Phenomenology of flavor-mediated supersymmetry breaking

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Abstract

The phenomenology of a new economical supersymmetric model that utilizes dynamical supersymmetry breaking and gauge-mediation for the generation of the sparticle spectrum and the hierarchy of fermion masses is discussed. Similarities between the communication of supersymmetry breaking through a messenger sector, and the generation of flavor using the Froggatt-Nielsen (FN) mechanism are exploited, leading to the identification of vector-like messenger fields with FN fields, and the messenger $U(1)$ as a flavor symmetry. An immediate consequence is that the first and second generation scalars acquire flavor-dependent masses, but do not violate flavor changing neutral current bounds since their mass scale, consistent with “effective supersymmetry”, is of order $10$ TeV. We define and advocate a “minimal flavor-mediated model” (MFMM), recently introduced in the literature, that successfully accommodates the small flavor-breaking parameters of the standard model using order one couplings and ratios of flavon field vevs. The mediation of supersymmetry breaking occurs via two-loop log-enhanced gauge-mediated contributions, as well as several one-loop and two-loop Yukawa-mediated contributions for which we provide analytical expressions. The MFMM is parameterized by a small set of masses and couplings, with values restricted by several model constraints and experimental data. Full two-loop renormalization group evolution is performed, correctly taking into account the negative two-loop gauge contributions from heavy first and second generations. Electroweak symmetry is radiatively broken with the value of $\mu$ determined by matching to the $Z$ mass. The weak scale spectrum is generally rather heavy, except for the lightest Higgs, the lightest stau, the lightest chargino, the lightest two neutralinos, and of course a very light gravitino. The next-to-lightest sparticle (NLSP) always has a decay length that is larger than the scale of a detector, and is either the lightest stau or the lightest neutralino. Similar to ordinary gauge-mediated models, the best collider search strategies are, respectively, inclusive production of at least one highly ionizing track, or events with many taus plus missing energy. In addition, $D^0 \leftrightarrow \overline{D}^0$ mixing is also a generic low energy signal. Finally, the dynamical generation of the neutrino masses is briefly discussed.
1 Introduction

Two central problems pervade the standard model (SM), namely understanding the how the scalar Higgs mass is stabilized against radiative corrections and the origin of the disparate fermion masses. Weak scale supersymmetry is well-known as the premiere solution to the first problem. The most successful attempts on the second are made primarily through flavor symmetries. It is natural to ask if we can construct a theory that can explain the fermion mass hierarchy simultaneously with an explanation of the gauge hierarchy problem.

Moreover, there are reasons to suggest supersymmetry breaking and flavor symmetry breaking ought to be closely related in supersymmetric models. The main impetus arises from the supersymmetric nonrenormalization theorem that protects superpotential couplings from radiative corrections. In particular, couplings that vanish at one scale (unbroken flavor symmetry) remain zero for all scales at all orders in perturbation theory, unless one of the following occurs: (i) a mass scale, put into the theory “by hand”, explicitly or spontaneously breaks the flavor symmetry, (ii) nonperturbative effects generate Yukawa couplings (i.e., the flavor symmetry is anomalous), or (iii) supersymmetry is broken \[1\]. Because we are interested in dynamically generating all scales below the Planck scale, we discuss exclusively the latter possibility.

Building a model that is both supersymmetric and incorporates a flavor symmetry is strongly constrained, particularly due to processes that can be enhanced by the presence of superpartners. In general, any successful supersymmetric flavor model must have a spectrum of scalar partners of quarks and leptons that do not contribute to low energy processes beyond experimental bounds. It is possible to respect limits on flavor changing neutral currents (FCNC) and CP violation \[2\], such as those from $K^0 - \bar{K}^0$ mixing, $\epsilon_K$, and lepton flavor violation, if the model contains one of the following spectra of squarks and sleptons:

- Their masses are generation independent (“degeneracy”).
- The scalar (mass)$^2$ matrices and the (mass)$^2$ matrices of their fermionic partners are both diagonal in the same basis (“alignment”) \[3\].
- The masses of the first two generations are much larger than the weak scale (“decoupling”) \[4, 5\].

To solve the flavor problem of the standard model, one must take care to respect the flavor problem of the minimal supersymmetric standard model as well.

One possibility is that the two solutions are decoupled from each other. The flavor symmetry could be broken at a high scale $M_F \gg M_Z$, below which the effective action contains the normal (MS)SM Yukawa couplings. Supersymmetry could then break in a hidden sector and the breaking be communicated via gravitational interactions \[6\] or gauge interactions \[7, 8, 9\]. Gauge-mediated supersymmetry breaking (GMSB), in it’s canonical form \[8\], leaves scalars of the same quantum numbers approximately degenerate, thus avoiding the flavor problem of the MSSM. In supergravity-mediated models this problem can be circumvented by choosing boundary conditions for soft-breaking parameters that leave the sfermions approximately degenerate.
or (as has been suggested recently) produce the decoupling solution. In these cases, diagonal soft masses would be generated, thereby respecting any flavor symmetry, and flavor breaking would only affect the form of supersymmetry breaking via the renormalization group.

The other possibility is that the flavor symmetry dictates the form of the soft masses. As in the models of Ref. \cite{3,12}, one or more U(1) flavor symmetries could restrict the quark and squark mass matrices to be approximately diagonal in the flavor basis. This approximate alignment of quark and squark masses is able to suppress the additional sparticle contributions to FCNC (though to satisfy experimental bounds, the models are rather complex). Similar efforts using a U(2) flavor symmetry that forced the first two generations of scalars to be degenerate at some scale was also able to suppress dangerous contributions \cite{13}. Alternatively, supersymmetry could be broken by an “anomalous” U(1) gauge symmetry. In this scenario, the first two generations carry equal non-zero charges and their scalar components become heavy and degenerate \cite{15}, and the U(1) could play the role of a flavor symmetry in a Froggatt-Nielsen mechanism \cite{17}. A completely different approach utilized a composite dynamical supersymmetry breaking, in which in some cases promising fermion textures and heavy first and second generations could result \cite{18}. Finally, several groups have recently studied the possibility of generating the Yukawa couplings radiatively \cite{19,20}.

In this paper, gauge-mediation is utilized for the communication of supersymmetry breaking, which has several benefits for flavor physics. First, there naturally appears a hierarchy of scales in the simplest class of models, due to the separation of the dynamical supersymmetry breaking sector (DSB), the messenger sector, and the MSSM. Furthermore, additional gauge symmetries are naturally required for the DSB fields to communicate their supersymmetry breaking to the messenger fields.

The model presented here \cite{21} is supersymmetric, incorporates a gauged U(1)$_F$ flavor symmetry, and utilizes a modified Froggatt-Nielsen mechanism to generate the necessary Yukawa coupling hierarchy. The Froggatt-Nielsen mechanism \cite{17} was invented two decades ago in the context of the SM as a way to reconcile the hierarchy of Yukawa couplings. The general principle is to forbid some tree-level Yukawa couplings, but introduce additional matter that when integrated out gives rise to the smaller Yukawa couplings. In our interpretation the third generation is considered “fundamental”, with nonzero Yukawa couplings allowed even with the flavor symmetry unbroken, while fermion masses for the first and second generations arise after integrating out heavy FN fields. We also insist that couplings are of order one, by which we mean quantitatively that no couplings differ by more than a factor of ten (i.e., all couplings are larger than about 0.1). In this way our quantitative improvement over an ordinary supersymmetry model is to reduce the hierarchy of Yukawa couplings $\sim 10^{-5}–1$ to $\sim 0.1–1$. We utilize the two-Higgs doublet structure of the MSSM and demand the Yukawa couplings for the third generation also satisfy this requirement. For third generation Yukawa couplings larger than 0.1, it follows that $\tan \beta > \sim 10$, which is the lower bound that we use in this paper.

Our analysis focuses almost entirely on one model \cite{21}, the minimal flavor-mediated model (MFMM), that has one set of fields and one parameterized superpotential. We use the term

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1. The U(1) is rendered anomalous below the Planck scale by the Green-Schwarz mechanism \cite{14}.
2. The upper bound is determined by the model dynamics, discussed in detail later in the paper.
“flavor-mediated” since the same gauge group that communicates supersymmetry breaking from the dynamical supersymmetry breaking sector also serves as the flavor symmetry. In this way supersymmetry breaking and flavor symmetry breaking are inseparably linked together, which is the precise meaning of “flavor-mediation”.

The main thrust of this paper is to derive the complete phenomenological spectrum of this model so that the most promising experimental signals can be extracted. Unlike ordinary gauge-mediation, there are several technical subtleties in accurately calculating the spectrum at the lowest scales. We believe we have overcome the vast majority of these difficulties. The model does have several new couplings and masses that are not precisely known, forcing us to parameterize this ignorance and show the results as a function of these high(er) scale quantities.

There are, however, several contortions that could be done to the model. Some of these could potentially address problems that we do not solve or only have partial solutions in the MFMM. For example, the $\mu$ problem persists in the MFMM, and we implement no particular solution (instead fixing $\mu^2$ using electroweak symmetry breaking (EWSB) constraints). Nevertheless, there are various ways that a $\mu$ term could be generated from the dynamics of the model. Since the details of such $\mu$ term generation dynamics are essentially irrelevant to the low energy phenomenology, we can rightly neglect this issue for all practical purposes.

The one aspect of the MFMM that may affect the weak scale phenomenology is the generation of neutrino masses. Several experiments, in particular those measuring atmospheric and solar neutrinos, have now convincingly established that some (if not all) generations of neutrinos are very likely mixing with one another. This mixing presumably arises due to nonzero neutrino masses. With certain assumptions, it is possible to modify and augment the MFMM to incorporate neutrino masses of a size that can generate mixings to explain both the atmospheric neutrino results [22] and the solar neutrino results [23]. However, no completely satisfactory solution is presented since specific assumptions about the physics at scales beyond the DSB scale are required. We will discuss this more fully in future work, but we are nevertheless encouraged by these preliminary results.

The outline of the paper is as follows. In Sec. 2 we describe in detail the sectors that make up the flavor-mediated model. The role and function of the flavor symmetry is given, along with the predictions for the fermion masses and CKM elements. The required and undetermined flavor charge assignments are given, as well as the resulting messenger–Froggatt-Nielsen superpotential. In Sec. 3 the mediation of supersymmetry breaking is discussed, and the resulting holomorphic and nonholomorphic contributions to the various fields are written. In Sec. 4 the renormalization group evolution is discussed in detail, including several important effects that arise in our particular model. These include consequences of an anomalous $U(1)_F$ below the scale of dynamical supersymmetry breaking, Yukawa-mediated supersymmetry breaking contributions, and two-loop gauge contributions induced by heavy first and second generations. In Sec. 5 we present many of the main results of our paper. We discuss in detail the relevant parameters of the model, and define a minimal parameter set associated with the MFMM. Several phenomenological constraints restrict the parameter space in highly nontrivial ways, and we illustrate these constraints with a series of graphs and discussion. In Sec. 6 the main signals of the model are discussed in detail. Our emphasis is on the collider signals of the weak-scale spectrum, that
depend ultimately on several factors (MFMM parameters). We have been able to apply to our scenario several recent results from the literature that used event-level simulations, and this has allowed us to be rather precise in the predictions for the MFMM. We also discuss signals in low energy flavor processes (i.e., FCNC and CP violation). Sec. 3 is devoted to discussing some variations of the MFMM. Our primary motivation is to briefly examine the impact of other choices of FN fields and superpotential couplings that result if the SU(5) ansatz is relaxed. In addition, we discuss some directions toward a solution of the $\mu$-problem, and perhaps most importantly we discuss ways to augment and modify the MFMM to accommodate nonzero neutrino masses. Finally, our conclusions appear in Sec. 8.

2 The model

There are essentially four sectors in this model. At the highest energy scales there is a DSB sector, of which we have certain requirements but we leave largely unspecified. Some DSB sector fields are charged under an additional nonanomalous U(1), which serves both as the messenger U(1) responsible for transmitting supersymmetry breaking to the messenger fields (among others), and also acts as the flavor U(1) responsible for specifying the necessary (or absent) couplings in the model. Vector-like messenger fields also serve in a dual capacity by transmitting supersymmetry breaking to the MSSM via ordinary gauge interactions\(^3\), and also as Froggatt-Nielsen (FN) fields that are needed to generate the effective Yukawa coupling hierarchy. To emphasize the dual role of this sector, we call this the messenger–Froggatt-Nielsen sector, or simply “FN sector”. Similarly, messenger–Froggatt-Nielsen fields are called simply “FN fields”. In order to generate the Yukawa hierarchy, several SM gauge singlets must also be present that are charged under the U(1)\(_F\) and obtain vacuum expectation values. These fields are called flavons, and in this model there are two distinct varieties. Three $\chi^{(i)}$ fields are responsible for generating the supersymmetric mass for the vector-like FN fields, and as such indirectly assist the generation of the Yukawa coupling hierarchy. Five $\phi^{(j)}$ fields are solely responsible to generate the effective Yukawa couplings, after the FN sector and the flavon sector is integrated out. Any given Yukawa coupling (except those of the third generation) or CKM element is ultimately proportional to the ratio $\langle \phi^{(j)} \rangle / \langle \chi^{(i)} \rangle$. Thus, judicious U(1)\(_F\) charge assignments with a properly generated flavon vev hierarchy give the required fermion masses and mixing angles. Calculating the flavon vev hierarchy with eight flavon fields amounts to a doing a complicated multi-dimensional effective potential minimization. It was shown in Ref. [21] that additional flavons are typically necessary to generate the required vevs, but they do not impact any other aspect of the model. We therefore simply assume flavons can be added such that the required local minimum is generated and is sufficient stable (lifetime of the universe). More concrete work detailing the generation of flavon vevs with the desired minimum can be found in Ref. [1]. Finally, the MSSM scalar fields acquire supersymmetry breaking masses from several sources. It is on this aspect of the model that we spend the most effort, since all signals of the model are crucially dependent on precise calculations of these weak scale quantities.

\(^3\)We define “MSSM gauge interactions” to mean only the SM gauge and the supersymmetrized gaugino interactions between sectors of the theory.
We have already remarked that the first and second generation fields are charged under the $U(1)_F$. There are several immediate consequences of this which we now discuss. The motivating reason is to forbid the first and second generation Yukawa couplings through the $U(1)_F$ flavor symmetry. These couplings are regenerated in the effective theory through FN ladder diagrams with FN vector-like matter coupling to flavons and the first and second generations. Since the first and second generations must be charged in this scenario, the scalar components acquire two-loop $U(1)_F$ gauge-mediated masses directly induced after integrating out the DSB sector. These are much larger than the contributions induced by ordinary two-loop gauge interactions after integrating out the FN sector. Thus, the MSSM fields also have a hierarchy: the first and second generations are expected to be much heavier than the other fields, due to the $U(1)_F$ coupling.

To generate the proper hierarchy of fermion masses without generating significant squark mixing, the first and second generations should be assigned different $U(1)_F$ charge for their component fields. Hence, the model generically has large differences between the first and second generation scalar masses which can induce large FCNC. This is precisely the supersymmetric flavor problem. However, their masses are expected to be much heavier than the other MSSM fields, as explained above. It is well known that the decoupling solution to the FCNC problem in a generic MSSM requires the first and second generation scalar masses to be so large they cause other phenomenological problems (e.g., unnatural contributions to the Higgs masses, and third generation scalar masses running negative [24, 25]). Thus, the decoupling solution must be augmented by some amount of degeneracy or alignment. Such an “effective supersymmetry” spectrum has been discussed in different contexts in Refs. [4, 5, 15, 26, 16, 11, 27] and indeed has been recently advocated as the most natural scenario [28, 11]. In our model, it is remarkable that the same mechanism that allows the generation of the Yukawa hierarchy and generates generically large splittings between the first and second generations, also solves the supersymmetric flavor problem since the first and second generations receive large supersymmetry breaking masses and the squark masses are approximately diagonal in the flavor basis. This one of the central motivations for pursuing this approach to flavor.

The messenger $U(1)_F$ is anomalous after the DSB sector is integrated out. This has several important consequences. A Fayet-Iliopoulos (FI) term is automatically generated at one-loop, but is calculable only by specifying a DSB model. Fields charged under the $U(1)_F$ acquire supersymmetry breaking masses near the DSB scale after the DSB fields are integrated out. Furthermore, $\text{tr} g_im_i^2$ is necessarily nonzero, where $g_i$ and $m_i$ are respectively the $U(1)_F$ charge and the mass of the scalar field $i$. This generates significant one-loop corrections to the $U(1)_F$ charged scalar (mass)$^2$, analogous to when hypercharge $D$-term is nonzero in other supersymmetric models.

To successfully accommodate the fermion masses and CKM angles with $O(1)$ couplings, we must construct a (modified) FN model. In Ref. [21], it was shown that the form of the fermion mass matrices is tightly constrained due to the restrictions on the first two generation masses and on the couplings of the up-type Higgs. Although we will not repeat the detailed arguments, we do present the up-type quark, down-type quark, and charged lepton mass matrices in the notation used in this paper to show how the fermion masses are generated.

$^4$ $U(1)_F$ charge is assigned to respect an SU(5) ansatz. This will be discussed below.

$^5$ Allowed variations of these matrices are discussed in Sec. 7.
After the $U(1)_F$ breaks and we are in the proper ground state, several flavon fields acquire vevs. These have the effect of giving large (supersymmetric) masses to the FN fields and mixing FN fields with MSSM fields of the same quantum numbers, leading to the mass matrices we present below. The generic form of the superpotential is

$$W \supset \chi F \overline{F} + \phi f \overline{F} + HfF,$$  \hspace{1cm} (1)$$

where the $F$ and $\overline{F}$ are FN fields, the $f$ are SM fields and $H$ is a Higgs field. Before any FN fields are integrated out, the up-type fermion mass matrix is

$$M^u = \begin{pmatrix}
0 & 0 & 0 & 0 & b_1^{(Q)} \langle \phi(1) angle \\
0 & 0 & 0 & 0 & b_2^{(Q)} \langle \phi(2) angle \\
0 & 0 & Y_1 \langle H^u \rangle & 0 & b_1^{(Q)} \langle \phi(1) angle \\
0 & 0 & Y_2 \langle H^u \rangle & 0 & b_2^{(Q)} \langle \phi(2) angle \\
b_1^{(u)} \langle \phi(1) angle & b_2^{(u)} \langle \phi(2) angle & \lambda_{2,1}^{(u)} \langle H^u \rangle & 0 & a_1^{(u)} \langle \chi(1) angle
\end{pmatrix} \hspace{1cm} (2)$$

where the columns represent ($u^1$, $u^2$, $u^3$, $u^{FN}$, $\overline{Q}^{FN}$) and the rows represent ($Q^1$, $Q^2$, $Q^3$, $Q^{FN}$, $\overline{\tau}^{FN}$). We have inserted the component superpotential parameters that are defined in Appendix A. The down-type fermion mass matrix is

$$M^d = \begin{pmatrix}
0 & 0 & 0 & 0 & \lambda_{1,1}^{(d)} \langle H^d \rangle & 0 & b_1^{(Q)} \langle \phi(1) angle \\
0 & 0 & 0 & 0 & \lambda_{2,1}^{(d)} \langle H^d \rangle & b_2^{(Q)} \langle \phi(2) angle & 0 \\
0 & 0 & Y_1 \langle H^d \rangle & 0 & \lambda_{2,1}^{(d)} \langle H^d \rangle & 0 & 0 \\
0 & 0 & Y_2 \langle H^d \rangle & 0 & \lambda_{2,1}^{(d)} \langle H^d \rangle & 0 & 0 \\
b_3^{(d)} \langle \phi(3) \rangle & b_5^{(d)} \langle \phi(5) \rangle & \lambda_{2,1}^{(d)} \langle H^d \rangle & 0 & a_2^{(d)} \langle \chi(2) \rangle & 0 & 0 \\
b_4^{(d)} \langle \phi(4) \rangle & \lambda_{2,1}^{(d)} \langle H^d \rangle & \lambda_{2,1}^{(d)} \langle H^d \rangle & 0 & a_3^{(d)} \langle \chi(3) \rangle & 0 & 0
\end{pmatrix} \hspace{1cm} (3)$$

where the columns represent ($d^1$, $d^2$, $d^3$, $d^{FN(1)}$, $d^{FN(2)}$, $\overline{Q}^{FN}$) and the rows represent ($Q^1$, $Q^2$, $Q^3$, $Q^{FN}$, $d^{FN(1)}$, $d^{FN(2)}$). Finally, for completeness, the charged lepton mass matrix is

$$M^e = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & b_3^{(L)} \langle \phi(3) \rangle & 0 \\
0 & 0 & 0 & 0 & 0 & b_4^{(L)} \langle \phi(4) \rangle & 0 \\
0 & 0 & Y_1 \langle H^d \rangle & \lambda_{4,2}^{(e)} \langle H^d \rangle & 0 & 0 & a_1^{(L)} \langle \chi(1) \rangle \\
0 & 0 & Y_2 \langle H^d \rangle & \lambda_{4,2}^{(e)} \langle H^d \rangle & 0 & 0 & a_3^{(L)} \langle \chi(3) \rangle \\
b_1^{(e)} \langle \phi(1) \rangle & b_2^{(e)} \langle \phi(2) \rangle & \lambda_{2,1}^{(d)} \langle H^d \rangle & 0 & 0 & 0 & 0
\end{pmatrix} \hspace{1cm} (4)$$

where the columns represent ($e^1$, $e^2$, $e^3$, $e^{FN}$, $L^{FN(1)}$, $L^{FN(2)}$) and the rows represent ($L^1$, $L^2$, $L^3$, $\overline{L}^{FN(1)}$, $\overline{L}^{FN(2)}$, $\overline{\tau}^{FN}$). Notice that all MSSM Yukawa couplings are forbidden other than among the third generation, just as the $U(1)_F$ flavor symmetry demands.

Assuming $a_i \langle \chi(i) \rangle \gg \langle \phi(j) \rangle$, $\langle H^u \rangle$, $\langle H^d \rangle$, we can integrate out the FN fields giving the $3 \times 3$...
These matrices predict the following quark masses

\[
M^u_{\text{eff}} = \langle H^u \rangle \begin{pmatrix}
0 & 0 & \lambda^{u}_{2,2} \epsilon^{(Q)}_{11} \\
0 & 0 & \lambda^{u}_{2,2} \epsilon^{(Q)}_{21} \\
\lambda^{u}_{2,1} \epsilon^{(u)}_{11} & \lambda^{u}_{2,1} \epsilon^{(u)}_{21} & Y_t
\end{pmatrix}
\]

(5)

\[
M^d_{\text{eff}} = \langle H^d \rangle \begin{pmatrix}
\lambda^{d}_{1,1} \epsilon^{(d)}_{32} & 0 & \lambda^{d}_{4,1} \epsilon^{(Q)}_{11} + \lambda^{d}_{4,1} \epsilon^{(d)}_{52} \\
0 & \lambda^{d}_{2,1} \epsilon^{(d)}_{43} & \lambda^{d}_{4,1} \epsilon^{(Q)}_{21} \\
0 & 0 & \lambda^{d}_{4,2} \epsilon^{(L)}_{11} + \lambda^{d}_{4,2} \epsilon^{(L)}_{52}
\end{pmatrix}
\]

(6)

\[
M^e_{\text{eff}} = \langle H^d \rangle \begin{pmatrix}
\lambda^{e}_{1,2} \epsilon^{(L)}_{32} & 0 & 0 \\
0 & \lambda^{e}_{2,2} \epsilon^{(L)}_{43} & 0 \\
\lambda^{e}_{4,2} \epsilon^{(e)}_{11} + \lambda^{e}_{4,2} \epsilon^{(e)}_{52} & \lambda^{e}_{4,2} \epsilon^{(e)}_{21} & Y_t
\end{pmatrix}
\]

(7)

where

\[
\epsilon_{ij}^{(f)} = -\frac{b_{ij}^{(f)} \langle \phi^{(i)} \rangle}{a_j^{(f)} \langle \chi^{(j)} \rangle}.
\]

These matrices predict the following quark masses

\[
\frac{m_d}{m_s} = \frac{\lambda^{d}_{1,1} \epsilon^{(d)}_{32}}{\lambda^{d}_{2,1} \epsilon^{(d)}_{43}},
\]

(9)

\[
\frac{m_s}{m_b} = \frac{\lambda^{d}_{2,1} \epsilon^{(d)}_{43}}{Y_b},
\]

(10)

\[
\frac{m_c}{m_t} = \frac{\lambda^{u}_{1,2} \lambda^{u}_{2,2} \epsilon^{(Q)}_{21} \epsilon^{(u)}_{21}}{Y_t^2},
\]

(11)

and CKM matrix

\[
V_{\text{CKM}} \simeq \begin{pmatrix}
1 & -\frac{\lambda^{d}_{4,1} \epsilon^{(d)}_{21}}{\epsilon^{(Q)}_{21}} & \frac{\lambda^{d}_{4,2} \epsilon^{(d)}_{52}}{\epsilon^{(Q)}_{21}} \\
\lambda^{d}_{4,2} \epsilon^{(d)}_{11} \epsilon^{(Q)}_{21} & 1 & \lambda^{d}_{4,1} \epsilon^{(d)}_{52} \\
\lambda^{d}_{4,2} \epsilon^{(d)}_{11} \epsilon^{(Q)}_{21} & \lambda^{d}_{4,1} \epsilon^{(d)}_{52} & 1
\end{pmatrix}.
\]

(12)

The one striking feature of these matrices is that \(m_u\) is zero\(^6\). This is a direct consequence of limiting ourselves to five \(q \bar{q}\) pairs of (color triplet) FN fields. Refs. \cite{29} suggest that \(m_u = 0\) is not excluded by experiment, and furthermore has the virtue of solving the strong CP problem, though this solution is as of yet controversial \cite{31}. Modifications to the FN sector which allow for \(m_u \neq 0\) are discussed in Sec. \cite{21,23,24}.

\(^6\)Our results differ from that in Ref. \cite{21} by expressing the mass matrices using superpotential couplings associated with the SU(3) \(_c\) \(\times \) SU(2)\(_L\) \(\times \) U(1)\(_Y\) "component" fields of the global SU(5) ansatz, which differentiate couplings in the mass matrices.

\(^7\)This is an exact statement as the matrix in Eq. \cite{8} has a zero eigenvalue which is protected by a chiral symmetry.
The charged lepton masses are equal to the down-type quark masses up to couplings of $\mathcal{O}(1)$. For example,

$$m_e \frac{\lambda_{1,2}^{d, (L)} (d)}{\lambda^{(d)}_{1,1} \epsilon^{(d)}_{32}} = \frac{\lambda_{1,2}^{d, (L)} (d)}{\lambda_{1,1}^{d, (d)} (d)} \sim \frac{1}{10} - \frac{1}{20}$$

is the smallest ratio between the two sectors. Thus ratios of individual couplings of $\sim 2/5$ can explain these mass differences.

To achieve this result the $U(1)_F$ charges must be judiciously chosen, although several degrees of freedom remain even after all the constraints are satisfied. We require the first and second generations are charged respecting an SU(5) ansatz, and all the other fields of the MSSM (third generation, Higgs, gauginos) are uncharged. Although we do not attempt to embed the model in an SU(5) GUT, there are four main motivations to enforce the charge assignments commute with SU(5): It is sufficient for gauge coupling unification to be at least naively preserved (to one-loop), $U(1)_F$ anomaly cancellation and $tr Y_i m_i^2 \simeq 0$ are relatively easy to achieve, and any assignment that did not compute with SU(5) (or SU(4)$\times$SU(2)$\times$SU(2), etc.) would immediately preclude any hope of embedding the model in a GUT at the unification scale. Having said this, we should also note that gauge coupling unification can be achieved with fields in representations that do not fit into SU(5) or any group $[31, 32]$ (although such a messenger sector cannot generate flavor) and there are numerous examples of free fermionic string constructions that break directly to the SM gauge group, never passing through a GUT in the first place.

Assuming the charge assignments to fields are distinct and that the Higgs and third generation fields are uncharged, the above mass matrices imply the following constraints on the charges:

$$q^{F,N(1)}_5 = -q^{10}_1$$
$$q^{F,N(2)}_5 = -q^{10}_2$$
$$q^{F,N(1)}_5 = -q^{(2)}_\chi + q^{10}_1$$
$$q^{F,N(2)}_5 = -q^{(3)}_\chi + q^{10}_2$$
$$q^{10,F N} = 0$$
$$q^{F N}_5 = -q^{(1)}_\chi$$
$$q^{(1)}_\phi = -q^{10}_1 + q^{(1)}_\chi$$
$$q^{(2)}_\phi = -q^{10}_2 + q^{(1)}_\chi$$
$$q^{(3)}_\phi = -q^{10}_1 - q^{(1)}_\chi + q^{(2)}_\chi$$
$$q^{(4)}_\phi = -q^{10}_2 - q^{(1)}_\chi + q^{(3)}_\chi$$
$$q^{(5)}_\phi = -q^{F N}_5$$

Furthermore, requiring the mixed $[SU(5)]^2 U(1)_F$ anomaly to vanish imposes the additional constraint

$$0 = 3(q^{10}_1 + q^{10}_2 - q^{(1)}_\chi) + q^{(1)}_\phi + q^{(1)}_\phi + q^{F N(1)}_5 + q^{F N(1)}_5 + q^{F N(2)}_5 + q^{F N(2)}_5.$$
We can rewrite the charges of $\chi^{(2)}$ and $\chi^{(3)}$ as

\[
q_{\chi^{(2)}} = \frac{3(q_{10}^{1} + q_{10}^{2} - q_{\chi^{(1)}}) + q_{7}^{1} + q_{5}^{2}}{q_{r}^{-1} + 1}, \\
q_{\chi^{(3)}} = \frac{3(q_{10}^{1} + q_{10}^{2} - q_{\chi^{(1)}}) + q_{7}^{1} + q_{5}^{2}}{q_{r} + 1}
\]

in terms of the ratio $q_{r} \equiv q_{\chi^{(2)}}/q_{\chi^{(3)}}$. Thus, the set of undetermined charges can be taken to be

$q_{10}^{1}, q_{10}^{2}, q_{7}^{1}, q_{5}^{2}, q_{\chi^{(1)}}, q_{r}$.

Naively the anomaly could be canceled with a particular choice of the first and second generations (i.e., applying the anomaly constraint on the four charges). However, this necessarily requires that some of the first and second generation charges have a different sign, which is phenomenologically excluded. (One of the (mass)$^{2}$ of the first or second generations would be negative.) Thus, at least one of $q_{\chi^{(2)}}$ or $q_{\chi^{(3)}}$ must be nonzero (and negative). It turns out that to construct a viable effective potential that yields the desired hierarchy of flavon vevs requires that either $q_{\chi^{(2)}}$ or $q_{\chi^{(3)}}$ be larger (in absolute value) than the first and second generation charges.

Hence, the remaining charges for the $\chi$ flavons are the ratio $q_{r}$ and $q_{\chi^{(1)}}$. We nominally take $q_{\chi^{(1)}}$ to vanish for our analysis (as it reduces contact between $H_{u}$ and $U(1)_{F}$ charged fields), however we discuss the consequences of a nonzero charge for $\chi^{(1)}$ in Sec. 7.

The full superpotential can be written using the SU(5) ansatz as

\[
W = a_{1}\chi^{(1)}_{10}^{F}N_{10}^{F} + a_{2}\chi^{(2)}_{5}N_{5}^{F} + a_{3}\chi^{(3)}_{10}^{F}N_{10}^{F} + b_{1}\phi^{(1)}_{10}^{F}, b_{2}\phi^{(2)}_{10}^{F}, b_{3}\phi^{(3)}_{5}^{F}, b_{4}\phi^{(4)}_{5}^{F}, b_{5}\phi^{(5)}_{5}^{F} + \lambda_{d}^{d}H_{d}^{10}5^{F}N^{1}, \lambda_{d}^{h}H_{d}^{10}5^{F}N^{2}, \lambda_{d}^{h}H_{d}^{10}5^{F}N^{3}, \lambda_{d}^{h}H_{d}^{10}5^{F}N^{5}, \lambda_{d}^{h}/2H_{d}^{10}5^{F}N^{5}, \lambda_{d}^{h}/2H_{d}^{10}5^{F}N^{5}.
\]

We have presented the complete superpotential in component form in Appendix A.

There are several comments that should be made regarding the form of Eq. (14):

- We have implicitly assumed $R$-parity ($R \equiv (-1)^{3B+L+S}$) with $B$, $L$, and $S$ representing baryon number, lepton number, and spin respectively) is exactly conserved, and thus we have not written any baryon or lepton number violating terms.

- The messenger–Froggatt-Nielsen sector consists of two $5 + \overline{5}$ pairs and one $10 + \overline{10}$ pair. For ordinary gauge-mediated models this implies that the gauge couplings encounter a Landau pole prior to their apparent unification near $M_{\text{unif}} = 10^{16}$ GeV. In this model, however, partial unification is still possible with semi-perturbative ($\alpha \lesssim 0.5$) couplings, as we will see.

- If $q_{10}^{F}N = 0$, then following term is allowed:

\[
W \subset a^{i}\chi^{(1)}H_{u}H_{d}
\]
which generates an effective $\mu$ term when the flavon $\chi^{(1)}$ acquires a vev. This term is too large to be consistent with electroweak symmetry breaking. Later we discuss to what extent it is possible to generate the $\mu$ term in this model.

3 Mediation of supersymmetry breaking

Supersymmetry is assumed to be dynamically broken in the DSB sector. This is communicated to the FN fields, flavons, and some MSSM fields in two important ways. First, the heavier $\chi$ flavons are assumed to acquire holomorphic ($F$-term) supersymmetry breaking vevs. Second, integrating out the DSB sector induces two-loop $U(1)_F$ gauge-mediated nonholomorphic contributions [soft (mass)$^2$] to the $U(1)_F$ charged scalar fields.

3.1 Holomorphic contributions

In ordinary models of GMSB [8, 9] there are one or more fields whose role is to communicate supersymmetry breaking to messenger quarks and leptons. These fields acquire non-zero vacuum expectation values for their scalar and auxiliary components, which give the messengers supersymmetric and supersymmetry breaking masses respectively. In our model, it is the flavons that play the role of these fields. While their holomorphic contribution to supersymmetry breaking in the FN sector is no longer the only contribution, it remains significant (and essential to generate nonzero gaugino masses, as we shall see).

The auxiliary components of the flavons are in general unknown parameters unless the interactions between flavons are specified. However, their “natural” sizes can be estimated. To produce the hierarchy of Yukawa couplings, the (flavon) superpotential must be such that only the $\chi$ flavons receive vevs at tree level. The $\phi$ fields would have non-zero vevs due to loop effects as discussed in Ref. [21]. In constructing toy models, we have found that additional flavons are required to produce the proper hierarchy, and that most $\chi$ and $\phi$ fields are not directly coupled. Thus, $|F_\chi|^2$ can in general be approximated by the vacuum energy density $V \sim \langle \phi \rangle^2$. The $F_\phi$ “turn on” at one or more loops. A rough guess, $F_\phi \sim \langle \phi \rangle \langle \chi \rangle$, agrees with the typical values produced in these toy models.

For these values, the $F_\phi$ do not contribute significantly to the low-energy spectrum, with two exceptions. One is the contribution of $F_\phi^{(5)}$ to the mass of $\bar{5}^3$. A non-zero $F_\phi^{(5)}$ mixes $\bar{5}^1$ and $5^{FN(1)*}$ giving a contribution to the lightest eigenvalue of $O((\phi^{(5)})^2)$. As we shall see, $\langle \phi^{(5)} \rangle$ may be relatively large (\textless; a few TeV) thus giving a large negative contribution to these third generation scalar (mass)$^2$s. This contribution is largely dependent on given superpotential couplings and unspecified flavon interactions. For simplicity, we assume $F_\phi^{(5)}$ is small enough such that we can ignore it, and take this as a mild constraint on the unspecified flavon superpotential.

The other exception is the contributions of $F$-terms to $CP$ violation. Non-zero $F_{\phi^{(1)}}$ and $F_{\phi^{(2)}}$ cause mixing between the first two generations and thus generating contributions to both $\Delta m_K$ and $\epsilon_K$. For $F_{\phi} \sim \langle \phi \rangle \langle \chi \rangle$, the contribution to $\Delta m_K$ is within experimental bounds, however with an $O(1)$ phase of $F_{\phi^{(1)}}F_{\phi^{(2)}}^*$, the contribution to $\epsilon_K$ is two orders of magnitude too
large. This effectively adds a constraint to the superpotential interactions. Either the phase of $F_{\phi(1)} F_{\phi(2)}^*$ is (unnaturally) small, or one (or both) of these $F$-terms happen to be small. In what follows, we assume at least one of these criteria are satisfied.

3.2 Gaugino masses

MSSM gaugino masses are induced at one-loop through ordinary gauge interactions after the FN fields are integrated out. The expressions for the masses are nearly identical to minimal messenger gauge-mediated models, except that the FN scalars have nonholomorphic contributions. These contributions can be written as

$$M_a(\mathrm{FN}) = \frac{a_a}{4\pi} \sum_i S(i) m_{f_i} \sin 2\theta_i \times \left[ \frac{y^i_1}{1-y^i_1} \ln y^i_1 - \frac{y^i_2}{1-y^i_2} \ln y^i_2 \right]$$

where $S(i)$ is the Dynkin index in a normalization where $S(5+\overline{5}) = 1$ and $S(10+\overline{10}) = 3$. The sum is over all the FN fields, for our model $i = 5^{FN(1)} + \overline{5}^{FN(1)}, 5^{FN(2)} + \overline{5}^{FN(2)}, 10^{FN} + \overline{10}^{FN}$. $m_{f_i} = a_i(\chi^{(i)})$ is the FN fermion mass and $y^i_{1,2} = (m^i_{1,2})^2/m^2_{f_i}$ are ratios of the light and heavy FN scalar masses. The FN scalar mass matrix is

$$\left( \begin{array}{c} F^{\dagger N} \end{array} \right) \left( \begin{array}{c} |a_i(\chi^{(i)})|^2 + m^2_{FN} \\ a_i F^{*}_i \\ a_i F^{*}_i \\ |a_i(\chi^{(i)})|^2 + m^2_{FN} \end{array} \right) \left( \begin{array}{c} F^{\dagger N} \end{array} \right)$$

where $m^2_{FN}$ and $m^2_{\overline{FN}}$ are the nonholomorphic (mass)$^2$, $FN = 5^{FN(1)}, 5^{FN(2)}, 10^{FN}$, and $F_{\chi}$ is vev of the auxiliary component of $\chi$. The mass eigenstates are

$$m^2_{f_i} + \frac{1}{2} (m^2_{FN_i} + m^2_{\overline{FN}_i}) \pm \frac{1}{2} \sqrt{(m^2_{FN_i} - m^2_{\overline{FN}_i})^2 + 4|a_i F_i|^2}$$

with a mixing angle

$$\tan 2\theta_i = \frac{2a_i F_i}{m^2_{FN_i} - m^2_{\overline{FN}_i}}.$$ 

In practice, $a(\chi) \gg m^2_{FN}, m^2_{\overline{FN}}$ and $2|aF| \gg m^2_{FN} - m^2_{\overline{FN}}$, thus the nonholomorphic correction to the gaugino masses is rather small.

We define $M_{FN}$ as the scale where the FN fields are integrated out. Roughly, this is an average of the FN fermion masses. This serves as the boundary condition for the renormalization scale dependent supersymmetry breaking masses for those contributions induced by the MSSM gauge interactions with FN fields.
If we take $F = F_{\chi^{(1)}} = F_{\chi^{(2)}} = F_{\chi^{(3)}}$ and define $M_{FN} \equiv a_1 \langle \chi^{(1)} \rangle = a_2 \langle \chi^{(2)} \rangle = a_3 \langle \chi^{(3)} \rangle$, then we find the gaugino masses at the weak scale to be about

$$M_1(M_1) \simeq (190 \text{ GeV}) \frac{F/M_{FN}}{28 \text{ TeV}} \quad (20)$$

$$M_2(M_2) \simeq (380 \text{ GeV}) \frac{F/M_{FN}}{28 \text{ TeV}} \quad (21)$$

$$M_3(M_3) \simeq (1000 \text{ GeV}) \frac{F/M_{FN}}{28 \text{ TeV}} \quad (22)$$

### 3.3 $U(1)_F$ charged scalar masses and gaugino mass

After the DSB sector is integrated out, $U(1)_F$ charged scalars and the $U(1)_F$ gaugino will acquire two-loop and one-loop contributions to their soft supersymmetry breaking masses respectively. In general they will take the form

$$m_i^2 = c \frac{g_F^2 g_1^4}{256 \pi^4} \Lambda_{\text{DSB}}^2 \quad (23)$$

$$M_F = C \frac{g_F^2}{16 \pi^2} \Lambda_{\text{DSB}} \quad (24)$$

where $\Lambda_{\text{DSB}}$ is the scale where the DSB field are integrated out, $g_F$ is the $U(1)_F$ gauge coupling, and $q_i$ is the $U(1)_F$ charge of the field $i$. The overall coefficients $C, c$ (as well as their signs) are not known without specifying a DSB sector\(^{13}\). However, we can parameterize the effective mass scale $\tilde{m}^2$ induced by the DSB fields as

$$\tilde{m}^2 = c \frac{g_F^2}{256 \pi^4} \Lambda_{\text{DSB}}^2 \quad (25)$$

giving\(^{14}\)

$$m_i^2 = q_i^2 \tilde{m}^2 \quad (26)$$

$$M_F \sim |\tilde{m}| \quad (27)$$

There is one additional subtlety. Since the $U(1)_F$ is anomalous after the DSB sector is integrated out, there is an additional contribution to the scalar (mass)\(^2\) from an effective Fayet-Iliopoulos term $\xi^2$ which appears at one-loop (and thus, in general, $\xi^2 \gg \tilde{m}^2$). This can be seen by examining the effective potential,

$$V_{\text{eff}} \supset \frac{g_F^2}{2} \left[ \xi^2 + \sum_i q_i |\phi_i|^2 \right]^2 + \tilde{m}^2 \sum_i q_i^2 |\phi_i^2| \quad (28)$$

When appropriate superpotential couplings are included, some flavons have non-zero expectation values which mostly cancel the FI term\(^{15}\). After shifting these fields by their vevs, a constant, $\xi^2 \sim \mathcal{O}(\tilde{m}^2)$, remains in the $U(1)_F$ D-term. The mass of the scalar fields at this minimum is

$$m_i^2 = q_i(q_i + q_\xi) \tilde{m}^2 \quad (29)$$

\(^{13}\)We do, however, expect the coefficient $c$ to be the same for all the scalars.

\(^{14}\)We neglect the overall coefficient in front of the $U(1)_F$ gaugino mass, which does not affect our results.

\(^{15}\)This in general occurs at a long-lived, local minimum.
where \( q_\xi \equiv g^2_2 \epsilon_2^2 / \tilde{m}^2 \). This simple result is central to the phenomenological analyses that we carry out in the remainder of the paper.

### 3.4 Uncharged scalar masses

The matter fields that are uncharged under the U(1)_F, namely the third generation and the Higgs, acquire supersymmetry breaking masses from several sources. Ordinary two-loop gauge-mediated contributions arise after integrating out the FN sector, however now the FN scalars acquire both holomorphic and nonholomorphic contributions after the DSB sector is integrated out. As we have emphasized above, these nonholomorphic contributions are soft masses that amount to shifts in the scalar masses away from the naive holomorphic calculation involving simply \( \langle \chi \rangle \) and \( F_\chi \). [See Eq. (18).] Unlike the one-loop calculation of the MSSM gaugino masses, however, the uncharged scalars acquire logarithmically enhanced contributions proportional to the supertrace over each messenger multiplet [34]. Explicitly, the result is [34, 25]

\[
\sum_i m^2_{f}\left(2y_1^i + 2y_2^i - 4\right) \times \left(2 + \gamma_E - \ln 4\pi - \ln \frac{\Lambda_{DSB}}{m_{f}}\right) + G(y_1^i, y_2^i, \theta_i) + G(y_2^i, y_1^i, \theta_i)
\]

\times \sum_{a} C_a(j)S_a(i) \left(\frac{\alpha_a}{4\pi}\right)^2
\]

where \( a = (U(1)_Y, SU(2)_L, SU(3)_c) \), \( C_a(j) \) is the quadratic Casimir of the group \( a \) for the scalar field \( j \), \( S_a(i) \) is the Dynkin index of the group \( a \) for the messenger field \( i \), and

\[
G(y_1, y_2, \theta) = + (\sin 2\theta)^2 \left[-\frac{1}{2} \ln^2 y_1 + y_1 \ln y_1 \ln y_2 - y_1 \text{Li}_2 \left(1 - \frac{y_1}{y_2}\right)\right]
\]

\[
+ \frac{1}{2} \ln^2 y_1 + y_1 \left[2 \ln y_1 + \ln^2 y_1 + \text{Li}_2 (1 - y_1) - \text{Li}_2 \left(1 - \frac{1}{y_1}\right)\right].
\]

where \( \text{Li}_2 \) is the dilogarithm function. This result is clearly a sum of two components: one proportional to the Str \( m^2 = m^2_{f}(2y_1^i + 2y_2^i - 4) \), and the other a threshold function \( G(y_1^i, y_2^i, \theta_i) + G(y_2^i, y_1^i, \theta_i) \). It straightforward to show that in the limit Str \( m^2 \) vanishes (and thus \( (\sin 2\theta)^2 = 1 \)), we recover the analytic result for the minimal messenger model calculated in Refs. [36, 31]. Conversely, in the limit \( m_{f} \to 0 \) but evaluating the logarithm at the scale \( Q \), we obtain a finite result proportional only to Str \( m^2 \). Indeed, the nonlogarithmic finite terms partially cancel, and thus to a good approximation the contribution to the (mass)^2 of the uncharged scalar fields at the scale \( Q \), in limit \( m_{f} \to 0 \), is

\[
m^2_j(Q) = -8 \sum_a C_a(j)(\text{tr}_i S_a(i)m^2_i) \left(\frac{\alpha_a}{4\pi}\right)^2 \ln \frac{\Lambda_{DSB}}{Q}
\]

which is immediately recognized as simply the two-loop renormalization of the uncharged fields due to MSSM gauge interactions with charged fields. This term was first calculated in Refs. [37, 38], and recognized to be of crucial importance to dynamical supersymmetry breaking models in Refs. [39, 40, 34].

\[16\] We employ the \( \overline{\text{DR}}' \) scheme [24, 23].
This result is actually more general than we have presented. Anytime gauge or Yukawa interactions connect apparently separate sectors of the theory, we can expect to find logarithmically divergent contributions to the scalar (mass)\(^2\) proportional to the supertrace of the other fields.

Uncharged scalars, therefore, receive contributions from logarithmically-enhanced two-loop gauge-interactions with FN fields, and also from the U(1)\(_F\) charged first and second generations. It is important to stress that since the logarithmically enhanced correction dominates, there is little distinction between FN fields and the first and second generations, despite the former being integrated out at \(M_{F_N}\).

The above contributions are generic in that they contribute to all scalar fields. In addition, some uncharged scalars also have couplings with charged scalar fields through one-loop, two-loop, or higher order Yukawa interactions. These interactions induce additional contributions to the uncharged scalar (mass)\(^2\) that will be calculated using the renormalization group in Sec. 4.1.

### 3.5 Scalar trilinear couplings

Once couplings between the FN fields and the MSSM fields are introduced, scalar trilinear couplings are generated [41]. They arise as a result of holomorphic supersymmetry breaking, and are proportional to \(\frac{F_{\chi}}{M_{F_N}}\) multiplied by a one-loop factor proportional to superpotential couplings. Since the scalar trilinear couplings enter the off-diagonal elements of mass matrices, we need only consider those fields where the diagonal elements are not tremendous, and the off-diagonal elements are not suppressed by a small fermion mass. Thus, only the scalar trilinear couplings associated with the top, bottom, or tan Yukawa coupling are relevant to this discussion.

The scalar trilinear couplings that arise after \(\chi^{(1)}\) acquires a vev for both its scalar and auxiliary components are [41]

\[
A_t = -\frac{1}{16 \pi^2} \left( \frac{\lambda_{u2,1}^2}{\langle \chi^{(1)} \rangle} \right) F_{\chi^{(1)}}
\]

\[
A_b = -\frac{1}{16 \pi^2} \left[ \left( \frac{\lambda_{u2,1}^2}{\langle \chi^{(1)} \rangle} \right) + \left( \frac{\lambda_{d4,1}^2}{\langle \chi^{(1)} \rangle} \right) \right] F_{\chi^{(1)}}
\]

\[
A_\tau = -\frac{1}{16 \pi^2} \left[ \left( \frac{\lambda_{u2,2}^2}{\langle \chi^{(1)} \rangle} \right) + \left( \frac{\lambda_{d4,2}^2}{\langle \chi^{(1)} \rangle} \right) \right] F_{\chi^{(1)}}
\]

These couplings enter the off-diagonal elements of the well-known sfermion mass matrices:

\[
M_\tilde{f} = \begin{pmatrix}
m_L^2 + m_f^2 + D_L & m_f(A_f - \mu a_\beta) \\
m_f(A_f - \mu a_\beta) & m_R^2 + m_f^2 + D_R
\end{pmatrix},
\]

where \(m_L^2, m_R^2\) are the SU(2)\(_L\) doublet and singlet soft (mass)\(^2\), \(m_f\) is the fermion mass, \(D_L\) and \(D_R\) are the MSSM D-terms, and \(a_\beta = 1/\tan \beta\) for up-type sfermions, and \(a_\beta = \tan \beta\) for down-type sfermions. If \(|\mu| \gtrsim A_f\), which is expected, \(A_b\) and \(A_\tau\) can be largely ignored since \(\tan \beta\) is large in the model, and thus the off-diagonal term is roughly \(-m_f \mu \tan \beta\). \(A_t\) is largely irrelevant since the diagonal components are expected to be much larger than \(m_t A_t\) (or \(m_t \mu / \tan \beta\)). Numerical calculations confirm these expectations, as we will see later in the paper.
4 Renormalization group evolution

There are two separate mass scales in this model where soft supersymmetry breaking masses are induced in light fields after the heavy fields of some sector of the theory are integrated out. Soft masses are induced for $U(1)_F$ charged fields after the DSB sector is integrated out (we assume this occurs at $\Lambda_{\text{DSB}}$), and soft masses are induced for MSSM fields after the FN sector is integrated out (we assume this occurs at $M_{FN} \simeq a_i(\chi^{(i)})$). The first and second generations receive contributions after both sectors are integrated out, although the $U(1)_F$ induced mass is always much larger than ordinary $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge-mediated contributions.

Once one sector of the model receives supersymmetry breaking masses, there are several ways in which masses can be induced for sparticles in other sectors of the model. We will be most concerned with the following effects: Two-loop MSSM gauge interactions between the heavy first and second generations with the lighter third generation. One-loop gauge interactions between the $U(1)_F$ charged fields due to the Fayet-Iliopoulos $D$-term. Yukawa interactions between $U(1)_F$ charged fields and uncharged fields leading to one-loop and two-loop contributions to scalar masses. All of these contributions arise from renormalization group evolution.

4.1 DSB scale to the FN scale

After the DSB sector is integrated out, it is assumed that all of the charged fields acquire masses which depend on their charge, i.e., Eq. (29) for the scalars and Eq. (27) for the $U(1)_F$ gaugino mass, at the scale $\Lambda_{\text{DSB}}$. All of the uncharged field masses are assumed to vanish at $\Lambda_{\text{DSB}}$. This serves as the boundary conditions for renormalization group evolution between the DSB scale and the FN scale. Note that the only parameters that enter the boundary conditions are the charges of the fields, the universal (mass)$^2$ term $\tilde{m}^2$, and the ratio $q_\xi$ of the FI $D$-term to $\tilde{m}^2$.

It is in the evolution to lower scales that a proliferation of parameters enter the calculations of the renormalization group evolution, and hence the masses. These include the gauge couplings $g_1, g_2, g_3$, and $g_F$, and numerous superpotential parameters [see Eq. (72)]. We have calculated the important renormalization group equations that are expected to evolve significantly. In particular, we can safely neglect the evolution of the superpotential parameters, gauge couplings, and the $U(1)_F$ gaugino mass between $\Lambda_{\text{DSB}}$ and $M_{FN}$, although the scalar (mass)$^2$ for all the matter fields (FN and MSSM) must be evolved. The one-loop RG evolution equations for the scalar (mass)$^2$ of all relevant fields are given in Appendix A.1.

We numerically integrated the coupled RG evolution for the calculations below. However, for several fields we provide approximate expressions for the dominant RG contributions for the scalar (mass)$^2$ applicable at the scale $M_{FN}$. The uncharged fields acquire masses from two-loop gauge-mediation, Eq. (30), as well as contributions from superpotential couplings at one-loop and two-loops to charged fields. The third generation component fields in the $\tilde{s}^0$ have the direct superpotential coupling $b_5$ to $U(1)_F$ charged fields, giving the one-loop contributions

\begin{equation}
\Delta m_{d_3}^2(M_{FN}) = -\frac{\tilde{m}^2}{8\pi^2}(b_{3}^{(d)})^2 q_{d_3}^2 \ln \frac{\Lambda_{\text{DSB}}}{M_{FN}}
\end{equation}
\[
\Delta m^2_{E \lambda}(M_{FN}) = -\frac{\tilde{m}^2}{8\pi^2} \left( b_5^{(L)} \right)^2 q_{\phi(5)}^2 \ln \frac{\Lambda_{DSB}}{M_{FN}}. \tag{38}
\]

The down-type Higgs field also has direct superpotential couplings to U(1)\( F \) charged fields, giving

\[
\Delta m^2_{H^d}(M_{FN}) = -\frac{\tilde{m}^2}{8\pi^2} \left[ 3\left( \lambda_{1,1}^d \right)^2 q_{101}^2 + \left( \lambda_{1,2}^d \right)^2 q_{101}^2 + 3\left( \lambda_{2,1}^d \right)^2 q_{102}^2 + \left( \lambda_{2,2}^d \right)^2 q_{102}^2 \right] \ln \frac{\Lambda_{DSB}}{M_{FN}}. \tag{39}
\]

Notice that these one-loop contributions are positive if \( \tilde{m}^2 < 0 \). The component fields \( Q^3 \) and \( e^3 \) do not have direct superpotential couplings to U(1)\( F \) charged fields. However, they do have couplings to uncharged fields that acquire masses at one-loop, namely \( \bar{d}^3, \pi^3 \), and \( H^d \). Thus, \( Q^3 \) and \( e^3 \) acquire effectively two-loop contributions,

\[
\Delta m^2_{Q^3}(M_{FN}) = \frac{Y^2 \tilde{m}^2}{128\pi^4} \left[ 3\left( \lambda_{1,1}^d \right)^2 q_{101}^2 + \left( \lambda_{1,2}^d \right)^2 q_{101}^2 + 3\left( \lambda_{2,1}^d \right)^2 q_{102}^2 + \left( \lambda_{2,2}^d \right)^2 q_{102}^2 \right. \\
+ \left. \left( b_5^{(d)} \right)^2 q_{\phi(5)}^2 \right] \times \left( \ln \frac{\Lambda_{DSB}}{M_{FN}} \right)^2. \tag{40}
\]

\[
\Delta m^2_{e^3}(M_{FN}) = \frac{Y^2 \tilde{m}^2}{64\pi^4} \left[ 3\left( \lambda_{1,1}^d \right)^2 q_{101}^2 + \left( \lambda_{1,2}^d \right)^2 q_{101}^2 + 3\left( \lambda_{2,1}^d \right)^2 q_{102}^2 + \left( \lambda_{2,2}^d \right)^2 q_{102}^2 \right. \\
+ \left. \left( b_5^{(L)} \right)^2 q_{\phi(5)}^2 \right] \times \left( \ln \frac{\Lambda_{DSB}}{M_{FN}} \right)^2. \tag{41}
\]

Notice that these two-loop superpotential-induced contributions are negative if \( \tilde{m}^2 < 0 \). Finally, \( u^3 \) and \( H^u \) acquire superpotential-induced couplings only at higher orders, which we do not present here. (These small contributions can be ignored since the two-loop gauge-mediated contributions dominate for these fields.)

There is also significant evolution among the charged scalar fields due to U(1)\( F \) gaugino interactions and the FI \( D \)-term. In both cases the RG contributions depend on the separation between \( M_{FN} \) and \( \Lambda_{DSB} \) and the U(1)\( F \) coupling strength \( g_F \). The gaugino contribution can be estimated

\[
\Delta m^2_i = \frac{g_i^2 g_F^2}{2\pi^2} |\tilde{m}| \ln \frac{\Lambda_{DSB}}{M_{FN}}. \tag{42}
\]

The contribution to the RG evolution from the FI \( D \)-term is shown in Eq. (74), and is really a sum of two distinct components. At \( \Lambda_{DSB} \) the masses of all charged fields are equivalent to \( q_i(q_i + q_\xi)\tilde{m}^2 \), and so we can write this term at \( Q = \Lambda_{DSB} \) quite simply:

\[
S'(\Lambda_{DSB}) = \tilde{m}^2 \sum_j S(j) q_j^3 + \tilde{m}^2 q_\xi \sum_j S(j) q_j^2. \tag{43}
\]

The first term is proportional to the \([U(1)F]^3 \) anomaly, while the second component is proportional to the FI \( D \)-term that results after the DSB fields are integrated out. A term proportional to the anomaly should not be surprising since the U(1)\( F \) is both anomalous and has a FI \( D \)-term after the DSB sector is integrated out. Since the anomaly contribution is a weighted sum over (charge)\(^3 \), several cancellations reduce this contribution to be typically much smaller than the
second term proportional to $q_\xi$. We emphasize that the second term is generated as a result of DSB sector dynamics, and in particular does not depend on whether, for example, the $U(1)_F$ anomaly is canceled among the light (non-DSB) fields (see Ref. [8]). We can approximate this contribution to $U(1)_F$ charged scalar fields as

$$
\Delta m_i^2 = -\frac{q_i g_\xi^2}{8\pi^2} S' \ln \frac{\Lambda_{DSB}}{M_{FN}},
$$

which is negative if $q_i \tilde{m}_i^2 > 0$, as is the case for the first and second generations.

The size of these terms is not known precisely, since we expect, for example, additional flavon fields charged under the $U(1)_F$ but uncharged under the SM gauge group to be present. If we assume that the anomaly term nearly vanishes, then the term proportional to $q_\xi$ is really a lower bound on the size of the RG contribution. In the sum we include only the fields charged under the $U(1)_F$ that we have stated in this paper, and thus it should be remembered that the contribution we include is really a lower bound on the FI $D$-term induced evolution.

4.2 FN scale to the weak scale

After the FN sector is integrated out, the remaining fields in the model are the MSSM fields and the light flavons. Since the light (and heavy) flavons are uncharged under the SM gauge group, they do not play any significant dynamical role in the low energy phenomenology. (They are, however, essential to generating the Yukawa coupling hierarchy.) The MSSM fields, in contrast, are precisely the ones that we expect to find in upcoming collider experiments if weak scale supersymmetry is indeed manifest in Nature.

For the evolution between the FN scale and the weak scale, we have used the full two-loop RG equations for all superpotential couplings and soft mass parameters in the MSSM [37, 38]. These include the dominant two-loop SM gauge interactions between the first and second generations with the third generation, as well as several subdominant terms.

One technical subtlety arises as to how (or at what scale) to precisely decouple the first and second generations from the third generation evolution. Unlike the FN sector (or any other heavy vector-like sector), it is more subtle to integrate out these contributions due to the light fermion masses. High precision demands two-loop matching conditions near the weak scale for the third generation sparticle masses, which could be comparable to three-loop log-enhanced renormalization group contributions. An alternative was performed in Ref. [25] by taking $m_f \to 0$ limit of Eq. (30) and finding that a nontrivial finite term proportional to the messenger supertrace is present. In particular, they found that Eq. (32) should be modified by letting

$$
\ln \frac{\Lambda_{DSB}}{Q} \to \left[ \ln \frac{\Lambda_{DSB}}{Q} + \frac{\pi^2}{6} \right],
$$

which we can rewrite as a shift of the scale

$$
Q \to \frac{Q}{\exp \frac{\pi^2}{6}} \sim \frac{1}{5} Q.
$$
In their approach, the scalars were decoupled at the first and second generation mass scale $Q = m_{1st,2nd}$, which is evidently approximately equivalent to performing log evolution only via Eq. (32), but down to a scale $Q \sim \frac{1}{5}m_{1st,2nd}$. Hence, a good approximation is to evolve the third generation including the two-loop effects of the first and second generations down to the weak scale, which is expected to be about a factor of 10 below $m_{1st,2nd}$. This is what we do in this paper.

Another issue that arises in the evolution from the FN scale to the weak scale is possible presence of the one-loop hypercharge $D$-term that is proportional to $\text{tr}Y_im_i^2$. Ordinarily this term is forbidden in supergravity-mediated models with common scalar masses since $m^2\text{tr}Y_i$ is simply proportional to the gravitational anomaly that is required to vanish. In gauge-mediated models, each scalar $m_i^2$ is a sum over several contributions proportional to its gauge charges, but again the hypercharge $D$-term cancels due to requiring the mixed and triangle anomalies to vanish. However, this problem would seem to be particularly acute for models with heavy first and second generations. Interestingly, $\text{tr}Y_im_i^2$ vanishes for matter in (for example) complete SU(5) multiplets, which is a strong motivation in our scenario to enforce that all of the first and second generations are heavy (see Sec. 7 for further discussion). Furthermore, the gauge anomaly argument also applies to our model for the two-loop gauge-mediated contributions, but does not apply to the Yukawa-induced contributions in Eqs. (37)–(41). This is obvious to see with, e.g., the contributions to $\overline{T}_3$ and $H_d^d$ that have identical gauge quantum numbers but distinct superpotential couplings. The effect of these contributions on the size of the hypercharge $D$-term can be enhanced if the SU(5) component superpotential couplings are widely split apart. In all cases the $\text{tr}Y_im_i^2$ contribution to the scalar (mass)$^2$ RGE for a given sparticle is suppressed by $g_1^2$, and thus the $D$-term term imparts its greatest relative one-loop effect on the stau mass. Depending on the sign of $\text{tr}Y_im_i^2$ (summed over the MSSM fields), the contribution can be either positive or negative, although a significant positive contribution is excluded by requiring proper EWSB, since one would also have an undesired significant positive contribution to $m_{H_u}^2$.

5 Dynamics of the model

The dynamics of the model that is of interest to us includes the full spectrum of both the FN fields and the MSSM fields. It should now be obvious that this is a highly interconnected model, and thus to extract e.g. weak scale observables with any accuracy requires a complete implementation of the model with as few approximations as possible. To this end, we have implemented the full set of RG evolution equations and the relevant boundary conditions in a computer program for numerical evaluation. The program uses a fourth order Runge-Kutta algorithm to evaluate the the coupled differential equations. The general procedure is to:

1. evolve the SM gauge couplings to the FN scale

2. set the DSB boundary conditions

\footnote{It is easy to show that as the superpotential parameters are taken to zero, the hypercharge $D$-term generated at the FN scale goes to zero.}
3. evolve all of the scalar (mass)\(^2\) from the DSB scale to the FN scale (neglecting evolution of the gauge couplings and the superpotential parameters)

4. integrate out the FN sector, adding the appropriate contributions to the soft mass parameters

5. evolve to the weak scale, decoupling the heavy states as appropriate

6. evaluate the one-loop effective potential to extract \(\mu^2\) and \(B_\mu\)

7. compute the mass eigenstates and appropriate mixing matrices of the weak scale sparticles

8. repeat steps 1 through 7 until one achieves convergence (about three or four repetitions is always sufficient)

Each one of these steps involves numerous technical subtleties, and we discuss several important ones in the following sections.

5.1 A minimal set of parameters

Applying no constraints, the minimal model has:

- four gauge couplings
- 30 superpotential parameters (plus phases\(^{18}\))
- two Higgs scalars that acquire vevs
- eight flavons with scalar component vevs
- three flavons with auxiliary component vevs (see Sec. 3.1)
- \(\tilde{m}^2\): universally induced scalar (mass)\(^2\) (we take \(M_{U(1)_F} \sim \sqrt{|\tilde{m}^2|}\))
- \(q_\xi\): ratio of FI D-term to \(\tilde{m}^2\)
- six undetermined \(U(1)_F\) charges

This may seem overwhelming, however consider that all of the parameters of the SM, namely the gauge couplings, quark and charged lepton Yukawa couplings, CKM matrix elements, the physical phases: two in each of the full down-quark and charged lepton mass matrices (with the same phases appearing in the sparticle matrices), one phase associated with the \(\mu\)-term and a phase for each flavon auxiliary component vev. One combination of these phases plays the role of the CKM phase while the remainder either contribute to \(B^0 - \bar{B}^0\) mixing significantly or are unimportant for phenomenology (except for one from the \(F\)-components of the \(\phi\)'s which must be constrained). This is true for the most part because all squarks in this model are \(\gtrsim 1\) TeV. We shall leave the phases out of the parameter counting and discuss their generic effects in Sec. 5.4.

\(^{18}\)When considering the tree-level potential below the DSB scale with flavons replaced with their vevs we find the following physical phases: two in each of the full down-quark and charged lepton mass matrices (with the same phases appearing in the sparticle matrices), one phase associated with the \(\mu\)-term and a phase for each flavon auxiliary component vev. One combination of these phases plays the role of the CKM phase while the remainder either contribute to \(B^0 - \bar{B}^0\) mixing significantly or are unimportant for phenomenology (except for one from the \(F\)-components of the \(\phi\)'s which must be constrained). This is true for the most part because all squarks in this model are \(\gtrsim 1\) TeV. We shall leave the phases out of the parameter counting and discuss their generic effects in Sec. 5.4.

\(^{19}\)Note that the phase in the CKM matrix will be discussed in Sec. 5.4, and \(\theta_{QCD}\) is not physical since the model has \(m_u = 0\) (see also Sec. 5).
Higgs vev, and Higgs mass are included in the above parameter set. In particular, the following is fixed by experiment:

- three gauge couplings \( g_1, g_2, \) and \( g_3 \)
- Higgs vevs determined by effective potential minimization up to \( \text{sign}(\mu) \) and the ratio \( \tan \beta \equiv \langle H^u \rangle / \langle H^d \rangle \)
- three superpotential parameters, \( Y_t, Y_b, \) and \( Y_\tau \)
- six fermion masses and three CKM elements that are determined by several superpotential parameters and ratios of light to heavy flavon vevs

Generally the superpotential parameters associated with the \( \chi \) flavons can be absorbed into the vevs (although one may still need to distinguish between quark and lepton couplings). Most of the other superpotential couplings feed into the quark and lepton mass matrices, but are otherwise unimportant. This is fortunate in that the number of parameters that influence the weak scale phenomenology is a much smaller set. The exceptions to this rule are the superpotential couplings that enter the higher order calculations of the uncharged scalar masses, namely \( b_{5}^{(d)}, b_{5}^{(L)}, \lambda_{1,1}^{d}, \lambda_{1,2}^{d}, \lambda_{2,1}^{d}, \) and \( \lambda_{2,2}^{d} \).

We can simplify the model by assuming

- \( M_{FN} = a_i \langle \chi^{(i)} \rangle \), the latter assumed to be roughly equal
- \( F_\chi = a_i F_{\chi^{(i)}} \), the latter assumed to be roughly equal
- \( q_{\chi^{(i)}} = 0; q_{\chi^{(2)}/\chi^{(3)}} \equiv q_{\xi} \sim 1 \)
- \( q = q_{10^1} \sim q_{10^2} \sim q_{5^1} \sim q_{5^2} \)
- \( b_5 = b_5^{(d)} \sim b_5^{(L)} \)
- \( \lambda_1^d = \lambda_{1,1}^d \sim \lambda_{1,2}^d \)
- \( \lambda_2^d = \lambda_{2,1}^d \sim \lambda_{2,2}^d \)

An overall \( U(1)_F \) charge can be absorbed in \( \tilde{m} \) (or equivalently in \( g_F \)), as is usual for an Abelian group. Thus if we assume all of the first and second generation charges are closely comparable (but not exactly equal), the only undetermined charge is the ratio \( q_\xi / q \).

Thus we arrive at the “minimal set of parameters” capable of adequately describing the model:

\[
M_{FN}, \ F_\chi, \ \Lambda_{DSB}, \ \tilde{m}^2, \ b_5, \ \lambda_1^d, \ \lambda_2^d, \ q_\xi / q, \ \text{sgn}(\mu), \ \tan \beta
\]  \hspace{1cm} (47)

We call this the “minimal flavor-mediated model” (MFMM), that is hereafter the set of free parameters we consider in the model.\(^{20}\) Some variations from this minimal parameter set are explored in more detail in Sec.\(^2\).

\(^{20}\)Note that in the limit \( \tilde{m} \to 0 \) the MFMM becomes an ordinary gauge-mediated model with \( n_{10^1+10^2} = 1 \) and \( n_{5^1+5^2} = 2 \). We explicitly verified that our calculations do indeed reproduce the spectrum of an ordinary gauge-mediated model \([42, 43, 44]\) in this limit.
5.2 General discussion

The MSSM gaugino masses are proportional to $F_\chi/M_{FN}$, and are well approximated by Eqs. (20)-(22). A rough lower bound is $F_\chi/M_{FN} \gtrsim 7$ TeV, since charginos lighter than about 90 GeV have not been seen at LEP. Imposing some degree of naturalness suggests the gluino ought not to be too heavy (for a recent discussion, see Ref. [45]). Requiring the gluino be lighter than $1.5\ltl 35$ TeV implies $F_\chi/M_{FN} \lesssim 28(35)$ TeV.

The universal (mass scale)$^2$ induced by the DSB sector, $\tilde{m}^2$, must be negative. This is evident from Eqs. (37),(38) that imply the one-loop Yukawa-induced (mass)$^2$ for $\tilde{d}^3$ and $\tilde{T}^3$ is positive only if $\tilde{m}^2 < 0$. One might speculate that such contributions could be smaller than positive log-enhanced two-loop gauge-mediated contributions from the FN scalars. However, $\tilde{m}^2 < 0$ (i.e., a negative messenger field supertrace) is required to generate positive contributions to the third generation masses via two-loop log-enhanced gauge-mediation. Self-consistency of the above condition requires $q < 0 < q_\xi$ and $|q| < q_\xi$ such that the (mass)$^2$ for the first and second generations is positive. Finally, preliminary work regarding the $U(1)_F$ charge relations that would generate the required flavon vev hierarchy suggested [21] that $|q_\chi| > |q|$, which we note is satisfied by the above relations.

The FN scale $M_{FN}$ is generically expected to be of order 100 TeV [21], which is roughly the vev of the heavier $\chi$ flavons. The ratios $(\phi)/\langle\chi\rangle$ are determined by the fermion mass hierarchy, and are thus largely fixed by experiment.

$\tilde{m}^2$ and the DSB scale $\Lambda_{DSB}$ are in principle related up to a group theory constant as shown in Eq. (25). However, the group theory factor is not known without specifying a DSB sector. In ordinary gauge-mediation the group theory factor that enters the two-loop expression for the MSSM scalar masses is $6n_{10}+10n_{5}+5$ (i.e., determined by the content of the messenger sector), so a reasonable estimate in our case is probably $1 \lesssim |c| \lesssim 10$. We can rewrite Eq. (25) as

$$\tilde{m}^2 \simeq (10 \text{ TeV})^2 \frac{c}{3} \left( \frac{g_F}{0.3} \right)^4 \left( \frac{\Lambda_{DSB}}{10^7 \text{ GeV}} \right)^2,$$

(48)

with some foresight into the next section where constraints on $g_F$ are discussed.

5.3 The first and second generation mass scale

The mass of the first and second generation scalars induced by the DSB sector at the scale $Q = \Lambda_{DSB}$ is given by Eq. (25). This is the physical mass under the approximation that RG evolution for the first and second generation scalar masses can be ignored. In Fig. 1 we show contour lines for various first and second generation masses within the allowed region of parameter space. Upper bounds on $\left| g^2 \tilde{m}^2 \right|$ (corresponding to the contours’ far left endpoint) arise from requiring that the flavon scalar (mass)$^2$ are positive. Upper bounds on $\left| q_\xi/q \right|$ (corresponding to the contours’ far right endpoint) arise from requiring the lightest stau (mass)$^2$ be positive. Notice that lowering $\tan \beta$ admits larger $\left| q_\xi/q \right|$, since lowering $\tan \beta$ also increases the stau mass. Thus,

\footnote{This generalizes to $q_{10_1}, q_{10_2}, q_{10_3}, q_{10_4}, q_{10_5}, q_{10_6}, q_{10_7}, q_{10_8}, q_{10_9}, q_{10_{10}} < q_\xi$ and $|q_{10_1}|, |q_{10_2}|, |q_{10_3}|, |q_{10_4}|, |q_{10_5}| < q_\xi$ for first and second generation charges that are not approximately degenerate.}
the internal dynamics of the model restrict the allowed range of input parameters, although the precise cutoffs do depend on other parameters including $M_{FN}$ and $F_X$.

There is, however, RG evolution of the first and second generation masses. Between the FN scale and the weak scale, there are ordinary gauge interactions coupling to the MSSM gauginos, but such contributions are very small since $M_a \ll m_{1st,2nd}$. There are also the effective first and second generation Yukawa interactions that result after the flavons acquire vevs, but these contributions are also tiny. It is the U(1)$_F$ interactions induced by the U(1)$_F$ gaugino mass, and even more importantly, the FI $D$-term that induce evolution among the U(1)$_F$ charged scalar masses [see Eq. (73)]. In Fig. 3 we illustrate the evolution of the first and second generation scalar masses as a function of the undetermined gauge coupling $g_F$, for a particular choice of other parameters (key among them $\Lambda_{DSB}/M_{FN} = 10^2$ and $\tan \beta = 10$). Note that if the content of the DSB sector is fixed, then the scale $\Lambda_{DSB}$ must also change inversely proportional to $g_F^2$. Other choices of the ratio $\Lambda_{DSB}/M_{FN}$ can be obtained by rescaling

$$g_F^2 \rightarrow \frac{g_F^2}{\ln 10^2} \ln \frac{\Lambda_{DSB}}{M_{FN}}.$$  

Obviously the first and second generation scalar masses are dramatically reduced as the gauge
Figure 2: RG evolution of $U(1)_F$ charged scalar masses resulting from interactions with the $U(1)_F$ gaugino and the FI $D$-term. The four lines correspond to $m_{1st,2nd}$ for a DSB model with $(-q^2 \tilde{m}^2)^{1/2} = 20, 15, 10, 7.5$ TeV from top to bottom.

coupling is increased, and this provides another phenomenological constraint on the model.

The reason to emphasize the precise first and second generation mass scale that is generated from the model is threefold. First, the first and second generation mass scale determines the size of supersymmetric FCNC contributions to e.g. $K^0 \leftrightarrow \bar{K}^0$ mixing, and so has indirect phenomenological relevance in FCNC processes. Second, increasing the first and second generation mass scale reduces the gauge couplings at a given high scale (i.e. $M_{unif} \sim 10^{16}$ GeV), avoiding the Landau pole that would be naively expected with the FN sector in the model. This will be discussed in greater detail in Sec. 5.6. Finally, and perhaps most important, the first and second generation mass scale determines the size of the two-loop negative gauge contributions to the third generation. Susinctly, increasing the first and second generation mass scale implies the third generation is reduced. This is important because it suggests a no-lose principle for this model: Either supersymmetric contributions to FCNC should be seen or the third generation (in particular, the lightest stau) should be seen in experiment. Simultaneously tightening these experimental bounds, even by relatively small factors, rapidly reduces the allowed parameter space for this model.
Figure 3: The stau mass is shown as a function of $\tan \beta$ with the contours corresponding to particular choices of for the the superpotential parameters $b_5$, $\lambda^d_1$, and $\lambda^d_2$ (taken to be equal for illustration). The (solid, dashed) lines correspond to taking the parameters $F_\chi/M_{FN} = (30, 20)$ TeV and $\tilde{m}^2 = (-30^2, -20^2)$ TeV$^2$.

5.4 Superpotential couplings

It is clear from Eqs. (37)-(41) that the superpotential couplings $b_5$, $\lambda^d_1$, and $\lambda^d_2$ determine both the positive one-loop and negative two-loop Yukawa-induced supersymmetry breaking contributions to the third generation. Of greatest concern is the negative two-loop contribution to the SU(2)$_L$ singlet stau (mass)$^2$, since the positive two-loop gauge-mediated log-enhanced contribution is proportional to the relatively small coupling $g_1^4$ multiplied by one power of $\ln(A_{DSB}/M_{FN})$. The negative two-loop Yukawa-mediated contributions are proportional to $\tilde{m}^2$ multiplied by a third generation down-type Yukawa coupling that can be well approximated by $Y_{b,\tau} = \sqrt{2}m_{b,\tau} \tan \beta/v$ (i.e., proportional to $\tan \beta$). The positive two-loop gauge-mediated log-enhanced contributions are also proportional to $\tilde{m}^2$, and thus one can always find an upper limit on $\tan \beta$ for fixed $b_5$, $\lambda^d_1$, $\lambda^d_2$ couplings that is essentially independent of $\tilde{m}^2$.

In Fig. 3 we show the stau mass as a function of $\tan \beta$, for several choices of the superpotential parameters $b_5$, $\lambda^d_1$, and $\lambda^d_2$ (taken to be equal for illustration). Requiring $O(1)$ couplings, i.e. couplings larger than about 0.1 (as explained in the introduction), implies $\tan \beta \gtrsim 10$ such that $Y_{b,\tau} \gtrsim 0.1$. If we impose the same restriction on the superpotential couplings, then $\tan \beta \lesssim 30$. This conclusion is not significantly modified by changing other parameters in the model. Hence, for the model to survive experimental constraints we are faced with couplings $\sim 0.1 \to 0.3$ in
either the third generation Yukawa couplings or the $b_5$, $\lambda^d_1$, $\lambda^d_2$ superpotential couplings.

In addition, the sizes of these couplings are interesting from a model building perspective, since their size (coupled with ratios of flavon vevs) impacts the predictions for the fermion masses. For example, if these superpotential couplings (and no others) were small, then $\epsilon^{(d)}_{52}$, which determines the CKM element $V_{ub}$, would be suppressed relative to $\phi^{(5)}/\chi^{(2)}$. Thus, while $\epsilon^{(d)}_{52}$ is required to be one of the smallest ratios, $\langle \phi^{(5)} \rangle$ would be comparable to the other flavon vevs. It is therefore possible to identify $\phi^{(5)} \equiv \phi^{(4)}$, giving one less flavon and an additional restriction on the $U(1)_F$ charges. In general, several other variations with additional couplings and other flavon identifications are possible, thereby reducing the number of flavon fields required for the model (however, one must verify the stability of the non color-breaking vacua, etc.). Since there is no immediate phenomenological impact of a reduction of the number of flavons, we do not pursue this model building exercise further in this paper.

5.5 Electroweak symmetry breaking

In ordinary supergravity-mediated models and minimal gauge-mediated models, electroweak symmetry breaking (EWSB) is radiatively induced due to the evolution of the up-type soft Higgs (mass)$^2$ into negative values near the weak scale. The negative one-loop contributions to $H^u$ arise from the top Yukawa coupling, although we note that there are analogous negative one-loop contributions to $H^d$ from the bottom and tau Yukawa couplings (that are large for large $\tan \beta$). In this model, EWSB occurs exactly as above, and we utilize the minimized Higgs effective potential at the weak scale to match to the $Z$ mass. The expressions are very well known, namely

$$\mu^2 = -\frac{1}{2} M_Z^2 + \frac{m^2_{H^u} - m^2_{H^d} \tan^2 \beta}{\tan^2 \beta - 1} + \cdots$$

$$m_A^2 = -\frac{2 B\mu}{\sin 2\beta} = \frac{1}{\cos 2\beta} \left( m^2_{H^u} - m^2_{H^d} \right) - M_Z^2 + \cdots$$

where $m_A$ is the CP-odd Higgs mass, $B\mu$ is the soft (mass)$^2$ parameter for $H^uH^d$, and the ellipsis stands for higher order corrections that we do not write here. We include one-loop corrections from all squarks and sleptons using the results of Ref. [46], but neglect the much smaller contributions from gauginos and Higgs scalars. We found that despite the large mass scale of the first and second generations in this model, their summed contribution to the effective potential is small compared with the third generation (which are dominated by the contributions from the stops).\footnote{In particular, the electroweak $D$-term contributions that are naively proportional to $M_Z^2 \ln(m_{1st,2nd}/Q)$ approximately cancel upon summing over the matter of each generation.}

Applying the constraint $m_A^2 > 0$ to Eq. (50) implies $m_{H^u}^2 > m_{H^d}^2$ is necessary to satisfy the EWSB conditions at tree-level (and generally also at one-loop). At large $\tan \beta$, the negative one-loop contributions to $H^d$ and $H^u$ are comparable, and thus the requirement $m_A^2 > 0$ establishes an upper bound on $\tan \beta$ in the model (for example, the endpoints of the lines on the far right of Fig. \[\alpha\] arise from this constraint). Since we have already argued that intermediate values of $\tan \beta$
lead to the most natural couplings, it follows that in general EWSB does not impose a severe constraint on the model.

Nevertheless, matching the Higgs vevs to the $Z$ mass using the EWSB relation Eq. (49) does allow us to determine the bilinear superpotential mass parameter $\mu$ (up to a sign) at the weak scale. In Fig. 4 we show the size of the $\mu$-term that results in the model as a function of the holomorphic supersymmetry breaking vev $F_\chi$. We stress that $\mu$ is within 1 TeV for typical parameters of the model, although a low value, i.e. $|\mu| \lesssim 500$ GeV, arises only for $m_{1st,2nd} \lesssim 5$ TeV which is difficult to reconcile with FCNC constraints. The left-hand side endpoints of the contours arise from the experimental bounds on the lightest chargino mass and the lightest stau mass. Notice that at larger $m_{1st,2nd}$ there is essentially no dependence on the holomorphic supersymmetry breaking vev $F_\chi$. Thus fine-tuning arguments that restrict the size of $|\mu|$ apply primarily to the DSB induced mass parameter $\tilde{m}^2$.

Finally, we note that although the EWSB conditions determine the value of $\mu$ at the weak scale, we have not specified a mechanism for generating $\mu$ from higher scale dynamics. We defer a discussion of this to Sec. 7.2.
5.6 Gauge coupling unification

One of the benefits of FN fields that fill complete SU(5) reps is that gauge coupling unification is preserved to one-loop. Unfortunately, this does not hold to two-loops (or higher), with the approximation increasingly poor as the gauge couplings become larger. It is well known that while the unification scale is unchanged to one-loop as the number of messenger fields in complete SU(5) reps is increased, the value of the unified gauge coupling increases. The upper bound on the number of messenger fields is usually taken to be $3n_{10+\overline{10}} + n_{5+\overline{5}} \leq 4$ for a messenger scale not too far removed from the weak scale\textsuperscript{23}. The model we have presented requires one $10+\overline{10}$ pair and two $5+\overline{5}$ pairs in order to generate the SM fermion masses, which nominally implies the gauge couplings encounter a Landau pole prior to unification.

In our model, however, the first and second generations are heavy. Using a decoupling scheme, the $\beta$-function coefficients of the gauge couplings are shifted from the MSSM values for scales between $M_Z$ and $m_{1st,2nd}$. The gauge $\beta$-function coefficient at one-loop $B_a^{(1)}$ is shifted by

$$\Delta B_a^{(1)} = -\frac{4}{3}$$

assuming all of the first and second generation scalars are decoupled. This gives a correction to the gauge couplings that is roughly

$$\Delta g_a \approx \frac{g_a^3}{16\pi^2} \Delta B_a^{(1)} \ln \frac{m_{1st,2nd}}{M_Z}$$

The relevance of this correction is apparent if we absorb this one-loop correction into a shift of the effective FN scale (where the gauge $\beta$-function coefficients are again changed due to the matter content of the FN fields)

$$\frac{M_{FN\text{eff}}}{M_{FN}} = \left( \frac{m_{1st,2nd}}{M_Z} \right)^{4/15}$$

which is nearly an increase of a factor of 4 for $m_{1st,2nd} \sim 10$ TeV. Hence, this one-loop analysis suggests that a model with heavy first and second generations allows the FN scale to be lowered by a nontrivial factor, relative to a model without heavy first and second generations.

In practice, one must use the full two-loop RG equations to calculate the evolution and determine if indeed the gauge couplings encounter a Landau pole prior to unification. In Fig.\textsuperscript{28} we illustrate the evolution of the three SM gauge couplings with the parameter choices $m_{1st,2nd} \simeq 10$ TeV and $M_{FN} = 2 \times 10^5$ GeV. It is obvious that the gauge couplings are diverging near the unification scale, although they remain semi-perturbative ($g_a \lesssim 2.5$) up to $M_{\text{unif}}$ where $g_2(M_{\text{unif}}) = g_3(M_{\text{unif}})$. Notice that $g_1(M_{\text{unif}})$ is also large but somewhat below the non-Abelian couplings, and therefore naive triple gauge coupling unification does not occur in the model. This conclusion appears to be robust in the model; other choices of parameters such as $M_{FN}$ or $m_{1st,2nd}$ do not affect this conclusion, as shown in Fig.\textsuperscript{28}. That $g_1$ is not unified is an intriguing

\textsuperscript{23}Increasing the messenger scale allows for more messenger fields in the messenger sector while avoiding a Landau pole at the unification scale, see e.g. Ref \textsuperscript{47}.\textsuperscript{28}
Figure 5: The evolution of the gauge couplings from the weak scale to $M_{\text{unif}} \simeq 10^{16}$ GeV where $g_2(M_{\text{unif}}) = g_3(M_{\text{unif}})$ is illustrated for a model with $m_{1,2,\text{nd}} \sim 10$ TeV and $M_{\text{FN}} = 2 \times 10^5$ GeV.

and unexpected result. We explicitly checked that this result is not significantly affected by large splittings between e.g., the colored and uncolored FN fields. However, unlike some hypothetical model with $g_2(M_{\text{unif}}) \neq g_3(M_{\text{unif}})$ or $g_1(M_{\text{unif}}) \gg g_2(M_{\text{unif}}) \sim g_3(M_{\text{unif}})$, several remedies can be applied in this case to restore full gauge coupling unification. As usual, modifications come at the expense of either an ad hoc assumption (such as postulating the $U(1)_Y$ normalization is not the usual $\sqrt{5}/3$ that results from the SU(5) GUT; although see Ref. [48]) or adding fields with special properties (in particular, fields that do not fill an SU(5) rep but instead are charged only under $U(1)_Y$). Since these modifications are unlikely to have any significant impact on the weak scale phenomenology of the model, we will not pursue this further in this paper.

6 Signals of the Model

There are numerous ways to search for signals of the flavor model, including through rare FCNC processes, collider phenomenology at the threshold of sparticle production, etc. We focus on several aspects of phenomenology that will be relevant at current or upcoming collider experiments. In particular, we provide a general overview of the model’s spectrum, the identity of the next-to-lightest sparticle (NLSP), the associated macroscopic decay length into a particle and a light gravitino, the prospects for stau production and the importance of taus in collider signals, and finally we examine $D^0 \leftrightarrow \bar{D}^0$ mixing as a window on a promising FCNC process that is
Figure 6: The variation of the values of the gauge couplings at $Q = M_{\text{unif}}$ as a function of the first and second generation mass scale $m_{1st,2nd}$ and $M_{FN}$. The solid, dotted, and dashed lines correspond to $M_{FN} = 10^5$, $2 \times 10^5$, and $4 \times 10^5$ GeV respectively.

expected in the model.

6.1 General comments

The weak scale spectrum of the model consists of the gauginos, the third generation scalars, and the Higgs scalars, with several of these fields expected to have $O$(1 TeV) masses. The chargino and neutralino mass eigenstates arise from mixing between the gaugino and Higgsino fields, as usual in the MSSM. In our model, $\mu$ is always significantly larger than the Bino mass $M_1$, and is also larger than the Wino mass $M_2$ in all but the most highly contrived scenarios. This implies that the two lightest neutralinos and the lightest chargino are dominantly gaugino-like,

\[
\tilde{N}_1 \simeq \tilde{B}, \quad \tilde{N}_2 \simeq \tilde{W}^0
\]

\[
\tilde{C}_1^\pm \simeq \tilde{W}^\pm
\]

with a mass spectrum that can be approximated by

\[
m_{\tilde{N}_1} \simeq M_1
\]

\[
m_{\tilde{N}_2} \simeq m_{\tilde{C}_1} \simeq M_2
\]

\[
m_{\tilde{N}_3} \simeq m_{\tilde{N}_4} \simeq m_{\tilde{C}_2} \simeq |\mu|.
\]
Some combination of the lightest chargino and the lightest two neutralinos are among the most likely sparticles to be produced with a large rate at a collider. Search strategies for these sparticles depend heavily on three factors: the identity of the NLSP, the decay length of the NLSP, and the mass of the lightest stau in relative comparison with the lighter gauginos. In virtually all of our model’s parameter space, there are only two possible mass hierarchies:

\begin{align*}
  m_{\tilde{N}_1} &< m_{\tilde{\tau}_1} < m_{\tilde{N}_2, \tilde{C}_1} \quad \text{(neutralino-NLSP scenario)} \\
m_{\tilde{\tau}_1} &< m_{\tilde{N}_2} < m_{\tilde{N}_2, \tilde{C}_1} \quad \text{(stau-NLSP scenario)}
\end{align*}

Thus, we expect that $\tilde{N}_2 \to \tilde{\tau}_1^+ \tau^+$ and $\tilde{C}_1^+ \to \tilde{\tau}_1^+ \nu_\tau$ to dominate throughout parameter space\footnote{These decays are expected to be at least comparable to (or possibly dominate over) $\tilde{C}_1^+ \to W^\pm \tilde{N}_1$ and $\tilde{N}_2 \to Z \tilde{N}_1$ even when the latter are kinematically open\cite{49}.}, regardless of the identity of the NLSP. Although a pure Wino-like chargino does not couple to $\tilde{\tau}_R$, there is sufficient mixing within $\tilde{\tau}_1 = \cos \theta_\tau \tilde{\tau}_L - \sin \theta_\tau \tilde{\tau}_R$ at moderate to large $\tan \beta$ such that $\tilde{C}_1$ can proceed through the 2-body decay into $\tilde{\tau}_1 \nu_\tau$, albeit suppressed by the mixing angle $\cos^2 \theta_\tau$. If the NLSP is the lightest stau, then there are potentially additional sources of taus in collider events, as we discuss below. In any case, the importance of searching for taus as signals of flavor-mediated supersymmetric models cannot be overemphasized.

The third generation scalar spectrum is roughly split into several heavy fields $\tilde{\tau}_2$, $\tilde{\nu}_\tau$, $\tilde{b}_1$, $\tilde{b}_2$, and one light field $\tilde{\tau}_1$. The heavier sleptons acquire their mass dominantly from $m_{\tilde{\tau}_L}$, hence $\tilde{\tau}_2 \approx \tilde{\tau}_L$, $m_{\tilde{\nu}_\tau} \approx m_{\tilde{\nu}_\tau}$, and thus $\tilde{\tau}_1 \approx \tilde{\tau}_R$. Since $\tilde{m}_3^0$ acquires a positive one-loop Yukawa-induced contribution, Eq. (37), while $Q^3$ acquires a negative two-loop Yukawa-induced contribution, Eq. (40), we find that $b_2 \approx \tilde{b}_R$ is the heaviest third generation scalar field. The other third generation squarks are roughly $b_1 \approx \tilde{b}_L$, $t_2 \approx \tilde{t}_L$, $t_1 \approx \tilde{t}_R$, and $m_{\tilde{t}_1} \approx m_{\tilde{t}_2} \approx m_{\tilde{b}_1}$. The stop mass eigenstates are not as well defined by their left- or right-handed squark eigenstates due to larger off-diagonal LR mixing (as compared with the sbottoms or the staus).

The mass scale of the third generation squarks is highly dependent on $\tilde{m}^2$ and the FI-term $q_\xi$, as shown in Fig. 7. The strong correlation between the mass scale of the first and second generations and the third generation arises due to the log-enhanced gauge- and Yukawa-mediated contributions proportional to $\tilde{m}^2$. For example, restricting the third generation to be less than about 1.5 TeV implies the first and second generation masses must be less than about 10 (15) TeV for $\tan \beta \lesssim 30$ (10).

The difference between the heavy third generation sleptons and the heavy third generation squarks is determined largely by the positive one-loop RG evolution induced by gauginos, just as in ordinary minimal gauge-mediated models. In particular, the colored sparticles receive one-loop gauge contributions proportional to $g_3^2 M_2^2$, as opposed to the left-handed sleptons which receive one-loop gauge contributions proportional to $g_2^2 M_2^2$. Thus the colored sparticles are significantly boosted in mass relative to their slepton counterparts by an amount that is ultimately proportional to the holomorphic supersymmetry breaking vev $F_X$. In typical cases, the heavier slepton masses for $\tilde{\tau}_2$ and $\tilde{\nu}_\tau$ are between about 0.6 to 0.9 times an average squark mass $(m_{\tilde{t}_1} + m_{\tilde{b}_1})/2$.

The Higgs scalars acquire masses that are dependent on the up-type and down-type Higgs
The average stop mass \( (\tilde{m}_t + \tilde{m}_{\tilde{t}})/2 \) is shown as a function of the ratio of the normalized FI \( D \)-term \( q_\xi \). The solid (dashed) lines are contours of the first and second generation mass scale \( m_{1st,2nd} \) in TeV, for \( \tan \beta = 10 \) (\( \tan \beta = 30 \)).

Soft masses \( m_{H_u}^2, m_{H_d}^2 \) and the conditions for EWSB that determine \( \mu^2 \) [see Eqs. (49), (50)]. At moderate \( \tan \beta \), and assuming the radiative corrections to the EWSB conditions are small\(^{25}\), we can make the approximations

\[
\mu^2 \simeq -m_{H_u}^2
\]

\[
m_A^2 \simeq m_H^2 \simeq m_{H\pm}^2 \simeq m_{H_d}^2 + \mu^2
\]

up to corrections of order \( M_Z^2/m_{H_u}^2 \), where the Higgs soft masses are evaluated at the “best minimization” scale that we take to be \( (\tilde{m}_t + \tilde{m}_{\tilde{t}})/2 \). Since \( H^d \) and \( \tilde{L}^3 \) have the same (local) gauge quantum numbers, they acquire identical gauge-mediated contributions to their masses at \( M_{FN} \), and have similar RG evolution to the weak scale. The one-loop Yukawa-mediated contributions are typically only slightly different [compare Eq. (39) with Eq. (38)], however in practice we find that subdominant negative two-loop Yukawa-mediated corrections reduce the mass of \( H^d \) by typically 10–25% relative to the mass of \( \tilde{L}^3 \). The up-type Higgs acquires a very small mass at the FN scale, and is driven to moderate-sized negative values via the top Yukawa coupling by one-loop RG evolution. For illustration, one can easily translate the values of \( \mu \)

\(^{25}\)This is a reasonable approximation to the accuracy we are considering in the following.

\(^{26}\)Since the relative size of \( m_{H_d} \) and \( m_{\tilde{L}^3} \) at the messenger scale (and at the weak scale) is determined by the Yukawa coupling contributions, these quantities are sensitive to the \( U(1)_F \) charge assignments for the first and second generations, as well as the relative size of the superpotential couplings \( b_5, \lambda_1^d, \) and \( \lambda_2^d \).
shown in Fig. 4 into good approximations of \((-m_H^2)^{1/2}\) using Eq. (61). For \(10 \lesssim \tan \beta \lesssim 30\), \(m_{H^+}\) and \(\mu\) are comparable, and so a rough approximation of the heavier Higgs scalar masses is \(\sqrt{2}\mu\). Thus, the heavier Higgs scalar fields have masses that are much larger than the weak scale (close to, for example, the heavier third generation sleptons).

The lightest Higgs mass is expected to fall within the bounds that have been well established for the MSSM [50, 51]. We performed one-loop [46] (and in some cases approximate two-loop [50]) calculations to extract its mass for several choices of parameters. We checked that at one-loop the lightest Higgs mass is not sensitive to the heavy first and second generation masses (considered in this paper), and for moderate \(\tan \beta\) depends largely on just the mass of the stops. Since the stops are relatively heavy in our model (see Fig. 7), the associated radiative corrections to the Higgs mass are significant. A precise calculation of the lightest Higgs mass requires careful treatment of the scale to evaluate the radiative corrections, and one must include the dominant two-loop corrections (which tend to reduce the Higgs mass relative to a naive one-loop calculation). Our estimates suggest that the lightest Higgs mass is near the upper end of the MSSM bounds, \(m_h = 110–125\) GeV, throughout the parameter space of the model. We have not attempted a high precision \(<\) few GeV) calculation, but do not expect our estimate to be in error by more than about 5\%. It is interesting to note that this mass safely evades the current bounds from LEP [52] due to the heavy stops that are an inevitable consequence of the model.

In Appendix B we present an example set of input parameters and some of the calculated sparticle masses to illustrate the generic spectra that are expected in the MFMM. The evolution of some of the weak scale soft masses is also shown as a function of scale.

### 6.2 Gravitino mass

In models of low energy supersymmetry breaking, the perhaps best-known generic prediction is that the gravitino mass should be rather small. Recall that after global supersymmetry is dynamically broken in the DSB sector, a massless spin-1/2 Goldstino is present. Ordinarily this would be phenomenological disaster, but in fact this would-be Goldstino becomes the longitudinal components of the spin-3/2 gravitino, after the model is embedded in supergravity at \(M_{Pl} = 2.4 \times 10^{18}\) GeV. The gravitino therefore acquires mass by this super-Higgs mechanism, of order

\[
m_{\tilde{G}} = \frac{\Lambda_{DSB}^2}{\sqrt{3}M_{Pl}} = (24\text{ keV}) \left(\frac{\Lambda_{DSB}}{10^7\text{ GeV}}\right)^2.
\]  

(63)

One of the central phenomenological consequences of a such a light gravitino is that every sparticle must ultimately decay into it.\(^{27}\) The characteristic scale associated with the two-body interaction of a gravitino and either a matter or vector supermultiplet scales as \(1/\Lambda_{DSB}^2\) for the longitudinal components (i.e. the Goldstino). Hence, the coupling is small in comparison to the gauge interactions but not suppressed by the Planck scale. All sparticles are therefore expected to decay into the next-to-lightest supersymmetric particle (NLSP) with very short decay lengths, and then the NLSP decays into the gravitino and a SM particle with a decay length that is

\(^{27}\)Assuming \(R\)-parity is (at least nearly) exact.
inversely proportional to the coupling strength, or directly proportional to the gravitino mass. This leads to an enormously rich phenomenology (for a review, see Ref. [9].)

As we discussed in the previous section, the lightest neutralino mass eigenstates are dominantly “gaugino-like”, with the lightest neutralino being composed dominantly of the Bino state. If the lightest neutralino is the NLSP, the dominant decay will always be $\tilde{N}_1 \to \gamma \tilde{G}$. Otherwise, if the lightest stau is the NLSP, the dominant decay will always be $\tilde{\tau}_1 \to \tau \tilde{G}$. The NLSP decay width $\Gamma(NLSP \to particle + \tilde{G})$ is virtually identical\(^{28}\) regardless of which sparticle is actually the NLSP. If the fundamental scale of supersymmetry breaking is indeed $\Lambda_{DSB}$, we can write the characteristic decay length $L = \Gamma^{-1}$ of the NLSP as\(^{44}\)

$$L(NLSP \to particle + \tilde{G}) = (1.7 \text{ km}) \left( \frac{m_{NLSP}}{150 \text{ GeV}} \right)^{-5} \left( \frac{\Lambda_{DSB}}{10^7 \text{ GeV}} \right)^4 \left( \frac{E_{NLSP}^2}{m_{NLSP}^2} - 1 \right)^{1/2}$$

(64)

where $E_{NLSP}$ is the energy of the NLSP that is produced.\(^{29}\)

It is obvious that if $m_{NLSP}$ and $\Lambda_{DSB}$ are of order the normalized values given in Eq. (64), the characteristic decay length is much larger than the scale of the detector, and thus virtually all produced NLSPs decay outside the detector. This is a well-known expectation of nearly all (multi-sector) DSB models. It has profound implications for the strategy to search for signals of our flavor model, depending on the identity of the NLSP, as we discuss in the next two sections.

Due to the very strong dependence on the NLSP mass and the DSB scale, it is natural to ask if there are regions of parameter space that might give characteristic decay lengths of order the scale of the detector. In such a case there are a host of fascinating, near background-free signals that could be exploited to probe the supersymmetric parameter space (see e.g. [44]). One trivial possibility is an $e^+e^-$ collider experiment operating at an energy slightly above the pair production threshold for NLSPs, thereby imparting very small energies to these sparticles. More nontrivial possibilities amount to shifts upward of $m_{NLSP}$ and/or shifts downward of $\Lambda_{DSB}$. These have a dramatic impact on the characteristic decay length due to the large power dependence in Eq. (64). Unfortunately it is difficult to lower $\Lambda_{DSB}$ by any significant factor, due to the direct connection to $\tilde{m}_2^2$ as shown in Eq. (48). The mass of the NLSP is simply $\min[m_{\tilde{N}_1}, m_{\tilde{\tau}_1}]$. If we restrict $M_3 \lesssim 1 \ (1.5) \text{ TeV}$, then $m_{\tilde{N}_1} \lesssim 190 \ (285) \text{ GeV}$. Although the stau mass is expected to be light, it is strongly dependent on the parameters of the model due to the several sources of supersymmetry breaking that contribute to its mass. In Fig. 3 we showed the mass of the stau as function of several parameters, with the result that $m_{\tilde{\tau}_1} \lesssim 300 \text{ GeV}$ is a conservative upper bound.\(^{30}\) Thus, unless the true parameters of the model are highly unnatural (couplings significantly less than 0.1 and/or a large $F_\chi$ vev giving a gluino mass significantly beyond 1.5 TeV), the characteristic decay length of the NLSP is almost certainly much larger than the scale of a collider detector.

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\(^{28}\)The decay length of any given NLSP is of course probabilistic. Eq. (54) is correctly interpreted as the length within which a fraction $1 - 1/e$ of an ensemble of NLSPs produced at the same energy would have decayed \(^{44}\).

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\(^{30}\)Since both the $F_\chi$ vev and $\tilde{m}_2^2$ are ultimately related to $\Lambda_{DSB}$ up to some factors, it is inappropriate to consider simultaneously increasing $m_{NLSP}$ while decreasing $\Lambda_{DSB}$. 

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6.3 Consequences of a neutralino NLSP

If the NLSP is the lightest neutralino, then the collider signals are expected to be very similar to the original supergravity-mediated models since virtually every $\tilde{N}_1$ escapes the detector carrying away missing energy. As we discussed above, $\tilde{C}_1$ and/or $\tilde{N}_2$ production is expected to dominantly produce taus in every event, due to Eq. (59). This is likely to make detection more difficult due to the prompt decay of the tau. A tau decays either leptonically into $\ell\nu_\tau$ or hadronically into several channels that include a $\nu_\tau$. The neutrino(s) from tau decay provide an additional source of missing energy, suggesting a larger missing energy cut may assist discerning signal from background without rejecting a tremendous amount of signal. Leptonic tau decays do yield “clean” events with only leptons and little hadronic activity, but the resulting leptons have significantly less energy than the parent tau due to the three-body decay, and one faces a branching ratio suppression of $0.35$ for every leptonic tau decay.

Searching for taus a signal of supersymmetry has recently been extensively studied, both in the context of supergravity-mediated models as well as gauge-mediated models [53, 42, 43, 54, 44, 55, 56, 49, 58, 59]. In the absence of any signal at LEP, attention has been turned to the prospects at the upgraded Tevatron, in several high luminosity versions. One of the most interesting signals of gaugino production is the “trilepton” signal arising from $\tilde{C}_1^\pm\tilde{N}_2$ production. If these gauginos do indeed decay dominantly into a stau, then the signal of this process in our model (with a neutralino NLSP) is three $\tau$’s plus missing energy. Matchev and Lykken [58] have analyzed several ways to detect this particular signal, and emphasized searching for modes with both some leptonic tau decays as well as some hadronic tau decays. They performed an event-level analysis folding in detector cuts and a semi-realistic detector simulation, and found that the best signature of tri-tau production is $\ell\ell\tau_h$ where two taus decay leptonically and one tau decays hadronically. In this process the Tevatron had the greatest reach, although at least 1–2 fb$^{-1}$ of data is required for a (weak) signal with low mass charginos ($m_{\tilde{C}_1} \lesssim 120$ GeV), and tens of fb$^{-1}$ of data is needed to exclude any mass chargino beyond the LEP bound of about 90 GeV.

Chargino production ($\tilde{C}_1^+\tilde{C}_1^{-}$) and stau production are also processes that could have a large rate at the Tevatron. The signature for both is expected to be two opposite-sign taus plus missing energy. Unfortunately, this signature suffers from a large Standard Model background forcing one to be clever about the event-level cuts (for a recent analysis, see e.g. [57, 59]).

Neutralino production ($\tilde{N}_2\tilde{N}_2$) has much more interesting signals comprising four taus plus missing energy. One interesting signal is a pair of like-sign dileptons plus two tau jets, where each neutralino decays ultimately into two taus, one of which decays hadronically. However, the pair production of the second-lightest neutralino is somewhat suppressed compared with chargino production or chargino-neutralino production, and requires a full event-level simulation to determine the prospects for discovery (or exclusion) at the Tevatron. If the results of Matchev and Lykken [58] can be roughly applied to this case, it is unlikely neutralino production will be detectable before the LHC becomes operational.
6.4 Consequences of a stau NLSP

If the NLSP is the stau, then every sparticle produced in a collider results in a charged stau track. Searches for heavy stable charged tracks have been performed at LEP and Tevatron (run I) without success, suggesting that the upgraded Tevatron has, once again, the most promising future prospects for detection.

An ionizing charged track resulting from a stau is a dramatic signal in a detector \[53, 42, 44, 60, 61\]. However, the only relevant distinguishing characteristic for a detector between a muon and a quasi-stable stau is the stau mass. If a produced stau is sufficiently relativistic (in the lab frame), it will behave as a “minimum-ionizing” particle and thus appear virtually identical to a muon. If a produced stau is slower, in particular \[\beta \gamma \equiv \left( \frac{E_\tau^2}{m_\tau^2} - 1 \right)^{1/2} \lesssim 0.85 \], then the stau has a greater-than-minimum ionization rate in the detector that can be distinguished from a muon \[12\]. The size of \(\beta \gamma\) depends on the mass of the produced sparticle, the energy of the collider, and whether the stau was directly produced or indirectly through heavier sparticle decay. Feng, Moroi \[60\], and Martin, Wells \[61\] analyzed in detail the detectability of quasi-stable staus at the Tevatron. They found that by searching for at least one highly-ionizing track (HIT) per event, stau production could be probed for stau masses up to roughly 150–200 GeV with between a few to tens of fb\(^{-1}\) of data. Similarly gaugino production is expected to yield several taus in the event (as described above) in addition to charged stau tracks. These authors found that chargino masses of up to 300–400 GeV could be probed for similar amounts of data. Here the process \(\tilde{N}_1\tilde{N}_1\) gives the interesting signal \(\tilde{\tau}\tilde{\tau}\tau\tau\), half of the time with same-sign staus and same-sign taus. We should note, however, that in some cases these authors’ analyses included contributions from (and discussed particular signatures associated with) first and second generation slepton production, that is of course absent in our model. In addition, we expect the lightest chargino and the second lightest neutralino to decay to a stau, generating plenty of taus in the final state, just as discussed in Sec. 6.3.

6.5 FCNC and CP violation: signals

It has been shown \[21\] that the additional contributions to \(\Delta m_K\) and \(\epsilon_K\) in the model under discussion do not exceed those of the SM and therefore agree with current experimental measurements. This result is not universal for generic weak-scale squark masses. The suppression of contributions to \(K^0 - \bar{K}^0\) mixing has two sources \[24\]. One is that the first two generations of scalars are heavy (\(\sim 7–20\) TeV) and that their (mass)\(^2\) matrices are approximately diagonal in the flavor basis. The other source of suppression arises due to the Cabibbo angle coming from the up sector. Thus the gluino box diagrams which contribute to \(K^0 - \bar{K}^0\) mixing are small

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\[31\] As discussed earlier, to avoid large contributions to \(\epsilon_K\) we require \(F_{\phi(1)} F_{\phi(2)}^{\ast}\) to have a small imaginary part (\(\sim \mathcal{O}(10^{-2}) \times \phi^{(1)} \phi^{(2)}(\chi^{(1)})^2\)).
because of both large scalar masses and small mixing.\footnote{Box diagrams with wino exchange do contain couplings with Cabibbo-like mixing. However, the suppression $(\alpha_w/\alpha_s)^2$ is enough to render these contributions sub-dominant.}

A definite signal from this model is enhanced $D^0 - \bar{D}^0$ mixing \cite{26} since first and second generation mixing at the quark-squark-gluino vertex is of order the Cabibbo angle (as expected since the Cabibbo angle comes from this sector). The largest contribution comes from the standard gluino box diagrams that give (in the “vacuum insertion” approximation)

$$
\Delta m_D \sim (7.0 \times 10^{-10} \text{ MeV}) \times Z_{u,LL}^{1i} Z_{u,LL}^{2i} Z_{u,LL}^{1j} Z_{u,LL}^{2j} \times \left( \frac{\alpha_s}{0.1} \right)^2 \times \left( \frac{f_D}{200 \text{ MeV}} \right)^2
$$

for first and second generation masses that are 10 TeV and a gluino mass that is 1 TeV. For typical values of the above constants, the Cabibbo-like mixing gives $x_D = \Delta m_D/\Gamma \sim 0.021$, which nearly saturates the current bound \cite{63}, and is two orders of magnitude above the SM estimate \cite{64}.

The $B$ system will also receive significant contributions from superpartners. The signals are similar to those outlined in \cite{26}. In the flavor basis, the mixing comes from the quark mass matrix. The squark mass matrix is nearly diagonal in this basis with respect to the third generation. The phase in the gluino box diagrams is different from the phase in the SM box diagrams ($W$-exchange) because equal portions of $V_{td}$ come from the up and down sectors. Here, quark-squark mixing is only due to the down matrix, and thus has a different phase than $V_{td}$.

At tree-level, left-right mixing matrices (the off-diagonal blocks of the squark (mass)$^2$ matrices) are of the same form as the corresponding quark mass matrices. Thus, left-right mixing is in general small in this model. Since the Cabibbo angle does not originate from the down sector \cite{65}, the model does not predict any significant supersymmetric contributions to the recent measurement of $\epsilon'$ \cite{33}.

7 \hspace{1cm} Modifying the model

The allowed superpotential terms in the FN sector of the MFMM were guided by several restrictions \cite{21}:

(i) U(1)$_F$ charge assignments must be compatible with an SU(5) GUT.

(ii) FN fields must be added in complete SU(5) multiplets.

(iii) The number of FN fields must be less than (the equivalent of) five $5^F N$, $\overline{5}^F N$ pairs to allow for at least semi-perturbative couplings at the GUT scale.

(iv) The ratio of the smallest to the largest superpotential couplings must not be less than 0.1.

Constraints from electroweak symmetry breaking and FCNC imposed additional restrictions on the model (e.g., $H^u$ could not couple to U(1)$_F$-charged fields in the superpotential, and the first
two generations had to be charged). In this section, we examine the consequences of lifting some of these constraints. In addition, we discuss two important extensions of the model: dynamical generation of both a $\mu$-term and neutrino masses.

There are, however, a few variations of the MFMM that do not require violating any of the above restrictions. One example is to take the charge of $\bar{5}F_N^{(1)}$ to vanish ($q_{\bar{5}F_N^{(1)}} = 0$), remove the superpotential coupling $H_d^{10}F_{N_{\bar{5}F_N^{(1)}}}$, and include the couplings $H_d^{10}F_{\bar{5}F_N}^{(1)}$ and $H_d^{10}F_{\bar{5}F_N}^{(11)}$. While this change does create a new parameter space, the low energy spectrum does not change dramatically in this new version, e.g., we still expect one light stau with all other squarks and sleptons to be at or above about 1 TeV.

7.1 Relaxing the SU(5) ansatz

Of the four criteria listed above, the first three involve unification, with criteria (iii) most relevant to ensuring the gauge couplings approximately unify at the high scale. Criteria (i) (charge assignments which commute with an SU(5) GUT) is required for standard SU(5) unification. Criteria (ii) (FN fields in complete SU(5) multiplets) is not necessarily required for an SU(5)-invariant high energy theory, however one would be faced with highly split multiplets that represent an additional hierarchy problem [analogous to the doublet-triplet splitting of the Higgs field that must occur with SU(5)]. Criteria (iii) (maximum number of FN fields to preserve unification) is a more general prediction of GUTs that need not be specific to SU(5), and could be imposed independent of the first two criteria. For example, some free fermionic string models break directly to the SM gauge group, also with possibly a nonstandard hypercharge normalization [48]. In the following, we explore models that do not satisfy criteria (i) and (ii), instead satisfying merely criteria (iii) and (iv), or just criteria (iv) only. One important consequence of these model variations is that the mass for the up quark is no longer necessarily zero, although we find that this occurs only for models that violate criteria (i)–(iii).

Relaxing the SU(5) ansatz, FN fields now come in three varieties: $(Q, \bar{Q})$, $(u, \bar{u})$ and $(d, \bar{d})$ pairs. One possible variation is to leave the field content alone and to allow the $U(1)_F$ charges to deviate from the SU(5) requirements. The benefit of this choice is that it allows the leptons to take different charges than the quarks, thus more easily accommodating their different masses. Moreover, this variation could lead to a simple mechanism for generating neutrino masses (see Sec. 7.3). However, the deviation from SU(5) comes at a cost: $\text{tr}Y_i m_i^2 \simeq 0$ is no longer automatically satisfied. This trace must approximately vanish to avoid a large contribution to a FI hypercharge D-term [4]. The soft masses are quadratic functions of the $U(1)_F$ charges, and therefore this requirement is highly restrictive. Regardless, it is possible to find charge assignments which violate SU(5), but in all of these variations the structure of the (full) quark mass matrices remains the same as in the MFMM case.

Variations in the particle content are perhaps more interesting. There are three viable FN sectors other than that of the MFMM that have $\leq 5$ pairs of additional quark triplets. All other possibilities are ruled out since they predict a zero eigenvalue in the down mass matrix or two zero eigenvalues in the up mass matrix. If $n_i$ (with $i = Q, u, d$) is the number of FN quark pairs, then these model variations can be labeled by the triplet $(n_Q, n_u, n_d)$. In this language, the four
allowed models satisfying criteria (iii) are (1, 1, 2) [the MFMM], (1, 0, 2), (1, 0, 3), and (2, 0, 1). The second of these is the most interesting as it is the only one with only four pairs of FN quarks. In this model, the gauge couplings remain completely perturbative up to the unification scale. The full up mass matrix for this (as well as the third) model is

\[
M^u = \begin{pmatrix}
0 & 0 & 0 & b_1^{(Q)} \langle \phi^{(1)} \rangle \\
0 & 0 & 0 & b_2^{(Q)} \langle \phi^{(2)} \rangle \\
0 & 0 & Y_t \langle H^u \rangle & 0 \\
0 & \lambda_1^u \langle H^u \rangle & \lambda_2^u \langle H^u \rangle & a_1^{(Q)} \langle \chi^{(1)} \rangle
\end{pmatrix}.
\] (67)

Notice this matrix requires the right-handed charm quark to be uncharged with respect to U(1)\(_F\). If the right-handed up quark is also uncharged, then the dominant contribution to the masses of the right-handed up-type squarks comes from the standard two-loop gauge interactions. For soft masses near 1 TeV (as is typical in the MFMM), these squarks are very nearly degenerate and contributions to \(D^0 - D^0\) mixing from this sector are far below experimental bounds. In a sense, this is a hybrid model between gauge-mediated and flavor-mediated supersymmetry breaking. However, with this field content, \(\text{tr} Y_i m_i^2 \gg 0\) because there is not a sufficient number of U(1)\(_F\) charged fields that have negative hypercharge to compensate the fields with positive hypercharge. The consequence of this large positive hypercharge \(D\)-term is typically that a scalar (mass)\(^2\), such as the stau, goes negative. In fact, all three of these variant field contents, (1,0,2), (1,0,3), (2,0,1), do not work for this reason. Thus, it is remarkable that the FN sector is completely fixed and unique simply by requiring \(n_Q + n_u + n_d \leq 5\).

In this framework, it is impossible to satisfy criteria (iii) and (iv) and have \(m_u \neq 0\). Giving up criteria (iv) and permitting \(H^u\) to couple to charged fields does allow the up quark to gain a mass, but radiative EWSB is very difficult (if not impossible) to achieve. Giving up criteria (iii) allows a somewhat reasonable model with a non-zero up mass (though it is still difficult to satisfy (iv) in such models). The minimal field content for this type of model is (2,0,2), although models with a larger field content need fewer small couplings. In these models the gauge couplings encounter a Landau pole prior to the unification scale, and therefore apparently require new physics to appear at some lower scale. Finally, while all of these models can produce \(m_u \neq 0\), the natural solution to the strong CP problem is lost.

### 7.2 The \(\mu\)-term

The one dimensionful parameter whose dynamical generation has not been accounted for is the superpotential mass parameter of the Higgs superfields. The difficulty of naturally generating the superpotential coupling \(\mu H^u H^d\) (the so-called “\(\mu\)-problem”), exists in this model in the same way as it does in ordinary gauge-mediated scenarios. The problem is not one of generating \(\mu\) of order the weak scale (required by naturalness); instead the problem is that most mechanisms that generate \(\mu\) also generate a soft supersymmetry breaking scalar bilinear mass term \(B_{\mu} H^u H^d\) that is much too large \([36]\). We do not have anything significantly new to add to the discussion, except to describe the situation in the MFMM.

As described in the conclusion of Ref. \([21]\), the \(\mu\)-term is naturally generated at one-loop
when the couplings $\lambda H^u 10^F N 10^F$ and $\lambda H^d 10^F N 10^F$ are added to the superpotential. The leading contribution is the right size: $\mu \sim (\lambda / 16\pi^2)(F_{\chi^{(1)}} / \langle \chi^{(1)} \rangle)$ times a group theory factor. However, there is a similar one loop contribution $[66]$ to the scalar bilinear $B_\mu = \mu F_{\chi^{(1)}} / \langle \chi^{(1)} \rangle$ which is much too large for EWSB to work naturally (if at all). The authors of Ref. $[66]$ did offer a solution that should work in our scenario, but it requires a new dimensionful parameter (although this could be dynamically generated as in Ref. $[67]$). An alternative approach using an extra $U(1)$ (unrelated to our flavor $U(1)_F$) whose breaking generates an effective $\mu$-term was pursued in the context of a chiral supersymmetric standard model in Refs. $[68, 69]$.

The simplest method to dynamically generate a $\mu$-term is to add a superpotential coupling $SH^u H^d$, where $S$ is a gauge singlet that may carry some global charge. There are several candidates in the model for $S$, namely the flavons. The MFMM offers $\chi^{(1)}$ as a gauge singlet, however the vev of this field is clearly too large to generate $\mu$. This provides sufficient motivation to give $\chi^{(1)}$ a non-zero $U(1)_F$ charge, although we note that there are phenomenological constraints on the size and sign of this charge that we do not discuss further here. A more natural possibility is $S \equiv \phi^{(3)}$. This choice simply requires the constraint $q^5 = q^{5F(1)}$, and is natural because $\langle \phi^{(3)} \rangle / \langle \chi^{(2)} \rangle \sim 10^{-2} - 10^{-3}$. However, again we face the problem of an overly large $B_\mu$ since, in general, we expect the $F$-term of $\phi^{(3)}$ to be of order $\langle \phi^{(3)} \rangle / \langle \chi^{(2)} \rangle$. A priori this is not guaranteed, and the flavon interactions might produce a small $F$-term, though it seems unlikely.

Finally, Nilles and Polonski $[70]$ produce a $\mu$-term via a gauge singlet that is coupled gravitationally to the DSB sector via the Kähler potential coupling $(S + S^\dagger)QQ^\dagger$, where $Q$ is a DSB sector chiral superfield. If $S$ has a self-cubic coupling in the superpotential, then for $\sqrt{F_Q} \simeq \Lambda_{\text{DSB}} \sim 10^7$ GeV, and $S$ has a vev of order the weak scale that can dynamically produce $\mu$ without producing a large $B_\mu$. This mechanism can be implemented in the MFMM by adding an additional singlet or by again taking $\phi^{(3)}$ to be the singlet (making the generation of its vev distinct from the other flavons).

7.3 Neutrino masses

The MFMM is apparently incomplete in one important sector of flavor: it fails to predict viable neutrino masses. Recent observations of zenith angle dependence in the number of atmospheric muon neutrino events $[22]$ together with the lower than expected measurements of solar neutrino flux $[23]$ suggest that (at least two) neutrinos have small non-zero masses. These experiments measure differences in (mass)$^2$ and find the relevant scales to be less than 1 eV. The MFMM cannot reproduce such a low scale in any reasonable way without invoking new interactions at a large mass scale. We find that it is possible to account for the observed neutrino oscillations in a flavor-mediated model by introducing the effects of heavy right-handed neutrinos $[71]$, or by introducing spontaneous $R$-parity breaking $[72, 73]$ and including Planck scale contributions. The latter solution does not work with the SU(5) restriction while the former solution does if an assumption is made about the right-handed neutrino spectrum.

Given the current experimental data with the assumptions that only the known three neutri-
nos are light, and the mixing scales are hierarchical (i.e., $\Delta m_{\text{atm}}^2 \gg \Delta m_{\text{solar}}^2$) one can establish significant constraints on neutrino mass and mixing parameters $^{73}$:

- The mass squared difference associated with atmospheric neutrinos is in the range $\Delta m_{\text{atm}}^2 \sim 1 \times 10^{-3} - 6 \times 10^{-3}$ eV$^2$ (90% C.L.).
- The mixing between $\nu_\mu$ and $\nu_\tau$ is large (i.e., the mixing angle $\theta$ is bounded by $\sin 2\theta > 0.8$ (90% C.L.).
- The $\nu_e$ fraction of the heaviest mass eigenstate is small ($\lesssim 0.1$ at 90% C.L.)$^{74}$

These constraints severely restrict the form of the neutrino mass matrix. If the neutrino mass matrices are to come from some symmetry, the leading contributions must be of the form $^{77}$:

$$M^{\nu_1} = \begin{pmatrix} 0 & B & A \\ B & 0 & 0 \\ A & 0 & 0 \end{pmatrix} \quad \text{or} \quad M^{\nu_2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & B^2/A & B \\ 0 & B & A \end{pmatrix}$$

(68)

where $A, B \sim O(\Delta m_{\text{atm}})$ and corrections to these forms are much smaller than $A$ and $B$. Flavor-mediated models that satisfy FCNC constraints and are consistent with SU(5) do not contain symmetries that could produce either of these matrices. The first matrix, however, could be produced if assumptions are made about the high scale physics that produced it. Either matrix can be produced once the SU(5) restriction is dropped.

There is an inherent difficulty producing viable neutrino masses in the SU(5)-symmetric case. Limits on FCNC require that the first two generations of right-handed down-type quarks $\overline{d}^1$, $\overline{d}^2$ must have non-zero $U(1)_F$ charges, while the third generation is constrained to be uncharged so that all the down-type fermions have nonzero masses. At the same time, the neutrino spectrum requires large mixing between the second and third generation lepton doublets. To naturally produce this large mixing, $\overline{\nu}_{\tau}$ and $\overline{\nu}_{\mu}$ should have the same charge (preferably zero). However, $\overline{\nu}_{\mu}$ and $\overline{\nu}_{\tau}$ must have the same charge, as required by SU(5). One can get around this problem (albeit in a slightly obtuse manner) by requiring that all leading order pieces to the neutrino mass matrix vanish. Using the SU(5) variation where $\overline{\nu}_{\tau}$ is uncharged, and choosing $U(1)_F$ charges such that $q_{\overline{\nu}_{\tau}} = -q_{\overline{\nu}_{\mu,\tau}}$, one can produce the following (Majorana) mass matrix for the left-handed neutrinos:

$$M^{\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 & \lambda_1^m m_{LL} & b_3^{(L)} \langle \phi^3 \rangle & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & b_4^{(L)} \langle \phi^4 \rangle \\ 0 & 0 & Y_{\nu_\tau} m_{LL} & \lambda_2^m m_{LL} & 0 & 0 & b_5^{(L)} \langle \phi^5 \rangle \\ 0 & 0 & \lambda_2^m m_{LL} & \lambda_3^m m_{LL} & 0 & a_2^{(L)} \langle \chi^2 \rangle & 0 \\ \lambda_1^m m_{LL} & b_3^{(L)} \langle \phi^3 \rangle & 0 & 0 & 0 & 0 & a_3^{(L)} \langle \chi^3 \rangle \\ b_4^{(L)} \langle \phi^4 \rangle & b_5^{(L)} \langle \phi^5 \rangle & 0 & 0 & 0 & 0 & 0 \\ a_2^{(L)} \langle \chi^2 \rangle & a_3^{(L)} \langle \chi^3 \rangle & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

(69)

$^{43}$The hierarchy $\Delta m_{\text{atm}}^2 \gg \Delta m_{\text{solar}}^2$ is actually required unless one ignores the data from one of the solar neutrino measurement techniques or ignores the solar models $^{76}$.

$^{44}$This bound includes results from the CHOOZ Collaboration $^{74}$. 

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where the rows and columns represent \((\mathcal{T}^1, \mathcal{T}^2, \mathcal{T}^3, \mathcal{T}^{FN(1)}, \mathcal{T}^{FN(2)}, L^{FN(1)}, L^{FN(2)})\), \(m_{LL} = \langle H_u \rangle^2 / M\), and \(M\) is the mass scale of right handed neutrinos (assuming couplings of order unity)\(^{35}\). After integrating out the FN fields, the \(3 \times 3\) neutrino mass matrix becomes:

\[
M_{\nu_{\text{eff}}} = m_{LL} \begin{pmatrix}
0 & \lambda_{\nu_1}^{(L)} \sqrt{\epsilon_{43}} & \lambda_{\nu_1}^{(L)} \sqrt{\epsilon_{53}} + \lambda_{\nu_2}^{(L)} \sqrt{\epsilon_{32}} \\
\lambda_{\nu_1}^{(L)} \sqrt{\epsilon_{43}} & 0 & Y_{\nu_\tau} \\
\lambda_{\nu_1}^{(L)} \sqrt{\epsilon_{53}} + \lambda_{\nu_2}^{(L)} \sqrt{\epsilon_{32}} & 0 & 0
\end{pmatrix} + O(\epsilon^2) \quad (70)
\]

This becomes \(M_{\nu_1}\) only if the (3,3) element vanishes. To see how this may be possible, consider a high energy theory containing the following superpotential couplings:

\[
W \supset \begin{array}{l}
N_1 b_5^T F_{N(2)} - b H_u + N_2 b_5^T b H_u + M N_1^b N_2^b,
\end{array} \quad (71)
\]

where the \(N_i\) are right handed neutrinos, the subscripts are \(U(1)_{FN}\) charges, and all order one couplings have been suppressed. Below the scale \(M\), the mass matrix Eq. \((69)\) would be produced without the couplings \(\lambda_{\nu_2}, \lambda_{\nu_3}, \text{ and } Y_{\nu_\tau}\). Similarly, a global symmetry could forbid these couplings. From atmospheric neutrino measurements and the other fermion mass matrices, the scale of right-handed neutrinos in this framework must be \(M \sim 10^{13}\) GeV. For solar neutrinos, either uncharged right-handed neutrinos should appear at the GUT scale (for a large angle MSW solution \(^{78}\)) or the global symmetry should be broken by Planck scale physics (for a vacuum oscillation solution).

The extension to the non-SU(5) invariant models allows one more flexibility. As we saw in Sec. \(7.2\) the field content cannot be varied from the MFMM in order to produce the correct phenomenology without encountering a Landau pole below the unification scale. However, allowing different charges in the lepton sector permits both \(\mathcal{T}^2\) and \(\mathcal{T}^3\) to be uncharged. In such a scenario, it is possible to produce either \(M_{\nu_1}\) or \(M_{\nu_2}\). The latter matrix could be produced via spontaneous violation of \(R\)-parity \(^{72, 73}\) in a different sector. It may be possible to use the DSB sector as the source of bilinear \(R\)-parity violation \(^{73}\), thus eliminating the need for an additional scale all together. We will not explore this option here, saving it for possible future work.

8 Conclusions

In this paper we have presented the phenomenology of a new economical model of flavor. Within the context of gauge-mediation, we utilize the hierarchy of scales, extra vector-like (messenger) matter, and additional gauge group structure to provide not only a means to communicate supersymmetry breaking to the MSSM fields, but also to generate the first and second generation fermion masses using a modified Froggatt-Nielsen mechanism. The identification of the messenger \(U(1)\) that communicates supersymmetry breaking from the DSB fields to the messenger-Froggatt-Nielsen fields is also the flavor symmetry in our model is essential. In particular, all of the first and second generations are charged under the \(U(1)_F\) and obtain flavor-dependent supersymmetry breaking contributions to their scalar masses. Ordinarily this would be disastrous, however it

\(^{35}\)Note that the coupling and \(U(1)_F\) charge of \(\phi^{(5)}\) in this scenario differ from those of the MFMM.
is precisely because these fields are charged under the U(1)$_F$ that implies these contributions are rather large, roughly equivalent to the DSB scale suppressed by a two-loop factor. Hence, squark-induced FCNCs are suppressed by the heavy mass scale, following the “more minimal supersymmetry” approach. Perhaps the most fascinating phenomenological aspect of this model is that the fermion mass hierarchy and heavy first and second generation squarks are inextricably linked together.

There are several subtle issues to construct a successful supersymmetric model of flavor along the lines we have discussed, and we merely summarize the results of these model-building efforts here. In Sec. 5 a simplified model was advocated as the “minimal flavor-mediated model”, that depends on four main scales: $\Lambda_{DSB}$, $M_{FN} \simeq \langle \chi \rangle$, $\tilde{m}$, and $\sqrt{F_{\chi}}$. Several other parameters play an important role, including the normalized U(1)$_F$ FI term $q_\ell$, the U(1)$_F$ gauge coupling $g_F$, three superpotential parameters ($b_5$, $\lambda_1^d$, and $\lambda_2^d$) that determine the size of Yukawa-induced supersymmetry breaking contributions, and also $\tan \beta$. We discovered that there is a tight connection between the weak scale masses, including $m_{\tilde{\tau}_1}$ and the mass scale of the other third generation scalars, and the heavy first and second generation mass scale. Increasing the (absolute) size of $\tilde{m}^2$ causes an increase in the first and second generation masses, but also causes a decrease in the mass of the lightest stau. This important result suggests a no-lose theorem for the model: Either the stau is light and should discovered in an upcoming collider experiment, or the first and second generations are “lighter”, and should be manifest in future measurements of FCNC processes.

The signals of the model depend heavily on three factors: The identity of the NLSP, the NLSP decay length, and the mass of the lightest stau. We showed that in the MFMM the decay length is always much larger than the scale of the detector (hence the NLSP always escapes the detector), and the mass of the stau is always less than the mass of the lightest chargino and the second lightest neutralino. The NLSP is therefore either the lightest neutralino or the lightest stau. In both cases the production of the lightest chargino and/or the second lightest neutralino is expected to result in about one tau per chargino, and two taus per neutralino due to the two-body decays $\tilde{C}_1^\pm \to \tilde{\tau}_1^\pm \nu_\tau$ and $\tilde{N}_2 \to \tilde{\tau}_1^\pm \tau_\pm$. If the NLSP is the lightest neutralino, searching for these taus is the best method of discovering supersymmetry. In this scenario the most promising production processes are $\tilde{C}_1^\pm \tilde{N}_2$, and $\tilde{N}_2 \tilde{N}_2$, that lead to the $3\tau$ and $4\tau$ plus missing energy final states. If the NLSP is the lightest stau, then the stau itself should manifest itself as a charged track in the detector. Depending on the energy of the stau, this could mimic a muon (if $\beta \gamma > 0.85$) or appear as a unique highly-ionizing track (HIT). Searching for signals with at least one HIT is the best search strategy for collider experiments. Indeed, the reach of a high luminosity Tevatron extends up to about two hundred GeV for the lightest stau, and several hundred GeV for the lightest chargino and second lightest neutralino [60, 61].

We should also remark that there are several other observables characteristic of our flavor model that we have not mentioned. The heavy first and second generation superfields are highly split in mass between their fermionic components and the the scalar components. It was shown in Refs. 73 that a consequence of nondegeneracy is an observable weak scale distinction between between the gauge coupling of fermions to gauge bosons, and the associated gauge coupling of a fermion and sfermion to a gaugino. Unfortunately, measuring the difference between a “gauge
coupling” and a “gaugino coupling” is not easy, especially for the electroweak couplings, since it requires plenty of observed sparticle production data. Another important class of signals that are perhaps much more promising for our model are more precise FCNC measurements. We have already mentioned the important \( D^0 \leftrightarrow \bar{D}^0 \) mixing process that is expected, although several other FCNC processes could provide a window on the (scalar) flavor structure of our model.

The two outstanding problems of the MFMM that are not definitively resolved are the dynamical origin of the \( \mu \)-term, and the generation of neutrino masses consistent with the recent atmospheric and solar neutrino experiments. The value of \( \mu^2 \) for any successful weak scale supersymmetric model must be matched to \( M_Z \), and thus a dynamical solution to the \( \mu \)-problem is really a model-building issue. Several possible methods to generate \( \mu \) were presented, although we remain agnostic about the mechanism that is best for this framework. Generating neutrino masses is more a glaring problem for the MFMM, and we presented a few possible ways that this could be accomplished. In all cases one must introduce new physics at scales larger than \( \Lambda_{\text{DDB}} \) to accomplish the usual see-saw mechanism. Combining this solution to flavor with a more natural mechanism for generating neutrino masses is an very interesting avenue of research shall be explored in future work.

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Appendix A: Superpotential in components

Here we present the full superpotential of the model in component form.

\[
W = + a_1^{(Q)} \chi^{(1)} Q^{FN} \bar{Q}^{FN} + a_1^{(u)} \chi^{(1)} u^{FN} \bar{u}^{FN} + a_1^{(e)} \chi^{(1)} e^{FN} \bar{e}^{FN} \\
+ a_2^{(d)} \chi^{(2)} d^{FN(1)} \bar{d}^{FN(1)} + a_2^{(L)} \chi^{(2)} L^{FN(1)} \bar{L}^{FN(1)} \\
+ a_3^{(d)} \chi^{(3)} d^{FN(2)} \bar{d}^{FN(2)} + a_3^{(L)} \chi^{(3)} L^{FN(2)} \bar{L}^{FN(2)} \\
+ b_1^{(Q)} \phi^{(1)} Q^{FN} + b_1^{(u)} \phi^{(1)} u^{FN} + b_1^{(e)} \phi^{(1)} e^{FN} \\
+ b_2^{(Q)} \phi^{(2)} Q^{FN} + b_2^{(u)} \phi^{(2)} u^{FN} + b_2^{(e)} \phi^{(2)} e^{FN} \\
+ b_3^{(d)} \phi^{(3)} d^{FN(1)} + b_3^{(L)} \phi^{(3)} L^{FN(1)} \\
+ b_4^{(d)} \phi^{(4)} d^{FN(2)} + b_4^{(L)} \phi^{(4)} L^{FN(2)} \\
+ b_5^{(d)} \phi^{(5)} d^{FN(1)} + b_5^{(L)} \phi^{(5)} L^{FN(1)} \\
+ \lambda_1^{d} H^{d} Q^{FN(1)} + \lambda_1^{d} H^{d} e^{FN(1)} \\
+ \lambda_2^{d} H^{d} Q^{FN(2)} + \lambda_2^{d} H^{d} e^{FN(2)}
\]

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\[ + Y_b H^d Q^3 \overline{d}^3 + Y_t H^d e^3 \overline{L}^3 \]
\[ + \lambda_{3,1}^d H^d Q^F N \overline{d}^3 + \lambda_{3,2}^d H^d e^F N \overline{L}^3 \]
\[ + Y_t H^u Q^3 u^3 + \lambda_{3,1}^u H^u Q^3 u^F N + \lambda_{3,2}^u H^u u^3 Q^F N. \]  

(72)

Since no Higgs triplets are present in the model, only some component terms remain after expanding the SU(5) tensor products. It is also possible to identify \( \lambda^u_i \) as the top Yukawa coupling \( Y_t \), and \( \lambda^d_{3,1}, \lambda^d_{3,2} \) as the bottom and tau Yukawa couplings \( Y_b, Y_\tau \).

To define our notation, the quantum numbers of these fields are given in Table [37]. We did not list the flavons since they are unchanged under the SM gauge groups.

### A.1 Renormalization group equations

The renormalization group equations of the (mass)\(^2\) for the scalars in the superpotential, Eq. (72), have been computed below using Ref. [37]. These are given to one-loop, which is sufficient for our purposes. (The two-loop gauge contributions to the uncharged scalars are incorporated via the boundary conditions at \( M_{F,N} \).)

In general, we write the \( \beta \)-function for the scalar (mass)\(^2\) as

\[
\frac{d}{dt} m_i^2 = \frac{1}{16\pi^2} \left( \beta_i - 8g_i^2 g_F^2 |M_F|^2 + 2q_i g_F^2 S' \right),
\]

(73)

where \( t \equiv \ln Q \), \( q_i \) is the U(1)\(_F\) charge of field \( i \), \( g_F \) is the U(1)\(_F\) gauge coupling, \( M_F \) is the U(1)\(_F\) gaugino mass, and we define the scalar (mass)\(^2\) function \( S' \) by

\[
S' = \sum_j S(j)q_j m_j^2,
\]

(74)

where \( S(j) \) is the Dynkin index for the field \( j \).

The Yukawa coupling-dependent components are given by

\[
\beta_{\chi^{(1)}} = +12 \left( a_1^{(Q)} \right)^2 \left[ m_{\chi^{(1)}}^2 + m_{Q,F,N}^2 + m_{Q,F,N}^2 \right] + 6 \left( a_1^{(u)} \right)^2 \left[ m_{\chi^{(1)}}^2 + m_{u,F,N}^2 + m_{u,F,N}^2 \right] \\
+ 2 \left( a_1^{(e)} \right)^2 \left[ m_{\chi^{(1)}}^2 + m_{e,F,N}^2 + m_{e,F,N}^2 \right]
\]

\[
\beta_{\chi^{(2)}} = +6 \left( a_2^{(d)} \right)^2 \left[ m_{\chi^{(2)}}^2 + m_{d,F,N(1)}^2 + m_{d,F,N(1)}^2 \right] + 4 \left( a_2^{(L)} \right)^2 \left[ m_{\chi^{(2)}}^2 + m_{L,F,N(1)}^2 + m_{L,F,N(1)}^2 \right] \\
+ 6 \left( a_3^{(d)} \right)^2 \left[ m_{\chi^{(3)}}^2 + m_{d,F,N(2)}^2 + m_{d,F,N(2)}^2 \right] + 4 \left( a_3^{(L)} \right)^2 \left[ m_{\chi^{(3)}}^2 + m_{L,F,N(2)}^2 + m_{L,F,N(2)}^2 \right]
\]

\[
\beta_{Q,F,N} = +2 \left( a_1^{(Q)} \right)^2 \left[ m_{Q,F,N}^2 + m_{\chi^{(1)}}^2 + m_{Q,F,N}^2 \right] + 2 \left( \lambda_{1,1}^{d} \right)^2 \left[ m_{Q,F,N}^2 + m_{H,d}^2 + m_{Q,F,N}^2 \right] \\
+ 2 \left( \lambda_{2,2}^{u} \right)^2 \left[ m_{Q,F,N}^2 + m_{H,u}^2 + m_{Q,F,N}^2 \right]
\]

\[
\beta_{u,F,N} = +2 \left( a_1^{(u)} \right)^2 \left[ m_{u,F,N}^2 + m_{\chi^{(1)}}^2 + m_{u,F,N}^2 \right] + 4 \left( \lambda_{2,1}^{u} \right)^2 \left[ m_{u,F,N}^2 + m_{H,u}^2 + m_{Q,F,N}^2 \right] \\
+ 2 \left( a_1^{(e)} \right)^2 \left[ m_{e,F,N}^2 + m_{\chi^{(1)}}^2 + m_{e,F,N}^2 \right] + 4 \left( \lambda_{d,2}^{i} \right)^2 \left[ m_{e,F,N}^2 + m_{H,d}^2 + m_{Q,F,N}^2 \right]
\]

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| field     | SU(5) component | SU(3) | SU(2) | U(1) \_Y | U(1) \_F |
|-----------|-----------------|-------|-------|-----------|-----------|
| $10^FN$   | $Q^FN$          | 3     | 2     | $\frac{1}{3}$ | 0         |
|           | $u^FN$          | $\overline{3}$ | 1     | $-\frac{2}{3}$ | 0         |
|           | $e^FN$          | 1     | 1     | 1         | 0         |
| $10^FN$   | $Q^FN$          | $\overline{3}$ | 2     | $-\frac{1}{6}$ | $q_{10}^{FN}$ |
|           | $\pi^FN$       | 3     | 1     | $\frac{2}{3}$ | $q_{10}^{FN}$ |
|           | $\sigma^FN$    | 1     | 1     | $-1$      | $q_{10}^{FN}$ |
| $5^FN(1)$ | $d^FN(1)$       | 3     | 1     | $-\frac{1}{3}$ | $q_{5}^{FN(1)}$ |
|           | $L^FN(1)$       | 1     | 2     | $\frac{1}{2}$ | $q_{5}^{FN(1)}$ |
| $5^FN(1)$ | $\overline{d}^FN(1)$ | $\overline{3}$ | 1     | $\frac{1}{3}$ | $q_{\overline{5}}^{FN(1)}$ |
|           | $\overline{L}^FN(1)$ | 1      | 2     | $-\frac{1}{2}$ | $q_{\overline{5}}^{FN(1)}$ |
| $5^FN(2)$ | $d^FN(2)$       | 3     | 1     | $-\frac{1}{3}$ | $q_{5}^{FN(2)}$ |
|           | $L^FN(2)$       | 1     | 2     | $\frac{1}{2}$ | $q_{5}^{FN(2)}$ |
| $5^FN(2)$ | $\overline{d}^FN(2)$ | $\overline{3}$ | 1     | $\frac{1}{3}$ | $q_{\overline{5}}^{FN(2)}$ |
|           | $\overline{L}^FN(2)$ | 1      | 2     | $-\frac{1}{2}$ | $q_{\overline{5}}^{FN(2)}$ |
| $10^1$    | $Q^1$           | 3     | 2     | $\frac{1}{6}$ | $q_{10^1}$ |
|           | $u^1$           | $\overline{3}$ | 1     | $-\frac{2}{3}$ | $q_{10^1}$ |
|           | $e^1$           | 1     | 1     | 1         | $q_{10^1}$ |
| $10^2$    | $Q^2$           | 3     | 2     | $\frac{1}{6}$ | $q_{10^2}$ |
|           | $u^2$           | $\overline{3}$ | 1     | $-\frac{2}{3}$ | $q_{10^2}$ |
|           | $e^2$           | 1     | 1     | 1         | $q_{10^2}$ |
| $5^4$     | $d^4$           | $\overline{3}$ | 1     | $\frac{1}{3}$ | $q_{5^4}$ |
|           | $\overline{L}^4$ | 1      | 2     | $-\frac{1}{2}$ | $q_{\overline{5}}^4$ |
| $5^2$     | $\overline{d}^2$ | $\overline{3}$ | 1     | $\frac{1}{3}$ | $q_{\overline{5}}^2$ |
|           | $\overline{L}^2$ | 1      | 2     | $-\frac{1}{2}$ | $q_{\overline{5}}^2$ |
| $10^3$    | $Q^3$           | 3     | 2     | $\frac{1}{6}$ | 0         |
|           | $u^3$           | $\overline{3}$ | 1     | $-\frac{2}{3}$ | 0         |
|           | $e^3$           | 1     | 1     | 1         | 0         |
| $5^3$     | $\overline{d}^3$ | $\overline{3}$ | 1     | $\frac{1}{3}$ | 0         |
|           | $\overline{L}^3$ | 1      | 2     | $-\frac{1}{2}$ | 0         |

Table 1: Charge assignments for the matter considered in this paper. Note that the $U(1)\_Y$ charges are in the GUT normalization.
\[
\begin{align*}
\beta^{\text{QFN}} &= +2 \left( a^{(Q)}_1 \right)^2 \left[ m^2_{Q_1} + m^2_{\phi(1)} + m^2_{\phi(2)} \right] + 2 \left( b^{(Q)}_1 \right)^2 \left[ m^2_{Q_2} + m^2_{\phi(2)} + m^2_{\phi(1)} \right] \\
&+ 2 \left( b^{(Q)}_2 \right)^2 \left[ m^2_{Q_2} + m^2_{\phi(2)} + m^2_{Q_1} \right] \\
\beta^{\text{F}} &= +2 \left( a^{(u)}_1 \right)^2 \left[ m^2_{\Phi} + m^2_{\chi(1)} + m^2_{\chi(2)} \right] + 2 \left( b^{(u)}_1 \right)^2 \left[ m^2_{\Phi} + m^2_{\chi(2)} + m^2_{\chi(1)} \right] \\
&+ 2 \left( b^{(u)}_2 \right)^2 \left[ m^2_{\Phi} + m^2_{\phi(2)} + m^2_{u_1} \right] \\
\beta^{\text{F}} &= +2 \left( a^{(e)}_1 \right)^2 \left[ m^2_{\Phi} + m^2_{\chi(1)} + m^2_{\chi(2)} \right] + 2 \left( b^{(e)}_1 \right)^2 \left[ m^2_{\Phi} + m^2_{\chi(2)} + m^2_{\chi(1)} \right] \\
&+ 2 \left( b^{(e)}_2 \right)^2 \left[ m^2_{\Phi} + m^2_{\phi(2)} + m^2_{e_1} \right] \\
\beta^{\text{F}}_{d(1)} &= +2 \left( a^{(d)}_2 \right)^2 \left[ m^2_{d_F(1)} + m^2_{\chi(2)} + m^2_{d_F(1)} \right] + 4 \left( \lambda^{(d)}_{1,1} \right)^2 \left[ m^2_{d_F(1)} + m^2_{d_F(1)} + m^2_{Q_1} \right] \\
\beta^{\text{F}}_{d(1)} &= +2 \left( a^{(d)}_2 \right)^2 \left[ m^2_{d_F(1)} + m^2_{\chi(2)} + m^2_{d_F(1)} \right] + 4 \left( \lambda^{(d)}_{1,1} \right)^2 \left[ m^2_{d_F(1)} + m^2_{d_F(1)} + m^2_{Q_1} \right] \\
\beta^{\text{F}}_{d(2)} &= +2 \left( a^{(d)}_3 \right)^2 \left[ m^2_{d_F(2)} + m^2_{\chi(3)} + m^2_{d_F(2)} \right] + 2 \left( b^{(d)}_4 \right)^2 \left[ m^2_{d_F(2)} + m^2_{d_F(2)} + m^2_{d_F(2)} \right] \\
\beta^{\text{F}}_{L(1)} &= +2 \left( a^{(L)}_2 \right)^2 \left[ m^2_{L_F(1)} + m^2_{\chi(2)} + m^2_{L_F(1)} \right] + 2 \left( b^{(L)}_3 \right)^2 \left[ m^2_{L_F(1)} + m^2_{\chi(2)} + m^2_{L_F(1)} \right] \\
\beta^{\text{F}}_{L(2)} &= +2 \left( a^{(L)}_3 \right)^2 \left[ m^2_{L_F(2)} + m^2_{\chi(3)} + m^2_{L_F(2)} \right] + 2 \left( b^{(L)}_4 \right)^2 \left[ m^2_{L_F(2)} + m^2_{\chi(3)} + m^2_{L_F(2)} \right] \\
\beta^{\text{F}}_{\phi(1)} &= +12 \left( b^{(Q)}_1 \right)^2 \left[ m^2_{\phi(1)} + m^2_{\phi(1)} + m^2_{\phi(1)} \right] + 6 \left( b^{(u)}_1 \right)^2 \left[ m^2_{\phi(1)} + m^2_{\phi(1)} + m^2_{\phi(1)} \right] \\
&+ 2 \left( b^{(e)}_1 \right)^2 \left[ m^2_{\phi(1)} + m^2_{\phi(1)} + m^2_{\phi(1)} \right] \\
\beta^{\text{F}}_{\phi(2)} &= +12 \left( b^{(Q)}_2 \right)^2 \left[ m^2_{\phi(2)} + m^2_{\phi(2)} + m^2_{\phi(2)} \right] + 6 \left( b^{(u)}_2 \right)^2 \left[ m^2_{\phi(2)} + m^2_{\phi(2)} + m^2_{\phi(2)} \right] \\
&+ 2 \left( b^{(e)}_2 \right)^2 \left[ m^2_{\phi(2)} + m^2_{\phi(2)} + m^2_{\phi(2)} \right] \\
\beta^{\text{F}}_{\phi(3)} &= +6 \left( b^{(d)}_3 \right)^2 \left[ m^2_{\phi(3)} + m^2_{\phi(3)} + m^2_{\phi(3)} \right] + 4 \left( b^{(L)}_3 \right)^2 \left[ m^2_{\phi(3)} + m^2_{\phi(3)} + m^2_{\phi(3)} \right] \\
\beta^{\text{F}}_{\phi(4)} &= +6 \left( b^{(d)}_4 \right)^2 \left[ m^2_{\phi(4)} + m^2_{\phi(4)} + m^2_{\phi(4)} \right] + 4 \left( b^{(L)}_4 \right)^2 \left[ m^2_{\phi(4)} + m^2_{\phi(4)} + m^2_{\phi(4)} \right] \\
\beta^{\text{F}}_{\phi(5)} &= +6 \left( b^{(d)}_5 \right)^2 \left[ m^2_{\phi(5)} + m^2_{\phi(5)} + m^2_{\phi(5)} \right] + 4 \left( b^{(L)}_5 \right)^2 \left[ m^2_{\phi(5)} + m^2_{\phi(5)} + m^2_{\phi(5)} \right] \\
\beta_{Q^1} &= +2 \left( b^{(Q)}_1 \right)^2 \left[ m^2_{Q^1} + m^2_{Q^1} + m^2_{Q^1} \right] + 2 \left( \lambda^{(d)}_{1,1} \right)^2 \left[ m^2_{Q^1} + m^2_{Q^1} + m^2_{Q^1} \right] \\
\beta_{Q^2} &= +2 \left( b^{(Q)}_2 \right)^2 \left[ m^2_{Q^2} + m^2_{Q^2} + m^2_{Q^2} \right] + 2 \left( \lambda^{(d)}_{2,1} \right)^2 \left[ m^2_{Q^2} + m^2_{Q^2} + m^2_{Q^2} \right]
\end{align*}
\]
\[
\beta_u = +2(b_1^{(u)})^2 \left[ m_{u_1}^2 + m_{\phi(1)}^2 + m_{\phi F N}^2 \right]
\]
\[
\beta_d = +2(b_2^{(u)})^2 \left[ m_{d_2}^2 + m_{\phi(2)}^2 + m_{\phi F N}^2 \right]
\]
\[
\beta_e = +2(b_1^{(e)})^2 \left[ m_{e_1}^2 + m_{\phi(1)}^2 + m_{\phi F N}^2 \right] + 4\left( \lambda_{1,2}^4 \right)^2 \left[ m_{e_1}^2 + m_{H^d}^2 + m_{L F N(1)}^2 \right]
\]
\[
\beta_e = +2(b_2^{(e)})^2 \left[ m_{e_2}^2 + m_{\phi(2)}^2 + m_{\phi F N}^2 \right] + 4\left( \lambda_{2,2}^4 \right)^2 \left[ m_{e_2}^2 + m_{H^d}^2 + m_{L F N(2)}^2 \right]
\]
\[
\beta_d = +2(b_3^{(d)})^2 \left[ m_{d_3}^2 + m_{\phi(3)}^2 + m_{d F N(1)}^2 \right]
\]
\[
\beta_d = +2(b_4^{(d)})^2 \left[ m_{d_4}^2 + m_{\phi(4)}^2 + m_{d F N(2)}^2 \right]
\]
\[
\beta_L = +2(b_4^{(L)})^2 \left[ m_{L_3}^2 + m_{\phi(3)}^2 + m_{L F N(1)}^2 \right]
\]
\[
\beta_L = +2(b_4^{(L)})^2 \left[ m_{L_4}^2 + m_{\phi(4)}^2 + m_{L F N(2)}^2 \right]
\]
\[
\beta_Q = +2(Y_t)^2 \left[ m_{Q_3}^2 + m_{Q_2}^2 + m_{u_3}^2 \right] + 2\left( \lambda_{2,1}^4 \right)^2 \left[ m_{Q_3}^2 + m_{H_4}^2 + m_{Q_5}^2 \right]
\]
\[
\beta_u = +4(Y_t)^2 \left[ m_{u_3}^2 + m_{H_4}^2 + m_{Q_4}^2 \right] + 4\left( \lambda_{2,2}^4 \right)^2 \left[ m_{u_3}^2 + m_{H_4}^2 + m_{Q_F N}^2 \right]
\]
\[
\beta_d = +2(Y_t)^2 \left[ m_{d_3}^2 + m_{H_4}^2 + m_{L_3}^2 \right]
\]
\[
\beta_d = +2(Y_t)^2 \left[ m_{d_4}^2 + m_{H_4}^2 + m_{L_4}^2 \right]
\]
\[
\beta_L = +2(Y_t)^2 \left[ m_{L_3}^2 + m_{H_4}^2 + m_{L_5}^2 \right] + 2\left( \lambda_{4,1}^4 \right)^2 \left[ m_{L_3}^2 + m_{H_4}^2 + m_{L_3}^2 \right]
\]
\[
\beta_H^u = +6(Y_t)^2 \left[ m_{H_4}^2 + m_{Q_3}^2 + m_{u_3}^2 \right] + 6\left( \lambda_{2,1}^4 \right)^2 \left[ m_{H_4}^2 + m_{Q_3}^2 + m_{u_3}^2 \right]
\]
\[
\beta_H^d = +6\left( \lambda_{1,1}^4 \right)^2 \left[ m_{H_4}^2 + m_{Q_1}^2 + m_{d_3}^2 \right] + 2\left( \lambda_{1,2}^4 \right)^2 \left[ m_{H_4}^2 + m_{e_1}^2 + m_{L F N(1)}^2 \right]
\]
\[
\beta_H^d = +6\left( \lambda_{2,1}^4 \right)^2 \left[ m_{H_4}^2 + m_{Q_2}^2 + m_{d_4}^2 \right] + 2\left( \lambda_{2,2}^4 \right)^2 \left[ m_{H_4}^2 + m_{e_2}^2 + m_{L F N(2)}^2 \right]
\]
\[
\beta_H^d = +6\left( Y_t \right)^2 \left[ m_{H_4}^2 + m_{Q_4}^2 + m_{d_3}^2 \right] + 2\left( Y_t \right)^2 \left[ m_{H_4}^2 + m_{e_3}^2 + m_{L_3}^2 \right]
\]
\[
\beta_H^d = +6\left( \lambda_{4,1}^4 \right)^2 \left[ m_{H_4}^2 + m_{Q F N}^2 + m_{d_4}^2 \right] + 2\left( \lambda_{4,2}^4 \right)^2 \left[ m_{H_4}^2 + m_{e_4}^2 + m_{L_3}^2 \right]
\]
Table 2: Example set of input parameters and some of the calculated weak scale masses and couplings for an example MFMM.

**Appendix B: Example parameter set and sparticle spectrum**

We present one example set of parameters of the MFMM, and the resulting weak scale spectra. The input parameters and calculated weak scale quantities are shown in Table 2. The importance of the RG evolution in this model was emphasized in Sec. 4, and thus we also show in Fig. 8 the evolution of several soft (mass)$^2$ as a function of the renormalization scale. Notice that there is significant evolution between both the DSB scale and the FN scale, as well as between the FN scale and the weak scale. Since all of the fields shown in the figure are uncharged under the $U(1)_F$, their mass at the DSB scale is assumed to vanish.

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Figure 8: Using the example set of parameters given in Table 2, the evolution of several soft scalar (mass)$^2$ is shown as a function of the renormalization scale. The (top, bottom) solid lines correspond to $(m_{H_d}^2, m_{H_u}^2)$, the (top, middle, bottom) dotted lines correspond to $(m_{Q_3}^2, m_{U_3}^2, m_{e_3}^2)$, and the (top, bottom) dashed lines correspond to $(m_{d_3}^2, m_{L_3}^2)$. The “break” in the evolution at $M_{FN} = 2 \times 10^5$ GeV corresponds to where the FN sector is integrated out, and thus additional supersymmetry breaking contributions are induced for the light MSSM fields.

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