Destruction of the orbital angular momentum in combined singular beams

Ya E Akimova, M V Bretsko

1 Physics and Technology Institute, V.I. Vernadsky Crimean Federal University, Simferopol, Republic of Crimea, Russia

E-mail: Akimova.yana@yandex.ru

Abstract. This paper shows the dependence of the orbital angular momentum (OAM) of combined singular beams on the magnitude of the local perturbation in the holographic grating. In the absence of a perturbation of the holographic grating, the OAM acquires a maximum value numerically equal to the topological charge of the singular beam. With weak disturbances of a regular holographic grating, dips in the orbital moment appear. The depth of the dips increases rapidly with increasing disturbance. It is shown that even a weak perturbation of the lattice leads to a sharp increase in the contribution of partial beams with other topological charges. The reliability of the results obtained confirms the high degree of correlation of the intensity distribution of the original and reconstructed combined beams.

1. Introduction

Among the various tasks of singular optics [1-4], the problem of the formation, transformation and destruction of vortex beams caused by weak disturbances is of paramount importance, for example, for optical data transmission and processing systems [5, 6], optical cryptography [7], and manipulations with micro- and macro-objects [8].

Special attention is paid to the problem of control of the orbital momenta (OAM) of single photons and polaritons. There is a well-developed method for the formation and control of OAM combined vortex beams by creating high-quality holographic grids [4, 9]. There are various methods of controlling OAM by electrically controlling the beam in crystals, using reflective metal surfaces, or by controlling the laser parameters. But is it possible to control the OAM of the vortex beam by introducing a weak local perturbation into the holographic lattice?

As a rule, the effect of a weak phase perturbation on a scalar singular beam carrying a single optical vortex leads to a change in its angular spectrum, which is conveniently represented as a superposition of a set of standard vortex beams with integer topological charges (orders). In particular, the perturbation of the phase pitch of the spiral phase plate leads to the appearance of an infinite set of vortex beams, which can be represented as a beam with a fractional topological charge [10]. The weak perturbation of the bifurcated holographic grating leads to a similar result [11]. The state of a complex combined beam is conveniently characterized by the amplitudes and phases of standard vortex beams, which form a vortex spectrum. Knowing the vortex spectrum, it is easy to form a holographic grating and restore the beam itself using fractional OAM [12].

Thus, the goal of our article is a theoretical and experimental study of controlled OAM resonances in combined vortex beams, reconstructed by a holographic grating with weak local perturbations.
2. Model of the vortex beam

In paper [11], Berry showed that small deviations of the parameters of a spiral wave plate from technical characteristics lead to a significant distortion of the structure of the diffracted paraxial beam due to the internal disintegration of the optical vortex. For example, if the deviation of the thickness of the wave plate from a multiple of the wavelength $\lambda$ of the original monochromatic radiation causes the birth of an optical vortex with a fractional topological charge $p$

$$\Psi(r, \varphi, z) = F(r, z)e^{ip\varphi},$$

(1)

then it will satisfy the equation only if the exponential factor $e^{ip\varphi}$, where $\varphi$ is the azimuthal coordinate, in the mathematical structure of the vortex can be decomposed into a Fourier series in integer values $m$

$$e^{ip\varphi} = e^{i\varphi}\sum_{m=-\infty}^{\infty} e^{im\varphi} = \sum_{m=-\infty}^{\infty} e^{im\varphi}.$$

(2)

The longitudinal component of the OAM can be found as

$$\ell_z = \frac{\sum_{m=-\infty}^{\infty} m^2}{\sum_{m=-\infty}^{\infty} (p-m)^2}.$$

(3)

The view of the dependence $\ell_z$ on the topological charge of the optical vortex of the partial beam $m$ is illustrated by a smooth curve in Fig.1. In the region of integer values of the topological charge $p \approx m$, OAM reaches a maximum $\ell_z = m$, the minimum value corresponds to half-integer $p$ values, which slightly differ from the maximum values.

Let us consider a new model of a combined beam endowed with resonant properties of an OAM. Imagine the angular spectrum of the combined beam in the form

$$U(k, \phi) = e^{i\varphi}\frac{\sin \pi p}{\pi} \sum_{m=-\infty}^{\infty} \sum_{M}^{\infty} \left(\Omega R k\right)^{m+2} M_m |m|! e^{im\varphi} \frac{e^{k^2/2}}{p-m}.$$

(4)

where $M_m$ – the amplitude coefficient, $k$ – the transverse wave number.

Then the complex amplitude of the beam is written as

$$\Psi_p(r, \varphi, z = 0) = e^{i\varphi}\frac{\sin \pi p}{2\pi} \sum_{m=-\infty}^{\infty} \sum_{M}^{\infty} \Omega |m|! r^{m+2} |m|! e^{im\varphi} e^{r^2/2}.$$

(5)

where $M_p = \Omega |m|^z$.

The orbital angular momentum of such a beam is written as

$$\ell_z = \sum_{m=-\infty}^{\infty} \frac{m \cdot \Omega |m|}{|m|^z (p-m)^z} + \sum_{m=-\infty}^{\infty} \frac{\Omega |m|^z}{|m|^z (p-m)^z}.$$

(6)

The OAM $\ell_z(p)$ spectrum is represented by curve in Fig. 1. (Curve 2) Small resonant bursts of OAM are observed already at small values of the topological charge $p$ in the interval $(0 \div 5)$. Then the spectral curve monotonously increases in the range of values $p \in (5, 13)$, where it almost merges
with curve , constructed according to expression (3) (Curve 1). A further increase in the parameter \( p \) is accompanied by a sequence of resonant bursts. It should be noted that the shape of the contour of the resonance curve changes with changes in both the topological charge \( p \) and the scale parameter \( \Omega \).

Figure 1. The dependence of the OAM \( \ell_z \) on a perturbation parameter \( p \) for the combined beams: curve 1 is calculated according to Eq. (3), curve 2 is plotted according to Eq. (6).

It should be expected that the sharp bursts and dips of the orbital angular momentum are due to the avalanche increase in the number of partial beams, forming an array of optical vortices with deviations of the topological charge \( p \) from the integer index values \( m \). However, these assumptions require experimental verification.

3. **The experiment and discussion**

The holographic amplitude grating is the main element of our experiment for the study of resonant bursts in the OAM spectrum; it forms a combined singular beam. Therefore, we first discuss the structure of the recovered beam field.

For our experiment, a beam was chosen, whose amplitude is described by expression (5), the squares of which coefficients are given as

\[
C_{m,mod}^2 = \frac{\Omega_{\ell}^{2|m|}}{|m|^2 (p-m)^2} \sum_{m=\infty} \frac{\Omega_{\ell}^{2|m|}}{|m|^2 (p-m)^2}.
\]

(7)

In order to check the presence of avalanche instability of the orbital angular momentum \( \ell_z(p) \), binary holographic grids were formed for the combined beams (5) in accordance with the expression

\[
T_p = \text{signum}\left[ \cos \left( \arg \Psi_p - Q r \cos \varphi \right) \right].
\]

(8)

where \( Q \) – is the scale parameter. The view of the holographic grid and the restored field are shown in the Fig.2.

Figure 2. Simulation of the holographic grating and intensity of the recovered combined beams with a small perturbation of the grating.
Experimental studies of the OAM resonant bursts in vortex arrays were carried out on an experimental setup, which was discussed in detail in [13]. Schematic diagram of the installation is shown in Figure 1, in this work. The spectrum of optical vortices and OAM of combined beams formed on a spatial light modulator (SLM) was measured. The method is based on the measurement of higher-order intensity moments $J_{p,q}$, which make it possible to determine the values of the squares of the coefficients (amplitudes) of partial vortex beams. The obtained values of the coefficients $C_m^2$ in the wave array allow us to estimate the OAM by the equation $\ell_z = \sum_{m=-N}^{N} m C_m^2$. For each value of the topological charge $p$, the spectra of the squares $C_m^2$ of the amplitudes shown in Fig. 3 and the spectrum of the OAM $\ell_z(p)$ in Fig. 4 were measured.

**Figure 3.** The intensity distribution of the beam obtained in the experiment and the dependence of the squares of the amplitudes $C_m^2$ on the topological charge $m$ of the combined beam modes (5) with the disturbance of states $\delta p = 0.5$.

**Figure 4.** Spectra $\ell_z(p)$ for disturbed states; curve 1 - theory, curve 2 – experiment.

To estimate the measurement error, we used the degree of image correlation, which lies in the limit $\eta = 0.9 \div 0.93$, which indicates the agreement between theory and experiment.

**Conclusion**

The paper presents a new model of a combined singular beam which consists of an array of optical vortices with OUM resonances. For this purpose, the spectral composition of optical vortices in combined beams consisting of a superposition of Laguerre-Gauss beams with a sequence of amplitudes that determines the fractional topological charge and the OAM of the combined beam as a whole were theoretically analyzed.

The considered resonant bursts of OUM in combined beams have two possible aspects of practical application. On the one hand, the use of beams carrying higher order optical vortices requires taking into account the quality of holographic gratings associated with these beams. Some types of such grids can lead to avalanche destruction of the OUM, although the appearance of the beam does not imply such an effect. On the other hand, the propagation of beams with OUM resonances through optical media can be accompanied by both a change in the shape of the spectral burst or dip, and a shift in its position. By fixing the change in the shape of the OUM resonances, one can argue about the nature of the medium irregularities, for example, with weak turbulence.

**Acknowledgments**

The reported study was funded by RFBR according to the research project № 19-29-01233. This work was partially supported by the V.I. Vernadsky Crimean Federal University Development Program for 2015 – 2024 № ВГ24/2018.
References

[1] Gbur G 2017 *Singular optics* (New York: CRC Press)

[2] Kotlyar V V, Kovalev A A and Porfirez A P 2018 *Vortex Laser Beams* (London: Taylor & Francis LCC)

[3] Allen L and Padget M 2011 *Twisted Photons: Applications of Light with Orbital Angular Momentum* (Weinheim: WILEY-VCH Verlag GmbH & Co. KGaA) p 288

[4] Wang J, Yang J-Y, Fazal I M, Ahmed N, Yan Y, Huang H, Ren Y X, Yue Y, Dolinar S, Tur M and Willner A E 2012 *Nat. Photonics* 6 488–96

[5] Soifer V and Golub M 1994 *Laser beam mode selection by computer-generated holograms* (Boca Raton: CRC Press) p 224

[6] Khonina S N, Kotlyar V V, Soifer V A, Jefimovs K and Turunen J 2004 *J. Mod. Opt.* 51 761–73

[7] Khonina S N, Kazanskiy N L and Soifer V A 2012 *Optical vortices in a fiber: mode division multiplexing and multimode self-imaging. Chapter in Recent Progress in Optical Fiber Research* (London: INTECH publisher) p 327-52

[8] Khonina S N, Kotlyar V V, Soifer V A, Paakkonen P and Turunen J 2001 *Optical Memory and Neural Networks* 10 241–55

[9] Abramochkin E and Volostnikov V 1991 *Optics Communications* 83 123–35

[10] Kirilenko M S and Khonina S N *Information Optics* 22 81–9

[11] Berry M 2004 *J. Opt. A* 6 259–69

[12] Wright E 2011 *Optical Vortex Cat States and their Utility for Creating Macroscopic Superpositions of Persistent Flows. Twisted Photons: Applications of Light with Orbital Angular Momentum* (Weinheim: WILEY-VCH Verlag GmbH & Co. KGaA) p 288

[13] Volyar A V, Bretsko M V, Akimova Ya E and Egorov Yu A 2018 *Opt. Lett.* 43 5635-38