Simplified Model and Response Analysis for Crankshaft of Air Compressor

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Abstract. The original model of crankshaft is simplified to the appropriateness to balance the calculation precision and calculation speed, and then the finite element method is used to analyse the vibration response of the structure. In order to study the simplification and stress concentration for crankshaft of air compressor, this paper compares calculative mode frequency and experimental mode frequency of the air compressor crankshaft before and after the simplification, the vibration response of reference point constraint conditions is calculated by using the simplified model, and the stress distribution of the original model is calculated. The results show that the error between calculative mode frequency and experimental mode frequency is controlled in less than 7%, the constraint will change the model density of the system, the position between the crank arm and the shaft appeared stress concentration, so the part of the crankshaft should be treated in the process of manufacture.

1. Introduction

CAD-CAE-CAM is a complete set of industrial production process. The finite element method in CAE has strong ability of analysis and calculation, and is widely used in different field. CAE analysis needs preprocessing the original model established by CAD. The progress mainly includes assigning material attributes, meshing, loading stress and so on, then converts it to finite element model. Many details are included in original CAD model. When the detail size is less than the finite element mesh size, dense grid or even meshes error is generated. Then contribute to long calculation time or even calculation error halfway. Therefore, it is necessary to simplify the geometric features of the original model under the premise of guaranteeing original CAD model characteristics. The main function of CAE is to realize the dynamic design of structure and to verify the rationality of design. Modal analysis is an important part of structural dynamic design, and it is the basis of vibration and noise prediction. Combining the theoretical modal analysis method with the experimental modal analysis method can solve the problems related to the dynamic design of complex structures to a great extent.

In this paper, an air compressor crankshaft is taken as the object of study. Firstly, the frequencies and shapes for original modal of the crankshaft between 0~2000 Hz are calculated by the finite element model, and compared with the experimental modal at the same time. Then, the CAD model is simplified and the mesh is retiling according to the previous dimensions. Natural frequency variation of the crankshaft in simplified process is compared. The response of the crankshaft reference point under certain constraint conditions is investigated, and the stress distribution of the original model is analysed.
2. Modal analysis

2.1. Vibration equation

The differential equation of crankshaft vibration is described as

\[
[M][\ddot{X}]+[C][\dot{X}]+[K][X] = [F]
\]

(1)

where \([M]\), \([C]\) and \([K]\) are the mass matrix, damping matrix and stiffness matrix respectively, \([\ddot{X}]\), \([\dot{X}]\) and \([X]\) are the acceleration matrix, velocity matrix and displacement matrix respectively, \([F]\) is the load matrix.

The influence of damping \([C]\) on modal shapes and modal frequencies can be neglected. In a free vibration status, we ignore the external fore in equation (1), namely \([F] = 0\). So the equation (1) can be written as

\[
[M][\ddot{X}]+[K][X] = 0
\]

(2)

When the structure at free vibration status is under natural frequency, substituting \([X] = [A] \sin(\omega t + \phi)\), we can get the basic equation of the modal analysis without damping as

\[
([K]-\omega^2[M])[A] = 0
\]

(3)

When the equation has nonzero solution, the condition \(\|([K]-\omega^2[M])\| = 0\) must be satisfied. Characteristic root \(\omega_i\) is the natural frequency of the structure, characteristic vector \([A_i]\) is the corresponding mode. Because of the irregular shape of the crankshaft, it is difficult to solve the differential equation. Therefore, the finite element method is used to solve the problem, and the experiment is used to verify it.

2.2. Original model modality

The air compressor is four stage compression, four connecting rods are connected side by side on a crank pin, the crankshaft axis is perpendicular to the ground, and the four cylinders are 90 degrees around the crank pin. The free suspension diagram of the crankshaft is shown in figure 1. The knocking point and the layout of the accelerometer is shown in figure 2. The modal test adopts the method of multi point knocking and single point response. The red arrow is the direction of sensor installation, the black small hammer indicates hammer point and direction, and knocking 27 points altogether. According to the data provided by the manufacturer, the density is \(37820 \text{ Kg/m}^3\), the elastic modulus is \(2.1 \times 10^{11} \text{ N/m}^2\), and the Poisson's ratio is 0.266. The mesh is divided into tetrahedral units, and the grid is refined at the corners and round holes. Analysis bandwidth is 0-2000Hz, and the mode shape contrast is shown in figure 3. The bending direction of the vibration mode is right, the error of first order frequency is 2.5%, and the error of two order frequency is 3.1%. Because of the influence of casting process, and residual oil in the oil hole, the error is less than 5%. Basically, it can meet the requirement of calculation.
2.3. Model simplification

In order to facilitate mesh division and save computer resources, we should simplify the model in the process of finite element analysis. To discuss the effect of simplification on modal frequencies, the simplified procedure of the model is shown in figure 4. The finite element modal analysis of the three cases is carried out, and the results are shown in table 1. The error of first order natural frequency between finite element model and the experimental is 6.6%, and the error of second order natural frequency between finite element model and the experimental is 5.6%. It can meet the computational requirements basically. For the simplified finite element, the higher order natural frequencies can be controlled less than 7% compared with the original finite element model.
| Table 1. Modal frequency under different degree of simplification (Hz). |
|-------------------------------------------------|
|                     | Order 1 | Order 2 | Order 3 | Order 4 | Order 5 | Order 6 |
| Original model      | 1322    | 1442    | 2198    | 2852    | 3211    | 4443    |
| Delete hole and     | 1321    | 1461    | 2205    | 2904    | 3263    | 4499    |
| chamfer model       |         |         |         |         |         |         |
| Modify peripheral   | 1266    | 1405    | 2081    | 2770    | 3142    | 4364    |
| detail model        |         |         |         |         |         |         |

3. Harmonic response analysis
Constraint and load addition is shown in figure 5, the short end of the shaft neck is fixed, the semi-circular surface connected to the coupling on the other end is applied 1Pa pressure, the direction point to the crankshaft axis, and the crank end labeled as a response point. The sine sweep frequency is set between 0~5000Hz, and the load step is set to 500, then the frequency response of the reference point is shown in figure 6. Each peak of the frequency correspond is the modal frequency under the constraint condition, which is understood as the installation state for the moment. It is obvious that the modal frequencies between 0Hz and 2000Hz are two more than those of free modes and the constraint condition will change the modal density. When the excitation frequency of the system is very low, the response is close to the static displacement under constant force, and belongs to the stiffness control zone. As the excitation frequency increases, the response frequency is shifted from one modal frequency to another.

4. Stress analysis
According to the corresponding formula for the exhaust pressure of the air compressor, the piston force variation of the four cylinders with the crank angle is obtained, as is shown in figure 7. The order of the gas pressure magnitude is $10^4$. With the two cylindrical surfaces of the crankshaft journal completely fixed, take the force is $10^3$, divided by the semi cylindrical area of the crank pin, then the pressure is acquired as $5.7 \times 10^3 \, Pa$, loaded on the crank pin, then get the stress distribution shown in figure 8. Generally, the stress concentration is easy to occur at the two sides of the crank arm. It can be seen that the stress concentration occurs in the part between the crank arm and the shaft neck, but the stress concentration is not very obvious besides crank pin. During the running of the crankshaft, cracks are easy to occur in the stress concentration area and rolling or nitriding can be carried out on the stress concentration part in the process of manufacture, and the transition fillet can also be increased.
Figure 7. Piston force of each stage.

Figure 8. Stress distribution of the crankshaft.

5. Conclusion
(1) Original finite element modal calculation results are closest to the experiment, but the calculation efficiency is the lowest. Simplified model greatly improve calculation efficiency, although the error is about two times of the original finite element model, compared with the experimental modal, the error can be controlled within 7%.
(2) Certain constraint conditions will change the modal density of the system, resulting in an increase in the modal density in some frequency bands.
(3) The stress analysis of the crankshaft shows that the stress concentration occurs at the place where the crank arm and the shaft neck are connected. It can roll, nitriding or increase the transition fillet in the stress concentration of the crankshaft during the manufacture.

6. Reference
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