Chiral Estimates of Strong CP Violation Revisited

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Abstract

The effects of the CP violating θ term in the QCD Lagrangian upon low energy hadronic phenomenology are reconsidered. Strong CP violating interactions among Goldstone bosons and octet baryons are incorporated into an effective chiral Lagrangian framework. The θ term’s impact upon the decays η → ππ and π⁰ → γγ is then investigated but found to be extremely small. A refined model independent estimate of nonanalytic contributions to the neutron electric dipole moment is also determined using velocity dependent Baryon Chiral Perturbation Theory. We obtain the approximate upper bound $|\theta| < 4.5 \times 10^{-10}$. 

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CP invariance is known to be a very good but inexact symmetry of the standard model. Violations of this discrete symmetry have been observed in neutral kaon decays and are hoped to be seen in bottom meson phenomena such as $B - \bar{B}$ mixing [1]. CP violation in these weak processes may be attributed to a complex phase in the Kobayashi-Maskawa matrix. However, this phase is not the only source of CP violation within the minimal six quark standard model. In the strong interaction sector, instanton effects generate a $G\tilde{G}$ term which preserves charge conjugation but breaks parity. This topological term enters into the QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^\mu\nu G_{\mu\nu} + \bar{q} i \slashed{D} q - \bar{q} M q + \theta \frac{g^2}{32\pi^2} G^\mu\nu \tilde{G}_{\mu\nu}$$  \hspace{1cm} (1)$$

with an undetermined coefficient $\theta$ that represents a fundamental parameter of the standard model. In this letter, we investigate the effects of the $\theta$ term upon low energy hadronic phenomenology.

To begin, we restrict the QCD Lagrangian to three light quarks and take the mass matrix $M$ to be real and diagonal without loss of generality:

$$M = \begin{pmatrix} m_u & m_d & m_s \end{pmatrix}.$$  \hspace{1cm} (2)$$

It is convenient to rotate the $\theta$ parameter away from the $G\tilde{G}$ term in (1) and into the quark mass matrix which subsequently becomes complex [2][3]:

$$\mathcal{L}_{QCD} \rightarrow -\frac{1}{4} G^\mu\nu G_{\mu\nu} + \bar{q} i \slashed{D} q - \bar{q} M q - i\theta \bar{m} \gamma^5 q$$

$$\hspace{2cm} = -\frac{1}{4} G^\mu\nu G_{\mu\nu} + \bar{q}_L i \slashed{D} q_L + \bar{q}_R i \slashed{D} q_R - \bar{q}_L (M + i\theta \bar{m}) q_R - \bar{q}_R (M - i\theta \bar{m}) q_L.$$  \hspace{1cm} (3)$$

If any of the current masses in (2) equals zero, the reduced quark mass

$$\bar{m} = \frac{m_u m_d m_s}{m_u m_d + m_d m_s + m_s m_u}$$

vanishes and the mass matrix remains real. However in the real world, the current quark masses have the small but nonvanishing values [4][5][6]

$$(m_u, m_d, m_s) \simeq (5, 9, 181) \text{ MeV}$$  \hspace{1cm} (4)$$

which imply $\bar{m} \simeq 3.2 \text{ MeV}$. So the $\theta$ parameter disappears from the topological term’s coefficient and reemerges in $\arg(\det(M + i\theta \bar{m})) = \theta + O(\theta^3)$. 

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At low energies, it is useful to replace Lagrangian (3) which describes QCD in terms of fundamental quarks and gluons with an effective Lagrangian of mesons and baryons. In particular, the self interactions of Goldstone bosons associated with the chiral symmetry breakdown \( G = SU(3)_L \times SU(3)_R \rightarrow H = SU(3)_{L+R} \) may be analyzed in a nonlinear chiral Lagrangian framework \([7]\). The Goldstone bosons appearing in the pion octet

\[
\pi = \frac{1}{\sqrt{2}} \begin{pmatrix}
\sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta \\
\pi^- \\
K^-
\end{pmatrix}
\begin{pmatrix}
\pi^+ \\
-\sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta \\
K^0
\end{pmatrix}
\begin{pmatrix}
K^+ \\
-\sqrt{\frac{2}{3}} \eta
\end{pmatrix}
\]

enter into the chiral theory through the combinations \( \Sigma = e^{2i\pi f} \xi = e^{i\pi f} \) where \( f = 93 \text{ MeV} \) represents the pion decay constant to lowest order. These exponentiated fields transform under \( G \) as

\[
\Sigma \rightarrow L \Sigma R^\dagger \\
\xi \rightarrow L \xi U^\dagger(x) = U(x) \xi R^\dagger
\]

where \( L \) and \( R \) denote global elements of \( SU(3)_L \) and \( SU(3)_R \) while local matrix \( U(x) \) is implicitly defined by \((6b)\). Goldstone terms in the effective Lagrangian are constructed from \( \Sigma \) and \( \xi \) in a derivative expansion with respect to the chiral symmetry breaking scale \( \Lambda_\chi \approx 1 \text{ GeV} \).

Incorporating baryons into this scheme seems problematic. The momentum expansion fails for baryons since their masses are not small compared to \( \Lambda_\chi \). However, a consistent approach for performing Baryon Chiral Perturbation Theory has recently been developed by Jenkins and Manohar \([8-11]\) using ideas and techniques familiar from the Heavy Quark Effective Theory \([12,13]\). At low energies, a baryon may be regarded as an almost on-shell heavy particle that travels along a straight worldline. In this kinematic regime, a baryon has four-momentum \( p = M_B v + k \) where the residual momentum \( k \) is small compared to its rest mass \( M_B \). As its four-velocity \( v \) is essentially unaffected by soft Goldstone boson absorption or emission, the baryon can be described by the velocity dependent field

\[
B_v(x) = e^{iM_B f u \cdot x} B(x)
\]

which has the rest energy removed from its definition. The use of such fields allows one to formulate a well-behaved derivative expansion in terms of the small quantity \( k/\Lambda_\chi \). This is the central idea behind velocity dependent Baryon Chiral Perturbation Theory.
The leading contributions to the effective Lagrangian
\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_\pi + \sum_v \mathcal{L}_B(v) \]
appear in \( d = 4 - \epsilon \) dimensions and at the renormalization scale \( \Lambda \) as
\begin{align}
\mathcal{L}_{\pi}^{(0)} &= \frac{\Lambda - \epsilon f^2}{4} \text{Tr}(\partial^\mu \Sigma \partial^\nu \Sigma) \\
\mathcal{L}_B^{(0)}(v) &= i \text{Tr} \overline{B}_v v \partial \mu B_v + D \text{Tr} \overline{B}_v \gamma^\mu \gamma^5 \{ A_\mu, B_v \} + F \text{Tr} \overline{B}_v \gamma^\mu \gamma^5 [A_\mu, B_v] \tag{7a} \\
&\text{Pions derivatively couple to the baryon octet through the Goldstone vector field}
\end{align}
\[ V^\mu = \frac{1}{2} (\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger) = \frac{\Lambda - \epsilon f^2}{4} [\pi, \partial^\mu \pi] - \frac{\Lambda^2}{24 f^4} \left[ \pi, [\pi, \pi, \partial^\mu \pi] \right] + O(\pi^6) \]
that resides within the covariant derivative
\[ \mathcal{D}^\mu B_v = \partial^\mu B_v + [V^\mu, B_v]. \]
In addition, they communicate via the axial current
\[ A^\mu = \frac{i}{2} (\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger) = -\frac{\Lambda^\epsilon / 2}{f} \partial^\mu \pi + \frac{\Lambda^3 \epsilon / 2}{6 f^3} \left[ \pi, [\pi, \partial^\mu \pi] \right] + O(\pi^5) \]
which explicitly appears in (7b). We adopt the best fit values \( D = 0.61 \) and \( F = 0.40 \) reported in refs. [10, 11] for the symmetric and antisymmetric axial current couplings. We also include electromagnetic interactions into the effective Lagrangian by gauging a \( U(1)_{EM} \) subgroup of the global \( SU(3)_L \times SU(3)_R \) symmetry group and treating photons as external fields.\footnote{In their original formulation of Baryon Chiral Perturbation Theory, Jenkins and Manohar introduced spin operators \( S_v^\mu \) that act on the velocity dependent baryon fields and incorporated them into the chiral Lagrangian. For the applications that we will consider, these spin operators offer no real advantage over conventional gamma matrices. We therefore express the derivative interactions of Goldstone bosons with baryons in our Lagrangian in terms of the latter rather than former objects.}
\footnote{The baryon decuplet may also be readily included into the low energy Lagrangian \[8, 11]. However, these fields do not influence CP violating baryon octet phenomena such as the neutron electric dipole moment at leading nontrivial order. We consequently neglect them here.}
fields. The partial derivatives acting on hadron fields in (7) are then promoted to covariant derivatives with respect to electromagnetism

\[ \partial^\mu \Sigma \rightarrow \partial^\mu \Sigma - i\Lambda^{\epsilon/2} eA^\mu [Q, \Sigma] \]

\[ \partial^\mu \xi \rightarrow \partial^\mu \xi - i\Lambda^{\epsilon/2} eA^\mu [Q, \xi] \]

\[ \partial^\mu B_v \rightarrow \partial^\mu B_v - i\Lambda^{\epsilon/2} eA^\mu [Q, B_v] \]

where

\[ Q = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \end{pmatrix} \]

denotes the quark charge matrix.

The symmetry breaking effects of the quark mass matrix \( M \equiv M + i\theta m \) in the underlying QCD Lagrangian can be systematically incorporated into the low energy theory via the spurion procedure. We first regard \( M \) as a fictitious field and assign it the transformation rules

\[ M \rightarrow \frac{G}{CMR^\dagger} \]

\[ M \rightarrow \frac{CP}{M^*} \]

which preserve chiral symmetry and discrete CP in eqn. (3). We then construct \( G \) and CP invariant terms in the chiral Lagrangian from the hadron fields and matrix \( M \). Finally, we set the spurion to its true constant value and thereby generate interactions that violate chiral symmetry, flavor and CP in the effective theory.

Working to linear order in \( M \), we add the following terms into the low energy Lagrangian:

\[ L_\pi^{(M)} = \frac{\Lambda - \epsilon f^2}{2} \mu \text{Tr}(\Sigma M^\dagger + M \Sigma^\dagger) \]

\[ L_{\mu}^{(M)}(v) = b_D \text{Tr} \bar{B}_v \{ \xi^\dagger M \xi^\dagger + \xi M \xi, B_v \} + b_f \text{Tr} \bar{B}_v [\xi^\dagger M \xi^\dagger + \xi M \xi, B_v] \]

\[ + \sigma \text{Tr} (\Sigma - 1) M^\dagger + M (\Sigma^\dagger - 1) \text{Tr} \bar{B}_v B_v. \]

Decomposing the mass matrix into its real and imaginary parts, we obtain CP conserving terms proportional to \( M \)

\[ L_\pi^{(M)} = \frac{\Lambda - \epsilon f^2}{2} \mu \text{Tr} M (\Sigma + \Sigma^\dagger) \]

\[ L_{\mu}^{(M)}(v) = b_D \text{Tr} \bar{B}_v \{ \xi^\dagger M \xi^\dagger + \xi M \xi, B_v \} + b_f \text{Tr} \bar{B}_v [\xi^\dagger M \xi^\dagger + \xi M \xi, B_v] \]

\[ + \sigma \text{Tr} M (\Sigma + \Sigma^\dagger - 2) \text{Tr} \bar{B}_v B_v. \]

\[ \text{A common baryon mass shift has been extracted from the } \sigma \text{ term in } (8b) \text{ and implicitly absorbed into the average octet mass parameter } M_B. \]
and CP violating interactions linear in $\theta m$

\[
\mathcal{L}_\pi^{(\theta)} = -i \frac{\Lambda^e f^2}{2} \mu \bar{\theta} \bar{m} \text{Tr}(\Sigma - \Sigma^\dagger)
\]

\[
\mathcal{L}_B^{(\theta)}(v) = -i \bar{\theta} m \left[ b_D \text{Tr} \bar{B}_v \{\Sigma - \Sigma^\dagger, B_v\} + b_F \text{Tr} \bar{B}_v [\Sigma - \Sigma^\dagger, B_v] + \sigma \text{Tr}(\Sigma - \Sigma^\dagger) \text{Tr} \bar{B}_v B_v \right].
\]

The leading terms in (9a, b) produce Goldstone boson masses and split the baryon octet multiplet. The parameters $\mu, b_D$ and $b_F$ as well as the mean baryon mass $M_B$ can therefore be fixed by a fit to the hadron mass spectrum using the assumed current quark masses in (4). We find the approximate tree level values

\[
\begin{align*}
\mu &= 1302 \text{ MeV} & M_B &= 1197 \text{ MeV} \\
 b_D &= 0.18 & b_F &= -0.54.
\end{align*}
\]

Having set up the necessary formalism, we can now investigate specific examples of strong CP violating phenomena. We first consider Goldstone boson processes. Expanding pion Lagrangian (10a), we isolate the self interaction terms

\[
\begin{align*}
\mathcal{L}_\pi^{(\theta)} &= -\frac{4 \Lambda^e/2}{f} \mu \bar{\theta} \bar{m} \text{Tr}(\pi^3) + O(\pi^5) \\
&= \frac{\Lambda^e/2}{f} \mu \bar{\theta} \bar{m} \left[ \frac{2\sqrt{3}}{3} \eta \pi^+\pi^- + \frac{\sqrt{3}}{3} \eta \pi^0 \pi^0 - \frac{\sqrt{3}}{3} \eta K^+ K^- - \frac{\sqrt{3}}{9} \eta K^0 \bar{K}^0 \right] + O(\pi^5).
\end{align*}
\]

From these trilinear vertices, one can easily compute the tree level rate for the CP violating decay $\eta \rightarrow \pi \pi$ [3, 14]:

\[
\Gamma(\eta \rightarrow \pi^+\pi^-) = \frac{1}{12\pi} \left( \frac{\mu \bar{\theta} \bar{m}}{f} \right)^2 \frac{\sqrt{m_\eta^2 - 4m_{\pi^+}^2}}{m_\eta^2} \approx 8.4 \times 10^{-2}\theta^2 \text{ MeV.}
\]

Comparison with the current experimental limit $\Gamma(\eta \rightarrow \pi^+\pi^-)_{\exp} < 1.79 \times 10^{-6} \text{ MeV}$ [15] implies that $|\theta| < 4.6 \times 10^{-3}$. However, much more stringent bounds on $\theta$ are available from electric dipole measurements. So it is of greater use to insert $|\theta| < 4.5 \times 10^{-10}$, the constraint which comes from the neutron electric dipole analysis that we will shortly present, into eqn. (13). We then obtain the prediction $\Gamma(\eta \rightarrow \pi^+\pi^-)_{\text{th}} < 1.7 \times 10^{-20} \text{ MeV.}$ This theoretical bound is unfortunately 14 orders of magnitude smaller than the experimental
upper limit. One may thus safely assume that the $\theta$ parameter will never be extracted from this Goldstone decay mode.

The CP violating interactions in (12) also contribute to loop processes. The simplest one-loop diagrams involving the Goldstone vertices are the $\pi^0$ and $\eta$ tadpoles displayed in fig. 1. The graphs’ $O(M^2)$ quadratic divergences automatically sum to zero as a result of the vanishing tadpole renormalization condition

$$\langle 0 | i \theta \bar{m} q \gamma^5 q | \pi^0 \rangle = 0$$

which we implicitly invoked when rotating $\theta$ into the quark mass matrix [3][16]. Terms that persist at higher orders in $M$ along with tadpole interactions in the effective Lagrangian starting at $O(M^3)$ may always be eliminated via an appropriate modification of the $\theta$ rotation.

While neutral Goldstone bosons do not disappear directly into the vacuum, they can decay into two photons. As illustrated in fig. 2, the process $\pi^0 \rightarrow \gamma \gamma$ may proceed through kaon triangle and “seagull” graphs that violate CP. A straightforward calculation yields the finite and gauge invariant on-shell amplitude

$$A(\pi^0(p+q) \rightarrow \gamma(p)\gamma(q)) = \frac{2\mu\theta mc^2}{16\pi^2 f} \left[ \frac{4m_K^2}{m_\pi^2} \left( \tan^{-1} \frac{m_\pi}{\sqrt{4m_K^2 - m_\pi^2}} \right)^2 - 1 \right] \left( g_{\mu\nu} - \frac{p_\mu q_\nu}{m_\pi^2} \right) \varepsilon^\mu(p) \varepsilon^\nu(q).$$

(14)

From the form of this expression, we see that the graphs in fig. 2 match onto an infinite string of nonrenormalizable operators that reduce via the equations of motion to $\pi^0 F^\mu\nu \tilde{F}_{\mu\nu}$. Since this operator does not respect CP, it cannot interfere with the one pion-two photon vertex in the Wess-Zumino action [17][18]

$$W(\Sigma, A^\mu) = \int d^4x \left\{ - \frac{N_c e^2}{48\pi^2 f} \pi^0 F^\mu\nu \tilde{F}_{\mu\nu} + \cdots \right\}.$$ 

(15)

The CP violating contribution to the decay rate

$$\Gamma(\pi^0 \rightarrow \gamma \gamma) = \frac{1}{64\pi} \left( \frac{\alpha_{EM}}{\pi} \right)^2 \left( \frac{\mu\theta m}{f} \right)^2 m_\pi^{-1} \left[ \frac{4m_K^2}{m_\pi^2} \left( \tan^{-1} \frac{m_\pi}{\sqrt{4m_K^2 - m_\pi^2}} \right)^2 - 1 \right]/2$$

(16)

is therefore suppressed by two powers of $\theta$ compared to its lowest order CP conserving counterpart

$$\Gamma(\pi^0 \rightarrow \gamma \gamma)_{WZ} = \frac{1}{64\pi} \left( \frac{\alpha_{EM}}{\pi} \right)^2 \left( \frac{m_\pi^2}{f} \right) m_\pi^{-1}. \quad (17)$$
Their ratio is consequently miniscule:

$$\frac{\Gamma(\pi^0 \rightarrow \gamma\gamma)\phi}{\Gamma(\pi^0 \rightarrow \gamma\gamma)_{WZ}} = 2.1 \times 10^{-6} \theta^2 < 4 \times 10^{-25}!$$

The preceding Goldstone boson examples clearly demonstrate that one cannot hope to observe strong CP violating effects unless they occur at linear order in $\theta$. Fortunately, there is a well-known phenomenon in the baryon sector which meets this criterion: the neutron electric dipole moment (NEDM). Previous attempts to determine the magnitude of this important observable have typically relied upon specific model calculations. We will reconsider this problem in the context of velocity dependent Baryon Chiral Perturbation Theory. While our investigation is clearly similar in spirit to the well-known current algebra NEDM analysis of Crewther, Di Vecchia, Veneziano and Witten [3], we believe that the effective field theory approach is more transparent and systematic. Furthermore, heavy hadron techniques are especially well suited for studying the electric dipole moment $d_N$ since it is a static property. Recall that $d_N$ is defined in terms of the form factor $F_3$ appearing in the interaction Lagrangian

$$\mathcal{L}_I = i\frac{F_3(q^2)}{2M_N} \overline{N}(p')\sigma^{\mu\nu}\gamma^5 N(p)F_{\mu\nu},$$

evaluated at zero momentum transfer ($q = p' - p = 0$) [19]:

$$d_N = \frac{F_3(0)}{M_N}.$$  

The neutron’s four-velocity is therefore conserved, and the assumptions underlying the static hadron picture are genuinely satisfied. So we view the NEDM problem as a particularly nice application of Baryon Chiral Perturbation Theory.

We first enumerate the leading terms which may appear in the low energy Lagrangian and directly contribute to $d_N$. Group theory counting indicates that there are ten independent terms which one can form from the octet fields $\overline{B}_\nu$ and $B_\nu$, the photon combination $\epsilon Q\sigma^{\mu\nu}\gamma^5 F_{\mu\nu}$, and the parity-odd mass term $\xi^\dagger M\xi^\dagger - \xi M\xi^\dagger$. A convenient basis for these
dimension-six operators is given below:

\[
O_1 = \frac{e}{\Lambda^2} \text{Tr} \left( \overline{\mathcal{B}}_v \sigma^{\mu \nu} \gamma^5 F_{\mu \nu} \{Q, B_v\} \right) \text{Tr} (\mathcal{M} \Sigma^\dagger - \Sigma \mathcal{M}^\dagger) \\
O_2 = \frac{e}{\Lambda^2} \text{Tr} \left( \overline{\mathcal{B}}_v \sigma^{\mu \nu} \gamma^5 F_{\mu \nu} [Q, B_v] \right) \text{Tr} (\mathcal{M} \Sigma^\dagger - \Sigma \mathcal{M}^\dagger) \\
O_3 = \frac{e}{\Lambda^2} \text{Tr} \left( \overline{\mathcal{B}}_v \sigma^{\mu \nu} \gamma^5 F_{\mu \nu} B_v \right) \text{Tr} \left( Q (\xi^\dagger \mathcal{M} \xi^\dagger - \xi \mathcal{M}^\dagger \xi) \right) \\
O_4 = \frac{e}{\Lambda^2} \text{Tr} \left( \overline{\mathcal{B}}_v \sigma^{\mu \nu} \gamma^5 F_{\mu \nu} (\xi^\dagger \mathcal{M} \xi^\dagger - \xi \mathcal{M}^\dagger \xi) \right) \text{Tr} (Q B_v) \\
O_5 = \frac{e}{\Lambda^2} \text{Tr} \left( \overline{\mathcal{B}}_v \sigma^{\mu \nu} \gamma^5 F_{\mu \nu} \right) \text{Tr} \left( (\xi^\dagger \mathcal{M} \xi^\dagger - \xi \mathcal{M}^\dagger \xi) B_v \right) \\
O_6 = \frac{e}{\Lambda^2} \text{Tr} \left( \overline{\mathcal{B}}_v \sigma^{\mu \nu} \gamma^5 F_{\mu \nu} \left[ (\xi^\dagger \mathcal{M} \xi^\dagger - \xi \mathcal{M}^\dagger \xi) Q, B_v \right] \right) \\
O_7 = \frac{e}{\Lambda^2} \text{Tr} \left( \overline{\mathcal{B}}_v \sigma^{\mu \nu} \gamma^5 F_{\mu \nu} \left[ Q, B_v \right] (\xi^\dagger \mathcal{M} \xi^\dagger - \xi \mathcal{M}^\dagger \xi) \right) \\
O_8 = \frac{e}{\Lambda^2} \text{Tr} \left( \overline{\mathcal{B}}_v \sigma^{\mu \nu} \gamma^5 F_{\mu \nu} \right) \text{Tr} \left( \left[ Q, (\xi^\dagger \mathcal{M} \xi^\dagger - \xi \mathcal{M}^\dagger \xi) B_v \right] \right) \\
O_9 = \frac{e}{\Lambda^2} \left( \text{Tr} \left( \overline{\mathcal{B}}_v \sigma^{\mu \nu} \gamma^5 F_{\mu \nu} \right) \text{Tr} \left( B_v \left[ (\xi^\dagger \mathcal{M} \xi^\dagger - \xi \mathcal{M}^\dagger \xi) Q \right] \right) \\
+ \text{Tr} \left( \overline{\mathcal{B}}_v \sigma^{\mu \nu} \gamma^5 F_{\mu \nu} \left[ (\xi^\dagger \mathcal{M} \xi^\dagger - \xi \mathcal{M}^\dagger \xi, QB_v \right] \right) \right) \\
O_{10} = \frac{e}{\Lambda^2} \text{Tr} \left( \overline{\mathcal{B}}_v \sigma^{\mu \nu} \gamma^5 F_{\mu \nu} \left\{ \left[ Q, (\xi^\dagger \mathcal{M} \xi^\dagger - \xi \mathcal{M}^\dagger \xi - \frac{1}{3} \text{Tr} (\xi^\dagger \mathcal{M} \xi^\dagger - \xi \mathcal{M}^\dagger \xi) B_v \right] \right\} \right). 
\]

Of these ten operators, only $O_1$ affects the electric dipole moment of neutral baryons. Its impact upon $d_N$ is unknown however since $O_1$ appears in the effective Lagrangian with an a priori undetermined coefficient. In principle, the coefficients of all the operators in (19) could be fixed from Goldstone boson-photoproduction data. But in practice, the challenge of performing such a fit appears formidable.

As first noted in ref. [3], the most important contributions to the neutron’s dipole moment actually do not come from tree level composite operators but rather from one-loop Goldstone boson diagrams. Such graphs generate infrared log renormalizations of $O_1$ which diverge in the chiral limit. Their nonanalytic dependence on $\mathcal{M}$ is exactly calculable unlike the analytic dependence of $O_1$’s coefficient. The nonanalytic terms consequently provide a rough but useful order-of-magnitude estimate for $d_N$.

There are a number of CP violating diagrams that enter into the neutron-neutron-photon 1PI Green’s function $\Gamma^{(NN\gamma)}$ at one-loop order. But only those listed in fig. 3 yield chiral log corrections to the NEDM. In addition, these graphs contain fractional power
terms starting at $O(M^{3/2})$ which result from the mass splitting between the external neutron and intermediate baryon. Summing the diagrams with internal $\pi^-$ and $P$ propagators as well as $K^+$ and $\Sigma^-$ lines, we obtain

$$\Gamma^{(NN\gamma)} = \frac{4\theta\pi e}{16\pi^2 f^2}\left\{ (D + F)(b_D + b_F)\left[ \log\frac{\Lambda^2}{m_{\pi}^2} - \frac{\pi(M_P - M_N)}{\sqrt{m_{\pi}^2 - (M_P - M_N)^2}} \right]ight.$$  

$$- (D - F)(b_D - b_F)\left[ \log\frac{\Lambda^2}{m_{K}^2} - \frac{\pi(M_\Sigma - M_N)}{\sqrt{m_{K}^2 - (M_\Sigma - M_N)^2}} \right]\right\}\times N_v(k')(\sigma^{\mu\nu}(k^\prime - k)_{\nu}N_v(k) + \cdots).$$

(20)

The Dirac algebra identity

$$\frac{1}{2} (v^\mu\gamma^\nu - v^\nu\gamma^\mu)\gamma^5 \frac{1}{2} + \frac{1}{2} = -i \frac{1}{2} \sigma^{\mu\nu}\gamma^5 \frac{1}{2}$$

transforms the velocity dependent expression into a manifest dipole operator. Numerically evaluating eqn. (20) at the scale $\Lambda = \Lambda_\chi = 1$ GeV, we find for its coefficient

$$\Gamma^{(NN\gamma)} = \left[ (2.76 - 0.08) \times 10^{-16} \theta e\cdot cm \right] i N_v(k')(\sigma^{\mu\nu}(k^\prime - k)_{\nu}N_v(k) + \cdots$$

where the pion and kaon contributions are separately displayed. The large disparity between these terms stems in part from their $SU(3)$ couplings and infrared logarithms. But more importantly, the discrepancy results from the near cancellation between the log $M$ and $M^{3/2}$ terms in the strange virtual hadron graphs. The total nonanalytic contribution to the neutron electric dipole moment is thus given by

$$d_N = (2.68 \times 10^{-16}\theta)e\cdot cm.$$  

Comparing with the current experimental upper limit $|d_N| < 1.2 \times 10^{-25}e\cdot cm$ at 95% CL [15], we deduce $|\theta| < 4.5 \times 10^{-10}$.

In conclusion, the results from our chiral Lagrangian investigation of the $\theta$ term’s effect upon low energy hadron phenomenology are in basic accord with earlier findings [20]. Its virtue is therefore not novelty but rather simplicity. Of course, as in any effective field theory analysis, the results are model independent and can be systematically improved by retaining higher order terms in the derivative expansion. In particular, nonanalytic $O(1/M_B)$ corrections to the NEDM could be determined. However given the uncertainty in the analytic contributions to $|d_N|$, such further refinement of our estimate for $\theta$ is not of much practical importance.
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Added note: After completion of this work, we learned that results similar to those presented here have been reported in ref. [21]. See also ref. [22].
References

[1] M. Wise, in Proceedings of the Banff Summer Institute, ed. A.N. Kamal and F.C. Khanna (World Scientific, 1988) p. 124.
[2] V. Bahuni, Phys. Rev. D19 (1979) 2227.
[3] R.J. Crewther, P. Di Vecchia, G. Veneziano and E. Witten, Phys. Lett. B88 (1979) 123; (E) Phys. Lett. B91 (1980) 487.
[4] J. Gasser and H. Leutwyler, Ann. Phys. 158 (1984) 142.
[5] J. Gasser and H. Leutwyler, Nucl. Phys. B250 (1985) 465.
[6] S. Weinberg, in A Festschrift for I.I. Rabi (New York Academy of Sciences, New York, 1977) 185.
[7] S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2239; C. Callan, S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2247.
[8] E. Jenkins and A. Manohar, Phys. Lett. B255 (1991) 558.
[9] E. Jenkins and A. Manohar, Phys. Lett. B259 (1991) 353.
[10] E. Jenkins, Nucl. Phys. B368 (1992) 190.
[11] E. Jenkins and A. Manohar, UCSD/PTH 91-30 (1991).
[12] H. Georgi, Phys. Lett. B240 (1990) 447.
[13] For reviews of HQET, see M. Wise, “New Symmetries of the Strong Interactions”, Lectures presented at the Lake Louise Winter Institute, Feb 17-23 1991, CALT-68-1721; H. Georgi, “Heavy Quark Effective Theory”, in Proc. of the Theoretical Advanced Study Institute (TASI) 1991, ed. R.K. Ellis, C.T. Hill and J.D. Lykken (World Scientific, Singapore, 1992) p. 589; B. Grinstein, “Lectures on Heavy Quark Effective Theory”, in High Energy Phenomenology, Proceedings of the Workshop, Mexico City 1-12 July 1991, eds. R. Heurta and M.A. Perez, (World Scientific, Singapore), SSCL-Preprint-17.
[14] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B166 493.
[15] Review of Particle Properties, Phys. Rev. D45, Part 2 (1992).
[16] J. Nuyts, Phys. Rev. Lett. 26 (1971) 1604.
[17] J. Wess and B. Zumino, Phys. Lett. B37 (1971) 95.
[18] E. Witten, Nucl. Phys. B223 (1983) 422.
[19] W. Bernreuther and M. Suzuki, Rev. Mod. Phys. 63 (1991) 313.
[20] H.-Y. Cheng, Phys. Rep. 158 (1988) 1, and references therein.
[21] A. Pich and E. de Rafael, Nucl. Phys. B367 (1991) 313.
[22] H.-Y. Cheng, Phys. Rev. D44 (1991) 166.
Figure Captions

Fig. 1. One-loop $\pi^0$ and $\eta$ tadpole diagrams. Large dots denote CP violating vertices. The cross in the $\pi^0\pi^0\eta$ graph represents the $O(m_u - m_d)$ off-diagonal mixing term in the neutral Goldstone boson propagator.

Fig. 2. CP violating graphs that contribute to $\pi^0 \rightarrow \gamma\gamma$. A virtual charged kaon runs around the loop.

Fig. 3. One-loop diagrams that contribute to the neutron electric dipole moment. Identical graphs with internal $\pi^-$ and $P_v$ propagators replaced by $K^+$ and $\Sigma_v^-$ lines are not pictured.