Induced scattering and two quantum absorption of Alfvén waves with arbitrary angles of propagation.

V.N.Zirakashvili

Institute for Terrestrial Magnetism, Ionosphere and Radiowave Propagation, 142190, Troitsk, Moscow Region, Russia

Abstract

The equation for temporary evolution of spectral energy of collisionless Alfvén waves is derived in framework of weak turbulence theory. The main nonlinear processes for such conditions are induced scattering and two quantum absorption of Alfvén waves by thermal ions. The equation for velocity distribution of thermal particles is derived that describes diffusion in momentum space due to this nonlinear processes. Comparison is done with the results of another authors. Results obtained are qualitatively differ from the ones obtained for the case of Alfvén waves propagation along mean magnetic field.

Introduction

It is well known that Alfvén waves are very important agent of cosmic plasma. It is really believed now that main part of energy of interplanetary and interstellar magnetohydrodynamic (MHD) turbulence is contained in Alfvén waves. This is because other types of low frequency waves - ionsound and magnetosonic waves are strongly damped due to linear Landau damping. For Alfvén waves such a damping is relatively small and nonlinear effects should be taken into account in order to investigate dissipation of MHD turbulence. It is well known now that the main nonlinear processes for such waves are induced scattering and two quantum absorption by thermal ions. Corresponding damping rates were calculated by many authors [1-7]. Nevertheless totally comprehensive study is absent now. Firstly induced scattering of Alfvén waves was considered in paper [1] but for $\beta \ll 1$. Here $\beta$ is the square of the ratio of thermal ion velocity $v^2_{Ti} = \sqrt{T_i/m_i}$ ($T_i$ - is ion temperature in energetic units, $m_i$ is ion mass) and Alfvén velocity $v_a = B_0/\sqrt{4\pi\rho_0}$ ($B_0$ is the mean magnetic field strength, $\rho_0$ is the mean plasma density). In papers [2-5] the case of wave propagation along mean magnetic field was considered. At last in papers [6,7] the oblique propagation of Alfvén waves was considered but random magnetic field component along mean magnetic field arising from magnetic pressure of Alfvén waves was not taking into account. As will be shown in this paper the last effect strongly changes resultant damping rates. In particular, the interaction of waves with the same signs of wavevector components along mean magnetic field is impossible.

Basic equations.

Kinetic approach is the most adequate method in nonlinear plasma theory. In framework of weak turbulence it assumes the expansion of velocity distribution of thermal particles in series on powers of random electric and magnetic fields and calculation of nonlinear electric currents which are substituted in Maxwell equations. Such a procedure applied for Alfvén waves results in cumbersome expressions containing multiple series of Bessel functions (cf. [1]). If the case of magnetized thermal particles is considered this functions should be expanded on powers of theirs small argument. These calculations are very tedious. We shall use another approach here that seems also more attractive from physical point of view. If wavelengths are much larger then thermal particles gyroradii it is possible to use equations averaged on gyrorotation. This

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means that we shall neglect by all dispersive effects related with finite particle gyroradius. The system of equations can be written as follows [8]:

\[ \rho \left( \frac{\partial u}{\partial t} + (u \nabla) u \right) = -\frac{1}{4\pi} [B \times [\nabla \times B]] - \nabla P_\perp - (\nabla b)(P_\parallel - P_\perp) \] (1)

\[ \frac{\partial F_\alpha}{\partial t} + (u_E + v_\parallel b) \nabla F_\alpha - \frac{v_\perp}{2} \frac{\partial F_\alpha}{\partial v_\perp} (v_\parallel \nabla b + \nabla u_E + u_E(b \nabla)b) + \frac{\partial F_\alpha}{\partial v_\parallel} \left( \frac{q_\alpha}{m_\alpha} F_\parallel + \frac{v_\perp^2}{2} \nabla b + u_E \left( v_\parallel (b \nabla)b + (u_E(b \nabla)b + \frac{\partial b}{\partial t}) \right) \right) = 0 \] (2)

\[ \frac{\partial B}{\partial t} = [\nabla \times [u \times B]] \] (3)

\[ \nabla B = 0 \] (4)

\[ \sum_\alpha q_\alpha \int d^3v F_\alpha = 0 \] (5)

where \( b = B/B \) is a unit vector along magnetic field \( B \), \( u_E = c[E \times B]/B^2 \) is electric drift velocity, \( E_\parallel \) is electric field component along magnetic field, \( \rho \) and \( u \) are plasma density and velocity, \( F_\alpha(v_\parallel, v_\perp) \), \( q_\alpha, m_\alpha \) are velocity distribution, electric charge and mass of thermal species \( \alpha \). Perpendicular and parallel pressures in Eq. (1) are given by the formulae

\[ P_\parallel = \sum_\alpha \int d^3v m_\alpha (v_\parallel - ub)^2 F_\alpha, \] (6)

\[ P_\perp = \sum_\alpha \int d^3v m_\alpha v_\perp^2 F_\alpha \] (7)

Guiding center equation (2) takes into account kinetic effects of thermal particles. For the case of frozen magnetic field that is considered here electric drift velocity \( u_E \) is simply perpendicular to magnetic field component of medium velocity \( u \).

**Expansion on powers of medium velocity.**

Let mean magnetic field \( B_0 \) is in \( z \)-direction. Let us write all quantities in a form \( B = B_0 + \delta B \), \( \rho = \rho_0 + \delta \rho \), etc. and expand them in Fourier series: \( \delta B = \sum_k B_k \exp(ikr - i\omega t) \), \( \delta \rho = \sum_k \delta \rho_k \exp(ikr - i\omega t) \) etc., \( k = (k, \omega) \). All these Fourier components should be expanded on powers of medium velocity due to presence of linear polarized Alfvén waves. As for linear theory of Alfvén waves \( \delta \rho_k = 0, E_\parallel k = 0, B_\parallel k = 0, u_{\perp k} = 0, u_{\parallel k} = 0 \), these quantities should be considered as second order ones. Performing expansion in Eq. (2) one can obtain second order \( \delta F_{\alpha k} \). Substitution of this quantity into quasineutrality equation (6) permits to find \( E_\parallel k \). As a result

\[ \delta F_{\alpha k} = \frac{B_{\parallel k}}{B_0} \left[ \frac{k_z}{v_\parallel k_z - \omega} \left( \frac{v_\perp^2}{2} - \frac{q_\alpha}{m_\alpha} \sigma_1(k) \right) \frac{\partial F_{0\alpha}}{\partial v_\parallel} - \frac{v_\perp}{2} \frac{\partial F_{0\alpha}}{\partial v_\perp} \right] + \]

\[ + \frac{1}{2} \sum_{k' k''} (u_{k}\cdot u_{k''}) \left[ -\frac{k_z' k_z''}{2\omega'\omega''} v_\perp \frac{\partial F_{0\alpha}}{\partial v_\perp} + \right. \]

\[ + \frac{k_z}{v_\parallel k_z - \omega} \frac{\partial F_{0\alpha}}{\partial v_\parallel} \left( \frac{v_\perp^2}{2} \frac{k_z' k_z''}{\omega' \omega''} + \frac{v_\parallel}{k_z} \left( \frac{k_z'^2}{\omega'} + \frac{k_z''^2}{\omega''} \right) - 1 + \frac{q_\alpha}{m_\alpha} \sigma_0(k', k'') \right], \] (8)
\[ E_{\parallel k} = i k_z \left[ \frac{B_{z k}}{B_0} \sigma_1(k) - \sum_{k=k'+k''} \frac{1}{2} \sigma_0(k', k'') (\mathbf{u}_{k'} \cdot \mathbf{u}_{k''}) \right] . \] (9)

For simplicity of notation we shall omit mean value of the product \( \mathbf{u}_{k'} \cdot \mathbf{u}_{k''} \) that should be extracted from correspondent product. Quantities \( \sigma_0(k', k'') \) and \( \sigma_1(k) \) in expressions (8) and (9) can be expressed in terms of velocity distribution of thermal particles:

\[
\sigma_0(k', k'') = \left[ \sum_\alpha \frac{q_\alpha^2}{m_\alpha} \int \frac{d^3v}{v_\parallel (k_z' + k_z'') - \omega' - \omega''} \frac{\partial F_{0\alpha}}{\partial v_\parallel} \right]^{-1} .
\]

\[
\sigma_1(k) = \left[ \sum_\alpha \frac{q_\alpha^2}{m_\alpha} \int \frac{d^3v}{v_\parallel k_z - \omega} \frac{\partial F_{0\alpha}}{\partial v_\parallel} \right]^{-1} \sum_\alpha q_\alpha \int \frac{d^3v}{v_\parallel k_z - \omega} \frac{v_\parallel^2}{2} \frac{\partial F_{0\alpha}}{\partial v_\parallel} .
\] (10)

Let us calculate \( z \)-component of random magnetic field \( B_{z k} \) now. In order to do this one should expand Fourier transform of Eq. (1,3) to the second order and multiply Eq. (1) on \( k_z \). The final result can be written as follows

\[
\frac{B_{z k}}{B_0} = \sum_{k=k'+k''} \frac{(\mathbf{u}_{k'} \cdot \mathbf{k}') (\mathbf{u}_{k''} \cdot \mathbf{k}'')}{v_\alpha \Delta(k)} \left( \frac{k_z' k_z''}{\omega' \omega''} - 1 \right) - \frac{1}{2} \sum_{k=k'+k''} (\mathbf{u}_{k'} \cdot \mathbf{u}_{k''}) \frac{k_z'}{\omega'} + \frac{k_z''}{\omega''} + k_z' \sum_\alpha \frac{m_\alpha}{\rho_0} \int \frac{d^3v k_z}{v_\parallel k_z - \omega} \frac{v_\parallel^2}{2} \frac{\partial F_{0\alpha}}{\partial v_\parallel} \left( \frac{v_\parallel}{k_z} \left( \frac{k_z'^2}{\omega'} + \frac{k_z''^2}{\omega''} \right) - 1 + \frac{q_\alpha}{m_\alpha} \sigma_2(k', k'') \right) .
\] (12)

Here quantity \( \sigma_2(k', k'') \) is given by the expression

\[
\sigma_2(k', k'') = \left[ \sum_\alpha \frac{q_\alpha^2}{m_\alpha} \int \frac{d^3v}{v_\parallel (k_z' + k_z'') - \omega' - \omega''} \frac{\partial F_{0\alpha}}{\partial v_\parallel} \right]^{-1} .
\]

\[
\cdot \sum_\alpha q_\alpha \int \frac{d^3v}{v_\parallel (k_z' + k_z'') - \omega' - \omega''} \frac{\partial F_{0\alpha}}{\partial v_\parallel} \left( 1 - \frac{v_\parallel}{k_z'} \left( \frac{k_z'^2}{\omega'} + \frac{k_z''^2}{\omega''} \right) \right) .
\] (13)

and denominator

\[
\Delta(k) = k_z^2 D_0(k) + (c_\alpha^2 k_z^2 - \omega^2) / v_\alpha^2
\] (14)
corresponds to dispersion relation of magnetosonic wave \( \Delta(k) = 0 \).

The velocity \( c_\alpha = \sqrt{v_\alpha^2 - (P_{\perp 0} - P_{\parallel 0}) / \rho_0} \) substitutes Alfvén velocity \( v_\alpha \) in plasma with anisotropic pressure. We consider here the case of firehose stable plasma when the expression under square root is positive. The quantity \( D_0(k) \) is given by the expression

\[
D_0(k) = 1 + \frac{8 \pi P_{\perp 0}}{B_0^2} + \frac{4 \pi P_{\parallel 0}}{B_0^2} \sum_\alpha m_\alpha \int d^3v \frac{k_z}{v_\parallel k_z - \omega} \left( \frac{v_\parallel^2}{2} - \frac{q_\alpha}{m_\alpha} \sigma_1(k) \right) \frac{v_\parallel^2}{2} \frac{\partial F_{0\alpha}}{\partial v_\parallel} .
\] (15)
Calculation of damping rates.

Standard method of calculation of damping rates uses the expansion of equations (1-3) up to the third order of medium velocity. We use here more simple method that gives the same result. We shall derive equations for thermal particles and find probability of scattering. Practically one should derive quasilinear equation from Eq. (2). The only difference is that derivatives on velocity in Eq. (2) are multiplied on factors of second or higher order. This equation can be written as follows:

\[
\frac{\partial F_{0\alpha}}{\partial t} = \frac{\partial}{\partial v} \sum_k \frac{B_{zk}}{B_0} \left( \frac{\nu^2}{2} - \frac{q_\alpha}{m_\alpha} \sigma_1(k) \right) + \frac{1}{2} \sum_{k=k'+k''} (u_k' u_{k''}) \left( \frac{\nu^2}{2} \frac{k'_z k''_z}{\omega' \omega''} + \nu \left( \frac{k'_z^2}{\omega'} + \frac{k''_z^2}{\omega''} \right) - 1 + \frac{q_\alpha}{m_\alpha} \sigma_0(k', k'') \right)^2 \frac{k_z^2}{i(v_k - \omega)} \frac{\partial F_{0\alpha}}{\partial v}
\]

Substituting expression (12) for \(B_{zk}\) and performing assemble averaging one can obtain

\[
\frac{\partial F_{0\alpha}}{\partial t} = \frac{\partial}{\partial v} \sum_{k', k''} W(k')W(k'') \left[ (k'_z + k''_z)^2 w(k', k'', \omega(k'), \omega(k'')) + (k'_z - k''_z)^2 w(k', -k'', \omega(k'), -\omega(k'')) \right] \frac{\partial F_{0\alpha}}{\partial v}.
\]

Here we use the expression

\[
< u_k u_{k'} > = \frac{\delta(k + k')}{2 \rho_0} \left( W(k) \delta(\omega - \omega(k)) + W(-k) \delta(\omega + \omega(k)) \right),
\]

where \(W(k)\) is spectral energy density of Alfvén waves with dispersion relation \(\omega(k) = c_\alpha |k_z|\). It’s normalized such that

\[
\frac{\langle \delta B^2 \rangle}{4\pi} \frac{c_\alpha^2}{v_a^2} = \sum_k W(k).
\]

First term in square brackets of expression (17) corresponds to two quantum absorption, the second one corresponds to induced scattering. The probability of two quantum absorption \(w(k', k'', \omega(k'), \omega(k''))\) is given by the formula

\[
w(k', k'', \omega(k'), \omega(k'')) = \frac{\pi}{4 \rho_0^2 v_a^4} \delta(\omega(k') + \omega(k'') - v_{||}(k'_z + k''_z)) \cdot \frac{1}{\Delta(k' + k'')} \left( \frac{\nu^2}{2} - \frac{q_\alpha}{m_\alpha} \sigma_1(k) \right) \left[ 2k'_z k''_z \sin^2 \varphi(k'_z, k''_z) \left( c_\alpha^2 k'_z k''_z \omega' \omega'' - 1 \right) - \cos \varphi(k'_z, k''_z) \left( \frac{k'_z k''_z}{\omega' \omega''} \left( (\omega' + \omega'')^2 - c_\alpha^2 (k'_z + k''_z)^2 \right) + (k'_z + k''_z)^2 \right) \right] \cdot \sum_{\alpha} \frac{m_\alpha}{\rho_0} \rho_0 \int \frac{d^3v}{v_{||}(k'_z + k''_z) - \omega' - \omega''} \frac{\nu^2}{2} \frac{\partial F_{0\alpha}}{\partial v_{||}} \left( \nu_\parallel \left( \frac{k'_z^2}{\omega'} + \frac{k''_z^2}{\omega''} \right) - 1 + \frac{q_\alpha}{m_\alpha} \sigma_2(k', k'') \right) + v_a^2 \cos \varphi(k'_z, k''_z) \left( \frac{\nu}{k'_z + k''_z} \left( \frac{k'_z^2}{\omega'} + \frac{k''_z^2}{\omega''} \right) - 1 + \frac{q_\alpha}{m_\alpha} \sigma_2(k', k'') \right)^2 \left( \omega' = \omega(k') \omega'' = \omega(k'') \right).
\]
Here $\varphi(k'_\perp, k''_\perp)$ is the angle between $k'_\perp$ and $k''_\perp$. The spectral energy density of Alfvén waves evolves according to the equation

$$\frac{\partial W(k)}{\partial t} = -2\Gamma(k)W(k)$$  \hfill (20)

where damping rate can be expressed in terms of velocity distribution of thermal particles and probabilities of scattering and two quantum absorption [9,10]:

$$\Gamma(k) = -\omega(k)\sum_{a,k'}m_{a}W(k')\int d^3v \left[(k_z + k'_z)w(k, k', \omega(k), \omega(k')) + (k_z - k'_z)w(k, -k', \omega(k), -\omega(k'))\right]\frac{\partial F_{0a}}{\partial v_{||}}$$ \hfill (21)

It is easy to see from expression (19) that waves with the same signs of $k_z$ can not interact. It is not so if one tend to zero $k'_\perp$ and $k''_\perp$. This means that one-dimensional case is particular one (see Discussion).

It is possible to transform damping rate (21) containing probability (19) to the more convenient for applications form:

$$\Gamma(k) = \frac{1}{4}\omega(k)Im\sum_{k'_\perp}W(k')\left[1 - \frac{1}{c^2_a}\left(\frac{\omega \pm \omega'}{k_z \pm k'_z}\right)^2\right] D_1(k \pm k') \cos^2 \varphi(k'_\perp, k''_\perp) +$$

$$+\frac{(k'_\perp \pm k''_\perp)^{-2}}{\Delta(k \pm k')} \left\{2k'_\perp k''_\perp \left(1 - \frac{c^2_a}{\omega \omega'} \frac{k_z k'_z}{k_z k''_z}\right) \sin^2 \varphi(k'_\perp, k''_\perp) \pm$$

$$\pm \cos \varphi(k'_\perp, k''_\perp) \left[1 - \frac{1}{c^2_a} \left(\frac{\omega \pm \omega'}{k_z \pm k''_z}\right)^2\right]\left(D_2(k \pm k')(k'_\perp \pm k''_\perp)^2 - \frac{c^2_a k_z k'_z}{\omega \omega'}(k_z \pm k_z)^2\right)\right\}^{\omega = \omega(k), \omega' = \omega(k')} \hfill (22)$$

and quantities $D_1(k)$ and $D_2(k)$ can be expressed in terms of velocity distribution of thermal particles:

$$D_1(k) = -v^2_\alpha \sum_{a} \frac{m_{a}}{\rho_0} \int \frac{d^3v_{k_z}}{v_{||}k_z - \omega} \left(1 - \frac{q_a}{m_{a}} \sigma_3(k)\right) \frac{\partial F_{0a}}{\partial v_{||}} \hfill (23)$$

$$D_2(k) = -\sum_{a} \frac{m_{a}}{\rho_0} \int \frac{d^3v_{k_z}}{v_{||}k_z - \omega} \left(1 - \frac{q_a}{m_{a}} \sigma_3(k)\right) \frac{v^2_{\perp}}{2} \frac{\partial F_{0a}}{\partial v_{||}} \hfill (24)$$

where $\sigma_3(k)$ is given by the expression

$$\sigma_3(k) = \left[\sum_{a} \frac{q^2_a}{m_{a}} \int \frac{d^3v}{v_{||}k_z - \omega} \frac{\partial F_{0a}}{\partial v_{||}}\right]^{-1} \sum_{a} q_a \int \frac{d^3v}{v_{||}k_z - \omega} \frac{\partial F_{0a}}{\partial v_{||}} \hfill (25)$$

It should be noted that damping rate (22) can be derived directly from expansion of Fourier transforms of Eq. (1-3) up to the third order of medium velocity but takes more tedious algebra.
Damping rates for Maxwellian plasma.

Strictly speaking collision term should be added to the right-hand side of equation (17). It tends to make the velocity distribution Maxwellian. It will be so if frequency of collisions is high enough. On the other hand frequency of thermal collisions can be small enough for validity of collisionless approximation. For Maxwellian plasma nonlinear damping rate (22) can be expressed in terms of function $J_+(x)$ [1]:

$$J_+(x) = -i \sqrt{\frac{\pi}{2}} x \exp \left( -\frac{x^2}{2} \right) + i \frac{x}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{dx'}{x-x'} \exp \left( -\frac{x'^2}{2} \right) \tag{26}$$

It contains principal value integral.

For the sake of simplicity we shall consider pure hydrogen plasma. Damping rate is mainly determined by thermal ions. Input of thermal electrons is $\sqrt{m_i/m_e}$ times smaller. The dependence of quantities $D_0(k), D_1(k), D_2(k)$ on frequency and wavenumber is then reduced to dependence on $x = \frac{\omega}{v_{Te}[k_z]}$:

$$D_0(x) = 1 + 2\beta J_+(x) + \frac{J_+^2(x)}{1 + \frac{T_i}{T_e} - J_+(x)} \tag{27}$$

$$D_1(x) = \beta^{-1} \frac{T_i}{T_e} \frac{1 - J_+(x)}{1 + \frac{T_i}{T_e} - J_+(x)} \tag{28}$$

$$D_2(x) = \left( 1 + \frac{T_i}{T_e} \right) \frac{1 - J_+(x)}{1 + \frac{T_i}{T_e} - J_+(x)} \tag{29}$$

It was assumed here that ions and electrons can be at different temperatures $T_i$ and $T_e$. We shall consider two extreme cases below.

$\beta \ll 1$.

For this case damping rate can be written in the form

$$\Gamma(k_z, k_\perp) = \frac{\omega(k)}{4} \sum_{k'} \theta(-k_z k'_z) \frac{W(k')}{B^2_\perp/4\pi} \left[ \frac{v_a^2 T_i^2}{v_T^2 T_e^2} \left( 1 - Re J_+(x) + T_i/T_e \right)^2 + \frac{\pi}{2} x^2 \exp(-x^2) \right]_{x = \frac{\omega(k)}{v_T}} +$$

$$+ \delta(4k_z k'_z + (k_\perp + k'_\perp)^2) \frac{\pi}{(k_\perp + k'_\perp)^2} \left( 4k_z \sin^2 \varphi(k_\perp, k'_\perp) - (k_\perp + k'_\perp)^2 \cos^2 \varphi(k_\perp, k'_\perp) \right)^2 \tag{30}$$

where $\theta(x)$ is step function.

The first term in expression (30) describes induced scattering of Alfvén waves which can be interpreted as generation of ionsound wave by pair of Alfvén waves that is absorbed by thermal particles. For the case considered scattering is differential, that is interaction of Alfvén waves with close absolute values of $k_z$ is possible only. The second term describes quantum absorption of Alfvén waves which can be interpreted as generation of magnetoionic wave that is absorbed by thermal particles also. Expansion of spectral energy density of Alfvén waves in the vicinity of $-k_z$ and change of summation to integration results in

$$\Gamma(k_z, k_\perp) = \frac{v_a k_z^2}{B^2_\perp/4\pi} \int d^2 k_\perp \left[ -\alpha_1 \left( W(-k_z, k'_\perp) + k_z \frac{\partial}{\partial k_z} W(-k_z, k'_\perp) \right) \cos^2 \varphi(k_\perp, k'_\perp) +$$

$$+ \delta(4k_z k'_z + (k_\perp + k'_\perp)^2) \frac{\pi}{(k_\perp + k'_\perp)^2} \left( 4k_z \sin^2 \varphi(k_\perp, k'_\perp) - (k_\perp + k'_\perp)^2 \cos^2 \varphi(k_\perp, k'_\perp) \right)^2 \right]$$
\[ + W \left( -\frac{(k_\perp + k'_\perp)^2}{4k_\perp^2}, k'_\perp \right) \frac{\pi k_\perp^{-2}}{16(k_\perp + k'_\perp)^2} \left( 4k_\perp k'_\perp \sin^2 \varphi(k_\perp, k'_\perp) - (k_\perp + k'_\perp)^2 \cos \varphi(k_\perp, k'_\perp) \right)^2 \],

and quantity \( \alpha_1 \) is given by integral

\[ \alpha_1 = \sqrt{\frac{\pi T_i^2}{2 T_e^2}} \int_{-\infty}^{+\infty} dx \frac{x^2 \exp(-x^2/2)}{(1 - ReJ_+(x) + T_i/T_e)^2 + \frac{7}{2} x^2 \exp(-x^2)} = \pi \] (32)

First term in expression (31) corresponds to result obtained by Livshits and Tsytovich [1]. The only difference is the value of factor \( \alpha_1 \) which depends on \( T_i/T_e \) in [1]. It is because in [1] simplifying assumption \( x = 0 \) in the denominator of integral (32) was used. The result obtained here is more correct and can be derived in framework of standard magnetohydrodynamics.

For this case expression (22) can be reduced to

\[ \Gamma(k_z, \mathbf{k}_\perp) = \frac{\omega(k)}{4} \frac{v_\perp^2}{v_\parallel^2} \sum_{\mathbf{k}'} \theta(-k_z k'_z) \frac{W(k')}{B_0^2/4\pi} \cos^2 \varphi(k_\perp, k'_\perp). \]

\[ \cdot Im \frac{1 - J_+(x)}{1 + \frac{T_i}{T_e} - J_+(x)} x^4 \left[ T_i \frac{1 + T_i}{T_e} J_+(x) \left[ 2 + 2T_i \frac{T_i}{T_e} \right] \right] \]

(33)

The damping rate (33) is determined by two quantum absorption that is again differential on absolute value of \( k_z \). Performing expansion similarly to previous case one can obtain

\[ \Gamma(k_z, \mathbf{k}_\perp) = \alpha_2 \frac{v_T k_z^2}{B_0^2/4\pi} \int d^2 k'_\perp W(-k_z, k'_\perp) \cos^2 \varphi(k_\perp, k'_\perp) \] (34)

where \( \alpha_2 \) is given by the expression:

\[ \alpha_2 = \int_0^{+\infty} dx Im \frac{1 - J_+(x)}{1 + \frac{T_i}{T_e} - J_+(x)} \left[ T_i \frac{1 + T_i}{T_e} J_+(x) \left[ 2 + 2T_i \frac{T_i}{T_e} \right] \right] \]

(35)

Numeric integration shows that this quantity practically does not depend on \( T_i/T_e \). Numeric value \( \alpha_2(T_i = T_e) = 2.25 \).

**Discussion.**

The main result of this paper is the expression for nonlinear damping rate (22). Results obtained differ from results [6,7] where oblique Alfvén waves were also considered. We take into account components of random field, corresponding to magnetosonic waves. As magnetosonic waves are strongly damped (especially in \( \beta > 1 \) plasma, exceptions are quasiparallel and quasiperpendicular propagation), these components should be considered as second order quantities and expressed in terms of medium velocity in Alfvén waves. Simply speaking we treat these quantities similarly to parallel electric field component \( E_\parallel \). This approach is correct if energy density of Alfvén waves is not concentrated in wavevectors at small angles:

\[ < \delta B^2 > /B_0^2 \ll k_\perp^2/k_z^2 \] (36)
For the opposite relation one-dimensional results [2-5] and results [6,7] are valid. The results of these papers can be readily obtained if one let $B_{zk} = 0$ in Eq. (16) or tend $k_\perp, k'_\perp$ to zero in expressions (19) and (22):

$$\Gamma(k) = \frac{1}{4} \omega(k) Im \sum_{k',\pm} \frac{W(k')}{{B_0^2/4\pi}} \cos^2 \varphi(k_\perp, k'_\perp) \left[ \left( 1 - \frac{1}{c^2_0} \left( \frac{\omega \pm \omega'}{k_z \pm k'_z} \right)^2 \right)^2 D_1(k \pm k') - \frac{v^4_0 k_z k'_z^2}{\omega^2 \omega'^2} D_0(k \pm k') - 2 \frac{v^2_0 k_z k'_z}{\omega \omega'} \left( 1 - \frac{1}{c^2_0} \left( \frac{\omega \pm \omega'}{k_z \pm k'_z} \right)^2 \right) D_2(k \pm k') \right]_{\omega = \omega(k)} \omega' = \omega(k')$$

(37)

In particular interaction of waves with the same signs of $k_z$ is possible in this case. It is described by the second term of damping rate (37). Derivation of damping rate (37) shows that in order to obtain validity condition for approach used in this paper one should compare terms $(k_z \pm k'_z)^2 - (\omega \pm \omega')^2/c^2_0$ and $(k \pm k')^2$, in denominator $\Delta(k \pm k')$ of damping rate (22). For the waves with the same signs of $k_z$ the fist term is equal to zero if dispersion relation for Alfvén waves is used. It does not equal to zero if one takes into account nonlinear shift of frequency that is of the same order as nonlinear damping rate. This procedure results in condition (36).

For $\beta \ll 1$ linear damping of magnetosonic waves is smaller than for the case $\beta > 1$ and is due to Landau damping on thermal electrons [11]:

$$\gamma = \sqrt{\frac{\pi}{8}} \frac{V{T_e}}{m_e} \frac{k_z^2}{|k_z|} \frac{m_e}{m_i}$$

(38)

and approach used in this paper is valid for stronger condition:

$$\frac{< \delta B^2 >}{B_0^2} \ll \sqrt{\frac{m_e T_e}{m_i T_i}} \beta \frac{k_z^2}{k_z^2}$$

(39)

For the opposite relation linear damping of magnetosonic waves can be disregarded and results [1] are valid. Three wave interactions of Alfvén and magnetosonic waves [11] should be also taken into account in this case.

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