Fractional factorial and D-optimal design for discrete choice experiments (DCE)

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Abstract. Discrete choice experiments (DCE) are a method to measure people's preferences in various fields of application that used to estimate the importance of attributes of the product or service based on the respondent's choice. The design of DCE consists of several choices called profiles in each choice set. The best DCE's design depends on how to divide the profiles into each choice set so it needs a strategy to construct the appropriate DCE's design. So, the purpose of our study is looking for the most appropriate strategy from two strategies used namely fractional factorial design as the first strategy and D-optimal design with a point-exchange algorithm as the second strategy. The quality of the two strategies is evaluated with the D-optimality criterion value. Both of them are simulated in the case that involves four attributes with two levels, respectively, so all possible profiles are $2^4 = 16$ profiles. The first strategy gained DCE's design that consisted of two choice sets and eight profiles, respectively. The second strategy gained two different DCE's designs. The first design consisted of four choice sets and four profiles, respectively. The second design consisted of eight choice sets and two profiles, respectively. Both the first and second strategies have the same D-optimality criterion values but a difference in structure. In terms of statistics, both strategies have the same quality. However, in terms of application, the second strategy is more appropriate in the structure than the first strategy. Hence, the D-optimal design is an alternative to construct DCE's design based on applications.

1. Introduction
Recent years, discrete choice experiments (DCE) have evolved rapidly in various fields of application especially in a marketing research application. According to Kessels et al. (2011) DCE analysis is widely used to study people's preferences for attributes of the products or services in different applications such as marketing, transportation, health economics, and environmental economics. Moreover, Burgess and Street (2005) also mentioned that DCE analysis is used for public welfare analysis. Thus, DCE is a method to measure people's preferences of the products or services based on respondent choice. Products or services in DCE are described by a combination of attribute levels namely profile that divided into several choice sets and respondents are asked to choose one of the most preferred profiles in each choice set. Thus, the design of DCE consists of several choice sets and each choice set consists of two or more profiles. Consequently, DCE analysis can estimate the effect of attributes of the products or services based on the respondent's choice to predict market demands or make public policies in the future.

One important step in DCE analysis is constructing a good design for DCE. Design of DCE has a good quality if the profiles are divided precisely into each choice set, its according to statement of Street et al. (2005) that the goodness of DCE analysis in estimating the effect of attributes of products or services depend on which profiles are used in choice experiment and how to divide the profiles into each choice set.
choice set. Therefore, a strategy is needed to construct the design that can divide the profiles into each choice set precisely. Hence, the focus of this study is constructing a design for DCE analysis using different strategies and evaluate which is the appropriate strategy based on some criteria. This study uses two different strategies, they were fractional factorial design as the first strategy and D-optimal design with a point-exchange algorithm as the second strategy.

The first strategy, the fractional factorial design will choose several profiles of all possible profiles that were constructed by full factorial design and then classify these profiles into the choice set. Numbers of the profile are defined by design fraction. So, the number of profiles in each choice set in the first strategy depends on the design fraction that should be used. Otherwise, the second strategy, numbers of profiles in each choice set is defined by the researchers but choice sets typically consist of two, three or four profiles.

The second strategy exchanges one by one the profiles in each choice set using point-exchange algorithm until getting an optimal design based design criteria. One of the most frequently used design criteria is the D-optimality criterion so the design is called D-optimal design. The D-optimality criterion is criteria that maximize the determinant of information matrix parameter estimation. So, the second strategy requires an appropriate model. DCE analysis that asks respondents to choose one of the most preferred products or services will result in a discrete response so using a linear model may not be suitable for DCE’s design. Sun (2012) mentioned that one of the models was widely used for DCE design which had a non-linear parameter form is a multinomial logit model (MNL). Hence, the purpose of this study is to get the most appropriate strategy in constructing the design of DCE from those two strategies based on the D-optimality criterion value.

2. Models for Discrete Choice Experiments (DCE)

The most commonly used model for data from discrete choice experiments (DCE) is model multinomial logit (MNL). If we assume that the utility of the respondents choose the profile $j$ in the choice set $i$, then the model is given by

$$U_{ij} = x_{ij}' \beta + \varepsilon_{ij} \quad (1)$$

In this model, $x_{ij}$ represents $q$ vector that containing attribute levels of the $j$th profile in the choice set $i$, $\beta$ represents the $q$ model coefficients that describe the respondents’ preferences and $\varepsilon_{ij}$ is random error term. Goos et al. (2010) assume that all error terms are independent and identically Gumbel distributed. So, as a result of the probability that profile $j$ is chosen in the choice set $i$ can be written by

$$p_{ij}^* = \frac{\exp(x_{ij}' \beta)}{\sum_{k=1}^{J} \exp(x_{ik}' \beta)} \quad (2)$$

where $J$ is the number of profiles in the choice set. In that case, we can derive the information matrix on the unknown model parameter $\beta$ is

$$M^* = \sum_{i=1}^{S} M_i^* = X' D' X \quad (3)$$

where $S$ is the number of choice sets in the DCE’s design, $X = [X_1', \cdots, X_S']'$, $D' = \text{diag}(D_1', \cdots, D_S')$, and $D_i' = P_i' - P_i' P_i''$.

3. D-optimality criterion

Statistically, the design is often compared to the variance-covariance matrix of parameter estimates and the best design is a design that is able to minimize the matrix of variance-covariance. The most commonly used approach to obtain a small variance-covariance matrix is to minimize its determinant, or equivalently, to maximize the determinant of the information matrix of parameter $\beta$ (Syafitri, 2015). In Kessels et al. (2012) assume that $\beta = \theta_q$ where $\theta_q$ is the $q$-dimensional vector of zeroes causes the probability of $p_{ij}^*$ of all $J$ profile in the choice set $i$ to be equal to $1/J$. So, the matrix in equation 3 to be equals to
\[
\mathbf{M}(\mathbf{X}) = \mathbf{X}'\mathbf{X} - \sum_{j=1}^{J} \mathbf{1}^\top \mathbf{X}'_{j} \mathbf{1}^\top \mathbf{X}_{j}
\]  

(4)

Where \( \mathbf{1}_j \) represents a \( J \)-dimensional vector of ones, respectively and \( J \) is the number of profiles in each choice set. Thus, the D-optimality criterion is given by

\[
\text{D-optimality criterion} = |\mathbf{X}'\mathbf{X} - \sum_{j=1}^{J} \mathbf{1}^\top \mathbf{X}'_{j} \mathbf{1}^\top \mathbf{X}_{j}|
\]  

(5)

4. The case study

This study uses an example case that involves several attributes and each attribute has some level attributes. The attribute is characteristics of products or services that are considered and attribute levels are characteristics that can be described by each attribute. The case of this study is limited on a case that involves several attributes with the same number of attribute levels and assumes that all of the choice sets are of the same size. As a real case example is the case of a student's preference for the bicycle lending system on Bogor Agricultural University (IPB) campus. We begin by considering four attributes of interest used to describe the bicycle lending system. These attributes and the corresponding attribute levels are as follows:

- Attribute 1 (\( A_1 \)): Lending system (leaving KTM and must return the bicycle to initial shelter, showing KTM and should not return the bicycle to the initial shelter)
- Attribute 2 (\( A_2 \)): Punishment (students have to change the bicycle if the bicycle is broke, the student must pay compensation if the bicycle is broke)
- Attribute 3 (\( A_3 \)): Operational hour (8.00 a.m. – 4.00 p.m., 6.00 a.m. – 4.00 p.m.)
- Attribute 4 (\( A_4 \)): Open day (weekday, weekend)

| Profile | \( A_1 \) | \( A_2 \) | \( A_3 \) | \( A_4 \) |
|---------|---------|---------|---------|---------|
| P1      | 0       | 0       | 0       | 0       |
| P2      | 0       | 0       | 0       | 1       |
| P3      | 0       | 0       | 1       | 0       |
| P4      | 0       | 0       | 1       | 1       |
| P5      | 0       | 1       | 0       | 0       |
| P6      | 0       | 1       | 0       | 1       |
| P7      | 0       | 1       | 1       | 0       |
| P8      | 0       | 1       | 1       | 1       |
| P9      | 1       | 0       | 0       | 0       |
| P10     | 1       | 0       | 0       | 1       |
| P11     | 1       | 0       | 1       | 0       |
| P12     | 1       | 0       | 1       | 1       |
| P13     | 1       | 1       | 0       | 0       |
| P14     | 1       | 1       | 0       | 1       |
| P15     | 1       | 1       | 1       | 0       |
| P16     | 1       | 1       | 1       | 1       |

The levels of attributes are usually coded by a number such as 0, 1, 2, and so on. In this paper, we use 0 for the first level and 1 for the second level of each attribute. So, all of the possible combinations of the attribute levels that called as profiles of the example case are \( 2^4 = 16 \). The combination of attribute levels can be generated by full factorial design. All of possible the profiles can be seen in Table 1 as the candidate set and these profiles which are chosen by respondents.

In practice, respondents usually are difficult to choose one of the preferred profiles in many profiles. That's why DCE analysis is one of the solutions to this problem because DCE analysis divides all of the profiles into several choices set so respondents should not evaluate so many profiles in each choice set. But the problem is how to divide the profile into each choice set appropriately because it can affect the
result of the analysis. So, the design is the important stage in constructing DCE analysis and the big problem in the design of DCE analysis is how to place the profiles into each choice set precisely. Therefore, we need the strategy to assign the profiles into each choice set appropriately so the profiles are placed in the right choice set. One of the simplest strategies is using a fractional factorial design. Commonly, this strategy gains the optimal design quality but there is some limitation in using this strategy, so we use another strategy to construct DCE's design. That is D-optimal design with a point-exchange algorithm and we evaluate which is the strategy is more appropriate to construct DCE's design based on the optimal design criteria, that is D-optimality criterion.

5. Strategy

5.1. The first Strategy: Fractional Factorial Design

The first strategy is the simplest strategy because the fractional factorial design just divides all of the possible profiles in the candidate set to be several parts that are two, three and so on based on the design fraction. In this example case, the design fraction should be used is $2^{-1} = \frac{1}{2}$. That means the design of DCE involves half of all profiles in the candidate set. So, it divides all profiles to be two choice sets and each choice set contains 8 profiles. The steps of constructing DCE’s design by fractional factorial design can be seen in Figure 1.

![Figure 1. Construct combinations by fractional factorial design.](image)

5.2. The second strategy: D-optimal design with a point-exchange algorithm

Different from the first strategy, the second strategy exchange one by one the profile in the candidate set into each choice set using the point-exchange algorithm. The candidate profile from the candidate set will be chosen into the choice set if it gains the maximum D-optimality criterion value. Our algorithm involves three steps to construct a DCE design. The first step, the candidate set is created. The candidate set contains all of possible the profiles. It can be generated by full factorial design as shown in Table 1. The second step, the starting design is created. The starting design is generated by randomizing the profiles that were arranged in the candidate set. The third step, improve the starting design by change
one by one the profiles in the starting design with the profiles in the candidate set. The result of the improvement step is the design that has the maximum D-optimality criterion value. So, to get the optimal design we should do our algorithm in several iterations. The number of iteration is defined by the researchers. Commonly, if we do our algorithm in a lot of iteration so the probability to get the maximum D-optimality criterion value is greater so in this study, we run our algorithm in 10000 iterations. The improvement step can be described by pseudo-code as shown in Figure 2.

\[ D_0 = \text{D-optimality criterion value from starting design} \]
\[ \text{Design} = \text{starting design} \]
\[ k = \text{candidate set 1, K} \]
\[ j = \text{profiles 1, J} \]

For \( j = 1, J \)

For \( k = 1, K \)

Exchange profile \( j \) by profile \( k \) in the candidate set
Compute the D-optimality criterion value (\( D_{\text{new}} \))

if \( D_{\text{new}} > D_0 \) then

Exchange profile \( j \) by candidate of profile \( k \)
\[ \text{Candidate set} = \text{reduce by selected profile candidate} \]
Save \( D_0 = D_{\text{new}} \)
Design = new design after being improved
else

Candidate set = candidate set
Save \( D_0 = D_0 \)
Design = starting design
End for
Save the best design of DCE
End for

Figure 2. Pseudo-code: improvement process with a point-exchange algorithm.

For the second strategy, we able to construct different structure designs based on the number of profiles in each choice set because the number of profiles in each choice set is defined by the researchers. It is an advantage in using the second strategy. So, in the second strategy, we construct two DCE's designs: (1) design consists of four choice sets and contains four profiles, respectively, and (2) design consists of eight choice sets and contains two profiles, respectively. Hence, we will have three designs of the two strategies.

6. Result and Discussion

6.1. The first design of DCE
The first DCE’s design was gained by a fractional factorial design strategy that consisted of two choice sets and contained eight profiles, respectively. The structure of the design can be seen in Table 2. Based on the structure of design in Table 2 so we can compute the D-optimality criterion value of the first DCE's design using equation (5) as follows
D-optimality criterion = $|X'X - \sum_{i=1}^{S} J^{-1}(X_i'1_j)(1_j'X_i)|$

\[
= \begin{bmatrix}
8 & 4 & 4 & 4 \\
4 & 8 & 4 & 4 \\
4 & 4 & 8 & 4 \\
4 & 4 & 4 & 8 \\
\end{bmatrix} - \begin{bmatrix}
4 & 4 & 4 & 4 \\
4 & 4 & 4 & 4 \\
4 & 4 & 4 & 4 \\
4 & 4 & 4 & 4 \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
4 & 0 & 0 & 0 \\
0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 \\
\end{bmatrix}
\]

\[= 4^4 = 256\]

Based on these calculations so the D-optimality criterion value of the first DCE's design is 256. This value is the maximum value of the D-Optimality criterion because from the matrix that has been gained, it shows that the matrix is orthogonal so the estimation of the main effect will be not correlated. Thus, DCE’s design that is generated is optimal design and it can be said that the profiles were divided appropriately into the choice set.

**Table 2. The first design.**

| Profile | $A_1$ | $A_2$ | $A_3$ | $A_4$ | Profile | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|---------|-------|-------|-------|-------|---------|-------|-------|-------|-------|
| P16     | 1     | 1     | 1     | 1     | P15     | 1     | 1     | 1     | 0     |
| P7      | 0     | 1     | 1     | 0     | P8      | 0     | 1     | 1     | 1     |
| P11     | 1     | 0     | 1     | 0     | P12     | 1     | 0     | 1     | 1     |
| P4      | 0     | 0     | 1     | 1     | P3      | 0     | 0     | 1     | 0     |
| P13     | 1     | 1     | 0     | 0     | P14     | 1     | 1     | 0     | 1     |
| P6      | 0     | 1     | 0     | 1     | P5      | 0     | 1     | 0     | 0     |
| P10     | 1     | 0     | 0     | 1     | P9      | 1     | 0     | 0     | 0     |
| P1      | 0     | 0     | 0     | 1     | P2      | 0     | 0     | 0     | 1     |

6.2. The second design of DCE

The second DCE’s design was gained by D-optimal design with a point-exchange algorithm strategy that consisted of four choice sets and contained four profiles, respectively. The structure of the second DCE’s design can be seen in Table 3. Actually, we shouldn’t compute manually the D-optimality criterion value for the second DCE’s design because the structure design was obtained by the maximization the D-optimality criterion so automatically the D-optimality criterion value of the design was maximum and it had been appearing in the output of the program. But, in order to be clear and know the structure of information matrix so we compute the D-optimality criterion value like the first DCE's design based on the structure design in Table 3 as follows.
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D-optimality criterion  

\[
D\text{-optimality criterion } = \left| X'X - \sum_{i=1}^{S} J^{-1}(X_iX_i) \right|
\]

\[
= \begin{bmatrix}
8 & 4 & 4 & 4 \\
4 & 8 & 4 & 4 \\
4 & 4 & 8 & 4 \\
4 & 4 & 4 & 8 \\
\end{bmatrix}
- \begin{bmatrix}
4 & 4 & 4 & 4 \\
4 & 4 & 4 & 4 \\
4 & 4 & 4 & 4 \\
4 & 4 & 4 & 4 \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
4 & 0 & 0 & 0 \\
0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 \\
\end{bmatrix}
\]

\[
= 4^4 = 256
\]

D-optimality criterion value of the second DCE's design is the same as the first DCE's design, even the matrix is the same. That means the second DCE's design is also optimal and the quality of the two design is the same. But, the structure of the design is different. In the second DCE’s design, the profiles in each choice set are fewer than the first DCE’s design.

**Table 3. The second design.**

| Choice set | Profile | A₁ | A₂ | A₃ | A₄ |
|------------|---------|----|----|----|----|
| Choice set 1 | P9 | 1 | 0 | 0 | 0 |
| | P4 | 0 | 0 | 1 | 1 |
| | P12 | 1 | 0 | 1 | 1 |
| | P13 | 1 | 1 | 0 | 0 |
| Choice set 2 | P5 | 0 | 1 | 0 | 0 |
| | P1 | 0 | 0 | 0 | 0 |
| | P16 | 1 | 1 | 1 | 1 |
| | P11 | 1 | 0 | 1 | 0 |
| Choice set 3 | P6 | 0 | 1 | 0 | 1 |
| | P15 | 1 | 1 | 1 | 0 |
| | P14 | 1 | 1 | 0 | 1 |
| | P3 | 0 | 0 | 1 | 0 |
| Choice set 4 | P10 | 1 | 0 | 0 | 1 |
| | P8 | 0 | 1 | 1 | 1 |
| | P7 | 0 | 1 | 1 | 0 |
| | P2 | 0 | 0 | 0 | 1 |

6.3. The Third design of DCE

The third design was also gained by the D-optimal design with point-exchange strategy. The third DCE’s design consisted of eight choice sets and contains two profiles, respectively. The structure of the third DCE’s design can be seen in Table 4. Same as the second DCE’s design we shouldn’t compute manually the D-optimality criterion value for the second DCE’s design but to make sure that the design that was gained optimal so better to compute the D-optimality criterion value like the first and second DCE’s design based on the structure design in Table 4 using equation (5) as follows
D-optimality criterion = \[ |X'X - \sum_{i=1}^{S} J^{-1}(X_i1_j)(1_j'X_i)| \]

\[
= \begin{bmatrix}
8 & 4 & 4 & 4 \\
4 & 8 & 4 & 4 \\
4 & 4 & 8 & 4 \\
4 & 4 & 4 & 8 
\end{bmatrix}
- \begin{bmatrix}
4 & 4 & 4 & 4 \\
4 & 4 & 4 & 4 \\
4 & 4 & 4 & 4 \\
4 & 4 & 4 & 4 
\end{bmatrix}
- \begin{bmatrix}
4 & 0 & 0 & 0 \\
0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4 
\end{bmatrix}
\]

= 4^4 = 256

D-optimality criterion value the same as the first and second DCE's design so does the matrix. That means the three of DCE's design have the same quality but different in the structure of the design. It shows that the D-optimal design with point exchange algorithm is more flexible to define profile numbers in each choice set that the fractional factorial design.

| Choice set | Profile | A1 | A2 | A3 | A4 |
|------------|---------|----|----|----|----|
| Choice set 1 | 1 | P6 | 0 | 1 | 0 | 1 |
| Choice set 2 | 2 | P11 | 1 | 0 | 1 | 0 |
| Choice set 3 | 1 | P3 | 0 | 0 | 1 | 0 |
| Choice set 4 | 2 | P14 | 1 | 1 | 0 | 1 |
| Choice set 5 | 1 | P7 | 0 | 1 | 1 | 0 |
| Choice set 6 | 2 | P8 | 0 | 1 | 1 | 1 |
| Choice set 7 | 1 | P16 | 1 | 1 | 1 | 1 |
| Choice set 8 | 2 | P1 | 0 | 0 | 0 | 0 |
| Choice set 9 | 1 | P12 | 1 | 0 | 1 | 1 |
| Choice set 10 | 2 | P5 | 0 | 1 | 0 | 0 |
| Choice set 11 | 1 | P2 | 0 | 0 | 0 | 1 |
| Choice set 12 | 2 | P15 | 1 | 1 | 1 | 0 |
| Choice set 13 | 1 | P13 | 1 | 1 | 0 | 0 |
| Choice set 14 | 2 | P4 | 0 | 0 | 1 | 1 |

6.4. Comparison of strategies
Commonly, an optimal design has an efficiency of 100%. A design nearly optimal if the efficiency of the design is high but there isn't a formal definition of how high the efficiency value is, so it will be ambiguous. But, we can consider the matrix information of the design to evaluate the quality of design.

Based on the result of the three design we can evaluate that both fractional factorial design and D-optimal design with a point-exchange algorithm as the strategy give a good performance in constructing DCE’s design. It is showed by the D-optimality criterion value from the three designs are the same and the matrix information of the three designs are orthogonal matrix so it can be said that the three designs have the same quality in terms of statistics. That means the two strategies gain the same quality of design, although the structure of the design is different. But, the fractional factorial design as the strategy to construct DCE’s design has limitations in terms of defining profile numbers in each choice set. Otherwise, the D-optimal design with a point-exchange algorithm as the strategy to construct DCE’s
design is more flexible to define profile numbers in each choice set. Hence, the second strategy is easier to be applied in practice than the fractional factorial design strategy.

7. Conclusions
Both of the strategies that are fractional factorial design and D-optimal design with a point-exchange algorithm can be used to construct DCE’s design. But, fractional factorial design as the first strategy has limitations to define the choice numbers in each choice set. Otherwise, D-optimal design with a point-exchange algorithm as the second strategy has the flexibility to define the choice numbers in each choice set. Even though in terms of statistics the two strategies have the same quality. But in terms of application, the second strategy is more appropriate in the structure of the design than the first strategy. Hence, the D-optimal design with the point-exchange algorithm is an alternative to construct profiles into each choice set based on application and it more recommended strategy to be used than fractional factorial design.

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