Adaptive Fuzzy Tracking Control With Global Prescribed-Time Prescribed Performance for Uncertain Strict-Feedback Nonlinear Systems

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Abstract—For strict-feedback systems with mismatched uncertainties, adaptive fuzzy control techniques are developed to provide global prescribed performance with prescribed-time convergence. First, a class of prescribed-time prescribed performance functions are designed to quantify the performance constraints of the tracking error. Additionally, a novel error transformation function is provided to eliminate the initial value limitations and resolve the singularity issue in previous research. To ensure the convergence of the tracking error into a prescribed bounded region within a prescribed time and satisfactory transient performance, controllers with or without approximating structures are established. Notably, the settling time and initial condition of the prescribed performance function are completely independent of the initial tracking error and system parameters, thereby improving upon existing results. Furthermore, the disadvantage of the semi-global boundedness of tracking error induced by dynamic surface control can be eliminated through the use of a novel Lyapunov-like energy function. Finally, the effectiveness of the proposed strategies is validated through numerical simulations performed on practical examples.

Index Terms—Adaptive fuzzy control, global prescribed performance, mismatched uncertainty, prescribed time, strict-feedback systems.

I. INTRODUCTION

TRAJECTORY tracking is a fundamental problem in the control community with wide applications in various fields, including tracking [1], [2], leader-following tracking [3], [4], formation–containment tracking [5], [6], bipartite tracking [7], [8], average tracking [9], [10], and complex networks [11], [12]. To guarantee the tracking performance, prescribed performance control [13] is commonly employed to force the tracking error into a sufficiently small prescribed region with a prescribed decay rate and a desired transient state performance. A lot of attention has been paid to prescribed performance control so far and fruitful results have been developed.

For uncertain multi-input–multi-output (MIMO) systems, a robust adaptive controller with exponential prescribed performance function was proposed in 2008 [13], which pioneered the methodology of prescribed performance control. Subsequently, the prescribed performance control for MIMO nonlinear systems [14], [15], [16], [17], [18], uncertain strict-feedback systems [19], [20], [21], [22], [23], [24] and high-power nonlinear systems [25] was studied through different control techniques. With hysteretic actuator nonlinearity and faults, the problem of adaptive fuzzy prescribed performance control for nonlinear systems was solved via the command filter theory [26]. In [27], the prescribed performance control was extended to the leader-following consensus for uncertain nonlinear strict-feedback multiagent systems under directed communication networks. Nussbaum-type functions and fuzzy logic systems were introduced to solve the problem of unknown control directions and nonlinearities, respectively. However, the Nussbaum-type function expands the dynamic order of the closed-loop systems, and fuzzy logic systems may lead to semi-global boundedness of all closed-loop signals. To this end, decentralized control laws of low complexity in the sense of no prior knowledge of system nonlinearity, no approximating structures, no complex calculations, and static control protocols, were proposed [28], [29].

Event-triggered control, known for its energy-saving capabilities, was introduced to explore prescribed performance control strategies for pure-feedback systems with unmeasured states and unknown nonlinearities [30]. Adaptive fuzzy dynamic surface control was employed in [31] to resolve the differential explosion problem in backstepping

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techniques [19], [30], and an error-driven nonlinear feedback function was designed to establish the semi-global boundedness of the closed-loop system. Via unifying control [32], the global prescribed performance control for high-order systems was achieved, improving the semi-global stability [15], [16], [17], [18], [26], [27], [30], [31], [33]. Moreover, the conventional prescribed performance control was significantly modified to handle discontinuities in desired trajectory [34] and time-varying delays in both state measurements and control inputs [35].

It should be stressed that only asymptotic or exponential convergence of the tracking error system can be guaranteed in all aforementioned results, which restricts their practical applicability under finite-time constraints. Accordingly, a finite-time performance function was defined in [36], and semi-globally practical finite-time tracking for a class of uncertain nonstrict feedback nonlinear systems was achieved using an adaptive neural network controller. Following this idea, the finite-time prescribed performance control was studied for nonstrict feedback nonlinear systems with adaptive fuzzy control [37], [38], MIMO nonlinear systems [39], multivariable strict-feedback nonlinear systems with neural network control [40], strict-feedback nonlinear systems with disturbance observer-based control [41], and stochastic nonlinear systems with adaptive backstepping control [42]. Based on exponential performance functions, finite-time adaptive fuzzy control with an event-triggered mechanism for uncertain strict-feedback nonlinear systems [43] and multiagent systems [44] was established. In addition, a fixed-time version was developed for uncertain robot systems [45]. Via novel integral sliding mode control techniques, the problem of finite-time exact tracking control with prescribed performance for uncertain strict-feedback nonlinear systems is addressed in [46]. Particularly, fuzzy logic systems [36], [37], [47], [48], [49] and dynamic surface control [38], [40], [41], [43], [44] were employed to address the differential explosion problem, thereby avoiding complex calculations.

Finite-fixed-time stability ensures higher convergence accuracy, faster convergence rate, and better anti-interference ability than asymptotic or exponential one. However, the settling-time function is independent of either the initial states or the system parameters. Therefore, the settling time generally cannot be prescribed in advance by users. It is even challenging to obtain the explicit convergence time under unavailable initial states or system parameters.

Currently, few results have been reported on the prescribed-time prescribed performance control. In [36], [48], and [50], a prescribed-time performance function was developed, leading to semi-global tracking. Using a skilful rate function and a self-tuning Nussbaum-type function, the problem was preliminarily solved for a second-order Euler–Lagrange system with full-state constraints and nonparametric uncertainties [51]. Then, this methodology was extended to uncertain strict-feedback nonlinear systems [52], [53], [54] and high-order multiagent systems [55], [56] via traditional backstepping control techniques. However, the self-tuning Nussbaum-type function multiplies the dynamic order of the closed-loop systems, which, together with the differential explosion problem, leads to higher computational complexity. Besides, the time-varying performance functions introduced in [53] and [54] exhibit singularity at the initial time, thus limiting their practical applicability. In conclusion, the challenge of achieving globally prescribed-time prescribed performance without excessive computational complexity for uncertain strict-feedback nonlinear systems remains an open problem deserving further investigation.

In general, there are three open issues that may result in semi-global boundedness of closed-loop signals in existing works.

1) A novel prescribed-time prescribed performance function is introduced. Contrary to finite-time prescribed functions [36], [37], [38], [39], [40], [41], [42], the settling time is independent of initial conditions and system parameters, and can be prescribed in advance by users. In addition, the issue raised in (A1) is resolved through the design of a novel error transformation function.

2) A novel Lyapunov-like energy function is proposed and a well-designed time-varying function derived from its derivative is employed to eliminate the influence of the error surface, which solves the problem in (A2). Besides, the problem in (A3) is addressed through the application of a generalized Lipschitz condition.

3) Two fuzzy control strategies, with or without approximating structures, are established. These strategies achieve global prescribed performance of tracking error, while ensuring the global uniform boundedness of all closed-loop signals. Specifically, both the singular phenomenon and the differential explosion are avoided in the control design. Consequently, the proposed controllers outperform those presented in [51], [52], [53], [54], and [55] in terms of reducing computational complexity and improving practical feasibility.
To summarize, the aforementioned limitations pertaining to the semi-global boundedness of closed-loop signals are effectively eliminated. Accordingly, the adaptive fuzzy tracking control with global prescribed performance for uncertain strict-feedback nonlinear systems is successfully achieved.

The remainder of this work is organized as follows. Preliminaries are presented in Section II. Control design and stability analysis are provided in Section III, and two practical examples are presented to verify the validity and effectiveness of the proposed methods in Section IV. Finally, Section V concludes this article.

Notation: In this work, $R_{\geq 0}$ and $R^n$ denote the set of non-negative real numbers and the $n$-dimensional Euclidean space, respectively. $I_n$ is an $n$-dimensional identity matrix, and $\mathbf{0}_n$ stands for a vector with all entries equal to 0.

II. PRELIMINARIES AND PROBLEM FORMULATION

In this section, the system model, fuzzy logic systems, some lemmas, and assumptions are provided.

A. System Descriptions

Consider a strict-feedback nonlinear system with mismatched uncertainties

$$
\begin{align*}
\dot{q}_i(t) &= f_i(\bar{q}_i(t)) + g_i(\bar{q}_i(t))q_{i+1}(t) + \omega_i(t), & i = 1, \ldots, n - 1 \\
\dot{q}_n(t) &= f_n(\bar{q}_n(t)) + g_n(\bar{q}_n(t))u(t) + \omega_n(t) \\
p(t) &= q_1(t)
\end{align*}
$$

where $q_i(t) \in R$ is the system state, and $\bar{q}_i(t) = (q_1(t), \ldots, q_i(t))^\top \in R^i$, $u(t) \in R$ and $p(t) \in R$ denote the control input and output trajectory of the system, respectively. $\omega_i(t) \in R$ ($i = 1, 2, \ldots, n - 1$) represent the mismatched uncertainties and $\omega_n(t) \in R$ is the matched uncertainty. $f_i(\bar{q}_i(t)) : R^i \rightarrow R$ and $g_i(\bar{q}_i(t)) : R^i \rightarrow R$ are unknown continuous nonlinear functions, called nonlinearities and control coefficients, respectively.

Define the reference signal as $p_r(t)$, which is continuous and differentiable. Let $\bar{p}_i(t)$ denote the $i$-dimensional vectorization of the reference signal $p_r(t)$, i.e.,

$$
\bar{p}_i(t) = (p_{r_1}(t), \ldots, p_{r_i}(t))^\top \in R^i,
$$

which can be obtained if $p_r(t)$ is available.

Remark 1: Compared with [19], [20], [21], [30], [32], [37], [41], [51], and [53], system (1) is more common and can describe many practical control plants including robot manipulators, mass-spring-damper systems, parallel active suspension systems, ship maneuvering systems, and switched RLC circuits. Therefore, it is of great significance to study the tracking control problem of system (1), especially with prescribed-time prescribed performance for more desired system response.

B. Fuzzy Logic Systems

Fuzzy control is a nonlinear intelligent control method based on fuzzy theory, fuzzy language, and fuzzy logic. It can incorporate the knowledge of human experts into the control scheme design to achieve the desired control performance. As the core of fuzzy control, fuzzy logic systems are capable of uniformly approximating any nonlinear functions defined on a compact set and are widely applied in adaptive control, pattern recognition, and decision analysis. Generally, a fuzzy logic system can be described by the fuzzy if-then rules [58]

$$
Q_k: \text{if } \sigma_1 = \Omega_k^1, \sigma_2 = \Omega_k^2, \ldots, \text{and } \sigma_n = \Omega_k^n \text{ then } \varsigma = \Delta_k^k, \; k = 1, 2, \ldots, m
$$

where $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)^\top \in R^n$, $\varsigma \in R$ and $m$ are the input, output variables and the number of fuzzy rules, respectively. $\Omega_k^n$ and $\Delta_k^k$ ($i = 1, 2, \ldots, n, k = 1, 2, \ldots, m$) are the fuzzy sets with membership functions $\mu_{\Omega_k^n}(\sigma_i)$ and $\mu_{\Delta_k^k}(\varsigma)$, respectively. In general, using Gaussian membership functions

$$
\mu_{\Omega_k^n}(\sigma_i) = \frac{1}{\sigma_i^k} e^{-\frac{(\sigma_i - \bar{\sigma}_i)^2}{\sigma_i^k}}
$$

and

$$
\mu_{\Delta_k^k}(\varsigma) = \frac{1}{\varsigma^k} e^{-\frac{(\varsigma - \bar{\varsigma})^2}{\varsigma^k}}
$$

product inference, singleton fuzzifiers, and center average defuzzification, fuzzy basis functions can be expressed as

$$
\varphi_k(\sigma) = \frac{\Pi_{i=1}^n \mu_{\Omega_k^n}(\sigma_i)}{\sum_{k=1}^m \Pi_{i=1}^n \mu_{\Omega_k^n}(\sigma_i)} (k = 1, 2, \ldots, m)
$$

where $\sigma_i > 0$, $\bar{\sigma}_i$ and $\sigma_i^k$ are some constants. The approximation can be described as

$$
\varsigma(\sigma) = \varphi^\top(\sigma) \vartheta = \sum_{k=1}^m \vartheta_k \varphi_k(\sigma)
$$

where $\varphi(\sigma) = (\varphi_1(\sigma), \varphi_2(\sigma), \ldots, \varphi_m(\sigma))^\top$, $\vartheta = (\vartheta_1, \vartheta_2, \ldots, \vartheta_m)^\top$, $\varsigma(\sigma)$ and $\vartheta_k$ are the output and optimal constant weights of the fuzzy logic system, respectively.

For fuzzy logic systems, three key points should be noted. First, fuzzy sets and membership functions, relying on the knowledge of human experts, significantly impact the approximation accuracy of fuzzy logic systems. Therefore, it is crucial to carefully select fuzzy sets and membership functions according to the control plants and practical requirements. Second, the number of fuzzy if-then rules can grow exponentially as the input of fuzzy logic systems increases, which is known as the rule explosion problem and will greatly increase the computational complexity. Finally, it should be emphasized that fuzzy logic systems usually cause semi-global stability of closed-loop systems due to the limitation of the input space within a compact set.

C. Assumptions and Lemmas

Before proceeding further, some common assumptions and lemmas are necessary.

Assumption 1: $p_r(t)$ and $\bar{p}_i(t)$ are bounded and available for control design.

Assumption 2: The sign of function $g_i(\bar{q}_i(t))$ is certain definite. Without losing generality, suppose that there exist constants $\bar{g}_i > 0$ and $\bar{g}_i > 0$ such that $\bar{g}_i < g_i(\bar{q}_i(t)) < \bar{g}_i$.

Assumption 3: $|\omega_i(t)| \leq \bar{\omega}_i$ for some positive constants $\bar{\omega}_i$ ($i = 1, 2, \ldots, n$).

Assumption 4: There exists a positive and continuous function $L_i(\bar{q}_i(t), \bar{p}_i(t), t)$ such that for any $\bar{q}_i(t) \in R^i$ and $\bar{p}_i(t) \in R^i$

$$
|f_i(\bar{q}_i(t)) - f_i(\bar{p}_i(t))| \leq L_i(\bar{q}_i(t), \bar{p}_i(t), t)\|\bar{q}_i(t) - \bar{p}_i(t)\|
$$

holds, where $L_i(\bar{q}_i(t), \bar{p}_i(t), t)$ is bounded if $\bar{q}_i(t)$ and $\bar{p}_i(t)$ are bounded ($i = 1, 2, \ldots, n$).
Remark 2: Assumption 1 is common and reasonable [23], [31], [48], [49], which implies that there exists a compact set \( \Omega \subset R^l \) such that \( \tilde{p}_i \in \Omega \). Hence, it is feasible to introduce fuzzy logic systems to deal with the unknown nonlinearity \( f(\tilde{p}_i(t)) \). Assumptions 2 and 3 are used in most existing literature [31], [34], [37], [59], [60]. Assumption 4 can be viewed as a generalized Lipschitz condition, which is less restrictive as compared with that in [52] and [53].

Lemma 1 [61]: Let \( t > 0, \kappa > 1, \varphi > 1 \) with \( (\kappa - 1)(\varphi - 1) = 1 \). Then, for all \( x_1, x_2 \in R \)
\[
x_1x_2 \leq \kappa |x_1|^\kappa + \frac{1}{\varphi^\varphi} |x_2|\varphi.
\]

Lemma 2 [62]: For any constants \( \alpha > 0 \) and \( \beta \in R \)
\[
0 \leq |\beta| - \frac{\beta^2}{\sqrt{\beta^2 + \alpha^2}} \leq \alpha.
\]

Lemma 3 [58], [63]: Let \( \Omega \subset R^l \) represent a compact set. For a continuous nonlinear function \( f(\tilde{p}(t)) : \Omega \rightarrow R \), there exists a fuzzy logic system with bounded optional weight \( \theta = (\theta_1, \theta_2, \ldots, \theta_m)^T \) and fuzzy basis function \( \varphi(\tilde{p}(t)) = (\varphi_1(\tilde{p}(t)), \varphi_2(\tilde{p}(t)), \ldots, \varphi_m(\tilde{p}(t)))^T \) such that
\[
f(\tilde{p}(t)) = \varphi^\top(\tilde{p}(t))\theta + \nu(t)
\]
where \( \tilde{v} > 0 \) is an arbitrarily positive accuracy, and \( \nu(t) \) is the approximation error satisfying \( |\nu(t)| \leq \tilde{v} \) for \( t \geq 0 \). In addition, for \( \varphi(\tilde{p}(t)) \) there holds
\[
\varphi(\tilde{p}(t))\varphi^\top(\tilde{p}(t)) \leq ml_m.
\]

III. MAIN RESULTS

This section presents main results, including prescribed performance function, control scheme design, and stability analysis.

A. Prescribed Performance Functions

The section introduces a novel prescribed-time performance function and further discusses some useful properties.

Definition 1: Let \( \rho > 0 \) be a positive constant and \( T > 0 \) be any user-prescribed time. A continuous and differentiable function \( \rho(t) \) is referred to as the prescribed-time performance function if:
1) \( \rho(t) > 0 \) and \( \dot{\rho}(t) \leq 0 \) for \( t \geq 0 \);
2) \( \lim_{t\rightarrow T} \rho(t) = \rho_0 \) and \( \rho(t) \equiv \rho \) for \( t \geq T \)
where \( T \) and \( \rho \) stand for prescribed time and prescribed accuracy, respectively.

Remark 3: Compared with the finite-time performance function [37], [41], [42], the settling time \( T \) of the prescribed-time performance function is independent of initial values and system parameters, which contributes to simplifying the design of performance function, improving convergence rate and achieving global performance. Therefore, it can better meet the practical application demands.

From Definition 1, a typical prescribed-time performance function can be designed as
\[
\rho(t) = \begin{cases} 
    a e^{-b(T-t)} + c, & 0 \leq t < T \\
    c, & t \geq T
\end{cases}
\]
where \( h > 0 \) and positive constants \( a, b, c \) satisfy \( \rho(0) = ae^{-b} + c = (\pi/2) \).

Define the tracking error \( e(t) = p(t) - \rho_0(t) \) and the error transformation function
\[
z_1(t) = \tan\left(\frac{\pi}{2} \arctan(\frac{e(t)}{\rho(t)})\right)
\]
which means that
\[
e(t) = \tan\left(\frac{2}{\pi} \rho(t) \arctan(z_1(t))\right).
\]

Therefore
\[
\dot{e}(t) = \frac{2}{\pi} \dot{\rho}(t) \arctan(z_1(t)) + \frac{2}{\pi} \rho(t) \frac{\dot{z}_1(t)}{1+z_1^2(t)}
\]

which is equivalent to
\[
\dot{z}_1(t) = \psi(t) \dot{z}_1(t) + \frac{2}{\pi} \rho(t) \arctan(z_1(t))
\]
where
\[
\psi(t) = \pi \left(1 + \frac{z_1^2(t)}{2\rho(t)}\right) > 0
\]
and
\[
\dot{\phi}(t) = \cos^2\left(\frac{2\rho(t) \arctan(z_1(t))}{\pi}\right) > 0.
\]

Proposition 1: The following properties hold for functions \( \rho(t) \) and \( z_1(t) \):
1) \( \rho(t) \) is continuous and infinitely differentiable with
\[
\dot{\rho}(t) = \begin{cases} 
    -\frac{abhT^2}{(T-t)^{3-p}} e^{-b(T-t)} \frac{h}{T}, & 0 \leq t < T \\
    0, & t \geq T
\end{cases}
\]
2) \( \dot{z}_1(0) = e(0) \) for any \( e(0) \in R \), and \( \dot{z}_1(t) \) is well defined if \( |e(t)| < \tan(\rho(t)) \).
3) If \( z_1(t) \) is bounded for \( t \in [0, +\infty) \), then there hold transient state performance \( |e(t)| < \tan(\rho(t)) \) for \( t \geq 0 \), and steady state performance \( |e(t)| < \tan(\rho(t)) \) for \( t \geq T \).

Remark 4: It is worth noting that in the existing exponential [17], [18], [20], [23], [25], [26], [27], [28], [29], [30], [32] and finite-time [39], [41], [42], [55] prescribed performance control methods, the error transformation is subject to the initial value constraint: \( |e(0)| < \rho(0) \). This constraint can potentially lead to semi-global stability of the tracking error and pose implementation difficulties of the error transformation when initial values are not available. In [53], an alternative error transformation is defined as
\[
s(t) = \frac{k_e^2(t)T^2}{(k_e(t) + \xi_1(t)) (k_e(t) - \xi_1(t))}
\]
where \( k_e(t) = \frac{1}{\xi_0(t) + \xi_1(t)}(k_e(t) + \xi_1(t)) \), \( \xi_0(t) = \beta(t) e(t), \beta(t) = (1/(1-b)t)k_e(t) + b), \kappa(t) = \left\{ \begin{array}{ll} 
    \frac{1}{b} (t^{-1})^{p+2}, & 0 \leq t < T \\
    0, & t \geq T
\end{array} \right. \)
and constants \( \xi_0 = 0, 0 < b_j < 1, 0 < T < +\infty \). It can be
observed that \( \lim_{t \to 0^+} k_c(t) = +\infty \) since \( \varepsilon_0 = 0 \). Therefore, \( -\infty = -[k_c(0)/\beta(0)] < \varepsilon(0) < [k_c(0)/\beta(0)] = +\infty \) holds for any \( \varepsilon(0) \in R \), which indicates that the initial value constraints are eliminated. However, the fact that \( \lim_{t \to 0^+} k_c(t) = +\infty \) leads to the singularity problem, and restricts the practical applicability. To this end, the novel prescribed performance function (2) and error transformation (3) are designed to address the above initial value constraint and singularity problems via tangent function \( \tan(\cdot) \) and its inverse. We provide Table I to explicitly demonstrate the comparison between our approach and existing methods.

According to Proposition 1, the control objective is given as follows.

Objective 1: Design controller \( u(t) \) to
1) drive error \( e(t) \) to converge into a prescribed region within a prescribed time, and satisfy transient state performance \( |e(t)| < \tan(\rho(t)) \) for \( t \geq 0 \) and steady-state performance \( |e(t)| < \tan(c) \) for \( t \geq T \);
2) guarantee global boundedness of all closed-loop signals.

B. Control Schemes

In the section, a modified dynamic surface control method is developed to design control schemes that ensure the global prescribed-time prescribed performance of the tracking error, effectively avoiding the differential explosion problem encountered in traditional backstepping techniques. To enhance clarity and eliminate ambiguity, the variable \( t \) will be omitted in the subsequent discussion.

Let \( \alpha_{i-1} \) be the virtual control, and define the intermediate error \( z_i = q_i - s_i \) and the error surface \( r_i = s_i - \alpha_{i-1} \) \( (i = 2, \ldots, n) \), where \( s_i \) is the filtering signal obtained by the first-order filter \( \lambda_i \hat{s}_i + s_i = \alpha_{i-1} \) with \( \lambda_i > 0 \) and initial condition \( s_i(0) = \alpha_{i-1}(0) \). Then, the control design is given as follows. Step 1: From (6), one obtains

\[
\dot{z}_1 = \psi \left( f_1(\tilde{q}_1) - f_1(\tilde{p}_1) + f_1(\tilde{p}_1) + g_1(\tilde{q}_1)q_2 \right.
+ \omega_1 - \hat{p}_1)\phi - \frac{2}{\pi} \hat{p} \arctan(z_1) \left.ight). \tag{7}
\]

According to Assumption 1 and Lemma 3, for any \( \tilde{v}_1 > 0 \), there exist \( \vartheta_1 \in R^m \) and \( \psi_1(\tilde{p}_1) : R \to R^m \) such that

\[
f_1(\tilde{p}_1) = \psi_1^\top(\tilde{p}_1)\vartheta_1 + v_1, \tag{8}
\]

where \( |v_1| \leq \tilde{v}_1 \). Let \( \vartheta_1 \in R^m \) be the estimate of the optimal weight \( \vartheta_1 \) and \( \tilde{\vartheta}_1 = \vartheta_1 - \vartheta_1 \) be the estimate error. Consider the Lyapunov-like energy function

\[
W_1 = \frac{1}{2} \tilde{z}_1^2 + \frac{1}{2\mu_1} \tilde{\vartheta}_1^\top \tilde{\vartheta}_1 \tag{9}
\]

where constant \( \mu_1 > 0 \). Differentiating \( W_1 \) along (7) results in

\[
\dot{W}_1 = z_1 \psi \left( f_1(\tilde{q}_1) - f_1(\tilde{p}_1) + \psi_1^\top(\tilde{p}_1)\vartheta_1 \right.
+ \vartheta_1^\top(\tilde{p}_1)\vartheta_1 + v_1 + g_1(\tilde{q}_1)q_2 \right.
+ \omega_1 - \hat{p}_1)\phi - \frac{2}{\pi} \hat{p} \arctan(z_1) \left.ight) - \frac{1}{\mu_1} \tilde{\vartheta}_1^\top \tilde{\vartheta}_1
\]

\[
= z_1 \psi \left( f_1(\tilde{q}_1) - f_1(\tilde{p}_1) + \psi_1^\top(\tilde{p}_1)\vartheta_1 \right.
+ \vartheta_1^\top(\tilde{p}_1)\vartheta_1 + v_1 + g_1(\tilde{q}_1)(z_2 + r_2 + \alpha_1) \right.
+ \omega_1 - \hat{p}_1)\phi - \frac{2}{\pi} \hat{p} \arctan(z_1) \left.ight) - \frac{1}{\mu_1} \tilde{\vartheta}_1^\top \tilde{\vartheta}_1
\]
where Assumption 2, Lemmas 1 and 2 are employed to derive the last inequality. Design

\[
\alpha_1 = - \frac{z_1 \phi \psi \bar{g}_1(\hat{q}_1)}{\bar{g}_1(\hat{q}_1)(z_1 \phi \psi \beta_1)^2 + \bar{\delta}_1^2} - \frac{z_1 \phi \psi \chi_1}{\bar{g}_1(\hat{q}_1)(z_1 \phi \psi \chi_1)^2 + \bar{\delta}_1^2} \leq \delta_1 - |z_1 \phi \psi \beta_1| + \sigma_1 - |z_1 \phi \psi \chi_1| + \sigma_1 - (z_1 \phi \psi)^2 - \frac{\sigma_1 \bar{z}_1^2}{2}
\]

where \( \sigma_1, \delta_1, \) and \( \alpha_1 \) are positive constants, \( \beta_1 \) and \( \chi_1 \) will be designed later. Therefore, the last term of (10) can be calculated as

\[
z_1 \phi \psi \bar{g}_1(\hat{q}_1) \alpha_1 = - \frac{g_1(\hat{q}_1)(z_1 \phi \psi \beta_1)^2}{\bar{g}_1(\hat{q}_1)(z_1 \phi \psi \beta_1)^2 + \bar{\delta}_1^2} - \frac{g_1(\hat{q}_1)(z_1 \phi \psi \chi_1)^2}{\bar{g}_1(\hat{q}_1)(z_1 \phi \psi \chi_1)^2 + \bar{\delta}_1^2} \leq \delta_1 - |z_1 \phi \psi \beta_1| + \sigma_1 - |z_1 \phi \psi \chi_1|
\]

From Assumption 4, (15) and (16), one has

\[
\tilde{W}_1 \leq |z_1 \phi \psi \bar{g}_1(l|z_2| + |r_2|) - \frac{\sigma_1 z_1^2}{2} + \frac{1}{2} (\bar{v}_1^2 + \bar{w}_1^2) + \delta_1 + \alpha_1 + \frac{\sigma_1 \bar{z}_1^2}{\mu_1} \alpha_1 \tilde{\theta}_1 \leq -\sigma_1 W_1 + \Delta_1 + |z_1 \phi \psi \bar{g}_1(l|z_2| + |r_2|) - \frac{\sigma_1 z_1^2}{2} + \frac{1}{2} (\bar{v}_1^2 + \bar{w}_1^2) + \delta_1 + \alpha_1 + \frac{\sigma_1 \bar{z}_1^2}{2 \mu_1} \alpha_1 \tilde{\theta}_1 \leq -\sigma_1 W_1 + \Delta_1 + \|z_1 \phi \psi \bar{g}_1(l|z_2| + |r_2|)
\]

where \( \Delta_1 = (1/2)(\bar{v}_1^2 + \bar{w}_1^2) + \delta_1 + \alpha_1 + \frac{\sigma_1 \bar{z}_1^2}{2 \mu_1} \alpha_1 \tilde{\theta}_1 \). 

Step 2: For \( i = 2, 3, \ldots, n - 1 \), differentiating \( z_i = q_i - s_i \) yields

\[
\dot{z}_i = \dot{q}_i - \dot{s}_i = f_i(\hat{q}_i) + g_i(\hat{q}_i)q_{i+1} + \omega_i - \dot{s}_i
\]

where \( f_i(\hat{q}_i) = q_i^T (\hat{p}_i) \hat{\theta}_i + v_i \) is employed with \( |v_i| \leq \tilde{v}_i \) under Assumption 1 and Lemma 3. Let \( \hat{\theta}_i \in \mathbb{R}^n \) be the estimate of the optimal weight \( \theta_i \), and \( \bar{\theta}_i = \hat{\theta}_i - \hat{\theta}_i \) be the estimate error. Consider the Lyapunov-like energy function

\[
W_i = W_{i-1} + \alpha_j(z_i) + \beta_i|z_i| + \frac{\bar{\delta}_i^T \bar{\theta}_i}{2 \mu_i}
\]

where \( \mu_i > 0 \) and \( \alpha_i > 1 \) are positive constants. By simple calculations, \( \alpha(z_i) + \beta_i|z_i| \geq 0 \) holds for all \( z_i \) and \( \lim_{|z_i| \rightarrow +\infty} \alpha(z_i) + \beta_i|z_i| = +\infty \). Therefore, function \( W_i \) is positive definite and radially unbounded. Denote \( \zeta_i = [1/(1 + z_i^2)] + \theta_i \cdot \text{sign}(z_i) \). Then, \( \zeta_i \neq 0 \) for all \( z_i \in \mathbb{R} \). Differentiating \( W_i \) along the solution of closed-loop system (18) yields

\[
\dot{W}_i = \dot{W}_{i-1} + \frac{\bar{\delta}_i^T \bar{\theta}_i}{\mu_i} - \frac{1}{\mu_i} \bar{\delta}_i^T \bar{\theta}_i
\]

where \( \zeta_i - \frac{1}{\mu_i} \bar{\delta}_i^T \bar{\theta}_i \)

Define

\[
\beta_1 = q_i^T (\hat{p}_i) \hat{\theta}_i - \dot{\tilde{p}}_r - \frac{2}{\mu_i} \rho \cdot \text{arctan}(z_i)
\]

where the last inequality is obtained via Lemma 1 and Assumption 2.
\begin{equation}
\alpha_i = -\frac{\xi_i \beta_i^2}{g_i \sqrt{\xi_i^2 \beta_i^2 + \delta_i^2}} - \frac{\xi_i \chi_i^2}{g_i \sqrt{\xi_i^2 \chi_i^2 + \sigma_i^2}} - \frac{\xi_i \gamma_i^2}{g_i \sqrt{\xi_i^2 \gamma_i^2 + \theta_i^2}} - \frac{\xi_i \xi_i^2}{g_i \sqrt{\xi_i^2 \xi_i^2 + \tau_i^2}} - \frac{\sigma_i (\arctan(z_i) + \varphi_i |z_i|)}{g_i} \xi_i \right) = \frac{\bar{g}_i (\bar{q}_i) (\arctan(z_i) + \varphi_i |z_i|)}{g_i} - \frac{\bar{g}_i (\bar{q}_i)^2 \xi_i^2}{g_i}
\end{equation}

Design

\begin{equation}
\hat{\vartheta}_i = -\sigma_i \hat{\vartheta}_i + \mu_i \xi_i \varphi_i (\bar{p}_i) \tag{22}
\end{equation}

where \( \sigma_i, \delta_i, \chi_i, \theta_i, \) and \( \tau_i \) are some positive constants, and

\begin{equation}
\beta_i = \psi_i^T (\bar{p}_i) \hat{\vartheta}_i - \frac{1}{\lambda_i} (\alpha_i - s_i) \tag{23}
\end{equation}

\begin{equation}
\gamma_i = \begin{cases} \frac{\bar{g}_i \psi_i (|z_i|)}{\bar{g}_i \bar{z}_i + 1 / 2 |z_i|}, & i = 2 \\ \frac{\bar{g}_i \psi_i (|z_i|)}{\bar{g}_i \bar{z}_i + 1 / 2 |z_i|}, & i = 3, 4, \ldots, n - 1 \\
\end{cases}
\end{equation}

\begin{equation}
\chi_i = L_i (q_i, p_i, r_i) \| \bar{q}_i - \bar{p}_i \| \tag{25}
\end{equation}

\begin{equation}
\xi_i = \begin{cases} \frac{\bar{g}_i \psi_i (|z_i|)}{\bar{g}_i \bar{z}_i + 1 / 2 |z_i|}, & i = 2 \\ \frac{\bar{g}_i \psi_i (|z_i|)}{\bar{g}_i \bar{z}_i + 1 / 2 |z_i|}, & i = 3, 4, \ldots, n - 1. \\
\end{cases}
\end{equation}

According to (21), the sixth term of (20) can be calculated as

\begin{equation}
\xi_i (\bar{g}_i (\bar{q}_i) \bar{q}_i \bar{q}_i \beta_i^2 - \frac{\bar{g}_i (\bar{q}_i) \bar{q}_i \chi_i \chi_i^2}{g_i} - \frac{\bar{g}_i (\bar{q}_i) \bar{q}_i \gamma_i \gamma_i^2}{g_i} - \frac{\bar{g}_i (\bar{q}_i) \bar{q}_i \xi_i \xi_i^2}{g_i} - \frac{\sigma_i (\arctan(z_i) + \varphi_i |z_i|)}{g_i} \xi_i \right) = \frac{\bar{g}_i (\bar{q}_i) \bar{q}_i \beta_i^2}{g_i \sqrt{\bar{g}_i^2 \beta_i^2 + \delta_i^2}} - \frac{\bar{g}_i (\bar{q}_i) \bar{q}_i \chi_i \chi_i^2}{g_i \sqrt{\bar{g}_i^2 \chi_i^2 + \sigma_i^2}} - \frac{\bar{g}_i (\bar{q}_i) \bar{q}_i \gamma_i \gamma_i^2}{g_i \sqrt{\bar{g}_i^2 \gamma_i^2 + \theta_i^2}} - \frac{\bar{g}_i (\bar{q}_i) \bar{q}_i \xi_i \xi_i^2}{g_i \sqrt{\bar{g}_i^2 \xi_i^2 + \tau_i^2}} - \frac{\sigma_i (\arctan(z_i) + \varphi_i |z_i|)}{g_i} \xi_i \right) = \frac{\bar{g}_i (\bar{q}_i) \bar{q}_i \beta_i^2}{g_i \sqrt{\bar{g}_i^2 \beta_i^2 + \delta_i^2}} - \frac{\bar{g}_i (\bar{q}_i) \bar{q}_i \chi_i \chi_i^2}{g_i \sqrt{\bar{g}_i^2 \chi_i^2 + \sigma_i^2}} - \frac{\bar{g}_i (\bar{q}_i) \bar{q}_i \gamma_i \gamma_i^2}{g_i \sqrt{\bar{g}_i^2 \gamma_i^2 + \theta_i^2}} - \frac{\bar{g}_i (\bar{q}_i) \bar{q}_i \xi_i \xi_i^2}{g_i \sqrt{\bar{g}_i^2 \xi_i^2 + \tau_i^2}} - \frac{\sigma_i (\arctan(z_i) + \varphi_i |z_i|)}{g_i} \xi_i \right) 
\end{equation}

where Assumption 2 and Lemma 2 are used to derive the first and second inequalities, respectively. Therefore, according to Assumption 4, combining (20), (21), (22), and (27) yields

\begin{equation}
\hat{W}_i \leq \hat{W}_{i-1} - \sigma_i (\arctan(z_i) + \varphi_i |z_i|) + \frac{\sigma_i \hat{\vartheta}_i}{\mu_i} \hat{\vartheta}_i + \bar{g}_i |z_i| (|z_i| + |r_i|) 
\end{equation}

\begin{equation}
- |\chi_i r_i| - |\xi_i r_i| + \frac{1}{2} (\frac{\bar{g}_i \bar{q}_i \bar{q}_i \beta_i^2}{g_i \sqrt{\bar{g}_i^2 \beta_i^2 + \delta_i^2}} - \frac{\bar{g}_i \bar{q}_i \bar{q}_i \beta_i^2}{g_i \sqrt{\bar{g}_i^2 \beta_i^2 + \delta_i^2}} - \frac{\bar{g}_i \bar{q}_i \bar{q}_i \beta_i^2}{g_i \sqrt{\bar{g}_i^2 \beta_i^2 + \delta_i^2}} - \frac{\bar{g}_i \bar{q}_i \bar{q}_i \beta_i^2}{g_i \sqrt{\bar{g}_i^2 \beta_i^2 + \delta_i^2}} - \frac{\sigma_i (\arctan(z_i) + \varphi_i |z_i|)}{g_i} \xi_i \right) = \frac{\bar{g}_i \bar{q}_i \bar{q}_i \beta_i^2}{g_i \sqrt{\bar{g}_i^2 \beta_i^2 + \delta_i^2}} - \frac{\bar{g}_i \bar{q}_i \bar{q}_i \beta_i^2}{g_i \sqrt{\bar{g}_i^2 \beta_i^2 + \delta_i^2}} - \frac{\bar{g}_i \bar{q}_i \bar{q}_i \beta_i^2}{g_i \sqrt{\bar{g}_i^2 \beta_i^2 + \delta_i^2}} - \frac{\bar{g}_i \bar{q}_i \bar{q}_i \beta_i^2}{g_i \sqrt{\bar{g}_i^2 \beta_i^2 + \delta_i^2}} - \frac{\sigma_i (\arctan(z_i) + \varphi_i |z_i|)}{g_i} \xi_i \right) 
\end{equation}

Design the actual controllers as follows:

\begin{equation}
u = -\frac{\xi_i \beta_i^2}{g_i \sqrt{\xi_i^2 \beta_i^2 + \delta_i^2}} - \frac{\xi_i \chi_i^2}{g_i \sqrt{\xi_i^2 \chi_i^2 + \sigma_i^2}} - \frac{\xi_i \gamma_i^2}{g_i \sqrt{\xi_i^2 \gamma_i^2 + \theta_i^2}} - \frac{\xi_i \xi_i^2}{g_i \sqrt{\xi_i^2 \xi_i^2 + \tau_i^2}} - \frac{\sigma_i (\arctan(z_i) + \varphi_i |z_i|)}{g_i} \xi_i \right) = -\frac{\xi_i \beta_i^2}{g_i \sqrt{\xi_i^2 \beta_i^2 + \delta_i^2}} - \frac{\xi_i \chi_i^2}{g_i \sqrt{\xi_i^2 \chi_i^2 + \sigma_i^2}} - \frac{\xi_i \gamma_i^2}{g_i \sqrt{\xi_i^2 \gamma_i^2 + \theta_i^2}} - \frac{\xi_i \xi_i^2}{g_i \sqrt{\xi_i^2 \xi_i^2 + \tau_i^2}} - \frac{\sigma_i (\arctan(z_i) + \varphi_i |z_i|)}{g_i} \xi_i \right) 
\end{equation}

\begin{equation}
\hat{\vartheta}_i = -\sigma_i \hat{\vartheta}_i + \mu_i \xi_i \varphi_i (\bar{p}_i) \tag{33}
\end{equation}

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strict-feedback nonlinear system
\[ p_n(t) = f_n(p_n(t)) + g_n(p_n(t))u_n(t) + \omega_n(t), \]
\[ q_n(t) = f_n(q_n(t)) + g_n(q_n(t))u_n(t) + \omega_n(t), \]
\[ p_t = p(t) - p_n(t), \]
\[ q_t = q(t) - q_n(t). \]

tracking errors and intermediate signals
\[ e(t) = p(t) - p_n(t), \]
\[ s_n(t) = q(t) - q_n(t), \]
\[ r_n(t) = s_n(t) - n(t), \]
\[ \Lambda_n(t) = 1 - n(t), \]
\[ \alpha_n(t) = \alpha_n - n(t). \]

virtual signals and actual controller
\[ \alpha(t) = \sum \alpha(t) \]
\[ \bar{p}_n(t) = \pi(t), \]
\[ \bar{q}_n(t) = \phi(t), \]
\[ \bar{r}_n(t) = \theta(t), \]
\[ \bar{t}_n(t) = \tau(t). \]

fuzzy adaptive parameter estimators
\[ \hat{\alpha}_n(t) = \alpha_n(t) \]
\[ \hat{\bar{p}}_n(t) = \bar{p}_n(t) \]
\[ \hat{\bar{q}}_n(t) = \bar{q}_n(t) \]
\[ \hat{\bar{r}}_n(t) = \bar{r}_n(t) \]
\[ \hat{\bar{t}}_n(t) = \bar{t}_n(t). \]

For better understanding, the schematic of the proposed control mechanism is shown in Fig. 1.

\[ \beta_n = \phi_n^T(\bar{p}_n) \hat{\bar{p}}_n - \frac{1}{\lambda_n}(\alpha_{n-1} - \sigma_n) \]
\[ \gamma_n = \frac{g_n - \lambda_n(\alpha_n + \xi_n)}{\zeta_n} \]
\[ \chi_n = L_n(g_{n+1}(\bar{p}_{n+1}, \bar{q}_{n+1}, \bar{r}_{n+1}, \bar{t}_{n+1})) \]
\[ \xi_n = \frac{g_n - \lambda_n(1 - \alpha_n)}{\zeta_n}. \]

Similar to (27), one has
\[ \zeta_n \sigma_n(g_n(\bar{p}_n)) \leq -\zeta_n \beta_n - [\zeta_n \chi_n - (\sigma_n \bar{r}_n - \bar{r}_n) - \sigma_n(\arctan(\sigma_n) + |\zeta_n|)] - \zeta_n^2 + \sigma_n + \theta_n + \tau_n. \]

Therefore, combining (31), (32), (33), and (38) yields
\[ \dot{W}_n \leq \dot{W}_{n-1} - [\zeta_n \gamma_n - \zeta_n \xi_n - \sigma_n(\arctan(\sigma_n) + |\zeta_n|)] + \frac{\hat{\bar{r}}_n^T \hat{\bar{r}}_n}{\mu_n} \]
\[ + \frac{1}{2}(\bar{r}_n^2 + \sigma_n^2) + \sigma_n + \theta_n + \tau_n \]
\[ \leq -2 \sigma_n \left( \frac{\sigma_n(\arctan(\sigma_n) + |\zeta_n|)}{2\lambda_n} + \frac{\hat{\bar{r}}_n^T \hat{\bar{r}}_n}{2\mu_n} \right) \]
\[ - \sigma_n W_{n-1} + \sum_{k=1}^n \Lambda_k \]
\[ \leq -\sigma W_n + \Lambda. \]

where \( \sigma = \min(\sigma_n, \sigma_2, \ldots, \sigma_n) \), \( \Lambda_n = (1/2)(\bar{r}_n^2 + \sigma_n^2) \), \( \delta_0 + \sigma_n + \theta_n + \tau_n \) and \( \Lambda = \sum_{k=1}^n \Lambda_k \).

For better understanding, the schematic of the proposed control mechanism is shown in Fig. 1.

The main results are presented in the following theorems.

**Theorem 1:** Using the virtual controllers (11) and (21), adaptive fuzzy update laws (12), (22), and (33), the actual controller (32) achieves Objective 1, if Assumptions 1–4 hold.

**Proof:** From (30) and (39), \( 0 \leq \dot{W}_n(t) \leq \dot{W}_{n-1}(t) e^{-\alpha t} + (\Lambda/\sigma) e^{-\alpha t} \). Therefore, both \( \dot{\bar{r}}_n \) and \( \hat{\bar{r}}_n \) are bounded. According to Proposition 1, Objective 1-1 is achieved.

From \( \dot{\bar{r}}_n = \bar{r}_n - \hat{\bar{r}}_n \), \( \dot{\hat{\bar{r}}}_n \) is bounded. Therefore, according to

\[ (\varphi_1^T(\bar{p}_1) \hat{\bar{p}}_1)^2 = (\varphi_1^T(\bar{p}_1) \hat{\bar{r}}_1)^2 \]
\[ = \bar{r}_1^T(\varphi_1(\bar{p}_1) \varphi_1^T(\bar{p}_1)) \hat{\bar{r}}_1 \]
\[ \leq m \bar{r}_1^T \hat{\bar{r}}_1 < +\infty \]

which implies that \( \beta_1 \) is bounded from (15). Furthermore, from \( e = p - p_e \) and Assumption 1, \( \chi_n \) in (16) is bounded. Therefore, the virtual control \( \alpha_1 \) is bounded based on (15) and (16), which implies that

\[ |s_2(t)| = \left| \alpha_1(0) e^{-(\frac{1}{\gamma})} + \int_0^t \alpha_1(\tau) e^{-(\frac{1}{\gamma})} d\tau \right| \]
\[ \leq |\alpha_1(0) e^{-(\frac{1}{\gamma})} + \gamma_1(1 - e^{-(\frac{1}{\gamma})}) | \]

where \( \alpha_1 = \sup_{t \in [0, +\infty)} |\alpha_1(t)| \). Consequently, the filtering signal \( s_2 \) is bounded. Recursively, \( \beta_i, \chi_i, \xi_i, r_i, q_i, \) and \( s_i \) (i = 2, 3, ..., n) are bounded according to \( z_i = q_i - s_i \) and \( r_i = s_i - \alpha_{i-1} \). Therefore, all system variables, including system states \( q_i(t) \) (i = 1, 2, ..., n), tracking error \( e(t) \), transformed error \( z(t) \), intermediate errors \( z_i(t) \) (i = 2, 3, ..., n), filtering signals \( s_i(t) \) (i = 2, 3, ..., n), and virtual signals \( \alpha_i(t) \) (i = 2, 3, ..., n) are bounded. Therefore, all system variables are bounded. Generally, employing adaptive fuzzy estimator to acquire the information of unknown nonlinearity may increase the computational complexity, which will cause unnecessary consumption of computational power and equipment wear. To avoid this, let

\[ \beta_1 = z_1 \varphi_i^T(\bar{p}_1) \varphi_i(\bar{p}_1) - \bar{r}_1 - \frac{1}{\pi} \rho \arctan(z_1) \]
\[ \beta_i = z_i \varphi_i^T(\bar{p}_1) \varphi_i(\bar{p}_1) - \frac{1}{\pi} \rho \arctan(z_i), \]
\[ \beta_n = z_n \varphi_n^T(\bar{p}_1) \varphi_n(\bar{p}_1) - \frac{1}{\pi} \rho \arctan(z_n) \]

Then, the results with prescribed-time prescribed performance without approximating structures can be developed.

**Theorem 2:** Under the virtual controllers (11) and (21) with \( \beta_1 \) and \( \beta_n \) defined in (41) and (42), respectively, the conclusions in Theorem 1 hold via the actual controller (32) with \( \beta_n \) in (43), if Assumptions 1–4 hold.

**Proof:** Let

\[ W = \sum_{i=1}^n W_i \]

where \( W_i = (1/2)z_i^2 + W_i = \arctan(z_i) + g_i|z_i| \) (i = 2, 3, ..., n). Differentiating \( W_i \) along system (7) results in

\[ \dot{W}_i = z_i \psi \left( f_i(\bar{q}_i) + f_i(\bar{p}_1) + \varphi_i^T(\bar{p}_1) \bar{r}_1 + v_1 \right) + \omega_i + g_i(\bar{q}_i)(z_i + r_2 + \alpha_1 - \bar{p}_1) \psi \]
\[ + \frac{2\varphi_i}{\rho} \arctan(z_i) \]
\[ \leq |z_i^2 \varphi_i f_i(\bar{q}_i) - f_i(\bar{p}_1))| \]
\[ + z_i^2 \varphi_i^2 (1 + \frac{1}{2} \varphi_i^2(\bar{p}_1) \varphi_i(\bar{p}_1)) \]
In this work, a novel Lyapunov-like energy function \( W_i \) is proposed. A skillful function, \( \varphi_i(t) \) in the first-order filter. Positive constants \( a_i, b_i, c_i, t \) and \( T \) are preassigned by users according to practical control demands under the constraint \( ac^{-b} + c = (\pi/2) \). From Proposition 1, \( \tan(c) \) is the steady-state maximum convergence threshold of the tracking error, and thus \( c \) should be selected according to practical requirements. According to (12), (22), and (33), parameters \( \mu_i \) and \( \sigma_i \) have an impact on the update rate of the adaptive fuzzy estimate \( \vartheta_i \), but do not require careful adjustment as long as \( \mu_i > 0 \) and \( \sigma_i > 0 \). It follows from (11), (21), and (32) that parameters \( \delta_i, \sigma_i, \theta_i, \tau_i \) and \( \gamma_i \) can adjust the amplitude of control signals but are irrelevant to the steady state performance. Therefore, these parameters can be chosen as some positive constants that are sufficiently large to reduce the values of virtual and actual controllers. According to the linear filter theory in sliding mode control [66], the filter parameter \( \lambda_i \) should be selected small enough to achieve satisfactory approximation accuracy.

Remark 7: According to (19) and (30), \( \dot{W}_i (i = 2, 3, \ldots, n) \) only exists when \( z_i \neq 0 \). Hence, the approach based on the differential inclusion theory in [64] and [65] is employed to prove the main results, which only needs the continuity and radial unboundedness of the Lyapunov-like energy function.

Remark 8: It follows from (8), (18), and (29) that under Assumption 4, the compact set constraint in fuzzy logic systems on the system state \( \hat{q}_i \) is transferred to that on the reference signal \( \hat{p}_i \) that is assumed to be bounded in Assumption 1. Hence, the system state \( \hat{q}_i \) no longer needs to be constrained in a compact set, and the global boundedness of closed-loop signals can be guaranteed. However, in some practical control systems, it might be difficult to obtain the positive function \( L_k(\hat{q}_i, \hat{p}_i, t) \) in advance. Therefore, some new control strategies need to be explored when \( L_k(\hat{q}_i, \hat{p}_i, t) \) is unavailable.

Remark 9: The design idea behind virtual control \( u(t) \) and actual control input \( u(t) \) is explained as follows. \( \beta_i(t) \) is designed to eliminate the approximation of fuzzy logic systems and the derivative of virtual signal \( s_i(t) \). \( \gamma_i(t) \) and \( \chi_i(t) \) are employed to handle the intermediate errors \( z_i(t)z_i(t) \) or \( \zeta_i(t)z_i(t) \) in the backstepping derivations and the unknown nonlinearity \( f_i(\hat{q}_i(t)) \), respectively. Finally, \( \hat{q}_i(t) \) eliminates the influence of the error surface defined in dynamic surface control on the energy function, ultimately contributing to obtaining the global boundedness of the tracking error.

IV. Simulations

In this section, some examples are provided to demonstrate the validity and performance of the proposed methods.

Consider an electromechanical system [67] shown in Fig. 2, whose dynamics are described as

\[
\begin{align*}
M\ddot{q} + B\dot{q} + N\sin(q) &= I \\
L\dot{I} + KB\dot{q} + RI &= V_e
\end{align*}
\]

where \( g = 9.81 \text{ N/s}^2 \) is the gravity coefficient, \( q \) is the angular motor position, \( I \) is the motor armature current and \( V_e \) is the input control voltage. \( M = (J/K_e) + [(m_0L_0^2)/3K_e] + [(m_0L_0^2)/K_e] + [(2m_0K_0^2)/5K_e] \), \( N = [(m_0L_0^2)/2K_e] + [(m_0L_0^2)/K_e] \), \( B = (B_0/K_e) \), whose meanings and values of system symbols are shown in Table II.

Take \( q_1 = q, q_2 = \dot{q}, q_3 = (I/M) \) and \( u(t) = (V_e/ML) \). Then, system (47) with mismatched uncertainties can be written as

\[
\begin{align*}
\dot{q}_1 &= q_2 + \omega_1(t) \\
\dot{q}_2 &= q_3 - \frac{N}{M}\sin(q_1) - \frac{B}{M}q_2 + \omega_2(t) \\
\dot{q}_3 &= u(t) - \frac{K_e}{M}q_2 - \frac{R}{M}q_3 + \omega_3(t)
\end{align*}
\]

Therefore, it follows from (48), Assumptions 2 and 4 that one can choose \( g = 0.1, \bar{g}_i = 10 \) (i = 1, 2, 3) and
Take $m = 11$ and set the Gaussian membership functions as

$$
\mu_{\Omega_i}(p_r) = 10e^{-\frac{(p_r-\mu_i)^2}{\sigma_i^2}},
$$

(49)

where $v_i = (v_i^1, v_i^2, \ldots, v_i^{11})^\top = (-20, \ldots, -4, 0, 4, \ldots, 20)^\top$ $(i = 1, 2, 3)$. In addition, let

$$
b = 0.1, c = 0.05, h = 1, T = 0.5, a = \frac{2b-c}{e^{-b}} = 1.6807
$$

$$
d_1 = \sigma_1 = 10^{10}, d_i = \sigma_i = \tau_i = 10^{10} (i = 2, 3)
$$

$$
\sigma_i = \mu_i = 10 (i = 1, 2), \mu_1 = 10, \lambda_i = 10^{-5} (i = 2, 3)
$$

$$
\sigma_3 = 5 \times 10^3, \mu_3 = 10
$$

and take two initial conditions as $\tilde{q}_1(0) = (5, 3, 2)^\top$ and $\tilde{q}_2(0) = -100q_2^i(0)$ with $\tilde{q}_i(0) = 0_{11}$ $(i = 1, 2, 3)$. Figs. 3 and 4 show the simulation results.

It can be observed from Figs. 3(a) and 4(a) that $-\rho(t) < \arctan(e(t)) < \rho(t)$ holds, equivalently, $|e(t)| < \tan(\rho(t))$ for $t \geq 0$, and $|e(t)| < \tan(0.05)$ for $t \geq 0.5$. Therefore, tracking control with prescribed-time prescribed performance is achieved via proposed controllers. It should be emphasized that the state performances without approximating structures in Fig. 4 are almost identical to those with approximating structures in Fig. 3, but the computational complexity without approximating structures is far below the latter. The control mechanism without approximating structures can thus save more computation resources and have more potential to practical application.

In what follows, a single-link manipulator [68] shown in Fig. 5 is employed to compare the proposed controller without approximating structures and the one designed in [41]. The single-link manipulator’s dynamics is

$$
I\ddot{q}(t) + B\dot{q}(t) + Mgl\sin(q(t)) = u(t)
$$

(50)

where $g = 9.81 \text{ N/s}^2$ is the gravity coefficient, $q(t)$ denotes the angles of the link, $u(t)$ stands for the controller, and the meanings of the rest notations can be found in [68]. For simplicity, take $I = 1 \text{ kg} \cdot \text{m}^2$, $B = 2 \text{ kg} \cdot \text{m/s}$, $M = 1 \text{ kg}$ and $l = 1 \text{ m}$.

$L_1(\dot{q}_1, \dot{p}_1, t) = 1, L_2(\dot{q}_2, \dot{p}_2, t) = [(N + B)/M], L_3(\dot{q}_3, \dot{p}_3, t) = [(K_B + R)/ML]$. In simulations, the disturbances $\omega_1(t) = 2\sin(5t), \omega_2(t) = 5\cos(2t), \omega_3(t) = 10\sin(t)$ and the reference signal $p_r(t) = \sin(10t) + 2$.

Denote $q_1 = q(t)$ and $q_2 = \dot{q}(t)$. Then, according to (50), the dynamics with external disturbance is

$$
\begin{align*}
\dot{q}_1 &= q_2 + \omega_1(t) \\
\dot{q}_2 &= -\frac{1}{T}(Bq_2 + Mgl\sin(q_1(t))) + \frac{1}{T}u(t) + \omega_2(t).
\end{align*}
$$

Let the reference signal $p_r(t) = \pi + 2\sin(10t)$, and disturbances $\omega_1(t) = 0, \omega_2(t) = 10\cos(5t)$. In this example, take $s_{L1} = s_{L2} = 0.5, \bar{s}_1 = \bar{s}_2 = 10, L_1(\dot{q}_1, \dot{p}_1, t) = 1, L_2(\dot{q}_2, \dot{p}_2, t) = [(B + Mgl)/I], \delta_1 = \delta_2 = \sigma_1 = \sigma_2 = \theta_2 = \tau_2 = 10^3, \sigma_2 = \varrho_2 = 5$ and $\lambda_2 = 0.001$. The parameters in prescribed performance functions are chosen as $b = 0.9, c = 0.05, T = 0.5, h = 1,$ and $a = 4.1340$. For the purpose of rigour in simulation, the settling time $t_r$ and initial value.
of prescribed performance function $\rho(0)$ in [41] are set as $t_r = T$ and $\rho(0) = \pi + 1$, and other parameters are the same as those in [41]. Therefore, $\psi = (\rho_0^2/\ell t_r) = 16.1466$, where $\ell = (2/13)$, $\rho_0 = \rho(0) - \rho_\ell$ and $\rho_\ell = \tan(\ell)$. For convenience, denote the proposed controller and the one in [41] by $u_1(t)$ and $u_2(t)$, respectively. In addition, Gaussian membership functions are the same as those in (49). For the sake of comparison, take two sets of initial values as $\bar{q}_{12}(0) = (0, 0)^T$ and $\bar{q}_{22}(0) = (10, 0)^T$, and all other initial values are chosen as zero. Figs. 6–9 present the simulation results.

It can be observed from Figs. 6–8 that controller $u_1(t)$ can guarantee the prescribed transient and steady-state performance of the tracking error within prescribed time under both $\bar{q}_{12}(0)$ and $\bar{q}_{22}(0)$. However, controller $u_2(t)$ fails to guarantee prescribed tracking performance under $\bar{q}_{22}(0)$ since the initial values constraint $|e(0)| < \rho(0)$ does not hold. Additionally, from Fig. 9, the value of the proposed controller $u_1(t)$ is generally less than that of controller $u_2(t)$ under $\bar{q}_{12}(0)$, which demonstrates the effectiveness and practicality of the proposed methods. It is worth noting that the controller $u_2(t)$ can only guarantee the semi-global boundedness of the tracking error for the constraints induced by $|e(0)| < \rho(0)$ and fuzzy logic systems, but these constraints can be removed when applying the controller $u_1(t)$ and the global boundedness is achieved.

V. CONCLUSION

Adaptive fuzzy tracking control with global prescribed-time prescribed performance for strict-feedback nonlinear systems with mismatched uncertainties has been studied. First, a class of prescribed-time prescribed performance functions...
Fig. 9. Control inputs.

independent of initial values, along with an error transformation function, are designed. Second, two adaptive fuzzy controllers, one with approximating structures and one without, are designed to guarantee prescribed-time prescribed performance of the tracking error and the global uniform boundedness of all system variables. With the modified dynamic surface control and a novel Lyapunov-like energy function, the differential explosion problem frequently occurring in backstepping techniques is effectively solved, and the global performance of the tracking error is guaranteed. It is worth noting that the control design does not involve any singular phenomena, enabling the avoidance of complex calculations can be avoided. Finally, some practical examples are employed to demonstrate the validity and effectiveness of the proposed methods. It is worth noting that it might be challenging to obtain \( L_i(q_i(t), \dot{p}_i(t), \dot{t}) \) in Assumption 4 for some practical systems with unknown nonlinearities. Additionally, the proposed control mechanism is not applicable to systems with unknown control directions, which limits its applicability and effectiveness in practical implementation to some extent. Therefore, future research will focus on the global performance-guaranteed tracking control for strict-feedback nonlinear systems with unknown control directions, without relying on prior knowledge of nonlinearity.

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