Study about the influence of cavitation on the dynamic characteristics for the sliding bearing

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Abstract. Sliding bearings are employed to support the rotor system and limit the vibration amplitude. In high speed rotor system, cavitation often occurs in the oil film and affects the dynamic characteristics of the sliding bearing greatly. In this paper, numerical method is adopted to simulate the cavitation in the oil film with homogeneous two-phase mixture flow using Singhal-et-al cavitation model in the commercial code FLUENT-solver. Cases without cavitation model were also calculated at the same time. Many computations with different frequency ratios were conducted. Then the rotor dynamic characteristics of the sliding bearing were retrieved. The results show that the cavitation has great influences on the pressure distribution in the oil film. As the rotational speed or whirling speed of the journal increases, the cavitation will become prominent. The dynamic coefficients of the bearing such as stiffness and damping with cavitation model considered are quite different from that without cavitation. So it is worth to pay attention to and do further study about the cavitation in the sliding bearing in the high speed rotor system.

1. Introduction
Sliding bearings has been widely used in rotating machinery which enables modern rotating machinery more stable and efficient. Oil is usually used in the bearing for lubricating the journal, supporting the shaft weight and taking away the viscosity friction heat. During the operation, the displacement perturbation will induce the rotor vibration which includes two kinds of motion, rotational speed around the shaft center and whirling speed around the balanced location. In this situation, the dynamic characteristics of the bearing such as direct and coupled stiffness and damping have great effects on the vibration amplitude. However, when the rotational speed or the whirling speed is too high, the local pressure in the oil film will fall below the specific value and induce the cavitation phenomenon, which will change the dynamic properties of the bearing and affect the vibration characteristics.

Cavitation around a hydrofoil has been thoroughly researched [1, 2], but the cavitation in the sliding bearing is been studied in recent years. Many experiments show the oil film doesn’t rupture completely, but presents thin strips [3]. Sun DC [4] did the experiment to show that the negative zone includes both air bubbles and oil vapors. Some numerical studies for the cavitation in the sliding bearing are also conducted. By solving the Reynolds equations, Li Qiang [5] put forwards a new method which can introduce the JFO boundary condition more easily, and compared the numerical...
results with the experimental results. Yang Ying [6] also solved the Reynolds equations with cavitation model using finite difference method. Furthermore, she united the energy equation to get the temperature distribution in the film. Guo Zenglin [7] first built the 3D CFD model for the sliding bearing. The results with CFD method matched the results by solving the Reynolds equation. However, the cavitation effects in the bearing was not considered. Song Yin [8] put forwards a new cavitation model based on air solubility. With this model, Computation for a journal bearing using CFD method was conducted. The numerical results matches well with the experiments. The new method shows better accuracy than the Half-Sommerfeld cavitation boundary condition. However, there are few papers that discuss the influence of the cavitation on the dynamic characteristics of the sliding bearing. It is meaningful for the rotor-bearing coupled system.

This paper focus on the cavitation in the sliding bearing. A 3D model of the oil film flow field was built. Homogeneous two-phase mixture flow with Singhal et al cavitation model was used in the commercial code FLUENT-solver. Cases without cavitation model were also calculated at the same time. Many computations with different frequency ratios were conducted. Then the rotor dynamic characteristics of the sliding bearing were retrieved. The influence of the cavitation on the dynamics properties of the sliding bearing were discussed in detail.

2. Numerical method

1.1. Governing equations
Mixture model is chosen for the cavitation computation in this paper. All fluids are in one flow field share one velocity field. The mixture model assumes that each quantity transported for all phases are same. The continuity and momentum equations are as follows:

1.1.1. Continuity equation. Mixture phase:
\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0
\] (1)

Bubble phase:
\[
\frac{\partial (\alpha_v \rho_v)}{\partial t} + \nabla \cdot (\alpha_v \rho_v \mathbf{u}) = R_e - R_c
\] (2)

1.1.2. Momentum equation:
\[
\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = \rho \mathbf{g} - \nabla p + \frac{1}{3} \nabla \left[ (\mu + \mu_t) \nabla \cdot \mathbf{u} \right] + \nabla \cdot \left[ (\mu + \mu_t) \nabla \mathbf{u} \right]
\] (3)

Where \( \rho = \alpha_v \rho_v + \alpha_l \rho_l \) is the density of mixture phase, \( \rho_v \) is bubble phase density, \( \rho_l \) is the liquid phase density, \( \alpha_v \) is the volume fraction of the bubble phase, \( \alpha_l \) is the volume fraction of the liquid phase, \( \mathbf{u} \) is the time mean velocity, \( \mu \) is the dynamic viscosity coefficient of the mixture phase, \( \mu_t \) is the eddy viscosity coefficient of the mixture phase, \( R_e \) is the bubble formation rate, \( R_c \) is the steam condensation rate.

1.2. Cavitation model
There are three cavitation model in ANSYS FLUENT including Singhal et al. model, Zwart-Gerber-Belamri model and Schnerr and Sauer model. Singhal-et-al cavitation model is used here which is based on the “full cavitation model”, developed by Singhal et al. [9]. It has the capability to account for multiphase flows or flows with multiphase species transport, the effects of slip velocities between the liquid and gaseous phases, and the thermal effects and compressibility of both liquid and gas phases.

Using the two-phase continuity equations, Singhal et al. derives an expression of the net phase change rate (R) as follows:
Here $R$ represents the vapour generation or evaporation rate, that is, the source term $R_e$. $n$ is the bubble number density. $\alpha$ is the vapour volume fraction. $l$ is liquid phase, $v$ is vapor phase, $\rho$ is mixture density. All terms, except $n$, are either known constants or dependent variables. In the absence of a general model for estimation of the bubble number density, the phase change rate expression is rewritten in terms of bubble radius ($R_B$), as follows:

$$R = \frac{3\alpha \rho_v \rho_l}{R_B} \sqrt{\frac{2}{3}} \frac{2(P_B - P)}{\rho_l}$$  \hspace{1cm} (5)

As for bubble collapse or the condensation process, though it is expected to be different from that of bubble growth, as a first approximation, also often used to model the bubble collapse by using the absolute value of the pressure difference and treating the right side as a sink term.

It may be noted that in practical cavitation model, the local far-field pressure $P$ is usually taken to be the same as the cell center pressure. The bubble pressure $P_B$ is equal to the saturation vapour pressure in the absence of dissolved gases, mass transport and viscous damping, that is, $P_B = P_v$. Where, $P_B$ is bubble pressure, $P_v$ is saturation vapor pressure.

Based on equation (5), Singhal et al. proposed a model where the vapor mass fraction is the dependent variable in the transport equation. This model accommodates also a single phase formulation where the governing equations is given by:

$$\frac{\partial}{\partial t} (f_v \rho) + \nabla \cdot (f_v \rho \bar{V}_v) = \nabla \cdot (\Gamma \nabla f_v) + R_e - R_c$$  \hspace{1cm} (6)

Where, $f_v$ is vapor mass fraction, $f_g$ is noncondensable gases, $\Gamma$ is diffusion coefficient.

The rates of mass exchange are given by the following equations:

If $P \leq P_v$

$$R_e = F_{vap} \frac{\max(1.0, \sqrt{k})(1 - f_v - f_g)}{\sigma} \frac{2(P_v - P)}{\rho_v \rho_l \sqrt{3} \rho_{rel}}$$  \hspace{1cm} (7)

If $P > P_v$

$$R_c = F_{cond} \frac{\max(1.0, \sqrt{k}) f_v}{\sigma} \frac{2(P_v - P)}{\rho_v \rho_l \sqrt{3} \rho_l}$$  \hspace{1cm} (8)

The saturation pressure is corrected by an estimation of the values of the turbulent pressure fluctuations:

$$P_v = P_{sat} + \frac{1}{2} (0.39 \rho k)$$  \hspace{1cm} (9)

The constants have the values $F_{vap} = 0.02$ and $F_{cond} = 0.01$. In this model, the liquid-vapor mixture is assumed to compressible. Also, the effects of turbulence and the noncondensable gases have been taken into account.

1.3. Approach for the dynamic characteristics of the journal bearing
Due to some factors such as unbalanced mass or axial misalignment, the journal usually rotates offset the bearing center (eccentricity, \( e \)) which can be resolved into two kinds of motion, rotation around the journal center \( O_1 \) with speed \( \Omega \), and the rotation around the bearing center \( O \) with speed \( \omega \), as shown in Figure 1. The forces that the oil film impacts to the rotor in a plane perpendicular to the axis of rotation are depicted in Figure 1, and decomposed into components in the direction \( x \) and \( y \), where this coordinate system is fixed in the framework of the rotor. The instantaneous forces are denoted by \( F_x(t) \), \( F_y(t) \), and the time-averaged values of these forces in the stationary frame are denoted by \( F_{0x}, F_{0y} \) which can be described using the following function:

\[
F = f(x, y, \dot{x}, \dot{y}, \ddot{x}, \ddot{y})
\]

(10)

Figure 1. Schematic diagram of the journal bearing.

It is usually assumed that this displacement is sufficiently small so that linear perturbation model is accurate. The fluid force can be expanded in linear Taylor’s series as following equation (11):

\[
\begin{bmatrix}
F_x \\
F_y
\end{bmatrix} = 
\begin{bmatrix}
F_{x0} \\
F_{y0}
\end{bmatrix} + 
\begin{bmatrix}
K_{xx} & K_{xy} \\
K_{yx} & K_{yy}
\end{bmatrix} \begin{bmatrix}
x(t) - x_0 \\
y(t) - y_0
\end{bmatrix} + 
\begin{bmatrix}
C_{xx} & C_{xy} \\
C_{yx} & C_{yy}
\end{bmatrix} \begin{bmatrix}
\dot{x}(t) - \dot{x}_0 \\
\dot{y}(t) - \dot{y}_0
\end{bmatrix} + 
\begin{bmatrix}
M_{xx} & M_{xy} \\
M_{yx} & M_{yy}
\end{bmatrix} \begin{bmatrix}
\ddot{x}(t) - \ddot{x}_0 \\
\ddot{y}(t) - \ddot{y}_0
\end{bmatrix}
\]

(11)

Where \([K]\) is the stiffness matrix, \([C]\) is the damping matrix, \([M]\) is the mass matrix, \(x(t)\) and \(y(t)\) are the displacement of the journal along direction \(x\) and \(y\). \(x_0\) and \(y_0\) are the displacement at the equilibrium position.

One particular feature of the rotor dynamic matrices, \([K]\), \([C]\) and \([M]\), deserves special note. There are many geometries in which the rotor dynamic forces should be invariant to a rotation of the \(x, y\) axes [10]. Such will be the case only if

\[
\begin{align*}
K_{xx} &= K_{yy} = K; \quad K_{xy} = -K_{yx} = k \\
C_{xx} &= C_{yy} = C; \quad C_{xy} = -C_{yx} = c \\
M_{xx} &= M_{yy} = M; \quad M_{xy} = -M_{yx} = m
\end{align*}
\]

(12)

Assumed that \(x(t)\) reaches to the maximum displacement when \(t\) is 0. So

\[
\begin{align*}
x(0) &= e, \quad y(0) = 0, \quad \dot{x}(0) = 0, \\
\dot{y}(0) &= e\omega, \quad \ddot{x}(0) = -e\omega^2, \quad \ddot{y}(0) = 0
\end{align*}
\]

(13)

Plug the equations (12) and (13) into formula (11), the following formulas can be obtained.
\[
\frac{F_x}{e} |_{t=0} = M \omega^2 - c \omega - K \\
\frac{F_y}{e} |_{t=0} = -m \omega^2 - C \omega + k
\]

(14)

Therefore, all six rotor dynamic coefficients can be directly evaluated from quadratic curve fits to the graphs of \( F_x \) and \( F_y \) against \( \omega \).

3. Numerical model and boundary conditions

To provide a better grid, hex grid was used to mesh the 3D flow field of the oil film in figure 2. The structure parameters of the sliding bearing is list in the Table 1. Here the eccentricity ratio is 0.2 long the direction X, and it is assumed that it is linear with the eccentricity. After grid independence check, the flow field of the water film was meshed with the grid number of 162,690 and 10 layers in the clearance. Because both the rotational effect and squeeze effect should be considered, rotating coordinate system was used in this study. In this method, a rotating coordinate system with the speed \( \omega \) same as the whirling speed is set for the whole flow region. The rotating speeds of the journal and bearing wall are set as \( \Omega - \omega \) and \( -\omega \) relative to the rotating coordinate system, respectively. The clearance is so small relative to the radius that the Reynolds number is very small, so the laminar model was chosen. Computation cases with difference combination of rotational speed and whirling speed are conducted. Here, 2000 Pa total pressure is used as the orifice inlet boundary. Static pressure (0 Pa) is set as the outlet boundary condition. The rotational speed changes from 1000 rpm to 5000 rpm. At each rotational speed, 7 whirling speeds were chosen as the frequency ratios (\( \omega/\Omega \)) are 0, 0.25, 0.50, 0.75, 1.00, 1.25, 1.50.

Figure 2. Computational grids for the sliding bearing.

Table 1. Some parameters of the bearing and oil.

| Item                  | Symbol | value  |
|-----------------------|--------|--------|
| Pad radius [mm]       | R₁     | 20.00  |
| Journal radius [mm]   | R₂     | 19.98  |
| Average gap [mm]      | c      | 0.02   |
| Bearing length [mm]   | L      | 15.00  |
| Oil orifice radius [mm]| r    | 4.00   |
| Eccentricity ratio [mm]| e   | 0.2    |
| Oil density [kg/m³]   | ρ      | 890    |
| Oil viscosity [kg/(m·s)] | ν  | 0.048  |
| Vaporization pressure [Pa] | P_vapor | 7550   |
| Surface Tension Coefficient [N/m] | ξ | 0.035  |
4. Results and analysis

4.1. Influence of the cavitation on the pressure in the oil film

Figure 3 shows the numerical result comparison between with cavitation and without cavitation model when rotational speed $\Omega = 3000$ rpm. In these cases the journal has an eccentricity ratio 0.2 along the direction X. Both the rotation speed and whirling speed rotate around the direction Z using the right-hand rule. Figure 3(a~c) show the results without whirl speed, while (e~f) show that of synchronous whirl. Comparing (a) with (b), the high pressure are formed in the convergence wedge. However, there is obvious zone of negative pressure in the divergence wedge in figure 3(a), while no negative pressure zone is observed in figure 3(b) considering the cavitation model. It can also be seen that the air volume fraction distribution concentrates in the divergence wedge where the local pressure usually falls down along the rotational direction, which explains why no negative pressure zone is observed in the figure 3(b). In figure 3(d), the high pressure zone is located in the divergence wedge while the negative pressure zone is in the convergence wedge because the squeezing effect of the whirl is greater than the rotational effect of the spinning in Synchronous whirl motion. Without cavitation model, high pressure is still located in the divergence wedge and no negative pressure exists as shown in figure 3(e) which can be explained by the air volume fraction distribution in figure 3(f).

![Figure 3. Computational result comparison with cavitation and without cavitation when $\Omega = 3000$ rpm: (a~c) $\omega = 0$ rpm, (e~f) $\omega = 3000$ rpm, (a, d) without cavitation, absolute pressure, (b, c) with cavitation, absolute pressure, (c, f) with cavitation, air volume fraction](image)

Figure 4 shows the influence of the whirl speed on the oil film force in X direction at different rotational speeds. Figure 4(a) shows the results without cavitation while figure 4(b) considers cavitation. Five cases with rotational speeds of 1000~5000 rpm are calculated where 7 frequency ratio ($\omega/\Omega$) of 0.0~1.5. $F_x$ increases almost linearly as the frequency ratio and rotational speed increase when no cavitation model is considered as shown in figure 4(a). In contrast, considering cavitation, $F_x$ shows obvious 2 order nonlinearity as the frequency ratio increases in figure 4(b).

Figure 5 shows the influence of the whirl speed on the oil film force in Y direction at different rotational speeds. Quite difference can also be observed between no cavitation model and cavitation. $F_y$ decreases almost linearly as the rotational speed increases in figure 5(a), while the $F_y$ shows 2 order nonlinearity in figure 5(b).
Figure 4. Oil film force on the journal in x direction with difference frequency ratio: (a) with cavitation (b) no cavitation

Figure 5. Oil film force on the journal in y direction with difference frequency ratio: (a) with cavitation (b) no cavitation

4.2. Influence of the rotational speed on the pressure properties in the oil film

Figure 6 shows the influence of the rotational speed on the pressure distribution in the oil film with no whirl speed, $\omega = 0$ rpm. The high pressure is formed in the convergence wedge. As the rotational speed increases from 1000 to 5000 rpm, the pressure increases are shown in the figure 6(a) ~ (b). Correspondingly, the cavitation area and air volume fraction increase in the divergence wedge with the rotational speed as shown in figure 7. Figure 8 shows the cavitation rate of the oil film. It can be seen that the cavitation rate increase as the rotational speed increases at each frequency rate except the rate 0.75. At each rotational speed, the cavitation rate first decreases, reaches the lowest point at frequency rate 0.5, and then increases as the frequency rate increases. Therefore, half-speed whirl can weaken the cavitation in the oil film.

4.3. Approach for the dynamic characteristics of the journal bearing

According to the equation (14), there are six unknown variables including direct and coupled stiffness, damping and added mass. One whirl speed can provide two equations for Force X and Force Y. So at least three whirl speed ($\omega$) need to be calculated for closing the equation sets. Here seven whirl speeds are used at each rotational speed to improve the calculation accuracy. The $F_x$ and $F_y$ at each set of rotational speed and whirl speed with cavitation and no cavitation are shown in figure 4 and 5.

Based on the equations (14) and the results in figure 4 and 5, the dynamics characteristics of the sliding bearing including stiffness, damping are calculated as shown in figure 9. In figure 9(a), the direct stiffness ($K$) and coupled stiffness ($k$) are nonlinear with the rotational speed. Direct stiffness with cavitation is larger than that without cavitation, while coupled stiffness with cavitation is smaller than that without cavitation. Figure 9(b) shows the damping coefficients of the bearing. The direct and
coupled damping without considering cavitation keep almost constant with the rotational speed, while the damping considering cavitation decreases as the rotational speed increases. The direct damping with cavitation is smaller than that without cavitation, similar to the coupled damping. Therefore the cavitation has great effects on the stiffness and damping.

Figure 6. Influence of rotational speed on the pressure distribution when $\omega = 0$ rpm 
(a) $\Omega = 1000$ rpm (b) $\Omega = 3000$ rpm (c) $\Omega = 5000$ rpm

Figure 7. Influence of rotational speed on the air volume fraction when $\omega = 0$ rpm 
(b) $\Omega = 1000$ rpm (b) $\Omega = 3000$ rpm (c) $\Omega = 5000$ rpm

Figure 8. Cavitation rate of the oil film.

According to the bearing-rotor coupled dynamic theory, the coupled stiffness and direct damping have great influences on the stability of the rotor system. Negative coupled stiffness and positive direct damping contribute to the stability of the rotor system, while positive coupled stiffness and negative direct damping have the opposite impact. However, if both the coupled stiffness and damping are
positive or negative, another method need. There is a stability criterion called whirl ratio ($\gamma$) combining both stiffness and damping effect described as the following formulation:

$$\gamma = \frac{k}{\Omega C}$$  \hspace{1cm} (15)

Where $\gamma$ is the whirl ratio, $k$ is the coupled stiffness, $\Omega$ is the rotational speed, and $C$ is the direct damping.

![Figure 9](image)

**Figure 9.** Dynamic characteristics of the sliding bearing with cavitation and no cavitation. (a) Direct and coupled stiffness (b) Direct and coupled damping.

If both $k$ and $C$ are positive, the rotor will be stable when $\gamma < 1$, and the smaller the $\gamma$ is, the more stable the rotor will be. If both $k$ and $C$ are negative, the rotor will be stable when $\gamma > 1$, and the larger $\gamma$ is, the more stably the rotor will be.

Because the coupled stiffness and direct damping are positive, the stability criterion (15) is used here. As listed in Table 2, the whirl ratio $\gamma$ without cavitation keeps almost constant as the rotational speed increases from 1000 rpm to 5000 rpm, while $\gamma$ with cavitation increases from 0.440 to 0.522. The whirl ratio is less than 1. So the bearing is stable when the rotational speed is less than 5000 rpm. And decreasing the rotational speed contributes to strengthen the stability of the rotor system.

| Rotational speed (rpm) | Stability ($\gamma$) |
|------------------------|----------------------|
|                        | No cavitation | Cavitation |
| 1000                   | 0.499        | 0.440      |
| 2000                   | 0.498        | 0.507      |
| 3000                   | 0.498        | 0.508      |
| 4000                   | 0.497        | 0.520      |
| 5000                   | 0.497        | 0.522      |

5. Conclusions

In this paper, numerical method is adopted to simulate the cavitation in the oil film with homogeneous two-phase mixture flow using Singhal-et-al cavitation model in the commercial code FLUENT-solver. Cases without cavitation model were also calculated at the same time. Some conclusions can be obtained.

1) Comparing with the cases without considering cavitation, the cavitation changes the pressure distribution in the oil film. No negative pressure zone exits in the film because of the film rupture.

2) As the rotational speed or whirling speed of the journal increases, the cavitation will be more obvious. The dynamic coefficients of the bearing such as stiffness and damping with cavitation model
are quite different from that without cavitation. So it is worth to pay attention to and do further study about the cavitation in the sliding bearing in high speed rotor system.

In the presented analysis, there are still some deficiencies in the computation. Because this paper focus on influence of the cavitation on the dynamics of the sliding bearing, so some computation processes are simplified. The rotating coordinate system and relative rotating speed are used to simulate the transient process approximately which is not the real transient computation and loses some transient terms. It is assumed that the oil force is linear with the eccentricity of the journal which is usually nonlinear. In addition, the selection of the cavitation model can also affect the numerical results. So future work should be done to improve the numerical method to obtain more accurate results.

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