Practical application of game theory based production flow planning method in virtual manufacturing networks

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Abstract. Modern enterprises have to react quickly to dynamic changes in the market, due to changing customer requirements and expectations. One of the key area of production management, that must continuously evolve by searching for new methods and tools for increasing the efficiency of manufacturing systems is the area of production flow planning and control. These aspects are closely connected with the ability to implement the concept of Virtual Enterprises (VE) and Virtual Manufacturing Network (VMN) in which integrated infrastructure of flexible resources are created. In the proposed approach, the players role perform the objects associated with the objective functions, allowing to solve the multi-objective production flow planning problems based on the game theory, which is based on the theory of the strategic situation. For defined production system and production order models ways of solving the problem of production route planning in VMN on computational examples for different variants of production flow is presented. Possible decision strategy to use together with an analysis of calculation results is shown.

1. Introduction

Market dynamics associated with technological advancement forces the manufacturers to change in the area of manufacturing processes organization that affect increasing flexibility level in functioning of enterprises on the market. Increasingly shorter product life cycles, high level of innovation and a multitude of new solutions cause that the producers are looking for more and more effective methods of supporting process of planning of production flow. For this type of demand (requiring increasing levels of flexibility, frequent changes in production process and resources efficiency and reducing costs) respond solutions associated with production processing in the form of Virtual Manufacturing (VM) [1-3]. Implementation of the production planning process (and the possibility of using conventional tools to support the planning process), the more complicated, the larger number of cooperating producers are concerned. Emerging problems among other things, are directed to methods of sharing available resources between producers. Producers are looking for flexible enough tools and methods that enable the development of VE concept in the direction of creating a very dynamic organization, formed depending on the current needs and market opportunities, and implementing collectively activities as long as these needs will not change. Virtual enterprise, if necessary, should be able to re-organize themselves by adding, removing or replacing some of its members at the same time with the rapid planning process of tasks [2-5].
The use of methods and formalism from the game theory to the formulation of the decision-making situation in the area of production planning is proposed in the paper. Considered issue concerns the selection of the route from the available alternative routes, which are a combination between all the participants in the network assigned to manufacture the new product. With a relatively easy implementation of mathematical notation to the production planning area and available models the game theory to production planning at the operational level was used [6-9].

2. Virtual enterprises

Nowadays, the solution applied more and more often by small and medium-sized enterprises is to combine a "group" for the purpose of producing a product, than work and often do not use the market opportunities through independent existence [2, 3]. Quoting [2]: virtual organization is an organisational structure based on the notion of collaborating entities, coming together to share their resources for the purpose of taking advantage of a particular opportunity. Each partner contributes complementary resources that reflect its strengths. An important aspect of virtual organization is the information technology, without which the producers could not cooperate with each other at the appropriate level, and which could entail the risk of failure in the execution of production order.

Key importance in the development of Virtual Enterprise (VE) concept is the solution of problems in exchange of information between existing heterogeneous and autonomous systems supporting production management at various levels in a distributed environment, including: planning, coordination and control of production activities. The possibilities offered by current IT technology create opportunities, what was once just a dream. This is even more important because each decision that can be taken at each organizational level has an influence on the performance indicators of the final product. Each organization participating in the execution of a production order should have access to critical data to make the best decisions. [1, 4, 5, 10, 11]. One of the concepts associated with VE is Virtual Manufacturing Network (VMN).

2.1. Virtual Manufacturing Network

Deciding to participate in the Virtual Manufacturing Network, small and large companies trying to take advantage of the opportunities resulting from the market. Depending on the size and complexity of the production order, in the network may be several or more manufacturers. They share their spare capacity and skills to produce good quality products. The network is created only for the purpose of the emerging order and resolved after completion of cooperation. Another important point of the network is, that these organizations could be geographically dispersed, so adequate planning between producers regarding appropriate allocation of resources, means of transport, deadlines, etc. is very important. [2-4]. Another important factor in the functioning of the producers in the network, as a temporary structure, is trust, without which it would be difficult to succeed, that is very easy to lose, but so hard to rebuild. In a virtual network manufacturers especially have to trust each other, because the possibility of immediate changes in response to the problems is limited due to geographical dispersion. Trust requires hard work of all the participants, non-stop and everyone should take care of its development. In extreme cases, where cooperation does not give a significant gain, it is possible to change the network members [7].

The other problem in VMN is connecting all of this companies on manufacturing resources level. There are a few barriers to overcome like manufacturing and transportation costs, depending on which groups of technological operations will be carried out at a particular member of VM network. To use the idea of VMN, producers should have access to a very flexible planning tools at operational level - in particular, decision support in the selection of production routes. In the paper practical implementation of the method of supporting production flow planning in VMN, in particular supporting the choice of routes from the permissible routes set between individual producers, based on methods derived from game theory is presented. The decision-making situation in discussed problem is considered as a non-cooperative, 2-person, non-zero-sum game with complete information. In our
approach, players are represented as objective functions: $F1$ - minimizing manufacturing costs and $F2$ - minimizing transportation cost between producers.

3. Application of game theory based production flow planning method in VMN
Considered decision problem concerns the decision-making at the operational level [9-14]. It is assumed the existence of independent production systems, which can be implemented interconnected production processes. Sets of production orders for which there are multiple possibilities of alternative routes for the production processes are given. Production orders are executed on chosen production resources from the available set (for individual producers). The proposed model of decision-making problem is therefore related to the production flow planning problem.

The production system is defined as:

$$S = (M, C, PP, B)$$  \hspace{1cm} (1)$$

where:

$S$ – the production system,

$M = \{M^\varepsilon, i = 1,2,\ldots,m_\varepsilon ; \varepsilon = 1,2,\ldots,R\}$ – production resources,

$m_\varepsilon$ – the number of resources owned by $\varepsilon$-th manufacturer,

$R$ – the number of manufacturer in the network,

$C = C^P + C^T$ – the unit production cost ($C^P$) and the unit costs of transport ($C^T$):

$$C^P = [C^P_{\varepsilon,k}] = \begin{bmatrix} 1 & \cdots & K \\
1 & \cdots & 1, \\
\vdots & \cdots & \vdots \\
R & \cdots & R_{K, K} \end{bmatrix},$$

where:

$K = \max (m_\varepsilon)$ – maximum number of resources available in the network members,

$C^P_{\varepsilon,k}$ - the unit cost of technological operation execution on $k$-th resource belonging to the $\varepsilon$-th manufacturer,

$$C^T = [C^T_{\varepsilon,k}] = \begin{bmatrix} 1 & \cdots & R \\
1 & \cdots & 1, \\
\vdots & \cdots & \vdots \\
R & \cdots & R_{K, K} \end{bmatrix},$$

where:

$C^T_{\varepsilon,k}$ - the unit cost of transportation between manufacturers belonging to the network,

The production order is defined as:

$$PP = (P, MP, N)$$  \hspace{1cm} (2)$$

$P = \{P_j, j = 1,2,\ldots,n\}$ – production processes,

$MP = \{MP^w_j, j = 1,2,\ldots,n; w = 1,2,\ldots,v_j\}$ – production processes matrix:

$n$ – the number of manufacturing processes,

$w$ – version of the process route, $v_j$ –the number of routes version for the $j$-th process,

$$MP^w_j = \begin{bmatrix} mp_{11} & mp_{12} & \cdots & mp_{1h_1} \\
mp_{21} & mp_{22} & \cdots & mp_{2h_2} \\
\vdots & \vdots & \ddots & \vdots \\
mp_{jh} & mp_{sh} & \cdots & mp_{sh} \end{bmatrix},$$

matrix for $w$-th version of the route for the $j$-th process,
$H_j^w$ – the number of technological operations in $w$-th route for $j$-th process,

$mp_{1h}$ – manufacturer No,

$mp_{2h}$ – production resource No for $h$-th technological operation,

$mp_{3h}$ – processing times for each operation.

$N_j$ – batch size,

$B=\{B_{lk} \mid l = 1,2,...,R; k = 1,2,...,R; R \neq k\}$ – storage capacities for individual producers.

3.1. The decision-making situation model

For the above-defined models of production systems and production orders, functions of the objective are defined:

$F1$: minimizing the production cost, equation (3):

$$F1(\sigma) = \sum_{j=1}^{H_j} \sum_{h=1}^{H_j} (C_{mp_{1h},mp_{1h}} \ast mp_{3h}) \rightarrow \min$$

$F2$: minimizing transportation costs between producers, equation (4):

$$F2(\sigma) = \sum_{j=1}^{H_j} \sum_{h=1}^{H_j} (C_{mp_{1h},mp_{1h}} \ast N_j) \rightarrow \min$$

where:

$\sigma$ - the route variant selected for implementation for $j$-th process, $\sigma = (p_j)$

In the paper notation for decision problem taken from the game theory to seek a solution is applied. Game theory is used in many fields of science: economics, sociology, psychology, military, etc. [7, 8, 15, 16]. The use of decision-making methods taken from the game theory requires defining the basic elements describing the game [8]:

- set of players,
- set of game rules, which are guided by the players,
- set of possible strategies for each player,
- set of possible outcomes,
- payment of players related to decisions.

The decision-making situation in discussed area is considered as a non-cooperative, 2-person, non-zero-sum game with complete information. Players are represented as objects of the objective function. The first player represents objective functions: $F1$ - minimizing manufacturing costs, and the second player $F2$ - minimizing transportation cost between producers. According to the theory of games, each player strives to choose the strategy with the possibility of solutions according to the available alternative routes for each production process.

With defined functions of the objective and players, the game can be written as:

$$G = (N, S, U)$$

where:

$N=\{N_g \mid g = 1,2\}$ – set of players corresponding to each of the objective function,

$S=(s')$ – set of strategies, corresponding to the execution of selected routes versions $s_j = \sigma_{p_j}$,

$U=(u')$ – payoff matrix corresponding to the values of the cost for the selected strategy ($C^p + C^T$).

3.2. Computational example

The virtual manufacturing network which consists of three producers is being considered. Manufacturers share their production resources in order to perform production order. Each
manufacturers offers three resources that can be used for the execution of the order. In the production order there are two production processes P₁ and P₂. Each production process has five different versions of its execution. Abbreviations P_jW_w are respectively j⁻th process and w⁻th variant (figure 1).

![Production flow diagram](image)

**Figure 1.** Production flow.

Processing times for each technological operation are recorded in table 1 for process P₁, and in table 2 for process P₂. M_εᵢ is an i⁻th resource of ε-th manufacturers. It was assumed the following batch sizes, which were respectively - 200 pieces for P₁ and 300 pieces for P₂.

**Table 1.** Processing time and cost of the P₁ process for the different routes variants.

| Variant | Route | Processing time (unit cost) |
|---------|-------|-----------------------------|
| 1       | M₁₁  | M₁₂                         |
| 2       | M₁₁  | M₂₁                         |
| 3       | M₃₁  | M₁₂                         |
| 4       | M₃₁  | M₂₁                         |
| 5       | M₃₄  | M₂₁                         |

**Table 2.** Execution time and cost of the P₂ process for the different routes variants.

| Variant | Route | Processing time (unit cost) |
|---------|-------|-----------------------------|
| 1       | M₁₁  | M₁₁                         |
| 2       | M₁₄  | M₂_; M₃; M₄; M₅; M₁₁; M₁₂ }
For the above processing times production and transportation costs between manufacturers participating in a virtual manufacturing network were calculated. Considering the cost of production and transport as the value of payments to players, the choice of the winning variant of the route can be made searching for saddle point between them. The obtained results are shown in figure 2.

![Figure 2. Production and transportation costs.](image)

The vertical axis describes the payment (production and transportation cost), the minimum value does not exceed 250 (for variant no 3). In turn, the horizontal axis of the values 1 to 5, means a specific variant of the production route. Obtained results in an exemplary game shows that for the players, there is no common values, i.e. for the minimum cost of one player, the other player does not bear the minimum costs. This means that although there is no equilibrium point in pure strategies, the choice can be made using mixed strategies.

4. Conclusions

In the article the possibilities of using production resources in enterprises through active participation in virtual manufacturing networks has been presented. Planning the production flow within such organizations, forcing manufacturers to seek more effective decision making methods relating to the selection of alternative routes. The possibilities of application of formalism taken from the game theory to define decision-making situation related to production planning and selection of routes also has been shown. Application of proposed planning methods should allow to increase the level of flexibility in the use of available capacity of production resources, which make it possible to increase the level of competitiveness and maintaining a business in a dynamic market. The subject of further research will be related to the verification of the applicability of a balance for both pure strategies and mixed together with an analysis of the applicability of one of the strategies to solve a certain class of the problem.
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