Predictions for the CMB from an anisotropic quantum bounce

Ivan Agullo,1 Javier Olmedo,1 and V. Sreenath2

1Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803, U.S.A.
2Department of Physics, National Institute of Technology Karnataka, Surathkal, Mangalore 575025, India.

We introduce an extension of the standard inflationary paradigm on which the big bang singularity is replaced by an anisotropic bounce. Unlike in the big bang model, cosmological perturbations find an adiabatic regime in the past. We show that this scenario accounts for the observed quadrupolar modulation in the temperature anisotropies of the cosmic microwave background (CMB), and we make predictions for the remaining angular correlation functions E-E, B-B and T-E, together with non-zero temperature-polarization correlations T-B and E-B, that can be used to test our ideas. We base our calculations on the bounce predicted by loop quantum cosmology, but our techniques and conclusions apply to other bouncing models as well.

Introduction. Anisotropies are generic features of homogeneous solutions to Einstein’s equations. This is manifest already for Bianchi I geometries, the simplest anisotropic spacetimes. There, in the absence of anisotropic sources, the contribution of shears to Friedmann equations dilutes with the expansion faster than that of matter and radiation. Therefore, unless anisotropies are exactly zero during the entire history of the cosmos, there must be a time in the past when they were dominant. From this viewpoint, the Friedmann-Lemaître-Robertson-Walker (FLRW) isotropic spacetimes are quite singular. Nevertheless, the standard model of cosmology appeals to a phase of slow-roll inflation, when the universe expanded exponentially fast, to argue that anisotropies were quickly diluted soon after the beginning of that phase, and from that time on one can just ignore them. However, the way this argument is applied contains a stronger assumption—that the quantum states describing cosmological scalar and tensor perturbations soon after the onset of slow-roll were also isotropic. Anisotropies in quantum fields do not dilute at the same rate as the shears of the homogeneous metric do. In fact, the only reason why they can be washed away is because the cosmic expansion red-shifts the wavelengths for which the perturbation fields are anisotropic, potentially shifting them out of the observable universe. There is no additional dilution [1]. But red-shift scales linearly with the expansion, while the dilution of the shear $\sigma^2$ scales with its sixth power (in absence of anisotropic sources). Hence, unless inflation is significantly longer than the minimum amount required, one cannot rule out that some of the anisotropic features that perturbations could have acquired in their pre-inflationary evolution can be imprinted in the CMB.

This argument, and the fact that the Planck satellite has observed anisotropic features in the CMB [2], has triggered our interest in studying anisotropic extensions of the standard cosmological model. However, within general relativity, one finds a major impediment: In a generic anisotropic universe, there are no preferred or universal initial states for the cosmological perturbations. In the theory of inflation, one uses the fact that the wave-lengths of the perturbations that we can probe in the CMB were much shorter than the Hubble radius at the onset of slow-roll. Then, the notion of adiabatic vacuum can be used to single out an initial quantum state, at least for these wavelengths. But this argument fails if the pre-inflationary spacetime is anisotropic (see e.g. [3]). In the absence of preferred initial data, the theory loses predictive power.

This paper proposes an extension of the standard model of cosmology beyond general relativity, where the big bang singularity is replaced by an anisotropic cosmic bounce. In short, we consider a framework in which the universe contracts in the remote past, according to Einstein’s theory, until matter and spacetime curvature reach a maximum value close to the Planck scale. Then, quantum gravity effects grow and dominate the dynamics, overwhelming the classical attraction and making the universe to bounce. In the far past, the universe isotropizes, and perturbations find an adiabatic regime. Therefore, in this scenario, one has preferred initial and final notions of vacuum and Hilbert spaces for perturbations. Our goal is to formulate this quantum theory and to solve the evolution, that in the Schrödinger picture reduces to compute the S-matrix and transition amplitudes between in and out states. We show that cosmic perturbations can retain memory of the anisotropic phase of the universe, and leave an imprint on the CMB, even though anisotropies in the spacetime are relevant only during a short period of time around the bounce. In order to isolate the effects of anisotropies, we work with Bianchi I spacetimes without anisotropic sources; they differ from spatially flat FLRW scenarios only by the presence of anisotropic shears.

The classical phase space. Loop quantum cosmology (LQC) uses canonical methods for quantization. Therefore, to incorporate perturbations we first need to formulate them in the Hamiltonian language. This task is significantly more tedious and complex than the FLRW counterpart [4], and to the best of our knowledge it has not been developed before (although classical gauge invariant perturbations in Bianchi I and their equations of motion have been derived in [3] by expanding Einstein’s
equations). We follow the geometric approach proposed in [6]. Gauge invariant perturbations can be obtained by finding a canonical transformation that makes four of the new momenta proportional to each of the four linear constraints of the theory, respectively—the scalar and vector constraints. This guarantees that the conjugate variables to these momenta are pure gauge, while the rest of fields are gauge invariant. The search for such transformation reduces to solving Hamilton-Jacobi-like equations for a generating function. There are multiple solutions, which correspond to different choices of gauge invariant fields. We have selected the choice that in the isotropic limit reduces to the familiar scalar perturbations and the two circularly polarized tensor modes (with helicity ±2), and denote them by $\Gamma_0$ and $\Gamma_{\pm 2}$, respectively.

The dynamics of gauge invariant perturbations is guaranteed to decouple from pure gauge fields, and is generated by a Hamiltonian $\mathcal{H}_{\text{pert}}$. Hamilton’s equations can be combined into the second order differential equations

$$\ddot{\Gamma}_s + 3H\dot{\Gamma}_s + \frac{k^2}{a^2}\Gamma_s + \frac{1}{a^2}\sum_{s'=0}^{2} U_{s,s'} \Gamma_{s'} = 0,$$

with $s = 0, \pm 2$; we have expanded the fields in Fourier modes $\Gamma_s(\vec{k}, t)$, and $\vec{k}$ is the comoving wavenumber. The functions $U_{s,s'}(\vec{k}, t)$ are effective potentials made of a complicated combination of the background variables (see [6] for details), $a(t)$ is the mean scale factor and $H = \dot{a}/a$ is Hubble rate. We have implemented this Hamiltonian theory in the symbolic language of Mathematica, and made the code publicly available in [7]. One important difference with FLRW spacetimes is that the potentials $U_{s,s'}$ are not diagonal (i.e., proportional to $\delta_{ss'}$) in presence of anisotropies. Therefore, the three fields $\Gamma_s$ are coupled and, because these couplings are time dependent, there is no way to diagonalize the equations of motion at all times by means of a local field redefinition.

**Quantum theory.** The classical phase space we are interested in is the product $V_{\text{BI}} \times V_{\text{pert}}$ of Bianchi I geometries and gauge invariant perturbations. At leading order in the perturbations, dynamics is implemented by first determining the evolution within $V_{\text{BI}}$, and then lifting the dynamical curves to $V_{\text{pert}}$ with the Hamiltonian $\mathcal{H}_{\text{pert}}$. We follow the same strategy in the quantum theory. Namely, the Hilbert space is the product $H_{\text{BI}} \otimes H_{\text{pert}}$. $H_{\text{BI}}$ has been described in [10] [11]. A good approximation for quantum states $\Psi_{\text{BI}} \in H_{\text{BI}}$ that at late times are sharply peaked on a classical geometry is provided by the so-called effective equations [12]. These are quantum corrected equations for the directional scale factors and their conjugate variables, whose solutions follow with precision the peak of the wave-function $\Psi_{\text{BI}}$. The physics of these spacetimes has been studied in detail in [13], and the main features are the following. All solutions contain a bounce of the mean scale factor $a(t)$, which is caused by quantum gravity effects. All strong curvature singularities are resolved, as long as the matter sector satisfies the null energy conditions. Energy densities and shears are bounded from above. Directional scale factors $a_s(t)$ bounce generically at different times, giving rise to a richer bounce than in the isotropic case. After the bounce, and in presence of an inflationary potential $V(\phi)$ [24], the evolution generically leads to a phase of slow-roll inflation, or in other words, such a phase is an attractor in the phase space of this quantum corrected theory.

To quantize the perturbations we follow the conceptual framework introduced in [14–16], and extend it to Bianchi I geometries. We obtain that the dynamics of quantum perturbations $\hat{\Gamma}_0$, $\hat{\Gamma}_{\pm 2}$ are described by the equations [1], with the background geometry given by a solution to the effective equations of LQC. The main difficulty arises from the interactions among the quantum fields $\hat{\Gamma}_0$, $\hat{\Gamma}_{\pm 2}$, induced by the anisotropies in the metric. To describe dynamics, we first define the $in$ and $out$ Hilbert spaces. The former is defined from an adiabatic vacuum in the past, that we take to be anytime before 10000 Planck seconds prior to the bounce. At this time anisotropies are already negligible in the Bianchi I geometries that we have explored, and all Fourier modes of interest are well inside the Hubble radius. The $out$ Fock space is the standard one built from the Bunch-Davies vacuum during inflation, when the anisotropies of the spacetime are negligible again. The quantum evolution is implemented by the $S$-matrix, that provides a unitary map between the $in$ and $out$ Fock spaces [18]. Its action on the $in$ vacuum produces

$$\hat{S}(\text{in}) = N \bigotimes_{\vec{k}} \exp \left[ \sum_s \sum_{s'=0, \pm 2} V_{ss'}(\vec{k}) \hat{a}_s^{\text{out}}(\vec{k}) \hat{a}_{s'}^{\text{out}}(-\vec{k}) \right] \langle \text{in} | \text{out} \rangle,$$

where $N$ is a normalization factor, and $V_{ss'}(\vec{k}) := \sum_s \frac{1}{2} \beta_{ss'}(\vec{k}) \alpha_{ss'}(\vec{k})$, with $\alpha_{ss'}(\vec{k})$ and $\beta_{ss'}(\vec{k})$ the Bogoliubov coefficients that relate the $in$ and $out$ vacua. They encode the information of the evolution of perturbations across the anisotropic bounce, and can be computed from the classical equations of motion. The operators $\hat{a}_s^{\text{out}}$, with $s = 0, \pm 2$, create quanta of the familiar scalar and tensor modes in inflation, respectively. The right hand side of (2) is the product of squeezing operators acting on $|\text{out}\rangle$. Consequently, the $in$ vacuum evolves to a state made of entangled pairs of quanta, one with wavenumber $\vec{k}$ and the other with $(-\vec{k})$—i.e., no net momentum is created. In the isotropic limit $V_{ss'}$ becomes diagonal, and the operator in (2) becomes the product of operators for scalar and each of the two tensor modes. This is not the case in presence of anisotropies, where the final state contains entanglement among the three types of perturbations. One can compute, e.g., the entanglement entropy, from the Bogoliubov coefficients [6].

**Constraints from observations.** We next analyze
constraints from current data on our parameter space. They come from Planck’s measurement of a quadrupolar direction-dependent modulation of the temperature anisotropies [2]. The freedoms in our model come from the choice of an effective Bianchi I quantum spacetime. One of these geometries is singled out by specifying the value of the shear squared $\sigma^2(t_B)$, the shear in one of the principal directions, say $\sigma_x(t_B)$, the value of the scalar field $\phi(t_B)$, and the sign of its time derivative, all at the time $t_B$ of the bounce. $\sigma^2(t_B)$ measures the total amount of anisotropies at $t_B$: $\sigma_x(t_B)$ indicates the way these anisotropies are distributed in the three principal directions, and $\phi(t_B)$ and the sign of $\dot{\phi}(t_B)$ control the number $N$ of e-folds of expansion from the bounce to the end of inflation ($\sigma^2(t_B)$ also affects this number, but in a sub-leading manner) [13]. Since our goal is to describe the largest possible signal that we can expect in the CMB, we choose $\sigma^2(t_B)$ close to its upper bound, and derive the constraints from observations on the other two parameters. Actually, the value of $\sigma_x(t_B)$ is not important in this task, since it can be modified by simply rotating the coordinate axes. Observations provide a lower bound for the number of e-folds $N$, which keeps anisotropic features in the CMB below the observed threshold. On the other hand, if this number happens to be very large, all anisotropies in perturbations would be red-shifted out of the observable patch of the universe. A representative example of our analysis is obtained by choosing $\sigma^2(t_B) = 5.75$ in natural units (this is half of its upper bound [13]), and $\sigma_x(t_B) = 0$. We have computed the quadrupolar modulation and compared it with data from Planck (see Figure 1). The result of this analysis is a lower bound for $N$ of 70.1. As we will shortly see, this value is not large enough to wash away all anisotropies in the CMB.

**Predictions for the CMB.** We compute the angular correlation functions $C_{TT,mm'}^{X,Y} \equiv \langle a_{tm}^XA_{tm'}^Y \rangle$, with

$$a_{tm}^X = \int d\Omega X(\hat{n}) Y_{tm}^*(\hat{n}),$$

where $X = T, E, B$ represents the temperature, electric and magnetic components of the polarization, respectively, of the anisotropies in the CMB.

(i) Temperature-Temperature (T-T). Our theory is invariant under translations and parity, but not under rotations. Parity invariance restricts $C_{TT,mm'}^{T,T}$ to vanish unless $\ell + \ell'$ is even (isotropy would have also imposed $\ell = \ell'$, $m = -m'$). We plot in Fig. 2 $C_{TT} = \frac{1}{2\pi^2} \sum_{m=-\ell}^{\ell} (-1)^m C_{TT,\ell,m}$, and compare it with the predictions of isotropic inflation. On the other hand, correlation functions for $\ell \neq \ell'$ are a smoking gun for anisotropies. In Fig. 2 we also show one of them, namely $C_{TT}^{T,T,\ell+2,00}$, as an illustrative example. Other values of $\ell, \ell', m, m'$ produce correlations with similar shape, and equal or smaller amplitudes. These plots show that, as expected, the effects of the pre-inflationary physics are larger for low multipoles (large angular scales). In $C_{TT,mm'}^{T,T}$ these effects translate to a modest enhancement of power, although small when compared to uncertainties coming from cosmic variance. The plot for the anisotropic correlations $C_{TT,\ell+2,00}$ is in agreement with the quadrupolar modulation observed by Planck.

(ii) E-E, B-B, and T-E correlations. For all these correlation functions the conclusions are similar than for the T-T case. Namely, they are different from zero only for $\ell + \ell'$ even, and the main departures from the isotropic model appear for low multipoles and for $\ell \neq \ell'$. As an example, we plot in Fig. 3 $C_{BB}^{BB} = \frac{1}{2\pi^2} \sum_{m=-\ell}^{\ell} (-1)^m C_{BB,\ell,m}$, and $C_{TT}^{B,B}$. The latter has an important contribution from the entanglement between tensor perturbations with different polarizations.

(iii) T-B and E-B. Because the B-polarization field is a pseudoscalar, while T and E are parity even, in a parity invariant theory these correlations vanish unless $\ell + \ell'$ is odd. Since isotropy would also imply $\ell = \ell'$, all these correlations vanish in the standard cosmological scenario. Fig. 4 shows $C_{TT}^{B,B}$ and $C_{TT}^{E,B}$ in our model. They originate exclusively from the entanglement between scalar and the two tensor modes.

The computational difficulty of these calculations comes from the need to resolve the angular dependence

\[ g_{2}(k) \equiv \sqrt{\sum_{m=\ell}^{\ell} g_{2M}^{T}(k)} \]

\[ g_{2}^{E}(k, q = -1) \]
of the T-T spectrum observed by Planck \cite{Planck} could be a remnant from an anisotropic pre-inflationary phase, rather than a statistical fluke. Furthermore, we predict that this modulation comes together with concrete effects in the E-E, T-E, B-B, T-B and E-B correlation functions, that provide a concrete way to test our ideas (further details omitted here can be found in \cite{details}).

**Discussion.** The merits of this work are (i) To introduce a Hamiltonian formulation of gauge invariant perturbations in Bianchi I spacetimes, and to implement the mathematical framework in a publicly available computational algorithm. (ii) To formulate an exact quantization of the coupled system of linear perturbations, and to use this formalism to compute the entanglement between scalar and tensor perturbations that anisotropies generate. (iii) To embed this theory within a quantization of the Bianchi I geometry, extending in this way previous studies on quantum cosmology to anisotropic scenarios, a task that has remained elusive due to the complexity of the system. (iv) To show that perturbations can retain memory of the pre-inflationary universe, although the anisotropies in the background geometry quickly dilute during inflation. This memory is codified in the form of anisotropic correlation functions and quantum entanglement between the different types of perturbations. (v) Finally, and most importantly, we have explained a possible origin for the non-zero quadrupolar modulation observed by Planck, and made concrete predictions for E-E, B-B, T-E, T-B and E-B correlation functions in the CMB. Although Planck’s observations of the T-T quadrupole alone are not significant enough to declare the detection of anisotropic physics, a detailed search for the effects we describe in the E-E, T-E correlations (that
Planck has already partially done), and particularly in B polarization, could boost the significance of the detection. Some of the values we predict, particularly the ones involving T-B and E-B correlations, are small, and probably difficult to observe. But others are not, and could be measured by a next generation space based mission dedicated to accurately measure CMB polarization, such as CORE [21].

Furthermore, although we have worked within loop quantum cosmology, we expect our conclusions to be valid for other theories that predict a similar bounce (see, e.g., [22, 23]). This is because perturbations are sensitive to the coarse grained aspects of the spacetime geometry mainly, and not to the finer details of a concrete scenario. Our techniques will also be useful to introduce anisotropies in bouncing scenarios that do not contain a phase of inflation, and where the primordial power spectra are generated in the contracting phase.

Acknowledgements.
We have benefited from discussions with Abhay Ashtekar, Mar Bastero-Gil, Brajesh Gupt, Guillermo A. Mena Marugán, Jorge Pullin, Parampreet Singh and Edward Wilson-Ewing. This work is supported by the NSF CAREER grant PHY-1552603, Project. No. FIS2017-86497-C2-2-P of MICINN from Spain and from the Hearne Institute for Theoretical Physics. V.S. was supported by Louisiana State University and Inter-University Centre for Astronomy and Astrophysics during different stages of this work. Portions of this research were conducted with high performance computing resources provided by Louisiana State University (http://www.hpc.lsu.edu).

* Electronic address: agullo@lsu.edu
† Electronic address: jolmedo1@lsu.edu
‡ Electronic address: sreenath@nitk.edu.in

[1] I. Agullo and L. Parker, Phys. Rev. D 83, 063526 (2011); Gen. Rel. Grav. 43, 2541-2545 (2011).
[2] Planck Collaboration, Planck 2018 results. X. Constraints on inflation. arXiv:1807.06211 (2018).
[3] T. S. Pereira, C. Pitrou and J. P. Uzan, JCAP 0709, 006 (2007); JCAP 0804, 004 (2008); Comptes rendus - Physique 16, 1027-1037 (2015).
[4] D. Langlois, Class. Quant. Grav. 11, 389 (1994).
[5] J. Goldberg, E. T. Newman, and C. Rovelli, On Hamiltonian systems with first-class constraints, J. Math. Phys. 32, 2739 (1991).
[6] I. Agullo, J. Olmedo and V. Sreenath, Hamiltonian theory of classical and quantum gauge invariant perturbations in Bianchi I spacetimes, to appear.
[7] J. Olmedo, I. Agullo and V. Sreenath, http://bitbucket.org/jolmedo/bianchi-perts/src/master/ (2019).
[8] A. Ashtekar and P. Singh, Class. Quant. Grav. 28, 213001 (2011).
[9] I. Agullo and P. Singh, Loop Quantum Cosmology: A brief review, in “Loop quantum Gravity: the first 30 years”, Edited by A. Ashtekar and J. Pullin, World Scientific (2017), arXiv:1612.01236.
[10] A. Ashtekar, and E. Wilson-Ewing, Phys. Rev. D 79, 083535 (2009).
[11] M. Martín-Benito, G. A. Mena Marugán, E. Wilson-Ewing, Phys. Rev. D 82, 084012 (2010).
[12] D. Chiu and K. Vandersloot, Phys. Rev. D 76, 084015 (2007).
[13] B. Gupt, P. Singh, Phys. Rev. D 86, 024034 (2012), Class. Quant. Grav. 30, 145013 (2013).
[14] A. Ashtekar, W. Kaminski and J. Lewandowski, Phys. Rev. D 79, 064030 (2009).
[15] I. Agullo, A. Ashtekar and W. Nelson, Phys. Rev. D 87, 043507 (2013); Class. Quant. Grav. 30, 085014 (2013); Phys. Rev. Lett. 109, 251301 (2012).
[16] M. Fernández-Méndez, G. A. Mena Marugán and J. Olmedo, Phys. Rev. D 86, 024003 (2012); Phys. Rev. D 88, 044013 (2013); L. Castelló Gomar, M. Fernández-Méndez, G. A. Mena Marugán and J. Olmedo, Phys. Rev. D 90, 064015 (2014); F. Bentez Martínez and J. Olmedo, Phys. Rev. D 93, 124008 (2016).
[17] A.E. Gumrukcuoglu, A. Himmetoglu, M. Pelosi Phys. Rev. D 81, 063528 (2010).
[18] I. Agullo, and A. Ashtekar, Phys. Rev. D 12, 124010 (2015).
[19] I. Olmedo, I. Agullo and V. Sreenath, http://bitbucket.org/jolmedo/cosmo-perts/src/master/ (2019).
[20] I. Agullo, J. Olmedo and V. Sreenath, to appear.
[21] http://www.core-mission.org.
[22] A. H. Chamseddine and V. Mukhanov, JCAP 1703, 009 (2017).
[23] D. Langlois, H. Liu, Karim Noui, and E. Wilson-Ewing, Class. Quant. Grav. 34, 225004 (2017).
[24] In our calculations, we use $V(\phi) = 1/2m^2\phi^2$, for the sake of simplicity. The anisotropies in our model are large close to the bounce only, and at that time $V(\phi)$ is subdominant in the solution we are interested in.