more diseases and over 100 tests) this time delay may be much longer. The computer used in the present study was relatively small, and it may well be that this problem would be solved by using the much larger time-sharing computers now available.

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Decision-making in Clinical Medicine

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For centuries physicians have been faced daily with difficult problems of decision-making under uncertainty and have found, in the practice of medicine, resolutions of these problems. The science of decision-making under uncertainty—statistical decision theory—is barely twenty years old (Wald, 1950). Have medical practitioners yet considered what advantages might accrue from their taking the younger discipline into partnership? In such a partnership, what role could the novice best play to ease the burden of medical practice? It is against the background of such general questions that this article examines one particular aspect of decision-making in medicine—the problem of treatment allocation.

While the application of statistical decision theory to real problems in
medicine clearly requires sophisticated mathematical and statistical tools, the basic concepts, structure, and inherent difficulties can be made apparent by study of a simple illustrative example. Since the purpose of this article is mainly expository, and since the oversimplification of a medical situation may prejudice the reader's appreciation of the analysis presented, the illustrative problem selected is one of treatment allocation from the author's own statistical practice. Nevertheless, physicians may recognise in it the pattern of their own problems and so be provoked to ask questions about whether such analyses are possible, practicable, and profitable in the field of medicine.

**ILLUSTRATIVE PROBLEM**

There is, in the student population, a well-known, easily diagnosed disease—statistophobia—most prevalent in the days preceding the first terminal examination in December. The symptoms are unmistakable: lassitude, presentation of negative variances, and so on. I will not distress the reader by enumerating all the horrific details. Like any physician I need not base my allocation of treatment solely on the symptoms presented. I may examine the patient to arrive at my full basic information about him, his present or initial state. Let us suppose that I recognise three possible outcomes of this examination:

(a) almost a pass,
(b) bad fail,
(c) catastrophic fail.

For simplicity I shall assume that all statistophobic students have common symptoms and, therefore, that differences in their initial states are effectively expressed by the classification \(a, b, c\).

Two remedial treatments are available for such students:

\[ (s) \] a course of supplementary lectures,
\[ (t) \] additional tutorials.

The object of treatment is to attempt to translate the student from the discomfort of his initial state to a more comfortable final state, that is, his state after the degree examination in June. There are three such final states:

(d) distinction, and so the production of a statistophile,
(e) effective performance, and so technically a pass or cure,
(f) fail, and so a kind of statistical death.

Experience has taught me that all students in the same initial state, say \(c\) (catastrophic fail) and given the same treatment, say \(t\) (tutorials), do not
necessarily reach the same final state. I must recognise that no prognosis can be made with certainty. But I have discerned a pattern or distribution of prognoses. Of c-students allocated to t, 10 per cent attain final state d, 30 per cent attain e, and 60 per cent attain f. When considering what to do with an individual c-student at the time of allocating treatment, I prefer to express this variability in a prospective form: if a c-student is assigned treatment t, the probabilities of his reaching final states d, e, and f are 0.1, 0.3, and 0.6 respectively, and so on for each of the six possible (initial state, treatment) combinations. These combinations and the probabilities attaching to the possible final states can be simply and completely displayed in a tree diagram (Fig. 1).

Knowledge of the prognosis distribution is a necessary ingredient of any analysis of the treatment allocation problem, but it is clearly not sufficient. Value judgements inevitably come into play. The usefulness of a treatment t
is to be measured in two ways: positively, as the 'advantage' of transforming a student from an initial state $x$ to (we hope, a more pleasant) final state $y$ and negatively, as the 'cost' of treatment—not only by the assessment of the differential monetary costs (tutorials are more expensive than supplementary lectures) but also in terms of the differential discomforts to the student (tutorials can be gruelling).

Suppose (and this is a supposition to which I shall certainly return) that I have been able to spell out utilities. For example, I assess that to allocate a $c$-student to $t$ and have him reach final state $d$ involves a gain of 38 (making due allowances for the costs of the tutorial treatment), whereas to have him reach final state $f$ involves a loss of 7, expressed as a gain of $-7$. The full utility structure is shown at the tips of the branches in Fig. 1.

Once this formulation is completed, the optimum allocation of treatment is easily accomplished. The basic principle of statistical decision theory, philosophically justified by a variety of fairly acceptable axioms of rational behaviour, is the following. At any stage where uncertainties lie ahead, and yet a decision has to be made, take that action which carries maximum expected utility. In other words, average the final utilities ahead, taking account of the weights of the probabilities associated with branches leading to these final utilities. For example, the expected utility from allocating a $c$-student to treatment $t$ is

$$0.1 \times 38 + 0.3 \times 23 + 0.6 \times (-7) = 6.5;$$

the expected utility from allocating a $c$-student to treatment $s$ is

$$0.1 \times 45 + 0.1 \times 30 + 0.8 \times 0 = 7.5.$$  

The entries of expected utilities associated with the six (initial state, treatment) combinations are shown in the boxes at the highest nodes in the tree. The appropriate optimum allocations are then clear. Since the entry 6.5 at the $(c, t)$ node is less than the entry 7.5 at the $(c, s)$ node, it is sensible to allocate $c$-students to treatment $s$. The return from such allocations can be measured in terms of the mean gain per student so assigned, the expected utility of 7.5. The optimum scheme of treatment allocation therefore emerges as in Table 1.

| Initial state | Optimum treatment          |
|---------------|----------------------------|
| $(a)$ Almost pass | $(s)$ Supplementary lectures |
| $(b)$ Bad fail          | $(t)$ Tutorials              |
| $(c)$ Catastrophic fail | $(s)$ Supplementary lectures |

Table 1
Some Extensions

Many of the simplifying restrictions of this model are more apparent than real.

1. The initial state is here a very simple, one-dimensional categorisation. 
This restriction can easily be removed to admit multi-dimensional states, 
containing information of a quantitative nature such as age and examination mark, as well as of a qualitative nature such as sex and previous courses attended.

2. The final state may also be complicated, and need not be of the same 
nature as the initial state. For example, it may contain information about 
whether the student wishes to continue his statistical studies.

3. There may be many more treatments available, for example, giving 
the student a 'programmed learning' text.

4. While the utility is shown assessed at the tips of the branches, the costs 
of the two treatments, which in my construction I took to be 10 for s and 17 
for t, could have been abstracted and placed along the lower branches, and 
the utilities at the tips adjusted.

5. The final state may not be the end of the story. A student who fails may 
be allowed to re-sit the degree examination in September, with the possibility 
of further therapy during the summer vacation. In fact, treatment allocation 
may be a sequential or continuing process. Again, the underlying principles 
of statistical decision theory extend to this complication.

The price of such extensions is, of course, the increased sophistication of 
the mathematics and the increased magnitude of the computations. But 
that is a matter for mathematicians, statisticians, and computer specialists 
to worry over. If the demand is strident enough they will overcome such 
difficulties.

With these extensions in mind, in what respect and to what extent does decision-making in medicine differ from the illustrative problem presented here?

The Problem of Diagnosis

The problem of diagnosis has been glossed over in the presentation. The 
effect of its introduction would be an enlargement of the decision tree at 
its base to accommodate this preliminary decision-making. While its exclusion 
has been largely due to limitations of space and to the knowledge that it is 
considered elsewhere in this Journal (Card, 1970; Taylor, 1970), the purpose 
is also to express the more positive viewpoint that the importance of diagnosis 
may be over-emphasised at the expense of treatment allocation. Although it
is intellectually satisfying to put initial states into compartments as a convenient guide to thinking for the physician, diagnosis should not become an obsession. As far as the patient is concerned, it is only a means to an end—his proper treatment. What diagnosis, right or wrong, is declared is irrelevant to the patient, though its name may cause him some mental distress. He wants a satisfactory treatment. He does not care whether he has a corn or a verruca on the sole of his foot. He wants the offending piece of himself cut out. Therefore, in my presentation I have assumed that the preliminary tradition of diagnosis has been completed and that we are faced with a stream of patients still in recognisably different initial states within this preliminary classification. The two basic ingredients of the formulation of treatment allocation problems are the prognosis distribution and the utility structure.

**The Prognosis Distribution**

The concept of a prognosis distribution is as old as medicine, but attempts at quantification are only recent (see, for example, Hughes et al., 1963; Ginsberg and Offensend, 1968; Norris et al., 1969). The traditional store for the prognosis distribution is the physician’s head, and is an accumulation of training, reading, and experience. While the courage and determination with which this phenomenal task of storage is undertaken invites unbounded admiration, it is difficult to disguise concern for the intolerable burden it places on the physician. The fact that such a store is mortal, and that it is still fed information by techniques little more sophisticated than those used by a tribal medicine man, prompts the question: How long can this informal storing of information in the human computer survive when highly efficient information retrieval systems develop?

In medical research, the most effective source of such prognosis distributions is in the controlled clinical trial, although this is less often the declared aim than one would expect. In comparing two treatments in this experimental way, how often do we ask the question, ‘Which, if either, of the two treatments is the better?’ rather than ‘For which initial states is this treatment better for which, that treatment?’

**The Utility Structure**

The second ingredient, the specification of the utility structure, is the more nebulous and controversial. For some problems of treatment allocation such a specification obviously requires the placing of a monetary value on human life, and how impossible a task that seems. Yet it is an undoubted fact that allocations to treatments are being repeatedly made by physicians under
circumstances similar to those of my illustrative example and in the face of great uncertainty. In these allocations they are surely, however informally and however subconsciously, making the equivalent of such value judgements. If their allocation behaviour is rational, they are acting as if they have a prognosis distribution and a utility structure, and choose that treatment which gives the greatest expected utility. An interesting question immediately poses itself: for an accepted prognosis pattern, to what extent is it possible to recover, or perhaps discover, the physician’s utility structure from the (presumably optimum for his implicit utility structure) allocations he makes for the particular patients or initial states he meets? That there is some hope of this recovery can be seen from a diagrammatic representation of the minimum data we could expect from such an exercise. Each point of Fig. 2 shows the (supposedly two-dimensional) initial state (for example,

![Diagram](image-url)

Fig. 2. Initial states of patients and their allocations to treatments, 1 represented by • and 2 by ○.
blood pressure, oral temperature) of a patient, and the treatment to which he has been allocated by the physician.

We have here data very similar to those of a diagnostic problem, the categories being treatments rather than diseases. The overlap of the treatment clusters suggests inconsistencies in decision-making or that the physician has not conveyed to us all the factors of the initial state on which he bases his allocation. There is clearly some critical dividing or discriminating line awaiting statistical estimation, and this gives us some vital information as to the nature of the underlying utility structure. At the very least we are able to simulate (that is, do as well as) the physician with this information. If the physician can provide more information than the optimum treatment—for instance, the utility he assesses will accrue from the use of this optimum treatment—for the particular cases presented to him, then deeper insight is given into his implicit utility structure and, indeed, it can be effectively recovered. Statistical techniques for the recovery of utility structure are discussed by Aitchison (1970).

**DISCUSSION**

What might be the rewards of such investigations? The physician confronted with his implicit utility structure may gain insight into the decision-making process. If he is disturbed by the form this takes he has at least been directed to the possibilities of modifying it. If he is satisfied by what is revealed he may be encouraged to consider using his estimated utility structure to automate the decision-making process. Since the statistical techniques are most likely to be successful when the problem concerns large numbers of patients, a situation where data are more easily come by, a first attempt at computerisation must be directed towards the more routine, and probably tiresome, decision-making. The result is surely that physicians would be released from some of their more routine chores and able to devote more time to more difficult decisions.

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