MOND rotation curves of very low mass spiral galaxies
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ABSTRACT

We present MOND analysis for several of the lowest mass disc galaxies currently amenable to such analysis—with (baryonic) masses below $4 \cdot 10^8 M_\odot$. The agreement is good, extending the validity of MOND and its predicted mass velocity relation, to such low masses.

Subject headings: dark matter galaxies: kinematics and dynamics

1. Introduction

The MOND paradigm (Milgrom 1983) has received a substantial boost with the recent advent of relativistic formulations of the theory [e.g., bekenstein(2004)]. MOND’s raison d’être remains, however, largely phenomenological. It is crucial then to continue to test the predictions of MOND in different contexts. The flagship of phenomenological testing of MOND is the analysis of rotation curves of disc galaxies because such data provides the most precise description of the radial dependence of force in an extragalactic context. Indeed there is a large body of work applying such analysis for about 100 disc galaxies with cogent success (Sanders & McGaugh 2002). Analysis of more galaxies with similar success will further buttress the case for MOND. It will also make it ever more unlikely that MOND is “just some clever way of summarizing tight relations between baryon and dark matter (DM) distributions”. MOND predicts a unique and history independent relation between the baryon distribution and the dynamics for each individual galaxy. It is inconceivable that the DM doctrine will reproduce this kind of predictions for individual galaxies: In the DM picture the relation between the distribution of the dissipative, interactive, and potentially explosive baryons and that of the inert DM should depend drastically on the unknown history of formation, intrinsic evolution, and interaction with the exterior of each individual galaxy.

Beside this quality that we can achieved by quantity, we should also strive to extend the range of galaxy types that we subject to rotation-curve analysis. For example, it is important to extend the analysis to include more high accelerations (high-surface-brightness) galaxies, for which MOND predicts initially declining rotation curves, and of which there are only a few examples currently in the literature. In another limit, that of low mass (and hence low velocity) spirals, there exist MOND analyses for quite a few galaxies, with particularly reliable data, having maximum rotational velocities in the range of $50 – 100 \text{km s}^{-1}$ (Milgrom & Braun 1988; Begeman Broeils & Sanders 1991; Broeils1992; de Blok & McGaugh 1998; Sanders & Verheijen 1998).
For spirals of even lower masses it becomes increasingly difficult to come by specimens with very reliable data. Among other problems such spirals tend to be of irregular shape and kinematics, kinematics that are more difficult to interpret as pure rotation (e.g. with misaligned kinematic and photometric axes). They are also characterized by increasing role of pressure support in comparison with the rotational support: Dynamical study of such galaxies may require then large asymmetric drift corrections, which are rather uncertain. [see e.g., (Côté Carignan & Freeman 2000; Begum & Chengalur 2004a; Begum & Chengalur 2004b)]. Obtaining accurate distances is also an issue since the nature of these galaxies constrain us to look at nearby ones for which Hubble distances are not applicable and Cepheid distances are, most often, unavailable.

Even so, these galaxies present a stringent test, not only because of their extremeness, but also because they are by and large fully in the deep MOND regime, showing very low accelerations at all radii. Also, their mass is generically dominated by gas, whose surface density distribution is determined directly and so the comparison with MOND is, in many cases, rather insensitive to the assumed or fitted M/L value. The MOND result in such cases is all but a pure prediction as opposed to a one parameter fit.

Here we test MOND on three galaxies with maximum rotational velocities of less than 50 kms$^{-1}$, which, according to MOND, corresponds to baryonic masses below 4 $\cdot$ 10$^8$ $M_\odot$. These have regular gas and velocity distributions. And, very importantly, all three have high inclinations, which minimizes errors due to inclination corrections. They also have relatively small asymmetric drift corrections. We also describe the results for one galaxy that is of even a lower mass, but isn’t all that ideal from the above points of view. We include it here more as a potential case study for the possible effects of the uncertainties inherent in small inclinations and large asymmetric-drift corrections.

Rotation curve predictions of MOND contain several sub predictions pertaining to partial aspects of the dynamics. One of these is an exact relation of the form $V_\infty^4 = MGa_0$ between the asymptotic rotational velocity, $V_\infty$, and the total (baryonic) mass, $M$, of a galaxy. This is not as probing a prediction as a full rotation curve analysis but requires less knowledge–e.g., of the exact mass distribution in a galaxy–and can thus be applied to larger samples. The galaxies we consider here have already featured in the mass-velocity plot shown by McGaugh (2005,2006) and all indeed fall on the predicted MOND mass-velocity relation, stretching its tested gamut to some five orders of magnitude in mass.

We describe the data in section 2 and the results of MOND analysis in section 3. These are discussed in section 4.

2. The Data

The data for the galaxies we consider come from recent work of Begum, Chengalur, and coworkers. These include NGC 3741 (Begum Chengalur & Karachentsev 2003), KK98 250 (=UGC
We do not analyze two other low mass galaxies, GR8 (Begum & Chengalur 2003) and CAM B (Begum Chengalur & Hopp 2003), because we deem them unfit for RC analysis (see reasons below).

NGC 3741 has an HI disk that is very extensive compared with the light distribution, and that exhibits a well ordered velocity field. It has an estimated distance of 3 Mpc based on the tip-of-the-red-giant-branch method (TRGB), and its inclination varies between 58° and 70° in a tilted-ring analysis. Begum and Chengalur (2005) attribute the sharp rise of rotational velocity at small radii to the bar (non-circular motions).

KK98 250 and KK98 251 also have well ordered velocity maps. The first has an inclination of 80°±4° based on the HI ellipticity (consistent with optical one of 79°). We use the HI rotation curve from Begum and Chengalur (20004a) and the $H\alpha$ measurements in the inner parts from McGaugh, Rubin, & de Blok (2001), provided in tabular form by Stacy McGaugh. KK98 251 has an inclination of 62° ± 5° from the axes ratio of the HI distribution (consistent with what is obtained from the kinematics in the tilted-ring fits: 65°); we know of no $H\alpha$ velocities for this galaxy. There are no direct distance measures to these two galaxies. The estimated distance to both, of 5.6 Mpc, is based on group membership (the NGC 6946 group) and on a mean distance to the group estimated from the brightest stars in eight members. The estimated asymmetric-drift correction for these two galaxies, which was included in the rotation curve we use, is small (Begum & Chengalur 2004a).

The distance to DDO 210, of 0.95 Mpc, is also based on the TRGB. The deduced rotation curve of DDO 210 is subject to two worrisome uncertainties. Its estimated inclination is small: 27°±7° as deduced from the ellipticity of the HI distribution which is rather irregular (a kinematic inclination could not even be derived from the velocity field). Also, while the estimated asymmetric drift correction for DDO 210 is not as large as that for CAM B it does require a substantial correction (see below).

The two galaxies from Begum and Chengalur we have not included are GR8 and CAM B. The data for GR8 is practically impossible to interpret as a rotating galaxy (e.g., its kinematic axis is almost perpendicular to the major axis of the HI distribution) and so no rotation curve is given. CAM B presents more regular kinematics than GR8, but the main problem we see in this case is that it is subject to a very large (and very uncertain) asymmetric-drift correction. So large, in fact, is the correction that it totally dominates the resulting curve. Taking the correction applied by Begum and Chengalur at face value it follows that CAM B is by and large supported by random motions, with the rotational support making only a small contribution. The deduced pressure force is 4 times the centripetal force at 0.1 kpc, with the ratio between the two increasing to about 9 at the last measured point. The asymmetric drift correction is not only uncertain, as always, but in this case the standard formula used is not really valid.
Fig. 1.— The MOND rotation curves (solid curves) for the four sample galaxies assumed to be at the distances adopted by Begum et al. The filled circles show the rotation curve obtained from the 21 cm line observations. In the case of KK98 250, the crosses show the rotation curve in the inner region derived from H$_\alpha$ observations. The dotted and dashed curves show the Newtonian rotation curves for the stars and the gas respectively.
3. Results

To calculate the MOND curve \(v(r)\) we use the algebraic MOND relation

\[
\mu(a/a_0)a = a_N
\]

with \(a = v^2/r\) the true (MOND) acceleration and \(a_N\) the Newtonian acceleration calculated from the mass distribution. We take \(a_0 = 10^{-8}\) cm \(s^{-2}\), and use \(\mu(x) = x(1 + x^2)^{-1/2}\). Note though that the exact form of \(\mu\) is rather immaterial here since all four galaxies are deep in the MOND regime so only the linear, \(x \leq 0.3\), part of \(\mu\) is probed. Also, for such galaxies the exact value of \(a_0\) is not well constrained as its effect is degenerate with that of the inclination and that of the distance, which is not well known; the three appearing in the fits in the combination \(a_0D^2sin^4i\) (Milgrom 1988).

As in previous analysis (Sanders & Verheijen 1998) both the stars and gas are assumed to be in a thin disk and the HI surface density is increased by a factor of 1.3 to account for the presence of primordial helium. The distances (all as adopted by Begum, Chengalur, and collaborators), luminosities, total gas mass, acceleration at the last measured point of the rotation curve, along with the fitted mass-to-light ratios, are given in Table 1.

Figure 1 shows the comparison of the measured rotation curve with that deduced from MOND for the four galaxies under study. In the case of KK98 250, the HI kinematic data has been supplemented with \(H\alpha\) data in the central regions.

4. Discussion

Due to the paucity of data, as well as the intrinsic uncertainties noted above, there have been very few studies of low rotation velocity, gas dominated dwarfs in the literature. Lake (1989) had in his sample two galaxies with nominally as low masses as ours, but their stated velocities, and

| Table 1: Adopted and derived properties of sample galaxies |
|-----------------------------------------------|
| Object | \(D\) (Mpc) | \(L\) \((10^8 L_\odot)\) | \(M_{gas}\) \((10^8 M_\odot)\) | \(a_f\) | \(\langle M/L \rangle_M\) |
|--------|-------------|-------------------|-------------------|--------|-------------------|
| KK98 250 | 5.6 | 1.15 (I) | 1.6 | 0.2 | 1.5 |
| NGC 3741 | 3.0 | 0.28 (B) | 2.0 | 0.12 | 1.5 |
| KK98 251 | 5.6 | 0.22 (B) | 1.0 | 0.2 | 0.5 |
| DDO 210 | 0.95 | 0.027 (I) | 0.036 | 0.19 | 0.2 |

The second column gives the distance as adopted by Begum, Chengalur, and collaborators; the third column gives the luminosity in either the I band or B band; the fourth column gives the total gas mass including primordial helium, the fifth column gives the centripetal acceleration at the last measured point on the rotation curve in acceleration units of \(10^{-8}\) cm \(s^{-2}\), and the last column gives the deduced MOND M/L value in solar units, for the band corresponding to the photometry.
hence masses, were based on low and uncertain inclinations (see discussion in Milgrom 1991). Côté Carignan & Freeman (2000) discusses SDIG, a galaxy with very low rotational velocities, but it is similar to GR8 and not amenable to a decent rotation curve analysis.

We show here, albeit with a very small sample, that, like their higher mass kin, the lowest mass spirals with reasonable data conform to the predictions of MOND. On the face of it, the agreement is not as nearly perfect as in many higher quality specimens, but considering the intrinsic uncertainties, the predicted rotation curves appear to be quite reasonable. The primary errors, not present in higher quality samples, result from uncertain distance determinations, non-symmetric or irregular gas distributions, ambiguous inclinations, and highly uncertain asymmetric drift corrections. Note also that here, as in all other data presented in the literature, only errors on the measured rotation curves are given, but there are no errors quoted on the mass distribution, which must be reflected as errors in the predicted MOND rotation curve.

We now discuss the results for individual galaxies.

KK98 250: The plotted rotation curve is from two sources: In the inner region, the higher resolution $H_\alpha$ data going up to about 2kpc (McGaugh Rubin & de Blok 2001), and also the lower resolution 21 cm observations that extend twice as far (Begum & Chengalur 2004b). In the inner parts the agreement with the MOND prediction is clearly better for the $H_\alpha$ data. This emphasizes the importance of higher resolution data in the inner parts especially for such small, low velocity galaxies. With the best fit MOND $M/L$ value, the stars and gas turn out to contribute similarly to the mass in this galaxy.

NGC 3741: This is a strongly gas dominated galaxy, with $M_{\text{gas}}/L \approx 7$; so that the adopted $M/L$ for the stellar disk hardly matters– the overall shape of the MOND rotation curve is determined essentially by the gas distribution. Indeed the principal uncertainty in this predicted curve results from the uncertainty of the azimuthally averaged gas density distribution. A 15% larger distance for this galaxy (consistent with the distance uncertainty) improves the agreement between the MOND and observed curves.

KK98 251: The predicted asymptotic rotation velocity agrees well with that observed. With the adopted stellar $M/L$ value the stars contribute only about 0.1 of the total mass; so, the deduced asymptotic MOND velocity is practically a prediction based only on the observed gas mass, and is rather insensitive to the assumed $M/L$ value (within reasonable range). As for KK98 250, the MOND curve lies above the observed 21 cm points in the inner regions. For KK98 250 this is rectified by the $H_\alpha$ data. The two galaxies are similar and we can expect, or predict, that the $H_\alpha$ rotation curve, which is still lacking for KK98 251, will better agree with the MOND curve in the inner parts.

DDO 210: This galaxy was already considered in light of MOND by Begum and Chengalur (2004b) who found good agreement with MOND, but leaving $a_0$ a free parameter they found a best fit value of $a_0 = 1.7 \cdot 10^{-8}$ cm s$^{-2}$, which is higher than the standard value. We notice that this is an artifact of a numerical error in the fitting program used and their correct best-fit value is in fact
\[ a_0 = 0.85 \cdot 10^{-8} \text{cm s}^{-2}, \] near to the value we use. We included this galaxy in our analysis mainly for the opportunity to demonstrate and discuss some of the sources of uncertainty besetting the analysis. The deduction of the gravitational acceleration in the plane of the galaxy (as expressed in terms of the published “rotation curve”) still requires a large asymmetric drift correction (although not as large as in CAM B): In the outer parts the “pressure” force is 4 times larger than centripetal force. The indicated errors stem only from the measured HI rotation curve and do not include uncertainties in the asymmetric drift correction. In addition, the adopted inclination is small and uncertain: The velocity field is clearly distorted and the tilted ring inclination fit failed to converge. For these reasons Begum and Chengalur (2004b) assumed a constant inclination estimated from the ellipticity of the HI distribution, which gave \( i = 27 \pm 7^\circ \) (this value totally disagrees with the optical inclination, which may be dominated by patches). This galaxy has \( M_{\text{gas}}/L \approx 1.3 \). With the stellar \( M/L \) value we find of about 0.2, the stellar mass is rather unimportant so the fit is not sensitive to \( M/L \). However, this value could be significantly higher than what we give, especially considering the above mentioned uncertainties.

A large sample of dwarf galaxies has been collected by Swaters (1999) and an analysis in terms of MOND is in preparation (Swaters and Sanders 2006). The predicted MOND rotation curves agree, in general, quite well with the observed curves particularly considering that galaxies in this sample are subject to similar uncertainties to those discussed here.

Note added: In a paper just posted on the net Gentile et al. (astro-ph/0611355) present new data, analysis, and MOND fits for NGC 3741. They find that to get a velocity curve from the velocity data one must include radial motions of considerable magnitude. Correcting for those is a rather uncertain, but necessary, procedure. This is in line with the worries expressed above regarding uncertainties in the analysis of such low mass galaxies. With their corrected rotation curve, Gentile et al. find a rather better agreement with MOND than we do, albeit for a somewhat larger distance of about 3.5 Mpc, compared with the pre-fixed value of 3 Mpc we use (our fit would also improve somewhat with a larger distance).

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