Canonical and Non-canonical Inflation in the light of the recent BICEP/Keck results

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Abstract. We discuss implications of the latest BICEP/Keck data release for inflationary models, with particular emphasis on scalar fields with non-canonical Lagrangians of the type $L = X^\alpha - V(\phi)$. The observational upper bound on the tensor-to-scalar ratio, $r \leq 0.036$, implies that the whole family of monomial power law potentials $V(\phi) \sim \phi^p$ are now ruled out in the canonical framework at 95% confidence, which includes the simplest classic inflationary potentials such as $\frac{1}{2}m^2\phi^2$ and $\lambda\phi^4$. Instead, current observations strongly favour asymptotically flat plateau potentials. However, working in the non-canonical framework, we demonstrate that monomial potentials, as well as the Higgs potential with its Standard Model self-coupling, can easily be accommodated by current CMB data. We find striking similarities between the $\{n_s, r\}$ flow lines of monomial potentials in the non-canonical framework and the T-model $\alpha$-attractors in the canonical framework. Significantly, $V(\phi)$ can originate from Planck scale initial values $V(\phi) \simeq m^4$ in non-canonical models while in plateau-like canonical inflation the initial value of the potential is strongly suppressed $V_{\text{plat}}(\phi) \leq 10^{-10}m^4$. This has bearing on the issue of initial conditions for inflation and allows for the equipartition of the kinetic and potential terms in non-canonical models.

Keywords: Inflation, Early Universe Theory
1 Introduction

The remarkable progress in Cosmology over the past three decades, reinforced by new theoretical insights and fostered by a plethora of precision cosmological missions, has resulted in the so-called ‘standard scenario’ of the universe in the form of the spatially flat $\Lambda$CDM model [1]. This model of ‘Concordance Cosmology’ successfully describes the evolution of our universe, both at the background as well as the perturbative level, commencing from almost a second after the hot Big Bang until the present epoch [2, 3] (however, see [4]). Nevertheless, the success of this standard model relies upon several (seemingly) unnatural initial conditions which include, at the background level: the spatial homogeneity, isotropy and near spatial flatness of the universe, and at the perturbative level: the existence of a spectrum of nearly scale-invariant adiabatic density fluctuations (which seed structure formation in the universe).

‘Cosmic Inflation’ has emerged as a leading prescription for describing the very early universe prior to the commencement of the radiative hot Big Bang Phase [5–11]. According to the inflationary paradigm, a transient epoch of at least 60-70 e-folds of rapid accelerated expansion suffices in setting natural initial conditions for cosmology in the form of spatial flatness as well as statistical homogeneity and isotropy on large angular scales [6–11]. Of equal significance is the fact that cosmic inflation generates a spectrum of initial scalar fluctuations (via quantum fluctuations of a scalar degree of freedom) which later seed the formation of structure in the universe [12–15]. In addition to scalar perturbations, quantum fluctuations during inflation also create a spectrum of almost scale invariant tensor perturbations which later form gravity waves [16–18].

The simplest models of inflation comprising of a single scalar field, called the ‘inflaton’, which is minimally coupled to gravity, make several distinct predictions [19, 20], most of which have received...
spectacular observational confirmation, particularly from the latest CMB missions [21]. Current observational data lead to a scenario in which the inflaton $\phi$ slowly rolls down a shallow potential $V(\phi)$ thereby giving rise to a quasi-de Sitter early stage of near-exponential expansion.

As mentioned earlier, both scalar and tensor perturbations are generated during inflation. The latter constitute the relic gravitational wave background (GW) which imprints a distinct signature on the Cosmic Microwave Background (CMB) power spectrum in the form of the B-mode polarization [21]. The amplitude of these relic GWs provides us information about the inflationary energy scale while their spectrum enables us to access general properties of the epoch of reheating, being exceedingly sensitive to the post-inflationary equation of state [17, 22]. The amplitude of inflationary tensor fluctuations, relative to that of scalar fluctuations, is usually characterised by the tensor-to-scalar ratio $r$. Different models of inflation predict different values of $r$ which is sensitive to the gradient of the inflaton potential $V'(\phi) = \frac{dV(\phi)}{d\phi}$ relative to its height $V(\phi)$. Convex potentials predict large values for $r$, while concave potentials predict relatively small values of $r$. While the spectrum of inflationary tensor fluctuations has not yet been observed, current CMB observations are able to place an upper bound on the tensor-to-scalar ratio on large angular scales. In particular, the latest CMB observations$^1$ of BICEP/Keck [23], combined with those of the PLANCK mission [21], place the strong upper bound $r \leq 0.036$ (at 95% confidence). In this work we discuss the implications of this observational bound on single field inflationary models, with particular emphasis on scalar fields with a non-canonical Lagrangian density.

This most recent upper bound on $r$, combined with the $2\sigma$ bound on scalar spectral index $n_s$, has important consequences for single field canonical inflation. In particular, given $r \leq 0.036$, all monotonically increasing convex potentials, including the whole family of monomial potentials $V(\phi) \propto \phi^p$, are completely ruled out in the canonical framework. Among these strongly disfavoured models are the simplest classic inflaton potentials $\frac{1}{2}m^2\phi^2$ and $\lambda\phi^4$. Instead, the observational upper bound on $r$ appears to favour asymptotically flat potentials possessing one or two plateau-like wings; see [24].

Despite their excellent agreement with observations, plateau potentials may face some theoretical shortcomings [25]. It is sometimes felt that, being a theory of the very early universe, inflation should commence at the Planck scale [9, 25–28] with $\rho_\phi \sim m^4_P$. By contrast plateau potentials are extremely flat and imply $V(\phi) \leq 10^{-10} m^4_P$. Although a large value of the kinetic term $V(\phi) \propto \dot{\phi}^2 \sim m^4_P$ can offset this difficulty, it implies that the universe cannot commence from equipartition initial conditions $V(\phi) \sim \dot{\phi}^2 \sim m^4_P$. Moreover the small value of the inflaton potential, $V(\phi) \leq 10^{-10} m^4_P$ precludes the presence of a large curvature term, $|\rho_K| \sim m^4_P$ at the commencement of inflation. Indeed, since $\rho_K = -3m^2_P/a^2 < 0$ in a closed universe, an initial value $|\rho_K| \gg V(\phi)$ would imply that the universe would begin to contract prior to the onset of inflation in plateau potentials.

These issues can be successfully tackled in non-canonical models in which a small value of $r$ can be accommodated by changes to the non-canonical kinetic term. Thus small values of $r \leq 0.036$ can easily arise for convex potentials having the form $V(\phi) \propto \phi^p$ and $V(\phi) \propto \phi^{-p}$ [29, 30]. In particular, we shall show that the Standard Model (SM) Higgs field, which cannot source CMB-consistent inflation in the canonical framework (owing to its large self-coupling), can easily source inflation in the non-canonical framework$^2$. Consequently equipartition initial conditions can easily be accommodated in non-canonical inflationary models and in canonical models based on the Margarita potential. Additionally, we discover striking similarities between the $\{n_s, r\}$ flow lines of the T-model $\alpha$-attractors in the canonical framework and monomial potentials in the non-canonical framework, which is one of the central results of our analysis. We also discuss the differences in the predictions of $\{n_s, r\}$ for these two classes of models. Similarly we show that the inverse power law potential $V(\phi) \sim \phi^{-p}$, which leads to power law inflation in the non-canonical framework, also satisfies the latest CMB bounds.

$^1$Note that BICEP stands for Background Imaging of Cosmic Extragalactic Polarization.

$^2$Note that the SM Higgs can also source inflation if it is non-minimally coupled to gravity [31–33]
Our paper is organised as follows: after a brief introduction of inflationary scalar field dynamics in section 2, we discuss inflation in the canonical framework in section 3 highlighting the implications of the latest CMB observations on plateau potentials. Section 4 demonstrates that some of the difficulties faced by plateau models can be circumvented in the Margarita family of potentials which interpolate between potentials which are convex at large $|\phi|$ and concave at moderate $|\phi|$. Inflation in the non-canonical framework is discussed in section 5. The concluding section 6 is dedicated to a discussion highlighting the major inferences drawn from this work.

We work in the units $c, \hbar = 1$. The reduced Planck mass is defined to be $m_p \equiv 1/\sqrt{8\pi G} = 2.43 \times 10^{18}$ GeV. We assume the background universe to be homogeneous and isotropic with the metric signature $(\mathbf{\text{-}}, \mathbf{\text{+}}, \mathbf{\text{+}}, \mathbf{\text{+}})$

## 2 Inflationary Dynamics

In the single field inflationary paradigm, inflation is sourced by a scalar field, called the inflaton field, which is minimally coupled to gravity and its dynamics is described by the action

$$S[\phi] = \int d^4x \sqrt{-g} \mathcal{L}(X, \phi),$$

where the Lagrangian density $\mathcal{L}(\phi, X)$ is a function of the field $\phi$ and the kinetic term

$$X = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi.$$  

Varying the action in Eq. (2.1) w.r.t $\phi$ results in the evolution equation of the inflaton field

$$\frac{\partial \mathcal{L}}{\partial \phi} - \left( \frac{1}{\sqrt{-g}} \right) \partial_{\mu} \left( \sqrt{-g} \frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \phi)} \right) = 0.$$  

While, the energy-momentum tensor associated with the inflaton field is given by

$$T^{\mu\nu} = \left( \frac{\partial \mathcal{L}}{\partial X} \right) \left( \partial^{\mu} \phi \partial^{\nu} \phi - g^{\mu\nu} \mathcal{L} \right).$$

In order to study the background dynamics during inflation, we specialize to a homogeneous scalar field in a spatially flat FLRW universe, described by the line element

$$ds^2 = -dt^2 + a^2(t) \left[ dx^2 + dy^2 + dz^2 \right],$$

and the energy-momentum tensor

$$T_{\mu\nu} = \text{diag} \left( -\rho_\phi, p_\phi, p_\phi, p_\phi \right),$$

where the energy density $\rho_\phi$, and pressure $p_\phi$, of the homogeneous inflaton condensate are given by

$$\rho_\phi = \left( \frac{\partial \mathcal{L}}{\partial X} \right) \left( 2X \right) - \mathcal{L},$$

$$p_\phi = \mathcal{L}. $$

The background kinetic term is given by $X = -\frac{1}{2} \dot{\phi}^2$. The evolution of the scale factor $a(t)$ is governed by the two Friedmann equations

$$\left( \frac{\dot{a}}{a} \right)^2 \equiv H^2 = \frac{1}{3m_p^2} \rho_\phi,$$

$$\frac{\ddot{a}}{a} \equiv \dot{H} + H^2 = -\frac{1}{6m_p^2} \left( \rho_\phi + 3p_\phi \right),$$

- 3 -
where $H \equiv \dot{a}/a$ is the Hubble parameter. The scalar field energy density and pressure satisfy the energy-momentum conservation equation

$$\dot{\rho}_\phi = -3H (\rho_\phi + p_\phi).$$

(2.11)

The aforementioned discussion is applicable to scalar fields with both canonical and non-canonical kinetic terms.

### 3 Inflation in the canonical framework

For a canonical scalar field $\phi$, with a suitable potential $V(\phi)$, the Lagrangian density in Eq. (2.1) becomes

$$\mathcal{L}(X, \phi) = -X - V(\phi),$$

(3.1)

Substituting Eq. (3.1) into Eqs. (2.7) and (2.8), we obtain

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi),$$

$$p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi),$$

(3.2)

and hence, the two Friedmann Eqs. (2.9) and (2.10), and the conservation Eq. (2.11) become

$$H^2 = \frac{1}{3m_p^2} \rho_\phi = \frac{1}{3m_p^2} \left[ \frac{1}{2} \dot{\phi}^2 + V(\phi) \right],$$

(3.3)

$$\dot{H} = \frac{\ddot{a}}{a} - H^2 = -\frac{1}{2m_p^2} \dot{\phi}^2,$$

(3.4)

$$\ddot{\phi} + 3H \dot{\phi} + V'(\phi) = 0.$$

(3.5)

Expansion of space during the inflationary evolution at an epoch with scale factor $a$ is usually described by the number of e-folds before the end of inflation, defined as

$$N_e(a) = \ln \left( \frac{a_e}{a} \right) = \int_t^{t_e} H(t) \, dt,$$

(3.6)

where $a_e$ denotes the scale factor at the end of inflation. Hence $N_e = 0$ corresponds to the end of inflation. Typically a period of near-exponential inflation lasting for at least 60-70 e-folds is required in order to address the fine tuning initial conditions of the standard hot Big Bang phase. Throughout the paper, we denote $N_*$ as the number of e-folds before the end of inflation when the CMB pivot scale $k_\ast = (aH)_\ast = 0.05 \text{ Mpc}^{-1}$ made its Hubble-exit. Usually, one considers $N_* \in [50, 60]$ depending upon the details of reheating history in the post-inflationary universe.

The slow-roll regime of inflation, facilitated by the presence of the Hubble-drag term in Eq. (3.5), is conveniently characterised by two kinematic Hubble slow-roll parameters $\epsilon_H, \eta_H$, defined by [11, 34]

$$\epsilon_H = -\frac{\dot{H}}{H^2} = \frac{1}{2m_p^2} \frac{\dot{\phi}^2}{H^2},$$

(3.7)

$$\eta_H = -\frac{\ddot{\phi}}{H \dot{\phi}} = \epsilon_H + \frac{1}{2\epsilon_H} \frac{d\epsilon_H}{dN_e},$$

(3.8)

where the slow-roll conditions are defined as

$$\epsilon_H, \eta_H \ll 1.$$

(3.9)
The slow-roll regime is also often characterised by the dynamical potential slow-roll parameters \([11]\), defined by
\[
\epsilon_V = \frac{m_p^2}{2} \left( \frac{V'}{V} \right)^2, \quad \eta_V = \frac{m_p^2}{2} \left( \frac{V''}{V} \right),
\] (3.10)
Under the slow-roll approximations, we have \(\epsilon - H \simeq \epsilon_V\) and \(\eta_H \simeq \eta_V - \epsilon_V\). Since \(H = \dot{a}/a\), using Eq. (3.7), we obtain
\[
\frac{\ddot{a}}{a} = \left(1 - \epsilon_H \right) H^2,
\] (3.11)
which demonstrates that the universe accelerates, \(\ddot{a} > 0\), when \(\epsilon_H < 1\). Using equation (3.3), the expression for \(\epsilon_H\) in (3.7) reduces to \(\epsilon_H \simeq \frac{3}{2} \dot{\phi}^2 V / \sqrt{3} \) when \(\dot{\phi}^2 \ll V\). In fact, under the slow-roll conditions (3.9), the Friedmann equations (3.3) and (3.5) take the form
\[
\dot{\phi} \simeq - \frac{V'(\phi)}{3H}.
\] (3.13)
Combining the slow-roll Eqs. (3.13) and (3.13) we obtain the slow-roll trajectory described by
\[
\dot{\phi} \simeq - \frac{m_p}{\sqrt{3} V(\phi)} V(\phi),
\] (3.14)
It is well known that the slow-roll trajectory is actually a local attractor for a variety of different inflationary potentials, see Refs. [33, 35].

Quantum fluctuations during inflation

In the single-field inflationary scenario, two gauge-invariant massless fields, one scalar and one transverse traceless tensor, get excited during inflation and receive quantum fluctuations that are correlated over super-Hubble scales [36] at late times. All cosmologically relevant fluctuations start out as being sub-Hubble, namely \(k \gg aH\), at early times. During inflation, as the comoving Hubble radius \((aH)^{-1}\) decreases with time, fluctuations eventually become super-Hubble, i.e \(k \ll aH\), where they remain frozen until their subsequent Hubble re-entry after the end of inflation. The gauge-invariant scalar degree of freedom \(R\) during inflation is called the comoving curvature perturbation\(^3\) whose dimensionless primordial power-spectrum at super-Hubble scales is given by [36]
\[
P_R \equiv \frac{k^3}{2\pi^2} \left| \mathcal{R}_k \right|^2 \bigg|_{k \ll aH},
\] (3.15)
where \(\mathcal{R}_k\) is the Fourier transformation of \(R\). During slow-roll inflation, imposing suitable Bunch-Davies initial conditions [37] in the sub-Hubble regime \(k \gg aH\), the subsequent computation of the super-Hubble power spectrum of \(R\) leads to the famous slow-roll approximation formula [11]
\[
P_R = \frac{1}{8\pi^2} \left( \frac{H}{m_p} \right)^2 \frac{1}{\epsilon_H}.
\] (3.16)
On large cosmological scales which are accessible to CMB observations, the scalar power spectrum typically takes the form of a power law represented by
\[
P_R(k) = A_s \left( \frac{k}{k_s} \right)^{n_s-1},
\] (3.17)
\(^3\)Note that the comoving curvature perturbation \(R\), is also related to the curvature perturbation on uniform-density hypersurfaces, \(\zeta\), and both are equal during slow-roll inflation as well as on super-Hubble scales, \(k \ll aH\), in general (see [11]).
where $A_s = P_R(k_*)$ is the amplitude of the scalar power spectrum at the pivot scale $^4$ $k = k_*$,

$$A_s = \frac{1}{8\pi^2} \left( \frac{H}{m_p} \right)^2 \frac{1}{\epsilon_H} \bigg|_{k=k_*} .$$  \(3.18\)

The scalar spectral tilt $n_s$, in the slow-roll regime is given by \[11\]

$$n_s - 1 \equiv \frac{d \ln P_R}{d \ln k} = 2\eta_H - 4\epsilon_H .$$  \(3.19\)

Similarly the tensor power spectrum, in the slow-roll limit, is represented by

$$P_T(k) = A_T \left( \frac{k}{k_*} \right)^{n_T} ,$$  \(3.20\)

with the amplitude of tensor power spectrum at the CMB pivot scale is given by \[11, 36\]

$$A_T \equiv P_T(k_*) = \frac{2}{\pi^2} \left( \frac{H}{m_p} \right)^2 \bigg|_{k=k_*} ,$$  \(3.21\)

and the tensor spectral index (with negligible running) is given by

$$n_T = -2\epsilon_H .$$  \(3.22\)

The tensor-to-scalar ratio $r$ is defined by

$$r \equiv \frac{A_T}{A_s} = 16\epsilon_H ,$$  \(3.23\)

yielding the single field consistency relation

$$r = -8n_T .$$  \(3.24\)

We therefore find that the the slow-roll parameters $\epsilon_H$ and $\eta_H$ play a key role in characterising the power spectra of scalar and tensor fluctuations during inflation. In the next section, we discuss the implications of the latest CMB observations for the slow-roll parameters and for other relevant inflationary observables. In order to relate the CMB observables to the inflaton potential $V(\phi)$, we work with the potential slow-roll parameters\(^5\) defined in equation (3.10).

### 3.1 Implications of the latest CMB observations for canonical inflation

Consider a canonical scalar field minimally coupled to gravity and having the potential

$$V(\phi) = V_0 f \left( \frac{\phi}{m_p} \right) .$$  \(3.25\)

The potential slow-roll parameters (3.10) are given by

$$\epsilon_V = \frac{m_p^2}{2} \left( \frac{f'}{f} \right)^2 ,$$  \(3.26\)

$$\eta_V = m_p^2 \left( \frac{f''}{f} \right) .$$  \(3.27\)

\(^4\)Note that in general, $k$ may correspond to any observable CMB scale in the range $k \in [0.0005, 0.5]$ Mpc\(^{-1}\). However, in order to derive constraints on the inflationary observables \{$n_s, r$\}, we mainly focus on the CMB pivot scale, namely $k \equiv k_* = 0.05$ Mpc\(^{-1}\).

\(^5\)While for slow-roll inflation, we obtain constraints on the potential slow-roll parameters $\epsilon_V, \eta_V$ from the CMB data, we also present constraints on the Hubble slow-roll parameters $\epsilon_H, \eta_H$ because they directly represent the evolution of the scale factor during inflation.
In the slow-roll limit $\epsilon_V, \eta_V \ll 1$, the scalar power spectrum is given by the expression (3.17) with the amplitude of scalar power at the CMB pivot scale $k \equiv k_* = 0.05 \text{ Mpc}^{-1}$ expressed as [11]

$$A_s \equiv P_R(k_*) \simeq \frac{1}{24\pi^2} \frac{V_0}{m_p^4} \frac{f(\phi_k)}{\epsilon_V(\phi_k)}_{k=k_*},$$

and the scalar spectral index (with negligible running) is given by

$$n_s \simeq 1 + 2 \eta_V(\phi_*) - 6 \epsilon_V(\phi_*),$$

where $\phi_*$ is the value of the inflaton field at the Hubble exit of CMB pivot scale $k_*$. Similarly the amplitude of tensor power spectrum at the CMB pivot scale is given by

$$A_T \equiv P_T(k_*) = \frac{2}{\pi^2} \left( \frac{H_k^{\text{inf}}}{m_p} \right)^2 \left[ \frac{2}{3\pi^2} \frac{V_0}{m_p^4} f(\phi_k) \right]_{k=k_*},$$

where $H_k^{\text{inf}}$ is the Hubble parameter during the Hubble exit of mode $k$. The tensor spectral index (3.22) becomes

$$n_T \simeq -2 \epsilon_V(\phi_*),$$

and the tensor-to-scalar ratio (3.23) can be written as

$$r \simeq 16 \epsilon_V(\phi_*),$$

satisfying the single field consistency relation (3.24). From the CMB observations of Planck 2018 [21], we have

$$A_s = 2.1 \times 10^{-9},$$

while the $2\sigma$ constraint on the scalar spectral index is given by

$$n_s \in [0.957, 0.976].$$

Similarly the constraint on the tensor-to-scalar ratio $r$, from the latest combined observations of Planck 2018 [21] and BICEP/Keck [23], is given by

$$r \leq 0.036,$$

which translates into $A_T \leq 3.6 \times 10^{-2} A_s$. Equation (3.30) helps place the following upper bound on the inflationary Hubble scale $H_k^{\text{inf}}$ and the energy scale during inflation $E_{\text{inf}}$

$$H_k^{\text{inf}} \leq 4.7 \times 10^{13} \text{ GeV},$$

$$E_{\text{inf}} \equiv \left[ \sqrt{3} m_p H_k^{\text{inf}} \right]^{1/2} \leq 1.4 \times 10^{16} \text{ GeV}.$$
The EOS \( w_\phi \) of the inflaton field is given by
\[
w_\phi = \frac{1}{2} \frac{\dot{\phi}^2 - V(\phi)}{\dot{\phi}^2 + V(\phi)} \simeq -1 + \frac{2}{3} \epsilon_V(\phi),
\]
(Eq. 3.41)

Therefore one finds from (3.38) the following constraint on the inflationary EOS at the pivot scale
\[
w_\phi \leq -0.9985,
\]
(Eq. 3.42)

implying that the expansion of the universe during inflation was near exponential (quasi-de Sitter like).

Given the latest CMB constraints on \( n_S \) and especially the upper bound on the tensor-to-scalar ratio \( r \) in (3.35), a number of famous models of inflation are strongly disfavoured. These include the simplest quadratic chaotic potential \( \frac{1}{2} m^2 \phi^2 \), along with other monotonically increasing convex potentials, see figure 1. It is important to stress that the entire family of power law potentials \( V(\phi) \propto \phi^p \) are now observationally ruled out as shown by the lime colour stripe in figure 1.

**Figure 1**: This figure plots the tensor-to-scalar ratio \( r \) versus the scalar spectral index \( n_S \) for a number of inflationary potentials (the thinner and thicker dots and curves correspond to \( N_*=50, 60 \) respectively). These include predictions of plateau potentials such as the Starobinsky model, the Standard Model Higgs inflation, the T-model and E-model \( \alpha \)-attractors as well as the D-brane KKLT inflation. The CMB 2\( \sigma \) bound \( 0.957 \leq n_S \leq 0.976 \) and the upper bound on the tensor-to-scalar ratio \( r \leq 0.036 \) are indicated by the shaded grey colour region. Given the upper bound on \( r \), it is easy to see that observations favour concave potentials over convex ones.

Indeed, CMB observations now favour potentials exhibiting asymptotically flat plateau-like wings. In the literature of inflationary model building [38], there exist a number of plateau potentials that satisfy the CMB constraints. These include single parameter potentials such as the Starobinsky
model [5, 33, 39], and the non-minimally coupled Standard Model (SM) Higgs inflation [31–33], along with a number of important plateau potentials with two parameters, such as the T-model and E-model $\alpha$-attractors [40, 41], and the D-brane KKLT potential [24, 42–44] (where KKLT stands for Kachru-Kallosh-Linde-Trivedi).

3.2 Important plateau potentials as future CMB targets

An asymptotically flat potential has the general functional form

$$V(\phi) = V_0 f\left(\frac{\lambda \phi}{m_p}\right),$$

(3.43)
such that $V(\phi) \rightarrow V_0$ at large field values $\lambda \phi \gg m_p$, where $\lambda$ is a free parameter. Such a potential, schematically represented in figure 2, usually features one or two plateau-like wings for large field values away from the minimum of the potential, depending upon whether the potential is asymmetric or symmetric. Additionally, an asymptotically flat potential might approach the plateau either exponentially or algebraically. Below we briefly discuss a number of important plateau potentials in light of the latest CMB observations, keeping one example of plateau models belonging to either of the following categories\(^6\) -

1. Symmetric or asymmetric plateau potentials.
2. Single or double parameter plateau potentials.
3. Potentials with exponential or algebraic approach to plateau.

\[\text{(a)}\]

\[\text{(b)}\]

**Figure 2**: This figure schematically depicts asymptotically flat inflationary plateau potentials. The left panel shows an asymmetric potential featuring only a single plateau-like wing which supports slow-roll inflation. While the right panel shows a symmetric potential possessing two plateau-like wings, both of which can support slow-roll inflation. Note that for large field values, some potentials approach plateau behaviour exponentially, while others approach algebraically.

\(^6\)Our primary intention here is to provide a few examples of plateau models whose predictions can be accommodated by the latest data, rather than reviewing all available plateau potentials in the inflationary literature. Similarly, we do not carry out a full parameter estimation program and likelihood comparison of different plateau models (which requires MCMC analysis and is out of scope of this paper). We again stress that we have chosen one potential from each of the above categories just to provide an example for the reader. We do not claim that these potentials are more favourable than other models that have not been discussed in this section.
1. Starobinsky model

The potential for Starobinsky inflation [5] in the Einstein frame, in which the scalaron\(^7\) takes the form of a canonical scalar field minimally coupled to gravity, is given by [33, 39]

\[
V(\phi) = V_0 \left(1 - e^{-\frac{2|\phi|}{\sqrt{6} m_p}}\right)^2.
\]  

(3.44)

This potential features a single parameter \(V_0\), which is related to the scalaron mass \(m\) by

\[
V_0 = \frac{3}{4} m^2 n^2 \text{p}^2 [33],
\]

and whose value is completely fixed by the CMB normalization (3.33). The prediction of \(\{n_s, r\}\) for the Starobinsky potential is given by

\[
n_s \simeq 1 - \frac{2}{N_\ast}, \quad r \simeq \frac{12}{N_\ast^2}, \text{ for } N_\ast \gg 1,
\]

(3.45)

which is shown by black colour dots in figure 1. As per standard convention, the smaller and larger black dots represent \(\{n_s, r\}\) predictions of Starobinsky potential corresponding to \(N_\ast = 50, 60\) respectively. It is important to note that the predictions of Starobinsky inflation lie at the centre of the observationally allowed region of \(\{n_s, r\}\) (shown in grey in fig. 1), making it one of the most popular inflationary models at present.

2. Non-minimally coupled Standard Model (SM) Higgs inflation

The SM Higgs inflation was originally formulated [31, 32] in the Jordan frame where the Higgs field is non-minimally coupled to the scalar (Ricci) curvature of space-time. One can however transform it to the Einstein frame by a conformal transformation of the metric [32, 33] so that the Einstein frame action describes a canonical scalar field minimally coupled to gravity. The corresponding SM Higgs inflaton potential, for large field values, takes the form

\[
V(\phi) \simeq V_0 \left(1 - e^{-\frac{2|\phi|}{\sqrt{6} m_p}}\right)^2.
\]

(3.46)

The above potential also features a single parameter \(V_0\), which is related to the strength of the non-minimal coupling \(\xi\) by \(V_0 = \frac{\lambda m^2}{4\xi^2}\) as shown in [32, 33], where \(\lambda = 0.1\) is the SM Higgs self-coupling. Its value is completely fixed by the CMB normalization (3.33). Given the similarity in the functional form of (3.44) and (3.46), the prediction of the non-minimally coupled SM Higgs inflation for \(\{n_s, r\}\) is similar to that of Starobinsky inflation\(^8\), given by (3.45).

3. T-model \(\alpha\)-attractor

The T-model \(\alpha\)-attractor potential [24] is a symmetric plateau potential of the functional form

\[
V(\phi) = V_0 \tanh^p \left(\frac{\phi}{m_p}\right),
\]

(3.47)

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\(^7\)Note that Starobinsky inflation was originally formulated as a modified gravity theory with the Jordan frame Lagrangian \(f(R) = R + R^2/6m^2\) which contains an additional scalar degree of freedom compared to general relativity. Upon conformal transformation of the Jordan frame metric, one can arrive at the Einstein frame Lagrangian where the extra scalar degree of freedom takes the form of a canonical scalar field, known as the ‘scalaron’.

\(^8\)Note that while the functional form of the Einstein frame potentials for Starobinsky inflation (3.44) looks similar to that of the non-minimally coupled SM Higgs inflation (3.46), however, the Starobinsky potential is asymmetric, while the Higgs potential is symmetric [33]. Similarly, the predictions of \(\{n_s, r\}\) for both the models can in principle be different, depending upon the details of reheating mechanism [45].
where $p$ is a positive real number. For a given value of $p$, the T-model potential has two parameters, namely $V_0$ and $\lambda$. As usual, the value of $V_0$ is fixed by the CMB normalization (3.33) while $\lambda$ determines the predicted value of $\{n_s, r\}$ for this potential.

\[
V(\phi) = V_0 \tanh^p \left( \frac{\lambda \phi}{m_p} \right)
\]

Figure 3: This figure is a plot of the tensor-to-scalar ratio $r$, versus the scalar spectral index $n_s$, for the T-model $\alpha$-attractor (3.47) (the thinner and thicker curves correspond to $N_\ast = 50, 60$ respectively). The latest CMB $2\sigma$ bound $0.957 \leq n_s \leq 0.976$ and the upper bound on the tensor-to-scalar ratio $r \leq 0.036$ are indicated by the shaded grey colour region. It is important to note that upon increasing the value of $\lambda$, the predictions of T-model potential (3.47) for different values of $p$ converge towards the cosmological attractor at the centre, described by equation (3.48). For $\lambda \gtrsim 0.1$, the predictions of T-model become compatible with the latest CMB $2\sigma$ bound (see table 2).

Predictions of the simplest T-model potential, which corresponds to $p = 2$ in equation (3.47), are shown by the red colour curves in figure 1. As we vary value of $\lambda$ from $\lambda : 0 \to \infty$, the $r$ versus $n_s$ values trace out continuous curves in figure 1 (the thinner and thicker red curves correspond to $N_\ast = 50, 60$ respectively). We notice that in one limit, namely $\lambda \ll 1$, the CMB predictions for $\{n_s, r\}$ of the T-model matches with that of the quadratic $\phi^2$ potential owing to the fact that the potential (3.47) for $p = 2$ behaves like $V(\phi) \propto \phi^2$ for $\lambda \phi \ll m_p$. In fact this is true in general for the T-model potential with any value of $p$ in (3.47), leading to the behaviour $V(\phi) \propto \phi^p$ for $\lambda \phi \ll m_p$, which is demonstrated in figure 3. However in the opposite limit, namely $\lambda \gtrsim 1$ (which results in $\exp (\lambda \phi / m_p) \gg 1$), the predictions of T-model become

\[
n_s \simeq 1 - \frac{2}{N_\ast}, \quad r \simeq \frac{2}{\lambda^2 N_\ast^2}, \quad \text{for } N_\ast \gg 1,
\]

(3.48)

---

9In the T-model potential, $\lambda$ is related to the $\alpha$ parameter of $\alpha$-attractors [41] by $\lambda^2 = \frac{1}{\alpha}$. 

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\[\text{Figure 3: This figure is a plot of the tensor-to-scalar ratio } r, \text{ versus the scalar spectral index } n_s, \text{ for the T-model } \alpha\text{-attractor (3.47) (the thinner and thicker curves correspond to } N_\ast = 50, 60 \text{ respectively). The latest CMB } 2\sigma \text{ bound } 0.957 \leq n_s \leq 0.976 \text{ and the upper bound on the tensor-to-scalar ratio } r \leq 0.036 \text{ are indicated by the shaded grey colour region. It is important to note that upon increasing the value of } \lambda, \text{ the predictions of T-model potential (3.47) for different values of } p \text{ converge towards the cosmological attractor at the centre, described by equation (3.48). For } \lambda \gtrsim 0.1, \text{ the predictions of T-model become compatible with the latest CMB } 2\sigma \text{ bound (see table 2).}

Predictions of the simplest T-model potential, which corresponds to } p = 2 \text{ in equation (3.47), are shown by the red colour curves in figure 1. As we vary value of } \lambda \text{ from } \lambda : 0 \to \infty, \text{ the } r \text{ versus } n_s \text{ values trace out continuous curves in figure 1 (the thinner and thicker red curves correspond to } N_\ast = 50, 60 \text{ respectively). We notice that in one limit, namely } \lambda \ll 1, \text{ the CMB predictions for } \{n_s, r\} \text{ of the T-model matches with that of the quadratic } \phi^2 \text{ potential owing to the fact that the potential (3.47) for } p = 2 \text{ behaves like } V(\phi) \propto \phi^2 \text{ for } \lambda \phi \ll m_p. \text{ In fact this is true in general for the T-model potential with any value of } p \text{ in (3.47), leading to the behaviour } V(\phi) \propto \phi^p \text{ for } \lambda \phi \ll m_p, \text{ which is demonstrated in figure 3. However in the opposite limit, namely } \lambda \gtrsim 1 \text{ (which results in } \exp (\lambda \phi / m_p) \gg 1), \text{ the predictions of T-model become [24]}

\[
n_s \simeq 1 - \frac{2}{N_\ast}, \quad r \simeq \frac{2}{\lambda^2 N_\ast^2}, \quad \text{for } N_\ast \gg 1,
\]

(3.48)

---

9In the T-model potential, $\lambda$ is related to the $\alpha$ parameter of $\alpha$-attractors [41] by $\lambda^2 = \frac{1}{\alpha}$. 

---
which is independent of $p$ (see [22]). Due to this property these models are called ‘cosmological attractors’ in [24]. In section 5.1, we will demonstrate that the $r$ versus $n_S$ flow lines of monomial potentials $V(\phi) \sim \phi^p$ in the non-canonical framework display striking similarities (see figure 8) with the predictions of T-model shown in figure 3.

4. E-model $\alpha$-attractor

The E-model $\alpha$-attractor potential [24] is an asymmetric plateau potential of the functional form

$$V(\phi) = V_0 \left( 1 - e^{-\lambda \phi/m_p} \right)^p,$$

where $p$ is a positive real number. For $p = 2$ and $\lambda = 2/\sqrt{6}$, the E-model potential coincides with Starobinsky potential\(^{10}\). In the limit $\lambda \geq 1$ (which results in $\exp (\lambda \phi/m_p) \gg 1$), the predictions of E-model become [24]

$$n_S \simeq 1 - \frac{2}{N_*}, \quad r \simeq \frac{8}{\lambda^2 N_*^2}, \quad \text{for } N_* \gg 1,$$

which again is independent of $p$ (see [22]). The CMB predictions $\{n_S, r\}$ of E-model potential are shown by the cyan colour curves in figure 1 for the case $p = 2$. (the thinner and thicker cyan curves correspond to $N_* = 50, 60$ respectively).

In all of the above models, the potential approaches plateau behaviour exponentially. In the following we briefly discuss KKLT potential which approaches plateau behaviour algebraically.

5. D-brane KKLT potential

The D-brane KKLT inflation [42, 43] potential (which has the same functional form as the polynomial $\alpha$-attractor model [46]) is given by [24, 44]

$$V(\phi) = V_0 \phi^n/M^n,$$

where $n$ is a positive integer and $M$ is a fundamental scale of the theory. This is a symmetric plateau potential, like the T-model $\alpha$-attractor (3.47). However the KKLT potential approaches plateau behaviour (saturates) algebraically, in contrast to the exponential approach to plateau behaviour exhibited by the T-model potential.

In the limit $M \gg m_p$, the potential asymptotes to a monomial power law form $V(\phi) \propto \phi^n$, the predictions of which are strongly disfavoured by observations. In the opposite limit, namely $M \ll m_p$, the potential is plateau-like, $V(\phi) \rightarrow V_0$, whose predictions satisfy the CMB data very well. Figure 4 shows the $r$ versus $n_S$ plot for KKLT potential (3.51) for three different values of $n$ (the thinner and thicker curves correspond to $N_* = 50, 60$ respectively). From this figure, it is easy to infer that the predictions of KKLT potential (3.51) for $\{n_S, r\}$ do not reach a common attractor regime for different values of $n$ (in contrast to those of the T-model and E-model), rather they cover a large horizontal portion of the observationally allowed region of $n_S$, as highlighted in [24, 44].

Note that the values of the parameter $\lambda$ in the T-model and E-model potentials, and the parameter $M$ in the KKLT potential, for which these models become compatible with the latest CMB data are discussed in appendix A.

\(^{10}\)In the E-model potential, $\lambda$ is related to the $\alpha$ parameter of $\alpha$-attractors [41] by $\lambda^2 = \frac{4}{\alpha}$.
In this section, we discussed the predictions for \( \{n_S, r\} \) of a number of plateau potentials which are consistent with current CMB data and are interesting target candidates for next generation CMB missions\(^{11}\). Despite their excellent agreement with observations, plateau potentials may face some theoretical shortcomings which relate to the issue of initial conditions. Although a large range of initial field values and field velocities \( \{\phi_i, \dot{\phi}_i\} \) results in adequate inflation for a plateau potential [33], the small height of the plateau \( V_0 \leq 10^{-10} m_p^4 \) does not allow for the equipartition of initial kinetic, potential and gradient terms in the Lagrangian density [25]. Similarly the presence of a non-negligible initial positive spatial curvature term might also prove problematic for plateau inflation especially if \( |\rho_{\text{initial}}| \gg V_0 \sim 10^{-10} m_p^4 \) (see [25]). Additionally, plateau potentials predicting a small value of the tensor-to-scalar ratio might exhibit relatively large running of the scalar spectral index \( n_S \) (see [47]).

These potentially important issues associated with plateau potentials can be addressed in two distinct ways\(^{12}\):

\(^{11}\)In passing, we would again like to stress that the set of plateau potentials discussed in section 3.2 is far from being exhaustive of all the inflationary models that are consistent with the CMB data, for which we refer the reader to more extensive literature on inflationary models [38].

\(^{12}\)Note that Starobinsky and SM Higgs inflation were originally formulated in their respective Jordan frames. However it is always possible to go to the Einstein frame by a conformal transformation of the metric, in which the action takes the form of a single canonical scalar field minimally coupled to gravity. Additionally, the T-model and E-model \( \alpha \)-attractors, as well as the D-brane KKLT inflation, can be formulated in a framework in which the Lagrangian density consists of a non-canonical kinetic term featuring a pole and a monomial potential [24]. However, it is possible to go to the canonical framework by a mere field redefinition, such that the kinetic term is canonical and
(i) Modifying the form of the potential $V(\phi)$ by making it convex at large values of $\phi$ while preserving its plateau-like form at moderate values of $\phi$ and keeping the canonical nature of the Lagrangian intact.

(ii) By exploring convex potentials, including $V \propto \phi^p$, in a non-canonical framework in which the Lagrangian density has the form $\mathcal{L} = X^\alpha - V(\phi)$.

The first of these possibilities will be examined in the next section while non-canonical scalars will be discussed in section 5.

4 Inflation with the Margarita potential

Most of the models discussed in the previous sections had plateau-like wings which prevented inflation from occurring at Planck scale values of the inflationary potential, and hence disfavoured the presence of equipartition initial conditions with $\dot{\phi}^2 \sim V(\phi) \sim m_p^4$. This minor flaw of plateau potentials is alleviated in a new family of potentials which we call the Margarita family. The Margarita potential introduces an intermediate flat wing on a potential which thereafter increases monotonically with $\phi$.

It therefore has properties similar to both plateau and monomial potentials. The Margarita potential allows for the presence of equipartition initial conditions while at the same time providing excellent agreement with current CMB constraints.

Margarita-type potentials\(^{13}\) have the following general features –

1. Monotonically growing wings support inflation at large values of the inflaton field.
2. An oscillatory quadratic asymptote, $V \propto \phi^2$, exists at small values of the inflaton field.
3. An intermediate plateau-like wing joins 1 and 2.

We propose the following simple functional form for Margarita-type potentials

$$V(\phi) = V_b(\lambda_1, \phi) V_c(\lambda_2, \phi), \quad \lambda_1 \gg \lambda_2.$$ (4.1)

Here $V_b(\phi)$ is an asymptotically flat plateau-like potential with height $V_0$, while $V_c(\phi)$ is the correction to the plateau-like wing at large inflaton field values. The two free parameters $\lambda_1$ and $\lambda_2$ in (4.1) satisfy $\lambda_1 \gg \lambda_2$. $V_c(\phi)$ increases monotonically for $\phi/m_p \gg 1/\lambda_2$ while $V_c(\phi = 0) = 1$. In this work we shall choose $V_c(\phi)$ such that $V_b V_c(\phi) \sim V_0 + \frac{1}{2} m_2^2 \phi^2$ for $\phi/m_p \ll 1/\lambda_2$, where $m_2$ depends on $\lambda_2$.

As a result, a generic Margarita potential will exhibit the following three asymptotic branches (see figure 5) –

Growing wing: $V(\phi) \simeq$ Monotonic, $\frac{\vert \phi \vert}{m_p} \gg \frac{1}{\lambda_2}$, (4.2)

Flat wing: $V(\phi) \simeq V_0 + \frac{1}{2} m_2^2 \phi^2$, $\frac{1}{\lambda_1} \ll \frac{\vert \phi \vert}{m_p} \ll \frac{1}{\lambda_2}$, (4.3)

Oscillatory region: $V(\phi) \simeq \frac{1}{2} m_1^2 \phi^2$, $\frac{\vert \phi \vert}{m_p} \ll \frac{1}{\lambda_1}$, (4.4)

where the mass parameters $m_1$ and $m_2$ depend on $\lambda_1$ and $\lambda_2$ respectively. Figure 5 illustrates the three asymptotic branches of the Margarita potential.

We first demonstrate the behaviour of Margarita potential by assuming $V_b$ to be the T-model based plateau potential [40, 41].

\(^{13}\)The Margarita potential is inspired by the shape of the Margarita cocktail glass. It was originally introduced in [48] to facilitate a period of transient acceleration which gave rise to dark energy. In this context, the Margarita potential featured a monotonically growing steep exponential wing at large field values. This allowed the small current value of the DE density to commence from a large basin of initial conditions.
Figure 5: This figure schematically illustrates a typical Margarita potential (4.5) with $\lambda_1 \gg \lambda_2$. For $\lambda_2|\phi| \gg m_p$, the potential exhibits monotonically growing convex inflationary wings (red shade). For small field values, namely $\lambda_1|\phi| \ll m_p$, the potential has a minimum (green shade). While for intermediate field values $\frac{1}{\lambda_1} < \frac{\phi}{m_p} < \frac{1}{\lambda_2}$, the potential displays plateau-like behaviour (grey shade).

4.1 T-model based exponential Margarita Potential

The Margarita potential in this case is given by

$$V(\phi) = V_0 \tanh^2 \left( \frac{\lambda_1 \phi}{m_p} \right) \cosh \left( \frac{\lambda_2 \phi}{m_p} \right).$$  \hspace{1cm} (4.5)

where the base potential

$$V_b(\phi) = V_0 \tanh^2 \left( \frac{\lambda_1 \phi}{m_p} \right),$$  \hspace{1cm} (4.6)

is the familiar T-model $\alpha$-attractor potential whose $r$ versus $n_s$ plot is shown by the red colour curve in figure 3. The correction potential, $V_c(\phi) = \cosh \left( \frac{\lambda_2 \phi}{m_p} \right)$, has exponential asymptotes for large field values $\lambda_2 \phi \gg m_p$.

In this Margarita potential (4.5), the exponential correction arising from $V_c(\phi) = \cosh \left( \frac{\lambda_2 \phi}{m_p} \right)$ to the base plateau potential modifies the nature of the $r$ versus $n_s$ plot, shifting this curve towards higher values of $n_s$ for identical values of $r$. Hence the $\{n_s, r\}$ predictions of the Margarita potential are, in principle, distinguishable from those of the base T-model potential. This shift is dependent upon the value of the correction parameter $\lambda_2$. For $\lambda_2 = 0$, one arrives at the base T-model potential while larger values of $\lambda_2$, give rise to a larger shift in the value of $n_s$ with respect to the base potential. Note that we assume $\lambda_2 < \sqrt{2}$ in order to realise inflation in the exponential asymptote to $V(\phi)$ at early times. We find that the Margarita potential (4.5) satisfies the CMB constraints on $n_s$ for $\lambda_2 \leq 0.1$ (while the CMB upper bound on $r$ can be satisfied for suitable range of values of $\lambda_1$, as in the case of T-model). Figure 6 shows the $r$ versus $n_s$ plot of this Margarita potential for $\lambda_2 = 0.04$. 
Scalar Spectral Index $n_S$

0.00 0.02 0.04 0.06 0.08 0.10 0.12 0.14 0.16
0.94 0.95 0.96 0.97 0.98 0.99 1.00

Tensor-to-scalar ratio $r$

Planck+BK18+BAO $r \leq 0.036$

Figure 6: This figure compares the $r$ versus $n_S$ plot of the margarita potential (4.5) with its asymptotically flat T-model base potential (4.6) for $\lambda_2 = 0.04$ (the tensor-to-scalar ratio $r$ decreases upon increasing the value of $\lambda_1$ and becomes compatible with the CMB bound $r \leq 0.036$ for $\lambda_1 \gtrsim 0.1$). As usual, the thinner and thicker curves correspond to $N_\star = 50, 60$, respectively.

4.2 Implications for initial conditions

As discussed in section 3.2, although recent CMB observations appear to favour asymptotically flat plateau potentials, the latter could face some theoretical shortcomings relating to the issue of initial conditions. In particular, owing to the fact that the height of the plateau saturates to a small value $V_0 \lesssim 10^{-10} m_p^4$ for large field values $\phi \gg m_p$, an equipartition of the initial inflaton kinetic energy density $\rho_{KE} = \frac{1}{2} \dot{\phi}^2$, potential energy density $\rho_V \simeq V_0$, and gradient term $\rho_{grd} \simeq \frac{1}{2} (\nabla \phi)^2$ will not be possible. Moreover, the possible presence of a large initial curvature density $\rho_K = -3 m_p^2/a_i^2$ (corresponding to positive spatial curvature, $K = +1$) might also be problematic for inflation.

To emphasise this point, let us imagine that we set initial conditions for inflation closer to the Planck scale, i.e., $\rho_{\phi_i} \simeq 0.1 m_p^4$. Suppose the height of the plateau is $V_0 \simeq 10^{-10} m_p^4$. In this case, starting from an initial state where the kinetic and curvature densities are of similar magnitude, namely $\rho_{KE} \simeq \rho_K \simeq \rho_{\phi_i}$, we notice that the universe begins to collapse before inflation begins. This has been demonstrated in the left panel of figure 7 which shows that even an initial curvature density as small as 1% of the initial inflaton energy density leads to the eventual collapse of an initially expanding universe. In fact the initial curvature density must be extremely sub-dominant compared to the initial inflaton energy density in order to prevent the collapse. A preliminary numerical analysis in a forthcoming paper [49] shows that $\rho_K < 10^{-6} \rho_{\phi_i}$ in order for inflation to begin. Hence, unless the spatial curvature is negative or, it is positive with a negligibly small density, plateau potentials are prone to the fine tuning of initial conditions.

Note that since inflation is supposed to address the flatness problem, we do not assume the spatial curvature of the universe to be zero initially.
Figure 7: This figure illustrates the implications of a relatively large initial curvature density $\rho_K = -3 m_p^2/a^2$ (corresponding to a positive spatial curvature, $K = +1$) at the commencement of inflation. The left panel shows the density evolution of the inflaton kinetic term $\rho_{KE}$ (red), potential term $\rho_V$ (blue) and the modulus of the spatial curvature term $|\rho_K|$ (green) for a plateau potential with $V(\phi) \simeq V_0 = 10^{-10} m_p^4$. The right panel shows the same for the Margarita potential $V(\phi)$ (4.5). From the left panel, we notice that for a plateau potential, even if $\rho_{KE} \gg |\rho_K|$ initially, an expanding universe collapses even before inflating due to the small value of $\rho_V$ in plateau potentials. By contrast the right panel shows that even though the initial value of $\rho_V$ is smaller than the curvature term, the universe keeps expanding and eventually begins to inflate. If one commences from an initial equipartition configuration in which all the three densities are comparable, then realising inflation in a Margarita potential is extremely easy.

However both the equipartition problem and the positive spatial curvature issue can be successfully addressed by a Margarita potential with monotonically growing inflationary wings. This has been demonstrated in the right panel of figure 7. We notice that, starting close to the Planck scale\(^{15}\) with a relatively large initial curvature density, the universe proceeds to inflate successfully and the collapse scenario can be completely avoided. It is easy to see that the equipartition condition can be easily satisfied in this case. Hence Margarita potentials could play a crucial role in addressing the initial (positive) curvature problem, as well as facilitating the equipartition of initial energy densities of different components, which were amongst the primary goals of inflation in its original formulation [6, 7].

4.3 General recipe for constructing a Margarita potential

The Margarita ‘cocktail glass’ shape of the potential in figure 5 is quite general and not restricted to the potential (4.5) discussed previously.

Indeed a Margarita potential $V(\phi)$ can easily be constructed using the prescription discussed earlier, namely:

- Multiply a plateau-like base potential $V_b(\phi)$ with a convex inflationary potential $V_c(\phi)$ to make the Margarita potential $V(\phi)$:

$$V(\phi) = V_b(\phi) \times V_c(\phi)$$

\(^{15}\)We keep the initial inflaton density $\rho_{\phi_i}$ to be one order smaller than the Planck scale in order to justify the usage of Einstein's GR in our numerical simulations.
Here \( V_b(\phi) \) must have the following properties:

1. It vanishes at the origin \( V_b(\phi = 0) = 0 \).
2. Its asymptotically flat: \( V_b(\phi \to \pm\infty) = V_0 \).

Examples of some widely studied plateau potentials are:

1. The KKLT potential \([24, 44]\)
   \[
   V_b(\phi) = V_0 - \frac{\phi^n}{\phi^n + M^n}.
   \] (4.8)

2. The T-model \( \alpha \)-attractor \([24, 44]\)
   \[
   V_b(\phi) = V_0 \tanh^{2n} \left( \frac{\alpha \phi}{m_p} \right).
   \] (4.9)

3. The (non-minimally coupled) standard model Higgs potential in the Einstein frame
   \[
   V(\phi) \simeq V_0 \left( 1 - e^{-2 \frac{|\phi|}{\sqrt{6} m_p}} \right)^2.
   \] (4.10)

The convex inflationary potential function \( V_c(\phi) \) should be dimensionless and preferably symmetric about the origin so that \( V_c(\phi) = V_c(-\phi) \) and \( V_c(\phi = 0) = 1 \).

| Plateau potential | Convex potential | Margarita potential |
|-------------------|------------------|---------------------|
| \( V_b(\phi) \)   | \( V_c(\phi) \)  | \( V(\phi) = V_b(\phi) \times V_c(\phi) \) |
| \( V_0 \tanh^{2n} (\alpha \phi) \) | \( \cosh (\beta \phi) \) | \( V_0 \tanh^{2n} (\alpha \phi) \cosh (\beta \phi) \) |
| \( V_0 \frac{\phi^n}{\phi^n + M^n} \) | \( 1 + \alpha \phi^2 \) | \( V_0 \frac{\phi^n}{\phi^n + M^n} \left( 1 + \alpha \phi^2 \right) \) |
| \( V_0 \left( 1 - e^{-\alpha |\phi| / m_p} \right)^2 \) | \( e^{\beta \phi^2} \) | \( V_0 \left( 1 - e^{-\alpha |\phi| / m_p} \right)^2 e^{\beta \phi^2} \) |

Table 1: This table lists three base potentials \( V_b(\phi) \) which are asymptotically flat and three convex potentials \( V_c(\phi) \). Note that any of the base potentials \( V_b \) in this table can be multiplied by any of the convex potentials \( V_c \) to construct the Margarita potential \( V(\phi) = V_b(\phi) \times V_c(\phi) \). \((m_p = 1)\) is assumed in this table.

A list of some plateau base potentials \( V_b \) and convex potentials \( V_c \) is given in table 1. Note that any of the base potentials \( V_b \) in table 1 can be multiplied by any of the convex potentials \( V_c \) to construct the Margarita potential \( V(\phi) = V_b(\phi) \times V_c(\phi) \). These examples are by no means exhaustive. However a general discussion of the Margarita potential both in the context of inflation and dark energy lies beyond the scope of the present paper. 

\[16\]In the context of the convex potential \( e^{\beta \phi^2} \) one should note that the resulting Margarita potential will satisfy equipartition initial conditions \( V(\phi) \sim \dot{\phi}^2 \sim m_p^4 \) but the extreme steepness of \( e^{\beta \phi^2} \) will prevent inflation from occurring while the inflaton rolls down the steep branch of the Margarita potential. Consequently the presence of a small initial (positive) curvature term could quite easily dominate over \( V(\phi) \) and make the universe contract before inflating.
5 Inflation in the non-canonical framework

A natural extension of the inflationary framework involves Lagrangian with a non-canonical kinetic term [50]. An attractive feature of this class of models is that the equations of motion remain second order [29, 50].

Non-canonical scalars have the Lagrangian density [50]

\[ L(X, \phi) = X \left( \frac{X}{M^4} \right)^{\alpha - 1} - V(\phi), \quad X = \frac{1}{2} \dot{\phi}^2, \]  

(5.1)

where \( M \) has dimensions of mass, while \( \alpha \geq 1 \) is a dimensionless parameter. When \( \alpha = 1 \) the Lagrangian (5.1) reduces to the usual canonical scalar field Lagrangian \( L(X, \phi) = X - V(\phi) \).

The energy density and pressure have the form

\[ \rho_\phi = (2 \alpha - 1) X \left( \frac{X}{M^4} \right)^{\alpha - 1} + V(\phi), \]

\[ p_\phi = X \left( \frac{X}{M^4} \right)^{\alpha - 1} - V(\phi), \quad X = \frac{1}{2} \dot{\phi}^2, \]

(5.2)

which reduces to the canonical expression \( \rho_\phi = X + V, \ p_\phi = X - V \) when \( \alpha = 1 \).

One should note that the equation of motion

\[ \ddot{\phi} + \frac{3H}{2\alpha - 1} + \left( \frac{V'(\phi)}{\alpha(2\alpha - 1)} \right) \left( \frac{2M^4}{\dot{\phi}^2} \right)^{\alpha - 1} = 0 \]

is singular at \( \dot{\phi} \to 0 \) and needs to be regularized so that the value of \( \ddot{\phi} \) remains finite in this limit. This can be done [29] by modifying the Lagrangian (5.1) to

\[ \mathcal{L}_R(X, \phi) = \left( \frac{X}{1 + \beta} \right) \left( 1 + \beta \left( \frac{X}{M^4} \right)^{\alpha - 1} \right) - V(\phi), \]

(5.4)

where \( \beta \) is a dimensionless parameter. In the limit when \( \beta \gg 1 \), equation (5.3) can be approximated as

\[ \ddot{\phi} + \frac{3H}{2\alpha - 1} + \left( \frac{V'(\phi)}{\epsilon + \alpha(2\alpha - 1)(X/M^4)^{\alpha - 1}} \right) = 0, \quad X = \frac{1}{2} \dot{\phi}^2, \]

(5.5)

where \( \epsilon \equiv (1 + \beta)^{-1} \) is an infinitesimally small correction factor when \( \beta \gg 1 \).

As shown in [29] for potentials behaving like \( V(\phi) \propto \phi^p \) near the minimum, the average EOS during scalar field oscillations is

\[ \langle \omega_\phi \rangle = \frac{p - 2\alpha}{p(2\alpha - 1) + 2\alpha} \cdot \]

For \( \alpha = 1 \) the above expression reduces to the canonical result

\[ \langle \omega_\phi \rangle = \frac{p - 2}{p + 2} \cdot \]

(5.6)

(5.7)

The inflationary slow-roll parameter \( \epsilon_{nc} \) for non-canonical inflation is given by [29]

\[ \epsilon_{nc} = \left( \frac{1}{\alpha} \right)^{\frac{1}{2\alpha - 1}} \left( \frac{3M^4}{V} \right)^{\frac{\alpha - 1}{2\alpha - 1}} (\epsilon_c)^{\frac{\alpha}{2\alpha - 1}}, \]

(5.8)

\( \epsilon_c \) being the canonical slow-roll parameter (3.26). Note that \( \epsilon_{nc} < \epsilon_c \) for \( 3M^4 \ll V \). This suggests that for a fixed potential \( V \), the duration of inflation can be enhanced relative to the canonical case \( (\alpha = 1) \), by a suitable choice of \( M \).

In the following, we will discuss the inflationary predictions for \( \{n_s, r\} \) in the non-canonical framework, starting with the family of monomial power law potentials \( V(\phi) \propto \phi^p \), followed by the inverse power law potentials \( V(\phi) \propto \phi^{-p} \).
5.1 Monomial power law potentials in the non-canonical framework

Monomial power law potentials in the non-canonical framework are described by the Lagrangian density

\[ L(X, \phi) = X \left(\frac{X}{M^4}\right)^{\alpha-1} - V_0 \left(\frac{\phi}{m_p}\right)^p. \]  

(5.9)

Expressions for the scalar spectral index \( n_s \), and the tensor-to-scalar ratio \( r \) are given by [29]

\[ n_s = 1 - 2 \left(\frac{\gamma + p}{2\gamma N_s + p}\right), \]

(5.10)

\[ r = \frac{1}{\sqrt{2\alpha - 1}} \left(\frac{16p}{2\gamma N_s + p}\right), \]

(5.11)

where the parameter \( \gamma \) is defined as

\[ \gamma = \frac{2\alpha + p(\alpha - 1)}{2\alpha - 1}. \]

(5.12)

The tensor spectral tilt for non-canonical potentials satisfies the single field consistency relation [29]

\[ r = -\frac{8}{\sqrt{2\alpha - 1}} n_T, \]

(5.13)

which differs from the canonical consistency relation (3.24) for \( \alpha > 1 \), and hence can be used as a smoking gun test for non-canonical inflation.

The plots of \( r \) versus \( n_s \) in the non-canonical framework for power law potentials \( V \propto \phi^p \) are shown in figure 8 for different values of \( p \). For \( \alpha = 1 \), the predictions of non-canonical power law potentials match with that of the canonical power law potentials, as expected. An increase in the value of \( \alpha \) leads to a decrease in the tensor-to-scalar ratio \( r \) for all values of \( p \). Hence in the non-canonical framework, power law potentials can be consistent with the CMB constraints for large enough values of \( \alpha \) (see table 5). In the large \( \alpha \) limit, the \( \{n_s, r\} \) predictions asymptote to

\[ n_s = 1 - \frac{3p + 2}{(p + 2)N_s + p}, \quad r = \frac{1}{\sqrt{2\alpha - 1}} \left(\frac{16p}{(p + 2)N_s + p}\right), \quad \text{for } \alpha \gg 1, \]

(5.14)

which is illustrated in figure 9.

From figure 8, we notice that the predictions of power law potentials in the non-canonical framework appear to be somewhat similar to that of the T-model potential (3.47) in the canonical framework (see figure 3), with the non-canonical parameter \( \alpha \) in (5.9) playing a role similar to that of \( \lambda \) in the T-model (3.47). However the \( r \) versus \( n_s \) plots in figure 8 are curved lines, in contrast to those in the case of T-model in figure 3 (where they are nearly straight lines). Another important difference is that the predictions of non-canonical power law potentials for different values of \( p \), as given in (5.14), do not converge towards a cosmological attractor in the large \( \alpha \) limit, in contrast to the predictions of the T-model which approach the cosmological attractor (3.48) independently of the value of \( p \). The absence of cosmological attractor behaviour for non-canonical power law potentials implies that they are capable of scanning through the entire parameter space of the observationally allowed region in the \( \{n_s, r\} \) plane, as can be seen from figures 8 and 9. Hence they are very important target candidates for future CMB missions.

Before proceeding to discuss inverse power law potentials in the non-canonical framework, we would like to briefly discuss the Standard Model Higgs inflation.
Figure 8: This figure is a plot of the tensor-to-scalar ratio $r$, versus the scalar spectral index $n_s$, for the monomial power law potentials, $V \propto \phi^p$, in the non-canonical framework (5.9) (the thinner and thicker curves correspond to $N^*_\phi = 50, 60$ respectively). The latest CMB $2\sigma$ bound $0.957 \leq n_s \leq 0.976$ and the upper bound on the tensor-to-scalar ratio $r \leq 0.036$ are indicated by the shaded grey colour region. The predictions of power law potentials in the non-canonical framework appear to be somewhat similar to those of the T-model potential (3.47) in the canonical framework (see figure 3), with the parameter $\alpha$ in (5.9) playing a role similar to that of $\lambda$ in (3.47) (note that $r$ decreases upon increasing the value of $\alpha$ in accordance with equation (5.11) and this model becomes compatible with CMB data for large enough values of $\alpha$ as shown in table 5 of appendix A). However, for small values of $r$, the $r$ versus $n_s$ curves of the non-canonical potential , $V \propto \phi^p$, do not converge towards a common cosmological attractor for different values of $p$ (in contrast to the attractor behaviour of the T-model, shown in figure 3). Instead, they span virtually the entire parameter space of the observationally allowed region in the $\{n_s, r\}$ plane.

5.2 Standard Model Higgs inflation in the non-canonical framework

The Standard Model (SM) Higgs potential is given by

$$V(\phi) = \frac{\lambda}{4} (\phi^2 - \sigma^2)^2 ,$$

(5.15)

where $\sigma$ is the vacuum expectation value of the SM Higgs field

$$\sigma = 246 \text{ GeV} = 1.1 \times 10^{-16} \ m_p ,$$

(5.16)

and the Higgs self-coupling constant has the value $\lambda \simeq 0.1$. It would be very interesting if inflation could be sourced by the SM Higgs field. Unfortunately, in the canonical framework, the self-interaction coupling of the Higgs field, $\lambda$ in (5.15), is far too large to be consistent with the small amplitude of scalar fluctuations observed by the CMB which suggest the much smaller value $\lambda \sim 10^{-13}$ [21]. However, this situation can be remedied if either of the following two possibilities is realized:
Figure 9: This figure plots the tensor-to-scalar ratio $r$, versus the scalar spectral index $n_S$, for monomial power law potentials, $V \propto \phi^p$, in the non-canonical framework (5.9). The latest CMB 2σ bound $0.957 \leq n_S \leq 0.976$ and the upper bound on the tensor-to-scalar ratio $r \leq 0.036$ are indicated by the shaded grey colour region. The tensor-to-scalar ratio $r$ in this figure has been plotted in the logarithmic scale in order to illustrate the asymptotic behaviour of \{n_S, r\} in the limit $\alpha \gg 1$, as given in (5.14), which shows that larger values of $\alpha$ result in smaller values of $r$. We notice that the \{n_S, r\} predictions in the large $\alpha$ limit are different for different values of $p$, demonstrating the absence of cosmological attractor behaviour which is present in the T-model (3.47) as shown in figure 3.

1. The Higgs couples non-minimally to gravity [31–33].

2. The Higgs field is described by a non-canonical Lagrangian [29, 33].

In section 3.2, we briefly discussed the first possibility and the corresponding predictions of \{n_S, r\}. Here we focus on SM Higgs inflation in the non-canonical framework.

Given that $\sigma = 246$ GeV $\ll m_p$, the Higgs potential (5.15) in the limit $\phi \gg \sigma$ takes the form

$$V(\phi) \simeq \frac{\lambda}{4} \phi^4,$$

which is the quartic monomial power law potential. In the non-canonical framework, the \{n_S, r\} predictions of SM Higgs potential becomes

$$n_S = 1 - \left( \frac{\gamma + 4}{N_\gamma + 2} \right),$$

$$r = \left( \frac{1}{\sqrt{2}\alpha - 1} \right) \left( \frac{32}{N_\gamma + 2} \right),$$

where

$$V(\phi) = V_0 \left( \frac{\phi}{m_p} \right)^p.$$
where
\[
\gamma \equiv \frac{2(3\alpha - 2)}{2\alpha - 1}.
\] (5.20)

Since \(\gamma\) increases from \(\gamma = 2\) for \(\alpha = 1\) to \(\gamma = 3\) for \(\alpha \gg 1\), therefore the scalar spectral index increases from the canonical value \(n_s = 0.951\) (\(\alpha = 1, N_* = 60\)) to \(n_s = 0.962\), in non-canonical models (with \(\alpha \gg 1\)). Similarly the tensor-to-scalar ratio \(r\) declines with the increase in \(\alpha\) as observed before in figure 8. Note that the black colour dots in figure 8 represent the \(\{n_s, r\}\) predictions of the non-minimally coupled SM Higgs inflation, while the green curves represent the same for SM Higgs inflation in the non-canonical framework.

Figure 10: This figure illustrates the relation between the non-canonical parameters \(M\) and \(\alpha\), given by equation (5.21), for SM Higgs inflation which results in the self-coupling value \(\lambda = 0.1\) in equation (5.15). Results for three different e-folding values \(N_* = 50, 55, 60\) are shown by blue, red, and green colour curves respectively. The shaded region corresponds to observationally allowed values of \(\alpha\) obeying the CMB constraint \(r \leq 0.036\).

The relation between the value of the Higgs self-coupling \(\lambda \simeq 0.1\) in the non-canonical framework and the corresponding canonical value \(\lambda_c\) is given by [33]

\[
\lambda = 4 \left[ \frac{32\lambda_c(N_* + 1)^3}{\sqrt{2\alpha - 1}} \left( \frac{\alpha}{4} \left( \frac{1}{M_*^4} \right)^{\alpha - 1} \right)^{\frac{2}{\alpha - 2}} \left( \frac{1}{N_*\gamma + 2} \right)^{\frac{\alpha - 2}{\alpha}} \right]^{\frac{\lambda_c - 2}{\alpha}},
\] (5.21)

where consistency with CMB observations suggests \(\lambda_c \sim 10^{-13}\).

Figure 10 describes the values of the non-canonical parameters \(\alpha\) and \(M\) that yield \(\lambda \simeq 0.1\) in (5.17) – the relation between \(M\) and \(\alpha\) being provided by equation (5.21). From our analysis, it is clear that SM Higgs field in the non-canonical framework can successfully source inflation, while obeying the CMB constraints, for large enough values of the non-canonical parameter \(\alpha\), as shown by the shaded region in figure 10.
6 Discussion

The latest CMB results from the BICEP/Keck collaboration [23], combined with the PLANCK 2018 data release [21], have imposed a stringent upper bound on the primordial tensor-to-scalar ratio \( r \leq 0.036 \) (at 95% confidence). Additionally, CMB observations constrain the scalar spectral tilt to be \( n_s \in [0.957, 0.976] \) at the 2\( \sigma \) level. In this paper, we have discussed the implications of these latest CMB constraints on inflationary models, with particular emphasis on inflation sourced by a scalar field with a non-canonical Lagrangian density.

In section 3, we discussed the implications of the latest CMB constraints on the primordial observables \( \{n_s, r\} \) in the framework of slow-roll inflation driven by a single canonical scalar field \( \phi \) with a potential \( V(\phi) \). The upper bound on the tensor-to-scalar ratio \( r \leq 0.036 \) strongly disfavourites monotonically increasing convex inflationary potentials within the canonical framework. In fact, one of the central inferences that can be drawn from the CMB constraints is the fact that the entire family of monomial power law potentials \( V(\phi) \sim \phi^p \) are now completely ruled out in the canonical framework. Among these, are the simplest classic inflationary models including the \( \frac{1}{2} m^2 \phi^2 \) and \( \lambda \phi^4 \) potentials. Instead, the CMB data now favours asymptotically flat plateau like potentials. In section 3.2, we discussed a number of important plateau potentials in light of the latest observational constraints, including the Einstein frame potentials associated with Starobinsky inflation and non-minimally coupled Higgs inflation, as well as the T-model and E-model \( \alpha \)-attractors and the D-brane KKLT inflation. The \( \{n_s, r\} \) predictions of these models are illustrated in figure 1. In the limit \( r \ll 1 \), the \( \{n_s, r\} \) predictions of \( \alpha \)-attractor potentials exhibit an attractor behaviour [24, 44] as illustrated in figure 3 for the T-model. We demonstrated that plateau potentials generically produce a small tensor-to-scalar ratio and hence are consistent with the CMB data, in agreement with earlier results [24, 44].

However, for large field values, the plateau potentials saturate to a constant value \( V_0 \) which is typically small, namely \( V_0 \leq 10^{-10} m^4 \) for \( r \ll 1 \). The small height of a plateau potential bears significant implications for the issue of initial conditions for inflation. Owing to the fact that the plateau height is typically very small compared to the Planck scale, an equipartition of the initial kinetic, potential, and gradient energy terms is not possible at very early times when initial conditions for inflation are usually set. Similarly, the initial presence of positive spatial curvature could also be problematic for inflation with plateau potentials. Additionally, for \( r \ll 1 \), plateau potentials might exhibit a large running of the scalar spectral tilt \( n_s \) as recently pointed out in [47]. Due to the aforementioned reasons, we focus on scenarios where monotonically increasing convex potentials can be accommodated by the CMB data. In this connection, we study two types of models. The first one is the Margarita potential formulated in the canonical framework, while the second scenario is associated with non-canonical Lagrangians of the form \( \mathcal{L} = X^\alpha - V(\phi), \quad \alpha \geq 1 \) \( (X = \dot{\phi}^2/2) \).

In section 4, we introduced the Margarita potential (4.1) which features a monotonically growing convex inflationary wing at the commencement of a plateau potential. We discussed the \( \{n_s, r\} \) predictions of a specific Margarita potential (4.5) constructed by incorporating extensions to the T-model potential at large field values. We demonstrated that Margarita potentials easily satisfy CMB constraints and also allow for an equipartition between initial energy densities. We also discussed a general recipe for constructing Margarita potentials.

Section 5 was dedicated to study of inflation in the non-canonical framework [29, 30] in the light of the recent CMB data release. In section 5.1, working with the family of power law potentials \( V(\phi) \sim \phi^p \), we demonstrated that inflation with monomial power law potentials can be successfully resurrected in the non-canonical framework. In particular, we compared and contrasted the \( \{n_s, r\} \) predictions of non-canonical power law potentials shown in figure 8 against those of the T-model \( \alpha \)-attractor potential in the canonical framework shown in figure 3. We found that CMB predictions of power law potentials in non-canonical inflation do not exhibit attractor behaviour, which stands in contrast to the predictions of \( \alpha \)-attractors. Instead, they span the entire parameter space of the observationally allowed region in the \( \{n_s, r\} \) plane. This is one of the central results of our analysis.
In section 5.2, we discussed the possibility of realising successful inflation with the Standard Model Higgs potential in the non-canonical framework. We showed that the npn-canonical Higgs with standard model parameters could easily satisfy CMB constraints.

Section B was devoted to a study of inflation with inverse power law (IPL) potentials $V(\phi) \sim \phi^{-p}$ in the non-canonical framework. IPL potentials within the non-canonical framework lead to power law inflation with $a(t) \sim t^q$ [30]. We demonstrated that the $\{n_s, r\}$ predictions of IPL potentials are consistent with the CMB data for sufficiently large values of the non-canonical parameter $\alpha > 20$, as shown in figure 11. We also discussed a remedy to the graceful exit problem for power law inflation in the non-canonical framework.

Before concluding, we succinctly summarise the analysis carried out in this paper for the sake of clarity.

- Given the latest CMB constraints on $\{n_s, r\}$, obtain explicit bounds on important inflationary parameters such as the Hubble slow-roll parameters $\epsilon_H$, $\eta_H$, the potential slow-roll parameters $\epsilon_V$, $\eta_V$, the Hubble parameter $H^\text{inf}_k$, equation of state of the universe during inflation $w_\phi$, energy scale of inflation $E^\text{inf}_{\phi}$ and the tensor tilt $n_T$ in the context of single field slow-roll inflation.

- We highlight that the latest upper bound on the tensor-to-scalar ratio, namely $r \leq 0.036$, not only rules out the classic convex potentials $V(\phi) \sim \phi^2, \phi^4$, but also disfavours the entire family of monomial potentials $V(\phi) \sim \phi^p$ for any $p > 0$ in the canonical framework (at 95% confidence). While this result was highlighted in [23, 24], we discuss it in section 3 in order to make comparison with the predictions of non-canonical monomial potentials in section 5.

- We stress that the latest CMB data favours asymptotically flat potentials\footnote{This result is already well-known in the inflationary literature, however, we discuss them for the continuity of the flow of the discussions in this paper} in the canonical framework and discuss a few examples on asymptotically flat potentials that belong to either of the list of categories - symmetric/asymmetric, one/two parameter models, exponential/algebraic approach to plateau behaviour.

- We then critically scrutinize the problem of initial conditions for these plateau potentials and propose the Margarita potential (which is, in fact, a class of potentials) that can address the issue of initial conditions. We also discuss a generic recipe to construct the Margarita potential phenomenologically.

- We then focus on inflation in the non-canonical framework and demonstrate that convex monomial potentials, that are disfavoured in the canonical framework, become compatible with the CMB data in the con-canonical framework for a suitable range of values of the non-canonical parameter $\alpha$. Similarly, in appendix B we show that the inverse power law potential $V(\phi) \sim \phi^{-p}$, which leads to power law inflation in the non-canonical framework, also satisfies the latest CMB bounds. While some of these results were earlier discussed in [29, 30], we reproduce the results and discuss their implications in light of the recent data.

- During our analysis in the non-canonical framework, we discover striking similarities between the $\{n_s, r\}$ flow lines of monomial potentials $V(\phi) \sim \phi^p$ in the non-canonical framework and the T-model $\alpha$-attractors canonical framework. We also discuss the difference between the predictions of both the models. This is one of the highlights of our analysis.

Finally we would like to draw attention to the fact that the analysis of initial conditions for inflation with plateau potentials [33] remains an interesting open area of research, especially in presence of a positive spatial curvature [28, 51]. We shall study this in more detail in a companion
work [49]. However, we would also like to emphasise that regardless of the issue of initial conditions, both power law and inverse power law potentials in the non-canonical framework (and the Margarita potential in the canonical framework), satisfy current CMB constraints and are therefore important target candidates for the next generation of CMB missions.

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Data Availability Statement: This work is entirely theoretical and has no associated data.

A Constraint on parameters of inflationary potentials

In this appendix, we present bound on the parameters of different inflationary potentials discussed in the paper, namely, $\lambda$ in T-model and E-model $\alpha$-attractors, $M$ in D-brane KKLT potential and the non-canonical parameter $\alpha$ in the context of non-canonical inflation with monomial potential $V(\phi) \sim \phi^p$. We obtain these constraints numerically by imposing the CMB 2$\sigma$ bound on $n_s$ and $r$ given in (3.34) and (3.35) -

$$n_s \in [0.957, 0.976], \quad r \leq 0.036.$$  

| $V(\phi) = V_0 \tanh^p \left( \frac{\lambda \phi}{m_p} \right)$ | $N_\ast = 50$ | $N_\ast = 60$ |
|-------------------------------------------------|-----------------|-----------------|
| $p = 1$                                          | $\lambda \geq 0.112$ | $\lambda \geq 0.086$ |
| $p = 2$                                          | $\lambda \geq 0.132$ | $\lambda \geq 0.108$ |
| $p = 3$                                          | $\lambda \geq 0.138$ | $\lambda \geq 0.113$ |
| $p = 4$                                          | $\lambda \geq 0.141$ | $\lambda \geq 0.116$ |

Table 2: CMB 2$\sigma$ bound on the parameter $\lambda$ of T-model $\alpha$-attractor potential (3.47) is presented in this table for different values of $p$.

- We begin with T-model $\alpha$-attractor potential (3.47)

$$V(\phi) = V_0 \tanh^p \left( \frac{\lambda \phi}{m_p} \right),$$

and present constraints on the parameter $\lambda$ for different values of $p$ in table 2.

- For E-model $\alpha$-attractor potential (3.49)

$$V(\phi) = V_0 \left( 1 - e^{-\lambda \frac{\phi}{m_p}} \right)^p,$$

and present constraints on the parameter $\lambda$ for different values of $p$ in table 3.
\[ V(\phi) = V_0 \left( 1 - e^{-\frac{\Delta \phi}{m_{p}}} \right)^p \]

| \( p = 1 \) | \( \lambda \geq 0.137 \) | \( \lambda \geq 0.094 \) |
| \( p = 2 \) | \( \lambda \geq 0.194 \) | \( \lambda \geq 0.158 \) |
| \( p = 3 \) | \( \lambda \geq 0.229 \) | \( \lambda \geq 0.182 \) |
| \( p = 4 \) | \( \lambda \geq 0.243 \) | \( \lambda \geq 0.195 \) |

**Table 3:** CMB 2\( \sigma \) bound on the parameter \( \lambda \) of E-model \( \alpha \)-attractor potential (3.49) is presented in this table for different values of \( p \).

- We then present the constraints on the parameter \( M \) of D-brane KKLT potential (3.51)

\[ V(\phi) = V_0 \frac{\phi^n}{\phi^n + M^n} \]

for different values of \( n \) in table 4.

| \( n = 2 \) | \( M \leq 7.07 \) | \( M \leq 9.16 \) |
| \( n = 4 \) | \( M \leq 10.17 \) | \( M \leq 12.21 \) |
| \( n = 6 \) | \( M \leq 13.51 \) | \( M \leq 15.82 \) |

**Table 4:** CMB 2\( \sigma \) bound on the parameter \( M \) of D-brane KKLT inflation potential (3.51) is presented in this table for different values of \( n \).

- Finally, we present constraints on the non-canonical parameter \( \alpha \) in the context of inflation in the non-canonical framework (5.9) with a monomial potential \( V(\phi) \sim \phi^p \) for different values of \( p \) in table 5. Note that for \( p = 4 \), the predictions for \( n_s \) of non-canonical monomial inflation is not compatible with CMB data for \( N_* = 50 \) (while it is compatible for \( N_* = 60 \)) as can be seen from figure 8. We find that monomial potential with \( p = 4 \) becomes compatible with CMB \( n_s \) bound for \( N_* \geq 54 \) and the constraint on tensor-to-scalar ratio, namely, \( r \leq 0.036 \) translates to \( \alpha \geq 15.52 \) (for \( N_* = 54 \)).

**B Inverse power law potentials in the non-canonical framework**

It is well known that within the canonical framework a spatially flat universe can expand as a power law with \( a(t) \sim t^n \), if inflation is sourced by an exponential potential [52]. However, owing to the fact that the tensor-to-scalar ratio \( r \) in such models turns out to be much larger than the upper bound set by the CMB observations, namely \( r \leq 0.036 \), the hope of realising power law inflation is dashed in the canonical framework. Instead, CMB constraints indicate that cosmic expansion is near-exponential if inflation is driven by a canonical scalar field, as discussed in section 3.1.
\[
V(\phi) = V_0 \left( \frac{\phi}{m_p} \right)^p,
\]

where \( p \) is related to the non-canonical parameter \( \alpha \) by [30]

\[
p = \frac{2 \alpha}{\alpha - 1},
\]

and the expansion rate, \( a(t) \sim t^q \), is related to the EOS of the scalar field by the usual relation\(^{18}\)

\[
q = \frac{2}{3} \left( \frac{1}{1 + w_\phi} \right).
\]

Expressions for the scalar spectral index \( n_S \), and the tensor-to-scalar ratio \( r \) in the non-canonical framework are given by [30]

\[
n_S = 1 - \frac{2}{q - 1},
\]

\[
r \simeq \frac{16}{q \sqrt{2 \alpha - 1}},
\]

where the ‘\( \simeq \)’ symbol in the expression of \( r \) refers to the fact that the above equation is valid in the slow-roll limit \( q \gg 1 \). From expressions (B.4) and (B.5), we notice that the scalar spectral tilt \( n_S \) does not depend upon the non-canonical parameter \( \alpha \), while the tensor-to-scalar ratio \( r \) decreases with an increase in \( \alpha \). Hence for large enough values of \( \alpha \), the IPL potential can satisfy the CMB constraints in the non-canonical framework, as shown in figure 11.

From expression (B.4), the CMB constraints (3.34) on the scalar spectral index \( n_S \) translate into \( q \in [47.5, 84.3] \). Figure 11 depicts the behaviour of \( r \) versus \( n_S \) for three different values of expansion exponent, namely \( q = 50, 60, 70 \), plotted in green, red, and blue curves respectively. The arrow mark on each plot indicates the direction of increase in the value of \( \alpha \). From this figure, it is easy to see that the CMB constraints, shown by the grey colour shaded region, can easily be satisfied for \( \alpha > 20 \). Hence power law inflation can be successfully sourced by the IPL potential in the non-canonical framework, without violating the CMB bounds on \( \{n_S, r\} \).

\(^{18}\)The exponent of power law expansion \( q \) is independent of the value of \( p \). Rather, the value of \( q \) is only dependent upon the amplitude \( V_0 \) of the IPL potential (B.1), in the sense that for a given value of \( \alpha \), a different value of \( V_0 \) leads to a different value of \( q \). Alternatively, one can keep \( q \) fixed by varying both \( \alpha \) and \( V_0 \) simultaneously (see [30]).

| \( p \) | \( \alpha \geq \) | 4.49 | 10.18 | 14.58 | 12.75 |
|---|---|---|---|---|---|
| 1 | \( \geq \) | 3.17 | 7.24 | 10.37 |  |
| 2 | \( \geq \) |  |  |  |  |
| 3 | \( \geq \) |  |  |  |  |
| 4 | \( \geq \) |  |  |  |  |
Figure 11: This figure is a plot of the tensor-to-scalar ratio $r$, versus the scalar spectral index $n_s$, for the inverse power law (IPL) potential (B.1) in the non-canonical framework, which leads to power law inflation $a(t) \sim t^q$, $q \gg 1$. The latest CMB $2\sigma$ bound $0.957 \leq n_s \leq 0.976$ and the upper bound on the tensor-to-scalar ratio $r \leq 0.036$ are indicated by the shaded grey colour region. The green, red, and blue curves correspond to $q = 50, 60, 70$ respectively. For a given value of $q$, while $n_s$ is insensitive to the value of the non-canonical parameter $\alpha$, the tensor-to-scalar ratio $r$ decreases with an increase in $\alpha$, which has been illustrated by the arrow marks on each plot. The purple lines represent contours of fixed $\alpha$ values. For $\alpha > 20$ and $q \in [47.5, 84.3]$, predictions of the non-canonical IPL potential are consistent with CMB constraints.

B.1 Graceful exit from power law inflation

During power law inflation, since the universe accelerates forever, it does not account for the end of inflation and the subsequent transition into the radiative hot Big Bang phase. This issue of ‘graceful exit’ from the inflationary phase turns out to be one of the central drawbacks of power law inflation. A possible way out is to assume that the potential driving power law inflation approximates a more general functional form which allows for the oscillations of the inflaton field at the end of inflation and hence results in a successful reheating scenario. An interesting general form of the potential in the context of non-canonical inflation is [30]

$$V(\phi) = V_0 \left[ \left( \frac{\phi}{m_p} \right)^{p/2} - \left( \frac{\phi}{m_p} \right)^{-p/2} \right]^2,$$

with $p = 2\alpha/(\alpha - 1)$ as given in (B.2). The potential is schematically shown in figure 12.

For $\phi < m_p$, the potential has the asymptotic form $V(\phi) \sim \phi^{-p}$ which leads to power law inflation in the non-canonical framework. While for $\phi > m_p$, the potential exhibits monomial behaviour $V(\phi) \sim \phi^p$ which leads to quasi-exponential inflation. After the end of inflation, the non-canonical

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Figure 12: This figure schematically depicts the inflationary potential (B.6) which can address the graceful exit problem of power law inflation in the non-canonical framework. For $\phi < m_p$, the potential exhibits inverse power law behaviour $V(\phi) \sim \phi^{-p}$ which results in power law inflation in the non-canonical framework. For $\phi > m_p$, the potential exhibits monomial behaviour $V(\phi) \sim \phi^p$ which results in quasi-exponential inflation. After the end of inflation, the non-canonical scalar field oscillates around the minimum of the potential at $\phi = m_p$, resulting in successful reheating of the universe.

scalar field oscillates around the minimum of the potential which is located at $\phi = m_p$, resulting in successful reheating.

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