Cordial Miners: A Family of Simple, Efficient and Self-Contained Consensus Protocols for Every Eventuality

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Abstract

Cordial Miners is a family of simple, efficient, self-contained, Byzantine Atomic Broadcast protocols, with optimal instances for asynchrony and eventual synchrony. Its simplicity-cum-efficiency stems from using the blocklace—a partially-ordered generalization of the totally-ordered blockchain—for all key algorithmic tasks, including block dissemination, equivocation exclusion, leader finality, block ordering, and for the identification and exclusion of faulty miners. The algorithm employs piecemeal topological sort of the partially-ordered blocklace into a totally-ordered sequence of blocks, excluding equivocations as well as the Byzantine miners perpetrating them along the way. The conversion process is monotonic in that the output sequence only extends as the input blocklace increases, which implies (i) safety – the outputs of two correct miners are consistent (one is a prefix of the other), and (ii) finality – any output of a correct miner is final.

The Cordial Miners protocols are self-contained, using simple all-to-all block communication to realize blocklace-based dissemination and equivocation exclusion. They promptly excommunicate equivocating Byzantine miners, and thus can reduce the supermajority required for finality and eventually enjoy equivocation-free execution. In contrast, state-of-the-art protocols such as DAG-Rider and its successor Bullshark employ reliable broadcast as a black box and thus allow Byzantine miners to participate and equivocate indefinitely.

We present two instances of the protocol family: One for the eventual synchrony model, employing deterministic/predicted leader selection and 3 rounds of communication to leader finality in the good case, which is three-quarters of the latency of state-of-the-art protocols. The second for the asynchrony model, employing retroactive random leader selection, 6 rounds to leader finality in the good case, and 9 rounds in the expected case, which is half the latency of state-of-the-art protocols in the good case and three-quarters of their latency in the expected case. In both protocols, message complexity is the same as the state-of-the-art.

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1 Introduction

1.1 Overview and Related Work

The problem of reaching consensus on the ordering of acts by participants in a distributed system has been investigated for four decades [35], with efforts in the last decade falling into two categories: Permissioned, where the set of participants is determined by some authority, and permissionless, where anyone may join and participate provided that they pass some
Table 1 Performance summary. Latency is measured in communication rounds. The Cordial Miners protocols for both models have better latency than Bullshark in both the good-case and the expected case. All protocols have the same concrete and asymptotic message complexity. Note that each round of Bullshark and DAG-Rider uses reliable broadcast, meaning two rounds of communication.

| Protocol                  | Latency (number of communication rounds) | Message complexity |
|---------------------------|------------------------------------------|--------------------|
|                           | Eventual synchrony | Asynchrony |                |
|                           | Good case | Expected case | Good-case | Expected case |          |
| Cordial Miners            | 3         | 4.5          | 6         | 9            | amortized $O(n)$ |
| Bullshark [22] (same protocol as DAG-Rider [25] in asynchrony) | 4         | Fallback to asynchrony | 8         | 12           | amortized $O(n)$ |

ˈsybil-proof’ test, notably proof-of-work [31] or proof-of-stake [26]. Two leaders-of-the-pack in the permissioned category are the State-Machine-Replication protocol (SMR, consensus on an ordering of proposals) for the eventual-synchrony model – Hotstuff [39] and its extensions and variations [12], and the Byzantine Atomic Broadcast protocol (BAB, consensus on an ordering of all proposals made by correct participants) for the asynchronous model – DAG-Rider [25] and its extensions and variations [22]. Since the emergence of Bitcoin [31], followed by Ethereum with its support for smart contracts [6], permissionless consensus protocols have received the spotlight.

Recent conceptual and computational advances, notably stake-based sampling, have allowed permissioned consensus protocols to join the cryptocurrency fray (e.g. Cardano [26] and Algorand [21]), offering much greater efficiency and throughput compared to proof-of-work protocols. According to this approach, in every epoch (which could be measured in minutes or weeks) a new set of miners is chosen in a random auction, where the probability of being an auction winner is correlated with the stake bid by the miner. Mechanism design ensures that miners benefit from performing the protocol well, benefit less if they perform the protocol less well, and lose their stake if they subvert the protocol.

With this in mind, the expectation is that miners will do their best, not their worst, to execute the protocol, and hence the focus of analyses of permissioned consensus protocols has shifted from worst-case complexity to good-case complexity [1, 22], where miners are generally expected to behave as well as they can, given compute and network limitations, as opposed to as bad as they can. Still, standard protections against a malicious adversary are needed, for example to prevent a double-spending, a hostile takeover, or a meltdown of the cryptocurrency supported by the consensus protocol.

The use of a DAG-like structure to solve consensus has been introduced in previous works, especially in asynchronous networks [30]. Hashgraph [2] builds an unstructured DAG, with each block containing two references to previous blocks, and on top of the DAG the miners run an inefficient binary agreement protocol. This leads to expected exponential time complexity. Aleph [20] builds a structured round-based DAG, where miners proceed to the next round once they receive $2f + 1$ DAG nodes from other miners in the same round. On top of the DAG protocols run a binary agreement protocol to decide on the order of vertices to commit. Nodes in the DAG are reliably broadcast. Blockmania [15] uses a variant of PBFT [10] in the eventual synchrony model.

DAG-Rider [25] is a BAB protocol for the asynchronous model. It assumes an adaptive adversary that eventually delivers messages between any two correct miners. In DAG-Rider the miners jointly build a DAG of blocks, with blocks as vertices and pointers to previously-created blocks as edges, divided into strong and weak edges. Strong edges are used for the commit rule, and weak edges are used to ensure fairness. The protocol employs an underlying
reliable broadcast protocol of choice, which ensures that eventually the local DAGs of all correct miners converge and equivocation is excluded. Each miner independently converts its local DAG to an ordered sequence of blocks, with the use of threshold signatures to implement a global coin that retrospectively chooses one of the miners as the leader for each round. The decision rule for delivering a block is if the vertex created by the leader is observed by at least $2f + 1$ miners three rounds after it is created. The DAG is divided to waves, each consisting of the nodes of four rounds. When a wave ends, miners locally check whether a decision rule is met, similar to our protocol. DAG-Rider has an expected amortized linear message complexity, and expected constant latency. Tusk \cite{16} is an implementation based on DAG-Rider. Bullshark \cite{22} is the current state-of-the-art dual consensus protocol based on DAG-Rider that offers a fast-track to commit nodes every two rounds in case the network is synchronous. Other DAG-based consensus protocols include \cite{11, 17, 37, 33}.

HotStuff \cite{39} is an SMR protocol designed for the eventual synchrony model. The protocol employs all-to-leader, leader-to-all communication: In each round, a deterministically-chosen designated leader proposes a block to all and collects from all signatures on the block. Once the leader has $2f + 1$ signatures, it can combine them into a threshold signature \cite{3} which it sends back to all. The decision rule for delivering a block is three consecutive correct leaders. This leads to a linear message complexity and constant latency in the good case. The protocol delivers a block if there are three correct leaders in a row, which is guaranteed to happen after GST. HotStuff is based on Tendermint \cite{5} and is also the core of several other consensus protocols \cite{21, 13, 19, 38}. In this model, there are number of leader-based protocols such as DLS \cite{18}, PBFT \cite{10}, Zyzzyva \cite{27}, and SBFT \cite{23}.

It is within this context that we introduce Cordial Miners – a family of simple, efficient, self-contained Byzantine Atomic Broadcast \cite{7} protocols, and present two of its instances: Retrospective random leader selection for the asynchronous model and deterministic leader selection for the eventual synchrony model (See Table 1 for their performance).

We believe that the simplicity-cum-efficiency of the Cordial Miners protocols stems from the use of the blocklace data structure and its analysis for all key algorithmic tasks (the following refers to correct miners):

1. **The Blocklace** \cite{34} is a partially-ordered generalization of the totally-ordered blockchain (Def. \cite{3}), that consists of cryptographically-signed blocks, each containing a payload and a finite number of cryptographic hash pointers to previous blocks. The blocklace induces a DAG, as cryptographic hash pointers cannot form cycles by a compute-bound adversary. The DAG induces a partial order $\succ$ (Def. \cite{8}) on the blocks that includes Lamport’s ‘happened-before’ causality relation \cite{28} among correct miners. The globally-shared blocklace is constructed incrementally and cooperatively by all miners, who cordially disseminate it to each other.

2. **Ordering**: The ordering algorithm (Algorithm \cite{2}) is used locally by each miner to topologically-sort its locally-known part of the blocklace into a totally-ordered output sequence of blocks, excluding equivocation along the way. This conversion is monotonic (Prop. \cite{22}) – the output sequence is extended as the miner learns of or produces ever-larger portions of the global blocklace, and in this sense every output block of each miner is final. We say that two sequences are **consistent** if one is a prefix of the other (Def. \cite{2}), a notion stronger than the common prefix property of Ouroboros \cite{26}. We assume that less than one-third of the miners are faulty, and prove that the following holds for the remaining correct miners of the Cordial Miners protocols:

   - **Safety**: Outputs of different miners are consistent (Prop. \cite{27}).
   - **Liveness**: A block created by one is eventually output by everyone (Proposition \cite{29}).
3. **Dissemination:** Any new block created by a miner \( p \) acknowledges blocks known to \( p \) by including pointers to the tips (DAG sources) of \( p \)'s local blocklace. Correspondingly, a miner \( p \) will buffer, rather than include in its blocklace, any received block with dangling pointers – pointers to blocks not known to \( p \). Hence, a block \( b \) by \( p \) informs any recipient \( q \) of blocks not known to \( p \) at the time of \( b \)'s creation. Thus \( q \), being cordial, when sending to \( p \) a new \( q \)-block, will include with it blocks \( q \) knows but, to the best of \( q \)'s knowledge, are not yet known to \( p \) and have not already been sent by \( q \) to \( p \), thus ensuring block dissemination (Prop. 28).

4. **Equivocation exclusion:** An equivocation (Def. 11) is a pair of blocks by the same miner that are not causally-related – have no path of pointers from one to the other; a miner that creates an equivocation is considered faulty and is referred to as an *equivocator*. The shared blocklace will eventually include any equivocating block known to a correct miner, and hence eventually known to all correct miners. The question is: What should miners do with this knowledge?

A block \( b \) *acknowledges* block \( b' \) if there is a (possibly empty) path from \( b \) to \( b' \), namely \( b \preceq b' \) (Def. 8). Let \([b]\) denote the set of blocks acknowledged by \( b \), also referred to as the closure of \( b \) (Def. 9). A block \( b \) *approves* block \( b' \) if it acknowledges \( b' \) and does not acknowledge any block \( b'' \) equivocating with \( b' \) (Def. 12, see Fig. 1.A). A key observation is that a miner cannot approve both blocks of an equivocation without being itself an equivocator (Ob. 14). Hence, if less than one-third of the miners are equivocators, then no equivocation will ever receive an approval from blocks created by a supermajority (at least two-thirds) of the miners. This is the basis of equivocation-exclusion by the blocklace: A miner finalizes a block \( b \) once its local blocklace includes blocks that approve \( b \) by a supermajority (Algorithm 2).

5. **Cordial Miners:** The *depth*, or *round*, of a block \( b \) is the maximal length of any path emanating from \( b \) (Def. 5). A *round* is a set of blocks of the same depth. Miners are cordial in two respects. First, as explained above, in informing other miners of blocks they believe the other miner lacks. Second, awaiting a supermajority of round \( d \) before producing a block of round \( d + 1 \) (Def. 17).

6. **Leader Selection:** The Cordial Miners protocol for the eventual synchrony model employs deterministic leader selection (e.g. via a shared pseudorandom function). In the asynchronous model, the adversary has complete control over the order of message delivery, indefinitely. The panacea to such an adversary, employed for example by DAG-Rider [25], is to use a shared random coin [8] and elect the leader retroactively.

7. **Ratified and Super-Ratified Leaders:** A block \( b \) of round \( r \) is *ratified* by block \( b' \) if \([b']\) includes a supermajority of round \( r + \alpha \) that approves \( b \). A leader block \( b \) of round \( r \) is *super-ratified* if there is a supermajority of round \( r + \beta \) that include the leader, each member of which acknowledges a supermajority of round \( r + \alpha \) that approves \( b \). In the case of eventual synchrony, we also require the acknowledging supermajority of round \( r + \beta \) to include the leader of that round. In asynchrony, the leader is only elected in the next round. The parameters are \( \alpha = 1, \beta = 1 \) for deterministic leader selection in the eventual synchrony model and \( \alpha = 2, \beta = 5 \) for retrospective random leader selection via a shared coin in the asynchronous model. Hence the *wave length* (gap between leaders) is 3 for eventual synchrony and 6 for asynchrony. (Def. 19, see Fig. 1).

8. **Common Core:** Since miners are cordial, the notion of *common core* [9] can be applied to prove a lower bound on the probability of a leader being super-ratified despite such a powerful adversary. This approach is applied in the Cordial Miners protocol for asynchrony, at the cost of delaying leader selection and hence expected finality to 4 rounds. As the
Figure 1 Acknowledgement, Approval, Equivocation, Ratification, Super-Ratification: (A) Observing an Equivocation: The `drop shape' depicts a blocklace with a block at the tip of the drop, where all the blocks that the tip block acknowledges being the entire volume of the drop. Initial blocks of the blocklace are at the bottom, and drop inclusion implies $\succ$. Assume $b_1, b_2$ are an equivocation (Def. 11) by the red miner. According to the figure, $b''$ approves $b_2$ (Def. 12) since it acknowledges $b_2$ (Def. 8, namely $b'' \succ b_2$) and does not acknowledge any conflicting red block, in particular it does not acknowledge $b_1$. However, since $b'$ acknowledges $b''$ it acknowledges both $b_1$ and $b_2$ and hence does not approve the equivocating $b_1$ (nor $b_2$). (B) Ratified: The block (black dot) at round $r$ is ratified by another block (gray dot), if the black block is approved by a supermajority (green horizontal line) that is acknowledged by the gray block. (C) Super-Ratified: A block (blue dot) at round $r$ is super-ratified if there is a supermajority (light green horizontal line) at round $r + \alpha$, each member of which ratifies the blue block at round $r$ by acknowledging a supermajority (dark green horizontal line) that approves the blue block. In the case of eventual synchrony, we also require the acknowledging supermajority of round $r + \beta$ to include the leader of that round (black dot). In asynchrony, the leader is only elected in the next round (grey dot). The parameters are $\alpha = 1, \beta = 2$ for deterministic leader selection in the eventual synchrony model and $\alpha = 2, \beta = 5$ for retrospective random leader selection via a shared coin in the asynchronous model. Hence the wave length (gap between leaders) is 3 for eventual synchrony and 5 for asynchrony.

approach uses threshold signatures, a secure method for key distribution is needed [14].

The common core property implies that there is a set $V$ of at least $2f + 1$ blocks in round $r$ such that every correct miner that issues block $b$ in round $r + 2$ has $b \succeq V$. Hence, if a miner issues a block at round $r + 3$ then it has a set $U$ of at least $2f + 1$ blocks in round $r + 3$ such that for $u \in U$ and $v \in V u \succ v$.

9. Ordering by Leaders: We assume a topological sort procedure that takes a blocklace as an input and produces a sequence of its blocks while respecting their causal partial order $\succ$. With it, a recursive ordering function $\tau$, applied to a leader block $b$, is defined as follows (Def. [21]): Let $b'$ be the highest-depth leader block in $[b]$ ratified by $b$. If there is no such $b'$, output the topological sort of $[b]$ and terminate. Else recursively call the ordering function with $b'$ and output its result following by the topological sort of $[b] \setminus [b']$.

10. Finality by Super-Ratified Leaders: The key insight to finality is this (Proposition 20): A super-ratified leader will be ratified by any subsequent leader (see Figure 2). Hence, the ordering algorithm (Algorithm 2) used by any miner is as follows: When identifying a new super-ratified leader $b$ in its blocklace, apply the ordering function $\tau$ to $b$ and output the newly added suffix since the previous super-ratified leader. As the ordering function is guaranteed to be called with the previous super-ratified leader in one
of its recursive calls, it can be optimized to return if it does, rather than recompute the prefix it has already delivered.

**Figure 2** Finality of a Super-Ratified Leader (Definition 18): Assume that the blue leader block of round $r$ (blue dot) is super-ratified. We show that it is ratified by any subsequent cordial leader. (A) If the cordial leader is at round $r + \beta$ (black dot, eventual synchrony instance) then it is part of the ratifying supermajority (light green) by definition. Else if the cordial red leader is at round $r + \beta + 1$ then it acknowledges a supermajority (red horizontal line) of the previous round $r + \beta$, which must have a correct agent (purple) in common with the ratifying supermajority at round $r + \beta$ (light green horizontal line), which acknowledges the approving supermajority at round $r + \alpha$, and via which the red leader at $r + \beta + 1$ ratifies the black leader at $r$. (B) Otherwise, there are two purple blocks $b_1, b_2$ by the same correct purple agent in the red $\hat{B}$ and light green $B'$ supermajorities, which are connected via a path since the purple agent is correct, connecting the red leader $\hat{b}$ to the blue leader $b$ via the approving supermajority $B$ (dark green horizontal line) at round $r + \alpha$. Namely, the red leader $\hat{b}$ ratifies the blue leader $b$.

11. **Identification and exclusion of faulty miners**: Any faulty $p$-block known to some correct miner will eventually be known to all, resulting in correct miners suspending any further communication with $p$. For example, any miner can easily verify whether another miner $p$ is cordial in the second sense by examining the blocklace. In addition, an equivocation by $p$, with each block of the pair known to a different correct miner, will eventually be known to all and result in the exclusion of $p$.

12. **Exclusion of nonresponsive miners**: A miner $p$ need not be cordial to a miner $q$ as long as $q$ has not acknowledged a previous block $b$ sent to $q$ by $p$. If $q$ fail-stopped, then $p$ should definitely not waste resources on $q$; if $q$ is only suspended or delayed, then eventually it will send to $p$ a block acknowledging $b$, following which $p$—being cordial—will send to $q$ all the backlog $p$ has previously refrained from sending it, and is not acknowledged by the new block received from $q$.

Miners accomplish all the above by simple and efficient analyses of their local blocklace.

The protocols are presented as algorithmic components written in pseudo-code: Algorithm 1 contains a description of a block in a blocklace and blocklace utilities. Algorithm 2 describes the local conversion each miner executes with the function $\tau$ converting its local copy of the partially-ordered blocklace into a totally-ordered sequence of blocks. Algorithm 3 incorporates Algorithms 1 and 2, describes how a miner creates a block and sends it to all correct miners.
and includes any backlog needed for dissemination. It has two instances that specify the conditions to create a block, one for asynchrony and one for eventual synchrony.

### 1.2 Models, Problem, Safety and Liveness Proof Outline

We assume \( n \geq 3 \) miners \( \Pi \), of which \( f < n/3 \) may be faulty (act arbitrarily, be Byzantine), and the rest are correct, and that a message sent from one correct miner to another will eventually arrive. We assume that every miner has a single and unique key-pair (PKI) and can cryptographically sign messages. Each miner \( p \in \Pi \) has an input call \( \text{payload}() \) that returns a payload (e.g., a proposal from a user or a mempool), and an output call \( \text{deliver}(b) \) where \( b \) is a block.

▶ **Definition 1.** We consider two models of communication with an adversary:

- Asynchrony, where an adversary controls the finite message delay of each message.
- Eventual Synchrony \( [18] \), where an adversary can delay arbitrarily an unknown but finite number of messages, beyond which messages arrive possibly under adversarial control but within a bounded delay.

In each model the protocol addresses Byzantine Atomic Broadcast \( [7] \).

▶ **Definition 2 (Prefix, \( \preceq \), Consistent Sequences).** A sequence \( x \) is a prefix of a sequence \( x' \), \( x \preceq x' \) if \( x' \) can be obtained from \( x \) by appending to it zero or more elements. Two sequences \( x, x' \) are consistent if \( x \preceq x' \) or \( x' \preceq x \).

▶ **Definition 3 (Safety and Liveness).** These are the requirements of a blocklace-based Byzantine Atomic Broadcast protocol:

- Safety: Outputs of correct miners are consistent.
- Liveness: A block created by a correct miner is eventually output by every correct miner with probability 1.

We note that these safety and liveness requirements, combined with the uniqueness of a block in a blocklace, imply the standard Byzantine Atomic Broadcast guarantees: Agreement, Integrity, Validity, and Total Order \( [4, 25] \).

The proofs of safety and liveness proceed as follows, assuming less than one-third of the miners are faulty, and referring to the remaining correct miners.

**Safety:**

1. Prove that the function \( \tau \) that converts a blocklace to a sequence of blocks is monotonic with respect to the superset relation (Prop. \( [22] \)).
2. Observe that if two sequences are each a prefix of a third sequence, then they are consistent (Ob. \( [23] \)). Given any two local blocklaces \( B, B' \) of miners \( p, p' \), then due to the monotonicity of \( \tau \), both \( \tau(B) \) and \( \tau(B') \) are prefixes of \( \tau(B \cup B') \). Therefore they are consistent (Corollary \( [24] \)).
3. Argue that Algorithm \( [2] \) correctly implements \( \tau \) and hence is safe (Prop. \( [27] \)).

**Liveness:**

1. Observe that the conversion function \( \tau \), applied to a super-ratified leader block \( b \), includes in the output sequence any block known to that leader, namely any block in \( [b] \) (Ob. \( [25] \)).
2. Given a block \( b \) known to a miner at some point \( t \) in the computation, argue that \( b \) will eventually be known to every miner at some later point \( t' \) in the computation (Prop. \( [28] \)).
3. Argue that eventually some leader block $b'$ of a miner will be ratified at a point later than $b'$ with probability 1. 

4. By construction, $b \in [b']$, hence $b$ is included in the output of $\tau$ applied to $b'$ (Prop. 29).

For asynchrony, we employ a shared random global coin.

Definition 4 (Shared random coin). We use a global perfect coin, which is unpredictable by the adversary. In round $r$, $r \in \mathbb{N}$, of the coin is invoked by miner $p_i \in \Pi$ by calling $\text{leader}_i(r)$. This call returns a miner $p_j \in \Pi$, which is the chosen leader for round $r$. Let $X_r$ be the random variable that represents the probability that the coin returns miner $p_j$ as the return value of the call $\text{leader}_i(r)$. The global perfect coin has the following guarantees:

Agreement If two correct miners call $\text{leader}_i(r)$ and $\text{leader}_j(r)$ with respective return values $p_1$ and $p_2$, then $p_1 = p_2$.

Termination If at least $f + 1$ miners call $\text{leader}(r)$, then every $\text{leader}(r)$ call eventually returns.

Unpredictability As long as less than $f + 1$ miners call $\text{leader}(r)$, the return value is indistinguishable from a random value except with negligible probability $\epsilon$. Namely, the probability $p_r$ that the adversary can guess the returned miner $p_j$ of the call $\text{leader}(r)$ is

$$p_r \leq \Pr[X_r = p_j] + \epsilon.$$ 

Fairness The coin is fair, i.e., $\forall r \in \mathbb{N}, \forall p_j \in \Pi: \Pr[X_r = p_j] = 1/n$.

Examples of such a coin implementation using a PKI and threshold signatures are in [3, 29, 36]. See DAG-Rider on details on how to implement such a coin as part of a distributed blocklace-like structure.

2 The Blocklace: A Partially-Ordered Generalization of the Totally-Ordered Blockchain

The blocklace was introduced in reference [34]. For completeness we include here needed definitions and results.

Definition 5 (Block). Given a set of miners $\Pi$ and a set of payloads $A$, a block is a triple $b = (p, a, H)$, referred to as a $p$-block, $p \in \Pi$, with $a \in A$ being the payload of $b$, and $H$ is a (possibly empty) finite set of hash pointers to blocks, namely for each $h \in H$, $h = \text{hash}(b')$ for some block $b'$. Such a hash pointer $h$ is a $q$-pointer if $b'$ is a $q$-block, and same-miner if $h$ is a $p$-pointer, in which case $b'$ is the immediate predecessor of $b$. The set $H$ may have at most one $q$-pointer for any miner $q \in \Pi$, and if $H$ has no same-miner pointer then $b$ is called initial. The depth of $b$, $\text{depth}(b)$, is the maximal length of any path emanating from $b$.

Note that hash being cryptographic implies that a cycle cannot be effectively computed.

Definition 6 (Dangling Pointer, Closed). A hash pointer $h = \text{hash}(b)$ for some block $b$ is dangling in $B$ if $b \notin B$. A set of blocks $B$ is closed if no block $b \in B$ has a pointer dangling in $B$.

The non-dangling pointers of a set of blocks $B$ induce finite-degree directed graph $(B, E)$, $E \subseteq B \times B$, with blocks $B$ as vertices and directed edges $(b, b') \in E$ if $b, b' \in B$ and $b$ includes a hash pointer to $b'$. We overload $B$ to also mean its induced graph $(B, E)$.

In the following we assume a given set of payloads $A$.

Definition 7 (Blocklace). Let $B$ be the maximal set of blocks over $A$ and hash for which the induced directed graph $(B, E)$ is acyclic. A blocklace over $A$ is a set of blocks $B \subseteq B$. 


The two key blocklace notions used in our protocols are acknowledgment and approval.

\begin{itemize}
  \item \textbf{Definition 8 (\(\succ\), Acknowledge).} Given a blocklace \(B \subseteq \mathcal{B}\), the strict partial order \(\succ_B\) is defined by \(b' \succ_B b\) if \(B\) has a non-empty path of directed edges from \(b'\) to \(b\) (\(B = \emptyset\)). Given a blocklace \(B\), \(b'\) acknowledges \(b\) in \(B\) if \(b' \succ_B b\). Miner \(p\) acknowledges \(b\) via \(B\) if there is a \(p\)-block \(b' \in B\) that acknowledges \(b\), and a group of miners \(Q \subseteq \Pi\) acknowledge \(b\) via \(B\) if for every miner \(p \in Q\) there is a \(p\)-block \(b' \in B\) that acknowledges \(b\).
  \item \textbf{Definition 9 (Closure, Tip).} The closure of \(b \in \mathcal{B}\) wrt \(\succ\) is the set \([b] := \{b' \in \mathcal{B} : b \preceq b'\}\). The closure of \(B \subset \mathcal{B}\) wrt \(\succ\) is the set \([B] := \bigcup_{b \in B} [b]\). A block \(b \in \mathcal{B}\) is a tip of \(B\) if \([b] = [B] \cup \{b\}\).
\end{itemize}

Note that a set of blocks is closed iff it includes its closure (and thus is identical to it):

\begin{itemize}
  \item \textbf{Observation 10.} \(B \subset \mathcal{B}\) is closed iff \([B] \subseteq B\).
\end{itemize}

With this, we can define the basic notion of equivocation (aka double-spending when payloads are conflicting financial transactions).

\begin{itemize}
  \item \textbf{Definition 11 (Equivocation, Equivocator).} A pair of \(p\)-blocks \(b \neq b' \in \mathcal{P}\), \(p \in \Pi\), form an equivocation of \(p\) if they are not consistent wrt \(\succ\), namely \(b' \neq b\) and \(b \neq b'\). A miner \(p\) is an equivocator in \(B\) if \([b]\) has an equivocation of \(p\).
\end{itemize}

Namely, a pair of \(p\)-blocks form an equivocation of \(p\) if they do not acknowledge each other in \(B\). In particular, two initial \(p\)-blocks constitute an equivocation by \(p\). As \(p\)-blocks are cryptographically signed by \(p\), an equivocation of \(p\) is a volitional fault of \(p\).

\begin{itemize}
  \item \textbf{Definition 12 (Approval).} Given a blocklace \(B\), a block \(b\) approves \(b'\) in \(B\) if \(b\) acknowledges \(b'\) in \(B\) and does not acknowledge any block \(b''\) in \(B\) that together with \(b'\) forms an equivocation. An miner \(p\) approves \(b'\) in \(B\) if there is a \(p\)-block \(b\) that approves \(b'\) in \(B\), in which case we also say that \(p\) approves \(b'\) via \(b\). A set of miners \(Q \subseteq \Pi\) approves \(b'\) via \(B\) in \(B\) if every miner \(p \in Q\) approves \(b'\) in \(B\) via some \(p\)-block \(b \in B', B' \subseteq B\).
  \item \textbf{Observation 13.} Approval is monotonic wrt \(\supset\).
\end{itemize}

Namely, if \(b\) or \(p\) approve \(b'\) in \(B\) they also approve \(b'\) in \(B' \supset B\).

A key observation is that a miner cannot approve an equivocation of another miner without being an equivocator itself (Fig. [I]A):

\begin{itemize}
  \item \textbf{Observation 14.} [Approving an Equivocation] If miner \(p \in \Pi\) approves an equivocation \(b_1, b_2\) in a blocklace \(B \subseteq \mathcal{B}\), then \(p\) is an equivocator in \(B\).
\end{itemize}

As equivocation is a fault, at most \(f\) miners may equivocate.

\begin{itemize}
  \item \textbf{Definition 15 (Supermajority).} A set of miners \(P \subset \Pi\) is a supermajority if \(|P| \geq 2f + 1\). A set of blocks \(B\) is a supermajority if the set \(\{p \in \Pi : b \in B\} \) is a supermajority.
  \item \textbf{Lemma 16 (No Supermajority Approval for Equivocation).} If there are at most \(f\) equivicators in a blocklace \(B \subseteq \mathcal{B}\) with an equivocation \(b, b' \in B\), then not both \(b, b'\) have supermajority approval in \(B\).
\end{itemize}

Blocklace utilities that realize these definition are presented in Algorithm [II].
### Algorithm 1 Cordial Miners: Blocklace Utilities

pseudocode for miner \( p \in \Pi \)

| Local variables: |
|------------------|
| \text{struct} block \( b \) |
| \hspace{1em} \text{b.creator} – the miner that created \( b \) |
| \hspace{1em} \text{b.payload} – a set of transactions |
| \hspace{1em} \text{b.pointers} – a possibly-empty set of hash pointers to other blocks |
| \text{blocks} \leftarrow \{ \} |

1. \textbf{procedure} create\_block(blocks') \hspace{1em} \triangleright \text{create into } b \text{ a new block pointing to the sources of } blocks' |
2. \hspace{1em} b.payload \leftarrow \text{payload}() \hspace{1em} \triangleright \text{e.g. dequeue a payload from a queue of proposals (aka mempool)} |
3. \hspace{1em} b.creator \leftarrow p |
4. \hspace{1em} b.pointers \leftarrow \{ \text{hash}(b') : b' \in \text{blocks}', b' \text{ has no incoming pointers in } \text{blocks}' \} |
5. \hspace{1em} \text{blocks} \leftarrow \text{blocks} \cup \{ b \} |
6. \hspace{1em} \text{deliver\_blocks}() |
7. \hspace{1em} \text{return } b |
8. \textbf{procedure} hash(b) \textbf{return} \begin{array}{c} \exists b_1, b_2, \ldots, b_k \in \text{blocks}, k \geq 1, \text{ s.t. } b_1 = b, b_k = b' \text{ and } \forall i \in [k-1]: b_{i+1} \in b_i, \text{ pointers} \end{array} |
9. \hspace{1em} \triangleright \text{ Returns a set if applied to a set } |
10. \textbf{procedure} depth(b, b') \textbf{return} \begin{array}{c} \exists b_1, b_2, \ldots, b_k \in \text{blocks}, k \geq 1, \text{ s.t. } b_1 = b, b_k = b' \text{ and } \forall i \in [k-1]: b_{i+1} \in b_i, \text{ pointers} \end{array} |
11. \hspace{1em} \triangleright \text{ Depth of block } b |
12. \textbf{procedure} depth(blocks) \textbf{return} \begin{array}{c} \{ \text{depth}(b) : b \in \text{blocks} \} \end{array} |
13. \hspace{1em} \triangleright \text{ Depth of set of blocks } |
14. \textbf{procedure} blocks\_prefix(d) \textbf{return} \{ b \in \text{blocks} : \text{depth}(b) \leq d \} |
15. \textbf{procedure} closure(b) \textbf{return} \{ b' \in \text{blocks} : \text{path}(b, b') \} \triangleright \text{also referred to as } [b], \text{ } b \text{ could also be a set of blocks } |
16. \textbf{procedure} leader(d) \textbf{return} \begin{array}{c} \text{Returns a leader at depth } d \text{ if there is one: Predetermined for every even } d \text{ for eventual synchrony; random/unpredictable using a shared coin for every } d \text{ divisible by 5 for asynchrony (Def. 5)} \end{array} |
17. \textbf{procedure} blocks\_leaders(d) \textbf{return} \{ b \in \text{blocks} : \text{block}(b) \} |
18. \textbf{procedure} equivocation(b_1, b_2) \textbf{return} \begin{array}{c} b_1.\text{creator} = b_2.\text{creator} \land b_1 \notin [b_2] \land b_2 \notin [b_1] \end{array} |
19. \hspace{1em} \triangleright \text{ See Figure 1A } |
20. \textbf{procedure} equivocator(q) \textbf{return} \begin{array}{c} \exists b_1, b_2 \in \text{blocks} \land b_1.\text{creator} = b_2.\text{creator} \land q \land \text{equivocation}(b_1, b_2) \end{array} |
21. \hspace{1em} \triangleright \text{ See Figure 1B } |
22. \textbf{procedure} approved(h_1, b) \textbf{return} \begin{array}{c} h_1 \in [b] \land \forall b_2 \in [b] : \text{equivocation}(h_1, b_2) \end{array} |
23. \hspace{1em} \triangleright \text{ See Figure 1A } |
24. \textbf{procedure} ratified(b_1, b_2) \textbf{return} \begin{array}{c} \exists h \in \text{blocks} \land h.\text{creator} = q \land \text{equivocation}(b_1, b_2) \end{array} |
25. \hspace{1em} \triangleright \text{ See Figure 1B } |
26. \textbf{procedure} cordial\_block(b) \textbf{return} \begin{array}{c} \{ b' \in [b] : \text{depth}(b') = \text{depth}(b) - 1 \} \geq 2f + 1 \end{array} |
27. \hspace{1em} \triangleright \text{ See Proposition 22 } |
28. \textbf{procedure} cordial\_round() \textbf{return} \begin{array}{c} \exists h_i, b_i \in \text{blocks} \land h_i.\text{creator} = q \land \text{equivocation}(b_i, b_i') \end{array} |
29. \hspace{1em} \triangleright \text{ Can be expanded } |
30. \textbf{procedure} argmax(d) \textbf{return} \begin{array}{c} \text{argmax}(d) \end{array} |
31. \textbf{return} \begin{array}{c} \text{argmax}(d) \text{ where } R = \end{array} |
32. \begin{array}{c} \{ r \in \text{depth(blocks)} : \{ h.\text{creator} = q \land \text{depth}(h) = r \land \text{equivocation}(h.\text{creator}) \} \geq 2f + 1 \land h.\text{creator} \in \text{blocks} : (h.\text{creator} = p \land \text{depth}(p) \geq r) \} \end{array} |

## 3 Converting a Blocklace into a Sequence of Blocks

Here we present a deterministic function \( \tau \) that incrementally converts a blocklace into a sequence of some of its blocks, respecting \( \succ \), and show that it is monotonic wrt to the subset relation, provided no more than \( f \) miners equivocate. Each miner in the Cordial Miners protocol employs \( \tau \) to locally compute the final output sequence of blocks from its local copy of the blocklace as input, as realized by Algorithm 2. The monotonicity of \( \tau \) ensures finality, as it implies that the output sequence will only extend while the input local blocklace increases over time. It also ensures the safety of a protocol that uses \( \tau \) (Proposition 22 below). To ensure liveness, one has to argue that every block in the input blocklace will eventually be in the output of \( \tau \); this argument is made in Proposition 29.
Definition 17 (Cordial). A block \( b \in B \) is cordial if the set \( \{ b' \in [b] : \text{depth}(b') = \text{depth}(b) - 1 \} \) is a supermajority. Miner \( p \) is cordial in blocklace \( B \subseteq B \) if every \( p \)-block \( b \in B \) is cordial.

Definition 18 (Round, Leader, Leader Block). Given a blocklace \( B \subseteq B \), then a round \( r \geq 1 \) in \( B \) is the set of blocks \( \{ b \in B : \text{depth}(b) = r \} \). We assume a leader selection function leader : \( \mathbb{N} \to \Pi \cup \{ \bot \} \) that is defined only for certain depths. If \( \text{leader}(r) = p \) then \( p \) is the leader of round \( r \), and if, in addition, \( b \in B \) is a \( p \)-block of depth \( r \), then \( b \) is a leader block of round \( r \) in \( B \).

See Figure 1.B for the following definition. We employ parameters \( \alpha \) and \( \beta \), where \( \alpha = 1, \beta = 1 \) for deterministic leader selection in the eventual synchrony model and \( \alpha = 2, \beta = 5 \) for retrospective random leader selection via a shared coin in the asynchronous model.

Definition 19 (Ratified and Super-Ratified Blocks). A block \( b \in B \) is ratified if there is a supermajority of blocks \( B' \) of depth \( d(b) + \alpha \) (light green in Figure 1.B) that approves \( b \); \( b \) is super-ratified if, in addition, there is a supermajority of blocks \( B'' \) of depth \( d(b) + \beta \) (dark green in Figure 1.B) such that each member of \( B'' \) acknowledges \( B' \). In the case of eventual synchrony, we also require the acknowledging supermajority of round \( r + \beta \) to include the leader of that round.

In asynchrony, the leader is only elected in the next round. Hence the wave length (gap between leaders) is 3 for eventual synchrony and 6 for asynchrony.

The following proposition ensures that a super-ratified block is ratified by any subsequent cordial leader block (Fig. 2).

Proposition 20 (Finality of Super-Ratification). If a leader block \( b \) is super-ratified in \( B \), then it is ratified by any subsequent cordial leader block \( b' \).

Proof. Assume a leader block \( b \) super-ratified via supermajorities \( B' \) and \( B \) (Fig. 1.B), and assume that \( b \) is a subsequent cordial leader block. Being cordial, \( b \) acknowledges a supermajority of the preceding round \( \hat{B} \). We consider three cases (Figure 2):

1. In the eventual synchrony instance, \( d(b') = d(b) + \beta \) (Figure 2 A), \( \hat{b} \) (black dot) is part of the supermajority that acknowledges \( B \).
2. Consider the asynchrony instance. If \( d(b') = d(b) + \beta + 1 \) (Figure 2 A), then \( \hat{B} \) and \( B' \) are supermajorities of the same depth and hence must have a block (purple dot Figure 2 A) in their intersection, via which \( b' \) acknowledges \( B \).
3. Else, \( d(b') > d(b) + \beta + 1 \) (Figure 2 B), then, by counting, at least one block \( b_1 \in B' \) and one block \( b_2 \in \hat{B} \) are both \( p \)-blocks by the same correct (purple) miner \( p \in \Pi \). Since \( p \) is not an equivocator \( b_2 \succ b_1 \). Hence \( \hat{b} \succ b_2 \succ b_1 \) and \( b_1 \) acknowledges \( B \); thus \( b \) acknowledges \( B \) and hence ratifies \( b \).

Proposition 20 ensures that given a blocklace \( B \), a super-ratified leader \( b \) in \( B \) will be ratified by any subsequent cordial leader, hence included in the sequence of final leaders. Hence, the final sequence up to a super-ratified leader \( b \) is fully-determined by \( b \) itself independently of the (continuously changing) identity of the last super-ratified leader. Hence the final sequence up to a super-ratified leader \( b \) can be ‘cached’ and will not change as the blocklace increases.

Super-ratified leaders are the anchors of finality in a growing chain, each ‘writes history’ backwards till the preceding ratified leader. We use the term ‘Okazaki fragments’ [32].
for the sequences computed backwards from each super-ratified leader to its predecessor, acknowledging the analogy with the way one of the DNA strands of a replicated DNA molecule is elongated via the stitching of backwards-synthesized Okazaki-fragments.

The following recursive ordering function $\tau$ maps a blocklace into a sequence of blocks. Formally, the entire sequence is computed backwards from the last super-ratified leader, afresh by each application of $\tau$. Practically, a sequence up to a super-ratified leader is final (Prop. 22) and hence can be cached, allowing $\tau$ to be computed one ‘Okazaki-fragment’ at a time, from the new super-ratified leader backwards to the previous super-ratified leader.

\begin{definition}[$\tau$.] We assume an fixed topological sort function (e.g. lexicographic) $s$ that maps a blocklace to a sequence of blocks consistent with $\triangleright$. The function $\tau : 2^B \rightarrow B^*$ is defined for a blocklace $B \subseteq B$ backwards, from the last element to the first, as follows: If $B$ has no super-ratified leaders then $\tau(B) := \Lambda$. Else let $b$ be the last super-ratified leader in $B$. Then $\tau(B) := \tau'(b)$, where $\tau'$ is defined recursively:

$$\tau'(b) := \begin{cases} s([b]) & \text{if } [b] \text{ has no leader ratified by } b, \text{ else} \\ \tau'(b') \cdot s([b] \setminus [b']) & \text{if } b' \text{ is the last leader ratified by } b \text{ in } [b] \end{cases}$$

Note that $\tau'$ uses the notion of a leader ratified by another leader, not a super-ratified leader.

Proposition 20 shows that if there are less than $f$ equivocators, a super-ratified leader is final, in that it will be ratified by any subsequent leader, super-ratified or not. Hence the following:

\begin{proposition}[Monotonicity and Finality of $\tau$.] If there are at most $f$ equivocators, the function $\tau$ is monotonic wrt $\subseteq$, namely $B \subseteq B'$ for two blocklaces with at most $f$ equivocators implies that $\tau(B) \preceq \tau(B')$, and in this sense membership in $\tau$ is final.

\end{proposition}

\begin{proof}
Assume given blocklaces $B \subseteq B' \subseteq B_{21}$. If $\tau(B) = \Lambda$ the claim holds vacuously. So let $b$ be the last super-ratified leader in $B$. If $B'$ has no super-ratified leader not in $B$, then $\tau(B) = \tau(B')$ and the claim holds vacuously. So let $b_1, \ldots, b_k, k \geq 1$, be the sequence of final leaders in $B' \setminus B$. Then by the definition of $\tau$, $\tau(B') = \tau(B) \cdot s([b_1] \setminus [b]) \cdot \ldots \cdot s([b_k] \setminus [b_{k-1}])$, namely $\tau(B) \preceq \tau(B')$.
\end{proof}

\begin{observation}[Consistent triplet.]
Given three sequences $x, x', x''$, if $x' \preceq x$ and $x'' \preceq x$ then $x'$ and $x''$ are consistent.
\end{observation}

The following Corollary ensures that the output sequences computed by miners based on their local blocklaces would be consistent as long as there are at most $f$ equivocators.

\begin{corollary}[Consistency of Output Sequences.]
If $B, B' \subseteq B$ are closed with at most $f$ equivocators in $B := B' \cup B''$ then $\tau(B')$ and $\tau(B'')$ are consistent.

\end{corollary}

\begin{proof}
By monotonicity of $\tau$ with at most $f$ equivocators (Prop. 22), $\tau(B') \preceq \tau(B)$ and $\tau(B'') \preceq \tau(B)$. By Observation 23 $\tau(B')$ and $\tau(B'')$ are consistent.
\end{proof}

While $\tau$ does not output all the blocks in its input, as blocks not acknowledged by the last super-ratified leader in its input are not delivered, the following observation reminds us of the half-full glass:

\begin{observation}[Fairness of $\tau$.]
If a block $b \in B$ is acknowledged by the last super-ratified leader in $B$, then it is included in $\tau(B)$.
\end{observation}
Algorithm 2 Cordial Miners: Conversion of Blocklace to Sequence of Blocks with \( \tau \)

pseudocode for miner \( p \in \Pi \)

Local Variable:
- \( \text{deliveredBlocks} \leftarrow {} \)
- \( \text{currentLeader} \leftarrow {} \)
- \( \alpha \) \( \triangleright \) A constant; \( \alpha \leftarrow 1 \) for eventual synchrony and \( \alpha \leftarrow 2 \) for asynchrony
- \( \beta \) \( \triangleright \) A constant; \( \beta \leftarrow 2 \) for eventual synchrony and \( \beta \leftarrow 5 \) for asynchrony

25: procedure \( \text{deliver\_blocks}() \)
26: if \( \text{super\_ratified\_leader()} \neq \text{currentLeader} \) then \( \tau \)(\( \text{super\_ratified\_leader()} \))

27: procedure \( \tau(b_1) \)
28: if \( b_1 \in \text{deliveredBlocks} \lor b_1 = \emptyset \) then return
29: \( b_2 \leftarrow \arg_{r \in R} \max \text{depth}(r) \) where \( R = \{ b \in \text{leaders}(\text{blocks}) : \text{ratified}(b, b_1) \} \)
30: \( \tau(b_2) \) \( \triangleright \) Recursive call to \( \tau \)
31: for every \( b \in [b_1] \setminus [b_2] \) s.t. \( \text{approved}(b, b_1) \), ordered by topological sort do \( \triangleright \) Deliver a new equivocation-free ‘Okazaki-fragment’
32: \( \text{deliver} (b) \)
33: \( \text{deliveredBlocks} \leftarrow \text{deliveredBlocks} \cup \{b\} \)

34: procedure \( \text{super\_ratified\_leader()} \) \( \triangleright \) Returns last super-ratified leader, if any, Figure 1B
35: \( \text{return} \arg_{u \in U} \max \text{depth}(u) \) where \( U = \{ b \in \text{leaders}(\text{blocks}) : B = \{ b' : \text{depth}(b') + \alpha = \text{depth}(b) + \alpha \} \land |B| \geq 2f + 1 \land |\{b' : \text{depth}(b') + \beta = \text{depth}(b) \} \land B \subseteq [b']| \geq 2f + 1 \land (\alpha = 1 \rightarrow B \subseteq [\text{leader\_block}(r + \beta)]) \}

If every block known to a correct miner will be known to all correct miners, and if every super-ratified leader has a successor, then every block will be eventually acknowledged by some super-ratified leader and therefore be delivered by \( \tau \).

Algorithm 2 is a fairly literal implementation of the mathematics described above: It maintained \( \text{deliveredBlocks} \) that includes the prefix of the output of \( \tau \) that it has already computed. Upon adding a new block to its blocklace (line 25), it computes the most-recent super-ratified leader \( b_1 \) according to Definition 19, and applies \( \tau \) to it, which is intended to be a literal realization of the mathematical definition of \( \tau \) (Def. 21), with the optimization, discussed above, that a recursive call with a delivered block is returned. Hence the following proposition:

Proposition 26 (Tau implements \( \tau \)). The procedure \( \tau \) in Algorithm 2 correctly implements \( \tau \) in Definition 21.

And based on it, we conclude the following, which ensures the safety of a protocol that employs Algorithm 2 for ordering a blocklace.

Proposition 27. Algorithm 2 satisfied the safety requirement of Definition 3.

Proof. Proposition 26 establishes that Algorithm 2 implements \( \tau \) correctly. Together with Corollary 24, we conclude that the outputs of all miners using Algorithm 2 are consistent, satisfying the safety requirement.

4 Cordial Miners Protocols

The Cordial Miners protocols employ the blocklace for dissemination. To do so, each miner maintains a history array that records its communication history with the other miners, and
Consider a correct miner $p$ with block $b$ it has created, which includes pointers to the blocks $p$ knows, to each non-faulty miner $q$, and if $q$ is responsive, it includes in the package all the blocks that $p$ knows but, to the best of $p$’s knowledge, $q$ does not, namely $[b] \setminus [\text{history}[q]]$. This package is constructed in line 50. Excluded from the closure of $b$ it is the closure of all $q$-blocks known to $p$ and all blocks $p$ has sent to $q$, both recorded in $\text{history}[q]$ of miner $p$. A miner $q$ is considered responsive (line 55) by $p$ if $q$ has responded to the last block $p$ has sent it.

Based on this, we argue the following:

**Proposition 28 (Algorithm 3 Dissemination).** In any run of Algorithm 3, if a correct miner knows a block $b$, then eventually every correct miner will know $b$.

**Proof.** Consider a correct miner $p$ with block $b \in \text{blocks}$, and miner $q$. If $q$ is correct then eventually it will send a block to $p$, and consider a subsequent round $r$. If at round $r$ the communication history of $p$ with $q$ shows that $q$ knows $b$, we are done. Else, $p$ will include $b$ in its package to $q$, together with all blocks in $[b]$ for which $p$ has no evidence that $q$ knows, based on their communication history. Then $q$ will eventually receive the package. The package has no blocks with dangling pointers since $p$ includes in the package everything $q$
might miss, hence $q$ can receive it and include it together with $b$ in its blocks.

Proposition 29 (Liveness of the Cordial Miners Protocol Instances for Asynchrony and Eventual Synchrony). The instances of Algorithm 3 for asynchrony and eventual synchrony satisfy the liveness requirement of Definition 3.

Proof. Consider a suffix of an infinite computation of Algorithm 3. According to Proposition 28, every block $b$ known to a correct miner will be known eventually to every correct miner. The probability of each leader block in this suffix being super-ratified is greater than some fixed $\epsilon$ (specifically, $\epsilon = \frac{2}{3}$ for asynchrony and for eventual asynchrony after GST). Hence the probability measure of an infinite computation in which no correct leader block being super-ratified is zero. Hence, for any block $b$ and point $t$ in the computation, some leader block $b'$ that acknowledges $b$ will be super-ratified at a point later than $t'$ with probability 1. Thus for any block $b$ known to a correct miner, there will be a subsequent super-ratified leader block $b'$ that acknowledges $b$ and hence delivers $b$, satisfying the liveness requirement.

5 Performance analysis

Latency (See Table 1). In the asynchronous instance of the protocol, each wave (rounds till leader finality) consists of six rounds. To calculate the probability that the decision rule is met in a wave, we use the common-core abstraction that is also used (and proved) in DAG-Rider and Bullshark.

Claim 30 (Blocklace common-core). Eventually, for every miner that completes round $r + 5$ there exists a set $V$ of blocks in round $r + 5$ and a set $U$ of blocks in round $r + 2$ such that $|V| \geq 2f + 1$, $|U| \geq 2f + 1$ and for any $u \in U, v \in V$ there exists a path from $u$ to $v$.

Therefore, for every wave $w$, any miner that completes round $r + 5$ has a supermajority (the blocks in $U$) that acknowledge a supermajority in round $r + 2$ (the blocks in $V$). Thus, the probability that the decision rule is met in wave $w$ is the probability that the blocks in $V$ acknowledge the retrospective elected leader block in round $r$. Since each block in $b \in V$ acknowledges at least $2f + 1$ blocks in round $r + 1$, which in turn acknowledge $2f + 1$ blocks in round $r$, then the probability of such a block $b$ to acknowledge a supermajority that approves a correct leader of round $r$ is $(1 - (\frac{f}{2f+1})^2f+1)^{2f+1}$, which is $1 - \epsilon$ where $\epsilon$ is a function of $f$ that converges exponentially to 0 as $f$ increases. Hence, the probability that each of the blocks in $V$ acknowledges the elected leader block is close to 1 if the elected leader is a correct process, and $\frac{2}{3} - \epsilon$ overall.

Therefore, in the good-case, where the decision rule is met in wave $w$, the latency is 6 rounds of communication (the length of a single wave in asynchrony). In the expected case, the decision rule is met on average every $1.5$ waves, and therefore the expected latency is $9$ rounds of communication.

Note that the adversary can equivocate up to $f$ times, but after each Byzantine process $p$ equivocates, all correct processes eventually detect the equivocation and do not consider $p$’s blocks as part of their cordial rounds when building the blocklace. Thus, in an infinite run, equivocations do not affect the overall expected latency.

In the eventual synchrony case, the probability that the decision rule is met in each wave is the probability that the elected leader is a correct miner, therefore, it is at least $2/3$. Thus, in the good case, the latency is 3 rounds of communication (the length of a single wave in eventual synchrony), and in the expected case it is $1.5$ waves, i.e., $4.5$ rounds of communication.
Bit complexity. Each block in the blockchain is linear in size, since it has a linear number of hash pointers to previous blocks. In the worst-case, each block is sent to all miners by all the other miners, i.e., in the worst-case the bit complexity is $O(n^3)$ per block. But, if we batch per block a linear number of transactions, when the decision rule is met, a quadratic number of transactions is committed each time. Thus, the amortized bit complexity per decision is $O(n)$. This is on par with the amortized bit complexity of DAG-Rider and Bullshark.

The concrete message complexity of the protocols depends on the security parameter of the hash function and the implementation of the threshold signatures in the shared random coin for the asynchronous protocol. In any case, using the same security parameters as DAG-Rider and Bullshark achieves the same concrete amortized bit complexity as those protocols as well, and not just the same asymptotic bit complexity.

6 Optimizations and Future Instances of the Cordial Miners Protocol Family

Several optimizations are possible to the protocol instances presented. First, as faulty miners are uncovered, they are excommunicated and therefore need not be counted as parties to the agreement, which means that the number of remaining faulty miners, initially bounded by $f$, decreases. As a result, the supermajority needed for finality is not $\frac{n+f}{2n}$ (namely $2f + 1$ votes in case $n = 3f + 1$), but $\frac{n+f-2f}{2(n-f)}$, where $f'$ is the number of uncovered faulty miners, which converges to simple majority ($\frac{1}{2}$) among the correct miners as more faulty miners are uncovered and $f'$ tends to $f$.

Secondly, once faulty miners are uncovered and excommunicated, their slots as leaders could be taken by correct miners, improving good-case and expected complexity.

Thirdly, a hybrid protocol in the spirit of Bullshark [22] can be explored. Such a protocol would employ two leaders per round—deterministic and random, try to achieve quick finality with the deterministic leader, and fall back to the randomly-selected leader if this attempt fails.

We also intend to explore the use of reliable broadcast [4] in the first round of each wave as it may improve the good case and expected case latency of our protocol in the asynchronous model.

7 Conclusions

The Cordial Miners protocols are simple and efficient. We believe simplicity has many ramifications when practical applications are considered: Simpler algorithms are easier to debug, to optimize, to make robust, and to extend.

An important next step towards making Cordial Miners a useful foundation for cryptocurrencies is to design a mechanism that will encourage miners to cooperate—as needed by these protocols—as opposed to compete, which is the current standard in mainstream cryptocurrencies.
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