Time Reversal MIMO Radar for Multi-targets DOA Estimation

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Abstract. For direction finding problem of multi-target in the multi-input multi-output (MIMO) radar, a multi-target direction of arrival (DOA) estimation method with time reversal (TR) MIMO radar is proposed. Firstly, according to the echo signal matrix of MIMO radar, we use TR theory to establish the echo signal model of TR MIMO radar. Then, the echo signal of TR MIMO is performed with matched filter and vectorized processing on the basis of the waveform diversity of MIMO radar, which extends the virtual aperture of MIMO radar. Finally, the Multiple Signal Classification (MUSIC) algorithm is used to estimate the DOAs, improving the estimation performance greatly. Compared with the conventional MIMO, TR MIMO can effectively improve the echo signal gain and increase the virtual aperture. The simulation results show that TR MIMO can accurately estimate the angles even when the (signal noise ratio) SNR is -20dB, which demonstrates the effectiveness of proposed configuration.

1. Introduction
Since the advent of the MIMO radar, it has been widely concerned and made great progress due to its unique performance advantages in targets detection, direction finding accuracy, anti-interference ability, anti stealth [1]-[2] and so on. It is useful to improve the estimation accuracy and angle resolution of radar system that MIMO radar transmits mutually orthogonal waveforms from transmitters and the signal is then performed with matched filter and vectorized processing in receivers, obtaining the extended virtual matrix. Several kinds of DOA algorithms in MIMO radar are introduced in [3].

The origin of time reversal (TR) is the study of phase conjugation method for optics, also called time inversion. In complex and multi scattering environment, TR technology [4]-[5] based on time symmetry and spatial reciprocity of wave equation in static medium, uses multipath components to coherently stack and performs incoherent superposition of other clutter signals, so as to achieve great space-time aggregation performance. At present, TR technology is widely used in the field of MIMO communication.

In recent years, more and more foreign scholars have combined TR technology with radar detection. The TR technology has been applied to radar in [6]-[8], showing unique performance advantages. TR MIMO radar is firstly extended to target location in [7], deducing the expressions of the deterministic Cramer-Rao bounds (CRB), having higher estimation accuracy in strong clutter environment proved by computer simulation. Compared with the conventional MIMO [9] radar, TR MIMO radars adapt the transmission waveforms to the multipath environment and thus provide a relatively straightforward procedure for adaptive waveform coding [10] and also have higher signal-to-noise ratio (SNR) and...
potential gain for the DOA and velocity estimation, resulting in the improved CBR and DOA estimation performance [11]. However, the internal research on TR technology used for DOA estimation in MIMO radar is relatively small. In [4], TR technology is applied to Ultra Wideband (UWB) MIMO radar, performing DOA and DOD estimation. According to the time domain expression, the paper uses MUSIC algorithm to estimate the DOAs, improving the estimation accuracy under the condition of low SNR. However, the paper does not give a complete signal model of TR MIMO radar based multi-target DOA estimation, which makes the establishment of TR MIMO radar target positioning system unsystematic.

In this letter, according to the echo signal model of MIMO radar and DOA estimation algorithm, combined with the characteristic of TR, a multi-target DOA estimation method with time reversal MIMO radar is proposed. The method can extend the virtual aperture of MIMO system by the echo signal of TR MIMO radar performed with matched filter and vectorized processing. The simulation results verify the authenticity and validity of the method.

2. MIMO Radar Array Signal Model

To establish the echo signal model of TR MIMO radar, we take monostatic MIMO radar system as an example in this paper. The system contains colocated transmit and receive arrays both equipped with uniform linear array, having \( M \) antennas and \( N \) antennas separated by \( d \) (To ensure the uniqueness of the angle, \( d = \lambda / 2 \)), respectively. The aperture and heights of uniform linear arrays are assumed far less than the target distance. The DOA for each target concerning the colocated receiver and transmitter is denoted by \( \theta_k \) (\( k = 1, 2, \ldots, K \)). Assume there are \( K \) far-field targets, which belong to incoherent targets, probed by narrowband orthogonal waveforms. The received signal can be expressed as

\[
X(t) = \sum_{i=1}^{K} a_i(\theta_i) \beta_i e^{j2\pi f_i t} a_i^T(\theta_i) s(t) + W(t)
\]

(1)

Where \( \beta_i \) is the radar cross section fading coefficient, \( f_i \) is Doppler frequency of the \( k \)th target. \( a_i(\theta_i) = [1, \ldots, e^{-j(\pi/2 \sin \theta_i)}]^T \), \( a_i(\theta_i) = [1, \ldots, e^{-j(\pi/2 \sin \theta_i)}]^T \), \( (\cdot)^T \) denotes transpose, and 
\( s(t) = [s_1(l), s_2(l), \ldots, s_K(l)]^T \), \( l = 0, \ldots, L \), \( L \) represent the number of snapshots. \( W(t) \) is an additive Gaussian white noise vector whose mean and covariance matrix is zero and \( \sigma^2 \), respectively. After matched filtering, we can have

\[
D = E[X(t)s^H(t)] = BAA^T + W
\]

(2)

Where \( B = [a_1(\theta_1), \ldots, a_K(\theta_K)] \), \( A = [a_1(\theta_1), \ldots, a_K(\theta_K)] \), \( A = \text{diag}(\beta_1 e^{j2\pi f_1}, \ldots, \beta_K e^{j2\pi f_K}) \), \( (\cdot)^H \) represents conjugate transpose. \( W \) denotes the noise matrix after matched filtering. Thus, after the vectorization operator of the matrix \( D \), a composite data vector can be expressed as

\[
\overrightarrow{D} = \text{vec}(D) = A \circ B\psi + \text{vec}(W)
\]

(3)

\[
A \circ B = [a_1(\theta_1) \otimes a_1(\theta_1), \ldots, a_K(\theta_K) \otimes a_K(\theta_K)]
\]

(4)

Where \( \text{vec}(\cdot) \) is the vectorization operator of matrix, \( \psi = [\beta_1 e^{j2\pi f_1}, \ldots, \beta_K e^{j2\pi f_K}]^T \), \( \circ \) is Kronecker product, and \( \otimes \) is Khatri-Rao product and Kronecker product, respectively.

3. TR MIMO Radar Array Signal Model

In the TR MIMO radar, the output vector \( X(t) \) is energy normalized by factor \( e \), phase conjugated, time reversed, and retransmitted into the medium. Then the TR transmitted signal is \( eX^*(-t) \), and the TR output can be given as

\[
\]
\[ Y(t) = e \sum_{i=1}^{K} a_i(\theta_i) e^{j4\pi f t} X^*(-t) + V(t) \]
\[ = e \sum_{i=1}^{K} [\beta_i e^{j4\pi f t} a_i(\theta_i) a_i^T(\theta_i) s^*] + \tilde{V}(t) \]

Where \( e = \sqrt{\int \|s(t)\|^2 dt / \int \|X(t)\|^2 dt} \), \( (*) \) represents conjugation; \( V(t) \) and \( \tilde{V}(t) \) are the observation noise for the TR stage and the accumulated noise from both the conventional and TR stages, respectively. Similar to the noises \( W(t) \), the noises \( V(t) \) are also spatially and temporally complex Gaussian white processes with zero-mean and variance \( \sigma^2 \). As described in [12], the accumulated noises can be proved as approximately white noise. Thus, after matched filtering, we can obtain

\[ Z = E\left[ Y(t)s^*(-t) \right] = \sum_{i=1}^{K} \beta_i e^{j4\pi f t} a_i(\theta_i) a_i^T(\theta_i) s^* + E[\tilde{V}(t)s^*(-t)] \]
\[ = N \sum_{i=1}^{K} \beta_i e^{j4\pi f t} a_i(\theta_i) a_i^H(\theta_i) + \tilde{V} \]
\[ = N \cdot A A^H + \tilde{V} \]

Where \( N \) are the number of MIMO radar receiving array elements, \( A = [a(\theta_1), \ldots, a(\theta_K)] \), \( A = \text{diag}([\beta_1 e^{j4\pi f t_1}, \ldots, \beta_K e^{j4\pi f t_K}]) \), \( \tilde{V} \) is an additive Gaussian white noise vector whose mean and covariance matrix is zero and \( \sigma^2 \). Similar to the vectorization operator of the matrix \( D \) for MIMO radar, the vector can be expressed as

\[ \tilde{Z} = \text{vec}(Z) = N \cdot \text{vec}(A A^H) + \text{vec}(\tilde{V}) = N \cdot A^* \circ A \eta + \nu \]
\[ A^* \circ A = [a_1(\theta_1) \otimes a_1(\theta_1), \ldots, a_K(\theta_K) \otimes a_K(\theta_K)] \]

Where \( \eta = [\| \beta_1 e^{j4\pi f t_1} \|^2, \ldots, \| \beta_K e^{j4\pi f t_K} \|^2]^T \), \( \nu = \text{vec}(\tilde{V}) \).

4. DOA Estimation Method based TR MIMO Radar

4.1 DOA Estimation Method

Firstly, calculate the covariance matrix of \( \tilde{Z} \)

\[ R_{zz} = E[\tilde{Z} \tilde{Z}^H] \]

In fact, the calculation formula of \( R_{zz} \) is

\[ \hat{R}_{zz} = \frac{1}{L} \sum_{l=1}^{L} \tilde{Z}(l) \tilde{Z}(l)^H \]

Where \( L \) are the sampling times for the virtual matrix, also called the number of snapshots.

Then, Perform eigenvalue decomposition of (10), and we can get

\[ \hat{R}_{zz} = U_\Sigma U_\Sigma^H + U_\Sigma U_\Sigma^H \]

Where \( \Sigma_\eta \) is a diagonal matrix consisting of the largest \( K \) eigenvalues; \( \Sigma_\eta \) stands for a diagonal matrix consisting of the smallest \( M \) \(- K \) eigenvalues; \( U_\Sigma \) and \( U_\Sigma \) are the signal subspace and the noise subspace, respectively. It is well known that the signal subspace is orthogonal to the noise subspace.

Finally, Assume \( a(\theta) = a^\dagger(\theta) \otimes a(\theta) \), using MUSIC algorithm, we can obtain

\[ P_{MUSIC}(\theta) = \frac{1}{a^H(\theta) U_\Sigma U_\Sigma^H a(\theta)} \]

According to (12), seeking the spectrum peak, we can get the DOAs.
4.2 Performance Analysis of the Proposed Method

(1) Estimation accuracy and resolution
Comparing (3) and (7), we can see that after the vectorization operator, TR MIMO radar can produce $N$ times gain. Thus, In the condition that the number of MIMO radar receiver and transmitter elements is properly set up ($M > N$), the DOA estimation accuracy can be improved in TR MIMO radar system when the SNR is low. Where $A \otimes B = [a_i(\theta_1) \otimes a_i(\theta_2), \ldots, a_i(\theta_k) \otimes a_i(\theta_k)]$ is the direction vector matrix of MIMO radar, $A' \otimes A = [a_i'(\theta_1) \otimes a_i'(\theta_2), \ldots, a_i'(\theta_k) \otimes a_i'(\theta_k)]$ is the direction vector matrix of TR MIMO radar. The dimension of $\hat{D}$ is $MN \times 1$, The dimension of $\hat{Z}$ is $M^2 \times 1$, showing that the free degree of MIMO radar is $M + N - 1$, the free degree of TR MIMO radar is $2M - 1$. So in the condition of $M > N$, TR MIMO radar extends the virtual aperture of the conventional MIMO, improving the angular resolution and estimation accuracy greatly.

(2) Computational complexity
The complexity of the proposed method mainly contains calculating the covariance matrix, eigenvalue decomposition and seeking the spectrum peak, so we can analyse from these three aspects. Where $\hat{Z}$ is $M^2 \times 1$ dimensional matrix, $\hat{D}$ is $MN \times 1$ dimensional matrix. Thus, the computational complexity of TR MIMO radar is about $O((M^4L + M^6 + n[M^2(M^2 - K) + M^2 - K])$, however, the computational complexity of the conventional MIMO is about $O((M^4N^2L + M^6N^3 + n[MN(MN - K) + MN - K])$, $n$ is the total search times. Assume $M > N$, the algorithm in this paper has a higher computational complexity.

5. Simulation Results
In the simulation experiment, we construct a MIMO radar system consisting of colocated receive and transmit arrays both equipped with uniform linear array (ULA), having 6 antennas and 8 antennas separated by half a wavelength, respectively. And the array elements transmit mutually orthogonal signals, the number of snapshots is $L = 200$. Three incoherent narrowband signal sources coming from $\theta_1 = -10^\circ, \theta_2 = 20^\circ, \theta_3 = 40^\circ$ are considered. We test the estimation performance by 300 Monte Carlo trials.

Example 1: Compare the estimation of TR MIMO and MIMO radar in the condition of low SNR. We compare the spatial spectrum of TR MIMO and MIMO when the SNR is 10dB and -20dB in Figure.1 and Figure.2, respectively. From Figure.1, in case of SNR=10dB, both TR MIMO and MIMO radar can estimate the DOAs, but the spectrum peaks of TR MIMO are higher. From Figure.2, in case of SNR=-20dB, the MUSIC algorithm of MIMO radar is invalid and can not estimate the DOAs. However, TR MIMO radar is still able to detect the angle of targets, showing the better performance advantages.

![Figure.1](image1.png) The spatial spectrum of TR MIMO and MIMO

![Figure.2](image2.png) The spatial spectrum of TR MIMO and MIMO
Example 2: In this example, we test the estimation performance by the RMSE versus SNR. As we can see from Figure 3, the RMSE of TR MIMO radar is smaller than MIMO radar, and TR MIMO still has a low error when the SNR is -20dB.

![Figure 3](image.png)

The RMSE of TR MIMO and MIMO versus the SNR

Example 3: In this example, we examine the estimation performance by the RMSE versus the number of snapshots. In Figure 4, we assume the SNR is 0dB. In the case of the different number of snapshots, the performance of TR MIMO is better than the conventional MIMO radar. Especially, when the number of snapshots is smaller than 50, the estimation performance of TR MIMO is not affected, showing that the proposed method is still superior performance under relatively low snapshots.

Example 4: In this example, we examine the estimation performance by the RMSE versus the number of arrays. Figure 5 shows the DOA estimation performance with different number of transmitter and receiver antennas. The more the number of transmitter or receiver antennas has, the better the performance of estimation will be gotten for TR MIMO radar.

Example 5: In this example, we examine the angle resolution by the spatial spectrum of TR MIMO and MIMO radar. Three close non-coherent narrowband signal sources coming from $\theta_1=22^\circ$, $\theta_2=25^\circ$, $\theta_3=28^\circ$ are considered. In the case of SNR=10dB, compared with the conventional MIMO radar, TR MIMO can still distinguish three targets in Figure 6, showing the better angle resolution.

![Figure 4](image.png)

The RMSE of TR MIMO and MIMO versus the number of snapshots

![Figure 5](image.png)

DOA estimation performance with different number of sensors

![Figure 6](image.png)

The spatial spectrum of TR MIMO
6. Conclusion
A new TR MIMO radar for multi-target DOA estimation is presented, which is optimized from MIMO radar and TR technology. As evident from the Monte Carlo simulations, in the condition of $M > N$, TR MIMO radar extends the virtual aperture of the conventional MIMO, improving the angle resolution and estimation accuracy greatly, increasing computational complexity.

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