Sampling high-dimensional images is challenging due to limited availability of sensors; scanning is usually necessary in these cases. To mitigate this challenge, snapshot compressive imaging (SCI) was proposed to capture the high-dimensional (usually 3D) images using a 2D sensor (detector). Via novel optical design, the measurement captured by the sensor is an encoded image of multiple frames of the 3D desired signal. Following this, reconstruction algorithms are employed to retrieve the high-dimensional data. Though various algorithms have been proposed, the total variation (TV) based method is still the most efficient one due to a good trade-off between computational time and performance. This paper aims to answer the question of which TV penalty (anisotropic TV, isotropic TV and vectorized TV) works best for video SCI reconstruction? Various TV denoising and projection algorithms are developed and tested for video SCI reconstruction on both simulation and real datasets.

Index Terms—Computational imaging, snapshot compressive imaging, coded aperture compressive temporal imaging, compressive sensing, total variation, FISTA, TwIST, FGP, ADMM, GAP.

1. INTRODUCTION

Snapshot compressive imaging (SCI) [1] refers to compressive imaging systems where multiple frames are mapped into a single measurement, with video SCI [2–10] and spectral SCI [11–15] as two representative applications. In video SCI shown in Fig. 1, high-speed frames are modulated at a higher frequency than the capture rate of the camera; in this manner, each captured measurement frame can recover a number of frames (usually 3D) images using a 2D sensor (detector). Via novel optical design, the wavelength dependent coding is implemented by a coded aperture (physical mask) and a disperser [12, 13]; more than 30 hyperspectral images have been reconstructed from a snapshot measurement. Though it is fair to say that SCI was inspired by compressive sensing (CS) [16, 17], the theory of SCI has just been developed in [18] due to the special structure of the sensing matrix.

Mathematically, the measurement in the SCI systems can be modeled by

\[ y = \Phi x + g, \]  

where \( \Phi \in \mathbb{R}^{n \times nB} \) is the sensing matrix, \( x \in \mathbb{R}^{nB} \) is the desired signal, and \( g \in \mathbb{R}^n \) denotes the noise. Unlike traditional CS, the sensing matrix considered here is not a dense matrix. In SCI, e.g., video CS as in CACTI [4, 5], the matrix \( \Phi \) has a very specific structure and can be written as

\[ \Phi = [D_1, \ldots, D_B], \]  

where \( \{D_k\}_{k=1}^B \) are diagonal matrices.

As in Fig. 1, consider that \( B \) high-speed frames \( \{X_k\}_{k=1}^B \in \mathbb{R}^{n_x \times n_y} \) (at timestamp \( t_1, \ldots, t_B \)) are modulated by the masks \( \{C_k\}_{k=1}^B \in \mathbb{R}^{n_x \times n_y} \), correspondingly. The 2D measurement \( Y \in \mathbb{R}^{n_x \times n_y} \) captured by the camera is given by

\[ Y = \sum_{k=1}^B X_k \odot C_k + G, \]

where \( \odot \) denotes the Hadamard (element-wise) product. For all \( B \) pixels (in the \( B \) frames) at position \((i, j), i = 1, \ldots, n_x; j = 1, \ldots, n_y\), they are collapsed to form one pixel in the measurement (in one shot) as

\[ y_{i,j} = \sum_{k=1}^B c_{i,j,k} x_{i,j,k} + g_{i,j}. \]

By defining

\[ x = [x_1^\top, \ldots, x_B^\top]^\top, \]

where \( x_k = \text{vec}(X_k) \), and \( D_k = \text{diag}((\text{vec}(C_k))) \), for \( k = 1, \ldots, B \), we have the vector formulation of Eq. (1), where \( n = n_x n_y \). Therefore, \( x \in \mathbb{R}^{n_x n_y B}, \Phi \in \mathbb{R}^{n_B \times (n_x n_y B)}, \)
and the compressive sampling rate in SCI is equal to \(1/B\),
which is defined by the hardware design. It has recently been
proved that even when \(B > 1\), reconstruction can be achieved
with overwhelming probability \([18, 19]\).

The following task for the algorithm is to reconstruct
the desired signal \(x\) given the measurement \(y\) and the
special sensing matrix \(\Phi\) determined by the physical masks
\(\{C_k\}_{k=1}^B\).

2. SOLVE SCI BY TOTAL VARIATION
REGULARIZATION

Obviously, Eq. (1) is an ill-posed problem and a regularizer
is usually utilized to confine the solution. In this paper, we
focus on the total variation (TV) regularization and thus solve
the following problem,

\[
\hat{x} = \arg\min_x \frac{1}{2} \|y - \Phi x\|_2^2 + \text{TV}(x),
\]

where \(\text{TV}(\cdot)\) denotes the TV regularizer. Since \(x\) inherently
is a 3D data-cube in SCI, various TV can be used. For ex-
ample, the Anisotropic TV (ATV) and the Isotropic TV (ITV)
and moreover the TV can be imposed on each 2D frame of
the video or on the entire 3D cube.

For the ease of notation, in the following, we first define
the operators:

\[
D_h x_k = X_k D_h^\top, \quad D_v x_k = D_v X_k,
\]

where \(\{D_h \in \mathbb{R}^{(n_y-1) \times n_y}, D_v \in \mathbb{R}^{(n_x-1) \times n_x}\}\)
are the gradient operator to perform differentiation on the desired frame
horizontally and vertically, respectively.

2.1. Different TV Formulations

Different TVs can thus be summarized as follows:

- ATV:

\[
\text{ATV}(x) = \sum_{k=1}^{B} \|D_h x_k\|_1 + \|D_v x_k\|_1.
\]  

Note that the formulation of ATV2D is the same as
ATV3D.

- ITV2D:

\[
\text{ITV2D}(x) = \sum_{k=1}^{B} \|D_h x_k\|_2^2 + \|D_v x_k\|_2^2.
\]  

- ITV3D:

\[
\text{ITV3D}(x) = \sqrt{\sum_{k=1}^{B} \|D_h x_k\|_2^2 + \|D_v x_k\|_2^2}.
\]

We thus have the following problems to solve the SCI re-
construction using various TV:

1) ATV:

\[
\hat{x} = \arg\min_x \frac{1}{2} \|y - \Phi x\|_2^2 + \lambda \text{ATV2D}(x_k).
\]  

2) ITV2D:

\[
\hat{x} = \arg\min_x \frac{1}{2} \|y - \Phi x\|_2^2 + \lambda \text{ITV2D}(x_k).
\]  

3) ITV3D:

\[
\hat{x} = \arg\min_x \frac{1}{2} \|y - \Phi x\|_2^2 + \lambda \text{ITV3D}(x).
\]  

2.2. Different Solvers

The previous section have presented different TV norms and
here we present different popular solvers (we are not seek-
ing for a thorough survey here) in the literature. The SCI re-
construction problem in Eq. (11)-Eq.(13) can be solved using
different frameworks.

- FISTA [20]: It consists the following steps

\[
z(t) = \theta(t) + \frac{1}{L(f)} \Phi^\top (y - \Phi \theta(t)),
\]

\[
x(t) = \text{TVdenoise}(z(t)),
\]

\[
\tau(t+1) = \frac{1 + \sqrt{1 + 4(\tau(t))^2}}{2},
\]

\[
\theta(t+1) = x(t) + \frac{\tau(t) - 1}{\tau(t+1)} (x(t+1) - x(t)),
\]

where \(\tau(1) = 1\) is introduced in FISTA and various TV
 norms in Sec. 2.1 (with solutions in Sec. 2.3) can be used.

- TwIST [21]: It consists the following steps

\[
z(t) = x(t) + \Phi^\top (y - \Phi x(t)),
\]

\[
\theta(t) = \text{TVdenoise}(z(t)),
\]

\[
x(t+1) = (1 - \alpha)x(t-1) + (\alpha - \beta) x(t) + \beta \theta(t),
\]

where \(\{\alpha, \beta\}\) are TwIST parameters and can be deter-
mined by the eigenvalues of \(\Phi^\top \Phi\).

- GAP [22]: It consists the following steps:

\[
x(t+1) = \theta(t) + \Phi^\top (\Phi \Phi^\top)^{-1} (y - \Phi \theta(t)),
\]

\[
\theta(t+1) = \text{TVdenoise}(x(t+1)).
\]  

- ADMM [23]: We derive the ADMM framework by formu-
lating the problem as

\[
\hat{x} = \arg\min_x \frac{1}{2} \|y - \Phi x\|_2^2 + \lambda \text{TV}(\theta), \text{ s.t. } \theta = x.
\]

This can be solved by the following sub-problems:

\[
x(t+1) = \arg\min_x \frac{1}{2} \|y - \Phi x\|_2^2 + \frac{\rho}{2} \|x - \theta(t) + u(t)\|_2^2,
\]

\[
\theta(t+1) = \text{TVdenoise}(u(t) + x(t+1)),
\]

\[
u(t+1) = u(t) + \lambda \text{ITV3D}(x).
\]

As derived in [24], since in SCI, \(\Phi \Phi^\top\) is a diagonal ma-
trix, Eq. (24) can be solved element-wise and thus very
efficiently and when \(\rho = 0\), it will degrade to GAP.

Note that in each framework, there is a “TVdenoise” step
and various TV priors in previous subsection can be used. In
the following, we present various solutions of different “TV-
denoise”.
2.3. Solutions of TV Denoising

We now present different solvers for various TV denoising.

- **ATV:**
  - Clip: The iterative clipping algorithm [25] was employed in GAP-TV [24]. It is derived from the min-max property and the majorization-minimization procedure and inspired by [26–28]. The full algorithm is listed in Algorithm 1. One key step is to introduce variables \( \{ w_h, w_v \} \), with \( |w_h| \leq 1, |w_v| \leq 1 \).
  - Chambolle: in [27, 29] (denoted as **ATV-Cham**).
  - FGP: (fast gradient projection) proposed in [26] (denoted as **ATV-FGP**) with solutions summarized in Algorithm 1. Note we have used \( \max(1, |w_h| + \delta t z_h^{(s+1)}) \) in the denominator of the update pf \( w_h \) and similar for \( w_v \), which is recommended in [27]. This can also be changed to \( 1 + \delta t |z_h^{(s+1)}| \) as originally derived in [29]. This also holds true for the following derivations on ITV2D and ITV3D.

- **ITV2D:**
  - **ITV2D-Cham:** Following ATV-Cham, Let
    \[
    \begin{align*}
    \tilde{w}_h^{(s+1)} &= w_h^{(s)} + \delta t z_h^{(s+1)}, \\
    \tilde{w}_v^{(s+1)} &= w_v^{(s)} + \delta t z_v^{(s+1)}.
    \end{align*}
    \]
    (27)
    Recall that \( \tilde{w}_h^{(s+1)} \) can be a 3D video and we reshape it to \( \tilde{w}_h \in \mathbb{R}^{n_h \times n_v \times B} \) by ignoring the boundary effects (and also dropping the index \( s + 1 \)), and similar to \( \tilde{w}_v \). We further let \( [\tilde{w}_h]_{i,j,k} \) denotes the \( (i,j) \)-th pixel or voxel in \( k \)-th frame and similar for \( [\tilde{w}_v]_{i,j,k} \). We now have the update equations for \( [\tilde{w}_h]_{i,j,k} \) and \( [\tilde{w}_v]_{i,j,k} \), which correspond to \( w_h \) and \( w_v \), respectively.

- **ITV2D-FGP:** Similar to ITV2D-Cham, we only need to change the update equations of \( p_h \) and \( p_v \) in ATV-FGP.

- **ITV3D:** The ITV3D denosing step can be solved by the algorithm proposed in [30] (denoted as **ITV3D-ATV**) or the FGP (fast gradient projection) proposed in [26] (denoted as **ITV3D-FGP**). Regarding the solution, the difference lies in Eqs. (29)-(30) and we now have

\[
\begin{align*}
[\tilde{w}_h]_{i,j,k} &= \max(1, \sqrt{z_h^{(s+1)}}, [\tilde{w}_h]_{i,j,k}^2 + [\tilde{w}_v]_{i,j,k}^2), \\
[\tilde{w}_v]_{i,j,k} &= \max(1, \sqrt{z_v^{(s+1)}}, [\tilde{w}_h]_{i,j,k}^2 + [\tilde{w}_v]_{i,j,k}^2).
\end{align*}
\]

(31)
(32)

Similar changes will happen for \( p_h \) and \( p_v \) for ITV3D-FGP.

Algorithm 1 gives the full algorithm of GAP with ATV using different denoising algorithms. It is easy to replace GAP with ADMM/TwIST/FISTA and replace ATV with ITV. We thus achieve the various compositions of frameworks for SCI reconstruction with different TV denoising algorithms summarized in Table 1.

3. EXPERIMENTAL RESULTS

Now, we apply various TV algorithms and projection frameworks to video SCI on both simulation and real data.

**Simulation:** We used four datasets, i.e., Kobe, Traffic,
Table 1. Different frameworks and various TV denoising algorithms to solve SCI. PSNR results of 4 datasets used in [1], in each cell, top-left: Kobe, top-right: Traffic, middle-left: Runner, middle-right: Drop, bottom: average. The bold number denotes the highest PSNR (based on the 0.001 precision) for each projection algorithm per video dataset. The red number denotes the highest PSNR for each dataset across all the algorithms. Italian denotes the highest average PSNR for each row and the blue Italian one is the highest average PSNR across all algorithms.

![Image](315x235 to 578x338)

**Fig. 2.** PSNR vs. Iteration Number for different algorithms.

**Fig. 3.** Real data results. A hand is moving in front of a SCI camera and 10 frames are reconstructed from a snapshot measurement (top-left). 3 masks out of 10 are shown in the first row.

**Fig. 4.** Results by plotting Frame 6 every 10 iterations.

### 4. CONCLUSIONS AND FUTURE WORK

We have investigated diverse total variation algorithms under different projection frameworks for video snapshot compressive imaging. GAP and ADMM are recommended for decent results while FISTA is a choice with limited running time. Regrading the total variation solver, FPG is recommended for different cases because it is faster. We are working on using these algorithms to initialize complicated algorithms [31–33] like DeSCI to get better results. Convergence results of these algorithms will also be derived. We also found that a recent research line of Plug-and-Play framework [34] is interesting to be investigated for SCI reconstruction.
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