Feynman rules in $N = 2$ projective superspace I:  
Massless hypermultiplets

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Abstract

Manifestly $N = 2$ supersymmetric Feynman rules are found for different off-shell realizations of the massless hypermultiplet in projective superspace. When we reduce the Feynman rules to an $N = 1$ superspace we obtain the correct component propagators. The Feynman rules are shown to be compatible with a “duality” that acts only on the auxiliary fields, as well as with the usual duality relating the hypermultiplet to the tensor multiplet.

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1 Introduction

Recently, there has been a great deal of interest in $N = 2$ Super-Yang-Mills theory. A number of explicit computations have been performed by reducing the $N = 2$ tree level superspace action to $N = 1$ component superfields whose Feynman rules are well known \[1\]. These calculations do not exhibit manifest $N = 2$ supersymmetry and it is not possible to fully determine the form of the $N = 2$ perturbative effective actions beyond the leading order terms. Here we present a path integral quantization in $N = 2$ superspace and construct very simple Feynman rules for the massless hypermultiplet in $N = 2$ projective superspace\[1\].

There are many different off-shell realizations of the hypermultiplet which all reduce to the same multiplet on-shell. All have either restricted couplings or infinite numbers of auxiliary fields. Here we focus on a class of representations introduced in \[4\], and compute the propagator and vertices. We first obtain the propagator in $N = 1$ superspace and then covariantize the result to find the corresponding $N = 2$ superspace propagator; subsequently, we derive the same result directly in $N = 2$ superspace.

For simplicity, we restrict our analysis to the massless hypermultiplet, and leave the discussion of massive hypermultiplet \[5\], couplings to the Yang-Mills multiplet \[6\] and quantization of $N = 2$ gauge multiplets \[7\] for the future. We prove the known non-renormalization theorems for the hypermultiplet.

2 Projective Superspace

We begin with a brief review of projective superspace \[8\]. The algebra of $N = 2$ supercovariant derivatives in four dimensions is\[9\]

$$
\{D_{ab}, D_{b\beta}\} = 0 , \quad \{D_{ab}, \bar{D}^b_{\dot{\beta}}\} = i\delta^b_a \delta_{a\dot{\beta}} .
$$

(1)

We define an abelian subspace of $N = 2$ superspace parameterized by a complex projective coordinate $\zeta$ and spanned by the supercovariant derivatives

$$
\nabla_\alpha (\zeta) = D_{1\alpha} + \zeta D_{2\alpha} , \quad \nabla_{\dot{\alpha}} (\zeta) = \bar{D}^{2\dot{\alpha}} - \zeta \bar{D}^{1\dot{\alpha}} .
$$

(2)

(3)

The conjugate of any object is constructed in this subspace by composing the antipodal map on the Riemann sphere with hermitian conjugation $\zeta^* \rightarrow -1/\zeta$ and multiplying by an appropriate factor. For example,

$$
\nabla_{\dot{\alpha}} (\zeta) = (-\zeta) (\nabla_\alpha)^* \left( -\frac{1}{\zeta} \right) = (-\zeta) \left( \bar{D}^{1\dot{\alpha}} + \left( -\frac{1}{\zeta} \right) \bar{D}^{2\dot{\alpha}} \right) .
$$

(4)

$^1$\textit{N} = 2 supersymmetric field theories with manifest $N = 2$ Feynman rules and supergraphs exist in harmonic superspace \[\[2\]. Projective superspace is believed to be related to the harmonic superspace, although the technology we develop is quite different. The $N = 2$ harmonic formalism has been used recently to calculate interesting physical effects and low energy effective actions \[3\].

$^2$\textit{We} will use the notation and normalization conventions of \[12\]; in particular we denote $D^2 = \frac{1}{2} D^a D_a$ and $\Box = \frac{1}{2} \partial^{\alpha a} \partial_{\alpha a}$.
Throughout the paper, all conjugates of fields and operators in projective superspace are defined in this sense.

To make the global $SU(2)$ transformation properties explicit we can introduce a projective isospinor $u^a = (1, \zeta)$

$$\nabla_a(\zeta) = u^a D_{aa}, \quad \bar{\nabla}_{\dot{a}}(\zeta) = \epsilon_{a\dot{a}} u^a \bar{D}_{\dot{a}}^b.$$  \hspace{2cm} (5)

Superfields living in this projective superspace obey the constraint

$$\nabla_\alpha \Upsilon = 0 = \bar{\nabla}_{\dot{a}} \Upsilon,$$  \hspace{2cm} (7)

and the restricted measure of this subspace can be constructed from the orthogonal operators

$$\Delta_\alpha = v^a D_{aa}, \quad \bar{\Delta}_{\dot{a}} = \epsilon_{a\dot{a}} u^a \bar{D}_{\dot{a}}^b$$

where $v_a = (\zeta^- 1, -1)$, $\epsilon_{a\dot{a}} u^a v^b = -2$. For constrained superfields, we can write an $N = 2$ supersymmetric action using this restricted measure

$$\frac{1}{32\pi i} \oint_C \zeta d\zeta dx \Delta^2 \bar{\Delta}^2 f(\Upsilon, \bar{\Upsilon}, \zeta),$$  \hspace{2cm} (8)

where $C$ is a contour in the $\zeta$-plane that generically depends on $f$; in all the examples below, it will be a small contour around the origin. Though our primary interest is in four dimensions, we write the measure as $dx$ since the equations we write are valid for all $d \leq 4$.

The algebra that follows from (1) is

$$\{\nabla, \nabla\} = \{\nabla, \bar{\nabla}\} = \{\Delta, \Delta\} = \{\Delta, \bar{\Delta}\} = \{\nabla, \Delta\} = 0,$$

$$\{\nabla_\alpha, \bar{\Delta}_{\dot{a}}\} = -\{\bar{\nabla}_{\dot{a}}, \Delta_\alpha\} = 2i\partial_{a\dot{a}}.$$  \hspace{2cm} (9)

For notational simplicity we write $D_{1\alpha} = D_\alpha$, $D_{2\alpha} = Q_\alpha$. Then the identities

$$\Delta_\alpha = \zeta^{-1}(2D_{\alpha} - \nabla_\alpha), \quad \bar{\Delta}_{\dot{a}} = 2\bar{D}_{\dot{a}} + \zeta^{-1}\bar{\nabla}_{\dot{a}},$$  \hspace{2cm} (10)

allow us to rewrite the action (8) in a form convenient for reducing it to $N = 1$ components:

$$\frac{1}{2\pi i} \oint_C d\zeta \partial \frac{d\zeta}{\zeta} D^2 \bar{D}^2 f(\Upsilon, \bar{\Upsilon}, \zeta).$$  \hspace{2cm} (11)

The constraints (7) can be rewritten as

$$D_\alpha \Upsilon = -\zeta Q_\alpha \Upsilon, \quad \bar{Q}_{\dot{a}} \Upsilon = \zeta \bar{D}_{\dot{a}} \Upsilon.$$  \hspace{2cm} (12)

The superfields obeying such constraints may be classified \cite{4} as: i) $O(k)$ multiplets, ii) rational multiplets, iii) analytic multiplets. We focus on $O(k)$ multiplets\cite{9}, which are polynomials in $\zeta$ with powers ranging from 0 to $k$, and on analytic multiplets, which are analytic in some

\footnote{O(2) multiplets were first introduced in \cite{3} and the O(k) generalization in \cite{10}. The harmonic superspace equivalent is given in \cite{11}.}
region of the Riemann sphere. Later it will be useful to denote the \( \zeta \) dependence of the product \( \zeta^i \times O(k) \) as \( O(i, i+k) \). The transformation properties of the \( O(k) \) multiplets under global \( SU(2) \) can be obtained from their parameterization in terms of projective spinors and \( SU(2) \) tensors

\[
\Upsilon = \sum_{n=0}^{k} \Upsilon_n \zeta^n \equiv u^{a_1} \ldots u^{a_k} L_{a_1 \ldots a_k}, \quad \bar{\Upsilon} = \sum_{n=0}^{k} \bar{\Upsilon}_n \left( -\frac{1}{\zeta} \right)^n .
\]  

(13)

For even \( k = 2p \) we can impose a reality condition with respect the conjugation defined above (see (4)). We use \( \eta \) to denote a real finite order superfield

\[
(-1)^p \zeta^{2p} \bar{\eta} = \eta \iff \eta = \frac{\eta}{\zeta^p} = \bar{\eta}.
\]  

(14)

This reality condition relates different coefficient superfields

\[
\eta_{2p-n} = (-)^{p-n} \bar{\eta}_n.
\]  

(15)

There are various types of analytic multiplets. The arctic multiplet can be regarded as the limit \( k \to \infty \) of the complex \( O(k) \) multiplet. It is analytic in \( \zeta \), i.e., around the north pole of the Riemann sphere.

\[
\Upsilon = \sum_{n=0}^{\infty} \Upsilon_n \zeta^n .
\]  

(16)

Its conjugate superfield (the antarctic multiplet)

\[
\bar{\Upsilon} = \sum_{n=0}^{\infty} \bar{\Upsilon}_n (\frac{1}{\zeta})^n ,
\]  

(17)

is analytic in \( \zeta^{-1} \), i.e., around the south pole of the Riemann sphere.

Similarly, the real tropical multiplet is the limit \( p \to \infty \) of the real \( O(-p,p) \) multiplet \( \eta(2p)/\zeta^p \). It is analytic away from the polar regions, and can be regarded as a sum of a part analytic around the north pole and part analytic around the south pole:

\[
V(\zeta) = \sum_{n=-\infty}^{+\infty} v_n \zeta^n , \quad v_{-n} = (-)^n \bar{v}_n .
\]  

(18)

The constraints (12) relate the different \( \zeta \)-coefficient superfields

\[
D_\alpha \Upsilon_{n+1} = -Q_\alpha \Upsilon_n , \quad \bar{D}_\alpha \bar{\Upsilon}_n = \bar{Q}_\alpha \bar{\Upsilon}_{n+1} .
\]  

(19)

For any real \( O(2p) \) multiplet these constraints are compatible with the reality condition (14). They also determine what type of \( N=1 \) superfields the \( \zeta \)-coefficients are.

We illustrate this with the real \( O(4) \) multiplet, which is the first example we consider because it has the simplest Feynman rules. Explicitly, it takes the form

\[
\eta = \Phi + \zeta \Sigma + \zeta^2 X - \zeta^3 \Sigma + \zeta^4 \Phi , \quad X = \bar{X} ,
\]  

(20)

\[\text{We are happy to thank Warren Siegel for suggesting this terminology.}\]
with $N=1$ superfield components:

i) an $N=1$ chiral superfield $\bar{D}_a \Phi = 0$, also obeying $Q_a \Phi = 0$;

ii) an $N=1$ complex linear superfield $\bar{D}^2 \Sigma = 0$, also obeying $Q^2 \Sigma = 0$;

iii) an $N=1$ real unconstrained superfield $X$.

For the complex $O(k)$ multiplet, the coefficients corresponding to the two lowest and highest orders in $\zeta$ are also $N=1$ constrained superfields, although they are not conjugate to each other. All the intermediate coefficient superfields are unconstrained in $N=1$ superspace

$$\Upsilon = \bar{\Phi} + \zeta \bar{\Sigma} + \zeta^2 X + \zeta^3 Y + \ldots + \zeta^{p-1} \bar{\Sigma} + \zeta^p \bar{\Phi}. \quad (21)$$

Note that a complex $O(k)$ multiplet has twice as many physical degrees of freedom as the real multiplet (i.e., it describes two hypermultiplets); nevertheless, we can write

$$\eta = \frac{1}{\sqrt{2}} (\Upsilon + (-\zeta^2)^p \bar{\Upsilon}) \quad (22)$$

to construct a real $O(2p)$ multiplet out of a complex $O(2p)$ one. Conversely, we can write a complex $O(2p)$ multiplet out of two real multiplets $\eta, \bar{\eta}$:

$$\Upsilon = \frac{\eta + i\bar{\eta}}{\sqrt{2}}, \quad \bar{\Upsilon} = (-\zeta^2)^{-p} \eta - i\bar{\eta} \frac{1}{\sqrt{2}}. \quad (23)$$

For the arctic multiplet, only the two lowest coefficient superfields are constrained. The other components are complex auxiliary superfields unconstrained in $N=1$ superspace.

$$\Upsilon = \bar{\Phi} + \zeta \bar{\Sigma} + \zeta^2 X + \zeta^3 Y + \ldots \quad (24)$$

Finally, for the real tropical multiplet all the $\zeta$-coefficient superfields are unconstrained in $N=1$ superspace.

3 $N=1$ superspace description

3.1 $N=1$ actions

In the previous section, we defined a graded abelian subspace of $N=2$ superspace and constructed both a measure and constrained superfields that can be used to form $N=2$ invariant actions (11).

For the real $O(2p)$ multiplet, the following action gives standard $N=1$ kinetic and interaction terms after performing the contour integral in $\zeta$ (although in general it is not $SU(2)$ invariant (11)):

$$\int dx \ D^2 D^2 \oint \frac{d\zeta}{2\pi i \zeta} \left[ \frac{1}{2} (-)^p \left( \frac{\eta}{\xi^p} \right)^2 + \mathcal{L}_I \left( \frac{\eta}{\xi^p} \right) \right]. \quad (25)$$
Note that the natural variable for the function in the integrand of (23) is the self-conjugate superfield $\eta/\zeta^p$. Since the measure is already real, this field allows us to construct manifestly real actions. The sign of the kinetic piece guarantees that after performing the contour integration we obtain the right kinetic terms for the chiral and linear component superfields. For the $O(4)$ multiplet, the free action in $N = 1$ components is

$$\int dx \ D^2 \bar{D}^2 \left( \Phi \bar{\Phi} - \Sigma \bar{\Sigma} + \frac{1}{2} X^2 \right).$$

(26)

We can use any real $O(2p)$ multiplet or the (ant)arctic multiplet to describe the physical degrees of freedom of the $N = 2$ hypermultiplet. The usual description in terms of two $N = 1$ chiral fields arises after a duality transformation that replaces the $N = 1$ complex linear superfield by a chiral superfield. As usual, this can be done by rewriting the action in terms of a parent action with a Lagrange multiplier $Z$, e.g., for the $O(4)$ case,

$$\int dx \ D^2 \bar{D}^2 \left( \Phi \bar{\Phi} - S \bar{S} + \frac{1}{2} X^2 + Z \bar{D}^2 \bar{S} + \bar{Z} D^2 S \right).$$

(27)

The field $S$ is unconstrained, but integrating out the field $Z$ imposes the linearity constraint on $S$. Alternatively we can integrate out $S$ and recover the kinetic term of a chiral field $\bar{D}^2 Z$. The two descriptions are dual formulations of the same physical degrees of freedom. Except for the $O(2)$ multiplet, the duality transformation merely changes the auxiliary fields of the theory, and for a nonlinear $\sigma$-model, induces a coordinate transformation in target space. The $O(2)$ case gives the four dimensional version of the well-known $T$-duality.

For a complex $O(k)$ or (ant)arctic multiplet, a real action necessarily involves both the field and its conjugate. The simplest free action we can construct obeying hermiticity and $N = 2$ supersymmetry is:

$$\int dx D^2 \bar{D}^2 \oint \frac{d\zeta}{2\pi i \zeta} \ U.$$  

(28)

The contour integration in the complex $\zeta$-plane produces the $N = 1$ kinetic terms of the coefficient superfields for the complex $O(k)$ multiplet

$$\int dx D^2 \bar{D}^2 \left( \phi \bar{\phi} - \Sigma \bar{\Sigma} + X \bar{X} - Y \bar{Y} + \ldots + (-)^{k-1} \bar{\Sigma} \bar{\Sigma} + (-)^k \phi \bar{\phi} \right),$$

(29)

and for the (ant)arctic multiplet

$$\int dx D^2 \bar{D}^2 \left( \phi \bar{\phi} - \Sigma \bar{\Sigma} + X \bar{X} - Y \bar{Y} + \ldots \right).$$

(30)

Note that the kinetic terms for fields of even order in $\zeta$ have opposite signs to those of odd order. Accordingly, the corresponding $N = 1$ propagators will also have opposite signs. Consequently, complex $O(k)$ multiplets for odd $k$ contain chiral ghosts as highest coefficient superfield, and are unphysical. Nevertheless, a formal calculation of an $N = 2$ propagator is still possible, as we see below. The large $k$ limit gives the (ant)arctic multiplet propagator independently of whether $k$ is odd or even.
3.2 \( N=1 \) Propagators

To quantize the theory in \( N = 1 \) components we need to know the propagators of chiral, complex linear and auxiliary \( N = 1 \) fields. The last is trivial, while the first is well known. We briefly review the calculation of the chiral field propagator to illustrate the general techniques that we use to compute the propagator of the linear superfield and the calculation in \( N = 2 \) superspace.

An \( N = 1 \) chiral superfield \( \Phi \) obeys the constraint \( \bar{D}_a \Phi = 0 \). To calculate the propagator for a chiral superfield we have the choice of adding either a constrained or an unconstrained source term to the kinetic action \([12]\). The former involves a chiral integral

\[
\int dx \left( \int d^4 \theta \, \Phi \Phi + \int d^2 \theta \, j \Phi + \int d^2 \bar{\theta} \, \bar{j} \Phi \right) .
\]

(31)

We convert the chiral integrals into full superspace integrals and rewrite the action with sources as

\[
\int dx \int d^4 \theta \left( \Phi \Phi + \Phi \frac{D^2}{\Box} j + \bar{\Phi} \frac{\bar{D}^2}{\Box} \bar{j} \right) .
\]

(32)

Completing squares is now trivial

\[
\int dx \int d^4 \theta \left[ \left( \Phi + \frac{\bar{D}^2}{\Box} \bar{j} \right) \left( \bar{\Phi} + \frac{D^2}{\Box} j \right) - \bar{j} \frac{1}{\Box} j \right] ,
\]

(33)

and the propagator is obtained by taking the functional derivative with respect to the sources. The functional derivative with respect to a chiral source \( j \) is \([12]\)

\[
\frac{\delta j(x, \theta)}{\delta j'(x', \theta')} = \bar{D}^2 \delta^4(\theta - \theta') \delta(x - x') .
\]

(34)

The antichiral-chiral propagator is therefore \(-D^2 \bar{D}^2 \delta/\Box\).

We can also use the most general unconstrained source field \( J \), with a nonchiral coupling to the chiral field

\[
\int d^4 \theta \left( J \Phi + \bar{J} \Phi \right) = \int d^4 \theta \left( J \frac{\bar{D}^2 D^2}{\Box} \Phi + \bar{J} \frac{D^2 \bar{D}^2}{\Box} \bar{\Phi} \right) ,
\]

(35)

where we have inserted projection operators for chiral and antichiral fields. Since \( j = \bar{D}^2 J \) is a solution to the chirality constraint of \( j \), we have effectively the same source coupling as above. Completing squares we find

\[
\int dx \, d^4 \theta \left[ \left( \Phi + \frac{\bar{D}^2 D^2}{\Box} j \right) \left( \bar{\Phi} + \frac{D^2 \bar{D}^2}{\Box} \bar{j} \right) - \bar{J} \frac{D^2 \bar{D}^2}{\Box} \bar{J} \right] ;
\]

(36)

taking the functional derivative with respect to the source

\[
\frac{\delta J(x, \theta)}{\delta J'(x', \theta')} = \delta^4(\theta - \theta') \delta(x - x') ,
\]

(37)

we obtain the same propagator.
\[ \langle \Phi(1)\Phi(2) \rangle = -\frac{D^2 \bar{D}^2}{\Box} \delta^4(\theta_1 - \theta_2)\delta(x_1 - x_2) . \] (38)

Constrained sources have been used to derive Feynman rules only for chiral superfields. For other fields we use unconstrained sources and insert projectors corresponding to the subspace where the fields live. This is how we compute propagators in \( N = 2 \) superspace.

Note that we include the supercovariant derivatives in the propagator. Equivalently, we could put them into the interaction vertices, and use \(-\delta^8_{12}/\Box\) as the propagator [12]. A similar choice is possible for the constrained \( N = 2 \) superfields we study.

All this is well known \( N = 1 \) technology. For the complex linear superfield \( \Sigma \) the propagator has not appeared in the literature (see, however, [13]). We now derive this propagator using an unconstrained superfield source \( J \), and introduce suitable projectors to complete squares in the action.

The free action with sources is
\[ \int dx \int d^4\theta \left( -\Sigma \bar{\Sigma} + \Sigma \bar{J} + J \Sigma \right) . \] (39)

A linear superfield obeys the constraint \( \bar{D}^2 \Sigma = 0 \), so we can insert the projector \( P = 1 - D^2 \bar{D}^2/\Box \) and its conjugate in the corresponding source terms. Integrating by parts and completing squares we find
\[ \int dx \int d^4\theta \left[ -(\Sigma - PJ)(\Sigma - \bar{P}J) + J\bar{P}J \right] , \] (40)

which yields the propagator
\[ \langle \Sigma(1)\Sigma(2) \rangle = \bar{P} \delta^4(\theta_1 - \theta_2)\delta(x_1 - x_2) = \left( 1 - \frac{\bar{D}^2 D^2}{\Box} \right) \delta_{12} . \] (41)

An alternative way to compute the complex antilinear-linear propagator is to perform the duality transformation in the \( N = 1 \) component action \textit{with sources}:
\[ \int dx \ D^2 \bar{D}^2 \left( -\bar{S} \bar{S} + \bar{Z} \bar{D}^2 S + \bar{D}^2 \bar{S} + J\bar{S} + S\bar{J} \right) . \] (42)

Completing squares and integrating out \( S \) gives a dual action
\[ \int dx \ D^2 \bar{D}^2 \left( \bar{D}^2 Z \bar{D}^2 \bar{Z} + J\bar{J} + \bar{J} \bar{D}^2 \bar{Z} + \bar{D}^2 J\bar{Z} \right) ; \] (43)

we see that the complex antilinear-linear propagator is equivalent to a chiral-antichiral propagator \textit{plus} a contact term:
\[ \langle \Sigma(1)\Sigma(2) \rangle = \frac{\delta}{\delta J_1} \frac{\delta}{\delta J_2} \ln Z_0 \]
\[ = \delta^4(\theta_1 - \theta_2)\delta(x_1 - x_2) - \left( \bar{D}^2 Z_1 D^2 \bar{Z}_2 \right) \]
\[ = \left( 1 - \frac{\bar{D}^2 D^2}{\Box} \right) \delta_{12} . \] (44)
Finally, the propagator for the real unconstrained superfield $X$ is simply

$$-\delta^4(\theta_1 - \theta_2)\delta(x_1 - x_2)$$

(45)

because completing squares for an unconstrained superfield is trivial.

4 $\mathbb{N}=2$ superspace propagators from $\mathbb{N}=1$ component propagators

4.1 The $O(4)$ multiplet

For simplicity, we begin with the $O(4)$ multiplet. We want to calculate the propagator

$$\langle \eta(\zeta_1, \theta_1, p) \eta(\zeta_2, \theta_2, -p) \rangle.$$  

Expanding in powers of $\zeta_1$ and $\zeta_2$, we can compute the two point functions of the $\mathbb{N}=1$ coefficient superfields

$$\langle \eta^{(1)} \eta^{(2)} \rangle |_{\theta_2^\alpha=0} = \langle \Phi(1)\Phi(2) \rangle \zeta_2^4 + \langle \Phi(1)\bar{\Phi}(2) \rangle \zeta_1^4 - \langle \bar{\Sigma}(2)\Sigma(2) \rangle \zeta_1 \zeta_2^3$$

$$-\langle \Sigma(1)\bar{\Sigma}(2) \rangle \zeta_1^3 \zeta_2 + \langle X(1)X(2) \rangle \zeta_2^2 \zeta_1^2.$$  

(46)

These are the only nonvanishing 2-point functions in the free theory. Substituting the $\mathbb{N}=1$ propagators from above, we find

$$\langle \eta^{(1)}\eta^{(2)} \rangle |_{\theta_2^\alpha=0} = \left( -\zeta_1 \zeta_2 (\zeta_1^2 + \zeta_1 \zeta_2 + \zeta_2^2) - \zeta_2 (-\zeta_1^3 + \zeta_2^3) \frac{D^2 D^2}{\Box} - \zeta_1 (\zeta_1^3 - \zeta_2^3) \frac{D^2 D^2}{\Box} \right)$$

$$\times \delta^4(\theta_1 - \theta_2)\delta(x_1 - x_2).$$  

(47)

Rewriting (47) with the identity

$$1 = \frac{\bar{D}^2 D^2}{\Box} + \frac{D^2 \bar{D}^2}{\Box} - \frac{D \bar{D}^2 D}{\Box},$$

(49)

and

$$\zeta_1^2 + \zeta_1 \zeta_2 + \zeta_2^2 = \frac{\zeta_1^3 - \zeta_2^3}{\zeta_1 - \zeta_2},$$

(50)

we find that the $\mathbb{N}=1$ projection of

$$-\frac{\zeta_3^3 - \zeta_2^3}{(\zeta_1 - \zeta_2)^3} \frac{\nabla_1^4 \nabla_2^4}{\Box} \delta^8(\theta_1 - \theta_2)\delta(x_1 - x_2)$$

(51)
reproduces (17). We note that the coefficient of this $N = 2$ propagator does not have a double pole at $\zeta_1 = \zeta_2$ because

$$\nabla_1^4 \nabla_2^4 = \nabla_1^2 \left( (\zeta_1 - \zeta_2)^2 \bar{D}^2 \bar{Q}^2 \right) \nabla_2^2 .$$

(52)

This ansatz for the $N = 2$ propagator generalizes to the real $O(2p)$ multiplet, and gives

$$\langle \eta(1) \eta(2) \rangle |_{\theta_2^a = 0} = (-)^{p+1} \left( (\Phi(1)\Phi(2)) \zeta_2^{2p} - (\Sigma(1)\Sigma(2)) \zeta_1 \zeta_2^{2p-1} \right.
\left. + (\bar{X}(1)X(2)) \zeta_1 \zeta_2^{2p-2} + \ldots \right)$$

$$- (\Sigma(1)\bar{\Sigma}(2)) \zeta_1^{2p-1} \zeta_2 + (\Phi(1)\bar{\Phi}(2)) \zeta_2^{2p} \right)$$

$$= (-)^{p+1} \left( \frac{\zeta_1^{2p-1} - \zeta_2^{2p-1}}{\zeta_1 - \zeta_2} \right) \nabla_1^4 \nabla_2^4 \delta(\theta_1 - \theta_2) \delta(x_1 - x_2) |_{\theta_2^a = 0} .$$

(53)

The real $O(2)$ multiplet has chiral, antichiral and real linear $N = 1$ superfield coefficients. The propagator for the last can be obtained by coupling to an unconstrained source and inserting the projector $-D\bar{D}^2/\Box$. The resulting real linear propagator is $-D\bar{D}^2\bar{D}\delta/\Box$. Despite the different off-shell degrees of freedom involved, the corresponding $N = 2$ propagator for the real $O(2)$ multiplet agrees with the general form (53)

$$\langle \eta(1) \eta(2) \rangle |_{\theta_2^a = 0} = \frac{D^2 \bar{D}^2}{\Box} \zeta_2^2 - \frac{D\bar{D}^2 D}{\Box} \zeta_1 \zeta_2 + \frac{\bar{D}^2 D^2}{\Box} \zeta_1^2$$

$$= \frac{1}{\Box(\zeta_1 - \zeta_2)^2} \nabla_1^4 \nabla_2^4 \delta_{12} |_{\theta_2^a = 0} .$$

(54)

4.2 The complex $O(k)$ and (ant)arctic multiplets

Just as in the $O(4)$ case we can expand the propagator $\langle \bar{Y}_1 Y_2 \rangle$ in powers of $\zeta_2/\zeta_1$ and compute the $N = 1$ projection of the two point function

$$\langle \bar{Y}(1) Y(2) \rangle |_{\theta_2^a = 0} = \langle \phi(1)\tilde{\phi}(2) \rangle - \left( \frac{\zeta_2}{\zeta_1} \right) \langle \Sigma(1)\bar{\Sigma}(2) \rangle + \left( \frac{\zeta_2}{\zeta_1} \right)^2 \langle X(1)\bar{X}(2) \rangle$$

$$+ \left( -\frac{\zeta_2}{\zeta_1} \right)^3 \langle Y(1)\bar{Y}(2) \rangle + \ldots$$

$$+ \left( -\frac{\zeta_2}{\zeta_1} \right)^{k-1} \langle \bar{\Sigma}(1)\bar{\Sigma}(2) \rangle + \left( -\frac{\zeta_2}{\zeta_1} \right)^k \langle \tilde{\phi}(1)\phi(2) \rangle .$$

(55)

As above, we insert the chiral-antichiral and linear-antilinear $N = 1$ propagators in the two lowest order terms and the conjugate ones in the two highest order terms of (53). The $N = 1$ auxiliary superfields give contributions $(\zeta_2/\zeta_1)^n \delta_{12}(x)\delta_{12}^4(\theta)$.
For finite $O(k)$ complex multiplets we find

$$
\langle \bar{\Upsilon}(1) \ U(2) \rangle |_{\theta^2 = 0} = - \frac{\bar{D}^2 D^2}{\Box} \delta_{12} - \frac{\zeta_2}{\zeta_1} \left(1 - \frac{D^2 \bar{D}^2}{\Box}\right) \delta_{12} - \left(\frac{\zeta_2}{\zeta_1}\right)^2 \sum_{n=0}^{k-4} \left(\frac{\zeta_2}{\zeta_1}\right)^n \delta_{12}
$$

$$
- \frac{\left(\frac{\zeta_2}{\zeta_1}\right)^{k-1}}{\zeta_1^k} \left(1 - \frac{D^2 \bar{D}^2}{\Box}\right) \delta_{12} - \frac{\zeta_2}{\zeta_1} \left(\frac{D^2 \bar{D}^2}{\Box}\right) \delta_{12}
$$

$$
= \left[\left(1 - \left(\frac{\zeta_2}{\zeta_1}\right)^{k-1}\right) \left(- \frac{D^2 \bar{D}^2}{\Box} + \frac{\zeta_2}{\zeta_1} \frac{D^2 \bar{D}^2}{\Box}\right) - \sum_{n=1}^{k-1} \left(\frac{\zeta_2}{\zeta_1}\right)^n\right] \delta_{12}
$$

$$
= \left[\zeta_1^{k-1} - \zeta_2^{k-1} \left(\frac{D^2 \bar{D}^2}{\Box} + \frac{\zeta_2 D^2 \bar{D}^2}{\zeta_1 \Box} - \frac{\zeta_2 \zeta_1}{\zeta_1 - \zeta_2}\right)\right] \delta_{12} . \tag{56}
$$

For the (ant)arctic multiplet, we do not have $\tilde{\Sigma}\Sigma$ and $\tilde{\phi}\phi$, so we have a geometric series in $\zeta_2/\zeta_1$:

$$
\langle \bar{\Upsilon}(1) \ U(2) \rangle |_{\theta^2 = 0} = \left[- \frac{\bar{D}^2 D^2}{\Box} - \frac{\zeta_2}{\zeta_1} \left(1 - \frac{D^2 \bar{D}^2}{\Box}\right) - \sum_{n=2}^{\infty} \left(\frac{\zeta_2}{\zeta_1}\right)^n\right] \delta_{12}
$$

$$
= \left[- \frac{D^2 \bar{D}^2}{\Box} + \frac{\zeta_2}{\zeta_1} \frac{D^2 \bar{D}^2}{\Box} - \sum_{n=1}^{\infty} \left(\frac{\zeta_2}{\zeta_1}\right)^n\right] \delta_{12} . \tag{57}
$$

In the region where $|\zeta_2/\zeta_1| < 1$ the series is convergent to $\zeta_2/(\zeta_1 - \zeta_2)$. If we use this propagator to connect $\bar{\Upsilon}_1 U_2$ lines from vertices at different points in the $\zeta$-plane and form a closed loop, when performing the contour integrals of each vertex the pole at $\zeta_1 = \zeta_2$ can lead to ambiguities\footnote{If the vertices of the theory depend on a real combination $\zeta^p \bar{\Upsilon} + (-\zeta)^{-p} \Upsilon$ for any $p$, the potential problem with the summed series disappears. We will see that such a dependence allows us to perform a duality transformation between the real $O(2p)$ multiplet action and the analytic multiplet action.}

Just as we did in the real $O(4)$ multiplet case, we find the $N = 2$ propagator of the complex $O(k)$ multiplet by using an ansatz to guess the $N = 2$ expression whose $N = 1$ reduction reproduces our result:

$$
\langle \bar{\Upsilon}(1) \ U(2) \rangle = - \frac{\sum_{n=0}^{k-2} \zeta_2^{k-2-n} \zeta_1^2}{\zeta_1 (\zeta_1 - \zeta_2)^2} \frac{\nabla_1^4 \nabla_2^4}{\Box} \delta_8(\theta_1 - \theta_2) \delta(x_1 - x_2) \tag{58}
$$

$$
= - \frac{\zeta_1^{k-1} - \zeta_2^{k-1}}{\zeta_1 (\zeta_1 - \zeta_2)^3} \frac{\nabla_1^4 \nabla_2^4}{\Box} \delta_8(\theta_1 - \theta_2) \delta(x_1 - x_2) \tag{59}
$$

and its conjugate

$$
\langle \Upsilon(1) \ \bar{\Upsilon}(2) \rangle = - \frac{\zeta_1^{k-1} - \zeta_2^{k-1}}{\zeta_2 (\zeta_1 - \zeta_2)^3} \frac{\nabla_1^4 \nabla_2^4}{\Box} \delta_8(\theta_1 - \theta_2) \delta(x_1 - x_2) . \tag{60}
$$

This result is consistent with our previous observation that a real $O(2p)$ superfield can be constructed from a complex $O(2p)$ multiplet and its conjugate.
\[
\langle\eta(1)\eta(2)\rangle = \frac{1}{2} \langle (\bar{\Upsilon}(1) + (-)^p \zeta_1^2 \Upsilon(1)) \ (\Upsilon(2) + (-)^p \zeta_2^2 \bar{\Upsilon}(2)) \rangle = \frac{1}{2} \left[ (-)^p \zeta_2^2 \langle \bar{\Upsilon}(1) \Upsilon(2) \rangle + (-)^p \zeta_1^2 \langle \bar{\Upsilon}(1) \Upsilon(2) \rangle \right]; \quad (61)
\]

likewise, the propagator of the complex \(O(2p)\) multiplet and its conjugate can be reobtained from that of a real \(O(2p)\) superfield by complexification.

The (ant)arctic multiplet propagator can also be reconstructed in this form and we find

\[
\langle \bar{\Upsilon}(1) \Upsilon(2) \rangle = -\frac{1}{\zeta_1^2} \sum_{n=0}^{\infty} \left( \frac{\zeta_2}{\zeta_1} \right)^n \frac{\nabla_1^4 \nabla_2^4}{(\zeta_1 - \zeta_2)^2} \delta^8(\theta_1 - \theta_2) \delta(x_1 - x_2)
= -\frac{1}{\zeta_1(\zeta_1 - \zeta_2)^3} \frac{\nabla_1^4 \nabla_2^4}{\square} \delta^8(\theta_1 - \theta_2) \delta(x_1 - x_2), \quad (62)
\]

\[
\langle \Upsilon(1) \bar{\Upsilon}(2) \rangle = -\frac{1}{\zeta_2^2} \sum_{n=0}^{\infty} \left( \frac{\zeta_1}{\zeta_2} \right)^n \frac{\nabla_1^4 \nabla_2^4}{(\zeta_1 - \zeta_2)^2} \delta^8(\theta_1 - \theta_2) \delta(x_1 - x_2)
= \frac{1}{\zeta_2(\zeta_1 - \zeta_2)^3} \frac{\nabla_1^4 \nabla_2^4}{\square} \delta^8(\theta_1 - \theta_2) \delta(x_1 - x_2). \quad (63)
\]

### 5 Feynman rules derived in \(N = 2\) superspace

#### 5.1 \(N = 2\) Propagators

We now derive the \(N = 2\) propagator of the complex \(O(k)\) multiplet directly in \(N = 2\) superspace and validate our ansatz. The real \(O(2p)\) multiplet propagator can be obtained using very similar arguments and does not provide any additional information. We introduce the operators

\[
M(\zeta) = \frac{1}{16 \square^2} \left( \Delta^4 + \frac{1}{\zeta} \nabla^a \Delta_a \Delta^2 + \frac{1}{\zeta^2} \nabla^2 \Delta^2 \right) = \frac{1}{4 \square^2} \frac{D^2 \Delta^2}{\zeta^2},
\]

\[
N(\zeta) = \frac{1}{16 \square^2} \left( \Delta^4 - \frac{1}{\zeta} \nabla^a \Delta_a \Delta^2 + \frac{1}{\zeta^2} \nabla^2 \Delta^2 \right) = \frac{1}{4 \square^2} Q^2 \Delta^2. \quad (64)
\]

that satisfy the following relations

\[
\nabla_1^4 M(\zeta) \nabla_2^4 = \frac{\left( \frac{\zeta_1}{\zeta_2} \right)^4}{(\zeta_1 - \zeta_2)^2} \frac{\nabla_1^4 \nabla_2^4}{\square}
\]

\[
\nabla_1^4 N(\zeta) \nabla_2^4 = \frac{1}{(\zeta_1 - \zeta_2)^2} \frac{\nabla_1^4 \nabla_2^4}{\square}. \quad (65)
\]
Inspired by the analogy to $N = 1$ chiral superfields, we introduce an unconstrained prepotential superfield and its conjugate

$$\Upsilon = \nabla^4 \psi = \nabla^4 \sum_{n=0}^{k-4} \psi_n \zeta^n, \quad \bar{\Upsilon} = \frac{\nabla^4}{\zeta^4} \bar{\psi} = \frac{\nabla^4}{\zeta^4} \sum_{m=0}^{k-4} \bar{\psi}_{-m} \zeta^{-m}. \quad (66)$$

In terms of $\psi$, the kinetic action can be rewritten with the full $N = 2$ measure

$$\int dx D^2 \bar{D}^2 \oint d\zeta^2 \pi i \zeta \nabla^4 \left( \nabla^4 \zeta^4 \bar{\psi} \psi \right) = \int dx d^8 \theta \oint d\zeta^2 \pi i \zeta \nabla^4 \zeta^2 \bar{\psi} \psi. \quad (67)$$

We derive the complex $O(k)$ and (ant)arctic multiplet propagators by completing squares in the full $N = 2$ action with sources. The simplest source is an unconstrained $O(k)$ superfield and its conjugate, which gives the action

$$S_0 + S_J = \int dx d^8 \theta \oint d\zeta^2 \pi i \zeta \left( \bar{\psi} \nabla^4 \zeta^2 \psi + \bar{J} \Upsilon + \bar{\Upsilon} J \right). \quad (68)$$

When we reduce this action to $N = 1$ components, every coefficient superfield $\Upsilon_n$ couples to an unconstrained source. The functional derivative of $\zeta$-dependent $N = 2$ superfields is defined by

$$\frac{\delta}{\delta J(x', \theta', \zeta')} \int dx d^8 \theta \oint d\zeta^2 \pi i \zeta JF = F(x', \theta', \zeta'). \quad (69)$$

For a generic $O(i,j)$ source $J = \sum_{n=i}^{j} \zeta^n J_n$,

$$\frac{\delta}{\delta J(x', \theta', \zeta')} J(x, \theta, \zeta) = \delta^{(j)}(\zeta, \zeta') \delta^8(\theta - \theta') \delta(x - x'), \quad (70)$$

where the delta function on the Riemann sphere

$$\delta^{(j)}(\zeta, \zeta') = \sum_{n=-j}^{j} \left( \frac{\zeta}{\zeta'} \right)^n = \frac{\zeta^i (\zeta')^{j-i+1} - \zeta^{j-i+1}}{\zeta' - \zeta} = \delta^{(-j)}(\zeta', \zeta) \quad (71)$$

projects onto the subspace of $O(i,j)$ functions when integrated with the measure $\oint d\zeta^2 \pi i \zeta'$.

Note that $\delta^{(j)}(\zeta, \zeta')$ is a function that approaches a well defined distribution when $i$ and (or) $j$ tends to infinity. For the $O(k)$ source in our action, the functional derivative

$$\frac{\delta}{\delta J(x', \theta', \zeta')} J(x, \theta, \zeta) = \delta^{(k)}(\zeta, \zeta') \delta^8(\theta - \theta') \delta(x - x'). \quad (72)$$

acting on the free theory path integral will give us the propagators. We now rewrite the action with sources in terms of the prepotential

$$S_0 + S_J = \int dx d^8 \theta \oint d\zeta^2 \pi i \zeta \left( \bar{\psi} \nabla^4 \zeta^2 \psi + \bar{J} \nabla^4 \psi + \bar{\psi} \nabla^4 \zeta^4 J \right). \quad (73)$$
We cannot complete squares directly on the prepotential because even the simplest sources have a different \( \zeta \)-dependence than the prepotentials. Hence, we must insert a projector with the following properties: i) it leaves the source coupling to \( \bar{\psi} (\psi) \) invariant; ii) it can be split in the product of the kinetic operator and some inverse operator that projects the source into the subspace of \( O(k - 4) \) functions.

We will proceed as follows: The identity \( 16 \Box^2 \nabla^4 = \nabla^4 \Delta^4 \nabla^4 \) defines a projection operator in \( N = 2 \) projective superspace. We may use it to rewrite the last source term as

\[
\mathcal{L}_J = \oint \frac{d\zeta}{2\pi i\zeta} \bar{\psi} \nabla^4 J = \oint \frac{d\zeta}{2\pi i\zeta} \bar{\psi} \frac{\nabla^4 \nabla^4}{16 \Box^2} J \, .
\]

(74)

However, since the operators \( \nabla^2 (\zeta) \Delta^2 (\zeta) \) and \( \nabla^\alpha (\zeta) \Delta^\alpha (\zeta) \Delta^2 (\zeta) \) are annihilated when inserted between two \( \nabla^4 (\zeta) \) operators, we may add any combination of those to the \( \Delta^4 \) operator. In particular we rewrite the source term as

\[
\mathcal{L}_J = \oint \frac{d\zeta}{2\pi i\zeta} \bar{\psi} \nabla^4 M(\zeta) \frac{\nabla^4}{\zeta^2} J = \oint \frac{d\zeta}{2\pi i\zeta} \bar{\psi} \nabla^4 N(\zeta) \frac{\nabla^4}{\zeta^2} J \, .
\]

(75)

Because \( \bar{\psi} \nabla^4 \) is \( O(-k + 2, 2) \), we can insert a \( \delta^{(k-2)}_{(-2)} \) and split the source term:

\[
\mathcal{L}_J = \oint \frac{d\zeta}{2\pi i\zeta} \oint \frac{d\zeta'}{2\pi i\zeta'} \bar{\psi}(\zeta) \frac{\nabla^4(\zeta)}{\zeta^2} \delta^{(k-2)}_{(-2)} (\zeta, \zeta') M(\zeta') \frac{\nabla^4(\zeta')}{\zeta'^2} J(\zeta') \, .
\]

(76)

To complete squares we still have to prove that the source is projected into a \( O(0, k - 4) \) field. Using the relations (65) we may rewrite the source term once more

\[
\mathcal{L}_J = \oint \frac{d\zeta}{2\pi i\zeta} \oint \frac{d\zeta'}{2\pi i\zeta'} \bar{\psi}(\zeta) \frac{\nabla^4(\zeta)}{\zeta^2} \delta^{(k)}_{(0)} (\zeta, \zeta') N(\zeta') \frac{\nabla^4(\zeta')}{\zeta'^2} J(\zeta') \, .
\]

(77)

where the extra factor of \( (\zeta/\zeta')^2 \) in (65) effectively shifts the range of the delta function two steps so that it now runs from 0 to \( k \). However, \( \bar{\psi} \nabla^4 \) is still \( O(-k + 2, 2) \) so the remaining contribution is

\[
\mathcal{L}_J = \oint \frac{d\zeta}{2\pi i\zeta} \oint \frac{d\zeta'}{2\pi i\zeta'} \bar{\psi}(\zeta) \frac{\nabla^4(\zeta)}{\zeta^2} \delta^{(k-2)}_{(0)} (\zeta, \zeta') N(\zeta') \frac{\nabla^4(\zeta')}{\zeta'^2} J(\zeta') \, .
\]

(78)

To see that there are no higher powers of \( \zeta \) than \( k - 4 \), we write

\[
\mathcal{L}_J = \oint \frac{d\zeta}{2\pi i\zeta} \oint \frac{d\zeta'}{2\pi i\zeta'} \bar{\psi}(\zeta) \frac{\nabla^4(\zeta)}{\zeta^2} \left[ \delta^{(k-4)}_{(0)} (\zeta, \zeta') N(\zeta') + \delta^{(k-2)}_{(k-3)} (\zeta, \zeta') N(\zeta') \right] \frac{\nabla^4(\zeta')}{\zeta'^2} J(\zeta') \, ,
\]

(79)

and use the relations (65) on the last term to get

\[
\mathcal{L}_J = \oint \frac{d\zeta}{2\pi i\zeta} \oint \frac{d\zeta'}{2\pi i\zeta'} \bar{\psi}(\zeta) \frac{\nabla^4(\zeta)}{\zeta^2} \left[ \delta^{(k-4)}_{(0)} (\zeta, \zeta') N(\zeta') + \delta^{(k-4)}_{(k-5)} (\zeta, \zeta') M(\zeta') \right] \frac{\nabla^4(\zeta')}{\zeta'^2} J(\zeta') \, ,
\]

(80)

Although the prepotential has a gauge invariance whose fixing may introduce ghosts for ghosts, we will not be concerned with it here, as in the absence of nonabelian gauge fields the ghosts decouple. See, however, 6.
which proves that \( \bar{\psi} \sum_{k}\frac{4}{\zeta} \) couples to a \( O(k - 4) \) projected source:

\[
J^{(k-4)}(\zeta) \equiv \oint \frac{d\zeta'}{2\pi i \zeta'} \left[ \delta^{(k-4)}(\zeta, \zeta')N(\zeta') + \delta^{(k-4)}_{(k-5)}(\zeta, \zeta')M(\zeta') \right] \frac{\nabla^4(\zeta')}{\zeta'^2} J(\zeta').
\]  

(81)

Note that

\[
\nabla^4 J^{(k-4)}(\zeta) = \nabla^4 \oint \frac{d\zeta'}{2\pi i \zeta'} \delta^{(k-2)}(\zeta, \zeta')N(\zeta') \frac{\nabla^4(\zeta')}{\zeta'^2} J(\zeta').
\]

Similarly

\[
\oint \frac{d\zeta'}{2\pi i \zeta'} \psi \nabla^4 J = \oint \frac{d\zeta'}{2\pi i \zeta'} \frac{\nabla^4 J}{\zeta'^2} \tilde{J}^{(4-k)}
\]

where

\[
\tilde{J}^{(4-k)}(\zeta) = \oint \frac{d\zeta'}{2\pi i \zeta'} \left[ \delta^{(0)}_{(4-k)}(\zeta, \zeta')M(\zeta') + \delta^{(5-k)}_{(4-k)}(\zeta, \zeta')N(\zeta') \right] \zeta'^2 \nabla^4(\zeta') \bar{J}(\zeta')
\]

(84)

with

\[
\nabla^4 \tilde{J}^{(4-k)}(\zeta) = \nabla^4 \oint \frac{d\zeta'}{2\pi i \zeta'} \delta^{(0)}_{(2-k)}(\zeta, \zeta')M(\zeta') \zeta'^2 \nabla^4(\zeta') \bar{J}(\zeta').
\]

(85)

Since we have shown that the prepotential and the projected source term are of the same type, we may now complete squares in the action \( S_0 \)

\[
S_0 + S_J = \int dx^8 \theta \oint \frac{d\zeta_0}{2\pi i \zeta_0} \left[ \bar{\psi}(\zeta_0) + \tilde{J}^{(4-k)}(\zeta_0) \right] \frac{\nabla^4(\zeta_0)}{\zeta_0^2} \left( \psi(\zeta_0) + J^{(k-4)}(\zeta_0) \right)
\]

(86)

\[
- \oint \frac{d\zeta d\zeta'}{(2\pi i)^2 \zeta \zeta'} \left[ \delta^{(0)}_{(2-k)}(\zeta_0, \zeta)M(\zeta) \zeta^2 \nabla^4(\zeta) \bar{J}(\zeta) \right] \frac{\nabla^4(\zeta_0)}{\zeta_0^2} \left[ \delta^{(k-2)}(\zeta_0, \zeta')N(\zeta') \frac{\nabla^4(\zeta')}{\zeta'^2} J(\zeta') \right].
\]

All the source dependence in the first term may be absorbed in a redefinition of the prepotential since they are both \( O(k - 4) \) multiplets. Using the same arguments backwards, we can rewrite the term quadratic in sources

\[
\ln Z_0[J, \bar{J}] = - \int dx^8 \theta \oint \frac{d\zeta}{2\pi i \zeta} \oint \frac{d\zeta_0}{2\pi i \zeta_0} \left( \delta^{(0)}_{(2-k)}(\zeta, \zeta_0) \bar{J}(\zeta_0) \nabla^4(\zeta_0) M(\zeta_0) \zeta_0^2 \right) \frac{\nabla^4(\zeta)}{\zeta^2} J^{(k-4)}(\zeta)
\]

(87)

\[
= - \int dx^8 \theta \oint \frac{d\zeta}{2\pi i \zeta} \bar{J}(\zeta) \frac{\nabla^4(\zeta) \Delta^4(\zeta) \nabla^4(\zeta)}{16 \Box^2} J^{(k-4)}(\zeta)
\]

Using (65) once more, we finally arrive at:
\[ Z_0[J, \bar{J}] = \exp \left( - \int dx d^8 \theta \oint d\zeta \frac{d\zeta'}{2\pi i\zeta'} \bar{J}(\zeta) \delta_{(0-2)}^{(k-2)}(\zeta, \zeta') \nabla^4(\zeta) \nabla^4(\zeta') \right). \] (88)

The complex \( O(k) \) multiplet propagator is obtained by functionally differentiating with respect to the unconstrained \( N = 2 \) sources

\[
\langle \bar{\Upsilon}(1) \Upsilon(2) \rangle = \frac{\delta^2}{\delta J(\zeta_1) \delta J(\zeta_2)} \ln Z_0
\]

\[
= \oint d\zeta \oint d\zeta' \frac{d\zeta}{2\pi i\zeta} \frac{d\zeta'}{2\pi i\zeta'} \delta^{(k)}_{(-2)}(\zeta_1, \zeta') \delta^{(0)}_{(0)}(\zeta_2, \zeta) \delta^{(k-2)}_{(0)}(\zeta, \zeta') \nabla^4(\zeta) \nabla^4(\zeta') \delta(\theta - \theta') \delta(x - x'). \] (89)

Evaluating the \( \zeta \) and \( \zeta' \) integrals after using (52) to cancel the double pole in \( \zeta - \zeta' \), we obtain our previous result (58). Note that this can be rewritten as

\[
\langle \bar{\Upsilon}(1) \Upsilon(2) \rangle = -\delta^{(k-2)}_{(0)}(\zeta_2, \zeta_1) \frac{\nabla^4_1 \nabla^4_2}{\zeta_1^2(\zeta_1 - \zeta_2)^2} \delta^8(\theta_1 - \theta_2) \delta(x_1 - x_2). \] (90)

The limit \( k \to \infty \) gives the (ant)arctic multiplet propagator. The derivation in \( N = 2 \) superspace follows exactly the same lines as the derivation for the complex \( O(k) \) multiplet. The only difference is that the prepotential \( \bar{\psi} \) is now antarctic. This implies that only the two lowest powers of \( \zeta \) in the projected source present a problem, the higher power being infinite. Therefore rewriting the projected source in terms of the operator \( \mathcal{N}(\zeta_2) \) is enough to obtain an arctic function coupled to \( \bar{\psi}(\zeta_1) \nabla^4_1 / \zeta_1^2 \). The propagator for the arctic multiplet derived in this way agrees with the form of the propagator found earlier in (62) and (63).

For completeness we will also calculate the propagator of the real \( O(2) \) multiplet in \( N = 2 \) superspace. A solution to the constraint (7) obeying the reality condition and with the correct global \( SU(2) \) transformation properties is:

\[
\eta = \nabla^4(\Delta^2 + \bar{\Delta}^2)\Psi. \] (91)

The product \( \nabla^2 \Delta^2 = 4D^2Q^2 \) is \( \zeta \)-independent, and therefore \( \Psi \) is a \( \zeta \)-independent dimensionless isoscalar. We can rewrite the free action with sources in terms of this prepotential

\[
S_0 + S_J = \int dx d^8 \theta \oint d\zeta \frac{d\zeta}{2\pi i\zeta} \left( \frac{1}{2} \Psi(\Delta^2 + \bar{\Delta}^2) \nabla^4(\Delta^2 + \bar{\Delta})^2 \Psi + \Psi(\Delta^2 + \bar{\Delta}^2) \nabla^4 J \right), \] (92)

and complete squares in \( \Psi \). This is particularly easy in this case because the kinetic operator of the prepotential acts as \( \Box^2 \) on the prepotential source

\[
(\Delta^2 + \bar{\Delta}^2) \nabla^4(\Delta^2 + \bar{\Delta}^2)(\Delta^2 + \bar{\Delta}^2) \nabla^4 J = 32 \Box^2(\Delta^2 + \bar{\Delta}^2) \nabla^4 J, \] (93)
and in addition this kinetic operator is $\zeta$-independent

$$\left(\Delta^2 + \bar{\Delta}^2\right) \nabla^4 \left(\Delta^2 + \bar{\Delta}^2\right) = 16D^2Q^2\bar{D}^2\bar{Q}^2 + 16\bar{D}^2Q^2D^2\bar{Q}^2 + 16D^2Q^2\Box + 16\bar{D}^2\bar{Q}^2\Box. \quad (94)$$

Defining the following $\zeta$-independent source

$$J = \oint d\zeta \frac{\zeta}{2\pi i \zeta} (\Delta^2 + \bar{\Delta}^2) \nabla^4 J(\zeta) \frac{(\zeta)}{\Box^2 \zeta^2} \quad (95)$$

the path integral can be expressed as a completed square

$$\ln Z_0[J] = \int dx \, d^8\theta \frac{1}{2}(\psi + J)(\Delta^2 + \bar{\Delta}^2) \nabla^4(\Delta^2 + \bar{\Delta}^2)(\psi + J)$$

$$- \frac{1}{2} \oint d\zeta_1 \oint d\zeta_2 \frac{J(\zeta_1)}{\zeta_1^2} \nabla_1^4(\Delta_1^2 + \bar{\Delta}_1^2) \frac{1}{32\Box^2}(\Delta_1^2 + \bar{\Delta}_1^2) \nabla_2^4 \frac{J(\zeta_2)}{\zeta_2^2}. \quad (96)$$

The resulting propagator is

$$\langle \eta(1)\eta(2) \rangle = \nabla_1^4(\Delta_1^2 + \bar{\Delta}_1^2)(\Delta_2^2 + \bar{\Delta}_2^2) \nabla_2^4 \frac{\delta_{12}}{32\Box^2} = \frac{1}{\Box(\zeta_1 - \zeta_2)^2} \nabla_1^4 \nabla_2^4 \delta_{12}, \quad (97)$$

in agreement with our previous calculation (54).

### 5.2 Vertices

We now define vertex factors that allow us to construct the diagrams of the interacting theory. As mentioned before, we can put the graded spinor derivatives $\nabla_1^4, \nabla_2^4$ of a propagator $\langle \eta(1)\eta(2) \rangle$ on the internal lines of interaction vertices. For the real $O(2p)$ multiplet, we would be working formally with propagators

$$\langle \eta(1)\eta(2) \rangle = (-)^{p+1} \frac{\zeta_{2p-1}^2 - \zeta_{2p-1}}{(\zeta_1 - \zeta_2)^3} \oint \frac{d\zeta}{2\pi i \zeta} \frac{1}{\Box} \nabla_1^4(\Delta_1^2 - \bar{\Delta}_1^2) \nabla_2^4 \frac{\delta_{12}}{32\Box^2} = \frac{1}{\Box(\zeta_1 - \zeta_2)^2} \nabla_1^4 \nabla_2^4 \delta_{12}, \quad (98)$$

connecting two $\eta$ internal lines. Accordingly, interactions such as

$$\int dx \, d^4\theta \oint d\zeta \frac{1}{2\pi i \zeta} \left(\frac{\eta}{\zeta^p}\right)^n \quad (99)$$

in which $q$ lines are external and $n - q$ lines are internal will contribute (with appropriate combinatorial normalization) as vertex factors of the form

$$\int d^4\theta \oint d\zeta \frac{1}{2\pi i \zeta} \left(\frac{\eta}{\zeta^p}\right)^q \left(\nabla_1^4 \right)^{n-q} \quad (100)$$

For the complex $O(k)$ multiplet and the (ant)arctic multiplet we also put the graded spinor derivatives $\nabla_1^4, \nabla_2^4$ of a propagator $\langle \Upsilon(1)\bar{\Upsilon}(2) \rangle$ in the internal lines of interaction vertices. The action will contain interactions.
\[
\int dx \, d^4 \theta \oint d\zeta \frac{1}{2\pi i \zeta} \frac{1}{n! m!} \left[ f(\zeta) \bar{\Upsilon}^n \Upsilon^m + \bar{f} \left( \frac{-1}{\zeta} \right) \bar{\Upsilon}^n \Upsilon^m \right]
\]  

(101)

giving vertex factors

\[
\int d^4 \theta \oint d\zeta \frac{d\zeta}{2\pi i \zeta} \left[ f(\zeta) \bar{\Upsilon}^q \left( \nabla^4 \right)^{n-q} \Upsilon^p \left( \nabla^4 \right)^{m-p} + \bar{f} \left( \frac{-1}{\zeta} \right) \bar{\Upsilon}^q \left( \nabla^4 \right)^{n-q} \Upsilon^p \left( \nabla^4 \right)^{m-p} \right].
\]  

(102)

5.3 Diagram construction rules

Once the explicit form of the propagator and interaction vertices is known, the rules for diagram construction are easily derived from the path integral definition of \(n\)-point functions. As usual, the overall numerical factors come from the differentiation of the path integral with respect to \(n\) sources, and the different combinatorial possibilities for connecting the lines of our vertices.

Once we have constructed a diagram by connecting vertex lines through propagators, we can extract an overall \(\nabla^4\) for each vertex \(i\) and use it to complete the restricted superspace measure on that vertex to a full \(N = 2\) superspace measure. This is analogous to completing the chiral measure of a superpotential interaction to a full \(N = 1\) superspace measure.

The strategy once we have constructed a given diagram and completed the \(N = 2\) superspace measure in all its vertices, is the same as in the \(N = 1\) case [12]: we integrate by parts the \(\nabla\) operators acting on some propagator to reduce it to a bare \(\delta^8(\theta_{ij})\). This can be integrated over \(\theta_i\) or \(\theta_j\) to bring the vertices \(i\) and \(j\) to the same point in \(\theta\)-space. Analogously to the \(N = 1\) formalism, we reduce

\[
\delta^8(\theta_{12}) \nabla_1^4 \nabla_2^4 \delta^8(\theta_{12}) = (\zeta_1 - \zeta_2)^4 \delta^8(\theta_{12}).
\]  

(103)

Also any number of \(\nabla\)'s less than 8 acting on two \(\delta_{12}\) vanishes

\[
\delta^8(\theta_{12}) \nabla_2^n \nabla_1^4 \delta^8(\theta_{12}) = 0, \quad n \neq 4
\]  

(104)

and any number larger than 8 has to be reduced to \(\nabla_1^4 \nabla_{m..n}^4\) by using the anticommutation relations

\[
\{\nabla^\alpha_1, \nabla^\dot{\alpha}_2\} = i(\zeta_1 - \zeta_2) \partial^{\alpha\dot{\alpha}}.
\]  

(105)

Transfer rules and integration by parts are used until the \(\theta\) dependence has been simplified. The result of these algebraic manipulations is the integral of a function local in the Grassmann coordinates. If we then perform the contour integrals corresponding to each vertex and integrate on the Grassmann coordinates of the second supersymmetry, the result must be the same as that that obtained using the \(N = 1\) formalism for the \(\zeta\)-coefficient superfields in the hypermultiplet. This is guaranteed by our construction of the \(N = 2\) formalism, since the calculations on that superspace only amount to an integration by parts of the second supersymmetry spinorial derivatives.
If we apply this reduction process to the diagrams of the massless hypermultiplet self-interacting theory, the final amplitude is a function of the projective hypermultiplet integrated with the full superspace measure. This is the basis of an important nonrenormalization theorem: there cannot exist ultraviolet corrections proportional to the original action \( \int d^4 \theta \eta^n \), which is integrated with a restricted superspace measure.

There might conceivably be infrared divergent corrections analogous to the \( N = 1 \) infrared divergences contributing to the superpotential

\[
\int d^4 \theta \phi^{n-1} \frac{D^2}{\Box} \phi = \int d^2 \theta \phi^{n-1} \frac{D^2}{\Box} \phi = \int d^2 \theta \phi^n .
\] (106)

For example, in the \( N = 2 \) case infrared divergent corrections to the \( O(4) \) multiplet kinetic action would be of the form

\[
\int d^8 \theta \iint d\zeta d\bar{\zeta} \frac{\eta \circ \eta}{2\pi i \zeta} = \int d^8 \theta \iint d\zeta d\bar{\zeta} \eta \circ \eta \circ \eta = \int d^8 \theta \iint d\zeta d\bar{\zeta} \eta \circ \eta \circ \eta = \int d^8 \theta \iint d\zeta d\bar{\zeta} \eta \circ \eta \circ \eta .
\] (107)

where the operator \( O \) satisfying this identity is

\[
O = \frac{1}{\zeta^2} \iint d\zeta_0 \frac{\nabla^4}{2\pi i \zeta} \delta_0 (0) \zeta_0 (0) \eta (\zeta_0 (0)) \eta (\zeta_0 (0)) \zeta_1 (0) .
\] (108)

It seems unlikely that we can produce such corrections, because the external \( \eta \) fields of \( n \)-point functions are evaluated at different positions in \( \zeta \)-space

\[
\langle \eta_1 \ldots \eta_n \rangle = \iint d\zeta_1 \ldots d\zeta_n \frac{\eta (\zeta_1) \ldots \eta (\zeta_n)}{2\pi i \zeta_1 \ldots 2\pi i \zeta_n} f (\zeta_1, \ldots, \zeta_n) .
\] (109)

To obtain a correction to the two point function which is local in \( \zeta \), we need a \( \delta \)-function in the complex plane as a result of the loop diagrams

\[
\iint d\zeta_2 \frac{\delta (0)}{2\pi i \zeta} \delta (0) (\zeta_1, \zeta_2) \eta (\zeta_1) \eta (\zeta_2) = \eta^2 (\zeta_1) .
\] (110)

From the propagators we obtain delta functions \( \delta (0) (\zeta_1, \zeta_2) \) and from reducing \( \delta \)-functions as in (103) we obtain factors \( (\zeta_1 - \zeta_2)^2 \). Using anticommutation relations (105) when there are more than eight \( \nabla \) operators acting on a \( \delta \)-function we also get factors \( (\zeta_1 - \zeta_2) \). With such factors it is not possible to obtain the proposed delta function, so it seems that no infrared corrections to the original action are possible. This arguments are also valid for the finite \( O(2p) \) multiplets, but for the arctic multiplets a more careful analysis is needed.

An interesting observation is that tadpole diagrams proportional to \( \eta \) and produced by a point-splitted three point vertex for example (or seagull diagrams coming from a four point vertex)

\[
\int d^4 \theta_1 \int d^8 \theta_2 \delta (0) (\theta_2 - \theta_1) \iint d\zeta_1 d\zeta_2 \delta (0) (\zeta_1, \zeta_2) \eta (\zeta_1) \eta (\zeta_2) = \eta^2 (\zeta_1) .
\] (111)

vanish upon performing the contour integral of the reduced propagator.

\( ^7 \)After having completed this manuscript we became aware of related work on [14].
\[
\int d^8 \theta_2 \oint \frac{d \zeta_2}{2 \pi i \zeta_2} \delta^{(2p)}(\zeta_1, \zeta_2) \sum_{n=0}^{2p-2} \zeta_1^n \zeta_2^{2p-2-n} \left( \delta^8(\theta_1 - \theta_2) \frac{\nabla^4 \nabla_2^4}{(\zeta_1 - \zeta_2)^2} \delta^8(\theta_2 - \theta_1) \right)
\]

\[
= \oint \frac{d \zeta_2}{2 \pi i \zeta_2} \delta^{(2p)}(\zeta_1, \zeta_2) \sum_{n=0}^{2p-2} \zeta_1^n \zeta_2^{2p-2-n} \left( \delta^8(\theta_1 - \theta_2) \frac{\nabla^4 \nabla_2^4}{(\zeta_1 - \zeta_2)^2} \delta^8(\theta_2 - \theta_1) \right)
\]

\begin{align*}
\text{(112)}
\end{align*}

This happens independently of the vanishing of the momentum loop integral in dimensional regularization. Thus the formalism automatically implements the absence of quadratic divergences in \( N = 2 \) supersymmetry.

The most striking novelty we find in the diagrams of complex multiplets corresponds to the additional pole in the convergent limit \( (12) \) of the (ant)arctic multiplet propagator. Integrating the complex coordinates of two vertices connected by a complex propagator, involves resolving an ambiguity that arises in the complex integration. That is because expanding the exponential of the interaction Lagrangian in the path integral to \( n \)th order, we find \( n \) complex integrations around the same complex contour. Since the contours are completely overlapping, we do not know if the additional pole of the (ant)arctic propagator is inside or outside the contour

\[
\oint_C \frac{d \zeta_1}{(2 \pi i) \zeta_1} \oint_C \frac{d \zeta_2}{(2 \pi i) \zeta_2} f(\zeta_1, \zeta_2) \frac{1}{\zeta_1 - \zeta_2}.
\]

\begin{align*}
\text{(113)}
\end{align*}

We can give a prescription to integrate on \( n \) infinitesimally separated and concentric contours, so that the arctic multiplet is always connected to the antarctic one of the next surrounding contour. The convergent limit of geometric series in the arctic-antarctic propagator is then justified for a tree level diagram. However if we try to construct a 1-loop diagram connecting the vertices of maximum and minimum contours through their respective arctic and antarctic multiplets, the propagator will have a divergent geometric series. A prescription to compute this apparently ill-defined expression will be presented in a future publication \( [6] \).

The alternative is to use the generic form of the propagator \( (12) \) where the geometric series has not been replaced by the convergent limit. We perform the diagram algebra and contour integrals using an \( O(k) \) propagator, and only at the end of the process we take the limit \( k \to \infty \). It is completely straightforward and well defined. It gives the same result as that obtained with the prescription mentioned before, up to an overall factor of two. This is easy to understand because in the complex \( O(k) \) propagator we have twice as many physical degrees of freedom as in the (ant)arctic multiplet.

### 6 Duality between the real \( O(2p) \) and (ant)arctic multiplet

As we have mentioned, the action for the real \( O(2p) \) multiplet and the action for a real combination of the arctic and antarctic multiplets can be made dual to each other. For \( p = 1 \) the duality between the tensor multiplet and the (ant)arctic hypermultiplet exchanges a physical real \( N = 1 \) linear superfield by a complex \( N = 1 \) chiral field and its conjugate,
and a physical chiral field by a complex linear one. For \( p > 1 \) the duality relates two off-shell descriptions of the hypermultiplet, in which only auxiliary fields are exchanged (see the analogous comment for the \( N = 1 \) duality after eq. (27)). Including self-interactions, in the case \( p > 1 \) the duality can be used to give different descriptions of the same \( \sigma \)-model, though there may be problems defining the correct contour of integration for the (ant)arctic multiplet \( \sigma \)-model.

The duality is manifest in the following parent action

\[
\int dx d^4 \theta \oint d\zeta \frac{d^2}{2\pi i \zeta} \left\{ \frac{1}{2} (-)^p X^2 + (\zeta^{p-1} \Upsilon + \frac{1}{\zeta} \bar{\Upsilon}) X + X \left( \frac{\nabla^4}{\zeta^2} J + \frac{\nabla^4}{\bar{\zeta}^2} \bar{J} \right) \right\} \tag{114}
\]

where \( \Upsilon \) and \( \nabla^4 J \) are arctic multiplets, while \( X \) is a tropical multiplet.

In the path integral of the theory, we can integrate out \( \Upsilon \) and \( \bar{\Upsilon} \). Performing the contour integral we obtain the following \( N = 1 \) constraints for the coefficient superfields:

\[
D_\alpha X_{-p} = 0 \quad D^2 X_{-p+1} = 0
\]

\[
\bar{D}_\dot{\alpha} X_p = 0 \quad \bar{D}^2 X_{p-1} = 0
\]

\[
X_n = 0, \forall |n| > p \tag{115}
\]

thus reproducing the real \( O(2p) \) multiplet free action for \( X \to \eta/\zeta^p \). As before, the source action gives \( N = 1 \) unconstrained sources coupled to the nonzero coefficient superfields of \( \eta \).

On the other hand, we can integrate out the real superfield \( X \) by completing squares on it. Using the real tropical multiplet and (ant)arctic multiplet prepotentials to write the action (114) with the full \( N = 2 \) superspace measure, the result is

\[
\int dx d^8 \theta \oint d\zeta \frac{d^2}{2\pi i \zeta} \left\{ \frac{(-)^p -1}{2} \left( \zeta^{p-1} \psi + \frac{\bar{\psi}(-)^{p-1}}{\zeta^4\zeta^{p-1}} + J + \bar{J} \right) \right\} \zeta^2 \nabla^4 \left( \zeta^{p-1} \psi + \frac{\bar{\psi}(-)^{p-1}}{\zeta^4\zeta^{p-1}} + J + \bar{J} \right).
\]

The contour integration selects the kinetic action of the (ant)arctic multiplet

\[
\int dx d^8 \theta \oint d\zeta \frac{d^2}{2\pi i \zeta} \psi \frac{\nabla^4}{\zeta^2} \psi = \int dx D^2 D^2 \oint d\zeta \frac{d^2}{2\pi i \zeta} \Upsilon \bar{\Upsilon}, \tag{116}
\]

a source term

\[
\int dx d^8 \theta \oint d\zeta \frac{d^2}{2\pi i \zeta} \Upsilon \bar{J}(\zeta) + \bar{J}(\zeta) \Upsilon; \quad \mathcal{J} = \oint \frac{d\zeta'}{2\pi i \zeta'} \delta^{(+\infty)}(\zeta, \zeta') \frac{J(\zeta') + \bar{J}(\zeta')}{\zeta^{p-1}}, \tag{117}
\]

\[
\bar{\mathcal{J}} = \oint \frac{d\zeta'}{2\pi i \zeta'} \delta^{(-\infty)}(\zeta, \zeta') \frac{(-\zeta')^{p-1} \left( \bar{J}(\zeta') + J(\zeta') \right)}{\zeta^{p-1}},
\]

and a term quadratic in sources

\[
\int dx d^8 \theta \oint d\zeta \frac{d^2}{2\pi i \zeta} (-)^{p-1} \frac{1}{2} (J + \bar{J}) \frac{\nabla^4}{\zeta^2} (J + \bar{J}). \tag{118}
\]
To find the path integral of this theory we complete squares on the (ant)arctic multiplet prepotential and integrate it out. The resulting path integral contains the term quadratic in sources, plus our expression (119) with an additional factor \((-\zeta/\zeta')^{p-1}\), and the arctic and antarctic sources replaced by a real tropical source. Inserting the \(N = 2\) projective superspace projector and the Riemann sphere delta distribution in the term quadratic in sources, we can also write it as a double complex integral

\[
\ln Z_0[J + \bar{J}] = \int dx d\theta \int \frac{d\zeta}{2\pi i \zeta} \int \frac{d\zeta'}{2\pi i \zeta'} \left[ (\bar{J}(\zeta) + J(\zeta)) \zeta^{p-1} \delta(+)\delta_{(0)}(\zeta, \zeta') \frac{\nabla^4(\zeta) \nabla^4(\zeta')}{\Box \zeta^2 (\zeta - \zeta')^2} \frac{J(\zeta') + J(\zeta')}{(-\zeta')^{p-1}} \right] + \frac{(-)^{p-1}}{2} (J(\zeta) + \bar{J}(\zeta)) \frac{\nabla^4(\zeta) \nabla^4(\zeta')}{\zeta^2 (\zeta - \zeta')^2} \left( J(\zeta') + \bar{J}(\zeta') \right) \delta(+)\delta_{(-\infty)}(\zeta, \zeta').
\]  

We can use this result to find a relation between the propagator of the real \(O(2p)\) multiplet and that of the (ant)arctic multiplet. Considering now the full path integral of the dual theory, we differentiate with respect the real source \(J + \bar{J}\)

\[
\langle X(1)X(2) \rangle = \left\{ \frac{\delta}{\delta J(1) + \bar{J}(1)} \right\} \left\{ \frac{\delta}{\delta J(2) + \bar{J}(2)} \right\} \langle \hat{Y}(1)\hat{Y}(2) \rangle + \left\{ \frac{-\zeta_1}{\zeta_2} \right\}^{p-1} \langle \hat{Y}(1)\hat{Y}(2) \rangle + \left\{ \frac{-\zeta_2}{\zeta_1} \right\}^{p-1} \langle \hat{Y}(1)\hat{Y}(2) \rangle 
\]

\[
\langle X(1)X(2) \rangle = \left\{ \frac{-\zeta_1}{\zeta_2} \right\}^{p-1} \left\{ -\delta(1-p)\delta_{(-\infty)}(\zeta_1, \zeta_2) - \delta(1-p)\delta_{(+\infty)}(\zeta_1, \zeta_2) + \delta(1-p)\delta_{(-\infty)}(\zeta_1, \zeta_2) \right\} + \frac{\nabla^4_1 \nabla^4_2}{(\zeta_1 - \zeta_2)^2} \delta^8(\theta_1 - \theta_2) \delta(x_1 - x_2) \]

Substituting the form of the (ant)arctic propagator in (121) and manipulating the Riemann sphere delta functions we obtain indeed \(\langle \eta(1)\eta(2) \rangle / \zeta_1^{p-1} \zeta_2^{p-1}\)

\[
\langle X(1)X(2) \rangle = \left\{ \frac{-\zeta_1}{\zeta_2} \right\}^{p-1} \left\{ -\delta(1-p)\delta_{(-\infty)}(\zeta_1, \zeta_2) - \delta(1-p)\delta_{(+\infty)}(\zeta_1, \zeta_2) + \delta(1-p)\delta_{(-\infty)}(\zeta_1, \zeta_2) \right\} \times \frac{\nabla^4_1 \nabla^4_2}{(\zeta_1 - \zeta_2)^2} \delta^8(\theta_1 - \theta_2) \delta(x_1 - x_2) \]

where equating the first line to the last one is understood to apply when we integrate on the Riemann sphere with the real measure \(\oint d\zeta / 2\pi i \zeta\). Thus duality gives the correct relation between the propagators.

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