The Power Grid Probabilistic Vulnerability Evaluation Considering the Stochastic Perturbations of Wind Farm

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Abstract. Considering the stochastic volatility of load and wind farm, stochastic outage rate of generator, based on the combination of Cumulant and Gram-Charlier series (CGC), a new evaluation method for probabilistic vulnerability of transmission network is proposed. Firstly, the three-parameter Weibull distribution is adapted to describe the random variation of wind speed and the probabilistic model for wind turbine generator system is established. And then, in order to avoid complicating convolution and improve the computational speed, the vulnerability index of nodes and branches in power grid is proposed with the combination of Cumulant and Gram-Charlier series. Furthermore, the K-means clustering algorithm is adapted to evaluate the vulnerability of nodes and branches. The simulation results in IEEE30 bus system show that the proposed method can lead to a reliable and effective evaluation result.

1. Introduction

In power system, the generator outage, power fluctuations of the wind farms or other renewable energy power generation connected cause the vulnerability of power transmission grid [1-3]. Power grid vulnerability refers to the weakening trend of voltage and transmission capability under the conditions of disturbances or faults [4]. Depending on the existing analytical methods, this characteristic has been divided into two types: state vulnerability and structural vulnerability. Evaluation of state vulnerability by utilizing methods such as energy function [5-6], scale-free graphs [7], artificial neuron [8] and sensitivity analysis [9]. As for the evaluation of structure vulnerability, the hybrid flow betweenness [10], Cascading Failure Model [11], electric betweenness [12] and small-world topological model [13] have been applied to identify vulnerable buses and branches. Based on the Max-Flow law and the complex network theory, reference [4] proposed a vulnerability assessment index to identify the vulnerable lines in the power grid. Reference [14] utilized graph theory and complex network theory to research the power grid vulnerability of Distributed Generation (DG) by evaluating the topological indexes of structural vulnerability analysis. Combining the complex network theory and the information entropy, reference [15] improved Electrical betweenness to investigate the power grid vulnerability. Reference [16] proposed a new continuous comprehensive evaluation index to evaluate the state vulnerability and structural vulnerability of branches, where the
energy function model of power system penetrated with the doubly-fed wind turbine is introduced and the electrical betweenness is combined.

The above studies have not analyzed the whole situation of power system voltage and power random fluctuation comprehensively. Obviously, compared with the conventional vulnerability analysis, probabilistic method, which takes uncertain factors into consideration, could reflect the uncertainties in the operation of power grid more clearly. On the basis of probabilistic Load Flow, stochastic volatility of load, stochastic outage of generator and randomness brought by wind farm is considered synthetically in this paper. The Gram-Charlier series expansion based on cumulant is introduced to calculate the distribution of random variables. According to the probabilistic distribution of each node voltage and each branch load flow, the probabilistic vulnerability index of power grid is proposed to identify the weak nodes and lines. The simulation analysis of IEEE-30 bus system containing wind power model has indicated the effectiveness and feasibility of the proposed method.

2. Stochastic probability model of wind farm

2.1. Probabilistic distribution model of wind turbine power

The Weibull (three parameter) model, which can better fit the probabilistic distribution of wind speed, is usually used to simulate the wind power system. The model is verified by a large number of measured data, which can well meet the average wind speed distribution in most areas. The output power of wind turbine generator can be normally expressed as

\[ P_w = \begin{cases} 0, & v \leq v_{ci} \\ k_1v + k_2, & v_{ci} < v \leq v_r \\ P_r, & v_r < v \leq v_{co} \\ 0, & v > v_{co} \end{cases} \]  

(1)

Where, \( P_w \) is the rated active power output of wind turbine; \( v_{ci} \) is the cut-in wind speed; \( v_r \) is the rated wind speed; \( v_{co} \) is the cut-out wind speed. \( k_1 = P_r / (v_r - v_{ci}) \), \( k_2 = -k_1v_{ci} \), \( v \) is wind speed.

The actual wind speed is mainly concentrated in the cut-in wind speed and rated wind speed, namely wind turbine active power and wind speed is linearly related. The probabilistic distribution of the active power produced by wind turbine can be expressed as

\[ F(P_w) = \int_{v_{ci}}^{v_r} f(v) dv + \int_{v_r}^{v_{co}} f(v) dv \]  

(2)

The probability density function of active power produced by wind turbine is

\[ f(P_w) = \frac{k}{k_c} \left( \frac{P_w - k_1v_{ci} - k_2}{k_c} \right)^{k-1} \exp \left[ -\left( \frac{P_w - k_1v_{ci} - k_2}{k_c} \right)^k \right] \]  

(3)

In China, asynchronous generators is used in most of the grid-connected wind power generation system. Asynchronous generators draw reactive power from the system while provide active power to establish the magnetic field. The wind turbine generator is usually simplified as a PQ node, and the power factor is assumed to be constant by automatic switching of the capacitor in the wind turbine [17]. The reactive power is expressed as the following equation

\[ Q = P \tan \varphi \]  

(4)

Where, \( \varphi \) is the power factor angle, which generally located in the fourth quadrant and \( \tan \varphi \) is negative.

3. Probabilistic analysis for power grid containing wind farms based on Cumulant method

3.1. Probabilistic distribution based on Cumulant and Gram-Charlier Expansion

Probabilistic load flow can not only reflect the influence produced by the random variation of various factors on the power system, but also describe the distribution characteristics of the system state...
variables accurately. The uncertainties of network topology, component parameters, node load value and generator output in the power system could be also synthetically considered [17]. The convolution of random variation occupies major part in the probabilistic flow computation. The method combining Gram-Charlier expansion with Cumulant is widely used to calculate the distribution of random variables, due to the high efficiency and accuracy.

The Gram-Charlier series expresses the distribution function of random variable as the series of derivations of normal random variable, as follows

\[
 f(z) = \varphi(z) + c_1 \varphi^{(1)}(z) + c_2 \varphi^{(2)}(z) + c_3 \varphi^{(3)}(z) + \cdots
\]

(5)

\[
 F(z) = \Phi(z) + c_1 \Phi^{(1)}(z) + c_2 \Phi^{(2)}(z) + c_3 \Phi^{(3)}(z) + \cdots
\]

(6)

Where, the coefficients \( c_i (i = 1,2,\ldots) \) of Gram-Charlier series can be determined by cumulant; \( \Phi(z) \) is the probability distribution function of random variable \( z \); \( f(z) \) is the probability density function of random variable \( z \). \( \varphi(z) \) and \( \Phi(z) \) are the distribution function and the probability density function of normal distribution with expectation \( m = 0 \) and standard deviation \( \sigma = 1 \), respectively.

\[
 \varphi(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} z^2\right)
\]

(7)

\[
 \varphi^{(r)}(z) = \left(\frac{d}{dz}\right)^r \varphi(z) = (-1)^r H_r(z) \varphi(z)
\]

(8)

Where, \( H_r(z) \) is Hermite polynomials. The Gram-Charlier series expansion of random variable distribution function can be obtained as

\[
 F(z) = \int_{-\infty}^{z} \varphi(z) dz + \varphi(z) \left[c_1 H_1(z) + c_2 H_2(z) + c_3 H_3(z) + \cdots\right]
\]

(9)

Where \( z \) is normalized random variable.

3.2. K-means cluster analysis

The K-means cluster algorithm is adopted here to analyze the vulnerability of nodes and branches. The samples are clustered into \( k \) clusters by calculating the distances between each other for the samples categories are unknown [18].

The probabilistic vulnerability evaluation index of nodes and branches are taken as samples. Set up \( x_{i_1}, \ldots, x_{i_k}, x_{j_1} \in \mathbb{R}, \ k \) cluster centroid are randomly selected as \( \mu_1, \mu_2, \ldots, \mu_k \in \mathbb{R} \). For each sample \( x_{i_1} \), the class which it belongs can be calculated as

\[
 c_{i_1} = \arg \min_j \left\| x_{i_1} - \mu_j \right\|
\]

(10)

For the \( j \)-th class, the centroid would be recalculated as

\[
 \mu_j = \frac{\sum_{i_{-1}}^{m} 1\{c_{i_1} = j\} x_{i_1}}{\sum_{i_{-1}}^{m} 1\{c_{i_1} = j\}}
\]

(11)

Where, \( k \) is the number of clusters; \( c_{i_1} \) is the class of all the points which are closest to the \( j \)-th centroid. The convergence of the mass center can be obtained through repeated iteration.

3.3. The branch probabilistic vulnerability based on Cumulant

When nodal injection power fluctuates to exceed the limited value, the running status of nodes would be unstable. The nodal injection power is the sum of generator injection power random variable \( \Delta W_g \), load injection power random variable \( \Delta W_l \) and wind turbine injection power random variable \( \Delta W_{\text{wind}} \) as follows

\[
 \Delta W = \Delta W_g + \Delta W_l + \Delta W_{\text{wind}}
\]

(12)
Supposing that each node power random variables is independent, the $k$th order cumulant $\Delta W^{(k)}$ of nodal injection power can be obtained from the additive and linear relationship of cumulant, which can be expressed as

$$\Delta W^{(k)} = \Delta W_g^{(k)} + \Delta W_f^{(k)} + \Delta W_{\text{wind}}^{(k)}$$  \hspace{1cm} (13)

Then, the $k$th order cumulant of branch injection power can be obtained as

$$\Delta Z^{(k)} = T_0^{(k)} \times \Delta W^{(k)}$$  \hspace{1cm} (14)

Where, $T_0^{(k)}$ is the sensitivity matrix; $T_0^{(k)} = G_0 \times J_0^{-1}$ is the transformation matrix; $G_0 = \frac{\partial Z}{\partial X}|_{x_0}$; $X_0$ is the expectation of state variable at the reference operating point. The branch vulnerability is dependent on range of branch active and reactive power. Thus, based on branch active power fluctuation, the probabilistic vulnerability evaluation index is proposed as

$$V_{\text{ap}} = \left| \frac{\Delta P}{P_{\text{max}} - P_{\text{min}}} \right|$$  \hspace{1cm} (15)

The probabilistic vulnerability evaluation index based on branch reactive power fluctuation is proposed as

$$V_{\text{aq}} = \left| \frac{\Delta Q}{Q_{\text{max}} - Q_{\text{min}}} \right|$$  \hspace{1cm} (16)

Where, $\Delta P = P - P_0$ and $\Delta Q = Q - Q_0$ is the branch active and reactive power fluctuation, $P_{\text{max}}, P_{\text{min}}$ and $Q_{\text{max}}, Q_{\text{min}}$ are the maximum and minimum value of branch active power and reactive power fluctuation, respectively. The fluctuation range of active and reactive power is normalized in respective branch constraint, and then is combined to evaluate the branch vulnerability.

### 3.4. The node voltage probabilistic vulnerability based on Cumulant

The $k$th order cumulant of state variable $\Delta X^{(k)}$ can be obtained as

$$\Delta X^{(k)} = J_0^{(k)} \times \Delta W^{(k)}$$  \hspace{1cm} (17)

Where, $\Delta X^{(k)}$ is composed of the $k$th order cumulant of node voltage amplitude variation $\Delta U^{(k)}$ and angle variation $\Delta \theta^{(k)}$, the probability density function of node voltage can be obtained by $\Delta U^{(k)}$. In actual operation of the system, the node voltage fluctuates randomly within a certain range, which determines vulnerability of the actual system. Thus, the node probabilistic vulnerability evaluation index is proposed as

$$V_{\text{node}} = \max \left| \frac{\Delta U}{U_{\text{max}} - U_{\text{min}}} \right|$$  \hspace{1cm} (18)

Where, $\Delta U = U - U_0$ is the amplitude variation of node voltage; $U_{\text{max}}$ and $U_{\text{min}}$ are the maximum and minimum value of node voltage, respectively.

### 4. Simulation results

In this paper, wind farms are connected to standard IEEE-30 bus system via transformer and 110 kV line, as is shown in figure 1. The IEEE-30 bus system is shown in figure 2.

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**Figure 1.** IEEE30 bus system with wind farm

The wind farm consists of 100 double-fed wind turbines with a rated capacity of 0.6MW. The wind turbines of Wind farms are divided into two rows. The row spacing is 50 m, the wheel hub is 50-meter high. Assuming that the air density of wind farm is 1.2245 kg/m³, the swept area of wind turbine is 1840 m². The cut-in wind speed, rated wind speed and cut-out wind speed of wind turbine are 3 m/s, 13.5 m/s, 25 m/s respectively. The equivalent impedance per unit of line is 0.1041 +
j0.2063Ω, the stator resistance is 0.00453+j0.0507Ω, the rotor impedance is 0.00486+j0.1491Ω, and the excitation reactance is 2.2059Ω. The three parameters of the Weibull distribution of wind speed are given below: \( v_0 = 2 \); \( k = 2 \); \( c = 11 \).

Figure 2. Schematic diagram of IEEE30 bus system  Figure 3. PDF comparison of node voltage

4.1. The node vulnerability distribution

With wind farms linking to all nodes in the IEEE-30 bus system respectively, the voltage fluctuation of each node can be obtained. The reactive power compensation, voltage support capability and the distance to linking point of each node are different, which results in that the under-voltage, over-voltage and node voltage fluctuations aren’t identical. The probability density functions (PDF) of partial nodes voltage are shown in figure 3 with the wind farm connected and unconnected to node 29.

As shown in figure 3, node 4 and 14 are the farther nodes away from wind farm. The voltage fluctuation range of node 29 become larger when the wind farm is connected to the system. Moreover, the voltage probability density curve shift to the smaller voltage. As for node 4, the voltage probability density is essentially constant, but the fluctuation range of voltage is the largest.

The K-means cluster is applied to analyze the node probabilistic vulnerability with the wind farm connected to different nodes. Partial results are shown in figure 3.

Figure 4. The K-means cluster analysis of vulnerable nodes. (a) Without wind farm. (b) Wind farms link on node 4. (c) Wind farms link on node 30

As shown in figure 4, the nodes are divided into 3 categories by K-means clustering for different vulnerability index. There’s only two nodes in the first category while most of the nodes are concentrated in the second and the third category. Due to limited space, this paper only presents a few representative link points in the grid. In addition, the third category is the remaining nodes of the first and second categories. The results are shown in table 1.
Table 1. The classifications of node vulnerability at different link point base on $V_{Node}$.

| Wind power link point | 1st category | 2nd category |
|-----------------------|--------------|--------------|
| Note 3                | 3, 4         | 6, 12, 14, 15, 18, 19, 23, 24, 26, 29, 30 |
| Note 15               | 3, 4         | 6, 12, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 26, 27, 29, 30 |
| Note 30               | 3, 4         | 24, 25, 26, 27, 29, 30 |
| No link               | 3, 4         | 6, 10, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30 |

The results show that different link points would cause different impact on the vulnerability of each node. The K-means clustering of vulnerable nodes and the sorting of vulnerability are diverse. However, node 3 or 4 is always the most fragile node and belongs to the first category. Furthermore, the sorting of second category nodes are relatively stable, the vulnerability of node 6 or 12 or 14 or 15 is also strong and top-ranking.

From Table 1, the most nodes show a stronger vulnerability when wind farms connected to node 3 or 15, for the node 1 or 2 or 5 or 8 or 11 or 13 is generator node, which undertake the task of injecting power into the system. When wind farms are linked on the nodes close to generator, the transmission tasks in that region increase obviously which cause a burden to the generator nodes and the neighbouring nodes. When wind farms are linked on nod 29 or 30, the impact on grid is relatively weaker, and the vulnerability of nodes are very small, for node 29 or 30 lies in the end of the line. When the wind farm is linked on these nodes with random disturbances, there is nearly no impact on the grid. Therefore, it can be seen that the impact on the vulnerability of the nodes in the end of the line is smaller when wind farm is linked.

When the wind farm is linked on the power system, the node voltage fluctuation increase. As a result, the node vulnerability becomes stronger. In addition, the closer to the wind farm, the stronger the vulnerability of the node.

4.2. The branch vulnerability distribution

The probability density functions (PDF) of active power and reactive power of some branches with the wind farms connected or unconnected to the node 30 are shown in figure 5.

![Figure 5. PDF comparison of active and reactive power flow. (a) active power flow of branch 2-4, (b) reactive power flow of branch 2-4](image)

As shown in figure 5, compared to the system unconnected to the wind farm, branch active and reactive power have a fluctuation range. Moreover, power probability density curve shift to the direction of smaller power. $V_{BP}$ and $V_{BQ}$ can be obtained by the PDF of branch active and reactive power, and then analyzed with K-means clustering, partial results are shown in figure 6.
shown in vulnerability of active power and reactive power of different branches. The closer to the link point of conclusion can be drawn: no matter what kind of classifications index is employed, the branches 4-6 strongly vulnerable region.

By combining these two index, the common vulnerability branches of 4-6, 4-12, 3-4, category of vulnerable nodes and branches, which are classified by K-means clustering, constitute a generator node. (2) The branch connected to a node with strong vulnerability, is also strongly degree of influence is related to the distance to the wind power link point or the distance from wind farm, the greater the branch power is. In addition, the greater the branch is affected, the greater the branch vulnerability.

As shown in figure 6, the branches are also divided into 3 categories, most of the branches lie in the third category while least of the branches lie in the first category. The first and second category are shown in Table 2.

Table 2. The classifications of branch vulnerability at different link point base on $V_{BP}$ and $V_{BQ}$.

| Wind power link point | 1st category (by $V_{BP}$) | 2nd category (by $V_{BP}$) | 1st category (by $V_{BQ}$) | 2nd category (by $V_{BQ}$) |
|-----------------------|----------------------------|----------------------------|-----------------------------|----------------------------|
| Note 3                | 3-4, 4-6                   | 1-2, 1-3, 6-7, 4-12        | 4-6, 4-12                   | 1-2, 3-4, 6-9, 9-11, 12-13, 12-15, 12-21 |
| Note 15               | 4-6                       | 1-2, 1-3, 3-4, 6-7, 4-12   | 4-12                       | 1-2, 3-4, 4-6, 6-8, 9-11, 12-13, 12-15, 10-21 |
| Note 29               | 3-4, 4-6, 4-12             | 1-2, 1-3, 2-4, 6-7, 8-28   | 4-12                       | 1-2, 3-4, 4-6, 9-11, 12-13, 12-15, 10-21, 22-24, 28-27, 37-39, 30-39, 8-28, 6-28 |
| No link               | 3-4, 4-6, 4-12             | 1-3, 2-4, 2-6, 6-7, 6-8, 6-9 | 4-6, 4-12                   | 1-2, 6-8, 6-9, 9-11, 12-13, 12-15, 10-21 |

From the comparison of the classifications and ranking of vulnerability index of each branches, this conclusion can be drawn: no matter what kind of classifications index is employed, the branches 4-6 and 4-12 are always the most vulnerable region. Because node 4 or 6 is close to the transformer node, which undertake the tasks of injecting power. The flow and operation state of power system change greatly due to wind farms linking. The greater change of power leads to an obvious increase in the vulnerability.

According to the above analysis, the link of wind farm has a different influence on the probabilistic vulnerability of active power and reactive power of different branches. The closer to the link point of wind farm, the greater the branch power flow is. In addition, the greater the branch power is affected, the greater the branch vulnerability.

5. Conclusion

In this paper, cumulant and Gram-Charlier expansion theory is used to analyze the probabilistic vulnerability of nodes and branches after wind farms link. The K-means clustering analysis is applied to obtain the vulnerable region of nodes and branches. The evaluation accuracy of node and branch vulnerability is greatly improved. The change of system vulnerability has been analyzed and compared in the case of different link points, the conclusion can be drawn as:

(1) The fluctuation of nodes and branches can be exacerbated due to the random probability of wind turbine generation. The vulnerability of nodes and branches is impacted by the wind farm, and the degree of influence is related to the distance to the wind power link point or the distance from generator node. (2) The branch connected to a node with strong vulnerability, is also strongly vulnerable. Vulnerable nodes and branches constitute a strong vulnerable region. The first and second category of vulnerable nodes and branches, which are classified by K-means clustering, constitute a strong vulnerable region.
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