Level Set Method-Based Identification of Locations and Shapes of Fuel Cell Defects

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Abstract: Nondestructive inspection of the sizes and positions of defect regions in a fuel cell is an indispensable task in order to ensure their practical use. In the present paper, a level set method for estimating the defects in a membrane electrode assembly (MEA) in fuel cells is proposed. Boundaries of the defects in the MEA are represented by a zero contour of a level set function. By updating the function, the shapes and positions of the defect regions can be identified. A cost function for updating the level set function consists of the mean squared error between the magnetic fields computed by a 2-D model and sensor data and the regularization terms for the estimated shapes. The magnetic fields were measured using magnetic impedance sensors. The effectiveness of the proposed method is demonstrated through numerical simulations and experiments.

Key Words: fuel cells, inverse problems, level set methods.

1. Introduction

Fuel cells are devices for converting chemical energy directly into electrical energy by electrochemical reactions. Because of their high efficiency and low emission of pollutants, the practical use of fuel cells has been actively researched [1],[2]. Figure 1 shows the structure of a polymer electrolyte fuel cell (PEFC), which is a typical fuel cell that is used mainly in households and vehicles.

However, the problem of the deformation or breakage of the cells should still be studied, and the fuel cells should be improved for their practical use [3]. Defects and the resultant deviation of currents in a fuel cell can cause serious accidents. In the structure of a PEFC, the membrane electrode assembly (MEA) plays a key role in generating electric currents from hydrogen and oxygen. The MEA can have defects due to external factors such as thermal stress, chemical expansion, and long-term use. Hence, a method for nondestructive detection of the defects in the MEA for monitoring the conditions of the fuel cells is needed.

Various studies have been conducted to examine the state of the fuel cells. Hauer et al. measured the three-dimensional magnetic field around a PEFC and then inversely reconstructed the current distribution inside the PEFC based on the Biot-Savart law [4]. However, there was a problem in that the estimated regions came to be oversmoothed due to regularization. Ifrek et al. examined the current density both in a healthy fuel cell and in a faulty fuel cell by singular value decomposition of a discretized Biot-Savart operator [5]. Although this method is able to model the current density of the fuel cell, it cannot precisely detect the size or shape of defects. Nara et al. calculated the external magnetic field from the current density inside the fuel cell using a two-dimensional (2-D) model and proposed a method for predicting the center positions of defects using a complex analysis based-method [6]. However, they were not able to obtain the spatial extent of the defects.

The present paper proposes an algorithm for estimating the positions and shapes of defects in the MEA using a level set method. This method is different from the conventional methods because prior information, such as the number of the defects is not required, and can directly determine the region of the defects, which is more effective for practical application.

The remainder of the paper is organized as follows. In Section 2, we propose to use a level set method for the identification of defects in an MEA. In Section 3, the proposed method is verified by numerical simulations and experiments. The paper is concluded in Section 4.

2. A Level Set Method for the Identification of Defects

In this section, we describe how to estimate fuel cell defects in the MEA.
2.1 Magnetic Flux Density Produced by a Current Flowing through a PEFC

The 2-D magnetic flux density outside a fuel cell is generated by a current flowing through the cell. In particular, it has been reported that a vertical current flowing perpendicular to the MEA dominantly generates the magnetic flux density. Nara et al. reported the validity of 2-D modeling for computing the magnetic flux density outside the fuel cells [6]. In the present paper, we also assume a 2-D current source model: let an MEA be on \( z = 0 \), and let the current \( I \) flow in the \(-z\) direction perpendicular to the MEA. We denote the region of the MEA by \( D \) and its area by \( S \). The permeability of all the domains is assumed to be equal to that of a vacuum, \( \mu_0 \). Magnetic sensors are located at \( r_i = (x_i,y_i)^T \) \( (i = 1,\ldots,N) \) in the \( xy \) plane outside of the PEFC. The \( x \)- and \( y \)-components of the magnetic flux density at the sensor position \( r_i \) can then be calculated by the Biot-Savart law as follows:

\[
B_0(r_i) = \int_D g(r_i, r) dS,
\]

where \( r = (x,y)^T \in D \) and \( g(r_i, r) \) is defined as

\[
g(r_i, r) = \frac{\mu_0 I}{2\pi S} \frac{1}{|r_i - r|^2} \left( y - y_i - x + x_i \right).
\]

When there are defects in the MEA, we assume that no current flows in the defective part and that the current flows uniformly in the other regions. Denoting the domains of the defects by \( \Omega \) and their areas by \( S_{\Omega} \), the magnetic flux density at the sensor position \( r_i \) is given by

\[
B(r_i) = \frac{S}{S - S_{\Omega}} \int_{\partial \Omega} g(r_i, r) dS.
\]

Then, the difference between \( B_0 \) and \( B \) is described as follows:

\[
\dot{B}(r_i) = B_0(r_i) - B(r_i) = \left(1 - \frac{S}{S - S_{\Omega}}\right) B_0(r_i) + \frac{S}{S - S_{\Omega}} \int_{\Omega} g(r_i, r) dS.
\]

We can observe the magnetic flux density for a normal MEA without defects \( B_0(r_i) \) and that for an MEA with defects \( \dot{B}(r_i) \) using the magnetic sensors. Therefore, \( \dot{B}(r_i) \) are the measurable quantities. See Fig. 2 for the relationship between \( B_0 \), \( B \) and \( \dot{B} \) and their corresponding currents.

2.2 Defect Estimation Using a Level Set Method

In the present study, we propose to use a level set method [7] that can estimate the domains without assuming their topologies.

First, we define a level set function \( \phi \), which is updated with a positive time step \( t \), as follows. Let \( \Omega_t \) be the estimated region of the defects at \( t \). Then, a level set function \( \phi \) is defined as

\[
\phi(r, t) = \begin{cases} 
< 0, & r \in \Omega_t, \quad r \notin \partial \Omega_t, \\
0, & r \in \partial \Omega_t, \\
> 0, & r \notin \Omega_t.
\end{cases}
\]

The boundaries of the defects are represented by the zero level set, \( \phi = 0 \). Next, we consider how to update \( \phi \). For a constant \( \alpha \in \mathbb{R} \), if a contour \( \phi(r, t) = \alpha \) moves to a contour \( \phi(r + dr, t + dt) = \alpha \) when updating \( \phi \) in a time step \( dt \), we have

\[
d\phi = \phi(r + dr, t + dt) - \phi(r, t) = 0,
\]

and hence it holds that

\[
\frac{\partial \phi}{\partial t} + V \cdot \nabla \phi = 0,
\]

where \( V = dr/dt \) is the velocity vector on the contour point \( r \). We define the speed of the contour in its normal direction by

\[
\gamma(r, t) = V \cdot \nabla \phi |_{\nabla \phi}.
\]

Then, we have

\[
\frac{\partial \phi}{\partial t} + \gamma(r, t)|_{\nabla \phi} = 0,
\]

which is an updating formula for \( \phi \).

In order for the zero level set of \( \phi \) to converge to the boundary shapes of the defects, we derive the normal velocity \( \alpha(r, t) \) in the updating formula (3). Note that \( S_t \) is denoted in terms of the Heaviside function \( H[-\phi] \) as follows:

\[
S_t = \int_D H[-\phi] dS,
\]

and the area integral \( g(r_i, r) \) on the region \( \Omega_t \) is written as

\[
\int_{\Omega_t} g(r_i, r) dS = \int_D g(r_i, r) H[-\phi] dS.
\]

Hence, the difference between \( B_0(r_i) \) and the magnetic flux density at time \( t \), \( B_t(r_i) \), is given by

\[
\dot{B}_t(r_i) = B_0(r_i) - B_t(r_i) = (1 - \gamma_t) \int_D g(r_i, r) dS + \gamma_t \int_D g(r_i, r) H[-\phi] dS,
\]

where \( \gamma_t = S_t/(S - S_t) \). Considering that \( \alpha(r, t) \) should be determined such that the squared error between \( \dot{B} \) and \( \dot{B}_t \) becomes smaller, we set the cost function as

\[
F = E_t + \mu L_t + \lambda K_t,
\]

where

\[
E_t = \frac{1}{2} \sum_{i=1}^{N} |\dot{B}(r_i) - \dot{B}_t(r_i)|^2,
\]

\[
L_t = \int_D |\nabla H[\phi]| dS,
\]

\[
K_t = \int_D \Psi_t(\kappa) |\nabla H[\phi]| dS.
\]
Here, is the error between the observation data and the model at time  and , and are the regularization terms and represent the length and the absolute value of the curvature of the boundaries of , respectively. We use , where is a non-negative value.

We decide the normal velocity such that the cost function decreases monotonically with respect to . To this end, by computing the time derivative of the cost function (5), we have

\[
\frac{dF}{dt} = -\int_D X(r, t)\alpha(r, t)\delta[-\phi]\nabla\phi dS,
\]

where

\[
X(r, t) = \sum_{i=1}^{N} -\frac{\gamma_i}{S} (B_i - B_i) - \int_{\Omega} g(r, r')dS',
\]

\[
+ \gamma_i g(r, r') \left[ (\dot{B}_i - \dot{B}_i) - \mu \nabla \left( \frac{\nabla \phi}{|\nabla \phi|} \right) + \frac{1}{|\nabla \phi|} (I - P) \left( \nabla \left( \nabla \phi \right) \right) \right].
\]

In the equations above, \( I(v) = v, P_u(v) = (v \cdot w)w \) and \( P = P_{\Delta} \) are used for simplicity. See Appendix for details. Hence, we take \( \alpha = X \), which guarantees \( dF/dt \) is always negative. With this \( \alpha, \phi \) can be updated by Eq. (3).

3. Experiment

In this section, we explain the setup of the experiments and evaluate the results of the simulations and the actual experiments.

3.1 Experimental Setup

In this experiment, we used a PEFC-type fuel cell. The PEFC consists of a copper end plate, a carbon separator, and an MEA. Figure 3 shows the arrangement of magnetic impedance sensors, a separator, and an MEA. Figure 4 shows (A) an overview of the experimental apparatus and (B) the MEA in a fuel cell with a square defect. The size of the MEA we used was 50 mm × 50 mm, and the dimensions of the electrode and the separator were 90 mm × 90 mm. The magnetic impedance sensors (HMC5983, Honeywell Microelectronics & Precision Sensors) were placed 20.8 mm from the side of the MEA, the interval of which were 10 mm. Nine sensors were placed on each side of the  plane. The total number of the sensors was 36. A total current we used was \( 3 \) A. In both the simulation and the actual experiment, we analyzed the above settings. In the simulation, we used Ansys Maxwell 2014 to compute the magnetic flux density using the finite element method (FEM) and added noise with a signal-to-noise ratio of 40 dB.

3.2 Settings of the Level Set Method

We used a signed distance function for the level set function. This function satisfies \( |\nabla \phi| = 1 \) on the defined regions, and this property alleviates numerical errors caused by the steep gradients of the level set functions [7]. Since the level set functions easily deviate from the signed distance function by updating  \( \phi \), we conducted the following reinitialization in order to preserve the property of the signed distance functions [7],[9].

\[
\frac{\partial \phi}{\partial t} = \frac{\phi_0}{\sqrt{\phi_0^2 + \Delta^2}} (1 - |\nabla \phi|).
\]

Here, \( \phi_0 \) is a level set function at time \( t_0 \), and \( \Delta \) is a discretized mesh size in the region of the MEA. In the analyses, we reinitialized the level set function for each timestep unless \( \max( |\nabla \phi| - 1 ) > 1.0 \times 10^{-2} \) or the number of repetition time is smaller than 200. An upwind differential scheme was used to calculate \( \nabla \phi \) in order to stabilize the update of  \( \phi \) [7]. We determined that the initial value of the region defined by \( \phi \) was a circle with a radius of 50/4 mm. \( \phi \) was updated satisfying the Courant-Friedrichs-Lewy condition [10] as follows:

\[
\Delta t = \frac{c}{\max(\alpha(\mathbf{r}, t))}, \quad 0 < c < 1
\]

and \( c = 0.95 \) was used. A third-order Runge-Kutta method was used to update \( \phi \). We chose the regularization parameters \( \mu \) and \( \lambda \) as follows:

\[
\mu = \eta_1 \frac{E_{r-1}}{L_{r-1}}, \quad \lambda = \eta_2 \frac{E_{r-1}}{K_{r-1}},
\]

where \( \eta_1 = \eta_2 = 1.0 \times 10^{-4} \). We are only interested in the zero level set because it expresses the boundaries of the defects. Hence, only a narrow band [7],[11], which consists of three pixels around the boundary of , is updated to reduce the computing costs. In order to calculate \( L_{r-1} \) and \( K_{r-1} \) in the discretized meshes, a smeared-out delta function was used

\[
\delta_{\rho}(\phi) = \begin{cases} 
0 & \text{if } |\phi| > \rho \\
\frac{1}{2\rho} + \frac{1}{2\rho} \cos(\frac{\pi|\phi|}{\rho}) & \text{if } |\phi| \leq \rho
\end{cases}
\]

where \( \rho = 1.5 \Delta \). Then, \( L_{r-1} \) and \( K_{r-1} \) are given as

\[
L_{r-1} = \int_D |\nabla H(\phi)|dS = \int_D \delta_{\rho}(\phi)|\nabla \phi|dS = \int_D \delta_{\rho}(\phi)|\nabla \phi|dS,
\]

\[
K_{r-1} = \int_D \Psi_{\epsilon(\mathbf{r}, t)} |\nabla H(\phi)|dS = \int_D |\Psi_{\epsilon(\mathbf{r}, t)}| \delta_{\rho}(\phi)|\nabla \phi|dS
\]
values of the estimated regions $S_i$ with respect to time (left) and its change $|S_{i+1} - S_i|$ (right).

\[
\approx \int_D \Psi_i(\kappa_i-1) \delta_1[\phi] \| \nabla \phi \| dS.
\]

Here, $\epsilon = 1.0 \times 10^{-1}$ was used for $\Psi_i(\kappa_i)$.

### 3.3 Results of Reconstruction with Numerical Simulations

First, we conducted the reconstruction with simulation data. As shown in Fig. 5, the proposed method was able to detect the defect positions and rough shapes even if there were multiple defects in the MEA. The left and right panels of Fig. 6 show the decrease in the cost function with respect to time steps and the level set function at cycle 1,500. What is important is that, even if the number of the defects is not known a priori, the two defects are identified: an initial zero level set given by a single circle has been automatically separated into the two domains.

It is noted here that, although the cost function looks almost converged after the 750 cycles in the left panel of Fig. 6, the estimated shapes still change in Fig. 5 (B) and (C). To judge when to stop the algorithm, the change of $S_j$ defined in Eq. (4) may be the other cue. The left and right panels of Fig. 7 show the values of $S_i$ and $|S_{i+1} - S_i|$ with respect to the cycles, respectively. The estimated shapes corresponding to the cycles denoted by (i) through (v) in Figs. 6 and 7 are shown in Fig. 8.

It is observed that the shapes hardly change after the 1300 cycles. Determining a threshold for $|S_{i+1} - S_i|$ is still a difficult issue, it gives a better cue for stopping the algorithm in order to identify the positions and the rough shapes of the defects.

Next, we performed the reconstruction using only 20 sensors on the $xy$ plane, the intervals of which were 20 mm. Figure 9 shows the results of reconstruction. These results show that even if the number of sensors is not sufficient, we can detect the rough shapes and positions of the defects using the level set method.

### 3.4 Results of the Reconstruction with Actual Experimental Data

Two cases were examined: case (a) an MEA with a single square-shaped defect centered at $(-0.02\, \text{m}, -0.02\, \text{m})$ with a side length of $0.01\, \text{m}$ as shown in Fig. 10, and case (b) an MEA with the same defect as in case (a) except that the center position is $(-0.01\, \text{m}, 0.01\, \text{m})$ as shown in Fig. 11. We compared (1) an imaging approach with $L_2$ regularizations [4], (2) an imaging approach with total variation (TV) regularizations, and (3) the proposed level set method. In the imaging approach with $L_2$ regularizations, we minimize the cost function as follows:

\[
F' = \sum_{j=1}^{N} |\tilde{B}(r_j) - G(r_j)j|^2 + \lambda' |j|^2
\]

s.t. $\sum_n j(r_n) = 0, \quad (7)$

where $j = (j(r_1'), j(r_2'), \ldots, j(r_M'))^T$ is the current density flowing in the $-z$ direction in the MEA to explain $\tilde{B}$ defined in Eq. (2). The matrix $G$ is given by

\[
G(r_j) = \frac{1}{T} \left( \begin{array}{cccc} g(r_j, r_1') & g(r_j, r_2') & \cdots & g(r_j, r_M') \end{array} \right),
\]

where $M$ is the number of the discretized meshes in the MEA, and $r_i'$ is the center position of the $i$ th element of the meshes. Note that, as shown in Fig. 2, the current density for $B$ is given.

![Fig. 5 Results of the reconstruction with the numerical simulation. The areas indicate the estimated regions and the two squares indicate the true defect regions. (A) Initial value. (B) Region after 750 iterations. (C) Region after 1,500 iterations.](image)

![Fig. 6 Values of the cost function $E_i$ with respect to time (left) and the level set function at the cycle 1500 (right).](image)

![Fig. 7 Values of the estimated regions $S_i$ with respect to time (left) and its change $|S_{i+1} - S_i|$ (right).](image)

![Fig. 8 Change of the shapes by the iterations.](image)

![Fig. 9 Results of reconstruction using only 20 sensors in the simulation. (A) Initial value. (B) Region after 750 iterations. (C) Region after 1,500 iterations.](image)
by \( j_0 - j \) where \( j_0 = I/S (1, \ldots, 1)^T \). By imposing Eq. (7), the total current in the \(-z\) direction which generates \( B \) is constrained to \( I \). The second term of Eq. (6) is for \( L_2 \) regularization in order to suppress the enhancement of the noise in the reconstruction results. Moreover, \( \lambda' \) is a regularization parameter, which can be determined by an L-curve method [12].

In an imaging approach with TV regularizations, we aim to obtain as uniform currents as possible inside and outside the defects. The cost function is given by

\[
F'' = \sum_{i=1}^{N} |\mathbf{b}(r_i) - G(r_i)j|^2 + \lambda''|J|_{TV},
\]

s.t. \( \sum_{a} j(r'_{a}) = 0, \)

where \( J \) is a matrix whose components correspond to the current density:

\[
J = \begin{bmatrix} j_{s,1} & \cdots & j_{s,1} \\ j_{s,2} & \cdots & j_{s,2} \\ \vdots & \ddots & \vdots \\ j_{s,w} & \cdots & j_{s,w} \\ j_{r,1} & \cdots & j_{r,1} \\ j_{r,2} & \cdots & j_{r,2} \\ \vdots & \ddots & \vdots \\ j_{r,w} & \cdots & j_{r,w} \end{bmatrix} = \begin{bmatrix} j'(r'_{1}) & \cdots & j'(r'_{1}) \\ j'(r'_{2}) & \cdots & j'(r'_{2}) \\ \vdots & \ddots & \vdots \\ j'(r'_{w}) & \cdots & j'(r'_{w}) \end{bmatrix},
\]

\( hw = M, \)

where \( w \) and \( h \) are the numbers of the discretized meshes of \( x \) and \( y \) directions in the MEA. \( |J|_{TV} \) is the total variation of \( J \) and it can be calculated using the following equation:

\[
|J|_{TV} = \sum_{a=1}^{h-1} \sum_{b=1}^{w-1} |\nabla j_{a,b}| \quad \text{with} \quad \nabla j_{a,b} = \begin{bmatrix} j_{a,b} - j_{a+1,b} \\ j_{a,b} - j_{a,b+1} \end{bmatrix},
\]

where \( \nabla \) is the discrete gradient operator and \( \lambda'' \) is a regularization parameter, which is given by the generalized cross validation [13].

The left and right panels of Fig. 12 show the current density \( j_0 - j \) in the MEA reconstructed by the imaging approach with the \( L_2 \) regularization for cases (a) and (b), respectively. The current density in the defect regions should be small because the currents do not flow there. In this method, the oversmoothed solutions were obtained because of the \( L_2 \) regularization.

The left and right panels of Fig. 13 show the current density \( j_0 - j \) in the MEA reconstructed by the imaging approach with the TV regularizations for cases (a) and (b), respectively. In case (a), the current pattern was closer to be square-shaped compared to the result by the \( L_2 \) regularization. In case (b), a polygon-like domain which was much larger than the defect size was obtained around the true defect. In contrast, as shown in Figs. 10 and 11, the proposed method was able to estimate the positions and rough shapes of the defects. Compared to the right-hand panel of Fig. 12, in which the existence of a defect is unclear, and that of Fig. 13, in which a defect larger than the true one is obtained, a single defect close to the true one can be obviously identified from Fig. 11 (C).

Figure 14 shows the magnetic fields calculated by the reconstructed current density by the imaging approach (dashed line in the left figure), by the final level set function (dashed line in the right figure), and by the sensor data (solid line). These results indicate that the calculated magnetic fields reconstructed by the imaging approach and the level set method are similar.

3.5 Discussion

Our experimental evaluation demonstrated that the locations and rough shapes of the defects were obtained. However, the fine shapes of the defects could not be estimated in both the simulations and the actual experiments. The first reason for this is ill-posedness of this inverse problem. The convolution kernel in the form of \( 1/r \) in Eq. (1) operates as a spatial low-pass filter to the current pattern. As a result, the magnetic fields based on the edge-shaped defects are almost the same as those based on the estimated defects.

The second reason for this is the difference between the ex-
Based on the above definitions, the smaller the MSE and MDE, the more similar the data.

Table 1 shows that both the MSE and the MDE between $\tilde{B}$ and $B_{\text{FEM}}$ are better than those between $\tilde{B}$ and $B_{2\text{D}}$. The FEM data were computed by the vertical current flowing in $-z$ direction and the horizontal current. Therefore, it is inferred that the horizontal current should be taken into consideration in representing the measured data more accurately, which is an important aspect of further studies. In the future, we will adapt the horizontal current in order to express the magnetic field precisely.

### 4. Conclusion

The present paper proposed an algorithm for estimating the locations and shapes of the defects in MEA using a level set method. Updating the level set function, the proposed method was able to estimate the regions of the defects in the fuel cell. Using the proposed method, the positions and rough shapes can be identified even when there are multiple defects. The proposed method was verified numerically and experimentally. In the future, we intend to estimate the fine shape of the defects using a full 3-D model that includes the horizontal current in the fuel cell.

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Appendix

The derivative of the cost function $F$ with respect to time is described in detail in the following. First, the time derivative of $E_t$ is given by

$$
\frac{dE_t}{dt} = - \sum_{i=1}^{N} \frac{d}{dt} \left( B(r_i) - \Phi(r_i) \right)
$$

$$
= - \sum_{i=1}^{N} \left( \frac{d\gamma_i}{dt} \int_D g(r_i, r)(1 - H[-\phi])dS - \gamma_i \int_D g(r_i, r) \alpha(r, t) |\nabla\phi[\phi - \phi]|dS \right) \left( B(r_i) - \Phi(r_i) \right).
$$

Since the time derivative of $\gamma_i$ is given by

$$
\frac{d\gamma_i}{dt} = \frac{\gamma_i^2}{S} \int_D \alpha(r, t) |\nabla\phi[\phi - \phi]|dS,
$$

the following equation is obtained:

$$
\frac{dE_t}{dt} = - \int_D \left( \sum_{i=1}^{N} \frac{\gamma_i^2}{S} \left( B_0(r_i) - \int_{\Omega} g(r, r')dS' \right) + \gamma_i g(r_i, r) \right) \left( B(r_i) - \Phi(r_i) \right) |\nabla\phi[\phi - \phi]|dS.
$$

Second, the time derivative of $L_t$ is

$$
\frac{dL_t}{dt} = \int_D \left( \frac{\nabla\phi}{|\nabla\phi|} \right) \alpha(r, t) |\nabla\phi[\phi]|dS.
$$

By Gauss’s divergence theorem, the first term of (A.1) is zero when $\delta[\phi] = 0$ on $\partial D$. Then, the following holds for the time derivative of $L_t$:

$$
\frac{dL_t}{dt} = \int_D \left( \nabla \left( \frac{\nabla\phi}{|\nabla\phi|} \right) \alpha(r, t) |\nabla\phi[\phi]|dS.
$$

Moreover, the time derivative of $K_t$ is given in [8] as

$$
\frac{dK_t}{dt} = \int_D \nabla \left( \frac{\nabla\phi}{|\nabla\phi|} \right) \alpha(r, t) |\nabla\phi[\phi]|dS.
$$

$$
- \frac{1}{|\nabla\phi|} \left( I - P \right) \left( \nabla(\Psi'(\kappa)) \nabla\phi \right) |\nabla\phi[\phi]|dS.
$$