Magnetic connection and current distribution in black hole accretion discs

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ABSTRACT

We discuss one of the possible origins of large-scale magnetic fields based on a continuous distribution of toroidal electric current flowing in the inner region of the disc around a Kerr black hole (BH) in the framework of general relativity. It turns out that four types of configuration of the magnetic connection (MC) are generated, i.e., MC of the BH with the remote astrophysical load (MCHL), MC of the BH with the disc (MCHD), MC of the plunging region with the disc (MCPD) and MC of the inner and outer disc regions (MCDD). It turns out that the Blandford-Znajek (BZ) process can be regarded as one type of MC, i.e., MCHL. In addition, we propose a scenario for fitting the quasi-periodic oscillations in BH binaries based on MCDD associated with the magnetic reconnection.

Key words: accretion, accretion discs — black hole physics — magnetic fields — stars: oscillations

1 INTRODUCTION

According to observations, large-scale magnetic fields exist in many astronomical cases, such as galaxies, young stellar objects, neutron stars and black hole (BH) X-ray binaries (Han & Qiao 1994; Livio Ogilvie & Pringle 1999). It is widely believed that magnetic fields play an important role in dynamics of accretion discs and jet formation. However, the origin of large-scale magnetic fields remains elusive (Blandford 2002).

Recently, much attention has been paid to the magnetic connection (MC) of a rotating black hole (BH) with its surrounding accretion disc (Blandford 1999; Li 2000; Wang, Xiao & Lei 2002; Uzdensky 2004, 2005). Not long ago, Li (2002) calculated the magnetic field configuration of MC by assuming a single electric current flowing in equatorial plane of a BH. Wang et al. (2007) discussed the MC between plunging region and disc (MCPD) as well as the MC between the BH horizon and the disc (MCHD). Very recently, Ge et al. (2008) analyzed the topology and feature of a magnetic field configuration arising from double counter oriented electric current-rings in an accretion disc around a Kerr BH.

Enlightened by the above works, in this paper, we intend to investigate the magnetic field configuration arising from toroidal electric current distributed continuously in a thin disc around a Kerr BH, and discuss the MC of the BH with its surrounding disc. In addition, we propose a scenario for fitting the quasi-periodic oscillations in BH binaries based on the MC associated with magnetic reconnection in the disc.

Throughout this paper the geometric units $G = c = 1$ are used.

2 MAGNETIC FIELD CONFIGURATION AND MAGNETIC FLUX

In this paper, the magnetic field configuration is produced by the toroidal electric current flowing in the inner region of an accretion disc based on the following assumptions.

(1) The disc is thin and Keplerian around a Kerr BH, and the concerned Kerr metric parameters are given by
\begin{align}
\rho^2 &= r^2 + a^2 \cos^2 \theta, \quad \Delta = r^2 - 2Mr + a^2, \\
A &= (r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta, \\
\alpha &= \left(\rho^2 \Delta / A\right)^{1/2}, \quad \varpi = \left(A / \rho^2\right)^{1/2} \sin \theta. 
\end{align}

(2) The toroidal electric current lies in the equatorial plane, varying continuously with the disc radius in a power-law,
\begin{align}
j &= j_0 \left(r / r_{ms}\right)^{-n} \delta(\cos \theta), \quad r_{ms} \leq r \leq \lambda r_{ms},
\end{align}

where $n$ is the power-law index for the variation of the electric current, $r_{ms}$ is the innermost stable circular orbit (ISCO), and $\lambda$ is a parameter to adjust the radial width of the current distribution.
Thus the four-current vector flowing on the circle of radius \( r' \) is given by

\[
J^\alpha = \frac{1}{r} \left( \frac{\Delta}{\Lambda} \right)^{1/2} \left( \frac{\partial}{\partial \varphi} \right) \Phi |_{r=r'}. 
\]

(3)

Based on the above distribution of the toroidal electric current we have the magnetic flux through a surface bounded by a circle with \( r = \text{const} \) and \( \theta = \text{const} \) as follows,

\[
\Psi (\alpha, n, \lambda, r, \theta) = 2\pi \int dA_{\varphi} (\alpha, n, \lambda, r, \theta; r').
\]

(4)

In equation (4) \( dA_{\varphi} (\alpha, n, \lambda, r, \theta; r') \) is the toroidal component of the electric vector potential, which can be determined by the electric current loop located at \( r' - r' + dr' \) (Znajek 1978; Linet 1979), and the position \((r, \theta)\) of the circle is described in the spherical coordinates.

A mapping relation between two circles with spherical coordinates \((r_1, \theta_1)\) and \((r_2, \theta_2)\) is given based on the conservation of magnetic flux as follows,

\[
\Psi (\alpha, n, \lambda, r_1, \theta_1) = \Psi (\alpha, n, \lambda, r_2, \theta_2).
\]

(5)

In equation (5) \( \Psi \) is the dimensionless magnetic flux, being defined as

\[
\Psi (\alpha, n, \lambda, r, \theta) = 2\pi A_{\varphi} (\alpha, n, \lambda, r, \theta) / B_0 M^2,
\]

(6)

where we have \( r \equiv r/M \) and \( B_0 \equiv 2j_0 \). Based on (6) we have the relation between the radii \( r_m \) and \( r_0 \) as follows,

\[
\Psi (\alpha, n, \lambda, r_0/2) = \Psi (\alpha, n, \lambda, r_m/2).
\]

(7)

Incorporating equations (1) – (7), we have the poloidal magnetic field configuration generated by continuous toroidal electric current with different values of the parameters \( \alpha, n \) and \( \lambda \) as shown in Fig.1.

As shown in Fig.1, the four types of MC configurations can coexist for different values of \( \alpha, n \) and \( \lambda \), where the characteristic field lines of Types I, II, III and IV are indicated by lines 1, 2, 3 and 4, respectively. These MC configurations are described as follows.

Type I: MC of the BH with the remote astrophysical load (MCHL);

Type II: MC of the BH with the disc (MCHD);

Type III: MC of the plunging region with the disc (MCPD);

Type IV: MC of the inner and outer regions of the disc (MCDD).

Among the above configurations MCHD and MCPD have been discussed by a number of authors (Blandford 1999; Krolik 1999; Gammie 1999; Li 2002; Wang et al. 2002; Wang et al. 2007), and these two MC processes are regarded as the variants of the Blandford-Znajek (BZ) process (Blandford & Znajek 1977). On the other hand, the BZ process can also be regarded as one kind of MC, i.e., MCHL, which was formulated with the MC process in a united model (Wang, Xiao & Lei 2002).

Based on equation (4) we have the magnetic flux \( \psi (r, \theta) \) penetrating the equatorial plane of a Kerr BH versus the disc radius \( \tilde{r} \) for different power-law index \( n \) as shown in Fig.2.

From Fig.2 we obtain the following results.

1) The magnetic flux \( \psi (r, \theta) \) varies with \( \tilde{r} \) non-monotonically, and attains its peak value outside ISCO. The

Figure 1. The magnetic field configuration generated by toroidal electric current distributed continuously over the inner region of a thin disc around a Kerr BH with (a) \( \alpha_s = 0.5, n = 3.0, \lambda = 3 \), (b) \( \alpha_s = 0.9, n = 3.0, \lambda = 3 \), (c) \( \alpha_s = 0.5, n = 5.0, \lambda = 3 \) and (d) \( \alpha_s = 0.5, n = 3.0, \lambda = 5 \).
greater the power-law index \( n \) is, the less the peak value is, and the closer the position of the peak value is to ISCO.

(2) Equation (7) provides a relation between the radii \( r_{ms} \) and \( r_0 \). For the given \( a_* \), the greater \( n \) corresponds to the less \( r_0 \), and the greater \( \lambda \) corresponds to the greater \( r_0 \) as shown in Figures 2a and 2b, respectively.

\[ \Omega_K = 2\nu_0(\xi^{3/2}\chi_{ms}^3 + a_*)^{-1}, \]  

where \( \nu_0 \) is defined as \( \nu_0 \equiv (m_{BH})^{-1}3.23 \times 10^{-4} \text{Hz} \) with \( m_{BH} \equiv M/M_\odot \), and parameter \( \xi \equiv r/r_{ms} \) is the disc radius in terms of \( r_{ms} \), and \( \chi_{ms} \equiv \sqrt{r_{ms}/M} \) is a function of the BH spin \( a_* \) (Novikov & Thorne 1973).

Since the inner footpoints of the magnetic field lines rotate faster than the outer ones, the rotating disc will twist the field lines of MCDD, resulting in an increasing toroidal component of the magnetic field. This configuration allows antiparallel segments of the field lines to be brought in contact, giving rise to magnetic reconnection in a similar way to that between a central star and the accretion disc (Montmerle et al. 2000). The magnetic energy thus liberated could heat the plasma and ignite a flare. In this way, successive reconnections and flaring may continue periodically for some time, providing a possible way for producing QPOs in BH accretion disc.

Considering that the magnetic reconnection arises from the twist of the closed field lines of MCDD, we infer that the flare occurs most probably in the critical field line, and the difference between the Keplerian frequencies at radii \( r_{ms} \) and \( r_0 \) as follows,

\[ \nu_{MCDD} = \frac{\Omega_K(r_{ms}) - \Omega_K(r_0)}{2\pi}, \]

\[ = \nu_0 \left[ \left( \frac{3}{\chi_{ms}^3 + a_*} \right)^{1/2} - \left( \frac{3}{\chi_{ms}^3 + a_*} \right)^{1/2} \right]. \]  

Thus we can fit the frequency \( \nu_{QPO} \) of single-component QPOs as \( \nu_{QPO} \) based on equation (9), from which we find that the frequency \( \nu_{QPO} \) depends on three parameters, \( m_{BH}, a_* \) and \( \xi_0 \). By using equation (9) we have an isosurface of \( \nu_{QPO} = \text{const} \) in a parameter space consisting of these parameters. Taking the BH binary GRO J1655-40 as an example, we have an isosurface of \( \nu_{QPO} = 300 \text{Hz} \) with \( \lambda = 3 \) as shown in Fig.3. The parameter \( \xi_0 \) approaches the maximum 1.554 at point A with \( m_{BH} = 6.6 \) and \( a_* = 0.65 \), and the minimum 1.338 at point B with \( m_{BH} = 6.0 \) and \( a_* = 0.8 \). Thus we have \( n = 5.284 \) and 6.656 by substituting \( \xi_0 = 1.554 \) and 1.338 respectively into (9).

We also have an isosurface of \( \nu_{QPO} = 300 \text{Hz} \) with a fixed value of \( n \), e.g., \( n = 3 \), and thus we can determine the minimum and maximum values of \( \xi_0 \), and obtain the corresponding values of the parameter \( n \). In this way we can fit the QPO frequencies of several sources based on MCDD as given in Table.1.

\subsection{3.2 Improving the fittings for 3:2 QPO pairs based on MCDD}

As is well known, the 3:2 QPO pairs have been observed in the BH binaries listed in Table 1, i.e., GRO J1655-40, XTE J1550+564 and GRS 1915+105 (MR06). Also, the 3:2 QPO pair appears in the BH candidate H1743-322, and its behavior resembles the BH binaries XTE J1550+564 and GRO J1655-40 in many ways (Homan et al. 2005; Kalemci et al. 2006; Remillard et al. 2002, 2006).

Abramowicz & Kluzniak (2001) explained the pairs as a resonance between orbital and epicyclic motion of accreting matter. Recently, the resonance model is presented in a more realistic context, in which “parametric resonance” concept...
is introduced to describe the oscillations rooted in fluid flow where there is a coupling between the radial and polar coordinate frequencies (Abramowicz et al. 2003; Kluzniak et al. 2004; Török et al. 2005).

Not long ago, Aschenbach (2004, hereafter A04) found that the BH mass and spin are strongly constrained by the 3:2 QPO pairs, and the ratio of the vertical epicyclic frequency to the radial epicyclic frequency is given by

$$\Omega_v(a_*,r_{31})/\Omega_R(a_*,r_{32}) = 3,$$

$$\Omega_v(a_*,r_{32})/\Omega_R(a_*,r_{32}) = 3/2.$$  \hfill (10)

In equation (10) $\Omega_v$ and $\Omega_R$ are the vertical and radial epicyclic frequencies, respectively (Nowak & Lehr 1998, Merloni et al. 1999). It has been found in A04 that equation (10) has only one solution for the two commensurable orbits, i.e., $r_{31} = 1.546$ and $r_{32} = 3.919$ with an extremely high spin $a_* = 0.99616$. As argued in A04, the radial gradient of the orbital velocity of a test particle changes sign in a narrow annular region when $a_* > 0.9953$, and the BH mass of these binaries can be constrained in a very narrow range, which are consistent with the dynamically determined masses within their measurement uncertainty range.

It has been pointed out that QPOs are generally associated with the steep power-law (SPL) state in BH X-ray binaries. Although the 3:2 QPO pairs could be interpreted in some epicyclic resonance models, there remain serious uncertainties as to whether epicyclic resonance could overcome the severe damping forces and emit X-rays with sufficient amplitude and coherence to produce the QPOs (e.g., see a review in MR06). The mechanism of producing QPOs based on MCDD might remedy the disadvantage of the epicyclic resonance models.

The 3:2 QPO pair has been observed in H1743-322 (Homan et al. 2005; Remillard et al. 2006; Kalemci et al. 2006), and its behavior resembles the BH binaries XTE J1550-564 and GRO J1655-40 in many ways (Remillard et al. 2002, 2006).

Based on the radial position of the vertical and radial epicyclic resonance given by A04, we set $r_0 = r_{32}$ by adjusting the parameters $n$ and $\lambda$ in equation (1) with $a_* = 0.99616$. Thus we expect that both the vertical and radial epicyclic resonance are stimulated and maintained due to energy transferred magnetically from the inner footpoint to the outer footpoint at $r_0$. According to the strict constraint required by the 3:2 QPO pair, we can estimate the BH mass as well as $\nu_{MCDD}$ as listed in Table.2.

In the above fitting the parameters $n$ and $\lambda$ are not independent, being related by equation (1) for $a_* = 0.99616$ and $r_0 = r_{32} = 3.919$. And we have the curve of $n$ versus $\lambda$ given in Fig.4. Inspecting Fig.4, we find that the power-law index $n$ increases very sharply with the parameter $\lambda$, as the latter is small, while $n$ increases very slowly with $\lambda$ as the latter is great. These results imply that the fittings of QPOs based on MCDD are very sensitive to the parameter $n$, if the radial width of current distribution is small, while the fittings are almost independent of $n$, provided that the current distribution is wide enough.

It is interesting to note that $\nu_{MCDD}$ for each source in Table 2 is about three times the corresponding upper frequency of the 3:2 QPO pair, i.e., $\nu_{MCDD} \approx 3\nu_{up}$. Based

### Table 1. Fitting QPO frequencies of BH X-ray binaries based on MCDD.

| Source       | Input $\nu_{QPO}$ | $m_{BH}$ | $a_*$ | $\xi_{max, \xi_{min}}$ | $\lambda = 3$ | $n = 3$ |
|--------------|------------------|---------|-------|------------------------|----------------|---------|
| GRO J1655-40 | 300              | 6.6     | 0.65  | 1.554                  | 5.284          | 1.547   |
|              | 450              | 6.6     | 0.65  | 2.287                  | 3.551          | 2.366   |
| XTEJ1550-564 | 184              | 10.8    | 0.71  | 1.485                  | 5.587          | 1.483   |
|              | 276              | 10.8    | 0.71  | 2.036                  | 3.868          | 2.083   |
| GRS 1915+105 | 113              | 18.0    | 0.98  | 1.221                  | 7.122          | 1.256   |
|              | 168              | 10.0    | 0.998 | 1.177                  | 8.714          | 1.182   |

Note. – The values of the BH mass and spin of the sources are adopted from Remillard & McClintock (2006), Davis et al. (2006) and McClintock et al. (2006). The quantities $\xi_{max}$ and $\xi_{min}$ are the maximum and minimum of $\xi_0$ on the isosurface of $\nu_{QPO} = const.$
Table 2. Fitting QPOs in BH binaries based on the epicyclic resonance model.

| Source         | $\nu_{up}/\nu_{down}$ | $m_{BH}$ | $a_*$, $\xi_0$ | $\nu_{MCDD}$ | $\nu_{MCDD}/\nu_{up}$ |
|----------------|------------------------|----------|----------------|--------------|------------------------|
| GRO J1655-40   | 450 / 300              | 6.77     | 6.75           | 1371.51      | 3.0478                 |
|                |                        |          |                | 1375.57      | 3.0568                 |
|                |                        |          |                | 839.921      | 3.0417                 |
|                |                        |          |                | 842.568      | 3.0528                 |
|                |                        |          | 0.99616, 2.9997|              |                        |
| XTEJ1550-564   | 276 / 184              | 11.06    | 11.02          | 502.169      | 2.9891                 |
|                |                        |          |                | 522.516      | 3.1102                 |
|                |                        |          | 11.02          | 522.516      | 3.1102                 |
| GRS 1915+105   | 168 / 113              | 18.49    | 18.49          | 731.111      | 3.0463                 |
|                |                        |          |                | 731.111      | 3.0463                 |
| H1743–322      | 240 / 160              | 12.70    | 12.70          | 733.421      | 3.0559                 |
|                |                        |          |                | 733.421      | 3.0559                 |

Note. – The BH masses of GRO J1655-40, XTEJ1550-564 and GRS 1915+105 are given by A04, and the the BH mass of H 1743–322 is determined by the same way given in A04. The parameter $\xi_0 = 2.9997$ is determined by $\xi_0 \equiv r_2/r_{ms} = r_0/r_{ms}$ for $a_* = 0.99616$.

Figure 4. The curve of the parameters $\nu$ versus $\lambda$ constrained by the 3:2 QPO pairs of the BH binaries.

on general relativity we have $\nu_{MCDD}/\nu_{up} = 3.05$, which is independent of the BH mass. This result seems consistent with the commensurate frequencies of the 3:2 QPO pair. In addition, the combination of MCDD with the epicyclic resonance might be helpful to interpret the association of the QPOs with the SPL states in BH binaries, because both the steep power–law component of radiation and the QPOs in BH binaries have the same origin of the magnetic reconnection. We shall address this issue in details in our future work.

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REFERENCES

Abramowicz, M. A., & Kluzniak, W. 2001, A&A, 374, L19
Abramowicz, M. A., Karas, V., Kluzniak, W., & Lee, W. H., Rebusco, P., 2003, PASJ, 55, 467
Aschenbach, B. 2004, A&A, 425, 1075 (A04)
Blandford R. D., Znajek R. L., 1977, MNRAS, 179, 433
Blandford, R. D., 1999, in ASP Conf. Ser. 160, Astrophysical Discs: An EC Summer School, ed. A. Sellwood & J. Goodman (San Francisco: ASP), 265
Blandford R. D., 2002, in Gilfanov M., Sunaev R., Churazov E., eds, Proc. MPA/ESO/MPE/USM Joint Astronomy Conference, Lighthouses of the Universe: The Most Luminous Celestial Objects and Their Use for Cosmology. Springer, Berlin, p. 381
Davis S. W., Done, C., Blaes O. M., 2006, ApJ, 647, 525
Gammie C. F., 1999, ApJ, 522, L57
Ge Z. J. Wang D.-X., Lei W.-H., 2008, Chin. Phys. Lett. 25, 2327
Han J. L., Qiao G. J., 1994, A&A, 288, 759
Homan J., et al. 2005, ApJ, 623, 383
Kalemci, E. et al. 2006, ApJ, 640, 55
Kluzniak W., Abramowicz, M. A., & Lee, W., 2004, AIPC, 714, 379
Krolik J. H., 1999, ApJ, 515, L73
Li L.-X., 2000, ApJ, 533, L115
Li L.-X., 2002, Phys. Rev. D, 65 084047
Linet B., 1979, J. Phys. A, 12, 839
Livio M., Ogilvie G. I., Pringle J. E., 1999, ApJ, 512, 100
McClintock J. E., Remillard R. A., 2006, in Lewin W. H. G., van der Klis M., eds, Compact Stellar X-Ray Sources. Cambridge Univ. Press, Cambridge, p. 157 (MR06)
McClintock J. E., Shafee R., Narayan R., Remillard R. A., Davis S. W., & Li, L.-X. 2006, ApJ, 652, 518
Merloni A., Nottaro S., Stella L., Bini D. 1999, MNRAS, 304, 155
Montmerle T., Grosso N., Tsuboi Y., Koyama K., 2000, ApJ, 532, 1097
Novikov I. D., Thorne, K. S., 1973, In : C. Dewitt, eds., Black Holes, Gordon and Breach, New York
Nowak, M. A. & Lehr, D. E. 1998, in Theory of Black Hole Accretion Discs, Cambridge University Press, eds. Abramowicz, M. A., Bjornsson, G., Pringle, J. E., 233
Remillard R. A., & Muno M. P., ApJ, 2002, 580, 1030
Remillard R. A., et al. 2006, ApJ, 637, 1002
Török G., 2005, A&A, 440, 1
Török G., Abramowicz, M. A., Kluzniak, W., & Stuchlik, Z., 2005, A&A, 436, 1
Uzdensky D. A., 2004, ApJ, 603, 652
Uzdensky D. A., 2005, ApJ, 620, 889
van der Klis, M., 2000, ARA&A, 38, 717
Wang D.-X., Xiao K., Lei W.-H., 2002, MNRAS, 335, 655
Wang D.-X., Ma R.-Y., Lei W.-H., Yao G.-Z., 2003, MNRAS, 344, 473
Wang D.-X., Ye Y.-C., Li Y., Liu D.-M., 2007, MNRAS, 374, 647
Wang D.-X., Ye Y.-C., Huang C.-Y., 2007, ApJ, 657, 428
Znajek R. L., 1978, MNRAS, 182, 639

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