Scalar-tensor cosmologies: general relativity as a fixed point of the Jordan frame scalar field

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Abstract

We study the evolution of homogeneous and isotropic, flat cosmological models within the general scalar-tensor theory of gravity with arbitrary coupling function and potential and scrutinize its limit to general relativity. Using the methods of dynamical systems for the decoupled equation of the Jordan frame scalar field we find the fixed points of flows in two cases: potential domination and matter domination. We present the conditions on the mathematical form of the coupling function and potential which determine the nature of the fixed points (attractor or other). There are two types of fixed points, both are characterized by cosmological evolution mimicking general relativity, but only one of the types is compatible with the Solar System PPN constraints.

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1 Introduction

Scalar-tensor theories (STT) of gravitation [1, 2, 3] have emerged in different contexts of theoretical physics, e.g. in Kaluza-Klein type unifications, supergravity, and low energy approximations of string theories. In cosmology STT has been invoked to model the accelerated expansion of inflation and dark energy. However, observations in the Solar System tend to indicate that in the intermediate-range distances the present Universe around us is successfully described by the Einstein tensorial gravity alone [4, 5, 6]. This means that only such models of scalar-tensor gravity are viable which in their late time cosmological evolution imply local observational consequences very close to those of Einstein’s general relativity (GR) [7].

The methods of dynamical systems provide natural tools to analyze the problem. In this paper, summarizing our recent work [8], we take the Jordan frame and consider general scalar-tensor theories which contain two functional degrees of freedom, the coupling function $\omega(\Psi)$ and the scalar potential $V(\Psi)$. We perform the dynamical systems analysis for the flat Friedmann-Lemaître-Robertson-Walker (FLRW) backgrounds with ideal barotropic fluid matter. Our strategy is to find the fixed points for the scalar field dynamics and compare these with the conditions of the limit of general relativity in the Solar System, as established by the parameterized post-Newtonian (PPN) formalism. Therefore, if the functional forms of $\omega(\Psi)$ and $V(\Psi)$ are specified from some considerations (e.g. the compactification manifold), our results allow to determine the fixed points along with their type and thus immediately decide whether general relativity is an attractor, i.e. whether the model at hand is viable or not.

2 Scalar-tensor cosmology as a dynamical system and the limit of general relativity

We consider a general scalar-tensor theory in the Jordan frame given by the action functional

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[ \Psi R(g) - \frac{\omega(\Psi)}{\Psi} \nabla^\mu \Psi \nabla_\mu \Psi - 2\kappa^2 V(\Psi) \right] + S_m(g_{\mu\nu}, \chi_m).$$

(1)

Here $\omega(\Psi)$ is a coupling function and $V(\Psi)$ is a scalar potential, $\nabla_\mu$ denotes the covariant derivative with respect to the metric $g_{\mu\nu}$ and $S_m$ is the matter part of the action as all other fields are included in $\chi_m$. In order to keep the effective Newtonian gravitational constant positive [9] we assume that $0 < \Psi < \infty$.

The field equations for the flat ($k = 0$) FLRW line element and perfect barotropic fluid matter, $p = w \rho$, read

$$H^2 = -H^2 + \frac{\dot{\Psi}^2}{\Psi^2} - \frac{\dot{\Psi}^2}{\Psi^2} \omega(\Psi) + \frac{\kappa^2}{3} \rho + \frac{\kappa^2}{3} V(\Psi),$$

(2)
\[
2\dot{H} + 3H^2 = -2H \frac{\ddot{\Psi}}{\dot{\Psi}} - \frac{1}{2} \frac{\Psi^2}{\dot{\Psi}^2} \omega(\Psi) - \frac{\ddot{\Psi}}{\dot{\Psi}} - \frac{\kappa^2}{\dot{\Psi}} w \rho + \frac{\kappa^2}{\dot{\Psi}} V(\Psi), \tag{3}
\]

\[
\ddot{\Psi} = -3H \dot{\Psi} - \frac{1}{2\omega(\Psi) + 3} \frac{d\omega(\Psi)}{d\Psi} \dot{\Psi}^2 + \frac{\kappa^2}{2\omega(\Psi) + 3} (1 - 3w) \rho + \frac{2\kappa^2}{2\omega(\Psi) + 3} \left[ 2V(\Psi) - \Psi \frac{dV(\Psi)}{d\Psi} \right], \tag{4}
\]

\[
\dot{\rho} = -3H (w + 1) \rho. \tag{5}
\]

Here \( H \equiv \dot{a}/a \) and we assume \( \rho \geq 0 \). Eqs. (2)–(5) are too cumbersome to be solved analytically, but useful information about the general characteristics of solutions can be obtained by rewriting (2)–(5) in the form of a dynamical system and finding the fixed points which describe the asymptotic behaviour of solutions.

The phase space of the system is spanned by four variables \( \{\Psi, \dot{\Psi}, H, \rho\} \). Defining \( \Psi \equiv x, \dot{\Psi} \equiv y \) the dynamical system corresponding to equations (2)-(5) can be written as follows:

\[
\dot{x} = y, \tag{6}
\]

\[
\dot{y} = -\frac{1}{2\omega(x) + 3} \left[ \frac{d\omega(x)}{dx} y^2 - \kappa^2 (1 - 3w) \rho + 2\kappa^2 \left( \frac{dV(x)}{dx} x - 2V(x) \right) \right] - 3H y, \tag{7}
\]

\[
\dot{H} = \frac{1}{2x(2\omega(x) + 3)} \left[ \frac{d\omega(x)}{dx} y^2 - \kappa^2 (1 - 3w) \rho + 2\kappa^2 \left( \frac{dV(x)}{dx} x - 2V(x) \right) \right] \\
- \frac{1}{2x} \left[ 6H^2 x + 2H y - \kappa^2 (1 - w) \rho - 2\kappa^2 V(x) \right], \tag{8}
\]

\[
\dot{\rho} = -3H (1 + w) \rho. \tag{9}
\]

Based on these equations we may make a couple of quick qualitative observations about some general features of the solutions. For example, the limit \( \Psi \to 0 \) in general implies \( |\dot{H}| \to \infty \), hence the solutions can not safely pass from positive to negative values of \( \Psi \) (from “attractive” to “repulsive” gravity), but hit a space-time singularity as the curvature invariants diverge. Similarly, the limit \( 2\omega + 3 \to 0 \) implies \( |\dot{H}| \to \infty \) with the same conclusion that passing through \( \omega(\Psi) = -\frac{3}{2} \) would entail a space-time singularity and is impossible. (Let us remark here, that these observations are quite general and do not preclude specially fine-tuned solutions in some fine-tuned models which may remain regular while crossing these points \[10\].)

The limit \( \frac{1}{2\omega + 3} \to 0 \) deserves a more closer examination. Let us define \( x_* \) by \( (2\omega(x_*) + 3)^{-1} = 0 \). Expressing \( H \) from the Friedmann constraint (2),

\[
H = -\frac{y}{2x} + \sqrt{(2\omega(x) + 3) \frac{y^2}{12x^2} + \frac{\kappa^2 (\rho + V(x))}{3x}}, \tag{10}
\]

makes clear that \( |H| \) diverges as \( x \to x_* \), unless also \( y \to 0 \) at the same time. What happens in the latter case is determined by the first term under the square root above. We can compute
its limit by Taylor expanding

\[
\lim_{y \to 0} (2\omega(x) + 3)y^2 = \lim_{r \to 0} \frac{r^2 \sin^2 \theta}{r \cos \theta + \frac{1}{2} \frac{d^2}{dx^2} \left( \frac{1}{2\omega(x) + 3} \right) x = x^*} \phi \cos \theta + \frac{1}{2} \frac{d^2}{dx^2} \left( \frac{1}{2\omega(x) + 3} \right) x = x^* r^2 \cos^2 \theta + \ldots
\]

\[
\begin{align*}
= 0, & \quad \text{if } \frac{d}{dx} \left( \frac{1}{2\omega(x) + 3} \right) x = x^* \neq 0, \\
\sim \tan^2 \theta, & \quad \text{if } \frac{d}{dx} \left( \frac{1}{2\omega(x) + 3} \right) x = x^* = 0, \quad \frac{d^2}{dx^2} \left( \frac{1}{2\omega(x) + 3} \right) x = x^* \neq 0, \\
= \infty, & \quad \text{if } \frac{d}{dx} \left( \frac{1}{2\omega(x) + 3} \right) x = x^* = 0, \quad \frac{d^2}{dx^2} \left( \frac{1}{2\omega(x) + 3} \right) x = x^* = 0,
\end{align*}
\]

where \(\Delta x = r \cos \theta, \Delta y = r \sin \theta\) was taken. (We have neglected the unphysical direction \(|\theta| = \frac{\pi}{2}\) that corresponds to approaching the point \((x = x^*, y = 0)\) along the line \(x = x^*\) where \(|H|\) is divergent.) So, in the limit \(x \to x^*, y \to 0\) the value of \(H\) is determined by the lowest non-zero derivative of \(\frac{1}{2\omega(x) + 3}\). If the both the first and second derivative vanish, then \(|H|\) diverges implying a spacetime singularity. If the first derivative vanishes but the second derivative is not zero,

\[
\frac{d}{dx} \left( \frac{1}{2\omega(x) + 3} \right) x = x^* = 0, \quad \frac{d^2}{dx^2} \left( \frac{1}{2\omega(x) + 3} \right) x = x^* \neq 0,
\]

then \(H\) is finite but (possibly) different for each solution as it depends on the angle of approach \(\theta\), while the Friedmann equation in this case acquires an extra term when compared to general relativity. If the first derivative is not zero,

\[
\frac{d}{dx} \left( \frac{1}{2\omega(x) + 3} \right) x = x^* = \frac{1}{(2\omega(x^*) + 3)^2} \frac{d\omega}{dx} x = x^* \neq 0,
\]

then \(H\) approaches the value \(H^2 = \frac{\kappa^2}{3x^*} (\rho + V(x^*))\), mimicking the Friedmann equation of general relativity with \(8\pi G = \frac{\kappa^2}{x^*}\) and \(\Lambda = \frac{\kappa^2}{x^*} V(x^*)\).

To summarize, we have just observed that in the limit (a) \(\frac{1}{2\omega(x) + 3} \to 0\), (b) \(y \to 0\) the Friedmann constraint \((11)\) tends to the form of general relativity if (c) \(\frac{1}{(2\omega(x^*) + 3)^2} \frac{d\omega}{dx} x = x^* \neq 0\). It must be also emphasized here that the process of taking the Taylor expansion \((11)\) hinges on the assumption that (d) \(\frac{1}{2\omega(x) + 3}\) is differentiable (derivatives do not diverge) at \(x^*\). In this context one may also ask when the full set of equations \((6)-(9)\) attains the form of general relativity. It is easy to see that besides (a)-(d) one must also impose

\[
\frac{1}{2\omega(x) + 3} \frac{d\omega}{dx} y^2 = \frac{1}{(2\omega(x^*) + 3)^2} \frac{d\omega}{dx} \left( (2\omega(x) + 3)y^2 \right) \to 0,
\]

but the latter is automatically satisfied if (c) holds, due to \((11), (13)\). Therefore we may tentatively call the conditions (a)-(d) ‘the general relativity limit of scalar-tensor flat FLRW cosmology’.
It is interesting to compare the cosmological GR limit to the GR limit obtained from PPN, which characterizes the slow motion approximation in a centrally symmetric gravitational field. Although the mathematical assumptions underlying the PPN formalism are clearly different from our cosmological reasoning above, we may still ask whether the results of both schemes agree with each other. In the context of PPN it is well established that the solutions of scalar-tensor theory approach those of general relativity when

\[
\frac{1}{2\omega(x) + 3} \to 0, \quad \frac{1}{(2\omega(x) + 3)^3} \frac{d\omega}{dx} \to 0.
\]  

Comparison shows that the cosmological conditions (a)-(d) are marginally stricter than the PPN condition (15), since (a), (c), (d) imply that (15) is satisfied, but (15) does not necessarily guarantee that (c) or (d) holds.

Let us also note that there is also another special case \( x_\bullet \), realized at

\[
\rho = 0, \quad y = 0, \quad x_\bullet \quad V(x_\bullet) - \frac{dV(x)}{dx} \bigg|_{x_\bullet} = 0,
\]  

when the cosmological equations (6)-(9) relax to those of general relativity featuring de Sitter evolution. However, as the value of \( \omega(x_\bullet) \) is not fixed by the condition (16), this case does not conform with the GR limit of PPN. Therefore, even when the limits (a)-(d) and (16) can be cosmologically indistinguishable, Solar System observations in the PPN framework can in principle reveal which of the two is actually realized. (In this paper when using the phrase ‘the GR limit of STT’ we mean the conditions (a)-(d), as these take the STT cosmological equations to those of general relativity and also guarantee that the PPN condition is satisfied. But note that some authors [11, 12] have not necessarily used the same definition.)

The general relativity limit of STT is purely given in terms of \( x \) and \( y \). In the following we extract from the full dynamical system (6)-(9) an independent subsystem for \( \{x, y\} \), find its fixed points and check whether the limit of general relativity matches to an attractive fixed point.

3 Fixed points for potential domination \((V \neq 0, \rho \equiv 0)\)

In the four phase space dimensions \( \{\Psi \equiv x, \dot{\Psi} \equiv y, H, \rho\} \) the physical trajectories (orbits of solutions) are those which satisfy the Friedmann constraint (2). In the limit of vanishing matter density the phase space shrinks to three dimensions \( \{x \equiv \Psi, y \equiv \dot{\Psi}, H\} \), where the Friedmann constraint restrains the physical trajectories to span two dimensions. We may solve the Friedmann constraint for \( H \), as (10), substitute it into Eq. (7), and thus in effect reduce the system 2-dimensional:

\[
\begin{align*}
\dot{x} &= y \\
\dot{y} &= \left( \frac{3}{2x} - \frac{1}{2\omega(x) + 3} \frac{d\omega}{dx} \right) y^2 - \frac{1}{2x} \sqrt{3(2\omega(x) + 3)y^2 + 12\kappa^2 x V(x)} y \\
&\quad + \frac{2\kappa^2}{2\omega(x) + 3} \left( 2V(x) - x \frac{dV}{dx} \right).
\end{align*}
\]  

(17)
This constitutes a projection of the trajectories on the original two-dimensional constraint surface in \((x, y, H)\) to the \((x, y)\) plane. The projection yields two “sheets”: the “upper sheet” marked by the \(-\) sign, and the “lower sheet” marked by the the \((+)\) sign in Eq. (17).

Standard procedure reveals that the dynamical system (17) is endowed with two fixed points, Table 1 lists their conditions and eigenvalues. The first fixed point \(\Psi_{\bullet}\) satisfies

\[
\left. \frac{dV}{d\Psi} \right|_{\Psi_{\bullet}} \Psi_{\bullet} - 2V(\Psi_{\bullet}) = 0, (18)
\]

which matches the second limit (16), discussed in Sec. 2. The second fixed point \(\Psi_{\star}\) satisfies

\[
\left. \frac{1}{2\omega(\Psi_{\star}) + 3} \right| = 0, \quad \left. \frac{1}{(2\omega(\Psi_{\star}) + 3)^2} \frac{d\omega}{d\Psi} \right|_{\Psi=\Psi_{\star}} \neq 0, (19)
\]
i.e. exactly the same conditions (a)-(d) as the limit of general relativity for flat FLRW STT cosmology, discussed in the end of Sec. 2.

From Eq. (10) it is straightforward to compute that the values of \(H\) corresponding to the fixed points \(\Psi_{\bullet}\) and \(\Psi_{\star}\) are \(H_{\bullet} = (\mp) \sqrt{\frac{\kappa^2 \Psi_{\bullet}}{3V_{\bullet}}} \) and \(H_{\star} = (\mp) \sqrt{\frac{\kappa^2 \Psi_{\star}}{3V_{\star}}} \), respectively. The result, which mimics de Sitter evolution in general relativity, was expected, since we saw in Sec. 2 that under the fixed point conditions (18) and (19) the full STT equations (6)-(8) reduce to the equations of general relativity.

### 4 Fixed points for matter domination \((V \equiv 0, \rho \neq 0)\)

In the case of cosmological matter \((\rho > 0)\) and vanishing scalar potential the Friedmann constraint restricts the solutions onto a three-dimensional surface in four phase space dimensions \((x, y, H, \rho)\). However, the system is amenable to a change of the time variable [7] that allows to combine the field equations into a dynamical equation for the scalar field which does not manifestly contain the scale factor or matter density. In the Jordan frame this amounts to defining a new time variable, \(dp = h_{\phi} dt \equiv \left| H + \frac{\dot{\Psi}}{2V} \right| dt\), and deriving from Eqs. (2)–(4) a “master” equation for the scalar field [15, 16]. It can be written in a form of a dynamical
system for variables $\Psi \equiv x$, $\Psi' \equiv z$ ($f' \equiv df/dp$)

$$\begin{cases}
x' = z + \frac{(2\omega(x)+3)(1-w)}{8x^2} \frac{3(1+w)}{4x} \frac{d\omega(x)}{dx} z^3 + \left(3(1+w) - \frac{1}{2\omega(x)+3} \frac{d\omega(x)}{dx}\right) \frac{3(1-w)}{2} z + \frac{3(1-3w)}{(2\omega(x)+3)x} x.
\end{cases}$$

(20)

The signs in Eq. (20) correspond to the “upper” and the “lower” sheet as before in accordance with the constraint equation (10).

The “master” equation can be relied on as long as $h_c = |H + \frac{\dot{\Psi}}{\Psi}|$ is finite. At $\Psi = 0$ the quantity $h_c$ diverges making the $t$-time to stop with respect to the $p$-time. Hence all $t$-time trajectories with finite $\dot{\Psi}$ get mapped to $\Psi' = 0$, giving a false impression of a fixed point there. However, in Sec. 2 we concluded that $\Psi = 0$ comes with a space-time singularity and exclude it from present analysis.

The other problematic points can be discussed by noticing that in terms of the new time variable $p$ the Friedmann constraint (2) can be written as

$$h_c^2 = \kappa^2 \rho \left(1 - \frac{\kappa^2 \rho}{12\Psi^2}\right).$$

(21)

To keep $h_c$ real, the right hand side of Eq. (21) must be nonnegative, thus constraining the dynamically allowed regions of the two-dimensional phase space ($\Psi, \Psi'$) of the scalar field. It is also important to verify that a fixed point in the $p$-time does indeed correspond to a fixed point in the cosmological $t$-time. But since on physical grounds it is reasonable to consider only the trajectories with finite $h_c$, it is immediate that $\Psi' = 0$ implies $\dot{\Psi} = 0$ due to redefinition $p = p(t)$.

Let us consider the cosmological matter behaving like dust ($w = 0$). An argument completely analogous to the one put forth for Eq. (11), reveals a single fixed point, satisfying the conditions (a)-(d) dubbed as the limit of general relativity for flat FLRW STT. The corresponding eigenvalues, to be evaluated at the fixed point coordinate, are given in Table 2. In particular, this point is an attractor on the “upper” sheet if $\frac{d\omega}{d\Psi} > 0$, while on the “lower” sheet attractor behavior is not possible. From Eq. (10) now it also follows that at the fixed point the evolution of the universe obeys the usual Friedmann equation from general relativity, $H_*^2 = \frac{\kappa^2 \rho}{3\Psi^2}$. This is expected, as the fixed point conditions were identical to the general relativity limit.

The dynamical system in the radiation dominated regime ($w = \frac{1}{3}$) has no fixed points. For small values of $\Psi'$ the system is ruled by friction on the “upper” sheet, as the $-$ sign of the dominating term forces the vector flow to converge to the $\Psi'' = 0$ axis. On the “lower” sheet, the the effect is the opposite (anti-friction).
| Case          | Fixed point | Condition                                                                 | Eigenvalues |
|--------------|-------------|---------------------------------------------------------------------------|-------------|
| $w = 0$      | $\Psi_*$    | $\frac{1}{2\omega(\Psi_*) + 3} = 0$, $\frac{1}{(2\omega(\Psi_*) + 3)^2} \frac{d\omega}{d\Psi} \neq 0$ | $\pm \frac{3}{4} \pm \frac{3}{4} \sqrt{1 - \frac{32}{9}} \frac{\Psi}{(2\omega + 3)^2} \frac{d\omega}{d\Psi} \Psi_*$ |
| $w = \frac{1}{3}$ | none        |                                                                           |             |

Table 2: Fixed points and their eigenvalues for the $\rho \neq 0$, $V \equiv 0$ case.

5 Conclusion

We have considered flat FLRW cosmological models in general scalar-tensor theories with arbitrary coupling function $\omega(\Psi)$ and scalar potential $V(\Psi)$ in the Jordan frame. Using the methods of dynamical systems we have found the scalar field fixed points in two distinct asymptotic regimes: potential domination ($V \neq 0$, $\rho \equiv 0$), and matter domination ($V \equiv 0$, $\rho \neq 0$). In nutshell there are two types of fixed points arising from different mechanisms: $\Psi_*$ from a condition on the potential and $\Psi_\star$ from the singularity of the scalar field kinetic term. Approaching both types of fixed points the cosmological equations coincide with those of general relativity, yielding de Sitter expansion in the potential domination case and Friedmann evolution in the matter domination case. However, for the Solar System experiments in the PPN framework only the fixed points of $\Psi_*$ type give predictions identical with those of general relativity. The nature of fixed points (attractor or otherwise) depends on the functional forms of $\omega(\Psi)$ and $V(\Psi)$ according to corresponding eigenvalues given in Tables 1 and 2. Therefore, in Jordan frame analysis, general relativity is an attractor for a large class of scalar-tensor models, but not for all.

Provided the transformation relating the Jordan and the Einstein frame is regular, there is an exact correspondence between the two frames and the Jordan frame phase space results should carry over to the Einstein frame [17, 18, 19, 20]. This is the case for the fixed point $\Psi_*$. However, as the transformation of the scalar field fails to be regular in the limit of general relativity, the properties of the $\Psi_\star$ fixed point may be altered in the Einstein frame [17]. To establish whether or how the correspondence holds in this case calls for a separate matching investigation in the Einstein frame.

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