Current driven magnetization dynamics in helical spin density waves

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A mechanism is proposed for manipulating the magnetic state of a helical spin density wave using a current. In this paper, we show that a current through a bulk system with a helical spin density wave induces a spin transfer torque, giving rise to a rotation of the order parameter. The use of spin transfer torque to manipulate the magnetization in bulk systems does not suffer from the obstacles seen for magnetization reversal using interface spin transfer torque in multilayered systems. We demonstrate the effect by a quantitative calculation of the current induced magnetization dynamics of Erbium. Finally we propose a setup for experimental verification.

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The possibility to manipulate the magnetization of a magnetic material using the spin transfer torque (STT) was proposed from theoretical considerations ten years ago by Slonczewski [1] and Berger [2]. Since then STT has been a very active area of research boosted by its potential technological application in magnetic random access memories [3]. The phenomenon has been studied in layered materials where it is considered to be an interface effect occurring when a current passes an interface between two regions with different magnetization direction [4]. Experimentally, it has been shown that the current induced torque is able to switch the magnetization of a ferromagnetic layer in a nano pillar containing a ferromagnetic/non magnetic/ferromagnetic trilayer [5]. An obstacle for using the STT in technological applications is the magnitude of the smallest current needed to switch the magnetization. Theoretical models of trilayer systems have shown that the size of this critical current is determined by the magnetic anisotropies [6].

In this work we consider materials with a helical spin density wave (SDW), a magnetic ordering which mathematically can be described as a spin spiral (SS). [5] In a SS the direction of the magnetization rotates as one moves along the SS wave vector. Without loss of generality we consider the magnetization rotation axis to be parallel to the SS wave vector and will refer to it as the spin spiral axis. In comparison with a multilayer a SS can be considered as a self assembled magnetic multilayer system where the atomic layers perpendicular to the spiral axis are ferromagnetic, but where the magnetization of the different layers are non parallel with respect to each other. We predict that a new kind of STT will be induced by a current parallel with the spiral axis. This phenomenon is a bulk effect in contrast to the STT in a multilayered nano pillar. An advantage with a bulk system is the possibility to better control the magnetic anisotropies that interfere with the STT in an artificial multilayer.

First we give a general argument for the occurrence of STT in SS systems and then we establish the effect by a quantitative calculation of the current induced STT in Er. Er is chosen since it is one of the many well known examples of a helical SDW in the rare earth (RE) series (see e.g. Ref. [8]). The calculation is performed by combining first principles calculations and semiclassical linear response theory.

The STT in multilayer systems can be understood by considering a trilayer system A/B/C of two ferromagnetic layers A and C, separated by a non magnetic layer B. If the two ferromagnetic layers A and C have non parallel magnetization, a current passing perpendicular to the layers, from layer A into layer C will change its spin polarization when passing into layer C. The change in spin polarization will cause a non zero spin flux into layer C, i.e. the spin current entering layer C carries a different spin than the spin current exiting layer C. The net spin flux into layer C acts as a torque on the magnetization of layer C. In a SS system a current along the spiral axis passes through atomic thick layers with non parallel magnetization. The spin of the electronic states that carry the current rotates coherently with the local magnetization of the spiral. The precessing motion of the spins induces a torque on the magnetization that tends to rotate the spin spiral. In the ground state the net STT from all the electronic states cancels out since there is an equal number of electronic states with a current parallel and anti parallel to the spiral axis. But if a net current flows on the spiral axis no cancellation will occur and a STT will act on the total angular momentum of the atoms in the material. The torque will cause the magnetization to rotate with a frequency proportional to the applied current, or equivalently make the SS slide along the spiral axis.

In order to quantify this effect we calculate the STT from a method similar to the method used in Ref. [4] for multilayer systems. The calculations are based on calculations of the spin flux using the spin current density tensor $Q$ (and the STT is investigated both for model systems and by using first principles calculations of real systems). $Q$ is given by

$$Q_{nk}(r) = \text{Tr} \text{Re} \left\{ \psi_{nk}^\dagger(r) S \otimes \hat{v} \psi_{nk}(r) \right\}, \quad (1)$$
where $S$ is the spin operator, $\dot{v}$ is the velocity operator and $\psi_{nk}$ are the spinor wave functions with band and wave vector indexes $n$ and $k$ respectively. The torque exerted by an electronic state with band index $n$ and wave vector $k$ on the angular momentum within a volume $V$ enclosed by the surface $S$ is given by its spin flux into $V$,

$$\int_S Q_{nk} \cdot dS = -\frac{\partial J_{nk}}{\partial t}. \quad (2)$$

In the above equation the torque is expressed as the change of total angular momentum $\partial J_{nk}/\partial t$, within volume $V$.

The volume of interest is arbitrary, for the STT in multilayer systems one considers the torque acting on a magnetic layer. For SS systems the STT on individual atoms can be calculated using non overlapping spheres centered at the atoms. The total torque on an atom is obtained by summing the torque from all the occupied electron states. If the system is in its equilibrium ground state the sum of all torques will be zero due to time invariance. However, if an external electric field $E$ is applied the occupation of the states at the Fermi surface (FS) will change and the total torque on an atom can be calculated from a FS integral derived from semi classical Boltzmann linear response theory,

$$\frac{\partial J}{\partial t} = V_C \frac{\tau e}{(2\pi)^3 \hbar} \sum_n \int_{FS} \frac{\partial J_{nk}}{\partial t} \left( \nabla_k \epsilon_{nk} \cdot E \right) \frac{dS_{nk}}{|\nabla_k \epsilon_{nk}|}, \quad (3)$$

where $V_C$, $\tau$, $e$ and $\epsilon_{nk}$ are the volume of the system, electron relaxation time, electron charge, and band energies respectively. The sum is over all bands crossing the Fermi level. The above equation defines a linear relation between the torque and the external field,

$$\frac{\partial J}{\partial t} = \tau \sum_n A_n \cdot E. \quad (4)$$

A similar expression is obtained for the resistivity, relating the current density with the external field,

$$j = \tau \sum_n B_n \cdot E \quad (5)$$

$$= \frac{-1}{(2\pi)^3 \hbar} \sum_n \int_{FS} \nabla_k \epsilon_{nk} \left( \nabla_k \epsilon_{nk} \cdot E \right) \frac{dS_{nk}}{|\nabla_k \epsilon_{nk}|} \cdot \nabla_k \epsilon_{nk} \cdot E.$$

Combining these two equations gives a linear relation between the torque and the current density where the unknown electron relaxation time $\tau$ has been canceled,

$$\frac{\partial J}{\partial t} = (\sum_n A_n)(\sum_m B_m)^{-1} j = C j. \quad (6)$$

The above equation for the torque current tensor $C$ will now be evaluated for a real SS system, the helical SDW in the RE metal Er. Erbium has a complex non-collinear magnetic structure which is strongly temperature dependent. Bulk Er has an hcp structure with $c=5.585$ Å and $a=3.56$ Å[11]. There is a rich variety of non-collinear magnetic structures and ordering vectors over different temperature ranges. Below 20K it has a conical SS, between 53.5 and 85K a longitudinal SDW and between 20K and 53.5K there is an intermediate magnetic structure (see Ref. [12]).

Although the formalism is valid for conical SS, in this work we will focus on planar spin spirals. All the material specific quantities of Er used in the calculation of the matrix $C$ were calculated from first principle density functional theory. The calculation of Er was made using the full-potential augmented plane wave plus local orbitals (FP-APW+lo) method as described in Ref. [13]. The local spin density approximation (LSDA) as parametrized by von Barth and Hedin was used without use of any shape-approximation to the non-collinear magnetization, i.e. charge and magnetization densities as well as their conjugate potentials are allowed to vary freely in space both regarding magnitude and direction. A set of 248 k-points was used for converging the electron density. The SS was treated using the generalised Bloch theorem. The 4f-electrons were treated as core electrons. A large set of SS wave vectors $q$ along he out-of-plane axis in the hcp lattice were calculated and an energy minimum was found for $q=0.20 \ 2\pi/c$. The results are in agreement with previous calculations made for helical SDW in RE[9].

The spin current density tensor was calculated at the surface of the augmentation spheres of the atoms where the APW expansion can be written as a sum of plane waves,

$$\psi_{nk}(r) = \sum_G \left( \alpha a_{nk,G} e^{i(G+k-q/2)r} + \beta b_{nk,G} e^{i(G+k+q/2)r} \right) \quad (7)$$

where $\alpha$ and $\beta$ are the up and down spinors respectively and $G$ are the reciprocal lattice vectors. The plane wave coefficients $a$ and $b$ are obtained from the first principles calculation. The spin flux into a sphere with radius $R$ centered at an atom at cite $r_n$ is for plane waves given by the expression
\[
\int_{S_{atom}} Q_{nk} \cdot dS = \frac{\hbar R^2}{m} \text{Re} \sum_{G,G'} -i4\pi \left[ \alpha_1 s_n^a a_{nk, G}^a a_{nk, G'}^e e^{-i(G-G')^R \tau_n} j_1((G - G'|R)(G' + k - q/2) \cdot (G - G') \right] \\
+ \alpha_1 s_n^b a_{nk, G}^b b_{nk, G'}^e e^{-i(G-G'-q)^R \tau_n} j_1((G - G' - q|R)(G' + k + q/2) \cdot (G - G' - q) \\
+ \beta_1 s_n^a b_{nk, G}^a a_{nk, G'}^e e^{-i(G-G'+q)^R \tau_n} j_1((G - G' + q|R)(G' + k - q/2) \cdot (G - G' + q) \\
+ \beta_1 s_n^b b_{nk, G}^b b_{nk, G'}^e e^{-i(G-G')^R \tau_n} j_1((G - G'|R)(G' + k + q/2) \cdot (G - G') \right] 
\] (8)

where \( m \) is the electron mass and \( j_1 \) is the first spherical Bessel function. In order to get a correct description of the FS used in Eqs. \( 3 \) and \( 5 \) a \( 41 \times 41 \times 41 \) k-point mesh was used to cover the first Brillouin zone.

The torque current matrix \( \mathbf{C} \) is evaluated for an Er atom situated at a site with magnetization direction \([100]\) and SS wave vector \([00q]\).

\[
\mathbf{C} = \hbar \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0.5 \\ 0 & 0 & 0 \end{array} \right) [\text{Å}^2].
\]

From the structure of the \( \mathbf{C} \) matrix we conclude that the torque induced by a current along the spiral axis causes the SS to translate along the spiral axis, which is equivalent to a rigid rotation of the spiral. In order to calculate the rotation frequency as a function of the current density we also need to consider the total angular momentum of the Er atoms \( J = 15/2 \) given by Hund’s rules as the sum of orbital and spin angular momentum \( L = 6 \) and \( S = 3/2 \). The torque calculated using the \( \mathbf{C} \) matrix will cause the total angular momentum of the atom as well as the SS to precess with 0.07 GHz if a current of \( 10^5 \text{A/cm}^2 \) flows along the spiral axis of bulk Er.

By analyzing the quantities in Eq. \( 6 \) one finds that the main contribution to the \( \mathbf{C} \) matrix comes from the the band whose FS is shown in Fig. \( 1 \). This surface is the remains of the FS which has the nesting features that drives some of the RE solids to have a helical SDW. \( 1 \)

The large contribution from this surface is due to that it has a large area being perpendicular to the spiral axis that contributes to the electron transport along the axis. An estimate of the STT induced by the electronic states on this FS can be obtained from the following considerations. As the conduction electrons flow along the spiral axis the component of their spins that are perpendicular to the SS axis i.e. for planar SS parallel to the local magnetization direction will be precessing around the SS axis. \( 1, 10 \)

The size of the parallel component \( P \) can be estimated by the expression

\[
P(k) = \left< \frac{\mathbf{m}(r) \cdot s_k(r)}{|s_k(r)|} \right>,
\]

where \( \mathbf{m}(r) \) is the local magnetization and \( \langle ... \rangle \) means space average. \( P \) is a measure of the local spin polarization and the value of \( P \) for the states at the FS in Fig. \( 1 \) is shown in Fig. \( 2 \). In Fig. \( 2 \) is the local spin polarization \( P \) calculated using the interstitial region between two atomic planes i.e. the space average in Eq. \( 9 \) is not done over the whole unit cell. From Fig. \( 2 \) we estimate that the spin of the electron states at the FS on average are tilted 30° from the spiral axis in the opposite direction of the local magnetization since the average spin polarization \( P \) for these states is -0.5. The current carried by an electronic state transfers spin upon passing an atomic layer perpendicular to the SS axis. This is since its parallel spin component will rotate with \( q \pi \) [rad] for each layer the current passes. For states on the FS a spin of \( (h/2)\pi Pq \) [Js] will be transfered per layer. A current of \( 1 [\text{A/m}^2] \) along the spiral axis induces a STT of \( (h/2)2\pi PqA/e [J] \) per unit cell, where \( e \) is the electron charge and \( A = a^2\sqrt{3}/2 \) is the cell area. The STT induced by \( 1 \text{A/m}^2 \) along the spiral axis causes the SS to rotate with \( (PqA)/(4Je) \) [Hz]. This estimate of the rota-
FIG. 2: Projection of the Fermi surface of Fig. on the x-y plane. The spin polarization parallel with the local magnetization direction $P$ is given by the colour code, 1 means that the spin polarization is parallel with the local magnetization direction, for details see text.

Spiral axis

SDW

Non magnetic

Ferromagnet

FIG. 3: Device for experimental verification of STT in a material with a helical SDW.

We suggest that the rotation frequency of the SS as a function of current density can be experimentally verified in the following way. Consider a nano pillar with a layer of a nonmagnetic material in between a SS layer, with its spiral axis perpendicular to the interface, and a ferromagnetic material as shown in Fig. 3. In principle all these three layers can be RE based, e.g. with Gd as the ferromagnet and Lu as the non-magnetic species. This system will in similarity with GMR and TMR systems have a conductance perpendicular to the layers dependent on the relative magnetization directions of the interfaces. If the magnetization of the ferromagnetic material is fixed and the magnetization of the SS layer precesses then the conductivity perpendicular to the layers will be time dependent. The oscillating conductivity can be used as a probe of the rotation frequency of the SS.

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