Exclusive $pp \rightarrow pp\pi^+\pi^-$ reaction:  
from the threshold to LHC

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Abstract

We evaluate differential distributions for the four-body $pp \rightarrow pp\pi^+\pi^-$ reaction which constitutes a irreducible background to three-body processes $pp \rightarrow ppM$, where $M$ are a broad resonances in the $\pi^+\pi^-$ channel, e.g. $M = \sigma, \rho^0, f_0(980), f_2(1275), f_0(1500)$. We include both double-diffractive contribution (both pomeron and reggeon exchanges) as well as the pion-pion rescattering contribution. The first process dominates at higher energies and small pion-pion invariant masses while the second becomes important at lower energies and higher pion-pion invariant masses. The amplitude(s) is(are) calculated in the Regge approach. We compare our results with measured cross sections for the ISR experiments at CERN. We make predictions for future experiments at PANDA, RHIC, Tevatron and LHC energies. Differential distributions in effective two-pion mass, pion rapidities and transverse momenta of pions are presented. The two-dimensional distribution in $(y_{\pi^+}, y_{\pi^-})$ is particularly interesting. The higher the incident energy, the higher preference for the same-hemisphere emission of pions. The processes considered constitute a sizeable contribution to the total nucleon-nucleon cross section as well as to pion inclusive cross section.

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I. INTRODUCTION

Diffractive processes although very difficult from the point of view of perturbative QCD are very attractive from the general point of view of the reaction mechanism. There are several classes of diffractive-type processes [1] in high-energy nucleon-nucleon collisions such as:

- (a) elastic scattering,
- (b) single-diffractive excitation of one of the nucleons,
- (c) double-diffractive excitation of both participating nucleons,
- (d) central (double)-diffractive production of a simple final state.

The energy dependence of the first three types of the reaction was measured and can be nicely described [2] in a somewhat academic two-state (but fulfilling unitarity) Good-Walker model [3]. The last case was not studied in too much detail either experimentally or theoretically. At not too high energies the dominant diffractive final state is the $\pi^+\pi^-$ and $\pi^0\pi^0$ continuum. The multi-pion and $KK$ continuum is expected to be smaller.

There is recently a growing interest in understanding exclusive three-body reactions $pp \rightarrow ppM$ at high energies, where the meson (resonance) $M$ is produced in the central rapidity region. Many of the resonances decay into $\pi\pi$ and/or $KK$ channels. The representative examples are: $M = \sigma, \rho^0, f_0(980), \phi, f_2(1275), f_0(1500), \chi_c(0^+)$. It is clear that these resonances are seen (or will be seen) "on" the background of a $\pi\pi$ or $KK$ continuum $^1$. Therefore a good understanding of the continuum seems indispensable. In the present analysis we concentrate on the $\pi^+\pi^-$ channel. Similar analysis can be done for $\pi^0\pi^0$ exclusive production.

At larger energies two-pomeron exchange mechanism dominates in central production (see [1] and references therein). In calculating the amplitude related to double diffractive mechanism for $pp \rightarrow pp\pi^+\pi^-$ we follow the general rules of Pumplin and Henyey [4] (for early rough estimates see also Ref.[5]).

At lower energies subleading reggeons must be included in addition to the pomeron exchanges. We include a new mechanism relevant at lower energies (FAIR, J-PARC) relying on the exchange of two pion. We shall call this mechanism pion-pion rescattering for brevity.

We discuss interplay of all the mechanisms in a quite rich four-body phase space.

II. THE $\pi N$ ELASTIC CROSS SECTION

At low energies, the total cross sections for $\pi^+p$ and $\pi^-p$ show a significant energy-dependent asymmetry defined as:

$$A_{\text{tot}}^{\pi p}(W) = \left[\frac{\sigma_{\text{tot}}^{\pi^+p}(W) - \sigma_{\text{tot}}^{\pi^-p}(W)}{\sigma_{\text{tot}}^{\pi^+p}(W) + \sigma_{\text{tot}}^{\pi^-p}(W)}\right].$$

The total cross section tests, via optical theorem, only imaginary part of the scattering amplitude. In our case of the $2 \to 4$ reactions $^2$ we should use rather full scattering amplitude.

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$^1$ In general, the resonance and continuum contributions may interfere. This may produce even a dip. A good example is $f_0(980)$ production (see [4],[7]).

$^2$ $2 \to 4$ reaction denotes a type of the reaction with two initial and four final particles.
In contrast to the total cross section the elastic scattering cross sections for $\pi^+p$ and $\pi^-p$ show at low energies rather small asymmetry defined as:

$$A_{el}^{\pi p}(W) \equiv \frac{|\sigma_{el}^{\pi^+p}(W) - \sigma_{el}^{\pi^-p}(W)|}{\sigma_{el}^{\pi^+p}(W) + \sigma_{el}^{\pi^-p}(W)}.$$  \hfill (2.2)

A reliably model should explain such details of the interaction.

Therefore to fix parameters of our double-diffractive model we consider first elastic pion-proton scattering. The amplitude for the elastic scattering of pions on nucleons is written in the simplified Regge-like form:

$$M^{\pi\pm p \rightarrow \pi\pm p}(s,t) = is C^{\pi\pm p}_{IP} \left(\frac{s}{s_0}\right)^{\alpha^{\pi\pm p}(t)-1} \exp\left(\frac{B^{\pi\pm p}_{\pi N} t}{2}\right)$$

$$+ \left(a_f + i\right) s C^f \left(\frac{s}{s_0}\right)^{\alpha^{\pi f}(t)-1} \exp\left(\frac{B^{\pi f}_{\pi N} t}{2}\right)$$

$$\pm \left(a_\rho - i\right) s C^\rho \left(\frac{s}{s_0}\right)^{\alpha^{\pi \rho}(t)-1} \exp\left(\frac{B^{\pi \rho}_{\pi N} t}{2}\right),$$  \hfill (2.3)

where $a_f = -0.860895$ and $a_\rho = -1.16158$. The strength parameters $C^{\pi\pm p}_{IP}, C^f, C^\rho$ are taken from the Donnachie-Landshoff model \[9\] for total cross section:

$$C^{\pi\pm p}_{IP} = 13.63 \text{ mb}, \quad C^f = 31.79 \text{ mb}, \quad C^\rho = 4.23 \text{ mb}.$$  \hfill (2.4)

This means that our effective phenomenological model describes the available total cross sections. The pomeron and reggeon trajectories determined from elastic and total cross sections are given in the form ($\alpha_i(t) = \alpha_i(0) + \alpha_i'(t)$):

$$\alpha^{\pi\pm p}_{IP}(t) = 1.088 + 0.25t, \quad \alpha^{\pi f}(t) = 0.5475 + 0.93t.$$  \hfill (2.5)

The values of the intercept $\alpha^{\pi\pm p}_{IP}(0)$ and $\alpha^{\pi f}(0)$ are also taken from the Donnachie-Landshoff model \[9\] for consistency. The effective slope parameter can be written as

$$B_{eff} \equiv B_{\pi N}(W_{\pi N}) = B_0 + 2\alpha_i' \ln\left(\frac{s}{s_0}\right).$$  \hfill (2.6)

We take $\alpha_i' = 0.25/0.93$ for pomeron and reggeon exchanges, respectively. The slope parameter $B_{\pi N}$, taken the same for the pomeron and reggeons, must be fitted to the data. From the fit to the data \[8\] we find $B_0 = 5.5 \text{ GeV}^{-2}$. The effective slope observed in $t$-distributions is of course much larger ($B_{eff} = 7-10 \text{ GeV}^{-2}$ for $P_{lab} = 3-200 \text{ GeV}$ \[8\]).

The differential elastic cross section is expressed with the help of the scattering amplitude as:

$$\frac{d\sigma_{el}}{dt} = \frac{1}{16\pi s^2} |M(s,t)|^2.$$  \hfill (2.7)

The differential distributions $d\sigma_{el}/dt$ for both $\pi^+p$ and $\pi^-p$ elastic scattering for three incident-beam momenta of $P_{lab} = 5 \text{ GeV}$, $P_{lab} = 50 \text{ GeV}$ and $P_{lab} = 200 \text{ GeV}$ are shown in Fig.\[1\]. Under a detailed inspection one can observe that the local slope parameter

$$B_{eff}(t) \equiv \frac{d}{dt} \ln\left(\frac{d\sigma_{el}}{dt}\right)$$  \hfill (2.8)
FIG. 1: Differential distributions for $\pi^+ p$ (left) and $\pi^- p$ (right) elastic scattering for different energies calculated with the amplitude (3.2) and parameters as given in the text. In this calculation the slope parameter was taken as $B_{\pi N} = 5.5$ GeV$^{-2}$ (dashed line). A fit to the world $\pi N$ elastic scattering data suggest that the pomeron and reggeon slopes may be slightly different. The solid line shows such a result. The details are explained when discussing Fig. 2. The experimental data are taken from Ref.[8].

is $t$-dependent and is slightly larger for $\pi^- p$ than for $\pi^+ p$. Such an effect was observed experimentally in Ref.[5]. The local slope decreases with increasing $t$. A rather good description of experimental $d\sigma_{el}/dt$ is achieved.

Our one-parameter ($B_{\pi N}$) model here is consistent with the simple Donnachie-Landshoff model for total cross section [9]. A more refined model should include absorption effects due to pion-nucleon rescatterings. The analysis of absorption effects clearly goes beyond the scope of the present paper. Our model sufficiently well describes the $\pi N$ data and includes absorption effects in an effective way. This has advantage for the $pp \rightarrow pp\pi\pi$ reaction discussed in the present paper where the $\pi N$ absorption effects do not need to be included explicitly. This considerably simplifies the calculation for the $2 \rightarrow 4$ reaction and actually this makes the calculation of the $2 \rightarrow 4$ reaction feasible.

Before we shall go to the $pp \rightarrow pp\pi^+\pi^-$ reaction, we have to discuss the parameters of the
πN interaction. The strength parameters of the pomeron and reggeon couplings are taken from the Donnachie-Landshoff analysis of the total cross section in several hadronic reactions as discussed above. The only free parameters – the slope parameters, are adjusted to the elastic π⁺p and π⁻p scattering. With \( B_{IP} = 5.5 \, \text{GeV}^{-2} \) and \( B_{IR} = 4 \, \text{GeV}^{-2} \) we nicely describe the existing experimental data for πp scattering as can be seen from Fig. 2 (solid lines). The long-dashed lines show pomeron (IP) and reggeon (IR) contributions and the short-dashed lines their interference term. In the Regge approach, high energy cross section is dominated by pomeron exchange. The reggeon exchange dominates in the resonance region. There is a region of energies where the interference term dominates. This is very different than for the total cross section where the cross section is just a sum of the pomeron and reggeon terms. We get a nice description of the data for \( \sqrt{s} > 2.5 \, \text{GeV} \). The region below contains resonances and is therefore very difficult for modeling.

![Graph showing integrated cross section for πN elastic scattering](image)

**FIG. 2:** The integrated cross section for πN elastic scattering. The experimental data are taken from Ref. [17].

Having fixed the parameters we can proceed to our four-body \( pp \rightarrow pp\pi^+\pi^- \) reaction.

### III. CENTRAL DOUBLE DIFFRACTIVE CONTRIBUTION

![Diagram showing dominant mechanisms of exclusive production of π⁺π⁻ pairs at high energies](image)

**FIG. 3:** A sketch of the dominant mechanisms of exclusive production of π⁺π⁻ pairs at high energies.

The general situation is sketched in Fig. 3. The corresponding amplitude for the \( pp \rightarrow \)
The $pp\pi^+\pi^-$ process (with four-momenta $p_a + p_b \rightarrow p_1 + p_2 + p_3 + p_4$) can be written as

$$\mathcal{M}^{pp\rightarrow pp\pi\pi} = M_{13}(t_1, s_{13}) F(t_a) \frac{1}{t_a - m_\pi^2} F(t_a) M_{24}(t_2, s_{24})$$

$$+ M_{14}(t_1, s_{14}) F(t_b) \frac{1}{t_b - m_\pi^2} F(t_b) M_{23}(t_2, s_{23}), \quad (3.1)$$

where $M_{ik}$ denotes ”interaction” between nucleon $i = 1$ (forward nucleon) or $i = 2$ (backward nucleon) and one of the two pions $k = 3$ ($\pi^+$), $k = 4$ ($\pi^-$). In the Regge phenomenology they can be written as

$$M_{ik}(t_i, s_{ik}) = i s_{ik} C_{I P} \left( \frac{s_{ik}}{s_0} \right)^{\alpha_{I P}(t_i) - 1} \exp \left( \frac{B_{I P}}{2} t_i \right)$$

$$+ (a_f + i) s_{ik} C_f \left( \frac{s_{ik}}{s_0} \right)^{\alpha_{I P}(t_i) - 1} \exp \left( \frac{B_{I P}}{2} t_i \right)$$

$$\pm (a_\rho - i) s_{ik} C_\rho \left( \frac{s_{ik}}{s_0} \right)^{\alpha_{I P}(t_i) - 1} \exp \left( \frac{B_{I P}}{2} t_i \right). \quad (3.2)$$

Above $s_{ik} = W_{ik}^2$, where $W_{ik}$ is the center-of-mass energy in the $(i, k)$ subsystems. The third term is with the sign plus if $k = 3$ and with the sign minus if $k = 4$. The normalization constants ($C_{I P}, C_f, C_\rho$) can be estimated from the fit to the total $\pi N$ cross section $(2.4)$. The values of the Regge trajectories $(2.5)$ are also taken from the Donnachie-Landshoff model [9]. The first term describes exchange of the leading (pomeron) trajectory while the next terms describe the subleading reggeon exchanges. At high $\pi N$ subsystem energies $W_{ik} > 20$ GeV only the pomeron exchange survive.

The extra form factors $F(t_a)$ and $F(t_b)$ ”correct” for off-shellness of the intermediate pions in the middle of the diagrams shown in Fig.3. In the following they are parametrized as

$$F(t_{1,2}) = \exp \left( \frac{t_{1,2} - m_\pi^2}{\Lambda_{off,E}^2} \right), \quad (3.3)$$

i.e. normalized to unity on the pion-mass-shell. In the following for brevity we shall use notation $t_{1,2}$ which means $t_1$ or $t_2$. In general, the parameter $\Lambda_{off,E}$ is not known but in principle could be fitted to the (normalized) experimental data. From our general experience in hadronic physics we expect $\Lambda_{off,E} \sim 1$ GeV. How to extract $\Lambda_{off,E}$ will be discussed in the result section.

The parametrization [9] can be used only for $W_{ik} > 2 - 3$ GeV. Below $W_{ik} = 2$ GeV resonances in $\pi N$ subsystems are present. In principle, their contribution could and should be included explicitly [3] [4]. The amplitude $(3.1)$ with $(3.2)$ is used to calculate the corresponding cross section including limitations of the four-body phase-space. To exclude resonance regions we shall ”correct” the parametrization $(3.1)$ with $(3.2)$ by multiplying by a purely phenomenological smooth cut-off correction factor:

$$f^{\pi N}_{cond}(W_{ik}) = \frac{\exp \left( \frac{W - W_2}{a} \right)}{1 + \exp \left( \frac{W - W_2}{a} \right)}. \quad (3.4)$$

3 The higher the center-of-mass energy the smaller the relative resonance contribution.

4 In the standard terminology the resonances belong to single-diffractive contribution to be distinguished from double-diffractive contribution discussed here.
The parameter $W_0$ gives the position of the cut and parameter $a$ describes how sharp is the cut off. The first parameter can have a significant influence on the results. We shall take $W_0 = 2 - 3$ GeV and $a = 0.1 - 0.5$ GeV. For large energies $f^N_{\text{cont}}(W_{ik}) \approx 1$ and close to kinematical threshold $W_{ik} = m_\pi + M_N : f^N_{\text{cont}}(W_{ik}) \approx 0$.

IV. PION-PION RESCATTERING

For the $pp \rightarrow pp\pi^+\pi^-$ or $p\bar{p} \rightarrow p\bar{p}\pi^+\pi^-$ reactions there is another type of semi-diffractive contribution shown in Fig.4.

![Fig. 4: A sketch of the high-energy pion-pion rescattering mechanisms.](image)

Similarly as for the $p\bar{p} \rightarrow N\bar{N} f_0(1500)$ reaction (see [13]) the amplitude squared - averaged over the initial and summed over the final state - for these processes can be written as:

$$|M|^2 = \frac{1}{4} \left[ \begin{array}{c} (E_a + m) (E_1 + m) \left( \frac{p_a^2}{(E_a + m)^2} + \frac{p_1^2}{(E_1 + m)^2} - \frac{2p_a \cdot p_1}{(E_a + m)(E_1 + m)} \right) \end{array} \right] \times 2$$

$$\times \frac{g^{2}_{\pi NN}}{(t_1 - m_\pi^2)^2} F^2_{\pi NN}(t_1) \times \left| M_{\pi^0\pi^0 \rightarrow \pi^+\pi^- (s_{34}, t_0; t_1, t_2)} \right|^2 \times \frac{g^{2}_{\pi NN}}{(t_2 - m_\pi^2)^2} F^2_{\pi NN}(t_2)$$

$$\times \left[ (E_b + m) (E_2 + m) \left( \frac{p_b^2}{(E_b + m)^2} + \frac{p_2^2}{(E_2 + m)^2} - \frac{2p_b \cdot p_2}{(E_b + m)(E_2 + m)} \right) \right] \times 2 .$$

(4.1)

In the formula above $m$ is the mass of the nucleon, $E_a, E_b$ and $E_1, E_2$ are energies of initial and outgoing nucleons, $p_a, p_b$ and $p_1, p_2$ are corresponding three-momenta and $m_\pi$ is the pion mass. The factor $g^{2}_{\pi NN}$ is the familiar pion nucleon coupling constant and is relatively well known [14] ($\frac{g^{2}_{\pi NN}}{4\pi} = 13.5 - 14.6$). In our calculations the coupling constants are taken as $g^{2}_{\pi NN}/4\pi = 13.5$.

At high-energies the pion-pion scattering amplitude of the subprocess $\pi^0\pi^0 \rightarrow \pi^+\pi^-$ with virtual initial pions can be written, similarly as for $\pi N$ scattering:

$$M_{\pi^0\pi^0 \rightarrow \pi^+\pi^- (s_{34}, t_0; t_1, t_2)} = (a_o - i) s_{34} \left( \frac{s_{34}}{s_0} \right)^{\alpha R(t_0) - 1} \exp \left( \frac{B_{\pi\pi}}{2} t_0 \right) F_{\pi^0\pi^0}(t_1) F_{\pi^0\pi^0}(t_2) .$$

(4.2)
This can be written in a useful form:

\[ F_{\pi NN}(t_{1,2}) = \frac{\Lambda^2 - m_{\pi}^2}{\Lambda^2 - t_{1,2}} . \quad (4.3) \]

Typical values of the form factor parameters are \( \Lambda = 1.2-1.4 \) GeV \[15\], however the Gottfried Sum Rule violation prefers smaller \( \Lambda \approx 0.8 \) GeV \[16\].

V. THE DIFFERENTIAL CROSS SECTION

The differential cross section for the \( 2 \rightarrow 4 \) reaction is given as

\[ d\sigma = \frac{1}{2s} |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_a + p_b - p_1 - p_2 - p_3 - p_4) \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \frac{d^3p_3}{(2\pi)^3 2E_3} \frac{d^3p_4}{(2\pi)^3 2E_4} . \quad (5.1) \]

This can be written in a useful form:

\[ d\sigma = \frac{1}{2s} |\mathcal{M}|^2 \delta^4(p_a + p_b - p_1 - p_2 - p_3 - p_4) \frac{1}{(2\pi)^8} \frac{1}{2^4} \times (dy_1 dp_1 dp_1 d\phi_1)(dy_2 dp_2 dp_2 d\phi_2)(dy_3 dp_3 dp_3 d\phi_3)(dy_4 dp_4) . \quad (5.2) \]

This can be further simplified:

\[ d\sigma = \frac{1}{2s} |\mathcal{M}|^2 \delta(E_a + E_b - E_1 - E_2 - E_3 - E_4) \delta^3(p_{1z} + p_{2z} + p_{3z} + p_{4z}) \frac{1}{(2\pi)^8} \frac{1}{2^4} \times (dy_1 dp_1 dp_1 d\phi_1)(dy_2 dp_2 dp_2 d\phi_2) dy_3 dp_3 ^2 p_m . \quad (5.3) \]

Above we have introduced an auxiliary quantity:

\[ \mathbf{P}_m = \mathbf{P}_{3t} - \mathbf{P}_{4t} . \quad (5.4) \]

We choose transverse momenta of the outgoing nucleons \( (p_{1t}, p_{2t}, p_{3t}, p_{4t}) \), azimuthal angles of outgoing nucleons \( \phi_1, \phi_2 \) and rapidity of the pions \( (y_3, y_4) \) as independent kinematically complete variables. Then the cross section can be calculated as:

\[ d\sigma = \sum_k J^{-1}(p_{1t}, \phi_1, p_{2t}, \phi_2, y_3, y_4, p_m, \phi_m) |\mathcal{M}(p_{1t}, \phi_1, p_{2t}, \phi_2, y_3, y_4, p_m, \phi_m)|^2 \frac{1}{2\sqrt{s - 4m^2}} \frac{1}{(2\pi)^8} \frac{1}{2^4} \times p_{1t} dp_{1t} dp_{1t} d\phi_1 \frac{1}{4} dy_3 dy_4 dp_m . \quad (5.5) \]
where the $\delta$ functions have been totally eliminated and $k$ denotes symbolically discrete solutions of the set of equations for energy and momentum conservation:

\[
\begin{cases}
\sqrt{s} - E_3 - E_4 = \sqrt{m_{1t}^2 + p_{1z}^2} + \sqrt{m_{2t}^2 + p_{2z}^2}, \\
-p_{3z} - p_{4z} = p_{1z} + p_{2z},
\end{cases}
\quad (5.6)
\]

where $m_{1t}$ and $m_{2t}$ are transverse masses of outgoing nucleons. The solutions of Eq. (5.6) depend on the values of integration variables: $p_{1z} = p_{1z}(p_{1t}, p_{2t}, p_{3t}, p_{4t}, \phi_1, \phi_2, y_3, y_4)$ and $p_{2z} = p_{2z}(p_{1t}, p_{2t}, p_{3t}, p_{4t}, \phi_1, \phi_2, y_3, y_4)$.

In Eq. (5.5) an extra Jacobian of the transformation $(y_1, y_2) \rightarrow (p_{1z}, p_{2z})$ has appeared:

\[
J_k = \left| \frac{p_{1z}(k)}{\sqrt{m_{1t}^2 + p_{1z}(k)^2}} - \frac{p_{2z}(k)}{\sqrt{m_{2t}^2 + p_{2z}(k)^2}} \right|.
\quad (5.7)
\]

In the limit of high energies and central production, i.e. $p_{1z} \gg 0$ (very forward nucleon1), $-p_{2z} \gg 0$ (very backward nucleon2) the Jacobian becomes a constant $J \rightarrow \frac{1}{2}$.

To calculate the total cross section one has to calculate the 8-dimensional integral (see Eq. (5.5)) numerically. This requires some care.

In the next section we shall show our predictions for several differential distributions in different variables.

**VI. RESULTS**

Before we go to our four-body reaction let us focus for a moment on $\pi^0\pi^0 \rightarrow \pi^+\pi^-$ on-shell scattering. In Fig. 5 we show the total (angle-integrated) cross section for the $\pi^0\pi^0 \rightarrow \pi^+\pi^-$ process. We include both the pion-pion rescattering contribution obtained from partial wave analysis [4] as well as contribution from the Regge phenomenology at higher energies. The parameters of the Regge amplitude for the $\pi\pi \rightarrow \pi\pi$ scattering were obtained in Ref. [10] from different isospin combinations of nucleon-(anti)nucleon, and pion-nucleon scattering assuming Regge factorization. For our case of $\pi^0\pi^0 \rightarrow \pi^+\pi^-$ reaction only the $\rho$-reggeon exchange is relevant. We show predictions for the Regge contribution for corrected $(W_0 = 1.5, 2$ GeV and $a = 0.2$ GeV in Eq. (3.4)) extrapolations to low energies and for different values of the slope parameter $B_{\pi\pi} = 4$ GeV$^{-2}$ (dotted lines), $B_{\pi\pi} = 5$ GeV$^{-2}$ (dashed line) and $B_{\pi\pi} = 6$ GeV$^{-2}$ (solid lines). A relatively good matching is achieved without extra fitting the model parameters. In the following we shall focus on the higher-$M_{\pi\pi}$ Regge component which dominates at higher energies (see a next figure).

In Fig. 6 we present the total cross section for the $pp \rightarrow pp\pi^+\pi^-$ reaction, i.e. the cross section integrated over full phase space, as a function of the center-of-mass energy. We show theoretical predictions from the models calculations with $\Lambda = 0.8$ GeV and $\Lambda_{off,E}^2 = 1$ GeV$^2$ (solid lines) and $\Lambda_{off,E}^2 = 0.5$ GeV$^2$ (dashed lines). The bottom dotted line was obtained with $\Lambda = 0.8$ GeV and $\Lambda_{off,E}^2 = 0.5$ GeV$^2$ while the top dotted line with $\Lambda = 1.4$ GeV and $\Lambda_{off,E}^2 = 2$ GeV$^2$. Details of the low-$M_{\pi\pi}$ rescattering contribution can be found in Ref. [6]. The search for a double pomeron exchange mechanism contribution leads to an upper limits of $\approx 20\mu$b for $M_{\pi\pi} \leq 0.7$ GeV [18], $(49 \pm 5.5)\mu$b [19], $(30 \pm 11)\mu$b [20] and $(44 \pm 15)\mu$b [21]. The experimental value of the cross section taken from [20] was obtained for $M_{\pi\pi} > 2$ GeV and no limitation on $M_{\pi\pi}$ reduces however to 9 $\mu$b for $M_{\pi\pi} \leq 0.6$ GeV [20]. For comparison we show the full cross sections for the $pp \rightarrow pp\pi^+\pi^-$ reaction (filled black
FIG. 5: The angle-integrated cross section for the reaction \( \pi^0 \pi^0 \rightarrow \pi^+ \pi^- \). We present contributions obtained from partial wave analysis \cite{6} and Regge phenomenology \cite{10} for corrected \((W_0 = 1.5, 2 \text{ GeV and } a = 0.2 \text{ GeV in Eq.}(3.4))\) extrapolations to low energies.

circles) from Ref.\cite{22} and for the \( p\bar{p} \rightarrow p\bar{p}\pi^+ \pi^- \) reaction (filled blue triangles) from Ref.\cite{23} which are more than 1 mb for \((2.5 < \sqrt{s} < 10) \text{ GeV}\). Clearly for low energies \((\sqrt{s} < 20 \text{ GeV})\) neither exclusive double diffraction nor pion-pion rescattering constitute the dominant mechanism. Here the production of single and double resonances is the dominant mechanism (see e.g. \cite{6}). The mechanism of the resonant production is rather complicated and will not be discussed in the present analysis.

FIG. 6: Cross section for the \( pp \rightarrow pp\pi^+ \pi^- \) reaction integrated over phase space as a function of the center-of-mass energy. We compare the pion-pion rescattering and double-diffractive contributions with the experimental data (open symbols represent DPE contribution from Refs.\cite{18, 19, 20, 21} and filled symbols show the cross sections for the \( pp \rightarrow pp\pi^+ \pi^- \) reaction (black circles) from Ref.\cite{22} and the \( p\bar{p} \rightarrow p\bar{p}\pi^+ \pi^- \) reaction (blue triangles) from Ref.\cite{23}). The theoretical uncertainties for these contributions are shown in addition.

The results depend on the value of the nonperturbative, a priori unknown parameter of the form factor responsible for off-shell effects. In Table I we have collected integrated cross

\begin{footnote}
\textsuperscript{5} This is a significant contribution to the total \( pp \) cross section.
\end{footnote}
TABLE I: Full-phase-space integrated cross section (in $\mu$b) for exclusive double diffractive $\pi^+\pi^-$ production at selected center-of-mass energies and different values of the off-shell-form factor parameters. Here $W_0 = 2$ GeV and $a = 0.2$ GeV in Eq.(3.4). No absorption effects were included explicitly.

| $F(t_{1,2})$ | $\Lambda_{off}^2$ (GeV$^2$) | $W = 5.5$ GeV | $W = 200$ GeV | $W = 1960$ GeV | $W = 14$ TeV |
|----------------|-----------------------------|----------------|----------------|----------------|----------------|
| $\exp\left(\frac{(t_{1,2} - m_{\pi}^2)}{\Lambda_{off,E}^2}\right)$ | 0.5 | 0.1 | 50.3 | 96.4 | 179.1 |
| 1 | 1.0 | 114.6 | 287.2 | 535.2 |
| $\frac{(\Lambda_{off,M}^2 - m_{\pi}^2)}{(\Lambda_{off,M}^2 - t_{1,2})}$ | 0.5 | 0.02 | 18.9 | 35.6 | 66 |
| 1 | 0.18 | 64.6 | 125.2 | 232.8 |
| $\left(\frac{(\Lambda_{off,D}^2 - m_{\pi}^2)}{(\Lambda_{off,D}^2 - t_{1,2})}\right)^2$ | 0.5 | 0.31 | 83.6 | 164.2 | 306.5 |
| 1 | 1.15 | 217.5 | 437.9 | 822.4 |

sections for selected energies and different values of model parameters. We show how the uncertainties of the form factor parameters affect our final results.

In Fig. 7 we show predictions for different values of the parameter $\Lambda_{off,E}^2 = 0.5$ GeV$^2$ (lower lines), $\Lambda_{off,E}^2 = 1$ GeV$^2$ (upper lines) and for naive (dashed lines) and corrected (solid lines with $W_0 = 2$ GeV and $a = 0.2$ GeV) extrapolations to low energies. The experimental cuts on the rapidity of the pions are included when comparing our results with existing experimental data. Although not all the data are in good agreement with the predictions, their general trend follows the theoretical expectations. No absorption effects were included in this calculation. In general, the higher energy the higher absorption effects. The bare cross section rises with energy. The absorption corrections are expected to lower or even stop the rise. Consistent inclusion of absorption effects is rather difficult and will not be studied here.

FIG. 7: Cross section for the $pp \rightarrow pp\pi^+\pi^-$ reaction integrated over phase space with cuts relevant for a given experiments $[19, 24, 25, 26]$. The experimental value from $[26]$ was obtained for the different cut $\Delta y = |y_p - y_\pi| > 2$. We show results for different values of the parameter $\Lambda_{off,E}^2 = 0.5$ GeV$^2$ (lower lines), $\Lambda_{off,E}^2 = 1$ GeV$^2$ (upper lines) and for the naive (dashed lines) and corrected (solid lines with $W_0 = 2$ GeV and $a = 0.2$ GeV) extrapolations to low energies.

The distribution in the $(y_3, y_4)$ space is particularly interesting. In Fig. 8 and Fig. 9 we
show distributions for the pion-pion rescattering and double-diffractive contributions, respectively. In this calculation the cut-off parameter $A_{\text{eff}, E}^2 = 1 \text{ GeV}^2$. The cross section for the pion-pion rescattering drops quickly with the center-of-mass energy. The rescattered pions are emitted preferentially in different hemispheres, $\pi^+$ at positive $y_3$ and $\pi^-$ at negative $y_4$ or $\pi^+$ at negative $y_3$ and $\pi^-$ at positive $y_4$. The bare (without absorption effects) cross section for the double-diffractive contribution grows with energy. At high energies the pions are emitted preferentially in the same hemispheres, i.e. $y_3, y_4 > 0$ or $y_3, y_4 < 0$. While at low energies (PANDA) both contributions (exclusive double diffraction and pion-pion rescattering) overlap, at high energies (RHIC, Tevatron, LHC) they are well separated, i.e. can, at least in principle, be measured.

![FIG. 8: Differential cross section in $(y_3, y_4)$ for the pion-pion rescattering contribution for different incident energies: $W = 5.5$ (PANDA), 200 (RHIC), 1960 (Tevatron), 14000 (LHC) GeV.](image)

At high energies the diffractive contribution seems more interesting. The camel-like shape of the $y_3, y_4$ distribution requires a separate discussion. In our calculation we include both pomeron and reggeon exchanges. In Fig. 10 we show the cross section in $y_\pi = y_3 = y_4$ for all ingredients included (thick solid line) and when only pomeron exchanges are included (long dashed line), separately for pomeron-reggeon and reggeon-pomeron exchanges (dotted lines) and when only reggeon exchanges are included (dashed line). In this calculation the cut-off parameter $A_{\text{eff}, E}^2 = 1 \text{ GeV}^2$. At low energies all individual cross sections when isolated are comparable. They strongly interfere leading to increase of the cross section. At higher energies each of the ”isolated” cross section peak in different region of $y_3$ or $y_4$. The $P \otimes IP$ cross section peaks at midrapidities of pions, while $P \otimes R$ and $R \otimes P$ at backward and forward pion rapidities, respectively. When interfering the three components in the amplitude produce significant (camel-like) enhancements of the cross section at forward/backward
rapidities. It would be desirable to identify the camel-like structure experimentally \(^6\). At even more forward/backward rapidities one may expect single-diffraction contributions (e.g. diffractive production of nucleon resonances and their decays) not included in the present analysis. This will be discussed elsewhere \(^28\).

In Fig. 10 we compare distributions of pion rapidities \(y_\pi\) for exclusive double diffraction and high-\(M_{\pi\pi}\) pion-pion rescattering at the PANDA, RHIC, Tevatron and LHC energies.

\(^6\) The ALICE experiment seems to be able to study the dependence because of the much lower threshold on pion transverse momenta.
FIG. 12: Differential cross section $d\sigma/dM_{\pi\pi}$ for diffractive and high-$M_{\pi\pi}$ pion-pion rescattering contributions at the PANDA, RHIC, Tevatron and LHC energies. The solid lines was obtained with $\Lambda^2_{off,E} = 1$ GeV$^2$ and dashed lines with $\Lambda^2_{off,E} = 0.5$ GeV$^2$.

respect to $\phi = \pi$ (relative azimuthal angle between charged pions) is caused by the disbalance of transverse momenta of exchanged pomerons and/or reggeons from both proton/antiproton lines. Whether the distributions can be measured at the LHC requires a Monte Carlo studies of the ALICE detector.

FIG. 13: Differential cross section $d\sigma/dp_{t\pi}$ for diffractive and pion-pion rescattering contributions at the PANDA, RHIC, Tevatron and LHC energies. The solid lines was obtained with $\Lambda^2_{off,E} = 1$ GeV$^2$ and the dashed lines with $\Lambda^2_{off,E} = 0.5$ GeV$^2$.

VII. OUTLOOK

A. Beyond the Born approximation

In the present, intentionally simplified, analysis we have performed calculation in the Born approximation with the form factor parameter roughly adjusted to existing "low-energy" experimental data. In a more microscopic approach one has to include higher-order diagrams shown in Fig. 14 and in Fig. 15.

The first type of the interaction was studied e.g. for three-body reactions. For the four-body reaction discussed here a similar effect is expected, i.e. large energy-dependent damping of the cross section which is often embodied in the soft gap survival probability.

When going from the Born (Fig. 3) to the diagrams with the pion-pion FSI (Fig. 15) the
FIG. 14: Diagrams representing the absorption effects due to proton-proton interaction.

FIG. 15: Diagrams representing pion-pion final state interaction.

The following replacement is formally required:

\[
\frac{F_{\text{off}}^A(k) F_{\text{off}}^B(k)}{k^2 - m_{\pi}^2} \rightarrow \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 - m_{\pi}^2} \frac{F_{\text{off}}^A(k, k_3) F_{\text{off}}^B(k, k_4)}{k_3^2 - m_{\pi}^2} \sum_{ij} M_{\pi\pi}^{\text{off-shell}} (k_3 k_4 \rightarrow p_3 p_4) ,
\]

(7.1)

where the sum runs over different isospin combinations of pions. In general the integral above is complicated (singularities, unknown elements), the vertex form factors (A and B) with two pions being off-mass-shell are not well known, and even the off-shell matrix element is not fully under control. Usually a serious simplifications are done to make the calculation useful on a practical level. Limiting to the S-wave \((L = 0)\) one can correct the Born amplitude by a phenomenological function which causes an enhancement close to the two-pion threshold and damping at \(M_{\pi\pi} \sim 0.8\) GeV. Dealing with higher partial waves is more complicated. At even larger \(M_{\pi\pi}\) the interaction becomes absorptive and was not much studied. Some work can be found in Ref. [10]. Clearly much more theoretical afford is required.

The second type of diagrams leads approximately to a redistribution of the strength but seems to modify the pion-pion integrated cross section very little [4]. The effect of pion-pion FSI must be, however, included if the spectrum of invariant mass is studied. At high invariant masses one may expect also a strong damping due to absorption in the pion-pion
subsystem. Only low-invariant-mass spectra were studied in the past experiments \[27\]. The experiments at LHC could study the potential damping of large-mass dipion production and therefore could shed more light on the not fully understood problem of absorption effects in a few-body hadronic systems, so important in understanding e.g. the exclusive production of the Higgs boson discussed recently in the literature.

B. Other not included processes at high energies

Up to now we have discussed only central double-diffractive (CDD) contribution to the \(pp \to pp\pi^+\pi^-\) reaction. In general, there are also contributions with diffractive single or double proton/antiproton excitations followed by the resonance decays shown in Figs.16 and 17. The first mechanism contribute both to the \(pp \to pp\pi^+\pi^-\) and \(p\bar{p} \to p\bar{p}\pi^+\pi^-\) reaction while the second mechanism only to the \(p\bar{p} \to p\bar{p}\pi^+\pi^-\) reaction at high energy \[7\].

Can these processes be separated from the CDD contribution. The general situation at high energy is sketched in Fig.18. The discussed in this paper CDD contributions lays along the diagonal \(y_3 = y_4\) and the classical DPE in the center \(y_3 \approx y_4\). The diffractive single resonance excitation (DSRE) contribution \[8\] is expected at \(y_3, y_4 \sim y_{\text{beam}}\) or \(y_3, y_4 \sim y_{\text{target}}\), i.e. situated at the end points of the CDD contribution. The diffractive double resonance excitation (DDRE) contribution is expected at \((y_3 \sim y_{\text{beam}}\) and \(y_4 \sim y_{\text{target}}\) or \((y_3 \sim y_{\text{target}}\) and \(y_4 \sim y_{\text{beam}}\)), i.e. well separated from the CDD contribution discussed in the present paper. The Tevatron is the only place where one could look at the DDRE contribution, never studied so far at high energies, when it is clearly separated from other mechanisms (CDD, DSRE).

\[7\] At low energy the double \(\Delta\) isobar excitation contribute to \(pp \to pp\pi^+\pi^-\).

\[8\] The Roper resonance excitation is a good example.
FIG. 18: A schematic localization of different mechanisms for the $pp \to pp\pi^+\pi^-$ or $pp \to pp\pi^+\pi^-$ reactions at high energies. The acronyms used in the figure are explained in the main text.

VIII. CONCLUSIONS

We have calculated several differential observables for the exclusive $pp \to pp\pi^+\pi^-$ and $pp \to pp\pi^+\pi^-$ reactions. Both double diffractive and pion-pion rescattering processes were considered. The full amplitude was parametrized in terms of subsystem amplitudes. Only continuum processes (classical DPE) were included in the present analysis.

In the first case the energy dependence of the amplitudes of $\pi N$ subsystems was parametrized in the Regge form which describes total and elastic cross section for $\pi N$ scattering. This parametrization includes both leading pomeron trajectory as well as subleading reggeon exchanges. Even at relatively high energies the inclusion of reggeon exchanges is crucial as amplitudes with different combination of exchanges interfere or/and $\pi N$ subsystem energies can be relatively small $W_{\pi N} < 10$ GeV. The latter happens when $y_{\pi^+}, y_{\pi^-} \gg 0$ or $y_{\pi^+}, y_{\pi^-} \ll 0$. In this region of the phase space one can expect a competition of single diffractive mechanism. In the literature mainly total single diffractive was calculated. We leave the estimation of the SD mechanism contributions to the $pp\pi^+\pi^-$ channel for a separate study.

The integrated cross section of the central double-diffractive component grows slowly with incident energy if absorption effects are ignored. In principle, the absorption effects may even reverse the trend.

In the second case the pion-pion amplitude was parametrized using a recent phase shift analysis at the low pion-pion energies and a Regge form of the continuum obtained by assumption of Regge factorization. The factorization assumption is made to estimate the process contribution.

The two contributions occupy slightly different parts of the phase space, have different energy dependence and in principle can be resolved experimentally. The interference of amplitudes of the both processes is almost negligible.

The energy dependence of the ”diffractive” central production of two-pions is quite different than the one for elastic scattering, single- or double-diffraction. This is due to the specificity of the reaction, where rather the subsystem energies dictate the energy dependence of the process.
At high energies we find a preference for the same hemisphere (same-sign rapidity) emission of $\pi^+$ and $\pi^-$. At ISR energies the same size emission is about 50% while at LHC energies the same hemisphere emission constitutes about 90% of all cases.

In the present analysis we have excluded several resonance contributions. Formally they belong to the category (c) and not (d) which we consider in the present analysis. But the distinction between the different categories is a bit arbitrary and may be quite involved experimentally. Further work is required to estimate contribution of such a process. This clearly goes beyond the scope of the present analysis but will be done in the future.

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[1] G. Alberi and G. Goggi, Phys. Rep. 74 (1981) 1.
[2] A.B. Kaidalov, Phys. Rep. 50 (1979) 157.
[3] M.L. Good and W.D. Walker, Phys.Rev. 120 (1960) 1857.
[4] J. Pumpkin and F.S. Heney, Nucl. Phys. B117 (1976) 377.
[5] Y.I. Azimov, V.A. Khoze, E.M. Levin and M.G. Ryskin, Sov. J. Nucl. Phys. 21 (1975) 21.
[6] P. Lebiedowicz, A. Szczurek and R. Kamiński, Phys. Lett. B680 (2009) 459.
[7] D. Alde et al. [GAMS Collaboration], Phys. Lett. 397 (1997) 350.
[8] A. Eide et al., Nucl. Phys. B60 (1973) 173; I. Ambats et al., Phys. Rev. D9 (1974) 1179; C.W. Akerlof et al., Phys. Rev. D14 (1976) 2864; D.S. Ayres et al., Phys. Rev. D15 (1977) 3105; A. Schiz et al., Phys. Rev. D24 (1981) 26.
[9] A. Donnachie and P.V. Landshoff, Phys. Lett. B296 (1992) 227.
[10] A. Szczurek, N.N. Nikolaev and J. Speth, Phys. Rev. C66 (2002) 055206.
[11] V.A. Khoze, A.D. Martin and M.G. Ryskin, Phys. Lett. B 401, 330 (1997); V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. C 23, 311 (2002).
[12] R.S. Pasechnik, A. Szczurek and O.V. Teryaev, Phys. Rev. D78 (2008) 014007.
[13] A. Szczurek and P. Lebiedowicz, Nucl. Phys. A826 (2009) 101.
[14] T.E.O. Ericson, B. Loiseau and A.W. Thomas, Phys. Rev. C66 (2002) 014005.
[15] R. Machleidt, K. Holinde and Ch. Elster, Phys. Rep. 149 (1987) 1; D. V. Bugg, R. Machleidt, Phys. Rev. C52 (1995) 1203.
[16] A. Szczurek and J. Speth, Nucl. Phys. A555 (1993) 249; B.C. Pearce, J. Speth and A. Szczurek, Phys. Rep. 242 (1994) 193; J. Speth and A.W. Thomas, Adv. Nucl. Phys. 24 (1997) 83.
[17] C. Amsler et al., (Particle Data Group), Phys. Lett. B667 (2008) 1, \url{http://pdg.lbl.gov/2009/hadronic-xsections/}.
[18] D. Denegri et al., [France-Soviet-Union Collaboration], Nucl. Phys. B98 (1975) 189.
[19] D.H. Brick et al., Z. Phys. C19 (1983) 1.
[20] M. Derrick et al., Phys. Rev. Lett. 32 (1974) 80.
[21] D.M. Chew, Nucl. Phys. B82 (1974) 422.
[22] E. Pickup et al., Phys. Rev. 125 (1962) 2091; E.L. Hart et al., Phys. Rev. 126 (1962) 742; A.M. Eisner et al., Phys. Rev. 138 (1965) B670; E. Gellert et al., Phys. Rev. Lett. 17 (1966) 884; G.
Alexander et al., Phys. Rev. 154 (1967) 1284; A.P. Colleraine and U. Nauenberg, Phys. Rev. 161 (1967) 1387; W. Chinowsky et al., Phys. Rev. 171 (1968) 1421; S.P. Almeida et al., Phys. Rev. 174 (1968) 1638; R. Ehrlich et al., Phys. Rev. Lett. 21 (1968) 1839; G. Kayas et al., Nucl. Phys. B5 (1968) 169; C. Caso et al., Nuovo Cim. A55 (1968) 66; C. Caso et al., Nuovo Cim. A33 (1976) 671; C.D. Brunt et al., Phys. Rev. 187 (1969) 1856; G. Yekutieli et al., Nucl. Phys. B18 (1970) 301; E. Colton et al., Phys. Rev. D3 (1971) 1063; J.G. Rushbrooke et al., Phys. Rev. D4 (1971) 3273; H. Boggild et al., Nucl. Phys. B27 (1971) 285; J. Le Guyader et al., Nucl. Phys. B35 (1971) 573; D.R.F. Cochran et al., Phys. Rev. D6 (1972) 3085; W. Burdett et al., Nucl. Phys. B48 (1972) 13; B.Y. Oh et al. [MFIM Collaboration], FERMILAB-PUB-77-114-E (1977); M. Derrick et al. Phys. Rev. D9 (1974) 1215; Zh.S. Takibaev et al., Yad. Fiz. 21 (1975) 1015; F.H. Cverna et al., Phys. Rev. C23 (1981) 1698; S.A. Azimov et al., Yad. Fiz. 34 (1981) 77; F. Shimizu et al., Nucl. Phys. A386 (1982) 571; L.G. Dakhno et al., Sov. J. Nucl. Phys. 37 (1983) 540; D.H. Brick et al., Z. Phys. C19 (1983) 1; J. Johanson et al. [PROMICE/WASA Collaboration], Nucl. Phys. A712 (2002) 75; S. Abd El-Bary et al. [COSY-TOF Collaboration], Eur. Phys. J. A37 (2008) 267.

[23] H. C. Dehne et al., Phys. Rev. 136 (1964) B843-B851; C. Walck et al., Nucl. Phys. B100 (1975) 61; M.A. Jabil et al., Nucl. Phys. B127 (1977) 365; C.K. Chen et al., Phys. Rev. D17 (1978) 42; D.R. Ward et al., Nucl. Phys. B172 (1980) 302; D.E. Zissa et al., Phys. Rev. D22 (1980) 2642; M.Yu. Bogolyubsky et al., Yad. Fiz. 43 (1986) 350; B.V. Batyunya et al., Sov. J. Nucl. Phys. 46 (1987) 650, Yad. Fiz. 46 (1987) 1117; L. Bertolotto et al. [JETSET Collaboration], Phys. Lett. B345 (1995) 325; A. Buzzo et al. [JETSET Collaboration], Z. Phys. C76 (1997) 475.

[24] R. Waldi, K.R. Schubert and K. Winter, Z. Phys. C18 (1983) 301.

[25] L. Baksay et al., [ACCGM Collaboration], Phys. Lett. B61 (1976) 89; H. De Kerret et al., [CHOV Collaboration], Phys. Lett. B68 (1977) 385; D. Drijard et al., [CCHK Collaboration], Nucl. Phys. B143 (1978) 61; B.Y. Oh et al., [MFIM Collaboration], FERMILAB-PUB-77-114-E (1977).

[26] M. Della Negra et al., [CCHK Collaboration], Phys. Lett. B65 (1976) 394.

[27] T. Akesson et al., Nucl. Phys. B264 (1986) 154.

[28] P. Lebiedowicz and A. Szczurek, to be presented in the future.
$\sigma_{\pi\pi}$ versus $p_{t\pi}$ for $W = 1960$ GeV. The graph shows the double-diffraction contribution and the high $\pi\pi$-mass rescattering effect.