Baryogenesis by Brane-Collision

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We present a new scenario for baryogenesis in the context of heterotic brane-world models. The baryon asymmetry of the universe is generated at a small-instanton phase transition which is initiated by a moving brane colliding with the observable boundary. We demonstrate, in the context of a simple model, that reasonable values for the baryon asymmetry can be obtained. As a byproduct we find a new class of moving-brane cosmological solutions in the presence of a perfect fluid.

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I. INTRODUCTION

An important feature of brane-world models which has attracted some attention [1] – [8] is the possibility of branes moving in the course of the cosmological evolution. In this paper, we would like to propose and analyse a new mechanism for creating the baryon asymmetry in the universe based on moving-brane cosmology.

We will be working in the context of heterotic M-theory [9–11] which constitutes the strong-coupling limit of $E_8 \times E_8$ heterotic string theory. More precisely, we will be interested in the five-dimensional heterotic brane-world models obtained by compactification on a Calabi-Yau three-fold [12,13]. In these models, five-dimensional space-time is bounded by two (3+1)-dimensional boundary planes, carrying the observable and the hidden sector, respectively. Additionally, these models may contain “bulk” three-branes, originating from M-theory five-branes, which can move along the fifth, transverse direction [11,14]. It is the motion of these bulk three-branes which we would like to use for our baryogenesis scenario. Another crucial ingredient is the small-instanton transition [15,16] which occurs when the bulk three-brane collides with one of the boundary planes. It then gets “absorbed” by the boundary plane and, at the same time, the properties of the four-dimensional theory on the affected plane is changed. In particular, the gauge group and/or the number of families can change due to the collision [17].

These small-instanton transitions constitute a type of phase transition with a property which is qualitatively new to cosmology. Normally the temperature of the gas of particles in the universe when a phase transition occurs is given by the mass scale associated with that transition. This is not the case for these brane collisions. The time at which the brane collides with an orbifold fixed point depends on kinematical factors relevant to the brane such as how fast it is moving and its initial position. However if the brane were to change the gauge group on the orbifold fixed point it hits into a smaller group then some of the gauge bosons would become heavy during the collision. The mass that these bosons would gain would be determined by the Calabi-Yau size - typically of order the GUT scale - not the temperature of the gas of particles on that fixed point at the time of collision. There are clearly many possible new cosmological scenarios that could be developed using such qualitatively new behavior. In this paper we shall restrict ourselves to giving one example of an exploitation of this phenomenon in order to give a detailed and focused analysis. We shall use this effect to develop a new scenario of baryogenesis.

Roughly, our mechanism for baryogenesis is then as follows. We start with a state in the early universe where the expansion is driven by a gas residing on the observable boundary plane and the kinetic energy of the brane which moves towards the observable brane. At this stage, the quasi massless spectrum on the observable brane is given by an $N = 1$ gauge theory with group $G_{\text{SM}} \times U(1)_B - L$, where $G_{\text{SM}} = SU(3) \times SU(2) \times U(1)$, three MSSM families of quarks and leptons plus three right-handed neutrinos (RHNs) and their scalar partners. All these particles are in relativistic equilibrium. When, eventually, the three-brane collides with the observable boundary, the gauge group is changed to $G_{\text{SM}}$ due to the small-instanton transition and, as a consequence, the RHNs become super-heavy. Their out-of-equilibrium decay then generates a lepton asymmetry which, via electroweak sphaleron processes is converted into a baryon asymmetry in the conventional way. Our mechanism is, in some ways, similar to a standard leptogenesis scenario [18]– [28]. However, instead of a GUT phase transition we are using a small-instanton phase transition, a genuine string effect. As we will see later, there are a number of other important differences, including a dependence of the baryon asymmetry on the parameters of the small-instanton transition and the decoupling of the transition temperature from the RHN mass.

The outline of the paper is as follows. In the next section, we will explain our scenario in detail but on an
informal level. In section three, an explicit realisation of this scenario in terms of a simple model is presented. The quantitative predictions of this model for the baryon asymmetry are analysed in section four and five. Section six contains a summary of our main results and an outlook. Finally, in the appendix, we present a new class of moving-brane cosmological solutions in the presence of a perfect fluid which are relevant to our baryogenesis scenario and, possibly, to a number of other applications, such as the inflationary scenario of Ref. [1].

II. THE SCENARIO

Before we explicitly describe our scenario let us briefly explain the theoretical framework which we will be using. Throughout this paper, we will be working in the context of Hořava-Witten theory [9–11], that is, M-theory on the orbifold $S^1/Z_2$. The low-energy limit of this theory is described by 11-dimensional supergravity coupled to two $E_8$ super-Yang-Mills multiplets each residing on one of the 10-dimensional orbifold fixed planes. More specifically, we will be dealing with the five-dimensional brane-world models that can be obtained by compactifying this theory on Calabi-Yau three-folds [12,13]. These models are described by gauged five-dimensional $N = 1$ supergravity theories in the bulk coupled to $N = 1$ gauge theories located on the now four-dimensional orbifold planes. As usual, we will interpret one of these orbifold planes as the observable sector and the other one as the hidden sector.

In addition, M-theory five-branes can be included in the compactification from 11 to five dimensions [11,14]. They wrap a two-dimensional curve in the Calabi-Yau space, stretch across the four uncompactified dimensions and are parallel to the orbifold planes. Hence, they appear as three-branes in the five-dimensional brane-world theory which are located somewhere in the bulk between the two orbifold planes. Each of these three-branes carries an additional $N = 1$ supersymmetric theory. A crucial feature for our purpose is that these three-branes are not fixed but, rather, can move along the orbifold direction.

The specific form of the $N = 1$ theory on, say, the observable orbifold plane is determined by the details of the compactification, that is, by the choice of Calabi-Yau manifold and internal vector bundle. The internal vector bundle can be thought of as instantons on the Calabi-Yau space which serve to break the $E_8$ gauge group to a phenomenologically more favorable subgroup. For different such instanton configurations one generally obtains different gauge groups and different sets of matter fields on the orbifold plane. It has been shown that phenomenologically interesting low-energy theories can be obtained in this way [29]–[35]. The dependence of the low-energy spectrum on the internal instanton configuration will be of particular importance for us.

A final theoretical ingredient which we need to discuss is the small-instanton transition [15–17]. This process occurs when one of the three-branes in the five-dimensional brane-world model collides with an orbifold fixed plane. From a five-dimensional viewpoint, this three-brane is then “absorbed” by the orbifold plane and disappears from the brane-world theory. For a more microscopical picture we recall that the three-brane originates from an M 5-brane which wraps a curve in the Calabi-Yau space and, hence, carries some internal structure. As the brane collides with the orbifold plane, this structure is being converted into an $E_8$ gauge instanton on the Calabi-Yau space. In other words, the internal instanton configuration, associated with the orbifold plane in question, is changed in such a collision. In accordance with the above discussion, this generally also implies that the low-energy gauge-group or the matter-field content on the affected orbifold plane is altered [17].

We are now ready to discuss our baryogenesis scenario. We will be working in the context of the five-dimensional brane-world model described above, where we consider a single three-brane in the bulk, for simplicity. Let us consider a period in the early universe after inflation where this three-brane moves along the orbifold. We consider
the existence of such a period a quasi-generic feature of our brane-world model. More precisely, we will analyse
the cosmological evolution of the model starting out from some initial configuration, specified at time $t_i$. At this
time, we assume the brane to be located at a specific point in the orbifold direction, possibly close to the hidden
orbifold plane, and having a certain initial velocity pointing towards the observable orbifold plane. In addition,
we assume that the energy density on the observable orbifold plane is dominated by a gas with temperature $T_i$
while there is no significant contribution to the energy density from the hidden orbifold plane. One may, for
example, think of this initial state as the result of reheating after inflation. At this stage, the observable orbifold
plane carries an $N = 1$ gauge theory with gauge group $G$ and a certain number of chiral matter fields. We assume
that this gauge group contains the standard model group $G_{SM} = SU(3) \times SU(2) \times U(1)_Y$, that is, $G_{SM} \subset G$.
For concreteness, let us, in the following, discuss the "minimal choice" $G = G_{SM} \times U(1)_{B-L}$, where $B$ and $L$
are baryon and lepton number, respectively. For simplicity, we also assume three standard model families plus
the same number of right-handed neutrinos (RHNs) and their scalar partners. However, our mechanism will
work for other groups $G$, such as grand unified groups, and a more general matter field content as well and the
subsequent discussion can be easily modified accordingly. All particles on the orbifold plane are exactly massless
perturbatively and it is plausible to assume that masses generated by non-perturbative effects will not exceed the
electroweak scale. We further assume that the temperature $T_i$ is much higher than that so that the gas on the
observable plane consists of all available particles each being in relativistic equilibrium. We have, schematically,
depicted this initial state at the top of Fig. 1.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig1.png}
\caption{Shown are the three main stages of our scenario. The three-brane starts its evolution at some initial time
$t_i$ when the observable plane carries a gas with temperature $T_i$ consisting of all degrees of freedom of an $N = 1$
gauge theory with group $G \subset G_{SM}$ plus matter fields (top). Shortly before the three-brane collides with the observable plane the
gas has cooled to a temperature $T_c$ (middle). After the collision, the theory on the observable plane has been changed to
the supersymmetric standard model due to the small-instanton phase transition. We now have a gas of standard model
particles with temperature $T_0$ plus right-handed neutrinos which were massless in the original $G$ theory and have now become
super-heavy (bottom). Their decay generates the lepton asymmetry.}
\end{figure}
Starting from this initial state, the three-brane moves towards the observable plane while the gas on this plane cools until it reaches the temperature $T_c$ shortly before collision. This is shown in the middle of fig. 1. In the next section, we will describe this evolution using the four-dimensional effective action associated with our brane-world model. As we will see [8], the three-brane motion necessarily implies a time evolution of the orbifold size as well as an evolution of the Calabi-Yau volume. We, therefore, have to consider three scalar fields contributing to the kinetic energy during this epoch.

When the three-brane finally collides with the observable plane, the theory on this plane is changed. We assume that the initial $N = 1$ gauge theory with group $G \subset G_{SM}$ plus matter fields is converted precisely into the supersymmetric standard model (MSSM) by the small-instanton transition. Later on, we will present explicit arguments that this can indeed be achieved for our concrete example $G = G_{SM} \times U(1)_{B-L}$. As a consequence, the right-handed (s)neutrinos, previously effectively massless, now become super-heavy with masses $M_i$, where $i$ is a generation index. What does this imply for the evolution of the gas on the observable plane during the transition? Before the collision, all MSSM particles are in thermal equilibrium at temperature $T_c$. Further, due to their $U(1)_{B-L}$ gauge interactions, the RHNs and their scalar partners are also in relativistic equilibrium [23]. After the collision, we have a gas of standard model particles in relativistic equilibrium at temperature $T_0$ which we assume to be much larger than the electroweak scale. It is plausible that a substantial number of now super-heavy RHNs are still present after the collision. In particular, if the characteristic time-scale of the collision is much shorter than the RHN decay time, as we will assume later on, one expects the number of RHNs to be basically conserved during the transition. Their number density after the collision is then given by the relativistic equilibrium distribution before collision. The decay of those super-heavy RHNs then creates a lepton asymmetry. Since the temperature $T_0$ is much higher than the electroweak scale, sphaleron processes can then be invoked as usual to convert this into a baryon asymmetry.

III. THE MODEL

We would now like to be more explicit and realize our scenario in terms of a simple model which will allow us to quantitatively estimate the generated baryon asymmetry.

First, we need to describe the three-brane motion across the orbifold which we will do in terms of the relevant four-dimensional $N = 1$ effective action. The minimal version of this action contains three moduli fields, namely the dilaton $S$, the universal $T$-modulus and the field $Z$ related to the position of the three-brane. The Kähler potential for these fields is given by [36,37]

$$K = -\ln \left( S + \bar{S} - q_5 \frac{(Z + \bar{Z})^2}{T + \bar{T}} \right) - 3 \ln (T + \bar{T}) ,$$

where $q_5$ is a constant. In terms of the underlying component fields, these superfields can be written as [37]

$$S = e^\phi + q_5 z^2 e^\beta - 2i(\sigma - q_5 z^2 \chi)$$

$$T = e^\beta + 2i\chi$$

$$Z = e^\beta z - 2i(\zeta - z\chi).$$

Here the three real scalars $\sigma$, $\chi$ and $\zeta$ are axionic fields, which can be set to zero consistently. We will do this, for simplicity, and work with the three remaining real scalar fields $\phi$, $\beta$ and $z$. From the above expression for the Kähler potential, the action for these component fields is given by
\[
S = -\frac{1}{2\kappa_p^2} \int \sqrt{-g} \left[ \frac{1}{2} R + \frac{1}{4} \partial_\mu \phi \partial^\mu \phi + \frac{3}{4} \partial_\mu \beta \partial^\mu \beta + \frac{q_5}{2} e^{\beta - \phi} \partial_\mu z \partial^\mu z \right].
\]  

The interpretation of these scalar fields is as follows. The size of the Calabi-Yau space and the orbifold are proportional to \(e^\phi\) and \(e^\beta\), respectively, while the position of the three-brane is given by \(z \in [0, 1]\) where \(z = 0\) corresponds to the observable (say) orbifold plane and \(z = 1\) to the hidden one. Perturbatively, these fields represent flat directions but at non-perturbative level a potential may have to be added to the above action. For simplicity, we will not consider such a potential explicitly which amounts to assuming that the energy density in the gas and the kinetic energy dominate the potential energy. As mentioned earlier, a moving three-brane necessarily implies time-evolution of the fields \(\phi\) and \(\beta\), as can be seen from the kinetic term of the \(z\) field in Eq. (3).

The cosmological solutions to the action (3) with a moving three-brane but without a gas have been found in Ref. [8]. Here, we will need the generalisation of those solutions to include the stress energy due to a gas with pressure \(p_{\text{gas}} = \frac{\rho_{\text{gas}}}{3}\) located on the observable brane at \(z = 0\). Remarkably, these solutions can be found exactly even for the more general equation of state \(p_{\text{fluid}} = w p_{\text{fluid}}\) where \(w\) is a constant. They are explicitly given in Appendix A. We stress that for our application to baryogenesis we will be using the positive-time branch of those solutions. Hence, unlike the inflationary scenario of Ref. [2] and pre-big-bang cosmologies in general, our model has no exit problem. What we need for our application is not so much the detailed form of the solution but, rather, the relation between the initial data, provided at time \(t_i\) and the data before collision at time \(t_c\). Let us define the ratio

\[
r(T) = \frac{\rho_{\text{kin}}}{\rho_{\text{gas}}}
\]

of the total kinetic energy of the three scalar fields and the energy density of the gas. In terms of our explicit model, both quantities are defined in Eq. (A25) and (A26). As we will see, for our simple description of the collision later on, the quantity \(r_c \equiv r(T_c)\) and the temperature \(T_c\) is all we really need to know as inputs right before the collision. The question is then how these quantities depend on the initial data and whether they are constrained in any way. Clearly, for all our solutions the ratio \(r(T)\) scales as

\[
\frac{T_i}{T_c} = \left[ \frac{r_i}{r_c} \right]^{1/2}
\]

where \(r_i \equiv r(T_i)\). This relation, of course, simply reflects the standard scaling properties of radiation and kinetic energy. Of course, there is no a priori information about the initial data although there may be plausible assumptions about their nature. However, these data are constrained by requirements to be imposed at time \(t_c\) before collision. First of all, we need to pick a solution where, at time \(t_c\), the three-brane indeed collides with the observable plane at \(z = 0\). In addition, we may require that the size of the Calabi-Yau space and the orbifold at time \(t_c\) are in the appropriate range for gauge unification in the sense of Ref. [11], thereby imposing constraints on \(\phi(t_c)\) and \(\beta(t_c)\). One can demonstrate from the explicit form of the solutions in App. A that all these constrains can be satisfied simultaneously by choosing appropriate initial conditions and that, by doing so,

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1 An example is to assume that \(r_i \sim 1\), so that kinetic and radiation energy density are of the same order, initially. Then, from Eq. (5), we have \(r_c \ll 1\) for only a moderate decrease in temperature and, hence, a case with small brane impact.

2 For a fully realistic model, one would have to include non-perturbative stabilising potentials for these fields.
no further constraints on $T_i$, $r_i$, $T_c$ and $r_c$ other than Eq. (5) are imposed. We will therefore use Eq. (5) as the single relation to link the initial state with the state before collision.

We should now discuss the brane collision. First, how do we realize the required transition of the gauge group from $G = \text{GSM} \times \text{UB}_L(1)$ to $\text{GSM}$? To obtain $G$ we need an internal bundle with structure group $\text{SU}(4) \times \text{Z}_n$ where $\text{Z}_n$ corresponds to a Wilson line. The $\text{SU}(4)$ part serves to break the original $\text{Es}$ to $\text{SO}(10)$ and the $\text{Z}_n$ Wilson line can be chosen to break $\text{SO}(10)$ precisely into $G$. To realize $\text{GSM}$ the required bundle structure is $\text{SU}(5) \times \text{Z}_m$ with an appropriate $\text{Z}_m$ Wilson line. From a purely group-theoretical viewpoint, the $\text{SU}(5)$-breaking Wilson line $\text{Z}_m$ can be chosen as the intersection of the $\text{SO}(10)$-breaking Wilson line $\text{Z}_n$ with $\text{SU}(5)$. This suggests that the Wilson lines can be viewed as “spectators” and that the required transition of the internal bundle is basically $\text{SU}(4) \rightarrow \text{SU}(5)$. Such transitions can indeed be obtained for suitable compactifications and explicit example have been given in Ref. [17].

Our main task is now to determine the basic initial conditions for leptogenesis, that is the initial number density of RHNs and the temperature at the beginning of leptogenesis, in terms of the parameters of our model. We have three generations of matter fields, in particular three RHNs with associated heavy mass scales $M_i$, where $i = 1, 2, 3$. For simplicity, in this and the following section, we will discuss the single-family case focusing on the first generation with corresponding RHN mass $M = M_1$. It should be noted that this mass is the mass of the RHN after the brane collision. Our results are easily generalised and all three flavours will be included in the numerical simulation, later on. Unfortunately, a detailed microscopical understanding of the dynamics of the small-instanton transition is well beyond present knowledge and we will not attempt to improve on this in the present paper. Instead we will rely on a “phenomenological” description mainly based on three simple assumptions in order to analyse our scenario. First of all, we assume continuity of the scale factor and its derivative across the transition. This allows us to match the total energy densities before and after. Secondly, we assume that the energy density after the collision is dominated by the gas of standard model particles and super-heavy RHNs. This asserts, among other things, that the scalars $\phi$ and $\beta$ do not carry significant kinetic energy after the collision. While a generalisation to include scalar field evolution after collision may be feasible, we would like to focus on the rather simpler case here. With these two assumptions, the energy density matching simply reads

$$\rho_{\text{gas}}(T_c) + \rho_{\text{kin}}(T_c) = \rho_{\text{gas}}(T_0) + \rho_{\text{N}}(T_0), \quad (6)$$

where $\rho_{\text{N}}$ is the energy density in RHNs and their scalar partners. An additional constraint is given on this energy density matching by the second law of thermodynamics. This means in the present situation that we should not allow thermal energy to be converted into some more ordered form. Thus we impose the constraint $T_0 > T_c$. Intuitively we would not expect to see situations where the energy from the five-brane position modulus is converted with perfect efficiency into RHN’s; we would expect $T_0$ to be at least a bit larger than $T_c$. However it takes very little extra effort to consider cases where the two temperatures are practically the same and so we shall not unnecessarily restrict the possibilities which we consider in our subsequent analysis. Our third and final assumption is that the transition time is much shorter than the decay time of the RHNs and, in fact, electroweak interaction rates. This implies that number densities of all particles in the initial gas are essentially unchanged across the transition. If the brane impact is sufficiently large, so that a significant amount of energy is transferred to the gas, one expects the gas to be out of equilibrium after the transition. Given our ignorance about the

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3The same is required for the bundle modulus which corresponds to the size of the small instanton.
details of the transition, such a situation will be hard to describe quantitatively. We, therefore, require that equilibrium is restored on a time-scale much shorter than the RHN decay time. We will identify the third stage of our scenario, corresponding to temperature $T_0$, with this particular time when equilibrium has been restored after the transition. Let us now analyse the conditions for such a swift thermalisation. The typical ratio of RHN decay rate $\Gamma_N$ and electroweak interaction rate $\Gamma_I$ is given by

$$\frac{\Gamma_N}{\Gamma_I} \sim \frac{|h_\nu|^2 M}{\alpha_{EW} T_0},$$

(7)

where $h_\nu$ is the RHN Yukawa coupling and $\alpha_{EW}$ is a typical standard model coupling. We should now distinguish the two cases $T_0 \gg M$ and $T_0 \ll M$. In the former case the ratio (7) is suppressed by $M/T_0$ and all particles including the RHNs get into equilibrium well before the RHNs decay. Hence, the RHNs are in relativistic equilibrium at temperature $T_0$. This determines the initial conditions for leptogenesis in the case $T_0 \gg M$ which are similar to the ones in the standard leptogenesis scenario. In particular, due to the thermalisation of RHNs after the collision, the initial number density of RHNs does not depend on the parameters of the small-instanton transition.

On the other hand, if $T_0 \ll M$, the RHNs are non-relativistic after collision and will not return to equilibrium. In order for the gas of MSSM particles to thermalise before the RHNs decay we should require, from Eq. (7), that $|h_\nu|^2 < \alpha_{EW} T_0$. This puts a mild constraint on the RHN Yukawa coupling which we will assume to be satisfied in the following. Then, the RHN number density at $T_0$ is given by its equilibrium value before collision, that is, by

$$n_N(T_0) = n_N(T_c) = \frac{3\zeta(3)}{4\pi^2} g_N T_c^3,$$

(8)

where $g_N$ is the number of degrees of freedom in the RHN supermultiplet. Further, we can use the standard expression

$$\rho_{gas}(T) = \frac{\pi^2}{30} g_*(T) T^4$$

(9)

for the energy density of a gas at temperature $T$. Applying the energy matching condition (6) to relate the temperatures $T_c$ and $T_0$ before and after collision one finds

$$\left(\frac{T_0}{T_c}\right)^4 = \left(1 + r_c - \delta \frac{M}{T_c}\right).$$

(10)

where we have defined the constant

$$\delta = \frac{45\zeta(3)g_N}{2\pi^4 g_*(T_c)} \approx 10^{-2}.$$

(11)

Eq. (10) represents the crucial matching condition for the case $T_0 \ll M$. The inequality $T_0 \ll M$, expressed in terms of the initial data at temperature $T_c$, takes the form

$$\frac{M}{T_c} \gg (1 + r_c)^{1/4}.$$

(12)

We recall, that we have assumed, for simplicity, that the number of families is unchanged by the transition and, hence, $g_*(T_c) \simeq g_*(T_0)$. If we had allowed for a change in the number of generations, the right-hand side of Eq. (10) would have to be multiplied by $g_*(T_c)/g_*(T_0)$. 

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4We recall, that we have assumed, for simplicity, that the number of families is unchanged by the transition and, hence, $g_*(T_c) \simeq g_*(T_0)$. If we had allowed for a change in the number of generations, the right-hand side of Eq. (10) would have to be multiplied by $g_*(T_c)/g_*(T_0)$.
The interpretation of the right-hand side of Eq. (10) in terms of an energy balance is straightforward. The first two terms represent the positive contributions from the energy density stored in the gas before collision and the kinetic energy of the scalar fields. Accordingly, the cases \( r_c \ll 1 \) and \( r_c \gg 1 \) correspond to a collision with small and large impact respectively. The third term is due to the RHNs becoming massive and, hence, it contributes with a negative sign. Of course, we have to ensure that the energy density before collision is sufficiently large to account for the masses of the RHNs given the restriction on energy density redistribution imposed by the second law. In other words the right-hand side of Eq. (10) must be greater than 1 \(^5\). An even more stringent constraint results in some cases from the fact that there should be enough energy so that the temperature after collision, \( T_0 \), exceeds a certain minimal temperature even if \( T_c \) does not. Since we would like sphaleron effects to convert the lepton asymmetry into a baryon asymmetry this minimal temperature should correspond to the electroweak scale \( T_{EW} \). This leads to the following constraint

\[
\left( \frac{T_{EW}}{T_c} \right)^4 < \left( 1 + r_c - \frac{\delta M}{T_c} \right).
\]

Depending on the parameters, after the collision, either the gas or the massive RHNs may dominate the energy density. It turns out that for

\[
T_c > M \left( \frac{2\delta}{1 + r_c} \right),
\]

the universe is radiation dominated after collision, that is \( \rho_{N}(T_0) < \rho_{gas}(T_0) \), and matter dominated otherwise. For the case \( T_0 \ll M \), we are now ready to express two crucial input quantities for leptogenesis in terms of the parameters of our model. These are the temperature \( T_0 \) at which the RHNs start to decay and the number of RHNs per entropy density \( Y_N(T_0) \). We find

\[
T_0 = \left[ \left( 1 + r_c - \frac{\delta M}{T_c} \right) T_c \right]^{1/4}, \quad Y_N(T_0) = \frac{3}{4} \delta \left( \frac{T_c}{T_0} \right)^3 = \frac{3}{4} \delta \left[ 1 + r_c - \frac{\delta M}{T_c} \right]^{-3/4},
\]

where we recall from Eq. (5) that

\[
r_c = \frac{T_i^2}{T_i^2 - r_i}.
\]

In the following section, we will use these input values for a simple analytic estimate of the baryon asymmetry if \( T_0 \ll M \). Such an estimate is possible since at these low temperatures lepton number violating scattering processes mediated by the RHNs are inoperative. For \( T_0 \sim M \) or \( T_0 \gg M \), on the other hand, the wash-out due to scattering can be significant and a quantitative description requires the numerical integration of the full set of Boltzmann equations. This will be discussed in further detail in section five.

We remark again that we are assuming the transition time to be shorter than the decay time of the RHNs. This excludes the possibility of significantly decreasing the number density of the RHNs across the transition. Nevertheless, it could still be possible to produce RHNs through parametric resonance, due to a non adiabatic change in their mass \([38]\). Particle production then takes place when the adiabaticity condition is violated and

\(^5\) If this condition is violated we would expect that our assumption of a fast phase transition would be invalid.
$M^2 \ll |\dot{M}|$. Typically, this happens over a short period of time $\delta t$, when the field dependent mass of the RHNs is very small. In our scenario this would occur at the beginning of the transition, when the mass is changing from zero to non-zero values; we do not expect further particle production afterwards. The number density of particles produced can be estimated as

$$n_N \sim \frac{(\dot{M}(t_*))^{3/2}}{8\pi^3}. \quad (18)$$

Comparing Eq. (18) with Eq. (8), our assumption that the number density for the RHNs remains unchanged through the transition implies that

$$\dot{M}(t_*) < \rho_{\text{gas}}(T_c)^{1/2}. \quad (19)$$

To evaluate this condition, we need information about the time variation $\dot{M}(t_*)$ of the RHN mass during the collision. Obviously, precise statements can only be made on the basis of a microscopic description of the transition. For some more qualitative information, however, we note that the mass of the RHNs is correlated with the size modulus of the small instanton. Hence, one expects $\dot{M}(t_*)$ to be small for small $\dot{z}(t_*)$. For small impact parameter, $r_c < 1$, the parametric resonance will, therefore, not significantly change the RHN number density. For large impact parameter, $r_c > 1$, we would need to know the precise relation between $M(t)$ and $z(t)$, that is, the details of the transition, to estimate the number density of the particles produced. With these details being unavailable, we cannot exclude the possibility of additional RHN production for large impact parameters. From a conservative viewpoint, our subsequent results for the baryon asymmetry at large brane impact should, therefore, be interpreted as lower bounds.

**IV. ESTIMATE OF THE BARYON ASYMMETRY**

In this section we focus on the case $T_0 < M$. The lepton asymmetry is generated in the decays of the RHNs due to the CP asymmetry $\epsilon_i$ in the decay into (s)leptons and anti-(s)leptons which arises due to interference between tree-level and one-loop diagrams [24]:

$$\epsilon_i = -\frac{1}{8\pi(h_\nu h_\nu^\dagger)_11} \sum_j \left( \text{Im} \left[ (h_\nu h_\nu^\dagger)_{ij} \right] \right)^2 f(M_j^2/M_i^2), \quad (20)$$

where $h_\nu$ are the Yukawa couplings and

$$f(x) = \sqrt{x} \left[ \log \left( \frac{1+x}{x} \right) + \frac{2}{x-1} \right]. \quad (21)$$

The second term in Eq. (21) originates from the one-loop self-energy, which can only be reliably calculated in perturbation theory for sufficiently large mass splittings [25]. In the case of small mass splittings this contribution could be enhanced [40]. However, since Eq. (21) has been calculated in standard zero temperature perturbation theory, the influence of thermal effects on the self-energy term is unclear. For simplicity we will therefore assume a hierarchy of the form $M_1 \ll M_2, M_3$ 6. Of course, all three generations of RHNs could still contribute to the

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6In principle, all masses, or at least Yukawa couplings, should be computable in a given M-theory model. While it would be interesting to analyse this in more detail in relation to our scenario, here we will simply treat those parameters as phenomenological quantities.
lepton asymmetry. However, if one assumes such a hierarchy there should not be strong cancellations between the different contributions. As in the previous section, we will focus on the first family, setting $\epsilon_{CP} = \epsilon_1$. Our result can then be easily generalised to apply to all flavours $i$ with $M_i/T > 1$. Eq. (21) also gives the CP asymmetry in the decays of scalar right-handed neutrinos into (s)leptons and anti-(s)leptons. At temperatures far above the electroweak scale, where SUSY breaking effects can be neglected, they will give the same contribution to the lepton asymmetry as the RHNs [26]. For simplicity, we will only mention RHNs in the following, but the contributions from their scalar partners will be included in our results.

$B + L$ violating sphaleron processes, which are in equilibrium for temperatures between $O(10^{12})$ GeV and $T_{EW}$, will partially convert the produced lepton asymmetry $Y_L$ into a baryon asymmetry $Y_B$, giving rise to the following relation between baryon and lepton asymmetries:

$$Y_B = -\frac{8}{23} Y_L .$$  

The observed value for the baryon asymmetry is given by $Y_B \sim 10^{-10}$.

In the case $M > T_0$, considered in this section, decay processes dominate over scattering processes. Hence, the scenario for leptogenesis simplifies to that of the out-of-equilibrium decay of a massive, non-relativistic species (RHNs) into light degrees of freedom. The evolution of the system composed of heavy RHNs and a gas of relativistic particles can be described by the equations [41]:

$$\dot{\rho}_N + 3H\rho_N + \Gamma_N\rho_N = 0 ,$$  

$$\dot{\rho}_{gas} + 4H\rho_{gas} - \Gamma_N\rho_N = 0 ,$$  

$$\dot{n}_L + 3Hn_L - \epsilon_{CP}\Gamma_N\frac{\rho_N}{M} = 0 ,$$  

where $n_L$ is the density of the net lepton number generated in the decays, $\Gamma_N = (h_\nu h_\nu^\dagger)_{11}M_1/(4\pi)$ is the decay rate, and $H$ is the Hubble rate, given by

$$H^2 = \frac{8\pi}{3M_P^2}(\rho_N(T) + \rho_{gas}(T)) .$$

From Eqs. (23-25) the evolution of the number per entropy densities $Y_N$, $Y_L$ can be written as

$$\dot{Y}_N = -Y_N(\Gamma_N + \frac{\dot{S}}{S}) ,$$  

$$\dot{Y}_L = \epsilon_{CP}\Gamma_N Y_N - Y_L\frac{\dot{S}}{S} ,$$

where $S$ is the entropy per comoving volume. The final value of the lepton asymmetry after the RHNs have decayed ($t_f \gg \Gamma^{-1}_N$) is then given by

$$Y_L = \epsilon_{CP}Y_N(T_0)\frac{S_0}{S_f} ,$$  

where $Y_N(T_0)$ is given in Eq. (16) in terms of the parameters of the model. The factor $S_0/S_f$ is the usual dilution factor due to entropy production during the decay. That is, assuming that the decay products rapidly thermalise, they heat up the Universe and contribute to the total entropy at a rate

$$\dot{S} = \Gamma_N a^3 \frac{\rho_N}{T} ,$$  

where $a$ is the scale factor. Entropy production will be significant if the total energy density is dominated by the RHNs, that is, when the decay products can make a non negligible contribution to the thermal bath. On the
other hand, if at $T_0$ the energy density was dominated by the gas of relativistic particles, the decay products will make almost no difference to the entropy. Therefore, the ratio $S_f/S_0$ is bounded between unity and the value [42]

$$\frac{S_f}{S_0} \simeq \left(1 + \frac{\bar{g}_*}{g_*(T_0)}\right)^{1/3} \left(\frac{\rho_N(T_0)}{\rho_{\text{gas}}(T_0)}\right) \left(1 + \frac{H(T_0)}{\Gamma_N}\right)^{2/3}$$

$$\simeq \left(\frac{\bar{g}_*}{g_*(T_0)}\right)^{1/4} \left(\frac{\rho_N(T_0)}{\rho_{\text{gas}}(T_0)}\right)^{3/4} \left(1 + \frac{H(T_0)}{\Gamma_N}\right)^{1/2},$$

(30)

where $\bar{g}_*$ is the average number of light degrees of freedom between $T_0$ and $T_f$, and

$$\frac{H(T_0)}{\Gamma_N} = \frac{1}{2} \left(\frac{T_c}{T_f}\right)^2 \sqrt{1 + r_c},$$

(31)

with $T_f \simeq 0.5g_*(T_0)^{-1/4} \sqrt{M_P \Gamma_N}$.

Let us first discuss the case where the RHNs dominate the energy density after collision, that is, when significant entropy is produced. This corresponds to the parameter range

$$\frac{\delta M}{r_c} < T_c < \frac{2\delta M}{1 + r_c}$$

(32)

of our model, where we recall that $\delta \approx 10^{-2}$ and $r_c \ll 1$ ($r_c \gg 1$) corresponds to a small (large) impact collision. We remind the reader that the upper bound in Eq. (32) guarantees that the universe is matter-dominated after the collision, see Eq. (14), while the lower bound is the kinematic limit which ensures that enough energy is available to account for the RHN mass while remembering that we must obey the second law of thermodynamics, see Eq. (13) in the limit $T_{\text{EW}}/T_c \ll 1$. Note that if $r_c$ is too small there is not enough energy available in this regime to account for the mass of the RHNs and the fact that the temperature of the gas cannot decrease during the collision. In other words if $r_c < 1$ the parameter range given by (32) closes up completely.

From Eqs. (28) and (30) we then find a lepton asymmetry of the order

$$Y_L \simeq \epsilon_{\text{CP}} \bar{g}_*^{-1/4} \sqrt{\frac{M_P \Gamma_N}{M}},$$

(33)

where the number of relativistic degrees of freedom $g_* \simeq g_*(T_0)$ is taken to be practically constant through the decay. Hence, for this case, the final baryon asymmetry does not depend on the parameters of the small instanton transition. In particular, it is independent of the initial number of RHNs given in Eq. (16). For typical values $\epsilon_{\text{CP}} \sim 10^{-6} - 10^{-8}$ one can clearly obtain an acceptable value for baryon asymmetry, in this case.

On the other hand, if there is no significant entropy production and, hence, the universe is radiation dominated after the collision, we should consider the parameter range

$$\frac{2\delta M}{(1 + r_c)} < T_c < \frac{M}{(1 + r_c)^{1/4}},$$

(34)

for the case $r_c \geq 1$, and

$$\frac{\delta M}{r_c} < T_c < \frac{M}{(1 + r_c)^{1/4}},$$

(35)

for $r_c < 1$. The upper bound is equivalent to $T_0 < M$, implying non-relativistic RHNs after the collision, see Eq. (12), while the lower bound guarantees a radiation-dominated universe after collision and that the energy...
matching requirements can be met, see Eq. (14) for the first of these. Note that if the brane is not moving fast enough (if \( r_c < \delta \)) then the energy matching requirements can not be met in this regime either. Using Eqs. (28) and (16) the lepton asymmetry is then given by

\[
Y_L \simeq \epsilon_{\text{CP}} Y_N(T_0) = \frac{3}{4} \epsilon_{\text{CP}} \delta \left( 1 + r_c - \frac{\delta M}{T_c} \right)^{-3/4}.
\]  

(36)

Given that \( \delta \approx 10^{-2} \) and \( \epsilon_{\text{CP}} \sim 10^{-6} - 10^{-8} \), it is possible to obtain the observed value for the baryon asymmetry for suitable choices of the parameters. However, unlike in the previous case, the result does depend on the parameters of the small-instanton transition, in particular on the parameter \( r_c \) which measures the brane impact. Specifically, for large impact, \( r_c \gg 1 \), we have

\[
Y_L \simeq \epsilon_{\text{CP}} \delta r_c^{-3/4},
\]

(37)

while for small impact, \( r_c \ll 1 \), we have instead

\[
Y_L \simeq \frac{3}{4} \epsilon_{\text{CP}} \delta \left( 1 - \frac{\delta M}{T_c} \right)^{-3/4} \approx \epsilon_{\text{CP}} \delta.
\]

(38)

The \( T_0 < M \) case can have another big advantage besides the ability to work analytically and the possibility of having information about the collision encoded in the baryon asymmetry. If \( T_0 \) is sufficiently low then one could envision our scenario taking place after some low scale inflationary mechanism, for example, which could be used to solve the gravitino problem. In other words we have the option of choosing a low reheat temperature, with all the benefits that brings, and still being able to produce baryons.

\[ \text{V.} \quad \text{NUMERICAL COMPUTATION OF THE BARYON ASYMMETRY} \]

If \( T_0 \gg M \) or \( T_0 \sim M \) then lepton number violating scatterings, which can reduce the generated lepton asymmetry by several orders of magnitude, can no longer be neglected, and one has to solve the full network of Boltzmann equations. In this case the expected asymmetry from Eq. (28) will be reduced by a washout factor \( \kappa \), that is, the generated asymmetry will be

\[
Y_L = \kappa \epsilon_{\text{CP}} Y_N(T_0) \frac{S_0}{S_f}.
\]

(39)

This has been studied previously in the standard scenario of thermal leptogenesis [26]. A characteristic feature of this scenario is that the generated baryon asymmetry mostly on the mass parameter

\[
\tilde{m}_1 = \left( h_\nu h_{\nu}^\dagger \right)_{11} \frac{v_2^2}{M_1},
\]

(40)

where \( v_2 \) is the vacuum expectation value of the MSSM Higgs field which gives Dirac masses to up-type quarks and neutrinos. This is due to the fact that all the scattering and decay terms entering in the Boltzmann equations are proportional to some power of \( \tilde{m}_1 \). Thermal leptogenesis is only possible in a rather narrow range of \( \tilde{m}_1 \). If \( \tilde{m}_1 \) is too low, the Yukawa interactions are too weak to produce a sufficient number of RHNs at high temperatures, whereas for large \( \tilde{m}_1 \) the lepton number violating scattering processes mediated by the RHNs are too strong and destroy any generated asymmetry [26].
FIG. 2. The washout parameter $\kappa$ as a function of $\tilde{m}_1$ for different values of the initial temperature $\frac{M}{T_0} = 0.1, 5, 10$ and 20 (from left to right).

In order to see whether this is also the case in our scenario we have numerically solved the set of Boltzmann equations, using initial conditions as discussed in section 3 and starting the simulation at different values of $T_0$. The results are shown in Fig (2), where we have plotted the washout factor $\kappa$ as a function of $\tilde{m}_1$, for initial temperatures $\frac{M}{T_0} = 0.1, 5, 10$ and 20. Further, we have assumed a hierarchy of RHN masses of the form $M_1 = 10^{10}$ GeV, $M_2 = 3 \times 10^{11}$ GeV and $M_3 = 10^{13}$ GeV.

Fig. 2 shows that $\kappa$ converges towards unity for small $\tilde{m}_1$, independently of the starting temperature $T_0$, since then the washout processes are suppressed and are out of equilibrium when the lepton asymmetry is produced. For larger $\tilde{m}_1$ the generated asymmetry starts to depend on $T_0$, since then the lepton number violating scattering processes can still be in thermal equilibrium at temperatures below the RHN neutrino mass, that is, even for $T_0 < M$ the generated asymmetry can be reduced by washout processes. Eventually however, the washout processes will freeze out, i.e. a smaller $T_0$ will result in a larger remaining asymmetry. If $\tilde{m}_1$ gets very large, above $\sim 3 \times 10^{-2}$ eV, the washout processes remain in thermal equilibrium down to very low temperatures, hence even for $T_0 \ll M$ the generated lepton asymmetry is strongly suppressed. In summary, we see that $\kappa$ approaches one for an increasing range in $\tilde{m}_1$ as $T_0$ decreases, which justifies our estimate for the $T_0 \ll M$ case in the previous section.

The discussion of this section should make one fact about this scenario clear; for certain possible parameter ranges in the MSSM our scenario is distinguishable experimentally from more conventional instances of leptoge-
If at future accelerator experiments, for example, the parameters of the MSSM were measured then they could be found to take values such that the conventional picture of leptogenesis is not phenomenologically viable due to problems with washout. In such a situation our scenario would not be ruled out and the conventional scenario would be - the two would have been distinguished by experiment.

VI. CONCLUSIONS AND OUTLOOK

Let us summarise the most important points and conclusions of this paper. We have proposed, in the context of heterotic M-theory brane-world models, a scenario for baryogenesis based on a small-instanton phase transition induced by a brane collision. Our scenario has a crucial difference to more standard scenarios, such as leptogenesis. It utilises the decoupling of the temperature after the phase transition ($T_0$) from the RHN mass, which is a phenomenon, seen in small instanton transitions, which is qualitatively new to cosmology. This allows, for example, the generation of a lepton asymmetry at temperatures significantly below the RHN mass. This could be of significant help in dealing with things like the gravitino problem, as mentioned at the end of section IV. We have found that, in some cases, the generated baryon asymmetry depends on characteristics of the brane-collision, such as the “impact” of the colliding brane. Most importantly, we have demonstrated that an acceptable value for the baryon asymmetry can be obtained under reasonable assumptions for the parameters in the model.

We have also performed a more detailed analytical as well as numerical analysis of our scenario, the latter based on the full set of Boltzmann equations. We have seen that the results crucially depend on the value $M/T_0$ (where $M$ is the RHN mass and $T_0$ is the temperature after brane collision) and the impact parameter $r_c$. For $T_0 > M$ the RHNs thermalise after the collision and before decaying. At the same time, scattering effects may be important and may wash out the baryon asymmetry. This case is, in fact, similar to standard leptogenesis as can be seen from the numerical results. If, on the other hand, $T_0 < M$ scattering effects are expected to become less important, which is indeed what our numerical results show. This allowed us to perform an analytic estimate in this case. It turns out that, between, roughly, $O(M/50) < T_0 < O(M)$ the universe is radiation-dominated after the collision. The baryon asymmetry then depends on the parameters of the collision such as the precise value of $T_0$ and the impact parameter $r_c$. If $T_0 < O(M/50)$, on the other hand, the energy density after collision is dominated by massive RHNs and, hence, the universe is matter-dominated. The baryon asymmetry is then diluted by significant entropy generation and becomes independent of the parameters $T_0$ and $r_c$.

Our description of the small-instanton transition was based on a number of “phenomenological” assumptions. It would clearly be desirable to carry out a more microscopical analysis. Unfortunately, a low-energy effective description of the transition, suitable for our purpose, is not available at present. Developing such a description and applying it to our proposal is an interesting challenge for future research.

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APPENDIX A: PERFECT FLUID COSMOLOGY WITH A MOVING BRANE

In this appendix, we present the moving-brane cosmological solutions to the action (3) in the presence of a perfect fluid with equation of state $p_{\text{fluid}} = w \rho_{\text{fluid}}$ where $w < 1$ is a constant. These solutions may have a wide range of applications in the context of moving-brane cosmologies. For example, for $w = -1$ they describe the motion of a brane in the background of a cosmological constant, a result relevant to the inflationary scenario of Ref. [1]. For the purpose of this paper, the solutions will be used to analyse the first stage of our baryogenesis scenario.

We start with the Ansatz

\begin{equation}
\begin{aligned}
\text{d}s^2 &= -e^{2\nu(\tau)} \text{d}\tau^2 + e^{2\alpha(\tau)} \text{d}x^2, \\
\beta &= \beta(\tau), \\
\phi &= \phi(\tau), \\
z &= z(\tau),
\end{aligned}
\end{equation}

(A1)

where we have chosen flat spatial sections, for simplicity. The energy density and pressure of the perfect fluid can be written as

\begin{equation}
\begin{aligned}
\rho_{\text{fluid}} &= \rho_0 e^{-3(1+w)\alpha}, \\
p_{\text{fluid}} &= w \rho_{\text{fluid}}
\end{aligned}
\end{equation}

(A5)

(A6)

where $\rho_0$ is a constant. We can integrate the equation of motion for $z$ derived from the action (3) to obtain

\begin{equation}
\dot{z} = ue^{-3\alpha + \nu - \beta + \phi}.
\end{equation}

(A7)

This result can be used to eliminate $z$ so that we remain with a closed set of equations for the fields $\alpha$, $\beta$ and $\phi$. Using the formalism of Ref. [43], these equations can be very elegantly summarised by an effective “moduli space” Lagrangian given by

\begin{equation}
\mathcal{L} = \frac{1}{2} E \alpha'^T G \alpha' - E^{-1} U.
\end{equation}

(A8)

Here we have arranged the fields into a vector $\alpha = (\alpha, \beta, \phi)^T$ and $G = \text{diag}(-3, \frac{3}{4}, \frac{1}{4})$ is a constant metric. The “Einbein” $E$ is a non-dynamical field defined by $E = e^{3\alpha - \nu}$ whose equation of motion leads to the Friedmann equation. Finally, the potential $U$ on the moduli space has the structure

\begin{equation}
U = \frac{1}{2} \sum_{r=0}^{1} u_r^2 e^{q_r^T \alpha}.
\end{equation}

(A9)

The first term originates from the perfect fluid where

\begin{equation}
u_0^2 = \rho_0, \quad q_0 = (3(1-w),0,0)^T
\end{equation}

(A10)

while the second one is due to the moving brane where

\begin{equation}
\begin{aligned}
u_1^2 &= \frac{1}{2} q_8 u^2, \\
q_1 &= (0,-1,1)^T.
\end{aligned}
\end{equation}

(A11)

The vectors $q_r$ are characteristic for the respective origin of the potential terms. Note that in our particular case we have
\begin{equation}
\langle \mathbf{q}_0, \mathbf{q}_1 \rangle = \mathbf{q}_0^T G^{-1} \mathbf{q}_1 = 0 \tag{A12}
\end{equation}

This implies that we are dealing with an SU(2)^2 Toda model which can be integrated exactly. Following Ref. [43], one can find the general solution in the gauge \( E = 1 \), that is, \( \nu = 3\alpha \). It is given by

\begin{align}
\alpha &= \frac{1}{\sqrt{3}} \sigma_0 \tag{A13} \\
\beta &= \frac{1}{\sqrt{3}} \sigma_1 + \sigma_2 \tag{A14} \\
\phi &= -\sqrt{3} \sigma_1 + \sigma_2 \tag{A15} \\
\nu &= 3\alpha \tag{A16}
\end{align}

with the modes

\begin{align}
\sigma_0 &= -q_0^{-1} \ln \left[ \frac{u_0^2}{k_0} \sinh^2(y_0) \right] \tag{A17} \\
\sigma_1 &= q_1^{-1} \ln \left[ \frac{u_1^2}{k_1} \cosh^2(y_1) \right] \tag{A18} \\
\sigma_2 &= k_2(|\tau| - \tau) \tag{A19}
\end{align}

and

\begin{equation}
y_r = \frac{1}{2} |q_r| \sigma_r (|\tau| - \tau) \tag{A20}
\end{equation}

where \( r = 0, 1 \). Further, the constants \( q_r \) are defined by

\begin{equation}
q_0 = \sqrt{3}(1 - w) \, , \quad q_1 = \frac{4}{\sqrt{3}} . \tag{A21}
\end{equation}

and \( \tau_i \), where \( i = 0, 1, 2 \), are arbitrary integration constants. The integration constants \( k_i \), where \( i = 0, 1, 2 \), are subject to the constraint

\begin{equation}
k_1^2 + k_2^2 = k_0^2 \tag{A22}
\end{equation}

which originates from the Friedmann equation. Inserting these results into Eq. (A7) we find that the brane motion is described by

\begin{equation}
z = z_0 + \frac{d}{2} \tanh(y_1) \tag{A23}
\end{equation}

where \( z_0 \) is an arbitrary constant and the maximal distance \( d \) by which the brane moves is given by

\begin{equation}
d = \frac{2\sqrt{3}|k_1|}{q_0 u} . \tag{A24}
\end{equation}

Let us discuss some properties of these solutions. It is straightforward to show that the total kinetic energy density \( \rho_{\text{kin}} \) and the energy density of the fluid \( \rho_{\text{fluid}} \) for our solutions are given by

\begin{align}
e^{2\nu} \rho_{\text{kin}} &= \frac{1}{4} \dot{\phi}^2 + \frac{3}{4} \dot{\beta}^2 + \frac{q_0}{2} e^{\beta - \phi} \dot{z}^2 = k_1^2 + k_2^2 \tag{A25} \\
e^{2\nu} \rho_{\text{fluid}} &= e^{2\nu} \rho = \frac{k_0^2}{\sinh^2(y_0)} . \tag{A26}
\end{align}
Defining the ratio $r$ of kinetic to fluid energy density one then finds in view of Eq. (A22)

$$r \equiv \frac{\rho_{\text{kin}}}{\rho_{\text{fluid}}} = \sinh^2(y_0).$$

(A27)

From the solution for $\alpha$, Eq. (A13), this leads to the simple scaling law

$$\frac{a(\tau_1)}{a(\tau_2)} = \left[ \frac{r(\tau_1)}{r(\tau_2)} \right]^{-1/(3(1-w))}.$$

(A28)

where $a = e^\alpha$ is the scale factor.

For a gas with temperature $T$ we have $T \sim a^{-1}$ (assuming isentropic evolution and unchanged number of degrees of freedom) which, using $w = \frac{1}{3}$, implies

$$\frac{T_1}{T_2} = \left[ \frac{r(T_1)}{r(T_2)} \right]^{1/2}.$$

(A29)

[1] G. Dvali and S. H. Tye, “Brane inflation,” Phys. Lett. B 450 (1999) 72 [hep-ph/9812483].

[2] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, “The ekpyrotic universe: Colliding branes and the origin of the hot big bang,” [hep-th/0103239].

[3] R. Kallosh, L. Kofman and A. Linde, “Pyrotechnic universe,” [hep-th/0104073].

[4] S. H. Alexander, “Inflation from D - anti-D brane annihilation,” Phys. Rev. D 65 (2002) 023507 [hep-th/0105032].

[5] C. P. Burgess, M. Majumdar, D. Nolte, F. Quevedo, G. Rajesh and R. J. Zhang, “The inflationary brane-antibrane universe,” [hep-th/0105204].

[6] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, “A brief comment on 'The pyrotechnic universe',” [hep-th/0105212].

[7] R. Kallosh, L. Kofman, A. Linde and A. Tseytlin, “BPS Branes in Cosmology,” [hep-th/0106241].

[8] E. J. Copeland, J. Gray and A. Lukas, “Moving five-branes in low-energy heterotic M-theory,” Phys. Rev. D 64 (2001) 126003 [hep-th/0106285].

[9] P. Horava and E. Witten, “Heterotic and type I string dynamics from eleven dimensions,” Nucl. Phys. B 460 (1996) 506 [hep-th/9510209].

[10] P. Horava and E. Witten, “Eleven-Dimensional Supergravity on a Manifold with Boundary,” Nucl. Phys. B 475 (1996) 94 [hep-th/9603142].

[11] E. Witten, “Strong Coupling Expansion Of Calabi-Yau Compactification,” Nucl. Phys. B 471 (1996) 135 [hep-th/9602070].

[12] A. Lukas, B. A. Ovrut, K. S. Stelle and D. Waldram, “The universe as a domain wall,” Phys. Rev. D 59 (1999) 086001 [hep-th/9803235].
[13] A. Lukas, B. A. Ovrut, K. S. Stelle and D. Waldram, “Heterotic M-theory in five dimensions,” Nucl. Phys. B 552 (1999) 246 [hep-th/9806051].

[14] A. Lukas, B. A. Ovrut and D. Waldram, “Non-standard embedding and five-branes in heterotic M-theory,” Phys. Rev. D 59 (1999) 106005 [hep-th/9808101].

[15] E. Witten, “Small Instantons in String Theory,” Nucl. Phys. B 460 (1996) 541 [hep-th/9511030].

[16] O. J. Ganor and A. Hanany, “Small $E_8$ Instantons and Tensionless Non-critical Strings,” Nucl. Phys. B 474 (1996) 122 [hep-th/9602120].

[17] B. A. Ovrut, T. Pantev and J. Park, “Small instanton transitions in heterotic M-theory,” JHEP 0005 (2000) 045 [hep-th/0001133].

[18] M. Fukugita and T. Yanagida, “Baryogenesis Without Grand Unification,” Phys. Lett. B 174 (1986) 45.

[19] P. Langacker, R. D. Peccei and T. Yanagida, “Invisible Axions And Light Neutrinos: Are They Connected?,” Mod. Phys. Lett. A1, (1986) 541

[20] M. Fukugita and T. Yanagida, “Sphaleron Induced Baryon Number Nonconservation And A Constraint On Majorana Neutrino Masses,” Phys. Rev. D 42 (1990) 1285.

[21] M. A. Luty, “Baryogenesis via leptogenesis,” Phys. Rev. D 45 (1992) 455.

[22] B. A. Campbell, S. Davidson and K. A. Olive, “Inflation, neutrino baryogenesis, and (S)neutrino induced baryogenesis,” Nucl. Phys. B 399 (1993) 111 [arXiv:hep-ph/9302223].

[23] M. Plüümacher, “Baryogenesis and lepton number violation,” Z. Phys. C 74 (1997) 549 [arXiv:hep-ph/9604229].

[24] L. Covi, E. Roulet and F. Vissani, “CP violating decays in leptogenesis scenarios,” Phys. Lett. B 384 (1996) 169 [arXiv:hep-ph/9605319].

[25] W. Buchmüller and M. Plüümacher, “CP asymmetry in Majorana neutrino decays,” Phys. Lett. B 431 (1998) 354 [arXiv:hep-ph/9710460].

[26] M. Plüümacher, “Baryon asymmetry, neutrino mixing and supersymmetric SO(10) unification,” Nucl. Phys. B 530 (1998) 207 [arXiv:hep-ph/9704231].

[27] R. Barbieri, P. Creminelli, A. Strumia and N. Tetradis, “Baryogenesis through leptogenesis,” Nucl. Phys. B 575 (2000) 61 [hep-ph/9911315].

[28] For a review, see W. Buchmüller and M. Plüümacher, “Neutrino masses and the baryon asymmetry,” Int. J. Mod. Phys. A15 (2000) 5047, [hep-ph/0007176].

[29] B. Andreas, “On vector bundles and chiral matter in $N = 1$ heterotic compactifications,” JHEP 9901 (1999) 011 [hep-th/9802202].

[30] G. Curio, “Chiral matter and transitions in heterotic string models,” Phys. Lett. B 435 (1998) 39 [hep-th/9803224].

[31] R. Donagi, A. Lukas, B. A. Ovrut and D. Waldram, “Non-perturbative vacua and particle physics in M-theory,” JHEP 9905 (1999) 018 [hep-th/9811168].

[32] R. Donagi, A. Lukas, B. A. Ovrut and D. Waldram, “Holomorphic vector bundles and non-perturbative vacua in
M-theory,” JHEP 9906 (1999) 034 [hep-th/9901009].

[33] R. Donagi, B. A. Ovrut, T. Pantev and D. Waldram, “Standard models from heterotic M-theory,” hep-th/9912208.

[34] R. Donagi, B. A. Ovrut, T. Pantev and D. Waldram, “Non-perturbative vacua in heterotic M-theory,” Class. Quant. Grav. 17 (2000) 1049.

[35] R. Donagi, B. A. Ovrut, T. Pantev and D. Waldram, “Standard-model bundles,” math.ag/0008010.

[36] J. Derendinger and R. Sauser, “A five-brane modulus in the effective N = 1 supergravity of M-theory,” Nucl. Phys. B 598 (2001) 87 [hep-th/0009054].

[37] M. Brandle and A. Lukas, “Five-branes in heterotic brane-world theories,” hep-th/0109173.

[38] J. H. Traschen and R. H. Brandenberger, “Particle Production During Out-Of-Equilibrium Phase Transitions,” Phys. Rev. D 42 (1990) 2491; L. Kofman, A. Linde and A. A. Starobinsky, “Reheating after inflation,” Phys. Rev. Lett. 73 (1994) 3195 [arXiv:hep-th/9405187]; L. Kofman, A. Linde and A. A. Starobinsky, “Towards the theory of reheating after inflation,” Phys. Rev. D 56 (1997) 3258 [arXiv:hep-ph/9704452]; G. F. Giudice, M. Peloso, A. Riotto and I. Tkachev, “Production of massive fermions at preheating and leptogenesis,” JHEP 9908 (1999) 014 [arXiv:hep-ph/9905242].

[39] G. Felder, L. Kofman and A. Linde, “Instant preheating,” Phys. Rev. D 59 (1999) 123523 [arXiv:hep-ph/9812289].

[40] For a discussion and references, see A. Pilaftsis, “Heavy Majorana neutrinos and baryogenesis,” Int. J. Mod. Phys. A 14 (1999) 1811 [hep-ph/9812256];

[41] E. W. Kolb and M. S. Turner, “The Early Universe”, Addison-Wesley (1989).

[42] R. J. Scherrer and M. S. Turner, ”Decaying particles do not ‘heat up’ the universe”, Phys. Rev. D 31 (1985) 681.

[43] A. Lukas, B. A. Ovrut and D. Waldram, “String and M-theory cosmological solutions with Ramond forms,” Nucl. Phys. B 495 (1997) 365 [hep-th/9610238].