We studied the weak field Hall voltage in 2D electron layers in Si-MOS structures with different mobilities, through the metal-insulator transition. In the vicinity of the critical density on the metallic side of the transition, we have found weak deviations (about 6-20 % ) of the Hall voltage from its classical value. The deviation does not correlate with the strong temperature dependence of the diagonal resistivity $\rho_{xx}(T)$. The smallest deviation in $R_{xy}$ was found in the highest mobility sample exhibiting the largest variation in the diagonal resistivity $\rho_{xx}$ with temperature (by a factor of 5).

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In the pioneering experiments by Fowler et al. [1] performed on Si-MOS structures in perpendicular magnetic field, the Shubnikov-de Haas oscillations of the conductivity were found to be periodic as a function of the gate voltage $V_g$. This is consistent with the quasiclassical approach [2], in which the charge of the inversion layer $Q_{inv}$ is proportional to the applied voltage, as in a plain capacitor formed by the metallic gate and the 2D layer. At low temperatures, when the bulk conductance is frozen out and the charge in the depletion layer does not vary with gate voltage, variation of the capacitor charge with $V_g$ is related to the inversion layer charge only:

$$Q_{inv} = C(V_g - V_i).$$

Here $C = dQ/dV_g$ is the geometric capacitance between the metallic gate and the 2D carrier layer, $V_i$ is determined by the difference in work functions of the Al- gate film and the 2D carrier layer, by the energy of the bottom of the lowest subband in the confining potential well (zero-point energy) and by the charge trapped in depletion layer and at the interface. The charge in the Si-MOS structure $Q_{inv}$ was measured directly by integrating the current entering (and quitting) the structure during its charging (and discharging) with time. The charge, $Q_{inv}$, was found to be equal (within 2% uncertainty) to the charge of the 2D carrier layer $Q_{2D} = e \times n_{ShdH}$, where $n_{ShdH}$ is the density of carriers participating in transport in the Shubnikov-de Haas or QHE regime, $n_{ShdH} = (eB/h) \times i$, and $i$ is the number of filled quantum levels in a given magnetic field $B$. The issue on the carrier density was raised recently again in connection with the metal-insulator (M-I) transition in 2D carrier systems at zero magnetic field. The transition was found earlier to occur at a critical gate voltage $V_{gc}$ (where $V_{gc} - V_i$). By analogy to the quasiclassical case and to the QHE-Insulator transitions, the critical gate voltage is interpreted as a quantity corresponding to a critical carrier density $n_c = (dn/dV_g)/(V_{gc} - V_i)$. Recently, an alternative interpretation was put forward where the density of carriers participating in zero-field transport was suggested to be equal to $n_{eff} = (dn/dV_g)V_g - n_e$, so that $n_{eff} = 0$ at the transition, i.e. at $V_g = V_{gc}$.

The effective number of carriers is not a well defined parameter close to the M-I transition, and may, a priori, be found different in different effects. One of the most relevant effects capable to provide information on the effective carrier density is the weak-field Hall effect where the Hall resistance in the single-particle approximation is inversely proportional to the number of carriers:

$$R_{xy} \approx (\omega_c \tau / \sigma_0)[1 - 1 / (\omega_c \tau_0)^2 \Delta G(\epsilon_F)]$$

Here $\sigma_0 = ne^2 \tau/m^*$ is the diagonal conductivity at $B = 0$, $\omega_c = eB/m^*$ is the cyclotron frequency, and $\tau$, the transport scattering time at $B = 0$. $G$ and $\Delta G$ are the monotonic and oscillatory parts of the density of states. According to the theory, electron-electron interaction affects $R_{xy}$ in the same order as $\sigma_{xx}$. No corrections are expected in $\sigma_{xy}$ due to interaction, and, thus $\delta R_{xy}/R_{xy} \approx 2(\delta \rho_{xx}/\rho_{xx}^0)$. On the experimental side, the Hall resistance was measured earlier across the QHE-insulator transition and found to remain close (within $\sim 10\%$) to its classical value $R_{xy} = B/(ne)$ with $n$ given by Eq. (1). Such behavior of the Hall resistance is consistent with the Hall insulator concept. Low frequency ($\sim 3$ Hz) ac-measurements...
dependence for shown in the inset to Fig. 2, demonstrates a critical behavior around $V_g$ between $n_{Hall}$ and $n_{ShdH}$, calculated from the measured $R_{xy}$ values for six temperatures is plotted in Fig. 3b. At high density and high temperature, the deviation in the Hall density tends to zero. As temperature decreases to 0.3 K, the deviation raises to almost 20%, and seems to saturate. Oscillations and scattering in the Hall density data (seen in the density of states between four-fold degenerate Landau levels) are equidistant in the gate voltage scale. The most disordered sample Si46 displayed an admixture of the longitudinal voltage produces large distortions (oscillating with gate voltage, calculated from the period of the Shubnikov-de Haas effect. As $V_g$ decreases, the effective Hall density slightly deviates from the classical linear dependence, and then falls quickly to zero, deep in the insulating state at $V_g = 12$ V. Just at the critical gate voltage $V_{gc} = 14.4$ V, the Hall-density is by 5% larger than the classical value given by Eq. (1).

For samples with higher mobility, the critical carrier density $n_c$ is lower and the critical resistivity $\rho_c$ is higher. By this reason, the admixture of the longitudinal voltage produces large distortions (oscillating with gate voltage) of the measured Hall voltage. As Fig. 3a shows, this admixture can be reduced by an order of magnitude by subtracting the results taken for opposite magnetic field directions. The Hall resistance for sample Si4/32 at low density is significantly larger than the quasiclassical value $B/(e\tau_{Hall})$. The deviation in the effective Hall-density, $\delta n_{Hall} = n_{Hall} - n_{ShdH}$, calculated from the measured $R_{xy}$ values for six temperatures is plotted in Fig. 3b. At high density and high temperature, the deviation in the Hall density tends to zero. As temperature decreases to 0.3 K, the deviation raises to almost 20%, and seems to saturate. Oscillations and scattering in the Hall density data (seen in the range of low $V_g$) are due to residual admixture of the large longitudinal $V_{xx}$ voltage into the small Hall voltage.

Finally, as shown in Fig. 4, the deviation in the Hall density for the high mobility sample Si22 depends on gate voltage and temperature qualitatively similar but 3 times weaker than for Si4/32. As gate voltage increases, the disagreement between $n_{Hall}$ and $n_{ShdH}$ becomes less than the measurement uncertainty. We must note, however, that the absolute value of $n_{ShdH}$ has the uncertainty about (1-2)% for Si22 and 4% for Si4/32. By this reason, the true position of
the “zero” on the vertical scales in Fig. 4 and Fig. 3b is defined within the uncertainty of (1 – 4)%$, correspondingly. Although the deviation of the Hall density in the vicinity of $V_{gc}$ is small for all samples, $\delta n_{Hall}/n_{ShdH} \ll 1$, it is much larger than the error bars. Due to the charge neutrality in the total Si-MOS structure, Eq. (1), the nonzero value of the $\delta n_{Hall}$ indicates either a lack of the Drude-Boltzmann interpretation of the Hall voltage in the vicinity of $V_{gc}$, or a noticable contribution of carriers exchange between the 2D layer and the shallow potential traps.

In the framework of the Drude-Boltzmann model, the effective carrier density $n_{Hall}$ for all samples remains close to the classical value through the metallic range of densities $V_g > V_{gc}$, where the resistivity $\rho_{xx}$ strongly varies with temperature. We conclude, in the same framework, that the strong exponential drop in $\rho_{xx}(T)$ as $T$ decreases $[3]$, is associated with an anomaly in the scattering time or in the transport mechanism, rather than with carrier density. The insets to figures 3a and 4 show that the variations of the diagonal resistivity with temperature (by a factor of 2.5 and of 5.5 for Si4/32 and Si22, correspondingly) are much larger than that in $R_{xy}$, over the same range of density and of temperature. The lack of a linear relationship between $\delta R_{xy}$ and $\delta \rho_{xx}$ in the vicinity of $V_{gc}$, indicates that at least one of these two quantities is not related to the interaction quantum corrections $[3]$ (with the reservation that the theory may be not valid for the strong interaction case, $r_s \sim 3 – 10$).

In conclusion, we have measured the weak field Hall resistance in $n$–Si-MOS samples through the metal-insulator transition. We found no strong changes in $R_{xy}$ (comparable to those in $\rho_{xx}(T)$) and found no signatures of a complete carriers freeze-out at $V_g = V_{gc}$. However, for low density and low temperatures, the Hall voltage in different samples was found to deviate from the classical value by about (6-20)%. The deviation in $R_{xy}$ does not correlate with the strong temperature dependence of the diagonal resistivity $\rho_{xx}(T)$. Particularly, among the samples studied, the smallest value of the deviation in $R_{xy}$ (by 6%) was measured in the high mobility sample Si22, where the diagonal resistivity $\rho_{xx}$ varies most strongly (by 5.5 times) in the same temperature range.

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FIG. 1. a) Shubnikov-de Haas oscillations in $\rho_{xx}$ measured on the sample Si22 vs gate voltage at $T = 0.29 K$ and $B = 2 T$. Dashed line and full dots demonstrate a linear dependence between the number of the quantum level and the gate voltage, from which the $n_{ShdH}$ density is calculated. Inset shows the sample geometry. b) $\omega_c \tau$ vs density at field $B = 0.2 T$. 

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FIG. 2. Resistivity at zero magnetic field (left Y-axis) vs gate voltage for the sample Si46 for 7 temperature values. Hall density (right Y-axis) as a function of gate voltage at $B = 0.3\,\text{T}$ and $T = 0.29\,\text{K}$. Dotted line depicts $V_{gc}$, dashed line is for the density $n_{\text{ShdH}}$ calculated from Shubnikov-de Haas oscillations.

FIG. 3. a) Hall resistance as a function of the density, measured at two opposite field directions with sample Si4/32 at $T = 2\,\text{K}$ and $B = 0.2\,\text{T}$. Dotted curve represents an averaged resistance, $< R_{xy} > = (R_{xy}(+B) - R_{xy}(-B))/2$. Bold dashed curve - classical dependence $B/ne$. b) Deviation in the “Hall-density”, $\delta n_{\text{Hall}} = n_{\text{Hall}} - n_{\text{ShdH}}$, measured at $B = \pm 0.2\,\text{T}$ for six temperatures. Vertical dashed lines mark the critical density $n_c$. The inset shows $\rho_{xx}$ vs temperature for 11 densities, 2.4, 2.5, 2.6, 2.7, 2.8, 3, 3.2, 3.4, 3.7, 4.7, 5.7, in unites of $10^{11}\,\text{cm}^{-2}$.

FIG. 4. Deviation in the “Hall-density”, $\delta n_{\text{Hall}} = n_{\text{Hall}} - n_{\text{ShdH}}$, measured at $B = \pm 0.2\,\text{T}$ with sample Si22 for four temperatures. Error bars indicate typical uncertainty for different temperatures and densities. Dashed line is a guide to the eye. Dotted vertical line marks the critical density $n_c$. The inset shows $\rho_{xx}$ vs temperature for 11 densities, 1.5, 1.7, 1.8, 1.9, 2.1, 2.4, 2.9, 3.34, 4.3, 5.5, 7.9, in unites of $10^{11}\,\text{cm}^{-2}$.
Fig. 1
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Fig. 2
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Fig. 4
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