Triangle Attack: A Query-efficient Decision-based Adversarial Attack

Xiaosen Wang¹², Zeliang Zhang¹, Kangheng Tong¹, Dihong Gong², Kun He¹, Zhifeng Li², Wei Liu²
¹School of Computer Science and Technology, Huazhong University of Science and Technology
²Data Platform, Tencent

Abstract

Decision-based attack poses a severe threat to real-world applications since it regards the target model as a black box and only accesses the hard prediction label. Great efforts have been made recently to decrease the number of queries; however, existing decision-based attacks still require thousands of queries in order to generate good quality adversarial examples. In this work, we find that a benign sample, the current and the next adversarial examples could naturally construct a triangle in a subspace for any iterative attacks. Based on the law of sines, we propose a novel Triangle Attack (TA) to optimize the perturbation by utilizing the geometric information that the longer side is always opposite the larger angle in any triangle. However, directly applying such information on the input image is ineffective because it cannot thoroughly explore the neighborhood of the input sample in the high dimensional space. To address this issue, TA optimizes the perturbation in the low frequency space for effective dimensionality reduction owing to the generality of such geometric property. Extensive evaluations on the ImageNet dataset demonstrate that TA achieves a much higher attack success rate within 1,000 queries and needs a much less number of queries to achieve the same attack success rate under various perturbation budgets than existing decision-based attacks. With such high efficiency, we further demonstrate the applicability of TA on real-world API, i.e., Tencent Cloud API.

1. Introduction

Despite the unprecedented progress of Deep Neural Networks (DNNs) [20, 21, 23], the vulnerability to adversarial examples [17, 39] poses serious threats to security-sensitive applications, e.g., face recognition [34], autonomous driving [16], etc. To securely deploy DNNs in various applications, it is necessary to conduct an in-depth analysis on the intrinsic properties of adversarial examples, which has inspired numerous researches on adversarial attacks [3–6, 9, 12, 14, 29, 30, 42] and defenses [19, 28, 36, 44, 45, 49]. Existing attacks could be split into two categories: white-box attack has full knowledge of the target model (often leveraging the gradient [5, 14, 17, 28]) while black-box attack could only access the model output, which is more applicable in real-world scenarios. The black-box attack could be implemented in different ways. Transfer-based attack [14, 26, 42, 47] adopts the adversarial examples generated on the substitute model to attack the target model. Score-based attack [2, 7, 22, 50] assumes that the attacker could access the output logits while decision-based (a.k.a. hard label) attack [4, 8, 9, 25, 29] only has access to the prediction (top-1) label.

Among various black-box attacks, decision-based attack is much more challenging and practical due to the minimum information requirement for attack. The number of queries on the target model often plays a significant role in decision-based attack, because the access to a victim model is usually restricted in practice. Though recent works manage to reduce the total number of queries from millions to thousands of requests [4, 25, 31], it is still insufficient for most of the practical applications [29].

Existing decision-based attacks [4, 25, 29, 31] first generate a large adversarial perturbation and then minimize
the perturbation while keeping its adversarial property by various optimization methods. As shown in Fig. 1, we find that at the \( t \)-th iteration, the benign sample \( x \), current adversarial example \( x_{t}^{adv} \), and next adversarial example \( x_{t+1}^{adv} \) could naturally construct a triangle for any iterative attacks. According to the law of sines, the adversarial example \( x_{t}^{adv} \) at the \((t+1)\)-th iteration should satisfy \( \beta_{t} + 2\alpha_{t} > \pi \) to guarantee that the perturbation decreases, i.e., \( \delta_{t+1} = \|x_{t+1}^{adv} - x\|_{p} < \delta_{t} = \|x_{t}^{adv} - x\|_{p} \) (when \( \beta_{t} + 2 \cdot \alpha_{t} = \pi \), it would be an isosceles triangle, i.e., \( \delta_{t+1} = \delta_{t} \)).

Based on the above geometric property, we propose a novel and query-efficient decision-based attack, called Triangle Attack (TA). Specifically, at the \( t \)-th iteration, we randomly select a directional line across the benign sample \( x \) to determine a subspace, in which we iteratively construct the triangle with the current adversarial example \( x_{t}^{adv} \), benign sample \( x \), learned angle \( \alpha_{t} \), and searched angle \( \beta_{t} \) until the third vertex of the constructed triangle is adversarial. Using the geometric information, we could conduct TA in the low frequency space generated by Discrete Cosine Transform (DCT) [1] for effective dimensionality reduction to improve the efficiency. And we further update \( \alpha_{t} \) to adapt to the perturbation optimization for each constructed triangle. Different from most existing decision-based attacks, there is no need to restrict \( x_{t}^{adv} \) on the decision boundary or estimate the gradient at each iteration in TA, making TA query-efficient.

Our contributions are summarized as follows:

- To the best of our knowledge, this is the first work that directly optimizes the perturbation in the frequency space via the geometric information, which significantly improves the query efficiency.
- Extensive evaluations on the ImageNet dataset show that TA exhibits a much higher attack success rate within 1,000 queries and needs a much less number of queries to achieve the same attack success rate under the same perturbation budget on various models than existing decision-based attacks [6, 9, 10, 25, 29, 31].
- TA generates more adversarial examples with imperceptible perturbations on Tencent Cloud API as shown in Sec. 4.3, showing its industrial-grade applicability.

2. Related Work

Since Szegedy et al. [39] identified adversarial examples, massive adversarial attacks have been proposed to fool DNNs. White-box attacks, e.g., single-step gradient-based attack [17], iterative gradient-based attack [12, 24, 28, 30], and optimization-based attack [3, 5, 39], often utilize the gradient and exhibit good attack performance. They have been widely adopted for evaluating the model robustness of defenses [11, 13, 28, 33, 49], but are hard to be applied in real-world limited information. To make adversarial attacks applicable in practice, various black-box attacks, including transfer-based attack [14, 41, 42, 46, 47], score-based attack [2, 7, 15, 22, 40, 48, 50], and decision-based attack [4, 6, 10, 29, 31], have gained increasing interest. Among them, decision-based attack is most challenging because it could only access the prediction label. In this work, we focus on boosting the query efficiency of decision-based attack by utilizing the geometric information and provide a brief overview of existing decision-based attacks below.

BoundaryAttack [4] is the first decision-based attack, which starts from a large adversarial perturbation and performs random walks on the decision boundary while keeping adversarial. Such a paradigm has been widely adopted in the subsequent decision-based attacks. OPT [9] formulates the decision-based attack as a real-valued optimization problem with zero-order optimization for adversarial examples. And SignOPT [10] further computes the sign of the directional derivative instead of the magnitude for fast convergence. HopSkipJumpAttack (HSJA) [6] boosts BoundaryAttack by estimating the gradient direction via binary information at the decision boundary. QEBA [25] enhances HSJA for better gradient estimation using the perturbation sampled from various subspaces, including spatial, frequency, and intrinsic components. To further improve the query efficiency, qFool [27] assumes that the curvature of the boundary is small around adversarial examples and adopts several perturbation vectors for efficient gradient estimation. GeoDA [31] approximates the local decision boundary by a hyperplane and searches the closest point to the benign sample on the hyperplane as the adversary. Surffree [29] iteratively constructs a circle on the decision boundary and adopts binary search to find the intersection of the constructed circle and decision boundary as the adversary without any gradient estimation.

Most existing decision-based attacks restrict the adversarial example at each iteration on the decision boundary and usually adopt different gradient estimation approaches for attack. In this work, we propose Triangle Attack to minimize the adversarial perturbation in the low frequency space by utilizing the law of sines without gradient estimation or restricting the adversarial example on the decision boundary for the efficient decision-based attack.

3. Methodology

In this section, we first provide the preliminaries and describe two geometry-inspired decision-based attacks. Then we introduce our motivation and the proposed Triangle Attack (TA).

3.1. Preliminaries

Given a target classifier \( f \) with parameters \( \theta \) and a benign sample \( x \in X \) with the ground-truth label \( y \in Y \), where \( X \) denotes all the legitimate images and \( Y \) is the output space.
The adversarial attack aims to find an adversarial example \( x^{adv} \) that misleads the target model:

\[
  f(x^{adv}; \theta) \neq f(x; \theta) = y \quad \text{s.t.} \quad \|x^{adv} - x\|_p < \epsilon,
\]

where \( \epsilon \) is the perturbation budget. Decision-based attacks usually first generate a large adversarial perturbation \( \delta \) and then minimize the perturbation as follows:

\[
  \min \|\delta\|_p \quad \text{s.t.} \quad f(x + \delta; \theta) \neq f(x; \theta) = y. \tag{1}
\]

Existing decision-based attacks \([9, 10, 25]\) often estimate the gradient or directly optimize the perturbation. Here we provide a detailed introduction of two geometry-inspired decision-based attacks.

**GeoDA** \([31]\) argues that the decision boundary at the vicinity of a data point \( x \) could be locally approximated by a hyperplane passing through a boundary point \( x_B \) close to \( x \) with a normal vector \( w \). Thus, the optimization problem in Eq. (1) could be locally linearized:

\[
  \min \|\delta\|_p \quad \text{s.t.} \quad w^\top (x + \delta) - w^\top x_B = 0.
\]

Here \( x_B \) is a data point on the boundary, which can be found by binary search with several queries, and GeoDA randomly samples several data points for estimating \( w \) to optimize the perturbation at each iteration.

**Surfree** \([29]\) assumes that the decision boundary could be approximated by a hyperplane locally around a boundary point \( x + \delta \). And Surfree adopts polar coordinates to represent the adversarial example at each iteration and searches an optimal \( \theta \) to update the perturbation as follows:

\[
  \delta_{t+1} = \delta_t \cos \theta (u \cos \theta + v \sin \theta),
\]

where \( u \) is the unit vector from \( x \) to \( x_t^{adv} \) and \( v \) is the orthogonal vector of \( u \).

### 3.2. Motivation

Different from most decision-based attacks with gradient estimation \([9, 10, 25, 31]\) or random walk on the decision boundary \([4, 29]\), we aim to optimize the perturbation using the geometric property without any queries for gradient estimation. After generating a large adversarial perturbation, the decision-based attacks move the adversarial example close to the benign sample, *i.e.*, decrease the adversarial perturbation \( \delta_t \), while keeping the adversarial property at each iteration. In this work, as shown in Fig. 1, we find that at the \( t \)-th iteration, the benign sample \( x \), current adversarial example \( x_t^{adv} \) and next adversarial example \( x_{t+1}^{adv} \) could naturally construct a triangle in a subspace for any iterative attacks. Thus, searching for the next adversarial example \( x_{t+1}^{adv} \) with smaller perturbation is equivalent to searching for a triangle based on \( x \) and \( x_t^{adv} \), in which the third data point \( x' \) is adversarial and satisfies \( \|x' - x\|_p < \|x_t^{adv} - x\|_p \). This inspires us to utilize the relationship between the angle and side length in the triangle to search an appropriate triangle to minimize the perturbation at each iteration. As shown in Sec. 4.4, however, directly applying such a geometric property on the input image leads to very poor performance.

Owing to the generality of such a geometric property, we optimize the perturbation in the low frequency space generated by DCT \([1]\) for effective dimensionality reduction.

Moreover, since Brendel et al. \([4]\) proposed Boundary-Attack, most decision-based attacks \([6, 9, 10, 29, 31]\) follow the setting in which the adversarial example at each iteration should be on the decision boundary. We argue that such a restriction is not necessary in decision-based attacks but introduces too many queries on the target model to approach the boundary. Thus, we do not adopt this constraint in this work and validate this argument in Sec. 4.4.

### 3.3. Triangle Attack

In this work, we have the following assumption for any deep neural classifier \( f \):

**Assumption 1.** Given a benign sample \( x \) and a perturbation budget \( \epsilon \), there exists an adversarial perturbation \( \|\delta\|_p \leq \epsilon \) towards the decision boundary which could mislead the target classifier \( f \).

This is a general assumption that we could find the adversarial example \( x^{adv} \) for the input sample \( x \), which has been validated by numerous works \([3–5, 17, 43]\). If this assumption does not hold, the target model is ideally robust so that we cannot find any adversarial example within the perturbation budget, which is beyond our discussion. Thus, we follow the framework of existing decision-based attacks by first randomly crafting a large adversarial perturbation and then minimizing the perturbation. To align with previous works, we generate a random perturbation close to the decision boundary with binary search \([25, 29, 31]\) and mainly focus on the perturbation optimization.
In two arbitrary consecutive iterations of the perturbation optimization process for any adversarial attacks, namely $t$-th and $(t+1)$-th iterations without loss of generalization, the input sample $x$, current adversarial example $x_{t}^{adv}$ and the next adversarial example $x_{t+1}^{adv}$ could naturally construct a triangle in a subspace of the input space $X$. Thus, as shown in Fig. 1, decreasing the perturbation to generate $x_{t+1}^{adv}$ is equivalent to searching for an appropriate triangle in which the three vertices are $x$, $x_{t}^{adv}$ and $x_{t+1}^{adv}$, respectively.

**Theorem 1** (The law of sines). Suppose $a$, $b$ and $c$ are the lengths of the sides of a triangle, and $\alpha$, $\beta$ and $\gamma$ are the opposite angles, we have $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$.

From Theorem 1, we can obtain the relationship between the side length and opposite angle for the triangle in Fig. 1:

$$\frac{\delta_t}{\sin \alpha_t} = \frac{\delta_{t+1}}{\sin (\pi - (\alpha_t + \beta_t))}. \quad (2)$$

To greedily decrease the perturbation $\delta_t$, the $t$-th triangle should satisfy that $\frac{\delta_{t+1}}{\delta_t} = \frac{\sin (\pi - (\alpha_t + \beta_t))}{\sin \alpha_t} < 1$, i.e., $\pi - (\alpha_t + \beta_t) < \alpha$. Thus, decreasing the perturbation at the $t$-th iteration can be achieved by finding a triangle constructed by the input sample $x$, current adversarial example $x_{t}^{adv}$ and the angles $\beta_t$ and $\alpha_t$, which satisfy $\beta_t + \alpha_t > \pi$ and the third vertex should be adversarial. We denote such a triangle as candidate triangle and $T(x, x_t^{adv}, \alpha_t, \beta_t, S_t)$ as the third vertex, where $S_t$ is a sampled subspace. Based on this observation, we propose a novel decision-based attack called Triangle Attack (TA) that searches the candidate triangle at each iteration and adjusts the angle $\alpha_t$ accordingly.

**Sampling the 2-D subspace $S$ of frequency space.** The input image often lies in a high-dimensional space, such as $224 \times 224 \times 3$ for ImageNet [23], which is too large for the attack to explore the neighborhood for minimizing the adversarial perturbation efficiently. Previous works [18, 25, 29] have shown that utilizing the information in various subspaces could improve the efficiency of decision-based attacks. For instance, QEBA [25] samples the random noise for gradient estimation in the spatial transformed space or the low frequency space but minimizes the perturbation in the input space with the estimated gradient. Surfree [29] optimizes the perturbation in the subspace of the input space determined by a unit vector randomly sampled in the low frequency space. In general, the low frequency space contains the most critical information for image. With the poor performance of TA in the input space as shown in Sec. 4.4 and the generality of the geometric property, as shown in Fig. 2, we directly optimize the perturbation in the frequency space at each iteration for effective dimensionality reduction. And we randomly sample a $d$-dimensional line across the benign sample in the low frequency space (top 10%) to determine a 2-D subspace $S$ for constructing the triangle to optimize the perturbation.

**Searching the candidate triangle.** Given a subspace $S$, the candidate triangle only depends on the angle $\beta$ since $\alpha$ is updated during the optimization. As shown in Fig. 3, if we search an angle $\beta$ without leading to an adversarial example ($x_{t+1}^{adv}$), we could further construct a symmetric triangle with the same angle in the opposite direction to check the data point $x_{t+1}^{adv}$ which has the same magnitude of perturbation as $x_{t+1}^{adv}$ but different directions. We denote the angle as $-\beta$ for the symmetric triangle without ambiguity. Note that with the same angle $\alpha$, the larger angle $\beta$ would make the third vertex closer to the input sample $x$, i.e., smaller perturbation. After determining the subspace $S$, we first check the angle $\beta_0 = \max(\pi - 2\alpha, \beta)$, where $\beta = \pi/16$ is a pre-defined small angle. If neither $T(x, x_t^{adv}, \alpha, \beta_0, S_t)$ nor $T(x, x_t^{adv}, \alpha, -\beta_0, S_t)$ is adversarial, we give up this subspace because it cannot bring any benefit. Otherwise, we adopt binary search to find an optimal angle $\beta^* \in [\max(\pi - 2\alpha, \beta), \min(\pi - \alpha, \pi/2)]$ which is as large as possible to minimize the perturbation. Here we restrict the upper bound of $\beta$ because $T(x, x_t^{adv}, \alpha, \beta_0, S_t)$ would be at the opposite direction w.r.t. $x$ for $\beta > \pi/2$ and $\pi - \alpha$ guarantees a valid triangle.
Adjusting the angle $\alpha$. Intuitively, the angle $\alpha$ balances the magnitude of perturbation and the difficulty to find an adversarial example:

**Proposition 1.** With the same angle $\beta$, a smaller angle $\alpha$ makes it easier to find an adversarial example while a larger angle $\alpha$ leads to smaller perturbation.

Intuitively, as shown in Fig. 4, the smaller angle $\alpha$ results in larger perturbation but is more likely to cross over the decision boundary, making it easier to search an adversarial example, and vice versa. It is hard to consistently find an optimal $\alpha$ for each iteration, letting alone various input images and target models. Thus, we adaptively adjust the angle $\alpha$ based on the crafted adversarial example:

$$
\alpha_{t,i+1} = \begin{cases} 
\min(\alpha_{t,i} + \gamma, \pi/2 + \tau) & \text{if } f(x^{adv}_{t,i+1}; \theta) \neq y \\
\max(\alpha_{t,i} - \lambda \gamma, \pi/2 - \tau) & \text{otherwise} 
\end{cases}
$$

where $x^{adv}_{t,i+1} = T(x, x^{adv}_{t,i}, \alpha_{t,i}, \beta_{t,i}, S_{t})$ is the adversarial example generated by $\alpha_{t,i}$, $\gamma$ is the change rate, $\lambda$ is a constant, and $\tau$ restricts the upper and lower bounds of $\alpha$. We adopt $\lambda < 1$ to prevent decreasing the angle too quickly considering much more failures than successes during the perturbation optimization. Note that the larger angle $\alpha$ makes it more difficult to find an adversarial example. However, the too small angle $\alpha$ results in a much lower bound for $\beta$, which also makes $T(x, x^{adv}_{t,i}, \alpha, \beta, S_{t})$ far away from the current adversarial example $x^{adv}_{t,i}$, decreasing the probability to find an adversarial example. Thus, we add the bounds for $\alpha$ to restrict it in an appropriate range.

TA iteratively searches the candidate triangle in the subspace $S_{t}$ sampled from the low frequency space to find the adversarial example and update the angle $\alpha$ accordingly. The entire algorithm of TA is summarized in Algorithm 1.

### 4. Experiments

To validate the effectiveness of the proposed TA, we conduct extensive evaluations on the standard ImageNet dataset using five models and Tencent Cloud API. In this section, we first specify the experimental setup, and then we compare TA with various decision-based attacks on offline and online models. Experimental results demonstrate that TA could achieve much higher attack success rate within 1,000 queries and needs much fewer number of queries to achieve the same attack success rate than existing decision-based attacks. Finally, we further provide ablation studies on the hyper-parameters and further discussions.

#### 4.1. Experimental Setup

**Dataset.** To validate the effectiveness of the proposed TA, we randomly sample 200 correctly classified images from the ILSVRC 2012 validation set [32] for evaluation on the corresponding models.

---

### Algorithm 1: Triangle Attack

**Input:** Target classifier $f$ with parameters $\theta$; Benign sample $x$ with ground-truth label $y$; Maximum number of queries $Q$; Maximum number of iteration $N$ in each subspace; Dimension of the directional line $d$; Lower bound $\beta$ for $\beta$.

**Output:** An adversarial example $x^{adv}_{t}$.

1. Initialize a large adversarial perturbation $\delta_{0}$;
2. $x^{adv}_{0} = x + \delta_{0}$, $q = 0$, $t = 0$, $\alpha_{0} = \pi/2$;
3. while $q < Q$ do
4.  sampling 2-D space $S_{t}$ in the low frequency space;
5.  $\beta_{t,0} = \max(\pi/2, \pi - \alpha_{0})$;
6.  if $f(T(x, x_{t}^{adv}, \alpha_{0}, \beta_{t,0}, S_{t}); \theta) = f(x; \theta)$ then
7.      $q = q + 1$, update $\alpha_{t,0}$ based on Eq. (3);
8.      if $f(T(x, x_{t}^{adv}, \alpha_{t,0}, -\beta_{t,0}, S_{t}); \theta) = f(x; \theta)$ then
9.         $q = q + 1$, update $\alpha_{t,0}$ based on Eq. (3);
10.        Go to line 3; \quad \triangleright \text{ give up this subspace }
11.        $\beta_{t,0} = \min(\pi/2, \pi - \alpha_{t})$;
12.    for $i = 0 \rightarrow N$ do \quad $\triangleright \text{ binary search for } \beta$
13.       $\beta_{t,i+1} = (\beta_{t,i} + \beta_{t,i}/2)$;
14.       if $f(T(x, x_{t}^{adv}, \alpha_{t,i}, \beta_{t,i+1}, S_{t}); \theta) = f(x; \theta)$ then
15.           $q = q + 1$, update $\alpha_{t,i}$ based on Eq. (3);
16.           if $f(T(x, x_{t}^{adv}, \alpha_{t,i}, -\beta_{t,i+1}, S_{t}); \theta) = f(x; \theta)$ then
17.              $\beta_{t,i+1} = \beta_{t,i+1}, \beta_{t,i+1} = \beta_{t,i};$
18.         $q = q + 1$, update $\alpha_{t,i+1}$ based on Eq. (3);
19.         $x^{adv}_{t,i+1} = T(x, x_{t}^{adv}, \alpha_{t,i+1}, \beta_{t,i+1}, S_{t}), t = t + 1;$
20.    return $x^{adv}_{t}$.

---

### Models

We consider five widely adopted networks, i.e., VGG-16 [35], Inception-v3 [37], ResNet-18 [20], ResNet-101 [20] and DenseNet-121 [21]. To validate the practical applicability, we also evaluate TA on Tencent Cloud API¹.

### Baselines

We take various decision-based attacks as our baselines, including the gradient estimation based attacks, i.e., OPT [9], SignOPT [10], HSJA [6], QEBA [25], and geometry-inspired attacks, i.e., GeoDA [31], Surfree [29].

### Evaluation metrics

Following the standard setting in QEBA [25], we adopt the root mean squared error (RMSE) between benign sample $x$ and adversarial example $x^{adv}$ to measure the magnitude of perturbation:

$$
d(x, x^{adv}) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x[i] - x^{adv}[i])^2}, \quad (4)
$$

where $N$ is the size of the input image. We also adopt the attack success rate, the percentage of adversarial examples which reach a certain distance threshold.

### Hyper-parameters

For fair comparison, all the attacks adopt the same adversarial perturbation initialization ap-

---

¹https://cloud.tencent.com/
Table 1. Attack success rate (%) on five models under different RMSE thresholds. The maximum number of queries is set to 1,000. We highlight the highest attack success rate in bold.

| Model       | VGG-16 | Inception-v3 | ResNet-18 | ResNet-101 | DenseNet-121 |
|-------------|--------|--------------|-----------|------------|--------------|
| RMSE        | 0.1    | 0.05         | 0.01      | 0.1        | 0.05         | 0.01        |
| OPT         | 76.0   | 38.5         | 5.5       | 34.0       | 17.0         | 4.0         |
| SignOPT     | 94.0   | 57.5         | 12.5      | 50.5       | 27.0         | 8.0         |
| HSJA        | 92.5   | 58.5         | 13.0      | 32.5       | 14.0         | 4.0         |
| QBA         | 98.5   | 86.0         | 29.0      | 78.5       | 54.5         | 17.0        |
| GeoDA       | 99.0   | 94.0         | 35.0      | 80.0       | 61.5         | 23.5        |
| Surfree     | 99.5   | 92.5         | 39.5      | 87.5       | 67.5         | 24.5        |
| TA (Ours)   | 100.0  | 95.0         | 44.5      | 96.5       | 81.5         | 30.0        |

Number of queries (× 1,000)

![Figure 5](image)

Figure 5. Number of queries to achieve the given attack success rate on ResNet-18 for the attack baselines and the proposed TA under various perturbation budgets. The maximum number of queries is 10,000.

4.2. Evaluation on Standard Models

To evaluate the effectiveness of the proposed TA, we first compare the attack performance on five popular models with different decision-based attacks and report the attack success rate under various RMSE thresholds, namely 0.1, 0.05 and 0.01.

The attack success rate within 1,000 queries is summarized in Table 1, which means the attack would fail to generate adversarial example for the input image if it takes 1,000 queries without reaching the given threshold. We could observe that TA consistently achieves much better attack success rate than existing decision-based attacks under various perturbation budgets on five models with different architectures. For instance, TA outperforms the runner-up attack with a clear margin of 1.0%, 7.5% and 13.0% under the RMSE threshold of 0.1, 0.05, 0.01 on ResNet-101, which is widely adopted for evaluating the decision-based attacks. In particular, the proposed TA significantly outperforms the two geometry-inspired attacks, i.e., GeoDA [31] and Surfree [29], which exhibit the best attack performance among the baselines. This convincingly validates the high effectiveness of the proposed TA. Besides, among the five models, Inception-v3 [38], which is rarely investigated in decision-based attacks, exhibits better robustness than other models under various perturbation budgets against both baselines and TA. Thus, it is necessary to thoroughly evaluate the decision-based attacks on various architectures instead of only ResNet models.

To further verify the high efficiency of TA, we investigate the number of queries to achieve various attack success rates under the RMSE threshold of 0.1, 0.05 and 0.01, respectively. The maximum number of queries is set to 10,000 and the results on ResNet-18 are summarized in Fig. 5. As shown in Fig. 5a and 5b, TA needs much less number of queries to achieve various attack success rates with RMSE threshold of 0.1 and 0.05, showing the high query efficiency of our method. For the smaller threshold of 0.01, as shown in Fig. 5c, our TA still needs less number of queries when achieving the attack success rate smaller than 50% but fails to achieve the attack success rate higher than 60%. Note that as shown in Fig. 6 and Table 1, RMSE threshold of 0.01 is very rigorous so that the perturbation is imperceptible but is also hard to generate the adversarial examples for decision-based attacks. Since we mainly focus on the query efficiency of attack only based on the geometric information, the attack performance under the RMSE threshold of...
0.01 is acceptable because it is impractical for such high number of queries when attacking real-world applications.

4.3. Evaluation on Real-world Applications

Recently, with the superior performance and unprecedented progress of DNNs, numerous companies have deployed DNNs for a variety of tasks and also provide commercial APIs (Application Programming Interfaces) for users. However, the vulnerability of DNNs to adversarial examples, especially the prosperity of decision-based attack, poses severe threats to these real-world applications. With the high efficiency of TA, we also validate the practical attack applicability using Tencent Cloud API. Due to the high cost of commercial APIs, we randomly sample 20 images from ImageNet validation set and the maximum number of queries is 1,000.

The numbers of successfully attacked images are summarized in Table 2. We could observe that TA successfully generates more adversarial examples than the attack baselines within 200, 500 and 1,000 queries under various RMSE thresholds. In particular, our TA could generate even more adversarial examples within 500 queries than the best attack baselines within 1,000 queries, showing the superiority of TA. We also visualize some adversarial examples generated by TA in Fig. 6. As we can see, TA could successfully generate high quality adversarial examples for various classes with few queries (≤ 200), validating the high applicability of TA in real-world. Especially when the number of queries is 200, the adversarial examples generated by TA are almost visually imperceptible for humans, which highlights the vulnerability of current commercial applications.

4.4. Ablation Study

We conduct a series of ablation studies to investigate the superior performance of the proposed TA on ResNet-18, including the subspace chosen by TA, the ratio for low frequency subspace and the change rate γ and λ for updating the angle α.

On the subspace chosen by TA. Different from existing decision-based attacks, the generality of geometric property used by TA makes it possible to optimize the perturbation in the frequency space. To investigate the effectiveness of frequency space, we implement TA in various spaces, namely

| RMSE | OPT | SignOPT | HSIA | QEBA | GeoDA | Surfice | TA (Ours) |
|------|-----|---------|------|------|-------|---------|-----------|
| 0.1  | 4/6/6 | 8/8/9 | 7/8/8 | 12/12/12 | 15/15/15 | 13/13/13 | 17/17/17 |
| 0.05 | 1/3/3 | 4/4/7 | 6/6/8 | 11/11/12 | 13/14/14 | 12/12/13 | 15/17/17 |
| 0.01 | 1/1/2 | 1/1/3 | 2/5/6 | 3/8/9 | 3/7/12 | 5/8/10 | 8/12/13 |

Table 2. The number of adversarial examples successfully generated by various attack baselines and the proposed TA on Tencent Cloud API within 200/500/1,000 queries. The results are evaluated on 20 randomly sampled images from the correctly classified images in ImageNet due to the high cost of online APIs.

| RMSE | TA | TA_F1 | TA_F | TA |
|------|----|-------|------|----|
| 0.1  | 39.5 | 97.5 | 98.5 | 100.0 |
| 0.05 | 17.5 | 73.0 | 85.0 | 94.0 |
| 0.01 | 3.0  | 22.5 | 25.5 | 51.5 |

Table 3. Ablation study on ResNet-18 for different spaces, i.e. input space (TA), frequency space for line sampling but input space for perturbation optimization (TA_F1), and full frequency space without mask (TA_F).

Figure 6. The adversarial examples of three benign samples generated by TA against Tencent Cloud API. #Q. denotes the number of queries for attack and RMSE denotes the RMSE distance between the benign sample and adversarial example. We report the correct label and the predicted label on the leftmost and rightmost columns, respectively. (Zoom in for details.) input space (TA), sampling the directional line in the frequency space but optimizing the perturbation in the input space (TA_F1) as in [29] and full frequency space (TA_F). As shown in Table 3, due to the high-dimensional input space, TA cannot effectively explore the neighborhood of the input sample to find good perturbation and shows very poor performance. With the information from frequency space to sample the subspace, TA_F1 exhibits much better results than TA. When optimizing the perturbation in the full frequency space, TA_F could achieve higher attack success rate than TA_F1, showing the benefit of frequency space. When utilizing the low frequency information for sampling the subspace in the frequency space, TA achieves much better performance than all the other attacks, supporting the necessity and rationality of the subspace chosen by TA.

On the ratio for low frequency subspace. The low frequency domain plays key role in improving the efficiency of TA. However, there is no criterion to identify the low frequency since it corresponds to high frequency, which is usually determined by the lower part of the frequency domain with a given ratio. Here we investigate the effect of this ratio
on the attack performance of TA. As shown in Fig. 7, the ratio has more significant influence on the attack success rate under the smaller RMSE threshold. In general, increasing the ratio roughly decreases the attack performance because it makes TA focus more on the high frequency domain, containing less critical information of the image. Thus, we adopt the lower 10% parts as the low frequency subspace for high efficiency, which also helps TA effectively reduce the dimension, making it easier for attack.

**On the change rate γ and λ for updating the angle α.** As stated in Sec. 3.3, the angle α plays a key role in choosing a better candidate triangle but it is hard to find a uniformly optimal α for different iterations and input images. We assume that the larger angle α makes it harder to find a candidate triangle but leads to smaller perturbation. As in Eq. (3), if we successfully find a triangle, we would increase α with γ. Otherwise, we would decrease α with λγ. We investigate the impact of various γ and λ in Fig. 8. Here we only report the results for RMSE=0.01 and the results for RMSE=0.1/0.05 exhibit the same trend. In general, γ = 0.01 leads to better attack performance than γ = 0.05/0.005. When we increase λ with γ = 0.01, the attack success rate increases until λ = 0.05 and then decreases. We also investigate the impact of τ which controls the bound for α in Eq. (3), which shows stable performance within 1,000 queries but takes effect for 10,000 queries and we simply adopt τ = 0.1. In our experiments, we adopt γ = 0.01, λ = 0.05 and τ = 0.1.

### 4.5. Further Discussion

BoundaryAttack [4] adopts random walk on the decision boundary for decision-based attack and the subsequent works often follow this setting to restrict the adversarial example on the decision boundary. We argue that such a restriction is not necessary and do not adopt it in our TA. To validate this argument, we also conduct binary search to move the adversarial example towards the decision boundary at each iteration after we find the candidate triangle to investigate the benefit of this restriction. As illustrated in Fig. 9, when the number of iterations for binary search (Nbs) is 0, it is vanilla TA that exhibits the best attack success rate. When we increase Nbs, the binary search takes more queries in each iteration which degrades the total number of iterations under the given total number of queries. In general, the attack success rate stably decreases when increasing Nbs especially for RMSE=0.01, which means the cost (i.e., queries) for binary search to restrict the adversarial example on the decision boundary is not worthy. Such restriction might also be not reliable and rational for most decision-based attacks, especially for geometry-inspired attacks. We hope this would inspire more attention to discuss the necessity of restricting the adversarial examples on the decision boundary and shed new light on the design of more powerful decision-based attacks.

### 5. Conclusion

In this work, we found that the benign sample, current and next adversarial examples could naturally construct a triangle in a subspace at each iteration for any iterative attacks. Based on this observation, we proposed a novel decision-based attack, called Triangle Attack (TA), which utilizes the geometric information that the longer side is opposite the larger angle in any triangle. Specifically, at each iteration, TA randomly samples a directional line across the benign sample to determine a subspace, in which TA iteratively searches a candidate triangle to minimize the adversarial perturbation. By solely utilizing the geometric property, TA optimizes the adversarial perturbation in the low frequency space generated by DCT with much lower dimensions than the input space, and significantly improves the query efficiency. Extensive experiments demonstrate that TA achieves a much higher attack success rate within 1,000 queries and needs much less queries to achieve the same attack success rate. The practical applicability on Tencent Cloud API also validates the superiority of TA. We hope that TA could shed light on improving the query efficiency.
via geometric information and more effective dimensionality reduction methods.

References

[1] Nasir Ahmed, T., Natarajan, and Kamisetty R Rao. Discrete cosine transform. *IEEE transactions on Computers*, 1974. 2, 3
[2] Abdullah Al-Dujaili and Una-May O’Reilly. Sign bits are all you need for black-box attacks. In *International Conference on Learning Representations*, 2020. 1, 2
[3] Anish Athalye, Nicholas Carlini, and David A. Wagner. Obfuscated gradients give a false sense of security: Circumventing defenses to adversarial examples. In *International Conference on Machine Learning*, 2018. 1, 2, 3
[4] Wieland Brendel, Jonas Rauber, and Matthias Bethge. Decision-based adversarial attacks: Reliable attacks against black-box machine learning models. In *International Conference on Learning Representations*, 2018. 1, 2, 3, 8
[5] Nicholas Carlini and David Wagner. Towards evaluating the robustness of neural networks. In *IEEE Symposium on Security and Privacy*, 2017. 1, 2, 3
[6] Jianbo Chen, Michael I Jordan, and Martin J Wainwright. Hopskipjumattack: A query-efficient decision-based attack. In *IEEE Symposium on Security and Privacy*, 2020. 1, 2, 3, 5
[7] Pin-Yu Chen, Huan Zhang, Yash Sharma, Jinfeng Yi, and Cho-Jui Hsieh. Zoo: Zeroth order optimization based black-box attacks to deep neural networks without training substitute models. In *ACM Workshop on Artificial Intelligence and Security*, 2017. 1, 2
[8] Weilun Chen, Zhaoxiang Zhang, Xiaolin Hu, and Baoyuan Wu. Boosting decision-based black-box adversarial attacks with random sign flip. In *European Conference on Computer Vision*, 2020. 1
[9] Minhao Cheng, Thong Le, Pin-Yu Chen, Huan Zhang, Jinfeng Yi, and Cho-Jui Hsieh. Query-efficient hard-label black-box attack: An optimization-based approach. In *International Conference on Learning Representations*, 2019. 1, 2, 3, 5
[10] Minhao Cheng, Simranjit Singh, Patrick H. Chen, Pin-Yu Chen, Sijia Liu, and Cho-Jui Hsieh. Sign-opt: A query-efficient hard-label adversarial attack. In *International Conference on Learning Representations*, 2020. 2, 3, 5
[11] Jeremy M. Cohen, Elan Rosenfeld, and J. Zico Kolter. Certified adversarial robustness via randomized smoothing. In *International Conference on Machine Learning*, 2019. 2
[12] Francesco Croce and Matthias Hein. Reliable evaluation of adversarial robustness with an ensemble of diverse parameter-free attacks. In *International Conference on Machine Learning*, 2020. 1, 2
[13] Yinpeng Dong, Qi-An Fu, Xiao Yang, Tianyu Pang, Hang Su, Zhihao Xiao, and Jun Zhu. Benchmarking adversarial robustness on image classification. In *Conference on Computer Vision and Pattern Recognition*, 2020. 2
[14] Yinpeng Dong, Fangzhou Liao, Tianyu Pang, Hang Su, Jun Zhu, Xiaolin Hu, and Jianguo Li. Boosting adversarial attacks with momentum. In *Conference on Computer Vision and Pattern Recognition*, 2018. 1, 2
[15] Jiawei Du, Hu Zhang, Joey Tianyi Zhou, Yi Yang, and Jiashu Feng. Query-efficient meta attack to deep neural networks. In *International Conference on Learning Representations*, 2020. 2
[16] Kevin Eykholt, Ivan Evtimov, Earlene Fernandes, Bo Li, Amir Rahmati, Chaowei Xiao, Atul Prakash, Tadayoshi Kohno, and Dawn Song. Robust physical-world attacks on deep learning visual classification. In *Conference on Computer Vision and Pattern Recognition*, 2018. 1
[17] Ian J. Goodfellow, Jonathon Shlens, and Christian Szegedy. Explaining and harnessing adversarial examples. In *International Conference on Learning Representations*, 2015. 1, 2, 3
[18] Chuan Guo, Jared S Frank, and Kilian Q Weinberger. Low frequency adversarial perturbation. *Uncertainty in Artificial Intelligence*, 2019. 4
[19] Chuan Guo, Mayank Rana, Moustapha Cissé, and Laurens van der Maaten. Countering adversarial images using input transformations. In *International Conference on Learning Representations*, 2018. 1
[20] Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. Deep residual learning for image recognition. In *Conference on Computer Vision and Pattern Recognition*, 2016. 1, 5
[21] Gao Huang, Zhuang Liu, Laurens van der Maaten, and Kilian Q. Weinberger. Densely connected convolutional networks. In *Conference on Computer Vision and Pattern Recognition*, 2017. 1, 5
[22] Andrew Ilyas, Logan Engstrom, Anish Athalye, and Jessy Lin. Black-box adversarial attacks with limited queries and information. In *International Conference on Machine Learning*, 2018. 1, 2
[23] Alex Krizhevsky, Ilya Sutskever, and Geoffrey E. Hinton. Imagenet classification with deep convolutional neural networks. In *Advances in Neural Information Processing Systems*, 2012. 1, 4
[24] Alexey Kurakin, Ian Goodfellow, and Samy Bengio. Adversarial examples in the physical world. In *International Conference on Learning Representations, Workshop*, 2017.
[25] Huichen Li, Xiaojun Xu, Xiaolu Zhang, Shuang Yang, and Bo Li. QEBA: query-efficient boundary-based blackbox attack. In *Conference on Computer Vision and Pattern Recognition*, 2020. 1, 2, 3, 4, 5
[26] Yanpei Liu, Xinyun Chen, Chang Liu, and Dawn Song. Delving into transferable adversarial examples and black-box attacks. In *International Conference on Learning Representations*, 2017. 1
[27] Yujia Liu, Seyed-Mohsen Moosavi-Dezfooli, and Pascal Frossard. A geometry-inspired decision-based attack. In *International Conference on Computer Vision*, pages 4890–4898, 2019. 2
[28] Aleksander Madry, Aleksandar Makelov, Ludwig Schmidt, Dimitris Tsipras, and Adrian Vladu. Towards deep learning models resistant to adversarial attacks. In *International Conference on Learning Representations*, 2018. 1, 2
[29] Thibault Maho, Teddy Furon, and Erwan Le Merrer. Surfree: a fast surrogate-free black-box attack. In Conference on Computer Vision and Pattern Recognition, 2021. 1, 2, 3, 4, 5, 6, 7

[30] Seyed-Mohsen Moosavi-Dezfooli, Alhussein Fawzi, and Pascal Frossard. Deepfool: A simple and accurate method to fool deep neural networks. In Conference on Computer Vision and Pattern Recognition, 2016. 1, 2

[31] Ali Rahmati, Seyed-Mohsen Moosavi-Dezfooli, Pascal Frossard, and Huaiyu Dai. GeoDA: a geometric framework for black-box adversarial attacks. In Computer Vision and Pattern Recognition, pages 8446–8455, 2020. 1, 2, 3, 5, 6

[32] Olga Russakovsky, Jia Deng, Hao Su, Jonathan Krause, Sanjeev Satheesh, Sean Ma, Zhiheng Huang, Andrej Karpathy, Aditya Khosla, Michael Bernstein, et al. Imagenet large scale visual recognition challenge. In International booktitle of computer vision, 2015. 5

[33] Ali Shafahi, Mahyar Najibi, Amin Ghiasi, Zheng Xu, John P. Dickerson, Christoph Studer, Larry S. Davis, Gavin Taylor, and Tom Goldstein. Adversarial training for free! In Advances in Neural Information Processing Systems, 2019. 2

[34] Mahmood Sharif, Sruti Bhagavatula, Lujo Bauer, and Michael K Reiter. Accessorize to a crime: Real and stealthy attacks on state-of-the-art face recognition. In ACM SIGSAC conference on Computer and Communications Security, 2016. 1

[35] Karen Simonyan and Andrew Zisserman. Very deep convolutional networks for large-scale image recognition. In International Conference on Learning Representations, 2015. 5

[36] Chuanbiao Song, Kun He, Liwei Wang, and John E. Hopcroft. Improving the generalization of adversarial training with domain adaptation. In International Conference on Learning Representations, 2019. 1

[37] Christian Szegedy, Vincent Vanhoucke, Sergey Ioffe, Jonathon Shlens, and Zbigniew Wojna. Rethinking the inception architecture for computer vision. In Conference on Computer Vision and Pattern Recognition, 2016. 5

[38] Christian Szegedy, Vincent Vanhoucke, Sergey Ioffe, Jonathon Shlens, and Zbigniew Wojna. Rethinking the inception architecture for computer vision. In Conference on Computer Vision and Pattern Recognition, 2016. 6

[39] Christian Szegedy, Wojciech Zaremba, Ilya Sutskever, Joan Bruna, Dumitru Erhan, Ian J. Goodfellow, and Rob Fergus. Intriguing properties of neural networks. In International Conference on Learning Representations, 2014. 1, 2

[40] Chun-Chen Tu, Paishun Ting, Pin-Yu Chen, Sijia Liu, Huan Zhang, Jinfeng Yi, Cho-Jui Hsieh, and Shin-Ming Cheng. Autozoom: Autoencoder-based zeroth order optimization method for attacking black-box neural networks. In AAAI Conference on Artificial Intelligence, 2019. 2

[41] Xiaosen Wang and Kun He. Enhancing the transferability of adversarial attacks through variance tuning. In Conference on Computer Vision and Pattern Recognition, 2021. 1, 2

[42] Xiaosen Wang, Xuanran He, Jingdong Wang, and Kun He. Admix: Enhancing the transferability of adversarial attacks. In International Conference on Computer Vision, 2021. 1, 2

[43] Xiaosen Wang, Jiadong Lin, Han Hu, Jingdong Wang, and Kun He. Boosting adversarial transferability through enhanced momentum. In British Machine Vision Conference, 2021. 3

[44] Eric Wong, Leslie Rice, and J. Zico Kolter. Fast is better than free: Revisiting adversarial training. In International Conference on Learning Representations, 2020. 1

[45] Boxi Wu, Heng Pan, Li Shen, Jindong Gu, Shuai Zhao, Zhifeng Li, Deng Cai, Xiaofei He, and Wei Liu. Attacking adversarial attacks as a defense. arXiv preprint arXiv:2106.04938, 2021. 1

[46] Weibin Wu, Yuxin Su, Michael R Lyu, and Irwin King. Improving the transferability of adversarial samples with adversarial transformations. In Conference on Computer Vision and Pattern Recognition, 2021. 2

[47] Cihang Xie, Zhishuai Zhang, Yuyin Zhou, Song Bai, Jianyu Wang, Zhou Ren, and Alan L. Yuille. Improving transferability of adversarial examples with input diversity. In Conference on Computer Vision and Pattern Recognition, 2019. 1, 2

[48] Zhewei Yao, Amir Gholami, Peng Xu, Kurt Keutzer, and Michael W. Mahoney. Trust region based adversarial attack on neural networks. In Conference on Computer Vision and Pattern Recognition, 2019. 2

[49] Hongyang Zhang, Yaodong Yu, Jiantao Jiao, Eric P. Xing, Laurent El Ghaoui, and Michael I. Jordan. Theoretically principled trade-off between robustness and accuracy. In International Conference on Machine Learning, 2019. 1, 2

[50] Pu Zhao, Pin-Yu Chen, Siyue Wang, and Xue Lin. Towards query-efficient black-box adversary with zeroth-order natural gradient descent. In AAAI Conference on Artificial Intelligence, 2020. 1, 2