FAST TRACK COMMUNICATION

Mechanical memory for photons with orbital angular momentum

H Shi and M Bhattacharya

School of Physics and Astronomy, Rochester Institute of Technology, 84 Lomb Memorial Drive, Rochester, NY 14623, USA
E-mail: mxbsps@rit.edu

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Abstract
We propose to use an acoustic surface wave as a memory for a photon carrying orbital angular momentum. We clarify the physical mechanism that enables the transfer of information, derive the angular momentum selection rule that must be obeyed in the process and show how to optimize the optoacoustic coupling. We theoretically demonstrate that high fidelities can be achieved, using realistic parameters, for the transfer of a coherent optical Laguerre–Gaussian state, associated with large angular momentum, to a mechanical shear mode. Our results add a significant possibility to the ongoing efforts towards the implementation of quantum information processing using photonic orbital angular momentum.

Significant developments in contemporary optics have resulted from the realization that photons can carry orbital angular momentum (OAM) encoded in the spatial distribution of the corresponding electromagnetic field [1, 2]. Distinct from the photon polarization, which can provide a maximum angular momentum of $\hbar$, the field profile can be engineered to endow the photon with virtually unlimited OAM, $l\hbar$. Recent experiments have produced up to $l = 300$ [3]. For high $l$, each photon can carry a large amount of information in a multi-dimensional Hilbert space. This capability makes photonic OAM states attractive for quantum information processing tasks [4–8], some of which are more efficiently [9], stably [10] or securely [11] accomplished in higher dimensional spaces.

In this communication, we propose to store OAM states in a simpler, more robust and readily scalable memory: acoustic modes on the surface of an optical mirror [33]. When the mirror is part of a Fabry–Perot cavity, the acoustic modes can phase-modulate an intracavity optical mode in a manner identical to a global displacement of the mirror. The optoacoustic interaction responsible for this phase modulation can be traced back to radiation pressure similar to the many optomechanical systems being realized currently [34–36].

Extending the analogy further, we suggest that this optoacoustic coupling can be used to store and retrieve optical information from a mechanical degree of freedom, as demonstrated recently for optomechanical systems [37]. We note that earlier optomechanical proposals involving optical OAM have relied on torsional oscillators [38–40], which, not being free rotors, offer states rather localized in angle, which provide only small spatial overlap with optical OAM states. In this case, however, as we show below, a high degree of mode-matching can be achieved between radiation and matter, leading to a large coupling and good memory fidelity.

Our specific system of interest is shown in figure 1, which consists simply of a two-mirror cavity. We consider surface acoustic modes on one of the mirrors, as observed in an earlier experiment [33]. The acoustic modes consist of...
modes of equation (1) with the unprimed indices

$$\mu_p(r, \theta) = \frac{4p!}{(1 + \delta_0)\pi (|l| + p)!} \int \frac{r^l}{w} \frac{2^l}{w^2} \exp \left(-\frac{r^2}{w^2}\right) \cos(l\theta),$$  \hspace{1cm} (1)

where the index \(l\) corresponds to the number of azimuthal nodes, the index \(p\) determines the number of radial nodes and \(w\) equals the acoustic beam waist \(w_a\). In equation (1), the index \(l\) can be either a positive or negative integer, while the index \(p\) can only assume positive integer values [2]. The intensity of an optical cavity field consisting of the superposition of two counter-rotating Laguerre–Gaussian modes can also be described in terms of equation (1), i.e. as \(\left|\mu_p(r, \theta)\right|^2\), where \(l, p\) and \(w = w_a\) now describe the optical parameters. Below we will consider the transfer of this optical counter-rotating superposition state \(|+l\rangle + |-l\rangle\) to a shear mechanical mode [33].

We expand the quantized electromagnetic field using the modes of equation (1) [41] with the unprimed indices \(l\) and \(p\), and similarly the acoustic displacement [33] but using mode functions with the primed indices \(l'\) and \(p'\). We find the optoacoustic interaction Hamiltonian to be of the form of the standard ‘linearly’ coupled optomechanical system [33, 36]

$$\hat{H} = -i\hbar g\chi_{l'pp'} \hat{a}^\dagger \hat{a} \hat{b}^\dagger \hat{b},$$  \hspace{1cm} (2)

where \(\hat{a} (\hat{a}^\dagger)\) and \(\hat{b} (\hat{b}^\dagger)\) are the annihilation (creation) operators for the optical and acoustic modes, respectively. The constant \(g\) is given by

$$g = \frac{x_0 \delta_0}{L}$$  \hspace{1cm} (3)

where \(L\) is the optical cavity length, \(x_0\) is the zero-point-momentum amplitude of the surface acoustic mode and \(\omega_0\) is the frequency of the free cavity field. The dimensionless constant \(\chi_{l'pp'}\), which is the overlap integral between the optical and acoustic modes,

$$\chi_{l'pp'} = \mu_p \int_0^\infty \int_0^{2\pi} \left|\mu_p(r, \theta)\right|^2 \mu_{p'}(r, \theta) \, d\theta \, dr,$$  \hspace{1cm} (4)

is analytically found to be

$$\chi_{l'pp'} = \xi_{l'pp'} \delta |l'|, \hspace{1cm} \gamma$$

where

$$\xi_{l'pp'} = \frac{p!}{(1 + \delta_0)(|l| + p)!} \frac{p!}{(1 + \delta_0)(|l| + p')!} \times F_1 \left[-p - 2k; -2k - 2|l| - 2|l'|; \frac{1 + \gamma / 2}{1 - \gamma / 2} \right],$$  \hspace{1cm} (6)

with

$$\gamma = \left(\frac{w_a}{w}\right)^2$$  \hspace{1cm} (7)

In equation (6), \(\Gamma(p + |l| + 1)\) is a Gamma function and \(F_1\) is a hypergeometric function. We note that \(\chi_{l'pp'}\) is only non-zero if the angular momentum conservation rule,

$$|l'| = 2|l|,$$  \hspace{1cm} (8)

is satisfied. The factor of 2 can be traced to the quadratic dependence of the integral of equation (4) on the optical mode function as compared to its linear dependence on the acoustic mode, and even further back to the bilinear dependence of the interaction on the optical operators and its linear dependence on the acoustic displacement, see equation (2). Physically, it represents the fact that the mechanical displacement couples to the optical intensity. Henceforth, we will assume that equation (8) is satisfied and will deal only with the quantity \(\xi_{l'pp'}\). Similarly, we will consider \(w_a\), which is determined by the mirror geometry [33], to be a fixed parameter, and consider variations only in \(w_a\), or equivalently in \(\gamma\), below.

We now examine the simple case \(p = p' = 0\), in which equation (6) reduces to

$$\xi_{000} = \frac{\gamma |l|}{(1 + \delta_0)|l|} \Gamma(|l| + 1) \left(1 + \frac{\gamma}{2}\right)^{(2|l| + 1)}.$$  \hspace{1cm} (9)

The value of \(\gamma\) that maximizes the overlap \(\xi_{000}\) is given by

$$\gamma_{\text{max}} = \frac{2|l|}{|l| + 1},$$  \hspace{1cm} (10)
In the case of sub-optimal control of larger nontrivial OAM couplings for large values of OAM photonic states. We will now show that non-zero values in-principle infinite-dimensional Hilbert space spanned by mode for large values of \( l \). The use of non-zero \( p \) and \( p' \) enables the generation of a significant coupling for optical states with high values of \( l \).

Before proceeding, we note that the effect of non-zero values of \( p(p') \) on the mode profile is twofold. First, non-zero values of \( p(p') \) introduce additional local maxima in the radial direction. This means that the intensity distributed to the central peaks decreases as \( p(p') \) become larger, for the same value of total intensity. Secondly, the radius at which the central intensity peaks occur is reduced. In figure 3, we have shown \( \xi_{pp'} \) as a function of \( p \) and \( p' \) for different values of \( l \), with \( \gamma = 0.1 \). For \( l = 0 \), the optimal arrangement of the optical and acoustic mode profiles occurs at \( p = p' = 0 \), where the two peaks are exactly aligned. However, for \( l > 0 \), non-zero values of \( p \) or \( p' \) help to increase the coupling constant to about the same order of magnitude as that in the case \( l = p = p' = 0 \). In figure 4(a), a comparison of the coupling strengths between the cases of \( p = p' = 0 \) (left) and non-zero values of \( p \) and \( p' \) (right) has been exhibited. As can be seen, even though the coupling constant still decreases as \( l \) gets larger, the rate of decrease is much slower than that in figure 2. In fact, for quite high OAM, \( \xi_{pp'} \) can be adjusted to be of the same order of magnitude as that for \( l = 0 \).

We have also calculated the fidelity for transferring a coherent optical state \( |\alpha\rangle \) in the OAM superposition \( |+l\rangle + |-l\rangle \) to the mechanical shear mode, accounting for the effects of noise and dissipation on both degrees of freedom. We have adapted an earlier treatment to calculate the fidelity of the transfer as a function of the indices \( l \), \( p \) and \( p' \) \([44]\). Assuming a strongly driven optical cavity and small single photon optomechanical coupling \( g \), the fidelity for the transfer of a coherent state is given by

\[
F_{pp'} = \frac{1}{1 + n_{pp'}} \exp \left( -\frac{\lambda_{pp'}^2}{1 + n_{pp'}} \right),
\]

where

\[
n_{pp'} = \frac{\gamma_{m}N_{m}}{2} \frac{\pi}{2g\sqrt{\xi_{pp'}}}
\]
Figure 4. Comparison of the (a) dimensionless coupling constant $\xi_{lpp'}$ and (b) state transfer fidelity with $p = p' = 0$ (left panel) and with $p \neq 0$ and $p' \neq 0$ (right panel), with $\gamma = 0.1$. The right panel is generated by finding the maximum coupling (fidelity) that can be achieved for a given value of $l$ without restrictions on $p$ and $p'$. It can be seen that the use of non-zero values of $p$ and $p'$ makes it possible to achieve the significant optoacoustic coupling and high state transfer fidelity for high values of $l$.

Figure 5. Fidelity for the transfer of a coherent optical state $|\alpha\rangle$ to the mechanical mode, with $g = 2\pi \times 0.2$ Hz, $n_c = 2 \times 10^{18}$, $\gamma = 0.1$, $\alpha = 1$, $\kappa = 2\pi \times 50$ kHz, $\gamma_m = 2\pi \times 50$ kHz, and $T = 1$ K [44].

is the effective number of thermal quanta responsible for heating the state during transfer and

$$\lambda_{lpp'} = \alpha \left( \frac{k + \gamma_m}{4} \right) \frac{\pi}{2g\sqrt{n_c}\xi_{lpp'}}$$

(13)

is a damping parameter accounting for dissipative effects. In equations (12) and (13), $\gamma_m$ is the acoustic mode decay rate, $k$ is the optical cavity decay rate, $N_m$ is the number of phonons in the environment, $\alpha$ is the coherent amplitude of the state to be transferred and $n_c$ is the number of intracavity photons due to the optical drive. We have chosen experimentally relevant parameters for our calculation, with a cavity decay rate of $\kappa = 2\pi \times 50$ kHz, a mechanical decay rate of $\gamma_m = 2\pi \times 50$ kHz, a drive of power $\sim 1$ mW ($n_c \sim 10^{18}$) and an environmental temperature of $T = 20$ mK [33, 44, 36]. The fidelity for the transfer of a coherent state with $\alpha = 1$ has been plotted in figure 5 for a few values of $l$. The transfer fidelity for high photon angular momentum ($l \geq 3$) is quite low with $p = p' = 0$. However, the use of non-zero values of $p$ and $p'$ can restore the transfer fidelity for states with high values of $l$ to the same level that can only be achieved for $l = 0$ with $p = p' = 0$. A comparison for the state transfer fidelity between the case of $p = p' = 0$ and $p \neq 0, p' \neq 0$ is shown in figure 4(b).

In conclusion, we have demonstrated that the substantial optomechanical coupling can be achieved in a system composed of an acoustical mode and a Laguerre-Gaussian cavity mode, provided that the ratio of the acoustic and optical waists as well as the indices $p$ and $p'$ for the modes is chosen judiciously. We have also shown that a high fidelity transfer of a coherent optical state to the mechanical mode can be carried out for experimentally realistic parameters. Our results make it possible to exploit the in-principle infinite-dimensional Hilbert space spanned by the optoacoustic system and realize a mechanical memory for photons carrying OAM. Unlike
the previous work, which has focused on atomic vapours, the mechanical platform we have suggested can easily be miniaturized and readily scaled.

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