Explores the influence of bulk viscosity of QCD on dilepton tomography

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Abstract

Signals of collective behavior originating from hadronic and electromagnetic radiation in heavy-ion collisions has been widely used to study the properties of the the Quark-Gluon Plasma (QGP). Indeed this collectivity, as measured by anisotropic flow coefficients, can be used to constrain the transport properties of QGP. The goal of this contribution is to investigate further the influence of the specific bulk viscosity ($\zeta/s$) on dilepton production, both at Relativistic Heavy-Ion Collider (RHIC) and Large Hadron Collider (LHC) energies. As will be shown, dileptons are capable of exposing certain dynamical features that bulk viscosity induces on the evolution of a strongly-interacting medium, that have not been seen before and may be used in constraining the overall size of $\zeta/s$.

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I. INTRODUCTION

The Quark-Gluon Plasma (QGP), an intriguing state of matter composed of colored, quarks and gluon degrees of freedom, is believed to have been formed during the first few microseconds after the Big Bang. The goal of relativistic heavy-ion collisions is to re-create and study this system of deconfined partons in the laboratory, with the ultimate goal being the extraction of its properties. A fundamental property of the QGP that is vigorously sought after is its viscosity, including both the bulk and shear components.

The effects of bulk viscosity ($\zeta$) alone on the elliptic flow of hadronic observables has been studied before [1], which was soon after improved by simulations that include both bulk and shear ($\eta$) viscosity effects [2, 3]. These pioneering papers have highlighted some of the important features of bulk viscosity, in terms of how they affect the evolution of the hydrodynamical medium and how the distribution function of hadrons on the freeze-out hypersurface gets modified by the presence of bulk viscosity. The role of bulk and shear viscosity was further investigated through different hadronic observables, namely Hanbury-Brown-Twiss (HBT) correlation radii, in Ref. [4], where it was shown that in the presence of both viscosities an improved description of the HBT radii is possible.

Those earlier simulations however were based on Israel-Stewart hydrodynamics [5, 6] which did not include bulk to shear coupling in the equations of motion for the bulk viscous pressure and the shear stress tensor. The mathematical form of this coupling was fully derived from a microscopic theory, namely from the Boltzmann equation, in Ref. [7], while its importance was studied in Ref. [8]. More recent hybrid calculations composed of hydrodynamical simulation of the QGP followed by the hadronic transport evolution, show that bulk viscosity, including bulk to shear couplings, is a key ingredient in improving the description of hadronic observables [9, 10].

Using the hydrodynamical simulations of Ref. [9], a direct photon calculation has also been carried out [11], showing that bulk viscosity in the hydrodynamical simulation affects the direct photon production at both Relativistic Heavy-Ion Collider (RHIC) and Large Hadron Collider (LHC) energies. The present contribution extends the earlier work of Ref. [9, 11] by looking at lepton pair (dilepton) production from the same hydrodynamical simulations.

The dilepton invariant mass $M$, or the center of mass energy of the lepton pairs, provides them with an advantage over photons whereby low invariant mass dileptons ($M \lesssim 1.1$ GeV) are dominated by light flavor hadronic contributions (composed of $u$, $d$ and $s$ quarks), while as invariant mass increases, partonic sources of lepton pairs radiation become more important. Specifically in the low invariant mass case, there are two contributions to dileptons: one from the in-medium decay of vector mesons and the other from the late hadronic direct and Dalitz decays. At intermediate invariant masses ($1.1 \lesssim M \lesssim 2.5$ GeV), the two main dilepton sources are direct QGP emission as well as semi-leptonic decays of open heavy flavor hadrons. Furthermore, the invariant mass degree of freedom has already been used to show that dileptons are sensitive various aspects of the shear viscous pressure [12, 13], and the present study continues in that direction by considering the effects of bulk viscous pressure on dileptons.

Isolating the thermal contribution of dilepton production is not an easy task, but can be done, as was first shown in the case of dimuons by the NA60 Collaboration at the Super Proton Synchrotron (SPS) at CERN. [14, 15]. At
RHIC, the STAR Collaboration has recently acquired new dimuon data using their Muon Telescope Detector (MTD) and Heavy Flavor Tracker (HFT) simultaneously [17]. Thus having both MTD and the HFT running at the same time is extremely useful as it permits to investigate the dilepton radiation coming directly from thermal radiation. At the LHC, the upgraded ALICE experiment is planning to measure dilepton yield and anisotropic flow, on the order of 10%, for the High Luminosity LHC run after 2020 [18]. Present work assumes that the open heavy-flavor contribution is removed and considers dilepton radiation coming from QGP and light hadronic sources in the low and intermediate mass region.

This paper is organized as follows: Section II provides details about the dynamical modeling of the strongly-interacting medium, with an emphasis on the fluid-dynamical equations of motion used. Section III gives the details of dilepton production originating from both partonic and hadronic sources. Section IV present results about how bulk viscosity influences dilepton yield and anisotropic flow, while Section V provides concluding remarks.

II. DYNAMICAL SIMULATION

The hydrodynamical equations of motion are based on the conservation laws for energy-momentum and net-charge. In the fluid-dynamical model of ultrarelativistic heavy-ion collisions, one is usually probing the low longitudinal momentum fraction ($x$) region of the nuclear parton distribution functions, which are gluon-dominated [19]. Therefore, the resulting hydrodynamical equations can neglect the conservation of net quark-flavor induced charge — i.e. net baryon number, electric charge and strangeness — near mid-rapidity [20]. Furthermore, the fact that the gluon distribution is highly populated in the low-$x$ region allows for the approximation that these gluon degrees of freedom can be dynamically evolved using classical Yang-Mills equations, which is precisely how the IP-Glasma models its early-time dynamical evolution [21]. Throughout this study, IP-Glasma will be used as a pre-hydrodynamical model of the strongly interacting medium, dynamically evolving the system for an assumed $\tau_0 = 0.4$ fm/c (see [10] and references therein for details), following which we start the hydrodynamical simulation.

The hydrodynamical equations of motion include dissipation, which is described by six dissipative degrees of freedom, $\Pi$ and $\pi^{\mu\nu}$, accounting for bulk and shear viscous effects, respectively. The energy-momentum conservation equation reads:

$$\frac{\partial}{\partial t} T^{\mu\nu} = 0,$$

$$T^{\mu\nu} = T_0^{\mu\nu} + \delta T_\Pi^{\mu\nu} + \delta T^{\mu\nu},$$

$$T_0^{\mu\nu} = \epsilon u^\mu u^\nu - P\Delta^{\mu\nu}, \quad \delta T_\Pi^{\mu\nu} = -\Pi\Delta^{\mu\nu}, \quad \delta T_\pi^{\mu\nu} = \pi^{\mu\nu}$$

(1)

where $\epsilon$ is the energy density, $u^\mu$ is the flow four velocity, $P$ is the thermodynamic pressure related to $\epsilon$ by the equation of state $P(\epsilon)$ [22], $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ projects on the spatial directions in the local fluid rest frame, and $g^{\mu\nu} = \text{diag}(1,-1,-1,-1)$ is the metric tensor. The dissipative degrees of freedom satisfy relaxation-type equations:

$$\tau_\Pi \Pi = -\zeta \Pi + \lambda_\Pi \pi^{\alpha\beta},$$

$$\tau_\pi \pi^{\mu\nu} = \frac{2\eta}{m^2} - \delta \pi^{\mu\nu} \theta + \lambda_\pi \pi^{\mu\nu} - \tau_\pi \pi^{(\mu}\pi^{\nu)} + \phi_\pi \pi^{(\mu}\pi^{\nu)},$$

(2)

(3)

where $\Pi = u^\alpha \partial_\alpha \Pi$, $\pi^{(\mu\nu)} = A^{\alpha\beta} u^\lambda \partial_\lambda \pi^{\alpha\beta}$, $\Delta^{\mu\nu} = (\Delta^\alpha_{\alpha'} \Delta_{\alpha\beta} + \Delta^\beta_{\alpha'} \Delta_{\alpha\beta})/2 - (\Delta_{\alpha\beta} \Delta^{\mu\nu})/3$, $\theta = \partial_\alpha u^\alpha$, $\sigma^{\mu\nu} = \delta^{\mu\nu}$, while $A^{\mu\alpha} \equiv A^{\mu\alpha} A^{\alpha\beta}$. Other than $\zeta$ and $\eta$, which will be discussed in a moment, the various transport coefficients present in Eq. (2) and Eq. (3) were computed assuming a single component gas of constituent particles in the limit $m/T \ll 1$ [7, 23], where $m$ is their mass and $T$ the temperature, respectively. These transport coefficients were also used in Refs. [9, 11] and are summarized in Table I where $c_s^2 = \partial P/\partial \epsilon$ is the speed of sound squared. The spatial resolution of the hydrodynamical simulation is $\Delta x = \Delta y = 0.17$ fm, while the temporal one is $\Delta \tau = 0.015$ fm/c.

| TABLE I: Transport coefficients in Eqs. (2) and (3). |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Bulk $\tau_\Pi = \zeta \left[15(\epsilon + P) \left(\frac{1}{\eta} - c_s^2\right)\right]^{-1}$ | $\delta_{\Pi\Pi} = \frac{2}{15} \tau_\Pi$ | $\lambda_{\Pi\Pi} = \frac{2}{15} \left(1 - c_s^2\right) \tau_\Pi$ | $\tau_{\Pi\Pi} = \frac{10}{15} \tau_\Pi$ | $\phi_\Pi = \frac{18}{15} \frac{\tau_\Pi}{\tau_\Pi}$ |
| Shear $\tau_\pi = 5\eta \left[\epsilon + P\right]^{-1}$ | $\delta_{\pi\pi} = \frac{2}{5} \tau_\pi$ | $\lambda_{\pi\pi} = \frac{2}{5} \tau_\pi$ | $\tau_{\pi\pi} = \frac{10}{7} \tau_\pi$ | $\phi_\pi = \frac{18}{7} \frac{\tau_\pi}{\tau_\pi}$ |

The specific shear viscosity $\eta/s$ — where $s$ is the entropy density — is here assumed to be temperature independent, while the specific bulk viscosity $(\zeta/s)$ is assumed to exhibit a strong temperature dependence in the vicinity of the
quark-hadron cross-over as shown in Refs. [9][11]. Indeed, assuming a medium that has both bulk and shear viscosity, a reasonable value of $\eta/s$ at LHC energy of $\sqrt{s_{NN}} = 2.76$ TeV is $\eta/s = 0.095$, obtained by fitting to hadronic observables measured by the ALICE and CMS Collaborations [9]. To narrow down some effects of bulk viscosity, two other simulations are run at LHC energy, one where bulk viscosity is removed while keeping $\eta/s = 0.095$, and another where we increase shear viscosity to $\eta/s = 0.16$ in order to better reproduce multiplicity as well as $v_2$ of hadrons [9]. However, this increase of $\eta/s$ comes at a price of a poorer description of the mean transverse momentum $\langle p_T \rangle$ of various hadronic species. All cases considered employ the same IP-Glasma initial conditions.

In order to fit hadronic observables with this model [9][10], it is necessary to modify the value of $\eta/s$ as one changes collision energy. In some sense, this is a way to take into account the temperature dependence of $\eta/s$. At the top RHIC energy, $\sqrt{s_{NN}} = 200$ GeV, a smaller value of $\eta/s$ (i.e. $\eta/s = 0.06$) is used in order to reproduce hadronic observables measured by the STAR Collaboration [9][10]. Given that the main goal of the current study is to explore how bulk viscosity affects dilepton production, two hydrodynamical simulations will be run at top RHIC energy, one with bulk viscosity and the other without. In both cases, shear viscosity is kept at $\eta/s = 0.06$ and the same IP-Glasma initial conditions are employed.

Lastly, hydrodynamical simulations are evolved until a switching temperature ($T_{\text{sw}}$) is reached, where fluid elements are converted to hadrons. As calibration of the model is done using hadronic observables, further hadronic dynamics are performed via UrQMD simulations [24][25]. However, no electromagnetic radiation from this stage is computed in this work. The switching temperature is also allowed to vary depending on collision energy. Within the model, the best description of the hadronic observables [9][10] at top RHIC collision energy is reached when $T_{\text{sw}}^{\text{RHIC}} = 165$ MeV, whereas at LHC energy a temperature $T_{\text{sw}}^{\text{LHC}} = 145$ MeV is used.

III. DILEPTON PRODUCTION

This study considers two categories of dilepton radiation: one is the hydrodynamical dilepton radiation from the underlying hydrodynamical simulation, which for the sake of brevity will often be called “thermal” dileptons, even though the medium and the dilepton rates account for non-equilibrium bulk and shear dissipation; the other source is cocktail dileptons, which consist of late Dalitz decays of pseudo-scalar mesons and vector mesons, as well as direct decays of vector mesons.

A. Hydrodynamical dileptons

The hydrodynamical or thermal dilepton production consists of emissions from the QGP as well as in-medium decays of vector mesons in the late hadronic evolution stage. The equation of state used throughout this study [22] smoothly interpolates between lattice QCD calculations (lQCD), which contain a cross-over transition, and the hadron resonance gas (HRG) model employed at lower temperatures. We use a smooth connection between lQCD and HRG in the temperature region $0.184 < T < 0.22$ GeV. The thermal dilepton rates will be interpolated within this temperature region as follows:

$$\frac{d^4R}{d^4q} = f_{QGP} \frac{d^4R_{QGP}}{d^4q} + (1 - f_{QGP}) \frac{d^4R_{HM}}{d^4q}$$

(4)

where $\frac{d^4R_{QGP}}{d^4q}$ is the partonic dilepton rate, $\frac{d^4R_{HM}}{d^4q}$ is the dilepton rate from the hadronic medium (HM), which are both defined in the following two subsections. The QGP fraction, denoted by $f_{QGP}$, is chosen such that $f_{QGP} = 1$ for temperature $T > 0.22$ GeV, $f_{QGP} = 0$ for $T < 0.184$ GeV and is linearly rising with temperature for $0.184 < T < 0.22$ GeV. Thermal dilepton rates are integrated for all temperatures above the switching temperature $T_{\text{sw}}$. Dileptons from the hadronic cocktail will be computed from the hypersurface of constant switching temperature specified in the previous section.

The dilepton production rate for an equilibrated system takes the following form, valid for both partonic and hadronic production sources:

$$\frac{d^4R^{+\ell^-}}{d^4q} = -\frac{L(M) \alpha^2_{EM}}{M^2} \frac{M^2}{\pi^3} \text{Im} \left[ \Pi_{EM}^R(M, |q|; T, \mu_i=Q,B,S) \right] e^{-u/T} - 1$$

(5)
where \( \mu_i = 0 \) in our hydrodynamical simulations\(^1\), \( L(M) = \left( 1 + \frac{2n}{3T} \right) \sqrt{1 - \frac{4n^2}{3T^2}} \), \( m_\ell \) is the lepton mass, \( M^2 = q_\mu q^\mu \), \( q^0 = \sqrt{M^2 + |q|^2} \), \( \alpha_{\text{EM}} = e^2 \gamma / 4\pi \approx 0.137 \), \( u^\mu \) is the local flow four velocity of the medium, while \( T \) is its temperature, and \( \text{Im} [\Pi^R_{\text{EM}}] \) is the imaginary part of the trace of the retarded (virtual) photon self-energy. A general form of the above expression, valid off-equilibrium, uses the real time formalism where the dilepton rate is proportional to \( \Pi^0_{\text{EM}} \) instead of \( \text{Im} [\Pi^R_{\text{EM}}] / \left[ e^{q \cdot u / T} - 1 \right] \). More theoretical details concerning off-equilibrium electromagnetic production have only been worked out for the case of real photons \(^{26,27}\).

1. High temperature dilepton production: partonic dilepton rates

Perturbative dilepton rates for an equilibrated QGP have been computed at next-to-leading (NLO) \(^{28-30}\) within a phenomenologically interesting kinematic region, while lattice calculations for in-equilibrium EM production \(^{31-33}\) as a function energy of the virtual photon, as well as its three momentum has been recently achieved \(^{34,35}\). However, as the effects of dissipation on NLO dilepton rates are still being worked out, this study will focus on the QGP dilepton rate within the Born approximation where dissipative corrections are included. Indeed, the main point of this work is to explore how the presence of bulk viscous pressure affects the hydrodynamical evolution and in turn dilepton production; thus it is important to have the effects of bulk dissipation consistently included into the calculation.

Assuming a non-dissipative fluid, the Born dilepton rate takes the following from:

\[
\frac{d^4R_0}{d^4q} = \int \frac{d^3k_1}{(2\pi)^3\lambda_1^3} \frac{d^3k_2}{(2\pi)^3\lambda_2^3} n_{0,k_1} n_{0,k_2} q^2 \sigma \delta^4(q - k_1 - k_2)
\]

\[
\sigma = \frac{16\pi \alpha_{\text{EM}}^2}{3q^2} \left( \sum f_i c_i^2 \right) N_c
\]

where \( n_{0,k_i} = \left[ \exp(k_i \cdot u / T) + 1 \right]^{-1} \) for \( i = 1, 2 \), \( u^\mu \) is the local flow velocity of the fluid, \( \sigma \) is the leading-order quark-antiquark annihilation (into a lepton pair) cross section, \( N_c = 3 \) is the number of colors in QCD, the number of flavors is labeled by \( f_i \), where only the low-mass ones are considered, i.e. \( f_i = u, d, s \). Extending the isotropic dilepton rate in Eq. (6) to include both bulk and shear-viscous effects, amounts to modifying the quark/anti-quark Fermi-Dirac distribution functions \( (n_{0,k}) \). Using the Israel-Stewart 14-moment approximation in \(^{36,37}\), the shear viscous correction to the QGP dilepton rate \( \delta R_\pi \), in the local rest frame of the medium, reads:

\[
\frac{d^4\delta R_\pi}{d^4q} = \frac{q^0 q^2 \sigma}{2T^2(\epsilon + P)} \left( C_q q^2 - \frac{\sigma}{2(2\pi)^3} \frac{T^5}{|q|^5} \int \frac{dE_k}{T} n_{0,k} \left[ 1 - n_{0,k} \right] n_0 \left( q^0 - E_k \right) D \right)
\]

where \( E_k = \frac{T}{\sqrt{2}} + \frac{q^0 |q|}{\sqrt{2}} \), \( n_{0,k} = \left[ \exp(E_k / T) + 1 \right]^{-1} \), \( n_0 \left( q^0 - E_k \right) = \left[ \exp \left( \frac{q^0 - E_k}{T} \right) + 1 \right]^{-1} \), \( D = T^{-4} \left[ (3q_0^2 - |q|^2)E_k^2 - 3q^0 E_k q^2 + \frac{3}{4} q^4 \right] \), and \( C_q = \frac{\pi^4}{6\zeta(5)} \approx 0.97 \). The bulk viscous correction to the quark distribution function used herein is obtained by solving the effective kinetic theory of quasiparticles described in Ref. \(^{38}\), where in the relaxation time approximation, the quasi-particle distribution function satisfies:

\[
k^\mu \partial_\mu n_k - \frac{1}{2} \frac{\partial (m^2)}{\partial x} \frac{\partial n_k}{\partial k} = -E_k \frac{\delta n_k}{\tau_R}
\]

In the local rest frame, using the Chapman-Eskog expansion in the relaxation time approximation, the leading order solution to \( \delta n_k \) reads \(^{11}\):

\[
\delta n_k = \Pi \frac{T_n}{\zeta} n_{0,k} \left[ 1 - n_{0,k} \right] \left[ \frac{E_k}{T} - \frac{n_{\bar{q},q}}{E_k T} \right] \left( \frac{1}{3} - e_s^2 \right)
\]

\(^{1}\) In Eq. (5), \( i \) can be the net electric change \( Q \), net baryon number \( B \), and/or net strangeness \( S \).
where \( \tau_{\Pi}/\zeta \) is given in Table I, \( c_s^2 \) is the speed of sound squared, \( m_{\rho,\omega}^2 \) is taken to be \( m_{\pi,\rho}^2 = g_s^2 T^2/3 \) with the coupling \( g_s = 2 \). Effects of running of the coupling are not considered in the above \( \delta n_k \). Using Eq. (9), the bulk viscous correction of the QGP dilepton rate \( \delta R_{\Pi} \) in the local rest frame is:

\[
\frac{d^3\delta R_{\Pi}}{d^3q} = \Pi_{\Pi} \frac{\tau_{\Pi}}{\zeta} \left[ A \left( \frac{q^0}{T} \frac{1}{|q|} \right) + B \left( \frac{q^0}{T} \frac{1}{|q|} \right) \right] \left( \frac{1}{3} - c_s^2 \right)
\]

\[
A \left( \frac{q^0}{T} \frac{1}{|q|} \right) = \frac{2T}{|q|} \int_{E_{\rho}^m}^{E_{\pi}^m} dE_k n_{0,k} |1 - n_{0,k}| n_0 (q^0 - E_k) E_k/T
\]

\[
B \left( \frac{q^0}{T} \frac{1}{|q|} \right) = \frac{2T}{|q|} \int_{E_{\rho}^m}^{E_{\pi}^m} dE_k n_{0,k} |1 - n_{0,k}| n_0 (q^0 - E_k) T/E_k
\]

The complete Born rate can therefore be expressed as \( \frac{d^4R}{d^4q} = \frac{d^4R_{\Pi}}{d^4q} + \frac{d^4R_{\delta}}{d^4q} + \frac{d^4R_{\nu}}{d^4q} \), where the first, second, and third terms are found in Eq. (6), Eq. (7), and Eq. (10), respectively.

2. Low temperature dilepton production: hadronic dilepton rates

In the hadronic sector, an important contribution to the dilepton production rate stems from decays of vector mesons in the QCD medium. In this work we leave out vector mesons made of charm and beauty quarks whose contribution to the medium properties has been ignored in the lattice QCD EoS used here. Only the low mass vector mesons composed of up, down and strange quarks, i.e. the \( \rho, \omega, \phi \), are included. The in-medium properties of these vector mesons are described via their spectral functions, while their connection to dilepton production is given by the Vector Dominance Model (VDM) first proposed by Sakurai [39]. Using VDM, relating the retarded virtual photon self-energy to the vector meson spectral function is done via:

\[
\text{Im} \left[ \Pi_{EM}^R \right] = \sum_{V=\rho,\omega,\phi} \left( \frac{m_V^2}{g_V^2} \right)^2 \text{Im} \left[ D_V^R \right]
\]

\[
= \sum_{V=\rho,\omega,\phi} \left( \frac{m_V^2}{g_V^2} \right)^2 \text{Im} \left[ \frac{1}{M^2 - m_V^2 - \Pi_V^R} \right]
\]

where \( m_V \) is the mass of the vector meson and \( g_V \) is the coupling to photons. An essential component needed to compute the spectral function \( \text{Im} \left[ D_V^R \right] \) is the in-medium self-energy of the vector mesons, while the Schwinger-Dyson equation [40] is used to construct the spectral function, once the self-energy is determined. The self-energy will be computed using the model first devised by Eletsky et al. [41]. In this model, the self-energy contains both the vacuum contribution and medium contribution [44,43], such that:

\[
\Pi_V^R = \Pi_{V,\text{vac}}^R + \Pi_{V,\text{med}}^R
\]

where \( \Pi_{V,\text{vac}}^R \) is computed via chiral effective Lagrangians [41,43], while the finite-temperature piece takes the form [37,41,43]:

\[
\Pi_{V,\text{med}}^R = -4\pi \int \frac{d^3k}{(2\pi)^3 k^0} n_{a,\text{med}}(\omega) \sqrt{s} f_{V,\text{a}}(s)
\]

where \( \omega = u \cdot k \), \( n_{a,\text{med}}(\omega) \) is the distribution of the scattering partners \( a \) of vector mesons \( V \), while \( f_{V,\text{a}}(s) \) is the forward scattering amplitude of \( V \) scattering onto \( a \) (see Ref. [41,43] for how \( f_{V,\text{a}}(s) \) is constructed). In a dissipative medium such as the one in the present study, there will be both an in-equilibrium and a dissipative contribution to \( n_a \), with the latter accounting for shear and bulk viscous effects. The shear viscous correction has been computed in [37] using the 14-moment approximation, while the bulk viscous correction obtained using the Chapman-Eskog expansion in the relaxation-time approximation [11] is:

\[
\delta n_{a,\Pi} = \Pi_{\Pi} \frac{\tau_{\Pi}}{\zeta} n_{0,a}(\omega) \left[ 1 \pm n_{0,a}(\omega) \right] \left[ \left( \frac{1}{3} - c_s^2 \right) \frac{\omega}{T} - \frac{n_a^2}{3mc_s^2T} \right]
\]

where \( \tau_{\Pi}/\zeta \) is found in Table I, while \( n_{0,a} \) is either a Fermi-Dirac or a Bose-Einstein distribution depending on whether \( a \) is a Boson or a Fermion. Following the procedure presented in Appendix B 2 of Ref. [37], substituting Eq. (14) into
Eq. (13) yields a correction to the vector meson self-energy owing to the bulk-modified distribution function, which reads:

$$\delta \Pi_{\gamma a,\Pi}^R = \Pi T \frac{2 - \xi}{\zeta} \left[ \left( \frac{1}{3} - \xi_s^2 \right) \mathcal{A}(|p|, T) + \mathcal{B}(|p|, T) \right]$$

$$\mathcal{A}(|p|, T) = -\frac{m_V m_a T}{\pi|p|} \int_{m_a}^\infty d\omega f_{\gamma a}^{s \text{rest}} \left( \frac{m_V}{m_a} \omega \right) \times \left\{ \frac{\omega_-}{T \exp \left( \frac{\omega_-}{T} \right)} \pm 1 - \frac{\omega_+}{T \exp \left( \frac{\omega_+}{T} \right)} \pm 1 \right\} \ln \left[ 1 \pm \exp \left( \frac{-\omega_-}{T} \right) \right] \ln \left[ 1 \pm \exp \left( \frac{-\omega_+}{T} \right) \right]$$

$$\mathcal{B}(|p|, T) = \frac{m_V m_a^2}{\pi|p|T} \int_{m_a}^\infty d\omega f_{\gamma a}^{s \text{rest}} \left( \frac{m_V}{m_a} \omega \right) \times \left\{ \frac{\omega_-}{T \exp \left( \frac{\omega_-}{T} \right)} \pm 1 - \frac{\omega_+}{T \exp \left( \frac{\omega_+}{T} \right)} \pm 1 \right\} \exp(\zeta)$$

where the upper (lower) signs refers to fermions (bosons), $f_{\gamma a}^{s \text{rest}}$ is the forward scattering amplitude evaluated in the rest frame of $a$, while $\omega_\pm = \frac{E \omega \pm |p||k'|}{m_V}$, $E = \sqrt{p^2 + m_V^2}$, $|k'| = \sqrt{(\omega')^2 - m_a^2}$. The shear correction to the self-energy $\delta \Pi_{\gamma a,\Pi}^R$ can be found in Appendix B 2 of Ref. [37], while the thermally equilibrated contribution $\Pi_{\gamma a,0}^R$ is given in Appendix B 1 of Ref. [37] as well as in Refs. [41][43].

### B. Cocktail dileptons

Once the hydrodynamical simulation reaches the switching temperature, the thermal dilepton production is stopped. However, dileptons are still being radiated from late decays of hadrons, which we will refer to as the dilepton cocktail. In the low invariant mass region $0.3 \lesssim M \lesssim 1.1$ GeV, this contribution mainly consists of late Dalitz decays of $\eta \rightarrow \gamma \gamma$, $\omega \rightarrow \pi^0 \gamma$, $\eta' \rightarrow \gamma \gamma$, $\phi \rightarrow \eta \gamma$ mesons, as well as late direct decays of vector mesons $\rho$, $\omega$, and $\phi$. An in-depth discussion about the sources of cocktail dileptons in the context of VDM – followed here throughout – is presented in Ref. [13]. We only summarize here the relevant results for our study. The invariant mass branching fraction of Dalitz decays, where the parent particle $a$ decays into the daughter particle $b$ and a virtual photon, reads [44]:

$$\frac{dB_{a \rightarrow b\gamma^*}}{d(M^2)} = N \frac{L(M)}{M^2} \left[ \left( 1 + \frac{M^2}{m_a^2 - m_b^2} \right)^2 - \frac{4M^2m_b^2}{(m_a^2 - m_b^2)^2} \right]^2 |F_{ab}(M)|^2$$

(16)

where $L(M)$ is defined below Eq. (6), $|F_{ab}(M)|^2$ is the form factor computed via VDM [44], while $N$ is an overall normalization such that:

$$\int_{4m_a^2}^{(m_a - m_b)^2} d(M^2) \frac{dB_{a \rightarrow b\gamma^*}}{d(M^2)} = B_{a \rightarrow b\gamma^*}$$

(17)

where $B_{a \rightarrow b\gamma^*}$ is the total branching fraction measured experimentally. The kinematics of the decay being determined in Eq. (16), one only needs to compute the distribution of virtual photons. In their local rest frame and at a given invariant mass $M$, the distribution of virtual photons can be obtained from the parent particle $a$, via [45]:

$$\frac{d^4N_{\gamma^* \rightarrow a}}{d^3q} \bigg|_{a \text{ is on-shell}} = \int d\Omega \frac{m_a^2}{4\pi} \frac{p^0 d^3N_a}{M^2 d^3p'}$$

(18)

where $(')$ denotes quantities in the rest frame of the virtual photon, with $d\Omega' = \sin \theta' d\theta' d\phi'$ being the usual solid angle in momentum space, while $\frac{d^3N_a}{d^3p}$ is the distribution of the meson $a$ obtained as explained in Ref. [10]. Thus, the virtual photon distribution originating from Dalitz decays is:

$$\frac{d^4N}{d^3p} = N \frac{L(M)}{M^2} \left[ \left( 1 + \frac{M^2}{m_a^2 - m_b^2} \right)^2 - \frac{4M^2m_b^2}{(m_a^2 - m_b^2)^2} \right]^2 |F_{ab}(M)|^2 \int d\Omega \frac{m_a^2}{4\pi} \frac{p^0 d^3N_a}{M^2 d^3p'}$$

$$\frac{d^4N_{\gamma^* \rightarrow a}}{d^3q} \bigg|_{a \text{ is on-shell}}$$

should be read as the virtual photon distribution obtained from decays of $a$. 

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2. $\frac{d^4N_{\gamma^* \rightarrow a}}{d^3q}$ should be read as the virtual photon distribution obtained from decays of $a$. 

As far as direct decays into dileptons are concerned, the branching ratio is obtained using VDM as well:

\[ MT_{\gamma \rightarrow \gamma^*} = \frac{\alpha^2}{3} \frac{m_{\rho}^4}{g^2/4\pi} \frac{L(M)}{M^2} \]

\[ MT_{\gamma} = -\text{Im} \left[ \Pi_{\gamma}^R \right] \]

thus, the branching fraction is:

\[ \frac{dN_{\gamma \rightarrow \gamma^*}}{d(M^2)} = \frac{\alpha^2}{3} \frac{m_{\rho}^4}{g^2/4\pi} \frac{L(M)}{M^2} \frac{1}{-\text{Im} \left[ \Pi_{\gamma}^R \right]} \]  (21)

With all the various branching fractions presented, only the distribution of the original hadrons remains to be specified. Other than the \( \rho \), all other mesons have a lifetime larger than the duration of the hydrodynamical simulations, thus they are treated using the Cooper-Frye (CF) prescription \([46]\) including resonance decays, which is how \( \frac{p^0 d^3N_{\rho}}{d^3p} \) in Eq. (18) is obtained \([10]\). The Cooper-Frye prescription reads:

\[ \frac{d^3N_{\rho}}{d^3p} = \int d^3\Sigma_{\mu} p^\mu d_\rho \rho d_n(\rho) \]

where \( d^3\Sigma_\mu \) is the freeze-out hypersurface element, \( d_\rho \) is the spin degeneracy of \( \rho \) with different isospin states being treated as separate particle species, while \( d_n(\rho) \) is a momentum distribution function. The CF distribution presented in Eq. (22) assumes that the spectral distribution of particles is \( \delta(p^2 - m^2) \Theta(p^0) \), which is valid for stable particles. To take into account short-lived particles, specifically the \( \rho \) meson, the CF distribution must be generalized as follows:

\[ N_{\rho} = \int d^3p \left( \frac{d^3p}{(2\pi)^3} \right)^4 \frac{d^3\Sigma_{\mu} p^\mu d_\rho \rho d_n(\rho)}{d^3p} \]

\[ = \int d^3p (2\pi)^4 2\delta(p^2 - m^2) \Theta(p^0) \int d^3\Sigma_{\mu} p^\mu d_\rho \rho d_n(\rho) \]

\[ \rightarrow \int d^3p (2\pi)^4 \rho_\rho(M) \int d^3\Sigma_{\mu} p^\mu d_\rho \rho d_n(\rho) \]  (23)

where \( \rho_\rho(M) \) is the spectral function of \( \rho \) mesons, which we write as \( \rho_\rho(M) = \frac{2 - \text{Im}[\Pi_{\rho}^R(M)]}{\pi} \), while \( M \) is assumed to be positive semi-definite. The latter form of \( \rho_\rho(M) \) reduces to the usual Breit-Wigner distribution if we assume the self-energy \( \Pi_{\rho}^R(M) \) is a complex number, while \( 2\delta(p^2 - m^2) \Theta(p^0) \) is recovered in the limit where \( \text{Im}[\Pi_{\rho}^R(M)] \rightarrow 0 \).

Combining Eq. (23) and Eq. (21) yields:

\[ \frac{d^4N_{\rho \rightarrow \gamma^*}}{d^4p} = \frac{\alpha^2}{\pi^3} \frac{m_{\rho}^4}{g^2/4\pi} \frac{L(M)}{M^2} \left| D_{\rho}^R(M) \right|^2 \int d^3\Sigma_{\mu} p^\mu n_\rho(\rho) \]  (24)

Note that the invariant mass dependence is present in both \( \left| D_{\rho}^R(M) \right|^2 \) as well as in the phase-space distribution \( n_\rho(M) \). The latter reduces to the Bose-Einstein distribution for a medium in thermal equilibrium, while bulk and shear viscous corrections used in computing \( n_\rho(M) \) are those of Ref. \([10]\). Thus, since the \( \rho \) meson is a broad resonance, its contribution from the hydrodynamical switching hypersurface will be different depending on whether its subsequent production from that hypersurface uses the vacuum version of the \( \left| D_{\rho}^R(M) \right|^2 \) distribution, or its in-medium counterpart. In order to quantify these differences, we will perform three calculations: (i) where we will use the in-medium distribution of \( \left| D_{\rho}^R(M) \right|^2 \) as well as allow for \( n_\rho(M) \) to depend on the invariant mass, (ii) where we set the \( \left| D_{\rho}^R(M) \right|^2 \) to its vacuum value while still letting \( n_\rho(M) \) be invariant mass dependent, and in (iii) we set the \( \left| D_{\rho}^R(M) \right|^2 \) to its vacuum value and evaluate on the mass shell \( n_{\rho}(M = m_{\rho}) \). Using method (iii) allows us to include the contribution to the multiplicity of \( \rho \) mesons coming from resonances decaying into \( \rho \) mesons. Indeed, if contributions from resonance decays were to be included to the \( \rho \) distribution calculated via method (ii), one must calculate the spectral functions of all the parent resonances contributing to \( \rho \)-production, as well include off-shell dynamics in all decay channels, which is beyond the scope of this study.

As far as \( \omega \) and \( \phi \) vector mesons are concerned, since their vacuum lifetime is significantly larger than that of the hydrodynamical medium, we approximate their cocktail contribution to the dilepton spectrum via

\[ \frac{d^4N_{\omega,\phi \rightarrow \gamma^*}}{d^4p} = \frac{\alpha^2}{\pi^3} \frac{m_{\omega,\phi}^4}{g^2_{\omega,\phi}/4\pi} \frac{L(M)}{M^2} \left| D_{\omega,\phi}^R(M) \right|^2 \int d^3\Sigma_{\mu} p^\mu n_{\omega,\phi}(M = m_{\omega,\phi}) \]  (25)
thus employing the $\omega, \phi$ distribution using the on-shell CF integral $\int d^3\Sigma_{n, \omega, \phi}$, including resonance decays $^{10}$, while keeping a non-trivial invariant mass dependence in form factor $|D^R_\omega(M)|^2$ and $|D^R_\phi(M)|^2$.

IV. RESULTS

Following the procedure of recent dilepton studies $^{12}$ $^{13}$, the scalar product method will be used to compute dilepton anisotropic flow coefficients$^3$. Within a centrality class, the anisotropic flow coefficients are computed using the method outlined in $^{11}$$^{13}$, namely:

$$v_n^+(X) = \frac{1}{N_{\text{ev}}} \sum_{i=1}^{N_{\text{ev}}} v_{n,i}^+(X) v_{n,i}^h \cos \left[ \frac{n}{N_{\text{ev}}} \left( n \left( \Psi_{n,i}^+(X) - \Psi_{n,i}^h \right) \right) \right]$$

$$= \frac{1}{N_{\text{ev}}} \sqrt{\sum_{i=1}^{N_{\text{ev}}} (v_{n,i}^h)^2} \left( \left( v_{n,i}^h \right)^2 \right)_{\text{ev}, i}$$

$$= \left( \left( v_{n,i}^h \right)^2 \right)_{\text{ev}, i}$$

(26)

where $N_{\text{ev}}$ is the number of IP-Glasma events, $X$ is any dynamical variable such as $M$ or $p_T$, and $\langle \ldots \rangle_{\text{ev}, i}$ is the average over events $i$. In a single event $i$, the hadronic $v_{n,i}^h$ and $\Psi_{n,i}^h$ are given by

$$v_{n,i}^h e^{in \Psi_{n,i}^h} = \frac{\int dp_T dy d\phi \rho \left( p_0 \frac{d^3N}{dp} \right) e^{in \phi}}{\int dp_T dy d\phi \rho \left( p_0 \frac{d^3N}{dp} \right)}$$

(27)

where the charged hadron distribution is integrated over the entire rapidity acceptance of the STAR detector at the RHIC and ALICE detector at the LHC, while all particles having $p_T > 0.3$ GeV are used when computing $v_{n,i}^h$ and $\Psi_{n,i}^h$. The dilepton $v_{n,i}^+$ and $\Psi_{n,i}^+$ are computed using the same approach, with the more general distribution $\frac{d^3N}{dx}$. Having computed the $v_{n}^+$ in 10% centrality sub-bins, bins are combined as follows,

$$v_{n}^+(X) [20 - 40\%] = \frac{dN_{\gamma^+} [20 - 30\%] v_{n}^+(X) [20 - 30\%] + dN_{\gamma^+} [30 - 40\%] v_{n}^+(X) [30 - 40\%]}{dN_{\gamma^+} [20 - 30\%] + dN_{\gamma^+} [30 - 40\%]}$$

(28)

where $\frac{dN_{\gamma^+}}{dx}$ is the dilepton multiplicity in a bin, while $v_{n}^+(X)$ is the corresponding anisotropic flow coefficient.

Results are discussed in several subsections. We first explore the effects of viscous corrections on partonic and hadronic dilepton production radiated during the hydrodynamical evolution. In the second subsection, we examine the effects of bulk viscosity on thermal dilepton yield and $v_2$ at RHIC and LHC collision energy. We shall see that bulk viscosity introduces some novel and quite unexpected effects on dilepton $v_2$, relative to hadron $v_2$, which may be used to better constrain $\zeta/s$. The last subsection is devoted to the calculation of the dilepton cocktail with special emphasis given on the treatment of the $\rho$ on the switching hypersurface at $T_{\text{sw}}$. This section also serves to highlight the invariant mass region were the novel effects of bulk viscosity can be seen in our dilepton $v_2$, calculation, and emphasizes that having a measurement with a good invariant mass resolution of dilepton elliptic flow is useful for constraining $\zeta/s$.

A. Effects of viscous corrections on partonic and hadronic dilepton emissions

In Fig. 1 we investigate the effects of bulk and shear viscous corrections to the dilepton emissions rates, by looking at the $p_T$-differential yield and $v_2$ at low invariant mass. Recall that the dilepton production rate changes from partonic to hadronic sources as the temperature decreases. The two different rates are smoothly interpolated according to

$^3$ In previous studies $^{12}$ $^{13}$ all the events in the 20-40% centrality bin were put together in one bin, while in the present study, the 20-40% centrality class is separated into two bins with 10% intervals, 20-30% and 30-40%, which are later recombined into 20-40%.
Eq. (4) in the temperature interval $0.184 \text{ GeV} < T < 0.22 \text{ GeV}$. This interpolation range is chosen to yield a smooth temperature dependence but should not be misinterpreted as suggesting the existence of hadronic matter up to temperatures above 200 MeV: The lattice-based equation of state used in our work, which controls the cooling rate and development of hydrodynamic flow in our dynamical simulations, encodes a crossover transition from partonic to hadronic matter at a temperature $T \approx 0.184 \text{ GeV}$. For notational simplicity we will denote in the following the lower-temperature dileptons emitted according to the rate based on hadronic medium sources by “HM” and the higher-temperature dileptons emitted with a rate calculated from partonic sources by “QGP”. The HM and QGP labels thus only serve to identify which rate formula was used to calculate the emissions.

At $M = 0.9 \text{ GeV}$ depicted in Fig. 1, the yield is dominated by radiation from the lower temperature medium — i.e. hadronic medium (HM), in which both yield and $v_2$ of dileptons are essentially unaffected by viscous corrections to the dilepton emission rate in our calculation. The higher temperature partonic (QGP) dilepton yield is affected more significantly by viscous corrections to the rate, especially for $p_T > 2 \text{ GeV}$. To better appreciate the effects of viscous corrections, the bottom panel of Fig. 1 displays the ratio of viscous over inviscid (ideal) dilepton production. As far as the $v_2$ of the higher temperature (QGP) region is concerned, Fig. 1b shows that viscous corrections are more pronounced there and essentially affect the entire $p_T$-distribution, while the $v_2$ of lower temperature dileptons from the hadronic sector are unaffected by viscous corrections to the emissions rate. For thermal (HM+QGP) dileptons, viscous emission rate corrections affect both dilepton yield and $v_2(p_T)$, with dilepton $v_2$ being slightly more sensitive to the rate modifications than the yield. Indeed, since $v_2$ of thermal dileptons is a yield-weighted average of the low and high temperature (HM and QGP) sources, the high temperature (QGP) contribution becomes significant on the total $v_2$ at $p_T > 1.5 \text{ GeV}$, and this is where viscous correction effects on $v_2$ can be more readily appreciated. At a higher invariant mass of $M = 1.5 \text{ GeV}$, the higher temperature (QGP) dilepton yield dominates any lower temperature (HM) contribution at all $p_T$, as clearly shown in Fig. 2a; and thus we only show the thermal $v_2$ in Fig. 2b as it closely follows radiations from higher (QGP) temperatures. At this invariant mass, the effects of viscous corrections can be seen, and they both reduce the $v_2(p_T)$ of dileptons. Figure 3 summarizes these findings by displaying the invariant mass distribution of dileptons as affected by bulk and shear viscous corrections explored herein. Note that the increase in the $v_2(M)$ of QGP dileptons at low invariant mass, under the influence of both viscosities relative to just shear, mirrors what is expected from the energy dependence of the bulk correction $\delta n_k \propto \frac{H}{\varepsilon} \left( \frac{E_k}{T} - \frac{m_k^2}{2kT} \right)$ in Eq. (9) which changes sign as $E_k$ (or invariant mass) increases, while $\frac{H}{\varepsilon}$ is typically negative\(^4\) as is displayed in Fig. 3c.

\(^4\) This was also shown in earlier hydrodynamical calculations, e.g. [3].
FIG. 2: (Color online) Similar to Fig. 1 but for larger dilepton invariant mass $M = 1.5$ GeV.

FIG. 3: (Color online) Effects of viscous corrections to the dilepton emission rate on the invariant mass distribution of $v_2$ for the hadronic (a) and partonic (b) dileptons [see Eq. (4)]. Note that in the top panel of (a), the black dash-dotted line is covered by the pink and red lines of the same type, and one needs to look at the ratio between the viscous over the ideal HM dilepton $v_2$, presented in the bottom panel of (a), to tell those curves apart. (c) Enthalpy density normalized bulk viscous pressure at two locations in the x-y plane.

B. Dynamics with shear and bulk viscosity as probed by thermal dileptons

As the effects of bulk viscosity on dilepton production are rather intricate, the discussion presented here is going to be separated into three subsections. Subsection [IV B 1] explores the manner in which dilepton $v_2(M)$ is affected by the presence of specific bulk viscosity at LHC collisions energy. Focus will specifically be given to the role played by the dilepton yield in obtaining thermal (HM+QGP) $v_2(M)$, as the thermal $v_2$ is a yield weighted average of the individual...
(HM/QGP) contributions. In subsection IV B 2, we will be investigating how the evolution of the temperature is affected by the presence of bulk viscosity, which in turn will affect the dilepton yield. Finally, subsection IV B 3 discusses the effects \( \zeta/s \) has on the invariant mass dependent dilepton yield and \( v_2 \) at top RHIC collision energy. To explain the unexpected results that we have found at RHIC, an exploration of how the hydrodynamical momentum anisotropy is modified once \( \zeta/s \) is included in the evolution will be done. Finally, a brief study of the sensitivity of our RHIC results to the particlization temperature \( T_{sw} \) is presented.

1. Effects of bulk viscosity on dilepton \( v_2 \) at the LHC

![Figure 4: Invariant mass distribution of \( v_2 \) for the hadronic (a) and partonic (b) dileptons under the influence of media having different \( \eta/s \) as well as a medium with non-zero values for both \( \zeta/s \) and \( \eta/s \). The definition of what constitutes hadronic (HM) versus partonic (QGP) dilepton radiation is presented in Eq. (4).](image)

To understand better the \( v_2(M) \) of thermal (HM+QGP) dileptons, one needs to focus on what is happening with
the yield under the influence of various viscous effects. To that end, we first focus on the \( v_2 \) at \( M > 0.8 \) GeV. In that invariant mass region, the dilepton yield goes from being HM dominated to being QGP dominated. So, even though bulk viscosity decreases the \( v_2 \) of lower temperature (HM) dileptons relative to any medium without \( \zeta/s \), at the same time bulk viscosity increases the yield of those dileptons. The invariant mass yield of higher temperature (QGP) dileptons is little affected by the various values of \( \eta/s \) and \( \zeta/s \) explored in our study. So, after performing a yield weighted average to compute the thermal \( v_2(M) \) in Fig. 5b for \( M > 0.8 \) GeV, the increase in the lower temperature (HM) dilepton yield dominates over the decrease in the \( v_2 \) of those dileptons, thus giving a increase to the \( v_2(M) \) of thermal dileptons in that invariant mass range. For \( M \leq 0.8 \) GeV, the lower temperature (HM) yield dominates over the higher temperature (QGP) yield, and a partial cancellation between the increase in the HM yield and the reduction in the HM \( v_2 \) is responsible for the thermal \( v_2 \) result seen in this invariant mass range.

2. Exploring the effect of bulk viscosity of the temperature evolution at LHC collisions energy

Recalling that the dilepton yield 
\[
\frac{dN}{dM dy} = \int d^2p_\perp \int d^4x \frac{d^2N}{\sqrt{s}dy},
\]
the increase in the lower temperature (HM) dilepton invariant mass yield for the medium with \( \zeta/s \) is most directly related to the increase in the hydrodynamical spacetime volume at a fixed temperature \( T \), especially for \( T_{sw} < T < 0.18 \) GeV, once \( \zeta/s \) was present in our hydrodynamical evolution. To understand the origin of this increase in volume a better, we first explore how the temperature in Fig. 6 is affected by the evolution of the fluid focusing a small portion of the hydrodynamical medium (having an extent in the \( x \) and \( y \) direction of \( \Delta x = \Delta y = 0.17 \) fm) located at the center of the simulation.

Figure 6 shows that, at late times, the temperature at the center of the medium is larger for a medium that has \( \zeta/s \) relative to the ones without. To understand this, let us examine the evolution of the temperature, as mainly governed by two competing effects: entropy production rate that tends to heat up the medium and expansion rate that cools it down. The temperature profile seen in Figs. 6a-c is split into three regions. Figure 6b corresponds to a section where purely QGP dilepton rates are being used [recall Eq. (4)]. In Fig. 6c, the transition between the QGP dilepton rates and the HM dilepton rates occurs, while Fig. 6d is dominated by HM dilepton radiation. The expansion rate of the system responsible for cooling the medium presented in Figs. 6e-f, and entropy production heating the system up is depicted in Figs. 6g-i. Figs. 6j-l displays shear and bulk viscous pressures normalized by enthalpy density. We now explore the evolution of the medium by investigating it in different time slices, each occupying its own column of Fig. 6.

At early times, displayed in the first column, there is no significant difference in neither the temperature nor the expansion rate evolution for all media considered, see Figs. 6a and d, respectively, while entropy production displayed in Fig. 6b is greatest for the medium with \( \eta/s = 0.16 \). This can be anticipated from Fig. 6, since entropy production is given by 
\[
\partial_{\mu}S^\mu = \frac{\pi^{\mu\nu}T_{\mu\nu}}{2(\eta/s)(\epsilon + P)} + \frac{\Pi^2}{(\zeta/s)(\epsilon + P)}
\]
and the largest \( \pi^{\mu\nu}/(\epsilon + P) \) is for the medium with \( \eta/s = 0.16 \). Figure 6 also shows how small \( \Pi/(\epsilon + P) \) is at early times, where \( \zeta/s \) is small given the high temperatures present there.

For \( 2.7 \leq \tau - \tau_0 \leq 4.7 \) fm/c depicted in the second column of Fig. 6, the systems continue to evolve and cool, while the magnitude of \( \zeta/s \) and consequently bulk viscous pressure becomes larger, thus increasing the entropy production of the medium having both bulk and shear viscosity relative to media without \( \zeta/s \). So, for \( 2.7 \leq \tau - \tau_0 \leq 4.7 \) fm/c, a competition between the expansion rate and the entropy production rate largely compensate each other such that no significant difference in the temperature profile can be seen. However, at later times depicted in the last column of panels in Fig. 6 one enters into the temperature region where HM dilepton radiation is dominating, and the entropy production of the medium with \( \zeta/s \) overshadows that of the other two media, while \( \zeta/s \) also acts to slow down the expansion rate at \( \tau - \tau_0 \geq 4.7 \) fm/c. These two effects combined lead to significantly larger temperatures displayed in Fig. 6 for the medium with \( \zeta/s \). So, the presence of \( \zeta/s \) in the hydrodynamical evolution drives larger temperatures and smaller expansion rates at late times, which in turn are responsible for generating larger spacetime volumes at a fixed temperature \( T \) — for \( T_{sw} < T \lesssim 0.18 \) GeV in our calculations — as depicted in Fig. 7. Figure 7 also depicts the entropy production as a function of temperature. The reduction in radial flow at \( T < 0.18 \) GeV has already been shown in Ref. 11, and thus will not be repeated here, as the same hydrodynamical simulation are employed in both calculations. Furthermore, since the invariant mass dilepton yield is a Lorentz invariant quantity (and so is \( \frac{dN}{dM dy} \) in our boost-invariant simulations), it is not directly sensitive to radial flow. The entropy production per temperature
FIG. 6: (Color online) Event-averaged temperature for the central cell during the first $\sim 2.5$ fm/c of evolution (a) and at later stages (b–c). Event-averaged expansion rate $\theta$ during the first few fm/c of evolution (d) and at later stages (e–f). Entropy production rate $\partial_\mu S^\mu$ rescaled by $\tau$ during the first few fm/c of evolution (g) and at later stages (h–i). Enthalpy density normalized shear and bulk viscous pressure during the first few fm/c of evolution (j) and at later stages (k–l).

bin and the associated volume shown in Fig. 7 are computed via

$$\Delta S = \frac{1}{\Delta T} \frac{1}{N_{ev}} \sum_{i=1}^{N_{ev}} (\partial_\mu S^\mu)_T$$

$$\Delta V_{T+1} = \frac{1}{\Delta T} \frac{1}{N_{ev}} \sum_{i=1}^{N_{ev}} (1)_T$$

$$(A)_{T} = \int \tau d\tau dy dx B(T) A$$

$$B(T) = \begin{cases} 1 & T(\tau, x, y) \in [T_j - \frac{\Delta T}{2}, T_j + \frac{\Delta T}{2}] \\ 0 & \text{otherwise} \end{cases}$$

(29)

where $A$ is any quantity one wants to bin in temperature, $T_j$ is the temperature in the center of the bin $j$, with the temperature bin-width being $\Delta T$. The area under the curves in Fig. 7 would give the total entropy production and
volume occupied by a medium of a given $\zeta/s$ and $\eta/s$.

![Graphs](image)

**FIG. 7:** (Color online) Development of the entropy within a temperature bin at LHC (a) and at RHIC (b). Hydrodynamical spacetime volume within a temperature bin at LHC (c) and at RHIC (d). The details regarding the way these quantities were computed are presented in Eq. (29).

Figure 7c,d clearly depicts that the presence of bulk viscosity in the hydrodynamical medium generates a larger spacetime volume at lower temperatures, thus affecting the lower temperature (HM) dilepton invariant mass yield seen in Fig. 5a. This larger spacetime volume is a consequence of larger entropy production (see Fig. 7a,b) and smaller expansion rate ultimately leading to a smaller radial flow at $T_{sw} < T < 0.18$ GeV, the latter being shown before in Ref. [11]. The effects of bulk viscosity on the evolution presented above, are directly comparable (as the underlying hydrodynamical evolution is the same) with the finding of Ref. [11], where a softer photon spectrum was obtained once $\zeta/s$ was included in the hydrodynamical evolution.

The same phenomenon that increases the lower temperature (HM) dilepton invariant mass yield at the LHC, will also increase the $dN/dM_{y=0}$ of HM dileptons at top RHIC energy, see Fig. 9a below. However at RHIC, the dilepton $v_2(M)$ in Figs. 8 and 9b doesn't always behave in the same way as at the LHC.

3. Exploring the $v_2$ at the RHIC and the LHC through the hydrodynamical momentum anisotropy

The dilepton $v_2(M)$ of higher temperature ("partonic") and lower temperature ("hadronic") sources at RHIC are shown in Fig. 8, where an interesting behavior can be noticed. While $v_2(M)$ of higher temperature (QGP) dileptons behaves similarly across the two collision energies studied here, the $v_2(M)$ for dileptons radiation at lower temperatures behaves differently: its anisotropic flow appears to be modestly increased under the influence of $\zeta/s$. This seemingly slight increase is enhanced once two contributions are combined into thermal dileptons (see Fig. 9b), for reasons we have explained in subsection IV B 1. To explore this phenomenon further, Fig. 10 compares how the hydrodynamical momentum anisotropy is developed at RHIC and LHC energies. The hydrodynamical momentum
anisotropy is computed as follows:

$$
\varepsilon_{p,X}(T) = \frac{1}{N_{ev}} \sum_{i=1}^{N_{ev}} \left\{ \sqrt{\frac{\langle T_{X,i}^{xx} - T_{X,i}^{yy} \rangle_T^2}{T_{X,i}^{xx} + T_{X,i}^{yy}}^2} + \frac{2 \langle T_{X,i}^{xy} \rangle_T^2}{T_{X,i}^{xx} + T_{X,i}^{yy}} \right\}
$$

(30)

where $\langle \cdot \rangle_T$ is defined in Eq. (29). $T_{X}^{\mu\nu}$ can be $T_0^{\mu\nu}$, $T_\pi^{\mu\nu} = T_0^{\mu\nu} + \delta T_{\pi}^{\mu\nu}$, or $T_{\pi+\Pi}^{\mu\nu} = T_0^{\mu\nu} + \delta T_{\pi}^{\mu\nu} + \delta T_{\Pi}^{\mu\nu}$, with $T_0^{\mu\nu}$, $\delta T_{\pi}^{\mu\nu}$, and $\delta T_{\Pi}^{\mu\nu}$ being defined in Eq. (1).

Figure 10 shows that around the temperature where $\zeta/s$ peaks, there is an enhancement in $\varepsilon_p$ owing to bulk viscous effects. As temperatures drop however, the system with bulk viscosity suppresses $\varepsilon_p$ development as well as the expansion rate.

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5 Referring to $\varepsilon_p$ without specifying $X$ implicitly implies that the statement is valid for all $X$.

6 Note that the extra entropy production that is present near the peak of $\zeta/s$, occurring for temperatures between 175–220 MeV, is also correlated with a localized increase in both expansion rate of the system in Fig. 9 and anisotropic flow development in Fig. 10. This phenomenon requires further study.
As dileptons are emitted throughout the entire history of the evolution, one must consider the size of the spacetime volume present under the different temperature bins, in order to appreciate how much of the enhancement in $\varepsilon_p$ generated around the peak of $\zeta/s$, translates to an increase of $v_2(M)$ of lower temperature (HM) dileptons. So, recalling that in our calculations HM dileptons are not particularly sensitive to viscous corrections to their emission rates, we will be estimating the volume present under the peak of $\varepsilon_{p,0}$ by computing the following quantity:

$$\varepsilon_{p,0}(\tau) = \frac{1}{N_{ev}} \sum_{i=1}^{N_{ev}} \sqrt{\left[ \frac{\langle T^{x,y}_{0,i} \rangle_{\tau}}{\langle T^{xy}_{0,0} \rangle_{\tau}} \right]^2 + \left[ \frac{2 \langle T^{xy}_{0,i} \rangle_{\tau}}{\langle T^{xy}_{0,0} \rangle_{\tau}} \right]^2}$$

$$\langle A \rangle_{\tau} = \int_{\tau_0}^{\tau} \int_{\tau_0}^{\tau} dy' dz' (1 - f_{QGP}) \Theta (T - T_{sw}) A$$

where $A$ is any quantity that one wants to integrate over $\tau$, $\Theta$ is a Heaviside function, while $f_{QGP}$ is defined in Eq. (31). Inspecting the hydrodynamical momentum anisotropy in the temperature region where HM dileptons are emitted reveals a clearer picture [see Fig. 11]. Indeed, Fig. 11b allows us to appreciate how much of the enhancement seen in $\varepsilon_{p,0}(T)$ translates into dilepton $v_2(M)$ by looking at $\varepsilon_{p,0}(\tau)$ at late time, say $(\tau - \tau_0) \sim 7.5 \text{ fm/c}$. By that point in time, one has effectively integrated over the entire spacetime volume in the HM sector [defined in Eq. (4)] and $\varepsilon_{p,0}(\tau)$ has reached its maximal value. Comparing the three simulations in Figs. 11a and 10a, one notices that not only is the $\zeta/s$-induced enhancement of $\varepsilon_{p,0}(T)$ smaller at LHC compared to at RHIC in a given temperature

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7 We can already see that hydrodynamical events without $\zeta/s$ start to freeze-out beyond $(\tau - \tau_0) \sim 7.5 \text{ fm/c}$ which affects the average $\varepsilon_{p,0}(\tau)$, as can be seen in Fig. 11b. Thus comparisons between red and blue curves in Fig. 11b become unreliable much past $(\tau - \tau_0) \sim 7.5 \text{ fm/c}$. 

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FIG. 10: (Color online) Development of the hydrodynamical momentum anisotropy as a function of temperature at LHC and at RHIC.
bin, but more importantly, there is a significant amount of spacetime volume sitting away from the peak in $\varepsilon_{p,0}(T)$ as the red curve in $\varepsilon_{p,0}(\tau)$ for $(\tau - \tau_0) \sim 7.5 \text{ fm}/c$ is smaller than the blue curves. Conversely, if there was a significant spacetime volume near the peak of $\varepsilon_{p,0}(T)$, that should show up in $\varepsilon_{p,0}(\tau)$ at $(\tau - \tau_0) \sim 7.5 \text{ fm}/c$ by making the red curve be larger that the blue curves in Figs. [11]. The latter is not the case, and thus the enhancement seen in $\varepsilon_{p,0}(T)$ does not translate to the final $v_2(M)$ of lower temperature (HM) dileptons at the LHC. At RHIC on the other hand, Fig. [11] shows that the enhancement in $\varepsilon_{p,0}(T)$ persists even after summing over temperature bins and show up in $\varepsilon_{p,0}(\tau)$ for $(\tau - \tau_0) < 3 \text{ fm}/c$, where the red curve of Fig. [11] is larger than the blue curve there. At $(\tau - \tau_0) \sim 4 \text{ fm}/c$ the earlier enhancement seen in the red relative to blue curves of $\varepsilon_{p,0}(\tau)$ seems to have disappeared as the curves are now within uncertainty of each other\(^8\). Assuming that lower temperature (HM) dileptons with $M > 0.8 \text{ GeV}$ are predominantly emitted at earlier times while HM dileptons with $M < 0.8 \text{ GeV}$ are mostly emitted at later times, our calculations shows that this correlation between $v_2(M)$ of hadronic dileptons and $\varepsilon_{p,0}(\tau)$ holds within uncertainty. So the enhancement seen in $\varepsilon_{p,0}(\tau)$ at $(\tau - \tau_0) < 3 \text{ fm}/c$ may be present in the $v_2(M)$ of lower temperature hadronic dileptons at $M > 0.8 \text{ GeV}$, however our current uncertainties do not allow us to draw a more definite statement. At the LHC, similar correlations can be seen in the HM sector between early-time $\varepsilon_{p,0}(\tau)$ and high-$M$ $v_2$, as well as late-time $\varepsilon_{p,0}(\tau)$ and low-$M$ $v_2$.

The modest increase in the $v_2(M)$ of hadronic and thermal dileptons is, however, sensitive to the switching temperature, which was obtained from a tune of the hydrodynamical simulations with UrQMD to hadronic observables presented in Refs. [9] [10]. For better comparison of dileptons emitted at RHIC and LHC collision energy, we have decided to lower the switching temperature for RHIC $\sqrt{s_{NN}} = 200 \text{ GeV}$ energy from 165 MeV to 150 MeV, in order to remain within ~ 5% agreement to the best fit (i.e. the one including bulk viscosity) obtained at 165 MeV [9] [10]. The corresponding dilepton yields and $v_2$ are presented in Fig. [12]. At lower switching temperature for RHIC collisions implemented here, the $v_2(M)$ of lower temperature (HM) dileptons at RHIC in Fig. [12] follows a similar pattern as at the LHC energy in Fig. [4]. After performing a yield weighted average, the thermal dilepton $v_2$ shown in Fig. [12] still displays an inversion in the ordering between the different runs of $v_2(M)$ around $M \sim 0.9 \text{ GeV}$ and $M \gtrsim 1.1 \text{ GeV}$, similarly to what was seen at the LHC.

In order to determine whether the bulk viscosity induced increase in thermal dilepton $v_2(M)$ at RHIC may be observed experimentally, cocktail dileptons should be included. Indeed, cocktail dileptons will have a sizable contribution to dilepton yield and, more importantly, will contribute with a significant $v_2$ to the total dilepton $v_2$. One could try to estimate cocktail dilepton production by letting the hydrodynamics evolve to a lower switching temperature. However, running a hydrodynamical simulation to lower temperature is an approximation of the actual dileptons production, as a hydrodynamical simulation of the hadronic stage generates stronger radial and anisotropic flow than a hadronic cascade, and in fact one should instead compute the dilepton production from a hadronic transport evolution. Furthermore, there are additional dilepton production channels in the hadronic transport models, coming from e.g. Dalitz decays, that are not accounted for in the thermal dilepton emission rates applied during the hydrodynamical evolution.

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\(^8\) Note that hydrodynamical simulations without $\zeta/s$ start freezing out past $(\tau - \tau_0) \sim 4 \text{ fm}/c$ at which point comparisons between red and blue $\varepsilon_{p,0}(\tau)$ curves become less reliable.
as the lifetime of parent hadrons decaying into dileptons (e.g. via the Dalitz channel) is much longer than the average time the medium spends evolving hydrodynamically. The most important of these dileptons channels, contributing to our dilepton cocktail, will be included as outlined below. A more complete calculation that includes dynamical dilepton production from hadronic transport will follow in an upcoming publication, where SMASH\textsuperscript{9} \cite{47–50} will be used to calculate dilepton generation from the hadronic cascade. In the mean time, given that the lifetime of all mesons contributing to our dilepton cocktail is large (except for the $\rho$), while their branching fraction to dileptons is small, a portion of the decays of cocktail mesons will happen during the later stages of a hadronic transport simulation, which is well captured by free-streaming\textsuperscript{10}. So, in order to get a better grasp on the production of dileptons after the hydrodynamical simulation, a calculation of cocktail dileptons using the free-streaming assumption will be shown. The results of this cocktail calculation are presented in the next section, and include direct decays of vector mesons as well as late Dalitz decays of both pseudoscalar and vector mesons contributing to our dilepton cocktail.

C. Cocktail dileptons and the $\rho$ spectral function

The main goal of this section is to compute both the yield and anisotropic flow of cocktail dileptons. The first part of the cocktail calculation in this section excludes the contribution from the $\rho$ meson. Indeed, except the $\rho$ which will be taken into account later, all other mesons tallied in our calculation of the dilepton cocktail have a long lifetime, i.e. they are narrow resonances. Therefore, their $\frac{d^2\sigma}{d^2p}$ distribution was obtained from the Cooper-Frye formula, including resonance decays, using the on-shell approximation as detailed in Ref. \[10\]. Their subsequent decays into

\textsuperscript{9} SMASH stands for Simulating Many Accelerated Strongly-interacting Hadrons.

\textsuperscript{10} How well this free-streaming approximation holds will be revisited in an upcoming publication where a comparison between free-streaming and SMASH will be done.
dileptons was computed through Dalitz decays as prescribed in Eq. (19), while direct vector meson decays follow Eq. (24), and Eq. (25).

Figure 13 presents the first event-by-event calculation, based on a realistically expanding medium, of cocktail dilepton $v_2$ at both RHIC and LHC collision energies, excluding the contribution from the $\rho$. The $v_2$ of the dilepton cocktail at RHIC and LHC collisions energies behaves similarly, which is expected from the hydrodynamical momentum anisotropy. Indeed, inspecting Figs. 10 at the switching temperature, one sees that the elliptic flow of cocktail dileptons from a medium that is only affected by shear viscosity is larger than the one featuring both $\eta/s$ and $\zeta/s$. This is also observed in $v_2$ of hadrons, as shown in Ref. [9][10]. Furthermore, the inversion in the order of the curves for cocktail and thermal (HM+QGP) dilepton $v_2(M)$ at RHIC [see Fig. 13a relative to Fig. 9a], though unexpected at first sight, is better understood from our discussion in the previous section about the $v_2$ thermal (HM+QGP) dileptons, while inspecting Fig. 10 can explain cocktail dileptons/hadrons. We now combine cocktail and thermal dileptons to investigate how much of the bulk viscosity-induced inversion seen in thermal dileptons shows up in the total $v_2$, and hence may have experimental signatures.

Focusing first on the results at RHIC in Fig. 14, one can see that after combining cocktail and thermal (HM+QGP) dileptons using the yield in Fig. 14b), shows again the same pattern as in Fig. 14c. Given the pattern of how bulk viscosity affects $v_2$ of dileptons in our calculation at both RHIC and LHC collision energies, an interesting quantity that highlights the effects of $\zeta/s$ in our study is the ratio of $v_2$ at $M \sim 0.9$ GeV relative to lower invariant masses. Specifically for our calculation, inspecting $\frac{v_2(M=0.9 \text{ GeV})}{v_2(M=0.3 \text{ GeV})}$ one sees that the presence of bulk viscosity, in addition to shear viscosity, yields $\frac{v_2(M=0.9 \text{ GeV})}{v_2(M=0.3 \text{ GeV})} > 1$ while shear viscosity alone makes it less than 1 in our calculations [see Table II for details].

TABLE II: The $\frac{v_2(M=0.9 \text{ GeV})}{v_2(M=0.3 \text{ GeV})}$ ratio as a tool to measure the effects of bulk viscosity (excluding the $\rho$ contribution to the dilepton cocktail)

|               | LHC $\left[ (\zeta/s)(T) + [\eta/s = 0.095] \right]$ | RHIC $\left[ (\zeta/s)(T) + [\eta/s = 0.06] \right]$ |
|---------------|------------------------------------------------------|-----------------------------------------------------|
| $\eta/s = 0.095$ | 1.09                                                | 1.09                                                |
| $\eta/s = 0.16$ | 0.881                                               | 0.920                                               |
| $\eta/s = 0.06$ |                                                     | 0.733                                               |

To complete the investigation of the influence bulk viscosity has on dilepton $v_2$, the contribution of the $\rho$ will be incorporated in two steps. The first computes the contribution of the $\rho$ on the switching hypersurface while the second includes $\rho$’s produced by resonance decays.

There are three different approaches to calculate the contribution of the $\rho$ on the switching hypersurface. As outlined in subsection III B the first approach consists of assuming that the $\rho$ meson width is broadened on the switching hypersurface compared to its vacuum value, thus will employ the in-medium $\rho$ distribution $|D_R|^2$ in Eq. (24), while using the invariant mass dependent version of the Cooper-Frye (CF) integral, namely $\int d^3 \Sigma \rho_\eta n_\rho(M)$. Note that apart for the invariant mass dependence, $n_\rho$ is otherwise the same as in Ref. [10]. The second option uses
FIG. 14: (Color online)
(a) Invariant mass distribution of dilepton yield at RHIC for thermal (HM+QGP) dileptons as well as cocktail dileptons excluding $\rho$.
(b) Invariant mass distribution of dilepton yield at LHC for thermal (HM+QGP) dileptons as well as cocktail dileptons excluding $\rho$.
(c) Invariant mass distribution of dilepton $v_2$ at RHIC for the thermal (HM+QGP) contribution as well as the total composed of thermal and cocktail (excl. $\rho$).
(d) Invariant mass distribution of dilepton $v_2$ at LHC for the total composed of thermal (HM+QGP) and cocktail (excl. $\rho$).

the vacuum description for the $\rho$ meson, i.e. neglecting in-medium contributions to $|D_R^\rho|^2$, while still computing the Cooper-Frye integral with a $\rho$ meson density $n_\rho(M)$ that varies with the invariant mass of the dilepton. The last option employs the vacuum description of $|D_R^\rho|^2$, and also enforces the on-shell condition in the CF integral, namely $\int d^3\Sigma p_\mu n_\rho(M=m_\rho)$. The results of these three prescriptions are compared in Fig. 15 where, for the moment, all contributions to the $\rho$ coming from resonance decays are neglected.

The total dilepton $v_2$ at RHIC (see Fig. 15b) is more affected by the dilepton cocktail than at the LHC (displayed in Fig. 15d). The reason for this is two-fold: First, the $v_2$ of the cocktail at RHIC is much larger than the thermal (HM+QGP) $v_2$ across all $M$. At the LHC, the contribution of the cocktail is more pronounced compared to thermal (HM+QGP) $v_2$ once $M \lesssim 0.65$ GeV. Second, at LHC collision energy, the thermal (HM+QGP) dilepton yield is larger than the cocktail dilepton yield over a wider range of invariant masses (see Fig. 15c) compared to RHIC (see Fig. 15a). This is expected, as the higher collision energy at the LHC produces a larger spacetime volume of the (hydrodynamical) medium compared to RHIC.\(^{11}\) So, the combination of these two effects explains why the total dilepton $v_2$ at RHIC is more sensitive to the dilepton cocktail than it is at the LHC.

Focusing more specifically on the contribution from the $\rho$ along the hypersurface of constant $T_{sw}$, we see that its

\(^{11}\) it may also produce a larger spacetime volume for the late hadronic rescattering stage but since in this work we do not follow that stage dynamically, rather letting the hadrons freeze-out kinetically directly on the switching surface with $T_{sw}$, it is premature to discuss about dileptons emitted during hadronic rescattering.
Whether or not the width of the ρ meson must be treated as an off-shell particle; specifically, the ρ must have a mass distribution in the Cooper-Frye integral \( \int d^3p' p'_\mu n_\rho(M) \) which enters through a mass-dependent density \( n_\rho(M) \). Indeed since \( \int d^3\Sigma' p_\mu n_\rho(M) \propto \exp(-M/T) \), the exponential suppression in invariant mass controls the convergence of the total dilepton \( v_2 \) as \( M \) increases, more so than its form factor \( |D^R_\rho|^2 \). This is clearly illustrated in both Fig. 15b and d. By comparing Figs. 15b,c with Figs. 15c,d, we see that the precise way we do or do not include the medium effects on the spectral function of the ρ has much smaller effects on the dilepton mass spectra than on their \( v_2 \) in the high-mass region \( M \geq 1.1 \) GeV. This study shows that evaluating the contribution from \( \rho \) decays to cocktail dileptons with invariant mass \( M \) must properly account for the exponential suppression \( \propto \exp(-M/T) \) to the number of contributing \( \rho \) mesons, especially to avoid overpredicting the dilepton \( v_2 \) for invariant masses above

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\(^{12}\) Whether or not the width of the ρ is broadened along the hypersurface of constant \( T_{sw} \) is not something that can be easily distinguished in our calculations, as can be seen from the thin black and gray lines in Fig. 15.
about 1 GeV.

FIG. 16: (Color online) Invariant mass distribution of the dilepton $v_2$ including the contribution from the $\rho$ on the switching hypersurface at RHIC (a) and LHC (b) collision energies under the influence of bulk and shear viscosities. The contribution from resonance decays to the $\frac{d^3N}{dp}$ at RHIC and LHC is in (c) and (d), respectively.

Using the vacuum off-shell description of the $\rho$ emanating from the switching hypersurface, Fig. 16a,b show the effects of $\eta/s$ and $\zeta/s$ on the combined dilepton $v_2$. In Fig. 16c,d we complete the calculation by also including cocktail dileptons from $\rho$ mesons produced by the decays from higher-mass resonances. As the $\rho$ mesons emerging from resonance decays are on their mass shell in our calculation, we focus on the invariant mass window $0.3 < M < 1.1$ GeV. In that invariant mass window, one sees that including the $\rho$ into the cocktail does not substantially change the pattern that was observed in Fig. 14.

| | LHC | | \(v_2(M=0.9 \text{ GeV})\) | | \(v_2(M=0.3 \text{ GeV})\) |
|---|---|---|---|---|---|
| | (\(\zeta/s\)(T)+\(\eta/s=0.095\)) | \(\eta/s=0.095\) | \(\eta/s=0.16\) | | \(\eta/s=0.06\) | \(\eta/s=0.06\) |
| incl. off-shell vac. $\rho$ | 1.12 | 0.922 | 0.954 | incl. off-shell vac. $\rho$ | 1.18 | 0.859 |
| incl. off-shell vac. $\rho$ & res. decay $\rho$ | 1.13 | 0.938 | 0.967 | incl. off-shell vac. $\rho$ & res. decay $\rho$ | 1.23 | 0.920 |

Going back to the ratio \(\frac{v_2(M=0.9 \text{ GeV})}{v_2(M=0.3 \text{ GeV})}\), Table III shows that this ratio continues to be sensitive to the effects of bulk viscosity even after resonance decay contributions are included. Thus it is useful quantity to highlight the effects of bulk viscosity in our study. Additional studies will be necessary to clarify if this $v_2$ ratio is a robust observable to constrain the bulk viscosity of QCD. Nevertheless the calculations presented in this work show that the invariant mass distribution of $v_2$ exhibits a clear sensitivity to the presence of bulk viscosity in our simulations. For the purposes of constraining bulk viscosity in data, a measurement of the entire invariant mass distribution of $v_2$ is needed. In fact, one should be combining hadron and dilepton anisotropic flow observables together, within the context of a Bayesian model to data comparisons, to put more robust constraints on bulk viscosity present inside hydrodynamical...
simulations. For this proposed study to yield the best possible outcome, a more precise measurement of dilepton $v_2$ is needed, and such measurement are currently being planned [18].

V. CONCLUSIONS

In the present study we have explored how bulk viscosity influences dilepton production at both RHIC and LHC collision energies. The total $v_2(M)$ in our calculations, which is composed of thermal and cocktail contributions, at RHIC and at LHC collisions energies behaves similarly under the influence of bulk viscosity. Indeed, bulk viscosity affects most directly the dilepton invariant mass yield, through the increase in the spacetime volume occupied at lower temperatures\textsuperscript{13}, thus increasing the HM and cocktail dilepton yield, while leaving QGP dileptons yield essentially unaffected\textsuperscript{14}. As the dilepton $v_2(M)$ is a yield-weighted average of the individual contributions, the effects of bulk viscosity also manifests itself in the total dilepton $v_2$ exhibiting similar features at both RHIC and LHC collisions energies. However, though the final result is similar, interesting features in bulk-induced dynamics appear once the thermal dilepton $v_2$ is isolated, and that is where differences between results at RHIC and at LHC are the most striking.

Bulk viscosity was found to have a rather unexpected dynamical effect on the generation of the hydrodynamical momentum anisotropy ($\varepsilon_p$) both as a function of temperature and as a function of proper time $\tau$. Investigating the development of $\varepsilon_p$ as a function of temperature first, starting from high temperatures and proceeding to lower temperatures, the medium with both bulk and shear viscosity develops $\varepsilon_p$ faster than any medium without bulk viscosity, and reaches its maximum as the temperature approaches the peak in $\xi/s$. However, once lower temperatures are reached, bulk viscosity acts to reduce the amount of hydrodynamical momentum anisotropy. These features are imprinted in thermal dilepton production and are affecting RHIC and LHC dileptons differently, owing to the differently-sized spacetime volumes present at those two collision energies, especially in the region of HM dilepton radiation. For this reason, the enhancement in $\varepsilon_p$ has a modest increase on the $v_2(M)$ of HM dileptons at RHIC, while reducing the $v_2(M)$ of HM dileptons at the LHC.

Whether or not the effects of bulk viscosity on thermal dileptons can be detected in experiment depends upon how well the cocktail dileptons can be removed. Of course, the contribution from semi-leptonic decays of open heavy flavor hadrons onto the dilepton spectrum needs to be removed as well. While the effects of open heavy flavor will be studied in an upcoming publication, that source can potentially be removed by using the Heavy Flavor Tracker installed in the STAR detector at RHIC, for example. So, this investigation concentrated more on dilepton production from thermal and cocktail sources, focusing on the invariant mass dependence of the dilepton yield and $v_2$ with a particular attention given to the contribution of the $\rho$ meson inside the dilepton cocktail. Given its large width, it was found that the $\rho$ meson is sensitive to the invariant mass distribution assumed while calculating its cocktail contribution from the hypersurface of constant $T_{sw}$. Therefore, if the cocktail $\rho$ is to be removed in experimental data in the process of isolating thermal dilepton radiation, its invariant mass distribution should be carefully taken into account. Combining all sources, we have found that the ratio $v_2(M=9.9\mathrm{GeV})$ is useful to highlight the effects of bulk viscosity in our calculation, while experimental measurement of $v_2(M)$ should be done at multiple invariant mass points to constrain the effects of bulk viscosity.

The upcoming dilepton calculation using SMASH will allow to investigate two effects present in dynamical dilepton production from hadronic transport. First, it opens the possibility to study through dileptons, the effects of collisional broadening on the in-medium properties of the parent hadrons generating the lepton pairs. Collisional broadening effects on in-medium vector mesons are already included in our dilepton production from the hydrodynamical medium, and it would be intriguing to investigate how important those are inside of hadronic transport. Furthermore, SMASH also includes long-range interactions that can modify the in-medium properties of the parent hadrons, which are different from the collisional broadening, while the effects of those interactions may leave interesting features in the invariant mass yield and $v_2$ of dileptons. If found to be significant, all those modifications would open a window to study the in-medium properties of hadrons inside a hadronic transport evolution. In addition to this, calculating dilepton production dynamically through the SMASH hadronic transport will generate additional anisotropic flow, following the hydrodynamical simulation, that our current calculation of the dilepton cocktail, using the free-streaming assumption, does not have.

\textsuperscript{13} Recall that the increase in spacetime volume for a medium with $\xi/s$ is caused by an increase entropy production and a reduction in the expansion rate of the system at later times.

\textsuperscript{14} Note that this finding depends on the form of the bulk viscous modification $\delta n$ used in Eqs. (9,14), and future studies will investigate how different parametrization of $\delta n$ affect dilepton production.
This study complements earlier investigations \cite{12,13} about the sensitivity of dileptons towards various transport coefficients of hydrodynamical simulations and together they show just how valuable of a probe dileptons are. Thus, the simultaneous use of dileptons and hadronic observables will yield a much better constraint on the properties of strongly interacting media than any of those observables alone.

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