The boundary condition at the valve for numerical modelling of transient pipe flow with fluid structure interaction

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Abstract. Transient flows in pipes (water hammer = WH) do appear in various situations and the accompanying pressure waves may involve serious perturbations in system functioning. To model these effects properly in the case of elastic pipe the dynamic fluid–structure interaction (FSI) should be taken into account. Fluid-structure couplings appear in various manners and the junction coupling is considered to be the strongest. This effect can be especially significant if the pipe can move as a whole body, which is possible when all its supports are not rigid. In the current paper a similar effect is numerically modelled. The pipe is fixed rigidly, but the valve at the end has a spring-dashpot mounting system, thus its motion is possible when WH is excited by the valve closing. The boundary condition at the moving valve is modelled as a differential equation of motion. The valve hydraulic characteristics during closing period are assumed by a time dependence of its loss factor. Preliminary numerical tests of that algorithm were done with an own computer program and it was found that the proper valve fixing system may produce significant lowering of WH pressures.

1. Introduction

Water hammer (WH) is a phenomenon produced by sudden change in steady pipe flow conditions due to valves operation, hydraulic machinery load variation or other reasons [1,2]. When the pipe is elastic the dynamic fluid-structure interaction (FSI) influences the flow and should be taken into account [3,4,5]. Three main fluid-structure coupling mechanisms are pointed in literature. The friction between the pipe wall and the liquid is the weakest one. The Poisson effect is responsible for the pipe-wall longitudinal stress wave generated by pressure-induced circumferential stresses and known as the precursor wave. The strongest FSI effect is the junction coupling which appears at pipe ends, bends, valves and other places where strong forces between the pipe and the liquid exist. This effect can be especially important if the pipe can move as a whole structure on the supports [6]. Such a motion forced by shock of the foundation can be even the source of WH event.

In general it could be expected that elastic pipe supports should influence in lowering of transient pressures due to the energy transfer from the liquid to the structure. This conclusion is however not unambiguous as reported by scientists [7] who pointed that elastic pipe supports and the junction coupling effect may also produce increase in the transient pressure magnitude. Thus it is important to determine the limitations for the supports parameters for which the pressure reduction is possible. In
the current paper the pipe supports system will not be the main goal, however a similar problem is examined. Valves operation is known to be one of the possible, and frequently used in the analyses, source of WH event. It is a known and utilized behaviour that setting the proper time dependence of the valve closuring process may highly reduce the transient pressures (usually elongation of the closuring time). In various scenarios, like check valves, such possibility may not exist, thus another solution can be valuable. An idea of a valve fixed in the pipe with a spring-dashpot system seems to be a natural concept, if the physics of WH phenomenon is considered. Such a possibility is mentioned in [1] but only some general remarks with no detailed solution nor conclusions are presented there. In the current paper the boundary condition (BC) at the valve is formulated as a differential equation of the valve motion and its solution for numerical application is found and preliminary tested.

2. Mathematical model
In the current study the four equations (4E) mathematical model [4,5,8] of WH-FSI is used. In this model one-dimensional (1D) liquid flow is assumed and only the longitudinal pipe motion is taken into account. The 4E model is reported [5] to give a good description of many real hydraulic systems. Moreover, the 1D treatment is justified, as the longitudinal effects at the valve are predominant.

2.1. Assumptions and governing equations
The standard assumptions of the WH-FSI 4E model are used herein. The pipe of the length L, inner diameter D and wall-thickness e is straight, slender (D/L<<1), thin-walled (e/D<<1) and prismatic of circular cross section. Its material of Young modulus E and the density $\rho_s$ is linearly elastic, which produces the standard relation for the celerity of longitudinal elastic waves in it:

$$c_s = \left(\frac{E}{\rho_s}\right)^{1/2}$$  \hspace{1cm} (1)

The liquid of bulk modulus K is weakly compressible, linearly elastic and has the density $\rho$. The density changes are small which is the consequence of the low pressure assumption (p/K<<1). The flow velocity $v$ is of little relativity to the celerity c of the pressure wave in the liquid ($v$<<c) given in:

$$c = \left(\frac{K}{\rho}\right)^{1/2}\left(1+(1-\nu^2)\frac{KD}{Ee}\right)^{-1/2}$$  \hspace{1cm} (2)

In the equation above $\nu$ is the Poisson coefficient and the second right-side factor is responsible for the influence of pipe wall elasticity. The cavitation is assumed not to be present. The liquid - pipe-wall friction shear stresses $\tau_s$ are assumed to be given with the quasi-steady formula ($\lambda$ is the Darcy-Weisbach friction factor and $w$ is the pipe cross-section velocity):

$$\tau_s = \frac{\lambda \rho}{8} |v-w||v-w|$$  \hspace{1cm} (3)

The 4E model is described with four hyperbolic partial differential equations of the first order [3,4,5]. Two of them govern the 1D liquid flow:

$$\frac{\partial v}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} = -g \sin \alpha - \frac{4}{\rho D} \tau_s$$  \hspace{1cm} (4)

$$\frac{\partial v}{\partial x} + \frac{1}{\rho e^2} \frac{\partial p}{\partial t} = 2v \frac{\partial w}{\partial x}$$  \hspace{1cm} (5)

The other two equations govern the pipe longitudinal motion:
\[
\frac{\partial w}{\partial t} - \frac{1}{\rho_s} \frac{\partial \sigma}{\partial x} = - g \sin \alpha + \frac{\tau_s}{\rho_s} e \tag{6}
\]

\[
\frac{\partial w}{\partial x} - \frac{1}{\rho_s c_s^2} \frac{\partial \sigma}{\partial t} = - \frac{\nu}{D} \frac{\partial^2 p}{\partial t} \tag{7}
\]

In the equations above \(x\) and \(t\) are the position and time coordinates, \(g (= 9.81 \text{ m/s}^2)\) is the gravitational acceleration and \(\alpha\) is the angle of the pipe inclination against the horizon. The pipe longitudinal stresses are \(\sigma = Q/A_s\) (\(Q\) is the longitudinal force and pipe cross section area is \(A_s = \pi De\)). For the solution of that model the method of characteristics (MOC) is frequently used \([3,5,8,9]\) and will be also applied in the current approach.

2.2. Method of characteristics

The standard MOC transformation of the governing equations produces two pairs of compatibility equations (CE) governing of two elastic waves. The WH wave is defined with a pair (+/-) of equations C0 presented below in the form proposed by the author \([10,11]\):

\[
\frac{d}{dt} \left[ v + Sw \right] \pm \frac{d}{dt} \left[ r - 3 q \right] = - \left[ 1 + S \right] g \sin \alpha - \left[ 1 - R \right] \frac{4 \tau_s}{\rho D} \tag{8}
\]

They are valid for positive (+) or negative (-) characteristic slope and \(x(t)\) dependence given with the relation \(dx/dt = \pm c_0\). The WH celerity \(c_0\) is slightly changed relative to (2) due to the Poisson coupling:

\[
c_0 = c A^{1/2} \tag{9}
\]

The precursor wave is governed by the following pair of equations C1:

\[
\frac{d}{dt} \left[ Rv - w \right] \pm \frac{d}{dt} \left[ \tilde{R} r + q \right] = - \left[ 1 - R \right] g \sin \alpha - \left[ 1 + S \right] \frac{\tau_s}{\rho_s e} \tag{10}
\]

They are valid respectively for the characteristics \(dx/dt = \pm c_1\) and the wave celerity \(c_1\) is:

\[
c_1 = c A^{1/2} \tag{11}
\]

In the above equations the unified variables were introduced. The normalized pressure [m/s] is defined as \(r = p/(\rho c_0)\) and normalized stresses [m/s] as \(g = \sigma/(\rho c_1)\). The above MOC transformation is valid for most practical cases, but in fact the condition \(1 - \chi > 0\) is necessary. The parameters \(\gamma\) and \(\chi\) are:

\[
\gamma = \frac{K \rho_s}{(E \rho)} \tag{12}
\]

\[
\chi = \frac{KD}{(E e)} \tag{13}
\]

The Poisson coupling constants \(S, R\) and the parameter \(A\) are defined with the following formulas:

\[
S = 4 \gamma \sqrt{\left[ (1 - \gamma + \chi) + \sqrt{(1 - \gamma + \chi)^2 + 4 \gamma \chi \nu^2} \right]^{-1}} \tag{14}
\]

\[
R = \frac{S \chi}{(4 \gamma)} \tag{15}
\]

\[
A = 1 + S \chi \nu \left[ 2 \gamma - S \chi \nu \right]^{-1} \tag{16}
\]

For water in steel pipe and \(D/e = 20\) these parameters are \(A = 1.016\), \(S = 0.042\), \(R = 0.027\). The “tilde” parameters have the following definition (\(\eta = c_0/c_1\)):

\[
\tilde{R} = R \eta \tag{17}
\]
\[
\hat{S} = \frac{S}{\eta} \tag{18}
\]

2.3. The solution scheme

In the equations (8) and (10) two types of fluid-structure couplings can be identified. The first one occurs due to the friction term on their right side. But the friction is weak, thus it can be taken into account iteratively and each of the waves C0 and C1 can be considered to be independent in its own “wave variables” kept in the parenthesis on the left side. The real physical quantities – pressure, stresses and velocities - are finally calculated as the combination of the proper wave variables producing superposition of WH and PC waves due to the Poisson coupling. To solve each of the waves a standard scheme is used. Integrating all the CE within the same time step \( \Delta t \) along their characteristic produces two pairs of finite difference equations (FDE) that can be solved marching in time on the properly designed space-time grid.

The pipeline may consist of a number of straight pipe reaches joint at junctions where proper BC exists. At such boundaries only half of the FDE are valid for each sub-pipe and to find the solutions additional equations of BC have to be formulated and taken into account. The basic BC for the liquid is the continuity equation which can be formulated in the following form (L=left, R=right):

\[
v_L - w_L = v_R - w_R \tag{19}
\]

The structure velocity \( w \) is in fact the same for both edges of the junction, which is rigid in itself, thus the above condition can be simplified to the equality of liquid velocities. At the close-end the BC is:

\[
v - w = 0 \tag{20}
\]

The pressure balance at the junction is the result of minor losses defined with the coefficient \( \zeta \):

\[
p_L = p_R + 0.5 \zeta \rho (v - w)^2 \tag{21}
\]

If the junction is fixed rigidly to the foundation the BC for the structure is \( w=0 \) and should be put to the above equations. However, if it is fixed with elastic mounting or is unfixed, the differential equation of motion should be formulated and solved concurrently with the CE.

3. Boundary condition at the valve

For a rigidly fixed valve the BC problem is limited to resolving the valve hydraulic characteristics during closing process. In the current study however, the major importance is devoted to the valve fixing system and its influence on WH run parameters.

3.1. Piping physical model

For the current analyses a pipeline physical model is composed of a pressure tank at the beginning, a straight pipe fixed to the foundation with a number of supports and a valve at the end. The flow is driven by the constant pressure of the tank. The valve is being closed with a certain time dependence to excite a water hammer. The valve is fixed to the pipe with an elastic mounting, which is modelled as a spring-dashpot system (Kelvin-Voigt model). The scheme of the pipeline is presented in figure 1.

3.2. Valve characteristics

In the equation of pressure balance at the valve it is assumed the outside pressure is zero, thus:

\[
p = 0.5 \zeta(t) \rho (v - w)^2 \tag{22}
\]

The valve characteristics are determined as the dependence of its pressure losses coefficient \( \zeta \) against the valve closing rate \( \phi \), \( \zeta = f(\phi) \). In fact \( \phi \) is a conventional parameter and it can be the closing angle (for a globe valve) or just a relative value equal 0 for a complete opening and 1 for a complete closure.
To model the time dependence of the closuring process the rate $\phi$ is a function of time $\phi=g(t)$, for which the time $\tau$ of the total closure is especially important, but the detail shape $g(t)$ could be also valid. For the valve completely closed the losses coefficient becomes infinite so the condition (22) has to be changed to (20). The characteristic $f(\phi)$ can be defined in an analytical way [8], but the most general could be a numerical description, as usually such characteristics are the result of experiments [12]. In the author’s approach the valve characteristic is defined numerically in several $(N+1)$ equally spaced points of closing rate $\phi$ and interpolation is used between them. Lin-lin interpolation is applied at the first interval (0..1/N), lin-hyp at the last one (1-1/N..1) and lin-log within the inner intervals.

3.3. Equation of motion

We can assume various types of support systems for the pipeline. A quite general situation is when the valve of mass $m$ is fixed to the pipe with a spring-dashpot mounting and the pipe itself is also fixed to the foundation with elastic supports. If $y(t)$ is the valve movement and $z(t)$ is the motion of the pipe-end the following equation was formulated for the valve ($A_c=\pi D^2/4$ is a liquid cross-section area):

$$m \ddot{y} + b(\dot{y} - \dot{z}) + k(y - z) = -b_s(\dot{y} - \dot{z}) + p A_c$$  \hspace{1cm} (23)

In the above equation $k$ and $b$ are respectively the stiffness and damping coefficients of the valve spring and $b_s$ models the friction between the valve and the pipe. Another equation is valid for the pipe-end. Now $m_p$ is the mass of that junction, $b_p$ and $k_p$ are the coefficients of the pipe-end spring.

$$m_p \ddot{z} + b_p \dot{z} + k_p z = -Q - b_s(\dot{z} - \dot{y}) - b(\dot{z} - \dot{y}) - k(z - y)$$  \hspace{1cm} (24)

Other mounting configurations can be also considered, but for the current analyses let us focus on the upper one simplified with the assumption of pipe-end rigidly fixed to the foundation. Thus $z(t)=0$ and the equation (23) can be presented in a new form. In fact, we have to measure the pressure at the valve relative to the initial, steady-state one as we measure the motion $y(t)$ relative to the initial, steady-state valve position. Dividing also the equation of motion by $(A_s \rho_s c_1)$ and using the “divided” mass, stiffness and damping coefficients (with hats) we can write it in the following way, in which $r_0$ is the initial, steady-state pressure (normalized) at the valve:

$$\hat{m} \ddot{y} + (\hat{b} + \hat{b}_s) \dot{y} + \hat{k} y = \hat{R} S^{-1}(r - r_0)$$  \hspace{1cm} (25)

To transform the differential equation to the finite form the Newmark method is used to the left side which allows to express it as a function of the valve velocity $u=dy/dt$ in a new time instant. To
transform the right side of the equation the two left FDE, C0 and C1 are used. Out of the four system
variables \( v, w, r, q \) the pipe-end velocity is zero \((w=0)\), thus \( r \) and \( q \) can be calculated as a function of
\( v \). But liquid velocity \( v \) and the valve velocity \( u \) are related with a formula similar to (22) \((u \text{ has to be}
\text{used instead of } w)\), which allows to determine \( v \) \((v \geq u)\) with:

\[
v = u + 2 \frac{c_0}{\zeta(t)} \sqrt{r}
\]

Thus, the pressure \( r \) can be expressed as a function of \( u \) and putting it to (25) transformed with the
Newmark method, the valve velocity \( u \) can be found. Then, all the system variables \( r, v, q \) can be
calculated and the BC is solved. For the current approach an assumption is made that the BC at the
valve is valid at a fixed point in space being the initial valve position, in spite of the valve motion. But
this motion is small, so the assumption is considered to be acceptable.

4. Numerical results
The above algorithms were implemented in an own computer program and preliminary numerical tests
of the proposed method in various scenarios were done.

4.1. Assumptions for computations
For the numerical analyses a steel pipe of the length \( L=50 \text{m} \), inner diameter \( D=50 \text{mm} \) and pipe-wall
thickness \( e=2.5 \text{mm} \) is assumed. It is fixed rigidly at 0, 14, 28, 42 \([\text{m}]\) from the pressure tank and at the
end. The flow is driven by constant pressure of 1.014 MPa supplied by the tank. The valve at the end
is partially closed to keep the velocity at the steady state at \( v_0=0.5 \text{m/s} \) and to avoid cavitation during
WH. The valve characteristic is defined with the following function \( f(\phi)= (0.1, 0.2, 0.5, 1.2, 3, 10, 30,
100, 400, 1800, \infty) \). The closing time is 6 ms and the function \( g(t) \) is linear. The mass of the valve
mechanism was assumed at \( m=0.5 \text{kg} \). The Darcy-Weisbach friction factor is constant \( \lambda=0.03 \). To
focus on the effects produced by the valve fixing system the Poisson phenomenon is neglected \((v=0)\).
The damping coefficient \( b \) of the valve dashpot is estimated with a formula valid for simple oscillator:

\[
b = 2 \xi \sqrt{kM}
\]

\( M \approx 98 \text{ kg} \) is the total mass of water in the pipe (and the valve) and \( \xi \) is a conventional damping
parameter corresponding to a damping ratio of simple oscillator.

4.2. Results
In the following figures valve displacements and pressure records at a distance of 1m before the valve
are presented. In figure 2 the results for zero damping and various stiffness \( k \) of the spring (7.5N/mm,
25N/mm or 75 N/mm) are plotted. For these parameters one can observe that the WH pressure
magnitudes decrease with lowering the stiffness of the valve spring. For rigidly fixed valve (classic
WH) the pressure amplitude is 0.67MPa. In figure 3 the influence of damping is presented. For higher
damping the transient decays faster due to energy dissipation in the valve damper.

4.3. Analyses and conclusions
The analyses of numerical results has allowed to conclude the presented numerical method works correctly
– the analytical solutions for a simplified model of the scenarios presented in figure 2 gave similar results.
Some behaviours however may still require explanation. Such a specific effect in the current results are the
very small and short peaks in the pressure records appearing twice a basic WH period \((f_{WH}=c_0/4L\approx 6.7 \text{Hz})\).
They could be physical effects as they are smaller for massless valve. It is possible however they are of
numerical origin being, e.g. the result of approximation that the point of the valve BC does not move.

The preliminary analyses of the system behaviour allows to conclude that lowering of transient
pressures is possible for the proper valve fixing system. For the presented scenarios decreasing of the
mounting stiffness produces WH pressure reduction but larger magnitudes of the valve motion. It was also observed within other results that pressure increase (relative to classic WH) for specific valve mounting parameters may happen. All these effects depend also on the damping properties of the valve fixing system. In general, energy dissipation produce reduction of the transient pressures and magnitudes of the valve motion, however this may not happen for the first peak in the pressure record which is visible in figure 3. All these effects are going to be tested more thoroughly.

Figure 2. Pressures and valve motion for various stiffness and zero damping of the valve spring.

Figure 3. Pressures and the valve motion for damping parameter $\xi=0.1$ or $\xi=0.4$ (k=25N/mm).
5. Summary
In the paper a solution for numerical application of the BC at the valve during WH-FSI is presented. The 4E model of WH-FSI is applied and shortly discussed. The numerical approach of the solution of this model is used in a form proposed by the author. Two issues of the BC problem are pointed. The valve closing hydraulic characteristics are modelled in a numerical way but the main problem analysed and solved within the BC is the idea of fixing the valve in the pipe with viscoelastic mounting. Though this proposal may seem to be a natural concept if the physics of WH is considered, the author does not know other works where this problem has been examined. The proper BC was formulated, solved and implemented in an own computer program. The preliminary numerical tests were done and it was found that WH pressure magnitudes can be significantly reduced in comparison with the traditional design of rigidly fixed valve. The idea of fixing the valve with viscoelastic mounting is expected to be applicable in practice and may be especially valuable in systems where rapid valve closing may happen (e.g. check valves) and transient pressure variation is undesired. Further studies within this subject are intended with the use of the developed software. The job will be also extended towards examination of the influence of viscoelastic pipe supports, including 3D effects.

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