Mobility Equity from a Game-Theoretic Perspective

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Abstract—In this letter, we consider a multi-modal mobility system of travelers, and propose a game-theoretic framework to efficiently assign each traveler to a mobility service (e.g., different modes of transportation). Our focus in this framework is to maximize the “mobility equity” in the sense of respecting the mobility budgets of the travelers. Each traveler seeks to travel using only one service (e.g., car, bus, train, bike). The services are capacitated and can serve up to a fixed number of travelers at any instant of time. Thus, our problem falls under the category of many-to-one assignment problems, where the goal is to find the conditions that guarantee the stability of assignments. We formulate a linear program of maximizing the mobility equity of travelers and we fully characterize the optimal solution. We also show that our framework under the proposed pricing scheme induces truthfulness from the strategic travelers, while they have incentives to voluntarily participate under informational asymmetry.

I. INTRODUCTION

Commuters in big cities have continuously experienced the frustration of congestion and traffic jams. Travel delays, accidents, and altercations have vastly increased damage to the economy, the society, and the environment just by the sheer number of idling engines [1]. In addition, one of the pressing challenges of our time is the increasing demand for energy, which requires us to make fundamental transformations in how we use and access transportation. Thanks to evolutionary developments that are currently afoot, it is highly expected that we will be able to entirely eliminate congestion and significantly increase mobility efficiency in terms of fuel consumption and travel time [2]. Self-driving cars offer a most intriguing opportunity that will allow us to travel safely and efficiently anywhere and anytime [3]. Several studies have shown the benefits of emerging mobility systems (e.g., ride-hailing, on-demand mobility services, shared vehicles, autonomous vehicles) to reduce energy and alleviate traffic congestion in a number of different transportation scenarios [4], [5].

Recently it was shown that when daily commuters were offered a convenient and affordable taxi service for their travels, a change of behavior was noticed, namely these commuters ended up using taxi services more often compared to when they drove their own car [6]. Along with other studies [7], [8] this shows us that emerging mobility systems will affect people’s tendency to travel and incentivize them to use cars more frequently which potentially can also lead to a shift away from public transit. Our approach, in this letter, is to study the game-theoretic interactions of travelers seeking to travel in a multi-modal mobility system, where each traveler has a unique mobility budget.

Wide accessibility to transportation in emerging mobility systems can be impacted by the socioeconomic background of the travelers, i.e., whether a traveler can afford it. For example, travelers with a low-income background may be unable to use any or all transportation travel options available to them in a city. A key characteristic of our approach, which allows us to differentiate from other studies on fairness and efficiency [9], [10], is that we adopt the Mobility-as-a-Service (MaaS) concept, which is a system of multimodal mobility that handles user-centric information and provides travel services (e.g., navigation, location, booking, payment) to a number of travelers. The goal is to guarantee mobility as a seamless service across all modes of transportation accessible to all in a socially-efficient and fair way.

One of the standard approaches to alleviate congestion in a transportation system has been the management of demand size due to the shortage of space availability and scarce economic resources in the form of congestion pricing. Such an approach focuses primarily on intelligent and scalable traffic routing, in which the objective is to guide and coordinate users in path-choice decision-making. For example, one computes the shortest path from a source to a destination regardless of the changing traffic conditions [11]. Interestingly, by adopting a game-theoretic approach, advanced systems have been proposed to assign users concrete routes or minimize travel time and study the Nash equilibria under different tolling mechanisms [12]–[16]. Partly related to our work are matching models which describe markets in which there are agents of disjoint groups and have preferences regarding the “goods” of the opposite agent they associate with. Notable examples are mechanisms for assigning students to schools, interns to hospitals or worker-firm contracts [17], [18]. It is easy to see that matching markets are quite practical as they offer insights into the more general economic and behavioral real-life situations. These examples are all centralized approaches of determining who gets assigned to whom at what cost and benefit. Given the natural usefulness of matching markets, various extensions of the assignment game have been developed focusing either on different behavioral settings or information structures [19]–[22]. Assignment games and matching markets have also been used and studied extensively in auction theory [23], [24]. For a complete and thorough overview of assignment games and matching models, see [25].

The main contribution of this letter is the development of a game-theoretical framework to study the economic inter-
actions of travelers in a multi-modal mobility system. Our analysis can be divided into two parts. First, we formulate our problem as a linear program to compute the assignments between travelers and mobility services in a socially-efficient and fair way, i.e., no one will seek to deviate from their match and their budget will not be violated. Second, we consider an asymmetric informational structure, where the social planner has no knowledge of the individual valuations of the travelers for each service. We provide a pricing mechanism and show how we can elicit the private information truthfully (Theorem 1) while ensuring monetary fairness (Theorem 2). We also show that every traveler voluntarily participates in our proposed framework (Theorem 3).

The remainder of the letter is structured as follows. In Section II, we present the mathematical formulation of the proposed game-theoretic framework. In Section III we derive the theoretical properties of the proposed framework, and finally, in Section IV, we draw conclusions and offer a discussion of future research.

II. Modeling Framework

In this section, we present the mathematical formulation of our game-theoretic framework for mobility equity.

A. Mathematical Formulation

We consider a setting where $I \subseteq N$ travelers, indexed by $i \in I$, $|I| = I$, are interested in a set of $J \subseteq N$ mobility services, indexed by $j \in J$, made available by a social planner (central authority in a city). Each traveler $i \in I$ has a private valuation $v_{ij}$ associated with each of the services $j \in J$, which is not known to the social planner. Additionally, the travelers are also budget constrained with budgets $\{b_1, b_2, \ldots, b_I\}$, $b_i \in \mathbb{R}_{\geq 0}$.

For the purposes of this work, we assume that the budgets of each traveler are known to the social planner. Part of our justification here is that it is reasonable to expect travelers to submit their travel budget on a mobility app [26], [27]. In future work, we plan to introduce a probabilistic model and consider private budgets.

For each service $j \in J$, we model the social planner’s beliefs on the realization of the private valuations for service $j$ as real values from some subset of real values.

**Definition 1.** For each traveler $i \in I$, the traveler $i$’s valuation profile of all mobility services is $v_i = (v_{i1}, v_{i2}, \ldots, v_{ij})$, $v_{ij} \in \mathbb{R}$. We write $v_{-ij} = (v_{i1}, \ldots, v_{i-1j}, v_{i+1j}, \ldots, v_{ij})$ for the valuations profile of all travelers except $i$ for service $j$ and denote by $v_{-i} = (v_{-i1}, \ldots, v_{-ij})$ the profile of valuations of all services of all travelers except traveler $i$.

**Definition 2.** Let $v = (v_i, v_{-i})$ denote the valuation profile of all travelers of all mobility services. Then the social planner’s beliefs on the valuations of the travelers are derived from set $\mathcal{V} \subseteq \mathbb{R}^{|I| \times |J|}$.

For an arbitrary traveler $i$, the valuation $v_{ij}$ can represent the realization of a satisfaction function that captures, for example, the maximum amount of money that traveler $i$ is willing to pay for mobility service $j$.

**Remark 1.** We can construct explicitly the set $\mathcal{V}$ in multiple ways depending on the type of information that we have access to. For example, we could use data gathered from a mobility app used by all travelers over a finite time period.

**Remark 2.** The travelers request to use at most one service to satisfy their mobility needs, i.e., to reach their destination, via a smartphone app available to all travelers. The social planner (e.g., a central computer) compiles all travelers’ origin-destination requests and other information (e.g., preferred travel time, value of time, and maximum willingness-to-pay) in order to provide a travel recommendation to each traveler.

Mathematically, the allocation of the finite number of mobility services to travelers can be described by a vector of binary variables.

**Definition 3.** The traveler-service assignment is a vector $a = (a_{ij})_{i \in I, j \in J}$, where $a_{ij}$ is a binary variable of the form:

$$a_{ij} = \begin{cases} 1, & \text{if } i \in I \text{ is assigned to } j \in J, \\ 0, & \text{otherwise}. \end{cases} \tag{1}$$

Note that the assignment $a_{ij}$ between traveler $i$ and service $j$ will depend on the valuation $v_i$ of traveler $i$ and the valuations of all other travelers, i.e., $v_{-i}$.

Naturally, each service can accommodate up to a some number of travelers, different for each type of services. So, we expect the “physical traveler capacity” of each service to vary significantly.

**Definition 4.** Each service $j \in J$ is associated with a current traveler capacity, denoted by $\varepsilon_j \in \mathbb{N}$ and $\varepsilon_j \leq \bar{\varepsilon}_j$, where $\bar{\varepsilon}_j$ denotes the maximum traveler capacity of service $j$.

For example, a bus can provide travel services to a hundred travelers (seated and standing) compared to a bike-sharing company’s bike in a generic city (one traveler per bike).

**Definition 5.** A feasible assignment is a vector $a = (a_{ij})_{i \in I, j \in J}$, $a_{ij} \in \{0, 1\}$ that satisfies

$$\sum_{j \in J} a_{ij}(v) \leq 1, \quad \forall i \in I, \quad \forall v \in \mathcal{V}, \tag{2}$$

$$\sum_{i \in I} a_{ij}(v) \leq \varepsilon_j, \quad \forall j \in J, \quad \forall v \in \mathcal{V}, \tag{3}$$

where (2) ensures that each traveler $i \in I$ is assigned to at most one mobility service $j \in J$, and (3) ensures that the traveler capacity of each service $j$ is not exceeded while it is shared by multiple travelers.

Next, we represent the preferences of each traveler with a utility function consisted of two parts: the traveler’s valuation of the mobility outcome and the associated payment required for the realization of that outcome. In other words, any traveler is expected to pay a toll or ticket fee for the mobility service used.

**Definition 6.** Each traveler $i$’s preferences are summarized by a utility function $u_i : \mathcal{V} \times \mathbb{R} \rightarrow \mathbb{R}$ that determines the
monetary value of the overall payoff realized by traveler $i$ from their assignment to service $j$. Let $p_i \in \mathbb{R}$ denote traveler $i$'s mobility payment. Thus, traveler $i$ receives a total utility

$$u_i(v_i, v_{-i}) = \sum_{j \in \mathcal{J}} v_{ij}a_{ij}(v_i, v_{-i}) - p_i(v_i, v_{-i}).$$  

(4)

Inspired from [28], we present the following definition for measuring the mobility equity in the mobility system.

**Definition 7.** The level of mobility equity is a linear function $w: \mathcal{V} \times \mathbb{R} \rightarrow \mathbb{R}_{\geq 0}$ that depends on the valuations, assignments, and individual budgets of all the travelers. We say that the proposed mechanism satisfies the property of mobility equity if

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} w_{ij}(v_{ij}, a_{ij}, b_i) - \sum_{i \in \mathcal{I}} p_i(v) \leq 0, \quad \forall v \in \mathcal{V},$$  

(5)

To the best of our knowledge, there is no single way to construct such a function $w_i$. However, using data [29, 30] and performing an “equity analysis” for transportation systems can give us an explicit function [31].

**Definition 8.** For the travelers to have no incentive to misreport their valuations to the social planner, we need

$$\sum_{j \in \mathcal{J}} v_{ij}a_{ij}(\bar{v}_i, v_{-i}) - p_i(\bar{v}_i, v_{-i})$$

$$- \sum_{j \in \mathcal{J}} v_{ij}a_{ij}(v_i, v_{-i}) + p_i(v_i, v_{-i}) \leq 0,$$  

(6)

for all $v = (v_i, v_{-i}) \in \mathcal{V}$, any $\bar{v}_i \in \mathcal{V}$, and for all travelers $i \in \mathcal{I}$ using any mobility service $j \in \mathcal{J}$. We call $\bar{v}_i$ traveler $i$’s reported valuation that deviates from the true valuation $v_i$. If (6) holds, then we say that the mechanism induces truthfulness.

**Definition 9.** The travelers in the mobility system voluntarily participate if, for any traveler $i \in \mathcal{I}$,

$$p_i(v_i, v_{-i}) \leq \sum_{j \in \mathcal{J}} v_{ij}a_{ij}(v_i, v_{-i}), \quad \forall v \in \mathcal{V},$$  

(7)

We say then that the proposed mechanism induces voluntary participation from all travelers.

**Definition 10.** The mechanism induces on individual level monetary fairness, if for any traveler $i \in \mathcal{I}$, we have

$$p_i(v) \leq b_i, \quad \forall v \in \mathcal{V},$$  

(8)

**B. The Optimization Problem**

**Problem 1.** The maximization problem of mobility equity is formulated as follows

$$\max_{a_{ij}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} w_{ij}(v_{ij}, a_{ij}, b_i),$$

subject to: [5], [6], [7], [8], [2], [3],

where $a_{ij} \in \{0, 1\}$ for all $i \in \mathcal{I}$ and all $j \in \mathcal{J}$.

We can relax the binary variable constraint to a non-negativity constraint variable in Problem [1].

**Problem 2.** The linear program formulation is

$$\max_{a_{ij}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} w_{ij}(v_{ij}, a_{ij}, b_i)$$

subject to: [5], [6], [7], [8], [2], [3], and

$$a_{ij} \geq 0, \quad \forall i \in \mathcal{I}, \quad \forall j \in \mathcal{J},$$

(11)

where [11] transforms the (binary) assignment problem to a (continuous) linear program.

In previous work [32], we showed that constraint relaxation does not affect the optimal solutions of Problem [1] as we can ensure all optimal solutions of the equivalent linear program are binary valued (only 0s and 1s).

**Remark 3.** Intuitively, $a_{ij}$ in Problem [2] can be interpreted as the probability that traveler $i \in \mathcal{I}$ is assigned to service $j \in \mathcal{J}$.

Inspired from Myerson’s auction [23] and the Vickrey-Clarke-Groves (VCG) auction mechanism [33], we introduce two key variables that can help us solve Problem [2], i.e., nominal assignments and reservation payments.

**Definition 11.** For any traveler $i$, there is a reservation payment for each mobility service $j$, denoted by $r_{ij} \in \mathbb{R}_{\geq 0}$, representing the minimum necessary mobility payment of traveler $i$ to get assigned to mobility service $j$.

**Definition 12.** The final assignment $\tilde{a}_{ij}(v)$ evaluated at the realized valuation profile $v$ is computed as the sum of the nominal assignment $\bar{a}_{ij}$ and the adapted assignment $\tilde{a}_{ij}(v)$, i.e., we have

$$a_{ij}(v_i, v_{-i}) = \bar{a}_{ij} + \tilde{a}_{ij}(v_i, v_{-i}).$$  

(12)

We provide the exact methodology of computing $\bar{a}_{ij}$ and $\tilde{a}_{ij}(v)$ in the next section.

Finally, we present the pricing scheme for any traveler $k \in \mathcal{I}$ of our proposed mechanism:

$$p_k(v) = \sum_{j \in \mathcal{J}} \bar{a}_{kj}(v)r_{kj} + \sum_{j \in \mathcal{J}} \bar{a}_{kj}r_{kj} + \sum_{j \in \mathcal{J}} \bar{a}_{kj}r_{kj} + \sum_{i \in \mathcal{I} \setminus \{k\}} \sum_{j \in \mathcal{J}} \bar{a}_{ij;k}(v_k)(v_{ij} - r_{ij})$$

$$- \sum_{i \in \mathcal{I} \setminus \{k\}} \sum_{j \in \mathcal{J}} \bar{a}_{ij}(v)(v_{ij} - r_{ij}),$$

(13)

where $\bar{a}_{ij;k}$ represents a “temporary” assignment of travelers to mobility services expecting traveler $k$. We formally present how to compute such an assignment in Theorem [2]. The term $\sum_{i \in \mathcal{I} \setminus \{j\}} \bar{a}_{ij}(v)(v_{ij} - r_{ij})$ represents the “social welfare” of all travelers based on the valuations of each mobility service $j$ and the reservation mobility payments $r_{ij}$. Using these reservation payments, we then introduce a mobility payment $p_k$ for traveler $k$ that charges the minimum required payment for traveler $k$ to get assigned to mobility service $j$ while keeping all other travelers’ reported valuations fixed.
III. Analysis and Properties of the Mechanism

In this section, we show formally that the proposed mechanism satisfies the desired properties of truthfulness, voluntary participation, and it is monetary fair for all travelers. We start our exposition by presenting some preliminary results.

Lemma 1. The dual problem of Problem 2 is

\[ \min \sum_{v \in \mathcal{V}} \left[ \sum_{i \in \mathcal{I}} \xi_i^1(v)\bar{b}_i + \sum_{j \in \mathcal{J}} \xi_j^2(v) \right] \]  
subject to:  
\[ \xi_j^1(v) + \xi_j^2(v) + \sum_{v_i \in \mathcal{V}} \bar{v}_{ij} \xi_i^3(v, \bar{v}_i) - v_{ij} \xi_i^4(v) \geq 0, \quad \forall v \in \mathcal{V}, \]  
\[ \sum_{v_i \in \mathcal{V}} \xi_i^4(v, \bar{v}_i) - \sum_{v_i \in \mathcal{V}} \xi_i^4(\bar{v}_i, v_{i-}) - \xi_1(v) \]  
\[ + \xi_i^5(v_{i-}, v_{i-}) + \xi_i^6(v_{i-}) = 0, \quad \forall v \in \mathcal{V}, \]  
\[ \xi_i^1(v), \xi_i^2(v), \xi_i^3(v), \xi_i^4(v), \xi_i^5(v), \xi_i^6(v) \geq 0, \]  
where \( \xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \) and \( \xi_6 \) are the dual variables for constraints (2), (3), (4), (6), (7), and (8), respectively.

Proof. The computations here are straightforward following standard techniques from [34], hence we omit them due to space limitations.

Next, we show that our mechanism induces truthfulness from all travelers, i.e., no traveler has an incentive to misreport or lie to the social planner.

Theorem 1 (Truthfulness). The proposed mechanism with assignments given by Definition 12 and payments given by (13) satisfy the property of truthfulness defined by Definition 9.

Proof. Consider traveler \( k \) with a true valuation \( v_{kj} \) for each service \( j \in \mathcal{V} \). By reporting \( v'_{kj} \), traveler \( k \) is assigned service \( j \) with probability \( \hat{a}_{kj}(v'_k, v_{-k}) \). We formulate the following optimization problem

\[ (\hat{a}_{ij}(v))_{i \in \mathcal{I}, j \in \mathcal{J}} = \arg \max_{\hat{a}_{ij}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \hat{a}_{ij}(v_{ij} - r_{ij}) \]  
where \( \mathcal{A} \) is the set of positive values for \( \hat{a} \) that satisfies the following two constraints:

\[ \sum_{i \in \mathcal{I}} \hat{a}_{ij} \leq 1 - \sum_{i \in \mathcal{I}} \hat{a}_{ij}, \quad \forall j \in \mathcal{J}, \]  
\[ \sum_{j \in \mathcal{J}} \hat{a}_{ij} \hat{v}_{ij} \leq b_i - \sum_{j \in \mathcal{J}} \hat{a}_{ij} r_{ij} \]  
\[ + \sum_{j \in \mathcal{J}} \hat{a}_{kj} \xi_{ij}^1 \gamma_{ij}, \quad \forall \bar{v} \in \mathcal{V}, \quad \forall i \in \mathcal{I}, \]  
where \( \gamma_{ij} = \arg \min_{v \in \mathcal{V}} \sum_{j \in \mathcal{J}} \hat{a}_{ij} \hat{v}_{ij} \). Since \( A \) does not depend on any specific valuation profile, we have

\[ \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \hat{a}_{ij}(v_k, v_{-k})(v_{ij} - r_{ij}) \geq \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \hat{a}_{ij}(v'_k, v_{-k})(v_{ij} - r_{ij}). \]  

Using Definition 6 we now compare the utilities of traveler \( k \) under the two different valuations. So, we have

\[ u_k(v_k, v_{-k}) = \sum_{j \in \mathcal{J}} \hat{a}_{kj}(v_k, v_{-k})v_{kj} - \hat{p}(v_k, v_{-k}), \]  
which, by Definition 12 and (13), we can expand as follows

\[ u_k(v_k, v_{-k}) = \sum_{j \in \mathcal{J}} \hat{a}_{kj}(v_k, v_{-k})v_{kj} + \sum_{j \in \mathcal{J}} \hat{a}_{kj}v_{kj} \]  
\[ - \sum_{j \in \mathcal{J}} \hat{a}_{kj}r_{kj} - \sum_{j \in \mathcal{J}} \hat{a}_{kj}r_{kj} + \sum_{j \in \mathcal{J}} \hat{a}_{kj} \xi_{ij} \]  
\[ - \sum_{i \in \mathcal{I} \setminus \{k\}} \sum_{j \in \mathcal{J}} \hat{a}_{ij}(v_k)(v_{ij} - r_{ij}) \]  
\[ + \sum_{i \in \mathcal{I} \setminus \{k\}} \sum_{j \in \mathcal{J}} \hat{a}_{ij}(v_k, v_{-k})(v_{ij} - r_{ij}) \]  
\[ = \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \hat{a}_{ij}(v_k, v_{-k})(v_{ij} - r_{ij}) - \sum_{i \in \mathcal{I} \setminus \{k\}} \sum_{j \in \mathcal{J}} \hat{a}_{ij}(v_k, v_{-k})(v_{ij} - r_{ij}) \]  
\[ + \sum_{i \in \mathcal{I}} \hat{a}_{ij}v_{kj} - \sum_{j \in \mathcal{J}} \hat{a}_{kj}r_{kj} + \sum_{j \in \mathcal{J}} \hat{a}_{kj} \xi_{ij} \]  
\[ \geq \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \hat{a}_{ij}(v'_k, v_{-k})(v_{ij} - r_{ij}) - \sum_{i \in \mathcal{I} \setminus \{k\}} \sum_{j \in \mathcal{J}} \hat{a}_{ij}(v_k)(v_{ij} - r_{ij}) \]  
\[ + \sum_{i \in \mathcal{I}} \hat{a}_{ij}v_{kj} - \sum_{j \in \mathcal{J}} \hat{a}_{kj}r_{kj} + \sum_{j \in \mathcal{J}} \hat{a}_{kj} \xi_{ij} \]  
\[ = \sum_{j \in \mathcal{J}} \hat{a}_{kj}(v'_k, v_{-k})v_{kj} + \sum_{j \in \mathcal{J}} \hat{a}_{kj}v_{kj} \]  
\[ - \sum_{j \in \mathcal{J}} \hat{a}_{kj}(v'_k, v_{-k})r_{kj} - \sum_{j \in \mathcal{J}} \hat{a}_{kj}r_{kj} \]  
\[ - \sum_{i \in \mathcal{I} \setminus \{k\}} \sum_{j \in \mathcal{J}} \hat{a}_{ij}(v_k)(v_{ij} - r_{ij}) \]  
\[ + \sum_{i \in \mathcal{I} \setminus \{k\}} \sum_{j \in \mathcal{J}} \hat{a}_{ij}(v_k, v_{-k})(v_{ij} - r_{ij}), \]  
where the last equality (27) is equal to \( u_k(v'_k, v_{-k}) \). Hence, the result follows.

Theorem 2. The proposed mechanism with assignments given by Definition 12 and payments given by (13) satisfy the property of mobility equity defined by Definition 10.
Proof. The mobility payment \((13)\) of any traveler \(k\) can be written as
\[
p_k(v) = \sum_{j \in J} \hat{a}_{kj}(v)r_{kj} + \sum_{j \in J} \bar{a}_{kj}r_{kj} - \sum_{j \in J} \bar{a}_{kj}x^k_j \gamma_{ij}
+ \sum_{i \in I \setminus \{k\}} \sum_{j \in J} \tilde{a}_{ij,j:k}(v_{-k})(v_{ij} - r_{ij})
- \sum_{i \in I \setminus \{k\}} \sum_{j \in J} \tilde{a}_{ij}(v)(v_{ij} - r_{ij}).
\] (28)

Next, we formulate the following optimization problem:
\[
(\hat{a}_{ij,j:k}(v_{-k}))_{i \in I \setminus \{k\}, j \in J} = \arg \max_{\hat{a}_{ij} \in \mathcal{A}_k} \sum_{i \in I \setminus \{k\}} \sum_{j \in J} \hat{a}_{ij}(v_{ij} - r_{ij}),
\] (29)

where \(\hat{a}_{ij,j:k}(v_{-k})\) takes positive values from set \(\mathcal{A}_k\) with constraints:
\[
\sum_{i \in I \setminus \{k\}} \hat{a}_{ij} \leq 1 - \sum_{i \in I \setminus \{k\}} \bar{a}_{ij}, \quad \forall j \in J
\] (30)
\[
\sum_{j \in J} \hat{a}_{ij} \bar{v}_{ij} \leq b_i - \sum_{j \in J} \bar{a}_{ij}r_{ij},
\] (31)

where (31) holds for all \(\tilde{v} \in \mathcal{V}\) and \(i \in I \setminus \{k\}\). Thus, we have \((\hat{a}_{ij,j:k}(v_{-k}))_{i \in I \setminus \{k\}, j \in J} = \mathcal{A}_k\), which yields
\[
\sum_{i \in I \setminus \{k\}} \hat{a}_{ij,j:k}(v_{-k}) \leq 1 - \sum_{i \in I \setminus \{k\}} \bar{a}_{ij}, \quad \forall j \in J
\] (32)
\[
\sum_{j \in J} \hat{a}_{ij,j:k}(v_{-k}) \bar{v}_{ij} \leq b_i - \sum_{j \in J} \bar{a}_{ij}r_{ij},
\] (33)

where (33) holds for all \(\tilde{v} \in \mathcal{V}\) and for all \(i \in I \setminus \{k\}\). If we construct an assignment \(\alpha = (\alpha_{ij})\) such that
\[
\alpha_{ij} = \begin{cases} 
\hat{a}_{ij,j:k}(v_{-k}), & \text{if } i \neq k, \quad \forall j \in J, \\
0, & \text{if } i = k, \quad \forall j \in J,
\end{cases}
\] (34)

then we have
\[
\sum_{i \in I} \alpha_{ij} \leq 1 - \sum_{i \in I} \bar{a}_{ij}, \quad \forall j \in J,
\] (35)
\[
\sum_{j \in J} \alpha_{ij} \bar{v}_{ij} = \sum_{j \in J} \hat{a}_{ij,j:k}(v_{-k}) \bar{v}_{ij} 
\leq b_i - \sum_{j \in J} \bar{a}_{ij}r_{ij},
\] (36)

where (36) holds for all \(\tilde{v} \in \mathcal{V}\) and for all \(i \in I \setminus \{k\}\). Similarly, for traveler \(k\), we have
\[
\sum_{j \in J} \alpha_{kj} \bar{v}_{kj} = 0 \leq b_k - \sum_{j \in J} \bar{a}_{kj}r_{kj}.
\] (37)

Thus, it follows immediately that \(\alpha \in \mathcal{A}\) which results to the following inequality:
\[
\sum_{i \in I} \sum_{j \in J} \tilde{a}_{ij}(v)(v_{ij} - r_{ij}) \geq \sum_{i \in I} \sum_{j \in J} \alpha_{ij}(v_{ij} - r_{ij})
= \sum_{i \in I \setminus \{k\}} \sum_{j \in J} \hat{a}_{ij,j:k}(v_{-k})(v_{ij} - r_{ij}).
\] (38)

So, we obtain
\[
p_k(v) \leq \sum_{j \in J} \tilde{a}_{kj}(v)r_{kj} + \sum_{j \in J} \bar{a}_{kj}r_{kj} - \sum_{j \in J} \bar{a}_{kj}x^k_j \gamma_{ij},
\] (39)

and by (39), we have \(p_k(v) \leq b_k\). \qed

Theorem 3. The proposed mechanism with assignments given by Definition 12 and payments given by (13) satisfy the property of voluntary participation defined by Definition 9.

Proof. By Theorem 1 we have
\[
u_i(v_k, v_{-k}) = \sum_{j \in J} \tilde{a}_{ij}(v_k, v_{-k})(v_{ij} - r_{ij})
- \sum_{j \in J} \sum_{j \in J} \tilde{a}_{ij:j:k}(v_{-k})(v_{ij} - r_{ij})
+ \sum_{j \in J} \tilde{a}_{kj}v_{kj} - \sum_{j \in J} \tilde{a}_{kj}r_{kj} + \sum_{j \in J} \tilde{a}_{kj}x^k_j \gamma_{ij},
\] (40)

which leads to
\[
u_i(v_k, v_{-k}) \geq \sum_{j \in J} \tilde{a}_{kj}v_{kj} - \sum_{j \in J} \tilde{a}_{kj}r_{kj} + \sum_{j \in J} \tilde{a}_{kj}x^k_j \gamma_{ij},
\] (41)

where we have used (38). By Lemma 1 we have
\[
\sum_{j \in J} \tilde{a}_{kj}v_{kj} \geq \sum_{j \in J} \tilde{a}_{kj}z_{kj}
= \sum_{j \in J} \tilde{a}_{kj}r_{kj}
\geq \sum_{j \in J} \tilde{a}_{kj}r_{kj} - \sum_{j \in J} \tilde{a}_{kj}x^k_j \gamma_{ij}.
\] (42)

Note that \(\tilde{a}_{kj}, x^k_{-j}, \text{ and } \gamma_{ij}\) are non-negative. Thus, we have, for any traveler \(i \in I\), \(u_i(v_k, v_{-k}) \geq 0\). \qed

IV. DISCUSSION

A. Implementation

In this subsection, we outline how our proposed framework could be potentially implemented as illustrated in Fig. 1. We consider a typical major metropolitan area with an extensive road and public transit infrastructure; a good example is Boston. Several key areas in Boston are connected by roads, busses, light rail, and bikes, thus any traveler has easy access to any of the four available modes of transportation, namely car, bus, light rail, or bike. By applying the MaaS concept, a social planner (a central computer) can offer travel services (e.g., navigation, location, booking, payment) to all passing travelers at certain travel hub locations (e.g., train stations with bus stops and taxi waiting line). Information can be shared among all travelers via a “mobility app,” which allows travelers to access the services offered by the social planner. Using this app, travelers will be able to pay for their travel needs while providing their individual budget and valuations. This can be done using a “preferences” questionnaire in the mobility app. Each mode of transportation offers different benefits in utility; for example, a car is
more convenient than a bus and is expected to be in high demand. This justifies our modeling choice of each traveler having valuations for each mobility service. Our framework guarantees that by the design of the payments, no traveler has an incentive to misreport these preferences (Theorem 1). In addition, travelers are incentivized to use the mobility app multiple times for their travels and interact with each other more than once (Theorem 3). Hence, our framework provides an efficient and fair way for travelers to travel using different modes of transportation from one place to another while competing with many other travelers and pay a ticket or a toll always within their individual budget using the mobility app (Theorem 2).

Fig. 1. An illustration of the proposed game-theoretic framework.

B. Concluding Remarks

In this letter, we have provided a game-theoretic framework for a multi-modal mobility system where travelers can travel using different modes of transportation and each has a different and unique mobility budget. We have showed that our framework ensures monetary fairness in the sense that no budget is violated. Thus, we can ensure mobility equity in the system, i.e., travelers can reach their destinations with their preferred service without being charged/taxed more than they are willing to spend. Under informational asymmetry, we have also showed that no traveler has an incentive to misreport and they voluntarily participate. Thus, our framework guarantees a socially-efficient and monetarily fair solution.

Ongoing work includes relaxing our assumption of linearity in the utility functions and also investigating our framework under the prospect theory behavioral model [35]. Ongoing work includes relaxing our assumption of linearity in the utility functions and also investigating our framework under the prospect theory behavioral model [35].

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