Cascaded Double $\kappa$-$\mu$ Shadowed Fading Channels

Mehmet Bilim (mbilim@nny.edu.tr)
Nuh Naci Yazgan Universitesi  https://orcid.org/0000-0003-2518-3125

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Cascaded Double $\kappa - \mu$ Shadowed Fading Channels

Mehmet Bilim*

Department of Electrical & Electronics Engineering, Nuh Naci Yazgan University, Kayseri, Turkey

*Corresponding Author:

*Tel: +90 (352) 324 00 00 / 2254

E-mail: mbilim@nny.edu.tr

Abstract

In this study, the cascaded double $\kappa - \mu$ shadowed fading model is mathematically introduced. A novel expression is obtained for the cumulative distribution function (CDF) of the cascaded double $\kappa - \mu$ shadowed fading channels by using Meijer’s $G$ function and Laplace transform. Based on the obtained CDF expression, the exact and asymptotic expressions are derived for the outage probability (OP), the bit error rate (BER) for various modulations of the considered channel model. The OP and the average BER curves with exact simulations obtained by the proposed expressions are presented for different parameters and modulation schemes of the considered channel model. Finally, the accuracy of the derived OP and the average BER expressions is confirmed.

Index Terms: Cascaded fading channels, performance evaluation, and $\kappa - \mu$ shadowed fading.
1 INTRODUCTION

In the last decades, cascaded fading channels have been a subject of interest due to it can be applied for various wireless communication systems. For this reason, the cascaded fading studies have been presented by considering many different fading channel models such as Nakagami, $\kappa-\mu$, $\eta-\mu$, $\alpha-\mu$, Beaulieu-Xie, and fluctuating two-ray [1]-[5]. While the statistical properties of $N$ independent random variables (RVs) which have Nakagami-$m$ distribution was introduced in [1], two different $\kappa-\mu$ RVs such as independent and non-identically was considered in [2]. In [3], a comprehensive work related to the product of two independent and non-identically distributed $\alpha-\mu$, $\eta-\mu$, and $\kappa-\mu$ variates was presented. The studies in [2] and [3] did not report the cascaded double $\kappa-\mu$ shadowed fading channels. These works analyzed only $\kappa-\mu$ fading without shadow effect. In another study in [4], Beaulieu-Xie which is more recent fading model was investigated for double cascaded fading scenario. For the same purpose, a different fading model, called by fluctuating two-ray fading, was analyzed [5]. It is noteworthy that the generalized fading models including traditional fading patterns such as Nakagami, Rayleigh and Rician are considered, especially when the recent studies are examined.

Moreover, the $\kappa-\mu$ shadowed model that contains the Rayleigh, Rician, one-side Gaussian, $\kappa-\mu$, Rician shadow, and Nakagami-$m$ was proposed [6]. The introduced channel model is considered different from the $\kappa-\mu$ fading in that it also includes the shadowing effect in the $\kappa-\mu$ channel model. It should be mentioned that the $\kappa-\mu$ shadowed fading is appropriate for different wireless communication applications [7]-[12]. In [7], coverage evaluation in cellular system for the $\kappa-\mu$ shadowed conditions was investigated. The author in [8] presented the extensive study for quadrature amplitude modulation analysis over the $\kappa-\mu$ shadowed channels. In [9], a statistical characterization investigation related to the $\kappa-\mu$ shadowed and Beckmann fading models were presented. In [10], the energy detection characteristic analysis for the $\kappa-\mu$ shadowed fading
channels were proposed. In [11], a mixture Gamma shadowed case was considered by using inverse Nakagami-$m$. Then, the proposed unified distribution model was applied to double $\kappa-\mu$ shadowed fading channels. However, this work did not consider cascaded double $\kappa-\mu$ shadowed fading. In another study in [12], the secrecy analysis for the multiple-input multiple-output network for the $\kappa-\mu$ shadowed conditions was presented. However, none of these studies ([1]-[12]) addressed the cascaded double $\kappa-\mu$ shadowed fading conditions.

Motivated by this, in this study, a cascaded double $\kappa-\mu$ shadowed fading model is proposed. To the best of the authors’ knowledge, in the literature, the statistical properties for a cascaded double $\kappa-\mu$ shadowed conditions has not been reported by means of using the Meijer’s $G$ function and Laplace transform. The reason of focusing the $\kappa-\mu$ shadowed fading in this study is that this fading channels physically characterizes a wide range of propagation circumstances. The main contributions are: (i) The novel and new cumulative distribution function (CDF) expression of the instantaneous signal-to-noise ratio (SNR) for the cascaded double $\kappa-\mu$ shadowed condition is derived. (ii) With the help of the derived CDF, the outage probability (OP) of the network model is analyzed. (iii) Based on CDF approach, the average bit error rate (BER) expression is also obtained for the considered fading channels.

This paper is organized as follows. Section II gives the properties of the considered fading model. Section III provides the OP and the average BER analysis for the cascaded double $\kappa-\mu$ shadowed environment. In section IV, some numerical findings are discussed. Finally, in section V, the concluding remarks are presented.
2 FADING MODEL

Let define $Z = a_1 a_2$ which is the product of the two RVs. The envelopes $r_1 = |a_1|$ and $r_2 = |a_2|$ are modeled by the $\kappa-\mu$ distribution which is proposed in [13]. Otherwise, the $\kappa-\mu$ shadowed model is proposed by the author in [6]. For $\kappa-\mu$ shadowed model, the probability density function (PDF) of the instantaneous SNR is written as

$$f_\gamma(\gamma) = \frac{\gamma^{\mu_i-1} \mu_i^\mu_i m_i^m (1+\kappa_i)^{\mu_i}}{\tilde{\gamma}_i^\mu_i (\mu_i \kappa_i + m_i)^m} e^{-\frac{\mu_i (1+\kappa_i) \gamma}{\tilde{\gamma}_i}} \frac{1}{\Gamma(\mu_i)} {}_1F_1 \left( m_i ; \mu_i ; \frac{\kappa_i (1+\kappa_i) \mu_i^2}{(m_i + \mu_i \kappa_i) \tilde{\gamma}_i} \right)$$ (1)

where $i \in \{1, 2\}$, ${}_1F_1(\cdot)$ and $\Gamma(\cdot)$ are the hypergeometric and the Gamma functions, respectively. $\kappa_i$ and $\mu_i$ are the parameters for the total power ratios between heavy components and the diffused waves. $m_i$ and $\tilde{\gamma}_i$ are the shape parameter for the Nakagami-$m$ random variable and average SNR, respectively. Using the identity ${}_1F_1(\alpha; \theta; x) \sum_{j=0}^{\infty} \frac{(\alpha)_j x^j}{j!}$ which is the infinite series representation of the ${}_1F_1(\cdot)$, ($a)_n$ is Pochhammer function) the PDF expression in (1) can be rewritten as

$$f_\gamma(\gamma) = \sum_{j=0}^{\infty} g_{i,j} \frac{\exp(-\gamma \tilde{\lambda}_i^2 \tilde{\gamma}_i^{j+\mu_i})}{\Gamma(j+\mu_i)}$$ (2)

where

$$g_{i,j} = \frac{\Gamma(j+\mu_i) \left( \frac{\mu_i \kappa_i}{\mu_i \kappa_i + m_i} \right)^j \left( \frac{\Gamma(j+\mu_i)}{\Gamma(m_i)} \right) \left( \frac{m_i}{\mu_i \kappa_i + m_i} \right)^m}{j! \left( \frac{\Gamma(j+\mu_i)}{\Gamma(\mu_i)} \right) \Gamma(\mu_i)}$$ (3)
where $\lambda_i = \mu_i (1 + \kappa_i) / \gamma_i$. From (2) and the Gamma distribution, we get

$$f_{\gamma_i}(\gamma) = \sum_{j=0}^{\infty} g_{i,j} G(\lambda_i, \beta_i; \gamma)$$

(4)

where $G(\lambda_i, \beta_i; \gamma)$ is the PDF of the Gamma distribution and $\beta_i = j + m_i$.

Lemma: The CDF expression is derived as

$$F_{\gamma_i}(\gamma) = \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} g_{i,k} g_{2,j} \frac{1}{\Gamma(\beta_1)\Gamma(\beta_2)} G_{1,3}^{2,0} \left( \begin{array}{c} \gamma_1, \lambda_2 \\ \beta_1, \beta_2, 0 \end{array} \right)$$

(5)

Proof: See Appendix.

It should be noted that the proposed CDF expression includes Meijer’s G function that is available in commonly used computational software such as Matlab and Mathematica.

3 PERFORMANCE ANALYSES

This section presents the OP and the average BER analyses for the cascaded double $\kappa-\mu$ fading channels by using the CDF approach.

3.1 OP Analysis

The OP which is significant performance metric for wireless communications systems is described as the probability that the total instantaneous SNR is lower than a specified SNR threshold ($\gamma_{th}$).

Therefore, by employing (5), and after the suitable substitutions ($\gamma_{th} \rightarrow \gamma$), the OP term for the considered cascaded fading model is obtained follows as
Now, we propose an asymptotic OP expression in order to gain further insights on OP for the cascaded double $\kappa - \mu$ shadowed fading channels. For high SNR analysis, the sum expressions in (6) can be ignored. Thus, we truncated $k = 0$ and $j = 0$. So, the asymptotic OP expression is derived as

$$
OP_{asymp} = \left( \frac{m_1}{\mu_1 \kappa_1 + m_1} \right)^{m_1} \left( \frac{m_2}{\mu_2 \kappa_2 + m_2} \right)^{m_2} \frac{1}{\Gamma(m_1) \Gamma(m_2)} \left( \frac{\Gamma(\mu_1)}{\Gamma(m_1)} \right) \left( \frac{\Gamma(\mu_2)}{\Gamma(m_2)} \right) \\
\times \left( \frac{\Gamma(\beta_1)}{\beta_1} \right) \left( \frac{\Gamma(\beta_2)}{\beta_2} \right) \left( \gamma_{th} \right)^{\lambda_1 \lambda_2} \left( \beta_1, \beta_2, 0 \right) \tag{7}
$$

3.1 Average BER Analysis

For various binary modulations, the average BER is calculated as [17]

$$
P_e = \frac{q^p}{2\Gamma(p)} \int_0^{\infty} e^{-q\gamma} \gamma^{q-1} F_\gamma(\gamma) d\gamma \tag{8}
$$

where $p$ and $q$ are modulation constant. For example, $p = 0.5, q = 1$ for binary phase shift keying (BPSK) modulation, $p = 1, q = 1$ for differential binary phase shift keying (DBPSK) modulation. $F_\gamma(\gamma)$ is the CDF expression in (5). Substituting (5) into (8) and after some mathematical manipulations, we have
\[ P_e = \frac{q^p}{2\Gamma(p)} \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \frac{\vartheta_{k,j}}{\Gamma(\beta_1)\Gamma(\beta_2)} \frac{1}{\Gamma(p)} \int_{0}^{\infty} e^{-\gamma q^{-1}} G_{2,3}^{1,2} \left( \frac{\lambda_{k,j}}{\beta_1, \beta_2, 0} \right) d\gamma \]  

(9)

With the help of [18, eq. 7.813.1], the average BER expression for the considered channel is derived as

\[ P_e = \frac{q^p}{2\Gamma(p)} \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \frac{\vartheta_{k,j} q^{-q}}{\Gamma(\beta_1)\Gamma(\beta_2)} \left( \frac{\lambda_{k,j}}{\beta_1, \beta_2, 0} \right) \]

(10)

Similarly to (7), at high SNR, for \( k = 0 \) and \( j = 0 \), the asymptotic BER statement can be obtained as

\[ P_{e_{\text{asymp}}} = \frac{q^p}{2\Gamma(p)} \left( \frac{m_1}{\mu_1 \kappa_1 + m_1} \right)^{m_1} \left( \frac{m_2}{\mu_2 \kappa_2 + m_2} \right)^{m_2} \frac{1-q.1}{q} \]

(11)

4 NUMERICAL RESULTS

Here, we provide some numerical illustrations for the OP and the average BER of the cascaded double \( \kappa-\mu \) shadowed fading channels. We set as \( \kappa_1 = \kappa_2 = 1.2 \) and \( \mu_1 = \mu_2 = 2 \) for both OP and the average BER results. Exact simulations obtained by the exact expressions are also contained to confirm the analytical results.

The OP results are demonstrated in Figs. 1 and 2 as a function of the \( \bar{\gamma}_1 \) for various \( m \) parameter, \( \gamma_{dh} \) and \( \bar{\gamma}_2 \) values. Clearly, it can be observed from Figs. 1 and 2 that the simulations provide an excellent match to the theoretical and asymptotic results, verifying the correctness of the proposed derivations. Furthermore, from Figs. 1 and 2, we can see that the higher \( m \) and \( \bar{\gamma}_2 \) values provide leading to performance improvement. Conversely, the higher \( \gamma_{dh} \) values
appear to cause performance degradation, as expected. For instance, considering $m_1 = m_2 = 1$ and $\gamma_2 = 5$ dB for $10^{-2}$ OP, $\gamma_{sh}$ value is 10 dB, while approximately 27.5 dB is required, while $\gamma_{sh}$ value is 5 dB, approximately 22.5 dB is needed. This indicates a gain of about 5 dB. It should be noted that there is a similar performance improvement with the increase of $m$ parameter value. These observations show the roles of the shadowing effect and threshold impact for the OP evaluation.

In Fig. 3, the average BER of the cascaded double $\kappa-\mu$ shadowed fading channels is depicted versus the $\bar{\gamma}_1$ under different modulation schemes (BPSK and DPSK) and $\bar{\gamma}_2$ conditions. It is seen that the theoretical findings of the average BER are in fine agreement with the simulations. Interestingly, it can be seen from this figure that as the values of the $\bar{\gamma}_2$ increase, the average energy used for the second line connected cascade increases, and the quality of the received signal increases, resulting in an increase in the average BER. It is worth mentioning that the performance improvement for the average BER when we employ BPSK modulation. For instance, for $\bar{\gamma}_2 = 5$ dB and BER level of $10^{-3}$, as modulation scheme changes from BPSK to DPSK, $\bar{\gamma}_1$ requires to be increased by about 3 dB. Further, it can be observed that the asymptotic findings given in (11) match fully the theoretical expression proposed in (10) proving the certainty of the derived expressions.
5 CONCLUSION

In this study, based on Laplace transform methods, we have proposed a new CDF expression for the cascaded double $\kappa-\mu$ shadowed channels which the generalized fading model. The proposed new model is quietly simple and analytically tractable. Moreover, it is appropriate for the performance analysis of the cascaded double $\kappa-\mu$ fading channels. Therefore, we have derived elementary and precise closed-form derivations for primary network performance metrics such as the OP of different scenarios, the average BER of different modulation schemes by applying the CDF method. Finally, this study demonstrates some numerical examples to approve and shows the theoretical derivations proposed in this study.
APPENDIX

Proof of the Lemma

We assume that $a_i$, $i = 1, 2$ are independent RVs and the PDF is the Gamma distribution which is given by

$$f_{a_i}(x) = \sum_{j=0}^{\infty} \mathcal{G}_{i,j} \frac{\lambda_i^j x^{j-1} e^{-\lambda_i x}}{\Gamma(\beta_i)}$$  \hspace{1cm} (12)

The Laplace transform of $a_i$ is given as

$$\mathbb{E}[e^{-sa_i}] = \sum_{j=0}^{\infty} \mathcal{G}_{i,j} \frac{\lambda_i^j}{\Gamma(\beta_i)} \int_0^{\infty} e^{-sx} x^{j-1} e^{-\lambda_i x} dx$$ \hspace{1cm} (13)

Using $e^{-px} = G_{0,1}^{1,0} \left( px \left| \begin{array}{c} - \\ 0 \end{array} \right. \right)$, the expression in (13) is rewritten as

$$\mathbb{E}[e^{-sa_i}] = \sum_{j=0}^{\infty} \mathcal{G}_{i,j} \frac{\lambda_i^j}{\Gamma(\beta_i)} \int_0^{\infty} x^{j-1} G_{0,1}^{1,0} \left( sx \left| \begin{array}{c} - \\ 0 \end{array} \right. \right) G_{0,1}^{1,0} \left( \lambda_i x \left| \begin{array}{c} - \\ 0 \end{array} \right. \right) dx$$ \hspace{1cm} (14)

where $G_{p,q}^{m,n} \left( \cdot \right)$ is the Meijer’s $G$ function. With the help of [14, eq. 2.24.1.1], the Laplace transform of $a_i$ is obtained as

$$\mathbb{E}[e^{-sa_i}] = \sum_{j=0}^{\infty} \mathcal{G}_{i,j} \frac{1}{\Gamma(\beta_i)} G_{1,1}^{1,1} \left( \frac{s}{\lambda_i} \left| \begin{array}{c} 1-\beta_i \\ 0 \end{array} \right. \right)$$ \hspace{1cm} (15)

Now, the Laplace transform of $a_1a_2$ is written as
Following the same steps for (15), we have

\[
E[e^{-sa_{12}^2}] = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} g_{1,j} g_{2,k} \frac{\lambda_1 \lambda_2}{\Gamma(\beta_1) \Gamma(\beta_2)} \int_0^\infty x^{\beta_1-1} e^{-\lambda_1 x} dx 
\]

The expression in (17) is obtained by employing [15]. Then, by using [16, eq. 3.38.1], the inverse Laplace transform of (17) is obtained as

\[
f_\zeta(z) = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} g_{1,j} g_{2,k} \left\{ \Gamma(\beta_1) \Gamma(\beta_2) \right\}^{-1} \frac{1}{z} \left[ z \lambda_1 \lambda_2 \right]_{\beta_1, \beta_2} 
\]

Besides, the term in (18) is the PDF of \( a_1 a_2 \). Using \( F_x(x) = \int_0^x f_x(x) dx \) identity and [14, eq. 2.24.2.2], the CDF is derive as

\[
F_x(z) = \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} g_{1,k} g_{2,j} \frac{1}{\Gamma(\beta_1) \Gamma(\beta_2)} G_{1,3}^{2,0} \left[ z \lambda_1 \lambda_2 \right]_{\beta_1, \beta_2, 0} 
\]

The proof is complete.
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FIGURE CAPTIONS

**Figure 1.** OP curves with $\gamma_{th} = 10$ dB as a function of $\overline{\gamma}_t$

**Figure 2.** OP curves with $\gamma_{th} = 5$ dB as a function of $\overline{\gamma}_t$

**Figure 3.** Average BER curves as a function of $\overline{\gamma}_t$
Figure 1. OP curves with $\gamma_m = 10$ dB as a function of $\bar{\gamma}_1$. 
Figure 2. OP curves with $\gamma_a = 5$ dB as a function of $\bar{\gamma}_1$. 
Figure 3. Average BER curves as a function of $\bar{\gamma}_1$