Nuclear “diffuseness” probed by proton-nucleus diffraction

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Abstract. In this contribution, we propose a way to probe nuclear density profile, especially, nuclear surface diffuseness. We discuss the relationship between the nuclear surface diffuseness and the elastic scattering differential cross section at the first diffraction peak of high-energy nucleon-nucleus scattering as an efficient tool for extracting the nuclear surface information from limited experimental data involving short-lived unstable nuclei. The high-energy reaction is described by a reliable reaction theory, the Glauber model. Extending the idea of the black sphere diffraction model, we find that the nuclear radius and surface diffuseness are reflected in the proton-nucleus elastic scattering diffraction at the first peak position, that is, one can extract both the nuclear radius and diffuseness simultaneously, using the position of the first diffraction peak and its magnitude of the elastic scattering differential cross section. The reliability of this approach is confirmed by using realistic density distributions obtained by a microscopic mean-field model. The possibility of separating neutron and proton surfaces is also discussed.

1. Introduction
Probing the nuclear surface profile provides useful information on nuclear structure and is important to know properties of nuclear matter. Here we present our recent works for an approach how to access the nuclear surface information of short-lived unstable nuclei using proton-nucleus scattering. Recent systematic analyses of the total reaction cross sections show that a proton probe is advantageous to study the neutron and proton radii because the proton has more sensitivity to the neutrons at low-incident energy [1, 2]. However, it was found that the proton-nucleus total reaction cross sections was somewhat insensitive to the nuclear density distributions but only probe the nuclear radii. To know more than the nuclear radii, we study the proton-nucleus elastic scattering focusing on reactions of small scattering angles up to a few diffraction peaks. Extending the idea of the black sphere diffraction model [3, 4, 5], we can relate the proton-nucleus diffraction to the nuclear radius as well as properties of the nuclear surface. We show that the nuclear surface “diffuseness” can be extracted as robust structure information using high-energy proton-nucleus elastic scattering. The reader is referred to Ref. [6] for more detailed discussions.

The paper is organized as follows: In the next section, we briefly explain a setup of the reaction model employed in this paper. In Section 3, we show how the nuclear density profile
is reflected in the proton-nucleus diffraction by using a simple density distribution in Sec. 3.1. Then, we move onto a more realistic case. Section 3.2 presents a numerical experiment for extracting the nuclear radius and “diffuseness” from the theoretically obtained elastic scattering differential cross sections using realistic density distributions obtained from a microscopic mean-field model. Section 3.3 discusses the possibility of separating neutron and proton surfaces, also through the numerical experiment. The conclusion is made in Ref. 4.

2. Reaction model

The high-energy reaction is described by a reliable semi-classical reaction theory, the Glauber theory [7], in which the elastic scattering differential cross section is calculated by

$$\frac{d\sigma}{d\Omega}(\theta) = |F(\theta)|^2$$

with the elastic scattering amplitude

$$F(\theta) = \frac{iK}{2\pi} \int e^{-i\mathbf{q} \cdot \mathbf{b}} \left( 1 - e^{i\chi(b)} \right) d\mathbf{b},$$

where $K$ is the wave number in the relativistic kinematics, $\mathbf{q}$ the momentum transfer vector, and $\mathbf{b}$ is the impact parameter vector perpendicular to the beam direction ($z$), and thus $\mathbf{q} \cdot \mathbf{b} = 2Kb\sin^{2}\theta$. In this model, all the structure information and reaction dynamics within the adiabatic and eikonal approximations is included in the phase shift function, $e^{i\chi(b)}$. However, in general, evaluation of the phase shift function is difficult because the multiple integration with respect to all the nucleon coordinates has to be performed [7]. Though it could be possible to do by using a Monte Carlo integration [8, 9] or a factorization procedure by using a Slater determinant wave function [10, 11, 12, 13, 14], for the sake of simplicity, we employ the optical-limit approximation (OLA) as the leading order of the cumulant expansion of the full phase shift function [7, 15]

$$i\chi(b) \simeq -\int \rho(r)\Gamma_{NN}(b-s) \, dr,$$

where $\mathbf{r} = (s, z)$ with $s$ being a two-dimensional vector perpendicular to $z$. Inputs to the theory are nuclear density distributions $\rho(\mathbf{r})$ and the nucleon-nucleon profile function $\Gamma_{NN}$. The parameters of the profile function for a wide range of incident energies are tabulated in Refs. [16, 17]. The multiple scattering effect would be ignored, which is neglected in the OLA, and even becomes smaller for systems involving medium to heavy nuclei [12, 13]. Actually, the OLA works well for many cases of the nucleon-nucleus scattering [8, 9, 11, 12, 13]. We ignore the elastic and inelastic Coulomb contributions since the effects are negligible in scattering with a small charged probe, the proton-nucleus scattering [2].

3. Results

3.1. Nuclear diffuseness and nucleon-nucleus diffraction

Here we discuss what information the elastic scattering cross sections of the first diffraction peak has. In a simple black sphere picture, which assumes a nucleus is a completely absorptive object at a sharp-cut-square-well radius, the model is mathematically equivalent to the Fraunhofer diffraction model [7, 18], offering one-to-one correspondence between the nuclear radius and the diffraction peak position. Actually, the black sphere model explains fairly well a systematic trend of the proton-nucleus total reaction cross sections [3, 4, 5]. Here we show the relationship
between the elastic scattering differential cross sections and density profile. For the sake of simplicity, we firstly assume a two-parameter Fermi (2pF) distribution

\[ \rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-R}{a}\right)} \]  

for the nuclear matter density. This would be a reasonable choice to simulate the density distributions of medium- to heavy mass nuclei. With several choices of the diffuseness parameter \( a \), the radius parameter \( R \) is chosen to give the same root-mean-square (rms) matter radius

\[ r_m = \sqrt{\frac{4\pi}{3} \int_0^\infty dr^3 \rho(r)} \]  

By the normalization condition: \( 4\pi \int_0^\infty \rho(r)^2 dr = A \), where \( A \) is the mass number of a nucleus, the factor \( \rho_0 \) is uniquely determined for given \( R \) and \( a \). We remark that a sharp-cut square-well density distribution with a radius \( \sqrt{\frac{5}{3}} r_m \) is contained in Eq. (4) with the limit \( a \rightarrow 0 \).

We perform the Glauber model calculation with the 2pF density distribution of Eq. (4). The rms radius of the 2pF density distribution is taken as the empirical rms radius \( r_m = \sqrt{\frac{5}{3}} \frac{1.2 A^{1/3}}{\sqrt{5}} \) [19], so that the radius parameter \( R \) is determined for each given \( a \). We note that with a finite \( a \) value the resultant \( R \) is in general different from the radius obtained by the sharp-cut square-well density distribution, \( \sqrt{\frac{5}{3}} r_m \). To calculate the cross sections, averaged nucleon-nucleon profile function [16] is employed. As confirmed in Appendix A of Ref. [6], we can safely use this profile function at incident energies higher than \( \sim 300 \) MeV, where the difference between proton-neutron and proton-proton cross sections are not significant.

Figure 1 plots the calculated elastic scattering differential cross sections of a nucleon-\( ^{120}\text{Sn} \) system and the corresponding density distributions of \( ^{120}\text{Sn} \) with several set of the diffuseness parameter \( a \) in the 2pF density distribution. Again, we note that all the density distributions give the same rms matter radius. It is found that the first peak position does not change with respect to changes of \( a \) and that its magnitude increases with smaller \( a \) (sharper surface). We also observe the same trend at the different incident energies. The nuclear diffuseness is really reflected in the magnitude of the elastic scattering differential cross section at the first peak position as shown below.

3.2. Numerical experiment: Extraction of nuclear radius and diffuseness
In the previous section, we showed the one-to-one correspondence between

(i) The nuclear radius and the first peak position of the elastic scattering differential cross sections.

(ii) The nuclear diffuseness and the magnitude of the elastic scattering differential cross sections at the first peak position.

Here we perform a numerical experiment to deduce the nuclear “diffuseness” as well as the nuclear radius from the reaction data. For this purpose, we use theoretically obtained reaction data, namely, the elastic scattering differential cross sections obtained from the realistic nuclear density distributions, the Skyrme-Hartree-Fock (HF) + BCS method [20, 21] with the SkM* interaction [22]. The validity and interesting applications of these density distributions are given in Refs. [2, 23, 24]. In this analysis, we regard these elastic scattering differential cross sections as experimental data, and then we determine \( R \) and \( a \) in such a way that the elastic scattering differential cross sections with the 2pF density matches the first peak position and its magnitude of the elastic scattering differential cross sections obtained by the HF+BCS density
Figure 1. (Left) Elastic scattering differential cross sections of nucleon($N$)-$^{120}$Sn scattering calculated with 2pF density distributions at 325, 550, and 800 MeV. The cross sections are multiplied by $10^5$ and $10^{10}$ for those at 550 and 800 MeV, respectively. (Right) Corresponding 2pF density distributions of $^{120}$Sn with various diffuseness parameter $a$, giving the same root-mean-square radius.

distributions. In the cross section calculation, we use more realistic profile functions which distinguishes the proton-neutron and proton-proton profile functions [17]. Figure 2 displays the extracted $a$ values at different incident energies at 325, 550 and 800 MeV. The isotope dependence of the nuclear surface profile can clearly be seen in the figure. We find that the $a$ values are obtained independently the choice of the incident energy and they include a robust structure information such as shell evolution, weak binding, and nuclear deformation, which crucially change the density profile at around the nuclear surface. It is interesting to remark that Ref. [23] showed quite similar trend in the surface width extracted from the same density distributions. Systematic determination of the nuclear “diffuseness” can be a good direction to reveal the structure changes in isotopic chains.

3.3. Separation of proton and neutron surfaces
We have discussed how we extract the radius and diffuseness of the matter density distribution from the elastic scattering differential cross section data at a certain incident energy. Separating neutron and proton surfaces is important to extract detailed structure information of unstable nuclei because neutron diffuseness will be more sensitive to the ground-state structure of neutron-rich isotopes as it is dominated by the neutron motion at around the nuclear surface. We remind that the proton probe has a nice property that the proton-neutron cross section becomes much larger than the proton-proton one at low incident energies. We can use this incident energy dependence to extract the neutron and proton diffuseness separately by extending the idea for the total reaction cross section analyses proposed in Refs. [1, 2]. Again, we perform a numerical experiment by respectively assuming 2pF distributions for neutron and proton. Thus, the nuclear density distributions are specified by the two radius parameters, $R_n$ and $R_p$, and two diffuseness parameters, $a_n$ and $a_p$. We determine these four parameters so as to reproduce the four observables, i.e., the first peak positions and their elastic scattering differential cross sections.
at low and high incident energies. To test this method, we calculate the diffuseness parameters and rms radii of neutron and proton for $^{120}$Sn, $^{208}$Pb, and neutron-rich $^{132}$Sn with several sets of two incident energies among 200, 300, 550, and 800 MeV. As was used in the previous section, the parameter sets of the profile function [17] which reproduce the incident energy dependence of the proton-neutron and proton-proton cross sections are used for the Glauber calculation.

Figure 3 plots the extracted rms radii and diffuseness parameters for neutron and proton. Stable $^{120}$Sn, $^{208}$Pb and neutron-rich $^{132}$Sn isotopes are chosen as examples. We choose several sets of two incident energies among 200, 300, 550, and 800 MeV. The error bar indicates a range of each value extracted. For $^{120}$Sn and $^{208}$Pb, the extracted diffuseness parameters are scattered around 0.4-0.6 fm depending on the incident energies chosen, although the rms radii are converged within ~ 0.5%. In such cases where the neutron and proton surfaces are located at almost the same position, the separation of the neutron and proton surface profiles might be difficult, whereas in case of $^{132}$Sn, where the surfaces of the neutron and proton density distributions are well separated ~ 0.2 fm, all the extracted values are consistent with each other. Although the application of the method is limited only to such neutron(proton)-rich systems, this method can be more significant to extract the information on the neutron and proton surfaces from the proton-nucleus elastic scattering in the inverse kinematics, where the most of scattered particles are concentrated on the forward directions.

4. Summary and conclusion

In summary, we have seen the relationship between the nuclear density profile and proton-nucleus elastic differential cross section, especially, focusing on the nuclear “diffuseness”, and performed numerical experiments using the theoretically obtained elastic scattering differential cross sections of high-energy nucleon-nucleus scattering. Based on the analysis using a microscopic reaction model, the Glauber theory, we have shown that the first diffraction peak directly reflects the nuclear density profile at around the nuclear surface. We propose a way to quantify the nuclear diffuseness by assuming the two parameter Fermi function as an input density. The diffuseness parameter can be determined only by the proton-nucleus elastic scattering cross sections at the first peak position. This can also be applied for extracting the information of the neutron and proton surfaces of neutron(proton)-rich nuclei.

This method only needs the cross sections at the forward directions and is advantageous for application to studies of unstable nuclei in which the inverse kinematics is applied. A systematic measurement of the proton-nucleus elastic scattering differential cross sections gives useful information to understand the role of excess nucleons in the nuclear structure.
Figure 3. Nuclear rms radii and “diffuseness” extracted from the numerical experiment with several sets of incident energies. See text for details.

Acknowledgments
This work was in part supported by JSPS KAKENHI Grant Numbers 18K03635, 18H04569, and 19H05140, and the collaborative research program 2019, information initiative center, Hokkaido University.

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