Utilization of a hardware-in-the-loop-system for controlling the speed of an eddy current brake

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Abstract. Rapid prototyping with a hardware-in-the-loop (HiL) system significantly reduces the development time for controller-type testing and is widely used in various fields of engineering. In this discussion, a controller is developed for a speed control application utilizing a magnetic brake. A mathematical model is presented first that has been implemented in Matlab/Simulink. The controller development steps are described that will form the basis of a control system for a wind turbine. A test is carried out that simulates the wind turbine inertial load.

1. Introduction
Deploying a magnetic brake can be advantageous as it encounters less wear compared to a conventional friction brake. In the case of a magnetic retarder, the braking force is merely developed by two opposing magnetic fields and the braking energy is completely dissipated in the form of heat produced by the eddy currents. However, the process of magnetic braking cannot be described by common dynamic modelling techniques such as the Euler-Lagrange method or Bond graphs as it exhibits nonlinear behaviour. The nonlinearity stems from the brake torque being proportional to the square of the exciting current, from the hysteresis of the magnetic field strength in the iron core and also from the fact that the geometry of the system is almost unconstrained; i.e. the eddy currents can flow into any direction. The problem of magnetic braking has been covered analytically [1,2] and it has also been approximated using linear models for different speed regions [3]. Additionally, with the usage of modern computers, powerful simulation program suites such as Comsol [4] can be used to solve the eddy current brake problem numerically. However, since today’s controller design procedures demand short development cycles, a HiL evaluation board can be used to significantly accelerate the controller design process. In this work, a power-based modelling approach for the brake is presented, considering the geometry of the iron core’s cross section. Based on a reduced single input-single output model, a speed controller is developed for future deployment in a wind turbine. The brake and its speed controller will maintain the shaft speed at the required reference input that has been evaluated according to the current wind speed and the wind turbine characteristics. The variable speed control scheme is also referred to as maximum power point tracking (MPPT) and, in this case, it is implemented using a mechanical control approach.

The controller has been designed at the software level and, according to its performance in the actual hardware setup, the HiL Simulation will validate the controller and enable its further
refinement. All software development and simulations are carried out in Matlab/ Simulink and the Control System Toolbox/ Simulink Control design [5]; in addition, the HiL simulation will utilize the Quarc real-time environment from Quanser [6]. Prior to developing a speed controller, a linearization is needed that has been carried out with the Simulink Control design extension using the Control and Estimation Tools Manager. With a transfer function of the overall system available, a controller can be designed in the SISO-tool GUI that comes with the Simulink Control design and then tested in the non-linear model. A response time of $t = 1.5$ s and 15% overshoot, if required, have been demanded in this analysis. After development, the controller is also tested with an approximated load equivalent to the wind turbine inertia. The parameters of the brake are adapted to the target system and the DC motor characteristics modified in a way that the model generates enough dragging torque. The initial parameters come from a test bench that was built for the HiL simulation with a DC motor driving the brake disk. The setup is described in the next section.

2. Experimental setup

The magnetic brake prototype assembly consists of an excitation coil with a C-shaped steel core (S235JR) and an aluminium disk mounted on a shaft that is directly driven by the DC motor (Figure 1). Within the DC motor there is a drive train contained with a gear ratio of 30:1. The rotational speed is measured with an optical sensor that was installed below the bottom of the disk. It retrieves an alternating signal from a black and white coloured ring that is attached to the disk. For ease of interfacing, the voltage of the magnet is controlled by a pulse width modulated signal that serves as a signal input for a full bridge motor driver. To allow future improvement of the control system’s performance, an analogue hall sensor was included in the setup that can be used for measuring the magnetic field strength. The required simulation parameters of the setup can be found in the appendix. The HiL development board, a Q2-USB from Quanser, has 8 bidirectional pins that are used for connecting the sensor and the bridge driving circuit to the board. As mentioned before, Quanser provides the fully integrated software development suite Quarc and its Simulink blocks can be directly utilized in a simulation model. All simulation tests can be run and monitored within Simulink.

![Figure 1. Magnetic brake test bench [7].](image)
3. Magnetic brake modelling

3.1. Principle of magnetic braking

Magnetic braking occurs when a moving conductor is exposed to a magnetic field. At first, a voltage is induced due to a change in flux. According to Faraday’s law, the rate of the changing flux determines the magnitude of the generated voltage around a closed loop:

\[
\oint E \cdot dl = \oint q \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot \vec{n} \cdot da
\]  

(1)

As can be seen from equation (1), the enclosed path integral equals the negative rate of change in flux. This reflects the nature of the induced voltage to generate a current flow direction that would create a field opposing the change in magnetic flux. According to Lorentz’s law, the electric force of moving charges equals the magnetic Lorentz force exerted on the particles in question:

\[
\vec{E} \cdot q = q \cdot \vec{v} \times \vec{B}
\]  

(2)

The induced electric field is therefore perpendicular to both the velocity and the magnetic field strength. As in the case of a rotational motion, the electric field points radially towards the centre of the brake disc, as long as the original excitation field persists. The electric field causes the movement of charges and developing currents generate an opposing field that, according to Lentz’s law and by superposition, adds up to a repelling magnetic force, causing the conductor to slow down. The same principle can be observed in a simple DC motor where the initial field accelerates the current carrying armature while the generated voltage increasingly inhibits a further speed increase.

In the case of a thin wire, the superposition problem reduces to a linear current flow and the electric field path over which to integrate is the length of the conducting wire that moves at a velocity perpendicular to the magnetic field. Obviously, the analysis becomes increasingly more complex when dealing with a two or three dimensional geometry.

3.2. State space representation of the system

As already mentioned in the introduction, the magnetic braking force depends on the square of the dynamic coil current and, furthermore, on the rotational speed of the overall system. Since Laplace transfer functions allow only one input variable per transfer function, the state space representation has been chosen for the following model description. Also, for prospective efforts of refining and extending the model and its controller to a full-state feedback control system, two measureable state space variables \((i, \omega)\) are a reasonable choice. They represent the aforementioned inputs to the system. The differential equations of the system are presented in the next section.

Again, the induced voltage can be described by Faraday’s law of induction:

\[
V_{\text{ind}} = -\frac{d\Phi}{dt}
\]  

(2)

Since the magnetic flux \(\Phi\) equals the magnetic field strength \(B\) times the area \(A\) exposed to the field, the derivative of this equation is:

\[
V_{\text{ind}} = -\frac{d(B \cdot A)}{dt} = -B(t) \frac{dA(t)}{dt} - A(t) \frac{dB(t)}{dt}
\]  

(3)
The swept area becomes approximately a ring segment with the width of the magnet core (Figure 3). Its equation and the derivative are given by the following equation:

\[
A = \frac{1}{2} \cdot (r_o^2 - r_i^2) \cdot \theta \cdot K_C
\]

\[
A(t) = \frac{1}{2} \cdot (r_o^2 - r_i^2) \cdot K_C \cdot \omega(t) \cdot t
\]

\[
\frac{dA(t)}{dt} = c_1 \cdot \omega(t) \cdot t
\]

\[
\frac{d^2A(t)}{dt^2} = \frac{d\omega(t)}{dt} \cdot t + \omega(t)
\]

where \(r_o\) and \(r_i\) are the outer and inner radii, respectively and \(K_C\) was added to account for the approximation.

![Figure 2. Swept area.](image)

The magnetic H-field is directly proportional to the excitation current and the magnetic field strength B was also found to be approximately proportional to the coil current [7]. Therefore, the derivative has been assumed to be proportional, too:

\[
B(t) = c_2 \cdot i(t)
\]

\[
\frac{dB(t)}{dt} = c_2 \cdot \frac{di(t)}{dt}
\]

with \(c_2 = \mu_0^* N/\delta\) where \(N\) are the number of windings and \(\delta\) the distance of the air gap, respectively. The relationship between the H- and B-field can be viewed in Figure 3. It can be seen that only a small hysteresis effect occurs around the origin. The author refers to the hysteresis curve to as a Rayleigh curve within larger hysteresis loops where the outer loops are not reached due to the excitation current limit.
The state space equation of the excitation current is retrieved by summing the voltages around the R-L-circuit (Figure 4a) of the coil:

$$\frac{di(t)}{dt} = -\frac{R}{L} \cdot i(t) + \frac{1}{L} \cdot V_{ex}$$  \hspace{1cm} (6)

The induced voltage is therefore:

$$V_{ind} = -c_1 \cdot c_2 \cdot i(t) \cdot \left[ \frac{d\omega(t)}{dt} \cdot t + \omega(t) \right] - c_1 \cdot c_2 \cdot \omega(t) \cdot t \cdot \frac{di(t)}{dt}$$

$$V_{ind} = -c_1 \cdot c_2 \cdot i(t) \cdot \left[ \frac{d\omega(t)}{dt} \cdot t + \omega(t) \right] - c_1 \cdot c_2 \cdot \omega(t) \cdot t \cdot \left[ -\frac{R}{L} \cdot i(t) + \frac{1}{L} \cdot V_{ex} \right]$$  \hspace{1cm} (7)

The induced current is found by assuming a parameter for the current path length $K_L$. This leads to a resistance in the disk of:
\[ R_{\text{disk}} = \frac{\rho_{\text{Al}} \cdot K_L}{w \cdot d} \]
\[ R_{\text{disk}} = c_3 \cdot K_L \]

with \( \rho_{\text{Al}} \), \( w \) and \( d \) being the resistivity of aluminium, the width of the magnet core and the thickness, respectively. The braking force can be calculated as:

\[ F_{\text{disk}} (t) = i_{\text{ind}} (t) \cdot B(t) \cdot K_L \]

\[ F_{\text{disk}} (t) = \frac{V_{\text{ind}}}{R_{\text{disk}}} \cdot c_2 \cdot i(t) \cdot K_L \]

\[ F_{\text{disk}} (t) = \frac{V_{\text{ind}}}{c_3} \cdot c_2 \cdot i(t) \]

(9)

It can be seen that, with this assumption, the force is independent of the current path length.

The calculated force is the total force exerted on the disk and the braking torque can be found by using the radius to the centre of the core \( r_F \) as the point of action:

\[ T_{B_F} (t) = -c_1 \cdot c_2 \left( i(t) \cdot \left[ \frac{d\omega(t)}{dt} \cdot t + \omega(t) \right] - \omega(t) \cdot t \cdot \left[ -\frac{R}{L} \cdot i(t) + \frac{1}{L} \cdot V_x \right] \right) \cdot c_2 \cdot r_F \cdot i(t) \]

(10)

The state variable \( \omega \) depends on both the braking and the motor torque. Since the motor torque replaces the wind turbine torque, i.e. the input to the system, the motor is assumed to be rotating at its equilibrium state. This assumption requires the change of the armature current be zero, hence

\[ \frac{di_a (t)}{dt} = 0 = -c_M \cdot \omega(t) - \frac{R_a}{L_a} \cdot i_a (t) + \frac{1}{L_a} \cdot V_M \]

\[ i_a = -R_a \cdot c_M \cdot \omega(t) + R_a \cdot V_M \]

(11)

The equivalent circuit for the differential equation in (11) is shown in Figure 4b where \( V_G = c_M^* \omega \) and \( c_M \) is the generator constant of the motor:

![Figure 4b. Equivalent armature circuit for DC motor.](image)

The equation for the state variable \( \omega \) can be derived by summing up all the torques applied to the rotating disk:

\[ \frac{d\omega}{dt} = \frac{1}{J} \left( T_{B_F} + T_{F_R} + T_M \right) \]

(12)
\[
\frac{d\omega}{dt} = \frac{1}{J} \left( T_{Fr} + T_{Fr} + 30 \cdot c_M \cdot i_a \right)
\]

with \( T_{Fr} \) being the total dynamic friction torque. In a consistent set of units, the same generator constant as in equation (11) relates the armature current to the motor torque. The gear ratio enhances the motor torque by a factor of 30. Completed with equation (10) for the braking torque, the state space equation for \( \omega \) becomes:

\[
\begin{align*}
\frac{d\omega}{dt} &= \frac{1}{J} \left( \frac{-c_1 \cdot c_2}{c_3} \left( i(t) \cdot \frac{d\omega(t)}{dt} + \omega(t) \right) - \omega(t) \cdot t \cdot \left[ -\frac{R}{L} \cdot i(t) + \frac{1}{L} \cdot V_{ex} \right] \right) \cdot c_2 \cdot r_p \cdot i(t) + T_{Fr} + 30 \cdot c_M \cdot i_a \\
\frac{d\omega}{dt} &= \frac{1}{J} \left( \frac{-c_1 \cdot c_2}{c_3} \left( i \cdot \frac{d\omega}{dt} + \omega \right) - \omega \cdot t \cdot \left[ -\frac{R}{L} \cdot i + \frac{1}{L} \cdot V_{ex} \right] \right) \cdot c_2 \cdot r_p \cdot i + T_{Fr} + 30 \cdot c_M \cdot \left[ -R_a \cdot c_M \cdot \omega + R_a \cdot V_M \right]
\end{align*}
\]

Rearranging for \( d\omega/dt \) yields:

\[
\frac{d\omega}{dt} = \frac{1}{J} \left( T_R + 30 \cdot c_M \cdot \left[ -R_a \cdot c_M \cdot \omega + R_a \cdot V_M \right] - i^2 \cdot \omega + \frac{R}{L} \cdot i^2 \cdot \omega \cdot t - \frac{1}{L} \cdot i \cdot \omega \cdot t \cdot V_a \right)
\]

(13)

With: \( c = \frac{c_1 c_2^2 r_p}{c_3} \).

It can be seen from equation (14) that the system is neither linear nor time-invariant; hence it must be linearized prior to the controller design. A Jacobian linearization can be used for arbitrarily chosen inputs. The linearized Jacobian differential equation takes the form [8]:

\[
\delta_x(t) = A \cdot \delta_x(t) + B \cdot \delta_u(t)
\]

(15)

where \( \delta_x \) and \( \delta_u \) are small deviations from the equilibrium point \( \bar{x} \) and input \( \bar{u} \), respectively, and A and B are coefficient matrices. Equation (15) is only valid for a small region around the chosen operating point.

Matrices A and B contains the partial derivatives of the state space functions with respect to \( x \) and \( u \):

\[
A_{num} = \frac{\partial f}{\partial x} \bigg|_{x=\bar{x}}, B_{num} = \frac{\partial f}{\partial u} \bigg|_{u=\bar{u}}
\]

(16)

Matrix A and B are therefore:
\[ A = \begin{pmatrix} \frac{\partial f_1}{\partial i} & \frac{\partial f_1}{\partial \omega} \\ \frac{\partial f_2}{\partial i} & \frac{\partial f_2}{\partial \omega} \end{pmatrix} \]  

(17)

\[ B = \begin{pmatrix} \frac{\partial f_1}{\partial V_{ex}} & \frac{\partial f_1}{\partial V_M} \\ \frac{\partial f_2}{\partial V_{ex}} & \frac{\partial f_2}{\partial V_M} \end{pmatrix} \]  

(18)

with

\[ f_1 = -\frac{R}{L} \cdot i + \frac{1}{L} \cdot V_{ex}, \]

\[ f_2 = \frac{1}{J} \left( \frac{T_L}{c} + \frac{c_M \cdot (-R \cdot c_M \cdot \omega + R_\omega \cdot V_M)}{c} - i^2 \cdot \omega + \frac{R}{L} \cdot \omega \cdot t - \frac{1}{L} \cdot i \cdot \omega \cdot t \cdot V_{ex} \right) \]

The elements of matrix A and B are therefore:

\[ A = \begin{pmatrix} -\frac{R}{L} & 0 \\ E_1 & E_2 \end{pmatrix} \]

(19)

\[ B = \begin{pmatrix} \frac{1}{L} & 0 \\ -E_3 & E_4 \end{pmatrix} \]

(20)

Equations E1-E4 have been solved using the Symbolic Math Toolbox [9] in Matlab. For clarity, the equations are shown separately:

\[ E_1 = \frac{c_1 c_2^2 r_f \cdot (-2L J c_3 \omega + 2R J c_3 t i \omega + V_{ex} c_1 c_2^2 r_f i^2 - V_{ex} J c_3 t \omega + 2T_L L c_3 t i \omega + 60 R_\omega c_M^2 L c_3 t \omega \cdot t - 60 R_\omega c_M L c_3 V_{ex} t i)}{L(c_1 c_2^2 r_f t i^2 + J c_3)^2} \]

\[ E_2 = \frac{-c_1 c_2^2 r_f L i \omega + c_1 c_2^2 r_f R t i^2 - c_1 c_2^2 r_f V_{ex} t i - 30 R_\omega c_M^2 L c_3}{L(c_1 c_2^2 r_f t i^2 + J c_3)} \]

\[ E_3 = \frac{-c_1 c_2^2 r_f t \omega}{L(c_1 c_2^2 r_f t i^2 + J c_3)} \]

\[ E_4 = \frac{30 R_\omega c_M L c_3}{L(c_1 c_2^2 r_f t i^2 + J c_3)} \]
This system was designed using common Simulink blocks and embedded into a subsystem. A block diagram is shown in Figure 5. Input ports 1 and 2 are the excitation voltage and the angular speed, respectively. Output port 1 transfers the computed braking torque.

Figure 5. Simulink subsystem for the magnetic brake.

4. Controller design and simulation
As has been shown with the Jacobian equation, the model must be linearized prior to developing a controller. Using the Control and Estimation Tools Manager in Matlab/ Simulink, the system can be linearized about an operating point that was taken from a simulation snapshot at a known equilibrium time. The linearization process is done blockwise with an exact result for Simulink blocks and a Jacobian perturbation method for user-written functions (e.g. S-Functions).

The controller has been designed using a step test input in the SISO-tool development environment and the results for the linearization process as well as the controller design and the closed loop response are presented in the next paragraph. For the HiL validation tests, the controller still needs to be transformed to the z-domain with a sampling rate that was chosen for counting the digital encoder pulses within that time period.

4.1. Extracting a model to Matlab
The brake diagram is included in Figure 6 which was configured using a subsystem input-output structure. The latter is needed for an open loop analysis when linearizing the system. For simulating the response to a given speed reference input, a feedback path is still required that compares to the reference input. Not all blocks in Simulink are linearized; the open-loop response of the non-linear system differs in terms of the settling time, if compared to the linearized model. However, the transfer function was used for designing a controller which proved to meet the requirements for the non-linear model as shown in section 4.2.
The operating point was retrieved from the non-linear system when its response has reached a steady-state value. The linearization process showed the open-loop response depicted in Figure 7 and yielded the following transfer function:

\[
F(s) = \frac{1.571 \times 10^4 \cdot s^2 + 3.442 \times 10^7 \cdot s + 1.603 \times 10^5}{s^3 + 2458s^2 + 5.868 \times 10^5 \cdot s + 6.515 \times 10^5} \tag{21}
\]

Figure 6. Overall open loop system.

Figure 7. Open loop step response of linearized subsystem.

4.2. Controller design and test
With the transfer function and the operating point exported to workspace, Matlab’s SISO-tool can be started and the control system requirements are specified in the tool. A classic control system structure was chosen with the controller cascaded before the plant. As mentioned before, the initial compensator requirements were 15% overshoot and a settling time of 1.5s. These requirements were met with the transfer function shown in equation (13).
The Laplace transform of the chosen controller is

\[
\frac{1.1 \cdot s + 1}{s \cdot (1 + 3s)}
\]  

(13)

The general form of a lag filter shown in Equation (14) reveals that the controller contains a filter times an integral controller with \(\tau_1 = 1.1\) and \(\tau_2 = 3\) where \(\tau_1 > \tau_2\):

\[
\frac{\tau_1 s + 1}{\tau_2 s + 1}
\]  

(13)

The step response of the linearized and compensated closed loop system can be seen in Figure 8. It shows a rise time of approximately 0.1s which is very quick compared to the inertial non-linear response depicted in Figure 9. However, it has been mentioned before that the linearized system obviously lacks some of the system dynamics since Simulink blocks were omitted in the linearization process.

For a better distinction between the driving and the braking process, the step response to a chosen brake input in Figure 9 has been programmed such that the brake is only turned on at \(t = 5\)s.

The reference angular speed given in this case is \(\omega = 17/s\) which has been chosen according to the DC motor speed at \(V_{ex} = 12\) V (approximately \(\omega = 19/s\)).

![Figure 8. Linearized system/compensated design response with settling time.](image-url)
Figure 9. Non-linear system response.

Figure 9 shows a settling time of approximately 3s. The response time could be further improved by adding a proportional gain block of $K_p=2$ to the controller. With that addition, the settling time of the non-linear system was also within the required range ($t_s \approx 1.2s$).

In a final step, the wind turbine inertial load was integrated in the model with the DC motor characteristics modified, so that the DC motor would be strong enough to drag the load. The result is displayed in Figure 10. The first PT1-response reflects the motor’s transient behaviour and after the motor has reached its equilibrium speed, the brake is switched on at $t=7$. It can be seen that the response time is well within the required time limit.

Figure 10. System response with the wind turbine equivalent load.
5. Conclusion
In this work, the development of a speed controller has been presented that will be utilized in a wind turbine MPPT system using a magnetic brake. The speed control mechanism is implemented such that the controller will maintain the wind turbine shaft speed at a given optimum speed, possibly computed from a wind turbine model or a look-up table (LUT).

The controller requirements were specified with a settling time of $t = 1.5$s and 15% overshoot, if applicable. Using a linearized transfer function, the controller could be quickly designed using the Mathworks’ SISO-tool. Despite the different system behaviour between the linearized system and the actual non-linear system, a controller could be designed that met the requirements for the linearized model; only a small proportional gain controller was added so that the non-linear system would respond accordingly.

At last, the system parameters were altered matching the final brake geometry and the DC motor characteristics were adapted so that a stronger dragging torque can be applied. The braking time response for this system was within the specified limit.

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Appendix

Brake parameters of test bench design

\[
d=4\times10^{-3}; \quad \text{thickness of disk [m]}
\]
\[
w=4\times10^{-2}; \quad \text{width of current path (assumption) [m]}
\]
\[
g=6\times10^{-3}; \quad \text{length of air gap [m]}
\]
\[
r_i=6.25\times10^{-2}; \quad \text{distance from rot. axis to steel core [m]}
\]
\[
r_o=10\times10^{-2}; \quad \text{distance from rot. axis to end of steel core [m]}
\]
\[
r_F=8.25\times10^{-2}; \quad \text{distance from rot. axis to middle of iron core [m]}
\]
\[
rho_{Al}=0.0286\times10^{-6}; \quad \text{specific resistance of aluminium [Ohms*m]}
\]
\[
my_0=1.257\times10^{-6}; \quad \text{magnetic field constant [Vs/Am]}
\]
\[
N=500; \quad \text{number of windings [-]}
\]
\[
I=1.25; \quad \text{current of electro magnet [A]}
\]
\[
V=12; \quad \text{voltage of electro magnet [V]}
\]
\[
R=V/I; \quad \text{resistance of coil [Ohms]}
\]
\[
L=36\times10^{-3}; \quad \text{inductivity of coil [H]}
\]
\[
c_1=1/2*(r_o^2-r_i^2); \quad \text{swept area constant}
\]
\[
c_2=my_0*N/g; \quad \text{magnetic field strength constant}
\]
\[
c_3=rho_{Al}/(width*d); \quad \text{disk resistance constant}
\]

Motor parameters

\[
R_a=16.5; \quad \text{Armature Resistance}
\]
\[
L_a=7.53\times10^{-3}; \quad \text{Armature Inductivity (measured)}
\]
\[
T_{Fr}=50\times10^{-3}; \quad \text{friction torque [Nm] (measured)}
\]
\[
J=1.5\times10^{-3}; \quad \text{moment of inertia (rotor, disk, shaft, adapt.)}
\]
\[
c_m=0.55; \quad \text{calculated generator constant of motor [Vs]}
\]