Spin-\textit{M1} and \textit{E1} responses of nuclei probed by proton inelastic scattering

Atsushi Tamii$^1$ and Hiroaki Matsubara$^{1,2}$

$^1$ Research Center for Nuclear Physics (RCNP), Osaka University, 10-1 Mihogaoka, Ibaraki 567-0047, Japan
E-mail: tamii@rcnp.osaka-u.ac.jp

Abstract. We pick up two studies on the nuclear responses from the recent experiments of high-resolution proton inelastic scattering at the Research Center for Nuclear Physics, Osaka University; 1) study of the nuclear symmetry and the neutron skin thickness by the measurement of energy electric dipole (\textit{E1}) response of $^{208}$Pb, and 2) study of the tensor correlation in the ground state by the measurement of the spin-\textit{M1} responses of even-even self-conjugate nuclei in the \textit{sd}-shell nuclei.

1. Electric dipole response of $^{208}$Pb and the symmetry energy

The electric dipole (\textit{E1}) response is one of the fundamental responses of atomic nuclei to the electromagnetic external field. The electric giant dipole resonances (GDR) was the first observed \cite{1} collective motions in nuclei after prediction by Migdal \cite{2}. GDR is described by a picture, given by Goldhaber and Teller \cite{3}, as a relative dipole oscillation between the protons and the neutrons. The \textit{E1} strength above and below the neutron separation energy ($S_n$) were studied intensively by ($\gamma$, \textit{xn}) and nuclear resonance fluorescence (NSR) measurements, respectively. However, the strength around the neutron threshold could not been studied well. Recently \textit{E1} strength concentration around the neutron threshold has been found in neutron rich heavy nuclei. The strength is called the low-energy dipole resonance, or often pygmy dipole resonance (PDR), and is attracting a lot of experimental and theoretical studies. In several theoretical models, the PDR is visualized as a collective oscillation of the neutron skin against the core. Thus the PDR naturally carries information on the neutron skin thickness and isospin dependence of nuclear equation of state, although the structure of the PDR is still under much discussion.

The electric dipole polarizability is another promising quantity to study the neutron skin thickness and the isospin dependence of nuclear equation of state, especially for the slope parameter of the symmetry energy term \cite{4, 5}. The electric dipole polarizability is defined as an inversely energy weighted sum-rule of the \textit{E1} reduced transition probability $B(E1)$. Thus the determination of the complete \textit{E1} strength distribution is required including the region of neutron separation energy. In addition concentration of magnetic dipole (\textit{M1}) excitation also exists in the same excitation energy region \cite{6}. Hence the decomposition of the \textit{E1} and \textit{M1} strengths is crucial for the study.

\footnote{Present address: National Institute of Radiological Sciences (NIRS), Chiba 263-8555, Japan}
We have measured proton inelastic scattering from \(^{208}\)Pb at very forward angles including zero degrees at the Research Center for Nuclear Physics, Osaka University for studying the \(E1\) strength distributions in \(^{208}\)Pb. By applying missing mass spectroscopy without detecting decaying particles or photons, the \(E1\) strengths could be determined from low (5 MeV) to high (20 MeV) excitation energies regardless of the neutron threshold. The forward scattering angle is essential to enhance the \(E1\) Coulomb excitation. The spin-\(M1\) excitation strengths were separated out from \(E1\) by using polarization transfer coefficients or by angular distributions at angles close to zero degrees. The details of the experimental method and the analysis can be found in Refs. [7, 8, 9].

The overall \(B(E1)\) distribution determined by the measurement is shown in Fig. 1. The bump centered at \(\sim 13\) MeV corresponds to the giant dipole resonance, and the strength concentration at around 7–9 MeV to the pygmy dipole resonance. The figure has been adapted from a figure in Ref. [10].

A linear correlation between the slope parameter \((L)\) of the symmetry energy and the neutron skin thickness of \(^{208}\)Pb is predicted by self-consistent mean field model calculations with various sets of interaction parameters [12, 13]. Here, the neutron skin thickness \(\Delta r_{np}\) is defined as the difference of the root-mean-square radii of the neutrons and the protons. Thus, experimental data on the neutron skin thickness may provide constraints on the slope parameter. Roca-Maza et al. [5] have found that \(\alpha_D J\) has good correlation with \(\Delta r_{np}\). The correlation could be naturally explained by a macroscopic droplet model. Adopting the measured \(\alpha_D\) and \(J = 31 \pm (2)_{\text{est}}\) MeV as a realistic range of values for the symmetry energy [14, 15], they extracted the constraint on the neutron skin thickness of \(^{208}\)Pb as [5]

\[
\Delta r_{np} = 0.165 \pm (0.009)_{\text{expt}} \pm (0.013)_{\text{theor}} \pm (0.021)_{\text{est}} \text{ fm}, 
\]

where “expt” uncertainty is associated with the estimates on \(J\).

Roca-Maza et al. have also shown a strong correlation between \(\alpha_D J\) and \(L\) [5]. By adapting the theoretical correlation we have extracted a constraint band in the \(J-L\) plane of the symmetry

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**Figure 1.** \(B(E1)\) distribution in \(^{208}\)Pb deduced from the proton inelastic scattering measurement at forward angles. The bump centered at \(\sim 13\) MeV corresponds to the giant dipole resonance, and the strength concentration at around 7-9 MeV to the pygmy dipole resonance. The figure has been adapted from a figure in Ref. [10].
energy from the measured dipole polarizability data. We have taken the quadratic sum of the 99.9\% theoretical confidence level band from the work by Roca-Maza et al. [5] and the one-sigma experimental uncertainty of $\alpha_D$ [8]. The result is plotted in Fig. 2 for the one-sigma uncertainty in comparison with various constraints from other works. These include heavy ion collisions (HIC), pygmy dipole resonance (PDR), isobaric analog states (IAS), nuclear mass formula with finite range droplet model (FRDM), analysis of neutron star observation data (n-star). A throughout discussion of these methods and the corresponding references are given in Ref. [14]. Additionally, constraints from a chiral effective field theory ($\chi$EFT) calculation including 3N forces [16] and a quantum Monte-Carlo (QMC) calculation [17] are shown together. This work was performed by the RCNP-E282 collaboration.

![Figure 2](image_url)

**Figure 2.** Constraints on the symmetry energy parameters $J$ and $L$ from various methods. Besides the present work based on the $^{208}$Pb dipole polarizability (DP), the results in this figure are taken from Ref. [14] except the ones labeled ($\chi$EFT) from Ref. [16] and QMC (solid circles) from Ref. [17]. The figure has been adapted from a figure in Ref. [10].

2. Isoscalar and Isovector spin-$M1$ responses and the tensor correlation in the ground state

The tensor interaction between two nucleons is playing an essential role to bound atomic nuclei. In fact, a deuteron is the only bound two nucleon system owing to the tensor interaction. The tensor interaction gives rise to a correlation between nucleons in nucleus, tensor correlation. The tensor correlation is quite important to study the properties and structures of nuclei.

Experimental evidence on the tensor correlation in nuclei is quite limited. Due to the short-range nature of the tensor interaction, a nucleon wave-function component with high-momentum is induced especially for a pair of a proton ($p$) and a neutron ($n$). The high-momentum component has been probed by electron scattering [18], ($p,d$) reaction [19], or by ($p,2p$) reaction [20]. In this article, we show a completely different experimental approach to probe the tensor correlation in nuclear ground states. We focus on the correlation between spins of protons and neutrons in nuclei, which could be a clear signature of the tensor correlation.
We define a total proton (neutron) spin operator \( \vec{S}_p(n) \) as

\[
\vec{S}_p(n) = \sum_i \vec{s}_{p(n),i},
\]

where \( \vec{s}_{p(n),i} \) is the spin operator of the \( i \)th proton (neutron). We define a \( p-n \) spin-correlation operator as \( \vec{S}_p \cdot \vec{S}_n \), and a \( p-n \) spin-correlation function as an expectation value of the operator for the ground state, \( \langle \vec{S}_p \cdot \vec{S}_n \rangle \). The \( p-n \) spin-correlation function is considered to take a positive value for the tensor-correlated components, while it vanishes for the case without tensor-correlation.

The following equations can be derived by applying the completeness relation,

\[
\langle (\vec{S}_p - \vec{S}_n)^2 \rangle = \frac{1}{4} \sum |M(\vec{\sigma}\tau_z)|^2
\]

\[
\langle (\vec{S}_p + \vec{S}_n)^2 \rangle = \frac{1}{4} \sum |M(\vec{\sigma})|^2,
\]

where \( M(\vec{\sigma}\tau_z) \) and \( M(\vec{\sigma}) \) are isovector and isoscalar spin-matrix elements, respectively, and the sum is taken for the matrix elements between the ground state and all the excited states. The \( p-n \) spin-correlation function can be written by the spin-matrix elements as

\[
\langle \vec{S}_p \cdot \vec{S}_n \rangle = \frac{1}{4} \{ \langle (\vec{S}_p + \vec{S}_n)^2 \rangle - \langle (\vec{S}_p - \vec{S}_n)^2 \rangle \} = \frac{1}{16} \{ \sum |M(\vec{\sigma})|^2 - \sum |M(\sigma\vec{\tau}_z)|^2 \}.
\]

In the case of even-even self-conjugate (\( N=Z \)) nuclei in the \( sd \)-shell, the ground state spin-parity and isospin is \( 0^+ \) and 0, respectively, without exception. Thus the first sum of Eq. (5) will be naturally taken for isoscalar (\( T=0 \)) \( 1^+ \) excited states and the second sum for isovector (\( T=1 \)) \( 1^+ \) excited states, since all the other matrix elements vanish, assuming the isospin as a good quantum number.

We have measured the isoscalar and isovector \( 1^+ \) excitations of even-even self-conjugate nuclei in the \( sd \)-shell by high-resolution proton inelastic-scattering measurements to obtain the spin-M1 matrix elements [7, 21, 22]. The sum of the spin-M1 strengths have been taken up to 16 MeV excitation energy. The details of the analysis can be found in Ref. [23].

Although still preliminary, the experimental results showed that the \( p-n \) spin correlation function takes positive values around 0.1 for the measured nuclei of \( ^{24}\text{Mg}, ^{28}\text{Si}, ^{32}\text{S}, \) and \( ^{36}\text{Ar} \), showing a positive signature of the tensor correlation in the ground state. In contrast a shell-model calculation with USD-A or USD-B effective interactions [24] with bare spin-operators predicts the \( p-n \) spin-correlation function of close to 0. The predictions do not improve even by using effective spin-operators. It is intuitive to compare the result with the predictions for the case of \( ^4\text{He} \) nucleus, since high-precision calculations with realistic nucleon-nucleon interactions are available for \( ^4\text{He} \). A global vector method has been applied [25]. The calculated \( p-n \) spin correlation function with the AV8’, G3RS and Minesota potentials are 0.135, 0.109 and -0.020, respectively. The predictions of the former two potentials, which incorporates the tensor interaction, are positive and close to 0.1, while the prediction of the last potential, which does not incorporate the tensor interaction, is close to zero. Further analysis is still in progress. This work was performed by the RCNP-E299 collaboration.

Acknowledgments

We are indebted to the RCNP accelerator staff and operators for providing the excellent beams. We appreciate the discussions with W. Horiuchi. The above works were supported in part by JSPS with Grant Numbers of 20302804 and 25105509.
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