Direct CP Violation in K-decay 
and Minimal Left-Right Symmetry Scale

Panying Chen,1 Hongwei Ke,1,2 and Xiangdong Ji1,3
1Department of Physics, University of Maryland, 
College Park, Maryland 20742, USA
2Department of Physics, Nankai University, Tianjin, 300071, P. R. China
3Center for High-Energy Physics and Institute of Theoretical Physics, 
Peking University Beijing, 100080, P. R. China
(Dated: October 14, 2008)

Abstract
We calculate the new contribution to the direct CP-violation parameter $\epsilon'$ in $K \to \pi\pi$ decay in the minimal left-right symmetric model with the recently-obtained right-handed quark Cabibbo-Kobayashi-Maskawa mixing. We pay particular attention to the uncertainty in the hadronic matrix element of a leading four-quark operator $O_{LR}^{-}$. We find that it can be related to the standard model electromagnetic penguin operator $O_{8}$ through $SU(3)_{L} \times SU(3)_{R}$ chiral symmetry. Using the lattice and large $N_c$ calculations, we obtain a robust constraint on the minimal left-right symmetric scale $M_{WR} > 5$ TeV from the experimental data on $\epsilon'$. 
One of the much studied themes for particle physics beyond the standard model (SM) is left-right symmetry at high-energy, introduced many years ago by Mohapatra and Pati [1]. In a recent work, it has been shown that supersymmetric left-right theory arises naturally from duality cascade of a quiver in the context of intersecting D-branes [2]. The twin-Higgs model, introduced to explain the disparity between the new physics scale and the electroweak scale [3], also utilizes the idea of left-right symmetry. However, the direct collider search for the signatory right-handed $W$ gauge boson shows that it is at least 10 times heavier than its left-handed counterpart [4]. The most stringent limit on the right-handed scale has been obtained from low-energy data, with the most well-known example being the neutral kaon mass difference [5], which gives a lower bound of at least $2.0 - 2.5$ TeV.

More recently, a general solution for the right-handed Cabibbo-Kobayashi-Maskawa (CKM) quark mixing in the minimal left-right symmetric model (LRSM) has been found [6]. Particularly interesting is the CP(charge-conjugation-parity)-violating mechanisms in the model: Apart from the usual Dirac CP phase appearing in the left-handed CKM mixing, there is also a spontaneous symmetry-breaking phase $\alpha$ that contributes to CP-violating observables. Using the neutral kaon mixing parameter $\epsilon$, $\alpha$ can be constrained accurately. Therefore, one can make predictions on other CP-violating observables including the neutron electrical dipole moment (EDM) and direct CP-violating parameter $\epsilon'$ in kaon decay; the experimental data can then provide new constraints on the left-right symmetric scale [7]. Unfortunately, the intermediate steps involve unknown hadronic matrix elements, and the simple factorization or large $N_c$ (number of quark colors) assumption is usually adopted to make estimations in previous studies [7, 8]. As a consequence, the bounds suffer from unknown hadronic physics uncertainties, as exemplified in reproducing the $\Delta I = 1/2$ rule for the $K$ to $\pi\pi$ decay.

In this paper, we focus on a better estimation of the uncertainty associated with the leading hadronic matrix element, and hence a more accurate bound on the minimal left-right symmetry scale. In particular, we have found a relation between the dominating four-quark operator $O^{LR}$ in the new contribution and the SM electromagnetic penguin operator $O_8$ through $SU(3)_L \times SU(3)_R$ chiral symmetry. We use the existing knowledge on the matrix element of the latter to get information on the former [9]. With a reasonable estimate of the $O^{LR}$ matrix element, we find the lower bound for the right-handed scale in the range of 5-9 TeV, consistent with that from the neutron EDM data [7].

The direct CP-violation parameter in the neutral kaon to $\pi\pi$ decay is calculated via

$$
\epsilon' = \frac{i}{\sqrt{2}} \omega \left( \frac{q}{p} \right) \left( \frac{\text{Im} A_2}{\text{Re} A_2} - \frac{\text{Im} A_0}{\text{Re} A_0} \right) e^{i(\delta_2 - \delta_0)},
$$

where the decay amplitudes $A_0$ and $A_2$ are defined as the matrix elements of the $\Delta S = 1$ effective Hamiltonian between the neutral-K meson and the isospin $I = 0$ and 2 $\pi\pi$ states,

$$
\langle (2\pi)_I | (-i) \mathcal{H}_{\Delta S = 1} | K^0 \rangle = A_I e^{i\delta_I}.
$$

$\delta_I$ is the strong phase for $\pi\pi$ scattering at the kaon mass, $\omega \equiv A_2/A_0$, and $p, q$ are the mixing parameters for $K^0 - \bar{K}^0$. To an excellent approximation, $\omega$ can be taken as real and $q/p = 1$. We use the experimental value for the real parts of $A_0$ and $A_2$: $\text{Re} A_0 \simeq 3.33 \times 10^{-7}$ GeV and $\omega \simeq 1/22$. We focus on calculating the imaginary part of the decay amplitudes.

In the SM, the contributions to $\epsilon'$ come from both QCD and electromagnetic penguin diagrams [10]. The QCD penguin contributes exclusively to the imaginary part of $\Delta I = 1/2$
decay, whereas the electromagnetic penguin is mainly responsible for the imaginary part of \( \Delta I = 3/2 \) decay. Both contributions are important but have opposite signs. Therefore, the final result depends on delicate cancelations of hadronic matrix elements. The state-of-art chiral perturbation theory [11, 12, 13, 14] and lattice QCD calculations [15, 16] have not yet been sufficiently accurate to reproduce the experimental result [17]. On the other hand, a large-\( N_c \) approach with final-state rescattering effect taken into account seems to be able to reproduce the experimental result [18]. A nice review of the SM calculation can be found in Ref. [3, 16].

\[
\begin{align*}
\mathcal{H}_{\Delta S=1}^{\text{tree}} &= \frac{G_F}{2\sqrt{2}} \lambda^L \left[ \left( \frac{\alpha_S(\mu^2)}{\alpha_S(M^2_L)} \right)^{\frac{1}{b}} O_{+}^{LL}(\mu) + \left( \frac{\alpha_S(\mu^2)}{\alpha_S(M^2_L)} \right)^{\frac{1}{b}} O_{-}^{LL}(\mu) \right] \\
&+ \frac{G_F}{2\sqrt{2}} \lambda^R \left[ \left( \frac{\alpha_S(\mu^2)}{\alpha_S(M^2_R)} \right)^{\frac{1}{b}} O_{+}^{RR}(\mu) + \left( \frac{\alpha_S(\mu^2)}{\alpha_S(M^2_R)} \right)^{\frac{1}{b}} O_{-}^{RR}(\mu) \right] \\
&+ \frac{G_F}{\sqrt{2}} \sin \zeta \lambda^L e^{i\alpha} \left[ \left( \frac{\alpha_S(\mu^2)}{\alpha_S(M^2_L)} \right)^{\frac{1}{b}} O_{+}^{LR}(\mu) - \left( \frac{\alpha_S(\mu^2)}{\alpha_S(M^2_L)} \right)^{\frac{1}{b}} O_{-}^{LR}(\mu) \right] \\
&+ \frac{G_F}{\sqrt{2}} \sin \zeta \lambda^R e^{-i\alpha} \left[ \left( \frac{\alpha_S(\mu^2)}{\alpha_S(M^2_R)} \right)^{\frac{1}{b}} O_{+}^{RL}(\mu) - \left( \frac{\alpha_S(\mu^2)}{\alpha_S(M^2_R)} \right)^{\frac{1}{b}} O_{-}^{RL}(\mu) \right],
\end{align*}
\]

where we have taken into account the leading-logarithm QCD corrections with renormalization scale \( \mu \) taken to be around the charm quark mass \( m_c \sim 1.3 \text{ GeV} \), and \( b = 11 - 2N_f/3 \) with \( N_f \) the number of active fermion flavors. The left-right mixing parameter is

\[
\tan \zeta = 2r \frac{m_b}{m_t} \left( \frac{M_W}{M_{WR}} \right)^2,
\]

where \( r \) is a parameter less than 1. The mixing coupling \( \lambda^{AB} = V_{AUS}^\dagger V_{BUD}^\dagger \), \( A, B \) are \( L, R \). The right-handed CKM matrix has a form,

\[
V_R = P_U V_L P_D,
\]

in which \( P_U = \text{diag}(s_u, se^{2i\theta_2}, st e^{2i\theta_3}) \), \( P_D = \text{diag}(s_d e^{i\theta_1}, s_s e^{-i\theta_2}, s_t e^{-i\theta_3}) \), and

\[
\bar{V}_L = \begin{pmatrix}
1 - \lambda^2/2 & \lambda & A\lambda^2(\rho - i\eta) \\
-\lambda & 1 - \lambda^2/2 & A\lambda^2 e^{-2i\theta_2} \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 e^{-2i\theta_2} & 1
\end{pmatrix},
\]
where $\lambda$, $A$, $\rho$ and $\eta$ are Wolfenstein parameters and the new phases $\theta_i$ are all related to spontaneous CP phase $\alpha$,

$$
\theta_1 = -\sin^{-1}[0.31(s ds_c + 0.18s ds_t)r \sin \alpha],
\theta_2 = -\sin^{-1}[0.32(s s_c + 0.25s s_t)r \sin \alpha],
\theta_3 = -\sin^{-1}[s bs_t r \sin \alpha],
$$

where experimental quark masses have been used with possible $s_i = \pm 1$ signs. The four-quark operators are

$$
O^{LL,RR}_+ = (\bar{s}_i u_i)_{V+A}(\bar{u}_j d_j)_{V+A} \pm (\bar{s}_i d_j)_{V+A}(\bar{u}_j u_j)_{V+A},
$$

$$
O^{LR,RL}_+ = (\bar{s}_i u_i)_{V+A}(\bar{u}_j d_j)_{V+\pm} - \frac{1}{3}(\bar{s}_i u_j)_{V+A}(\bar{u}_j d_i)_{V+\pm},
$$

$$
O^{LR,RL}_- = -\frac{1}{3}(\bar{s}_i u_j)_{V+A}(\bar{u}_j d_i)_{V+A},
$$

where $i$ and $j$ are color indices and the subscript $V \pm A$ refers to a quark bilinear of the form $\bar{q}\gamma_\mu(1 \pm \gamma_5)q$.

As mentioned above, one has to include the penguin contributions in the SM calculation because the CKM matrix elements have non-zero CP phases only when the third family is introduced. The only detail we would like to point out about the SM contribution is that because the CKM matrix elements have non-zero CP phases only when the third family is

$$
\eta = \sin^{-1}\left[0.18(s ds_c + 0.18s ds_t)r \sin \alpha\right],
$$

where $\alpha$ is spontaneous CP phase.

The dominating new contribution is from the left-right W-boson interference. Due to the QCD running effect and chiral suppression, $O^{LR}_+$ operator is less important relative to $O^{LR}_-$.
and hence will be ignored. Therefore, we need to consider only the matrix element of $O_{LR}^-$ operator in the $I = 2$ state. Introduce the following $(8, 8)$ operators,

\[
O^{(8,8)}_{3/2} = (\bar{s}_i d_j)_{V-A}(\bar{u}_j u_i)_{V+A} + (\bar{s}_i u_j)_{V-A}(\bar{d}_j d_i)_{V+A},
\]
\[
O^{(8,8)}_{1/2A} = (\bar{s}_i d_j)_{V-A}(\bar{d}_j d_i)_{V+A},
\]
\[
O^{(8,8)}_{1/2S} = (\bar{s}_i d_j)_{V-A}(\bar{u}_j u_i)_{V+A} + (\bar{s}_i u_j)_{V-A}(\bar{d}_j d_i)_{V+A} + 2(\bar{s}_i d_j)_{V-A}(\bar{d}_j d_i)_{V+A} - 3(\bar{s}_i d_j)_{V-A}(\bar{s}_j s_i)_{V+A},
\]
where subscripts 3/2 and 1/2 indicate isospin. Using the above, one can express $O_{LR}^-$ as follows

\[
O_{LR}^- = \frac{1}{9} O^{(8,8)}_{3/2} - \frac{1}{18} O^{(8,8)}_{1/2A} + \frac{1}{6} O^{(8,8)}_{1/2S}.
\]

(12)

On the other hand, the electromagnetic penguin operator $O_8$ can be expressed as

\[
O_8 = \frac{1}{2} \left( O^{(8,8)}_{3/2} + O^{(8,8)}_{1/2A} \right).
\]

(14)

Therefore, we find the model-independent relation,

\[
\langle (\pi\pi)_{I=2} | O_{LR}^- | K_0 \rangle = -\frac{2}{9} \langle (\pi\pi)_{I=2} | O_8 | K_0 \rangle.
\]

(15)

FIG. 2: The new contribution in LRSM to $\epsilon'$ as a function of $M_{W_R}$ for $\sin \alpha = 0.1$, $r = 0.5$ with $s_d = s_s = -1$ and all other $s_q = 1$. The light shaded part is allowed by the experimental data, and the heavy-shaded area is 1/4 of the experimental data.

In the vacuum insertion approximation, one finds

\[
\langle (\pi\pi)_{I=2} | O_8 | K_0 \rangle = \sqrt{6} f_\pi \left( \frac{m_K^2}{m_s(\mu) + m_d(\mu)} \right)^2 i,
\]

(16)

which is about $0.95i$ GeV$^3$ ($f_\pi = 93$ MeV) if the strange quark mass is taken to be 120 MeV at the scale of $m_c$. On the other hand, the lattice QCD calculation in Ref. [15] gives
$1.4i$ GeV$^2$ at the scale of 1.9 GeV. This lattice calculation, however, does not reproduce the experimental data on $\epsilon'$. In Ref. [9], an extensive discussion has been made about the size of this matrix element. It is expected that the variation of the matrix element is between 1 to 2 of the factorization result.

Because the phase $\alpha$ in the factor $e^{i\alpha}$ is dominating, $\epsilon'$ is approximately a function of $r \sin \alpha$, rather than $r$ and $\sin \alpha$ independently. Since $r \sin \alpha$ has been fixed by $\epsilon$ and neutron EDM $d^n_e$ [4], $\epsilon'$ is approximately a function of $M_{W_R}$ only. In Fig. 2, we plot $\epsilon'$ as a function of $M_{W_R}$ for $\sin \alpha = 0.1$, $r = 0.5$ and $s_d s_u = 1$ which is required by the neutron EDM calculation. [All other $s_i = 1$.] We choose the renormalization scale at the charm quark mass and $\Lambda_{QCD} = 340$ MeV. The dashed curve shows the result with the large-$N_c$ matrix element, whereas the solid curve shows that from the lattice QCD [15].

If one uses that factorized matrix element and following the Refs. [9, 18] for other hadronic matrix elements, the experimental data is roughly reproduced by the SM calculation. Requiring the new contribution is less than 1/4 of the experimental data, we get a large lower bound of 8 TeV on the right-handed scale. On the other hand, if one takes the calculation in Ref. [15] seriously, the lattice QCD generates a small and negative contribution to $\epsilon'$. If then requiring that the experimental number is entirely reproduced by the new contribution, we find a limit on $M_{W_R}$ about 5 TeV. In any case, $\epsilon'$ gives a tighter lower bound on $M_{W_R}$ than the well-known neutral kaon mass difference. If on the other hand, we take $r \sin \alpha = 0.15$, as required by low $M_H$, the bound changes to 8.5 TeV. Therefore, we take the range 5-8 TeV as our final estimate.

Finally, we have also calculated the tree-level flavor-changing neutral Higgs contributions to $\mathcal{H}_{\Delta S=1}$. Since the relevant coupling is suppressed by either the Cabibbo angle or the quark masses, their contribution is negligible.

To conclude, we have found that a robust bound on the mass of the right-handed $W$-boson based on a relatively well-known estimate on the strong interaction matrix element of $O^{LR}_{R}$, which is known to within a factor of 2. The result is on the order of 5-8 TeV, which is just on the border for the Large Hadron Collider detection. This situation turns out to be better than the similar calculation in SM.

We thank R. Mohapatra and Y. Zhang for numerous discussions related to the subject of this paper. This work was partially supported by the U. S. Department of Energy via grant DE-FG02-93ER-40762. H. W. Ke acknowledges a scholarship support from China’s Ministry of Education.
[1] R. N. Mohapatra and J. C. Pati, Phys. Rev. D 11, 566 (1975); R. N. Mohapatra and J. C. Pati, Phys. Rev. D 11, 2558 (1975); G. Senjanovic and R. N. Mohapatra, Phys. Rev. D 12, 1502 (1975); Phys. Rev. D 23, 165 (1981); For a review, Rabindra N. Mohapatra, *CP Violation*, World Scientific Publ. Co., C. Jarlskog, Ed., 1989.

[2] J. J. Heckman, C. Vafa, H. Verlinde and M. Wijnholt, JHEP 0806, 016 (2008) [arXiv:0711.0387 [hep-ph]].

[3] Z. Chacko, H. S. Goh and R. Harnik, Phys. Rev. Lett. 96, 231802 (2006) [arXiv:hep-ph/0506256]. Z. Chacko, H. S. Goh and R. Harnik, JHEP 0601, 108 (2006) [arXiv:hep-ph/0512088].

[4] W. M. Yao *et al.* [Particle Data Group], J. Phys. G 33, 1 (2006).

[5] G. Beall, M. Bander and A. Soni, Phys. Rev. Lett. 48, 848 (1982).

[6] Y. Zhang, H. An, X. Ji and R. N. Mohapatra, Phys. Rev. D 76, 091301 (2007) [arXiv:0704.1662 [hep-ph]].

[7] Y. Zhang, H. An, X. Ji and R. N. Mohapatra, Nucl. Phys. B 802, 247 (2008) [arXiv:0712.4218 [hep-ph]].

[8] G. Ecker and W. Grimus, Nucl. Phys. B 258, 328 (1985); J. M. Frere, J. Galand, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Rev. D 46, 337 (1992).

[9] A. J. Buras and M. Jamin, JHEP 0401, 048 (2004) [arXiv:hep-ph/0306217].

[10] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Sov. Phys. JETP 45, 670 (1977) [Zh. Eksp. Teor. Fiz. 72, 1275 (1977)].

[11] A. J. Buras, [arXiv:hep-ph/9806471].

[12] G. Buchalla, A. J. Buras and M. K. Harlander, Nucl. Phys. B 337, 313 (1990).

[13] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996) [arXiv:hep-ph/9512380].

[14] S. Bosch, A. J. Buras, M. Gorbahn, S. Jager, M. Jamin, M. E. Lautenbacher and L. Silvestrini, Nucl. Phys. B 565, 3 (2000) [arXiv:hep-ph/9904408].

[15] T. Blum *et al.* [RBC Collaboration], Phys. Rev. D 68, 114506 (2003) [arXiv:hep-lat/0110075].

[16] D. Pekurovsky and G. Kilcup, Phys. Rev. D 64, 074502 (2001) [arXiv:hep-lat/9812019].

[17] H. Burkhardt *et al.* [NA31 Collaboration], Phys. Lett. B 206, 169 (1988); V. Fantí *et al.* [NA48 Collaboration], Phys. Lett. B 465, 335 (1999) [arXiv:hep-ex/9909022]; A. Alavi-Harati *et al.* [KTeV Collaboration], Phys. Rev. Lett. 83, 22 (1999) [arXiv:hep-ex/9905060].

[18] E. Pallante and A. Pich, Phys. Rev. Lett. 84, 2568 (2000) [arXiv:hep-ph/9911233]. E. Pallante, A. Pich and I. Scimemi, Nucl. Phys. B 617, 441 (2001) [arXiv:hep-ph/0105011].

[19] S. Bertolini, J. O. Eeg and M. Fabbrichesi, Phys. Rev. D 63, 056009 (2001) [arXiv:hep-ph/0002234].