7364        | 15956       | 32674       |
|           | RENB      | 2095        | 2854        | 7765        | 9728        | 20442       | 40964       |
Table 4: AIC and BIC values of different estimators when $N = 200$

| Criterion | Estimator | $T = 10$ | $T = 15$ | $T = 40$ | $T = 50$ | $T = 100$ | $T = 200$ |
|-----------|-----------|----------|----------|----------|----------|-----------|-----------|
| AIC       | FEP       | 2738     | 4471     | 13920    | 17758    | 36276     | 72970     |
|          | REP       | 3845     | 5725     | 15530    | 19439    | 38217     | 75201     |
|          | FENB      | 2661     | 4346     | 13516    | 17250    | 35247     | 70841     |
|          | RENB      | 3765     | 5594     | 15122    | 18927    | 37186     | 73065     |
| BIC       | FEP       | 2755     | 4489     | 13941    | 17780    | 36300     | 72995     |
|          | REP       | 3873     | 5755     | 15565    | 19475    | 38257     | 75244     |
|          | FENB      | 2683     | 4370     | 13544    | 17279    | 35279     | 70875     |
|          | RENB      | 3793     | 5624     | 15157    | 18964    | 37225     | 73108     |

| Criterion | Estimator | $T = 10$ | $T = 15$ | $T = 40$ | $T = 50$ | $T = 100$ | $T = 200$ |
|-----------|-----------|----------|----------|----------|----------|-----------|-----------|
| AIC       | FEP       | 2909     | 4832     | 14085    | 18282    | 39330     | 81501     |
|          | REP       | 4038     | 6117     | 15662    | 19961    | 41285     | 83724     |
|          | FENB      | 2690     | 4429     | 13009    | 16874    | 36129     | 74712     |
|          | RENB      | 3836     | 5749     | 14709    | 18708    | 38447     | 77729     |
| BIC       | FEP       | 2926     | 4850     | 14106    | 18304    | 39353     | 81526     |
|          | REP       | 4066     | 6147     | 15697    | 19998    | 41325     | 83767     |
|          | FENB      | 2712     | 4453     | 13037    | 16903    | 36161     | 74746     |
|          | RENB      | 3864     | 5779     | 14744    | 18744    | 38486     | 77772     |

| Criterion | Estimator | $T = 10$ | $T = 15$ | $T = 40$ | $T = 50$ | $T = 100$ | $T = 200$ |
|-----------|-----------|----------|----------|----------|----------|-----------|-----------|
| AIC       | FEP       | 3710     | 6338     | 18914    | 25275    | 50986     | 107382    |
|          | REP       | 4889     | 7677     | 20574    | 27026    | 52961     | 109621    |
|          | FENB      | 2245     | 3846     | 11703    | 15432    | 31654     | 65768     |
|          | RENB      | 3882     | 5984     | 15813    | 20520    | 40428     | 82363     |
| BIC       | FEP       | 3727     | 6356     | 18935    | 25296    | 51010     | 107407    |
|          | REP       | 4917     | 7707     | 20608    | 27062    | 53001     | 109664    |
|          | FENB      | 2268     | 3870     | 11731    | 15461    | 31686     | 65802     |
|          | RENB      | 3910     | 6014     | 15848    | 20556    | 40467     | 82406     |
Table 5: AIC and BIC values of different estimators when \( N = 300 \)

| Criterion | Estimator | \( T =10 \) | \( T =15 \) | \( T =40 \) | \( T =50 \) | \( T =100 \) | \( T =200 \) |
|-----------|-----------|-------------|-------------|-------------|-------------|-------------|-------------|
| \( \phi = 0.5 \) | | | | | | | |
| AIC | FEP | 4202 | 6800 | 20854 | 26174 | 54488 | 110267 |
| | REP | 5889 | 8696 | 23289 | 28693 | 57410 | 113565 |
| | FENB | 4083 | 6597 | 20223 | 25445 | 52884 | 107178 |
| | RENB | 5768 | 8489 | 22651 | 27961 | 55793 | 110467 |
| BIC | FEP | 4220 | 6819 | 20877 | 26197 | 54513 | 110294 |
| | REP | 5919 | 8728 | 23326 | 28731 | 57452 | 113610 |
| | FENB | 4107 | 6623 | 20253 | 25476 | 52917 | 107214 |
| | RENB | 5798 | 8521 | 22688 | 28000 | 55835 | 110512 |
| \( \phi = 1 \) | | | | | | | |
| AIC | FEP | 4431 | 7448 | 23003 | 28309 | 59628 | 119295 |
| | REP | 6146 | 9362 | 25432 | 30864 | 62582 | 122638 |
| | FENB | 4071 | 6850 | 21078 | 25968 | 54681 | 109616 |
| | RENB | 5811 | 8817 | 23707 | 28796 | 58194 | 114064 |
| BIC | FEP | 4449 | 7467 | 23025 | 28332 | 59653 | 119322 |
| | REP | 6176 | 9394 | 25469 | 30902 | 62624 | 122683 |
| | FENB | 4095 | 6875 | 21108 | 25998 | 54715 | 109652 |
| | RENB | 5841 | 8849 | 23744 | 28834 | 58236 | 114109 |
| \( \phi = 5 \) | | | | | | | |
| AIC | FEP | 5883 | 9147 | 29922 | 38116 | 77376 | 157627 |
| | REP | 7684 | 11133 | 32446 | 40751 | 80353 | 160991 |
| | FENB | 3511 | 5616 | 18192 | 23092 | 47518 | 96938 |
| | RENB | 6026 | 8748 | 24631 | 30809 | 60771 | 121400 |
| BIC | FEP | 5901 | 9166 | 29945 | 38139 | 77401 | 157654 |
| | REP | 7714 | 11165 | 32483 | 40789 | 80394 | 161036 |
| | FENB | 3535 | 5642 | 18221 | 23122 | 47551 | 96974 |
| | RENB | 6057 | 8781 | 24668 | 30847 | 60812 | 121445 |
Table 6: AIC and BIC values of different estimators when $N = 500$

| Criterion | Estimator | $T=10$ | $T=15$ | $T=40$ | $T=50$ | $T=100$ | $T=200$ |
|-----------|-----------|--------|--------|--------|--------|---------|---------|
|           |           |        |        |        |        |         |         |
| AIC       | FEP       | 6987   | 11517  | 34800  | 44163  | 90093   | 188100  |
|           | REP       | 9790   | 14666  | 38819  | 48395  | 94901   | 193680  |
|           | FENB      | 6790   | 11182  | 33758  | 42842  | 87566   | 182370  |
|           | RENB      | 9590   | 14324  | 37757  | 47064  | 92357   | 187937  |
| BIC       | FEP       | 7007   | 11537  | 34823  | 44187  | 90119   | 188128  |
|           | REP       | 9823   | 14700  | 38858  | 48436  | 94946   | 193728  |
|           | FENB      | 6816   | 11210  | 33789  | 42874  | 87601   | 182408  |
|           | RENB      | 9622   | 14359  | 37797  | 47105  | 92402   | 187984  |
|           |           |        |        |        |        |         |         |
|           |           |        |        |        |        |         |         |
| AIC       | FEP       | 7415   | 12082  | 37232  | 47737  | 95237   | 198462  |
|           | REP       | 10264  | 15275  | 41235  | 52005  | 100090  | 204052  |
|           | FENB      | 6822   | 11102  | 34245  | 43772  | 87761   | 182291  |
|           | RENB      | 9718   | 14374  | 38580  | 48461  | 93483   | 189719  |
| BIC       | FEP       | 7435   | 12103  | 37256  | 47761  | 95264   | 198491  |
|           | REP       | 10297  | 15310  | 41275  | 52045  | 100134  | 204100  |
|           | FENB      | 6848   | 11129  | 34277  | 43805  | 87797   | 182329  |
|           | RENB      | 9751   | 14409  | 38620  | 48502  | 93527   | 189767  |
|           |           |        |        |        |        |         |         |
|           |           |        |        |        |        |         |         |
| AIC       | FEP       | 9209   | 15020  | 46861  | 61780  | 128929  | 273759  |
|           | REP       | 12152  | 18304  | 51005  | 66135  | 133911  | 279428  |
|           | FENB      | 5610   | 9280   | 28915  | 37965  | 79252   | 165683  |
|           | RENB      | 9679   | 14416  | 39209  | 50474  | 101376  | 207999  |
| BIC       | FEP       | 9228   | 15040  | 46884  | 61804  | 128955  | 273788  |
|           | REP       | 12185  | 18339  | 51044  | 66176  | 133955  | 279476  |
|           | FENB      | 5636   | 9308   | 28947  | 37998  | 79287   | 165721  |
|           | RENB      | 9712   | 14451  | 39248  | 50514  | 101420  | 208046  |
Fig. 1: AIC and BIC values of different estimators when \( N = 5 \)

Fig. 2: AIC and BIC values of different estimators when \( N = 10 \)
6. EMPIRICAL STUDY: PATENTS IN ARAB COUNTRIES

There are many economic studies are interested with patent applications, e.g. [9, 13, 16, 29, 30, 31, 32]. In our application, we will follow the same methodology presented by Youssef et al. [13], their methodology is summarized the estimation steps and how to select the appropriate model for the data based on the Hausman test and the goodness-of-fit measures (AIC and BIC). Youssef et
al. [13] estimated the number of patents for seventeen high-income countries in the world over the period from 2005 to 2016, while in this application, the sample was chosen based on the available data on the number of patents in Arab countries in the World Bank website. Our sample contains seven Arab countries: Egypt, Algeria, Jordan, Morocco, Saudi, Tunisia, and Yemen over the period from 2000 to 2016.

In our study, the dependent variable is the number of patent applications, and three explanatory variables: GDPC, IMPO, and UNEM; where GDPC is the gross domestic product per capita (U.S. Dollar), IMPO is the information and communication technology goods imports (percentage of total goods imports), and UNEM is unemployment rate (percentage of total labor force).

We repaired the data before estimating the parameters of CPD models. The data contains some missing values in the number of patent and IMPO, these missing values were estimated using the mean-imputation method [21, 33]. We performed a unit root test for all variables, and the results indicated that the data are stationary in the level [34]. The variance inflation factor (VIF) is calculated to check the multicollinearity problem of the explanatory variables, the results indicated that the data not have multicollinearity problem because all values of VIF less than five. For more details on how to deal with the multicollinearity problem in regression models, see e.g. [20, 35, 36].

We estimated the parameters in fixed effects models using CML method, while the ML estimation method was used to estimate the random effects models. Table 7 presents the results of FEP and REP models, the two models are statistically significant because the P-value of the Wald test is less than 0.05. Based on the results of Hausman test, the P-value of chi-squared is greater than 0.05, then we can accept the null hypothesis, this means that REP model is more appropriate.

Table 8 presents the results of CML estimates of FENB model and ML estimates of RENB model. The two (FENB and RENB) models are statistically significant because the P-value of the Wald test is less than 0.05. Since the P-value of Hausman test is greater than 0.05, then the RENB model is more appropriate.
Table 7: Estimates of Poisson panel models

| Variable | Fixed Effects Poisson Model | Random Effects Poisson Model |
|----------|-----------------------------|-----------------------------|
|          | Estimate | Z-value | P-value | Estimate | Z-value | P-value |
| GDPC     | .0001355 | 48.07   | 0.001   | .0001354 | 48.05   | 0.001   |
| IMPO     | -6.586289 | -8.38   | 0.001   | -6.58876 | -8.38   | 0.001   |
| UNEM     | -7.792349 | -2.31   | 0.021   | -7.814001 | -2.32   | 0.021   |
| Intercept| -------- | -------- | -------- | 5.002777 | 13.15   | 0.001   |

\[ \chi^2 = 2350.23, \text{df} = 3, \text{P-value (}\chi^2\text{)} < 0.001 \]

Hausman Test

\[ \chi^2 = 0.23, \text{df} = 3, \text{P-value (}\chi^2\text{)} = 0.8896 \]

Table 8: Estimates of negative binomial panel models

| Variable | Fixed Effects NB Model | Random Effects NB Model |
|----------|------------------------|------------------------|
|          | Estimate | Z-value | P-value | Estimate | Z-value | P-value |
| GDPC     | .0000551 | 2.96    | 0.003   | .0000578 | 3.30    | 0.001   |
| IMPO     | -5.762925 | -1.28   | 0.201   | -4.332509 | -0.98   | 0.329   |
| UNEM     | .7660229 | 0.54    | 0.592   | .6834497 | 0.47    | 0.637   |
| Intercept| .9827058 | 3.21    | 0.001   | .9091168 | 2.98    | 0.003   |

\[ \chi^2 = 9.48, \text{df} = 3, \text{P-value (}\chi^2\text{)} = 0.0235 \]

\[ \chi^2 = 11.50, \text{df} = 3, \text{P-value (}\chi^2\text{)} = 0.0093 \]

Hausman Test

\[ \chi^2 = 2.26, \text{df} = 3, \text{P-value (}\chi^2\text{)} = 0.3225 \]

Based on the results from tables 7 and 8, we can conclude that REP and RENB models are more fit to this data than FEP and FENB models. Then we should use AIC and BIC to determine the best model (REP or RENB model). Table 9 shows that the RENB model has minimum values of AIC and BIC, and then the RENB model is the best model to fit the data.
Table 9: Goodness-of-fit measures of random effects models

| Measure         | Random Effects Poisson | Random Effects Negative Binomial |
|-----------------|------------------------|----------------------------------|
| Log likelihood  | -2701.233              | -666.526                         |
| AIC             | 5412.465               | 1345.051                         |
| BIC             | 5426.361               | 1361.726                         |

In the RENB model, we find that GDPC is statistically significant because the P-value of Z-value for this variable is less than 0.05, while IMPO and UNEM variables are not statistically significant.

7. CONCLUSION

In this paper, we used the Monte Carlo simulation for making a comparison study between the four estimation methods of CPD models. Furthermore, we examined the effect of some economic variables on the number of patent applications in seven Arab countries by applying four CPD models. We can summarize the main conclusions of our Monte Carlo simulation and the empirical study in the following points:

1. When the dispersion parameter equal one, the FENB estimator is better than FEP and REP estimators according to AIC and BIC values. Moreover, in case of increasing dispersion parameter value, the AIC and BIC values of FENB estimator is decreasing dramatically and the AIC and BIC values of FENB estimator is smaller than FEP, REP, and RENB estimators.
2. When the values of the number of units or time period are increased, the values of AIC and BIC of all CPD estimators are increasing in all simulation situations.
3. In general, simulation results indicated that the AIC and BIC values of FENB estimator is smaller than the AIC and BIC values of FEP, REP, and RENB estimators for all cases of the simulation. Thus, the FENB estimator is better than FEP, REP, and RENB estimators.
4. In our application, we examined the effect of some economic variables on the number of patents in seven Arab countries over the period from 2000 to 2016 by applying four CPD models to explore the main variables that effect on the number of patent applications in these countries. Based on the Hausman test and model-selection criteria (AIC and BIC), we found
that the RENB estimator is appropriate for this data, because it has minimum AIC and BIC values. RENB results indicated that the GDP per capita has a positive significant effect on the number of patents in Arab countries, and the other variables have not significant effect. In future work, we plan to study the efficiency of ML estimators in case of outliers [19, 21, 26] or missing data [21, 33] in CPD models. Moreover, we can study the impact of the COVID-19 pandemic [37] or the food and non-food expenditures [38, 39] on the number of patents in the Arab countries using modern CPD models.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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