DIFFERENCE IMAGE ANALYSIS OF GALACTIC MICROLENSING. I. DATA ANALYSIS

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ABSTRACT

This is a preliminary report on the application of Difference Image Analysis (DIA) to Galactic bulge images. The aim of this analysis is to increase the sensitivity to the detection of gravitational microlensing. We discuss how the DIA technique simplifies the process of discovering microlensing events by detecting only objects that have variable flux. We illustrate how the DIA technique is not limited to detection of so-called “pixel lensing” events but can also be used to improve photometry for classical microlensing events by removing the effects of blending. We will present a method whereby DIA can be used to reveal the true unblended colors, positions, and light curves of microlensing events. We discuss the need for a technique to obtain the accurate microlensing timescales from blended sources and present a possible solution to this problem using the existing Hubble Space Telescope color-magnitude diagrams of the Galactic bulge and LMC. The use of such a solution with both classical and pixel microlensing searches is discussed. We show that one of the major causes of systematic noise in DIA is differential refraction. A technique for removing this systematic by effectively registering images to a common air mass is presented. Improvements to commonly used image differencing techniques are discussed.

Subject headings: atmospheric effects — Galaxy: stellar content — gravitational lensing — techniques: image processing

1. INTRODUCTION

The study of microlensing has become well established after a number of groups followed up the ground-breaking proposal of Paczynski (1986). Over the past few years, well over 100 events that can only reasonably be attributed to microlensing have been discovered by the MACHO (Alcock et al. 1998a), EROS (Beaulieu et al. 1995), DUO (Alard & Guibert 1997), and OGLE (Udalski et al. 1994) groups, in their attempt to characterize the nature of the dark matter halo of our Galaxy.

Fits to the light curves from microlensing events do not give us a direct measure of the mass of the objects causing the magnification. What they do provide is a measure of the lensing timescale and amplification. The amplification of light from a point source due to microlensing is given by equation (1), where \( u(t) \) can be obtained from equation (2), \( u_{\min} \) is the impact parameter in terms of Einstein radii, and \( t_{\max} \) is the time at which maximum amplification is reached:

\[
A(t) = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}}, \quad (1)
\]

\[
u^2(t) = u_{\min}^2 + \left(\frac{2(t - t_{\max})}{\dot{t}}\right)^2. \quad (2)
\]

The timescale \( \dot{t} \) of a microlensing event is set by the time the Einstein ring takes to traverse the source star at velocity \( V_\parallel \). The projected size of the Einstein radius \( R_E \) (eq. [3]) is dependent upon \( D_\odot \), the observer-lens distance, \( D_s \), the source-observer distance, \( D_{ds} \), the lens source distance, and the lens mass \( M \) (in solar masses):

\[
R_E = \sqrt{\frac{4GM D_s D_{ds}}{c^2}}. \quad (3)
\]

To determine uniquely the mass of the lens causing any given event, one must determine its distance, the source distance, and its transverse velocities relative to our line of sight. This is usually not possible. However, for a small number of exotic events where binarity or parallax are found, the degeneracy of \( M \), \( D_\odot \), and \( V_\perp \) (e.g., Alcock et al. 1995) can be broken. One can instead extract the distribution of lens masses by assuming the distribution of lens transverse velocities and distances (Griest 1991). Because this is a statistical process, an increase in the number of events leads to increased accuracy in the determination of lens mass distribution.

The number of events detected in microlensing surveys is limited by the number of stars that it is possible to monitor and the overall detection efficiency. In terms of telescopes,
this means such surveys are limited by the size of the field of view of the survey telescope and its light-gathering power. Limitations to such surveys also come from the sampling rate and the seeing. These effects limit the accuracy of the photometry obtained and thus the event detection threshold. To maximize the number of events, one naturally chooses to observe fields with the greatest density of stars. This introduces crowding, which sets a seeing-dependent limiting magnitude for such surveys. To maximize the number of monitored stars, one can take an image in the best seeing conditions, where the crowding and sky background levels are at minimum. The detected stars can then be monitored even when the seeing is poor and sky background is high, by transforming the coordinates of stars from the reference image to each subsequent observation. This is the standard approach taken in such surveys.

The use of crowded fields limits the accuracy of photometry. For images with poor seeing conditions, faint stars become immersed in the flux from neighboring brighter stars. An important consideration when working with such fields is that only a small fraction of the stars are actually detected. In fact, within each seeing disk there are generally many stars. The determined stellar centroid is actually a flux-weighted mean centroid of the seeing disk. Any one of the stars within such a blend can be gravitationally lensed. Blending thus has a major effect on the actual number of stars that are monitored and hence the optical depth to microlensing. Such blending of microlensing events causes an amplification bias in the number of detected events (Bailion et al. 1993; Nemiroff 1994; Han 1997b; Woźniak & Paczyński 1997; Alard 1997). In past analyses such effects have been taken into account statistically (Alcock et al. 1997a), but corrections are difficult to apply on an event-by-event basis (Han 1997b). In this way the method of monitoring a star field using fixed positions, in some sense, does allow us to monitor some sources that were too faint to have been detected by virtue of their own flux. This does not, however, allow us to detect any microlensing events due to such source stars that are not blended with a neighboring star or stars bright enough to be detected.

New, large, ground-based telescopes are useful in increasing the number and frequency with which star fields can be monitored because of the shorter exposure times required. But such telescopes are still prone to the same intrinsic seeing limits as small telescopes and thus the same crowding limits of images. The effective seeing can be improved with the use of better observing sites, but this does not resolve the issue of blending completely when many stars are present per arcsec$^2$. Attention must be applied to improving the reduction techniques to remove blending effects. Such techniques would also ideally increase the number of events detected by increasing the effective number of stars monitored per image. To achieve both requires a technique that will overcome the crowding in the images and increase the monitored area within each image. With this intent we shall apply difference image analysis (DIA). This uses a similar technique to that first described by Crotts (1992) and Phillips & Davis (1995).

Lensing events where the source stars are only detectable during microlensing are usually designated “pixel lensing” events. Events where the source star is resolved are termed “classical microlensing” events (see Gould 1996, 1997 for full details). The term pixel lensing comes from an analogy with microlensing surveys in M31 (Ansari et al. 1997), where each pixel in this survey represents the sum of hundreds of unresolved stars that may be microlensed. This term is applicable to local microlensing surveys (bulge and LMC) to an extent, since even for the bulge, given arcsecond seeing, each pixel contains light from a number of stars. In this way we define local pixel lensing events as microlensing events where the sources are not associated with stars we monitor. Furthermore, to remove the ambiguity in what is a pixel lensing event and what is a classical lensing event, we will use the color and position information available with the DIA technique. The three properties flux, color, and position allow us to make a fairly robust separation between these event types.

In the following sections we will demonstrate how the DIA technique can be used to increase the number and quality of results from microlensing surveys. In the next section we will briefly outline the observational strategy. In § 3 we will also discuss how the DIA technique was employed on our set of data and what improvements to the standard method were made. In § 4 we will discuss the relevance of DIA to the issue of blending. In § 5 a comparison between point-spread function (PSF) photometry and this technique is made. In § 6 we outline how to determine source fluxes from the results. In the final section we shall make our concluding remarks. The light curves and parameters for the microlensing events discovered will be presented in Alcock et al. (1999, hereafter Paper II).

2. OBSERVATIONS

The MACHO observation database consists of over 70,000 individual observations of the Galactic bulge, the LMC, and the SMC. The present analysis considers only a single 42′ × 42′ field (MACHO 108) in the Galactic bulge centered at $\alpha = 18^h01^m20^s$, $\delta = -28^\circ17'39''$ (J2000). Observations were taken on the Mount Stromlo and Siding Spring Observatories' 1.3 m Great Melbourne Telescope with the dual-color (red, blue) wide-field “MACHO camera.” All bulge observations have 150 s exposure times.

The MACHO camera consists of a mosaic of eight 2k × 2k CCDs (four red and four blue) with 0.63 pixels. Each of the eight CCDs in the MACHO camera consists of two amps. Light passing through the telescope is separated into two passbands using a dichroic beam splitter allowing both red and blue images of a field to be taken simultaneously (Marshall et al. 1993).

Observations in this analysis span the dates from 1995 March 10 to 1997 August 2, with breaks from the end of October to the beginning of March each year when the bulge is unobservable at Mount Stromlo. Within this observing period 385 observations of the target field were taken. The seeing for the data set varies from 1″ to 6′, with a mean of 2′. The sky background level in the blue bandpass varies from 1000 to 3000 counts with a mean of ~2600 counts. Similar levels were observed for red images.

We chose to reject a number of observations from this data set, since good photometry was required to detect the microlensing of faint stars. Images with seeing FWHM > 4″ or a blue-band sky background greater than 8000 analog-to-digital converter units (ADU) were excluded from the reduction. These two cuts removed 42 observations from the data set. Because the usable area of a difference images is dependent on the differences in pointing
between observations, we also excluded a small number of images where the difference in pointing of the reference and subsequent observation was greater than 25°. With this criteria imposed, we rejected 19 more observations, leaving 324 before reduction was attempted.

3. DIA

Since the work of Crotts (1992) and Phillips & Davis (1995), there have been a number of applications of DIA-type techniques (Tomaney & Crotts 1996; Reiss et al. 1998; Alard & Lupton 1997). These papers show the rapid evolution of the technique to a near optimal case. The implementation of the technique as used here varies in several important aspects from these approaches. So we shall outline our technique, noting the similarities and differences to these applications.

In brief, DIA of our images involves registering a test image to a preselected, high signal-to-noise ratio (S/N), low air-mass, stacked reference image. Next, ~200 bright, uncrowded, so-called PSF stars, are selected to calculate the convolution required to map the template image to the test one. The convolution kernel for this mapping is calculated and applied to the reference. The test image is then photometrically normalized (scaled and offset) to match the convolved reference image. The resultant, registered, convolved, and normalized images are then differenced from the test image. The process is carried out for red and blue passbands, and objects are detected. The positions of these objects are matched in the two colors to separate real objects from spurious ones. The results are then sorted and characterized to separate microlensing events from variable stars.

3.1. Template Construction

To perform DIA the first requirement is to have a comparison reference image (template) to difference against. To minimize the noise contribution to the final difference images from such a reference image, it is an obvious step to use the highest S/N image. The MACHO project has a database of hundreds of observations of each field taken with the same telescope, under similar observing conditions. This database makes it possible to increase the S/N for a reference image simply by stacking matched images. Because the highest S/N is associated with the best seeing, we choose to degrade the reference image seeing to the test image. The registration process is accurate to about 0.1 pixels on average (compared with ~0.3; Reiss et al. 1998). We believe our accuracy is due to the inclusion of higher order transformation terms and the large number of stars used to constrain the fit. To show the importance of accurate image registration we present Figure 1. In this figure we approximate a standard stellar profile as a Gaussian. Using the average seeing of ~2.5 and an offset between two stellar profiles of 0.3, we expect the average residual (as seen perpendicular to the offset direction) to have the form of the long-dashed line given. For our average registration accuracy (0.07) the stellar and residual profiles are given by the short-dashed lines. For this figure we did not include the effects of pixelization or photon noise since this makes little difference to the observed effect.

3.3. Differential Refraction Corrections

The first mention of the deleterious effects of atmospheric refraction on difference images was made by Tomaney & Crotts (1996). They stressed that, for broadband filters, the
is only the smaller second-order effects, caused by differences in star color, which cause differential refraction offsets. The corrections for these effects are in the order of tenths of an arcsecond at blue-band wavelengths and so are still appreciable.

To correct for differential refraction the following approach has been taken: As a first approximation, the effective temperature for any given star can be associated with a color. We know that the position of the centroid of a star is dependent on the color of the star and the air mass. So if one knows the air mass, the color, and the observation parallactic angle, it should be possible to determine the change to the stellar flux distribution with air mass and parallactic angle.

To determine the degree of change we first modeled our Galactic bulge stars based on M. Bessell's (1998, private communication) tabulated color-temperature relations. We then assumed a blackbody spectrum approximation for these stars with no blanketing or spectral features.

These spectra were then integrated within the MACHO camera's blue ($B_m$) and red ($R_m$) passband responses to find the average centroid wavelength associated with a given star color. The $V-R$ colors for M. Bessell's (1998, private communication) tabulated stars were transformed to MACHO $R_m$ and $B_m$ colors using the results of Alves (1998). A star of a given color can thus be related to this centroid wavelength.

$$R_0 = \frac{(n^2 - 1)}{2n^2}, \quad (4)$$

$$R = z_t - z_a, \quad (5)$$

$$R \approx R_0 \tan z_t - 0.067 \tan^3 z_a. \quad (6)$$

The refraction of light at a given wavelength, temperature, and air mass is given by Filippenko (1982). Using the equations in Filippenko (1982) and equation (4), one may determine the constant of refraction, where $n$ is the refractive index of air. The degree of refraction is given by equation (5), where $z_t$ is the true zenith position and $z_a$ is the apparent zenith position. Under normal temperature and pressure conditions ($15^\circ C, 760 \text{ mm Hg}$), this reduces to equation (6), and $R_0$ becomes 58/3.

The positional shift of the centroid of the star is associated with its average blackbody wavelength. For a star of known color the centroid shift can be obtained. The offset in the centroid positions for the model stars are given in Figure 2 for three values of air mass. To make use of this information, one needs to know the color of each star and must be able to shift its centroid in proportion to this color. This task sounds more daunting than it really is. In fact, this can easily be achieved (at least approximately) for two images taken at a known air mass in the following way: Using the IRAF tasks GEOMAP and GEOTRAN, it is possible to map two images of different sizes, orientations, and even geometric distortions onto each other. We used these tasks to map the images from the two passbands of the MACHO camera onto each other. These mapped images were then used to form a quotient image that, after sky background subtraction, can serve as a color map of each pixel in the field. However, as the centroid wavelengths of each star within the two passbands varies with air mass, one must calculate this quotient near unit air mass in order not to have a color map that is affected by refraction.
To find the relationship between the pixel values of our color map and the $V-R$ colors of the stars present in the color map, a simple calibration is carried out. Having performed photometry on the individual images in the color map, one can associate a $B_m - R_m$, and hence a $V-R$ color, with a quotient image pixel value. We thus have a $V-R$ color for every pixel within the image (except for saturated or bad pixels). The relationship between the measured $B_m - R_m$ photometry values and the associated color map pixel value is given in Figure 3. The relationship is quite strong for most stars. A small number of points are scattered because of stars being blended with neighbors.

Because the degree of refraction is wavelength dependent, it is in fact only necessary for us to apply our corrections to images taken through the blue passband. In the red passband the centroid offset is less than one-quarter (<<0.1) that of the blue passband (see Fig. 4).

The final result is that each blue image pixel value in the reference image was interpolated based on the calibrated color map pixel values, the air mass of the observation relative to the reference image, and the parallactic angle of the observation. The flux corrections were carried out while imposing a condition of flux conservation within the image. We interpolated the flux in the reference image rather than in each observation since, for most observations, it is not possible to make a color map for these because of first-order refraction effects. Color maps made with individual observations would have much lower S/Ns and CCD defects.

The results from this reduction were used as an initial estimate, since it was known that the results for the differential refraction offsets would be dependent on the assumptions about the model stars (smooth blackbody emission) and the uncertainty in the form of the MACHO blue-band response function. This estimate was improved by differencing a number of images taken at a range of air masses to obtain a semiempirical result for the offset required with color and air mass (relative to the reference image).

The scale of our corrections can be seen in Figure 5. On the left is a difference image, where the effects of differential refraction have not been removed. On the right the differential-refraction-corrected images are shown. The test image used in producing these difference images is at an air mass of 2.4 (approximately the air-mass limit imposed on these observations). The template air mass is 1.01, so the corrections applied in this case are quite large. However, the same degree of differential refraction effects is observed at much lower air masses when the seeing quality of the test observation is better.

![Figure 2](image2.png)  
**Fig. 2.** Predicted differential refraction offsets between stellar centroids for a range of star colors and air masses. Offsets are simulated for three air masses relative to a reference air mass of 1.01 and $V-R$ color of zero. The three air-mass values are 1.9 (squares), 1.5 (circles), and 1.2 (triangles). Results are for the MACHO blue ($B_m$) passband.

![Figure 3](image3.png)  
**Fig. 3.** Relationship between pixel values in the red divided by blue quotient image (color map) and the standard MACHO passband colors $B_m$ and $R_m$. Values are used to determine and correct for offsets caused by differential refraction for each pixel (star) in the image.

![Figure 4](image4.png)  
**Fig. 4.** Difference in the offset induced by differential refraction as a function of air mass, for a 5 K (model) star and a 10 K (model) star. The two MACHO camera passbands $B_m$ and $R_m$ are shown. Offsets in the blue passband are much bigger than the red because of the strong wavelength dependence of refraction.
image is good. The residual is highly dependent on seeing, with the best seeing images being affected much more because of the steepness of the stellar profile.

The calibrated differential refraction corrections for the blue passband of the MACHO camera, as a function of air mass and color, are given by

\[ S = 0.29 \times \left( \tan \left[ \arccos \left( \frac{1}{A_R} \right) \right] - \tan \left[ \arccos \left( \frac{1}{A_T} \right) \right] \right) , \]  

(7)

\[ O = S(C - 0.6) , \]  

(8)

where \( S \) is related to the scale of the offset for air masses \( A_R \) and \( A_T \) of the reference and test images, respectively; \( O \) is the offset to apply to a given pixel (in arcseconds) of \( V-R \) color \( C \). This process compensates for the shift in the stars centroid position. It does not compensate for the dispersion of the PSF with air mass. However, we believe this effect to be quite small and, for our purposes, ignorable.

3.4. **PSF Matching**

The PSF shape of two images taken at different times is never exactly the same. If we were to match the photometric conditions of a reference and test image and then difference the images, we would not expect to obtain a resulting image without systematic noise. What one would invariably find from such a process is that at the position of each star there would be systematic residuals. The degree and structure of these residuals would be dependent on the form of the spatial difference between the PSFs. To achieve the best difference images one has to match the form of the PSFs. As we mentioned earlier, the template is constructed so that it has high S/N and seeing similar to the best observation.\(^{15}\) To match the PSFs we degrade the reference images' seeing to match that of the test observation.

\(^{15}\) All observations are well sampled with minimum FWHM of \( \sim 2.8 \) pixels.

The PSF-matching process is based on the fact that it is theoretically possible to match the profiles of stars observed under two different sets of conditions, with a simple convolution of the form given by equation (9). Here \( r \) characterizes the flux distribution of a star in good (better) seeing, \( t \) characterizes that in poor seeing, and \( k \) is the convolution kernel. A convolution in real space is equivalent to a multiplication in Fourier space. Therefore, in principle, the Fourier transform (FT) of the kernel required to match the good seeing (reference) image to the poor seeing (test) image should be the quotient of the FTs of the star. The inverse FT recovers the required matching kernel (eq. [10]):

\[ t(x, y, z) = r(x, y, z) \star k(x, y, z) , \]  

(9)

\[ k = \text{IFT} \left[ \frac{\text{FT}(t)}{\text{FT}(r)} \right] . \]  

(10)

In reality, a division of these FTs is very sensitive to the high-frequency, low-power noise component. This leads to a poor match of the images. To fix this, one can use the fact that the PSF of a star is roughly Gaussian and the Fourier transformation of a Gaussian is a Gaussian. We can then select a level in Fourier space below which the noise component is dominant and replace this with a Gaussian fitted to the FT (Ciardullo, Tamblyn, & Phillips 1990). This method is not always useful because in many cases the wings of the FT are not well modeled by a Gaussian.

To determine the best convolution kernel, the highest S/N and least blended PSF stars are required. We thus need these PSF stars to be unblended with neighboring stars in the bulge where effectively all stars are blended. Blended stars must be removed from those used in determining the PSF.

This situation was overcome by producing a list of a couple of hundred bright stars on a given image. Stars were culled from the list based on the relative proximity and brightness of neighboring stars. Blending of bright stars can
in part be determined from the shape of the PSF of each star. We thus selected stars based on their ellipticity, position angle, FWHM, and moment. The remaining stars were then combined to form a generalized PSF profile for the image.

A further complication to the ideal case arises from the fact that the form of the PSF varies across a frame because of poor focusing, telescope flexure, and temperature-dependent effects (see Tomaney & Crotts 1996). If a single star is used to PSF-match an entire image, the systematic noise in the difference image increases the farther one gets from that star. A solution to this is to split the image into subrasters and use a local PSF to match the subregions of the image. The subrasters can then be mosaicked back together to reform the image. Fortunately, the magnitude of the PSF variation for our observations is small, with modest sized effects due to differences only being seen on the scale of 500 × 500 pixels (or 5′ × 5′).

The actual matching of PSFs is accomplished using the IRAF task PSFMATCH. This task is similar to the implementation used by Riess et al. (1998), but has been updated to include new features. The convolution kernel is determined for subrasters 500 × 500, and each is convolved separately, then mosaicked to form 1k × 1k images. The entire observation field of the MACHO camera is 4k × 4k per bandpass, but larger images were not made because of problems with matching such as the scaling, color terms, differences in bias levels, and gain in the different CCDs (Reiss et al. 1998).

The process of replacing the noise with the Gaussian fit is not regarded as the optimal approach because there are real differences between the PSF shape and a Gaussian. To improve the difference image quality, we characterize the residuals in the difference image and remove them. This is accomplished in the following manner: By stacking subrasters of the difference image at the positions where the PSF stars were in the reference image, we determined the median systematic residual at each pixel of the PSF profile. This stacked residual image characterizes the real deviations of the PSF shape from that used in the matching process. This image was then scaled and subtracted from our test image PSF. We finally had an essentially empirical PSF and no need to replace the wings of the FT. The convolution kernel is now recalculated without the Gaussian replacement, and the difference images are recalculated. This final PSF appeared to give the best possible subtraction with the available stellar information. This step was found to reduce the systematic residuals by a factor of ~2 in most cases. The final resulting difference images had an average systematic noise component of ~1.3% in both colors, with some images having less than 0.5%.

3.5. Photometric Scaling

A standard approach to matching reference and test images photometrically is to make a linear fit to the pixel values for a single star found on both images (Reiss et al. 1998). The test image is then scaled and offset using the fitted values. We refer to this as a single-point calibration. Another approach is to use photometry taken on a matched set of stars in the two images and scale the image based on this information. In our analysis we apply this second approach, which we believe is superior.

The single-point technique has the advantage that it uses all the pixel information for this region. However, it suffers from the fact that most of the pixels have a low S/N since they come from the wings or background around a star. Because there are very few pixels with high S/N, the fitted slope can be skewed by the pixels near the background noise limit.

An important consideration when matching images is that the two images were probably taken under different seeing conditions. Although the PSF profiles have already been matched, real images can have gradients in sky brightness and transmission. Sky brightness gradients can often be attributed to the proximity of the observed field to the Moon and naturally are dependent on its phase. Differences in the transmission come with air mass, cloud cover, and dust extinction. Such differences in sensitivity can also be caused by problems with the flat-fielding.

A single-point calibration cannot compensate for the presence of these spatial variations. However, this can easily be achieved by performing photometry on matched pairs of stars across the two images. This photometry gives a scaling factor (which represents changes in transmission), and an offset term (which represents the change in the sky brightness), for each position where the photometry was performed. To deal with the possibility of gradients we fitted a low-order polynomial to the scaling and offset terms and determined a transformation for each pixel in the test image. Such a method requires a large number of points within the image to constrain the fit. This is not a problem because there are many thousands of stars per arcmin² in bulge fields.

The photometric scaling we used was always applied to the test image so that images were always registered to the same reference image. In this way no correction was required for transmission differences with air mass.

3.6. Object Detection

The fully matched images were differenced to reveal objects whose flux rate has varied in some way. The images were then searched for these sources. Variable stars are detected as positive or negative sources as they became brighter and fainter than they were in the reference image. However, microlensing events and asteroids generally only appear as positive excursions from the reference image baseline.

For source detection we applied a purpose-written program. We felt this approach was necessary because the nature of the noise distribution is unlike that in other images. The fundamental difference is that the noise varies from pixel to pixel even though there is no signal. This is due to the photon noise of the subtracted stars. With such a program we search for positive and negative source simultaneously.

To detect objects, we would ideally like to know the noise at each point in the image. To determine this, we simultaneously examine the reference and difference images. The significance of each detection is accessed based on its S/N. We calculate the noise at each source position using the flux in the reference image and the systematics from the difference image. The various noise components (photon, readout, and systematic) are well characterized in the

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16 We expect variations in sky level and transmission to be relatively smooth.

17 It is possible for microlensing events to appear with negative flux if they were amplified in the combined reference images.
reduction and used to provide a position-dependent noise threshold for the difference image. The photon noise of the test image dominates for faint objects, whereas for brighter objects the systematic noise from the difference image becomes important. Because the seeing in our data set is always greater than 2 pixels, we choose to determine the signal in a $3 \times 3$ pixel box at every point in the image. The amount of signal in this box is compared with the noise expected for the same pixels in the reference image. Results with a S/N greater than 3 were written to a file along with other noise parameters such as the systematic and readout noise of the difference image.

The detection process is carried out with images from both passbands. The positions of objects found in the two colors are taken to be matched if they were within 1". This color match provides a strong constraint that separates the real objects from the large number of detections due to cosmic rays, bad pixels, and saturation effects.

Candidate objects that pass these selection criteria were then checked against those obtained in previous reductions of the field. If no match is found, these new results were added to the database of object positions. If a result has previously been detected, the number of detections for this object is incremented. This provides us with an extra parameter in characterizing the nature of the object. Objects detected most often can usually be attributed to variable stars, whereas other detections can be due to asteroids or microlensing events.

3.7. Light Curves

Aperture photometry is performed on the red and blue difference images at the sources positions. We feel that PSF photometry is unnecessary because the difference images are no longer crowded with nonvariable objects. The noise is obtained from the initial images, as noted in the detection strategy. With this photometry we produce a database that is used to produce difference-flux light curves for microlensing event detection.

These positions where photometry is performed are independent of whether there is flux at this position in the reference image or not. This varies from the traditional approach, which has been to follow up stars detected in the template (reference) image only. With this technique we are searching among a database of variables for microlensing rather than among a database of all objects. The number of light curves that require scrutiny is thus around a factor of 20 less than the number of stars detected in the reference image. The removal of the positional dependence gives us greater sensitivity to detecting the microlensing of stars that were either too faint or too blended to be resolved in the reference image. This effectively provides us with a larger search area and a greater number of monitored stars.

4. BLENDING

First, let us define what we mean by blending. This is the case in which two or more stars are located within a few pixels of each other and the seeing disk is greater than a few pixels. Such blending is dependent on seeing and masks the true baseline flux of microlensing sources. In this analysis, we are not concerned with the case where the lens in a microlensing event emits flux that is blended with the flux from the source. In this case the flux of the lens is not amplified, and such blending is generally negligible for stars with masses consistent with microlensing results. The color shifts associated with this type of blending usually cannot be detected with the present level of photometry (Buchalter et al. 1996). This case is also generally inseparable from blending with objects that are not the sources. We are also unconcerned with blending where the source is in a binary association. Here the sources can be blended within the

Fig. 6.—Blending: (a) Position of an microlensing event (arrowed) in an 11" × 11" ground-based image MACHO database image. (b) Drizzled HST image of the same region with reference stars (A–E) to approximately the same scale. Note the number of objects blended within the ground-based images seeing disk. This result comes from an event in the LMC where the crowding is similar to the images analyzed in this analysis.
Einstein radius, but this constitutes a small number of events (Griest & Hu 1992; Dominik 1998), and in many cases one can detect such events from the microlensing light curve.

The presence of the apparent blending of the stars monitored in microlensing surveys has been known for some time (Nemiroff 1994), and it is inherent in the fact that the fields used for microlensing searches are crowded. These fields were chosen because of the large number of stars that could be monitored at one time. Crowding is demonstrated in Figure 6, where the number of stars blended within the seeing disk of the bright stars can be clearly seen.

The main effect of the blending is that we do not know the true source flux for most events. If such events are fitted as unblended sources, the amplitudes of the events and the event timescales are underestimated, and the number of stars monitored is underestimated (Han 1997a; Woźniak & Paczynski 1997). The overall consequence, if blending is unaccounted for, is that ~40% of events toward the bulge are affected by amplification bias, and thus the determined optical depth can be overestimated by a factor of ≈1.3–2.4 (Han 1997a). The blending effect is therefore a major factor in the determination of the true optical depth to microlensing toward the Galactic bulge, LMC, SMC, and M31. Statistical corrections have been made for the LMC and the bulge optical depths (Alcock et al. 1997a, 1997b), but corrections are really required on an event-by-event basis. One way to bypass the effects of blending would be to have Hubble Space Telescope (HST) images of all the events. With this approach we could resolve the source star and obtain an unblended source flux. Unfortunately, a large number of hours of HST time are needed to perform this (Han 1997b). We note, however, that we do have HST observations of microlensing sources for a number of important events.

To determine the blending in the case of classical microlensing, one must attempt to fit the light curves in order to find the unlensed, blended, flux component of the source baseline flux. In this regard Woźniak & Paczynski (1997) noted that one can only determine the blend fraction, with the present accuracy of photometry, when the impact parameter is small, \( u_{\text{min}} < 0.3 \). This is the case in only a small percentage of events. The situation is not quite the same with the DIA since we have different information. We shall outline this in the next section. In short, instead of the color, centroid, and light curve of the blend, we have the color and centroid of the source and the light curve of the source amplification.

5. DIA VERSUS PSF PHOTOMETRY

Our usual technique for detecting microlensing is to perform PSF photometry simultaneously at fixed positions in two passbands. With our DIA technique, photometry is carried out at the end of the reduction process at positions where excess flux was detected at some time during the reduction.

The main advantage of using traditional PSF photometry over DIA comes when dealing with fields where the stellar profiles are not blended together. In such situations the determination of the PSF from a few stars is relatively simple. PSF photometry, unlike DIA photometry, is not subject to either the addition of photon noise via the differing process or the systematics introduced in the image alignment and matching processes.

One common belief about difference image techniques is that they are not as powerful at constraining microlensing as standard PSF photometry because it does not give a baseline flux. This is not the case. For classical microlensing events, photometry can simply be done once on the reference image to determine the baseline flux. In our present method of analysis this image has a higher S/N than any individual image in the data set and has the best seeing and a well-defined PSF. The photometry baseline determined from this frame can simply be added to the individual difference frame photometry to provide the same flux baseline as the standard PSF photometry. However, this source flux baseline can still be blended as in the PSF case.

To produce difference images with the lowest possible systematic noise, the input images must be accurately photometrically matched. If the difference image has only a small contribution from systematics, then we know that the images were accurately matched photometrically. To perform PSF photometry we have to make corrections for air mass and differences in transmission between observations.

In Figure 7 we show the photometry performed on an object with DIA and with PSF photometry using the same set of images. A quite dramatic improvement in the photometry is seen. Proof that this demonstrates a real improvement in photometry is evident from the microlensing fit residuals shown on the same figure. Although only one color is shown, data points from both passbands were consistent within the uncertainties presented. The improvement in photometry comes partly from the fact that the source in this event is highly blended with a neighboring bright star. With DIA the nearby nonvariable star is more accurately removed than with the standard PSF photometry. The PSF photometry is also dependent on how well the centroid positions of the stars were initially determined in the tem-
The constant $C$ within a microlensing event can be represented as follows:

$$f_R(t) = f_{uR} + A(t)f_{RD}, \quad (11)$$

$$f_B(t) = f_{uB} + A(t)f_{BD} \quad (12)$$

(Alcock et al. 1997a), where $f_{uR}$ and $f_{uB}$ are the fluxes of the unlensed blended sources, and $f_{RD}$ and $f_{BD}$ are the baseline fluxes of the lensed star. All of these terms must be found from the microlensing fits. The major problem with this approach is that, even if we know the true color of the source, we do not know its true brightness. For bright, well-covered microlensing events, source fluxes can be found. However, when uncertainties of greater than 1% are associated with the photometry, the fit is practically degenerate with respect to the unlensed flux component (see Woźniak & Paczyński 1997). This means that we cannot accurately determine the value of the amplification for a large number of events.

In Figure 8 we show three models of the amplitude of a microlensing event with different values of the lensing parameters corresponding to different values of source flux (given in Table 1). In the top panel there are three curves that are almost indistinguishable. In the lower panel we plot the difference between the dashed curves and the solid one. One can see that even for such large differences in source flux, hence lensing amplitude, differences of less than a few percent occur in the form of the difference in curve shape. The difference between the curves increases slowly only as one moves away from the true values. This predicament is improved greatly with photometry taken in two or more colors because of the extra leverage this gives.

For DIA the flux in the two bandpasses is given simply by equations (13) and (14). Here we do not have blending to consider, so there is no unlensed flux term. The difference fluxes in the two passbands are given by $f_{RD}(t)$ and $f_{BD}(t)$. The constant $C$ is the ratio of blue to red flux, which can be obtained from the color of the source:

$$f_{RD}(t) = [A(t) - 1]f_{RD}, \quad (13)$$

$$f_{BD}(t) = [A(t) - 1]Cf_{RD}. \quad (14)$$

For this situation we only need to be able to measure the baseline flux in one color to determine the amplitude, since $Cf_{RD} = f_{BD}$. If this is not possible, the accuracy of the determination of the amplitude suffers from the same problems as the PSF photometry. For cases where we cannot determine the baseline flux (pixel lensing), it should be possible to estimate this quantity statistically.

6. SOURCE FLUXES

The case of determining the source flux with pixel microlensing is similar to the case of determining the blending for classical microlensing. In both situations we do not know the initial source flux. The standard way to determine the true amplitude of microlensing events is to use the shape of the light curve. The accurate determination of the source flux requires well-sampled light curves with small percentage errors. In many cases the present data do not meet this requirement.

With pixel lensing the source star of the event is initially unresolved, and the S/N is generally low. The task of determining an accurate source flux can be impossible in many cases. However, from the reference image we can determine an upper limit to this flux. In such cases, as for DIA of classical microlensing events, we still have an accurate position and color for the source. With this color (and the associated uncertainties), the HST luminosity function of the bulge (Holtzman et al. 1998), the upper limit of source flux, and the shape of the light curve, one can determine the probability distribution associated with the source flux for each pixel lensing event. This source flux probability dis-
distributions and the light-curve shape can be used to give the distribution of \( i \) for each event. The combination of these \( i \) distributions gives us an overall distribution of \( i \) from which the lens mass distribution can be extracted in the traditional way (Griest 1991).

For classical microlensing events where the sources are faint and the amplifications are moderate, the associated photometry errors are typically greater than a few percent. Typically, for these events, as for pixel lensing events, we cannot accurately determine the amount of unblended light because of blending. In order to use these events when determining the microlensing optical depth, one approach could be to use the same technique as outlined above for pixel lensing. This would help to constrain a large number of classical events. Indeed, this approach is desirable when we consider the effect of blending on measured optical depth.

Aside from the aspects mentioned above, we note that Griest & Hu (1992) and Dominik (1998) found that it was possible that Galactic binary sources are fitted well with a single source with blending. Because DIA does not suffer from blending, this caveat is removed, and the light curves should thus readily give unbiased results for binary sources. Again we remind the reader that the results of this analysis can be found in Paper II.

7. SUMMARY

We have outlined a detailed approach to the DIA scheme. A method of using multiple PSFs to address the shortfalls of matching images with a single PSF was discussed. We noted how gradients in transmission and sky level across an image can be important when producing difference images. A solution using spatially dependent photometric offsets and scaling was outlined. A method for reducing difference images noise by combining observations to form the reference image was presented.

The importance of using an accurate image registration was stressed. The effect of differential refraction on difference images has been examined and is shown to be crucial. A method for compensating for differential refraction by offsetting stars relative to their colors was presented.

A new method for determining the distribution of event timescales was discussed. This method would use existing HST color-magnitude diagrams to determine the distribution of possible event sources with color. The colors of events determined with DIA would then be used to determine a distribution of possible \( i \) values for each event. These would then be combined to determine the overall distribution of \( i \) values. From this lens mass distribution can be found in the usual way.

We have demonstrated how the DIA photometry can make a large improvement in the quality of light curves over PSF photometry because of blending. We discussed how there are fundamental differences between results from PSF and DIA photometry. Difference images naturally provide unblended colors and centroid positions and light curves for microlensing events, while PSF results are usually blended to some extent.

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