Transport properties near the Anderson transition

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Abstract. The electronic transport properties in the presence of a temperature gradient $\nabla T$ in disordered systems near the metal-insulator transition (MIT) are considered. The
d.c. conductivity $\sigma$, the thermoelectric power $S$, the thermal conductivity $K$ and the Lorenz
number $L_0$ are calculated for the three-dimensional (3D) Anderson model of localization
using the Chester-Thellung-Kubo-Greenwood formulation of linear response. We show that
$\sigma$, $S$, $K$ and $L_0$ can be scaled to one-parameter scaling curves with a single scaling parameter
$k_B T / |(\mu - E_c) / E_c|$. 

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1 Introduction

In this paper we shall demonstrate that in the Anderson model of localization near the
metal-insulator transition (MIT) the d.c. conductivity $\sigma$, the thermoelectric power $S$, the thermal conductivity $K$ and the Lorenz number $L_0$ obey scaling.

A fundamental model often used in dealing with transport phenomena of disordered
media is given by the Anderson Hamiltonian \[1\. Investigations of this model yield
that the energy $E$ dependent conductivity $\sigma(E)$ behaves as

$$\sigma(E) = \begin{cases} 
\sigma_0 \left| 1 - \frac{E}{E_c} \right|^\nu, & |E| < E_c, \\
0, & |E| \geq E_c,
\end{cases} \tag{1}$$

in energy regions near the mobility edge $E_c$ at which the MIT occurs \[3\. Here $\sigma_0$ is
a constant and $\nu$ is a universal critical exponent. Using Eq. \[1\] we have been able to
study the temperature $T$ dependence of $\sigma$, $S$, $K$ and $L_0$ near the MIT \[3\] using linear
response theory \[4\. Here we present our observations of their scaling features.

2 Calculating the transport properties

In the presence of a small temperature gradient $\nabla T$ in an open circuit, $S$ is the coefficient
of proportionality between $\nabla T$ and the electric field it induces. The coefficient $K$ relates $\nabla T$ to the heat current while the Lorenz number $L_0$ measures the ratio between $K$ and the product $\sigma T$. The transport properties are defined in the framework
of linear response theory as
\[ \sigma = L_{11}, \quad S = \frac{L_{12}}{|e|TL_{11}}, \quad K = \frac{L_{22}L_{11} - L_{21}L_{12}}{e^2TTL_{11}}, \quad \text{and} \quad L_0 = \frac{L_{22}L_{11} - L_{21}L_{12}}{(k_BTL_{11})^2}, \]

where \( e \) is the electron charge and \( k_B \) is Boltzmann’s constant. For a noninteracting system with no inelastic scattering processes present, such as phonon mediated hopping, the kinetic coefficients \( L_{ij} \) are given in the Chester-Thellung-Kubo-Greenwood formulation \([4]\) as
\[ L_{ij} = (-1)^{i+j} \int_{-\infty}^{\infty} A(E) [E - \mu(T)]^{i+j-2} \left[ -\frac{\partial f(E, \mu, T)}{\partial E} \right] dE, \]

where \( i, j = 1, 2, \mu \) is the chemical potential of the system, \( f(E, \mu, T) \) is the Fermi distribution function, and \( A(E) \) describes the system dependent features. In the Anderson model, one finds \( A(E) \) to be equal to the critical behavior \([5]\) of \( \sigma \) near the MIT \([5]\).

Previously, we have numerically determined \( \mu(T) \) for the 3D Anderson model from its density of states \([3]\). Using that result, i.e., \( \mu(T) \propto T^2 \), it is straightforward to determine the kinetic coefficients \([3]\) and, consequently, the transport properties \([4]\).

In order to compare our results with experiments we have chosen the energy units to be in electron volts. For the same reason, we measure \( \sigma \) in units of \( \Omega^{-1} \text{cm}^{-1} \). In this paper we only show the results obtained for disorder \( W = 12 \) where \( W \) is the width of the box distribution of randomly chosen potential energies in the Anderson model of localization. We have obtained similar results for other disorders not too close to the critical disorder \( W_c \approx 16.5 \) \([6, 7]\) and where there are no large fluctuations in the density of states. For \( W = 12 \), the corresponding mobility edge is at \( E_c = 7.5 \) \([6]\). The value of \( \nu \) is set to 1.3 in agreement with numerical results \([8]\). Note that this choice of \( \nu \) determines the magnitude of the transport properties but does not change their \( T \) dependence \([3]\) upon which their scaling is based.

3 Scaling Features

Wegner \([9]\) was the first to show for disordered noninteracting electron systems that \( \sigma \) can be written close to the MIT in a scaling form as
\[ \sigma(t, \Omega) = b^{2-d}F(b^{1/\nu}t, b^{\nu}\Omega). \]

Here \( F \) is a system dependent function of the dimensionless parameter \( t \) giving the distance from the critical point, \( \Omega \) is an external parameter such as the frequency, \( b \) is a scaling parameter, \( d \) is the dimension and \( \nu \) is the dynamical exponent. For the noninteracting case, \( z = d \) \([10]\). In this paper, the appropriate parameters are \( d = 3, \quad t = |(\mu - E_c)/E_c|, \quad \Omega = T \text{ and } b = t^{-\nu} \) and Eq. \([4]\) simplifies to
\[ \frac{\sigma(t, T)}{t^{\nu/\nuz}} = F \left( \frac{T}{t^{\nu/\nuz}} \right). \]
In Fig. 1 we show that the σ data from the metallic (|E_F| < E_c), the insulating (|E_F| > E_c) and the critical (E_F = E_c) energy regions collapse onto a single scaling curve when plotted as a function of k_BT/(μ − E_c)/E_c. In the right hand side of Fig. 1, we illustrate similar scaling of S. Note that there is no pre-factor in the scaling form such as in σ since S becomes independent of T at the MIT as T → 0. It was pointed out in Ref. 3 that S for high and low T might scale in terms of k_BT/|E_F − E_c|. This is approximately valid 3 although the proper scaling variable should be k_BT/(μ − E_c)/E_c. The scaling curve for S converges at the value 228.4 μV/K as predicted in Ref. 1 for ν = 1.3. In the metallic regime, our result agrees with the Sommerfeld expansion result at low T. We find that S grows infinitely large in the insulating regime. This can be attributed to the decrease in charge carriers which contribute to a current.

The corresponding scaling curves for K and L_0 are shown in Fig. 2. They also scale with respect to k_BT/(μ − E_c)/E_c. The normalization factor in K is due to the fact that K ∝ k_BT^{ν+1} as T → 0 at the critical regime. L_0 does not require a pre-factor since it is independent of T at the MIT. The result for L_0 shows that it approaches π^2/3 in the metallic regime as expected in the Sommerfeld free electron theory of metals. In the critical regime, L_0 converges to 2.414 which is the value for ν = 1.3 predicted in Ref. 1. Due to the exponential decay of the derivative of the Fermi function as T → 0, L_0 approaches ν + 1 = 2.3 in the insulating regime.

To summarize, we achieve scaling for the four transport quantities σ, S, K and L_0 with respect to a single parameter k_BT/(μ − E_c)/E_c. This factor stems from the form of L_{ij} and from the fact that the explicit T dependence enters via μ and the Fermi distribution function. Comparing our results from σ in Fig. 1 with Eq. 4 we obtain νz = 1. With ν = 1.3 this yields a dynamical exponent z = 0.77
which is to be compared to $z = d = 3$ as expected from scaling arguments. Moreover, our result that $\nu z = 1$ seems to violate the Harris criterion which requires $\nu z > 2$. However, this observation needs to be examined in more detail since the Harris criterion might be altered for our choice of $T$ dependence via the Fermi distribution.

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