Lower Limits on $\mu \to e\gamma$ from new Measurements on $U_{e3}$

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New data on the lepton mixing angle $\theta_{13}$ imply that the $e\mu$ element of the matrix $m_\nu m_\nu^\dagger$, where $m_\nu$ is the neutrino Majorana mass matrix, cannot vanish. This implies a lower limit on lepton flavor violating processes in the $e\mu$ sector in a variety of frameworks, including Higgs triplet models or the concept of minimal flavor violation in the lepton sector. We illustrate this for the branching ratio of $\mu \to e\gamma$ in the type II seesaw mechanism, in which a Higgs triplet is responsible for neutrino mass and also mediates lepton flavor violation. We also discuss processes like $\mu \to eee$ and $\mu \to e$ conversion in nuclei. Since these processes have sensitivity on the individual entries of $m_\nu$, their rates can still be vanishingly small.

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I. INTRODUCTION

The observation of lepton mixing in the form of neutrino oscillations shows without doubt that there is physics beyond the Standard Model of elementary particles. To be precise, the presence of lepton flavor violation (LFV) has been established. While being well-entrenched in the neutrino sector, the question arises how large LFV in the charged lepton sector is, and how it is connected to the quantities in the neutrino sector. The power of the Glashow-Iliopoulos-Maiani mechanism [1] in the Standard Model ensures that for instance observation of $\mu \to e\gamma$ will be unambiguously a sign of new physics beyond the presence of “only” massive neutrinos. If this new physics is connected to neutrino mixing parameters it is an extremely model-dependent question.

In this short note we point out an interesting new implication for scenarios in which LFV is governed by $m_\nu m_\nu^\dagger$, where $m_\nu$ is the neutrino mass matrix. In particular the $e\mu$ entry of this matrix is of interest, as it is often responsible for $\mu \to e\gamma$, $\mu \to 3e$, or muon-to-electron conversion in nuclei. The advantage of scenarios in which $m_\nu m_\nu^\dagger$ governs LFV is their predictivity: $m_\nu m_\nu^\dagger$ only depends on measurable neutrino oscillation parameters: both mass-squared differences including the sign of the atmospheric one, three mixing angles and the Dirac CP phase. Until very recently neutrino data allowed for the possibility that $(m_\nu m_\nu^\dagger)_{e\mu}$ vanishes, namely when the lepton mixing matrix element $|U_{e3}|$ takes a small value around 0.015. However, recent results from T2K [2], Double Chooz [3] and finally Daya Bay [4] imply a surprisingly large value of the lepton mixing matrix element $|U_{e3}|$ around 0.15:

$$|U_{e3}| = 0.153^{+0.014}_{-0.015}(0.039),$$

where we have given the 1σ and 3σ ranges. As we will see, this sizable value implies that $(m_\nu m_\nu^\dagger)_{e\mu}$ cannot vanish, and hence a lower limit on $(m_\nu m_\nu^\dagger)_{e\mu}$ arises. Correspondingly, lower limits on lepton flavor violating processes arise. Of course, the processes can still be unobservable because of too heavy masses of the additional particles which mediate the decays. However, the point here is that the flavor physics part of the problem cannot spoil observation anymore. Thereby, yet another possibility for LFV to hide from future experiments is ruled out.

A popular example for which the rates of LFV processes are functions of $m_\nu m_\nu^\dagger$ is the type II (or triplet) seesaw mechanism [5,6]. Here neutrino mass is generated by a Higgs triplet, which in turn can mediate LFV, and in

1 After completion of the paper, the RENO collaboration reported a new measurement [5], resulting in $|U_{e3}| = 0.163^{+0.014}_{-0.014}$ at 1σ. Our results hardly change by considering this range of values.
particular leads to a branching ratio of $\mu \to e\gamma$ depending on $(m_\nu m_\nu^\dagger)_{e\mu}$. We focus here on the triplet seesaw mechanism, but point out that $m_\nu m_\nu^\dagger$ governs LFV also in classes of theories in which “minimal flavor violation” in the lepton sector is realized \[12\]. Minimal flavor violation assumes that Standard Model Yukawa couplings are the only sources of flavor symmetry breaking. This very economical and elegant concept was originally invented for the quark sector \[13\], but can be applied to the lepton sector as well \[12\], predictions for LFV rates depending however on the explicit operator realization. Also for the supersymmetric triplet seesaw, with a very heavy triplet and universal boundary conditions \[14\], consequences of our observation arise, absolute rates depending however on a variety of additional parameters. Another explicit realization of $\text{Br}(\mu \to e\gamma) = f[(m_\nu m_\nu^\dagger)_{e\mu}]$ can be found in \[12\]: here neutrinos are Dirac particles within a particular two Higgs Doublet Model. There are presumably many more examples. For definiteness, we consider here only the triplet seesaw, where there are only two free parameters besides the ones governing neutrino oscillations, namely the mass of the triplet and the vacuum expectation value of its neutral component.

The same result for $|U_{e3}|$ implies that $(m_\nu m_\nu^\dagger)_{e\tau}$ cannot vanish anymore, and lower limits on $\tau e$ LFV processes arise. However, due to the approximate $\mu-\tau$ symmetry of lepton mixing, it holds that $(m_\nu m_\nu^\dagger)_{e\tau} \sim (m_\nu m_\nu^\dagger)_{e\mu}$. This implies that rates for $\tau e$ LFV processes are of the same order as rates for $\mu e$ LFV processes. Since future limits on the $\tau e$ sector are expected to be less stringent than present constraints on the $\mu e$ sector, those decay channel are not observable in this framework. This in turn implies that for instance observation of $\tau \to e\gamma$ will signal the presence of lepton flavor violation not depending on $m_\nu m_\nu^\dagger$.

The processes as $\mu \to 3e$ and $\mu - e$ conversion have some dependence on $(m_\nu m_\nu^\dagger)_{e\mu}$ as well. However, either the contribution of $(m_\nu m_\nu^\dagger)_{e\mu}$ is suppressed, or cancellations from other contributions can occur. Setting lower limits in the same sense as for $\mu \to e\gamma$ is not possible.

The paper is build up as follows: in Section II we quantify the fact that new oscillation data for large $|U_{e3}|$ imply non-vanishing $(m_\nu m_\nu^\dagger)_{e\mu}$. Section III introduces the type II seesaw and relevant expressions for lepton flavor violating processes. A numerical study of the various constraints is performed in Section IV before we conclude in Section V.

## II. NON-VANISHING $|U_{e3}|$ AND NON-VANISHING $(m_\nu m_\nu^\dagger)_{e\mu}$

In this section we note the simple yet consequential fact that large $|U_{e3}|$ implies non-vanishing $(m_\nu m_\nu^\dagger)_{e\mu}$. As stated in the introduction, a variety of scenarios and frameworks leads to LFV processes depending on the quantity $m_\nu m_\nu^\dagger$. Here $m_\nu$ is the neutrino mass matrix which is given as

$$m_\nu = U \, \text{diag}(m_1, m_2, m_3) \, U^T,$$

(2)

where $m_i$ are the three light neutrino masses and $U$ the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) lepton mixing matrix. Its standard parametrization is

$$U = \begin{pmatrix}
  c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta} \\
  -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i \delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i \delta} & s_{23} c_{13} \\
  s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i \delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i \delta} & c_{23} c_{13}
\end{pmatrix} \times \text{diag} \left( 1, e^{i \phi_1}, e^{i \phi_2} \right).$$

(3)

In Eq. (3), $\delta$ denotes the Dirac CP-phase, while $\phi_1, \phi_2$ denote two Majorana phases. The quantities $c_{ij}$ and $s_{ij}$ represent $\cos \theta_{ij}$ and $\sin \theta_{ij}$, respectively.

We consider here classes of theories in which LFV is governed by $m_\nu m_\nu^\dagger$. Note that this matrix is independent of the Majorana phases and the interesting off-diagonal entries are furthermore independent of the neutrino mass scale (recall that $m_\nu m_\nu^\dagger$ is the same quantity which appears in the classical Hamiltonian for neutrino oscillations). We plot in Fig. II the $\delta$ dependency of the off-diagonal elements of $m_\nu m_\nu^\dagger$, fixing the remaining parameters to their best-fit values. It is apparent (and well-known) that $|(m_\nu m_\nu^\dagger)_{\mu\tau}|$ is larger than the other entries by one order of magnitude, that $|(m_\nu m_\nu^\dagger)_{e\mu}| \sim
and the result for $(m_e m_{\mu})_{\mu e}$ with \( \delta \) is much smaller compared to that of the other two off-diagonal entries. Such studies have been performed several times in the literature before \cite{14, 16, 17} and also recently \cite{18}, and here we wish to focus on the implication of non-vanishing and sizable \(|U_{e3}| \) on \(|(m_e m_{\mu})_{\mu e}| \) and thus on \( \mu \rightarrow e\gamma \).

In those cases, in which LFV depends on \( m_e m_{\mu} \), the \( e\mu \) entry is of particular interest, as in the \( e\mu \) sector the strongest experimental limits on LFV exist, and even stronger limits are to be expected in the near future \cite{19, 20}. The crucial flavor physics quantity is therefore \(|(m_e m_{\mu})_{\mu e}| \). One might therefore wonder whether \(|(m_e m_{\mu})_{\mu e}| \) can vanish in principle. This is indeed possible, and the result for \( |(m_e m_{\mu})_{\mu e}| = 0 \) is a rather simple formula:

\[
|U_{e3}|(m_e m_{\mu})_{\mu e} = 0 \quad = \frac{1}{2} R \sin 2\theta_{12} \cot \theta_{23} \pm \frac{1}{2} R \sin 2\theta_{12} \cot \theta_{23}
\]

\[= \begin{cases} 
0.0135^{+0.004 (0.009)}_{-0.002 (0.005)} & \text{normal}, \\
0.0141^{+0.003 (0.005)}_{-0.002 (0.005)} & \text{inverted},
\end{cases}
\]

where the minus (plus) sign is for the normal (inverted) mass ordering and \( R \) is the positive ratio of the solar and the atmospheric mass-squared differences (\( \Delta m_{\text{atm}}^2 \) and \( \Delta m_{ee}^2 \), respectively)\(^2\). We have also given the implied value of \(|U_{e3}| \) when the best-fit values as well as 1\( \sigma \) and 3\( \sigma \) ranges of the oscillation parameters from Ref. \cite{22} are inserted. The value of \(|U_{e3}| \) for which \(|(m_e m_{\mu})_{\mu e}| \) vanishes is rather small, being of order 0.014. It has to be compared to the value of \(|U_{e3}| = 0.153^{+0.039}_{-0.055} \) determined by Daya Bay, given in Eq. (1), which is significantly larger. This implies a non-zero lower limit on \(|(m_e m_{\mu})_{\mu e}| \), and hence on branching ratios for lepton flavor violating processes in a variety of scenarios. This is the main point of this paper, and we will quantify this for the example of Higgs triplets in the type II seesaw mechanism. Values of the oscillation parameters in the 1\( \sigma \) and 3\( \sigma \) range are given in Table II. Using those values, the explicit range at 1\( \sigma \) and 3\( \sigma \) of \(|(m_e m_{\mu})_{\mu e}| \) reads:

\[
|(m_e m_{\mu})_{\mu e}| \quad \text{[eV]} = \begin{cases} 
1.9 \times 10^{-4} - 4.5 \times 10^{-4} & (1\sigma), \\
1.0 \times 10^{-4} - 3.5 \times 10^{-4} & (3\sigma),
\end{cases}
\]

\[ \text{(5)} \]

\[ \text{TABLE I: best-fit, 1\( \sigma \) and 3\( \sigma \) ranges of the oscillation parameters. Values of all the parameters (except } \sin^2 \theta_{13} \text{) are taken from Ref. } \cite{21}. \text{ For } \theta_{13}, \text{ the results of Daya Bay } \cite{14} \text{ have been used. Results applying for the inverted mass ordering are in square brackets.} \]

| Parameters | best-fit | 1\( \sigma \) range | 3\( \sigma \) range |
|------------|-----------|----------------|----------------|
| \( \Delta m_{\text{atm}}^2 \) [eV\(^2\)] | 7.59 | 7.41 – 7.79 | 7.09 – 8.19 |
| \( \Delta m_{ee}^2 \) [eV\(^2\)] | 2.50 | 2.34 – 2.59 | 2.14 – 2.76 |
| \( \sin^2 \theta_{12} \) | 0.312 | 0.297 – 0.329 | 0.27 – 0.36 |
| \( \sin^2 \theta_{23} \) | 0.52 | 0.45 – 0.58 | 0.39 – 0.64 |
| \( \delta \) | –0.61\( \pi \) | –1.26\( \pi \) – 0.14\( \pi \) | –0.2\( \pi \) | 0 – 2\( \pi \) |

with differences between the normal and inverted ordering not showing up before the second decimal place. Using the recent RENO result \cite{23} would give minimal (maximal) values smaller (larger) by about 0.2 \( \times 10^{-4} \) eV\(^2\).

In the same spirit, the large value of \(|U_{e3}| \) has implications for LFV in the \( e\tau \) sector. The condition for \(|(m_e m_{\mu})_{\mu \tau}| = 0 \) gives the following result:

\[
|U_{e3}|(m_e m_{\mu})_{\mu \tau} = 0 \quad = \frac{1}{2} R \sin 2\theta_{12} \tan \theta_{23} \pm \frac{1}{2} R \sin 2\theta_{12} \tan \theta_{23}
\]

\[= \begin{cases} 
0.0146^{+0.004 (0.010)}_{-0.003 (0.006)} & \text{normal}, \\
0.0153^{+0.003 (0.009)}_{-0.003 (0.006)} & \text{inverted},
\end{cases}
\]

\[ \text{(6)} \]

Similar to \(|(m_e m_{\mu})_{\mu e}| \) one can evaluate the right-hand side of Eq. (3), giving similar numbers.

LFV processes in the \( \tau \mu \) sector also have lower limits, since the relevant flavor quantity \(|(m_e m_{\mu})_{\mu \tau}| \) cannot vanish. This was true even before the recent results on \( U_{e3} \). At leading order, one finds

\[
|(m_e m_{\mu})_{\mu \tau}| \quad \leq \frac{1}{2} \Delta m_{\text{atm}}^2 \sin 2\theta_{23} (1 - R \cos^2 \theta_{12}),
\]

\[ \text{(7)} \]

which is always non-zero. The order of magnitude of \(|(m_e m_{\mu})_{\mu \tau}| \) is always larger than the one of \(|(m_e m_{\mu})_{\mu e}| \):

\[
\frac{|(m_e m_{\mu})_{\mu e}|^2}{|(m_e m_{\mu})_{\mu \tau}|^2} \quad \geq \frac{|U_{e3}|^2}{\cos^2 \theta_{23}} + 2 |U_{e3}| \cot \theta_{23} \frac{\sin \theta_{12}}{\sin \theta_{23}} R.
\]

\[ \text{(8)} \]

We will continue with a study focusing on the decay \( \mu \rightarrow e\gamma \) in the type II seesaw, leaving a more detailed study of other decays and other scenarios for a future study. In general, however, the necessary existence of LFV in the \( e\mu \) (and \( e\tau \) sector) adds to the known existence of LFV in the \( \tau \mu \) sector, and guarantees the presence of all three channels.

\(^2\) Interestingly, the above condition on \(|U_{e3}| \) requires in addition CP conservation, i.e. \( \delta = 0 \) and \( \pi \), respectively. Note that with \( f = m_e m_{\mu} \) the Jarlskog invariant for leptonic CP violation in neutrino oscillations is proportional to \( \text{Im}[f_{\mu e} f_{\nu e} f_{\nu \mu}] \) \cite{22}. Hence, the vanishing of an off-diagonal element of \( m_e m_{\mu} \) implies CP conservation.
FIG. 1: Plots showing the effect of the Dirac CP-phase $\delta$ on various $x_{\alpha\beta}$ where $x \equiv |(m_\nu m_\nu^\dagger)|$. The remaining oscillation parameters are fixed at their best-fit values (see Table I).

III. NON-VANISHING BRANCHING RATIOS: EXAMPLE OF THE HIGGS TRIPLET

As mentioned before, we focus here on the type II or triplet seesaw mechanism. In this framework neutrino masses are generated by interactions of lepton doublets $L_\alpha$, with $\alpha = e, \mu, \tau$, with a weak triplet, hypercharge 2 scalar:

$$\mathcal{L} = h_{\alpha\beta} L_\alpha^c \tau_2 \Delta L_\beta + H.c., \text{ where }$$

$$\Delta = \begin{pmatrix} H^+ / \sqrt{2} & H^{++} \vspace{1mm} \\ H^0 & -H^+ / \sqrt{2} \end{pmatrix}. \tag{9}$$

Upon acquiring a vacuum expectation value (VEV) $\langle H^0 \rangle = v_\Delta / \sqrt{2}$, the neutrino mass matrix for light Majorana neutrinos is

$$\langle m_\nu \rangle_{\alpha\beta} = \sqrt{2} v_\Delta h_{\alpha\beta}, \tag{10}$$

where $h_{\alpha\beta}$ are the neutrino Yukawa couplings. The interesting and potentially substantiate part of this mechanism is that the members of the triplet induce LFV with couplings given in terms of Eqs. (9) and (10), i.e. in terms of in principle measurable parameters \[16\]. These parameters, together with the masses of the triplet members which are in principle accessible at colliders \[17\], allow for a scenario that is fully determinable and makes definite predictions for LFV.

Let us recapitulate the well-known formulas for the branching ratios \[16\]. For $\mu \to e\gamma$ one has

$$\text{Br}(\mu \to e\gamma) = \frac{27 \alpha}{256 \pi G_F^2 M_{H\pm}^4} \frac{|(m_\nu m_\nu^\dagger)_{\mu e}|^2}{v_\Delta^2} \text{Br}(\mu \to e\nu), \tag{11}$$

with $M_{H\pm}$ as the triplet mass and $\text{Br}(\mu \to e\nu) \simeq 100\%$. The branching ratio for $\tau \to e\gamma$ is given by

$$\text{Br}(\tau \to e\gamma) = \frac{27 \alpha}{256 \pi G_F^2 M_{H\pm}^4} \frac{|(m_\nu m_\nu^\dagger)_{\tau e}|^2}{v_\Delta^2} \text{Br}(\tau \to e\nu), \tag{12}$$

where $\text{Br}(\tau \to e\nu) = 17.82 \pm 0.04\%$ \[23\]. The analogous formula for $\text{Br}(\tau \to \mu\gamma)$ depends on $(m_\nu m_\nu^\dagger)_{\mu\tau}$.

At this stage, combining Eqs. (11) and (12), we can rewrite Eq. (12) as

$$\text{Br}(\tau \to e\gamma) = 0.1782 \times \frac{|(m_\nu m_\nu^\dagger)_{\tau e}|^2}{|(m_\nu m_\nu^\dagger)_{\mu\tau}|^2} \text{Br}(\mu \to e\gamma). \tag{13}$$

In general, as stated earlier, $\text{Br}(\mu \to e\gamma)$ and $\text{Br}(\tau \to e\gamma)$ are of the same order of magnitude since $(m_\nu m_\nu^\dagger)_{\mu e} \sim (m_\nu m_\nu^\dagger)_{\tau e}$ due to the approximate $\mu-\tau$ symmetry of lepton mixing. The current limit on $\text{Br}(\tau \to e\gamma)$ is $3.3 \times 10^{-8}$.
with a potential improvement to 3.0 \times 10^{-9} in the SuperB facility \cite{20}, still being way below the current \( \mu \rightarrow e\gamma \) limit. Recall that this was recently improved to \cite{24}.

\[
\text{Br}(\mu \rightarrow e\gamma) < 2.4 \times 10^{-12},
\]

and future limits to values down to 10^{-13} are foreseen \cite{19}. Exact \( \mu - \tau \) symmetry would result in \[
\frac{|(m_{\mu} m_{\tau})_{e\mu}|^2}{|(m_{\mu} m_{\tau})_{e\mu}|^2} = 1
\]
and thus \[
\frac{\text{Br}(\tau \rightarrow e\gamma)}{\text{Br}(\mu \rightarrow e\gamma)} \simeq 0.2.\]
In this case a limit on \( \text{Br}(\mu \rightarrow e\gamma) < 10^{-12} \) would correspond to \( \text{Br}(\tau \rightarrow e\gamma) < 10^{-13} \), beyond the reach of upcoming experiments (see Table \text{II}).

A careful study including the variation of the oscillation parameters shows that \[
\frac{\text{Br}(\tau \rightarrow e\gamma)}{\text{Br}(\mu \rightarrow e\gamma)} \simeq 0.15 - 0.21\] (both for 1\(\sigma \) and 3\(\sigma \)), and hence this conclusion remains valid. Thus any evidence of \( \tau \rightarrow e\gamma \) in near future experiment will rule out triplet seesaw models or any model in which \( m_\nu \leftarrow m_\tau \) governs LFV.

We should remark that \( \mu \rightarrow 3e \) is also a very interesting process, being mediated at tree level. The branching ratio for \( \mu \rightarrow 3e \) is given by

\[
\text{Br}(\mu \rightarrow 3e) = \frac{1}{16G_F^2 M_{H^\pm}^4} \frac{|(m_{\mu} m_{e})|(|m_{\mu} m_{e})|^2}{1 + \frac{\Delta m^2}{\Delta \lambda}} \text{Br}(\mu \rightarrow e\nu).\]

Unlike \( \mu \rightarrow e\gamma \), \( \tau \rightarrow e\gamma \) or \( \tau \rightarrow \mu\gamma \), the process \( \mu \rightarrow 3e \) can yield an experimentally inaccessible branching ratio even with recent \( \theta_{13} \) value and low triplet masses, namely when the \( ee \) or \( e\mu \) elements of the Majorana neutrino mass matrix vanish. In this case, one-loop diagrams can provide the dominant contribution, depending on \( (m_{\mu} m_{\nu})_{e\mu} \), the same flavour quantity that governs \( \mu \rightarrow e\gamma \). Assuming that the decay is generated by \( e^+e^- \) pair creation from a virtual photon, the following ratio of branching ratio is found:

\[
\frac{\text{Br}(\mu \rightarrow 3e)}{\text{Br}(\mu \rightarrow e\gamma)} = \frac{\alpha_{em}}{3\pi} \left[ \log \frac{m_{\mu}^2}{m_e^2} - \frac{11}{4} \right] \simeq 1.5 \times 10^{-3}.
\]

Finally the \( \mu \) to \( e \) conversion rate in nuclei is given by \cite{26}:

\[
R(\mu N \rightarrow eN^+) = \frac{\alpha^5 m_{\nu}^5 m_{\mu}^5 Z^4 |Z| F(q)|^2}{16\pi^4 M_{H^\pm}^4 \sqrt{\Delta \lambda}} \left[ \sum_{k = e, \mu, \tau} \frac{(m_{\nu}^5 m_{\mu}^5)_{ek} F(r, s_k)}{3} - \frac{3 (m_{\nu}^5 m_{\mu}^5)_{e\mu}}{8} \right]^2,
\]

where

\[
F(r, s_k) = \ln s_k + \frac{4 s_k}{r} + \left( 1 - \frac{2 s_k}{r} \right) \sqrt{\left( 1 + \frac{4 s_k}{r} \right) \ln \sqrt{\frac{1 + 4 s_k}{r} + 1}} - \sqrt{\frac{1 + 4 s_k}{r} - 1}.
\]

with \( r = -\frac{s_{\tau}^2}{M_{H^\pm}^2}, s_k = \frac{m_{\mu}^2}{M_{H^\pm}^2}, k = e, \mu, \tau \). For \( \mu N \rightarrow eN^+ \) in different nuclei corresponding values of
\(Z_{\text{eff}}, \Gamma_{\text{capt}}, F(q^2 \simeq -m_{\nu}^2)\) can be obtained from Ref. [27]. The best current limit on the \(\mu - e\) conversion ratio \(\text{R}(\mu \rightarrow e)\) is \(7 \times 10^{-13}\) for \(^{197}\text{Au}\) [23]. Future experiments (\(\text{Mu2e, COMET, using } {}^{79}\text{Al}\) [25]) are expected to reach a sensitivity of \(2 \times 10^{-17}\) in the near future. In the far future using \(^{48}\text{Ti}\), the ratio is expected to be probed down to values of \(10^{-18}\) [28]. As obvious from Eq. (17), there are two contributions to the process, and it turns out that setting a lower limit on the rate of \(\mu - e\) conversion is not possible, even with large \(|U_{e3}|\). While the second contribution in \(\text{R}(\mu N \rightarrow eN^*)\) is the same expression as in \(\mu \rightarrow e\gamma\) and has a lower limit, it can be cancelled by the more complicated first term, which depends in a complicated way on the individual neutrino masses and Majorana phases. In fact, the rate of \(\mu - e\) conversion under certain assumptions can vanish for certain parameter values, as recently shown in Ref. [23]. We will therefore not study this process anymore and will rather focus on the minimal \(\text{Br}(\mu \rightarrow e\gamma)\) as implied by recent data on \(U_{e3}\).

IV. RESULTS OF NUMERICAL ANALYSIS

Our observation is here that the large observed value of \(|U_{e3}|\) implies that the branching ratio of the decay \(\mu \rightarrow e\gamma\) cannot vanish, and hence a lower limit on its branching ratio arises. We quantify this finding now as a function of the triplet VEV \(v_\Delta\) and the triplet mass \(M_{H^{\pm\pm}}\). When evaluating the minimal (and maximal) value of \(\mu \rightarrow e\gamma\), we vary the neutrino oscillation parameters within the ranges given in Table I their 1\(\sigma\) and 3\(\sigma\) ranges are from Ref. [22], and for \(\theta_{13}\) we have considered the 1\(\sigma\) and 3\(\sigma\) ranges from Daya Bay [2]. The three CP phases were also varied in their allowed ranges. We took the current constraints on a large number of LFV processes into account, which are listed in Table I. Moreover, we also considered the case of when all processes obey limits obtainable in future experiments: most of the future limits have been taken from Ref. [20].

We have studied the variation of the lowest possible branching ratio for \(\mu \rightarrow e\gamma\) with the triplet mass \(M_{H^{\pm\pm}}\) for four different triplet VEVs, \(v_\Delta = 0.5\) eV, 1.0 eV, 5.0 eV and 10.0 eV. In the course of investigation we have also considered the impact of the absolute neutrino mass scale \((m_1\text{ for normal hierarchy and } m_3\text{ for inverted hierarchy})\) for three different values, namely 0.003 eV, 0.05 eV and 0.2 eV. These values are chosen in a fashion that not only they covered the pure normal and inverted hierarchical \((m_1^{(3)} = 0.003\text{ eV})\) scenarios, but also the quasi-degenerate and intermediate cases. While the branching ratio of \(\mu \rightarrow e\gamma\) does not depend on those masses, as well as on the Majorana phases, there is an indirect influence from the limits on the other LFV processes.

It is well understood from Eqs. (11), (12), (15) and (17) that the branching ratios will decrease for larger \(M_{H^{\pm\pm}}\) and \(v_\Delta\). Consequently, if we ask that the stronger future constraints are obeyed, larger \(M_{H^{\pm\pm}}\) and \(v_\Delta\) are more favorable. Further, with light \(v_\Delta\), larger triplet masses are favorable. Of course, for sufficiently large values of triplet mass and VEV, some of these branching ratios will be inaccessible to the ongoing and even to the future experiments. In addition there may arise situations when some of the processes remain unobserved while others have been seen. Such more complicated situations will be discussed elsewhere.

| Process | Present | Constraints |
|---------|---------|-------------|
| \(\text{Br}(\tau \rightarrow e\gamma)\) | \(2.7 \times 10^{-9}\) [23] | \(1.0 \times 10^{-9}\) [20] |
| \(\text{Br}(\tau \rightarrow e\mu)\) | \(1.8 \times 10^{-9}\) [23] | \(1.0 \times 10^{-9}\) [20] |
| \(\text{Br}(\tau \rightarrow e\mu)\) | \(1.7 \times 10^{-8}\) [23] | \(1.0 \times 10^{-9}\) [20] |
| \(\text{Br}(\tau \rightarrow \mu\mu)\) | \(2.1 \times 10^{-7}\) [23] | \(1.0 \times 10^{-9}\) [20] |
| \(\text{Br}(\tau \rightarrow \mu\mu)\) | \(1.8 \times 10^{-8}\) [23] | \(1.0 \times 10^{-9}\) [20] |
| \(\text{Br}(\tau \rightarrow \mu\mu)\) | \(1.5 \times 10^{-8}\) [23] | \(1.0 \times 10^{-9}\) [20] |
| \(\text{Br}(\tau \rightarrow v\gamma)\) | \(4.4 \times 10^{-8}\) [23] | \(2.0 \times 10^{-9}\) [20] |
| \(\text{Br}(\tau \rightarrow e\gamma)\) | \(3.3 \times 10^{-8}\) [23] | \(3.0 \times 10^{-9}\) [20] |
| \(\text{Br}(\mu \rightarrow e\gamma)\) | \(2.4 \times 10^{-12}\) [24] | \(1.0 \times 10^{-13}\) [20] |
| \(\text{Br}(\mu \rightarrow \mu\mu)\) | \(1.0 \times 10^{-12}\) [23] | \(1.0 \times 10^{-13}\) [20] |
| \(\text{Br}(\mu \rightarrow e\gamma)\) | \(7.0 \times 10^{-13}\) [24] | \(2.0 \times 10^{-17}\) [25] |

Varying over the oscillation parameters, one expects very similar behavior for the normal and inverted ordering (there are only tiny differences because the indirect constraints from other LFV processes depend on the mass ordering). Therefore, we only plot the normal ordering case in Fig. 2. As can be seen, with lighter \(v_\Delta = 0.5\) and 1.0 eV, the region with lighter triplet mass is excluded by the other LFV constraints. With the present constraints, there exists no allowed region for \(v_\Delta = 0.5\) eV and \(m_1^{(3)} = 0.2\) eV. Obviously with heavier triplet mass
FIG. 2: Plots showing the variation of the lowest possible Br($\mu \rightarrow e\gamma$) vs. $M_{H^\pm\pm}$ with different values of $v_\Delta$ for the normal neutrino mass ordering. The left plots are considering the present constraints on different LFV processes and the right ones are with the future constraints. Plots in the upper row are with the lightest neutrino mass $m_1 = 0.003$ eV, the middle row is for $m_1 = 0.05$ eV and the lower row is for $m_1 = 0.2$ eV. The solid (dotted) line corresponds to the $3\sigma$ ($1\sigma$) range of the oscillation parameters. The colored (dark) band corresponds to the exclusion region as suggested by present and future experimental bounds. All constraints are listed in Table VII. The corresponding plots for the inverted ordering look basically identical.

(M$_{H^\pm\pm} > 1$ TeV), such conclusion no longer remains valid. Nevertheless, scenario with a very heavy triplet has less appealing collider phenomenology. We have noted that throughout all the parameter space $\mu-e$ conversion poses the most stringent bounds. With the future constraints, exclusion of the entire region with any values of triplet mass and for $v_\Delta = 0.5$ and 1.0 eV, is solely due to the very stringent future $\mu-e$ conversion constraint 22. As can be seen from Fig. 2 pushing the branching ratio of $\mu \rightarrow e\gamma$ down to $10^{-15}$ makes it possible to definitely probe regions of parameter space of $v_\Delta$ and $M_{H^\pm\pm}$. Examples are if $v_\Delta \lesssim 5$ eV and $M_{H^\pm\pm} \lesssim 200$ GeV, or when $v_\Delta \sim 1$ eV and $M_{H^\pm\pm} \lesssim 700$ GeV. We stress again that before the recent results on large $U_{e3}$ were obtained this was not possible. The effects of the constraints of the other LFV modes on the minimal value of Br($\mu \rightarrow e\gamma$) can be seen in Fig. 3. Two implications are resulting when one switches on the other LFV limits: (i) the scale of $M_{H^\pm\pm}$ is set to larger values, and (ii) the lower limit on the branching ratio is increased by a moderate amount.

V. SUMMARY

Lepton Flavor Violation (LFV) may be connected directly or indirectly to neutrino oscillation parameters. In
\[ \sigma, m_1 = 0.003 \text{ eV, } v_\Delta = 0.5 \text{ eV, Normal ordering, Lowest of } Br(\mu \rightarrow e\gamma). \]

\[ Br(\mu \rightarrow e\gamma). \]

\[ no - constraints applied \]

\[ \text{only } \tau \rightarrow e\gamma \text{ applied} \]

\[ \text{only } \tau \rightarrow \mu\gamma \text{ applied} \]

\[ \text{only } \mu N \rightarrow eN' \text{ applied} \]

\[ \text{only } \mu \rightarrow ee \text{ applied} \]

FIG. 3: Plots showing the effects of the constraints on other LFV processes on the minimal branching ratio of \( \mu \rightarrow e\gamma \). The colored (dark) band corresponds to the exclusion region as suggested by the present experimental bound.

this paper we worked in scenarios with presumably the most direct connection, in which the quantity \( m_\nu m_\nu^\dagger \) is responsible for LFV in the charged lepton sector. Minimal flavor violation in the lepton sector, as well as other frameworks and scenarios, has such a feature. We noted that recent results on the lepton mixing parameter \( U_{e3} \) imply that the \( (m_\nu m_\nu^\dagger)_{e\mu} \) cannot vanish. Consequently, lower limits on lepton flavor violation arise, and we have quantified this with the example of \( \mu \rightarrow e\gamma \) in the type II seesaw mechanism, in which a Higgs triplet is responsible for neutrino mass. We stress that many more examples in which our finding applies can be discussed.

We also shortly discussed processes as \( \mu \rightarrow 3e \) and \( \mu - e \) conversion, where \( (m_\nu m_\nu^\dagger)_{e\mu} \) is also of relevance. However, either the contribution of \( (m_\nu m_\nu^\dagger)_{e\mu} \) is suppressed, or cancellations from other contributions can occur. Setting lower limits in the same sense as for \( \mu \rightarrow e\gamma \) is not possible.

While searches for lepton flavor violation do not need further motivation, we feel that our observation closes yet another loophole that would allow LFV to hide, and adds additional interest to study LFV in the \( e\mu \) sector.

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