Cosmological effects of multivacua theories: gravitational waves from first order transitions during inflation

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Abstract. We study the gravitational waves emitted from the physical processes related to first order phase transitions during inflation. The spectrum shows features that differ both from the case of waves generated via quantum vacuum oscillations of the metric and both from the case of transitions occurring outside inflation. We also study the detectability of the waves.

1. Introduction
Primordial gravitational waves generated during inflation can open a window of opportunities for research, as they decouple quite immediately from their surrounding and carry valuable information about the physics that produced them.

Many important and interesting sources arise when first order phase transitions occur: in this short communication we will deal with the emission, features and detectability of gravitational waves produced by this kind of transitions if they occur during inflation. We will mainly follow the analysis and results of [1].

This scientific investigation has different motivations. Most notably, the discovery of many metastable minima in the effective theories of high-energy physics and quantum gravity [2] raises the interest in studying scenarios where phase transitions occur at early times, such as during inflation. It has also been recently fully investigated a model where inflation is entirely driven by a series of first order phase transitions: chain inflation [3].

Gravitational waves generated by first order transitions have been a field of intense research, documented in the literature, but only for what concerns transitions at reheating or at the electroweak and QCD scales [4].

2. Setup and approach
Our scenario is a period of inflation where some first order phase transitions occur. The underlying theoretical model can be very complicated, with a potential exhibiting many metastable minima, multiple fields and various dynamical phases while the fields pass through the minima. There are different possibilities for inflation: for example it can take place via a mechanism such as chain inflation or also in other ways, for example by slow-roll, depending on the behaviour of the fields of the model.

The first order phase transitions occur via nucleation of bubbles of the new phases within the old ones. The latent heat is released by the collisions of the bubbles and the decays of their
walls into a radiation-dominated fluid. Many sources of gravitational waves appear thanks to this dynamics.

We simplify the description of this setup by describing it through a set of physical parameters. The emission and features of the gravitational waves can also be studied using these parameters and avoiding the complicated field theory description. The analysis has also the advantage of obtaining general results, which can then be adapted to the various specific models by computing the physical parameters from first principles in the model of interest. In this way the model can be tested.

3. Analysis of the setup

We list here the physical parameters we will use in our analysis of wave production. We also investigate and discuss the most important bounds on these parameters, which ensure the success of the transitions, reaching percolation and large scale thermalization, and the efficiency of inflation (small backreaction).

**Background.** For our purposes, we can describe the background evolution with only two parameters: the Hubble parameter $H$ and a parameter $\varepsilon = -\frac{\dot{H}}{H^2}$ yielding its time evolution. We consider a quasi-de Sitter evolution, for which $\varepsilon < 1$.

**Transitions.** As we will see, only a few features of the transitions suffices for studying the wave emission during inflation. The relevant physical parameters are: the nucleation rate per unit time $\Gamma_n$, the transition time-scale $\beta_n^{-1}$, the nucleation radius of bubbles $r_n$, the energy density $\Delta \epsilon_n$ released by the transition.

These parameters are constrained by the requirement of self-consistency of the scenario. In particular, the transitions must be successful in reaching percolation and large scale thermalization without backreacting too heavily on the background. This forces the ratio of the typical transition scale and the Hubble scale to be bounded as

$$10 < \frac{\beta_n}{H} < \left( \frac{\pi^2}{S_E^{(n)}} \right)^{\frac{1}{4}} 10^7. \quad (1)$$

**Radiation fluid.** A radiation component of the Universe necessarily appears as a consequence of the collisions and decay of the bubble walls. Although it is (and must be) a subdominant component compared to the vacuum energy driving inflation, the radiation is in general relevant when discussing perturbations, see for example [3].

In order to be compatible with a FRW description of inflation, at least at certain scales, the fluid must rapidly thermalize\(^{2}\). Its most important physical parameter will then be its temperature $T$. Furthermore, as in general the fluid constituents will be charge carriers, the (gauge) coupling(s) $g$ will be important parameter(s).

The fluid temperature must be bounded if we want backreaction not to stop inflation. In [1], using the value of the spectrum of density perturbations measured by WMAP [7] to relate the Hubble scale to the Planck mass, it was found

$$T < 10^{-2} \kappa^{-\frac{1}{2}} T^{\frac{1}{2}} M_{\text{Planck}}, \quad (2)$$

where $\kappa = \kappa(T)$ counts the number of relativistic degrees of freedom at temperature $T$. It was also found that $\frac{T}{r_n} < 1$, which indicates that bubble nucleation via vacuum tunneling (describable in terms of some order parameter) dominates over thermal nucleation [6].

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1. We allow for more than one transition to occur, therefore indicating each of them with a suffix $n$, $1 \leq n \leq N$, $N$ kept generic.

2. As this depends on the specific features of the model and investigating it would go beyond the scope of our analysis, we assume that thermalization occurs sufficiently rapidly.
4. Gravitational wave solutions
Gravitational waves are solution of the linearized Einstein equation (in a suitable gauge)

\[ h''_ij + 2\mathcal{H}h'_ij - \nabla^2 h_{ij} = 16\pi G a^2 \pi^T_{ij}. \]  

where \( \mathcal{H} = aH \) and \( \pi^T_{ij} \) is the traceless symmetric transverse tensor part of the anisotropy stress tensor, obtained by projection from the total energy-momentum tensor. Here\(^3\), \( h(k, \eta) \) is the mode function of the graviton field \( h(\vec{x}, \eta) \) expanded in eigenfunctions of the Laplace-Beltrami differential operator with eigenvalues \( -k^2 \). The wavenumber \( k \) is related to the physical momentum by \( k = a(t)p \). The ‘( )’ indicates derivative by \( \eta \) (t).

During inflation the solution for a generic source is

\[ h = \left( c_1 + i\frac{\pi}{4} \int_{\eta} h'(\eta')^{1-\nu^T} H^{(2)}_{\nu^T}(-k\eta') S(k, \eta') \right) (-\eta)^{\nu^T} H^{(1)}_{\nu^T}(-\eta) + \left( c_2 - i\frac{\pi}{4} \int_{\eta} h'(\eta')^{1-\nu^T} H^{(1)}_{\nu^T}(-k\eta') S(k, \eta') \right) (-\eta)^{\nu^T} H^{(2)}_{\nu^T}(-\eta), \]  

where \( H^{(1,2)}_{\nu^T} \) are Hankel functions, \( \nu^T = \frac{3}{2} + \varepsilon \sim \frac{3}{2} \) and \( S(k, \eta) = 16\pi G a^2 \pi^T(k, \eta) \).

We will also need the solution during matter or radiation domination for modes within the horizon \( (k > \mathcal{H}) \):

\[ h = \frac{A_+ (k)}{a\sqrt{k}} e^{-ik\eta} + \frac{A_- (k)}{a\sqrt{k}} e^{ik\eta}. \]  

The two quantities that are relevant when discussing the production, features and detectability of gravitational waves are the spectrum

\[ \langle h^*(k, t)h(k', t) \rangle = \delta(k - k') \frac{2\pi^2}{k^3} P_h(k, t) \]  

and the energy density radiated per octave

\[ k^3 \frac{\rho_h(k, t)}{dk} = k^3 \frac{\langle h^*(k, t)h(k', t) \rangle}{8\pi G} = k^3 \frac{\langle h^\nu(k, \eta)h^\nu(k', \eta) \rangle}{8\pi Ga^2}. \]  

5. Sources and emission
In presence of first order phase transitions many different sources of gravitational waves can appear.

5.1. Transitions and bubble collisions
The energy density released by a transition goes partly in the acceleration of the bubble walls and partly into the velocity spectrum of the fluid coming from the collision of bubbles of a preceding transition. The features of the gravitational waves emission are different depending on how the energy is divided into these two channels.

The fraction of energy going into the fluid velocity can be estimated looking for the solutions of the continuity equations for energy and momentum across the bubble wall. It is found that there are no stationary solutions compatible with inflation \([1]\). This means that the energy released by the transition goes predominantly into the acceleration of the wall (near-vacuum description).

The collisions of bubbles are complicated non-equilibrium processes which must be studied with the help of numerical simulations. The best simulations available today are performed in

\(^3\) We will avoid writing polarizations when not strictly necessary, to avoid cluttering of formulas.
the setup of static universe [7]. They show that only two quantities are effectively important for
the spectrum of gravitational waves: the overall released energy and the completion time \( \beta_n^{-1} \)
for the phase transition, which sets the peak frequency of emission. The complicate small scale
dynamics add up incoherently and are sub-dominant.

The emission from phase transitions occurring during inflation exhibits three important
differences: i) the static quality of the background is an approximation valid for scales shorter
than the horizon, ii) the total released energy is
\[
\Delta \epsilon_n \sim -\frac{d\rho}{dt} \beta_n^{-1} \simeq 6H^3M_{\text{Planck}}^2 \beta_n^{-1} \varepsilon ,
\]
smaller by the factor \( 2\frac{H}{\beta_n} \varepsilon \) than the total energy density \( \rho \) dominated by the vacuum component,
iii) the evolution of the waves during inflation is peculiar as modes can exit the horizon and the
wave amplitude freezes (as we can see from (4)).

The spectrum of superhorizon modes at time \( t \) after the emission time \( t_n \) is
\[
P_{h}^{\text{sup}} = \sum_{n=1}^{N} \frac{9}{2 \pi^4} \left( \frac{H}{\beta_n} \right)^6 \varepsilon^2 \chi_n^2 \left( \frac{f_{\beta n}}{p} \right)^5 \begin{cases} \frac{f_{H_n}}{p} & p < f_{\beta n} \\ \frac{f_{H_n}}{p} & p > f_{\beta n} \end{cases}
\]
\[
= \sum_{n=1}^{N} \frac{2}{\pi^2} A_n \left( \frac{H}{M_{\text{Planck}}} \right)^2 \left( \frac{\beta_n}{k} \right)^5 \begin{cases} a(t_n)H & k < \tilde{\beta}_n \\ \frac{H}{\beta_n} & k > \tilde{\beta}_n \end{cases},
\]
where we have taken into account the redshift of the waves between the emission time \( t_n \) and
the time \( t_{ex} \) of exit from horizon for the mode \( k \)
\[
a(t_{ex}) = \chi_n \beta_n \frac{p_n}{\beta_n} \chi_n = \frac{H}{\beta_n} .
\]
Superhorizon modes are the most interesting ones as they are not suppressed after they exit
the horizon. Note how (9) differs both from the spectrum of tensor modes generated by
quantum oscillations of the metric in vacuum (in particular, no scale invariance), and both from
the spectrum sourced by first order transitions occurring outside inflation (different frequency
dependence). Finally, the backreaction of (9) is strong only for a huge number of transitions
\( N \sim 10^{12} \).

5.2. From turbulence, (hyper)magnetic fields, viscosity

There are other sources related to first order phase transitions which could generate a sizable
spectrum of gravitational waves [8]. However, we will show that during inflation they appear to
be very suppressed or even not to occur.

Turbulence

Turbulence can be an important source of waves. However it is possible to prove that it
cannot occur for the range of values of the physical parameters that are in accordance with
inflation (see section 3). Indeed, turbulence is signalled by large Reynold numbers, but from
(1), (2), we find
\[
Re = v f(\beta_n) \beta_n^{-1} \nu \sim v f(\beta_n) \frac{T}{\beta_n} g^4 \log(g^{-1}) \leq 1
\]
where \( v f(\beta_n) \) is the characteristic velocity of the fluid flow at the injection scale, which is the
bubble size at collision \( \sim \beta_n^{-1} \).

4 With the attribute hyper we denote generically the magnetic fields associated with some U(1) gauge symmetry
at high energy, here sourced by the phase transitions.
Plasma physics and gauge fields

In the dynamical range interesting for our study, the relativistic fluid of charge carriers can be described as a plasma. Gravitational waves could be efficiently sourced by hypermagnetic fields generated by the local currents arising because of the bubble dynamics and collisions. The hypermagnetic fields are generated at the typical scale of the bubbles at collision and then amplified by hypermagnetic turbulence. However, the latter does not occur when the physical parameters lie within the bounds we have found to be consistent with inflation (see section 3). Indeed, the magnetic Reynold number is

\[ Re_\mu \sim v_f (\beta_n) \frac{T}{\beta_n g^2 \log(g^{-1})}, \]  

which, even for reasonably small couplings \( g \sim 0.1 \), is much less than 100.

Fluid viscosity

The anisotropic stress tensor generated by viscosity of a fluid with velocity \( \bar{u} \) is \[ \pi_{ij}^{(\text{visc})} = -\varsigma \left( \partial_j u_i^{(f)} + \partial_i u_j^{(f)} - \frac{2}{3} \nabla \cdot \bar{u}^{(f)} \delta_{ij} \right) - \zeta \nabla \cdot \bar{u}^{(f)} \delta_{ij}. \] (14)

The bulk viscosity \( \zeta \) is vanishing for a relativistic fluid. The shear viscosity \( \varsigma \) could instead be large, as it is given by \( \varsigma = \zeta_0 \frac{T^3}{\alpha^2 \log(g^{-1})} \) for a weakly coupled fluid [10].

One can however calculate \( \Omega^{(\text{visc})}/\Omega^{(\text{coll})} \), the energy density radiated per octave normalized to the total energy density, at the time of emission, by using (7) and normalizing it. However, for scales compatible with the plasma description \( (k > \mathcal{H}) \), one finds that this radiated energy is much smaller than the one emitted by the collision of bubbles at the same (hydrodynamical) scales:

\[ \frac{\Omega^{(\text{visc})}}{\Omega^{(\text{coll})}} \lesssim g^4 \log(g^{-1}) \nabla^2 k = \lambda_y^{-1} \ll 1. \] (15)

6. Detection

We will discuss the possible detection of the gravitational waves via the temperature anisotropy of the CMBR or directly by interferometers.

6.1. CMBR

We choose to concentrate on the temperature anisotropies and not the polarization, although both are affected by gravitational waves emission. In particular, we focus our discussion on the scalar-to-tensor ratio \( r \).

For illustrative reasons, we now make the simplifying assumption that \( \frac{\beta_n}{P(t_n)} \) does not depend too much on \( n \), and we consider only multipoles \( \ell_k = \frac{2k}{P(t_n)} \) for wavenumbers \( k_\beta = a_n \beta_n \) and \( k_H \sim a(t_n)H \), as they are respectively the typical comoving scale of the transitions and the Hubble scale. Using (9), the bounds in section 3 and the typical scalar density spectrum, we obtain

\[ c_s 10^{-4} N \varepsilon^2 \left( \frac{S_E}{\pi^2} \right)^\frac{2}{5} \frac{\ell_\beta}{\ell_*} \leq r \leq c_s 10 \frac{9}{4 \pi^2} N \varepsilon^2 \frac{\ell_\beta}{\ell_*}. \] (16)

A value of \( r \sim 0.07 \) is detectable. We see that \( N \sim 3 \cdot 10 \) transitions at the lowest allowed rate \( \frac{\beta_n}{P(t)} \sim 10 \) would leave detectable marks at multipoles \( \sim \ell_{k_H} \). The signal however rapidly weakens for higher multipoles and/or faster transitions.
6.2. Interferometers

To discuss direct detection of these primordial waves by interferometers such as LIGO, LISA, DECIGO, we need to evolve them after horizon re-entering. Frequencies falling within the sensitivity of those three interferometers must have been within the horizon before matter-radiation equality and nucleosynthesis. From (5) we see that \( h \) evolves as \( \sim a^{-1} \) and therefore we can compute the strain produced by the waves and compare it with the sensitivity of the interferometers at the relevant frequencies.

The detailed results in [1] show that the detectability of the emitted waves depends on the number of transitions, their timescales and the number of e-foldings that must have occurred to sufficiently redshift their frequencies after the waves were emitted.

To give an example, we can specialize to the case of chain inflation, which is characterized by fast transitions. One can find that in the case of many transitions (\( N \gg \beta H_{\text{infl}} \)), where \( H_{\text{infl}} \) is the Hubble parameter during inflation, the waves would be detectable at DECIGO for all allowed values of \( \frac{\beta}{H_{\text{infl}}} \), detectable at LISA for \( \beta H_{\text{infl}} < 11 \), but not detectable at LIGO.

Note that waves that re-enter the horizon before nucleosynthesis must not interfere with the latter. There is therefore a constraint on the wave amplitude (or equivalently the strain) [11]. In [1] it was shown that the constraint is easily satisfied by the waves we discuss, thanks to the bounds in section 3.

7. Conclusion

We have studied the emission and features of gravitational waves from first order phase transitions during inflation. On the positive side, we have shown that the spectrum of waves presents features that are very different from those of the waves generated by vacuum quantum oscillations of the metric or by transitions occurring outside inflation, allowing to clearly distinguish them. On the negative side, the intensity of the sources is highly constrained by the requests of small backreaction, homogeneity and isotropy at large scales, and this makes the amplitude of the spectrum of a single transition quite suppressed.

However, the accumulation due to a moderate number of slow but successful transitions (which could occur in settings such as the string landscape) would leave a detectable imprint both in the CMBR and at interferometers. For faster transitions, the signal rapidly weakens.

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