High-Gain Extended State Observer Based Adaptive Sliding Mode Path Following Control for An Underactuated Vessel Sailing in Restricted Waters

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Received: 20 January 2019; Accepted: 11 March 2019; Published: 15 March 2019

Abstract: This paper studied the path following problem for an underactuated vessel sailing in restricted waters with varying water depths. A novel high-gain extended state observer based adaptive sliding mode path following control scheme was proposed. The high-gain extended state observer based line-of-sight guidance law was designed according to vessel kinematics in the horizontal plane, which achieved accurate guidance in spite of time-varying sideslip angles. In the guidance system, a guidance angle was calculated to serve as a reference input for the yaw tracking control system. The sliding mode yaw tracking control system was designed, which can deal with model uncertainties and external disturbances. Since it is hard to obtain the exact model parameters in advance, an adaptive technique was adopted to estimate the unknown parameters, and an adaptive sliding mode control was designed to make the yaw tracking errors globally and asymptotically converge to zero in spite of unknown model parameters, model uncertainties, and external disturbances. Furthermore, the global uniformly asymptotically stability of the closed-loop system was proven based on the cascade system theory. Lastly, simulation experiments were conducted to validate the analysis results and to demonstrate the superiority of the proposed scheme.

Keywords: underactuated vessel; shallow water effect; path following; line-of-sight guidance; high-gain extended state observer; adaptive sliding mode control

1. Introduction

A typical motion control problem for an underactuated vessel is path following, which is concerned with the design of control laws that force a vessel to reach and follow a time-independent geometric path. Pioneering works [1–6] in this area proposed many solutions for a vessel sailing in open waters. When a vessel sails in restricted waters (e.g., coastal areas), vessel-bank and vessel-bottom interactions have significant influences on the maneuverability and make it difficult to maintain a steady course [7,8]. Many studies have pointed out that a vessel sailing in restricted waters is unstable in the absence of reasonable control [9]. Therefore, the motion controller design for an underactuated vessel sailing in restricted waters is required. Some researchers [7,10–15] made extensive studies on vessel behaviors in restricted waters, including the detailed hydrodynamic coefficients in the mathematical models of vessel motion and analysis of directional stability. Their research laid the foundation for the study and design of the controller for an underactuated vessel sailing in restricted waters. Several control methods were introduced and satisfactory results were obtained assuming...
the water depth being constant [7,14–16]. While in real life, the actual seabed usually has a varying bottom, resulting in varying water depths. The straight-line path following control problem with varying water depths is needed, considering the poor maneuverability in restricted waters.

The straight-line path following control for an underactuated surface vessel without sway actuator is studied. The absence of sway actuator brings about significant challenges on the path following control design. To overcome this difficulty, an effective way is to implement the path following control scheme, which consists of a guidance system and a control system. In the guidance system, the reference signals for the successive control system are produced according to the path information and environmental conditions. A commonly used guidance system is designed based on the traditional line-of-sight (LOS) guidance law. Despite its simplicity and effectiveness, it has limitations when the vessel is subject to model uncertainties and external disturbances acting in the underactuated lateral direction. Variations in the vessel’s lateral position generate a nonzero sideslip angle for the vessel. Using the traditional LOS, the desired path cannot be followed by an underactuated vessel subjecting to the effect of sideslip angle.

To solve this problem, scholars have devoted many efforts to the field in the past. Hac and Bevly measured the sideslip angle directly using sensors [17,18], while the data from sensors is affected by noise and sensor errors, which is not precise enough. Another method to solve this problem is the integral line-of-sight (ILOS) guidance. It was first put forward by Borhaug et al. [19], and the integral term was added to the traditional LOS to compensate for the constant current to achieve precise path following. Furthermore, Fossen et al. [20] adopted the adaptive control theory to design the guidance law, they assumed the sideslip angle to be an unknown constant. They also proposed both direct and indirect adaptive LOS (ALOS) guidance laws in [21] to deal with the time-varying current. Nevertheless, choosing adaptive gains is a tough problem because of the intrinsic tradeoff between the convergence speed and the chattering. A reduced order state observer was proposed to design the guidance law to conquer the intrinsic weakness of the ALOS guidance [22]. However, the sideslip angle is assumed small, and only one design parameter is available in the observer design, it is hard to guarantee better estimation performance. Furthermore, none of the above work considered designing a guidance system for a vessel sailing in restricted waters.

In the guidance system, a guidance angle is calculated to serve as a reference input for the yaw tracking control system. For a vessel sailing in restricted waters with varying water depths, challenges in the yaw tracking control system lay in the inherent nonlinearity, unknown parameters and external disturbances. To overcome these difficulties, a back-stepping approach was adopted to design the nonlinear yaw tracking control system [2,23]. However, the back-stepping method requires repeated differentiation of the virtual control function, which results in a complicated control law. Alternatively, the sliding mode technology was proposed to design the sliding mode yaw tracking control system [21,24], while imperfections in switching devices and delays lead to the system suffering from chattering. Furthermore, up till now, very few researchers considered the yaw tracking control for a vessel sailing in restricted waters with varying water depths.

Motivated by the above studies, a new high-gain extended state observer based adaptive sliding mode path following control (HGESO-ASMPFC) scheme is proposed in this essay, to obtain the straight-line path following for a vessel sailing in restricted waters with varying water depths. The high-gain extended state observer is adopted to design the guidance law, which can exactly estimate time-varying sideslip angles, and guarantee system stability and performance in spite of vessel-bank interaction and varying water depths. In the yaw tracking control system, the vessel’s steering dynamics is described by Nomoto equations. The equations’ parameters vary with water depths and vessel-bank distance. According to the Nomoto equations, an adaptive continuous proportional integral sliding mode control (PI SMC) is put forward to make the yaw converge to the guidance angle despite unknown parameters, model uncertainties, and external disturbances. The proposed yaw tracking control system can reduce chattering and relax assumptions in [25], such as the existences of first derivatives of disturbances and reference input. In the end, the closed-loop system is proven to be
global uniform asymptotic stable (GUAS), the path following error converges to zero asymptotically. Simulation experiments are carried out to validate the proposed path following control scheme and demonstrate its superiority.

2. Preliminaries and Problem Formulation

2.1. Preliminaries

Some key definitions and lemmas adopted by the control system design and stability analysis are collected and introduced here.

Definition 1 [26]. Given a system
\[
\dot{x} = f(t, x),
\]
its equilibrium \( x = 0 \) is GUAS if and only if
\[
\|x(t)\| \leq \zeta(\|x(0)\|, t), (t > 0)
\]
where \( \zeta(\cdot) \) is a class KL function.

Definition 2 [26]. Given a system
\[
\dot{x} = f(t, x, u),
\]
where \( f : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^p \) is continuously differentiable, the system is input-to-state stable (ISS) if
\[
\|x(t)\| \leq \zeta(\|x(0)\|, t) + \gamma(\|u\|), (t > 0)
\]
for any initial state and any bounded input, in which \( \zeta(\cdot) \) is a class KL function, \( \gamma(\cdot) \) is a class K function.

Lemma 1 [27]. Given the system (1) with the equilibrium \( x = 0 \), \( f(\cdot) \) is locally Lipschitz in \( x \) uniformly in \( t \). There exists a continuously differentiable, positive definite and radially unbounded function \( V : [0, \infty) \times \mathbb{R}^n \rightarrow \mathbb{R} \), it satisfies
\[
\dot{V} = \frac{\partial V}{\partial x} f(x, t) \leq -W(x) \leq 0, \forall t \geq 0, \forall x \in \mathbb{R}^n,
\]
in which \( W \) is continuous. Then, all solutions of system (1) are globally uniformly bounded and satisfy
\[
\lim_{t \to \infty} W(x(t)) = 0.
\]
In addition, if \( W(x) \) is positive definite, then the equilibrium \( x = 0 \) is GUAS.

Lemma 2 [26]. Let \( V : [0, \infty) \times \mathbb{R}^n \rightarrow \mathbb{R} \) be continuously differentiable and satisfy
\[
\alpha_1(\|x\|) \leq V(t, x) \leq \alpha_2(\|x\|),
\]
\[
\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x, u) \leq -W(x), \forall \|x\| \geq \rho(\|u\|) > 0,
\]
where \( \forall (t, x, u) \in [0, \infty) \times \mathbb{R}^n \times \mathbb{R}^m \), \( \alpha_1, \alpha_2 \) are class \( K_\infty \) functions, \( \rho \) is a class K function, and \( W(x) \) is a continuous positive definite function on \( \mathbb{R}^n \). Then, the system \( \dot{x} = f(t, x, u) \) is ISS with \( \gamma = \alpha_1^{-1} \circ \alpha_2 \circ \rho \).

The following lemmas establish the stability of a cascade system.

Lemma 3 [26]. Consider the cascade system
\[
\dot{x}_1 = f_1(t, x_1, x_2),
\]
\[
\dot{x}_2 = f_2(t, x_2),
\]
if the system (8) with \( x_2 \) as input is ISS and the equilibrium point of the system (9) is GUAS, then the equilibrium point of the cascade systems (8) and (9) is GUAS.

**Lemma 4** [27]. Consider the cascade system

\[
\dot{x}_1 = f_1(t, x_1, x_2), \quad (10)
\]

\[
\dot{x}_2 = f_2(t, x_2, u), \quad (11)
\]

if the system (10) with \( x_2 \) as input is ISS, and the system (11) with \( u \) as input is ISS, then the cascade systems (10) and (11) are ISS.

2.2. Vessel Kinematics and Dynamics

The schematic diagrams for a vessel sailing in restricted waters are presented in Figures 1 and 2. Here restricted waters with sloped bottoms are considered since the term “slope bottom” is a simple but quite realistic description of the near-shore area. Two right-handed reference frames, the inertial reference frame \( o_x = x_3y_3z_3 \), and the body-fixed reference frame \( o_{xb} = x_{yb}y_{yb}z_{yb} \), are adopted. The plane \( o_x = x_3y_3 \) is fixed at the undisturbed free surface, with \( o_x \) being the vessel’s initial position, \( y_{yb} \)-axis being perpendicular to the bank with the positive direction pointing to the bank, and the positive direction of \( x_{yb} \)-axis pointing toward the vessel’s course; the plane \( o_{yb} = x_{yb}y_{yb} \) is located at the undisturbed free surface, with the origin \( o_{yb} \) at the midship section, \( x_{yb} \)-axis directing from aft to fore, and \( y_{yb} \)-axis directing to starboard. \( D \) presents the initial distance of the vessel away from the bank, and \( D = 44.5 \text{ m} \).

![Figure 1](image1.png)

**Figure 1.** Coordinate systems for a vessel sailing in restricted waters.

![Figure 2](image2.png)

**Figure 2.** Sketch maps of a vessel sailing in restricted waters with varying water depths. (a) Side view; (b) rear view.

The kinematic equations for the vessel’s 3-DOF motion in the horizontal plane are:

\[
\dot{x} = u \cos \psi - v \sin \psi, \\
y = u \sin \psi + v \cos \psi, \\
\dot{\psi} = r, \quad (12)
\]
where $[x, y, \psi]^T$ denotes the position and orientation of the vessel in the inertial reference frame, $[u, v, r]^T$ is the velocity vector including surge and sway velocities and yaw rate in the body-fixed reference frame.

It is assumed that (i) the vessel is regarded to be rigid with its mass uniformly distributed; (ii) the vessel is symmetrical about the longitudinal center plane; (iii) the vessel’s speed $U$ is constant, the yaw angle $\psi$ is small, therefore $u = U$; and only the sway and yaw motions are considered; (iv) the bank is straight.

According to the above assumptions, the dynamics of a vessel sailing in restricted waters is denoted:

$$
\begin{align*}
(m + m_{22}) \ddot{v} + (x_G m + m_{26}) \dot{r} + (m + m_{11}) r = Y(v, r, y, h, \delta), \\
(I_z + x_G^2 m + m_{66}) \ddot{r} + (x_G m + m_{26}) \dot{v} + x_G m r = N(v, r, y, h, \delta),
\end{align*}
$$

where $x_G$ is the longitudinal position of the vessel’s center of gravity. $m$ is the vessel’s mass and $I_z$ is the inertia moment about $o_{obz}$; $m_{ij}$ is the added mass. $Y(\cdot)$ and $N(\cdot)$ are the sway force and yaw moment acting on the vessel excepting the hydrodynamic inertia terms, the item $h$ in $Y(\cdot)$ and $N(\cdot)$ represents the influence of water depth on the vessel motion. The detailed expressions for $Y(\cdot)$ and $N(\cdot)$ refer to Sano et al. [7]. According to Equation (13), the only available control is the rudder angle $\delta$. To solve the path following problem of an underactuated vessel, a sideslip observer based path following control scheme was designed, to make the vessel converge to the desired path.

2.3. Path Following Error Dynamics and Control Target

To design the sideslip observer, the path following error dynamics are calculated. Since the desired path is a straight-line, the path tangential angle is zero. For a vessel located at $(x, y)$, the cross-track error is defined as

$$
y_e = y - y_d.
$$

Therefore, the path following goal turns into stabilizing $|y_e|$ to zero.

Taking the time derivative of $y_e$, it follows from Equation (12):

$$
\dot{y}_e = u \sin \psi + u \cos \psi \tan \beta,
$$

where $\beta = \atan\left(\frac{v}{u}\right)$ is the sideslip angle (see Figure 1), denotes the angle between the vessel’s velocity vector and the $x_G$-axis, the positive direction is defined according to Figure 1, which is unknown in practice. Without compensating the unknown sideslip angle for an underactuated vessel, it will cause a large tracking error. When a vessel sails along the bank in restricted waters, bank effect with varying water depths in shallow water results in time-varying sideslip angles. To accurately estimate the time-varying sideslip angles, a HGESO was designed.

The control goal is to design a HGESO-ASMPFC scheme, such that the vessel with kinematics (12) and dynamics (13) converges to and precisely follows the desired straight-line path subjecting to vessel-bank interaction with varying water depths in shallow water, namely fulfills $\lim_{t \to \infty} y_e = 0$.

3. Guidance Law and Analysis

3.1. Sideslip Angle Observer

In this subsection, the high-gain extended state observer was designed to exactly estimate the time-varying sideslip angles. Before starting, the cross-track error dynamics (15) is rewritten as:

$$
\ddot{y}_e = u \sin \psi + g(u, \psi, \beta),
$$

where

$$
g(u, \psi, \beta) = v \cos \psi = u \cos \psi \cdot a,
$$

where $\alpha = \tan \beta$.

**Proposition 1.** The nonlinearity $g(\cdot)$ is $C^1$ differentiable and satisfies $|\dot{g}| \leq G < \infty$ ($G$ is a positive constant).

**Proof.** The derivative of $g$ is

\[
\dot{g} = \dot{v} \cos \psi - \dot{v} \sin \psi \cdot r
\]

where $f_v(\cdot)$ is continuous, and is determined by the dynamics of the vessel. According to extreme value theorem, $f_v(\cdot)$ is bounded for $v, r, y$ and $\delta$ are bounded for a vessel sailing along the bank. Therefore, there exists a positive constant $G$ satisfying $|\dot{g}| \leq G < \infty$. □

Design the high-gain extended state observer as:

\[
\begin{align*}
\dot{\hat{y}}_e &= u \sin \psi + \hat{g} + a_1 \epsilon (y_e - \hat{y}_e), \\
\dot{\hat{g}} &= a_2 \epsilon^2 (y_e - \hat{y}_e).
\end{align*}
\]

where $\hat{y}_e$ and $\hat{g}$ are estimation value of $y_e$ and $g$ respectively. Define the estimation error as

\[
\begin{align*}
\tilde{y}_e &= \hat{y}_e - y_e, \\
\tilde{g} &= \hat{g} - g,
\end{align*}
\]

whose derivatives combining with Equations (16) and (19) can be expressed as

\[
\begin{align*}
\dot{\tilde{y}}_e &= -a_1 \epsilon \tilde{y}_e + \tilde{g}, \\
\dot{\tilde{g}} &= -a_2 \epsilon^2 \tilde{y}_e - \tilde{g}.
\end{align*}
\]

**Lemma 5.** The system (20), with the states being $(\tilde{y}_e, \tilde{g})$ and the input being $\dot{g}$, is ISS.

**Proof.** Rewrite Equation (20) into a compact form

\[
\dot{x}_o = A_o x_o + B_o g,
\]

where $x_o = (\tilde{y}_e, \tilde{g})^T$, $A_o = \begin{bmatrix} -a_1 \epsilon & 1 \\ -a_2 \epsilon & 0 \end{bmatrix}$, $B_o = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$.

Consider a Lyapunov function candidate

\[
V_1 = \frac{1}{2} x_o^2.
\]

Taking the time derivative of $V_1$, it follows from Equation (21):

\[
\dot{V}_1 = x_o \dot{x}_o
= x_o (A_o x_o + B_o \dot{g})
\leq \lambda_{\max}(A_o) \cdot \|x_o\|^2 + \|B_o\| \|x_o\| \|\dot{g}\|
= \lambda_{\max}(A_o) \cdot (1 - \theta_1) \|x_o\|^2 + \theta_1 \lambda_{\max}(A_o) \cdot \|x_o\|^2 + \|B_o\| \|x_o\| \|\dot{g}\|,
\]

where $0 < \theta_1 < 1$. By properly choosing $a_1$ and $a_2$, the eigenvalues of $A_o$ meet $\lambda_{\max}(A_o) < 0$, then it follows:

\[
\|x_o\| \geq \frac{\|B_o\| \|\dot{g}\|}{\lambda_{\max}(A_o) \cdot \theta_1},
\]

leads to

\[
\dot{V}_1 \leq \lambda_{\max}(A_o) \cdot (1 - \theta_1) \|x_o\|^2 \leq 0
\]
Choosing \( \alpha_0(s) = \alpha_{02}(s) = \frac{1}{2}s^2 \), \( \rho_0(s) = \frac{\|R_0\|s}{s_{\text{max}}(A_0)\theta_1} \), by Definition 2 and Lemma 2, it follows from Equation (25) that
\[
\|x_0(t)\| \leq \zeta_0(\|x_0(0)\|, t) + \gamma_0(\|\dot{y}\|),
\]
where \( \zeta_0(\cdot) \) is a class KL function, \( \gamma_0(\cdot) \) is a class K function. This concludes the proof. \( \square \)

### 3.2. Guidance Law Design

Based on the high-gain extended state observer in (19), the guidance law is designed as
\[
\psi_d = \arctan\left(-\frac{y_e}{\Delta} - \hat{\alpha}\right),
\]
where Delta is the designed look ahead distance, and
\[
\hat{\alpha} = \frac{\dot{\gamma}}{u \cos \psi}.
\]

Noting that
\[
\begin{align*}
\sin \psi_d &= \sin(\arctan(-\frac{y_e}{\Delta} - \hat{\alpha})) = -\frac{y_e + \Delta \hat{\alpha}}{\sqrt{\Delta^2 + (y_e + \Delta \hat{\alpha})^2}}, \\
\cos \psi_d &= \cos(\arctan(-\frac{y_e}{\Delta} - \hat{\alpha})) = \frac{\Delta}{\sqrt{\Delta^2 + (y_e + \Delta \hat{\alpha})^2}}.
\end{align*}
\]

It is assumed that \( \psi = \psi_d \). The cross-track error dynamics is denoted as
\[
\dot{y}_e = u \sin \psi + u \cos \psi \tan \beta = \frac{-u y_e}{\sqrt{\Delta^2 + (y_e + \Delta \hat{\alpha})^2}} + \frac{u \Delta \cdot \hat{\alpha}}{\sqrt{\Delta^2 + (y_e + \Delta \hat{\alpha})^2}},
\]
where \( \hat{\alpha} = \alpha - \hat{\alpha} \).

**Lemma 6.** The system (30), with the state being \( y_e \) and the input being \( \hat{\alpha} \), is ISS.

**Proof.** Consider a Lyapunov function candidate
\[
V_2 = \frac{1}{2}y_e^2.
\]

Taking the time derivative of \( V_2 \), it follows from Equation (30):
\[
\dot{V}_2 = \frac{-u y_e^2}{\sqrt{\Delta^2 + (y_e + \Delta \cdot \hat{\alpha})^2}} + \frac{u \Delta \cdot \hat{\alpha} \cdot y_e}{\sqrt{\Delta^2 + (y_e + \Delta \cdot \hat{\alpha})^2}}.
\]

Let
\[
c = \frac{u}{\sqrt{\Delta^2 + (y_e + \Delta \cdot \hat{\alpha})^2}} > 0.
\]

It follows:
\[
\dot{V}_2 = -c \cdot y_e^2 + c \cdot \Delta \cdot \hat{\alpha} \cdot y_e
\]
\[
= -c \cdot (1 - \theta_2) \cdot y_e^2 - c \cdot \theta_2 \cdot y_e^2 + c \cdot \Delta \cdot \hat{\alpha} \cdot y_e,
\]
where \( 0 < \theta_2 < 1 \), then it gives
\[
|y_e| \geq \frac{\Delta \cdot \hat{\alpha}}{\theta_2},
\]
leading to
\[
\dot{V}_2 \leq -c \cdot (1 - \theta_2) \cdot y_e^2 \leq 0.
\]
Choosing \( a_{11}(s) = a_{12}(s) = \frac{1}{2}s^2 \), \( p_1(s) = \frac{\Delta}{\tilde{z}^2} \), by Definition 2 and Lemma 2, it follows from Equations (36) that there exists a class KL function \( \zeta_1(\cdot) \) and a class K function \( \gamma_1(\cdot) \) such that
\[
\|y_e(t)\| \leq \zeta_1(\|y_e(0)\|, t) + \gamma_1(\|\tilde{r}\|).
\] (37)

This concludes the proof. \( \square \)

3.3. Cascade Stability of the HGESO and the Guidance Law

The closed-loop systems (20) and (30) are rewritten as a cascade form
\[
\sum_1: \dot{y}_e = -cy_e + \tilde{g},
\] (38)
\[
\sum_2: \begin{cases}
\dot{\tilde{y}}_e = -\frac{a_1}{\tau} \tilde{y}_e + \tilde{g}, \\
\dot{\tilde{g}} = -\frac{a_2}{\tau} \tilde{y}_e - \tilde{g}.
\end{cases}
\] (39)

where the state \( \tilde{g} \) of the system \( \Sigma_2 \) is the only input of the system \( \Sigma_1 \), therefore the closed-loop system \( \Sigma_1 - \Sigma_2 \) forms completely cascade structure.

**Theorem 1.** Applying the HGESO based LOS guidance schemes (19) and (27) to the path following system (15), it gives the closed-loop system ISS.

**Proof.** Lemmas 5 and 6 have shown that system \( \Sigma_1 \) with state \( y_e \) and exogenous input \( \tilde{g} \) and system \( \Sigma_2 \) with the states \( (\tilde{y}_e, \tilde{g}) \) and the exogenous input \( \tilde{g} \) are ISS, respectively. By Lemma 4, it can be concluded that the closed-loop system \( \Sigma_1 - \Sigma_2 \) with states \( (y_e, \tilde{y}_e, \tilde{g}) \) and exogenous input \( \tilde{g} \) is ISS. \( \square \)

4. The Yaw Tracking Control System

The steering dynamics for a vessel sailing in restricted waters is described by the following Nomoto equations
\[
\begin{align*}
\dot{\phi} &= r, \\
\dot{r} &= a_v \dot{v} + a_r r + a_y y + b\delta + \Lambda(v, r, y, \delta),
\end{align*}
\] (40)

where \( a_v, a_r, \) and \( a_y \) are the linear hydrodynamic coefficients; \( y \) is the lateral distance off the original course line, to present the bow outward moment due to the vessel-bank interaction. When a vessel sails close to a bank, the vessel moves with its bow being repelled from the bank. It is hard to obtain precise motion models for a vessel sailing in restricted waters, not to mention the external disturbances. The item \( \Lambda(\cdot) \) is used to denote the combination of model uncertainties and external disturbances, which meets \( |\Lambda(\cdot)| \leq \Lambda_0 \), where \( \Lambda_0 \) is the upper bound of \( \Lambda(\cdot) \). The rudder angle \( \delta \) acts as the only control input for the system (40).

The sliding mode control (SMC) is a well-known robust method to control nonlinear and uncertain systems. To achieve the objective of \( \lim_{t \to \infty} \dot{\psi} = \lim_{t \to \infty} (\psi - \psi_d) = 0 \), the sliding surface in the standard SMC is selected as
\[
s_0 = \lambda \dot{\psi} + \ddot{\psi},
\] (41)

where \( \lambda > 0 \).

It is clear that if a control law is designed to make the trajectories of the closed-loop system (40) driving on and evolving along the sliding surface (41), the stabilization goal \( \lim_{t \to \infty} \ddot{\psi} = 0 \) can be reached. To accomplish it, the reaching law is chosen
\[
\dot{s}_0 = -\tau_0 s_0 + \Lambda(\cdot) - \rho_0 \text{sgn}(s_0),
\] (42)
where $\tau_0 > 0$, the reaching law can guarantee the convergence of the trajectory of the closed-loop system [28]. With this reaching law, the control law can be obtained

$$\delta_0 = \frac{1}{b} \left( \lambda r - \lambda \dot{\psi}_d + a_v v + a_r r + a_y y - \dot{\phi}_d + \tau_0 s_0 + \rho_0 \text{sgn}(s_0) \right)$$  \hspace{1cm} (43)

where $\rho_0 > \Lambda_0$.

Consider the following Lyapunov function candidate

$$V_3 = \frac{1}{2} s_0^2$$  \hspace{1cm} (44)

Its time derivative is

$$\dot{V}_3 = s_0 \dot{s}_0$$

$$= s_0 [ - \tau_0 s_0 + \Lambda(\cdot) - \rho_0 \text{sgn}(s_0) ] < 0.$$  \hspace{1cm} (45)

Thus the control objective $\lim_{t \to \infty} \dot{\phi} = 0$ can be achieved. However, the standard SMC has two drawbacks. One is that the model coefficients ($a_v$, $a_r$ and $a_y$), the control coefficient $b$ and the bound of uncertainty $\Lambda(\cdot)$ cannot be precisely known in advance. The other one is that the expressions for $\dot{\phi}_d$ and $\dot{\psi}_d$ in Equation (43) are complicated, which makes the control law hard to be applied in the real system.

To overcome these difficulties, an adaptive PI SMC is proposed in this paper. To design the adaptive PI SMC, the steering dynamics in Equation (40) are reconstructed as

$$\begin{aligned}
\dot{\psi} &= r,
\dot{J} &= a_v v + a_r r + a_y y + \delta + \Gamma(v, r, y, \delta),
\end{aligned}$$  \hspace{1cm} (46)

where $J = \frac{1}{p}$, $a_v = \frac{\dot{a}_v}{p}$, $a_r = \frac{\dot{a}_r}{p}$ and $a_y = \frac{\dot{a}_y}{p}$, they are unknown in advance; $\Gamma(\cdot) = \frac{\Lambda(\cdot)}{p}$ meets $|\Gamma(\cdot)| \leq \Gamma_0$, where $\Gamma_0$ is the upper bound of $\Gamma(\cdot)$ which is unknown in advance.

**Remark 1.** Equation (46) is derived from Equation (40) assuming that $b \neq 0$. It is easily met for the real vessel.

In the proposed adaptive PI SMC, the sliding surface is selected as

$$s_a = \lambda_1 \int \tilde{\phi} dt + \lambda_2 \tilde{\phi} + r,$$  \hspace{1cm} (47)

where $\lambda_1 > 0$, $\lambda_2 > 0$, which fulfill that the eigenvalues of $\nabla^2 + \lambda_2 \nabla + \lambda_1 = 0$ ($\nabla$ is Laplace operator) are in the left half complex plane.

Combining with the sliding surface (47), a control law can be obtained to make the system states moving from the outside to the inside of the sliding surface, and finally, they remain inside the surface despite unknown parameters, model uncertainties, and external disturbances. Therefore, the control law in the proposed adaptive PI SMC is

$$\delta_a = \delta_p - (\lambda_1 \tilde{\phi} + \lambda_2 r) \hat{J} - \dot{\phi}_d v - \dot{\phi}_d r - \dot{\phi}_d y - \tau_0 s_a,$$  \hspace{1cm} (48)

where

$$\delta_p = -\text{sgn}(s_a) \dot{\Gamma}_0.$$  \hspace{1cm} (49)

The adaptive law is

$$\begin{aligned}
\dot{J} &= \frac{1}{p_1} s_a (\lambda_1 \tilde{\phi} + \lambda_2 r), \\
\dot{a}_v &= \frac{1}{p_2} s_a v, \\
\dot{a}_r &= \frac{1}{p_3} s_a r, \\
\dot{a}_y &= \frac{1}{p_4} s_a y, \\
\dot{\Gamma}_0 &= \frac{1}{p_3} |s_a|.
\end{aligned}$$  \hspace{1cm} (50)
Therefore, all problems existing in the standard SMC can be solved by the modified control.

Remark 4. The control law (48) is discontinuous when crossing the sliding surface \( s_a = 0 \), which may lead to undesirable chattering. A solution to eliminate chattering is to introduce a bounded layer around the sliding surface, replace the signum function by a high-slope saturation [30]:

\[
\delta_p = -\Gamma_0 \text{sat}\left( \frac{s_a}{\varepsilon} \right),
\]

Substituting Equations (48)–(50) into Equation (52) yields

\[
\dot{V}_4 = s_a \left( J_1 \dot{\psi} + J_2 \dot{r} + J_\Gamma \right) + s_0 \left( \lambda_1 \dot{\psi} + \lambda_2 \dot{r} + \alpha_v \dot{s}_a + \alpha_r \dot{r} + \alpha_y \dot{y} + \delta \right) + \Gamma_0 \left( \dot{s}_a \right)
\]

Thus, the yaw tracking control system is GUAS. According to Barbalat’s Lemma [29], the system states move to and stay on the sliding surface \( \lim_{t \to \infty} s_a(t) = 0 \). In addition, the objective \( \lim_{t \to \infty} \dot{\psi} = 0 \) can be achieved. □

Remark 2. The control law (48) is independent of the model coefficients and the control coefficient. Thus, the proposed control law can be used to design the yaw tracking control system with large model uncertainties. Moreover, the controller does not depend on the disturbance bound and the derivative terms (\( \dot{\psi}_d \) and \( \dot{\phi}_d \)). Therefore, all problems existing in the standard SMC can be solved by the modified control.

Remark 3. The parameters \( \lambda_1 > 0, \lambda_2 > 0 \) in the control law (48) are very important, for they determine the convergence rate of the sliding surface. It is clear that the farther the eigenvalues of \( \nabla^2 + \lambda_2 \nabla + \lambda_1 = 0 \) away from the imaginary axis on the left half complex plane, the faster the convergence to the origin. While fast convergence speeds need very high control input, which is always bounded in real life, thus \( \lambda_1 > 0, \lambda_2 > 0 \) must be selected reasonably.

Remark 4. The control law (48) is discontinuous when crossing the sliding surface \( s_a = 0 \), which may lead to undesirable chattering. A solution to eliminate chattering is to introduce a bounded layer around the sliding surface, replace the signum function by a high-slope saturation [30]:

\[
\delta_p = -\Gamma_0 \text{sat}\left( \frac{s_a}{\varepsilon} \right),
\]
where \( \varepsilon > 0 \) is the bounded layer; \( \text{sat}(\cdot) \) is the saturation function defined by

\[
\text{sat}(u_i) = \begin{cases} 
  u_i, & \text{if } |u_i| \leq 1 \\
  \text{sgn}(u_i), & \text{if } |u_i| > 1 
\end{cases}
\]  

(55)

The benefit of the smooth technique is that it offers a continuous approximation to the discontinuous SMC law inside the bounded layer, and guarantees the output tracking error within any neighborhood of zero.

5. Stability Analysis of the Path Following Control System

In this section, the stability analysis of the HGESO-ASMPFC scheme as presented in Figure 3 was carried out.

![Figure 3. The block diagram of the HGESO-ASMPFC scheme.](image)

Substituting \( \psi = \psi_d + \tilde{\psi} \) into Equation (15), the cross-track error dynamics satisfies

\[
\dot{y}_e = -c \cdot y_e + c \cdot \Delta \cdot \tilde{\alpha} + D_{y_e}
\]  

(56)

where

\[
D_{y_e}(\tilde{\psi}) = -c(y_e - \Delta \tilde{\alpha}) \cdot (\cos \tilde{\psi} - 1) + c[\Delta + \alpha \cdot (y_e + \Delta \cdot \tilde{\alpha})] \cdot \sin \tilde{\psi},
\]  

(57)

which implies that \( D_{y_e}(0) = 0 \).

The guidance system can be presented as

\[
\begin{aligned}
\dot{y}_e &= -c y_e + c \cdot \Delta \cdot \tilde{\alpha} + D_{y_e}, \\
\dot{\tilde{y}}_e &= -\frac{a_1}{\varepsilon} \tilde{y}_e + \tilde{g}, \\
\dot{\tilde{g}} &= -\frac{a_2}{\varepsilon} \tilde{y}_e - \tilde{g}.
\end{aligned}
\]  

(58)

According to Theorem 1, the nominal system

\[
\begin{aligned}
\dot{y}_e &= -c y_e + c \cdot \Delta \cdot \tilde{\alpha} \\
\dot{\tilde{y}}_e &= -\frac{a_1}{\varepsilon} \tilde{y}_e + \tilde{g} \\
\dot{\tilde{g}} &= -\frac{a_2}{\varepsilon} \tilde{y}_e - \tilde{g}
\end{aligned}
\]  

(59)

is ISS.

To analyze the stability of the entire path following control system, the cross-track error dynamics and the tracking error dynamics are written as:

\[
\sum' : \dot{y}_e = -c y_e + c \cdot \Delta \cdot \tilde{\alpha} + D_{y_e},
\]  

(60)
where \( \tilde{\psi} = f_\psi(\psi, \psi_d) \).

Theorem 3. The guidance system, given by Equations (19) and (27) together with the control law (48), renders the origin of the entire closed-loop system GUAS.

Proof. According to Theorem 2, the yaw tracking control system renders the equilibrium point \( \tilde{\psi} = 0 \) of the subsystem \( \Sigma'_2 \) GUAS. According to Lemma 4, the cascade system of \( \Sigma'_1 - \Sigma'_2 \) is GUAS. Since the external input \( \tilde{g} \) meets \( \lim_{t \to \infty} \tilde{g} = 0 \), it follows \( \lim_{t \to \infty} y_e = 0 \). □

6. Simulation Results

To validate the theoretical results, simulations were carried out in this part. The developed strategy was applied to an underactuated vessel in the simulations. The main parameters of the vessel are given in Table 1, the dynamics of the vessel are given by Sano et al. [7]. The control objective is to make the vessel follow a straight-line path parallel to the bank with varying water depths. The surge speed of the vessel is assumed to be constant. The initial kinematics and dynamics of the vessel are \([x_0, y_0, \psi_0]^T = [0, 0, 0]^T\), \([u_0, v_0, r_0]^T = [3.075 \text{ m/s}, 0, 0]^T\). The rudder saturation and rate limits \((|\delta| \leq 35^\circ, |\dot{\delta}| \leq 5^\circ/\text{s})\) are considered.

Table 1. Main parameters of the vessel.

| Parameter                              | Value       |
|----------------------------------------|-------------|
| Length (m)                             | 136.7       |
| Breadth (m)                            | 22.0        |
| Mean draft (m)                         | 10.5        |
| Longitudinal position of the vessel’s center of gravity (m) | -0.89       |
| Displacement volume (m\(^3\))          | 15,317      |

Case 1. The desired lateral position is set as \( y_d = 0.1L \) (\( L \) denotes vessel length), the water depth to draft ratio is set as 2.03 in the beginning. When the vessel sails 100 \( L \) (\( t = 4446 \) s), the water depth to draft ratio begins to decrease gradually until its value reaches 1.72; then the water depth is maintained unchanged until the vessel sails 200 \( L \) (\( t = 8892 \) s), the water depth to draft ratio begins to decrease gradually again until its value reaches 1.37, then the water depth is maintained unchanged till the end of the simulation. In the process of varying water depths, the slope of the varying bottom is set as 1.

The initial values for the high-gain extended state observer is \([-y_d, 0]\), the design parameters of the proposed guidance law are \( \alpha_1 = 10, \alpha_2 = 2, \epsilon = 0.2 \). Comparison studies with ILOS, ALOS and ELOS guidance laws are conducted to give a more vivid demonstration of the proposed guidance law. An ILOS guidance law proposed by Borhaug et al. [19] is

\[
\begin{align*}
\psi_d &= -\tan^{-1}\left(\frac{y_e + \epsilon_\text{ILOS} y_{\text{int}}}{\Delta_\text{ILOS}}\right), \\
y_{\text{int}} &= \frac{\Delta_\text{ILOS} \psi_d}{(y_e + \epsilon_\text{ILOS} y_{\text{int}})^2 + \Delta_\text{ILOS}^2}.
\end{align*}
\]

An ALOS guidance law proposed by Fossen et al. [20] is

\[
\begin{align*}
\psi_d &= -\tan^{-1}\left(\frac{1}{x_{\text{ALOS}} y_e + \hat{\beta}}\right), \\
\hat{\beta} &= \gamma_{\text{ALOS}} \frac{\Delta_\text{ALOS} y_e}{\sqrt{\Delta_\text{ALOS}^2 + (y_e + \Delta_\text{ALOS} \hat{\beta})^2}}.
\end{align*}
\]
An ELOS guidance law proposed by Liu et al. [22] is

\[
\begin{align*}
\psi_d &= -\tan^{-1}\left(\frac{\psi_e \Delta_{ELOS}}{\sigma_{ELOS}} + \hat{\beta}\right), \\
\hat{\beta} &= \frac{\hat{\theta}}{U \cos \psi}, \\
\hat{\theta} &= p + k_{ELOS} y_e, \\
\dot{p} &= -k_{ELOS} p - k_{ELOS}^2 y_e - k_{ELOS} U \sin \psi.
\end{align*}
\]

The design parameters in these guidance laws are properly selected by a trial and error approach to guarantee that the closed-loop system, under the corresponding guidance law, performs better. The look-ahead distance in ILOS is \(\Delta_{ILOS} = 4L\), the integral coefficient is \(\sigma_{ILOS} = 0.08\). The look-ahead distance in ALOS is \(\Delta_{ALOS} = 4L\), the adaptive coefficient is \(\gamma_{ALOS} = 0.01\). The look-ahead distance in ELOS is \(\Delta_{ELOS} = 5.5L\), the observer gain is \(k_{ELOS} = 0.12\), the initial value of ELOS is set as \(p(t_0) = -k_{ELOS} y_e(t_0)\).

In Figure 4, comparison of lateral position with different guidance laws at \(y_d = 0.1L\) is presented. It can be seen that the vessel with all the guidance laws can track the desired straight-line path in the steady state, while the response in the dynamic process is significantly different. In the proposed guidance law, the lateral position suffers the smallest overshoot with the fastest convergence speed, which demonstrates the superiority of the proposed guidance law. When the water depth varies, the vessel-bank interaction becomes more challenging.

The quantitative measure of the overshoot and settling time under different guidance laws are compared in Table 2, it can be seen that the overshoot is the least and the settling time is the shortest using the proposed guidance law when the water depth changes.

![Figure 4](image-url)
Table 2. Comparison of overshoot and settling time at \( y_d = 0.1L \).

| Guidance Law | Overshoot (m) | Overshoot (%) | Settling Time (s) | Settling Time (s) |
|--------------|--------------|---------------|-------------------|-------------------|
| ILOS         | 8.43         | 61.67%        | 1937              | 2108+             |
| ALOS         | 7.68         | 56.18%        | 2674              | 2108+             |
| ELOS         | 6.77         | 49.52%        | 3140              | 2108+             |
| HGESO        | 3.43         | 25.09%        | 1937              | 1208              |

The time histories of the rudder angle are presented in Figure 5. It can be seen that the rudder operations with different guidance laws meet the rudder saturation and rate limits. A larger rudder angle is needed when the water depth decreases. The vessel–bank interaction becomes aggravated as the water depth decreases and makes control more challenging.

![Figure 5. Comparison of rudder angle with different guidance laws at \( y_d = 0.1L \).](image)

The desired heading angle and the true heading angle in the proposed path following control scheme are presented in Figure 6. It can be seen that the true heading angle can asymptotically converge to the desired heading angle without steady-state errors, which illustrates that the designed yaw tracking control system is robust to model uncertainties, unknown parameters, and external disturbances.
Case 2. The desired lateral position is set as $y_d = 0.05L$; other conditions are the same as those in Case 1. The time histories of the lateral position with different guidance laws are presented in Figure 7. It can be seen that the vessel under all the guidance laws can converge to the desired path asymptotically, and it converges to the desired path with the smallest overshoot and the fastest speed in the proposed guidance law, which demonstrates the superiority of the proposed guidance law. From the comparisons of Figures 4 and 7, it can be seen that the dynamic process of the lateral position greatly changes when $y_d$ turns from 0.1$L$ to 0.05$L$ using the ILOS, ALOS and ELOS guidance laws. However, it only slightly changes in the proposed guidance law, which demonstrates that the proposed guidance law is much more robust to different desired paths than the other guidance laws. In Table 3, the quantitative measure of the overshoot and settling time under different guidance laws are compared, while the benefits of the proposed guidance law are objectively evaluated.
Table 3. Comparison of overshoot and settling time at $y_d = 0.05L$.

|       | Overshoot | Settling Time (s) | Overshoot | Settling Time (s) |
|-------|-----------|-------------------|-----------|-------------------|
| ILOS  | 3.675 m   | 53.77%            | 1.496 m   | 21.89%            |
| ALOS  | 3.855 m   | 56.4%             | 2.95 m    | 18.95%            |
| ELOS  | 3.975 m   | 58.16%            | 1.051 m   | 15.38%            |
| HGESO | 2.493 m   | 36.47%            | 0.465 m   | 6.8%              |

Figures 8 and 9 present the rudder angle and heading angle respectively. It can be seen that the rudder operations in different guidance laws satisfy the saturation and rate limits. From Figure 8, it can be seen that the rudder angle increases as the water depth decreases. Since the vessel-bank interaction becomes aggravated as the water depth decreases, a larger yaw angle is required to conquer the vessel-bank interaction in shallower water. From the comparisons of Figures 5 and 8, it can be seen that the rudder angle increases as the desired path increases. Since a larger desired path means the vessel is closer to the bank, leading to a greater vessel–bank interaction, a larger angle is needed. From Figure 9, it can be seen that the true heading angle can asymptotically converge to the desired heading angle without a steady-state error, which illustrates that the designed yaw tracking control system is robust to model uncertainties, unknown parameters, and external disturbances.

![Figure 8. Comparison of rudder angle in different guidance laws at $y_d = 0.05L$.](image-url)
Case 3. To illustrate the ability of the proposed guidance law in following the varying desired straight-line paths, the desired straight-line path is set as 0.1 L in the beginning, and when the vessel sails 100 L (t = 4446 s), the desired straight-line path is changed to 0.05 L. The water depth to draft ratio is set as 2.03. Other conditions are the same as those in Case 1. Simulation results are presented in Figures 10–12. The lateral position with different guidance laws are compared in Figure 10. It can be seen that the vessel can follow the varying desired straight-line paths under all the guidance laws, while the vessel under the proposed guidance law can reach the desired path with the smallest overshoot and the fastest speed. In Figure 11, the time histories of the rudder angle with different guidance laws are compared. It can be seen that the rudder operations with different guidance laws meet the rudder saturation and rate limits. The rudder angle is fluctuating at t = 4446 s, and the fluctuations in the proposed guidance law are more drastic than those in other guidance laws. It indicates that the good performance of the lateral position in the proposed guidance law is at the cost of the high demanding rudder operation. The comparisons of the desired heading angle and the true heading angle in the proposed guidance law are presented in Figure 12. It can be seen that the true heading angle can track the desired heading angle without steady-state error, which illustrates that the designed yaw tracking control system is robust enough to model uncertainties, unknown parameters, and external disturbances in following varying desired straight-line paths. In Table 4, the quantitative measure of the overshoot and settling time using different guidance laws are compared, and it can be seen that the overshoot is the least and the settling time is the shortest using the proposed guidance law when the vessel following the varying desired straight-line paths.
of the high demanding rudder operation. The comparison of the desired heading angle and the true heading angle in the proposed guidance law are presented in Figure 12. It can be seen that the true heading angle can track the desired heading angle without steady-state error, which illustrates that the designed yaw tracking control system is robust enough to model uncertainties, unknown parameters, and external disturbances in following varying desired straight-line paths.

In Table 4, the quantitative measure of the overshoot and settling time using different guidance laws are compared, and it can be seen that the overshoot is the least and the settling time is the shortest using the proposed guidance law when the vessel following the varying desired straight-line paths.

Figure 10. Comparison of lateral position with different guidance laws at varying desired paths.

Figure 11. Comparison of rudder angle with different guidance laws at varying desired paths.
Figure 12. The desired heading angle vs. the true heading angle in the proposed guidance law at varying desired paths.

Table 4. Comparison of overshoot and settling time at varying desired paths.

|                   | Overshoot (m) | Settling Time (s) |
|-------------------|---------------|-------------------|
| ILOS              | 5.333         | 78.02%            |
| ALOS              | 6.077         | 88.91%            |
| ELOS              | 6.205         | 90.78%            |
| HGESO             | 5.009         | 73.28%            |

7. Conclusions

In this paper, an HGESO-ASMPFC scheme was proposed to design a path following controller for an underactuated vessel sailing in restricted waters with varying water depths. First, a sideslip observer-based guidance law is designed based on a high-gain extended state observer. Theoretical analysis was carried out for the kinematic system based on the cascade system theory; the results showed that the proposed guidance system could guarantee ISS, and effectively compensate the drift force due to the vessel–bank interaction in varying water depths. Then, an adaptive PI SMC was designed based on the Nomoto equations considering model uncertainties, unknown parameters, and external disturbances. Analysis based on Lyapunov theory has proven that the yaw tracking control system is GUAS, and the stabilization objective \( \lim_{t \to \infty} \dot{\psi} = 0 \) can be achieved.

Finally, the full kinematic-dynamic closed-loop system was analyzed, and the result showed that the closed-loop system was GUAS, which indicated that the entire HGESO-ASMPFC scheme could accurately follow the desired straight-line path in varying water depths. The analysis results were validated via simulation experiments. In particular, comparisons with ILOS, ALOS, and ELOS demonstrated the superiority of the proposed guidance law, while the lateral position in the proposed guidance law could converge to the different desired straight-line paths with the smallest overshoot and the fastest convergence speed in the varying water depths.

**Author Contributions:** J.W. put forward the original concept, proposed the control strategy and wrote the article. Z.Z. and T.W. gave their valuable suggestions on the research design. Further, J.W., Z.Z. and T.W. analyzed and discussed the experimental results.

**Funding:** This work is supported by the National Natural Science Foundation of China (Grant No. 51479156).

**Conflicts of Interest:** The authors declare no conflict of interest.
Nomenclature

- **ALOS**: adaptive line-of-sight
- **GUAS**: global uniformly asymptotically stable
- **HGESO-ASMPFC**: high-gain extended state observer based adaptive sliding mode path following control
- **ILOS**: integral line-of-sight
- **LOS**: line-of-sight
- **SMC**: sliding mode control
- $$a_v, a_r, a_y$$: linear model coefficients in Nomoto equations
- $$b$$: control coefficient in Nomoto equations
- $$D$$: the initial distance of the vessel away from the bank
- $$I_v$$: vessel’s inertial moment about the vertical axis
- $$m$$: vessel’s mass
- $$m_{ij}$$: the added mass
- $$o_x$$-$$x$$,$$y$$,$$z$$: the body-fixed reference frame
- $$o_s$$-$$x$$,$$y$$,$$z$$: the inertial reference frame
- $$u, v, r$$: surge and sway velocities and yaw rate of the vessel in the body-fixed reference frame
- $$x, y, \psi$$: longitudinal position, lateral position and heading angle of the vessel in the inertial reference
- $$x_G$$: the longitudinal position of the vessel’s center of gravity
- $$y_d$$: the desired path
- $$y_e$$: the cross-track error
- $$\alpha_1, \alpha_2, \epsilon$$: the designed positive constants in high-gain extended state observer
- $$\beta$$: the sideslip angle
- $$\delta$$: rudder angle
- $$\psi_d$$: the guidance angle
- $$\Delta$$: the designed look ahead distance
- $$\hat{}$$: the estimation value of the corresponding variable
- $$\sim$$: the error between the real value and the estimation value of the corresponding variable

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