Numerical Solution of the Navier-Stokes Equations for Steady Magnetohydrodynamic Flow Between Two Parallel Porous Plates with an Angular Velocity

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Abstract. The Steady Laminar stream of an electrically directing thick, incompressible liquid between two parallel permeable plates of a divert within the sight of a transverse attractive field with an angular velocity when the liquid is being pulled back through both the dividers of the channel at a similar rate with a precise speed is examined. Numerical arrangement is acquired for various estimations of R (Suction Reynolds number) utilizing R-K Gill's technique and the diagrams of dimensionless functions f' and f have been drawn.

1. Introduction
The Steady Magnetohydrodynamic stream between two parallel permeable plates is a traditional issue in liquid flow whose arrangement has numerous applications in Magnetohydrodynamic (MHD) control generators, magnetohydrodynamic pumps, quickening agents, streamlined warming, polymer innovation, oil industry, diffusive detachment of issue from liquid, refinement of unrefined petroleum and liquid beads and splashes.

Hartmann and Lazarus [1] contemplated the impact of a transverse uniform attractive field on the stream of a leading liquid between two unbounded parallel, stationary and protected plates. Hassanien and Mansour [2] talked about the precarious attractive course through a permeable medium between two unending parallel plates. There has been a reestablished intrigue appeared by Makinde and Sibanda [3] in examining Magnetohydrodynamic (MHD) stream and warmth move in permeable media in view of the impact on attractive fields on the execution of numerous frameworks. Aboul-Hassan and Attia [4] talked about the stream of a leading Visco versatile liquid between two flat permeable plates within the sight of a transverse-attractive field.

Attia [5] has considered the insecure Hartmann stream with warm exchange of a viscoelastic liquid considering the Hall impact. Krishnambal and Ganesh [6] examined the temperamental Stokes stream of thick liquid between two parallel permeable plates. Hayat, Wang and Hutter [7] contemplated the Hall impacts on the shaky hydromagnetic oscillatory stream of a moment review liquid. Hazeem Ali Attia [8] contemplated the precarious laminar stream of an incompressible gooey liquid and warmth exchange between two parallel plates within the sight of a uniform suction and infusion thinking about factor properties. Ganesh and Krishnambal [9] examined the Magnetohydrodynamic stream of thick liquid between two parallel permeable plates. Ganesh and Krishnambal [10] talked about the flimsy Magnetohydrodynamic Stokes stream of thick liquid between two parallel permeable plates.
The goal of this investigation is to dissect the Steady Magnetohydrodynamic stream of thick liquid between two parallel permeable plates with a precise speed when the liquid is being pulled back through both the dividers of the channel at a similar rate.

2. Mathematical Formulation of the Model
The Steady laminar stream of an incompressible thick liquid between two parallel permeable plates with a rakish speed is considered within the sight of a transverse attractive field of quality \( \mathbf{H}_0 \) connected opposite to the dividers. The inception is taken at the focal point of the channel and let \( x \) and \( y \) be the arrange tomahawks parallel and opposite to the channel dividers. The length of the channel is thought to be \( L \) and \( 2h \) is the separation between the two plates. Give \( u \) and \( v \) a chance to be the speed segments in the \( x \) and \( y \) headings individually, \( \Omega \) is the angular velocity.

The continuity equation is

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(1)

Equations of momentum are

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + 2\Omega u - \frac{\sigma_e B_0^2 u}{\rho}
\]

(2)

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\mu}{\rho} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - 2\Omega v - \frac{\sigma_e B_0^2 v}{\rho}
\]

(3) Where \( \sigma_e \) is the electrical conductivity and \( B_0 = \mu_0 \mathbf{H}_0 \), \( \mu_0 \) being the magnetic permeability.

The limit conditions are \( u = 0 \) on \( y = h \) and \( y = -h \); \( v = v_0 e^{i\omega t} \) on \( y = h \) and \( v = -v_0 e^{i\omega t} \) on \( y = -h \), where \( v_0 \) is the velocity of suction at the walls of the channel.

In Steady State \( \frac{\partial u}{\partial t} = 0 \) and \( \frac{\partial v}{\partial t} = 0 \), \( \therefore \) equations (2) and (3) becomes,

\[
\frac{u}{h} \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + 2\Omega u - \frac{\sigma_e B_0^2 u}{\rho}
\]

(4)

\[
\frac{u}{h} \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\mu}{\rho} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - 2\Omega v - \frac{\sigma_e B_0^2 v}{\rho}
\]

(5)

Let the dimensionless variable \( \eta = \frac{v}{h} \), \( u = u(x,y) e^{i\omega t} \), \( v = v(x,y) e^{i\omega t} \), \( p = p(x,y) e^{i\omega t} \), where \( \omega \) is the frequency.

hence equations (1), (4) and (5) converted into,

\[
\frac{\partial u}{\partial x} + \frac{1}{h} \frac{\partial v}{\partial \eta} = 0
\]

(6)

\[
\frac{u}{h} \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial \eta} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \left( \frac{\partial^2 u}{\partial x^2} + \frac{1}{h^2} \frac{\partial^2 u}{\partial \eta^2} \right) + 2\Omega u - \frac{\sigma_e B_0^2 u}{\rho}
\]

(7)

\[
\frac{u}{h} \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial \eta} = -\frac{1}{\rho h} \frac{\partial p}{\partial \eta} + v \left( \frac{\partial^2 v}{\partial x^2} + \frac{1}{h^2} \frac{\partial^2 v}{\partial \eta^2} \right) - 2\Omega v - \frac{\sigma_e B_0^2 v}{\rho}
\]

(8)
where \( \nu = \frac{\mu}{\rho} \) = Kinematic Viscosity, \( \rho \) the density of the fluid, \( \mu \) the coefficient of viscosity and \( p \) the pressure.

The limit conditions are converted into

\[
\begin{align*}
u(x,1) &= 0, \quad u(x,-1) = 0, \quad \text{and} \quad \nu(x,1) = v_0, \quad v(x,-1) = -v_0
\end{align*}
\]

(9) \( v(x,1) \)

Let \( \psi \) be the stream function such that

\[
\begin{align*}
u &= \frac{1}{h} \frac{\partial \psi}{\partial \eta} \\
u &= -\frac{\partial \psi}{\partial x}
\end{align*}
\]

(11) \( \nu \)

(12) \( \nu \)

The equation of continuity can be satisfied by a stream function of the form

\[
\Psi(x,\eta) = (hu(0) - v_0 x) f(\eta)
\]

(13) \( \Psi(x,\eta) \)

where \( u(0) \) is the average entrance velocity at \( x = 0 \).

From equation (13), the velocity components (11) and (12) are given by

\[
\begin{align*}
u &= \frac{1}{h} (hu(0) - v_0 x) f'(\eta) \\
u &= v_0 f(\eta)
\end{align*}
\]

(14) \( \nu \)

(15) \( \nu \)

where the prime denotes the differentiation with respect to \( \eta = \frac{y}{h} \). Since the liquid is being pulled back at consistent rate from both the dividers, \( v_0 \) is free of \( x \).

Using (14) and (15) in (7) and (8), then the equation of momentum reduces to

\[
\begin{align*}
&- \frac{1}{\rho} \frac{\partial p}{\partial x} = \left( u(0) - \frac{v_0 x}{h} \right) \left( \frac{\sigma_e B_0^2}{\rho} - 2\Omega \right) f' - \frac{\nu}{h^2} f'' - \frac{v_0 (ff'' - f'^2)}{h}
\end{align*}
\]

(16) \( \nu \)

and

\[
\begin{align*}
&- \frac{1}{h \rho} \frac{\partial p}{\partial \eta} = \left( 2\Omega v_0 + \frac{\sigma_e B_0^2}{\rho} v_0 \right) f - \frac{\nu \cdot v_0 f''}{h^2} + \frac{v_0^2 f'}{h}
\end{align*}
\]

(17) \( \nu \)

Differentiating equation (17) w.r.t ‘x’, we get

\[
- \frac{1}{\rho} \frac{\partial^2 p}{\partial x \partial \eta} = 0
\]

(18) \( \nu \)

Differentiating equation (16) w.r.t ‘\eta’ we get,

\[
- \frac{1}{\rho} \frac{\partial^2 p}{\partial x \partial \eta} = \left( u(0) - \frac{v_0 x}{h} \right) \frac{d}{d\eta} \left( \frac{\sigma_e B_0^2}{\rho} - 2\Omega \right) f' - \frac{\nu}{h^2} f'' - \frac{v_0 (ff'' - f'^2)}{h}
\]

(19) \( \nu \)

Comparing equation (16) and (17) we get,

\[
\frac{d}{d\eta} \left( \frac{\sigma_e B_0^2}{\rho} - 2\Omega \right) f' - \frac{\nu}{h^2} f'' + \frac{v_0 (ff'' - f'^2)}{h} = 0
\]

(20) \( \nu \)

Let \( R \) (Suction Reynolds number) = \( \frac{hv_0}{\nu} \) and \( M_1 = B_0 h \left( \frac{\sigma_e}{\nu \rho} - \frac{2\Omega}{\sqrt{B_0^2}} \right)^\frac{1}{2} \)

Integrating equation (20) w.r.t \( \eta \) and using the above expressions we get,
\[
\left(\frac{\sigma B_0^2}{\rho} - 2\Omega\right)f' - \frac{v}{h^2} f''' + \frac{v_0 (ff'' - f'^2)}{v} = K
\]

where \( K \) is an arbitrary constant.

Dividing by \(-v / h^2\), it becomes

\[
f'' = \left(\frac{\sigma B_0^2}{\rho \nu} - \frac{2\Omega h}{v_0}\right) f' - \frac{v h}{v_0} (ff'' - f'^2) = K
\]

i.e.,

\[
f'' - aR f' - R (ff'' - f'^2) = K
\]

where \( a = \left(\frac{\sigma B_0^2}{\rho \nu} - \frac{2\Omega h}{v_0}\right) \), \( aR = \left(\frac{\sigma B_0^2 h^2}{\rho \nu} - \frac{2\Omega h^2}{v_0}\right)\)

hence equation (21) becomes

\[
f'' + R (f'^2 - ff'') - aR f' = K
\]

\[
f'' + M_1^2 f' + R (f'^2 - ff'') = K
\]

where \( M_1^2 = aR \)

Let \( A = M_1^2 \)

\[
f'' - Af' + R (f'^2 - ff'') = K
\]

Differentiating equation (23) w.r.t \( \eta \), it is seen that

\[
f'''' + R (f' f'' f'''') - Af''' = 0
\]

Boundary conditions on \( f(\eta) \) are

\[
f(1) = 1, \ f(-1) = -1, \ f'(1) = 0 \quad \text{and} \quad f'(-1) = 0
\]

Hence the solution of the equations of motion and continuity is given by a nonlinear fourth order differential equation (24) subject to the boundary conditions (25).

3. Numerical Solution

To acquire the point by point data on the idea of the stream for various estimations of \( R \) and \( A \), a numerical answer for the overseeing conditions is important. For various scopes of parameters \( R \) and \( A \), the two limit pace issue communicated by conditions (24) and (25) has been incorporated by utilizing R-K Gill’s strategy (Jeffrey Winicour 1997) and the diagrams have been drawn for the dimensionless capacity \( f' \) and \( f \). These are appeared in Figures 1 to 8.

Figures 1 to 4 speak to the variety of the dimensionless capacity \( f' \) for various estimations of \( A \) to be specific \( A=1, 3, 5 \) and 10 and for \( R>0 \). These profiles diminish in the focal area and increment close to the dividers of the channel with the reduction of \( R \). It is likewise observed that as \( A \) increments impressively the speed profiles have a tendency to end up plainly level in the focal locale and soak close to the dividers as found in Figures 3 and 4. This demonstrates for the bigger estimations of \( A \), the liquid moves like a square with a type of unbending nature. This affirms in leading liquids, attractive field acquires inflexibility the liquid. Consequently it is watched that \( f(\eta) \) diminishes with the abatement in the estimations of \( R \) and the profile is explanatory.

In Figure 5, we see that diagrams for variety of the dimensionless capacity \( f' \) is drawn for \( A=1 \) and for various estimations of \( R \) in particular \( R<0 \). From the above diagram we see that \( f' \) diminishes as \( R \) diminishes. The capacity \( f(\eta) \) (\( \eta \)-speed profiles) is plotted against \( \eta \) for different estimations of \( R \) in Figure 6. It is watched that if \( R > 0 \) and for various estimations of \( A \) in \( -1 \leq \eta \leq 0, f \) diminishes with increment of \( R \) while in the area \( 0 \leq \eta \leq 1, f \) increments with increment of \( R \).
Figure 1. Variation of the dimensionless function $f'$ when $A=1$ and $R > 0$

Figure 2. Variation of the dimensionless function $f'$ when $A=3$ and $R > 0$
Figure 3. Variation of the dimensionless function $f'$ when $A=5$ and $R > 0$

Figure 4. Variation of the dimensionless function $f'$ when $A=10$ and $R > 0$
Figure 5. Variation of the dimensionless function $f'$ when $A=1$ and $R < 0$

Figure 6. Variation of the dimensionless function $f$ when $A=0.75$ and $R > 0$
Figure 7. Variation of the dimensionless function $f$ when $A=0.5$ and $R > 0$

Figure 8. Variation of the dimensionless function $f$ when $A=1$ and $R > 0$

4. Conclusion

In the above investigation, it is seen that within the sight of a transverse attractive field with an angular velocity the class of arrangements of Steady Magnetohydrodynamic stream of thick liquid between two parallel permeable plates with a rakish speed is displayed when the liquid is being pulled back through both the dividers of a channel at a similar rate.
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