Applications of $S_\mu/S_{em}$ showers universality for mass composition and hadronic interactions studies

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Abstract: We present the first results of the application of the recently found universality of behavior of muon signal $S_\mu$ to electromagnetic (EM) signal $S_{em}$ ratio with respect to the vertical depth of showers maximum $X_{max}$ for mass composition and hadronic interaction studies. Making use of the fact that for zenith angles $>45$ degrees the dependence of $S_\mu/S_{em}$ on $X_{max}$ is very similar for QGSJET II and EPOS 1.99 we show that this provides the possibility to estimate muon shower content in almost interaction model independent way. To evaluate the excess of signal in the data in respect to Monte-Carlo predictions we propose to use mass independence of the electromagnetic signal. Using the simulations with EPOS 1.99 as a fake data we show that one can determine the absolute scaling factor between these fake data and the interaction model under test (QGSJET II in our case). Applying this scaling factor to the total and muon signals of QGSJET II one can make accurate conclusions on the primary mass of samples prepared with EPOS 1.99.

Keywords: shower universality, muon signal, electromagnetic signal, hadronic interactions, mass composition

Introduction

In this paper we apply for mass composition and hadronic interaction studies the recently found shower universality property, stating that the ratio of the muon signal to the EM one $S_\mu/S_{em}$ is the same for all hadronic showers having the maximum at the same vertical depth $X_{max}$\textsuperscript{[1,2]}. This property provides a very simple parametrization for the muon signal \textsuperscript{[1]}

$$S_\mu^\text{fit} = S_{1000}/(1 + 1/((X_{\text{max}}/A)^{1/b} - a)),$$ \text{(1)}

where $A$, $a$ and $b$ are the fit parameters and $S_{1000}$ is the total ground plane signal in water Cherenkov detectors similar to the detectors of the Pierre Auger Observatory \textsuperscript{[3]}.

We use the large set of CORSIKA showers for interaction models QGSJET II/Fluka and EPOS 1.99/Fluka, described in detail in \textsuperscript{[2]}, to demonstrate that for zenith angles above 45 degrees, where EM halo from muon decays and interactions composes large part of the EM signal, the discussed universality allows to find the muon signal in almost interaction model independent way. This is done using QGSJET II muon signal parameterizations on EPOS 1.99 simulations which serve in the given case as a fake data. Once the muon signal is found the independence of the $S_\mu/S_{em}$ on interaction model properties and independence of the EM signal on the primary mass are used to determine the scaling factor between QGSJET II and EPOS 1.99. It is shown that such scaling procedure allows to extract with a good accuracy primary mass composition from EPOS 1.99 samples with the use of the QGSJET II model.

1 $S_\mu/S_{em}$ universality in inclined $\theta > 45^\circ$ showers

In \textsuperscript{[1]} it was demonstrated that the parametrization in the form (1) provides unbiased estimate of the muon signal with RMS of 8\% and 5\% for protons and iron correspondingly if the $0^\circ - 65^\circ$ angular range is considered. Here we would like to study in more detail only inclined $\theta > 45^\circ$ showers, since $S_\mu/S_{em}$ here should be very similar for both QGSJET II and EPOS 1.99 \textsuperscript{[2]}.

From Fig. 1 one can note that $S_\mu/S_{em}$ in proton showers at some fixed $X_{\text{max}}$ is slightly larger than in iron ones for $45^\circ - 65^\circ$ angular range. As discussed in \textsuperscript{[1]} proton showers having the same average depth of maximum $X_{\text{max}}$ with iron showers should be more inclined since they have deeper $X_{\text{max}}$. This in turn means that particles in proton showers should cross larger slant distance along shower axis from the shower maximum to the observation level and the fraction of $\pi^0$ EM component in them should be smaller than in iron showers, bringing to larger $S_\mu/S_{em}$. For this reason, as our calculations show, muon signal for $p$ and Fe showers obtained with the fit (1) on average slightly ($<2\%$) differ from the simulated signals, and for the EM signals this difference is within 7\% with RMS deviations.
from the simulated signal reaching 15% and 12% for \( p \) and Fe correspondingly. It is possible to improve the muon and EM signals recovery. Taking into account that \( S_\mu/S_{em} \) in proton showers at fixed \( X_{max}^p \) is larger than in iron showers and that at the same time proton showers are slightly more inclined than iron ones it is enough to multiply \( S_\mu/S_{em} \) by \( \cos^\alpha(\theta) \) finding \( \alpha \) which allows to reduce to minimum the difference in \( S_\mu/S_{em} \cos^\alpha(\theta) \) between \( p \) and Fe showers. We have found that this is achieved for \( \alpha \approx 1 - 1.5 \) and eventually we have chosen \( \alpha = 1.2 \). In this case the muon signal parametrization looks like this

\[
S_\mu^{fit} = \frac{S_{1000}}{1 + \cos^\alpha(\theta)/(X_{max}/A)^{1/6 - \alpha}}.
\]

we have performed the fits with (2) in three energy ranges \( \lg(E/eV) = 18.5 - 19.0, 19.0 - 19.5, 19.5 - 20.0 \) and the fit parameters are given in the Table 1. The use of this approach brings to unbiased estimates of both muon and EM signals with RMS around 10% for EM signals and 5% (3%) for muon signal in proton (iron) showers.

In Fig. 2 we present the result of application of formula (2) with the fit parameters for QGSJET II to the dataset simulated with EPOS 1.99. The increasing role of the EM halo, that brings to almost the same scaling of total and muon signals [2], and the similarity of \( S_\mu/S_{em} \) behavior on \( X_{max}^p \) allow to derive muon signal from EPOS 1.99 simulations with errors below 6% for protons and 3% for iron showers applying formula for QGSJET II (and vice versa). It is seen that with the increase of the energy interaction model invariance becomes more violated due to the increasing fraction of \( \pi^0 \) EM component arriving at the observation level.

2 Determination of the absolute signal scaling factor

The muon signal is one of the most powerful shower characteristics for the mass composition analysis. The procedure of determination of the muon signal proposed in the previous section provides the way to get the muon signal from hybrid data in Auger-like experiments, but due to well-known problem of muon deficit in Monte-Carlo (MC) shower simulation codes [4–6] the retrieved signal can not be used in mass composition studies. The latter could become possible only if one would be able to find absolute signal scaling factor \( S(p, real\ data)/S(p, MC) \). We would like to propose a possible way to solve this problem using as a fake data EPOS 1.99 simulations and QGSJET II as a test interaction model. Of course, for our fake data produced with EPOS 1.99 we know precisely the total, muon and EM signals and this will help us to estimate the accuracy of the proposed procedure.

Since primary mass composition is unknown the only way to determine the scaling factor is to use mass composition independent shower characteristics and the only appropriate candidate to this role is the EM signal. In Fig. 3 one can see its mass independence for QGSJET II and EPOS 1.99 interaction models.

Table 1: Fit parameters in (2) for QGSJET II and EPOS 1.99 interaction models.

| \( \lg(E/eV) \) | QGSJET II | EPOS 1.99 |
|----------------|-----------|-----------|
| 18.5 – 19.0    | 2070      | 10550     |
| 19.0 – 19.5    | 1045      | 2028      |
| 19.5 – 20.0    | 742       | 871       |
iron. One can see also that with the increase of the energy $E$ the error on the EM signal extracted from the data to the QGSJET II one. From Fig. 2 it is shown the ratio of the EPOS 1.99 EM signals to QGSJET II ones and the average of this ratio is equal to 1.25. Hence, it is reasonable to limit the scaling by the energy range $10^{18.5} - 10^{19.5}$ eV and in this case one gets the scaling factors equal to 1.34, 1.28 and 1.27 for $p$, $O$ and Fe correspondingly (Fig. 4, normalization is done in respect to QGSJET II oxygen). The true ratio of the muon signals is 1.26 and in case of pure iron or mixed primary composition after scaling both models should give very close predictions of the muon signals, for pure primary proton flux the discrepancy between the true EPOS 1.99 signals and scaled QGSJET II signals can exceed 5%. Evidently, this error is due to deep proton showers and if one applies $X_{\mu \text{ max}}^N < 500$ g/cm$^2$ cut only during scaling procedure this will affect almost exclusively proton showers and will change the scaling factor for them to 1.26. Hence, using the cut $X_{\mu \text{ max}}^N < 500$ g/cm$^2$ the scaling factor that one gets will be within 1.26 - 1.28 range independently on the mass composition, in very good agreement with the true muon signals ratio. In Fig. 4 the muon signal obtained from EPOS 1.99 ‘data’ with (2) and parameters for QGSJET II is compared with the scaled by 1.26 QGSJET II muon signal. One can see that after the entire procedure one gets quite consistent picture with the tendency to underestimation of the ‘true’ primary mass, since e.g. the ‘true’ proton sig-

Figure 3: Top: EM signals for QGSJET II (closed symbols) and EPOS 1.99 (open symbols) normalized by primary energy and $f(\theta) = 1 - 4.3 \cos(\theta) + 4.9 \cos^2(\theta)$ to reduce spread caused by the spread of zenith angles) in the energy range $10^{18.5} - 10^{19.5}$ eV. At the bottom it is shown the ratio of the EPOS 1.99 EM signals to QGSJET II ones and the average of this ratio is equal to 1.23. Due to increasing role of the EM halo in the considered angular range $45^\circ - 65^\circ$ $S_\mu/S_\text{em}$ becomes less dependent on the hadronic interaction properties [2] providing the following approximate equality which holds true for any primary nuclei ($p$, $O$, Fe etc.):

$$\frac{S_\text{EPOS}}{S_\text{QGSJET II}} \approx \frac{S_\mu}{S_\text{em}} \approx \frac{S_{\mu \text{ EPOS}}}{S_{\text{em}}} \quad (3)$$

The ratios of the total and muon signals confirm these relation with quite good accuracy: their average values for all primaries (1.25 and 1.26) are quite the same as the value for the EM signals. The ratios of the total and muon signals remain constant across the entire considered energy range.

Now it remains only to try to scale the QGSJET II model predictions using EPOS 1.99 set as a fake data. To find EM and muon signals in the ‘data’ one should simply apply the parametrization (2) with coefficient for QGSJET II to EPOS 1.99 simulations and to find the ratio of the EM signal from the ‘data’ to the QGSJET II one. From Fig. 2 one can see that the error on the EM signal extracted from EPOS 1.99 can reach 10% for proton primaries and 6% for iron. One can see also that with the increase of the energy

Figure 4: Top: ratio of the parametrized EPOS 1.99 EM signals to the EM signal for QGSJET II oxygen for energies below $10^{19}$ eV fitted with flat line. Mean values are 1.34, 1.28 and 1.27 for $p$, $O$ and Fe correspondingly. Bottom: scaled by 1.26 QGSJET II muon signals and muon signals obtained from EPOS 1.99 simulated dataset with (2) and coefficients for QGSJET II.
Figure 5: Top: reconstructed with QGSJET II and MTA on the basis of Fisher’s variables distributions proton (red squares) and iron (blue crosses) abundances in the EPOS 1.99 samples with known primaries content. Bottom: same but for proton–oxygen (brown diamonds) mixture. Lines mark the true primary fractions. $\lg(E/eV) = 18.90 - 19.10$

We have reconstructed a mass composition of proton-oxygen and proton-iron mixtures prepared with EPOS 1.99 using scaled by 1.26 QGSJET II model signals. The muon signal from EPOS 1.99 samples was retrieved with the use of the fit (2) with parameters for QGSJET II. To discriminate primaries ($X_{\text{max}}, S_{\mu}^\text{fit}/(E X_{\text{max}}^\nu)$) variables have been used and the approach has been the same as in [7], i.e. with consequent application of the Fisher’s discriminant analysis and Multiparametric Topological Analysis (MTA) [8]. From Fig. 5 one can see that for proton-iron mixtures the method gives excellent results, while for proton-oxygen mixture the reconstructed composition is lighter than the original one and errors grow with the increase of the oxygen fraction from 10 to 17%. Let us note that these errors are almost completely due to errors in muon signal and scaling factor determination (see Fig. 4), while accuracy provided by MTA itself is better than 2%. More precise results for $p$–Fe samples are explained by very good separation of the Fisher’s variable distributions for these primaries [7] and small errors on the scaling factor do not influence significantly the events misclassification rate. For the scaling factor of 1.28 the accuracy of reconstruction is $2 - 4\%$ and $15 - 20\%$ for $p$–Fe and $p$–O mixtures correspondingly.

**Conclusions**

In this paper we have demonstrated that the universality of $S_{\mu}/S_{\text{em}}$ ratio in $45^\circ - 65^\circ$ angular range in respect to the interaction model properties allows to get the muon signal from hybrid data with accuracy of $3 - 5\%$. The application of this approach to the data of the Pierre Auger Observatory can be found elsewhere [9]. Further, using the independence of the EM signal on the primary mass we have proposed a procedure giving a possibility to find absolute scaling factor between real data and MC simulated signals. Using EPOS 1.99 simulations as a fake data we have found a scaling factor for QGSJET II signals with an accuracy of few percents. Application of the scaled QGSJET II muon signals allowed to reconstruct mass composition of the samples prepared with EPOS 1.99 with errors below $4\%$ for proton-iron mixtures, while for proton-oxygen ones the accuracy is around $10 - 20\%$. Hence, the use of the both models in the proposed way for reconstruction of the real primary mass composition will give closely agreeing results. The preference to results obtained with one of the models can be given on the basis of the comparison with measurements of $X_{\text{max}}$ and $S_{\mu}$ distributions.

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