Fields of an ultrashort tightly-focused laser pulse

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Analytic expressions for the electromagnetic fields of an ultrashort, tightly focused, laser pulse in vacuum are derived from scalar and vector potentials, using on equal footing two small parameters connected with the waist size of the laser beam and its duration. Compared with fields derived from a complex-source-point approach and a Lax series expansion approach, the derived fields are shown to be well-behaved and accurate even in the subcycle pulse regime. Terms stemming from the scalar potential are shown to be non-negligible and could significantly influence laser-matter interactions, in particular, direct electron acceleration in vacuum by an ultrashort laser pulse.

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With the rapid development of high-energy ultrashort laser pulses [1-6], laser-matter interactions involving a pulse of subcycle duration have recently been the subject of much interest [7-9]. Theoretical efforts aimed at describing such pulses have also been evolving to meet the need to model the electric and magnetic fields of the required pulses [10,11].

In the study of many laser-matter interactions, the fields of a fundamental Gaussian beam describe the radiation adequately. For processes that employ a tightly focused beam, this description is improved upon further using solutions to the wave equation in the paraxial approximation [20]. Higher order solutions in ascending powers of the square of the diffraction angle $\epsilon \equiv w_0/z_r$, known as a Lax series [15,21,22], provide a good description for a focused laser beam of long duration. Here $w_0$ and $z_r$ are the waist radius at focus and the Rayleigh length, respectively. However, the use of a Lax series expansion approach (LSEA) is limited in that the pulse temporal shape is factorized in the field expressions and, consequently, it does not take into account distortion of the pulse during propagation (the pulse temporal shape is independent of the propagation coordinate $z$). While this may work well for some applications, in which the change in pulse-shape is not appreciable, yet for many important current and future developments [23,24] involving ultrashort and tightly-focused pulses of laser light, space-time evolution and distortion [25] of the pulse-shape ought to be taken into account.

Another solution of Maxwell’s equations for a focused laser beam can be obtained [17,27] using the complex-source-point approach (CSPA) [26,28]. However, this solution is singular at the point $r = \pm (z + i z_0)$ which limits its utility in numerical simulations.

About two decades ago, Esarey et al. [10] proposed a way to describe the spatio-temporal evolution of an ultrashort and tightly-focused laser beam by solving the wave equation with the use of two small parameters: the spatial diffraction angle $1/k w_0$ and the temporal spreading parameter $1/\omega_0^2$, where $\tau_0$ is the laser pulse duration, with $\omega$ the frequency, and $k$ the wavenumber ($\omega = ck$ in vacuum). In [10], the laser pulse propagating in a plasma was considered and the transverse fields (with respect to the propagation direction) were calculated from a transverse vector potential alone. However, description of the longitudinal components of the laser field requires introduction of a scalar potential. Moreover, when the laser field is described by scalar and vector potentials, the scalar potential will alter the transverse fields, as well. The aim of this Letter is to determine the spatio-temporal evolution of the transverse and longitudinal fields of an ultrashort and tightly-focused laser pulse propagating in vacuum. We derive the fields from the appropriate combination of vector and scalar potentials using the approach of [10].

Expressions for the electromagnetic fields propagating in vacuum may be found from scalar and vector potentials $\Phi$ and $A$, respectively, which satisfy the equations $\nabla^2 \Phi - (1/c^2) \partial^2 \Phi / \partial t^2 = 0$, and $\nabla^2 A - (1/c^2) \partial^2 A / \partial t^2 = 0$, provided the Lorentz condition (SI units are used), $\nabla \cdot A + (1/c^2) \partial \Phi / \partial t = 0$, is satisfied, simultaneously. They are equivalent to the full set of Maxwell equations in the absence of any charges or currents [29]. Note that, assuming $\Phi = \phi_0 \phi(x,y,z,t) \exp[i k_0 (z - ct)]$, where $\phi_0$ is a constant and $k_0 = 2\pi/\lambda_0$ is a central wavenumber, corresponding to the wavelength $\lambda_0$, the Lorentz condition will give $\Phi = c^2 (\nabla \cdot A) / [i c k_0 - (1/\phi) \partial \phi / \partial t]$. More importantly, in the special case of a vector potential polarized linearly, say along the $x$-direction, the vector and scalar potentials satisfy identical (scalar) wave equations. As such, the potentials can only differ by multiplicative constants. We may, therefore, write $A = x \alpha_0 a(x,y,z,t) \exp[i k_0(z - ct)]$, where $\vec{x}$ is a unit vector in the polarization direction and $\alpha_0$ is a constant. Now, setting $\phi_0 \phi = a_0 a$ the scalar potential is turned into $\Phi = c^2 (\nabla \cdot A) / [i c k_0 - (1/a) \partial a / \partial t]$.

So, the problem of finding the scalar and vector potentials reduces to finding a solution for $a(x,y,z,t)$. Then, expressions for the fields $E$ and $B$ may, respectively, be found from

$$E = -\frac{\partial A}{\partial t} - \nabla \Phi, \quad B = \nabla \times A. \quad (1)$$

In the absence of a scalar potential function $\Phi$, the elec-
tric field \( E \) will have the same polarization as the vector potential \( A \), whereas the magnetic field \( B \) will not be altered by introduction of the scalar potential.

Following Esarey et al [10], a change of variables is first made to \( \zeta = z - ct, \eta = (z + ct)/2 \) and \( \rho = \sqrt{x^2 + y^2}/w_0 \) for a pulse propagating along \( z \). Assuming a linearly polarized vector potential, with the ansatz \( A = \hat{x}A \), \( A = a_0 a(\rho, \zeta, \eta) \exp(ik_0 \zeta) \), the equation for \( a \) is solved by employing a Fourier transformation

\[
a(\rho, \zeta, \eta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} a_k(\rho, k, \eta) e^{ik\zeta} dk.
\]

The Fourier components \( a_k \) are determined by the following equation

\[
\left( \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} + 4ikz_{\rho} \frac{\partial}{\partial \eta} \right) a_k(\rho, k, \eta) = 0,
\]

where \( z_{\rho} = (k_0 + k)w_0^2/2 \). This equation is solved in the paraxial approximation [21] and the solution reads \( a_k(\rho, k, \eta) = f_k \exp(\psi_k) \), where \( \psi_k = -\ln(1 + \alpha_k^2)/2 - \rho^2/(1 + \alpha_k^2) - i\tan^{-1}\alpha_k \). \( f_k \) is the initial axial envelope profile, with the focus taken at \( \eta = 0 \). Our analytic investigations will be continued below based on the specific choice \( f_k = N(1 + k/k_0)(L/\sqrt{2}) \exp(-k^2L^2/4) \), where \( N \) is a normalization factor. This choice corresponds to the lowest-order Gaussian mode of pulse length \( L \). It also has the advantage of riding us of the unphysical situation arising from \( k = -k_0 \).

For further analytic work to be done below, \( \psi_k \) viewed as a function of \( k' = k + k_0 \), is next Taylor-series expanded around the central wavenumber \( k_0 \), for the sake of finding the Fourier transform \( a(\rho, \zeta, \eta) \),

\[
\psi_k = \sum_{n=0}^{\infty} \psi^{(n)}(n)k^n/n!; \quad \psi^{(n)}(n) = \frac{\partial^n\psi_k}{\partial k^n}.
\]

This expansion is valid as long as \( k/k_0 \ll 1 \). Properties of the Fourier transform pair, Eq. (2), imply \((\Delta k)^2(\Delta \zeta)^2 \sim 1 \). Taking \( \Delta k \sim (k' - k_0) = k \) and \( \Delta \zeta \sim c\tau_0 = L \), one gets \( k \sim 1/L \). Consequently, \( k/k_0 \sim k_0/(2L) \ll 1 \). Therefore, the fields derived from this approach are valid when \( L \gg \lambda_0/2\pi \), which is fulfilled for a pulse length of at least half a cycle.

Thus, to \( m \)th order,

\[
A \approx A^{(m)} = a_0 e^{ik_0 \zeta} a^{(m)}(\rho, \zeta, \eta),
\]

where

\[
a^{(m)}(\rho, \zeta, \eta) = \int_{-\infty}^{\infty} dk f_k e^{ik\zeta} \prod_{n=0}^{m} \exp \left( \psi^{(n)}(n)k^n/n! \right).
\]

Note that, to order \( m \), the \( a \) in the expression for the scalar potential, \( \Phi \), is to be replaced with \( a^{(m)} \).

Furthermore, according to Eqs. (5) and (6), the zeroth-order vector potential is

\[
A = \frac{N_0 a_0 e^{ik_0 \zeta}}{\sqrt{1 + \alpha^2}} \left( 1 + \frac{2i\zeta}{k_0 L^2} \right) \exp \left[ -\frac{\zeta^2}{L^2} - \frac{\rho^2}{1 + \alpha^2} \right].
\]

where, the total phase is \( \varphi = \varphi_0 + k_0 \zeta + \alpha \rho^2/(1 + \alpha^2) - \tan^{-1}\alpha \), and \( \varphi_0 \) is an initial (constant) phase. Up to here known results have been reproduced for completeness [10].

In what follows, deviating from Ref. [10], the fields may be obtained via Eqs. (1) including the scalar potential \( \Phi \) explicitly. The Cartesian components of a linearly-polarized zeroth-order electric and magnetic fields, derived from combined vector and scalar potentials, are

\[
E_x = \frac{TRS}{\omega_0} + \frac{Tc}{z_r} \left( \frac{R}{RS} - \frac{2Q}{k_0^2L^2} \right) + \frac{i\omega_0 x}{2z_r(\omega_0 RS)^2},
\]

\[
E_y = \frac{Txy\omega_0}{z_r^2} + \frac{ic}{2z_r(\omega_0 RS)^2},
\]

\[
E_z = \frac{T}{z_r} \left[ \frac{i\omega_0 x}{S} \left( P - \frac{R + Q}{k_0^2L^2} \right) + \frac{2\omega_0^2 x}{L^2z_r S^2} \right] + \frac{2\omega_0^2 x}{L^2z_r S^2} \left( 1 - \frac{L^2(2Q - R)}{8S^2 z_r R^3} - \frac{2P^2}{k_0^2L^2} \right).
\]
curves are for fields derived from CSPA, and the blue dashed curves are based on the VSP fields of this work.

\[ B_x = 0, \quad B_y = i \frac{TR}{c} \left( \frac{P}{2} - \frac{Q}{2k_0} \right), \quad B_z = \frac{T}{c} \frac{y}{z}, \]

where, \( S = i\omega_0 \left( P + Q \right) \left( 2k_0^2 z_r^2 \right) \), and \( T = E_0 Nw_0 P / w R \exp \left[ -\left. \frac{\rho^2}{\alpha^2} / \left( 1 + \alpha^2 \right) + i\varphi \right] \right], \)

\( w = u_0 \sqrt{1 + \alpha^2}, \quad P = 1 + 2i\zeta \left( k_0 L^2 \right), \quad Q = 1 + i\alpha - \rho^2, \quad R = 1 + i\alpha. \) We take a Gaussian pulse envelope with a FWHM of \( L^* / (2\sqrt{2\ln 2}) \).

Based on the vector potential alone \( \mathbf{E} \), the corresponding fields are transverse, with

\[ E_x = \frac{TRS}{\omega_0}; \quad N = \frac{\omega_0}{S_0}, \]

and \( E_y = E_z = 0; \) whereas, the magnetic field components would be the same as in Eqs. (11) – (12). Thus we see that the scalar potential, which is necessary for describing the longitudinal fields, also modifies the transverse fields.

Typical distributions of the electric field components are shown in Fig. 1. Figs. 1(a)–(c) are based on Eqs. (8)–(10), derived from combined vector and scalar potentials (VSP) while (d) is based on Eq. (13) or fields derived from a vector potential (VP). Note that Figs. 1(a) and (d) are hardly distinguishable for the parameter set used, on account of the fact that the second term in Eq. (8) is a very small correction to Eq. (13). Easily visible deviations may be brought out when subcycle pulses are used instead. On the other hand, \( E_z \) and \( E_y \), illustrated by Figs. 1(b) and (c) are solely due to the VSP approach and are absent from the VP analysis. Those components, although more than two orders of magnitude weaker than their associated \( E_x \) component, yet they play a significant role in the interaction of matter with such a laser pulse in vacuum.

Figure 2 compares and contrasts our VSP fields at focus with those of the CSPA \( \mathbf{E} \) and LSEA. In Fig. 2(a), it is hard to distinguish the curves stemming from all three approaches, for the case of a long laser pulse \( L = 5\lambda \). However, in the case of a subcycle pulse \( L = \lambda / 2 \) shown in Fig. 2(b) variations of \( E_x \) at focus with time based on the CSPA and VSP are almost identical, whereas those based on the LSEA are markedly different, due to the temporal diffraction effects (in the LSEA no limitations are imposed on the pulse duration).

Results of further CSPA and VSP comparison and contrast efforts are exhibited in Fig. 3. Here, variations of \( E_x \) with \( x/\lambda \) are displayed for different pulse durations and at different longitudinal positions. For the subcycle case illustrated in Figs. 3(a)–(c), the numerical error induced by the singularity as \( r \approx \pm i(z + iz_r) \) in the CSPA fields is quite large near the focus and decreases with increasing \( z \). Also, the effect of the singularity diminishes with increasing pulse length as may be inferred from Figs. 3(d)–(f).

In all cases, the VSP fields are consistently well-behaved and, hence, may be judged as more reliable than those of the CSPA.

To push reliability of our VSP fields even further, their utility in vacuum electron acceleration calculations will now be illustrated in Fig. 4. In Fig. 4(a), the energy gain of an electron from the fields of counter-propagating short tightly-focused laser pulses, derived from VSP and VP, are shown vs. \( \eta' \). The figure shows clearly that the end gain from the VSP fields is several orders of magnitude larger than from the VP ones. The obvious explanation to this is that the VSP fields have an axial (accelerat-
ing) component $E_z$, while the VP fields do not, even though the longitudinal components are about two orders of magnitude weaker than the main transverse one, $E_x$. Results of calculations similar to those leading to Fig. 4(a) are shown in (b) for gain from a 1 TW laser pulse. The black dashed curve in (b) shows that the electron seems to continue to gain energy from the CSPA fields, of close to 100 MeV, even after the pulse has left it behind.

The abnormal behaviour exhibited in Fig. 4(b) is induced by the singularity in the field expressions. The results of Fig. 4 could be investigated by the Extreme Light Infrastructure (ELI) [5, 6] or other high energy ultrashort facilities [3, 4] in the near future.

Firmly based on the statement that the scalar and vector potential wave equations, together with the Lorentz gauge, are entirely equivalent to the full set of Maxwell equations in empty space, expressions for the fields that accurately model an ultrashort, tightly focused, laser pulse, propagating in vacuum, have been explicitly obtained. Such expressions may be of utility for current and future applications in laser-acceleration [30] and other ultrafast physics [31]. It has been demonstrated that, with the proper introduction of a scalar potential, the fields develop new axial components and modify the transverse ones quite significantly. Moreover, deviation of the derived fields from those based on the paraxial approximation is shown to be non-negligible when the pulse duration is less than a few cycles, and substantial for subcycle pulses.

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