Hall effect on MHD Jeffrey fluid flow with Cattaneo–Christov heat flux model: an application of stochastic neural computing

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Received: 5 June 2021 / Accepted: 15 April 2022 / Published online: 9 May 2022 © The Author(s) 2022

Abstract
Exploration and exploitation of intelligent computing infrastructures are becoming of great interest for the research community to investigate different fields of science and engineering offering new improved versions of problem-solving soft computing-based methodologies. The current investigation presents a novel artificial neural network-based solution methodology for the presented problem addressing the properties of Hall current on magneto hydrodynamics (MHD) flow with Jeffery fluid towards a nonlinear stretchable sheet with thickness variation. Generalized heat flux characteristics employing Cattaneo–Christov heat flux model (CCHFM) along with modified Ohms law have been studied. The modelled PDEs are reduced into a dimensionless set of ODEs by introducing appropriate transformations. The temperature and velocity profiles of the fluid are examined numerically with the help of the Adam Bashforth method for different values of physical parameters to study the Hall current with Jeffrey fluid and CCHFM. The examination of the nonlinear input–output with neural network for numerical results is also conducted for the obtained dataset of the system by using Levenberg Marquardt backpropagated networks. The value of Skin friction coefficient, Reynold number, Deborah number, Nusselt number, local wall friction factors and local heat flux are calculated and interpreted for different parameters to have better insight into flow dynamics. The precision level is examined exhaustively by mean square error, error histograms, training states information, regression and fitting plots. Moreover, the performance of the designed solver is certified by mean square error-based learning curves, regression metrics and error histogram analysis. Several significant results for Deborah number, Hall parameters and magnetic field parameters have been presented in graphical and tabular form.

Keywords Cattaneo–Christov heat flux · Artificial neural network · MHD flow · Jeffery fluid · Hall current · Heat transfer · Variable thickness · Stretching sheet

List of symbols

\[ x, y, z \] Cartesian coordinate system
\[ D_B \] Brownian diffusion coefficient
\[ a, b, c, m \] Positive constants

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Heterogeneous porous materials, which are composed of different materials with distinct physical properties, are crucial in numerous applications such as energy storage, filtration, and medical devices. The physical properties of these porous materials are significantly influenced by the interplay between the solid, liquid, and gaseous phases present in the microstructure. Among these properties, thermal conductivity is a critical factor that determines the efficiency of the material in various applications. Therefore, understanding and predicting the thermal conductivity of heterogeneous porous materials is essential for optimizing their performance.

The thermal conductivity of a material is a measure of its ability to transfer heat through itself. It is a fundamental property that is influenced by several factors, including the material's microstructure, temperature, and pressure. In heterogeneous porous materials, the thermal conductivity is a function of the solid phase's thermal conductivity, the liquid and gas phase's thermal conductivity, and the porosity of the material. The porosity, in turn, is determined by the material's microstructure, which can be highly variable depending on the fabrication process.

To predict the thermal conductivity of heterogeneous porous materials, a variety of models have been developed. These models include empirical equations, continuum mechanics-based models, and micromechanical models. Empirical equations are simple and efficient but lack the ability to capture the microstructural effects on thermal conductivity. Continuum mechanics-based models, on the other hand, consider the material's microstructure but can be computationally expensive. Micromechanical models, such as the Mori-Tanaka method, provide a detailed description of the microstructure and its effect on thermal conductivity but can be complex to implement.

In recent years, computational fluid dynamics (CFD) has emerged as a powerful tool for predicting the thermal conductivity of heterogeneous porous materials. CFD models can simulate the flow of fluids through the microstructure and provide insights into the heat transfer mechanisms. However, the accuracy of these models depends heavily on the quality of the microstructural data and the assumptions made in the model.

To enhance the accuracy of these models, there has been a growing interest in using machine learning techniques, such as artificial neural networks (ANNs), to predict the thermal conductivity of heterogeneous porous materials. ANNs can learn complex patterns from the data and provide accurate predictions even when the underlying physics is not well understood. This approach has the potential to revolutionize the field by providing a powerful tool for predicting the thermal conductivity of heterogeneous porous materials without the need for extensive experimental data.

In conclusion, the thermal conductivity of heterogeneous porous materials is a critical property that is influenced by multiple factors. To predict this property accurately, a combination of experimental methods, theoretical models, and computational tools, such as ANNs, is required. Further research in this area is needed to develop more accurate and efficient methods for predicting the thermal conductivity of heterogeneous porous materials.
numerous industrial and technological applications including wire coating, dyeing, polymer productions, food dispensation, geophysics, chemical and petroleum, plastic manufacturing, biological fluid, etc. Sreelakshami et al. [10] present the relation of Jeffrey fluid for non-Newtonian fluids. The power-law fluid with the effect of MHD and entropy generation was studied by Khan et al. [11]. Noreen et al. [12] examined Dufour and Soret effects on Jeffrey fluid. The MHD boundary layer with Jeffrey fluid was examined by Shahzad et al. [13]. Researchers investigate a Jeffery fluid under different circumstances. Patel and Maher [14] work on the Jeffery–Hemal flow with a magnetic field. Vaidya et al. [15] look at the peristaltic Jeffery liquid with heat movement in an opposite permeable layer. Nazeer et al. [16] give the impact of non-linear thermal radioactivity on the 3D Jeffery fluid over shrinking/stretching surface in the occurrence of heterogeneous–homogeneous reactions, and injection/suction. Some potential studies about Jeffery fluid are found in these Refs. [17–20]. Asha and Sunitha [21] studied the effect of heat transfer and hall current on peristaltic blood flow on MHD with Jeffery fluid in a permeable channel. More features of Jeffery fluid with Hall current over 3D are investigated by Sinha et al. [22]. In literature, a lot of research have been done on the transportation of heat and mass theories such as enhancing the heat transfer rates and pressure loss reduction using compact heat exchanger [23]. Cattaneo model was further modified by Christov by changing time-derivative with Oldroyd-B variant [24, 25] and stability is reported in [26]. Further relevant studies on Cattaneo–Christov heat flux model (CCHFM) can be seen in [27–29]. Alamir et al. [30] work on the perspective of CCHFM. Shah et al. [31] used this model for micropolar ferrofluid on a stretched sheet. Cattaneo-Cristov heat flux model incorporated with slip condition is studied by Ahmad et al. [32]. Khan et al. [33] worked on numerical and analytical solutions of Maxwell fluid on a stretched cylinder with CCHFM.

The objective of the present research is to explore the effects of Hall current on MHD with Jeffery fluid over a nonlinear stretchable sheet. We examined the applications of Hall current on MHD with a different perspective. We analyze the characteristics of CCHFM over the variable stretchable sheet with varied thickness. In this regards, we consider the influences of heat transfer, temperature, velocity, stretching sheet with variable thickness, effects of the electric field, induced magnetic field, Hall current parameter, Jeffrey parameters, Deborah number, Nusselt number, skin friction coefficient, shear stress are computed. Mathematical modeling will be presented to construct the nonlinear coupled ordinary differential equations. Similarity transformations are applied and transformed governing equations using Adams Bashforth method. Further, the experimental data will be analyzed using the Artificial Neural Network model with the Levenberg Marquardt method (ANN-LMM). Artificial intelligence techniques-based stochastic approaches are based on machine learning mechanism which works on the pattern of human behavior to find the stiff and valuable solutions to various types of important problems related to face identification, device management system, radar assembling, cancer diagnostic mechanism, and virus deification. The main components in the intelligent system work with the setting and adjustments of neurons and layers which play a vital and significant role for the best modeling of the designed networks and for their optimizations through different local and global heuristics. system etc. Robbins and Monro [34] analyzed intelligent computing infrastructure for the mathematical system. Mehmoond et al. [35] examined the thermal transfer through a fluid flow via the design of a stochastic intelligent computing system. The ANN technique for heat transfer rate are analyzed by Sheikhholeslami et al. [36]. An exclusive description made via investigators on this regime consists of [37–39]. Transformed governing equations are analyzed numerically using Adams Bashforth method [40–43].

### Description of the fluid flow system

Consider the electrically conducted, unsteady Jeffrey fluid which passing over a stretching surface with fluctuating thickness. The stretching is by the side of the axial direction (x-axis) and y-axis which is perpendicular to the stretchable surface. The applied magnetic field \( B = [0, 0, 0] \) is taken along y axis. The low magnetic Reynolds number is taken so that the induced magnetic field is negligible. The modified ohm’s law by adding Hall’s current effect are taken into account. The expressions of the Jeffrey model for the non-Newtonian fluids are given as:

\[
\tau = -PI + \frac{\mu}{1 + \lambda_1} + S \tag{1}
\]

\[
S = \frac{\mu}{1 + \lambda_1} \left[ R_1 + \lambda_2 \left( \frac{\partial R_1}{\partial t} + \nabla \cdot V \right) \right] \tag{2}
\]

where \( P \) denotes the pressure, \( \tau \) be the Cauchy stress tensor, \( S \) is the stress tensor, \( R_1 \) is the Rivlin–Ericken tensor, \( \lambda_1 \) stands for the ratio of relaxation to the retardation times while the \( \lambda_2 \) is a retardation time (see Fig. 1).

Governing equations after boundary layer approximation are reduced to [5, 10, 13, 18, 28]:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3}
\]
Physical configuration of the geometry

\[ \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0, \quad v = 0, \quad w = 0 \quad \text{at} \quad y = A(x + b)^{\frac{1}{n+1}} \quad \text{as} \quad y \to \infty. \]  

(6)

In the above expression, \( u, v, w \) and \( x, y \) represent the velocity components and Cartesian coordinates, respectively. \( \nu \) represents the kinematic viscosity, \( \mu \) is the dynamic viscosity and \( \rho \) is the density. The stretching rate \( U(x) = U_0(x + b)^n \), with \( U_0 \) be the reference velocity, \( b \) represents the relative stretching parameter and \( n \) denotes the velocity exponent. The sheet is non-flat, and its surface is taken at \( y = A(x + b)^{\frac{1}{n+1}} \) where \( A \) is the stretching coefficient while the quantities are assumed to be constant along \( z \)-axis.

Modified Ohm’s law with Hall’s current effect is defined as:

\[ J = \sigma \left( E + V \times B - \frac{1}{\epsilon n_e} J \times B + \frac{1}{\epsilon n_e} \nabla \rho_e \right). \]  

(7)

Here \( J \) denotes a current density, \( \sigma \) denotes electric conductivity parameter, \( \rho_e \) is called electronic pressure parameter and \( B_0 \) denotes the magnetic field parameter. The components of \( J \) can be given as follows:

\[ J_x = \frac{\sigma B_0^2}{(1 + m^2)} (mu - w), \quad J_z = \frac{\sigma B_0^2}{(1 + m^2)} (u + mw), \]  

(8)

where \( m \) is the Hall parameter \( m = \frac{\sigma B_0^2}{\epsilon n_e} \). The heat equation of steady viscous flow is defined as:

\[ \rho c_p V_\cdot \nabla T = -\nabla q. \]  

(9)

where \( \rho \) is the density, \( T \) the temperature, \( c_p \) stands for the specific heat while \( q \) be the heat flux. Cattaneo–Christov heat flux law is defined as:

\[ q + \lambda(V_\cdot \nabla q - q_\cdot \nabla V + V_\cdot V)q = -\kappa \nabla \cdot T. \]  

(10)

Here \( \kappa \) represents the thermal conductivity, \( \lambda \) be the thermal relaxation factor. For incompressible flow

\[ q + \lambda(V_\cdot \nabla q - q_\cdot \nabla V)q = -\kappa \nabla \cdot T. \]  

(11)

From Eqs. (9) and (10) we have

\[ \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial^2 y} - \lambda \left[ \left( \frac{\partial u}{\partial x} + \nu \frac{\partial v}{\partial y} \right) \frac{\partial T}{\partial x} + \left( \frac{\partial u}{\partial y} + \nu \frac{\partial v}{\partial x} \right) \frac{\partial T}{\partial y} + 2 \nu \frac{\partial^2 T}{\partial x \partial y} \right] + \mu \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right). \]  

(12)

Conditions for fluid temperature are

\[ T = T_w \quad \text{at} \quad y = A(x + b)^{\frac{1}{n+1}} \quad \text{and} \quad T \to T_\infty \quad \text{as} \quad y \to \infty. \]  

(13)

The expression \( \alpha = \frac{\kappa}{\rho c_p} \), \( T_w, T_\infty \) defined respective thermal diffusivity parameter, surface temperature and ambient temperature.

**Similarity Conversion system**

The best suitable transformation system for the presented model [44] is proved as follows:

\[ u = U(x)F'(\xi) = U_0(x + b)^n F'(\xi), \quad v = -\sqrt{\frac{n + 1}{2}} \nu U_0(x + b)^{\frac{n-1}{n}} F'(\xi) \left( \frac{n - 1}{n + 1} \right), \quad w = U(x)G(\xi) = U_0(x + b)^n G(\xi), \]  

(14)
Using these transformations, we have:

\[ F'''(\xi) - (1 + \lambda_1) \left( \frac{2n}{n + 1} F'' - f F' \right) \]
\[ + \beta \left[ \frac{n + 1}{2} F F' + \frac{3n - 1}{2} (F')^2 + (n - 1) F'' \right] \]
\[ - \frac{2M}{(1 + n)(1 + m^2)} (1 + \lambda_1)(F' + mG) = 0, \quad (15) \]

\[ G''(\xi) - (1 + \lambda_1) \left[ \frac{2n}{n + 1} F G - FG' \right] \]
\[ + \beta \left[ (1 - n) G'' F + \frac{3n - 1}{2} F'' G' - \frac{n + 1}{2} FG'' \right] \]
\[ + \frac{2M}{(1 + n)(1 + m^2)} (1 + \lambda_1)(mF' - G) = 0, \quad (16) \]

\[ \Theta'' + \text{Pr} \left[ F \Theta' + \gamma \left( \frac{n - 3}{2} F F' \Theta' - \frac{n + 1}{2} F^2 \Theta'' \right) \right] = 0 \quad (17) \]

With the transformed boundary conditions:

\[ F(\alpha) = \alpha \frac{1 - n}{1 + n}, \quad F'(\alpha) = 1, \quad G(\alpha) = 0, \quad \Theta(\alpha) = 0, \quad (18) \]

In which differentiation is with respect to \( \xi \). We further assume the following [44]:

\[ F(\xi) = f(\xi - \alpha) = f(\eta), \]
\[ G(\xi) = g(\xi - \alpha) = g(\eta), \]
\[ \Theta(\xi) = \theta(\xi - \alpha) = \theta(\eta), \]

where \( \xi - \alpha = \eta \), and \( \alpha = A \sqrt{\frac{n + 1}{2} \frac{U_0}{v}} \). Then Eqs. (15)–(17) becomes:

\[ f'''(1 + \lambda_1) \left[ \frac{2n}{n + 1} (f')^2 - f f'' \right] \]
\[ + \beta \left[ \frac{n + 1}{2} f f' + \frac{3n - 1}{2} (f')^2 + (n - 1) f'' \right] \]
\[ - \frac{2M}{(1 + n)(1 + m^2)} (1 + \lambda_1)(f' + mG) = 0, \quad (20) \]

\[ g''(1 + \lambda_1) \left[ \frac{2n}{n + 1} f' g - f g' \right] \]
\[ + \beta \left[ (n - 1) g'' f + \frac{3n - 1}{2} f'' g' - \frac{n + 1}{2} f g'' \right] \]

\[ + \frac{2M}{(1 + n)(1 + m^2)} (1 + \lambda_1)(m f' - g) = 0, \quad (21) \]

\[ \theta'' + \text{Pr} \left[ f \theta' + \gamma \left( \frac{n - 3}{2} f f' \theta' - \frac{n + 1}{2} f^2 \theta'' \right) \right] = 0 \quad (22) \]

Here \( M \) denotes the magnetic field parameter, \( \gamma \) is the thermal relaxation parameter, \( \beta \) is the Deborah number, \( \text{Pr} \) is the Prandtl number, \( \alpha \) is the thermal diffusivity and \( m \) is the Hall parameter. In boundary conditions \( \alpha \) is the wall thickness parameter, \( \eta \) corresponds to the surface of the sheet which non-dimensional similarity variable, \( f \) is the dimensionless stream function, \( T \) is the temperature of the fluid, \( T_w \) is the surface temperature, \( T_{\infty} \) is the ambient temperature. \( \alpha \) is the dimensionless temperature. Where prime denotes the derivative with respect to \( \eta \). Parameters involved in the non-dimensional equations are:

\[ M = 2 \sigma (x + b) \nu B^2 / \rho U(x) \]

represent the magnetic field parameter, \( \nu = \rho \epsilon / k \), represents Prandtl number \( \gamma = \lambda U_0 (x + b)^{\alpha - 1} / \nu \) is the thermal relaxation parameter, \( \alpha = A \sqrt{(n + 1) U_0 / 2 \nu} \) is wall thickness parameter where \( A = \gamma (x + b)^{\alpha} \) and \( \beta = \lambda_2 A \) is the Deborah number where \( A = \gamma / \lambda \).

**Skin friction factor and Nusselt number**

The skin friction coefficient at the stretched surface is written as:

\[ C_f = \frac{\tau_{w_1}}{\rho u_w^2}, \quad \text{where} \]
\[ \tau_{w_1} = \frac{\mu}{1 + \lambda_1} \left[ \frac{\partial u}{\partial y} + \lambda_2 \left( \frac{\partial^2 u}{\partial x \partial y} + \nu \frac{\partial^2 u}{\partial y^2} \right) \right]_{y = A(x + b) \frac{1 - n}{1 + n}}, \quad (24) \]

\[ C_f = \frac{\tau_{w_z}}{\rho u_w^2}, \quad \text{where} \]
\[ \tau_{w_z} = \frac{\mu}{1 + \lambda_1} \left[ \frac{\partial w}{\partial y} + \lambda_2 \left( \frac{\partial^2 w}{\partial x \partial y} + \nu \frac{\partial^2 w}{\partial y^2} \right) \right]_{y = A(x + b) \frac{1 - n}{1 + n}}. \quad (25) \]

Here \( \tau_{w_1}, \tau_{w_z} \) are shear-stress of the surface in the horizontal direction and shear stress is perpendicular to the
horizontal direction, accordingly. Dimensionless form is:

\[ C_f \frac{Re_x}{2} = \frac{1}{1 + \lambda_1} \left[ f'' + \beta \left( \frac{3n - 1}{2} f' f'' - \frac{n + 1}{2} f f''' \right) \right] \]

(26)

\[ C_f \frac{Re_x}{2} = \frac{1}{1 + \lambda_1} \left[ g' + \beta \left( \frac{3n - 1}{2} f' g' - \frac{n + 1}{2} f g'' \right) \right] \]

(27)

where \( C_f, C_{f_1} \) denotes skin friction \( Re_x = \frac{(x+b)U(x)}{v} \) is the Local Reynold number.

The heat transfer rate relations are written as follows:

\[ Nu_x = \frac{xq_w}{k(T_W - T_\infty)}, \quad q_w = -k \frac{\partial T}{\partial y}_{y=A(x+b)+\frac{b}{2}} \]

(28)

Here \( q_w \) is the surface heat flux. Non-dimensional form is:

\[ Nu_x \frac{1}{Re} = -\sqrt{1 + \frac{n}{2}} \theta''(0), \]

(29)

where \( Nu_x \) represents the local Nusselt Number.

**Structure of the designed intelligent network**

Stat of the art Adams Bash-forth numerical method is incorporated with the assistance of ND-Solve command exploited through Mathematica software. The considered numerical is the best suitable numerical computing technique for the generation of the dataset for further designing of the artificial neural networks. The diagram of the designed intelligent network is described in Fig. 2.

The above-mentioned network is a mathematical system inspired by biological neural networks, which is dependent upon the collection of neurons. Neurons are the integral and important component of the designed soft computing-based intelligent network that transformed the data obtained through any deterministic-based method like Adam’s numerical method and then gives the result in the output layer. Data traveled from the input layer to the layer connected with the output setting of the network. Different layers are also incorporated into the designed networks for finding the best possible outcomes. The total data set is classified into 70% training, 15% validation, 15% testing. The present article carries out the Hall current with Jeffrey fluid and CCHFM. In this regards the experimental data will be analyzed by using the Artificial Neural Network model with Leven-berg Marquardt method (ANN-LMM).

**Numerical results and discussion**

In a current research article, reference numerical result and ANN is applied for the estimate of the Hall effect on MHD flow with Jeffrey fluid and heat transfer with CCHFM. Numerical solutions with the help of ND-Solve command and Artificial Neural Network (ANNs) are investigated. Table 1 is constructed for all variants of the presented MHD flow of the Jeffrey fluid system under the impact of heat. Velocity profile \( f'(\eta) \) is represented through case study 1, and another velocity profile \( g(\eta) \) is shown via case study 2, whereas case study 3 represents the temperature profile \( \theta(\eta) \). Scenarios 1, 2, 3 denotes the variables. Scenarios 1, 2 and 3 of case study 1 represent the Hall current parameter (m), wall thickness parameter (\( \alpha \)), ratio of relaxation to the retardation time (\( \lambda_1 \)). Case study 2 is about Hall current parameter (m), Deborah number (\( \beta \)). Case study 3 represents the thermal relaxation parameter (\( \gamma \)), Prandtl number (Pr), velocity exponent parameter (n).

**Performance analysis of numerical solution:**

We obtained the non-dimensional velocities and temperature profiles for emerging parameters. Using ND-solve command in MATHEMATICA software with ADAM BASHFORTH method, we obtained solutions for profiles \( f'(\eta), g(\eta) \) and \( \theta(\eta) \) for various cases. Figures 3, 4 and 5 investigate the impact of parameter \( m, M, \lambda_1 \) with the ranges \( 0.3 \leq m \leq 0.12, 0.0 \leq M \leq 1.5, 0.0 \leq \lambda_1 \leq 0.12 \) for \( f'(\eta) \). Figure 3 indicates the behavior of hall current parameter (m). The velocity profile \( f'(\eta) \) increases for a large value of \( 0.3 \leq m \leq 0.12 \) because the effect of electric conductivity of
the fluid enhances the molecular movement which results in an increase in fluids velocity. According to this relation \( \sigma/(1 + m^2) \), effective conductivity decreases when we increase the values of \( m \). By increasing the Hall current parameter \( m \), the factor \( 1/(1 + m^2) \) becomes smaller so the resistivity of the fluid decreases whereas it shows the same behavior in the case of velocity profile \( g(\eta) \) which is observed in Fig. 6. When we increase the value of Hall current parameter \( m \), the velocity profile also increases. Figure 4 shows the impact of the magnetic field parameter \( (M) \). When we increase the value of \( M \) the velocity component \( f'(\eta) \) shows a reduction. This is due to the fact that magnetic field \( M \) induces the resistive force which is also called a Lorentz force while the velocity profile reduces because when the Lorentz force becomes weaker, the motion of the fluid reduces and the fluid become to rest. It is because of the fact the magnetic field acts as retarding/controlling agent and has the ability to control the fluids velocity upto desired value. Figure 5 influences the ratio of relaxation to the retardation time \( \lambda_1 \) on the velocity profile \( f'(\eta) \) along with the boundary layer because the physical ratio of relaxation to the retardation time depends upon the retardation time. As \( \lambda_1 \) increases the relaxation time and

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**Table 1** Values of parameters associated with the fluid flow system

| Case study | 1 | 2 | 3 | 2 | 3 |
|------------|---|---|---|---|---|
| Scenario   | 1 | 2 | 3 | 1 | 2 |
| Case: 1    | 0.3 | 0.0 | 0.0 | 0.1 | 0.3 | 0.2 | 0.71 | 0.1 |
| Case: 2    | 0.6 | 0.3 | 0.4 | 0.3 | 0.4 | 0.6 | 1.0 | 0.3 |
| Case: 3    | 0.9 | 1.0 | 0.8 | 0.6 | 0.6 | 0.7 | 1.5 | 0.6 |
| Case: 4    | 0.12 | 1.5 | 0.12 | 0.9 | 0.8 | 0.8 | 2.0 | 0.9 |
it reduces the retardation time. Jeffrey fluid parameter was the reason for the variation of the momentum boundary layer, so the velocity reduces in the variable sheet. Figure 7 exhibit the effect of Deborah number $\beta$ on the velocity profile $g(\eta)$. As it was observed from Fig. 7 that for the larger value of the Deborah number $\beta$, velocity profile $g(\eta)$ reduces. Physically, Deborah number depends upon retardation time so with the enhancement of the retardation time Deborah number decreases but it increases for the gradient of the velocity profile of the Jeffrey fluid. Figures 8, 9 and 10 exhibit the influence of $\gamma$, Pr, $n$ with the ranges $0.2 \leq \gamma \leq 0.8$, $0.71 \leq \text{Pr.} \leq 2.0$, $0.1 \leq n \leq 0.9$ respectively on the temperature profile $\theta(\eta)$. Figure 8 shows that an increment in the value of thermal relaxation parameter $\gamma$ results into a reduction of the temperature profile $\theta(\eta)$ because the temperature of variable sheet decreases with the enhancement of the thermal relaxation $\gamma$. In case when the thermal relaxation parameter reduces to zero i.e. ($\gamma = 0$) the CCHFM becomes the classical Fourier law of heat conduction. Figure 9 presents the impact of Prandtl number Pr. On the temperature profile $\theta(\eta)$, it is observed from the figure that the temperature profile $\theta(\eta)$ reduces for the larger values of Pr. Number. Physically, the Prandtl number depends upon the thermal diffusivity and thermal diffusivity becomes lower with the enhancement of the Prandtl fluid because of the fluid with high pr. number shows less conduction. Due to this reason when we increase the value of pr. number thermal diffusivity reduced and temperature profile reduces. Figure 10 demonstrated the influence of the velocity exponent parameter ($n$) on $\theta(\eta)$. For the larger $n$, the profile $\theta(\eta)$ increases. The positive value of $n$ i.e ($n > 0$) shows that the variable sheet is stretching. For the transverse velocity distributions, it shows the same behavior. Variation for $M$, Pr., $\alpha$, $\lambda_1$, $m$, $\gamma$, $\beta$ of and skin friction coefficient and Nusselt number are presented in Table 2.
Performance analyses on outcomes of the networks

The elaborative numerical solution of transformed system of ODEs by ANN is presented for various parameters of profiles \( f'(\eta), g(\eta) \) and \( \theta(\eta) \). Solution by ANN with Levenberg Marquardt method (ANN-LMM) interpreted through error histogram, plot fit, training states, performance and regression. Performances of three scenarios of all the cases of case study 1, 2 and 3 are presented. Result will be analyzed by comparison.

Case study 1

The performances of 3 scenarios (m, M, \( \lambda_1 \)) for case study 1 (CS1) for different \( f'(\eta) \) are illustrated in Figs. 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24 and 25. Sub-Fig. 11a–c present the error histogram which exhibits the data fitting error for all the cases which are near to zero error reference line. More positive error is exhibited for case 1 whereas negative error for case 2 and case 3. Figure 12a–c delineate the data of fitness based on difference between the target and network outputs after training a neural network, i.e., the difference between the predicted value and target value. The range of the absolute zero in three cases of scenario 1 is \((-2 \times 10^{-4} \text{ to } 2 \times 10^{-4})\). The error is found to be very close to zero which shows the fitness of the method with good accuracy. Means square error (MSE) of three cases of scenario 1 is illustrated in sub-Fig. 13a–c. Minimum of MSE is achieved at epoch (349, 66, 264) with respective best validation performance (8.262e−10, 3.15671e−10, 1.5309e−9). Figure 14a–c present algorithm execution states of the networks. Training states describe the outcomes of controlling indices of Mu and gradient. For all the cases best validation performance depends upon epoch weights. The values of the gradient for all the cases at (349, 66, 264) are given respectively as (9.96e−8, 9.51e−8, 9.99e−8) that verified networks performance. Sub-Figure 15a–c gives the regression plots of the data for different outputs. The regression measure, i.e., correlation \( R = 1 \), exhibits a strong correlation consistently.

Figures 16, 17, 18, 19 and 20 describe the interpretation of three cases of scenario 2 of case study 1 (CS1) graphically. Figure 16a–c portray plots for the error histogram studies. The error is consistently close to reference with higher error for cases 1 and 2 and relatively less for case 3 which shows the reasonable performance. Sub-Figure 17a–c present the fitness plots. Interval of the absolute error with the ranges \((-5 \times 10^{-4} \text{ to } 5 \times 10^{-4}), -2 \times 10^{-4} \text{ to } 2 \times 10^{-4}, -5 \times 10^{-4} \text{ to } 5 \times 10^{-4})\) respectively are analyzed which are close to zero. Figure 18a–c demonstrate the MSE for all the cases of scenario 2. The MSE gradually decreases with epochs, it is observed at epoch (73, 89, 298) with the best respective validation performance are (1.33312e−8, 6.41000e−10, 9.79812e−10). Training states depend upon Mu, gradient and validation check as shown in Fig. 19a–c. The magnitude of the gradient (1.78e−5, 9.99e−8, 9.99e−8) at (73, 89, 298). The sub-Fig. 20a–c present the regression plots with a value of \( R = 1 \) for scenario 2 consistently for each dataset of the model.

Performance analysis of ANNs model for each variation of scenario 3 is graphically provided in Figs. 21, 22, 23, 24 and 25. The sub-Fig. 21a–c displayed the error histograms of scenario 3 for three cases. The data set points with an error close to zero having less errors for cases 1 and 3 whereas bit more errors for case 2. The sub-Fig. 22a–c present the fitness plots which show that the error is close to zero i.e., the predicted value suits to experimental data values. The error for all the cases lies in the range of \((-2 \times 10^{-4} \text{ to } 2 \times 10^{-4}), -1 \times 10^{-5} \text{ to } 1 \times 10^{-5}, -2 \times 10^{-4} \text{ to } 2 \times 10^{-4})\). Figure 23 demonstrated the performance analysis graphically After training the data set the least value of MSE is obtained. The best validation performance at epochs (284, 293, 249) is given as (1.6407e−9, 22.23755e−11, 8.87428e−10) which shows that the data set is well trained. Further, the training states give Mu, gradient, validation checks observed in Fig. 24a–c. The gradient is (9.96e−8, 9.97e−8, 9.83e−8) for the three cases of scenario 3 which gives consistently viable results. Additionally, the value of Mu (1e−9, 1e−9, 1e−9) is found close to zero for scenario 3 in each case. Sub-Figure 25a–c also show the regression analysis of the predicted and target value of scenario 3 with regression index \( R = 1 \) that presents the rationality of the accurate performance of the ANN network model.

Case study 2

The analysis of networks for velocity profile \( g(\eta) \) of different scenarios for various emerging parameters of case study 2 (CS2) are illustrated in Figs. 26, 27, 28, 29, 30, 31, 32, 33, 34 and 35. Figure 26a–c depicts accurately our model through the prediction of the data set after training. Higher positive error exhibits for case 1 and negative error for cases 2 and 3. Figure 27a–c demonstrates the fitting plots graphically for scenario 1 of three cases. Absolute errors for the presented fluid system show good performance analysis as its most of the values lie in the ranges \((-5 \times 10^{-5} \text{ to } 5 \times 10^{-5}, -2 \times 10^{-4} \text{ to } 2 \times 10^{-4}, -1 \times 10^{-4} \text{ to } 1 \times 10^{-4})\) respectively with better precision. The sub-Fig. 28a–c presents the performance analysis with the best validation performance of the given data set for three cases of scenario 1. The minimum value of MSE are (1.57918e−10, 1.41400e−10, 2.98581e−10) at the epoch (12, 21, 39) with good validation. The sub-Fig. 29a–c depict the training states of all variants associated with scenario 1. The magnitudes of the gradient (6.14e−8, 9.60e−8, 9.35e−8) at the epoch (12, 21, 39). Further, the value of Mu for all the cases of scenario 1 is (1e−12, 1e−11, 1e−11) which shows the convergence of the ANN.
Table 2 Outcomes on Skin friction coefficients $-C_f x Re_{0.5}$, $C_f x Re_{0.5}$ and local Nusselt number $Nu x Re_{0.5}$ various values of parameters

| M  | Pr | $\alpha$ | $\lambda_1$ | m   | $\gamma$ | $\beta$ | $-C_f x Re_{0.5}$ | $C_f x Re_{0.5}$ | $Nu x Re_{0.5}$ |
|----|----|----------|-------------|-----|---------|---------|------------------|------------------|-----------------|
| 0.1| 0.71| 0.1      | 0.12        | 0.1 | 0.1     | 0.3     | 0.882806         | 0.00867397       | 0.499076        |
| 0.5|     |          |             |     |         |         | 1.76644          | 0.0350695        | 0.490395        |
| 1.0|     |          |             |     |         |         | 1.48126          | 0.0573754        | 0.48217         |
| 1.5|     |          |             |     |         |         | 1.7403           | 0.0739294        | 0.475888        |
| 0.5| 0.71| 0.1      |             |     |         |         | 1.17644          | 0.0350695        | 0.490395        |
| 1.0|     |          |             |     |         |         | 1.17644          | 0.0350695        | 0.542903        |
| 1.5|     |          |             |     |         |         | 1.17644          | 0.0350695        | 0.636657        |
| 2.0|     | 0.1      |             |     |         |         | 1.17643          | 0.0350695        | 0.732635        |
| 0.5| 0.71| 0.1      | 0.11        | 0.1 | 0.3     |         | 1.17674          | 0.352166         | 0.491761        |
| 0.5|     |          |             |     |         |         | 1.15536          | 0.0347679        | 0.491071        |
| 1.0|     |          |             |     |         |         | 1.14012          | 0.0344409        | 0.490563        |
| 1.5|     | 0.21     |             |     |         |         | 1.12552          | 0.034218         | 0.490064        |
| 0.12| 0.1  | 0.3      |             |     |         |         | 1.17644          | 0.350695         | 0.490395        |
| 0.3 |     | 1.15457  |             |     |         |         | 1.09684          | 0.165504         | 0.492517        |
| 0.6 |     | 1.03398  |             |     |         |         | 0.195308         | 0.494294         |                 |
| 0.9 |     | 1.17644  |             |     | 0.1     |         | 0.350695         | 0.490395         |                 |
| 0.1 |     | 1.17644  |             |     | 0.5     |         | 0.350695         | 0.467594         |                 |
| 1.0 |     | 1.17644  |             |     |         |         | 0.350695         | 0.43867          |                 |
| 2.0 |     | 1.17644  |             |     |         |         | 0.0350695        | 0.379288         |                 |
| 0.3 | 1.2  | 0.3      | 0.1         | 0.1 | 0.1     |         | 1.63931          | 0.0622123        | 0.803698        |
| 0.3 |     | 1.73033  |             |     |         |         | 0.350695         | 0.490395         |                 |
| 0.5 |     | 1.81492  |             |     |         |         | 0.694501         | 0.806599         |                 |
| 0.8 |     | 1.93405  |             |     |         |         | 0.723667         | 0.810269         |                 |

Fig. 11 Error histogram of Scenario 1. a Case: 1, b Case: 2, c Case: 3
Fig. 12  Plot fit of scenario 1. a Case: 1, b Case: 2, c Case: 3

Fig. 13  Performance curve of scenario 1. a Case: 1, b Case: 2, c Case: 3

Fig. 14  Training state of scenario 1. a Case: 1, b Case: 2, c Case: 3
Fig. 15 Regression plot of scenario 1. a Case: 1, b Case: 2, c Case: 3

Fig. 16 Error histogram of scenario 2. a Case: 1, b Case: 2, c Case: 3

Fig. 17 Plot fit of scenario 2. a Case: 1, b Case: 2, c Case: 3
Fig. 18 Performance curve of scenario 2. a Case: 1, b Case: 2, c Case: 3

Fig. 19 Training state of scenario 2. a Case: 1, b Case: 2, c Case: 3

Fig. 20 Regression plot of scenario 2. a Case: 1, b Case: 2, c Case: 3
Fig. 21 Error Histogram of scenario 3. a Case: 1, b Case: 2, c Case: 3

Fig. 22 Plot fit of scenario 3. a Case: 1, b Case: 2, c Case: 3

Fig. 23 Performance curve of scenario 3. a Case: 1, b Case: 2, c Case: 3

model. Figures 30a–c presents the regression metric \( R = 1 \) for the three cases of scenario 1 with good accuracy of the ANN model.

Figures 31, 32, 33, 34 and 35 influence the scenario for three cases of case study 2 (CS2) Fig. 31a–c depicts that most of the values give reasonable accuracy for case 1, whereas higher negative zero error for case 2 and 3. Figure 32 shows the plot fitness of all variants associated with scenario 1. The error was found close to \((-2 \times 10^{-4} \text{ to } 2 \times 10^{-4})\) for all the cases. Figure 33a–c shows the mean square error and the best validation performance at epoch (7, 9, 9) are given (6.29335e-9, 1.96818e-9, 2.22200e-9), respectively, which shows that the data set is well trained. Moreover,
Fig. 24 Training state of scenario 3. a Case: 1, b Case: 2, c Case: 3

Fig. 25 Regression plot of scenario 3. a Case: 1, b Case: 2, c Case: 3

Fig. 26 Error histogram of scenario 1. a Case: 1, b Case: 2, c Case: 3
the magnitude of controlling parameters, i.e., Mu and gradient, are presented in Fig. 34a–c. The value of gradient for three cases (9.66e⁻⁸, 8.17e⁻⁸, 5.07e⁻⁸) with best fitting data sets. The magnitude of Mu for all the cases is (1e⁻¹⁰, 1e⁻¹², 1e⁻¹⁲). Figure 35a–c shows the best regression analysis (R = 1) plot for variants associated with scenario 2 with good accuracy between the target and output values.

Case study 3

The network is designed to plot the temperature profile θ(η) for two scenarios (γ, Pr.) for all variants associated with the Jeffrey fluid flow system and are presented graphically in Figs. 36, 37, 38, 39, 40, 41, 42, 43, 44 and 45. Figure 36a–c shows the histogram analysis of the ANN model of scenario 1 for three cases. The zero error line close to zero
Fig. 30 Regression plot of scenario 1. a Case: 1, b Case: 2, c Case: 3

Fig. 31 Error histogram of scenario 2. a Case: 1, b Case: 2, c Case: 3

Fig. 32 Plot fit of scenario 2. a Case: 1, b Case: 2, c Case: 3
Fig. 33  Performance curve of scenario 2. a Case: 1, b Case: 2, c Case: 3

Fig. 34  Training state of scenario 2. a Case: 1, b Case: 2, c Case: 3

Fig. 35  Regression plot of scenario 2. a Case: 1, b Case: 2, c Case: 3
with negative zero error is observed in case 1, 2 and positive zero error for case 3. The error line which is close to zero represents the accuracy of data sets. Figure 37a–c demonstrates the accuracy of the data for all variants associated with scenario 1. Absolute errors values present the fitness of the network and lie in the range \((-5 \times 10^{-0} to 5 \times 10^{-0}, -1 \times 10^{-0} to 1 \times 10^{-0}, -1 \times 10^{-5} to 1 \times 10^{-5})\), Further, training of data of 3 cases of scenario 1 is illustrated in Fig. 38a–c which presents the best validation performance. The least mean square error MSE with validation performance \((1.098096e^{-10}, 7.21843e^{-14}, 1.11346e^{-11})\) at the corresponding epoch (466, 98, 381) is observed. The training states of data set are presented in Fig. 39a–c, Optimized value of the weight with gradient value \((9.99e^{-8}, 9.67e^{-8}, 9.95e^{-8})\). The value of Mu for three cases of scenario 1 are \((1e^{-8}, 1e^{-13}, 1e^{-9})\) respectively. Figure 40a–c shows the best fitness analysis through regression plots with good accuracy.
The analysis of designed ANNs for scenario 2 of all variants is depicted in Figs. 41, 42, 43, 44 and 45. Figure 41a–c shows the plotted error histograms with negative absolute error for case 1,3 and positive absolute error for case 2. Figure 42a–c analyzed the fitting plot which shows the best error analysis and lies in the range \((-5 \times 10^{-5} to 5 \times 10^{-5}, -5 \times 10^{-5} to 5 \times 10^{-5}, -1 \times 10^{-5} to 1 \times 10^{-5})\) which authenticates the fitness of the proposed neural network. Figure 43a–c presents MSE with the best validation performances of scenario 2 for three cases. The best validation performance with minimum MSE \((1.63600e^{-12}, 8.44640e^{-11}, 9.51037e^{-14})\) at corresponding epoch \((318, 374, 187)\) respectively with good validation. Figure 44a–c shows the training states of all variants of scenario 2. As it is observed from Fig. 44 training states depend upon Mu, gradient, validation checks. The value of the gradient for three cases are \((9.98e^{-8}, 9.97e^{-8}, 9.99e^{-8})\) at the corresponding epoch \((318, 374, 187)\), respectively. The magnitude of Mu for three values are \((1e^{-10}, 1e^{-8}, 1e^{-13})\) which is close to zero with good accuracy. Figure 45 shows the best regression analysis of scenario 2 for all variants with good error fitness of ANN showing the closeness of output and target values.

Tabular description for case studies 1–3

The results presented in Tables 3, 4 and 5 describe the trials of performance of the networks for the three cases studies. Results of different scenarios of all the cases with the least value of MSE at the respective epoch are presented in tables. The value of regression \((R)\) for all the cases are 1. Different values of Mu and gradient are presented. Time analysis are presented in tables where the maximum time for an accurate result is 6 s (see Table 6).

Concluding remarks

In the presented investigation, a Hall current effect on Magnetohydrodynamics flow with Jeffrey fluid and Heat transfer with CCHFM over a stretchable sheet with varied thickness.
The results are effectively analyzed through designed ANN-LMM using error histogram, plot fit, performance, training states, regression plot. Major outcomes of the present study are summarized below:

1. Both velocity components $f'(\eta)$ and $g(\eta)$ along with the skin friction coefficient in the horizontal as well as in and $z$-axis direction are accelerated with the increase in Hall current parameter ($m$). Actually, it happens due to the controlling mechanism of electric conductivity for the fluid system, which accelerates molecular movement.

2. Magnetic field parameter reduces the thickness of momentum boundary layer along x-axis, while an increment in $M$ will tend to reduce fluid velocity as magnetic field parameter is the ratio of electromagnetic force to the viscous force and due to this fact drag force is enhanced resulting in the increment in the skin friction coefficients along with $x$ and $z$ axes directions, respectively.
Fig. 44 Training state of scenario 2. a Case: 1. b Case: 2. c Case: 3

Fig. 45 Regression plot of scenario 2. a Case: 1. b Case: 2. c Case: 3

Table 3 Convergence analysis presentation for all variants of Jeffery fluid-related CS

| S | Case | Neurons | MSE-based fitness | Gradient | $R$ | Epochs | Mu | Running time |
|---|------|---------|-------------------|----------|----|--------|----|--------------|
|   |      |         | Training          | Validation| Testing |        |    |              |
| 1 | 1    | 10      | 6.8527e−10        | 8.2625e−10| 2.878e−09 | 9.96e−08 | 1  | 349          | 1.00e−08 | 0:00:04 |
| 2 | 2    | 10      | 6.9184e−10        | 3.1567e−10| 1.7468e−09| 9.51e−08 | 1  | 66           | 1.00e−10 | 0:00:01 |
| 3 | 3    | 10      | 2.2067e−09        | 1.5309e−09| 1.6088e−09| 9.99e−08 | 1  | 264          | 1.00e−08 | 0:00:04 |
| 2 | 1    | 10      | 4.7119e−09        | 1.3331e−10| 5.9424e−09| 1.78e−05 | 1  | 73           | 1.00e−08 | 0:00:01 |
| 2 | 2    | 10      | 7.0416e−10        | 6.4100e−10| 7.4295e−10| 9.99e−08 | 1  | 89           | 1.00e−10 | 0:00:01 |
| 3 | 3    | 10      | 3.1285e−10        | 4.3772e−10| 7.6601e−10| 9.798e−09| 1  | 298          | 1.00e−08 | 0:00:03 |
| 3 | 1    | 10      | 5.1799e−10        | 1.6407e−09| 5.8558e−09| 9.96e−08 | 1  | 284          | 1.00e−09 | 0:00:03 |
| 2 | 2    | 10      | 1.8271e−11        | 2.2375e−11| 2.5498e−11| 9.97e−08 | 1  | 293          | 1.00e−09 | 0:00:03 |
| 3 | 3    | 10      | 1.0590e−09        | 8.8742e−10| 1.8105e−09| 9.83e−08 | 1  | 48           | 1.00e−10 | 0:00:02 |
Table 4 Convergence analysis presentation for all variants of Jeffery fluid-related CS-2

| Scenario | Case | Neurons | MSE (Training) | MSE (Validation) | MSE (Testing) | Gradient | $R$ | Epochs | Mu | Running time |
|----------|------|---------|---------------|-----------------|--------------|----------|-----|--------|----|-------------|
| 1        | 1    | 10      | 1.8441e–10    | 1.5791e–10      | 1.2956e–10   | 6.14e–08 | 1   | 12     | 1.00e–12 | 0:00:00    |
| 2        | 1    | 10      | 1.6421e–10    | 1.4140e–10      | 1.2045e–10   | 9.60e–08 | 1   | 21     | 1.00e–11 | 0:00:00    |
| 3        | 10   | 10      | 1.1878e–10    | 2.9858e–10      | 1.2045e–10   | 9.35e–08 | 1   | 39     | 1.00e–11 | 0:00:00    |
| 2        | 1    | 10      | 7.0042e–09    | 6.2933e–09      | 9.4006e–09   | 9.66e–08 | 1   | 07     | 1.00e–10 | 0:00:00    |
| 2        | 10   | 10      | 1.5642e–09    | 1.9681e–09      | 1.8550e–09   | 8.17e–08 | 1   | 09     | 1.00e–12 | 0:00:00    |
| 3        | 10   | 10      | 1.4827e–09    | 2.2220e–09      | 1.0160e–09   | 5.07e–08 | 1   | 09     | 1.00e–12 | 0:00:00    |

Table 5 Convergence analysis presentation for all variants of Jeffery fluid-related CS-3

| Scenario | Case | Neurons | MSE (Training) | MSE (Validation) | MSE (Testing) | Gradient | $R$ | Epochs | Mu | Running time |
|----------|------|---------|---------------|-----------------|--------------|----------|-----|--------|----|-------------|
| 1        | 1    | 10      | 1.0614e–10    | 1.0980e–10      | 1.2006e–10   | 9.99e–08 | 1   | 466    | 1.00e–08 | 0:00:06    |
| 2        | 10   | 10      | 5.6532e–14    | 7.2184e–14      | 7.3058e–14   | 9.67e–08 | 1   | 98     | 1.00e–13 | 0:00:01    |
| 3        | 10   | 10      | 1.0913e–11    | 1.1134e–11      | 1.2658e–11   | 9.95e–08 | 1   | 381    | 1.00e–09 | 0:00:05    |
| 2        | 1    | 10      | 1.1531e–12    | 1.6360e–12      | 1.7100e–12   | 9.98e–08 | 1   | 318    | 1.00e–10 | 0:00:04    |
| 2        | 10   | 10      | 8.0637e–11    | 8.4464e–11      | 1.0036e–10   | 9.97e–08 | 1   | 347    | 1.00e–08 | 0:00:06    |
| 3        | 10   | 10      | 4.8363e–14    | 9.5103e–14      | 8.1864e–14   | 9.99e–08 | 1   | 187    | 1.00e–13 | 0:00:04    |

Table 6 Comparison of present results with published data [45] for different values of $n$

| $n$ | Present results $f''(0)$ when $\alpha = 0.25$ [45] | Present results $f''(0)$ when $\alpha = 0.25$ [45] |
|-----|-------------------------------------------|-------------------------------------------|
| −0.6 | 0.8503                                   | −1.4522                                   |
| −0.5 | −0.0833                                  | −1.1667                                   |
| −0.33| −0.5000                                  | −1.0000                                   |
| 0.0  | −0.7843                                  | −0.9576                                   |
| 0.5  | −0.9338                                  | −0.9799                                   |
| 3.0  | −1.0905                                  | −1.0359                                   |
| 5.0  | −1.1186                                  | −1.0486                                   |
| 7.0  | −1.1323                                  | −1.0550                                   |
| 10.0 | −1.1433                                  | −1.0603                                   |

Whereas along $x$-axis and $z$-axis the skin friction coefficient increases for $M$.

3. The temperature profile for $\theta(\eta)$ shows a reduction with an increment of Pr as is the ratio of momentum diffusivity to the thermal diffusivity as due to the large value of Pr, the thermal diffusivity becomes low which declines the temperature profile.

4. The velocity component $f'(\eta)$ tends to increase with an increment in Deborah number $\beta$, while the opposite behavior is found for $g(\eta)$.

5. Velocity profile $f'(\eta)$ decreases for larger value of ratios of relaxation to the retardation time ($\lambda_1$) while $g(\eta)$ shows opposite behavior.

6. With an increment in relaxation time of the heat flux $\gamma$ tend to decrease temperature profile $\theta(\eta)$.

7. Local Nusselt number Nu$_x$ increase with increment in the Pr, $\alpha$, $m$, $\beta$ and decreases with increase in $M$, $\lambda_1$, $\gamma$/As the larger value of the Nu$_x$ corresponds to more effective convection in the fluid flow system.

8. In the Artificial neural network, the error between the target and output value after training are analyzed by an error histogram. The Regression ($R$) of the trained data set for all the cases is 1.i-e ($R = 1$).

In the future, one may exploit/investigate the strength of the proposed ANN-LMM in various applications arising
in the studies of nanofluids [46–52] and nonlinear systems [53–55].

Acknowledgements Prof M. Y. Malik extends his appreciation to the Deanship of Scientific Research at King Khalid University, Abha, 61413, Saudi Arabia for funding this work through the research group program under number RGP-2-110-43.

Declarations

Conflict of interest The authors declared that they have no conflict of interest.

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