THE COLD DARK MATTER HALOS OF LOCAL GROUP DWARF SPHEROIDALS

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ABSTRACT

We use N-body simulations to study the evolution of dwarf spheroidal galaxies (dSphs) driven by galactic tides. We adopt a cosmologically motivated model in which dSphs are approximated by a King model embedded in an NFW halo. We find that these NFW-embedded King models are extraordinarily resilient to tides; the stellar density profile still resembles a King model even after losing more than 99% of the stars. As tides strip the galaxy, the stellar luminosity, velocity dispersion, central surface brightness, and core radius decrease monotonically. Remarkably, we find that the evolution of these parameters is solely controlled by the total amount of mass lost from within the luminous radius. Of all parameters, the core radius is the least affected: after losing 99% of the stars, \( R_c \) decreases by just a factor of \( \sim 2 \). Interestingly, tides tend to make dSphs more dark matter dominated because the tightly bound central dark matter “cusp” is more resilient to disruption than the “cored” King profile. We examine whether the extremely large mass-to-light ratios of the newly discovered ultrafaint dSphs might have been caused by tidal stripping of once brighter systems. Although dSph tidal evolutionary tracks parallel the observed scaling relations in the luminosity-radius plane, they predict too steep a change in velocity dispersion compared with the observational estimates hitherto reported in the literature. The ultrafaint dSphs are thus unlikely to be the tidal remnants of systems like Fornax, Draco, or Sagittarius. Despite spanning four decades in luminosity, dSphs appear to inhabit halos of comparable peak circular velocity, lending support to scenarios that envision dwarf spheroidals as able to form only in halos above a certain mass threshold.

Subject headings: galaxies: dwarf — galaxies: evolution — galaxies: fundamental parameters — galaxies: halos — galaxies: kinematics and dynamics — Local Group

Online material: color figures

1. INTRODUCTION

Ever since the first measurements of the velocity dispersion of dwarf galaxies in the vicinity of the Milky Way became available (Aaronson 1983; Aaronson & Olszewski 1987), they have been a difficult puzzle to piece together in models of galaxy formation. Their relatively large size, low luminosity, and sizable velocity dispersion suggest the presence of large amounts of dark matter (Armandroff et al. 1995; see the review of Mateo 1998, and for more recent work consult Klypin et al. 2005; Muñoz et al. 2005, 2006, and references therein). However, the absence of clear correlations between inferred dark matter content and the structural properties of the luminous component have led to arguments about the true physical nature of these objects and to various proposals to explain their large mass-to-light ratios.

Velocity dispersions unduly affected by binary stars (Olszewski et al. 1996), line-of-sight alignment of unbound stars (Kroupa 1997), and remnants of stellar clusters severely disrupted by tides (Metc & Kroupa 2007) have all been considered in the literature, but consensus now seems to have been reached. Dwarf spheroidal galaxies (dSphs) are widely regarded as dark matter–dominated systems sampling the extreme low-mass end of the dark halo mass function. Nevertheless, a number of issues regarding the dark matter content of these elusive systems, as well as the spatial extent of their dark halos, remain largely unresolved.

These issues were brought into focus when cosmological N-body simulations revealed the presence of substantial substructure in galaxy-sized cold dark matter (CDM) halos (Klypin et al. 1999; Moore et al. 1999). These simulations indicate that hundreds of self-bound CDM halos massive enough (in principle) to harbor a dwarf galaxy are expected to populate the halo of the Milky Way, in sharp contrast with the mere tens of dwarf galaxies known to orbit the Galaxy. This realization rekindled interest in explaining the detailed correspondence between dark halos and luminous galaxies at the faint end of the luminosity function, an issue that had long been highlighted as a challenge for hierarchical galaxy formation models (White & Rees 1978; Kauffmann et al. 1993).

The leading scenario for reconciling the discrepancy between “luminous” and “dark” substructure in the Milky Way halo envisions dwarf galaxies as able to form only in halos above a certain mass threshold. The threshold is determined by the need to retain gas and to sustain continuing star formation despite the effects of feedback from evolving stars and the heating from photoionizing radiation (Efstathiou 1992; Bullock et al. 2000; Somerville 2002; Benson et al. 2002). Adjusting the mass threshold appropriately, the scarcity of luminous dwarfs may be explained by the relatively few massive substructure halos that exceed the threshold (Stoehr et al. 2002; Hayashi et al. 2003; Kazantzidis et al. 2004). This is an appealing and elegant solution, and offers a relatively clean prediction that may be tested observationally: most dwarfs at the extreme faint end of the luminosity function should inhabit relatively massive halos, of mass comparable to that defined by the threshold.

For example, given the large spread in luminosity of dwarfs in the Local Group\(^2\) and the expectation that they should all inhabit

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2 We consider in this paper the “traditional” dwarfs brighter than \( M_l \sim -8 \), but recent discoveries based on the SDSS have uncovered the presence of many fainter dwarfs, extending the galaxy luminosity function to as faint as a few thousand \( L_\odot \) (e.g., Irwin et al. 2007 and references therein).
halos of similar mass, this proposal implies that there should be little correlation between the dark matter and luminous content of a dwarf. Massive dark halos would also be more affected by dynamical friction (Peñarrubia et al. 2002, 2006; Zenitner & Bullock 2003; Peñarrubia & Benson 2005), leading to a bias in the spatial distribution of dSphs within the halo of the Milky Way relative to the bulk of substructure halos—an effect that may be potentially observable.

Finally, their relatively massive halos would make these extremely faint galaxies more resilient to tidal disruption, suggesting that tidal tails and other obvious evidence of tidal stirring should be present only in systems that have lost most of their original mass by the action of tidal forces. In these models, the “tidal” radii attached by King-model fitting to the surface brightness profiles of dSphs, as well as the “break” radii identified in the outskirts of some dSphs, should reflect just the edge of the luminous component rather than a feature of dynamical significance (e.g., UMi: Martínez-Delgado et al. 2001; Palma et al. 2003; Carina: Majewski et al. 2005). Establishing the present dark matter content of dwarf galaxies in the Local Group thus promises to be a fruitful enterprise.

Validating (or refuting) these theoretical expectations through observation, however, is not straightforward. One reason is that few dwarfs have gas on circular orbits, and therefore stars are the only viable dynamical tracer. The interpretation of the observations is thus complicated by degeneracies between orbital shapes, their radial dependence, and the overall mass profile (Wilkinson et al. 2002, 2004; Kazantzidis et al. 2004; Mashchenko et al. 2006; Kleya et al. 2004; Tolstoy et al. 2004; Muñoz et al. 2005, 2006; Wang et al. 2005; Walker et al. 2006a, 2006b; Sohn et al. 2007). The second reason concerns the fact that these dynamical tracers lie, by definition, within the luminous radius of the dwarfs. Since dark halos are expected to be extended objects reaching far beyond the luminous confines of a galaxy, a certain amount of uncertain extrapolation appears inevitable. In this respect, mass-follows-light models (Richstone & Tremaine 1986) only provide an approximate estimate of the amount of dark matter within the luminous radius that cannot be safely extrapolated to larger distances. Finally, Galactic tides may affect dark matter and stars differently, especially if stars are, as expected, strongly segregated relative to the dark halo. Thus, a dwarf may today inhabit a relatively low mass halo (say, below the threshold mentioned above), even though it actually formed in a massive one that has since seen much of its mass stripped away by tides (Kravtsov et al. 2004).

This paper is the first in a series that attempts to address these issues by interpreting available observational data on Local Group dwarfs in the cosmological context defined by the leading paradigm of structure formation: the cold dark matter theory. We start by considering the constraints on CDM halos placed by the structural and dynamical properties of the dwarfs, and use these results to try to interpret the origin of structural differences in the population of dwarfs that orbit M31 and the Milky Way, respectively. A future paper on the subject will analyze in detail the effects of Galactic tides, both in the survival of the stellar components and on the observational signatures imprinted by stripping (Peñarrubia et al. 2008).

The paper is organized as follows. Section 2 introduces the relevant observations and the modeling procedure, whereas § 3 applies the models to the Milky Way dwarfs. We apply these results to M31 dwarfs in § 4 and conclude with a brief summary in § 5.

2. PRELIMINARIES

2.1. Summary of Observations

Table 1 lists the observational parameters of the Local Group dwarfs that we study here. We split the dwarfs into two groups and list each in order of decreasing luminosity. The first set of objects are Milky Way (MW) dwarf spheroidals, whereas the second set are all dSph-type systems for which accurate core and tidal radii from King-model fitting are available. Most of the latter satellites orbit around M31, except for Cetus, which is one of only two isolated dSphs in the Local Group.

Table 1 lists the Galactic coordinates of each dwarf (l and b), as well as its heliocentric distance, D. Distances are taken from McConnachie et al. (2004, 2005) where available, otherwise, from the compilation by Mateo (1998). Absolute magnitudes (M), core (Rc), and tidal (Rt) radii for the best-fitting King profiles to the surface brightness distribution are taken from Irwin & Hatzidimitriou (1995) for the MW satellites and from McConnachie & Irwin (2006) for Cetus and the M31 population, and are quoted after rescaling to the adopted distance. One exception is And IX, with core and tidal radii taken from Harbeck et al. (2005), distance from McConnachie et al. (2005), and the absolute magnitude from Zucker et al. (2004). The other is And II, the one dSph in the Local Group whose surface brightness profile shows evidence for the presence of more than one dynamical component (McConnachie & Irwin 2006; McConnachie et al. 2007) and for which single King-model fits provide a poor description of the structure of the dwarf. In this case, we quote the And II core radius used by Côté et al. (1999), since this is consistent with the traditional definition of core radius as the distance from the center where the surface brightness drops by a factor of 2.

The central velocity dispersions ($\sigma_v(0)$) for the MW population in Table 1 are taken from the compilation by Mateo (1998; his Table 7), although we note that since the publication of that review velocities for hundreds of stars in these dSphs have been measured with the aid of multiobject spectrographs. These data constrain the velocity dispersion profiles, $\sigma_v(R)$, in these systems, which are found to be approximately flat out to the nominal tidal radius (see, e.g., Fornax: Walker et al. 2006a; Battaglia et al. 2006; Leo I: Koch et al. 2007; Sculptor: Tolstoy et al. 2004; Westfall et al. 2006; Leo II: Sohn et al. 2007; Sextans: Walker...
et al. 2006b; Carina: Muñoz et al. 2006; Draco and Ursa Minor: Wilkinson et al. 2004; Muñoz et al. 2005). Note that the presence of several distinct components may affect the measured kinematics of a dwarf, depending on the spatial distribution of the tracers for which velocities are available (see, e.g., McConnachie et al. 2006 and references therein). We neglect here complications that arise from this issue, although we plan to address this issue in future work.

Kinematic data for the M31 dwarf population is scarce and of comparatively of poor quality. For example, Chapman et al. (2005) find that the velocity dispersion of And IX is comparatively of poor quality. For example, Chapman et al. (2005) find that the velocity dispersion of And IX is based on just seven stars but note that adding a single (possible member) star to their sample raises this estimate to ~12 km s\(^{-1}\). Similarly, the velocity dispersion for And II (Côté et al. 1999) is based on just seven stars and is thus subject to sizable uncertainty. The \(\sigma_p(0)\) estimate for Cetus (Lewis et al. 2007) is based on a larger sample of stars and is thus more reliable. We present these data in Table 1 between parenthesis to emphasize that these data are not of the same quality as is available for the MW dSphs. We return to this issue in § 4.

The past few years have seen a dramatic increase in the number of dwarf satellites discovered around M31 and the MW, in particular as a result of the completion of the Sloan Digital Sky Survey (SDSS; Zucker et al. 2004, 2006a, 2006b, 2007; Willman et al. 2005; Belokurov et al. 2006, 2007; Martin et al. 2006; Irwin et al. 2007). Many of these systems are morphologically unlike the ones listed in Table 1, but they are typically much fainter and of much lower surface brightness, which has precluded robust fits to their surface brightness profiles.

The sample of Milky Way dwarf galaxies used here only includes dwarf spheroidal galaxies (i.e., nonrotating systems with little or no detectable gas). These galaxies must have: (1) King profile fit parameters available from the literature and (2) measured central velocity dispersion. Thus, our sample excludes many of the recently discovered dSph galaxies in the Milky Way and M31. We have also excluded the Canis Major dSph (Martin et al. 2004) because of its uncertain nature. A lively debate can be found in the literature, with arguments in favor of this system being a new galaxy (Martinez-Delgado et al. 2005; Bellazzini et al. 2006) and against it (e.g., Momany et al. 2006; Moitinho et al. 2006). The Sagittarius dSph was also removed from our sample because it is a clear case in which tidal stripping has altered the luminous component as well as the dark matter halo. This particular system will be studied in detail in subsequent papers in this series, which will address the effects of tides on the results presented here.

2.2. Modeling

Our dwarf galaxies models assume the presence of two components in dynamical equilibrium: (1) a stellar component approximated by a King (1966) model and (2) a dark matter halo, which we approximate using a Navarro-Frenk-White profile (Navarro et al. 1996, 1997, hereafter NFW). We assume that the dark matter dominates the dynamics of the system and that stars may be regarded as massless tracers of the potential.

2.2.1. Luminous Component

The density profile of a King model may be written as (King 1962)

\[
\rho_\ast(r) = \frac{K}{r^2} \left[ \frac{\cos^{-1}(x)}{x} - \sqrt{1 - x^2} \right],
\]

where

\[
x = \left[ 1 + \left( \frac{r}{r_k} \right)^2 \right]^{-1/2},
\]

Here \(\rho_\ast(r > r_k) = 0\), \(r_k\) is an inner radial scale, \(r_i\) is the King “tidal” radius, and \(K\) is an arbitrary normalizing constant.

In the absence of rotation, the stellar kinematics is determined by the total gravitational potential, \(\Phi(r)\), through Jeans’ equations. In particular, the radial velocity dispersion of stars, \(\sigma_r\), is given by (Binney & Tremaine 1987)

\[
\sigma_r^2 = \frac{1}{\Sigma(R)} \int_r^{\infty} \frac{\rho_\ast \sigma_r^2}{\sqrt{r^2 - R^2}} \frac{d\Phi}{dr} \, dr,
\]

(3)

where \(\beta\) is the velocity anisotropy. We assume hereafter that the stellar velocity distribution is isotropic (\(\beta = 0\)), which implies that the (observable) stellar line-of-sight velocity dispersion, \(\sigma_p(R)\), is given by

\[
\sigma_p^2(R) = \frac{2}{\Sigma(R)} \int_0^R \frac{\rho_\ast \sigma_r^2}{\sqrt{r^2 - R^2}} \, dr,
\]

(4)

We follow traditional convention and define the (projected) core radius, \(R_c\), by the condition \(\Sigma(R_c) = \Sigma(0)/2\). The core radius, defined in this way, depends on both of the King-model parameters, \(r_k\) and \(r_i\). For example, \(R_c \approx 0.73 r_k\) for \(r_i/r_k = 10\).

2.2.2. Dark Matter Component

We assume that dark matter halos may be approximated by NFW profiles (Navarro et al. 1996, 1997). The density profile may be written as

\[
\rho_{\text{NFW}} = \frac{M_{\text{vir}}}{4\pi r_{\text{vir}}^3} \left( \frac{r}{r_{\text{vir}}} \right)^{-1} \left[ \ln(1 + c) - c/(1 + c) \right],
\]

(6)

where \(M_{\text{vir}}\) is the mass within the virial radius, \(r_{\text{vir}}, r_s\) is the scale radius, and \(c\) is the concentration \((c \equiv r_{\text{vir}}/r_s)\). The virial radius is defined so that the mean overdensity relative to the critical density is \(\Delta_{\text{vir}}\)

\[
\frac{M_{\text{vir}}}{(4/3)\pi r_{\text{vir}}^3} = \Delta_{\text{vir}} \rho_{\text{crit}} = \frac{3H(z)^2}{8\pi G}.
\]

(7)

We follow Bryan & Norman (1998) and define the overdensity by

\[
\Delta_{\text{vir}}(z) = 18\pi^2 + 82f(z) - 39f(z)^2,
\]

(8)

where

\[
f(z) = \frac{\Omega_0 (1 + z)^3}{\Omega_0 (1 + z)^3 + \Omega_\Lambda} - 1.
\]

(9)

We assume throughout the paper a \Lambda CDM universe by fixing the cosmological parameters to \(\Omega_0 = 0.3, \Omega_\Lambda = 0.7, h = 0.7\).
vertical units are arbitrary in this plot. The corresponding velocity dispersion profiles are shown in Figure 2. Velocities have been scaled to the peak circular velocity of the NFW halo, \(V_{\text{max}}\), and show clearly the anticipated behavior: the deeper the stars are embedded within the halo, the smaller the stellar velocities. For example, if \(r_c \sim 0.1r_s\), the stellar central velocity dispersion, \(\sigma_r(0)\), is only about 40% of \(V_{\text{max}}\), but this value rises to 80% for \(r_c \sim r_s\). There is also a weak dependence on the tidal radius, but this is minor, as discussed below.

This finding implies that a “family” of NFW halos is consistent with a King model of given \(r_c\) and \(\sigma_r(0)\) (see Strigari et al. 2006 for a similar argument applied to the Fornax dSph). The more embedded we assume the King model to be within the NFW halo, the more massive (i.e., higher \(V_{\text{max}}\)) the halo must be in order to explain a given \(\sigma_r(0)\). This “King-NFW degeneracy” is illustrated in Figure 3, where the thick lines show the correspondence between \(V_{\text{max}}/\sigma_r(0)\) and \(r_c/r_s\). The three thick lines (almost indistinguishable from one another in this panel) correspond to three different values of the tidal-to-core radius ratio chosen for the King model, and confirm the result anticipated above regarding the weak dependence on \(R_t\) of these results.

Any NFW halo whose circular velocity peaks somewhere along the thick curve in Figure 3 is therefore consistent with a King model of given \(r_c\) and \(\sigma_r(0)\). A few examples of NFW halos belonging to this family are shown by the thin lines in Figure 3. Note that essentially all the circular velocity profiles of these halos, despite having very different masses and concentrations, cross each other at \(r \approx r_c\). This implies that the total enclosed mass within the core radius of a King model is robustly determined given our assumptions: in particular, we find \(V_c(r_c) \approx 1.2\sigma_r(0)\) and \(M(r_c) \approx 1.44r_c\sigma_r(0)^2/G\). Physically, this means that, although the total mass and radial extent of the halo are not well pinned down, the mass within the luminous core radius may be constrained even in the presence of a nearly universal radial velocity dispersion profile.

3. KING MODELS EMBEDDED IN NFW HALOS

3.1. Spatial Segregation and Velocity Dispersion

Stars that follow a King model and inhabit an NFW halo have their kinematics dictated principally by the spatial segregation of stars relative to dark matter, which may be quantified by the ratio of King “core” radius to NFW scale radius, \(r_c/r_s\). The more concentrated the stars are relative to the dark matter, the smaller the velocity of the stars will be relative to the characteristic circular velocity of the dark matter-dominated potential.

Figure 1 shows the projected density profile of King models as a function of \(r_c/r_s\) and compares them to an NFW profile. The radial units are scaled to the NFW scale radius, \(r_s\), and the
Fig. 3.—King-NFW degeneracy. The thick lines show the halo peak circular velocity, $V_\text{max}$, in units of the central velocity dispersion and the radius of the peak, $r_\text{max}$, in units of the King-model core radius. Solid, dotted, and dashed lines denote different King-model concentrations ($R_\text{c}/R_\text{s}$). Any NFW halo whose circular velocity peaks along this curve is consistent with the King-model structure and kinematics. A few of these NFW models are shown for illustration by the thin curves. Note that all these NFW models cross each other at approximately $R \approx R_\text{s}$ and $V_\text{c} \approx 1.2\sigma_0(0)$. This implies that, given our assumptions, the mass within the core radius of the stellar component is robustly constrained to be $M(R_s) \sim 1.44\sigma_0^2(0)/G$. [See the electronic edition of the Journal for a color version of this figure.]

region sampled by the tracers is. (See also Strigari et al. 2007 for a similar approach and conclusion.)

A further constraint may be gleaned from Figure 2, which shows that the shape of the stellar velocity dispersion profile also depends on the degree of segregation between stars and dark matter. For $R_0 0.1r_s (= 0.05r_\text{max})$ the velocity profile remains approximately flat well outside the core radius and declines abruptly only at the “tidal” radius. On the other hand, less segregated King models show a steep velocity decline noticeable near the core radius; for example, for $R_0 \sim r_s$, the velocity dispersion declines by roughly 70% at the core radius from the central value. The velocity dispersion profiles of all dwarfs for which such data are available show little sign of declining outside $R_0$ (see references in §2.1). In the context of our modeling, this suggests that the stellar component is deeply embedded within its parent CDM halo (i.e., $R_0r_\text{max}$), an issue to which we return below.

3.2. Application to Milky Way Dwarfs

One way of breaking the degeneracy illustrated in Figure 3 is to appeal to the results of cosmological $N$-body simulations. These show that there is a strong correlation between the mass and concentration of a CDM halo or, equivalently, between $r_\text{max}$ and $V_\text{max}$. As discussed by NFW, this correlation arises because the characteristic density of a halo is proportional to the density of the universe at the time of its assembly. In practice, and for galaxy-sized systems, the mass-density dependence is quite weak, implying that $r_\text{max}$ is roughly proportional to $V_\text{max}$. This correlation is now well established, and a number of authors provide simple formulae to compute it once the cosmological parameters are specified (NFW; Eke et al. 2001; Bullock et al. 2001).

The relation between $V_\text{max}$ and $r_\text{max}$ implies that, of all NFW halos in the family of models allowed for a given dwarf by the degeneracy illustrated in Figure 3, a single one will be consistent with the parameters expected in a given cosmogony. This is shown in Figure 4, where the curved lines show, in different panels and for each of the eight MW dSphs, the King-NFW “degeneracy” relation. The straight lines in each panel delineate the $V_\text{max}-r_\text{max}$ relation consistent with the ΛCDM cosmogony. The set of three straight lines correspond to NFW halos identified at various redshifts; we adopt the $z = 0$ models here, but note that our conclusions are unlikely to be severely affected by this choice. [See the electronic edition of the Journal for a color version of this figure.]

In each panel a solid dot indicates, for reference, the core radius and the central velocity dispersion of each galaxy, as listed in Table 1. All of these points lie well to the left of the cosmological relations, confirming our earlier suggestion that the luminous components are significantly segregated within their dark halos (i.e., they are substantially smaller for given characteristic velocity).

The intersection between the King-NFW degeneracy and the cosmological relation is marked by gray symbols for each dwarf. This indicates the parameters of the NFW halo model that is consistent with the ΛCDM cosmogony and, at the same time, matches the kinematics and structure of each dwarf galaxy. The circular velocity profiles of the eight NFW halos satisfying these criteria are shown in Figure 5, labeled from top to bottom in order of decreasing halo mass.

This ranking shows interesting peculiarities. For example, it shows that the peak circular velocities of dwarf halos vary from ~17 to ~35 km s$^{-1}$, corresponding to a spread of about 8 in mass, much narrower than the factor of ~70 spanned by dSph luminosities. Note as well that, as anticipated in § 1, halo mass is not monotonically related to luminosity. Intriguingly, Draco,
one of the faintest dwarfs in our sample, is assigned the most massive halo, whereas Fornax, despite being 70 times brighter, is assigned a halo 5 times less massive.

The lack of correlation between luminosity and halo mass is shown explicitly in Figure 6, where we plot, as a function of total luminosity, the virial mass of the halo (gray symbols at the top), as well as the mass within the core radius of each dwarf (black symbols at bottom). Symbols are the same as those used in Figure 5 to denote different galaxies.

Note that the total mass within the core radius—a fairly robust measure according to our discussion of Figure 3—is approximately independent of luminosity.\footnote{This is not a new result, and it is consistent with Mateo’s (1998) conclusion that simple dynamical mass estimates of dwarf galaxies are independent of luminosity.} This implies that the average density of dark matter will be higher in physically smaller systems such as Draco than in more extended dwarfs such as Fornax. Why does this matter? Because more massive halos are denser than less massive ones at all radii in the CDM cosmogony. Indeed, note that the circular velocity profiles of the NFW halos shown in Figure 5 do not cross, which means that measuring the halo circular velocity (or mass) at any radius leads to a well-defined estimate of the total mass of the halo. Because Draco, despite being faint, has a circular velocity comparable to Fornax’s at a much smaller radius, it requires a denser, and therefore more massive, halo to satisfy the observational constraints.

Of the eight dwarfs, the lowest halo mass corresponds to Sextans, whose relatively large radius and small velocity dispersion is inconsistent with a very massive CDM halo.

3.3. Effects of Tidal Stripping

The results obtained in the previous section assume that the structure of dark matter halos is well approximated by an NFW profile. Although this assumption may be appropriate for isolated halos, it is unlikely to hold in detail for “substructure” halos of dSphs orbiting within the main halo of the Milky Way. Recently, Stoehr et al. (2002), Hayashi et al. (2003), and Kazantzidis et al. (2004) have examined the modifications undergone by an NFW halo as it is tidally stripped inside a more massive system. Stripping affects principally the outer regions of the halo, and therefore deeply embedded stellar structures may survive unscathed the removal of large fractions of their halos.

This process is illustrated in Figure 7, which shows the circular velocity profiles of an NFW halo after being tidally stripped of 50%, 75%, and 90% of its mass, respectively. The profiles are taken from N-body simulations of NFW halos orbiting within the potential of a much larger system, as in Hayashi et al. (2003). Figure 7 indicates that NFW halos must lose at least 90% of their original mass before regions within $0.1r_{\text{max}}$ are significantly affected. Interestingly, this is precisely the region that our analysis suggests is populated by the stars of MW dwarfs. According to Figure 5 the average $R_c/r_{\text{max}}$ for all MW dwarfs is 0.054, and they all have, with the possible exception of Sextans, $R_{\text{core}}/r_{\text{max}} = 0.1$.

Tidal stripping is clearly affecting the structure of the Sagittarius dSph (which is thus not in the sample considered here), but the evidence is less clear-cut in the case of other dwarfs, despite a rich literature on the topic. Much of this work is based on detecting stars associated with a dwarf beyond the King “tidal radius” or on the interpretation of “breaks” in the surface density profile in the outer regions of a dwarf (e.g., Ursa Minor: Martinez-Delgado et al. 2001; Carina: Majewski et al. 2005).

The presence of a break in the outer profile, however, does not imply that the system is necessarily losing stars. Such feature might just be intrinsic to the dSph, as shown, for example, by the multiple-component models presented in McConnachie et al. (2006). These authors show that systems with multiple stellar components (such as And II) may have overall density profiles that resemble those of dwarf galaxies with outer “tidal” breaks.
The presence of well-defined and kinematically distinct populations of stars within some dSphs (Sculptor: Tolstoy et al. 2004; Fornax: Battaglia et al. 2006; Canis Venatici: Ibata et al. 2006) indeed argues against tides as a major driver in the evolution of a dSph, since tidal stirring would tend to render uniform such distinctions.

On similar grounds, one cannot rule out that other features in the light profile usually ascribed to tides, such as the presence of lumps (UMi: Olszewski & Aaronson 1985), shells (Fornax: Coleman et al. 2005), and aspherical isopleths (UMi: Martínez-Delgado et al. 2001) may actually reflect complexities in the formation process and/or the internal evolution of dwarf galaxies.

The lack of overwhelming evidence for the ongoing stripping of stars from most dSphs may thus be taken to imply that dwarf halos have retained at least 10% of their original mass at present. This is important, because even for such sizeable loss, the peak circular velocity of the halo is barely affected.

The thin line in Figure 7 tracks the position of the circular velocity peak of an NFW halo as it is tidally stripped and shows that halos that have lost 75% of their mass to stripping see only a 10% decrease in \( V_{\text{max}} \). Even after losing 90% of its mass, a halo sees its \( V_{\text{max}} \) reduced by only \( \sim 30\% \).\(^4\) We conclude, therefore, that unless dSphs have suffered catastrophic tidal losses, our estimates of the peak circular velocities for the MW dSphs (Fig. 5) should be relatively robust, even if the amount of dark matter beyond the luminous radius remains unknown.

We emphasize, however, that tides do pose a number of interesting questions that our modeling fails to address. For example, do King profiles survive strong tidal stripping? Do we expect the presence of “extratidal” stars beyond a break radius if the luminous component is still surrounded by dark matter? These issues are best addressed by self-consistent N-body simulation of the tidally driven evolution of multicomponent dSph models. This is beyond the scope of the present paper, but will be dealt with in a forthcoming paper of our series.

3.4. Application to the “Satellite Crisis”

The analysis of the preceding subsections suggests that the eight MW dwarfs chosen for our analysis inhabit halos with peak circular velocities in the range 17–35 km s\(^{-1}\), a factor of \( \sim 3 \) higher than their central (stellar) velocity dispersions. As discussed by Stoehr et al. (2002), Hayashi et al. (2003), and Kazantzidis et al. (2004), this correction is enough to alleviate substantially the satellite crisis highlighted by Klypin et al. (1999) and Moore et al. (1999). This result is illustrated in Figure 8, which compares the estimates of the peak circular velocities obtained in this contribution (filled circles) with those that adopt mass-follows-light models (filled triangles).

In rough terms, \( N \)-body simulations show that there are typically about 20–30 substructure halos with peak circular velocities exceeding 10% of the main halo’s virial velocities. That is shown in Figure 8 from the results of Diemand et al. (2007) for two different Milky Way–like halos (dotted and dashed lines). According to our analysis, there are about a dozen dwarfs with \( V_{\text{max}} \geq 17 \) km s\(^{-1}\) (adding to our sample brighter satellites such as the Magellanic Clouds). This velocity corresponds to between 8% and 11% of the MW virial velocity, assuming, as seems likely, that the latter is in the range 150–220 km s\(^{-1}\). This factor of \( \sim 2–3 \) discrepancy between the number of massive dark matter substructures and luminous satellites does not appear extravagant given the uncertainties.

\(^4\) The location of the peak, \( r_{\text{max}} \), on the other hand, is more significantly affected and shifts inward by almost a factor of \( \sim 3 \) after a halo loses 90% of its mass.
Furthermore, the discrepancy may disappear altogether if (as the scenario outlined in §1 would suggest) the ultrasound MW dwarfs identified in SDSS data (Fig. 8, open symbols) turn out to inhabit halos of masses comparable to those of the eight dSphs we consider here. Since these newly discovered dwarfs have, on average, similar physical size to the dSphs in Table 1, we expect that their velocity dispersions will also be comparable, falling in the range $\sim$6–10 km s$^{-1}$, a prediction of this CDM-motivated modeling that should be testable in the near future.

Indeed, further progress on this subject seems imminent, as observational campaigns secure velocity dispersions for many of the newly discovered dwarfs and extend available data to allow velocity dispersion profiles to be measured for more systems (see, e.g., the recent preprint by Simon & Geha 2007). One also expects that numerical simulations will improve to the point that a reliable direct estimate of the number of substructure halos satisfying the $M(R_c)$ constraints shown in Figure 6 will be possible, obviating the need to extrapolate results out to the peak of the halo circular velocity curve. The latter goal requires simulations able to resolve convincingly the inner $\sim$100 pc of substructure halos, an order of magnitude improvement over simulations published so far. This is a numerical challenge that will likely be met soon but that will nevertheless require the investment of massive computational resources. (See the recent preprint by Kuhlen et al. 2007 for a report of recent progress on this issue.)

4. MILKY WAY VERSUS M31 DWARFS

Photometric studies of the dwarf galaxies in the Local Group (Irwin & Hatzidimitriou 1995; McConnachie & Irwin 2006) have shown that, at fixed luminosity, M31 dwarfs are considerably more extended than their Milky Way counterparts (see Table 1). We illustrate this in Figure 9, where King-model fits to the surface brightness profiles of three pairs of dwarfs of similar luminosity are shown. Each pair consists of one M31 satellite and one MW satellite, respectively. Clearly, the M31 dwarfs are systematically different: they are about 1.5 mag fainter in central surface brightness and about a factor of 2 larger in radial extent compared with MW dwarfs (see Fig. 9.

The origin of this difference is unclear, but it nonetheless offers a way of assessing the general validity of the modeling proposed in the previous sections. Indeed, under the plausible assumption that the dark matter halos of dwarfs in M31 and in the Milky Way are not systematically dissimilar, the arguments of §3.1 imply that the difference in size of M31 dwarfs should be reflected in their kinematics through significantly higher velocity dispersions. Note that this prediction depends explicitly on the presence of dark matter halos whose mass and extent are largely independent from that of the stars, as implicitly assumed in our modeling so far. Other (also plausible?) assumptions, such as mass traces light, would lead to the opposite trend and would predict systematically lower velocity dispersions.

In order to estimate quantitatively the velocity dispersion of M31 dwarfs we need to know how spatially segregated the stars are from the dark matter in these systems. Once this is fixed, for example, by assuming a value for the ratio $R_c/r_{max}$, we can compute $r_{max}$ for each dwarf and from that result infer $V_{max}$ assuming the cosmological relation between these parameters. Since the ratio $R_c/r_{max}$ also fixes $V_{max}/\sigma_p(0)$ according to the King-NFW degeneracy shown in Figure 3, an estimate can then be made of the central velocity dispersion, $\sigma_p(0)$, for each M31 dwarf.

We have followed this procedure, assuming $R_c/r_{max} = 0.05$, consistent with the average segregation measure derived for MW dwarfs (see Fig. 7), as well as with the flat $\sigma_p(R)$ constraint mentioned in §3.1. This leads to the estimates of the velocity dispersions of M31 dwarfs plotted in the bottom panel of Figure 10 (open symbols). Error bars show the variation induced in the estimate by allowing $R_c/r_{max}$ to vary between 0.025 and 0.1. Clearly, the velocity dispersion of M31 dSphs is significantly larger (by about a factor of $\sim$2) than that of their MW counterparts, as
shown by the shift between filled circles and open symbols in Figure 10.

We emphasize that these “predictions” for the velocity dispersions of M31 dwarfs should be taken with caution; indeed, the open symbols in Figure 10 are best interpreted as indicative of the relative shift in the velocity dispersion of the population of M31 dwarfs, rather than as separate individual predictions.

So far, the few available observations are consistent with the predicted trend. Of the three dwarfs outside the MW for which velocity dispersions have been published (see Table 1), the most reliable is Cetus, which, at 17 km s$^{-1}$, also has the highest $\sigma_v$ in the sample, consistent with the predicted trend (our naive prediction yields $11.4^{+3}_{-2}$ km s$^{-1}$). Likewise, the higher of the two estimates (12 km s$^{-1}$) derived by Chapman et al. (2005) for And IX is also consistent with this trend, although the very few stars in this galaxy (as well as in And II) with measured velocities suggest that it would be premature to draw firm conclusions until the data improve substantially.

It is clear, however, that should future observations fail to confirm the predicted trend, one or many of our assumptions would need to be revised. As an illustrative example, we consider how our interpretation would change if the velocity dispersion of M31 dwarfs were to show no systematic departure from that of their MW counterparts (i.e., if the open symbols were found to align with the dashed line in the bottom panel of Fig. 10). In this case, one would be forced to conclude that M31 and MW dwarfs of similar luminosity inhabit systematically different halos. In particular, M31 dwarf halo hosts would need to be significantly less massive (i.e., lower $V_{\text{max}}$ and smaller $r_{\text{max}}$) in order to accommodate their larger physical size but similar kinematics.

This possibility is shown in Figure 11, implying that essentially all M31 dwarfs considered here would have $V_{\text{max}}$ 15 km s$^{-1}$ (with the possible exception of And VII). Since, in all likelihood, M31 inhabits a halo at least as massive as the Milky Way’s, it should have at least as many massive substructures as our own Galaxy, and one would need to explain why those substructures have not “lit up” in M31 as they have in the Milky Way. We consider this a compelling argument against this interpretation and in support of higher velocity dispersions for M31 dwarfs as the most natural prediction of CDM-motivated models of the Local Group dSphs.

One last possibility should be mentioned, namely, that the structural differences between M31 and MW dwarfs reflect differences in the dynamical evolution driven by tides in M31 and the Galaxy. Although we have argued in § 3.3 that this is unlikely, a definitive assessment requires high-resolution, self-consistent $N$-body simulations of how tides affect multiple-component models of dSph galaxies. We plan to address this issue in the following paper of this series (Peñarrubia et al. 2008).

5. SUMMARY

We have considered in this paper the observational constraints placed on the mass and spatial extent of cold dark matter halos surrounding Local Group dwarf spheroidal galaxies. Assuming that the luminous component may be approximated by King models and that dark halos follow the NFW mass profile, we conclude that estimates of the halo mass of a dSph depend principally on the degree of spatial segregation between stars and dark matter: the more embedded the stars within the dark halo, the more massive the halo must be in order to explain the observed stellar kinematics. This degeneracy may be broken by appealing to the results of cosmological $N$-body simulations—which indicate a strong correlation between mass and size of cold dark matter halos. The procedure results in reasonably robust estimates of the dark halo mass of a dSph and of its peak circular velocity.

Our analysis indicates that the mass within the luminous radius of the dwarf is well constrained by the velocity dispersion of the stars and that it is approximately independent of the luminosity of the system. Within the context of cosmologically motivated CDM halos, this finding implies that dSphs are surrounded by halos with peak circular velocities a factor of $\sim 3$ times larger than their stellar velocity dispersion. These results are consistent with previous work and substantially alleviate the “missing satellites” problem, particularly if most of the newly discovered ultrafaint satellites are confirmed to be bona fide dwarf galaxies akin to the more luminous, classical systems analyzed here.

We also find that stars in dSphs are deeply embedded within their dark matter halos, with core radii typically of order 5% the radius where the original halo circular velocity peaks. This result yields velocity dispersion profiles that are nearly flat out to the nominal King tidal radius, as observed for most of the Milky Way dSphs with suitable kinematical data. The deep segregation of the stellar component also implies that the stars of dwarf galaxies are fairly resilient to tidal disruption; a halo would need to lose more than 90% of its mass before stars begin to be affected. Even in this case, the peak circular velocity of a halo would only drop by $\sim 30\%$, suggesting that our $V_{\text{max}}$ estimates are robust even if halos have been heavily (but perhaps not catastrophically) affected by tidal stripping.

Applied to the dSph population of M31, this analysis suggests that the systematic difference in size between M31 dwarfs and their MW counterparts should be reflected in their kinematics. Under the plausible assumption that substructure halos
in M31 are similar to those of the Milky Way, we conclude that as a population, the velocity dispersion of M31 dwarfs should be ~50%–100% higher than that of dwarfs in our own Galaxy.

This prediction is likely to be validated (or challenged) soon by the results of the many ongoing observational projects devoted to obtaining accurate spectra and radial velocities for stars in these galaxies. Whether Local Group dwarfs conform with the expectations of the prevailing CDM paradigm or throw down a further gauntlet we should know in the near future.

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