STOCHASTIC RESONANCE IN A SYSTEM OF FERROMAGNETIC PARTICLES

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Abstract

We show that a dispersion of monodomain ferromagnetic particles in a solid phase exhibits stochastic resonance when a driven linearly polarized magnetic field is applied. By using an adiabatic approach, we calculate the power spectrum, the distribution of residence times and the mean first passage time. The behavior of these quantities is similar to their corresponding ones in other systems in which stochastic resonance has also been observed.
I. INTRODUCTION

The phenomenon known as stochastic resonance (SR) was first predicted by Benzi et al. \[?,\] and consists of the coherent response of a multistable stochastic system to a driven periodic signal. Up to now, SR has been observed in diverse physical situations as in lasers, in electron paramagnetic resonance, or in free standing magnetoelastic beams. The description of the phenomena as well as its fundamentals and applications are included in the recent reviews by F. Moss \[?,\] and K. Wiesenfeld and F. Moss \[?] (see also in this context refs. \[?\] and \[?\]).

Our purpose in this paper is to show theoretical predictions about the occurrence of the phenomenon in a system of ferromagnetic monodomain particles dispersed in a solid phase (a crystalline polymer, for example) when an alternating magnetic field is imposed. In a ferromagnet the interdomain walls are of the order \(10^{-6}\) cm, then particles whose size is of this order of magnitude or less may be considered monodomains \[?\]. Such a particles are always magnetized to the spontaneous magnetization \(M_s\). For these fine ferromagnetic particles the energy consists of contributions coming from two competing mechanisms which tend to orient the magnetic moment: potential energy due to the field and energy of anisotropy. The energy is therefore a nonlinear function of the orientation angle, which is precisely the stochastic variable. This fact was already taken into account by Néel \[?] to estimate the relaxation time for the magnetic moment in magnetic powders. As we will show, under certain conditions the ferromagnetic particle constitutes a bistable stochastic system, with the external field providing a periodic contribution.

The stochastic behavior of systems of ferromagnetic particles was discussed in ref. \[?] where a Fokker-Planck equation for the probability density of the orientations of the particles was derived. This equation is related to the Landau-Gilbert equation \[?\], \[?] in which a stochastic source accounting for Brownian motion of the magnetic moment is added. The procedure is, however, restricted to the case in which the external magnetic field is constant.
in time. This theory provides the framework for our subsequent analysis and consequently must be extended to the case of a time-dependent magnetic field.

We have distributed the paper in the following way. In section II we introduce our model and from the Gilbert-Landau equation we establish the kinetic equation for the probability distribution of the magnetic moment. In section III, by using an adiabatic approximation, we compute the power spectrum of the fluctuations of the magnetic moment, whereas, in section IV, and within the framework of the same approach, we calculate the probability distribution of residence times and the mean first passage time. Finally, in section V we give numerical values of the characteristic parameters of our system and discuss our main results. Additionally, we show that due to the very short time scale that rules the relaxation of the system, the adiabatic approach is justified.

II. THE DISPERSION OF FERROMAGNETIC PARTICLES

We consider an assembly of single-domain ferromagnetic particles dispersed in a solid phase at a concentration which is assumed to be low enough to avoid magnetic interactions among them. When we apply an external uniform a.c. magnetic field \( \vec{H}(t) = \vec{H}_0 \sin \omega_0 t \), \( \vec{H}_0 \) being the magnetic field strength and \( \omega_0 \) its angular frequency, the energy of each particle splits up into contributions coming from the external field and the crystalline anisotropy \( ? \) and is given by

\[
U(t) = -\vec{m} \cdot \vec{H}(t) + K_a V_p (\vec{m} \cdot \hat{s})^2 .
\]

Here \( \vec{m} = m_s \hat{m} \) is the magnetic dipole moment, \( m_s = M_s V_p \) the magnetic moment strength, with \( M_s \) being the saturation magnetization and \( V_p \) the volume of the particle, \( K_a > 0 \) is the anisotropy constant and \( \hat{s} \) is a unit vector perpendicular to the symmetry axis of the particle.
The dynamics of the magnetic moment $\vec{m}$ is governed by the Gilbert equation [?], [?]

$$\frac{1}{\gamma} \frac{d\vec{m}}{dt} = \vec{m} \times (\vec{H}_{eff} + \vec{H}_d) ,$$

(2)

where $\gamma(= -e/mc)$ is the gyromagnetic ratio. From (2) one may identify the two mechanisms responsible for the variation of $\vec{m}$. The effective field

$$\vec{H}_{eff} \equiv \frac{\partial U}{\partial \vec{m}} = \vec{H}(t) - 2 \frac{K_a V_p}{m_s} (\vec{m} \cdot \hat{s}) \hat{s}$$

(3)

which implies a Larmor precessional motion of $\vec{m}$ and the mean field

$$\vec{H}_d \equiv -\eta \frac{d\vec{m}}{dt}$$

(4)

that introduces a damping whose microscopic origin lies in the collisions among the electrons participating in the formation of the magnetic moment of the domain. In eq. (4) $\eta$ is a damping coefficient.

Eq. (2) can be solved self-consistently giving

$$\frac{d\vec{m}}{dt} = \vec{\omega}_L \times \vec{m} + h\vec{m} \times \vec{H}_{eff} \times \vec{m} ,$$

(5)

where $\vec{\omega}_L = -m_s g \vec{H}_{eff}$ is the Larmor angular frequency of the precessional motion executed by the magnetic moment of a dipole. Moreover, we have introduced the quantities

$$g = \frac{\gamma}{m_s(1 + \eta^2 m_s^2 \gamma^2)} ,$$

(6)

and

$$h = -\frac{\eta \gamma^2}{(1 + \eta^2 m_s^2 \gamma^2)} .$$

(7)
When the external field is constant, the precessional motion is extinguished in a time scale
\( \tau_0 = (m_s h H_{\text{eff}})^{-1} \), obtained by comparison of the left hand term and the second right hand term in (5). Thus when Brownian motion is absent, \( \vec{m} \) become parallel to \( \vec{H}_{\text{eff}} \) for times larger than \( \tau_0 \). Keeping only first order terms in the damping coefficient, one obtains the Landau equation [?]

\[
\frac{d\vec{m}}{dt} = -\gamma \vec{H}_{\text{eff}} \times \vec{m} + \lambda \vec{m} \times \vec{H}_{\text{eff}} \times \vec{m} ,
\]

where \( \lambda \) may be identified as \( \eta \gamma^2 \).

The presence of thermal noise was considered by Brown [?] by simply adding the random field \( \vec{H}_r \) to the Gilbert equation (2). This equation thus becomes a nonlinear Langevin equation with multiplicative noise

\[
\frac{1}{\gamma} \frac{d\vec{m}}{dt} = \vec{m} \times (\vec{H}_{\text{eff}} + \vec{H}_d + \vec{H}_r) .
\]

The random term constitutes a gaussian stochastic process with zero mean and a fluctuation-dissipation theorem given by

\[
\langle \vec{H}_r(t') \vec{H}_r(t' + t) \rangle = 2K_B T \eta \overline{\delta}(t) ,
\]

where \( \overline{\delta} \) is the unit tensor.

Following the standard procedure, it is possible to derive the Fokker-Planck equation related to (9). One then obtains [?], [?]

\[
\frac{\partial \psi}{\partial t} - \vec{S} \cdot \vec{L} \psi = -\frac{1}{2\tau} \vec{S} \cdot \psi \overline{S} \left( \frac{U(t)}{K_B T} + \log \psi \right) ,
\]

where \( \psi(\hat{m}, t) \) is the distribution function for the orientations of the vector \( \hat{m} \) and \( \overline{S} = \hat{m} \times \frac{\partial}{\partial \hat{m}} \) is the rotational operator. From this equation we infer the appearance of the relaxation time

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\[ \tau = (-2K_B T h)^{-1}, \] corresponding to the time scale in which one achieves the stationary state where the probability flux is constant.

If the external magnetic field is applied along the direction of the easy axis of magnetization, the problem posed by eq. (11) has axial symmetry. In this case, the energy of the system can be written as

\[ U = -m_s H_0 \omega_0 t \cos \theta + K_a V_p \sin^2 \theta \]  \hspace{1cm} (12)

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