Axial masses in quasielastic neutrino scattering and single-pion neutrino production on nucleons and nuclei.

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We analyse available experimental data on the total charged-current \( \nu N \) and \( \bar{\nu} N \) cross sections for quasielastic scattering and single-pion neutrino production. Published results from the relevant experiments at ANL, BNL, FNAL, CERN, and IHEP are included dating from the end of sixties to the present day, covering \( \nu_\mu \) and \( \bar{\nu}_\mu \) beams on a variety of nuclear targets, with energies from the thresholds to about 350 GeV. The data are used to adjust the poorly known values of the axial masses.

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1. Introduction

It is well known that the theoretical description of the cross sections for CC and NC (quasi)elastic neutrino-nucleon scattering (QES) and single-pion neutrino production through baryon resonances (RES) are very sensitive to the shape of the weak axial-vector elastic and transition form factors. By adopting the standard dipole parametrization for these form factors, their shapes can be described with the two phenomenological parameters \( M_{\text{QES}}^A \) and \( M_{\text{RES}}^A \), the so-called axial (dipole) masses. In general, these masses are different and, moreover, the numerical value of \( M_{\text{RES}}^A \) is vastly dependent of the particular dynamic model for the resonance production.

The experimental values for both \( M_{\text{QES}}^A \) and \( M_{\text{RES}}^A \) coming from measurements of (quasi)elastic neutrino and antineutrino scattering off protons and nuclei and from the more involved and model-dependent analyses...
of charged pion electroproduction off protons, spread within rather wide ranges. In this study we attempt to fine-tune the axial masses by fitting all available data on the CC QES (with $\Delta Y = 0$) and RES $1\pi$ total cross sections for $\nu_{\mu}$ and $\bar{\nu}_{\mu}$ scattering off different nuclear targets from experiments at ANL [1–5], BNL [6, 7], FNAL [8–12] CERN [13–26], and IHEP [27–33]. In our opinion, this procedure is more selfconsistent in comparison with the usual straightforward averaging over the experimental values of the axial mass (see, e.g., Ref. [34]) extracted under different assumptions about the other badly known factors involved into the analyses of each experiment.

2. Axial mass from the data on quasielastic scattering

Figure 1 shows a compilation of the QES data from experiments at ANL [1, 3, 4], BNL [6], FNAL [10–12], CERN [13–17, 20, 26], and IHEP [27–30, 32, 33] performed with a variety of nuclear targets. The cross sections reported in the earlier experiments [1, 3, 13–16] exhibit uncontrollable systematic errors and fall well outside the most probable range determined through the fit to the full dataset of about 200 datapoints; the value of $\chi^2$ evaluated for each subset of these data exceeds $\sim 5$ ndf. Hence, following the (nonstringent) selection criterion $\chi^2/\text{ndf} < 4.5$, they were excluded from the final fit.

For the $\nu n \to \mu^- p$ and $\bar{\nu} p \to \mu^+ n$ cross sections we use the result of Ref. [35] neglecting possible second-class current contributions (see Appendix); under this standard assumption it coincides with that of Strumia and Vissani [36]. For the elastic electromagnetic form factors we apply the QCD VM model of Gari and Krümpelmann [37] extended and fine-tuned by Lomon [38] (“GKex(02S)” version) and the most current inverse polynomial parametrization by Budd et al. [39] (“BBBA2006”) obtained through a global fit to the world data on the Sachs form factors. For the axial and pseudoscalar form factors we use the conventional representations [40]

$$F_A (Q^2) = F_A(0) \left( 1 + \frac{Q^2}{M_A^2} \right)^{-2}, \quad F_P (Q^2) = \frac{2M_N^2}{m_N^2 + Q^2} F_A (Q^2),$$

(1)

with $F_A(0) = g_A = -1.2695 \pm 0.0029$ [41] (assuming $g_V = 1$) and $M_A \equiv M_A^{QES}$ being a free parameter of our fit.

The nuclear effects for the data obtained for deuterium [3, 4, 6, 10, 26] and neon-hydrogen [11] targets were subtracted by the authors of the experiments. Therefore these data are fitted by the cross sections evaluated for free nucleons. To describe the remaining experimental data we apply the relativistic Fermi gas model by Smith and Moniz [43] with the kinematics and values of binding energies and Fermi momenta of the target proton and neutron determined by the composition of each target quoted in Fig. 1.
Fig. 1. Total quasielastic $\nu_\mu n$ and $\nu_\mu p$ cross sections measured for different nuclear targets by the experiments ANL 1969 [1], ANL 1973 [3], ANL 1977 [4], BNL 1981 [6], FNAL 1983 [10], FNAL 1984 [11], NuTeV 2004 [12], CERN BC 1965 [13], CERN HLBC 1966 [14], CERN HLBC 1967 [15], GGM 1973 (Gargamelle, CERN) [16], GGM 1977 [17], GGM 1979 [20], BEBC 1990 (CERN) [26], IHEP SKAT 1981 [27], IHEP 1982 [28], IHEP-ITEP 1985 [29], IHEP SKAT 1988 [30], IHEP SKAT 1990 [32], and IHEP SKAT 1992 [33]. The curves and bands correspond to the world average value of $M_A^{QES} = 0.95 \pm 0.03$ GeV obtained with the GKex(02S) model for the vector form factors from the fit to the subset of these data (160 datapoints). The data of Refs. [1,3,13–16,28] (39 grey datapoints) are rejected from the fit being either superseded or not satisfying our selection criterion $\chi^2/\text{ndf} < 4.5$. 
The resulting world average obtained are

\[ M_{\text{QES}}^{\text{GKex(02S)}} = 0.95 \pm 0.03 \text{ GeV} \quad (\chi^2/\text{ndf} = 0.92), \]  
\[ M_{\text{QES}}^{\text{BBBA2006}} = 0.96 \pm 0.03 \text{ GeV} \quad (\chi^2/\text{ndf} = 0.91). \]  

The errors correspond to the usual one-standard-deviation errors (MINUIT default [42]) plus the systematic errors, added quadratically, which account for the uncertainties in the data on the vector form factors, nuclear effects (within the adopted model) and radiative corrections. The fit performed, for a comparison, with the naive dipole model for the vector form factors yields \( M_{\text{A}}^{\text{QES}} = 0.93 \pm 0.03 \text{ GeV} \) with \( \chi^2/\text{ndf} = 0.95 \).

The obtained world average values of the axial mass are in strong contradiction with the recently published result of the K2K Collaboration [44]:

\[ M_{\text{A}}^{\text{QES}}[\text{K2K}] = 1.20 \pm 0.12 \text{ GeV} \]  

This value has been determined for a water target through fitting the \( Q^2 \) distributions of muon tracks reconstructed from neutrino-oxygen quasielastic interactions by using the combined K2K-I and K2K-IIa data from the Scintillating Fiber detector in the KEK accelerator to Kamioka muon neutrino beam.\(^1\)

In Fig. 2 we show the \( \nu_\mu n \rightarrow \mu^- p \) cross section recalculated from the fit values of \( M_{\text{A}}^{\text{QES}} \) obtained in Ref. [44] for the shape of the \( Q^2 \) distribution for each reconstructed neutrino energy.\(^2\) The calculation was performed with our default inputs that introduces an uncertainty of at most 2\% which is added to the quoted error bars quadratically. Also shown are the cross sections evaluated by using the world average value, the K2K best fit, and the value of 1.1 GeV used as a default in the recent neutrino oscillation analyses of K2K [45,46] and Super-Kamiokande I [47].

\(^1\) Data from the continuation of the K2K-II period were not used in the analysis [44].
\(^2\) The authors underline that the result for each energy should not be considered a measurement, but rather a consistency test.
3. Axial mass from the data on single pion neutrinoproduction

Figures 3-5 show a compilation of the data on single pion neutrinoproduction cross sections from experiments at ANL [2,5], BNL [7], FNAL [8,9], CERN [18,19,21–26], and IHEP [31]. The nuclear targets are listed in the legends. All the data, as well as the theoretical curves, are classified through the panels corresponding to the experimental cut-offs in invariant hadronic mass $W$ ranging from 1.4 to 2.55 GeV and including the measurements without cuts in $W$.

![Fig. 3. Total $\pi^-$ and $\pi^+$ production cross sections measured for different nuclear targets by the experiments ANL 1982 [5], BNL 1986 [7], GGM 1979 [21], BEBC 1983 [23], BEBC 1990 [26], and IHEP SKAT 1989 [31]. The data are classified according to the cuts in $W$. The curves and bands correspond to the world average value of $M_A^{RES} = 1.12 \pm 0.03$ GeV obtained from the fit to a subset (196 points) of the full data presented in this and two next figures (see text for more details).}
Fig. 4. Total $\pi^-$, $\pi^0$, and $\pi^+$ production cross sections measured for different targets by the experiments ANL 1973 [2], ANL 1982 [5], BNL 1986 [7], FNAL 1978 [8], GGM 1978 [19], GGM 1979 [21], BEBC 1980 [22], BEBC 1986 [24], BEBC 1989 [25], BEBC 1990 [26], and IHEP SKAT 1989 [31]. See Fig. 3 and text.
Fig. 5. Total $\pi^0$ and $\pi^-$ production cross sections measured for different targets by the experiments ANL 1982 [5], BNL 1986 [7], FNAL 1980 [9], GGM 1978 [18], GGM 1979 [21], BEBC 1983 [23], BEBC 1986 [24], BEBC 1989 [25], BEBC 1990 [26], and IHEP SKAT 1989 [31]. See Fig. 3 and text.

For the theoretical description of the single-pion neutrino production through baryon resonances we apply an extended version of the Rein-Sehgal (RS) model [48]. Our extension [49] is based upon a covariant form of the charged leptonic current with definite lepton helicity and takes into account the lepton mass. In the present calculations, we use the same set of 18th nucleon resonances with central masses below 2 GeV and the same ansatz for the nonresonance background as in the original RS model. With that, all relevant parameters are updated according to the current data [41]. Significant factors (normalization coefficients etc.) estimated in Ref. [48] numerically are recalculated by using the new data and a more accurate integration algorithm. The relativistic quark model of Feynman, Kislinger,
and Ravnadal [50] adopted in the RS approach unambiguously determines the structure of the transition amplitudes involved into the calculation and the only unknown structures are the vector and axial-vector transition form factors $G^{V,A}(Q^2)$. In the RS model, they are assumed to have the form

$$G^{V,A}(Q^2) \propto \left(1 + \frac{Q^2}{4M_N^2}\right)^{1/2-n} \left(1 + \frac{Q^2}{M_{V,A}^2}\right)^{-2}$$

with the "standard" value of the vector mass $M_V = 0.84$ GeV (that is the same as in the dipole parametrization of the elastic vector form factor). The integer $n$ in the first ("ad hoc") factor of Eq. (5) is the number of oscillator quanta present in the final resonance. The axial mass $M_A = M_A^{RES}$ (fixed to be 0.95 GeV in the RS model) is the free parameter of our fit. In order to compensate for the difference between the experimental value of the nucleon axial-vector coupling $g_A$ and the $SU_6$ predicted value ($g_A(SU_6) = -5/3$), Rein and Sehgal introduced a renormalization factor $Z = 0.75$. For adjusting the renormalization to the current world averaged value $g_A = -1.2695 \pm 0.0029$ [41] we use $Z = 0.762$ and assume $g_V = 1$.

The nuclear effects for all nuclear targets different from hydrogen and deuterium are taken into account through the standard Pauli blocking factor (see, e.g., Ref. [51] and references therein). The estimated relevant uncertainty is taken into account in the fit and in the error of its output.

Almost all the data (196 points) shown in Figs. 3-5 participate in the fit. Several data subsets are excluded since they are superseded in the posterior reports of the same collaborations (e.g., the data from Refs. [2, 23]), or are transformation of the others derived from the same experimental samples (e.g., the data of Refs. [21] with no cut on $W$). Note that all the data included into the fit satisfy the criterion $\chi^2/\text{ndf} < 4.5$. The resulting world average obtained in the fit is

$$M_A^{RES} = 1.12 \pm 0.03 \text{ GeV} \quad (\chi^2/\text{ndf} = 1.14).$$

As in the QES case, the error is the combination of the 1σ deviation given by MINUIT and estimated systematic uncertainties. The obtained world average is in agreement with the recent analysis by Furuno et al. [52] of the BNL 7-foot bubble chamber deuterium data\(^3\) as well as with the value

\(^3\) The analysis of Ref. [52] is based on the total event sample of 1.8 M pictures and holds two periods of runs in 1976-77 and 1979-80. The outputs of the analysis are $M_A^{RES} = 1.08 \pm 0.07$ GeV (statistical error only) – from the fit of the $Q^2$ distributions of $p\pi^+n_s$ events and $M_A^{RES} = 1.15^{+0.08}_{-0.06}$ GeV (both statistical and QES errors are included) – from the $1\pi$ and QES cross sections ratio. The best-fit value of $M_A^{QES}$ obtained in the same analysis assuming the dipole model for the vector form factors (with the standard $M_V$) is $1.07 \pm 0.05$ GeV that is well above our result (see Sect. 2).
$M^\text{RES}_A = 1.1$ GeV (the same as $M^\text{QES}_A$) adopted in the most recent K2K neutrino oscillation analysis [46] but considerably lower than the value $M^\text{RES}_A = 1.2$ GeV used for the atmospheric neutrino analysis of the Super-Kamiokande Collaboration [47].

Figure 6 shows a comparison between our calculations and the result of Ref. [52] for the ANL and BNL data on the ratios of the one-nucleon normalized $1\pi$ and QES $\nu_\mu D_2$ cross sections (calculated and measured with no cut on $W$). Being transformations of the others, these data are not included into the fit. The narrow bands indicate the uncertainties in the values of the axial masses (2) and (6). The agreement is reasonably good.

**Fig. 6.** The ratios of $1\pi$ and QES cross sections evaluated in Ref. [52] from the data of ANL and BNL deuterium experiments. The curves and bands are calculated with the world average values (2) and (6) for $M^\text{QES}_A$ and $M^\text{RES}_A$, respectively.

4. Conclusions

To summarise, we performed a statistical study of the QES and $1\pi$ neutrino-production total cross section data in order to extract the best-fit values of the parameters $M^\text{QES}_A$ and $M^\text{RES}_A$. Our results given by Eqs. (2), (3), and (6) are, of course, model dependent and can be recommended for use only within the same (or numerically equivalent) model assumptions as in the present analysis. We are planning to extend the analysis by employing more sophisticated treatments of the nuclear effects and including additional experimental information.

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Appendix

The most general formula for the QES $\nu N$ cross section is

$$\frac{d\sigma_{\text{QES}}}{dQ^2} = \frac{G_F^2 \cos^2 \theta_C M^2}{2\pi E_\nu^2} \left(1 + \frac{Q^2}{M_W^2}\right)^{-2} \left[A + \left(\frac{s-u}{4M^2}\right) B + \left(\frac{s-u}{4M^2}\right)^2 C\right],$$

where $s = (k+p)^2$, $u = (k' - p)^2$, $Q^2 = -q^2$; $k$, $k' = k - q$, and $p$ are the 4-momenta of (anti)neutrino, final lepton, and initial nucleon, respectively; the coefficient functions $A$, $B$, and $C$ are given by

$$A = 2 \left[(x' + r^2)(2x' + x^2) - x^4\right] \text{Re} (F_V^s F_M^s)$$

$$-4x^2 \left\{r \text{Re} [F_A^s (F_V + F_M)] + (x' + r^2 + x^2) \text{Re} (F_A^s F_P)\right\}$$

$$+ \left[(x' + x^2)(x' - 1 + r^2 - x^2) - r^2\right] |F_V|^2$$

$$+ \left[(x' + x^2)(x' + 1 - r^2 - x^2) - r^2\right] |F_A|^2$$

$$- [x'(x' + r^2)(x' - 1 + x^2) + x^4] |F_M|^2$$

$$+ 4x^2 (x' + x^2)(x' + r^2) |F_M|^2$$

$$\pm 4r (x' + r^2) \left[(x' + 1 + x^2) \text{Re} (F_T^s F_A) + 2x^2 \text{Re} (F_T^s F_P)\right]$$

$$\pm 4r x^2 \left[(x' + 1 + x^2) \text{Re} (F_T^s F_V) + x^2 \text{Re} (F_T^s F_M)\right]$$

$$-4 (x' + r^2) \left[(x' + x^2)(x' + 1 + r^2) + r^2\right] |F_T|^2$$

$$+ 4x^2 (x' + 1)(x' + x^2) |F_S|^2,$$

$$B = \pm 4x \text{Re} [F_A^s (F_V + F_M)] \pm 2r x^2 \left[|F_M|^2 + \text{Re} (F_V^s F_M + 2F_A^s F_P)\right]$$

$$+ 4x^2 \text{Re} \left\{F_T^s \left[F_A^s - 2(x' + r^2) F_P\right] - F_S^s \left(F_V - x' F_M\right)\right\},$$

$$C = |F_V|^2 + |F_A|^2 + x'|F_M|^2 + 4r \text{Re} (F_T^s F_M) + 4(x' + r^2) |F_T|^2,$$

with the upper (lower) signs corresponding to neutrino (antineutrino) scattering. The six form factors $F_i$ involved are functions of $Q^2$;

$$x = \frac{Q^2}{2(pq)}, \quad x' = \frac{Q^2}{4M^2}, \quad \varkappa = \frac{m}{2M}, \quad r = \frac{M_n - M_p}{2M}, \quad M = \frac{M_p + M_n}{2},$$

and the remaining notation is standard. In the limit $M_n = M_p$, the general formula reduces to that of Ref. [40] and by putting $F_S = F_T = 0$ it coincides with the result of Ref. [36] derived for the inverse $\beta$ decay, taking account the proton-neutron mass difference. In this paper, we apply the Standard Model assumptions ($T$ and $C$ invariance + CVC), thus neglecting the scalar and tensor form factors $F_{S,T}$ induced by the second-class currents, as well as the imaginary parts of the first-class form factors $F_{V,M,A,P}$.

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4 In fact, the latter effect is insignificant for the present analysis, although it is included for completeness, together with the exact kinematics.
REFERENCES

[1] R.L. Kustom et al., Phys. Rev. Lett. 22, 1014 (1969).
[2] J. Campbell et al., Phys. Rev. Lett. 30, 335 (1973).
[3] W.A. Mann et al., Phys. Rev. Lett. 31, 844 (1973).
[4] S.J. Barish et al., Phys. Rev. D16, 3103 (1977).
[5] G.M. Radecky et al., Phys. Rev. D25, 1161 (1982); erratum – *ibid*. D26, 3297 (1982).
[6] N.J. Baker et al., Phys. Rev. D23, 2499 (1981).
[7] T. Kitagaki et al., Phys. Rev. D34, 2554 (1986).
[8] J. Bell et al., Phys. Rev. Lett. 41, 1008 (1978).
[9] S.J. Barish et al., Phys. Lett. 91B, 161 (1980).
[10] T. Kitagaki et al., Phys. Rev. D28, 436 (1983).
[11] A.E. Asratyan et al., Yad. Fiz. 39, 619 (1984) [Sov. J. Nucl. Phys. 39, 392 (1984)]; Phys. Lett. B137, 122 (1984).
[12] N. Suwonjandee, “The measurement of the quasi-elastic neutrino-nucleon scattering cross section at the Tevatron,” Ph. D. Thesis, University of Cincinnati, Cincinnati, 2004; FERMILAB-THESIS-2004-67 (Illinois, February 2004).
[13] H. Burmeister et al., in Proc. of the Informal Conf. on Experimental Neutrino Physics (CERN, Geneva, January 20-22, 1965), edited by C. Franzinetti, CERN Yellow Report 65-32 (Geneva, 1965), p. 25.
[14] C. Franzinetti, a lecture at the Chicago Meeting of the American Physical Society, (Chicago, October 28, 1965), CERN Report 66-13 (Geneva, 1966).
[15] E.C.M. Young, CERN Yellow Report 67-12 (Geneva, 1967).
[16] T. Eichten et al., Phys. Lett. B46, 274 (1973).
[17] S. Bonetti et al., Nuovo Cim. A38, 260 (1977).
[18] W. Krenz et al., Nucl. Phys. B135, 45 (1978); the datapoint is recalculated by E.A. Hawker, a talk at the 2nd International Workshop on Neutrino-Nucleus Interactions in the few-GeV Region (“NuInt’02”) (University of California, Irvine, December 12–15, 2002).
[19] W. Lerche et al. Phys. Lett. 78B, 510 (1978).
[20] N. Armenise et al., Nucl. Phys. B152, 365 (1979); M. Pohl et al., Lett. Nuovo Cim. 26, 332 (1979).
[21] T. Bolognese et al., Phys. Lett. 81B, 393 (1979); T. Bolognese, “Etude des interactions d’antineutrinos avec production d’un pion en courant charge,” Ph.D. Thesis, University of Strasbourg, Strasbourg, 1978; CRN/HE 78-22. The data are converted to free nucleon following the prescription of Ref. [23].
[22] P. Allen et al. Nucl. Phys. B176, 269 (1980).
[23] D. Allasia et al. Z. Phys. C20, 95 (1983).
[24] P. Allen et al. Nucl. Phys. B264, 221 (1986).
[25] G.T. Jones et al. Z. Phys. C43, 527 (1989).
[26] D. Allasia et al., *Nucl. Phys.* B343, 285 (1990).
[27] V.V. Makeev et al., *Pisma Zh. Eksp. Teor. Fiz.* 34, 418 (1981) [JETP Lett. 34, 397 (1981)].
[28] S.V. Belikov et al., *Yad. Fiz.* 35, 59 (1982) [Sov. J. Nucl. Phys. 35, 35 (1982)].
[29] S.V. Belikov et al., *Z. Phys.* A320, 625 (1985).
[30] H.J. Grabosch et al., *Yad. Fiz.* 47, 1630 (1988) [Sov. J. Nucl. Phys. 47, 1032 (1988)].
[31] H.J. Grabosch et al., *Z. Phys.* C41, 527 (1989).
[32] J. Brunner et al. *Z. Phys.* C45, 551 (1990).
[33] V.V. Ammosov et al., *Fiz. Elem. Chast. Atom. Yadra* 23, 648 (1992) [Sov. J. Part. Nucl. 23, 283 (1992)].
[34] V. Bernard, L. Elouadrhiri, Ulf-G. Meißner, *J. Phys.* G28, R1 (2002).
[35] K.S. Kuzmin, V.V. Lyubushkin, V.A. Naumov, *Nucl. Phys. B* (Proc. Suppl.) 139, 154 (2005) [hep-ph/0408107].
[36] A. Strumia, F. Vissani, *Phys. Lett.* B564, 42 (2003) [astro-ph/0302055].
[37] M.F. Gari, W. Krüempelmann, *Phys. Lett.* B274, 159 (1992); erratum – ibid. 282, 483 (1992).
[38] E.L. Lomon, *Phys. Rev.* C66, 045501 (2002) [nucl-th/0203081].
[39] R. Bradford, A. Bodek, H. Budd, J. Arrington, *Phys. Rept.* 3C, 261 (1972).
[40] S. Eidelman et al. (Particle Data Group), *Phys. Lett.* B592, 1 (2004).
[41] F. James, “MINUIT, Reference Manual, Version 94.1,” CERN Program Library Long Writeup D506 (Geneva, 1994).
[42] R.A. Smith, E.J. Moniz, *Nucl. Phys.* B43, 605 (1972); erratum – ibid. B101, 547 (1975).
[43] R. Gran et al., *Phys. Rev.* D69, 051106 (2004) [hep-ex/0408104].