Scalar graviton in the healthy extension of Hořava-Lifshitz theory

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In this note we study the linear dynamics of scalar graviton in a de Sitter background in the infrared limit of the healthy extension of Hořava-Lifshitz gravity with the dynamical critical exponent $z = 3$. Both our analytical and numerical results show that the non-zero Fourier modes of scalar graviton oscillate with an exponentially damping amplitude on the sub-horizon scale, while on the super-horizon scale, the phases are frozen and they approach to some asymptotic values. In addition, as the case of the non-zero modes on super-horizon scale, the zero mode also initially decays exponentially and then approaches to an asymptotic constant value.

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\section{I. INTRODUCTION}

Recently, a power-counting renormalizable ultraviolet (UV) complete quantum gravity theory was proposed by Hořava \cite{1,2}. This theory is characterized by the anisotropic scaling between time and space, so the complete diffeomorphism invariance of general relativity (GR) is lost, instead the Hořava-Lifshitz (HL) gravity is invariant under the so-called “foliation-preserving” diffeomorphism Diff($M,\mathcal{F}$). Since this theory was proposed, a great deal of efforts have been made, including studies of cosmology \cite{3–17} and black hole physics \cite{18–25}, among others \cite{26–28}. Because of the differences of diffeomorphism groups between in HL and in GR, one expects to see some new dynamical degrees of freedom of gravitational fields in HL gravity. In the minimal realization \cite{2}, because the lapse function $N(t)$ introduced in the Arnowitt-Deser-Misner (ADM) formalism respects the “projectability condition”, i.e. it is a function of time only, the usual local Hamiltonian constraint becomes into a global one which will not affect the local dynamics. So, the absence of local Hamiltonian constraint leads to a new scalar degree of freedom in addition to the usual helicity-2 polarizations of the graviton at the linear perturbation level \cite{2,29–38} (see \cite{39} for a review on the “projectable model”). However, further studies on the non-linear dynamics show that the extra scalar mode suffers several pathologies, such as instability and strong coupling problem \cite{40,41}. A naive extension to the Hořava’s original proposal, which is often refereed to the “nonprojectable model” in literatures \cite{1,2}, is to restore the full dependence of the lapse function on space and time $N(t,x)$. In this model the scalar graviton becomes non-dynamical, because the equation of motion for the lapse function gives the local Hamiltonian constraint \cite{42,43}. Unfortunately, the “nonprojectable model” also confronts several conceptual difficulties \cite{41,44–46}. Recently, a new “projectable extension” with gauged $U(1)$ symmetry is proposed by Hořava and Melby-Thompson \cite{47}.

In this note we will investigate another extension of the HL theory, which is called healthy extension of HL gravity in literatures \cite{48} (see a nice review in \cite{49}). In this model, a new ingredient $a_i \equiv \nabla_i N/N$, which transforms under Diff($M,\mathcal{F}$) as a spatial vector and a time scalar, is introduced into the action and the Hamiltonian constraint becomes into the second-class one. As a result, one extra degree of freedom should appear in the healthy extension. A preliminary parameterized post-Newtonian study in \cite{48} shows that the extra scalar graviton is free from ghost instabilities in some parameter regions. Furthermore, the cosmological evolutions of metric and density scalar perturbations in both radiation and matter dominated eras are investigated in \cite{50}. They found that, although the system has two scalar degrees of freedom, corresponding to a scalar graviton and an adiabatic matter fluctuation, the late-time evolution of perturbations can be sufficiently specified by the value of one gauge-invariant variable. So, it is natural to ask how the scalar graviton behaves in the early universe, i.e. de Sitter or inflationary phase. In this note we study the linear dynamics of HL scalar graviton in a flat universe with a positive cosmological constant. Both our analytical and numerical results show that the non-zero Fourier modes of scalar graviton oscillate with an exponentially damping amplitude on the sub-horizon scale, while on the super-horizon scale, the phases are frozen and amplitudes continuously decays until they approach to their asymptotic constant values. In addition, as the case of the non-zero modes on super-horizon scale, the zero mode also decays exponentially with respect to time initially, and then approaches to an asymptotic constant value.

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This note is organized as follows. In section II, we briefly review the “foliation-preserving” gauge symmetry and the setup of the healthy extension of Hořava-Lifshitz gravity in 3 + 1 dimensions. Then we construct some gauge-invariant variables and derive the background equations in a spatially flat universe in section III. In section IV, we present both analytical and numerical studies on the cosmological linear perturbations. Finally, we conclude in section V.

II. THE HEALTHY EXTENSION OF HOŘAVA GRAVITY

In this section, we firstly present the gauge transformations compatible with the “foliation-preserving” diffeomorphism $\text{Diff}(M,F)$, then briefly review the setup of the healthy extension of HL gravity in 3 + 1 dimensions.

A. Gauge symmetry

The field contents in the healthy extension of HL gravity are

\[ \text{lapse} : N(t,x), \text{ shift} : N^i(t,x), \text{ 3d spatial metric} : g_{ij}(t,x), \]

where we abandon the “projectability condition” on lapse function $N(t,x)$. In terms of these fields we can cast the 4-dimensional line element in the ADM formalism as

\[ ds^2 = -N^2c^2dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \]

where $x^0 = ct$ and the light speed $c$ is restored explicitly in order to obtain the nonrelativistic theory from relativistic one by taking $c \to \infty$ limit.

In the Hořava’s proposal the local Lorentz invariance is violated due to the anisotropic scaling between space and time. For instance, in 3 + 1 dimensions the coordinates $(t,x)$ scale as

\[ t \to \ell^z t, \quad x \to \ell x, \]

where $z$ is called dynamical critical exponent. In the case of general $z$, the classical scaling dimensions of the fields are

\[ [N^i] = [c] = \frac{[dx]}{[dt]} = z - 1, \quad [g_{ij}] = [N] = 0. \]

In order to investigate the spacetime diffeomorphisms in the nonrelativistic theory, it is convenient to start with the relativistic metric $g_{\mu\nu}$ but with $c$ restored

\[ g_{\mu\nu} = \begin{pmatrix} -N^2 + N_i N^i / c^2 & N_i / c \\ N_i / c & g_{ij} \end{pmatrix}, \]

\[ g^{\mu\nu} = \begin{pmatrix} -1/N^2 & N^i N^{-2} / c \\ N^i N^{-2} / c & N^i N^{-2} / c^2 \end{pmatrix}, \]

where we only keep the leading terms in $1/c$ expansion. Correspondingly, the covariant generators $\tilde{x}^\mu = x^\mu + \xi^\mu$ of spacetime diffeomorphisms are also expanded formally with respect to the small parameter $1/c$

\[ \xi^0 = cf(t) + O(1/c), \quad \xi^i = \xi^i(t,x) + O(1/c^2). \]

As stated above, the nonrelativistic transformation rules are easily obtained by taking $c \to \infty$ limit of the relativistic diffeomorphisms

\[ \delta g_{ij} = -g_{jk} \nabla_i \xi^k - g_{ik} \nabla_j \xi^k - f \tilde{g}_{ij}, \]

\[ \delta N_i = -\nabla_i \xi^j N_j - \xi^j \nabla_j N_i - \xi^j \tilde{g}_{ij} - f \tilde{N}_i - f \tilde{N}_i, \]

\[ \delta N = -\xi^i \nabla_i N - \tilde{f} N - f \tilde{N}, \]

where $\nabla_i$ is the 3-dimensional covariant derivative compatible with spatial metric $g_{ij}$ and the over dot denotes the derivative with respect to the nonrelativistic time $t$. 
B. Model setup

Comparing with the naive promoting the lapse function to a spacetime field in the “nonprojectable model” [1, 2], a new ingredient for constructing gauge-invariant terms in the action is introduced in the healthy extension of HL gravity [41, 48]

\[ a_t = \nabla_i \ln N(t, x) . \]  

(11)

In fact, the appearance of this term in the action is compulsory to make theory be free of the pathologies, which has been mentioned in the previous section. The key point of the healthy extension is that once terms with this new ingredient are introduced in the action, the Hamiltonian is no longer linear in the lapse, and the equation of motion for \( N(t, x) \) becomes into the second-class constraint. By using the standard Hamiltonian analysis [51, 52], the number of degrees of freedom in the \( D + 1 \) dimensional healthy HL gravity is

\[ \mathcal{N} = \frac{1}{2}(\dim \mathcal{P} - 2C_1 - C_2) = \frac{1}{2}D(D - 1) , \]

(12)

where \( \dim \mathcal{P} = (D + 1)(D + 2) \) is the dimension of phase space, \( C_1 = 2D \) is the number of first-class constraints, and \( C_2 = 2 \) is the number of second-class constraints. So, in the \( 3 + 1 \) spacetime this model exhibits an extra degrees of freedom (dof) in addition to the usual transverse traceless helicity-2 gravitons.

Because our primary purpose is to investigate the linear dynamics of scalar perturbations on a flat universe, we write the most general form of the action relevant to our interests

\[ S_H = \frac{M^2_{\text{pl}}}{2} \int dt d^3 x \sqrt{g} N \left\{ \mathcal{L}_K - \mathcal{V}[g_{ij}, a_i] \right\} , \]

(13)

where the kinetic term takes usual form

\[ \mathcal{L}_K = K_{ij} K^{ij} - \lambda K^2 , \]

(14)

with the extrinsic curvature defined as \( K_{ij} = (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i)/2N \). The most general potential up to the terms with dynamical critical exponent \( z = 3 \) is given by \( \mathcal{V}[g_{ij}, a_i] = \sum_{z=1}^{3} \mathcal{V}_z \),

\[ \mathcal{V}_1 = -\eta_1 R - \eta_0 a^i + \sigma , \]

(15)

\[ \mathcal{V}_2 = M^2_{\text{pl}}(\eta_2 R^2 + \eta_3 R_{ij} R^{ij} + \eta_4 R \nabla_i a^i + \eta_5 R \nabla_i a^i) , \]

(16)

\[ \mathcal{V}_3 = M^4_{\text{pl}}(\eta_4 R \Delta R + \eta_5 \nabla_i R_{ij} \nabla^i R^{jk} + \eta_6 R_{ij} \Delta a^i + \eta_7 R \nabla_i a^i) , \]

(17)

where \( R_{ij} \) and \( R \) are the Ricci tensor and Ricci scalar, respectively. In our convention the Laplace operator reads \( \Delta = \nabla_i \nabla^i \), its square and cubic are \( \Delta^2 = \Delta \cdot \Delta \) and \( \Delta^3 = \Delta \cdot \Delta \cdot \Delta \). A parity-violating term \( \epsilon^{ijk} R_{il} \nabla_j R^{jk} \) is absent in our paper because it only affects the tensor cosmological perturbations at the linear order. However, this term will contribute to the scalar perturbations at the high-order level due to the appearance of coupling vertices between scalar and tensor modes.

Now we would like to write out the equation of motion for this system of gravity. Variation with respect to the lapse \( N(t, x) \) gives the local Hamiltonian constraint

\[ \mathcal{L}_K + \mathcal{V} + 2\eta \nabla_i a^i - 2\eta_2 M^2_{\text{pl}} \Delta \nabla_i a^i + \frac{\eta_3}{M^2_{\text{pl}}} \Delta R - 2\eta_4 M^4_{\text{pl}} \Delta^2 \nabla_i a^i + \frac{\eta_5}{M^4_{\text{pl}}} \Delta^2 R = 0 . \]

(18)

Equation of motion (eom) for \( N_i \) gives the ordinary momentum constraint

\[ \nabla_j \pi^{ij} = 0 , \]

(19)

with \( \pi^{ij} = K^{ji} - \lambda K g^{ij} \). In principle, the propagating equations of this system can be obtained by the variation of gravitational action with respect to \( g_{ij} \), i.e. \( \delta S_H / \delta g_{ij} = 0 \). However, the explicit expressions of eom for \( g_{ij} \) are very lengthy, and one can find them in [50]. In the section IV, we will derive the linear approximations of these equations through a perturbative approach.
III. GAUGE-INvariant VARIABLES AND BACKGROUND EVOLUTIONS IN A FLAT UNIVERSE

In this section we will firstly study the “foliation-preserving” gauge transformations of linear scalar modes in a flat universe, and then demonstrate that the dynamics of scalar perturbations at the linear level can be analyzed without complete gauge fixing due to the novel properties of “foliation-preserving” gauge transformations, finally derive the set of background equations by using the linearized action.

The background line element of a flat Friedmann-Robertson-Walker (FRW) universe reads

$$ds^2 = -dt^2 + a^2\delta_{ij}dx^idx^j,$$

where the scale factor $a(t)$ is a function of time only. In our convention we linearize the metric scalar perturbations as

$$\delta g_{00} = -2\phi(t, x),$$
$$\delta g_{0i} = a^2\partial_i B,$$
$$\delta g_{ij} = -2a^2(\psi\delta_{ij} - \partial_i\partial_j E),$$

where $\partial_i$ is the ordinary spatial derivative compatible with Kronecker delta function $\delta^{ij}$ and $\partial^2 = \partial^i\partial_i$. The transformation rules under “foliation-preserving” diffeomorphism for these modes can be obtained by virtue of (8)-(10),

$$\delta \phi = -\dot{f}(t),$$
$$\delta B = -\dot{\epsilon}(t, x)$$
$$\delta \psi = f(t)H(t),$$
$$\delta E = -\epsilon(t, x),$$

with $\zeta^i = \partial_i \epsilon + \epsilon^i$ and $\partial_i \epsilon^i = 0$.

Armed with these transformation rules, we construct some useful gauge invariant variables as follows

$$\Phi = \phi + \frac{\psi}{H},$$
$$\beta = B - \dot{\epsilon}$$
$$\Psi = \psi + H \int_{t_0}^t \phi \dot{\epsilon} dt,$$

as pointed out in [30], $\Psi$ in fact is the integral version of $\Phi$.

From (22), we can see that the gauge variations for the quantities $\phi(t, x)$ and $\psi(t, x)$ are two functions of time only. This novel feature of gauge transformations allows us to analyze the dynamics of linear perturbations by using the gauge dependent variables, which will not lead to any gauge artifacts even though we do not fix the gauge completely. To see this more clearly, here we give a simple example to illustrate our method. We consider a system consisting of two fields $\chi$ and $\gamma$

$$S[\chi, \gamma] = -\frac{1}{2} \int \left\{ \chi^2 - \partial_i \chi \partial^i \chi - m^2 \chi^2 - \mathcal{L}^{(c)}[\chi, \gamma] \right\},$$

where we explicitly write down the free sector for $\chi$, but denote both the $\gamma$ sector and interaction terms by an implicit form $\mathcal{L}^{(c)}$. Furthermore, we assume that the action (26) is invariant under the transformation

$$\chi(t, x) \rightarrow \tilde{\chi}(t, x) = \chi(t, x) + A(t),$$
$$\gamma(t, x) \rightarrow \tilde{\gamma}(t, x) = \gamma(t, x) + B(t).$$

Under the “gauge” ($A = 0$, $B = 0$), the com for $\chi$ reads

$$\ddot{\chi} - \partial^2 \chi + m^2 \chi + \mathcal{C}[\chi, \gamma] = 0,$$

with the coupling term $\mathcal{C} = \frac{1}{2}\delta \mathcal{L}^{(c)}/\delta \chi$. On the other hand, we can also write the action (26) in terms of tilde quantities in (27) and (28) as

$$\tilde{S}[\chi, \gamma] \equiv S[\tilde{\chi}, \tilde{\gamma}] = -\frac{1}{2} \int \left\{ \tilde{\chi}^2 - \partial_i \tilde{\chi} \partial^i \tilde{\chi} - m^2 \tilde{\chi}^2 - \mathcal{L}^{(c)}[\tilde{\chi}, \tilde{\gamma}] \right\},$$
$$= S[\chi, \gamma] - \frac{1}{2} \int (\dot{A}\dot{\chi} - m^2 A\chi) - \frac{1}{2} \int \tilde{\mathcal{L}}^{(1)}[\chi, \gamma, A, B] - \frac{1}{2} \int F[A, B],$$
with

\[ \mathcal{L}^{(c)}[\chi, \gamma] = \mathcal{L}^{(c)}[\chi, \gamma] + \tilde{\mathcal{L}}^{(1)}[\chi, \gamma, A, B] + \tilde{\mathcal{L}}^{(2)}[A, B] , \]  
\[ \mathcal{F}[A, B] \supset \tilde{\mathcal{L}}^{(2)}[A, B] , \]

where \( \tilde{\mathcal{L}}^{(1)}[\chi, \gamma, A, B] \) contains the linear terms of \( \chi \) only, while \( \tilde{\mathcal{L}}^{(2)}[A, B] \) and \( \mathcal{F}[A, B] \) contains the quadratic terms of \( A, B \) only. Thus, the variation of tilde action (30) with respect to \( \chi \) gives

\[ \delta \tilde{\mathcal{L}}^{(1)}[\chi, \gamma, A, B] \]

\[ = \mathcal{S}[A, B] , \]

with \( \mathcal{S} = \frac{1}{2} \delta \tilde{\mathcal{L}}^{(1)}/\delta \chi - \bar{A} - m^2 A . \) Comparing with the result in (29), equation (34) has an extra source term on the right hand side, but it depends on time only. Given this fact, if we turn to the Fourier space, we can easily see that such only time dependent source term vanishes for all \( k \neq 0 \) modes, while it contributes a Dirac delta function to \( k = 0 \) mode. So, we conclude that the only time dependent gauge transformations do not affect the dynamics of \( k \neq 0 \) mode, i.e. the eom for \( k \neq 0 \) modes is the same in all gauges. These features of Hořava-type theory allow us to study the dynamics of non-zero Fourier modes by using the gauge dependent variables \( \phi \) and \( \psi \) directly without leading to any gauge artifacts.

In the rest part of this section, we list the background equations which can be obtained by varying the linearized action with respect to the scalar perturbations. In details, eom for \( \phi \) gives Hamiltonian constraint

\[ \sigma + 3(1 - 3\lambda)H^2 = 0 , \]  

(35)
eom for \( \psi \) or \( \partial^2E \) gives the evolution equation

\[ \dot{H} = 0 . \]  

(36)

Finally, the momentum constraint, i.e. eom for \( \partial_t B \), is trivially satisfied on the background. Note that (35) and (36) give a de Sitter solution.

\[ \text{IV. DYNAMICS OF SCALAR MODES IN LINEAR COSMOLOGICAL PERTURBATIONS} \]

In this section we will investigate the linear dynamics of scalar perturbations without any gauge fixing. First, we have to derive the second order action

\[ S^{(2)}_H = \frac{M_{pl}^2}{2} \int dtd^3x a^3 \left\{ (1 - 3\lambda) \left[ 3(\psi + H\phi)^2 + 2(\psi + H\phi)\partial^2(B - \dot{E}) \right] \right. 
\]

\[ + (1 - \lambda)\partial^2(B - \dot{E}) \cdot \partial^2(B - \dot{E}) - \eta a^{-2}\dot{\phi}\partial^2\phi + 2g_1 a^{-2}(2\phi - \psi)\partial^2\psi \]

\[ - M_{pl}^{-2}a^{-4} \left[ 16g_2 + 6g_3 \right] \partial^2\psi \cdot \partial^2\psi - \eta_2 \partial^2\phi \cdot \partial^2\phi + 4\eta_3 \partial^2\psi \cdot \partial^2\psi \]

\[ - M_{pl}^{-2}a^{-6} \left[ 16g_4 - 6g_5 \right] \partial^2\psi \cdot \partial^2\psi - \eta_4 \partial^2\phi \cdot \partial^4\phi + 4\eta_5 \partial^2\phi \cdot \partial^4\psi \right\} , \]  

(37)

where we have used the background equations and dropped all surface terms. Hamiltonian constraint is given by eom for \( \phi \), i.e. \( \delta_\phi S^{(2)}_H = 0 \)

\[ (1 - 3\lambda) \left[ 6H^2 \dot{\phi} + 2H \partial^2(B - \dot{E}) + 6H \dot{\psi} \right] - 2\eta a^{-2}\partial^2\phi + 4g_1 a^{-2}\partial^2\psi \]

\[ + 2M_{pl}^{-2} \eta_2 a^{-4} \partial^4\phi - 4M_{pl}^{-2} \eta_3 a^{-4} \partial^4\psi + 2M_{pl}^{-4} \eta_4 a^{-6} \partial^6\phi - 4M_{pl}^{-4} \eta_5 a^{-6} \partial^6\psi = 0 . \]  

(38)

The eom for \( \partial^2(B - \dot{E}) \) produces the momentum constraint

\[ (1 - 3\lambda)(H\phi + \dot{\psi}) + (1 - \lambda)\partial^2(B - \dot{E}) = 0 . \]  

(39)

Both the Hamiltonian and momentum constraint equations are consistent the linear approximations of (18) and (19). Finally, the propagating equations is obtained by eom for \( \psi \)

\[ (1 - 3\lambda) \left[ -6H \partial^2(B - \dot{E}) - 20(\dot{B} - \dot{E}) - 18H^2 \phi - 6H \phi - 6H \dot{\phi} - 18H \dot{\psi} - 6\dot{\psi} \right] + 4g_1 a^{-2}\partial^2(\phi - \psi) \]

\[ - M_{pl}^{-2}a^{-4} \left[ 2(16g_2 + 6g_3) \partial^4\psi + 4\eta_3 \partial^4\phi \right] - M_{pl}^{-4}a^{-6} \left[ 2(16g_4 - 6g_5) \partial^6\psi + 4\eta_5 \partial^6\phi \right] = 0 . \]  

(40)
A. Non-propagation of the scalar graviton in the original non-projectable version

In order to check the method presented in the section III, we need to recover the results in the original “non-projectable model” by using this method. So, we turn off all $a^i$ terms in action (13) and (37). For further simplification, we turn off all operators with dimension higher than 2 in the gravity potential (16) and (17), i.e. merely keep $R$ and $\sigma$ terms in the potential (15), without losing any information. Thus, the second order gravity action (37) becomes

$$S_H^{(2)} = \frac{M^2_{pl}}{2} \int dt d^3x \; a^3 \left\{ (1 - 3\lambda) \left[ 3(\dot{\psi} + H\phi)^2 + 2(\dot{\psi} + H\phi)\partial^2 (B - \dot{E}) \right] + (1 - \lambda) \partial^2 (B - \dot{E}) \cdot \partial^2 (B - \dot{E}) + 2g_1 a^{-2} (2\phi - \psi)\partial^2 \psi \right\}.$$  \hspace{1cm} (41)

Solving the Hamiltonian and momentum constraints gives

$$\phi = \frac{1}{H} \dot{\psi} - g_1 \frac{1 - \lambda}{1 - 3\lambda H^2} a^{-2} \partial^2 \psi,$$ \hspace{1cm} (42)

$$\partial^2 \beta = g_1 \frac{a^{-2}}{H} \partial^2 \psi,$$ \hspace{1cm} (43)

and plugging them back into (41), we arrive at

$$S_H^{(2)} = \frac{M^2_{pl}}{2} \int dt d^3x \; a^3 \left( -4g_1 a^{-2} \frac{\dot{\psi}}{H} \partial^2 \psi - 2g_1 a^{-2} \partial^2 \psi - 2g^2_1 \frac{1 - \lambda}{1 - 3\lambda H^2} a^{-4} \partial^2 \psi \cdot \partial^2 \psi \right).$$ \hspace{1cm} (44)

From the above expression we can see that the kinetic term $\dot{\psi}^2$ is absent, so at the linear perturbation level there does not exist the dynamical scalar graviton in the “nonprojectable model”. This result agrees with the observations made in [42, 43]. Beyond the non-linear order, however, the conclusion is changed. As shown in [47], this mode becomes a dynamical one if the terms of the fluctuation beyond the quadratic order are taken into account in the action. In addition, the absence of the usual quadratic time derivative term leads to an unacceptably strong-coupling problems on the small scale [53].

B. Linear dynamics of scalar graviton in a de Sitter universe

As be expected from Hamiltonian analysis (12), there should exist one extra scalar graviton in the pure gravitational system for the healthy extension of the HL theory. Now, we will study the linear dynamics of scalar graviton in a flat de Sitter universe with the healthy extension of the HL gravity.

Solving the momentum and Hamiltonian constraints formally, we obtain

$$\partial^2 \beta = -\frac{1 - 3\lambda}{1 - \lambda} (\psi + H\phi),$$ \hspace{1cm} (45)

and

$$\phi = 2 \left( -\frac{1 - 3\lambda}{1 - \lambda} \partial_t - g_1 a^{-2} \partial^2 + M_{pl}^{-2} \eta_3 a^{-4} \partial^4 + M_{pl}^{-4} \eta_5 a^{-6} \partial^6 \right) \left( \frac{2H^2}{1 - 3\lambda} - \eta \partial^2 \psi + M_{pl}^{-2} \eta_2 a^{-4} \partial^4 + M_{pl}^{-4} \eta_4 a^{-6} \partial^6 \right) \psi.$$ \hspace{1cm} (46)

Firstly, we plug the momentum constraint into action (37) and reduce it into a functional of two scalar fields $\phi$ and $\psi$

$$S_H^{(2)} = \frac{M^2_{pl}}{2} \int dt d^3x \; a^3 \left\{ \frac{2(1 - 3\lambda)}{1 - \lambda} (\psi + H\phi)^2 - \eta a^{-2} \partial^2 \phi + 2g_1 a^{-2} (2\phi - \psi) \partial^2 \psi - M_{pl}^{-2} a^{-4} \left[ (16g_2 + 6g_3) \partial^2 \psi \cdot \partial^2 \phi - \eta_2 \partial^2 \phi \cdot \partial^2 \psi + 4\eta_3 \partial^2 \phi \cdot \partial^2 \psi \right] - M_{pl}^{-4} a^{-6} \left[ (16g_4 - 6g_3) \partial^2 \psi \cdot \partial^4 \phi - \eta_4 \partial^2 \phi \cdot \partial^2 \phi + 4\eta_5 \partial^2 \phi \cdot \partial^4 \phi \right] \right\},$$ \hspace{1cm} (47)

where $\phi$ field is non-dynamical dof and its em provides a constraint

$$\frac{1 - 3\lambda}{1 - \lambda} 4H (\psi + H\phi) - 2\eta a^{-2} \partial^2 \phi + 4g_1 a^{-2} \partial^2 \psi + 2M_{pl}^{-2} \eta_2 a^{-4} \partial^4 \phi - 4M_{pl}^{-2} \eta_3 a^{-4} \partial^4 \psi + 2M_{pl}^{-4} \eta_4 a^{-6} \partial^6 \phi - 4M_{pl}^{-4} \eta_5 a^{-6} \partial^6 \psi = 0.$$ \hspace{1cm} (48)
The eom for \( \psi \) is
\[
-\frac{4(1-3\lambda)}{1-\lambda} \left[ \psi + 3H(\dot{\psi} + H\phi) + \dot{H}\phi + H\dot{\phi} \right] + 4g_1a^{-2}\partial^2(\phi - \psi) - M_{pl}^{-2}a^{-4} \left[ 2(16g_2 + 6g_3)\partial^4\psi + 4\eta_2\partial^4\phi \right] \\
-M_{pl}^{-4}a^{-6} \left[ 2(16g_4 - 6g_5)\partial^6\psi + 4\eta_5\partial^6\phi \right] = 0 .
\] (49)

In the Fourier space, the above two partial differential equations reduce into the ordinary one
\[
\frac{1}{1-3\lambda} \left[ \ddot{\psi} + 3H(\dot{\psi} + H\phi) + \dot{H}\phi + H\dot{\phi} \right] + 2\eta a^{-2}k^2\phi - 4g_1a^{-2}k^2\psi + 2M_{pl}^{-2}\eta_2a^{-4}k^4\phi \\
-4M_{pl}^{-2}\eta_3a^{-4}k^4\psi - 2M_{pl}^{-4}\eta_4a^{-6}k^6\phi + 4M_{pl}^{-4}\eta_5a^{-6}k^6\psi = 0 ,
\] and
\[
-\frac{4(1-3\lambda)}{1-\lambda} \left[ \ddot{\psi} + 3H(\dot{\psi} + H\phi) + \dot{H}\phi + H\dot{\phi} \right] - 4g_1a^{-2}k^2(\phi - \psi) \\
-M_{pl}^{-4}a^{-4}k^4 \left[ 2(16g_2 + 6g_3)\psi + 4\eta_3\phi \right] + M_{pl}^{-4}a^{-6}k^6 \left[ 2(16g_4 - 6g_5)\psi + 4\eta_5\phi \right] = 0 ,
\] (50)

where we have suppressed the momentum \( k \) implicitly in the Fourier modes \( \phi_k \) and \( \psi_k \). Before solving the above differential equations, we would like to discuss the ghost-free condition which comes from the positiveness of kinetic term \( \dot{\psi}^2 \) in the Lagrangian (47). Because the field \( \phi \) is not a dynamical one, in order to analyze the dynamics of the extra scalar graviton easily we need to solve the Hamiltonian constraint equation (50) and plug it into the Fourier form of the action (47), then the action becomes a functional of \( \psi \) only. In the following discussions on the ghost-free condition we will take this approach. Firstly, let us focus on the ultraviolet limit \( (k \rightarrow \infty) \), in which \( k^{4,6} \) terms are far more important than \( k^{0,2} \) terms. Because there are some coupling constants \( (\eta_2, \eta_3, \eta_4, \eta_5) \) of higher spatial derivative terms, the ghost-free condition can be easily satisfied, i.e., the parameter constraints to avoid ghost instability are rather loose, here we will not write them down explicitly. On the other hand, the ghost-free condition in the infrared limit \( (k \rightarrow 0) \) is controlled by the parameter \( \eta \) only, one can obtain an explicit constraint on \( \eta \). In the infrared limit, we can safely neglect \( O(k^{4,6}) \) terms, thus (50) reduces into
\[
\phi = \frac{4g_1\tilde{k}^2\psi - \frac{1-3\lambda}{1-\lambda}4H\psi}{1-3\lambda4H^2 + 2\eta k^2} \simeq -\frac{\dot{\psi}}{H} + \frac{(1-\lambda)g_1\tilde{k}^2\psi}{(1-3\lambda)H^2} + \frac{(1-\lambda)\eta\tilde{k}^2\dot{\psi}}{2(1-3\lambda)H^3} ,
\] (52)

where we have denoted the physical wavelength by \( \tilde{k} = k/a \). The relevant part for the kinetic term in the action reads
\[
S_{H}^{(2)} \supset \frac{M_{pl}^2}{2(2\pi)^3} \int dt \int d^3k a^3 \left\{ \frac{2(1-3\lambda)}{1-\lambda} |\dot{\psi} + H\phi|^2 + \frac{4\eta k^2|\phi|^2}{2(1-3\lambda)H^3} + \cdots \right\} .
\] (53)

Here we emphasize that the Fourier components \( \phi, \psi \) are complex fields, in order to keep the action real we have the absolute value symbol for each field component, or we can also write \( |\phi|^2 \) as \( \phi(t,k)\phi^*(t,k) \). Plugging (52) into (53), one can easily see that the leading term in the kinetic term reads
\[
\text{Kinetic term} \simeq \frac{\eta \tilde{k}^2}{H^2} |\dot{\psi}|^2 ,
\] (54)

from which we can see that the ghost-free condition in the infrared limit is \( \eta > 0 \). Our result agrees with the stability constraint in the Minkowskian background [48], but contradicts with the one in [55] where by Cerioni and Brandenberger find the ghost-free condition is \( \eta < 0 \). At the end of the paper, we will have more to say on this point.

Due to the cosmological interests, in the rest of this section we will investigate both analytically and numerically the dynamics of scalar graviton in the infrared limit, i.e. we turn off all coupling constants \( (\eta_2, g_2, \eta_3, g_3, \cdots) \) in the front of the operators with dimension larger than 2. Firstly, we will solve the above equations numerically with the initial condition \( \psi(t_0,k) = 1, \ d\psi/dN|_{t_0,k} = 0, \) (see Fig. 1). In the numerical calculations, we plot the dynamical evolutions of six different Fourier modes which cross horizon \( (k_* \sim a_*H) \) at the eolding number \( N_* = \ln(a_*/a_0) = 1 \) (Top left), \( N_* = 2 \) (Top right), \( N_* = 3 \) (Middle left), \( N_* = 4 \) (Middle right), \( N_* = 5 \) (Bottom left), and \( N_* = 6 \) (Bottom right), where star symbol represents for the moment of crossing horizon and the subscript zero for the initial time of de Sitter phase. The numerical results show that the Fourier modes oscillate with damping amplitudes on the sub-horizon scale, while freeze to some asymptotic values on the super-horizon scale.
FIG. 1: This figure shows the infrared behaviors of different Fourier modes of the scalar graviton $\psi$ in a de Sitter universe, where the horizontal axis denotes the efolding number $N = \ln(a/a_0)$. The six curves are the Fourier mode which exits Hubble horizon ($k \sim a_* H$) at the efolding number $N_* = 1$ (Top left), $N_* = 2$ (Top right), $N_* = 3$ (Middle left), $N_* = 4$ (Middle right), $N_* = 5$ (Bottom left), and $N_* = 6$ (Bottom right), respectively. In our numerical calculations, we set the initial scale factor $a_0 = 1$, Planck mass $M_{pl} = 1$, cosmological constant $\sigma = 1$, gravitational coupling parameters $\lambda = 1.05$, $\eta = 0.1$ and $g_1 = 1$. The initial conditions are given by $\psi(t_0, k) = 1$ and $d\psi/dN|_{t_0, k} = 0$.

Secondly, we will try to find some analytical solutions which are able to explain the numerical behaviors. For this, it is convenient to use the conformal time $\tau = -(aH)^{-1}$. Combining (50) with (51), we obtain the propagating equation for the scalar graviton in conformal time

$$ F_1 (H k \tau) \psi'' + F_2 (H k \tau) \frac{1}{\tau} \psi' + F_3 (H k \tau) \frac{1}{\tau^2} \psi = 0 , $$

(55)

where prime denotes the derivative with respect to conformal time $' = d/d\tau$. Notice that the equation (55) is obtained non-perturbatively, and the expressions for coefficients $F_1 \ F_2$ and $F_3$ are given in Appendix A. From the above expressions we can see that, the coefficients $F_1 \ F_2$ vanish if all $a_1$ terms are absent in the action, i.e. $\eta_i = 0$. So the solution of (55) is trivially $\psi = 0$, which means that in this case the scalar graviton does not exist in the case of absence of matter sources. This result is consistent with our observations for the “nonprojectable model” in the subsection IV A.

Now, we will turn to the healthy extension model in which not all coupling constants $\eta_i$ vanish simultaneously. Because the dimensionless quantity $k \tau$ is much less than unit on the super-horizon region, we can use it to expand the equation (55) perturbatively, and solve them order by order. For the super-horizon modes ($k \tau \ll 1$), the propagating equation (55) reduces to

$$ \psi'' + \mu k^2 \tau \psi' + \epsilon_k^2 k^2 \psi = 0 , $$

(56)

with

$$ \mu = \frac{1 - \lambda}{3\lambda - 1} \eta , $$

(57)
and sound speed
\[ c_s^2 = \frac{\lambda - 1}{3\lambda - 1} \left( \frac{2g^2}{\eta} + g_1 \right), \tag{58} \]
where we have neglected the contributions from coupling terms \( \eta_2, g_2, \eta_3, g_3 \) etc. The dynamical stability can be satisfied provided the sound speed is positive definitely \( (c_s^2 > 0) \). Considering the constraint on the coupling constant \( 0 < \eta < 2 \) from Minkowskian space \([48]\), such stability condition can be achieved in the parameter regions \( \lambda \in (-\infty, 1/3) \cup (1, +\infty) \) and \( g_1 \in (-\infty, -1) \cup (0, +\infty) \).

In order to compare with our numerical results, we rewrite the equation (56) in terms of the e-folding number \( N = \ln(a/a_0) \)
\[ \frac{d^2 \psi}{dN^2} + \left( 1 - \mu \frac{k^2}{a^2 H^2} \right) \frac{d\psi}{dN} + c_s^2 k^2 \frac{1}{a^2 H^2} \psi = 0, \tag{59} \]
where the initial scale factor \( a_0 \) can be set to unit. For super-horizon modes, \( k/aH \ll 1 \), the above equation reduces into a simple form
\[ \frac{d^2 \psi}{dN^2} + \frac{d\psi}{dN} = 0. \tag{60} \]
The general solution of the above equation reads
\[ \psi = C_1 e^{-N} + C_2, \tag{61} \]
with two integrate constants \( C_1 \) and \( C_2 \). From the above solution, we can see that the phases for super-horizon modes are frozen and the amplitudes experience an exponential suppression initially, and then approach to some asymptotic constant values. These behaviors are in agreement with our numerical calculations.

For the sub-horizon mode \( (k\tau \gg 1) \), (55) becomes into
\[ \psi'' = \frac{2}{\tau} \psi' + k^2 c_s^2 \psi = 0, \tag{62} \]
with another sound speed
\[ c_s^2 = \frac{\lambda - 1}{3\lambda - 1} \left( \frac{2g^2}{\eta} - g_1 \right). \tag{63} \]
To avoid the exponential instability, the parameters must satisfy \( \lambda \in (-\infty, 1/3) \cup (1, +\infty) \) and \( g_1 \in (-\infty, 0) \cup (1, +\infty) \). The solution for (62) reads
\[ \psi = \frac{\tau^{3/2}}{\sqrt{c_s k \tau}} \left[ C_1 \cos(c_s k \tau) - \frac{\sin(c_s k \tau)}{c_s k \tau} \right] + C_2 \left( \sin(c_s k \tau) + \frac{\cos(c_s k \tau)}{c_s k \tau} \right), \tag{64} \]
where \( C_1, C_2 \) are another two integration constants. If we rewrite the equation (62) with the e-folding number
\[ \frac{d^2 \psi}{dN^2} + 3 \frac{d\psi}{dN} + \frac{c_s^2 k^2}{a^2 H^2} \psi = 0. \tag{65} \]
Then the general solution can be expressed as
\[ \psi = C_1 \left[ \frac{c_s k}{aH} \cos \left( \frac{c_s k}{aH} \right) - \sin \left( \frac{c_s k}{aH} \right) \right] + C_2 \left[ \cos \left( \frac{c_s k}{aH} \right) + \frac{c_s k}{aH} \sin \left( \frac{c_s k}{aH} \right) \right]. \tag{66} \]
From (66) we can easily see that the sub-horizon Fourier modes have two oscillating solutions. The amplitude of one solution is large, but it is decaying with the scale factor \( a^{-1} \); the amplitude of the other is constant, but it is smaller than the former by a factor \( aH/c_\tau \ll 1 \). The Middle right, Bottom left and Bottom right panels in Fig. (1) show that the former exponentially suppressed modes dominate the sub-horizon solutions in our numerical calculations.

Finally we would like to make a comment on the zero mode, for which the above analysis becomes invalid. Fortunately, we can rewrite the action (47) into a gauge-invariant form, if we neglect all spatial gradient terms by using the gauge-invariant variable \( \Psi \) (25)
\[ S_H^{(2)} \simeq \frac{M_{pl}^2}{2} \int dt d^3x a^3 \left\{ \frac{2(1 - 3\lambda)}{1 - \lambda} (\dot{\psi} + H\dot{\psi})^2 \right\}, \tag{67} \]
Consequently, $\psi$ is given by
\[ \ddot{\Psi} + 3H\dot{\Psi} = 0, \] (68)
with the general solution
\[ \Psi = Ae^{-3Ht} + B. \] (69)
we see that as the case of the super-horizon non-zero modes, the general solution of $\Psi$ also contains two parts, one of them decays exponentially with respect to time, the other one is a constant.

V. CONCLUSION AND DISCUSSION

In this note we investigated both analytically and numerically the linear dynamics of scalar graviton in the healthy extension of the HL theory in a de Sitter background. We found that due to the “foliation-preserving” diffeomorphism $\text{Diff}(M, \mathcal{F})$, the gauge transformations of metric perturbations $\phi$ and $\psi$ are two functions of time only. These novel features of gauge transformations allow us to analyze the dynamics of linear perturbations by using the gauge dependent variables, which will not lead to any gauge artifacts even though we do not fix the gauge completely. Given these observations, we studied the linear dynamics of scalar perturbations without any gauge fixing in section IV.

Firstly, we used our method to study the dynamics of scalar graviton in the original “nonprojectable model” and found that the scalar mode does not exist in the absence of matter sources, which is consistent with the existing results in the literatures. Secondly, we analyzed the dynamics of non-zero Fourier modes of scalar graviton in the infrared limit of the healthy extension of HL gravity by our method. Both our numerical and analytical solutions show that on the sub-horizon scale, the non-zero Fourier modes oscillate with exponentially damping amplitudes; but on the super-horizon scale, the phases are frozen and amplitudes decay continuously until they approach to their asymptotic values, i.e. the Fourier modes are conserved once they cross the horizon. Finally, the dynamics of zero Fourier mode of scalar graviton is also presented by using the gauge-invariant variables. As the case of the non-zero modes on super-horizon scale, the zero mode also decays exponentially with respect to time initially, and then approaches to an asymptotic value.

In this note we have only investigated the dynamics of scalar graviton in a de Sitter universe in the healthy extension of the HL theory without matter sources. For a more realistic case, we should take some matter sectors into account. In particular, in the early universe, it is natural to introduce other scalar field(s) to drive the inflation. Thus, in such a system there will exist two scalar perturbations at least, one is the HL scalar graviton and the others are the fluctuations of matter fields. So, it is natural to ask among all scalar perturbations which one is responsible for the generation of primordial adiabatic perturbations. Some other cosmological aspects, such as primordial non-Gaussianities and trans-planckian problems, in the healthy extension of HL gravity are also worthy to further investigate.

Note added: At the last stage of this work, two papers [54, 55] appeared in the arXiv, which contain some relevant discussions. The authors of both [54] and [55] consider a system within the healthy extension of HL gravity coupled minimally with a Lifshitz-like inflaton field $\phi$. In such a system, there exist two dynamical scalar degrees of freedom on the linear perturbation level around a cosmological background, one is the scalar graviton $\psi$ which newly appears in the healthy extension, the other is the inflaton fluctuation $\delta\phi$. The authors in both [54] and [55] found that the extra scalar graviton couples with the matter field in a general FRW universe. But in the infrared limit, these two modes decouple. However, the reasons given in the two papers for the decouple phenomenon is a little bit different. In [54] Koh and Shin argued that the prefactor of kinetic term $\dot{\psi}^2$ vanishes at the leading order $\mathcal{O}(k^0)$ when the momentum ($k \to 0$), i.e. the extra scalar graviton becomes non-dynamical due to the absence of kinetic term. On the other hand, Cerioni and Brandenberger [55] performed the calculation to the next order and they found that the kinetic term reads $\ddot{\psi}^2/H^2$ at $\mathcal{O}(k^2)$ order, but the mass of the extra scalar graviton becomes infinitely heavy when one recast the kinetic term into the canonical form $(\dot{\psi}^2/2)$, because the mass term takes the form of $(m_0k^0 + m_2k^2 + m_4k^4 + \cdots)H^2\dot{\psi}^2/k^2$. Although the kinetic term does not vanish at the sub-leading order, the scalar graviton still becomes non-dynamical due to the infinitely heavy mass. Hence, at late times, it will decouple from low energy physics and will not contribute to cosmological perturbations on scales relevant to current observations.

In our present work, we merely considered a pure gravity theory and found that the scalar graviton is still dynamical in the infrared limit. This result is consistent with those in [54, 55], because when one turn off the matter sector related with the inflaton $\phi$ in the Lagrangian, both $m_0$ and $m_2$ vanish simultaneously, and the leading term becomes $m_4k^4\dot{\psi}^2$. Furthermore, the kinetic term still takes the form of $\ddot{\psi}^2/H^2$, so after expressing the Lagrangian in the canonical form the mass term of $\psi$ reads $m_4k^2H^2\dot{\psi}^2$, i.e. the extra scalar graviton keeps massless in the infrared limit of the de Sitter background. This feature is the most important difference from the theory coupled with scalar matter fields [54, 55]. Finally we would like to mention that the ghost free condition in a de Sitter background in our derivation
(\eta > 0) are in contradiction from those obtained by Cerioni and Brandenberger (\eta < 0) \cite{55}. However, our result is consistent with the one from the consideration in a Minkowskian background \cite{48}. So, it is necessary to fix this disagreement, but it is out of the scope of the present work.

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Appendix A:

In this appendix we list the coefficients in (55), which is obtained through a non-perturbative method. As a general consideration, we keep all terms in the action (47)

\[ F_1 (x) = \frac{4\sigma}{3(1 - \lambda)} G^{-1} (x) \left( G(x) - \frac{2\sigma M_{pl}^2}{3(1 - \lambda)} \right), \]

\[ F_2 (x) = \frac{8\sigma}{9(1 - \lambda)^2} G^{-1} (x) \left[ 3 \left( \frac{1 - \lambda}{M_{pl}^2} - 2\sigma \right) \eta_4 x^6 + \left( 4\sigma M_{pl}^2 - 3(1 - \lambda) \right) \eta_2 x^4 + (2\sigma M_{pl}^2 - 3(1 - \lambda) M_{pl}^2) \eta x^2 \right], \]

\[ F_3 (x) = 8G^{-1} (x) \left[ M_{pl}^{-6} \eta_5 x^{12} - 2M_{pl}^{-4} \eta_3 \eta_5 x^{10} + M_{pl}^{-2} \left( \eta_3^2 - 2g_1 \eta_5 \right) x^8 + \left( 2g_1 \eta_3 + M_{pl}^{-2} \sigma \right) \eta x^6 \right. \]

\[ + \left( g_2 M_{pl}^2 - \frac{1}{3} \frac{\sigma}{1 - \lambda} \eta_3 \right) x^4 + \frac{1}{3} \frac{\sigma}{1 - \lambda} \eta_1 M_{pl}^2 x^2 \]

\[ + 8\sigma(1 - \lambda)^{-1} G^{-2} (x) \left[ 2M_{pl}^{-4} \eta_5 \eta_4 x^{12} - \frac{4}{3} \eta_5 \eta_2 + 2\eta_3 \eta_4 \right) M_{pl}^{-2} x^{10} \]

\[ + \left( \frac{4}{3} \eta_3 \eta_2 - \frac{2}{3} \eta_5 \eta - 2g_1 \eta_4 \right) x^8 + \left( \frac{2}{3} \eta_3 \eta + \frac{4}{3} g_1 \eta_2 \right) M_{pl}^2 x^6 + \frac{2}{3} g_1 M_{pl}^4 \eta x^4 \right] \]

\[ - \frac{32g_4 - 12g_5}{M_{pl}^4} x^6 + \frac{32g_2 + 12g_3}{M_{pl}^2} x^4 - 4g_1 x^2, \]

\[ G(x) = \left( -\frac{\eta_4}{M_{pl}^2} x^6 + \eta_2 x^4 + \eta_4 M_{pl}^2 x^2 + \frac{2\sigma M_{pl}^2}{3(1 - \lambda)} \right). \]

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