The Optimal Investment Strategy of P2G Based on Real Option Theory

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This work was supported in part by the China Scholarship Council and British Council through the Newton Fund, and in part by the Electric Power Research Institute, Yunnan Power Grid Co., Ltd., under Grant 0562002018030301GZ00004.

ABSTRACT This paper initially proposes a real options model for the investment of the Power-to-gas (P2G) plant based on uncertain operating cost which mainly refers to the price of electricity. Through the analysis of the uncertainty parameters affecting the operation of the P2G project, the mathematical model expressing the relations between the parameters of P2G operation cost, electricity price, sunk cost, and other parameters are established. The Brownian motion is utilized to describe the operation cost, based on which, the option value and the project value models of P2G are derived in detail. According to these two models, the optimal investment timing of the P2G device and the corresponding optimal investment capacity can be determined. The above models are verified by numerical simulation. In addition, the influence of the change of parameters on the investment timing and investment capacity in the real options model is studied. The results show that the volatility of electricity price has a greater impact on the option value of P2G project than that of other parameters. When there is a high operating cost uncertainty, waiting is a better option, and the investment can be performed when the operating cost falls to the cost with reference to the optimal investment timing.

INDEX TERMS Power-to-gas, real option, investment opportunity, investment capacity, operation cost uncertainty.

I. INTRODUCTION

Energy storage device can effectively solve the unbalance matching between source and load at the aspect of the time and geographic location in power system. P2G device which can convert redundant power into hydrogen or natural gas, compared with batteries, compressed-air, flywheel and capacitors, has the characteristics of long-term energy storage [1]. The power system is facing the severe problem of volatility and intermittency especially with a high penetration of renewable energy in the future. Germany, for example, will generate 80% of its electricity from renewable sources by 2050. The UK aims to generate 30% of its energy from renewable sources by 2020 and reduce its greenhouse gas emissions by 80% by 2050 compared with 1990 [2]. While alleviating the pollution caused by energy use, the safety and stability of power grid with its volatility and uncertainty is threatened by renewable energies [3]. In addition, there is a situation that renewable energy has low load demand in its peak output period, and the concomitant problems of wind, solar and water curtailment need to be solved urgently [4]. On the other hand, many industries need a lot of hydrogen, such as petroleum refining, metal smelting, food processing and other industries [5]. Natural gas needs to be imported in large quantities in many countries, such as Mexico, Korea, Japan, China, and India [6]. The redundant renewable energy can be transformed into hydrogen or natural gas by applying P2G technology for the above industries, which can effectively improve the utilization rate of renewable energy, and solve the pollution problem caused by energy use from the origin.

The operation economy of P2G has been discussed in many literatures. Before the advent of P2G technology, power grids and gas networks only relied on gas units as energy conversion devices, and the energy flow was one-way. After the emergence of P2G, the energy transmission between power and gas networks becomes bidirectional, thus increasing resources available to power network and gas network. Literature [7] establishes the heating value model of mixed hydrogen and natural gas, and studies the impact that
injecting hydrogen into the natural gas pipelines on the wind
abandoning, gas sources, gas pressure and carbon dioxide
emission reduction of the power grid; Literature [8] proposes
a robust optimization model considering the coordinated
operation of P2G devices to coordinate the optimal operation
of power and natural gas systems with uncertainties; Litera-
ture [9] establishes a scenario-based operation optimization
model to analyze the micro-integrated power, natural gas
and heating system with the consideration of the P2G and carbon
capture technologies; Literature [10] analyzes the optimal
Day-Ahead scheduling of power grid and gas network on
the background of the transformation of redundant electricity
power into natural gas for direct sales and combining the price
information of power and natural gas market; Literature [11]
studies the improvement of renewable energy utilization and
carbon dioxide emission reduction effect of the system based
on the carbon dioxide recycling system composed of gas
turbine and electric gas; Literature [12] introduces the syn-
ergistic optimization of P2G device and fuel cells.

Some literatures have also studied the planning of the
power network and gas network including the P2G device.
A robust optimization programming model with uncertainty
is proposed for long-term cooperation between power and gas
systems in literature [13]. Based on this model, generators,
transmission lines, gas transfer devices, natural gas suppliers,
pipelines and compressor stations are simultaneously opti-
mized in this work. In literature [14], electrolyzers as an
effective means is taken to reduce the uncertainty of power
grid dispatching. The control strategy of ensuring the mini-
mum number of gas turbines is implemented in the running
state under a given total output, and the optimal installation
capacity of electrolyzers is determined.

Planning is the basis of operation, while it also needs to
analyze the economy of investment. The above literatures
mainly studied on the operation and planning of the electricity
and gas networks with the P2G device, and few literatures
investigated the investment economy of P2G. Literature [15]
is one of the few to analyze the investment economy of P2G.
The electricity is converted into hydrogen via electrolyzers in
the P2G system. That work analyzes two scenarios, the hydro-
gen is converted back to power system and is directly sold.
Based on the above scenarios, the economy of investing in
electrolyzers as a wind farm energy storage system is ana-
yzed. Further, Brownian motion is used to model the value
of energy storage configuration system and a mathematical
model between the benefits of energy storage system and vari-
ous uncertain factors is established. Monte carlo method
is used for simulation, discounted cash flow is calculated and
taken as the premise of necessity of the real option analysis,
and the sensitivity of related uncertain parameters is simu-
lated to analyze its impact on option value. The analytical
perspective is novel and the yield analysis of hydrogen is
more comprehensive, that is, the hydrogen gains from various
possible approaches are synthesized. However, it does not
analyze the optimal investment capacity of hydrogen, and
its hydrogen capacity is artificially set at 50MW. It can be
expected that the option value of the investment project will
vary with the investment capacity. In addition, the value of
the project is based on the benefits of the hydrogen storage
system. The profit-based option investment model does not
have a clear trace of the relationship among the operating
and maintenance costs of hydrogen energy storage, the price,
the capacity, etc., so it cannot conduct in-depth analysis of the
basic variables.

The real option theory applied in literature [15] has been
applied in various aspects of investment in power system:
In literature [16], the application of the path-dependent sce-
nario tree technology in the real option theory is researched
to evaluate wind power projects by solving the uncertainty
of wind resource assessment, more accurate wind resource
estimation is expected to enhance the value of the wind power
investment. Based on the assumption that wind speed fol-

Literature [17] applies real options to the carbon market
to estimate the large incremental return of grid-connected
renewable energy in the carbon market, namely the approved
emission reduction-CER. The writing background of his the-
thesis is based on whether the registration of clean development
mechanism in Brazil has the value of investment. Incremen-
tal returns need to determine the carbon emission reduction
under the baseline scenario, and the baseline measurement
standard is the Brazilian document ACM0002. The present
value of the proceeds from the sale of CER is considered as
the value of the option, and the cost of registering CER with
the United Nations commission for grid-connected power
generation projects and the additional fees involved are con-
considered as the strike price. In addition, the operation period
of the project is regarded as the expiration date of the option.
The price fluctuation of CER sold is taken as the volatility of
the option, and the binomial option model is applied to determine
the investment value of the CER project.

Literature [18] regards the portfolio contract of demand
response as an investable project, evaluates the value of the
demand response contract by applying the real option model,
and takes the load aggregator and demand response user as
both parties of the contract. Its essence is the power market
trading based on the spot price of power and considering the
option contract. Load aggregators of demand response (DR)
can participate in the balancing market, use DR contracts
to balance energy portfolios, reduce unbalanced costs, and
thus hedge risks related to real-time price fluctuations. When
the spot price exceeds the strike price, the option holder
will exercise the option contract to purchase power from the
demand response user at a lower strike price, thereby reducing
the user load in the short term, and when the spot price is
less than the strike price, the option will not be exercised and
the option holder will purchase the power at the spot price.
In addition, the situation that the aggregator’s participation
in the day-ahead energy market as an energy retailer is also analyzed.

In literature [19], the option model is applied to determine the investment proportion of wind power and small hydropower, and the timing of investment is also determined. The optimization model in this paper is a two-layer model of Max-Min. The upper model is the investment option model of renewable energy projects considering the risks of energy purchase and sale, while the lower model is the purchase and sale costs of energy contracts in the worst case. The two-layer model is solved by robust optimization. The core idea of the work is to make portfolio project decisions based on the revenue level of energy contracts. Its innovation is reflected in the application of option model on the field of energy, which extends the application scenarios of investment decisions, namely, it is applied to the energy contracts of energy companies, and robust optimization is considered.

In literature [20], the optimal investment timing in irreversible engineering in which investment income and investment cost following continuous time stochastic process is studied, and the optimal investment rules and the formula of investment option value are derived. These formulas can be used to accurately calculate the investment time rules of a project and the loss value of investment in sub-optimal time.

With the increased confidence of countries in the application of P2G equipment, many countries have launched demonstration projects for P2G devices, and there is an urgent need to conduct research on the investment strategies of P2G project. Many countries in the world have built demonstration plants for P2G, including Germany, the United Kingdom, the United States, Spain, Canada, France, Denmark and so on. The power sources of P2G can be public power grid [21], photovoltaic power generation [22], [23], wind power [24], [25], hydropower [26], [27], etc.

The investment decision is of great importance on account of the irreversibility of the investment in the P2G equipment. Once invested, there will be sunk costs because of the specificity of the application of the P2G device. The P2G plants convert power into hydrogen, which has high conversion efficiency and economy compared to other forms. To this end, this paper mainly studies the investment optimization of electrolyzer plants. The most important factor affecting the economics of hydrogen investment via electrolyzers is the price of electricity. The price of electricity has a strong volatility, and it is the cost of hydrogen production. Therefore, the operating cost of hydrogen investment is uncertain. In the face of increasing uncertainty, real options are particularly suitable for investment analysis of P2G projects because a relatively complete analytical framework for uncertain environments can be provided. However, the uncertainty parameter of the investment model of traditional real options is generally the price of the output of the investment device. Under this circumstance, the idea of real option investment modeling in literature [28] is applied in this work and the model is also rewritten and expanded to fit the investment of P2G. An option investment model of the electrolyzer device based on the uncertainty of operating cost is established, and the optimal investment timing and optimal investment capacity of the device are determined in this work. The main contributions of this paper are as follows:

1. Instead of output price volatility consideration in traditional models, a real option investment model with production cost uncertainty is established, which is right fit the case of P2G.
2. The proposed real option investment model is capable of determining the optimal investment timing and the associated optimal investment capacity simultaneously.
3. In case study, sensitivity analysis is conducted with reference to different electricity price fluctuation rates, different investment costs, and prices of different output products. Furthermore, with minor adjustment, the application of the model can be expanded to other physical assets with high uncertainty in production cost and relatively stable output price.

The remainder of this paper is structured as follows. In section II, the operation cost model of the P2G and, based on the cost uncertainty, the option value and the project value are proposed so as to calculate the optimal investment opportunity and the optimal investment capacity of the project. In Section III, the solving steps of the mathematical model proposed in the second part are described. Numerical Simulation are introduced in Section IV to demonstrate the feasibility and effectiveness of the proposed real option mathematical model of the P2G project. Finally, a brief conclusion is given in Section V.

II. OPTION VALUE MODELING FOR P2G PLANT INVESTMENT

Real options can be used to adjust the project’s investment decisions of the larger uncertainties currently facing by modeling the investment flexibility.

The P2G project investment has greater uncertainty since it takes the electricity price as a cost. However, traditional cash flow (NPV) investment methods are difficult to flexibly adjust to unforeseen market changes and correct subsequent decisions. For example, after starting an investment project, it will always operate according to the basic scale until the end of the project life. While the real option (ROA) theory can be used to model the uncertainty, and with the advent of new information, the relevant uncertainty gradually becomes clear. The real option theory can be utilized to change the initial business strategy and adapt to the changing market. The decision-makers can invest in opportunities that will be profitable in the future, and can delay, expand, shrink, abandon or even change projects at various stages of the project’s business cycle [29]. It shows the difference between the NPV and ROA methods in Figure 1. The ROA method allows waiting for uncertain information about the project investment. The project is only invested when the uncertain information disappears and the investment environment shows a good signal. Thereby, the volatility $\sigma$ of the uncertain information is decreased, and the expected value $\mu$ of the
project is also increased. NPV method has lower returns and higher risks to an investment project, relatively speaking, ROA can achieve a win-win situation that avoids risks and gets higher returns.

The decision maker of the P2G project can choose to invest immediately or postpone the investment when the right to build the plant is obtained. When a P2G plant has been set up, the decision maker needs to make a decision on its operation in response to the changing power market: put it into operation immediately or wait for the operation opportunity. It needs to weigh the benefits and risks to make a choice between the two strategies. Postponing the investment is equivalent to holding an unlimited call option, but policymakers face uncertain risks from electricity prices, hydrogen prices and so on. The historical price of hydrogen is relatively stable. Thus, the price of hydrogen is set at a constant value in this paper. It is assumed that the option to invest in the P2G project has been decided to be implemented. The project can be temporarily shut down costlessly if the sales price of hydrogen falls below the operating cost during production, and it can be restarted later without any cost if the sales price of hydrogen rises above the operating cost thereafter. Based on the above situation, the investment decisions for electrolyzer plants is made in this paper. It is necessary to determine the investment strategy, including the investment capacity of the device and the timing of investment.

A. ABBREVIATIONS BROWNIAN MOTION MODEL OF P2G PROJECT OPERATION COST

In the power market environment, the power required for P2G operation is completely purchased from the electricity market. At this time, the mathematical model of $E(t)$ is described in equation (1).

$$E(t) = \frac{P_I}{\eta_{P2G}(t)} C_M(t) + C_F$$  \hspace{1cm} (1)

where $C_M$ is the market price of the electricity, $P_I$ is the electricity energy consumption in theory of 1kg of hydrogen generated by the P2G device, $\eta_{P2G}$ is the investment efficiency of P2G plant, $C_F$ refers to other costs required by the operation of the P2G device.

Mean reversion model can be used for power price modeling, but geometric Brownian motion modeling will not produce large errors in the results [30]. In particular, the Brownian motion model can be used to obtain satisfactory results compared with the model having slow mean recovery. For simplicity, electricity price $C_M(t)$ follows geometric Brownian motion. The model of Brownian motion of $C_M(t)$ is as follows:

$$dC_M(t) = \alpha_C C_M(t) dt + \sigma_C C_M(t) dz(t)$$  \hspace{1cm} (2)

where, $\alpha_C$ is the expected relative drift of $C_M(t) per unit of time, $\sigma_C$ is the standard deviation per unit of time.

For the convenience of analysis, let $C_F = 0$, when the electricity price $C_M(t)$ follows the Brownian motion, $E(t)$ also follows the Brownian motion with the same drift rate and volatility. The following is the derivation process.

Using Ito’s Lemma according equation (1), expand $dE$ in equation (3):

$$dE = \left( \frac{\partial E}{\partial C_M} \alpha_C C_M + \frac{\partial E}{\partial t} + \frac{\partial^2 E}{\partial C^2_M} \sigma_C^2 C_M^2 \right) dt + \frac{\partial E}{\partial C_M} \sigma_C C_M dz$$  \hspace{1cm} (3)

The equation (4) can be derived after inserting $dE/\partial C_M = P_I/\eta_{P2G}, dE/\partial t = 0, \partial^2 E/\partial C^2_M = 0$ to equation (3):

$$dE = \frac{P_I}{\eta_{P2G}} C_M(t) \alpha_C dt + \frac{P_I}{\eta_{P2G}} C_M(t) \sigma_C dz$$  \hspace{1cm} (4)

$dE$ can be written as follows:

$$dE = \alpha_E E(t) dt + \sigma_E E(t) dz$$  \hspace{1cm} (5)

Then, the equation (6) can be derived:

$$\begin{aligned} \alpha_E &= \alpha_C \\ \sigma_E &= \sigma_C \end{aligned}$$  \hspace{1cm} (6)

The model of Brownian motion of $E(t)$ is as follows:

$$dE(t) = \alpha_E E(t) dt + \sigma_E E(t) dZ(t)$$  \hspace{1cm} (7)

where, $\alpha_E$ is the expected relative drift of $E(t)$ per unit of time, $\sigma_E$ is the standard deviation per unit of time. And $dZ(t)$ is the increment of Wiener process. $dZ(t) = \varepsilon(t) \sqrt{dt}$, $\varepsilon(t) \sim N(0, 1)$, $E(0) = E_0$.

Since $dE(t)/E(t)$ follows normally distribution, thus $dE(t)$ follows lognormally normally distributed. $G = lnE, \partial G/\partial t = 0, \partial G/\partial E = 1/E, \partial^2 G/\partial E^2 = -1/E^2$.

The mathematical model of income and profit of P2G output is as follows.

$$Y(t) = R - E(t)$$  \hspace{1cm} (8)

$$R = R_p \times W_{P2G}$$  \hspace{1cm} (9)

$Y(t)$ is the profit of the investment of P2G, and $R$ is the overall income of the production. $R_p$ is the unit price of the production. $W_{P2G}$ is the installation capacity of P2G device.

The mathematical expression of the investment of P2G is as follows.

$$Q(t) = \eta(t) W_{P2G}$$  \hspace{1cm} (10)
where $Q(t)$ is the annual output of P2G device, $N(t)$ is its utilization function, and it is assumed to be a constant here.

The capacity cost model of P2G plant adopts the exponential function model in literature [31]:

$$C_{P2G} = K_{P2G}e^{n_1W_{P2G}}$$

(11)

$C_{P2G}$ is the investment cost when the capacity of the P2G device is equal to $W_{P2G}$. $n_1$ is the exponential constant of the power function, and $K_{P2G}$ is the constant coefficient.

**B. ATHEMATIC MODELING OF OPTIMAL INVESTMENT TIMING BASED ON ROA-INVESTMENT PROJECT VALUE $V(E)$**

The decision maker who invests in the P2G device holds an option similar to a financial call option, which gives the right, but not the obligation, to invest in the device at a time of its choosing. When a company decides to invest in a P2G device, the investing entity executes the option but at the cost of abandoning the option, that is, the possibility of waiting for new information that may affect the willingness or timing of the investment is given up. The lost option value is the opportunity cost of investment, which must be included in the investment cost [32].

The value of P2G project is $V(E)$. It can determine the expected rate of return by constructing a risk-free portfolio and equating the expected rate of return to the risk-free interest rate.

Next an investment portfolio can be built. Let the option of holding a project with a value of $V(E)$ obtain a short position $n = V'(E)$. The dynamic investment value $\Phi$ of the P2G device can be expressed as follows.

$$\Phi = V(E) - V'(E)E$$

(12)

where $\pi(E)$ is the profit from an investment project during the $dt$ period.

The short-term portfolio’s return required during the $dt$ period is $(r - \alpha_E)EV'(E)$, which is required to be paid by the short position in the portfolio, so the total return on holding the portfolio during the $dt$ period is described as follows.

$$\varphi = \pi(E) + dV(E) - V'(E)dE - (r - \alpha_E)EV'(E)dt$$

(13)

The following equation can be obtained from Ito’s lemma.

$$dV(E) = V'(E)dE + \frac{1}{2}V''(E)(dE)^2$$

(14)

Equation (15) can be obtained from equation (13) and equation(14).

$$\varphi = \pi(E) + \frac{1}{2}V''(E)(dE)^2 - (r - \alpha_E)EV'(E)dt$$

(15)

Since $E(t)$ follows Brownian motion, equation (16) can be obtained from (7).

$$(dE)^2 = \sigma_E^2E^2dt$$

(16)

Equation (17) can be obtained from equation (15) and equation (16).

$$\varphi = \pi(E) + \frac{1}{2}\sigma_E^2E^2V''(E)dt - (r - \alpha_E)EV'(E)dt$$

(17)

According to the no arbitrage principle, equation (18) can be derived.

$$\varphi = r\Phi dt$$

(18)

Equation (19) can be obtained by solving the simultaneous equations (12), (17) and (18).

$$\frac{1}{2}\sigma_E^2E^2V''(E)dt + \alpha_EEV'(E) - rV(E) + \pi(E) = 0$$

(19)

The solution form of the Equation (19) is as follows.

$$V(E) = A_1E^{\beta_1} + A_2E^{\beta_2} + M_C$$

(20)

$M_C$ is a constant term here. After simple transformation, it can be known that the special solution of equation (19) is $(-\frac{E}{r - \alpha_E} + \frac{E}{\sigma_E^2})$, and the equation has the following form:

$$V(E) = A_1E^{\beta_1} + A_2E^{\beta_2} - \frac{E}{r - \alpha_E} + \frac{R}{r}$$

(21)

where $A_1$, $A_2$ and $\beta_1$, $\beta_2$ are undetermined coefficients. The first two items of equation (21) can be seen as the option value, while the latter two items can be seen as the cash flow of the investment project.

The solution process of equation (19) is divided into two steps, namely, the situations of $E > R$ and $E < R$:

1) When $E > R$, $\pi(E) = 0$, that is, the operation of the project is postponed in this case, and the project does not have any profit flow.

The equation at this time is a homogeneous differential equation with the following form:

$$V(E) = K_1E^{\beta_1} + K_2E^{\beta_2}$$

(22)

where, $K_1$ and $K_2$ are undetermined coefficients, and $\beta_1$ and $\beta_2$ are solutions of the following equation:

$$\frac{1}{2}\sigma_E^2E^2(\beta_1^2 + \frac{1}{2}\sigma_E^2)\beta_2 - r = 0$$

(23)

Solving Equation (23) can get the following solutions.

$$\beta_1 = \frac{1}{2} - \frac{\alpha_E}{\sigma_E^2} + \sqrt{\frac{\alpha_E}{\sigma_E^2} - \frac{1}{2} - \frac{2r}{\sigma_E^2}}$$

(24)

$$\beta_2 = \frac{1}{2} - \frac{\alpha_E}{\sigma_E^2} - \sqrt{\frac{\alpha_E}{\sigma_E^2} - \frac{1}{2} - \frac{2r}{\sigma_E^2}}$$

(25)

Since $\beta_1 > 1$, $E^{\beta_1} \rightarrow \infty$ when the operation cost $E \rightarrow \infty$, $E^{\beta_2} \rightarrow \infty$. So multiplied by this item equal to zero. This constant term is represented by the symbol $K_1$. The value of the project can be described as follows.

$$V_1(E) = K_2E^{\beta_2}$$

(26)
2) When \( E < R, \pi (E) \neq 0 \). Therefore, equation (19) has the following form of solution.

\[
V(E) = B_1 E^{\beta_1} + B_2 E^{\beta_2} - \frac{E}{r - \alpha E} + \frac{R}{r} \tag{27}
\]

where, \( B_1 \) and \( B_2 \) are undetermined coefficients.

Since \( \beta_2 < 0 \), the value of the option cannot be increased indefinitely with the extension of time. Therefore, when \( E \rightarrow 0, R^{\beta_2} \) has no economic significance. The solution form in this case is as follows.

\[
V_1(E) = B_1 E^{\beta_1} - \frac{E}{r - \alpha E} + \frac{R}{r} \tag{28}
\]

where \( B_1 \) and \( \beta_1 \) are undetermined coefficients.

3) Determine the undetermined coefficients \( K_2 \) and \( B_1 \) in equations (26) and (28) at point \( E = R \):

The solution to \( V(R) \) must be a continuously differentiable value, so the following equation can be derived.

\[
\begin{align*}
V_1(0) &= V_1'(0) = V_1''(0) = V_1'''(0) \\
K_2 &= \frac{R^{1-\beta_2}}{\beta_2_1 - \beta_1} (\beta_1 - \beta_1 - 1) \\
B_1 &= \frac{R^{1-\beta_1}}{\beta_1 - \beta_2} (\beta_2 - \beta_2 - 1)
\end{align*}
\]

1) MATHEMATICAL MODELING OF OPTIMAL INVESTMENT CAPACITY BASED ON ROA–OPTION VALUE F(E)

The net present value of the investment is shown in equation (32).

\[
NPV = V(W_{P2G}\vert E_*) - C_{P2G}(W_{P2G}) \tag{32}
\]

When investing in the P2G device, the maximum net present value can be used to find the optimal investment capacity. The schematic diagram of the principle is shown in FIGURE 2.

\[
\frac{\partial V}{\partial W_{P2G}} = \frac{\partial C_{P2G}}{\partial W_{P2G}} \tag{33}
\]

The project will be invested only when \( E < R \). Equation (34) can be obtained from equation (9), (11) and (28).

\[
B_1 \beta_1 E_0 E^{\beta_1-1} W_{P2G}^{r} - \frac{E_u}{r - \alpha E} + \frac{R_u}{r} = K_2 P_{P2G} e^{\eta_1 W_{P2G}} \tag{34}
\]

It can be seen from the above equations that the optimal investment capacity \( W_{P2G}^* \) is a function of the operating cost \( E_u \). Although \( W_{P2G}^* \) cannot write out an analytical expression about \( E_u \), the numerical solution of the optimal investment capacity \( W_{P2G}^* \) can be derived if \( E_u \) is given.

\[
Y_1(W_{P2G}\vert E_u) = B_1 \beta_1 E_0 E^{\beta_1-1} W_{P2G}^{r} - \frac{E_u}{r - \alpha E} + \frac{R_u}{r} = K_2 P_{P2G} e^{\eta_1 W_{P2G}} \tag{35}
\]

The expression of the optimal investment capacity \( W_{P2G}^* \) is as follows.

\[
W_{P2G}^* = Arg\{Y_1(W_{P2G}\vert E_u) = 0\} \tag{36}
\]

The investment cost under the optimal investment capacity is described in equation (37).

\[
C_{P2G} = K_2 P_{P2G} e^{\eta_1 W_{P2G}} \tag{37}
\]

The operating cost \( E \) of P2G follows the geometric Brownian motion, so the option value of its investment satisfies the following form:

\[
F(E) = M_1 E^{\beta_1} + M_2 E^{\beta_2} \tag{38}
\]

\( F(E) \) should satisfy the following three constraints:

\[
\begin{align*}
F(\infty) &= 0 \\
F(E_*) &= V(E_*) - C_{P2G}(E_*) \\
F_y(E_*) &= \frac{\partial V(E_*, Q_*)}{\partial E}
\end{align*}
\]

In order to satisfy \( F(E) = 0 \) when \( E \rightarrow \infty \), it is necessary to make \( M_1 = 0 \), and the following equations are obtained during the operation period \( E < R \).

\[
M_2(E_*)^{\beta_2} = B_1(E_*)^{\beta_1} - \frac{E_*}{r - \alpha E} + \frac{R}{r} - C_{P2G} \tag{40}
\]

\[
\beta_2 M_2(E_*)^{\beta_2-1} = \beta_1 B_1(E_*)^{\beta_1-1} - \frac{1}{r - \alpha E} \tag{41}
\]

Equation (42) is obtained from (40) and (41).

\[
B_1 (\beta_2 - \beta_1) E_0 W_{P2G}^{r} + \frac{(1 - \beta_2) E_0 W_{P2G}^{r}}{r - \alpha E} + \frac{R}{r - \alpha E} = 0 \tag{42}
\]

It can be seen from the above equation that the value of \( E_* \) of the optimal investment timing can be obtained if the optimal investment capacity \( W_{P2G}^* \) is given. Further, \( E_* \) has
no analytical expression, but the numerical solution can be obtained.

\[ Y_2(E_u^*|W_{P2G}^*) = B_1(\beta_2 - \beta_1)(E_u^* W_{P2G}^*)^{\beta_1} + \frac{(1 - \beta_2)E_u^* W_{P2G}^*}{r - \alpha_E} + \beta_2 \left( \frac{R}{r} - C_{P2G} \right) \]  

(43)

Then, the optimal investment rule is described in equation (44).

\[ E_u^* = \text{Arg}\{Y_2(E_u^*|W_{P2G}^*) = 0\} \]  

(44)

Equation (44) does not have an analytical solution, but there is a numerical solution.

Therefore, the optimal investment capacity \( W_{P2G}^* \) and the \( E_u^* \) value of the optimal investment timing can be derived from solving the equations (34) and (42).

\[ M_2 = (E^*)^{-\beta_2}(B_1(E^*)^{\beta_1}) - \frac{E^*}{r - \alpha_E} + \frac{R}{r} - C_{P2G} \]  

(45)

\[ F(E) \] can be derived by substituting (25) and (45) into equation (38).

III. THE SOLUTION METHOD OF THE PROPOSED REAL OPTIONS MODEL OF P2G PROJECT

The operating cost sequence \( E(t) \) of the P2G device is affected by the investment capacity \( W_{P2G} \), and the investment capacity \( W_{P2G} \) is determined according to the drift rate \( \alpha_E \) and the volatility \( \sigma_E \) of the running cost sequence. In the case that the power energy used by the P2G device is completely obtained from the power market, the optimal timing and capacity of the investment and operation of the P2G project are determined using the above option model. The drift rate \( \alpha_E \) and the volatility \( \sigma_E \) can be obtained by equation (6).

The Brownian motion is used to model the electricity price, and the drift rate \( \alpha_E \) and the volatility \( \sigma_E \) can be calculated by equations (46) and (47):

\[ Y_3 = \alpha_E = \frac{1}{n} \sum_{i=1}^{n} [(E_{i+1} - E_i)/E_i] \]  

(46)

\[ Y_4 = \sigma_E = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} [(E_{i+1} - E_i)/E_i - \frac{1}{n} \sum_{i=1}^{n} ((E_{i+1} - E_i)/E_i)]^2} \]  

(47)

The solving steps are summarized in FIGURE 3. The solution steps of this mode are as follows:

Applying the above mathematical model can get the value of \( V(E) \) and \( F(E) \). The \( E(t) \) value of each moment can be obtained according to the formula (1). \( E(t) \) sequence can be described by Brownian motion. Through the analysis of massive data of electricity price, drift rate \( \alpha_E \) and the volatility \( \sigma_E \) can be obtained through formula (6). Based on the historical data of sunk cost of P2G investment and the above mathematical model, relevant parameters are obtained, such as \( K_{P2G} \) and \( n_1 \). In combination with other parameters \( r \) and hydrogen price \( R_u \) etc., the optimal investment capacity and timing can be derived according to the derivation of the above mathematical models.

IV. NUMERICAL SIMULATION

This section takes the solution method and the solution process given in Part III to the application and analysis of the real option mathematical model of the P2G project established in Part II.

The energy conversion relationship between the power energy and the hydrogen in P2G is generally 48 ~ 60 kWh/kg, which is 54 kWh/kg in the calculation, and the theoretical value is 39 kWh/kg. In actual operation, the operation efficiency of electrolyzers is generally between 67% and 76%, the efficiency is 72% used in this work, the operating cost of hydrogen is generally between 2.7 and 3.5 $/kg in general, and the annual operating hours of P2G is 6000 [33]–[37]. Based on the data set above, the proposed mathematical model can be used to analyze the investment of the P2G project. It can be regressed of the historical data of the investment cost of the P2G, and the result is obtained, \( K_{P2G} = 51.77, n_1 = 0.02084 \). Assuming that the fitting functional relationship between the investment capacity and the cost is accurate, using the real option model established above, the operating options of the P2G project are analyzed as follows.

A. APPLYING THE REAL OPTION MODEL TO DERIVE THE OPTIMAL INVESTMENT TIMING AND OPTIMAL INVESTMENT CAPACITY OF THE P2G PROJECT

The flow chart shown in FIGURE 3 can be used to solve the problem. The parameters in the real option model are as follows: \( r = 0.04, \alpha_E = 0, \sigma_E = 0.2, h = 6000, R_u = 5\epsilon/kg \). In FIGURE 4, the black line is the option...
value \( F(E) \) of the P2G project, the red line represents \( V_1 - C_{P2G} \), and the blue line represents \( V_1 - C_{P2G} \). The optimal investment timing \( E^*_u = 2.3202 \, \text{€/kg} \) is derived at tangent point of the \( F(E) \) curve and the \( V(E) - C_{P2G} \) curve, and the corresponding optimal investment capacity is \( W_{P2G}^n = 215.8241 \, \text{kg/h} \), the total sunk cost is \( C_{P2G}^n = 46.50 \) million euros.

The option value \( F(E) \) is greater than the net present value \( V(E) - C_{P2G} \) of the immediate investment when \( E_u > E^*_u \). The investment project encourages waiting before the operation cost value of the P2G investment project reaches \( E^*_u \). In addition, the project arrives at the optimal investment timing when the net present value of the investment is equal to the value of waiting.

Then, the sensitivity of the parameters in the mathematical real option model of the P2G device is analyzed under full load operation to find the optimal investment timing and the optimal investment capacity \( W_{P2G}^n \). The sensitivity analysis of the parameters affecting P2G project investment in the real option model is conducted in the following part. Therefore, the mathematical model proposed in this paper can be applied to invest in the P2G project in different situations.

**B. THE SENSITIVITY ANALYSIS OF PARAMETERS IN REAL OPTION MODEL**

1) THE CASE WITH THE CHANGE OF HYDROGEN PRICE \( R_u \)

This part investigates the change of P2G device investment strategy when the sales price of P2G output changes. The parameters in the real option model are as follows: \( r = 0.04, \alpha_E = 0, \sigma_E = 0.2, h = 6000, R_u = 5.5/6.5/7.5/\text{€/kg} \). In this case, the optimal investment timing \( E^*_u \) are 2.58, 3.11 and 3.65 euro/kg respectively which can be seen from FIGURE 5. And the corresponding optimal investment capacities \( W_{P2G}^n \) are 220.41, 228.46 and 235.36 kg/h respectively.

It can be seen from FIGURE 5 that the optimal investment opportunity moves to the right, and the operation of P2G investment can be started at a higher cost as the sales price of hydrogen increasing. That is, when the sales price of hydrogen is high, it can engage in investment and operation activities even if the electricity price is high. On the other hand, if the high-end market for hydrogen can be found, the sales price of hydrogen can be kept high, and the investment and operation can be carried out when the operation cost of P2G is high.

2) THE CASE WITH THE CHANGE OF SUNK COST \( C_{P2G} \)

This part investigates the change of P2G device investment strategy when the sunk cost of P2G (\( C_{P2G} \) changes. The parameters in the real option model are as follows: \( r = 0.04, \alpha_E = 0, \sigma_E = 0.2, h = 6000, R_u = 6.5/\text{€/kg} \). The sunk cost of investment \( C_{P2G} \) is going to be 100%, 80%, 60% of the base case. Through calculation, the optimal investment timing \( E^*_u \) is 3.1124, 3.1876 and 3.2768€/kg respectively which can be seen from FIGURE 6. It can be seen that the investment cost has little influence on the optimal investment timing. The optimal operation cost only increases by 5.28% even if the investment sunk cost decreases by 40%.

3) THE CASE WITH THE CHANGE OF VOLATILITY \( \sigma_E \)

The change of P2G device investment strategy is investigated in this part when the volatility of investment cost (\( \sigma_E \)) changes. The parameters in the real option model are as follows: \( r = 0.04, \alpha_E = 0, h = 6000, R_u = 6.5/\text{€/kg}, \sigma_E = 0.1/0.2/0.3. \) It can be seen from FIGURE 7 that
FIGURE 7. The value of investment opportunities with the change of volatility $\sigma_E$.

FIGURE 8. The sample sequences of $E(t)$ with $\sigma_E = 10\%$.

FIGURE 9. The sample sequences of $E(t)$ with $\sigma_E = 20\%$.

FIGURE 10. The sample sequences of $E(t)$ with $\sigma_E = 30\%$.

FIGURE 11. Schematic diagram of hydrogen price and optimal investment timing, the ratio of operation cost, and hydrogen price.

FIGURE 12. Schematic diagram of the relationship between hydrogen price and optimal investment capacity, and investment cost.

due to the higher volatility of electricity price in the case of high uncertainty, the price sequence will have greater volatility in the future. As a result, there may be a possibility of sharp rise in electricity price in the future. Therefore, decision makers of P2G project are encouraged to wait for a longer time until lower investment cost can be determined. In this way, operation options can be exercised to carry out the investment and operation of P2G project. It can be used to explain why the option value of the P2G project will grow with the volatility. That is, the operation risk of P2G device is greater due to the higher volatility. Therefore, waiting has greater value to avoid future risks. The bigger the $\sigma_E$ is, the option value is higher.

In FIGURE 8, FIGURE 9 and FIGURE 10, the initial value of $E(t)$ is 4$\€$/kg. It can be seen from them that the greater
the volatility $\sigma_E$, the more possibility is that $E(t)$ sequences will decline under the same drift rate. The drift rate of $E(t)$ and $C_M(t)$ is set at 0.09, and the data trend is downward, the volatility is 10%, 20% and 30% respectively.

4) THE RELATIONSHIP BETWEEN HYDROGEN PRICE $R_H$ AND OPTIMAL INVESTMENT COST $E_Q$

The P2G operating cost ($E_u$) and the ratio of operation cost and hydrogen price ($E_u/R_H$) with different hydrogen prices at optimal investment timing are shown in FIGURE 11. It can be seen from the figure that the optimal investment timing of hydrogen is basically linear with the price of hydrogen, and the operating cost increases with the hydrogen price under the premise that other parameters are unchanged. The relationship between hydrogen price and the optimal investment capacity and investment sunk cost of P2G is shown in FIGURE 12. The optimal investment capacity of P2G gradually increases with the hydrogen price, but its increment gradually decreases, and its investment sunk cost is basically linear with the price.

V. CONCLUSIONS

The largest proportion of electricity costs in the operation of the P2G plant is the most important factor affecting the economics of the plant operation. In the electricity market environment, there is great uncertainty in electricity prices. A real option investment model based on the uncertainty of operating costs of P2G device is established in this work. Brownian motion model is used to describe the price of electricity in this work, and the economic efficiency of P2G in power market is modeled and analyzed. On this basis, the sensitivity analysis of the relevant parameters which affect the investment of P2G project is carried out.

From the analysis results, the P2G investment project is affected by the sales price of the output production, the cost of electricity price, while the sunk cost of the investment in P2G project is less affected. Especially when the operating cost volatility is high, the option has a greater value, that is, to encourage waiting rather than immediately investment operations. Therefore, there is a higher waiting incentive in this case, and investment will have greater value until the uncertainty is reduced, or the investment cost can be kept low.

The mathematical model of the real option applied in the P2G project in this paper is different from the traditional real option in terms of the price of the output product as the uncertainty variable, it is modeled by the cost as the uncertainty variable, and the valuation of optimal investment capacity suitable for the optimal production cost is also considered. The model proposed in this work can be used to determine the optimal investment timing and optimal investment capacity of the P2G device under different electricity price fluctuation rates, different investment costs, and sales prices of different output products. The mathematical model based on the real option theory established above can be adjusted to apply in physical assets that have high uncertainty in production cost and relatively stable output price which will have broad applicability.

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