Inclusive electron scattering off $^4$He

S. Bacca$^a$, H. Arenhövel$^b$, N. Barnea$^c$, W. Leidemann$^d$ and G. Orlandini $^d$

$^a$Gesellschaft für Schwerionenforschung, Planckstr. 1, 64291 Darmstadt, Germany

$^b$Institut für Kernphysik, Johannes Gutenberg-Universität, Becher-Weg 45, 55099 Mainz, Germany

$^c$Racah Institute of Physics, Hebrew University, 91904 Jerusalem, Israel

$^d$Dipartimento di Fisica, Università di Trento and Istituto Nazionale di Fisica Nucleare, Gruppo Collegato di Trento, I-38050 Povo, Italy

Inclusive electron scattering off $^4$He is calculated exactly with a complete treatment of the final state interaction within a simple semirealistic potential model. We discuss results for both the longitudinal and the transverse response functions, at various momentum transfers. A consistent meson exchange current is implemented. Good agreement with available experimental data is found for the longitudinal response function, while some strength is still missing in the transverse response function.

1. INTRODUCTION

Inclusive electron scattering is governed by two response functions: the longitudinal $R_L(\omega, \mathbf{q})$ and the transverse response $R_T(\omega, \mathbf{q})$. They are induced by the electromagnetic charge $\hat{\rho}(\mathbf{q})$ and current $\hat{J}(\mathbf{q})$ operators, respectively. We study this process on the nucleus of $^4$He, for which an exact calculation of the response function can be performed, including a consistent treatment of the electromagnetic excitation operator. The final state interaction of the continuum four-body wave function is fully taken into account via the Lorentz Integral Transform (LIT) method [1], which leads to a Schrödinger-like equation with bound-state-like asymptotic. We solve it by making use of a spectral resolution method based on the construction of an effective interaction in the hyperspherical harmonics basis (EIHH) [2]. For the present calculation we take the simple semirealistic Malfliet-Tjon (MTI-III) [3] as nucleon-nucleon (NN) interaction.

In a non-relativistic approach the electromagnetic charge is given by a one-body operator, while the current by both a one-body and a two-body operator, the meson exchange current (MEC). We firstly show our calculation of the longitudinal response function and then we present our result for the transverse response function, where we consider also a consistent two-body current.

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2. LONGITUDINAL RESPONSE FUNCTION

The longitudinal response function is defined as

\[
R_L(\omega, \mathbf{q}) = \sum_{f} \left| \langle \Psi_f | \hat{\rho}(\mathbf{q}) | \Psi_0 \rangle \right|^2 \delta (E_f - E_0 - \omega),
\]

where \( |\Psi_{0/f}\rangle \) and \( E_{0/f} \) are the wave functions and the energies of ground and final states, respectively. The complication of the explicit calculation of all final states in Eq. (1) is circumvented via the LIT method, where one has to solve the bound-state-like equation

\[
(\hat{H} - E_0 - \sigma_R + i\sigma_I) \hat{\Psi} = \hat{\rho}(\mathbf{q}) \hat{\Psi}_0 \text{ with } L(\sigma_R, \sigma_I) = \left\langle \hat{\Psi} | \hat{\Psi} \right\rangle,
\]

where \( \sigma_R \) and \( \sigma_I \) are the parameters of the transform \( L \). For the calculation we expand the charge operator into Coulomb multipoles [4], separating them into isoscalar and isovector parts. The expansion is truncated when convergence is achieved. In Fig. 1 we show the LIT of the isovector multipoles for momentum transfers \( q = 300 \) and \( 500 \) MeV/c: one readily notes that for the lower momentum transfer five multipoles are enough to reach convergence, while for the higher momentum transfer two additional multipoles need to be considered. In Fig. 2 we present the result for the longitudinal response function for \( q = 300 \) and \( 500 \) MeV/c, which is achieved by inverting the transform for each multipole. As in a previous calculation of \( R_L \) with the Trento (TN) potential [5], one can note that the semirealistic interaction leads to a rather good overall description of the experimental data from Bates [6] and Saclay [7] for the longitudinal response function. The only deviation we observe is a pronounced peak close to threshold, which is due to the monopole excitation of \(^4\)He.

Figure 1. The LIT of the various isovector Coulomb multipoles, consecutively summed, as a function of the parameter \( \sigma_R \) with \( \sigma_I = 20 \) MeV fixed.
3. TRANSVERSE RESPONSE FUNCTION

The transverse response function is defined as

$$R_{T}(\omega, q) = \sum_{f} \left| \langle \Psi_f | \hat{J}_T(q) | \Psi_0 \rangle \right|^2 \delta (E_f - E_0 - \omega),$$

(3)

where $J_T(q)$ is the transverse electromagnetic current operator. The corresponding bound-state-like equation is the same as in Eq. (2), where $\hat{\rho}(q)$ is replaced by $\hat{J}_T(q)$. The transverse current includes one-body and two-body operators. A two-body current is required in order to satisfy the continuity equation, and has therefore to be consistent with the NN interaction used. We derive a consistent MEC for the MTI-III potential, which is based on the exchange of two effective scalar mesons [3]. The two-body current takes the form

$$J_2(q) = \frac{1}{4\pi^3} e^{i\mathbf{R} \cdot \mathbf{q}} (\nabla_r I_m(q, r)),$$

(4)

where the function $I_m$ contains the meson propagator (for details see Ref. [8]), $\mathbf{r}$ is the relative distance between the two particles and $\mathbf{R}$ is the center of mass of the two-body sub-system. In our calculation we neglect the $\mathbf{R}$ dependence for the sake of numerical simplicity, setting $e^{i\mathbf{R} \cdot \mathbf{q}} \approx 1$. We therefore restrict ourselves to the case of low momentum transfer $q$, where this approximation is valid.

In Fig. 2 we show the transverse response function for the different parts of the current operator. The spin current strongly dominates at the higher momentum transfer $q = 300$ MeV/c (therefore we do not show the convection current contribution separately), while the effect of the convection current is still important at $q = 100$ MeV. One can see that MEC plays an important role at $q = 100$ MeV/c, but is very small at $q = 300$ MeV/c, about 2 – 3% in the peak. At a momentum transfer of $q = 300$ MeV/c the additional contribution of the two-body current is not enough to describe satisfactorily the experimental strength in the quasi-elastic peak.

Figure 2. $R_L$ with the MTI-III potential as function of the laboratory energy for momentum transfers $q = 300$ and 500 MeV/c.
Figure 3. Transverse response function: effect of one-body and two-body currents for \( q = 100 \) and 300 MeV/c in comparison with the available experimental data from Bates [6] and Saclay [7].

4. CONCLUSIONS

We have presented the first calculation of the inclusive longitudinal and transverse response functions of \(^4\text{He}\) within the LIT and EIHH methods. Good agreement with available experimental data is found for the longitudinal response function, while some strength is still missing in the transverse response function. Strong MEC effect are found at low momentum transfer, where unfortunately no experimental data are available. Within our semirealistic potential model and consistent MEC we do not find a strong two-body current effect at \( q = 300 \) MeV/c as obtained in Ref. [9]. This is probably due to the missing explicit pionic degrees of freedom in our model.

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