Wormhole Time Machines and Multiple Histories

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Abstract

In a previous paper, we showed that a class of time travel paradoxes which cannot be resolved using Novikov’s self-consistency conjecture can be resolved by assuming the existence of multiple histories or parallel timelines. However, our proof was obtained using a simplistic toy model, which was formulated using contrived laws of physics. In the present paper we define and analyze a new model of time travel paradoxes, which is more compatible with known physics. This model consists of a traversable Morris-Thorne wormhole time machine in 3+1 spacetime dimensions. We define the spacetime topology and geometry of the model, calculate the geodesics of objects passing through the time machine, and prove that this model inevitably leads to paradoxes which cannot be resolved using Novikov’s conjecture, but can be resolved using multiple histories. An open-source simulation of our new model using Mathematica is available for download on GitHub. We also provide additional arguments against the Novikov self-consistency conjecture by considering two new paradoxes, the switch paradox and the password paradox, for which assuming self-consistency inevitably leads to counter-intuitive consequences. Our new results provide more substantial support to our claim that if time travel is possible, then multiple histories or parallel timelines must also be possible.
Contents

1 Introduction 3
  1.1 Time travel and its paradoxes .................................................. 3
  1.2 Resolving paradoxes with multiple histories ................................ 4
  1.3 Our old and new paradox models .............................................. 5

2 In-depth comparison of the old and new models 6
  2.1 The original toy model ......................................................... 6
  2.2 The new and improved model ................................................. 8

3 The wormhole time machine 10
  3.1 The metric ................................................................. 10
  3.2 The geodesics ............................................................... 12
  3.3 Converting the wormhole to a time machine ............................... 13

4 Creating time travel paradoxes 13
  4.1 Consistency paradoxes ....................................................... 13
  4.2 Bootstrap paradoxes ......................................................... 15
  4.3 Resolving the paradoxes using multiple histories ......................... 17

5 Simulation of the model using Mathematica 18

6 Additional arguments against Novikov’s conjecture 20
  6.1 Consistency paradoxes: the switch paradox ............................... 20
  6.2 Bootstrap paradoxes: the password paradox .............................. 22

7 Conclusions and discussion 23
  7.1 Our main argument .......................................................... 23
  7.2 Are our assumptions valid? .................................................. 24
  7.3 Future plans ................................................................. 24

8 Acknowledgments 25

References 25
1 Introduction

1.1 Time travel and its paradoxes

Einstein’s theory of general relativity has been around for more than 100 years, and has been verified experimentally to very high accuracy. However, much about this theory is still not well-understood. One particularly interesting question that has so far remained unanswered is whether it is possible, within this theory, to violate causality by traveling back in time.

Indeed, there are many spacetime geometries within general relativity\[1, 2, 3, 4\] which seem to allow time travel, defined more precisely by the existence of closed causal (timelike or null) curves. In this paper we will focus on one such example: a wormhole\[5, 6\].

A wormhole can be roughly defined as a “shortcut” from one point in spacetime to another, allowing a traveler to go from point A directly to point B without traversing the distance between them. If a wormhole can connect two points in space, then it stands to reason that it could perhaps also connect two different points in time, thus forming a time machine.

In fact, even if the wormhole originally only connected two points in space which correspond to the same moment in time, one might be able to use relativistic time dilation to make time at one mouth pass slower than time in the other mouth, thus converting it into a time machine\[7\].

But if time travel is possible, it could lead to paradoxes\[8, 9, 10\]. This includes two main types of paradoxes:

- **Consistency paradoxes**, where the chain of events is inconsistent. For example, imagine that Alice goes into her time machine, travels back 5 minutes to the past, and then destroys the time machine. Since the machine is now destroyed, Alice won’t be able to travel back and destroy the machine. But this means the machine will be in working order, so Alice will be able to travel back in time and destroy it. In other words, the machine exists if and only if it is destroyed, leading to an inconsistency.

- **Bootstrap paradoxes**, where an event causes itself, or something is created out of nothing. For example, imagine that Bob receives plans for a time machine from his future self, who arrives in a similar time machine. Bob then builds the machine, goes back in time, and gives himself the plans. Everything is perfectly consistent, so this is not a consistency paradox. However, Bob’s construction of the time machine causes itself to happen, and the plans for the time machine were created from nothing\[1]\.

How are these paradoxes resolved? One option is to assume that time travel is simply impossible in the first place, as Hawking’s chronology protection conjecture claims\[11, 12, 13\]. However, at the

\[1\] We will discuss in more detail why this is considered a paradox in section 6.2.
moment, this remains merely an unproven conjecture \[14\]. Indeed, a conclusive proof of this conjecture would require a consistent and experimentally-proven theory of quantum gravity, which does not yet exist.

Still, there have been some attempts to prove this conjecture using quantum field theory on curved spacetime and semi-classical gravity. For example, in \[15\] it was shown that the renormalized energy-momentum tensor of a certain quantum field diverges in the presence of a time machine. However, the authors commented that such divergences should get cut off by quantum gravity effects, and \[16\] later presented spacetimes containing time machines where the energy-momentum tensor in fact does not diverge.

Similarly, \[17\] introduced theorems which show that the renormalized expectation value of a quantum scalar field and its energy-momentum tensor are ill-defined or diverge in the presence of time machines, but \[18\] later showed that this can be avoided by replacing some assumptions. We thus see that Hawking’s conjecture is, at present, still far from being proven, and therefore time travel remains a possibility.

If time travel is possible, then it must be possible without paradoxes. One certainly cannot imagine a universe where the time machine is simultaneously both destroyed and not destroyed\footnote{One may wonder if perhaps this can be achieved by allowing the time machine to be in a superposition of destroyed and not destroyed. Indeed, we will discuss how quantum mechanics, in the context of the Everett (“many-worlds”) interpretation, can be used to resolve time travel paradoxes in section \ref{sec:7.3}.}. The simplest way to avoid paradoxes, while still allowing for time travel to occur, is via the Novikov self-consistency conjecture \[19\], which suggests that one can simply never make any changes to the past.

According to this conjecture, any attempts to change the past will necessarily either fail, or bring about the very future they tried to prevent. If the past cannot be changed, then there is also no possibility of paradoxes, no matter how many times you travel to the past.

Bootstrap paradoxes, however, still remain even if Novikov’s conjecture is correct, as we will discuss in section \ref{sec:6.2}. For example, the bootstrap paradox scenario we described above is 100% compatible with Novikov’s conjecture, as is has no inconsistencies.

### 1.2 Resolving paradoxes with multiple histories

It has been shown \[20\] that Novikov’s conjecture can be applied to certain simple consistency paradoxes. However, in a previous paper \[21\], we analyzed a physical system for which it was previously proven \[8\] that the Novikov conjecture cannot apply under any circumstances.

\footnote{The Novikov self-consistency conjecture (sometimes also called the Novikov self-consistency principle) is named after physicist Igor Novikov, and should not be confused with another “Novikov conjecture”, named for mathematician Sergei Novikov, which is related to topology.}

\footnote{More precisely, one of us, Barak Shoshany, along with his student Jacob Hauser.}
In the same paper, we also showed that the same paradox that cannot be resolved using Novikov can, in fact, be completely resolved by assuming the existence of multiple histories, or equivalently, parallel timelines. Since the histories are independent, the paradoxical chain of events breaks:

- Alice steps into the time machine in history 1, but emerges from it in history 2.
- When Alice destroys the time machine in history 2, there is no inconsistency, as this is not the same machine she used to travel back in time, which is still intact – in history 1.

Furthermore, unlike Novikov’s conjecture, multiple histories can also resolve bootstrap paradoxes. For example, the bootstrap paradox described above is paradoxical because Bob gave himself the plans for the time machine, but never created the plans in the first place. This paradox may easily be resolved by considering that in this scenario, there are in fact three Bobs in three separate histories:

- Bob 1 creates the plans for the time machine in history 1, then travels back in time, emerges in history 2, and gives Bob 2 the plans.
- Bob 2 then builds the machine, travels back in time, emerges in history 3, and gives Bob 3 the plans.

While neither Bob 2 nor Bob 3 created the plans, we now see that the plans were, in fact, created by Bob 1, so there is no paradox here.

1.3 Our old and new paradox models

The idea of resolving time travel paradoxes using multiple histories is, of course, not new; see for example the discussion in [6] (chapter 19), [22], [23], and [24]. However, to our knowledge, this idea remained strictly abstract for a long time, with no concrete examples of explicit paradoxes resolved by multiple histories presented in the literature – until the publication of our previous paper, [21], where we introduced such a concrete example for the first time. However, our previous model was not without issues.

The main problem (and source of criticism [25]) of our previous paper is that the physical model we used to analyze the time travel paradoxes was a very simplistic toy model. Therefore, one might argue that perhaps this model cannot be used to prove results about the real world, so maybe our conclusions were invalid, and Novikov’s conjecture is safe after all. In the present paper, we remedy this problem by performing a similar analysis, and reproducing the same proof, with a more realistic model, as detailed in section 2.2.

Of course, we do not yet know if wormholes can exist in reality – and even if they do, we do not yet know if they can be used for time travel. However, the new model is completely “realistic” in the
sense that if (traversable) wormholes did exist, and could be used for time travel, then the theory of general relativity, as we understand it today, appears to predict that one could in principle, using sufficiently advanced technology, construct the physical system described by our model and perform real experiments with it, and its behavior will be governed by familiar and well-tested laws of physics.

In this paper, we prove that this new and more realistic model inevitably leads to paradoxes which cannot be resolved using Novikov’s conjecture, but can be resolved using multiple histories. We thus further strengthen our claim that time travel necessarily implies multiple histories. If multiple histories do not exist, then time travel would inevitably lead to paradoxes, and thus to inconsistent physics, which would make it impossible.

2 In-depth comparison of the old and new models

2.1 The original toy model

Let us recall the toy model used in the previous paper [21], which was based on a previous model by Krasnikov [8]. The model is formulated in the twisted Deutsch-Politzer (TDP) space in 1+1 spacetime dimensions, with coordinates \((t, x)\). This space is constructed by associating the interval \((1, x)\) with \((-1, -x)\) for \(-1 < x < 1\). TDP space is meant to be a toy model of a wormhole time machine, but as we will see, the two are not quite analogous.

A particle entering the interval at \(t = 1\) will travel back in time to \(t = -1\). Furthermore, it will be “twisted”, with its spatial orientation inverted. The reason for the twisting is to ensure that the particle emerging at \(t = -1\) will collide with its past self and potentially create a paradox; if both particles moved in the same direction, they would never collide.

To obtain time travel paradoxes in this model, we impose a few simple physical laws:

1. The particles are all massless, and move along lightlike (or null) paths.
2. Each particle can have one of two colors, for example blue and green.
3. Whenever two particle worldlines intersect, the particles interact. Each particle flips both its direction of motion and its color, independently of the color of the other particle. A blue particle turns green and a green particle turns blue.

The last law is crucial, as without it, time travel paradoxes could be easily avoided using Novikov’s conjecture [20][8]. By allowing the particles to have colors, which make them distinct from one another,
and by having them interact in this particular way, we ensure that there cannot be a consistent evolution – and a paradox is always created.

\[(1, -1) \quad (1, 1)\]

\[(-1, -1) \quad (-1, 1)\]

Figure 1: Example of a paradox in the toy model.

For example, in Figure 1 a blue particle is approaching the system from the right, and collides with its future self. This collision results in the two colliding particles switching colors. The particle that emerged from the time machine then continues, enters the time machine at \(t = 1\), and exits at \(t = -1\). If the particles did not have colors, then this would be perfectly consistent, and we could have said that Novikov’s conjecture applies to this model.

However, since the particles do have colors, and since the particle that goes into the time machine at \(t = 1\) is the same particle that comes out of it at \(t = -1\), the color of the two gray lines in the figure must be the same. The reader should verify that there is no consistent choice of color for the two gray lines, since our imposed physical laws force both particles to switch colors in the collision.

In other words, the two gray lines must have the same color since they are the same particle, but they must also have different colors due to the imposed interaction vertex. Therefore, we have a consistency paradox which cannot be resolved by Novikov’s conjecture.

This result is independent of the initial conditions; the particle can come from any point in space, and can have any initial color. As long as it eventually goes into the time machine, there is a paradox. This means that Novikov’s conjecture cannot be applied to this model under any circumstances, and we must resolve the paradox in another way.
In [21], we proved that multiple histories provide a suitable resolution to the paradoxes for all possible initial conditions. This is illustrated in Figure 2. The blue particle starts in history \( h = 1 \) and goes into the time machine undisturbed; it did not “yet” travel through the time machine, so there is no future particle for it to collide with.

In history 2, the path of the blue particle is the same up to the point where it collides with its copy from history 1. At that point, there is an interaction, and the particle is prevented from going into the time machine. Instead, the particle from history 1, which has now turned green, goes into the time machine. The color that goes into the time machine is now different from the color that came out of it, but that is fine, because the particle that goes in is not the same particle that came out. Thus, there is no paradox. Note that this requires an infinite number of histories, since this process will continue indefinitely. In [21] we considered some other scenarios which only require a finite number of histories, but we will not repeat that discussion here.

Finally, we note that, in the case where there are no colors, Novikov’s conjecture can resolve the consistency paradox, but not the bootstrap paradox. Since the particles collide elastically, the particle that comes out of the time machine is the same one that goes into it, so it has no point of origin – it only exists without the closed causal curve, and was essentially created from nothing. However, in the case of multiple histories, the particle that comes out of the time machine in history 2 originated from history 1, so there is no bootstrap paradox.

### 2.2 The new and improved model

In this paper, we will present a more realistic paradox model using the Morris-Thorne traversable wormhole metric [5]. The main differences between the two models were summarized in section 1.3.
Let us go over these differences in more detail now.

**1+1 vs. 3+1 dimensions**

The toy model was formulated in 1+1 spacetime dimensions. This greatly simplified the analysis, but the real universe has 2 additional spatial dimensions that are unaccounted for. Furthermore, in 1+1 dimensions, general relativity is trivial (as the Einstein tensor is identically zero), so that the toy spacetime is not a true general-relativistic spacetime. By increasing the number of spacetime dimensions to 3+1, we make the model more realistic and permit studying the effects of gravity.

We will use spherical coordinates, with the objects moving only along radial geodesics. This means that effectively, the objects are still moving in a 1+1-dimensional hypersurface – but that is to be expected, as there is no reason for them to make turns at any point. While the model could be further generalized by allowing non-radial geodesics, that would merely complicate the model without adding any further insights into time travel paradoxes.

**Color vs. temperature**

The particle “colors” used in the toy model can be thought of as a discrete property similar to electric charge or QCD color charge. In this sense, the colors themselves are not unrealistic. However, the interaction vertex where each particle simply flips its color in each collision is artificial, and does not have an analogue in any known laws of physics; in fact, if the colors are charges, then this vertex clearly violates conservation of charge.

In the new model, we replace color with temperature. In thermodynamics, two systems in mutual contact will exchange heat until they reach thermodynamic equilibrium, with heat flowing from the hotter object to the colder one. Thus, we replace the particles with objects that have temperature, and assume that the environment is sufficiently cold that the objects will continuously lose heat over a sufficiently long amount of time.

By replacing particles and colors with objects and temperatures, we exchange the contrived physical laws of the toy model with the well-established laws of thermodynamics. As we will see below, the gradually decreasing temperature provides essentially the same mechanism for generating inconsistencies that the colors provided in the toy model.

**Massless vs. massive**

In the toy model, we only considered massless particles moving along lightlike (null) trajectories. This made the analysis easier, as the particles only moved in 45° angles in the spacetime diagrams.
However, it also severely limited the applicability of the model, as one usually wants to send massive objects through time machines, not just photons.

In the new model we remedy this by considering massive objects moving along timelike trajectories, with any speed $v < 1$ (in units where $c \equiv 1$). Massless “objects” can also be considered, either by taking the limit as $m \to 0$ and $v \to 1$, or by taking the “object” to be a gas of photons. In this way, we ensure that the new model is as general as possible and can handle both massive and massless objects.

**Flat hole vs. wormhole**

The TDP space used in the toy model is a 1+1-dimensional flat spacetime with a “hole”. There is no geometry, only topology, so nothing interesting is happening in terms of gravity. The entire analysis is performed using special relativity – in fact, essentially using just Newtonian mechanics – and particles do not experience any gravity.

The TDP time machine itself is an idealized one – merely a topological identification of two lines, without any physical structure. The TDP space also has another problem which, for simplicity, we chose to ignore in our previous paper: the four points at $(t, x) = (\pm 1, \pm 1)$ are singularities, and this turns out to have some bizarre consequences [2, section 3.3].

In the new model, we instead construct a time machine using a physical wormhole. This means that we are no longer using a purely topological time machine, but a fully geometric one, where the effects of gravity can be explored. This allows us to accurately describe how objects would realistically move in this spacetime if it was possible to construct it, by considering solutions to the geodesic equations.

The wormhole time machine we will explore exists eternally, and sends any object that enters it at any time a fixed duration back in time. This is in contrast to the TDP space time machine, where the wormhole only comes into existence at two particular points in time, sends objects from the future point to the past point, and does not exist at any other times. As we will show, the new model still results in inevitable paradoxes that can only be removed by assuming multiple histories.

### 3 The wormhole time machine

#### 3.1 The metric

It is now finally time to introduce the mathematical details of the new model. Let us consider a simplified version of the (static and spherically symmetric) Morris-Thorne traversable wormhole metric [5,6]:

$$ds^2 = -dt^2 + dl^2 + (l_0^2 + l^2)(d\theta^2 + \sin^2 \theta \, d\phi^2),$$

(1)
where \( t \in \mathbb{R}, l \in \mathbb{R}, \theta \in [0, \pi], \) and \( \phi \in [0, 2\pi) \) are the coordinates, and \( b_0 \in [0, \infty) \) is a constant. The coordinate \( l \) acts as a radial coordinate, except that it can also be negative. The positive range corresponds to one region, and the negative range corresponds to another region. Both regions are asymptotically flat in the limit \( |l| \to \infty \), and they are connected at the point \( l = 0 \), called the throat of the wormhole. The metric is symmetric under \( l \mapsto -l \).

Objects can travel from one region, through the wormhole’s throat, into the other region. The choice of regions is up to us. They can be two completely separate universes, or they can be two parts of the same universe. In the latter case, the wormhole can be a shortcut through space, connecting two very distant locations and thus potentially allowing faster-than-light travel.

We can also introduce a different radial coordinate \( r \) defined by \( r^2 \equiv b_0^2 + l^2 \). With this coordinate, the metric takes the form

\[
\text{d}s^2 = -\text{d}t^2 + \frac{\text{d}r^2}{1 - b_0^2/r^2} + r^2 \left( \text{d}\theta^2 + \sin^2\theta \, \text{d}\phi^2 \right).
\]

Note that \( r \) is strictly positive, and the smallest value it can have is \( b_0 \) (corresponding to \( l = 0 \)), which means the region \([0, b_0]\) is inaccessible. \( b_0 \) represents the radius of the wormhole throat. Furthermore, while \( l \) covers both regions, \( r \) only covers one region, so copies of the coordinate system \((t, r, \theta, \phi)\) are needed to cover the entire wormhole. The embedding diagram for the wormhole (see [5] for derivation) can be seen in Figure 3.

![Embedding diagram of the Morris-Thorne wormhole metric with \( \theta = \pi/2 \).](image)

Each circle represents a slice of a sphere with radius \( r \). The orange surface represents positive \( l \), while the blue surface represents negative \( l \). A possible geodesic, crossing through the wormhole from one region to the other, is highlighted in black in the diagram.
Morris-Thorne wormholes have been studied extensively in the literature, as summarized in the books [6], [3], and [2]. One interesting question that remains unanswered is whether such wormholes are stable under perturbations, such as those produced due to sending matter through the wormhole.

The stability of the wormhole largely depends on the properties of the matter used to construct it. Solving the Einstein equation for the Morris-Thorne wormhole metric (1) suggests that it must be supported by “exotic matter”, which violates some of the energy conditions [7, 26, 27]. It is currently unknown whether such matter exists in our universe in sufficient quantities to build a wormhole.

For example, Armendáriz-Picón [28] considered exotic matter in the form of a minimally-coupled massless scalar field with a reversed-sign kinetic term, and concluded that wormholes constructed using it are indeed stable under arbitrary perturbations. However, it is unclear whether such a scalar field can exist in nature.

More recently, some authors have analyzed the stability of wormholes using modified theories of gravity. For example, Kuhfittig [29] showed that Morris-Thorne wormholes with zero tidal forces are in stable equilibrium, as defined by the Tolman-Oppenheimer-Volkov (TOV) equation. That analysis was performed in \( f(R) \) gravity with \( f(R) \sim R^{1+\epsilon} \), which reproduces Einstein gravity in the limit \( \epsilon \to 0 \), but it is unclear whether the results still hold in that limit.

Undoubtedly, much more work is needed in order to determine whether Morris-Thorne wormholes can be stable in pure Einstein gravity. However, for the purposes of the time machine model presented in this paper, we will be working under the assumption that the wormhole is indeed stable.

### 3.2 The geodesics

In this paper we will only be interested in radial geodesics; allowing for non-radial geodesics will complicate the calculations while not adding any new insights into time travel paradoxes, which is our main goal in the paper. As we are only considering geodesics along \( l \), we have \( d\theta = d\phi = 0 \), and the metric simplifies to

\[
ds^2 = -dt^2 + dl^2, \tag{3}\n\]

which is a simple 1+1-dimensional flat metric. Thus we immediately see that the geodesic equation is given by

\[
\ddot{l}(t) = 0. \tag{4}\n\]

This equation has the solution

\[
l(t) = l_0 + vt, \tag{5}\n\]

where \( v \in [0, 1] \) is the object’s velocity and \( l_0 \) is the initial position at \( t = 0 \). Although the geodesics are just straight lines in the \( l \) coordinate, they become more complicated when considering the \( r \) coordinate, as we will see in the simulations in section 5.
3.3 Converting the wormhole to a time machine

Let us now assume that both sides of the wormhole, positive $l$ and negative $l$, are not only in the same universe, but in fact in the exact same position, chosen without loss of generality to be the spatial origin. To convert the wormhole into a time machine, we simply assume that the origin of one region is shifted by 2 along the $t$ axis with respect to the origin of the other region. This then means that any object entering the wormhole in the future region, at time $t$, will exit it in the past region, at time $t - 2$.

There is a slight complication here, as the future and past wormholes are located on top of each other, so it is unclear if objects entering the spatial origin are supposed to be sent 2 time units to the past or to the future. However, we can simply assume that incoming objects always start at positive $l$, and that negative $l$ is in the past, so that objects are always sent to the past.

The last property we need to define is the relative spatial orientation of the two regions, that is, how the direction of the object entering the wormhole in the future is related to that of the object exiting the wormhole in the past. Recall from section 2.1 that in the TDP space, we had to twist the particle as it went through the time machine, so that when it comes out it goes back the way it came from, and is thus guaranteed to collide with itself and create a paradox.

To reproduce this behavior in the wormhole model, we simply place the two mouths of the wormhole such that they both have the same orientation. Both the positive $l$ values and the negative $l$ values are now mapped to the same spatial coordinate patch (rather than two separate coordinate patches, as we assumed before), except shifted in time by 2 units.

Importantly, the $\theta$ and $\phi$ coordinates remain unchanged after passing through the wormhole. This means that objects coming in from any radial direction with decreasing $r$ (and $l$ decreasing from $+\infty$ to 0) will come out with increasing $r$ (and $l$ decreasing from 0 to $-\infty$) in the same direction, and are thus guaranteed to create a paradox by colliding with themselves.

In other words, we are restricting both sides of the wormhole to be in the same 1+1-dimensional subsurface of the entire 3+1-dimensional spacetime; this also substantially simplifies the following analysis.

4 Creating time travel paradoxes

4.1 Consistency paradoxes

The crucial component in creating time travel paradoxes that cannot be resolved by Novikov’s conjecture is introducing a distinguishing property that all objects going through the time machine have, such that this property will end up having different values at the entrance and exit of the time machine, resulting in an inconsistent evolution and thus an inevitable paradox.
In the toy model, this property was the particle’s color – an artificial property invented specifically for this model. In the new wormhole model, we instead use a property that any macroscopic object already has: temperature. We assume that the objects are hot and that the environment is cold enough to ensure that the objects transfer heat to the environment continuously for the entire duration of the experiment.

More precisely, for a massive object, we assume that it is a black body satisfying the Stefan-Boltzmann law for radiative cooling:

\[ P = \frac{dE}{d\tau} = \epsilon\sigma A \left(T^4 - T_{\text{ambient}}^4\right), \]  

where \( P \) is the total power radiated from the object, \( E \) is the energy of the object, \( \tau \) is time, \( \epsilon \) is the emissivity (\( \epsilon = 1 \) for a black body), \( \sigma \) is the Stefan-Boltzmann constant, \( A \) is the surface area of the object, \( T \) is the temperature of the object, and \( T_{\text{ambient}} \) is the temperature of the environment. The specific heat capacity \( c \) of the object is defined by the relation

\[ c = \frac{1}{m} \frac{dE}{dT} \quad \Rightarrow \quad mc = \frac{dE}{dT}, \]  

where \( m \) is the object’s mass. Multiplying by \( dT/\tau \) and using the chain rule, we get:

\[ mc \frac{dT}{d\tau} = \frac{dE}{dT} \frac{dT}{d\tau} = \frac{dE}{d\tau} = P = \epsilon\sigma A \left(T^4 - T_{\text{ambient}}^4\right). \]  

Assuming that the temperature \( T \) is much greater than the ambient temperature, we may neglect \( T_{\text{ambient}} \) and rewrite this relation as follows:

\[ \frac{1}{T^4} dT = \frac{\epsilon\sigma A}{mc} d\tau. \]  

Integrating this equation from \( T \) to the initial temperature \( T_0 \), we get

\[ \frac{1}{3} \left( \frac{1}{T^3} - \frac{1}{T_0^3} \right) = \int_T^{T_0} \frac{1}{T^4} dT = \frac{\epsilon\sigma A}{mc} \int d\tau = \frac{\epsilon\sigma A}{mc} \tau. \]  

Isolating \( T \), we find that the temperature of the object at time \( \tau \) is

\[ T(\tau) = \left( \frac{3\epsilon\sigma A \tau + \frac{1}{T_0^3}}{mc} \right)^{-1/3}. \]  

Therefore, a massive object’s temperature is a monotonically decreasing function \( T(\tau) \), where \( \tau \) is

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6 Defined since 2019 to have the exact value \( \sigma \equiv \frac{2\pi^5k^4}{15\sqrt{\gamma}} \) where \( k \) is the Boltzmann constant, \( c \) is the speed of light, and \( h \) is the Planck constant.

7 Note that \( T < T_0 \), so we must integrate from \( T \) to \( T_0 \) in order for the integral to be positive.
the proper time along a timelike geodesic.

It is interesting, although not crucial for our purposes, to allow for massless “objects” as well. In this case, the geodesic is lightlike, and \( \tau \) is an affine parameter. We will assume that the “object” is a photon gas, which satisfies the equation \( N \sim VT^3 \) where \( N \) is the number of photons, \( V \) is the volume, and \( T \) is the temperature. The temperature is still monotonically decreasing, as it is reduced by the gradual absorption of photons into atoms in the air as the gas travels.

To prove that there is a paradox, we must first prove that the objects always collide; if they don’t, then there is no way for the object to change its past trajectory, and thus there will be no paradoxes. An object comes in radially from infinity towards decreasing values of \( r \), enters the time machine at time \( t \), and then exits it at \( t - 2 \), now moving to infinity towards increasing values of \( r \). We thus have two half-infinite lines that are not parallel and therefore must cross, so there will always be a collision.

Now, let’s say that the object arrives in the vicinity of the time machine at some initial temperature \( T_0 \). Since the temperature is monotonically decreasing, after traversing a finite distance the object enters the time machine at a strictly lower temperature \( T_1 < T_0 \). Since the throat length is zero, the object exits the time machine in the past at the same temperature \( T_1 \). It moves a bit away from the time machine until it reaches an even lower temperature \( T_2 < T_1 \), and then collides with its past self.

In this collision, the incoming (past) object will reverse its direction of movement, so it is the time-traveling object that will go (again) into the time machine. It will reach the time machine at some temperature \( T_3 < T_2 \), but since we already know that the object that left the time machine had temperature \( T_1 \), for consistency we must have \( T_3 = T_1 \). This is impossible, since \( T_3 < T_2 < T_1 \) and all the inequalities are strict, so we have a consistency paradox.

Importantly, if the objects did not have temperatures, then we could have applied Novikov’s conjecture here, and argued that there is no way to tell whether it is the future or past object that enters the time machine, so there is no inconsistency.

The addition of temperature, much like the addition of color in the old toy model, allows us to create a consistency paradox by forcing an object to have two different temperatures at the same time. In fact, the temperature could be replaced with any other measurable property of the object that is monotonically decreasing or increasing, and a similar inconsistency will be achieved.

### 4.2 Bootstrap paradoxes

Note that in addition to a consistency paradox, that is, a “grandfather”-like paradox, there is also a bootstrap paradox here. The object that comes in from infinity never actually enters the time machine. The other object comes out of the time machine and then ends up entering the time machine again. Therefore, the object seems to be created out of nothing, and only exists within the time loop.
One way to avoid a bootstrap paradox, while maintaining the consistency paradox, is to create a situation where the two objects “pass through” each other instead of colliding. In the case of a massless photon gas, this is already what happens – therefore there is no bootstrap paradox in that case.

To avoid a bootstrap paradox in the case of a massive object, we could, in principle, consider a situation where the object is actually composed of two disconnected pieces moving side by side in unison, so that one pair of pieces can pass through the other pair without touching it.

The temperature of the past pair will nevertheless decrease as a result of “passing through” the future pair, as there will be some heat exchange (however short) via radiation between the two pairs. Therefore, there will still be a consistency paradox, since the object that will enter the time machine will have a lower temperature than the object that left the time machine.

To illustrate this, assume that a pair of pieces arrives in the vicinity of the time machine at some temperature $T_0$. It moves towards the entrance of the time machine, reaching some temperature $T_1 < T_0$ somewhere along the path, and then enters the time machine at some temperature $T_2 < T_1$. It exits in the past at the same temperature $T_2$. It moves a bit away from the time machine until it reaches some temperature $T_3 < T_2$, and then passes through the other pair.

At the time of interaction, when the two pairs pass through each other, the future (older) pair is at temperature $T_3 < T_2$, and the past (younger) pair is at temperature $T_1 > T_2$. Therefore, the past pair is the hotter one, and during the short interaction time, it will transfer a small amount of heat to the colder future pair. Thus, the past pair will now be at some temperature $T'_1 < T_1$, and will then continue and enter the wormhole at some temperature $T'_2$.

We know that on the way from that particular point along the path to the time machine, the pair originally decreased its temperature from $T_1$ to $T_2$. Hence, regardless of the precise definition of the monotonically decreasing function $T(\tau)$, since the pair started at a temperature $T'_1 < T_1$ it must enter the time machine at a temperature $T'_2 < T_2$.

But since we already know that the pair that left the time machine had temperature $T_2$, we must have $T'_2 = T_2$, in contradiction with the fact that $T'_2 < T_2$; even if the temperature difference is extremely small, the temperatures are still different. We thus still have a consistency paradox, but we managed to avoid a bootstrap paradox.

This has been an interesting exercise, but unfortunately, it is a very contrived way of resolving the bootstrap paradox, as it requires delicate fine-tuning of the precise arrangement of the pieces, and that the objects are composed of two pieces in the first place.

Since our stated goal in this paper is to avoid contrived situations (such as the interaction vertices in our toy model), and instead get comprehensive results from which we can learn about time travel paradoxes in general, we do not consider the bootstrap paradox to be resolved, except in this very special case – at least not without introducing multiple histories.
4.3 Resolving the paradoxes using multiple histories

We have proven that, with our wormhole time machine model, paradoxes are inevitable, and cannot be resolved using Novikov’s conjecture. Consistency paradoxes are always created, and – except in the special contrived case where the objects can pass through each other – bootstrap paradoxes are always created as well. However, both types of paradoxes can be resolved by introducing the notion of multiple histories (or timelines), as we did with the toy model in [21].

We assume that the universe can have many histories, each distinguished by a different value of a label \( h \). Upon traveling back in time, the universe “branches” into two separate histories. The original history is left unchanged; the new history is the same up to the point in time when the branching took place, but can be different from that point on.

Any changes made to the new history will not influence the old history, and thus no paradoxes can be created. We further impose that it is impossible for the time traveler to go back from the new history to the one they originally came from, with the possible exception of going back to a point in time after they left, which does not violate causality.

In this paper, we are not proposing a specific mechanism for creating the new histories – we are merely assuming that there is such a mechanism already in place. The exact meaning of \( h \), and the set from which its values are taken (whether \( \mathbb{N}, \mathbb{Z}, \mathbb{R} \), or an even larger set) would depend on the particular mechanism.

For example, if the multiple-history model is a consequence of the Everett interpretation (see section 7.3), then different values of \( h \) could represent different paths along the branching tree of quantum possibilities.

The same – as yet unknown – mechanism that allows wormholes to somehow connect two points in space and/or time, or even two different universes, could presumably also be used to connect two different histories.

Thus, in addition to the time shift \( t \mapsto t - 2 \), it is a simple matter to introduce a conjectured “history shift” \( h \mapsto h + 1 \) (assuming, for simplicity, that \( h \) is an integer). Note that this does not mean we are treating \( h \) as a 5th spacetime dimension; it is simply a label, and nothing more.

The reader is referred to [21] for a discussion of some interesting nuances of multiple-history models we did not cover here, such as cyclic histories. Below, we will show that multiple histories can resolve all of the paradoxes created by our model, both consistency and bootstrap.
5 Simulation of the model using Mathematica

The GitHub repository for this paper, at https://github.com/bshoshany/WormholeParadoxSimulation, contains the Mathematica notebook WormholeParadoxSimulation.nb, which simulates the wormhole time machine model presented in this paper.

Note that purchasing Wolfram Mathematica is not required to run the simulation; it can be viewed and interacted with using Wolfram Player (version 12 and above), which can be freely downloaded at https://www.wolfram.com/player/.

Our time machine consists of the Morris-Thorne metric [1], with the wormhole at the spatial origin: 
\((t, l, \theta, \phi) = (t, 0, \pi/2, 0)\), and with the two mouths separated in \(t\) by 2 time units. Let the object (whether a ball, a photon gas, or something else) begin at \(t = 0\) in the initial position \(l = l_0 > 0\). It then follows a radial geodesic towards the wormhole, given by \(l(t) = l_0 - vt\) with a constant velocity \(v \in (0, 1]\).

The object reaches \(l = 0\) at time \(t = \frac{l_0}{v}\), at which point it traverses the wormhole and continues to negative values of \(l\). Equivalently, since we are identifying the region of negative \(l\) with the exact same spacetime, except 2 units of time in the past, we can simply interpret this as switching the sign of \(v\).

The object will exit at the origin, with a time shift \(t \mapsto t - 2\), that is, at time

\[ t = \frac{l_0}{v} - 2. \]  

(12)

The new worldline will thus be given by

\[ l(t) = v \left( t - \left( \frac{l_0}{v} - 2 \right) \right) = v \left( t + 2 \right) - l_0. \]  

(13)

The old and the new worldlines will intersect when

\[ \underbrace{l_0 - vt}_{\text{old}} = \underbrace{v \left( t + 2 \right) - l_0}_{\text{new}}, \]  

(14)

that is, at time

\[ t = \frac{l_0}{v} - 1. \]  

(15)

The collision will always happen; there is no way for the new worldline to go “around” the old one, or to never reach it in the first place, since they are both in the same plane and can be extended to infinity.

This scenario is simulated in our Mathematica notebook, as shown in Figure [4]. The wormhole is represented in the figure by the green cylinder. Shown is a slice of the Morris-Thorne spacetime with time \(t\) as the vertical axis, the radial coordinate \(r \in [b_0, \infty)\) orthogonal to the \(t\) axis, and the azimuthal angle \(\phi\) going around the \(t\) axis.
Figure 4: The user interface of WormholeParadoxSimulation.nb showing a wormhole with a single history. Note the gray lines, indicating a paradox. The user may use the sliders to adjust the initial position of the object, its velocity (in units of the speed of light), the viewing angle of the plot, and the wormhole radius.

The polar angle $\theta$ is suppressed, such that each circle in the cylinder represents a sphere at a particular instance in time. Note that in the Morris-Thorne spacetime, the region $r \in [0, b_0)$ – the interior of the cylinder in the plot – is inaccessible.

The incoming and outgoing paths of the object are also shown in the figure, as calculated using the geodesic equations. Note that the geodesics are shown in the $r$ coordinate, defined by $r^2 \equiv b_0^2 + l^2$, instead of the $l$ coordinate – which is why they are not straight lines. The colors along the paths indicate temperature, with red being the hottest.

The object starts at the red temperature and approaches the wormhole. Upon traversal, the object exits the wormhole in the past, in the opposite spatial direction, and then collides with its past self. In the figure, we can see that some of the path is colored gray, which means there is no consistent choice of temperature along the path, hence there is a paradox.

By switching from a single history to multiple histories, the paradox can be resolved. Two histories will be shown on the screen, with the first history on the left and the second (new) history on the right, as in Figure 5.
Notice that now there are no gray lines; there is a fully consistent temperature evolution along the paths, and thus we have resolved the consistency paradox. Furthermore, the object that emerges from the time machine in the new history came from the old history, so it was not “created from nothing”, and thus the bootstrap paradox is also resolved, without having to assume the objects pass through each other.

6 Additional arguments against Novikov’s conjecture

6.1 Consistency paradoxes: the switch paradox

In this paper we took great care to formulate a precise mathematical model of a paradox and prove that Novikov’s self-consistency conjecture does not apply to it. For completion, we would now like to present some other arguments which are more philosophical in nature, but nonetheless worth mentioning.

Imagine that the wormhole time machine has an on/off switch. Upon setting the switch to “on”, matter is shifted in space such that the spacetime geometry is that of a wormhole. Upon setting the
switch to “off”, matter is shifted back to its original position, and the wormhole disappears.

As above, the wormhole entrance is located 2 units of time in the future of the wormhole exit. Alice intends to create a paradox using the switch in the following way:

- At time $t = +1$, Alice goes into the wormhole entrance.
- At time $t = -1$, Alice emerges from the wormhole exit and turns off the switch, preventing herself from going into the wormhole at $t = +1$, thus creating an inconsistency.

Assume that Alice carefully watches the switch from $t = -1$ until $t = +1$. During that time, does she see her future self appear and turn off the switch, or not?

If Alice never sees her future self, and then goes into the time machine at $t = +1$, then we must be in a multiple-history scenario and Alice must have been in the first history, “before” she traveled through the time machine. There is no self-consistent chain of events where Alice turns off the switch at $t = -1$ without her past self seeing it happening, so Novikov self-consistency cannot apply.

If Alice does see her future self, but her future self does not turn off the switch, then one must ask why she did not turn it off. Clearly, not turning off the switch will avoid the paradox and create a fully consistent chain of events. But we said above that Alice is determined to turn off the switch in an attempt to create a paradox, no matter what.

If, despite her determination, she did not turn off the switch, then that seems to imply that Alice has no free will. While this is, by itself, perfectly consistent with all known laws of physics, some people may be concerned that accepting Novikov’s conjecture means we must reject the notion of free will.

If Alice does see her future self, and her future self does turn off the switch, then the only self-consistent scenario is that something mysteriously goes wrong that is beyond her control, such as the switch suddenly malfunctioning.  

Assuming that the switch was carefully constructed to be as reliable as possible, the probability of it malfunctioning is presumably very low, and yet to ensure self-consistency, an event with a seemingly very high probability (the switch successfully turning off) must never happen, and an event with a seemingly very low probability (the switch malfunctioning) must happen with certainty.

This means that a selection process must be happening behind the scenes. The local laws of physics, which were used to calculate the probabilities of these events, must somehow be modified by an unseen hand to make things consistent globally, selecting some events over others in a way that would not make sense to a local observer unaware of the existence of the time machine.

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8In fact, Alice changing her mind can also fall into this category. While she is determined to turn off the switch, there is always a small probability that she changes her mind after all. The issue isn’t that she changed her mind, but rather that she had to change her mind regardless of how low the probability for that should have been. Treated this way, we can avoid involving the controversial notion of free will in the discussion.
Indeed, one way to formulate Novikov's principle of self-consistency is that “the only solutions to the laws of physics that can occur locally are those which are globally self-consistent” \[19\]. This seems to contradict the local nature of the known laws of physics.

### 6.2 Bootstrap paradoxes: the password paradox

As a final argument, we note that while Novikov self-consistency can be used to resolve some consistency paradoxes, it cannot be used to resolve bootstrap paradoxes. The reason is obvious from simply noting the name self-consistency: this principle, by definition, only guarantees consistency. But there are many perfectly consistent scenarios that nonetheless suffer from bootstrap paradoxes – where something is created from nothing, or an event is its own cause.

Some may argue that bootstrap paradoxes are not really paradoxes, and are instead “pseudo-paradoxes”, as there is no reason for the paradoxical chain of events to happen in the first place. However, we will now argue that under certain conditions, Novikov’s self-consistency principle may guarantee that such a chain of events happens.

We turn the switch paradox into a bootstrap paradox as follows. The switch starts at \( t = -1 \) in the on position, and at \( t = 0 \), it turns off automatically. In order to turn it back on, a secret password must be entered. This password is not known to Alice. Furthermore, for reasons that will become clear shortly, we assume that if the switch is activated with the correct password, the person who activated it is automatically (and inescapably) killed by the switch mechanism.

This seems to inevitably lead to a consistency paradox. Since Alice doesn’t know the password, she will not be able to re-open the wormhole. But if the wormhole entrance is not open at \( t = +1 \), this is inconsistent with the fact that the wormhole exit was open at \( t = -1 \).

This inconsistency can be solved using Novikov self-consistency as follows. At \( t = -1 \), future Alice emerges from the wormhole exit. At \( t = 0 \), after the switch turns itself off, future Alice enters the password “42”, and activates the switch, which then kills her instantly and opens the wormhole entrance.

With the wormhole entrance now open, past Alice enters it at \( t = +1 \), and returns to \( t = -1 \). Knowing now that the password is “42”, she enters it and activates the switch. We have resolved the consistency paradox, but in the process, we have created a bootstrap paradox – Alice could not have known that the password is “42” had she not seen herself type it in the first place.

What made this bootstrap paradox happen? One could, perhaps, reason as follows. The local laws of physics allow, with extremely low probability, for an object to spontaneously emerge out of the wormhole exit at \( t = -1 \) due to random quantum fluctuations. There are a potentially infinite number of possible objects that can be created in this way, but only one of them is globally self-consistent: an Alice with the knowledge that the password is “42”.
Indeed, if anyone else comes out of the wormhole with the knowledge that the password is “42”, then this is inconsistent, because once they enter the password, they die— and only Alice remains in the room, so only she can enter the wormhole at \( t = +1 \), and therefore only she can exit it at \( t = -1 \).

In all other cases—either Alice or someone else appears at the wormhole exit without knowledge of the password, or some random object appears, or even no object appears at all—the wormhole entrance will remain closed, and this is again inconsistent, since the exit cannot exist without the entrance.

While the event where Alice spontaneously appears at the wormhole exit knowing that the password is “42” obviously has a very low probability to happen, we have already established in the previous section that Novikov self-consistency has the ability to make such low-probability events happen with certainty in order to guarantee consistency. Hence, we conclude that the self-consistency principle will inevitably lead to a bootstrap paradox in the process of resolving the password paradox.

7 Conclusions and discussion

7.1 Our main argument

Let us now summarize the main argument of our paper. We considered a universe where time travel is possible, and in particular, it is possible via Morris-Thorne wormholes. In this universe, we assumed that the only two potential methods of resolving time travel paradoxes are Novikov’s self-consistency conjecture and multiple histories.

By proving that Novikov’s self-consistency conjecture cannot be applied to certain time travel paradoxes, we see that while it may well be applicable to some time travel paradoxes, it cannot apply to all paradoxes, and therefore, if time travel is indeed possible, Novikov’s conjecture alone is not sufficient to allow time travel without any paradoxes.

Since we cannot simply sweep these Novikov-incompatible paradoxes under the carpet, we conclude that multiple histories (or parallel timelines) are a mandatory feature of any universe where time travel is possible.

7.2 Are our assumptions valid?

It is worthwhile to make some comments regarding the validity of our assumptions. Of course, the most radical assumption is that time travel is possible. We do not yet have enough data to determine whether this assumption is valid, and it may take many decades to get anywhere near a concrete answer.
While some exotic metrics in general relativity seem to, at least in principle, allow for the existence of closed causal curves, time travel may still be forbidden, for example by some version of Hawking’s chronology protection conjecture \[11, 12, 13\], as discussed in more detail in section 1.1.

However, this simply does not concern us here, as we are merely interested in proving that multiple histories must exist if time travel is possible; we make no claims whatsoever regarding the existence of time travel itself.

Even if time travel is indeed possible in our universe, the assumption that it is possible via Morris-Thorne wormholes may still be incorrect. First of all, we do not know if wormholes themselves can exist in nature. It is often claimed that, since wormholes violate the energy conditions \[11, 27\], they are unrealistic, and cannot exist in our universe.

However, the energy conditions are seemingly arbitrary conditions imposed by hand, and there are many known examples of both hypothetical and real forms of matter which may violate them \[3\]. Still, if some day it is proven – from first principles – that the violations of the energy conditions required to create a wormhole are unrealistic, then this would be enough to rule wormholes out. Furthermore, even if wormholes are in fact realistic, at least given sufficiently advanced technology, this does not by itself guarantee that they can be used for time travel.

However, since in our proof we did not use any properties that are unique to this particular spacetime, our argument should still stand even if time travel is achieved by other means – as long as the objects that travel through time can have temperature and the initial conditions can be set up such that collisions are inevitable.

Finally, there could be other potential methods of resolving time travel paradoxes besides Novikov self-consistency and multiple histories\[^9\]. However, to our knowledge, no other methods have been suggested so far in the literature.

### 7.3 Future plans

If multiple histories must indeed exist, there must be some mechanism for creating them and physical laws governing their existence. Our goal in this paper was only to show that multiple histories are a necessary consequence of time travel, but we make no claims regarding the nature of these histories.

Although the possibility of resolving time travel paradoxes using multiple histories has been occasionally mentioned before in the literature \[11, 6, 3\], an actual mechanism for creating them has never, to our knowledge, been suggested. Developing such a mechanism will not only provide concrete proof

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\[^9\] Interestingly, in \[21\] we showed that in the case of cyclic histories (there is a “last” history, and it connects back to the first) one may combine Novikov’s conjecture and multiple histories into a “hybrid” method; but this is a special case that will not be relevant to our discussion.
for the theoretical possibility of paradox-free time travel; it will also allow us to better understand the
type of time and causality in our universe.

Classically, non-Hausdorff manifolds [6, 30, 31, 32] or non-locally-Euclidean manifolds [33] were
suggested as a possible underlying mathematical model for branching spacetimes. However, these
models have many unresolved issues, mostly regarding how to do differential geometry at points
where the Hausdorff or locally-Euclidean conditions are violated. Furthermore, no concrete model has
so far been suggested for relating such manifolds to time travel.

However, in quantum mechanics, the Everett (“many-worlds”) interpretation seems to provide a
natural and “built-in” way to account for multiple histories. Deutsch [23] famously considered
quantum mechanics in the vicinity of closed causal curves and found that quantum mechanics itself
must be modified in order to resolve paradoxes. However, in an upcoming paper [34], we will provide
a general framework for resolving time travel paradoxes using unmodified quantum mechanics.

Quantum mechanics alone may not be sufficient, though. To illustrate this, consider again the switch
paradox described in section 6.1. In this scenario, the spacetime geometry changes when the switch is
turned on or off. To resolve this paradox using multiple histories, one would need to allow the second
history to have a different spacetime geometry than the first.

This means that the quantum state of spacetime itself must be considered, and that cannot be done
properly until a consistent and experimentally-verified quantum theory of gravity becomes available.
In a future publication, we plan to discuss this problem in more detail.

In addition, it would be interesting to construct models for time travel paradoxes which do not involve
wormholes. Such models may be based on other proposed forms of faster-than-light travel, such as
warp drives [35] or hyperspace [36].

This would allow us to study time travel paradoxes in scenarios very different from both the TDP
space model and the wormhole model, and ensure that our conclusions regarding the necessity of
multiple histories for resolving time travel paradoxes are universally valid.

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