The partial decay width of $P_c(4312)$ as a $\bar{D}\Sigma_c$ molecular state

Yong-Jiang Xu$^1$, Chun-Yu Cui$^2$, Yong-Lu Liu$^1$, and Ming-Qiu Huang$^{1\dagger}$

$^1$Department of Physics, College of Liberal Arts and Sciences, National University of Defense Technology, Changsha, 410073, Hunan, China and
$^2$Department of Physics, Third Military Medical University (Army Medical University), Chongqing, 400038, China

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In the present work, we investigate the partial decay width of $P_c(4312) \to J/\psi p$ under the assumption that $P_c(4312)$ is a $\bar{D}\Sigma_c$ molecular state via QCD sum rule method. Firstly, we calculate the spectral parameters, mass and the residual of mass, which are two of the input parameters as we investigate the strong decay form factors in the next step. After obtaining the numerical values of the two form factors, we finally give the partial decay width of $P_c(4312) \to J/\psi p$ which are compatible with the total width of $P_c(4312)$. Our results suggest that it is reasonable to assign $P_c$ to be a $\bar{D}\Sigma_c$ molecular state.

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I. INTRODUCTION

As early as the birth of the quark model, the concept of the multiquark states appeared, whose quark substructure may be $qq\bar{q},qqq\bar{q}$ and so on. There have been many theoretical and experimental study on the multiquark states since the observation of $X(3872)$ in 2003 by the Bell Collaboration (see review articles [1] for details).

Pentaquark states, one kind of the multiquark states, are the focus of the research on the multiquark states, especially after the discovery of the $P_c(4380)$ and $P_c(4450)$ states in 2015 by the LHCb Collaboration [2]. These studies based on different models and assumptions about the quark configurations in the hadrons, including meson-baryon molecules [3–9], diquark-diquark-antiquark pentaquarks [10–13], compact diquark-triquark pentaquarks [14, 15], the topological soliton model [16], genuine multiquark states other than molecules [17], and kinematical effects related to the triangle singularity [18–20], etc.

Recently, a new pentaquark state $P_c(4312)$ was discovered by the LHCb Collaboration in the $J/\psi p$ invariant mass spectrum of the $\Lambda_b \to J/\psi pK$ decay [21]. Triggered by this observation, there are many theoretical investigation on the properties of $P_c(4312)$ [22–24].
However, to gain a deep understanding on their nature and substructure, which are still not certain yet, it is necessary to do more experimental and theoretical investigations which may help us to learn more about their properties. Studying their possible decay channels may provide valuable insights in this respect.

In this article, we study the spectral parameters and strong decay property of $P_c(4312)$ in the QCD sum rule method promoted by M.A. Shifman, A.I. Vainshtein and V.I. Zakharov in 1979 [40]. The basic idea of QCD sum rule is that we can calculate the correlation function of the interpolating currents of hadrons we are interested in from both phenomenological and QCD sides, then match the two representation and extract the physical quantities of the considered hadron. One of the most important steps during the calculation is making a Borel transform which can simultaneously improve the convergence of OPE on the QCD side and suppress the contributions of higher and continuum states. The QCD sum rule method has extensively been used to investigate the X,Y,Z states (see the review article [41] for the detail). For the ground pentaquark states, we also can use this method to study their relevant properties. In fact, there are some related works on the pentaquark states in the QCD sum rule method [22, 38, 42–45] based on the meson-baryon molecular configuration assumption and [13] assuming the diquark-diquark-antiquark substructure.

The rest of the paper is organized as follows. In section II, we give the sum rules for the spectral parameters and the strong decay form factors of $P_c(4312)$. Section III is devoted to the numerical analysis, the partial decay width and a short summary.

II. QCD SUM RULES OF THE SPECTRAL PARAMETERS AND STRONG DECAY FORM FACTORS OF $P_c(4312)$

In this section, we first give the spectral parameters of $P_c(4312)$, which are the input parameters of the calculation of the form factors.

In the QCD sum rule method, the starting point of the calculation of the spectral parameters, mass and residual of mass, is the 2-point correlation function:

$$\Pi(p) = i \int dx^4 e^{ipx} \langle 0 | T(J(x))J(0) | 0 \rangle = \Pi_1(p^2) + \Pi_2(p^2)$$

where $J(x)$ is the interpolating current of $P_c(4312)$ viewed as $(\bar{D}\Sigma_c)$ molecular states in the present work. According to ref. [43], $J(x)$ can take the form

$$J(x) = [\bar{c}(x)i\gamma_5d(x)][\epsilon^{ijk}(u^T_1(x)C\gamma_\mu u_3(x))\gamma^\mu\gamma_5c_k(x)]$$

where $T$ denotes the matrix transposition, $C$ means charge conjugation, and $i,j,k$ are color indices.

By inserting into the correlation function $\Pi(p)$ a full set of relevant states having the same quantum numbers as $J(x)$, we get the phenomenological representation of $\Pi(p)$ in
terms of the hadronic parameter,

\[ \Pi^{\text{phe}}(p) = \lambda_{Pc}^2 \frac{p^4 + M_{Pc}^2}{m_{Pc}^2 - p^2} + \frac{1}{\pi} \int_{s_0^{Pc}}^\infty ds \frac{\text{Im} \Pi_1^{\text{phe}} + \text{Im} \Pi_2^{\text{phe}}}{s - p^2}, \]

(3)

where \( m_{Pc} \) is the hadron mass, \( \lambda_{Pc} \) is defined as \( \langle 0 | J(0) | P_c(p, s) \rangle = \lambda_{Pc} u(p, s) \) and \( s_0^{Pc} \) is the threshold parameter.

On the other hand, \( \Pi(p) \) can be calculated from QCD via OPE method in terms of the quark propagators. To this end, we insert the interpolating current \( J(x) \) into the correlation function and contract the relevant quark fields and find

\[ \Pi^{\text{OPE}}(p) = -2i \epsilon_{abc} \epsilon_{a'b'c'} \int d^4 x e^{ipx} \gamma^\mu \gamma_5 S_{cc}^{(4)}(x) \gamma^\nu \gamma_5 Tr[i \gamma_5 S_{dd}^{(d)}(x) (i \gamma_5) S_{cc}^{(c)}(-x)] \\
Tr[\gamma_\mu S_{bb}^{(u)}(x) \gamma_\nu CS_{aa}^{(u)T}(x) C], \]

(4)

with \( S_{c}(x) \) and \( S_{u(d)}(x) \) are the charm- and up(down)-quark propagator, which are given by

\[ S_{ij}^q(x) = \frac{i}{2\pi^2 x^4} \delta_{ij} - \frac{m_q}{4\pi^2 x^2} \delta_{ij} - \frac{\langle \bar{q}q \rangle}{12} \delta_{ij} + i \frac{\langle \bar{q}q \rangle}{48} m_q \delta_{ij} - \frac{x^2}{192} (g_s \bar{q}q Gq) \delta_{ij} + \frac{x^2}{1152} \frac{m_q}{32\pi^2 x^2} (g_s \bar{q}q Gq) \delta_{ij} - i \frac{g_s t^a_i G^a_{\mu\nu}}{32\pi^2 x^2} (\gamma \sigma^{\mu\nu} + \sigma^{\mu\nu} \gamma) + \cdots, \]

(5)

for light quarks, where \( t^a = \frac{\lambda^a}{2} \) and \( \lambda^a \) are the Gell-Mann matrix, \( g_s \) is the strong coupling constant, and \( i, j \) are color indices. Through dispersion relation, \( \Pi^{\text{OPE}}(p) \) can be written as

\[ \Pi^{\text{OPE}}(p) = \frac{1}{p^4} \int_{4m_Q^2}^\infty ds \rho_1(s) \frac{s}{s - p^2} + \int_{4m_Q^2}^\infty ds \rho_2(s) \frac{s}{s - p^2}, \]

(7)

where \( \rho_i(s) = \frac{1}{2} \text{Im} \Pi_i^{\text{OPE}}(s), i = 1, 2 \) are the spectral densities. We will give the explicit expression of \( \rho_1(s) \) up to dimension-9 and \( \alpha_s \) order in the following.

Matching the phenomenological and QCD representations, using the quark-hadron duality and making a Borel transform, we obtain the sum rules,

\[ \lambda_{Pc}^2 e^{-\frac{m_{Pc}^2}{M_B^2}} = \int_{4m_Q^2}^{s_0^{Pc}} ds \rho_1(s) e^{-\frac{s}{M_B^2}}, \]

(8)

and

\[ \lambda_{Pc}^2 m_{Pc} e^{-\frac{m_{Pc}^2}{M_B^2}} = \int_{4m_Q^2}^{s_0^{Pc}} ds \rho_2(s) e^{-\frac{s}{M_B^2}}, \]

(9)
where $M_B^2$ is the Borel parameter. To get the sum rules for the mass and the residual of mass, taking derivative of Eqs. (8) and (9) with respect to $\frac{1}{M_B}$ and dividing by the equation itself, the results are

$$m_{Pc}^2 = \int_{4m_Q^2}^{m_P^2} ds \rho_1(s)e^{-\frac{s}{M_B^2}} / \int_{4m_Q^2}^{\infty} ds \rho_1(s)e^{-\frac{s}{M_B^2}}, \quad (10)$$

and

$$m_{Pc}^2 = \int_{4m_Q^2}^{m_P^2} ds \rho_2(s)e^{-\frac{s}{M_B^2}} / \int_{4m_Q^2}^{\infty} ds \rho_2(s)e^{-\frac{s}{M_B^2}}. \quad (11)$$

Substituting the obtained value of the mass into Eqs. (8) or (9), we can give the value of the residual of mass.

Now, we give the explicit expression of the spectral density,

$$\rho_1(s) = \rho_1^0(s) + \rho_1^{(\bar{q}q)}(s) + \rho_1^{(q_2GG)}(s) + \rho_1^{(g_\sigma q G q)}(s) + \rho_1^{(q q)^2}(s) + \rho_1^{(g q q G q)}(s) + \rho_1^{(g q q G G)}(s) + \rho_1^{(q q)^3}(s), \quad (12)$$

with

$$\rho_1^0(s) = \frac{-1}{20480\pi^8} \int_{a_{\text{min}}}^{a_{\text{max}}} \frac{da}{a^4} \int_{b_{\text{min}}}^{1-a} \frac{db}{b^4} (1 - a - b)^3((a + b)m_c^2 - abs)^5, \quad (13)$$

$$\rho_1^{(\bar{q}q)}(s) = \frac{\langle \bar{q} q \rangle}{256\pi^5} m_c \int_{a_{\text{min}}}^{a_{\text{max}}} \frac{da}{a^2} \int_{b_{\text{min}}}^{1-a} \frac{db}{b^2} (1 - a - b)^2((a + b)m_c^2 - abs)^3, \quad (14)$$

$$\rho_1^{(q_2GG)}(s) = -\frac{\langle g_2^2 GG \rangle}{24576\pi^8} m_c \int_{a_{\text{min}}}^{a_{\text{max}}} \frac{da}{a^4} \int_{b_{\text{min}}}^{1-a} \frac{db}{b^4} (a^3 + b^3)(1 - a - b)^3((a + b)m_c^2 - abs)^2 \text{ and }$$

$$-\frac{\langle g_2^2 GG \rangle}{16384\pi^8} \int_{a_{\text{min}}}^{a_{\text{max}}} \frac{da}{a^4} \int_{b_{\text{min}}}^{1-a} \frac{db}{b^3} (2a + b)(1 - a - b)^2((a + b)m_c^2 - abs)^3, \quad (15)$$

$$\rho_1^{(g_\sigma q G q)}(s) = \frac{3\langle g_\sigma q G q \rangle}{512\pi^6} m_c \int_{a_{\text{min}}}^{a_{\text{max}}} \frac{da}{a} \int_{b_{\text{min}}}^{1-a} \frac{db}{b^2} (1 - a - b)((a + b)m_c^2 - abs)^2$$

$$-\frac{3\langle g_\sigma q G q \rangle}{512\pi^6} m_c \int_{a_{\text{min}}}^{a_{\text{max}}} \frac{da}{a} \int_{b_{\text{min}}}^{1-a} \frac{db}{b^3} (1 - a - b)^2((a + b)m_c^2 - abs)^2, \quad (16)$$

$$\rho_1^{(q q)^2}(s) = \frac{\langle q q \rangle^2}{64\pi^4} \int_{a_{\text{min}}}^{a_{\text{max}}} \frac{da}{a} \int_{b_{\text{min}}}^{1-a} \frac{db}{b} ((a + b)m_c^2 - abs)^2, \quad (17)$$

$$\rho_1^{(g q q G q)}(s) = \frac{\langle g q q G q \rangle}{3072\pi^6} m_c \int_{a_{\text{min}}}^{a_{\text{max}}} \frac{da}{a^2} \int_{b_{\text{min}}}^{1-a} \frac{db}{b^4} (a^3 + b^3)(1 - a - b)^2$$

$$+\frac{\langle g q q G q \rangle}{1024\pi^6} m_c \int_{a_{\text{min}}}^{a_{\text{max}}} \frac{da}{a} \int_{b_{\text{min}}}^{1-a} \frac{db}{b^3} (1 - a - b)^2((a + b)m_c^2 - abs)$$

$$+\frac{\langle g q q G q \rangle}{1024\pi^6} m_c \int_{a_{\text{min}}}^{a_{\text{max}}} \frac{da}{a} \int_{b_{\text{min}}}^{1-a} \frac{db}{b} (1 - a - b)((a + b)m_c^2 - abs), \quad (18)$$
\begin{align}
\rho_1^{(q\bar{q})(g^2GG)}(s) &= -\frac{\langle\bar{q}q \rangle}{128\pi^4} \int_{a_{\text{min}}}^{a_{\text{max}}} da \int_{b_{\text{min}}}^{1-a} db ((a+b)m_c^2 - ab) \\
&- \frac{\langle\bar{q}q \rangle}{64\pi^4} \int_{a_{\text{min}}}^{a_{\text{max}}} da (m_c^2 - a(1-a)s), \\
\rho_1^{(q\bar{q})^3}(s) &= -\frac{\langle\bar{q}q \rangle^3}{24\pi^2 m_c} \int_{a_{\text{min}}}^{a_{\text{max}}} d aa,
\end{align}

where \( a_{\text{max}} = \frac{1+\sqrt{1-4a^2}}{2} \), \( a_{\text{min}} = \frac{1-\sqrt{1-4a^2}}{2} \) and \( b_{\text{min}} = \frac{am_c^2}{as-m_c^2} \).

With the above results about the spectral parameters of \( P_c(4312) \), we now turn to the calculation of the form factors of the strong decay \( P_c(4312) \rightarrow J/\psi p \). To this end, we begin with the following 3-point correlation function,

\begin{align}
\Gamma_\mu(p, p', q) = i^2 \int d^4x d^4y e^{ip'x + iqy} \langle 0 | T J^N(x) J^{J/\psi}_\mu(y) P_c(4312)(0) | 0 \rangle,
\end{align}

where \( p = p' + q, J^P_c(x) \) is the interpolating current of \( P_c(4312) \) defined in Eq. (2), \( J^N(x) \) and \( J^{J/\psi}_\mu(x) \) are the interpolating currents of proton and \( J/\psi \) respectively. The interpolating currents take the following form,

\begin{align}
J^N(x) &= \epsilon_{abc}[u_a^T(x)C \gamma_\mu u_b(x)] \gamma_5 \gamma_\mu d_c(x), \\
J^{J/\psi}_\mu(x) &= \bar{c}(x) \gamma_\mu c(x).
\end{align}

In order to get the physical representation of the 3-point correlation function [21], we insert complete sets of states having the same quantum numbers with the interpolating currents and use the following definitions,

\begin{align}
\langle 0 | J^N | N(p') \rangle &= \lambda_N u^N(p'), \\
\langle 0 | J^{J/\psi}_\mu | J/\psi(q) \rangle &= f_{J/\psi} m_{J/\psi} \epsilon_\mu(q), \\
\langle N(p') J/\psi(q) | P_c(p) \rangle &= \epsilon_\mu^*(q) u^N(p') [f_1 \gamma_\mu - if_2 \frac{\sigma^{\mu\nu} q_{\nu}}{m_N + m_{P_c}}] i\gamma_5 u^{P\mu}_c(p),
\end{align}

where \( f_{J/\psi} \) and \( \epsilon_\mu(q) \) are the decay constant and polarization vector of the \( J/\psi \) state, \( \lambda_N \) and \( u^N(p') \) are the residual and spinor of the proton, and \( f_1 \) and \( f_2 \) are the strong decay form factors, respectively. Finally, we obtain the phenomenological side of the sum rules,

\begin{align}
\Gamma_\mu(p, p', q) &= \frac{\lambda_N \lambda_{P_c} f_{J/\psi} m_{J/\psi}}{(m^2_{P_c} - p'^2)(m^2_N - p^2)(m^2_{J/\psi} - q^2)} \left[ (f_1 \frac{2m^2_{J/\psi} - p^2 + p'^2}{m^2_{J/\psi}} + f_2 \frac{m_N - m_{P_c}}{m_N + m_{P_c}}) \not{p'} \gamma_5 q_\mu \\
&\quad - (f_1 (m^2_N + m_{P_c}) + f_2 \frac{p'^2 - p^2}{m_N + m_{P_c}}) \not{p'} \gamma_\mu \gamma_5 \right] + \text{other structures} + \cdots
\end{align}

where we only remain the two Lorentz structures we are interested in, \( \not{p'} \gamma_5 q_\mu \) and \( \not{p'} \gamma_\mu \gamma_5 \).
On the theoretical side, inserting the interpolating currents into the 3-point correlation function and contracting the quark fields, we obtain the following representation of the correlation function,

\[ \Gamma_\mu(p, p', q) = 2i^2 \epsilon_{abc} \epsilon_{a'b'c'} \int d^4x d^4y e^{i p' x + i q y} \gamma_\alpha S_{cd}^{(d)}(x) i \gamma_5 S_{ad}^{(c)}(y) \gamma_\beta \gamma_5 \]

\[ Tr[\gamma_\alpha S_b^{(u)}(x) \gamma_\beta C S_a^{(u)}(x) C] = \Gamma_1(P^2, p'^2, q^2) \rho' \gamma_\beta q_\mu + \Gamma_2(P^2, p'^2, q^2) \rho' \gamma_\mu \gamma_5 + \cdots. \tag{26} \]

where \( P^2 = -p^2 \).

Following the same steps as the two-point case, we can get the sum rules for the form factors \( f_1(P^2) \) and \( f_2(P^2) \),

\[ f_1(P^2) = \left[ \frac{m_{J/\psi}^2 + P^2}{\lambda_1 \lambda_2 f_{J/\psi}(m_{J/\psi}^2 + m_N^2 + 2P^2 + (m_N^2 + P^2)^2)} \right] \left[ \frac{m_{J/\psi}^2 + (m_{J/\psi}^2 + (m_N^2 + P^2)^2)}{m_{J/\psi}^2 + (m_{J/\psi}^2 + m_N^2 + 2P^2 + (m_N^2 + P^2)^2)} \right] \tag{27} \]

\[ f_2(P^2) = -\left[ \frac{m_{J/\psi}^2 + (m_N^2 + m_j^2)}{\lambda_1 \lambda_2 f_{J/\psi}(m_{J/\psi}^2 + m_N^2 + 2P^2 + (m_N^2 + P^2)^2)} \right] \left[ \frac{m_{J/\psi}^2 + (m_N^2 + m_j^2)}{m_{J/\psi}^2 + (m_{J/\psi}^2 + m_N^2 + 2P^2 + (m_N^2 + P^2)^2)} \right] \tag{28} \]

where \( M_{B_1}^2 \) and \( M_{B_2}^2 \) are the Borel parameters corresponding to \( q^2 \) and \( p^2 \) respectively. The coefficients \( \Gamma_1(P^2) \) and \( \Gamma_2(P^2) \) can be written as via double dispersion relation,

\[ \Gamma_i(P^2) = \int_{4m_c^2}^{\infty} ds \int_0^{\infty} du \rho_i^{(3)}(P^2, s, u), \tag{29} \]

where \( \rho_i^{(3)}(P^2, s, u), i = 1, 2 \) are the spectral densities whose explicit expressions, up to dimension-6 and \( \alpha_s \) order, are with

\[ \rho_i^{(3)}(P^2, s, u) = \frac{(g q)_{576 \pi^4 s^2}}{s(s - 4m_c^2)}(s + 2m_c^2)(P^2 + s + u) \]

\[ -\frac{9216 \pi^6 s}{s(s - 4m_c^2)} \]

\[ + \frac{288 \pi^4 s}{s(s - 4m_c^2)} \]

\[ (g_s, q, Gq) m_c^2(P^2 + s + u) \]

\[ -\frac{(g_s, q, Gq) \sqrt{s(s - 4m_c^2)}(s + m_c^2)(P^2 + s + u)}{1152 \pi^4 s^2} \delta(u) \]

\[ -\frac{(g q)^2 m_c \sqrt{s(s - 4m_c^2)}}{24 \pi^4 s^2} \delta(u), \tag{30} \]
and

$$\rho_{2}^{(3)}(P^2, s, u) = -u^2 \frac{\sqrt{s(s - 4m^2_c)(s + 2m^2_c)}}{6144\pi^6 s} - \frac{\langle \bar{q}q \rangle m_c \sqrt{s(s - 4m^2_c)}(P^2 + s + u)}{192\pi^4 s}$$

$$+ \frac{\langle g^2_s GG \rangle s u^2(s - 3m^2_c)}{36864\pi^6 M^4_B \sqrt{s(s - 4m^2_c)}} - \frac{\langle g^2_s GG \rangle u^2(2s - 5m^2_c)}{36864\pi^6 M^2_B \sqrt{s(s - 4m^2_c)}}$$

$$- \frac{\langle g^2_s GG \rangle \sqrt{s(s - 4m^2_c)(s + 2m^2_c)}}{147456\pi^6 s \sqrt{s(s - 4m^2_c)}} - \frac{\langle g_s \bar{q}\sigma Gq \rangle m_c (s - m^2_c)(P^2 + s + u)}{12288\pi^6 s}$$

$$- \frac{\langle g_s \bar{q}\sigma Gq \rangle m_c \sqrt{s(s - 4m^2_c)}(P^2 + s + u)}{576\pi^4 s \sqrt{s(s - 4m^2_c)}} + \frac{\langle g_s \bar{q}\sigma Gq \rangle m_c \sqrt{s(s - 4m^2_c)}(P^2 + s + u)}{384\pi^4 s} \delta(u),$$

where $\delta(u)$ is the Dirac $\delta$-function.

### III. NUMERICAL ANALYSIS AND THE PARTIAL DECAY WIDTH

The QCD sum rules for the spectral parameters and the strong decay form factors contain some input parameters which are required to obtain the numerical values of these quantities. We present them in Table I. Beside these input parameters, there are a few auxiliary parameters introduced during the calculations: the continuum thresholds and the Borel parameters. These are not physical quantities, hence the physical observable should be approximately insensitive to them. Therefore, we look for working regions of these parameters such that the dependence of the mass on these parameters are weak. The continuum thresholds are related to the square of the first exited states having the same quantum numbers as the interpolating currents, while the Borel parameters are determined by demanding that both the contributions of the higher states and continuum are sufficiently suppressed and the contributions coming from higher dimensional operators are small.

Firstly, we analyze the spectral parameters: mass and the residual of mass. In Fig I, we compare the various OPE contributions as functions of $M^2_B$ with $s_0^{P_c} = 4.8GeV$ and represent the ratio of the pole and continuum contribution, the ratio of the $\langle \bar{q}q \rangle^3$ term and total contribution varying with $M^2_B$ with $s_0^{P_c} = 4.8GeV$. From the figure, we can see that it is needed to limit $M^2_B$ from 2.3GeV$^2$ to 2.7GeV$^2$ in order to simultaneously satisfy the requirements of pole dominance at the phenomenological side (the pole contribution is bigger than the continuum contribution) and convergence of the operator product expansion.
TABLE I: Some input parameters needed in the calculations.

| Parameter | Value |
|-----------|-------|
| $\langle \bar{q}q \rangle$ | $-(0.24 \pm 0.01)^3 GeV^3$ |
| $\langle g_s \bar{q}\sigma Gq \rangle$ | $(0.8 \pm 0.1) \langle \bar{q}q \rangle GeV^2$ |
| $\langle g_s^2 GG \rangle$ | $0.88 \pm 0.25 GeV^4$ |
| $m_c$ | $1.275^{+0.025}_{-0.035} GeV [46]$ |
| $m_{J/\psi}$ | $3096.900 \pm 0.006 MeV [46]$ |
| $m_N$ | $938.272081 \pm 0.00006 MeV [46]$ |
| $\lambda_N^2$ | $0.0011 \pm 0.0005 GeV^6 [47]$ |
| $f_{J/\psi}$ | $481 \pm 36 MeV [48]$ |

FIG. 1: Fig.(a) denotes the various OPE contributions as functions of $M_B^2$ with $\sqrt{s^P_c} = 4.8 GeV$; Fig.(b) represents the ratio of the pole and continuum contribution, the the ratio of the $\langle \bar{q}q \rangle^3$ term and total contribution varying with $M_B^2$ with $s^P_c = 4.8 GeV$.

After determining the interval of $M_B^2$, we can turn to the analysis of the mass and the residual of the mass. The results are represented in Fig.2 from which it is obvious that the sum rules for the mass and residual vary weakly with the continuum threshold parameter $s^P_c$. Borel parameter $M_B^2$ in the interval determined above. As a result, we can reliably read the values of the mass and residual: $m_{P_c} = 4.1 \pm 0.1 GeV^2$ which is agreement with the experimental value $m_{P_c} = 4311.9 \pm 0.7^{+0.8}_{-0.6} MeV [21]$ considering the accuracy of QCD sum rule method and $\lambda_{P_c} = (1.4 \pm 0.2) \times 10^{-3} GeV^6$.

Now it is time to study the form factors $f_1(P^2)$ and $f_2(P^2)$ of the strong decay $P_c(4312) \to J/\psi p$. Similar to the 2-point case, we should first determine the allowed ranges of the Borel parameters $M_{B_1}^2$ and $M_{B_2}^2$. To this end, the various OPE contributions of the Lorentz structure $\not p'\gamma_5 q_\mu$ as functions of the Borel parameters $M_{B_1}^2$ with $M_{B_2}^2 =$
FIG. 2: Figs. (a) and (b) shows the dependence of the mass sum rule and the residual sum rule on the Borel parameter $M^2_B$ in the interval determined above, respectively.

$1.4 GeV^2, P^2 = 4 GeV^2$ and $M^2_{B_2}$ with $M^2_{B_1} = 3 GeV^2, P^2 = 4 GeV^2$ are showed in Figs. (a) and (b), and Fig. (c) and (d) represents the ratio of the pole and total contributions and the ratio of the $\langle \bar{q}q \rangle^2$-term contribution and the total OPE series for the same Lorentz structure. Visually, when $1 GeV^2 \leq M^2_{B_1} \leq 3.8 GeV^2$ and $1 GeV^2 \leq M^2_{B_2} \leq 1.7 GeV^2$, both of the criteria: pole dominance at the phenomenological side and convergence of the operator product expansion can be met. Similar analysis can be done for the structure $\not\!P \gamma_\mu \gamma_5$. Finally, we obtain the following intervals of the Borel parameters: $1.6 GeV^2 \leq M^2_{B_1} \leq 3.3 GeV^2$ and $1 GeV^2 \leq M^2_{B_2} \leq 1.7 GeV^2$. In Fig. (d), the numerical results of the sum rules for the form factors $f_1$ and $f_2$ are presented. It is obvious that our sum rules for the form factors depend weakly on the Borel parameters and threshold parameters, indicating that our results are reliable.

With the working intervals of the auxiliary parameters and other input parameters, we can now obtain the dependence of the form factors on the $P^2$. Because of the limitation of the value of $P^2$ on the OPE side, it is necessary to make a fit in order to obtain the physical values of the form factors. In the present work, we apply the following fit function

$$f_i(P^2) = \frac{f_0}{1 + aP^2 + bP^4}, i = 1, 2$$

(32)

where $f_0, a$ and $b$ are the fit parameters having the values presented in Table III. To show the consistency of the fit function with the QCD sum rule results, we give the dependencies of the form factors on $P^2$ obtained from both sum rules and fit results in Fig. (e), which indicates that our fit function represents QCD sum rule results well in the region where the sum rule results are reliable. Substituting $P^2 = -m^2_{Pc}$ in the fit functions we can obtain the values of the form factors which are presented in Table III.

After determining all of the needed parameters, we turn to the main task of this work investigating the decay width of $P_c(4312) \to J/\psi p$. Using the transition matrix element
FIG. 3: The coefficients of the Lorentz structure $p'\gamma_5 q_\mu$ as functions of the Borel parameters $M^2_{B_1}$ with $M^2_{B_2} = 1.4 GeV^2$, $P^2 = 4 GeV^2$ and $M^2_{B_2}$ with $M^2_{B_1} = 3 GeV^2$, $P^2 = 4 GeV^2$ are compared in Figs.(a) and (b) respectively; Figs.(c) and (d) shows the ratio of the pole and total contributions and the ratio of the $\langle \bar{q}q \rangle^2$-term contribution and the total OPE series for the same Lorentz structure.

TABLE II: Fit parameters

| Form Factor | $f_0$  | $a$    | $b$   |
|-------------|--------|--------|-------|
| $f_1$       | -1.836 | -0.1489| 0.09816|
| $f_2$       | 3.388  | -0.1033| 0.04496|

TABLE III: Values of the form factors.

| Form Factor | value         |
|-------------|---------------|
| $f_1$       | $-0.292507^{+0.005}_{-0.015}$ |
| $f_2$       | $0.219414^{+0.15}_{-0.06}$    |
defined in Eq.24 and following the standard method, we obtain the decay width,

$$\Gamma(P_c(4312) \to J/\psi p) = \frac{(m_{p_c} + m_N)^2 - m_{J/\psi}^2}{16\pi m_{p_c}^2 m_{J/\psi}^2 (m_{p_c} + m_N)^2} \sqrt{(m_{p_c}^2 + m_N^2 - m_{J/\psi}^2)^2 - 4m_{p_c}^2 m_N^2}$$

$$[J_1^2(m_{p_c} + m_N)^2(2m_{J/\psi}^2 + m_{p_c} - m_N)^2 - 6f_1 f_2 m_{J/\psi}^2(m_{p_c}^2 - m_N^2) + f_2^2 m_{J/\psi}^2 (m_{J/\psi}^2 + 2m_{p_c} - m_N)^2].$$

Substituting the values of the parameters involved in the above formula, we find

$$\Gamma(P_c(4312) \to J/\psi p) = 6.5^{+3.7}_{-2.9}(MeV),$$

(34)

which are compatible with the total width of $P_c(4312)$ reported in [21] supporting the assignment of $P_c(4312)$ as the $D\Sigma_c$ molecular state.

To sum up, we study the partial decay width of $P_c(4312) \to J/\psi p$ under the assumption that $P_c(4312)$ is a $D\Sigma_c$ molecular state. Firstly, we calculate the spectral parameters, mass
and the residual of mass, which are two of the input parameters as we investigate the strong
decay form factors in the next step. After obtaining the numerical values of the two form
factors, we finally give the partial decay width of $P_c(4312) \rightarrow J/\psi p$ which are compatible
with the total width of $P_c(4312)$. Our results suggest that it is reasonable to assign $P_c$ to
be a $\bar{D}\Sigma_c$ molecular state.

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