A multiphase approach for modelling the shock response of composite materials

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Abstract
The shock response of unidirectional fiber reinforced composite materials is inherently anisotropic due to their microstructural geometric configuration. Unlike typical elastic-plastic materials, composite materials form the observed two-wave structure under longitudinal shocks due to a precursor wave travelling through the fibers ahead of a bulk wave in the resin constituent. The nature of this response presents a problem in traditional hydrocode frameworks where each cell or material point tracks only a single velocity field. This paper outlines an adaptation of the Baer and Nunziato multi-phase model in CTH where a mixture rule is used to determine the velocity field of each constituent (fiber and matrix) of the composite material. The model modifies the momentum exchange term to represent the frictional drag forces between the fiber and matrix constituents, while assuming no mass or energy exchange. The momentum drag model is dependent not only upon the pressure difference between the constituents but also the directional dependence of the shock response. Finally, the model is implemented and the sensitivity of the solution to the interaction parameters demonstrated.

1. Introduction
The shock response of fiber reinforced composite materials is inherently anisotropic due to their microstructural configurations. Their shock response is also a function of the constituents (fiber and matrix), which comprise the composite mixture. Therefore, to capture the overall mixture response one must also be able to describe the constituent responses. The work detailed herein focuses on adapting a multiphase mixture model that can capture both the mixture and composite response along with the individual constituent responses.

The Baer and Nunziato multiphase mixture model [1,2] was first developed for reactive materials to describe the initiation and combustion of granular energetics. The model considers two phases; a solid granular reactant materials and a gaseous product material. For each of these phases Baer and Nunziato write the conservation equations as:
Conservation of Mass:
\[ \rho_i' = -\rho_i \frac{\partial v_i'}{\partial x} + c_i^+ \]  
(1)

Conservation of Momentum:
\[ \rho_i v_i' = -\frac{\partial (\tau_i)}{\partial x} + \rho_i b_i + m_i^+ - c_i^+ v_i \]  
(2)

Conservation of Energy:
\[ \rho_i e_i' = -\tau_i \frac{\partial v_i'}{\partial x} - \rho_i q_i + m_i^+ v_i - c_i^+ \left( e_i - \frac{1}{2} v_i^2 \right) \]. 
(3)

Where \( c_i^+ \), \( m_i^+ \) and \( e_i^+ \) represent the mass, momentum and energy exchange terms between the phases and \( \rho_i \) are the material partial densities.

To adapt the model to non-reactive composite materials, it is assumed that both the mass and energy exchange terms are zero. The mass exchange term is removed based on the observation that the constituents in a traditional structural composite material are non-reactive and therefore do not change phase under high-pressure shock loading. The energy exchange term between the constituents in the composite is also removed, as this exchange term is a function of the temperature difference between the phases. Specifically, the only source of heating for the non-reactive constituents of the composite is shock heating which occurs on a microsecond timescale, which is much shorter than the second timescale that the heat exchange between phases occurs.

The only remaining exchange term is then the momentum exchange. Baer and Nunziato give the constitutive momentum exchange relationship as:
\[ m_i^+ = \rho g \frac{\partial \alpha_i}{\partial x} - D \].  
(4)

In equation (4), \( D \) represents the drag force per unit volume between the two phases. The relationship for the drag force is given in equation (5), where \( \delta_{sg} \) is the drag coefficient between the two constituents of the mixture.
\[ D = \delta_{sg} (v_i - v_g) \].  
(5)

Equation (5) can also be written in tensor form. Equation (6) gives the tensor form of equation (5) as presented by Kashiwa and Rauenzahn [3].
\[ D_{kl} = \sum_i \phi_i \phi_k \delta_{kl} (v_i - v_k) \].  
(6)

2. Composite Drag Model

To adapt the multiphase model to a composite material mixture, a Coulomb friction drag law as given in equation (7) was assumed between the fiber and matrix constituents.
\[ F_f = \mu F_N = \mu (P_{fiber} - P_{matrix}) A N \]  
(7)

In equation (7), \( F_f \) is the frictional force exerted between the fiber and matrix due to the coefficient of friction (\( \mu \)) and normal force (\( F_N \)) acting at the fiber/matrix interfaces. For the CTH framework where this model was implemented, the normal force is represented by a difference in pressure between the two constituents multiplied by an interaction surface area (A) and the number of interfaces comprising the area of interest (N).

To form the drag coefficient used for this work it was assumed that each individual fiber could be represented as a cylinder. Using this assumption, the drag can be approximated as a fluid flowing over a cylinder. To begin, the fiber volume fraction of a unit volume of composite material is written as:
\[ \phi_{\text{fiber}} = \pi r^2 LN. \]  

(8)

Where \( L \) is the length of the fibers and \( N \) is the number of fibers within the unit volume. Using equation (8) and equation (7), the force per unit volume within the unit cell can be written as

\[ f_{\text{fiber/matrix}} = \frac{2\mu(P_{\text{fiber}} - P_{\text{matrix}})\phi_{\text{fiber}}}{r}. \]  

(9)

Setting equation (9) equal to the frictional drag force per unit volume of equation (6), the following relationship for the drag exchange coefficient between the fiber and matrix can be arrived at:

\[ \delta_{\text{fiber/matrix}} = \frac{2\mu(P_{\text{fiber}} - P_{\text{matrix}})}{\phi_{\text{matrix}}(v_{\text{fiber}} - v_{\text{matrix}})r}. \]  

(10)

3. Model Results and Demonstration

Demonstration of the modified multiphase model for composite materials is done via symmetric impact problems. The first configurations that were studied were simulating the symmetric impact problem assuming a composite fiber volume fraction (FVF) of 99%. This condition is used to demonstrate that at the limits of the mixture volume fraction the results approach the expected homogeneous symmetric impact response.

Figure 1(a) and 1(b) show a comparison of the pressure (top) and velocity (bottom) response for both the homogeneous fiber impact simulation and the 99% FVF multiphase simulation. It is noted that in this figure and others that follow the material boundary between the flyer plate and the stationary target plate is marked with a vertical black dashed line. Figure 1(a) and 1(b) show that the multiphase model as compared to the homogenous fiber impact response achieves excellent agreement. The first thing to note is that the multiphase pressure and velocity response for the mixture and the fiber constituent converge at the same value as expected based on the FVF. Second, the pressure in the fiber constituent for both simulations is approximately 45 Mdyn/cm² and the velocity for both simulations is 7.5 km/s as is expected in a symmetric impact problem where the impact velocity is 15 km/s.

Figure 1(c) and 1(d) show similarly good correlation for the inverse problem where the multiphase model is assumed nearly all resin or a FVF of 1%. Again, figures 1(c) and 1(d) show that the pressure and velocity for the multiphase mixture and matrix constituent are essentially converged. Finally, the pressure in the matrix constituent for both simulations is approximately 35 Mdyn/cm² and the velocity for both is again 7.5 km/s.

The second simulations performed for demonstration of the multiphase model for composite materials was to again perform symmetric impact problems. However, these simulations varied the coefficient of friction between the fiber and resin constituent to demonstrate the influence of this parameter. For these simulations the impact velocity was kept the same at 1.5 km/s, with a FVF of 50%.

Figure 2 shows the response of the multiphase composite drag model using three different drag coefficients: 1e-9, 0.015 and 0.3. Figure 2 shows that for each of these coefficients, the pressure response was unchanged. However, the velocity response is significantly different as expected. For the lower coefficient of friction (\( \mu = 1e-9 \)), the velocity responses of the fiber and matrix are different from the mixture velocity and remain at their difference throughout the shock propagation. As the coefficient of friction is increased, the momentum exchange is increased and the fiber and matrix begin to converge upon the mixture velocity. This behaviour is very evident in the higher coefficient of friction (\( \mu = 0.3 \)) simulation where the fiber and matrix response very rapidly converge on the mixture velocity.
Figure 1. Symmetric impact response of the multiphase model. (a) homogeneous fiber response; (b) 99% FVF multiphase response; (c) homogenous matrix response; (d) 1% FVF multiphase response.
The final simulations performed demonstrate the response of the multiphase composite model under varying relaxation time scales ($\tau$). This parameter comes from the compaction or volume fraction evolution relationship given by equation (11).

$$\dot{\phi}_i = \phi_{fiber} \phi_{matrix} \tau (P_{fiber} - P_{matrix})$$  \hspace{1cm} (11)

In equation (11) the relaxation time scale drives the time scale for the constituent’s pressure equilibrium (volume fraction evolution). A detailed discussion of this parameter can be found in [4]. Figure 3 shows the response of the multiphase composite model for decreasing relaxation time scales. As intended, the overall response of the multiphase composite model to this parameter shows that the pressure response of the composite constituents converge quicker on the mixture pressure as the relaxation time scale decreases. Figure 3 also shows that the inverse compaction coefficient somewhat influences the constituent velocity responses.
4. Conclusions
The work presented herein demonstrates a set of modifications to a multiphase mixture model that allows for the simulation of non-reactive structural composite materials. The model provides a simple momentum exchange relationship that is based on the coefficient of friction between the fibers and matrix. The drag model also allows for the drag coefficient to be defined as a tensor rather than a scalar to provide a directionally dependent momentum exchange that can capture the anisotropic shock response that is typically seen in these types of materials. The model also allows for control of the convergence of the pressure and velocity fields with the mixture response through the relaxation time scale and the coefficient of friction.

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