Dynamic modelling of overhead crane

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Abstract. This paperwork deals with dynamics of overhead crane. Dynamically speaking, I considered the overhead crane beams as continuous medium with distributed elasticity and weight, simple supported at ends, stressed by distributed as well as focused forces. The distributed forces are the own weight and forces of inertia (mass forces) and focused forces are those produced by the carriage and transported load. Focused forces may travel along the overhead crane, so they have variable position and are, in their turn, variable in time as the carriage, always considered in contact with overhead crane, is, on one hand subject to the same accelerations as the beams oscillate in vertical plane in the place where the carriage is at the said moment, and produce variable inertia forces, and, on the other hand, a load is hanging on it, sustained by elastic cables, pulling down with a variable force, depending on how oscillations of the load and overhead crane beam mix up together. The overhead crane is a beam with variable profile subject to static and dynamic, focused and distributed loads. This makes the mathematic model of the overhead crane dynamics to be very complicated and the presentation that follows shall prove it. Analysing the results presented in this study it may come to the conclusion that monitoring and analyse of overhead crane dynamics is extremely important because it may highlight the potentially dangerous situations in operation of this equipment.

1. Introduction
From mechanical point of view, an overhead crane is a two parallel beams assembly movable on a rolling way whereon a car bearing a lifting mechanism for focused loads moves, figure 1.

![Figure 1. Process flow diagram.](image_url)

The overhead crane travels on a suspended way seated on its own pillars that support the roof and walls of the hall wherein is installed. The beams of overhead crane, rolling way and pillars are subject...
to some static loads when in rest moments, while in use moments dynamic loads appear showing variable values in time, their maximum exceeding static loads [1, 2].

2. Dynamics of overhead cranes

Dynamically speaking, we consider the overhead crane beams as a continuous medium with distributed resiliency and mass, simple seated at ends, subject both to distributed and focused forces. The distributed forces are their own weight and forces of inertia (mass forces) and focused forces are those produced by the car and carried load. The focused forces are able to travel along the overhead crane so they have variable position and, in their turn, are variable in time as the car, always considered in contact with overhead crane, is subject, on one hand, to the same accelerations the beams are oscillating in vertical plane in the place where the car is at that certain moment and produce variable forces of inertia, and, on the other hand, it has a hanging load suspended on elastic cables, load that pulls down with a variable force as the oscillations of the load and overhead crane beam combine together. So, we can say that the overhead crane is a beam with variable profile subject to static and dynamic loads, focused and distributed [2]. This makes the mathematic model of the overhead crane dynamics to be very complicated.

In addition, the following hypothesis are considered:
- the rolling way is rigid, as only the behavior of overhead crane itself is of interest; an elastic rolling way can be solutioned in the same manner as the overhead crane but only after get used to the model of overhead crane that is already complicated enough;
- vertical drive of the load is done by electric motor that has kinematic and dynamic features known, as also proceeded in the previous situations;
- overhead crane load is suspended on elastic cables, so it can oscillate on vertical and oriental directions as a pendulum;
- the car is always in contact with rolling way of the overhead crane and moves on vertical direction jointly with it; in other words, under no circumstances the car detaches from overhead crane and permanently acts by pressing [3, 4].

Mathematic model is based on the equation of transversal oscillations of a continuous beam with variable section:

\[
\frac{\partial^2 v(x)}{\partial x^2} \left( x(x) \frac{\partial^2 v(x,t)}{\partial x^2} + \rho(x) A(x) \frac{\partial^2 v(x,t)}{\partial t^2} = q(x,t) \right) \tag{1}
\]

where, besides notations seen on figure 1, there are also \( E(x) = \) elasticity module of crane material on coordinate \( x, I(x) = \) axial moment of de inertia of the crane cross section on coordinate \( x, v(x,t) = \) vertical sag of the crane on coordinate \( x \) and moment \( t, \rho(x) = \) crane material density on coordinate \( x, A(x) = \) cross section area of crane beams on coordinate \( x, q(x,t) = \) vertical loading distributed on crane on coordinate \( x \) and moment \( t \). We will consider that the elasticity module is the same on entire overhead crane, \( E(x) = \) constant but the moment of inertia obviously is variable. So, the differential equation (1) becomes:

\[
E \left( \frac{\partial^4 v}{\partial x^4} + \frac{\partial v}{\partial x} \frac{\partial^3 v}{\partial x^3} + I \frac{\partial^2 v}{\partial t^2} \right) + \rho A \frac{\partial^2 v}{\partial t^2} = q(x,t) \tag{2}
\]

In equation (2) we did no longer indicate that the terms \( I, v, \rho, A \) are functions of \( x \) or \( t \) in order to write simpler. In area of overhead crane ends the moment of inertia of a beam is given by expression:

\[
I_{ref}(x) = \frac{b(h+x \frac{h-x}{y})^3 -(h-2) \left[ h_0 + \frac{h-x}{y} \right]^3}{12} \tag{3}
\]
and the overhead crane moment of inertia will be double than shown by expressions (3) and (4) as there are 2 parallel beams. Between the two areas, the moment of inertia for a single beam is constant:

\[ I(x) = \frac{bh^3-(b-2s)(h-2s)^3}{12} \]  

(5)

Even without using complicated enough expressions for dynamic loading \( q(x,t) \), as will be seen furtheron, thus admitting a loading only by own weight, it is clearly seen that the differential equation (1) is impossible to be analytically solved. There are analytic solutions only for simple cases, when the equation is homogenous, meaning \( q(x,t)=0 \) and for condition I=constant. This is why solving is done by numerical way and, once this choice is made, there is no restraint to complicate things for bringing them close to reality. Dynamic loading \( q(x,t) \) is expressed as follows:

\[ q(x,t) = q_g(x,t) + q_c(x,t) + q_s(x,t) \]  

(6)

where \( q_g(x,t) \)=weight force of beams on length unit, \( q_c(x,t) \)=weight force of car on length unit, \( q_s(x,t) \)=force caused by losd M distributed on length unit. Written in detail, we will have:

\[ q_g(x,t) = \rho_{\text{beam}} \cdot A_{\text{beam}}(x) \cdot g, \quad g=9.81 \text{ ms}^{-2} \]  

(7)

\[ q_c(x,t) = \frac{m \cdot g}{l_3}, \quad x \in x_{\text{car}} \]  

(8)

\[ q_s(x,t) = \frac{k_{el}}{l_s} [y(t) - v(x,t)], \quad x \in x_{\text{car}} \]  

(9)

In above relations we have: \( m=\)car mass, \( l_3=\)car length (considered in contact with the crane along entire length \( l_3 \); I could have considered the car weight focused in seating points of the wheels but the results are not significantly different), \( k_{el}=\)elastic constant of the load cables, \( v(x,t)=\) solution of equation (1), \( y(t)=\)solution of cable oscillator (mass M, elastic constant kel and frictions in the cable), \( F_{cf}=\)centrifugal force produced by pendulum created by load M and suspending cables. The value \( y(t) \) resulted from integrating the differential equation:

\[ M\ddot{y} + F_{fr} + F_{el} = Mg \]  

(10)

where \( F_{fr} = \) cable internal friction force, \( F_{el}=\)elastic force in the cable; these are found by means of known relations already used in other occasions

\[ F_{fr} = c_y \cdot |v(x_{\text{car}},t) - y(t)| \cdot \text{sgn}[v(x_{\text{car}},t) - y(t)] \]  

(11)

\[ F_{el} = \begin{cases} k_{el} \cdot |y(t) - v(x_{\text{car}},t)| & \text{if } y(t) - v(x_{\text{car}},t) > 0 \\ 0 & \text{if } y(t) - v(x_{\text{car}},t) < 0 \end{cases} \]  

(12)

with \( c_y=\)internal friction coefficient in the cable [3,4].

Pendulum movement \( x_M(t) \) considered on horizontal direction is found from homogenous differential equation:

\[ M\ddot{x}_M + F_{fr} + F_{el} = 0 \]  

(13)

Under hypothesis of small angle oscillations, below 50, for which we have
\begin{align}
F_{fr} &= c_f \cdot \dot{x}_M \quad (14) \\
F_{el} &= M \cdot \frac{\dot{x}_M}{g} \quad (15)
\end{align}

with \( c_f \) = friction coefficient with medium, \( l_p \) = pendulum length, variable if we take into account potential move of the load on vertical direction during lifting or lowering. Centrifugal force from (9) will be:

\[ F_{cf} = M \cdot \frac{\ddot{x}_M}{l_p} \quad (16) \]

Initial conditions to integrate the differential equations (2), (10) and (13) are:

\begin{align*}
v(x,0) &= 0; \\
y(0) &= 0; \\
y'(0) &= \frac{v_{cy}}{g}; \\
\dot{x}(0) &= 0; \\
\dot{x}'(0) &= -\frac{v_{cx}}{g}
\end{align*}

wherein \( v_{cy} \) and \( v_{cx} \) are load speeds on vertical and horizontal directions. Differential equations can only be integrated by numerical methods [5]. I used two different methods, given that we have different differential equations, respectively ordinary (10) and (13) and with partial derivatives (2). For equation with partial derivatives I used the method of finite differences with centered differences and on previous moment for partial derivatives in relation to space and finite differences to the left in relation to time, these for the solution to result directly and simple from an explicit algebraic equation.

Finding solution \( v(x,t) \) is not the only goal of this paperwork. The solution found as numerical function serves further to find the reactions on seats of the overhead crane \( V_1 \) and \( V_2 \). Following the figure 1, the moment equations against each seat can be written:

\begin{align}
V_1 \cdot n &= \sum_{i=1}^{n} m_i \left(g - \ddot{v}_{i,3}\right) \cdot (n - i) \\
V_2 \cdot n &= \sum_{i=1}^{n} m_i \left(g - \ddot{v}_{i,3}\right) \cdot i
\end{align}

wherefrom the reactions \( V_1 \) and \( V_2 \) are found. In (18) and (19) \( n \) = number of knots the crane was divided into along its length, \( \ddot{v}_{i,3} \) is the vertical acceleration of the knot from \( i \) position in current moment (current moment has the value 3, previous moment the value 2, pre-previous moment the value 1 in finite differences related to time), \( m_i \) = knot \( i \) mass, including crane beams and, where needed, the car [6,7].

3. Simulation of overhead crane behaviour in dynamic mode during load handling

Modelling of overhead crane dynamics was done under established conditions by the following operating hypothesis, meaning:

- The crane has a structure consisting of only one box type beam;
- Load in the hook is exclusively handled on vertical direction;
- Mobile car is positioned at the middle of overhead crane opening, and its speed is null during load handling;
- The beam of overhead crane was modelled like a system with distributed mass, with constant geometric parameters, simply seated at ends, subject to dynamic actions exclusively on transversal direction;
- Taking into account the car speed is null during simulation, this was modelled as a concentrated mass attached to the beam at its middle point;
- Lifting system of the load (load included) was modeled like a dynamic system with one freedom degree, attached to overhead crane beam at its middle point, concentrated mass being given by the
load that should be handled, and elastic constant being equal to equivalent value of the stiffness of the hoist cables assembly;

- The two models are connected by the fact that the dynamic force in the lifting system acts upon the beam of the overhead crane, connection term being present both within the mathematic model of the beam and the one of lifting system;
- Assembly excitation is of kinematic type, through the load speed, required in initial moment of simulation; the other conditions are null;
- Border conditions, needed to solve the mathematic model of the beam, take into account its geometric configuration and seating conditions.

Numerical values (constant for all simulated situations) of parameters involved in mathematic model of the overhead crane assembly are the following, namely:

- Box type beam with height 0.2 m, width 0.1 m and plate thickness 0.02 m;
- The material is steel with density 7850 kg/m³ and longitudinal elasticity module $E = 2.1 \cdot 10^{11}$ N/m²;
- Overhead crane beam opening is 20 m;
- Seating at ends of the crane beam is of stiff type (less for the last studied case, that assume seating is of elastic type, having stiffness coefficient $10^{12}$ N/m);
- Value of handled load is 1000 N;
- Hoist cable is made of steel with diameter 0.010 m, and the hoist is made of three cables;
- Hoist length is, 10m, for the basic model; for studying the influence of this parameter upon dynamics of assembly, hoist length values 3 and respectively 16m were considered in addition (the values of this parameter shall be explicitly presented for each studied case).

Initial load speed is the main parameter whose influence was analysed in this study. By this reason, its value is different for each case studied and will be presented accordingly. Taking into account the usual values of handling speed within 1.5 - 16 m/min interval, which in International System means $0.025 - 0.27$ m/s, within this study the following values were considered, namely: case 1: $v = 0.03$ m/s (lower limit of usual speeds range); case 2: $v = 0.30$ m/s (upper limit of usual speeds range); case 3: $v = 0.60$ m/s (incidental situation involving the double of upper limit of usual speeds range); case 4: $v = 0.90$ m/s (limit situation involving the triple of upper limit of usual speeds range).

All four cases previously presented assume that the sheave length is 10m, and the beam is single stiff seated at ends.

For analyzing the influence of hoist length upon dynamics of entire assembly of overhead crane, the following two additional cases were considered, namely: case 5: $v = 0.30$ m/s; $L_{\text{cable}} = 3$ m; case 6: $v = 0.30$ m/s; $L_{\text{cable}} = 16$ m.

The overhead crane beam’s manner of seating was kept identical also for these two additional cases. If assumed that in reality the configuration of seating system shows some elasticity, it is obvious that this has direct implications on global dynamics of entire overhead crane assembly. To evaluate this situation, previously considered parameters were fixed to initial values ($v = 0.30$ m/s and $L_{\text{cable}} = 10$ m), and seating of crane beam was considered of elastic type, with stiffness coefficient having the value $10^{12}$ N/m. This is case 7 analysed in this study. It is to be mentioned that all simulations were done for 5 seconds period, less the last case (case 7), when the time for analyse was increased to 8 seconds (due to the need to evaluate certain transition of the system, completely, to stabilized mode).

During simulations, the following parameters were monitored:
- vertical travel of the mobile car (vertical travel of crane beam, at its middle, in connection point of load lifting system);
- vertical travel of the load in the hook;
- to facilitate comparative study between the two travel signals, these were comparatively presented on the same diagram;
- the reaction in the seating system of the crane beam (taking into account the modelled system is symmetrical, the two reactions have equal values, for presentation only one reaction being kept);
- spectral componence of beam travels (in middle point) and, respectively, load travels; also the amplitudes as well as the phases of the two travels were studied, and their presentation was
comparatively done, to underline the coherence of simulation results and interdependency of the two models components of the assembly.

Numerical application developed for the purpose to perform simulations shows availability to assess geometrical configuration of the overhead crane beam in different moments of time within simulated range, as well as their successive graphic presentation, with a certain refreshing rate, so that to simulate the movement gained by the said system on entire time of study. Figure 2 shows some sequences from the said animation, with the purpose to reveal the geometric configuration got by the beam under dynamic loads influence.
The results obtained further to simulations, for the seven cases previously considered, are shown in figures 3 to 9.

In each figure, the diagrams were presented in the same sequence, their meaning being the following: reaction (a), travels (b), amplitude (c) and, respectively, phase (d) of travels in frequency range. In case of multiple diagrams, the meaning of represented sizes are mentioned in each diagram attached legend.

Figure 2. Evolution of crane beam for different moments of simulation (the time value is indicated in the title of each diagram).
Figure 3. Results of simulations for case 1: reaction (a), travels (b), amplitude (c) and, respectively, phase (d) of travels; the meaning of represented sizes are mentioned in each diagram attached legend.
Figure 4. Simulations result for case 2: reaction (a), travels (b), amplitude (c) and, respectively, phase (d) of travels; the meaning of represented sizes are mentioned in each diagram attached legend.
Figure 5. Simulations result for case 3: reaction (a), travels (b), amplitude (c) and, respectively, phase (d) of travels; the meaning of represented sizes are mentioned in each diagram attached legend.
Figure 6. Simulations result for **case 3**: reaction (a), travels (b), amplitude (c) and, respectively, phase (d) of travels; the meaning of represented sizes are mentioned in each diagram attached legend.

Figure 7. Simulations result for **case 5**: reaction (a), travels (b), amplitude (c) and, respectively, phase (d) of travels; the meaning of represented sizes are mentioned in each diagram attached legend.
Figure 8. Simulations result for case 6: reaction (a), travels (b), amplitude (c) and, respectively, phase (d) of travels; the meaning of represented sizes are mentioned in each diagram attached legend.
Figure 9. Simulations result for case 7: reaction (a), travels (b), amplitude (c) and, respectively, phase (d) of travels; the meaning of represented sizes are mentioned in each diagram attached legend.

Extreme values (minimum and maximum) got by reaction at crane beam end were assessed for each and every case, and systematic presentation of these values is done in table 1. In the table, there were marked (in red color) the cases for which the maximum value of reaction exceeds the limit value corresponding to detaching of rolling system of the crane beam from the related rolling way. For studied situation in this paperwork, according to work hypothesis previously mentioned, the said limit value was assessed as being half of total load (beam+car+load) acting in static conditions. This value is 16200N. For quality and quantity like assessment of dangerous situations, when there is the danger that rolling system of the overhead crane beam to detach from related rolling way, the maximum values of reactions were graph represented, together with suitable limit value. The obtained diagram is shown in figure 10. From analysis of data in table 1, as well as from analysis of diagram in figure 10, results that the potentially dangerous situations are those corresponding to cases 3, 5 and 7. A detailed analysis of these cases reveals that the danger for detachment from rolling way may come up.

- For limit situations that involve using some much upper values of load lifting/lowering speed in comparison to usual upper limit;
- When using some big values of operating speed (in upper area of the usual range), but for reduced values of the hoist length;
- In “normal” situations, for relatively high speeds (in upper area of the usual range), but when, from various reasons, the seating/rolling system of the crane beam gets an elastic type behaviour.
Figure 10. Study of maximum values of reaction, in the seating point of overhead crane beam, in comparison with limit value for detachment (R_{\text{limit}} = 16200 \text{ N}).

Analyzing the results presented in this study, we may conclude that monitoring and analyze of overhead crane dynamics is extremely important as it may emphasize the situations with potential danger in operation of such equipment.

Table 1. Extreme values of reaction in the seating system of overhead crane beam.

| Studied case | Speed v [m/s] | L_cable [m] | Beam seating type | Value of reaction in beam seat [N] |
|--------------|--------------|-------------|-------------------|----------------------------------|
|              |              |             |                   | min                              | max                               |
| 1            | 0.03         | 10          | Rigid             | 15676.531                      | 15720.833                        |
| 2            | 0.30         | 10          | Rigid             | 15365.267                      | 15908.200                        |
| 3            | 0.60         | 10          | Rigid             | 15230.533                      | 16116.399                        |
| 4            | 0.90         | 10          | Rigid             | 13995.792                      | 16323.291                        |
| 5            | 0.30         | 3           | Rigid             | 15216.301                      | 16201.120                        |
| 6            | 0.30         | 16          | Rigid             | 15552.115                      | 15837.983                        |
| 7            | 0.30         | 10          | Elastic           | 15073.209                      | 16376.390                        |

Although the work hypothesis used in development of this study are very restrictive as to limit situations modelled and simulated, comparing to the range of limit situations that may be met in current practice, however it can be noticed that for certain values or combination of values of some specific parameters, the system trend is to get an operation mode outside of certain safety limit.

4. Conclusions

Overhead cranes travel on rolling ways located on pillars alongside the working hall, lift and transport the most various loads, ensuring service on a rectangular surface from technological surface of the hall.

On metallic structure of the overhead cranes (main longitudinal beams connected in a stiff way at ends with cross, end beams) load lifting mechanism and car travel mechanism are mounted and this transmits loads due to loading, own weight, etc., to supporting pillars of crane’s rolling way. Precise knowing the reactions $V_1$ and $V_2$ that are transmitted to the rolling way and strength elements of the structure supporting the rolling way of the crane, result in a more precise dimensioning of strength structures. Also it is necessary to now the total vertical sag of metallic structure, dimensions of section for the chosen beam (chosen profile). The stud of sags is statically done and without taking into account the car travel, moment of inertia variation, elasticity constant of the cable, while the dynamic model previously presented, takes into consideration all these situations. By their precise knowing and by knowing the maximum disturbance cases (sudden catch of the load or its incidental release, when $\Delta t \to 0$) we will be able to obtain a most precise relation to calculate the sag.
Studying the dynamics of an overhead crane with mobile car under dynamic load, the conclusion is reached that we deal with an increase-decrease process on big periods, due to periodical combination of some dynamic effects and power transfer through coupling of the two oscillators.

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