A transition between bouncing hyper-inflation to $\Lambda$CDM from diffusive scalar fields

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We consider the history of the universe from the big bang to the late period in the context of a unified interacting dark energy - dark matter model. The model is based on the Two Measures Theories, which introduces metric independent volume elements and this allows to construct unified Dark Matter Dark Energy, where the cosmological constant appears as an integration constant associated to the equation of motion of the measure fields. The Dynamical space time Theories generalize the Two Measure Theories by introducing a vector field, whose equation of motion guarantees the conservation of a source of a certain Energy Momentum tensor, which may be related, but in general is not the same as the gravitational Energy Momentum tensor. By considering the dynamical space field appearing in another part of the action or by demanding that the dynamical space time vector be a gradient of a scalar, we obtain a diffusive non conservation of an energy momentum tensor. These generalizations leads at the end to a formulation of interacting DE-DM dust models in the form of a diffusive type interacting Dark Energy and Dark Matter scenario. A numerical solution of the theories shows that in some cases the evolution of the very early universe is governed by Stiff equation of state, and undergoes to $\Lambda$CDM. In another cases the universe bounces to hyper inflation. But all of these solutions have the another final transition to $\Lambda$CDM as a fixed point for the late universe.

INTRODUCTION

The best explanation, and fitting with data for the accelerated expansion of our universe, is the $\Lambda$CDM model, which tells us that our universe contains 0.68 of dark energy, and 0.27 of dark matter$^{[1,2]}$. This model present two big questions: The Cosmological Constant problem$^{[3,4]}$ - why there is a large disagreement between the vacuum expectation value of the energy momentum tensor which comes from Quantum Field Theory and the observable value of dark energy density? and the Coincidence problem - why observable values of dark energy and dark matter densities in the late universe are of the same order of magnitude? In order to solve this problem, many approaches emerged. The basic models for our universe is for a perfect fluid. One interesting suggestion was a diffusive exchange of energy between dark energy and dark matter made by Calogero$^{[5,6]}$, Haba$^7$, and others, with some solution to cosmic problems. The basic notion is that diffusion equation (or more exactly - Fokker Planck equation$^{[5,9]}$), implies a non-conserved stress energy tensor $T^\mu\nu$, which has some current source $j^\mu$:

$$\nabla_\mu T^{\mu\nu} = 3\sigma j^\nu \tag{1}$$

where $\sigma$ is the diffusion coefficient of the fluid. This generalization is Lorentz invariant and fit for curved space time. The current $f^\mu$ is a time-like covariantly conserved vector field and its conservation tells us that the number of particles in this fluid is constant. However, in the gravitational equations, the Einstein tensor is proportional to a conserved stress energy tensor $\nabla_\mu T^{\mu\nu}(g) = 0$, we labeled with "G"$^{[10,11]}$. So Calogero come up with what he called $\phi$CDM-model, which achieves a conserved total energy momentum tensor appearing in the right hand side of Einstein’s equation. But for the dark energy and dust stress tensors there is some source current for those tensors (however the sum is conserved). But as Calogero mentioned$^{[5]}$, the diffusion model introduced in his paper lacks an action principle formulation. Therefore we develop from a generalization of Two Measure Theories$^{[12-21]}$, a "diffusive energy theory" which can produce on one hand a non-conserved stress energy tensor $(T^\mu\nu(\chi))$, as in$^{[1]}$, and on the other hand a conserved stress energy tensor $(T^\mu\nu(G))$ that we know from the right hand side of Einstein’s equation. Our theory has some similarities to $\phi$CDM, but are not equivalent, furthermore approaches much faster a $\Lambda$CDM behavior for the late universe.

TWO MEASURES THEORIES AND $\Lambda$CDM

In addition to the regular measure of integration in the action $\sqrt{-g}$, also with another measure which is also a density and which is also a total derivative. In this case, one can use for constructing this measure 4 scalar fields $\varphi_a$, where $a = 1,2,3,4$. Then we can define the density $\Phi = \varepsilon^{abc\delta} \varepsilon_{abcd} \partial_\alpha \varphi_\beta \partial_\beta \varphi_\gamma \partial_\gamma \varphi_\delta \partial_\delta \varphi_\delta$, and then we
can write an action that uses both of these densities:
\[ S = \int d^4x \Phi L_1 + \int d^4x \sqrt{-g} L_2 \]  
(2)

As a consequence of the variation with respect to the scalar fields \( \varphi_a \), assuming that \( L_1 \) and \( L_2 \) are independent of the scalar fields \( \varphi_a \), we obtain that:
\[ A^\alpha_a \partial_\alpha \varphi_a = 0 \]  
(3)

where \( A_\alpha^a \equiv \varepsilon^{\alpha \beta \gamma \delta} \varepsilon_{abcd} \partial_\beta \varphi_a \partial_\gamma \varphi_b \partial_\delta \varphi_d \). Since \( \det[A^\alpha_a] \sim \Phi^3 \) as one easily sees, then that for \( \Phi \neq 0 \) \( \text{[3]} \) implies that \( L_1 = M = \text{Const.} \)

**Unified dark energy - dark energy**

A simple example for this kind of modified gravity approach is a model of unified dark energy and dark matter from one kinetic scalar field term \[ \text{[22-23-24]} \]. The action is:
\[ S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R + \int d^4x (\Phi + \sqrt{-g}) L(X, \phi) \]  
(4)

where \( L(X, \phi) \) could be any order of k-essence term, and \( X \) is the kinetic term of some scalar field \( X = \frac{\Phi}{\sqrt{\phi}} \). For simplicity we take only the first order, but at \[ \text{[4]} \] shows that the order of the k-essence terms does not change the results.

From the equation of motion to the scalar field \( \varphi_a \) gives us a constraint on the actual value of the term \( X = \alpha_1 \). In addition, the variation according to the scalar field \( \phi \) gives a conserved current \( j_\alpha^a = 0 \), which can be presented as:
\[ j_\alpha = \left( \frac{\phi}{\sqrt{-g}} + 1 \right) \phi_\alpha \]  
(5)

In order to perform the correct integration by parts we have to make use the scalar field \( \chi = \frac{\phi}{\sqrt{-g}} \) which is invariant under continuous general coordinate transformations, instead of the scalar density \( \Phi \). And finally, we have the variation according to the metric, which gives us the stress energy tensor that appears in Einstein equation:
\[ T^\mu_\nu = g^\mu_\nu X + j_\mu \phi_\nu \]  
(6)

For FRWM metric the solution for the new measure variation gives a constant value for the scalar field \( \dot{\phi}^2 = \alpha_1 \), which is the value for dark energy. The conserved current gives another constant of integration \( \alpha_2 \), which divided by the measure of integration \( a^3 \), and therefore the fraction between the two measures gives:
\[ \frac{\alpha_2}{a^3} = (\chi + 1) \dot{\phi} \]  
(7)

The stress energy tensor gives the density and the pressure of that "unified scalar fluid". With the equations \[ \text{[5]}-\text{[7]} \] we get:
\[ \rho_{DE} = \dot{\phi}^2 = \alpha_1 \]  
(8)
\[ \rho_{Dust} = \frac{\sqrt{\alpha_2 \alpha_1}}{a^3} \]  
(9)

And the pressure is \( p = -\rho_{DE} \). This solution is a unified picture of DE-DM, and gives precisely LCDM model. From comparing to the LCDM solution, we can obtain how the observables values related to the constant of integration that come from the solution of the theory:
\[ \Omega_\Lambda = \frac{\alpha_1}{H_0} \text{; } \Omega_m = \frac{\alpha_2 \sqrt{\alpha_1}}{H_0} \]  
(10)
where \( H_0 \) is Hubble constant for the late universe. A generalization of this mathematical approach, gives the diffusive energy action.

**A Diffusion from Dynamical Time Theories**

The constraint on the action \( L_2 \) as \[ \text{[2]} \], can expressed as a covariant conservation of a stress energy momentum of the form \( T^{\mu_\nu}_{(\phi)} = L \gamma^{\mu_\nu} \), and using the 2nd order formalism, where the covariant derivative of \( g^{\mu_\nu} \) is zero, we obtain that \( \nabla_\mu T^{\mu_\nu}_{(\phi)} = 0 \), implies \( \partial_\alpha L_1 = 0 \). This suggests generalizing the idea of the Two Measures Theory, by imposing the covariant conservation of a more nontrivial kind of energy momentum tensor, which we denote as \( T^{\mu_\nu}_{(\chi)} \) \[ \text{[25]} \]. Therefore, we consider an action of the form:
\[ S = S_{(\chi)} + S_{(R)} = \int d^4x \sqrt{-g} \chi_{\mu_\nu} T^{\mu_\nu}_{(\chi)} + \frac{1}{16\pi G} \int d^4x \sqrt{-g} R \]  
(11)

where \( \chi_{\mu_\nu} = \partial_\nu \chi_\mu - \Gamma^\lambda_{\mu_\nu} \chi_\lambda \). If we assume \( T^{\mu_\nu}_{(\chi)} \) to be independent of \( \chi_\mu \) and having \( T^{\mu_\nu}_{(\chi)} \) being defined as the Christoffel Connection Coefficients, then the variation with respect to \( \chi_\mu \) gives a covariant conservation:
\[ \nabla_\mu T^{\mu_\nu}_{(\chi)} = 0 \]. The energy density is the canonically conjugated variable to \( \chi^0 \), which is what we expect from a dynamical time (here represented by the dynamical time \( \chi^0 \)).

An interesting particular case, which will be expended below, is obtained when \( T^{\mu_\nu}_{(\chi)} \) is taken to be of the form \( T^{\mu_\nu}_{(\chi)} = g^{\mu_\nu} L_m \), then introducing in \[ \text{[11]} \] gives:
\[ \int d^4x \sqrt{-g} \chi^\mu T^{\mu_\nu}_{(\chi)} = \int d^4x \sqrt{-g} \chi^\mu L_m = \int d^4x \partial_\mu (\sqrt{-g} \chi^\mu) L_m = \int d^4x \sqrt{-g} \chi^\mu L_m \]  
(12)
As \([3]\), the variation with respect to the scalar field gives \(\partial_{\mu} L_m = 0\), for dynamical time theories, the variation with respect to the dynamical time vector field gives also this constraint, and therefore \(L_m = Const\). Notice that this action is like a regular contribution to a standard gravity theory except that instead of \(\sqrt{-g}\), in that part of the action the measure of integration is the total derivative \(\Phi = \partial_{\mu}(\sqrt{-g}\chi^{\mu})\). These kind of contributions have been considered in the Two Measures Theories, which are of interest in connection with the Cosmological Constant Problem. This new definition of the measure seemingly is not made from the scalar fields as TMT, but this definition does not change the results.

Some cosmological solutions of \([11]\) have been studied in \([20]\), in the context of spatially flat radiation like solutions, and considering gauge field equations in curved space time. For example, for Maxwell stress energy tensor, the dynamical space time becomes Conformal Killing vector:

\[
\mathcal{L}_\chi g_{\mu\nu} \equiv \chi_{\mu;\nu} + \chi_{\nu;\mu} = \frac{\chi^\lambda}{2} g_{\mu\nu}
\]  \hspace{1cm} (13)

In a power law universe \((a(t) = t^\alpha)\), the Conformal Killing vector is a homothetic Killing vector \((\chi^\lambda = Const)\), and the dynamical space vector components time looks like a real space time vector field:

\[
\chi^\mu = \frac{\chi^\lambda}{4} (t, (1 - \alpha) \vec{x})
\]  \hspace{1cm} (14)

**Dynamical time action with diffusive source**

For the diffusive stress energy tensor action, in a 4 dimensional case, where there is a coupling between a scalar field \(\chi^\mu\), and a stress energy momentum tensor \(T^{\mu\nu}\),

\[
S_{(\chi A)} = \int d^4x \sqrt{-g} \chi_{\mu;\nu} T_{(\chi)}^{\mu\nu} + \frac{\sigma}{2} \int d^4x \sqrt{-g} (\chi_{\mu} + \partial_{\mu} A)^2
\]  \hspace{1cm} (15)

where \(A\) is another scalar field. Then from a variation of respect to \(\chi_{\mu}\) we obtain:

\[
\nabla_\nu T_{(\chi)}^{\mu\nu} = \sigma (\chi^\mu + \partial^\mu A) = f^\mu
\]  \hspace{1cm} (16)

where the source is: \(f^\mu = \sigma (\chi^\mu + \partial^\mu A)\). From the variation with respect to \(A\), we indeed obtain that the current \(\chi^\mu + \partial^\mu A\) is conserved, which means that:

\[
\nabla_\mu f^\mu = \nabla_\mu (\chi^\mu + \partial^\mu A) = 0
\]  \hspace{1cm} (17)

which it is the source of the stress energy momentum tensor. This stress energy tensor is substantially different from stress energy tensor we all know, which is defined as \(\frac{8\pi G}{c^4} T^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R\). In this case, the stress energy momentum tensor \(T_{(\chi)}^{\mu\nu}\) is not conserved (but there is some conserved current \(f^\nu\), which is the source to this stress energy momentum tensor non conservation), here there is some conserved stress energy tensor \(T_{(G)}^{\mu\nu}\) which comes from variation of the action according to the metric:

\[
T_{(G)}^{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta (\sqrt{-g} L_M)}{\delta g^{\mu\nu}} \nabla_\mu T_{(G)}^{\nu\mu} = 0
\]  \hspace{1cm} (18)

The lagrangian \(L_M\) could be the modified term \(\chi_{\mu;\nu} T_{(\chi)}^{\mu\nu}\), but as we will see, additional terms can be added. Using different expressions for \(T_{(\chi)}^{\mu\nu}\) which depend on other variables, will give the connection between the dynamical space time vector field \(\chi_{\mu}\), and those other variables.

**Higher derivatives action**

Another family of theories that are also Diffusive energy Theories but higher derivative where \(\sigma\) goes to plus or minus infinity. In this case the contribution of the current \(\frac{\sigma}{2} (\chi_{\mu} + \partial_{\mu} A)^2\) in the equations of motion goes to zero, and then \(\chi_{\mu} = -\partial_{\mu} A\), or the vector field becomes to a gradient of the scalar \(A\). The theory \([20]\) becomes:

\[
S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} \Lambda(\phi, X) + \int d^4x \sqrt{-g} T^{\mu\nu}_{(\chi)} \nabla_\mu \nabla_\nu A
\]  \hspace{1cm} (19)

The equations of motion are the same as \([22, 23]\), but with no the terms with the coefficient \(\frac{1}{2}\). This leads to the densities as \([28, 29]\) with no the \(a^{-6}\) terms, which in the early universe probably be non-negligible. This corresponds to the "dynamical space time" theory \([11]\), where the dynamical space time 4-vector \(\chi_{\mu}\) is replaced by a gradient of a scalar field \(\chi\). In the "dynamical space theory" we obtain 4 equations of motion, by the variation of \(\chi_{\mu}\), which correspond to covariant conservation of energy momentum tensor \(\nabla_\mu T_{(\chi)}^{\nu\mu}\), \(\nabla_\mu T^{\mu\nu}_{(\chi)}\) to asymptotic conservation of energy momentum tensor \((T^{\mu\nu}_{(\chi)})\), which corresponds to a conservation of a current.

However because of Ostragovsky instabilities, which proofed that theories with high derivatives are not stable, we take \(\sigma\) to be very big, but still using a theory with no high derivatives.

**SCALAR GRAVITY WITH DIFFUSIVE BEHAVIOR**

Our starting point is the following non-conventional gravity-scalar-field action, which will produce a diffusive
type of interacting DE-DM theory \[27,28\]:

\[
S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g}\Lambda(\phi, X) + \int d^4x \sqrt{-g} \chi_{\mu\nu} T^{\mu\nu}_{(\chi)} + \frac{\sigma}{2} \sqrt{-g}(\chi_{\mu} + \partial_{\mu} A)^2
\]  

(20)

with the following explanations for the different terms: \( R \) is the Ricci scalar which appears in: Einstein-Hilbert action. \( \Lambda(\phi, X) \) a simple kinetic scalar field \( \Lambda = \partial_{\mu} \phi \partial^{\mu} \phi \). As we will see, this last action will produce a diffusive interaction between DE-DM type theory. For the ansatz of \( T^{\mu\nu}_{(\chi)} \) we choose to use some tensor which is proportional to the metric, with a proportionality function \( \Lambda(\phi, X) \):

\[
T^{\mu\nu}_{(\chi)} = g^{\mu\nu} \chi_{\lambda}(\phi, X) \Rightarrow S(\chi) = \int d^4x \chi\Lambda
\]  

(21)

From the variation of the scalar field \( \Lambda \) and the vector field \( \chi_{\mu} \) we get: \( \square \Lambda = 0 \), whose solution will be interpreted as a dynamically generated Cosmological Constant with diffusive source. From the variation according to the scalar field we get a conserved stress energy tensor:

\[
T^{\mu\nu}_{(\chi)} = g^{\mu\nu} \Lambda + \frac{1}{\sigma} \chi_{\mu} \chi^{\nu} + \frac{1}{\sigma^2} \chi_{\mu} \chi^{\nu}
\]  

(23)

For cosmological solutions the interpretation for dark energy is for term proportional to the metric \( -\Lambda + \frac{1}{\sigma^2} \chi_{\mu} \chi^{\nu} \), and dark matter dust from the '00' component of the tensor \( j^{\mu}\phi^{\nu} - \chi^{\mu} \Lambda^{\nu} - \chi^{\nu} \Lambda^{\mu} + \frac{1}{\sigma} \Lambda^{\mu} \Lambda^{\nu} \). Let’s undertake the important analysis of the diffusion model under the assumption of spacial homogeneity and isotropy, i.e. a spactime with Friedman Robertson Walker Metric:

\[
ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right]
\]  

(24)

The scalar field becomes \( \Lambda = \partial_{\mu} \phi \partial^{\mu} \phi = -\dot{\phi}^2 \). Notice that there are high derivative equation, but all such type of equations, correspond to conservation laws. For example, we get that the variation of the scalar field \( \dot{\phi} \) will give \( \frac{d}{dt}(2\dot{\phi} \phi a^3) = 0 \), which can be integrated to:

\[
2\dot{\phi} = \frac{C_2}{a^3} \Rightarrow \phi^2 = C_1 + C_2 \int \frac{dt}{a^3}
\]  

(25)

The conserved current from eq \(22\) gives us the relation:

\[
2\dot{\phi}(\chi^\lambda_{\lambda} + 1) = \frac{C_3}{a^3}
\]  

(26)

which can be also integrated to give:

\[
\chi_0 = \frac{1}{a^3} \int a^3 dt + \frac{C_4}{a^3} - \frac{C_3}{2a^3} \int \frac{dt}{\phi}
\]  

(27)

which provides the solution for the scalar field \( \chi \). From \(23\) we get the terms for DE-DM densities:

\[
\rho_{de} = \dot{\phi}^2 + \frac{\chi_0 C_2}{a^3} + \frac{1}{2\sigma} \frac{C_2}{a^3}
\]  

(28)

\[
\rho_{dm} = \frac{C_3}{a^3} \phi - \frac{2\chi_0 C_2}{a^3} - \frac{1}{\sigma} \frac{C_2}{a^3}
\]  

(29)

and the pressure of DE: \( \rho_{de} = -\rho_{de} \) and DM: \( \rho_{dm} = 0 \). This leads to the Friedman equations with \(28\) \(29\). For simplicity, we take the limit \( \sigma \to \infty \). For the late universe this change won’t be dominant at all, but for the early universe may have some changes.

**SOLUTIONS FOR THE THEORY**

**Asymptotic solution and \Lambda CDM fixed point**

We can solve asymptotically and by the way show the basic stability of the theory (which should eliminate any concerns related to the formal unboundedness of the action). First we solve for \( \chi_0 \) \(27\). We see that the leading term is the fraction \( \frac{1}{\sigma} \int a^3 dt \). For asymptotically De-Sitter space, where \( a(t) \approx a_0 \exp(H_0 t) \), then we obtain that there is a unique asymptotic value:

\[
\lim_{t \to \infty} \chi_0 = \frac{1}{3H_0}
\]  

(30)

This is in accordance with our expectations that the expansion of the universe will stabilize the solutions. With this information we can check what is the asymptotic value of DE-DM, from \(25\) \(28\). We see that in this limit, the non-constant part of \( \dot{\phi}^2 \) is canceled by \( \frac{2\chi_0 C_2}{a^3} \), and then asymptotically:

\[
\rho_{de} = C_1 + O\left(\frac{1}{a^3}\right)
\]  

(31)

\[
\rho_{dm} = (C_3 \sqrt{C_1} - \frac{2C_2}{3H_0}) \frac{1}{a_3} + O\left(\frac{1}{a^3}\right)
\]  

(32)

As the Friedman equation provide a relation between \( C_1 \) and \( H_0 \) (the asymptotic value of Hubble constant) which is \( H_0^2 = \frac{8\pi G}{3} C_1 \). For negative \( C_2 \) we have decaying dark energy, the last term of the contribution for dark energy density is positive (and the opposite). This behavior, where \( C_2 < 0 \), has a chance of explaining the coincidence problem, because unlike the standard \( \Lambda CDM \) model, where the dark energy is exactly constant, and
the dark matter decreases like \( a^{-3} \), in our case, dark energy can slowly decrease, instead of being constant, and dark matter also decreases, but not as fast as \( a^{-3} \). As advanced, this behavior can be understood by the observation that in an expanding universe a non-covariant conservation of an energy momentum tensor, which may imply that some energy density is increasing in the locally inertial frame, does not mean a corresponding increase of the energy density in the co-moving cosmological frame, here in particular the non-covariant conservation of the dust component of the universe will produce a still decreasing dust density, although for \( C_2 < 0 \), there will be a positive flow of energy in the inertial frame to the dust component, but the result of this flow of energy in the local inertial frame will be just that the dust energy density will decrease a bit slower than the conventional dust (but still decreases). This is yet another example where potential instabilities are softened or in this case eliminated by the expansion of the universe. As it is known in the case of the Jeans Gravitational instability which is much softer in the expanding universe and also in other situations as well [32]. Another application for this mechanism could be to use it to explain the particle production, "taking vacuum energy and converting it into particles" as expected from the inflation reheating epoch. May be this combined with a mechanism that creates standard model particles. As we see, the expansion of the universe stabilizes the solutions, such that for large times all of them become indistinguishable to \( \Lambda \)CDM, which appears as an attractor fixed point of our theory, showing a basic stability of the solutions at large times. Choosing \( C_1 \) as positive is necessary, because of the demand that the terms with \( \sqrt{C_1} \) won’t be imaginary. But for the other constants of integration, there is only the condition \( C_3 \sqrt{C_1} > \frac{2 H_0^2}{3 a_0} \), which gives a positive dust density at large times.

A transition between a bouncing hyper-inflation to \( \Lambda \)CDM from numerical solution

Before we solve the theory numerically, we normalized all the constants of integrations, and the variables from (25)-(29). Basically, the dimensions of the constant of integration are depending on dimensions of density and time. Therefore the normalization could used the critical density \( \rho_c = \frac{3H^2}{8\pi G} \) and the proper value of Hubble constant \( H_0 \). The normalized constants are:

\[
C_1 := \frac{C_1}{\rho_c}, C_2 := \frac{C_2}{H_0 \rho_c} \tag{33a}
\]

\[
C_3 := \frac{C_3}{\rho_c}, C_4 := H_0 C_4 \tag{33b}
\]

From the asymptotic solution, we can understand that \( \Lambda \)CDM has those values: The diffusion constant is zero \( C_2 = 0 \). The value of \( C_4 \) is not affected by the evolution of the universe, because it does not appear in the density equation (28)-(29) where \( C_2 = 0 \). The value of \( C_1 = 0.68 \) is the ratio of dark energy \( \Omega_\Lambda \), and \( C_3 = 0.327 \) is the fraction between the ratio of dark matter and the square of the ratio of dark energy \( \Omega_m^2 \). This case is of \( C_2 = 0 \) equivalent to TMT with unified dark energy dark matter from scalar fields [3]-[9].

For the remaining solutions, \( a(t) \) we assume at first that is decreasing toward the past and the time variable can be replaced by the cosmological redshift variable:

\[
a(t) = \frac{a_0}{z(t) + 1} \tag{34}
\]

In the case of a bounce, the assumption of \( a(t) \) being decreasing toward the past breaks down. The time derivative:

\[
dt = - \frac{a}{a H(z)} \frac{dz}{d\nu} \tag{35}
\]

The equations (25), (27) can rewrite as

\[-H(z) \frac{d}{dz} \phi^2 = \frac{C_2}{a^2} \tag{36}\]

\[H(z) \frac{d}{dz}(a^3 \chi) = -a^4 + \frac{C_3 a}{2\phi} \tag{37}\]

and the Fridmann eq. (28)-(29) as

\[H(z) = \sqrt{\dot{\phi}^2 + \frac{C_3 \phi - C_2 \chi}{a^3}} - \frac{k}{a^2} \tag{38}\]

where \( k \) is the normalized spatial curvature of the universe. As we study from the asymptotic solution, we demand that the diffusion constant be very small \( C_2 \ll 1 \) (in dimensionless terms). From the numerical solution we obtain that there are two different cases, for negative and positive diffusion constants. All of the solution asymptotically go to \( \Lambda \)CDM at large times (low red shifts). In addition to the \( C_2 \) initial condition, we have the \( C_4 \) constant, which determines \( \chi(z = 0) \). For understanding the evolution of this kind of universe, we solved numerically the deceleration rate:

\[q = -1 - \frac{\ddot{H}}{H^2} = \frac{1}{2}(1 + 3\omega)(1 + \frac{K}{aH^2}) \tag{39}\]

which can teach us the nature of the universe: \( q = -1 \) is the standart inflation (\( \rho = -p \)), and \( q < -1 \) means hyper inflation. \( q = \frac{1}{2} \) is for dust dominant (\( p = 0 \)),
FIG. 1: Evolution of the deceleration parameter for different positive diffusion constant $C_2 = 10^{-3}$, and different values of $\dot{\chi}(0)$. We can see at that the split points of the solutions is strongly depend on value of $C_2$ more then $\dot{\chi}(0)$.

FIG. 2: Evolution of the deceleration parameter for negative diffusion constant $C_2 = -10^{-30}$, and different values of $\dot{\chi}(0)$. In addition we can see a zoom on the point which split to different values of $\dot{\chi}(0)$, and another zoom on the hyper inflation part. The solution that approach $a = 0$ are big bang - big crunch solutions, separate from the hyper inflation bouncing solution.

and $q = 2$ for massless scalar field, which is called Stiff equation of state ($\rho = p$).

As we can see in figure 1 and figure 2, there is a different behavior for positive and negative $C_2$. For the case $C_2 > 0$, we can see a smooth change from $\Lambda$CDM at the late universe, to Stiff equation of state at the early universe directly. However, for $C_2 < 0$ we can see also the same behavior of transition, but for some values of $C_4$ we get a transition to $q = -2$ for a period of red shifts, which means hyper inflation, that from $\dot{\chi}(0)$ means that $H > 0$. In particular as we see here, we can produce a bounce which means $H(z)$ goes from negative values to positive values, as in figure 3, which means $a(t)$ is not a monotonic function of time.

A very important point is the dependence of the red shift we can see the transition of the scalar field. The actual point of the transition can change by the values of $C_2$ and $C_4$, but is strongly dependent on $C_2$ then $C_4$, as one can see at figure 2. This conclusion has a big influence on the constraint on the components. From what we know about the hot and early universe, there wasn’t this kind of transition from hyper inflation to $\Lambda$CDM in the nucleosynthesis. Therefore a big constraint on the values of $C_2$ and $C_4$ is coming from requirement that this kind of phase transition would happen only before BBN ($z = 10^8$).

In the case we restrict ourselves to $k = 0$ the value of $C_2$ is in the order of $C_2 \lesssim 10^{-15}$ and $\dot{\chi}(0)$ effects the duration of the hyper-inflation, but there are no 60 e-foldings needed to solve the flatness problem. One has to point out however, that the importance of the super acceleration period is not so much to produce many e-foldings, but to produce the bouncing of the universe (see for example [35][36][37]), and therefore the avoidance of the initial singularity. In a future publication we will see how to extend the period of inflation to obtain 60 e-foldings. In addition to hyper inflation there are separate different big bang - big crunch solutions, where the universe starts at $a = 0$ and goes back to $a = 0$. We depict that prediction in figure 2.

We also obtain that even for small spacial curvature, the first possibility of bounce happens for different red shifts. Indeed, a prediction of standard inflation theory, the curvature of our universe should be a exactly zero, however from measurements there are only strict constraints on the spacial curvature. And for our model, if there is a small curvature, the red shift where the phase transition takes place changes.
DISCUSSION

In this paper we have generalized the TMT and the dynamical space time theory, which imposes the covariant conservation of an energy momentum tensor. By coupling the dynamical time vector field to another scalar field, we obtain a covariant conservation of a current source for diffusive energy momentum tensor that is introduced in the action. This current that drives the non-conservation of the energy momentum tensor, is dissipated in the case of an expanding universe. So we get an asymptotic conservation of this energy momentum tensor. This energy tensor, in not the gravitational energy tensor which appears in the right hand side of the Einstein tensor, in the gravity equations, but the non-covariant conservation of the energy momentum tensor that appears in the action induces an energy momentum transfer between the dark energy and dark matter components, of the gravitational energy momentum tensor, in a way that resembles the ideas in \[7\]. But the $\phi$CMD mode of ref \[7\] arise from an action principle. Although the mechanism is similar, our formulation and theirs are not equivalent.

From the asymptotic solution we obtain that when $C_2 < 0$, unlike the standard $\Lambda$CDM model, where the dark energy is exactly constant, and the dark matter decreases like $a^{-3}$, in our case, dark energy can slowly decrease, instead of being constant, and dark matter also decreases, but not as fast as $a^{-3}$. This special property, is different in the $\phi$CMD model, where the exchange between DE and DM is much stronger in the asymptotic limit.

This behavior, where $C_2 < 0$, has a chance of explaining the coincidence problem, because unlike the standard $\Lambda$CDM model, where the dark energy is exactly constant, and the dark matter decreases like $a^{-3}$, in our case, dark energy can slowly decrease, instead of being constant, and dark matter also decreases, but not as fast as $a^{-3}$. This behavior can be understood by the observation that in an expanding universe a non-covariant conservation of an energy momentum tensor, which may imply that some energy density is increasing in the locally inertial frame, does not mean a corresponding increase of the energy density in the co moving cosmological frame, here in particular the non-covariant conservation of the dust component of the universe will produce a still decreasing dust density, although for $C_2 < 0$, there will be a positive flow of energy in the inertial frame to the dust component, but the result of this flow of energy in the local inertial frame will be just that the dust energy density will decrease a bit slower that the conventional dust (but still decreases).

From the numerical solution of the theory we obtain that there are two interesting cases. One case is when the universe starts from Stiff equation of state, and make a transition to $\Lambda$CDM. In a few solutions also the universe becomes to a ”dark radiation” dominant, before it evolves into $\Lambda$CDM. In another solutions, the bounces take place at very high red shifts if $C_2$ small enough, and therefore they are singularity free. The bounce is consequence of the hyper inflation period, where the deceleration parameter is -2, and $H > 0$ (so H goes from negative values to $H = 0$ and then to a positive value). The hyper inflation itself does not give enough e-foldings in the most simple version of the theory (not more than 4 e-foldings), but although inflation address many questions like the horizon and the flatness problem, it does not solve the singularity problem, which the bounce solves. In the future we will try to generalize this theory, and give a more complete picture that fit for our universe.

In ref. \[44\] a similar model has been studied, but there $\Lambda$ is fundamental field (not defined a in terms of a scalar field $\phi$), and dark matter - dark energy unification is not discuss, but some type of bounce behavior is also found.

More about TMT

TMTs also have many points of similarity with the ‘Lagrange Multiplier Gravity (LMG)’ \[38, 39\]. The Lagrange multiplier field in LMG enforces the condition that a certain function be zero. In the TMT this is equivalent to the constraint that requires some Lagrangian to be constant. The two measure models presented here, are different to the LMG models of \[38, 39\], and provide us with an arbitrary constant of integration for the value of a given Lagrangian, this constant of integration, if non zero, can generate spontaneous symmetry breaking of scale invariance, which is present in the theory for example. Recently a lot of interest has been attracted by the so called ”mimetic” dark matter model proposed in \[40\]. The latter employs a special covariant isolation of the conformal degree of freedom in Einstein gravity, whose dynamics mimics cold dark matter as a pressureless ”dust”. Important questions concerning the stability of of ”mimetic” gravity are studied in \[41, 42\], also a formulates a generalized mimetic tensor-vector-scalar ”mimetic” gravity which avoids those problems is studied. In \[43\] the idea is applied to inflationary scenarios.

Non-covariantly conserved Stress energy tensor

An equivalent expression for \([1]\), when $T^{\mu\nu}_{\text{(x)}}$ is formulated as a perfect fluid in FRWM space is:

$$\dot{\rho} + 3 \frac{\dot{a}}{a} (\rho + p) = \frac{C_2}{a^3}$$

when $C_2 = 0$, the stress energy tensor is conserved, and there is no diffusive effect. For late times, where the scale
parameter goes to infinity, we obtain that the diffusive effect vanishes.

Most versions of the mimetic gravity, except for \[\text{[41]}\], appears equivalent to a special kind of Lagrange multiplier theory or TMT models that were known before, where the simple constraint that the kinetic term of a scalar field be constant. This of course gives identical results to a very special TMT, where the Lagrangian that couples to the new measure is the kinetic term of this scalar field.

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