Stripes disorder and correlation lengths in doped antiferromagnets

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For stripes in doped antiferromagnets, we find that the ratio of spin and charge correlation lengths, $\xi_s/\xi_c$, provide a sharp criterion for determining the dominant form of disorder in the system. If stripes are disordered predominantly by topological defects then $\xi_s/\xi_c \lesssim 1$. In contrast, if stripes correlations are disordered primarily by non-topological elastic deformations (i.e., a Bragg-Glass type of disorder) then $1 < \xi_s/\xi_c \lesssim 4$ is expected. Therefore, the observation of $\xi_s/\xi_c \approx 4$ in (LaNd)$_{2-x}$Sr$_x$CuO$_4$ and $\xi_s/\xi_c \approx 3$ in La$_{2/3}$Sr$_{1/3}$NiO$_4$ invaribly implies that the stripes are in a Bragg glass type state, and topological defects are much less relevant than commonly assumed. Expected spectral properties are discussed. Thus, we establish the basis for any theoretical analysis of the experimentally observed glassy state in these materials.

I. INTRODUCTION

Recent experiments [1–3] shed new light and conjure up new questions on the glassy state of doped antiferromagnets. There is growing experimental evidence and theoretical arguments that electronic charge and spin “topological stripes” density wave correlations are a generic feature in lightly doped strongly correlated antiferromagnetic materials, which may not necessarily be superconducting (as evidenced in both Copper-Oxides and non-superconducting Nickel-Oxides systems).

Moreover, there is preliminary evidence for stripes throughout the glassy state region in La$_{2-x}$Sr$_x$CuO$_4$ [4], which extends into part of the superconducting phase. Thus, our analysis of stripe disorder is relevant for understanding the general phase diagram of High-Tc materials. (Though the role that stripes play in the particular phenomenon of superconductivity is still controversial).

In all experimental systems, where the full charge & spin stripes structure was observed, it was found that both spin and charge order are glassy [1–3]. An intrinsic finite stripes correlation length signature of disorder is consistently observed [1–4]. In this paper, based on our analysis of general types of disorder in stripes, we are able to determine unambiguously the nature of the glassy state in these systems. Our analysis is rather simple, based on elementary general principles, and independent of model details and parameters.

The canonical two dimensional stripes structure comprise of hole rich lines (charge stripes) which form anti-phase domain walls between antiferromagnetic (AFM) correlated spin regions (see fig.2a). As a result, the spin periodicity $\alpha_s$ is twice the charge periodicity $\alpha_c$. The charge and spin correlations (in the periodic direction) are hence inexorably connected. It is thus quite surprising that, experimentally, the correlation lengths of charge and spin differ substantially; e.g., in Copper-Oxides (LaNd)$_{2-x}$Sr$_x$CuO$_4$ (with $x > 0.1$) the ratio of the stripes spin correlation length $\xi_s$ to the charge correlation length $\xi_c$ is $\xi_s/\xi_c \approx 4/1$ [4].

We highlight a distinction between two ways by which disorder potential can lead to finite correlation length. Conventional models attribute glassiness to the existence of topological defects. In contrast, Giamarchi & LeDoussal recently argued for the existence of a thermodynamically distinct zero temperature glassy state - labeled “Bragg Glass” - due purely to elastic deformation [4]. In two dimensions it is reduced to a ”quasi Bragg glass” state, were disorder by elastic deformations result with correlation length $\xi$ smaller than typical distance $R_D$ between unpaired defects; $\xi \ll R_D$. Since the typical distinctions between the two glassy states are not easily probed by experiments, the Bragg glass state has been a subject of continuing controversy.

Kivelson&Emery speculated that a stripes Bragg glass state may occur as part of a general stripes phase diagram [4]. In stripes, due to the unique feature of having two distinct and coupled periodic order parameters (spin and charge), we argue that the ratio of spin and charge correlation lengths, $\xi_s/\xi_c$, provide a novel sharp criterion for determining the dominant form of disorder in the system. We conclude that all current experiments on stripes are compatible only with the conjecture that stripe correlations are disordered primarily by non-topological elastic deformations (see fig.2c) [4].

The line of argument is the following: We find that disorder dominated by topological defects lead only to the possibility of having the charge stripes correlation length larger than the spin stripes correlation length $\xi_s$, i.e., $\xi_s/\xi_c < 1$. Consequently, we establish a robust qualitative distinction between topological defects leading always to $\xi_s/\xi_c < 1$, while predominant elastic deformations leading always to $\xi_s/\xi_c > 1$ (and expectantly $\xi_s/\xi_c = 4$). On this basis, we argue that the observation of $\xi_s/\xi_c \approx 4$ in (LaNd)$_{2-x}$Sr$_x$CuO$_4$ unequivocally indicates a quasi Bragg glass type state. It implies that topological defects are much less relevant than commonly assumed for glassy properties of Copper-Oxide systems [4]. Expected spectral properties are discussed.

The charge stripes are modeled as classical elastic charge density wave structures. Similarly, the AFM spin regions are discussed as a classical two-dimensional Heisenberg model. Disorder is then incorporated as acting directly on these classical objects. The observed cor-
relation length should not be confused with the size of the stripes domains, where stripes orientation in distinct domains differ by 90°. All of our analysis concerns the disorder within a single stripes domain (i.e., all stripes have the same general orientation).

II. DISORDER BY TOPOLOGICAL DEFECTS

The topology of the stripes charge lines is similar to that of smectic liquid crystals. Since in this paper we are not interested in significant changes of the stripes orientation, we ignore disclination defects and focus on dislocations (fig.1). Though each topological defect in the charge lines order induces also a topological defect in the spin order, the respective disordering effects of the defects are different.

The charge lines are antiphase domain walls of the spin order, i.e., the local AFM order parameter undergoes a local π phase slip across each charge stripe line. Therefore, if a round trip goes through an odd number of charge lines then it is enclosing a π vortex of the spin order. Such is the case for going around a line ending or a line splitting dislocation defects, which are highlighted by colored circles in fig.1 (The defects are caricaturistically drawn with rather sharp line angles, but more realistic softening of the curves will not soften our conclusions). One should not confuse dislocations in the charge stripes lines with any kind of dislocations in the 2D Heisenberg spin lattice points (i.e., the underlying Cu-O plane). The magnetic π-vortex defects are associated purely with rotations in spin space. The resulting spin texture can be either an XY model vortex (if spins remain confined to the plane), or otherwise Skyrmions in an O(3) model.

A connection with the more familiar spin glass model of frustrated/non-frustrated plaquettes in a 2D Heisenberg model can be made by mapping onto a square lattice where, every bond which is cut by a charge stripe is a ferromagnetic bond and otherwise an AFM bond. Frustrated/non-frustrated plaquettes are those with an odd/even number of AFM bonds respectively.

To recapitulate, every topological defect in the charge smectic order is simultaneously associated with a topological defect in the spin order. Therefore, it seems at first that disorder by topological defects would inevitably lead to having the same charge and spin correlation lengths (i.e., \(\xi_s/\xi_c \approx 1\)).

In fig.1, we show a possible arrangement of two dislocation defects (the full 2D plane can be understood as composed of similar arrangements at a given density of defects). The associated π-vortex spin defects are known to limit the spin correlation length in the plane, yet the effect on charge correlations is less trivial. While any dislocation defect in a smectic topology destroys the correlation along the lines (z-axis in fig.1), the effect on correlations perpendicular to the lines (z-axis in fig.1) is more subtle. The experimentally measured stripes correlations which are discussed throughout this paper are the ones perpendicular to the stripe lines.

To be precise, imagine an x-ray scattering experiment performed on the "window" of charge stripes configuration depicted in fig.1. If the beam’s spot size is narrow (e.g., like the dashed yellow line) then obviously the observed peak width will not differ from that of perfectly ordered stripes (within the size of the depicted stripes figure). If the beam’s spot size is widened to size of the window of fig.1 then there will be additional second harmonic satellite peaks, but the width of all peaks will still remain that of the perfectly ordered stripes system (with an added weak background due to the relatively small deformation regions).

In conclusion, due to the different character of the charge and spin order parameters, the distance between topological defects limits the spin correlation length but the charge stripes correlation length (in the modulation direction) is less affected. Therefore, predominant topological defects disorder always lead to having \(\xi_s/\xi_c < 1\).

![FIG. 1. Possible configuration of two topological defects](image)

III. DISORDER BY ELASTIC DEFORMATIONS

In the previous section we established the effect of topological defects. We now examine the extreme opposite case, where topological defects are excluded and only elastic deformations dominate the stripe’s disorder (fig.2c). i.e., a perfect Bragg glass stripes state. We show that, unlike the case of disorder by topological defects, a Bragg glass stripe state allows for \(\xi_s/\xi_c > 1\), and moreover it is expected that \(\xi_s/\xi_c = 4\).

A. Correlation length ratio \(\xi_s/\xi_c\)

First, we elucidate how a finite correlation length arise due to static elastic deformations. Elastic deformations of the stripes look like fig.2c. For clarity, we elaborate an approximate simpler model of disorder (depicted in fig.2b): Picture the charge stripes as an array of parallel rigid rods (fig.2a) with period \(a_c = u_j^{j+1} - u_j^j\) (in the absence of disorder), and that the effect of external disorder potential is only to locally perturb the stripes separation.

The disorder potential induces deviations \(\Delta u_j^c\) in the local separation between stripes; \(u_j^{j+1} - u_j^j = a_c + \Delta u_j^c\),

\[\Delta a_j^c = \Delta u_j^c\]
with equal probability of local compression and extension, i.e., $\Delta a^j = \pm |\Delta a_c(P)|$, and which may have a distribution of variable size deviations $|\Delta a_c(P)|$ with probability $P$. The typical local deformation may be small, $|\Delta a_c(P)| \ll a_c$. Yet, when statistically accumulated over a length of $N$ stripes, the average size of fluctuation in the distance $u^{L} = u^j$ from its ordered state (where $u^{j+1} = u^j = N a_c$) grows typically like $\sqrt{N} |\Delta a_c|$, where $|\Delta a_c| \equiv \sqrt{\langle |\Delta a_c(P)|^2 \rangle}$.

The Bragg glass correlation length $\xi_c$ is the scale on which relative displacements of the stripes from their ordered position becomes of order of the bare stripes spacing $a_c$. In the ordered state (fig.2a), a Bragg peak results from the constructive interference of reflections from each stripe line (at the stripes wave-vector). Due to disorder, when the accumulated deviation from the ordered state $\sum_{j=1}^{N} \Delta a^j$ gets to be about half the bare periodicity (see fig.2b), the constructive interference is lost which give rise to a finite peak width in a scattering experiment by which the correlation length is defined. Thus, we define

$$\xi_c \equiv N_a a_c = \left( \frac{a_c}{2 |\Delta a_c|} \right)^2 a_c \quad (1)$$

where $N_a$ is given by the condition

$$\frac{a_c}{2} \approx \sqrt{\sum_{j=1}^{N_a} \Delta a^j} = \sqrt{N_a |\Delta a_c|} \quad (2)$$

In the relevant materials, the disorder potential (due mostly to the dopant Sr charge impurities out of the stripe plane) couples directly only to the charge stripes. What is the effect on the spin correlation length? The key observation is that, in the stripe structure, the spin order deviations are forced to follow the charge lines, since the charge lines are anti-phase domain walls irrespective of their position (elsewhere we explicitly defend this statement). In other words, the spin order is enslaved to the charge order and has no independent intrinsic elastic stiffness (in real space motion, not in spin space rotations). This is an essential outcome of the frustrated phase separation view of stripes.

Therefore, for the resulting spin correlation length, it is only the spin period $a_s$ that replace $a_c$ on the left side of (2) in the defining condition which relate the bare period to the associated finite correlation length due to elastic deformations, while the average disorder "steps" remain $|\Delta a_c|$ per charge line also for the spin order. Thus,

$$\xi_s = \left( \frac{a_s}{2 |\Delta a_c|} \right)^2 a_c \quad (3)$$

$$\xi_s/\xi_c = \left( \frac{a_s}{a_c} \right)^2 = 4. \quad (4)$$

Which is exactly the experimental observation, with no free adjustable parameters. The above conclusion is insensitive to details of the proper elastic model of stripes (and also insensitive to the characteristic deformation size $|\Delta a_c|$). It is the outcome of the generic structure where $a_s/a_c = 2$, and the generic origin of finite correlation length due to random small deformations in periodic systems.

**FIG. 2.** (a) Canonical stripes structure. (b) Model of elastic deformations of parallel stripes with disordered spacings. (c) "Realistic" image of elastic stripes deformations with no topological defects.

Once the correct framework for analyzing the glassy state is identified, we are in position to derive various implications. For example, we estimate the strength of disorder as characterized by the ratio $\left( \frac{v a_c^2}{2 |V_q|} \right)^{-1}$, where $v$ is the elastic coefficient associated with compressing or stretching the spacing of the stripes with respect to their preferred spacing $a_c$, and $V_q$ is the disorder potential strength.

Properly, the charge stripes should be described by some form of anisotropic 2D elastic model (and moreover coupled to an underlying lattice which breaks the rotation symmetry in the plane). The full elastic model is very complicated, but a qualitative estimate can be derived using the limit model of stripes as a one dimensional CDW (as in fig.2b). Using the method of for weak disorder, the ensuing correlation length $\xi_c$ is estimated,

$$\frac{\xi_c}{a_c} = \left( \frac{v a_c^2}{2 |V_q|} \right)^{2/3}. \quad (5)$$

For $(\text{LaNd})_{7/8} \text{Sr}_{1/8} \text{CuO}_4$, where $a_c \approx 16\text{Å}$ and $\xi_c \approx 100\text{Å}$, we find $\left( \frac{v a_c^2}{2 |V_q|} \right)^{-1} \approx 1/15$, which indicates that the assumption of weak disorder is at least self-consistent.

**B. Charge and spin glassiness without topological defects?**

Since the disorder potential due to the dopant Sr impurities couples only to the charge stripes, it is not a-priori clear how a concomitant spin disorder character can be obtained other than due to topological defects.
A glassy state is characterized by a wide distribution of activation energies \( \sigma(\omega) \). For the frequency dependent conductivity \( \sigma(\omega) \) associated with various fluctuation modes of the charge stripes density wave, we can use the calculations of \( \sigma(\omega) \). \( \sigma(\omega) \) has the general form of a peak at energy \( \omega_0 \approx v/\xi_c \) and peak-width of similar magnitude \( \Delta \approx \omega_0 \). \( v/\xi_c \) can be interpreted as the excitation of a fluctuation mode on the order of the pinning localization length "domain". The peak shape is not Gaussian, and has power-law tails \( \xi \). In contrast with the case of topological defects, there is no contribution to the glassy spin characteristics from static non-topological deformations of the charge stripes. Yet, spin dynamics is affected through its coupling to the charge stripes dynamics. Though an explicit microscopic theory of spin-charge coupling in stripes is still missing \( \xi \), it is expected that the distribution of charge fluctuation frequencies translates into a distribution of local spin fluctuation energies (possibly with similar characteristics). Thus, when topological defects are dilute, glassy spin behavior may be dominated by the contribution of non-topological stripe fluctuations.

IV. CONCLUSIONS

To summarize, our main conclusion is the following: In stripes, disorder purely by topological defects configurations lead to \( \xi_s/\xi_c < 1 \). Therefore, the seemingly minute detail of experimental finding that \( \xi_s/\xi_c \approx 4 \) in \( (LaNd)_{7/8}Sr_{1/8}CuO_2 \) is enough to exclude the possibility of disorder dominated by topological defects. Moreover, we find that for disorder by pure elastic deformations it is expected that \( \xi_s/\xi_c \approx 4 \) quite insensitively to details of the elastic stripe models. Thus, any observation of \( 1 < \xi_s/\xi_c < 4 \) implies that there is a relatively dilute concentration of topological defects (with typical distance \( R_D \)) where \( \xi_s < R_D < 4\xi_c \), which cut the spin correlation length \( \xi_s \) to be below its expected value in the absence of defects.

Hence we proclaim that \( (LaNd)_{7/8}Sr_{1/8}CuO_2 \) at low temperatures is a quasi Bragg glass type disordered system, in the sense that it is disordered mainly by elastic deformations resulting in stripes correlation length \( \xi_c \) much smaller than the typical distance \( R_D \) between unpaired topological defects \( \xi \), i.e., \( 4\xi_c < R_D \). Thus, any theoretical approaches and interpretation of the glassy properties of the known experimental stripe systems should not be blindly done by fitting parameters to conventional theory based on topological defects (dressed in stripes costume).

Our analysis can be applied, for example, to the effect of Zinc doping on stripes. It is not clear whether the main effect is of local pinning of the stripes or also as nucleation centers for stripe defects. We propose that an examination of the ratio \( \xi_s/\xi_c \) will give the answer.

Following our analysis, non-superconducting \( (LaNd)_{7/8}Sr_{1/8}CuO_2 \) becomes the first doped Copper-Oxide material whose electronic ground-state, including interactions and disorder, is delineated. It is also one of the first strong evidence for realization of the controversial quasi Bragg glass state \( \xi \).

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