Phenomenology of charmed mesons in the extended Linear Sigma Model

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We study the so-called extended linear sigma model for the case of four quark flavors. This model is based on global chiral symmetry and dilatation invariance and includes, besides scalar and pseudoscalar mesons, vector and axial-vector mesons. Most of the parameters of the model have already been determined in a previous work by fitting hadron properties involving three quark flavors. Only three new parameters, all related to the current charm quark mass, appear when introducing charmed mesons. Our results for the (hidden and open) charmed meson masses, weak decay constants, and strong (OZI-dominant) decay widths turn out to be in good agreement with experimental data. This is remarkable because it shows that chiral symmetry, although strongly explicitly broken by the current charm quark mass, still has a strong influence on hadronic interactions, even in the charm sector.

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I. INTRODUCTION

Quantum chromodynamics (QCD) describes the fundamental interactions of quarks and gluons. However, due to confinement, in the low-energy domain the physical degrees of freedom are hadrons (mesons and baryons). In the last decades, effective low-energy approaches of the strong interaction have been developed by imposing chiral symmetry which is one of the basic symmetries of the QCD Lagrangian in the limit of vanishing quark masses (the so-called chiral limit) \[ SU(2) \times SU(2) \]. Chiral symmetry is explicitly broken by the nonzero current quark masses, but it is also spontaneously broken by a nonzero quark condensate in the QCD vacuum \[ \langle \bar{q}q \rangle \neq 0 \]. As a consequence, pseudoscalar (quasi-)Goldstone bosons emerge. In a world with only u and d quarks (i.e., for \( N_f = 2 \) quark flavors), these are the pions, while for \( N_f = 3 \), i.e., when also the strange quark s is considered, these are the pions, kaons, and the \( \eta \) meson. [The \( \eta' \) meson is not a Goldstone boson because of the chiral anomaly \( \{4, 5\} \).]

Chiral symmetry can be realized in hadronic approaches in the so-called nonlinear or in the linear representation. In the nonlinear case, only the Goldstone bosons are considered \([6, 7]\) in recent extensions also vector mesons are added, see e.g. Ref. [8]. On the contrary, in the linear case also the chiral partners of the Goldstone bosons, the scalar mesons, are retained \([9, 11]\). When extending this approach to the vector sector, both vector and axial-vector mesons are present \([10, 11]\). Along this line, recent efforts have led to the construction of the so-called extended linear sigma model (eLSM), first for \( N_f = 2 \) \([12, 14]\) and then for \( N_f = 3 \) \([15]\). In the eLSM, besides chiral symmetry, a basic phenomenon of QCD in the chiral limit has been taken into account: the symmetry under dilatation transformation and its anomalous breaking (trace anomaly), see e.g. Ref. \([16]\). As a result, the eLSM Lagrangian contains only a finite number of terms. In the case \( N_f = 3 \) it was for the first time possible to describe (pseudo)scalar as well as (axial-)vector meson nonets in a chiral framework \([14]\); masses and decay widths turn out to be in very good agreement with the results listed by the Particle Data Group (PDG) \([17]\).

In this work we investigate the eLSM model in the four-flavor case \( N_f = 4 \), i.e., by considering mesons which contain at least one charm quark. This study is a straightforward extension of Ref. \([15]\); the Lagrangian has the same structure as in the \( N_f = 3 \) case, except that all (pseudo)scalar and (axial-)vector meson fields are now parametrized in terms of \( 4 \times 4 \) (instead of \( 3 \times 3 \)) matrices. These now also include the charmed degrees of freedom. Since low-energy (i.e., nonstrange and strange) hadron phenomenology was described very well \([12, 14]\), we retain the values for the parameters that already appear in the three-flavor sector. Then, extending the model to \( N_f = 4 \), three additional parameters enter, all of which are related to the current charm quark mass (two of them in the (pseudo)scalar sector and one in the (axial-)vector sector).

Considering that the explicit breaking of chiral symmetry by the current charm quark mass, \( m_c \simeq 1.275 \text{ GeV} \), is quite large, one may wonder whether it is at all justified to apply a model based on chiral symmetry. Related to this, the charmed mesons entering our model have a mass up to about 3.5 GeV, i.e., they are strictly speaking no longer part of the low-energy domain of the strong interaction. Naturally, we do not expect to achieve the same precision as refined potential models \([18, 19]\) and lattice-QCD calculations \([20]\) [see also the review of Ref. \([21]\) and refs. therein]. Nevertheless, it is still interesting to see how a successful model for low-energy hadron phenomenology fares when extending it to the high-energy charm sector. Quite remarkably, a good agreement with experimental...
values for twelve (hidden and open) charmed meson masses is obtained by fitting just the three additional parameters mentioned above.

In addition to meson masses, we study the OZI-dominant decays of the heavy charmed states into light mesons. In this way, our model acts as a bridge between the high- and low-energy sector of the strong interaction. It turns out that the OZI-dominant decays are in agreement with current experimental results. This, in turn, means that these decays are largely governed by chiral symmetry. As a by-product of our analysis, we also obtain the value of the charm-anticharm condensate and the values of the weak decay constants of $D$ mesons. Moreover, in the light of our results we shall discuss the interpretation of the enigmatic scalar strange-charmed meson $D_{S0}^*(2317)$ and the axial-vector strange-charmed mesons $D_{S1}(2460)$ and $D_{S1}(2536)$.

This work is organized as follows: in Sec. 2 we present the eLSM for $N_f = 4$. In Sec. 3 we discuss the results for the masses and decay widths, and in Sec. 4 we give our conclusions. Details of the calculations are relegated to the Appendices. Our units are $\hbar = c = 1$, the metric tensor is $g^{\mu\nu} = \text{diag}(+,-,-,-)$.

II. THE LAGRANGIAN OF THE ELSM FOR $N_f = 4$

In this section we extend the eLSM to the four-flavor case. To this end, we introduce four matrices which contain, in addition to the usual nonstrange and strange mesons, also charmed states. The matrix of pseudoscalar fields $P$ (with quantum numbers $J^{PC} = 0^{-+}$) reads

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}(\eta_N + \pi^0) & \pi^+ & K^+ & D^0 \\ K^- & \sqrt{2}(\eta_N - \pi^0) & K^0 & D^- \\ \bar{D}^0 & \bar{K}^0 & \eta_S & \bar{D}_S^0 \\ D^+ & \bar{D}_S^0 & \eta_c & \bar{c}\bar{c} \end{pmatrix} \sim \frac{1}{\sqrt{2}} \begin{pmatrix} \bar{u}\Gamma_u d\Gamma_u & \bar{s}\Gamma_u s\Gamma_u & \bar{c}\Gamma_c c\Gamma_c \\ \bar{u}\Gamma_d d\Gamma_d & \bar{s}\Gamma_d s\Gamma_d & \bar{c}\Gamma_c c\Gamma_c \\ \bar{u}\Gamma_s s\Gamma_s & \bar{s}\Gamma_s s\Gamma_s & \bar{c}\Gamma_c c\Gamma_c \\ \bar{u}\Gamma_c c\Gamma_c & \bar{s}\Gamma_c c\Gamma_c & \bar{c}\Gamma_c c\Gamma_c \end{pmatrix},$$

where, for sake of clarity, we also show the quark-antiquark content of the mesons (in the pseudoscalar channel $\Gamma = i\gamma^5$). In the nonstrange-strange sector (the upper left 3 $\times$ 3 matrix) the matrix $P$ contains the pion triplet $\pi$, the four kaon states $K^+$, $K^-$, $K^0$, $\bar{K}^0$, and the isoscalar fields $\eta_N = \sqrt{1/2}(\bar{u}u + \bar{d}d)$ and $\eta_S = \bar{s}s$. The latter two fields mix and generate the physical fields $\eta$ and $\eta'$ [see details in Ref. 15]. In the charm sector (fourth line and fourth column) the matrix $P$ contains the open charmed states $D^+$, $D^-$, $D^0$, $\bar{D}^0$, which correspond to the well-established $D$ resonance, the open strange-charmed states $D_S^0$, and, finally, the hidden charmed state $\eta_c$, which represents the well-known pseudoscalar ground state charmonium $\eta_c(1S)$.

The matrix of scalar fields $S$ (with quantum numbers $J^{PC} = 0^{++}$) reads

$$S = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}(\sigma_N + a_0^0) & a_0^+ & K_0^+ & D_0^{0+} \\ a_0^- & \sqrt{2}(\sigma_N - a_0^0) & K_0^- & D_0^{0-} \\ K_0^{*+} & K_0^{*-} & \sigma_S & D_{S0}^{*+} \\ \bar{D}_0^{*+} & \bar{D}_0^{*+} & \bar{c}\bar{c} & \chi_{cc} \end{pmatrix}.$$

The quark-antiquark content is the same as in Eq. (1), but using $\Gamma = 1$. A long debate about the correct assignment of light scalar states has taken place in the last decades. Present results [28, 29], which have been independently confirmed in the framework of the eLSM [15], show that the scalar quarkonia have masses between 1-2 GeV. In particular, the isotriplet $a_0$ is assigned to the resonance $a_0(1450)$ (and not to the lighter state $a_0(980)$). Similarly, the kaonic states $K_0^{*+}$, $K_0^{*-}$, $K_0^{*+}$, $\bar{K}_0^{*0}$ are assigned to the resonance $K_0^{*+}(1430)$ (and not to the $K_0^{*+}(800)$ state). The situation in the scalar-isoscalar sector is more complicated, due to the presence of a scalar glueball state $G$, see Ref. [14] and below. Then, $\sigma_N$, $\sigma_S$, $G$ mix and generate the three resonances $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$. While in all realistic mixing scenarios $f_0(1370)$ comes out to be predominantly a $\sqrt{1/2}(\bar{u}u + \bar{d}d)$ state, it is not yet clear if $f_0(1500)$ or $f_0(1710)$ is predominantly a glueball state. As a consequence, the light scalar states $f_0(500)$ and $f_0(980)$ are not quarkonia (but, arguably, tetraquark or molecular states) [31, 32]. In the open charm sector, we assign the charmed states $D_0^+$ to the resonances $D_{S0}^+(2400)^0$ and $D_{S0}^+(2400)^\pm$ (the latter state has not yet been unambiguously established). In the strange-charm sector we assign the state $D_{S0}^+$ to the only existing candidate $D_{S0}^+(2317)^\pm$; it should, however, be stressed that the latter state has also been interpreted as a tetraquark or molecular state because it is too light when compared to quark-model predictions, see Refs. [18, 21, 24, 25]. In the next section, we investigate in more detail the possibility that a heavier, very broad (and therefore not yet discovered) scalar charmed state exists. In the hidden charm sector the resonance $\chi_{cc}$ corresponds to the ground-state scalar charmonium $\chi_{cc}(1P)$.

The matrices $P$ and $S$ are used to construct the matrix $\Phi = S + iP$, which transforms under the group $U(4)_R \times U(4)_L$ as $\Phi \rightarrow U_L \Phi U_R^\dagger$, where $U_L$ and $U_R$ are independent $4 \times 4$ unitary matrices. Due to this property, the matrix $\Phi$ is used
as a building block for the construction of a Lagrangian which is invariant under the chiral group $U(4)_R \times U(4)_L$, see below.

We now turn to the vector sector. The matrix $V^\mu$ which includes the vector degrees of freedom is:

$$
V^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix}
\sqrt{2} (\omega_N + \rho^0) & \rho^+ & K^+(892)^+ & D^{*0} \\
\rho^- & \sqrt{2} (\omega_N - \rho^0) & K^+(892)^0 & D^{*-} \\
K^+(892)^0 & K^+(892)^0 & \omega_S & D_S^{*+} \\
D^{*+} & D_S^{*+} & J/\psi & \end{pmatrix}^\mu
$$

(3)

The quark-antiquark content is that shown in Eq. (1), setting $\Gamma = \gamma^\mu$. The isotriplet field $\vec{\rho}$ corresponds to the $\rho$ meson, the four kaonic states correspond to the resonance $K^+(892)$, the isoscalars $\omega_N$ and $\omega_S$ correspond to the $\omega$ and $\phi$ mesons, respectively. [No mixing between strange and nonstrange isoscalars is present in the eLSM; this mixing is small anyway.] In the charm sector, the fields $D^{*0}$, $D^{*-}$, and $D^{*0}$ correspond to the vector charmed resonances $D^*(2007)^0$ and $D^*(2010)^\pm$, respectively, while the strange-charmed $D_S^{*\pm}$ corresponds to the resonance $D_S^{*\pm}$ (with mass $M_{D_S^{*\pm}} = (2112.3 \pm 0.5)$ MeV; note, however, that the quantum numbers $J^P = 1^-$ are not yet fully established). Finally, $J/\psi$ is the very well-known lowest vector charmonium state $J/\psi(1S)$.

The matrix $A^\mu$ describing the axial-vector degrees of freedom is given by:

$$
A^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix}
\sqrt{2} (f_{1,N} + a_1^0) & a_1^+ & K_1^+ & D_1^0 \\
ar_1^- & \sqrt{2} (f_{1,N} - a_1^0) & K_1^0 & D_1^- \\
K_1^0 & K_1^0 & f_{1,S} & D_{S1}^0 \\
D_{S1}^0 & D_{S1}^0 & D_{S1}^0 & \chi_{c,1} \end{pmatrix}^\mu
$$

(4)

The quark-antiquark content is that shown in Eq. (1), setting $\Gamma = \gamma^\mu \gamma^5$. The isotriplet field $\vec{a}_1$ corresponds to the field $a_1(1260)$, the four kaonic states $K_1$ correspond (predominantly) to the resonance $K_1(1200)$ [but also to $K_1(1400)$], because of mixing between axial-vector and pseudovector states, see Ref. [35] and refs. therein. The isoscalar fields $f_{1,N}$ and $f_{1,S}$ correspond to $f_1(1285)$ and $f_1(1420)$, respectively. In the charm sector, the $D_1$ field is chosen to correspond to the resonances $D_1(2420)^0$ and $D_1(2420)^\pm$. (Another possibility would be the not yet very well established resonance $D_1(2430)^0$, or, due to mixing between axial- and pseudovector states, to a mixture of $D_1(2420)$ and $D_1(2430)$. Irrespective of this uncertainty, the small mass difference between these states would leave our results virtually unchanged.) The assignment of the strange-charmed doublet $D_{S1}^\pm$ is not yet settled, the two possibilities listed by the PDG are the resonances $D_{S1}(2460)^\pm$ and $D_{S1}(2536)^\pm$. According to various studies, the latter option is favored, while the former can be interpreted as a molecular or a tetraquark state [37, 38]. Thus, we assign our quark-antiquark $D_1$ state to the resonance $D_{S1}(2536)^\pm$. Finally, the charm-anticharm state $\chi_{c,1}$ can be unambiguously assigned to the charm-anticharm resonance $\chi_{c,1} (1P)$.

From the matrices $V^\mu$ and $A^\mu$ we construct the left-handed and right-handed vector fields $L^\mu = V^\mu + A^\mu$ and $R^\mu = V^\mu - A^\mu$. Under chiral transformations they transform as $L^\mu \rightarrow U_L L^\mu U_L^\dagger$ and $R^\mu \rightarrow U_R R^\mu U_R^\dagger$. Due to this transformation property they are used as building blocks of the chirally invariant Lagrangian, see below.

The last field entering the model is the scalar glueball $G$, described by the dilaton Lagrangian

$$
\mathcal{L}_{dil} = \frac{1}{2} (\partial_\mu G)^2 - \frac{1}{4} \frac{m_G^2}{\Lambda^2} \left( G^4 \log \frac{G}{\Lambda} - \frac{G^4}{4} \right),
$$

(5)

which mimics the trace anomaly of QCD [15, 16, 32]. The dimensionful parameter $\Lambda$ sets the energy scale of low-energy QCD; in the chiral limit it is the only dimensionful parameter besides the coefficient of the term representing the axial anomaly. All other interaction terms of the Lagrangian are described by dimensionless coupling constants. The parameter $m_G$ is the glueball mass in the quenched approximation (no quarks), which is about 1.5-1.7 GeV [41]. As mentioned above, the identification of $G$ is still uncertain, the two most likely candidates are $f_0(1500)$ and $f_0(1710)$ and/or admixtures of them. Note that, while we include the scalar glueball because it is conceptually important to guarantee dilatation invariance of the model (thus constraining the number of possible terms that it can have), just as in Ref. [13] we do not make an assignment for the glueball. This is an ongoing project [42] which, however, does not affect the results of the present study. [The pseudoscalar glueball $\bar{G}$ has been also added to the eLSM [42], but it is also not relevant here and therefore omitted.]

The full Lagrangian is constructed by requiring chiral symmetry, dilatation invariance (besides the exceptions
the charm quark mass is large. We describe them separately:

\[ \mathcal{L} = \mathcal{L}_{\text{det}} + \text{Tr}[D^\mu \Phi]^4[D^\mu \Phi] - m_0^2 \left( \frac{G}{G_0} \right)^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 + \text{Tr}[H(\Phi + \Phi^\dagger)] \]

+ \text{Tr}\left\{ \left( \frac{G}{G_0} \right)^2 \frac{m_0^2}{2} + \Delta \right\} [(L^\mu)^2 + (R^\mu)^2] - \frac{1}{4} \frac{\text{Tr}[(L^\mu)^2 + (R^\mu)^2]}{2} - 2 \text{Tr}[\epsilon^\dagger \Phi] + c(\text{det}\Phi - \text{det}\Phi^\dagger)^2

+ i \frac{g_2}{2} \text{Tr}(L^{\mu\nu}[L^\mu, L^\nu]) + \text{Tr}(L^{\mu\nu}[L^\mu, L^\nu]) + h_1 \frac{1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}[(L^\mu)^2 + (R^\mu)^2] + h_2 \text{Tr}[(\Phi R^\mu)^2 + (L^\mu \Phi)^2]

+ 2 h_3 \text{Tr}(\Phi R^\mu \Phi^\dagger L^\mu) + \ldots, \quad (6) \]

where \( \mathcal{L}_{\text{det}} \) is the dilaton Lagrangian of Eq. \( \Box \), \( D^\mu \Phi \equiv \partial^\mu \Phi - ig_1 (L^\mu \Phi - \Phi R^\mu) \), \( L^{\mu\nu} \equiv \partial^\mu L^\nu - \partial^\nu L^\mu \), and \( R^{\mu\nu} = \partial^\mu R^\nu - \partial^\nu R^\mu \). The dots refer to further chirally invariant terms listed in Ref. [15]; these terms do not affect the masses and decay widths studied in this work and are therefore omitted.

The terms involving the matrices \( H, \epsilon, \) and \( \Delta \) describe the breaking of dilatation and chiral symmetry due to nonzero current quark masses. They are of particular importance when the charmed mesons are considered, because the charm quark mass is large. We describe them separately:

(i) The term \( \text{Tr}[H(\Phi + \Phi^\dagger)] \) with

\[ H = \frac{1}{2} \begin{pmatrix} h_U & 0 & 0 & 0 \\ 0 & h_D & 0 & 0 \\ 0 & 0 & \sqrt{2} h_S & 0 \\ 0 & 0 & 0 & \sqrt{2} h_C \end{pmatrix} , \quad (7) \]

describes the usual explicit symmetry breaking (tilting of the Mexican-hat potential). The constants \( h_i \) are proportional to the current quark masses, \( h_i \propto m_i \). Here we work in the isospin limit, \( h_U = h_D = h_N \). The pion mass, for instance, turns out to be \( m_\pi^2 \propto m_u \), in agreement with the Gell-Mann–Oakes–Renner (GOR) relation [43]. The parameter \( h_C \) is one of the three new parameters entering the \( N_f = 4 \) version of the model when compared to the \( N_f = 3 \) case of Ref. [15].

(ii) The term \( -2 \text{Tr}[\epsilon^\dagger \Phi] \) with

\[ \epsilon = \begin{pmatrix} \epsilon_U & 0 & 0 & 0 \\ 0 & \epsilon_D & 0 & 0 \\ 0 & 0 & \epsilon_S & 0 \\ 0 & 0 & 0 & \epsilon_C \end{pmatrix} , \quad (8) \]

where \( \epsilon_i \propto m_i^2 \), is the next-to-leading order correction in the current quark-mass expansion. In the isospin-symmetric limit \( \epsilon_U = \epsilon_D = \epsilon_N \) one can subtract from \( \epsilon \) a matrix proportional to the identity in such a way that the parameter \( \epsilon_N \) can be absorbed in the parameter \( m_0^2 \). Thus, without loss of generality we can set \( \epsilon_N = 0 \). Following Ref. [15], for the sake of simplicity we shall here also set \( \epsilon_S = 0 \), while we keep \( \epsilon_C \) nonzero. This is the second additional parameter with respect to Ref. [15].

(iii) The term \( \text{Tr} [\Delta (L^\mu L^\mu + R^\mu R^\mu)] \) with

\[ \Delta = \begin{pmatrix} \delta_U & 0 & 0 & 0 \\ 0 & \delta_D & 0 & 0 \\ 0 & 0 & \delta_S & 0 \\ 0 & 0 & 0 & \delta_C \end{pmatrix} , \quad (9) \]

where \( \delta_i \sim m_i^2 \), describes the current quark-mass contribution to the masses of the (axial-)vector mesons. Also in this case, in the isospin-symmetric limit it is possible to set \( \delta_U = \delta_D = \delta_N = 0 \) because an identity matrix can be absorbed in the term proportional to \( m_0^2 \). The parameter \( \delta_S \) is taken from Ref. [15]. The third new parameter with respect to Ref. [15] is \( \delta_C \).

Another important term in the Lagrangian \( \Box \) is \( c(\text{det}\Phi - \text{det}\Phi^\dagger)^2 \), which describes the axial anomaly and is responsible for the large \( \eta' \) mass. Care is needed, because the determinant changes when the number of flavors changes. The relation between \( c \) and its counterpart in the three-flavor case \( c_{N_f = 3} \) of Ref. [15] is given by:

\[ c = \frac{2 c_{N_f = 3}}{\phi_C} . \quad (10) \]
Thus, the parameter $c$ can be determined once the condensate $\phi_C$ is obtained, see the next section.

The Lagrangian (6) induces spontaneous symmetry breaking if $m_0^2 < 0$: as a consequence, the scalar-isoscalar fields $G, \sigma_N, \sigma_S$, and $\chi_{C0}$ develop nonzero vacuum expectation values. One has to perform the shifts $G \to G + g_0$, $\sigma_N \to \sigma_N + \phi_N$, $\sigma_S \to \sigma_S + \phi_S$, and $\chi_{C0} \to \chi_{C0} + \phi_C$. The quantity $g_0$ is proportional to the gluon condensate [14], while $\phi_N$, $\phi_S$, and $\phi_C$ correspond to the nonstrange, strange, and charm quark-antiquark condensates.

Once the shifts are performed, one also has to get rid of mixing between (axial-)vector and (pseudo)scalar states by a proper shift of axial-vector fields and renormalization of the (pseudo)scalar ones. Then, the physical masses and interaction terms can be calculated. We present the detailed expressions for the masses in Appendix A.

III. RESULTS

A. Masses

The Lagrangian (6) contains the following 15 free parameters: $m_0^2$, $\lambda_1$, $\lambda_2$, $m_1$, $g_1$, $c_1$, $h_1$, $h_2$, $h_3$, $\delta_S$, $\delta_C$, $\varepsilon_C$, $h_N$, $h_S$, and $h_C$. For technical reasons, instead of the parameters $h_N$, $h_S$, and $h_C$ entering Eq. (7), it is easier to use the condensates $\phi_N$, $\phi_S$, and $\phi_C$. This is obviously equivalent, because $\phi_N$, $\phi_S$, and $\phi_C$ form linearly independent combinations of the parameters.

In the large-$N_c$ limit, one sets $h_1 = \lambda_1 = 0$. Then, as shown in Ref. [13], for the case $N_f = 3$, ten parameters can be determined by a fit to masses and decay widths of mesons below 1.5 GeV as shown in Table 1. In the following we use these values for our numerical calculations. As a consequence, the masses and the decay widths of the nonstrange-strange mesons are – by construction – identical to the results of Ref. [15] (see Table 2 and Fig. 1 in that ref.). Note also that, in virtue of Eq. (10), the parameter combination $\phi_C^2c/2$ is determined by the fit of Ref. [15].

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $m_0^2$ | $0.413 \times 10^6$ MeV$^2$ | $m_0^2$ | $-0.918 \times 10^6$ MeV$^2$ |
| $\phi_C^2c/2$ | $450 \times 10^{-6}$ MeV$^{-2}$ | $\delta_S$ | $0.151 \times 10^6$ MeV$^2$ |
| $g_1$ | $5.84$ | $h_1$ | $0$ |
| $h_2$ | $9.88$ | $h_3$ | $3.87$ |
| $\phi_N$ | $164.6$ MeV | $\phi_S$ | $126.2$ MeV |
| $\lambda_1$ | $0$ | $\lambda_2$ | $68.3$ |

For the purposes of the present work, we are left with three unknown parameters: $\phi_C$, $\varepsilon_C$, $\delta_C$. We determine them by performing a fit to twelve experimental (hidden and open) charmed meson masses listed by the PDG [17]

$$\chi^2 \equiv \sum_{i}^{12} \left( \frac{M_i^{th} - M_i^{exp}}{\xi M_i^{exp}} \right)^2,$$

(11)

where $\xi$ is a constant. We do not use the experimental errors for the masses, because we do not expect to reach the same precision with our effective model which, besides other effects, already neglects isospin breaking. In Ref. [15], we required a minimum error of 5% for experimental quantities entering our fit, and obtained a reduced $\chi^2$ of about 1.23. Here, we slightly change our fit strategy: we choose the parameter $\xi$ such that the reduced $\chi^2$ takes the value $\chi^2/(12 - 3) = 1$, which yields $\xi = 0.07$. This implies that we enlarge the experimental errors to 7% of the respective masses.

The parameters (together with their theoretical errors) are:

$$\phi_C = (176 \pm 28) ~\text{MeV}, \quad \delta_C = (3.91 \pm 0.36) \times 10^6 ~\text{MeV}^2, \quad \varepsilon_C = (2.23 \pm 0.71) \times 10^6 ~\text{MeV}^2.$$

(12)

In Table 2 we present the results of our fit by comparing the theoretically computed with the experimentally measured masses [see also Ref. [14] for preliminary results]. For the nonstrange-charmed states we use the masses of the neutral members of the multiplet in the fit, because the corresponding resonances have been clearly identified and the masses have been well determined for all quantum numbers. In view of the fact that the employed model is built as a low-energy chiral model and that only three parameters enter the fit, the masses are quite well described. The mismatch grows for increasing masses because Eq. (11) imposes, by construction, a better precision for low masses.
Table 2: Masses of charmed meson used in the fit.

| Resonance | Quark content | Our Value [MeV] | Experimental Value [MeV] |
|-----------|---------------|-----------------|--------------------------|
| $D^0$     | $u, ar{u}$  | $0^+$           | 1981 ± 73               |
| $D_s^-$   | $s, ar{s}$  | $0^+$           | 2004 ± 74               |
| $\eta_c(1S)$ | $c, ar{c}$ | $0^+$           | 2673 ± 118              |
| $D_s^0(2400)^0$ | $u, ar{u}$ | $0^+$           | 2414 ± 77               |
| $D_s^0(2317)^0$ | $s, ar{s}$ | $0^+$           | 2467 ± 76               |
| $\chi_{c0}(1P)^0$ | $c, ar{c}$ | $0^+$           | 3144 ± 128              |
| $D^*(2007)^0$ | $u, ar{u}$ | $1^+$           | 2168 ± 70               |
| $D_s^*(2007)^0$ | $s, ar{s}$ | $1^+$           | 2203 ± 69               |
| $J/\psi(1S)$ | $c, ar{c}$ | $1^+$           | 2947 ± 109              |
| $D(2420)^0$ | $u, ar{u}$ | $1^+$           | 2429 ± 63               |
| $D_s(2460)^0$ | $s, ar{s}$ | $1^+$           | 2480 ± 63               |
| $\chi_{c1}(1P)^0$ | $c, ar{c}$ | $1^+$           | 3239 ± 101              |

Two remarks about the scalar states are in order: (i) the experimental value for the mass of the charged scalar state $D_s^0(2400)^\pm$, which is $(2403 \pm 14 \pm 35)$ MeV, is in good agreement with our theoretical result, although the existence of this resonance has not yet been unambiguously established. (ii) The theoretically computed mass of the strange-charmed scalar state $D_s^0(2317)^0$ turns out to be larger than that of the charmed state $D_s^0(2400)^0$. In this respect, the experimental result is puzzling because the mass of $D_s^0(2317)^0$ is smaller than that of $D_s^0(2400)^0$. A possibility is that the resonance $D_s^0(2317)$ is not a quarkonium, or that the current mass of the quarkonium field is diminished by quantum fluctuations [see e.g. Ref. [45]].

(iii) The theoretical mass of the axial-vector strange-charmed state $D_s^0$ reads 2480 MeV, which lies in between the two physical states $D_s(2460)^0$ and $D_s(2536)^0$. We shall re-analyze the scalar and the axial-vector strange-charmed states in light of the results for the decay widths, see next subsection.

Interesting quantities are the following mass differences in which the explicit dependence on the parameters $\varepsilon_C$ and $\delta_C$ cancels:

$$m_{D_s}^2 - m_{D_s^*}^2 = \sqrt{2}(g_1^2 - h_3)\phi_N \phi_C, \quad m_{\chi_{c1}}^2 - m_{J/\psi}^2 = 2 (g_1^2 - h_3)\phi_C^2, \quad m_{D_s^0}^2 - m_{D_s^*}^2 = 2 (g_1^2 - h_3)\phi_S \phi_C.$$  \hspace{1cm} (13)

Using Eq. (12), the theoretical results are

$$m_{D_s^0}^2 - m_{D_s^*}^2 = (1.2 \pm 0.6) \times 10^6 \text{MeV}^2, \quad m_{\chi_{c1}}^2 - m_{J/\psi}^2 = (1.8 \pm 1.3) \times 10^6 \text{MeV}^2, \quad m_{D_s^0}^2 - m_{D_s^*}^2 = (1.2 \pm 0.6) \times 10^6 \text{MeV}^2,$$

while the experimental values are (the experimental errors are omitted because, being of the order of $10^3 \text{MeV}^2$, they are very small when compared to the theoretical ones):

$$m_{D_s^0}^2 - m_{D_s^*}^2 = 1.82 \times 10^6 \text{MeV}^2, \quad m_{\chi_{c1}}^2 - m_{J/\psi}^2 = 2.73 \times 10^6 \text{MeV}^2, \quad m_{D_s^0}^2 - m_{D_s^*}^2 = 1.97 \times 10^6 \text{MeV}^2.$$

The agreement is fairly good, which shows that our determination of the charm condensate $\phi_C$ is compatible with the experiment, although it still has a large uncertainty.

B. Decays

In this subsection we study the decays of charmed mesons. As a first step we evaluate the weak-decay constants of the pseudoscalar mesons $D$ and $D_S$. Their analytic expressions read [see Appendix A and also Ref. [44]]

$$f_D = \frac{\phi_N + \sqrt{2}\phi_C}{\sqrt{2}Z_D}, \quad f_{D_S} = \frac{\phi_S + \phi_C}{Z_{D_S}}, \quad f_{\eta_c} = \frac{\sqrt{2}\phi_C}{Z_{\eta_c}}.$$  \hspace{1cm} (14)

Using the parameters of the fit we obtain

$$f_D = (254 \pm 17) \text{MeV}, \quad f_{D_S} = (261 \pm 17) \text{MeV}, \quad f_{\eta_c} = (222 \pm 28) \text{MeV}.$$  \hspace{1cm} (15)

The experimental values $f_D = (206.7 \pm 8.9)$ MeV, $f_{D_s} = (260.5 \pm 5.4)$ MeV [17] show a good agreement for $f_{D_s}$ and a slightly too large theoretical result for $f_D$. The quantity $f_{\eta_c}$ is slightly smaller than the experimental value $f_{\eta_c} = (335 \pm 75)$ MeV [48] and the theoretical results $f_{\eta_c} = (292 \pm 25)$ MeV [17] and $f_{\eta_c} = (300 \pm 50)$ MeV [48].

We now turn to the (OZI-dominant) strong decay widths of the resonances $D_0$, $D_s^*$, $D_s$, and $D_{s1}$, whose results are summarized in Table 3. For the calculation of the decay widths we have used the physical masses listed by the PDG.
This is necessary in order to have the correct phase space. With the exception of $D_{S1}(2536)^+ \rightarrow D^* K$, all values are in agreement with the mean experimental values and upper bounds. This is remarkable if one considers that the decay amplitudes depend on the parameters of the three-flavor version of the model determined in Ref. 15. The theoretical errors have been calculated by taking into account the uncertainty in the charm condensate $\phi_C = (176 \pm 28)$ MeV. The lower theoretical value corresponds to $\phi_C = (176 - 28)$ MeV, while the upper one to $\phi_C = (176 + 28)$ MeV. The explicit expressions for the decay widths are reported in Appendix B.

Here we do not study the decay of other (hidden and open) charm state because we restrict ourselves to OZI-dominant processes. The study of OZI-suppressed decays which involve the large-$N_c$ suppressed parameters $\lambda_1$ and $h_1$ is left for future work. Then, also the decays of the well-known charmion states (such as $\chi_{c0}$ and $\eta_c$) will be investigated.

Table 3: Decay widths of charmed mesons

| Decay Channel | Theoretical result [MeV] | Experimental result [MeV] |
|---------------|---------------------------|---------------------------|
| $D_0^+(2400)^0 \rightarrow D\pi = D^+\pi^- + D^0\pi^0$ | $139_{-113}^{+242}$ | $D^+\pi^-$ seen; full width $\Gamma = 267 \pm 40$ |
| $D_0^+(2400)^+ \rightarrow D\pi = D^+\pi^- + D^0\pi^0$ | $51_{-51}^{+52}$ | $D^+\pi^0$ seen; full width: $\Gamma = 283 \pm 24 \pm 34$ |
| $D^*(2007)^0 \rightarrow D^0\pi^0$ | $0.025 \pm 0.003$ | seen: $< 1.3$ |
| $D^*(2007)^+ \rightarrow D^+\pi^+$ | $0$ | not seen |
| $D^*(2010)^+ \rightarrow D^+\pi^+$ | $0.018_{-0.003}^{+0.002}$ | $0.029 \pm 0.008$ |
| $D^*(2420)^0 \rightarrow D^\ast\pi^- + D^\ast\pi^+$ | $65_{-37}^{+136}$ | $D^\ast\pi^-$ seen; full width: $\Gamma = 27.4 \pm 2.5$ |
| $D_1(2420)^0 \rightarrow D^0\pi^- + D^\ast\pi^+$ | $0.59 \pm 0.02$ | seen |
| $D_1(2420)^0 \rightarrow D^+\pi^- + D^\ast\pi^+$ | $0.21_{-0.015}^{+0.004}$ | seen |
| $D_1(2420)^0 \rightarrow D^\ast\pi^- + D^\ast\pi^+$ | $0$ | not seen; $\Gamma(D^\ast\pi^-)/\Gamma(D^\ast\pi^+) < 0.24$ |
| $D_1(2420)^0 \rightarrow D^\ast\pi^- + D^\ast\pi^+$ | $65_{-36}^{+136}$ | $D^\ast\pi^+$ seen; full width: $\Gamma = 25 \pm 6$ |
| $D_1(2420)^+ \rightarrow D^+\pi^- + D^\ast\pi^+$ | $0.56 \pm 0.02$ | seen |
| $D_1(2420)^+ \rightarrow D^\ast\pi^- + D^\ast\pi^+$ | $0.22 \pm 0.01$ | seen |
| $D_1(2420)^+ \rightarrow D^\ast\pi^- + D^\ast\pi^+$ | $0$ | not seen; $\Gamma(D^\ast\pi^-)/\Gamma(D^\ast\pi^+) < 0.18$ |
| $D^\ast(2536)^+ \rightarrow D^\ast K = D^\ast(0)K^0 + D^\ast(+)K^0$ | $25_{-15}^{+14}$ | seen; full width $\Gamma = 0.92 \pm 0.03 \pm 0.04$ |
| $D_{S1}(2536)^+ \rightarrow D^\ast K = D^\ast(0)K^0 + D^\ast(+)K^0$ | $0$ | not seen |

The following comments are in order:

(i) The decay of $D_0^0(2400)$ into $D\pi$ has a large theoretical error due to the imprecise determination of $\phi_C$. A qualitative statement is, however, possible: the decay channel $D_0^0(2400) \rightarrow D\pi$ is large and is the only OZI-dominant decay predicted by our model. This decay channel is also the only one seen in experiment (although the branching ratio is not yet known). A similar discussion holds for the charged counterpart $D_0^+(2400)^+$.

(ii) The decay widths of the vector charmed states $D^*(2007)^0$ and $D^*(2010)^+$ are in very good agreement with the experimental results and upper bounds.

(iii) The results for the vector strange-charmed state $D_1(2420)^0$ and $D_1(2420)^+$ are compatible with experiment. Note that the decay into $D^\ast\pi$ is the only one which is experimentally seen. Moreover, the decays $D_1(2420)^0 \rightarrow D^\ast\pi^-$ and $D_1(2420)^+ \rightarrow D^0\pi^+$, although kinematically allowed, do not occur in our model because there is no respective tree-level coupling; this is in agreement with the small experimental upper bound. Improvements in the decay channels of $D_1(2420)$ are possible by taking into account also the multiplet of pseudovector quark-antiquark states. In this way, one will be able to evaluate the mixing of these configurations and describe at the same time the resonances $D_1(2420)$ and $D_1(2430)$.

(iv) The decay of the axial-vector strange-charmed state $D_{S1}(2536)^\pm \rightarrow D^* K$ is too large in our model when compared to the experimental data of about 1 MeV. This result is robust upon variation of the parameters, as the error shows. We thus conclude that the resonance $D_{S1}(2536)^\pm$ is not favored to be (predominantly) a member of the axial-vector multiplet (it can be, however, a member of the pseudovector multiplet). Then, we discuss two possible solutions to the problem of identifying the axial-vector strange-charmed quarkonium:

Solution 1: There is a ‘seed’ quark-antiquark axial-vector state $D_{S1}$ above the $D^* K$ threshold, which is, however, very broad and for this reason has not yet been detected. Quantum corrections generate the state $D_{S1}(2460)^\pm$ through pole doubling [53]. In this scenario, $D_{S1}(2460)$ is dynamically generated but is still related to a broad quark-antiquark seed state. In this way, the low mass of $D_{S1}(2460)$ in comparison to the quark-model prediction [18] is due to quantum corrections [43, 50]. Then, the state $D_{S1}(2460)$, being below threshold, has a very small decay width.

Solution 2: Also in this case, there is still a broad and not yet detected quark-antiquark field above threshold, but solution 1 is assumed not to apply (loops are not sufficient to generate $D_{S1}(2460)$). The resonance $D_{S1}(2460)^\pm$ is not
a quark-antiquark field, but a tetraquark or a loosely bound molecular state and its existences is not related to the quark-antiquark state of the axial-vector multiplet.

(v) For the state $D_{S0}^*(2317)$ similar arguments apply. If the mass of this state is above the $DK$ threshold, we predict a very large ($\gtrsim 1$ GeV) decay width into $DK$ (for example: $\Gamma_{D_{S0}^*-DK} \approx 3$ GeV for a $D_{S0}^*$ mass of 2467 MeV as determined in Table 2). Then, the two solutions mentioned above are applicable also here:

**Solution 1:** A quark-antiquark state with a mass above the $DK$ threshold exists, but it is too broad to be seen in experiment. The state $D_{S0}^*(2317)$ arises through the pole-doubling mechanism.

**Solution 2:** Loops are not sufficient to dynamically generate $D_{S0}^*(2317)$. The latter is not a quarkonium but either a tetraquark or a molecular state.

In conclusion, a detailed study of loops in the axial-vector and scalar strange-charm sector mixing with a pseudovector quark-antiquark state should also be included. These tasks go beyond the tree-level analysis of our work but are an interesting subject for the future.

### IV. CONCLUSION AND OUTLOOK

In this work we have developed a four-flavor extended linear sigma model with vector and axial-vector degrees of freedom and we have calculated masses and decay widths of charmed mesons.

By using the parameters determined in the low-energy study of mesons in Ref. [13] and listed in Table 1, we have performed a fit of the three remaining free parameters (which are related to the current charm quark mass) to twelve masses of hidden and open charmed mesons. The results are shown in Table 2 and agree well with the experimental values. Then, we have calculated the weak-decay constants of the pseudoscalar states $D$, $D_S$, and $\eta_c$, which are in good agreement with the experimental values and, as a last step, we have evaluated the OZI-dominant decays of charmed mesons (Table 3). The result for $D_{S0}^0(2400)^0$, $D_{S0}^0(2400)^+$, $D^*(2007)^0$, $D^*(2010)^+$, $D_1(2420)^0$, and $D_{14}(2420)^+$ are compatible with the results and the upper bounds listed by the PDG [17].

The fact that a good description is obtained by using a chiral model and, more remarkably, by using the parameters determined by a study of $N_f = 3$ mesons, means that a remnant of chiral symmetry is present also in the sector of charmed mesons. Chiral symmetry is (to a large extent) still valid because the parameters of the eLSM do not vary too much as a function of the energy at which they are probed. Besides mass terms which describe the large contribution of the current charm quark mass, all interaction terms are the same as in the low-energy effective model of Refs. [13, 14, 15] which was built under the requirements of chiral symmetry and dilatation invariance. As a by-product of our work we also evaluate the charm condensate which is of the same order as the nonstrange and strange quark condensates. This is also in accord with chiral dynamics enlarged to the group $U(4)_R \times U(4)_L$.

Concerning the assignment of the scalar and axial-vector strange-charmed quarkonium states $D_{S0}$ and $D_{S1}$, we obtain the following: If the masses of these quarkonia are above the respective thresholds, we find that their decay widths are too large, which probably means that these states, even if existent, have escaped detection. In this case, the resonances $D_{S0}^*(2317)$ and $D_{S1}(2460)$ can emerge as dynamically generated companion poles (alternatively, they can be tetraquark or molecular states). Our results imply also that the interpretation of the resonance $D_{S1}(2536)$ as a member of the axial-vector multiplet is not favored because the experimental width is too narrow when compared to the theoretical width of a quarkonium state with the same mass. An investigation of these resonances necessitates the calculation of quantum fluctuations and represents a topic of future work.

A further important future project is the study of OZI-suppressed decays of charmonium states. To this end, one should allow for a nonzero value of the large-$N_c$ suppressed parameters. Due to the large amount of existing data, this is an interesting project to test our chiral approach in more detail in the realm of hidden and open charmed states.

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Appendix A: Shifts and masses

1. Shifts

Due to spontaneous symmetry breaking the scalar-isoscalar fields $\sigma_N$, $\sigma_S$, and $\chi_{C0}$ are shifted by their vacuum expectation values $\phi_N$, $\phi_S$, and $\phi_C$ as:

$$\sigma_N \to \sigma_N + \phi_N \ , \ \sigma_S \to \sigma_S + \phi_S \ , \ \chi_{C0} \to \chi_{C0} + \phi_C .$$  \hspace{1cm} (A1)

As a consequence, bilinear mixing terms involving the mesons $\eta_{N,f1N} - f_{1}^{\pm}, \eta_{S,f1S}, K_0^* - K^*$, and $K - K_1$ arise:

$$- g_1 \phi_N (f_{1 N}^\mu \partial_\mu \eta_N + \vec{\pi} \cdot \vec{\pi} - \vec{d} \cdot \vec{d}) - \sqrt{2} g_1 \phi_S (f_{1 S}^\mu \partial_\mu \eta_S$$

$$- i \frac{g_1}{2} (\phi_N - \phi_S) (K^{*0}_0 \partial_\mu K^0_0 + K^{*+}_0 \partial_\mu K^+_0 + K^{*-}_0 \partial_\mu K^-_0)$$

$$+ i \frac{g_1}{2} (\phi_N - \sqrt{2} \phi_S) (K^{*0}_1 \partial_\mu K^0_1 + K^{*+}_1 \partial_\mu K^+_1 + K^{*-}_1 \partial_\mu K^-_1).$$  \hspace{1cm} (A2)

In addition, for charmed mesons similar mixing terms of the type $\eta_{Cf1C1}, D宋-D_s S_1, D_s S_0 - D_s S_1, D_0^* - D^*$, and $D-D_1$ are present:

$$- g_1 \phi_C \mu \partial_\mu \eta_C - \frac{g_1}{\sqrt{2}} g_1 \phi_S (D_{S1}^{\mu-} \partial_\mu D_{S1}^+ + D_{S1}^{\mu+} \partial_\mu D_{S1}^-)$$

$$+ i \frac{g_1}{\sqrt{2}} \phi_S (D_{S0}^{\mu-} \partial_\mu D_{S0}^+ - D_{S0}^{\mu+} \partial_\mu D_{S0}^-)$$

$$+ i \frac{g_1}{2} \phi_N (D^{\mu-} \partial_\mu D^{\mu+} - D^{\mu+} \partial_\mu D^{\mu-} + D^{\mu0} \partial_\mu D^{\mu0} - D^{\mu0} \partial_\mu D^{\mu0})$$

$$- \frac{g_1}{2} \phi_N (D_1^{\mu0} \partial_\mu D_0^0 + D_1^{\mu0} \partial_\mu D_0^0 + D_1^{\mu+} \partial_\mu D^{\mu-} + D_1^{\mu-} \partial_\mu D^{\mu+}) .$$  \hspace{1cm} (A3)

These mixing terms are removed by performing the following field transformations:

$$f_{1N,S}^\mu \to f_{1N,S}^\mu + w_{f1N,S} Z_{\eta_{N,S}} \partial^\mu \eta_{N,S} ,$$  \hspace{1cm} (A4)

$$\vec{d}_1^\mu \to \vec{d}_1^\mu + w_{1} Z_{\phi} \partial^\mu \phi ,$$  \hspace{1cm} (A5)

$$K^{*+0} \to K^{*+0} + w_{K^*} Z_{K^*_0} \partial^\mu K^{*0}_0 ,$$  \hspace{1cm} (A6)

$$K^{*-0} \to K^{*-0} + w_{K^*} Z_{K^*_0} \partial^\mu K^{*-0}_0 ,$$  \hspace{1cm} (A7)

$$K^{0\pm,0} \to K^{0\pm,0} + w_{K} Z_{K} \partial^\mu K^{0\pm,0} ,$$  \hspace{1cm} (A8)

$$\chi_{C1} \to \chi_{C1} + w_{\chi_{C1}} Z_{\phi_C} \partial^\mu \phi_C ,$$  \hspace{1cm} (A9)

$$D_{S1}^{\mu-} \to D_{S1}^{\mu-} + w_{D_{S1}} Z_{D_{S1}} \partial^\mu D_{S1}^+ ,$$  \hspace{1cm} (A10)

$$D_{S1}^{\mu+} \to D_{S1}^{\mu+} + w_{D_{S1}} Z_{D_{S1}} \partial^\mu D_{S1}^- ,$$  \hspace{1cm} (A11)

$$D_{S0}^{\mu+} \to D_{S0}^{\mu+} + w_{D_{S0}} Z_{D_{S0}} \partial^\mu D_{S0}^+ ,$$  \hspace{1cm} (A12)

$$D_{S0}^{\mu-} \to D_{S0}^{\mu-} + w_{D_{S0}} Z_{D_{S0}} \partial^\mu D_{S0}^- ,$$  \hspace{1cm} (A13)

$$D_{S0}^{\mu0} \to D_{S0}^{\mu0} + w_{D_{S0}} Z_{D_{S0}} \partial^\mu D_{S0}^0 ,$$  \hspace{1cm} (A14)

$$D_0^{*0} \to D_0^{*0} + w_{D_0} Z_{D_0} \partial^\mu D_0^0 ,$$  \hspace{1cm} (A15)

$$D_{1}^{\mu\pm,0} \to D_{1}^{\mu\pm,0} + w_{D_1} Z_{D_1} \partial^\mu D_1^{\pm,0} .$$  \hspace{1cm} (A16)
The quantities appearing in the previous expressions are:

\[ w_{f_{1N}} = w_{a_{1}} = \frac{g_{1}\phi_{N}}{m_{a_{1}}^{2}}, \quad w_{f_{1S}} = \frac{\sqrt{2}g_{1}\phi_{S}}{m_{f_{1S}}^{2}}, \quad (A18) \]
\[ w_{K^{*}} = \frac{ig_{1}(\phi_{N} - \sqrt{2}\phi_{S})}{2m_{K}^{2}}, \quad w_{K_{1}} = \frac{g_{1}(\phi_{N} + \sqrt{2}\phi_{S})}{2m_{K_{1}}^{2}}, \quad (A19) \]
\[ w_{\chi_{c1}} = \frac{\sqrt{2}g_{1}\phi_{C}}{m_{\chi_{c1}}^{2}}, \quad w_{D_{S1}} = \frac{g_{1}(\phi_{S} + \phi_{C})}{\sqrt{2}m_{D_{S1}}^{2}}, \quad (A20) \]
\[ w_{D_{S}}^{*} = \frac{ig_{1}(\phi_{N} - \sqrt{2}\phi_{C})}{2m_{D_{S}}^{2}}, \quad w_{D_{1}}^{*} = \frac{g_{1}(\phi_{N} + \sqrt{2}\phi_{C})}{2m_{D_{1}}^{2}}, \quad (A21) \]
\[ w_{D_{o^{*}}} = \frac{ig_{1}(\phi_{N} - \sqrt{2}\phi_{C})}{2m_{D_{o^{*}}}}. \quad (A22) \]

Moreover, one has to rescale the (pseudo)scalar fields as:

\[ \pi^{\pm,0} \rightarrow Z_{\pi}\pi^{\pm,0}, \]
\[ \eta_{N/S/C} \rightarrow Z_{\eta_{N/S/C}}\eta_{N/S/C}, \]
\[ D^{\pm,0,\bar{0}} \rightarrow Z_{D}D^{\pm,0,\bar{0}}, \]
\[ D_{0}^{\pm,0,\bar{0}} \rightarrow Z_{D_{0}}D_{0}^{\pm,0,\bar{0}}, \]

where the wave-function renormalization constants read:

\[ Z_{\pi} \equiv Z_{\eta_{N}} = \frac{m_{a_{1}}}{\sqrt{m_{a_{1}}^{2} - g_{1}^{2}\phi_{N}^{2}}}, \]
\[ Z_{K} = \frac{2m_{K_{1}}}{\sqrt{4m_{K_{1}}^{2} - g_{1}^{2}(\phi_{N} + \sqrt{2}\phi_{S})^{2}}}, \]
\[ Z_{\eta_{C}} = \frac{m_{\chi_{c1}}}{\sqrt{m_{\chi_{c1}}^{2} - 2g_{1}^{2}\phi_{C}^{2}}}, \]
\[ Z_{D_{S0}} = \frac{\sqrt{2}m_{D_{S1}}^{2}}{\sqrt{2m_{D_{S1}}^{2} - g_{1}^{2}(\phi_{S} - \phi_{C})^{2}}}, \]
\[ Z_{D_{o0}} = \frac{2m_{D_{o}}^{2}}{\sqrt{2m_{D_{o}}^{2} - g_{1}^{2}(\phi_{N} - \sqrt{2}\phi_{C})^{2}}}, \]
\[ Z_{D^{0}} = \frac{2m_{D_{1}}^{2}}{\sqrt{2m_{D_{1}}^{2} - g_{1}^{2}(\phi_{N} + \sqrt{2}\phi_{C})^{2}}}. \quad (A29) \]

The nonstrange, strange, and charm condensates read:

\[ \phi_{N} = Z_{\pi}\phi_{N}, \quad (A30) \]
\[ \phi_{S} = \frac{2Z_{K}f_{K} - \phi_{N}}{\sqrt{2}}, \quad (A31) \]
\[ \phi_{C} = \frac{2Z_{D}f_{D} - \phi_{N}}{\sqrt{2}} = \sqrt{2}Z_{D}f_{D} - \phi_{S} = \frac{Z_{\eta_{C}}f_{\eta_{C}}}{\sqrt{2}}, \quad (A32) \]

The quantities \( f_{\pi} = 92.4 \text{ MeV} \) and \( f_{K} = 155/\sqrt{2} \text{ MeV} \) are the pion and kaon decay constants, while \( f_{D} \) and \( f_{D_{s}} \) are the decay constants of the pseudoscalar \( D \) and \( D_{s} \) mesons, respectively.
2. Tree-level masses

After having performed the transformation above, we obtain the tree-level masses of nonstrange-strange mesons in the eLSM:

(i) Pseudoscalar mesons:

\[
m^2_{\pi^0} = Z^2_{\pi^0} \left[ m_0^2 + \left( \lambda_1 + \frac{\lambda_2}{2} \right) \phi_N^2 + \lambda_1 \phi_S^2 + \lambda_1 \phi_C^2 \right],
\]
\[
m^2_K = Z^2_K \left[ m_0^2 + \left( \lambda_1 + \frac{\lambda_2}{2} \right) \phi_N^2 - \frac{\lambda_2}{\sqrt{2}} \phi_N \phi_S + (\lambda_1 + \lambda_2) \phi_S^2 + \lambda_1 \phi_C^2 \right],
\]
\[
m^2_{\eta N} = Z^2_{\eta N} \left[ m_0^2 + \left( \lambda_1 + \frac{\lambda_2}{2} \right) \phi_N^2 + \lambda_1 \phi_S^2 + \lambda_1 \phi_C^2 + \frac{c}{2} \phi_N^2 \phi_S^2 \phi_C^2 \right],
\]
\[
m^2_{\eta S} = Z^2_{\eta S} \left[ m_0^2 + \lambda_1 \phi_N^2 + (\lambda_1 + \lambda_2) \phi_S^2 + \lambda_1 \phi_C^2 + \frac{c}{8} \phi_N^4 \phi_C^2 \right].
\]

(ii) Scalar mesons:

\[
m^2_{\omega_0} = m_0^2 + \left( \lambda_1 + \frac{3}{2} \lambda_2 \right) \phi_N^2 + \lambda_1 \phi_S^2 + \lambda_1 \phi_C^2 ,
\]
\[
m^2_{K^0} = Z^2_{K^0} \left[ m_0^2 + \left( \lambda_1 + \frac{\lambda_2}{2} \right) \phi_N^2 + \frac{\lambda_2}{\sqrt{2}} \phi_N \phi_S + (\lambda_1 + \lambda_2) \phi_S^2 + \lambda_1 \phi_C^2 \right],
\]
\[
m^2_{\sigma N} = m_0^2 + 3 \left( \lambda_1 + \frac{\lambda_2}{2} \right) \phi_N^2 + \lambda_1 \phi_S^2 + \lambda_1 \phi_C^2 ,
\]
\[
m^2_{\sigma S} = m_0^2 + \lambda_1 \phi_N^2 + 3(\lambda_1 + \lambda_2) \phi_S^2 + \lambda_1 \phi_C^2 .
\]

(iii) Vector mesons:

\[
m^2_p = m^2_{\omega N},
\]
\[
m^2_{\omega N} = m_1^2 + 2 \delta_N + \frac{\phi_N^2}{2} \left( h_1 + h_2 + h_3 \right) + \frac{h_1}{2} \phi_S^2 + \frac{h_1}{2} \phi_C^2,
\]
\[
m^2_{\omega S} = m_1^2 + 2 \delta_S + \frac{h_1}{2} \phi_N^2 + \left( \frac{h_1}{2} + h_2 + h_3 \right) \phi_S^2 + \frac{h_1}{2} \phi_C^2,
\]
\[
m^2_{K^*} = m_1^2 + \delta_N + \delta_S + \frac{\phi_N^2}{2} \left( g_1^2 + h_1 + \frac{h_2}{2} \right) + \frac{1}{\sqrt{2}} \phi_N \phi_S (h_3 - g_1^2) + \frac{\phi_S^2}{2} (g_1^2 + h_1 + h_2) + \frac{h_1}{2} \phi_C^2.
\]

(iv) Axial-vector mesons:

\[
m^2_{J_N} = m^2_{\eta N},
\]
\[
m^2_{J_1} = m_1^2 + 2 \delta_N + g_1^2 \phi_N^2 + \frac{\phi_N^2}{2} \left( h_1 + h_2 - h_3 \right) + \frac{h_1}{2} \phi_S^2 + \frac{h_1}{2} \phi_C^2,
\]
\[
m^2_{J_{1S}} = m_1^2 + 2 \delta_S + \frac{h_1}{2} \phi_N^2 + \frac{h_1}{2} \phi_C^2 + 2 g_1^2 \phi_S^2 + \phi_S^2 \left( \frac{h_1}{2} + h_2 - h_3 \right),
\]
\[
m^2_{K_{1N}} = m_1^2 + \delta_N + \delta_S + \frac{\phi_N^2}{2} \left( g_1^2 + h_1 + \frac{h_2}{2} \right) + \frac{1}{\sqrt{2}} \phi_N \phi_S (g_1^2 - h_3) + \frac{\phi_S^2}{2} (g_1^2 + h_1 + h_2) + \frac{h_1}{2} \phi_C^2,
\]

All previous expressions coincide with Ref. [17]. The masses of (open and hidden) charmed mesons are as follows.

(i) Pseudoscalar charmed mesons:

\[
m^2_{\eta C} = Z^2_{\eta C} \left[ m_0^2 + \lambda_1 \phi_N^2 + \lambda_1 \phi_S^2 + (\lambda_1 + \lambda_2) \phi_C^2 + \frac{c}{8} \phi_N^4 \phi_S^2 + 2 \varepsilon_C \right],
\]
\[
m^2_D = Z^2_D \left[ m_0^2 + \left( \lambda_1 + \frac{\lambda_2}{2} \right) \phi_N^2 + \lambda_1 \phi_S^2 - \frac{\lambda_2}{\sqrt{2}} \phi_N \phi_C + (\lambda_1 + \lambda_2) \phi_C^2 + \varepsilon_C \right],
\]
\[
m^2_{D_S} = Z^2_{D_S} \left[ m_0^2 + \lambda_1 \phi_N^2 + (\lambda_1 + \lambda_2) \phi_S^2 - \lambda_2 \phi_C \phi_S + (\lambda_1 + \lambda_2) \phi_C^2 + \varepsilon_C \right].
\]
(ii) Scalar charm mesons:

\[ m_{N_{C0}}^2 = m_0^2 + \lambda_1 \phi_N^2 + \lambda_1 \phi_S^2 + 3(\lambda_1 + \lambda_2)\phi_C^2 + 2\varepsilon C, \quad (A52) \]

\[ m_{D_0}^2 = Z_{D_0}^2 \left[ m_0^2 + \left( \frac{\lambda_1 + \lambda_2}{2} \right) \phi_N^2 + \lambda_1 \phi_S^2 + \frac{\lambda_2}{\sqrt{2}} \phi_C \phi_N + (\lambda_1 + \lambda_2)\phi_C^2 + \varepsilon C \right], \quad (A53) \]

\[ m_{D_0^o}^2 = Z_{D_0^o}^2 \left[ m_0^2 + \left( \frac{\lambda_1 + \lambda_2}{2} \right) \phi_N^2 + \lambda_1 \phi_S^2 + \frac{\lambda_2}{\sqrt{2}} \phi_C \phi_N + (\lambda_1 + \lambda_2)\phi_C^2 + \varepsilon C \right], \quad (A54) \]

\[ m_{D_{s0}}^2 = Z_{D_{s0}}^2 \left[ m_0^2 + \lambda_1 \phi_N^2 + (\lambda_1 + \lambda_2)\phi_S^2 + \lambda_2 \phi_C \phi_S + (\lambda_1 + \lambda_2)\phi_C^2 + \varepsilon C \right]. \quad (A55) \]

(iii) Vector charm mesons:

\[ m_{D^*}^2 = m_1^2 + \delta_N + \delta_C + \frac{\delta^2}{2} \left( \frac{g_1^2}{2} + h_1 + h_2 \right) + \frac{1}{\sqrt{2}} \phi_N \phi_C (h_3 - g_1^2) + \frac{\phi_C^2}{2} (g_1^2 + h_1 + h_2) + \frac{h_1}{2} \phi_N^2, \quad (A56) \]

\[ m_{J/\psi}^2 = m_1^2 + 2\delta_C + \frac{h_1}{2} \phi_N^2 + \frac{h_1}{2} \phi_S^2 + \left( \frac{h_1}{2} + h_2 + h_3 \right) \phi_C^2, \quad (A57) \]

\[ m_{D_{s*}}^2 = m_1^2 + \delta_S + \delta_C + \frac{\delta^2}{2} \left( g_1^2 + h_1 + h_2 \right) + \phi_S \phi_C (h_3 - g_1^2) + \frac{\phi_C^2}{2} (g_1^2 + h_1 + h_2) + \frac{h_1}{2} \phi_N^2. \quad (A58) \]

(iv) Axial-vector charm mesons:

\[ m_{D_{s1}}^2 = m_1^2 + \delta_S + \delta_C + \frac{\delta^2}{2} \left( g_1^2 + h_1 + h_2 \right) + \phi_S \phi_C (g_1^2 - h_3) + \frac{\phi_C^2}{2} (g_1^2 + h_1 + h_2) + \frac{h_1}{2} \phi_N^2, \quad (A59) \]

\[ m_{D_{s1}}^2 = m_1^2 + \delta_N + \delta_C + \frac{\delta^2}{2} \left( \frac{g_1^2}{2} + h_1 + h_2 \right) + \frac{1}{\sqrt{2}} \phi_N \phi_C (g_1^2 - h_3) + \frac{\phi_C^2}{2} (g_1^2 + h_1 + h_2) + \frac{h_1}{2} \phi_N^2, \quad (A60) \]

\[ m_{D_{s1}}^2 = m_1^2 + 2\delta_C + \frac{h_1}{2} \phi_N^2 + \frac{h_1}{2} \phi_S^2 + 2g_1 \phi_C^2 + \phi_C \left( \frac{h_1}{2} + h_2 - h_3 \right). \quad (A61) \]

Appendix B: Decay widths

1. General formula

The decay width of a particle \(A\) decaying into particles \(B\) and \(C\) has the general expression

\[
\Gamma_{A \to BC} = \frac{S_{A \to BC} K(m_A, m_B, m_C)}{8\pi m_A^2} |M_{A \to BC}|^2, \quad (B1)
\]

where \(K(m_A, m_B, m_C)\) is the modulus of the three-momentum of one of the outgoing particles:

\[
K(m_A, m_B, m_C) = \frac{1}{2m_A} \sqrt{m_A^4 + (m_B^2 - m_C^2)^2 - 2m_A^2 (m_B^2 + m_C^2) \theta(m_A - m_B - m_C)}, \quad (B2)
\]

and where \(M_{A \to BC}\) is the corresponding decay amplitude. The quantity \(S_{A \to BC}\) refers to a symmetrization factor (it equals 1 if \(B\) and \(C\) are different and 2 if they are identical).

The decay width of \(A\) into three particles \(B_1, B_2,\) and \(B_3\) takes the general expression \([17]\)

\[
\Gamma_{A \to B_1 B_2 B_3} = \frac{S_{A \to B_1 B_2 B_3}}{32(2\pi)^3 m_A^3} \int_{(m_1 + m_2)^2}^{(m_A - m_3)^2} \int_{(m_2)_{\text{min}}}^{(m_2)_{\text{max}}} \int_{(m_1)_{\text{min}}}^{(m_1)_{\text{max}}} | - iM_{A \to B_1 B_2 B_3} |^2 \, dm_{23}^2 \, dm_{12}^2, \quad (B3)
\]

where

\[
(m_{23})_{\text{min}} = (E_2^* + E_3^*)^2 - \left( \sqrt{E_2^{*2} - m_2^2} + \sqrt{E_3^{*2} - m_3^2} \right)^2, \quad (B4)
\]

\[
(m_{23})_{\text{max}} = (E_2^* + E_3^*)^2 - \left( \sqrt{E_2^{*2} - m_2^2} - \sqrt{E_3^{*2} - m_3^2} \right)^2, \quad (B5)
\]
The decay width for the charged scalar state $D^+_S$ is
\[
\Gamma_{D_S^+ \rightarrow \phi \pi^-} = \frac{1}{8\pi m_S} \left[ \frac{(m_S^2 - m_{\pi^+}^2 - m_{\pi^-}^2)^2 - 4m_{\pi^+}^2m_{\pi^-}^2}{4m_S^2} \right]^{1/2} \left[ A_{D_S^0 D_S^+} + (C_{D_S^0 D_S^+} + E_{D_S^0 D_S^+}) \right]
\]
where $A_{D_S^0 D_S^+}$, $B_{D_S^0 D_S^+}$, $C_{D_S^0 D_S^+}$, and $E_{D_S^0 D_S^+}$ are all different, it equals 2 for two identical particles in the final state, and it equals 6 for three identical particles in the final state).

2. Decay widths of charmed scalar mesons

In the scalar sector we report the expressions for the decay widths of $D_0^{*0}$, $D_0^{*+}$, and $D_0^{*0}$. The scalar state $D_0^{*0}$ decays into $D^0\pi^0$ and $D^+\pi^-$. The explicit expression for the process $D_0^{*0} \rightarrow D^0\pi^0$ reads
\[
\Gamma_{D_0^{*0} \rightarrow D^0\pi^0} = \frac{1}{8\pi m_S} \left[ \frac{(m_S^2 - m_{\pi^0}^2 - m_{\pi^0}^2)^2 - 4m_{\pi^0}^2m_{\pi^0}^2}{4m_S^2} \right]^{1/2} \left[ A_{D_0^{*0} D^0} + (C_{D_0^{*0} D^0} + E_{D_0^{*0} D^0}) \right]
\]
where $S$, $P_1$, and $P_2$ refer to the scalar meson $D_0^{*0}$ and to the pseudoscalar mesons $D^0$ and $\pi^0$ and where

\begin{align}
A_{D_0^{*0} D^0} &= -\frac{Z_D Z_D D_0^{*0}}{\sqrt{2}} \lambda_2 \phi_C, \\
B_{D_0^{*0} D^0} &= \frac{Z_D Z_D D_0^{*0}}{4} \left[ g_1^2(3\phi_N + \sqrt{2}\phi_C) - 2g_1 \frac{w_{a_1} + w_{D^0}}{w_{a_1} w_{D^0}} + h_2(\phi_N + \sqrt{2}\phi_C) - 2h_3\phi_N \right], \\
C_{D_0^{*0} D^0} &= \frac{Z_D Z_D D_0^{*0}}{2} \left[ \sqrt{2}g_1^2 \phi_C - g_1 \frac{w_{D^0} + iw_{D^0}}{w_{D^0} w_{D^0}} - \sqrt{2}ih_3\phi_C \right], \\
E_{D_0^{*0} D^0} &= \frac{Z_D Z_D D_0^{*0}}{4} \left[ ig_1^2(3\phi_N - \sqrt{2}\phi_C) + 2g_1 \left( \frac{w_{a_1} - iw_{D^0}}{w_{D^0} w_{a_1}} \right) + ih_2(\phi_N - \sqrt{2}\phi_C) - 2ih_3\phi_N \right].
\end{align}

The decay width for $D_0^{*0} \rightarrow D^+\pi^-$ has the same expression but is multiplied by an isospin factor 2. The positively charged scalar state $D_0^{*+}$ decays into $D^+\pi^0$ and $D^0\pi^+$. The decay width $D_0^{*+} \rightarrow D^+\pi^0$ has the same form as Eq. (B7) upon identifying $S$, $P_1$, and $P_2$ with $D_0^{*+}$, $D^+$, and $\pi^0$. A similar expression holds for the decay width for $D_0^{*+} \rightarrow D^0\pi^+$, where an overall isospin factor 2 is also present.

Finally we turn to the scalar state $D_0^{*0}$ which decays into $D^+K^0$ and $D^0K^+$. The decay width $D_0^{*+} \rightarrow D^+K^0$ is obtained from Eq. (B7) upon identifying $S$, $P_1$, and $P_2$ with $D_0^{*+}$, $D^+$, and $K^0$ and upon replacing $A_{D_0^{*0} D^0}$, $B_{D_0^{*0} D^0}$, $C_{D_0^{*0} D^0}$, $E_{D_0^{*0} D^0}$, $P_1$, and $P_2$ with $D_0^{*0}$, $D^+$, and $K^0$ and where:

\begin{align}
A_{D_0^{*0} D^0 K^0} &= \sqrt{2} \lambda_2 \left[ \phi_N + \sqrt{2}(\phi_S - \phi_C) \right], \\
B_{D_0^{*0} D^0 K^0} &= \frac{Z_D Z_D D_0^{*0}}{2} \left[ -\sqrt{2}g_1 \frac{w_{K^0} + w_{D^0}}{w_{K^0} w_{D^0}} + \sqrt{2}(g_1^2 - h_3)\phi_N + (g_1^2 + h_2)(\phi_S + \phi_C) \right], \\
C_{D_0^{*0} D^0 K^0} &= \frac{Z_D Z_D D_0^{*0}}{2} \left[ \sqrt{2}g_1 \frac{w_{D^0} + i w_{D^0}}{w_{D^0} w_{D^0}} - i \left( g_1^2 + h_2 \right) \phi_N + i (g_1^2 + h_2) \phi_S + 2i(h_3 - g_1^2)\phi_C \right], \\
E_{D_0^{*0} D^0 K^0} &= \frac{Z_D Z_D D_0^{*0}}{2} \left[ \sqrt{2}g_1 \frac{w_{K^0} - i w_{D^0}}{w_{K^0} w_{D^0}} + i \left( g_1^2 + h_2 \right) \phi_N + 2i(g_1^2 - h_3) \phi_S - i(g_1^2 + h_2)\phi_C \right].
\end{align}

The decay of the scalar state $D_0^{*0}$ into $D^0K^+$ has an analogous analytic expression.
3. Decay widths of charmed vector mesons

The neutral state $D^{*0}$ decays into $D^{0}\pi^0$. The corresponding expression is:

$$
\Gamma_{V\to P_1 P_2} = \frac{1}{24\pi} \left[ \frac{(m^2_{V} - m^2_{P_1} - m^2_{P_2})^2 - 4m^2_{P_1}m^2_{P_2}}{4m^4_{V}} \right]^{3/2} (A_{D^* D\pi} - B_{D^* D\pi} + C_{D^* D\pi} m^2_{V})^2, 
$$

(B16)

where

$$
A_{D^* D\pi} = \frac{i}{2} Z_\pi Z_D \left[ g_1 + \sqrt{2}w_{D_1}(h_3 - g^2_1)\phi_C \right],
$$

(B17)

$$
B_{D^* D\pi} = -\frac{i}{2} Z_\pi Z_D \left[ 2g_1 - w_{a_1}(3g^2_1 + h_2 - 2h_3)\phi_N + \sqrt{2}w_{a_1}(g^2_1 + h_2)\phi_C \right],
$$

(B18)

$$
C_{D^* D\pi} = \frac{i}{2} Z_\pi Z_D w_{a_1} w_{D_1} g_2.
$$

(B19)

and where $V, P_1, and P_2$ refer to $D^{*0}, D^0, and \pi^0$.

For the decay $D^{*+} \to D^+ \pi^0$, Eq. (B16) holds upon multiplication by an isospin factor 2.

4. Decay widths of charmed axial-vector mesons

The neutral nonstrange axial-vector meson $D_1^0$ decays into $D^{*0}\pi^0, D^{*+}\pi^-, D^{0}\pi^0\pi^0, D^{0}\pi^+\pi^-, and D^{-}\pi^+\pi^0$. The decay width for $D_1^0 \to D^{*0}\pi^0$ is

$$
\Gamma_{A\to VP} = \frac{K(m_A,m_V,m_P)}{12\pi m^2_A} \left[ \frac{|h^{\mu\nu}_{D_1,D^*\pi}|^2}{m^2_A} - \frac{|h^{\mu\nu}_{D_1,D^*\pi} k_A,\mu|^2}{m_V^2} - \frac{|h^{\mu\nu}_{D_1,D^*\pi} k_{V,\nu}|^2}{m^2_V} + \frac{|h^{\mu\nu}_{D_1,D^*\pi} k_A,\mu k_{V,\nu}|^2}{m_A^2 m^2_V} \right],
$$

(B20)

where $A, V, and P$ refer to the mesons $D_1^0, D^{*0}, \pi^0$ and $h^{\mu\nu}_{D_1,D^*\pi}$ is

$$
h^{\mu\nu}_{D_1,D^*\pi} = i \left\{ A_{D_1,D^*\pi} g^{\mu\nu} + B_{D_1,D^*\pi} [k^\mu_A k^\nu_P + k^\nu_A k^\mu_P - (k_A \cdot k_P) g^{\mu\nu} - (k_V \cdot k_P) g^{\mu\nu}] \right\},
$$

(B21)

with

$$
A_{D_1,D^*\pi} = \frac{i}{\sqrt{2}} Z_\pi (g^2_1 - h_3)\phi_C,
$$

(B22)

$$
B_{D_1,D^*\pi} = \frac{i}{2} Z_\pi g_2 w_{a_1}.
$$

(B23)

The quantities $k^\mu_A = (m_A,0), k^\mu_V = (E_V,k), and k^\mu_P = (E_P,-k)$ are the four-momenta of $D_1^0, D^{*0}, and \pi^0$ in the rest frame of $D_1^0$, respectively. The following kinematic relations hold:

$$
k_V \cdot k_P = \frac{m^2_A - m^2_V - m^2_P}{2},
$$

$$
k_A \cdot k_V = m_A E_V = \frac{m^2_A + m_V - m^2_P}{2},
$$

$$
k_A \cdot k_P = m_A E_P = \frac{m^2_A - m_V + m^2_P}{2}.
$$

The terms entering in Eq. (B20) are given by

$$
|h^{\mu\nu}_{D_1,D^*\pi}|^2 = 4A^2_{D_1,D^*\pi} + B^2_{D_1,D^*\pi} \left[ m^2_V m^2_P + m^2_A m^2_P + 2(k_V \cdot k_P)^2 + 2(k_A \cdot k_P)^2 + 6(k_V \cdot k_P)(k_A \cdot k_P) \right]
$$

- $6A_{D_1,D^*\pi} B_{D_1,D^*\pi} (k_V \cdot k_P + k_A \cdot k_P),
$$

(B24)

$$
|h^{\mu\nu}_{D_1,D^*\pi} k_A,\mu|^2 = 4A^2_{D_1,D^*\pi} m^2_A + B^2_{D_1,D^*\pi} \left[ ([k_A \cdot k_V]^2 m^2_P + (k_V \cdot k_P)^2 m^2_A - 2(k_A \cdot k_V)(k_A \cdot k_P)(k_V \cdot k_P) \right]
$$

+ $2A_{D_1,D^*\pi} B_{D_1,D^*\pi} ([k_A \cdot k_V](k_A \cdot k_P) - (k_V \cdot k_P)m^2_A),
$$

(B25)
\[ |h^{\mu}_{D_{1}D_{\pi}} k_{V,\nu}|^2 = A^2_{D_1D_{\pi}} m_{V}^2 + B^2_{D_1D_{\pi}} [(k_A \cdot k_V)^2 m_{V}^2 + (k_A \cdot k_P)^2 m_{P}^2 - 2(k_A \cdot k_V)(k_A \cdot k_P)(k_V \cdot k_P)] + 2 A_{D_1D_{\pi}} B_{D_1D_{\pi}} [(k_A \cdot k_V)(k_V \cdot k_P) - (k_A \cdot k_P)m_{V}^2], \]  
(B26)

\[ |h^{\mu}_{D_1 k_{A,\nu}} k_{V,\nu}|^2 = |A_{D_1}(k_A \cdot k_V)|^2 \]

with \( E_V = \sqrt{K^2(m_A, m_V, m_P) + m_V^2} \) and \( E_P = \sqrt{K^2(m_A, m_V, m_P) + m_P^2} \).

The decay width for \( D_1^0 \to D^{*+} \pi^- \) is still given by Eq. (B20) upon substituting the fields and upon multiplication by an isospin factor 2.

Now we are turning to the three body-decay of the axial-vector meson \( D_1^0 \), which decays into three pseudoscalar mesons \((D\pi\pi)\). First, the decay width for \( D_1^0 \to D^{0}_0 \pi^0 \pi^0 \) can be obtained as

\[ \Gamma_{A \to P_1P_2P_3} = \frac{2}{96(2\pi)^3 M^2} \int_{(m_1+m_2)^2}^{(M-m_3)^2} \int_{(m_3)^{\text{max}}}^{(m_23)^{\text{max}}} \left[ -|h^{\mu}_{D_1D\pi}|^2 + |h^{\mu}_{D_1D\pi} k_{\mu}|^2 \right] dm_{23}^2 dm_{12}^2 , \]  
(B27)

where \( h^{\mu}_{D_1D\pi} \) is the vertex following from the relevant part of the Lagrangian

\[ h^{\mu}_{D_1D\pi} = -[A_{D_1D\pi} g^\mu k_1^\mu + B_{D_1D\pi} k_2^\mu] , \]  
(B28)

with the coefficients

\[ A_{D_1D\pi} = \frac{1}{4} Z^2 \ Z_D \ w_{D_1}(g_1^2 + 2h_1 + h_2), \]  
(B29)

\[ B_{D_1D\pi} = \frac{1}{4} Z^2 \ Z_D \ w_{D_1}(3g_1^2 + 2h_1 - 2h_3), \]  
(B30)

and \( k^\mu = (M, 0), k_1^\mu = (E_{P_1}, k), k_2^\mu = (E_{P_2}, -k) \) are the four-momenta of \( D_1^0, D^0, \pi^0, \) and \( \pi^0 \) in the rest frame of \( D_1^0 \), respectively. Using the following kinematic relations

\[ k_1 \cdot k_2 = \frac{m_{12}^2 - m_1^2 - m_2^2}{2}, \]
\[ k \cdot k_1 = m_1^2 + \frac{m_{12}^2 - m_1^2 - m_2^2}{2} + \frac{m_{13}^2 - m_1^2 - m_3^2}{2}, \]
\[ k \cdot k_2 = m_2^2 + \frac{m_{12}^2 - m_1^2 - m_2^2}{2} + \frac{m_{23}^2 - m_2^2 - m_3^2}{2}, \]

where the quantities \( M, m_1, m_2, m_3 \) are the masses of \( D_1^0, D^0, \pi^0, \pi^0 \), respectively. The decay of the scalar state \( D_1^+ \to D^{+} \pi^0 \pi^0 \) has an analogous analytic expression. The decay of the scalar states \( D_1^0 \to D^{0}_{0} \pi^0 \pi^0 \) and \( D_1^+ \to D^{0}_{0} \pi^0 \pi^- \) have the same formula as presented in Eq. (B27) but multiplied by an isospin factor 2 as given by the Lagrangian (6).

The decay width \( D_1^0 \to D^+ \pi^- \pi^0 \) reads

\[ \Gamma_{A \to P_1P_2P_3} = \frac{F^2_{D_1D\pi}}{96(2\pi)^3 M^2} \int_{(m_1+m_2)^2}^{(M-m_3)^2} \int_{(m_3)^{\text{max}}}^{(m_23)^{\text{max}}} \left[ \frac{k_1 \cdot k_2 - k \cdot k_3}{M^2} - (m_3^2 + m_2^2 + 2k_2 \cdot k_3) \right] dm_{23}^2 dm_{12}^2 , \]  
(B31)

where the quantities \( A, P_1, P_2, \) and \( P_3 \) refer to the fields \( D_1^0, D^+, \pi^-, \) and \( \pi^0 \), respectively. Moreover:

\[ F_{D_1D\pi} = \frac{\sqrt{2}}{4} Z^2 \ Z_D \ w_{A}(g_1^2 - h_2 - 2h_3), \]  
(B32)

and

\[ k_2 \cdot k_3 = \frac{m_{23}^2 - m_2^2 - m_3^2}{2}, \]
\[ k \cdot k_3 = m_3^2 + \frac{m_{13}^2 - m_1^2 - m_3^2}{2} + \frac{m_{23}^2 - m_2^2 - m_3^2}{2}. \]
The charged state $D^*_K$ decays into $D^{*+}\pi^0$, $D^{*0}\pi^+$, $D^{+}\pi^0\pi^0$, $D^{+}\pi^+\pi^-$, and $D^{0}\pi^0\pi^+$. Analogous expressions hold.

As a last step we turn to the strange-charmed axial-vector state $D^{**}$. It decays predominantly into the channels $D^{*+}K^0$ and $D^{*0}K^+$. The formula for the decay width $\Gamma_{D^{**}\rightarrow D^{*+}K^0}$ and $\Gamma_{D^{**}\rightarrow D^{*0}K^+}$ is as in Eq. (B20) upon replacing the fields:

$$h_{D^{**}\rightarrow D^*K}^{\mu\nu} = i \left\{ A_{D^{**}D^*K} g^{\mu\nu} + B_{D^{**}D^*K} \right\}$$

where

$$A_{D^{**}D^*K} = \frac{i}{4} Z_K \left[ g_1^2 (2\phi_N - 2\phi_S - 4\phi_C) + h_2 (2\phi_N - 2\phi_S) + 4h_3 \phi_C \right],$$

$$B_{D^{**}D^*K} = - \frac{i}{\sqrt{2}} Z_K g_2 w_{K1},$$

where $g_1, g_2, g_3$ are the coupling constants.

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