Inflation in $R + R^2$ gravity with torsion

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Abstract

We examine an inflationary model in $R + R^2$ gravity with torsion, where $R^2$ denotes five independent quadratic curvature invariants; it turns out that only two free parameters remain in this model. We show that the behavior of the scale factor $a(t)$ is determined by two scalar fields, axial torsion $\chi(t)$ and the totally anti-symmetric curvature $E(t)$, which satisfy two first-order differential equations. Considering $\dot{\chi} \approx 0$ during inflation leads to a power-law inflation: $a \sim (t + A)^p$ where $1 < p \leq 2$, and the constant $A$ is determined by the initial values of $E, \chi$ and the two parameters. After the end of inflation, $\chi$ and $E$ will enter into an oscillatory phase.

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1. Introduction

Since cosmic microwave background radiation (CMB) was discovered, the standard hot big bang cosmological model has become widely accepted. However, the hot big bang universe leads to several problems, e.g. horizon and flatness problems [9]. An inflationary scenario was then proposed to resolve these problems (see [10]). The standard approach to the inflationary universe is to introduce scalar inflaton fields without modifying general relativity (GR). However, these inflaton fields will violate the strong energy condition. Instead of introducing inflatons, it has been found that several alternative theories of gravity can also provide the inflationary universe. A well-known pure gravity inflationary model is based on a fourth-order gravity theory $\hat{R} + \epsilon \hat{R}^2$, i.e., adding a quadratic pseudo-Riemannian scalar curvature $\hat{R}^2$ to the Einstein–Hilbert action without introducing any scalar inflaton field [15, 21, 23]. In the following, we put a hat on any geometric object $W$ to indicate that $\hat{W}$ is defined from the Levi-Civita connection $\hat{\nabla}$. In [23] (see [22] for further generalization), it provides a better understanding of $\hat{R}^2$ inflation by showing that the Lagrangian $-2\Lambda + \hat{R} + \epsilon \hat{R}^2$ is conformally related to Einstein gravity plus a scalar field, where $\Lambda$ denotes the cosmological
constant. Furthermore, it has been demonstrated that the Lagrangian \(-2\Lambda + \hat{R} + \epsilon\hat{R}^2\) can be considered as a general quadratic curvature Lagrangian for the homogeneous, isotropic cosmological model, i.e., the pseudo-Riemannian curvature square \(\hat{R}_{abcd}\hat{R}^{abcd}\) and the Ricci curvature square \(\hat{R}_{ab}\hat{R}^{ab}\) can be expressed in terms of \(\hat{R}^2\) by using the Gauss–Bonnet theorem and a homogeneous, isotropic assumption [2]. It should be stressed that this demonstration will not be true in a Riemann–Cartan spacetime, i.e., the metric-compatible connection \(\nabla\) with torsion. 

Since magnitudes of curvature are expected to be very large in the early universe, the effects of quadratic curvature invariants may dominate. So it is reasonable to include them in a Lagrangian, which is a linear function of scalar curvature. However, it is not satisfactory to restrict these investigations in pseudo-Riemannian geometry, since the connection \(\nabla\) and the metric \(g\) are fundamentally independent [2]. Therefore, the fundamental variables of gravity should be considered as \([g, \nabla]\). It motivates us to investigate quadratic curvature effects, which is based on Riemann–Cartan spacetime, during an inflationary epoch. In this paper, we extend the Einstein–Cartan theory to include all of quadratic curvature invariants, i.e. five independent invariants, in the early universe. Since the dimensions of torsion and curvature are \(L^{-1}\) and \(L^{-2}\) respectively, quadratic torsion effects can be neglected by comparing them to the quadratic curvature in the early universe. So we do not include quadratic torsion terms in the Lagrangian. It turns out to be a degenerate case of the Poincaré gauge theory of gravity (PGT), which does have three quadratic torsion invariants [6]. A homogeneous, isotropic cosmological model in the PGT has been developed and many special cases and solutions have been discovered [8, 16, 17, 20]. These special cases have been used to explain dark energy problems [17, 20] and inflation [16]. In [16], the authors reduced the PGT field equations into second-order differential equations with respect to the Hubble radius by certain restrictions on indefinite parameters and also introduced a scalar field and matter fields during inflation. However, our inflationary model is completely different from [16]. We start from a general quadratic curvature Lagrangian in the Riemann–Cartan spacetime and then obtain a pure gravity inflationary model without introducing any scalar or matter fields.

Our purpose in this paper is to investigate quadratic curvature effects in the early universe and also to analyze whether these effects will generate inflation. Since our analysis is based on the Riemann–Cartan spacetime, there are five independent quadratic curvature invariants in the Lagrangian. It is worth pointing out that our work is not a special case of \(f(\hat{R})\) gravity with torsion [4], where \(\hat{R}\) is a scalar curvature. In the \(f(\hat{R})\) gravity with torsion, it does not include all possible curvature invariants, e.g. Ricci square \(\hat{R}_{ab}\hat{R}^{ab}\), in its Lagrangian. There is no fundamental reason for us to neglect these terms in the early universe. Moreover, we do find that these neglected terms can yield a power-law inflation. The field equations of \(f(\hat{R})\) gravity with torsion give totally antisymmetric torsion tensor, i.e. axial-vector torsion, vanishing. Conversely, our inflationary solution is purely determined by the time component of the axial-vector torsion \(\chi\) called axial torsion.

The plan of this paper is as follows. In section 2, we give a brief review of Riemann–Cartan geometry. Moreover, we start from a general quadratic curvature Lagrangian and obtain an effective quadratic curvature Lagrangian, which contains only three quadratic curvature invariants, for a homogeneous, isotropic cosmological model. A similar argument has been done in a pseudo-Riemannian spacetime [2, 15]. In section 3, we derive the equations of motion for inflationary cosmology. Since we assume that there is no matter field during inflation, it further reduces one more parameter. The equations of motion will contain only two parameters. In section 4, we find an inflationary solution which corresponds to a power-law inflation. The e-folding number \(N\), which is determined by one of the two parameters,
can satisfy the standard requirement $N > 60$. At the end of inflation, $\chi$ will enter into an oscillatory phase. In this paper, $\hbar = c = 1$ and the metric signature is $(- + + +)$.

2. Riemann–Cartan geometry and the quadratic curvature Lagrangian

Since Einstein developed GR in terms of a pseudo-Riemannian spacetime, it was argued that GR has a pre-geometry, especially when one discovered that intrinsic spins can naturally generate torsion tensor. A directly extended theory of GR to include torsion is called Einstein–Cartan theory, which is based on a Riemann–Cartan spacetime. If the spin densities of matter sources vanish, the Einstein–Cartan theory will return to GR. Since torsion effects generated by spin sources are too small, the current experiments still cannot distinguish their differences. However, these two theories may become significantly different in the early universe due to quadratic curvature effects.

The geometry of a Riemann–Cartan spacetime is described by the metric $g$ and a metric-compatible connection with torsion $\nabla$. The curvature 2-forms $R_{ab}$ and torsion 2-forms $T^a$ can be expressed in terms of connection 1-forms $\omega_{ab}$ and an orthonormal co-frame $e^a$:

$$R_{ab} = d\omega_{ab} + \omega_{ac} \wedge \omega^c_b,$$

$$T^a = de^a + \omega^a_b \wedge e^b = De^a,$$

where $D$ is the covariant exterior derivative [3]. In an orthonormal frame, $\omega_{ab}$ satisfy the antisymmetric condition

$$\omega_{ab} + \omega_{ba} = 0,$$

where the indices are lowered by $\eta_{ab} = \text{diag}(-1, 1, 1, 1)$. In the following, Latin indices are raised and lowered by $\eta_{ab}$.

It has been shown that the curvature 2-forms $R_{ab}$ can be irreducibly decomposed into the following pieces [14]: $R_{ab} = \frac{1}{6} R_{ab}$(Weyl) + $\frac{2}{5} R_{ab}$(Paircom) + $\frac{3}{4} R_{ab}$(Pseudoscalar) + $R_{ab}$(SymRicci) + $R_{ab}$(AntiRicci) + $R_{ab}$(Scalar). Moreover, the irreducible components of torsion 2-forms yield $T^a = \frac{1}{5} T^a$(Tensor) + $\frac{2}{3} T^a$(Vector) + $T^a$(Axitor). It should be pointed out that one has $\frac{2}{5} R_{ab} = \frac{1}{6} R_{ab} = \frac{1}{5} R_{ab} \equiv 0$ and $T^a \equiv 0$ in the pseudo-Riemannian spacetime. In terms of $R_{ab}$, the most general quadratic curvature action (neglected any possible derivative on curvature) in the Riemann–Cartan spacetime is given by

$$S[e^a, \omega_{ab}] = \frac{1}{2\kappa} \int_M \Lambda \ast 1 + c_0 R_{ab} \wedge \ast e^a \wedge e^b + R_{ab}^{\alpha} \wedge \sum_{\alpha=1}^{6} c_{\alpha} \ast R_{ab},$$

where $\kappa = 8\pi G$ and $e^{\cdot \cdot \cdot \cdot}_{c \cdot \cdot \cdot \cdot}$ denotes $e^a \wedge \cdots \wedge e^b \wedge e_c \wedge \cdots \wedge e_d$. $\Lambda$ is the cosmological constant and $\ast$ denotes the Hodge map associated with $g$. $c_0$ is a dimensionless parameter and $[c_{\alpha}] = L^2$. In the PGT, the most general gauge invariant action should also include three irreducible pieces of torsion square in its Lagrangian [6, 8]. Although $R_{ab}$ have six irreducible components, there are only five independent quadratic curvature invariants due to the generalized Gauss–Bonnet theorem, which gives [19]

$$\varepsilon_{abcd} R^{ab} \wedge R^{cd} = \text{an exact form}.$$
the Newtonian theory in the weak field approximation [5, 18], we should fix $c_0 = 1$, which actually gives Einstein–Cartan action with $\Lambda$ if quadratic curvature invariants vanish.

It is straightforward to obtain field equations by varying $S$ with respect to $\{e^a\}$ and $\{\omega_{ab}\}$ and these equations should contain five independent parameters. However, one can further reduce two parameters in the field equations according to an assumption of homogeneous, isotropic universe. It can be verified that the homogeneous and isotropic assumption yields $R_{ab} = R_{ab} = 0$ by using (9) and (10), so quadratic $R_{ab}$ and $5R_{ab}$ do not contribute to field equations for homogeneous, isotropic cosmology. It is worth pointing out that they may participate in the perturbed field equations. Since our current work is only concentrated on the homogeneous and isotropic universe, we can start from the following effective action:

$$S_E[e^a, \omega_{ab}] = \frac{1}{2\kappa} \int_M \left[ \Lambda + R_{ab} \wedge e^{ab} + b_3 R_{ab} \wedge e^{ab} \wedge * (R_{ab} \wedge e^{ab}) \right]$$

$$+ b_4 P_a \wedge * P^a - b_5 R_{ab} \wedge e^{ab} \wedge * (R_{ab} \wedge e^{ab})],$$

where $P_b = i_a R_{ab}$ denotes Ricci 1-forms. So it only involves three indefinite parameters. In the following, we set $\Lambda = 0$ since the effects of $\Lambda$ in the early universe are expected to be small.

Varying $S_E + S_{\text{matter}}$ with respect to the orthonormal co-frames $\{e^a\}$ and connection 1-forms $\{\omega_{ab}\}$ gives the following field equations:

$$R_{ab} \wedge e^{ab} = b_2 E \left( E \wedge e + 2i_t R_{ab} \wedge e^{ab} \right) + b_4 \left( i_c P^b \wedge * P_b \right)$$

$$+ 2i_t R^{ab} \wedge t_a + b_5 R_{ab} \wedge e^{ab} \wedge * (R_{ab} \wedge e^{ab}) = -2\kappa \tau_c,$$

$$T^c \wedge e^{ab} = 2b_3 D(E e_{ab}) - b_4 D * (P^a \wedge e^a - b_5 \wedge e^b)$$

$$+ 2b_5 D(R * e^{ab}) = -2\kappa S^{ab},$$

where $R = i_a P^a$ is the scalar curvature and $E = \frac{1}{2} R_{abcd} e^{abcd}$ is the totally antisymmetric curvature. $i_a$ denotes the interior derivative and $e^{abcd}$ is the Levi-Civita antisymmetric $\varepsilon$-symbol. $\tau_c$ and $S^{ab}$ are stress–energy and spin density 3-forms of the matter fields.

3. Equations of motion for inflationary cosmology

In an isotropic, homogeneous cosmological model, the spacetime metric and torsion 2-forms become [8]:

$$g = -dt \otimes dt + a^2(t) \left( 1 + \frac{1}{4}k r^2 \right)^{-2} \sum_{A=1}^{3} dx^A \otimes dx^A,$$

$$T^a = f(t) e^a \wedge e^0 + 2\chi(t) * (e^a \wedge e^0),$$

where curvature constant $k = -1, 0$ or 1. For simplicity, we only concentrate on the flat universe $k = 0$. Moreover, the non-vanishing components of $\tau_a$ and $S^{ab}$ yield

$$\tau_a = \eta_0 \rho(t) + p(t) \wedge e_0,$$

$$S^{ab} = -q(t) e^{[a} \wedge * (e^{b]} \wedge e^0) + s(t) e^0 \wedge e^a \wedge e^b.$$
In standard inflation models, the energy density $\rho$ and pressure $p$ of an inflaton field are given by $\rho = \frac{1}{2} \dot{\phi}^2 + V(\phi)$ and $p = \frac{1}{2} \dot{\phi}^2 - V(\phi)$ [9]. It still lacks a fundamental theory which can explain the origin of the inflaton fields and also derive the potential $V(\phi)$. Moreover, there is no observational evidence to yield any feature of matter fields during an inflationary epoch. In this paper, we will assume $\tau_a = S^0b = 0$ in the early universe and then obtain a pure gravity inflationary model in the Riemann–Cartan spacetime.

Using (9)–(12), the field equations (7)–(8) with $k = 0$ yield

$$\{ (H + f)^2 - \dot{\chi}^2 \} + \frac{b_3}{6} E^2 - 4b_3 E \dot{\chi} (H + f)$$

$$= \frac{(b_4 + 3b_6)}{18} R^2 + \frac{2(b_4 + 3b_6)}{3} R \{(H + f)^2 - \dot{\chi}^2 \} = \frac{\kappa \rho}{3}, \quad (13)$$

$$R = \kappa (\rho - 3p), \quad (14)$$

$$\left( b_6 + \frac{b_4}{3} \right) (R - 2R f) + 2b_4 \dot{\chi} (H + F) + 2b_3 E \dot{\chi} - f = -\frac{\kappa q}{2}, \quad (15)$$

$$b_3 (E - 2f E) - \frac{(b_4 + 6b_6)}{3} R \dot{\chi} - 2b_4 \dot{\chi} \{(H + f)^2 - \dot{\chi}^2 \} - \dot{\chi} = -\kappa s, \quad (16)$$

where

$$R = 6(\dot{H} + 2H^2 + 3H f + \dot{f} + f^2 - \dot{\chi}^2), \quad (17)$$

$$E = 6(\dot{\chi} + 3H \chi + 2f \dot{\chi}). \quad (18)$$

and $H = \frac{\dot{a}}{a}$ denotes the Hubble parameter. It can be shown that (13)–(16) form a complete system to describe the evolution of the isotropic, homogeneous flat universe once the equation of state $p = p(\rho)$ is given. In particular, $\rho = p = 0$ (inflationary epoch) or $p = \frac{\rho}{3}$ (radiation domination) both yield $R = 0$ (see (14)). It is not difficult to see that (13) yields the Friedmann equations when the quadratic curvature terms and the spin density vanish. By substituting $R = 0$ into (13), (15) and (16), one can easily verify that only $b_1$ and $b_2$ are left in these equations.

Since we consider $\rho = p = q = s = 0$ during inflation, (13)–(16) with (18) then become

$$\dot{E} = \frac{\chi}{b_3} + 2(\Phi - H)E + \frac{2b_4}{b_3} \chi (\Phi^2 - \chi^2), \quad (19)$$

$$\dot{\chi} = \frac{E}{6} - H \chi - 2\chi \Phi, \quad (20)$$

with the two algebraic equations

$$\Phi = 2b_3 E \chi \pm A, \quad (21)$$

$$H = -\frac{b_4}{3} E \chi + 8b_3 b_4 E \chi^3 \pm (1 + 4b_4 \chi^2) A, \quad (22)$$

where $A = \sqrt{(2b_3 E \chi)^2 + \chi^2 - \frac{b_4 E}{6}}$ and $\Phi = H + f$. We further simplify (19) and (20) by substituting (21) and (22) into them, which gives

\[\begin{align*}
(13) & \text{–} (16) \text{ are a degenerated case of the field equations (4.2)–(4.5) in [8].}
\end{align*}\]
\[ \dot{E} = \frac{\chi}{b_3} + \left( 4b_3 + \frac{b_4}{3} \right) E^2 \chi, \quad (23) \]

\[ \dot{\chi} = \frac{E}{6} + \left( \frac{b_4}{3} - 4b_3 \right) E \chi^2 - \left( 3 + 4b_4 \chi^2 \right) \mathcal{A} \chi - 8b_3 b_4 E \chi^4. \quad (24) \]

It turns out that the evolution of the inflationary universe is characterized by \( b_3, b_4 \) and the initial values \( (t = 0) \) of \( E \) and \( \chi \). We should point out that here \( t = 0 \) denotes the beginning of inflation. It is reasonable to require \( b_3 \equiv -b < 0 \) in order to ensure that \( \mathcal{A} \) is a real function.

Moreover, we assume that the initial values \( E_0 \) and \( \chi_0 \) are both less than or on the order of the Planck scale to ensure the classical validity of the evolution [15].

4. Power-law inflation

We first note that (22)–(24) has a similarity to hybrid inflation models [11, 12], with the effective potential given by

\[ V( E, \chi) = \frac{3}{\kappa} - \frac{1}{H^2}. \]

In hybrid inflation models, the effective potentials \( V( \sigma, \phi) \) are constructed in such a way that one scalar field \( \sigma \) during slow-roll inflation, and starts to roll down to its true vacuum when the other scalar field \( \phi \) falls to a critical value \( \phi_c \).

We adopt a similar argument by considering \( |\chi| \approx |\chi_0| \ll \frac{1}{l_{ph}} \) during inflation and \( |E_0| \sim \frac{1}{l_{ph}^2} \). By neglecting \( \chi^2 \) in \( \mathcal{A} \), (24) reduces to an algebraic cubic equation

\[ \left( 32\alpha + \frac{8\alpha^2}{3} \right) \tilde{\chi}^3 - \left( 20 + \frac{4\alpha}{3} - \frac{\alpha^2}{9} \right) \tilde{\chi}^2 - \left( \frac{1}{6} + \frac{\alpha}{9} \right) \tilde{\chi} + \frac{1}{36} \approx 0, \quad (25) \]

where \( \tilde{\chi} = b \chi_0^2 > 0 \) and \( \alpha = -\frac{b}{\tilde{\chi}} \), which are both dimensionless parameters. From numerically and analytically analyzing (25), we find that the condition \( \alpha \sim \tilde{\chi} \sim O(1) \), which is consistent with neglecting \( \chi^2 \) in \( \mathcal{A} \), can naturally provide the sufficient number of e-foldings (see below), so we will assume the condition being satisfied in the following discussions.

Since \( b = \frac{b}{\tilde{\chi}} \gg \frac{1}{l_{ph}^2} \), one can neglect \( \chi b_3 \) in (23), and it becomes

\[ \dot{E} \approx -\left( 4 + \frac{\alpha}{3} \right) b \chi_0 E^2. \quad (26) \]

(26) then yields a general solution

\[ E \approx B \left( t + \frac{B}{E_0} \right)^{-1}, \quad (27) \]

where \( B = \frac{3}{12\alpha b_3 \chi_0} \). By substituting (27) into (22), we obtain

\[ H \approx p(t + \frac{B}{E_0})^{-1}, \quad \text{where} \]

\[ p = \frac{3}{12 + \alpha} \left( \frac{a}{3} + 8\alpha \tilde{\chi} \pm (1 - 4\alpha \tilde{\chi}) \sqrt{4 + \frac{1}{6\tilde{\chi}}} \right) \quad (28) \]

for the case \( \chi_0 E > 0 \), and \( \pm \) should be changed to \( \mp \) for the other case \( \chi_0 E < 0 \). So a solution of \( a(t) \) for \( a(t) > 0 \) gives

\[ a \approx A_0 \left( t + \frac{B}{E_0} \right)^p, \quad (29) \]

where \( A_0 = a_0 \left( \frac{B}{E_0} \right)^{-p} > 0 \), and \( a_0 > 0 \) denotes the initial value of \( a \).

The requirement of positive kinetic energy in the spin 0\(^{-}\) mode of linearized PGT also gives \( b_1 < 0 \) [5].
It is known that a general criterion for inflation is $\ddot{a} > 0$, which may be sufficient to solve the horizon and flatness problems. It turns out that to require $p > 1$ in (29) corresponds to a new type of the power-law inflation [1, 13]. From (28), one can see that $p > 1$ will limit the value of $\alpha$. For example, if we consider a degenerate case $\alpha = 0$, it gives $p = \frac{1}{2}$ which does not have inflation. However, in the $\alpha = 1$ case, which yields $\chi \approx 0.623$, the numerical calculations of (28) while choosing the negative sign in the round bracket yield $p \approx 1.938$, which gives power-law inflation. From numerical calculations, we also find that $1 \leq p \leq 2$ when $\alpha > 0$. In addition to $\ddot{a} > 0$, the amount of inflation is usually required to satisfy $N \geq 60$, where $N \equiv \ln \frac{a(t_{\text{end}})}{a_0}$ denotes the number of e-foldings, and $t_{\text{end}}$ means the end of inflation. In order to estimate $N$, we consider that the inflation comes to an end when (25) is no longer valid, and it occurs when $\chi^2$ in $\mathcal{A}$ cannot be neglected, i.e.

$$|bE_{\text{end}}^2| \sim \chi_0^2 \quad \text{or} \quad |b^2E_{\text{end}}^2| \sim O(1). \quad (30)$$

From (27) and (30) with the help of $\alpha \sim \chi \sim O(1)$, the values $E_{\text{end}}$ and $t_{\text{end}}$ can be estimated as $E_{\text{end}} \approx \frac{3}{12\pi\omega m}$ and $t_{\text{end}} \approx \frac{1}{\omega m}$. By substituting $t_{\text{end}} \approx \frac{1}{\omega m}$ into the number of e-foldings

$$N = \ln \left( \frac{E_0}{Bt_{\text{end}} + 1} \right)^p, \quad (31)$$

we obtain $e^N \approx (4 + \frac{1}{2})E_0b^p$. It turns out that the value of $N$ is determined by the parameter $b$. In the $\alpha = 1$ case, $N \geq 60$ requires $b > 10^{15.5} \sim 10^{-55}m^2$, which satisfies the tests of solar experiments [7]. After the end of inflation, $\chi$ and $E$ both become very small, and the linear terms in (19) and (20) should finally dominate. In the linearized equations of (23) and (24), one can easily obtain oscillatory solutions: $\chi \sim \sin \frac{\sqrt{B}}{\sqrt{m}}$ and $E \sim \cos \frac{\sqrt{B}}{\sqrt{m}}$. Whether these oscillatory phases can provide a reheating process is still in progress.

5. Conclusion

In conclusion, we first discovered a pure gravity inflationary model in $R + R^2$ gravity theories with torsion, and presented a detailed analytic analysis of this model. It turns out that the dynamics of the inflationary model is purely determined by two parameters $b$ and $\alpha$ with two initial values $E_0$ and $\chi_0$. We further found that $|\chi_0| \ll l_{\text{ph}}^{-1}$ and $|E_0| \sim l_{\text{ph}}^{-2}$ yields a power-law inflation, if $\alpha > 0$. In this power-law inflation solution, $\chi$ is near a constant $\chi_0$ and $E \sim t^{-1}$. Since $\chi$ cannot be near $\chi_0$ after $E$ decays to satisfy the condition $|bE_{\text{end}}^2| \sim \chi_0^2$, it can be considered as the end of inflation. The period of inflation can be estimated to be $\frac{1}{\chi_0}$. Moreover, the e-folding number $N$ is completely determined by $b$ and also satisfies the requirement $N > 60$ when $b > 10^{15.5} \sim 10^{-55}m^2$.

After the end of inflation, the system may enter into an asymptotical regime which yields oscillatory phases. These oscillatory phases are expected to give a reheating process. Since CMB anisotropy has been discovered, a further understanding of the reheating process and primordial perturbations will provide more restricted constraints on $b$, $\alpha$, $E_0$, and $\chi_0$. These issues will be considered as our future work.

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References

[1] Abbott L F and Wise M B 1984 Nucl. Phys. B 244 541
[2] Barrow J D and Ottewill A C 1983 J. Phys. A: Math. Gen. 16 2757
[3] Benn I M and Tucker R W 1987 An Introduction to Spinors and Geometry with Applications to Physics (Bristol: Institute of Physics Publishing)
[4] Capozziello S, Cianci R, Stornaolo C and Vignolo S 2007 Class. Quantum Grav. 24 6417
[5] Hayashi K and Shirafuji T 1980 Prog. Theor. Phys. 64 866
Hayashi K and Shirafuji T 1980 Prog. Theor. Phys. 64 883
Hayashi K and Shirafuji T 1980 Prog. Theor. Phys. 64 1435
Hayashi K and Shirafuji T 1980 Prog. Theor. Phys. 64 2222
[6] Hehl F W, von der Heyde P, Kerlick G D and Nester J 1976 Rev. Mod. Phys. 48 393
[7] Gladchenko M S and Zhytnikov V V 1994 Phys. Rev. D 50 5060
[8] Goenner H and Müller-Hoissen F 1984 Class. Quantum Grav. 1 651
[9] Liddle A R and Lyth D 2000 Cosmological Inflation and Large-Scale Structure (Cambridge: Cambridge University Press)
[10] Linde A D 1990 Particle Physics and Inflationary Cosmology (Chur: Harwood Academic)
[11] Linde A D 1990 Phys. Lett. B 249 18
[12] Linde A D 1991 Phys. Lett. B 259 38
[13] Lucchin F and Matarrese S 1985 Phys. Rev. D 32 1316
[14] Mijíc M B, Morris M M and Suen W-M 1986 Phys. Rev. D 34 2934
[15] Minkevich A V and Garkun A S 2006 Class. Quantum Grav. 23 4237
[16] Minkevich A V, Garkun A S and Kudin V I 2007 Class. Quantum Grav. 24 5835
[17] Müller-Hoissen F 1983 Gen. Rel. Grav. 15 1051
[18] Nich T B 1980 J. Math. Phys. 21 1439
[19] Shie K F, Nester J M and Yo H J 2008 Phys. Rev. D 78 023522
[20] Starobinsky A A 1980 Phys. Lett. B 91 99
[21] Wands D 1994 Class. Quantum Grav. 11 269
[22] Whitt B 1984 Phys. Lett. B 145 176