Resonant energy conversion of 3-minute intensity oscillations into Alfvén waves in the solar atmosphere

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Abstract

Nonlinear coupling between 3-minute oscillations and Alfvén waves in the solar lower atmosphere is studied. 3-minute oscillations are considered as acoustic waves trapped in a chromospheric cavity and oscillating along transversally inhomogeneous vertical magnetic field. It is shown that under the action of the oscillations the temporal dynamics of Alfvén waves is governed by Mathieu equation. Consequently, the harmonics of Alfvén waves with twice period and wavelength of 3-minute oscillations grow exponentially in time near the layer where the sound and Alfvén speeds equal i.e. \( c_s \approx v_A \). Thus the 3-minute oscillations are resonantly absorbed by pure Alfvén waves near this resonant layer. The resonant Alfvén waves may penetrate into the solar corona taking energy from the chromosphere. Therefore the layer \( c_s \approx v_A \) may play a role of energy channel for otherwise trapped acoustic oscillations.

Key words: solar atmosphere; magnetohydrodynamic waves

1 Introduction

Waves play an important role in the dynamics of the solar atmosphere. They are believed as carriers of photospheric energy into the corona leading to plasma heating. Waves are intensively observed in the solar chromosphere and corona by SOHO (Solar and Heliospheric Observatory) and TRACE (Transition Region and Coronal Explorer) (Nakariakov & Verwichte 2005). Dominant intensity oscillations in a chromospheric network have a period of \( \sim 3\)-min. The

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3-min oscillations are believed to be either standing waves in a chromospheric cavity (Leibacher & Stein 1981) or propagating waves, which arise as the response of the atmosphere to a general disturbance set at its base (Fleck & Schmitz 1991). Coupling of these oscillations to the Alfvén waves is a crucial process in both cases, as the Alfvén waves may easily pass through the transition region taking the energy into the corona.

On the other hand, recent numerical (Rosenthal et al., 2002), analytical (Zaqarashvili & Roberts, 2006) and observational (Muglach et al., 2005) studies point out the importance of $\beta \sim 1$ region of the solar atmosphere in the wave coupling phenomena.

Here we consider the 3-min oscillations as the standing acoustic waves oscillating along an uniform vertical magnetic field and study their weakly nonlinear coupling to Alfvén waves near the chromospheric $\beta \sim 1$ regions.

2 The wave coupling

We use Cartesian coordinate system $(x, y, z)$, where spatially inhomogeneous (along the $x$ axis) magnetic field is directed along the $z$ axis, i.e. $B_0 = (0, 0, B_z(x))$. Unperturbed plasma pressure $p_0(x)$ and density $\rho_0(x)$ are also assumed to vary along $x$. In the equilibrium, magnetic and hydrodynamic pressures satisfy the transverse balance, i.e. $p_0(x) + B_z^2(x)/8\pi = \text{const}$. Plasma $\beta$ is defined as $\beta = 8\pi p_0(x)/B_z^2(x) = 2c_s^2/\gamma v_A^2(x)$, where $c_s = \sqrt{\gamma p_0/\rho_0}$ and $v_A(x) = B_z/\sqrt{4\pi \rho_0}$ are the sound and Alfvén speeds respectively and $\gamma$ is the ratio of specific heats. For simplicity, temperature is suggested to be homogeneous so that the sound speed does not depend on the $x$ coordinate.

We consider wave propagation along the $z$ axis (thus along the magnetic field) and wave polarisation in $yz$ plane. Then in this case only sound and linearly polarised Alfvén waves arise. The velocity component of sound wave is polarised along the $z$ axis and the velocity component of Alfvén wave is polarised along the $y$ axis. Then the ideal MHD equations can be written as

\[
\frac{\partial b_y}{\partial t} + u_z \frac{\partial b_y}{\partial z} = -b_y \frac{\partial u_z}{\partial z} + B_z(x) \frac{\partial u_y}{\partial z},
\]

\[
\rho \frac{\partial u_y}{\partial t} + \rho u_z \frac{\partial u_y}{\partial z} = \frac{B_z(x)}{4\pi} \frac{\partial b_y}{\partial z},
\]

\[
\frac{\partial \rho}{\partial t} = -\rho \frac{\partial u_z}{\partial z} - u_z \frac{\partial \rho}{\partial z},
\]

\[
\rho \frac{\partial u_z}{\partial t} + \rho u_z \frac{\partial u_z}{\partial z} = -\frac{\partial p}{\partial z} - \frac{\partial (b_y^2)}{\partial z 8\pi}.
\]
\( \frac{\partial p}{\partial t} = -\gamma p \frac{\partial u_z}{\partial z} - u_z \frac{\partial p}{\partial z}, \quad (5) \)

where \( p = p_0 + p_1 \) and \( \rho = \rho_0 + \rho_1 \) denote the total (unperturbed plus perturbed) hydrodynamic pressure and density, \( u_y \) and \( u_z \) are velocity perturbations (of the Alfvén and sound waves, respectively), \( b_y \) is the perturbation in the magnetic field. Note, that the \( x \) coordinate stands as a parameter in these equations.

Eqs. (1)-(5) describe the fully nonlinear behavior of sound and linearly polarized Alfvén waves propagating along the magnetic field. However the sound waves may be trapped in a chromospheric cavity between the photosphere and transition region leading to standing patterns. Therefore here we consider the sound waves oscillating along the \( z \) axis and bounded at \( z = 0 \) and \( z = l \) points. Thus there are the standing patterns

\[
u_z = v(t) \sin(k_n z), \quad \rho_1 = \tilde{\rho}(t) \cos(k_n z), \quad (6)\]

where \( k_n \) is the wavenumber of sound wave such that \( k_n l = 2\pi l/\lambda_n = n \pi \), so \( l/\lambda_n = n/2 \), where \( n = 1, 2, ... \) denotes the order of corresponding harmonics.

It is natural that almost whole oscillation energy in bounded systems is stored in a fundamental harmonic. Therefore here we consider the first \((n = 1)\) harmonic of acoustic oscillations, however the same can be applied to harmonics with arbitrary \( n \). Recently, Zaqarashvili and Roberts (2006) found that the harmonics of acoustic and Alfvén waves are coupled when the wave length of acoustic wave is half of Alfvén wave one. Therefore let express the Alfvén wave components as

\[
u_y = u(t) \sin(k_A z), \quad b_y = b(t) \cos(k_A z), \quad (7)\]

where \( k_A \) is the wavenumber of the Alfvén waves and the condition \( k_1 = 2k_A \) is satisfied.

Then the substitution of expressions (6)-(7) into Eqs. (1)-(5) and averaging with \( z \) over the distance \((0, l)\) leads to

\[
\frac{\partial b}{\partial t} = k_A B_0 u + k_A \nu b, \quad (8)\\
\frac{\partial u}{\partial t} = -\frac{k_A B_0}{4\pi \rho_0} b - k_A \nu v, \quad (9)\\
\frac{\partial v}{\partial t} = \frac{k_1 c_s^2}{\rho_0} \tilde{\rho} + \frac{k_A}{8\pi \rho_0} b^2, \quad (10)
\]
\[
\frac{\partial \tilde{\rho}}{\partial t} = -\rho_0 k_1 v. \quad (11)
\]

Here Eqs. (10)-(11) describe the time evolution of acoustic oscillation forced by ponderomotive force of Alfvén waves, while Eqs. (8)-(9) governs the dynamics of Alfvén waves forced by acoustic waves in a parametric way. It must be noted that the coupling between sound and Alfvén waves at \( \beta \sim 1 \) has been recently studied by Zaqarashvili & Roberts (2006). They consider the general case of propagating waves and show an alternate energy exchange between the waves during the propagation. Here we consider the coupling between standing patterns of the waves, which is the particular case of their results.

Substitution of \( u \) from Eq. (8) into Eq. (9) and neglecting of all third order terms leads to the second order differential equation for Alfvén waves

\[
\frac{\partial^2 b}{\partial t^2} + k_A^2 v_A^2 \left[ 1 - \frac{2\tilde{\rho}}{\rho_0} \right] b = 0. \quad (12)
\]

This equation reflects the parametric influence of standing acoustic wave due to the density variation. Then the particular time dependence of density perturbation determines the type of equation and consequently its solutions. If we consider the initial amplitude of Alfvén waves smaller than the amplitude of acoustic waves, then the term with \( b^2 \) in Eq. (10) can be neglected. Physically it means that the back reaction of Alfvén waves due to the ponderomotive force is small. Then the solution of Eqs. (10)-(11) is just harmonic function of time \( \tilde{\rho} = \alpha \rho_0 \cos(\omega_1 t) \), where \( \omega_1 \) is the frequency of the first harmonic of standing acoustic wave and \( \alpha > 0 \) is the relative amplitude. Here we consider the small amplitude acoustic waves \( \alpha \ll 1 \), so the nonlinear steepening due to the generation of higher harmonics is negligible. Then the substitution of this expression into Eq. (12) leads to Mathieu equation

\[
\frac{\partial^2 b}{\partial t^2} + k_A^2 v_A^2 \left[ 1 - 2\alpha \cos(\omega_1 t) \right] b = 0. \quad (13)
\]

The solution of this equation with the frequency of \( \omega_1/2 \) has an exponentially growing character, thus the main resonant solution occurs when

\[
\omega_A = v_A k_A = \frac{\omega_1}{2}, \quad (14)
\]

where \( \omega_A \) is the frequency of Alfvén waves. Since \( k_A = k_1/2 \), the resonance takes place when \( v_A = c_s \). Since the Alfvén speed \( v_A(x) = B_0(x)/\sqrt{4\pi \rho_0(x)} \) is a function of the \( x \) coordinate, then this relation will be satisfied at particular locations along the \( x \) axis. Therefore the acoustic oscillations will be resonantly
Fig. 1. Numerical simulations of wave conversion in $c_s = v_A$ region. Upper panels show the density and velocity components of standing acoustic oscillations, while lower panels show the magnetic field and velocity component of Alfvén waves. Relative amplitude of acoustic oscillations is 0.1. Alfvén waves with twice the period of acoustic oscillations show rapid exponential increase in time.

transformed into Alfvén waves near this layer. We call this region *swing layer* (see similar consideration in Shergelashvili et al. 2005).

Under the resonant condition (14) the solution of Eq. (13) is

$$b(t) = b_0 \exp \left( \frac{\alpha \omega_1}{4} t \right) \left[ \cos \frac{\omega_1}{2} t - \sin \frac{\omega_1}{2} t \right],$$  \hspace{1cm} (15)$$

where $b_0 = b(0)$.

The solution (15) has a resonant character within the frequency interval $|\omega_A - \omega_1/2| < |\alpha \omega_1/2|$. This expression can be rewritten as $|v_A/c_s - 1| < |\alpha|$. Thus the thickness of the resonant layer depends on the acoustic wave amplitude. Therefore the acoustic oscillations are converted into Alfvén waves not only at the surface $v_A = c_s$ but in the region where $c_s (1 - \alpha) < v_A < c_s (1 + \alpha)$. Thus the resonant layer can be significantly wider for stronger amplitude acoustic oscillations. Note, that the resonant Alfvén waves expressed by Eqs. (7) and (15) are standing patterns with the velocity node at the bottom boundary ($z = 0$) and antinode at the top boundary ($z = l$); the wave length of Alfvén waves is twice than that of the acoustic oscillations due to the condition $k_A = k_1/2$. Therefore the oscillation of magnetic field lines at the upper boundary may excite the waves in an external plasma, which carry energy away.
It must be noted that the Alfvén waves may undergo phase mixing due to the dependence of the Alfvén speed (Tsiklauri & Nakariakov 2002). However the aim of this paper is to show the energy conversion from 3-min oscillations into Alfvén waves only, therefore we do not consider the effect of phase mixing here.

Numerical solutions of Eqs. (8)-(11) (here the back reaction of Alfvén waves, i.e. the second term of right hand side in Eq. (10), is again neglected) are presented on Fig.1-2. Figure 1 shows the wave dynamics in \( c_s = v_A \) region and we see the rapid growth of Alfvén waves with twice the period of acoustic oscillations. However Fig.2 shows the wave dynamics away from the resonant layer (in \( c_s = 0.7v_A \) region) and we see no energy exchange between the waves. Thus there is a good agreement between numerical results and analytical solutions.

It must be mentioned that the equilibrium used in this paper is simplified as gravitational stratification, which is important in the solar chromosphere, is ignored. The stratification leads to Klein-Gordon equation for propagating waves with a cut-off for wave frequencies (Roberts, 2004). The 3-min oscillations and Alfvén waves are above the cut-off, so they may propagate in the chromosphere. Unfortunately, the stratification greatly complicates mathematical description of the non-linear coupling. On the other hand, plasma \( \beta \) can be constant along a vertical magnetic tube even in the case of stratification (Roberts, 2004); this is the case when sound and Alfvén speeds vary with height but their ratio remains constant. Therefore if the waves are coupled at \( \beta \sim 1 \) region in an unstratified atmosphere, then the same can be expected in a stratified medium. However the wave coupling in the stratified atmosphere needs further detailed study, but is not the scope of present paper. Our goal is
just to show that the 3-min oscillations may transfer energy into incompressible Alfvén waves in \( \beta \sim 1 \) region, which was recently observed by Muglach et al. (2005).

### 3 Conclusions

Here we show that acoustic oscillations trapped in transversally inhomogeneous medium can be resonantly absorbed by Alfvén waves near the layer of \( v_A \approx c_s \). The spatial width of the layer depends on amplitude of acoustic oscillations and can be significantly wider for strong amplitude oscillations.

We consider the observed 3-min oscillations as the standing fundamental harmonic of acoustic waves trapped in the solar chromospheric cavity with vertical magnetic field and show that their nonlinear coupling to Alfvén waves may take place near \( \beta \sim 1 \) layer. The coupling may explain the recent observational evidence of compressible wave absorption near \( \beta \sim 1 \) region of solar lower atmosphere (Muglach et al., 2005). The amplified Alfvén waves with the period of \( \sim 6 \) min may carry energy into the corona. There they may deposit the energy back to density perturbations leading to observed intensity variations in coronal spectral lines.

Thus the layer of \( \beta \sim 1 \) may play a role of energy channel for otherwise trapped acoustic oscillations guiding the photospheric energy into the solar corona. Therefore the process of wave coupling can be of importance for coronal heating problem, but requires further study especially for a stratified atmosphere.

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