A Novel Reluctance Network Model Applicable for Open Magnetic Circuits

Y. Hane, K. Sugahara*, and K. Nakamura
Graduate School of Engineering, Tohoku Univ., 6-6-11 Aoba Aramaki, Aoba-ku, Sendai 980-8579, Japan
*Graduate School of Science and Engineering, Kindai Univ., 3-4-1 Kowakae, Higashiosaka 577-8502, Japan

In previous research, we have developed a reluctance network analysis (RNA) to calculate various electric machines’ characteristics, including electric motors, with high accuracy and high speed. However, those researches were limited to the closed magnetic circuits. This paper presents a novel RNA model for open magnetic circuits with the Kelvin transformation concept. The proposed method applies to analyzing the particle accelerator magnets, wireless power transfer systems, etc.

Key words: reluctance network analysis (RNA), open magnetic circuit, Kelvin transformation

1. Introduction

The finite element method (FEM) is widely used in many fields of engineering. Since its application in electrical engineering in the 1960s, it has been an essential tool in electromagnetic design and analysis. However, this method generally requires a long calculation time and a large computer memory which is a significant problem in practical use. The authors proposed a reluctance network analysis (RNA) to overcome this issue, which expresses an analytical object by one reluctance network. All the reluctances can be determined by the B-H curve of the material and dimensions. The RNA has some advantages: simple model, fast calculation, easy coupling with external electric circuits and motion equation, etc. The RNA was applied to calculate various electric machines’ characteristics, including transformers and motors. However, the previous researches on the RNA were limited to the closed magnetic circuits, and it has never been studied for the open magnetic circuits, that is, in the case of including unlimited domain.

A wireless power transfer (WPT) system is a typical example that has open magnetic circuits. In reference 8), the RNA is applied to the WPT system analysis; however, the calculation accuracy is insufficient due to the limited analytical domain. Another example is a particle accelerator. Its electromagnet usually has a gapped iron core, which does not essentially have open magnetic circuits. However, since very high calculation accuracy is required in particle accelerators, the analytical region must be broad enough.

In order to expand the scope of application of the RNA for the WPT systems and particle accelerator magnets, this paper presents a novel RNA model for open magnetic circuits with the concept of the Kelvin transformation, which has been utilized in the open boundary problems in the FEM. When using the Kelvin transformation in the FEM, the analytical region is truncated by circular and spherical boundaries in two-dimensional (2-D) and three-dimensional (3-D) analysis, respectively, and connected to another exterior region. The Kelvin transformation can calculate for all the space elements, including the point at infinity, with high accuracy. Furthermore, the Kelvin transformation can have a high affinity with the RNA by extending its concept which only requires simple region transformation since the RNA has no boundary. By incorporating the Kelvin transformation into the RNA, there is no restriction on the shapes of a circle and sphere, and it is possible to analyze objects with various shapes. This paper proposes applying the Kelvin transformation to the RNA by expanding its concept, and its validity is proved by comparing the calculation results of some examples with the FEM.

2. Derivation of RNA Model for Open Magnetic Circuits

In this chapter, first, the Kelvin transformation principle is described by using a 2-D analysis as an example. Next, a method for deriving the proposed RNA model incorporating the Kelvin transformation concept is described in the 2-D and 3-D analysis (here, the 3-D analysis is in the case of axisymmetric).

2.1 Principle of Kelvin transformation

Fig. 1 shows a schematic diagram of a 2-D FEM model using the conventional Kelvin transformation. As shown in this figure, an analytical region including the point at infinity is divided into interior and exterior regions truncated by a circular boundary, and the unknown boundary condition connects both. Although the interior and exterior regions have the same mesh as just an example in Fig. 1, the actual analytical model does not necessarily have to be made in this way. In the figure, the interior and exterior regions’ coordinate systems are (x, y) and (x’, y’), respectively, where each origin is the center of the circle. Here, x’ and y’ are given by the following equations:

Corresponding author: Y. Hane (e-mail: yoshiki.hane.e2@tohoku.ac.jp).
\[ x' = \frac{a^2}{x^2 + y'^2} x, \quad (1) \]
\[ y' = \frac{a^2}{x^2 + y'^2} y, \quad (2) \]

where a radius of interior and exterior regions is \( a \).

In the previous papers, it was demonstrated that the open boundary problems can be solved by using the Kelvin transformation \(^9,10\). Moreover, this method is known to be extended to a 3-D analysis \(^11-14\). Besides, in this method, the interior and exterior regions can be separated in magnetic materials, though they are generally separated in air region\(^15\).

### 2.2 Derivation of RNA model for open magnetic circuits

The following formulation is common in 2-D and 3-D axisymmetric analysis and can be extended to 3-D analysis in general. To facilitate the RNA formulation, let the coordinate systems of the interior and exterior regions be \((r, \theta, z)\) and \((r', \theta', z')\) in a 2-D analysis, and \((r, \theta, \phi)\) and \((r', \theta', \phi')\) in a 3-D axisymmetric analysis, respectively. Here, each origin is the center of the circle, and each divided element is fan-shaped.

Fig. 2 shows a method for deriving the proposed RNA model for open magnetic circuits. First, as shown in Fig. 2(a), the analytical region, including the point at infinity, is divided into interior and exterior regions truncated by a circular boundary, and each is divided into multiple elements. As shown in Fig. 2(b), each divided element can be expressed in a unit magnetic circuit composed of four reluctances in the \( r \)- and \( \theta \)-axis directions. The following equation gives each reluctance:

\[ R_m = \frac{l}{\mu \mu_0 S}, \quad (3) \]

where the relative permeability of the material is \( \mu_r \), and the vacuum permeability is \( \mu_0 \), and the average cross-sectional area and magnetic flux path length of each element are \( S \) and \( l \), respectively. \( S \) and \( l \) can be determined by \( l_1, l_2, l_3, l_4, l_5, l_6, l_7 \), and \( l_8 \), which are dimensions of each element (as shown in Fig. 2(b)). Among them, \( l_1, l_2, l_3, \) and \( l_8 \) are given by the following equations, respectively:

\[ l_1 = |r_1 - r_2|, \quad (4) \]
\[ l_2 = r_1 |\theta_1 - \theta_2|, \quad (5) \]
\[ l_3 = \frac{a^2}{r_1^2} |\theta_1 - \theta_2|, \quad (6) \]
\[ l_4 = \frac{a^2}{r_1^2} |\theta_1 - \theta_2|, \quad (7) \]

Even when the element includes the point at infinity, each element’s reluctances can be calculated by the above equations. Thus, in this way, the point at infinity can be expressed by only a few reluctances.

On the other hand, \( l_5, l_6, l_7, \) and \( l_9 \) are the thicknesses of each element in the interior and exterior regions in the 2-D and 3-D models, respectively. Among them, \( l_5 \) and \( l_9 \) are given by the following equations, respectively:

\[ l_5 = 2\pi r^2 \sin \frac{|\theta_1 - \theta_2|}{2}, \quad (8) \]
\[ I_\rho = 2\pi r^2 \sin \left( \frac{\theta_1 + \theta_2}{2} \right). \] (9)

By using the above-derived RNA model, it is possible to analyze considering infinity. However, the RNA has a lower degree of freedom in the element shape than the FEM since a unit magnetic circuit represents each element in the RNA. Thus, it is generally difficult to construct the RNA model with only fan-shaped elements. Therefore, it is necessary to extend the above method to apply the proposed RNA model to the analytical objects with arbitrary shapes.

For example, when the analytical object has a rectangular shape, the element division, shown in Fig. 3(a), is desirable in the RNA. The RNA allows any shapes of mesh as long as the boundaries between the interior and exterior regions are common, due to the nature of the circuit. This figure reveals that the RNA has a higher degree of freedom in the model topology since it does not have complicated shape functions, unlike the FEM. In this model, there is a rectangular region at the center, and quadrant ones sandwich both sides.

In the quadrant regions, each element can be expressed in the same manner as the unit magnetic circuit shown in Fig. 2(b). On the other hand, in the rectangular region, the unit magnetic circuit can be represented as shown in Fig. 3(b). In this figure, \( l_1, l_2, l_3, l_4, l_5, l_6, l_7, l_8 \) and \( l_9 \) are dimensions of each element. Among them, \( l_1 \) and \( l_2 \) are given by the following equations, respectively:

\[
l_1 = |x_1 - x_2|, \quad l_2 = |y_1 - y_2|. \] (10)

\[
l_1' = |x_1' - x_2'|, \quad l_2' = |y_1' - y_2'|. \] (11)

Same as in the quadrant regions, in the elements including the point at infinity, the reluctances in the \( y \)-axis direction can be calculated by taking the limit values of \( l_1' \), while the ones in the \( x \)-axis direction, which are connected to the point at infinity, are open. On the other hand, \( l_1 \) and \( l_2 \) are determined by only the number of divisions in the \( y \)-axis direction, not the coordinates on the \( x \)-axis. \( l_3, l_4, l_5, l_6, l_7, l_8 \) and \( l_9 \) are the thicknesses of each element in the interior and exterior regions in the 2-D and 3-D models, respectively. Among them, \( l_9 \) and \( l_9' \) are given by the equations (8) and (9), respectively.

3. Simulation Results by Using Proposed Method

In this chapter, the validity of the proposed RNA model described in chapter 2 are proved. First, the analytical object composed of a bar magnet and iron is analyzed as a primary study. Next, a gapped iron core is analyzed to indicate the proposed method’s applicability to particle accelerator magnets.

3.1 Analysis of object composed of bar magnet and iron

Fig. 4 and Fig. 5 show the analytical object’s specifications and its RNA model when \( a = 50, 40, 30 \) mm, respectively. As mentioned in the section 2.1, the interior and exterior regions can be connected via not only the air but also the magnetic core. Here, the MMF \( f_c \) of the magnet is expressed by the following equation using the coercive force \( H_c \) and the average magnet length \( l_m' \):

\[ f_c = H_c l_m', \] (12)

and inserted in series for each reluctance in the magnet in the \( y \)-axis direction.

In this section, the validity of the proposed method’s calculation results is proved by comparing with those of the FEM by using the FEMM, which is general-purpose electromagnetic simulation software. In the FEM, the Kelvin transformation is used as the boundary condition to consider the influence of the exterior region. Here, the radius of the interior and exterior regions is \( a = 100 \) mm, which is large enough to obtain highly accurate calculation results. The distribution of the \( x \)-component of the magnetic flux density on the dashed line (\( x = 20 \) mm) shown in Fig. 4 is compared between the RNA and FEM.

Fig. 6(a) and (b) show the comparison of the
calculation results of the magnetic flux distribution of the RNA and FEM in the cases of 2-D and 3-D axisymmetric analysis. The RNA is carried out for three cases of $a = 50$ mm, 40 mm, and 30 mm. Besides, the RNA and FEM when there is only the internal region are also carried out for reference. From these figures, it is clear that the calculation results of the RNA hardly change even if the value of $a$ changes within the range of the necessary and sufficient spatial resolution. Moreover, when analyzing only the internal region, the calculation results are quite different from those when considering the exterior region for both the RNA and FEM. Furthermore, comparing the RNA and FEM in the 2-D analysis, the maximum error is 1.16%, which indicates that the calculation accuracy of the proposed method is high. On the contrary, in the 3-D axisymmetric analysis, the minimum and maximum errors are 0.88% and 11.2%, respectively, which indicates that the calculation accuracy is not necessarily sufficient and has a considerable variation depending on the position. The cause of the errors is considered discretization due to constant values of $l_\phi$ and $l_\phi'$ in each element.

3.2 Analysis of gapped iron core

Fig. 7 and Fig. 8 show dimensions of a gapped iron core used for consideration and its RNA model in the core and air regions, respectively. In the air region, the point at infinity is considered by using the Kelvin transformation concept in the same way as in the previous section. In this model, the number of reluctances is 190 in the core region and 2567 in the air one. Here, the RNA model is halved in the $z$-axis direction due to the symmetry.

In this section, the validity of the proposed RNA model is verified by comparing the calculation results with the FEM by using the OPERA 3D \cite{17}, which is a general-purpose electromagnetic simulation software. In the FEM, the Improved Absorbing Boundary Condition \cite{18, 19} is used as the boundary condition. The number of layers is eight so that the effect of the boundary edge can be reduced with necessary and sufficient accuracy. Here, as analysis conditions, the relative permeability of the iron core is set to be three cases of 100, 1000, and 10000, and the MMF of a coil is

Fig. 4 Specifications of analytical object.

| Relative permeability of iron | 1000  |
|-------------------------------|-------|
| Relative permeability of a magnet | 1     |
| Coercive force of a magnet (kA/m) | 300   |

Fig. 5 Constructed RNA model.
Fig. 6 Comparison of calculation results of magnetic flux distribution between RNA and FEM.

Fig. 7 Dimensions of a gapped iron core used for consideration.

Fig. 8 Comparison of calculation results of magnetic flux distribution between RNA and FEM.
2000 A.

In this section, the calculation results of the magnetic flux density at the center of the air gap are compared between the RNA and FEM. The relevant value is defined as the x-component of the average magnetic flux density of the elements, including the center point of the air gap in the RNA, while it is obtained from integration by using the Biot-Savart’s law in the post-processing for the electromagnetic field analysis in the FEM.

Fig. 9 shows the comparison of the calculated magnetic flux density at the center of the air gap between RNA and FEM. As shown in the figure, the maximum error is 4.9%, which is in almost good agreement. Moreover, the calculation accuracy of the RNA with only internal region reduced to 12.8% error when the relative permeability is 100. From these results, the validity of the proposed model is apparent.

4. Conclusion

This paper presented a novel RNA model for open magnetic circuits by incorporating the Kelvin transformation concept. To prove the validity of the proposed method, the analytical object, which consists of a bar magnet and iron, was analyzed as an introductory study. By comparing the calculated values between the RNA and FEM, it was clear that the errors are always about 1% or less, which is extremely high accuracy in the 2-D analysis. On the contrary, in the 3-D asymmetric analysis, although the overall tendency is almost in good agreement, the errors at several points are more than 10% due to the discontinuity of dimensions of elements.

Next, a gapped iron core is analyzed by using the proposed method to verify the applicability to the particle accelerator magnets. As a result, it was clear that the errors of the calculated values of the gap flux density between the RNA and FEM are less than 5%, which is almost in good agreement in different permeabilities.

In the future, we aim to apply the proposed method to analyze the wireless power transfer systems and particle accelerator magnets.

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