Evolution of the fractal structure

G M Kravchenko, L I Pudanova*
Don State Technical University, 1 Gagarin sq., Rostov-on-Don, 344000, Russia

E-mail: lpudanova@yandex.ru

Abstract. The principles of fractal geometry are applicable in the development of new designs based on fractal shaping and the evolution of fractal structures. It’s necessary to use new materials to design fractal structure. The evolution of the structure is characterized by optimization of the initial geometry in the process of iteration. Thus, the fractal structures of older generations from innovative material have the best indicators of reliability. The aim of this study is to develop new designs using the principles of fractal shaping. To reduce the weight of the elements of the building frame in parallel with the evolution of the structure, it is necessary to explore innovative materials with certain material properties. The development of fractal structures requires the creation of new technologies and materials.

Introduction

Design of building and construction has a need to use fundamentally new structural elements and materials in both bearing and enclosing structures.

Bar elements are mostly homogeneous and prismatic in shape. The optimization resource of such constructions is almost exhausted. The evolution of bar spaceframes requires the development of a new paradigm, principles and methods for designing modern fractal structures. The complication of geometric forms and use of inhomogeneous effective structures lead to optimal geometry with the use of the criterion of equal tension.

We need to use modern innovative technologies and materials to design new fractal structures. Such structures are characterized by improved reliability index, and stiffness. They are also more environmentally friendly and aesthetically pleasing.

Various software systems for modeling unique elements and structures, innovative building materials, as well as new types of facilities are developing to implement the idea of a unique building of a new generation. Innovative methods and facilities aim to create structures made of strong and light materials.

The synthesis of parametric design and fractal geometry represents the possibility of investigating the evolution of the framework elements. Also, this synthesis gives the opportunity for creation of the unique fractal structure [1].

In general, the fractal structure is a structure with a self-similarity property [2, 3]. It means that the structure consists of fragments with structural composition that repeats during the scale changing. This is just one of the many interesting properties of fractal structure. Another important property is self-organization, that can be observed during the evolution process of the structure and its fragments and elements.

The fractal approach is one of the effective ways to analyze various architectural and structural shapes. Fractal proposes to consider mathematical abstraction that has certain specific properties. The
combination of fractal geometry and architecture has unimaginable potential determined by infinite possibilities.

The aim of the investigation is the development of new unique structures with use of the fractal shaping principles.

The objectives of the investigation:
- improvement of the building modeling process;
- introduction of innovative materials and technologies in the construction process;
- implementation of the concept of comfortable existence inside the building.

Applying the principles of fractal shaping to structures is an innovative modeling method.

**Fractal shaping**

Fractal shaping ideas underlie the fractal architecture [4]. The appearance of fractal-like forms is due to intuition, talent and a sense of harmony of architects. The Eiffel Tower is built by using only one type of metal rod, which is used in various sizes throughout the tower to minimize the weight of the whole structure. Such an execution can be considered one of the first conscious realizations of fractal geometry in a construction structure.

The fractal theory is a unique tool which is used for development of various scientific industries. Conscious and comprehensive application of the fractal theory has become possible because of the computer technology development. Fractals have a complex self-developing structure and are perceived as complex object for human repetition. Therefore, it is necessary to use specialized tools and facilities for modeling and creating fractal structure [5].

The paradigm of the fractal shaping based on the possibility to use standard methodology, which includes fractal elements, for creating new unique structures, rather than borrowing fractal ideas and unsystematic addition of the typical elements [6]. It’s important to use basic principles and definitions of fractal geometry in the process of model building structures and investigation of the fractal structure evolution [7, 8].

In this investigation we consider the “classic” hinged beam and explore fractal structure evolution based on the “classic” beam. To analyze the arising forces in the structure we assume that structures of different generation have the same self-weight. Adaptation with constant weight is the simplest way to analyze the fractal structure in the initial consideration of the fractal shaping hypothesis.

Specially developed by the authors program “Fractal structure generations” is used to model and investigate the fractal structures.

The program is designed for modeling of flat fractal structure with use of the finite element method. The main idea of the program consists on assembling the structure based on the set of points of the structure occurred during the iterations the original element – axiom. “Fractal structure generations” is applicable for creation of the flat fractal structures and based on the fractal shaping principles.

A rod of a given length is taken as an axiom. The replacement of the axiom by the group of several elements is taken as a rule in the process of fractal structure evolution. In this case, the group of elements form a rhombus. The program has initial data input such as base points for the axiom and rule, the number of iterations or generations.

As a result of the calculations at each generation we have the points coordinates of the structure and a set of rod finite elements that form a unique flat fractal structure.

**Fractal structure evolution**

The zero-generation structure is taken as “classic” hinged beam of fixed length. Because during the initial analysis of fractal shaping all structures have the same weight, the cross section of the structural elements varies depending on the generation and the total length of the group of structural elements. Steel is taken as the initial material for structures of all generations.

The structure of the first generation is a rule used to form a fractal structure. A fixed length beam is replaced by the structure that consist of four elements forming a rhombus. The structures of next
generations may turn out to be unstable with general rule like this. Furthermore, it was decided to add a special connecting rod in mid-span of the structure to the rule. The connecting rod is not replaced at subsequent iterations, i.e. it does not participate in the generate process. This idea ensures the stiffness and stability of the fractal structures.

Authors propose to consider structures with connecting rods during the investigation of the second and older generation structures. Also, this is significantly affecting the appearance of the fractal structures of older generations (Figure 1). Considering the fractal structure evolution with applying the reasonable algorithm contributes to the creation of new kind of construction.

![Figure 1. Initial generations of fractal structure without and with connecting rods](image1)

Second generation structure does not fully reflect the advantages of fractal shaping principles. At this stage the structure is look more like a simple space frame than a unique fractal structure.

The fractal structure evolution implies that with older generation structure becomes more stable due to a logical assembling of numerous of self-similar elements. The different generations of fractal structure are in Figure 2.

![Figure 2. Generations of the fractal structure: 2 to 7](image2)
The analysis of the forces in elements of fractal structures different generations (Table 1). It was considered combined effect of self-weight of the construction and distributed load over the upper belt, which is taken to be 1 kN. This value of the distributed load is close to the limit load for the generation zero structure.

**Table 1.** Result of calculations for fractal structures different generations with the same weight

| Generation | Max z, [mm] | Generation | Max My, [kH/m] | Generation | Max Qz, [kN] |
|------------|------------|------------|---------------|------------|--------------|
| 0          | 163.5648   | 0          | 18.7200       | 0          | 6.2400       |
| 1          | 17.4859    | 1          | 3.9787        | 1          | 3.4779       |
| 2          | 3.4739     | 2          | 0.9592        | 2          | 1.7164       |
| 3          | 0.2122     | 3          | 0.2472        | 3          | 0.8688       |
| 4          | 0.2607     | 4          | 0.0646        | 4          | 0.4434       |
| 5          | 0.3272     | 5          | 0.0172        | 5          | 0.2282       |
| 6          | 0.4181     | 6          | 0.0051        | 6          | 0.1212       |
| 7          | 0.5423     | 7          | 0.0021        | 7          | 0.0776       |

Despite the fact that the fractal structures of the older generation are laced and thin, they are able to bear a significant load which is in many times exceed the capabilities of the original beam. The resulting self-similar structure of high generation consists of thin elements, and, therefore, requires the use of the unique thin durable materials.

As a result, the construction based on complex c fractal structure is almost impossible to create by using classical methodologies and materials such as concrete or steel, or it will be extremely laborious and economically unprofitable.

**Variation of materials**

To investigate the influence of the chosen material, it was decided to consider the constructions based on the third and seventh generation of fractal structure.

In the study of structures of the third generation, the following materials are used: steel, aluminum, titanium and carbon fiber. For all the structures of this generation, there were adopted circular sections with a diameter of 30 mm. It was considered as a calculated load combining the effect of the self-weight of the construction and the distributed load over the upper belt, which is taken to be 1 kN. The results are presented in Table 2.

**Table 2.** Result of calculations for fractal structures third generation with the different materials

| Material     | Max z, [mm] | Material | Max My, [kH/m] | Material | Max Qz, [kN] |
|--------------|------------|----------|---------------|----------|--------------|
| Steel        | 0.526099   | Steel    | 0.2612        | Steel    | 0.9112       |
| Aluminum     | 1.237507   | Aluminum | 0.2521        | Aluminum | 0.8834       |
| Titanium     | 0.869963   | Titanium | 0.2553        | Titanium | 0.8931       |
| Carbon fiber | 0.00806    | Carbon fiber | 0.2502    | Carbon fiber | 0.8776      |

It is necessary to consider that the structures under investigation have the same geometrical characteristics and similar bearing capacity. Accordingly, the chosen material plays an important role simultaneously with the structure's own weight. It should be noted that the steel fractal structure is five times heavier than the identical carbon fiber structure, almost three times heavier than the aluminum structure and two times heavier than the titanium structure.

The same types of materials as were used earlier are also used in the study of structures of the seventh generation. A circular section with a diameter of 10 mm. was adopted for all the structures. The results of the action of the structures' own weights combined with the distributed load on the upper belt are presented in Table 3.
Table 3. Result of calculations for fractal structures seventh generation with the different materials

| Material     | Max z, [mm] | Material     | Max My, [kH/m] | Material     | Max Qz, [kN] |
|--------------|-------------|--------------|----------------|--------------|--------------|
| Steel        | 1.296456    | Steel        | 0.00194        | Steel        | 0.07536      |
| Aluminum     | 3.028533    | Aluminum     | 0.00177        | Aluminum     | 0.07161      |
| Titanium     | 2.136598    | Titanium     | 0.00183        | Titanium     | 0.07292      |
| Carbon fiber | 0.019673    | Carbon fiber | 0.00173        | Carbon fiber | 0.07083      |

As a result of the analysis, we can say that with the application of carbon plastic the main ideas of the fractal structure evolution are fully realized in high-generation structures. Especially it showed that the subtle elements of the light fractal structure are capable of perceiving considerable efforts.

Summary
It’s important that the material for the structure is selected based primarily on the technical and technological capabilities as well as economic feasibility.

Carbon plastics are the most widely used structural polymer materials. CFRPs differ from classical metals due to a combination of characteristics: high specific strength and rigidity, low linear thermal expansion and friction coefficients, significant wear resistance and resistance to aggressive media, and high fatigue strength under static and dynamic loads.

3D printing technology is needed for the implementation of structures developed according to the principles of fractal shaping. It means the creation of numerous types of objects with different geometry, colors, properties, and materials.

Modern 3D printing technology allows the implementation of any ideas of the designer and implementation of projects of any degree of complexity and detail. Such printers create real objects based on the computer volumetric model with use of the specialized methodologies. The advantages of the technology include low cost of the fabricated structure, efficiency and quality, reduced labor costs, and reduced waste generation. Currently, genetic algorithms are used to solve problems of optimization of structures [7]. First of all, this is a parametric synthesis, which results in minimizing the mass of deformable objects.

Consequently, it is necessary to develop innovative materials used in the synthesis with modern technologies to create constructions of new generation based on the fractal structures.

References
[1] Jencks Ch 2002 The New Paradigm in Architecture, (seventh edition of The Language of Post-Modern Architecture) (Yale University Press, London, New Haven).
[2] Mandelbrot BB 1982 The Fractal Geometry of Nature (San Francisco).
[3] Peitgen H-O, Richter P H 1986 The beauty of fractals (Springer-Verlag, Heidelberg).
[4] Kravchenko G M, Vasil’ev S E, Pudanova L I 2017 Parametric architecture. The concept of sustainable development of science in modern conditions (Collection of papers of International scientific-practical conference, Ufa) 264 – 267 (rus).
[5] Kravchenko G 2017 Modeling the External Structure of a Fractals (IOP Conference Series, Earth and Environmental Science) 90 012100.
[6] Kravchenko G M, Vasil’ev S E, Pudanova L I 2017 Fractal structure paradigm (Inženernyj vestnik Dona (Rus)) 4.
[7] Shanshan J, Jinxi Z, Baoshan H 2013 Fractal analysis of effect of air void on freeze–thaw resistance of concrete (Construction and Building Materials) 47 126–130.
[8] Rayneau-Kirkhope D, Mao Y and Farr R 2012 Ultralight fractal structures from hollow tubes (Phys. Rev. Lett) 109 204301.
[9] Optimization of fractal space frames under gentle compressive load 2013 (Phys. Rev.) E 87 063204.