1 Introduction

When searching for physics beyond the Standard Model (BSM), including in CP-violating observables, one often encounters multi-hadron final states, e.g., in semi-leptonic, D- and B-meson decays. In addition, the identification of exotic resonances in such final states requires control over rescattering effects. Especially for heavy particles decaying with net strangeness, it thus becomes increasingly important to also describe the abundantly appearing final-state interactions of kaons and pions up to high energies. In particular a consistent description of πK scattering and production can serve as a test of SM physics, be used to search for exotic hadronic states in crossed channels, and improve the spectroscopy of excited kaon resonances.

To be more specific, in the quest for CP violation beyond the SM the inelastic contributions to the πK channel enter in the CP asymmetry in τ → K_S π ν_τ generated by a tensor operator, as the elastic contributions cancel by Watson’s theorem [2]. Further, in the hunt for exotic hadrons the Z_c(4430) was discovered by Belle and LHCb in the reaction B → ψ’πK in the ψ’π subsystem [3,4]. Since, to describe such a crossed process, the different partial waves of the πK subsystem interfere, a high control over especially their phases is compulsory, which
cannot be achieved by a simple Breit–Wigner (BW) model. Hence, generally speaking a better understanding of the $\pi K$ form factors is needed to describe all these processes appropriately.

For these purposes, we constructed a representation of the $\pi K$ $S$-wave form factor using the elastic $\pi K$ scattering phase shifts via dispersion relations in the elastic region, as demanded by Watson’s theorem, and extended this model into the inelastic region using resonance exchange, while maintaining unitarity and the correct analytic structure [5, 6]. As a first application, we successfully described the $\tau \to K_S \pi \nu_\tau$ spectrum, including the highly overlapping $S$-wave resonance $K^*_0(1430)$ and $P$-wave resonance $K^*(1410)$. In contrast to common BW parameterizations, which violate unitarity, our parameterization has the correct phase behavior built in and fulfills unitarity by construction. For an improved separation of these resonances using future measurements, we further calculated forward–backward (FB) asymmetries for the different fit scenarios. In addition, we could use our results to refine the estimate of the $C P$ asymmetry generated by a tensor operator. Finally, we were able to extract the resonance properties of the $K^*_0(1430)$ and $K^*_0(1950)$ via Padé approximants. Here, we provide a summary of the main ideas and applications, while deferring a more detailed discourse to Ref. [1].

## 2 Formalism

Our formalism for the $T$-matrix that fulfills the criteria of unitarity and analyticity is built upon the Bethe–Salpeter equation, which in channel space in matrix form reads
\[
T_{if} = V_{if} + V_{im}G_{mm}T_{mf},
\]
and fulfills unitarity as long as $V_{if} \in \mathbb{R}$ and $\text{disc} G_{mm} = 2i \rho_m$, where $\rho_m$ denotes the two-body phase space in channel $m$. Furthermore, we employ the so-called two-potential formalism [7], which starts by splitting the scattering potential $V$ into two pieces
\[
V = V_0 + V_R.
\]
Accordingly, this results in a corresponding splitting of the $T$-matrix
\[
T = T_0 + T_R,
\]
where $T_0$ fulfills the Bethe–Salpeter equation that has $V_0$ as input, $T_0 = V_0 + V_0 GT_0$. In our application we assume $T_0$ to be purely elastic and consider the additional channels to couple only through the resonance exchange in $T_R$, as motivated by the phenomenologically successful isobar model [8–10]. Employing a two-channel setup, corresponding in our application to $\pi K$ and $\eta' K$, we therefore have
\[
T_0 = \begin{pmatrix} \frac{1}{\rho_1} \sin \delta_0 e^{i\delta_0} & 0 \\ 0 & 0 \end{pmatrix},
\]
which only depends on the scattering phase $\delta_0$ and makes any explicit parameterization of $V_0$ obsolete, using an empirical parameterizations of $\delta_0$ instead.

By means of dispersion theory we can use the given constraints to calculate $T_R$ and consequently the full scattering $T$-matrix, which is given as
\[
T = T_0 + T_R = T_0 + \Omega [\mathbb{1} - V_R \Sigma]^{-1} V_R \Omega^T,
\]
with
\[
\Omega = \begin{pmatrix} \Omega_{11} & 0 \\ 0 & 1 \end{pmatrix}, \quad \Omega_{11} = \exp \left( \frac{s}{\pi} \int_{s_{th}}^{\infty} dz \frac{\delta_0(z)}{z(z-s)} \right).
\]
the Omnès function for the $\pi K$ channel and
\[
\Sigma_{ij}(s) = \frac{s}{2\pi i} \int_{s_{th}}^{\infty} \frac{\Omega^T_{im}(z) \text{disc } G_{mn}(z) \Omega_{mf}(z)}{z(z-s)} dz,
\]
(7)

the dressed loop operator also called self energy. Furthermore, we parameterize the resonance potential $V_R$ as
\[
V_R(s)_{ij} = \sum_r g_{i(r)} \frac{s-s_0}{(s-\tilde{M}^2_r)(s_0-\tilde{M}^2_r)} g_{j(r)},
\]
(8)

which is subtracted at $s_0 = (M_K + M_\eta)^2$ to remove low-energy contributions already considered via $T_0$. Here the mass $\tilde{M}_r$ is the bare mass of resonance $r$ and $g_{i(r)}$ is the bare coupling of the resonance $r$ to channel $i$. The scalar form factor $f_\pi$, given in channel space via
\[
(f_\pi)_i = M_i + T_{im}G_{mn}M_m,
\]
(9)
can now be expressed as
\[
f_\pi(s) = \Omega(s) [\mathbb{1} - V_R(s)\Sigma(s)]^{-1} M(s),
\]
(10)

where $M$ is a reparameterized source term, which can be written as
\[
M_i = \sum_{k=0}^{k_{\text{max}}} c_i^{(k)} s^k - \sum_r g_{i(r)} \frac{s-s_0}{(s-\tilde{M}^2_r)(s_0-\tilde{M}^2_r)} \alpha^{(r)}.
\]
(11)

Here, the coefficients $c_i^{(k)}$ and the resonance couplings $\alpha^{(r)}$ are parameters of the model that depend on the source.

3 Fit to scattering data

We fit our parameterization of the $\pi K I = \frac{1}{2} S$-wave in combination with an elastic $I = \frac{3}{2}$ parameterization from Ref. [11] to the data of Ref. [12]. As phase input we use the elastic part of Ref. [11], where we turn off all resonant contributions and guide the phase smoothly to $\pi$ at $\sqrt{s_{\text{m}}} = 1.52$ GeV, thus only including the lowest resonance $K_0^*(700)$ within the phase. As we aim at a description from the $\pi K$ threshold up to 2.5 GeV the additional scalar resonances $K_0^*(1430)$ and $K_0^*(1950)$ are included explicitly via $V_R$. We consider a two-channel setup, incorporating only the $\pi K$ and $\eta' K$ channel, as the $\eta K$ channel turns out empirically to effectively decouple in that energy range. Figure 1 shows the results of the combined fit of argument and absolute value. Considering the modest quality of the data, we find the fit suitable up to about 2.3 GeV. An extension to higher energies would require the inclusion of yet another $K_0^*$ resonance, which however would require reliable data up to even higher energies.

4 Application to $\tau$ decays

Using the scattering parameterization acquired in the previous section we now have, using Eq. (10), a parameterization of the $\pi K I = \frac{1}{2}$ scalar form factor, with free parameters only contained within the source term (11). For now we use this parameterization to describe the
scalar form factor within the decay spectrum of $\tau^- \to K\pi^- \nu_\tau$ as measured by the Belle collaboration [14]. We fix the scalar form factor using our parameterization via $(f_s)_1$, see Eq. (10), while we use for the vector form factor a more conventional parameterization from resonance chiral perturbation theory [15–19], where we follow the conventions from Ref. [19]. There we choose the subtraction constants fixed to their central values (as determined independently from $K_{13}$ decays) and allow the bare mass and width as well as the mixing parameter to vary in such a way that the shape of the generated $\pi K$ $P$-wave remains phenomenologically viable.

We find four representative fit scenarios, which turn out to be visibly indistinguishable in the total decay rate. However, the actual scalar form factors contained within the four parameterizations, see Fig. 2, differ more substantially, especially above the $\eta'K$ threshold, where the contribution to the decay rate is very suppressed by the phase space and the data points have large uncertainties. All fits are equally well suitable to describe the decay rate and have similar fit statistics with a reduced $\chi^2 \approx 1$. Despite the strong overlap of the $K^*(1410)$ and $K^*_0(1430)$ in the total decay rate, which in the past was usually solved by discarding one of the resonances, we were able to get distinct parameters for both resonances, as in our application the resonance couplings to the $K^*_0(1430)$ are already fixed by the scattering data. The $K^*_0(1950)$, on the other hand, is difficult to constrain from this fit, as it lies above the $\tau$ mass and its influence on the decay region is quite limited, which can also be seen by the huge contributions in the scalar form factor in Fit 1 and 2, above the boundary of the phase space with a free $\alpha^{(2)}$. However, even the parameterizations without a source term coupling to the $K^*_0(1950)$ (Fit 3 and 4 with $\alpha^{(2)} = 0$) show resonant structures around its mass, which nicely shows the built-in unitarity indirectly including the knowledge of the scattering phase about...
5 Implications and results

5.1 Forward–backward asymmetry

One such possibility to further distinguish the different fit scenarios would be to examine the FB asymmetry \([20,21]\) as can be measured by Belle II \([21]\):

\[
A_{FB}(s) = \frac{\int_0^1 dz \left[ \frac{df}{dz}(z) - \frac{df}{dz}(-z) \right]}{\int_0^1 dz \left[ \frac{df}{dz}(z) + \frac{df}{dz}(-z) \right]} = \frac{-2 \text{Re}(f_0^*f_+^*) \Delta_{\pi K} q_{\pi K} \sqrt{s}}{|f_0|^2 \Delta_{\pi K}^2 + \frac{4}{3} |f_+|^2 q_{\pi K}^2 \left( \frac{2s^2}{m_c^2} + s \right)},
\]

(12)

all resonances. Furthermore, by construction the phase of the scalar form factor is fixed up to the \(\eta'K\) threshold to the scattering phase as demanded by Watson’s theorem and elastic unitarity. This is in marked contrast to the BW parameterizations by Belle \([14]\), which show very inconsistent phases at low energies and unphysical structures below the \(\pi K\) threshold.

Including a linear term \(c_1^{(1)}\) in the source term (Fit 2 and 4) gives some slight improvements to the fit quality, but changes the high-energy behavior of the scalar form factor to approach a constant instead of falling of like \(1/s\), as expected by perturbative-QCD arguments. As the fit statistics are even without this linear term sufficient, we conclude that all fits are basically equivalent. To further distinguish the different fit scenarios, additional data are required.
5.2 Branching ratio

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Figure 3: FB asymmetry as defined in Eq. (12) for the four fit variants.

where \( z \) denotes the cosine of the \( \pi K \) helicity angle. This quantity would separate vector and scalar components and make it easier to distinguish the \( K^*(1410) \) and \( K_S^*(1430) \) contributions. We show the FB asymmetry for the four fit scenarios in Fig. 3. Major differences can be seen above the \( \eta'K \) threshold, as expected due to the different phase motion in that energy region.

5.2 Branching ratio

By integrating over the differential decay rate we calculate the branching ratio for \( \tau \to K_S \pi \nu_\tau \). Averaging over all four fit scenarios with its spread as systematic uncertainty (sys) we get

\[
\text{BR}(\tau \to K_S \pi \nu_\tau) = 4.35(6)_{\text{st}}(3)_{\text{norm}}(7)_{\text{sys}} \times 10^{-3} = 4.35(10) \times 10^{-3},
\]

where we also included the statistical error (st) propagated from the fit parameters and the uncertainty from the normalization constants (norm). Although this result is 2\( \sigma \) above the original Belle result \( \text{BR}(\tau \to K_S \pi \nu_\tau) |^{[14]} = 4.04(13) \times 10^{-3} \), it agrees at 1.5\( \sigma \) with the more recent \( \text{BR}(\tau \to K_S \pi \nu_\tau) |^{[22]} = 4.16(8) \times 10^{-3} \) as well as the Particle Data Group average \( \text{BR}(\tau \to K_S \pi \nu_\tau) |^{[23]} = 4.19(7) \times 10^{-3} \).

5.3 \( CP \) asymmetry

We were further able to improve the estimate of the BSM \( CP \) asymmetry produced by a tensor operator with Wilson coefficient \( c_T \) interfering with the vector operator. Due to the absence of a scalar–vector interference this is the only option to generate a \( CP \) asymmetry with new heavy degrees of freedom. As shown in Ref. \([2]\), by Watson’s theorem it follows that in this case all elastic contributions to the \( CP \) asymmetry cancel identically, as vector and tensor operators follow the same unitarity condition and thus have to have the same strong phase in the purely elastic case. Hence inelastic effects, included in our parameterization via resonance exchange, are mandatory to obtain a non-vanishing \( CP \) asymmetry. Within the SM, a \( CP \) asymmetry is generated by \( K^0-\bar{K}^0 \) mixing, but the corresponding prediction shows a 2.8\( \sigma \) tension with the 2012 BaBar measurement \([24]\), which could point to \( CP \) violation beyond the SM. Using our parameterization we find

\[
A_{CP}^{\tau, \text{BSM}} = -0.034(14) \text{ Im } c_T,
\]

which supports the simpler estimate of Ref. \([2]\). Unfortunately, limits on \( \text{Im } c_T \) from the neutron electric dipole moment and \( D-\bar{D} \) mixing rule out this mechanism to explain the tension.
5.4 Pole extraction

We were able to extract the pole positions as well as residues of the two scalar resonances \( K_0^* (1430) \) and \( K_0^* (1950) \) using Padé approximants as shown in Fig. 4. For both resonances our results are in reasonable agreement with previous extractions, indicating a \( K_0^* (1430) \) mass towards the lower end, see Ref. [1] for more details on the extraction of the pole parameters and residues.

Furthermore, we extracted the coupling of the \( R = K_0^* (1430) \) resonance to the \( \bar{s} \gamma^\mu u \) current in a model-independent way via its residue \( C_R^{1\mu} \), which can then be re-interpreted in a narrow-width sense as a decay rate

\[
\Gamma(\tau \rightarrow R \nu_\tau) = \frac{6 \pi^2 c_\tau \Delta^2_{\pi K}}{M_R^4} \left( 1 - \frac{M_R^2}{m_\tau^2} \right)^2 |C_R^{1\mu}|^2.
\]  

We find an upper bound on the branching ratio of \( \text{BR}(\tau \rightarrow K_0^* (1430) \nu_\tau) < 1.6 \times 10^{-4} \) (at 95% confidence level), which is by a factor of 3 better than the current literature values [23, 34]. Our residues (from the four fit scenarios) are scattered around the values from other theoretical investigations, including Refs. [35, 36], which however are more rigid in the fit function. For instance, Ref. [36] uses a coupled-channel Omnès matrix, which requires inputs for scattering phases and elasticity parameters for all channels that are not available at the moment. The implementation from Ref. [36] circumvents this issue by relying on further chiral constraints, with the scalar form factor uniquely determined from the \( T \)-matrix. Our parameterization, on the other hand, also fulfills the chiral constraints, but allows for more flexibility by adding further terms in the resonance potential.

6 Conclusion

In conclusion, the formalism proposed in Ref. [1] has proven adequate for the description of \( \pi K \) S-wave scattering as well as the scalar form factor up to the \( K_0^* (1950) \). It should thus also allow for a meaningful description of the \( \pi K \) form factors in future analyses of semileptonic \( D \) - and \( B \)-meson decays and transfer to higher partial waves. Future measurements
expected from Belle II, for the $\tau \rightarrow K_\text{S} \pi \nu_\tau$ spectrum, FB and $CP$ asymmetry, will improve the phenomenology presented here, especially in the inelastic region, and thereby provide valuable input for controlling $\pi K$ final-state interactions in more complicated systems.

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