Resonant effect of the ultrarelativistic electron–positron pair production by gamma quanta in the field of a nucleus and a pulsed light wave

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Abstract

Resonant electron–positron pair production by a high-energy gamma quantum $\omega_i \gtrsim 10^2$ GeV in the field of a nucleus and a quasi-monochromatic laser wave with the intensities $I \lesssim 10^{16} - 10^{17}$ W cm$^{-2}$ was theoretically studied. Under the resonant condition, an intermediate virtual electron (positron) in the laser field becomes a real particle. Due to this fact, the initial process of the second order in the fine structure constant of a laser field effectively reduces into the two successive processes of the first order: the laser-stimulated Breit–Wheeler process and the laser-assisted process of an intermediate electron (positron) scattering by a nucleus. It is shown that there is a threshold energy for the initial gamma quantum, which significantly depends on the number of absorbed wave photons. At the resonance, the electron–positron pair energies are determined by the outgoing angle between the momenta of the initial gamma quantum and the positron (for the channel A) or the electron (for the channel B). The differential cross-sections for the first few resonances with simultaneous registration of the energy and the outgoing angle of the positron or the electron were obtained. For the initial gamma quantum energy $\omega_i = 125$ GeV the resonant electron–positron pair energies for the case of the first three resonances can be measured with a very high magnitude of the differential cross-section: from $\sim 10^{13}$ for the first resonance to $\sim 10^8$ (in the units of $\alpha Z^2 r_e^2$) for the third resonance.

Keywords: electron–positron pair, nucleus, pulsed laser field, gamma quantum

(Some figures may appear in colour only in the online journal)

1. Introduction

Nowadays, plenty of powerful laser radiation sources are widely used in physical experiment practice [1–5]. Therefore, the theoretical study of quantum electrodynamics (QED) processes in a strong light field is one of the great priority trends which has intensively developed [6–53]. The main results have been systematized in monographs [6–9] and reviews [10–14].

One of the main peculiarities of the QED processes of higher than the first order in the fine structure constant in a laser field (the QED processes assisted by a laser field) can occur through the resonant channels. In a laser field the so-called Oleinik resonances [17, 18] can take place due to the fact that in an external light field the first-order processes in the fine structure constant (the laser-stimulated processes of QED) are allowed [6]. We accentuate that the probability of a resonant process in a laser field significantly (by the several orders of magnitude) exceeds the corresponding one for a process without a laser field.

The resonant behavior of the photoproduction of a pair within the external electromagnetic plane wave field can be accounted for in the following manner [17, 18]. Due to the
Fourier transform, there are poles in the Green’s function of a fermion that emerge:

\[ \hat{E} + r\omega = \pm \sqrt{\left(\hat{\mathbf{p}} + r\mathbf{k}\right)^2 + m^2}; \quad r = 0, \pm 1, \pm 2, \ldots \] (1)

Here, \( \hat{\mathbf{p}} = (\hat{E}, \mathbf{p}) \) and \( m^2 \) are the four-quasimomentum and the effective mass of a fermion (see the following equation (16)) in the field of a plane electromagnetic wave [6], accordingly. The plus and minus signs correspond to the electron and positron states, respectively. According to the conventional interpretation of the poles of the Green’s function [34], the quantities \( E_{\text{pr}} = \hat{E} + r\omega \) and \( \mathbf{P}_r = \hat{\mathbf{p}} + r\mathbf{k} \) can be considered as the energy and the momentum of a quasiparticle that correspond to a system consisting of an electron (positron) and a plane electromagnetic wave. Consequently, although the potential of the external field depends on the time, one can introduce a discrete energy spectrum of the system under study, and this spectrum consists of an infinite number of levels. Therefore, the physical nature of resonances in the system under consideration is the same as in the resonant transition in a discrete spectrum.

There are many previous researches [8, 9, 14–16], which are dedicated to the problem of the resonant photoproduction of pairs (PPP). For example, there was consideration of the resonance only for one of the possible channels, when in the field of a wave the initial gamma quantum produces a positron and an intermediate electron, which is then scattered by a nucleus at large angles (the channel A) [8, 9, 14]. The second channel, when the initial gamma quantum produces an electron and an intermediate positron, which is then scattered by a nucleus (the channel B), has not been studied. In the article [15] authors investigated the first resonance (with the absorption of the one wave photon) of the PPP with the ultrarelativistic energy of a pair. Herewith, the positron and the electron propagate in a narrow cone along the initial gamma quantum momentum. This process was studied in the plane monochromatic wave field. It should be noted that the crossed channel of the considered process is a spontaneous bremsstrahlung of an ultrarelativistic electron in the field of a nucleus and a plane monochromatic wave [19]. In addition, we would like to point out some works where the authors draw attention to the non-resonant PPP and discuss related issues as well [20–28].

In the present paper, we develop the theory of the several first resonances (with the absorption of one, two, three and so on photons of a wave) for the process of the electron–positron pair production by the high-energy gamma quantum in the field of a nucleus and a pulsed laser wave.

There are two characteristic parameters in the problem of PPP on a nucleus in the field of a plane wave. The classical relativistically invariant parameter is [6]:

\[ \eta_0 = \frac{eF_0}{mc} \] (2)

which numerically is equal to the ratio of the work of a field at a wavelength to the electron rest energy (\( e \) and \( m \) are the charge and the electron mass, \( F_0 \) and \( \lambda = c/\omega \) are the strength and the electromagnetic wavelength, \( \omega \) is the wave frequency).

The quantum multiphoton parameter [7, 29] (Bunkin–Fedorov parameter) is:

\[ \gamma_0 = \eta_0 \frac{mve}{\hbar\omega} \] (3)

Herein \( v \) is the electron (positron) velocity and \( c \) is the light speed. Within the optical frequency range (\( \omega \sim 10^{15} \text{ s}^{-1} \)) the classical parameter is of the order of unity \( \gamma_0 \sim 1 \) for the fields \( F_0 \sim 10^{10}–10^{11} \text{ V cm}^{-1} \). At the same time, the quantum parameter has the order of \( \gamma_0 \sim 1 \) for the fields \( F_0 \sim \left(10^5 – 10^6\right) \left(c/\nu\right) \text{ V cm}^{-1} \). Hence, in the fields with \( \gamma_0 \sim 1 \) the quantum parameter \( \gamma_0 \) may be large. However, this is true only if the electrons (positrons) are scattered by the nucleus at large angles. In such a situation, the quantum parameter \( \gamma_0 \) defines multiphoton processes [7, 29]. We underline that for the process of PPP when the electrons (positrons) are scattered by a nucleus at small angles, the quantum parameter (3) does not appear [15, 30]. Consequently, the main parameter that determines the multiphoton processes is the classical relativistically invariant parameter (2). This case will be considered in the present article. Throughout this paper, we will use the relativistic system of units: \( \hbar = c = 1 \).

2. The amplitude of the process

Let us choose the four-potential of a plane electromagnetic pulse wave propagating along the \( z \) axis in the following form:

\[ A(\varphi) = \left( \frac{F_0}{\omega} \right) \cdot g \left( \frac{\varphi}{\omega\tau} \right) \cdot \left( e_x \cos \varphi + \delta e_y \sin \varphi \right), \]

\[ \varphi = kx = \omega (t - z), \] (4)

where \( k = (\omega, \mathbf{k}) \) is the wave four-vector and \( \delta \) is the ellipticity parameter of the wave. Also, here \( e_x = (0, e_x) \) and \( e_y = (0, e_y) \) are the polarization four-vectors of the wave, particularly \( e_x \cdot e_y = -1 \). \( (e_x, k) = K^2 = 0 \). In expression (4) the function \( g(\varphi/\omega\tau) \) is the envelope of our potential. We require \( g(0) = 1 \) and the exponential decrease of the function \( g \to 0 \) when \( |\varphi| \gg \omega\tau \). Hereinafter, we assume that the duration of a pulse significantly exceeds the characteristic oscillation time of a wave:

\[ \omega\tau \gg 1. \] (5)

In this case, we can consider \( \tau \) as a duration of a laser pulse. We treat the effect of the nuclear Coulomb potential in the Born approximation. This consideration leads to the restriction on the nuclear charge (\( Z e^2 / v \ll 1 \), \( Z \) is the nuclear charge).

The process of the resonant PPP on a nucleus in the presence of an external electromagnetic field is the second-order process in the fine structure constant, and it is described by two Feynman diagrams (see figure 1).

The envelope function is selected as a function of variable \( \varphi \); therefore, the electromagnetic field (4) can be treated as a plane wave. It is known that there is an exact solution of the Dirac equation for an electron in the field of a plane wave with an arbitrary spectral composition (Volkov function) [55]. Also,
there is the expression for the Green’s function of an electron in the field of a plane wave [56, 57]. This allows us to write a general expression for the amplitude due to the standard procedure of the perturbation theory [54]:

\[
S = -ie^2 \int d^4x_1d^4x_2 \times \left[ \bar{\psi}_{p_+}(x_1|A) \tilde{\gamma}_0 A_0 \left( |x_2]\right) G(x_2,x_1|A) \hat{A}_i \left( x_1, k_i \right) \times \psi_{p_+}(x_1|A) + \bar{\psi}_{p_-}(x_1|A) \hat{A}_i \left( x_1, k_i \right) G(x_1,x_2|A) \tilde{\gamma}_0 A_0 \left( |x_2]\right) \times \psi_{p_-}(x_2|A) \right].
\] (6)

Hereinafter the notations with hats imply the dot product of the corresponding four-vector with the Dirac gamma matrices: \( \tilde{\gamma}_\mu = (\tilde{\gamma}_0, \tilde{\gamma}_\mu) \), \( \mu = 0, 1, 2, 3 \) (for example, \( \hat{A}_i = A^\mu_\nu \tilde{\gamma}_\mu \tilde{\gamma}_0 - A^0_\nu \tilde{\gamma}_0 \)). In equation (6) \( A^\mu_\nu \left( x_1, k_i \right) \) is the four-potential of the initial gamma quantum and \( A^0 \left( |x_2]\right) \) is the Coulomb potential of the nucleus that has the following expressions:

\[
A^0 \left( |x_2]\right) = \frac{Ze}{|x_2|}. \] (8)

Herein \( \epsilon^\mu \) and \( k_i = (\omega_i, k_i) \) are the polarization four-vector and the four-momentum of the initial gamma quantum correspondingly. Besides that, \( \psi_{p_+} \) and \( \psi_{p_-} \) are the Volkov’s functions of the electron and the positron, \( G(x_2,x_1|A) \) and \( G(x_1,x_2|A) \) are the Green’s functions of the intermediate electron and positron within the external pulsed field correspondingly:

\[
\psi_{p_+}(x_n|A) = D(-p_+,x_n) \frac{\nu_{p_+}}{\sqrt{2E_+}}, \quad \psi_{p_-}(x_n|A) = \frac{\tilde{u}_{p_-}}{\sqrt{2E_-}} D(p_-,x_n),
\] (9) (10)

\[
G(x_n,x_n',|A) = -\int \frac{d^4p}{(2\pi)^4} D(p_n,x_n) \frac{\tilde{p} + m}{p^2 - m^2} D(p_n,x_n')
\] (11)

\[
D(p_n,x_n) = \left[ 1 + \frac{e}{2(kp)} \tilde{k} \tilde{A} (k_n) \right] \exp \{iS(p_n,x_n)\}; \quad \tilde{D} = \tilde{\gamma}_0 D^\dagger \tilde{\gamma}_0
\] (12)

\[
S(p_n,x_n) = -(p_n \cdot x_n) - \frac{e}{(kp)} \int \left[ (p \tilde{A} (\varphi')) - \frac{eA^2(\varphi')}{{2}} \right] d\varphi'.
\] (13)

Here \( p_{\pm} = (E_{\pm}, \pm p_{\pm}) \) are the four-momenta of the positron and the electron, \( \nu_{p_+} \) and \( \tilde{u}_{p_-} \) are the Dirac bispinors of the positron and electron, respectively. Expression (13) represents the classical action for a fermion within the external plane wave field [54].

For the onward analysis we have to integrate equation (13) assuming the certain potential of the external pulse field. We perform the integration with accuracy to the zeroth order of the value \( 1/\omega \tau \ll 1 \), for example:

\[
\int_{-\infty}^{\varphi} \frac{\varphi'}{\omega \tau} \cdot d(\sin \varphi') = \sin \varphi g \left( \frac{\varphi'}{\omega \tau} \right)_{-\infty}^{\varphi}
\] (14)

Eventually, the expression for equation (13) takes the form:

\[
S(p_n,x_n) = \int \tilde{p}_\xi dx'_{n}\xi
- \eta(\varphi_n) \frac{m}{(kp_n)} [p_n e_i \sin \varphi_n - \delta (p_n e_i) \cos \varphi_n]
- \frac{1 - \delta^2}{ \eta(\varphi_n) } \frac{m^2}{8(kp_n)} \sin 2\varphi_n; \quad \varphi_n = kx_n,
\] (15)
We introduce here the quasi-momentum of a particle \( (\varphi_n = kx_n; \ n = 1, 2) \):

\[
\tilde{p}_n = p_n + (1 + \delta^2) \eta^2 (\varphi') \frac{m^2}{4(kp_x)} k'; \quad \varsigma = +, -
\]

(16)

\[
\eta(\phi_n) = \eta_0 g(\phi_n), \quad n = 1, 2.
\]

Here \( \eta_0 \) is the intensity of a wave at a pulse peak (2), and \( g(\phi_{1, 2}) \) is the envelope function of a laser wave (4).

Therefore, the classical action (33) within the external classical field (4) due to assumption (5) fully matches with the case of the monochromatic plane wave, accepting the only difference that the four-momenta \( \tilde{p}_n \) and amplitudes of the potential depend on slowly varies variable \( \varphi \). Such a structure of the classical action corresponds to separation of classical fermion motion into systematic drift along a certain smooth trajectory and rapid oscillation in the vicinity of it. The dependence of \( \tilde{p}_n \) on the variable \( \varphi \) is the effect of ponderomotive scattering [31]. This dependence no longer allows us to consider \( \tilde{p}_n \) as the quasi-momenta of the particles, unlike the case of monochromatic plane waves.

Using relations (9)–(13) we obtain the expression for the amplitude of the PPP within the external field:

\[
S = -i \frac{Ze_0 \sqrt{\pi}}{\sqrt{2\omega_0 E_0 \varepsilon_0}} \bar{u}_{\mu} B \nu_{\mu,},
\]

(18)

where

\[
H_{s,1}^{(1)} \left( \frac{\varphi_1}{\omega \tau}, \frac{\varphi_2}{\omega \tau} \right) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int_{\varphi_1}^{\varphi_1 + 2\pi} d\varphi'_{1} \int_{\varphi_2}^{\varphi_2 + 2\pi} d\varphi'_{2} \times P_1 \left( \frac{\varphi'}{\omega \tau}, \frac{\varphi'}{\omega \tau}, \frac{\varphi'}{\omega \tau}, \frac{\varphi'}{\omega \tau} \right) \times \exp \left( -is\varphi'_{1} - is'\varphi'_{2} \right).
\]

(23)

We apply the Taylor expansion to the slowly varied part of the functions \( P_1 \) and \( P_2 \) (21) within the vicinity of the point \( \varphi_{1, 2}' = \varphi_{1, 2} \):

\[
P_1 \left( \varphi_{1}', \varphi_{2}', \varphi_{1}', \varphi_{2}' \right) = P_1 \left( \varphi_{1}', \varphi_{2}', \varphi_{2}', \varphi_{1}' \right) + \frac{\varphi_{1}' - \varphi_{1}}{\omega \tau} \frac{\partial P_1(\varphi_{1}')}{\partial g} g' \left( \frac{\varphi_{1}}{\omega \tau} \right) + \frac{\varphi_{2}' - \varphi_{2}}{\omega \tau} \frac{\partial P_1(\varphi_{2}')}{\partial g} g' \left( \frac{\varphi_{2}}{\omega \tau} \right) + \cdots.
\]

(24)

It follows that in the frame of the zeroth order accuracy with respect to \( 1/\omega \tau \) Fourier coefficients depends on the initial point of the interval only through the slowly varied function \( g(\varphi/\omega \tau) \). Due to this fact, the expressions for the Fourier coefficients may be presented in the following way:

\[
H_{s,1}^{(1)} \left( \frac{\varphi_1}{\omega \tau}, \frac{\varphi_2}{\omega \tau} \right) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int_{0}^{2\pi} d\varphi_{1}' d\varphi_{2}' P_1 \left( \frac{\varphi_{1}'}{\omega \tau}, \frac{\varphi_{1}'}{\omega \tau}, \frac{\varphi_{2}'}{\omega \tau}, \frac{\varphi_{2}'}{\omega \tau} \right) \times \exp \left( -is\varphi'_{1} - is'\varphi'_{2} \right).
\]

(25)
Here, to perform integration in (26) we have to change the set of variables:

\[ t_n = \frac{(x_n + x_{n-})}{2}, \quad z_n = \frac{(x_n - x_{n-})}{2}, \quad n = 1, 2. \]  

For further calculations, we consider the process in the case of a circular polarization (\( \delta^2 = 1 \)) and the field intensity of the pulse peak is:

\[ \eta_0 \ll 1. \]  

Taking this consideration into account [9, 14], we obtain the expression for the amplitude of the PPP process on a nucleus in the field of a plane quasi-monochromatic (5) weak electromagnetic wave (28):

\[ S = \sum_{l = -\infty}^{+\infty} S(l). \]

Herein \( S(l) \) is a partial amplitude of the process with emission (absorption) of \( |l| \)-photons of a wave:

\[ S(l) = \frac{2\pi^2 \omega Z e^3}{\sqrt{2\alpha E E^L + (q^2 + q_0 (q_0 - 2\xi))}}, \]

where

\[ B'_{(l)} = B'_{+(l)} + B'_{-(l)}, \quad q = p_+ + p_+ - k_+ + \ell k. \]  

Here \( q = (q_0, q) \) is a transferred four-momentum, \( B'_{+(l)} \) and \( B'_{-(l)} \) are the amplitudes for the channels A and B:

\[ B_{+(l)} = \sum_{r = -\infty}^{+\infty} \int_{-\infty}^{\infty} d\phi \int_{-\infty}^{\infty} d\phi_2 \exp(iq_0 r \phi_2) M_{(+)}^{(l, r)} (p_- q_-, \phi_2) \times G(q_-, \phi_2 - \phi_1) \left[ \epsilon_{\mu} F_{(-)}^{(l, r)} (q_-, p_+, \phi_1) \right], \]

\[ B_{-(l)} = \sum_{r = -\infty}^{+\infty} \int_{-\infty}^{\infty} d\phi \int_{-\infty}^{\infty} d\phi_2 \exp(iq_0 r \phi_2) M_{(+)l}^{(l, r)} (p_+ q_+, \phi_2) \times G(q_+ \phi_2 - \phi_1) \left[ \epsilon_{\mu} F_{(-)}^{(l, r)} (q_+, p_-, \phi_1) \right], \]

\[ G(q_\pm, \phi_1 - \phi_2) = \int_{-\infty}^{\infty} d\xi \left[ \left( \frac{\hat{q}_\pm + m}{\sqrt{q_\pm^2 - m^2}} \right)^2 + 2\xi (kq_\pm) \right] \times \exp[i(\omega \tau \xi) (\phi_2 - \phi_1)], \]

\[ q_- = -p_+ + k_+ + \ell k, \quad q_+ = -p_- + k_+ + \ell k. \]

In the equations (31)–(34) we introduced \( \phi_0 = \varphi_n/\omega \tau \) (\( n = 1, 2 \)), also we defined \( q_- \) and \( q_+ \) as the four-momenta of the intermediate electron and positron for the channels A and B (see figure 1 and equation (35)). In the expressions (32) and (33) there is the matrix \( M_{(+/)}^{(l, r)} (p'_+, p, \phi_2) \), which under the resonant conditions and with the expression in square brackets in the denominator (30) taken into account represents the amplitude of the intermediate electron (positron) scattering on a nucleus (\( p \rightarrow p' \)) with emission (absorption) of \( |\ell + r| \)-photons of a wave (the laser-assisted Mott process) [7, 29]. At the same time, \( F_{(-)}^{(l, r)} (p', p, \phi_1) \) represents the amplitude of an electron–positron pair production (with momenta \( p \) and \( p' \)) by a gamma quantum \( k \), with absorption of \( r \)-photons of a wave (the laser-stimulated Breit–Wheeler process) [6]:

\[ M_{(+/)}^{(l, r)} (p', p, \phi_2) = \frac{\c^0 L_{l+} (p', p, \phi_2)}, \]

\[ F_{(-)}^{(l, r)} (p', p, \phi_1) = \bar{\c}^\mu L_{r-} (p', p, \phi_1) \]

\[ + b_{p, p}^{(l, r)} (\phi_1) L_{r-1} + b_{p, p}^{(l, r)} (\phi_1) L_{r+1}. \]

Here we introduce matrices \( b_{p, p}^{(l, r)} \) and special functions \( L_{r-} (p', p, \phi_1) \) which are studied in detail in the following article [58]:

\[ b_{p, p}^{(l, r)} (\phi_1) = \eta (\phi_1) \left[ \frac{m}{4 (kp)} \bar{e}_\pm \hat{c}_L \right] \left[ \frac{m}{4 (kp)} \bar{e}_\pm \hat{c}_L \right]. \]

\[ L_{r} (p', p, \phi_0) = \exp (-i\xi p' p_0) J_s (\eta p_0 (\phi_0)). \]

Herein \( J_s \) is the Bessel function of an integer index. The parameters \( \gamma_{p, p}^{(l, r)}, \chi_{p, p}^{(l, r)} \) and four-vectors \( e_\pm \) are defined as follows:

\[ \gamma_{p, p}^{(l, r)} (\phi_0) = \eta (\phi_0) \sqrt{-Q_{p, p}}. \]

\[ \chi_{p, p}^{(l, r)} = \frac{p'}{(kp)} - \frac{p_0}{(kp)} \]

\[ \tan \chi_{p, p} = \frac{\delta (Q_{p, p} e)}{(Q_{p, p} e)} e_\pm = e_\pm \pm i d e_\gamma. \]

Herein, the magnitudes of the four-momenta \( p \) and \( p' \) in the expressions (36)–(41) are defined by the corresponding expressions in the amplitudes (32) and (33).

We want to underline that the envelope function \( g(\phi_1) \) is contained only in the amplitudes \( M_{(+)}^{(l, r)} (36) \) and \( F_{(-)}^{(l, r)} (37) \) through the parameter \( \eta (\phi_1) (17) \). From the relation (34) we can see that the substantiae range of the variable \( \xi \), which makes the main contribution to the integrals (34) is defined by the condition:

\[ |\xi| \lesssim \frac{1}{\omega \tau} \ll 1. \]

It is noteworthy that the dependence on the variable of integration in the denominator of the expression (34) is a consequence of the pulsed behavior of the laser wave [9, 14]. We emphasize that there is no such term in the case of a monochromatic wave and as a consequence that leads to a resonant infinity in the amplitude of the PPP process on a nucleus in the field of a wave [8, 15]. Since when |\xi| \gg 1/\omega \tau, due to the high-frequency oscillations, these integrals would be small. Thus, we can neglect \( \xi \) in the numerator of the expression (34) in comparison with \( (q_\pm + m) \). As a result, the integral (34) is easily calculated.
and the expression for the channels A and B takes the following form:

\[ G(q_{\pm}, \phi_1 - \phi_2) = \frac{\pi i (q_{\pm} + m)}{2(kq_{\pm})} \exp[-2i\beta_{\pm}(\phi_1 - \phi_2)] \times \text{sgn}(\phi_1 - \phi_2). \tag{43} \]

Herein \( \beta_- \) and \( \beta_+ \) are the resonant parameters for the channels A and B \([9, 14]\):

\[ \beta_{\pm} = \frac{(q_{\pm}^2 - m^2)}{4(kq_{\pm})} \omega \tau. \tag{44} \]

Taking (43) into account we finally deduce the expressions for the amplitudes (32) and (33):

\[ B_{+}^{(i)} = \sum_{r=-\infty}^{\infty} \frac{\pi i}{(kq_{+})} \int_{-\infty}^{\infty} d\phi_1 \int_{-\infty}^{\infty} d\phi_2 \exp(-2i\beta_{-}\phi_1) \]
\[ \times \exp[i(q_{0\tau} + 2\beta_{-}\phi_2)] \times \text{sgn}(\phi_1 - \phi_2) M_{1+(r)}(p_-, q_-, \phi_2)(q_{\pm} + m) \times \left[ e_{r}^{*} F^{i}_{-r}(q_+, p_+, \phi_1) \right], \tag{45} \]

\[ B_{-}^{(i)} = \sum_{r=-\infty}^{\infty} \frac{\pi i}{(kq_{+})} \int_{-\infty}^{\infty} d\phi_1 \int_{-\infty}^{\infty} d\phi_2 \exp(-2i\beta_{+}\phi_1) \]
\[ \times \exp[i(q_{0\tau} + 2\beta_{+}\phi_2)] \times \text{sgn}(\phi_1 - \phi_2) M_{1+(r)}(p_+, q_+, \phi_2)(q_{\pm} + m) \times \left[ e_{r}^{*} F^{i}_{-r}(q_+, p_+, \phi_1) \right]. \tag{46} \]

The expressions for the amplitudes (29)–(31), (45) and (46) are valid for the circular polarized quasi-monochromatic weak laser wave (5) and (28).

From now on we will consider the high-energy initial gamma quanta and the ultrarelativistic energies of the produced electrons and positrons, when all particles propagate in a narrow cone along the direction of the initial gamma quantum momentum \([15]\):

\[ \omega_i \gg m, \quad E_{\pm} \gg m, \tag{47} \]

\[ \theta_{\pm} = \angle(k, p_{\pm}) \ll 1, \quad \bar{\theta}_{\pm} = \angle(p_{+}, p_{-}) \ll 1, \tag{48} \]

\[ \theta_{i} = \angle(k, k) \sim 1, \quad \theta_{\pm} = \angle(p_{\pm}, k) \sim 1. \tag{49} \]

### 3. Poles of the PPP amplitude

Resonant behavior of the amplitudes (29)–(31), (45) and (46) is explained by quasi-discrete structure: an electron (a positron) + a plane electromagnetic wave. It follows that due to the approximate fulfillment of the energy-momentum conservation law, the four-momentum of the intermediate electron (positron) lies near the mass shell. In this way, the following conditions take place for the channels A and B in the resonance \([9, 14]\) (see (45) and (46), and also figure 2):

\[ |\beta_{\mp}| \ll 1 \Rightarrow \frac{(q_{\pm}^2 - m^2)}{4(kq_{\mp})} \ll \frac{1}{\omega \tau} \ll 1. \tag{50} \]

The resonant parameters \( \beta_{\pm} \) (44) for the case of a weak field (28) and the kinematical conditions (47)–(49) have the following form:

\[ \beta_{\mp} = \frac{r}{4} \frac{(1 - x_{\pm}) x_{\pm} \varepsilon_r - 4x_{\pm}^2 \theta_{\pm}^2 - 1}{16 \varepsilon_r (1 - x_{\pm}) x_{\pm}} \omega \tau, \tag{51} \]

\[ \delta_{\pm} = \frac{\omega \delta_{\pm}}{2m}, \quad x_{\pm} = \frac{E_{\pm}}{\omega_i}, \tag{52} \]

\[ \varepsilon_r = \frac{\omega_r}{\omega_{\text{thr}(r)}}, \quad \omega_{\text{thr}(r)} = \frac{m^2}{r \omega \sin(\theta_i/2)}. \tag{53} \]

Herein \( r = 1, 2, 3, \ldots \) is the number of resonances (the number of wave photons, which are absorbed in the laser-stimulated Breit–Wheeler process), and \( \omega_{\text{thr}(r)} \) is the threshold energy for the rth-resonance (see (55)–(59)). This energy is defined by the rest energy of an electron, the total energy of the absorbed wave photons and the angle between the momenta of the initial gamma quantum and the laser wave. Within the optical frequency range, the threshold energy for the first resonance is of the order of \( \omega_{\text{thr}(1)} \sim 10^{2} \text{ GeV} \). With an increase in the number
of resonances (absorbed photons of a wave), the threshold energy decreases by a factor of \( r \) \( (53) \). In the expression \( (53) \) we denoted the energy of the initial gamma quantum in the units of the threshold energy of \( r \)-th resonance as \( \varepsilon_r \). We recall that in the article \([15]\), only the first resonance \( (r = 1) \) of the process of the ultrarelativistic electron–positron pair photo-production in the field of a nucleus and a weak monochromatic laser wave was studied. Wherein, the resonant infinity is eliminated by the Breit–Wigner procedure with the introduction of a radiation width. In the present article, we study the process in the field of a pulsed laser wave for the first several resonances. Herewith, the resonant width, associated with the pulsed nature of a wave, arises as a consequence of the used mathematical apparatus (see \((85)\)–\((87)\), \((113)\) and \((114)\)).

Using the expressions for the resonant parameters \( (51) \) we rewrite the resonant conditions \( (50) \) for the channels A and B:

\[
\frac{r^4(1-x_{\pm}) x_{\pm} \varepsilon_r - 4 x_{\pm}^2 \delta_{\pm}^2 - 1}{16 \varepsilon_r (1-x_{\pm}) x_{\pm}} \lesssim \frac{1}{\omega^r} \ll 1. \tag{54}
\]

We can see that the expressions in the modulus have to be close to zero. Hence, we have quadratic equations that define the resonant energies of the positron and the electron. The solutions to these equations for the channels A and B have the following form:

\[
x_+ \left( \delta_+^2 \right) \approx x_{+\left(1,2\right)} \left( r \right) = \frac{\varepsilon_r \pm \sqrt{\varepsilon_r \left( \varepsilon_r - 1 \right) - \delta_+^2}}{2 \left( \varepsilon_r + \delta_+^2 \right)}, \tag{55}
\]

\[
x_- \left( \delta_-^2 \right) \approx x_{-\left(1,2\right)} \left( r \right) = 1 - x_{+\left(1,2\right)} \left( r \right), \tag{56}
\]

\[
x_- \left( \delta_-^2 \right) \approx x_{-\left(1,2\right)} \left( r \right) = \frac{\varepsilon_r \pm \sqrt{\varepsilon_r \left( \varepsilon_r - 1 \right) - \delta_-^2}}{2 \left( \varepsilon_r + \delta_-^2 \right)}, \tag{57}
\]

\[
x_+ \left( \delta_+^2 \right) \approx x_{+\left(1,2\right)} \left( r \right) = 1 - x_{-\left(1,2\right)} \left( r \right), \quad x_{\pm\left(1,2\right)} \left( r \right) = \frac{E_{\pm\left(1,2\right)} \left( r \right)}{\omega_i}. \tag{58}
\]

From the derived relations we can conclude that the resonant energies of a positron and an electron for each of the two channels depend on the parameter \( \varepsilon_r \) and take two different values for a certain positron (the channel A) or electron (the channel B) outgoing angle (see figures \( 3 \)–\( 5 \)). For clarity, we introduce here an additional upper subscription for the resonant energies of the positrons and the electrons to distinguish the expressions with different signs in front of the square root in the numerator of the relations \( (55) \) and \( (57) \). Herewith, the parameter \( \varepsilon_r \) has to satisfy the condition:

\[
\varepsilon_r \geq 1 \rightarrow \omega_i \geq \omega_{thr\left(r\right)}, \quad r = 1, 2, 3, \ldots \tag{59}
\]

It follows that value \( \omega_{thr\left(r\right)} \) \( (53) \) is the threshold energy for the initial gamma quantum. This threshold energy decreases with increase in the number of absorbed wave photons. From the expressions \( (55) \) and \( (57) \) we can see that the possible outgoing angles depend on the value of the parameter \( \varepsilon_r \) and are enclosed in the interval:

\[
0 \leq \delta_\pm^2 \leq \delta_\pm^2_{\text{max}}, \quad \delta_\pm^2_{\text{max}} = \varepsilon_r \left( \varepsilon_r - 1 \right), \quad \varepsilon_r \geq 1. \tag{60}
\]

**Figure 3.** Dependence of the positron resonant energy (in the units of the initial gamma quantum energy) on the square of the positron outgoing angle (the parameter \( \delta_+^2 \) ) (figure 3(a), for the channel A) and on the square of the electron outgoing angle (the parameter \( \delta_-^2 \) ) (figure 3(b), for the channel B) for the case \( \omega_i > \omega_{thr\left(1\right)} \) and the first three resonances \( \left( \omega_i = 125 \text{ GeV}, \omega_{thr\left(1\right)} = 100 \text{ GeV} \right) \).

In figures 3(a) and (b), we represent possible values of the positron energy for the channels A and B for the first, second and third resonances in the case when the initial gamma quantum energy is greater than the threshold energy for the first resonance \( \omega_i > \omega_{thr\left(1\right)} \). In figures 4(a) and (b), we represent the possible values of the positron energy for the channels A and B for the second, third and fourth resonances in the case when the initial gamma quantum energy is greater than the threshold energy for the second resonance \( \omega_i > \omega_{thr\left(2\right)} \). In figures 5(a) and (b), we depict the possible values of the positron energy for the channels A and B for the third, fourth and fifth resonances in the case when the initial gamma quantum energy is greater than the threshold energy for the third resonance \( \omega_i > \omega_{thr\left(3\right)} \). We would like to emphasize that all the curves (figures 3–5) are presented.
Figure 4. Dependence of the positron resonant energy (in the units of the initial gamma quantum energy) on the square of the positron outgoing angle (the parameter $\delta_2^+$) (figure 4(a), for the channel A) and on the square of the electron outgoing angle (the parameter $\delta_2^-$) (figure 4(b), for the channel B) for the case $\omega_{thr}(1) > \omega_i > \omega_{thr}(2)$ and for the second, third and fourth resonances ($\omega_i = 75$ GeV, $\omega_{thr}(2) = 50$ GeV).

Figure 5. Dependence of the positron resonant energy (in the units of the initial gamma quantum energy) on the square of the positron outgoing angle (the parameter $\delta_2^+$) (figure 5(a), for the channel A) and on the square of the electron outgoing angle (the parameter $\delta_2^-$) (figure 5(b), for the channel B) for the case $\omega_{thr}(2) > \omega_i > \omega_{thr}(3)$ for the third, fourth and fifth resonances ($\omega_i = 40$ GeV, $\omega_{thr}(3) = 33.3$ GeV).

It is very important to emphasize that the electron–positron pair energies for the channel A depend only on the outgoing angle of a positron (parameter $\delta_2^+$) and for the channel B depend only on the outgoing angle of an electron (parameter $\delta_2^-$). We can also see that in the frame of one of the channels (A or B) different resonances do not interfere (see figures 3–5, there are no intersections of the curves with different $r$). Due to this fact, we can ignore the interference between different resonances in the frame of a certain channel when calculating the resonant differential cross-section. Herewith, we will exclude the case of the electron (positron) scattering at a zero angle ($\delta_2^--\delta_2^+=0$).

Finally, we would like to summarize that the presence of the considered resonances depends on the initial gamma quantum energy as well as the kinematics (outgoing angles) of
the produced electron–positron pair. Besides that, an important role is played by the number of absorbed photons in the external field (see (53)).

4. Resonant cross-section of the PPP

We simplify amplitudes of the process (29), (31), (45) and (46) for the case of the resonant kinematics (47)–(49), (55) and (57). Consequently, the arguments of the Bessel function (39) in the amplitudes \(M_{l+r}^\mu\) (36) and \(F_{l-r}^\mu\) (37) are small \((\gamma_{l+r} \lesssim \eta_0 \ll 1)\). So we can put:

\[
M_{l+r}^\mu(p',p,\phi_2) \approx \tilde{\gamma}^0 L_0 (p',p,\phi_2) \approx \tilde{\gamma}^0 (l = -r), \tag{61}
\]

\[
F_{l-r}^\mu(p',p,\phi_1) = \tilde{\mu} L_{l-r}(p',p,\phi_1) + L_{l+r+1}(p',p,\phi_0) \eta(\phi_1) \times \left[ \frac{m}{4(kp')} \hat{e}_r \tilde{\gamma}^\mu - \frac{m}{4(kp)} \tilde{\gamma}^\mu \hat{e}_r \right]. \tag{62}
\]

The further analysis will be carried out for a certain type of the envelope function. Let us choose the envelope function in a Gaussian-like form:

\[
g(\phi_1) = \exp \left\{- \frac{(2\phi_1)^2}{2} \right\}. \tag{63}
\]

Here we want to note that the chosen envelope function does not quite correspond to the real shape of the laser pulse. However, it allows us to carry out a further investigation analytically and gives us the opportunity to obtain the results that qualitatively explain the behavior of the resonances.

Next, we apply the series expansion for the Bessel function with respect to the small parameter in the amplitudes (62) and also integrate it with respect to \(\phi_1\). Finally, the amplitude of the PPP for the resonance with number \(n\) takes the following form:

\[
S_{l-1} = \frac{\pi^2 \gamma^2 \omega |E|^3}{2\omega |E_\gamma| E_+} \left\{ \frac{n_{\mu, B_{l-1}}(q_{l-1})}{q^2 + q_0 (q_0 - 2q_z)} \right\}, \tag{64}
\]

where

\[
B_{l-1} = B_{l+1} + B_{l-1}. \tag{65}
\]

Here \(B_{l+1}\) and \(B_{l-1}\) are the resonant amplitudes for the channels A and B:

\[
B_{l+1} = U_{l+1}(q_{l+1}) \left[ \tilde{\gamma}^0 \left( \hat{q}_{l+1} + m \right) \left( \tilde{\gamma}^\mu \hat{e}_r \right) \right], \tag{66}
\]

\[
B_{l-1} = U_{l-1}(q_{l-1}) \left[ \tilde{\gamma}^0 \left( \hat{q}_{l-1} + m \right) \left( \tilde{\gamma}^\mu \hat{e}_r \right) \right], \tag{67}
\]

\[
U_{l+1}(q_{l+1}) = \frac{\eta_0}{\sqrt{r(kq_{l+1})}} \exp \left\{ -\frac{\beta^2_{l+1}}{4r} \right\} V_r(q_0, \beta_{l+1}), \tag{68}
\]

\[
V_r(q_0, \beta_{l+1}) = \int_{-\infty}^{\infty} d\phi_2 \exp \left\{ i (q_0 \tau + 2\beta_{l+1} \phi_2) \right\} \times \text{erf} \left( 2\sqrt{r} \phi_2 + \frac{i\beta_{l+1}}{2\sqrt{r}} \right). \tag{69}
\]

We point out that there is no summation over \(r\) in the amplitudes (66) and (67) due to the absence of the interference of resonances with different \(r\). The expressions \(c_{l\pm(r)}^\mu\) have the following form:

\[
c_{l+1}^\mu(r) = \gamma^\mu c_{l+1}^\mu + c_{l+1}^{-1}(q_{l+1}) \left[ \frac{m}{4(kq_{l+1})} \hat{e}_r \tilde{\gamma}^\mu - \frac{m}{4(kp_{l+1})} \tilde{\gamma}^\mu \hat{e}_r \right], \tag{70}
\]

\[
c_{l-1}^\mu(r) = \gamma^\mu c_{l-1}^\mu + c_{l-1}^{-1}(q_{l-1}) \left[ \frac{m}{4(kq_{l-1})} \hat{e}_r \tilde{\gamma}^\mu - \frac{m}{4(kp_{l-1})} \tilde{\gamma}^\mu \hat{e}_r \right]. \tag{71}
\]

Herein we introduced the following notation:

\[
c_{l\pm} = (-1)^{1/2} \frac{r}{\pi} \exp (ir\chi_{\pm}) \tag{72}
\]

\[
\gamma_{l\pm} = r \sqrt{\frac{u_{l\pm}}{u_r}} \left( 1 - \frac{u_{l\pm}}{u_r} \right). \tag{73}
\]

In the expression (73) the relativistically invariant parameters for the channels A and B are defined in the following way [6]:

\[
u_{l\pm} = \frac{(k_{l\pm})^2}{4(kp_{l\pm})(kq_{l\pm})}, \quad u_r = \frac{(k_{r})}{2m^2}. \tag{74}
\]

It is crucial to emphasize that the applicability of the obtained expansions for the Bessel functions (39) is valid when the following inequality holds for the number of resonance and wave intensity:

\[
\frac{r}{\sqrt{r+1}} \ll \eta_0^{-1}. \tag{75}
\]

Right after, we obtained the probability of the resonant process for the entire observation time in the interference absence of the channels A and B (first and second terms in (65)):

\[
dw_{l\pm(r)} = \frac{\pi^2 \omega^2 \gamma^2 Z^2 e^4}{2\omega |E_\gamma| E_+} \left\{ \frac{n_{\mu, B_{l\pm}}(q_{l\pm})}{q^2 + q_0 (q_0 - 2q_z)} \right\} \frac{d^3 p_+ d^3 p_-}{T(2\pi)^6}. \tag{76}
\]

Herein \(T\) is some relatively large observation time (\(T \gg \tau\)). This observation time must be brought into accord with the duration of the pulse as well. The reason is that in the absence of the laser field there is another process, which could possibly have an impact on the magnitude of the cross-section, the so-called Bethe–Heitler process [59]. As the non-resonant production of the electron–positron pairs on the nucleus by the initial gamma quantum may be held in the absence of the external field, so that the observation during the time greater, then the pulse width will result in the suppression of the influence of the pulsed wave. The ratio between the observation time and pulse width is determined by the conditions of a particular experiment. In the case when the detection time of kinematics and energy characteristics of the produced fermions comports with the laser pulse width, the effects, which are induced by such a pulse, may be found. Also, for the case of successive pulses...
one can think about the value $T/\tau$ as the ratio of the distance between neighboring pulses to the characteristic duration of the pulse [31].

The differential cross-section of the PPP process on a nucleus in the field of a pulsed light wave is obtained from the probability per unit of time by dividing it by the density of the incident particle flux. The flux density is equal to $j = 1 \text{ (cm}^{-2} \text{s}^{-1})$ for the case of the incident gamma quanta. Then, averaging over the initial gamma quanta polarization and summation over the polarization of the final electrons and positrons is carried out in the standard way [54]. Thus, after all the calculations, we derived the resonant differential cross-section of the PPP (for example, for the channel A) in the following form:

$$
d\sigma_{\gamma}\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!0.0
Figure 6. Dependence of the resonant profile function $P_{\text{res}}(r)$ (85) on the resonant parameter $\beta_{\pm}$ (the deviation of the position for the channel A) or electron (for the channel B) energies from their values in exact resonance, see (90)) for the first three resonances.

For the first five resonances we have: $a_{(r)} \sim 1, \epsilon_{(r)} \sim 1, b_{(r)} \sim 10^{-2}$. From the relation (86) it follows that the maximum of the function is determined by the parameter $a_{(r)}$ (88), which depends on the number of resonances and the ratio between the observation time and the duration of a laser pulse (the parameter $\rho$). In figure 6 we can see that resonances take place if the resonant parameters satisfy $|\beta_{\pm}| < 1$. When $|\beta_{\pm}| \geq 1$ the resonant profile function exponentially decreases [9, 14].

Additionally, it would be useful to mention that the parameters $\beta_{\pm}$ are fully defined by the resonant kinematics as well as the experimental conditions. One can easily expound their physical essence by rewriting them due to the expressions (55) and (57) in the following form:

$$\beta_{\pm} \sim \omega \tau \left( x_{\pm}^{(1,2)} - x_{\pm}^{(r)} \right). \quad (90)$$

In as much as the value $\omega \tau \gg 1$ then the positron energy (for the channel A) and the electron energy (for the channel B) have to be close to their resonant values $|x_{\pm}^{(r)} - x_{\pm}^{(0)}| \ll 1/\omega \tau \ll 1$ ($j = 1, 2$).

5. Analysis of the resonant differential cross-section of the PPP by a high-energy gamma quantum

We rewrite the resonant differential cross-section of the PPP (81)–(85) accordingly to the resonant kinematic (47)–(49):

$$d\sigma_{+}(r) = \alpha Z_{e}^{2} r_{0}^{2} \omega^{2} r_{0}^{2} \left( \frac{1 - x_{+}(r)}{x_{+}(r)} \right) \frac{K_{+}(r) P_{\text{res}}^{\phi}(r) K_{-}(r) P_{\text{res}}^{\phi}(r)}{e^{2}} \frac{1 - x_{+}(r)}{e^{2}} \frac{1 - x_{-}(r)}{e^{2}} \times d\delta_{+}^{2} d\delta_{-}^{2} dx_{+}(r) dx_{-}(r), \quad (91)$$

$$d\sigma_{-}(r) = \alpha Z_{e}^{2} r_{0}^{2} \omega^{2} r_{0}^{2} \left( \frac{1 - x_{-}(r)}{x_{-}(r)} \right) \frac{K_{-}(r) P_{\text{res}}^{\phi}(r) K_{+}(r) P_{\text{res}}^{\phi}(r)}{e^{2}} \frac{1 - x_{-}(r)}{e^{2}} \frac{1 - x_{+}(r)}{e^{2}} \times d\delta_{+}^{2} d\delta_{-}^{2} dx_{-}(r) dx_{+}(r). \quad (92)$$

Herein $\varphi$ is an angle between the planes $(k_{+}, p_{+})$ and $(k_{-}, p_{-})$. The resonant profile functions $P_{\text{res}}^{\phi}(r)$ are determined by the expression (85), where the resonant parameters are given by the relation (51). Functions $K_{\pm}(r)$ are defined by the probability of the laser-stimulated Breit–Wheeler process:

$$K_{\pm}(r) = \left( r_{0}^{2} \right)^{2} \cdot \frac{4x_{\pm}(r) \delta_{\pm}^{2}}{\left( \delta_{\pm}^{2} + 1 \right)^{2}} \times \left[ \frac{16x_{\pm}(r) \delta_{\pm}^{2}}{\delta_{\pm}^{2} + 1} + \frac{x_{\pm}(r) + 1}{1 - x_{\pm}(r)} \right]. \quad (93)$$

The values $d_{\pm}(r)$ are defined by the square of the transferred momentum:

$$d_{\pm}(r) = d_{0} \left( x_{\pm}(r) \right) + \left( \frac{m}{2\omega r_{0}} \right)^{2} \times \left( \delta_{\pm}^{2} \left( x_{\pm}(r) \right) + \frac{4\epsilon_{r}}{\sin(\theta/2)} \right) \left[ 4\epsilon_{r} - g_{0} \left( x_{\pm}(r) \right) \right]. \quad (94)$$

where

$$d_{0} \left( x_{\pm}(r) \right) = \delta_{\pm}^{2} + \delta_{\pm}^{2} + 2\delta_{\pm} \delta_{\pm} \cos \varphi. \quad (95)$$

$$g_{0} \left( x_{+}(r) \right) = \frac{1 + \delta_{+}^{2}}{x_{+}(r)} + \frac{1 + \delta_{+}^{2}}{1 - x_{+}(r)}, \quad (96)$$

$$g_{0} \left( x_{-}(r) \right) = \frac{1 + \delta_{+}^{2}}{x_{-}(r)} + \frac{1 + \delta_{+}^{2}}{1 - x_{-}(r)}, \quad (97)$$

In the same kinematic region, the differential cross-section of the PPP in the absence of the external laser field $d\sigma_{+}$ has the following expression [15, 54]:

$$d\sigma_{+}(r) = \frac{128}{\pi} Z_{e}^{2} r_{0}^{2} r_{0}^{2} \left( 1 - x_{\pm}(r) \right)^{3} \times \left( \frac{D_{0} \left( x_{\pm}(r) \right) + \left( m/\omega_{r} \right)^{2} D_{1} \left( x_{\pm}(r) \right)}{D_{0} \left( x_{\pm}(r) \right) + \left( m/\omega_{r} \right)^{2} D_{1} \left( x_{\pm}(r) \right)} \right) \frac{1}{2 \delta_{\pm}^{2}} dx_{\pm}(r). \quad (98)$$

Here the top sign “+” means that the cross-section without a field is integrated with respect to the electron final energy and the only dependence on the positron energy remains.
The bottom sign “−” corresponds to the integration of the cross-section with respect to the positron final energy and the dependence on the electron energy remains. In the expression (98) we denoted:

\[
D_0 (x_\pm) = - \frac{2 \delta_\pm}{(1 + \delta_\pm)^2} + \frac{\delta_\pm^2}{(1 + \delta_\pm)^2} + \frac{1}{2x_\pm(1 - x_\pm)} \left( \frac{x_\pm}{1 - x_\pm} + \frac{1 - x_\pm}{x_\pm} \right) \frac{\delta_\pm}{(1 + \delta_\pm)^2} \cos \varphi,
\]

(99)

\[
D_1 (x_\pm) = \left( \frac{1}{x_\pm} + \frac{1}{(1 - x_\pm)^2} \right) b_\pm (x_\pm), \quad \delta_\pm = 2x_\pm \delta_\pm,
\]

(100)

\[
b_\pm (x_\pm) = \frac{\delta_\pm^2}{12(1 + \delta_\pm)^2} \xi_\pm,
\]

(101)

\[
\xi_\pm = 2 \left( 1 - \delta_\pm \right) \left( 3 - \delta_\pm \right) - \frac{9 + 2 \delta_\pm^2 + \delta_\pm^4}{x_\pm (1 - x_\pm)} + \left( \frac{x_\pm}{1 - x_\pm} + \frac{1 - x_\pm}{x_\pm} \right) \left( 9 + 4 \delta_\pm^2 + 3 \delta_\pm^4 \right).
\]

(102)

The important peculiarity of the deduced differential cross-sections (91), (92), (94) and (98) is that we introduced the small corrections, which are proportional to \( \sim (m/\omega_i)^2 \ll 1 \). These corrections make a dominant contribution to the corresponding differential cross-sections under the conditions:

\[
|\varphi - \pi| \leq \frac{m}{\omega_i}, \quad |\delta_+ - \delta_-| \ll \frac{m}{\omega_i}.
\]

(103)

These conditions lead to \( D_0 (x_\pm) \rightarrow 0 \), \( d_0 (x_\pm) \rightarrow 0 \) and as a result, the corresponding differential cross-sections have sharp maxima [15]. Consequently, the differential cross-section without the external field (98) and in the field of a wave (91) in the kinematic region (103) will have the following order of magnitude:

\[
\frac{d\sigma_{\omega_\pm}(\varphi)}{d\delta_\pm^2 d\delta_-^2 dx_\pm d\varphi} \sim Z^2 \alpha r_\gamma^2 (\omega_i/m)^2 (\eta_\varphi \omega)^2 \frac{d\sigma_{\omega_\pm}(\varphi)}{d\delta_\pm^2 d\delta_-^2 dx_\pm d\varphi} \sim Z^2 \alpha r_\gamma^2 (\omega_i/m)^2 (\eta_\varphi \omega)^2.
\]

(104)

We integrate the resonant differential cross-sections (91), (92) and also the cross-section without the external field (98) with respect to the azimuth angle \( \varphi \):

\[
d\sigma_{\omega\pm}(\varphi) = \alpha Z^2 r_\gamma^2 r_\gamma^2 \left( \delta_\pm^2 + \delta_-^2 \right) x_{\omega\pm}(1 - x_{\omega\pm}),
\]

\[
\times K_{\omega\pm}(\varphi) P_{\omega\pm}(\beta_-) dx_{\omega\pm} d\delta_-^2 d\delta_-^2,
\]

(106)

\[
d\sigma_{\omega\omega\pm}(\varphi) = \alpha Z^2 r_\gamma^2 r_\gamma^2 \left( \delta_\pm^2 + \delta_-^2 \right) x_{\omega\omega\pm}(1 - x_{\omega\omega\pm}),
\]

\[
\times K_{\omega\omega\pm}(\varphi) P_{\omega\omega\pm}(\beta_-) dx_{\omega\omega\pm} d\delta_-^2 d\delta_-^2,
\]

(107)

where

\[
G_{\omega\pm}(\varphi) = \left( \delta_\pm^2 - \delta_-^2 \right) + \frac{m}{2 \omega_i} \left( \delta_\pm^2 + \delta_-^2 \right)
\]

\[
\times \left( g_0^2 (x_\pm(\varphi) + \frac{4 \epsilon_\gamma}{\sin (\theta / 2)} \left[ 4 \epsilon_\gamma - g_0^2 (x_\pm(\varphi)) \right] \right).
\]

(108)

Herewith, the differential cross-section of PPP without a laser field takes the following form [15]:

\[
d\sigma_{\omega\pm}(\varphi) = 64Z^2 \alpha r_\gamma^2 x_\pm^3 (1 - x_\pm)^3
\]

\[
\times \left( \frac{\delta_\pm^2 + \delta_-^2}{\delta_\pm^2 + \delta_-^2} \right) dx_\pm d\delta_-^2 d\delta_-^2,
\]

(109)

where \( G_{\omega\pm}(\varphi) \) can be derived from the expression for \( G_{\omega\pm}(\varphi) \) (108) if we put \( \epsilon_\gamma = 0 \). Also, we introduced the following expressions:

\[
D_2 (x_\pm) = D_0 (x_\pm) + (m/\omega_i)^2 D'_1 (x_\pm),
\]

(110)

\[
D'_1 (x_\pm) = - \frac{\delta_\pm^2}{(1 + \delta_\pm)^2} - \frac{\delta_\pm^2}{(1 + \delta_\pm)^2}
\]

\[
+ \frac{1}{2x_\pm(1 - x_\pm)} \left( \frac{x_\pm}{1 - x_\pm} + \frac{1 - x_\pm}{x_\pm} \right) \frac{\delta_\pm}{(1 + \delta_\pm)^2}
\]

\[
\times \left( \frac{\delta^2_\pm + \delta^2_-}{\delta^2_\pm + \delta^2_-} \right) \left( \frac{1 + \delta^2_-}{1 + \delta^2_-} \right) \left( 1 + \delta^2_- \right),
\]

(111)

\[
D'_1 (x_\pm) = D_1 (x_\pm) + \left( \frac{x_\pm}{1 - x_\pm} + \frac{1 - x_\pm}{x_\pm} \right)
\]

\[
\times \frac{g_0^2 (x_\pm)}{2 \left( \delta^2_\pm + \delta^2_- \right) \left( 1 + \delta^2_- \right) \left( 1 + \delta^2_- \right)}.
\]

(112)
The expression for the resonant profile function $P^{res}_{\pm}(\beta_{\pm})$ in (106), when $|\beta_{\pm}| \ll 1$, can be rewritten in the following manner:

$$P^{\text{res}}_{\pm}(r) \approx \frac{a(r)^2}{\left[\left(\delta^2_{\pm} - \delta^2_{\pm}(r)\right) + \Gamma^2_{\pm}(r)\right]} \approx P_{\text{max}}^{\text{res}}(r) = a(r). \quad (113)$$

Here the parameters $\delta^2_{\pm}(r)$ satisfy the corresponding equations for the resonant frequency for the first three resonances and the initial gamma quantum energy $\omega_1 = 125 \text{ GeV}$ ($\omega_i > \omega_{\text{max}(r)} = 100 \text{ GeV}$).

The obtained condition (118) and introduced restriction (5) together determine the possible intervals of a laser pulse, where the derived differential cross-sections are valid.

Let us investigate the ratio of the maximum resonant differential cross-section $d\sigma^{\text{max}}_{\pm}(r) \approx d\sigma_{\pm}(r)/(|\beta_{\pm}| \ll 1)$ (106) and (113) to the corresponding differential cross-section without a laser field (83):

$$R_{\pm}^{\text{max}}(r) = \frac{d\sigma^{\text{max}}_{\pm}(r)}{d\sigma_{\pm}(r)} = \frac{a(r)^2}{\left[\left(\delta^2_{\pm} - \delta^2_{\pm}(r)\right) + \Gamma^2_{\pm}(r)\right]} \times \frac{K_{\pm}(r)}{x^2_{\pm}(1 - x_{\pm}(r))^2 D_2(x_{\pm}(r))} \frac{G_{\pm}(0)}{|G_{\pm}(r)|}^{3/2}. \quad (119)$$

The expression (119) determines the magnitudes of the resonant differential cross-section of the PPP (in the units of the corresponding differential cross-section of the PPP in the absence of a laser field) for the channels A and B with simultaneous registration of the outgoing positron and electron angles (the parameters $\delta^2_{\mp}$ and $\delta^2_{\pm}$) and also the positron energy in the interval from $E_{\pm}(r)$ to $[E_{\pm}(r) + dE_{\pm}(r)]$ (for the channel A) and the electron energy in the interval from $E_{-}(r)$ to $[E_{-}(r) + dE_{-}(r)]$ (for the channel B). It is important to emphasize that for the channel A, the positron outgoing angle (parameter $\delta^2_{\mp}$) determines as the positron resonant energy $E_{\pm}(r)$, so the electron resonant energy $E_{-}(r) \approx \omega_1 = E_{\pm}(r)$. Herewith, these values do not depend on the electron outgoing angle (parameter $\delta^2_{\pm}$). Meanwhile, for the channel B, we have the opposite situation. Herein the electron outgoing angle (parameter $\delta^2_{\pm}$) determines as the electron resonant energy $E_{-}(r)$, so the positron resonant energy $E_{\pm}(r) \approx \omega_1 = E_{-}(r)$ and these values do not depend on the parameter $\delta^2_{\mp}$.

In figure 7 we represent the dependencies of the functions $R_{\pm}^{\text{max}}(r)$ (119) on the electron or positron outgoing angles (parameters $\delta^2_{\mp}$) with the certain value of the initial gamma quantum energy and the fixed outgoing angle of a positron (for the channel A) or an electron (for the channel B). We can conclude that for the channel A, $\delta^2_{\mp} = \delta^2_{\pm}$ (97) (for the channel A) and $\delta^2_{\mp} = \delta^2_{\pm}$ (for the channel B) there is a sharp maximum in the resonant differential cross-section. The resonant differential cross-section significantly exceeds the corresponding one of the PPP without a laser field. For the first resonance $R_{\pm}^{\text{max}}(1) \gtrsim 10^{10}$, for the second resonance $R_{\pm}^{\text{max}}(2) \gtrsim 10^{14}$ and for the third resonance $R_{\pm}^{\text{max}}(3) \gtrsim 10^{16}$ (see (104)). We accentuate that in figure 7 all curves are depicted for the sign “+” in front of the square root in the expression for the positron energy (55) (the channel A) or the electron energy (57) (the channel B). The second solution with sign “−” suppressed and does not make a significant contribution to the value of the resonant differential cross-section.
Further, we integrate the resonant differential cross-section with respect to the parameter $\delta^2_e$ (for the channel A) and parameter $\delta^2_\mu$ (for the channel B):

$$d\sigma_e^+(r) = (\alpha r^2 Z^2) g_e F^+(r) dx_+(r) d\delta^2_e, \quad (120)$$

$$d\sigma_\mu^-(r) = (\alpha r^2 Z^2) g_\mu F^-(r) dx_-(r) d\delta^2_\mu, \quad (121)$$

where

$$g_r = \left( \frac{\omega_{hr}(r)}{m} \right)^2 \left( \eta_0 \omega_r \right)^2, \quad (122)$$

$$F^+_e(r) = \frac{\rho}{\varepsilon_r} \left( \frac{x_+(r)}{1-x_+(r)} \right) K_+ P^\text{res}_+(r) (\beta_-), \quad (123)$$

$$F^-_\mu(r) = \frac{\rho}{\varepsilon_r} \left( \frac{x_-(r)}{1-x_-(r)} \right) K_- P^\text{res}_-(r) (\beta_+). \quad (124)$$

The relation (120) defines the resonant differential cross-section of the PPP with simultaneous registration of the positron outgoing angle and the positron resonant energy (irrespective to the electron outgoing angles). The relation (121) defines the resonant differential cross-section of the PPP with simultaneous registration of the outgoing electron angle and the electron resonant energy (irrespective to the positron outgoing angles). We underline that the function $g_r$, (96) is the same for both channels. Its magnitude is governed by the two factors. One is large enough and associated with the small transferred momentum and the threshold energy of resonant resonance. The other is associated with the transit width. For instance, when $\omega_{thr}(1) = 100\text{ GeV}$, $\eta_0 = 10^{-1}$, $(\omega_r) = 10^2$ we can calculate from (122) that for the first resonance $g_1 \approx 4 \times 10^{12}$, for the second resonance $g_2 \approx 10^{10}$, for the third resonance $g_3 \approx 4.4 \times 10^7$, for the fourth resonance $g_4 \approx 2.5 \times 10^5$, for the fifth resonance $g_5 \approx 1.6 \times 10^7$. Here the function $g_r$ essentially depends on the wave intensity and also on the resonance number. There is suppression of the laser-stimulated Breit–Wheeler process with the increase in the number of absorbed photons and the decrease in the wave intensity. It is important to point out that for the certain resonance (certain $r$) the quantitative difference of the resonant differential cross-sections for the channels A and B is determined by the functions $F^+_r$ (123) and (124). When $|\beta_-| < 1$, $|\beta_+| < 1$ these functions and the corresponding resonant differential cross-sections (120) and (121) are of the maximum values:

$$d\sigma^\text{max}_e^+(r) = (\alpha r^2 Z^2) \Phi^\text{max}_e(r) dx_+(r) d\delta^2_e, \quad (125)$$

and for the fifth resonance $g_5 \approx 1.6 \times 10^7$. Here the function $g_r$ essentially depends on the wave intensity and also on the resonance number. There is suppression of the laser-stimulated Breit–Wheeler process with the increase in the number of absorbed photons and the decrease in the wave intensity. It is important to point out that for the certain resonance (certain $r$) the quantitative difference of the resonant differential cross-sections for the channels A and B is determined by the functions $F^+_r$ (123) and (124). When $|\beta_-| < 1$, $|\beta_+| < 1$ these functions and the corresponding resonant differential cross-sections (120) and (121) are of the maximum values:

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$$d\sigma^\text{max}_e^+(r) = (\alpha r^2 Z^2) \Phi^\text{max}_e(r) dx_+(r) d\delta^2_e, \quad (125)$$

and for the fifth resonance $g_5 \approx 1.6 \times 10^7$. Here the function $g_r$ essentially depends on the wave intensity and also on the resonance number. There is suppression of the laser-stimulated Breit–Wheeler process with the increase in the number of absorbed photons and the decrease in the wave intensity. It is important to point out that for the certain resonance (certain $r$) the quantitative difference of the resonant differential cross-sections for the channels A and B is determined by the functions $F^+_r$ (123) and (124). When $|\beta_-| < 1$, $|\beta_+| < 1$ these functions and the corresponding resonant differential cross-sections (120) and (121) are of the maximum values:

$$d\sigma^\text{max}_e^+(r) = (\alpha r^2 Z^2) \Phi^\text{max}_e(r) dx_+(r) d\delta^2_e, \quad (125)$$
Table 1. The most probable values of the electron–positron pair energies and the corresponding resonant differential cross-sections.

| r | Channel | $\delta_{\pm}^2(r)$ | $E_+, \text{GeV}$ | $E_-, \text{GeV}$ | $\Phi_{\pm}^\text{max}(\eta = 0.1)$ |
|---|---------|-----------------|-----------------|-----------------|-------------------------------|
| 1 | A       | $0 \leq \delta_{\pm}^2 \leq 0.3125$ | $34.55 \leq E_+ \leq 90.45$ | $E_- = (125 - E_+)$ | $4 \times 10^{12} \leq \Phi_{\pm}^\text{max}(1) \leq 1.7 \times 10^{13}$ |
|    | B       | $0 \leq \delta_{\pm}^2 \leq 0.3125$ | $E_+ = (125 - E_-)$ | $34.55 \leq E_- \leq 90.45$ | |
| 2 | A       | $\delta_{\pm}^2(2) = \delta_{\pm}^2(1) = 0.103$ | 105.88 | 19.12 | $\Phi_{\pm}^\text{max}(2) \approx 6.95 \times 10^{10}$ |
|    | B       | $\delta_{\pm}^2(2) = \delta_{\pm}^2(1) = 0.103$ | 19.12 | 105.88 | |
| 3 | A       | $\delta_{\pm}^2(3) = \delta_{\pm}^2(1) = 0.147$ | 111.28 | 13.72 | $\Phi_{\pm}^\text{max}(3) \approx 5.89 \times 10^{8}$ |
|    | B       | $\delta_{\pm}^2(3) = \delta_{\pm}^2(1) = 0.147$ | 13.72 | 111.28 | |

Table 2. The most probable values of the electron–positron pair energies and the corresponding resonant differential cross-sections.

| r | Channel | $\delta_{\pm}^2(r)$ | $E_+, \text{GeV}$ | $E_-, \text{GeV}$ | $\Phi_{\pm}^\text{max}(\eta = 0.1)$ |
|---|---------|-----------------|-----------------|-----------------|-------------------------------|
| 1 | A       | $\delta_{\pm}^2 = \delta_{\pm}^2(2) = 0.118$ | 53.19 | 21.81 | $\Phi_{\pm}^\text{max}(2) \approx 4.05 \times 10^{10}$ |
|    | B       | $\delta_{\pm}^2 = \delta_{\pm}^2(2) = 0.118$ | 21.81 | 53.19 | |
| 2 | A       | $\delta_{\pm}^2 = \delta_{\pm}^2(3) = 0.167$ | 60.15 | 14.85 | $\Phi_{\pm}^\text{max}(3) \approx 4.33 \times 10^{8}$ |
|    | B       | $\delta_{\pm}^2 = \delta_{\pm}^2(3) = 0.167$ | 14.85 | 60.15 | |
| 3 | A       | $\delta_{\pm}^2 = \delta_{\pm}^2(4) = 0.19$ | 63.6 | 11.4 | $\Phi_{\pm}^\text{max}(4) \approx 5.02 \times 10^{6}$ |
|    | B       | $\delta_{\pm}^2 = \delta_{\pm}^2(4) = 0.19$ | 11.4 | 63.6 | |

Table 3. The most probable values of the electron–positron pair energies and the corresponding resonant differential cross-sections.

| r | Channel | $\delta_{\pm}^2(r)$ | $E_+, \text{GeV}$ | $E_-, \text{GeV}$ | $\Phi_{\pm}^\text{max}(\eta = 0.1)$ |
|---|---------|-----------------|-----------------|-----------------|-------------------------------|
| 1 | A       | $\delta_{\pm}^2(3) = \delta_{\pm}^2(5) = 0.145$ | 22.41 | 17.59 | $\Phi_{\pm}^\text{max}(3) \approx 9.29 \times 10^{7}$ |
|    | B       | $\delta_{\pm}^2(3) = \delta_{\pm}^2(5) = 0.145$ | 17.59 | 22.41 | |
| 2 | A       | $\delta_{\pm}^2(4) = \delta_{\pm}^2(5) = 0.241$ | 26.6 | 13.4 | $\Phi_{\pm}^\text{max}(4) \approx 2.41 \times 10^{6}$ |
|    | B       | $\delta_{\pm}^2(4) = \delta_{\pm}^2(5) = 0.241$ | 13.4 | 26.6 | |
| 3 | A       | $\delta_{\pm}^2(5) = \delta_{\pm}^2(6) = 0.256$ | 29.43 | 10.57 | $\Phi_{\pm}^\text{max}(5) \approx 3.84 \times 10^{4}$ |
|    | B       | $\delta_{\pm}^2(5) = \delta_{\pm}^2(6) = 0.256$ | 10.57 | 29.43 | |

\[d\sigma_{-}(r) = (\alpha r^2 Z^2) \Phi_{\pm}^\text{max}(r) dx_{-}(r) d\delta_{-}^2,\] (126)  
\[
\Phi_{\pm}^\text{max}(r) = g_r F_{\pm}^\text{max}(r),\] (127)  
\[
F_{\pm}^\text{max}(r) = \frac{ra(r)}{c^2_r} \left( \frac{x_{\pm}(r)}{1 - x_{\pm}(r)} \right) K_{\pm}(r),\] (128)

It is worth noting that the obtained expressions (120), (121) and (125), (126) are true for the case of the one initial gamma quantum. To derive the relations for the case of the gamma quantum flux one has to multiply the corresponding equations by the concentration $n_y$.

In figures 8–10 we represent the dependencies of the functions $F_{\pm}^\text{max}(r)$ (102) on the corresponding parameters $\delta_{\pm}^2$ (the positron and the electron outgoing angle) for the first three possible resonances for the different initial gamma quantum energies. We can see that the all resonances, except the first, $(r = 2, 3, 4, 5, \ldots)$ have maxima in the distribution functions. These maxima correspond to the most probable values of the electron–positron pair resonant energies for the channels A and B at the corresponding outgoing angles $\delta_{\pm}^2 = \delta_{\pm}^2(r)$ or

It is not difficult to see from these tables that for all the initial gamma quanta ($\omega_i = 125 \text{ GeV}, 75 \text{ GeV}, 40 \text{ GeV}$) the most probable resonant energies of the positron and the electron in the frame of the same resonance are different. When we transfer from the channel A to the channel B these resonant energies replace each other ($E_{-}(A) \rightarrow E_{-}(B), E_{-}(A) \rightarrow E_{-}(B)$). Moreover, for the channel A, the positron resonant energy is greater than the electron resonant energy. However, for the channel B, the
electron resonant energy is greater than the positron resonant energy. This fact gives us the opportunity to determine the appropriate channel in an experiment with measurement of the electron–positron pair resonant energies. It is important to note that there are different sets of the electron–positron pair resonant energies for the different channels. In addition, the corresponding differential cross-sections are large enough (see value of $\Phi_{\pi_{\pm}}^{\max}(r)$ in tables 1–3). The order of the magnitudes for the obtained resonant differential cross-sections (in the units of $\alpha r^2 Z^3$) varies across a wide range: from $\sim 10^{13}$ to $\sim 10^8$ (for $\omega_i = 125$ GeV). The magnitude of the resonant differential cross-section decreases with the decrease in the initial gamma quantum energy. Nevertheless, it still remains large enough. For example, for the initial gamma quantum energy $\omega_i = 40$ GeV we have $\Phi_{\pi_{\pm}}^{\max}(r) \sim 10^8$ (for the third resonance) and $\Phi_{\pi_{\pm}}^{\max}(5) \sim 10^4$ (for the fifth resonance).

Herein we want to underline that if we integrate the differential cross-section of PPP without the external field (109) with respect to the electron outgoing angle (for the channel A) or to the positron outgoing angle (for the channel B) we will obtain $\delta x_{\pi_{\pm}}(\pm) \sim (Z^2 r^2) dx_i / d \omega_i^2$. Taking this fact into account as well as the relations (125) and (126) we can conclude that the excess of the resonant cross-section over the corresponding one in the absence of the external laser field is defined by the functions $\Phi_{\pi_{\pm}}^{\max}(r)$ (127). Therefore, the background from the process without laser field would be less by the many orders magnitude than the impact from the resonant process within the laser field.

6. Conclusion

The study of the resonant process of the PPP by a high-energy gamma quantum in the field of a nucleus and a weak quasi-monochromatic laser wave allows us to formulate the main results.

The process of the resonant PPP for the channels A and B effectively reduces into the two processes of the first order in the fine structure constant: the production process of the ultrarelativistic electron–positron pair by the gamma quantum energy. Nevertheless, it still remains large enough. For example, for the initial gamma quantum energy $\omega_i = 40$ GeV we have $\Phi_{\pi_{\pm}}^{\max}(r) \sim 10^8$ (for the third resonance) and $\Phi_{\pi_{\pm}}^{\max}(5) \sim 10^4$ (for the fifth resonance).

Herein we want to underline that if we integrate the differential cross-section of PPP without the external field (109) with respect to the electron outgoing angle (for the channel A) or to the positron outgoing angle (for the channel B) we will obtain $\delta x_{\pi_{\pm}}(\pm) \sim (Z^2 r^2) dx_i / d \omega_i^2$. Taking this fact into account as well as the relations (125) and (126) we can conclude that the excess of the resonant cross-section over the corresponding one in the absence of the external laser field is defined by the functions $\Phi_{\pi_{\pm}}^{\max}(r)$ (127). Therefore, the background from the process without laser field would be less by the many orders magnitude than the impact from the resonant process within the laser field.

The outgoing angle of the positron (for the channel A) or the electron (for the channel B) relative to the initial gamma quantum momentum defines the electron–positron pair resonant energy, which significantly depends on the number of absorbed wave photons (the number of resonance). Herewith, for the channel A, the most likely situation is when the positron resonant energy is greater than the electron one. For the channel B, we have the opposite situation: the most probable electron resonant energy is greater than the positron one. This fact allows us to distinguish the corresponding reaction channel by the resonant–positron pair.

The distributions of the resonant differential cross-sections over the positron or electron outgoing angles of the higher resonances ($r = 2, 3, \ldots$) have clearly defined maxima, in contrast to the corresponding distribution of the first resonance. The most probable positron and electron resonant energies significantly differ from each other as for the frame of the same resonance, so for the different resonances.

For the intensities of a laser wave $I \lesssim 10^{16}–10^{17}$ W cm$^{-2}$ and the initial gamma quantum energy $\omega_i = 125$ GeV, the resonant energies of the electron–positron pair for the channels A and B in the case of first, second and third resonances may be measured with an extremely large magnitude of the differential cross-section: from $\sim 10^3$ for the first resonance to $\sim 10^8$ (in the units of $\alpha Z^2 r^2$) for the third resonance (see table 1). The obtained results may be experimentally verified using the facilities of pulsed laser radiation (SLAC, FAIR, XFEL, ELI, XCELs).

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