Performance-Based Optimization of Reinforced Ductile Concrete Frames with Asymmetric Reinforcement in Columns Using the ISR Analogy

Luis Fernando Verduzco Martínez1*, Jaime Moisés Horta Rangel1, Miguel Ángel Pérez Lara y Hernández1 and Juan Bosco Hernández Zaragoza1

1 Faculty of Engineering, Campus Centro Universitario, Autonomous University of Queretaro, Cerro de las Campanas, 76010, Santiago de Querétaro, Querétaro, México.

Authors’ contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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ABSTRACT

Aims/ Objectives: The present work exposes the design of optimization procedures both with the “Particle Swarm Optimization” (PSO) algorithm and the “Genetic Algorithm” (GA) for the design of reinforced concrete frames, making comparisons in cost, weight of the structure and predicted damage. The optimization procedures are built up using the “Idealized Smeared Reinforcement” (ISR) analogy for each element of the structural model frames considered for this work.

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*Corresponding author: E-mail: lf.verduzcomartinez@ugto.mx;
Methodology: Two different numerical structural plane-frame models were created for the application and comparison of the performance of the optimization design procedures hereby proposed. The optimization procedures were mono-objective with a cost-objective function, taking on account steel reinforcement and concrete for the cost computation. Two different design approaches were carried on for this work, one proposing asymmetrical reinforcement for columns and the other with symmetrical reinforcement. In order to compute the damage indices considered for this study a non-linear Pushover structural analysis is performed.

Results: Results show that asymmetrical reinforcement in columns may reduce concrete volumes, although such reduction in material might not be quite proportional with construction cost, given that asymmetric reinforcement in columns is more expensive than symmetrical, per unit-cost. The bigger the structure, the more likely is to obtain lighter structures by using asymmetrical reinforcement. Regarding damage of the structure, results show that when using asymmetrical reinforcement in columns, it is more likely to obtain smaller values for the expected damage with no great difference on the estimated collapse Safety Factors for the seismic loads. In general, the proposed methodology hereby proposed enhances quite good optima results, requiring only a few adjustments of clash-free and slap reinforcement after the optimization procedure terminates.

Conclusion: When designing reinforced concrete frames with asymmetric reinforcement in columns, an increase in construction costs of as much as 25% as that obtained for symmetric reinforcement could be achieved. In general, with the proposed methodology to optimally design reinforced concrete frames, savings of as much as 20% in construction costs from an initial structural proposal can be reached.

Keywords: ISR analogy; reinforced concrete frames; mono-objective optimization; asymmetrical reinforcement; collapse safety-factors; damage indices.

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1 INTRODUCTION

Every day is more remarkable the tendency to perform optimal designs in civil engineering to reduce material volumes and environmental impact. Regarding reinforced concrete buildings, the material wastes both for concrete and reinforcing bars are of great relevance. Gan et al [1] demonstrated that as much of 18.24% of $CO_2$ emissions could be saved after performing an optimization structural procedure. When dealing with concrete frames, many aspects could be taken for optimization, either mechanical behaviour aspects as well as construction aspects, among others; therefore there’s been extensive research regarding each of them. As for instance: cuts-clashes-slaps detailing optimization procedures [2],[3] to minimize wastes using “Building Information Modelling (BIM) technology, or Performance-Based optimization procedures to reduce damage and repairing-cost of structures [4], focusing on stiffness, ductility or resistance efficiency of its elements, taking on account weight of the structure [5] or soil-structure interaction [6]. However, most of these current studies dealing with complex structural frames leave aside the account of the mere rebar optimization on its elements, considering only a certain percentage quantity of steel reinforcement based on code specifications [7], nevertheless to consider solely the arrangement of rebar over a cross-section element may define completely its dimensions based on separation requirements, and may influence the whole optima convergence; besides, when considering the reinforcing steel rebar in an optimization design procedure, no further modifications would be needed to be done [8] after the optimization process is terminated. Therefore, it is of great importance to integrate this aspect on future research for optimization of reinforced concrete frames [7], as it could enhance a much wider range of design possibilities with a higher grade of accuracy and reliability, in any field in which one may incur.
However, the formulation of such Optimization Procedures is not quite simple to develop nor to compute, as it would be required to perform an integrated optimization procedure of rebars for each proposal of dimensions of each element based on their respective load conditions, which is the main reason why this aspect is rarely taken on account on optimization of RC frames as it could make the design procedures a bit robust, computationally speaking. In [9] a new method was proposed to integrate into the design process of RC frames both the structural modelling analysis as well as the rebar design into the same pre-processing itself, considering the structural elements as composed material elements taking their respective pre-designed reinforcing steel rebar on account with the aid of the ISR analogy [10], enhancing more accurate models and more realistic structural analysis results. This such design-analysis approach turned out to be easy to adopt and to replicate through optimization design procedures based on computing programming leaving no excuses for designers and researchers to consider from now on more accurate structural models on their optimization experimentations.

On the other hand, another aspect to consider is the distribution of such rebars with different design approaches, not only by rebar separation constraints. Columns are usually symmetrically reinforced, due to simplicity requirements for construction; however, when referring to asymmetrical reinforcement on structural elements subject to biaxial bending, it has been demonstrated that as much as 50% of reinforcing steel could be saved in relation with symmetrical reinforcement for both circular and rectangular cross-sections [11],[12] and that when dealing with optimization designs asymmetrical reinforcement is more likely to take place, having influence not only on resistance of the elements but also a slight-moderate effect on their ductility curves [13]. In [9] a comparison of performance of asymmetrical and symmetrical reinforcement in columns was also done, regarding construction costs, reinforcing area, computing execution time as well as structural resistance efficiency by the creation of several algorithms under certain criteria for rebar optimization design, and it was found that really simple asymmetrical designs could be obtained by stating simple construction assumptions, enhancing lighter columns with almost same resistance efficiency as with symmetric reinforcement. Even though construction costs may result slightly higher for the reinforcement it could still enhance smaller cross-sections for columns through an stochastic optimization process, which may reduce at the same time concrete volumes and concrete costs on Reinforced Concrete Structure buildings.

Nevertheless, code specifications do not contain enough criteria to design at this point concrete elements with asymmetrical reinforcement, not even for columns or pillars subject to biaxial bending where it would be of more relevance. The main objective of this research work is to study the influence of integrating design criteria of asymmetrical reinforcement in columns in a design optimization process of RC frames through comparisons with design criteria of symmetrical reinforcement regarding the final design optima convergence, taking on account aspects of ductility, estimated damage, weight and cost of the structures. This such study could enhance the definition of parameters and new design criteria related to asymmetrical reinforcement in columns when designing ductile RC frames. According to some authors [7] this is a vital research area for the improvement of performance-based optimization procedures for RC structures to enhance more sustainable solutions in building construction.

2 THE ISR ANALOGY FOR DESIGN OPTIMIZATION OF REINFORCEMENT

The ISR analogy ‘Idealized Smeared Reinforcement’ is an idealization for reinforcing steel over cross-section concrete elements, such that a minimum required reinforcement area may be estimated (either equal for each section boundary-edge or for each of the section boundaries) so that later such optimal reinforcement area may be transformed to reinforcing bars. There are different computational methodologies for the application
3 STRUCTURAL OPTIMIZATION

During the earliest stages of design of a construction project it is recommended to take into account as many variables and influential factors as possible; generating the best family of potential solutions for their assessment through optimization procedures. The way in which an optimization problem is approached and formulated constitutes the mere essence of the analysis procedure. The main purpose consists on minimizing or maximizing one or more objective functions from initial values for the design variables, such that the conjunction of pre-established constraints and restrictions may be complied at any moment. In structural design of reinforced concrete structures it is common to minimize the weight of the structure, as well as its costs, although by minimizing the weight may not necessarily lead to the minimization of costs. The variables usually considered are the elements cross-section (dimensions), material (type of concrete) and topological variables (number of members, location of members, lengths of members). A mono-objective optimization problem (where only one feature is minimized) may be formulated as (3.1); where \( f(x) \) is the objective function to optimize (cost, stiffness, weight, etc.), \( x \) is the compound of design variables (material, cross-section dimensions, length, etc.), \( k \) is the number of design variables, \( g_i(x) \) are the design constraints, \( m \) is the number of total design constraints, \( x_j^L, x_j^U \) are the upper and lower range limits of variable \( j \).

\[
F = f(x) = \begin{cases} 
  g_i(x) \geq 0 & i = 1, 2, ..., m \\
  x_j^L \leq x_j \leq x_j^U & j = 1, 2, ..., k
\end{cases}
\]

It is of extreme importance the definition of the cost function of an optimization procedure, so that it could represent the most influential cost components, matching simultaneously the explicit constraints given by formulae in design codes. If the reciprocal relationships between the cross-sectional design variables and the design action effects are established, the cost function could then be formulated as a function of the design action effects \([17]\) in an iteratively manner taking the critical design cross-sections for each structural element. Even though this approach may be time-saving by only re-analysing the structure instead of re-designing, it may not really take into account the mere optimization of rebars over each critical design cross-section for each iteration, which may have the greater influence over the cross-section dimensions rather than just the mechanic actions, only by the mere restrictions of rebar minimum separation; ever more when there are several design cross-sections to take over an element, such as a beam. Thus, in the present work, the reinforcement steel is taken as a preponderant factor which may define the final optimal outcome, so that the cost function takes on account not only the volume of steel rebars but assembly costs (according to the type of rebar involved, the complexity of such reinforcement arrangement, as well as the type of structural element in question). The cost of concrete (transportation, pouring, pumping, etc.) are also considered.

4 SEISMIC DESIGN AND ANALYSIS FOR RC FRAMES

Due to the soil movement by which a structure is supported, inertial forces acting over the structure may be developed. A lot of analytical simplifications are made in order to simulate the complexity and irregularity of such effects over a structure. In general, mostly only...
horizontal vibrations for buildings and frames are considered as critical ones. The stiffness $K$ of a structure as function of its mass $M$ and topology influences the way in which a structure vibrates, so that the acting inertial forces may depend on the dynamic properties of such structure (4.1).

$$\text{Modal - analysis} \rightarrow \begin{cases} \det[K - \omega^2M] = 0 \\ (K - \omega^2M)[\phi_i] = 0 \\ |f_{max}| = \left(\frac{[\phi_i^TM][1]}{M}\right)SaM\phi_i \\ i = 1, 2, ..., n \rightarrow \text{floors} \end{cases} \quad (4.1)$$

Where $\phi$ are the modal vibrations, $\omega$ is the vibration frequency of the structure, $f_{max}$ are the lateral inertial forces and $Sa$ is the response acceleration taken as the maximum value from a response spectrum. For this study, $Sa = 200\,m/s^2$ according to the CFE-15 [18] corresponding to Zone “D of very high seismic intensity without considering site effects by type of soil. Such analysis follows the mechanism of an inverted pendulum according to a System of Degrees of Freedom. An inverted triangular load was applied to the frames of this work according to the first modal of vibration.

5 PERFORMANCE-BASED DESIGN

Performance Based Design approaches for structures under seismic hazards are a major focus for earthquake engineering. Its main objective is to design structures such that constraints of lateral deformations and structural damage are met, according to the given specifications for various levels of seismic actions operational requirements for a building. This way, a cost-design of a structure may be correlated with the expected damage that such structure may undergo during its life-cycle. An optimal design of a structure could then have the minimum weight as possible but strong enough to resist earthquake loads and prevent critical damage.

5.1 Lateral Drift

Lateral deflection of a building may cause human discomfort and damage of components. Severe earthquakes may generate extreme inelastic lateral deflections that may cause not only failure of the non-structural engineering systems of a building but also instability of the building itself. Therefore, lateral drift constraints are imposed at various performance levels. By [19] three building performance levels may be stated in terms of lateral drift for Reinforced Concrete Frames as (5.1), where (IO) Immediate Occupancy, (LS) Life Safety and (CP) Collapse Prevention. $H$ is the Frame height.

$$\text{Performance - Levels} \rightarrow \begin{cases} \text{IO Level} \Delta_{IO} \leq 1.0\% H \\ \text{LS Level} \Delta_{LS} \leq 2.0\% H \\ \text{CP Level} \Delta_{CP} \leq 4.0\% H \end{cases} \quad (5.1)$$

5.2 Pushover Structural Analysis

In order to evaluate peak dynamic deformation demand of structures at various performance levels, non-linear static analysis are required, so that a good estimation of damage may be determined for the structure life-cycle. For this work, a plastic-hinge analysis with the Pushover method is used, evaluating the gradual plastic formations of cross-sections under flexure at the ends of the structural elements until the collapse mechanism is reached. With the aid of the Pushover analysis many damage indices can be determined, based on lateral drifts, ductility factors and evolution of the fundamental period of the structure.
The Pushover analysis consists of analysing a structure by incremental loads imposed, defining step by step plastic-formations on cross-section of the elements. The analysis is terminated when the collapse mechanism is reached, either by stating a certain state of damage for the structure or a degree of stiffness degradation. This way, Safety Factors of seismic applied loads until collapse can be found, which is one of the main reasons why this method is hereby used, so that the evolution of the Safety Factor along the optima convergence could be assessed. A condition of stiffness degradation was imposed to stop the analysis. Where $K_j$ is the last stiffness matrix and $K_0$ the initial under elastic behaviour.

$$\frac{\text{det}(K_j)}{\text{det}(K_0)} < 0.003 \quad (5.2)$$

### 5.3 Damage Assessment

The assessment of damage performance in structures is of great importance in the design stage of a structure, as well as when repairing one, since it can be related with potential losses not only regarding the performance of the building itself but also economical losses (cost of repairing, rehabilitation and such). One of the most effective tools to estimate damage performance is through Damage Indices (DI). A vast number of Damage Indices have been proposed and developed based on structural properties (response) or dynamical properties of the structure. Such indices may predict the level of degradation state of a structure and therefore its vulnerability. A DI could even be related to the economic loss due to restoration to compensate a damage imposed on a structure. For this study, the following three non-cumulative DI were considered in order to analyse the relation between the evolution of the optima structure with its estimated DI along its life cycle: the Inter-story Drift DI (5.3), the Plastic Inter-story Drift DI (5.4) and the Deformation Based DI (5.5) proposed by Powell et al [20]. Such DI can reflect quite well the state of a structure at his last stage of collapse and are among the most detailed DI used and of quite simplicity for application [21].

$$DI_{\text{drift}} = \frac{\Delta_{\text{max}} - \Delta_y}{H} \quad (5.3)$$

$$DI_{\mu} = \frac{\Delta_{\text{max}} - \Delta_u}{\Delta_u - \Delta_y} \quad (5.5)$$

Where $H$ is the floor height, $\Delta_y$ is the yielding lateral deformation of the floor, $\Delta_{\text{max}}$ represents the maximum lateral deformation of the floor, $\Delta_u$ denotes the ultimate displacement at failure. Classification of the damage state for deformation based DI can be made according to [22].

### 5.4 Optimization Methods

Choosing the appropriate optimization method is an important issue. Meta-heuristic algorithms are the most fit and therefore used for these such tasks in which many variables are involved. There has been extensive research of methods to apply for this topic regarding structural frames, varying not only from the method itself but even from choosing the objective functions, constraints, performance and damage assessment. On the other hand, classical optimization methods or mathematical programming require derivatives of the objective function which make them limited in potential their application in this problems. Nevertheless, the Steepest Gradient Descent Optimization Method is actually used in this work to optimize the reinforcement for beams, symmetric-reinforced columns and footings design cross-sections as done by [9], whereas for asymmetric-reinforced columns the PSO is used. On the other hand, by refering to the Frames Optimization the PSO algorithm and the GA are be used, given that they are of the most used for structural optimization problems and of great performance [7].

### 6 FORMULATION OF THE DESIGN OPTIMIZATION PROCEDURES FOR RC FRAMES

In order to formulate an optimization design process for RC frames, the design and analysis
mechanisms have to be clearly established so that each optimization design process with the ISR analogy for each structural element may be encapsulated into one task, for each iteration during the global optimization procedure, updating the required data for each run-model. For this such scenario, not only restrictions of the design of each element have to be taken on consideration, but also design restrictions and analysis criteria involving whole RC frames. All of these such design and analysis criteria parameters are presented in the next sections.

6.1 Restrictions and Constraints for Structural Elements

Concrete frames of high ductility (Q=4) are considered for this work, according to code specifications ACI 318-19 [23] and the NTC-17 [24]. The following restrictions have to be complied:

6.1.1 Beams and Columns

Dimension Requirements ACI 318-19/NTC-17 for highly ductile structures

Beams:

- \( b_{min} = 25 \text{cm} \)
- \( \frac{b}{h} \leq 3 \)
- \( \frac{L}{h} > 4 \)

Columns

- \( b_{min} = 30 \text{cm} \)
- \( \frac{b}{h} \geq 0.4 \)
- \( \frac{L}{h} \leq 15 \)
- \( A_g \geq \frac{P_0}{\nu f'c} \)

Minimum separation of rebars ACI 318-19/NTC-17

Beams:

\[
s_{\text{cp,min}} = \begin{cases} 
\frac{4}{3}d_{ag}, & d_{ag} = \frac{3}{4} \text{in} 
\end{cases}
\]

(6.1)

Columns:

\[
s_{\text{cp,min}} = \begin{cases} 
\frac{2}{7}d_b, & d_{ag} = \frac{3}{4} \text{cm} 
\end{cases}
\]

(6.2)

Minimum reinforcement area ACI 318-19/NTC-17
Beams:

\[ 0.7 \sqrt{\frac{f_t}{f_y}} bd \leq \rho \leq 0.7 \sqrt{\frac{f_t}{f_y}} \frac{6000 \beta_1}{f_y + 6000} \]  \hspace{1cm} (6.3)

Columns:

\[ 0.01 \leq \rho \leq 0.04 \]  \hspace{1cm} (6.4)

Rebar disposition ACI 318-19/NTC-17

Beams:

At least four rebars should be placed (one on each cross-section corner). When a beam is analysed, three main sections are taken along its length (left, middle, right) according to nature of the mechanic elements present. A good simplification was made in the present work to design as accurate as possible the reinforcement and to compute based on acceptable parameters the weight of the structure including such reinforcement as well as the inertia properties of each element (See [9]).

Columns:

At least four rebars should be placed (one on each cross-section corner) regardless of the type of reinforcement over such cross-sections.

When it comes to symmetric reinforcement in columns, it was considered to allow only one type of rebar over the whole section. As for asymmetric reinforcement, as much as four different types of rebar are allowed to be placed over a cross-section, one along each boundary of the cross-section.

Ductility requirement ACI 318-19/NTC-17:

Beams:

The minimum strain deformation for the steel reinforcement (grade 42) in tension \( \epsilon_t \) was set to 0.004 in order to comply with the ductility requirements proposed in ACI 318-19 [23] for ductile reinforced concrete beams, so that the neutral axis may be restricted with values (6.5).

\[ \epsilon \geq \frac{d}{0.004 + 1} \]  \hspace{1cm} (6.5)

And the reduction resistance factor \( \phi \) may be then calculated as (6.6):

\[ \phi = \begin{cases} 0.65 + (\epsilon_t - 0.002) \frac{250}{3} & \text{if } 0.004 \leq \epsilon_t < 0.005 \\ 0.9, & \text{if } \epsilon_t > 0.005 \end{cases} \]  \hspace{1cm} (6.6)
Columns:

For columns there has to be established if the section is tension-controlled or compression-controlled, so that the resistant reduction factor \( \phi \) is calculated as (6.7):

\[
\phi = \begin{cases} 
0.75, & \text{...tension} \\
0.65, & \text{...compression}
\end{cases}
\] (6.7)

These factors are applied to the nominal interaction diagrams, based on the balanced condition.

6.1.2 Beam-Column connections

- All width dimensions of beams’ cross-section must be smaller or equal than those of columns
- The upper column’ dimensions must be small or equal than those of the lower column’ dimensions

6.1.3 Column-Footing connections dimensions ratio

For the design of footings, the minimum dimensions relation between column-footing are considered as \( B - b_c \geq 60\text{cm} \) and \( L - b_c \geq 60\text{cm} \) for a good development of the design mechanisms of flexure and shearing, where \( B \) is the dimension of the footing in-plane with the greater dimension \( b_c \) of the column the dimension \( L \) in-plane with the smaller dimension \( b_c \).

6.1.4 Isolated footings

Minimum reinforcement area ACI 318-19/NTC-17

\[
0.7 \sqrt{f_y} \leq \rho \leq 0.7 \sqrt{f_y} \frac{6000\beta_1}{f_y + 6000}
\] (6.8)

The minimum reinforcement percentage by temperature of 0.0018 will be regarded for steel in compression.

Rebar disposition ACI 318-19/NTC-17

A stated in ACI 318-19 and NTC-17 for rectangular isolated footings, the reinforcement bars over the longer transversal dimension is distributed non-uniformly with the quantity of steel at the center of such longitudinal dimension calculated as \( A'_s = A_s \left( \frac{20}{L} \right) \) and the rest at the ends as \( A''_s = A_s - A'_s \). Only one type of rebar is allowed to be placed on a horizontal layer.

Minimum rebar separation

For this work, it was considered a \( \text{sep}_{\text{min}} = 10\text{cm} \) for construction practicality. Besides, given the requirements of shearing and contact pressures, the dimensions of isolated footings give it a lot of resistance without even considering the reinforcement.

6.1.5 Weak beam-Strong column criteria

\[
\sum M_{\text{col}} \geq 1.5 \sum M_b
\]

wherein \( M_{\text{col}} \) for each columns the factorized axial load is taken to enhance the minor resistant bending moment.

6.2 Limit States Design of Structural Elements

6.2.1 Beams

Beams were analysed only in flexure, a minimum of reinforcing area according to codes ACI 318-19 and NTC-17 was considered over the compression zone. The depth of the neutral axis is determined iteratively until \( T - C - C_s = 0 \) complies, or expressed as \( \sum_{i=1}^{n_{\text{bars}}} A_{si} E_y \epsilon_i + \beta_c b(h) f_y = 0 \), following a linear distribution of stresses according the Hooke’s elasticity law (where \( T \) is the resistance of the steel reinforcement in tension, \( C \) the resistance of the concrete zone in compression and \( C_s \) the resistance of the steel in compression).
6.2.2 Columns

The Bresler’s formula “Inverse load method” was employed in order to reduce the biaxial problem to a single symmetry plan problem to compute only interaction diagrams for the calculation of resistance for small values of the axial load as \( \frac{P_{nc}}{P_{ot}} \leq 0.1 \) so that a structural resistance efficiency may be calculated as 
\[
\text{Efficiency} = \frac{P_{nc}}{P_{ot}} < 1.0.
\]
Where \( P_{nc} \) and \( P_{ot} \) are the net resistant axial force in compression and tension respectively. For higher values of axial loads, the Contour Load Method with the Bidirectional Interaction equation is used for which the structural efficiency would be then determined as 
\[
\text{Efficiency} = \frac{M_{nx}}{M_{Rx}} + \frac{M_{ny}}{M_{Ry}} < 1.0.
\]

When asymmetric reinforcement is considered in columns, the variation of the location of the Plastic-Centroid (PC) is calculated, contrary to what happens for symmetric reinforcement where the Geometrical-Centroid and the Plastic-Centroid coincide.

Slenderness effects were considered for the design of columns of each frame-model, given that all frames are classified as Non-restricted. In order to compute the amplified moment loads two structural seismic analysis are performed for each evaluation during the optimization process, one with seismic forces acting to the left and the other to the right; this way a maximum displacement \( \Delta \) may be computed for each story’s columns with fixed supports at each end taking a slender factor of \( k = 0.5 \) which is quite acceptable for the analysis of slenderness effects.

Given that only plane frames were built up for this work, a minimum load eccentricity was considered out-of-plane for each of the columns according to codes NTC-17 \[24\] taken as \( e_{min} = [0.05h > 2cm]. \)

6.2.3 Isolated footings

It was considered for all structural experimental frame models a \( F.S. = 2.0 \) for the bearing load capacity. Eccentrically loaded isolated footings were considered, with a linear stress distribution from the footing to the soil, such that 
\[
q_{real} < q_{max} < q_{adm}.
\]

For the design of reinforcement in isolated beams each transversal cross section is considered as a beam section with the steel in tension over the lower boundary and compression over the upper boundary. For this study the steel area in compression remains constant as stated in code ACI 318-19 \[23\] with a minimum by temperature, on the other hand steel in tension must be between the limits imposed as for beam s in flexure.

6.2.4 Nodes column-beam resistance

For the Plane-Frame models built for this work, four main types of nodes-connections were considered Fig. 1 according to their topology. The nodes were considered as Type 2 according to NTC-17 \[24\] for seismic actions. During each frame design process, all nodes shear resistance are checked until all nodes comply.

![Fig. 1. Types of nodes considered for the plane-frame models built for this work: a) Interior, b) Exterior, c) Exterior-Roof, d) Interior-Roof.](image-url)
For interior nodes, two beams intersect two columns. The beam with the higher height is taken to compute the forces in Tension (outwards from the node $T$). For this work, the upper column cross-section $(h_c, b_c)$ is considered for calculation of resistant shearing (given that the upper column cross-section dimensions are designed equal or smaller than the lower one’s).

## 7 Optimization ISR Formulations for Rebar Detailing

The ISR optimization process encompasses the first step to determine an optima of arrangement of rebars. Based on [9] in which a study of different methodologies of ISR formulations for different structural elements was carried on, the most efficient ISR formulations were taken for this research for beams, columns and isolated footings.

### 7.1 Beams

The idealized cross-section adopted in [9] with one $t$ width variable for the steel in tension is considered also for this project, whereas for the steel in compression the width remains constant with the minimum reinforcement area. The alternative option of reinforcement of two-rebar packs was computed in case the usual option of one-layer of rebars may not comply the minimum separation restriction. The algorithm in pseudo-code can therefore be found in [9].

### 7.2 Columns

#### 7.2.1 Symmetric Reinforcement

For symmetric reinforcement, the ISR analogy could apply using the Steepest Gradient Descent method, allowing only one rebar type to be placed over the cross-section through a Simple-Search process (given the limited number of possible options), enhancing quite acceptable results as proven in [9]. No rebar packs of two were considered. The algorithmic process in pseudo-code is presented next Algorithm 7.1:

**Algorithm 7.1:** General algorithmic process for symmetric rebar optimization for a column cross-section

```
BEGIN
1.- Execute Steepest Gradient Descent method to find $t$ optimum $A_t$ to begin rebar optimization
2.- Determine the maximum number of rebars horizontally and vertically for each type of rebar
   For $i=1$ to $n$-rebar-types=7
   \[
   \text{max Rebar Horizontal} = \frac{(b - 2(\text{cover}) + \text{sep}_{\min} + 2\text{d}}{\text{sep}_{\min} + \text{d}}
   \]
   \[
   \text{max Rebar Horizontal} = \frac{(h - 2(\text{cover}) + \text{sep}_{\min} + 2\text{d}}{\text{sep}_{\min} + \text{d}}
   \]
   Determine the minimum number of rebars horizontally
   \[
   \text{min Rebar Horizontal} = \left[\frac{1}{2} (n - 2(\text{max Rebar Horizontal})) \geq 2\right]
   \]
   Search over all the possible rebar arrangements
   For $j = \text{min Rebar Horizontal to max Rebar Horizontal}$
   Evaluate structural efficiency $Eff < 100$
   Evaluate cost < $cost_{\text{min}}$
   End For
   Save the option with lowest structural efficiency among the most economical
   End For
   If $sep \geq sep_{\min}$, is not complied for any option, then:
   The column height is increased $h = h + 5cm$
   End if
END
```
7.2.2 Asymmetric Reinforcement

For asymmetric reinforcement, an algorithm adapted from [9] was used, also using the Steepest Gradient Descent method with the ISR analogy with a single $t$ from which to transform to rebars through the PSO algorithm in this case, instead of using Simple-Search. The pseudo-code of the algorithmic process is presented next Algorithm 7.2:

Algorithm 7.2: General algorithmic process for asymmetric rebar optimization for a column cross-section

BEGIN
1.- Execute Steepest Gradient Descent method to find $t$ optimum $A_{t,1}$, $A_{t,3}$ to begin rebar optimization
2.- Determine all possible rebar combinations
   For each $A_{t,i}$:
   Check $n_{j}a_{j} \geq A_{t,i}$
   $sep \geq sep_{min}$
   $face_{upper-lower} = rebar - type : [4, 5, 6, 8, 9, 10, 12]$
   $number - rebars : [n_{1}, n_{2}, n_{3}, n_{4}, n_{5}, n_{6}, n_{7}]$
   $n \geq 1$
   $face_{left-right} = rebar - type : [4, 5, 6, 8, 9, 10, 12]$
   $number - rebars : [n_{1}, n_{2}, n_{3}, n_{4}, n_{5}, n_{6}, n_{7}]$
   $n \geq 1$
3.- Optimize the combination
   PSO-algorithm
   For $i=1:numberParticles$
   Initial positions for each particle ($combo - rebar = [k_{1}, k_{2}, k_{3}, k_{4}]$)
   $k_{j}$ takes a value between [1,7] (number of available rebars)
   $x_{ij} = combo_{ij} = combo_{min} + r(7 - 1)$
   $v_{ij} = \frac{\Delta t}{2} \left( -\frac{7}{2} + r(7 - 1) \right)$
   End For
   For $i=1:numberParticles$
   Evaluate $Eff < 100\%$ and $sep_{j} \geq sep_{min}$
   Minimize cost $cost_{i} < cost_{min}$
   End For
   For $i=1:numberParticles$
   Update positions and velocities
   $v_{ij} = v_{ij} + c_{1}q \left( combo_{best - combo_{ij}} \right) + c_{2}r \left( combo_{best - combo_{ij}} \right)$
   $x_{ij} = combo_{ij} + v_{ij} \Delta t$
   End For
   End For
   End PSO-algorithm
   If best - position $\neq 0$
   $h = h + 5$
   Repeat step 3
   End if
4.- Extract best combo $combo_{best} = [k_{1}, k_{2}, k_{3}, k_{4}]$
END
7.3 Isolated Footings

The algorithmic process for rebar optimization of isolated footings is similar as for beams, except that here only one layer of rebars is allowed. When rectangular footings are to be designed two-rebar packs are allowed at the ends of the longer dimension cross-section (when $sep \geq sep_{\text{min}}$ over such ends of length $\frac{1}{2}(L-B)$) Fig. 2.

![Fig. 2. Rebar layout options for isolated footings: a) uniformly distributed, b) non-uniformly distributed in two-rebar packs at the ends](image)

8 NUMERICAL MODEL FORMULATIONS

8.1 Modified Momentum of Inertia of Elements

Two different formulations for numerical structural modelling are formulated: a) taking on account symmetric reinforcement steel in columns and b) taking asymmetric reinforcement steel in columns. For such formulations, a transformed cross section will be regarded as shown next for both beams cross-sections and columns cross-sections, where the moment of inertia of each element’s cross-section is updated in each iteration of the optimization process once the optimum arrangement of rebars has been found.

For beams a cracked cross-section mechanism is taken. It is to stress that the application of the transformed section method for columns differs from that applied to beams or section under pure flexure stress.

For cracked beams the modified moment of inertia could be calculated as (8.1), where the neutral axis $y = k d$ is determined as:

$$y = \frac{-nA_s + \sqrt{(nA_s)^2 + 2bnA_s d}}{b}$$

$$I_{ag} = \frac{by^3}{12} + \frac{by^2}{4} + nA_s (d - y)^2 \quad (8.1)$$

Whereas for columns, a non-cracked cross section is assumed. Assuming symmetric reinforcement Fig. 3 (Left) the modified momentum of inertia can be determined as (8.2):

![Fig. 3. Left-Non-cracked cross-section for symmetrical reinforced columns, Right-Non-cracked cross-section for asymmetrical reinforced columns](image)
\[ I_x = \frac{bh^3}{12} + 2\frac{(b - 2(\text{cover}))((n - 1)t_1)^3}{12} + 2(n - 1)t_1(b - 2(\text{cover}))((\frac{h}{2} - \text{cover}) + \frac{2(n - 1)t_2(h - 2(\text{cover}))^3}{12} \right) \]

(8.2)

On the other hand, for asymmetric reinforcement four different widths are considered Fig. 3 (Right) so that the modified momentums of inertia are defined as (8.3):

\[ I_x = \frac{bh^3}{12} + bh(\frac{h}{2} - PC)^2 + (n - 1)t_1(b - 2\text{cover})(\text{cover} - CP)^2 + \ldots \]
\[ (n - 1)t_2(b - 2\text{cover})(h - \text{cover} - CP)^2 + \frac{(n - 1)t_3(h - 2(\text{cover}))^3}{12} + \ldots \]
\[ ((n - 1)t_3)(h - 2(\text{cover}))((\frac{h}{2} - CP)^2 + \frac{(n - 1)t_4(h - 2\text{cover})^3}{12} + \ldots \]
\[ (n - 1)t_4(h - 2\text{cover})(\frac{h}{2} - PC)^2 \]

(8.3)

8.2 Structural Analysis Procedure

When executing the structural analysis for each potential numerical model, not only the momentum of inertia of each element’s cross-section is modified with the reinforcement steel detailing but also the self-weight. Concrete unit weight was set to \(2400 \text{ Kg/m}^3\) and reinforcement steel unit weight to \(7800 \text{ Kg/m}^3\). Load factor were applied according to the NTC-17 \[24\] set to 1.1 for all loads, considering for all structural frames a Live Load of \(LL = 100 \text{ Kg/m}^2\).

Thus, the structural analysis and modelling was incorporated to the design process itself, and vice-versa according to the method proposed by \[9\] as shown in the flow-diagram Fig. 4:

Fig. 4. Flow-diagram for the structural analysis and numerical modelling process for RC frames. (Adapted from [9])

9 FORMULATION OF THE MONO-OBJECTIVE OPTIMIZATION DESIGN PROCEDURES

Both Optimization formulations (with the GA and PSO) have a similar algorithmic structure. The “evaluate-individual function and the “evaluate-particle function, respectively for each algorithm are basically the proposed coupled analysis-design method shown in the previous section Fig. 4. The whole algorithmic process for both methods are shown next as flow-diagrams Fig. 5 and described in detail in the following subsections.
9.1 Objective Function

For both the GA and the PSO algorithm, the objective function was set as (9.1), where $C_C$ and $C_S$ are the total cost of concrete and reinforcement steel, respectively:

$$\text{Cost} = C_C + C_S \quad (9.1)$$

9.2 Variables

For both optimization processes, the variables were set as the width dimension $b$ of each structural element (only beams and columns) so that for each element width $b_e$, the height dimension $h_e$ may be increased in case the reinforcement restrictions of max-min reinforcement area and minimum separation were not complied. For each iteration such height dimension $h$ is increased as $h \to h + 5$ indefinitely. Although in order for a frame model to be selected a possible one, the dimensions ratio constraints established in Section 6.1 must be complied. For both Optimization formulations, the maximum variable values were set to $[b_{max} = 50]$ for beams and $[b_{max} = 65]$ for columns.

9.3 Initial Values and Parameters

Structural parameters such as $f'_c$, $E$, node coordinates, length of elements $L$, supports and $\text{LiveLoad}$ are set initially and remain constant. On the other hand, when generating the $b$ variables for each optimization method, the height dimensions is set initially as $h = b$ for columns and $h = 2b$ for beams.

9.4 Formulation with the GA

9.4.1 The Evolutionary Algorithm - GA

The GA was the first evolutionary algorithm to be developed [25] based on the Theory of Species of Darwin, which main steps are chromosome decoding where the variables are computed to generate individuals, evaluation of individuals where a fitness value is assigned to each individual through the objective function, sexual reproduction where the most fit individuals are selected through selection to be reproduced through mutation and crossover, so that more fit individuals are generated in the following generations by replacement.

9.4.2 Algorithmic Design Process

During each individual evaluation (frame), all restrictions are checked. After the decoding of all variables ($b_e$ dimensions) a process of uniformity is carried on so that lower adjacent columns width dimensions may be greater or equal than the upper ones, and beams widths to be also equal or smaller than the intersecting columns. These modified values will remain constant through the node and rebar design process. After such design and revising, another uniformity process is carried on for the height dimensions.
of the elements, so that lower adjacent columns’ heights $h_e$ may be greater or equal to the upper ones. These adjustments of the initial decoded $b_e$ variables do not necessarily affect negatively the reach of an optima (by making the variables dependent on one another), but instead could enhance better arrangements for the whole frame model (taking all factors on consideration for the objective cost function). The decoding of chromosomes is then formulated as (9.3), rounding values to the closer multiple of 5:

$$x = \sum_{j=1}^{2(n_{\text{ele}})} (2^{-j}g_{j+(n_{\text{bar}}-1)}(2(n_{\text{ele}})))$$

$$b_e = b_{\text{min}} + \frac{b_{\text{max}} - b_{\text{min}}}{2^{n_{\text{ele}}}} \cdot x$$  

9.5 Formulation with the PSO Algorithm

9.5.1 The PSO algorithm

The PSO algorithm, inspired by the social behaviour of bird flocking. The first swarm model was developed in the 80’s by Craig Raynolds [26] and then improved by Eberhart and J.Kennedy [27] in the 90’s in its standardized form, in which potential solutions are regarded as particles with respective positions and velocities in a given time $dt$. Each particle is evaluated through its position to assign it a performance value based on the objective function. The position and velocities are updated for each iteration and the ones with the best performances are stored (globally and locally) until a termination condition is reached.

9.5.2 Algorithmic design process

Similar to the GA algorithm, during each particle evaluation (frame), all restrictions are checked, also $b_e$ variables are modified, so that all adjacent upper columns’ width dimensions $b_e$ have to be equal or smaller than the lower columns dimension. At the end, after node and rebar design, a final modification of the resultant height dimensions is also carried on. The particles positions are then computed initially as (9.4), and then updated as (9.5), rounding values to the closer multiple of 5.

$$b_e = b_{\text{min}} + \text{rand}_{\text{number}}(b_{\text{max}} - b_{\text{min}})$$

$$b_e = b_e + v_e dt$$

10 CONSTRUCTION COSTS

The following types of rebar are to be accepted (taking the most common types): #4, #5, #6, #8, #9, #10, #12

For asymmetric reinforcement due to the complexity factor of reinforcement 0.7 the assembly performances were assumed a bit lower than for symmetric reinforcement for each type of rebar, thus higher unit-costs were computed. The following Table 1 displays both approaches assembly performances and costs assuming only one type of rebar for each column cross-section:

When more than one type of rebar is placed asymmetrically over a column cross-section, the assembly performance is set to 110 Jor Kg and a unit cost of 36.00 MXN Kg.

The concrete costs considered for this work were taken assuming Pumping.

11 STRUCTURAL EXPERIMENTAL MODELS

Two different frame structural models were used Fig. 6. Model 1 was taken to simulate a non-regular short structure and Model 2 represents a slender heavy structure. It was expected that through Model 2 a more sensible analysis regarding weight an cost could be obtained in order to make better conclusions about these two aspects.
Table 1. Performance for each available type of rebar and construction unit cost for symmetric and asymmetric reinforcement in columns

| Type (#) | performance (asymmetric) | Unit – Cost (asymmetric) | performance (symmetric) | Unit – Cost (symmetric) |
|---------|---------------------------|--------------------------|-------------------------|-------------------------|
| #4      | 150                       | 32.23                    | 212                     | 29.19                   |
| #5      | 152                       | 32.10                    | 216                     | 29.06                   |
| #6      | 154                       | 31.96                    | 220                     | 28.93                   |
| #8      | 154                       | 31.96                    | 220                     | 28.93                   |
| #9      | 154                       | 31.96                    | 220                     | 28.93                   |
| #10     | 154                       | 31.96                    | 220                     | 28.93                   |
| #12     | 154                       | 31.96                    | 220                     | 28.93                   |

Fig. 6. Structural Models taken for experimentation: a) Short irregular frame, b) Slender frame

All of the structural models were restrained under the Preventing-Collapse Performance Level of lateral drift (PC) (5.1), meaning that for every potential model to be accepted as a possible optima during an optimization process, such restrictions of lateral drift must be complied.

12 RESULTS AND DISCUSSION

12.1 GA Frame Optimization

Damage Level \( \Delta \leq [\Delta_{CP} = 4.0\%H] \)

GA parameters:

\( Crossover_p = 0.6, \ population_{size} = 10, \ mutation_p = 0.01, \ number_{gen} = 20(n – bars), \ tournament_{selection} – \ parameter = 0.6, \ tournament_{size} = 2, \ number – generations = 100. \)
12.1.1 Frame model 01

Experimentation resume:

Table 2. Resume of experiments for the Frame Model 01, using the GA, for symmetric and asymmetric reinforcement

| Experiment          | Optimal Cost (MXN) | Structure weight (Kg) | Steel Rebar weight (Kg) | Safety Factor | DI-Drift | DI-Deformation | DI-Plastic Drift |
|---------------------|--------------------|-----------------------|-------------------------|---------------|----------|----------------|------------------|
| Symmetric Reinforcement | 33,336             | 26,952                | 1,722                   | 1.02          | 0.6196 | 1.0           | 0.5955           |
| Asymmetric Reinforcement | 44,200             | 34,950                | 1,942                   | 1.04          | 0.607  | 1.0           | 0.543            |

12.1.2 Frame model 02

Experimentation resume:

Table 3. Resume of experiments for the Frame Model 02, using the GA

| Experiment          | Optimal Cost (MXN) | Structure weight (Kg) | Steel Rebar weight (Kg) | Safety Factor | DI-Drift | DI-Deformation | DI-Plastic Drift |
|---------------------|--------------------|-----------------------|-------------------------|---------------|----------|----------------|------------------|
| Symmetric Reinforcement | 95,458             | 105,632               | 5,266                   | 1.03          | 1.16    | 1.0           | 1.126            |
| Asymmetric Reinforcement | 122,320            | 101,861               | 5,563                   | 1.03          | 0.873  | 1.0           | 0.816            |

12.2 PSO Frame Optimization

Damage Level $\Delta \leq [\Delta^{CP} = 4.0\%H]$

PSO parameters:

$\alpha = 1.0, c_1 = 2, c_2 = 2, dt = 1.0, \text{inertia}_{\text{weight}} = 1.3, \text{number}_{\text{dimension} - \text{space}} = n_{\text{items}}, \beta = 0.99, \text{number}_{\text{particles}} = 15, n_{\text{iterations}} = 30.$
12.2.1 Frame model 01

Experimentation resume:

| Experiment          | Optimal Cost (MXN) | Structure weight (Kg) | Steel Rebar weight (Kg) | Safety Factor | DI-Drift | DI-Deformation | DI-Plastic Drift |
|---------------------|--------------------|-----------------------|-------------------------|---------------|----------|----------------|------------------|
| Symmetric Reinforcement | 32,751             | 24,154               | 1,699                    | 1.03          | 1.14     | 1.19           | 1.13             |
| Asymmetric Reinforcement | 40,377             | 27,318               | 1,709                    | 1.03          | 1.72     | 1.0            | 1.67             |

12.2.2 Frame model 02

Experimentation resume:

| Experiment          | Optimal Cost (MXN) | Structure weight (Kg) | Steel Rebar weight (Kg) | Safety Factor | DI-Drift | DI-Deformation | DI-Plastic Drift |
|---------------------|--------------------|-----------------------|-------------------------|---------------|----------|----------------|------------------|
| Symmetric Reinforcement | 103,215            | 82,638                | 5,467                   | 1.03          | 0.697    | 1.0            | 0.667            |
| Asymmetric Reinforcement | 113,115            | 103,688               | 5,449                   | 1.03          | 0.185    | 0.25           | 0.142            |

12.3 Comparison of Results

For both optimization algorithmic design processes (PSO and GA), it is more likely to obtain smaller values of estimated damage by using asymmetrical reinforcement (See Appendix and previous Tables 2, 3, 4, 5). In most cases the whole structure weight results higher for asymmetric reinforcement contrary as what it was supposed initially: this fact is greatly influenced by the algorithmic design process of asymmetric reinforcement of each column and the nature of the biaxial loads over the columns’ cross-sections, and also it may be influenced by the different distribution of stresses that happen to take place when using symmetric or asymmetric reinforcement, given that the elements are considered as composed structures. The optimal cost, on the other hand, is as higher for asymmetric reinforcement as it was initially predicted to be, nevertheless for larger and 3D structure the savings in concrete volumes may compensate such increment in cost.

Regarding optimization convergence for both meta-heuristic algorithms used, it seems that the mere stochastic algorithmic nature of asymmetric reinforcement design (using the PSO algorithm) affects directly the convergence optimization of frames with the GA, not so much as with the PSO (See Appendix), where for the latter, a faster and more uniform optimization convergence
is thereby enhanced, aside of a tendency to obtain slightly lighter structures as well as more economical, although that could be compensated by just increasing the number of generations for the GA.

For every scenario, the global optima Frame carried very low values of the Safety Seismic Loads Factors (quite close to 1.0) (see Tables 2, 3, 4, 5) with a tendency to decrease as the optimal cost evolved, even though the design optimization processes were built up from ductile specifications. But this fact would just imply that in order to really design optimally ductile frames, certain measures have to be integrated to take on account locations of plastic formations over the elements.

### 12.4 Additional Commentaries and Recommendations

A better performance of optimal design for frames with asymmetrical reinforcement in columns is supposed to be obtained for robust 3D frames where the biaxial moment ratio becomes a crucial factor, given that for plane frames the preponderant moment is acting only in one direction for every column.

The optimization design process for asymmetric reinforcement could be improved so that it could adapt better to load conditions by considering perhaps the cross-sections in columns as **cracked ones**, this way the designs could be more sensible to load eccentricities.

In order to maximize Safety Seismic Load Factors it would be recommended to carry on a multi-objective optimization procedure, given the tendency of such Safety Factors to decrease as the weight and cost of the structure decreases. Such optimization procedure could implicitly take on account locations of plastic hinges formations over the elements to not only design optimally lighter structures, but also more ductile ones.

Regarding the quality of final results, it would be required to make a final adjustment in the detailed reinforcement layout in order to make the optimum designs practical for construction, given that rebar overlaps were not considered (see Fig. 7). It is thus recommended for future research to integrate in the optimum design process free-clash and overlap design criteria to reach therefore, better results. For this work, ANSYS SpaceClaim was used in order to better visualize the reinforcement results for each optimization case, using parametric visual programming (See Appendix). Such tool could be used further for such implementation of free-clash and overlaps rebar criteria, obtaining almost automatically the reinforcement detailing drawings in CAD.

![Fig. 7. Final detailed results of reinforcement in elements through ANSYS SpaceClaim](image-url)
13 CONCLUSIONS

When designing reinforced concrete frames with asymmetric reinforcement in columns, an increase in construction costs of as much as 25% as that obtained for symmetric reinforcement could be enhanced.

In general, with the proposed methodology to optimally design reinforced concrete frames, savings of as much as 20% in construction costs from an initial structural proposal can be reached.

The proposed approach for designing optimally reinforced concrete frames based on performance produces also quite good results regarding practicality in construction, given that only a few adjustments may be applied after the optimization design process in order to get to the final detailing stage of a project.

It was demonstrated that the influence of taking on account different steel reinforcement disposition on the elements through the design and analysis process in an optimization procedure is of great relevance on the optimal convergence.

This methodology could enhance more accurate optimization design processes for reinforced concrete structures in further research projects in which as many variables as possible may be considered, opening a new range space of possibilities for the way such structures are modelled and designed.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

[1] Vincent J. L. Gan, Wong CL, Tse KT, Jack C. P. Cheng, Irene M. C. Lo, Chan CM. Parametric modelling and evolutionary optimization for cost-optimal and low-carbon design of high-rise reinforced concrete buildings. Journal of Advanced Engineering Informatics. 2019;42. DOI:https://doi.org/10.1016/j.jaei.2019.100962

[2] Eleftheriadis S, Duffour P, Stephenson B, Mumovic D. Automated specification of steel reinforcement to support the optimisation of RC floors. Autom. Con Struct. 2018;96:366-377. DOI:https://doi.org/10.1016/j.autcon.2018.10.005

[3] Zahra S. Moussavi, Ahmed W. A. Hammad, Xiao J, Ali Akbarnezhad. Minimizing cutting wastes of reinforcing steel bars through optimizing lap splicing within reinforced concrete elements. Journal of Construction and Building Materials. 2018;185:600-608. DOI:https://doi.org/10.1016/j.conbuildmat.2018.07.023

[4] Siamak Talatahari. Optimum performance-Based Seismic design of frames using meta-heuristic optimization algorithms. Metaheuristic Applications in Structures and Infrastructures Journal. 2013;419-437. DOI:https://doi.org/10.1016/B978-0-12-398364-0.00017-6

[5] Herlan Al. Leyva, Eden Bojrquez, Juan Bojrquez, Alfredo Reyes-Salazar, Jos H. Castorena, Eduardo Fernndez, Barraza MA. Earthquake design of reinforced concrete buildings using NSGA-II. Advances in Civil Engineering; 2018. Article ID 5906279. DOI:https://doi.org/10.1155/2018/5906279

[6] Negrn A, Chagoyn E. Optimización metaheuristica de conjuntos estructurales de hormigón armado. Universidad Central “Marta Abreu de las Villas, Villa Clara, Cuba. Spanish; 2019.
[7] Afzal M, Liu Y, Jack C. P. Cheng, Gan V. Reinforced concrete structural design optimization: A critical review. Journal of Cleaner Production. 2020;260. DOI:https://doi.org/10.1016/j.jclepro.2020.120623

[8] Akin A, Saka M. P. Harmony search algorithm based optimum detailed design of reinforced concrete plane frames subject to ACI 318-05 provisions. Computers and Structures. 2015;147:79-95. DOI:https://doi.org/10.1016/j.compstruc.2014.10.003

[9] Verduzco LF, Horta J. Optimization of reinforced concrete structures with the ISR analogy. XVII International Engineering Congress (CONIIIN-21), Universidad Autonoma de Queretaro, Revista del Congreso Internacional de Ingeniera (CONIIIN); 2021. (In press)

[10] Verduzco Martinez L. The Idealized Smeared Reinforcement (ISR) method for the optimization of concrete sections: a survey of the state-of-the-art and analysis of potential computational approaches. Revista Mexicana de Métodos Numéricos. 2021;5:1. DOI: https://www.scipedia.com/public/Verduzco_Martinez_2021a

[11] Gil-Martín LM, Hdez-Montes E, Aschheim M. Optimal reinforcement of RC columns for biaxial bending. Materials and Structures Journal. 2010;43:1245-1256. DOI: 10.1617/s11527-009-9576-x

[12] Juan F. Carbonell-Mrquez, Luisa M. Gil-Martín, Alejandro Fernandez-Ruz M, Enrique Hernandez-Montes. Asymmetrically reinforced concrete piles in earth retaining systems. 37th IABSE Symposium Madrid. 2014;102. DOI: 10.2749/222137814814068058

[13] Hernandez-Montes E, Aschheim M, Gil-Martín LM. Impact of optimal longitudinal reinforcement on the curvature ductility capacity of reinforced concrete column section. Magazine of Concrete Research. 2004;56(9). DOI: 10.1680/macr.2004.56.9.499

[14] Verduzco Martinez L, Hernandez-Montes E. Optimization of reinforcing steel for the design of concrete columns. Bachelor Thesis, University of Guanajuato, Mexico; 2019. Available:http://repositorio.ugto.mx/handle/20.500.12059/2684

[15] Aschheim M, Hernandez-Montes E, Gil-Martín LM. Design of optimally reinforced RC beam, column and wall section. J Struct Eng. 2018;134(2):169-188. DOI: 10.1061/(ASCE)0733-9445(2008)134:2(231)

[16] Ho Jung Lee, Mark Aschheim, Hernandez Montes E, Gil-Martín LM, Pasadas-Fernandez M. Optimum RC column reinforcement considering multiple load combinations. Struct Multidiscip Optim. 2009;39:153-170. DOI: 10.1007/s00158-008-0318-4

[17] Sharafi P, Muhammad N. S. Hadi, Lip H. The. Cost optimization of column design of reinforced concrete buildings. Metaheuristic Applications in Structures and Infrastructures. 2013;129-146. DOI:https://doi.org/10.1016/B978-0-12-398364-0.00006-1

[18] Comisión Federal de Electricidad, Manual de Diseño de Obras Civiles, Capítulo C.1.3 Diseño Por Sismo, México. Spanish; 2015.

[19] FEMA 273. NEHRP Guidelines for the seismic rehabilitation of Buildings. NEHRP; 1997.

[20] Powell GH, Allahabadi R. Seismic damage prediction by the deterministic method: Concepts and procedures. Earthquake Engineering and Structural Dynamics. 1988;16(5):719734. DOI: 10.1002/eqe.4290160507

[21] Makhloof DA, Ibrahim AR, Xiaodan Ren. Damage assessment of reinforced concrete structures through damage indices: a state-of-the-art review. Computer Modeling in Engineering & Sciences. 2021;128(3). DOI: doi:10.32604/cmes.2021.016882
[22] Ladjinovic D, Folic R. Application of improved damage index for designing of earthquake structures. 13th World Conference on Earthquake Engineering, Vancouver, Canada; 2004. Available: https://www.researchgate.net/publication/265621539

[23] Building Requirements for structural concrete and commentary. American Concrete Institute ACI 318; 2019.

[24] Gaceta Oficial de la Ciudad de México. Normas Técnicas Complementarias. Spanish; 2017.

[25] Holland JH. Adaptation in natural and artificial systems. An introductory analysis with applications to biology, control and artificial intelligence, University of Michigan Press, Ann Arbor, Mich; 1975.

[26] Craig W. Reynolds, Flocks, Herds and Schools: A distributed behavioural model. Computer Graphics. 1987;21(4).

[27] James Kennedy y Russell Eberhart. Particle Swarm Optimization, Proceedings of ICNN’95 - International Conference on Neural Networks. 1995;4:1942-1948. DOI: 10.1109/ICNN.1995.488968
APPENDIX

GA Frame Optimization

Model Frame 01
Asymmetric Reinforcement

Fig. 8. Final Results for the optimal Frame Model 01, with asymmetric reinforcement in columns, using the GA. Left-Cost Optimization convergence, Middle-Evolution of the weight of the Structure, Right-Evolution of the weight of the Steel Rebar

Fig. 9. Final results and dimensions for the optimal Frame model 02, with asymmetric reinforcement in columns, using the GA. Left-Evolution of the Safety Factor for collapse against seismic loading, Middle-Evolution of the Damage Indices, Right-Optimal Dimensions of elements for the Frame

Symmetric Reinforcement

Fig. 10. Final results for the optimal Frame Model 01, with symmetric reinforcement in columns, using the GA. Left-Cost Optimization convergence, Middle-Evolution of the weight of the Structure, Right-Evolution of the weight of the Steel Rebar
Fig. 11. Final results and dimensions for the optimal Frame model 01, with symmetric reinforcement in columns, using the GA. Left-Evolution of the Safety Factor for collapse against seismic loading, Middle-Evolution of the Damage Indices, Right-Optimal Dimensions of elements for the Frame.

Model Frame 02

Asymmetric Reinforcement

Fig. 12. Final results for the optimal Frame Model 02, with asymmetric reinforcement in columns, using the GA. Left-Cost Optimization convergence, Middle-Evolution of the weight of the Structure, Right-Evolution of the weight of the Steel Rebar.

Fig. 13. Final results and dimensions for the optimal Frame Model 02, with asymmetric reinforcement in columns, using the GA. Left-Evolution of the Safety Factor for collapse against seismic loading, Middle-Evolution of the Damage Indices, Right-Optimal Dimensions of elements for the Frame.
Symmetric Reinforcement

Fig. 14. Final results for the optimal Frame Model 02, with symmetric reinforcement in columns, using the GA. Left-Cost Optimization convergence, Middle-Evolution of the weight of the Structure, Right-Evolution of the weight of the Steel Rebar

Fig. 15. Final results and dimensions for the optimal Frame Model 02, with symmetric reinforcement in columns, using the GA. Left-Evolution of the Safety Factor for collapse against seismic loading, Middle-Evolution of the Damage Indices, Right-Optimal Dimensions of elements for the Frame.

PSO Frame Optimization

Frame Model 01

Symmetric Reinforcement

Fig. 16. Final results for the optimal Frame Model 01, with symmetric reinforcement in columns, using the PSO. Left-Cost Optimization convergence, Middle-Evolution of the weight of the Structure, Right-Evolution of the weight of the Steel Rebar
Fig. 17. Final results and dimensions for the optimal Frame model 01, with symmetric reinforcement in columns, using the PSO. Left-Evolution of the Safety Factor for collapse against seismic loading, Middle-Evolution of the Damage Indices, Right-Optimal Dimensions of elements for the Frame.

Asymmetric Reinforcement

Fig. 18. Final results for the optimal Frame Model 01, with asymmetric reinforcement in columns, using the PSO. Left-Cost Optimization convergence, Middle-Evolution of the weight of the Structure, Right-Evolution of the weight of the Steel Rebar

Fig. 19. Final results and dimensions for the optimal Frame model 01, with asymmetric reinforcement in columns, using the PSO. Left-Evolution of the Safety Factor for collapse against seismic loading, Middle-Evolution of the Damage Indices, Right-Optimal Dimensions of elements for the Frame.
Frame Model 02
Symmetric Reinforcement

Fig. 20. Final results for the optimal Frame Model 02, with symmetric reinforcement in columns, using the PSO. Left-Cost Optimization convergence, Middle-Evolution of the weight of the Structure, Right-Evolution of the weight of the Steel Rebar

Fig. 21. Final results and dimensions for the optimal Frame model 02, with symmetric reinforcement in columns, using the PSO. Left-Evolution of the Safety Factor for collapse against seismic loading, Middle-Evolution of the Damage Indices, Right-Optimal Dimensions of elements for the Frame.

Asymmetric Reinforcement

Fig. 22. Final results for the optimal Frame Model 02, with asymmetric reinforcement in columns, using the PSO. Left-Cost Optimization convergence, Middle-Evolution of the weight of the Structure, Right-Evolution of the weight of the Steel Rebar
Fig. 23. Final results and dimensions for the optimal Frame model 02, with symmetric reinforcement in columns, using the PSO. Left-Evolution of the Safety Factor for collapse against seismic loading, Middle-Evolution of the Damage Indices, Right-Optimal Dimensions of elements for the Frame.