Study on Band Gaps Characteristics of Local Resonance Phononic Crystal with Four-Core Structure

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Abstract. In this paper, a new type of local resonance phononic crystal structure is proposed. The band gap's formation mechanism and vibration characteristics are analyzed with finite element methods COMSOL5.3. The structure’s "mass-spring" simplified model is also established. Furthermore, the influences of the structure parameters on the bandgap are studied. It is shown that, the proposed structure is lighter in weight and can generate wider band gap ranging from 220.17~620.34 Hz at the same time. This new type locally resonant structure could have a potential application in reducing noise and vibration of aircraft or submarine’s cabin.

1. Introduction

The low frequency noise has stronger penetration capability in dissemination process, so it is hard to be attenuated by traditional sound absorption or insulation methods. Therefore, the low frequency noise can easily make adverse effects on industrial equipment and people's life and become one of the main forms of environmental pollution [1]. In the military field, with the increasing demands of modern warfare, the new generation high-tech weapons and equipment such as of UAVs, stealth fighters, and stealth submarines have placed higher requirements for their stealth performance. Due to the limitation of the noise reduction mechanism, the traditional silencing tile is not valid for low-frequency noise, especially those below 300 Hz. This makes the submarine easily exposed to the enemy's low-frequency sonar [2]. Especially in infrasound range, the low frequency noise could resonate with body organs easily and cause physical and mental harm to the soldiers’ health [3]. Therefore, how to effectively control low frequency noise has become a hot topic. The existence of band gap in phononic crystal, the artificial composite structure with periodic changes in materials, provides a new idea. In 2000, Liu [4] first proposed a three-dimensional local resonance phononic crystal with three components. The wavelength of the incident wave matching to the bandgap reached more than 300 times the unit size, which meant this local resonance phononic crystal could realize "using small size to control large wavelength". Due to the outstanding performance of local resonance phononic crystal in noise control and isolation, rod, shaft, beam, plate, grid, membrane and petal structures with subwavelength size characteristics have been studied to analyze formation mechanism and vibration characteristics of the structures’ band gap [5-17].

In this paper, a new type of multi-resonance phononic crystal structure is proposed. The formation mechanism of the structure’s band gap and its vibration characteristics are analyzed with finite element method COMSOL5.3. And the structure’s "mass-spring" simplified model is also established. Furthermore, the influences of the structure parameters on the bandgap are studied.
2. Design of local resonance phononic crystal
The unit structure of local resonance phononic crystal designed in this paper is shown in figure 1. And it consists of three parts: A (Epoxy base frame), B (silastic coating layer) and C (gold core). The outer length of the square base frame is \( a \) and the inner length is \( b \). The radius of small holes that are evenly distributed in the silastic coating layer is \( r \). The length of the square gold core is \( c \). The entire unit structure is strictly symmetrical. The structural and material parameters of designed structure are shown in Table 1 and Table 2 respectively.

![Figure 1. Unit Structure drawing.](image1)
![Figure 2. Irreducible Brillouin zone (shaded part).](image2)

| \( a/\text{mm} \) | \( b/\text{mm} \) | \( c/\text{mm} \) | \( r/\text{mm} \) |
|---|---|---|---|
| 20 | 16 | 4 | 2 |

3. Band gap diagram of phononic crystal
In this paper, the eigenfrequency of the two-dimensional infinite periodic structure is obtained by using the finite element software COMSOL 5.3. Based on Bloch theorem, Bloch-Floquet condition is applied. By varying the value of wave vector \( k \) in the Irreducible Brillouin zone (figure 2) and solving the spectral problem, the band gap diagram of local resonant phononic crystal can be obtained as shown in figure 3. Where X-axis is the wave vector \( k \), Y-axis represents the calculated eigenfrequency in Hz.

As shown in figure 3, in the frequency range below 200 Hz, the dispersion curve branches at point \( \Gamma \). This is because the wavelength of elastic wave in the local resonance phononic crystal structure is much larger than the lattice constant of the structure unit. Here, the propagation of the elastic wave in the structure is the same as propagation in the homogeneous medium, so the dispersion curve is linear. When the propagation of elastic waves is prohibited, the frequency range between dispersion curves is called band gap, as shown in the green part of figure 3. The designed local resonance phononic crystal structure has several complete band gaps nearly between 220.17 and 620.34 Hz.

4. Analysis of vibration characteristics and Simplified model of the structure
In order to further explain the formation mechanism of the band gap, six highly symmetric points of Irreducible Brillouin Zone (Point A-F) are selected, and their corresponding vibration characteristics are analyzed, as shown in figure 4.

The local resonance structure could be equivalent to a "mass-spring" system. When the elastic wave propagates in the local resonance unit, it can be equivalent to applying an external force \( F \) to the
"mass-spring" system. The spring will generate a reaction force $F'$, and the base frame will vibrate under the combined action of two forces.

![Figure 3. Band gap diagram of phononic crystal (A and B indicate the location of vibration modal in higher symmetry point $X$; C-F indicates the location of vibration modal in higher symmetry point $\Gamma$).](image)

When the frequency of the elastic wave is close to eigenfrequency of the local resonant unit, the resonant mode is activated. The elastic wave and the local resonant unit are strongly coupled and the energy is rapidly attenuated due to localization in the resonant unit. So the elastic wave is impossible to propagate, thus a band gap is generated. At this time, $F$ and $F'$ are equal in size and opposite in direction. So the resultant force of the external force on the base frame is 0 and the base frame remains basically stationary.

![Figure 4. Local vibration mode diagram (arrows indicate relative quantity and direction of displacement).](image)

The points A and B's vibration mode appear as the translational state of the gold core (figure3 (a), (b)). At this time, the mass block moves in translation, one side of the silastic is extruded, the other side of the silastic is stretched, and the outer base frame remains basically static. The local oscillator in the adjacent cell has opposite vibration phase, which makes the whole system keep dynamic balance.

Point C corresponds to the rotational resonance mode (figure3 (c)). Point D corresponds to the torsional resonance mode of the silastic coating layer (figure3 (d)). At this time, the shear deformation of the silastic coating layer only produces torsional force on the base frame, and there is no resultant
force in the X or Y direction. Therefore, it is difficult for the elastic wave propagating in the base frame to interact with the local resonance unit, so a flat band will be generated. The flat band is located in other band gaps and traverses the entire Brillouin region.

The points E and F's vibration mode appear as the translational state of the base frame (figure 3 (e), (f)). At this time, the base frame and the corresponding local oscillator (silastic coating layer) reverse vibration, and the inner gold core remains basically static. The local oscillator in the adjacent cell has the same vibration phase, which makes the whole system keep dynamic balance. When the frequency of the elastic wave is close to the cut off frequency of the local resonant unit, base frame forms an anti-phase vibration through the connection of the spring. The resultant force in the x or y direction is reduced, the coupling effect of the structure is weakened, so that the band gap is cut-off.

The simplified model corresponding to initial frequency of the structure is shown in figure 5 (a), where k is the equivalent stiffness of part of silastic coating layer, which acts like a spring. And m1 is the equivalent mass of four gold cores and part of silastic coating layer.

The simplified model corresponding to cut-off frequency of the structure’s band gap is shown in figure 5 (b). Where m2 is the equivalent mass of base frame and part of silastic coating layer, which vibrates with the base frame, m1 and k have the same meanings as above. At this time, the local resonance unit is equivalent to a "mass-spring-mass" system with two-degree of freedom. And there is a position in the spring that will remain stationary, that is, the static point.

The initial frequency \( f_1 \) is

\[
f_1 = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m_1}}
\]  

(1)

The cut-off frequency \( f_2 \) is

\[
f_2 = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k(m_1 + m_2)}{m_1m_2}}
\]  

(2)

5. Influences of factors on band gap

When elastic wave propagates in the base frame, the local oscillator vibrates, they have a strong coupling of each other that makes elastic wave impossible to continue to propagate, thus the band gap is generated. The initial frequency and bandwidth of the band gap are directly related to the effect of the coupling. Therefore, the size of the structure inevitable affects the band gap. In order to deeply analyze the influence of structural parameters on the local resonance band gap, further circumstances have been discussed.
5.1 Influence of outer length of the square base frame "a" on band gap
The outer length of the square base frame "a" is increased from 18 mm to 22 mm, while maintaining the other parameters the same. Band gap diagrams of local resonance phononic crystal structure with different "a" are obtained and the range of band gap varies as shown in figure 6.

It can be seen from the figure 6 that as the outer length of the square base frame increases, both the initial and cut off frequencies of the band gap decrease. And the width of band gap decreases as well. This is because when the outer length of the square base frame increases, the mass of the base frame increases, the effect of couple between elastic wave and the local resonance phononic crystal structure is weakened, so both the initial and cut off frequencies of the band gap decrease. As in equation (2), cut off frequency \(f_2\) is directly related with the mass of base frame. So the rate at which the cut off frequency decreases is significantly larger than that of the initial frequency.

5.2 Influence of radius of holes in silastic coating layer "r" on band gap
The radius of holes in silastic coating layer "r" is increased from 1.25mm to 2.5mm, while maintaining the other parameters the same. Band gap diagrams of local resonance phononic crystal structure with different "r" are obtained and the range of band gap varies as shown in figure 7.

It can be seen from the figure 7 that when the outer length of the square base frame increases, both the initial and cut off frequencies of the band gap decrease. And the width of band gap decreases as well. This is because when the outer length of the square base frame increases, the mass of the base frame increases, the effect of couple between elastic wave and the local resonance phononic crystal structure is weakened, so both the initial and cut off frequencies of the band gap decrease. As in equation (2), cut-off frequency \(f_2\) is directly related to the mass of base frame. So the rate at which the cut-off frequency decreases is significantly larger than that of the initial frequency.

5.3 Influence of length of the gold core "c" on band gap
The length of the gold core "c" is increased from 2 mm to 4 mm, while maintaining the other parameters the same. Band gap diagrams of local resonance phononic crystal structure with different "c" are obtained and the range of band gap varies as shown in figure 8.

It can be seen from the figure 8 that when the length of the gold core increases, the initial frequency of the band gap decreases at first, then maintains nearly still. And the cut-off frequency of the band gap increases sharply. As a result, the width of band gap broadens. This is because as the length of the gold core increases, the mass of the gold core increases, and \(m_1\) increases. As in equation (1), when \(m_1\) increases, the initial frequency \(f_1\) is reduced. But in the other way, when the mass of the gold core increases, the effect of couple is enhanced, so cut off frequency of the band gap increases.
6. Conclusion
In this paper, a new type of multi-resonance phononic crystal structure with four gold cores is proposed. The formation mechanism of the structure’s band gap and its vibration characteristics are analyzed with finite element method COMSOL5.3. This structure can generate a band gap ranging from 220.17~620.34 Hz, and the band gap width is 400.17 Hz. The structure’s simplified model is also established. Furthermore, the influences factors of band gap are fully discussed. By changing the structural parameters, the position of the band gap could be adjusted, achieving the purpose of reducing the load of the aircraft or submarine and making effective control of the low frequency at the same time. Thus the proposed phononic crystals structure has a practical application in vibration isolation and noise reduction, especially in low frequency noise reduction area.

Acknowledgements
This research was funded by the National Nature Science Foundation of China (Grant No.11504429).

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