Optimization on Paddy Crops in Central Java (with Solver, SVD on Least Square and ACO (Ant Colony Algorithm))

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Abstract. Several algorithms and objective functions on paddy crops have been studied to get optimal paddy crops in Central Java based on the data given from Surakarta and Boyolali. The algorithms are linear solver, least square and Ant Colony Algorithms (ACO) to develop optimization procedures on paddy crops modelled with Modified GSTAR (Generalized Space-Time Autoregressive) and nonlinear models where the nonlinear models are quadratic and power functions. The studied data contain paddy crops from Surakarta and Boyolali determining the best period of planting in the year 1992-2012 for Surakarta where 3 periods for planting are known and the optimal amount of paddy crops in Boyolali in the year 2008-2013. Having these analyses may guide the local agriculture government to give a decision on rice sustainability in its region. The best period for planting in Surakarta is observed, i.e. the best period is in September-December based on the data 1992-2012 by considering the planting area, the cropping area, and the paddy crops are the most important factors to be taken into account. As a result, we can refer the paddy crops in this best period (about 60.4 thousand tons per year) as the optimal results in 1992-2012 where the used objective function is quadratic. According to the research, the optimal paddy crops in Boyolali about 280 thousand tons per year where the studied factors are the amount of rainfalls, the harvested area and the paddy crops in 2008-2013. In this case, linear and power functions are studied to be the objective functions. Compared to all studied algorithms, the linear solver is still recommended to be an optimization tool for a local agriculture government to predict paddy crops in future.

1. Introduction
Optimization techniques are not yet compactly and frequently written and implemented in agriculture data analysis software in Indonesia. On the other hand, there have been lots of problems on decision analysis to predict necessity for food suppliers especially rice in some regions in Indonesia. One normally uses experiences for doing an optimization in this case. Therefore, errors may occur in food management in Indonesia, particularly rice. According to the e-news (Solo post) for instance, Boyolali as one of districts in Central Java Indonesia has failed to be predicted its paddy production in the year 2015, i.e. it was expected that this district obtained 300,957 tons whereas the real value was 287,000 tons. In 2016, the local government expects to have 300,322 tons of rice. Hence an optimization procedure is still required.
Since rice is the major food in Indonesia, the sustainability of rice must be correctly guaranteed. In order to have better future planning of food sustainability especially rice, an optimization method should be implemented in management of agriculture by the local government. Japan for instance, the optimization techniques have been established supporting decision-making on agricultural management by a farming organization using mixed 0-1 programming [1]. Thailand has explored the technical and social mechanisms for maximizing paddy yields to ensure quality of life of farmers and other stakeholders in agriculture system [2] and Philippine reduced CH4 emission from rice field to maintain C allocation in rice plants since climate may affect methane emission indirectly through effects on rice growth [3]. Unfortunately, there exists not enough software and knowledge in this direction for local agriculture administrators in Indonesia, and hence an effort for it must be done. Hence the objective of this research is providing an optimization software for agriculture data. The prototype G2A is a small software that has been developed for optimization agriculture data [4] where this software is a combination between a solver output from excel and GIS (Geographical Information System). The solver is used to solve GSTAR (Generalized Space-Time Autoregressive) models for optimization of paddy crops in Boyolali (districts in Central Java, Indonesia). However this software has not been checked and analyzed into detail with more exercises data. Hence, the results of prototype G2A must be compared with more established software in optimizations such as MATLAB and more modern algorithm such as Ant Colony Algorithm (ACO). Therefore the purpose of this paper is to show the study with several optimization procedures that have been available such that the appropriate algorithm can be chosen for practices.

Mathematical methods here rely on solving of linear system and nonlinear systems obtained from parameters determination as traditional methods in optimization. Compared to the latest development in optimization techniques [4], the methods here are well known. However, the methods have not much implemented in local agriculture management in Indonesia. Since data are typically from Indonesia, the techniques directly introduced through samples data from agriculture in Central Java. Since data are mostly from villages collected by local governments and the administrators have only very basic background in data analysis, the data will not need highly computational techniques. Thus the knowledge introduced here can be applied easily with only a single computer.

Note that most literatures have posed the objective functions [5][6]. Since we have only data, the derivation of objective functions must be done firstly. There are two objective functions required to set up an optimization problem based on the given data, i.e. according to an optimization for parameter determination and an optimization of the objective functions. Before solving the second optimization problem, one need to have some basic properties of the chosen objective functions such as a convex function in a convex domain leading to a minimum value of the objective function and a concave function in a concave domain leading to a maximum value. However two properties may be hold locally in some sub domains. These situations are found in this paper since the given data provide these two properties, i.e. convexity and concavity of the obtained objective functions. Hence to model a minimization problem one needs a convexity of an objective function and vice versa for a maximization problem. Therefore some clues are given here for designing an optimization problem, i.e.:

- **Dimensional Analysis**
  Before using all mathematical tools, the data must be transformed into dimensionless since different units may lead to different perception. Additionally, the mathematical theory is defined without any units.

- **Visualize the predicted objective function**
  If the problem only has 1 independent variable, the objective function is easily to be illustrated. This can be done with the same way for 2 independent variables.

- **Choose the appropriate variables to model for an objective function and designing related constraints.** For more than 2 independent variables and a big number of variables, visualization may not work for predicting a suitable objective function. Instead, employ statistics to choose
dominant variables [7]. If possible to reduce the number of variables, one may use only dominant variables for designing an objective function.

- Choose an objective function by the known properties of the function. For example, if the optimization problem is a minimization problem, the objective function must a convex function and vice versa for a maximization problem.

With the suggestions above some basic procedures in the code are presented here including the used data and the analysis. The discussion is limited to a single objective function with multivariable. For more than 1 objective function, one is suggested to include goal programming which is not discussed here.

2. Optimization methods for agriculture analysis
One should be aware that before an optimization theory is applied, one needs to analyze whether the given data support enough to achieve an optimal solution (a maximum or a minimum). There are several necessary conditions must be satisfied. Therefore, the definition of maximum and minimum must be defined mathematically. Additionally some keywords in optimization procedures must also be known. Hereafter some basic knowledge in mathematics is rewritten for beginners in optimization.

2.1. General Procedures in the code
Suppose the function is denoted by The condition for critical points commonly requires that the gradient of must be zero, i.e.

$$\nabla f = 0.$$  

(1)

The equation (1) may lead to a linear or nonlinear system and the solutions are the critical solutions. The critical solutions can be minimizers if the Hessian of on these points is positive (semi) definite[5]. Recall that the components of Hessian matrix of is defined by

$$H_f = \left[ \frac{\partial^2 f}{\partial x_i \partial x_j} \right]_{i,j=1,2,\ldots,n}.$$  

(2)

Notation of $H_f|\vec{x}^*$ denotes the Hessian matrix is defined on $\vec{x}^*$. As mention above that a positive definite Hessian means that the eigenvalues of this matrix on the point $\vec{x}^*$ are all positives and vice versa. Other definitions are possible mathematically, however the definition here is the most applicable definition used in this paper. Additionally a least square method is commonly implemented to find the best parameters meaning one must minimize the error governed by sum of square residual between data and the approximation function.

2.2. Procedures on least square
Most software in statistics is dealing with linear cases whereas optimization procedures are not taken into account in Indonesia. In our code, mathematics and statistics will be combined. In this paper, we introduce linear and nonlinear procedures since nonlinear cases occur frequently. The linear cases have been presented in the previous researches [8]. Parameters of an objective function must be determined by minimizing errors. In general case, one has a vector $\vec{y} \in D, D \subset \mathbb{R}^n$ and its approximation is modelled by a continuous $\hat{y}_{model}$ as its model where the sum squared distances are minimized, i.e.

$$\min \, R = \sum_{i=1}^{n} \left( y_{i,\text{data}} - y_{i,\text{model}}(\vec{x}, \vec{\alpha}) \right)^2$$  

(3.a)

with $\vec{x} \in \mathbb{R}^n, \vec{\alpha} \in \mathbb{R}^m, m << n$. 

Singularity often occurs for \( m \geq n \), where \( m \) denotes the number of parameters and \( n \) denotes the number of variables, see examples [9]. Hence the number of parameters should be much less than the number of independent variables. Since \( \bar{y} \in D \) is assumed to be a dependent variable, the critical condition is designed satisfying \( \nabla _{a} R = \bar{0} \in \mathbb{R} ^{m} \). As a result, a nonlinear system occurs and its solution \( \bar{a} \) must be the optimizer of \( R \). Since a numerical approach will be used, the vector \( \bar{a} \) must be analyzed its optimality before the optimization of the objective function is proceeded. This is done by evaluating the Hessian of \( R \) at \( \bar{a} \). Let us denote 
\[
R = \bar{g} ^{T} \bar{g} , \quad \bar{g} = \left[ g_{1} , g_{2}, \ldots , g_{n} \right] ^{T} , \quad g_{i} = y_{i} - y_{i, \text{model}} \left( \bar{x} , \bar{a} \right) , \quad \text{for } i = 1, \ldots , n .
\]
Therefore 
\[
\nabla _{a} R = \left[ \bar{g} R_{a_{i}} , \bar{g} R_{a_{i}} , \ldots , \bar{g} R_{a_{i}} \right] ^{T} , \quad \bar{g} R_{a_{i}} = 2 \bar{g} \cdot \partial _{a_{i}} \bar{g} .
\]
The notion \( \nabla _{a} R \) expresses a vector derivative of \( R \) with respect to each parameter \( a_{i} \). Each its component \( \left( \bar{g} R_{a_{i}} \right) \) describes a dot product between a vector \( \bar{g} \) and a vector of gradient \( \bar{a} \). The diagonal components of the Hessian \( R \) and its off diagonals contain \( \frac{\partial ^{2} R}{\partial a_{i} ^{2}} \) and \( \frac{\partial ^{2} R}{\partial a_{i} \partial a_{j}} \) respectively. Note that
\[
\frac{\partial ^{2} R}{\partial a_{i} ^{2}} = 2 \left( \partial _{a_{i}} \bar{g} \cdot \partial _{a_{i}} \bar{g} + \bar{g} \cdot \partial _{a_{i}} \left( \partial _{a_{i}} \bar{g} \right) \right) ,
\]
\[
\frac{\partial ^{2} R}{\partial a_{i} \partial a_{j}} = 2 \left( \partial _{a_{i}} \bar{g} \cdot \partial _{a_{j}} \bar{g} + \bar{g} \cdot \partial _{a_{j}} \left( \partial _{a_{i}} \bar{g} \right) \right) .
\]
Nonlinear least squares frequently fail to have optimizers since non convexity of \( R \) may occur (see [9] for instance). A hybrid global optimization algorithm has been developed to handle such problems
[10]. The function \( R \) convex if and only if
\[
R ( \bar{a} ) + \nabla _{a} R \left( \bar{a} - \bar{b} \right) \leq R ( \bar{b} ) , \quad \forall \bar{a} , \bar{b} \in D \subset \mathbb{R} ^{m} .
\]
This definition is difficult for a practical purpose since one has no information of \( D \) in general. Instead, the condition of Hessian matrix of \( R \) is considered for an initial analysis of the obtained parameters \( \bar{a} \). The Hessian matrix must be positive (semi) definite (nonnegative definite) to get minimum (locally) of \( R \), i.e. the eigenvalue of Hessian matrix of \( R \) must be positive (nonnegative) on the critical points.

Note that we seek the optimal \( \bar{a} \) minimizing \( R \). This is done by solving \( \nabla _{a} R = \bar{0} \) which is mostly solved numerically. Clever initial guess of \( \bar{a} \) may reach an optimal solution, i.e. its Hessian of \( R \) is positive definite. Additionally, the obtained parameter \( \bar{a} \) should practically reasonable. Otherwise, one has to adjust another least square by choosing another assumption of an objective model before a further optimization. Thus, one may use Hessian matrix to select data such that optimization procedures fit to the used theories. Some examples have been studied to select data based on the Hessian matrix [9] since the critical conditions (satisfying \( \nabla _{a} R = \bar{0} \)) are not enough.

2.3. Solving linear system with singular value decomposition (SVD)

After data are inserted to the model (linear and nonlinear), one mostly faces a linear system or nearly a singular matrix \( A \). The linear problem \( A \bar{V}_{\alpha} = \bar{B} \) must be solved. A good approximation for this is SVD method. The SVD method initially writes
\[
A = U \Sigma V ^{T} .
\]
where \( A \in \mathbb{R} ^{nxn} \) has rank \( r \). The method constructs \( U \) matrix containing eigenvectors as the column vectors where \( U \in \mathbb{R} ^{nxn} \). The \( \Sigma \) matrix is a diagonal matrix where its components are the eigenvalues.
of $AA^T$ and $\Sigma \in \mathbb{R}^{nm \times nm}$. The $V$ matrix is the matrix with its column vectors are the eigenvectors of $AA^T$ where $V \in \mathbb{R}^{nm \times nm}$. Inserting Eq.(8) into $AVA = \bar{b}$ and rewriting as

$$\sum V^T \bar{v}_a = U^T \bar{b}.$$ 

The usual error bound is used after the parameters are found (norm of the residual of objective functions) and the conditional number of the matrix $A$ is computed, i.e.

$$\kappa(A) = \begin{cases} ||A^{-1}||A||_{\infty} \text{if } A \text{ nonsingular} \end{cases}.$$ 

A maximum bound of conditional number of the matrix $A$ is a condition for the success of solving a linear system problem above. Finally, we will use these methods for optimizing our data.

3. Numerical Code for Agriculture

One available program is solver included in excel and the prototype of algorithm called G2A [8] as a combination between solver result and GIS (Geographical Information System). Therefore the solver is also used to introduce for solving optimization problems.

3.1. Available functions from MATLAB

There are functions provided by MATLAB such as lsqnonlin.m, fmincon.m, fsolve.m and fminsearch.m that one just inserts the objective functions. Unfortunately these codes are not easily translated or known by agriculture administrators in Indonesia. This code is compared with fsolve.m in MATLAB for solving.

3.2. Ant Colony Algorithm code

The Ant Colony Algorithm (ACO) has been developed individually for several optimization problems such as paddy optimization [11] and stevioside syrup production [12]. The code is based on the natural behaviour of ant colonies where the food target is considered to the optimal destination. The content of pheromone and its evaporation are the basic properties in the algorithm. In this paper we only present the result of the code since the ACO has been explained into detail in another literature [12]. The convergence of ACO based on the Gaussian distribution has been discussed to guarantee the optimal solutions [13].

Thus this paper will use several codes such as fmincon.m, prototype G2A meaning a combine of solver and GIS, and ACO to do optimization of paddy crops in Surakarta and Boyolali.

4. Paddy Crops Analysis Result

4.1. Optimization planting paddy period in Surakarta

There are common 3 periods for planting on paddy fields, i.e. January – April, period II May – August, and period III September – December where the period III is known naturally as the best period. Though climate change may give impact on paddy productivity as noted in Punjab [14], the planting area, cropping area and the amount of paddy crops are considered to be the most common variables. Due to the given data from Surakarta productivity of paddy fields in 20 years (1992-2012), the code here has shown agreement. The following paragraphs show the results. Suppose:

$x_{ij} = \text{planting area (PA) } i\text{-th year on } j\text{-th period (ha)}$

$y_{ij} = \text{cropping area (CA) } i\text{-th year on } j\text{-th period (ha)}$

$S_{ij} = \text{paddy crops } i\text{-th year of } j\text{-th period (ton)}$

$i = 1, 2, ..., n, \quad j = 1, 2, 3$. A quadratic function is chosen to be the objective function, i.e.

$$s(x, y) = \frac{1}{2} \begin{bmatrix} a_1 x^2 + a_2 y^2 + a_3 x y + a_4 x + a_5 y + a_6 \end{bmatrix}.$$
The vector parameters must be tested for the studied data. The matrix $A$ is obtained by using Eq.(3.a) based on the given data through least square method. The study is shown in Table I.

| The Measured Quantity | Period | II | III |
|-----------------------|--------|----|-----|
| Determinant of matrix $A$ | -0.0029 | -0.0014 | -0.0047 |
| Error                 | 9.4%   | 13.8342% | 18.5405% |
| Conditional number    | 23568  | 37013  | 22201 |
| $H_f$ (Hessian matrix) | positive semi definite | positive semi definite | positive semi definite |

(Source: Vina, et.al,2013)

The exercise data have shown that the code provides the study on parameter properties before the objective functions are optimized. One is allowed to decide for the next process, i.e. optimizations of objective functions. The conditional number is considerable good enough since it is less than 67108864 (according to MATLAB manual). A comparison between data period I and the quadratic function is shown in Fig.1. The function and the given data have error about 18 %. Since the conditional number and Hessian matrix have satisfied the necessary conditions for minimizing the residual function, the error can be acceptable. Though the least square method works well and the Hessian matrix has provided good result, we will compare with the SVD method for having better results. This is shown in Table II.

**Figure 1.** Comparison between the data period I and the approximation; horizontal : index, horizontal : paddy (dimensionless)
Table 2. Coefficients Properties

| The measured quantity | Period | Method | I       | II       | III       |
|----------------------|--------|--------|---------|----------|-----------|
| Period               |        |        |         |          |           |
| Error                | Least square | 9.4%  | 13.8342%| 18.5405%|
| Error                | SVD    | 7.27%  | 11.89%  | 8.43%    |
| Conditional number   | Least square | 23568 | 37013   | 22201    |
| Conditional number   | SVD    | 162.86 | 221.188 | 160.35   |

(Source: Vina, et.al, 2013)

The improvement is done by SVD since less errors and smaller conditional numbers for all data on each period according to the result in Table II. Finally, we choose the SVD result for the next optimization procedure. Thus, the objective functions can be optimized subject to the constraints by defining Lagrange function. The model is still quadratic. Additionally the constraints are only the lower and upper bounds each variable. The Ant Colony Algorithm code is also implemented to have comparison results as shown in Table III. Each period is compared with the ACO solutions where the quadratic program is preferable due to smaller errors. PTIMIZATION RESULTS FOR EACH PERIOD (EACH number is multiplied by 1 thousand)

Table 3. Optimization Results for Each Period (Each Number is Multiplied by 1 Thousand)

| Variables           | Method   | Period I     | Period II    | Period III   |
|---------------------|----------|--------------|--------------|--------------|
| Planting area (ha)  | Quadratic| 132.9505     | 54.6966      | 120.4908     |
|                     | ACO      | 118.944      | 109.55       | 142.33       |
| Cropping area (ha)  | Quadratic| 120.4864     | 105.2448     | 76.1586      |
|                     | ACO      | 98           | 103.9        | 63.99        |
| Paddy crops (ton)   | Quadratic| 61.7237      | 55.8306      | 60.4087      |
|                     | ACO      | 29.18        | 34.31        | 43.8         |

(Source: Vina, et.al, 2013)

Since the 3rd period is the best period, one may rely on its optimization results. We conclude that the optimal amount of paddy crops is about 60.4 thousand tons.

4.2. Optimization paddy related to 19 locations in Boyolali

Governments normally should have analysis of local productivity of agriculture and use the analysis for checking the local food sustainability. The paper here proposes the analysis method through modelling. The study is shown here. The used model is semi auto nonlinear regression since the present of the same variable in the previous time for paddy. The model is given by

\[ Z_k(t) = \alpha_1 Z_k(t-1)^{\alpha_2} Y_k(t)^{\alpha_3} R_k(t)^{\alpha_4} \]  

(P.1)

\[ Z_k(t) \] : the amount of paddy in k-th location at time t ,  
\[ Z_k(t-1) \] : the amount of paddy in k-th location at time t-1,  
\[ Y_k(t) \] : the harvested area in k-th location at time t,  
\[ R_k(t) \] : the amount of rainfalls in k-th location at time t.

The function shown by Eq.(P.1) cannot be visualized since it has 3 independent variables. The convexity or concavity is not easily captured. Therefore, we simply assume the critical solutions may occur. Additionally, the residual function is always a quadratic function (Eq.(3.a) ) yielding possibility its minimum exists. Thus one solves Eq.(3.a) for having the parameters in the model. Following the least square procedure above, the system has been obtained, i.e.

\[ \nabla_{\alpha} R = \vec{0} \]  

(P.2)
where each component is in the following statements, i.e.
\[ \partial R_{aj} = 2\tilde{g} \cdot \partial_{aj} \tilde{g} \quad \text{or} \quad \partial R_{ai} = 2\tilde{g} \cdot \partial_{ai} \tilde{g} = 2\left[ (Z - \tilde{Z}) \right] - \left\{ \partial_{ai} \tilde{Z} \right\} \]

where
\[ Z - \tilde{Z} = \left[ Z_1 - \tilde{Z}_1 \ldots Z_n - \tilde{Z}_n \right] \quad \text{and} \quad \left\{ \partial_{ai} \tilde{Z} \right\} = \left[ \partial_{ai} \tilde{Z}_1 \ldots \partial_{ai} \tilde{Z}_n \right]. \]

Note that
\[ \partial R_{ai} = 2(Z_1 - \tilde{Z}_1) \partial_{ai} \tilde{Z}_1 + \ldots + 2(Z_n - \tilde{Z}_n) \partial_{ai} \tilde{Z}_n. \]
\[ \partial_{ai} \tilde{Z}_k = Z_k(t-1)^{a_k} Y_k(t)^{a_k} R_k(t)^{a_k} \quad \partial_{ai} \tilde{Z}_k = \alpha_i Z_k(t-1)^{a_k} \ln Z_k(t-1) Y_k(t)^{a_k} R_k(t)^{a_k}, \]
\[ \partial_{ai} \tilde{Z}_k = \alpha_i Z_k(t-1)^{a_k} \ln Y_k(t)^{a_k} R_k(t)^{a_k}, \quad \partial_{ai} \tilde{Z}_k = \alpha_i Z_k(t-1)^{a_k} Y_k(t)^{a_k} R_k(t)^{a_k} \ln R_k \]

for \( k = 1, \ldots, n \).

Finally the model Eq.(P.1) above is used to determine optimal productivity due to the given data. The data are collected from Statistical Bureau of Boyolali since 2008-2013. There are 19 sub-districts in Boyolali where some productivities analyses of these sub-districts will be shown in the next paragraphs.

On the hand, the modified GSTAR model (Generalized Space-Time Autoregressive) have been used to present paddy crops and the optimal values are computed by 3 algorithms, i.e. solver using simplex algorithm through G2A code [4], fmincon.m using least square and ACO. Note that the used objective models here are modified GSTAR models underlying the computation on triple locations simultaneously. The map is presented in Fig.2 to show the definition of triple locations. The blue, red and black boundaries show regions called A, B, C containing 3 sub-districts to adjust modified GSTAR models.

![Figure 2. Boyolali map (source : private collection); example of triple locations denoted by the boundaries and regions A, B, C.](image-url)
The optimal parameters are determined by fsolve.m function, i.e. solving the gradient of residual to be zero nonlinear models. In this paper, the optimal values are compared with nonlinear models Eq.(P.1). Table IV shows that the residual gradients tend to zero for all locations identifying Eq. (P.2) is satisfied.

Table 4. Coefficients Properties According to $\|\nabla R\|$  

| Location | Initial parameter | Optimal Parameter | $\|\nabla R\|$ |
|----------|-------------------|-------------------|----------------|
| Simo     | [1.5; 2; 1; 1]    | [0.9995; -0.0325; -0.0051 0.0008] | $7.7053 \times 10^{-10}$ |
| Karanggede | [10; 1.5; 5; 1] | [0.9982; 0.0639; 0.0551; -0.0046] | $1.0232 \times 10^{-11}$ |
| Klego    | [1; 2.5; 1; 0]   | [0.9947; -0.0984; -0.0897; -0.0050] | $3.7473 \times 10^{-9}$ |

Table 5. Coefficients Properties According to Hessian and Error Functions  

| Location | Eigenvalues | Hessian properties | Error(%) |
|----------|-------------|--------------------|----------|
| Simo     | 0.3384      | Positive Definite  | 4.2734   |
|          | 0.6674      |                    |          |
|          | 121.4665    |                    |          |
| Karanggede| 0.8022     | Positive Definite  | 7.0747   |
|          | 1.0438      |                    |          |
|          | 127.7353    |                    |          |
|          | 254.6961    |                    |          |
| Klego    | 4.0133      | Positive Definite  | 13.7322  |
|          | 4.0723      |                    |          |
|          | 124.2692    |                    |          |
|          | 239.2924    |                    |          |

The analysis of these parameters is done by studying the properties of Hessian matrices (Eq.(2)) on these optimal parameters displayed by Table V. These 2 tables allow us to work further optimization process, i.e. to determine optimal paddy crops. The algorithms provided by solver (excel), fmincon.m (MATLAB) and ACO (individual programmed) are used and the results have been presented in Table VI. Other possible triple locations may occur, e.g. Simo, Karanggede and Klego can be defined as the new triple.

In nonlinear cases modelled by Eq.(P.1) for all 19 sub-districts, the analyses are listed for each triple sub-districts to have easier comparison with results of the GSTAR models. The first study refers to this triple, i.e. Simo, Karanggede and Klego. We have obtained the parameters to define the objective function for each sub district shown in Table IV and the properties are also shown in Table V according to the Hessian matrices in these parameters and the error functions. Finally, the objective functions must be optimized and compared to the previous approximation (GSTAR models). The optimization of paddy crops in dimensionless is listed in Table VI. Similarly, the other triple locations are computed and listed in Table VII.
Table 6. Optimization of PADDY Crops (Dimensionless)

| Location  | Solver | fmincon | ACO  | Nonlinear model |
|-----------|--------|---------|------|-----------------|
| Simo      | 1.0887 | 0.8976  | 1.0904 | 1.0035          |
| Karanggede| 1.1303 | 0.8397  | 1.1506 | 1.0387          |
| Klego     | 1.2996 | 0.7032  | 1.3080 | 1.0681          |

Table 7. Optimization of PADDY Crops (Dimensionless)

| Location  | Solver | Fmincon | ACO  | Nonlinear model |
|-----------|--------|---------|------|-----------------|
| Klego     | 1.2256 | 0.7064  | 1.2417 | 1.0747          |
| Andong    | 1.0541 | 0.9447  | 1.0615 | 1.0162          |
| Kemusu    | 1.2938 | 0.6740  | 1.2082 | 1.0716          |

Note that a sub-district may be computed twice according to the geographical neighbouring locations. Up to now, there exists no detail study according to the precision related to different triple objective functions.

Table VIII exhibits the parameters properties for nonlinear model Eq.(P.1) for 6 sub-districts. The forth column shows the norms of $\nabla R$ tend to zero indicating the parameters are well defined. The Hessian and error functions are not displayed here for simplify the paper here. All locations have been studied partially leading to a conclusion that the nonlinear model (P.1) can be used to be an objective function for paddy as a function of paddy produced in previous time, the harvested area and the amount of rainfalls. Finally all optimal values are scaled into dimensional. Some results are shown in Table IX for 6 sub-districts (Sambi, Ngemplak, Nogosari, Selo, Ampel and Cepogo). The 2nd-4th columns are governed by modified GSTAR models and 5th column is the optimal solution for the nonlinear model. We compare the results with the maximum data in the last column. Practically, the local agriculture government will use the total productivity and rounded values. For instance, Sambi has roughly 30,000 tons with the first 3 algorithms where 25,500 tons is suggested by the nonlinear model computed by ACO (see 1st row).

The optimal values here express the optimal paddy crops during 2018-2013 in Boyolali. It is concluded that about 278,528 tons are the optimal productivities based on the given data 2008-2013 by summing up paddy productivities from 19 sub-districts and provided by solver. Authors suggest that 280,000 tons will be the optimal values of paddy crops per year in Boyolali. Compared to the data in the introduction, this value is more reasonable than the district government has predicted (about 300,000 tons). Increment 20,000 tons is considerable too large different. The study here has given an improvement to predict optimal paddy crops for future. Generally, the optimal values given by 3 algorithms are near to each other. On the other hand, the nonlinear models have almost all lower values except the result from Selo sub-district.

Table 8. Coefficients Properties for Sambi Ngemplak, Nogosari, Selo, Ampel, Cepogo

| Location  | Initial Parameter | Parameter Optimal       | Norm (VR)       |
|-----------|-------------------|-------------------------|----------------|
| Sambi     | [0; 0.1; 0.1; 0.1]| [0.9998; -0.0717; -0.1476; 0.0002] | 1.7760 10^{-14}|
| Ngemplak  | [0; 0.1; 0.5; 0]  | [0.9966; 0.1951; 0.0292; -0.0099] | 9.1097 10^{-15}|

The optimal values here express the optimal paddy crops during 2018-2013 in Boyolali. It is concluded that about 278,528 tons are the optimal productivities based on the given data 2008-2013 by summing up paddy productivities from 19 sub-districts and provided by solver. Authors suggest that 280,000 tons will be the optimal values of paddy crops per year in Boyolali. Compared to the data in the introduction, this value is more reasonable than the district government has predicted (about 300,000 tons). Increment 20,000 tons is considerable too large different. The study here has given an improvement to predict optimal paddy crops for future. Generally, the optimal values given by 3 algorithms are near to each other. On the other hand, the nonlinear models have almost all lower values except the result from Selo sub-district. COEFFICIENTS PROPERTIES FOR SAMBI, NGEMPLAK, NOGOSARI, SELO, AMPEL, CEPOGO.
Table 8. Cont.

| Location | Solver     | Fmincon    | ACO         | Nonlinear model | Data (max) |
|----------|------------|------------|-------------|-----------------|------------|
| Sambi    | 29826      | 29320.2674 | 30050       | 25511           | 29048      |
| Ngemplak | 22179      | 22017.2768 | 22894       | 20293           | 22633      |
| Nogosari | 38131      | 37656.5172 | 39111       | 34097           | 36202      |
| Selo     | 84         | 84.0345    | 82.1194     | 54.7405         | 613        |
| Ampel    | 6676       | 6704.8228  | 7263.7      | 6614.7          | 6731       |
| Cepogo   | 644        | 644.1149   | 663.1135    | 555.4898        | 613        |

Table 7. Optimization of PADDY Crops (Dimensionless)

| Location | Solver     | Fmincon    | ACO         | Nonlinear model | Data (max) |
|----------|------------|------------|-------------|-----------------|------------|
| Sambi    | 29826      | 29320.2674 | 30050       | 25511           | 29048      |
| Ngemplak | 22179      | 22017.2768 | 22894       | 20293           | 22633      |
| Nogosari | 38131      | 37656.5172 | 39111       | 34097           | 36202      |
| Selo     | 84         | 84.0345    | 82.1194     | 54.7405         | 100        |
| Ampel    | 6676       | 6704.8228  | 7263.7      | 6614.7          | 6731       |
| Cepogo   | 644        | 644.1149   | 663.1135    | 555.4898        | 613        |

5. Conclusion
The paper has shown the study of paddy crops from Surakarta in 1992-2012 and Boyolali in 2008-2013. The data from Surakarta are considered to be quadratic functions. The given data have provided the necessary conditions for optimization procedures such as convexity domains such that the minimum errors are guaranteed yielding the parameters in the quadratic functions as the objective functions are well defined. Therefore the optimization for searching the best period is allowed theoretically. The best period for paddy planting that commonly known by farmers is in September-December. The climate change has not been considered in the analysis that may affect this period for planting. Based on this research, the optimal paddy crops in Surakarta from this period is about 60.4 thousand tons per year.

The given data from Boyolali are studied by linear and nonlinear models, i.e. the modified GSTAR models and the nonlinear power functions optimized by solver, least square and ACO algorithms. The modified GSTAR models are optimized by 3 algorithms yielding good agreements to the given data whereas the nonlinear models roughly have less optimal values. According to this research, 280 thousand tons per year are the optimal amount of paddy crops from Boyolali. Additionally, authors suggest that the local agriculture government may use pessimistic values provided by nonlinear models to give encouragement for farmers to increase paddy crops in the coming years.

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