On the collectivity of Pygmy Dipole Resonance within schematic TDA and RPA models

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Within schematic models based on the Tamm-Dancoff Approximation and the Random-Phase Approximation with separable interactions, we investigate the physical conditions which determine the emergence of the Pygmy Dipole Resonance in the E1 response of atomic nuclei. We find that if some particle-hole excitation manifests a different, weaker residual interaction, an additional mode will appear, with an energy centroid closer to the distance between two major shells and therefore well below the Giant Dipole Resonance. This state, together with Giant Dipole Resonance, exhausts all the transition strength in the Tamm-Dancoff Approximation and all the Energy Weighted Sum Rule in the Random-Phase Approximation. These features suggest a collective nature for this mode which we identify with the Pygmy Dipole Resonance.

In spite of their apparent simplicity schematic physics models are always very insightful as they provide in a transparent way the essential physical content which determines a specific feature that is shaping an otherwise complex phenomenon. A quite successful class of such models is that devoted to explain within a quantum approach in the absence of ground-state correlations, one usually defines the particle-hole vacuum $|0\rangle$ and the particle (hole) energies associated with the single-particle excitations $\epsilon_p (\epsilon_n)$. The unperturbed particle-hole excitation energies are obtained as $\epsilon_i = \epsilon_p - \epsilon_h$, where $i$ labels the specific particle-hole configurations. If the interaction among these quasiparticles is expressed in terms of the difference between the direct and exchange term as $A_{ij} = V_{ph'}^{ij} = V_{ph'}^{ij} - V_{ph',p'h}$, then, using the linear approximation of the equation-of-motion method \cite{17}, the equations of the Tamm-Dancoff Approximation (TDA) take the form:

\[
\sum_j (\epsilon_i \delta_{ij} + A_{ij}) X_{ij}^{(n)} = E_n X_i^{(n)}. \tag{1}
\]

Together with the normalization condition $\sum_j |X_j^{(n)}|^2 = 1$ they will determine the energy $E_n$ of the state $|n\rangle = \Omega_{TDA}^{(+n)}|0\rangle$, as well as the amplitudes which define the excitation operator:

\[
\Omega_{TDA}^{(+n)} = \sum_{p,h} X_{ph}^{(n)} a_p^+ a_h. \tag{2}
\]

As a next step, the exchange term is neglected and a separable particle-hole interaction $A_{ij} = \lambda D_{ph} D_{p'h}^{(*)} = \lambda Q_i Q_j^*$ is introduced for the direct one. One then arrives at the dispersion relation:

\[
\sum_i \frac{|Q_i|^2}{E_n - \epsilon_i} = \frac{1}{\lambda}, \tag{3}
\]
which can be solved for $E_n$. From a simple graphical analysis one notices that for positive (negative) $\lambda$ one solution is pushed up (down) in energy from the solutions corresponding to the unperturbed energies. This state $|n_c\rangle$ has a collective nature, as it can be easily seen from equation 43 if the degenerate case $\epsilon_i = \epsilon$ is considered. Indeed, for this situation the energy of the collective state is given by $E_{n_c} = \epsilon + \lambda \sum_i |Q_i|^2$, while for all others (non-collective) states one finds that $E_n = \epsilon$. Moreover, the transition probability $|\langle n_c | Q | 0 \rangle|^2 = \sum_i |Q_i|^2$, i.e., the collective state exhausts all the energy-independent sum rules, while the transition probability to other states cancel $\langle n | Q | 0 \rangle = 0$. Allowing for correlations in the ground state, the TDA treatment is upgraded to the Random Phase Approximation (RPA). The amplitudes which appear in the excitation operator

$$
\Omega^{(n)}_{\text{RPA}} = \sum_{p,h} X^{(n)}_{p h} a_p^+ a_h + Y^{(n)}_{p h} a_h^+ a_p,
$$

and which obey the normalization conditions

$$
\sum_j |X_j^{(n)}|^2 - |Y_j^{(n)}|^2 = 1
$$

are obtained from the RPA equations

$$
\epsilon_i X_i^{(n)} + \sum_j (A_{ij} X_j^{(n)} + B_{ij} Y_j^{(n)}) = E_n X_i^{(n)},
$$

$$
\epsilon_i Y_i^{(n)} + \sum_j (B_{ij}^* X_j^{(n)} + A_{ij}^* Y_j^{(n)}) = -E_n Y_i^{(n)},
$$

with $B_{ij} = \tilde{V}_{ph} h^i$. The amplitudes $Y_j$ are a measure of ground state correlations and by setting all $Y_j = 0$ we recover the TDA equations. For separable particle-hole interactions $A_{ij} = \lambda Q_i Q_j^*$ and $B_{ij} = \lambda Q_i Q_j$ we arrive at the dispersion relation

$$
\sum_i \frac{2\epsilon_i |Q_i|^2}{E_n - \epsilon_i} = \frac{1}{\lambda},
$$

which, unlike the TDA treatment, admits a double set of solutions, $\pm E_n$. In the degenerate limit the collective state $|n_c\rangle$ has the energy

$$
E_{n,RPA} = \epsilon + 2\lambda \sum_i |Q_i|^2 = E_{n,TDA} - \lambda^2 \left(\sum_i |Q_i|^2\right)^2.
$$

A very specific feature of the RPA collective state is that it exhausts the whole Energy Weighted Sum Rule (EWSR) gathered in the unperturbed case by the transitions towards unperturbed states $\epsilon \sum_i |Q_i|^2$, i.e.,

$$
E_{n,RPA} |\langle n_c | Q | 0 \rangle|^2 = \epsilon \sum_i |Q_i|^2. \quad \text{Here} \ |0\rangle \text{denotes the correlated ground-state.}
$$

Summing up, we notice that the repulsive residual particle-hole interaction builds up a state which is a coherent sum of the $|ph\rangle$ states. This is characterized by an energy which is pushed upwards from the unperturbed value and carries all the dipole strength.

The expression of the coupling constant $\lambda$ can be obtained from considerations based on the self-consistency between the vibrating potential and the induced density variations $^{18}$. In the case of the GDR this quantity is determined by the potential contribution to the symmetry energy at saturation. In the expression of energy per nucleon the symmetry energy $E_{\text{sym}}/A$ defines the contribution determined by the isospin $I = \frac{N-Z}{A}$ degree of freedom, i.e., by the difference between the number of neutrons and protons: $E/A (\rho, I) = \frac{E}{A} (\rho, I = 0) + \frac{E_{\text{sym}}}{A} (\rho, I^2)$ and contains both a kinetic contribution associated with Pauli correlations, as well as a potential contribution determined by the nuclear interaction:

$$
\frac{E_{\text{sym}}}{A} = \tilde{b}_{\text{sym}} (\rho) I^2
$$

where $\langle r^2 \rangle$ is the mean square radius of the nucleus and $\rho_0$ is the saturation density. Considering this value for $\lambda$ and accounting for the sum-rules satisfied by the matrix elements $|Q_i|^2$, the energy centroid and the EWSR exhausted by the GDR were successfully reproduced within these models.

**TDA treatment for Pygmy Dipole Resonance.** For finite systems, however, part of the nucleons can be at a lower density. Since the symmetry energy decreases with density, one expects a smaller value for the coupling constant for these particle-hole interactions. Similar arguments were promoted in phenomenological models $^{21}$ when three coupled fluids (i.e., protons, blocked neutrons and excess neutrons) were considered to describe various normal modes in a hydrodynamical description. We shall implement this idea in a schematic approach by relaxing the condition to have a unique coupling constant for all particle-hole pairs. To this end, we assume that for a subsystem of particle-hole pairs, namely $i, j \leq i_c$, the interaction is $A_{ij} = \lambda_1 Q_i Q_j^*$, with $\lambda_1 = \lambda$ corresponding to the potential symmetry energy at saturation density, while for the other subsystem, namely $i, j > i_c$, the interaction is characterized by a weaker strength $A_{ij} = \lambda_2 Q_i Q_j^*$, associated to a symmetry energy at a much lower density. If $i \leq i_c, j > i_c$ or $i > i_c, j \leq i_c$, i.e., for the coupling between the two subsystems, we consider $A_{ij} = \lambda_2 Q_i Q_j^*$ with $\lambda_1 > \lambda_2 > \lambda_3 > 0$. The TDA equations for the corresponding amplitudes $X_i^{(n)}$ can be generalized straightforward as

$$
\epsilon_i X_i^{(n)} + \lambda_1 Q_i \sum_{j \leq i_c} Q_j^* X_j^{(n)} + \lambda_2 Q_i \sum_{j > i_c} Q_j^* X_j^{(n)} = E_n X_i^{(n)}
$$

if $i \leq i_c$, (9)

$$
\epsilon_i X_i^{(n)} + \lambda_2 Q_i \sum_{j \leq i_c} Q_j^* X_j^{(n)} + \lambda_3 Q_i \sum_{j > i_c} Q_j^* X_j^{(n)} = E_n X_i^{(n)}
$$

if $i > i_c$, (10)
with the solutions
\[
X_i^{(n)} = \frac{N^c}{E_n - \epsilon_i} Q_i \quad \text{if} \quad i \leq i_c, \tag{11}
\]
\[
X_i^{(n)} = \frac{N^c}{E_n - \epsilon_i} Q_i \quad \text{if} \quad i > i_c. \tag{12}
\]
Here the normalization factors are given by
\[
N^c = \lambda_1 \sum_{j \leq i_c} Q_j^* X_j^n + \lambda_2 \sum_{j > i_c} Q_j^* X_j^n, \tag{13}
\]
\[
N^c = \lambda_2 \sum_{j < i_c} Q_j^* X_j^n + \lambda_3 \sum_{j > i_c} Q_j^* X_j^n. \tag{14}
\]
Using equations (11) and (12) we observe that \(N^c\) and \(N^e\) satisfy the homogeneous system of equations:
\[
\left(\lambda_1 \sum_{i \leq i_c} \frac{|Q_i|^2}{E_n - \epsilon_i} - 1\right) N^c + \lambda_2 \sum_{i > i_c} \frac{|Q_i|^2}{E_n - \epsilon_i} N^c = 0, \tag{15}
\]
\[
\lambda_2 \sum_{i \leq i_c} \frac{|Q_i|^2}{E_n - \epsilon_i} N^c + \left(\lambda_3 \sum_{i > i_c} \frac{|Q_i|^2}{E_n - \epsilon_i} - 1\right) N^c = 0. \tag{16}
\]
If we restore to the degenerate case \(\epsilon_i = \epsilon\), with \(\alpha = \sum_{i \leq i_c} |Q_i|^2, \beta = \sum_{i > i_c} |Q_i|^2\), from the condition to have non-trivial solutions
\[
(E_n - \epsilon)^2 - (\lambda_1 \alpha + \lambda_3 \beta)(E_n - \epsilon) + (\lambda_1 \lambda_3 - \lambda_2^2) \alpha \beta = 0, \tag{17}
\]
we obtain the TDA collective energies
\[
E_n^{(1)} = \epsilon + \frac{(\lambda_1 \alpha + \lambda_3 \beta)}{2} \left(1 - \sqrt{1 - \frac{4(\lambda_1 \lambda_3 - \lambda_2^2) \alpha \beta}{(\lambda_1 \alpha + \lambda_3 \beta)^2}}\right), \tag{18}
\]
\[
E_n^{(2)} = \epsilon + \frac{(\lambda_1 \alpha + \lambda_3 \beta)}{2} \left(1 - \sqrt{1 - \frac{4(\lambda_1 \lambda_3 - \lambda_2^2) \alpha \beta}{(\lambda_1 \alpha + \lambda_3 \beta)^2}}\right). \tag{19}
\]
It is obvious from equation (17) that by setting \(\lambda_1 = \lambda_2 = \lambda_3 = \lambda\) we return to the standard situation with only one collective energy. Simple expressions for \(E_n^{(1)}\) and \(E_n^{(2)}\) are obtained if we assume that \(\lambda_1 \alpha >> \lambda_3 \beta:\)
\[
E_n^{(1)} \approx \epsilon + (\lambda_1 \alpha + \lambda_3 \beta), \tag{20}
\]
\[
E_n^{(2)} \approx \epsilon + \frac{(\lambda_1 \lambda_3 - \lambda_2^2) \alpha \beta}{(\lambda_1 \alpha + \lambda_3 \beta)}. \tag{21}
\]
One of the solutions, \(E_n^{(1)}\), is closer to the value associated to the collective mode obtained in the usual TDA approach while the other one, \(E_n^{(2)}\), is much closer to unperturbed value \(\epsilon\). The corresponding amplitudes \(X_{n_1}^{(2)}\) and \(X_{n_2}^{(2)}\) will define the two operators whose action on ground state will generate the two collective states \(|n_{c,1}\rangle\) and \(|n_{c,2}\rangle\). It is interesting to observe that now the strength of all the transition probabilities towards the unperturbed states \(|ph\rangle\) is distributed only between these two states, i.e.,
\[
|\langle n_{c,1} | Q | 0 \rangle|^2 + |\langle n_{c,2} | Q | 0 \rangle|^2 = \alpha + \beta = \sum_i |Q_i|^2. \tag{22}
\]
We therefore conclude that both states manifest the feature expected for a collective behavior. The validity of equation (22) is quite transparent if one observes that
\[
\langle n_{c,k} | Q_i | 0 \rangle = \sum_i Q_i X_i^{(n_{k})} = \frac{\alpha + x_k \beta}{\sqrt{\alpha + x_k \beta}}, \tag{23}
\]
where \(k = 1, 2\) and \(x_k = ((E_n^{(k)}) - \epsilon - \lambda_1 \alpha)/\lambda_2 \beta\). When all coupling constants become equal the transition amplitude of the state with higher energy goes to \(\alpha + \beta\) as expected, while the transition amplitude of the state with energy \(\epsilon\) becomes zero.

RPA treatment for Pygmy Dipole Resonance. Including the ground state correlations does not change the main conclusions obtained within the TDA treatment. Also in this case we shall find the appearance of a second collective state if the unique coupling constant condition is relaxed. The equations for forward and backward amplitudes become
\[
\epsilon_i X_i^{(n)} + \lambda_1 Q_i (\sum_{j \leq i_c} Q_j^* X_j^n + \sum_{j > i_c} Q_j Y_j^n) + \lambda_2 Q_i (\sum_{j < i_c} Q_j^* X_j^n + \sum_{j > i_c} Q_j Y_j^n) = \alpha \psi_i \psi_i, \tag{24}
\]
\[
\epsilon_i Y_i^{(n)} + \lambda_1 Q_i (\sum_{j \leq i_c} Q_j^* Y_j^n + \sum_{j > i_c} Q_j Y_j^n) + \lambda_2 Q_i (\sum_{j < i_c} Q_j^* Y_j^n + \sum_{j > i_c} Q_j X_j^n) = -\alpha \psi_i \psi_i. \tag{25}
\]
with the solutions
\[
X_i^{(n)} = \frac{M^c}{E_n - \epsilon_i} Q_i; \quad Y_i^{(n)} = -\frac{M^c}{E_n + \epsilon_i} Q_i^* \quad \text{if} \quad i \leq i_c, \tag{26}
\]
\[
X_i^{(n)} = \frac{M^c}{E_n - \epsilon_i} Q_i; \quad Y_i^{(n)} = -\frac{M^c}{E_n + \epsilon_i} Q_i^* \quad \text{if} \quad i > i_c. \tag{27}
\]
The normalization factors
\[
M^c = \lambda_1 \sum_{j \leq i_c} (Q_j^* X_j^n + Q_j Y_j^n) + \lambda_2 \sum_{j > i_c} (Q_j^* Y_j^n + Q_j X_j^n), \tag{28}
\]
\[
M^c = \lambda_2 \sum_{j \leq i_c} (Q_j^* X_j^n + Q_j Y_j^n) + \lambda_3 \sum_{j > i_c} (Q_j^* Y_j^n + Q_j X_j^n). \tag{29}
\]
verify the homogeneous system of equations:

\[
\left( \lambda_1 \sum_{i \leq i_e} \frac{2\epsilon_i |Q_i|^2}{E_n - \epsilon_i^2} - 1 \right) M^c + \lambda_2 \sum_{i > i_e} \frac{2\epsilon_i |Q_i|^2}{E_n - \epsilon_i^2} M^c = 0, \tag{30}
\]

\[
\lambda_2 \sum_{i \leq i_e} \frac{2\epsilon_i |Q_i|^2}{E_n - \epsilon_i^2} M^c + \left( \lambda_3 \sum_{i > i_e} \frac{2\epsilon_i |Q_i|^2}{E_n - \epsilon_i^2} - 1 \right) M^c = 0. \tag{31}
\]

In the degenerate case \( \epsilon_i = \epsilon \) nontrivial solutions are obtained if

\[
(E_n^2 - \epsilon^2)^2 - 2\epsilon (\lambda_1 \alpha + \lambda_3 \beta) (E_n^2 - \epsilon^2) + 4\epsilon^2 (\lambda_1 \lambda_3 - \lambda_2^2) \alpha \beta = 0. \tag{32}
\]

Then the collective RPA energies are:

\[
E_{n,RPA}^{(1)} = \epsilon^2 + 2\epsilon (E_{n,TDA}^{(1)} - \epsilon) = \epsilon (2E_{n,TDA}^{(1)} - \epsilon), \tag{33}
\]

\[
E_{n,RPA}^{(2)} = \epsilon^2 + 2\epsilon (E_{n,TDA}^{(2)} - \epsilon) = \epsilon (2E_{n,TDA}^{(2)} - \epsilon), \tag{34}
\]

where \( E_{n,TDA}^{(1)} \) and \( E_{n,TDA}^{(2)} \) are the corresponding energies in the TDA approximation given by [18,19]. It is interesting to notice that within the RPA treatment the total EWSR is shared only by these two states

\[
E_{n,RPA}^{(1)} |Q_{v,1}^i\rangle^2 + E_{n,RPA}^{(2)} |Q_{v,2}^i\rangle^2 = \sum_i |Q_i|^2, \tag{35}
\]

therefore both of them manifest a collective nature. The last relation can be easily deduced observing that \( k = 1,2 \)

\[
|Q_{v,k}\rangle = \sum_i (Q_{k,i}^* Y_{i}^{(nk)*} + Q_{i}^* Y_{i}^{(nk)*} - \sqrt{\alpha + z_k^2 \beta}) E_{n,RPA}^{(k)} - \frac{\lambda_1 \alpha}{\lambda_2 \beta} \tag{36}
\]

To discuss the predictions of these schematic models for specific nuclear systems, we shall employ the EWSR associated to the dipolar field corresponding to the unperturbed case: \( m_1 = \hbar \omega_0 (\alpha + \beta) = \frac{\hbar^2 N Z}{2m A} \). The separated values for \( \alpha \) and \( \beta \) are related to the number of protons \( Z_e \) and neutrons \( N_e \) which belongs to core \( \times \) (i.e., nucleons at saturation density, \( A_e = N_e + Z_e \)) and the number of neutrons considered in excess \( N_e \) (i.e., nucleons at lower density), respectively. We consider \( N_e \) as a parameter in the following analysis \( (N_e + N_e = N) \) but a more precise value can be fixed from arguments based of density distributions of protons and neutrons. We then obtain \( \hbar \omega_0 \alpha = \frac{\hbar^2 N Z}{2m A_e} \) and \( \hbar \omega_0 \beta = \frac{\hbar^2 N_e Z^2}{2mA_e} \). \[22, 23\]

In what regards the coupling constants we observe that in the presence of the dipolar field the charges of protons and neutrons are considered to be \( N/A \) and \( -Z/A \), respectively. Then \( \lambda_1 = \frac{A^2}{N Z} 10^5 \text{MeV} \). For \( \lambda_3 \) we adopt a constant value \( \lambda_3 = 0.2 \lambda_1 \) which corresponds to the lower density associated to the skin region and investigate the influence of \( \lambda_2 \) when varied from \( \lambda_3 \) (a weak coupling between the two subsystems) to \( \lambda_1 \) (a strong coupling between the two subsystems).

We consider first the nucleus \( ^{68}\text{Ni} \) and determine the position of the energy centroids corresponding to the two collective states both in TDA (black lines) and RPA (red lines) calculations, see Figure 1(a). Two values were chosen for the number of excess neutron, namely \( N_e = 12 \) which corresponds to the extreme case \( N_e = N - Z \) (dashed lines) and \( N_e = 6 \) (solid lines). We observe that the ground state correlations are influencing stronger the GDR peak and that the RPA predictions are closer to the experimental values (around 17.8 MeV). The PDR energy centroid does not change too much neither when we modify the value of \( N_e \) nor when we include the ground state correlations. The experimental value recently reported in [14] is \( E_{PDR}^{exp} = 9.55 \text{ MeV} \), while in our study, for \( N_e = 6 \), it changes from \( E_{PDR} = 10.2 \text{ MeV} \) to 9.3 MeV, when \( \lambda_2 \) increases from \( N_e = 12 \) to \( N_e = 6 \).

We report the same type the calculations for the \( ^{132}\text{Sn} \) in Figure 1(b) considering the cases \( N_e = 32 \) (dashed lines) and \( N_e = 12 \) (solid lines). In this case for \( N_e = 12 \) the position of the PDR energy centroid changes from \( E_{PDR} = 8.5 \text{ MeV} \) to 7.5 MeV when \( \lambda_2 \) is varied as before. A steeper decrease is observed for a greater value of \( N_e \).

In Figure 2 we plot the fraction of EWSR exhausted by the GDR (\( f_{GDR} \)) and PDR (\( f_{PDR} \)) as predicted by the RPA calculations for the same systems: \( ^{68}\text{Ni} \) Fig. 2 (a) and (b) and \( ^{132}\text{Sn} \) Fig. 2(c) and (d). A greater value of \( N_e \) determines a larger value of the EWSR fraction exhausted by PDR. Moreover, \( f_{PDR} \) is strongly influenced by the value of the coupling constant \( \lambda_2 \) at variance with...
the $E_{\text{PDR}}$ position. It goes towards zero when $\lambda_2$ is approaching to $\lambda_1$. In the case of $^{68}\text{Ni}$, for $\lambda_2/\lambda_1 = 0.4$, $f_{\text{PDR}}$ varies from 2.4% to 5.2% when $N_e$ changes from 6 to 12. The experimental values are spanning a domain between 2.8% and 5% [13, 14].

In summary, we introduced in this work schematic models based on separable interactions where the condition to have a unique coupling constant for all particle-hole interactions was relaxed. Since this quantity for the dipole response can be related to the potential part of the symmetry energy, which is density dependent, the model is well suited to describe the situations when part of nucleons are at a lower density, as is the case of the neutron skin. The correlations induced by this type of interaction induce the coherent superposition of particle-hole states which generate two collective states sharing all the EWSR. For realistic values of the parameters we reproduce simultaneously the basic experimental features of the Giant Dipole Resonance and the Pygmy Dipole Resonance which in this description manifest a collective nature.

We would like to complete these final remarks by mentioning that our approach allows also an analysis of the role of the symmetry energy. While the value of $\lambda_1$ is determined by the properties of the symmetry energy at saturation, the values of $\lambda_{2,3}$ can be related to the slope parameter $L$ characterizing the density dependence of the symmetry energy around saturation $E_L$: a greater $\lambda_{2,3}$ means a larger symmetry energy below saturation and this corresponds to a smaller slope parameter $L$. Therefore, from the results reported in Fig. 2 our model predicts that $f_{\text{PDR}}$ will decrease if the slope parameter $L$ is smaller (greater $\lambda_2$). In other words, a stronger coupling between the core and the skin does not favour a PDR response. Finally, let us mention that the observations concerning the position of $E_{\text{PDR}}$, close to $h\omega_0$, the weak dependence of PDR peak on $\lambda_2$ (or equivalently on $L$) as well as the increase of the fraction $f_{\text{PDR}}$ with $L$ are consistent with the findings based on a Vlasov approach [24] to the study of dipolar modes in nuclear systems.

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[1] U. Fano, Rv. Mod. Phys. 64, 313 (1992).
[2] D.M. Brink, Nucl. Phys. A 4, 215 (1957).
[3] G.E. Brown, Unified Theory of Nuclear Models (North-Holland Publishing Company, Amsterdam, 1964).
[4] G.E. Brown, M. Bolsterli, Phys. Rev. Lett. 3, 472 (1959).
[5] G.E. Brown, J.A. Evans, D.J. Thouless, Nucl. Phys. A 24, 1 (1961).
[6] M.N. Harakeh, A. van der Woude, Giant Resonances (Clarendon Press, Oxford, 2001).
[7] V.V. Balashov, V.G. Shevchenko, N.P. Yudin, Sov. Phys. JETP 14, 1371 (1962).
[8] V.V. Balashov, Nucl. Phys. A 40, 93 (1963).
[9] A.M. Green, Prog. Rep. Phys. 28, 113 (1965).
[10] T. Aumann and T. Nakamura, Phys. Scr. T 152, 014012 (2013).
[11] D. Savran, T. Aumann and A. Zilges, Prog. Part. Nucl. Phys. 70, 210 (2013).
[12] P. Adrich et al., Phys. Rev. Lett. 95, 132501 (2005).
[13] O. Wieland et al., Phys. Rev. Lett. 102, 092502 (2009); O. Wieland, A. Bracco, Prog. Part. Nucl. Phys. 66, 374 (2011).
[14] D.M. Rossi et al., Phys. Rev. Lett. 111, 242503 (2013).
[15] N. Paar, D. Vretenar, E. Khan, G. Colo, Rep. Prog. Phys. 70, 691 (2007).
[16] N. Paar, J. Phys. G: Nucl. Part. Phys. 37, 064014 (2010).
[17] T.T. S. Kuo, E. Osnes, in Collective Phenomena in Atomic Nuclei (International Review of Nuclear Physics, Vol 2, 1984) p.79, Edited by T. Engeland, J. Rekstad, J.S. Vaagen (World Scientific, 1984).
[18] A. Bohr, B. R. Mottelson, Nuclear Structure vol II, p. 481 (World Scientific, Singapore, 1998).
[19] V. Baran, M. Colonna, M. Di Toro, V. Greco, Phys. Rep. 410, 335 (2005).
[20] P. Ring, P. Schuck, The Nuclear Many-Body Problem (Springer, New-York, 1980).
[21] R. Mohan, M. Danos, L.C. Biedenharn, Phys. Rev. C 3, 1740 (1971).
[22] V. Baran et al., Rom. J. Phys. 57, 36 (2012).
[23] V. Baran, B. Frecus, M. Colonna, M Di Toro, Phys. Rev. C 85, 034322(R) (2012).
[24] V. Baran, M. Colonna, M Di Toro, A. Croitoru, D. Dumitru, Phys. Rev. C 88, 044610 (2013).