THE SOLAR ABUNDANCE PROBLEM: THE EFFECT OF THE TURBULENT KINETIC FLUX ON THE SOLAR ENVELOPE MODEL

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ABSTRACT

Recent three-dimensional (3D) simulations have shown that the turbulent kinetic flux (TKF) is significant. We discuss the effects of TKF on the size of the convection zone and find that the TKF may help solve the solar abundance problem. The solar abundance problem is that, with new abundances, the solar convection zone depth, the sound speed in the radiative interior, the helium abundance, and the density in the convective envelope are not in agreement with helioseismic inversions. We have performed Monte Carlo simulations on solar convective envelope models with different profiles of TKF to test its effects. The solar abundance problem is revealed in the standard solar convective model with AGSS09 composition, which shows significant differences (∼10%) in density from the helioseismic inversions, but the differences in the model with the old composition GN93 is small (∼−0.5%). In the testing models with a different TKF imposed, it is found that the density profile is sensitive to the value of TKF at the base of the convective envelope and insensitive to the structure of TKF in the convection zone. The required value of turbulent kinetic luminosity at the base is about −13% to −19% L⊙. Comparing with the 3D simulations, this value is plausible. This study is for the solar convective envelope only. Evolutionary solar models with TKF are required to investigate the effects of TKF on the solar interior structure below the convection zone and the whole solar abundance problem, but the profile of the TKF in the overshoot region is necessary.

Key words: convection – Sun: abundances – turbulence

1. INTRODUCTION

Recent photospheric analyses (e.g., Asplund et al. 2005, 2009; Caffau et al. 2011) have indicated that the solar photospheric metallicity is significantly lower than older values (Grevesse & Noels 1993; Grevesse & Sauval 1998). This results in the solar abundance problem, in which standard solar models with revised compositions show serious deviations from the helioseismic inversions, i.e., the depth of the convection zone (CZ) rbc, the surface helium abundance Y, and the sound speed and density in the solar interior (Basu & Antia 2004; Bahcall et al. 2005, 2006; Yang & Bi 2007). Many models have been proposed to modify the solar model, including enhanced diffusion (Guzik et al. 2005), the accretion model (Guzik et al. 2005; Guzik & Mussack 2010; Serenelli et al. 2011), particles with an axion-like interaction (Vincent et al. 2013), etc. However, none of these models has succeeded in solving the problem. The accretion model has shown improvements over the solar model, but the inconsistency remains because rbc and Y cannot fit the helioseismic restrictions simultaneously (Guzik et al. 2005; Guzik & Mussack 2010; Serenelli et al. 2011). It is expected that the opacity at the base of the convection zone (BCZ) could be adjusted upward, since tests with increased opacity reduce the discrepancies between the model and helioseismic inversions (Basu & Antia 2004; Bahcall et al. 2005; Christensen-Dalsgaard et al. 2009).

The turbulent kinetic flux (TKF) is usually ignored in modeling stars because it is thought to be small. Another reason is that there is no widely accepted non-local convection theory to describe the TKF. However, three-dimensional (3D) simulations of stellar convection envelopes have shown that TKF cannot be ignored in some cases (Tian et al. 2009; Hotta et al. 2014). It should be noted that the TKF at BCZ is negative because the turbulent convection transports kinetic energy from the CZ to the radiative region. A negative TKF requires a larger sum of the radiative flux and convective flux, which thus leads to a deeper convective boundary. This could help improve the solar model. Based on this ideal, we test the TKF in solar convective envelope (CE) models.

2. EFFECTS OF TURBULENT KINETIC FLUX ON CONVECTIVE BOUNDARY

The TKF is the transport of turbulent kinetic flux, which is a non-local effect of turbulent convection. When TKF is taken into account, the stellar energy equation is as follows:

$$\frac{\partial[4\pi r^2 (F_R + F_C + F_K)]}{\partial m_r} = \varepsilon_N - \varepsilon_{\text{neu}} + \varepsilon_g,$$  \hspace{1cm} (1)

where $F_R$, $F_C$, and $F_K$ are the radiative flux, the convective flux, and the TKF, respectively, and $\varepsilon_N$, $\varepsilon_{\text{neu}}$, and $\varepsilon_g$ are the energy contributed by nuclear burning, neutrino loss, and entropy variation, respectively. Their sum is the total flux $F$. According to turbulence theories (e.g., Xiong 1981, 1985; Li & Yang 2007; Meakin & Arnett 2010), TKF satisfies the following equation:

$$\frac{\partial(4\pi r^2 F_K)}{\partial m_r} = \frac{\delta g F_C}{\rho c_p T} - \varepsilon_{\text{turb}},$$ \hspace{1cm} (2)

where $\varepsilon_{\text{turb}}$ is the turbulent dissipation and the first term on the RHS is the turbulence produced by buoyancy. Accordingly, the stellar energy equation can be rewritten as

$$\frac{\partial[4\pi r^2 (F_R + F_C)]}{\partial m_r} = \varepsilon_N - \varepsilon_{\text{neu}} + \varepsilon_g + \varepsilon_{\text{turb}} - \frac{\delta g F_C}{\rho c_p T},$$ \hspace{1cm} (3)

which shows the processes in which turbulent dissipation converts kinetic energy to thermal energy and buoyancy converts...
thermal potential energy to kinetic energy. Equations (1), (2), and (3) are the equations of the total energy, the kinetic energy, and the thermal energy, respectively.

The TKF $F_K$ should be determined by the non-local convection theory. In the local convection theory, e.g., the widely used mixing length theory (MLT), turbulence is assumed to be in local equilibrium, and thus $F_K$ is always 0 there. However, little is known about non-local convection theories at present. On the other hand, recent studies have shown that TKF cannot be ignored because it is comparable with the total flux in some cases. Tian et al. (2009) have shown that TKF is as large as $\sim 10\%$ or more of the negative total flux in their simulations of downward overshoot in red giant branch (RGB) stars (see Figure 2 in their paper). Hotta et al. (2014) have shown that, in most of the solar CZ, TKF is comparable with the total flux, i.e., $F_K \sim -F_{tot}$. However, in the simulations by Hotta et al. (2014), the real value of $F_K$ at the BCZ cannot be revealed since the bottom boundary was set at the BCZ. Here, we simply discuss the effects of TKF on the size of the CZ.

The total flux is comprised of three parts, thus,

$$\frac{\lambda T}{H_P} \nabla_R = F = F_R + F_C + F_K = \frac{\lambda T}{H_P} \nabla_C + F_K,$$

where $\nabla_R$ is the radiative temperature gradient and $\nabla$ is the temperature gradient in the stellar interior. In the CZ in the stellar interior (e.g., $\log T > 6$), $\nabla \approx \nabla_{ad}$ can be ensured because of a high Péclet number. Therefore, the convective flux satisfies

$$F_C \approx \frac{\lambda T}{H_P} \left( \nabla_R - \nabla_{ad} - \frac{H_P F_K}{\lambda T} \right).$$

The CZ is defined by $F_C > 0$, which means that the buoyancy works on fluid elements i.e., the convective instability. Accordingly, the convective criterion is

$$\nabla_R - \nabla_{ad} - \frac{H_P F_K}{\lambda T} > 0. \quad (6)$$

At the boundaries of the CZ, turbulent flows transport turbulent kinetic energy from the CZ to the overshoot region. Therefore, $F_K > 0$ in the top boundary of the CZ (e.g., the convective core boundary) and $F_K < 0$ in the BCZ. In the case of the convective core, $F_K > 0$ leads to a deeper convective boundary and a small convective core compared with the Schwarzschild criterion. In the case of stellar CE, $F_K < 0$ leads to a deeper convective boundary and a larger CE comparing with the Schwarzschild criterion.

This effect may help solve the solar abundance problem. In the standard solar model with a revised composition (Asplund et al. 2009), the BCZ is too shallow to fit the helioseismic value since the metallicity is low. In order to enlarge the CE, the opacity at the BCZ is expected to be larger. It has been found that a required upward adjustment on the opacity is about 10–30% (Basu & Antia 2004; Bahcall et al. 2005; Christensen-Dalsgaard et al. 2009). It should be noted that taking TKF into account leads to a similar effect as increasing the opacity. It is convenient to define another radiative temperature gradient for thermal flux:

$$\nabla_{R, Therm} = \frac{H_P (F_R + F_C)}{\lambda T} = \nabla_R - \frac{H_P F_K}{\lambda T}, \quad (7)$$

which describes the required temperature gradient if the thermal energy is transported only by radiation. The convective criterion is $\nabla_{R, Therm} > \nabla_{ad}$ and it is not difficult to find

$$\nabla_{R, Therm} \propto \kappa (F_C + F_R) \propto \kappa \left( 1 - \frac{F_K}{F} \right). \quad (8)$$

Therefore, a negative TKF results in an effect similar to increasing the opacity.

Arnett et al. (2010) have stated that some discrepancies between the standard solar model and helioseismic inversions may be caused by ignoring some significant aspects of convection, which includes the TKF. Arnett et al. have proposed that the internal gravity wave excited at the BCZ transports energy inward and changes the solar structure in a similar way as increasing the opacity (Guzik 2006; Guzik & Mussack 2010). This effect is similar to the TKF because the gravity wave below the BCZ also shows a negative energy flux.

3. THE MODEL OF THE PRESENT SOLAR CONVECTIVE ENVELOPE

Studying the solar CE is easier than studying the solar evolutionary models. The solar CE is homogeneous and the composition has been determined via observations and the helioseismic technique. The luminosity in the solar CE can be regarded as a constant since there is no nuclear burning that significantly releases energy and the gravitational energy can be ignored for the main-sequence Sun. Additionally, the solar radius and the solar luminosity are also determined. As a consequence, the structure of the present solar CE satisfies the differential equations with initial conditions as follows:

$$\frac{d(P + P_{turb})}{dr} = -\rho g, \quad (9)$$

$$\frac{dm_r}{dr} = 4\pi r^2 \rho, \quad (10)$$

$$\frac{d\ln T}{d\ln P} = \nabla, \quad (11)$$

$$P = P(\rho, T, Y, Z), \quad (12)$$

$$4\pi R^2 \sigma T^4 = L_c = L_r = 4\pi r^2 (F_C + F_R + F_K), \quad (13)$$

$$m_\gamma = M_\odot, \quad (14)$$

and

$$\frac{d\ln \rho}{d\tau} = \delta \left[ \frac{g}{P_k} \left( \frac{\partial \ln T}{\partial \ln P} \right) - \frac{d\ln T}{d\tau} \right]. \quad (15)$$

Equation (12) is the equation of state and Equation (15) is the integral of the atmosphere determining the density at the surface $\rho_s$. Krishna Swamy’s (1966) $T$–$\tau$ relation is adopted in Equation (15). Equations (13) and (14), and the density at the surface determined by Equation (15) are three initial conditions. $P_{turb} = m u^2 / 2$ is the turbulent pressure where $u^2$ represents the radial kinetic energy estimated from the adopted convection theory. $\nabla$ is the temperature gradient determined from the adopted convection theory. An advantage of studying the solar CE is that we can exclude the uncertainties in modeling the radiative core and the chemical evolutionary history.

Calculating the properties of the non-local convection via non-local convection theories is difficult, and there is no widely accepted non-local convection theory. For this reason, the $F_K$...
is imposed externally in our calculations to test the effects. The temperature gradient is calculated via the local convection theory MLT. Both the local and non-local convection theories show the similar result that the solar CE has adiabatical settling may not be removed. The MLT parameter $L_{K,bc}$ is imposed externally in our calculations to test the effects. The imposed TKF $F_K (= L_K/(4\pi r^2))$ is as follows:

$$L_K = \begin{cases} 
L_{K,bc} + (L_{K,cz} - L_{K,bc}) \frac{r_{bc} - r}{r_{bc}}, & (r_{bc} \leq r \leq r_{bc} + r_0) \\
L_{K,cz}, & (r \geq r_{bc} + r_0, \lg T \geq b) \\
L_{K,S} + (L_{K,cz} - L_{K,S}) \frac{\lg T - a}{b-a}, & (a \leq \lg T \leq b) \\
L_{K,S}, & (\lg T \leq a) 
\end{cases}$$

where $L_{K,bc}$, $L_{K,cz}$, $L_{K,S}$, $r_0$, $a$, and $b$ are parameters. The motivations for these parameters are as follows. $L_{K,bc}$ and $L_{K,S}$ provide energy for the downward and upward overshoots. $L_{K,cz}$ represents the average kinetic luminosity in most ranges of the CZ, motivated by the simulation (Hotta et al. 2014), which shows the kinetic luminosity being approximately constant ($L_{K,cz} \sim -L_{K,bc}$) in most of the CZ. The swap region between $L_{K,bc}$ and $L_{K,cz}$ with length $r_0$ near the BCZ is set because turbulent diffusion and dissipation dominates here. The swap region between $L_{K,S}$ and $L_{K,cz}$ between $a < \lg T < b$ (typically we adopt $a \leq 3.9 \leq b$) is set because the most violent turbulence is near the temperature about $\lg T = 3.9$ and the turbulent diffusion varies quickly.

We integrate Equations (9), (10), and (11) from the surface down to the BCZ (i.e., where $V_{R,\text{Therm}} = V_{ad}$) with the corresponding chemical composition. We do not integrate equations into the overshoot region because the chemical abundance is unknown. The overshoot mixing is of low efficiency (Zhang 2013), and thus the chemical abundance gradient caused by settling may not be removed. The MLT parameter $\alpha$ is iteratively adjusted to ensure that $r_{bc}$ is consistent with the helioseismic inversion. The state of the envelope is interpolated from the OPAL equation of state (EOS) tables (Rogers & Nayfonov 2002) and the opacity is interpolated from the OPAL tables (Iglesias & Rogers 1996) with corresponding metal compositions and the low-temperature opacity table by Ferguson et al. (2005). The solar CE models calculated based on the above scheme have the correct helium abundance and $r_{bc}$. We must compare the sound speed and the density with the helioseismic inversions to check whether the envelope models are consistent with the helioseismic restrictions.

4. NUMERICAL RESULTS

We have performed Monte Carlo simulations with about 21,000 solar CE models. The results are shown in Figure 1. The helioseismic inversions, density, and sound speed are referenced from Basu et al. (2009). Two kinds of models are calculated: the black points with fixed $r_{bc}$ and $Y$ are uniformly distributed for the parameters of the TKF and the factor of $P_{turb}$, and the gray points, which take into account the uncertainties of $r_{bc}$ and $Y$. The parameter spaces are described in the table comments. Two standard CE models without the TKF and turbulent pressure, GN93 and AGSS09, are also shown for comparison. It is found that, with the helioseismic $r_{bc}$ and $Y$, the standard CE model AGSS09 with revised composition (Asplund et al. 2009) shows significant inconsistency (about 10%) in the density. The GN93 CE model with the old composition (Grevesse & Noels 1993) seems much better than the AGSS09 model, since the rms difference in the density is about 0.5%. The inconsistency of the AGSS09 CE model indicates that when the standard stellar structure equations and standard input physics (e.g., equation of state, opacity, etc.) are used in the solar CE, it is impossible to obtain a solar model with the AGSS09 composition fitting all helioseismic restrictions regardless of which modifications are
adopted in the solar radiative core or in the chemical evolution equations. This may be the reason why \( r_{bc} \) and \( Y_1 \) cannot fit the helioseismic restrictions simultaneously in the accretion model (Guzik et al. 2005; Serenelli et al. 2011).

The correlation coefficients for rms differences of density and sound speed are listed in Table 1. It is shown that some parameters can affect the sound speed. However, the resulting sound speeds are always consistent with the helioseismic inversions (\( c^2 \) with an accuracy of 0.2%) as shown in Figure 1, and the sound speed differences are also affected by the \( T−\tau \) relation in the atmosphere and the interpolations of EOS. Therefore, we cannot give meaningful “best” value ranges of the parameters based on the sound speed differences.

On the other hand, the correlation coefficients for the density show significant dependency on \( L_{K, bc} \), and are insensitive to other parameters of TKF. The effects of the value of \( L_{K, bc} \) on the density are shown in Figure 2. It is found that low \( L_{K, bc} \) results in low density in the solar CE. We can estimate \( L_{K, bc} \) from Figure 1 and Figure 2: for \( Y = 0.2485 \) and \( r_{bc} = 0.7135 R_\odot \), the best value is about \( L_{K, bc} = -0.164 L_\odot \), or, at least, \( L_{K, bc} \) should be in the range of \((-0.19 L_\odot, -0.13 L_\odot) \) taking into account the uncertainties of \( Y \) and \( r_{bc} \). However, the other parameters cannot be estimated. It can be seen by comparing the gray points with the black points in Figure 1 that the uncertainties of \( Y \) and \( r_{bc} \) lead to significant dispersion. For fixed \( L_{K, bc} \), the black points show that the other parameters lead to a dispersion of about 0.5% in the rms error of density, and the gray points, which take into account the uncertainties of \( Y \) and \( r_{bc} \), show a dispersion of about 3%. This indicates that the uncertainties of \( Y \) and \( r_{bc} \) lead to a dispersion of about 2.5%, which is much larger than the dispersion caused by the parameters of the TKF except \( L_{K, bc} \), thus the uncertainties of \( Y \) and \( r_{bc} \) prevent us from estimating the best values of other parameters. In a word, \( L_{K, bc} \) is the only parameter of TKF sensitive to the density of the CE. The reason why other parameters are insensitive may be that the CE is in adiabatical stratification in most regions, thus the shape of \( L_K \) and the turbulent pressure hardly affects the structure of most regions of the CE. The turbulent pressure is important only in a thin layer near the solar surface, i.e., \( P_{\text{turb}}/P \approx 15\% \) at \( \log T \approx 3.9 \) but less than 0.1% in \( r < 0.995R_\odot \). There is no data in the Basu et al. (2009) helioseismic inversions in \( r > 0.995R_\odot \) to detect the turbulent pressure.

### Table 1

| \( L_{K, bc} \) | \( L_{K, c} \) | \( L_{K, S} \) | \( r_0 \) | \( a \) | \( b \) | \( P_{\text{turb}} \) | \( Y \) | \( r_{bc} \) |
|---|---|---|---|---|---|---|---|---|
| 0.3636 | -0.3765 | -0.05255 | 0.001676 | -0.1559 | -0.1369 | 0.5704 | 0.1031 | -0.04167 |

Notes. The correlation coefficient between row name and column name are shown in corresponding rows and columns. The samples are the gray points in Figure 1. The column \((\delta\rho/\rho)_A\) is for \( L_{K, bc} < -0.164 L_\odot \) (about 6000 models), and the column \((\delta\rho/\rho)_B\) is for \( L_{K, bc} < -0.164 L_\odot \) (about 5000 models). This split is because the density of the CE models is higher than the helioseismic inversions for \( L_{K, bc} < -0.164 L_\odot \) and lower for \( L_{K, bc} < -0.164 L_\odot \). The rms errors are absolute values, and thus its tendency changes near \( L_{K, bc} = -0.164 L_\odot \). In this case, performing statistics respectively is more reasonable.

5. CONCLUSIONS AND DISCUSSIONS

In this Letter, we discussed the effects of the TKF on the size of the CZ and test the effects of TKF on the solar CE models. The main conclusions are as follows.

1. The presence of TKF modifies the convective criterion and makes convective boundaries shift downward, thus the convective core becomes smaller and the CE becomes larger.

2. The solar abundance problem is revealed in the solar CE models. The standard solar CE model with revised composition (Asplund et al. 2009) shows a significant difference (~10%) in density from the helioseismic inversions. This makes it impossible to obtain a solar model with the AGSS09 composition fitting all helioseismic restrictions if the standard stellar structure equations and standard input physics are used in the solar CE.

3. Taking TKF into account could improve the solar CE model. The density structure of the solar CE is sensitive to the value of the TKF at the BCZ and insensitive to its profile in the CZ. The required turbulent kinetic luminosity at the BCZ is \(-13\% L_\odot < L_{K, bc} < -19\% L_\odot \) taking into account the uncertainties of \( Y \) and \( r_{bc} \).

In this Letter, we have performed a limited test with the TKF on the solar abundance problem, i.e., the testing required TKF to construct the CE of the solar model (with AGSS09 composition) fitting all helioseismic restrictions. However, we cannot show the effect of the TKF in complete solar evolutionary models. The TKF profile below the BCZ is required to do that. However, there is no simulation showing the profile. The standard model with AGSS09 abundance shows lower helium abundance and shallower BCZ compared with the helioseismic inversions. Guzik et al. (2005) have shown that the sound speed differences below the BCZ cannot be removed in some non-standard models even with the correct BCZ. We believe that taking into account some important aspects of convection (the overshoot mixing and TKF) could help to solve the solar abundance problem. The incomplete mixing caused by the convective overshoot (Zhang 2013) partially compensates the settling and thus increases the helium abundance.
The incomplete mixing below the BCZ is favored by the sound speed when the BCZ is in the correct location (Brun et al. 1999; Zhang & Li 2012; Zhang 2013). An issue caused by incomplete mixing leads to a low \( Z \) when the surface \( Z/X \) is fixed, thus the BCZ becomes shallow (Brun et al. 1999; Zhang & Li 2012). However, the TKF could compensate for it.

The required \( L_{K, bc} \) seems to be too high because it is comparable with the total luminosity. According to Tian et al. (2009) and Hotta et al. (2014), that is plausible. Xiong’s turbulent convection model gives \( L_{K, bc} \sim -1\% L_\odot \) (Xiong & Deng 2001). However, Tian et al. (2009) have shown that the gradient type model of the TKF in Xiong’s model is too imprecise to be acceptable.

A serious problem that could significantly affect the stellar models arises if the TKF in fact cannot be ignored. The possibly effects may be in H/He main-sequence stars with convective core and RGB/AGB stars with convective dredge up, since the variation of the boundary of the convective core/envelope changes the profile of chemical abundance in the stellar interior.

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