Incommensurate spin resonance in URu$_2$Si$_2$.

A. V. Balatsky, A. Chantis, Hari P. Dahal, David Parker, and J.X Zhu

1Theoretical Division and Center for Integrated Nanotechnologies, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
2Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
3U.S. Naval Research Laboratory, 4555 Overlook Ave. SW, Washington, DC 20375, USA

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The nature of the hidden order (HO) in URu$_2$Si$_2$ below $T_{HO} = 17.5K$ has been a puzzle for a long time. Neutron scattering studies of this material reveal a rich spin dynamics. We focus on inelastic neutron scattering in URu$_2$Si$_2$ and argue that observed gap in the fermion spectrum naturally leads to the spin feature observed at energies $\omega_{res} = 4 - 6meV$ at momenta $Q^* = (1 \pm 0.4, 0, 0)$. We discuss how spin features seen in URu$_2$Si$_2$ can indeed be thought of in terms of spin resonance that develops in HO state and is not related to superconducting transition at 1.5K. In our analysis we assume that the HO gap is due to a particle-hole condensate that connects nested parts of the Fermi surface with nesting vector $Q^*$. Within this approach we can predicted the behavior of the spin susceptibility at $Q^*$ and find it to be is strikingly similar to the phenomenology of resonance peaks in high-$T_c$ and heavy fermion superconductors. The energy of the resonance peak scales with $T_{HO} \sim 4kB T_{HO}$. We discuss observable consequences spin resonance will have on neutron scattering and local density of states. Moreover, we argue how establishment of spin resonance in URu$_2$Si$_2$ and better characterization of susceptibility, temperature, pressure and Rh doping dependence would elucidate the nature of the HO.

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I. INTRODUCTION

The problem of the nature of the hidden order below $T_{HO} = 17K$ and the superconducting order below $T_c = 1.5K$ in URu$_2$Si$_2$ has perplexed the condensed matter physics community for over two decades.\cite{1}

The heavy-fermion (HF) superconductor URu$_2$Si$_2$ exhibits an order of an unknown origin which sets in at $T_{HO} = 17.5K$. Thermodynamic measurements revealed a rather large jump of approximately 300 mJ/mol-K$^2$ in the linear specific heat coefficient $\gamma$ at 17.5 K. This material contains a linear specific heat coefficient $\gamma$ measured at 70-180 mJ/mol-K$^2$, placing it as a moderately HF material. Below $T_{HO}$, the specific heat follows an exponentially activated behavior $\exp(-\Delta/T)$ with $\Delta$ estimated at 148 K. This gap also appears in optical measurements and vacuum tunneling and is comparable to that observed in inelastic neutron scattering experiments. Anomalies in the DC resistivity, Hall coefficient, thermal expansion and linear and nonlinear susceptibilities are also seen at $T_{HO}$, suggesting a substantial reordering of the conduction electrons. Neutron-scattering experiments found an antiferromagnetic order below 17.5 K but with a staggered magnetization of only 0.03 $\mu_B$ per U atom, which is far too small to account for the observed specific heat anomaly. This anomaly correspond to an unobserved order which is therefore termed ”hidden”. Yet there are physical fields that clearly destroy hidden order. It is believed to be destroyed by an applied magnetic field of $\sim 40$ T, suggesting a possible magnetic origin, but it was shown that there are two distinct field-independent energy scales, with opposite tendencies with magnetic field. Therefore, any magnetic origin of this order must not couple directly to field in the same manner as the small AFM order. The application of pressure\cite{2} and Rh doping also suppress the HO.\cite{3}

Along with the determination of the experimental facts, there have been many theoretical attempts to understand this hidden order. Theories proposed include spin density waves of either unconventional or higher angular momentum character,\cite{4} orbital antiferromagnetism,\cite{5} staggered quadrupolar order,\cite{6} and Jahn-Teller distortions,\cite{7} multispin correlated order,\cite{8} AFM states with anomalous g factors,\cite{9} valence admixture,\cite{10} octupole order,\cite{11} and helicity order.\cite{12} Determining the hidden order is complicated by possible phase separation into a magnetic moment phase and regions of hidden order, as argued by.\cite{13} To date no theory has shown conclusive agreement with the above experimental facts, and there exists no consensus as to the origin of the hidden order.

Recently, Wiebe et al conducted an inelastic neutron scattering (INS) study of URu$_2$Si$_2$, in conjunction with specific-heat measurements above and below the 17.5 K onset temperature. Wiebe et al found that above the ordering temperature $T_{HO}$, gapless (with velocity $\sim v_F$) spin wave excitations centered on incommensurate wavevectors $Q^* = (1 \pm 0.4, 0, 0)$ appeared. But below this temperature these excitations were gapped, with an approximate gap at 1.5 K of 4-6 meV. Wiebe also estimated the specific heat coefficient of these gapless excitations and found a fair agreement with the experimental value. It was concluded that the reduction in specific heat below $T_{HO}$ resulted from the gapping of these spin-wave excitations; however, the order parameter responsible for this gapping remained indeterminate.
The effect of opening a HO gap on spin excitations appears remarkably similar to the phenomenon of spin resonance in INS, seen in the superconducting state in cuprate materials and in the CeCoIn superconductor. For example, in the cuprates this resonance in the susceptibility $\chi(\mathbf{q}, \omega)$ is centered at the commensurate wavevector $\mathbf{q} = (\pi, \pi)$, and can be interpreted as a bosonic mode transferring $\mathbf{q} = (\pi, \pi)$ from the neutron to the Cooper pair. For completeness of the discussion we also point the case of $\text{Sr}_2\text{RuO}_4$ where resonance was predicted but not observed to date. One might therefore expect that a similar effect of gapping on the spin excitations can occur in a state with hidden order, even if the exact nature of HO is not yet settled.

In this paper we propose that $\text{URu}_2\text{Si}_2$ should exhibit an incommensurate spin resonance based on an analogy with the inelastic neutron scattering resonance observed at 41 meV in the cuprates. We argue that:

1) The observation by Wiebe et al. of the substantial changes of spin susceptibility below and above $T_{HO}$ at an incommensurate momentum is indicative of the gapping of spin excitations due to the gapping of the electronic spectrum below $T_{HO}$. We have developed a microscopic theory of the spin susceptibility, outlined below. This theory is based on the estimate of changes in susceptibility due to gap in fermionic spectrum. The gap in spin susceptibility we estimate to be twice as large as a gap in single spin excitation as explained below. As a result we estimate $\omega_{res} \simeq 4kbT_{HO}$ opening that allows us to estimate the energy of the spin resonance to be in the range $\omega_{res} = 4 - 6$ meV and the momentum to be $\mathbf{Q}^* = (1 \pm 0.4, 0, 0)$. Changes in spin susceptibility due to the HO gap $\Delta_{Q^*}$ will naturally change spin excitation spectrum. Given the mean field character of the HO gap opening as seen in the specific heat data, we expect that the intensity of the resonance scales as $|\Delta_{Q^*}|^2 \sim (T_{HO} - T)$ below $T_{HO}$.

2) Multiple orders were proposed as an explanation of HO. We argue that the experimental observations are consistent with a specific particle-hole order that has a finite incommensurate momentum $\mathbf{Q}^* = (1 \pm 0.4, 0, 0)$ (and related by $k_z \leftrightarrow k_y$ permutation) and leads to a gap in the spectrum $\Delta_{Q^*}$. The exact nature of this hidden order is likely be a hybridization gap $\Delta_{Q^*}$ that opens up due to the nesting of different parts of Fermi surface separated by $\mathbf{Q}^*$. For our analysis of the spin susceptibility we focus on terms of second order in $\Delta_{Q^*}$ that would contribute to the spin susceptibility and therefore we do not need to know the exact details of the HO. Nevertheless our conclusion is that the data on INS and specific heat are consistent with the particle hole excitation being gapped below $T_{HO}$. Recent neutron scattering work by Janik et al. and theory proposal by Oppeneer does point to the nesting phenomenon as a possible source of HO and is consistent with our proposal.

3) The HO leads to spectral weight changes that produce a peak in the spin susceptibility which we call a spin resonance with energy $\omega_{res} = 4 - 6$ meV at momentum $\mathbf{Q}^*$. In the previous cases where a resonance peak has been seen in the ordered state, opening up a partial gap at the Fermi surface, this resonance peak has been observed at commensurate momenta. We point that complicated spin dynamics that is affected by the HO, in addition to already established spin gapping, should exhibit a phenomena of spin resonance peak in $\text{URu}_2\text{Si}_2$. Main difference with respect to previous discussion on spin resonance is that this resonance occurs at the incommensurate momentum $\mathbf{Q}^*$ in the nonsuperconducting state.

To support our claim about fermion spectrum gapping, we will provide fits to the specific heat based on a mean field gap in the spectrum with the ratio $\Delta_{Q^*}/k_bT_c = 2.5$ that give a reasonably good fit to the data. We also address the density of states that can be measured by a scanning probe as another observable that might reveal the existence of an energy feature at $\omega_{res}$.

We present arguments that naturally lead to the prediction of the spin resonance in $\text{URu}_2\text{Si}_2$ in Sec II. Then we discuss observables such as the specific heat and the local density of states due to this resonance in Sec III. We conclude with a discussion section.

II. SPIN RESONANCE IN $\text{URu}_2\text{Si}_2$

In the cuprates, the resonance in the susceptibility $\chi(\mathbf{q}, \omega)$ is centered at the commensurate wavevector $\mathbf{q} = (\pi, \pi)$, and can be interpreted as a bosonic mode transferring $\mathbf{q} = (\pi, \pi)$ from the neutron to the Cooper pair. The energy of this resonance is independent of temperature, while its intensity depends strongly on temperature and vanishes at $T_c$. Within the SO(5) theory linking superconductivity and magnetism in the cuprates, an excitation bearing these properties can arise naturally in the particle-particle superconducting channel, and leads to a resonant susceptibility $\chi((\mathbf{q} = \pi, \pi, \omega) \propto \Delta^2/\omega - \omega_{res}^2 + i\Gamma)$, where $\Delta$ is the superconducting order parameter and $\Gamma$ is a damping constant. This resonance peak appears only below $T_c$ because it is only below this temperature that the mixing of electrons and holes that occurs in the superconducting state allows coupling of magnetic excitations via particle-hole and particle-particle channel coupling. In the cuprates, this interaction is active within the superconducting particle-particle channel, but as we shall see it can be extended under suitable conditions to the particle-hole channel, leading to a similar result. In this case, however, the resonance occurs at an incommensurate wavevector, putting constraints on the origin of this resonance.

In a more recently investigated case of CeCoIn, a similar resonance is seen at $(\pi, \pi, \pi)$ and has been interpreted as evidence for d-wave symmetry. On the other hand, a spin resonance has been observed in the p-nitride superconductor $\text{Ba}_{0.6}\text{K}_{0.4}\text{Fe}_2\text{As}_2$ where the pairing symmetry could be different.

We point out here that conflicting opinions on the
Green’s functions that capture the appearance of incommensurate response of incommensurate magnets, also have been discussed\textsuperscript{22,23}. The relevance for the present discussion is that we do not see a need to have a superconducting reference state as a prerequisite for spin resonance. The gapping of the spectrum is essential but the gap does not have to be superconducting. This is an important difference we stress: most of the cases of spin resonance were discussed with regards to superconductors. We do not imply here that URu$_2$Si$_2$ has superconducting correlations in the HO phase.

A. Spin Susceptibility

The suggestions of the previous section quickly lead to another option for connecting the formation of the hidden order with the spin dynamics. We propose a relatively simple explanation, consistent with the spin/hidden order coexistence, namely that a resonance peak in the susceptibility $\chi(Q^*, \omega = \omega_{\text{res}})$ appears as a result of the appearance of a particle-hole condensate, although more complex than the usual density-wave condensate. In particular, we argue that the Fermi surface geometry, as depicted in Figure 1, is such as to allow an incommensurate nesting between the central $\Gamma$ Fermi surface pocket and the pocket, separated by $Q^*$. This nesting is not complete and would require a strong interaction to produce an instability. This fact is in accord with our observation that we need to use a strong coupling version of mean field specific heat to fit specific heat data, see below.

We start with the calculation of spin-spin susceptibility, assuming that the particle hole ordering gaps the FS, which is nested with momentum $Q^*$. Assuming that the gap opens up below $T_{\text{HO}}$ we will argue that the change in susceptibility will have a term that is proportional to $\Delta_{Q^*}$. As such this second order correction will occur regardless of the detailed nature of the HO. Similar second order terms in the spin susceptibility for superconducting gap were argued for in earlier work\textsuperscript{18}.

We begin with the spin-spin susceptibility of itinerant electrons in $URu_2Si_2$ at $T=0$ signatures are seen in zz component $\chi^{zz}(Q^*, \omega) = i\langle TS^z(Q^*, t)S^z(-Q^*, 0) \rangle$:

$$
\chi^{zz}(Q^*, \omega) = i \sum_{kk'} \int \langle Tc_{k'Q^*, \sigma}^\dagger(t)c_{kQ^*, \sigma}(0)\sigma^z_{\alpha\beta}c_{kQ^*, \beta}(0) \rangle e^{i\omega t} dt
$$

$$
= -i \sum_{kk'} \int \langle Tc_{k'Q^*, \sigma}^\dagger(t)c_{kQ^*, \alpha}(0)\sigma^z_{\alpha\beta}c_{kQ^*, \beta}(0) \rangle e^{i\omega t} dt.
$$

To make the next step we introduce the anomalous Green’s functions that capture the appearance of incommensurate order at $Q^*$:

$$
F_{kQ^*}(\omega) = i \langle Tc_{kQ^*, \sigma}^\dagger(0)c_{kQ^*, \sigma}(t)\delta_{\alpha\sigma} \rangle = \frac{\Delta_{Q^*}}{\omega^2 - E_{kQ^*}^2 + i\delta},
$$

function $F$ describes particle hole density order that represents HO and is nonmagnetic. This order partially gaps excitations at the Fermi surface. $F$ relates regions of the Fermi surface that are connected by nesting vector $Q^*$. We choose it to have a typical mean-field form. As we have argued, with the focus on second order terms in $\Delta_{Q^*}$, the detailed structure of the propagators is not critical. The same conclusions can be drawn from Ginzburg-Landau theory for the HO state. Hereafter we will ignore smooth terms in the susceptibility. Then the susceptibility related to the appearance of anomalous order is

$$
\chi^{zz}(Q^*, \omega) = -i \sum_k \int F_{kQ^*}(\omega_1)F_{-k,-Q^*}(\omega + \omega_1) d\omega_1.\quad (3)
$$

the integral in $\chi^{zz}(Q^*, \omega)$ can be written as,

$$
\chi(Q^*, \omega) = \Delta_{Q^*}\Delta_{-Q^*} \sum_k \frac{1}{2E_{kQ^*}}(E_{kQ^*}^2 + \omega)^2 - E_{kQ^*}^2 \quad (4)
$$

$$
+ \Delta_{Q^*}\Delta_{-Q^*} \sum_k \frac{1}{2E_{kQ^*}}(-\omega + E_{kQ^*})^2 - E_{kQ^*}^2 \quad (5)
$$

$$
= \Delta_{Q^*}\Delta_{-Q^*} \sum_k \frac{1}{E_{kQ^*}}\frac{\omega^2}{\omega^2 - 4E_{kQ^*}^2} \quad (6)
$$

FIG. 1: A depiction of the calculated Fermi surface geometry of URu$_2$Si$_2$, taken from\textsuperscript{22}, with potential $Q^*$ nesting vectors indicated entered at the corner of the Brillouin zone. However, this nesting is between two bands of with the same spin, so that there is little or no magnetic signal and the order is "hidden".
We took (see below) \( N(E) = N(0) \frac{E}{\sqrt{E^2 - \Delta^2_{Q^*}}} \) as is appropriate for a gapped spectrum, then

\[
\chi^{zz}(Q^*, \omega) = |\Delta_{Q^*}|^2 \int \frac{1}{\sqrt{E^2 - \Delta^2_{Q^*}}} \frac{1}{\omega^2 - 4E^2} dE. \quad (7)
\]

Thus the susceptibility indeed acquires a term that scales quadratically with the HO gap. The details of the integral over energy in Eq.(7) depend on the band structure. For any density of states that is smooth, simple analysis shows that for \( \omega << \Delta_{Q^*} \), \( \chi^{zz}(Q^*, \omega) \propto |\Delta_{Q^*}|^2 \omega^2 \), and for \( \omega \Delta_{Q^*} \), \( \chi^{zz}(Q^*, \omega) \propto \frac{|\Delta_{Q^*}|^2}{\omega} \), with the crossover at \( \omega \sim |\Delta_{Q^*}| \). We therefore immediately conclude that there is a resonance contribution to spin susceptibility \( \sim \Delta^2_{Q^*} \), and that contribution will have a peak at \( \omega \sim \Delta_{Q^*} \).

Finally, we give an argument on why the spin resonance energy \( \omega_{res} = 4 - 6 \text{meV} \) should appear in INS below \( T_{HO} \), and the intensity of this peak should increase quasilinearly, as in prior work \cite{13},

\[
\delta \chi^{zz}(Q^*, \omega = \omega_{res}, T) \sim \Delta^2_{Q^*} \sim |T - T_{HO}| \quad (10)
\]

with decreasing temperature before saturating at low temperature \((< 0.6 T_{HO})\). This peak should be centered at the incommensurate wavevectors \((1 \pm 0.4, 0, 0)\),

\[
\delta \chi^{zz}(q, \omega = \omega_{res}, T \ll T_{HO}) \sim \frac{\Delta^2_{Q^*}}{|q - Q^*|^2 + \xi^2}. \quad (11)
\]

The energy, momentum and temperature dependence of the resonance peak is illustrated in Fig(2) with a width \( 1/\xi \) depending on the microscopic details of the theory. From Ref[27] we estimate \( \xi^{-1} \sim 0.1 \text{meV} \).

From this analysis we would expect both intensity and resonance energy be temperature dependent. At temperatures below \( T_{HO} \) resonance energy will evolve with temperature \( \omega_{res} \sim \Delta_{Q^*} \).

In the same region intensity of resonance peak with change as a function of temperature \( I(Q^*, \omega_{res}, T) \sim \Delta^3_{Q^*} \). Broholm et.al \cite{29} have shown that gap in the neutron scattering peak at \( Q^* \) does indeed depends on \( T - T_{HO} \) in a mean field manner. They also argued that intensity of neutron scattering feature does increase below \( T_{HO} \). Another mean to test dependence of resonance on HO gap is investigate effects of pressure or \( Rh \) doping.\cite{30,31}. Resonance peak energy would be suppressed with \( Rh \) doping. These estimates can be tested experimentally.

### III. EXPERIMENTAL CONSEQUENCES

We now focus on the experimental observables that can be used to test the prediction of a resonance peak in \( URu_2Si_2 \). We will consider neutron scattering and local density of states features. In addition we will address electronic specific heat features due to HO gap to illustrate that we can achieve a reasonable fit using a simple mean field description.

#### A. Inelastic Neutron Scattering

We expect a resonance peak with \( \omega_{res} = 4 - 6 \text{meV} \) should appear in INS below \( T_{HO} \), and the intensity of this peak should increase quasilinearly, as in prior work \cite{13},

\[
\delta \chi^{zz}(Q^*, \omega = \omega_{res}, T) \sim \Delta^2_{Q^*} \sim |T - T_{HO}| \quad (10)
\]

with decreasing temperature before saturating at low temperature \((< 0.6 T_{HO})\). This peak should be centered at the incommensurate wavevectors \((1 \pm 0.4, 0, 0)\),

\[
\delta \chi^{zz}(q, \omega = \omega_{res}, T \ll T_{HO}) \sim \frac{\Delta^2_{Q^*}}{|q - Q^*|^2 + \xi^2}. \quad (11)
\]

The energy, momentum and temperature dependence of the resonance peak is illustrated in Fig(2) with a width \( 1/\xi \) depending on the microscopic details of the theory. From Ref[27] we estimate \( \xi^{-1} \sim 0.1 \text{meV} \).

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#### B. Specific Heat

The gapping of fermions on part of the Fermi surface directly results in the loss of entropy observed below \( T_{HO} \), and we will demonstrate an excellent quantitative fit to the experimental specific heat data.

In Figure[2] we plot the specific heat data of Wiebe\cite{12}, and our fit, assuming a \( \Delta_{Q^*}/T_c \) ratio of 2.5 and a “strong-coupling” temperature dependence of \( \Delta_{Q^*}(T) \). We work by analogy with the BCS theory of superconductivity, which shares many of the same expressions with this gapping of the Fermi surface\cite{26,27}. In particular, the specific heat of the gapped portion of the system is given by

\[
C(T) = 2k_B \beta \sum_k f_k (1 - f_k) \left( E_k^2 + \beta \frac{d \Delta^2}{d \beta} \right) \quad (12)
\]

where \( E_k \) is the quasiparticle energy in the gapped state given by

\[
E_k = \sqrt{\epsilon_k^2 + \Delta^2_{Q^*}} \quad (13)
\]

\( \epsilon_k \) is the normal state dispersion, \( \beta = 1/k_B T \), and \( \alpha \) is the gapped fraction of the Fermi surface. The jump in
FIG. 2: (Color online) a) The spin susceptibility at the resonance momentum $Q^*$ and at the resonance energy $\omega_{\text{res}}$ is plotted as a function of temperature normalized to the hidden order transition temperature, $T_{HO}$. The temperature dependence is shown to be determined by the temperature dependence of the "hidden-order" order parameter. b) The intensity plot of spin susceptibility near $Q^*$ and $\omega_{\text{res}}$ is shown. It is clearly seen that the spectral weight of the susceptibility is transferred to the resonance momentum and the resonance energy. The spin susceptibility c) at the resonance energy as a function of the momentum and d) at the resonance momentum as a function of the energy are shown.

the specific heat at $T_{HO}$ is caused by the second term of the above equation. The effect of this term is enhanced both by the $\Delta Q^*(0)/T_{HO}$ value of 2.5 exceeding the BCS weak-coupling value of 1.76 and by the assumed "strong-coupling" form of $\Delta Q^*(T)$, in which the quasiparticle gap develops more rapidly below $T_{HO}$ than in standard BCS theory. Such a rapid gap opening is well-known from studies of the cuprates, and can occur due to the rapid suppression of bosonic excitations below $T_{HO}$. The gap still retains a square-root singularity at $T_{HO}$, and hence a mean-field, second-order phase transition at this temperature. Comparing the numbers from strong coupling theory with the data we see a reasonable agreement: $\Delta_{HO} \sim 4 - 6 \text{meV}$, $T_{HO} = 17K$ and $\Delta Q^*(0)/T_{HO} \sim 2.3$.

To the gapped specific heat must be added a term from the ungapped portion of the Fermi surface, given simply by $C_n = (1 - \alpha)\gamma T$, with $\gamma$ the Sommerfeld specific heat coefficient. For this calculation approximately 60 percent of the Fermi surface was assumed to be gapped.

We have not included in the calculation the effects of the phonon specific heat or of the apparently correlation-induced rise in $C/T$ at very low temperature; these effects have opposite temperature dependencies and are of comparable magnitude, so that the overall effect on the fit of neglecting these effects is expected to be small.

C. Local density of states

Here we present a simple qualitative argument that shows how the Local Density of States (LDOS) can be used to reveal the resonance peak. The energy of the resonance makes its observation relatively simple with a Scanning Tunneling Microscope (STM). The main feature that we focus on is the LDOS at the tunneling bias that reveals the energy gap in the electron spectrum in the range $2 - 4\text{meV}$. We begin by assuming a typical
ordered state self-energy

\[ \Sigma(k, \omega) = \frac{|\Delta_{Q^*}|^2}{\omega - \epsilon_{k+Q^*}} \]

which can then be combined with Dyson’s equation:

\[ G(\omega, k) = \frac{1}{\omega - \epsilon_k - \Sigma(k, \omega) + i\delta} \]

Solving for the poles of the Green’s function gives the quasiparticle dispersion relation as

\[ \omega = \frac{\epsilon_k + \epsilon_{k+Q^*} \pm \sqrt{(\epsilon_k - \epsilon_{k+Q^*})^2 + 4|\Delta_{Q^*}|^2}}{2} \]

which is the dispersion for a density-wave nested at \( Q^* \). In particular, if \( k \) and \( k + Q^* \) are on the gapped portion of the Fermi surface, we obtain a simple gapped spectrum

\[ \omega = \pm \Delta_{Q^*} \]

from the usual density-of-states relationship of a gapped spectrum,

\[ N(\omega) = N_0 \frac{\omega}{\sqrt{\omega^2 - |\Delta_{Q^*}|^2}} \]

Such a feature should be readily observable by low-temperature STM for \( E = 2-4 \text{ meV} \), although the effects of impurities and inhomogeneities will tend to broaden this peak.

### IV. RELATION TO SUPERCONDUCTORS THAT EXHIBIT RESONANCE PEAK

There is an interesting correspondence between the energy of the resonance peak in U\( Ru_2Si_2 \) and in superconductors. Assuming that our prediction about the temperature dependence and the energy of the resonance peak is supported by experiment, we expect the resonance energy to be in the range \( \omega_{res} = 4-6 \text{ meV} \) and it to occur below \( T_{HO} \).

The relation between energy and critical temperature for HO phase is remarkably similar to the relation between resonance energy and \( T_c \) for unconventional superconductors. For \( URu_2Si_2 \) HO state we find the ratio

\[ h\omega_{res} \approx 4k_BT_{HO} \]

that is very similar to superconducting relation:

\[ h\omega_{res} = 4k_BT_c \]

We do not know the specific reason for this close correspondence other than the general observation that a gapped spectrum could also produce suppressed spectral weight in the spin susceptibility.

Uemura noticed a universal scaling between resonance energy and critical temperature for unconventional superconductors like high-\( T_c \) cuprates, CeCoIn5, and in the pnictide superconductor \( B_{0.6}K_{0.4}Fe_2As_2 \). He proposed an analogy of resonance mode with rotons in superfluid \( ^4 \text{He} \) using a plot shown in Fig. 4. We note that datum for HO phase is remarkably close to this relation for superconductors and \( \text{He} \), as demonstrated by a new point for \( URu_2Si_2 \) added in Fig. 4. This analogy, while appealing, can only go up to a point, since HO state in \( URu_2Si_2 \) is non-superconducting and resonance feature is incommensurate.

### V. DISCUSSION AND CONCLUSION

In conclusion, we propose to search for the spin resonance in \( URu_2Si_2 \) at \( \omega_{res} = 4-6 \text{ meV} \) at the incommensurate wavevector \( Q^* = (1 \pm 0.4, 0, 0) \). We expect that this spin resonance will set in at temperatures below the HO transition and the intensity of this peak will scale as

\[ \Delta_{HO}^2 \sim (T_{HO} - T) \]
FIG. 4: (Color online) The relation between the resonance energy $\omega_{\text{res}}$ and $T_c$ is shown for a variety of superconductors in this Uemura roton plot. At a lower left corner we have added the point, indicated by arrows, that marks HO relation between expected resonance peak and $T_{HO}$. The graph for superconductors and superfluid He is taken from [30].

The resonance peak is known to occur in the states with superconducting gap and results in the gapping of the electronic spectrum [18,19,21]. In the case of HO the gap $\Delta_{HO}$ results in the partially gapped electron spectrum. That appears to be a sufficient condition, as shown by Wiebe et al. [17] to produce a gap in spin excitation spectrum.

There are few ways one can further experimentally test the predicted relation between $T_{HO}$ and resonance energy with temperature, impurity doping, pressure and with magnetic field. Resonance energy $\omega_{\text{res}} \sim \Delta_{Q^*}$ is a monotonic function of temperature and Rh doping. Similarly intensity of resonance peak in critical region will scale as $\Delta_{Q^*}^2$. Upon adding Rh and Th into $URu_2Si_2$ one can suppress HO and respective transition temperature and we would expect that the resonance energy $\omega_{\text{res}}$ would track $T_{HO}(x)$. Similarly one can measure changes in transition temperature and in resonance energy as a function of pressure and magnetic field, if this is feasible.

The resonance discussed here is, to the best of our knowledge, the first case where the spin resonance occurs at an incommensurate vector $Q^*$. The authors thank C. Batista, E. Bauer, C. Broholm, P. Chandra, P. Coleman, N. Curro, J.C. Davis, J. Floquet, M. Graf, G. Kotliar, J. Lashley G. Luke, P. Oppeneer, F. Ronning, J. Sarrao, Y.J. Uemura, and C. Wiebe for useful discussions. This work was supported by US DOE at Los Alamos. D.P. is grateful to LANL and the T4 group for hospitality.

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