Super-Eddington accretion disc around a Kerr black hole

A. M. Beloborodov

Astro Space Centre of P.N. Lebedev Physical Institute, 84/32 Profsoyuznaya Street, Moscow 117810, Russia

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ABSTRACT

We calculate the structure of accretion disc around a rapidly rotating black hole with a super-Eddington accretion rate. The luminosity and height of the disc are reduced by the advection effect. In the case of a large viscosity parameter, $\alpha > 0.03$, the accretion flow strongly deviates from thermodynamic equilibrium and overheats in the central region. With increasing accretion rate, the flow temperature steeply increases, reaches a maximum, and then falls off. The maximum is achieved in the advection dominated regime of accretion. The maximum temperature in the disc around a massive black hole, $M = 10^8 M_\odot$, with $\alpha = 0.3$ is of order $3 \times 10^8$ K. Discs with large accretion rates can emit X-rays in quasars as well as in galactic black hole candidates.

Key words: accretion, accretion discs – black hole physics – hydrodynamics – relativity – radiation mechanisms

1 INTRODUCTION

Black hole accretion discs are believed to be the mechanism of energy release in active galactic nuclei and in some X-ray binaries. Their luminosity is constrained by the Eddington limit, $L_E = 4\pi GMm_p c/\sigma_T$ ($M$ is the black hole mass), and the accretion rate, $\dot M$, is usually expressed in the Eddington units, $\dot m = M c^2/L_E$. The luminosity becomes equal to the Eddington limit at $\dot m_{cr} = \eta^{-1}$, where $\eta$ is the radiative efficiency of the disc. Accretion rates of order $\dot m_{cr}$ are likely to feed the huge energy release in quasars, and $\dot m \sim \dot m_{cr}$ is implied in many models of the soft X-ray excess observed in quasar spectra (e.g. Czerny & Elvis 1987; Dörer et al. 1996; Szuszkiewicz, Malkan & Abramowicz 1996).

According to the standard model (Shakura 1972; Shakura & Sunyaev 1973), the effective temperature of the disc surface scales as $T_{eff} \propto (\dot m/M)^{1/4}$. A special feature of discs with $\dot m \sim \dot m_{cr}$ is that their temperature can be much greater than $T_{eff}$ provided the viscosity parameter $\alpha$ is large enough (Shakura & Sunyaev 1973). In the inner region of such a disc, the inflow time-scale is shorter than the time-scale for relaxation to thermodynamic equilibrium and the accretion flow is overheated. The overheating might be strong enough for a transition from the standard radiation dominated disc to the hot two-temperature ion-pressure supported regime of accretion proposed by Shapiro, Lightman & Eardley (1976). This transition has been discussed in recent works (Liǎng & Wandel 1991; Artemova et al. 1996; Björnsson et al. 1996).

The standard model can be a good approximation only if $\dot m < \dot m_{cr}$. When the accretion rate approaches $\dot m_{cr}$, the main assumption of the model, that the viscously released energy is radiated away locally, becomes inadequate. Inside some radius $r_t$, the produced radiation is trapped by the flow and advected eventually to the black hole instead of being radiated away. This radius can be estimated by taking the internal energy density, $\varepsilon$, and the density, $\rho$, of the disc from Shakura & Sunyaev (1973). Then one finds that the radial flux of internal energy $= M \varepsilon/\rho$ exceeds the total flux of radiation emitted between $r$ and $2r$ at

$$ r_t \approx \tilde{m} r_g \sqrt{1 - \left(\frac{3 r_g}{r_t}\right)^{1/2}} , $$

where $r_g = 2GM/c^2$ is the gravitational radius. For $\dot m = \dot m_{cr} = 12$ equation (1) yields $r_t \approx 7.1 r_g$. A substantial portion of the released energy must be swallowed by the black hole even when $\dot m < \dot m_{cr}$. On the other hand, the relative height of the Shakura-Sunyaev disc, $H/r$, equals $\dot m/27$ at the maximum. So, advection becomes essential before the accretion flow becomes quasi-spherical, and this effect can be investigated in an extended version of the standard model retaining the vertically integrated approximation. The corresponding set of equations was proposed by Paczyński & Bisnovatyi-Kogan (1981). Numerical solution of these equations gives adequate description for the inner transonic edge of the accretion flow and shows that the disc remains relatively thin at moderately super-Eddington accretion rates (Abramowicz et al. 1988; Chen & Taam 1993). This made natural the extension of the standard model to the super-Eddington advection dominated regime, called ”slim” accretion disc.

All the models of the super-Eddington slim disc employed the pseudo-Newtonian approximation to the black
hole gravitational field (Paczyński & Wiita 1980) which is
good only in the case of Schwarzschild black hole. Recently,
the equations of relativistic slim disc has been derived by
Lasota (1994, see also Abramowicz et al. 1996) and applied
to another type of advection dominated accretion flow, op-
tically thin and hot.

In the present paper, we solve the relativistic equations
of the super-Eddington disc. In the case of a rapidly ro-
tating black hole relativistic effects become especially im-
portant. The sonic radius is typically inside the ergosphere,
close to the black hole horizon. The released power steeply
increases when the hole spin approaches its extreme value,
\( a_* = Jc/M^2G \rightarrow 1 \) (\( J \) is the angular momentum of the black
hole). In this paper, we calculate the disc structure for the
case \( a_* = 0.998 \) and compare it with the case of non-rotating
black hole \( a_* = 0 \).

We pay particular attention to the case of a large vis-
cosity parameter, \( \alpha \). In the previously investigated pseudo-
Newtonian models, the super-Eddington disc was assumed to
be in thermodynamic equilibrium which is a good ap-
proximation in the case of small \( \alpha \). We find that a signifi-
cant deviation from the equilibrium occurs when \( \alpha \gtrsim 0.03 \).
Then the flow overheats in the central region. We find that
the temperature of the flow steeply rises at \( \tilde{m} > \tilde{m}_{\text{eq}} \) and
reaches a maximum in the advection dominated regime of
accretion. The maximum temperature is especially high in
the case of a rapidly rotating black hole.

In next Section, we write down the equations of the rel-
ativistic advective disc. The equations describe a radiation
-dominated flow and neglect the thermal pressure of the ac-
creting plasma. Their solution yields the radiation density,
\( \Sigma \), surface rest mass density \( \Sigma \), and

\( \Sigma = \frac{\dot{M}}{2\pi} \left( \frac{M^2 c^2}{2 G} + 2 \nu \Sigma r \sigma^2 \right) = \frac{F^-}{c^2} r u_\phi, \)

where \( \nu \) is the kinematic viscosity, and \( \sigma^2 \) is the shear,
\( \sigma^2 = \frac{1}{2} g^{\alpha\beta} g_{\rho\varphi} \sqrt{-g^{\mu\nu}} \gamma^2 \frac{d\Omega}{dr} \).

iii) First law of thermodynamics

\( F^+ - F^- = c u^{\alpha} \left( \frac{d\Pi}{dr} - \frac{\Pi + P}{\Sigma} \frac{d\Sigma}{dr} \right), \)

where \( \xi \approx 1 \) is a numerical factor accounting for non-
homogeneity of the disc in the vertical direction, hereafter
we set \( \xi = 1 \). The surface heating rate is given by

\( F^+ = 2 \nu \Sigma \mu \sigma^2 c^2, \quad \sigma^2 = \frac{1}{2} g^{\alpha\beta} g_{\rho\varphi} \left( -g^{\mu\nu} \right)^2 \frac{d\Omega}{dr} \).

iv) Radial Euler equation

\[
\begin{align*}
\frac{1}{2} \frac{d}{dr} (u_r u_r') &= -\frac{1}{2} \frac{\partial g_{\varphi\varphi}}{\partial r} (\Omega - \Omega_K^{+}) (\Omega - \Omega_K^{-})
\frac{1}{c^2 \Sigma \mu} \frac{dP}{dr} F^+ \frac{u_r}{c^2 \Sigma \mu},
\end{align*}
\]

where \( \Omega_K^{+} \) are the Keplerian angular velocities,
\( \Omega_K^{+} = \pm \frac{c}{r(2r/r_g)^{1/2}} \pm a \).

The set of disc structure equations becomes closed when
the viscosity, \( \nu \), and radiative cooling, \( F^- \), are specified. The
standard \( \alpha \)-prescription for viscosity is \( \nu = \alpha c H \), where \( \alpha \)
is a constant, \( c_s = c(P/U)^{1/2} \) is the isothermal sound speed.
The half-thickness of the disc, \( H \), should be estimated from
the vertical balance condition.

v) Vertical balance

Near the black hole, relativistic effects become import-
ant and the tidal force contracting the disc in vertical di-
rection depends on \( \Omega \). At \( \Omega = \Omega_K^{+} \), the vertical tidal acer-
lation in the comoving tetrad equals (e.g. Riffert & Herold
1995)

\[
a_{(z)} = \frac{z r_g (2r - a + \sqrt{2r_g r} + 0.75a^2/r_g^2)}{r^3 (2r^2 - 3r_g r + a r_g + \sqrt{2r_g r})} = \frac{z r_g}{2r^3} J(a_*, r),
\]

where \( z = \sqrt{\bar{g}_{\phi\phi} (\theta - \pi/2) \gamma^2} \) is the height in the disc, \( J \) is
a relativistic correction factor becoming unity at \( r \gg r_g \).
Then the typical half-thickness of the disc can be estimated as

$$H^2 = \frac{P}{U} \frac{2\sigma^3}{r_g J}.$$  \hspace{1cm} (7)

More accurate expression accounts for a deviation $\Delta \Omega = \Omega - \Omega_K$ of the gas rotation from Keplerian (Abramowicz, Lanza & Percival 1997). We will consider only the disc region outside the sonic radius where the correction to estimation (7) connected with $\Delta \Omega$ does not exceed several percent, and hereafter we use this estimation.

vi) The radiative losses

The time-scale for photon diffusion from the interior of the disc to its surface equals $t_D = H\tau_0/c$ where $\tau_0 = \sigma T \Sigma/2m_p$. This is a typical leaking time for the radiation trapped inside the disc, and the radiative losses can be written as

$$L^- = \chi \frac{m_u c \Pi}{\sigma T \Sigma H},$$  \hspace{1cm} (8)

where $\chi \sim 1$ is a numerical factor. In the standard model with the vertical structure governed by the radiation diffusion this factor equals $2/\sqrt{3}$. In the advection dominated region, $t_D$ exceeds the inflow time-scale $t_a$ and a detailed treatment of the stationary diffusion is not relevant: $t_a < t_D$ means that the gas accretes faster than a stationary vertical distribution of the trapped radiation could establish. However, the estimation (8) gives the weight limit $F^- \ll F^+$ at $r \ll r_t$. Indeed, when $t_a < t_D$ we have $\Pi \approx F^+ t_a$, hence $F^-/F^+ \approx t_a/t_D \ll 1$. The radiative losses are "turned off" inside $r_s$, and the exact value of $F^-$ is not important for hydrodynamical behaviour of the flow. A detailed calculation of $F^-$ would require 2D simulation of the radiation diffusion in the disc with a specified vertical distribution of the viscous energy release.

For definiteness we hereafter choose $\chi = 2/\sqrt{3}$ in equation (8).

vii) Global energy conservation

The luminosity of the disc is related to the accretion rate by (Beloborodov et al. 1997)

$$L^- = \frac{2\pi}{c} \int_{r_s}^{r_\infty} u_\phi F^- r dr \approx \frac{M c^2}{(1 + \mu_s u_\phi^2/c)},$$  \hspace{1cm} (9)

where index "s" refers to the inner transonic edge of the disc. The advection effect results in that the luminosity is less than the total power released in the disc,

$$L^+ = \frac{2\pi}{c} \int_{r_s}^{r_\infty} u_\phi F^+ r dr.$$

The power "swallowed" by the black hole equals $L_{adv} = L^+ - L^-$.  

3 NUMERICAL SOLUTION

To solve numerically the disc structure equations, we choose three independent variables $\Omega, c_s$, and $\zeta = \mu(2\nu \Sigma r \sigma_\varphi + \dot{M} u_\phi/2\pi)$. From equations (3),(4) we have

$$\frac{d\zeta}{dr} = \frac{F^-}{c^2} ru_\varphi,$$  \hspace{1cm} (10)

$$\frac{P}{\Sigma^2} \frac{d\Sigma}{dr} - \frac{3}{d} \left( \frac{P}{\Sigma} \right) = \frac{2\pi r c}{M} (F^+ - F^-).$$  \hspace{1cm} (11)

Expressing $u_\varphi, F^\pm, P, \Sigma$ in terms of $\Omega, c_s, \zeta$ we get two differential equations for the three unknowns. The third differential equation is the radial equation (5). We solve the equations (5,10,11) with external boundary conditions $c_s^{\text{out}}, \Omega^{\text{out}}, \zeta^{\text{out}}$ at a radius $r_{\text{out}}$ such that $r_t < r_{\text{out}} < r_{eq}$. The values of $c_s^{\text{out}}$ and $\Omega^{\text{out}} = \Omega_K^{\pm}$ are taken from the relativistic version of the standard model (Novikov & Thorne 1973; Page & Thorne 1974) with the corrected vertical balance (Riffert & Herold 1995). The value of $\zeta^{\text{out}}$ is the eigen value of the problem, see below.

We use the relaxation technique. As first approximation we assume $\Omega(r) = \Omega_K^+(r)$ and solve equations (10,11) for $c_s, \zeta$. Substituting the obtained solution to the radial equation (5), we calculate $\Omega'(x)$ needed to fulfill this equation. Then we perform the relaxation step changing $\Omega(r)$ to $\Omega(r) + \epsilon \Omega'(r)$ where $\epsilon = \Omega' - \Omega$ and $\epsilon < 1$ is the relaxation parameter. When new $\Omega(r)$ we solve again equations (10,11), and so on until $\Omega'(r)$ converges to zero at all $r$. This method is numerically unstable unless some smoothing procedure is used to suppress numerical oscillations in $\Omega'(r)$. We have found the needed procedure as a combination of two-point and three-point smoothing, and determined $\Delta \Omega = \Omega - \Omega_K^{\pm}$ with an accuracy of $\sim 0.1\%$.

This technique allows to obtain a solution of equations (5,10,11) for any given $\zeta^{\text{out}}$. Then we adjust $\zeta^{\text{out}}$ to fulfill the regularity condition at the sonic radius, $r_s$, and find the transonic solution. The regularity condition can be written as $N = D = 0$ where $N$ and $D$ are the numerator and denominator in the radial equation with explicitly expressed derivative of velocity, $d u'/dr = N/D$. In this way, we find $r_s$ with an accuracy of $\sim 1\%$ and calculate the disc structure at $r > r_s$ (inside $r_s$ the gas is almost in free fall).

Example solutions outside $r_s$ are shown in Fig. 1 for the cases of $\alpha = 0$ and $\alpha = 0.998$ (the represented magnitudes are independent of the black hole mass). In these models $\alpha = 0.1$. To a good accuracy, solutions with different $\alpha$ can be obtained from the solutions with $\alpha = 0.1$ using a simple scaling law: $\Sigma' = \Sigma \alpha/\alpha', P' = P \alpha/\alpha'$, with the same $H(r)$, $\Delta \Omega(r)$, and $c_s(r)$. Essential deviation from this law appears only when $\alpha \rightarrow 1$. Then the sonic radius moves outwards being beyond the radius of the Keplerian marginally stable orbit, $r_{ms}$. At small $\alpha$, $r_s < r_{ms}$.

The relativistic version of the standard model is shown by the dashed lines in Fig. 1. The accretion flow deviates from this model at the trapping radius, $r_t$. Inside $r_t$, the condition $Mc^2 \mu > 2\pi r^2 F^+$ takes place which means that the radial flux of internal energy exceeds the local radiative losses, i.e., the flow is advection dominated. The trapping radius is practically independent of $\alpha$, and the dependence on $m$ is given in Fig. 3. From Fig. 1 one can see that: 1) the advection effect reduces the height, $H$, below the standard value, so that the relative disc height, $H/r$, is kept modest even at $m \gtrsim 10 m_{ch}$. 2) The Thomson optical depth of the disc, $\tau_0$, has a minimum of $\sim 100 (\alpha/0.1)^{-1}$ at $m \sim m_{ch}$, and 3) the maximum deviation from Keplerian rotation is $\sim 20\%$. 

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The structure of the disc around a black hole with $a_* = 0$ (a–c) and $a_* = 0.998$ (d–f) in the case of viscosity parameter $\alpha = 0.1$. The upper panels show the relative height of the disc, the panels in the middle show the Thomson optical depth of the flow, and the bottom panels show the deviation from Keplerian rotation. All these quantities are independent of the black hole mass. Curves with different numbers correspond to different accretion rates: 1 – $\dot{m} = \dot{m}_{cr}$ ($\dot{m}_{cr} = 17.5$ in the case of $a_* = 0$ and $\dot{m}_{cr} = 3.11$ in the case of $a_* = 0.998$), 2 – $\dot{m} = 100$, 3 – $\dot{m} = 1000$. The dashed lines display the relativistic standard model.

The critical accretion rate $\dot{m}_{cr} = 17.5$ in the case of $a_* = 0$ and $\dot{m}_{cr} = 3.11$ in the case of $a_* = 0.998$. At $\dot{m} > \dot{m}_{cr}$ a substantial fraction of the total energy released in the disc is advected into the black hole, see Fig. 2.

Note that in the standard disc with $\dot{m} < \dot{m}_{cr}$ the density, $n = \Sigma/2H\rho$, scales as $\dot{m}^{-2}$. In the advection dominated regime ($\dot{m} > \dot{m}_{cr}$, $r \ll r_\text{g}$), the density scales as $\dot{m}$.

It means that at $\dot{m} \sim \dot{m}_{cr}$ there must be a minimum of the density. At this minimum a strong overheating of the flow is possible as discussed in next Section.
w free emission capability, i.e., the flow is far from the black body state. 

If the heating rate, \( \dot{\alpha} \), exceeds the heating rate, \( \dot{\alpha}_{\text{eff}} \), estimated from the condition between the black body and the overheated regions can be expressed as

\[ \dot{\alpha} = \dot{\alpha}_{\text{eff}} \]

where \( T \) and \( \dot{\alpha} \) are temperature and heating rate, respectively. Then the radiation is concentrated in the inner region of the disc.

A strong deviation from the equilibrium occurs in the inner region of the disc. In this case, the accreting plasma has a low density and therefore, a low emission capability. As a result, the plasma does not manage to reprocess the released energy into Planckian radiation, and the balance between heating and heat-removal through radiative cooling maintains a temperature \( T \) less than the corresponding Planckian value, \( T_{\text{eff}} \). Then the radiation is concentrated in the Wien peak of temperature \( T \) and its density, \( w \), is much less than the corresponding Planckian value, \( \rho_{\text{pl}} = \rho_{\text{pl}} T^4 \), i.e., the flow is far from the black body state.

\[ a) \text{Radius of the overheated region} \]

In the outer "black body" region of the disc, the free-free emission capability, \( \dot{w}_{\text{ff}} \), of the plasma with \( T = T_{\text{eff}} \) exceeds the heating rate, \( \dot{w}^+ = F^+/2H \). The boundary \( r \) between the black body and the overheated regions can be estimated from the condition

\[ \dot{w}^+ = \dot{w}_{\text{ff}}(T_{\text{eff}}) \quad \text{at} \quad r = r_*, \]

\[ \dot{w}_{\text{ff}} = 1.6 \times 10^{-27} n^2 \sqrt{T}. \]

At exact thermodynamic equilibrium, free-free emission would be balanced by free-free absorption, and \( \dot{w}_{\text{ff}} \) can be written as \( \dot{w}_{\text{ff}} = c n \sigma_{\text{ff}} w_{\text{pl}} \), \( \sigma_{\text{ff}} \) introduced in this way has the meaning of an effective cross section for absorption of the Planckian radiation; it is \( 4 \) times the Rosseland averaged \( \sigma_{\text{ff}} \).

Radius \( r_* \) is usually estimated from the condition that the effective optical depth of the flow \( \tau_* = (\tau_{\text{ff}})^{1/2} = 1 \) where \( \tau_{\text{ff}} = H_{\text{eff}} \sigma_{\text{ff}} \) (Rosseland \( \sigma_{\text{ff}} \) is also often taken instead of \( \sigma_{\text{ff}} \)). For the standard disc this condition is equivalent to equation (12). In that case, the radiation density inside the disc equals \( w \approx \dot{w}^+ t_D \) where \( t_D = \tau_0 H/c \) is the diffusion time-scale. Then equality (12) with \( w \approx w_{\text{pl}} \) gives \( \tau_* \approx 1 \), and the decoupling of \( w \) below \( w_{\text{pl}} \) happens if \( \tau_* < 1 \). The condition \( \tau_* < 1 \) means that a deviation from thermodynamic equilibrium in the standard disc occurs if the bulk of radiation diffuses out without absorption and re-emission.

This condition, however, is not relevant in the advection dominated regime. As the radiation is advected rather than escape, the time-scale for free-free absorption should be compared with the inflow time-scale rather than with the diffusion time-scale. Then the condition \( \tau_* < 1 \) should be replaced by \( \tau_* < (t_D/t_\text{in})^{1/2} \sim r_i/r \) that follows from equation (12).

We have calculated \( r_* \) from equation (12) for the case of \( \alpha = 0.3, M = 10^5 M_\odot, \alpha_\star = 0.998 \). In Fig. 3, \( r_* \) is shown versus \( \dot{m} \) and compared with the trapping radius, \( r_t \). One can see that the entire advection dominated region is overheated at \( \dot{m} < 300 \).

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**Figure 2.** The fraction of the total power released in the disc that is eventually advected into the black hole (see item vii of Section 2). Curves 1 and 2 correspond to \( \alpha_\star = 0 \) and \( \alpha_\star = 0.998 \) respectively.

**Figure 3.** The radius of the overheated region \( r_* \) versus \( \dot{m} \) in the case of \( \alpha = 0.3, M = 10^5 M_\odot, \alpha_\star = 0.998 \). For comparison the trapping radius, \( r_t \), is also plotted.
The overheated plasma inside $r_*$ is cooled by Comptonized free-free emission with rate
\[ \dot{w}_{\text{cool}} = A \dot{w}_{\text{ff}}, \quad r \ll r_*, \]
(13)
where $A(n, T)$ is an amplification factor due to Compton upscattering of photons emitted at low energies $h\nu < kT$. This factor is given by (e.g. Rybicki & Lightman 1979)
\[ A = 1 + \frac{3}{4} \ln^2 x_{\text{coh}}, \]
(14)
where $x_{\text{coh}} = h\nu_{\text{coh}}/kT$ is the dimensionless photon energy below which the absorption time-scale is shorter than the time-scale for shifting in frequency due to Comptonization. $x_{\text{coh}}$ is determined by the equation $\sigma_{\text{ff}}(x_{\text{coh}}) = 8\theta\sigma_T$, where $\theta = kT/mc^2$, and $\sigma_{\text{ff}}(x) = 1.8 \times 10^{-33} \ln(2.25/x)(1 - e^{-x})x^{-3/2}n\sigma_T$. Expression (14) for the Compton enhancement factor assumes that photons with $x > x_{\text{coh}}$ upscatter to the Wien peak before they can escape the disc or advect to the black hole. The time-scale for upscattering to the Wien peak is $t_C = \ln(x_{\text{coh}}^{-1})/8\eta\sigma_T$. Hence, $t_C/t_D \sim \ln(x_{\text{coh}})/y$ where $y = 46\tau_0$ is the Kompaneets’ parameter. In the considered situation, $y \gg 1$ and photons with $x > x_{\text{coh}}$ do comptonize to the Wien peak before they can escape. Inside the trapping radius, $t_C$ should be compared with $t_a$. We check in the calculated models that $t_C$ is shorter than $t_a$ as well.

c) Heating=cooling balance

Everywhere in the disc the plasma temperature is determined by the heating=cooling balance $\dot{w}^+ = \dot{w}_{\text{cool}}$. Inside $r_*$, $\dot{w}_{\text{cool}}$ is given by equation (13). Outside $r_*$, the cooling rate is equal to a difference between free-free emission and absorption, being proportional to a small deviation of $w_{\text{pl}}$ from $w$: $\dot{w}_{\text{cool}} = \dot{w}_{\text{ff}}(1 - w/w_{\text{pl}})$. The transition at $r \sim r_*$ can be smoothly described by the following interpolation for the heating=cooling balance,
\[ \dot{w}^+ = \dot{w}_{\text{ff}} \left[ \frac{w}{w_{\text{pl}}} + A \left( 1 - \frac{w}{w_{\text{pl}}} \right) \right] \left( 1 - \frac{w}{w_{\text{pl}}} \right). \]
(15)
In the limit $w \approx w_{\text{pl}}$ this equation yields $\dot{w}^+ = \dot{w}_{\text{ff}}(1 - w/w_{\text{pl}})$ while in the limit $w \ll w_{\text{pl}}$ it transforms into $\dot{w}^+ = A\dot{w}_{\text{ff}}$.

d) Results

Using the balance (15), we have calculated the temperature in the disc around a massive black hole $M = 10^8M_\odot$ with spin $a_* = 0$ and $a_* = 0.998$. The results are shown in Fig. 4, 5 for the case of $\alpha = 0.3$. The strong overheating, $T \gg T_{\text{eff}}$, occurs inside the radius $r_*$. The maximum temperature is reached in the innermost region. It is $\sim 3 \times 10^8$ K in the case of a rapidly rotating black hole and $\sim 10^7$ K in the case of a Schwarzschild black hole. In Fig. 6, 7 we show the dependence of the maximum temperature on the accretion rate in the cases of $a_\alpha = 0.03$ and $a_\alpha = 0.3$.

A question of interest is whether the protons can be thermally decoupled from the electrons at such temperatures. The time-scale for Coulomb energy exchange between protons of temperature $T_p$ and electrons of temperature $T_e$...
is given by (Landau & Lifshitz 1981)

\[ t_{ep} = \sqrt{\frac{\pi m_p}{2 m_e} \left( \frac{kT_e}{m_e c^2} \right)^{3/2}} \frac{1}{\ln \Lambda n \sigma T c} \approx 12.5 \frac{T_e^{3/2}}{n}, \]

where \( \ln \Lambda \approx 20 \) is the Coulomb logarithm. Even in the most hot models with \( \alpha = 0.3 \) and \( \alpha_* = 0.998 \), \( t_{ep} \ll t_0 \) and the accreting plasma can be well described in the one-temperature approximation \( T_e \approx T_p \approx T \).

Then we have compared the thermal plasma pressure, \( p_{gas} = 2nkT \), with the radiation pressure, \( p_{rad} = \nu/3 \). Even in the hottest region of the disc with \( \alpha = 0.3 \), the plasma pressure does not exceed \( \sim 10^{-3} p_{rad} \). So, the calculated radiation dominated models are self-consistent. Note, that accretion flows with \( \alpha \to 1 \) are very unlikely (it would imply a turbulence of scale \( H \) with sound speed). Anyway, such flows could not be much hotter because at higher temperatures additional cooling mechanisms become important: Comptonization of cyclotron radiation and collective waves in the plasma, as noted by Shakura & Sunyaev (1973). Cyclotron radiation and collective waves are additional sources of soft photons which can be upscattered to the Wien peak and cool the plasma efficiently. Roughly, the cyclotron source starts to be important when the gyrofrequency, \( \nu_B = eB/2\pi m_e c \), exceeds \( \nu_{coh} \) (a more detailed treatment including higher cyclotron harmonics is given in Gnedin & Sunyaev 1973).

In a similar way, Comptonization of collective waves can be important when \( \nu_{pl} \gtrsim \nu_{coh} \), where \( \nu_{pl} = (ne^3/\pi m_e) \) is the plasma frequency. In case the magnetic field is comparable to the equipartition value \( B_{eq} = (3\pi n)^{1/2} \), the gyrofrequency exceeds \( \nu_{coh} \) in the disc sooner than the plasma frequency does. In Fig. 5, the heavy line shows the temperature at which the gyrofrequency equals \( \nu_{coh} \). The corresponding maximum temperature in the disc is shown by the dashed line in Fig. 7.

5 CONCLUSIONS

We have investigated the relativistic accretion disc around a Kerr black hole with a super-Eddington accretion rate. The disc has a modest relative height, \( H/r < 0.4 \), and a luminosity near the Eddington limit. The bulk of the energy released in the inner region of the disc is advected into the black hole. The angular velocity of rotation deviates from Keplerian up to 20 %. We paid particular attention to the case of a large viscosity parameter \( \alpha > 0.03 \). In this case, the accretion flow deviates from thermodynamic equilibrium and overheats in the central region. The hottest flow has accretion rate \( \dot{m} \sim 3\dot{m}_{cr} \). We have calculated the maximum temperature in the disc around a massive black hole, \( M = 10^8 M_\odot \), with \( \alpha = 0.3 \). For a Schwarzschild black hole it equals \( \sim 10^7 \) K. For a rapidly rotating black hole, the maximum temperature is of order \( \sim 3 \times 10^8 \) K. It far exceeds the effective temperature corresponding to thermodynamic equilibrium. However, it is not large enough for thermal decoupling of the protons and transition to the ion pressure dominated regime of accretion.

The employed vertically integrated model gives approximate characteristics of the bulk of the gas flowing in the disc, and does not describe physical conditions in the upper layers (in the "skin" of the disc) where the spectrum of emerging radiation is formed. To evaluate the spectrum of the super-
Eddington disc a more detailed treatment of the vertical energy transfer is needed which is likely to demand a 2D simulation of the advective region. Besides, the disc spectrum may be strongly affected by a corona activity above the surface. It is clear, however, that the surface of the flow with large $\alpha$ and a large accretion rate must be much hotter than the effective surface temperature, $T_{\text{eff}} = (F^* / 2\sigma)^{1/4}$, and an activity of the corona may make the spectrum only harder. X-ray emission of the innermost region of the super-Eddington disc may help to reconcile the observed quasar spectra with accretion disc models.

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REFERENCES

Abramowicz M.A., Chen X.-M., Granath M., Lasota J.-P., 1996, ApJ, 471, 762
Abramowicz M.A., Czerny B., Lasota J.-P., Szuszkiewicz E., 1988, ApJ, 332, 646
Abramowicz M.A., Lanza A., Percival M.J., 1997, ApJ, 479, 179
Artemova I.V., Björnsson G., Bisnovatyi-Kogan G.S., Novikov, I.D., 1996, ApJ, 456, 119
Beloborodov A.M., Abramowicz M.A., Novikov I.D., 1997, ApJ, 491, 267
Björnsson G., Abramowicz M.A., Chen X., Lasota J.-P., 1996, ApJ, 467, 99
Chen X., Taam R.E., 1993, ApJ, 412, 254
Czerny B., Elvis M., 1987, ApJ, 321, 305
Gnedin Yu.N., Sunyaev R.A., 1973, MNRAS, 162, 53
Dörner T., Riffert H., Staubert R., Ruder H., 1996, A&A, 311, 69
Landau L.D., Lifshitz E.M., 1981, Course of Theoretical Physics: Vol. 10, Physical Kinetics. Pergamon Press, Oxford, p. 174
Lasota J.-P., 1994, in Duschl W.J., Frank J., Meyer F., Meyer-Hofmeister E., Tscharnuter W.M., eds, Theory of Accretion Disks-2. Kluwer, Dordrecht, p. 341
Liang E.P., Wandel A., 1991, ApJ, 376, 746
Misner C.W., Thorne K.S., Wheeler J.A., 1979, Gravitation. Freeman, San Francisco
Novikov, I.D., Thorne, K.S. 1973, in de Witt C., de Witt B.S., eds, Black Holes. Gordon & Breach, New York, p. 343
Paczyński B., Bisnovatyi-Kogan G.S., 1981, Acta Astron., 31, 3
Paczyński B., Wiita P.J., 1980, A&A, 88, 23
Page D.N., Thorne K.S., 1974, ApJ, 191, 499
Riffert H., Herold H., 1995, ApJ, 450, 508
Rybicki G.B., Lightman A.P., 1979, Radiative Processes in Astrophysics. Wiley, New York
Shakura N.I., 1972, Soviet Astron., 16, 756
Shakura N.I., Sunyaev R.A., 1973, A&A, 24, 337
Shapiro S.L., Lightman A.P., Eardley D.M., 1976, ApJ, 204, 187
Szuszkiewicz E., Malkan M.A., Abramowicz M.A., 1996, ApJ, 458, 474

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