Simultaneous Solution to $B \to \phi K$ CP Asymmetry and $B \to \eta' K, B \to \eta K^*$ Branching Ratio Anomalies from R-Parity Violation

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Abstract

The branching ratios for the neutral and charged B decay channels to $\eta' K$ and $\eta K^*$ are well above the Standard Model expectations. Moreover, the mixing-induced CP asymmetry in $B \to \phi K_S$ is incompatible to that found in $B \to J/\psi K_S$. We investigate whether a flavour-specific tree-level $b \to s \bar{s}s$ operator coming from R-parity violating supersymmetry can resolve both these discrepancies, without jeopardizing those results which are in agreement with the Standard Model. We found that it is possible to have a parameter space satisfying all these requirements, including that of a low strong phase difference compatible with the color transparency argument. Furthermore, we put a robust bound on the relevant coupling, which is two orders of magnitude better than the existing one.

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1 Introduction

The flavour structure of the Standard Model (SM) is, arguably, the best proof that there is new physics lurking above the electroweak scale. We do not know how this new physics should manifest itself, so one is forced to consider all possible ways to uncover it. Moreover, any experimental result that deviates from the SM expectation is bound to receive close scrutiny for new physics. Apart from the direct production of new particles, one can look for their indirect effects, i.e., how the operators at a high energy affect the low-energy observables. In this respect, the $e^+e^-$ B factories at Cornell, SLAC, and KEK are doing a commendable job by churning out a huge amount of data on various B decay modes, including the branching ratios (BR) and CP asymmetries.

Even at this stage, there exist several hints that everything is not as one should expect from the SM, if we take the experiments seriously. The SM is in no way ruled out, since there is much scope for theoretical uncertainty in low-energy QCD, but it will be under considerable strain if the experimental results persist over the next few years. In a large set of data, most of which are in perfect agreement with the SM, there are three so-called sore thumbs sticking out: (i) The direct and mixing-induced CP-asymmetries for the mode $B \rightarrow \pi^+\pi^-$ where Belle and BaBar are not consistent [1, 2], (ii) The abnormally high branching ratios for the $B \rightarrow \eta'K$ and $B \rightarrow \eta K^*$ modes, incompatible with the SM prediction with factorization [3], and (iii) The measured value of $\sin 2\beta$ from $B \rightarrow \phi K_S$ [4, 5] 1. (Last two points are closely related; see, e.g., [7].) Let us briefly comment on these anomalies. But first we wish to make our notation clear: we use $B$ as a general shorthand for both $B^0$ and $\bar{B}^0$, and also for the charged $B$ mesons where the mention of charge is either unnecessary or self-evident, and the BRs are assumed to be averaged over the CP conjugate states. Moreover, we use the symbol $\eta K$ to denote the final states with either an $\eta$ or $\eta'$ and a $K$ or $K^*$ (neutral or charged).

The first anomaly exists only for Belle — BaBar data is completely consistent with the SM expectation, if one uses perturbative QCD calculations [8] or QCD-improved factorization model [9]. However, if we take the Belle data seriously, existence of new physics is hinted. A possible solution is discussed in [10].

The second and third anomalies are the main theme of this paper. Before one confronts the data with any specific model, three things are obvious. First, even if there is new physics, it is very much flavour-sensitive, since modes with the same topologies are not equally affected. Secondly, the effect of new physics is of the order of unity, considering the change in the BRs of $B \rightarrow \eta K$ modes, and the CP asymmetry from the $B \rightarrow \phi K_S$ mode. Thus, the effective operators generated by the new physics should better be tree-level at the high scale, unless one prefers a strong-coupling theory. Thirdly, the new physics amplitude should have weak (and possibly strong) phases different from the corresponding SM amplitude to generate the observable CP asymmetries, particularly in the $B \rightarrow \phi K_S$ channel [11].

These constraints severely limit the choice of plausible new physics options 2. Among those that survive, one of the most respectable options is R-parity violating supersymmetry (RPV) [13, 14]. RPV automatically generates flavour-specific tree-level operators which can leave their signals in some, but not all, B decay modes.

1 There are some other anomalies, e.g., lack of a consistent fit of the BRs of the $B \rightarrow PV$ channel [6]. It is not clear at this moment whether this is a feature of QCD factorization models, which was used in the analysis, or is true in general.

2 There may be new physics with loop-induced and/or flavour-blind operators, but their effects will be very hard to detect in the B factories. However, some supersymmetric resolutions of the $\phi K$ puzzle have been proposed [12].
The effect of RPV on B decays has been extensively discussed in the literature [10, 15, 16]. It has been proposed as a solution to the $B \to \phi K_S$ CP asymmetry puzzle [17], and even a simultaneous solution to both the $\eta' K$ and $\phi K_S$ anomalies has been proposed [18].

What is new, then, in this paper? There are two main points where our analysis differs from [18]. This is the first time that one takes all the relevant parameters, including the final-state strong phases, into account, and tries to explain all available data, including the direct CP asymmetry results. Secondly, we have not looked at isolated points in the parameter space as possible solutions, but have made a complete scan over the whole parameter space, and find out the regions which satisfy all the existing data. In fact, we introduce only one new physics operator, which, we find, does the fitting quite well. Moreover, we get a stronger bound on the relevant coupling, two orders of magnitude better than the existing one.

The discrepancies from the SM expectations show up in the BRs of different $B \to \eta K$ modes (see our convention before), and the value of $\sin(2\beta)$ extracted from $B \to \phi K_S$ decay. The numerical values are given in Section 2. By itself, the CP asymmetry data has nothing wrong in it, but it is definitely not consistent with the value of $\sin(2\beta)$ extracted from $B \to J/\psi K_S$ (and other charmonium channels) even at $2\sigma$.

That the experimental numbers are not consistent with the SM predictions for $\eta' K$ and $\eta K^*$ modes are known for a long time. Many solutions for this are proposed, ranging from introducing a charm content of $\eta'$ or anomalous $\eta'$ coupling to gluons to the introduction of new physics like RPV [15, 19]. For our calculation we take the charm contents in both these mesons to be zero. To have an estimate of the SM BRs for these modes, we assume naive factorization (NF) to hold good. Note that the BRs are more or less stable with the variation of the number of effective colors, $N_c$, which takes into account the nonfactorizable effects [20]. This, however, is not true for the generic $B \to \phi K$ modes, which is a penguin and the BR depends sensitively on $N_c$, falling quickly as $N_c$ increases. Thus, if one takes a fixed value of $N_c$, the BR predictions (within the context of the SM) for the $\eta K^*$ and $\eta' K$ modes are sort of reliable, but that is not the case for the $\phi K$ mode. In fact, it has been shown that dynamical enhancement of the penguin operators may lead to a twofold increase of the BR over the expectation from the NF model [21]. Keeping in mind such sensitivity which results from a numerical cancellation between different Wilson coefficients (WC), we think it prudent to present the analysis both by including $\text{Br}(B \to \phi K)$ as a constraint and by discarding it. We will show that the tighter constraints on the parameter space come from the CP asymmetry data and the $B \to \eta K$ BRs, and relaxation of the $B \to \phi K$ BR constraint does not affect the solutions in any perceptible way.

The paper is arranged as follows. In the next section we enlist all our input parameters, relevant formulae and experimental data. In Section 3, we introduce the RPV effective hamiltonian, and show how it affects the modes we are interested in. Section 4 deals with the numerical results, and we display the allowed regions of the parameter space that satisfy all data. In Section 5 we summarize and conclude.
2 Input Parameters

We consider only those modes which are, even in the SM, governed by the $b \to s \bar{q} q$ transitions, with $q = u, d$ or $s$. The relevant effective four-Fermi hamiltonian reads

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{us}^* \sum_{i=1,2} C_i(\mu) O_i(\mu) - V_{tb} V_{ts}^* \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right]$$

(1)

where the operators $O_i$ have the standard form

$$O_1 = (\bar{\sigma}_\alpha \gamma^\mu P_L u_\beta)(\bar{b}_\beta \gamma^\mu P_L b_\alpha),$$

$$O_{3,5} = (\bar{\sigma}_\alpha \gamma^\mu P_L b_\alpha) \sum_q (\bar{\eta}_\beta \gamma^\mu P_L q_\beta),$$

$$O_{7,9} = \frac{3}{2} (\bar{\sigma}_\alpha \gamma^\mu P_L b_\alpha) \sum_{q=u,d,s} (e_q \bar{q}_\beta \gamma^\mu P_{L,R} q_\beta),$$

(2)

with $\alpha$ and $\beta$ being the color indices, and $e_q$ the charge of the corresponding quark. The operators $O_{2n}$ are obtained from $O_{2n-1}$ by color-singlet $\leftrightarrow$ color-octet transformation. The projection operators are $P_L(P_R) = 1 - (+) \gamma_5$.

The effective WCs for the transition $b \to s$ are evaluated at the scale $\mu = m_b/2$ at next-to-leading-log (NLL) precision in naive dimensional regularization (NDR) scheme, with $m_t = 170$ GeV, $\alpha_s(m_Z) = 0.118$, $\alpha(m_Z) = 1/128$, and the QCD scale $\Lambda^{(5)}_{\text{MS}} = 225$ MeV. They typically change by 10% if we vary the QCD scale by about 60 MeV. The values, which are taken from [20], read

$$C_1 = -0.33, \quad C_2 = 1.16, \quad C_3 = 0.022 + 0.003i, \quad C_4 = -0.051 - 0.009i,$$

$$C_5 = 0.016 + 0.003i, \quad C_6 = -0.063 - 0.009i, \quad C_7 = -(1.2 + 1.3i) \times 10^{-4},$$

$$C_8 = 5 \times 10^{-4}, \quad C_9 = -(101 + 1.3i) \times 10^{-4}, \quad C_{10} = 20 \times 10^{-4}. \quad (3)$$

At $\mu = m_b/2$, the current quark masses (in GeV) are taken to be

$$m_u = 0.0042, \quad m_d = 0.0076, \quad m_s = 0.122, \quad m_c = 1.5, \quad m_b = 4.88. \quad (4)$$

The masses for the mesons $B^0, B^-, \pi, \eta, \eta', K$ and $K^*$ are the corresponding central values as given in [3].

The meson decay constants (in GeV) are:

$$f_\pi = 0.133, \quad f_K = 0.158, \quad f_{K^*} = 0.214, \quad f_\phi = 0.233. \quad (5)$$

The $\eta$ and $\eta'$ decay constants are obtained from $f_{\eta_1} = 1.10 f_\pi$ and $f_{\eta_8} = 1.34 f_\pi$ [15]:

$$f_{\eta_1}^u = \frac{f_{\eta_8} \cos \theta}{\sqrt{6}} - \frac{f_{\eta_1} \sin \theta}{\sqrt{3}}, \quad f_{\eta_1}^s = \frac{2 f_{\eta_8} \cos \theta}{\sqrt{6}} - \frac{f_{\eta_1} \sin \theta}{\sqrt{3}},$$

$$f_{\eta_1}^{u*} = \frac{f_{\eta_8} \sin \theta}{\sqrt{6}} + \frac{f_{\eta_1} \cos \theta}{\sqrt{3}}, \quad f_{\eta_1}^{s*} = \frac{2 f_{\eta_8} \sin \theta}{\sqrt{6}} + \frac{f_{\eta_1} \cos \theta}{\sqrt{3}}. \quad (6)$$

The mixing angle $\theta$ is taken to be $-22^\circ$. Thus the numerical values are $f_{\eta_1}^u = 0.099, f_{\eta_1}^s = -0.103, f_{\eta_1}^{u*} = 0.051, f_{\eta_1}^{s*} = 0.133$. This shows why the strange quark plays a dominant role in decays.
involving an $\eta'$. These values differ slightly from those given in [20] using a two-angle mixing scheme taking into account the coupling of gluons to $\eta$ and $\eta'$; the corresponding numbers are $0.077$, $-0.112$, $0.063$ and $0.141$ respectively. Our results do not change appreciably if we use the latter set.

The magnitude of the CKM elements are taken from [22] which uses a fit based only on the unitarity of the mixing matrix. The error limits are at 95% CL.

\[
|V_{ud}| = 0.97504 \pm 0.00094, \quad |V_{us}| = 0.2221 \pm 0.0042, \quad |V_{ub}| = 0.00352 \pm 0.00103, \\
|V_{cd}| = -0.2220 \pm 0.0042, \quad |V_{cs}| = 0.97422 \pm 0.00102, \quad |V_{cb}| = 0.0407 \pm 0.0028, \\
|V_{td}| = 0.0079 \pm 0.0016, \quad |V_{ts}| = -0.0403 \pm 0.0030, \quad |V_{tb}| = 0.99917 \pm 1.2 \times 10^{-4}.
\]

(7)

We could even have used the numbers from the standard CKM fits with $\Delta m_B$ as one of the inputs, since, as we will show later, the RPV couplings considered here do not affect the $B^0 - \bar{B}^0$ mixing amplitude. This, however, is not true in general; \textit{e.g.}, see ref. [10].

The transition formfactors [23] at $q^2 = 0$ are given by

\[
F_0(B \to K) = 0.38; \quad F_0(B \to \eta) = 0.145; \quad F_0(B \to \eta') = 0.135; \quad A_0(B \to K^*) = 0.32,
\]

(8)

and $F_0(0) = F_1(0)$. One could have used the so-called ‘hybrid’ formfactors using both lattice QCD and light-cone QCD [20]. However, they are completely consistent with the numbers quoted above, and the $B \to \eta K$ BRs are only mildly affected by the latter choice. Moreover, the SM spread in these BRs is more or less taken care of by a scan over $N_C$.

The CP asymmetry for $B \to J/\psi K_S$ is not modified by our choice of RPV parameters (more on this in the next section). This helps us to take the SM value of the angle $\beta$ to be given by [24]

\[
\sin(2\beta) = 0.79 \pm 0.10.
\]

(9)

We show all our results with the central value of $\sin(2\beta)$, since the uncertainty has negligible effect over the final results.

For the $\eta K$ modes, the data reads [3, 6, 25, 26]:

\[
\begin{align*}
Br(B^+ \to \eta' K^+) &= (75 \pm 7) \times 10^{-6} \\
Br(B^+ \to \eta K^{*+}) &= (25.4 \pm 5.6) \times 10^{-6} \\
Br(B^0 \to \eta' K^0) &= (58_{-13}^{+14}) \times 10^{-6} \\
Br(B^0 \to \eta K^{*0}) &= (16.41 \pm 3.21) \times 10^{-6} \\
Br(B^+ \to \eta K^+) &< 6.9 \times 10^{-6} \\
Br(B^+ \to \eta' K^{*+}) &< 35 \times 10^{-6} \\
Br(B^0 \to \eta K^0) &< 9.3 \times 10^{-6} \\
Br(B^0 \to \eta' K^{*0}) &< 24 \times 10^{-6} \\
A_{CP}(B^0 \to \eta' K^\pm) &= 0.11 \pm 0.11 \pm 0.02 \ (\text{BaBar}) \\
A_{CP}(B^0 \to \eta K^\pm) &= 0.015 \pm 0.070 \pm 0.009 \ (\text{Belle}) \\
A_{dir}(B \to \eta' K_S) &= -0.13 \pm 0.32_{-0.09}^{+0.06} \ (\text{Belle}) \\
A_{mis}^{mff}(B \to \eta' K_S) &= -0.28 \pm 0.55_{-0.08}^{+0.07} \ (\text{Belle})
\end{align*}
\]
where

\[ A_{CP}(B^\pm \to \eta' K^\pm) = \frac{\Gamma(B^+ \to \eta' K^+) - \Gamma(B^- \to \eta' K^-)}{\Gamma(B^+ \to \eta' K^+) + \Gamma(B^- \to \eta' K^-)} \]

\[ A_{CP}^{\text{dir}} = \frac{1 - |\lambda|^2}{1 + |\lambda|^2} \]

\[ A_{CP}^{\text{mix}} = \frac{2Im \lambda}{1 + |\lambda|^2} \]

(11)

with

\[ \lambda = e^{-i\phi_M} \frac{\langle \eta' K_S | B^0 \rangle}{\langle \eta K_S | B^0 \rangle} \]

(12)

\( \phi_M \) being the mixing phase in the \( B^0 - \overline{B^0} \) box (\( 2\beta \) in the SM). Note that BaBar uses a convention which differs by a minus sign from our convention of \( A_{CP} \) (see eq. (1) of [25]), and the convention of Belle differs by a minus sign too, in both direct and mixing-induced CP asymmetries. Also note that if \( B \to \eta' K_S \) were a pure \( b \to s \) penguin, the mixing-induced CP asymmetry would have been \( \sin(2\beta) \); but the presence of a tree-level \( b \to u \) us term makes the calculation more complicated. That is why we have used the data from [26] which gives directly the direct and mixing-induced CP asymmetries, rather than [5], which quotes an effective value of \( \sin(2\beta) = 0.76 \pm 0.36 \).

Among these data, the measured BRs are definitely higher than the theoretical predictions; the discrepancy is about a factor of 2 to 3 for the \( \eta' K \) modes and almost a factor of 10 for the \( \eta K^* \) modes within the NF model (this depends on the choice of the regularization scale, and the effective number of colors). The expressions for the amplitudes can be found in the appendix, and the numerical values of the theoretically expected BRs in tables 8 and 10, of [20]. However, there are other models, as the perturbative QCD [21], which predict a dynamical enhancement of the penguin amplitude due to the higher-twist corrections. Judging by the enhancement one may get in these models for charmless modes, one expects at most to gain a factor of 2 over the factorization amplitude. This, evidently, does not serve our purpose. Let us just point out again that we take neither any charm content in \( \eta \) or \( \eta' \) nor any anomalous \( \eta(\eta') \) coupling with gluon. The CP asymmetry data are consistent with the SM prediction; if one assumes these decays to be penguin dominated as a first approximation, the direct CP asymmetry should be zero, whereas the mixing-induced CP asymmetry should be just \( \sin(2\beta) \). The tree-level charged current operators may change that prediction.

The modes \( B \to \phi K \) (both neutral and charged) have also been measured, and the averaged BRs are [6]:

\[ Br(B^+ \to \phi K^+) = (8.58 \pm 1.24) \times 10^{-6}, \]

\[ Br(B^0 \to \phi K^0) = (8.72 \pm 1.37) \times 10^{-6}, \]

(13)

where the near equality is expected from the isospin symmetry. The direct CP asymmetry is measured by BaBar [25] for the charged mode and by Belle [5] for the neutral mode:

\[ A_{CP}(B^{\pm} \to \phi K^{\pm}) = 0.05 \pm 0.20 \pm 0.03 \]

\[ A_{CP}(B \to \phi K_S) = 0.56 \pm 0.41 \pm 0.12. \]

(14)

Note again that both BaBar and Belle conventions differ from ours in an extra minus sign. The mixing-induced CP asymmetry, which should give \( \sin(2\beta) \) to a very good approximation, is (at 1\( \sigma \)) [4, 5]:

\[ \sin(2\beta)_{B \to \phi K_S} = -0.19^{+0.52}_{-0.50} \pm 0.09 \quad \text{(BaBar)} \]

\[ \sin(2\beta)_{B \to \phi K_S} = -0.73 \pm 0.64 \pm 0.18 \quad \text{(Belle)}. \]

(15)
Though their central values are not compatible, one may still make an average:

$$\sin(2\beta)_{\text{ave}} = -0.39 \pm 0.41.$$  \hspace{1cm} (16)

We use the averaged values of BR and mixing-induced CP asymmetries as our input parameters. For the direct CP asymmetry, we use the Belle data, but it may be observed later that the in the entire allowed parameter space the direct CP asymmetry is rather small (between 0.13 and 0.25) so that the BaBar numbers are not in trouble.

The BR for the $B \to \phi K$ mode is a sensitive function of $N_c$. It has also been shown in [21] that one needs to take into account the annihilation and nonfactorizable contributions, which push the BR up by almost a factor of 2. While the BR is in perfect agreement with the NF model, there is no model which can explain the data on mixing-induced CP asymmetry without invoking new physics. For this purpose, in our analysis, we use the NF model to calculate the BR keeping $N_c$ a free parameter, which we take to be the same for both $\phi K$ and $\eta K$ modes just for simplicity. We will also comment on what happens when one relaxes the BR constraint on $\phi K$.

\section{R-parity Violating Supersymmetry}

R-parity is a global quantum number, defined as $(-1)^{3B+L+2S}$, which is $+1$ for all particles and $-1$ for all superparticles. In the minimal version of supersymmetry and some of its variants, R-parity is assumed to be conserved \textit{ad hoc}, which prevents single creation or annihilation of superparticles. However, models with broken R-parity can be constructed naturally, and such models have a number of interesting phenomenological consequences. The crucial point is that unlike most extensions of the SM, RPV contributes to $B$ decay amplitudes at the tree level. Moreover, the current bounds [14] on sparticle masses and couplings leave open the possibility that such contributions can indeed be comparable to or even larger than the SM amplitude. It may be noted that the presence of two interfering amplitudes of comparable magnitude is essential for a large deviation of CP asymmetries from the SM prediction.

It is well known that in order to avoid rapid proton decay one cannot have both lepton number and baryon number violating RPV model, and we shall work with a lepton number violating one. This leads to slepton/sneutrino mediated $B$ decays. Since the current lower bound on the slepton mass is weaker than that on squark mass, larger effects within the reach of current round of experiments are more probable in this scenario. We start with the superpotential

$$W_N' = \lambda'_{ijk} L_i Q_j D^c_k,$$  \hspace{1cm} (17)

where $i,j,k = 1,2,3$ are quark and lepton generation indices; $L$ and $Q$ are the $SU(2)$-doublet lepton and quark superfields and $D^c$ is the $SU(2)$-singlet down-type quark superfield respectively. This leads to a four-Fermi hamiltonian relevant for $B$ decays [15]:

$$H_R = \frac{1}{4} d^R_{jkn}(\bar{d}_n \gamma^\mu P_L d_j) \bar{b}(\bar{d}_k \gamma_\mu P_R b) + \frac{1}{4} d^L_{jkn}(\bar{d}_n \gamma^\mu P_L b) \bar{d}(\bar{d}_k \gamma_\mu P_R d_j) + \frac{1}{4} u^R_{jnk}(\bar{u}_n \gamma^\mu P_L u_j) \bar{b}(\bar{d}_k \gamma_\mu P_R b) + \text{H.c.}$$  \hspace{1cm} (18)

where

$$d^R_{jkn} = \sum_i \lambda'_{i j k} \lambda'_{i m n} \frac{1}{2m^2_{\nu_{Li}}}, \quad d^L_{jkn} = \sum_i \lambda'_{i j k} \lambda'_{i m n} \frac{1}{2m^2_{\nu_{Li}}}, \quad u^R_{jnk} = \sum_i \lambda'_{i j k} \lambda'_{i m n} \frac{1}{2m^2_{\nu_{Li}}}.$$  \hspace{1cm} (19)
and the subscript 8 indicates that the currents are in color SU(3) octet-octet combination.

Following the standard practice we shall assume that the RPV couplings are hierarchical i.e., only one combination of the couplings is numerically significant. Let us note that both the transitions \( B \rightarrow \eta K^{(*)} \) and \( B \rightarrow \phi K \) are controlled by the quark-level transitions \( b \rightarrow s \bar{s}s \). Thus, let us assume, to start with, only \( d_{R,222}^{L} \) and \( d_{L,222}^{R} \) to be nonzero, as has been done in [18]. Of course, \( \eta K \) modes can be fed by \( b \rightarrow u \bar{s}s \) and \( b \rightarrow d \bar{d}s \) transitions. Since they affect other decay modes like \( B \rightarrow \pi K \) where there is no apparent discrepancy with SM expectations, we assume those operators to be vanishing.

Next let us discard \( d_{R,222}^{L} \) too. The reason for this is that \( d_{R,222}^{L} \) and \( u_{R,222}^{R} \) are related by SU(2) symmetry, and are the same if we neglect the electroweak D-term that causes the sneutrino-slepton mass splitting (on the other hand, \( d_{R,222}^{L} \) and \( d_{R,222}^{L} \) are completely unrelated, and unless there is some underlying texture in the RPV couplings, there is no reason why they should be equal). However, presence of \( u_{R,222}^{R} \) generates \( b \rightarrow c \bar{s}s \) transition, which in turn affects the modes like \( B \rightarrow J/\psi K \) which is used as a standard to extract \( \sin(2\beta) \). Since the values of \( \sin(2\beta) \) extracted from different charmonium modes, as well as from the \( J/\psi \pi^0 \) mode with a different quark-level process, are almost the same [5], it is a safe assumption, also compatible with the principle of Occam’s razor, to have no RPV contribution to that channel. Thus, the value of \( \beta \) extracted from \( B \rightarrow J/\psi K \) can be taken to be the SM value for that angle. The product coupling \( d_{L,222}^{R} \) does not contribute to the \( B^0 - \bar{B}^0 \) box, so that there is no scope to have an extra box amplitude, in contrast to the situation, e.g., in [10] (but \( d_{L,222}^{R} \) contributes to the \( B_s \) box; this is discussed later). The QCD corrections are easy to implement: the short-distance QCD corrections enhance the \((S - P) \times (S + P)\) RPV operator by approximately a factor of 2 while running from the slepton mass scale (assumed to be at 100 GeV) to \( m_b \) [27].

The RPV amplitude for \( B \rightarrow \eta' K \) is given by

\[
M_{\eta' K}^R = \frac{1}{4} d_{222}^{R} R_1 \left( A_{\eta' K}^u - A_{\eta' K}^u \right) - \frac{1}{N_c} A_{\eta' K}^u
\]

and that for \( B \rightarrow \eta K^* \) is

\[
M_{\eta K^*}^R = \frac{1}{4} d_{222}^{L} R_2 \left( A_{\eta K^*}^u - A_{\eta K^*}^u \right) - \frac{1}{N_c} A_{\eta K^*}^u
\]

where

\[
R_1 = \frac{m_{\eta'}^2}{m_s(m_b - m_s)}
\]
\[
R_2 = -\frac{m_{\eta}^2}{m_s(m_b + m_s)}
\]

and

\[
A_{\eta' K}^u = f_{\eta'}^u \Gamma_0^{B \rightarrow K} (m_{\eta'}^2)(m_B^2 - m_K^2)
\]
\[
A_{\eta K^*}^u = 2f_{\eta}^u m_K \cdot A_0^{B \rightarrow K^*} (m_{\eta}^2)(\epsilon_{K^*} \cdot \eta).
\]

For \( B \rightarrow \phi K \) (both neutral and charged channels), the RPV amplitude is

\[
M_{\phi K}^R = \frac{1}{4N_c} d_{222}^{L} A_{\phi K}
\]
with

\[ A_{\phi K} = 2f_0 m_\phi F_0^{B \to K}(m_\phi^2)(e_{\phi \cdot pK}) \] (25)

All these amplitudes are calculated at the slepton mass scale, and, as stated earlier, should be multiplied roughly by a factor of 2 when we compute their effects at \( m_b \).

The product coupling \( d_{222}^L \) can in general be complex, which we write as

\[ d_{222}^L = |d_{222}^L| \exp(i\phi_R). \] (26)

In our analysis we vary this phase over the range 0 to \( \pi \), and include the effects of \( \pi \leq \phi_R \leq 2\pi \) by allowing \( |d_{222}^L| \) to take both positive and negative values.

For generic \( B \to \eta K \) modes, even the SM has two factorizable amplitudes, tree and penguin. Following the color transparency argument which predicts the strong phase difference \( \Delta \delta_{SM} \) between them to be small [9, 28], we take \( \Delta \delta_{SM} = 0 \). We vary the strong phase difference \( \Delta \delta \) between the SM and the RPV amplitudes over a range of 0 to 2\( \pi \), but expect to find solutions allowing \( \Delta \delta \) to be near 0 or 2\( \pi \), which is theoretically pleasing. Note that if all strong phase differences are exactly zero, there should not be any direct CP violation; indeed, with color transparency expectations, we should get a small \( A_{dir}^{CP} \), which anyway is perfectly allowed by data. For simplicity we take \( \Delta \delta \) to be the same for all channels to be considered.

4 The Analysis

Our input parameters are specified in Sections 2 and 3. We scan the CKM element \( V_{ub} \), which has an almost 30% uncertainty, over its entire range. We also vary \( 1/N_c \) from 0.1 to 1. The weak phase \( \phi_R \) associated with \( d_{222}^L \) is scanned over 0 to \( \pi \) and the strong phase difference \( \Delta \delta \) between SM (tree or penguin) and RPV over 0 to 2\( \pi \) (we, however, present our results for the range \( -\pi/6 < \Delta \delta < \pi/6 \), motivated by the color-transparency argument). The CKM angle \( \gamma \) is varied between 0 and \( \pi - \beta \) where \( \beta = 0.5 \arcsin(A_{CP}(B \to J/\psi K_S)) \).

The following constraints were applied: (i) BR for the modes \( \eta'K^+, \eta'K^0, \eta K^{*+}, \eta K^{*0} \) and \( \phi K_S \), (ii) the direct CP asymmetry for \( B^\pm \to \eta'K^\pm \) from BaBar, (iii) the direct CP asymmetry for \( B \to \phi K_S \) from Belle, and (iv) the average value of BaBar and Belle for sin(2\( \beta \)) extracted from \( B \to \phi K_S \). Constraints (ii) and (iii) are applied with the rationale that they have larger error bars and we wish to check whether the data from the other experiment with smaller errors can be accomodated. The direct and mixing-induced CP asymmetries for \( B \to \eta'K_S \) are not imposed as constraints but one can easily check from the figures that most of the allowed region is perfectly compatible with the data.

Our results are shown in figures 1-4. Let us note the salient features of the analysis.

• It is known that the nonfactorizable effects in these decays can be substantial. To account for that, we have taken \( N_c \) as a free parameter, and not stuck to its QCD value of 3. However, the Wilson coefficients are evaluated with \( N_c = 3 \). It appears that there is a significant nonfactorizable contribution in \( B \to \phi K \), since we have obtained the fit only for \( 0.15 \leq 1/N_c \leq 0.25 \). Note that \( \eta K \) channels are \( N_c \) stable, so the constraint only comes from the \( \phi K \) mode. This is in conformity with the analysis in [21].

• There are two possible bands of solutions, as can be seen from Fig. 1. The left-hand side band, with more points and more width, is for negative values of \( d_{222}^L \) and \( \phi_R \) in the first
Figure 1: The allowed parameter space for $\lambda_{i32}^\prime \lambda_{i22}^\prime$ and $\phi_{R}^k$. For more details, see text.

Figure 2: One can indeed have solutions with small difference in strong phases of the SM and the RPV amplitudes.
Figure 3: The range of $\sin(2\beta)$ as extracted from the $B \rightarrow \phi K_S$ decay.

Figure 4: The mixing-induced CP asymmetry for $B \rightarrow \eta' K_S$. Note that upper half of Band II is disallowed from the Belle data.
quadrant (this we will call Band I). There is a second narrow band (Band II) for positive values of $d_{22}^L$ and $\phi_R$ in the second quadrant. This shape is essentially controlled by the BRs and CP asymmetries of different $\eta K$ channels.

We found a much stronger constraint on the product coupling $|\lambda_{i32}^* \lambda_{i22}^*|$

\[
|\lambda_{i32}^* \lambda_{i22}^*| \leq 2.3 \times 10^{-3}.
\]  

(27)

If we take all these experimental data seriously, it is possible to get even an upper bound:

\[
|\lambda_{i32}^* \lambda_{i22}^*| \geq 1.3 \times 10^{-3}.
\]  

(28)

The present bound, as quoted in [29] for the third slepton generation, is only 0.23, and that too assuming squarks at 100 GeV. If one has 300 GeV squarks, the bound gets weaker by a factor of 9. However, one gets a comparatively better constraint from $B_s - \overline{B_s}$ mixing, since nonzero values of $\lambda_{i32}^*$ and $\lambda_{i22}^*$ can generate a second amplitude for the box, with two sneutrinos and two right-handed strange quarks flowing inside the box. Taking $\Delta m_{B_s}$ to be completely saturated by the RPV contribution, and using the experimental lower limit, one gets a bound on the product coupling which is approximately $1.5 \times 10^{-2}$. The main loophole in the analysis is the fact that only an experimental upper bound can generate an upper bound on some unknown parameter; moreover, it is questionable to neglect the SM contribution completely. Even then we have an improvement by an order of magnitude.

• Figure 2 shows that $\Delta \delta$ should be positive for Band I and negative for Band II. It can be checked easily that this ensures a positive direct CP asymmetry in all channels, and a negative mixing-induced CP asymmetry in $B \to \phi K_S$. Figure 3 shows the range of $\sin(2\beta)$ in this model, which is between the upper limit of 0.02 and $-0.6$. The central value of Belle, however, cannot be reproduced.

• Figure 4 shows the mixing-induced CP asymmetry for $B \to \eta' K_S$. Note that imposition of the Belle data means a significant portion (the region in the upper right-hand corner) of the already weak Band II is ruled out, while Band I is completely allowed. Still, one cannot rule out Band II completely.

• The CP asymmetry in $B^\pm \to \eta' K^\pm$ is found to lie between 0.115 and 0.22 (the upper limit) for Band I, and between 0.14 and 0.22 for Band II. The direct CP asymmetry in the neutral channel lies between 0.13 to 0.24. This is definitely compatible with the Belle data, but again the central value is far away. One must wait for the error bars to come down.

• The direct CP asymmetry for $B \to \phi K_S$ lies between 0.13 and 0.25 for Band I and between 0.13 and 0.22 for Band II. This is in perfect harmony with the BaBar data too.

Let us try to understand why we get a nonzero $A_{CP}^{dir}$. This is due to the constraint put by $A_{cp}^{mix}(B \to \phi K)$. To see this qualitatively, let us assume that there is only one SM amplitude and both weak and strong phase differences vanish so that there is no direct CP asymmetry. It is easy to see that in that case there is no change in the prediction for $\sin(2\beta)$, which is given only by the phase in the $B^0 - \overline{B^0}$ box.

• The angle $\gamma$ can take any value upto $\pi - \beta$ for Band I. On the other hand, it can be only in the second quadrant for Band II. The origin of such a pattern can easily be traced back to the expressions of BRs and CP asymmetries of the $\eta K$ modes. Ref. [22] estimates $40^\circ < \gamma < 78^\circ$ at 95% CL from different $B \to \pi\pi$ and $B \to K\pi$ modes. These modes are not affected.
by $d_{222}^R$, so one concludes that if there are no other nonzero RPV couplings, Band II is completely ruled out. Thus, if the value of $\sin(2\beta)$ extracted from $B \to \eta'K_S$ converges towards that found from $B \to J/\psi K_S$, this solution will be in trouble. However, the error bars are too large to draw any definite conclusion right now.

- As a check, we redo the analysis switching the $B \to \phi K$ BR constraint off. There is no substantial change in the result, except that the lower bound on $A_{CP}(B^\pm \to \eta'K^\pm)$ marginally decreases to 0.11. Thus we consider our result to be fairly robust with respect to hadronic uncertainties. However, there is no upper limit on $1/N_c$ anymore.

We have also explicitly checked that BRs of the so far unobserved $\eta K$ channels remain below their experimental upper bounds.

5 Summary and Conclusion

We found that a minimal set of RPV couplings, compatible with all present data, can explain the BR anomalies for $B \to \eta'K$ and $B \to \eta K^*$, and the unusual value of $\sin(2\beta)$ from $B \to \phi K_S$. This is mainly due to the fact that RPV contributes to B decays at tree-level. A nice feature is that one can have points where the strong phase difference between RPV and SM is small, which is what one expects from the color transparency argument.

With more data pouring in, one can significantly shrink the allowed parameter space for RPV. However, even at present we get sufficiently strong bound on the relevant product coupling — better by two orders of magnitude at least.

What other effects are mediated by the product coupling $\lambda_{322}^i \lambda_{i22}^j$? This generates a top decay channel $t \to c \bar{s}s$, whose strength is, unfortunately, only a few per cent of the corresponding SM channel. Thus, the signal is essentially unobservable. A better signal may come from the strange squark mediated semileptonic top decays $t \to c \ell^+ \ell^- (\ell = e, \mu, \tau)$. This also generates $b \to s u \bar{\nu}$ or $B_s \to \nu \bar{\nu}$ decays. The third effect is a new box amplitude in $B_s - \overline{B_s}$ mixing. Here, also, the amplitude is only at a few per cent level compared to the SM amplitude, so we do not expect any significant CP asymmetry in channels like $B_s \to J/\psi \phi$ even if $\phi_R$ is large.

We have not discussed the product coupling $d_{222}^R$ which can also explain the anomalies that are studied in this paper. As we mentioned before, that coupling also generates the SU(2) conjugate transition $b \to c \bar{s}s$. This may jeopardize the predictions from the $B \to J/\psi K_S$ channel, for example. However, a nonzero phase in $d_{222}^R$ should show up in the $B_s$ box, i.e., one may get a significant CP-asymmetry in the channel $B_s \to J/\psi \phi$, contrary to the SM expectation. The reason is simple: RPV contributes to both the box amplitude and to the decay $b \to c \bar{s}s$. The rare decays $b \to s \ell^+ \ell^-$ or $B_s \to \ell^+ \ell^-$ also receive a tree-level contribution from $d_{222}^R$ and may be pushed up to the observable level.

At present, one has to wait for the errors to come down. This may rule out part or all of one or both bands (existence of Band II is already under threat from the fit of $\gamma$, as we have shown). However, if the bands still remain allowed, one should look for any unexpected CP asymmetry signal in $B_s$ decays. Only such correlated studies can unravel the exact nature of new physics.

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