Analysis of Contact force optimization for a hybrid manipulator

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Abstract: A novel hybrid manipulator is designed. Based on the fact that the sum of the external force vector for the object is zero, the criterion of the stability of the object is derived. The formula for calculating contact force of fingers was deduced by matrix theory. The maximum frictional force was taken as boundary condition, then the contact force of fingers was optimized. The advanced CAD software was used to simulate the process of grasping the object. The stability of the object at a certain time was verified. Finally, the contact force between fingers and objects was optimized. It provides a theoretical basis for the application of the hybrid manipulator.

1. Introduction
When fingers are equipped on the moving platform of a parallel manipulator (PM), it becomes a hybrid manipulator. It has played an important role in equipment attitude adjustment, medical equipment, aerospace detection, fire rescue. At present, fingers are generally designed according to specific tasks. This kind of fingers have the advantages of simple structure, high reliability and easy control \cite{1, 2}. However, the manipulator has the following defects also: 1) Poor versatility, when the shape of the object changes, the end effector needs to be replaced. 2) The hybrid manipulator grasp the object by exerting a large positive pressure on the surface of the object, that is lack of accurate force control. 3) The driving efficiency is low and the dynamic response is poor \cite{3, 4}. Taking several fingers as the end effector of PMs can solve the above problems. Depending on the driving of fingers, it can adjust the pose of the object and realize the object moving efficiently and quickly. When the shape of the object changes, there is no need to replace the end effector. The contact force between fingers and object can be controlled by force sensor. Therefore, a hybrid manipulator with compound joints is proposed in the paper.

In the process of grasping the object, the object must bear the contact force, external load and gravity. In order to keep the object stable, the sum of all external force vectors which are loaded on object must be zero. The contact force not only depends on the external load, but also depends on the number of contact points and the layout of the contact points between the fingers and the object. Mechanical analysis of the hybrid manipulator is an important part of hybrid manipulator theory. In recent years, the solution of contact force for hybrid manipulator has been widely concerned by scholars at home and abroad. Yoshikawa\textsuperscript{5} established the grasping matrix by the pose relationship of the contact points, and solved the contact force based on the generalized inverse matrix of the grasping
matrix. Liu Qingyun [6] based on the friction point contact constructed the planning algorithm of contact force, which integrated the layout of contact points and the distribution of contact force into a unified process. According to the above methods, the solution of contact force is not unique, and it is necessary to optimize the contact force. Lippiello [7] proposed an algorithm solution which is suitable for online implementation for calculation of contact force for dexterous fingers under the premise of considering joint constraint force / moment. Wang [8] proposed a simple and practical optimization algorithm for the contact force of multi-fingered dexterous hand by using Lagrange multiplier method in the case of rigid body contact and soft finger contact. Up to now, the studies on the contact force optimization for hybrid manipulator have not been found. But it is significant to obtain optimal contact force in order to avoid the damage of the target object. For this reason, the analytical equation of the contact force is derived, and the optimal design model of the contact force for the hybrid manipulator is established by taking the maximum friction constraint condition as the boundary condition.

2. The Prototype of the hybrid manipulator
The hybrid manipulator mechanism is composed of 5-DOF parallel manipulator (PM) and three fingers, as shown in Figure 1. The 5-DOF PM consists of a base, a moving platform, two composite driving branches and one SPR (spherical joint-active prismatic joint-revolute joint) active leg. The compound driving branch includes: 1 upper beam, 2 cylinders and 2 piston. In the compound driving branch, two pistons are connected with both ends of the upper beam through revolute joint, and two cylinders are connected with the base through two universal joints. The axes of four revolute joints at both ends of the two driving legs are parallel to each other. So the upper beam, pistons rod and cylinders are in the same plane. The upper beam is connected with the moving platform through a universal joint. Here, two active legs are connected with the moving platform by the same joint which is a composite joint. In the SPR active legs, the cylinder is connected with the base through the spherical joint, and the piston is connected with the moving platform through a revolute joint. The finger mechanism is composed of one gripper, one piston, one cylinder and one connector. The gripper are connected with piston and connector by revolute joint respectively. In the active leg, the cylinder and gripper, the piston and connector are connected by revolute joint in the same plane.

3. Basic concept and condition of contact force
There are three types of contact between fingers and the object: point contact, line contact and surface contact. Point contact can be divided into smooth point contact and friction point contact. For smooth
point contact, there is only normal pressure between the fingers and the object, so it is difficult for the object to keep static balance, and it is difficult to apply in engineering practice. The solution of constraint force and constraint moment is complex for the line contact and surface contact, so it is rarely used in engineering practice also. In the article normal pressure and friction are studied based on the point contact. That is, the point contact between fingers and object is a fixed contact position, regardless of the rolling and sliding. Let \{Coi\} be the contact coordinate system which is fixed on the contact point, as shown in Figure 2. In the contact coordinate system, zi is perpendicular to the contact tangent plane and points to the outside, while yi is perpendicular to zi in the contact tangent plane. The xi is determined according to the right-hand rule. In the contact coordinate system \{Coi\}, let \(CoiF_i\) be the contact forces of three fingers. They can be expressed as follows:

\[
\mathbf{C}_{oi} F_i = \begin{bmatrix} \mathbf{C}_{oi} F_{i1} \\ \mathbf{C}_{oi} F_{i2} \\ \mathbf{C}_{oi} F_{i3} \end{bmatrix}, \quad (i=1,2,3)
\]

(1)

Here, \(F_{Czi}\) is the normal force between the fingers and the object, \(F_{Cix}, F_{Ciy}\) are the friction forces in xi and yi directions respectively. According to Coulomb's theory, the frictional force must be less than the maximum statics friction force \(F_{ZQi}\). They must satisfy:

\[
\mu_i F_{Czi} \leq F_{Cix}, F_{Ciy}, \quad (i=1,2,3)
\]

(2)

Here, \(\mu_i (i=1,2,3)\) be the friction coefficient between the fingers and object. Let \(B\) be the base coordinate frame \(O-XYZ\) which is fixed on \(B\). Let \(\mathbf{RR}_{BC}\) be rotation matrix from \{Coi\} to \{B\}. The contact force in \{B\} can be expressed as follows:

\[
\mathbf{F}_{B} = \mathbf{GR}_{BC} \mathbf{F}_{C}, \quad (i=1,2,3)
\]

(3)

The contact forces can be transformed to the center of the object. Let \(r_{bi}\) be the vector from the center of the object \(o_m\) to the contact point \(coi\). Let \(f\) be the generalized force between the fingers and the object. Let \(t\) be the generalized moment. They are expressed as follows:

\[
\mathbf{F} = \begin{bmatrix} \mathbf{f} \\ \mathbf{t} \end{bmatrix}, \quad \mathbf{f} = \mathbf{F}_{C} + \mathbf{R}_{BC} \mathbf{F}_{C}, \quad \mathbf{t} = \mathbf{r}_{i} \times \mathbf{F}_{C1} + \mathbf{r}_{i} \times \mathbf{F}_{C2} + \mathbf{r}_{i} \times \mathbf{F}_{C3}
\]

(4)

Substituting equation (3) into equation (4), the generalized force \(\mathbf{F}\) can be expressed as:

\[
\mathbf{F} = \mathbf{G}_{C} \mathbf{F}_{C}, \quad \mathbf{G}_{RB} = \begin{bmatrix} \mathbf{c}_{1}^{T} R & \mathbf{c}_{2}^{T} R & \mathbf{c}_{3}^{T} R \\ \hat{r}_{1} \times \mathbf{c}_{1} & \hat{r}_{2} \times \mathbf{c}_{2} & \hat{r}_{3} \times \mathbf{c}_{3} \end{bmatrix}
\]

(5)

Here, \(\mathbf{G}\) is a 6×9 matrix. Let \(f_{6}, \mathbf{l}_{6}\) to be the external load force and the external load moment respectively, and \(\mathbf{F}_{6}\) be the generalized force that is loaded on the object in the coordinate system \{B\}. In order to keep the object stable, the sum of all external force vectors loaded on the object must be zero.

\[
\mathbf{F}_{6} = \begin{bmatrix} f_{6} \\ \mathbf{l}_{6} \end{bmatrix}, \quad \mathbf{F}_{6} + \mathbf{F}_{6} = 0, \quad \mathbf{F}_{6} = -\mathbf{G}_{C} \mathbf{F}_{C}
\]

(6)

Based on the fingers contact force \(\mathbf{G}_{C} \mathbf{F}_{C}\), a symmetric matrix \(\mathbf{P}(\mathbf{Coi} \mathbf{F}_{C})\) is constructed as:

\[
\mathbf{P}(\mathbf{Coi} \mathbf{F}_{C}) = \begin{bmatrix} \mu_{i} F_{Cix} & 0 & F_{Ciy} \\ 0 & \mu_{i} F_{Cix} & F_{Ciy} \\ F_{Cix} & F_{Ciy} & \mu_{i} F_{Cix} \end{bmatrix}, \quad (i=1,2,3)
\]

(7)

The eigenvalues of the matrix \(\mathbf{P}_{i}(\mathbf{Coi} \mathbf{F}_{C})\) are \(\delta_{i} = \mu_{i} F_{Cix} + \sqrt{\mu_{i}^{2} F_{Cix}^{2} + F_{Ciy}^{2}}, \quad \delta_{i} = \mu_{i} F_{Cix} - \sqrt{\mu_{i}^{2} F_{Cix}^{2} + F_{Ciy}^{2}}\). Obviously, when the eigenvalues of the matrix \(\mathbf{P}_{i}\) are nonnegative, it is not difficult to derive the maximum friction constraint condition. Therefore, it can be concluded that the maximum friction constraint condition is equivalent to the semi-positive of the matrix \(\mathbf{P}_{i}\). Combining with equation (5), the condition of object balance can be constructed as:

\[
\begin{align}
(1) & \text{rank}(\mathbf{G}) = 6 \\
(2) & \text{There are column vectors which satisfy:} \quad \mathbf{P}(\mathbf{Coi} \mathbf{F}_{C}) \succeq 0 \quad \text{and} \quad \mathbf{G}_{C} \mathbf{F}_{C} = 0
\end{align}
\]

(8)
In the condition (1) of equation (8), the matrix $G$ is a row full rank matrix that is embodied in the layout requirements of the contact points between fingers and object. The condition (2) of equation (8) shows that the contact force of fingers must satisfy the constraint condition of maximum friction.

4. Solution of contact force for the hybrid manipulator

When fingers grasp the object, the object is mainly affected by the contact force $C_0F_C$ and external load. The contact force $C_0F_C$ can be divided into two parts: internal force $C_0F_q$ and operating force $C_0F_p$. They can be expressed as follows:

$$C_0F_q = C_0F_o + C_0F_e, \quad (9)$$

The main function of internal force $C_0F_q$ is to grasp the object and prevent the object from sliding. The vector sum of internal forces must zero, so the internal force does not affect the movement of the object. Operating force $C_0F_p$ is used to balance external force $F_o$ applied on object. According to the related knowledge of matrix theory, let $G^+$ be the generalized inverse of matrix $G$. Let $W$ be the fundamental matrix of zero space of matrix $G$. The vector $P_m$ is a three-dimensional column vector. The general solution of equation (6) is derived as follows:

$$C_iF_p = -G^+F_o + \lambda WP_m, \quad \lambda = \frac{1}{m} \left( \frac{-G^+F_o}{\lambda} \right) \quad (10)$$

Here, $\lambda$ is an arbitrary constant that does not change the general solution of the equation $GP=b$. The contact internal force $C_iF_q$ is the set of zero vector sum of fingers acting on the object. According to $\lambda WP_m=0$ the contact internal force $C_iF_q$ can be derived from equations (9) and (10).

$$C_iF_q = -G^+F_p, \quad C_iF_p = \lambda WP_m \quad (11)$$

Let $C_iF_{q1}$ be the contact internal force of the ith finger. The matrix $W$ is divided into three 3×3 submatrices $W_1$, $W_2$ and $W_3$. The matrix $W_i$ ($i=1, 2, 3$) is decomposed into three 3×1 sub matrices $W_{ix}$, $W_{iy}$, $W_{iz}$. They can be expressed as follows:

$$C_iF_q = \lambda WP_m, \quad C_iF_p = \lambda WP_m, \quad (i=1,2,3) \quad (12)$$

Here, $F_{q1}$, $F_{q2}$, $F_{q3}$ are the components of contact internal force for $z$-axis, $x$-axis and $y$-axis respectively. The contact internal force $C_0F_q$ is a linear combination of column vectors of the matrix $W$ and it has no unique solution. Equation (12) is a statically indeterminate system. The contact force $C_0F_C$ satisfies the maximum friction constraint condition, then one of the internal force components $C_0F_q$ also satisfies the maximum friction constraint condition. Thus, there is:

$$F_{q1}^2 + F_{q2}^2 + F_{q3}^2 \leq \mu^2 F_{q1}^2, \quad F_{q1} \geq 0, \quad (i=1, 2, 3) \quad (13)$$

By substituting equation (12) into equation (13), they are deduced that:

$$\begin{align*}
\left[ P_m (W_{ix}W_{ix}^T + W_{iy}W_{iy}^T - \mu^2 W_{iz}W_{iz}^T) P_m \leq 0 \\
-2W_{iz} P_m \leq 0, \quad (i=1, 2, 3)
\end{align*} \quad (14)$$

Equation (14) is the constraint condition of maximum friction for the solution of contact force. In the solution of equation (11), if $P_m$ satisfies the constraint condition of maximum friction in equation (14), there will be no sliding between the object and the fingers. Then, the contact force between the finger and the object can be calculated.

5. Optimization of contact force for the hybrid manipulator

According to the relevant knowledge of matrix theory, the solution of the contact force obtained by equation (11) is not unique, so the contact force should be optimized. When the contact force applied onto object becomes too large, the hybrid manipulator or object may be damaged. At the same time, when the contact force applied onto object becomes too small, it is easy to slide or roll between the fingers and the object. Therefore, the optimal solution of the contact force should meet the situation that there is no sliding between the fingers and the object, and the contact force should not be too large. The boundary condition of optimal design is established by using the constraint condition formula of maximum friction. Let $\eta$ be the objective function, $P_m$ be the design variable. Let the
constraint condition of maximum friction be the constraint condition. The objective of optimization
design is to find the $P_m$ which satisfies the maximum friction constraint. The optimal design
expression can be derived as follows:
\[
\min \eta \quad \eta < 0, -W_a^T P_a \leq \eta, \quad (i = 1, 2, 3), \quad P_m^T (W_a^T W_a^T + W_p^T W_p^T - \mu_i^2 W_a^T W_a^T) P_a \leq \eta
\]
(15)
Substituting the result of optimization design $P_0$ into equation (12), it leads to:
\[
C_{i0} F_p = \lambda_i W_a P_a, \quad (i = 1, 2, 3), \quad F_{q0} = \lambda_i W_a P_a, F_{q0} = \lambda_i W_a P_a, F_{q0} = \lambda_i W_a P_a
\]
(16)
The contact force $C_{i0} F_C$ is equal to the sum of contact internal force and operating force. Therefore, there is:
\[
C_{i0} F_C = C_{i0} F_{pi}, \quad (i = 1, 2, 3), \quad C_{i0} F_{pi} = \begin{bmatrix}
C_{i0} F_{p1} \\
C_{i0} F_{p2} \\
C_{i0} F_{p3}
\end{bmatrix}, \quad C_{i0} F_{pi} = \begin{bmatrix}
F_{p1} \\
F_{p2} \\
F_{p3}
\end{bmatrix}
\]
(17)
Here, $C_{i0} F_{pi}$ is the operating force of the ith finger. $F_{pix}, F_{piy}, F_{piz}$ are the components of operating
force for $z$-axis, $x$-axis and $y$-axis respectively. Similarly, the contact force $C_{i0} F_C$ satisfies the maximum
static friction constraint. They can be represented as follow:
\[
F_{p1}^2 + F_{p2}^2 + F_{p3}^2 \leq \mu_i^2 (F_{p1}^2 + F_{p2}^2)^2, F_{p1} + F_{p2} + F_{p3} \geq 0 \quad (i = 1, 2, 3)
\]
(18)
It is further deduced. It leads to:
\[
\begin{vmatrix}
\mu_i^2 F_{p1}^2 - F_{p1}^2 - F_{q0}^2 + 2(\mu_i^2 F_{p1} F_{q0} - F_{p1} F_{q0} - F_{p1} F_{q0}) + \mu_i^2 F_{p1}^2 - F_{p1}^2 - F_{p1}^2 \geq 0
\end{vmatrix}
\]
(19)
By substituting equation (16) into equation (19), it is deduced that:
\[
\begin{vmatrix}
C_{i0}^2 \lambda_i^2 + 2D_{i0} \lambda_i + E_{i0} \geq 0 \\
F_{p1} + \lambda_i W_a P_a \geq 0, \quad (i = 1, 2, 3), \quad D_{i0} = (\mu_i^2 F_{p1} W_a^T - F_{p1} W_a^T - F_{p1} W_a^T) P_a \\
E_{i0} = \mu_i^2 F_{p1}^2 - F_{p1}^2 - F_{p1}^2
\end{vmatrix}
\]
(20)
In equation (20), $\lambda_i (i = 1, 2, 3)$ are solved which satisfy the constraint condition of maximum friction.
\[
\lambda_i = \max \left(\frac{-D_{i0} + \sqrt{D_{i0}^2 - C_{i0} E_{i0}}}{C_{i0}}, \frac{-F_{p1}}{W_a P_a}\right), \lambda_i = \max (\lambda_1, \lambda_2, \lambda_3), \quad (i = 1, 2, 3)
\]
(21)
Substituting the $\lambda$ into the equation (16), the contact internal force is solved. The contact internal
force is substituted into equation (12) and the optimized contact force can be solved.

6. Solved example and verification

6.1. Verification of force closure for the object
The geometric parameters and the input kinematics of hybrid manipulator are given as follows: $m_g=10$kg, $f_o=[0.0, -120]$ Nm, $t_o=[0.0, 100]$ Nm, $\mu=0.3$, $r=60$mm. The object is a cylinder. In order to ensure that there is no rolling and sliding between the finger and the object, the driving speed of the fingers is zero. In order to keep the stability of the object, the sum of contact force vectors which is loaded on the object must be zero. When the simulation time is 3 seconds, the force closure of the object is verified. According to the pose between $\{C_{oi}\}$ and $\{B\}$, the transformation matrix $G$ of equation (5) are solved as follows:
\[
\begin{bmatrix}
-0.202 & 0.782 & 0.118 & -0.202 & -0.383 & -0.901 & -0.202 & -0.588 & 0.782 \\
-0.016 & -0.603 & 0.992 & -0.016 & 0.921 & -0.388 & -0.016 & -0.797 & -0.603 \\
0.979 & 0.151 & 0.041 & 0.979 & -0.063 & -0.193 & 0.979 & -0.135 & 0.151 \\
-39.13 & -7.026 & 0 & 16.503 & -8.100 & 0 & 22.62 & -8.105 & 0 \\
4.967 & -0.581 & 0 & -36.85 & -0.697 & 0 & 31.88 & -0.846 & 0 \\
-7.252 & 33.923 & 0 & -2.859 & 39.16 & 0 & 10.11 & 38.44 & 0
\end{bmatrix}
\]
\[
W = \begin{bmatrix}
0 & 0 & 0 & 0.581 & 0 & -0.581 \\
1 & 1521 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]
(22)
\[
F_s = \sum_{i=1}^{3} k_i W_i
\]
It is easy to judge that the rank of formula matrix $G$ is 6, that satisfies the first condition of equation (8). According to the matrix $G$ and formula (8), the zero space fundamental matrix $W$ of matrix $G$ can
be easily obtained. The contact internal force vector is a linear combination of the \( W \)-column vectors of the matrix, as shown in equation (22). According to the second constraint conditioning equation (8), if there is a vector \( P_m = [k_1 \ k_2 \ k_3]^T \) that satisfied \( P^T \! \! F_q \) to be a positive semi-definite matrix, then the solution \( F = k_1 W_1 + k_2 W_2 + k_3 W_3 \) meets the maximum friction constraint.

6.2. Solution and optimization of contact force
In advanced CAD software environment, the process of hybrid manipulator grasping object is simulated, and the contact force between the fingers and the object is obtained. The contact force simulation curves of the first, second and third fingers are shown in Figure 3. The boundary conditions of optimal design are established by the maximum friction constraint condition, and the evaluation function is constructed based on the function minimization method. The design variables are optimized by iterative method by MATLAB software. Let \( k \) be the number of iterations in the process of contact force optimization. The optimization process of the first finger for normal contact force are shown in Figure 4(a).

![Figure 3 Simulation value of contact forces](image1.png)

![Figure 4 Optimization process and results of contact force for three finger](image2.png)

It can be seen from Figure 4 (a) that in the process of optimization. From the fourth time, and the optimization result of contact force for the first finger began to satisfy the maximum friction constraint condition. With the development of optimization, the normal component of the contact force for the first finger decreases gradually. At the 11th iteration, when the difference between two adjacent objective functions is less than the optimization accuracy, the iteration stopped. The normal component of the contact force is optimal. According to the normal component, the tangential force of three fingers is calculated, and the optimized curve of contact force for three fingers is obtained, as shown in Fig. 4 (b). According to the optimized results of the contact force in Fig. 4 (b), it can be seen: the normal component of the optimized contact forces are smaller than that of the simulation contact forces. This not only ensures that there will be no sliding or rolling between the object and the fingers, but also the deformation of the object is avoided.

7. Conclusions
(1) A compound joint is designed, and a hybrid manipulator with compound joint is constructed. (2) The criterion of zero sum of external force vector for the object is derived. Based on the matrix theory. The optimum contact force model of the hybrid manipulator is established, and the formulæ for solving the optimum contact forces are derived.(3) The process of grasping object is simulated by advanced CAD software, and the vector sum of the contact force at a certain moment is verified to be closed. The contact force between the fingers and the object is optimized by MATLAB software.
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