Realistic shell-model calculations and exotic nuclei around $^{132}$Sn

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Abstract. We report on a study of exotic nuclei around doubly magic $^{132}$Sn in terms of the shell model employing a realistic effective interaction derived from the CD-Bonn nucleon-nucleon potential. The short-range repulsion of the latter is renormalized by constructing a smooth low-momentum potential, $V_{\text{low}-k}$, that is used directly as input for the calculation of the effective interaction. In this paper, we focus attention on proton-neutron multiplets in the odd-odd nuclei $^{134}$Sb, $^{136}$Sb. We show that the behavior of these multiplets is quite similar to that of the analogous multiplets in the counterpart nuclei in the $^{208}$Pb region, $^{210}$Bi and $^{212}$Bi.

Keywords: Realistic shell model; $^{132}$Sn region; $^{208}$Pb region
PACS: 21.60.Cs, 21.30.Fe, 27.60.+j, 27.80.+w

INTRODUCTION

We have recently studied [1, 2, 3, 4] several nuclei beyond doubly magic $^{132}$Sn within the framework of the shell model employing realistic effective interactions derived from the CD-Bonn nucleon-nucleon ($NN$) potential [5]. A main difficulty encountered in this kind of calculations is the strong short-range repulsion contained in the bare $NN$ potential $V_{NN}$, which prevents its direct use in the derivation of the shell-model effective interaction $V_{\text{eff}}$. As is well known, the traditional way to overcome this difficulty is the Brueckner $G$-matrix method. Instead, in the calculations mentioned above we have made use of a new approach [6] which consists in deriving from $V_{NN}$ a low-momentum potential, $V_{\text{low}-k}$, that preserves the deuteron binding energy and scattering phase shifts of $V_{NN}$ up to a certain cutoff momentum $\Lambda$. This is a smooth potential which can be used directly to derive $V_{\text{eff}}$, and it has been shown [6, 7] that it provides an advantageous alternative to the use of the $G$ matrix.

In this paper, we shall focus attention on the proton-neutron multiplets in the two odd-odd Sb isotopes $^{134}$Sb and $^{136}$Sb, which are most appropriate for testing the matrix elements of the proton-neutron interaction between valence nucleons outside $^{132}$Sn. Note that $^{136}$Sb with an $N/Z$ ratio of 1.67 is at present the most exotic open-shell nucleus beyond $^{132}$Sn for which information exists on excited states.

While it is very difficult to obtain information on neutron-rich nuclei around $^{132}$Sn, which lie well away from the valley of stability, this is not the case for the $^{208}$Pb neighbors, which have long been the subject of experimental and theoretical studies. The new data which are becoming available for the $^{132}$Sn region make it possible to investigate more quantitatively the resemblance between the spectroscopy of this region
and that of nuclei around $^{208}\text{Pb}$, which has been pointed out in several recent papers \cite{8, 9, 10}.

With the above motivation, we have calculated the proton-neutron multiplets in $^{210}\text{Bi}$ and $^{212}\text{Bi}$, which are the counterparts of $^{134}\text{Sb}$ and $^{136}\text{Sb}$ in the Pb region.

We start by giving a brief description of the theoretical framework in which our realistic shell-model calculations are performed and then present and discuss our results. A short summary is given in the last section.

**OUTLINE OF THEORETICAL FRAMEWORK**

The shell-model effective interaction $V_{\text{eff}}$ is defined, as usual, in the following way. In principle, one should solve a nuclear many-body Schrödinger equation of the form

$$H\Psi_i = E_i\Psi_i,$$

(1)

with $H = T + V_{\text{NN}}$, where $T$ denotes the kinetic energy. This full-space many-body problem is reduced to a smaller model-space problem of the form

$$P H_{\text{eff}} P \Psi_i = P (H_0 + V_{\text{eff}}) P \Psi_i = E_i P \Psi_i.$$

(2)

Here $H_0 = T + U$ is the unperturbed Hamiltonian, $U$ being an auxiliary potential introduced to define a convenient single-particle basis, and $P$ denotes the projection operator onto the chosen model space.

As pointed out in the Introduction, we “smooth out” the strong repulsive core contained in the bare $NN$ potential $V_{\text{NN}}$ by constructing a low-momentum potential $V_{\text{low}−k}$. This is achieved by integrating out the high-momentum modes of $V_{\text{NN}}$ down to a cut-off momentum $\Lambda$. This integration is carried out with the requirement that the deuteron binding energy and phase shifts of $V_{\text{NN}}$ up to $\Lambda$ are preserved by $V_{\text{low}−k}$. A detailed description of the derivation of $V_{\text{low}−k}$ from $V_{\text{NN}}$ as well as a discussion of its main features can be found in Refs. \cite{6, 11}.

Once the $V_{\text{low}−k}$ is obtained, we use it as input interaction for the calculation of the matrix elements of the shell-model effective interaction. The latter is derived by employing a folded-diagram method, which was previously applied to many nuclei using $G$-matrix interactions \cite{12}. Since $V_{\text{low}−k}$ is already a smooth potential, it is no longer necessary to calculate the $G$ matrix. We therefore derive $V_{\text{eff}}$ following the same procedure as described, for instance, in Refs. \cite{13, 14}, except that the $G$ matrix used there is replaced by $V_{\text{low}−k}$. More precisely, we first calculate the so-called $\tilde{Q}$-box \cite{15} including diagrams up to second order in the two-body interaction. The shell-model effective interaction is then obtained by summing up the $\tilde{Q}$-box folded diagram series using the Lee-Suzuki iteration method \cite{16}.

**CALCULATIONS AND RESULTS**

In our calculations for $^{134}\text{Sb}$ and $^{136}\text{Sb}$ we assume that the valence neutrons occupy the six levels $0h_{9/2}$, $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, and $0i_{13/2}$ of the 82-126 shell, while for the
odd proton the model space includes the five levels $0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, and $0h_{11/2}$ of the 50-82 shell. Similarly, for the two Bi isotopes we take as model space for the valence proton the six levels of the 82-126 shell and let the valence neutrons occupy the seven levels $1g_{9/2}$, $0i_{11/2}$, $0j_{15/2}$, $2d_{5/2}$, $3s_{1/2}$, $1g_{7/2}$, and $2d_{3/2}$ of the 126-184 shell.

As regards the choice of the single-proton and single-neutron energies, we have proceeded as follows. For Sb isotopes, we have taken them from the experimental spectra of $^{133}$Sb and $^{133}$Sn, with the exception of the proton $s_{1/2}$ and the neutron $i_{13/2}$ levels, which are still missing. The values of $\varepsilon_{s_{1/2}}$ and $\varepsilon_{i_{13/2}}$ have been taken from Refs. [17] and [18], respectively, where it is discussed how they are determined. In the same way, for Bi isotopes, we have made use of the experimental spectra of $^{209}$Bi and $^{209}$Pb. The adopted values of the single-particle energies are reported in Refs. [1] and [19] for Sb and Bi isotopes, respectively.

As already mentioned in the Introduction, in our shell-model calculations we have made use of effective interactions derived from the CD-Bonn $NN$ potential renormalized through the $V_{\text{low-k}}$ procedure. As in our previous studies [1, 2, 3, 4, 19], the cutoff momentum $\Lambda$ is given the value $2.2 \text{ fm}^{-1}$. The computation of the diagrams included in the $\hat{Q}$ box is performed within the harmonic-oscillator basis using intermediate states composed of all possible hole states and particle states restricted to the five proton and neutron shells above the Fermi surface. The oscillator parameter is $7.88 \text{ MeV}$ for the $A = 132$ region and $6.88 \text{ MeV}$ for the $A = 208$ region, as obtained from the expression $\hbar \omega = 45A^{-1/3} - 25A^{-2/3}$.

Let us now come to the results of our calculations, which have been performed by using the NushellX shell-model code [20].

In Fig. 1 we show the energies of the first eight calculated states, which are the members of the $\pi g_{7/2} \nu f_{7/2}$ multiplet, and compare them with the eight lowest-lying experimental states [21]. The wave functions of these states are characterized by very little configuration mixing, the percentage of the leading component having a minimum value of $88\%$ for the $J^\pi = 2^-\text{ state}$ while ranging from $94\%$ to $100\%$ for all other states.

![FIGURE 1](image-url)  
**FIGURE 1.** Proton-neutron $\pi g_{7/2} \nu f_{7/2}$ multiplet in $^{134}$Sb. The theoretical results are represented by open circles and the experimental data by solid triangles.

As regards $^{136}$Sb, with two more valence neutrons, its ground state was identified as $1^-\text{ in the early } \beta\text{-decay study of Ref. [22] while the spectroscopic study of Ref. [23] led to the observation of a } \mu s \text{ isomeric state, which was tentatively assigned a spin and
parity of $6^-$. Very recently, new experimental information has been obtained on the $\mu$s isomeric cascade, leading to the identification of two more excited states.

This achievement was at the origin of our realistic shell-model calculation for this nucleus [4], whose results we are now going to present. In Fig. 2, we show the four observed levels together with the calculated yrast states having angular momentum from $0^-$ to $7^-$, which all arise from the $\pi g_{7/2}v(f_{7/2})^3$ configuration. These states may be viewed as the evolution of the $\pi g_{7/2}v f_{7/2}$ multiplet in $^{134}$Sb.

From Figs. 1 and 2 we see that the agreement between theory and experiment is very good, the discrepancies being in the order of a few tens of keV for most of the states. The largest discrepancy occurs for the $7^-$ state in $^{134}$Sb, which lies at about 130 keV above its experimental counterpart. It is an important outcome of our calculation for $^{134}$Sb that we predict almost the right spacing between the $0^-$ ground state and first excited $1^-$ state. In fact, the latter has been observed at 13 keV excitation energy, our value being 53 keV.

In Fig. 3 the calculated $\pi h_{9/2}v g_{9/2}$ multiplet in $^{210}$Bi is reported and compared with
FIGURE 4. Proton-neutron states in $^{212}$Bi arising from the configuration $\pi h_9/2 \nu (g_9/2)^3$. The theoretical results are represented by open circles while the experimental data by solid triangles.

the experimental data [24]. This multiplet is the counterpart of the $\pi g_{7/2} f_{7/2}$ multiplet in $^{134}$Sb, with the substitution $n, l, j \rightarrow n, l + 1, j + 1$. Comparison of Figs. 1 and 3 shows evidence of the similarity between the behavior of the two multiplets.

Let us now come to $^{212}$Bi, which is the counterpart of $^{136}$Sb. In Fig. 4 we show the calculated yrast states having angular momentum from $0^-$ to $9^-$, which all arise from the $\pi h_9/2 \nu (g_9/2)^3$ configuration, and compare them with the available experimental data [24]. From Fig. 4 we see that the pattern of the multiplet is substantially different from that in $^{210}$Bi, but quite similar to that predicted for $^{136}$Sb. In particular, as compared to the case of the one proton-one neutron nuclei $^{134}$Sb and $^{210}$Bi, we see that the curves of Figs. 3 and 4 are no longer concave downwards and that $0^-$ state is raised at about 380 keV and 200 keV excitation energy in $^{136}$Sb and $^{212}$Bi, respectively.

SUMMARY

We have presented here the results of a shell-model study of neutron-rich nuclei around $^{132}$Sn, focusing attention on proton-neutron multiplets in the odd-odd isotopes $^{134}$Sb and $^{136}$Sb. We have compared the results obtained for these two nuclei far from stability with those for $^{210}$Bi and $^{212}$Bi, which are their counterparts in the region of stable doubly magic $^{208}$Pb. In both cases, the two-body effective interaction has been derived by means of a $\hat{Q}$-box folded-diagrams method from the CD-Bonn $NN$ potential, the short-range repulsion of the latter being renormalized by use of the low-momentum potential $V_{\text{low-k}}$.

Our results for all four nuclei are in very good agreement with the available experimental data and show evidence of a striking resemblance between the behavior of the multiplets in the $^{132}$Sn and $^{208}$Pb regions. The observation of the missing states of the multiplets in $^{136}$Sb and $^{212}$Bi is certainly needed to further verify the soundness of our findings.
ACKNOWLEDGMENTS

This work was supported in part by the Italian Ministero dell’Istruzione, dell’Università e della Ricerca (MIUR).

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