Observable Manifestation of an Electron-Positron Plasma Created by the Field of an Optical Laser

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Abstract. Electron-positron vacuum pair creation in the quasi-periodic electric field of a standing wave is investigated for field strengths and frequencies corresponding to modern optical lasers using a quantum kinetic equation with a non-markovian source term. For a field $E \sim 5 \cdot 10^{11} \text{V/cm}$, the instantaneous quasiparticle electron-positron plasma is created which maximum density of order $10^{20} \text{cm}^{-3}$. The mean value of the pair density per period $\langle n \rangle$ exceeds by many orders of magnitude the residual density $n_r$ which is taken over an integer number of field periods. The value $\langle n \rangle$ is proportional to the squared field intensity and does not depend on the frequency at $\nu \ll m$. Under conditions of presently available optical lasers with intensities exceeding $I \approx 10^{20} \text{W/cm}^2$, the mean number of created pairs in the volume of a wavelength cubed is about $10^7$ which corresponds to $5 - 10$ two-photon annihilation events per one laser pulse. The generated $\gamma$-quanta are suggested to be an observable signal of $e^+e^-$ pair vacuum creation in experiments with counter-propagating optical laser beams.

1. Introduction
The vacuum pair creation effect by a classical electric field has been predicted in QED a long time ago [1, 2]. A complete theoretical description of this effect has been obtained [3]-[6], but there is still no experimental proof. The main obstacle is the high value of the critical electric field strength for the pair creation, $E_{cr} = 1.3 \times 10^{16} \text{ V/cm}$ for electron-positron case. According to the Schwinger formula [2], the pair creation rate in a constant electric field is suppressed exponentially when $E \ll E_{cr}$. However, a very different situation occurs when the field acts only in a finite time interval (dynamical Schwinger effect) [3, 7, 8, 9]. In this case, the Schwinger formula as well as its analog for a monochromatic field (Brezin-Itzykson formula [10]) become inapplicable in the weak field regime.

A few examples of physical situations have been discussed where despite the high critical field strength the Schwinger effect could occur, e.g., relativistic heavy ion collisions [11], neutron stars [12] and focussed laser pulses [13]. It is well known [2] that no pairs are created when both invariants of the electromagnetic field vanish, $E^2 - B^2 = 0$, $EB = 0$. The field produced by focussed laser beams is very close to such a configuration [14] and therefore the pair creation is expected to be essentially suppressed. It should be possible in the background of a spatially
uniform field, which may be created in an antinode of the standing wave produced by the superposition of two coherent counter-propagating laser beams [15].

Since the Schwinger effect is non-perturbative and it requires an exact solution of the dynamical equations it is customary to approximate the complicated structure of a real laser field by a spatially uniform time-dependent electric field. According to different estimates [9]-[17] the effect of vacuum creation should not be observable with presently available laser parameters.

The recent development of laser technology, in particular the invention of the chirped pulse amplification method, resulted in a huge increase of the light intensity in the laser focal spot [18, 19]. As the construction of X-ray free electron lasers XFELs [20] is now planned, the possibility of an experimental proof of the Schwinger effect attracts attention again. Under conditions of short pulses the non-stationary effects become important. We use in our work the kinetic equation approach, which allows us to consider the dynamics of the vacuum creation process while taking into account the initial conditions properly [7]. This approach has been applied already to the periodical field case [8] with near-critical values of the field strength and X-ray frequencies. The method [7] found also application for description of the pre-equilibrium evolution of quark-gluon plasma in the conditions of ultrarelativistic heavy ion collisions at RHIC and LHC [21, 22].

In the present work, we consider the other region of field parameters already achievable nowadays at the currently operating laser systems [23, 24], namely $\nu^2 \ll E \ll E_{cr}$, where $\nu$ is the laser field frequency. We use as a criterion for the creation efficiency the mean density $\langle n \rangle$ per period and the residual density $n_r$, which is taken over an integer of the field periods [25]-[27]. Our main result is that optical lasers can generate a greater number of pairs than X-ray ones in the volume of a cubed wavelength $\lambda^3$ which could be observable, e.g., by means of registration of coincident $\gamma$ pairs with mean total energy $\approx 1$ MeV from electron-positron annihilation.

2. The kinetic equation approach

The basic quantity used in the kinetic approach [7] is the distribution function of quasiparticles in the momentum representation $f(p, t)$ [3]-[6]. The kinetic equation for this function is derived from the Dirac equation in an external time-dependent electric field by the canonical Bogoliubov transformation method [3], or by the help of an oscillator representation [28]. This procedure is exact but one is valid only for the simplest field configurations, e.g., for the uniform time dependent electric field with fixed direction $E(t) = (0, 0, E(t))$. It is assumed that the electric field vanishes at the initial time $t = t_0$, where real particles are absent (“in-vacuum” state). The corresponding kinetic equation has the form [7] (in the collisionless limit)

$$\frac{\partial f(p, t)}{\partial t} + eE(t) \frac{\partial f(p, t)}{\partial p} = \frac{1}{2} \Delta(p, t, t) u(p, t)$$

$$u(p, t) = \int_{t_0}^{t} dt_1 \Delta(p, t_1, t_1) \left[ 1 - 2f(p, t_1) \right] \times \cos \left[ 2 \int_{t_1}^{t} dt_2 \varepsilon(p, t_2, t) \right],$$

where $p$ is the kinematic momentum, $p(t_1, t_2)$ - characteristic

$$p(t_1, t_2) = p - e \int_{t_1}^{t_2} E(t') dt',$$

$$\Delta(p, t_1, t_2) = eE(t_1) \sqrt{m^2 + p_{\perp}^2} / \varepsilon^2(p, t_1, t_2),$$

$$\varepsilon(p, t_1, t_2) = \sqrt{m^2 + p_{\parallel}^2 + p_{\perp}^2(t_1, t_2)},$$

(2)
and \( m \) is the electron mass. The total field \( E(t) \) is defined as the sum of the external (laser) field \( E_{ex} \) and the self-consistent internal field \( E_{in} \) which can be found from the Maxwell equation

\[
\dot{E}_{in} = -\frac{e}{(2\pi)^3} \int \frac{d\mathbf{p}}{\varepsilon_0} \left\{ 2p_1 f(p,t) + u(p,t)\sqrt{m^2 + p^2} \right\},
\]

where \( \varepsilon_0 = \varepsilon(p,t,t) = \sqrt{m^2 + p^2} \). The total current density on the r.h.s. of Eq. (3) is the sum of the conductivity and vacuum polarization contributions, respectively.

The kinetic equation Eq.(1) contains two small parameters \( E \ll m^2 \) and \( \nu \ll m \), but we cannot construct any perturbation theory because of the memory effects present in the argument of the cosine. These memory effects can be neglected only when

\[
\frac{eE}{m\nu} \ll \frac{\nu}{m}.
\]

This condition would contradict the quasiclassical condition for the electric field \( E \gg \nu^2 \). The system of equations (1), (3) is integrated numerically with the initial condition \( f(p,t_0) = 0 \).

The total quasiparticle number density can be found afterwards as a moment of the distribution function

\[
n(t) = 2 \int \frac{d\mathbf{p}}{(2\pi)^3} f(p,t).
\]

We consider the two simplest models for laser field, which can be formed in the focus of two counter-propagating laser beams: the harmonic field which acts during an integer \( z \) of the periods

\[
E_{ex}(t) = E_m \sin \nu t, \quad 0 \leq t \leq zT
\]

and the Gaussian-like pulse with the maximum at \( t = a \) and the width \( 2b \)

\[
E_{ex}(t) = E_m \exp \left\{ -\left[ (t-a)/b \right]^2 \right\} \sin \nu t.
\]

Fig. 1 depicts the time dependence of the number density of quasiparticles, which are generated by the pulse field. The efficiency of plasma production is present on an example.
of the working optical Ti:sapphire laser [23] with \( E_m \approx 3.54 \cdot 10^{-5} E_{cr} \), \( \lambda = 795 \) nm. For the monochromatic field (6), the instantaneous density oscillates with twice the field frequency \( \nu \).

The residual density \( n_r \) which corresponds to an integer \( n \) of field periods, \( n_r = n(zT) \), is negligible in comparison with mean density \( \langle n \rangle \) for optical lasers and it does not depend on \( z \). The ratio \( \langle n \rangle/n_r \) is approximately \( 10^{11} \) for the field (6) in the considered case [23]. As a consequence, in spite of the residual density for the X-ray laser exceeds the one for the optical laser by a large factor, the situation is different regarding the mean density: the optical laser produces more pairs in the volume of \( \lambda^3 \) than the X-ray one. According to Fig. 1, the pair density reaches \( \sim 10^{20} \) cm\(^{-3}\) what corresponds to of \( \approx 10^7 \) pairs in a volume of \( \lambda^3 \) on average. This sufficiently dense plasma exists during a laser pulse duration but vanishes almost completely after switching off the field.

The momentum spectrum of created quasiparticle pairs is represented in Figs. 2, 3. The momentum distribution of the quasiparticle pairs has a width of order of electron mass for both transverse and longitudinal momenta as against the standard assumption about zero longitudinal momentum of e\(^+\)-e\(^-\) pairs [11]. The shape of momentum distribution is changed essentially as the length of a wave decreases up to values 1-10 nm. When the electric field takes zero value, a stratum-like structure is formed with a characteristic scale corresponding to a field frequency, see Fig. 3. This peculiar momentum distribution defines the residual density of e\(^+\)-e\(^-\) pairs.

### 3. Possible ways to detect the created plasma

The quasiparticle plasma created in the weak field case can be manifested in various physical effects, such as nonlinear Thomson scattering [29], damping of electromagnetic waves [17], one [4] and two-photon annihilation, the non-linear Breit-Wheeler process [30] etc. As an example, we estimate the intensity of two-photon annihilation in the plasma volume. The corresponding \( \gamma \)-quanta with the mean total energy \( \approx 1 \) MeV can be registered outside the focus of the counter-propagating laser beams. The production rate of this process is defined by relation

\[
\frac{dN}{dV dt} = \int d\mathbf{p}_1 d\mathbf{p}_2 \sigma(\mathbf{p}_1, \mathbf{p}_2) f_1(\mathbf{p}_1, t) f_2(\mathbf{p}_2, t) \times \sqrt{(\mathbf{v}_1 - \mathbf{v}_2)^2 - |\mathbf{v}_1 \times \mathbf{v}_2|^2},
\]

(8)
Figure 5. The time dependence of the pair density for the case of the two-mode field (11) with \( k = 2, 3 \) and \( E_1 = E_2 = 3.54 \times 10^{-5} E_{cr} \).

Figure 6. The Fourier components of the selfconsistent electric field \( E_{in} \) for the case of the two-mode field with \( k = 3 \).

where \( v \) - the particle velocity, \( \sigma \) - the cross-section of two-photon annihilation

\[
\sigma(p_1, p_2) = \frac{\pi \epsilon^4}{2m^2 \tau^2 (\tau - 1)} \left( (\tau^2 + \tau - 1/2) \ln \left( \sqrt{\tau + \sqrt{\tau - 1}} \right) - (\tau + 1) \sqrt{\tau (\tau - 1)} \right). \tag{9}
\]

The t-channel kinematic invariant \( \tau \) is given by

\[
\tau = \frac{(p_1 + p_2)^2}{4m^2} = \frac{1}{4m^2} [(\epsilon_1 + \epsilon_2)^2 - (p_1 + p_2)^2]. \tag{10}
\]

We have made an estimate of the number of annihilation events (8) per one laser pulse with the following parameters: pulse intensity \( I = 10^{20} \) W/cm\(^2\), pulse duration \( \tau_L \sim 85 \) fs, wavelength \( \lambda = 795 \) nm, cross size of laser beams \( \approx 2.5 \) \( \mu \)m [23]. The estimate results is approximately \( 5-10 \) annihilation events per laser pulse. The wavelength dependence of the discussed quantities is shown in Fig. 4.

Another detection of the created plasma creation is possible in an experiment with two-mode laser beams [31]. Let us consider the two-mode field

\[
E_{ex}(t) = E_1 \sin \nu t + E_2 \sin kv t. \tag{11}
\]

According to the work [31], the value \( k = 3 \) is preferential for the observation of interference effects. Fig. 5 depicts the time dependence of the pair density for the cases of \( k = 2, 3 \). The mean pair density for the two-mode field is twice that of the one-mode field and it is independent of the value \( k \). Fig. 6 presents the frequency spectrum of the self-consistent electric field \( E_{in} \); the intensity of the 2-nd mixed harmonic is at a level of \( 10^{-10} \) of the external field strength and very sharply decreases upon reduction of one of the modes.

4. Conclusion

We have shown that the simplest model of the laser field predicts the creation of a dense quasiparticle plasma in the foci of counter-propagating optical laser beams with parameters corresponding the operating ones [23, 24]. The plasma lives during the laser pulse and vanishes almost completely after switching off the field. The mean density is defined by the field strength
and does not depend on frequency reaching values $\sim 10^{20}$ cm$^{-3}$ for the achieved field strength of $10^{11}$ V/cm. A possible manifestation of the plasma can be the emission of pairs of $\gamma$-quanta with a spectrum peaked in the vicinity of a total energy of 1 MeV with an intensity of $5 \times 10^6$ events per laser pulse. This would be a non-linear transformation of the soft laser photons to $\gamma$-quanta with a frequency ratio of about $10^6$.

Another possibility to observe the created plasma is its diagnostics with the help of a probing monochromatic signal which crosses the laser focus. The nonlinear interaction of the probing signal with the electron-positron plasma results in the generation of higher harmonics of the laser frequency, which can be observable.

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