Reducing Estimation Bias via Weighted Delayed Deep Deterministic Policy Gradient

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Abstract—The overestimation phenomenon caused by function approximation is a well-known issue in value-based reinforcement learning algorithms such as deep Q-networks and DDPG, which could lead to suboptimal policies. To address this issue, TD3 takes the minimum value between a pair of critics, which introduces underestimation bias. By unifying these two opposites, we propose a novel Weighted Delayed Deep Deterministic Policy Gradient algorithm, which can reduce the estimation error and further improve the performance by weighting a pair of critics. We compare the learning process of value function between DDPG, TD3, and our proposed algorithm, which verifies that our algorithm could indeed eliminate the estimation error of value function. We evaluate our algorithm in the OpenAI Gym continuous control tasks, outperforming the state-of-the-art algorithms on every environment tested.

Index Terms—Deep reinforcement learning, estimation bias, neural networks

I. INTRODUCTION

The goal of reinforcement learning (RL) is to learn good policies for sequential decision-making problems through optimizing the cumulative delayed reward signals. Combined with deep learning, a lot of achievements have been produced in a wide range of field such as playing Atari games [1], playing chess, go and shoji [2], [7], beating human players in StarCraft [3], [4], controlling robotic manipulation [5], playing cube [6] and so on. However, there still exist several severe issues that prevent deep reinforcement learning (DRL) from being applied to a wider range of tasks. One of the trickiest issues is the systematic estimation bias of value function in value-based reinforcement learning algorithms, such as Deep Q-networks [24] and DDPG [17].

In a typical RL setting with discrete action space, the overestimation bias of value function has been well-studied. However, the study of the underestimation bias in continuous control tasks seems to be forgotten by researchers. In this paper, we focus on the problem of underestimation of value functions in continuous action space. We first discuss the overestimation of value functions, and then we theoretically prove that taking the minimum value of a pair of critics would lead to underestimation bias when solving the overestimation bias of value function, and then we verify our hypothesis with experiments.

The overestimation phenomenon occurs when the value estimated by a function approximation which is larger than the true value. Overestimation bias is a property of the max operator of Q-learning, where maximization of value function estimation with noise leads to consistent overestimation bias [9]. In function approximation, noise is unavoidable which may be caused by model bias, inaccurate approximation error function or system error caused by computer precision. This error is further amplified by the nature of temporal difference learning, where the value function is updated by the estimates of subsequent value, which is known as the accumulation of error [8]. It is possible to have a relatively high value for any state, such as bad states or states with few visits due to the overestimation, thus leading to sub-optimal policies or divergence.

To solve this problem, many researchers try to minimize the accumulation of errors through the idea of an average function [10], adding a penalty term or a correction term to the learning process of policy [11] [31], or using a smooth value function approximation approach [12]. On the other hand, Hasselt et al. notice that the overestimation problem often happens when using a single action-value function, so they introduce a pair of action-value functions to solve the overestimation problem and they propose double DQN algorithm [15]. Double DQN reduces the overestimation of the state-action value function in discrete action space, which improves performance by using two decoupled functions. Unfortunately, Double DQN still overestimates action-value for continuous control tasks [18]. Fujimoto et al. propose TD3 to reduce overestimation by taking the minimum value of a pair of action-value functions. This min operator is efficient for reducing overestimation bias, which makes the agent have a stable and robust performance. However, the min operator
presents underestimation issue, i.e., the action-value output by function approximation is lower than the true value. Although this error does not show propagation when the value function is updated, it still makes agents pessimistic about the future for the underestimation of the action-value function, thus harming the performance of algorithms. Fujimoto et al. proposed Batch-Constrained deep Q-learning (BCQ) [26], which still utilizes two action-value functions updated by a weighted target Q-function. However, they concentrated on how to learn from a fixed dataset produced by reinforcement learning without more discussion about the underestimation phenomenon.

Previous work on value-based RL pays little attention to underestimation bias, such as DDPG, TD3, BCQ. In this paper, we focus on the impact of underestimation on value function. We prove the underestimation problem of the recently proposed value-based learning algorithm TD3 even though given the true value of a pair of action-value functions. Furthermore, our experiments show that such underestimation bias does occur, thus hurting the performance of algorithms.

As mentioned above, overestimation problems often occur in algorithms that use only one critic, but when utilizing a pair of critics at the same time and take the minimum of them, underestimation problems arise. Can we combine these two opposites to get a more accurate estimation? To address estimation bias, we explore how to combine the two opposite bias to make value function estimation more accurate. In this paper, we propose a novel algorithm that offers a more accurate estimation of the value function called Weighted Delayed Deep Deterministic Policy Gradient (WD3), which offers a kind of convex joint of underestimation and overestimation and thus offers a kind of trade-off. We evaluate WD3 on OpenAI gym continuous control tasks [21], and WD3 matches or outperforms all other algorithms.

The major contributions are summarized as follows:

- We prove that underestimation bias occurs when taking the minimum of a pair of action-value functions. Furthermore, we experimentally verify that this phenomenon does occur and hurt performance.
- A novel convex connection mechanism for a pair of action-value functions is incorporated into the TD3 algorithm to reduce the estimation bias both in theory and practice for continuous control tasks.
- Our method achieves better performance than the state-of-the-art algorithms on all OpenAI gym environments tested through more accurate action-value estimation.

II. PRELIMINARIES

In this paper, we consider a standard reinforcement learning paradigm that an agent interacts with an environment in discrete timesteps. We formalize the standard reinforcement learning as a Markov Decision Process (MDP), which is defined by a tuple \((S, A, R, \rho_0, \gamma)\) that consists of a state space \(S\), an action space \(A\), a reward function \(R : S \times A \rightarrow \mathbb{R}\), a transition probability function \(p\), an initial state distribution \(\rho_0\) and a discount factor \(\gamma \in [0, 1]\). At each time step \(t\), the agent is given an state \(s \in S\) and selects an action \(a \in A\) with respect to its policy \(\pi : S \rightarrow A\), receiving a reward \(r\) and a new state \(s'\) of environment. The return is defined as the discounted cumulative reward \(R_t = \sum_{i=t}^{T} \gamma^{i-t}r(s_i, a_i)\) with a discounted factor \(\gamma\) determining the priority of short-term rewards. Note that the return depends on the actions, and thus on the policy \(\pi\), deterministic or stochastic. A trajectory \(\tau = (s_0, a_0, s_1, a_1, ...)\) is a sequence of states and actions where \(s_0 \sim \rho_0\) and \(a_i \sim \pi\). A transition is a tuple \((s, a, r, s')\), where action \(a\) is performed at state \(s\), getting reward \(r\) and next state \(s'\). The goal of reinforcement learning is to find an optimal policy that maximize the discounted cumulative reward \(R_t\). In value-based reinforcement learning algorithms, the action-value function, a.k.a. Q-function, critic, is defined as \(Q(s, a) = \mathbb{E}_{\tau \sim \pi}[R(\tau)|s_0 = s, a_0 = a]\) which measures the quality of an action \(a\) given a state \(s\). State-value function, a.k.a value function, is defined as \(V(s) = \mathbb{E}_{\tau \sim \pi}[R(\tau)|s_0 = s]\) that measures the quality of a specific state \(s\). Both value function and action-value function can be used to evaluate the policy and further guide the algorithm to learn a higher quality value, that is, a better policy. So accurate estimation of the value function is of vital importance.

When the transition probability function is unknown, the state-value function can be recursively estimated by Bellman equation [25] with a transition tuple \((s, a, r, s')\):

\[
Q(s, a) = r + \gamma \mathbb{E}_{\tau \sim \pi}[Q(s', a')].
\]  

However, when using the function approximation method to estimate the action-value function, especially when using the neural network, there is a tendency to have large variance due to the property of generalization of the neural network. Besides, there is the problem of estimation bias, which is composed of three factors, model bias, function approximation error, and data noise. In the following, we discuss the mathematical background of some algorithms that related to our work: DQN, Double DQN, DDPG, and TD3.

A. Deep Q-networkS (DQN)

DQN uses a multi-layered neural network to approximate the action-value function that for a given state \(s\) outputs a vector of action-value \(Q(s, \cdot; \theta)\) where \(\theta\) represents the parameters of the neural network. For addressing the instability from the combination between neural network and Q-learning, Mnih et al. (2015) propose two important technologies: target network and experience replay [1]. The optimal action-value function \(Q^*(s, a)\) can be learned by minimizing the following loss function with respect to the neural network parameters \(\theta\) according to Bellman equation (1):

\[
L(\theta) = \mathbb{E}_{(s, a, r, s') \sim \mathcal{B}}[ (y - Q(s, a; \theta))^2],
\]

where \(y = r + \gamma \max_{a' \in A} Q(s', a'; \theta')\) is target action-value which is computed by the frozen and separated network parameters \(\theta'\) which is copied from online learning parameters \(\theta\) for every fixed time steps to decouple correlation between the online learning action-value and the target action-value. And \(\mathcal{B}\) is the replay buffer that stores the past transitions, which also reduces the correlation of sampled transitions. Both the
target network and the experience replay dramatically improve the performance of DQN.

Although DQN can achieve human-level control in many real-world tasks, e.g. playing Atari game, there still are some issues in this algorithm. For achieving better performance, plenty of remarkable methods have been proposed to improve the DQN, such as recurrent neural network [28], better exploration [33][32], various replay buffer [29][30], function regularization [19] and etc. These algorithms are no longer just learning a value function, but use an actor to learn the policy, where the actor is used to select actions which can process continuous action space. And policy gradient [34] algorithm is used to optimize the actor. The algorithms mentioned above are almost all stochastic policy, that is, the action is sampled from a distribution $\pi$. Silver et al. proposed a deterministic policy gradient algorithm [20].

B. Double DQN

A well known issue of DQN is overestimate the action-value. The DQN algorithm involves max operator in the construction of its target policy, which makes it more likely to select overestimated action-value, resulting in overoptimistic value estimates. The max operator not only estimates the action-value function, but also involves the process of action selection, from this view we can see the overestimation problem simply. Double DQN [15] decouples the selection from the evaluation. Suppose we have two action-value functions, $Q_1(\cdot, a)$ and $Q_2(\cdot, a)$, each of which is a true estimate of all actions. We can use one $Q_1(\cdot, a)$ to determine the action that maximizes the action-value function, $a^* = \arg\max_a Q_1(\cdot, a)$, and another $Q_2(\cdot, a)$ to estimate the action-value function, $Q_2(\cdot, a) = Q_2(\arg\max_a Q_1(\cdot, a))$. We repeat this process so that we decouple the evaluation from the selection. This is the idea behind Double DQN. Double DQN is still updated using equation (2), but the target is

$$y = r + \gamma Q(s', \arg\max_a Q(s', a; \theta_1); \theta_2).$$  

Note that we still use online parameters $\theta_2$ to select an action. And we use the second set of parameters $\theta_2$ to fairly evaluate the value of the policy. Double DQN is unbiased for action-value functions approximation. Although the performance of Double DQN is better than that of DQN in discrete action space, it still suffers when applied to continuous action setting [18].

C. Deep Deterministic Policy Gradient (DDPG)

Silver et al. proposed a Deterministic Policy Gradient method (DPG) [20] to optimize the expected reward which uses a deterministic policy $\pi : S \rightarrow A$ instead of typical stochastic policy in actor-critic setting with continuous action space. DPG does not need to integrate actions, so it is a more efficient method to estimate value functions than stochastic policy. Inspired by the DQN [1], Lillicrap et al. combined DPG algorithm with deep neural network and proposed the Deep Deterministic Policy Gradient algorithm (DDPG) [17]. This algorithm utilizes the learned action-value function to update the policy. We use $\phi, \phi', \theta$ and $\theta'$ to mark the parameters of actor, target actor, critic and target critic respectively. The critic is still updated in the same way as the DQN:

$$L(\theta) = \mathbb{E}_{(s, a, r, s') \sim \mathcal{B}} \left[ (y - Q(s, a; \theta))^2 \right], \tag{4}$$

where $y = r + \gamma Q(s', \pi(s'; \phi); \theta')$ is the target value, based on the independent target networks. The updating mode of the policy network is based on the online learning critic $\theta$ and updated by the chain rule of gradient propagation:

$$\nabla_\phi J(\phi) = \mathbb{E}_{s \sim \mathcal{P}_n} \nabla_a Q(s, a; \theta)_{|a = \pi(s; \phi)} \nabla_\phi \pi(s; \phi), \tag{5}$$

where $Q(s, a) = \mathbb{E}_{s' \sim \mathcal{P}_d}[R_t|s, a]$ is the expected return when performing action $a$ in state $s$ and following policy $\pi$ after, which is a parameterized function. After updating online learning parameters, the target network parameters, $\theta'$, $\phi'$, are soft-updated in a different way from DQN:

$$\theta' = \eta \theta + (1 - \eta) \theta', \quad \phi' = \eta \phi + (1 - \eta) \phi',$$  

where $\eta$ is a enough small constant, which greatly improves the stability of the learning process because of the slow update. A significant issue with the DPG algorithm is the lack of exploration capability. Lillicrap et al. claimed that adding noise drawn from Ornstein-Uhlenbeck process [35] to actions can help DDPG explore better. However, Fujimoto et al. found that the same performance could be achieved using Gaussian noise [18].

D. Twin Delayed Deep Deterministic Policy Gradient (TD3)

TD3 [18] algorithm is an improved version of the DDPG algorithm and is also a deterministic policy gradient algorithm. TD3 algorithm, which takes the minimum value in a pair of critics as the target value, is called clipped double Q-learning. TD3 still utilizes the same form of loss function (4), but $y = r + \gamma \min_{i=1,2} Q'_i(s', \pi(s'; \phi_i); \theta'_i)$, where $Q'_1$ and $Q'_2$ represent the two target critics with respect to two independent critics $Q_1$ and $Q_2$, which is the first improvement over DDPG. With this improvement, the TD3 algorithm can simultaneously train two critics and pick the minimum value of critics, thus alleviating the overestimation phenomenon. The second improvement is to delay the actor update until the critic network is updated after a specific time step. This update rule decouples actor from critic and reduces function estimation error [18]. Although the TD3 algorithm achieves a better performance than the DDPG algorithm, the TD3 algorithm still estimates the value function inaccurately. Because of the existence of the min operator, the estimation of value function tends to underestimated, thus harming the performance.

III. THE UNDERESTIMATION PHENOMENON

In this section, we begin with a theoretical analysis of the underestimation bias that the occurrence of the min operator in the learning of action-value functions leads to. Then we empirically show that using the minimum value of two critics can cause underestimation bias and thus harm performance in the recently proposed TD3 algorithm. According to the
Bellman equation \([1]\), the learning process of the action-value function involving the min operator can be expressed as

\[
Q(s, a) \leftarrow r + \gamma \min_{i=1,2} Q_i(s', \pi(s'; \phi'); \theta'_i).
\]

(7)

To better understand the learning process. We assume that \(Q_i\) is an estimate of the true action-value \(Q^*\), with an estimated error of \(Z_i = Q_i - Q^*\) due to noise, where \(Z_i\) is drawn from a specific independent identical distribution. The minimization operator acts on \(Q_i\). By assuming that \(Z_i\) satisfies different distributions, we explain theoretically that the minimization operator can cause the underestimation problem of value functions.

**Theorem 1.** Let \(Q^*\) denote the true state-action value, suppose that there are 2 estimate value \(\hat{Q}_i\) for \(i = 1,2\). Denote the estimate error \(G_i = \hat{Q}(s, a) - Q^*(s, a)\) are independently Gaussian distribution \(\mathcal{N}(0, \sigma^2)\). Then,

\[
\mathbb{E}[\min_{i=1,2}\{G_i\}] = -\sigma \frac{1}{\sqrt{\pi}}
\]

**Proof.** Obviously, we have \(\min\{G_1, G_2\} = \frac{1}{2}(G_1 + G_2 - |G_1 - G_2|)\). Denote \(Y = |G_1 - G_2|\), hence, \(Y \sim \mathcal{N}(0, 2\sigma^2)\),

\[
\mathbb{E}[Y] = \int_{-\infty}^{+\infty} |y| \phi(y) dy
= \int_{+\infty}^{+\infty} \frac{2y}{\sqrt{2\pi}\sqrt{2\sigma}} \exp\{-\frac{y^2}{2 \cdot 2\sigma^2}\} dy
= \frac{\sigma}{\sqrt{\pi}}
\]

where \(\phi(y) = \frac{1}{\sqrt{2\pi}\sqrt{2\sigma}} \exp\{-\frac{y^2}{2 \cdot 2\sigma^2}\}\).

This implies that \(\mathbb{E}[\min\{G_1, G_2\}] = \mathbb{E}\left[\frac{1}{2}(G_1 + G_2 - |G_1 - G_2|)\right] = -\sigma \frac{1}{\sqrt{\pi}}.\)

The expectation is \(-\gamma \frac{1}{\sqrt{\pi}} < 0\). Note that even the function approximation is unbiased, there still exits underestimation issue. In the next theorem, we borrow heavily from the proof of Theorem 2 in double DQN [15].

**Theorem 2.** Consider a state \(s\) where true optimal action values is \(Q^*\), suppose that there are \(N\) estimate value \(\hat{Q}_i\) for \(i = 1, \cdots, N\). Denote the estimate error \(Z_i = \hat{Q}_i(s, a) - Q^*(s, a)\) are independently distribution uniformly in interval \([-\delta, \delta]\). Then,

\[
\mathbb{E}\left[\min_{i=1,\cdots,N}\{Z_i\}\right] = -\frac{N - 1}{N + 1} \delta
\]

**Proof.** We denote the probability density function of \(Z_i\) for \(i = 1, \cdots, N\) as \(f(x)\):

\[
f(x) = \begin{cases} 
\frac{1}{2\delta} & x \in (-\delta, \delta) \\
0 & \text{else}
\end{cases}
\]

Then, we can derive that the cumulative distribution function for all variables \(Z_i\), where \(i = 1, \cdots, N\):

\[
P\{Z_i > x\} = \begin{cases} 
\frac{1}{2\delta} & x \leq -\delta \\
0 & x \geq \delta
\end{cases}
\]

Since \(Z_i\) is a uniformly random variable in \([-\delta, \delta]\), the probability that \(\min_{i=1,\cdots,N} Z_i \geq x\) for \(x\) is equal to the probability that \(Z_i \geq x\) for all \(i = 1, \cdots, N\) simultaneously, we can derive:

\[
P\{\min_i Z_i \geq x\} = P\{Z_1 \geq x, Z_2 \geq x, \cdots, Z_N \geq x\}
= \prod_{i=1}^{N} P\{Z_i \geq x\}
= \begin{cases} 
\frac{1}{2\delta} & x \leq -\delta \\
0 & x \geq \delta
\end{cases}
\]

This implies that we can get the cumulative density function (CDF):

\[
P\{\min_i Z_i < x\} = 1 - P\{\min_i Z_i \geq x\}
= \begin{cases} 
1 - \left(\frac{1}{2\delta}\right)^N & x \leq -\delta \\
1 & x \geq \delta
\end{cases}
\]

Then, we can get the probability density function of this variable by using the derivative the CDF:

\[
f_{\min}(x) = \frac{d}{dx} P\{\min_i Z_i < x\}
= N \left(\frac{1}{2\delta}\right)^N \frac{d}{dx} \left(\frac{1}{2\delta}\right)^N - 1
= \frac{N}{2\delta} \int_{-\delta}^{x} N \left(\frac{1}{2\delta}\right)^{N-1} \frac{d}{dh} h^{N-1} dh
= \frac{N - 1}{N + 1} \delta
\]

where \(h = x - \delta\). Its expectation can be written as an integral

\[
\mathbb{E}\left[\min_{i=1,\cdots,N} Z_i\right] = \int_{-\delta}^{x} f_{\min}(x) dx
= \int_{-\delta}^{x} \frac{N}{2\delta} \left(\frac{1}{2\delta}\right)^N \frac{d}{dh} h^{N-1} dh
= \frac{N - 1}{N + 1} \delta
\]

When \(N = 2\), the expectation is \(-\frac{1}{3} \delta < 0\).

**A. Will the underestimation problem occur in practice?**

We conduct experiments to verify whether the underestimation occurs in practice or not. We utilize Ant-v0 of the pyBullet suite [37] on OpenAI gym environments to verify that overestimation occurs in DDPG and TD3 does underestimate the action-value. More details of the experiments are discussed in section V. In Fig. 1 we graph the average value estimate where every data point is based on 50 trajectories and compare it to an estimation of the true value. At the beginning
Algorithm I Weighted Delayed Deep Deterministic Policy Gradient (WD3)

1: Initialize actor network \( \pi \), and critic network \( Q_i \) for \( i = 1, 2 \) with random parameters \( \phi, \theta_i \)
2: Initialize target networks \( \theta'_i \leftarrow \theta_i \), \( \phi' \leftarrow \phi \)
3: Initialize replay buffer \( B \)
4: Initialize \( \beta, d, \sigma, \tilde{\sigma}, \eta, c \) total steps \( T \), and \( t = 0 \)
5: Reset the environment and receive initial state \( s \)
6: while \( t < T \) do
7:   Select action with noise \( a = \pi(s; \phi) + \epsilon, \epsilon \sim \mathcal{N}(0, \sigma^2) \), and receive reward \( r \), new state \( s' \)
8:   Store transition tuple \( (s, a, r, s') \) to \( B \)
9:   Sample mini-batch of \( N \) transitions \( (s, a, r, s') \) from \( B \)
10: \( \tilde{a} \leftarrow \pi(s'; \phi') + \epsilon, \epsilon \sim \text{clip}(\mathcal{N}(0, \tilde{\sigma}^2), -c, c) \)
11: \( y \leftarrow r + \gamma(\beta \min_{i=1,2} Q_i(s', \tilde{a}; \theta'_i) \right) + \frac{1-\beta}{2} \sum_{i=1}^{2} Q_i(s', a; \theta'_i) \)
12: Update critic \( \theta \leftarrow N^{-1} \sum (y - Q_\phi(s, a))^2 \)
13: if \( t \) mod \( d \) then
14:   Update \( \phi \) by the deterministic policy gradient:
15:   \( \nabla_\phi J(\phi) = N^{-1} \sum \nabla_\phi Q_i(s, a; \theta'_i) |_{a=\pi(s; \phi)} \nabla_\phi \pi(s; \phi) \)
16: Update target networks:
17: \( \theta'_i \leftarrow \eta \theta_i + (1-\eta) \theta'_i \)
18: \( \phi' \leftarrow \eta \phi + (1-\eta) \phi' \)
19: end if
20: \( t \leftarrow t + 1 \)
21: \( s \leftarrow s' \)
22: end while

![Chart](image1.png)

Fig. 1. Measuring the estimation bias on continuous control tasks: (a) overestimation bias in DDPG, (b) underestimation bias in TD3, and accurate estimation in WD3. Our method WD3 achieves a balance of these two opposites. The shaded area stands for a standard deviation of the average evaluation over 10 random seeds.

of the DPG algorithm learning process, the Q-function is greatly overestimated, and then the Q-function slowly declines. Together with Fig. 3, we find that the overestimation of the Q-function does make the DPG algorithm unable to learn skills. With the decrease of overestimation, the performance of DPG is increasing gradually. Due to the instability of the reinforcement learning environment, the TD3 algorithm also presents the problem of greatly overestimating the initial learning process, which is then controlled by the min operator. In the later learning stage, the learning curve of value function becomes stable, and the Q-function is underestimated and maintains for a long time. We unify these two opposites and propose the Weighted Delayed Deep Deterministic Policy Gradient algorithm (WD3). By weighted averaging target critic, WD3 enables the Q-function to reach a balance between overestimation and underestimation, which makes the learning process of value function estimation more stable and accurate.

IV. WEIGHTED DELAYED DEEP DETERMINISTIC POLICY (WD3)

To address estimation bias, Double DQN introduces separated Q-function which still overestimates action value in continuous control tasks. TD3 takes the minimum value of two critics as a target to update value function which results in underestimation bias. Based on TD3, we propose a novel Weighted Delayed Deep Deterministic Policy Gradient algorithm which alleviates the estimation bias by introducing a weighted smooth update mechanism that can be applied to any actor-critic algorithms.

A. Weighted target update

In Double DQN, greedy value function update is deconstructed by keeping two Q functions, \( Q_1 \) and \( Q_2 \) can be used to update each other. However, the purpose of decoupling cannot be achieved in continuous control tasks due to vast action space. The slow change of the policy learning process makes the two Q networks coupled due to the slow learning process in continuous action space and the early exploration of the agent, which cannot reflect the principles that informed its development. The huge variance of Q-function estimation brought by the continuous action space makes the learning of the value function unstable compared with the discrete action state, thus leading to the overestimation problem \([18]\). The DDPG algorithm tends to overestimate the action-value function on the continuous control task. Fujimoto et al. proposed to use the minimum value of a pair of critics as the target for updating, resulting in an underestimation bias as discussed above. The overestimation problem of DDPG algorithm and the underestimation problem of TD3 are exactly the two opposites. We propose the WD3 algorithm to achieve the balance between overestimate and underestimate by weighting a pair of target critic. We utilize a pair of critics, \( Q_1 \) and \( Q_2 \) and a policy network \( \pi \). The parameterized Q functions is updated by:

\[
Q_i \leftarrow r + \gamma(\beta \min_{i=1,2} Q_i(s', a'; \theta'_i) + \frac{1-\beta}{2} \sum_{i=1}^{2} Q_i(s', a'; \theta'_i))
\]  

(8)

Where \( \beta \in [0, 1] \) controls the balance between overestimation and underestimation. When \( \beta = 1 \), the algorithm decays to TD3. The parameters of actor is update by

\[
\hat{J}(\phi) = N^{-1} \sum_{s, a} \nabla_\phi Q_i(s, a; \theta'_i) |_{a=\pi(s; \phi)} \nabla_\phi \pi(s; \phi)
\]

(9)

In the following section we discuss the form of the target action.
functions and environment settings without any changes to
without any modification. Besides, we use the default reward
work as we originally intended. We use the original low-
simply without any engineering tricks to make the algorithm
A. Implementation Details

B. Action noise smoothing

A well-known issue of deterministic policy gradient algo-
rithms is a lack of exploration capability because they directly
output a certain action rather than a distribution of the action.
To solve this problem, the original DDPG proposed to increase
the exploration ability by adding noise to the action which is
drawn from the Ornstein-Uhlenbeck process [35]. However,
Fujimoto et al. found that this kind of noise has no additional
benefit to the exploration, and the same performance can be
achieved with Gaussian noise. Matthias et al. added noise to
the parameters of the neural network, but the method has no
significant advantage over the former one in the continuous
control tasks[36]. To ensure exploration, we add Gaussian noise
to actions when the agent interacts with the environment and
target actions. Therefore, our target action \( \hat{a} \) is:

\[
\hat{a} = a + \epsilon,
\]

where \( a = \pi(s; \phi), \epsilon \sim \mathcal{N}(0, \sigma^2) \). We summarize the entire
algorithm on Algorithm 1

V. Experiments

To evaluate our algorithm, we measure the performance of
WD3 on the suite of pyBullet (Fig. 2) [37] continuous control
tasks. The state dimension and action dimension of the suite is
show in Table I. By using the modifications discussed in
section IV, we increase the stability and accuracy of the
Q-function learned by considering the estimation problem.
The WD3 algorithm still maintains a pair of critics, and we
update the Q-function with a weighted average [8]. The policy
network is updated by equation (5). Utilizing a specific action-
value function, we can update the parameters of the policy
network through the chain rule of gradient propagation. Every
d time step, the policy network is updated with the action-
value function, according to the deterministic policy gradient
algorithm [20]. To increase the stability and performance of the
algorithm, the soft update method is adopted when updating
target networks.

A. Implementation Details

Given the recent concerns about algorithms reflect the prin-
ciples that informed its development [38], we implement WD3
simply without any engineering tricks to make the algorithm
work as we originally intended. We use the original low-
dimensional state vector provided by the environment as input
without any modification. Besides, we use the default reward
functions and environment settings without any changes to
achieve a fair comparison of algorithms performance.

Due to recent concerns about the reproducibility crisis of
deep reinforcement learning algorithms [22], we run all the
tested algorithms over 10 random seeds. For WD3, we use
two-layer feedforward neural network, each layer has 256
units respectively, using the rectified linear units (ReLU) as
the activation function of each layer for all actors and critics,
but the last layer of the actor is followed by a tanh activation
function to keep the output in the action space of agent. To
minimize the loss function of the algorithm, Adam [39] is
used as the default optimizer for all neural networks to update
the samples randomly and uniformly collected for mini-batch
100 with a learning rate of 3e−4. The actor network and
two target critic networks perform delayed soft updates every
d = 2 iterations, where \( \tau = 0.005 \). To balance overestimate
and underestimate and to fairly evaluate our algorithm, we
use \( \beta = 0.45 \) on all tasks when computing the target critic.
The same setting is applied to OurDDPG to fairly compare
between the estimation of value function and that of WD3.

To balance exploration and exploitation, the Gaussian noise
of \( \epsilon \sim \mathcal{N}(0, 0.1) \) is added to the actions when an agent selects
actions to interact with the environment, and then the actions
with noises are clipped in the action space of the agent. When
updating the value function, we add the Gaussian noise of \( \epsilon \sim \mathcal{N}(0, 0.2) \)
to the action selected according to the target actor,
which is clipped to \([-0.5, 0.5]\). To eliminate the dependence
of the policy network on the initial parameters, we used the
pure exploration policy for all environments for the first 25,000
time steps.

Each task runs on 1 million time steps, with evaluations
conducted every 5,000 time steps. All algorithms are run and
evaluated on ten random seeds. In the evaluation process,
there is no exploration noise, and the transitions from the
evaluation will not be carried over to the experience replay
buffer. Furthermore, all our experiments are reported based on
ten random seeds.

We compared our algorithm with the TD3 algorithm and the
state-of-the-art policy gradient algorithms PPO, TRPO [14],
and DDPG, which are implemented by the OpenAI Baselines
[40]. For the TD3 algorithm, we use the author’s implemen-
tation. Besides, considering that there are some engineering
skills in the DDPG implementation of OpenAI, we implement
the DDPG algorithm by ourselves, called OurDDPG, without
adding any tricks on the original DDPG that affects the
performance of the algorithm.

The learning performance curves are graphed in Fig. 3.
The results show that WD3 matches or outperforms all other
algorithms without fine-tuning. We evaluate the influence of \( \beta \)
on WD3, and the results are graphed in Table II which shows
that WD3 is robust for \( \beta \).

B. Q-function estimation

We evaluate the Q-functions of OurDDPG and TD3 in Ant
environment over 10 random seeds. Every 5,000 time steps we
get the average action-value of current agent and the true value
estimated by the Monte Carlo method. We take 50 trajectories,
and each trajectory contains 1,000 transitions to approximate.
true action-value. All the experiments are carried out on ten random seeds. So each data point is based on 500,000 values. We present the results in Fig. 1. The results shows that WD3 does achieve our purpose. In the initial training stage of an agent, the estimation of the value function is stable and then rises gradually, which is close to the real value function. In the learning process of the OurDDPG, the action-value function is greatly overestimated, which makes the performance of the algorithm suffer. The TD3 algorithm overestimates the action-value in the initial stage as OurDDPG. With the progress of learning, the algorithm begins to underestimate action-value. Note that the value function of TD3 appears a small magnitude of overestimation in some stages of action-value function learning. We argue that the overestimation bias in this curve is caused by the delay effect of neural networks. When an agent explores a new space, the neural network has been updated for a period, which makes the output of the Q-function network higher than that of the previous network. For \((s, a)\) pair not explored by the agent, the neural network will have the problem of overestimation, which is caused by the poor generalization ability of the neural network. Specifically, neural networks generalize learned high action-value to unseen one. The value of unseen state-action pairs is usually less than explored one. Because the agent does not explore this

| Environment | Ant | HalfCheetah | Hopper | InvertedDouble | InvertedPendulum | Swingup | Reacher | Walker2D |
|-------------|-----|-------------|--------|----------------|------------------|--------|---------|---------|
| State Dimension | 28  | 26  | 15  | 9  | 5  | 22 | 9 | 2 |
| Action Dimension | 8  | 6  | 3  | 1  | 1  | 6 | 1 | 2 |

\(a\) This is short for InvertedDoublePendulum

\(b\) This is short for InvertedPendulumSwingup

**TABLE I**

Quantitative descriptions of pyBullet suite for version 0. The first row represents the state dimension and the second represents the action dimension, where both of them are continuous in each dimension.

**TABLE II**

The last 5 return over 10 trials of 1 million time steps for various \(\beta\). Maximum value for each task is bolded. Corresponds to a single standard deviation over trials.

| Beta | Ant | HalfCheetah | Hopper | InvertedDouble | InvertedPendulum | Swingup | Reacher | Walker2D |
|------|-----|-------------|--------|----------------|------------------|--------|---------|---------|
| 0.15 | 2810.89±183.66 | 2616.87±300.32 | 2113.79±292.63 | 9321.41±153.8 | 985.84±70.8 | 887.93±13.97 | 20.14±2.39 | 1760.08±298.9 |
| 0.30 | 2948.46±178.79 | 2551.4±253.1 | 2101.9±248.49 | 8691.23±1718.2 | 1000.0±0.0 | 870.81±42.02 | 19.53±2.87 | 1773.36±230.79 |
| 0.45 | 2868.36±257.58 | 2630.83±139.69 | 2115.72±282.72 | 9023.9±1223.6 | 845.31±294.0 | 886.67±13.63 | 19.03±2.64 | 1943.26±190.86 |
| 0.50 | 2964.19±128.02 | 2261.6±176.84 | 2215.27±200.02 | 8775.14±1346.84 | 969.62±151.92 | 889.3±0.83 | 19.08±3.24 | 1798.14±263.92 |
| 0.60 | 2813.11±206.33 | 2476.65±204.58 | 1768.56±742.98 | 9022.89±1244.94 | 942.89±204.14 | 885.13±19.08 | 18.45±4.26 | 1841.39±227.03 |
| 0.75 | 3042.37±100.38 | 2285.7±296.99 | 1978.8±189.19 | 9323.49±166.4 | 1000.0±0.0 | 886.2±11.89 | 19.65±3.36 | 1625.85±407.89 |
| TD3  | 2741.59±277.94 | 2358.92±227.99 | 1850.97±590.81 | 6544.22±4239.66 | 773.24±411.3 | 887.64±8.47 | 18.71±3.17 | 1674.36±340.4 |

Fig. 3. Performance curves for OpenAI gym continuous control tasks in pyBullet suite. The shaded region represent a standard deviation of the average evaluation over 10 seeds. The curves are not smoothed at all.
space, the actions outputted by the policy network are not as good as explored space, thus the action-value network cannot match the returns, then the outputs of the action-value function are overestimated. Once the agent learns for a while in this space of this state, the underestimation problem will occur again, which is consistent with our theoretical analysis. The Q-function learning process of the WD3 algorithm is more stable than DDPG and TD3, without underestimation or huge overestimation. WD3 has a good property for value function learning, which is reflected by the performance curves.

VI. CONCLUSION

The estimation bias is a crucially important challenge in value-based reinforcement learning. In this paper, we prove that the estimation error exists widely in the deterministic policy gradient algorithm in theory and practice, overestimation, and underestimation. We discuss the overestimation issue of the combination of action-value function learning and neural network, which is caused by the max operator. And we prove that underestimation issue does occur both in theory and in practice. In order to reduce the estimation bias of the Q-function, we propose the WD3 algorithm, which makes the updating process of the Q-function more stable and accurate utilizing the weighted average target critic, thus improving the performance. We prove experimentally that WD3 is indeed more stable for updating value functions. Furthermore, experiments show that our algorithm matches or outperforms the state-of-the-art algorithms on continuous control tasks.

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