Weak lensing reconstruction through cosmic magnification. II: Improved power spectrum determination and map-making

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ABSTRACT
The existence of galaxy intrinsic clustering severely hampers the weak lensing reconstruction from cosmic magnification. In paper I \cite{Yang2011}, we proposed a minimal variance estimator to overcome this problem. By utilizing the different dependences of cosmic magnification and galaxy intrinsic clustering on galaxy flux, we demonstrated that the otherwise overwhelming galaxy intrinsic clustering can be significantly suppressed such that lensing maps can be reconstructed with promising accuracy. This procedure relies heavily on the accuracy of determining the galaxy bias from the same data. Paper I adopts an iterative approach, which degrades toward high redshift. The current paper presents an alternative method, improving over paper I. We prove that the measured galaxy clustering between flux bins allows for simultaneous determination of the lensing power spectrum and the flux dependence of galaxy bias, at this redshift bin. Comparing to paper I, the new approach is not only more straightforward, but also more robust. It identifies an ambiguity in determining the galaxy bias and further discovers a mathematically robust way to suppress this ambiguity to non-negligible level ($\sim 0.1\%$). The accurately determined galaxy bias can then be applied to the minimal variance estimator proposed in paper I to improve the lensing map-making. The gain at high redshift is significant. These maps can be used to measure other statistics, such as cluster finding and peak statistics. Furthermore, by including galaxy clustering measurement between different redshift bins, we can also determine the lensing cross power spectrum between these bins, up to a small and correctable multiplicative factor.

Key words: cosmology: theory – cosmological parameters – gravitational lensing – dark matter

1 INTRODUCTION
Cosmic magnification \cite{Gunn1967, Blandford1992, Bartelmann1995, Dolag1997, Hamana2001, Menard2002, Menard2002b, Menard2003}, the lensing induced coherent fluctuation in galaxy number distribution, offers an attractive alternative to cosmic shear \cite[for reviews see][]{Bartelmann2001, Hoekstra2006, Hoekstra2008, Munshi2008}, to reconstruct the matter distribution of the universe. (1) It does not require galaxy shape measurement and hence avoids all potential problems associated with it. (2) It is even insensitive to photometry errors. This is quite surprising, given that weak lensing reconstruction through cosmic magnification indeed requires galaxy/quasar flux measurement. This point will be further explained in the appendix.

A formidable task in weak lensing reconstruction through cosmic magnification is to reduce contamination caused by the galaxy intrinsic clustering, which is in general overwhelming. \cite{Zhang2005} argued that such contamination can be removed by the distinctive flux dependences of the cosmic magnification signal and the intrinsic clustering noise. In a companion paper \cite{Yang2011}, we showed that such separation is indeed doable. We constructed a minimal variance estimator for the weak lensing map reconstruction. It not only extracts the lensing signal from the observed galaxy number distribution, but also introduces the stochasticity between galaxy number density distributions of different fluxes. However, this reconstruction requires no prior knowledge on the galaxy bias, other than that the stochasticity between galaxy number density distributions of different fluxes is not overwhelming.

Nevertheless, we noticed in paper I that the reconstruction accuracy degrades at high redshift. It is hence worthwhile to explore new approaches. The current paper proposes a promising alternative. It is a two-step procedure. Firstly, we start with the measured galaxy angular power spectra between different flux bins (but of the same redshift bin). These are heavily reduced data comparing to the
“raw” maps of galaxy number density distribution on the sky and are much easier to analyze than the “raw” maps. They are the mixtures of the galaxy intrinsic clustering (power spectra), the lensing power spectrum and cross terms. We prove that, due to the different dependence of cosmic magnification and galaxy intrinsic clustering on galaxy flux, we can separate these components and solve for the lensing power spectrum. This improves on paper I, especially at high redshift.

The galaxy bias can also be determined simultaneously, with significantly improved accuracy at high redshift. In particular we find a degeneracy in determining the galaxy bias. This degeneracy likely degrades the weak lensing map reconstruction at high redshifts in paper I (Fig. 6). Fortunately, now we find a mathematically robust remedy to minimize its impact to 0.1% on the determined galaxy bias. This allows us to construct lensing maps with the minimal variance estimator proposed in paper I. Due to the improved galaxy bias determination, the quality of maps is improved, especially at high redshift. This is the second step. These maps can be used for cluster finding, peak statistics and other non-Gaussian statistics. In particular, they are useful for cross correlating other cosmic fields such as CMB lensing (Seljak & Zaldarriaga 1999; Hu & Okamoto 2002; Hirata & Seljak 2003; Smidt et al. 2011; Das et al. 2011; Van Engelen 2012; Bleem et al. 2012; Das et al. 2013; Ade et al. 2013), 21cm background lensing (Cooray 2004; Pen 2004; Zahn & Zaldarriaga 2006; Mandel & Zaldarriaga 2006), cosmic shear (Van Waerbeke et al. 2000; Bacon et al. 2000; Hoekstra et al. 2002, 2006; Massey et al. 2007; Fu et al. 2008; Lin et al. 2012; Jee et al. 2013), galaxy distribution (Kaiser 1992; Menard, Bartelmann, & Mellier 2003; Jain, Scranton, & Sheth 2003; Scranton et al. 2005; Zhang & Pen 2006; Hildebrandt et al. 2009; Van Waerbeke et al. 2010; Menard et al. 2010; Hildebrandt et al. 2011; Ford et al. 2012), the thermal Sunyaev Zel’dovich effect (e.g. the thermal SZ tomography, Shao et al. 2011), the integrated Sachs-Wolfe effect (Loverde et al. 2007 and references therein), and other cosmic fields.

The paper is organized as follows. In § we present our method to directly determine the lensing auto power spectrum and galaxy bias. We make a performance of this approach to SKA. We also find that the lensing cross power spectrum between different redshift bins can be determined with the determined galaxy bias previously. In § we show how the improved galaxy bias from the lensing power spectrum determination approach is to significantly improve the κ map-making (paper I). We discuss and summarize in §. In appendix we prove the uniqueness of the direct power spectrum determination (appendix A) and discuss why the reconstruction from cosmic magnification is insensitive to the photometry errors (appendix B). The adopted specifications of SKA and the fiducial model are the same as paper I.

2 DIRECT DETERMINATION OF LENSING POWER SPECTRUM THROUGH GALAXY POWER SPECTRUM MEASUREMENTS

For any given redshift bin, we can further split galaxies into different flux bins and measure the galaxy number density correlations between these flux bins. Since the galaxy intrinsic clustering and cosmic magnification depend on the galaxy luminosity in different ways, naively we expect that it is possible to measure the two simultaneously by directly fitting the measured correlations. However, further investigation presented in this section found a degeneracy between the intrinsic clustering and cosmic magnification. Fortunately we found a simple but efficient remedy. It is able to render this degeneracy irrelevant for realistic cases and enables direct determination of the lensing power spectrum feasible. In this section, § § and § focus on correlations/power spectra within the same redshift bin. In § we will discuss the extension to cross correlation between different redshift bins.

2.1 Direct determination of lensing auto power spectrum

For convenience, we will work in Fourier space. For a flux limited survey, we divide galaxies into flux and redshift bins. Throughout the paper, we use subscript “i” and “j” to denote the flux bins. For a given redshift bin and a given scale ℓ, δ_{ij}^L (ℓ) is the Fourier transform of the observed galaxy over-density of the i-th flux bin. For brevity, we simply denote it as δ_{ij}^L hereafter. With the observed galaxy power spectra between all flux bins we are able to perform a direct weak lensing power spectrum determination without any priors on the galaxy bias except that it is deterministic.

The cosmic magnification effect changes the galaxy number over-density to δ_{ij}^L = δ_i + g_iκ. Here δ_i is the galaxy intrinsic clustering, g_i ≡ 2(α_i - 1) and α_i is defined as the negative logarithmic slope of the differential luminosity function minus one (Bartelmann 1999; Broadhurst et al. 1993; Scranton et al. 2005; Zhang & Pen 2005; Yang & Zhang 2011), and κ is the convergence (Jain & Seljak 1997; Bartelmann & Schneider 2001). It is worth noting that, δ_{ij}^L and g_i are measurable quantities, but δ_i and κ are unknowns. The observed galaxy power spectrum between the i-th and j-th flux bins (but of the same redshift bin) is

\[ C_{ij}(\ell) = \left\langle \delta_{i}^L(\ell)\delta_{j}^L(-\ell) \right\rangle \]

Here \( C_{im}, C_{mn} \text{ and } C_{m} \) are the matter power spectrum, matter-lensing cross power spectrum and lensing power spectrum, respectively. They are all unknowns. \( C_{m} \) is the signal that we want to directly reconstruct. The unknown galaxy bias \( b_i \) and the associated intrinsic clustering \( b_i b_i C_{im} \text{ are the major uncertainties that we want to mitigate errors (appendix B). The adopted specifications of SKA and the fiducial model are the same as paper I.
The new set of unknowns is

\[ \bar{\lambda} = \left( C_n, m^2 C_m, b \right) \equiv \lambda \left( b(j = 1, \ldots, N_l), b_i C_m \right) . \]  

(2)

Naively speaking, when the number of measurements is larger than the number of unknowns \((N_l \geq 3)\), in principle we can solve these equations for all the unknowns and extract \(C_n\) rather model-independently.

However, there exists a strict degeneracy among these unknowns, which prohibits us to solve for all of them. Specifically, Eq. (1) is invariant under the following transformation:

\[ \begin{align*}
  b_i & \to Ab_i + Bg_i, \\
  C_m & \to A^{-2} C_m, \\
  C_{\text{max}} & \to A^{-1} C_{\text{max}} - A^{-2} B C_m, \\
  C_n & \to A^{-2} B^2 C_m - 2A^{-1} B C_{\text{max}} + C_n .
\end{align*} \]  

(3)

Here, the parameters \(A\) and \(B\) are arbitrary (flux independent) constants.

It turns out that this degeneracy arises from those cross terms \((gbC_{\text{max}})\). We can eliminate these cross terms by switching to new variables \(\bar{b}_i\) and \(\bar{C}_n\). Here,

\[ \bar{b}_i \equiv \sqrt{C_m} \left( b_i + g_i \frac{C_{\text{max}}}{C_m} \right) , \]  

(4)

and

\[ \bar{C}_n \equiv C_n (1 - r_{\text{max}}^2) ; \quad r_{\text{max}}^2 \equiv \frac{C_{\text{max}}^2}{C_m C_n} . \]  

(5)

Under these new notations,

\[ \bar{C}_{ij} = \bar{b}_i \bar{b}_j + \bar{C}_n g_i g_j . \]  

(6)

The new set of unknowns is \(\bar{\lambda}_{\text{new}} = \left\{ \bar{C}_n, \bar{b}_i (i = 1, 2, \ldots, N_l) \right\} \), and the number of these new unknowns accounts to \(N_l + 1\).

In Appendix A we mathematically prove that, when \(\bar{b}\) and \(g\) have different flux dependences and \(N_l \geq 2\), the solution of \(\lambda_{\text{new}} = \left\{ \bar{C}_n, \bar{b} \right\} \) is unique. Furthermore, for all cases we evaluated, the Fisher matrix inversion is stable under the two conditions, so the uniqueness of the solution is numerically guaranteed too. Therefore, we can solve for \(\bar{C}_n\) and \(\bar{b}\) from the measured \(\bar{C}_{ij}\), but not \(C_n\) and \(b\).

\(\bar{C}_n\) is a biased measure of the true lensing signal \(C_n\), subject to a multiplicative factor \(1 - r_{\text{max}}^2\). But in practice this factor is of little importance, for two reasons. (1) Firstly, \(r_{\text{max}}^2 \ll 1\) for a sufficiently narrow redshift bin, since the efficiency of matter in this bin to lens a source in the same redshift bin is low. For example, for a reasonable \(\Delta z = 0.2\), \(r_{\text{max}}^2 \approx 1\%\) at \(z \approx 0.5\) and \(r_{\text{max}}^2 \lesssim 0.1\%\) at \(z \gtrsim 1\).

So the induced systematic error is of the order 1% or less, much smaller than other errors in weak lensing measurement (Fig. 1). For this reason, we can safely neglect this error and safely treat \(C_n = \bar{C}_n\). (2) Furthermore, since by definition \(r_{\text{max}}^2\) is independent of galaxy bias, \(r_{\text{max}}^2\) can be robustly calculated given a cosmology and the multiplicative correction \(1 - r_{\text{max}}^2\) can be appropriately taken into account in theoretical interpretation. So it in principle does not cause any systematic error in cosmological parameter constraints.

On the other hand, \(\bar{b}\) is a biased measure of the galaxy bias \(b\). The prefactor \(\sqrt{C_m}\) is flux-independent. So its absolute value is irrelevant in the lensing map making (paper I). For this reason, we often neglect this prefactor where it does not cause confusion. The additive error \(g \bar{C}_{\text{max}} / C_m \approx \bar{C}_{\text{max}} / C_m\) is flux-dependent and indeed biases the \(\kappa\) map-making. This effect will be quantified later in appendix C. Nevertheless, since the error \(g \bar{C}_{\text{max}} / C_m \sim 10^{-3} \bar{g}\) (Fig. 3) and since both \(b\) and \(g\) are of order unity, to an excellent approximation this additive error is negligible and \(\bar{b}\) has virtually identical flux dependence as \(b\). Hence \(\bar{b}\) offers an excellent template to construct the minimal variance estimator for the weak lensing map reconstruction in paper I. Later in (5) we show that this improved estimation of galaxy bias indeed improves the map reconstruction significantly.

This direct power spectrum determination does not rely upon priors on galaxy bias other than it is deterministic. In this sense, it is robust. We now proceed to quantify its performance in galaxy redshift surveys.

### 2.2 Error sources

We derive the likelihood function and adopt the Fisher matrix analysis to estimate the errors of the direct lensing power spectrum determination. This determination is from the measurements of galaxy-galaxy power spectra between all pairs of flux bins and based on the validity of Eq. (1) so statistical and systematic deviations from this equation will all bias the determination.
The galaxy power spectra measured in a real survey are contaminated by measurement noise, which is denoted as $\Delta C_{ij}$ and throughout the paper we only consider shot noise. On the other hand, our modeling of the galaxy power spectra may be imperfect, which will cause systematic error $\delta C_{ij}$. So the real observed power spectra are given by

$$C_{ij} = \bar{C}_{ij} + \Delta C_{ij} + \delta C_{ij} \quad (i \leq j).$$

### 2.2.1 Statistical error forecast

Measurement error $\Delta C_{ij}$ propagates into the weak lensing power spectrum reconstruction and causes statistical error in the reconstructed power spectrum. In the current paper we do not attempt to make forecast on its cosmological constraining power. Instead we focus on the accuracy of the determined power spectrum, with respect to the true power spectrum in the given survey volume instead approximated by a gaussian distribution thanks to the central limit theorem.

The galaxy distribution as the source of statistical error in the measured shot noise. The Fisher matrix under this simplified condition is (Zhang et al. 2010),

$$F_{\mu\nu} = \sum_{i,j} \frac{C_{ij,\mu} C_{ij,\nu}}{\sigma_{ij}^2}.$$  

Throughout the paper, we use the subscript “$\mu$” and “$\nu$” to denote the unknowns ($\lambda_{\mu}^{\text{new}}, \lambda_{\nu}^{\text{new}}$, etc.). The variance of statistical error in the observed angular power spectrum $C_{ij}$ is

$$\sigma_{ij}^2 = \langle \Delta C_{ij}^2 \rangle = \frac{1 + \delta_{ij}}{(2f + 1) \Delta \ell} \frac{1}{\bar{n}_i \bar{n}_j}.$$  

Here $\bar{n}_i$ is the average galaxy surface number density of the $i$-th flux bin. $\delta_{ij}$ is the delta function: $\delta_{ij} = 1$ when $i = j$ and 0 when $i \neq j$. In this paper, we adopt $\Delta \ell = 0.2 \ell$. The statistical error on the parameter $\lambda_{\mu}^{\text{new}}$ is

$$\Delta \lambda_{\mu}^{\text{new}} = \sqrt{(F^{-1})_{\mu\mu}}.$$  

### 2.2.2 Systematic error forecast

Systematic deviations from Eq. [1] can induce systematic errors into the reconstructed parameters, $\delta \lambda^{\text{new}} \equiv \lambda^{\text{true}} - \lambda^{\text{fit}}$. Here $\lambda^{\text{true}}$ is the set of parameters to maximize the likelihood and $\lambda^{\text{fit}}$ is the set of their fiducial values. The Fisher matrix can also estimate this kind of error. We have (Huterer & Takada 2005; Zhang et al. 2010)

$$\delta \lambda_{\mu}^{\text{new}} = F_{\mu\nu}^{-1} J_{\mu} = \sum_{i,j} \frac{1}{\sigma_{ij}^2} \delta C_{ij} \frac{\partial \bar{C}_{ij}}{\partial \lambda_{\mu}^{\text{new}}}.$$  

Here we discuss three main sources of systematic error.

1. The first one arises from the galaxy stochasticity. A reasonable and widely adopted approximation is a deterministic bias (no stochasticity). Nevertheless, since the lensing signal is much weaker than the noise of galaxy intrinsic clustering, we have to be careful of the stochasticity, even if it is small. The stochasticity, at two-point statistics level, can be completely described by the cross correlation coefficient $r_{ij}$ between the $i$-th and $j$-th flux bins. It biases the galaxy power spectrum modeling by

$$\delta C_{ij} = b_i b_j \Delta r_{ij} C_{mn} ; \Delta r_{ij} \equiv 1 - r_{ij}.$$  

Plugging it into Eq. [10] we obtain the induced bias in $C_{\lambda \mu}$. To proceed, we need a model of $r_{ij}$, which in general depends on redshift, angular scale, flux and galaxy type. Such modeling is beyond the scope of this work and will be postponed until we analyze mock catalogues and observational data. For consistency, we adopt the same toy model as in paper I: $\Delta r_{ij} = 1\%$. Notice that by definition it has $\Delta r_{ii} = 0$. Readers can conveniently scale the resulting systematic error to their favorite models by multiplying a factor $100 \Delta r_{ij}$. This systematic error turns out to be the dominant in many cases of the direct power spectrum determination.

2. The determination also requires precision measurement of $g$, the prefactor of cosmic magnification. It relies on precision measurement of the galaxy luminosity function, which could be biased by photometry errors or errors in redshift measurement. If $g$ is systematically biased by $\delta g$, we have

$$\delta C_{ij} = g_i g_j (C_{mn} + g_i C_\alpha) + \delta g (C_{mn} g_i g_\alpha) + \delta g_i \delta g_\alpha C_{\alpha \alpha}.$$  

This $\delta C_{ij}$ is usually much smaller than that induced by the galaxy stochasticity, because $C_{mn} \ll C_\alpha$ and $C_\alpha \ll C_m$. So unless at very large scale where $\Delta r_{ij} \ll 1\%$, we can neglect the $\delta g$ induced error. Furthermore, in this paper we will target at $C_{\lambda \mu}$. We can then neglect the cosmological parameter constraint.

3. Dust extinction and photometry calibration error both bias the flux measurement and both induce extra fluctuations in galaxy number density. The induced fluctuation is $\alpha \propto \delta g$ instead of $\propto (\alpha - 1)$, because unlike gravitational lensing, dust extinction and photometry error do not change the surface area. For two reasons we do not consider such type of errors in this paper. Firstly, for radio survey SKA, it is free of dust extinction. Secondly, due to the different flux dependence ($\alpha$ vs. $\alpha - 1$), they can be distinguished from the cosmic magnification. Nevertheless, we caution the readers that weak lensing reconstruction from optical surveys may need to take this complexity into account.

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1. It is noticed that when we compare the determined weak lensing power spectrum with its ensemble average predicted by theory to constrain the cosmological parameters, we must consider the statistical error from cosmic variance. Another point we address is that the cosmic variance still influences our results presented in Table 1 through entering the fiducial power spectra. Since what enters into the key equation is not the ensemble average power spectra from theoretical prediction, but the actual values with the right cosmic variance in the observed cosmic volume. In the performance of the proposed method, we neglect the cosmic variance in these real fiducial power spectra and use the ensemble average ones instead of them. Fortunately, this approximation is a subdominant source of error, since the cosmic variance of each power spectrum is usually much smaller than the ensemble average one with a large survey volume (e.g. SKA).

2. There are $N_{\lambda \mu} (N_{\lambda \nu} - 1)/2$ independent $r_{ij}$. If we have to treat all of them as unknowns, the total number of unknowns will be larger than the number of independent measurements, for any $N_{\lambda \mu}$. The lensing power spectrum determination would fail in this extreme case. Fortunately, in reality we know that $r_{ij}$ vanishes toward large scales. So we can carry out the determination at the limit $r_{ij} = 0$, but with extra work to quantify the associated systematic bias. Nevertheless, this stochasticity problem prohibits the weak lensing reconstruction through cosmic magnification at sufficiently small scales, where the stochasticity becomes large.
Figure 3. Predicted accuracy of the direct lensing power spectrum determination for a given redshift bin. The expected lensing auto power spectrum $C_\kappa$ (solid line) increases with redshift. Its systematic error $\delta C_\kappa$ (dashed line) from stochasticity is dominant at low redshift (e.g. $z \sim 0.5$). It decreases with redshift and then becomes negligible at $z \sim 2$. For SKA with specifications described in paper I, the predicted statistical error $\Delta C_\kappa$ (solid error bars) is 5%-10% at low redshift $z \sim 0.5$ and $10 \lesssim \ell \lesssim 10^3$. At redshift $z \sim 1$, it can be controlled to 0.5%-5% level at $10 \lesssim \ell \lesssim 10^3$. At these scales and high redshift $z \sim 2$, it is still under control at redshifts $z \gtrsim 1$. Comparing this direct power spectrum determination with the minimal variance $\kappa$ map reconstruction (see Fig. 6 in paper I), we find that both reconstructions fail at $z < \sim 0.5$ (Fig. 3), for the fiducial value of $\Delta r_{ij} = 0.01$. Nevertheless, it works well at $z \gtrsim 1$. For $\Delta r_{ij} = 0.01$, the induced systematic error is $\sim 10\%$ at $z \sim 1$ and $\sim 1\%$ at $z \sim 2$. Possibilities remain to further suppress this systematic error. For example, we can utilize the spectroscopic redshift information to disregard pairs close along the line of sight, which are mostly responsible for this systematic error. This removal is known to be efficient (Zhang & Pen 2006), and when needed, can be applied to precision lensing reconstruction through cosmic magnification.

2.3 The performance

In order to compare with the minimal variance $\kappa$ map reconstruction presented in paper I, we also target at SKA to investigate the feasibility of the proposed direct lensing power spectrum determination. Details of SKA specification are given in paper I.

Fig. 3 shows the forecasted statistical and systematic errors in the lensing power spectrum $C_\kappa$ determination, at four redshift bins ($0.4 < z < 0.6$, $1.0 < z < 1.2$, $1.2 < z < 1.4$ and $2.0 < z < 2.2$). For SKA, statistical error induced by shot noise is well under control at all the four redshift bins and scales $\ell \lesssim 10^4$. Even if we reduce the number density of galaxies by a factor of 10, corresponding to an artificial survey with the same sky coverage as SKA but 90% less galaxies, the shot noise induced error is still insignificant at $z \sim 1$ and $\ell \sim 10^3$.

The stochasticity induced bias is more severe. The lensing power spectrum increases with redshift while the galaxy intrinsic clustering decreases with redshift. For this reason, the same amount of galaxy stochasticity induces larger systematic errors at lower redshifts. Consequently, the direct power spectrum determination fails at $z \lesssim 0.5$ (Fig. 3), for the fiducial value of $\Delta r_{ij} = 0.01$. Nevertheless, it works well at $z \gtrsim 1$. For $\Delta r_{ij} = 0.01$, the induced systematic error is $\sim 10\%$ at $z \sim 1$ and $\sim 1\%$ at $z \sim 2$. Possibilities remain to further suppress this systematic error. For example, we can utilize the spectroscopic redshift information to disregard pairs close along the line of sight, which are mostly responsible for this systematic error. This removal is known to be efficient (Zhang & Pen 2006), and when needed, can be applied to precision lensing reconstruction through cosmic magnification.

We make a comparison between the results of the direct lensing power spectrum determination and those recovered from the
reconstructed $\kappa$ map in paper I (Fig. 6). Statistical error from shot noise in Fig. 5 should be compared to the minimized shot noise in Fig. 6 of paper I. The systematic error $\delta C_\kappa$ of Fig. 5 should be compared to the systematic error $\delta C_{\kappa(b)}$ in paper I, both from the galaxy stochasticity. At $z \lesssim 1$, the two sets of result agree with each other reasonably well. Due to the overwhelming error from galaxy stochasticity, both reconstructions fails at $z \lesssim 0.5$. At redshift $z \sim 1$, this kind of error is dominant. Nevertheless both reconstructions can achieve an accuracy of $\sim 10\%$.

However, at high redshift $z \sim 2$, the situation is different. The direct lensing power spectrum determination works even better than at lower redshift. To the opposite, the reconstruction presented in paper I fails. We suspect that this failure arises from the wrong determination of galaxy bias. In paper I, we adopted a recursive procedure to solve the galaxy bias, with the initial guess of it

$$
(b_i^{(1)})^2 = \tilde{C}_{\kappa i}/C_m = b_i^2 + (g_i^2 C_{\kappa i} + 2b_i g_i C_{\kappa m})/C_m .
$$

At low redshift, this initial guess is nearly perfect. But at high redshift, due to increasing $C_{\kappa}$ and decreasing $C_m$, the deviation from the true value increases. Because of the degeneracy presented by Eq. 2, the recursive procedure may not converge at the true value of galaxy bias. The lensing map reconstructed based on the obtained false bias is then biased. Naively we expect that the problem becomes more severe at higher redshift.

To the opposite, the direct power spectrum determination is free of this problem. We have proved that $\tilde{C}_{\kappa i} = C_{\kappa i}(1 - r_{\kappa m})$ is what we can solve strictly and we have numerically shown that $r_{\kappa m} \ll 1$ at all redshifts.

This approach also provides better determination of the galaxy bias. We have proved that we can solve $b_i$ whose flux dependence differs from that of $b$ only by a small additive error $g_i C_{\kappa m}/C_m$. For example, this error is far below 1% for all four redshift bins plotted in Fig. 3. Furthermore, it is sub-dominant to the systematic error from galaxy stochasticity arising from a conservative $\Delta r_{\kappa i} = 0.01$, which is under 1% at redshift up to $z \sim 2.2$. The same amount of galaxy stochasticity induces smaller error at lower redshift. As to the statistical error caused by shot noise, when we choose $\Delta \ell = 0.2\ell$, it is below 1% at scales $10 \lesssim \ell \lesssim 10^3$ and $0.2 < z < 1.6$. This statistical error increases with increasing galaxy number density. However even at $2.0 < z < 2.2$, it is controlled to better than 25% level for the bin with highest flux and hence lowest number of galaxies. Fig. 4 shows the statistical error of the first flux bin at four redshift bins. In the plot, $b$ determination becomes worse with redshift. For the highest redshift bin, the statistical error is below 10% at scales $\ell \lesssim 10^4$. This bin has quite large $g = 7.0$, since we only observe galaxies at the bright end.

Finally we want to emphasize that, to separate the galaxy intrinsic clustering from cosmic magnification, $b$ and $g$ must have different flux dependences. $g$ changes from positive to negative with decreasing luminosity, but $b$ remains positive (see Fig. 3 in paper I). So deeper surveys have advantage to perform cosmic magnification.

### 2.4 Direct determination of lensing cross power spectrum between different redshift bins

So far we focus on determining the lensing auto power spectrum of a given redshift bin. However, a larger portion of cosmological information is encoded in the lensing cross power spectrum between different redshift bins. For $N_z$ redshift bins, there are $N_z (N_z - 1)/2$ cross power spectra, but only $N_z$ auto power spectra. So the information encoded in these cross power spectra is usually richer than that in the auto power spectra. In particular, such information is essential to perform weak lensing tomography and to measure the structure growth rate of the universe.

We are then well motivated for a more ambitious project, namely to determine these $N_z (N_z - 1)/2$ cross power spectra. To achieve this goal, we need not only the measurements of the galaxy clustering within the same redshift bin $(\tilde{C}_{\kappa i})$, but also those between different redshift bins. We denote the redshift bins with Greek letters “$\alpha$”, “$\beta$” and flux bins with “$r$”, “$j$”. We have $N_z$ redshift bins centered at $z_\alpha$ $(\alpha = 1, \cdots, N_z)$ and each redshift bin has $N_l$ flux bins. So the available measurements are $C_{ij}^{\alpha\beta}$ $(i, j \in [1, N_z]$ and $\alpha, \beta \in [1, N_z])$.

For the $\alpha$-th redshift bin, the available measurements are

$$
\tilde{C}_{ij}^{\alpha\beta} = b_i^{\alpha} b_j^{\beta} C_{\kappa i}^{\alpha\beta} + g_i^{\alpha} g_j^{\beta} C_{\kappa m}^{\alpha\beta} + (b_i^{\alpha} g_j^{\beta} + b_j^{\beta} g_i^{\alpha}) C_{\kappa m}^{\alpha\beta} .
$$

We choose the independent ones with $i \leq j$. We also have cross correlation measurements between the $i$-th flux bin of the $\alpha$-th redshift bin and the $j$-th flux bin of the $\beta$-th redshift bin,

$$
\tilde{C}_{ij}^{\alpha\beta} = b_i^{\alpha} g_j^{\beta} C_{\kappa m}^{\alpha\beta} + g_i^{\alpha} g_j^{\beta} C_{\kappa m}^{\alpha\beta} .
$$

The above equation assumes that $z_\alpha < z_j$ $(\alpha < \beta)$. By requiring $z_\beta - z_\alpha \gtrsim 0.1$, we safely neglect a term $\propto C_{\kappa m}^{\alpha\beta}$. We still assume a deterministic galaxy bias, $C_{\kappa m}^{\alpha\beta}$ is the lensing cross power spectrum.

Finally, we want to emphasize that, to separate the galaxy intrinsic clustering from cosmic magnification, $b$ and $g$ must have different flux dependences. $g$ changes from positive to negative with decreasing luminosity, but $b$ remains positive (see Fig. 3 in paper I). So deeper surveys have advantage to perform cosmic magnification.

![Figure 4](image-url)

**Figure 4.** Predicted accuracy of the direct galaxy bias $\tilde{b}$ determination. Here, we divide $\tilde{b}$ by the flux independent scaling $\sqrt{C_m}$ (solid line). We plot the first flux bin ($i = 1$) of all four redshift bins. For a fixed redshift bin, $\tilde{b}$ strongly changes with flux (Fig. 3 of paper I). Here we give the average value $g_i$ at corresponding flux bin. Since the fiducial galaxy bias $b_i$ we adopt is scale independent (see details on paper I) and the additive error (flux and scale dependence) $g_i C_{\kappa m}/C_m$ is negligible, $\tilde{b}/\sqrt{C_m}$ is almost scale independent in the plot. At these four redshift bins, systematic error $\delta b_i$ from galaxy stochasticity by adopting $\Delta r_{\kappa i} = 0.01$ is below 1% (this line is too low and hence does not show up in the plot), and statistical error $\Delta \tilde{b}_i$ (error bars) can be controlled to better than $10\%$ at scales $10 \lesssim \ell \lesssim 10^4$. In the plot, $b$ determination is getting worse with redshift.
are invariant under the following transformation,
\begin{align}
\tilde{b}_i^\alpha &\rightarrow Ab_i^\alpha + Bg_i^\alpha , \\
C_{\alpha\alpha,\min}^{\alpha\alpha} &\rightarrow A^{-2}C_{\alpha\alpha,\min}^{\alpha\alpha} , \\
C_{\alpha\alpha,\max}^{\alpha\alpha} &\rightarrow A^{-1}C_{\alpha\alpha,\max}^{\alpha\alpha} - A^{-2}BC_{\min}^{\alpha\alpha} , \\
C_{\alpha\beta,\max}^{\alpha\beta} &\rightarrow A^{-2}BC_{\alpha\max}^{\alpha\alpha} - 2A^{-1}BC_{\max}^{\alpha\beta} + C_{\beta\beta,\max}^{\beta\beta} , \\
C_{\alpha\beta,\min}^{\alpha\beta} &\rightarrow A^{-1}C_{\beta\beta,\min}^{\beta\beta} , \\
C_{\alpha\beta}^{\alpha\beta} &\rightarrow C_{\alpha\beta}^{\alpha\beta} - A^{-1}BC_{\min}^{\alpha\beta} .
\end{align}

The parameters \(A\) and \(B\) are arbitrary (flux independent) constants.

Due to this degeneracy, we are not able to uniquely solve \(C_{\alpha\beta}\), the lensing cross power spectrum. Nevertheless, following discussions in [21] and the appendix \(A\) we find the solution to the following combinations is unique,
\begin{align}
\tilde{C}_{\alpha\beta}^{\alpha\beta} &\equiv C_{\alpha\beta,\max}^{\alpha\beta} \left(\frac{C_{\alpha\alpha,\max}^{\alpha\alpha}}{C_{\alpha\alpha,\min}^{\alpha\alpha}}\right)^2 , \\
\tilde{C}_{\alpha\beta,\min}^{\alpha\beta} &\equiv C_{\alpha\beta,\min}^{\alpha\beta} \frac{C_{\alpha\max}^{\alpha\beta}C_{\alpha\max}^{\beta\alpha}}{C_{\alpha\alpha,\min}^{\alpha\alpha}} , \\
\tilde{C}_{\alpha\beta}^{\beta\beta} &\equiv \frac{C_{\alpha\beta}^{\beta\beta}}{\sqrt{C_{\alpha\alpha,\min}^{\alpha\alpha}}}, \\
\tilde{b}_i^\alpha &\equiv b_i^\alpha \sqrt{C_{\alpha\alpha,\min}^{\alpha\alpha}} + g_i^\alpha \frac{C_{\alpha\max}^{\alpha\alpha}}{\sqrt{C_{\alpha\alpha,\min}^{\alpha\alpha}}} .
\end{align}

From the appendix \(A\) we know that measurements \(C_{\alpha\beta,\min}^{\alpha\beta}\) allow for unique determination of \(\tilde{C}_{\alpha\beta}^{\alpha\beta}\) and \(\tilde{b}_i^\alpha\). For \(\tilde{C}_{\alpha\beta}^{\beta\beta}\) and \(\tilde{C}_{\alpha\beta,\min}^{\alpha\beta}\), we rewrite Eq. [16] as
\begin{align}
\tilde{C}_{\alpha\beta}^{\alpha\beta} = y^\beta \left[b_i^\alpha \tilde{C}_{\alpha\beta,\min}^{\alpha\beta} + g_i^\alpha \tilde{C}_{\alpha\beta}^{\beta\beta}\right].
\end{align}

Since \(\tilde{b}_i^\alpha\) is uniquely solved through measurements \(\tilde{C}_{\alpha\beta}^{\alpha\beta}\) and \(\tilde{g}_i^\alpha\) is measurable, there are only two flux-dependent unknowns, \(\tilde{C}_{\alpha\beta,\min}^{\alpha\beta}\) and \(\tilde{C}_{\alpha\beta}^{\beta\beta}\). As long as \(b_i/b_j \neq g_i/g_j\), the measurements \(\tilde{C}_{\alpha\beta}^{\beta\beta}\) uniquely determine \(\tilde{C}_{\alpha\beta,\min}^{\alpha\beta}\) and \(\tilde{C}_{\alpha\beta}\).

\(\tilde{C}_{\alpha\beta}\) differs from the true lensing cross power spectrum \(C_{\alpha\beta}\) by a multiplicative factor \(1-y\),
\begin{align}
\tilde{C}_{\alpha\beta}^{\alpha\beta} &\equiv C_{\alpha\beta,\max}^{\alpha\beta} (1-y) ,
\end{align}

where
\begin{align}
y &\equiv C_{\alpha\max}^{\alpha\beta} \frac{C_{\alpha\max}^{\alpha\alpha}}{C_{\alpha\alpha,\min}^{\alpha\alpha}} = \frac{C_{\alpha\min}^{\alpha\beta}C_{\alpha\max}^{\beta\alpha}}{C_{\alpha\alpha,\min}^{\alpha\alpha}} .
\end{align}

The three cross correlation coefficients \(r_{\max}^{\alpha\alpha}, r_{\max}^{\alpha\beta}\) and \(r_{\min}^{\alpha\beta}\) are defined respectively as
\begin{align}
r_{\max}^{\alpha\alpha} &\equiv \frac{C_{\alpha\min}^{\alpha\alpha}}{\sqrt{C_{\alpha\max}^{\alpha\alpha}C_{\alpha\alpha,\min}^{\alpha\alpha}}}, \\
r_{\max}^{\alpha\beta} &\equiv \frac{C_{\alpha\max}^{\alpha\beta}C_{\alpha\max}^{\beta\alpha}}{\sqrt{C_{\alpha\max}^{\alpha\alpha}C_{\alpha\alpha,\max}^{\beta\beta}}}, \\
r_{\min}^{\alpha\beta} &\equiv \frac{C_{\alpha\max}^{\alpha\beta}}{\sqrt{C_{\alpha\alpha,\max}^{\alpha\alpha}C_{\alpha\beta}^{\beta\beta}}} .
\end{align}

Their values are sensitive to the lensing kernel. So \(y\) changes with the choice of foreground and background redshift bins.

By definition, \(y\) is insensitive to galaxy bias and can be calculated given a cosmology according to Eq. [21]. This is a desirable property, meaning that we can safely use \(\tilde{C}_{\alpha\beta}\) to constrain cosmology without introducing uncertainties from galaxy formation. Fig. 5 shows \(y\) as a function of scales at different foreground and background redshift bins. It can vary from less than \(10^{-2}\) to \(0.2\).

Figure 5. Directly determined \(\tilde{C}_{\alpha\beta}\) differs from the true lensing cross power spectrum \(C_{\alpha\beta}\) by a multiplicative factor \(1-y\). Here, \(y\) is determined by three cross correlation coefficients \(r_{\min}^{\alpha\alpha}, r_{\max}^{\alpha\beta}\) and \(r_{\min}^{\alpha\beta}\). It induces error of the order \(2\%\) or less at intermediate foreground redshifts \(z_{\alpha} \sim 1\). At lower foreground redshift \(z_{\alpha} \sim 0.5\), the induced error reaches up to \(\sim 20\%\). Nevertheless, \(y\) is correctable, since it is free of galaxy bias and thus can be calculated given a cosmology. So the direct determination of lensing cross power spectrum works.

So we have to take it into account when interpreting the measured \(\tilde{C}_{\alpha\beta}\).

We address that \(\tilde{C}_{\alpha\beta}\) is also free of deterministic galaxy bias. Therefore the measured \(\tilde{C}_{\alpha\min}^{\alpha\beta}\) is also useful for cosmology. It is beyond the scope of this paper to fully quantify the measurement accuracy of all these quantities and to quantify cosmological information encoded in these statistics.

Now we have demonstrated that the direct lensing power spectrum determination indeed works. Both the lensing auto power spectra and cross power spectra can be robustly determined from the measured galaxy angular power spectra between different redshift and flux bins. It hence provides a promising alternative to lensing power spectrum measurement through cosmic shear. Even better, next section will demonstrate that, we can improve the lensing map-making with the improved galaxy bias determination.

### 3 IMPROVING THE \(\kappa\) MAP-MAKING

The minimal variance estimator of weak lensing, proposed in paper I, requires the galaxy bias as input. It turns out to be the limiting factor for the \(\kappa\) map-making. Fortunately, previous section shows that the galaxy bias can be determined to high precision through direct fitting against the galaxy power spectrum measurements. We can then improve the \(\kappa\) map-making.

In paper I, we derived the unbiased minimal variance linear estimator for the \(\kappa\) reconstruction,
\begin{align}
\hat{\kappa} = \sum_i w_i (\delta_i) \delta_i^I ,
\end{align}

with \(\hat{b}\) as the estimated galaxy bias. The subscript “I” denotes the \(i\)-th flux bin of the given redshift bin. \(w_i\) is the value of the weighting function \(w\) at the \(i\)-th flux bin.

By design, these errors in the reconstructed lensing map are additive,
\begin{align}
\tilde{\kappa} = \kappa + \delta\kappa .
\end{align}
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Figure 6. Quality of the reconstructed $\kappa$ maps, quantified by the error power spectrum of each individual error sources. Shot noise due to discrete galaxy distribution causes statistical errors, shown as error bars on top of the expected lensing auto power spectrum (bold solid line). We have identified four main sources of systematic error, whose power spectra are labeled as $\delta C^{(1,2,3),\text{new}}$ respectively, with detailed explanation in appendix C. The galaxy stochasticity induced error ($\delta C^{(2)}$, dotted line) is the most severe. It is comparable to the lensing signal for any pixel size (angular scale) of interest, at $z \sim 0.5$. The situation improves with increasing redshift. At $z \sim 1$, lensing signal dominates over noises for pixel size of arc minute and above. Comparing with the old $\kappa$ map-making (Fig. 6 of paper I), we find that both reconstructions fail at low redshift $z < \sim 0.5$, due to overwhelming $\delta C^{(2)}$. At intermediate redshift $z \sim 1$, the lensing signal overwhelms all errors and can be measured to $\sim 10\%-20\%$ accuracy. At $z \geq 1.2$, since the dominant error $\delta C^{(1)}_{\text{bb}}$ from wrong determination of galaxy bias in paper I is correctable in present paper, the new measurement is robust than the old one. One can directly measure the lensing power spectrum from these maps, with accuracy comparable to the direct power spectrum determination (Fig 3). This means that these maps are close to optimal.

We can then define an error power spectrum $\delta C(\ell)$ for each source of error $\delta \kappa$. It shows the contamination to the lensing signal as a function of angular scale $\ell$. Different error sources can be correlated, so cross terms between different sources of error exist. It turns out that there are many of these terms. For brevity, we only show the error power spectrum corresponding to each single source of error.

Furthermore, errors in maps of two redshift bins may also be correlated. Such kind of contamination to the lensing maps can be quantified by the cross error power spectrum $\delta C_{\text{cross}}(\ell)$. Derivations and explanations of these error sources are a little bit technical and may not be of general interest. So we move detailed derivations to appendix C. In Fig. 6 we show four error power spectra in the reconstructed lensing maps, denoted as $\delta C^{(1)}, \delta C^{(2)}, \delta C^{(3)}, \delta C^{\text{new}}$. The first three have correspondences in paper I and represent the systematic errors caused by the $b-g$ degeneracy, the galaxy stochasticity and the statistical error in galaxy bias, respectively. But the last term is new, for that we include the galaxy cross power spectra measured between different flux bins to determine the galaxy bias, while in paper I only the auto power spectra

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Footnote 3: We caution that the error power spectra discussed here and shown in Fig. 6 are to demonstrate their contaminations in the reconstructed $\kappa$ maps. They are measures of the map quality. But they should not be regarded as the errors in the lensing power spectrum measured through cosmic magnification. Correct errors should be estimated following §2 (and Fig. 4).
Improved lensing reconstruction

Figure 7. Quality of the reconstructed κ maps, as quantified by the cross error power spectrum between two maps at different redshifts. We plot the expected lensing cross power spectrum by a solid line. The error bars and dotted line show its statistical error and systematic error, respectively. Some errors in the maps are uncorrelated between two redshifts, such as the one arising from the galaxy stochasticity. Systematic error arising from the b-g degeneracy is dominant.

Comparing with the old κ reconstruction (Fig. 7 in paper I), we find that these two reconstructions have similar quality at low foreground redshift (e.g. \( z_f \sim 0.5 \)). At higher foreground redshift (e.g. \( z_f \sim 1 \)), the new maps have better quality. Hence we shall use these improved maps to measure lensing statistics other than the power spectrum.

Some of the errors in Fig. 6 can be compared with that in Fig. 6 of paper I. But detailed comparison is not necessary since there is no exact connection between them. Nevertheless, we notice that the total error power spectrum is comparable to that of the direct fitting method (Fig. 3). Since in the direct fitting we use the Fisher matrix to quantify the error and since the error determined in this way represents the lower limit of the true error, we conclude that the improved map-making method is close to be optimal.

A major uncertainty in quantifying the map-making performance is the galaxy stochasticity. As discussed in appendix C, it contributes two errors, \( \delta C^{(2)} \) and \( \delta C^{\text{new}} \). Fig. 5 shows that, \( \delta C^{(2)} \) not only dominates over \( \delta C^{\text{new}} \) at all redshifts and all angular scales, but also dominates over other systematic errors at almost all redshifts/scales. As a reminder, \( \delta C^{(2)} \propto \Delta r \). We have adopted a fiducial value \( \Delta r_{ij} \equiv 1 - r_{ij} = 0.01 \). It already forbids the lensing measurement at \( z \leq 0.5 \). This value is reasonable at \( k \sim 0.1h/\text{Mpc} \) (e.g. Bonoli & Pen (2009)). But it can be much larger at smaller scales (e.g. Bonoli & Pen (2009)) and hence severely degrades the power of weak lensing reconstruction through cosmic magnification. Nevertheless, higher redshifts are still promising, even if the stochasticity is a factor of 10 larger than the fiducial value chosen in here.

Furthermore, contamination induced by galaxy stochasticity is
less severe in the determined lensing cross power spectrum between different redshift bins, because there is no intrinsic clustering between two widely separated redshift bins. Fig 7 forecasts the error cross power spectra between different redshift bins, which quantify correlation strength of errors in maps of two redshifts. Among the errors, $\delta C^{(1)}_{\ell_{mg}}$ arising from the $b-g$ degeneracy is dominant. Comparing with Fig. 7 of paper I, we find that the new measurement has comparable accuracy at low foreground redshift (e.g. $z_1 \sim 0.5$), but considerably higher accuracy at higher foreground redshift. We again confirm that the new method supersedes the old one. Hence the quality of the maps is good for cross correlation analysis.

The reconstructed lensing maps can be cross correlated with external maps, such as lensing maps from cosmic shear or from CMB lensing (Das et al. 2011; Van Engelen 2012; Bleem et al. 2012; Das et al. 2013; Ade et al. 2013). These external correlations can have extra advantages. For example, a major systematic error in cosmic shear is the galaxy intrinsic alignment. But its correlation with the $\kappa$ reconstructed from cosmic magnification of the same redshift bin is weak, due to the vanishing lensing kernel. Major systematic error in cosmic magnification is the residual galaxy clustering ($\delta \alpha_{mg}$). But its correlation with cosmic shear is weak, again due to the vanishing lensing kernel. So if we cross correlate the lensing map reconstructed from cosmic magnification and the lensing map from cosmic shear, of the same redshift bin, major systematic errors can be significantly suppressed.

4 CONCLUSIONS AND DISCUSSIONS

In this paper we improve the lensing reconstruction through cosmic magnification (Zhang & Pen (2005) & paper I). It is a two step process.

- Step one. For a given redshift bin, directly fitting against the measured galaxy cross power spectra between different flux bins to solve for the lensing auto power spectrum and the galaxy bias simultaneously. This can be extended to include the galaxy cross power spectra between different redshift bins to simultaneously solve for the lensing cross power spectrum between two redshifts.

- Step two. Applying the fitted galaxy bias to the minimal variance estimator derived in paper I to construct the lensing $\kappa$ maps. These maps then can be used to measure other statistics such as the lensing peak abundance and lensing bispectrum, to cross correlate with cosmic shear or CMB lensing, etc.

We have estimated its performance for the SKA survey and demonstrated its great potential. This method has superb performance and hence supersedes our previous works (Zhang & Pen (2005) & paper I). Here we summarize its advantages.

- Our method differs from the traditional cosmic magnification measurement through foreground galaxy-background galaxy (quasar) cross correlation (Scranton et al. 2005; Hildebrandt et al. 2009; Van Waerbeke 2011; Menard et al. 2010; Hildebrandt et al. 2011; Ford et al. 2012). What the later measures is actually the galaxy-galaxy lensing and has limited cosmological implications due to the unknown foreground galaxy bias. To the opposite, what our method measures is free of galaxy bias and can be used for cosmological parameter constraints the same way as cosmic shear.

- It is in principle applicable to all galaxy surveys with reasonable redshift information. This is especially valuable for spectroscopic surveys, for which cosmic shear methods do not apply.

- It uses the extra flux information that comes for free in galaxy surveys. It does not require priors on the galaxy bias other than it is deterministic. Even better, the induced systematic errors from stochastic bias are under control for the expected level of galaxy stochasticity.

- It is insensitive to dust extinction and photometry calibration error. This property is quite surprising given the fact that both dust extinction and photometry error affect the galaxy flux measurement. The reason is that, these two effects only alter the galaxy flux, but do not change the surface area as lensing does. For this reason, the induced galaxy density fluctuation $\propto \alpha$, instead of $\propto \alpha - 1$ as lensing does. In the appendix B we will further show that, our approach is not only insensitive to random photometry errors, but also insensitive to systematic bias in photometry, as long as we stick to the observed $\alpha$. Dust extinction not only induces fluctuations in the galaxy brightness but also systematically dims the galaxies. From the same argument against the photometry error, it does not induce systematic error in the weak lensing reconstruction through cosmic magnification.

Weak lensing reconstruction through cosmic magnification through our method is highly complementary to other approaches of weak lensing reconstruction. (1) It can be used to check and control systematic errors arising from PSF and galaxy intrinsic alignment in cosmic shear measurement. (2) The reconstructed lensing maps can be cross correlated with those reconstructed from CMB lensing and 21cm lensing to improve the lensing tomography. (3) It helps to diagnose the impact of dust extinction in lensing reconstruction through type Ia supernova (Jonsson et al. 2010), galaxy fundamental plane (FP) (Bertin & Lombardi 2004; Huff & Graves 2011), the Tully-Fisher relation for late-type galaxies (Kronborg et al. 2010) and the average flux method (Schmidt et al. 2012). All suffer from dust extinction, especially the extinction by intergalactic gray dust, which can not be corrected through reddening (Zhang & Corasaniti 2007). In contrast, our method is insensitive to dust extinction. Comparison of the two provides a promising way to infer the elusive intergalactic gray dust.

Despite the great potential of weak lensing reconstruction through cosmic magnification that we have demonstrated, there is a long list of further studies to consolidate its role in precision cosmology. Here we list three of them in our immediate research plan.

- The galaxy stochasticity. We have identified the galaxy stochasticity as the dominant source of systematic errors. The induced systematic error can be further reduced. Researches (Tegmark & Bromley 1999; Bonoli & Pen 2009) show that the covariance matrix of halo clustering between different mass bins and of different galaxy populations can be well described by the first two principal components. If it is applied in general, it means that only $N_L - 1$ parameters, instead of $N_L(N_L - 1)/2$, are required to describe the galaxy correlation coefficient $\tau_{ij}$ ($i,j = 1, \ldots , N_L$). Our method can incorporate this improved understanding of galaxy stochasticity into account straightforwardly. It is hence promising to solve for the lensing power spectrum, galaxy bias and its stochasticity simultaneously. Details will be discussed in a future paper (Yang et al. 2013, in preparation).

- Tests against mock catalog. This will be done using the existing simulation data at Shanghai Astronomical Observatory (SHAO) ($\sim 100 \mathrm{Gpc}^3$ volume in total).

- Application to real data. As to this aspect, CFHTLS and COSMOS are promising targets. Both of them are sufficiently deep, with photometric redshift information. We emphasize that the photo-z error may be a main source of systematic error, since it could bias
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the measurement of $\alpha(s, z)$. In the current paper and in paper I we target at the spectroscopic survey SKA, so it is irrelevant.

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APPENDIX A: SOLUTION TO EQ. 6 IS UNIQUE

Here we prove that the solution to Eq. 6 is unique. Suppose that there is another set of solution $(f_1, f_2, \cdots, \sigma)$ to Eq. 6 so that

$$ f_i f_j + g_i g_j \sigma = \tilde{C}_{ij}. \quad (A1) $$

Comparing to Eq. 6 we have

$$ f_i f_j = b_i b_j + (\tilde{C}_i - \sigma) g_i g_j. \quad (A2) $$

Squaring it, we have

$$ (f_i f_j)^2 = \left(b_i b_j + (\tilde{C}_i - \sigma) g_i g_j\right)^2. \quad (A3) $$

Eq. A2 also tells us

$$ f_i^2 = b_i^2 + (\tilde{C}_i - \sigma) g_i^2, \quad f_j^2 = b_j^2 + (\tilde{C}_j - \sigma) g_j^2. \quad (A4) $$

Substituting Eq. A2 into Eq. A3 we have

$$ (\tilde{C}_i - \sigma)(b_i g_j - b_j g_i)^2 = 0. \quad (A5) $$

Plugging the $b$-$b$ relation (Eq. 6), we obtain

$$ (\tilde{C}_i - \sigma)(b_i g_j - b_j g_i)^2 = 0. \quad (A6) $$

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In general, \(b\) and \(g\) have different dependences on flux and hence \(b_i/b_j \neq g_i/g_j\). For example, \(g\) for faint galaxies can be negative while \(b\) remains positive. We then obtain \(\sigma = C_\kappa\), and \(f_i = b_i\).

Hence we prove that the solution to Eq. (B3) is unique, as long as that the number of flux bin \(N_i \geq 2\) and that the galaxy bias \(b\) and \(g\) have different flux dependences. In the main text, we have shown that the solution \(C_\kappa\) only differs from the true lensing power spectrum \(C_\kappa\), by a negligible multiplicative bias. We can then draw the conclusion that by directly fitting the measured galaxy clustering between different flux bins (but of the same redshift bin), the lensing auto power spectrum can be determined uniquely.

**APPENDIX B: PHOTO METRY ERROR AND DUST EXTINCTION**

Photometry error operates onto the lensing magnified flux. For a galaxy with intrinsic flux \(s\) and given magnification \(\mu\), the observed flux \(s^O\) is

\[
s^O = s\mu \times (1 + p)
\]

Here we denote \(p\) as the photometry error. It may have a nonzero mean \((\langle p \rangle \neq 0)\) and non-negligible fluctuations around the mean.

Through the galaxy number conservation, we have

\[
n(s)dsdA = n^O(s^O)d^Osd\mu
\]

Here \(A\) is the surface area, which is amplified by lensing by a factor \(\mu\). We then have the relation between the observed galaxy distribution \(n^O\) and the intrinsic distribution \(n\),

\[
n^O(s^O) = \frac{1}{\mu^2(1 + p)} n\left(\frac{s^O}{\mu(1 + p)}\right)
\]

Here, we have neglected the flux dependence in \(p\). This approximation is reasonable if the photometry error does not strongly depend on the flux.

We can Taylor expand \(n\) around \(s^O\) to linear order in \(p\) and \(\mu - 1 \approx 2\kappa\). In this way, the coefficients in front of \(p\) and \(\kappa\) are functions of \(n\). However, since \((p) \neq 0\), even to the first order approximation, \((n^O) \neq (n)\). This means that we can not directly calculate these coefficients from observables (e.g. \(n^O\)).

We circumvent this problem by defining another galaxy flux distribution function \(n^P\), given by \(n(s)ds = n^P(s^P)ds^P\) in which \(s^P \equiv s(1 + p)\). We then have

\[
n^P(s^O) = \frac{1}{\mu^2} n^P\left(\frac{s^O}{\mu}\right) \approx n^P(s^O)\left(1 + g^P\kappa\right)
\]

Here, \(g^P \equiv 2(-d\ln n^P/d\ln s^P)|_{s^P = 0} - 2\).

It is now clear that the cosmic magnification expression is still applicable, as long as we replace the intrinsic galaxy distribution \(n\) with \(n^P\) and replace \(g\) with \(g^P\). Furthermore, since we have \(\langle \mu \rangle = 1\) and \((\mu - 1)^2 = O(10^{-5}); \langle n^O(s^O) \rangle = \langle n^P(s^O) \rangle\) is a good approximation. Under this limit, \(g^P \approx 2(-d\ln n^P/d\ln s^O)|_{s^O = 0} - 2\), an observable.

Hence photometry error does not bias the magnification coefficient \((g^P)\) and in this sense does not bias cosmic magnification measurement. But it indeed introduces new fluctuations in the galaxy density distribution. Taylor expanding the relation \(n^P(s^O) = n(s^O/(1 + p))/\langle 1 + (g^P\kappa + \langle 1 + g^P\kappa \rangle)\rangle\), we obtain

\[
n^O(s^O) \approx n(s^O)\left(1 + \delta + g^P\kappa + \left(1 + \frac{g^P}{2}\right)p\right)
\]

or equivalently,

\[
\delta^O_g \simeq \delta + g^P\kappa + \left(1 + \frac{g^P}{2}\right)(p - \langle p \rangle)
\]

Notice that \(g \neq g^P\) with the presence of photometry error. Since the photometry error \(p\) is a random number, it causes fluctuation in galaxy distribution. This fluctuation can be distinguished from the cosmic magnification in two ways. Firstly, they have different flux dependences \((1 + g^P/2 vs. g^P)\). Secondly, they have different spatial clustering. \(p - \langle p \rangle\) may resemble a shot noise like spatial clustering, although its amplitude can vary across the sky due to spatial variation in photometry calibration accuracy.

Since we allow \(\langle p \rangle \neq 0\) and allow \(p\) to fluctuate across the sky, the above discussion also applies to dust extinction. Then we conclude that both photometry error and dust extinction do not bias our weak lensing reconstruction through cosmic magnification.

**APPENDIX C: ERROR SOURCES IN THE RECONSTRUCTED LENSING MAPS**

A number of errors in the map making comes as follows. In the limit of deterministic bias \((b_i = b_0\delta_m)\), we have

\[
\hat{\kappa} \to \kappa + \epsilon\delta_m
\]

where \(\epsilon \equiv \sum_i w_i(\hat{b})b_i\).

We require \(\sum_i w_i(b_i)b_i = 0\) in order to eliminate the galaxy intrinsic clustering. \(w_i\) satisfying this condition while minimizing the rms error in the map-making is derived in paper I. This optimal weighting function has an analytical expression. It is uniquely fixed by the galaxy luminosity function and the galaxy bias \(b\). Notice that \(w_i\) depends not only on the bias at the \(i\)-th flux bin, but also biases at other bins. We highlight this dependence by explicitly showing \(b\), instead of \(b_i\), as the argument of \(w_i\).

\(w_i\) is invariant under a flux independent scaling in the galaxy bias \(b\) (paper I). In previous sections we show that we can determine \(b_i \equiv \sqrt{C_{\kappa m}(b_i + g_iC_{\kappa m}/C_{\kappa m})}\) to high accuracy. So in deriving the optimal estimator \(w\), we do not need to worry about the absolute value of \(C_{\kappa m}\), which is flux-independent. Hereafter we will ignore this prefactor.

By reconstruction, our estimator guarantees \(\sum_i w_i(\hat{b})b_i = 0\), but not the desired \(\sum_i w_i(b_i)b_i = 0\). Errors in \(\hat{b} (\hat{\kappa} \neq b)\) cause \(\epsilon \neq 0\) and induce additive errors in the reconstructed lensing maps. Taylor expanding \(w(b)\) around the true value \(b\), we have

\[
\epsilon = \sum_{ij} \left[ \frac{\partial w_i}{\partial b_j}(b_j - b_j)b_i \right] + \frac{1}{2} \sum_{ijk} \left[ \frac{\partial^2 w_i}{\partial b_j \partial b_k}(b_j - b_j)(b_k - b_k)b_i \right]
\]

A systematic error has \((b - b) \neq 0\). So we just keep the linear term above to evaluate \(\epsilon\). A statistical error has \((b - b) = 0\), but \((b-b)^2 \neq 0\). Since \(w(b)\) is nonlinear in terms of \(b\), \(\epsilon \neq 0\). So even a statistical error in \(b\) can be rendered into systematic error in the \(\kappa\) reconstruction.

\(^4\) There are errors which can not be described by \(\epsilon\).
We then have
\[ \delta C = \epsilon^2 C_m + 2\epsilon C_{max}. \]  
(C4)

Notice that although usually \( \epsilon \ll 1 \), \( \epsilon^2 C_m \) is not necessarily smaller than \( 2\epsilon C_{max} \), because \( C_m \gg C_{max} \) by one or two magnitude (see Fig. 1 of paper I).

We also have
\[ \delta C_{cross} = \epsilon f C_{m(i)j}. \]  
(C5)

Here, following notations in paper I, we use the superscript "b" to denote the background (higher redshift) bin and the superscript "f" to denote the foreground (lower redshift) bin. In the expression, we have neglected the correlation \( C_{m(i)j} \) between foreground and background matter distributions. It is natural for non-adjacent redshift bins with separation \( \Delta z > 0.1 \), since foreground and background galaxies have no intrinsic correlation. For two adjacent redshift bins (e.g. the left-upper panel in Fig. 7), there is indeed a non-vanishing matter correlation. However, this correlation is also safely neglected since both the foreground and background intrinsic clustering are sharply suppressed by factors \( 1/\epsilon_{l,h} \), respectively. For this reason, stochasticity no longer causes a term like \( \delta C^{(2)} \) (discussed later).

For the cross power spectrum between different bins, the error power spectrum shown in Fig. 8 is the errors in the lensing cross power spectrum measured through cosmic magnification. Since the cross terms between different sources of error no longer exist in the cross power spectrum measurement.

### C1 Systematic error caused by the b-g degeneracy

In the ideal case of no other sources of error, the b-g degeneracy (Eq. 3) still causes a systematic error in the determined galaxy bias \( \tilde{b}_i \equiv b_i \propto b_i + g_i C_{max}/C_m \neq b_i \). As we discussed earlier and as in paper I, flux-independent scaling in the galaxy bias (e.g., \( \sqrt{C_m} \)) does not affect the map-making. So we will ignore the \( \sqrt{C_m} \) prefactor in \( b \). Following the notation in paper I, we denote such error with superscript "(1)"
\[ \epsilon^{(1)} \equiv \frac{C_{max}}{C_m} \left( \sum_{ij} \frac{\partial w_i}{\partial b_j} \bigg|_{b_j} g_j b_i \right). \]  
(C6)

Since \( \epsilon^{(1)} \sim C_{max}/C_m = O(10^{-3}) \ll 1 \) (Fig. 3), the galaxy intrinsic clustering is heavily suppressed.

### C2 Systematic errors induced by galaxy stochasticity

Stochasticity biases the \( \kappa \) reconstruction in two ways. The first has been identified in paper I (Eq. 26). Even if we correctly figure out the deterministic component of galaxy bias, stochasticity does not allow us to completely remove the intrinsic galaxy clustering. The residual part is
\[ \delta C^{(2)} = \left[ \sum_{ij} w_i(b_j)w_j(b_i)b_ib_j \Delta r_{ij} \right] C_m. \]  
(C7)

Following the notation in paper I, we denote this error with a superscript "(2)". This error does not affect the cross correlation measurement.

The galaxy stochasticity also causes systematic bias in the determined galaxy bias (Eq. 11) and hence biases the \( \kappa \) reconstruction through \( w(b) \). It arises since we include the cross power spectra between different flux bins to measure galaxy bias. This one does not have counterpart in paper I, where only the auto power spectra of the same flux bin are utilized to infer the galaxy bias. We will denote this new type of error with a superscript "new". The corresponding \( \epsilon^{new} \) can be calculated with Eqs. [10] & [11] given by
\[ \epsilon^{new} \equiv \sum_{ij} \frac{\partial w_i}{\partial b_j} \bigg|_{b_j} \delta b_i. \]  
(C8)

### C3 Systematic error caused by statistical error in galaxy bias

As discussed earlier, statistical error in galaxy bias also induces systematic error in the \( \kappa \) reconstruction. The induced systematic error in \( \kappa \) is
\[ \epsilon^{(3)} \equiv \sum_{ij} \frac{\partial w_i}{\partial b_j} \bigg|_{b_j} \Delta \tilde{b}_j b_i + \sum_{ijk} \left[ \frac{\partial^2 w_i}{\partial b_j \partial b_k} \bigg|_{b_j} \right] B_{jk}^{-1} b_i. \]  
(C9)

Here \( B^{-1} \) is a sub-matrix of the Fisher matrix \( F^{-1} \) corresponding to parameters of galaxy bias. Although the ensemble average of the first term is zero, this term does contribute to \( \epsilon^{(3)} \) in Eq. [C4]. This error has a counterpart in paper I. Following the notation there, we denote it with a superscript "(3)".