Development of a control system to maintain steady transition boiling

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Abstract. Steady transition boiling offers opportunities to observe fluid behavior and to measure transient and local heat flux as the surface dries and wets. This report discusses temperature control in transition boiling. Each component in the control system is either measured or estimated, and the controller parameters are determined along with the optimum depth of the temperature feedback point. Experiments are performed to verify the theoretical stability limit.

1. Introduction
In applications of boiling, the most important characteristic is the critical heat flux, because the heat-transfer surface may be damaged if the applied heat load exceeds the critical heat flux. To make a compact steam generator or cool a large heater wire, electric insulator, and the heater block. We measured the signal response of the temperature feedback point estimated by the controller parameters are determined along with the optimum depth of the temperature feedback point. Experiments are performed to verify the theoretical stability limit.

2. Temperature control system
To maintain steady transition boiling, a temperature control system is needed. The system’s stability is determined from the roots of the system’s characteristic equation \(1+G_0(s) = 0\), where \(G_0(s)\) is the open-loop transfer function. For optimal control system performance, the transfer functions of all the system elements should be specified, namely, a thermocouple, temperature controller, power supply, and an assembly of heater wire, electric insulator, and the heater block. We measured the signal responses of the thermocouple, controller, and power supply and specified that of the assembly analytically.

2.1. Thermocouple
Treat thermocouple as a first-order lag system and write its governing equation and transfer function as

\[
\frac{dT}{dt} = \frac{1}{\tau}(T_M - T) \quad \text{and} \quad G_{TC}(s) = \frac{1}{1+\tau s}, \quad \text{respectively.} \quad (1) \quad (2)
\]
Here, \( T \) is the temperature sensed by the thermocouple, \( T_M \) is the surface temperature of the hole in which the thermocouple is inserted (feedback point), and \( \tau \) is the time constant determined from the heat capacity of the thermocouple and the heat conductance between it and the feedback point. If \( T = T_i \) at \( t = 0 \) and \( T_M \) changes as Eq. (3) for \( t > 0 \) with constant \( k_H \), then \( T \) changes as Eq. (4):

\[
T_M = T_i + k_H t, \quad T = T_i + k_H (t - \tau) + k_H \tau e^{-t/\tau}. \tag{3} \tag{4}
\]

After the exponential term diminishes, \( T \) follows \( T_M \) exactly with delay \( \tau \).

We prepared a \( 15 \times 16.5 \times 3 \) mm copper test piece (Fig. 1) containing an insertion hole for the sheathed measurement thermocouple and two bare-wire thermocouples, the outer surfaces of which were coated with heat-resistant paint except at the silver-soldered (BAg-1A) tip \( \sim 0.5 \) mm from the surface. The delay due to conduction in copper is \( \rho c \Delta x^2 / (2 \lambda) \approx 5 \) ms, where \( \lambda / \rho c \) is the thermal diffusivity and \( \Delta x \) is the distance between the two types of thermocouples. The temperature response of a \( 0.5 \)-mm-diameter sheathed thermocouple is shown in Fig. 2, and the time constant is estimated to be \( \sim 75 \) ms.

\[\text{Figure 1. Test piece for measuring thermocouple response}\]

\[\text{Figure 2. Thermocouple response}\]

### 2.2. Temperature controller

We use an Omron E5AC temperature controller. Because its response is not in the public domain, we evaluated its transfer function experimentally. First, we measured the response for a square-wave input of \( 0.4995 \) Hz in proportional operation. This showed that the output is renewed every \( 51 \) ms and that the input circuit is an integral type with an integral period of \( \tau_{AD}=40 \) ms.

The transfer function is determined to be

\[
G_C(s) = G_I(\omega) \frac{e^{-L_c s}}{1 + \tau_C s} K_C \left[ 1 + \frac{1}{T_I s} + e^{-L_D s} T_D s \right] \quad \text{with} \quad G_I(\omega) = \frac{2}{\tau_{AD} \omega} \sin \left( \frac{\tau_{AD} \omega}{2} \right) \, , \tag{5} \tag{6}\]

with \( L_c=83 \) ms, \( \tau_C=34 \) ms from the frequency response at \( 0.1-10 \) Hz in proportional operation. Dead time \( L_D=25.5 \) ms results from the time differential having calculated as the changing rate between two successive measurements. Eq. (6) comes from the integral-type input, and \( K_C \) is measured in V/K.

### 2.3. Power supply and heater wire

From the measured frequency response at \( 0.1-10 \) Hz, we specify the power supply transfer function as

\[
G_{PS}(s) = \frac{e^{-L_{PS} s}}{1 + \tau_{PS} s} K_{PS} \, , \tag{7}\]

with \( L_{PS}=0.8 \) ms and \( \tau_{PS}=15.5 \) ms. Because heat generation in the heater wire is proportional to the output voltage squared, we inserted an analog square-rooting circuit between the controller and power supply, whereupon the Joule heat generated in the heater wire is proportional to the input. \( K_{PS} \) in Eq. (7) is the constant of the transfer function from controller output to heat flux at the heater wire (heat generation in the heater wire divided by the heater block area) in W/(m\(^2\)\(\text{V}\)).
2.4. Heater wire to local temperature at feedback point

The transfer function from the heat generated in the heater wire to the temperature of the copper block at feedback thermocouple ($Hs$ from the surface) was obtained by solving the 1-D heat conduction equation for the heater block, which comprises a flat 0.12-mm-thick 2.4-mm-wide nichrome wire, 50–65-µm-thick mica insulator on both sides of the wire, 0.85-mm-thick 2.8-mm-high copper comb teeth, and a 10-mm-thick copper block. We apply the following boundary conditions: (1) spatially uniform internal heat generation in the wire, (2) the center cross-sectional face of the wire is adiabatic, (3) temperature and heat flow are continuous at the boundaries of the neighboring components, and (4) heat transfer at the boiling surface is assumed as

$$-\lambda \frac{\partial T}{\partial z} = \Gamma (T - T_\infty),$$

where $\lambda$ is the thermal conductivity of the copper block, $\Gamma$ is the local slope of the boiling curve (negative in transition boiling), and $T_\infty$ is a constant that is higher than $T$ in transition boiling.

The coefficients of the general solution $e^{st} e^{\pm k z}$ with $k = \sqrt{\rho c / \lambda}$ for the unsteady heat conduction equation were determined to satisfy conditions (1)–(4); then we found transfer function $G_B(s)$ in m$^2$/K/W. Here, $\rho c$ is the volumetric heat capacity, and Table 1 shows the properties used.

| Table 1. Thickness, cross-sectional area, and properties [4] used in the analysis |
|-----------------------------------|
| Flat nichrome wire | Mica sheet | Copper block |
| Thickness, mm                      | 0.06$^a$ | 0.065 | 1.5$^a$ | 10.0 |
| Cross-sectional area relative to that of block | 4.36 | 4.36 | 0.773 | 1 |
| Volumetric heat capacity $\rho c$, MJ/(m$^3$K) | 3.85 | 1.76 | 3.54 |
| Thermal conductivity $\lambda$, W/(mK) | 13.2 | 0.137 | 393 |

$^a$ Half of actual thickness

2.5. Finding roots of characteristic equation and root locus

We used the Newton–Raphson method with complex numbers to find the roots of the characteristic equation $1 + G_0(s) = 0$, where $G_0(s) = G_{TC}(s)G_C(s)G_{PS}(s)G_B(s)$. Because the part $G_I(\omega)$ in $G_C(s)$ is real and a function of the real angular frequency $\omega$, roots are found for the product $G_I(\omega)K_CK_{PS}$, which is then divided by $G_I(\omega)$ for $K_CK_{PS}$ using $\omega$, the imaginary part of $s$.

Blum et al. [1] judged stability using the root locus—the locus of the roots on the complex plane when the controller gain is changed from zero to infinity. Roots in the 1st or 4th quadrant give unstable modes. The root loci obtained were similar to those obtained by Blum et al. The locus near the origin of the complex plane starts as two positive real numbers that merge and then separate into the 1st and 4th quadrants. When they enter the 2nd and 3rd quadrants, the gain at which the roots cross the imaginary axis is the lower limit of stable gain. Some loci go back to the 1st and 4th quadrants, while others become negative real numbers. The former gives the upper limit of stable gain. The other loci start on the far left of the complex plane and enter 1st or 4th quadrant. When the nearest locus does not return to the 1st or 4th quadrant, the second-nearest locus to the origin gives the upper limit of stable gain.

2.6. Stable range of proportional gain for given $\Gamma$

The ranges of stable gain obtained for various values of the boiling-curve slope $\Gamma$ and other parameters are plotted in Fig. 3. For small $H_S$ or $T_D$, the stable range of $K_CK_{PS}$ shrinks on both sides as $\Gamma$ increases (absolute $\Gamma$ increases). However, when either $H_S$ or $T_D$ is large, the upper limit of $K_CK_{PS}$ increases. Note that the stable upper limit is the minimum value of $K_CK_{PS}$, e.g., taken near $\Gamma = -40$ kW/(m$^2$K) for $H_S = 4.0$ mm and $T_D = 0.3$ s, because the system must be stable for all $\Gamma$ greater than the minimum.
3. Experiments

We made a heater block assembled from five parts (Fig. 4). Three parts were 10 mm thick excluding the comb teeth, and the other two were 12.8 mm thick. Each part’s temperature was controlled individually using the thermocouple installed at a depth of 4 mm. We set an integration time $T_I$ of 10 s and a time differential $T_D$ of 0.3 or 0.5 s for the thinner or thicker blocks, respectively.

The boiling test was performed at 1 atm using purified water. The set value of the controller was increased stepwise, and the temperatures and outputs of the power supplies were sampled every 1 s. Wall temperatures were calculated assuming a linear temperature profile, and the heat flux was from heat generation. Results for the central square part are plotted in Fig. 5, along with the fluctuation of heat generation at two points. A steady state was maintained up to a negative slope of $-60\text{ kW/}(\text{m}^2\text{K})$. The actual maximum negative slope seems much steeper than what the control system can stabilize.

4. Concluding remarks

Following control theory, a temperature feedback system was constructed and steady transition boiling was maintained for the negative slope of the boiling curve up to the designed value.

References

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