NON-GAUSSIAN STATISTICS AND STELLAR ROTATIONAL VELOCITIES OF MAIN-SEQUENCE FIELD STARS

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ABSTRACT

In this Letter, we study the observed distributions of rotational velocity in a sample of more than 16,000 nearby F and G dwarf stars, magnitude complete, and presenting high-precision $V\sin i$ measurements. We show that the velocity distributions cannot be fitted by a Maxwellian. In addition, an analysis based on both Tsallis and Kaniadakis power-law statistics is by far the most appropriate statistics and gives a very good fit. It is also shown that single and binary stars have similar rotational distributions. This is the first time, to our knowledge, that these two new statistics have been tested for the rotation of such a large sample of stars, pointing solidly to a solution of the puzzling problem of the function governing the distribution of stellar rotational velocity.

Key words: stars: evolution – stars: fundamental parameters – stars: rotation – stars: statistics

1. INTRODUCTION

Rotation is a fundamental observable for the study of evolution of stars, also providing valuable information on stellar magnetism, mixing of chemical abundance in stellar interior, and tidal interaction in close binaries. In addition, it can be argued that the value of the rotational velocity of a star at a given evolutionary stage may be a measure of its original angular momentum. In that case, the distribution of rotational velocity may also be used to study some of the characteristics of the physical processes controlling star formation. In spite of a large observational effort carried out by different authors over the past 50 years, the problem concerning the nature of the statistical law controlling the distribution of stellar rotational velocity has turned into a very puzzling question. In addition, there is widespread recognition that stellar rotation axes have a random orientation (Struve 1945).

The first to derive analytically the distribution of stellar projected rotational velocity, on the basis of a Gaussian distribution, were Chandrasekhar & Munch (1950). These authors first assumed a parametric form for a function $f(v)$, where $v$ is the true rotational velocity, then computed the corresponding distribution of the projected rotational velocity $V\sin i$, and finally adjusted a set of stellar parameters to reproduce the $V\sin i$ measurements. Although Deutsch (1970) claimed that the distribution of stellar rotational velocities follows a Maxwellian–Boltzmann law, a number of studies have shown a clear discrepancy between theory and observations, where observed distributions are not well fitted by a Gaussian or Maxwellian function (e.g., Wolff et al. 1982; de Medeiros et al. 1996). More recently, Soares et al. (2006) have shown, based on an analysis of the rotation of low-mass stars in the Pleiades open cluster, that the question of the nature of the distribution of stellar rotational velocity is not simply a question of which mathematical function model is used. It depends on the statistical mechanics applied.

Here, we investigate the effects of power-law statistics on the observed distribution of projected rotational velocity measurements of a sample of more than 16,000 nearby F and G dwarf stars from the Nordström et al. (2004) catalog. We show clearly that the velocity distributions cannot be fitted by a Maxwellian.

Our analysis is based on both Tsallis and Kaniadakis non-Gaussian statistics and it is shown that these are by far the most appropriate statistics, giving a very good fit. We also show that single and binary stars have essentially similar rotational distributions.

This Letter is organized as follows. In Section 2, based on the formalism presented by Deutsch (1970), we present a generalization of the rotational velocity distribution in the spirit of Tsallis and Kaniadakis statistics. A brief discussion on the stellar sample is made in Section 3. Our main results are discussed in Section 4, and we summarize the main conclusions in Section 5.

2. TSALLIS AND KANIADAKIS DISTRIBUTION FUNCTIONS

During the last two decades, great interest has arisen in the nonextensive Tsallis statistical mechanics (Gell-Mann & Tsallis 2004) and, more recently, on the Kaniadakis extensive generalized power-law statistics (Kaniadakis 2002, 2005) motivated by various restrictions to the applicability of classical, extensive statistical mechanics.

From the mathematical point of view, the Tsallis statistics is based on the $q$-exponential and $q$-logarithm functions, which are defined by

$$
\exp_q(f) = (1 + (1 - q) f)^{1/(1-q)},
$$

$$
\ln_q(f) = \frac{f^{1-q} - 1}{1 - q},
$$

whereas the $q$-entropy associated with the $q$-statistics is given by (Gell-Mann & Tsallis 2004)

$$
S_q = -\int d^3p \, f \, \ln_q f = -\langle \ln_q(f) \rangle.
$$

The expressions above reduces to the standard results in the limit $q = 1$.

Similarly, the $\kappa$-framework is based on $\kappa$-exponential and $\kappa$-logarithm functions, defined as

$$
\exp_\kappa(f) = (\sqrt{1 + \kappa^2 f^2} + \kappa f)^{1/\kappa},
$$
\[ \ln_x(f) = \frac{f^\kappa - f^{-\kappa}}{2\kappa}. \]  
\[ S_\kappa = -\int d^3p \ln x(f) = -(\ln x(f)). \]  

The \( \kappa \)-entropy is given by (Kaniadakis 2002, 2005)

Again, the standard results are attained in the limit \( \kappa = 0 \).

The Tsallis statistics has been investigated in a wide range of problems in physics. In the astrophysical domain, the first applications of this power-law statistics studied stellar polytropes (Plastino & Plastino 1993) and the peculiar velocity function of galaxy clusters (Lavagno et al. 1998). More recently, Kaniadakis statistics has also been studied in the theoretical and experimental context, however the first application with a possible connection with astrophysical system has been the simulation in relativistic plasmas (Lapenta et al. 2007). More recently, by considering the distribution of stellar rotational velocity for low-mass stars in the Pleiades open cluster, Carvalho et al. (2008) showed that \( \kappa \) and \( q \) distributions give a good fit for the observed distribution.

Following the model given by the authors (Soares et al. 2006; Carvalho et al. 2008), it is possible to show that the Kaniadakis and Tsallis distributions are given by

\[ F(\Omega) = \exp_x\left(-\frac{\Omega^2}{\sigma_x^2}\right), \quad F(\Omega) = \exp_q\left(-\frac{\Omega^2}{\sigma_q^2}\right). \]

where \( \Omega \) is the nondimensional quantity \( \omega j, j \) being a parameter with the dimension \( \omega^{-1} \).

Here, it is worth mentioning that the standard distribution of the true rotational velocity \( V \) for a star sample is \( F(V) \sim V^2 \exp(-V^2) \). As shown by Deutsch (1970), the standard observed distribution of the projected rotational velocity \( V \sin i \), for a random orientation of axes, must be given by \( \phi(y) \sim y \exp(-y^2) \) (Kraft 1970), with \( y = V \sin i \). Henceforth, the \( \kappa \)-distributions \( \phi_\kappa(y) \) (\( q \)-distributions \( \phi_q(y) \)) should reproduce the standard one, in the same way as \( F_x(w)F_q(v) \) recovers \( F(V) \) in the \( \kappa = 0 \) \( q = 1 \) limiting case. Therefore, by considering these arguments, we introduce the following distribution function for the observed stellar rotational velocities:

\[ \phi_x(y) = y \exp\left(-\frac{y^2}{\sigma_x^2}\right), \quad \phi_q(y) = y \exp\left(-\frac{y^2}{\sigma_q^2}\right). \]

3 For a complete and updated list of references, see http://tsallis.cat.cbpf.br/biblio.htm.
for the Kaniadakis fit compared with the Tsallis one. Also, the uncertainties in the parameters of the Kaniadakis distribution are smaller than in the case of Tsallis distribution.

In Figure 2, we present the fitting to the complete sample using the parameter given in Table 1. The results are similar to the original sample. The $q$- and $\kappa$-distribution give the best fit with the Kaniadakis function being slightly better.

Finally, we have found no noteworthy difference in the distribution of the projected rotational velocity between single and binary stars (see Table 1). Despite the fact that the values of $q$ and $\sigma_q$ and $\kappa$ and $\sigma_\kappa$ are slightly larger for binary stars, the difference are, in most case, within the statistical fluctuation.

5. CONCLUSIONS

In this work, we have used non-Gaussian statistics to investigate the observed distribution of projected rotational velocity of a magnitude complete sample of more than 16,000 nearby F and G dwarf stars. We have fitted a standard Maxwellian and $q$-Maxwellian based on the generalization of the power-law statistics proposed by Tsallis and Kaniadakis. We conclude that the $V \sin i$ distribution deviates significantly from a standard Maxwellian and the best fit is attained for Tsallis distribution with $q = 1.521 \pm 0.043$ and Kaniadakis distribution with $\kappa = 0.667 \pm 0.033$. These values correspond to all stars (single + binary) in the complete sample.

As far as we are aware, this is the first time that the Tsallis and Kaniadakis statistics are tested for such an exceptionally large sample of stars. This gives us an excellent degree of confidence on the fact that the distribution of stellar rotational velocity does not follow a Maxwellian–Boltzmann law as suggested by the pioneer studies by Chandrasekhar & Munch (1950) and Deutsch (1970). The present results show clearly that for the stellar

|                  | $N$           | $q$            | $\sigma_q$ (km s$^{-1}$) | Reduced-$\chi^2$ | $\kappa$ | $\sigma_\kappa$ (km s$^{-1}$) | Reduced-$\chi^2$ |
|------------------|---------------|----------------|--------------------------|-------------------|----------|-------------------------------|-----------------|
| **Original sample**               |               |                |                          |                   |          |                               |                 |
| All stars        | 11,818        | 1.525 ± 0.041  | 4.25 ± 0.21              | 4493              | 0.667    | 4.95 ± 0.15                   | 3088            |
| Single           | 6,888         | 1.484 ± 0.053  | 4.13 ± 0.25              | 422               | 0.636    | 4.74 ± 0.17                   | 400             |
| Binary           | 4,930         | 1.551 ± 0.037  | 4.63 ± 0.21              | 223               | 0.687    | 5.47 ± 0.17                   | 207             |
| **Complete sample**              |               |                |                          |                   |          |                               |                 |
| All stars        | 4,473         | 1.521 ± 0.043  | 5.08 ± 0.25              | 550               | 0.667    | 5.92 ± 0.19                   | 424             |
| Single           | 2,382         | 1.507 ± 0.046  | 4.92 ± 0.26              | 196               | 0.655    | 5.69 ± 0.19                   | 142             |
| Binary           | 2,091         | 1.529 ± 0.049  | 5.34 ± 0.31              | 149               | 0.672    | 6.26 ± 0.25                   | 133             |
rotation the nonextensivity holds, and that the distribution of the observed rotational velocity is explained by a generalized power-law statistics in the spirit of Tsallis and Kaniadakis statistical mechanics.

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REFERENCES

Carvalho, J. C., Silva, R., do Nascimento, J. D., Jr, & De Medeiros, J. R. 2008, Europhys. Lett., 84, 59001
Chandrasekhar, S., & Munch, G. 1950, ApJ, 111, 142
De Medeiros, J. R., Da Rocha, C., & Mayor, M. 1996, A&A, 314, 499
Deutsch, A. J. 1970, in Proc. IAU Colloq. 4, Stellar Rotation, ed. A. Slettebak (Dordrecht: Reidel), 207
Gell-Mann, M., & Tsallis, C. (ed.) 2004, in Nonextensive Entropy—Interdisciplinary Applications (New York: Oxford Univ. Press)
Holmberg, J., Nordström, B., & Andersen, J. 2007, A&A, 475, 519
Kaniadakis, G. 2002, Phys. Rev. E, 66, 056125
Kaniadakis, G. 2005, Phys. Rev. E, 72, 036108
Kraft, R. P. 1970, in Spectroscopic Astrophysics: An Assessment of the Contributions of Otto Struve, ed. G. H. Herbig (Berkeley, CA: Univ. California Press), 385
Lapenta, G., Markidis, S., Marocchino, A., & Kaniadakis, G. 2007, ApJ, 666, 949
Lavagno, A., Kaniadakis, G., Rego-Monteiro, M., Quariti, P., & Tsallis, C. 1998, Astrophys. Lett. Commun., 35, 449
Nordström, B., et al. 2004, A&A, 418, 989
Plastino, A., & Plastino, A. R. 1993, Phys. Lett. A, 174, 384
Soares, B. B., Carvalho, J. C., do Nascimento, J. D., Jr, & De Medeiros, J. 2006, Physica A, 364, 413
Struve, O. 1945, Pop. Astron., 53, 202
Wolff, S. C., Edwards, S., & Preston, G. W. 1982, ApJ, 252, 322