Cosmological constraints on a Peccei-Quinn flatino as the lightest supersymmetric particle

Eung Jin Chun\textsuperscript{a,b}, Hang Bae Kim\textsuperscript{a} and David H. Lyth\textsuperscript{a}

\textsuperscript{a}Department of Physics, Lancaster University, Lancaster LA1 4YB, UK
\textsuperscript{b}Korea Institute for Advanced Study, Seoul 130-012, Korea

(Dated: November 13, 2018)

Abstract

In an interesting class of models, non-renormalizable terms of the superpotential are responsible for the spontaneous breaking of Peccei-Quinn (PQ) symmetry as well as the generation of the $\mu$ term. The flaton fields which break PQ symmetry are accompanied by flatinos, and the lightest flatino (the LSP) can be the stable, while the decay of the lightest neutralino (the NLSP) might be visible at colliders with a low axion scale. We examine the cosmology of these models involving thermal inflation just after the PQ phase transition. The branching ratio of flatons into axions must be small so as not to interfere with nucleosynthesis, and flatons must not decay into the LSP or it will be over-abundant. We explore a simple model, with light flatons which can decay into $Z$ or $W$ bosons, or into a light Higgs ($h^0$) plus a $Z$ boson, to show that such features can be realized in a wide range of parameter space. The mass of the NLSP can be as low as $(m_{h^0} + m_Z)/2$, with an axion scale of order $10^{10}$ GeV and a final reheat temperature typically of order $10$ GeV. Then, the flatino LSP is a good dark matter candidate because the reheat temperature can be high enough to allow its production from the decay of the thermalized LSP, while low enough to prevent its overproduction from the decay of sfermions.
I. INTRODUCTION

In supersymmetric extensions of the Standard Model, both the $\mu$ problem and the strong CP problem can be resolved through a simple extension of the Higgs sector, which implements spontaneously broken Peccei-Quinn (PQ) symmetry [1]. Suppose that the Higgs bilinear term in the minimal supersymmetric standard model (MSSM) superpotential takes the non-renormalizable form:

$$W_{\mu\text{-term}} = h \prod_i \frac{\phi_i^{l_i}}{M_p^{-1}} H_1 H_2$$  \hspace{1cm} (1)

where $M_P \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass, $p = \sum_i l_i \geq 2$, and $W_{\mu\text{-term}}$ respects a PQ symmetry. Then, nonzero vacuum expectations values (vevs) $\langle \phi_i \rangle$ can spontaneously break PQ symmetry, providing the axion solution to the strong CP problem [2], while generating the right order of magnitude for the $\mu$ parameter [3];

$$\mu = h \frac{\prod_i \langle \phi_i \rangle^{l_i}}{M_p^{-1}}.$$  \hspace{1cm} (2)

In the case of a polynomial of the second order ($p = 2$), with $h \sim 1$, this gives $\mu \sim F_a^2/M_P \sim 100$ GeV if the axion scale $F_a \sim \langle \phi_i \rangle$ is of order $10^{10}$ GeV, its lowest feasible value. With a higher power $p > 2$, one would need a larger value of $F_a$ to get the right size of $\mu$.

From now on, we consider the case of supergravity with gravity-mediated supersymmetry breaking [4]. There are two ways of spontaneously breaking PQ symmetry. One is to introduce a renormalizable tree-level superpotential. In this case, the particles corresponding to the fields $\phi_i$, and their spin $1/2$ partners, all have mass of order $F_a$ except for a supermultiplet comprising the axion (the pseudo-Goldstone boson of the PQ symmetry), the spin zero saxion, and the spin $1/2$ axino. The mass of the axino in this case is generically of order 100 GeV, but it can be as small as $O(keV)$ [5, 6]. If it is the lightest supersymmetric particle (LSP), there follow some interesting consequences in cosmology and collider physics. If the axino mass is $\gtrsim 10$ GeV, it can be the cold dark matter of the universe if its (non-thermal) production comes mostly from the decay of the next-to-lightest supersymmetric particle (NLSP) [7]. For this to happen, of course, the primordial LSP relic density has to be diluted away, which would be the case when the reheat temperature after inflation is sufficiently low [8]. (In contrast, the axino mass is required to be less than a few keV if its primordial abundance is not washed out [9].)

The alternative to a renormalizable superpotential is a non-renormalizable superpotential with soft supersymmetry breaking, leading to a flaton model of of PQ symmetry breaking [10]–[22]. (A flaton [16, 23] is a scalar field with vev much bigger than its mass, corresponding to a very flat potential.) For simplicity, the superpotential is supposed to be dominated by a single term,

$$W_{\text{PQ}} = \frac{f}{M_p^{n-3}} \prod_i \phi_i^{k_i}$$  \hspace{1cm} (3)

where $n = \sum_i k_i \geq 4$. If the soft mass-squared of one or more of the singlet fields $\phi_i$ is negative, e.g., through a radiative mechanism [15], there exists always a nontrivial global minimum. The $\phi_i$ with nonzero vevs are then flatons. The axion scale is $F_a \sim \langle \phi_i \rangle \sim (m_0 M_p^{n-3})^{1/n-2}$, which is naturally of the right size. For instance, for $n = 4$, $f \sim 1$ and $m_0 \sim 100$ GeV, the vacuum expectation value is of order $10^{10}$ GeV, but it can be bigger if $n$ is bigger.
In a flaton model, the masses of the particles corresponding to the fields \( \phi_i \), and their spin 1/2 partners, all have mass of order 100 GeV, with the sole exception of the axion. The saxion and axino, defined as the superpartners of the axion, have no special significance and are in general not mass eigenstates. Instead, with \( N \geq 2 \) fields breaking PQ symmetry, there are \( 2N - 1 \) spin zero flaton particles, and \( N \) spin 1/2 flatinos. These play the same role as, respectively, the saxion and the axino of renormalizable models. In particular, the lightest flatino may be the LSP and hence a dark matter candidate.

With either an axino or a flatino LSP, the NLSP, which is typically the lightest neutralino in the MSSM, decays to the LSP with the rate proportional to \( 1/F_a^2 \). An interesting implication for collider experiments has been pointed out in Ref. [24]. Although the NLSP decays are very weak, and thus the corresponding decay lengths can be much larger than a collider size, there is an opportunity to observe these decays in future colliders with a large number of supersymmetric events so as to provide direct information about physics at very high energy scale.

In this paper, we study the cosmology of models with a flatino LSP, identifying various bounds on the parameters which are required for a viable cosmology. The paper may regarded as a sequel to [22], in which all of the flatinos were supposed to be unstable. We shall focus particularly on the case of a low axion scale, \( F_a \sim 10^{10} \) GeV, which increases the possibility of collider signatures.

II. COSMOLOGY OF FLATINO LSP

Flaton models lead to very interesting cosmological effects. On the reasonable assumption that the flaton fields are nonzero in the early Universe, they lead to thermal inflation which can dilute away all the unwanted relics [16, 21]. In a flaton model of PQ symmetry breaking, some restrictions have to be put on the model in order not to overproduce unwanted relics again from flaton decay. As a (scalar) flaton field \( \phi \) has always the decay channel \( \phi \to aa \) with a rate \( \Gamma_{\phi \to aa} \sim m_\phi^3/32\pi F_a^2 \) [6], flaton decay may produce too many (unthermalized) axions and upset standard nucleosynthesis. In the scenario of the flatino LSP under consideration, it is also required that the LSP is not overproduced from flaton decay. In general, a flaton \( \phi \) can decay into ordinary particles and their superpartners \( X \), the axion \( a \) and the flatino LSP \( \tilde{F}_1 \). After flaton decay, the energy densities are given by \( \rho_X = B_X \rho_\phi = (\pi^2/30)g_{RH}T_{RH}^4 \), \( \rho_a = B_a \rho_\phi \) and \( \rho_{\tilde{F}_1} = B_{\tilde{F}_1} (m_{\tilde{F}_1}/m_\phi) \rho_\phi \) where \( \rho_\phi \) is the energy density of the flatons before they decay and the \( B \)'s are essentially branching ratios of the flaton decay. The precise relation between \( B_a \) and the flaton branching ratios is given in [22]. As is well-known, nucleosynthesis (NS) puts a constraint on the extra amount of relativistic energy density, which is conveniently given in terms of the equivalently number of neutrino species, \( \delta N_\nu \). At present [25], the bound is something like \( \delta N_\nu < 0.3 \) in the favored ‘low deuterium’ scenario. This can be translated to upper limits on \( B_a \) and \( B_{\tilde{F}_1} \). Applying \( (\rho_a/\rho_\nu)_NS < \delta N_\nu \) for the energy densities at the time of nucleosynthesis, one finds [19, 22]

\[
B_a < \frac{7}{43} \left( \frac{g_{RH}}{43/4} \right)^{1/3} \delta N_\nu .
\]
In the similar way, the energy density of the stable and massive LSP $\tilde{F}_1$ has to be constrained; 
\((\rho_{\tilde{F}_1}/\rho_\nu)_{NS} < \delta N_\nu\), which leads to 
\[ B_{\tilde{F}_1} < \frac{7/4}{g_{RH} m_{\tilde{F}_1}} \frac{T_{NS}}{T_{RH}} \delta N_\nu. \]  
(5)

To get a rough estimate of the reheat temperature, let us take the rate of the decay $\varphi \to X$ to be $\Gamma_X \sim \Gamma_a/B_a$ resulting in
\[ T_{RH} \approx 1.2 g_{RH}^{-1/4} \sqrt{M_p \Gamma_X} \sim 19 \text{ GeV} \left( \frac{m_\varphi}{100 \text{ GeV}} \right)^{3/2} \left( \frac{10^{10} \text{ GeV}}{F_a} \right) \]  
where $g_{RH} \sim 100$ is the effective number of particle degrees at $T_{RH}$ and we took the numerical factor $B_a = 0.1$.

For $T_{RH} \sim 10$ GeV, we get $B_{\tilde{F}_1} \lesssim 10^{-4}$ with $B_a \lesssim 0.1$. A more stringent limit on $B_{\tilde{F}_1}$ comes from the flatino contribution to the present energy density. That is, in order to avoid the overclosure by the LSP, 
\[ \left( \rho_{\tilde{F}_1}/\rho_c \right)_0 < 1, \]  
one gets
\[ B_{\tilde{F}_1} \lesssim \frac{4}{3} \frac{m_\varphi}{m_{\tilde{F}_1}} \left( \frac{3.55 \text{ eV}}{T_{RH}} \right) B_X. \]  
(7)

Taking $T_{RH} \sim 10$ GeV, one obtains $B_{\tilde{F}_1} \lesssim 10^{-10}$. This is a very strong constraint. It appears impossible to get such a small number without relying on a severe fine-tuning of parameters in the model once flaton decay into flatino is allowed kinematically. Therefore, we have to forbid the decay of flatons into the flatino LSP imposing
\[ m_\varphi < 2m_{\tilde{F}_1}. \]  
(8)

Of course, the condition (7) can be invalidated in the case that the flatino is heavy enough to decay into ordinary particles, e.g., a neutralino and a light Higgs boson [22] which is opposite to our consideration.

Forbidding the decay $\varphi \to \tilde{F}_1$ kinematically, we now have to fulfill the condition (4). Important thermalizable decay modes of a flaton to achieve $B_a \ll B_X \approx 1$ include the decays into two top quarks, two light stops, two light Higgs bosons $h^0$, two $Z/W$ gauge bosons and a $Z$ boson plus a Higgs boson. In this paper, we take the flatons to be as light as possible, so as to allow the lightest possible masses for the flatino LSP and for the NLSP. We therefore assume that only the last two decay modes are kinematically allowed. The decay mode into Higgs bosons comes from direct couplings of Higgs bosons and flatons given by Eq. (1) and has been considered in Ref. [22] in the context of an ordinary neutralino LSP. The other modes listed above come from mixing between Higgs bosons and flatons induced by the same $\mu$ term interaction (1), which is the bosonic counterparts of mixing between neutralinos and flatinos which has been worked out in Ref. [24]. (The qualitative features of the decay modes into top quarks or stops have also been considered in Ref. [19].)

Under the condition of no CP violation, scalar flatons (denoted by $F$) can decay into two gauge bosons, e.g., $F \to WW$, and pseudoscalars (denoted by $F'$) into a $Z$ boson and a scalar Higgs, $F' \to h^0Z$, as we describe below. The former mode gives a lower limit on the scalar flaton masses, $2m_W < m_F$. Combining it with Eq. (8) we find
\[ 2m_W < m_F < 2m_{\tilde{F}_1} < 2m_{\tilde{\chi}_1^0}. \]  
(9)
where $\tilde{\chi}_1^0$ is the NLSP neutralino. (When $m_F < 2m_W$ for instance, flatons can of course decay into three ordinary light fermions mediating a sfermion. But the corresponding decay rates have a large phase space suppression which makes it hard to dominate over the decay mode $F \to aa$.) Concerning pseudoscalar flatons, there are more possibilities. When $m_{F'} > m_F$, the decay mode $F' \to aF$ is open and has to be suppressed against the mode $F' \to h^0Z$. This requires

$$m_{h^0} + m_Z < m_{F'} < 2m_{\tilde{F}_i} < 2m_{\tilde{\chi}_1^0}. \quad (10)$$

On the other hand, if $m_{F'} < m_F$, $F'$ has no two body decay mode into an axion, and thus $F'$ can be so light as to have the decay modes into light fermions; $F' \to f\bar{f}(Z)$ or $f\bar{f}h^0$. In this case, the corresponding decay rate has a large Yukawa-coupling or phase-space suppression factor leading to lower reheat temperature than in Eq. (6), and only the bound (9) is applied.

Without resorting to a specific model, let us describe how the flaton couplings to gauge bosons arise. If we assume no CP violation, scalar flatons mix with the two CP-even Higgs bosons $h^0, H^0$ and pseudoscalar flatons mix with the CP-odd Higgs boson $A$, through the $\mu$-term potential as in Eq. (1). Quantifying small mixtures of flatons in Higgs bosons by $\varepsilon_{h^0F}, \varepsilon_{H^0F}$ and $\varepsilon_{AF}$ in a self-explaining notation, it is straightforward to write down the flaton-Higgs couplings from the MSSM Lagrangian:

$$L = -g_m W_W \varepsilon_{VVF}[FW_μ^+ W_μ^\alpha + \frac{1}{c_W^2}FZ_μ Z^α]$$

$$- \frac{g}{c_W} \varepsilon_{AF'}[\cos(\alpha - \beta)h^0 + \sin(\alpha - \beta)H^0][\bar{f}_μ F'Z^μ] \quad (11)$$

where $\varepsilon_{VVF} \equiv \sin(\alpha - \beta)\varepsilon_{h^0F} - \cos(\alpha - \beta)\varepsilon_{H^0F}$, $\alpha$ is the diagonalization angle of CP-even Higgses, $\beta$ is defined by $\tan \beta = \langle H_2 \rangle/\langle H_1 \rangle$ and $c_W = \cos \theta_W$ with $\theta_W$ being the weak mixing angle.

Let us now consider the flatino–neutralino mixing [24] which arises at tree level due to the $\mu$ term (1). For typical parameters, this mixing will give the main contribution to the flatino interactions with ordinary particles and their superpartners, dominating the one-loop processes [7] which are otherwise responsible for these interactions. The relevant interactions for our purpose are the decay of the NLSP to the flatino LSP, and the decays of sfermions to the flatino LSP.

The term (1) leads to the mass matrix that mixes $\tilde{F}_i$ and $(\tilde{B}, \tilde{W}_3, \tilde{H}_1, \tilde{H}_2)$ as follows;

$$
\begin{pmatrix}
0 & 0 & \mu s_β δ_i & \mu c_β δ_i \\
0 & 0 & 0 & 0 \\
\mu s_β δ_i & 0 & 0 & 0 \\
\mu c_β δ_i & 0 & 0 & 0
\end{pmatrix}
\quad (12)
$$

where $c_β \equiv \cos β, s_β \equiv \sin β$, and the coefficients $δ_i \sim v/F_a$ depend on the specific form of the superpotential (1). Here $v = 264$ GeV is the Higgs vev. Let $N$ be the diagonalization matrix of the MSSM neutralino mass matrix $M_N$: $\tilde{M}_N = N^T M_N N$ where $\tilde{M}_N$ is the diagonalized neutralino mass matrix for the eigenstate $\tilde{\chi}_j^0$. Then, further diagonalizing (12) can be done by the small mixing elements;

$$\varepsilon_{\tilde{\chi}_j^0 \tilde{F}_i} = \delta_i \frac{\mu(s_β N_{3j} + c_β N_{4j})}{m_{\tilde{\chi}_j^0} - m_{\tilde{F}_i}} \quad (13)$$

which describe the small content of the flatino $\tilde{F}_i$ in the neutralino $\tilde{\chi}_j^0$. Note that we can have an enhancement in the mixing (13), $\varepsilon_{\tilde{\chi}_j^0 \tilde{F}_i} \gtrsim 10v/F_a$, when $m_{\tilde{\chi}_j^0} - m_{\tilde{F}_i} = O(10)$ GeV.
Flaton-Higgs mixing parameters and flaton/flatino masses are dependent on specific forms of the terms (1) and (3). As we discussed, in order for the flatino LSP scenario to be consistent with cosmological considerations, the model should fulfill the mass relations (8), (9) and (10), and have sufficiently large mixing elements $\varepsilon$ in Eq. (11) to suppress the axion decay modes. However, note that mixing between the flaton sector and the MSSM sector is determined by the term (1) and flaton/flatino mass spectrum by the term (3) and its soft-breaking term. Therefore, arrangement for the mass relations (8,9,10) and large mixing elements $\varepsilon$ can be done by independent parameters in two separate sectors, which implies that such an arrangement can be easily achieved in generic flaton models.

As the flatino LSP has a mass in the 100 GeV region, its slight regeneration after the thermal inflation can provide a sizable contribution to the matter density at present. There are two important sources of the flatino regeneration. The first is the neutralino decay [7] after its decoupling from the thermal bath. In the flaton models, the NLSP neutralino can decay into $Z(h^0)\tilde{F}_1$ or $\tilde{f}\bar{f}\tilde{F}_1$ within 0.01 sec [24] and thus without affecting nucleosynthesis. Then, the ratio of the flatino LSP density to the critical density is given by

$$\Omega_{\tilde{F}_1} = \frac{m_{\tilde{F}_1}}{m_{\tilde{\chi}_1}} \Omega_{\tilde{\chi}_1}.$$  \hspace{1cm} (14)

For our case, $m_{\tilde{F}_1}/m_{\tilde{\chi}_1} = O(1)$ and thus we can have $\Omega_{\tilde{F}_1} \sim \Omega_{\tilde{\chi}_1} \sim 0.1 - 1$ which is valid in a wide range of the MSSM parameter space [26]. For this mechanism to work, the reheat temperature (6) after thermal inflation should be larger than the neutralino decoupling temperature $\sim m_{\tilde{\chi}_1}/20$. As can be seen from Eq. (6), this condition holds for low $F_a \sim 10^{10} - 10^{11}$ GeV which increases collider signals as well.

A potentially more important source is thermal regeneration [27]. Since our reheat temperature is typically below 100 GeV, decay processes dominate over scattering processes for the thermal regeneration. To get a qualitative calculation of the flatino population, let us take a sfermion–fermion–flatino interaction arising from the neutralino–flatino mixing,

$$\mathcal{L} = -g\varepsilon_{\tilde{f}\tilde{f}\tilde{F}_1} \tilde{f}\tilde{f}\tilde{F}_1 + h.c.$$  \hspace{1cm} (15)

where $\varepsilon_{\tilde{f}\tilde{f}\tilde{F}_1} \propto v/F_a$ contains the factors from gauge quantum numbers and neutralino–flatino mixing and $\tilde{f}/f$ denotes a left-handed or right-handed sfermion/fermion field. The thermally regenerated flatino population is given by $\Omega_{\tilde{F}_1} h^2 = m_{\tilde{F}_1} Y_{\tilde{F}_1}/3.55$ eV with the factor [27, 28]:

$$Y_{\tilde{F}_1} \approx 2 \times 10^{-5} \frac{M_P \Gamma_f}{T_{RH}^2} F(x_f).$$  \hspace{1cm} (16)

where $\Gamma_f = g^2 \varepsilon_{\tilde{f}\tilde{f}\tilde{F}_1} m_f/8\pi$ is the decay rate and $F(x_f)$ is the Boltzmann suppression factor as a function of $x_f = m_f/T_{RH}$ given by

$$F(x_f) \equiv \frac{1}{x_f^2} \int_{x_f}^{\infty} \frac{dr}{e^r - 1} \left[ \left( \frac{\pi}{2} - \tan^{-1} \frac{x}{\sqrt{x^2 - x_f^2}} \right) x^4 + x_f \left( x^2 - 2x_f \sqrt{x^2 - x_f^2} \right) \right].$$

We have defined the function $F(x_f)$ to remove the dependence on the axion scale $F_a$ for the remaining factor; $M_P \Gamma_f/T_{RH}^2 \propto \Gamma_f/\Gamma_\varphi$ where $\Gamma_\varphi$ is the flaton decay rate. To a very good
approximation for the range of \( x \gtrsim 5 \), the function \( F(x) \) can be expressed in terms of an analytic function; \( F(x) \approx 36e^{-0.98x} \) giving

\[
\Omega_{\tilde{F}_1}h^2 \approx 5 \times 10^4 \left( \frac{m_{\tilde{F}_1}}{m_{\tilde{f}}} \right)^2 \left( \frac{\mu_{f}}{10^{-8}} \right)^2 x^2 e^{-0.98x}. \tag{17}
\]

From the above equation, one finds that the requirement \( \Omega_{\tilde{F}_1}h^2 \lesssim 1 \) gives \( x_{\tilde{f}} \gtrsim 17 \) with \( m_{\tilde{F}_1} = m_{\tilde{f}} \) and \( \varepsilon_{f}\tilde{f}\tilde{F}_1 = 10^{-8} \). This consideration gives some meaningful bounds on sfermions masses, \( m_{\tilde{f}} > 17T_{\text{RH}} \) given the reheat temperature (6), or vice versa. The thermally regenerated population of the flatino LSP can also be the cold dark matter which, however, needs a fine arrangement for the sfermion mass and reheat temperature as \( \Omega_{\tilde{F}_1}h^2 \) is a very sensitive function of \( x_{\tilde{f}} \). Note that the thermal regeneration becomes easily negligible as \( F_a \) becomes large. For instance, for \( F_a = 10^{11} \) GeV, one has \( T_{\text{RH}} \sim 2 \) GeV (6) and thus \( x_{\tilde{f}} \gtrsim 50 \) for \( m_{\tilde{f}} > 100 \) GeV.

### III. A MODEL

To illustrate our discussion more explicitly, we consider the minimal case of two fields and \( n = 4 \), which has been analyzed in Ref. [22]. This model contains two scalar flatons, one pseudoscalar flaton, and two flatinos. The flaton superpotential is

\[
W_{PQ} = \frac{f}{M_P} P^3 Q \tag{18}
\]

with \( U(1)_{PQ} \) charges of \( P \) and \( Q \) being 1 and \(-3\), respectively. Analyzing the scalar potential including soft supersymmetry breaking terms

\[
V_{\text{soft}} = \frac{f A_f}{M_P} P^3 Q + h.c., \tag{19}
\]

one finds scalar flatons \( F_{2,1} \) with masses-squared

\[
m_{F_{2,1}}^2 = \frac{1}{2} f^2 \mu_0^2 \left( 3(12 - \xi) + x^2(12 + \xi) \pm |12 - \xi| \sqrt{x^4 + 42x^2 + 9} \right) \tag{20}
\]

and a pseudoscalar flaton \( F' \) with mass-squared

\[
m_{F'}^2 = f^2 \mu_0^2 \xi (x^2 + 9) \tag{21}
\]

where the parameters are defined by

\[
x \equiv \frac{\langle P \rangle}{\langle Q \rangle}, \quad \xi \equiv -\frac{A_f}{f \mu_0} > 0, \quad \mu_0 \equiv \frac{\langle P \rangle \langle Q \rangle}{2M_P}.
\]

In the above, the flatons \( P, Q \) are expanded as

\[
P = \frac{1}{\sqrt{2}} \left( \langle P \rangle + P' - 3i \frac{\langle Q \rangle}{F_a} F' \right)
\]

\[
Q = \frac{1}{\sqrt{2}} \left( \langle Q \rangle + Q' - i \frac{\langle P \rangle}{F_a} F' \right) \tag{22}
\]
where \( F^2_a \equiv \langle P \rangle^2 + 9 \langle Q \rangle^2 \). The diagonalization matrix of the scalar flatons \( P' \) and \( Q' \) is
\[
\begin{pmatrix} P' \\ Q' \end{pmatrix} = \begin{pmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{pmatrix} \begin{pmatrix} F_2 \\ F_1 \end{pmatrix}
\] (23)
where the mixing angle \( \varphi \) is determined by
\[
\begin{align*}
\cos 2\varphi &= \text{sgn} (12 - \xi) \frac{x^2 + 3}{\sqrt{x^4 + 42x^2 + 9}} \\
\sin 2\varphi &= \text{sgn} (12 - \xi) \frac{6x}{\sqrt{x^4 + 42x^2 + 9}}.
\end{align*}
\] (24)

From the superpotential (18), one also finds the flatino masses
\[
m_{\tilde{F}_2, \tilde{F}_1} = 3f\mu_0 [\sqrt{x^2 + 1} \pm 1]
\] (25)
The light flatino \( \tilde{F}_1 \) is supposed to be the LSP. The rotation matrix from the flavor states \( \tilde{P}, \tilde{Q} \) to the mass eigenstates \( \tilde{F}_2, \tilde{F}_1 \) is given by
\[
\begin{pmatrix} \tilde{P} \\ \tilde{Q} \end{pmatrix} = \begin{pmatrix} \cos \tilde{\varphi} & -\sin \tilde{\varphi} \\ \sin \tilde{\varphi} & \cos \tilde{\varphi} \end{pmatrix} \begin{pmatrix} \tilde{F}_1 \\ \tilde{F}_2 \end{pmatrix}
\] (26)
with the mixing angle \( \tilde{\varphi} \) satisfying
\[
\begin{align*}
\cos 2\tilde{\varphi} &= -\frac{1}{\sqrt{x^2 + 1}}, \\
\sin 2\tilde{\varphi} &= -\frac{x}{\sqrt{x^2 + 1}}.
\end{align*}
\] (27)

As all the masses contain the overall factor \(|f\mu_0|\), it is now straightforward to find the region of the parameters \( x \) and \( \xi \) satisfying the desired mass relations. It turns out that \( m_{\tilde{F}_1, \tilde{F}} < 2m_{\tilde{F}_1} \) requires

\[
\begin{align*}
(A) & \quad \xi < 12; \quad x > 2.6, \quad \xi_{a,b} < \xi < \xi_c \\
(B) & \quad \xi > 12; \quad x > 3.5, \quad 12 < \xi < \xi_c
\end{align*}
\] (28)
with
\[
\begin{align*}
\xi_a &= 12 - \frac{x^4 + 42x^2 + 9 - x^2 - 3}{\sqrt{x^4 + 42x^2 + 9 + x^2 - 3}} \\
\xi_b &= 12 - \frac{x^4 + 42x^2 + 9 + 12\sqrt{x^2 + 1} - 5x^2 - 9}{\sqrt{x^4 + 42x^2 + 9 + x^2 - 3}} \\
\xi_c &= 36x^2 + 2 - 2\sqrt{x^2 + 1} \\ & \quad x^2 + 9
\end{align*}
\]
We always have \( m_{\tilde{F}_1} < m_{\tilde{F}'}, m_{\tilde{F}_2} \) in the regions (A,B) and also \( m_{\tilde{F}_2} < m_{\tilde{F}'} \) in the region (B). The requirement (10) has to be fulfilled in both regions.

The important interactions of flatons (flatinos) and MSSM fields comes from the mixings between flatons (flatinos) and Higgs bosons (neutralinos) due to the \( \mu \)-term superpotential of our choice
\[
W_{\mu\text{-term}} = h \frac{PQ}{M_P} H_1 H_2
\] (29)
with \( U(1)_{PQ} \) charge of \( H_1 H_2 \) being +2. Note that \( \mu = h \langle PQ \rangle / 2M_P = h\mu_0 \). First of all, Eq. (29) determines the flatino–Higgsino mixing elements \( \delta_i \) defined in Eq. (12) as follows:
\[
\begin{align*}
\delta_1 &= \frac{v}{F_a} \sqrt{x^2 + 9} \left( c_{\tilde{\varphi}} + xs_{\tilde{\varphi}} \right), \\
\delta_2 &= \frac{v}{F_a} \sqrt{x^2 + 9} \left( -s_{\tilde{\varphi}} + xc_{\tilde{\varphi}} \right),
\end{align*}
\] (30)
with \( c_\varphi \equiv \cos \varphi \) and \( s_\varphi \equiv \sin \varphi \). Including the soft term of (29), we get the Higgs-flaton scalar potential,

\[
V = |H_1|^2 \left( m_{H_1}^2 + \left| h \frac{PQ}{M_P} \right|^2 \right) + |H_2|^2 \left( m_{H_2}^2 + \left| h \frac{PQ}{M_P} \right|^2 \right) \\
+ \left\{ h H_1 H_2 \left( A_h \frac{PQ}{M_P} + 3 f^* \frac{P^2 |Q|^2}{M_P^2} + f^* \frac{P^2 |P|^2}{M_P^2} \right) + c.c. \right\} + \frac{1}{8} (g^2 + g'^2) \left( |H_1|^2 - |H_2|^2 \right)^2. 
\]

We note that the CP-odd Higgs boson has mass-squared

\[
m_A^2 = -\frac{2}{\sin 2\beta} \mu^2 \left( \frac{A_h}{\mu} + \frac{f}{h} (x^2 + 3) \right).
\]

It is then straightforward to find the following flaton–Higgs mixing:

\[
\varepsilon_{h^0 F_1} = \frac{v}{F_a} \frac{\sqrt{x^2 + 9}}{x} \frac{\mu^2}{m_{h^0}^2 - m_{F_1}^2} \left\{ -\sin(\alpha - \beta)[2 s_\varphi - 2 x c_\varphi] \right. \\
+ \cos(\alpha - \beta)[\frac{f}{h} (4x^2 s_\varphi + 6 s_\varphi - 6 x c_\varphi) + \frac{A_h}{\mu} (s_\varphi - xc_\varphi)] \right\} 
\]

\[
\varepsilon_{H^0 F_1} = \frac{v}{F_a} \frac{\sqrt{x^2 + 9}}{x} \frac{\mu^2}{m_{H^0}^2 - m_{F_1}^2} \left\{ \cos(\alpha + \beta)[2 s_\varphi - 2 x c_\varphi] \\
+ \sin(\alpha + \beta)[\frac{f}{h} (4x^2 s_\varphi + 6 s_\varphi - 6 x c_\varphi) + \frac{A_h}{\mu} (s_\varphi - xc_\varphi)] \right\} 
\]

\[
\varepsilon_{h^0 F_2} = \frac{v}{F_a} \frac{\sqrt{x^2 + 9}}{x} \frac{\mu^2}{m_{h^0}^2 - m_{F_2}^2} \left\{ \sin(\alpha - \beta)[2 c_\varphi + 2 x s_\varphi] \\
- \cos(\alpha - \beta)[\frac{f}{h} (4x^2 c_\varphi + 6 c_\varphi + 6 x s_\varphi) + \frac{A_h}{\mu} (c_\varphi + xs_\varphi)] \right\} 
\]

\[
\varepsilon_{H^0 F_2} = \frac{v}{F_a} \frac{\sqrt{x^2 + 9}}{x} \frac{\mu^2}{m_{H^0}^2 - m_{F_2}^2} \left\{ -\cos(\alpha + \beta)[2 c_\varphi + 2 x s_\varphi] \\
- \sin(\alpha + \beta)[\frac{f}{h} (4x^2 c_\varphi + 6 c_\varphi + 6 x s_\varphi) + \frac{A_h}{\mu} (c_\varphi + xs_\varphi)] \right\} 
\]

\[
\varepsilon_{AF'} = \frac{v}{F_a} \frac{x^2 + 3}{x} \frac{\mu^2}{m_A^2 - m_{F'}^2} \left\{ \frac{A_h}{\mu} - \frac{6 f}{h} \right\}
\]

where \( v \equiv \sqrt{\langle H_1 \rangle^2 + \langle H_2 \rangle^2} \). The flaton decay rates for the processes \( F \to WW \) and \( F' \to h^0 Z \) of interest are given by

\[
\Gamma(F \to WW) = \alpha_2 |\varepsilon_{VV F}|^2 \frac{m_W^2}{m_F} \sqrt{1 - 4 \frac{m_W^2}{m_F^2}} 
\]

\[
\Gamma(F' \to h^0 Z) \approx \frac{\alpha_2}{8 c_W^2} \cos(\alpha - \beta)^2 |\varepsilon_{AF'}|^2 m_{F'} \left( 1 + \frac{m_{h^0}^2 - \frac{1}{2} m_Z^2}{m_{F'}^2} \right)
\]

9
where $\varepsilon_{\nu F_1} \equiv \sin(\alpha - \beta)\varepsilon_{h^0 F_1} - \cos(\alpha - \beta)\varepsilon_{H^0 F_1}$. The above rates have to be compared with the rates for the decay processes $F \rightarrow aa$ and $F' \rightarrow aF_i$ [restricting ourselves to the region (B) in Eq. (28)] which are given by [22]

\[
\begin{align*}
\Gamma(F_1 \rightarrow aa) &= \frac{1}{32\pi} \frac{m_{F_1}^3}{F_a^2} \frac{(-x s_\phi + 9 c_\phi)^2}{(x^2 + 9)} \\
\Gamma(F_2 \rightarrow aa) &= \frac{1}{32\pi} \frac{m_{F_2}^3}{F_a^2} \frac{(x c_\phi + 9 s_\phi)^2}{(x^2 + 9)} \\
\Gamma(F' \rightarrow aF_i) &= \frac{1}{16\pi} \frac{m_{F'}^3}{F_a^2} \left(1 - \frac{m_{F_i}^2}{m_{F'}^2}\right)^3 \frac{(3c_\phi - 3xs_\phi)^2}{(x^2 + 9)}.
\end{align*}
\]

(39)

Let us define $R(F_i) \equiv \Gamma(F_i \rightarrow aa)/\Gamma(F_i \rightarrow WW)$ and $R(F') \equiv \Gamma(F' \rightarrow aF_i)/\Gamma(F' \rightarrow h^0Z)$ which behave like

\[
R(F_i) \propto \frac{m_{F_i}^4}{v^2 m_W^2} \frac{m_{A}^4}{\mu^4}, \quad R(F') \propto \frac{m_{F'}^2(m_A^2 - m_{F'}^2)^2}{v^2}\frac{1}{\mu^4}
\]

(40)

apart from the other numerical factors. Eq. (40) shows that small $R$ can be easily obtained with small flaton masses and large $\mu$. In the case of small $\mu$, one would need to have small $m_A$ as well as $m_{F'} < v$.

Having calculated all of the relevant quantities, we could in principle identify the region of parameter space which gives a viable cosmology as well as viable particle physics. Here, we present instead two sets of parameters, with (i) a bino-like and (ii) Higgsino-like NLSP, which satisfy all the requirements. The sets give an NLSP mass around the lowest possible value $(m_{h^0} + m_Z)/2$. The flatino LSP is only a little lighter, while the other flatino as well as the flatons have masses around 200 GeV, features which appear to be typical for the region of parameter space corresponding to a light NLSP.

In making the choice of parameters, we focussed on the region $2m_W < m_{F_1}, m_{F_2} < 2m_Z$, which can be arranged for $\xi \sim 12$ [see Eq. (20)]. Both parameter sets have

\[
\tan \beta = 3, \quad m_{h^0} = 110\text{GeV}, \quad M_2 = 2M_1,
\]

and the other parameters are listed in in Table I along with the important output quantities. In both cases, we have $R(F_i, F') < O(0.01)$ and the reheat temperature $T_{RH} = 16$ GeV using the decay rates (38). The neutralino NLSP decouples after reheating, and can decay into the flatino LSP to provide the cold dark matter of the universe. To check that potential overproduction of the flatino LSP from the decay of thermal sfermions can be avoided, let us take as an example the right-handed stau. From $\varepsilon_{\nu F_1}$ in Table I, the coupling in Eq. (17) is given by $\varepsilon_{\tau_R \nu F_1} = 4.7 \times 10^{-8}$ which gives $\Omega_{\tau_R} h^2 \lesssim 1$ for $m_{\tau_R} \gtrsim 300$ GeV for the case (i) and a similar figure is obtained for the case (ii).

IV. CONCLUSION

In an interesting class of models, a non-renormalizable term of the superpotential is responsible for the spontaneous breaking of PQ symmetry, while another such term generates the $\mu$ term of the MSSM. The flaton fields which break PQ symmetry are accompanied by flatinos, and the lightest flatino can be the LSP. Through the $\mu$ term of the superpotential, the NLSP neutralino decay might then be visible at colliders.
In this paper, we have examined the cosmology of this kind of model, following our earlier work [22] which explored the same flaton models on the assumption that all of the flatinos were unstable. We make the reasonable assumption that the flaton fields are nonzero in the early Universe, giving thermal inflation which eliminates pre-existing relics and leads to a rather well-defined cosmology. The decay of flatons into relativistic axions will interfere with nucleosynthesis unless the branching ratio is small, while the decay of flatons into flatinos should be forbidden altogether so as to avoid overproduction of the flatino LSP. We have argued that these combined requirements lead to the bounds

\[ m_{\tilde{\chi}_1^0} > m_{\tilde{F}_1} > m_W \]  

or

\[ (m_Z + m_{h^0})/2 \]  

on the masses of the neutralino NLSP \( \tilde{\chi}_1^0 \) and the flatino LSP \( \tilde{F}_1 \). In order to have the lightest possible NLSP, one can focus on the case of light scalar (pseudoscalar) flatons, which can decay only into two \( W \) bosons (a light Higgs and a \( Z \) boson). These decays, as well as the neutralino NLSP decay into the flatino LSP, come from the mixing between flatons (flatinos) and Higgses (Higgsinos) due to the \( \mu \) term of the superpotential.

We focus on the case of a low axion scale, \( F_a \sim 10^{10} \) GeV, which increases the feasibility of observing NLSP decays in future colliders. While the axion can hardly be the dark matter with such a low scale, the flatino LSP becomes a good dark matter candidate, because the reheat temperature can be high enough to generate the NLSP in thermal equilibrium, but low enough to suppress flatino production from the decay of thermally produced sfermions.

To make some definite statements, we studied an explicit model with \( F_a \sim 10^{10} \) GeV. We verified that flaton decays into \( WW \) or \( h^0Z \) can be large enough compared to their decays into axions in a wide range of parameter space, giving rise to the reheat temperature \( T_{RH} \approx O(10) \) GeV. In this model, the NLSP can be as light as \( \sim 120 \) GeV with \( m_{h^0} = 110 \) GeV, while the other flatino and the flatons have masses \( \gtrsim 200 \) GeV. We further notice that the parameter space admits the interesting mass region, \( m_{\tilde{\chi}_1^0} - m_{\tilde{F}_1} = O(10) \) GeV, where an enhancement of the flaton–neutralino mixing, \( \varepsilon_{\tilde{\chi}_1^0\tilde{F}_1} \sim 10^{-7} \), results and thus the NLSP decays becomes faster.

[1] R.D. Peccei and H.R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977); Phys. Rev. **D16**, 1791 (1977).
[2] J.E. Kim, Phys. Rep. **150**, 1 (1987); R.D. Peccei, in “CP Violation”, ed. C. Jarlskog (WSPC, Singapore, 1989) p.503.
[3] J.E. Kim and H.P. Nilles, Phys. Lett. **B138**, 150 (1984); E.J. Chun, J.E. Kim and H.P. Nilles, Nucl. Phys. **B370**, 105 (1992).
[4] H.P. Nilles, Phys. Rep. **110**, 1 (1984); H. Haber and G. Kane, Phys. Rep. **117**, 75 (1985); S.P. Martin, hep-ph/9709356.
[5] T. Goto and M. Yamaguchi, Phys. Lett. **B276**, 103 (1992); E.J. Chun, J.E. Kim and H.P. Nilles, Phys. Lett. **B287**, 123 (1992).
[6] E.J. Chun and A. Lukas, Phys. Lett. **B357**, 43 (1995).
[7] L. Covi, J.E. Kim and L. Roszkowski, Phys. Rev. Lett. **82**, 4180 (1999).
[8] E.J. Chun, H.B. Kim and A. Lukas, Phys. Lett. **B328**, 346 (1994); E.J. Chun Phys. Lett. **B348**, 111 (1995); S. Chang and H.B. Kim, Phys. Rev. Lett. **77**, 591 (1996).
[9] K. Rajagopal, M.S. Turner and F. Wilczek Nucl. Phys. **B358**, 447 (1991).
[10] P. Moxhay and K. Yamamoto, Phys. Lett. **B151**, 363 (1985).
[11] G. Lazarides, C. Panagiotakopoulos and Q. Shafi, Phys. Rev. Lett. **56**, 557 (1986).
[12] P. Binetruy and M.K. Gaillard, Phys. Rev. **D34**, 3069 (1986).
[13] J.S. Casas and G.G. Ross, Phys. Lett. B192, 119 (1987).
[14] K. Yamamoto, Phys. Lett. B194, 390 (1987).
[15] H. Murayama, H. Suzuki and T. Yanagida, Phys. Lett. B291, 418 (1992).
[16] D.H. Lyth and E.D. Stewart, Phys. Rev. Lett. 75, 201 (1995); Phys. Rev. D53, 1784 (1996).
[17] T. Barreiro, et al, Phys. Rev. D54, 1379 (1996).
[18] E.D. Stewart, M. Kawasaki and T. Yanagida, Phys. Rev. D54, 6032 (1996).
[19] K. Choi, E.J. Chun and J.E. Kim Phys. Lett. B403, 209 (1997).
[20] L. Hui and E.D. Stewart, Phys. Rev. D60, 023518 (1999).
[21] G. Lazarides and Q. Shafi, hep-ph/0006202.
[22] E.J. Chun, D. Comelli and D.H. Lyth, hep-ph/0008133.
[23] K. Yamamoto, Phys. Lett. B161, 289 (1985).
[24] S.P. Martin, hep-ph/0005116.
[25] S. Sarkar, ‘Big Bang Nucleosynthesis Reprise’, in Dark Matter in Astrophysics and Particle Physics, ed. H. V. Klapdor and L. Baurdis (IOP Publishing, 1999) pp. 108-130; K. A. Olive, G. Steigman and T. P. Walker, Phys. Rep. 333, 389 (2000).
[26] For a recent study and references, see M. Drees, et al, hep-ph/0007202.
[27] L. Covi, H.B. Kim, J.E. Kim and L. Roszkowski, work to appear.
[28] K. Choi, K. Hwang, H.B. Kim and T. Lee, Phys. Lett. B467, 211 (1999).
|                  | Bino-like NLSP | Higgsino-like NLSP |
|------------------|----------------|--------------------|
| **Inputs**       |                |                    |
| $M_1$            | 120 GeV        | 220 GeV            |
| $\mu$            | 287 GeV        | 130 GeV            |
| $m_A$            | 300 GeV        | 150 GeV            |
| $m_{H^0}$        | 305 GeV        | 162 GeV            |
| $x$              | 4              | 4                  |
| $f/h$            | $1/24$         | $1/11$             |
| $A_h/\mu$        | -1             | -2.2               |
| $A_f/\mu$        | $13/24$        | $13/11$            |
| **Outputs**      |                |                    |
| $F_a$            | $6.6 \times 10^{10}$ GeV | $4.4 \times 10^{10}$ GeV |
| $m_{F'}$         | 216 GeV        | 223 GeV            |
| $m_{F_2}$        | 175 GeV        | 181 GeV            |
| $m_{F_1}$        | 162 GeV        | 168 GeV            |
| $m_{\tilde{F}_2}$| 184 GeV        | 190 GeV            |
| $m_{\tilde{F}_1}$| 112 GeV        | 116 GeV            |
| $m_{\tilde{\chi}^0}$ | 121 GeV        | 123 GeV            |
| $\epsilon_{\tilde{\chi}^0 F_1}$ | $-6.4 \times 10^{-8}$ | $37 \times 10^{-8}$ |
| $\epsilon_{\chi^0 F_a}$ | $(15, -1.7, 15.6, -4.8) v/F_a$ | $(10, -6.1, -44, -41) v/F_a$ |
| $T_{RH}$         | 16 GeV         | 16 GeV             |

**TABLE I:** Two representative parameter sets taking $\tan \beta = 3$, $m_{h^0} = 110$ GeV, $M_2 = 2M_1$ and $\xi = 13$. Note that $\chi^0 = (\tilde{B}, \tilde{W}_3, H_1^0, H_2^0)$ and $v = 264$ GeV