THE QUEST FOR THE CABIBBO KOBAYASHI MASKAWA MATRIX

M. Artuso

Syracuse University, Syracuse, NY 13244

ABSTRACT

A piece of the Standard Model presently undergoing intense experimental scrutiny is the Cabibbo Kobayashi Maskawa matrix. Several different measurements are planned to enrich the spectrum of experimental constraints and thus provide one of the most stringent tests of Standard Model validity. The success of this program is closely related to theoretical progress in evaluating QCD matrix elements in a non-perturbative regime, as we need to extract fundamental quark properties from observations on decays involving hadrons. This interplay between experimental and theoretical progress will be illustrated in the context of the present knowledge of the magnitudes of the quark mixing parameters $|V_{cb}|$ and $|V_{ub}|$.

1artuso@phy.syr.edu
1 Introduction

In the framework of the Standard Model the gauge bosons, $W^\pm$, $\gamma$ and $Z^0$ couple to mixtures of the physical $d$, $s$ and $b$ states. This mixing is described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \quad (1)$$

Since the CKM matrix must be unitary, it can be expressed as a function of only four parameters. A commonly used approximate parameterization was originally proposed by Wolfenstein. It reflects the hierarchy between the magnitude of matrix elements belonging to different diagonals. In the form accurate to $\lambda^3$ for the real part and $\lambda^5$ for the imaginary part, it is given by:

$$\begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(1 - \lambda^2/2) \\ \lambda & 1 - \lambda^2/2 - i\eta A^2 \lambda^4 & A\lambda^2(1 + i\eta \lambda^2) \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \quad (2)$$

This matrix has already several experimental constraints [1], and both the number of measurements and their accuracy will see remarkable improvements in the next few years and may eventually lead to evidence for new physics. In order to accomplish this goal, precision measurements are needed, as well as sophisticated theoretical calculations able to evaluate QCD matrix elements in a regime where non-perturbative effects are important. The interplay between theoretical and experimental errors will be discussed with reference to the two quark mixing parameters $|V_{cb}|$ and $|V_{ub}|$.

2 Experimental determination of the quark mixing parameter $|V_{cb}|$

This parameter enables us to determine the Wolfenstein parameter $A$. It strongly affects the effectiveness of the $CP$ violation parameter in the $K^0 - \bar{K}^0$ system, $\epsilon_K$ [2], in constraining $\rho$ and $\eta$, as $\epsilon_K$ depends on $A^4$. $|V_{cb}|$ is measured by studying semileptonic decays of the type $B \to X_c \ell \nu$, where $X_c$ is a charmed hadron. Two approaches have been taken: an “inclusive” method, focusing on the lepton momentum spectrum, and an “exclusive” method, studying a specific channel, most notably the dominant decay $B \to D^* \ell \nu$. 

1
The decay $B \to D^* \ell \nu$ has received considerable attention, especially after the theoretical development known as Heavy Quark Effective Theory (HQET), that has offered the opportunity to replace quark models of various nature with an effective theory, exact in the limit of infinite quark masses and where non-perturbative effects can be expressed in powers of $1/m_Q$. In this approach, the distribution $d\Gamma/dw$ is given by:

$$d\Gamma/dw = \frac{G_F^2|V_{cb}|^2}{48\pi^3}K(w)F(w)^2,$$

where $w$ is the inner product of the $B$ and $D^*$ meson 4-velocities, $K(w)$ is a known phase space factor and the form factor $F(w)$ is generally expressed as the product of a normalization factor $F(1)$ and a shape function, $g(w)$, constrained by dispersion relations [4]. There are several different corrections to the infinite mass value $F(1) = 1$:

$$F(1) = \eta_{QED}\eta_A [1 + \delta_{1/m^2} + ...]$$

Note that, by virtue of Luke’s theorem [3], the first term in the non-perturbative expansion in powers of $1/m_Q$ vanishes. QED corrections up to leading logarithmic order give $\eta_{QED} \approx 1.007$, QCD radiative corrections to two loops give $\eta_A = 0.960 \pm 0.007$ and different estimates of the $1/m^2$ corrections, involving terms proportional to $1/m_b^2$, $1/m_c^2$ and $1/m_b m_c$, give an average value $\delta_{1/m^2} = -0.55 \pm 0.035$ [4]. These corrections give $F(1) = 0.913 \pm 0.007 \pm 0.024 \pm 0.011$, where the first error represents uncertainties in radiative corrections, the second uncertainties in $1/m^2$ corrections and the last one is related to higher order power corrections. Adding the errors linearly, we get $F(1) = 0.913 \pm 0.042$, that will be used to extract $|V_{cb}|$ from data. The preliminary value from a quenched Lattice HQET calculation is $0.931 \pm 0.035$ [3], in good agreement with the previous estimate. This is a situation that is good: several different approaches have been used to evaluate a crucial input parameter and they have close central values and comparable uncertainties.

Experiments determine the product $F(1)|V_{cb}|$ by fitting the measured $d\Gamma/dw$ distribution. Fig. [4] shows the recent CLEO measurement [7] of $F(w)|V_{cb}|$ as a function of $w$, based on a data sample of 3.33 millions $B\bar{B}$ pairs. They obtain $F(1)|V_{cb}| = 0.0424 \pm 0.0018 \pm 0.0019$, where the first error is statistical and the second is systematic, dominated by the uncertainty in slow π finding efficiency. The LEP experiments have also studied $F(w)|V_{cb}|$
with different experimental approaches \cite{8}. Table 1 summarizes the data available so far. The agreement between these measurements is far from perfect. In the LEP case, the dominant source of error is the subtraction of the so called “D***” contribution to the $d\Gamma/dw$ distribution.

An alternative approach is to use the measured semileptonic width to charmed hadrons and relate it to $|V_{cb}|$ through the Heavy Quark Expansion (HQE) \cite{9}, where the semileptonic width is expressed in terms of the $b$ quark mass $m_b$ and a parameter $\mu_\pi$, related to the average kinetic energy of the $b$ quark moving inside the $B$ hadron. They obtain:

$$|V_{cb}| = \frac{0.0411\sqrt{1.55}}{0.105} \Gamma(B \rightarrow X_c\ell\nu)(ps^{-1}) \left[ 1 - 0.025 \left( \frac{\mu_\pi^2-0.5\text{GeV}^2}{0.2\text{GeV}^2} \right) \right] \left[ 1 \pm 0.01|m_b \pm 0.01|_{\text{pert}} \pm 0.015\frac{1}{m_b^3} \right].$$

The semileptonic width depends effectively upon $m_b^2(m_b - m_c)^3$ \cite{9}. Thus $|V_{cb}| \approx m_b(m_b - m_c)^{1.5}$. The $m_b$ errors are taken from the most recent extractions of the so called ‘kinetic’ $m_b$ (of the order of 1-1.5%). The term $(m_b - m_c)$ is related to the spin averaged mass of the $B$ and $D$ mesons:

$$m_b - m_c = \langle M_B \rangle - \langle M_D \rangle + \mu_\pi^2 \left( \frac{1}{m_c} - \frac{1}{m_b} \right) + O(1/m_{b,c}^2).$$

The uncertainty in $\mu_\pi^2$ gives an error of $\pm 0.025$ in $|V_{cb}|$ and is mostly related to the $m_b - m_c$ dependence upon this parameter. No error is given for the $1/m^2$ term in eq. 6. The theoretical errors quoted above, added linearly, amount to about 6.0% \cite{9}, a figure comparable to the 4.6% in the theoretical extraction with the exclusive method. The most recent value of $|V_{cb}|_{\text{incl}}$ combining information from the four LEP experiments is $(40.76 \pm 0.41 \pm 2.0) \times 10^{-3}$. The first error is the combined statistical and systematic error in the measurement, added in quadrature, and the latter is the theoretical error, assumed to be 4.9%. An issue that has gained considerable attention in recent years \cite{10} is a possible sizeable source of errors related to the assumption of quark-hadron duality, crucial to the calculation that lead to eq. 5. More experimental checks on the applicability of the quark hadron duality ansatz and measurements of $m_b$, $m_b - m_c$ and $\mu_\pi$ need to be performed before we can claim a full understanding of the uncertainties in this extraction.
3 Experimental determination of $|V_{ub}|$

The hurdles along the path towards a precise determination of the parameter $V_{ub}$ are even more challenging than in the $V_{cb}$ case. The reason is again deeply rooted in the difficulties that we need to overcome to obtain a good estimate of the relevant hadronic matrix elements. In this case there is no effective theory like HQET to provide a reliable form factor normalization. A variety of calculations of such form factors exist, based on lattice gauge theory [11], light cone sum rules (LCSR) [12], and quark models [13]. Most of them focus on the lightest charmed hadrons recoiling against the lepton-ν pair, the π and ρ mesons. In this case they are far from saturating the semileptonic decay width to charmless hadrons. Moreover, because of the light mass of the “ground state” hadrons recoiling against the lepton-neutrino pair, a much wider $q^2$ region is spanned by these decays, adding a strong sensitivity to the $q^2$ dependence of the form factors involved. Lattice gauge calculations are progressing, but they will produce reliable results for exclusive decays only in the vicinity of $q^2_{max}$ [14].

The first experimental evidence for a non-zero value of the parameter $|V_{ub}|$ was provided by CLEO [15], and soon corroborated by ARGUS [16]. It was based upon an excess of leptons with momentum greater than the maximum allowed in the decay $B \rightarrow X_c \ell \nu$. Although subsequent CLEO data [17] provided a very good measurement of the endpoint lepton yield, there are very convincing theoretical arguments that point to a substantial limitation posed by the stringent lepton momentum cut. The Operator Product Expansion (OPE) cannot give reliable predictions because the momentum region considered is of the order of $\Lambda_{QCD}$ and thus an infinite series of terms in this expansion may be relevant.

The CLEO collaboration reported the first convincing evidence for the decays $B \rightarrow \rho \ell \nu$ and $B \rightarrow \pi \ell \nu$ [18]. CLEO subsequently performed a measurement of the decay $B \rightarrow \rho \ell \nu$ with a different technique and a bigger data sample [19]. They used several different models to extract the value of $|V_{ub}|$. Their results are summarized in Table 2. The first three calculations are based on quark models and their uncertainties are guessed to be in the 25-50% range in the rate, corresponding to a 12.5-25% uncertainty for $|V_{ub}|$. The other approaches, light cone sum rules and lattice QCD, estimate their errors in the range of 30%, leading to a 15% error in $|V_{ub}|$. We can conclude that the average value of $|V_{ub}|$ extracted with this method is $|V_{ub}| = (3.25 \pm 0.14^{+0.21}_{-0.31} \pm 0.5) \times 10^{-3}$. This corresponds to a value of
\[ |V_{ub}/V_{cb}| = 0.080 \pm 0.014. \] The statistical and systematic errors have been added in quadrature and the theoretical error has been added linearly to be conservative. Note that the theoretical error is somewhat arbitrary.

Recently, interest has been stirred by a new approach to the extraction of \( |V_{ub}| \) based on the OPE approach. The idea is that if the semileptonic width \( \Gamma_u \) is extracted by integrating over the hadronic mass \( X_u \) recoiling against the lepton neutrino pair in a sufficiently large region of phase space, the relationship between \( |V_{ub}| \) and the measured value of the charmless semileptonic branching fraction can be reliably predicted. Early estimates ([20], [21]) showed the potential of this method and assessed the errors on \( |V_{ub}| \) to be of the order of 10-15\% if a sufficient \( X_u \) range was considered. A subsequent analysis [26] gave the following assessment of the theoretical uncertainties:

\[
|V_{ub}| = 0.00442 \left( \frac{B_{d}(B^{0} \to X_u e \nu)}{0.002} \right)^{0.5} \left( \frac{1.55\text{ps}}{\tau_B} \right)^{0.5} (1 \pm 0.025_{QCD} \pm 0.035_{mb}) \tag{7}
\]

The first error is a lumped estimate of perturbative and non-perturbative QCD corrections. Adding the errors linearly, one would assume a 6\% error in the theory extrapolation. A recent analysis [25] favors a more conservative but still optimistic 10\% theoretical error.

All the LEP experiments but OPAL attempted to use this technique to determine \( |V_{ub}| \). The three experiments combine their analyses and quote \( |V_{ub}| = (4.13^{+0.42}_{-0.47}(\text{stat+det})^{+0.43}_{-0.48}(b \to c)^{+0.24}_{-0.25}(b \to u) \pm 0.02(\tau_b) \pm 0.20(\text{HQE}) \times 10^{-3} \) [24]. These measurements have significant \( b \to c \) background that needs to be understood very well, given the small value of \( |V_{ub}/V_{cb}|^2 \) (\( \approx 1\% \)). Moreover, predictions to validate the precision of the method to measure \( |V_{ub}| \) are needed. The authors of one of the original papers [20] include an interesting statement in their abstract: “\( |V_{ub}| \) can be extracted [with the method proposed] in a largely model-insensitive way. This conclusion is based on the applicability of the OPE to actual semileptonic B decays. A direct cross-check of this assumption and a determination of the required basic parameters of the heavy quark expansion will be possible in the future with more experimental data.” I think that this is a very important program, not yet completed. Important tests include the extraction of \( m_b \) and \( \mu_2^{2} \) from moments of the lepton energies and hadron invariant mass in semileptonic decays [27]. Comparing the two sets of values for \( m_b \) and \( \mu_2^{2} \) from the two different moment analyses among themselves and with the theoretical evaluation would provide an important check. An early preliminary analysis from
CLEO seemed to yield inconsistent results [28], but we do not have yet a definitive answer.

Much work needs to be done to achieve a precise measurement of $|V_{ub}|$. This is a quite important element of our strategy to pin down the CKM sector of the Standard Model. On the theoretical side, large efforts are put in developing more reliable methods to determine the heavy to light form factors. A combination of several methods [29], all with a limited range of applicability, seem to be the strategy more likely to succeed. For instance, lattice QCD can provide reliable estimates of the form factors at large momentum transfer, where the discretization errors are under control. HQET predicts a relationship between semileptonic $D$ decays and semileptonic $B$ decays. To check these predictions and apply them to $|V_{ub}|$ estimates, large data sample with reconstructed neutrino momentum are necessary.

4 Conclusions

This discussion has been focused only on a partial set of the information used to constrain the $\rho$ and $\eta$ Wolfenstein parameters. My goal is to urge the community to be cautious in drawing conclusions on the most probable value of $\rho$ and $\eta$ and their uncertainties obtained from global fits using averaged quantities with aggressively low errors. The present knowledge is a the first important step along the way towards a rich experimental and theoretical program involving refinements on these measurements and important additions like CP violation observables in $B$ decays. Much more work is needed to provide a meaningful test of the Standard Model.

5 Acknowledgements

I thank I.I. Bigi, Z. Ligeti and S. Stone for interesting discussion and K. Ecklund and T. Skwarnicki for useful comments. I also want to express my appreciation for the excitement and the spirit of adventure provided by Y. Rozen to “Beauty 2000” and my gratitude to P. Schlein, who makes each conference in this series a truly remarkable experience. This work was supported by NSF.
References

[1] F.J. Gilman, K. Kleinknecht and B. Renk, The. Eur. Phys. Jou. C 15 (2000) 110.

[2] C. Hamzaoui, J.L. Rosner and A.I. Sanda, in Proceedings of the workshop on High Sensitivity Beauty Physics at Fermilab (Fermilab, Batavia, IL, 1988).

[3] M.E. Luke Phys. Lett. B 252 (1990) 447.

[4] C. Glenn Boyd, B. Grinstein and R.F. Lebed Phys. Lett. B 353 (1995) 306.

[5] P.F. Harrison and H.R. Quinn, editors The BaBar Physics Book (1998).

[6] J.N. Simone et al. Nucl. Phys. Proc. Suppl. 83 (2000) 334; hep-lat/9910026.

[7] J.P. Alexander et al. CLEO CONF 00-3; hep-ex/0007053 (2000).

[8] http://lepvcb.web.cern.ch/LEPVCB/Osaka00.html.

[9] I.I. Bigi Contributed Paper to Workshop on the Derivation of $|V_{cb}|$ and $|V_{ub}|$, CERN; hep-ph/9907270 (1999).

[10] N. Isgur Jlab-Thy-98-03; hep-ph/9809279 (1998).

[11] Del Rebbio L et al. Phys. Lett. B 436 (1998) 392.

[12] Ball P and Braun V M Phys. Rev. D 58 (1998) 094016.

[13] Isgur N and Scora D Phys. Rev. D 52 (1995) 2783.

[14] C. Sachrajda, contribution to this conference.

[15] R. Fulton et al. Phys. Rev. Lett. 64 (1990) 16.

[16] H. Albrecht et al. Phys. Lett. B 234 (1990) 409.

[17] J. Bartelt et al. Phys. Rev. Lett. 71 (1993) 4111.

[18] J.P. Alexander et al. Phys. Rev. Lett. 77 (1996) 5000.
[19] B. H. Behrens et al. Phys. Rev. D 61 (2000) 052001.

[20] I.I. Bigi, N. Dikemann and N. Uraltsev, Eur. Phys. J. C4 (1998) 453.

[21] A. Falk, Z. Ligeti and M. Wise, Phys. Lett. B 406 (1997) 225; hep-ph/9705233.

[22] M. Beyer and D. Melikhov Phys. Lett. B 436 (1998) 344.

[23] Z. Ligeti and M. Wise Phys. Rev. D 53 (1996) 4937.

[24] http://home.cern.ch/a/abbooneo/www/vub.

[25] M. Neubert, hep-ph/0006068 (2000).

[26] N. Uraltsev Contributed Paper to Workshop on the Derivation of $|V_{cb}|$ and $|V_{ub}|$, CERN; hep-ph/9905520 (1999).

[27] A. Falk, M. Luke and M. Savage, Phys. Rev. D 53 (1996) 2491.

[28] J. Bartelt et al., CLEO CONF 98-21 (1998)

[29] M. Neubert in Heavy Flavours II (Singapore: World Scientific) (1997).
Table 1: Summary of $|V_{cb}|$ determinations from the decay $B \to D^* \ell \nu$. The parameter $|V_{cb}|$ has been evaluated using $\mathcal{F}(1) = 0.913 \pm 0.042$ and the last error reflects the uncertainty in this parameter.

| Experiment   | $\mathcal{F}(1)|V_{cb}| \times 10^3$ | $|V_{cb}| \times 10^3$ |
|--------------|---------------------------------|---------------------|
| CLEO         | 42.4 ± 1.8 ± 1.9               | 46.4 ± 2.0 ± 2.1 ± 2.1 |
| ALEPH        | 32.3 ± 2.1 ± 1.3               | 35.4 ± 2.3 ± 1.4 ± 1.6 |
| DELPHI       | 36.5 ± 1.4 ± 2.4               | 40.0 ± 1.5 ± 1.8 ± 1.8 |
| OPAL (excl)  | 36.6 ± 1.7 ± 1.8               | 40.1 ± 1.9 ± 2.0 ± 1.8 |
| OPAL (incl)  | 37.5 ± 1.3 ± 2.4               | 41.1 ± 1.4 ± 2.6 ± 1.9 |
| LEP AVE      | 34.9 ± 0.7 ± 1.6               | 38.1 ± 0.8 ± 1.8 ± 1.7 |
| WORLD AVE    | 37.0 ± 1.3 ± 0.9               | 40.5 ± 1.4 ± 1.0 ± 1.8 |

Table 2: Values of $|V_{ub}|$ using $B \to \rho \ell \nu$ and some theoretical models. The $|V_{ub}|$ data include the results of a recent CLEO analysis [19] and a previous CLEO result on exclusive charmless semileptonic decays [18]. The average $|V_{ub}|$ includes an additional contribution representative of the theoretical uncertainty in the measurement.

| Model          | $|V_{ub}| \,(\times 10^{-3})$ |
|----------------|-------------------------------|
| UKQCD [11]     | 3.32 ± 0.14±0.24              |
| LCSR [12]      | 3.45 ± 0.15±0.22              |
| ISGW2 [13]     | 3.24 ± 0.14±0.22              |
| Beyer-Melikhov  | 3.32 ± 0.15±0.30              |
| Wise/Ligeti    | 2.92 ± 0.13±0.19              |
| Average        | 3.25 ± 0.14±0.31 ± 0.5        |
Figure 1: $\bar{B}^0 \to D^+ \ell^- \bar{\nu}$ from CLEO. The data have been fit to a functional form suggested by dispersion relations [4]. The abscissa gives the value of the product $|F(w) \cdot V_{cb}|$. 