D0-branes with non-zero angular momentum

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Abstract.
In my talk I shall consider the mechanism of self-expansion of a system of N D0-branes into high-dimensional non-commutative world-volume investigated by Harmark and Savvidy in [1]. Here D2-brane is formed due to the internal angular momentum of D0-brane system. The idea is that attractive force of tension should be cancelled by the centrifugal motion preventing a D-brane system from collapse to a lower-dimensional one. I shall also present a new extended solution where a total of 9 space dimensions is used to embed a D0-brane system. In the last section, by performing linear analysis, the stability of the system is demonstrated.

INTRODUCTION

In the last years there has been increasing interest in dimensionally reduced supersymmetric Yang-Mills theories [2, 3, 4]. One of the reasons is that the reduction of ten-dimensional theory to \( p + 1 \) dimension is relevant for the description of Dp-branes, \( p \)-dimensional extended objects carrying Ramond-Ramond (RR) charges in type II superstring theories [5]. In the extreme reduction to zero dimensions it is believed to describe D0-branes, fundamental pointlike objects in type IIA superstring theory [6]. In certain energy regimes the dynamics of \( N \) such particles can be described by the supersymmetric quantum mechanics of \( N \times N \) Hermitian matrices obtained from dimensional reduction of \( \mathcal{N} = 1, D = 10 \) super-Yang-Mills theory down to \( 0 + 1 \) dimensions \(^1\). It is believed that supersymmetric quantum mechanics of many D0-branes in type IIA superstring theory is equivalent to a partonic description of light-front M-theory [11], a more fundamental underlying M-theory [8, 9, 10]. The existence of matrix formulation of \( M \)-theory [11], the BRSS-conjecture, crucially relies on the existence within type-IIB string theory of a tower of massive BPS particles electrically charged with respect to the RR 1-form. These particles originally described as black holes in IIA supergravity were identified with D0-branes and can be interpreted as Kaluza-Klein particles of eleven-dimensional \( M \)-theory compactified on a circle. The existence of the \( M \)-theoretic Kaluza-Klein tower of states is equivalent to the statement that supersymmetric Yang-Mills quantum mechanics has exactly one bound state for each \( N \) [12, 13, 14].

Remarkable aspect of matrix theory is that not only classical gravitational interactions can be produced in the large N-limit, but also the appearance of the superstring extended objects in terms of pointlike fundamental degrees of freedom. One example of such

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\(^1\) It was also proposed that \( 0 + 0 \)-dimensional matrix model should give a Poincare invariant non-perturbative definition of IIB superstring theory, the so-called IKKT model [7].
phenomena is the observation that a IIA superstring and a system of N D0-branes can be blown-up to a D2-brane by placing it in background field [15, 16].

In my talk I shall consider another mechanism of self-expansion of a system of N D0-branes into high dimensional non-commutative world-volume investigated by Harmark and Savvidy in [1], here D2-brane is formed due to the internal angular momentum of D0-brane system. The idea is that attractive force of tension should be cancelled by the centrifugal motion preventing a D-brane system from collapse to a lower-dimensional one ². I shall also present a new extended solution where a total of 9 space dimensions is used to embed a D0-brane system. In the last section, by performing linear analysis, the stability of the system is demonstrated.

In the D-brane formulation the dynamics of N D0-branes can be described by the supersymmetric quantum mechanics of $N \times N$ Hermitian matrices obtained from dimensional reduction of $\mathcal{N} = 1, D = 10$ super-Yang-Mills theory to 0+1 dimensions [6, 16, 20, 21] (the quantum mechanical model was originally studied in [4, 22, 23]). The effective action of N D0-branes is the non-abelian SU(N) Yang-Mills action plus the Chern-Simons action

$$S_{YM} = -T_0 (2\pi l_s^2)^2 \int dt \Tr \left( \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right), \quad (1)$$

where $F_{\mu\nu}$ is the non-abelian SU(N) field strength in the adjoint representation and $T_0 = (g_s l_s)^{-1}$ is the D0-brane mass. To write this action in terms of coordinate matrices $X^i$, one has to use the dictionary [1]

$$A_i = \frac{1}{2\pi l_s^2} X^i, \quad F_{0i} = \frac{1}{2\pi l_s^2} \dot{X}^i, \quad F_{ij} = -\frac{i}{(2\pi l_s^2)^2} [X^i, X^j] \quad (2)$$

with $i, j = 1, 2, ..., 9$ and in $A_0 = 0$ gauge we have

$$S_{YM} = T_0 \int dt \Tr \left( \frac{1}{2} \dot{X}^i \dot{X}^i - \frac{1}{4 (2\pi l_s^2)^2} [X^i, X^j] [X^i, X^j] \right). \quad (3)$$

The equations of motion are

$$\ddot{X}^i = -\frac{1}{(2\pi l_s^2)^2} [X^i, [X^i, X^i]] \quad (4)$$

and should be taken together with the Gauss constraint

$$[X^i, X^i] = 0. \quad (5)$$

The Chern-Simons action derived in [16, 20, 21] for the coupling of N D0-branes to bulk RR $C^{(1)}$ and $C^{(3)}$ fields is

$$S_{CS} = T_0 \int dt \Tr \left( C_0 + C_3 \dot{X}^i + \frac{1}{2\pi l_s^2} \left[ C_0^{(3)} [X^i, X^j] + C_3^{(3)} [X^i, X^j] \dot{X}^k \right] \right) \quad (6)$$

² Similar phenomena appear in the cases of rotating branes on spheres [17, 18], "giant gravitons", and in the cases when angular momentum is generated by crossed electric and magnetic BI fields [19].
and describes the interaction of \( N \) D0-branes with slowly varying background fields of Type IIA supergravity. Even though the D0-brane world-volume is only one-dimensional a multiple D0-brane system can couple to a brane charges of higher dimension. Myers [16] considers the system of \( N \) D0-branes in a constant external 4-form RR field strength \( F_4 = dC(3) \) and has found a stable solution, where the D0-branes are polarized and arranged into a static spherical configuration. A lower-dimensional object under the influence of higher-form RR fields may nucleate or be ’blown-up’ into D2-brane. The background field imposes an external force that prevents the collapse of the D-brane to a lower-dimensional one.

Another way to get self-support against collapse is to allow D0-brane system to carry mechanical angular momentum [1]. This new kind of rotating solution of the system of \( N \) D0-branes was constructed by Troels and Konstantin in [1] and in the subsequent articles [24, 25] it was demonstrated that this solution describes a stable D2-brane configuration. Below I shall concentrate mostly on this solution and on it generalizations presented in [25]. The basic idea in their construction is that the attractive force of tension should be cancelled by the centrifugal repulsion force. The earlier work where membrane solution appeared with non-zero angular momentum was [26].

**D2-BRANE FROM MULTIPLE ROTATING D0-BRANES**

I shall briefly review the spherical D2-brane configuration of type IIA string theory since a new solution for a rotating system of \( N \) D0-branes presented below uses the essential elements of this construction. The solution is equivalent to the spherical membrane solution of M(atrix) theory [11, 29].

With the aim to construct a membrane with an \( S^2 \) geometry we shall embed the \( S^2 \) in a three-dimensional space spanned by the 123 directions and consider the ansatz

\[
X_i(t) = \frac{2}{\sqrt{N^2 - 1}} L_i r_i(t), \quad i = 1, 2, 3
\]  

(7)

where the \( N \times N \) matrices \( L_1, L_2, L_3 \) are the generators of the \( N \) dimensional irreducible representation of \( SU(2) \), with algebra

\[
[L_i, L_j] = i \varepsilon_{ijk} L_k.
\]  

(8)

and with the quadratic Casimir \( \sum_{i=1}^{3} L_i^2 = \frac{N^2 - 1}{4} \), so that \( \text{Tr}(L_i^2) = \frac{N(N^2 - 1)}{12} \). For vanishing background fields the Hamiltonian is

\[
H = \frac{NT_0}{2} \left[ \frac{1}{2} \sum_{i=1}^{3} \dot{r}_i^2 + \frac{\alpha^2}{2} \left( r_1^2 r_2^2 + r_1^2 r_3^2 + r_2^2 r_3^2 \right) \right],
\]

where we have introduced a convenient parameter \( \alpha = \frac{2}{\sqrt{N^2 - 1}} \frac{1}{2\pi l_s} \). This gives the equations of motion

\[
\dot{r}_1 = -\alpha^2 (r_2^2 + r_3^2) r_1, \quad \dot{r}_2 = -\alpha^2 (r_1^2 + r_3^2) r_2, \quad \dot{r}_3 = -\alpha^2 (r_1^2 + r_2^2) r_3.
\]  

(9)

This system is otherwise known as \( 0+1 \) dimensional classical \( SU(2) \) YM mechanics [22, 23]. Let us for simplicity take all radii to be equal to each other: \( r_1 = r_2 = r_3 = r \).
With this we have from (7) the physical radius of the membrane \( R^2 = X_1^2 + X_2^2 + X_3^2 = \mathbf{I} \ r^2 \), where \( \mathbf{I} \) is the \( N \times N \) identity matrix. The last formula shows that the \( N \) D0-branes are constrained to lie on an \( S^2 \) sphere of radius \( r \). The equations of motion (9) in this case reduce to the equation \( \ddot{r} = -2\alpha^2 \dot{r}^2 \) and the solution is \( r(t) = R_0 \sin(\alpha t + \phi) \) oscillating between \( R_0 \) and \( -R_0 \). One can trust this solution if \( |r(t)| \ll l_s \sqrt{N} \), \( |\dot{r}(t)| \ll 1 \), \( |\ddot{r}(t)| \ll l_s^{-1} \). Since we also require \( |r(t)| \gg l_s \) we must have \( N \gg 1 \). Thus it is necessary in order to have a large amount of D0-branes to build a macroscopic spherical membrane.

In order to construct the rotating ellipsoidal membrane, viewed as a non-commutative collection of moving D0-branes we shall take previous configuration of the non-commutative fuzzy sphere in the 135 directions, and set it to rotate in the transverse space along three different axis, i.e. in the 12, 34 and 56 planes. We thus use a total of 6 space dimensions to embed our D0-brane system. The corresponding ansatz is [1]

\[
\begin{align*}
X_1(t) &= \frac{2}{\sqrt{N^2 - 1}} L_1 r_1(t) \quad X_2(t) = \frac{2}{\sqrt{N^2 - 1}} L_1 r_2(t) \\
X_3(t) &= \frac{2}{\sqrt{N^2 - 1}} L_2 r_3(t) \quad X_4(t) = \frac{2}{\sqrt{N^2 - 1}} L_2 r_4(t) \\
X_5(t) &= \frac{2}{\sqrt{N^2 - 1}} L_3 r_5(t) \quad X_6(t) = \frac{2}{\sqrt{N^2 - 1}} L_3 r_6(t) 
\end{align*}
\]

where the \( N \times N \) matrices \( L_1, L_2, L_3 \) are the generators of the \( N \)-dimensional irreducible representation of \( SU(2) \). In this ansatz the matrix structure is such that the coordinate matrices are proportional to the \( SU(2) \) generators in pairs and the Gauss constraint (5) is identically satisfied. It is also the only finite-dimensional subalgebra of the group of diffeomorphisms of \( S^2 \), the \( \text{SDiff}(S^2) \) [28]. That is why the \( SU(2) \) ansatz is in some sense unique: it is the only type of solution that carries over to the supermembrane without modification [30].

Substituting the ansatz into (3) gives the Hamiltonian \( H = \frac{N T_0}{3} \left( \frac{1}{2} \sum_{i=1}^{6} \dot{r}_i^2 + \frac{\alpha^2}{2} \left[ (r_1^2 + r_2^2) (r_3^2 + r_4^2) + (r_1^2 + r_3^2) (r_2^2 + r_4^2) + (r_1^2 + r_4^2) (r_2^2 + r_3^2) \right] \right) \) and the corresponding equations of motion

\[
\begin{align*}
\dot{r}_1 &= -\alpha^2 (r_3^2 + r_4^2 + r_5^2 + r_6^2) r_1 \\
\dot{r}_2 &= -\alpha^2 (r_3^2 + r_4^2 + r_5^2 + r_6^2) r_2 \\
\dot{r}_3 &= -\alpha^2 (r_1^2 + r_2^2 + r_4^2 + r_6^2) r_3 \\
\dot{r}_4 &= -\alpha^2 (r_1^2 + r_2^2 + r_3^2 + r_5^2) r_4 \\
\dot{r}_5 &= -\alpha^2 (r_1^2 + r_2^2 + r_3^2 + r_4^2) r_5 \\
\dot{r}_6 &= -\alpha^2 (r_1^2 + r_2^2 + r_3^2 + r_4^2) r_6
\end{align*}
\]

The special solution of these equations, describing a rotating ellipsoidal membrane with three distinct principal radii \( R_1, R_2 \) and \( R_3 \) is [1]

\[
\begin{align*}
r_1(t) &= R_1 \cos(\omega_1 t + \phi_1) \\
R_2(t) &= R_1 \sin(\omega_1 t + \phi_1)
\end{align*}
\]

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3 Clearly the spherical membrane will be classically point-like at the nodes of the \textit{sinus}. Thus the membrane solution will break down after a finite amount of time, since the classical solution may not be valid at substringy distances, and possibly decay into a Schwarzschild black hole [29].
\[
\begin{align*}
    r_3(t) &= R_2 \cos(\omega_2 t + \phi_2), \\
    r_4(t) &= R_2 \sin(\omega_2 t + \phi_2), \\
    r_5(t) &= R_3 \cos(\omega_3 t + \phi_3), \\
    r_6(t) &= R_3 \sin(\omega_3 t + \phi_3).
\end{align*}
\]

(12)

This particular functional form of the solution ensures that the highly non-linear equations for any of the components \( r_i \) are reduced to a harmonic oscillator. The solution (12) keeps \( r_1^2 + r_2^2 = R_1^2 \), \( r_3^2 + r_4^2 = R_2^2 \) and \( r_5^2 + r_6^2 = R_3^2 \) fixed, which allows us to say that the object described by (12) rotates in six spatial dimensions as a whole without changing its basic shape. At any point in time one can always choose a coordinate system in which the object spans only three space dimensions.

Using the equations of motion (11), the three angular velocities are determined by the radii, and do not necessarily have to coincide: \( \omega_1 = \alpha \sqrt{R_2^2 + R_3^2} \), \( \omega_2 = \alpha \sqrt{R_1^2 + R_3^2} \), \( \omega_3 = \alpha \sqrt{R_1^2 + R_2^2} \). This dependence of the angular frequency on the radii is such that the repulsive force of rotation has to be balanced with the attractive force of tension in order for (12) to be a solution. Thus the radii \( R_1 \), \( R_2 \) and \( R_3 \) parameterize (12) along with the three phases \( \phi_i \) to produce altogether a six-parameter family of solutions. In order to exhibit the properties of the solution (12) one shall evaluate the energy, the components of angular momentum and to find out nonzero D-brane currents. The non-zero components of \( M_{ij} \) are \( M_{12} = -M_{21} \), \( M_{34} = -M_{43} \) and \( M_{56} = -M_{65} \) and correspond to rotations in the 12, 34 and 56 planes respectively. Their values fit with the interpretation of the solution as \( N \) D0-branes rotating as an ellipsoidal membrane in that they are time-independent due to conservation law and proportional to \( N \): \( M_{12} = \frac{1}{2} NT_0 \omega_1 R_1^2 \), \( M_{34} = \frac{1}{2} NT_0 \omega_2 R_2^2 \), \( M_{56} = \frac{1}{2} NT_0 \omega_3 R_3^2 \), thus

\[
    M^2 = \frac{1}{9} (NT_0)^2 \left( \omega_1^2 R_1^4 + \omega_2^2 R_2^4 + \omega_3^2 R_3^4 \right), \quad E = \frac{NT_0}{4} \left( \omega_1^2 R_1^2 + \omega_2^2 R_2^2 + \omega_3^2 R_3^2 \right).
\]

(13)

Let us now compute the coupling of the solution (12) to the \( C^{(3)} \) RR potential. The interaction with the \( C^{(3)} \)-field is governed by the action (6). We denote the corresponding current by \( J^{ijk} \)

\[
    J^{ijk} = \frac{1}{2\pi l_s^2} \text{Tr} \left( [X^i, X^j] X^k \right),
\]

(14)

and we have to impose anti-symmetrization with respect to \( ijk \) indices. For our solution the non-zero components of \( Q \) are: \( Q_{135} = \frac{1}{2} NT_0 R_1 R_2 R_3 \cos \omega_1 t \cos \omega_2 t \cos \omega_3 t \), together with \( Q_{246}, Q_{146}, Q_{136}, Q_{235}, Q_{236}, Q_{145}, Q_{245} \). From this it is easy to obtain the corresponding \( J \)’s by differentiation with respect to time. Thus the Chern-Simons action (6) shows that the coupling of this system to \( F_{0123} \) is non-vanishing and that the spherical membrane solution has a D2-brane dipole moment. The higher currents are equal to zero and our system does not carry D4-D8-charges.

In addition I shall present "breathing" brane solutions. For that it is convenient to introduce polar coordinates \( (\rho, \cos \phi, \rho \sin \phi) \) so that the Hamiltonian \( \hat{H} = \frac{3M}{NT_0} \) takes the form: \( \hat{H} = \frac{1}{2} \sum_{i=1}^{3} \left[ \dot{\rho}_i^2 + \rho_i^2 \dot{\phi}_i^2 \right] + \frac{1}{2} \left[ \rho_1^2 \dot{\rho}_2^2 + \rho_2^2 \rho_3^2 + \rho_3^2 \rho_1^2 \right] \). The conservation integrals are: \( \rho_1^2 \dot{\phi}_1 = \dot{M}_1, \quad \rho_2^2 \dot{\phi}_2 = \dot{M}_2, \quad \rho_3^2 \dot{\phi}_3 = \dot{M}_3 \), where \( \dot{M}_i = \frac{3M}{NT_0} \rho_i^2 \) and the Hamiltonian takes the
form: $\dot{H} = \frac{1}{2} \sum_{i=1}^{3} \left[ \rho_i^2 + \frac{\tilde{M}_i^2}{\rho_i^3} + \frac{1}{2} [\rho_i^2 \rho_j^2 + \rho_2^2 \rho_3^2 + \rho_3^2 \rho_1^2] \right]$. The equations of motion are:

$$\ddot{r}_i = -\rho_i (\dot{r}_i^2 - \dot{r}_j^2) + \tilde{M}_i^2 / \rho_i^3, \quad i = 1, 2, 3$$

(15)

and our previous solution (12) is $r_i = R_i = \text{Const}, i = 1, 2, 3$ and $\phi_i^* = \omega_i^* R_i = \dot{R}_i = \phi(t)$ can depend on time, the "breathing" brane solution, then $\dot{H} = \frac{3}{2} [\rho^2 + \tilde{M}_i^2 + \tilde{M}_j^2]$ and corresponding equation can be integrated. The new solution is elliptic function $\rho = \rho(t)$ [24].

In order to increase the number of parameters of the rotating N D0-brane system, I shall take previous configuration and set it to rotate in the transverse spaces along three different axis, i.e. in the 123, 456 and 789 planes. Thus I shall use a total of 9 space dimensions to embed D0-brane system. The corresponding ansatz is

$$X_i(t) = \frac{2}{\sqrt{N_i - 1}} L_i r_i(t), \quad X_2(t) = \frac{2}{\sqrt{N_2 - 1}} L_1 r_2(t), \quad X_3(t) = \frac{2}{\sqrt{N_3 - 1}} L_1 r_3(t),$$

$$X_4(t) = \frac{2}{\sqrt{N_4 - 1}} L_2 r_4(t), \quad X_5(t) = \frac{2}{\sqrt{N_5 - 1}} L_2 r_5(t), \quad X_6(t) = \frac{2}{\sqrt{N_6 - 1}} L_2 r_6(t),$$

$$X_7(t) = \frac{2}{\sqrt{N_7 - 1}} L_3 r_7(t), \quad X_8(t) = \frac{2}{\sqrt{N_8 - 1}} L_3 r_8(t), \quad X_9(t) = \frac{2}{\sqrt{N_9 - 1}} L_3 r_9(t).$$

(16)

The coordinate matrices are again proportional to the $SU(2)$ generators and the Gauss constraint (5) is identically satisfied. Substituting the last ansatz into (3) gives the Hamiltonian: $H = \frac{NT_0}{3} \left( \frac{1}{2} \sum_{i=1}^{9} \dot{r}_i^2 + 2 \left[ (\dot{r}_1^2 + \dot{r}_2^2 + \dot{r}_3^2 + \dot{r}_4^2 + \dot{r}_5^2 + \dot{r}_6^2 + \dot{r}_7^2 + \dot{r}_8^2 + \dot{r}_9^2) (r_1^2 + r_2^2 + r_3^2 + r_4^2 + r_5^2 + r_6^2 + r_7^2 + r_8^2 + r_9^2) \right] \right)$ and the equations

$$\ddot{r}_i = -\alpha^2 (r_4^2 + r_5^2 + r_6^2 + r_7^2 + r_8^2 + r_9^2) r_i, \quad i = 1, 2, 3,$$

$$\ddot{r}_j = -\alpha^2 (r_1^2 + r_2^2 + r_3^2 + r_4^2 + r_5^2 + r_6^2 + r_7^2 + r_8^2 + r_9^2) r_j, \quad j = 4, 5, 6,$$

$$\ddot{r}_k = -\alpha^2 (r_1^2 + r_2^2 + r_3^2 + r_4^2 + r_5^2 + r_6^2 + r_7^2 + r_8^2 + r_9^2) r_k, \quad k = 7, 8, 9.$$ \hspace{0.5cm} (17)

The special solution of these equations, which ensures that the highly non-linear equations for any of the components $r_i$ are reduced to a harmonic oscillator, is:

$$r_1(t) = R_1 \cos(\omega_1 t + \phi_1), \quad r_2(t) = R_1 \sin(\theta_1) \cdot \sin(\omega_1 t + \phi_1), \quad r_3(t) = R_1 \cos(\theta_1) \cdot \sin(\omega_1 t + \phi_1),$$

$$r_4(t) = R_2 \cos(\omega_2 t + \phi_2), \quad r_5(t) = R_2 \sin(\theta_2) \cdot \sin(\omega_2 t + \phi_2), \quad r_6(t) = R_2 \cos(\theta_2) \cdot \sin(\omega_2 t + \phi_2),$$

$$r_7(t) = R_3 \cos(\omega_3 t + \phi_3), \quad r_8(t) = R_3 \sin(\theta_3) \cdot \sin(\omega_3 t + \phi_3), \quad r_9(t) = R_3 \cos(\theta_3) \cdot \sin(\omega_3 t + \phi_3).$$

The object described by (18) rotates in nine spatial dimensions as a whole without changing its basic shape. The radii $R_1$, $R_2$ and $R_3$ parameterize (18) along with

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4 A somewhat similar ansatz was proposed in [31], where some of the features of the solution were foreseen.
the six phases $\theta_i$ and $\phi_i$, to produce altogether a nine-parameter family of solutions. The energy and the angular momentum are the same (13). The interaction with the $C^{(5)}$-field is governed by the Chern-Simons action derived in [16, 20, 21] 
\[
\frac{T_0}{(2\pi l_s^2)^{2}} \int dt S \text{Tr} \left( C^{(5)}_{0ijk} [X^i, X^j] [X^k, X^l] + C^{(5)}_{ijklm} [X^i, X^j] [X^k, X^l] X^m \right).
\]
We denote the corresponding current by $J^{ijklm}$
\[
J^{ijklm} = \frac{1}{(2\pi l_s^2)^{2}} S \text{Tr} \left( [X^i, X^j] \dot{X}^k [X^l, X^m] \right).
\]
(19)
One can be convinced that all components of $J$ are equal to zero. Thus the Chern-Simons action shows that the coupling of this system to $F_{ijklm}$ is vanishing and that the extended solution does not carry D4-brane charge.

**STABILITY ANALYSIS WITHIN SU(2) AND FULL SU(N) GROUP**

The purpose of this section is to present a complete stability analysis of the fluctuations in the neighborhood of the rotating D0-brane solution of [1]. Initially in [1] were analyzed perturbations that do not modify the original SU(2) ansatz. In [25, 24] this analysis was extended to the case when perturbations are in the full SU(N) algebra directions. In the full SU(N) case there are exactly $N^2 + 12$ zero-modes, of which $N^2 - 1$ are the consequence of the global color rotation symmetry of the solution, and 6 are associated with global space rotations. All other modes are completely stable and execute harmonic oscillations around the original trajectory.

Let me present the stability analysis of the SU(2) ($l = 1$) perturbation of the system [1] and then move to more general SU(N) ($l = 2, 3, 4, \ldots$) perturbations [25, 24]. The equations of variation which follow from equations in polar coordinates (15) are:
\[
\begin{align*}
\delta \rho_1 &= -4(R_2^2 + R_3^2) \delta \rho_1 - 2R_1 R_2 \delta \rho_2 - 2R_1 R_3 \delta \rho_3 \\
\delta \rho_2 &= -2R_1 R_2 \delta \rho_1 - 4(R_1^2 + R_3^2) \delta \rho_2 - 2R_2 R_3 \delta \rho_3 \\
\delta \rho_3 &= -2R_1 R_3 \delta \rho_1 - 2R_3 R_2 \delta \rho_2 - 4(R_1^2 + R_2^2) \delta \rho_3,
\end{align*}
\]
and have only positive modes [24]
\[
\Omega_1^2 = 4(R_1^2 + R_2^2 + R_3^2), \quad \Omega_{2,3}^2 = 2(R_2^2 + R_2^2 + R_3^2) \pm \sqrt{2((R_2^2 - R_3^2)^2 + (R_2^2 - R_3^2)^2 + (R_2^2 - R_3^2)^2)}. \quad (24)
\]
(24)
To consider perturbations in all directions of underlying SU(N) group we should represent SU(N) generators $Y^l_m$ as higher order monomials in the $N \times N$ matrix generators $L_i$, $i = 1, 2, 3$ of the SU(2) group
\[
Y^l_m = \sum_{i_1, \ldots, i_l} c^l_{m (i_1, \ldots, i_l)} L_{i_1} \cdots L_{i_l}.
\]
(25)
The total number of generators is then $\sum_{l=1}^{N-1} (2l + 1) = N^2 - 1$ as it should be for SU(N). The general explicit construction of the $Y^l_m$ is due to Schwinger (see also [32, 33, 27, 34]).
Let us represent the rotating D0-brane solution in a more convenient form: \( X'(t) = L_i R \cos(\omega t), \quad \dot{X}'(t) = X'^{i+1} = L_i R \sin(\omega t), \quad i = 1, 2, 3.\) In what follows we will set \( R = 1, \) with \( \omega \) equal to \( \omega'^2 = 2R^2 = 2.\) In the basis provided by the spherical operators \( Y^l_m \) (25) we have [28, 30],

\[
\begin{align*}
\left[ L_z, Y^l_m \right] &= m Y^l_m \quad \text{for } l = 1, \ldots, N - 1 \\
\left[ L_{\pm}, Y^l_m \right] &= \sqrt{(l \mp m)(l \pm m + 1)} Y^l_{m \pm 1}.
\end{align*}
\]

(26)

We will not use the explicit form of these matrices, as the defining relations (26) is all that is needed. The properties under Hermitian conjugation can be summed up as

\[
\eta^* = \xi, \quad \eta^* = \eta^T = \eta_m^* = (-1)^m \eta_{-m} \quad \text{for all } m = -l, \ldots, l.
\]

The variational equations of motion are

\[
-\delta \ddot{X}^i = \left[ \delta X^j, [X^j, X^i] \right] + [X^j, \left[ \delta X^j, X^i \right]] + [X^j, \left[ X^j, \delta X^i \right]] + \left[ \delta X^j, [X^j, X^i] \right] + \left[ \dot{X}^j, \left[ \delta X^j, X^i \right] \right] + \left[ X^j, \left[ \dot{X}^j, \delta X^i \right] \right].
\]

(28)

The constraint equation looks like

\[
\sum_{i, m} \left[ \delta X^j, X^i \right] + [X^j, \delta X^i] + [\delta X^j, \dot{X}^i] + [X^j, \delta \dot{X}^i] = 0.
\]

(29)

Using the commutation relations (26) we get for the constraint \( \sum_i L^j_{nm} (\cos(\omega t) \xi^i_m + \cos(\omega t) \eta^i_m + \sin(\omega t) \eta^i_m) = 0,\) where \( L^j_{nm} \) are now the \( SU(2) \) generators in the \( (2l + 1) \times (2l + 1) \) representation. In the co-moving coordinates

\[
u^i_m = \cos(\omega t) \xi^i_m + \sin(\omega t) \eta^i_m \quad \text{and} \quad v^i_m = -\sin(\omega t) \xi^i_m + \cos(\omega t) \eta^i_m
\]

(30)

the constraint looks simpler,

\[
\sum_{i, m} L^j_{nm} (\dot{u}^i_m - 2\omega v^i_m) = 0.
\]

(31)

The variational equation of motion (28) after substituting the fields (27) is

\[
\dot{\xi}^i_m + l(l + 1) \xi^i_m = \cos(\omega t) \left( L^j_{nm} L^i_{nm} + i \varepsilon jk L^k_{nm} \right) (\cos(\omega t) \xi^i_m + \sin(\omega t) \eta^i_m).
\]

(32)

The decoupling of the modes with different \( l \) is seen to be a direct consequence of (26), and more fundamentally, of the pure \( SU(2) \) structure of the original background.
solution (12). The equation for \( \eta \) is gotten by exchanging cosines for sines and \( \xi \) for \( \eta \).

In the co-moving coordinates (30) the equation becomes a linear system with constant coefficients:

\[
\begin{align*}
\ddot{u}_n^i + (l(l + 1) - 2) u_n^i - 2\omega v_n^i &= \left( L_{i}^{j}L_{i}^{k} + i\epsilon_{jik}L_{n}^{k} \right) u_m^i, \quad (33) \\
\ddot{v}_n^i + (l(l + 1) - 2) v_n^i + 2\omega u_n^i &= 0. \quad (34)
\end{align*}
\]

Thus we shall analyze the system of equations (31) (33), (34). The \( \text{rhs} \) of (33) is a matrix acting on a \( 3(2l + 1) \) component vector and the eigenvalues \( \Lambda \) of this block matrix are given in the table, together with their multiplicity [25]. Choose a fixed frequency ansatz \( u_n^i(t) = e^{i\Omega t} u_n^i, \quad v_n^i(t) = e^{i\Omega t} v_n^i \). The second equation (34) can be solved as

\[
\dot{v}_n^i = \frac{-2\sqrt{2}i\Omega^2}{l(l+1) - 2 - \Omega^2} \dot{u}_n^i,
\]

and then substituted back into the first equation (33). In the basis in which the matrix on the \( \text{rhs} \) is diagonalized, it can be replaced with its respective eigenvalue \( \Lambda \), resulting in an algebraic equation for the \( \Omega \):

\[
(l(l + 1) - 2 - \Omega^2)^2 - 8\Omega^2 = \Lambda (l(l + 1) - 2 - \Omega^2). \]

Finally, this quadratic equation can be solved, \( \Omega_{1,2}^2 = -\frac{1}{2}\Lambda + l(l + 1) + 2 \pm \frac{1}{2} \sqrt{\Lambda^2 - 16\Lambda + 32l(l + 1)} \) and the corresponding modes are given in the table

| \( \Lambda \)    | \( \Omega_1^2 \) | \( \Omega_2^2 \) | multiplicity |
|----------------|------------------|-----------------|--------------|
| \( l(l + 1) - 2 \) | 0                | \( l^2 + l + 6 \) | 2l + 1       |
| 2l             | \( l^2 - 3l + 2 \) | \( l^2 + 3l + 2 \) | 2l + 3       |
| -(2l + 2)      | \( l^2 - l \)    | \( l^2 + 5l + 6 \) | 2l - 1       |

Note that the number of zero modes changes from 9 for the case \( l = 1 \) and 12 for \( l = 2 \), to \( 2l + 1 \) for arbitrary \( l > 2 \). Thus the total number of zero modes is the sum \( 9 + 12 + \sum_{n=3}^{N} (2l + 1) = N^2 + 12 \). Now I shall consider perturbations that are in the directions 789, if we had oriented the original solution along 123456. The perturbations \( \delta X_k = \sum_m Y_m^i \epsilon_{m}^{k} \) for \( k = 7, 8, 9 \) satisfy the simple harmonic equation \( \ddot{\epsilon}_{m}^{k} + \Lambda_{m}^{k} = 0 \). This clearly has only positive frequencies and is therefore stable. For \( l = 1 \) all the 9 modes have the same frequency as the original solution, corresponding to infinitesimal global rotations of the system into the 789 hyperplane. The counting goes as follows, there are \( 9 \times 2 = 18 \) first order degrees of freedom here, which coincides with the dimensionality of the Grassmannian manifold of embeddings of a 6-hyperplane into \( R^9 \), i.e. \( SO(9) \times SO(3) \). From these results it follows that, zero-modes notwithstanding, all the frequencies in the system are positive, and arbitrary small perturbation will remain bounded for all times.\(^5\)

\(^5\) The same problem was considered also in the paper [35]. However the authors of [35] initially arrived at the Mathieu equation instead of the equations (33), (34), (31) and therefore to the opposite conclusion,
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