Regular Reissner-Nordström black hole solutions from linear electrodynamics

J. Ponce de Leon
Laboratory of Theoretical Physics, Department of Physics
University of Puerto Rico, P.O. Box 23343, San Juan,
PR 00931, USA
June 13, 2017

Abstract

In recent years there have appeared in the literature a large number of static, spherically symmetric metrics, which are regular at the origin, asymptotically flat, and have both an event and a Cauchy horizon for certain range of the parameters. They have been interpreted as regular black hole (BH) spacetimes, and their physical source attributed to electric or magnetic monopoles in a suitable chosen nonlinear electrodynamics. Here we show that these metrics can also be interpreted as exact solutions of the Einstein equations coupled to ordinary linear electromagnetism—i.e., as sources of the Reissner-Nordström (RN) spacetime—provided the components of the effective energy-momentum tensor satisfy the dominant energy condition (DEC). We use some well-known regular BH metrics to construct nonsingular RN black holes, where the singularity at the RN center is replaced by a regular perfect fluid charged sphere (whose charge-to-mass ratio is not greater than 1) which is inside the RN inner horizon.

PACS numbers: 04.50.+h, 04.20.Cv, 98.80. Es, 98.80 Jk
Keywords: Regular black holes in linear electrodynamics.

*E-mail: jpdel1@hotmail.com
1 Introduction

In general relativity spacetime singularities have been present from the beginning, starting with the first solutions of Einstein’s equations such as the Schwarzschild solution and the Friedmann solution. At first it was believed that these were mathematical artifacts induced by the requirement of spherical symmetry and the simplifying assumptions invoked to obtain the solutions. Today it is generally accepted—due to the famous singularity theorems (see, e.g., [1])—that spacetime singularities are an inevitable feature for most of the “physically reasonable” models of Universe and gravitational systems within the framework of the Einstein theory of gravity (for a review see, e.g., [2]).

However, the presence of singularities is usually regarded as indicating the breakdown of the classical theory, requiring modifications in the regions where the spacetime curvature becomes sufficiently high. The common opinion is that the problem of singularities could be solved by a consistent quantum theory of gravity. In the absence of such a theory, the issue of the resolution of singularities as produced by classical gravity remains open.

The validity of singularity theorems is established under the hypothesis that certain conditions, which can be roughly interpreted as causality and energy conditions, are met. If we leave aside the conditions that require causality, then the only possibility to avoid singularities is a violation of energy conditions. In this regard, one way to eliminate the singularity during gravitational collapse was proposed by Gliner [3]. He suggested that at very high densities, below some length scale, matter somehow makes a transition into a vacuumlike state, leading to the formation of a central core with de Sitter geometry [4]. This hypothetical transition avoids the conclusion of the Geroch-Hawking-Penrose theorems by violating the assumption that matter obeys the strong energy condition (SEC).

Two years after Gliner’s proposal, Bardeen [5] presented a metric for an asymptotically flat, static, spherically symmetric spacetime with a central de Sitter core, which—at that time—served to establish the conditions for the existence of singularities. Henceforth, that metric has been interpreted as describing a regular black hole (BH), since it is regular everywhere and has an inner and outer Killing horizon for certain values of the parameters. From then on a number of models for regular BHs have been proposed, which are exact solutions to the Einstein field equations with different physical sources: (a) anisotropic fluids [6–11]; (b) nonlinear electrodynamics [12–20]; (c) scalar fields [21–23]; and (d) modified gravity [24–26]. As expected, the discussion has been expanded to include rotating regular BHs [27–30].

After reviewing the literature, it is rather curious to note that ordinary linear electrodynamics is seldom considered for seeking regular BHs, with the notable exceptions of the models having a de Sitter interior which is joined to the exterior Reissner-Nordström (RN) field through a charged shell (see e.g., [31,32]), and the ones made out of charged phantom matter [33]. In this work we ask whether the BH metrics sourced by nonlinear electrodynamics can be used to construct regular RN black holes within the context of linear electrodynamics. We will see that the answer to this question is positive if the components of the effective energy-momentum tensor (EMT) satisfy the dominant energy condition (DEC). We use some well-known regular RN BH metrics to construct nonsingular RN black holes, where the singularity at the RN center is replaced by a regular perfect fluid charged sphere (whose charge-to-mass ratio is not greater than 1) which is inside the RN inner horizon.

The paper is organized as follows. In section 2 we (i) introduce the notation and present the relevant field equations, (ii) discuss the conditions under which a static, spherically symmetric solution of Einstein’s equations coupled to nonlinear electrodynamics can be interpreted as a perfect-fluid solution of Einstein’s equations coupled to ordinary linear electromagnetism, and (iii) develop the boundary conditions. In section 3 we consider the general class of nonsingular BHs recently considered by Fan and Wang [19] and construct the appropriate solutions to the ordinary Einstein-Maxwell (EM) equations, we find that for a range of parameters, such solutions represent BHs with the central singularity replaced by a charged perfect fluid sphere located inside the RN horizon. Similar results are obtained when we extend the discussion to include the celebrated Dymnikova’s vacuum nonsingular BH [6]. Finally, in section 4 we give a summary of the paper.

Throughout the paper, we use a number of acronyms, e.g., BH (black hole), EM (Einstein-Maxwell), RN (Reissner-Nordström), EMT (energy-momentum tensor), DEC (dominant energy condition), SEC (strong energy condition). However, we avoid the use of expressions like “the EMT satisfies the DEC but not the SEC,” which make the paper difficult to read.
2 Field equations

Throughout this work we use relativistic units where \( c = G = 1 \) and the sign conventions are those of Landau and Lifshitz [34]. For nonlinear electrodynamics in general relativity we consider the action

\[
S = -\frac{1}{16\pi} \int \sqrt{-g} \left[ R + L(F) \right] d^4x,
\]

where \( R \) is the scalar curvature, \( L \) is a function of \( F = F_{\alpha\beta} F^{\alpha\beta} \), and \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) is the electromagnetic field tensor. The Lagrangian density of the electromagnetic field is \( \Lambda = -\frac{1}{16\pi} L(F) \). Thus, the field equations for gravity are

\[
G_{\lambda\rho} = R_{\lambda\rho} - \frac{1}{2} g_{\lambda\rho} R = 8\pi T_{\lambda\rho},
\]

where \( T_{\lambda\rho} \) represents the EMT associated with the nonlinear electromagnetic field, which is

\[
T_{\lambda\rho} = \frac{1}{4\pi} \left( \frac{L}{4} g_{\lambda\rho} - L_\nu F_{\mu\lambda} F_{\nu\rho} g^{\mu\nu} \right), \quad L_\nu \equiv \frac{dL}{dF}.
\]

The tensor \( F^{\mu\nu} \) is governed by the equations

\[
(L_\nu F^{\mu\nu})_{;\mu} = 0, \quad F_{\mu\nu;\lambda} + F_{\nu\lambda;\mu} + F_{\lambda\mu;\nu} = 0.
\]

In the case where \( L = F \) as well as in the Maxwell weak-field limit, when \( L(F) \xrightarrow{F \to 0} F \), the above equations reduce to the familiar set of (linear) EM equations, as expected.

The most general static spherically symmetric metric can be written as

\[
ds^2 = f(u) dt^2 - h(u) du^2 - r^2(u) d\Omega^2,
\]

where \( d\Omega^2 = (d\theta^2 + \sin^2 \theta d\phi^2) \) and \( u \) is a radial coordinate. Due to the spherical symmetry, the EMT for an arbitrary kind of matter can be written as

\[
T^\mu_\mu = \text{diag} (\epsilon, -p_r, -p_\perp, -p_\perp),
\]

where the energy density \( \epsilon \), the radial pressure \( p_r \) and the transverse pressure \( p_\perp \) are functions of \( u \). A physically reasonable EMT must be free of singularities, have nonnegative energy density and satisfy the local conservation of stress-energy \( \nabla_\mu T^\mu_\nu = 0 \). In addition, ordinary/baryonic matter is expected to obey the energy conditions\(^2\).

By virtue of the spherical symmetry the only nonvanishing components of \( F_{\mu\nu} \) are \( F_{01} = -F_{10} \) and \( F_{23} = -F_{32} \). If we set

\[
F_{01} F^{01} = -E^2, \quad F_{23} F^{23} = B^2,
\]

where \( E \) and \( B \) only depend on the radial coordinate, then

\[
F = 2(B^2 - E^2),
\]

\[
T^0_0 = T^1_1 = \frac{1}{4\pi} \left( \frac{L}{4} + L_\nu E^2 \right),
\]

\[
T^2_2 = T^3_3 = \frac{1}{4\pi} \left( L - L_\nu B^2 \right).
\]

\(^2\)For the EMT \( T^\mu_\mu \) the dominant energy condition (DEC) requires \( \epsilon \geq |p_r|, \epsilon \geq |p_\perp| \); the weak energy condition (WEC) requires \( \epsilon > 0, \epsilon + p_r \geq 0, \epsilon + p_\perp \geq 0 \); the null energy condition (NEC) requires \( \epsilon + p_r \geq 0, \epsilon + p_\perp \geq 0 \); the strong energy condition (SEC) requires \( \epsilon + p_r + 2p_\perp \geq 0, \epsilon + p_r \geq 0, \epsilon + p_\perp \geq 0 \). These are not mutually independent; if the DEC is satisfied, then the weak and the null energy conditions are automatically satisfied as well. Also, the NEC is implied by the strong energy condition.
As a consequence of (10), the equation $G^0_0 = G^1_1$—evaluated for the line element (6)—can be easily integrated to obtain

$$h(u) = \frac{\text{constant}}{f(u)} \left( \frac{dr}{du} \right)^2. \quad (12)$$

Substituting this into (6), setting the constant of integration equal to 1, and using $r = r(u)$ as the new radial coordinate we arrive at the simplified line element

$$ds^2 = f(r) dt^2 - \frac{dr^2}{f(r)} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (13)$$

In these coordinates the nonvanishing components of the generic EMT (7) are

$$\epsilon = -p_r = \frac{1}{8 \pi r^2} [1 - (r f)'], \quad (14)$$
$$p_\perp = \frac{(r^2 f')'}{16 \pi r^2}, \quad (15)$$

where a prime denotes differentiation with respect to $r$. We note that

$$p_\perp = -\epsilon - \frac{r \epsilon'}{2}, \quad (16)$$

which implies that (i) $p_\perp \approx -\epsilon$ near the (regular) center, and (ii) $\epsilon' < 0 \Leftrightarrow (\epsilon + p_\perp) > 0$. Thus, when $\epsilon > 0$ and $\epsilon' < 0$ the weak and null energy conditions are automatically satisfied. The SEC is violated in the central region, because it stipulates that $p_\perp \geq 0$ and $\epsilon + p_\perp \geq 0$ when $p_r = -\epsilon$. The DEC, which now reduces to $\epsilon \geq |p_\perp|$, is not necessarily satisfied.

In terms of the mass function

$$m(r) = 4 \pi \int_0^r \bar{r}^2 \epsilon(\bar{r}) d\bar{r}, \quad (17)$$

the field equation (14) can be integrated as

$$f(r) = 1 - \frac{2m(r)}{r}, \quad (18)$$

where, to avoid a singularity at the origin, the constant of integration has been set equal to zero. Equivalently, the mass function can be written as

$$m(r) = \frac{r}{2} (1 - f). \quad (19)$$

However, the active gravitational mass inside a volume $V$ is given by the Tolman-Whittaker (TW) formula

$$M = \int (T^0_0 - T^1_1 - T^2_2 - T^3_3) \sqrt{-g} dV, \quad (20)$$

which in the case under consideration reduces to

$$M(r) = \frac{r^2 f'}{2}. \quad (21)$$

Clearly the TW mass calculated between the center and any point inside (outside) the first horizon (event horizon) is negative (positive). In the region where $f < 0$ the coordinate $r$ is timelike and $t$ is spacelike. Therefore, (20) in that region no longer has the direct physical meaning of active gravitational mass.
2.1 Charged perfect fluid interpretation

In general relativity the same geometry can be engendered by different material distributions. For example, under some circumstances the EMT of a generic anisotropic fluid can be represented as a multicomponent fluid. Recently, it has been shown that static spheres of anisotropic fluid can be represented as a linear combination of perfect fluid, electromagnetic field, and minimally coupled scalar field [36]. Typically the decomposition procedure is not unique because the number of independent functions in the multicomponent model is greater than the one in the anisotropic one-fluid model. Below we will see that the matter distribution supporting the line element (13) can be interpreted as a charged perfect fluid, with good physical properties, if the DEC \( \epsilon \geq |p_\perp| \) holds.

Thus, we set

\[
T_{\mu\nu} = \tau_{\mu\nu} + E_{\mu\nu},
\]

where \( \tau_{\mu\nu} \) and \( E_{\mu\nu} \) represent the EMT for perfect fluid and linear \((L = F)\) electromagnetism, respectively. Namely,

\[
\tau_{\mu\nu} = (\rho + p) u_\mu u_\nu - p g_{\mu\nu},
\]

\[
E_{\mu\nu} = \frac{1}{4\pi} \left( -F_{\mu\lambda} F_{\nu\sigma} g^{\lambda\sigma} + \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right).
\]

Here \( u^\mu, \rho \) and \( p \) are the four-velocity, energy density and isotropic pressure of the fluid, respectively. From \( 24 \) we find

\[
E^{\mu}_{\nu, \nu} = -F^\mu_{\nu} J^\nu,
\]

where the four-vector \( J^\mu \) is defined by the equation

\[
F^{\mu\nu}_{\rho, \nu} = \frac{1}{\sqrt{-g}} \frac{\partial(\sqrt{-g} F^{\mu\nu})}{\partial x^\rho} = -4\pi J^\rho.
\]

Since \( J^\mu_{\rho, \rho} = 0 \), it can be interpreted as an (effective) current density four-vector. Thus, \( 26 \) is equivalent to the second pair of Maxwell equations [34]. Consequently, the conservation equation \( T^{\mu\nu}_{\rho, \rho} = 0 \) can be written as

\[
\tau^{\mu\nu}_{\rho, \nu} = F^{\mu\nu} J^\nu.
\]

Finally, the proper electrical charge density \( \bar{\rho}_e \) is introduced through the relation

\[
J^\mu = \bar{\rho}_e u^\mu.
\]

Thus, by definition \( \bar{\rho}_e^2 = (J^\mu J^\mu) \).

Let us now go back to the case of a spherical distribution of matter. The EMT \( 22 \) in the comoving frame, where \( u^\mu = (\delta^\mu_0 / \sqrt{g_{00}}) \), reduces to

\[
T^\mu_\mu = \text{diag} (\rho + W, -p + W, -p - W, -p - W),
\]

with

\[
W = \frac{E^2 + B^2}{8\pi},
\]

which is the energy density of the electromagnetic field. Equating term by term the components of tensors \((7)\) and \((29)\) we obtain a system of three equations in three unknowns from which we get

\[
\rho = \epsilon - \frac{1}{2} (p_\perp - p_r),
\]

\[
p = \frac{1}{2} (p_r + p_\perp),
\]

\[
W = \frac{1}{2} (p_\perp - p_r).
\]
In the case under consideration \( p_r = -\epsilon \), as a result of \((7)\) and \((10)\). Therefore, the above equations reduce to

\[
\rho = -p = \frac{1}{2} (\epsilon - p_\perp), \quad W = \frac{1}{2} (\epsilon + p_\perp).
\]

(34)  

(35)

Now we can show that in the present interpretation there is no room for linear EM magnetic monopoles, even if the original spacetime \((13)\) is attributed to a magnetic monopole in a nonlinear electromagnetic theory. In fact, from \((8)\) we obtain \( F_{23} = \pm r^2 \sin \theta B(r) \). Substituting this into \((5)\) – with \( \mu = 1, \nu = 2, \lambda = 3 \) – we get

\[
B(r) = \frac{Q_m}{r^2},
\]

where \( Q_m \) is a constant of integration. Then, from \((30)\) and \((35)\) it follows that

\[
E^2 \to 0 - \to -\frac{Q_m^2}{r^4}
\]

for any spacetime satisfying regularity conditions at the origin. To avoid this unwanted consequence in what follows we set \( Q_m = 0 \). Thus,

\[
E^2 = 4 \pi (\epsilon + p_\perp).
\]

(36)

According to \((16)\), at the center \( \rho = \epsilon \) and \( E = 0 \). The DEC \( (\epsilon \geq |p_\perp|) \) ensures that both \( \rho \) and \( E^2 \) are nonnegative.

The first zero of the equation \( \epsilon = p_\perp \) (the closest to the center, say \( r = r_s \)) represents the boundary surface of the distribution since both the matter density \( \rho \) and pressure \( p \) vanish there. Then \((13)\) can be used to represent the interior of a charged perfect fluid sphere of coordinate radius \( r = r_s \). Otherwise, if \( \epsilon > |p_\perp| \) holds everywhere, the charged perfect fluid occupies the whole space.

In the static case, from \((26)\) we get

\[
E(r) = \frac{4 \pi}{r^2} \int_0^r \frac{\rho_e(\bar{r}) \, d\bar{r}}{\bar{r}^2} \equiv \frac{q(r)}{r^2},
\]

(37)

where \( E(r) = -\sqrt{-g_{00} g_{11}} F^{01} \) is the usual radial electric field intensity, \( q(r) \) is the electric charge inside a sphere of coordinate radius \( r \) and \( \rho_e \) is the charge density which is related to the proper charge density \( \bar{\rho}_e \) by

\[
\rho_e = \sqrt{-g_{11}} \bar{\rho}_e.
\]

(38)

Similarly, \((27)\) reduces to the Tolman-Oppenheimer-Volkoff (TOV) equation of hydrostatic equilibrium for a charged perfect fluid sphere, viz.,

\[
p' + (\rho + p) \left( \ln \sqrt{-g_{00}} \right)' = E \rho_e.
\]

(39)

For the models under consideration \( p = -\rho \). Therefore this equation reduces to \( p' = E \rho_e \). This means that the pressure gradient – which exerts a force towards the center – is only balanced by the electrostatic repulsion.

Finally, the components of the EMT \((29)\) in terms of the metric \((13)\) are given by

\[
\rho = -p = \frac{1}{16 \pi r^2} \left( 1 - f - 2 r f' - \frac{r^2 f''}{2} \right),
\]

(40)

\[
E^2 = \frac{1}{2 r^2} \left( 1 - f + \frac{r^2 f''}{2} \right).
\]

(41)

2.2 Boundary conditions

The solution of the EM equations for \( r > r_s \), outside the sphere, is given by the RN field which in curvature coordinates has the form

\[
ds^2 = f_{RN}(r) \, dt^2 - \frac{dr^2}{f_{RN}(r)} - r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right),
\]

(42)

3From \((8)\) it follows that \( E(r) = \pm \sqrt{-g_{00} g_{11}} F^{01} \), we choose the negative sign in order to get the familiar equations in electrodynamics.
with
\[ f_{RN}(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}. \] (43)

The radial electric field is
\[ E(r) = \frac{Q}{r^2}. \] (44)

In the above expressions \( M \) and \( Q \) are the total mass and charge, respectively, which are related to the parameters of the internal solution through the boundary conditions. Later we will need the expressions for the mass function and the TW mass in the external region—say \( m_{RN}(r) \) and \( M_{RN}(r) \), respectively. They are given by
\[ m_{RN}(r) = M - \frac{Q^2}{2r}, \] (45)
\[ M_{RN}(r) = M - \frac{Q^2}{r}. \] (46)

These equations show that both \( m_{RN}(r) \) and \( M_{RN}(r) \) become negative when \( r \) is sufficiently small.

To match the internal and external solutions across the boundary \( r = r_s \), we require continuity of the first and second fundamental forms. For this, we need continuity of \( f(r) \) and \( T^1_1(r) \), respectively. Since \( p(r_s) = 0 \), in the absence of surface concentration of charge the latter condition demands continuity of \( E(r) \), which in turn—by virtue of (40) and (41)—requires continuity of \( f' \). In summary, at the boundary we require
\[ f_s = f(r_s) = f_{RN}(r_s), \quad f'_s = f'(r_s) = f'_{RN}(r_s). \] (47)

Using (43), these expressions constitute a system of two algebraic equations which allow us to obtain \( M \) and \( Q \) in terms of \( f_s \) and \( f'_s \). The solution is
\[ M = r_s (1 - f_s - \frac{r_s f'_s}{2}), \] (48)
\[ Q^2 = r_s^2 (1 - f_s - r_s f'_s). \] (49)

These expressions guarantee the continuity of the mass function [Eqs. (19) and (45)] and TW mass [Eqs. (21) and (46)] across the surface of the sphere.

Restrictions on \( r_s \): As a consequence of the DEC the possible values of \( r_s \) are bounded below by
\[ r_{min} = \frac{2Q^2}{3M}. \] (50)

In fact, from (19) it follows that \( \epsilon' \leq 0 \) if \( \epsilon \geq -p_\perp \), which means \( \epsilon(r) \geq \epsilon(r_s) \). Combining this with \( f_s = f_{RN}(r_s) \) we obtain
\[ \frac{8\pi}{r_s} \int_0^{r_s} r^2 \left( \rho + \frac{E^2}{8\pi} \right) dr = \frac{2M}{r_s} - \frac{Q^2}{r_s^2} \geq \frac{Q^2}{3r_s^2}. \]

Solving the inequality for \( r_s \) we obtain \( r_s \geq r_{min} \), given by (50). This guarantees that \( m_{RN}(r) \) is positive for all values of \( r_s \), while \( M_{RN}(r) \) is negative inside and in the vicinity of the sphere when \( r_{min} \leq r_s < 3r_{min}/2 \).

In addition, the charged spheres can be inside the RN horizon if
\[ \alpha \equiv \frac{|Q|}{M} \in \left( \frac{\sqrt{3}}{2}, 1 \right] \approx (0.866, 1]. \] (51)

Indeed, when \( |Q| = M \) the Killing horizon is located at \( r = r_s = M \), so that \( r_{min} = \frac{2}{3} r_s \). Similarly, when \( |Q| < M \) the inner horizon is located at \( r = r_- = M \left[ 1 - \sqrt{1 -(Q/M)^2} \right] \). Consequently, \( r_{min} < r_- \) when (51) is satisfied.
3 Regular RN black holes

In this section, we apply the above equations to some well-known regular BH spacetimes and show that they can be interpreted as exact solutions to the EM equations describing spherical distributions of charged perfect fluid. The coordinate radius of the spheres is determined by the equation \( \epsilon = p_\perp \). They have several interesting properties:

(i) the charge-to-mass ratio \( \alpha = |Q|/M \) is bounded below; (ii) the spheres having \( \alpha \leq 1 \) are hidden behind the RN horizons and are gravitationally repulsive inside as well as in their vicinity, although the repulsive region is covered by the event horizon; (iii) when \( \alpha > 1 \) the vicinity of the spheres can be gravitationally attractive or repulsive, depending on the model; (iv) the energy density at the (regular) center is proportional to \( \alpha^{-6} M^{-2} \), for an object one solar mass it is about \( 10^{20} \text{ kg/m}^3 \)—although an increase (decrease) in \( M \) leads to a decrease (increase) of this quantity.

3.1 General class of nonsingular RN black holes

First we consider the line element (13) generated by the metric function

\[
f(r) = 1 - \frac{2 m r^{\sigma-1}}{(r^\beta + K)^{\sigma/\beta}},
\]

where \( \sigma > 1, \beta > 0, K \geq 0 \) and \( m \geq 0 \) are constants. It reduces to the Schwarzschild vacuum solution with mass \( m \) for \( K = 0 \). This line element has recently been discussed in the context of nonlinear electrodynamics by Fang and Wang [19]. It includes the Bardeen solution [5] for \( \sigma = 3 \) and \( \beta = 2 \), the Hayward solution [9] for \( \sigma = \beta = 3 \), as well as a new class of solutions considered by Fan and Wang for \( \beta = 1 \). For certain values of the parameters these can be interpreted as asymptotically flat BHs.

Our aim is to show that (52) can be used to represent perfect fluid charged spheres in ordinary EM theory. To begin with we use (14) and (15) to evaluate the effective EMT. We obtain

\[
\epsilon = -p_r = \frac{\sigma m K r^{\sigma-3}}{4 \pi (r^\beta + K)^{(\sigma+\beta)/\beta}},
\]

\[
p_\perp = \frac{\epsilon [(1 - \sigma) K + (1 + \beta) r^\beta]}{2 (r^\beta + K)}.
\]

The parameter \( \sigma \) is specified by the equation of state \( (p_\perp/\epsilon) \) near the origin as

\[
\sigma = 1 - 2 \left. \frac{p_\perp}{\epsilon} \right|_{r=0}.
\]

If \( (p_\perp/\epsilon)|_{r=0} \in (0, -1] \), then \( \sigma \in (1, 3] \). It determines the behavior of the energy density: when \( \sigma \leq 3 \), \( \epsilon \) is positive everywhere and monotonically decreases outward; when \( \sigma > 3 \), \( \epsilon \) increases from zero at the origin up to a maximum value and then decreases to zero for large values of \( r \). When \( \sigma = 3 \) the central region is de Sitter-like with \( \epsilon = -p_r \approx -p_\perp = \frac{3 m}{4 \pi} K^{-3/\beta} \).

The DEC is satisfied in different regions depending on the choice of \( \beta \). Namely,

4 A horizon corresponds to a zero of \( f(r) \); the outermost zero is the event horizon of the BH. The function [22] has a minimum at \( r = r_{\text{min}} = [K (\sigma - 1)]^{1/\beta} \), viz.,

\[
f_{\text{min}} = f(r_{\text{min}}) = 1 - \left( \frac{K_{\text{crit}}}{K} \right)^{1/\beta},
\]

where

\[
K_{\text{crit}} = \frac{(2 m)^\beta}{(\sigma - 1)^\beta} \left( \frac{\sigma - 1}{\sigma} \right)^\sigma.
\]

Thus, [22] has no zeros if \( K > K_{\text{crit}} \), one double zero at \( r = r_* = 2 m \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma/\beta} \) if \( K = K_{\text{crit}} \), and two simple zeros if \( K < K_{\text{crit}} \).
\[ \beta \leq 1 : r^\beta \geq \frac{K(\sigma - 3)}{3 + \beta}, \]  
\[ \beta > 1 : \frac{K(\sigma - 3)}{3 + \beta} \leq r^\beta \leq \frac{K(1 + \sigma)}{\beta - 1}. \]

Thus, \( \epsilon \geq |p_\perp| \) everywhere only if \( \sigma \leq 3 \) and \( \beta \leq 1 \). Otherwise it only holds in the region \((57)\).

Consequently, the matter distribution supporting the metric \((13)\) with the function \(f(r)\) given by \((52)\) can be interpreted as a charged perfect fluid when \(\sigma \leq 3\), which includes the origin for any \(\beta > 0\). The corresponding energy density, isotropic pressure, and electric field are

\[ \rho = -p = \frac{\epsilon [K(\sigma + 1) - (\beta - 1) r^\beta]}{4(r^\beta + K)}, \]  
\[ E^2 = \frac{2 \pi \epsilon [(3 - \sigma) K + (3 + \beta) r^\beta]}{(r^\beta + K)}. \]

**Boundless charged configuration:** When \(0 < \beta \leq 1\) and \(1 < \sigma \leq 3\) the DEC \((50)\) is satisfied everywhere and the pressure \((58)\) never vanishes. Therefore, \((58)-(59)\) can be interpreted as an unbounded spherical distribution of charged perfect fluid. However, the total charge \(|Q| = [r^2 E(r)] \to \infty\) is finite—and equal to \(\sqrt{2K\sigma m}\)—only when \(\beta = 1\). As expected, this is consistent with the limiting expression

\[ f(\infty) \to 1 - \frac{2m}{r} + \frac{2K\sigma m}{r^2} - O\left(\frac{1}{r^3}\right), \]

which also shows that \(m\) is the total mass of the configuration. We arrive at the same conclusion by using the TW mass, viz., \(M(r) \to M = m\).

For this case, the metric function \((52)\) can be parametrized by mass and charge as

\[ f(u) = 1 - \frac{2}{u} \left(1 + \frac{u^2}{2\sigma u}\right)^{-\sigma}, \]

where \(u = \frac{r}{M}\), and \(0 \leq u < \infty\). The condition \(f(u) > 0\) imposes a lower limit on \(\alpha\), say \(\alpha_{\text{min}},\) viz.,

\[ \alpha > \alpha_{\text{min}} = 2 \left(\frac{\sigma - 1}{\sigma}\right)^{(\sigma-1)/2}. \]

By virtue of \((53)\), \(\alpha_{\text{min}}\) is determined by the equation of state at the center. Note that \(\alpha_{\text{min}}\) is greater than 1 for all values of \(\sigma \in (1, 3]\).

When \(\sigma = 3\), the density at the center is completely determined by \(Q\) and \(M\). Indeed, for the case under consideration, when \(\beta = 1\) (which implies \(K = \frac{Q^2}{6M}\)) from \((53)\) and \((58)\), we get

\[ \rho_c = \frac{162}{\alpha^6 \pi M^2}, \]

where \(\rho_c\) is the energy density at the center. In general, \(\rho_c M^2\) is bounded above as \(\alpha\) is bounded below. For \(\sigma = 3\), we find \(\alpha_{\text{min}} = 4/3\) and thus \(\rho_c M^2 < 9.178\), approximately. If we apply this inequality to an object of one solar mass \((M = M_\odot)\) we find \(\rho_c < \rho_{\text{max}} \approx 5.683 \times 10^{21} \text{kg/m}^3\).

\[ \text{For } 0 < \beta < 1 \text{ the total electric charge diverges because } [r^2 E^2] \to \infty \text{ as } r \to 1 - \beta. \]
Charged perfect fluid spheres: When \( \beta > 1 \) and \( 1 < \sigma \leq 3 \) the DEC (57) is met from the origin up to a maximum value of \( r \) which determines the boundary of the sphere, where the density and pressure vanish.

Thus, from (63) \( K \) can be expressed in terms of \( r_s \), the coordinate radius of the sphere, as

\[
K = \frac{(\beta - 1) r^2}{1 + \sigma}.
\]

(63)

Using the boundary conditions (48) and (49) at \( r = r_s \) we find

\[
r_s = \frac{M (1 + \sigma) \beta a^2}{2 \sigma (\beta - 1)}, \quad m = \frac{M}{\beta} \left( \frac{\sigma + \beta}{1 + \sigma} \right)^{(\sigma + \beta)/\beta}.
\]

(64)

(65)

Note that \( m \) is no longer the total mass of the charged configuration.

Taking these results into account, the metric function (52) can be written as

\[
f(x) = 1 - \frac{4 \sigma (\beta - 1) \left( \sigma + \beta \right)^{(\sigma + \beta)/\beta} x^{\sigma - 1}}{\alpha^2 \beta (1 + \sigma)^2 \left[ (1 + \sigma) x^\beta + \beta - 1 \right]^{\sigma/\beta}}, \quad x = \frac{r}{r_s}.
\]

(66)

This function must be positive in the whole range \( x \in [0, 1] \). Leaving aside the details of the analysis, we find that this condition imposes a lower bound on \( \alpha \), viz.,

\[
\alpha > \tilde{\alpha}_{\text{min}} = \frac{2\sqrt{\sigma}}{1 + \sigma}.
\]

(67)

Otherwise, when \( \alpha \leq \tilde{\alpha}_{\text{min}} \) it cannot be satisfied by any \( \beta \). Note that unlike the unbounded case (61), now \( \tilde{\alpha}_{\text{min}} \) is less than 1 for all \( \sigma \in (1, 3) \). We also find that \( f(x) > 0 \) is satisfied by any \( \beta > 1 \) and \( \sigma \in (1, 3) \) if

\[
\alpha \geq \alpha_* = 2 \left( \frac{\sigma - 1}{\sigma} \right)^{(\sigma - 1)/2}.
\]

(68)

Consequently, the condition \( f(x) > 0 \) only restricts the values of \( \beta \) when \( \tilde{\alpha}_{\text{min}} < \alpha < \alpha_* \), e.g., \( \sqrt{3}/2 < \alpha < 4/3 \) for \( \sigma = 3 \). Parenthetically, we point out that \( \alpha_* \) is equal to \( \alpha_{\text{min}} \) introduced in (61), although they refer to different physical scenarios. Obviously, \( \alpha_* > \tilde{\alpha}_{\text{min}} \) for \( \sigma \in (1, 3) \).

In the range \( \alpha \in (1, \alpha_*) \)—e.g., \( \alpha \in (1, 4/3) \) for \( \sigma = 3 \)—the condition \( f(x) > 0 \) leads to a cumbersome transcendental inequality between \( \alpha, \sigma \), and \( \beta \), namely,

\[
1 < \alpha < \alpha_* : \quad \alpha^2 - 4 \beta^{-2} \left[ \sigma^{(\beta - \sigma)} (\beta - 1)^{(\beta - 1)(\sigma + \beta)} (\sigma - 1)^{\sigma - 1} (1 + \sigma)^{-(\sigma - 1 + 2\beta)} \right]^{1/\beta} > 0.
\]

(69)

However, the range of interest of \( \alpha \) is \( \alpha \in (\tilde{\alpha}_{\text{min}}, 1] \), which allows the presence of RN black holes. In that range we find that \( f(x) > 0 \) only if

\[
\tilde{\alpha}_{\text{min}} < \alpha \leq 1 : \quad \beta > \frac{2 \sigma [(\sigma - 1) + (\sigma + 1) \sqrt{1 - \alpha^2}]}{\alpha^2 (1 + \sigma)^2 - 4 \sigma},
\]

(70)

which is one of the solutions to the inequality \( f(1) > 0 \). Note that the denominator here is positive by virtue of (67). The last inequality has several interesting consequences:

- **Static spheres hidden behind RN horizons:** From (70) it follows that the charged spheres with \( \alpha \leq 1 \) are located inside the RN horizons. To illustrate this we consider the cases \( \alpha = 1 \) and \( \alpha < 1 \) separately.

(i) If \( \alpha = 1 \), then from (70) we obtain \( \beta > \frac{2 \sigma}{\alpha} \). It is easy to verify that this also follows from the inequality \( r_s < r_* = M \), where \( r_s \) is given by (64). Thus, the spheres with \( \alpha = 1 \) are inside the Killing horizon \( r = r_* = M \). In fact, from (64)—with \( \alpha = 1 \) and \( \frac{2 \sigma}{\alpha} < \beta < \infty \)—we get
\[
\frac{1 + \sigma}{2 \sigma} < \left( \frac{r_s}{r_*} \right) < 1
\]  
(71)

(ii) If \( \alpha < 1 \), then (70) is also the solution to the inequality
\[
r_s < r_- = M \left( 1 - \sqrt{1 - \alpha^2} \right),
\]  
(72)

where \( r_s \) is given by (63), provided (67) holds. Therefore, all configurations with \( \frac{2 \sqrt{\pi}}{1 + \sigma} < \alpha < 1 \) are inside the inner horizon. In fact, from (64) we get
\[
\frac{(1 + \sigma) \left( 1 + \sqrt{1 - \alpha^2} \right)}{2 \sigma} < \left( \frac{r_s}{r_-} \right) < 1,
\]  
(73)

This inequality for \( \alpha = 1 \) reduces to (71), as expected. For a fixed \( \sigma \), it narrows as \( \alpha \) approaches its minimum (67) in such a way that \( r_-(r_s) \to 0^+ \), with \( r_- \to \frac{2M}{\sigma + 1} \), when \( \alpha \to \tilde{\alpha}_{\text{min}} \) (in this limit \( r_+ \to \sigma r_- \)).

- **Central density:** When \( \sigma = 3 \) the central density is finite; using (63), (64) and (65) it can be expressed as
\[
\rho_c = \frac{81 (3 + \beta) (3 + \beta)/\beta (\beta - 1) ^ {3(\beta - 1)/\beta}}{128 \pi \alpha^6 \beta^4 M^2},
\]  
(74)

which reduces to (62) for \( \beta = 1 \). For any given \( M \) and \( Q \) this expression gives the range of values for \( \rho_c \) allowed by \( \beta \), which for \( \alpha \in (\tilde{\alpha}_{\text{min}}, 1) \) is determined by
\[
\tilde{\beta} < \beta < \infty, \quad \tilde{\beta} = \frac{3 \left( 1 + 2 \sqrt{1 - \alpha^2} \right)}{4 \alpha^2 - 3}.
\]  
(75)

From the last two expressions, we get
\[
1 < \frac{128 \pi \rho_c M^2 \alpha^6}{81} < \frac{1}{\beta^4} \left[ \left( 3 + \tilde{\beta} \right) (3 + \tilde{\beta}) \left( \beta - 1 \right) ^ {3(\beta - 1)/\beta} \right] ^ {1/\beta}.
\]  
(76)

To obtain an order of magnitude for \( \rho_c \) we take \( M = M_0 \). Then, from (70), we get
\[
\begin{align*}
\alpha &= 1.00 : \quad \rho_c \in (1.25, 2.22) \times 10^{20} \text{ kg/m}^3, \\
\alpha &= 0.99 : \quad \rho_c \in (1.32, 1.80) \times 10^{20} \text{ kg/m}^3, \\
\alpha &= 0.90 : \quad \rho_c \in (2.34, 2.46) \times 10^{20} \text{ kg/m}^3, \\
\alpha &= 0.87 : \quad \rho_c \in (2.88, 2.88) \times 10^{20} \text{ kg/m}^3.
\end{align*}
\]

Thus, the central density is of the order of \( 10^{20} \text{ kg/m}^3 \) in the whole range of \( \alpha \). We recall that \( \sigma = 3 \) requires \( \tilde{\alpha}_{\text{min}} = \sqrt{3}/2 \) so that \( \alpha \in (\sqrt{3}/2, 1) \), which incidentally saturates the inequality (51) obtained on general grounds.

- **Tolman-Whittaker mass:** In the present case the TW mass (21) inside the spheres can be expressed as
\[
M(x) = \frac{M x^{\sigma} \left[ (1 + \sigma) x^{\beta} - (\sigma - 1) (\beta - 1) \right]}{\beta (1 + \sigma)} \frac{\sigma + \beta}{\beta + 1} = \frac{M x^{\sigma} (\sigma - 1)}{\beta + 1} \left( \frac{\alpha + \beta}{\sigma + \beta} \right)^{\alpha + \beta}/\beta.
\]  
(77)

Evaluating this at the boundary surface \( (x = 1) \) we get
\[
M(r_s) = -\frac{M (\sigma - 1)}{\beta (1 + \sigma)} \left( \beta - \frac{2 \sigma}{\sigma - 1} \right).
\]  
(78)

As expected, a sphere with \( \alpha \leq 1 \) (for which \( \beta > \frac{2 \sigma}{\sigma - 1} \)) is gravitationally repulsive not only in its interior but also and its vicinity, although it is covered by an horizon.

If \( \alpha > 1 \), then the vicinity of the spheres can be gravitationally attractive or repulsive, depending—respectively—on whether \( \beta \) is less or greater than \( 2 \sigma/(\sigma - 1) \). This can be detected by an external observer since there are no horizons.
3.2 Dymnikova’s nonsingular black hole

Next, we consider Dymnikova’s spacetime [6], which—in our notation—is generated by the function

\[ f(r) = 1 - \frac{a b}{r} \left(1 - e^{-r^3/a^3}\right), \quad (79) \]

where \( a \) and \( b \) are positive constants; \( a \) has dimensions of length and \( b \) is dimensionless. This line element has widely been discussed in the literature with about 272 citations (adsabs.harvard.edu/abs/1992GReGr..24..235D); it is de Sitter-like near the center and resembles the Schwarzschild metric with total mass \( (a b)/2 \) at large distances \( (r \gg a) \). The presence of horizons is governed by the parameter \( b \). Namely, (79) has no zeros if \( b < b_{\text{crit}} \approx 1.456 \), one double zero at \( r \approx 1.235 a \) if \( b = b_{\text{crit}} \), and two simple zeros if \( b > b_{\text{crit}} \). It can be interpreted as an exact solution of the Einstein equations coupled to nonlinear electrodynamics describing a magnetic monopole.\(^6\)

Below we will see that Dymnikova’s spacetime can be used to describe the interior of regular charged spheres, whose properties are essentially the same as those considered in the preceding subsection.

The effective EMT supporting (79) is

\[
\begin{align*}
\epsilon &= -p_r = \frac{3b}{8 \pi a^2} e^{-r^3/a^3}, \\
p_{\perp} &= \epsilon \left(-1 + \frac{3}{2 a^3} r^3\right).
\end{align*}
\quad (80)\]

(81)

We note that the DEC \( (\epsilon \geq |p_{\perp}|) \) is only satisfied in the interior region where \( (r^3/a^3) \leq (4/3)^{1/3} a \). If we restrict the use of (79) to that region, then the matter content that generates (79) can be interpreted as a charged perfect fluid, whose energy density \( \rho \), pressure \( p \) and electric field intensity \( E \) are given by

\[
\begin{align*}
\rho(r) &= -p(r) = \frac{3b}{32 \pi a^2} \left(4 - \frac{3}{a^3} r^3\right), \\
E^2(r) &= \frac{9b r^3}{4 a^5} e^{-r^3/a^3},
\end{align*}
\quad (82)\]

which have been obtained by substituting (80)–(81) into (34) and (36).

The energy density \( \rho(r) \) is positive and drops continuously from its maximum value at the center to zero at the surface \( r_s = (\frac{3}{4})^{1/3} a \). Therefore, \( r_s \) represents the outer boundary of the charged fluid sphere, i.e.,

\[ a = \left(\frac{3}{4}\right)^{1/3} r_s. \quad (84) \]

The boundary conditions (48) and (49) provide two equations from which we get \( b \) and \( r_s \),

\[
\begin{align*}
b &= \frac{16 \times 6^{2/3} \times e^{4/3}}{3 a^2 (3 + e^{4/3})^2}, \\
r_s &= \frac{M a^2}{8} (3 + e^{4/3}).
\end{align*}
\quad (85)\]

(86)

In this parametrization the original function (79) becomes

\^6\ As a matter of fact, from [19] it follows that the Einstein field equations \( (2) \) evaluated for the metric \( (13) \) are automatically satisfied for any function \( f(r) \) if the EMT given by \( (9), (10) \) and \( (11) \) describes a magnetic dipole, i.e., when \( E(r) = 0 \) and \( B(r) = Q_m/r^2 \), where \( Q_m \) is a constant coming from the integration of the Bianchi identities \( (5) \). In which case the function \( L(F) \), where \( F = 2 B^2 \), is given by \( L = 2 G_0^0 \) with \( r = \left(\frac{2 Q_m^2}{a^4}\right)^{1/4} \). For Dymnikova’s spacetime (79) one can easily find \( L(F) = \frac{\alpha b}{\pi} \) \( \exp \left[\left(-\frac{2 Q_m^2}{\alpha^2 + 1}\right)^{3/4}\right]\).
\[ f(x) = 1 - \frac{16 e^{4/3}}{\alpha^2 x (3 + e^{4/3})^2} \left( 1 - e^{-4x^3/3} \right), \quad x = \frac{r}{r_s}, \quad 0 \leq x \leq 1. \]  

(87)

As in the previous case, the charge-to-mass parameter \( \alpha \) is bounded below by the condition \( f(x) > 0 \). Since \( (87) \) is a decreasing function of \( x \), this condition is fulfilled if \( f(1) > 0 \), which in turn requires

\[ \alpha > \bar{\alpha}_{\text{min}} = \frac{4 \sqrt{\frac{4}{3}} - 1}{(3 + e^{4/3})} \approx 0.98. \]

(88)

It is easy to verify that this inequality is the solution to (72) with \( r_s \) given by (86).

- Consequently, when \( \bar{\alpha}_{\text{min}} < \alpha \leq 1 \) the charged spheres are located inside the horizon. In fact, when \( \alpha = 1 \) from (86) we obtain

\[ \left( \frac{r_s}{r} \right) = \frac{3 + e^{4/3}}{8} \approx 0.85. \]

Similarly, when \( \bar{\alpha}_{\text{min}} < \alpha < 1 \) we get

\[ \left( \frac{r_s}{r} \right) = \frac{3 + e^{4/3}}{8} (1 + \sqrt{1 - \alpha^2}) \approx 0.85 \left( 1 + \sqrt{1 - \alpha^2} \right), \]

which is approximately \( 0.85 < \left( \frac{r_s}{r} \right) < 1 \) for \( \alpha \in (\bar{\alpha}_{\text{min}}, 1) \).

- Regarding the central density \( \rho_c \); from (82), (84), (85) and (86) we find

\[ \rho_c = \frac{2^{9/2} e^{4/3}}{\pi (3 + e^{4/3})^4 \alpha^6 M^2}. \]

(89)

Thus, for \( \alpha \in (\bar{\alpha}_{\text{min}}, 1) \) we get \( 0.29 < \rho_c M^2 < 0.32 \), approximately. Consequently, for a body of \( M = M_\odot \) the central density is about \( \rho_c = (1.79-1.98) \times 10^{20} \text{ kg/m}^3 \).

- The TW mass inside the spheres is given by

\[ M(x) = \frac{M}{1 + 3e^{-4x^3/3}} \left[ 1 - (1 + 4x^3) e^{(-4x^3/3)} \right], \]

(90)

where we have used (21), (86), and (87). On the other hand, from (46) and (86) it follows that \( M_{R_N}(r) \) is negative for \( r < \left( \frac{r_s}{3x e^{4/3}} \right) \approx 1.18 r_s \). Consequently, unlike the previous case, all these charged spheres possess negative TW mass not only inside but also in their neighborhood, regardless of the choice of \( \alpha \). When \( \alpha < 1 \) the region of negative TW mass is hidden behind the horizon, but not when \( \alpha > 1 \) because there are no horizons.

### 4 Final comments and summary

The popular belief is that at the core of a BH there is a singularity covered by an event horizon. This notion is a cumulative result of various far-reaching developments, among which the more influential are (a) the understanding that the gravitational collapse of a homogeneous spherical dust cloud—as it evolves in time—leads to the formation of a singularity covered by an event horizon [37]; (b) the singularity theorems of Geroch, Hawking, and Penrose [1] which—in the framework of general relativity and the assumption that certain general conditions hold—prove that a sufficiently massive collapsing object will undergo continual gravitational collapse, resulting in the formation of a gravitational singularity; (c) the (weak) cosmic censorship hypothesis which asserts that these singularities are always hidden inside a BH (see e.g., [38] and references therein).

However, by definition a BH is a region of an asymptotically flat spacetime from which it is impossible to send signals to infinity. Therefore, its characterizing feature is the appearance of an event horizon, not the presence (or absence) of a spacetime singularity. As mentioned in the Introduction, there are several solutions of Einstein’s field
equations—with different sources—which have horizons but no spacetime singularities. Most of them are represented by static, spherically symmetric metrics of the form (13) which are (i) regular everywhere, (ii) asymptotically flat, and (iii) \( f(r) \) possesses a minimum, say \( f_{\text{min}} \), such that, for \( r > 0 \), it has no zeros if \( f_{\text{min}} > 0 \), two simple zeros if \( f_{\text{min}} < 0 \) and one double zero if \( f_{\text{min}} = 0 \). When such spacetimes are used in the whole range \( 0 \leq r < \infty \) these three cases describe, respectively, a regular spacetime, a regular non-extreme BH with both outer and inner Killing horizons, and a regular extreme BH with degenerate Killing horizon.

If the Einstein equations (2) are used to evaluate the effective EMT, these regular BHs are supported by anisotropic matter with finite energy density and pressures which have a de Sitter-like behavior in the central region, as envisaged by Gliner [3]. Therefore, in that region the components of the EMT obey the dominant but not the strong energy condition. Some of these spacetimes have been interpreted in terms of electric or magnetic monopoles in a suitable chosen nonlinear electrodynamics.

In this work we have shown that these regular BH metrics can also be interpreted as exact solutions of the Einstein equations coupled to ordinary linear electromagnetism—i.e., as sources of the RN spacetime. We have constructed regular RN black holes, where the central singularity is replaced by a regular perfect fluid charged sphere which, for the case where \( |Q| < M \) (\( Q = M \)), is located inside the inner (Killing) horizon. The coordinate radius of the sphere is determined by the equation \( \epsilon = p_{\perp} \) and is expressed in terms of \( M, Q \) and the parameters of the solutions. It is important to emphasize that the condition \( f > 0 \) is fulfilled if and only if the spheres are inside the inner horizon (Killing horizon when \( |Q| = M \)). This condition also imposes a lower bound on the charge-to-mass parameter \( \alpha \). In Newtonian terms, it provides the electrostatic repulsive force, required by the TOV equation (39), to balance the inward hydrostatic force produced by the gradient of the negative pressure; if \( \alpha \) were less than the required minimum, then \( f \) would be negative in the outermost layers and the perfect fluid sphere could not be in static equilibrium. Regarding the central mass density, we have seen that it is practically insensitive to the concrete value of \( \alpha \); for an object of one solar mass it is about \( 10^{20} \text{kg/m}^3 \), which is thousands times greater than the approximate density of an atomic nucleus. However, an increase (decrease) of \( M \) leads to a decrease (increase) of this quantity.

It should be mentioned that in the present interpretation there is no room for linear (Einstein-Maxwell) magnetic monopoles, even if the original spacetime (13) is attributed to a magnetic monopole in a nonlinear electromagnetic theory. Also, the lower limit on \( \alpha \) heavily depends on the model. For example, the general BH metrics [42]—as well as the thin-shell models [31,32]—saturate the requirement [51]; meanwhile for the Dymnikova’s spacetime it is considerably more restricted [55], although it can still be less than 1, allowing the existence of static charged spheres inside the RN horizons. For the Beato and Garcia solution [13] (not discussed here), \( \alpha > 1 \).

The inner horizon in the standard RN solution is unstable under “small” external perturbations [39]. The full nonlinear nature of this instability—dubbed “mass-inflation”—was discovered by Poisson and Israel [40] and has been confirmed in a number of scenarios (see, e.g., [41–43]). On the other hand, Dymnikova and Galaktionov [44] demonstrated that any configuration described by a spherically symmetric geometry with a de Sitter center is stable to axial perturbations, and—in the case of the polar perturbations—they found the criteria for stability of Dymnikova’s nonsingular black hole [6]. At first glance, both results do not appear to be mutually consistent. Therefore, given that the predicted charged spheres are located inside the inner RN horizon and are de Sitter at the center, the next step of this research would be to study their stability. However, that is beyond the scope of the present work.

References

1. S.W. Hawking and G.F. Ellis, *The Large Scale Structure of Space-Time*. Cambridge Univ. Press 1973.

2. Pankaj S. Joshi, “Spacetime Singularities” arXiv:1311.0449.

3. E. B. Gliner, “Algebraic properties of the energy-momentum tensor and vacuum-like states of matter”, Sov. Phys. JETP 22, 378 (1966).

4. A nice short description of the efforts of a number of Soviet cosmologists to eliminate the initial singularity is provided by C. Smeenk, “False Vacuum: Early Universe Cosmology and the Development of Inflation” ⟨http://publish.uwo.ca/~csmeenk2/files/FalseVacuum.pdf⟩.
[5] J. Bardeen, *Proceedings of GR5*, Tiflis, U.S.S.R (1968). This paper is not readily available, but a thorough discussion of Bardeen’s model can be found in A. Borde, Phys. Rev. D50 (1994) 3692.

[6] I. Dymnikova, “Vacuum nonsingular black hole”, Gen. Rel. Grav. 24 (1992) 235.

[7] M. Mars, M.M. Marta-Prats and J.M.M. Senovilla, “Models of regular Schwarzschild black holes satisfying weak energy conditions”, Class. Quant. Grav. 13 (1996) L51.

[8] A. Borde, “Regular Black Holes and Topology Change”, Phys. Rev. D55 (1997) 7615 [arXiv:gr-qc/9612057].

[9] S. A. Hayward, “Formation and evaporation of non-singular black holes”, Phys.Rev.Lett. 96 (2006) 031103 [arXiv:gr-qc/0506126].

[10] M.R. Mbonye and D. Kazanas, “A non-singular black hole model as a possible end-product of gravitational collapse”, Phys.Rev. D72 (2005) 024016 [arXiv:gr-qc/0506111].

[11] S. Ansoldi, “Spherical black holes with regular center: a review of existing models including a recent realization with Gaussian sources” [arXiv:0802.0330].

[12] E. Ayón-Beato and A. García, “The Bardeen Model as a Nonlinear Magnetic Monopole”, Phys. Lett. B493 (2000) 149 [arXiv:gr-qc/0009077].

[13] E. Ayón-Beato and A. García, “Regular black hole in general relativity coupled to nonlinear electrodynamics”, Phys. Rev. Lett. 80 (1998) 5056 [arXiv:gr-qc/9911046].

[14] E. Ayón-Beato and A. García, “New regular black hole solution from nonlinear electrodynamics”, Phys. Lett. B464 (1999) 25 [arXiv:hep-th/9911174].

[15] E. Ayón-Beato and A. García, “NonSingular Charged Black Hole Solution for NonLinear Source”, Gen. Rel. Grav. 31 (1999), 629 [arXiv:gr-qc/9911084].

[16] E. Ayón-Beato and A. García, “Four Parametric Regular Black Hole Solution”, Gen.Rel.Grav. 37 (2005) 635 [arXiv:hep-th/0403229].

[17] K.A. Bronnikov, “Regular magnetic black holes and monopoles from nonlinear electrodynamics”, Phys. Rev. D63 (2001) 044005 [ArXiv:gr-qc/0006014].

[18] I. Dymnikova, “Regular electrically charged structures in nonlinear electrodynamics coupled to general relativity”, Class. Quantum Grav. 21 (2004) 4417 [arXiv:gr-qc/0407072].

[19] Zhong-Ying Fan and Xiaobao Wang, “Construction of Regular Black Holes in General Relativity”, Phys. Rev. D94 (2016) 124027 [arXiv:1610.02630].

[20] S. Fernando, “Bardeen-de Sitter black holes” [arXiv:1611.05337].

[21] A. Bogojevic, and D. Stojkovic, “A Non-Singular Black Hole”, Phys.Rev. D61 (2000) 084011 [arXiv:gr-qc/9804070].

[22] K.A. Bronnikov and J.C. Fabris, “Regular phantom black holes” Phys. Rev. Lett. 96 (2006) 251101 [arXiv:gr-qc/0511109].

[23] K.A. Bronnikov, H. Dehnen and V.N. Melnikov, “Regular black holes and black universes”, Gen.Rel.Grav. 39 (2007) 973 [arXiv:gr-qc/0611022].

[24] V. P. Frolov, “Notes on non-singular models of black holes”, Phys.Rev. D 94 (2016) 104056 [arXiv:1609.01758].

[25] W. Berej, J. Matyjasek, D. Tryniecki and M. Woronowicz, “Regular black holes in quadratic gravity”, Gen. Rel. Grav. 38 (2006) 885 [ArXiv:hep-th/0606185].
[26] M.E. Rodrigues, E. L. B. Junior, G.T. Marques, and V. T. Zanchin, “Regular black holes in $f(R)$ gravity coupled to nonlinear electrodynamics”, Phys. Rev. D94 (2016) 049904.

[27] C. Bambi and L. Modesto, “Rotating regular black holes”, Phys.Lett. B721 (2013) 329 arXiv:1302.6075.

[28] S.G. Ghosh and S.D. Maharaj, “Radiating Kerr-like regular black hole”, Eur. Phys. J. C75 (2015) 7 arXiv:1410.4043.

[29] I. Dymnikova and E. Galaktionov, “Regular rotating electrically charged black holes and solitons in nonlinear electrodynamics minimally coupled to gravity”, Class. Quant. Grav. 32 (2015) 165015 arXiv:1510.01353.

[30] A. Abdurjabarov, M. Amir, B. Ahmedov, S. G. Ghosh, “Shadow of rotating regular black holes”, Phys. Rev. D93 (2016) 104004 arXiv:1604.03809.

[31] J.P.S. Lemos and V.T. Zanchin, “Regular black holes: Electrically charged solutions, Reissner-Nordström outside a de Sitter core”, Phys.Rev. D83 (2011) 124005 arXiv:1104.4790.

[32] N. Uchikata, S. Yoshida and T. Futamase, “New solutions of charged regular black holes and their stability”, Phys. Rev. D86 (2012) 084025 arXiv:1209.3567v3.

[33] J.P.S. Lemos and V.T. Zanchin, “Regular black holes: Guilfoyle’s electrically charged solutions with a perfect fluid phantom core”, Phys. Rev. D93 (2016) 124012 arXiv:1603.07359.

[34] L.D. Landau and E.M. Lifshitz, The Classical Theory of Fields, Fourth Edition: Volume 2 (Course of Theoretical Physics Series). Butterworth-Heinemann (2002).

[35] O.B. Zaslavskii, “Regular black holes and energy conditions”, Phys.Lett. B688 (2010) 278 arXiv:1001.2362.

[36] P. Boonserm, T. Ngampitipan and M. Visser, “Mimicking static anisotropic fluid spheres in general relativity”, Int.J.Mod.Phys. D25 (2016) 1650019 arXiv:1501.07044.

[37] J.R. Oppenheimer and H. Snyder, “On Continued Gravitational Contraction”, Phys. Rev. 56 (1939) 455.

[38] S. Hod, “Weak Cosmic Censorship: As Strong as Ever”, Phys.Rev.Lett. 100 (2008) 121101 arXiv:0805.3873.

[39] S. Chandrasekhar and J.B. Hartle, “On crossing the Cauchy horizon of a Reissner-Nordström black hole”, Proc. Roy. Soc. London A384 (1982) 301.

[40] E. Poisson and W. Israel, “Internal structure of black holes”, Phys. Rev. D41 (1990) 1796.

[41] S. Droz, “Numerical investigation of black hole interiors”, Phys. Rev. D55 (1997) 3575.

[42] M. Dafermos, “The interior of charged black holes and the problem of uniqueness in general relativity”. Commun. Pure Appl. Math. 58 (2005) 445 arXiv:gr-qc/0307013.

[43] A.J.S. Hamilton and P.P. Avelino, “The physics of the relativistic counter-streaming instability that drives mass inflation inside black holes”, Phys.Rept. 495 (2010) 1 arXiv:0811.1926.

[44] I. Dymnikova and E. Galaktionov, “Stability of a vacuum nonsingular black hole”, Class.Quant.Grav. 22 (2005) 2331 arXiv:gr-qc/0409049.