An effective approach for the solution of fully fuzzy transportation problems

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Abstract. In this paper, a simple method is proposed to solve fully fuzzy transportation problems whose parameters are triangular fuzzy numbers. We express all the triangular fuzzy numbers in their parametric form, applying the new arithmetic and new ranking function and solve the fully fuzzy transportation problems without converting in its crisp form. A numerical example is provided to illustrate the theory developed in this article.

Keywords: Transportation problem, Triangular fuzzy numbers, new ranking, Arithmetic operations.

1. Introduction

Optimization means minimizing costs, maximizing benefit, minimizing the occurrence of errors in a model and achieving the best design. It is a basic tool in all fields of applied mathematics, engineering, medicine, economics and other sciences.

Transportation problems are one such optimization problems and are subclass of Linear Programming Problems. It involves distribution of varying quantities of a homogeneous product from the producer to the consumer through various modes of distribution like traders, retailers and so on. A destination may receive its demand from many sources. However, a shipping plan is executed in such a way that it reduces the cost of transportation of goods. To prepare a shipping plan, availability of resources at each source, requirements at each destination and unit transportation cost from a source to a destination must be known well in advance. Execution of such shipping plan will result in better optimization of the transportation problem.

In real life circumstances, due to lack of exact information or errors in measurement, the decision parameters such as cost of transportation, available supply, required demand, etc, may not be known precisely. They may be available as approximate quantities. Hence the classical set theory and its
methodologies are not suitable to solve such problems with imprecise data. Zadeh’s fuzzy set theory is an important tool to solve such kind of transportation and assignment problems in imprecise nature.

The transportation problem was first recommended by Hitchcock (1941) and later Charnes et al. [4] introduced solution methods for the same. Further, Russell [16] applied Dantzig’s algorithm extension for the transportation problem and obtained an initial near-optimal basis. Afterwards, several authors like Pandian et.al. [15], Korkoglu et.al. [8], Babu et.al. [3], Ahmed et.al. [2], Morade [13] and Kumar et.al. [10] applied different approaches to solve the transportation problems.

Fegad et.al. [6] used interval and triangular membership functions to obtain the optimal solution of fuzzy transportation problem. Narayanamoorthy et.al. [14] applied fuzzy Russells method, WaliUllah et.al. [19] applied Modified Vogel's Approximation method, Dinesh et.al. [5] used one-point conventional model and Kenan Karagaul et.al. [7] used a novel approximation method to solve fuzzy transportation problem. Amarpreet Kaur et.al [1], Shugani Poonam et.al. [18], Mathur et.al. [12], Maheswari et.al. [11] have discussed fuzzy transportation problems with different ranking methods.

In this paper, we minimize the transportation cost in the easiest way by representing the decision parameters (fuzzy numbers) in its parametric form with location index and fuzziness index concept.

In this paper, we minimize the transportation cost in the easiest way by representing the decision parameters (fuzzy numbers) in its parametric form with location index and fuzziness index concept.

The rest of our work is organized as follows. Fuzzy set and its Basics, arithmetic operations of triangular fuzzy numbers are discussed in section 2. Algorithm for the proposed model is discussed in section 3. A real-life example is dedicated to understand the fuzzy transportation problem in section 4. Conclusion part in Section 5.

2. Basic concepts

Definition 2.1. A fuzzy set \( \tilde{a} \) defined on the set of real numbers \( R \) is said to be a fuzzy number, if its membership function \( \tilde{a} : R \rightarrow [0,1] \) has the following characteristics:

(i) \( \tilde{a} \) is convex, (i.e.) \( \tilde{a}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\tilde{a}(x_1),\tilde{a}(x_2)\}, \lambda \in [0,1] \), for all \( x_1, x_2 \in R \)

(ii) \( \tilde{a} \) is normal, (i.e.) there exists an \( x \in R \) such that \( \tilde{a}(x) = 1 \)

(iii) \( \tilde{a} \) is piecewise continuous.

Definition 2.2. A fuzzy number \( \tilde{a} \) on \( R \) is a triangular fuzzy number if its membership function \( \tilde{a} : R \rightarrow [0,1] \) has the following characteristics:

\[
\tilde{a}(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\
\frac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3 \\
0, & \text{otherwise}
\end{cases}
\]

We denote this triangular fuzzy number as \( \tilde{a} = (a_1, a_2, a_3) \). We use \( F(R) \) to denote the set of all triangular fuzzy numbers defined on \( R \).
Definition 2.3. A triangular fuzzy number \( \tilde{a} = (a_1, a_2, a_3) \in F(R) \) can also be represented as an pair \( \tilde{a} = (\underline{a}, \overline{a}) \) of functions \( \underline{a}(\alpha), \overline{a}(\alpha) \), for \( 0 \leq \alpha \leq 1 \) which satisfies the following requirements:

(i) \( \underline{a}(\alpha) \) is a bounded monotonic increasing left continuous function.

(ii) \( \overline{a}(\alpha) \) is a bounded monotonic decreasing left continuous function.

(iii) \( \underline{a}(\alpha) \leq \overline{a}(\alpha), \quad 0 \leq \alpha \leq 1. \)

It is also represented by \( \tilde{a} = (a_0, a_*, a^*) \) where \( a_* = (a_0 - \underline{a}), \quad a^* = (\overline{a} - a_0) \) are called the left fuzziness index function and the right fuzziness index function respectively. For an arbitrary triangular fuzzy number \( \tilde{a} = (\underline{a}, \overline{a}) \) the number \( a_0 = \left( \frac{\underline{a}(1) + \overline{a}(1)}{2} \right) \) is said to be a location index number of \( \tilde{a} \).

2.1. Arithmetic Operations and Ranking of Triangular Fuzzy Numbers

For arithmetic operations and ranking of triangular fuzzy numbers (See [9] )

3. Fuzzy Transportation Problem

We have taken a fuzzy transportation problem with \( m \) sources and \( n \) destinations involving trapezoidal fuzzy numbers.

Let \( a_i, (a_i \geq 0) \) be the availability at source \( i \) and \( b_j, (b_j \geq 0) \) be the requirement at destination \( j \).

Let \( c_{ij} \) be the unit fuzzy transportation cost from source \( i \) to destination \( j \).

Let \( x_{ij} \) denote the number of fuzzy units to be transported from source \( i \) to destination \( j \).

The problem is to determine a feasible way of transporting where the available cost is minimized.

Minimize \( Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij} \)

\[ \sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, 3, \ldots m \]

\[ \sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, 3, \ldots n \]

subject to

\[ \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j, \quad i = 1, 2, 3, \ldots m, \quad j = 1, 2, 3, \ldots n \]

\[ x_{ij} \geq 0 \]
3.1 Algorithm for Fuzzy Transportation Problem

Step 1: Construct the transportation table and examine whether total demand equals total supply.

Step 2: Represent all the fuzzy numbers in its parametric form.

Step 3: Difference between maximum and minimum element of each row is calculated and is divided by the number of columns of the cost matrix. The calculated value is considered as the row-wise difference.

Step 4: Difference between maximum and minimum element of each column is calculated and is divided by the number of rows of the cost matrix. The calculated value is considered as the column-wise difference.

Step 5: Find the maximum of the resultant values and find the corresponding minimum cost value, allocate that particular cell of the given matrix. Suppose there is more than one maximum consequent value one can be selected.

Step 6: Iterate Step (1) through Step (5) till all the allocations are completed.

4. Numerical Example

Consider the fuzzy transportation problem discussed by Srinivasan et.al. [17] where the parameters are triangular fuzzy numbers.

|       | D1          | D2          | D3          | D4          | D5          | D6          | Supply       |
|-------|-------------|-------------|-------------|-------------|-------------|-------------|--------------|
| S1    | (10,12,14)  | (3,4,5)     | (11,13,15)  | (16,18,20)  | (8,9,10)    | (1,2,3)     | (100,120,140)|
| S2    | (8,9,10)    | (14,16,18)  | (9,10,11)   | (6,7,8)     | (13,15,16)  | (10,11,12)  | (65,80,95)   |
| S3    | (3,4,5)     | (8,9,10)    | (9,10,11)   | (6,8,10)    | (8,9,10)    | (6,7,8)     | (40,50,60)   |
| S4    | (8,9,10)    | (2,3,4)     | (10,12,14)  | (3,6,9)     | (3,4,5)     | (3,5,7)     | (80,90,100)  |
| S5    | (6,7,8)     | (10,11,12)  | (3,5,7)     | (16,18,20)  | (1,2,3)     | (6,7,8)     | (80,100,120) |
| S6    | (14,16,18)  | (6,8,10)    | (3,4,5)     | (3,5,7)     | (0,1,2)     | (9,10,11)   | (55,60,65)   |
| Demand| (55,75,95)  | (80,85,90)  | (120,140,160)| (30,40,50)  | (90,95,100) | (55,65,75)  |              |

Converting all fuzzy numbers in parametric form, we have
Table 2. Fuzzy transportation problem in parametric form

|      | $D_1$                      | $D_2$                      | $D_3$                      | $D_4$                      | $D_5$                      | $D_6$                      | Supply                  |
|------|----------------------------|----------------------------|----------------------------|----------------------------|----------------------------|----------------------------|-------------------------|
| $S_1$| $(12,2\alpha,2\alpha)$    | $(4,1\alpha,1\alpha)$     | $(13,2\alpha,2\alpha)$    | $(18,2\alpha,2\alpha)$    | $(9,1\alpha,1\alpha)$     | $(2,1\alpha,1\alpha)$     | $(120,20\alpha,20\alpha)$ |
| $S_2$| $(9,1\alpha,1\alpha)$     | $(16,2\alpha,2\alpha)$    | $(10,1\alpha,1\alpha)$    | $(7,1\alpha,1\alpha)$     | $(15,2\alpha,2\alpha)$    | $(11,1\alpha,1\alpha)$    | $(80,15\alpha,15\alpha)$ |
| $S_3$| $(4,1\alpha,1\alpha)$     | $(9,1\alpha,1\alpha)$     | $(10,1\alpha,1\alpha)$    | $(8,2\alpha,2\alpha)$     | $(9,1\alpha,1\alpha)$     | $(7,1\alpha,1\alpha)$     | $(50,10\alpha,10\alpha)$ |
| $S_4$| $(9,1\alpha,1\alpha)$     | $(3,1\alpha,1\alpha)$     | $(12,2\alpha,2\alpha)$    | $(6,3\alpha,3\alpha)$     | $(4,1\alpha,1\alpha)$     | $(5,2\alpha,2\alpha)$     | $(90,10\alpha,10\alpha)$ |
| $S_5$| $(7,1\alpha,1\alpha)$     | $(11,1\alpha,1\alpha)$    | $(5,2\alpha,2\alpha)$     | $(18,2\alpha,2\alpha)$    | $(2,1\alpha,1\alpha)$     | $(7,1\alpha,1\alpha)$     | $(100,20\alpha,20\alpha)$ |
| $S_6$| $(16,2\alpha,2\alpha)$    | $(8,2\alpha,2\alpha)$     | $(4,1\alpha,1\alpha)$     | $(5,2\alpha,2\alpha)$     | $(1,1\alpha,1\alpha)$     | $(10,1\alpha,1\alpha)$    | $(60,5\alpha,5\alpha)$   |
| Demand| $(75,20\alpha,20\alpha)$ | $(85,5\alpha,5\alpha)$   | $(140,20\alpha,20\alpha)$| $(40,10\alpha,10\alpha)$  | $(95,5\alpha,5\alpha)$   | $(65,10\alpha,10\alpha)$  |                         |

Table 3. Fuzzy transportation problem after giving the allocation

|      | $D_1$                      | $D_2$                      | $D_3$                      | $D_4$                      | $D_5$                      | $D_6$                      | Supply                  |
|------|----------------------------|----------------------------|----------------------------|----------------------------|----------------------------|----------------------------|-------------------------|
| $S_1$| $(12,2\alpha,2\alpha)$    | $(4,1\alpha,1\alpha)$     | $(13,2\alpha,2\alpha)$    | $(18,2\alpha,2\alpha)$    | $(9,1\alpha,1\alpha)$     | $(2,1\alpha,1\alpha)$     | $(120,20\alpha,20\alpha)$ |
| $S_2$| $(9,1\alpha,1\alpha)$     | $(16,2\alpha,2\alpha)$    | $(10,1\alpha,1\alpha)$    | $(7,1\alpha,1\alpha)$     | $(15,2\alpha,2\alpha)$    | $(11,1\alpha,1\alpha)$    | $(80,15\alpha,15\alpha)$ |
| $S_3$| $(4,1\alpha,1\alpha)$     | $(9,1\alpha,1\alpha)$     | $(10,1\alpha,1\alpha)$    | $(8,2\alpha,2\alpha)$     | $(9,1\alpha,1\alpha)$     | $(7,1\alpha,1\alpha)$     | $(50,10\alpha,10\alpha)$ |
| $S_4$| $(9,1\alpha,1\alpha)$     | $(3,1\alpha,1\alpha)$     | $(12,2\alpha,2\alpha)$    | $(6,3\alpha,3\alpha)$     | $(4,1\alpha,1\alpha)$     | $(5,2\alpha,2\alpha)$     | $(90,10\alpha,10\alpha)$ |
| $S_5$| $(7,1\alpha,1\alpha)$     | $(11,1\alpha,1\alpha)$    | $(5,2\alpha,2\alpha)$     | $(18,2\alpha,2\alpha)$    | $(2,1\alpha,1\alpha)$     | $(7,1\alpha,1\alpha)$     | $(100,20\alpha,20\alpha)$ |
| $S_6$| $(16,2\alpha,2\alpha)$    | $(8,2\alpha,2\alpha)$     | $(4,1\alpha,1\alpha)$     | $(5,2\alpha,2\alpha)$     | $(1,1\alpha,1\alpha)$     | $(10,1\alpha,1\alpha)$    | $(60,5\alpha,5\alpha)$   |
| Demand| $(75,20\alpha,20\alpha)$ | $(85,5\alpha,5\alpha)$   | $(140,20\alpha,20\alpha)$| $(40,10\alpha,10\alpha)$  | $(95,5\alpha,5\alpha)$   | $(65,10\alpha,10\alpha)$  |                         |
Transportation cost =

\[
(4,1-\alpha,1-\alpha)(55,20-20\alpha,20-20\alpha) + (2,1-\alpha,1-\alpha)(55,20-20\alpha,20-20\alpha) + (9,1-\alpha,1-\alpha)(25,20-20\alpha,20-20\alpha) + (10,1-\alpha,1-\alpha)(55,20-20\alpha,20-20\alpha) + (4,1-\alpha,1-\alpha)(50,20-20\alpha,20-20\alpha) + (3,1-\alpha,1-\alpha)(30,10-10\alpha,10-10\alpha) + (12,2-2\alpha,2-2\alpha)(20,20-20\alpha,20-20\alpha) + (6,3-3\alpha,3-3\alpha)(40,20-20\alpha,20-20\alpha) + (5,2-2\alpha,2-2\alpha)(5,20-20\alpha,20-20\alpha) + (2,1-\alpha,1-\alpha)(95,20-20\alpha,20-20\alpha) + (4,1-\alpha,1-\alpha)(60,20-20\alpha,20-20\alpha) + (2,1-\alpha,1-\alpha)(95,20-20\alpha,20-20\alpha) + (4,1-\alpha,1-\alpha)(60,20-20\alpha,20-20\alpha)
\]

\[
= (2350,20-20\alpha,20-20\alpha)
\]

\[
= (2330+20\alpha,2350,2370-20\alpha)
\]

5. Results and Discussions

The optimum solution obtained by the proposed method are tabulated for different values of \( \alpha \).

**Table 4: Some of the fuzzy optimum solutions**

| Value of \( \alpha \) | Proposed Method |
|-----------------------|-----------------|
| \( \alpha = 0 \)      | (2330,2350,2370) |
| \( \alpha = 0.5 \)    | (2340,2350,2360) |
| \( \alpha = 1 \)      | (2350,2350,2350) |

For the same problem Srinivasan et.al. [17] have obtained (1825,2455,3085) which is with more vagueness comparing with the solution obtained by the proposed method.

6. Conclusion

In this paper, a fully fuzzy transportation problem involving triangular fuzzy numbers is considered. A simple and effective Max-min method was introduced to solve fully fuzzy transportation problem with triangular fuzzy numbers without affecting the fuzzy nature of the given problem. A numerical example is solved and a vagueness reduced optimum solution is obtained by applying the proposed method. It is also important to note that the proposed method gives flexibility to the decision maker to choose their preferable solution by selecting suitable \( \alpha \in [0,1] \).

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