Decomposition of sensitivity of the symmetry energy observables

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To exactly answer which density region that some frequently used symmetry-energy-sensitive observables probe, for the first time, we make a study of decomposition of the sensitivity of some symmetry-energy-sensitive observables. We find that for the Au+Au reaction at incident beam energies of 200 and 400 MeV/nucleon, frequently used symmetry-energy-sensitive observables mainly probe the density-dependent symmetry energy around 1.5 times saturation density. Effects of the symmetry energy from the low-density region is small but observable.

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In the last 20 years, great progress has been made in the study of a new branch of nuclear physics, i.e., isospin nuclear physics \cite{1–4}. Theoretical studies have shown that, within the parabolic approximation, the energy per particle in asymmetric nuclear matter can be approximately expressed as $E(\rho, \delta) = E(\rho, \delta = 0) + E_{\text{sym}}(\rho)\delta^2$, where $\delta \equiv (\rho_n - \rho_p)/(\rho_n + \rho_p)$ is the isospin asymmetry parameter and $E_{\text{sym}}(\rho)$ is the density-dependent nuclear symmetry energy. The latter has been studied for decades due to its great importance in both nuclear physics and astrophysics \cite{17, 18}. Although significant progress has been made, the symmetry energy is still subject to uncertainties especially at high density region \cite{8}. Nowadays, many sensitive observables have been identified as promising probes of the symmetry energy, such as the $\pi^-/\pi^+$ ratio \cite{4,15}, the neutron-proton ratio \cite{16,18}, $t/He$ \cite{19,20}, the isospin fractionation \cite{17,21,22} and the neutron-proton differential flow \cite{24,25}. However, one only knows these observables are overall sensitive to the high-density or low-density behavior of the symmetry energy at certain beam energy whereas none knows in which density region the symmetry energy has maximal effects. In this study, based on an isospin-dependent IBUU transport model, we addressed the above question.

In this study, the isospin-dependent single particle potential adopted reads

$$U(\rho, \delta, \tau) = U_0(\rho) + U_{\text{sym}}(\rho, \delta, \tau), \quad (1)$$

where the isoscalar potential

$$U_0(\rho) = -356u + 303u^{7/6}. \quad (2)$$

This soft nuclear isoscalar potential (SBKD) with $K_0 = 200$ MeV, was firstly introduced by Bertsch, Kruse and Das Gupta \cite{26}. For the isovector potential $U_{\text{sym}}(\rho, \delta, \tau)$, we use the following two forms \cite{27}

$$U_{\text{sym}}^{x=0}(\rho, \delta, \tau) = 4\tau\delta(1.27 + 25.4u - 9.31u^2 + 2.17u^3 - 0.21u^4) - \delta^2(1.27 + 9.31u^2 - 4.33u^3 + 0.63u^4), \quad (3)$$

and

$$U_{\text{sym}}^{x=1}(\rho, \delta, \tau) = \begin{cases} 4\tau\delta(3.08 + 39.6u - 29.2u^2 + 5.68u^3 - 0.52u^4) - \delta^2(3.08 + 29.2u^2 - 11.4u^3 + 1.57u^4), & \text{for neutrons (protons)} \end{cases} \quad (4)$$

where $u = \rho/\rho_0$ is the reduced baryon density and $\tau = 1/2 (-1/2)$ for neutrons (protons). As shown in figure \ref{fig:1} the two symmetry potentials roughly stand for two kinds of frequently used symmetry potentials ($U_{\text{sym}}^{x=0}$ and $U_{\text{sym}}^{x=1}$, corresponding to stiff and soft symmetry energies, respectively) in nuclear transport models \cite{28}.

To probe the density-dependent symmetry energy, it is instructive to know the time evolution of maximal baryon density reached in heavy-ion collisions. Figure \ref{fig:2} shows the evolution of the maximal baryon density reached in $^{197}$Au+$^{197}$Au reaction at beam energies of 400 and 200 MeV/nucleon with an impact parameter of $b=1$ fm. One can see that the maximal baryon density reached is about $2.5\rho_0$ for the incident beam energy of 400 MeV/nucleon.

FIG. 1: Density dependences of nucleon isovector potential for neutrons and protons. $\rho_0 = 0.168 fm^{-3}$ stands for normal nuclear matter density.

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\end{itemize}
FIG. 2: Evolution of the maximal baryon density reached in the reaction of $^{197}\text{Au} + ^{197}\text{Au}$ at beam energies of 400 MeV/nucleon and 200 MeV/nucleon with an impact parameter of $b=1$ fm.

FIG. 3: Evolution of the distribution probability of baryon number at different density regions in the central reaction of $^{197}\text{Au} + ^{197}\text{Au}$ at beam energies of 400 MeV/nucleon and 200 MeV/nucleon.

FIG. 4: Decomposition of sensitivity of the symmetry-energy-sensitive observable $\pi^-/\pi^+$ in different density regions.

FIG. 5: Decomposition of sensitivity of the symmetry-energy-sensitive observable free n/p in different density regions.

and about $2.0\rho_0$ for 200 MeV/nucleon case. And one can also see that the supra-density nuclear matter exists longer for 200 MeV/nucleon case than that for 400 MeV/nucleon case.

In order to better understand our target of studying the sensitivity of symmetry-energy-sensitive observables at different density regions, as shown in Figure 3, we provide the evolution of distribution probability of baryon number at different density regions. We can see that distribution probability of baryon number at different density regions changes with reaction time. More non-free baryons are located at density around $1.5\rho_0$ in the whole reaction process. In general, a small proportion of baryon is located at the highest density region. In the final stage, almost all baryons are free.

Since most of baryons are not always located at certain high-density region, it is thus necessary to study in which density region the symmetry-energy-sensitive observables shows maximal sensitivity. For this purpose, we divide the whole density region into several parts and use $U_x=0_{\text{sym}}$ as the standard calculation. To study the sensitivity of observable in certain density region, we also make calculation by switching the symmetry potential from standard input $U_x=0_{\text{sym}}$ to $U_x=1_{\text{sym}}$ only if baryons are located at that density region. In the following, we decompose the sensitivity of the frequently mentioned observables (charged $\pi^-/\pi^+$ ratio, free nucleonic n/p ratio, neutron-proton differential transverse flow $F_{n-p}^x$ as well as the difference of nucleon flow $F_{n}^x-F_{p}^x$) in the literature in different density regions.

The $\pi^-/\pi^+$ ratio was first proposed as a probe of n-
clear symmetry energy in 2002 by Li [9]. It is generally considered to be sensitive to the high-density behavior of the symmetry energy. Shown in Figure 4 is the decomposition of sensitivity of the symmetry energy observable $\pi^-/\pi^+$ in different density regions. For convenience, we define $R_0$ as the standard calculation of $\pi^-/\pi^+$ and $R_i$ as the value of $\pi^-/\pi^+$ for switching the symmetry potential in certain density region. The $(R_i - R_0)/R_0$ thus stands for the sensitivity of charged pion ratio $\pi^-/\pi^+$ in each density region. From Figure 4 we clearly see that the maximal sensitivity of $\pi^-/\pi^+$ to the density-dependent symmetry energy is around $1.25 \sim 1.5 \rho_0$. Above $1.5 \rho_0$, sensitivity of $\pi^-/\pi^+$ to the symmetry energy is roughly equal to that below $\rho_0$. With the increase of density, collision effect becomes larger. Thus one can see that the maximal sensitivity is not at the maximal density region reached. Due to rescatterings of pion meson in low-density region, charged pion ratio $\pi^-/\pi^+$ is also affected by the low-density behavior of the symmetry energy in some degree. Overall, the charged pion ratio $\pi^-/\pi^+$ is sensitive to the high-density behavior of nuclear symmetry energy. This can be seen by integrating the sensitivities at each density regions below and above $\rho_0$, respectively.

shown in Figure 5 are the decomposition of sensitivity of the symmetry energy observable free neutron to proton ratio $n/p$ in different density regions. From Figure 5, we can clearly see that the maximal sensitivity of free neutron to proton ratio $n/p$ at incident beam energy of 400 MeV/nucleon is around $1.5 \rho_0$. And overall the symmetry energy at low density has minor effect.

It is of interest to make similar study at incident beam energy below 400 MeV/nucleon since such experiments of probing the symmetry energy and related are doing at RIKEN (Japan) [29]. Shown in Figure 6 is the decomposition of sensitivity of the symmetry energy observable $\pi^-/\pi^+$ in different density regions at the beam energy of 200 MeV/nucleon. We can also see that the maximal sensitivity of $\pi^-/\pi^+$ to the density-dependent symmetry energy is around $1.25 \rho_0$. And it is sensitive to the high-density behavior of the symmetry energy but the low-density symmetry energy has effect in some degree. More interestingly, we see that the sensitivity of charged pion ratio $\pi^-/\pi^+$ to the symmetry energy is about one time larger than that for 400 MeV/nucleon beam energy. Therefore, using charged pion ratio $\pi^-/\pi^+$ to probe high-density behavior of the symmetry energy, it is better to do experiments at relative low incident beam energy [30].

As for the observable free neutron to proton ratio at incident beam energy of 200 MeV/nucleon, it is shown in Figure 7. The maximal sensitivity of free neutron to proton ratio $n/p$ at incident beam energy of 200 MeV/nucleon also locates around $1.5 \rho_0$. And overall the symmetry energy at low density region has minor effect.

The neutron-proton differential transverse flow, which was first proposed as a probe of nuclear symmetry energy by Li [24] in 2000, was defined as

$$F_{n-p}^x(y) = \frac{1}{N(y)} \sum_{i=1}^{N(y)} p_i^x(y) \omega_i, \quad (5)$$

where $N(y)$, $N_n(y)$ and $N_p(y)$ are the total number of free nucleons, free neutrons and free protons at rapidity $y$, $p_i^x$ is the momentum in $x$ direction of nucleon $i$ in the reaction plane, $\langle p_i^x(y) \rangle$ and $\langle p_i^y(y) \rangle$ are the average transverse momenta of free neutrons and free protons, respectively. $\omega_i$ is +1 (-1) for neutrons (protons). In the case of neutrons and protons have the same multiplicity ($N_n(y) = N_p(y)$) but different average transverse momenta ($p_n^x(y) \neq p_p^x(y)$), Eq. (5) is reduced to

$$F_{n-p}^x = \frac{1}{2} \left( \langle p_n^x(y) \rangle - \langle p_p^x(y) \rangle \right) = \frac{1}{2} (F_n^x - F_p^x). \quad (6)$$

The decomposition of sensitivity of flow $F_{n-p}^x$ and $F_n^x - F_p^x$ in different density regions at incident beam energy
of 400 MeV/nucleon is shown in Figure 8. We can see that stiffer symmetry energy ($U_{sym}^{\mu=0}$) leads to a stronger $F_{n-p}^{x}$ and inverse case happens for $F_{\pi}^{+} - F_{\pi}^{-}$. Again, we see that maximal sensitivity of $F_{n-p}^{x}$ at incident beam energy of 400 MeV/nucleon also roughly locates around 1.5$\rho_0$.

As shown in Figure 8 owing to baryons transitorily locate at the highest-density region coupled with larger collision effect, the symmetry-energy-sensitive observable usually does not sensitive to the symmetry energy at maximal baryon density reached. Therefore it is a challenge to probe the density-dependent symmetry energy at higher-density region. More studies are need on how to probe the symmetry energy at supradensities.

In summary, within an isospin dependent IBUU transport model, we studied the Au+Au reaction at incident beam energies of 200 and 400 MeV/nucleon. It is found that the symmetry-energy-sensitive observables including charged pion ratio $\pi^-/\pi^+$, free neutron-proton ratio $n/p$, neutron-proton differential transverse flow $F_{n-p}^{x}$ as well as the difference of the average nucleon transverse flow $F_{n-p}^{x}$ mainly probe the density-dependent symmetry energy around 1.5 times saturation density. At such incident beam energies, effects of the symmetry energy from the low-density region is small but observable. It is thus a challenge to probe the symmetry energy at higher-density region.

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