An Inverted Neutrino Mass Hierarchy for Hot Dark Matter 
and the Solar Neutrino Deficit

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ABSTRACT

The cosmological model in which 20\% of the dark matter is shared by
two nearly equal mass neutrinos fits the structure of the universe on all
scales. This has been motivated by a \( \nu_\mu \rightarrow \nu_\tau \) oscillation explanation of
the deficit of atmospheric \( \nu_\mu \)'s. If the observed atmospheric \( \frac{\nu_\mu}{\nu_e} \) ratio has
an alternative explanation, the cosmological model can be retained if the
deficit of solar neutrinos is explained by \( \nu_e \rightarrow \nu_\tau \) oscillation. In this case, an
inverted neutrino mass hierarchy is required with \( m_{\nu_\mu} \ll m_{\nu_e} \approx m_{\nu_\tau} \approx 2.4 \)
eV. We show that if there exists an \( L_e - L_\tau \) symmetry in nature, then
both the near mass degeneracy of \( \nu_e \) and \( \nu_\tau \), as well as the consistency of
the above mass values for neutrino masses with the negative results of the
neutrinoless double beta decay search experiments are easily understood.
We show that this symmetry implemented in the context of a high-scale
left-right symmetric model with the see-saw mechanism can lead to a simple
theoretical understanding of the desired form of the mass matrix.

∗ Work supported by the Department of Energy grant No. DE-EG03-91ER40618
† Work supported by the National Science Foundation grant No.PHY9421385
1. Introduction

The standard electroweak model of Glashow, Weinberg and Salam predicts that the neutrinos are massless. There are, however, strong indications from solar neutrino data that the only way to reconcile all four experimental results[1] with the calculations of the standard solar model[2] is to assume that the neutrinos have mass and they mix among themselves. There are also indications of a nonzero neutrino mass from attempts to understand the observed large scale structure in the universe. The best fit of any available model to the structure on all scales, such as the anisotropy of cosmic microwave background, galaxy-galaxy angular correlations, velocity fields on large and small scales, correlations of galaxy clusters, etc. seems to be provided by the assumption that the dark matter in the universe is made up of about 75% cold (CDM) and about 20% hot dark matter (HDM). The obvious HDM candidate is a neutrino with mass in the few eV range. It has recently been argued that actually two neutrino species degenerate in mass ($\approx 2.4$ eV) provide a better detailed fit to data than a single species with the same total mass[3].

Another observation in support of a nonzero neutrino mass comes from attempts to understand a possible deficit of muon neutrinos in the data on cosmic ray neutrinos observed in several recent experiments[4]. The conclusion of a neutrino mass from these latter experiments may be on a somewhat weaker footing, since all experiments are not in agreement, and also there exist arguments that the deficit may have an alternative explanation (see, for example, Ref.5). The atmospheric neutrino deficit, explained by $\nu_\mu \rightarrow \nu_\tau$ oscillations with two nearly degenerate $\nu_\mu$ and $\nu_\tau$ was the original motivation for the two-neutrino cold plus hot dark matter model[3]. Because of the success of that model, we pursue here an alternative neutrino mass scenario which could preserve the model even if the observed $\frac{\nu_\mu}{\nu_e}$ ratio does not require a neutrino mass explanation. It will turn out that the neutrino mass hierarchy needed may have some advantages.

2. Input information

Of the variety of constraints on possible neutrino masses and mixings that come from accelerator searches for neutrino oscillations as well as cosmological and astrophysical considerations, we briefly summarize those
that are directly relevant for our considerations.

Solar neutrino deficit:

Because two out of the three types of solar neutrino experiments have to be wrong for an astrophysical explanation of the deficit to work\cite{6}, it is likely that the explanation of these data involves the oscillation of the $\nu_e$ to another species of neutrino. The required mass-squared difference is $\Delta m_{ei}^2 \approx 10^{-5} \text{ eV}^2$ where $i = \mu$ or $\tau$. The mixing angles favored by data\cite{7} are $\sin^2 2\theta \approx 0.007$, the so-called small-angle Mikheyev-Smirnov-Wolfenstein(MSW) solution and $\sin^2 2\theta \approx 0.8 - 1.0$, the MSW large-angle solution. There is also a vacuum large-angle solution, which requires $\Delta m_{ei}^2 \approx 10^{-10} \text{ eV}^2$.

Neutrinoless double beta decay:

The latest results from the search for neutrinoless double beta decay in $^{76}\text{Ge}$ seem to imply (with a possibly optimistic set of nuclear matrix element evaluations) that $\langle m_{\nu_e} \rangle \leq .68 \text{ eV}$ at the 68\% confidence level\cite{8}. Other matrix elements could increase that limit by up to a factor of two. The effective neutrino mass measured in that process is

$$\langle m_{\nu} \rangle \approx \Sigma \eta_i U_{ei} m_i, \quad (1)$$

where each neutrino of mass $m_i$ contributes to the total through the mixing matrix element $U_{ei}$, but with a sign $\eta_i = \pm 1$ determined by its CP-eigenvalue, so that cancellations can occur.

Supernova r-process:

It has been noted by Qian et al\cite{9} that for mass-squared difference $|m_{\nu_e}^2 - m_{\nu_X}^2|$ (where $X = \mu$ or $\tau$) in the $\geq \text{eV}^2$ range, an MSW resonance condition can be met in the supernova environment causing rapid transition of $\nu_\mu$ and $\nu_\tau$ to $\nu_e$, provided the mixing angles satisfy the constraint $\sin^2 2\theta \geq 10^{-5}$ or so. Since the $\nu_\mu$ and $\nu_\tau$ energies are about a factor of two larger than the $\nu_e$ energies, such transitions generate extra energetic $\nu_e$’s which have a larger cross-section for $\nu_e + n \rightarrow p + e^-$, depleting neutrons. The rapid capture of neutrons (r-process) in the neutrino-heated ejecta of supernovae is believed to be responsible for much of the production of the heavy elements; so one infers that for mass differences in the above range, the mixing angles must
be severely restricted. This constraint, if taken seriously, is therefore very relevant to the discussion of consistent neutrino mass matrices. One way to avoid this bound is to have $m_{\nu_e} \gg m_{\nu_\mu}$ so that the MSW resonance condition is not met. However this possibility has the apparent drawback that in this situation the MSW resonance condition occurs for $\nu_\mu \rightarrow \nu_e$ leading to contradictions with the celebrated $\nu_e$ data of IMB and Kamiokande experiments from SN1987A. In a recent paper[10], Fuller, Primack and Qian have shown that this effect is not as strong as one might have suspected, leaving this way to avoid the SN r-process bound as a viable mechanism.

An arrangement of neutrino masses which satisfies the three inputs above, as well as providing the two-neutrino version of the cold plus hot dark matter, would have $m_{\nu_\mu} \ll m_{\nu_e} \approx m_{\nu_\tau} \approx 2.4 \text{eV}$, with $|\Delta m^2_{\nu_e\nu_\tau}| \approx 10^{-5} \text{eV}^2$ for the large-angle MSW resolution to the solar neutrino deficit. The r-process constraint is avoided by the inverted mass hierarchy. If the $\nu_e$ was a Dirac particle, either of the two MSW solutions could be used, but in theoretical frameworks driven by elegant symmetry considerations, one is forced to use the large-angle solution, which then leads to an automatic cancellation between the $\nu_e$ and $\nu_\tau$ contributions in the neutrinoless double beta decay amplitude in equation (1). In this Letter, we show that the relevant symmetry is a global $L_e - L_\tau$ symmetry implemented in the framework of the left-right symmetric model, which not only leads to near degeneracy between the $\nu_e$ and $\nu_\tau$ masses, but also it automatically satisfies the neutrinoless double beta decay constraint, regardless of the absolute values of those masses. The scale of left-right symmetry in the simplest version of the model has to be in the $10^{12} \text{GeV}$ range. In the low energy limit, this model coincides with the standard electroweak theory with the additional feature that the neutrinos have the desired mass pattern. We later point out some phenomenological implications of the model.

3. $L_e - L_\tau$ symmetry and $\nu_e - \nu_\tau$ degeneracy implemented in a gauge model

To see the main points of this discussion, let us ignore the muon neutrino temporarily and consider only the $\nu_e$ and the $\nu_\tau$ with the following mass matrix:
\[ M_{e\tau} = \begin{pmatrix} \delta_1 & m \\ m & \delta_2 \end{pmatrix}. \] (2)

If we assume that \( \delta_{1,2} \ll m \), then \( L_e - L_\tau \) becomes a good symmetry of the model and the eigenvalues of this matrix become nearly degenerate. Furthermore, the effective mass observed in neutrinoless double beta decay in this case becomes

\[ \langle m_{\nu_e} \rangle \simeq \delta, \]

where \( \delta \) is equal to \( (\delta_1 + \delta_2)/2 \). Thus the double beta decay constraint is easily satisfied. We can extend this discussion to the case of three neutrinos easily as long as \( m_{\nu_\mu} \) is very small. The full \( 3 \times 3 \) matrix in this case would be

\[ M = \begin{pmatrix} -m_\beta - \delta & -\mu_1 & m + \delta \\ -\mu_1 & \mu & -\mu_1 \\ m + \delta & -\mu_1 & m_\beta - \delta \end{pmatrix}. \] (3)

In the above matrix, we choose \( m = 2.4 \text{ eV} \) to fit the HDM; \( \delta \sim 10^{-5} \text{ eV} \) to fit the large-angle MSW solution; \( \beta \simeq (1 - \sin^2 \theta_{e\tau}) \) where \( \theta_{e\tau} \) is the angle required by the large-angle MSW solution to the solar neutrino problem. The ratio \( \mu_1/m \) denotes the \( \nu_\mu \nu_e \) mixing angle that can be measured in the \( \nu_\mu \) to \( \nu_e \) oscillation experiments; finally, the parameter \( \mu \) (which is \( \ll m \)) is the \( \nu_\mu \) mass. Let us now proceed to the derivation of the full \( 3 \times 3 \) mass matrix in a left-right symmetric gauge model.

We will work within the conventional \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) models with the see-saw mechanism supplemented by a physical global symmetry \( U(1)_{L_e - L_\tau} \) which we will assume to be softly broken. Let us start by displaying only the leptonic and the relevant Higgs sectors. We denote the lepton doublet by \( \psi_i^T \equiv (\nu_i, e_i) \) (where \( i \) denotes the generation index) and assign the \( \psi_L \) and \( \psi_R \) to the left- and right-handed doublets under the gauge group. For the Higgs sector, we choose the conventional multiplets of the usual left-right symmetric model[11]; i.e., \( \Delta_L(3,1,+2) \oplus \Delta_R(1,3,+2) \) with \( L_e - L_\tau \) quantum number zero; we choose two sets of bi-doublets denoted by \( \phi(0,1)(2,2,0) \) with \( L_e - L_\tau \) quantum numbers 0 and 1. As in Ref. 11, the VEV of \( \Delta_R^0 \) breaks the \( SU(2)_R \) symmetry. We will assume this scale to be in the range of \( 10^{11} \) to \( 10^{12} \) GeV. The VEV’s of \( \phi_a \) break the standard model gauge symmetry. The important point for us is that
the potential minimization[11] leads to an induced VEV for $\Delta^0_L$ given by $v_L \simeq \lambda \langle \phi \rangle^2 / v_R$ which is now of order of a few eV’s for $\lambda \approx 10^{-2}$ to $10^{-1}$, where $v_L$ and $v_R$ are the VEV’s of $\Delta^0_L$ and $\Delta^0_R$, respectively, and $\lambda$ is a scalar self coupling in the Higgs potential.

To see the detailed structure of the neutrino masses, let us write down the Yukawa couplings of the leptons and the Higgs multiplets:

$$L_Y = \sum_{i=1,2,3} h_{ii} \bar{\psi}_iL \phi_0 \psi_iR + h_{12} \bar{\psi}_eL \phi_1 \psi_\mu R + h_{23} \bar{\psi}_\mu L \phi_1 \psi_\tau R +$$

$$[L \rightarrow R \text{ and } \phi_0 \rightarrow \phi_0^T \text{ and } \phi_1 \rightarrow \tau_2 \phi_1^T \tau_2]$$

$$+ [f_{1eL} \psi_\tau L \Delta_L + f_{2\muL} \psi_\mu L \Delta_L + L \rightarrow R] + h.c. \quad (4)$$

It is now easy to see that after symmetry breaking, one obtains the following $6 \times 6$ mass matrix in the basis $[\nu_e, \nu_\mu, \nu_\tau, (N_e, N_\mu, N_\tau)]$ (where we have denoted the right-handed neutrinos by $N_i$):

$$M_B = \begin{pmatrix} m_{LL} & m_{LR} \\ m_{LR}^T & M_{RR} \end{pmatrix}, \quad (5a)$$

where

$$m_{LL} = \begin{pmatrix} 0 & 0 & f_{1L} v_L \\ 0 & f_{2L} v_L & 0 \\ f_{1L} v_L & 0 & 0 \end{pmatrix}; \quad (5b)$$

$$m_{LR} = \begin{pmatrix} h_{11}\kappa_0 & h_{12}\kappa_1 & 0 \\ h_{12}\kappa_1' & h_{22}\kappa_0 & h_{23}\kappa_1 \\ 0 & h_{23}\kappa_1' & h_{33}\kappa_0 \end{pmatrix}; \quad (5c)$$

and

$$M_{RR} = \begin{pmatrix} 0 & 0 & f_{1R} v_R \\ 0 & f_{2R} v_R & 0 \\ f_{1R} v_R & 0 & 0 \end{pmatrix}. \quad (5d)$$

Here, we have denoted

$$\langle \phi_i \rangle = \begin{pmatrix} \kappa_i \\ 0 \\ \kappa_i' \end{pmatrix}.$$  

Note that in deriving the above $m_{LR}$, we used the fact that under left-right transformation we must have $\phi_1 \rightarrow \tau_2 \phi_1^T \tau_2$, since $\phi_1$ possesses a non-zero $L_e - L_\tau$ quantum number.

We see that the see-saw mechanism is fully operative for all three neutrino generations, and the dominant mass for the light neutrinos arises from the $v_L$ contributions. Since it conserves $L_e - L_\tau$ symmetry, the mixing
between the $\nu_e$ and $\nu_\tau$ is maximal. The $\nu_\mu$ also gets a mass from the $v_L$ term at this level. We choose $f_2 \ll f_1$ so that $m_{\nu_\mu} \ll eV$. The $\nu_e - \nu_\tau$ mass degeneracy is split by the $L_e - L_\tau$ violating contributions that come from the Dirac mass sector after the $\phi_{1,2}$ acquire VEV. An interesting point is that if we choose the right-handed mass scale to be of order $10^{12}$ GeV, then the $\nu_e\nu_\tau$ mass splitting is naturally of order $\approx \frac{m_e m_\tau}{v_R}$ which is $\approx 10^{-5}$ eV (for $v_R \simeq 10^{11}$ GeV) as required by the large-angle MSW solution. These $L_e - L_\tau$ violating terms also lead to the parameter $\beta$ in Eq.(3), which can make the $\sin^2 2\theta_{e\tau}$ slightly less than one. The $\nu_e - \nu_\mu$ mixing angle, however, comes naturally of the right order of magnitude if we choose the parameters $h_{ij}$ suitably and $\kappa_1 \ll \kappa'_1$.

4. Phenomenology

In this gauge model neutrinoless double beta decay is completely unobservable, but since the $\nu_e$ mass is about half the present limit from measurements at the endpoint energy of $^3$H beta decay[12], perhaps future work could test this hypothesis. The required large-angle MSW resolution of the solar neutrino puzzle could also be tested by the day-night effect in the solar neutrino flux when Super Kamiokande becomes operative.

Since the $\nu_\mu - \nu_\tau$ mass splitting is large, (i.e., $\Delta m_{\mu\tau}^2 \simeq 6 eV^2$) it is possible that the currently operating CHORUS and NOMAD experiments at CERN or the future E803 experiment at Fermilab would detect neutrino oscillations. It is especially intriguing that the preliminary LSND observation of $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$[13], for which also $\Delta m_{e\mu}^2 \simeq 6 eV^2$, may be compatible with this model. Indeed, it was the primary motivation of two recent preprints[10,14] in proposing this neutrino mass scheme to reconcile the LSND result result with the r-process constraint. We wish to emphasize that the LSND data could provide corroboration, but the mass matrix model is motivated without this information.

The Raffelt-Silk preprint[14] also suggested a variation of our scenario of three nearly degenerate neutrinos ($m_{\nu_e} \approx m_{\nu_\mu} \approx m_{\nu_\tau} \approx 1.6 eV$)[15,16] but with the $\nu_e$ and $\nu_\mu$ masses inverted. While this avoids the r-process bound and also provides for a $\nu_\mu \rightarrow \nu_\tau$ explanation of the atmospheric neutrino deficit, it does not satisfy the constraint from neutrinoless double beta decay, unless one chooses a maximal mixing angle scenario, as ad-
vocated in ref.17, with long-wavelength vacuum oscillation to resolve the solar neutrino puzzle. This is also unlikely to be borne out by LSND. The other preprint[14] also introduced a version of the four-neutrino scheme (suggested by us earlier[15]) to provide an explanation of the atmospheric neutrino problem but again inverting the $\nu_e$ and $\nu_\mu$ masses, which makes the $\nu_e$ a Dirac neutrino. To accommodate a hot dark matter component, the atmospheric neutrino deficit and a solar $\nu_e$ deficiency, it is necessary to introduce a sterile neutrino, if the scheme with three nearly degenerate neutrinos does not work. The four-neutrino version we presented before[15] and which has theoretically favored Majorana masses, might still avoid the supernova constraints if the sterile neutrino alters the supernova dynamics sufficiently.

5. Conclusion

In summary, we have discussed the theoretical and phenomenological consequences of an inverted mass hierarchy scenario for the three known neutrinos that can account for the constraints on neutrino masses and mixings: the solar neutrino deficit, mixed dark matter picture of the universe, r-process generation of heavy elements, and the lower limits on the lifetime for neutrinoless double beta decay. This scenario does not account for the atmospheric neutrino deficit, which would have to have some alternative explanation. We show that the conventional left-right symmetric models with a softly broken global $L_e - L_\tau$ symmetry and a high scale for the right-handed symmetry breaking very naturally generate such mass matrices. The day-night effects in the solar neutrino flux, as well as neutrino data from future supernovae[18], can test such models. Earlier evidence could come from the neutrino oscillation experiments such as LSND, CHORUS and NOMAD.

One of the authors (D. O. C.) would like to thank G. Fuller, S. T. Petcov, J. R. Primack and G. Raffelt for discussions.

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