Analysis of the nature of features of heat and mass transfer in discrete bulk of food products

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Abstract. Despite the importance of resolving issues related to predicting the formation of heat-moisture fields during storage of grain, seeds and bulk-stored roots and tubers and the changes in technological properties of food raw materials that depend on them, currently, the mathematical descriptions of these associated phenomena do not suffice. This is due to the complexity and poor knowledge of the storage processes for bulk-stored food products at both the micro and macro levels. Such masses are a system with distributed parameters. In this regard, the possibility of predicting the fields of heat and moisture content depends, on the one hand, on the completeness of the necessary information about the moisture and temperature of the components of the mass at the initial moment, and on the other hand, on the reliability of the mathematical model capable of describing the real processes of heat transfer in bulk based on this information, which is used to set boundary conditions and thermophysical constants. An analysis of the nature of the features of heat and mass transfer in a discrete bulk of food product can be carried out only on the basis of the developed analytical models. This is the subject of this paper.

1. Introduction
The known mathematical models of temperature conditions for storing bulk-stored roots and tubers were developed for the purpose of constructing systems for automatic control of the agricultural product storage conditions. Moreover, systems with distributed parameters are as a rule used as models of the temperature field of the self-heating zone, and very significant restrictions are imposed on the process under consideration. In most papers, bulk-stored product is considered as a continuous thermophysical medium, in which heat transfer is carried out by the mechanism of conductive heat diffusion. The development of the fields of heat and moisture content in the seed mass is modelled by the laws of the phenomenological theory of heat and mass transfer in continuous media. In [1], equations of heat distribution in bulk-stored grain of a self-heating zone in the form of plates, a cylinder, and a ball were obtained.

2. Formulation of the problem
We consider the indicated problem on the assumption that a change in temperature at an arbitrary point in time at any point in the 2 R thick heating layer and in the surrounding mass having its own thermal background is found by solving the differential heat conduction equations:
for the layer

$$\rho_1 c_1 \frac{\partial t_1}{\partial \tau} = \frac{\partial}{\partial z} \left( \lambda_1 \frac{\partial t_1}{\partial z} \right) + q_u$$

(1)

for the surrounding bulk mass

$$\rho_2 c_2 \frac{\partial t_2}{\partial \tau} = \frac{\partial}{\partial z} \left( \lambda_2 \frac{\partial t_2}{\partial z} \right) + q_0$$

(2)

where $\tau$ is the duration of the process;

$z$ is the distance from the middle plane of the heating layer;

$\rho$ is the density of the bulk of roots and tubers;

$c$ is the specific heat of the bulk of roots and tubers;

$\lambda$ is the coefficient of heat conduction of the bulk of roots and tubers;

$t$ is the temperature at the considered point of the bulk of roots and tubers;

$q_0$ is the rate of heat release (thermal background) of the bulk of roots and tubers surrounding the heating layer;

$q_u$ is the rate of heat release of the heating layer with increased physiological activity.

Index 1 refers to the heating layer, index 2 refers to the environment.

The system of equations (1)–(2) is solved based on the principle of superposition under the following simplifying premises. The temperature of the heating layer at the initial time is the same throughout the volume and equal to $t(z, t_0) = t_0$, and in the bulk of roots and tubers, which surrounds the layer, it is equal to $t(z, t) = t_1$; a bulk of roots and tubers of rather large size is considered (unlimited plate); thermal interaction between the self-heating zone and the surrounding mass occurs according to the thermal conduction law; the heat release rates of the layer $q_u$ and of the surrounding mass $q_0$ are constant in time; the thermophysical characteristics in the bulk of roots and tubers are assumed to be constant, but generally different from each other.

Let us obtain a solution to the problem with the corresponding boundary conditions under the assumption $t_0 = t_1$ for the heating layer

$$t_1(z_i, \tau) - t_0 = \left( \frac{q_u - q_0}{\rho c_1} \right) \frac{\tau}{\rho c_1} \left( 1 - \frac{4}{1 + k_c} F_1 \right) + \frac{q_0 \tau}{\rho c_2} \right) ,$$

(3)

where the sum is denoted by $F_1$

$$F_1 = \sum_{n=1}^{\infty} (-h)^{n-1} - \left[ \text{erf} \left( \frac{z}{\sqrt{2} \sqrt{Fo_1}} \right) + \text{erf} \left( \frac{z}{\sqrt{2} \sqrt{Fo_1}} \right) \right]$$

(4)

for the bulk of roots and tubers

$$t_2(z_i, \tau) - t_0 = \left( \frac{q_u - q_0}{\rho c_1} \right) \frac{\tau}{\rho c_1} \left( 1 + k_c \right) \left[ -\text{erf} \left( \frac{z}{\sqrt{2} \sqrt{Vo_2}} \right) - \frac{2}{1 + k_c} F_2 \right] + \frac{q_0 \tau}{\rho c_2} \right) ,$$

for the layer

$$\rho_1 c_1 \frac{\partial t_1}{\partial \tau} = \frac{\partial}{\partial z} \left( \lambda_1 \frac{\partial t_1}{\partial z} \right) + q_u$$

(1)

for the surrounding bulk mass

$$\rho_2 c_2 \frac{\partial t_2}{\partial \tau} = \frac{\partial}{\partial z} \left( \lambda_2 \frac{\partial t_2}{\partial z} \right) + q_0$$

(2)

where $\tau$ is the duration of the process;

$z$ is the distance from the middle plane of the heating layer;

$\rho$ is the density of the bulk of roots and tubers;

$c$ is the specific heat of the bulk of roots and tubers;

$\lambda$ is the coefficient of heat conduction of the bulk of roots and tubers;

$t$ is the temperature at the considered point of the bulk of roots and tubers;

$q_0$ is the rate of heat release (thermal background) of the bulk of roots and tubers surrounding the heating layer;

$q_u$ is the rate of heat release of the heating layer with increased physiological activity.

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The system of equations (1)–(2) is solved based on the principle of superposition under the following simplifying premises. The temperature of the heating layer at the initial time is the same throughout the volume and equal to $t(z, t_0) = t_0$, and in the bulk of roots and tubers, which surrounds the layer, it is equal to $t(z, t) = t_1$; a bulk of roots and tubers of rather large size is considered (unlimited plate); thermal interaction between the self-heating zone and the surrounding mass occurs according to the thermal conduction law; the heat release rates of the layer $q_u$ and of the surrounding mass $q_0$ are constant in time; the thermophysical characteristics in the bulk of roots and tubers are assumed to be constant, but generally different from each other.

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$$F_1 = \sum_{n=1}^{\infty} (-h)^{n-1} - \left[ \text{erf} \left( \frac{z}{\sqrt{2} \sqrt{Fo_1}} \right) + \text{erf} \left( \frac{z}{\sqrt{2} \sqrt{Fo_1}} \right) \right]$$

(4)

for the bulk of roots and tubers

$$t_2(z_i, \tau) - t_0 = \left( \frac{q_u - q_0}{\rho c_1} \right) \frac{\tau}{\rho c_1} \left( 1 + k_c \right) \left[ -\text{erf} \left( \frac{z}{\sqrt{2} \sqrt{Vo_2}} \right) - \frac{2}{1 + k_c} F_2 \right] + \frac{q_0 \tau}{\rho c_2} \right) ,$$
where

\[ F_2 = \sum_{n=1}^{\infty} (-h)^{n-1}(-1)^{n}e_{rc}c \frac{2nk_0^{\frac{1}{2}} + \frac{z}{R} - 1}{2\sqrt{F_0}_2} \]

In the written equations, \(a\) is the coefficient of thermal diffusivity of the bulk of roots and tubers;

\[ k_c = \frac{\varepsilon_1}{\varepsilon_2} \sqrt{\frac{\rho c_1 \lambda_1}{\rho_2 c_2 \lambda_2}} \] is the heat absorption criterion;

\[ h = \frac{1-k_c}{1+k_c} ; k_a = \frac{a_1}{a_2} \] is the criterion characterizing the thermal inertia properties of the medium relative to the plate;

\[ Fo_1 = \frac{a_1 \tau}{R^2} \] and \(Fo_2 = \frac{a_2 \tau}{R^2} \) are the Fourier numbers for the plate and medium, respectively;

\[ er\int cx = \int x = \frac{2}{\sqrt{\pi}} \int x \exp(-x^2) dx, \]

where \( er\int cx = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-x^2) dx \) is the Gaussian error function.

3. Results and discussion

From the solution (3), (4) we obtained special simplified cases.

In [2], the author carried out a theoretical analysis of heat and mass transfer in the bulk of roots and tubers in the presence of a heating layer with a constant heat release rate.

To carry out such an analysis, it was assumed that in the bulk of roots and tubers, there is a 2R thick layer with elevated temperature and moisture content (Figure 1).

![Figure 1](image)

**Figure 1.** Graphical interpretation of the initial conditions.

A, B – distance from the base of the storage bin; 2R, – thickness of the heating layer; h – distance from the centre of the heating layer.
According to [3,4], heat and moisture transfer in the bulk of roots and tubers with elevated temperature for h < R is described by the following system of equations:

\[
\begin{align*}
\frac{\partial t_1(h, \tau)}{\partial \tau} &= \left( a + a_m \delta \nu \frac{c_m}{c_q} \right) \frac{\partial^2 t_1(h, \tau)}{\partial h^2} + \alpha \nu \frac{c_m}{c_q} a_m \frac{\partial^2 u_1(h, \tau)}{\partial h^2} + \frac{q_1}{c_q \rho} \\
\frac{\partial u_1(h, \tau)}{\partial \tau} &= a_m \frac{\partial^2 u_1(h, \tau)}{\partial h^2} + a_m \delta \frac{\partial^2 t_1(h, \tau)}{\partial h^2}
\end{align*}
\]

where \( t_1 \) is the temperature of the bulk of roots and tubers;
\( u_1 \) is moisture content of the bulk of roots and tubers;
\( a \) is thermal diffusivity of the bulk of roots and tubers;
\( a_m \) is the coefficient of diffusion of moisture of the bulk of roots and tubers;
\( \delta \) is the thermal-gradient coefficient of the bulk of roots and tubers;
\( \epsilon \) is the phase transformation number;
\( \nu \) is the heat of phase transformation;
\( c_m \) is specific isothermal moisture capacity of the bulk of roots and tubers;
\( \rho \) is the density of the bulk of roots and tubers;
\( q_1 \) is the amount of heat released by the unit volume of the heating layer per unit time;
\( \tau \) is time.

The heat and moisture transfer in the bulk of roots and tubers for a distance h > R from the centre of the heating layer can be described by the following system of equations on the basis of the above argument and simplifications about the heat insulation of the side walls of the storage bin

\[
\begin{align*}
\frac{\partial t_2(h, \tau)}{\partial \tau} &= \left( a + a_m \delta \nu \frac{c_m}{c_q} \right) \frac{\partial^2 t_2(h, \tau)}{\partial h^2} + \alpha \nu \frac{c_m}{c_q} a_m \frac{\partial^2 u_2(h, \tau)}{\partial h^2} + \frac{q_2}{c_q \rho} \\
\frac{\partial u_2(h, \tau)}{\partial \tau} &= a_m \frac{\partial^2 u_2(h, \tau)}{\partial h^2} + a_m \delta \frac{\partial^2 t_2(h, \tau)}{\partial h^2},
\end{align*}
\]

where \( t_2 \) is the temperature of the bulk of roots and tubers surrounding the heating layer;
\( u_2 \) is the moisture content of the bulk of roots and tubers surrounding the heating layer;
\( q_2 \) is the amount of heat released by the unit volume of the bulk of roots and tubers surrounding the heating layer per unit time.

The initial conditions for the above systems of equations:
\[
\begin{align*}
t_1(h,0) &= t_1; \ t_2(h,0) = t_2; \\
u_1(h,0) &= u_1; \ u_2(h,0) = u_2;
\end{align*}
\]

The boundary conditions for these systems are:
\[
\begin{align*}
\frac{\partial t_1(0, \tau)}{\partial h} &= 0; \quad \frac{\partial u_1(0, \tau)}{\partial h} = 0; \\
\frac{\partial t_2(0, \tau)}{\partial h} &= \frac{\partial u_2(0, \tau)}{\partial h} = 0; \\
t_1(R, \tau) &= t_2(R, \tau); \ u_1(R, \tau) = u_2(R, \tau); \\
\frac{\partial t_1(R, \tau)}{\partial h} &= \frac{\partial t_2(R, \tau)}{\partial h}; \quad \frac{\partial u_1(R, \tau)}{\partial h} = \frac{\partial u_2(R, \tau)}{\partial h}.
\end{align*}
\]
It is obvious that the thermophysical characteristics of the heating layer and the bulk of the root and tubers surrounding it are constant and the same.

When solving the above problems, the following conditions were observed:
1) the sides of the cylinder are thermally insulated, and the heat and moisture transfer between the bulk of roots and tubers and the air through the bases of the cylinder is insignificant, therefore it is neglected;
2) the heat and moisture transfer inside the bulk of roots and tubers occurs only through heat conduction, and between the bulk of roots and tubers and the air in the bulk it is insignificant, therefore it is neglected.

It is understood that the assumptions made are very serious.

For the corresponding initial and boundary conditions for the above systems of differential equations, we have obtained a solution that describes the temperature field of the bulk:

\[ t(z, \tau) = t_2(h, \tau) = t_2 + \frac{q_2 \tau}{c \rho} + \frac{2\tau(q_2 - q_1)}{c \rho} \left[ t_2 \operatorname{erfc} \frac{R + |h|}{2\sqrt{a \tau}} - t_1 \operatorname{erfc} \frac{|h| - R}{2\sqrt{a \tau}} \right]. \]

Based on the heat and mass transfer problem formulated, a simplified solution for a bulk of food products was obtained, assuming thermal insulation of the side walls of the storage bin and neglecting heat and moisture transfer through its base. In addition, it was assumed that heat and moisture transfer between the individual elements is carried out only through heat conduction. For storage conditions that do not meet the accepted assumptions, the solution may have significant errors in comparison with the actual distribution of the temperature field, which requires additional complication of the problem formulation [5-6] for its practical use.

It can be assumed, for example, that for storing a bulk of product accepted as a homogeneous and isotropic medium in a cylindrical-shaped storage, the side surface is heat insulated, and the temperatures of the base and top surface are changed arbitrarily. Then the heat transfer problem can be formulated as follows: it is necessary to solve the inhomogeneous heat conduction equation for an unlimited plate (the height of the bulk of product is smaller than the diameter of the storage bin), or for a limited rod (the height of the bulk of product is much larger than the diameter of the storage bin)

\[ \frac{\partial t(z, \tau)}{\partial \tau} = a \frac{\partial^2 t(z, \tau)}{\partial z^2} + \frac{q_0}{c} \exp(k_1 \tau - k_2 z), \]

under the initial condition

\[ t(z, 0) = t_0 = \text{const}, \]

and boundary conditions:

\[ t(0, \tau) = f_1(\tau), \]
\[ t(h, \tau) = f_2(\tau), \]

where \( a \) is the temperature conductivity coefficient, \( h \) is the height of the bulk, \( t_0 \) is the temperature of the product at the beginning of the storage process, \( c \) is the specific heat capacity of the bulk of product, \( f_1(\tau) \) and \( f_2(\tau) \) are the specified bounded and continuous functions. By substituting [7]

\[ t(z, \tau) = v(z, \tau) + \frac{q_0}{c \left( k_1 - a k_2 \right)} \exp(k_1 \tau - k_2 z), \]

where \( v(z, \tau) \) is the new required function, we reduce the equation (2) to homogeneous one, that is, without a heat source,
\[
\frac{\partial t(z, \tau)}{\partial \tau} = a \frac{\partial^2 t(z, \tau)}{\partial z^2}
\]

but with new initial and boundary conditions:

\[
v(z, 0) = t_0 - \frac{q_0}{c(k_1 - ak_2^2)} \exp(-k_2 z) = \varphi(z)
\]

\[
v(0, \tau) = f_1(\tau) - \frac{q_0}{c(k_1 - ak_2^2)} \exp(k_1 \tau) = \varphi_1(\tau)
\]

\[
v(h, \tau) = f_2(\tau) - \frac{q_0}{c(k_1 - ak_2^2)} \exp(k_1 \tau - k_2 h) = \varphi_2(\tau)
\]

Using the integral finite sine transform method, an analytical solution of the recorded boundary value problem for the distribution of the temperature field in the bulk of food product was obtained.

For a special case of constant temperatures of the base of the bulk, that is, for \(t(0, \tau) = t_1 = \text{const}, t(h, \tau) = t_2 = \text{const}\), and taking into account the known relations [8–9], we obtained a solution in the form:

\[
t(z, \tau) = \frac{q_0}{c(k_1 - ak_2^2)} \exp(k_1 - k_2 z) + t_1 + (t_2 - t_1) \frac{z}{h} + \left( \frac{2}{\pi} \right) \sum_{n=1}^{\infty} t_0 - t_1 - (-1)^{n} \frac{(t_0 - t_2)}{n\pi} \sin \frac{n\pi}{h} \exp \left( \frac{an^2 \pi^2 \tau}{h^2} \right) -
\]

\[
\sum_{n=1}^{\infty} \frac{2q_0}{c(k_1 - ak_2^2)} \sum_{n=1}^{\infty} \frac{1 - (-1)^{n} \exp(-k_2 h)}{n\pi} \sin \frac{n\pi}{h} \exp \left( \frac{-an^2 \pi^2 \tau}{h^2} \right) \exp(k_1 \tau - \frac{k_2 h}{a} + \left( n\pi \right)^2)
\]

The obtained dependence was used to model the storage process in the bulk. As experimental data, operational data on storage of sunflower seeds in a warehouse were used. Since the difference in humidity across the layers during the placing of the batch (\(x=0\)) did not fluctuate significantly (±0.2%), the value of \(k_2\) was taken equal to zero. The value of moisture content of the seeds in the layer at the initial moment of time \(U(Z, 0) = 8.4\%\), the temperature value \(t_0 = 16^\circ\text{C}, k_1 = 0.8 \times 10^{-7}\text{c}^4\), \(q_0 = 0.013\text{W/kg}\). This corresponds to the estimates of the heat release of seeds with such humidity and temperature obtained according to the data of [10].

The calculation results for the bulk with a height of 3m are given in Table 1.

**Table 1.** Experimental and calculated values of temperatures during storage of sunflower seeds (\(t_1 = 24^\circ\text{C}; t_2 = 22^\circ\text{C}\)).

| \(F_{0a} \times 10^4\) | \(X=0.17\) | \(X=0.5\) | \(X=0.83\) |
|-----------------|-------------|-------------|-------------|
|                 | \(t_{cop}\) | \(t_{cok}\) | \(t_{cop}\) | \(t_{cok}\) | \(t_{cop}\) | \(t_{cok}\) |
| 1               | -           | 20.6        | -           | 20.1        | -           | 19.2        |
| 3               | 22          | 21.4        | 20          | 21.2        | 13          | 20.8        |
| 6               | -           | 21.9        | -           | 21.5        | -           | 21.0        |
| 9               | -           | 22.3        | -           | 21.8        | -           | 21.3        |
| 13              | 23          | 22.6        | 21          | 22.0        | 20          | 21.5        |
| 17              | -           | 23.0        | -           | 22.2        | -           | 21.7        |
| 20              | 23          | 23.2        | 31          | 22.3        | 29          | 21.9        |
| 25              | -           | 23.4        | -           | 22.4        | -           | 21.9        |
| 30              | -           | 23.5        | -           | 22.5        | -           | 21.9        |
Here \( F_0 = \frac{a \tau}{h^2} \) is the Fourier number, \( X = \frac{z}{h} \) is the dimensionless coordinate.

It is possible to formulate heat and mass transfer problems under more complicated boundary conditions. For a two-layer bulk of food products (roots and tubers, grain or seeds), modelled by a spherical capillary-porous “shell-core” medium, the system of differential equations of the combined heat and mass transfer is applicable [8]:

\[
\frac{\partial [u_k(r, \tau)]}{\partial \tau} = a_{mk} \frac{\partial^2 [u_k(r, \tau)]}{\partial r^2} + c_{mk} \cdot \delta_k \frac{\partial^2 [u_k(r, \tau)]}{\partial r^2},
\]

Here the following notation is accepted:
- \( k \) is the layer number, \( k=1 \) (core): \( 0<r<R_1 \); \( k=2 \) (shell): \( R_1<r<R_2 \);
- \( t_k(r, \tau) \) is the temperature of the \( k \)-th layer of the bulk;
- \( r \) is the coordinate; \( R_k \) is the radius of the ball;
- \( \tau \) is time, \( \tau>0 \);
- \( u_k(r, \tau) \) is the moisture content;
- \( a_{mk} \) is the thermal conductivity;
- \( a_{mk} \) is the coefficient of potential conductivity (moisture conductivity);
- \( c_{mk} \) is specific heat;
- \( \rho_k \) is the specific heat of phase transformation;
- \( \delta_k \) is the thermal-gradient coefficient.

For the zonal calculation system, the thermophysical characteristics of the material are constant values, which are different for different layers.

In the process of conditioning of the bulk, the temperature and moisture content of the environment vary insignificantly. In addition, in the process under consideration, the thermal mass transfer is not significant, i.e. \( \delta_k = O \). By the time the conditioning of the bulk of the above products begins, the products have uniform distribution of temperature and moisture.

All the noted features of drying and wetting the bulk of product lead to the solution of the system of differential equations

\[
\frac{\partial t_k}{\partial \tau} = a_{mk} \frac{\partial^2 t_k}{\partial r^2} + c_{mk} \cdot \delta_k \frac{\partial^2 t_k}{\partial r^2},
\]

Under the following boundary conditions:

\[
\begin{align*}
& t_k(r, 0) = t_0 = const; \\
& u_k(r, 0) = u_0 = const; \\
& t_k(R_1, \tau) = t_z(R_1, \tau); \\
& u_k(R_1, \tau) = u_z(R_1, \tau); \\
& \lambda_{q1} \frac{\partial t_1(R_1, \tau)}{\partial r} = \lambda_{q2} \frac{\partial t_z(R_1, \tau)}{\partial r};
\end{align*}
\]
\[
\frac{\partial u_1(r, \tau)}{\partial r} = \frac{\partial u_2(r, \tau)}{\partial r};
\]
\[
\frac{\partial t_1(0, \tau)}{\partial r} = \frac{\partial u_1(0, \tau)}{\partial r};
\]
\[
t_1(0, \tau) < \infty, u_1(0, \tau) < \infty;
\]
\[
t_2(R_2, \tau) = t_c = \text{const};
\]
\[
u_2(R_2, \tau) = u_c = \text{const},
\]
where \(t_0\) is the initial temperature of the seeds; \(t_c\) is ambient temperature; \(u_0\) is the initial moisture content; \(u_c\) is equilibrium moisture content; \(R_k\) is the radius of the ball; \(\lambda_q\) is the thermal conductivity coefficient; \(\lambda_m\) is the mass conductivity coefficient.

The above equations represent the initial conditions and boundary conditions of the fourth kind, which consist in the equality of temperatures, moisture contents, heat fluxes and moisture flows at the interface between the core of an individual element of the bulk and its shell.

The last three relations define the conditions of symmetry and physical limitation of temperatures and moisture content, as well as boundary conditions of the first kind, which specify the temperature \(t_c\) and moisture content \(u_c\) on the surface of the element of the bulk.

The problem is solved by the method of Laplace integral transforms. Distributions of the fields of mass and heat transfer potentials were obtained in the following form:

\[
\Theta_{k1} = \frac{u_k(r, \tau) - u_c}{u_0 - u_c} = \frac{2R}{X} \sum_{n=1}^{\infty} c_n z_n \exp(-\mu_n^2 F_0);
\]
\[
\Theta_{k2} = \frac{t_k(r, \tau) - t_c}{t_0 - t_c} = \frac{2c_2 K_0 L_2 R}{X(L - L_2)} \times \left[ \sum_{n=1}^{\infty} c_n \exp(-\mu_n^2 F_0) - \sum_{n=1}^{\infty} \sum_{i=1}^{2} c_n z_n \exp(-\mu_n^2 F_0) \right]
\]
where \(\mu_n\) – consecutive positive roots of the characteristic equation
\[
\sqrt{a_m u_c r_g} \left( \sqrt{a_m (R - 1) \mu_n} + 1 \right) + \lambda_m (\mu e r g - 1) = 0;
\]
\[
c_n = \frac{1}{\varphi_n} \sin \mu_n;
\]
\[
z_1 = \sin \left( \sqrt{a_m (R - 1) \mu_n} \sin(\mu_n X) \right);
\]
\[
z_2 = \sin \mu_n \sin \left( \sqrt{a_m (R - X) \mu_n} \right);
\]
\[
\varphi_n = k_m \mu_n \sin^2 \left( \sqrt{a_m (R - 1) \mu_n} \right) + \sqrt{a_m (R - 1) \mu_n} \sin^2 \mu_n + \frac{1 - \sqrt{a_m k_m}}{\sqrt{a_m \mu_n}} \sin^2 \mu_n \sin^2 \left( \sqrt{a_m (R - 1) \mu_n} \right)
\]
\(\mu_n\) – consecutive positive roots of the characteristic equation.
\[
\sqrt{a_L u_t \mu \text{ctg} \left( a_L u_t (R-1) \mu \right)} + 1 + \lambda_q \times \left( \sqrt{u_t \mu \text{ctg} \left( u_t \mu \right)} - 1 \right) = 0;
\]

The obvious complication of the solution of the problem does not always lead to a significant increase in the accuracy of its solution. Thus, the specificity of setting the boundary conditions is always determined by the requirements of their applicability in practice.

4. Conclusion
When considering various formulations of the problem of heat and mass transfer in a bulk of food products, it was found that the nature of the boundary conditions very significantly affects the complexity of its solution. The adequacy of the somewhat complicated model to the real process is obvious for small heights of the bulk of seeds and for the Fourier number \( F_0 < 0.02 \). In other cases, discrepancies between the calculated and experimental values are observed, which can be explained by several reasons. On the one hand, the function of the heat source does not consider the change in humidity during storage, which amounted to 0.6 to 1.0% as a result of storage. In addition, the accelerating effect of temperature on the heat release rate is not taken into account in the model. The temperature difference at the beginning and at the end of storage process reached 15°C for the middle layer. We can conclude that it is necessary to move on to models that take into account combined heat and moisture transfer on the one hand, and on the other, nonlinearity in temperature. The proposed mathematical models make it possible to predict and control the temperature fields of bulks of food products and thereby affect their quality and storage period.

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