Effects of $e^+e^-\nu_e$ Decays of
Tau Neutrinos Near A Supernova

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Abstract

We revisit the constraints implied by SN 1987A observations on the decay rate of a multi-MeV $\nu_\tau$ decaying into the visible channel $\nu_\tau \to e^+e^-\nu_e$, if its lifetime is more than 10 sec. We discuss its implication for the minimal left-right symmetric model with see-saw mechanism for neutrino masses. We also speculate on the possible formation of a “giant Capacitor” in intergalactic space due to the decay of “neutronization” $\nu_\tau$’s and spin allignment possibility in the supernova.

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I. Introduction

One of the most important questions in particle physics today is whether
the neutrinos have mass[1]. A non-zero neutrino mass would indicate rich
new physics beyond the standard model. Several attractive scenarios for
such physics already exist, predicting neutrino masses much smaller than
those of the corresponding charged fermions via the see-saw mechanism[2].
In these theories, the neutrino mass provides an estimate of the scale \( (V_{BL}) \)
above which global or local B-L symmetry will manifest. In the usual see-saw
models with local B-L symmetry[3], the neutrino masses scale quadratically
with the charged lepton (or quark) masses of the corresponding generation.
If we entertain the possibility that the scale of local B-L symmetry is in
the TeV range, (a possibility completely consistent with all known data ),
then quadratic mass formulae mentioned above predict the tau neutrino
mass to be in the MeV range (and the \( \nu_{\mu} \) and \( \nu_{e} \) in the KeV and eV ranges
respectively). Accelerator data[4] provides an upper limit of 31 MeV on \( m_{\nu_{\tau}} \).
A tau neutrino in the MeV range has the unique property that it can decay
to visible channels such as \( \nu_{\tau} \rightarrow \nu_{e} e^{+} e^{-} \). This decay channel has interesting
implications in many situations. In this paper, we study one of them, which
arises near SN 1987A, and discuss the constraints on this decay mode of \( \nu_{\tau} \)
using SN 1987A observation.

It is well-known that if the mass of the tau neutrino \( \nu_{\tau} \) lies in the six
decade range:

\[
30 \text{ eV} \leq m_{\nu_{\tau}} \leq 31 \text{ MeV} \quad ,
\]  

(1)
then it should decay in order to be consistent with the standard big bang cosmology. This decay must be fast enough \(^5\) (for \(m_{\nu_\tau} \leq \text{few} \ MeV\))
\[
\tau_{\nu_\tau} \leq 5.4 \times 10^3 \left( \frac{\text{MeV}}{m_{\nu_\tau}} \right)^2 \sec ,
\]
in order that the relic \(\nu_\tau\) decay products, even if massless and unobservable, red shift sufficiently to have their contribution to the energy density of the universe remain below the critical density \(i.e \rho(\nu_\tau \text{ decay products}) \leq \rho_{\text{crit}}\).

Demanding in addition that the \(\nu_\tau\) decay will not disturb structure (Galaxy) formation leads to the more model dependent (hence less reliable) bound \(^6\),
\[
\tau_{\nu_\tau} \leq 10^4 \sec \left( \frac{1 \text{ MeV}}{m_{\nu_\tau}} \right)^2 .
\]

The possible decay modes of \(\nu_\tau\) in the extensions of standard model are \(^1\):
\[
\nu_\tau \rightarrow \nu_i \ \nu_j \ \nu_k , \quad (i, j, k = e, \mu) ,
\]
the three neutrino mode, the radiative mode
\[
\nu_\tau \rightarrow \nu_i + \gamma ,
\]
and if \(m_{\nu_\tau} \geq 2 \ m_e \simeq 1\text{MeV}\), the three body charged mode [F.1],

[F.1] In certain models with spontaneous breaking of global B-L[7], we have massless singlet majorons \(J\) and \(\nu_\tau \rightarrow \nu_i + J\) is possible. In some versions of these models, \(\nu_\tau\) lifetime could be made very short (\(i.e. \leq 1\ sec\)). We will not consider such models here; instead we discuss models, where \(\nu_\tau\) lifetime is bigger than 10 sec, which, for example, may happen in the minimal singlet Majoron model (see later).
\[\nu_\tau \rightarrow \nu_e \ e^- \ e^+ \ . \tag{6}\]

There are no standard model tree diagrams contributing to the decay mode in eq.(4) into the neutrinos. In the left-right symmetric model, exchange of Higgs bosons lead to this decay mode at the tree level with a rate given by

\[\Gamma(\nu_\tau \rightarrow \nu_i \nu_j \nu_k) = \tilde{G}_F^2 m_{\nu_\tau}^5 / 192\pi^3 \ , \tag{7}\]

where \(\tilde{G}_F \simeq \sqrt{2} f_{\tau i} f_{jk} / 4 m_{\Delta_0}^2\). Making the reasonable assumption that \[\tilde{G}_F \leq G_F \ , \tag{8}\]

with \(G_F = 10^{-5}/m_N^2\) the ordinary Fermi constant, we find that if the rate of the three neutrinos decay mode satisfies the bounds (2) and (3), we need to have respectively\[9, 10\]

\[m_{\nu_\tau} \geq 0.035 \text{ MeV and } m_{\nu_\tau} \geq 0.1 \text{ MeV} \ . \tag{9}\]

Turning to the radiative decay, it involves the transition of magnetic moments \(\mu_{\nu_e \nu_\tau}\). Defining \(\mu_{\nu_e \nu_\tau} \equiv \kappa_{\epsilon\tau}\epsilon h / 2m_e c\), we know that they are bounded by \(\kappa_{\epsilon\tau} \leq 10^{-9}\) from direct \(\nu_i \ e^-\) scattering experiments\[11\]. Since the radiative decay rate are \(\Gamma(\nu_\tau \rightarrow \nu_i \gamma) \simeq \kappa_{\epsilon\tau}^2 \alpha m_{\nu_\tau}^3 / 16\pi^2 m_e^3\), one finds that eq.(2) is satisfied for all allowed values of \(m_{\nu_\tau}\) (down to the eV range) whereas eq.(3) requires \(m_{\nu_\tau} \geq .3 \text{ KeV}\). However, once the \(\nu_\tau\) mass is in the range of few KeV’s, the astrophysical constraints on \(\mu_{\nu_\tau \nu_e}\), which are much more stringent (\(i.e. \kappa_{\tau e} \leq 10^{-12}\)) become operative\[12\]. Thus, the effective lower bound
on $m_{\nu_\tau}$ from radiative decays is in the few KeV range. One must of course realize that from a theoretical standpoint $\kappa_{\tau 1} \simeq 10^{-9}$ may not be so easy to obtain as $m_{\nu_\tau}$ becomes lighter than few KeV’s. In any case, once $m_{\nu_\tau} \geq$ MeV, the $\nu_\tau \rightarrow e^+ e^- \nu_e$ decay mode arises and we focus on this from now on. Such a decay could result from a standard model W exchange diagram provided one adds the right-handed neutrinos to the model and allows for neutrino mixing:

$$\Gamma_{SM}^{(\nu_\tau \rightarrow \nu_e e^+ e^-)} \simeq G_F^2 m_{\nu_\tau}^5 V_{\tau e}^2 / 192 \pi^3 ,$$

(10)

where $V_{\tau e}$ is a CKM-like $\nu_e - \nu_\tau$ mixing and the above expression, valid for $m_{\nu_\tau} >> m_e$, is slightly modified in the $m_{\nu_\tau} \simeq 1 - 2$ MeV region by a phase space factor. In general there could be other non-standard model mechanisms contributing to $\Gamma(\nu_\tau \rightarrow e^- e^+ \nu_e)$ so that, barring accidental cancellation, $\Gamma_{SM}^{(\nu_\tau \rightarrow \nu_e e^+ e^-)}$ is effectively a lower bound to the rate of the process of interest.

It has been pointed out some time ago[13] that positron accumulation in the Galaxy due to $\nu_\tau \rightarrow e^- e^+ \nu_e$ in all previous galactic supernovae could lead to a strong 512 keV annihilation line. Demanding that this will not exceed its observed value excluded $\tau_{\nu_\tau} \geq 10^4 sec$ if $\nu_\tau \rightarrow \nu_e e^+ e^-$ was the dominant decay mode. More generally these results imply that

$$\left( \frac{\tau_{\nu_\tau}}{10^4 sec} \right)^2 B(\nu_\tau \rightarrow e^+ e^- \nu_e) \leq 1 \ \text{if} \ \tau_{\nu_\tau} \leq 2.7 \times 10^6 sec ; \quad (11.a)$$

$$B(\nu_\tau \rightarrow e^+ e^- \nu_e) \leq 10^{-5} \ \text{otherwise} , \quad (11.b)$$
with $B(\nu_\tau \to e^+e^-\nu_e)$ the branching ratio for the $e^+e^-$ mode. In this paper, we consider the constraints on the $\nu_\tau \to e^+ + e^- + \nu_e$ decay mode that arise from the SN 1987A observations.

The observation of the neutrino pulses from Supernova 1987A (with a known progenitor) has nicely confirmed the expectation for time scale, spectra and overall energetics of the collapse and emitted neutrinos[14]. It puts the above discussions on much firmer footing. The fact that $\gamma$ ray fluence was measured in an SMM satellite at the time of the neutrino burst and shortly thereafter[15] has put stringent upper bounds[16] on the radiative $\nu_\tau$ decays and also on the $e^+e^-$ decay.

In section II, we consider in some detail various photon generating mechanisms related to the decay $\nu_\tau \to e^+e^-\nu_e$ in the vicinity of a type II Supernova progenitor. These could be used to improve the bounds in eqs.(11) found by considering the comulative effects of past Supernovae. Our discussion in section II of photons emerging from annihilation of positrons and electrons originating from $\nu_\tau$ decay parallels the analysis of Dar and Dado[17] with which we are in basic agreement. In particular, the bounds are further improved in section II-c, in which annihilation of positrons on ambient electrons are considered. The analysis of II-b and II-c implies that even if the $\nu_\tau$ decay occurs fairly near the star, there is only a small probablity of $e^+, e^-$ annihilation.

In section III, we speculate on the possibility that a $\nu_\tau$ excess, coupled with asymmetric $e^+, e^-$ spectra in the $\nu_\tau \to e^-e^+\nu_e$ decay can lead to a large scale charge separation and strong electric fields outside the progenitor.
This new “Capacitor effect” may have interesting observational implications, unless $\Gamma(\nu_\tau \to e^+e^+\nu_e)$ is strongly bounded. Section IV contains the theoretical implications of our results. In section V, we mention another possible effect of global neutrino spin alignment around supernova. We conclude in section VI.

II. Constraints from $\gamma$-ray observation:

There are three mechanisms via which $\gamma$ rays can be emitted in associated with $\nu_\tau \to e^- e^+\nu_e$ decays near a type II supernova;

a) We have the radiative $\nu_\tau \to e^- e^+\nu_e \gamma$ decay;

b) Positrons and electrons emerging from the decays could annihilate, and finally

c) Positrons from $\nu_\tau$ decays could annihilate on the ambient interstellar material near the progenitor.

The lack of observation of such photons in the case of SN 1987A will limit the space of the relevant parameters $m_{\nu_\tau} \equiv m$, $\tau_{\nu_\tau} \equiv \tau$ and $\Gamma(\nu_\tau \to e^- e^+\nu_e) \equiv \Gamma$ (for convenience we omit the suffixes). The latter width can also be expressed in terms of the branching ratio $B$ of the $e^- e^+$ mode

$$\Gamma(\nu_\tau \to e^- e^+\nu_e) \equiv \Gamma = \frac{B}{\tau}, \quad (12)$$

We will a priori limit the range of $m$ and $\tau$ to

$$m \geq 2 \ m_e \approx 1 \ \text{MeV}, \quad (13)$$

( so that $e^+e^-$ decay occurs ) and

$$\tau \geq 10 \ \text{sec}, \quad (14)$$
(so that \(\nu_\tau\) can decay outside the star). The lifetime should not also be too long, so that \(\nu_\tau\) has time to decay before it arrives on the earth.

We point out here that cosmological arguments based on nucleosynthesis appears to rule out \(m_{\nu_\tau} \geq 0.3\) to \(0.5\) MeV if \(\tau_{\nu_\tau} \geq 10^2 \sim 10^3\) sec, provided \(\delta N_\nu \leq 0.6[18]\) if one assumes the standard big bang model of the universe. This overlaps with most of the range we are interested in. If these arguments are taken literally, our bounds will be useful only for \(10\) sec \(\leq \tau_\tau \leq 10^2 \sim 10^3\) sec. We nonetheless present our discussions as an independent piece of information based on more direct observations and no specific assumptions about the early universe.

During about 10 seconds when neutrinos are diffusing out of the core, i.e., during the time of the neutrino pulses, we expect a total number of tau neutrinos and antineutrinos \(N_{\nu_\tau} \simeq 10^{58}\) with average energies \(E_\nu \simeq 20\) MeV, to be emitted. The average \(\nu_\tau\) decay will occur at laboratory lifetime \(\tau/\gamma\) after collapse and at a distance

\[
R = \beta c \frac{\tau}{\gamma} \simeq 6 \times 10^{11} \frac{\tau}{m} \ (cm),
\]

where \(\gamma = E_\nu/m\) with \(m\) in MeV and \(\tau, \Gamma^{-1}\) in second in all the subsequent, and \(\beta = \frac{v}{c}\) with \(v\) being the neutrino velocity.

If at the time of decay, or shortly there after, a photon is emitted towards the earth, it will be delayed relative to the arrival of neutrino pulse by

\[
\delta t = (1-\beta)\frac{\gamma\tau}{2} \simeq \frac{1}{2\gamma^2} \frac{\tau}{\gamma} = \frac{\tau}{2\gamma}.
\]

Having set the general framework we now proceed to discuss the specific photon generating mechanisms (a - c) above.
II-a. Photons from $\nu_\tau \to \nu_e e^+ e^- \gamma$ decay:

The rate of the radiative process $\nu_\tau \to e^+ e^- \nu_e \gamma$ is expected to be that of $\nu_\tau \to e^- e^+ \nu_e \gamma$ times a factor $\frac{\alpha}{2\pi} \simeq 10^{-3}$. Since we have altogether $N_{\nu_\tau} B \ e^+ e^-$ decays, $10^{-3} N_{\nu_\tau} B \simeq 10^{55} \ B$ photons should arrive at earth during $\delta t \simeq \frac{\tau}{2\pi}$, where $B$ is the branching ratio for $\nu_\tau \to \nu_e \ e^+ e^-$. The local flux of photons would then be given by:

$$\Phi_\gamma = \frac{10^{55} \ B}{4\pi d^2} \frac{1}{\delta t} \simeq \frac{2}{9\pi} \ 10^9 \ \frac{\gamma}{\Gamma} \ ,$$

where we use $d = 1.5 \times 10^{23} \ cm$ for the distance from the earth to SN 1987A and $\Gamma \equiv \Gamma(\nu_\tau \to \nu_e \ e^+ e^-) = B \ \tau^{-1}$. Using $\gamma = 20/(m \ in \ MeV)$, we have

$$\Phi_\gamma \simeq \frac{10^9}{(m \ in \ MeV)} \ (\Gamma \ in \ sec^{-1}) \ (cm)^{-2} \ (sec)^{-1} \ ,$$

The SMM bound restriction[15] is $\Phi_\gamma \leq 0.1 (cm)^{-2} \ (sec)^{-1}$; hence it implies

$$\Gamma \leq \left( \frac{10^{-10}}{m \ in \ MeV} \right) \ (sec)^{-1} \ .$$

Note that the bound pertains directly to the rate of $\nu_\tau \to e^+ e^- \nu_e$, as expected, since all other decay modes do not contribute to the photons of interest.

II-b. Photons from $e^+ e^-$ annihilation following $\nu_\tau \to \nu_e e^+ e^-$ decay:

In this subsection, we discuss photons originating from mutual annihilation of decay positrons and electrons. The probability that a $\nu_\tau$ will decay at a distance between $r$ and $r + dr$ away from the star, $e^{-r/R} \frac{dr}{R}$, is roughly uniform for $r < R$. The total number of electrons (positrons) from $\nu_\tau$ decays occurring inside a sphere $r \ (r \leq R)$ is therefore (using eq.15),

$$N^{\pm}(r) = B \ N(1 - e^{-r/R}) \simeq \frac{r}{R} \ B \ 10^{58} \ = \frac{B \ r \ 10^{58}}{\beta \ c \ \tau \ \gamma} \simeq \frac{\Gamma \ 10^{58} \ r}{\gamma \ c} \ ,$$
where we have assumed \( r \ll R \) and the unit of \( \Gamma \) is \((sec)^{-1}\).

However, the original \( \nu_\tau \) and descendant electrons move in the fixed steller frame with a Lorentz factor \( \gamma \). Recalling that to a good approximation, all \( \nu_\tau \) are emitted at \( t = 0 \), the above \( N^\pm (r) \) electrons and positrons will be within a shell of thickness \( \delta r \) given (in analogy with eq.(16)) by:

\[
\delta r \simeq \frac{r}{2 \gamma^2} .
\]

The average number density in this shell is

\[
n^\pm (r) = \frac{N^\pm (r)}{4 \pi r^2 \delta r} \simeq \frac{10^{58}}{2 \pi} \frac{\gamma \Gamma}{c r^2} .
\]

The cross section for \( e^+e^- \) annihilation, \( e^+e^- \rightarrow \gamma \gamma \), at center mass energy \( \sqrt{s} \) is related to \( \sigma_{Th} = 6 \times 10^{-25} (cm)^2 \), the Thompson cross section via:

\[
\sigma_{ann}(s) = \sigma_{Th} \frac{4m_e^2}{s} = 6 \times 10^{-25} \frac{(MeV)^2}{(cm)^2} .
\]

For an electron and positron moving with energies \( E^-, E^+ \), (which for simplicity we take \( E^\pm \geq m_e \)) and at a relative angle \( \theta_{1,2} \), \( s = 2E^+E^- (1 - \cos \theta_{1,2}) \). If \( \gamma \gg 1 \) all momenta tend to align in the outward radial direction to within \( \theta_i \approx 1/\gamma \). Hence we can expand

\[
s \approx E^+E^+ \theta_{1,2}^2 \simeq \frac{E^+E^-}{\gamma^2} \simeq \frac{100 \ (MeV)^2}{\gamma^2} ,
\]

and substituting in eq.(23) we finally have

\[
\sigma_{ann} \approx 6 \times 10^{-27} \frac{\gamma^2}{\gamma} \ (cm)^2 .
\]
The probability that any given positron among the \( \mathcal{N}^\pm(r) \) in the shell will annihilate while traversing (in essentially radial direction) the shell of thickness \( \delta r \) is

\[
P_{\text{ann}}(\delta r) \simeq \delta r \ n^\pm(r) \ \sigma_{\text{ann}} \ \frac{1}{2 \sqrt{\gamma}} .
\] (26)

The last factor, \( (2 \sqrt{\gamma})^{-1} \), reflects the fact that the \( e^+e^- \) are “chasing each other” on their joint outward drift. This reduces the relative flux, in the \( \gamma \gg 1 \) limit, by \( 1/2 \sqrt{\gamma} \) which precisely compensates for the increase of density due to the squeezing from \( r \) to \( \delta r \) in eq.(22) by the same factor. Combining eqs.(22) (25) and (26) we obtain

\[
P_{\text{ann}}(r) \simeq 0.8 \times 10^{20} \ \frac{\Gamma}{\sqrt{\gamma} \ r} \simeq \frac{0.4 \times 10^{19}}{r} \ (\frac{m}{\text{MeV}}) .
\] (27)

We are only interested in annihilations occurring outside the progenitor, \( i.e \ r \geq R_{st} \simeq 3 \times 10^{12} (cm) \). Hence eq.(27) restricts \( P_{\text{ann}} \):

\[
P_{\text{ann}}(r) \leq 10^6 \ \Gamma \ m \quad (\text{all } r \geq R_{st}) .
\] (28)

In particular, \( P_{\text{ann}}(r) \leq 1 \) if

\[
\Gamma \leq \frac{10^{-6}}{m} \ (\text{sec})^{-1} .
\] (29)

If eq.(27) yields an annihilation probability \( P_{\text{ann}}(r) \leq 1 \) we can use this equation in computing the total number of photons emerging from annihilations of \( e^+e^- \) originating in sphere with radius \( r \) and thickness \( \delta r \)

\[
\mathcal{N}_\gamma(r) = P_{\text{ann}}(r) \ \mathcal{N}^\pm(r) \simeq 2.5 \times 10^{67} \ \frac{\Gamma^2}{\sqrt{\gamma}} .
\] (30)
As expected $N_\gamma$ scales with $\Gamma^2$. Its independence of $r$ can be qualitatively understood as follows: if $r \to 2r$ the number of decays and the length for annihilation doubles but at the same time the original density of electrons and positrons decreases as $1/r^2$ due to geometrical divergence.

The important point, in so far as putting stringent bounds on $\Gamma$ is concerned, is that $N_\gamma \simeq 2.5 \times 10^{67} \Gamma^2/\gamma^2$ photons are emitted within the times required for the $\nu_\tau$’s to travel up to $2R_{st}$ (which we use as our running $r$ value), and for the $e^+e^-$ generated in $\nu_\tau$-decay partially annihilate, yielding the above $N_\gamma$ photons. These photons will arrive over a time spread

$$\delta t \sim 10 \sec ,$$

( recall that there is a minimal 10 seconds width due to the duration of the original $\nu$ burst. )

Arguing, as in the derivation of eq.(19) above, that the expected fluence during 10 seconds after $t = 0$ at earth

$$\int_0^{\delta t} \Phi_\gamma dt \simeq \frac{N_\gamma}{4\pi d^2} \simeq 10^{20} \left( \frac{\Gamma}{\gamma} \right)^2,$$

should be smaller than the SMM bound of 0.6, we find

$$\Gamma \leq 0.8 \times 10^{-10} \frac{m}{(m/1\ MeV)(sec)^{-1}}.$$  

This last bound is the main result of this section. Since $\Gamma$ in eq.(33) satisfies requirement for eq.(29) (i.e $P_{ann} \leq 1$), our argument is self consistent. We can indeed show directly that condition (29), i.e $\Gamma \leq \frac{10^{-6}}{m}(sec)^{-1}$, should always be satisfied. If it is not satisfied there is a range of $r$ values

$$R_{st} \leq r \leq r_{max} \equiv \left( \frac{\Gamma}{10^{-6}/m} \right) R_{st} ,$$
for which $P_{ann} = 1$. By considering the photons emerging from this volume (where all $\mathcal{N}^\pm(r_{max})$ annihilating to photon) over a time interval $\Delta t = r_{max}/2\gamma^2$, we readily find out that the SMM bound is violated.

II-c. Photons from positron annihilation in pre-supernova stellar debris:

There is evidence[19] that the blue giant progenitor sanduleak 6902 was emitting, at the time of collapse, a wind of velocity $v_B = 500 \text{ km/sec}$ leading to mass loss at a rate $\dot{\mu}_B = 3.10^{-6} M_\odot/\text{yr}$. The resulting density near the steller surface is

$$\rho_B|_{r=R_{st}} = \frac{\dot{\mu}_B}{4\pi \frac{r_{st}^2}{v_B}} \simeq \frac{3 \times 10^{-12} \text{gr}}{(\text{cm})^3}. \quad (35)$$

Since this density falls roughly like $1/r^2$ we expect an integrated column density of $\int_{R_{st}}^{\infty} \rho \, dr \simeq \rho_B|_{r=R_{st}} R_{st} \simeq 1 \frac{\text{gr}}{(\text{cm})^2}$, for $R_{st} \simeq 3.10^{12} \text{cm}$. [Actually gravity diminishes somewhat the velocity of escaping particles, the density $\rho(r)$ falls slower than $1/r^2$ and the column density could be slightly higher (by $\simeq 2$).] This will introduce an uncertainty of almost a factor of two in our estimates below. $\nu_\tau$ decays occur between $R_{st}$ and $2 R_{st}$ (say) at a rate

$$\dot{N}_{\nu_\tau \rightarrow e^+e^-\nu_e} \simeq 10^{58} \frac{\Gamma}{\gamma} e^{-\frac{R_{st}}{\gamma} \frac{\text{cm}}{\text{sec}}} (\text{sec}^{-1}), \quad (36)$$

and this lasts for $\Delta t \simeq 10 \sim 100 \text{ sec}$. The fraction of positron annihilating in the $2 \text{ gr}/(\text{cm})^2$ (or $10^{24}/(\text{cm})^2$) column is $f = \sigma_{ann} \int n \, dr \simeq 0.1$ and the annihilated photons would arrive within $100/2 \gamma^2 \text{sec}$ of the collapse (for $\gamma \leq 3$ only otherwise $\delta t \simeq 10 \text{ sec}$). Hence the $\gamma$ ray flux on earth during this period is

$$\frac{\dot{N}_{\nu_\tau \rightarrow e^+e^-\nu_e} f}{4\pi d^2 \delta t} \leq 0.1 \text{ sec}^{-1} \text{ cm}^{-2}, \quad (37)$$
Taking $\delta t \simeq 10 \text{ sec}$, then we have for the lifetime satisfying $\gamma \tau \geq 100 \text{ sec}$

$$\Gamma \leq \left( \frac{10^{-10}}{m \text{ in MeV}} \right) \text{ sec}^{-1}.$$  \quad (38)

We note that, as we proceed to further distance, the added column density and annihilation probability do fall off rather rapidly. The space telescope discovered a “ring” at a radius of $R_{\text{ring}} \simeq 6 \times 10^{17} \text{ cm}$ of thickness of $\Delta R \simeq 10^{17} \text{ cm}$ and particle number density of $n = 2 \times 10^4/(\text{cm})^3$. Presumably this ring results from the hot blue star wind catching up with previous red giant ejecta. Note however that the extra total column density in the ring $\Delta R n \simeq 2 \times 10^{21}/(\text{cm})^2 = 3 \times 10^{-3} \text{ gr}/(\text{cm})^2$ is negligible.

Thus even if the $\nu_\tau$ decay occurs very near to the stellar surface most electrons and positrons will escape. In this connection we would like to comment on an early work of R. Cowsik, D.N. Schramm and P. Höfflich[20] who claimed a much stronger bound:

$$\Gamma \leq 2.5 \times 10^{-16} \ (\text{sec})^{-1} \ (\text{for } m \leq kT).$$  \quad (39)

Their derivation assumes that the positrons and electrons lose all their energy in the supernova “debris”. However the main source of the “debris” is the stellar envelope ejected in the SN 1987A explosion. The velocity of the envelope is low ( $\simeq 3000 \ \text{km/sec} = 10^{-2} \ c$ ). It will therefore never catch up with the $\nu_\tau$ and/or their $e^+ e^-$ decay products which are moving effectively with the speed of light. Cowsik et al[20] suggest that there could be trapping of the decay electrons and positrons in the galactic magnetic fields of order $5 \ \mu G$. We fully agree that there will be magnetic trapping on a galactic scale.
There cannot, however, even for $\Gamma$ is as small as $10^{-11}$, be a local trapping of the $e^+ e^-$ in the neighbourhood of the supernova so that the “debris” could eventually catch up with them, allowing the stringent bound to be reinstated. To see it, let us assume that:

$$\Gamma \simeq 10^{-11} .$$

(40)

The total number of $\nu_\tau$'s decaying to $e^+ e^-$ within a year would be $10^{58} \cdot 10^{-11} \cdot 3 \times 10^7 \simeq 3 \times 10^{54}$ and the total $e^+ e^-$ energy would be $\simeq 2 \times 10^{54} \cdot 20 \text{ MeV} = 6 \times 10^{49} \text{ erg}$. Because of their large energy and momentum, the flux of positrons will simply blow the magnetic field away. Indeed if the distance scale over which trapping occurs is denoted by $R^*$, then $R^*$ can be estimated by requiring that the total B field energy swept satisfies:

$$\int E \cdot dV = \frac{4\pi}{3} R^*^3 B^2 \simeq \frac{1}{6} R^*^3 10^{-12} = 6 \times 10^{49} \text{ ergs} ,$$

leading to $R^* \simeq 10^{21} \text{ cm}$ (while the nearby steller field is intense, say $B_{\text{steller}} \simeq 10^3 \text{ Gauss}$, its total energy $\frac{4\pi}{3} R_{\text{st}}^3 \frac{B^2}{8\pi} = 10^{44}$ is smaller). Thus, in our opinion the stringent bounds suggested in ref.20 is unlikely to hold.

II-d. $\nu_\tau$ decays inside the steller envelope:

In this subsection we consider the case where $\nu_\tau$ decays inside the star. If most of the decays occure inside the star, the bound in eq.(38) will be weaken:

$$\Gamma \leq \left( \frac{10^{-10}}{m \text{ in MeV}} \right) e^{\frac{100}{\tau}} (sec)^{-1} .$$

(42)

But, if $\tau = 10 \text{ sec}$ and $\frac{\tau}{\tau} = 2$, we still have an appreciable bound $\Gamma \leq 10^{-9}$.

It is amusing to note that if the decay distance $R \simeq \frac{\tau}{c} \tau$ falls somewhat near the surface $R = R_{\text{st}}$ there could be yet another independent effect and
certain ranges of $\Gamma$ could be excluded even without appealing to the SMM results. The point is that all models of the progenitor (see e.g. Barkat and Wheeler[21]) suggest that near the steller surface say between $R_{st}/2$ and $R_{st}$ there is, thanks to a much reduced intensity, a relatively small mass $[.5 \ M_\odot]$. 

The total energy deposited into the layer is 
\[ W_{dep} \simeq 10^{53} \text{ erg} \ B \ \frac{R_{st}}{2} \frac{R_{st}}{c \tau} e^{-x} \frac{R_{st}}{c \tau} . \] 
It should be compared to the gravitational binding of the layer 
\[ B.E. = -G_N \frac{M_{st} \Delta M}{R_{st}^{3/4}} \simeq 4 \times 10^{47} \text{ erg} \] 
where we use $M_{st} \simeq 10 M_\odot$ and $R_{st} = 3 \times 10^{12} \text{ cm}$. 

If $W_{dep} \geq B.E.$ i.e. if $B \ x \ e^{-x} \geq 4 \ 10^{-6}$ with $x = R_{st}/2 \ c \tau$, the whole layer would be blown off. Since this process would start right away after the $\nu$ pulse, we should have spotted dramatic changes in the steller luminosity during period between the $\nu$ pulses and the supernovae explosion. 

We therefore conclude that $Bxe^{-x} \leq 4 \times 10^{-6}$, which gives an upper limit 
\[ \Gamma \leq \frac{10^{-6}}{(m \ \text{in MeV})} . \] 
This is a much weaker constraint on $\Gamma$ than what we got before.

**III. Capacitor effect induced by $\nu_\tau \rightarrow \nu_e e^+ e^-$ :**

It is amusing to note that the $\nu_\tau \rightarrow e^- e^+ \nu_e$ decay of the many $\nu_\tau$’s produced in a type II supernova, coupled with a plausible $\nu_\tau - \overline{\nu}_\tau$ asymmetry could lead to a rather fascinating possibility. Specifically, we could have a large scale coherent phenomenon of charge separation in the neighborhood of the
star and a corresponding build-up of large scale strong radial electric fields. We will investigate this phenomenon in more detail in the future, but would like to briefly mention the essential features here. There are three main ingredients which conspire to make such an effect conceivable:

i) The likely $\nu_{\tau} - \bar{\nu}_{\tau}$ asymmetric emission;

ii) The different energy spectra of electrons and positron emerging from a decay of tau neutrinos due to the V-A nature of the interaction;

iii) The likely survival of most electrons and positrons from $\nu_{\tau}$ and $\bar{\nu}_{\tau}$ decays in the vicinity of the progenitor star.

Let us elaborate on these points:

i) The collapsing stellar core of mass $\simeq 2 \, M_{\odot}$ has a large electron (lepton) number

$$N_L \simeq N_e \simeq 10^{57}.$$ (45)

The collapse is facilitated by a copious production of electron neutrinos via the electron capture (weak) process:

$$e^- \, p \rightarrow n \, \nu_e \quad ,$$ (46)

which lowers the Fermi energy of the electrons. Some portion of these neutrinos are emitted right away during the few mili-seconds of the collapse as a “neutronization burst” preceeding the main $\sim 10 \, sec$ emission of the thermal neutrinos. However as the star quickly collapses and core densities $\rho \geq 10^{11} \, gr/(cm)^3$ are achieved, the electron neutrinos become trapped and we have thermal equiliblium of reaction in eq.(46) and its inverse $n \, \nu_e \rightarrow e^- \, p$. 

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If we have processes which conserve overall lepton number but allow the interconversion of different leptonic flavors, then the free energy of the system could be further lowered by having the trapped lepton excess reside in $\nu_\mu$, $\nu_\tau$ as well.

Thus for massive mixing neutrinos we could have along with reaction in eq.(46) also:

$$e^- p \rightarrow \nu_\tau n \quad \text{(or } e^- p \rightarrow \nu_\mu n) \quad ,$$

with cross sections reduced by the appropriate $\theta^2_{e\tau}$ (or $\theta^2_{e\mu}$) factors due to CKM-like mixing angles. While $\theta_{e\mu}$ and $\theta_{e\tau}$ are likely to be small, there are many weak interaction reactions of the type $e^- p \rightarrow n \nu$. Indeed for density $\rho \simeq 10^{15} gr/(cm)^3$ and energy $E \geq 30$ MeV involved, the mean free path for weak reactions is very short $l \simeq 30$ cm. As neutrinos diffuse out of the core of radius $r_c \simeq 10$ km they suffer many collisions

$$N_c \simeq \left( \frac{r_c}{l} \right)^2 \simeq 10^9 \, .$$

This is also the relevent number of $e^- p \rightarrow n \nu$ reactions. Thus we expect an initial electron flavor excess $N_e$ to build up gradually a $\nu_\tau$, $(\nu_\mu)$ excess of magnitude

$$\Delta N_{\nu_\tau} = N_{\nu_\tau} - N_{\nu_\mu} \simeq N_c \theta^2_{e\tau} N_e \simeq (N_c \theta^2_{e\tau}) \, 10^{57} \, ,$$

( and $\Delta N_{\nu_\mu} \simeq N_c \theta^2_{e\mu} \, 10^{57} \, )$.

Obviously these expression hold only if $N_c \theta^2 < 1$, otherwise we would have equilibration of the different flavor excess

$$\Delta N_{\nu_\tau} = \Delta N_{\nu_\mu} = \ldots = \frac{1}{3} N_e \simeq 0.3 \, 10^{57} \, ,$$

(50)
Defining in general \( \epsilon = \frac{(N_{\nu_\tau} - N_{\bar{\nu}_\tau})}{(N_{\nu_\tau} + N_{\bar{\nu}_\tau})} \), we see that we have appreciable asymmetry \( \epsilon \geq 10^{-3} \) even if \( \theta_{e\tau}^2 \simeq 10^{-12} \).

Evidently, new physics, flavor violating processes such as \( \nu_\mu + \nu_\mu \rightarrow \nu_\mu + \nu_\tau \), if present, could even enhance the \( \nu_\tau \) flavor asymmetry via \( \nu_e \rightarrow \nu_\mu \rightarrow \nu_\tau \). In any case, a build up of a \( \nu_\tau \) excess from an initial \( \nu_e \) excess is quite plausible. Let us study the consequence of this \( \nu_\tau \) excess, if the decay \( \nu_\tau \rightarrow \nu_e e^+ e^- \) occur with a significant branching ratio.

ii) The decay \( \nu_\tau \rightarrow e^- e^+ \nu_e \) is mostly likely caused by V-A interactions (such as W-exchange or \( \Delta^0 \) Higgs exchange), and leads to momenta of final \( e^- \), which are approximately twice of that of \( e^+ \). This asymmetry is preserved under boosts of the original \( \nu_\tau \) from rest to \( E_{\nu_\tau} \simeq 20 \) MeV (say). Only if we have equal number of \( \nu_\tau \) and \( \bar{\nu}_\tau \) and with equal energy spectra, e.g. in the ideal thermal case, could we argue, on general (CP) symmetry grounds, that the generated \( e^+ \) and \( e^- \) population will have the same features. Otherwise, we will in general have \( e^+ \) and \( e^- \) from the \( \nu_\tau \) decay moving with different momenta.

On average an \( e^- (e^+) \) emerging from a \( \nu_\tau \) decay will have a velocity

\[
\beta^\pm = 1 - \frac{1}{2 \gamma^\pm},
\]

with \( \gamma^\pm \equiv E^\pm / m \) having an overall average value \( \gamma = \frac{\gamma^+ + \gamma^-}{2} \simeq \frac{E^+ + E^-}{2} \simeq 7 \text{ MeV}^{1/2} \text{ MeV} = 14. \) (Recall that \( E_{\nu_\tau} = 20 \) MeV is the initial energy, shared by the three light leptons, and averaging over the asymmetry yields then \( E \simeq E_{\nu_\tau} / 3 \simeq 7 \) MeV.) The energy-momentum asymmetry implies that the
\( e^+ e^- \) originating from \( \nu_\tau \)'s have on average a velocity difference

\[
\Delta \beta = \beta_{e^-} - \beta_{e^+} \approx \frac{1}{2} \frac{1}{\gamma^2} \approx 2 \times 10^{-3} .
\]  

\( (52) \)

iii) The discussion in section II-b and II-c implies that the probability of positron annihilation is smaller than \( \simeq 10^{-1} \).

Let us proceed to show then how the ingredients i) and ii) above can be synthesized to make a scenario for large scale charge separation and corresponding strong electric fields. The basic point is simple. There is a systematic tendency of the \( \Delta N_- = \epsilon B N_{\nu_\tau} \) electrons coming from the decay of the excess \( \nu_\tau \) to move radially out with a velocity exceeding that of the corresponding positrons by \( \Delta \beta \approx 2 \times 10^{-3} \). Let us make the crude approximation that all electrons and positrons emerged from \( \nu_\tau \) decay at \( t = \beta \gamma \tau \) and \( r = R = ct \). Further, let us ignore, in the first approximation, the response of the (almost) neutral plasma of \( n_{e^+} + n_{e^-} \). After time \( \Delta t \) we have the two layers of charge \( \pm Q \), \( (Q = \Delta N^{-(+)} e) \), separated by a distance \( \Delta r = \Delta \beta c \Delta t \). Even for \( \Delta N^{-(+)} \ll N_{\nu_\tau} \) \( i.e \) for \( \epsilon B \ll 1 \) such putative separation of charges will generate large electrostatic fields and energies. This is so because the magnitude of the separating charges

\[
Q = \epsilon B 10^{58} e = \frac{\epsilon B}{2} 10^{49} \text{ esu} ,
\]  

\( (53) \)

is very large. There will therefore be strong responses (in particular of the neutral \( e^- e^+ \) plasma ) which will tends to quench these fields and the charge separation. However in the process the “asymmetry” energy ( \( i.e \) the energy of the collective relative \( e^- , e^+ \) motion ) \( W_A \approx \Delta N_- \times 7 \text{ MeV} \approx \epsilon B 10^{53} \text{ erg} \)
would be dissipates via some e.m., fluorescence effect and could allow for yet another sensitive signature of the $\nu_\tau \to e^- e^+ \nu_e$ decays. We do not derive any specific upper bound on $\Gamma$ from these considerations, due to a lack at present of a quantitative study of the charge separation.

IV. Theoretical implication of the bound on $\Gamma(\nu_\tau \to \nu_e e^+ e^-)$:

Let us now discuss the theoretical implications of the stringent bounds on the decay rate for $\nu_\tau \to \nu_e e^+ e^-$. This decay can occur if the standard model is extended by the inclusion of three right-handed neutrinos (one per family), so that the neutrinos have masses and there will be mixings between different generations. The decay rate is given in eq. (10). The upper bound on $\Gamma$ in eq.19 implies that

$$V_{\tau e} \leq 4 \times 10^{-3} \left( \frac{\text{MeV}}{m_{\nu_\tau}} \right)^3.$$  \hfill (54)

In general, there must be mechanism for an invisible decay $\nu_\tau$ with lifetime in the range $10^3 \text{ sec}$ to $10^8 \text{ sec}$ (so that $\nu_\tau$ decays outside the supernova core and before reaching the earth). This can be the case in the minimal singlet Majoron model[7].

It must also be noted that, a Majorana $\nu_\tau$ will contribute to neutrinoless double beta decay via the mixing $V_{\tau e}$. For $m_{\nu_e}$ in the few MeV range, we have the bound[22] $V_{\tau e}^2 m_{\nu_\tau} \leq 1 \text{ eV}$. This bound is consistent with eq.54 for $m_{\nu_\tau} > 2 \text{ MeV}$.

A specific model where neutrinos are massive is the left-right symmetric model with the see-saw mechanism for neutrino masses[1]. For a $W_R$-mass in the TeV range, this theory predicts a tau neutrino mass in the MeV range
as already mentioned. Therefore, the decay $\nu_\tau \to \nu_e \ e^+ \ e^-$ is kinematically allowed in this model. In addition to the already mentioned $W_L$-exchange contribution that arises from $\nu_e - \nu_\tau$ mixing, there are also Higgs contributions to the process $\nu_\tau \to \nu_e \ e^+ \ e^-$ in this model. Furthermore, if certain Yukawa couplings are chosen to be of order $\leq 10^{-1}$, $\nu_\tau$ will have a lifetime longer than $10 \text{ sec}$, so that, we can apply the discussions of the previous sections. The SN 1987A upper bound on this process therefore leads to upper bounds on these Higgs couplings, which in turn imply upper bounds on the rare process $B(\tau \to 3 \ e)$ for $m_{\nu_\tau} \geq 2 \text{ MeV}$ since the same couplings are involved in the $\tau \to 3 \ e$ decay. To discuss this, let us write down the coupling of the left-handed triplet Higgs in the basis where all leptons are mass eigenstates:

$$\mathcal{L}_Y = \nu_L^T K F K^T C^{-1} \nu_L \Delta^0 - \sqrt{2} \nu_L^T K F C^{-1} E_L \Delta^+$$

$$- E_L^T C^{-1} F E_L \Delta^{++} + h.c$$

where $\nu = (\nu_e, \nu_\mu, \nu_\tau); E = (e, \mu, \tau)$ and $F$ is the a real symmetric $3 \times 3$ matrix and $K$ is the neutrino mixing matrix. In a low scale $M_{W_R}$ theory ($M_{W_R} \simeq \text{TeV}$), the masses of the left-handed triplets $\Delta^0_L, \Delta^+_L$ and $\Delta^{++}_L$ can be in the 100 GeV range without unnatural fine tuning of the scalar self couplings. We set the $F_{e\mu} = 0$ in order to prevent $\mu \to 3e$ decay, whose branching ratio has an experimental upper limit of $10^{-12}$\cite{23}. The present experimental upper limit on $\mu \to e\gamma$ also imposes strong constraints on the product $F_{e\tau} F_{\mu\tau} \leq 2 \times 10^{-5}$. The process $\nu_\tau \to \nu_e \ e^+ \ e^-$ arises in this model
at the tree level from $\Delta^+_L$ exchange and leads to:

$$\Gamma(\nu_\tau \to \nu_e e^+ e^-) \simeq \frac{(F_{\tau e} + K_{13} F_{\tau\tau})^2 F^2_{ee} m^5_{\nu_\tau}}{1536\pi^3 M^4_{\Delta^+_L}}.$$  \hfill (56)

Using the bound in eq.19, we then get

$$(F_{\tau e} + K_{13} F_{\tau\tau})^2 F^2_{ee} \leq 4.6 \times 10^{-14} \left(\frac{\text{MeV}}{m_{\nu_\tau}}\right)^6 \left(\frac{M_{\Delta^+_L}}{\text{GeV}}\right)^4.$$  \hfill (57)

Barring unnatural cancellation between $F_{\tau e}$ and $K_{13} F_{\tau\tau}$, we conclude that,

$$F^2_{\tau e} F^2_{ee} \leq 4.6 \times 10^{-14}\left(\frac{M_{\Delta^+_L}}{\text{GeV}}\right)^2 \left(\frac{\text{MeV}}{m_{\nu_\tau}}\right)^6.$$  \hfill (58)

A question that now arises is whether this constraint suppresses the $\nu_\tau$ decay rate so much that an MeV range $\nu_\tau$ fails to satisfy the mass density constraints.

Let us therefore study the total decay rate for $\nu_\tau$. In a general left-right models, eq.(54) implies that,

$\nu_\tau \to \bar{\nu}_\mu \nu_\mu \nu_e, \ \bar{\nu}_\mu \nu_\mu \nu_\mu, \ \bar{\nu}_e \nu_\mu \nu_\mu, \ \bar{\nu}_e \nu_e \nu_e$ amplitudes are governed by the following combination of couplings:

$$\bar{\nu}_\mu \nu_e \nu_e : \ \simeq F_{ee} (F_{\tau\mu} + K_{32} F_{\mu\mu} + K_{23} F_{\tau\tau}) ; \ \ \ \ (59.a)$$

$$\bar{\nu}_\mu \nu_\mu \nu_\mu : \ \simeq F_{\mu\mu} (F_{\tau\mu} + K_{32} F_{\mu\mu} + K_{23} F_{\tau\tau}) ; \ \ \ \ (59.b)$$

$$\bar{\nu}_e \nu_\mu \nu_\mu : \ \simeq F_{\mu\mu} (F_{\tau e} + K_{31} F_{ee} + K_{13} F_{\tau\tau}) ; \ \ \ \ (59.c)$$

$$\bar{\nu}_e \nu_e \nu_e : \ \simeq F_{ee} (F_{\tau e} + K_{31} F_{ee} + K_{13} F_{\tau\tau}) . \ \ \ \ (59.d)$$
Note that if we assume the diagonal F-couplings $F_{ee}$, $F_{\mu\mu}$, $F_{\tau\tau}$ are un-suppressed, supernova constraints require only $F_{\tau e} + K_{13}F_{\tau\tau}$ to be highly suppressed; however, dominant contributions to $\nu_\tau \rightarrow 3 \nu$ decay can still arise from $F_{\tau\mu}$, $K_{31}F_{ee}$, $K_{32}F_{\mu\mu}$ terms in the above equation to make the MeV $\nu_\tau$ life time consistent with cosmological mass density constraints. For instance, we require

$$F_{aa} F'_{bc} \geq 5.7 \times 10^{-9} \left( \frac{10 \text{ MeV}}{m_\nu_\tau} \right)^{3/2} \left( \frac{M_{\Delta_L^0}}{\text{GeV}} \right)^2, \quad (60)$$

where $a = e, \mu$ and $F'_{bc}$ is either $F_{\tau\mu}$, $K_{31}F_{ee}$ and $K_{32}F_{\mu\mu}$. These lower bounds are consistent with the upper bounds on the neutrino mixing angles.

Let us now discuss some other implications. We note that the combination of couplings $F_{e\tau}F_{ee}$ also leads to the rare decay of tau-lepton, $\tau \rightarrow 3 \, e$, via $\Delta_{L}^{++}$ exchange. We can therefore derive the following inequality:

$$B(\tau \rightarrow 3 \, e) \leq 7 \times 10^{-6} \left( \frac{\text{MeV}}{m_{\nu_\tau}} \right)^6 \left( \frac{M_{\Delta_L^+}}{M_{\Delta_{L}^{++}}} \right)^4. \quad (61)$$

In the left-right model, we have

$$\left( \frac{M_{\Delta_{L}^{++}}^2}{M_{\Delta_L^+}^2} \right)^2 = \frac{1 + \alpha}{1 + 2\alpha} \leq 1, \quad (62)$$

where $\alpha$ is a real positive parameter. Using this, we get an upper bound on $B(\tau \rightarrow 3 \, e) \leq 7 \times 10^{-6}(\text{MeV}/m_{\nu_\tau})^6$ for $m_{\nu_\tau} \geq \text{MeV}$. In order to see the significance of this bound, we observe that, if the upper limit of the branching ratio $B(\tau \rightarrow 3 \, e)$ keeps going down, no conclusion can be derived from it; on the other hand, if evidence for $\tau \rightarrow 3 \, e$ is found, then, it would rule out an MeV range $\nu_\tau$ in the framework of the minimal left-right symmetric model.
V. Possible neutrino spin alignment during neutronization:

In this section, we speculate on yet another possibility where the unique parity violating character of the weak interactions conspires to make macroscopic spin allignment of neutrinos. For this purpose, let us consider the initial “neutronization” pulse of electron neutrinos and focus our attention on some region in the collapsing core corresponding to a small solid angle and bounded between two radial shells $R_1 \leq r \leq R_2$. Let us assume that most of the neutronization neutrino traversing this region have originated in a smaller core region yet, $r << R_1$. The trajectories of the neutrinos will tend therefore to predominantly align in the (outward) radial direction. Because, all these are left-handed neutrinos (rather than an equal mixture of neutrinos and anti-neutrinos) of negative helicity, the spins of these neutrinos will align too. Thus if we have at a given time $N_\nu$ neutrinos in this region, the total spin of the neutrinos will be coherently added to a large, macroscopic, total angular momentum:

$$S_\nu = N_\nu \frac{\hbar}{2}.$$  \hfill (63)

The corresponding densities $s_\nu, n_\nu$ are obtained by dividing by

$$(R_2 - R_1)R_1^2d\Omega \simeq V$$ the volume of the region in question. Assume that the neutronization pulse containing altogether $\sim 0.5 \times 10^{57}$ neutrinos lasts for a millisecond and originates from a region of corresponding size say $= c\delta t \simeq 10^8$ cm then

$$n_\nu \sim \frac{5 \times 10^{56}}{4\pi[10^8]^3} \simeq 4 \times 10^{31} / (cm)^3.$$  \hfill (64)

Let us consider possible implications of such a macroscopic spin alignment.
Two possibilities come to mind: one, if the neutrino has an intrinsic magnetic dipole $\mu_{\nu_e}$, then, naively, we would have expected a build-up of a local “neutrino magnetization” magnetic field of magnitude $B = 4\pi n\mu_{\nu}$. This however is not the case. The would be field is radial and Maxwell equation $\text{div} \vec{B} = 0$ excludes such fields implying a complete cancellation of the spherically symmetric field[F.2].

Another interesting possibility concerns the singlet Majoron model. In this model we have a massless boson namely the Majoron $\chi$. The Majoron couplings to ordinary light neutrinos are small:

$$L \simeq \frac{m_i}{V_{BL}} \chi \nu \gamma_5 \nu,$$

where $m_i$ the neutrino mass and $V_{BL}$ the scale of B-L symmetry. For $\nu_e$, $m_{\nu_e} \leq 10\text{eV}$ and $V_{BL} \geq 10^2 \sim 10^3 \text{GeV}$, we have $\frac{m_i}{V_{BL}} \leq 10^{-10} \sim 10^{-11}$, which is very small. Furthermore the latter coupling involves, in the non-relativistic limit of soft Majoron emission (from neutrinos), momentum dependent spin flip interaction (and correspondingly generates a $1/r^3$ spin-dependent $\nu\nu$ potential). Hence such interactions are almost virtually undetectable. The situation might be more favorable in the case when all neutrino spins are aligned. Even in this case the $1/r^3$ fall off makes the effect of Majoron mediated $\nu_e \nu_e$ interaction negligible.

VI. Conclusion:

[F.2] In principle the $\nu_e$ could have also an electric dipole moment and radial electric fields are allowed. However for this to be the case it needs also CP violation in the neutrino sector.
In this paper, we have studied the possible effects connected with $\nu_\tau$’s emitted in SN 1987A if their masses are in the MeV range and the electroweak theory allows the visible decay $\nu_\tau \rightarrow \nu_e e^+ e^-$. We derive constraints on this decay mode of $\nu_\tau$, if the $\nu_\tau$-lifetime is longer than 10 sec. As mentioned in the text, our results agree (where they overlap) with the earlier work of Dar and Dado[17]. We explain the physics underlying the derivation of these constraints. We study the implications of these constraints on the Higgs couplings of the left-right symmetric models with a low scale see-saw mechanism. We then speculate on two new effects, which could be generated during the early “neutronization” moment of the supernovae, if there is non-negligible $\nu_e - \nu_\tau$ mixing: one of them has to do with the formation of a “giant capacitor” in intergalactic space and another with a possible large neutrino spin alignment near the supernova core.

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