Numerical investigation of refrigeration machine compressor operation considering single-phase electric motor dynamic characteristics

Y Baidak\textsuperscript{1} and V Smyk\textsuperscript{2}

\textsuperscript{1}Professor of Thermodynamics and Renewable Energetics Department, Odessa National Academy of Food Technologies, Odessa, Ukraine
\textsuperscript{2}post-graduate student, Odessa National Maritime Academy, Odessa, Ukraine

E-mail: kozak_admin@ukr.net

Abstract. Using as the base the differential equations system which was presented in relative units for generalized electric motor of hermetic refrigeration compressor, mathematical model of the software for dynamic performance calculation of refrigeration machine compressors drive low-power asynchronous motors was developed. Performed on its ground calculations of the basic model of two-phase electric motor drive of hermetic compressor and the proposed newly developed model of the motor with single-phase stator winding, which is an alternative to the industrial motor winding, have confirmed the benefits of the motor with innovative stator winding over the base engine. Given calculations of the dynamic characteristics of compressor drive motor have permitted to determine the value of electromagnetic torque swinging for coordinating compressor and motor mechanical characteristics, and for taking them into consideration in choosing compressor elements construction materials. Developed and used in the process of investigation of refrigeration compressor drive asynchronous single-phase motor mathematical and software can be considered as an element of computer-aided design system for design of the aggregate of refrigeration compression unit refrigerating machine.

1. Introduction

Simulation of dynamic processes that affect the operation of key facilities involved in the automated production, namely electromagnetic devices, electric motors, electromagnetic switchgear, transformers that form the modern automated electric drive of compression refrigeration equipment and determine the conditions of their operation while changing modes of operation – is an important basis for creation of more advanced production process automated control systems. At the stage of dynamics simulation the importance should be given to solve issues which lay the field of electromagnetism, thermal physics, and mechanics. The mechanics of the object, despite the fact that determines the final characteristics of automated production, is the most influential (Dote, 1990). Structural materials from which the elements of automated devices are made have to be pushed nearer to their limits. Simulation of dynamic processes that define the operation of production facilities is a multi complex, multifactorial problem and therefore the issue of solving tasks of dimensions number reducing by
identifying the most influential output / input criteria and factors as well as disclosure of connections and relationships between them is a priority one (Guinee, R.A. & Lyden, C., 1998; 2001).

Mathematical model used as a basis for the dynamic characteristics determination of the electromechanical energy conversion process in time in the electrical motor of the refrigeration machine compressor drive, despite of its complexity, allows deeper address matters related to processes, operational factors and modes of any electromechanical energy converter. The solution of differential equations granted for instantaneous values of variables can give answers to questions that can not be obtained through the use of equivalent circuits, vector variables and their graphs (Persson, E.K. & Buric, M., 1976).

Recently in the practice of engineering calculations a wide variety of multipurpose software for calculation of dynamic processes in automated electric drive is applied, e.g. MatLab - Simulink addition of Power System Blockset manufacturer Math Work (Beucher, O., Weeks M. (2008), Ong, Chee-Mun. (1998); Chaturvedi Devenndra, K. (2010), Giurgiutiu, V., Lychevski, S. E., (2009), Karris, Steven T. (2008). However, despite their extensive capabilities, the licensed product usage is always a difficulty. Mastering software products of leading world manufacturers requires much time, knowledge and skills.

Electric asynchronous motor with short circuit rotor is the most widespread actuator in many automated systems and a lot of researchers were involved into the development of its mathematical or "generalized" theory, with its further implementation to the study of dynamic performance characteristics (Ansari A. et al. (2010), Galan, N., Mammadov A. I., (2006), Leonhard, W. (2001). The issue of "generalized" theory development arose as an attempt to combine in a single unit previously known and innovative methods of electric machines theoretical analysis: the method of symmetrical components, two reactions, rotating magnetic fields. "Generalized" theory clearly demonstrated that a set of circuits that are mutually and relatively moving may be replaced by equivalent mutually fixed electric circuits. Since mathematical tools at this level are special, it is useful to consider the relationships between the general theory of electrical machines and classical approach to the analysis of dynamic characteristics of single-phase induction motor with short circuit rotor on the example of refrigeration machine compressor motor drive.

2. Simulation of refrigeration compressor drive electrical motor operational characteristics in dynamics
2.1. Differential equations of the dynamics of short-asynchronous motor

The actual two-phase asynchronous electromotor with short circuit rotor rotating relatively fixed coils of the stator winding \( W_a, W_d \), made of unequal number of turns and arranged in the stator bore under the solid angle of their magnetic axes shift \( \alpha, \beta \) at 90 electrical degrees, Fig.1, while simulation of operating dynamic characteristics can be provided as a joint system of electrical balance equations, which take into consideration the rotor rotation. Bringing rotor winding resistances to the stator winding should also be considered, as well as currents and voltages of one stator coil winding to another through transformation ratio of turns of the coils \( k = w_a / w_d \).

The equations system of electrical balance of asynchronous motor with short circuit rotor, which is provided relatively to voltages and currents vectors in natural or converted coordinates - with respect to the fixed stator coordinate system \((a, j\beta)\), and with respect to the movable coordinate system of the rotor \((d, jq)\), in which the voltage of short circuit rotor \( \bar{U}_{p, q} = 0 \), is given by

\[
\begin{align*}
\bar{U}_{e, \beta} &= i_{e, \beta} r_e + \frac{d\bar{\Psi}_{e, \beta}}{dt}; \\
0 &= i_{p, \beta} r_p + \frac{d\bar{\Psi}_{p, \beta}}{dt}.
\end{align*}
\]

(1)
In equation (1) \( \overrightarrow{U_{c_\alpha,\beta}} \) — vector of coils voltage supply \( W_{d_\alpha}, W_{d_\beta} \) windings \( W_c \) of the stator; \( \hat{i}_{c_\alpha,\beta}, \hat{i}_{p_\alpha,\beta} \) — vectors of currents in coils of the stator winding and rotor winding rods; \( r_c, r_p \) — active resistances of stator and rotor. The joint solution of equations system (1), from which those relating to the stator are given in a fixed coordinate system, and for rotor - in the moving one, is impossible and therefore it is appropriate to rearrange the equation of rotating rotor electrical circuit voltages for stator fixed coordinate system. In this case, it is considered conventionally that the electric motor rotor is rotating, for example, counterclockwise, and solid angle of shift of rotor and stator winding axis \( \varphi_{d_\alpha,\alpha} \), Fig.1, at any time is considered by multiplying equation components of rotor voltages on the phase operator of its axis rotation relative to stator axis, i.e. \( 1 \cdot e^{-j\varphi_{d_\alpha}} \). Then the equation of voltages of short circuit rotor, reduced to fixed stator axes is given by

\[
0 = \hat{i}_{p_\alpha,\alpha} \cdot e^{-j\varphi_{d_\alpha}} r_p + e^{-j\varphi_{d_\alpha}} \frac{d\Psi_{p_\alpha,\alpha}}{dt}.
\]

(2)

In turn, the rotor winding flux linkage \( \Psi_{p_\alpha,\beta} \), which is provided relatively to coordinate system \((d, jq)\), in the coordinates \((\alpha, j\beta)\) is \( e^{j\varphi_{d_\alpha}} \Psi_{p_\alpha,\beta} \) and, accordingly, the rotor voltage equation (2) is converted to the form

\[
0 = \hat{i}_{p_\alpha,\alpha} e^{-j\varphi_{d_\alpha}} r_p + e^{-j\varphi_{d_\alpha}} \frac{d(e^{j\varphi_{d_\alpha}} \Psi_{p_\alpha,\beta})}{dt} = \hat{i}_{p_\alpha,\alpha} e^{-j\varphi_{d_\alpha}} r_p + \frac{d\Psi_{p_\alpha,\beta}}{dt} + j\Psi_{p_\alpha,\beta} \frac{d\varphi_{d_\alpha,\alpha}}{dt},
\]

or

\[
0 = \hat{i}_{p_\alpha,\beta} r_p + \frac{d\Psi_{p_\alpha,\beta}}{dt} + j\Psi_{p_\alpha,\beta} \frac{d\varphi_{d_\alpha,\alpha}}{dt}.
\]

Figure 1. Two-phase asynchronous electromotor with stator and rotor coils in rectangular system of coordinates
Thus the system of equations of electrical balance of asynchronous motor with short circuit rotor, written for vectors of voltages and currents in the transformed coordinates to stator fixed coordinate system \((\alpha, j\beta)\), is given by

\[
\begin{align*}
\vec{U}_c &= \ddot{i}_c r_c + \frac{d\vec{\Psi}_c}{dt}; \\
0 &= \ddot{i}_p r_p + \frac{d\vec{\Psi}_p}{dt} + j\bar{\Psi}_p \omega_p;
\end{align*}
\]

where \(\omega_p = \frac{d\varphi_{d,\alpha}}{dt}\) – the angular speed of the rotor rotation; \(j\bar{\Psi}_p \omega_p\) – electromotive force of rotor winding, which allows its rotation in the stator fixed coordinate system.

To simplify the expressions, the voltages concerning the stator and rotor are given in the joint coordinate system of the stator \((3)\), and reducing the number of unknown variables we replace the flux linkage vectors included in their composition, by corresponding dependencies from creating currents, namely by expressions:

\[
\begin{align*}
\vec{\Psi}_c &= \ddot{i}_c L_c + \ddot{i}_p e^{-j\varphi_{d,\alpha}} L_m; \\
\vec{\Psi}_p &= \ddot{i}_p L_p + \ddot{i}_c e^{j\varphi_{d,\alpha}} L_m,
\end{align*}
\]

where \(L_c = L_{c\sigma} + L_m, L_p = L_{p\sigma} + L_m\) – full inductances of stator and rotor windings with taking into consideration the self- and mutual inductance \(L_m\) at the coincidence of their axes \((\alpha, j\beta)\) i \((d, jq)\). In the same way as at bringing equations of rotor electrical circuit voltages to stator fixed coordinate system, we perform reduction of rotor winding flux linkage \((4)\), included in the moving coordinate system \((d, jq)\) to the fixed stator coordinate system \((\alpha, j\beta)\) by multiplying by the phase rotation operator \(1 \cdot e^{-j\varphi_{d,\alpha}}\) coordinates of windings of rotor and stator. We obtain

\[
\begin{align*}
\vec{\Psi}_p e^{-j\varphi_{d,\alpha}} &= \ddot{i}_p e^{-j\varphi_{d,\alpha}} L_p + \ddot{i}_c e^{j\varphi_{d,\alpha}} e^{-j\varphi_{d,\alpha}} L_m,
\end{align*}
\]

or in general form and in the common coordinate system \((\alpha, j\beta)\)

\[
\begin{align*}
\vec{\Psi}_c &= \ddot{i}_c L_c + \ddot{i}_p L_m; \\
\vec{\Psi}_p &= \ddot{i}_p L_p + \ddot{i}_c L_m,
\end{align*}
\]

Making the replacement of values vectors of stator windings voltage supply, currents and flux linkages by their projections on the axes \((\alpha, j\beta)\), as in \(\ddot{i}_c = i_{ca} + ji_{cb}\), we obtain the ultimate system of equations of two-phase short-asynchronous motor \((3)\) with five indeterminate - four current projections of the stator, the rotor on the axes and the angular velocity of the rotor, and known - supply voltage and windings resistance:
\[
\begin{align*}
U_{c_a} &= i_{c_a} r_c + \frac{d\Psi_{c_a}}{dt}; \\
U_{c_p} &= i_{c_p} r_c + \frac{d\Psi_{c_p}}{dt}; \\
0 &= i_{p_a} r_p + \frac{d\Psi_{p_a}}{dt} + j\Psi_{p_a} \omega_p = i_{p_a} r_p + \frac{d\Psi_{p_a}}{dt} + \Psi_{p_a} \omega_p; \\
0 &= i_{p_p} r_p + \frac{d\Psi_{p_p}}{dt} + j\Psi_{p_p} \omega_p = i_{p_p} r_p + \frac{d\Psi_{p_p}}{dt} - \Psi_{p_p} \omega_p,
\end{align*}
\]  
(6)

in which the projections of flux linkage vector of stator and rotor windings on the axis at a fixed stator coordinate system are

\[
\begin{align*}
\Psi_{c_a} &= i_{c_a} L_c + i_{p_a} L_m; \\
\Psi_{c_p} &= i_{c_p} L_c + i_{p_p} L_m; \\
\Psi_{p_a} &= i_{p_a} L_p + i_{c_a} L_m; \\
\Psi_{p_p} &= i_{p_p} L_p + i_{c_p} L_m.
\end{align*}
\]  
(7)

The system (6) consisting of four equations can be solved provided the addition of a fifth equation to determine the angular speed of the rotor rotation \( \omega_p \).

2.2. Mathematical model of direct start of asynchronous motor compressor drive

Two-phase asynchronous electromotor is an electromechanical transducer of electrical energy, the rotor rotation of which is characterized by the motion equation of the type

\[
J \frac{d\omega_p}{dt} + M_{om} = M_{em},
\]

where \( J = 0.0862 \, GD^2 \, n^2 \) - moment of inertia of the rotor and associated with it, the reduced moment of mechanism inertia; \( GD^2 \) - non-jumping moment, reduced to the motor shaft; \( n \) - rotor speed; \( M_{om} \) - torque of resistance on the rotor shaft, which includes loading torque and mechanical losses; \( M_{em} \) - electromagnetic torque.

Electromagnetic torque is determined by vector addition of solid vector of stator winding flux linkage and solid vector of stator current, i.e. in the form

\[
M_{em} = \text{Re} \left[ j\Psi_c \times \hat{i}_c \right]
\]

where \( \hat{i}_c \) - complex conjugate vector of stator current. If vector addition is provided as a projection of vectors on the axes of the stator winding \((a,j\beta)\), will be given by

\[
M_{em} = \left[ \Psi_{c_a} + j\Psi_{c_j} \right] \left[ i_{c_a} + j i_{c_j} \right] = \Psi_{c_a} i_{c_p} [1,j] + \Psi_{c_j} \left[ i_{c_p} [j,1] = \Psi_{c_a} i_{c_p} + \Psi_{c_j} i_{c_p} \right].
\]  
(8)

Transforming regarding the current components the equations system of flux linkage rotor and stator winding vector projections (7), shown on the axes of the stator fixed coordinate system, we obtain
Through expressions substitution of currents (9) to the electrical balance equations system (6) recorded according to the second Kirchhoff’s rule, and to the electromagnetic torque expression (8), we obtain a system of five inhomogeneous differential equations of the first order, which already have their own solution, i.e.

\[
\begin{align*}
\frac{di_{ca}}{dt} &= U_{ca} - r_e (\Psi_{ca} L_p - \Psi_{pa} L_m)/(L_c L_p - L_m^2), \\
\frac{di_{cp}}{dt} &= U_{cp} - r_e (\Psi_{cp} L_p - \Psi_{pp} L_m)/(L_c L_p - L_m^2), \\
\frac{dp_{pa}}{dt} &= -\omega_p \Psi_{pa} - r_p (\Psi_{pa} L_c - \Psi_{ca} L_m)/(L_c L_p - L_m^2), \\
\frac{dp_{pp}}{dt} &= \omega_p \Psi_{pp} - r_p (\Psi_{pp} L_c - \Psi_{cp} L_m)/(L_c L_p - L_m^2), \\
\frac{d\omega_p}{dt} &= \frac{1}{J} \left[ \Psi_{pp} \left( \Psi_{pp} L_c - \Psi_{cp} L_m \right)/(L_c L_p - L_m^2) - \Psi_{pa} \left( \Psi_{pa} L_c - \Psi_{ca} L_m \right)/(L_c L_p - L_m^2) - M_{e,ib} \right]
\end{align*}
\]

(10)

Given in the form of expression of the derivative \( y_i' = f(y_i) \) with five unknown values \( i = 5 \), i.e.: \( \Psi_c, \Psi_p, \Psi_{pa}, \Psi_{pp}, \omega_p \) and expected decision in the form of time dependence \( y = f(t) \), where \( t \) – time, the system can be solved by any method, including a numerical method of Runge-Kutta according to the scheme

\[
y_{i(n+1)} = y_{in} + (K_{i1} + 2K_{i2} + 2K_{i3} + K_{i4})/6,
\]

(11)

where \( n \) – integration step; \( i \) – target value, so component ratios are related by:

\[
\begin{align*}
K_{i1} &= h \cdot f(y_m), \\
K_{i2} &= h \cdot f(y_m + K_{i1}/2), \\
K_{i3} &= h \cdot f(y_m + K_{i2}/2), \\
K_{i4} &= h \cdot f(y_m + K_{i3}).
\end{align*}
\]

where \( h \) – period of time or integration step. At the beginning of equation (11) solution, for the moment of time \( t = 0, n_0 = 0 \), the initial values of unknown values \( y_{i1} \) should be chosen which correspond to starting of the engine – i.e. zero ones, because the currents, flux linkage are absent, but are those according to which later instantaneous values of their speeds \( y_{i(n+1)} \) will be calculated.
3. Results

3.1. The numerical calculations of dynamic characteristics and analysis of transition processes in the motor of refrigeration machine compressor drive

Determination of dynamic characteristics of an asynchronous motor with short circuit rotor based on the mathematical model in the form of equations system (10), using the Runge-Kutta method (11) is performed on the base of developed program in the Visual Fortran Pro, v.6.1. software (Baidak Yu., 2011). Time dependences were built for stator and rotor currents, rotating torque depending on the angular speed of the stator windings magnetic field rotation and acceleration characteristics of the base motor with two-phase double-layer winding and the new one, with improved power characteristics taking into consideration the calculations results of asynchronous capacitor motor magnetic equilibrium inhomogeneous differential equations system were determined. On the Figure 2 and Figure 3 summarized in time calculation results for currents, electromagnetic torque and rotor speed for new and base engines are shown.

**Figure 2** Summarized in time results of calculation of currents, electromagnetic torque and rotor speed of the base motor

**Figure 3** Summarized in time results of calculation of currents, electromagnetic torque and rotor speed of the new motor

General view of the characteristics allows to determine that time dependencies of stator and rotor currents as well as of electromagnetic torque reach the default values at significant influence of their too complex free components.
At the beginning of simulation of dependencies when \( t = 0, \omega = 0 \), the program is performing calculations on direct start of the motor without taking into consideration winding inductance, i.e. when \((\Psi_a, \Psi_r, \Psi_{pa}, \Psi_{pr}, \alpha_p) = 0\). Later, an abrupt increase of stator and rotor currents is observed (9), dependent on them flux linkage (10) and dependent on both of them electromagnetic torque (8).

Their dynamic dependencies have at the beginning of the movement a large amplitude of oscillations that affect the angular speed of the rotor, braking it. Speed characteristics of both motors have several sites with the loss of momentum dialed by rotor, that is connected not only with a sharp decrease of electromagnetic torque, but even with a change in its direction to opposite braking and this is while simultaneous increasing of current in the stator and rotor. The reason for this phenomenon should be considered as a sharp decrease of flux linkages of stator and rotor windings, equation (10).

Moreover, the dynamic electromagnetic torque at some moments of time significantly exceeds the maximum moment of static mode. But unlike the static moment, it takes into consideration not only the equivalent circuit parameters of the motor, but also the time of load and inertia of the rotor. The length of electromagnetic transient and the dispersal of the motor largely depend from them. In contrast, the length of the electromechanical process mainly depends on the winding inductance and remains unchanged. Thus, we should define that an essential impact on the duration of the transition process during rotor rotational speed increase has instantly undamped magnetic field generated by currents of stator and rotor and is determined by their flux linkage.

Further analysis of the dynamic characteristics of electromagnetic torque and rotor angular frequency shows that the number of torque oscillations in the new motor with a single-phase winding is less than in the basic one, despite the fact that their amplitude is slightly higher. This makes high-speed angular frequency characteristics of the new engine more rigid, and the transition process shorter. The amplitude of the current oscillations in the new motor is less, this fact positively affects on the operation of the power supply. Reaching 75% of the rotor speed in the new motor is performed for 0.03 seconds, while at the base one - with 0.036 sec.

If we assume that the quadratic current pulse during the transition process - starting of electric motor results in adiabatic temperature increase of its windings, and the additional coil of base motor, which is made of winding wire with diameter of 0.41 mm with cross-section of 0.132 mm² and a valid current 6 A/mm², for the period of time 0.036 seconds, at the start of the motor will be hotter on 19°C. Provided that the stipulated duration of motor start is 0.3 seconds - with a nominal static moment of load, excess of temperature of additional windings will approach to its critical value in the class of wire insulation thermal resistance, namely 154°C.

At the same time, additional coil of new motor, made of wire of cross section 0.26 mm² according to the scheme given in (Baidak Yu., 2011), will increase its temperature only on 3.85°C or in long mode start-up - on 38.5°C that almost does not affect its insulation properties and, as a consequence, the reliability of the engine. Speed characteristics also show that achieving the 99% nominal revolutions of base motor rotor is carried out within 0.065 seconds and 0.055 seconds for the new motor that should be considered as an advantage. In general, except above-mentioned, that is necessary to consider during the implementation of energy saving measures, both motors on dynamics satisfy the conditions of their use as a drive of refrigeration machine compressor.

Conclusion

Based on a system of differential equations, suggested in relative units for generalized electric machine, mathematical model of the programs for calculation of the dynamic performance of refrigeration machine compressors low-power asynchronous motors drive was developed and proposed. This model is invariant for any asynchronous motors, a numerical model of which is adequate to the test sample. Performed on its basis comparative analyses of the basic model of two-phase electric motor of hermetic compressor drive and a new model of motor with single-phase stator
winding, which is an alternative to industrial motor winding, confirmed the benefits of the new motor with single-phase stator winding over the base engine.

The calculations of the dynamic characteristics of the motor of compressor drive permit even at the theoretical level determine the value of electromagnetic torque jumps to coordinate mechanical characteristics of the compressor and the motor, and for taking them into account in choosing construction materials for the compressor elements. Developed and used at research of single-phase asynchronous motor of compressor drive math- and software should be considered as a constituent part of computer-aided design of refrigeration equipment.

References
[1] Dote Y 1990. Servo Motor and Motion Control using Digital Signal Processors, PHI.
[2] Guinee R and Lyden C 1998 Accurate Modelling and Simulation of a High Performance Brushless DC Motor Drive System for Industrial Applications, Proc. of the IASTED International Conference Applied Modelling and Simulation, Honolulu, Hawaii

More references
[3] Guinee R and Lyden C 2001 Motor Parameter Identification using Response Surface Simulation and Analysis, Proc. of American Control Conference, ACC-2001, VA, USA.
[4] Persson E and Buric M 1976 Mathematical Modelling and Simulation of High Performance Brushless DC Motors, IMCSDE 4th annual symp.
[5] Beucher O and Weeks M 2008 Introduction to MATLAB & Simulink: a project approach. Hingham, Massachusetts New Delhi, Inf. Sci. Press LLC Publ., 390 p.
[6] Ong Chee-Mun 1998 Dynamic simulation of electric machinery using MATLAB/Simulink. Upper Saddle River, New Jersey, Prentice Hall PTR Publ., 627 p.
[7] Chaturvedi Devenndra K 2010 Modeling and simulation of systems using MATLAB and Simulink. New York, CRC Press Taylor & Francis Group Publ., 711 p.
[8] Giurgiutiu V and Lyshevski S 2009 Micromechatronics modeling, analysis, and design with MATLAB. New York, CRC Press Taylor & Francis Group Publ., 920 p.
[9] Karris S 2008 Introduction to Simulink with engineering applications. Orchard Publications., 716 p
[10] Ansari A and Deshpande D 2010 Mathematical Model of Asynchronous Machine in MATLAB Simulink” I J E S T Vol. 2(5)
[11] Galan N Mammadov A 2006 The building system and the mathematical model of axial air-gap three-phased asynchronous motor, Taormina – Italy, SPEEDAM.
[12] Leonhard W 2001. Control of Electrical Drives, 3rd ed., SpringerVerlag, New York
[13] Baydak Yu et.al 2013 Energy Efficient Single-Phase Electric Motor of Domestic Refrigeration Devices Motor-Compressor Aggregate. Proceedings of the 8th International Conference on Compressors and Coolants, Papernicka – Smolenice, Slovak Republic.
[14] Baydak Yu 2011 Asynchronous condenser engine Pat. 63322 Ukraine, МПК7 HO2K 17/08. №у201101642; appl. 14.02.11; published. 10.10.11, Bul. №19.