Economics from a Physicist’s point of view: Stochastic Dynamics of Supply and Demand in a Single Market. Part I

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Abstract

Proceeding from the concept of rational expectations, a new dynamic model of supply and demand in a single market with one supplier, one buyer, and one kind of commodity is developed. Unlike the cob-web dynamic theories with adaptive expectations that are made up of deterministic difference equations, the new model is cast in the form of stochastic differential equations. The stochasticity is due to random disturbances ("input") to endogenous variables. The disturbances are assumed to be stationary to the second order with zero means and given covariance functions. Two particular versions of the model with different endogenous variables are considered. The first version involves supply, demand, and price. In the second version the stock of commodity is added. Covariance functions and variances of the endogenous variables ("output") are obtained in terms of the spectral theory of stochastic stationary processes. The impact of both deterministic parameters of the model and the random input on the stochastic output is analyzed and new conditions of chaotic instability are found. If these conditions are met, the endogenous variables undergo unbounded chaotic oscillations. As a result, the market that would be stable if undisturbed loses stability and collapses. This phenomenon cannot be discovered even in principle in terms of any cobweb deterministic model.

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Only the simple theories that can be explained
to an intelligent outsider (one’s wife)
turn out to hold up in economics.

P. A. Samuelson

1 Introduction

It is universally accepted wisdom that supply and demand interaction is the
main driving force of a market economy. The word ”interaction” means that
supply and demand, as well as the price they depend upon, change with
time and are therefore not static phenomena but dynamic processes. These
price-quantity processes can develop in different ways. They can converge to
an equilibrium state, oscillate, collapse or explode, all in a regular or chaotic
manner. To gain insight into this complex behavior, supply-and-demand
problems should be treated not only statically, as in economics textbooks,
but also dynamically.

Yet surprisingly, the dynamic consideration of these problems has, almost
completely, given way to a static analysis. As a result, the inadequate static
approach has led to a limited and sometimes even incorrect understanding of
the price-quantity interaction. A case in point is the single one-commodity
market with one seller and one buyer. There are some quasi-dynamic models,
to be briefly discussed below, expressly designed for a study of stability of
supply-and-demand equilibrium price in such a market.

No wonder that such specific models fail to deal with the whole interactive
process of supply, demand, and price. Instead, this process is routinely dis-
cussed from the static point of view based on the well-known oversimplified
assumptions:

1. Supply $S$ and demand $D$ are functions only of a price $P$ and do not
depend on time $t$. The variables $S, D, P$, which are included in the analysis,
constitute an endogenous set (a market) $E = \{S, D, P\}$.

2. The plots of $S(P)$ and $D(P)$ have respectively positive (”upward”) and
negative (“downward”) slopes everywhere. These curves intersect at a
point of equilibrium $E^e = \{S^e, D^e, P^e\} \subset E$ where supply equals demand
($S^e = D^e$) and the equilibrium price $P^e$ clears the market.
Suppose that a set $G$ of all exogenous variables which can disturb the market equilibrium is excluded from the analysis. This is equivalent to an assumption that $G = \emptyset$ where $\emptyset$ is an empty set. Yet there still may be an endogenous force, say, the seller who raises the price $P$ that can displace the market $E$ from the equilibrium point $E^e$ to a non-equilibrium state $E^n = \{S^n, D^n, P^n\} \neq E^e$. In the end, however, as the static analysis shows, the market $E$ will automatically return from $E^n$ to $E^e$. Hence, according to the static consideration, the equilibrium of the simplest market $E = \{S, D, P\}$ is always stable provided $G = \emptyset$.

This discovery stands so high in economics that it is frequently called the "law" of supply and demand (e.g., [1]). Even its introductory part,—the existence of an equilibrium price $P^e$, the fact already known in the 18-th century,—was praised by Thomas R. Malthus as the "first, greatest, and most universal principle" of economics. To a certain extent this is true since the "law" of supply and demand can not only give us information about the equilibrium state $E^e$, but also predict in some instances the impact of the exogenous variables $G \neq \emptyset$ on the market $E = \{S, D, P\}$.

For example, a static analysis of an aggregate one-commodity market (macroeconomics) enabled one to gain an initial insight into the enigmatic phenomena of unemployment, inflation, etc., and explain how they appear when either the aggregate supply ($AS$) or the aggregate demand ($AD$) curve shifts in response to the variations in the exogenous set $G$. Despite, however, its merits, the famous "law" of supply and demand has to be taken with a grain of salt because actually it is not a universal principle.

Firstly, the change in the exogenous variables $G$ often leads to the "shifts on both the demand and supply side at the same time, so the simple rules [the above explanations] ... don't work" [2]. What happens in this case to, say, macroeconomics, or a market of commodities, depends on the shifts' magnitude which the famous "law" cannot predict.

Secondly, contrary to the conventional assumption, the $AS$ and $AD$ static curves can be "perverse." This means that "$AS$, at least in some situations, slope downward and/or $AD$ may slope upward" and "the usual implications of the macro $AS - AD$ analysis may be misleading." [3]

Thirdly, and most importantly, even if the set of exogenous variables $G \neq \emptyset$ the static "law" of supply and demand does not work either as long as it disregards the time-dependent nature of the endogenous variables $E$. 
Indeed, let us instead of the familiar static set \( E = \{S, D, P\} \) consider a new dynamic set \( E(t) = \{S(t), D(t), P(t)\} \) where the endogenous variables \( S, D, P \) (supply, demand, and price) are functions of time \( t \).

Now, since all the three variables change with time, so will do both the curves of supply and demand as functions of the price. Consequently, at different points the curves will have, generally speaking, different slopes which can be positive, or negative, or zero. These dynamic phenomena undermine the fundamental feature of the static ”law” of supply and demand,-the fixed positive (negative) slope of the supply (demand) curve,-and hence invalidate the ”law’s” assertion that the market equilibrium is always stable if the exogenous variables \( G \neq \emptyset \). Because of this fault, the static ”law” becomes impractical and can perhaps serve, as it does, only as a first step in understanding supply-and-demand problems.

Such ”myopic preoccupations of traditional equilibrium analysis” [4] persisted for a long time. Only the last few decades have seen a monotonic increase in studies of price-quantity dynamics. The respective dynamic models have been developed on the basis of (1) differential and (2) difference (cobweb) time-dependent equations.

The first (”differential”) direction of research goes back to Hicks [5] and Samuelson [6]. They focused on the study of stability of supply-and-demand equilibrium in a single and multiple markets in terms of differential equations. Because of such a narrow objective, Hicks’ and Samuelson’s models were too limiting. They were incapable of dealing with the general problem of price-quantity evolution although the time-dependent differential equations did make it possible. Consequently, by a strange coincidence, the ”differential” direction in price-quantity dynamics gave way to the cobweb models. Such models were introduced in the 1930’s (e.g., [7]) and have since then became the prevailing theoretical tools for studying price-quantity dynamics.

One of the first who employed a special cobweb model for analyzing the price stability was Samuelson [6]. His model came to be known as a naive expectations theory. Later on a more sophisticated (and up to now widely used) cobweb model was proposed based on a concept of adaptive expectations. The model is reviewed in detail in our forthcoming paper. Now we only point out some of the shortcomings of the adaptive expectations theory (see, e.g., [8]).
1. In this theory, the curves of supply \( f_S \) and demand \( f_D \) are supposed to be "rigid," their shape is fixed and time-independent. This brings us back to the simplistic static assumption adopted in textbooks. But in price-quantity dynamics there are no "rigid" supply and demand curves. As has been already mentioned, both the curves are generally dependent on and changing with time.

2. The governing equations of the adaptive expectations theory are explicitly deterministic because no stochastic components are included. Only by a special choice of the key deterministic functions \( f_S \) and \( f_D \), the cobweb model may be able to reveal some stochastic features of the price-quantity dynamics. But because there are in fact no "rigid" functions \( f_S \) and \( f_D \), this particular approach is in general overly restrictive. Therefore a better way to study the chaotic behavior of the price-quantity process is to address it directly, by incorporating stochastic functions into governing equations. Now we can proceed with our new theory.

2 The Stochastic Dynamic Model

We begin with two assumptions.

1. It is clear that a profit-minded supplier should acutely aware of his/her costs. He/she will therefore control the actual output of commodity \( S(t) \) in view of the difference \( P_{St} = P(t) - P_S \) between the market price \( P(t) \) and some characteristic price \( P_S \) which includes all costs and the desired profit. It is reasonable to believe that the actual supply \( S(t) \) will increase if the net price \( P_{St} > 0 \), or decrease if \( P_{St} < 0 \). So, \( P_S \) can be interpreted as the seller’s borderline price above which the supply of commodity will rise or below which it will fall.

2. It is also clear that a sensible buyer will adjust his/her actual demand for commodity \( D_t \) based on the difference \( P_{Dt} = P(t) - P_D \) between the market price \( P(t) \) and a characteristic price \( P_D \). The latter is formed by the buyer’s needs and financial opportunities, as well as by his/her tastes and other unspecified psychological, physiological, etc. factors. The higher the price \( P_D \), the higher is the buyer’s willingness to purchase the commodity. In other words, the actual demand \( D(t) \) is likely to increase if the net price \( P_{Dt} < 0 \), or decrease if \( P_{Dt} > 0 \). Hence, \( P_D \) can be viewed as the buyer’s
borderline price below which the demand for commodity $D(t)$ will rise or above which it will fall.

All the factors influencing the price $P(D)$, the buyer’s needs, financial opportunities, tastes, etc., are, to a different degree, random by nature. We may therefore express $P(D)$ as a sum of two components $P(D) = \langle P_D \rangle + \overline{P_D}$ where $\langle P_D \rangle \neq 0$ is a deterministic part (a mean) of $P_D$ while $\overline{P_D}$ is a stochastic disturbance of $P_D$ with a zero mean $\langle \overline{P_D} \rangle = 0$. Besides, the seller’s borderline price $P_S$ is similar to $P_D$ and is therefore also stochastic. Yet in the present paper, for the sake of simplicity, the price is taken to be a deterministic quantity.

Applying these two assumptions (as well as some others) to a rather inclusive rational expectations model (e.g., [9]), we have derived governing relations of our dynamic theory:

\[
\begin{align*}
\dot{S}(t) &= a[P(t) - P_S] + k[D(t) - S(t)] \\
\dot{D}(t) &= b[< P_D > - P(t)] \\
\dot{P}(t) &= c[D(t) - S(t)] \\
&+ \begin{cases} 0 & \phi_S(t) \\ bP_D & \phi_D(t) \\ c & \phi_P(t) \end{cases} \\
&= \begin{cases} 0 & \phi_S(t) \\ bP_D & \phi_D(t) \\ c & \phi_P(t) \end{cases} \\
&= (2.1)
\end{align*}
\]

It is a system of three stochastic first-order linear differential equations for the set of three endogenous variables $E(t) = \{S(t), D(t), P(t)\}$. In Eqs. (2.1), dots denote differentiation with respect to time $t$; the quantities $a, b, c, k$ are non-negative constants; $\phi_S(t), \phi_D(t), \phi_P(t)$ represent exogenous deterministic functions influencing supply, demand, and price.

Now a natural question arises: What is the economic interpretation of the governing equations (2.1)? Let us first look at the first of these equations which we designate as (2.1)$_1$. Its meaning seems quite clear:

(i) If the market price exceeds the seller’s borderline price, supply will rise. If the seller’s borderline price exceeds the market price, supply will fall.

(ii) If demand exceeds supply, supply will rise. If supply exceeds demand, demand will fall. The meaning of next equation designated as (2.1)$_2$ is no less clear:

(iii) If the buyer’s borderline price exceeds the market price, demand will
rise. If the market price exceeds the buyer’s borderline price, demand will fall.

Lastly, let us the third of Eqs.(2.1) designated as (2.1). Its meaning has been given by Samuelson’s dictum [6]:

(iv) "If at any price demand exceeds supply, price will rise. If supply exceeds demand, price will fall."

We thus see that the paraphrase of Eqs. (2.1) is very simple. If we had started from this literal interpretation (that is, by inverting our approach), Eqs. (2.1) could have easily been written "toutdesuite. Yet we have avoided this path and derived the dynamic model (2.1), as mentioned before, differently, - in terms of the more fundamental rational expectations model. This has been done on purpose, in order to show that our model (2.1) not only differs from the rational expectations model, but also has a certain affinity with it which would otherwise have been difficult, if not impossible, to see.

There is also the other side of the coin. Once the stochastic equations (2.1) are obtained, they lend themselves well to developing more inclusive dynamic models of supply-and-demand to be dealt with elsewhere. It is important to note that unlike Eqs. (2.1) and without them such advanced models are not directly derivable from the rational expectations model (cf. [9]).

3 A Closed Deterministic Market

First we consider a closed deterministic market, that is, a market without exogenous variables and stochastic disturbances. This means that in the right-hand side of the model (2.1) we should omit the second and the third vector components and thus obtain

\[
\begin{align*}
\dot{S}(t) & = a[P(t) - P_S] + k[D(t) - S(t)] \\
\dot{D}(t) & = b[P_D > -P(t)] \\
\dot{P}(t) & = c[D(t) - S(t)]
\end{align*}
\]

(3.1)

It is a system of three deterministic first-order linear differential equations which can easily be solved by using any traditional techniques (see, e.g., [10],[11]). Analytic solutions of Eqs. (3.1) are simple but rather cumbersome. Therefore, in what follows, we restrict ourselves to the illustrations of typical results.
Case 1

Suppose that there is no damping, that is, the constant \( k = 0 \). In this particular case, the solution of (3.1) is comparatively compact:

\[
S(t) = a + b \frac{S(0) - D(0)}{A_1} \cos(\beta t) + \frac{a}{b} [P(0) - A_1] \sin(\beta t) + A_2 t + A_3 \tag{3.2}
\]

\[
D(t) = -b \frac{S(0) - D(0)}{A_1} \cos(\beta t) + \frac{b}{\beta} [P(0) - A_1] \sin(\beta t) + A_2 t + A_3 \tag{3.3}
\]

\[
P(t) = [P(0) - A_1] \cos(\beta t) - \frac{\beta}{a + b} [S(0) - D(0)] \sin(\beta t) + A_1 \tag{3.4}
\]

where \( S(0), D(0), P(0) \) stand for initial values of supply, demand, and price, and the parameters \( A_1, A_2, A_3, \beta \) are defined as

\[
\begin{bmatrix}
A_1 \\
A_2 \\
A_3 \\
\beta
\end{bmatrix} =
\begin{bmatrix}
(aP_S + b(P_D))(a + b)^{-1} \\
ab(\langle P_D \rangle > P_S)(a + b)^{-1}
\end{bmatrix}
\]

We see that according to Eqs. (3.2)-(3.5),

- If \( \langle P_D \rangle > P_S \), then \( A_2 > 0 \). Consequently, both supply and demand increase in time, and the market is booming.
- If in addition \( S(0) \neq D(0) \) and/or \( P(0) \neq A_1 \), then the monotonic increase of supply and demand is accompanied by undamped oscillations.
- If \( \langle P_D \rangle < P_S \), then \( A_2 < 0 \). As a result, both supply and demand decrease in time and the market goes south.
- If simultaneously \( S(0) \neq D(0) \) and/or \( P(0) \neq A_1 \), then the monotonic decrease of supply and demand is followed by undamped oscillations.
- If \( \langle P_D \rangle = P_S \), then \( A_2 = 0 \). Accordingly, both supply and demand either oscillate [when \( S(0) \neq D(0) \) and/or \( P(0) \neq A_1 \)] or remain equal to \( A_3 = \text{constant} \).
- If \( P(0) \neq A_1 \) and/or \( S(0) \neq D(0) \), then the price \( P(t) \) oscillates.
- If both \( P(0) = A_1 \) and \( S(0) = D(0) \), then the price \( p(t) \) does not change and remains equal to the initial value \( P(0) = A_1 \).

The foregoing brief analysis leads to important conclusions.

1. The supply-and-demand dynamics is mainly influenced by the ratio \( \alpha = \langle P_D \rangle / P_S \). This ratio defines what may be called the market asymmetry.
2. If $\alpha = 1$, the market is symmetric since both the seller and the buyer adhere to the same borderline price $\langle P_D \rangle = P_S$. As a result, the market will neither expand nor collapse, i.e., it is in a sense stable.

3. If $\alpha \neq 1$, the market is asymmetric because the seller and the buyer adhere to different borderline prices. Consequently, the asymmetric market is, in the same sense, unstable, -it will either boom or collapse.

The above new criteria of stability are drastically different from the corresponding stability conditions established by such figures as L. Walras, A. Marshall, J.R. Hicks, and P.A. Samuelson [12]. The new criteria also show that, contrary to the static”law” of supply and demand, the equilibrium of a single market is not always stable even if the set of endogenous variables is empty $[G(t) = \emptyset]$ as has already been mentioned in Introduction.

It is also interesting to observe that the seller’s borderline price $P_S$ is usually hidden from the buyer and, vice versa, the buyer’s borderline price $\langle P_D \rangle$ is generally unknown to the seller. The obvious inference is that incomplete (asymmetric) information about the characteristic prices $P_S$ and $\langle P_D \rangle$ available respectively to the seller and to the buyer may have either a beneficial or an adverse effect on the market. This phenomenon has a direct bearing on a theory of markets with asymmetric information [13].

Particular examples of the closed deterministic market described by (3.1) will be considered in the next paper.

4 References

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