Children building and having fun while they learn geometry

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Abstract
Geometry is a basic discipline in STEM education. Recent educational reports, however, suggest that geometry is one of the subjects that sees the lowest levels of performance in the math curriculum. This paper presents a gamified itinerary through digital activities designed to teach geometry. Our aim is to attract Primary School children into the world of geometry. First, we scaffold geometry outcomes to take students progressively from basic geometric shapes to a stronger understanding of complex three-dimensional (3D) properties. Second, we use gamification to introduce fun into the learning process. Third, we propose digital activities in virtual environments, with which students can, in particular, develop 3D spatial perception. Concretely, children are immersed in a two-dimensional (2D)–3D world in which they play the role of architects who manipulate 2D and 3D shapes to create buildings. Results obtained in the evaluation of the system with 60 children showed an improvement in both their learning and their interest in math. Both children and teachers rated the experience very positively.

KEYWORDS
2D–3D geometry learning, gamification

1 | INTRODUCTION
Geometry is a difficult subject for students at primary and secondary levels. Indeed, the latest OECD Programme for International Student Assessment [38] pointed to the existence of significant difficulties in the assimilation of mathematical concepts, with geometry being one of the lowest-performing subjects in the maths curriculum. Specifically, four sub-areas—quantity, uncertainty and data, change and relationships, and space and form—were evaluated and overall performance was clearly worse in the latter two. This continued under-performance highlights the importance of improving geometry teaching, especially the curricular blocks dealing with space, form, and measurement.

In general, poor academic performance can be explained by external and internal factors. External factors put the focus on the home environment, the school, and the learning-teaching methods [35,36], while internal factors are related to children’s attitudes and emotions.
Specifically, in the context of geometry learning, several studies have identified the main reasons for low performance, focusing mainly on learning-teaching methods [5,10] involving an excessive reliance on memorized procedural processes, an imbalance in the implementation of the curriculum marked by an excess of calculation, a lack of focus on geometry in other blocks of non-strictly geometric contents, and above all, a lack of experimental activities in the classroom. The introduction of experimental activities in the classroom can be facilitated by an approach known as learning-by-doing, either physically or digitally [6,9,23]. On the one hand, schools are opting for physical and manipulative activities to practice shapes and their spatial relationships in Two-dimensional (2D) and three-dimensional (3D), which serve to better assimilate the concepts while keeping students motivated [8]. Furthermore, science museums are increasingly including this kind of activity in their informal learning programs [41,45]. On the other hand, digital activities may support learning theories such as situated cognition—which argues that learning takes place in a meaningful context [29]—and cognitive disequilibrium—which challenges students when they cannot fit new experiences into their existing knowledge schemes [42]. Digital activities may further assist the learning of geometry since the manipulation of 2D and 3D virtual shapes help students to perceive and analyze something that may be difficult to mentally imagine and put down on paper. Nevertheless, the state-of-the-art approaches that provide students with digital activities for geometry learning focus strongly either on the cognitive side [18] or on the fun side [32].

The two cornerstones of this study work rely on these two sides: scaffolded learning activities, which is the introduction of complex concepts and skills in a graduated fashion, and gamification, which is the use of game mechanics in nongame contexts. Therefore, our research hypothesis is that “Scaffolded gamified activities facilitate a motivated and successful learning experience of geometry,” and our contribution is an understanding of the design and evaluation of scaffolded gamified activities in the context of 2D–3D geometry learning. The designed gamified experience was evaluated with basic education students between 10 and 14 years of age at a school in the city of Huesca (Spain). It consisted of pre-experience and postexperience learning assessments, and questionnaires, and interviews with stakeholders (teachers and children).

This paper is structured as follows. Section 2 reviews approaches that gamified the learning of geometry, as well as the existing guidelines (or frameworks) focused on the design of gamified activities. In Section 3 we detail our gamification proposal based on the LEGA framework [26]. Section 4 describes the deployment of our proposal by means of a digital game. Section 5 presents the evaluation. Finally, Section 7 details our conclusions and the future work.

2 RELATED WORK

The complexity of learning geometry at an early age has been the subject of study of several pedagogical models proposed over the years [31,42], none of which have lost their validity with regard to the design of geometry courses nowadays [24,34,52]. Specifically, the model proposed by van Hiele [31] advocates the following principles: the learning of geometry is accomplished by passing through certain levels of thought and knowledge, these levels are not associated with ages, and only when one level is reached can one move on to the next. Dina and Pierre Van Hiele defined five levels which are summarized as Level 0: Visualization, where students perceive geometric shapes visually and identify figures, Level 1: Analysis, where students perceive geometric figures as sets of mathematical properties and elements, without yet making logical relations between them, Level 2: Informal deductions, where students understand dependency relationships between mathematical properties, Level 3: Formal deductions, and Level 4: Rigour, where students reason correctly about axiomatic systems. High achieving primary school students usually reach Level 2. At this point, it should be noted that any person, when faced with learning new geo-metric content, goes through all these levels and their greater or lesser mastery of geometry will influence whether they do so more or less quickly (Figure 1).

Following this idea of ordering these activities by levels, several experiments have been carried out in face-to-face classes [24,52] demonstrating the validity of these learning models. Recently, to reach out more to an audience that is digitally native, a small number of studies have made proposals based on digital platforms such as geoGebra [34] and VRMath [3], based on web-based online activities (such as Java-Applets zur Mathematik [16] and WisWeb applets [53]), and mobile apps with Augmented Reality [14] or VR [20]. Table 1 shows screenshots of these digital activities. They have generally been used to generate isolated activities to reinforce certain specific geometry concepts, and some of them have been used in learning pathways with encouraging results when evaluated with users [21].

In addition to these digital learning environments, in recent years a wide range of gamified approaches have been applied to motivate and engage students in the field of Geometry. Some of them used basic gamification
mechanics (GMs), such as Points, Badges, and Leader-Boards [14,44]. Others, such as [28] used two well-known badging platforms (ClassDojo and Class-Badges) and evaluated the effectiveness of the gamification approach.

The authors reported positive effects on the engagement of students, those who received the most rewards from the instructor achieved significantly better average performances. Moreover, the performance of the teacher

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**FIGURE 1**  Examples of interactive digital activities for learning geometry

**TABLE 1**  Intended learning outcomes (ILOs) classified according to the Bloom’s Revised Taxonomy (BT) and Van Hiele theory

| Bloom’s categories | Van Hiele levels | ILOs                                                                 |
|--------------------|-----------------|----------------------------------------------------------------------|
| BT3: Applying      | **Level 2 (Abstraction):** Students understand that properties are related and that one set of properties may imply another property. | - Build a complex target figure using basic ones. |
|                    |                 | - Differentiate movements in the 3D space: translation and rotation. |
|                    |                 | - Distinguish the projection of 3D objects on a plane and dihedral views. |
|                    | **Level 1 (Analysis):** Students can discuss the properties of the basic figures and recognize them from these properties. | - Predict polyhedrons spatially (solids in 3D with flat polygonal faces, straight edges and sharp corners or vertices), by means of their faces and their unfolding on the plane. |
|                    |                 | - Examine the relationship between faces of a 3D object. |
|                    |                 | - Calculate areas and perimeters of 2D polygons and compare them with other 2D shapes. |
| BT2: Understanding | **Level 0 (Visualization):** Students must identify the forms of basic geometrical figures (triangle, circle, square) by focusing on their holistic appearance, rather than their properties. | - Identify 2D shapes and recognize types of symmetry and angles. |
|                    |                 | - Identify 3D objects, and recognize 2D shapes (i.e. faces) that form them. |
| BT1: Remembering   |                 | - Recognize basic 3D objects that form a complex 3D object. |
using the tools was reported as being fundamental for the results of this study.

Others proposed serious games to play with geometry directly embedded in a videogame and these include (DragonBox Elements [15], MathToons Geometry Pro [25], and Math Playground [43] among others). Moreover, there have been preliminary proposals to produce authoring tools that allow teachers to modify the game [33,1]. Some examples of serious games related to geometry can be seen in Table 2.

These games are popular among students but are not easily transferable to the classroom due to a lack of adaptability to the current curricula, and to the skills of each student. In addition, they are external applications lacking assessment tools (Figure 2).

Last but not least, recent approaches recommend activities automatically according to students' progress during the course. Chungho et al. proposed an Adaptive Learning Path Recommendation System (ALPRS) that included a gamified prototype [12]. The gamified application contains a set of mini-games shown as a visual challenge with interaction and continuous feedback. As well as obtaining good results regarding the practice of geometry, different lineal learning pathways of 13 fixed activities were generated for the practice of primary school geometry concepts. These itineraries are different for each of the learning profiles. These 13 activities have been proposed by experienced geometry educators.

Following these ideas, in this paper, we propose the design of a game based on geometry-related missions in line with the desired learning outcomes of Primary School's current geometry curricula. These challenges are organized into a scaffolded itinerary based on the levels proposed in the Van Hiele learning model. The level of difficulty of each challenge will be adapted to a student's progression throughout the experience. We will employ the LEGA framework [26], which will help us to organize and identify the game mechanics as well the learning activities to be introduced in the challenges.

## Proposal for the Gamification of Geometry Learning

LEGA [26] is an iterative gamification design framework based on the Outcome-Based Education (OBE) theory [48], that focuses on learning goals (or outcomes). Thus, LEGA establishes a sequence of stages for the design, deployment, and evaluation of gamified learning activities (see Figure 3).

In the first stage, LEGA recommends determining the set of Intended Learning Outcomes (ILOs) to be achieved, establishing how to assess them, and defining how to measure the effectiveness of the gamification. In the second stage, LEGA proposes identifying the students' learning styles (LSs) [39], and the player types [11].

### Table 2 Summary of the identified Learning styles, their associated teaching/learning activities (TLAs), the identified learning mechanics (LMs), and the selected gamification mechanics (GMs) from the LEGA framework

| Learning style | TLAs | LMs | GMs |
|----------------|------|-----|-----|
| Activist (who learn by doing) | Puzzles | Instruction-Repetition; Action/task | LEGAL: On-boarding, progress, feedback, theme, time pressure, customization, surprise. ACH: Challenges/Quests, levels-progression PLA: Role-playing, Points, leaderboards, badges/achievements |
| Pragmatist (who need to see how to put the learning into practice) | Problem-solving | Instruction-Repetition; Questions/answers, Action/task, Demonstration, Imitation, Simulation | LEGAL: On-boarding, progress, feedback, time pressure, customization, surprise. SOC: Social Status FR: Unlockable content ACH: Challenges, quests, levels, and progression PH: Sharing knowledge PLA: Role-playing, Points, leaderboards, badges/achievements DIS: Alter the system |

Note: The acronyms GRAL, ACH, PLA, SOC, FR PH, and DIS refer to the Player Types: General—contains all types of players—, Achiever, Player, Socializer, Free Spirit, Philanthropist, and Disruptor, respectively.
The identification of LSs makes it easier to define the most suitable learning activities for each profile. Similarly, identifying player types can help to determine the most desirable GMs to be applied. At this point, in the third stage, the gamification design can be deployed either as manipulative activities in the classroom or as digital activities. Finally, in the last stage, the system is evaluated in terms of both learning and gamification aspects. On the basis of these LEGA stages, our proposal for gamifying the learning of geometry is detailed in the following sections.

3.1 Goals identification

This stage identifies, on the one hand, the ILOs in Geometry, that is, what the students should know or be able to do at the end of the course—, and on the other hand, the gamification objectives (see the first column in Figure 3). LEGA proposes classifying ILOs using Bloom’s revised taxonomy [2], which is a general learning theory. Focusing on geometry, Van Hiele [7,13], as introduced in Section 2 proposed five specific levels for understanding geometry that can be used to guide instruction and assess students’ abilities. Taking into account this proposal for promoting students’ geometric thinking, and considering that our audience consists of 10–13 years old students, we identified the following Learning Outcomes in the geometry curricula:

- Students are able to identify basic 2D and 3D geometric figures (Van Hiele Level 0).
- Students are able to identify the properties of geometric figures and their relationships (Van Hiele Level 1).
- Students are able to predict the relationship between the 2D faces of a 3D object and how to fold them from 2D to 3D.
- Students are able to identify and predict simple movements of 3D solids between two concrete positions, make decisions based on the movements made, and observe the results in 3D visualizations (Van Hiele Level 2).

Table 1 details these Learning Outcomes and their classification according to both of the above-mentioned taxonomies by Bloom and Van Hiele. Basically, students develop their skills in a progression marked by different milestones, from the identification of 2D and 3D shapes and their relationships to the application of the properties needed to build 3D figures from 2D polygons. Identifications correspond to Level 0 of Van Hiele’s model (and the first and the second Bloom Categories), and the 2D and 3D manipulations using unfolding, translations,
rotations, and projections correspond to Levels 1 and 2 in Van Hiele (and the third Bloom category). Moreover, as these Learning Outcomes cover both 2D and 3D concepts, we aim to move students progressively towards knowledge acquisition. This scaffold instruction allows us to break down the learning process into several phases. Thus, we introduce an itinerary in which students can progress from very basic 2D concepts through explorations of the relationships between the 2D and 3D spaces, to exploring the properties of solids and 3D projections.

On the other hand, the gamification objectives are to involve students in the process, increasing their motivation, interest, and knowledge in topics related to geometry. Moreover, the gamified activities are intended to reinforce students' cross-disciplinary competencies, including problem-solving, decision making, and becoming more autonomous and creative.

Additionally, in this first design phase, we define the metrics to evaluate the achievement of the ILOs and the effectiveness of the gamification. Evaluating competencies is not easy, and although there are different proposals in the educational community, new questions arise about what should be evaluated and how to do it [27]. In our design, we adopt metrics based on different approaches:

- Conventional paper-based exams in which students demonstrate their knowledge.
- Tracking their individual progress during the learning process, measuring the completion of the tasks and their successes and their errors.
- Measuring their perception of the knowledge acquired.

Thus, we will measure the student’ skills using a paper-based exam before and after the gamified learning experience (see the last column in Figure 3). Additionally, we will track their progress during the experience. Finally, we will measure students’ perceptions of the gamified experience by using satisfaction surveys.

### 3.2 Know the users

As we mentioned before, our design is focused on students between 10 and 13 years of age who are in the final stage of their primary studies and the first year of their secondary studies. Their profiles are very diverse. While some are on the way to becoming adults, others are still playing in the schoolyard. Despite this diversity, all of them can be characterized by their learning preferences (their so-called LS). LSs have been widely studied and applied in the field of education and can be grouped in terms of four labels (Active, Pragmatic, Theoretical, and Reflective) [39]. It should be noted that generally speaking, we can identify a predominant personal trend, though all LSs are present to some degree in all human beings [48].

The LEGA framework recommends identifying the main LSs of our audience to better design appropriate teaching/learning activities (called TLAs in the second row of Figure 3). Additionally, LEGA proposes the use of predefined questionnaires that help to identify the students’ LSs [40]. Moreover, in geometry learning, some works have recommended that learning activities should be performed in virtual environments, in which students can develop especially their 3D spatial perception [12,22].

Related to gamification, learners become players and, again, they can be described in terms of different types (Free Spirit, Achiever, Philanthropist, Player, Socialiser, and Disruptor). As happens with the LSs, while each person does not have one unique way to play [46], we can identify the most predominant profiles. Once more, LEGA recommends the use of a player type predefined questionnaire [11] to get to know our target audience, and also to include suitable GMs of the prevailing Player Types.

The most predominant LSs and Player Types were defined by means of the two respective questionnaires conducted on a prospective audience made up of 24 students from five different schools. The subjects were distributed equally in terms of gender and age. We aggregate all the LSs and the Player Types, and, in summary, we establish the main distribution of the LSs as Active (35%), Pragmatic (35%), Theoretical (20%), and Reflective (10%). Thus, following the LEGA recommendations based on [49], puzzles and problem-solving activities are the most suitable TLAs for the main identified styles (Activist and Pragmatist). The two first columns in Table 2 show the predominant LSs found in our prospective population along with the corresponding TLAs.

Furthermore, the distribution of Player Types indicates a uniform tendency among three labels—Achiever (25%), Player (25%), Socialiser (25%)—, and a few from the other profiles—Philanthropist (10%), Disruptor (10%), and Free Spirit (5%)—, though without any predominant trend. Therefore, we will include GMs that cover the most common identified player types.

### 3.3 The design of gamified learning activities

At this stage, we designed the gamification of learning activities based on puzzles and problem-solving exercises,
according to GMs, suitable for all player types. At this point, LEGA defines the most suitable GMS for each activity. Thus, for instance, if we select “Puzzles” as a teaching/learning activity, LEGA constrains the list of selectable GMs for an “Achiever Player” to “Points, leaderboards, and badges/achievements” (see Table 2). From among all the LEGA recommendations, we only selected the GMs shown in Table 2, taking into account the need to cover all player types.

Once GMs were selected, and as we mentioned in Section 3.1, we used scaffolding to progressively introduce complexity in the learning experience (see Figure 4). Thus, we planned to start with specific 2D puzzles, in which students had to identify 2D figures, types of symmetry, angles, and areas and perimeters, highlighting the following: (i) The identification of different types of triangles, quadrilaterals, and pentagons, (ii) The recognition of reflective symmetries and rotational symmetries, (iii) The identification of right, obtuse, and acute angles in 2D polygons, and (iv) Calculate the areas and perimeters of 2D polygons and compare them with other 2D shapes.

The second level of scaffolding requires the students to be able to construct a 3D figure from a set of 2D polygons. It is a problem-solving activity that involves joining 2D polygons, thereby performing polyhedral unfolding on the plane. This activity allows students to work on the concepts of space and shape and to make predictions and search for relationships between 2D faces in the unfolding of a 3D object (see Figure 4).

The third level of the scaffolding is a problem-solving activity in which students must build a complex 3D object using pieces of different shapes and sizes. The basic 3D pieces are cubes, tetrahedrons, squared pyramid, octahedrons, triangular prisms, and pentagonal prisms. First, students should identify the pieces needed in the construction, and then, by using 3D movements such as translations and rotations, they must assemble a targeted 3D object. This activity allows students to work on concepts of space and shapes in 3D, as well as make predictions and searches for relationships between polyhedra to compose a complex solid.

Finally, we propose an optional activity in which students are creators of shapes, that is, they are able to construct new artifacts. This kind of activity is considered to be a development tool by which students have the opportunity to alter the system, and contribute to it with new content, as well as having the opportunity to teach their peers.

Using the GM’s identified in Table 2, we propose the following design. We include an opening scene with a narrative (theme) to engage students to start the learning experience (on-boarding), and a brief tutorial that tells them how to play. The students play the role of builders who construct artifacts (or 3D objects). When they select an artifact, the first set of 2D puzzles is activated as a mission. This mission has a set of challenges related to 2D concepts. Once students have collected all of the 2D faces of the artifact, the next mission, which is related to the 2D-3D activity, is unlocked. Similarly, when this mission is accomplished, the 3D activity will be activated. All missions and challenges are time-pressured, with a countdown visible during the mission. Students can try to solve the challenges several times until achieving the mission objective. We also propose the addition of some surprise elements, such as Easter eggs as hidden bonuses, and penalizations. At each attempt, students receive feedback on their successes and failures. The level of complexity of the subsequent mission or challenge depends on their performance in the previous one. Additionally, students can win coins or points for each accomplished mission, as well as badges recognizing their improvements (for instance, they are awarded a badge if they end the challenge under the time limit, and they receive a badge if they have not made any mistakes).

Additionally, students can view their progress, their achievements, their coins, and their inventory at any time. Moreover, they can exchange coins for the polygons that they need. Furthermore, to promote relatedness and competition, each activity includes leaderboards, highlighting the social status of the players.

### 4 | GAMIFICATION DEPLOYMENT

This gamified design can be implemented in different ways: (i) using physical materials in practical sessions with students: cardboard shapes that can be selected, cut, folded and, used to create geometric constructions and
(ii) with digital activities integrated into a digital game that challenges the student to build virtual artifacts.

We chose the second option, which follows the DGBL (Digital Game-Based Learning) approach [30]. Indeed, the use of video games for learning has proved its potential for the teaching of more didactic, fun, and motivating curricular content [30,51].

### 4.1 System architecture

Our system incorporates people, data, and processes that interact to provide a playful and effective learning experience. Stakeholders are students and teachers, who play the game and who track their progress, respectively. Moreover, as introduced in Section 3, students have the possibility of building a new 3D complex figure and adding it to the system as a new challenge for their peers. Therefore, they play the role of creators of game content.

The managed data comprise the game state, that is, completed missions, current missions, collected objects, scores, and users’ accounts. Processes are mainly those required to execute the game logic, to facilitate students’ and teachers’ login, and to perform game persistence and tracking tasks.

The system uses a client-server architecture. On the one hand, the game is implemented using the Unity game engine, which is in charge of executing the game logic, acquiring user’s input, and providing the user with appropriate textual, visual, and auditory feedback. On the other hand, the web-based interface allows the teachers to monitor students’ progress.

Figure 5 depicts the software infrastructure. A Django server features a REST API that allows external applications, to access its services, such as database management and user authentication. On the left-hand side of Figure 5, the player downloads and starts the game. Then, the game obtains game session data and restores the current player status (including her inventory and accumulated points, etc.). On the right-hand side of Figure 5, teachers track students’ progress through a web-based GUI.

### 4.2 Digital game

The activities designed in Section 3 were integrated in a digital game. In the following, we use italics to indicate how the GMs (which were designed to be gender-neutral) referred to in that section were actually incorporated in the game.

The game addresses geometry competencies in the final stage of primary studies, which are usually revisited in the first year of secondary studies. Therefore, it is aimed at children between 10 and 13 years of age. The students play the role of architects who construct buildings in a small town (role-play, theme) in different campaigns. Concretely, one building is constructed per campaign.

One campaign consists of three missions, which correspond to three gamified activities (see Figure 4). Each mission is composed of one or more challenges.

As shown in Figure 8, when the player selects a campaign (i.e., a building), the system shows the list of shapes needed to construct the building and, next to them, the buttons that launch the different missions. For example, the construction of a fountain requires the player to collect 10 squares, 20 triangles, three pentagons, two triangular prisms, and one pentagonal prism. Concretely, three visual elements inform the player about
collected shapes: an icon representing the shape, two numbers (5/10 means the player has collected five squares from a total of 10), and green checks and red crosses indicating that the needed shapes have already been collected or not yet collected, respectively.

In the first mission, which consists of several challenges (or quests), the student collects all of the 2D polygons (faces) that define the borders of basic building pieces. Then, she accesses the challenges that define the second mission. There, the student defines the unfolding of 2D polygons to build 3D elementary pieces. Finally, in the third mission, the player faces a single big challenge, which is to create the 3D building, that is, to join those basic 3D pieces together into more complex 3D shapes, using rotations and translations. Note that each mission has locked content, which is unlocked when the previous mission is completed. For example, whenever the student gathers the eight squares needed to build a cube, she can undertake the mission of unfolding on the plane to effectively obtain a cube.

The completion of challenges provides a certain level of experience to the player in a specific competence (such as the identification of figures, symmetry, areas, and perimeters), which allows for the automatic adaptation of the game according to the progress of the player. As the players complete challenges, these become more complicated, showing more figures, applying restrictions to the construction of the figure, or involving the identification of objects with rotations or animations. In each challenge, the players get coins (points) with which they can buy more pieces and advance faster in the game, as

FIGURE 7  First row: 2D challenges, the second row 2D–3D: shapes folding in the plane, and third row: construction of complex 3D shapes from simpler ones. 2D, two-dimensional; 3D, three-dimensional

FIGURE 8  The fountain campaign was selected. The shapes that the player should achieve through different 2D, 2D–3D, 3D missions to build the fountain. 2D, two-dimensional; 3D, three-dimensional
well as earning access to more types of avatars for personalization (customization). Additionally, players win stars (badges) when they finish the challenge before the time limit or when they do not make mistakes.

**Mission 1: collect 2D faces**

The player is challenged to identify polygons of a specific type and with a specific area or angles. Polygons are situated in the cells of a matrix (see Figure 9). Depending on the correctness of the player’s actions, the game provides feedback in the form of traffic lights: the correct identification of polygons get a green light, incorrectly identified polygons get a red light, while polygons yet to be identified are given an amber light. Thus, the player has several opportunities to successfully overcome the challenge within a certain time limit (time pressure). The time is shown above the matrix of polygons. Traffic lights are located in the bottom right-hand part of the screen (if there are no traffic lights on a screen, it means that the player has not yet clicked the “Check result” button yet). Additionally, we included hidden “Extra Bonus” points in some of the cells in the matrix, as well as “Bombs” that explode and force the restart of the challenge (surprise).

The difficulty level of the next 2D challenge will increase or decrease depending on players’ progress. The difficulty level is graduated in different ways: by the number of cells in the matrix of polygons, by rotating the polygons within the matrix, and by including movement in the polygons. For example, if the player uses more time than estimated, the next challenge will have a matrix with fewer cells (see the 2 × 2 matrix for shapes identification and a 3 × 3 matrix for the identification of symmetries in Figure 9).

The game also keeps track of the missions accomplished by the player to decide on the pedagogical content of the next mission (progression). When players complete polygon identification missions, they are presented with area and perimeter, and symmetry missions to reinforce other concepts.

**Mission 2: collect basic 3D figures.**

The player is challenged to create basic 3D shapes using the 2D polygons collected in the previous mission. These (basic) 3D pieces can be used to construct the (complex) village building. The number of squares, triangles, and pentagons needed for the folding is indicated in the bottom part of the screen (over a red background as seen in Figure 10).

The player first selects and then situates different 2D polygons on the plane. The game shows visual guides (feedback), indicating the target positions for new polygons. Afterward, she then clicks on the Ready button and the system validates the proposed unfolding by showing an animation. See Figure 10 for an invalid and a valid unfolding. If the player changes her mind, she can click on the Reset button and start again.

When the player has mastered the unfolding technique, some of the positions in which the player can situate 2D polygons are blocked (see the bottom part of Figure 11) with the objective of increasing the difficulty of the challenge. The face that has just been situated is shown in orange, possible positions for new faces are shown in green, and blocked positions appear in red.

The following two characteristics are checked for a valid unfolding:

- The number of polygons of each type forming the target polyhedron (i.e., building).
- Face rotation rules define the animation and allow for error detection. The rules are defined as “X–X: α”, where X indicates the type of face at each end of the

![FIGURE 9 2D missions: from left to right, and top to bottom, shapes, symmetry, angles, and area and perimeter identification respectively. Note the different size of the matrices. 2D, two-dimensional](image-url)
edge (S = Square, T = Triangle, P = Pentagon) and \( \alpha \) is the allowed dihedral angle between faces.

This characterization allows the teacher to easily extend the game with additional figures, containing different types of faces.

**Mission 3: build complex 3D figures**

Finally, the campaign finishes when the player is challenged to construct the village building (see Figure 12). Given the basic pieces collected so far, the player should select the adequate ones, and translate and rotate them correctly. Pieces should join together to create the building. During this challenge, the student also manages projections in 3D as the display shows dihedral views (top, side, front) of the building.

Figure 12 shows the 3D environment the player interacts with during this mission. The player’s inventory, that is, the pieces acquired in the two previous missions, together with their number appears on the left-hand side of the image. The building is constructed in the center of the display, and the three 2D planes correspond to the dihedral views. The Ready button serves the purpose of checking whether the building was constructed correctly. The Save button in the bottom part of the screen allows the player to save the created building. In this way it is incorporated in the inventory, and, if approved by the teacher, can become another building (knowledge sharing) in the game.

**5 | GAMIFICATION EVALUATION**

**5.1 | Methodology**

We evaluated the design in terms of both learning and gamification metrics as defined in the first stage of the development (see Section 3).
Participants

A between-subject study was conducted with 60 students between 10 and 13 years of age, who had been selected from among volunteers at a school in the city of Huesca (Spain). They were in the last year of elementary school or the first year of secondary school. Teachers were strongly interested in targeting these two consecutive levels during which 2D and 3D geometry concepts must be assimilated. We also recruited two teachers who introduced the activities to the participants. We collected the parental consent forms informing parents of the anonymity and confidentiality of their data. We performed a randomized controlled study to compare student achievement in the gamified activity with traditional instruction, by dividing the participants randomly into two equally numbered groups: a control group whose members did not play the game, but rather they did exercises and an experimental group whose participants played the game. Students at both elementary and secondary levels were distributed uniformly between the control and the experimental groups.

5.1.1 | Procedure

We conducted the evaluation over a period of 2 weeks. During the first week, teachers explained geometrical concepts through lectures and direct instruction to all participants. After that, we measured student learning using an objectively scored paper-based test. In the second week, the students in the experimental group played the builder' game in the lab rooms for 2 h. We tracked their actions while playing the game. Meanwhile, students in the control group practiced in-class exercises. At the end of the second week, both groups did a second paper-based test (final exam). Students in the experimental group, who played the game, also filled in a questionnaire regarding their perception of the experience in terms of learning and fun.

The paper-based tests used before and after the game consisted of a 15-min questionnaire with six multi-answer questions covering 2D and 3D geometric competencies. They were scored between 0 and 10. Questionnaires on students’ perception of the learning process and satisfaction included Likert-type questions with responses varying from “1”: completely disagree, to “5”: completely agree.

Additionally, to complement this evaluation, we also gathered opinions from the teachers who participated in the experience. In the following, we measure the real impact of our proposed activities by comparing the results of both students groups.

5.2 | Results

In the following, we analyze the data gathered from the control and the experimental groups, with 22 and 31 subjects (N), respectively. Table 3 summarizes the minimum, maximum, median, and average values of grades. The first two rows show data obtained in the initial exam (before the experience) and in the final exam (at the end of the experience), while the last three rows show the progress achieved as the difference between the grades achieved in both exams (total, 2D and 3D progress). Overall, grades are rather low, which is in line with the results of the PISA report [38]. Negative values in some measures, mainly in the control group, maybe due to several reasons including geometry-related deficiencies carried over from previous courses, the lack of interest of students in the control group in the tasks, and again their disinterest while performing both the initial and final exams. In any case, as we will see in this section, the
comparison of both conditions (exercises vs game) shows significant differences in favor of those who played the game (Table 4).

If we compare the average grade achieved in the initial exam by both groups, we obtain the results shown in Figure 13. The average grade in the initial exam by the control group \( (M = 3.78, SD = 1) \), was slightly lower than the one achieved by the experimental group, \( (M = 4.23, SD = 1.31) \). Nevertheless, this difference is not significant, \( t(51) = -1.348, p = .184 \), so we can say that the starting point was similar in both groups.

When it comes to analysing the grades achieved in the final exam, the 31 participants who played the game \( (M = 5.02, SD = 1.39) \) performed significantly better than the 22 participants in the control group \( (M = 3.5, SD = 1.18) \), \( t(31) = -4.160, p = .000 \), Hedges’ \( g = 0.38 \), which represents a medium effect size (ES).

Significant differences appeared between the means values for the progress achieved by the experimental \( (M = 0.79 SD = 1.14) \) and control groups \( (M = -0.28 SD = 1.37) \) \( t(51) = -3.086; p = 0.003 \), strengthening the idea that the digital activity may provide a positive reinforcement of learning. Indeed, analyzing the progress in 2D and 3D outcomes in the experimental group, we observe a significant difference between average 2D progress \( t(30.97) = -2.829; p = 0.008 \). However, 3D progress is not statistically different between both groups \( t(51) = -1.040; p = 0.303 \). We think that this fact may be due to the inherent complexity of the 3D concepts, but it may also be due to the order of the locked missions. Each building starts with 2D missions and, some students did not have enough time to play the 3D missions, and were therefore unable to practice the challenges of polyhedron composition (Figure 14).

Finally, we compared these measurements within subjects in each group before and after the experience, rather than across groups. Whilst in the control group significant differences between before \( (M_{before} = 4, SD_{before} = 1.0) \) and after the experience \( (M_{before} = 3.5, SD_{after} = 1.18) \) were not found, the differences seemed significant in the experimental group \( (M_{before} = 4, SD_{before} = 1.31, M_{after} = 4.75, SD_{after} = 1.39) \). Indeed, using a paired differences \( t \) test at the two different points in time, we obtained differences in the experimental group \( (t(31) = -3.827; p = 0.001) \), although no differences were found in the control group \( (t(21) = 0.967; p = 0.345) \), Cohen’s \( d = 0.387 \), which represents a large ES.

To analyze the evolution of students’ grades (initial exam vs. final exam) in both conditions, we explored the data to uncover clustering patterns (see Figure 15).

Figure 15A shows clusters in the control group. Those students who obtained a low grade in the initial exam, the blue cluster (C1), improved more than those who obtained a medium grade, the red cluster (C2). The latter maintained almost the same grade. We think that the (low) degree of difficulty of the initial and final exams may have helped in the improvement of students with low grades in the initial exam and may have led to the observed lack of progress by the medium-grade students.

Furthermore, Figure 15B shows the data for students in the experimental group, that is, those who played the game. We observe that two groups (clusters) of students achieved better scores after the experience: those students with a previously low grade, the blue cluster (C1), and those students with a medium-high grade in the initial exam, the green cluster (C3). We believe that the low and fixed difficulty of the game at the very beginning of the game, and the dynamic difficulty adjustment (DDA) of the 2D and 2D-3D missions suited best low and high profiles of students, which suggests the need for a DDA redesign for considering medium level profiles. Of course, the fun factor may explain the good results. To conclude, our findings provide further evidence on the game’s impact on students’ learning. These results also align with the statistical difference found between mean scores before and after the experience in the experimental group. In line with the literature [37,50], our results support our research hypothesis “Scaffolded gamified activities facilitate a successful learning experience of geometry” and support the use of serious games as a complementary tool in geometry education.

### Table 3: Data from control (did exercises) and experimental (played the game) groups

|                     | Min  | Max  | Median | Average (SD) |
|---------------------|------|------|--------|--------------|
| Control group \( (N = 22) \) |      |      |        |              |
| Initial exam        | 1.5  | 5.5  | 4      | 3.78 (1.0)   |
| Final exam          | 1.5  | 5.5  | 3.5    | 3.5 (1.18)   |
| Progress            | −3.5 | 2.25 | −0.25  | −0.28 (1.37) |
| 2D Progress         | −2.25| 2.5  | 0      | 0.01 (1.24)  |
| 3D Progress         | −1.5 | 1    | −0.25  | −0.3 (0.73)  |
| Experimental group \( (N = 31) \) |      |      |        |              |
| Initial exam        | 2.25 | 7    | 4      | 4.23 (1.31)  |
| Final exam          | 2.25 | 8    | 4.75   | 5.02 (1.39)  |
| Progress            | −1.75| 2.63 | 0.5    | 0.79 (1.14)  |
| 2D Progress         | −0.75| 2.25 | 0.75   | 0.85 (0.72)  |
| 3D Progress         | −2   | 2    | 0      | −0.06 (0.88) |

1Our data samples were normally distributed \( (Shapiro(53) = 0.977; p = .405) \), with equal variances \( (Levene's test \( F = 1.355; p = .25) \).

2Our data samples were normally distributed \( (Shapiro(53) = 0.891; p = .558) \), with equal variances \( (Levene's test \( F = 0.660; p = .420) \).
Table 4: Questionnaire. 5-point Likert-type: scoring frequencies from Strongly agree to Strongly disagree

| Question                                                                 | Strongly Disagree | Disagree | Somewhat Agree | Agree | Strongly Agree |
|--------------------------------------------------------------------------|-------------------|----------|----------------|-------|----------------|
| Q1-Gameplay: I achieved the goals of the game in an easy and fun way     | 1                 | 5        | 6              | 9     | 2              |
| Q2-Motivation: I liked to learn geometry by playing                      | 3                 | 3        | 8              | 5     | 4              |
| Q3-Satisfaction: I would like to use games again to learn maths and other subjects | 4                 | 0        | 0              | 1     | 18             |
| Q4-Learning: I learned by playing the game                                | 0                 | 0        | 0              | 8     | 15             |

Figure 13: Box plot of initial exam (control and experimental groups)

Figure 14: Box plot of the final exam (control and experimental groups)

Figure 15: Unsupervised classification of students in control (left) and experimental (right) groups (k-means algorithm)

Clusters in the control group.  Clusters in the experimental group.
With the aim of gathering data on gameplay, perceived learning, and overall satisfaction, we designed a questionnaire for the 30 students in the experimental group. Twenty-three out of the 30 participants filled it in. The questionnaire contained five points Likert-type questions (Strongly Disagree, Disagree, Somewhat Agree, Agree, Strongly Agree) on both the UXs (User eXperiences). Additionally, qualitative questions allowed users to comment on the 2D, 2D–3D, and 3D challenges, and to give suggestions.

Overall, the responses were positive. More than 50% perceived the game to be easy to play (Q1-Gameplay—73, 91%). The game motivated to the majority of students (73.9%) to learn geometry (Q2-Motivation), and more than 82.6% said they would like to use games again to learn maths and other subjects (Q3-Satisfaction). Related to the perception of learning, 34.8% of participants thought that they had learned a lot about geometry (Q4-Learning), and the rest believed they had improved a little, while nobody believed that they had not learned anything with the activity.

Moreover, the students wrote observations and suggestions to improve the game. When asked about the 2D challenges, students found the interaction and the feedback clear, although some students were not entirely satisfied with the use of traffic lights for the interpretation of errors. Regarding the 2D–3D challenges, players found the interaction difficult but they especially liked the folding animations, which they believed helped them to understand their mistakes. Finally, the main criticism of the 3D challenges was the movement of the viewpoint needed to move the 3D pieces correctly. Additionally, they liked the customization of their avatar; the music, the bombs, the semaphores, the animation of 3D objects (repeatedly), and winning stars. However, they would have preferred a more complex storyline to motivate the construction of the town and a chatbot to interact with when they needed help during the missions.

Finally, teachers valued the experience very positively. They had no doubts about using games in the future as reinforcement for classroom activities. Although their school had few digital resources, our research encouraged them to push forward with similar initiatives.

Although we obtained positive responses to this questionnaire, they are preliminary and should be taken cautiously. We defined a simple questionnaire so as not to overwhelm students and also few students filled it in. However, the results encourage us to plan a study focused only on motivation and thereby further analyze the research hypothesis “Scaffolded gamified activities facilitate a motivated learning experience of geometry.”

6 | DISCUSSION

The literature reports tests to assess the Van Hiele levels of geometry reasoning of students with ages between 14 and 17 years [52,54]. Those tests proved to be good predictors of students’ performance in geometry and also confirmed that Van Hiele levels 0–3 (0-Visualization, 1-Analysis, 2-Informal deductions, 3-Formal deductions) were easily testable, except the fourth, 4-Rigour. Our gamification study was addressed to younger students aged 10–13, and thus our activities target the first three levels of the Van Hiele model.

Indeed, the analysis of students’ abilities in these levels can help us consider the use of new game mechanics in the game. For example, students at “Van Hiele level 2-Informal deductions” join 3D shapes together based on evident mathematical properties and relationships between parts (faces, edges, vertices), and move a solid from an original position to match a targeted position [4]. Nevertheless, those students at “Van Hiele level 3 and 4” reason based on the mathematical structure of the solids and their elements. Including properties not seen but formally deduced from the definitions or other properties. Students at these levels have a great ability in visualization and this ability allows them to make economic and accurate use of movements. For instance, rotation angles depend on the angles between particular faces or edges. Students can also transform a non-available movement into a sequence of available ones. These abilities at level 4 suggest us the possibility of including mechanics that reward, for example, students that reach the target figure in a few, or the optimal number of movements.

It is worth noting that our scaffolding design for gamified activities strictly follows the property of Van Hiele levels, as they were defined by the theory “a student must go through the levels in order” [31]. Moreover, our results agree with others that used the virtual manipulation of shapes for geometry learning at 10th grade [21], and also with those that proved the usefulness of games to enhance the learning of geometry concepts [17,47]. Nevertheless, our study has limitations. First, regarding the scale and timings of the evaluation, stronger conclusions can be extracted with a larger sample and also by conducting a long-term study. Results in the 2D and 2.5D gamified activities are definitively encouraging, but future studies should reveal further insights into 3D challenges.

Finally, we can draw a number of important conclusions that can help other researchers. First, the game design should smartly interleave learning and gaming concepts. That is, gamified activities should be conceived
to let the children connect mathematical representations (shapes, angles, coordinates) with images and figures in the game (characters, buildings, the positioning of elements). Similarly, the design should let children act on game elements so that they can apply geometrical operations on game elements (e.g., rotations, translations, and projections). Second, it is very important to know the students we are designing for because different factors (educational and cultural context, age, interests) should be considered when designing the game. Finally, dynamic adjusting of difficulty is an important feature of any game but is especially important in the learning context. Students' engagement will be much better with a balance between difficulty and skill. For example, low-performance students can progressively advance across the game levels without being frustrated because of their gaps in knowledge, and conversely, high-performance students should play at their own pace, even letting them become content creators, i.e. incorporating new challenges to the game.

7 | CONCLUSIONS

This study proposes the gamification of geometry learning, considering the scaffolding of content from 2D to 3D competencies. The design of gamified activities is based on a gamification design framework called LEGA [26]. It includes the teaching activities, the learning mechanics (LMs), and the GMs identified by LEGA for the characteristics of the target students (children between 10 and 13 years of age). These gamified activities were deployed as a digital game that challenges the student to play the role of an architect who constructs buildings in a small village. The results showed that the gamified activities helped to increase the performance of students in the experimental group. Additionally, the game helped to engage and motivate children, as the questionnaires revealed. Nevertheless, a small number of students (17.4%) disliked the experience. That encourages us to study the possible reasons and, if necessary, re-design the experience. Moreover, teachers were enthusiastic about the experience.

As future work, we plan to incorporate an authoring platform to allow teachers to configure (i.e. customize) some of the basic educational parameters of a personalized learning experience, and include new buildings. Additionally, as the questionnaires revealed, 3D interactions can be improved with the inclusion of new interaction styles (e.g., conversational). The multi-player version of the system is another line of development.

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CONFLICT OF INTERESTS
The authors declare that there are no conflict of interests.

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