Modern WLAN Fingerprinting Indoor Positioning Methods and Deployment Challenges

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Abstract—Wireless Local Area Network (WLAN) has become a promising choice for indoor positioning as the only existing and established infrastructure, to localize the mobile and stationary users indoors. However, since WLAN has been initially designed for wireless networking and not positioning, the localization task based on WLAN signals has several challenges. Amongst the WLAN positioning methods, WLAN fingerprinting localization has recently achieved great attention due to its promising results. WLAN fingerprinting faces several challenges and hence, in this paper, our goal is to overview these challenges and the state-of-the-art solutions. This paper consists of three main parts: 1) Conventional localization schemes; 2) State-of-the-art approaches; 3) Practical deployment challenges. Since all the proposed methods in WLAN literature have been conducted and tested in different settings, the reported results are not equally comparable. So, we compare some of the main localization schemes in a single real environment and assess their localization accuracy, positioning error statistics, and complexity. Our results depict illustrative evaluation of WLAN localization systems and guide to future improvement opportunities.

Index Terms—Indoor positioning, WLAN fingerprinting, real time processing, clustering, sparse recovery, outlier detection.

I. INTRODUCTION

LOCATION-based services (LBSs) are currently in high demand and strongly drive the development of location-computing technologies [1]. In particular, indoor LBS will significantly improve network management and security [2], emergency personnel navigation [3], healthcare monitoring [4], [5], personalized information delivery [6], context awareness [8] and enable other applications. While US Global Positioning System (GPS) and other similar global navigation satellite systems (GNSS) provided good quality for outdoor positioning [9]–[11], robust indoor positioning is still an open problem. The GPS and similar localization networks do not work indoors as they need direct Line-of-Sight (LOS) between the satellites and user which is not a case indoors as shown in Fig. [1]

Various techniques have been proposed for indoor positioning. From signaling perspective these approaches can be divided into two categories [12], [13]: (1) radio-based positioning such as radio frequency (RF) proximity sensors [14]–[18], also called radio-frequency identification (RFID), Ultra Wide Band (UWB) methods [19], [20], Bluetooth-based methods [21]–[23], ZigBee-based methods [24], [25], Frequency Modulation (FM) methods [26], [27], and IEEE 802.11 Wireless Local Area Networks (WLAN) based methods; and (2) non-radio-based positioning methods which utilize infrared (IR) [28], ultrasonic and sound techniques [29]–[34], visible light [35], [36], inertial systems [37], [38] and magnetic field exploitation [39], [40].

Many of the proposed technologies including RFID assume massive transceiver and infrastructure deployments and incur high maintenance costs. However, IEEE 802.11 WLANs are already broadly deployed to render a ubiquitous and continuous wireless network coverage which is exploited for localization purposes as well. These networks operate in the several unlicensed bands such as 5-GHz (IEEE 802.11a) and 2.4-GHz (IEEE 802.11b/g) and others. Since these bands are unlicensed, several networks may transmit simultaneously and coexist with some interference distortions [41], [42].

A. Indoor Localization Approaches

Historically, from position computation perspective of radio-based signaling systems, the known approaches for WLAN positioning are of three main categories: (1) Angle of Arrival (AOA) and related Direction of Arrival (DOA) methods; (2) Time of Arrival (TOA) and related Time Difference of Arrival (TDOA) techniques; and (3) RSS exploitation methods (fingerprinting). These methods are shown in Fig. [2] and will be reviewed next.

In AOA, the angle between the incident wave and a reference direction, known as orientation, is measured from at least two APs. The APs are equipped with an antenna array to be capable of determining the angle of the received signal. The

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intersection of the two virtual lines heading in the direction of the angles defines the user position \([43]–[45]\).

TOA techniques use the travel time that a wave takes from the transmitter to the receiver and transform it to range distance. At least three APs measure the TOA from a mobile device. For this positioning technique, normally, trilateration is applied \([37], [46]\). In trilateration technique, the APs coordinates are known. Considering an AP as the locus, the range distance defines a circle of certain radius. The intersection of these circles associated with several loci allows to estimate the user’s position. However, there is a great probability that the circles do not intersect precisely at a point due to noisy measurements and the position is estimated with a limited accuracy. The localization based on TOA is shown in Fig. 3.

To find the user’s location, the following non-linear system of equations should be solved

\[
r_i = d_i + \epsilon_i = \sqrt{(x_i - x)^2 + (y_i - y)^2} + \epsilon_i
\]

where \(r_i\) is the range distance computed from TOA, \(\epsilon_i\) is the corresponding noise, \(p = (x, y)\) is the user’s location to be estimated, and \(p_i = (x_i, y_i)\) is the \(i\)-th AP location. As illustrated in Fig. 3, due to the range measurement noise, the location of the user cannot be computed exactly and more sophisticated algorithms that minimize the mean square error (MSE) of the noise, such as least squares (LS), are applied.

TDOA is a variation of TOA, in which a source signal is selected and the time difference of arrival between several spatially distributed APs are measured with respect to the source signal. Since the signal is received from several APs, the system locates the sending device on a hyperboloid \([47], [48]\).

The above approaches need direct AP-user LOS. Although some enhancements have been proposed for Non-LOS (NLOS) conditions \([49], [50]\), the localization errors are high \([46]\). In addition, the location of the APs should also be known which is a non-realistic assumptions as the location of the APs are generally unknown and is subject to change regularly for the purpose of providing maximum network coverage.

WLAN fingerprinting methods which use Received Signal Strength (RSS), i.e. the power of received signals from WLAN Access Points (APs), have recently captured a lot of attentions. The reason is twofold: 1) WLANs are widely deployed in offices, business buildings, shopping malls, airports, home environments, etc. and provide ubiquitous coverage for the area. 2) The mobile and wireless receivers all contain the NICs to provide the RSS measurements, and thus, there is no need to install any additional hardware, leading to a reduction in infrastructure installation, equipment and labor costs. Usually, NICs are able to capture distinct RSS magnitudes at a rate of either 0.5 or 1 samples per second.

In general, the RSS exploitation approaches are divided into two broad categories: model-based (path loss) and model-free (radio map) approaches.

The model based approaches use the collected RSS fingerprints to train the parameters for the predefined propagation models \([14], [33], [51], [52]\). These techniques assume a priori path loss model for the indoor propagation which is a logarithmic decay function of the distance from the APs as \([53]\)

\[
PL = PL_0 + 10\gamma \log_{10}\frac{d}{d_0}
\]

where \(PL\) is the path loss measured in dB, \(d\) is the length of the path, \(d_0\) is the reference distance, and \(\gamma\) is the path loss parameter. Using the collected RSS, the distance \(d\) between the AP and the user is computed and the location of the user is estimated using the trilateration which incurs knowing the AP locations. To render a more accurate modeling and decrease the discrepancy between the RSS measurements and the model, a random component is added to the model to compensate for the RSS variations \([54]\). However, the underlying assumption of symmetric signal power decaying in indoor environments is questionable as the RSS attenuations decay at different rates in different directions due to asymmetric indoor structure.

The radio map based techniques, also called fingerprinting techniques, make the use of dense AP deployments in indoor areas. A set of RSS or other measurements serve as fingerprint which should be more or less unique for each location. In most cases, WLAN fingerprinting consists of offline and online phases. A schematic of typical WLAN fingerprinting localization is depicted in Fig. 3. First of all, a set of predefined points, referred to as Reference Points (RPs), also called landmarks, grid or survey points, are selected. So, these terms may be used interchangeably throughout this paper. During an offline phase, a survey is conducted and multiple copies of RSS measurements are read at each RP from available APs throughout a time interval. The database of fingerprints for all RPs makes a radio map for the whole area. Then during online phase, user observes RSS measurements at his location and applies algorithms to associate these measurements to the radio map entries finding similar fingerprints, and using associated

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**Fig. 2.** WLAN localization schemes.

**Fig. 3.** Localization based on TOA.
B. Criteria for the Categorization of Fingerprinting Methods

WLAN fingerprinting methods differ in computational requirements. Since the computational complexity of the localization systems are high, some approaches assign the localization task on high-power servers, and hence, called server-based localization. Quite the opposite, client-based approaches minimize the computational complexity of the localization procedure and the positioning computations are performed in resource-limited hand-held wireless devices. Client-based methods are sometimes considered more preferable for users as privacy issues are typically associated with the server-based techniques. Without loss of generality, client-based approaches are considered in the following.

The user is required to carry a wireless device such as laptop, tablet, and smart phone. These devices are required to capture the RSS measurements for the localizations. However, some methods have been recently proposed that does not require the user to carry any device, known as passive (device-free) localization [56]–[59]. In passive localization, RSS measurements are taken from wireless devices available in the area which basically measure the changes in RSS profile in the presence of the user at different positions. The passive methods are not discussed in this paper. A summary of abovementioned methods is provided in Fig. 5.

C. Fingerprinting Localization Challenges

Several challenges face the WLAN fingerprinting localization schemes. RSS measurements are distorted by shadowing and NLOS propagation due to the presence of walls, doors, furniture, objects, human. [60]–[63]. Fig. 6 shows a typical office environment and a wireless router signals which travel different paths to the wireless devices. So, the propagated signal encounters with severe frequency selective multipath fluctuations and hence cannot be considered wide sense stationary (non-WSS) [64]. Moreover, WLAN operates on unlicensed frequencies of 2.4GHz and 5GHz, open to cordless phones, microwaves and the resonance frequency of water. These lead to interference from such devices and signal absorption by the human body. These phenomena make RSS densities non-Gaussian and time varying. In addition, there are various AP networks in a typical area which add extra interference. Also, it is possible that the wireless network coverage degrades due to AP failures [65], [66].

There are also logistical problems in fingerprinting WLAN localization. First of all, the surveying stage is very time consuming as the surveyor needs to carry the recording device to each RP and record the RSS for a time period. Furthermore, pre-processing techniques are usually exploited to reduce search area of the user location to a smaller region rather than the whole area. The smaller region is usually selected by clustering the area. This eliminates the need for a comparison of the online measured RSS with all the RPs fingerprints and hence, the computation time decreases significantly. In the online phase, a subset of RSS measurements should be selected as not all measurements provide beneficial information. Normally, an assessment of measurement sanity is conducted and a subset of APs are selected for positioning. An elaborate discussion on these methods are provided in this paper.

D. What This Paper Brings to the Scene

Wireless indoor localization has been previously reviewed [23], [67]–[76]. Though impressive, most of the previous
surveys did not comprehensively cover all stages of a complete fingerprinting localization system. More importantly, these surveys have discussed the current localization approaches generally and did not dig into technical aspects.

This paper provides a technical overview of the state-of-the-art WLAN fingerprinting positioning approaches and practical implementation issues. The discussion of this paper is divided into three main parts which are overviewed next. Table I lists the related sections and contents of each part:

1) Problem Formulation and Conventional Approaches (Section II): In this part, we discuss the fundamental fingerprinting concepts and unify the different notations used in the literature. Then, the conventional approaches, which have been proposed in the early stages of Wi-Fi based localization, are organized in three general categories.

2) State-of-the-Art Approaches: The wide variety of recent trends toward Wi-Fi-based localization can be organized along the following paths:

   - Refinements of the Conventional Localization: Since the conventional methods cannot achieve the necessary localization accuracy and the online running time cannot pace with the user’s motion, refinements have been introduced. They have focused on RP clustering (Section III), exploitation of APs (Section IV), and advanced density and weight estimation methods (Section V). These techniques are direct modifications of the conventional approaches.

   - Sparsity-based Localization (Section VI): A reformulation of WLAN localization has been recently introduced which exploits sparse recovery methods.

   - Assisted Localization (Section VII): Aside from merely utilizing WLAN fingerprints for localization, some methods gain assistance from available resources in the environment and user’s device to achieve superior localization accuracy. These methods may integrate sensory signatures built in the modern wireless devices, track the user’s motion, exploit the available environment landmarks, or utilize the peer-to-peer collaboration between devices, and collectively fall under assisted localization.

3) Deployment Challenges: Localization schemes face laborious deployment challenges which constrain their applicability as real positioning systems. Even with advanced localization techniques, practical systems should account for several challenges listed next.

   - Radio-map Construction (Section VIII): An existing problem with fingerprinting methods is the need for dense survey of the area. Previous works attempt to decrease the time and cost of fingerprinting tasks through crowd-sourcing, implicit or unlabeled data collection, and radio map interpolation.

   - Outlier Detection (Section IX): APs are easily prone to infrastructure problems that render faulty readings. These faulty readings are called outliers. Outliers can occur both on fingerprints during the survey process and more importantly during the online phase. The fingerprint outliers are easier to detect. The presence of outliers during the online phase implies that the user’s location should be estimated using faulty measurements.

   - Heterogeneous Devices (Section X): Wireless devices obtain RSS fingerprints through their Network Interface Cards (NICs). The sensitivity of the wireless devices differ as the NICs chipsets are different and the position of the antenna on the device affects the RSS readings.

After the theoretical discussion, we provide a numerical evaluation of the representative approaches based on localization accuracy and positioning error statistics in Section XI. The methods are tested on the same set of fingerprints collected at the University of Texas at San Antonio (UTSA). These comparisons provide illustrative guidelines for future improvements. A critical summary and future directions are provided in Section XII.

II. WLAN FINGERPRINTING LOCALIZATION: PROBLEM FORMULATION AND CONVENTIONAL APPROACHES

This section provides the definitions and formulation of the WLAN fingerprinting localization, and a description of the conventional localization methods comes in sequel.
A. Problem Formulation

In fingerprinting, the area is divided into a set of RPs \( \mathcal{P} = \{\mathbf{p}_j = (x_j, y_j)\mid j = 1, \ldots, N\} \) where \( \mathcal{P} \) defines the set of RP Cartesian coordinates, which are not necessarily set apart in equal distances. The mobile device records RSS fingerprints at time instants \( t_m \), \( m = 1, \ldots, M \), with RSS magnitudes \( (r_j^i(t_1), \ldots, r_j^i(t_M)) \) at each RP, where \( i \) indicates the AP index from the set of APs, \( \mathcal{L} = \{AP^1, \ldots, AP^L\} \). It is typical to take the same number of training samples, \( M \), at each RP. The RSS fingerprints from all APs at time \( t_m \) at \( \mathbf{p}_j \) are organized in a vector \( \mathbf{r}_j(t_m) = [r_j^1(t_m), \ldots, r_j^L(t_m)]^T \).

The entire radio map at recording instant \( t_m \) is represented as

\[
\mathbf{R}(t_m) = (\mathbf{r}_1(t_m), \ldots, \mathbf{r}_N(t_m)) = \\
\begin{pmatrix}
\psi_1^1 & \cdots & \psi_1^N \\
\vdots & \ddots & \vdots \\
\psi_L^1 & \cdots & \psi_L^N
\end{pmatrix},
\]

(3)

where \( \psi_j = [\psi_j^1, \ldots, \psi_j^N]^T \) and \( \psi_j^i = \frac{1}{M} \sum_{m=1}^M r_j^i(t_m) \).

Let also \( \mathbf{r}_j = [r_j^1(t_1), \ldots, r_j^M(t_M)]^T \) and \( \mathbf{r}_j(t_m) = [r_j^1(t_m), \ldots, r_j^L(t_m)]^T \) indicate a vector of RSS fingerprints for different time instants, different RPs, and different APs, respectively. If the time sequence of radio maps, \( \mathbf{R}(t_m) \), is averaged over the recording time, the time averaged radio map is denoted as

\[
\mathbf{\Psi} = (\mathbf{\psi}_1, \ldots, \mathbf{\psi}_N) = \\
\begin{pmatrix}
\psi_1^1 & \cdots & \psi_1^N \\
\vdots & \ddots & \vdots \\
\psi_L^1 & \cdots & \psi_L^N
\end{pmatrix}
\]

(4)

where \( \mathbf{\psi}_j = [\psi_j^1, \ldots, \psi_j^N]^T \).

A subset of RPs with the most similarity to the online measurement is denoted by \( \mathcal{K} \) where \( |\mathcal{K}| = K \). This similarity is defined differently in each localization method and will be discussed in detail later.

In the online phase, the mobile user receives the online RSS measurements, \( \mathbf{y} = (y_1, \ldots, y_N)^T \). The goal of a localization scheme is to find the user’s location, \( \hat{\mathbf{p}} = (\hat{x}, \hat{y}) \), based on a rule that compares the received online measurements against radio map fingerprints as:

\[
\hat{\mathbf{p}} = f(\mathbf{R}, \mathbf{y}).
\]

(5)

where \( \mathbf{R} \) denotes the collection of radio maps at all recording instances. Some techniques (especially the advanced probability methods in Section II-B) need multiple online measurements which are indexed by time instants \( t_m \), as \( \mathbf{y}(t_m) = (y^1(t_m), \ldots, y^N(t_m))^T \). Next, three conventional localization approaches are discussed.

B. Conventional Localization Approaches

In this section we elaborate on the early WLAN fingerprinting localization approaches [69, 71, 91]. A diagram summarizing these approaches is shown in Fig. 7 along with a categorization of the related works in Table II.

Fig. 7. Conventional localization approaches.

1) Deterministic Approaches: In deterministic approaches, the general form of position estimation is achieved through selecting RPs whose fingerprints are the closest to the online RSS measurements as

\[
\hat{\mathbf{p}} = \arg\min_{j=1,\ldots,N} d(\hat{\mathbf{r}}_j, \mathbf{y})
\]

(6)

where \( \hat{\mathbf{r}}_j \) is the representative fingerprint value at RP \( j \) [14], [80] and \( d(\hat{\mathbf{r}}_j, \mathbf{y}) \) defines a typical distance metric [92]. In case of time-average, the representative value is \( \hat{\mathbf{r}}_j \). Euclidean distance is a well-known distance metric for (6) defined as

\[
d(\hat{\mathbf{r}}_j, \mathbf{y}) = \|\mathbf{y} - \hat{\mathbf{r}}_j\|_2, \quad j = 1, \ldots, N.
\]

(7)

A solution which finds the RP with the minimum Euclidean distance among measurements is known as the nearest neighbor (NN) method.

Median filtering has also been used to improve the robustness of the KNN method to unusual fingerprint readings [81]. In this case, the \( i \)-th entry of \( \hat{\mathbf{r}}_j \) is \( \hat{r}_j^i = \text{med}\{r_j^i(t_m), \quad m = 1, \ldots, M\} \).

If instead of selecting a single RP with the least distance, a set of closest RPs are selected, the method is known as K-nearest neighborhood (KNN) [14], [68], [74], [77]. In KNN, the user position is usually the centroid of a set of \( K \) RPs with the least distances \( d(\hat{\mathbf{r}}_j, \mathbf{y}) \)

\[
\hat{\mathbf{p}}_{KNN} = \frac{1}{K} \sum_{j \in \mathcal{K}} \mathbf{p}_j.
\]

(8)

The weighted KNN approach differentiates RPs by assigning weights in (8) proportional to the inverse of their corresponding \( d(\hat{\mathbf{r}}_j, \mathbf{y}) \). So, RPs that are similar to the online measurement, receive higher weights. KNN weights can also be computed based on the inverse of the RSS variance at each RP [79], or cosine similarity [93]. An RP can be excluded from engaging in positioning if its total RSS variation is above a predefined threshold as it is unreliable [54].

Since the number and availability of APs varies across the localization area, (7) is typically estimated over the common visible APs and missing APs' readings are replaced by a boundary number indicating weak signal (usually -95 dBm).
2) Probabilistic Approaches: A single RSS fingerprint may not be a sufficient representation of the data because of the time-varying nature of indoor propagation. The performance of deterministic localization approaches can be improved if instead of a single representative RSS fingerprint, all fingerprints are used. In probabilistic approaches, the whole ensemble of RSS fingerprints are utilized to provide statistical characteristics of the area.

The underlying approach in probabilistic localization is the Maximum A Posteriori (MAP) estimation [70], [94]. The MAP estimates the location of the user based on maximizing the conditional probability of the location given the received online measurement

\[
\hat{p} = \text{argmax}_{j=1,\ldots,N} f(p_j | y)
\]

(9)

where \(f(p_j | y)\) is the conditional probability that the user is in \(p_j\) given the received online vector \(y\). The equivalent reformulation of (9) is achieved through the Bayes rule

\[
f(p_j | y) = \frac{f(y | p_j) f(p_j)}{\sum_{j=1}^{N} f(y | p_j) f(p_j)}
\]

(10)

The probability \(f(p_j)\) is the distribution of the user location over the area and is usually assumed to be uniform, i.e. \(f(p_j) = \frac{1}{N}\) since there is no prior knowledge regarding the user location and all survey points are equally probable. Therefore, \(f(p_j)\) can be ignored in the maximization problem (9). Likewise, the denominator in (10) is the same for all \(j = 1,\ldots,N\). Therefore, the MAP estimation in (9) is equivalent to the following problem

\[
\hat{p} = \text{argmax}_{j=1,\ldots,N} f(y | p_j)
\]

(11)

known as Maximum Likelihood (ML) estimation [95]. Another alternative to the ML estimate of (11) is to select three non-collinear RPs with the highest probability. The user’s location can be estimated through an interpolation between these RPs by solving a system of two equations with two unknowns [96].

ML estimate picks the RP with the maximum statistical similarity to the online measurement, however, if the user is in between the RPs only a single RP is not a suitable location estimate of the user. To this end, the convex hull of the RPs that surround the user’s location provides a suitable estimate. Therefore, (11) can be replaced by an estimate that utilizes all (or a subset of) RPs with corresponding weights as follows [97], [98]

\[
\hat{p} = \sum_{j=1}^{N} w_j p_j, \quad w_j = \frac{f(y | p_j)}{\sum_{j=1}^{N} f(y | p_j)} .
\]

(12)

The ML estimate of (11) renders the most similar RP as the user’s location. This leads to a high localization error if the user is not exactly at one RP. The supremacy of (12) over (11) is that it renders the user’s location as the weighted convex combination of the RPs that own the most similar fingerprints to the online measurements.

The previous discussion reveals that the task of positioning relies on estimating the prior density \(f(y | p_j)\). There are two main approaches regarding fingerprint distribution estimation: parametric and non-parametric estimation. Parametric estimation methods try to map the data to known analytical distributions, e.g., Gaussian, to approximate temporal RSS characteristics [82], [83]. This assumption has been questioned in several works, e.g., [62]. Early approaches consider the RSS distribution as log-normal [99]. However, it is shown that the distribution is not typically log-normal but left skewed, stationary only over small time frames, and the user’s presence makes it multi-modal [64], [84], [100]. Moder density estimation methods use kernel functions, and an overview is provided in Section V.

Non-parametric estimation methods do not assume any known distribution matches with the RSS fingerprints. Instead, the fingerprint distributions are generated using histogram matching of radio-map fingerprints [82], [83], [101]. In histogram matching, the whole data is quantized into multiple levels and the frequency of each bin is calculated for the estimation of \(f(y | p_j)\). The histogram consists of the concatenation of these bins. However, a large number of time samples are needed at each RP to generate a histogram. Besides, the histogram is primarily dependent on bin width and the choice of origin [34], [102], [103].

3) Pattern Recognition Techniques: The basic idea of pattern recognition methods is based on classifiers, that are trained using surveyed fingerprinting data and then used to discriminate unknown RSS measurements during the online phase. In the training phase, the system tunes the internal classifier model knowing a radio map database. In the testing phase, the received RSS data from unknown locations are processed by the classifier by estimating the most likely location. The difference between pattern recognition approaches is in their pattern-matching techniques. The outcome of the pattern recognition algorithm is typically a likelihood of various locations given observed measurements, which allows to estimate the centroid of the all candidate positions as the solution. Support Vector Machine (SVM), Canonical Correlation Analysis (CCA), Neural Network, and linear discriminant analysis are examples of contemporary pattern recognition schemes [87]–[90].
The conventional WLAN fingerprinting localization methods face several challenges which degrade the positioning accuracy and introduce biased estimations. These challenges motivate all remaining parts and are listed next in this paper and are shown in Fig. 8 with corresponding state-of-the-art solutions detailed as follows:

- **Challenge c1**: The number of RPs increases with the area size, which increases the required memory needed to store the surveyed data and the computing resources.
- **Challenge c2**: APs do not necessarily provide independent information and the fingerprints can be correlated.
- **Challenge c3**: APs have limited coverage area and may not be accessible to all RPs in the surveyed area. Utilizing distant APs with weak signals at the user location degrades positioning accuracy.
- **Challenge c4**: Possible faulty RSS measurements may incur biased position estimates.
- **Challenge c5**: The distribution of RSS fingerprints is non-Gaussian, skewed, multimodal, and time-varying.
- **Challenge c6**: Most proposals for conventional methods give low accuracy guarantees.
- **Challenge c7**: The radio map construction is labor intensive and time consuming.
- **Challenge c8**: The difference between surveying device and the user’s device leads to heterogeneity of fingerprints readings which impose a great error on localization.

Practical positioning schemes have attempted to address these issues [75], [104], [105]. Fig. 8 maps the challenges with corresponding solutions. Note that one solution may address several challenges simultaneously.

To address challenges c1 and c6, offline RP clustering and online coarse localization have been proposed. In RP clustering, the RPs are divided into groups (clusters) based on a similarity metric. Then, the localization coarsely estimates the user location in a subset of RPs and then the fine location of the user is estimated within this subset. The RP clustering reduces the computational burden, and guides the fine localization step.

Challenges c2 and c3 are addressed through AP selection, in which an evaluation metric assigns scores to APs. Generally, the score defines the suitability of each AP for localization considering the online measurement of the user. Then, the best set of APs that can provide distinguishable information are used in localization. AP selection discards the APs that do not provide independent information, and biased location estimation due to distant APs.

Challenges c4, c5, and c6 should be treated with accurate metrics that measure the distance between the fingerprints and online measurements. In recent probabilistic methods, the RSS fingerprints distributions are estimated through more sophisticated schemes that account for the multimodality of the distribution. Also, advanced techniques have been introduced for weight estimation in [12]. In addition, the Wi-Fi fingerprints can be integrated with additional environmental features, inertial device sensors, and collaboration between devices to use all the available information and deliver more accurate location estimations. Furthermore, recent approaches have introduced a new solution to the WLAN fingerprinting problem via sparse recovery methods. Above all, inordinate readings in online measurements are treated with outlier detection methods.

To tackle challenge c7, recent techniques propose to record the radio map with the help of users or at a coarser grid with subsequent interpolation in between RPs at a finer grid.

Challenges c8 is treated with approaches that match the online measurements with the offline fingerprints. These methods are discussed in Section X.

Fig. 2 categorizes the state-of-the-art solutions (part II) that come as refinements and enhancements to conventional approaches. The shaded box denotes the three tasks that a typical modern localization system performs. Sparsity-based localization and assisted localization may also be combined with these tasks to improve the localization accuracy. The corresponding literature is summarized in Table III.

Fig. 10 categorizes the deployment challenges (part III) with the localization methods. These challenges and the methods to address them are the contents of part III of the paper. Related works are listed in Table IV.

### III. RP Clustering and Coarse Localization

In WLAN positioning, the characteristics of RSS fingerprints highly depend on environmental features and available APs. This motivated the recent works to constraint the positioning algorithm to a subset of RPs that show similar characteristics [106]. In other words, a coarse localization stage reduces the search space of the user location to a smaller number of RPs, which is followed by a finer search on the refined set of RPs [107]. This procedure is typically called radio map clustering or spatial filtering. These terms are used interchangeably in this context. The clustering is an offline process where the members of a cluster are grouped...
TABLE III  
STATE-OF-THE-ART APPROACHES AND RELATED LITERATURE

| Methods                        | Related Works         |
|--------------------------------|------------------------|
| Exploitation of APs            | 891, 101, 112, 118    |
| Advanced density and weight estimation | 84, 119, 120      |
| Sparsity-based localization    | 105, 111, 116, 121   |
| Assisted localization          | 12, 110, 126, 128    |

Together based on a similarity metric. A representative value of fingerprints shows the characteristics of each cluster and is used for coarse localization. Specific clustering methods are surveyed next.

A. Clustering Using AP Coverage

One spatial filtering method is based on the assumption that neighbor RPs receive similar RSS fingerprints [84], [108]. The intuition is that neighboring RPs should receive RSS readings from the same set of APs. The scheme relies upon defining the continuous coverage of an AP over a subset of RPs. First, the set of time slots for which the fingerprints corresponding to each AP are above a threshold $\gamma$ is computed:

$$\mathcal{T}_j^i = \{ m \in \{1, \ldots, M \} | r_j^i(t_m) \geq \gamma \},$$

$$i = 1, \ldots, L, \ j = 1, \ldots, N.$$  

(13)

An AP is considered reliable for RP $j$ if its RSS fingerprints are above a threshold “most of the time.” The indicator $I^j_i$ denotes the APs whose readings satisfy (13) for, e.g., 90% of the time during the fingerprint phase:

$$I^j_i = \begin{cases} 1 & |\mathcal{T}_j^i| \geq 0.9 M \\ 0 & \text{otherwise} \end{cases} \ i = 1, \ldots, L, \ j = 1, \ldots, N.$$  

(14)

where $| \cdot |$ denotes the cardinality and $I^j_i$ is called the coverage indicator of AP $i$ at RP $p_j$. Next, let $\mathcal{L}_j$ be the set of APs that satisfy (14) for RP $p_j$, i.e.

$$\mathcal{L}_j = \{ i \in \{1, \ldots, L \} | I^j_i = 1 \}, \ j = 1 \ldots, N.$$  

(15)

A binary coverage vector, $\mathbf{I}_j = [I^1_j, \ldots, I^L_j]$, is assigned to each RP as an indication of the difference between RPs. Likewise a coverage vector $\mathbf{I}_y = [I^1_y, \ldots, I^L_y]$ is defined for online measurement $y$ where the $i$-th entry is given by

$$I^i_y = \begin{cases} 1 & \text{if } y^i \geq \gamma \\ 0 & \text{otherwise} \end{cases} \ i = 1, \ldots, L.$$  

(16)

The coarse localization is performed by selecting a subset of $\mathcal{P} \subseteq \mathcal{P}$ RPs whose coverage vector $\mathbf{I}_y$’s has distance from $\mathbf{I}_y$ below a threshold as follows

$$\mathcal{P} = \{ j \in \{1, \ldots, N \} | d_H(\mathbf{I}_y, \mathbf{I}_j) \leq \eta \}$$  

(17)

where $d_H$ is the Hamming distance between $\mathbf{I}_y$ and $\mathbf{I}_j$ is defined as

$$d_H(\mathbf{I}_y, \mathbf{I}_j) = \sum_{i=1}^L |I^i_y - I^i_j|, \ j \in \{1, \ldots, N\}.$$  

(18)

B. Affinity Propagation

The affinity propagation considers that the set of all RPs are the nodes $\mathcal{V}$ of a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. The set of edges consists of all pairs $(j, j')$, $j, j' = 1, \ldots, N$. This method is based on an iterative message exchange between the nodes to find clusters and an exemplar (cluster head) for each cluster [116]. The messages are simply the negative of the Euclidean distance between the fingerprints $\psi_j^i$ and $\psi_{j'}^i$, at two different RPs $j$ and $j'$. Message passing between nodes reaches to a decision convergence in which the exemplars and corresponding clusters are defined.

In the online phase, the online measurement is compared against the cluster heads and a set of clusters whose cluster
heads have the least distances are selected as the coarse location of the user.

C. K-means Clustering

K-means clustering also finds the clusters and their associated cluster centroids iteratively, however, the centroid of each cluster is updated at each iteration \([112], [113]\). In this method, the number of clusters should be defined a priori through a training set. Block-based weighted clustering is a further evolution of K-means clustering proposed in \([157]\), where a weighted least squares objective function is used. The distance between an RP and a centroid is minimized and the weights are obtained through a polynomial function of this distance.

D. Splitting-based Clustering

Unlike the conventional clustering schemes where a similarity measure groups RPs into a cluster, splitting-based clustering starts from the whole area and at each iteration (level) splits the area into four clusters. Then, the mean and variance of each cluster defines a score, which signifies the distinction among the current level subclusters for AP \(i\) \([118]\):

\[
\xi_k^i = \frac{\sum_{k,k' \in C} (\rho_k^i - \rho_k'^i)^2}{\sum_{k=1}^{[C]} \sigma_k^2}, \quad i = 1, \ldots, L
\]  

(19)

where \(C\) is the set of clusters, \(k, k' \in C\) are two distinct clusters, and \(\rho_k^i\) and \(\sigma_k^2\) are the mean and variance of fingerprints \(\psi_j^i\) of AP \(i\) in cluster \(k\), respectively. Each subcluster is labeled with a subset of \(L'_k \subseteq L\) of APs that provide the largest score in \([19]\). The \(\rho_k^i, i \in L'_k\) is stored for subcluster \(k\). If \(\xi_k^i\) is above a threshold for AP \(i\), the cluster is divided into subclusters again. The clustering process ends when no subcluster satisfies this criterion.

In coarse localization, the comparison with the online measurement \(y\) is started from the first level of clusters. An Euclidean metric is computed between the online measurements and each subcluster’s mean, where the difference is computed only on the labeled APs for sub-cluster \(k\).

E. Weighted clustering

The weighted connection (edge) between two nodes is regulated by a similarity measure between the nodes \([109]\). This similarity is based on the fact that spatially close RPs should receive similar readings from the same set of APs. The similarity metric that reveals this feature is

\[
s(j, j') = \begin{cases} \frac{1}{d_H(I_j, I_{j'})}, & \text{if } d_H(I_j, I_{j'}) \neq 0 \\ \Lambda, & \text{otherwise} \end{cases}
\]

(20)

\(\forall j, j' = 1, \ldots, N\), \(j \neq j'\)

which is proportional to the inverse of Hamming distance between two different RPs, and \(\Lambda\) is a sufficiently large number.

The trust on an RP includes the stability of the fingerprint readings through the fingerprinting time. Hence, the variance of readings for all RPs is also computed as

\[
\Delta_j^i = \frac{1}{M - 1} \sum_{m=1}^{M} (r_j^i(t_m) - \psi_j^i)^2
\]

(21)

\(i = 1, \ldots, L, j = 1, \ldots, N\).

The variance of \(j\) is the average of variances in the set of APs that obey \([15]\), i.e. \(L_j^i\):

\[
\Delta_j = \frac{1}{|L_j^i|} \sum_{i \in L_j^i} \Delta_j^i, \quad j = 1, \ldots, N.
\]

(22)

The cluster, \(C(k)\), is the set comprising the cluster head, CH(\(k\)), and its followers, \(F\mathcal{L}(k)\):

\[
C(k) = \{\text{CH}(k)\} \cup F\mathcal{L}(k).
\]

(23)

An RP is randomly selected as the cluster head CH(\(k\)). The criteria for RP \(j\) to be in its cluster, i.e. being assigned as
The measurement coverage vector and that of each RP is computed. Let \( y \) vector as in (14) and (16) for the offline fingerprints and online per-formance in the online phase. After defining the AP coverage reading. So, unlike the previous methods, the clustering is performed in the online phase. The minimum of the Hamming distance over all the cluster members:

\[
CH(k) = \{ j \in C(k) | \Delta_j = \min \{ \Delta_l \}, l \in C(k) \} 
\]

(25)

The coarse localization is performed by selecting the cluster whose CH has the least distance from the online measurement \( y \). If the cluster with the minimum distance has RPs common with other clusters, then the neighbor cluster RPs are also included.

F. Layered Clustering

Another method for clustering RPs using the AP coverage vector is proposed in [110], [111]. This method is a layered clustering of RPs based on their similarity to the online reading. So, unlike the previous methods, the clustering is performed in the online phase. After defining the AP coverage vector as in [14] and [16] for the offline fingerprints and online vector \( y \), the Hamming distance \( d_H(\mathbf{I}_y, \mathbf{I}_j) \) between the online measurement coverage vector and that of each RP is computed.

The minimum and maximum of the Hamming distance over the area is defined as

\[
d_H^{\text{min}} = \min_{j=1,...,N} d_H(\mathbf{I}_y, \mathbf{I}_j) \]
\[
d_H^{\text{max}} = \max_{j=1,...,N} d_H(\mathbf{I}_y, \mathbf{I}_j). \]

(26)

Then, the group Hamming range is defined, as follows

\[
r = \frac{d_H^{\text{max}} - d_H^{\text{min}}}{K} \]

(27)

where \( K \) is the number of groups (clusters) and is defined experimentally or from a training set. RPs are clustered with respect to their Hamming distances to the online measurement. Specifically, the distance range \([d_H^{\text{min}}, d_H^{\text{max}}]\) is partitioned in \( K \) groups collected in set \( D \)

\[
D = \{ [d_{k-1}, d_k] | d_k = d_H^{\text{min}} + kr, k = 1, \ldots, K \} \]

(28)

where \( d_0 = d_H^{\text{min}}. \) Then, \( j \) is assigned to group \( k \) if and only if

\[
d_{k-1} \leq d_H(\mathbf{I}_y, \mathbf{I}_j) \leq d_k. \]

(29)

It could happen that \( d_H(\mathbf{I}_y, \mathbf{I}_j) = d_k, \) so, \( j \) may belong to groups \( k \) and \( k+1. \) In this case, \( j \) is randomly assigned to one of these groups. The corresponding weight for each group is the inverse of the average of group Hamming distance

\[
w_k = \frac{2}{d_{k-1} + d_k} \quad \forall k = 1, \ldots, K. \]

(30)

During the fine localization, all groups accompany in localization through corresponding weights. This clustering scheme is not for coarse localization, but is used (together with the weights) for the group sparsity based localization (Section [VI]) that combines coarse and fine localization in a single step.

G. Spectral Clustering

The similarity measure in spectral clustering is the pairwise cosine similarity between two RPs \( j, j' \) as

\[
s(j, j') = \frac{\langle \psi_j, \psi_{j'} \rangle}{\| \psi_j \| \| \psi_{j'} \|} \]

(31)

RPs are grouped into a predefined number of clusters, \( K \), so that the similarity of RSS vectors within the cluster is maximized [126], i.e.,

\[
\max_k \sum_{j=1}^{K} \sum_{j \in C(k)} s(\psi_j, \rho_k) \]

(32)

where \( \rho_k \) is the average of fingerprint vectors \( \psi_j \) within cluster \( k. \)

IV. EXPLOITATION OF APs FOR LOCALIZATION

The complexity of the indoor propagation environment causes several challenges associated with APs. We first elaborate on these challenges and then discuss approaches to address them.

A. Challenges Related to APs

The challenges with APs can be generally divided into three main categories: 1) the unavailability of APs; 2) large set of available APs, from which a subset of APs should be selected; and 3) faulty APs. The first two issues are elaborated in this section and the last one is discussed Section [IX].

The main reason for unavailability of APs is the range limitation. For instance, a typical IEEE 802.11b AP provides a coverage of less than 100m at 5.5 Mbps. To provide a ubiquitous network coverage, multiple APs are installed in buildings. So, not all APs can provide RSS signatures for a single RP. In large areas, a subset of APs is visible on the user’s device, however, if the user moves far from the previous location, another subset of APs become visible. For instance, Fig. [11] shows the RSS profile for a single AP in a real environment. The RPs are numbered as specified by the horizontal axis index. This figure indicates that the device cannot receive signals from the AP when located at the RPs beyond 140.

In addition, due to the wide deployment of APs in indoor settings, including all APs in positioning is not recommended due to the following issues:
The number of available APs is usually more than the minimum needed (3 APs for 2D localization and 4 APs for 3D localization) for positioning. Advanced APs can transmit in different channels with different Media Address Control (MAC) settings. Including all MAC addresses for one AP does not add information to the system. For example, during an experiment in a typical real office environment, we found a total of 268 MAC addresses. APs usually provide correlated readings. This correlation may occur in three ways: 1) Neighbor RPs may receive correlated fingerprints from a specific AP because the RSS fingerprints are obtained from the same signal received in close locations. This prevents the distinguishability between RPs. 2) A pair of APs may provide correlated fingerprints for an RP. This issue occurs when the APs are located close to each other but belong to different networks. So, APs from different networks produce similar measurements and engaging all may introduce biased position estimates, incur overfitting, and impose time and computational complexity. 3) Fingerprints at one RP may be correlated during the fingerprinting time. This incurs a large difference between the fingerprints and online measurements.

To mitigate the above effects two major tasks are usually performed, namely, 1) feature selection that maps the information of APs to other domains to obtain more distinguishable representation; and 2) AP selection, whereby a subset of APs that better represents the characteristics of the environment is selected. The focus of this paper is on AP selection.

### B. AP Selection Methods

AP selection can be performed in both offline and online phases. If the AP selection is performed in the offline phase, a subset of APs is selected using only the radio map regardless of the online measurements. However, if the characteristics of the environment are different from the online localization phase, this selection mechanism fails to choose a suitable subset of APs. Hence, the RSS readings from specific APs are selected in the online phase utilizing the online measurements explicitly or implicitly. In explicit utilization, a subset of the APs are selected considering only the online measurement. In implicit utilization, the selection of APs is performed exploiting both online measurements and radio map. One method is to select a subset of APs from the online measurements and apply the offline AP selection techniques on this subset. Another method is to apply the offline AP selection methods on the radio map using only the RPs that have been selected at the coarse localization stage. This way, the fingerprints of the RPs that are the most similar to the online measurements assist in AP selection.

To formulate the process, define an AP selection matrix \( \Phi \) which selects a subset of APs \( \mathcal{L}' \subseteq \mathcal{L} \). Let \( \mathcal{L}' \subseteq \mathcal{L} \) be the cardinality of \( \mathcal{L}' \). The \( i \)-th row of \( \Phi \), i.e., \( \Phi^i \), is a \( 1 \times L \) vector that defines the selected AP through zeroing out all indices except the selected AP index as

\[
\Phi^i = [\ldots, 1, \ldots, 0, \ldots], \quad i = 1, \ldots, \mathcal{L}'.
\]  

Hence, the modified localization problem of (5) is

\[
\hat{p} = f(\Phi \mathbf{R}, \Phi \mathbf{y}).
\]

and the localization methods of Section II are performed on \( y = \Phi \mathbf{y} \) instead of \( y \).

Although a plethora of methods have been already introduced in [113], the following provides a summary of recently introduced AP selection methods:

1) **Strongest APs (MaxMean):** The early studies advocate to select APs based on their signal strengths in the online phase and select the same set of APs from the radio map fingerprints [101]. The intuition is that the strongest APs provide most coverage time and render more accurate measurements. Different set of APs are selected if the user travels into different locations. The strongest AP selection scheme, however, may not always render a suitable criterion [112].

2) **Fisher Criterion:** The Fisher criterion is a metric that quantifies the discrimination ability of each AP across RPs and takes into account the stability of AP fingerprints. This metric uses the statistical properties of the radio map fingerprints and selects APs based on their performance during the offline fingerprinting period. A score is assigned to each AP separately as

\[
\zeta_i = \frac{\sum_{j=1}^{N} (\bar{\psi}_j^i - \bar{\bar{\psi}}^i)^2}{\frac{1}{M-1} \sum_{m=1}^{M} \sum_{j=1}^{N} (\bar{\psi}_j^i - \bar{\psi}_j^m)^2}, \quad i = 1, \ldots, \mathcal{L}.
\]

where

\[
\bar{\psi}_j^i = \frac{1}{N} \sum_{j=1}^{N} \psi_j^i.
\]

This criterion is based on the fact that APs with higher variance should receive smaller scores as they are less reliable. This score is sorted decreasingly for all APs and a number of APs with the highest scores are selected [84]. [98].
However, the Fisher discriminant analysis for AP selection considers the offline fingerprints only. If the APs are not available in the online phase or provide faulty online measurements, then this criterion is not a suitable one. This issue is discussed in Section [13].

3) Bhattacharyya distance: This AP selection method is better suited to methods that utilize statistical properties of the radio map as it measures the distance between the probability densities of the fingerprints from two APs $i, i'$ at RP $j$: 

$$d_B(r_j^i, r_j^{i'}) = -\ln \left( \int \sqrt{f_j^i(r_j^i)f_j^{i'}(r_j^{i'})} dr_j^i dr_j^{i'} \right)$$  

(37)

where $f_j^i(r_j^i)$ and $f_j^{i'}(r_j^{i'})$ are the fingerprint distributions at APs $i$ and $i'$. Although the fingerprints are not Gaussian distributed, computing (37) under the Gaussian assumption provides an acceptable distance measure [84]. This measure gives a score to each pair of APs. Unlike the previously discussed methods, this method selects a subset $L'$ of APs by choosing for each RP $j$ the pairs of APs with the highest scores. To find these APs, it needs an exhaustive search over $(\binom{L}{2})$ pairs to find the ones with the smallest distance according to (37).

4) Information Potential (IP): This AP selection method also measures the distance between the RSS fingerprints [84] 

$$d_I(r_j^i, r_j^{i'}) = -\ln \left( \frac{1}{M} \sum_{m=1}^{M} \sum_{m'=1}^{M} k(r_j^i(t_m), r_j^{i'}(t_{m'})) \right)$$  

(38)

where $k(\cdot, \cdot)$ is a kernel function of the distance between each single fingerprint at APs $i, i'$ and RP $j$. The selection of APs is similar to the one under the Bhattacharyya distance. This criterion also selects pairs of APs and hence, suffers from the exhaustive search set.

5) Information Gain (InfoGain): This offline criterion selects the APs with the highest discriminative power. The discriminative power of AP $i$ is measured through the mutual information between two random variables [112] as follows 

$$\text{InfoGain}(r_j^i) = H(p) - H(p|r_j^i)$$  

(39)

where $H(p) = -\sum_{j=1}^{N} f_j^i(p_j) \log f_j^i(p_j)$, $H(p|r_j^i) = -\sum_{j=1}^{N} f_j^i(p_j) \sum_{v} f_{j}^i(r_j^i = v|p_j) \log f_{j}^i(r_j^i = v|p_j)$, and $v$ is one possible value of signal strength for AP $i$. The distributions $f_{j}^i(r_j^i = v|p_j)$ and $f_{j}^i(r_j^i = v)$ are estimated analytically, through histograms, or using kernels. A modification that ranks APs jointly by InfoGain and mutual correlation has also been introduced [117].

6) Entropy Maximization: The entropy maximization for AP $i$ discretizes the RSS range into $u \in \{1, \ldots, U\}$ levels. The probability of occurrence for level $u$ is 

$$f^i(u) = \frac{N^i_u}{N^i}$$  

(40)

where $N^i_u$ is the number of RPs whose RSS is in level $u$ and $N^i$ is the number of RPs that detect AP $i$. The entropy of AP $i$ is given by 

$$H^i = -\sum_{u=1}^{U} f^i(u) \log_2 f^i(u).$$  

(41)

A subset of the APs with the maximum entropy are selected for localization.

7) Group Discrimination (GD): The idea is that a group of APs that provides the maximum discrimination is selected, rather than choosing the APs independently. This method finds the best subset $L' \subset L$ of APs that produce the least score for the subset. The score is defined as follows [89]: 

$$\xi_{j,j'} = \sum_{m=1}^{M} \sum_{m'=1}^{M} \alpha_j \alpha_{j'} k(r_j^i(t_m), r_{j'}^i(t_{m'})), \quad i \in L'$$  

(42)

Score $L'$: 

$$\sum_{j,j' \in L'} \xi_{j,j'}$$  

(43)

A subset of $L'$ APs with the least scores is selected.

V. ADVANCED DENSITY AND WEIGHT ESTIMATION METHODS

The fine localization accuracy highly depends on the distance between online measurements and RSS radio map fingerprints. An incorrect metric may not lead to a representative difference and can cause a biased estimation towards specific RPs. To better exploit the features in the offline fingerprints and provide a more accurate comparison with the online measurements, more advanced techniques have been proposed for fingerprint density estimation—as a modification to [10], from which the weight for each RP is computed—or for directly computing weight for each RP—as a modification to [12]. In this section, we elaborate on these methods.

A. Kernel Density Estimation (KDE) Method

As discussed previously, one of the approaches is the non-parametric estimation of the RSS prior distribution. Since the usual analytical assumptions on the prior probability, such as Gaussianity, do not necessarily hold, the parametric estimation cannot exactly capture the empirical characteristics of the fingerprints [158]. An alternative approach is to estimate the empirical fingerprint distributions non-parametrically.

An approach to estimate the empirical distributions is to use kernel density estimation (KDE) as follows [84], [159]: 

$$\hat{f}(y|p_j) = \frac{\sigma^{-L}}{M} \sum_{i=1}^{M} k \left( \frac{y - r_j^i(t_m)}{\sigma} \right)$$  

(44)

where $k(\cdot)$ is the kernel function, $\sigma$ is the kernel width estimated through either training sequence or analytical solutions provided for Gaussian kernels [119], and $L'$ is the
number of the APs used for localization. The KDE is based on a superposition of kernel functions centered around the fingerprints. The kernel functions can also be used for weight computations. Consider first that the weights are obtained through an average normalized inner product between the fingerprints and online measurements

\[ w_j = \frac{1}{M} \sum_{i=1}^{M} \frac{\langle y, \varphi_i(r_j(t_m)) \rangle}{\|y\| \|r_j(t_m)\|} \]  

(45)

where \( \langle \cdot \rangle \) denotes the inner product. This metric basically measures the angles between the online measurements and radio map fingerprints. As discussed earlier at the beginning of this section, if the AP readings are correlated, this angle is small, and hence, not a representative metric.

For the sake of better differentiability between APs, an alternative approach is to map the data to another space where the difference between these angles becomes larger and more distinguishable. Let \( \varphi \) be a nonlinear mapping such that \( \varphi : \mathbb{x} \mapsto \varphi(\mathbb{x}) \). The transformed weights of (45) take the form

\[ w_j = \frac{1}{M} \sum_{i=1}^{M} \frac{\langle \varphi(y), \varphi_i(r_j(t_m)) \rangle}{\|\varphi(y)\| \|\varphi_i(r_j(t_m))\|} \]

\[ = \frac{1}{M} \sum_{i=1}^{M} \frac{k(y, \varphi_i(r_j(t_m)))}{\sqrt{k(y, y)k(\varphi_i(r_j(t_m)), \varphi_i(r_j(t_m)))}} \]  

(46)

One should note that in (46) the kernel function computes the inner product between the mapped online measurement and fingerprints and thus, the specific definition of the nonlinear mapping is circumvented.

**B. Principal Component Analysis (PCA) Method**

An alternative approach for computing \( \hat{f}(\mathbb{y}|p_j) \) is to map the online measurements to the domain of its principal components (PCs) [120]. First, the sample covariance of the fingerprints at RF \( j \) is computed as

\[ C_{\mathbb{y}|p_j}(i, i') = \frac{1}{M} \sum_{m=1}^{M} (r_j(i_m) - \psi_j(i))(r_j(i_m) - \psi_j(i'))^{T} \]  

\[ i, i' = 1, \ldots, L, \ j = 1, \ldots, N \]  

(47)

and the global covariance matrix has entries

\[ C_{\mathbb{y}}(i, i') = \frac{1}{MN} \sum_{j=1}^{N} \sum_{m=1}^{M} (r_j(i_m) - \psi_j(i))(r_j(i_m) - \psi_j(i'))^{T} \]  

\[ i, i' = 1, \ldots, L. \]  

(48)

The eigenvectors of the global covariance matrix are defined as follows:

\[ \mathbf{C}_{\mathbb{y}} \cdot \mathbf{v}_\ell = \lambda_\ell \cdot \mathbf{v}_\ell, \ \ell = 1, \ldots, L. \]  

(49)

A transformation of the data to its PCs is achieved through concatenating the eigenvectors corresponding to the eigenvalues sorted decreasingly as

\[ \mathbf{A} = [\mathbf{v}_1, \ldots, \mathbf{v}_L], \ \lambda_1 \geq \lambda_2 \geq \ldots, \lambda_L. \]  

(50)

The fingerprints and the online measurements should be transformed to the PC domain. To this end, the online measurement, the radio map, and the covariance matrix are mapped to the PC domain through multiplication with the transformation matrix \( \mathbf{A} \) as

\[ \mathbf{q} = \mathbf{A} \mathbf{y}, \ \mu_{q_i|p_j} = \mathbf{A} \psi_j, \ C_{q_i|p_j} = \mathbf{A} C_{\mathbb{y}|p_j} \mathbf{A}^T, \ j = 1, \ldots, N. \]  

(51)

The posterior probability of (10) is computed using only the first \( L' \leq L \) PCs as

\[ \hat{f}(\mathbb{q}|p_j) = \prod_{i=1}^{L'} \frac{1}{\sqrt{2\pi C_{q_i|p_j}(i, i')}} \exp \left( -\frac{1}{2} \frac{(q_i - \mu_{q_i|p_j})^2}{C_{q_i|p_j}(i, i')} \right) \]  

where \( \mathbb{q}' \) is the first \( L' \) entries of \( \mathbb{q} \) as \( \mathbb{q} = [q^1, \ldots, q^{L'}]^T \).

**C. KL-Divergence Method**

The Kullback-Leibler (KL) divergence is fundamentally a distance between two probability density functions—namely of the online measurements \( f^i(y) \) and RSS fingerprints \( f_j(r_j) \)—written as a kernel function [154], [160]. To obtain the probability density function of the online measurement, the user should remain at his/her location to get multiple online measurements. The symmetrized KL divergence between two probability functions is computed as

\[ D(f^i(y)^r, f_j(r_j)^r) = KL(f^i(y)^r||f_j(r_j)^r) + KL(f_j(r_j)^r||f^i(y)^r) \]  

(53)

where \( KL(\cdot, \cdot) \) is the KL divergence

\[ KL(\mathbb{X}|Y) = \sum_v f(\mathbb{X} = v) \log \left( \frac{f(\mathbb{X} = v)}{f(\mathbb{Y} = v)} \right). \]  

(54)

The KL divergence is combined with a kernel function in order to yield the weights for location estimation:

\[ w_j = \exp \left( -\sigma \sum_{i=1}^{L} D(f^i(y)^r, f_j(r_j)^r) \right). \]  

(55)

**D. Geometry-based Localization**

Tilejunction [161], Sectjunction [162], and Contour-based trilateration [163] are recent methods that exploit the geometry of the area to come up with weights which are found by solving a convex optimization problem with environmental constraints such as presence of walls. Define

\[ \Gamma_j^i = (y^i - \psi_j^i)^2 + (\sigma^i)^2 + (\Delta_j^i)^2 \]  

(56)

where \( \sigma^i \) is the variance of \( \psi_1^i, \ldots, \psi_N^i \) and \( \Delta_j^i \) was defined in (21). The difference between the offline fingerprints and the online measurements are computed through the expected signal difference as

\[ \Gamma_j = \frac{1}{N} \sum_{i=1}^{L} \Gamma_j^i \]  

(57)
The user’s location is estimated as \( \hat{p} = \sum_{j=1}^{N} w_j p_j \), where the weights are computed from the following linear program:

\[
\begin{align*}
\text{argmin}_{\{w_j\}_{j=1}^{N}} & \quad \sum_{j=1}^{N} w_j \Gamma_j \\
\text{s.t.} & \quad \text{Environment constraints (58)} \\
& \quad \sum_{j=1}^{N} w_j = 1, \ w_j \geq 0.
\end{align*}
\]

VI. SPARSITY-BASED LOCALIZATION

As the computational complexity of the probabilistic approaches is high and the localization accuracy of the deterministic approaches is low, a new reformulation of the WLAN localization problem has been proposed. This section elaborates on the sparse reformulation of the WLAN localization problem and introduces the methods that solve the sparsity-based localization problems.

A. Measurement Model Enabling Sparse Recovery

The localization problem can be interpreted as finding only one location among all RPs, which is the closest to the user position. The localization can be transformed into a sparse recovery problem with only one selection out of many options [16], [121]. Let the location vector be recast as a sparse vector as

\[
\theta = [0, \ldots, 0, 1, 0, \ldots, 0]^T
\]

where all entries of \( \theta \) correspond to radio-map RPs and 1 corresponding to the index of the RP to which the user is the closest. The equivalent measurement model that enables sparse recovery is

\[
y = \Phi \Psi \theta + \epsilon
\]

(60)

where \( \Phi \) is the AP selection matrix, i.e., the matrix that selects certain elements of \( \Psi \) corresponding to selected APs (Section IV-B). \( \Psi \) is the modified radio map matrix, \( \epsilon \) is the error vector, and \( y \) is the online captured RSS vector from specific APs as

\[
y = \Phi y
\]

(61)

Since the dimension of \( y \) is less than that of \( \theta \), (61) is an under-determined problem. Next, we discuss the sparsity-based localization methods that solve this problem.

B. CS-based Localization

Although under-determined problems may have infinite solutions, the location vector \( \theta \) in (60) is sparse as the user can only be in one of the RP locations. This type of problems can be addressed through Compressive Sensing (CS), and may have unique solutions if certain conditions are satisfied. The CS problem can be solved via the convex optimization

\[
\hat{\theta} = \text{argmin}_{\theta} ||\theta||_1
\]

s.t. \( y = \Phi \Psi \theta \)

(62)

where \( ||\theta||_1 \) is the \( \ell_1 \)-norm of \( \theta \). Using the \( \ell_1 \)-norm, the CS renders a sparse vector. Under certain conditions listed shortly, problem (62) has a unique solution. Several algorithms have been proposed to solve this problem, e.g. greedy algorithms [164], iteratively re-weighted linear least-squares (IRLS) [163], and basis pursuit [166].

The CS formulation faces several challenges. We enumerate these challenges next and provide improvements in the ensuing subsections.

1) In order to obtain a unique sparse solution in CS formulation, the sensing matrix \( \Phi \) and the basis matrix \( \Psi \) should obey two criteria [167]:

- **Restricted Isometry Property (RIP):** This property states that the multiplication of the sensing and the basis matrix, i.e., \( \Phi \Psi \), should approximately preserve the Euclidean norm of the positioning vector \( \theta \). Mathematically, this condition is expressed as

\[
(1 - \delta_s) ||\theta||_2^2 \leq ||\Phi \Psi \theta||_2^2 \leq (1 + \delta_s) ||\theta||_2^2.
\]

(63)

where \( \delta_s \) is a small positive number. The above condition for example would be satisfied if the matrix \( \Phi \Psi \) were orthonormal.

- **Mutual Incoherence:** This requires that the rows of \( \Phi \) cannot sparsely represent the columns of \( \Psi \) and vice versa. Smaller coherence leads to a better chance to reconstruct a unique and optimum sparse solution [168].

To induce the above conditions, an orthonormalization procedure on the radio map is applied [166]. Nonetheless, this procedure does not make \( \Phi \Psi \) completely orthonormal, as it is not square.

2) The computational complexity of the optimization algorithm increases with the size of area and hence makes the positioning impractical in small hand-held devices. Hence, a preprocessing step is required to reduce the searching area and this is done through the radio map clustering algorithms (Section III).

3) The CS optimization formulation assumes that the model (60) does not contain the measurement error \( \epsilon \) and attempts to find the RPs whose fingerprints match the online measurements exactly.

C. LASSO-based Localization

The shortcomings of the CS localization are overcome by recent sparse recovery methods which do not need the orthogonalization step, and not rely on special properties of the matrix \( \Phi \Psi \), which may not be valid in practice. In addition to recovering a sparse vector, the proposed localization methods use (60) as the model, and thus, work better with noisy measurements.

The localization accuracy can be improved if the sparse recovery problem also suppresses the error between the online measurement vector and radio map fingerprints. The LASSO localization minimizes the \( \ell_1 \)-norm of the location vector and the \( \ell_2 \)-norm of the residuals [109]. The convex optimization problem for localization is reformulated as

\[
\hat{\theta} = \text{argmin}_{\theta} \left[ \frac{1}{L} ||y - H\theta||_2^2 + \lambda ||\theta||_1 \right]
\]

(64)
where $\mathbf{H} = \Phi \Psi$ and $\lambda \geq 0$ is a tuning parameter. This problem is also known as $\ell_1$-penalized least squares, which incorporates feature and model selection into the optimization process [169]. The first component seeks coefficients that minimize the residuals, and the second one promotes a sparse $\theta$. LASSO has been shown to be more indifferent to correlated RSS fingerprints. The parameter $\lambda$ is a tuning parameter that regularizes between minimizing the residuals and the sparse vector solution. This parameter can be tuned experimentally or using cross validation (CV) [52].

D. GLMNET-based Localization

Suppose there are correlated predictors in the modified radio map. If the user is exactly at an RP, the online measurement is supposed to be very similar to the fingerprints of that RP. Another possible case is when the user is between two RPs with similar environmental features. The location estimation problem in both cases is expected to assign higher coefficients to the points with correlated fingerprints. Hence, the correlated predictors should be allowed to jointly borrow strength from each other. GLMNET-based localization incorporates the above features as follows [109]:

$$
\hat{\theta} = \arg\min_{\theta} \left[ \frac{1}{L} \|y - \mathbf{H}\theta\|_2^2 + P_\alpha \right]
$$

$$
P_{\alpha} = \lambda (1 - \alpha) \|\theta\|_2^2 + \alpha \|\theta\|_1
$$

where $\lambda \geq 0$ is a tuning parameter and $0 \leq \alpha \leq 1$ is a compromise between ridge regression and LASSO. Ridge regression promotes the shrinkage of the coefficients of correlated radio map columns towards each other and is expressed by the $\|\theta\|_2^2$ objective. Hence, we take advantage of the correlation between the radio map readings. If $\alpha = 0$, the above formulation amounts to the ridge regression. As $\alpha$ increases from 0 to 1 for a given $\lambda$ the sparsity of the solution increases monotonically from 0 to the sparsity of the LASSO solution. This formulation therefore jointly considers the correlated predictors and finds a sparse solution for the user’s pose. This estimator is known as GLMNET [170].

The computational complexity of the above optimization problem grows with the number of predictors (size of radio map). Therefore, the previously mentioned coarse localization schemes in Section III reduce the size of the area that the optimization problems seek for the solution and hence reduce the computation time. This allows these procedures to be executed on resource-limited devices.

E. Group Sparsity (GS)-based Localization

Since there is no guarantee that the cluster within which the solution is searched is the correct cluster, Group Sparsity (GS)-based localization is proposed which utilizes all the clusters, each with a different weighting in the following optimization [110]

$$
\hat{\theta} = \arg\min_{\theta} \left[ \frac{1}{L} \|y - \mathbf{H}\theta\|_2^2 + \lambda_1 \|\theta\|_1 + \lambda_2 \sum_{k=1}^{K} w_k \|\theta_k\|_2 \right]
$$

where $\theta_k$ is a segment of position vector corresponding to group $k$, $w_k$ is the weight assigned to group $k$, $K$ is the total number of groups, and $\lambda_1, \lambda_2 \geq 0$ are tuning parameters. The weights $w_k$ can be obtained from any of the coarse localization methods discussed in Section III. The first component minimizes the impact of online measurement noise considering that the RSS fingerprint noises have already been minimized through time-averaging of the fingerprints. The second component promotes sparsity in the position vector $\theta$. The last term provides the sparsity among the groups (clusters) so that the recovered vector’s nonzero elements are concentrated within a single group. This term basically plays the role of coarse localization. This minimization is known as Sparse Group Lasso (SGL) [171], [172].

VII. ASSISTED LOCALIZATION

In this Section, different techniques that employ additional information from environmental and common wireless device sensors to assist the Wi-Fi fingerprinting localization are detailed.

A. Sensor Fusion Assistance for Localization

Although Wi-Fi signals provide redundant wireless RSS data for fingerprinting, these networks are not originally designed for localization and the measurements captured by wireless devices can be distorted due to various phenomena. wireless devices are untenable to reduce the unobtrusiveness of cues or increase comprehension. Most of wireless devices such as smartphones encompass other sensors that can provide additional assistance to RSS-based fingerprinting. Fig. shows some of the additional sensor data which can be extracted from smart mobile devices and fused with RSS-based localization. Several examples are listed in the following.

1) Sound or Ambient color/light: Nearly all of the wireless devices contain speakers and most of them accommodate microphones. The ambient sound renders coarse location specific features if a dataset of sound fingerprints for different places are available. For instance, the ambient sound of a restaurant is different from that of shopping malls. Hence, the ambient sound can help in defining the area that the user is and so, prevent from large localization errors [107].

For a case example, consider shopping malls where ambient light and color conditions are brand-specific. So, these thematic colors along with the lighting styles may provide location-specific signatures which can be fused with the Wi-Fi RSS fingerprints [107]. However, the lights and lightening styles are subject to frequent changes.

While useful, such light/color based fingerprints may often change, and it was shown that floor imagery provides more reliable measurements.

2) RSSI from cellular base stations [122]: Although the use of Wi-Fi fingerprints helps to achieve finer localization accuracy due to dense deployments, RSSI measurements from other networks, such as cellular, can serve as additional fingerprints, especially in areas with low density Wi-Fi deployments, in weak signal conditions and when Wi-Fi fingerprinting cannot resolve location ambiguities.
3) RF-ID [18]: Different from Wi-Fi signals, the RFID tags should be installed and thus require infrastructure upgrades. However, they provide independent location estimation with the RSS fingerprints. The question then arises on how optimally integrate the location estimations from two different sources. Let \( \hat{p} = \{ \hat{p}_1, \ldots, \hat{p}_n \} \) be the estimated locations of the user from \( n \) different localization procedures. The final user’s location, \( \hat{p} = \sum_{j=1}^{n} \beta_j \hat{p}_j \), can be estimated through a weighted combination of the individually estimated locations where the weights should be assigned so that the variance of the final estimated location is minimized. This variance is

\[
\text{Var}(\hat{p}) = \text{Var}\left( \sum_{j=1}^{n} \beta_j \hat{p}_j \right) = \beta^T \text{diag}(\sigma_1^2, \ldots, \sigma_n^2) \beta
\]

(67)

where \( \beta = (\beta_1, \ldots, \beta_n) \) aggregates weights corresponding to location estimation procedures, and \( \text{diag}(\sigma_1^2, \ldots, \sigma_n^2) \) is an \( n \times n \) with diagonal matrix. The optimal \( \beta \) can be obtained through the following minimization problem:

\[
\beta = \arg\min_{\beta} \beta^T \text{diag}(\sigma_j^2) \beta \\
\text{s.t. } \|\beta\|_1 = 1, \quad \beta \succeq 0
\]

(68)

and the closed form optimal solution is

\[
\beta^*_j = \left( \sigma_j^2 \sum_{j=1}^{n} \frac{1}{\sigma_j^2} \right)^{-1}
\]

(69)

B. Motion Assisted Localization

Through the widespread deployment of Micro Electro-Mechanical System (MEMS) sensors in the smart wireless devices, the Pedestrian Dead Reckoning (PDR) is proliferating as a feasible option for indoor tracking. The set of the sensors that are being used for indoor tracking are called the Inertial Measurement Units (IMUs). These sensors underpin the localization through providing additional details regarding the user’s motion such as counting user’s steps, inertial navigations, and heading directions [123]. The Dead-reckoning systems use these sensors and estimate the change of the user’s motion such as counting user’s steps, inertial navigations, and heading directions [123]. The Dead-reckoning systems use these sensors and estimate the change of the user’s motion such as counting user’s steps, inertial navigations, and heading directions [123]. The Dead-reckoning systems use these sensors and estimate the change of the user’s motion such as counting user’s steps, inertial navigations, and heading directions [123]. The Dead-reckoning systems use these sensors and estimate the change of the user’s motion such as counting user’s steps, inertial navigations, and heading directions [123]. The Dead-reckoning systems use these sensors and estimate the change of the user’s motion such as counting user’s steps, inertial navigations, and heading directions [123]. The Dead-reckoning systems use these sensors and estimate the change of the user’s motion such as counting user’s steps, inertial navigations, and heading directions [123]. The Dead-reckoning systems use these sensors and estimate the change of the user’s motion such as counting user’s steps, inertial navigations, and heading directions [123]. The Dead-reckoning systems use these sensors and estimate the change of the user’s motion such as counting user’s steps, inertial navigations, and heading directions [123]. The Dead-reckoning systems use these sensors and estimate the change of the user’s motion such as counting user’s steps, inertial navigations, and heading directions [123].

1) Available Motion Sensors: The sensors that are available in smart devices which can support the localization are:

- **Barometer:** Measures the atmospheric pressure. The readings of the atmospheric pressure may indicate a special location.
- **Accelerometer:** Shows the 3-D acceleration of the user while carrying the device. When the user lifts her foot the acceleration increases and when the foot is planted the acceleration decreases, all leading to a cyclic peak-valley motion pattern.
- **Gyroscope:** Measures the angular velocity of the device and show the orientation of the user.
- **Magnetometer:** Provides the strength and direction of the earth magnetic field in the environment through which we can know the heading direction of the user.

2) Sensor Exploitation: The sensors in smart mobile devices are used to collect various user’s motion patterns [123]. Through the detection of motion patterns, the following information can be obtained:

- **Walking direction [124]:** It is needed to compute location in the first place which leverages application-specific opportunities such as crowd-sourcing of the Wi-Fi data and knowing the user’s facing direction.
- **Walking detection [123]:** Although the motion sensors can be exploited to deliver user’s motion, utilizing these sensors for long time consumes a great portion of battery. A smarter localization procedure is to turn on the signals only when the user moves.
- **Step counting [125]:** The most common user’s location detection though the motion sensors is to estimate the user’s location through the distance that the user has passed from a starting point. Accelerometers provide the 3-D acceleration of the wireless device, and although the obtained data depends on the position and orientation of device with respect to the user, it provides useful information on the user’s step length. The accelerometers are triggered based on the lifting and planting of the user’s foot. The passed distance of the user is detected from counting the strides along with the stride length. Several methods have been proposed to detect the number of passed strides such as peak detection, zero-crossing, cycle detection, correlation analysis, and fast Fourier transform.

These techniques help to estimate the vicinity of the user’s location through techniques such as Kalman Filters, Information Potential (IP) [172]–[175], Conditional Random Fields (CRF) [134], [176], nonlinear filters [177], etc..

C. Land-mark Assisted Localization

The landmark assisted localization helps to harness certain locations in indoor environment which represent identifiable signatures of their surrounding area. These landmarks trace to two types of assistance: 1) calibrating the dead-reckoning schemes, thereby curbing the error growth; 2) assistance in coarse localization, which warrants more attainable precision.

Landmarks provide specific features to the user depending on the sensors that the user is using for localization. Basically, there are two different landmarks in a typical environment:

- **Seed landmarks (SLMs):** These are the physical landmarks which can be associated with their actual locations, such as elevators, escalators, and stairs. For instance, using a camera, the user can match the images of the environment with a database of the available environment images.
- **Organic landmarks (OLMs):** These are the landmarks that are associated with detecting sensory signatures that are area-specific and confined to a small area. For instance, an elevator affects the z-dimensional pattern of the phone’s accelerometer.
As an example, UnLoc [126] looks for certain structure in the buildings- stairs, elevators, escalators, entrances- that force the user to have predictable motion patterns. For instance, the method checks the confidence level of the GPS as an indicator of user entrance from outdoors to indoors. SemanticSLAM [127] also checks the gyroscope angular readings readings to recognize the turns at the end of corridors, classrooms, etc.

D. Collaborative Localization

The idea of collaborative localization is to exploitation the sensors in mobile phones to find the distance between wireless devices, through which the relative locations between neighboring devices are obtained. These relative locations serve as additional constraints in location estimation and improves the localization accuracy [13], [32], [128]. The range (distance) between wireless devices can be obtained through the following sensors:

- **Acoustic ranging:** If the wireless devices contain the speaker-microphone, the distance between the devices can be obtained through transmitting acoustic signals and use the TOA to compute the ranges (distances) between the wireless devices [178]. The ranges help in reducing the search space of the user’s location and solve a system of equations to find relative users locations in the collaborative constellation.

- **Bluetooth:** The efficacy of the Bluetooth for proximity estimation has been shown in [179] for collaborative localization and offers accuracies up to 1.5 m. The fingerprints from the Bluetooth of wireless devices are collected in a data base which train the coefficients of a RSSI-distance model.

VIII. Radio Map Construction

This section starts part III of the paper, whose structure is provided in Fig. [10] along with the related works in Table [IV].

A major problem in WLAN positioning systems is the surveying scale in terms of collecting RSS data at large number of RPs for high accuracy positioning. With large scale deployments, the upfront cost of the deployment effort becomes tremendous. Furthermore, the radio-map changes over the time and should be periodically calibrated. The size of this dataset is increasing with the size of the area, granularity of the RPs, the number of APs, and the recording length. As this process is labor intensive, some works have focused on reducing the efforts of data collection such as model-based map generation [129], Simultaneous Localization And Mapping (SLAM) techniques [130], and dynamic radio map construction [131].

Crowdsourcing approaches introduce the participatory role of the user during localization [55], [132], [133], [142]. A dedicated surveyor does not collect the fingerprints, but the users help to update the radio map if they volunteer. The tedious task of fingerprinting is split between involved users. However, the accuracy of the data decreases as the fingerprinting time is short and the location of the fingerprints cannot be guaranteed.

Another simplified data collection tasks resides on implicit data collection, in which the users help with collecting the data through their daily life routines. For instance, mobile devices can be configured to implicitly collect surveying data without direct involvement of the users. If part of the data is labeled with its corresponding locations, users can also collect some data without any location association (label), called unlabeled data collection. Then the unlabeled data can be associated with locations through some algorithms such as Hybrid Generative/Discriminative Learning [134].

AP power profiling has been addressed in [135]. In this approach, the fingerprints (location, RSS) are considered as Gaussian Processes (GP) and a model is used to define the relation between the locations and the fingerprints. The coefficient matrix of the regression model is estimated using different learning methods such as linear regression, nonlinear GP, Gaussian Kernel Learning, and augmented path-loss model. Once the coefficient matrix has been estimated using a training set, the RSS values of an unknown location is estimated using a zero-mean GP regression [136], [137].

Linear interpolation has also been used for interpolation of RSS measurements between RPs [138], [141]. With the assumption that three non-colinear RPs \( j_1, j_2, j_3 \) have been chosen, RSS values for an RP that is inside the convex hull of these RPs is computed as

\[
\begin{align*}
    r^j_i(t) &= \lambda_1 r^j_{i1}(t) + \lambda_2 r^j_{i2}(t) + \lambda_3 r^j_{i3}(t) \\
    \text{where } \lambda_1 + \lambda_2 + \lambda_3 &= 1
\end{align*}
\]

Other interpolation methods that use the minimum and mean of the RSS values of the three non-colinear RPs \( j_1, j_2, j_3 \) have also been introduced [141]. The RSS fingerprint of the nearest RP may also be used as the RSS of the virtual RP [142]. A more sophisticated method is to use a weighted average of the close RPs [142].

Sparse recovery methods can also be used in the offline phase to reconstruct the radio map from a lower number of RSS fingerprints. Let \( F \) be the \( N \times N \) Fourier transform matrix that linearly transforms the vector of radio map fingerprints to its equivalent representation in the frequency domain as

\[
\psi_i^j = F \psi_i, \quad i = 1, \ldots, L.
\]

The vector \( \psi_i^j \) is sparse; that is, most of the frequency components are zero; see e.g., [89]. This observation helps to reconstruct the radio map in the subsequent discussions. Then, consider a matrix that defines the relation between all RPs and those over which fingerprints have been taken. To this end, we define an \( S \times N \) matrix \( A \) whose rows are 1-sparse vectors \( \alpha^i = [0, \ldots, 1, \ldots, 0] \) denoting the index of the RP that is measured during radio map fingerprinting. Let \( S < N \) be the total number of RPs where fingerprints are recorded. In essence, \( A \) selects the RPs in which actual fingerprints are recorded.

The model for the offline radio map interpolation corresponding to AP \( i \) can be represented as

\[
b^i = A \psi^i = A F^{-1} \psi_i^j \quad \forall i = 1, \ldots, L.
\]

The model in (72) is an under-determined system of equations because \( S < N \). However, since \( \psi_i^j \) is sparse, a unique
solution exists for it. Two methods have been proposed to find the unique solution for (72). The CS theory has been used for the interpolation [116] as

\[ \hat{\psi}_j = \arg\min_{\psi_j} \| \psi_j \|_1 \]

\[ \text{s.t. } b^j = AF^{-1}\psi_j. \]  

(73)

The LASSO has also been used for radio map interpolation [109] as

\[ \hat{\psi}_j = \arg\min_{\psi_j} \left[ \frac{1}{2} \| b^j - AF^{-1}\psi_j \|_2^2 + \lambda_1 \| \psi_j \|_1 \right] \]

which has the form of the group sparse recovery (4) with \( \lambda_2 = 0 \). The above formulation minimizes the error between the measured RSS fingerprints and the interpolated fingerprints, while the second term promotes sparsity of the RSS fingerprints in the Fourier domain.

The previous optimizations (73) or (74) are solved for all APs. The reconstructed radio map rows are computed as

\[ \hat{\psi}^j = F^{-1}\hat{\psi}_j. \]  

(75)

Using (75), RSS fingerprints can be measured on a smaller number of RPs, and the radio map is interpolated in between RPs at a finer granularity.

IX. OUTLIER DETECTION

In this section we first discuss the possible causes of outliers and then an overview of outlier detection and mitigation methods is provided. APs may experience faults during their operations due to the following causes:

- Some APs become intermittently unavailable or provide erroneous RSS measurements due to unexpected failures, jamming, power outages, or intentional adversary attacks that may weaken or strengthen the AP signals.
- The indoor obstacles introduce a multipath profile to the traveling signals.
- There is no guarantee that the APs that have been visible during the fingerprinting time are visible during the online localization phase.
- Modern APs are able to adapt their transmit power based on the traffic.

Due to the previous reasons, the AP characteristics in the fingerprinting phase may not match those in the online phase. In such cases, online readings of APs are not trustable. These inordinate online measurements are called outliers.

An outlier occurs when the online measurement from an AP is significantly different than any fingerprint in the area. This hurdle has surprisingly received little attention in the literature. Note that existing AP selection schemes select the APs based on the AP performance during the fingerprinting period, and are therefore not well-suited to mitigate outliers which occur in the online phase.

Outliers may also occur during the fingerprinting period. However, some post-sanitary measures such as authentication of beacon nodes, radio map collection over various periods, validation, and attack detection help to remedy any impersonation and data corruption [180], [181].

Next, an overview of the schemes for the detection of outliers in the online measurements is provided. Some approaches focus on outlier detection and improve the localization performance of conventional methods [65], [143], [144]. A categorization of the recently proposed outlier detection schemes for WLAN localization is depicted in Fig. 10.

A. Hampel Filter

Hampel filter has been extensively used for outlier detection in statistical data [145]–[148] and has been introduced as an offline and online outlier detection procedure in [66]. It replaces the outlier-sensitive mean and standard deviation estimates with the outlier-resistant median and median absolute deviation from the median (MAD). The latter is defined as

\[ R_j^2 = 1.4826 \times \text{median}\left\{ \| r_j^y(t_m) - \text{median}(r_j^y) \| \right\}. \]  

The factor 1.4826 was chosen so that the expected value of \( R_j^2 \) is equal to the standard deviation for normally distributed data. The MAD-scale substitute of the data is

\[ \text{MAD}_j(t_m) = \frac{\| r_j^y(t_m) - \text{median}(r_j^y) \|}{R_j^2}. \]  

(77)

B. Modified Distance-based Outlier Detection

A modified KNN method has been proposed as an alternative fault tolerant localization method [149]. The Euclidean distance between the online measurements and fingerprints over a modified subset of APs is defined as

\[ d_{\text{Euc}}(\psi_j, y) = \sqrt{\sum_{i \in A' \cup A'_y} (y^i - \psi_j^i)^2 + \sum_{i \in A'_y \setminus A'} (y^i - \psi_j^i)^2}, \]

\[ j = 1, \ldots, N. \]  

(78)

where \( A' \) and \( A'_y \) are respectively the subsets of APs available during fingerprinting at \( p_j \), and in \( y \). The first summation component is on a subset of APs that are available in both fingerprinting and online phase while the second term sums over the APs that are available only in the online period and not in the fingerprinting period.

A more comprehensive model of outliers has been proposed [150], which considers different causes of outliers as

\[ y^j = b_1 y^j + b_2 (y^j + n(i)) + b_3 y^j_{\text{bog}} + b_4 c_{\text{NaN}} \]  

(79)

where \( b_k \in \{0, 1\}, k = 1, \ldots, 4 \), and \( \sum_{k=1}^4 b_k = 1 \), which means only one of the components is active at a time. The second term models the extra noise due to jammed APs, where \( n(i) \sim \mathcal{N}(0, \sigma^2) \), \( y^j_{\text{bog}} \) models the bogus APs that imitate an actual AP, and \( c_{\text{NaN}} \) models the unavailability.

The localization procedure contains a modified distance which switches between the Euclidean and median distances as

\[ d_{\text{mod}}(\tilde{r}_j, y) = \min\{d_{\text{Euc}}(\tilde{r}_j, y), d_{\text{med}}(\tilde{r}_j, y)\} \]  

(80)

where \( d_{\text{Euc}} \) and \( d_{\text{med}} \) are given by (7) and \( \tilde{r}_j \) is replaced by the average or median fingerprint, as explained in Section II-B.
C. Sparsity-based Outlier Detection

Localization in the presence of outliers via sparse recovery methods has also considered. The main idea is that outliers are modeled exactly by augmenting $\Phi$. Specifically, with $\kappa$ denoting the outlier vector, the online measurements adhere to the following model:

$$y = \Phi \Psi \theta + \kappa + \epsilon.$$  \hfill (81)

The advantage of the previous model is that the outliers vector $\kappa$ will be sparse as long as the number of corrupted APs is small, and can therefore be estimated jointly with the position indicator vector $\theta$ via $\ell_1$-minimization. The premise of explicitly modeling the outliers for robust regression in a general statistical setting has been previously analyzed in [151] and [152]. In what follows, the CS, LASSO, and GLMNET approaches are modified so that the outlier vector $\kappa$ can be estimated alongside the user position vector $\theta$.

The modified CS (M-CS) approach minimizes the weighted combination of the $\ell_1$ norms of $\theta$ and $\kappa$ [109]

$$\begin{align*}
(\hat{\theta}, \hat{\kappa}) &= \arg\min_{\theta, \kappa} ||\theta||_1 + \mu ||\kappa||_1 \\
\text{s.t. } y &= \Phi \Psi \theta + \kappa.
\end{align*}$$  \hfill (82)

where $\mu > 0$ is a tuning parameter.

The modified LASSO (M-LASSO) minimizes the squared residuals, in addition to the $\ell_1$ norms of the sparse vectors:

$$\begin{align*}
(\hat{\theta}, \hat{\kappa}) &= \arg\min_{\theta, \kappa} \frac{1}{|L|} ||y - H \theta - \kappa||_2^2 + \lambda ||\theta||_1 + \mu ||\kappa||_1
\end{align*}$$  \hfill (83)

Finally, The modified Group-Sparsity (MGS)-based regression is formulated as [110]

$$\begin{align*}
(\hat{\theta}, \hat{\kappa}) &= \arg\min_{\theta, \kappa} \frac{1}{|L|} ||y - H \theta - \kappa||_2^2 + P_\alpha
\end{align*}$$  \hfill (84)

where $P_\alpha = \lambda[||\theta||_1 + \alpha ||\kappa||_1]$.

In the previous joint localization and outlier detection formulations, the outlier vector, $\kappa$, enables the optimization algorithms to discard the outliers in the online measurement vector. The terms promoting sparsity of the user’s location vector and the outlier indicator vector have the weights $\lambda$ and $\mu$, respectively. Optimization problems (82)–(85) are convex problems which can be efficiently solved [182], [183].

X. HETEROGENEOUS DEVICES

One of the issues related to the deployment of fingerprinting approaches is that wireless devices do not read equal RSS measurements if they are located in the same position, primarily, due to heterogeneous reception characteristics of embedded NICs. Rapid growth of wireless devices from different manufacturers caused hardware variations amongst devices or even across models (same manufacturer), such as the receiving antenna gain, position of the antenna on the device, sensitivity, and Operating System (OS) characteristics.

Hardware variation can significantly degrade the positional accuracy of RSS-based WiFi localization systems. RSS data (fingerprints and online measurements) can be transformed using linear regression, expectation maximization, and neural networks [153]. The Pearson correlation coefficient has also been used to find the similarity between RSS fingerprints and online measurements [153]. Device-invariant fingerprints can be derived from RSS measurements by proper normalization such as using signal strength ratios between pairs of APs instead of absolute RSS values. The rank-ordering of APs can also serve as device invariant measure [154].

Some works have also used the Signal Strength Difference (SSD) instead of dealing with RSS fingerprints directly to compensate for different devices’ hardware readings of RSS signals [155], [156].

XI. EXPERIMENTAL EVALUATION AND COMPARISONS

In this section, we provide an illustration of the localization performance for some the approaches in the previous sections on a real indoor environment. The results render beneficial insights as all the localization approaches are compared within a single environment.

The results are based on data collected at the second floor of the Applied Engineering and Technology (AET) building at the University of Texas at San Antonio which has an area of 576ft $\times$ 35ft. The map of the surveying area is provided in Fig. 12. The area represents a typical office environment as it includes several research labs, offices, library, study area, and break rooms.

The localization approaches have been assessed through their localization accuracy. Let $N_t$ be the number of the test points (online measurements taken at different positions). The Mean Absolute Error (MAE) is a measure of the localization accuracy defined as [84], [89], [116], [121]

$$\text{MAE} = \frac{1}{N_t} \sum_{n=1}^{N_t} \sqrt{(\hat{p}(n) - p(n))^T (\hat{p}(n) - p(n))}.$$  \hfill (86)

where $p(n)$ and $\hat{p}(n)$ are the true and estimated positions, respectively. To define the spread of the localization errors, the cumulative distribution function (CDF) of the localization errors is also evaluated.

First, we assess the performance of the localization approaches without clustering and coarse localization. The performance of localization methods is then evaluated together with one of the coarse localization techniques of Section III.

The localization approaches that have been selected are as follows: KNN, KDE, CS, LASSO, GLMNET, GS, and Contour-based localization. Table V shows the formula based on which the user’s location is estimated.

A. Localization Error Without Coarse Localization

The methods in this subsection have been implemented without utilizing any coarse localization. However, for reduc-
Fig. 12. The map of experimental environment. The green dots indicate the RP locations.

| Methods    | Related Computing Formula |
|------------|---------------------------|
| KNN        | (8)                       |
| KDE        | (44) and (12)             |
| CS         | (62)                      |
| GLMNET     | (65)                      |
| LASSO      | (64)                      |
| GS         | (66)                      |
| Contour-based | (55)                  |

Fig. 13 illustrates the localization error versus an increasing number of APs. For the KNN method, $K = 10$ RPs have been selected. The kernel widths for KDE approach have been computed through the recommendations given in [84]. The probability density of the RSS fingerprints had to be estimated in the online phase because the APs engaging in the localization should be known for the KDE approach. The GS approach needs the corresponding weight for each cluster which is computed through the layered clustering method ($K = 10$). The results show high localization errors for all approaches although the errors decrease as the number of APs increases. However, the sparse recovery methods show higher accuracy, among which the GS-based localization shows the highest localization accuracy if less than 10 AP are used. The GS accuracy slightly improves if more APs are used. Overall, LASSO-based localization shows the least localization error if more APs are used.

The localization error distribution is shown in Fig. 14 when 10 APs have been used for localization. The contour-based approach introduces the largest errors because it needs an estimation of the path loss parameters. These parameters are assumed uniform for an AP along all directions which is not a suitable assumption in complex indoor environments. The KNN and KDE techniques do not show satisfactory performance either.

B. Localization Error With Coarse Localization

As shown in the previous subsection, the localization accuracy is low without coarse localization in large surveying areas. To enhance the performance, the user’s location is first estimated in the coarse localization stage, and the fine localization step is applied afterwards. To show that the localization accuracy is enhanced with coarse localization, the clustering using the AP coverage vector has been utilized for the KNN and KDE approaches as in [84], weighted clustering has been used for CS, LASSO, and GLMNET, and layered clustering has been used for GS.

Fig. 15 shows the average localization error for an increasing number of APs. Increasing the number of APs improves the KNN, KDE and GS approaches, however, the localization error decreases from 10 ft to 2 ft for LASSO and GLMNET if the number of engaged APs is increases from 4 and 29. However, it is evident that the localization error for CS, LASSO, and GLMNET has overall been decreased.
Fig. 14. The CDF of the localization error for 10 APs without clustering.

Fig. 15. Localization error comparison for different number of APs with clustering.

Fig. 16. The CDF of the localization error with 10 APs and clustering.

The distribution of the localization error is depicted in Fig. 16 when only 10 APs are utilized in localization. Comparing Figs. 16 and 14 reveals that the errors of CS, LASSO, and GLMNET are greatly decreased and the 80% of the errors are less than 20 ft. However, the KNN and KDE methods render unacceptably high localization errors.

XII. CRITICAL SUMMARY AND RECOMMENDATIONS FOR FUTURE WORK

A. Critical Summary

The WLAN indoor localization has attracted great attention due to the low cost deployment, existing infrastructure, and ease of implementation. The WLAN fingerprinting approach became very popular as proven performance was in real environments. As indoor propagation is a very complex phenomenon distorted by multipath and signal blockages, traditional techniques such as trilateration do not show good performance. The research has become very broad and extensively branched due to necessity to address various issues. This paper attempts to systematize various aspects of the state-of-the-art.

First, the paper categorizes conventional localization approaches at early stages. Then, the challenges that are associated with the fingerprinting approaches and conventional problems are enumerated. The state-of-the-art solutions to these challenges are categorized and the related works for each category has been overviewed. A key issue was to unify the misleading concepts and notations that varied among approaches and introduce them in a single trackable package. Recent approaches enhance the conventional methods, utilize the peculiarities of available environments and sensors, and leverage sparse recovery methods.

Since localization approaches in the literature have been evaluated in different settings, representative fingerprinting approaches are implemented in a typical office environment for illustration purposes. In parallel, the details of some of the fingerprinting approaches are listed in Table V. A qualitative comparison over these methods is also included in Table VI. Table V shows the RP clustering method, AP selection method, fine localization technique, reported accuracy and details about the implemented setting. The comparison over the reported accuracies is difficult as the methods have been implemented in different testbeds which differ in the size of the area, number of RPs, granularity of RPs, and number of training samples. It is also commonly understood that the RP clustering and AP selection schemes have great impact on improving the accuracy.

In addition, if one compares the accuracy of approaches with coarser granularity, such as Tilejunction [86], the accuracy seems to be degraded compared to approaches with finer granularity. However, all methods should be implemented in a comparable granularity in order to extract safe conclusions. Therefore, comparison of many diverse localization techniques
TABLE VI
COMPARISON OF REPRESENTATIVE FINGERPRINTING LOCALIZATION APPROACHES AND TECHNICAL DETAILS

| Scheme          | RP Selection Technique | AP Selection Technique | Fine Localization Technique | Reported Accuracy at 50 % | Testbed Information |
|-----------------|------------------------|------------------------|-----------------------------|---------------------------|---------------------|
| RADAR [14]      | –                      | Strongest AP           | KNN                         | ~ 8.16m                   | 70 RPs, N/A         |
|                 |                        |                        |                             |                           | > 20 samples/RP area: 43.5m x 22.5m |
| Cosine Similarity [93] | –                      | –                      | WNN                         | ~ 3.5m                    | 213 RPs, 0.5m apart |
|                 |                        |                        |                             |                           | 100 samples/RP area: 35.9m x 11.8m |
| Horus [86]      | Incremental Triangulation | Strongest AP            | Weighted probabilistic      | ~ 0.6m                    | 110 RPs, 2.13m apart |
|                 |                        |                        |                             |                           | 100 samples/RP area: 35.9m x 11.8m |
| Tilejunction [86] | Entropy Maximization | Spectral Clustering    | Linear programming          | ~ 6m                      | 183 RPs, 5m apart   |
|                 |                        |                        |                             |                           | 15 samples/RP area: 2000m² |
| PCA [120]       | –                      | –                      | Weighted probabilistic      | ~ 1.6m                    | 45 RPs, 2m apart,   |
|                 |                        |                        |                             |                           | 100 samples/RP area: 24.6m x 17.6m |
| Kernel-based [84] | AP coverage            | Bhattacharyya distance Information potential | Kernel density estimation | ~ 1.8m                   | 66 RPs, 2m apart,   |
|                 |                        |                        |                             |                           | 4 – 200 samples/RP area: 36m x 42m |
| CS [116]        | Affinity propagation   | Strongest APs Fisher criterion Random combination | Compressive sensing        | ~ 1.5m                   | 72 RPs, 1.5m apart, |
|                 |                        |                        |                             |                           | 50 samples/RP area: 30m x 46m |
| ACS-based [118] | Splitting-based Joint selection | ML                     |                             | ~ 0.8m                   | 16384 RPs, 1.56m apart, |
|                 |                        |                        |                             |                           | 100 samples/RP area: 200m x 200m |
| CaDet [112]     | K-means                | InfoGain               | Decision tree               | ~ 0.8m                   | 99 RPs, 1.5m apart, |
|                 |                        |                        |                             |                           | 100 samples/RP area: N/A |
| LASSO [109]     | Weighted clustering    | Fisher Criterion       | LASSO sparse recovery       | ~ 0.52m                  | 192 RPs, 0.91m apart, |
|                 |                        |                        |                             |                           | 100 samples/RP area: 300m x 25m |
| GLMNET [109]    | Weighted clustering    | Fisher Criterion       | Elastic net sparse recovery | ~ 0.96m                  | 192 RPs, 0.91m apart, |
|                 |                        |                        |                             |                           | 100 samples/RP area: 300m x 35m |
| GS [110]        | Layered clustering     | Fisher Criterion       | GS sparse recovery          | ~ 1.24m                  | 192 RPs, 0.91m apart, |
|                 |                        |                        |                             |                           | 100 samples/RP area: 300m x 35m |

TABLE VII
OVERVIEW OF STRENGTHS AND WEAKNESSES OF REPRESENTATIVE FINGERPRINTING LOCALIZATION APPROACHES

| Scheme          | Strengths                                      | Weaknesses                                           |
|-----------------|------------------------------------------------|------------------------------------------------------|
| RADAR [14]      | Ease of implementation                         | No efficient AP selection; Low localization accuracy |
| Cosine Similarity [93] | Enhanced metric between fingerprints and online measurements | No coarse localization; No AP selection |
| Horus [86]      | High localization accuracy                     | Time-consuming implementation                         |
| Tilejunction [86] | Accounts for the constraints                    | Complex implementation; Needs model-based parameter estimation |
| PCA [120]       | Suitable feature extraction                    | Complex decomposition implementation                 |
| Kernel-based [84] | Enhanced metric between fingerprints and online measurements | Complex kernel implementation                        |
| CS [116]        | High localization accuracy                     | Optimization’s equality constraint; Needs to satisfy special properties |
| ACS-based [118] | Area-based AP selection                        | Low accurate metric                                   |
| CaDet [112]     | Enhanced AP selection technique                | Complex Probabilistic AP selection                   |
| LASSO [109]     | High localization accuracy; Enhanced optimality condition | Needs parameter tuning                               |
| GLMNET [109]    | High localization accuracy; Enhanced optimality condition | Needs parameter tuning                               |
| GS [110]        | High localization accuracy with low number of APs; Integrates coarse and fine localization in a multicomponent optimization problem | Needs parameter tuning                               |
is hindered by the lack of standardized representative data that can be used for fair comparisons. To this end, we plan to create an open repository of our data that can be used by the community for comparative studies.

B. Recommendations for Future Work

Emerging fields of Wi-Fi fingerprinting-based localization includes the following directions:

- The future practical localization approaches should greatly care about the multipath effects of the indoor fingerprints. The fingerprinting profile may include a multipath profile of fingerprints instead of time collection of single fingerprints. This needs the access to the physical layer of the wireless front-ends. As far as the authors know, the smart devices do not yet allow to this access due to security issues. The team is working on a software defined radio implementation that can provide such capability.

- The fingerprinting profile of an RP may also include the fingerprints of the user along with his trajectory. This associates a vector of the RSS to one RP and improves the available information in the system.

- Localization approaches should care about the real environments performance when the infrastructure experiences intentional faults or in emergency scenarios when the navigation of people is of great importance.

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