A POSSIBLE ORIGIN OF LOGIC

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1. INTRODUCTION

An origin is often an intriguing issue. It becomes doubly intriguing when the logical form of thinking is considered. In this paper we will investigate exactly that: we will conjecture on the origin of basic instruments of logical thinking.

As the starting point we consider a very general case of a subject interacting with an environment. The subject might often have a choice of actions to take. Additionally, it is of fundamental importance that as a rule the subject’s internal representations of (beliefs about) the environment are inaccurate.

Beginning with this very general schema we will eventually arrive at a possible mechanism of how Logic could have emerged. Our reasoning will be supported with some in-depth case studies.

To formalize the settings we introduce two partially ordered sets \((M, \leq_m)\) of environmental positions that the subject can occupy with its "internal" ordering of their values. A partially ordered set \((L, \leq_l)\) of "external" estimates can be used to characterize the objective value of an environmental position. For that we have an estimate function \(\psi : (M, \leq_m) \rightarrow (L, \leq_l)\). So the partial order \(\leq_m\) describes the subject’s internal (and therefore subjective) representations of desirability or reachability of positions from \(M\). Then \((\psi(M), \leq_l)\) yields the objective ("external") values of the positions.

A crucial task of the subject is to find a position \(m \in M\) maximizing the estimate \(\psi(m)\) in the poset \((\psi(M), \leq_l)\).

No logical inference is needed to find an extremum with respect to its subjective ("internal") ordering \((M, \leq_m)\). It is possible to reach such a maximum by means of a greedy algorithm. The same is true when \(\psi\) is a monotonical function (i.e., the condition \(x \leq y \implies \psi(x) \leq \psi(y)\) is satisfied).

However, the task becomes more complex when the estimate function is not monotonical. It is then most naturally to explore some version of the successive approximation method.

The rest of the paper is organized as follows. We will start with the general theory, move onto a special case, and then apply the introduced approach to the classical two-valued propositional logic and modal propositional logics. A conclusion section
will follow. We will use the standard mathematical logic and partial order set theory notation [1, 2].

2. General theory

The suggested version operates on a particular yet very general representation of the operator \( \psi \) in the so-called "approximating form". It uses three axiomatically defined operations \( \sqcup, \sqcap, \odot \) based only on general properties of the posets \( (M, \leq_m) \), \( (L, \leq_l) \) as follows.

For every poset \( (R, \leq_r) \) the standard mappings \( (\cdot)_{\Delta}, (\cdot)_{\nabla} : R \to 2^R \) are defined by
\[
t_{\Delta} = \{ t' \in R | t' \leq_r t \}, t_{\nabla} = \{ t' \in R | t \leq_r t' \}.
\]

Let us suppose a binary operation \( \boxplus : L \times L \to L \) and unary operations \( \boxminus : 2^L \to L, \odot : L \to L \) are defined in such a way that the following system \( \mathcal{A} \) of axioms holds.

\[
\begin{align*}
\mathcal{A}_1 : (\forall S \subseteq M)(\exists \tilde{S} \subseteq S)[(\forall s \in S)(\exists \tilde{s} \in \tilde{S})[\tilde{s} \leq_m s] \& (\forall s, \tilde{s}' \in \tilde{S})[\tilde{s}' \not\leq_m \tilde{s}]], \\
\mathcal{A}_2 : (\forall l', L'' \subseteq L)(\forall x \in L')[[x \leq_l \Box(l') \& (L' \subseteq L'' \implies \Box(l') \subseteq_l \Box(L''))], \\
\mathcal{A}_3 : (\forall \nu \in L)[\Box(l, \circ(l)) = l \& (l \leq_l l' \implies \circ(l) \leq_l \circ(l'))], \\
\mathcal{A}_4 : (\forall \nu \in L)[l \leq_l l' \implies (\exists \nu'' \in L)[\Box(l', \nu'') = l \& (l' \leq_l \nu'')].
\end{align*}
\]

For every operator \( \nu : M \to L \) we call set \( n(\nu) = \{(m, m')|(m \leq_m m') \& (\nu(m) \leq_l \nu(m'))\} \) non-monotonicity domain of \( \nu \). If \( n(\nu) = \emptyset \) then \( \nu \) is called monotonical operator.

**Theorem 1.** Let all axioms of the system \( \mathcal{A} \) be satisfied for \( (M, \leq_m), (L, \leq_l) \) and \( (M, \leq_m) \) have only finite increasing chains. Then for every \( \psi : M \to L \) there exists a representation \( \psi = \Box(\varphi_1, \Box(\varphi_2, \Box(\varphi_3, \ldots))) \) where all \( \varphi_i, i = 1, 2, 3, \ldots \) are monotonical mappings from \( (M, \leq_m) \) to \( (L, \leq_l) \).

The number of occurrences of the operation \( \Box \) in this representation does not exceed the maximal length among lengths of all increasing chains in poset \( (M, \leq_m) \).

**Proof.** Let us reduce the problem for given operator \( \psi \) to the same problem for an simpler operator \( \psi_1 \) such that the following holds \( \psi = \Box(\varphi_1, \psi_1) \) and \( n(\psi_1) \subseteq n(\psi) \).

First we define \( M_1 = \{ x \in M | n(\psi) \cap (x_{\Delta} \times x_{\Delta}) \neq \emptyset \}, M^1 = M_1 \)

\[
\varphi_1(x) = \begin{cases} 
\Box(\psi(x_{\Delta})) & x \in M_1, \\
\psi(x) & x \in M^1.
\end{cases}
\]

Then we set \( \psi_1(x) \) to any such \( z \in L \) that \( \Box(\varphi_1(x), z) = \psi(x) \& \circ(\varphi_1(x)) \leq_l z \) if \( \varphi_1(x) \neq \psi(x) \). Otherwise we set \( \psi_1(x) = \circ(\psi(x)) \).

Existence of the element \( z \) in the definition is guaranteed by axioms \( \mathcal{A}_3, \mathcal{A}_4 \). Now equality \( \psi(x) = \Box(\varphi_1(x), \psi_1(x)) \) is true because of the definitions of \( \varphi_1, \psi_1 \).

Let us prove that operator \( \varphi_1 : (M \leq_m) \to (L, \leq_l) \) is monotonical one.
First, \( \varphi_1 = \psi \) on \( M^1 \) and we may use condition \( x, y \in M^1 \& x \leq_m y \implies \psi(x) \leq \psi(y) \). Indeed, otherwise \( \psi(x) \nleq \psi(y), x \leq_m y, \psi(x) \neq \psi(y) \) and therefore \( y \in M_1 \cap M^1 \). However, \( M^1 \cap M_1 = \emptyset \) which leads to a contradiction.

Second, \( \varphi_1 \) maps \( (M_1, \leq_m) \) into \( (L, \leq_1) \) monotonically in accordance with \( A_2 \).

Finally, let us consider the "mixed" case when \( x \in M^1, y \in M_1 \) and all elements of \( M \) are comparable with respect to \( \leq_m \). It is clear \( y \leq_m x \) is impossible since condition \( z \in M_1 \implies z^\varphi \subseteq M_1 \) follows from the definition of \( M_1 \) immediately.

Thus, it remains to consider the possibility of \( x \leq_m y \). In this case \( \varphi_1(y) = \Box(\psi(y)) \geq_1 \psi(x) \) in accordance to \( A_2 \). On the other hand, \( \psi(x) = \varphi_1(x) \) on \( M^1 \) follows from the definition of \( \varphi_1 \). Hence operator \( \varphi_1 \) is monotonic.

We are now ready to prove the last assertion of the theorem. For that is is sufficient to show the inclusion \( M^1 \cup M_1 \subseteq M^2 \). Here \( M^2, M_2 \) are defined for \( \psi \) in the same way as \( M^1, M_1 \) were defined for \( \psi \) above. \( \tilde{M}_1 \) is the set of all minimal elements of set \( M_1 \), see \( A_1 \). Namely: \( M^2 = M_2 \) and \( M_2 = \{ x \in M | n(\psi) \cap (x^\varphi \times x^\varphi) \neq \emptyset \} \).

From here we have \( M_2 \subseteq (M_1 \setminus \tilde{M}_1) \) and \( n(\psi) \subseteq n(\psi) \setminus \tilde{M}_1 \times M_1 \). So the sequence \( M_1 \supseteq M_2 \supseteq M_3 \supseteq \ldots \) interrupts on a step with the number that can not be higher the highest of lengths of the increasing chains in poset \( (M, \leq_m) \). Indeed, since \( \tilde{M}_2 \subseteq M_1 \setminus \tilde{M}_1 \) then in accordance with \( A_1 \) for every element \( y \in \tilde{M}_2 \) there exists some \( x \in \tilde{M}_1 \) such that \( x \ll_1 y \). Therefore, one can choose some increasing chain of representatives of sets \( \tilde{M}_1, \tilde{M}_2, \tilde{M}_3, \ldots \) which are mutually disjoint sets.

We will now prove \( M^1 \cup \tilde{M}_1 \subseteq M^2 \). First, \( \varphi_1(x) = \psi(x) \) is true for every \( x \in M^1 \). From here \( \psi_1(x) = \oplus(\psi(x)) \). However, mapping \( \psi_1 \) is monotonical on \( M^1 \) in view of \( A_3 \) and since \( \psi \) is monotonous on \( M^1 \). So \( (M^1 \times M^1) \cap n(\psi_1) = \emptyset \) and therefore \( M^1 \subseteq M^2 \).

Further, let \( x, y \in M^1 \cup \tilde{M}_1 \) and \( x \leq_m y \). Then we can show that \( \psi_1(x) \leq_1 \psi_1(y) \). Indeed, the case \( x, y \in M^1 \) was considered above. The case \( x, y \in \tilde{M}_1 \) is impossible since all elements of \( \tilde{M}_1 \) are incomparable by the definition. We saw above that \( x \in M_1 \& x \leq_m y \implies y \in M_1 \). Besides \( M^1 \cap M_1 = \emptyset \). Therefore, \( x \in M^1, y \in \tilde{M}_1 \) is the only case remaining to consider. By definition \( \psi_1(x) = \oplus(\psi(x)) \) and relation \( \Box(\varphi_1(y), \psi_1(y)) = \psi(y) \) holds. Moreover, \( \psi(y) <_1 \varphi_1(y) \). In accordance with \( A_4 \) we have \( \oplus(\varphi_1(y)) \leq y \psi_1(y) \). Hence \( \psi_1(x) \leq_1 \psi_1(y) \) takes place since \( \oplus \) is a monotonical operation in view of \( A_3 \) and \( \psi(z) \leq_1 \varphi_1(z), z \in M \) in accordance to \( A_2 \) and the construction.

Instead of or together with \( \mathcal{A} \) the dual axiom system \( \mathcal{A}^* \) can be fulfilled. It is obtained by replacing \( \leq \) with \( \geq \) and \( \Box, \Box, \oplus \) with \( \Box^*, \Box^*, \oplus^* \) correspondingly:

\[
\mathcal{A}^*_1: (\forall S \subseteq M)(\exists \tilde{S} \subseteq S)[(\forall s \in S)(\exists \tilde{s} \in \tilde{S})[\tilde{s} \geq_m s] \& (\forall \tilde{s}, \tilde{s}' \in \tilde{S})[\tilde{s} \not\leq_m \tilde{s}']].
\]
Then the dual theorem holds:

**Theorem 1.** Let all axioms of the system $A^*$ be fulfilled for posets $(M, \leq_m)$, $(L, \leq_i)$, and operators $\Box^*, \Diamond^*, \circ^*$ and $(M, \leq_m)$ have only finite decreasing chains. Then for every operator $\psi : M \rightarrow L$ there exists a representation $\psi = \Box^*(\ldots \Box^* (\Box^*(\varphi_{n+1}, \varphi_n), \varphi_{n-1}) \ldots \varphi_1)$ where all $\varphi_i, i = 1, \ldots, n, n + 1$, are monotonic mappings from $(M, \leq_m)$ to $(L, \leq_i)$.

The number $n$ of occurrences of operators $\Box^*$ in the representation does not exceed the highest length among the lengths of decreasing chains in $(M, \leq_m)$.

We call the representing forms from these theorems approximating forms. Another way to obtain approximating forms is suggested in theorem 2 below.

Let us suppose a binary operation $\boxtimes, \forall : L \times L \rightarrow L$ and unary operations $\circ : L \rightarrow L$ are defined in such a way that the following system $B$ of axioms takes place.

\[ B_1: (\forall S \subseteq M)(\exists \tilde{S} \subseteq S)((\forall s \in S)((\exists \tilde{s} \in \tilde{S})[s \leq_m \tilde{s}] \& (\forall s, \tilde{s}' \in \tilde{S})[\tilde{s} \neq_m \tilde{s}']]). \]

\[ B_2: (\forall x, y \in L)[x, y \leq_i \forall(x, y)]. \]

\[ B_3: (\forall l, l' \in L)[\Box(l, \circ(l)) = l \& (l \leq_i l' \implies \circ(l) \leq_i \circ(l'))]. \]

\[ B_4: (\forall l, l' \in L)[l \leq_i l' \implies (\exists l'' \in L)[\Box(l', l'') = l \& \circ(l') \leq_i l'']]. \]

**Theorem 2.** Let all axioms of the system $B$ be satisfied for $(M, \leq_m), (L, \leq_i)$ and $(M, \leq_m)$ have only finite increasing chains. Then for every $\psi : M \rightarrow L$ there exists a representation $\psi = \Box(\varphi_1, \Box(\varphi_2, \Box(\varphi_3, \ldots)))$ where all $\varphi_i, i = 1, 2, 3, \ldots$ are monotonic mappings from $(M, \leq_m)$ to $(L, \leq_i)$.

The number of occurrences of the operation $\Box$ in this representation does not exceed the maximal length among the lengths of all increasing chains in poset $(M, \leq_m)$.

**Proof.** First, in the case when $(\forall x \in M)[|x^\Delta| < \infty$ is true we can prove our theorem using theorem 1. For that we only need to note that in this case it is possible to replace $\Box(\psi(x^\Delta))$ with any expression of kind $\forall(\psi(z_1), \uplus(\cdots \forall (\psi(z_n), \psi(z_n), \ldots))$. Here $z_1, \ldots, z_n$ is an enumeration of the finite set $x^\Delta$. Indeed, in the proof of theorem 1 we used axiom $A_2$ only for subsets of $L$ of the form $\psi(x^\Delta)$. Thus, it is sufficient to check only that axiom $A_2$ is true for sets of kind $\psi(x^\Delta)$. This check is a trivial one on the base of axiom $B_2$ for operation $\forall$. 

Otherwise, when there are infinite sets \( x^\Delta \) we can make use of the condition of finiteness of increasing chains in \( (M, \leq_m) \). Let us associate every non-minimal element \( x \in M \) with some maximal with respect to the inclusion relation \( \subseteq \) increasing chain \( x_1 <_m x_2 <_m \cdots <_m x_{k+1} = x \). So \( x_1 \) is the minimal element of \( (M, \leq_m) \) and for any \( y, j \in \{1, k\} \) if \( x_j \leq_m y <_x j + 1 \) then \( x_j = y \lor y = x_{j+1} \). Let us then denote the previous element \( x_k \) of the chain by \( \hat{x} \).

Now we replace the definition of operator \( \varphi_1 \) from the proof of theorem 1 above with the following inductive definition:

**Basis:** \( x \in M^1 \). Then \( \varphi_1(x) = \psi(x) \).

**Induction Step:** \( x \in M_1 \) and \( \hat{x} \) is defined. Then we set \( \varphi_1(x) = \hat{\psi}(\psi(x), \varphi_1(\hat{x})) \).

From this we evidently have that \( \varphi_1 \) is a monotonical operator and \( \psi(x) \leq_l \varphi_1(x), x \in M \). The remaining part of the proof follows the corresponding part of theorem 1 proof. \( \square \)

The dual theorem relates with the dual axiom system \( \mathcal{B}^* \).

\[ \mathcal{B}_1^*: (\forall S \subseteq M)(\exists \hat{S} \subseteq S)[(\forall s \in S)(\exists \hat{s} \in \hat{S})[\hat{s} \geq_m s] \& (\forall \hat{s}, \hat{s}' \in \hat{S})[\hat{s} \neq_m \hat{s}']] \]

\[ \mathcal{B}_2^*: (\forall x, y \in L)[x, y \geq_l \psi^*(x, y)] \]

\[ \mathcal{B}_3^*: (\forall l, l' \in L)[\psi^*(\circ^*(l), l) = l \& (l \geq_l l' \implies \circ^*(l) \geq_l \circ^*(l'))] \]

\[ \mathcal{B}_4^*: (\forall l, l' \in L)[l \geq_l l' \implies (\exists l'' \in L)[\psi^*(l'', l') = l \& \circ^*(l'') \geq_l l'']] \]

Then the dual theorem holds:

**Theorem 2**. Let all axioms of the system \( \mathcal{A}^* \) be fulfilled for posets \( (M, \leq_m), (L, \leq_l) \), operators \( \psi^*, \exists^*, \circ^* \) and \( (M, \leq_m) \) have only finite decreasing chains. Then for every operator \( \psi : M \to L \) there exists representation \( \psi = \boxplus^*(\cdots \boxplus^*(\varphi_{n+1}, \varphi_n), \varphi_{n-1}, \ldots, \varphi_1) \) where all \( \varphi_i, i = 1, \ldots, n, n + 1 \), are monotonical mappings from \( (M, \leq_m) \) to \( (L, \leq_l) \).

The number \( n \) of occurrences of operations \( \boxplus^* \) in the representation does not exceed the highest length among the lengths of decreasing chains in \( (M, \leq_m) \).

**2.1. A special case.** Sometimes it is possible to choose another type of operators \( \varphi_i \) in the previous theorems. We suggest some conditions for that in the following new axiom system \( \mathcal{B}^+_5 \) which consists of the above-introduced system \( \mathcal{B} \) completed with the following axiom:

\[ \mathcal{B}_5^+: (\forall l \in L)(\exists \hat{l} \in L)[l \leq_l \hat{l} \& (\exists l' \in L)[\hat{l} <_l l']] \]

This axiom postulates that for any element \( l \in L \) there exists at least one maximal element \( \hat{l} \) of \( L \) greater than \( l \).
Let us denote by $\min M$ the class of all minimal elements of $(M, \leq_m)$, and by $\max L$ the class of all maximal elements of $(L, \leq_l)$. Then let us denote by $\theta : M \to L$ any such function that for every increasing chain $m_1 <_m m_2 <_m \cdots <_m m_t$ where $m_1 \in \min M$ the following conditions are satisfied.

1) $\theta(m_i) = \odot(m_i) \lor \theta(m_i) \in \max L$ & $m_i \leq_l \theta(m_i)$;
2) $\theta(m_i) \in \max L$ & $i \leq j \implies \theta(m_j) \in \max L$. This condition means that $\theta(m) \in \max L \implies (\forall x \in M^\gamma)[\theta(x) \in \max L]$.

We call these mappings $\theta$-mappings.

At last, let $K$ be a class consisting of $\theta$-mappings such that for any pair $(y, x) \in \mathbb{M}_m$ there exists a $\theta$-mapping $\theta_{y,x}$ obeying the conditions $\forall y'[y' \notin x^\gamma \implies \theta_{y,x}(y') = \odot(y')]$, $\forall x'[x' \in x^\gamma \implies \theta_{y,x}(x') \in \max L]$. We refer to such functions as special $\theta$-functions.

**Theorem 3** Let all axioms of the system $B^+$ be satisfied for $(M, \leq_m), (L, \leq_l)$, $K$ satisfy the condition above, and $(M, \leq_m)$ be finite. Then for every $\psi : M \to L$ there exists a formula $\Phi(z_1, \ldots, z_n)$ that only operations $\sqcup, \sqcap$ occur and there exists a substitution $p : \{z_1, \ldots, z_n\} \to K$ such that $\psi = Sb^1\cdots^1\Phi(z_1, \ldots, z_n)$.

**Proof.** We will follow theorem 1 proof but re-define $\varphi_i$. First, we choose a pair $(y, x) \in \mathbb{M}_m$ such that $y$ is a maximal in $M^1$ and $x$ immediately follows $y$ in $(M, \leq_m)$. Then define $\varphi_{y,x}$ as:

$$\varphi_{y,x}(z) = \begin{cases} \sqcup(\psi(z), \theta_{y,x}), & z \in M_1, \\ \psi(z), & z \in M^1. \end{cases}$$

Then we set $\psi_{y,x}(u)$ equal to any such $z \in L$ that $\sqcup(\varphi_{y,x}(u), z) = \psi(u) \& \sqcap(\varphi_{y,x}(u)) \leq_l z$ if $\varphi_{y,x}(u) \neq \psi(u)$. Otherwise we set $\psi_{y,x}(u) = \odot(\psi(u))$.

Analogously to the proof of theorem 2 it can be shown that $\psi = \sqcup(\varphi_{y,x}, \psi_{y,x})$ where $n(\psi_{y,x}), n(\varphi_{y,x}) \subseteq n(\psi)$.

On further steps we handle $\psi_{y,x}, \varphi_{y,x}$ in the same way and so forth. Since $M$ is a finite set and we use special $\theta$-functions this reduction converges in a finite number of steps. \qed

Of course the last theorem can be reformulated in the dual form.

**3. Consideration of the classical two-valued propositional logic from the developed approach**

It is easy to arrive at the classical two-valued propositional logic now. For that it is sufficient to choose $(\{0, 1\}, 0 \leq 1$ as $(L, \leq_l)$ and the standard poset $(B^n, \preceq)$ on
boolean cube $B^n$ as poset $(M, \leq_m)$. It is well known that every finite poset can be isotonically included into $(B^n, \preceq)$ for the appropriate $n$.

Also it is well known that poset $(B^n, \preceq)$ is a self-dual poset for any $n$. Therefore, both above-introduced representations take place in this case.

**Lemma.** 1) The system of posets $(B^n, \preceq), (B, \leq)$ as $(M, \leq_m), (L, \leq_l)$ correspondingly and operation $\rightarrow$ as $\exists^*$, operation $1 : B^n \rightarrow \{1\}$ as $\ominus^*$, and operation $\&$ as $\oplus^*$ fulfil axiom set $A^*$.

2) The system of posets $(B^n, \preceq), (B, \leq)$ as $(M, \leq_m), (L, \leq_l)$ correspondingly and operation $\rightarrow$ as $\exists^*$, operation $1 : B^n \rightarrow \{1\}$ as $\ominus^*$, and operation $\&$ as $\oplus^*$ fulfil axiom set $B^+$.

**Proof.** This can done via a routine check of the axioms.

The direct corollary of this lemma and theorems above is

**Theorem 4.** In the special cases of finite "internal" orders $(M, \leq_m)$ and linear "external" orders $(L, \leq_l), |L| = 2$, approximating forms from every of theorems 1,2,3 and their dual ones generate all formulas of the classical propositional logic (within logical equivalence).

Also the following interesting statement follows.

**Corollary.** Every $n$-argument logical (boolean) function $f$ can be represented by the implicative normal form $f = P_k \rightarrow P_{k-1} \rightarrow \cdots \rightarrow P_1$, where $k \leq n$, and $P_i, i = 1, k$, are monotonical boolean function.

It is remarkable that just the dual approximating form presents the usual propositional implication or that operation $\rightarrow^*$ is not presented in the natural language. In our opinion, the main reason is that our dual approximating forms of theorems 1*, 2* start from a given operator $\psi$ and approximate it by means of successive simplifications: $\psi_1 = \exists^*(\psi, \varphi_1), \psi_2 = \exists^*(\psi_1, \varphi_2), \ldots$ while $\psi_i$ is not a monotonical operator (i.e. not an "easy" one). Thus, the approximation starts from the target unlike in the case of the approximating form from theorems 1,2.

Now one can consider the classical two-valued propositional logic merely as a realization of the above-mentioned principle of successive approximations for the decision-making problems within subject-environment survival framework.

Thus, from this viewpoint, the classical propositional logic can take its beginning from the survival problem. It is also important that this hypothetical origin of logic appears quite natural.
4. ABOUT MODAL PROPOSITIONAL LOGICS

Following this idea, various types of logic can be viewed as theories of such reductions for chosen classes of the operators. Here we suggest the following result concerning modal logic. Its demonstration follows the expounded above method.

**Theorem 5.** Every propositional extension of the classical propositional logic can be obtained by addition of one-place logical functions to the classical list →, &, ∨, ¬.

**Proof.** Indeed, given $L = \{l_1, \ldots, l_q\}$ and $M = L^n = L \times L \times \ldots L$ we can construct one-argument functions $\Gamma_i : L \rightarrow L$ where:

$$\Gamma_i(x) = \begin{cases} 1, & i \leq x, \\ 0, & x < i. \end{cases}$$

Furthermore, we consider these functions $\Gamma_i, i = \overline{1, q}$, as functions $\theta_i^j(x_1, \ldots, x_j, \ldots, x_n), i = \overline{1, q}, j = \overline{1, n}$ such that $(\forall x_1 \ldots x_n)[\theta_i^j(x_1, \ldots, x_j, \ldots, x_n) = \Gamma_i(x_j)]$. (Thus every $\theta_i^j$ has only one essential variable $x_j$.) These functions $\theta_i^j$ satisfy the conditions of special $\theta$-functions above. Then we may take the closure relatively & of set $\{\theta_i^j | i = \overline{1, q}, j = \overline{1, n}\}$ as the class $K$ from theorem 3. Hence we can use the theorem (as well as theorem 3*) to represent an arbitrary function $\psi : M \rightarrow L$ by a formula constructed from standard operators →, &, ∨ and one-place functions $\Gamma_i, i = \overline{1, q}$. □

5. CONCLUSIONS

As the research demonstrates, the classical two-valued propositional logic can be viewed merely as a realization of the above-mentioned principle of successive approximations for the decision-making problems within subject-environment survival framework.

From this viewpoint, the classical propositional logic can take its beginning from the survival problem. It is also very important that this hypothetical origin of logic appears quite natural.

Then the approach can serve as a background for consideration of other families of mappings from one poset to another with a chosen notion of simplicity of mapping. Any such case generates a corresponding logic.

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