Chronology Protection and Quantized Fields: Complex Automorphic Scalar Field in Misner Space

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Abstract

The renormalized stress-energy tensor $\langle T_{\mu\nu} \rangle$ of the quantized complex massless scalar field which obeys the automorphic condition in Misner space is obtained. It is shown that there exists the special value of the automorphic parameter for which $\langle T_{\mu\nu} \rangle$ is regular on the chronology horizon and, so, can not act as a protector of chronology through a back reaction on a spacetime metric. However, it is shown that, at the same time, the value of field square $\langle \phi^2 \rangle$, which characterizes the quantum field fluctuations, is divergent on the chronology horizon. The assumption is suggested that the infinitely growing quantum field fluctuations, which appear if a (self)interaction of the scalar field is taken into account, would prevent the chronology horizon formation.

1 Introduction

It has been known that classical general relativity permits spacetime to develop closed timelike curves (CTCs) and, as a consequence, violate causality (see [1] and references therein). Believing in the causality conservation, recently, Hawking has proposed a chronology protection conjecture [2], which states that the laws of physics will always prevent the formation of closed timelike curves. Hawking has argued that this conjecture might be fulfilled quite generally due to vacuum polarization effects which will cause the expectation value of the stress-energy tensor $\langle 0|T_{\mu\nu}|0 \rangle$ to be divergent at any chronology horizon where CTCs are trying to form, and will prevent the formation of CTCs through a back reaction on the spacetime metric.

So far no proof of the chronology protection conjecture has been given. So it seems important to study the vacuum polarization for different physical fields on a chronology horizon in various spacetimes with CTCs. In a number of previous papers [3-13] such investigation has been done.

In papers [3-11] it has been shown that $\langle 0|T_{\mu\nu}|0 \rangle$ for a non-twisted massless scalar field diverges at the chronology horizon in various spacetimes with CTCs. The back reaction of the metric to this diverging stress-energy through the Einstein equations may be able to prevent the formation of CTCs. However, in Boulware’s work it has been obtained that $\langle 0|T_{\mu\nu}|0 \rangle$ for a massive scalar field remains finite on the chronology horizon in Gott space [7]. The similar result in Grant space was obtained by Tanaka and Hiscock. They have found that $\langle 0|T_{\mu\nu}|0 \rangle$ is finite on the chronology horizon provided the mass of the scalar field is above a lower limit which depends on the topological identification scale lengths of the spacetime [13]. Thus, in these examples the metric backreaction caused by a massive quantized field may not be large enough to significantly change the space geometry and prevent the formation of CTCs.

One may see that the mass of a field could be a mechanism which provides a regular behavior of $\langle 0|T_{\mu\nu}|0 \rangle$ on the chronology horizon. Another possibility has been pointed out by Frolov in Ref. [4]. He has stressed that the sign of the vacuum energy density may depend on the spin of a field. In particular one may expect that the contribution of fermions to the energy density has an opposite sign than the contribution of bosons. And, in principle, the situation could be when there is an exact cancellation of the leading contributions of all fields (as it happens for the vacuum energy density in flat spacetime in a supersymmetric theory). In the later case $\langle 0|T_{\mu\nu}|0 \rangle$ could be finite on the chronology horizon.

In the previous paper [12] I have considered two-dimensional model of "time machine" with an automorphic complex scalar field. The automorphic fields $\phi(X)$ ($X$ is a spacetime point) are those ones which obey the
generalized periodic (or automorphic) condition \( \phi(\gamma X) = a(\gamma)\phi(X) \), where \( a^2(\gamma) = 1 \), and \( \gamma \) are elements of the discrete group of isometry \( \Gamma \) on a spacetime; operators \( \gamma \) act in the following way: \( \gamma X = \bar{X} \), where points \( X \) and \( \bar{X} \) are identified. Note that the Lagrangian \( \mathcal{L}[\phi(X)] \) of free fields is quadratic in \( \phi \) so it is invariant under the symmetry transformations \( \gamma \) of spacetime. In the case of the complex scalar field the automorphic condition takes the form
\[
\phi(\gamma X) = e^{2\pi i \alpha} \phi(X), \quad 0 \leq \alpha \leq \frac{1}{2},
\]
where \( \alpha \) is an automorphic parameter. There are two particular cases \( \alpha = 0 \) and \( \alpha = \frac{1}{2} \) for which the condition \[\phi(\gamma X) = \phi(X) \] (a non-twisted field) and \( \phi(\gamma X) = -\phi(X) \) (a twisted field), respectively. (For more details about automorphic fields, see Refs. 15 and 16.)

In the paper [12] it has been shown that \( \langle 0|T_{\mu\nu}|0 \rangle \) for the complex scalar field remains finite on the chronology horizon if the automorphic parameter has some specific values. This result is not a surprise. Moreover, one may expect to obtain the similar results for automorphic fields in any spacetimes with CTCs. Really, it is known that the sign of the vacuum energy density is different for the non-twisted (\( \alpha = 0 \)) field and the twisted (\( \alpha = \frac{1}{2} \)) one. So one may suppose that near the chronology horizon a diverging part of \( \langle 0|T_{\mu\nu}|0 \rangle \) will have different signs for cases of a non-twisted field and twisted one. And one may also expect that in the case of some intermediate value of the automorphic parameter \( \alpha \) the diverging part of \( \langle 0|T_{\mu\nu}|0 \rangle \) will vanish, so that the vacuum expectation values of \( \langle 0|T_{\mu\nu}|0 \rangle \) will be finite on the chronology horizon.

In this paper we continue an investigation of a quantized complex scalar field in a spacetime with CTCs in order to understand better a role of automorphic fields in a mechanism of the chronology protection. In previous work [12] the two-dimensional particular model of a time machine has been considered. Here we shall consider the Misner space which is convenient for next reasons: (i) Misner space is a flat spacetime with non-trivial topology so it does not need matter fields with the negative energy density as a wormhole spacetime does. (ii) Misner space has a simple mathematical structure so it becomes possible to carry out the exact calculation of the two-dimensional particular model of a time machine [12]. Here we shall consider the Misner space metric becomes identical with the Minkowski space metric, in the Misner coordinates \((t, x^1, x^2, x^3)\) the metric is given by
\[
\begin{align*}
\mathcal{L}[\phi(X)] & = \frac{1}{2} \left( \partial_\mu \phi \partial^\mu \phi - m^2 \phi^2 \right), \\
|T_{\mu\nu}| & = 0
\end{align*}
\]
By transforming to a new set of coordinates \( \{y^\alpha\} \) defined by
\[
y^0 = t \cosh(x^1), \quad y^1 = t \sinh(x^1), \quad y^2 = x^2, \quad y^3 = x^3,
\]
the Misner space metric becomes identical with the Minkowski space metric,
\[
ds^2 = -(dy^0)^2 + (dy^1)^2 + (dy^2)^2 + (dy^3)^2.
\]
The unique properties of Misner space originate in its topology. In the Misner coordinates, the spacetime is taken to be periodic in the \( x^1 \) direction with period \( a \). Thus, the following points are identified with one another:
\[
(t, x^1, x^2, x^3) \leftrightarrow (t, x^1 + na, x^2, x^3).
\]
In the Minkowski coordinates, the above topological identification takes the form
\[
(y^0, y^1, y^2, y^3) \leftrightarrow (y^0 \cosh(na) + y^1 \sinh(na), y^0 \sinh(na) + y^1 \cosh(na), y^2, y^3).
\]
From (1) one may see that the Misner space identifications are equivalent to a discret Lorentz boost by a velocity equal to \( \tanh(na) \) in the \( y^1 \) direction in the Minkowski space.

Consider the \( t-x^1 \) plane of the Misner space and the light cone in the two-dimensional Minkowski space. The lower half-plane \( (t < 0) \) of the Misner space is mapped onto the lower quadrant (the past light cone) of the Minkowski space. The Misner metric (2) has an apparent singularity at \( t = 0 \). However, one can extend it by introducing new coordinates

\[ \tau = t^2, \quad v = \ln t + x^1. \]

The metric then takes the form

\[ ds^2 = -dv d\tau + \tau dv^2. \]

This can then be extended through \( \tau = 0 \). This corresponds to extending from the lower quadrant into the left-hand quadrant. However, at \( \tau = 0 \), the light cone tip over and a closed null geodesic appears. For negative \( \tau \), closed timelike curves appear. The same result is obtained for the upper half-plane \( (t > 0) \) of the Misner space and the future light cone in the Minkowski space. Thus, the \( t = 0 \) surface in the extended Misner space separates the region with CTCs from the region without them. This surface is a chronology horizon.

3 \( \langle T_{\mu\nu} \rangle^{ren} \) and \( \langle \phi^2 \rangle^{ren} \) for complex scalar field in Misner space

Consider a complex massless scalar field \( \phi \) with the stress-energy tensor

\[
T_{\mu\nu} = (1 - 2\xi)\nabla(\mu)\nabla(\nu)\phi + (4\xi - 1)g_{\mu\nu}\nabla\alpha\phi\nabla\alpha\phi - 2\xi(\phi\nabla\mu\nabla\nu\phi + \bar{\phi}\nabla\mu\nabla\nu\bar{\phi}) + \frac{1}{2}\xi g_{\mu\nu}(\phi\bar{\phi} + \bar{\phi}\phi).
\]

(7)

The scalar field \( \phi \) satisfies the field equation

\[ \Box \phi = 0. \]

(8)

(The complex conjugate field obeys the same equation \( \Box \bar{\phi} = 0 \).) In the Minkowski coordinates \( \{ y^\alpha \} \), a general positive-frequency solution of this equation can easily be written in the form

\[
\phi(y^0, y^1, y^2, y^3) = \iiint dk_1dk_2dk_3A(k_0, k_1, k_2, k_3)e^{i(-k_0y^0 + k_1y^1 + k_2y^2 + k_3y^3)},
\]

(9)

where \( k_0 = \sqrt{k_1^2 + k_2^2 + k_3^2} \), and \( A(k_0, k_1, k_2, k_3) \) is an arbitrary "spectral" function. Now let us demand that this solutions obey the automorphic condition (1), which in Misner space takes the form

\[
\phi(y^0 \cosh a + y^1 \sinh a, y^0 \sinh a + y^1 \cosh a, y^2, y^3) = e^{2\pi i\alpha} \phi(y^0, y^1, y^2, y^3).
\]

(10)

Positive-frequency solutions (3) will be automorphic provided

\[
A(k_0 \cosh a - k_1 \sinh a, k_1 \cosh a - k_0 \sinh a, k_2, k_3) = e^{-2\pi i\alpha} A(k_0, k_1, k_2, k_3).
\]

(11)

The general solution of this functional equation is

\[
A(k_0, k_1, k_2, k_3) = \sum_n C_n(k_2, k_3)(k_0 - k_1)^\nu,
\]

(12)

where \( \nu = -2\pi a^{-1}(n + \alpha) \), \( C_n(k_2, k_3) \) are arbitrary functions of \( k_2 \) and \( k_3 \), and the summation is taken over all integer numbers \( n \). Substituting the expression for \( A(k_0, k_1, k_2, k_3) \) in (3) and carrying out the integration over \( k_1 \) one can obtain the following representation for a general positive-frequency automorphic solution:

\[
\phi(y^0, y^1, y^2, y^3) = \sum_n \iint dk_2dk_3\bar{C}_n(k_2, k_3)\phi_J(y^0, y^1, y^2, y^3),
\]

(13)
where $J$ is a multi-index $\{n, k_2, k_3\}$, and modes $\phi_J(y^0, y^1, y^2, y^3)$ is given by

$$
\phi_J(y^0, y^1, y^2, y^3) = D_n(y^0 + y^1)^{i\nu/2}(y^0 - y^1)^{-i\nu/2} \times
$$

$$
H^{(2)}_{1\nu} \left( \kappa \sqrt{(y^0)^2 - (y^1)^2} \right)  e^{i(k_2 y^2 + k_3 y^3)}
$$

(14)

$\kappa = \sqrt{(k_2)^2 + (k_3)^2}$, and $D_n$ are normalizing coefficients. Below it will be convenient to use the Misner coordinate set $\{X^\alpha\} = \{t, x^1, x^2, x^3\}$. Using the relation (4) between Minkowski and Misner coordinates one can rewrite the solutions (14) as follows:

$$
\phi_J(t, x^1, x^2, x^3) = D_n H^{(2)}_{1\nu}(\kappa t) e^{i(\nu x^1 + k_2 x^2 + k_3 x^3)}.
$$

(15)

Let us introduce the following scalar product $\langle \phi_1, \phi_2 \rangle$ in the space of automorphic solutions (13)

$$
\langle \phi_1, \phi_2 \rangle = i \int_\Sigma [\phi_1(X) \partial_\mu \bar{\phi}_2(X) - \bar{\phi}_2(X) \partial_\mu \phi_1(X)]d\sigma^\mu,
$$

(16)

where $d\sigma^\mu$ is a surface element of a Cauchy surface $\Sigma$ in Misner space. As the value of this scalar product does not depend on the particular choice of $\Sigma$, one may choose the surface $\Sigma$ defined by the equation $t = \text{constant}$. Taking into account the periodicity of Misner space in $x^1$ direction, one may represent the scalar product as follows:

$$
\langle \phi_1, \phi_2 \rangle = -it \int_0^\alpha \int_{t=\text{const}} [\phi_1(X) \partial_t \bar{\phi}_2(X) - \bar{\phi}_2(X) \partial_t \phi_1(X)] dx^1 dx^2 dx^3.
$$

(17)

The set of positive-frequency automorphic solutions (13) with the scalar product (17) forms a Hilbert space $H$. The solutions (15) form an orthonormal basis in $H$: $\langle \phi_n, \phi_m \rangle = \delta_{nm}$ provided

$$
D_n = \frac{e^{\pi \nu/2}}{4\sqrt{\pi a}}.
$$

(18)

The Hadamard function $G^{(1)}(X, \bar{X})$ is defined by

$$
G^{(1)}(X, \bar{X}) = \sum_J [\phi_J(X) \phi_J(\bar{X}) + \phi_J(\bar{X}) \bar{\phi}_J(X)],
$$

(19)

where $\sum_J = \sum_k \int dk_2 dk_3$. Substituting the solutions (15) into the last expression, one can obtain after calculations the following expression for the Hadamard function $G^{(1)}$:

$$
G^{(1)}(X, \bar{X}) = \frac{1}{4\pi^2 \sigma} + \frac{1}{\pi^2 t} \int \sum_{n=1}^\infty \cosh[n(x^1 - \bar{x}^1)] \sin(n \arccos \chi) \Psi_n(\alpha, a),
$$

(20)

where

$$
\sigma = \frac{1}{2} \left[ -(t - \bar{t})^2 + 2t \bar{t} (\cosh(x^1 - \bar{x}^1) - 1) + (x^2 - \bar{x}^2)^2 + (x^3 - \bar{x}^3)^2 \right],
$$

(21)

and

$$
\Psi_n(\alpha, a) = \frac{e^{\alpha a} \cos 2\pi a - 1}{e^{2\alpha a} - 2e^{\alpha a} \cos 2\pi a + 1}.
$$

(22)

To obtain the renormalized Hadamard function for the Misner space we must subtract from the expression (20) the divergent Minkowski vacuum state term

$$
G_0^{(1)}(X, \bar{X}) = \frac{1}{4\pi^2 \sigma_0},
$$

(23)

where $\sigma_0 = \frac{1}{2} g_{\alpha \beta}(X^\alpha - \bar{X}^\alpha)(X^\beta - \bar{X}^\beta)$ is a geodetic interval, which in the Misner coordinates is given by

$$
\sigma_0 = \frac{1}{2} [-(t - \bar{t})^2 + t^2(x^1 - \bar{x}^1)^2 + (x^2 - \bar{x}^2)^2 + (x^3 - \bar{x}^3)^2].
$$

(24)
So, the renormalized Hadamard function $G^{(1)}_{ren}(X, \tilde{X})$ is
\[
G^{(1)}_{ren}(X, \tilde{X}) = \frac{1}{4\pi^2} \left( \frac{1}{\sigma} - \frac{1}{\sigma_0} \right) + \frac{1}{\pi^2 t i \sqrt{1 - \chi^2}} \sum_{n=1}^{\infty} \cosh(n(x^1 - \tilde{x}^1)) \sin(n \arccos \chi) \Psi_n(\alpha, a). \tag{25}
\]

Now we may find the renormalized vacuum expectation values of the stress-energy tensor $\langle 0|T_{\mu\nu}|0 \rangle$ and the field square $\langle 0|\phi^2|0 \rangle$ (hereafter $T_{\mu\nu}$ and $\langle \phi^2 \rangle$).

The value of $\langle \phi^2 \rangle$, which characterize the vacuum fluctuations, is defined as follows:
\[
\langle \phi^2 \rangle = \lim_{\tilde{X} \to X} G^{(1)}_{ren}(X, \tilde{X}), \tag{26}
\]

Hence, using the expression (25) for $G^{(1)}_{ren}$, we can obtain
\[
\langle \phi^2 \rangle = \frac{K}{t^2}, \quad \text{where} \quad K \equiv K(\alpha, a) = \frac{1}{\pi^2} \sum_{n=1}^{\infty} n \Psi_n(\alpha, a). \tag{27}
\]

The vacuum expectation value of the stress-energy tensor for a complex massless scalar field can be written as
\[
\langle T_{\mu\nu} \rangle = \lim_{\tilde{X} \to X} \{ (1 - \xi) \nabla_\mu \nabla_\nu + (2\xi - \frac{1}{2}) g_{\mu\nu} \nabla_\alpha \nabla^\alpha \\
-2\xi \nabla_\mu \nabla_\nu + \frac{1}{2} \xi g_{\mu\nu} \Box \} G^{(1)}_{ren}(X, \tilde{X}). \tag{28}
\]

Using the expression (25) for $G^{(1)}_{ren}$, computing the derivatives and taking the limit as the separated points are brought together, one can obtain the following expressions for the components of $\langle T_{\mu\nu} \rangle$ with arbitrary curvature coupling:
\[
\langle T_{\mu\nu} \rangle = \frac{1}{t^4} \text{diag}(L, 3L, M, M), \tag{29}
\]

where
\[
L \equiv L(\xi, \alpha, a) = \frac{1}{\pi^2} \sum_{n=1}^{\infty} \left( \frac{n - n^3}{3} - \frac{3\xi n}{2} \right) \Psi_n(\alpha, a), \tag{30}
\]
\[
M \equiv M(\xi, \alpha, a) = \frac{1}{\pi^2} \sum_{n=1}^{\infty} \left( \frac{2n + n^3}{3} - \frac{9\xi n}{2} \right) \Psi_n(\alpha, a). \tag{31}
\]

For conformal coupling ($\xi = \frac{1}{6}$) the stress-energy tensor $\langle T_{\mu\nu} \rangle$ has the form
\[
\langle T_{\mu\nu} \rangle = \frac{1}{t^4} \text{diag}(N, 3N, -N, -N), \tag{32}
\]

where
\[
N \equiv N(\alpha, a) = L(\frac{1}{6}, \alpha, a) = -M(\frac{1}{6}, \alpha, a) = \frac{1}{\pi^2} \sum_{n=1}^{\infty} \left( \frac{n}{12} - \frac{n^3}{3} \right) \Psi_n(\alpha, a). \tag{33}
\]

4 Behavior of $\langle T_{\mu\nu} \rangle$ and $\langle \phi^2 \rangle$ near the chronology horizon

As has been shown in section 2 the chronology horizon in Misner space is the $t = 0$ surface. One may see that the expressions (27), (29) and (33) for $\langle \phi^2 \rangle$ and $\langle T_{\mu\nu} \rangle$ are generally diverged on the chronology horizon, when $t$ goes to zero. However, it may be possible that the factors $K$, $L$, $M$, or $N$ become equal to zero. In this case the corresponding components of $\langle T_{\mu\nu} \rangle$ and/or $\langle \phi^2 \rangle$ are equal to zero too and they remain regular on the chronology horizon.

Consider the case of non-conformal coupling ($\xi \neq \frac{1}{6}$). In this case $\langle T_{\mu\nu} \rangle$ is given by the expression (28). Coefficients $L$ and $M$ are the functions $L(\xi, \alpha, a)$ and $M(\xi, \alpha, a)$ of parameters $\xi$, $\alpha$ (automorphic parameter) and $a$ (periodicity parameter). Let us fix two parameters $\xi$ and $a$. The graphs of $L$ and $M$ as functions of $\alpha$ is
shown in the figure 1. One may see that the functions $L$ and $M$ have alternating signs. They become equal to zero at some values of $\alpha$, which are different for $L$ and $M$, i.e. the coefficients $L$ and $M$ are not equal to zero simultaneously. Note that this behavior of the functions $L$ and $M$ remains qualitatively the same for all values of $\xi$ and $a$. Thus, we can do the conclusion that the components of $\langle T_{\mu\nu} \rangle$ (at least, the some of them) diverge at the chronology horizon for all values $\xi \neq \frac{1}{6}$, $\alpha$ and $a$.

Figure 1: Graphs of $L(\xi, \alpha, a)$ (solid line) and $M(\xi, \alpha, a)$ (dashed line) as functions of $\alpha$.

Now consider the case of conformal coupling $\xi = \frac{1}{6}$. In this case $\langle T_{\mu\nu} \rangle$ is given by the expression (32). We may see that the behavior of $\langle T_{\mu\nu} \rangle$ is only determined by the coefficient $N$. The family of the graphs of $N(\alpha, a)$ as a function of $\alpha$, corresponding to the various values of $a$, is shown in the figure 2 (the solid lines). From figure 2 one can see the next characteristic features of the behavior of $N$. The function $N(\alpha, a)$ has alternating signs, and, for any value of $a$, it becomes zero at the point $\alpha = \alpha^* \approx 0.24$.

We obtain that the coefficient $N$ is equal to zero if $\alpha = \alpha_*$. But if $N = 0$, then all components of $\langle T_{\mu\nu} \rangle$ are identically equal to zero too. Thus, we can conclude that there exists the special case with the automorphic parameter $\alpha = \alpha_*$, when the stress-energy tensor $\langle T_{\mu\nu} \rangle$ of the complex massless scalar field vanishes in the whole Misner space. Of course, this "everywhere null" stress-energy tensor is also equal to zero on the chronology horizon, and so it can not prevent closed timelike curves from appearing through the back reaction on the spacetime metric.

Does it means that this field configuration can not act as a protector of chronology? To answer, let us consider the behavior of $\langle \phi^2 \rangle$. $\langle \phi^2 \rangle$ is given by the expression (27) and fully determined by the coefficient $K$. The family of the graphs of $K(\alpha, a)$ as a function of $\alpha$, corresponding to the various values of $a$, is shown in the figure 2 (the dashed lines). One may see that $K$ is a function with alternating signs, which becomes zero at some point $\alpha = \alpha_*$, It is important that $\alpha_* \neq \alpha_*$. Hence, when the components of $\langle T_{\mu\nu} \rangle$ are equal to zero, at the same time $\langle \phi^2 \rangle$ is distinct from zero. Thus, we may conclude that $\langle \phi^2 \rangle$ will be divergent on the chronology horizon, and, as a consequence, the vacuum field fluctuations will be infinitely growing as the chronology horizon try to form. This infinitely growing vacuum fluctuations may lead to an appearance of the infinite vacuum energy density if a (self)interaction of the scalar field is taken into account. As real physical fields are non-free, one may suppose that the vacuum fluctuations could be a mechanism that prevents the chronology horizon from forming.

Figure 2: Family of graphs of $K(\alpha, a)$ (dashed lines) and $N(\alpha, a)$ (solid lines) as functions of $\alpha$. 

6
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