EVOLUTION OF MASSIVE BLACK HOLE BINARIES

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ABSTRACT

We present the results of large-scale $N$-body simulations of the stellar-dynamical evolution of massive black hole binaries at the center of spherical galaxies. We focus on the dependence of the hardening rate on the relaxation timescale of the parent galaxy. A simple theoretical argument predicts that a binary black hole creates a “loss cone” around it. Once the stars in the loss cone are depleted, the hardening rate is determined by the rate at which field stars diffuse into the loss cone. Therefore, the hardening timescale becomes proportional to the relaxation timescale. Recent $N$-body simulations, however, have failed to confirm this theory, and various explanations have been proposed. By performing simulations with sufficiently large $N$ (up to $10^6$) for sufficiently long time, we found that the hardening rate does indeed depend on $N$. Our result is consistent with the simple theoretical prediction that the hardening timescale is proportional to the relaxation timescale. This dependence implies that massive black hole binaries are unlikely to merge within a Hubble time through interaction with field stars and gravitational wave radiation alone.

Subject headings: black hole physics — galaxies: interactions — galaxies: nuclei — methods: $n$-body simulations — stellar dynamics

On-line material: color figure

1. INTRODUCTION

1.1. Massive Black Hole Binaries and the Central Structure of Ellipticals

The possibility of the formation of massive black hole binaries in the cores of elliptical galaxies was first pointed out by Begelman, Blandford, & Rees (1980, hereafter BBR). At that time, the possibility of such events was purely theoretical, derived from two hypotheses. One is that QSOs are driven by massive central black holes, with masses of the order of $10^8 M_\odot$ or larger. The second is that most elliptical galaxies, which might contain such massive black holes, are formed through the merging of two galaxies. If both of the progenitor galaxies contain massive black holes, these black holes sink to the center of the merger product through dynamical friction and form a binary.

Although this scenario was purely theoretical at the time it was first proposed, a considerable amount of observational support has been provided in the last two decades. First of all, there is now plenty of evidence that many, if not most, giant ellipticals contain massive central black holes (Magorrian et al. 1998). Also, it has been suggested that the black hole mass, $M_{\text{BH}}$, shows a tight correlation with the spheroidal mass and the central velocity dispersion (Gebhardt et al. 2000; Ferrarese & Merritt 2000). The most straightforward and natural explanation of the observed correlations is that massive galaxies are formed by the merging of less massive galaxies and that the central black also grows through merging (Kauffmann & Haehnelt 2000).

This merging scenario has an additional important advantage: it nicely explains the observed structure of the central region of massive elliptical galaxies. In the 1970s and 1980s, observations revealed that large ellipticals had large “cores,” with a good linear correlation between the size of the core and the size of the galaxy. At that time, this correlation was thought to be very strong counterevidence against the merger hypothesis. The larger core size of the larger galaxy implies that the central phase space density is lower for larger galaxies. However, if larger galaxies had been formed by collisionless merging of smaller galaxies, the central phase space density would have been roughly conserved, and therefore the core size would also have been roughly conserved.

Strictly speaking, the phase space argument only guarantees that the central phase space density would not increase and does not imply that it is conserved. Nevertheless, numerical simulations of mergers (Farouki et al. 1983; White 1979; Okumura et al. 1991) all demonstrated that the core size would not increase significantly through merging. And if some dissipational process implicit in the dynamics of gas clouds is taken into account, most likely the central density would increase rather than decrease. Thus, it seemed very difficult to explain the observed correlation between the core size and the size of the galaxy within the framework of the merger hypothesis.

Ebisuzaki et al. (1991) proposed that the formation of a black hole binary might solve this difficulty. If both progenitors in a collision between two galaxies contain central massive black holes, these black holes would form a binary, as suggested by BBR. The back reaction of the sinking of black holes through dynamical friction and the subsequent hardening of the black hole binary cause a heating of the stars in the core, leading to an expansion of the core. Using $N$-body simulations of spherical galaxies with and without central black holes, they demonstrated that the core size, defined as the density-weighted distance from the density center (Casertano & Hut 1985), increases in the case of a merger of two galaxies with central black holes.

In the mid-1990s, high-resolution observations by the Hubble Space Telescope revealed that the “cores” of the giant elliptical galaxies are not really cores with a flat volume.

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density but rather very shallow cusps in which the density continues to rise toward the center as \( \rho \propto r^{-\alpha} \), where \( \rho \) is the volume luminosity density and \( r \) is the distance from the center, with power index \( \alpha = 0.5 \sim 1 \) (Gebhardt et al. 1996; Byun et al. 1996).

Around the same time, the completion of GRAPE-4 (Makino et al. 1997) made it possible to perform N-body simulations of merging galaxies with central black holes using far more particles than had been previously possible. For example, Makino & Ebisuzaki (1996) performed a simulation of repeated mergers of galaxies with central black holes in which the final product of one merger simulation is used as the progenitor for the next simulation. They found that the central structure of the merger remnant converges to a unique profile in the form of a central cusp around the black hole with a slope of approximately \(-0.5\); the total mass of the stars in the cusp region is comparable with the mass of the black hole binary. Furthermore, Nakano & Makino (1999a, 1999b) showed, by a combination of N-body simulations and analytic arguments, that this shallow cusp can be explained by the fact that the distribution function of stars has a lower cutoff energy, \( E_0 \). If there is a lower cutoff in energy distribution, one can show in a simple derivation that the density profile close to the central black hole must have a slope with a power of \(-0.5\). Such a cutoff is therefore naturally formed when black holes sink to the center and form a binary.

In this theory, the radius of the cusp region relative to the effective radius of the galaxy is proportional to the mass of the black hole relative to that of the parent galaxy. Thus, the correlation between the spheroidal mass and the black hole mass (Magorrian et al. 1998) nicely explains the correlation between the cusp radius and the effective radius. Indeed, Milosavljević et al. (2002) estimated “the mass ejected from the central cusp” of observed ellipticals and found that this shallow cusp can be explained by the fact that the distribution function of stars has a lower cutoff energy, \( E_0 \). If there is a lower cutoff in energy distribution, one can show in a simple derivation that the density profile close to the central black hole must have a slope with a power of \(-0.5\). Such a cutoff is therefore naturally formed when black holes sink to the center and form a binary.

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### 1.2. The Fate of a Black Hole Binary

As we reviewed in the previous section, a merger of galaxies with central black holes can account for the observed characteristics of the central structure of elliptical galaxies quite well. However, we are still faced with one fundamental question: what will happen to the binary black hole?

Once formed, a binary black hole further hardens through interactions with nearby stars, much in the same way as hard binaries in globular clusters continue to harden (Spitzer 1987; Heggie & Hut 2003). If we can assume that the stellar density around the black holes is roughly constant in time, the hardening rate of the binary, \( dE_0/dt \), is also constant.

However, there are two significant differences. One is that the black hole binary is far more massive than the field stars. Therefore, it stays at the center of the galaxy with only a very small random velocity. The second is that a galaxy contains far more stars than globular clusters do, and therefore the central two-body relaxation time is much longer for a galaxy than for a globular cluster. BBR argued that because of these two differences, stars that can interact with the binary will sooner or later be depleted and the hardening rate of the black hole binary will drop to practically zero. Once the nearby stars (stars with sufficiently small angular momentum to approach to the black hole binary) are scattered into different orbits, in other words, after the “loss cone” is depleted, the hardening rate drops to a quite small value. In this stage, the hardening rate is determined by the timescale in which the loss cone is refilled by thermal relaxation. Thus, in real ellipticals with very long central relaxation times, the timescale of refilling of the loss cone is very long and therefore the hardening timescale of the binary would be much longer than the Hubble time.

Whether or not this loss-cone depletion actually occurs has been one of the key issues in the theoretical and numerical studies of massive black hole binaries. Early simulations (Makino et al. 1993; Mikkola & Valtonen 1992) could not cover a large enough range for the number of particles (and thus for the relaxation time) to see the corresponding change in the hardening timescale. Makino (1997) performed merger simulations with a number of particles \( N \) in the range 2K–256K and found that the hardening timescale showed a clear increase for increasing \( N \). From numerical results, Makino estimated that the hardening timescale was proportional to \( N^{1/3} \). This dependence is notably weaker than BBR’s theoretical prediction that the hardening timescale should be proportional to the relaxation time, or \( N \). Quinlan & Hernquist (1997) performed similar calculations with up to 200K stars and found a qualitatively similar dependence of the hardening timescale on the number of stars. However, they observed that the evolution of the black hole binary was almost the same for the runs with 100K stars and the run with 200K stars and concluded that for large enough \( N \) the hardening rate would become independent of \( N \). Chatterjee et al. (2003) repeated calculations similar to those performed by Quinlan & Hernquist (1997), but using up to 400K stars, and reached the same conclusion. Both Quinlan & Hernquist (1997) and Chatterjee et al. (2003) used a combination of the SCF (self-consistent field) technique (Hernquist & Ostriker 1992) and direct calculation in which interactions between field stars are treated using the SCF, while Makino (1997) relied on a fully direct calculation of all forces using the GRAPE-4. Although the SCF approximation does not eliminate two-body relaxation (Hernquist & Barnes 1990; Hernquist & Ostriker 1992), it may have affected the dependence of the relaxation timescale on \( N \) in a complex way.

Milosavljević & Merritt (2001) performed simulations of mergers with central black holes, by again using a composite method, but here composite in the time domain and not in the spatial domain. Before the formation of the black hole binary, they employed a parallel tree code with individual time steps (Springel et al. 2001). Just before the binary formation, they then switched over to a direct calculation using a general-purpose parallel computer. The use of a general-purpose parallel computer for direct calculations limited the number of particles to 32K and fewer, but their simulation is unique in that they started from a self-consistent model with a central cusp of slope \(-2\). They did not find any dependence of the hardening rate on the number of stars.

To summarize, there is no single accepted view for the evolution of massive black hole binaries in the center of galaxies. The simple analytic theory (BBR) predicts that the hardening timescale should be determined by the timescale for refilling the loss-cone and therefore should be proportional to the relaxation time, or roughly speaking, the number of stars in the system. The results of numerical studies range from no dependence (Milosavljević & Merritt 2001) to a dependence \( \propto N^{1/3} \) (Makino 1997).

In this paper, we give a clear and decisive answer to the question of whether the hardening timescale depends on the number of particles used in the simulation, and if so, in what
way. In § 2 we describe the initial model and the numerical method used. In § 3 we give the result. Section 4 contains the discussion.

2. NUMERICAL METHOD AND INITIAL MODELS

2.1. Numerical Method

We performed $N$-body simulations of galaxies with massive black holes. For all calculations, we used a program with direct force calculation and a fourth-order Hermite integrator with individual (block) time steps (Makino & Aarseth 1992). For gravitational interaction between field stars, we apply the usual Plummer softening. The size of the softening is described in § 2.2. The gravitational interactions between black holes and that between black holes and field particles are calculated with very small softening, $\epsilon_{BH} = 10^{-6}$. Since we do not apply any regularization technique, we need to guarantee that the gravitational force does not diverge.

For calculation of gravitational forces from field particles (to both the field particles and black holes), we used the special-purpose hardware GRAPE-6 (Makino et al. 2003). The calculation of the forces from black holes was done on the host computer to maintain sufficient accuracy, in a way similar to Makino & Ebisuzaki (1996). The relative accuracy of the pairwise force calculated with “high-accuracy” versions of GRAPE hardware (GRAPE-2, -4 and -6) is roughly single precision (24 bit mantissa). This accuracy is usually sufficient, since the errors of the forces from many particles partially cancel each other. However, single-precision is not quite enough for forces from black hole particles. The reason is that some particles, and most notably the black hole particles themselves after they formed a binary, orbit around a black hole particle a number of times, essentially feeling only the force from one black hole particle. In this case, the round-off error accumulates and the error in the total energy becomes alarmingly large. Moreover, all the errors are generated from the black hole binary and stars that are close to them, the behavior of which we are interested in. In order to guarantee sufficient accuracy, we chose to calculate all gravitational forces from black hole particles on the host computer, using full double-precision (53 bit mantissa) numbers.

No relativistic effect was taken into account. Our calculation is purely Newtonian. We do not model the accretion to black holes or collisions between stars, since their cross sections are small.

2.2. Initial Models

In this paper we consider a simple model in which we place two massive point-mass particles in a spherical galaxy. For our standard set of runs, the initial galaxy model is a King model with nondimensional central potential $W_0 = 7$. We use Heggie units, where the mass $M$ and the virial radius $R_v$ of the initial galaxy model and gravitational constant $G$ are all unity. In these units, the binding energy of the initial galaxy is $E = -\frac{1}{2}$.

The mass of the black hole particles is $M_{BH} = 0.01$. They are initially placed at $(\pm 0.5, 0, 0)$ with velocity $(0, \pm 0.1, 0)$. Thus, they are initially outside the core, at the apocenter of nearly radial orbits.

We varied the number of particles $N$ from 2000 to 1,000,000. To investigate the possible dependence of the results on the softening length, we tried three different choices for the softening length: (a) $\epsilon = 0.01$, (b) $\epsilon = 0.01/(N/2000)^{1/3}$, and (c) $\epsilon = 20/N$. All give the same $\epsilon = 0.01$ for $N = 2000$, but they have a different dependence on $N$.

The largest calculation (1 million particles for up to $t = 300$) took about 1 month on a single-host, four-processor-board GRAPE-6 system with a peak speed of four Tflops. The total number of individual time steps was $1.2 \times 10^{11}$. In other words, the (harmonic) average time steps is around $2.5 \times 10^{-3}$. In comparison, the total number of block time steps is $2.1 \times 10^9$. Thus, the typical time steps size of the particle with the smallest time steps (generally the black hole particles themselves or particles close to them) is $1.4 \times 10^{-6}$, more than 1000 times smaller than the average step size. Without the use of individual time steps, this calculation would have required a fraction of a Petaflops-year, rather than $\frac{1}{5}$ Teraflops-year.

For all calculations, the total energy is conserved to better than 0.1%, or better than 1% of the binding energy of the BH binary. We did several test calculations with both higher and lower accuracy criteria but found no systematic difference in the final results.

3. RESULTS

3.1. The $N$-Dependence

Figure 1 shows the time evolution of the specific binding energy (per unit of reduced mass) of the black hole binary. The relation between the semimajor axis $a$ and the binding energy is

$$a = \frac{M_{BH}}{E_b} = -\frac{1}{100E_b}.$$ (1)

The relation between the orbital velocity of the black holes and $E_b$ is given simply by

$$v_{BH} = \sqrt{E_b},$$ (2)

where $v_c$ is the three-dimensional velocity dispersion of the field stars, in a central region of the galaxy, chosen to be large.
enough not to be affected significantly by the black holes. Thus, if we interpret the host galaxy as a galaxy with a velocity dispersion of 300 km s\(^{-1}\), the black hole binary with \(E_b = -1\) has an orbital velocity of 300 km s\(^{-1}\).

From Figure 1 it is clear that the evolution timescale continues to depend on \(N\) for the entire range of \(N\) for which we performed our simulations, with no indication whatsoever of even an onset toward convergence.

Figure 2 shows the early evolution. Initially the binding energy shows large oscillations, simply because the black hole particles are not bound to each other but orbit within the parent galaxy in highly eccentric orbits. As their orbits shrink through dynamical friction, the amplitude of the oscillations becomes smaller and eventually the two black holes become bound. We can see that the early evolution of the black hole binary, before the specific binding energy reaches \(-1\), is almost independent of \(N\). However, after \(E_b\) reaches \(-2\), the evolution timescale shows a strong dependence on the number of particles.

To quantitatively evaluate the dependence of the hardening rate on the number of particles, we calculate the hardening rate \(\beta\), defined as

\[
\beta = \left| \frac{\Delta E_b}{\Delta t} \right|.
\]

Here \(\Delta t = t_1 - t_0\), where \(t_0\) and \(t_1\) are the times at which \(E_b\) reached the values \(E_{b,0}\) and \(E_{b,0} + \Delta E_b\), respectively. We use \(\Delta E_b = -0.5\) for all values of \(E_{b,0}\). Figure 3 shows the result, for \(E_{b,0} = -1, -3, -5, -7\). When \(E_b = -1\), the hardening rate is almost independent of \(N\). However, as the binary becomes harder, \(\beta\) decreases, and the decrease is larger for larger \(N\). Thus, the hardening rate \(\beta\) for large values of \(|E_b|\) shows a strong dependence on the number of particles \(N\).

This result is exactly what is expected from the simple loss-cone argument: after the loss cone is depleted, \(\beta\) should be inversely proportional to the relaxation time, which is proportional to \(N\). Note that we used a constant softening, for which the Coulomb logarithm does not depend on \(N\).

If the loss-cone argument were correct, writing \(\beta \propto N^{-\gamma}\) would let \(\gamma\) approach unity for large enough \(N\). Figure 4 shows \(\gamma\), calculated using the value of \(\beta\) for \(N = 20,000\) and 1,000,000. The choice of these values of \(N\) is somewhat arbitrary, but as one can see from Figure 3, we obtained similar figures with different choices of \(N\). We could use a least-square fit but decided against it since there is no obvious reason to assign equal weights to results with different \(N\).

We can see that \(\gamma\) indeed increases as \(|E_{b,0}|\) increases. Even at \(E_b = -8\), \(\gamma\) has not converged to a final value. If we could extend the calculation to higher values of \(E_b\), we would be able to determine whether or not \(\gamma\) really approaches unity. Unfortunately, it would be too time consuming, even on a
GRAPE-6, to further extend the calculations, given that the hardening rate is so small for large $N$.

From the current simulations, we can safely conclude that the hardening rate $\beta$ depends on the number of particles $N$, and the power index of the dependence $\gamma$ numerically obtained is larger than 0.7. The numerical result is consistent with the simple loss-cone argument, which predicts $\gamma = 1$.

### 3.2. Dependence on the Softening Length

The results given in the previous subsection demonstrate clearly that the hardening rate depends on the number of particles. In this and the following subsections, we will check whether this result is really reliable. First we look at the effect of the softening. The relatively large and constant softening used in our standard runs has the effect of suppressing two-body relaxation. In particular, in the core of the galaxy, the effective Coulomb logarithm might be very small, resulting in unphysical suppression of the relaxation effect. Since the timescale of the loss-cone refill is related to the relaxation timescale, it is crucial to express the relaxation with a reasonable accuracy. To test the effect of the softening, we performed several runs with a much smaller softening length. Figure 5 shows the result. For $N = 2 \times 10^5$, reducing $\epsilon$ by a factor of 100 resulted in only a small increase in the hardening rate. Furthermore, in the case of runs with $N = 10^6$, the change in the softening has practically no effect on the hardening rate. Thus, we can safely conclude that the effect of the softening, if any, is sufficiently small that it does not affect the results in the previous section.

### 3.3. Dependence on the Initial Models

Before the loss cone is depleted, the hardening rate is expected to be proportional to the central density of the parent galaxy. After the loss cone is depleted, the hardening rate is determined by the timescale at which the stars close to the loss cone diffuse into the cone. Thus, here again the hardening timescale is expected to depend on the initial central density.

Figure 6 shows the results of runs with different initial galaxy models. As expected, the hardening is faster for models with deeper central potential (higher central density). However, for all runs the hardening rate depends on the number of particles.

Figure 7 shows the growth rate $\beta$ as a function of the initial central density of the parent galaxy, $\rho_{0,0}$. In the early phase ($E_b = -1$), $\beta$ is roughly proportional to $\rho$ for $\rho < 10^2$. For higher $\rho$ or for later phases the dependence is somewhat...
weaker, presumably because the black hole binary has already ejected nearby stars, thereby reducing the central density.

3.4. Loss-Cone Depletion and Wandering of the Binary

In the previous subsections we have seen that the behavior of the hardening rate is consistent with the loss-cone argument. In this subsection, we directly investigate whether or not the loss cone is actually depleted.

Figure 8 shows the distribution of particles in the \((E, J)\) plane, where \(E\) is the specific energy and \(J\) is the specific total angular momentum. We use the coordinate origin as the reference point for the angular momentum. We also tried to use the center of mass of the black hole binary, but that resulted in practically indistinguishable figures. Here we can clearly see that the number of particles with \(J < 0.01\) is more and more depleted as time proceeds. At \(T = 10\), only particles with low energy \((E < -2)\) are affected. However, the depletion reaches higher energy levels as time proceeds, and by \(T = 80\) s only stars with nearly zero energy are left with low angular momentum. Note that there were no particles with nearly zero energy at \(T = 10\). These barely bound particles have been kicked to high-energy, long-period orbits by the binary black hole.

Figure 9 shows the radial density profiles for the same snapshots as used for Figure 8. Here we took the center of mass of the black hole binary as the center of the coordinate. Although the loss-cone depletion is clearly visible in the distribution of particles in the \((E, J)\) plane, the density profiles show no clear sign of the existence of a loss cone. This result is not at all surprising. If the distribution of particles in the \((E, J)\) plane is the same as that of the initial King model, adding the central black hole potential would result in a density cusp with a slope of \(-0.5\) (Nakano & Makino 1999b). Thus, the fact that the density does not increase toward the

![Fig. 8.—Distribution of particles in the \((E, J)\) plane at times \(T = 10, 20, 40\) and \(80\) (top left to bottom right). The number of particles is \(10^6\).](image-url)
center actually means that the particles close to the black hole binary are strongly depleted.

Figure 10 shows the distribution of particles in the $(E, J)$ plane for runs with a smaller number of particles. The distribution for $N = 10^4$ does not show any sign of depletion, while the distribution for $N = 10^5$ shows a weak indication of depletion. It also shows an enhancement of nearly zero-energy stars with small angular momentum.

Note that Figure 8 shows that the periastron distance of the particles that are depleted, a measure for the effective radius of the loss cone, is 0.01 or larger. This radius is much bigger than the semimajor axis of the binary, which is around 0.001 at $T = 80$. The reason that the effective size of the loss cone is much bigger than the semimajor axis of the black hole binary is the wandering of the binary. Figure 11 shows the time variation of the $x$-coordinate of the center of mass of the binary. The typical wandering distance is roughly proportional to $N^{-1/2}$, as theoretically predicted and as demonstrated by previous numerical work (Makino (1997); Milosavljević & Merritt 2001). For $N = 10^5$, the typical wandering distance is around 0.01, roughly consistent with the size of the loss cone.

4. DISCUSSION

4.1. Comparison with Previous Works

As we summarized in the introduction, several researchers have performed $N$-body simulations of the evolution of massive black hole binaries in the center of a galaxy, and nobody has obtained a result that could be interpreted as being consistent with the simple loss-cone argument by Begelman et al. (1980). In the present work, however, we have obtained a result that is consistent with the loss-cone argument. In order to understand the cause of the discrepancy, we first discuss our own previous work (Makino 1997) and then work by others (Quinlan & Hernquist 1997; Milosavljević & Merritt 2001; Chatterjee et al. 2003).

Makino (1997) obtained a value for the slope $\gamma$ of the dependence of the binary hardening rate $\beta$ on the number of particles $N$ of $\gamma \sim \frac{1}{2}$ from merger simulations with $N = 2,048$ to 262,144. If we compare his Figure 1 and our Figures 1 and 2, the reason that the value of $\gamma$ was small is obvious: the value of the binding energy at which $\beta$ was measured was simply too small. Therefore, $\beta$ had not yet reached the value determined by the relaxation timescale. As shown in Figures 3 and 4, $\gamma$ increases as $|E_b|$ increases. In hindsight, it is clear that previous simulations did not cover a long enough time.

Quinlan & Hernquist (1997) performed simulations very similar to those presented here. They found that $\beta$ depended on $N$, for $N < 10^5$. However, the hardening rate was practically the same for the runs with $N = 10^5$ and $N = 2 \times 10^5$. They explained this result as follows. In their calculation, they had used the SCF method to evaluate the gravitational interaction between field stars. Therefore, the two-body relaxation
had been strongly suppressed, and the dependence of the hardening rate on the number of particles should come only through the wandering (Brownian motion) of the black hole binary. The random velocity of the black hole binary would be proportional to \( \sqrt{N} \). However, the distance the wandering covers would not become arbitrarily small: since the black hole binary would deplete the loss cone and create a kind of vacuum around it, the black hole binary would not feel a restoring force as long as it remained in the vacuum.

The above explanation looks plausible, but unfortunately Quinlan & Hernquist (1997) gave no evidence that the distance covered by the black hole binary became independent of \( N \). Our simulations demonstrate that the excursion distance is proportional to \( 1/\sqrt{N} \) for up to \( 10^6 \) particles.

An additional complication with their calculations is that it is difficult to estimate the effects of two-body relaxation and wandering, because they used a mass for the particles that depended on their initial angular momenta (or, precisely, the periastron distances). They used this trick to increase the mass resolution near the center of the galaxy. However, because of this trick, the typical mass of stars that interact with the black hole binary would become larger as the black hole binary kicks out more and more of the nearby (low-mass) particles. This implies that the strength of the Brownian motion depends on how much mass has been kicked out of the core. Also, even though the effect of direct two-body encounters of field particles is suppressed, these particles can still indirectly exchange energy and angular momentum since all field stars directly interact with the black hole binary, which has a finite random velocity. Even distant massive particles do interact significantly with the black holes. Thus, the way the two-body relaxation scales is quite difficult to evaluate for their scheme.

Quinlan & Hernquist tried one simulation in which the center of mass of the binary is fixed at the origin of the coordinates and found that the hardening rate dropped dramatically. They argued that this drop is evidence for wandering being important. However, there is another, equally plausible explanation. In their SCF expansion, they incorporated only spherically symmetric terms. Thus, when they fixed the center of mass of the binary to the origin, the gravitational potential calculated became strictly spherically symmetric, except for the multipole moment of the binary black hole. Thus, the angular momentum of field stars with periastron distance larger than the semimajor axis of the binary is conserved. In other words, the loss cone, once depleted, can never be refilled. Once they allow the center-of-mass motion of the binary to occur, however, the gravitational potential is no longer spherically symmetric and the angular momenta (as well as energies) of field particles change through interactions with the center-of-mass motions of the binary. This process effectively works as a relaxation mechanism and causes the refilling of the loss cone.

To summarize, although Quinlan & Hernquist (1997) observed that in their calculation the hardening rate became independent of \( N \) for large \( N \), it is somewhat difficult to generalize their result, obtained with a spherically symmetric potential expansion code and radius-dependent particle mass, to real \( N \)-body system or real elliptical galaxies.

Chatterjee et al. (2003) performed calculations similar to that presented in Quinlan & Hernquist (1997), using essentially the same calculation code. However, they did use a few nonspherical terms in their potential expansion, and they used equal-mass particles instead of particles having radius-dependent mass. Thus, their results can be directly compared with our results without much complication. In the text, they stated that the hardening rate became constant for the value of \( N \) around \( 2 \sim 4 \times 10^3 \).

For their 400K run, \( M_{\text{BH}} = 0.00125 \), which is \( 1/4 \) of the value we used. Unfortunately, the simulation was stopped before we would expect to see whether the loss-cone depletion would or would not occur.

Unlike the previous two papers, Milosavljević & Merritt (2001) calculated the actual merger of two galaxies with central black holes, as Makino (1997) did. The difference between Milosavljević & Merritt (2001) and Makino (1997) is that the former started with a galaxy model with a density cusp with \( \rho \propto r^{-2} \) around the central black hole, while the latter used a King model with a finite core as the initial model. For simulations of mergers of elliptical galaxies with massive central black holes (10\(^8\) \( M_\odot \) or larger), a galaxy model with finite-size core would be more appropriate, since the shallow “cusp” of such large ellipticals corresponds to a cutoff of the distribution function at finite energy. The stellar distribution in the central region of spiral galaxies is consistent with a cusp of \( \rho \propto r^{-2} \). Therefore, for simulations of mergers of spiral galaxies, the initial model used by Milosavljević & Merritt (2001) may be more appropriate.

Milosavljević & Merritt (2001) performed three runs, with 8K, 16K and 32K stars, and found that the hardening rate was independent of the number of particles. As discussed in their paper, this result stemmed from the fact that the loss cone was not depleted in their simulations for two reasons. The first is that the initial central density of their model was high, since they tailored the distribution function so that the progenitor galaxies had central cusps around the black holes. This is a situation very different from that used in other studies, where black holes were placed off-center in a single galaxy or placed at the centers of galaxies with relatively large cores. Since the initial stellar density around the black holes is very high, the binary initially hardens very rapidly. A binary with a small semimajor axis has a small interaction cross section, which implies that it takes a long time to deplete the nearby stars.
The second reason is the relatively small number of particles employed in their simulation, which resulted in rather large random velocities. Therefore, Brownian motion of the binary covered a fairly large radius. Even in their largest calculation, with 32K particles, the total mass of the stars that can enter the region covered by the Brownian motion of the black hole binary is much larger than the mass ejected by the binary. Thus, the very high initial central density and the relatively small number of particles conspired together to prevent the loss cone from being depleted.

If Milosavljević & Merritt (2001) could have used a much larger number of particles, the wandering distance would have shrunk and the loss cone would have been formed early on. According to their own estimate (Milosavljević & Merritt 2001), for \( N > 2 \times 10^5 \) the loss-cone depletion becomes important.

To summarize, there is no real discrepancy among the results of full N-body simulations. Makino (1997) observed weaker dependence of the hardening rate than obtained in the present study simply because his calculations were not long enough. Milosavljević & Merritt (2001) found no dependence on \( N \), essentially because the number of particles they used was too small for the loss cone to be depleted.

4.2. Merging Timescale of Massive Black Hole Binaries in Ellipticals

As first suggested by BBR and confirmed by a number of follow-up works, if the hardening timescale of the black hole binary is proportional to the relaxation timescale of the parent galaxy, the evolution timescale of typical black hole binaries in elliptical galaxies exceeds the Hubble time by many orders of magnitude. In other words, binaries are unlikely to merge through encounters with field stars and gravitational wave radiation.

Our results strongly suggest that the hardening timescale is indeed determined by the relaxation timescale for large enough \( N \) and after the binary becomes sufficiently hard. This implies that gravitational interaction with field stars is insufficient to let the binary merge.

There are a number of alternative mechanisms that may lead to the merger of the black hole binary. If there is a significant amount of gas left at the center, or if gas is supplied from the disk during the merger event, it would certainly change the entire picture. However, in the case of a merger of two ellipticals, there is not much cold gas left in the resulting galaxy. In this case, the most likely outcome is that the binary, stuck at a certain semimajor axis, stays at the center of the loss cone.

If the eccentricity of the binary goes up, the timescale of orbital evolution by gravitational wave radiation is reduced significantly. Roughly speaking, if the eccentricity reaches \( 0.9 \), a fair fraction of the binary black holes would merge in a time less than the Hubble time. Some of the early \( N \)-body simulations and scattering experiments have focused on this possibility (Makino et al. 1993; Mikkola & Valtonen 1992). However, the general consensus seems to be that the eccentricity does not change much during the hardening.

This situation can change if there are more than two massive black holes. If the binary black hole has a long lifetime, it is quite natural to assume that some of the galaxies that contain binary black holes will undergo a further merger with another galaxy with a central massive black hole or a binary. If we regard the black holes as point-mass particles interacting through Newtonian gravity, then with three (or more) black holes we expect at most one black hole binary to be left in the galaxy, having ejected all other black holes by the gravitational slingshot mechanism (Saslaw et al. 1974). However, here the eccentricity effects might play an important role. Simple estimates assuming a thermal distribution of eccentricities (Makino & Ebisuzaki 1994) suggest that, during repeated three-body interactions, the eccentricity of the binary can reach a very high value, resulting in quick merging through gravitational wave radiation. In principle, it is possible that multiple black holes form a stable hierarchical system, where evolution of the outer binary is halted because of the loss-cone depletion. In this case, however, the inner binary would typically have a semimajor axis of order \( 1/10 \) of that of the outer binary, and the gravitational wave radiation timescale of the inner binary would generally be short. Also, the Kozai mechanism could play a role in increasing the eccentricity of the inner binary (Blaes et al. 2002).

So far, our primary attention has been directed to supermassive black holes in large ellipticals, since observational evidence for massive dark objects is strongest for large ellipticals. If low surface brightness (LSB) galaxies had central black holes (which would be more like intermediate-mass black holes than massive black holes), some of them must have experienced major merger events and therefore are likely to have contained multiple black holes at some point. Here, again, the crucial question is whether or not these black holes can merge. Since the black hole masses are smaller, if the central regions of LSB galaxies are dominated by normal stars, loss-cone depletion is less effective and black holes can become very tight. Indeed, if we regard the field particles in our simulations as normal stars with typical masses around a solar mass, we have performed simulations of BH binaries with masses up to \( 10^4 M_\odot \), for which we have seen that the hardening rate is still pretty high. However, if the central region is dominated by cold dark matter, as suggested by both theory and observations, the thermal relaxation time would be practically infinite and loss-cone depletion would occur instantly. In the case of LSB galaxies, it is unlikely that two black holes merge during triple interactions, because the central potential is shallow. All black holes would be ejected from the parent galaxy before they would get a chance to merge. Thus, unlike large ellipticals, LSB galaxies are unlikely to display something like a Magorrian relation or a \( M_{\text{BH}}-\sigma \) relation.

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REFERENCES

Begelman, M. C., Blandford, R. D., & Rees, M. J. 1980, Nature, 287, 307
Blaes, O., Lee, M. H., & Socrates, A. 2002, ApJ, 578, 775
Byun, Y.-I., et al. 1996, AJ, 111, 1889
Casertano, S., & Hut, P. 1985, ApJ, 298, 80
Chatterjee, P., Hernquist, L., & Loeb, A. 2003, ApJ, 592, 32
Ebisuzaki, T., Makino, J., & Okumura, S. K. 1991, Nature, 354, 212
Farouki, R. T., Shapiro, S. L., & Duncan, M. J. 1983, ApJ, 265, 597
Ferrarese, L., & Merritt, D. 2000, ApJ, 539, L9

Chatterjee, P., Hernquist, L., & Loeb, A. 2003, ApJ, 592, 32
Ebisuzaki, T., Makino, J., & Okumura, S. K. 1991, Nature, 354, 212
Farouki, R. T., Shapiro, S. L., & Duncan, M. J. 1983, ApJ, 265, 597
Ferrarese, L., & Merritt, D. 2000, ApJ, 539, L9
Gebhardt, K., et al. 1996, AJ, 112, 105
———. 2000, ApJ, 539, L13
Heggie, D. C., & Hut, P. 2003, The Gravitational Million-Body Problem: A Multidisciplinary Approach to Star Cluster Dynamics (Cambridge: Cambridge Univ. Press)
Hernquist, L., & Barnes, J. E. 1990, ApJ, 349, 562
Hernquist, L., & Ostriker, J. P. 1992, ApJ, 386, 375
Kauffmann, G., & Haehnelt, M. 2000, MNRAS, 311, 576
Makino, J. 1997, ApJ, 478, 58
Makino, J., & Aarseth, S. J. 1992, PASJ, 44, 141
Makino, J., & Ebisuzaki, T. 1994, ApJ, 436, 607
———. 1996, ApJ, 465, 527
Makino, J., Fukushige, T., Koga, M., & Namura, K. 2003, PASJ, 55, 1163
Makino, J., Fukushige, T., Okumura, S. K., & Ebisuzaki, T. 1993, PASJ, 45, 303
Makino, J., Taiji, M., Ebisuzaki, T., & Sugimoto, D. 1997, ApJ, 480, 432
Mikkola, S., & Valtonen, M. J. 1992, MNRAS, 259, 115
Milosavljević, M., & Merritt, D. 2001, ApJ, 563, 34
Milosavljević, M., Merritt, D., Rest, A., & van den Bosch, F. C. 2002, MNRAS, 331, L51
Nakano, T., & Makino, J. 1999a, ApJ, 510, 155
———. 1999b, ApJ, 525, L77
Okumura, S. K., Ebisuzaki, T., & Makino, J. 1991, PASJ, 43, 781
Quinlan, G. D., & Hernquist, L. 1997, NewA, 2, 533
Saslaw, W. C., Valtonen, M. J., & Aarseth, S. J. 1974, ApJ, 190, 253
Spitzer, Lyman, J. 1987, Dynamical Evolution of Globular Clusters (Princeton: Princeton Univ. Press)
Springel, V., Yoshida, N., & White, S. D. 2001, NewA, 6, 79
White, S. D. M. 1979, MNRAS, 189, 831