Modulated Inflation (@SUSY08)

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Abstract

We consider cosmological perturbations caused by modulated inflaton velocity. During inflation, the inflaton motion is damped and the velocity is determined by the parameters such as couplings or masses that may depend on light fields(moduli). The number of e-foldings is different in different patches if there are spatial fluctuations of such parameters. Based on this simple idea, we consider “modulated inflation” in which the curvature perturbation is generated by the fluctuation of the inflaton velocity. This talk is based on our recent papers

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1 Introduction

Let us first discuss common ideas for cosmological perturbations in terms of the $\delta N$ formalism. Assuming that $H$ is a constant during inflation, the formula for $\delta N$ is given by $\delta N \simeq H \delta t$, where $\delta t$ is the fluctuation related to the time passed after horizon crossing. According to the traditional inflationary scenario, the curvature perturbation is generated at the horizon crossing. Another idea for the cosmological perturbation is “at the end” scenario. In the “at the end” scenario, the curvature perturbation is generated at the end, where the fluctuation of the goal-line is induced by the light field ($M$); $\delta \phi_e \simeq \phi_e' \delta M$. Of course, considering the fluctuation of the distance ($\delta \phi$) to obtain $\delta t$ is very natural, as it obviously causes the fluctuation of the number of e-foldings $\delta N$. However, we know that the time elapsed after horizon crossing $\sim |t_N - t_e|$ depends not only on the distance $\sim |\phi_N - \phi_e|$ but also on the inflaton velocity $\dot{\phi}$. Therefore, if the inflaton velocity depends on a light field, the fluctuation of the light field may lead to the fluctuation of the inflaton velocity; $\delta \dot{\phi} \sim (\dot{\phi})' \delta M$, which eventually causes $\delta t$ and $\delta N$. The effect would be obvious when (1) many massless degrees of freedom appear in the inflaton trajectory, or (2) the inflaton mass is slightly larger than the Hubble parameter and there is no significant perturbation from $\delta \phi_N$. Based on the above simple speculation, we will focus on modulated inflaton velocity, where spatial fluctuation of the time elapsed during inflation $\delta t$ is caused by the fluctuation of the inflaton velocity. Moreover, if the fundamental parameters are determined by the moduli in the underlying (string) theory, it would be natural to think that the Planck scale may also depend on moduli. What happens if the Planck scale is determined by moduli fields that may have fluctuations during inflation? We will see that the perturbation related to $\delta M_p(M)$ is explained in terms of $\delta \dot{\phi}$. Let us see more details of this “modulated velocity scenario”, considering simple examples.

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2 See the first picture in Fig.1
3 See the second picture in Fig.1
4 See the red-dotted line in the last picture in Fig.1
In the first (Traditional) and the second (At the end) scenarios, $\delta t$ is caused by the fluctuation of the distance between the start-line at $\phi_N$ and the goal-line at $\phi_e$, while in the last scenario $\delta t$ is caused by $\delta \dot{\phi}$.

1.1 Modulated velocity from the inflaton potential

For the first example, we consider standard kinetic terms and conventional interaction term in the potential, $V(\phi, M)$. In this case, the definition of the number of e-foldings for constant $H$ is

$$N = \int H dt = \int H \frac{\dot{\phi} d\phi + \dot{M} dM}{\dot{\phi}^2 + M^2}. \tag{1.1}$$

If there is no bend in the trajectory, the perturbation related to the inflaton velocity is expanded as

$$\delta N \simeq - \int_{\phi_e}^{\phi_N} H \frac{\dot{\phi}}{\dot{\phi}^2} \left( \delta \dot{\phi} - \dot{\phi} A \right) d\phi, \tag{1.2}$$

where we consider linear scalar perturbations of the metric, $ds^2 = -(1 + 2A)dt^2 + 2aB_i dx^i dt + a^2 [(1 - 2\psi)\delta_{ij} + 2E_{ij}] dx^i dx^j$. From the conventional energy and momentum constraints we can find

$$\dot{\phi} \left( \delta \phi - \dot{\phi} A \right) - \ddot{\phi} \delta \phi \simeq \dot{\phi} \left( \delta \phi - \dot{\phi} A \right) \propto \frac{k^2}{a^2}, \tag{1.3}$$
which suggests that the factor $\delta \dot{\phi} - \dot{\phi} A$ appearing in the delta-N formula decays after horizon crossing. Therefore, the factor $e^{-2Ht}$ must be included in the calculation, even if the perturbation $\dot{\phi} \delta \dot{\phi}$ itself is supposed to be a constant. Of course, $e^{-2Ht}$ in the integral does not lead to the exponential suppression after the integration. It is easy to see that there is no exponential suppression for large $N(\sim Ht)$,

$$\int_0^{t_c} \delta CHe^{-2Ht} dt \simeq \frac{1}{2} \delta C,$$

(1.4)

where the actual suppression factor is not significant. In fact, the corrections from terms proportional to $k^2/a^2$ have been disregarded in previous studies, since $k^2/a^2$ is obviously small at a distance. However, if they appear in the equation of $\dot{R}$, these terms may yield significant correction to $R$ after integration, as we can see easily from the above equation.

1.2 Modulated velocity from the inflaton kinetic term

Next, we consider moduli-dependent kinetic term $\sim \frac{1}{2} \omega(M) g^{ab} \dot{\phi} \dot{\phi}$. In this case, the number of e-foldings is

$$N = \int H dt \simeq \int \frac{H}{\dot{\phi}} d\phi \simeq \int \frac{3H^2}{V_\phi} \omega d\phi.$$  

(1.5)

Again, we assume no bend. $\delta N$ that is caused by the moduli perturbation is

$$\delta N \simeq \int \frac{3H^2}{V_\phi} \omega' \delta M d\phi \simeq \frac{\omega'}{\omega} \frac{\omega N}{\omega} \delta M.$$  

(1.6)

Note that unlike the perturbation caused by the potential, the constraints from the energy and momentum does not yield $\frac{k^2}{a^2}$ factor for the perturbation related to the kinetic term [5].

2 Pure case

Let us see what happens if both boundaries are completely flat while there is modulated velocity. Here we consider fast-roll inflation with hybrid-type potential [6, 7]. Inflaton may have large mass ($m_\phi \simeq O(1)H$) due to the $\eta$-problem. Then $\delta \phi$ decays or cannot cross the horizon during inflation. Even in this case, non-oscillatory (fast-roll) inflation is possible if the friction is significant. We consider the hybrid-type potential

$$V(\phi, \sigma) = \lambda (\sigma^2 - v^2)^2 + \frac{1}{2} g^2 \phi^2 \sigma^2 + V(\phi),$$  

(2.1)
where $\phi$ is the inflaton and $\sigma$ is the trigger field. Here the end of inflation expansion occurs at

$$\phi_e = \frac{\sqrt{\lambda v}}{g},$$

(2.2)

and the number of e-foldings is given by

$$N = \frac{1}{F_{\phi}} \log \frac{\phi_N}{\phi_e},$$

(2.3)

where $F \equiv \frac{3}{2} \left( 1 - \sqrt{1 - 4m^2/9H^2} \right)$. Considering the factor $\sim k^2/a^2$, we find for $m \simeq H$,

$$\delta N(\mathcal{M}) \sim m' \delta \mathcal{M},$$

(2.4)

where $m'$ is the derivative of $m$ with respect to $\mathcal{M}$. More specific result is obtained for

$$m^2(\mathcal{M}) \equiv m_0^2 \left[ 1 + \beta \log(\mathcal{M}/M_*) \right],$$

(2.5)

where $\delta N$ is given by

$$\delta N(\mathcal{M}) \simeq \beta \left( \frac{\delta \mathcal{M}}{\mathcal{M}} \right).$$

(2.6)

Since the mass of $\mathcal{M}$ is smaller than $H_I$ during inflation, we find the condition

$$m^2_{\mathcal{M}} \simeq \beta m_0^2 \left( \frac{\phi_N}{\mathcal{M}} \right)^2 < H_I^2.$$  

(2.7)

In this case, the non-Gaussianity parameter is

$$f_{nl} = -\frac{5}{6} \frac{N''}{(N')^2} \propto \frac{1}{\beta},$$

(2.8)

which can be large and may take either sign.

### 3 Adding significant non-Gaussianity

Our question is “Is it possible to add significant non-Gaussianity to the standard inflationary perturbation after horizon crossing?” Our answer is “Yes”. To show how to add non-Gaussianity to the conventional perturbation, we consider hybrid inflation with standard $\sim g^2 \phi^2 \mathcal{M}^2$ interaction. During inflation, hybrid inflation has the effective potential

$$V(\phi, \mathcal{M}) = V_0 + \frac{1}{2} m_{\phi}^2 \phi^2 + g^2 (\phi - \phi_{ESP})^2 \mathcal{M}_i^2,$$

(3.1)

Curvaton and inhomogeneous preheating may lead to curvature perturbations with significant non-Gaussianity[8, 9].
where $M_i$ become massless near the enhanced symmetric point (ESP) at $\phi_{ESP}$. The perturbation of the inflaton velocity caused by the number of $n$ massless excitation is

$$
\delta \dot{\phi} \simeq \frac{2ng^2(\phi - \phi_{ESP})(\delta M)^2}{3H}
$$

(3.2)

where the first order perturbation vanishes. The second order perturbation at the ESP adds significant non-gaussianity to the perturbation;

$$
\hat{f}_{NL} \simeq \frac{\delta \dot{\phi} / \dot{\phi}}{(H^2 / \dot{\phi})^2} \simeq \frac{m_{\phi} \eta_s \phi}{\phi - \phi_{ESP}}.
$$

(3.3)

Since the non-gaussianity is uncorrelated, the usual non-linear parameter $f_{NL}$ is given by $f_{NL} \sim (\hat{f}_{NL}/1300)^3$. We conclude that we can add significant non-gaussianity to the standard perturbation after horizon crossing.

4 Modulated Planck scale

Finally, we briefly mention the consequence of modulated Planck scale in terms of the delta-N formalism. We consider a light scalar field $\mathcal{M}$ coupled to gravity. The model is

Figure 2: ESP may appear in the inflaton trajectory.
given by the action
\[
S = \int d^4x \sqrt{-g} \left[ f(\dot{\phi}^2) R - g(\phi, \dot{\phi}) \left( \nabla \phi \right)^2 - \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right].
\]

(4.1)

Considering (for simplicity) Jordan-Brans-Dicke theory, the action in the Einstein frame has the moduli-dependent kinetic term \( \sim \frac{1}{2} \omega(M) g^{ab} \phi_a \phi_b \), where
\[
\omega(M) = \exp \left( -\beta \kappa M \right)
\]

(4.2)

and the potential
\[
V = \omega^2 W(\phi).
\]

(4.3)

Note that there are both sources for the velocity perturbation, from the potential and the kinetic term. The perturbation caused by the potential has the factor \( k^2/a^2 \), while the one from the kinetic term does not.

5 Summary

We considered cosmological perturbations caused by modulated inflaton velocity. The velocity perturbation has been disregarded in previous studies for multi-field inflation. However, the perturbation may lead to significant results, as we have shown in this presentation. The sources of such perturbations are clear in the \( \delta N \) formalism. Important results are:

- Small deviation from the standard perturbation may be explained by the modulated velocity.
- The curvature perturbation can be generated after horizon crossing even if the both boundaries are completely flat.
- It is possible to add significant non-gaussianity to the conventional perturbations after horizon crossing.

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