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Prevention and Control of Tuberculosis Relying on a Tuberculosis Dynamic Model Based on the Cases of American

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Abstract

Background: Tuberculosis (TB) which is a preventable and curable disease, is claimed as the second largest number of fatalities and there are 9,029 cases of American in 2018. Many researches have done many study to control TB and had evident effects, but TB is also a serious problem for human being. So the study is always improving.

Methods: In the paper, we propose a new dynamic model to study the transmission dynamic and prevalence of TB, then use global differential evolution and local sequential quadratic programming (DESQP) optimization algorithm to estimate parameters of the model. Next, we use Latin hypercube sampling (LHS) to sample and partial rank correlation coefficients (PRCC) to analyze the influence of parameters on the basic reproduction number ($R_0$) and the total infectious (including the diagnosed, undiagnosed and incomplete treatment infectious), respectively.

Results: With the PRCC and p-value of the parameters, we find how the factors affect the outbreaks of TB. Chemoprophylaxis, treatment and vaccination have positive effects, relatively, the vaccine expiry date, diagnostic techniques and the contact ratio have negative effects. Especially, chemoprophylaxis is the most sensitive factors controlling TB.

Conclusion: With results, we give some suggestions to control the prevalence of TB, such as prolonging the duration of the vaccine by researching new and better vaccines to prevent TB, persuading people infected with TB in the latent stage to use chemoprophylaxis to treat and do not contact with the infected and instructing people take care of themselves and be treated in time when they are infected with TB. By doing these, we can effectively control and prevent the prevalence of TB according to the epidemiological characteristics of tuberculosis transmission.

Trial Registration: Not applicable.

Keywords: Tuberculosis (TB); Latin hypercube sampling (LHS); Partial rank correlation coefficients (PRCC); Basic reproduction number; Parameter estimation; Prevention and control measures
1 Introduction

To date, tuberculosis (TB) is seen as one of the world’s deadliest single infectious disease, second only to acquired immunodeficiency syndrome (AIDS) caused by human immunodeficiency virus (HIV) [1, 2]. TB is caused by Mycobacterium tuberculosis (MTB) which can spread through the air when the infected people cough, talk, sneeze or sing [3]. In most situation, MTB usually can affect the lungs of the infected individuals among the susceptible population. Tuberculosis is so highly contagious that there is a high probability of contracting tuberculosis when a susceptible person inhale tiny particles with MTB. Since the MTB are very small, they can keep themselves in the air for a long time and constantly move. The immune system is an important line of defense that limits the growth and spread of MTB. If the immune system cannot suppress their growth, they will most likely spread throughout the body [4].

It is reported in Global Tuberculosis Report 2018 that there are 1.3 million deaths caused by TB [2]. People of all ages and in all countries may be infected with TB, but overall 90% were adults (aged \( \geq 15 \) years), 9% were people living with HIV (72% in Africa). And most cases happened in following eight countries: India (27%), China (9%), Indonesia (8%), the Philippines (6%), Pakistan (5%), Bangladesh (4%), Nigeria (4%), South Africa (3%). All of these countries and 22 other countries are listed by WHO as the top 30 countries with the high burden of TB, and they account for 87% of the world’s cases. On the contrary, the global cases of the WHO European Region (3%) and the WHO American Region (3%) account for only 6%.

In American, there are about 10 to 20 thousands new cases of TB in the last 20 years (see Table 1 ) [5]. Although the number of people infected with TB has been dropping, there are also thousands and thousands of cases every year in American and the death rate is from 0.05 to 0.07. Therefore, it is essential for researchers to explore the factors which are related to the infection, outbreak and prevalence of tuberculosis and then we could take more measures to protect people from TB.

From 1945 to 1955, the widespread use of antibiotics reduced tuberculosis mortality in the United States by 70%, even though the United States had previously achieved a significant reduction in tuberculosis mortality [4, 6]. Based on decades of technology and experience, most active and latent tuberculosis can be effectively treated, and latent tuberculosis can be treated with isoniazid, but treatment can be effective only if the course of cures last at least 6 months [4]. Active tuberculosis can be eliminated with a complex treatment regimen and treated with multiple drugs (isoniazid, rifampicin, pyrazinamide) for nine months to achieve the course of treatment [4, 6].

Bacillus Calmette-Guerin (BCG) is a vaccine which have been used to prevent TB for a long time. BCG duration varies widely, ranging from 10 to 20 years [7]. BCG prevents about 20 percent of children from getting infected, while the vaccine protects about half of those already infected from getting worse [8]. In some areas where TB is highly endemic, BCG for newborns is important, but should not be a major measure of TB control. BCG is widely used in countries with high burden of TB, but for small burden countries may be different. Most countries have preferred to use BCG, but in the US, BCG has not been in general use and even though the govern cancelled the policy of BCG for newborn, and in its place, chemoprophylaxis
is widely favoured. Chemoprophylaxis does provide protection to compliant subjects from selected groups, is cheap and is easy to take, it is not strikingly effective. And for the majority of tuberculin reactors the risk of developing tuberculosis is small and the potential benefit offered by chemoprophylaxis is not great enough to justify its use. But for some country with little burden for TB, chemoprophylaxis is a good chance to control TB [9].

In fact, not all people infected with TB will become sick. Some people will have a latent period which may be 1 year to forever, and some people will become sick at once. However, the cost of treatment for active tuberculosis patients is so high that it is difficult to achieve effective implementation in most developing countries. When the treatment is unfinished, the drug-resistant strains will reproduce which may seriously increase the difficulty of treatment [4, 10, 11]. According to the latest treatment outcome data for new cases in 2016, 82% people who are able to be successfully healed. In contrast, this is another reduction from 86% in 2013 to 83% in 2015 [2].

On the dynamic model study, many researchers have devoted big efforts for the research of the epidemic regularity and transmission dynamics of tuberculosis. In 1962, Waaler et al. established the first dynamic model of tuberculosis which is based on a susceptible-infected-recovered (SIR) model [12]. More and more TB models have been constructed with a variety of factors, just like exogenous reinfection, vaccinated, co-infected with HIV, reinfection, relapse and the different infection and so on [9, 13–16]. Revelle et al. considered prophylaxis, cure and BCG vaccination to research the optimal strategy to fight against TB, which was then extensively used to study the epidemic model of transmission for infectious disease in 1967 [17]. Buonomo et al. studied the global behavior of a non-linear susceptible-infectious-removed (SIR)-like epidemic model with a non-bilinear feedback mechanism [18]. The SEI model proposed by Bowong et al. exhibits the traditional threshold behavior [16]. Whang et al. use a SEIR model with the time-dependent parameters to develop a dynamic model for tuberculosis (TB) transmission in South Korea [19]. A mathematical model was proposed to understand the spread of tuberculosis disease in human population for both pulmonary and drug-resistant subjects by Mishra et al. [20]. Three control factors must be considered simultaneously to decrease the threat of TB by Gao et al., as the following: a preventive measure in the form of vaccination and two treatment measures aiming at the susceptible and individuals infected TB in the active stage and latent stage [21].

In developing countries, the increase of TB cases by a high level of undiagnosed infectious population and incomplete treated population is one of the greatest challenges to control TB. These people are more possible to develop multi-drug resistance relative than the diagnosed infectious population [22, 23]. Based on the model proposed by Moualeu et al. who considered recurrence of disease, exogenous reinfection, cases of primary active tuberculosis, traditional medicine, natural rehabilitation or self-medication, as well as undiagnosed infectious and lost-sight people [22, 24], our model also considers the vaccinated and the relapsed that the recovered population come into susceptible class. In fact, recovered people have a chance to be infected with TB and even though the state haves cancelled the BCG vaccination for newborns in American, there are also some will be vaccinated to prevent TB.
The aim of our study is to analyze the factors affecting tuberculosis based on the dynamic model and give some measures to control and prevent the TB. The structure of this paper is as follows. In the Section 2, we introduce our tuberculosis model expressed by ordinary differential equations (ODE) and the definition of parameters. Then one describe the model hypothesis and modeling idea in detail. The disease-free equilibrium and basic reproduction number is given in Section 3. Section 4, the model is simulated by global differential evolution and local sequential quadratic programming (DESQP) \[25, 26\] optimization algorithm based on the US cases. We analyze the fitting effect by the root mean square percentage error (RMSPE) and the mean absolute percentage error (MAPE). In the fifth Section, we make the uncertainty and sensitivity analysis of the parameters for our model by Latin hypercube sampling (LHS) and partial rank correlation coefficients (PRCC). We have analyzed the sensitivity of each parameter on the basic reproduction number respectively and the total infected. Last but not least, we give some suggestion to prevent TB by the result we get from our research.

2 Methods

In this section, we introduce our new mathematical model and shortly explain the structure of our model. The total population is denoted by \(N(t)\) which is sub-divided into the following seven mutually exclusive sub-populations: vaccinated \((V(t))\), susceptible \((S(t))\), exposed \((E(t))\), latently infected and exposed to TB but not infectious), diagnosed infectious \((I(t)) \) infected with TB and diagnosed by hospital), undiagnosed infectious \((J(t)) \) infected with TB but undiagnosed by hospital), incomplete treated \((L(t)) \) have been diagnosed with active TB and begun their treatment but quitted before the end) and recovered \((R(t)) \), recovered after treatment).

2.1 Model Introducing

We give some definition of the parameters in Table 2, and according to Moualeu et al. \[22, 24\] and characteristics of epidemiology, we determined the range of the parameters of Table 2. We propose a mathematical model to understand the transmission dynamics and prevalence of TB which is represented by the following system of ordinary differential equations:

\[
\begin{align*}
\frac{dV}{dt} &= \Lambda - \psi V - \mu V - \varepsilon V, \\
\frac{dS}{dt} &= \Lambda(1 - \chi) + \psi V + (1 - q_1 - q_2)\gamma R - \phi S - vS - \mu S, \\
\frac{dE}{dt} &= \varepsilon vV + (1 - p_1 - p_2)vS + q_2\gamma R + \rho J - (1 - r)kE - \mu E, \\
\frac{dI}{dt} &= p_1vS + h(1 - r)kE + \theta J + q_1\gamma R + \alpha L - (\delta + q)I - (\mu + d_1)I, \\
\frac{dJ}{dt} &= (1 - h)k(1 - r)E + p_2vS - (\theta + \rho)J - (\mu + d_2)J, \\
\frac{dL}{dt} &= \delta I - (\alpha + \omega)L - (\mu + d_3)L, \\
\frac{dR}{dt} &= gI + \omega L - (\gamma + \mu)R,
\end{align*}
\]

where

\[ N(t) = V(t) + S(t) + E(t) + I(t) + J(t) + L(t) + R(t), \tag{2} \]

and we display our dynamic model with a flow diagram shown in Fig. 1.
The susceptible and vaccinated are infected with tuberculosis from individuals with active TB at a transmission rate $v(I, J, L)$ given by:

$$v = \frac{\beta_1 I}{N} + \frac{\beta_2 J}{N} + \frac{\beta_3 L}{N},$$

(3)

where $\beta_i$, $i = 1, 2, 3$ are the rates of which the diagnosed, undiagnosed infectious and incomplete treated sufficiently and effectively transmit TB to the susceptible or the vaccinated [1].

2.2 Model Instruction

As the Fig. 1 depicted, there is a mind map of the spread of TB from left to right and the following is the explication:

- For the dynamic system, there will be a certain population $\Lambda$ coming into the system, which have a proportion $\chi$ come into the vaccinated $V(t)$ and a proportion $(1 - \chi)$ become the susceptible $S(t)$.
- For the $V(t)$, BCG is ineffective for some newborns, and the vaccinated $V(t)$ may turn into the exposed $E(t)$ when they are exposed to tuberculosis. Even if it effective for others, it’s not protected for life, and the vaccinated $V(t)$ may turn into the susceptible $S(t)$ after 10 to 20 years [7].
- For the $S(t)$, they may become the vaccinated $V(t)$ through vaccination and become the exposed $E(t)$, diagnosed $I(t)$ and undiagnosed $J(t)$ by contacting with the diagnosed $I(t)$, undiagnosed $J(t)$ or incomplete treated $L(t)$.
- For $E(t)$, some of them are converted into the diagnosed $I(t)$ with obvious symptoms and the undiagnosed $J(t)$ with mild symptoms.
- Fifthly, one consider that susceptible, exposed, undiagnosed infectious, incomplete treated and recovered are all likely to become diagnosed infectious.
- For $I(t)$, some can be completely treated and become $R(t)$. Some people who have been diagnosed with tuberculosis and cured, but they unfininished the course of treatment will become $L(t)$.
- For $J(t)$, some may be recovered and become exposed by their immunity system and some will go to diagnosing and be treated.
- For $L(t)$, a part of them still have possibility to be cured but some of them will be exacerbated and have to be treated again.
- For $R(t)$, some may relapse and become exposed or diagnosed infected and it’s also possible to become susceptible because of the loss of immunity.

In the last, considering natural death, it is inevitable that a certain number of people will die in every part of the system for all causes except for TB. Affected by TB, the death toll will increase significantly. In our article, the main feature of our model is to consider the transmission route and mechanism of tuberculosis in the context of reality in a comprehensive way, which has a strong practical significance. Therefore, the conclusion will be closer to reality.

3 Basic Reproduction Number

Our model is a biological system model, so it must meet the biological conditions. The biologically feasible region of model (1) is:

$$\Omega = \{(V, S, E, I, J, L, R) \in \mathbb{R}_+^7 : V, S, E, I, J, L, R \geq 0, V + S + E + I + J + L + R \geq 0\},$$
which can be confirmed as positively invariant (i.e., given non-negative initial values in \( \Omega \), all solutions to model (1) have non-negative components and stay in \( \Omega \) for \( t \geq 0 \)) and globally attracting in \( \mathbb{R}_+^7 \) with respect to model (1). Therefore, we restrict our attention to the dynamics of model (1) in \( \Omega \).

It is easy to see that the model (1) always has a disease-free equilibrium \( P_0 \), and the disease-free equilibrium are the solutions of the algebraic equations:

\[
\begin{align*}
\Lambda \chi + \varphi S - (\phi + \mu) V - \varepsilon v V &= 0, \\
\Lambda (1 - \chi) + \phi V + (1 - q_1 - q_2) \gamma R - \varphi S - vS - \mu S &= 0, \\
\varepsilon v V + (1 - p_1 - p_2) v S + q_2 \gamma R + \rho J - (1 - r) k E - \mu E &= 0, \\
p_1 v S + h (1 - r) k E + \theta J + q_1 \gamma R + \alpha L - (\delta + g) I - (\mu + d_1) I &= 0, \\
(1 - h) k (1 - r) E + p_2 v S - (\theta + \rho) J - (\mu + d_2) J &= 0, \\
\delta I - (\alpha + \omega) L - (\mu + d_3) L &= 0, \\
g I + \omega L - (\gamma + \mu) R &= 0, \\
I &= 0.
\end{align*}
\]

The disease-free equilibrium \( P_0 \):

\[
P_0 = ((\varphi \Lambda + \mu \chi \Lambda)/(\mu \varphi + \mu \phi + \mu^2), (\mu \Lambda + \phi \Lambda - \mu \chi \Lambda)/(\mu \varphi + \mu \phi + \mu^2), 0, 0, 0, 0, 0).
\]

The next-generation matrix approach is applied to calculate the basic reproduction ratio \( R_0 \) [27]. For this purpose, we can write the right-hand side of model (1) as \( F \) and \( V \) with:

\[
F = \begin{pmatrix} 0, & 0, & (1 - p_1 - p_2) v S + \varepsilon v V, & p_1 v S, & p_2 v S, & 0, & 0 \end{pmatrix}^T,
\]

and

\[
V = \begin{pmatrix}
-(\Lambda \chi + \varphi S - (\phi + \mu) V - \varepsilon v V) \\
-(\Lambda (1 - \chi) + \phi V + (1 - q_1 - q_2) \gamma R - \varphi S - vS - \mu S) \\
-(q_2 \gamma R + \rho J - (1 - r) k E - \mu E) \\
-(h (1 - r) k E + \theta J + q_1 \gamma R + \alpha L - (\delta + g) I - (\mu + d_1) I) \\
-(1 - h) k (1 - r) E - (\theta + \rho) J - (\mu + d_2) J) \\
-(\delta I - (\alpha + \omega) L - (\mu + d_3) L) \\
-(g I + \omega L - (\gamma + \mu) R)
\end{pmatrix}.
\]

Jacobian matrices \( F \) and \( V \) at the disease-free equilibrium of \( F \) and \( V \) are calculated respectively. The basic reproduction number \( R_0 \) is the spectral radius of \( F V^{-1} \). Because the expression of \( R_0 \) is too complex, we only give the calculation method. In the following Sections, we will use Matlab to calculate it. Therefore \( R_0 \) gives the number of secondary infectious cases produced by an infectious individual during his or her effective infectious period when introduced in a population of susceptible [10].
4 Parameter Estimation and Prediction

Based on the reported data from 1984 to 2017 by WHO [5] and the model (1), an global differential evolution and local sequential quadratic programming (DESQP) optimization algorithm and least squares fitting were conducted to estimate the undetermined parameters [19, 26, 28]. DESQP which combine differential evolution (DE) [25] and local sequential quadratic programming (SQP) [29, 30] is a method used to search for optimal solution of DE. In the method DE is used as a base level search and SQP is used as a local search. DE is first applied to short term of problem to find best solution. This best solution is given to SQP as an initial condition to fine tune the solution to reach the global optimum or near global optimum.

We get the estimated value, standard deviation, confidence interval, p-value and t-statistic of the parameters which are listed in Table 4 (Appendix A). Based on the estimated results, the basic reproduction number \( R_0 \) can be calculated \( R_0 = 2.3597 \).

By its biological significance, for threshold system, \( R_0 > 1 \) means the disease will break out and become a endemic disease, while \( R_0 < 1 \) means the disease is under control and will be eliminated. In order to control and prevent TB, the smaller \( R_0 \) is, the easier to control TB [1, 27]. The real data and the model results are shown in the following Fig. 2.

We evaluate the fitting effect of our established model through the root mean square percentage error (RMSPE) and the mean absolute percentage error (MAPE) which are significant evaluation indicators. The RMSPE and the MAPE are defined as:

\[
MAPE = \left( \frac{1}{n} \sum_{t=1}^{n} \left| \frac{I(t)^* - I(t)}{I(t)} \right| \right) \times 100\%,
\]

\[
RMSPE = \sqrt{\frac{1}{n-1} \sum_{t=1}^{n} \left( \frac{I(t)^* - I(t)}{I(t)} \right)^2} \times 100\%,
\]

where \( I(t)^* \) is the real value at time \( t \) and \( I(t) \) is its fitting value and \( n \) is the number of data used for prediction. The criteria of MAPE and RMSPE are shown in Table 3 [31, 32]. We use model (1) to simulate the number of the infected, where \( MAPE = 4.93\% \) and \( RMSPE = 5.92\% \) which means the fitting effect is very well and our system have strong prediction ability and high prediction accuracy.

5 Sensitivity Analysis

In our model (1), the precise estimation of parameter values is one of the greatest challenge. The values of parameters are often estimated by experiments, the fitting of model simulation or experimental data, and the specific parameter’s values measured are rare and difficult in vivo [33]. Therefore, parameters estimation is always associated with uncertainty and sensitivity analysis. It is vital to study the influence of the uncertainty of these parameters on the model, which will help us to successfully use the mathematical and computational models of biological systems as prediction tools and to comprehend the functions of biological systems.
In our article, each parameter with different mean and variance value and it is tedious and especially complex to study their influence on the outcome of the model. What’s more, some factors are which we cannot control. Generally, we used to choose the factors which we can control to analysis the sensitivity, so that we can find good measures to eliminate TB. Because we can’t control the death rate $d_1, d_2$ and $d_3$. So in our analysis, we only did sensitivity analysis for 22 parameters which we can control or eliminate TB. In order to find how the parameters impact on the outcome, a general and better way is to do the sensitivity analysis for each parameter. Concretely, it is ideal to use Latin hypercube sampling (LHS), and partial rank correlation coefficients (PRCC) to study the dependence of parameters of the model on basic reproduction number [34, 35].

5.1 Uncertainty analysis
Generally, many mathematical and computational model input factors have parameters and initial conditions for model variables. Thanks to natural variation, lack of current techniques and so on, the parameters are not always known with enough certainty [35]. The purpose of uncertainty analysis (UA) is to solve these problems. UA can quantify the degree of confidence in the experimental data and the estimation value of parameters [35].

In the article, the most popular and efficient Latin hypercube sampling-LHS which belongs to Monte Carlo (MC) class of sampling methods and was introduced by Mckay et al. was used to perform UA [35]. MC method is a common algorithm to solve various computational problems, and can realize the evaluation of multiple models and the results can not only be used to perform SA, but also to determine the uncertainty of model inputs. LHS can unbiased estimate the average output of the model, and fewer samples are required to achieve the same accuracy as simple random sampling [35].

The remaining 22 parameters are difficulty to estimate correctly and have been chosen to do uncertainty analysis. We assume each parameter to be a random variable with normal distribution to analyze the uncertainty in the value of these parameters. Normal distribution for all parameters with the mean (i.e., estimated value) and variance value (i.e., square of standard deviation) are given in the Table 4. Latin hypercube sampling has been used to sample for these parameters which are considered for sensitivity analysis. Here, we set the sample size $N=2000$. Using Latin hypercube sampling method, probability density function for each parameter is stratified into 2000 equiprobable (1/2000) serial intervals. Then a single value is chosen randomly from each interval. This produces 2000 sets of values for each parameters, and we can compute 2000 sets of values for $R_0$ from 2000 sets of different parameters values mixed randomly and get the distribution hist of $R_0$ which is shown in Fig. 3. By these we see the property of the parameters and $R_0$, which can help us further our research.

5.2 Sensitivity analysis
Sensitivity analysis (SA) is a quantitative way in staying effects of the parameter uncertainty on the model’s outputs. SA is applied for field from environmental science to software engineering, which is identified a condition precedent for model
building [36]. Based on the parameters, we are able to raise presumptions about the biological system that actuate the system behavior which can be measured by conducting experiments [37]. Local SA techniques, one class of SA, research the effects of small variations in individual parameters around some nominal point and have been applied to a number of signal transduction and metabolic pathway models [38, 39]. Due to confirmed the most influential parameters, the model’s predictive can be enhanced to a great extent. Meanwhile, it’s helpful to eliminate TB.

In the section, we compute PRCC to analyze the sensitivity of the factors to the $R_0$ and the total infected to identify which parameters have great effect on the variability in the outcome and how the parameters affect $R_0$ and the total infected. Here we compute the PRCC of $R_0$ and the total infected based on the LHS matrix, the result can be seen from Table 5.

In our research, with the LHS method, we get the sample matrix LHS matrix and based on the model equation (1), we compute the PRCC to analyze the sensitivity of the factors to $R_0$ (see Fig. 4, Table 5). In our experiment, we assume that the parameters have a significant effect on the $R_0$ and total infected when p-value $< 0.01$.

From Fig. 4, we can easily see that different parameters have different degree and different effect on $R_0$ and the total infectious which may be complex to take proper measures to control TB respectively. In order to better control TB, we put emphasis on analyzing the parameters whose PRCC $> 0.2$. And we assume these parameters have high degree and significant effect on $R_0$ and the total infectious. Expect for the uncontrollable factor (which we cannot take relative measure to control TB) from Fig. 4, we can easily see that $r$, $\omega$, $g$ and $\phi$ have significant and positive affect on the $R_0$ and the total infectious, and $\varepsilon$, $k$, $\psi$, $\delta$ and $\theta$ have significant and negative affect on the $R_0$ and the total infectious.

6 Results and Discussion

In this section, we present the results of simulation for the model and discuss the sensitivity of parameters to the $R_0$ and the total infectious. Furthermore, we give some measures to prevent TB.

6.1 Results

In generally, parameter estimation is an iterative process, which we use the current parameter values as the initial values of the next iteration [1]. All the parameter values of the first iterative process are set to be their initial guess values which are estimated with the lowest sub-condition. Then parameters estimation is carried out with a limited list of parameters which are previously non-identifiable. Finally, repeat the estimation process and check all the estimated parameters to see whether the new value of the previously unrecognized parameter affects the value of the identifiable parameter. We use the data of the diagnosed infectious in American from 1984 to 2017 published by Centers for Disease Control and Prevention (CDC) to estimate the parameters of the model (1).

In our model, some parameters have been estimated by WHO and some parameters have been estimated by the TB researchers and others are uncertainty. In the following we specify some parameter values.
(1) The natural mortality $\mu$: It is assumed to be equal to the inverse of the life expectancy at birth and $1/\mu = 79.30$ is the average lifespan of human, then $\mu = 0.0126$ [40].

(2) Rate of progression to infectious individuals (including diagnosed and undiagnosed infectious) $k$: Based on the parameter estimation $k = 0.0421$, the incubation period of tuberculosis $1/k = 23.7529$ years. The latent period of TB is 1 year to forever in generally. [1, 9, 41].

(3) Recovery rate of the diagnosed infectious $g$: In our simulation, $g = 0.5022$, and the course of recovery for the diagnosed infection is estimated $1/g = 1.9912$ years. By 2010, the course of treatment of the tuberculosis patient that first time is commonly treated in 6 months, and the course of treatment of the tuberculosis patient that is resistant to drug is commonly 24 months, the course of treatment of the tuberculosis patient that is extensively resistant to drug is 36 months [42].

(4) Diagnosis rate $\theta = 0.6082$ per year which means the undiagnosed individual will be diagnosed with active TB after $1/\theta = 1.6442$ years, generally, some people with active TB is difficult to be diagnosed.

(5) Progression rate at which diagnosed infectious people become incomplete treated $\delta$: It has been estimated as $\delta = 0.5698$ per year which means the diagnosed people may give up treatment after $1/\delta = 1.7550$ years. Generally, the average convalescence period of tuberculosis is about 1 year which means when people are treated 1 years, they will think they have recovered, but not [27].

(6) Relapse ratio $q_i$, $i = 1, 2$: It has been estimated as $q_1 = 2.40\%$, $q_2 = 54.25\%$, this shows that relapse for most people is slow progress, $(1 - q_1 - q_2) = 43.35\%$, which means the most people will lost the immunity and become the susceptible.

(7) Progression rate at which incomplete treated people become diagnosed infectious $\alpha$: It has been estimated as $\alpha = 0.1002$ which means that the incomplete treated may be treated again after $1/\alpha = 9.9800$ years.

(8) The natural vaccination rate of the newborn babies $\chi$: It has been estimated as $\chi = 13.88\%$. In American, the newborns haven’t been asked to be vaccinated [43]. The United States and other western countries with low TB burden have abolished BCG vaccination for newborns, some Americans will be vaccinated at the doctor’s advice [43, 44].

(9) Rate of the vaccinated become the susceptible $\psi$. It has been estimated as $\psi = 0.0510$ which means the vaccination may be invalid after $1/\psi = 19.6078$ years. BCG vaccine duration varies widely, ranging from 10 to 20 years [7].

(10) Relapse rate of the recovered individuals $\gamma$. The average relapse rate of recovered individuals is estimated as $\gamma = 0.1444$ per year which means the recovery individuals may be re-infected after $1/\gamma = 6.9252$ years. It’s reported that the recurrence rate was 92.96\% within 5 years after cure, 32.39\% within 1 year, and 7.04\% after 5 years [45].

(11) Progression rate at which the undiagnosed become the exposed $\rho$. It has been estimated as $\rho = 0.1993$ which means it takes about $1/\rho = 5.0176$ years for the undiagnosed to become a non-infectious individuals (exposed).

(12) Detection rate of active TB $h$. It is estimated as $h = 39.98\%$. This shows that 60.02\% of tuberculosis patients will not be diagnosed or will not be diagnosed for a short time.
(13) Vaccination coverage \( \phi \): It has been estimated as \( \phi = 0.0500 \) which means after an average of \( 1/\phi = 20.0000 \) years, people will lose antibodies to tuberculosis and be vaccinated again. Generally, BCG is not widely used in American, and the adult also will not choose to vaccinate if it’s unnecessary [43, 44].

(14) Chemoprophylaxis rate \( r \): It has been estimated as \( r = 0.9219 \). Bhunu et al. have estimated \( r = 0.7 \) [9].

(15) Recovery rate of the incomplete treated \( \omega \): It has been estimated as \( \omega = 0.1986 \) which means some incomplete treated will naturally recovery after \( 1/\omega = 5.0352 \) years. Bacaër et al. have estimated that the natural recovery for HIV-negative TB and HIV-positive TB cases as 0.1390 and 0.2400 per year, respectively, HIV-negative TB and HIV-positive TB will be recovered after 7.1900 years and 4.1700 years without treatment [46].

(16) The rate of the susceptible become the diagnosed and undiagnosed infectious \( p_1, p_2 \): Based on the infectious data [47], we estimate that \( p_1 = 2.5\% \) and \( p_2 = 34.14\% \), which means \( 2.5\% + 34.14\% = 36.54\% \) of the people will be sick at once after being infected TB, while 2.5\% have serious symptom and 34.14\% have mild symptom which do not go to diagnosing. The remained \( (1 - p_1 - p_2) = 63.46\% \) who enter a slow progression of TB infection become the exposed.

6.2 Discussion

TB is a high-prevalence infectious disease in the world and the infectious are widespread worldwide. It is vital to seize the main cause and find the best treatment according to the cause to treat the disease. In the article, we have constructed a TB model to study the transmission dynamic system and provide some measures to reduce and prevent TB infection in US. To find more ways to prevent TB, we analysis many factors which may have effect on \( R_0 \) and the total infectious. The sensitivity analysis of the parameters with \( R_0 \) and the total infectious have been done (see Fig. 4, Table 5). When we take measures to control TB, the result is shown in Fig. 5. In generally, we find we can control the factors with \( \alpha, g, \psi, r, \beta_i, i = 1, 2, 3, \phi \) and \( \delta \). From the result in Fig. 5, we can clearly find that \( r \) has the greatest effect on the total infected, then the \( g \) has the secondly greatest effect, and others have the similar effect.

Strategy 1: We can find \( r \) is significantly negatively correlated with \( R_0 \) which means it’s wise to use chemoprophylaxis to control TB. And with our control, the TB can be greatly control from Fig. 5. For American with little burden of TB, vaccine only have little or no help for control TB, and American govern have cancelled the policy of BCG for newborn. In order to prevent the TB outbreak, we can encourage people to have chemoprophylaxis in time by the media, and research new and more efficient chemoprophylaxis to improve the effect, reduce the harm to human body, improve the therapeutic effect [48].

Strategy 2: The parameter \( g \) have a negative effect on the \( R_0 \), and with our control, it has clear reduction on the total infectious. By doing these, we should take much money and energy to research new medicine and therapy to reduce the period of treatment [49]. Nowadays, even though there has been a big improvement, but not enough, in the treatment of tuberculosis, still TB is the second leading cause of death in the world. So we still have a long way to go to end TB.
Strategy 3: From Fig. 4, we can see $\beta_{i}, i = 1, 2, 3$ have positive effect on $R_0$. With our control, we can greatly reduce the total infectious by reducing the contacting rate. Despite the emphasis we have placed on treating TB patients in isolation, the majority of people contract TB from contact each year [50]. We should strictly monitor this, do protective measures, away from tuberculosis. Check for outsiders to avoid contact with tuberculosis patients [51]. If we decrease half of $\beta_{i}, i = 1, 2, 3$, we can find the total infectious will decrease in some content even though it’s little (see Fig. 5).

Strategy 4: It is also clearly shown that $\psi$ is positively correlated with $R_0$ and the total infectious which means the longer the vaccine lasts, the easier it is to control TB. We make $0.1 \times \psi$, and find the total infectious decreased greatly which means we can decrease it to prevent TB. We can delay the duration of the BCG by researching new and better vaccination to prevent infecting TB. Currently, one of major categories of health intervention which are available for TB prevention is vaccination of children with the BCG. But for countries with low burden of TB, it may be not.

Strategy 5: The parameter $\delta$ have positive effect on the $R_0$, and $\alpha$ have negative effect on the $R_0$. With our assumption, controlling them can help us end TB. From one hand, we can encourage people finish the treatment to cure completely by reducing the cost [52]. The state can make more medical insurance to relieve the financial burden and help people heal. For another, educate people to know more about TB treatment and to follow the doctor’s plan, let they know health is more important than everything.

Strategy 6: It is clearly shown that $\phi$ is negatively correlated with $R_0$ and the total infectious which means increase the people of vaccination per year can protect people from TB and control TB. Even the state have cancelled the policy of BCG for newborn, BCG also can help to prevent TB. We should increase awareness of protection, prevent it before it happens, and vaccinate as often as possible.

7 Conclusion
In generally, from the analysis, it is clearly that the prevalence of TB in the United States is heavily influenced by exposure, vaccination and treatment effectiveness which are similar to TB studies elsewhere. In our study, we also found that chemoprophylaxis affects the prevalence of TB more than all others. However, each coin have two sides, chemoprophylaxis has certain harm to human body [48], so we should research new medicine to cure TB. In some content, it does more good than harm. Based on the analysis, we give some tragedies to control and end TB in two ways: prevent and treatment. Especially, chemoprophylaxis can greatly control and cure TB in some content with some side effect, so we should do much research to find better chemoprophylaxis.

In fact, because the recovered infectious may be relapse, it’s difficult to diagnose TB and treat, TB is hard to control. For the measures we proposed, these may cannot eliminate TB, but these are critical useful for the infectious. According to the latest report, in the announcement came at the first WHO Global Ministerial Conference on Ending Tuberculosis in the Sustainable Development Era, there are 75 ministers agreed to take urgent measures to end tuberculosis (TB) by 2030 [53].
From our analysis, it is difficult to end the tuberculosis till to 2030 under the existing conditions (Fig. 5). So we should find more and efficient methods to end TB. It will be difficult to eliminate TB in a short period of time, but we believe that in the future, with advanced technologies, TB can be eliminated completely.

Abbreviations
TB: Tuberculosis; DESQP: global differential evolution and local sequential quadratic programming; LHS: Latin hypercube sampling; PRCC: partial rank correlation coefficients; ODE: ordinary differential equations; MAPE: the mean absolute percentage error; RMSPE: the root mean square percentage error; AIDS: acquired immunodeficiency syndrome; HIV: human immunodeficiency virus; MTB: Mycobacterium tuberculosis; BCG: Bacillus Calmette-Guerin; China’s CDC: Chinese Center for Disease Control and Prevention; WHO: The World Health Organization; SQP: sequence quadratic program; SA: sensitivity analysis; UA: uncertain sensitivity analysis;

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Availability of data and materials
The data that support the findings of this study are available from the Center for Disease Control and Prevention (CDC) (https://www.cdc.gov/tb/statistics/reports/2017/table1.htm), these network direct data are completely open, and we count these data month by month.

Authors’ contributions
WY and LY conceptualized and designed the study, drafted the initial manuscript, and approved the final manuscript as submitted. WY analyzed the data and simulated parameters. HM, WX, JL and YY carried out the initial analyses, reviewed and revised the manuscript. All authors read and approved the final manuscript.

Ethics approval and consent to participate
Not applicable.

Consent for publication
Not applicable.

Competing interests
The authors declare that there is no conflict of interests regarding the publication of this article. No authors have potential conflicts of interest with reference to this work.

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Figures

Figure 1 process.eps The flow diagram for compartment model of the transmission dynamics system of TB.

Figure 2 Simulation.eps The comparison of real data and fitted data and the projection for the future status of TB.

Figure 3 Hist_R0.eps The distribution of the basic reproduction number $R_0$.

Figure 4 R0.eps and Total.eps (a) show the PRCC of parameters with and $R_0$ and (b) show the PRCC of parameters with the total infected. Here, we assume that when p-value < 0.01, the parameters have significant effect $R_0$ and the total infectious. In order to better control TB, we put emphasis on analyzing the parameters whose PRCC > 0.2.

Figure 5 control.eps Simulation of the total infectious with all parameters $1.03 \times r = 0.9496$, $1.5 \times \alpha = 0.1503$, $1.5 \times g = 0.7533$, $1.5 \times \phi = 0.0750$, $0.9 \times \delta = 0.5128$, $0.99 \times \psi = 0.0505$, $0.7 \times \beta_1 = 3.1067$, $0.7 \times \beta_2 = 0.2096$ and $0.7 \times \beta_3 = 4.1063$ from the first column of Table 4, respectively, when one parameters takes a specific value, others take the value of the first column in Table 4. Differently, we synthesize the effects of three contact rates $\beta_i$, $i = 1, 2, 3$ into one contact rate $\beta$ effect. 'With all control' means that we let all parameters specific values simultaneously. 'Without control' is the situation which we take no measures. We can find $\psi$ has a mild effect on the total infected, with its line overlapping with 'without control' approximately.
Table 1  Population, tuberculosis Cases, Case Rates per 100,000 Population, Deaths, and Death Rates per 100,000 Population, and Percent Change: United States, 1984-2017. Where, \[ \text{Rate}_j^{(3)} = \frac{\text{Number}_j^{(3)}}{\text{Number}_j^{(5)}}, \text{Number}_j^{(2)} = \frac{\text{Number}_j^{(3)} - \text{Number}_{j-1}^{(3)}}{\text{Number}_{j-1}^{(5)}}, \text{Rate}_j^{(2)} = \frac{\text{Rate}_j^{(3)} - \text{Rate}_{j-1}^{(3)}}{\text{Rate}_{j-1}^{(5)}} = \frac{\text{Number}_j^{(3)} - \text{Number}_{j-1}^{(3)}}{\text{Number}_{j-1}^{(5)}}/\text{Number}_{j-1}^{(2)} \] (i = 1, 2) and j is the year. For example 2016, \[ 2.9 = \frac{9253}{3.2307 \times 10^3}; -3.1 = \frac{9253 - 9547}{9547}; -3.8 = \frac{9253/3230.7 - 9547/3207.4}{9547/3207.4}. \] The data of population from [54] and others from [5].

| Year | Population (\(\times 10^6\)) | Original Number | Number | Rate | Change rate | Original Number | Number | Rate | Change rate |
|------|---------------------------------|-----------------|--------|------|-------------|-----------------|--------|------|-------------|
| 1984 | 2.3583                          | 22255           | 9.4    | -6.7 | -7.5        | 1729            | 0.7    | -2.8 | -3.6        |
| 1985 | 2.3792                          | 22201           | 9.3    | -0.2 | -1.1        | 1752            | 0.7    | 1.3  | 0.4         |
| 1986 | 2.4013                          | 22768           | 9.5    | 2.6  | 1.6         | 1782            | 0.7    | 1.7  | 0.8         |
| 1987 | 2.4229                          | 22517           | 9.3    | -1.1 | -2.0        | 1755            | 0.7    | -1.5 | -2.4        |
| 1988 | 2.4450                          | 22436           | 9.2    | -0.4 | -1.3        | 1921            | 0.8    | 9.5  | 8.5         |
| 1989 | 2.4682                          | 23495           | 9.5    | 4.7  | 3.7         | 1970            | 0.8    | 2.6  | 1.6         |
| 1990 | 2.4962                          | 25701           | 10.3   | 9.4  | 8.2         | 1810            | 0.7    | -8.1 | -9.2        |
| 1991 | 2.5298                          | 26283           | 10.4   | 2.3  | 0.9         | 1713            | 0.7    | -5.4 | -6.6        |
| 1992 | 2.5651                          | 26673           | 10.4   | 1.5  | 0.1         | 1705            | 0.7    | -0.5 | -1.8        |
| 1993 | 2.5902                          | 25102           | 9.7    | -5.9 | -7.1        | 1631            | 0.6    | -4.3 | -5.6        |
| 1994 | 2.6313                          | 24206           | 9.2    | -3.6 | -4.7        | 1478            | 0.6    | -9.4 | -10.5       |
| 1995 | 2.6628                          | 22725           | 8.5    | -6.1 | -7.2        | 1336            | 0.5    | -9.6 | -10.7       |
| 1996 | 2.6939                          | 21210           | 7.9    | -6.7 | -7.8        | 1202            | 0.4    | -10.0 | -11.1      |
| 1997 | 2.7266                          | 19751           | 7.2    | -6.9 | -8.0        | 1166            | 0.4    | -3.0  | -4.2        |
| 1998 | 2.7585                          | 18286           | 6.6    | -7.4 | -8.5        | 1112            | 0.4    | -4.6  | -5.7        |
| 1999 | 2.7904                          | 17499           | 6.3    | -4.3 | -5.4        | 930             | 0.3    | -16.4 | -17.3      |
| 2000 | 2.8216                          | 16308           | 5.8    | -6.8 | -7.8        | 776             | 0.3    | -16.6 | -17.5      |
| 2001 | 2.8497                          | 15945           | 5.6    | -2.2 | -3.2        | 764             | 0.3    | -1.5  | -2.5        |
| 2002 | 2.8763                          | 15055           | 5.2    | -5.6 | -6.5        | 784             | 0.3    | 2.6   | 1.7         |
| 2003 | 2.9011                          | 14835           | 5.1    | -1.5 | -2.3        | 711             | 0.2    | -9.3  | -10.1       |
| 2004 | 2.9281                          | 14499           | 5.0    | -2.3 | -3.2        | 657             | 0.2    | -7.6  | -8.4        |
| 2005 | 2.9552                          | 14065           | 4.8    | -3.0 | -3.9        | 648             | 0.2    | -1.4  | -2.3        |
| 2006 | 2.9838                          | 13727           | 4.6    | -2.4 | -3.3        | 652             | 0.2    | 0.6   | 0.3         |
| 2007 | 3.0123                          | 13280           | 4.4    | -3.3 | -4.2        | 554             | 0.2    | -15.0 | -15.8       |
| 2008 | 3.0409                          | 12889           | 4.2    | -2.9 | -3.9        | 585             | 0.2    | 5.6   | 4.6         |
| 2009 | 3.0677                          | 11514           | 3.8    | -10.7 | -11.4 | 529         | 0.2    | -9.6  | -10.4       |
| 2010 | 3.0933                          | 11100           | 3.6    | -3.6 | -4.4        | 569             | 0.2    | 7.6   | 6.7         |
| 2011 | 3.1158                          | 10504           | 3.4    | -5.4 | -6.1        | 539             | 0.2    | -5.3  | -6.0        |
| 2012 | 3.1387                          | 9935            | 3.2    | -5.4 | -6.1        | 510             | 0.2    | -5.4  | -6.1        |
| 2013 | 3.1606                          | 9561            | 3.0    | -3.8 | -4.4        | 555             | 0.2    | 8.8   | 8.1         |
| 2014 | 3.1839                          | 9398            | 2.9    | -1.7 | -2.4        | 493             | 0.2    | -11.2 | -11.8       |
| 2015 | 3.2074                          | 9547            | 3.0    | 1.6  | 0.8         | 470             | 0.1    | -4.7  | -5.4        |
| 2016 | 3.2307                          | 9253            | 2.9    | -3.1 | -3.8        | 528             | 0.2    | 12.3  | 11.5        |
| 2017 | 3.2515                          | 9105            | 2.8    | -1.6 | -2.3        | –               | –      | –    | –           |
Table 2 The definition and value range of parameters for model (1).

| Parameter | Definition |
|-----------|------------|
| $\mu$    | Natural mortality of human |
| $\Lambda$ | New individuals coming into the system |
| $\alpha$ | The rate for incomplete treated going to diagnosed infectious |
| $\beta_1$ | Transmission rate of diagnosed infectious |
| $\beta_2$ | Transmission rate of undiagnosed infectious |
| $\beta_3$ | Transmission rate of incomplete treated |
| $\chi$ | The natural vaccination rate of the newborn babies |
| $d_1$ | TB-related mortality of diagnosed infectious |
| $d_2$ | TB-related mortality of undiagnosed infectious |
| $d_3$ | TB-related mortality of incomplete treated |
| $\delta$ | Rate for diagnosed infectious coming into the incomplete treated |
| $g$ | Indicate the reduction in risk of infection due to vaccination |
| $\gamma$ | Recovery rate of the diagnosed infectious |
| $h$ | Relapse rate of recovered individuals |
| $k$ | Detection ratio of active TB |
| $p_1$ | The proportion of susceptible individuals who become diagnosed infectious |
| $p_2$ | The proportion of susceptible individuals who become undiagnosed infectious |
| $\psi$ | Loss of vaccination rate |
| $\phi$ | Vaccine coverage rate |
| $q_1$ | The proportion of recovered who become diagnosed infectious due to relapse |
| $q_2$ | The proportion of recovered who become exposed due to relapse |
| $r$ | Chemoprophylaxis of the exposed |
| $\rho$ | Rate of progression from undiagnosed infectious to exposed |
| $\theta$ | Rate of progression from undiagnosed infectious to diagnosed infectious |
| $\omega$ | Recovery rate of the incomplete treated |
Table 3 The criteria of MAPE and RMSPE.

| MAPE and RMSPE | Forecasting power       |
|----------------|-------------------------|
| < 10%          | Highly accurate forecasting |
| 10-20%         | Good forecasting         |
| 20-50%         | Reasonable forecasting   |
| > 50%          | Inaccurate forecasting   |
Table 4  The estimated result, property value of parameters and initial condition of each compartment of model (1).

| Parameter | Value   | Standard deviation | CI Low Bound | CI High Bound | p-Value | t-statistic |
|-----------|---------|--------------------|--------------|--------------|---------|-------------|
| $\alpha$  | 0.1002  | 0.0340             | -0.0081      | 0.2085       | 0.0603  | 2.9434      |
| $\beta_1$ | 4.4381  | 0.2390             | 3.6774       | 5.1987       | 0.0003  | 18.5687     |
| $\beta_2$ | 0.2994  | 0.0312             | 0.2002       | 0.3985       | 0.0024  | 9.6079      |
| $\beta_3$ | 5.8662  | 0.1946             | 5.2469       | 6.4855       | 0.0001  | 30.1466     |
| $\chi$    | 0.1388  | 0.0425             | 0.0036       | 0.2740       | 0.0469  | 3.2664      |
| $d_1$     | 0.0506  | 0.0240             | -0.0256      | 0.1269       | 0.1251  | 2.1126      |
| $d_2$     | 0.7257  | 0.1033             | 0.3968       | 1.0546       | 0.0059  | 7.0222      |
| $d_3$     | 0.0947  | 0.0572             | -0.0875      | 0.2769       | 0.1966  | 1.6543      |
| $\delta$  | 0.5698  | 0.1622             | 0.0535       | 1.0861       | 0.0391  | 5.3121      |
| $d$       | 0.5539  | 0.1130             | 0.1944       | 0.9134       | 0.0162  | 4.9029      |
| $e$       | 0.2022  | 0.1049             | 0.1682       | 0.8362       | 0.0173  | 4.7856      |
| $\gamma$  | 0.1444  | 0.0388             | 0.0210       | 0.2679       | 0.0337  | 3.7225      |
| $h$       | 0.3998  | 0.0687             | 0.1811       | 0.6184       | 0.0101  | 5.1814      |
| $k$       | 0.0421  | 0.0154             | -0.007       | 0.0913       | 0.0721  | 2.7281      |
| $\Lambda$ | 3.3510 $\times 10^3$ | 6.6920               | 3.3300 $\times 10^3$ | 3.3723 $\times 10^3$ | 1.7562 $\times 10^{-8}$ | 5.0076 $\times 10^2$ |
| $p_1$     | 0.025   | 0.0125             | -0.0146      | 0.0647       | 0.1384  | 2.0072      |
| $p_2$     | 0.3414  | 0.0538             | 0.1702       | 0.5125       | 0.0079  | 6.3472      |
| $\phi$    | 0.0500  | 0.0255             | -0.0311      | 0.1311       | 0.1444  | 1.9630      |
| $\psi$    | 0.0510  | 0.0123             | 0.0120       | 0.0901       | 0.0252  | 4.1617      |
| $q_1$     | 0.0240  | 0.0077             | 6.7543 $\times 10^{-4}$ | 0.0486       | 0.0535  | 3.0952      |
| $q_2$     | 0.5425  | 0.0390             | 0.4183       | 0.6667       | 0.0008  | 13.9023     |
| $r$       | 0.9219  | 0.0259             | 0.8394       | 1.0045       | 0.0200  | 35.5518     |
| $\rho$    | 0.1993  | 0.0769             | -0.0456      | 0.4442       | 0.0811  | 2.5903      |
| $\theta$  | 0.6082  | 0.1642             | 0.0856       | 1.1308       | 0.0342  | 3.7037      |
| $\omega$  | 0.1986  | 0.0233             | 0.1245       | 0.2726       | 0.0034  | 8.5350      |
| $V_0$     | 9.2311 $\times 10^5$ | 172.1804           | 9.2256 $\times 10^5$ | 9.2366 $\times 10^5$ | 1.4311 $\times 10^{-11}$ | 5.3613 $\times 10^3$ |
| $S_0$     | 4.6139 $\times 10^6$ | 101.8849           | 4.6136 $\times 10^6$ | 4.6142 $\times 10^6$ | 2.3746 $\times 10^{-14}$ | 4.5286 $\times 10^4$ |
| $E_0$     | 1.1199 $\times 10^6$ | 164.3129           | 1.1194 $\times 10^6$ | 1.1204 $\times 10^6$ | 6.9652 $\times 10^{-12}$ | 6.8157 $\times 10^3$ |
| $I_0$     | 2.2255 $\times 4$ | -                   | -             | -             | -       | -           |
| $J_0$     | 4.8511 $\times 10^4$ | 21.2908            | 4.8443 $\times 10^4$ | 4.8579 $\times 10^4$ | 1.8643 $\times 10^{-10}$ | 2.2785 $\times 10^2$ |
| $L_0$     | 5.7244 $\times 10^2$ | 2.5604             | 5.6423 $\times 10^2$ | 5.8065 $\times 10^2$ | 2.0199 $\times 10^{-7}$ | 2.2184 $\times 10^2$ |
| $R_0$     | 6.9875 $\times 10^2$ | 1.9114             | 6.9267 $\times 10^2$ | 7.0483 $\times 10^2$ | 4.5139 $\times 10^{-8}$ | 3.6557 $\times 10^2$ |
Table 5 The value of PRCC between each parameter and $R_0$ and the total infectious.

| Parameters | PRCC | $R_0$ P-value | PRCC | Total P-value |
|------------|------|---------------|------|---------------|
| $\alpha$  | -0.1025 | $4.8981 \times 10^{-10}$ | -0.1248 | $2.5216 \times 10^{-8}$ |
| $\beta_1$ | 0.0281 | $2.1229 \times 10^{-1}$ | 0.1756 | $2.5743 \times 10^{-15}$ |
| $\beta_2$ | 0.0306 | $1.7314 \times 10^{-1}$ | 0.0580 | $9.9135 \times 10^{-3}$ |
| $\beta_3$ | 0.0096 | $6.7013 \times 10^{-1}$ | 0.1614 | $5.1136 \times 10^{-13}$ |
| $\chi$    | -0.0050 | $8.2461 \times 10^{-1}$ | -0.0259 | $2.5024 \times 10^{-1}$ |
| $\delta$  | 0.2269 | $1.5761 \times 10^{-24}$ | 0.3822 | $7.6609 \times 10^{-70}$ |
| $\varepsilon$ | 0.0437 | $5.2022 \times 10^{-2}$ | 0.3028 | $3.0568 \times 10^{-43}$ |
| $\gamma$  | -0.5037 | $7.7684 \times 10^{-128}$ | -0.6457 | $6.9717 \times 10^{-234}$ |
| $\eta$    | -0.0247 | $2.7261 \times 10^{-1}$ | -0.0220 | $3.2766 \times 10^{-1}$ |
| $h$       | 0.2023 | $1.0612 \times 10^{-19}$ | 0.2636 | $8.0894 \times 10^{-33}$ |
| $k$       | 0.8883 | 0.0000 | 0.7931 | 0.0000 |
| $\Lambda$ | -0.0015 | $9.4530 \times 10^{-1}$ | -0.0051 | $8.2052 \times 10^{-1}$ |
| $p_1$     | 0.0713 | $1.4980 \times 10^{-3}$ | 0.2591 | $9.9110 \times 10^{-32}$ |
| $p_2$     | 0.1699 | $2.7490 \times 10^{-14}$ | 0.4083 | $2.3152 \times 10^{-80}$ |
| $\phi$    | -0.3874 | $6.9735 \times 10^{-1}$ | -0.7513 | 0.0000 |
| $\psi$    | 0.2276 | $1.1225 \times 10^{-24}$ | 0.4032 | $3.1138 \times 10^{-78}$ |
| $\gamma_1$ | 0.0423 | $5.9895 \times 10^{-2}$ | 0.0776 | $5.4967 \times 10^{-14}$ |
| $\gamma_2$ | -0.0341 | $1.2968 \times 10^{-1}$ | 0.0046 | $8.3880 \times 10^{-10}$ |
| $r$       | -0.8593 | 0.0000 | -0.7511 | 0.0000 |
| $\rho$    | -0.1269 | $1.4744 \times 10^{-8}$ | -0.1633 | $2.6748 \times 10^{-13}$ |
| $\theta$  | 0.3151 | $7.2127 \times 10^{-47}$ | 0.6273 | $5.8105 \times 10^{-217}$ |
| $\omega$  | -0.2294 | $4.7563 \times 10^{-25}$ | -0.3289 | $3.9428 \times 10^{-51}$ |

Table Captions
Table 1 - Population, tuberculosis Cases, Case Rates per 100,000 Population, Deaths, and Death Rates per 100,000 Population, and Percent Change: United States, 1984-2017. Where, $Rate_{ij} = \frac{Number_{ij} - Number_{ij+1}}{Number_{ij} + Number_{ij+1}}$. $Rate_{ij} = \frac{Rate_{ij} - Rate_{ij+1}}{Rate_{ij} + Rate_{ij+1}} = \frac{Number_{ij} - Number_{ij+1}}{Number_{ij} + Number_{ij+1}} (i = 1, 2)$ and $j$ is the year. For example 2016, $2.9 = \frac{9547/3230.7 - 9547/3230.4}{(9547/3230.7 + 9547/3230.4)}$. The data of population from [5] and others from [6].
Table 2 - The definition and value range of parameters for model (1).
Table 3 - The criteria of MAPE and RMSPE.
Table 4 - The estimated result, property value of parameters and initial condition of each compartment of model (1).
Table 5 - The value of PRCC between each parameter and $R_0$ and the total infected.

Figure Captions
Figure 1 - The flow diagram for compartment model of the transmission dynamics system of TB.
Figure 2 - The comparison of real data and fitted data and the projection for the future status of TB.
Figure 3 - The distribution of the basic reproduction number ($R_0$).
Figure 4 - (a) show the PRCC of parameters with and $R_0$ and (b) show the PRCC of parameters with the total infected. Here, we assume that when p-value $< 0.01$, the parameters have significant effect $R_0$ and the total infectious. In order to better control TB, we put emphasis on analyzing the parameters whose PRCC $> 0.2$.
Figure 5 - Simulation of the total infectious with all parameters $1.03 \times r = 0.9496, 1.5 \times \alpha = 0.1503, 1.5 \times \gamma = 0.7533, 1.5 \times \phi = 0.0750, 0.9 \times \delta = 0.5128, 0.99 \times \psi = 0.0505, 0.7 \times \beta_1 = 3.1067, 0.7 \times \beta_2 = 0.2096$ and $0.7 \times \beta_3 = 4.1063$ from the first column of Table 4, respectively, when one parameters takes a specific value, others take the value of the first column in Table 4. Differently, we synthesize the effects of three contact rates $\beta_i, i = 1, 2, 3$ into one contact rate $\beta$ effect. 'With all control' means that we let all parameters specific values simultaneously. 'Without control' is the situation which we take no measures. We can find $\psi$ has a mild effect on the total infected, with its line overlapping with 'without control' approximately.
Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- control.eps
- Simulation.eps
- reference.bib
- bmcarticle.tex
- R0.eps
- process.eps
- HistR0.eps