Adjoint Numerical Method for a Multiphysical Inverse Problem of Two-Phase Well Testing in Petroleum Reservoirs

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Abstract. Inverse problem solution is an integral part of data interpretation for well testing in petroleum reservoirs. In case of two-phase well tests with water injection, forward problem is based on the multiphase flow model in porous media and solved numerically. The inverse problem is based on a misfit or likelihood objective function. Adjoint methods have proved robust and efficient for gradient calculation of the objective function in this type of problems. However, if time-lapse electrical resistivity measurements during the well test are included in the objective function, both the forward and inverse problems become multiphysical, and straightforward application of the adjoint method is problematic. In this paper we present a novel adjoint algorithm for the inverse problems considered. It takes into account the structure of cross dependencies between flow and electrical equations and variables, as well as specifics of the equations (mixed parabolic-hyperbolic for flow and elliptic for electricity), numerical discretizations and grids, and measurements in the inverse problem. Decomposition is proposed for the adjoint problem which makes possible step-wise solution of the electric adjoint equations, like in the forward problem, after which a cross-term is computed and added to the right-hand side of the flow adjoint equations at this timestep. The overall procedure provides accurate gradient calculation for the multiphysical objective function while preserving robustness and efficiency of the adjoint methods. Example cases of the adjoint gradient calculation are presented and compared to straightforward difference-based gradient calculation in terms of accuracy and efficiency.

1. Introduction
Well tests (WT) are an essential part of petroleum reservoir studies. They are crucial for evaluation of formation properties in-situ, at actual flow conditions. Traditional WT data interpretation is based on analytical and numerical models of underground fluid flow. Inverse problems are formulated for these models to determine reservoir properties, such as permeability, skin-factor, reservoir pressure and others [1, 2]. Both forward and inverse problems of traditional WT are reduced to single-physics (flow in porous media) and, in most cases, single-phase (flow of oil or gas) formulations.

Multiphase WT are less common and more complicated both in field implementation and data interpretation. However, they can provide important information about multiphase flows, e.g. oil displacement by water injection – such as displacement efficiency (or residual oil saturation) and relative permeabilities (RP) as functions of fluid saturations. In situ data of multiphase WT help to overcome scale and wettability issues associated with core studies typically used to determine these parameters.

In this paper, we consider two-phase WT with water injection described in [3-6]. The forward problems are based on numerical models of two-phase oil-water flows in 1D or 2D with optional
simulation of salt transport in the water phase [7]. Inverse problems are based on weighted least squares (WLS) or maximum likelihood (ML) formulations. Though a variety of methods are available for inverse problem solution in petroleum applications [8], gradient-based optimization techniques combined with the adjoint method to calculate the objective function gradient have proved robust and efficient in this type of problems [9, 7].

Though essentially multiphase, these forward and inverse problems still remain single-physics (flow and transport in porous medium), and thus allow for simultaneous solution of the underlying set of equations on a single grid both within the forward and adjoint problems. However, in practical applications, one has to consider near-wellbore saturation measurements provided by well logging. These measurements are not direct and also imply interpretation, which typically relies on some assumptions not quite accurate for dynamic conditions of the multiphase WT.

To overcome this issue, a joint forward problem for the two-phase WT with resistivity logging was presented in [10]. Now the formulation is multiphysical: it combines a dynamic 1D problem for two-phase flow with salt transport – nearly hyperbolic in saturation and salt concentration, and parabolic in pressure, – with the quasi-stationary 2D elliptic problem for electrical potential. Though different in dimensionality, numerical schemes and grids, these problems remain strongly coupled through the dependency of local reservoir electrical resistivity on water saturation and salt concentration. Hence, straightforward application of the adjoint method is not possible within the inverse problem. That’s why trial-and-error or assisted techniques are usually chosen in practical solution of similar problems [11].

In this paper we present an efficient yet accurate adjoint algorithm for the coupled flow-electrical problem of the two-phase WT. It honors the individual specifics of the numerical schemes and grids for flow and electrical subproblems, and makes use of the structure of coupled equations.

2. The forward problem
The joint forward problem of two-phase WT with resistivity logging was described in detail in [10]. Here we recall some points principal for the following discussion.

The flow subproblem consists of the mass balance (continuity, or flow) equations for oil and water, and mass balance equation for salt (salt transport equation). In 1D it takes the form:

\[
\frac{\partial}{\partial u} \left( \frac{k \kappa \rho \alpha}{\mu \alpha} \frac{\partial p \alpha}{\partial u} \right) = R_b^2 e^{2u} \frac{\partial}{\partial t} (\phi S \alpha \rho \alpha), \quad \alpha = o, w. \tag{1}
\]

\[
\frac{\partial}{\partial u} \left( \frac{k k r \rho w l}{\mu w} \frac{\partial p w}{\partial u} \right) = R_b^2 e^{2u} \frac{\partial}{\partial t} (\phi S w \rho w l + M_{sw} l_{sw}), \tag{2}
\]

where \( u = \ln(r/R_b) \) is the logarithmic radial coordinate, \( r \) is the natural radial coordinate, \( R_b \) is the external radius of the drainage area; \( t \) is the time; \( p \alpha \) is the pressure in phase \( \alpha \) (\( \alpha = o \) corresponds to the oil phase, and \( \alpha = w \) – to the aqueous/water phase); \( S \alpha \) is the saturation of the porous medium with the phase \( \alpha \) (normalized by effective pore volume); \( \rho \alpha \) and \( \mu \alpha \) are the density and viscosity of the phase \( \alpha \); \( \phi \) and \( k \) are the effective porosity and effective permeability of the porous medium (reservoir); \( k r \alpha \) is the relative permeability to the phase \( \alpha \) (normalized by effective porosity); \( l \) is the salt concentration in the mobile aqueous phase (salinity); \( l_{sw} \) is the salt concentration in bound water; \( M_{sb} \) is the mass of bound water per unit volume of the reservoir.

The set of three equations (1)-(2) is supplemented with initial conditions – distributions of oil pressure, water saturation and salinity at the beginning of the well test. Boundary conditions are no-flow at the external boundary \( r = R_b \), and vary in time (specified liquid rate / water injection rate and salinity / no-flow) with changing regimes at the inner boundary (well radius) \( r = r_w \).

The closing relations include of the normalization condition for saturations, and the dependencies for fluid densities and viscosities on pressure, and capillary pressure and relative permeabilities on water saturation (capillary pressure equals to the difference between the pressures in the oil and aqueous phases).
The flow subproblem based on the partial differential equations (PDEs) (1)-(2) is solved for $p_a, S_w$ and $l$ with a fully implicit control-volume scheme. The problem is parabolic in pressure and nearly-hyperbolic in water saturation and salt concentration. Upwind weighting is used for saturations and salt concentration.

The coupled forward problem of resistivity logging (electrical subproblem) is 2D and quasi-stationary. It is solved at certain moments in time corresponding to the resistivity measurements performed at the well during the well test. Reservoir thickness is divided into a set of layers with identical resistivity distributions obtained using water saturation and salinity values calculated from the solution of the flow subproblem. Extra layers are added above the top and below the bottom of the reservoir with formation resistivity typical for clays. The grid along the radial coordinate is supplemented with “borehole” cells for $r_s \leq r \leq r_w$, where $r_s$ is the radius of the sonde (logging tool), with resistivity values reflecting the fluid and casing effects in the wellbore.

The PDE for forward electrical subproblem is elliptical and takes the form:

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \sigma(r,z,t) \frac{\partial U}{\partial r} \right) + \frac{\partial}{\partial z} \left( \sigma(r,z,t) \frac{\partial U}{\partial z} \right) = -g$$

with the boundary conditions:

$$\frac{\partial U}{\partial r}|_{r=r_s} = U|_{r=R_b} = U|_{z=\pm h} = 0.$$  

(4)

Here $z$ is the vertical coordinate measured from the mean depth of the reservoir; $h$ is the total thickness of the modeled area; $U(r,z)$ is the electric potential; $g$ is the source term equal to density of current at the sonde source electrode and null elsewhere; $\sigma = 1/\rho_{res}$ is the electrical conductivity. The resistivity of the saturated medium $\rho_{res}$ is related to local water saturation and salinity by specified analytical relations [10].

For a given moment of well logging, the problem (3)-(4) is repeatedly solved for different vertical positions of the sonde with a control-volume numerical scheme. Again, logarithmic transformation of the radial axis is used. The computed potential values at the points of measuring electrodes are used to calculate the apparent resistivity (AR) using one of the analytical formulas depending on the sonde type.

Thus, the coupled forward problem of the two-phase WT with resistivity logging combines the solution of the flow and electrical subproblems which differ in physics, dimensionality, equation types and numerical grids.

### 3. The inverse problem and the adjoint method

The inverse problem and the adjoint method for the flow problem of two-phase WT were discussed in [7]. The formulation is based on the WLS objective function, and the adjoint method is applied to the discretized system of the flow and transport equations. The discretized adjoint problem is solved on the same spatial and temporal grid with the same numerical scheme as the forward problem (but backwards in time). Calculated adjoint variables are used to compute the gradient of the objective function with respect to control (identified) model parameters.

For the joint inverse problem of two-phase WT with resistivity logging, the objective function $J$ is a weighted sum of the objective functions for “flow” measurements $J_f$ and “electric” (resistivity) measurements $J_e$:

$$J = J_f + \omega \cdot J_e$$

(5)

$$J_f = \sum_{j=1}^{N} \varphi_f^j \left( \tilde{x}_f^j(\bar{u}), \bar{u} \right)$$

(6)

$$J_e = \sum_{j=(j_{es},...,j_{em})} \varphi_e^j \left( \tilde{x}_e^j(\bar{u}) \right)$$

(7)
\[
\phi_f^j = (\tilde{y}_f^j(\tilde{x}_f^j(\tilde{u})) - \tilde{y}_f^j)^T \Omega_f (\tilde{y}_f^j(\tilde{x}_f^j(\tilde{u})) - \tilde{y}_f^j)
\]

Here \(\tilde{u}\) is the vector of control (identified) parameters (including reservoir permeability, permeability of the near-wellbore (skin) zone, residual oil saturation, parameters of relative permeabilities as functions of water saturation); \(\omega\) is a weighting factor; \(N\) is the number of measurement moments during the study, and superscript \(j\) corresponds to a specified moment of measurements; \(\tilde{y}_f^j\) and \(\tilde{y}_f^j\) are the vectors of calculated (simulated) and actually recorded values of "flow" measurements at the \(j\)-th moment (including well bottomhole pressure, well watercut or oil/water rate, and produced water salinity); \(\tilde{x}_f^j\) is the vector of the flow forward problem variables at the \(j\)-th moment \((p_a, S_w, l)\) in every grid cell; \(\Omega_f\) is a diagonal weighting matrix; \(\{f_{el1}, \ldots, f_{elm}\}\) is the set of measurement moments with resistivity logging; \(\tilde{x}_e^j\) is the vector of the electric forward problem variables at the \(j\)-th moment \((U\) in every grid cell\); \(\tilde{p}^j\) and \(\tilde{R}^j\) are the vectors of calculated (simulated) and actually recorded values of AR at the \(j\)-th moment for different positions of the sonde; \(n_A\) is the number of sonde positions, and subscript \(k\) corresponds to the values measured or computed for a specified sonde position.

Formulas (5)-(9) can be rearranged to the general form of the objective function:

\[
J = \sum_{j=1}^N \phi_f^j(\tilde{x}_f^j(\tilde{u}), \tilde{u}),
\]

where \(\tilde{y}_f^j\) and \(\tilde{y}_f^j\) combine all the measurements (flow and electric) and \(\tilde{x}_f^j\) – all the variables for the moment \(j\).

Let’s also combine all the discretized equations (flow and electric) of the forward problem at \(j\)-th timestep in the following manner:

\[
\tilde{F}^j(\tilde{x}_f^j, \tilde{x}_f^{j-1}, \tilde{u}) = \tilde{0}
\]

Formally, expressions (10)-(12) are the same as in the ordinary flow inverse problem [7]. Applying the standard derivation for the adjoint method, we obtain the expression for the gradient of the objective function (10) (vector of partial derivatives with respect to the control parameters \(\tilde{u}\)) and the discretized adjoint equations [7, 9]:

\[
\nabla J = X^T \tilde{F}_x^T \tilde{y}^j \Delta t^1 + \sum_{j=1}^N \frac{\partial J}{\partial \tilde{u}} \tilde{y}_f^j(\Delta t^1),
\]

\[
\tilde{F}_x^j \nabla \tilde{y}_f^j = -\frac{\Delta t_{j+1}}{\Delta t_j} \tilde{F}_x^{j+1} \tilde{y}_f^{j+1} - \frac{1}{\Delta t_j} \phi_f^j(\tilde{x}_f^j), \quad j = N, \ldots, 1.
\]

Here \(\nabla \tilde{y}_f^j\) is the vector of adjoint variables at \(j\)-th time layer; \(\tilde{F}_x^j\) is the matrix of partial derivatives (Jacobi matrix) of (12) with respect to the control parameters \(\tilde{u}\); \(\tilde{F}_x^j\) is the Jacobi matrix with respect to the forward problem variables \(\tilde{x}_f^j\), and \(\tilde{F}_x^{j-1}\) is the Jacobi matrix with respect to the variables of the previous time layer \(\tilde{x}_f^{j-1}\); \(\phi_f^j\) and \(\phi_f^j\) are the vectors of partial derivatives of (11) with respect to \(\tilde{u}\) and \(\tilde{x}_f^j\), respectively; \(\Delta t_j\) is the \(j\)-th timestep; and \(X\) is the matrix relating the variation of initial conditions of the forward problem to the variation of the control parameters: \(\delta \tilde{x}_0 = X \delta \tilde{u}\).

For the joint flow-electric problem, expressions (13)-(14) do not provide a straightforward way to compute the adjoint variables and the objective function gradient. This is due to very different structures of the flow and electric equations, variables and grids hidden in (12).
Let's consider the structure of matrices in (13)-(14) in more detail.

Figure 1 schematically shows the structure of the \( \vec{F}^j \) matrix. The upper part of it corresponds to the flow equations, and the lower one – to the electric equations. Similarly, the left part corresponds to the flow variables, and the right part – to the electric variables. Note that the electric variables and equations each consist of \( n_A \) sets, and each set corresponds to all the gridblocks in the electric problem for a specified sonde position. The flow equations and variables are combined to a block structure, with 3×3 blocks corresponding to water-oil-salt equations × \( p_o-S_w-l \) variables.

![Figure 1. The schematic structure of the \( \vec{F}^j \) matrix for the joint flow-electric problem](image)

The key point in Figure 1 is that the flow equations do not depend on the electric variables. Thus, the combined system of linear equations for the forward problem (12) takes the form:

\[
A_{ff}^j \vec{x}_f^j = \vec{b}_f^j \tag{15}
\]

\[
A_{ef}^j \vec{x}_f^j + A_{ee}^j \vec{x}_e^j = \vec{b}_e^j, \quad k = 1, \ldots, n_A, \tag{16}
\]

where \( A_{ff}^j \) is the quadratic block-tridiagonal matrix of the flow equations, \( A_{ee}^j \) is the quadratic 5-diagonal matrix of the electric equations, \( A_{ef}^j \) is the sparse matrix of derivatives of the electric equations with respect to the flow variables; \( \vec{b}_f^j \) and \( \vec{x}_f^j \) are the right-hand side (RHS) and the solution of the flow equations; \( \vec{b}_e^j \) and \( \vec{x}_e^j \) are the RHS and the solution of the electric equations for the \( k \)-th position of the sonde.

Note that equations (15) are formal because the flow problem is nonlinear. However, same matrix \( A_{ff}^j \) appears in the system of linear equations at iterations of the Newton’s method and hence specifies the matrices in the inverse problem.

Now let’s examine the structure of the matrices and vectors in the adjoint problem (14). Figure 2 shows their schematic representation. Here non-zero blocks are shown in blue, and null blocks – in white.
As seen from Figure 2, the adjoint problem (14) can be effectively decomposed. The adjoint equations corresponding to the electric forward problem are independent of the flow adjoint variables. They form a set of independent systems of quasi-stationary equations, each corresponding to a specific position of the sonde, with same matrix and different RHS.

As for the adjoint equations corresponding to the flow forward problem, they become very similar to those of the flow-only inverse problem [7]. The correction is in the RHS: additional cross-terms come from the product of the $A_{ef}^T$ matrix with the solutions of the electric adjoint equations. Finally we get:

$$
\begin{align*}
A_{ff}^T \psi_f^j &= - \frac{1}{\Delta t_j} \sum_{k=1}^{n_A} A_{ef}^T \psi_{e,k}^j - \frac{\Delta t_{j+1}}{\Delta t_j} A_{ff}^T \psi_f^{j+1} - \frac{1}{\Delta t_j} \phi_{x,f}^j \\
A_{ee}^T \psi_{e,1}^j &= - \phi_{x,e,1}^j \\
A_{ee}^T \psi_{e,k}^j &= - \phi_{x,e,k}^j \\
A_{ee}^T \psi_{e,n_A}^j &= - \phi_{x,e,n_A}^j \\
A_{ef}^T \psi_{f}^j &= - \phi_{x,f}^j \\
A_{ef}^T \psi_{f}^{j+1} &= - \phi_{x,f}^{j+1}
\end{align*}
$$

Thus, the proposed decomposition for the adjoint problem makes possible independent solution of the electric adjoint equations at the moments of resistivity logging measurements, just like in the forward electric problem. For every sonde position, the solution found is used to compute a cross-term which is added to the RHS of the flow adjoint equations at this timestep. In other aspects, the flow adjoint equations are formed and solved in the same manner as in the flow-only inverse problem.
The adjoint variables from (17) are used to compute the objective function gradient. Figure 3 shows the structure of the matrix and vector from (13) corresponding to the derivatives with respect to the control parameters.

\[ \frac{\partial J}{\partial u} = \begin{bmatrix} \frac{\partial f}{\partial u} \\ \frac{\partial e}{\partial u} \end{bmatrix} \]

\[ \frac{\partial J}{\partial \omega} = \begin{bmatrix} \frac{\partial \omega}{\partial \omega} \end{bmatrix} \]

**Figure 3.** The schematic structure of the matrix and vector in (13)

Both electric forward equations and electric terms in the objective function are independent of the control parameters. Substitution of the matrices and vectors from Figures 2 and 3 into (13) results in the same expression for the gradient of the objective function as in the flow-only inverse problem.

Note that that electric adjoint variables do not contribute directly to the \( \nabla J \). But their indirect contribution is essential, as given by the cross-term in the first equation of (17).

4. **Test case and computational efficiency**

To study the efficiency and accuracy of the proposed adjoint algorithm, we consider a synthetic test case of a two-phase WT with resistivity logging. Key input data are presented in Table 1. More details of the case can be found in [10] (see the case with constant initial water salinity).

The WT consists of 6 stages: oil production – 50 m³/day for 240 hours, pressure build-up (the well is shut) – 240 hours, first water injection (salinity 0.04 g/g) – 100 m³/day for 80 hours, first two-phase liquid production – 75 m³/day for 176 hours, second water injection (salinity 0.1 g/g) – 100 m³/day for 80 hours, second two-phase liquid production – 75 m³/day for 176 hours. Flow measurements in the objective function include bottomhole pressure, near-wellbore water saturation, oil rate and water salinity. Resistivity logging data are represented by a single measurement in the middle of the first two-phase liquid production. AR values for 38 sonde positions (1 m step in depth) form the electric part of the objective function \((n_A = 38)\). AR data correspond to the lateral device marked A2M0.5N (see [10] for details), the current at the source is I=0.5 A. Weighting factor in (5) is \( \omega = 1 \), for which the contribution of the electric part to the objective function is \( \sim 15\% \).

Table 2 shows two sets of values for control parameters \((\bar{u})\): true and initial. True values were used to generate the "actually measured" data for the objective function from the corresponding solution of the forward problem. Initial data form the point at which the objective function gradient is to be computed. AR profiles for the true parameters ("measured values") and initial parameters ("calculated values") are compared in Figure 4. They correspond to the \( \mathbf{R} \) and \( \mathbf{f}(\mathbf{x}_e(\bar{u})) \) vectors in (9).

Tables 3 and 4 show a comparison of the gradient values calculated using the presented adjoint algorithm or by numerical differentiation. Note that numerical differentiation for a certain parameter with a certain value of increment requires an extra solution of the flow and electric forward problems:

\[ \nabla J_i = \frac{\partial f}{\partial u_i} \approx \frac{f(u_{i-\Delta u,\ldots,u_{i+\Delta u}}) - f(u_{i-\Delta u,\ldots,u_{i+\Delta u}})}{\Delta u_i} \]  

(18)

Table 3, as an example, demonstrates the accuracy of the adjoint calculation for the objective function derivative with respect to a single control parameter – residual oil saturation (parameter 5 in Table 2). For this calculation, all the other control parameters were assigned their true values. As expected, the adjoint-based value of the derivative falls within the range of numerically computed values with different increments.
Table 1. Initial data for the synthetic test case

| Reservoir properties                          |        |
|-----------------------------------------------|--------|
| Configuration of the reservoir (drainage zone) | circular, piecewise homogeneous (with a skin zone) |
| Radius of the external boundary, m            | 500    |
| Well radius, m                                | 0.1    |
| Top depth of the formation, m                 | 2000   |
| Formation thickness (with clay intervals), m   | 40     |
| Oil-saturated reservoir thickness, m           | 15     |
| Resistivity of clays, ohm·m                   | 2      |
| Effective porosity of the reservoir           | 0.16   |
| Effective permeability of the reservoir, μm²  | 0.5    |
| Reservoir temperature, °C                     | 50     |
| Initial reservoir pressure, bar (1 bar=10⁵ Pa)| 200    |
| Assumed radius of the skin zone, m            | 0.032  |
| Assumed permeability of the skin zone, μm²    | 0.025  |
| Equivalent skin factor                        | 5.275  |

| Fluid properties                              |        |
|------------------------------------------------|--------|
| Oil viscosity at reservoir conditions, mPa·sec | 5      |
| Water viscosity in reservoir conditions, mPa·sec | 1      |
| Oil density at standard conditions, kg/m³     | 810    |
| Water density at standard conditions, kg/m³   | 1300   |
| Initial and bound water saturation – normalized by total / effective pore volume | 0.2 / 0 |
| Residual oil saturation – normalized by total / effective pore volume | 0.2 / 0.25 |
| Formation water salinity, g (salt)/g (water)  | 0.0062 |

| Parameters of the Archie-Dakhnov formula for reservoir resistivity |
|----------------------------------------------------------------------|
| a / m / n                                                            | 1.0 / 2.0 / 2.0 |

Table 2. True and initial values of the control parameters

| Parameter                                           | True value | Initial value |
|-----------------------------------------------------|------------|---------------|
| 1 Effective permeability of the reservoir, μm²      | 0.5        | 0.8           |
| 2 Permeability of the skin zone, μm²                 | 0.025      | 0.1           |
| 3 Oil relative permeability factor                   | 1.0        | 0.3           |
| 4 Oil relative permeability exponent                 | 2.0        | 2.5           |
| 5 Residual oil saturation (normalized by effective pore volume) | 0.25 | 0.4 |
| 6 Water relative permeability exponent               | 2.0        | 1.5           |
Figure 4. AR profiles for the true ("measured values") and initial ("calculated values") sets of control parameters (horizontal axis in log scale)

Table 3. Comparison table for the objective function derivative with respect to a single control parameter (residual oil saturation)

| Calculation method | Value of the derivative |
|--------------------|-------------------------|
| Adjoint            | 151397                  |
| Numerical differentiation with the relative increment: |
| -0.01              | 148746                  |
| -0.001             | 151643                  |
| -0.0001            | 152553                  |
| 0.001              | 151702                  |
| 0.01               | 152424                  |
|                    | 155210                  |

Table 4. Comparison table for the components of the objective function gradient (6 parameters)

| Objective function derivative with respect to | Calculation method |
|-----------------------------------------------|--------------------|
| Objective function derivative with respect to | Adjoint | Numerical differentiation |
| 1. Effective permeability                      | -155450 | -155576 |
| 2. Skin-zone permeability                      | -51078  | -51000  |
| 3. Oil relative permeability factor            | -435403 | -430650 |
| 4. Oil relative permeability exponent          | 81317   | 80134   |
| 5. Residual oil saturation                     | 727159  |         |
| 6. Water relative permeability exponent        | 39679   |         |
Table 4 compares the objective function gradient for 6 parameters computed by the adjoint algorithm and by numerical differentiation. Here a single relative increment was used in numerical differentiation for every parameter, so the numerical values might be not very accurate. However, good correspondence of the gradients is observed.

Efficiency of the adjoint method is demonstrated by Table 5. We compare the average timings required to compute the derivatives with respect to 1, 2 or 6 parameters. For the adjoint method, computational time increased by only 14% for 6 parameters as compared to 1 parameter. For the numerical differentiation, timings increase almost proportional to the number of parameters. They are about 2 times larger than the timings of the adjoint method for 1 parameter, 3 times larger – for 2 parameters, and more than 8 times larger – for 6 parameters.

The method was tested on a synthetic case of the joint flow-electrical two-phase WT problem. Derivatives of objective function computed with the adjoint algorithm were compared to results of numerical differentiation. The method proved accurate and efficient, with less than 15% increase in computational time for 6 parameters as compared to 1 parameter. Gain over the numerical differentiation was more than 8 times for 6 parameters.

### 5. Conclusions

An adjoint method was presented for the multiphysics inverse problem of two-phase WT with resistivity logging. It honors the specifics of the flow and electric subproblems involved – different equation types, numerical schemes and grids, and makes use of the coupling structure between them in terms of derivative matrices and vectors.

Efficient decomposition of the adjoint problem was proposed which enables independent solution of the electric adjoint problem at each moment of measurements and for each position of the sonde. Electric adjoint variables are used to compute cross-terms which are added to the RHS of the flow adjoint equations at current timestep. Adjoint flow variables contribute directly to the gradient of the objective function, while the contribution of the electric adjoint variables is through the cross-term coupling.

The method was tested on a synthetic case of the joint flow-electrical two-phase WT problem. Derivatives of objective function computed with the adjoint algorithm were compared to results of numerical differentiation. The method proved accurate and efficient, with less than 15% increase in computational time for 6 parameters as compared to 1 parameter. Gain over the numerical differentiation was more than 8 times for 6 parameters.

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