BARYONIC MIXING AND PRODUCTION OF HYPERNUCLEI

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We address mixing of different baryonic states in hypernuclei and formation of strange nuclei of various kinds. First, production of neutron-rich Λ hypernuclei by the $(\pi^-, K^+)$ reaction is considered, and one-step production via baryonic admixtures in the final as well as in the initial state is discussed. Then, the $\Lambda\Lambda - \Xi N$ mixing in $p$ shell $\Lambda\Lambda$ hypernuclei exemplified by $^{12}_\Lambda\Lambda$Be is studied. The last topic is devoted to production of $\Theta$ pentaquark nuclei (pentanuclei). We suggest a type of reactions with recoilless $\Theta$ production.

1. Production of neutron-rich $\Lambda$ hypernuclei by the $(\pi^-, K^+)$ reaction

Interest to neutron-rich $\Lambda$ hypernuclei was recently stimulated by the KEK experiment. In this experiment, production of neutron-rich hypernuclei $^{10}_\Lambda$Li and $^{12}_\Lambda$Be by the $(\pi^-, K^+)$ reaction was observed at the first time.

We considered two mechanisms of this reaction. The first one is the two-step production with meson charge-exchange: $\pi^- p \rightarrow \pi^0 n, \pi^0 p \rightarrow K^+ \Lambda$ or $\pi^- p \rightarrow K^0 \Lambda, K^0 p \rightarrow K^+ n$. The second mechanism is the one-step production $\pi^- p \rightarrow K^+ \Sigma^-$ proceeding via a $\Sigma^-$ doorway state. The $\Sigma^-$ admixture arises from the $\Lambda n - \Sigma^- p$ coupling interaction. The results of our study can be summarized as follows.

- The cross sections of the $(\pi^-, K^+)$ reaction are typically smaller by about three orders of magnitude than those of the “usual” $(\pi^+, K^+)$ reaction.
- The two-step mechanism is dominant.
- Pion and kaon charge exchanges give comparable contributions.
- The cross sections of the $^{10}$B$(\pi^-, K^+)^{10}_\Lambda$Li reaction are higher than those of the $^{12}$C$(\pi^-, K^+)^{12}_\Lambda$Be reaction.

The first and the last conclusions are confirmed qualitatively by the
Comparison of our predictions\textsuperscript{a} with the measured data is shown in Table 1. Quantitatively the agreement is relatively poor, which is not, however, so discouraging in view of rather complicated nature of the process and many uncertainties in the input information. It should be noted also that the compared quantities in Table 1 are not of the same meaning strictly, since the theoretical cross sections were summed over all the neutron bound states of the final hypernuclei whereas integration over the whole interval $0 < B_\Lambda < 15$ MeV has been performed in the experiment.

| reaction        | $p_\pi$, GeV/c | $\frac{d\sigma}{d\Omega}$, nb/sr |
|-----------------|----------------|-----------------------------------|
| $^{10}$B($\pi^-, K^+)_\Lambda$ | 1.05           | 38                               |
|                 | 1.2            | 22                               |
|                 | 1.2            | 12 ± 2                           |
| $^{12}$C($\pi^-, K^+)_\Lambda$ | 1.05           | 5                                |
|                 | 1.2            | 2.5                              |
|                 |                 | 7                                |

The main qualitative disagreement is in the energy dependence of the cross sections. We predicted the cross sections at $p_\pi = 1.05$ GeV/c to be about twice as large as those at $p_\pi = 1.2$ GeV/c according to the sharp peak of the cross section of the elementary $\pi N \rightarrow K \Lambda$ reaction at $p_\pi = 1.05$ GeV/c. The experimental ratio is nearly inverse.\textsuperscript{1} The energy dependence is still an open problem now.

Here, we would like to suggest a new mechanism of the reaction. It is also a one-step process like production via the $\Sigma^-$ admixture but it is originated from a baryonic admixture in the initial (nuclear) rather than in the final (hypernuclear) state. It is known that $\Delta$ baryonic admixtures occur in ordinary nuclei. Then, the $\pi^- \Delta^{++} \rightarrow K^+ \Lambda$ process on this admixture leads just to the reaction considered.

Under usual approximations, the cross section of this process can be estimated as

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_{\text{elem}}}{d\Omega} \cdot p_{\Delta^{++}} \cdot N_{\text{eff}}. \quad (1)$$

Here $\frac{d\sigma_{\text{elem}}}{d\Omega}$ is the cross section of elementary process $\pi^- \Delta^{++} \rightarrow K^+ \Lambda$. Of course, it can not be measured experimentally. We use the theoretical

\textsuperscript{a}The calculated values in Table 1 are somewhat different from those in Ref. 2 due to a more accurate treatment of relevant elementary cross sections.
estimation\(^3\) based on a quark model.\(^b\) Then, \(p_{\Delta^{++}}\) is the probability of the \(\Delta^{++} + A - 1(Z - 2)\) state, where the nucleus is in some definite, for instance, the ground, state. Recent experiments\(^3\) attempted to measure \(\Delta\) admixture probabilities in nuclei. From these data, we adopt \(N_{\Delta^{++}} \equiv 0.4\%\) for \(^{12}\)C. It should be noted that this is the total \(\Delta^{++}\) probability (not related to a certain \(^{10}\)Be state), so \(p_{\Delta^{++}} < N_{\Delta^{++}}\). Thus, using \(N_{\Delta^{++}}\) instead of \(p_{\Delta^{++}}\), we obtain an upper limit for the cross section.

Typical effective numbers \(N_{\text{eff}}\) for specific states of final \(\Lambda\) hypernuclei are known to be \((2–3)\times10^{-2}\) for the \(^{12}\)C target from the theory of the \((\pi^-, K^+)\) reaction. Assuming the same values for the reaction considered, we finally estimate \(d\sigma/d\Omega \sim (10–15)\) nb/sr (recall that this is an upper bound) for the \(^{12}\)C\((\pi^-, K^+)\)\(^{12}\)Λ\(^{12}\)Be reaction at forward angles.

Comparing this value with the cross sections exhibited in Table 1, we see that the suggested mechanism is at least non-negligible. Of course, existence of the additional mechanism makes the problem still more complicated. On the other hand, it might be rather interesting if \(\Delta\) admixtures in ordinary nuclei are closely related to the production of neutron-rich \(\Lambda\) hypernuclei, and this mechanism deserves further study.

2. Double-\(\Lambda\) \(p\) shell hypernuclei and \(\Lambda\Lambda - \Xi N\) mixing

The \(\Lambda\Lambda - \Xi N\) mixing in \(\Lambda\Lambda\) hypernuclei is possibly the most significant among baryonic mixings in various nuclear systems due to the small (about 25 MeV) mass splitting between \(\Lambda\Lambda\) and \(\Xi N\) pairs. Here, we study the effect in \(p\) shell hypernuclei, specifically, \(^{12}\)\(^{10}\)Be attainable by the \(^{12}\)C\((K^-, K^+)\) reaction. We adopt the coupled-channel model similar to that previously applied\(^5\) to \(s\) shell hypernuclei \(^{5}\)\(^{10}\)H and \(^{5}\)\(^{10}\)He.

We treat the following coupled channels. The main channel is \(^{10}\)Be\(_{gs}\) + 2\(\Lambda\). Here, we consider \(1s^2\)\(_L\)\((L = 0)\) and \(1p^2\)\(_L\)\((L = 0, 1, 2)\) states. The second channel is \(^{11}\)B\(_{gs}\) + \(\Xi^-\), where \(\Xi^-\) is in a \(p\) state. Also the third channel was considered, namely, \(^{11}\)Z\(_{gs}\) + \(N\), where the \(\Xi N\) pair possesses zero isospin, and the nucleon is in an \(s\) state. The role of the third channel is found to be typically small if the orthogonalization of the \(s\) nucleon state to the \(1s\) nucleons within \(^{11}\)\(_\Xi\)Z is performed. On the other hand, the precise value of the probability of this channel depends strongly on input parameters, so we present below the results obtained in the two-channel approximation.

\(^b\)This calculation predicts a smooth energy dependence of the elementary cross section in the relevant energy range, so the mechanism unlikely can give an explanation of the puzzling energy dependence of the hypernuclear production.
Figure 1. $\Delta B_{\Lambda\Lambda}$ (top) and $\Xi$ admixture probabilities (bottom) in $^{12}\Lambda\Lambda$Be as functions of \( \int d^3rV(\Lambda\Lambda - \Xi N) \). Diagonal $\Lambda\Lambda$ and coupling $\Lambda\Lambda - \Xi N$ potentials are from Nijmegen models\(^6\) (from left to right) NHC-D, NSC97f, NSC97e, NSC89, ESC03 and NHC-F.

We use phenomenological Woods-Saxon potentials for $\Lambda$-nucleus and $\Xi$-nucleus interactions. For diagonal $\Lambda\Lambda$ and coupling $\Lambda\Lambda - \Xi N$ interactions, potentials\(^5\) derived from $G$ matrix calculations with various meson-exchange Nijmegen models\(^6\) are employed.
In Fig. 1, $\Delta B_{\Lambda \Lambda}$ values (upper panel) and $\Xi$ channel probabilities $p_\Xi$ (lower panel) are presented for various Nijmegen models as functions of volume integral $\int d^3r V(\Lambda \Lambda - \Xi N)$. First, it is seen that the contribution of the coupling interaction to the energy of the ground ($1s^2_\Lambda$) state may be considerable. For instance, the $\Delta B_{\Lambda \Lambda}$ values for the weakest coupling (Nijmegen hard-core D model, the leftmost point) and for the strongest coupling (Nijmegen hard-core F model, the rightmost point) differ by about 2 MeV. In the latter case, $\Delta B_{\Lambda \Lambda}$ is as large as almost 4 MeV (note that all the interactions are fitted to $\Delta B_{\Lambda \Lambda}(^6\Lambda \Lambda\text{He}) = 1$ MeV, see Ref. 5). On the other hand, the $\Xi$ probability is not substantially greater than 1% anyway.

A stronger coupling effect may be expected in the $1p^2_\Lambda$ states since they lie much closer in energy to the $\Xi$ hypernuclear spectrum. It is seen in Fig. 1 that this is just the case for the $L = S = 0$ state: $\Delta B_{\Lambda \Lambda}(1p^2_\Lambda) \equiv B_{\Lambda \Lambda}(1p^2_\Lambda) - 2B_{\Lambda}(1p_\Lambda)$ for strong coupling potentials exceeds the ground state value, though loosely bound hyperons move rather far from each other. For the strongest coupling, $\Delta B_{\Lambda \Lambda}$ reaches a huge value about 6 MeV. Contrary to the ground state, the $\Xi$ probabilities may be also rather great.

On the other hand, the coupling effect is relatively small for the $L = 2$ state and extremely small for the $L = 1$ state (the latter is not shown in Fig. 1). So the coupling can split levels almost degenerated without it.

The most interesting point observed here is probably the possibility of rather large mixing in the $1p^2_\Lambda (L = 0)$ state. Due to small energy separation from the $\Xi$ hypernuclear spectrum, excited double-$\Lambda$ hypernuclei can be essentially states with comparable weights of the $\Lambda \Lambda$ and $\Xi N$ (and possibly also $\Sigma \Sigma$ not considered here) components. Large $\Xi$ admixtures generally imply relatively large production rates in the ($K^-, K^+$) reaction via $\Xi^-$ doorway states and the one-step $K^- p \to K^+ \Xi^-$ process similar to the mechanisms considered in Sec. 1. Note that the $\Xi^-$ probabilities obtained for $^{12}\Lambda \Lambda\text{Be}$ are greater by several orders of magnitude than the $\Sigma^-$ probabilities in the neutron-rich $\Lambda$ hypernuclei. However, the production rate for the specific $1p^2_\Lambda (L = 0)$ state in $^{12}\Lambda \Lambda\text{Be}$ is evidently small since the $0^+ \to 0^+$ transition is strongly suppressed in the ($K^-, K^+$) reaction due to a large momentum transfer. Search for $\Lambda \Lambda$ hypernuclear states with considerable $\Xi$ admixtures and yet producible with reasonable rates is a challenging problem.
3. How to inject the $\Theta$ pentaquark into a nucleus?

The recent discovery of positive-strangeness baryon $\Theta^+$ (Ref. 8; further references are collected in Ref. 9) is one of the most exciting events in particle/nuclear physics. So far, most of experimental activities, either performed or planned, are focused on static properties of the pentaquark.

Several theoretical groups studied $\Theta$ interaction with nuclei and possibility of $\Theta$ nuclei existence. By analogy with hypernuclei, such systems may be named pentanuclei. It is notable that, while a number of different approaches was applied, attraction between $\Theta$ and nucleons was deduced in all the papers, though nuclear wells vary from very shallow to a deep one enough even to stabilize the system with respect to strong decays.

We do not deal with any specific model of $\Theta$-nucleus interaction. Instead, just the question formulated in the heading of this Sec. is addressed.

It is well known that the key point for production of “usual” hypernuclei is the kinematical conditions. For instance, the main ways to form $\Lambda$ hypernuclei are the $(K, \pi)$ reaction, where $\Lambda$ can be produced recoilessly (with zero momentum), as well as the $(\pi, K)$ and $(\gamma, K)$ reactions providing $\Lambda$ momenta nonzero, but comparable to the nucleon Fermi momentum.

Since $\Theta$ is heavier than hyperons, the momenta transferred to $\Theta$ produced from a single nucleon $(\gamma N \to \bar{K}\Theta$, $\pi N \to \bar{K}\Theta$, $KN \to \pi\Theta$) are rather high (upper curves in Fig. 2). The minimal momentum transfer in these reactions is 635 MeV/c reached in the $(K, \pi)$ reaction at 900 MeV/c. The momenta transfer in the $\gamma$- and $\pi$-induced reactions (as well as for nucleon-induced production, which is not shown in Fig. 2) are still higher.

We suggest another type of reactions, namely, production of $\Theta$ from a nucleon pair. It is seen from Fig. 2 that reactions $\gamma NN \to \Theta Y$, $\pi NN \to \Theta Y$ and $KNN \to \Theta N$, where $Y$ is $\Lambda$ or $\Sigma$, provide really recoilless kinematics. It is important that the momenta transfer remain small compared to the nucleon Fermi momentum in a wide range of the incident momentum. The relevant nuclear reactions are as follows: $^{A}Z(\gamma, Y)^{A-1}_{\Theta}Z'$, $^{A}Z(\pi, Y)^{A-1}_{\Theta}Z'$ and $^{A}Z(K, N)^{A-1}_{\Theta}Z'$, so the reactions are binary, like the usual reactions of hypernuclear production, but detection of baryons (instead of mesons) is needed.

Usually cross sections of hypernuclear production are factorized into

\[^{6}\text{Note that kinetic energy } T_{K} = 300 \text{ MeV of the incident kaon suggested in Ref. 11 as the “optimal” condition for the } (K^+, \pi^+) \text{ reaction lies well below the threshold of the elementary reaction } K^+ p \to \pi^+ \Theta, \text{ which is about } T_{K} = 410 \text{ MeV } (p_K = 760 \text{ MeV/c}). \text{ At } T_{K} = 300 \text{ MeV, the reaction is possible only as a subthreshold many-body effect.}\]
Figure 2. The momentum transferred to pentaquark Θ as a function of the projectile momentum for various reactions at zero angle. The curves for γ- and π-induced reactions are close to each other, therefore only the latter ones are drawn.

To get some rough estimation, we deal with pentanucleus production on deuteron clusters and consider cross sections summed over all final states using the closure approximation in lines of Ref. 12. We have

\[
\frac{d\sigma_{\text{sum}}}{d\Omega}[^AZ(a, B)^{A-1}Z', \theta_B = 0] = \frac{d\sigma}{d\Omega}[d(a, B)\Theta, \theta_B = 0] \cdot N_d \cdot S_{\text{dist}},
\]

where \(N_d\) is the deuteron cluster number taken from a cluster model\(^13\), \(S_{\text{dist}}\) is the distortion factor derived using the eikonal approximation.

Any experimental information on the reactions \(ad \to B\Theta\), where \(B\) is a baryon, is still absent. The reaction \(\gamma d \to \Theta Y\) was considered theoretically.\(^14\) So we performed estimations for the \(^{12}\text{C}(\gamma, \Lambda)_\Theta^{11}\text{C}\) reaction.
Using the cross sections from Ref. 14, we estimate the summed cross section as follows: \( \frac{d\sigma}{d\Omega}^{12C(\gamma, \Lambda)}_{\Theta} = 29, 10 \) and 2 nb/sr at \( E_\gamma = 1.2, 1.6 \) and 2.0 GeV, respectively. Such cross sections are evidently measurable at existing facilities. It should be noted, however, that the elementary cross sections in the model\(^{14}\) are proportional to the unknown \( \Theta \) width, and the calculations\(^{14}\) have been performed at an arbitrary value of 5 MeV. If the \( \Theta \) width is about 1 MeV as argued recently,\(^{15}\) the cross sections must be reduced by the factor of 5.

The summed cross section is not directly related to production of low-lying pentanuclear states, but gives rather an upper bound. From the theory of the \(^A\)Z(K\(^-\), \(\pi^-\))\(^A\)Z reaction (also recoilless) it has been known\(^{16}\) that about a half of the summed cross section is contributed by low-lying substitutional states for the \(^{12}\)C target. So one may expect that the fraction of the low-lying pentanuclear states in the summed cross section is also substantial.

Of course, it is rather problematic now to predict the elementary cross sections reliably, so the experimental data on the \( ad \to B\Theta \) reactions are strongly needed. The \( g10 \) experiment at TJNAF, in which the \( \gamma d \to \Lambda\Theta \) reaction is studied,\(^{17}\) is encouraging from this point of view. The reaction \( \gamma + ^3\)He with \( \Lambda \) emission\(^{17}\) is also rather interesting since pentadeuteron \(^2\)He can be produced, if it exists. On the other hand, the meson-induced reactions deserve study as well.

To conclude this section, the pentaquark production from a nucleon pair is probably the most promising way for formation of \( \Theta \) nuclei (pentanuclei).

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