FCNC transitions of $\Lambda_b$ to neutron in Bethe-Salpeter equation approach

Liang-Liang Liu $^a$, Chao Wang $^b$, Xian-Wei Kang $^c$, and Xin-Heng Guo $^c$

(a) College of Physics and Information Engineering, Shanxi Normal University, Linfen 041004, People’s Republic of China

(b) Center for Ecological and Environmental Sciences, Key Laboratory for Space Bioscience and Biotechnology, Northwestern Polytechnical University, Xi’an, 710072, People’s Republic of China

(c) College of Nuclear Science and Technology, Beijing Normal University, Beijing 100875, People’s Republic of China

In a covariant quark-diquark model, we investigate the rare decay of $\Lambda_b \to n l^+ l^-$ and $\Lambda_b \to n \gamma$ in the Bethe-Salpeter equation approach. In this model the baryons are treated as bound states of a constituent quark and a diquark interacting via a gluon exchange and the linear confinement. We find that the ratio of form factors $R$ is varies from $-0.90$ to $0.25$ and the branching ratios $\text{Br}(\Lambda_b \to n l^+ l^-) \times 10^8 = 6.79 (l = e)$, $4.08 (l = \mu)$, $2.9 (l = \tau)$ and the branching ratio $\text{Br}(\Lambda_b \to \gamma) \times 10^7$ = 3.69 in central values of parameters.

I. INTRODUCTION

The flavor changing neutral current (FCNC) decays of $b$-quark such as $b \to s \gamma(l^+ l^-)$ can provide constrains on new physics, give essential information about the quark structure of heavy baryons and give more model-independent information such as CKM matrix elements. Significant experimental progresses about rare decays of the $\Lambda_b$ baryon have been achieved at LHCb [1–4]. The rare decay $\Lambda_b \to \Lambda \mu^+ \mu^-$ first observed by CDF collaboration in 2011 [5]. The radiative decay $\Lambda_b \to \Lambda \gamma$ was observed at LHCb in 2019 [2]. There have been also many theoretical works on the rare decays $\Lambda_b$ induced by $b \to s$ transition [6–25]. Ref. [26] gave the branching ratios $\text{Br}(\Lambda_b \to n l^+ l^-) \times 10^8 = 3.19 \pm 0.46 (l = e)$, $3.76 \pm 0.42 (l = \mu)$, $1.65 \pm 1.09 (l = \tau)$ in the context of light cone QCD sum rules (LCSR). The form form factors (FFs) of $\Lambda_b(\Lambda^*_b) \to N l^+ l^-$ were given in Ref. [28] in LCSR and taking into account the contribution of $\Lambda^*_b$ the branching ratios $\text{Br}(\Lambda_b \to N l^+ l^-) \times 10^8 = 8 \pm 2 (l = e)$, $7 \pm 2 (l = \mu)$, $2 \pm 0.4 (l = \tau)$ were obtained. Ref. [29] gave the branching ratios $\text{Br}(\Lambda_b \to n \mu^+ \mu^-) \times 10^8 = 3.75 \pm 0.38$ and $\text{Br}(\Lambda_b \to n \gamma) \times 10^7 = 3.7$ in the relativistic quark-diquark picture in the QCD-motivated interquark potential model. Ref. [30] studied the $\Lambda_b \to N^* l^+ l^-$ decay in LCSR and gave the branching ratios $\text{Br}(\Lambda_b \to N^* l^+ l^-) \times 10^8 = 4.82 \pm 1.85 (l = e)$, $4.25 \pm 1.5 (l = \mu)$, and $0.25 \pm 0.09(l = \tau)$. Ref. [27] gave analysed CP-violation in polarized $b \to d l^+ l^-$. With the experiment development, the transition $\Lambda_b \to n$ will be detected in the near future, so it is

$^*$Electronic address: corresponding author. liu0604@sxnu.edu.cn

$^†$Electronic address: corresponding author. xhguo@bnu.edu.cn
necessary to study $\Lambda_b \to n$ theoretically.

In this work, we will calculate the FFs of $\Lambda_b \to n$ in the Bethe-Salpeter equation approach in a covariant quark-diquark model. This model has been used to study nucleon electromagnetic form factors and N-$\Delta$ transition form factors \[36\]. In the previous works, heavy baryon properties have been studied extensively in this model \[37–42, 44, 45\]. The possible existence of diquark within baryons has been studied for a long time \[31–32, 35\]. The negative neutron mean square charge radius can be explained by diquark model, which cannot be explained in pure SU(6) quark model \[32\].

In our model, $\Lambda_b$ can be regarded as a bound state of two particles: one is a heavy quark $b$ and the other is a scalar diquark $(ud)$. Using the $SU(6)$ wave function of baryons, we can get the neutron wave function in the quark-diquark model \[33, 34\].

The negative neutron mean square charge radius can be explained by diquark model, which cannot be explained in pure SU(6) quark model \[32\].

This paper is organized as follows. In Section II, we will establish the BS equation for $q(ud)_{00}$ system ($q = b, d$). In Section III we will derive the FFs for $\Lambda_b \to n$ in the BS equation approach. In Section IV the numerical results for the decay FFs of $\Lambda_b \to n l^+ l^-$ will be given. Finally, the summary and discussion will be given in Section V.

### II. BS EQUATION FOR $Q(ud)_{00}$ SYSTEM

Following our previous work, the BS equation of the $q(ud)_{00}$ system in momentum space satisfies the following homogeneous integral equation \[37, 42, 44\]:

$$\chi_F(p) = i S_F(p_1) \int \frac{d^4p}{(2\pi)^4} \left[ I \otimes IV_1(p, q) + \gamma_\mu \otimes (p_2 + q_2)^\mu V_2(p, q) \right] \chi_F(q) S_D(p_2),$$

where $S_F(p_1)$ and $S_D(p_2)$ are propagators of the $q$ quark and the $(ud)$ scalar diquark, respectively, $p_1 = \lambda_1 P + p$ and $p_2 = \lambda_2 P - p$ correspond to the momenta of the quark and the diquark, respectively. $P$ is the momentum of the baryon. $V_1$ and $V_2$ are the scalar confinement and one-gluon-exchange terms in the kernel, respectively. Generally, the $q(ud)_{00}$ system needs two scalar functions to describe its BS wave function \[37, 38, 41\]:

$$\chi_F(p) = (f_1(p_i^2) + \psi_i f_2(p_i^2)) u(P),$$

where $f_i$ ($i = 1, 2$) are the Lorentz-scalar functions of $p_i^2$, $u(P)$ is the spinor of the baryons, $p_i$ is the transverse projection of the relative momenta along the momentum $P$, $p_i^\mu = p^\mu - (v \cdot p)v^\mu$, and $p_1 = \lambda_2 M - v \cdot p$ (where we defined $v^\mu = P^\mu / M$). We use $M$, $m$, and $m_D$ to represent the masses of the baryons, the $q$-quark and the $(ud)$ diquark, respectively.

According to the potential model, $V_1$ and $V_2$ have the following forms in the covariant instantaneous approximation ($p_i = q_i$) \[39, 40, 44, 45\]:

$$\tilde{V}_1(p_t - q_t) = \frac{8\pi\kappa}{[(p_t - q_t)^2 + \mu^2]^2} - (2\pi)^2 \delta^3(p_t - q_t) \int \frac{d^3k}{(2\pi)^3} \frac{8\pi\kappa}{(k^2 + \mu^2)^2},$$
where $q_t$ is the transverse projection of the relative momenta along the momentum $P$ and defined as $q^\mu_t = q^\mu - (v \cdot q)v^\mu$, $q_t = \lambda_2 M - v \cdot q$. The second term of $\tilde{V}_1$ is introduced to avoid infrared divergence at the point $p_t = q_t$, and $\mu$ is a small parameter to avoid the divergence in numerical calculations.

\begin{equation}
\tilde{V}_2(p_t - q_t) = -\frac{16\pi}{3} \frac{\alpha_{seff}^2 Q_0^2}{[(p_t - q_t)^2 + \mu^2][(p_t - q_t)^2 + Q_0^2]},
\end{equation}

It was found that $Q_0^2 = 3.2$ GeV$^2$ can lead to consistent results with the experimental data by analyzing the electromagnetic FFs of the proton \cite{6}. The parameters $\kappa$ and $\alpha_{seff}$ are related to the scalar confinement and the one-gluon-exchange diagram, respectively.

The quark and diquark propagators can be written as the follows:

\begin{equation}
S_F(p_1) = i\psi \left[ \frac{\Lambda^+_q}{M - p_t - \omega_q + i\epsilon} + \frac{\Lambda^-_q}{M - p_t + \omega - i\epsilon} \right],
\end{equation}

\begin{equation}
S_D(p_2) = \frac{i}{2\omega_D} \left[ \frac{1}{p_t - \omega_D + i\epsilon} - \frac{1}{p_t + \omega_D - i\epsilon} \right],
\end{equation}

where $\omega_q = \sqrt{m^2 - p^2_t}$ and $\omega_D = \sqrt{m^2_D - p^2_t}$. $\Lambda^\pm$ are the projection operators which have the following relations:

\begin{equation}
\begin{align*}
2\omega_q \Lambda^\pm_q &= \omega_q \pm \phi(p_t + m), \quad (7) \\
\Lambda^\pm_q \Lambda^\pm_q &= \Lambda^\pm_q, \quad (8) \\
\Lambda^\pm_q \Lambda^\mp_q &= 0. \quad (9)
\end{align*}
\end{equation}

Following our previous work, in order more precisely calculate the FFs of $\Lambda_b \rightarrow n$, we can take $E_0 = -0.14$ GeV (where $E_0 = M - m - m_D$ is the binding energy) and $\kappa$ to be about $0.05 \pm 0.005$ GeV$^3$ for $\Lambda_b \rightarrow n$ \cite{43}. Defining $\tilde{f}_1(2) = \int \frac{dp_t}{2\pi} f_1(2)$, and using the covariant instantaneous approximation, $p_t = q_t$, the scalar BS wave functions satisfy the following coupled integral equation:

\begin{equation}
\tilde{f}_1(p_t) = \int \frac{d^3q_t}{(2\pi)^3} M_{11}(p_t, q_t) \tilde{f}_1(q_t) + M_{12}(p_t, q_t) \tilde{f}_2(q_t),
\end{equation}
\[ \tilde{f}_2(p_t) = \int \frac{d^3q_t}{(2\pi)^3} M_{21}(p_t, q_t) \tilde{f}_1(q_t) + M_{22}(p_t, q_t) \tilde{f}_2(q_t), \] 

where

\[
M_{11}(p_t, q_t) = \frac{(\omega_q + m)(\tilde{V}_1 + 2\omega_D \tilde{V}_2) - p_t \cdot (p_t + q_t) \tilde{V}_2}{4\omega_D \omega_q (-M + \omega_D + \omega_q)} - \frac{(\omega_q - m)(\tilde{V}_1 - 2\omega_D \tilde{V}_2) + p_t \cdot (p_t + q_t) \tilde{V}_2}{4\omega_D \omega_q (M + \omega_D + \omega_q)},
\]

\[
M_{12}(p_t, q_t) = \frac{-(\omega_q + m)(q_t + p_t) \cdot q_t \tilde{V}_2 + p_t \cdot q_t (\tilde{V}_1 - 2\omega_D \tilde{V}_2)}{4\omega_D \omega_q (-M + \omega_D + \omega_c)} - \frac{(m - \omega_q)(q_t + p_t) \cdot q_t \tilde{V}_2 - p_t \cdot q_t (\tilde{V}_1 + 2\omega_D \tilde{V}_2)}{4\omega_D \omega_q (M + \omega_D + \omega_q)},
\]

\[
M_{21}(p_t, q_t) = \frac{-(\tilde{V}_1 + 2\omega_D \tilde{V}_2) - (\omega_q + m) \frac{(p_t + q_t) \cdot p_t}{p_t^2} \tilde{V}_2}{4\omega_D \omega_q (-M + \omega_D + \omega_q)} - \frac{-(\tilde{V}_1 - 2\omega_D \tilde{V}_2) + (\omega_q + m) \frac{(p_t + q_t) \cdot p_t}{p_t^2} \tilde{V}_2}{4\omega_D \omega_q (M + \omega_D + \omega_q)},
\]

\[
M_{22}(p_t, q_t) = \frac{(m - \omega_q)(\tilde{V}_1 + 2\omega_D \tilde{V}_2) \frac{(p_t + q_t) \cdot p_t}{p_t^2} - (q_t^2 + p_t \cdot q_t) \tilde{V}_2}{4\omega_D \omega_q (-M + \omega_D + \omega_q)} - \frac{(m + \omega_q)(-\tilde{V}_1 - 2\omega_D \tilde{V}_2) \frac{(p_t + q_t) \cdot p_t}{p_t^2} + (q_t^2 + p_t \cdot q_t) \tilde{V}_2}{4\omega_D \omega_q (M + \omega_D + \omega_q)}.
\]

When \( \frac{1}{m} \rightarrow 0 \) \([39]\), the quark propagator can be written as following,

\[
S_F(p_1) = i \frac{1 + \gamma_5}{2(E_0 + m_D - p_l + i\epsilon)},
\]

considering the Dirac equation for \( \Lambda_b \) we have

\[
\phi(p) = - \frac{i}{(E_0 + m_D - p_l + i\epsilon)(p_l^2 - \omega_D^2)} \int \frac{d^4q}{(2\pi)^4} (\tilde{V}_1 + 2p_l \tilde{V}_2) \phi(q),
\]

where the BS wave function of \( \Lambda_b \) was given in the previous work \([39]\) and has the form \( \chi_P(v) = \phi(p)u_{\Lambda_b}(v, s) \) with \( \phi(p) \) being the scalar BS wave function.

Generally, the BS wave function can be normalized under the condition of the covariant instantaneous approximation \([43]\):

\[
i\delta_{j_1j_2}^{i_1i_2} \int \frac{d^4q d^4p}{(2\pi)^8} \chi_P(p, s) \left[ \frac{\partial}{\partial P_0} I_p(p, q)^{i_1i_2j_1j_2} \right] \chi_P(q, s') = \delta_{ss'},
\]

where \( i_{1(2)} \) and \( j_{1(2)} \) represent the color indices of the quark and the diquark, respectively, \( s^{(i)} \) is the spin index of the baryon, \( I_p(p, q)^{i_1i_2j_1j_2} \) is the inverse of the four-point propagator written as follows

\[
I_p(p, q)^{i_1i_2j_1j_2} = \delta^{i_1j_1} \delta^{i_2j_2} (2\pi)^4 \delta^4(p - q) S_F^{-1}(p_1) S_D^{-1}(p_2).
\]
III. MATRIX ELEMENT OF $\Lambda_b \to nl^+l^-$ AND $\Lambda_b \to n\gamma$ DECAYS

In the standard model, the $\Lambda_b \to nl^+l^-$ transition is described by $b \to dl^+l^-$ at the quark level. The effective Hamiltonian describing the electroweak penguin and weak box diagrams related to this transition is given by

$$
\mathcal{H}(b \to dl^+l^-) = \frac{G_F\alpha}{2\sqrt{2\pi}}V_{tb}V_{ts}^* \left[ C_9^{\text{eff}}d\gamma_\mu(1-\gamma_5)b\bar{l}\gamma^\mu l - iC_7^{\text{eff}}d\sqrt{2m_\mu q^\mu q^\mu}(1+\gamma_5)b\bar{l}\gamma^\mu l \right] + C_{10}d\gamma_\mu(1-\gamma_5)b\bar{l}\gamma^\mu\gamma_5 l, 
$$

where $G_F$ is the Fermi coupling constant, $\alpha$ is the fine structure constant at Z mass scale, $e^\nu$ is the polarization vector of photon, respectively. $q$ is the total momentum of the lepton pair and $C_i^{\text{eff}}$ ($i = 7, 9, 10$) are the Wilson coefficients, $C_7^{\text{eff}} = -0.313$, $C_9^{\text{eff}} = 4.334$, $C_{10} = -4.669$. The amplitude is obtained by sandwiching the effective Hamiltonian between the initial and final states. The matrix element for $\Lambda_b \to n$ can be parameterized in terms of the FFs as the following:

$$
\langle n(P')|\bar{d}\gamma_\mu|\Lambda_b(P)\rangle = \bar{u}_n(P')(g_1\gamma^\mu + ig_2\sigma_{\mu\nu}q^\nu + g_3p_\mu)u_{\Lambda_b}(P),
\langle n(P')|\bar{d}\gamma_\mu\gamma_5|\Lambda_b(P)\rangle = \bar{u}_n(P')(t_1\gamma^\mu + it_2\sigma_{\mu\nu}q^\nu + t_3p^\mu)\gamma_5u_{\Lambda_b}(P),
\langle n(P')|\bar{d}\gamma_\mu\gamma_5q^\nu|\Lambda_b(P)\rangle = \bar{u}_n(P')(s_1\gamma^\mu + is_2\sigma_{\mu\nu}q^\nu + s_3q^\mu)\gamma_5u_{\Lambda_b}(P),
\langle n(P')|\bar{d}\sigma_{\mu\nu}q^\nu|\Lambda_b(P)\rangle = \bar{u}_n(P')(d_1\gamma^\mu + id_2\sigma_{\mu\nu}q^\nu + d_3q^\mu)\gamma_5u_{\Lambda_b}(P),
$$

where $P'$ and $P$ are the momenta of the neutron and $\Lambda_b$ respectively, $q = P - P'$, $u_n$ and $u_{\Lambda_b}$ are the spinors of the initial and final baryons respectively, $g_i$, $t_i$, $s_i$, and $d_i$ ($i = 1, 2$ and 3) are the transition FFs which are Lorentz scalar functions of $q^2$. When working in the limit $m_b \to \infty$, the number of independent FFs is reduced to 2. The $\Lambda_b \to n$ matrix element with an arbitrary matrix $\Gamma$ is given by

$$
\langle n(P')|\bar{d}\Gamma b|\Lambda_b(v)\rangle = \bar{u}_n(P')(F_1(\omega) + F_2(\omega)\gamma_5)u_{\Lambda_b}(v),
$$

where $\Gamma = \gamma_\mu$, $\gamma_\mu\gamma_5$, $q^\nu\sigma_{\nu\mu}$, $q^\nu\sigma_{\nu\mu}\gamma_5$. $F_1$ and $F_2$ can be expressed as functions solely of $\omega = v \cdot P'/m_{\Lambda_b}$, which is the energy of the neutron in the $\Lambda_b$ rest frame. The baryons states can be normalized as follows,

$$
\langle n(P')|n(P)\rangle = 2E_n(2\pi)^3\delta^3(P - P'),
$$

$$
\langle \Lambda_b(v', P')|\Lambda_b(v, P)\rangle = 2v_0(2\pi)^3\delta^3(P - P').
$$

Comparing Eq. (21) with Eq. (22), we obtain the following relations:

$$
g_1 = t_1 = s_2 = d_2 = \left( F_1 + \sqrt{r}F_2 \right),
g_2 = t_2 = g_3 = t_3 = \frac{1}{m_{\Lambda_b}}F_2,
s_3 = F_2(\sqrt{r} - 1),
d_3 = F_2(\sqrt{r} + 1),
s_1 = d_1 = F_2m_{\Lambda_b}(1 + r - 2\sqrt{r}\omega),
$$

(26)
where $r = m_n^2/m_{\Lambda_b}^2$. On the other hand, the transition matrix for $\Lambda_b \rightarrow n$ can be expressed in terms of the BS wave functions of $\Lambda_b$ and $n$,

$$
\langle n(P')|d\Gamma b|\Lambda_b(P)\rangle = \int \frac{d^4p}{(2\pi)^4} \bar{\chi}_\nu^{\Lambda_b}(p') \Gamma \chi^{\Lambda_b}_\mu(p) S_D^{-1}(p_2).
$$

where the $\chi^{\Lambda_b}_\nu$ are the BS wave functions of neutron and $\Lambda_b$ respectively.

Define

$$
\int \frac{d^4p}{(2\pi)^4} f_1(p') \phi(p) S_D^{-1}(p_2) = k_1(\omega),
$$

$$
\int \frac{d^4p}{(2\pi)^4} f_2(p') p_\mu' \phi(p) S_D^{-1}(p_2) = k_2(\omega) v_\mu + k_3(\omega) v_\mu',
$$

where $v' = P'/m_n$, then we find the following relations when $\omega \neq 1$:

$$
k_3 = -\omega k_2,
$$

$$
k_2 = \frac{1}{1 - \omega^2} \int \frac{d^4p}{(2\pi)^4} f_2(p') p_\mu' \phi(p) S_D^{-1},
$$

$$
F_1 = k_1 - \omega k_2,
$$

$$
F_2 = k_2.
$$

The differential decay rate of $\Lambda_b \rightarrow n l^+ l^-$ is obtained as:

$$
\mathcal{M}(\Lambda_b \rightarrow n l^+ l^-) = \frac{G_F \lambda_t}{2\sqrt{2} \pi} \left[ \bar{u}_n[\gamma_\mu(A_1 + B_1 + (A_1 \cdot B_1) \gamma_5)] u_{\Lambda_b} \\
+ i\sigma^\mu p_\nu(A_2 + B_2 + (A_2 \cdot B_2) \gamma_5) u_{\Lambda_b} \right] \left[ \bar{u}_n[\gamma_\mu(D_1 + E_1 + (D_1 \cdot E_1) \gamma_5)] u_{\Lambda_b} \\
+ i\sigma^\mu p_\nu(D_2 + E_2 + (D_2 \cdot E_2) \gamma_5) u_{\Lambda_b} \right]
\left[ p^\mu(D_3 + E_3 + (D_3 \cdot E_3) \gamma_5) u_{\Lambda_b} \right],
$$

where $\lambda_t = |V_{tb} V_{ts}^*|$, the parameters $A_i, B_i$ and $D_j, E_j$ ($i = 1, 2$ and $j = 1, 2, 3$) are defined as

$$
A_i = \frac{1}{2} \left\{ C_9^{\text{eff}}(g_i - t_i) - \frac{2C_7^{\text{eff}} m_b}{p^2} (d_i + s_i) \right\},
$$

$$
B_i = \frac{1}{2} \left\{ C_9^{\text{eff}}(g_i + t_i) - \frac{2C_7^{\text{eff}} m_b}{p^2} (d_i - s_i) \right\},
$$

$$
D_j = \frac{1}{2} C_{10}(g_j - t_j), \quad E_j = \frac{1}{2} C_{10}(g_j + t_j).
$$

In the physical region $(4m_i^2 \leq q^2 \leq (m_{\Lambda_b} - m_n)^2)$, the decay rate of $\Lambda_b \rightarrow n l^+ l^-$ is obtained as

$$
\frac{d\Gamma(\Lambda_b \rightarrow n l^+ l^-)}{dq^2} = \frac{G_F^2 \alpha^2}{2^{13} 3 \pi^5 m_{\Lambda_b}} |V_{tb} V_{td}^*|^2 v_\ell \sqrt{\lambda(1, r, s)} \mathcal{M}(s),
$$
where \( s = 1 + r - 2\sqrt{r}\omega\), \( (1, r, s) = 1 + r^2 + s^2 - 2r - 2s - 2rs\), and \( v_l = \sqrt{1 - \frac{4m_l^2}{s^2m_{\Lambda_b}^2}}\), and the decay amplitude is given as [21]

\[
M(s) = M_0(s) + M_2(s), \tag{33}
\]

where

\[
M_0(s) = 32m_l^2m_{\Lambda_b}^4s(1 + r - s)(|D_3|^2 + |E_3|^2) + 64m_l^2m_{\Lambda_b}^3(1 - r - s)\text{Re}(D_1^*E_3 + D_3E_1^*) + 64m_l^2m_{\Lambda_b}^3\sqrt{r}(6m_l^2 - M_{\Lambda_b}^2)s\text{Re}(D_1^*E_1) + 64m_l^2m_{\Lambda_b}^3\sqrt{r}(2m_{\Lambda_b}s\text{Re}(D_3^*E_3) + (1 - r + s)\text{Re}(D_1^*D_3 + E_1^*E_3)) + 32m_l^2(2m_l^2 + m_{\Lambda_b}^2)(1 - r + s)m_{\Lambda_b}\sqrt{r}\text{Re}(A_1^*A_2 + B_1^*B_2) - m_{\Lambda_b}(1 - r - s)\text{Re}(A_1^*B_2 + A_2^*B_1) - 2\sqrt{r}(\text{Re}(A_1^*B_1) + m_{\Lambda_b}^2s\text{Re}(A_2^*B_2))\right\} + 8m_{\Lambda_b}^2\left\{ 4m_l^2(1 + r - s) + m_{\Lambda_b}^2((1 + r)^2 - s^2) \right\}(|A_1|^2 + |B_1|^2) + 8m_{\Lambda_b}^4\left\{ 4m_l^2[\lambda + (1 + r - s)s] + m_{\Lambda_b}^2[(1 - r)^2 - s^2] \right\}(|A_2|^2 + |B_2|^2) - 8m_{\Lambda_b}^2\left\{ 4m_l^2(1 + r - s) - m_{\Lambda_b}[(1 - r)^2 - s^2] \right\}(|D_1|^2 + |E_1|^2) + 8m_{\Lambda_b}^5s^2\left\{ -8m_{\Lambda_b}s\sqrt{r}\text{Re}(D_2^*E_2) + 4(1 - r + s)\sqrt{r}\text{Re}(D_1^*D_2 + E_1^*E_2) - 4(1 - r - s)\text{Re}(D_1^*E_2 + D_2^*E_1) + m_{\Lambda_b}[(1 - r)^2 - s^2](|D_2|^2 + |E_2|^2) \right\}, \tag{34}
\]

\[
M(s) = 8m_{\Lambda_b}^6sv_l^2\lambda(|A_1|^2 + |B_2|^2 + |C_2|^2 + |D_2|^2) - 8m_{\Lambda_b}^4v_l^2\lambda(|A_1|^2 + |B_1|^2 + |C_1|^2 + |D_1|^2). \tag{35}
\]

Similarly, the Hamiltonian for exclusive rare radiative decay \( \Lambda_b \rightarrow n\gamma \) with \( \gamma \) as a real photon is given by

\[
\mathcal{H}(b \rightarrow d\gamma) = -\frac{iG_F\Gamma}{4\sqrt{2}\pi^2}V_{tb}V_{td}^*C_7^{eff}\left[ m_b\bar{d}\sigma_{\mu\nu}q^\mu(1 + \gamma_5)b + m_d\bar{d}\sigma_{\mu\nu}q^\mu(1 - \gamma_5)b \right] \epsilon^\nu, \tag{36}
\]

where \( \epsilon^\nu \) is the polarization vector of the photon. Then, the decay width is given by

\[
\Gamma(\Lambda_b \rightarrow n\gamma) = \frac{\alpha G_F^2m_b^2m_{\Lambda_b}^3}{2^9\pi^4}V_{tb}V_{td}^*|C_7^{eff}|^2[s_2^2(0) + d_2^2(0)]\left(1 - \frac{m_n^2}{m_{\Lambda_b}^2}\right)^3. \tag{37}
\]
IV. NUMERICAL ANALYSIS AND DISCUSSION

In this section we present a detailed numerical analysis of the rare decay $\Lambda_b \to n l^+ l^-$ and radiative decay $\Lambda_b \to n \gamma$. In our calculations, we take the masses of baryons as $m_{\Lambda_b} = 5.62$ GeV, $m_n = 0.94$ GeV \[53\], and the masses of quarks, $m_b = 5.02$ GeV and $m_d = 0.34$ GeV \[38, 40, 41\]. The variable $\omega$ varies from 1 to 3.073, 3.069, 1.89 for $e$, $\mu$, $\tau$, respectively.

Solving Eq. (10), (11) and (17) for the neutron and $\Lambda_b$ with the parameters we have taken, we get the numerical solutions of BS wave functions. In Table I, we give the values of $\alpha_{seff}$ with different values of $\kappa$ for the neutron and $\Lambda_b$ and in Fig. 2 and 3, we give the BS wave functions for the neutron and $\Lambda_b$.

| $\kappa$ (GeV$^3$) | 0.045 | 0.047 | 0.049 | 0.051 | 0.053 | 0.055 |
|-------------------|-------|-------|-------|-------|-------|-------|
| neutron           | 0.829 | 0.811 | 0.793 | 0.775 | 0.758 | 0.741 |
| $\Lambda_b$       | 0.775 | 0.777 | 0.778 | 0.780 | 0.782 | 0.784 |

TABLE I: The values of $\alpha_{seff}$ for the neutron and $\Lambda_b$.

![FIG. 2: (color online) The BS wave functions for the neutron.](image)

It can be seen from Table I that the dependence of $\alpha_{seff}$ for the neutron on $\kappa$ is obviously stronger than that for $\Lambda_b$. From the figures in Figs. 2 and 3, we find that BS wave functions of neutron is very similarly on different $\kappa$, the values of $f_1(p_t)$ is about from 0 to 0.14 $f_2(p_t)$ varies about from 0 to 0.06 and $\phi(p_t)$ varies from 0 to 0.17. In Fig. 2 we plot the FFs and $R(\omega) = F_2/F_1$ for different $\kappa$. From this figure, we find that $F_1(\omega)$ increases with the increase of $\kappa$, but the value of $R(\omega)$ is not sensitive to the change of the value of $\kappa$. The value of $R(\omega)$ varies from $-0.9$ to $-0.25$ when $\omega$ changes from 1 to 3.1.

In the heavy quark limit, assuming the same shape for $F_1$ and $F_2$, the ratio $R = -0.35 \pm 0.04$ (stat) $\pm 0.04$ (syst) was previously measured by the CLEO Collaboration using the experimental data for the semileptonic decay $\Lambda_c \to \Lambda e^+ \nu_e$ when $q^2$ changes from $m_{\Lambda_c}^2$ to $m_{\Lambda_c}^2$ \[47\]. In the same region, we find that $R(\omega)$ varies from $-0.32$ to $-0.25$ in our model. In Ref. 12 $R(\omega)$ varies from $-0.42$ to $-0.83$ when $q^2$ change from 0 to $(M_{\Lambda_b} - M_{\Lambda})^2$, and in our model $R(\omega)$ change from $-0.25$ to $-0.75$ in the same region. However, in Ref. 13 gives the
behaviour \( R(q^2) \propto -1/q^2 \), which agrees with the pQCD scaling law \([46, 53, 56]\). Therefore, using the CLEO Collaboration experimental data \([47]\), we can estimate that the value of \( R(\omega) \) should change from to \(-0.91 \pm 0.03\) to \(-0.3 \pm 0.03\) approximately, which agrees with our result as shown in Fig. 4. From the data in Ref. \([26]\), we find that \( R(\omega_{max}) = -2.75 \) in LCSR and \( R(\omega_{max}) = -2.33 \) by fit the data from LQCD \([59, 60]\). From the data with the contribution of \( \Lambda_b^* \) being considered Ref. \([28]\), we find that \( R(\omega_{max}) = -3.47 \) in LCSR. These results are much larger than experimental data \( R(\omega_{max}) = -0.35 \) \([47]\) and do not agree with our result.

In Fig. 5, we give the \( \omega \)-dependence of the decay widths of \( \Lambda_b \to nl^-l^+ \) for different parameters. For the central values of parameters, we find that the branching ratio are \( BR(\Lambda_b \to nl^-l^+) \times 10^8 = 6.79 \) \((l = e)\), \( 4.08 \) \((l = \mu)\), \( 2.90 \) \((l = \tau)\) and \( BR(\Lambda_b \to n\gamma) \times 10^7 = 3.69 \). Our result for the branching ratios of \( BR(\Lambda_b \to nl^-l^+) \) and \( BR(\Lambda_b \to n\gamma) \) are listed in Table II together those in other approaches.

|                   | present work | LCSR \([26]\) | LQCD \([26]\) | LCSR \([28]\) | Ref. \([29]\) |
|-------------------|--------------|--------------|--------------|--------------|-------------|
| \( Br(\Lambda_b \to ne^+e^-) \times 10^8 \) | 6.79\pm0.66\_1.82 | 3.79\pm0.46 | 3.19\pm0.32 | 8\pm2 | 3.81 |
| \( Br(\Lambda_b \to n\mu^+\mu^-) \times 10^8 \) | 4.08\pm0.44\_1.19 | 3.76\pm0.42 | 3.15\pm0.29 | 7\pm2 | 3.75 |
| \( Br(\Lambda_b \to n\tau^+\tau^-) \times 10^8 \) | 2.9\pm0.37\_0.78 | 1.65\pm0.19 | 1.42\pm0.13 | 2\pm0.4 | 1.21 |
| \( Br(\Lambda_b \to n\gamma) \times 10^7 \) | 3.69\pm0.19\_0.95 | - | - | - | 3.7 |

TABLE II: The values of the branching ratios of \( \Lambda_b \to nl^-l^+ \) and \( \Lambda_b \to n\gamma \) and compare with other model.

In Ref. \([26]\), the authors use the parameters from LCSR \([58]\) and LQCD \([59, 60]\) to fit the FFs of \( \Lambda_b \to n \) and gave the branching ratio of \( \Lambda_b \to nl^-l^+ \). In Ref. \([28]\), the authors also calculated the FFs of \( \Lambda_b \to n \) in the framework of LCSR, but their results were different. Our results for the branching ratios of \( \Lambda_b \to nl^-l^+ \) \((l = e, \tau)\) are very similar to those in Ref. \([28]\), and our result for \( BR(\Lambda_b \to n\mu^+\mu^-) \) agrees with that in Ref. \([26]\). Our radiative decay result \( BR(\Lambda_b \to n\gamma) \)
FIG. 4: (color online) The Values of FFs (the lines become thicker with the increases of $\kappa$) and $R(\omega)$ with different values of $\kappa$.

FIG. 5: (color online) The decay widths of $\Lambda_b \to nl^+l^-$ (the values of the decay width increase with the increase of $\kappa$ from 0.045 to 0.055) for the lines with the same color.

agrees with Ref. [29].
V. SUMMARY

In our work, we calculated the FFs between baryons states induced by the rare $b \to d$ transition in the BS equation approach in a covariant quark-diquark model. In our model, $\Lambda_b$ is regarded as a bound state of the $b$-quark and the scalar $ud$ diquark, thus only the $d'(du)_{00}/\sqrt{3}$ component of the neutron contributes to the FFs. We established the BS equations for the $q(ud)_{00}$ $(q = b, d)$ system and derived the FFs for $\Lambda_b \to n$ in the BS equation approach. We solved the BS equation of $q(ud)_{00}$ $(q = b, d)$ system and then we calculated the FFs and $R$ numerically. Using these FFs, we obtained the branching ratios of $\Lambda_b \to n\ell^+\ell^-$ and $\Lambda_b \to n\gamma$. Comparing with other works we found that our FFs are very different with other model [26, 29], but the branching fractions of the semileptonic decay are of the order $10^{-8}$ and the radiative branching ratio is of the order $10^{-7}$.

In the near future, our results can be tested at LHCb. Our model can be used to study the forward-backward asymmetries and CP violation in the rare decays of $b$ baryons to check our FFs.

Acknowledgments

This work was supported by National Natural Science Foundation of China under contract numbers 11775024,11575023,11847052,11981240361 and 11905117.

[1] R. Aaij et al.,LHCb collaboration, Phys. Lett. B 725, 25 (2013).
[2] R. Aaij et al.,LHCb collaboration, Phys. Rev. Lett. 123, 031801 (2019).
[3] R. Aaij et al.,LHCb collaboration. JHEP 09, 146 (2018).
[4] R. Aaij et al.,LHCb collaboration, JHEP 06115 (2017); 09, 145 (2018).
[5] T. Altonen et al.,CDF collaboration, Phys. Rev. Lett. 107, 201802 (2011).
[6] T. Mannel and S. Recksiegel, J. Phys. G: Nucl. Part. Phys. 24, 979 (1998).
[7] R. Mohanta, et al., Prog. Theor. Phys. 102, 645 (1999).
[8] Y.M. Wang, M.J. Aslam, C.D. Lü, Eur. Phys. J. C 59, 847 (2009).
[9] T. Gutsche et al., Phys. Rev. D 87, 074031 (2013).
[10] R.N. Faustov, V.O. Galkin, Phys. Rev. D 96, 053006 (2017).
[11] R.F. Alnahdi, T. Barakat, H.A. Alhendi, Prog. Theor. Exp. Phys. 073B04 (2017).
[12] C.S. Huang and H.G Yan, Phys. Rev. D 59, 114022 (1999).
[13] X.H. Guo and T. Huang, Phys. Rev. D 53, 4946 (1996).
[14] C.S. Huang, C.Q. Geng, Phys. Rev. D 63, 114024 (2001).
[15] T.M. Aliev, A. Özpineci, M. Savcı, C. Yüce, Phys. Lett. B 542, 229 (2002); Phys. Rev. D 67, 035007 (2003).
[16] T.M. Aliev, V. Bashiry, M. Savcı, Eur. Phys. J. C 38, 283 (2004); Nucl. Phys. B 709, 115 (2005).
[17] T.M. Aliev, M. Savcı, JHEP 05, 001 (2006); Eur. Phys. J C 48, 117 (2006).
[18] T.M. Aliev, M. Savcı, B.B. Şircanlı, Eur. Phys. J. C 52, 375 (2007).
[19] T.M. Aliev, K. Azizi, M. Savcı, Phys. Rev. D 81, 056006 (2010).
[20] C.H. Chen and C.Q. Geng, Phys. Lett. B 516, 327 (2001); Phys. Lett. B 725, 25 (2013).
[21] A.K. Giri, R. Mohanta, Eur. Phys. J C 45, 151 (2006).
