Exergetic review on passive and active systems for ventilation

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Abstract. The conceptualization of warm/cool and wet/dry exergies for a better understanding of various heating/cooling systems has been found very useful for describing the whole thermodynamic process of various built-environmental systems, from building envelopes to mechanical heating and cooling systems. In parallel to these sub-concepts of exergy, the conceptualization of “high-pressure/low-pressure” exergies, which is associated with mechanical non-equilibria, is also necessary for full exergetic description of flow and circulation within the built-environmental systems, from natural and mechanical ventilation to the circulation of hot or cold water between heat exchangers. This paper describes the essence of “high-pressure/low-pressure” exergies and also demonstrates a couple of example calculations, which clarify how natural ventilation systems function from the exergetic viewpoint. The conclusion reached by the present analyses is that the exergy input and consumption for natural ventilation is much smaller than those for mechanical ventilation: e.g., for the number of air change at the rate of 2 h⁻¹ in a room whose floor area is 36 m², the exergy inputs for wind-driven and buoyancy-driven ventilation are approximately 0.1% of the electricity input for the same air change rate with mechanical ventilation.

1. Introduction

Over the last twenty-five years or so, exergetic approach for the better understanding of various built-environmental conditioning systems such as lighting, heating and cooling has progressed very much. In due course, the conceptualization of warm/cool exergies was found very useful for describing the whole thermodynamic process of built environmental systems, from building envelopes to mechanical heating and cooling systems; it has become possible to articulate what is really consumed, where and how the consumption emerges, and what is the priority of the exergy-consumption minimization [1][2][3]. In addition to warm/cool exergies, the conceptualization of wet/dry exergies was also found necessary, in particular, to describe the human-body exergetic process for thermal comfort [2][3].

In parallel to the concepts of warm/cool and wet/dry exergies, the former of which is associated with thermal non-equilibria and the latter with chemical non-equilibria, the conceptualization of “high-pressure/low-pressure” exergies, which is associated with mechanical non-equilibria, is also necessary to describe fully a variety of flow and circulation within the built-environmental systems such as natural and mechanical ventilation and also the circulation of hot or cold water between heat exchangers [2]. This paper describes the essence of “high-pressure/low-pressure” exergies and also demonstrates a couple of example calculations, which clarify how natural ventilation systems function from the exergetic viewpoint.

2. High-pressure and low-pressure exergies

So-called Bernoulli theorem that was fully developed by Euler in 1755 with the assumption of no friction may be rewritten as follows so that the effect of pressure decrease is inclusive within the pressure balance. Assuming a flow system, whose inlet and outlet heights are the same as each other,

\[ P_i + \frac{1}{2} \rho v_i^2 - \Delta P_i = P_o + \frac{1}{2} \rho v_o^2 \]  

(1)
where $P_1$ and $P_2$ are pressure at inlet and outlet, respectively [N/m²], $\rho$ is air density [kg/m³], $v_1$ and $v_2$ are air velocities at inlet and outlet, respectively [m/s], and $\Delta P$ is the decrease in total pressure due to friction occurring between inlet and outlet [Pa]. Multiplying the volumetric flow rate, $V$ [m³/s], to both sides of eq.(1) yields energy balance equation.

$$PV + \left(\frac{1}{2}\rho v_1^2\right)V - \Delta PV = P_2V + \left(\frac{1}{2}\rho v_2^2\right)V \tag{2}$$

Since the units of pressure is equivalent to J/m³ (= N/m² = (N·m)/(m²·m)) and the unit of volumetric flow rate is m³/s, each term of eq.(2) is in the unit of J/s, that is, W. Therefore, the last term of the left-hand side of eq.(2), $\Delta PV$, is exactly the rate of heat generation due to friction. Having the exergy-consumption theorem described in [1][2][3] in mind, $\Delta PV$ should correspond to the rate of exergy consumption. This implies that we should be able to come up with exergy balance equation from eq.(2). For this purpose, we include the effect of the surrounding pressure, $P_o$, into eq.(2). By adding $(P_0 V)$ to both sides of eq.(2),

$$\left(P_1 - P_o\right)V + \left(\frac{1}{2}\rho v_1^2\right)V - \Delta PV = (P_2 - P_o)V + \left(\frac{1}{2}\rho v_2^2\right)V \tag{3}$$

Eq.(3) can be read as exergy balance equation. The last term in the left-hand side, $\Delta PV$, is nothing other than exergy consumption rate, $X_c$ [W], to be expressed as follows.

$$X_c = \Delta PV = S_g T_o \tag{4}$$

where $S_g$ is entropy generation rate [W/K] due to friction and $T_o$ is environmental temperature [K].

The two terms in front of $\Delta PV$ of eq.(3) are exergy inputs and the two terms in the right-hand side are exergy outputs. The second term in each side of eq.(3) is kinetic exergy. The first term in each side of eq.(2) is work to be performed due to the difference in pressure between the air inside the duct and the environment. The manner that they are expressed in relation to the pressure difference is similar to how thermal exergy and chemical exergy are determined, respectively [1][2][3]. They are the exergy to be determined by the difference in pressure, which turns all the flow and circulation into action.

In order to clarify the nature of exergy in relation to the pressure difference between a system in question and its environment, let us take a look at a duct system comprising two subsystems A and B as shown in Figure 1-(a). Within subsystem A, there is a wind turbine to extract work from the flow of air inside the duct at the rate of $W_{out}$ [W], while on the other hand, within subsystem B, there is a fan to be operated with the input of work by electricity at the rate of $W_{in}$ [W] to let the bulk of air move from left to right. The fan pressurizes the air moving forward so that the air pressure at outlet 2, $P_2$ [Pa], is higher than the environmental pressure, $P_o$ [Pa], while on the other hand, since the fan sucks the air from the rear side, the air pressure at inlet 1, $P_1$ [Pa], is lower than $P_o$. The duct surface is assumed to be thermally perfect-insulated, but a portion of the surface of respective subsystems is thermally conductive so that heat transfer can take place at the rates of $Q_{in}$ [W] in subsystem A and $Q_{out}$ [W] in subsystem B.

In order to make the present discussion simple, let us assume that $Q_{in}$ and $Q_{out}$ are negligible; in other words, the fan and turbine efficiencies are close to unity and also the sectional areas are the same throughout the duct; that is, $v_1 = v_2 = v_o$ according to the mass-balance relationship. With these assumptions in mind, exergy balance equation for subsystem B can be set up as follows.
in 1 2 o CB oW P PV X P PV − − =− ,                                    (5)

B B C f goX PV S T = ∆= ,                                               (6)

where B CX is the exergy consumption rate within subsystem B [W], B fP is the pressure difference between inlet 1 and outlet 2 [Pa], and B gS is the entropy generation rate within subsystem B [W/K].

The reason that there is no thermal exergy flow with in Q and out Q in eq.(5) is due to the assumption of negligible heat generation in the fan. Since 12 oPPP << and 0 V < , ( ) 1 0 oP PV − < and ( ) 20 oP PV < − . Namely, the exergy in relation to pressure becomes either positive or negative. Such feature of exergy in relation to pressure, negativity, has already been pointed out by Ishigai [4] and Bejan [5], but a further explanation on its implication has not yet been given. Therefore, let us do it here. For this purpose, we first add ( ) − ( ) 1 oPPV − − to both sides of eq.(5) and find the following equation.

( ){ } ( ) in 1 2 CB o oW X P P V P PV − = −− +− .                                      (7)

Eq.(7) is the final form of exergy balance equation for subsystem B under the condition of negligible heat generation in the fan and v1 = v2 = v o .

What eq.(7) implies is that there is an exergy input as W in to subsystem B, its portion as XCB = ( ) ∆P fB V is consumed and thereby two exergy outputs as ( ) − ( ) 1 − P P o V and ( ) 2 − P P o V are produced. Here, let us call ( ) 2 − P P o V as “high-pressure” exergy, since 2 > P o , and ( ) − ( ) 1 − P P o V as “low-pressure” exergy, since 1 < P o . The characteristic of “high-pressure” exergy is similar either to warm exergy or wet exergy, while that of “low-pressure” exergy is either to cool exergy or dry exergy. The exergy balance equation for subsystem A to be consistent with eq.(7) for subsystem B can be expressed as follows.

( ){ } 1A o C outPPVX W −− − = ,                                              (8)

AA A C f goX PV S T = ∆= ,                                                (9)

where A CX is the exergy consumption rate within subsystem A [W], A fP is the pressure difference between inlet 0 and outlet 1 [Pa], and A gS is the entropy generation rate within subsystem A [W/K].
Eq.(8) implies that there is the input of “low-pressure” exergy, \( -\left( P_1 - P_o \right) V \), to subsystem A, its portion, \( X_{CA} = \Delta P_{AV} \), is consumed and thereby work is produced at the rate of \( W_{out} \) by the wind turbine. Figure 1-(b) schematically demonstrates the exergy balances in subsystems A and B. In summary, the fluid flow through subsystems A and B are realized by the exergy input into a fan in subsystem B, where both “high-pressure” and “low-pressure” exergies are produced with the exergy consumption rate at \( X_{CB} \). The wind turbine in subsystem A functions with the supply of “low-pressure” exergy provided by subsystem B and it produces work at the rate of \( W_{out} \) as the result of exergy consumption rate at \( X_{CA} \). Note that the flow of “high-pressure” exergy is in the same direction as the flow of air, while the flow of “low-pressure” exergy is in the opposite direction to the flow of air. The latter relationship between exergy flow and air flow is comparable to the relationship between cool exergy flow and heat flow or that between dry exergy flow and moisture flow. The characteristics of “high-pressure” and “low-pressure” exergies given here for a duct system with a fan and a wind turbine can be applied to a pipe system with a pump and a waterwheel. They can also be applied to describing how natural ventilation systems function either with wind effect or with buoyancy effect. The relationship between the air flow through window openings and “high-pressure” or “low-pressure” exergy in a case either of wind-driven or buoyancy ventilation can be outlined as shown in Figure 2.

3. Exergy balance of wind-driven ventilation

Assuming a room having two windows: one in windward and the other in leeward as shown in the drawing attached to Figure 3, the pressure balance across the respective windows can be expressed as follows referring to eq.(1).

\[
P_o + C_W \left( \frac{1}{2} \rho_o v_o^2 \right) - \Delta P_{f, w1} = P_r , \tag{10}
\]

\[
P_r - \Delta P_{f, w2} = P_o + C_L \left( \frac{1}{2} \rho_o v_o^2 \right) , \tag{11}
\]

where \( v_o \) is outdoor reference air velocity [m/s], \( C_W \) and \( C_L \) are windward and leeward dynamic pressure coefficients, respectively, \( \Delta P_{f, w1} = \xi_{w1} \left( \frac{1}{2} \rho_o v_{s1}^2 \right) \) and \( \Delta P_{f, w2} = \xi_{w2} \left( \frac{1}{2} \rho_o v_{s2}^2 \right) \), for which \( \xi_{w1} \) and \( v_{s1} \) are the dynamic pressure-decrease coefficient and air velocity at the windward window,
respectively, and $w_1^\xi$ and $w_2^\xi$ are those at the leeward window, respectively; $\rho_o$ and $\rho_r$ are the density of outdoor air and indoor air, respectively. $P_o$ and $P_r$ are static pressure outdoors and indoors at the floor level \([\text{Pa}]\), respectively.

If indoor air temperature is higher than outdoor air temperature, then $\rho_o > \rho_r$. It causes buoyancy effect provided that the heights of two windows are different. In such a case, $P_o$ and $P_r$ at windward and leeward windows should be taken at their respective heights. But, here we assume that the heights of windward and leeward windows are equal to each other so that the buoyancy effect does not matter.

The mass balance equation is,

$$\rho_o A_{w1} v_{w1} = \rho_r A_{w2} v_{w2}. \quad (12)$$

where $A_{w1}$ and $A_{w2}$ are windward and leeward window areas \([\text{m}^2]\), respectively.

If the values of $P_o$, $v_o$, $C_w$, $C_L$, $\xi_{w1}$, $\xi_{w2}$, $A_{w1}$, $A_{w2}$, $\rho_o$ and $\rho_r$ are assumed, then the unknown variables are the average room air pressure, $P_r$, and two air velocities, $v_{w1}$ and $v_{w2}$. Therefore, with three equations above, from eq.(10) to (12), the values of $P_r$, $v_{w1}$ and $v_{w2}$ can be determined.

Figure 3 demonstrates the results of example calculation for the outdoor reference air velocity from 0 to 1.0 m/s with the assumption of a room as follows: the floor area is 36 m$^2$; the ceiling height is 3 m; the windward-window area is 0.7 m$^2$, which is 2% of the floor area; the leeward-window area is 75% of the windward-window area; the room is located in the middle of a multi-story building \((C_w = 0.8\) and $C_L = -0.75$); and both windows are covered by insect screen \((\xi_{w1} = \xi_{w2} = 2.78)\).

Figure 3-(a) shows the relationship between the outdoor reference air velocity and air change rate together with the average air velocity at the windward window. As the outdoor reference air velocity increases, the air change rate linearly increases. The same tendency applies to the average air velocity.
at the windward window, which is a little lower than a half of the outdoor reference air velocity. Such a
decrease in air velocity is due mainly to the use of insect screen on the windows. Since the air velocity
from 0.3 to 0.8 m/s is sufficient for restoring thermal comfort in a naturally ventilated room, the insect
screen may be regarded as a devise to ease the incoming outdoor wind while preventing the insects [2].
The whole exergy balance for wind-driven ventilation can be expressed as follows.

\[ C_w \left( \frac{1}{2} \rho_o v^2 \right) V_{w1} + \left[ \frac{C_L}{2} \rho_o v^2 \right] V_{w1} - X_C = 0, \quad (13) \]

where \( V_{w1} (= A_{w1} v_{w1}) \) is the volumetric air flow rate through the windward window [m\(^3\)/s] and \( X_C \)
is the exergy consumption rate [W], which is equal to \( (\Delta P_{f,w1} + \Delta P_{f,w2}) V_{w1} \). The first term of eq.(13)
represents the input of “high-pressure” exergy to the windward window and the second term the input
of “low-pressure” exergy to the leeward window.

Figure 3-(b) shows how the rates of exergy inputs and consumption vary with the outdoor
reference air velocity. They increase in proportion to the third power of velocity, since the dynamic
pressure is proportional to the square of air velocity and the volumetric air rate is proportional to air
velocity. The reason that “high-pressure” exergy input is larger than “low-pressure” exergy input here
in this example is due to the difference in the absolute values of dynamic pressure coefficient between
windward and leeward, which were assumed to be 0.8 and -0.75, respectively. The exergy consumption
rate at the leeward window is larger than that at windward window; this is due to the leeward window
being smaller than windward window area, with which the average air velocity turns out to be higher at
the leeward window than at the windward window. The total of exergy input rates is in the order of 10
to 250 mW; this is 0.28 to 6.9 mW per floor area. The consumption of both “high-pressure” and “low-
pressure” exergy brings about the air change rate shown in Figure 3-(a).

So far discussed for wind-driven ventilation can be extended to buoyancy-driven ventilation
and also to mechanical ventilation as well [2]. In order to have air change rate of 2 h\(^{-1}\) with the size of
windows smaller than the ones shown in Figure 2, the exergy input in the case of wind-driven ventilation
and buoyancy ventilation is in the order of 20 mW, while on the other hand, that in the case of
mechanical ventilation is in the order of 20 W; the former is 0.1% of the latter [2]. This implies that
designing appropriate sized openable/closable windows is crucially important in the development of
rational passive systems for ventilation.

4. Conclusion
Findings in the present research work are summarized as follows:
1) The conceptualization of “high-pressure/low-pressure” exergies to be in parallel to warm/cool and wet/dry
exergies was made possible.
2) “High-pressure” exergy flows in the same direction to the corresponding air flow, while on the other hand,
“low-pressure” exergy flows in the opposite direction to the air flow.
3) Exergy balance of wind-driven ventilation was demonstrated and it was clarified that exergy consumption
occurs within the windows by friction and thereby brings about ventilation effects.
4) The exergy input to drive natural ventilation is in the order of 0.1% of mechanical ventilation so that designing
appropriate sized openable/closable windows was suggested to be crucially important.

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