Application of Roy’s equations to analysis of $\pi\pi$ experimental data

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The scalar-isoscalar, scalar-isotensor and vector-isovector $\pi - \pi$ partial wave amplitudes are analyzed. Preliminary results indicate that only the scalar-isoscalar amplitude fitted to the "down-flat" data satisfies Roy’s equations and consequently crossing symmetry.

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1 Motivations and theoretical constraints

Pions are produced in many reactions. A good experimental and theoretical information on low, medium and high-energy pion-pion interactions should give us a better understanding of non-perturbative QCD and of the chiral perturbation theory. It allows a better insight into $q\bar{q}$ vacuum condensate and then into the mechanism of spontaneous breaking of chiral symmetry. It should also give a better knowledge of the meson spectrum in particular of the $\sigma$-meson and glueballs.

Recently Kamiński et al. [1], using not only pion-exchange but also $a_1$-exchange, have reanalyzed the data obtained in the seventies on the $\pi^- p \rightarrow \pi^+ \pi^- n$ reaction at 17.2 GeV/c without and with a polarized target. Essentially, for $m_{\pi\pi}$ between 600 and 980 MeV, two solutions “up-flat” and “down-flat” (hereafter called up and down) were found for the pion-pion isoscalar S-wave.

Besides unitarity and analyticity, crossing symmetry plays a very important role in the $\pi\pi$ interactions as in each channel the same particles interact. Projection on partial waves of the twice subtracted fixed-t dispersion relations leads to Roy’s equations for the scattering amplitude of isospin I, viz.

$$Re f_\ell^I(s) = \left(\begin{array}{c} a_0^0 \\ a_0^2 \end{array}\right) \delta_{\ell 0} + (2a_0^0 - 5a_0^2) \frac{s - 4m_{\pi}^2}{12m_{\pi}^2} \left(\begin{array}{c} \delta_{10} \\ \delta_{12}/6 \\ -\delta_{00}/2 \end{array}\right) + \sum_{\ell' = 0}^2 \sum_{\ell'' = 0}^1 \frac{110m_{\pi}^2}{4m_{\pi}^2} \int K_{\ell'\ell''}^\prime(s, s') Im f_{\ell''}^\prime(s') ds' + d_{\ell}^I(s).$$

These equations are valid for $4m_{\pi}^2 \leq s \leq 68m_{\pi}^2$ (= 1.15 GeV) and express the real part of the scalar-isoscalar, scalar-isotensor and vector-iso vector partial waves as integrals on their imaginary parts. The two subtractions are expressed in terms of the scattering lengths $a_{\ell}^I$: $a_0^0$ and $a_0^2$. The kernels $K$ are known singular functions. The driving terms $d_{\ell}^I(s)$ contain the contributions of the partial waves $\ell' \geq 2$ as well as the high-energy contributions [2,3]. For the driving terms we use the parameterization of Basdevant et al. [4].

Pions are the quasi-Goldstone bosons of the chiral symmetry of strong interaction, so at low $m_{\pi\pi}$ we use constraints from chiral perturbation theory. For instance the two-loop calculations of the $\pi\pi$ amplitudes using $K_{\ell\ell}$ decay constraints lead to: $a_0^0 = 0.219 \pm 0.005 m_{\pi}^{-1}$, $a_0^2 = -0.042 \pm 0.001 m_{\pi}^{-1}$ and to the slope parameters of the phase shifts; $b_0^0 = 0.279 \pm 0.011 m_{\pi}^{-3}$, $b_0^2 = -0.076 \pm 0.002 m_{\pi}^{-3}$ [5].

2 Applications and outlook

For the scalar-isoscalar phase shifts $\delta^0_0$ we use the unitary three-channel model of Kamiński et al. [6] with chiral symmetry constraints on $a_0^0$, $b_0^0$ and on $\pi\pi \rightarrow K\bar{K}$ and $K\bar{K} \rightarrow K\bar{K}$ reactions at the $\pi\pi$ threshold as given by Donoghue et al. [7]. For $\delta^0_2$ we use chiral symmetry constraints on $a_2^0$ and $b_2^0$ and do a fit to Hoogland A data [8] using the Padé approximant parameterization of Schenk [9]. For $\delta^1_1$ we use the Schenk parameterization with $a_1^1 = 0.035 m_{\pi}^{-1}$ [10].
First we built up a three-channel fit to the solution down and to Roy’s equations. The quality of fit to Roy’s equations is judged by a comparison between the exact real part of the partial waves calculated from phase shifts and inelasticities (called input) and the output calculated from Roy’s equations. These equations are well satisfied by this fit called “best-down” fit. The input is close to the output with some small deviations above 900 MeV, in particular for the scalar-isotensor wave.

Let us try to solve the up-down ambiguity. The solution up and down differ mainly for $800 \leq m_{\pi\pi} \leq 980$ MeV. We use a Padé approximant with 8 parameters to fit $\tan \delta_0^0$ of the solutions up and down. The parameters are determined: i) to reproduce the “chiral” values $a_0^0$, $b_0^0$ and the $\delta_0^0$ of the “best-down” fit at 500 and 600 MeV, ii) to have a smooth junction to the “best-down” fit close to 970 MeV and iii) to obtain a best fit of the data between 680 and 950 MeV.

Results of different fits with their corresponding $\chi^2$ are shown in Fig. 1. The “Padé lower” and “Padé upper” denote the fits to the data shifted downwards and upwards according to their error bars. In Fig. 2 we check how this “upper” and “lower” fits satisfy Roy’s equations. For the S-wave and for $840 \leq m_{\pi\pi} \leq 950$ MeV (left panel) there is no overlap between the “upper” and “lower” input and output bands. Here the solution up does not satisfy Roy equation, i.e. it is not compatible with crossing symmetry. On the contrary for the solution down (right panel), for $m_{\pi\pi} \leq 950$ MeV, both bands overlap. The other waves satisfy relatively well Roy’s equations. The solution down is compatible with crossing symmetry.

Let us remark here that recent joint analysis by Kamiński et al. [11] of the CERN-Munich, the CERN-Cracow-Munich $\pi^+\pi^-$ data and of the $\pi^0\pi^0$ data of the Brookhaven E852 Collaboration at 18.3 GeV/c [12] eliminates the “up-flat” solution and leads to a solution compatible with solution down. Our study here shows that only the solution down satisfies crossing symmetry.

Currently we are studying the sensitivity of the driving terms to the parameterization of the resonant $f_2(1270)$ and $\rho_3(1690)$ amplitudes and to the high energy Regge contributions. We shall compare our results to those of Ananthanarayan et al. [3].

The present study shows that we can construct a three-channel $\pi\pi$, $K\bar{K}$ and effective $(2\pi)(2\pi)$ model which fulfills unitarity, crossing symmetry and chiral symmetry constraints. This will give
Figure 2: Tests of Roy’s equations for solutions up (left panel) and down (right panel).

more confidence in the parameters of the scalar-isoscalar mesons predicted by the model. This will also allow to check the amplitudes below the $K \bar{K}$ threshold. Let us finally mention that any new precise data on $\pi \pi$ are welcome.

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