The role of interfacial friction on the peeling of thin viscoelastic tapes

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Abstract

We study the peeling process of a thin viscoelastic tape from a rigid substrate. Two different boundary conditions are considered at the interface between the tape and the substrate: stuck adhesion, and relative sliding in the presence of frictional shear stress. In the case of perfectly sticking interfaces, we found that the viscoelastic peeling behavior resembles the classical Kendall behavior of elastic tapes, with the elastic modulus given by the tape high-frequency viscoelastic modulus. Including the effect of frictional sliding, which occurs at the interface adjacent to the peeling front, makes the peeling behavior strongly dependent on the peeling velocity. Also, at sufficiently small peeling angles, we predict a tougher peeling behavior than the classical stuck cases. This phenomenon is in agreement with recent experimental evidences indicating that several biological systems (e.g. geckos, spiders) exploit low-angle peeling to control attachment force and locomotion.

Keywords: peeling, viscoelasticity, interfacial friction, adhesion

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I. INTRODUCTION

The ability to control the mechanical behavior of real interfaces is one of the most challenging topics in modern industrial engineering, as witnessed by the effort made in the last decades in elastic [1-8], viscoelastic [9-16] and adhesive [17-24] contact mechanics studies. In this respect, the functionality of several real-life devices such as, for instance, pressure sensitive adhesives, modern touch screens, biomedical wound dressing and Band-Aids, is tightly connected to the possibility of tailoring the interfacial adhesion between these systems and the mating surfaces. Therefore, the attachment/detachment performances of such interfaces are of primary concern. Among the possible detachment mechanisms, peeling is usually regarded as one of the most important [25] as it offers the chance to independently variate both the peeling load \( P \) and the peeling angle \( \theta \). Due to its reliability, also the adhesive performance of industrial interfaces is often controlled by means of opportunely designed peeling tests [26]. For these reasons, a deep understanding of the physics behind peeling processes is of primary importance in modern engineering.

The basic understanding of the peeling behavior of thin elastic tapes is nowadays well established. Indeed, Kendall’s [27] studies set the theoretical framework to investigate the peeling evolution by relying on energy balance of the system. His pioneering results were valid under specific hypothesis, such as the linear elasticity of the tape, perfect interfacial backing between the tape and the substrate. However, in the last decades, most of the original assumptions have been relaxed in successive studies. This is the case, for instance, of the effect of bending stiffness in thick tapes, which has been investigated in Refs [28, 29], in the presence of viscous losses occurring in the adhesive layer and in the tape. In Refs [30, 31], the effect of the substrate viscoelasticity on the peeling behavior has been studied to show that the steady detachment speed can be tuned under specific conditions and ultra-tough peeling may occur at low peeling angles. The effect of the tape viscoelasticity has been investigated in Ref. [32-35], where it has been empirically shown that, in a limited range of peeling velocities [25], the peeling force \( P \approx kV^n \).

Also the tape-substrate interfacial interactions play a fundamental role in determining the evolution of the peeling process. Kendall showed that interfacial sliding between the tape and the substrate may trigger cyclic detachment and re-attachment of the tape [36, 37]. Nonetheless, such a relative sliding represents an additional source of energy dissipation due to frictional shear stress occurring at the interface [38, 39]. This has been investigated in details in the case of elastic tapes peeled away from rigid substrates under frictionless [40] and frictional [41] sliding. Theoretical and experimental investigations have concluded that, as expected, the presence of frictional sliding at the interface leads to significantly higher peeling force, which may theoretically diverge at vanishing peeling angle. It has been suggested that the enhancement of peeling force caused by the energy dissipated in frictional sliding plays a key role in biological applications. Indeed, several studies have identified this mechanism as one of the possible sources of the superior adhesive performance and the ability to switch between firm attachment and effortless detachment of geckos [42-44], insects [45-46] and frogs [47], together with the hierarchical configuration of the pad/toes fibrils [48-50], and the V-shaped peeling scheme often occurring at the interface [51-53]. Specifically, in the presence of interfacial sliding, the enhanced peeling force arises due to the combination of three main physical mechanisms [55]: (i) tape pre-strain due to partial sliding occurring during detachment [54, 55]; (ii) the friction losses associated with the slip at the interface [56]; (iii) the viscous dissipation [46]. Interestingly, in wet contact case, the
last contribution is usually ascribed to the viscous shear in the interfacial thin fluid film formed by the pad secretion \[59, 60\]; however, a comprehensive model on the effect of the pad tissue bulk viscoelasticity \[61, 62\] on the overall frictional peeling is lacking.

In this study, we present a theoretical model of the behavior of a thin viscoelastic tape peeled away from a rigid substrate. Specifically, we aim at investigating the combined effect of frictional interfacial sliding occurring during the detachment process and the energy dissipation associated with the viscoelastic behavior of the tape. In order to shed light on the interplay between these mechanisms of energy dissipation, we consider two different configurations: (i) firstly, we focus on stuck conditions, where a rigid constraint avoids any possible interfacial displacement between the tape and the substrate, so that no additional energy contribution is present at interface beside the change in the energy of adhesion; then (ii) we consider the sliding case, where energy dissipation occurs due to frictional relative sliding in the tape elongated region. Our findings may be of help to estimate the effect of the bulk viscoelasticity on the overall peeling behavior of bioinspired and natural systems (e.g. biological fibrils, industrial polymeric tapes, etc.) in the presence of interfacial frictional sliding.

II. FORMULATION

In this section, we present the mathematical model for the viscoelastic peeling assuming two different conditions at the interface between the tape and the rigid substrate: (i) stuck adhesion, where tape normal and tangential displacements are inhibited; (ii) relative frictional sliding, i.e. tape normal displacements are inhibited but tangential sliding occurs, which is opposed by frictional shear stresses. In both cases, the problem formulation builds on energy balance. We focus on a linear viscoelastic material with a single relaxation time \( t_c \). Further, we neglect any dynamic effect during the tape detaching process (i.e. we assume that the peeling front velocity is steady and far lower than the speed of sound in the tape).

Stuck interface

Consider a viscoelastic tape of thickness \( d \) and transverse width \( b \), baked onto a rigid substrate with no relative sliding at the interface. As shown in Fig. 1, the tape is peeled away at an angle \( \theta \) under a constant force \( P \). We assume the peeling front moves on the left at a constant velocity \( V_0 \) relative to the substrate. We observe the peeling process in the peeling front, so that the substrate moves on the right at speed \( V_0 \) as shown in Fig. 1.

Under steady state conditions, the energy balance per unit time of the viscoelastic tape is given by

\[
W_E + W_I + W_S = 0
\]  

where \( W_E \) is the work per unit time of the external forces acting on the tape, \( W_I \) is work per unit time done by tape internal stresses, which takes into account for both the change in the stored elastic and viscous energy dissipation in the tape, and \( W_S \) is the work per unit time done by interfacial forces. Notably, in this formulation we neglect any other source of energy dissipation, such as acoustic or thermal emissions.

The term \( W_E \) in Eq. (1) can be calculated considering the external forces acting on the system, which are the remote load \( P \) acting on the detached tape tip, and the corresponding
FIG. 1: The scheme of the peeling process of a thin viscoelastic layer from a rigid substrate in the presence of stuck adhesion at the interface, so that no relative sliding occurs. In the lower part, qualitative diagrams of the tape stress $\sigma$ (red) and deformation $\varepsilon$ (blue) are shown.

opposite substrate reaction force $-P \cos \theta$. We have

$$W_E = PV_\infty - PV_0 \cos \theta = \sigma_0 V_0 bd \left(1 + \frac{\sigma_0}{E_0} - \cos \theta \right)$$  \hspace{1cm} (2)

where we defined $\sigma_0 = P/bd$, and $E_0$ is the low frequency viscoelastic modulus. Moreover, the mass balance of the tape gives $V_\infty = V_0(1 + \sigma_0/E_0)$. Notably, in Eq. (2) we assumed that the tape tip (where the force $P$ is applied) is located sufficiently far from the peeling front, so that complete viscoelastic relaxation occurs in the detached tape.

The surface term $W_S$ in Eq. (1) represents the energy per unit time associated with the rupture of interfacial adhesive bond, with $\Delta \gamma$ being the work of adhesion needed to detach a unit surface of the tape from the substrate. Although $\Delta \gamma$ may also depend on the viscoelastic energy dissipation occurring very close to the crack tip (i.e. small-scale viscoelasticity), for the sake of simplicity, here we neglect such effects, thus assuming a constant $\Delta \gamma$ value. Nonetheless, this effect could be straightforwardly introduced in the present model by opportuneely modifying the value of $\Delta \gamma$ as described in Refs. [63, 64]. Therefore, we can write

$$W_S = -\Delta \gamma b V_0$$  \hspace{1cm} (3)

As mentioned above, the term $W_I$ in Eq. (1) takes into account for both the elastic energy stored in the tape, and the bulk energy dissipation occurring due to viscoelastic creep in the detached strip. Moreover, observing that the bending stiffness of the tape depends on the third power of thickness $d$, and considering that we focus on very thin tapes, the bending contribution to $W_I$ can be neglected (see also Refs. [52, 53]). Hence we write

$$W_I = -V_0 bd \int_{-\infty}^{+\infty} \sigma(x) \varepsilon'(x) \, dx$$  \hspace{1cm} (4)

where $\varepsilon'(x)$ is the spatial derivative of the strain $\varepsilon(x)$. Note that, in Eq. (4), we used $\dot{\varepsilon}(x) = V_0 \varepsilon'(x)$, with $\dot{\varepsilon}(x)$ being the time derivative of $\varepsilon(x)$. 

4
Since in this section we assume no interfacial sliding between the adhering tape and the rigid substrate, the stress distribution in the viscoelastic tape is given by

\[ \sigma(x) = \sigma_0 H(x) \]

with \( H(x) \) being the Heaviside step function (see the diagram in Fig. 1). Moving from the linear viscoelastic constitutive equation \([63]\), in the framework of steady state conditions, the deformation field can be calculated as

\[ \varepsilon(x) = \int_{-\infty}^{x} J(x-s)\sigma'(s) \, ds \]  \hspace{1cm} (5)

where \( J(x) \) is the spatial transformation of the viscoelastic creep function given by

\[ J(x) = H(x) \left[ \frac{1}{E_0} - \frac{1}{E_1} \exp \left( -\frac{x}{\lambda} \right) \right] \]  \hspace{1cm} (6)

where \( \lambda = V_0 t_c \) is the relaxation length, and \( E_1^{-1} = E_0^{-1} - E_\infty^{-1} \) with \( E_0 \) and \( E_\infty \) being the low and very high frequency viscoelastic moduli, respectively.

For the case at hand, Eq. (5) gives \( \varepsilon(x) = \sigma_0 J(x) \), which substituting into Eq. (1), after some algebra, gives

\[ W_I = -V_0 b d \sigma_0^2 \left( \frac{1}{E_0} - \frac{1}{2 E_\infty} \right) = -V_0 b d \sigma_0^2 \left[ \frac{1}{E_0} + \frac{1}{E_1} \right] \]  \hspace{1cm} (7)

where we used \( \int_{-\infty}^{\infty} \delta(x)H(x) = 1/2 \), which gives \( \varepsilon(0) = \frac{1}{2} [\varepsilon(0^-) + \varepsilon(0^+)] = \frac{1}{2} \varepsilon(0^+) \). Note that the viscoelastic dissipated energy per unit time is

\[ D_s = \frac{V_0 b d \sigma_0^2}{2 E_1} \]  \hspace{1cm} (8)

Finally, substituting Eqs. (2,3,7) into Eq. (1) we have

\[ \frac{\sigma_0^2}{2 E_\infty} + \sigma_0(1 - \cos \theta) = \frac{\Delta \gamma}{d} \]  \hspace{1cm} (9)

which represents the peeling equilibrium condition. Interestingly regardless of the peeling velocity \( V_0 \), Eq. (9) is identical to the Kendall equation with the elastic modulus given by the high frequency viscoelastic modulus \( E_\infty \). Notice that Eq. (9) can be rephrased as

\[ \frac{1}{2} \left( \frac{\sigma_0}{E_0} \right)^2 + \kappa \frac{\sigma_0}{E_0}(1 - \cos \theta) = \kappa \frac{\Delta \gamma}{E_0 d} \]  \hspace{1cm} (10)

where we have defined \( \kappa = E_\infty / E_0 \). For \( \theta = 0 \), we get

\[ \sigma_K = \sqrt{ \frac{2 \kappa E_0 \Delta \gamma}{d} } = \sqrt{ \frac{2 E_\infty \Delta \gamma}{d} } \]  \hspace{1cm} (11)

so that, in this case, the peeling is much more tough than in the (low frequency) elastic case as the effective work of adhesion is \( \kappa \)-times larger than \( \Delta \gamma \).

In order to explain the appearance of the high-frequency viscoelastic modulus \( E_\infty \) in Eq. (9) we note that, because of the stuck condition assumption (no relative sliding at the tape-substrate interface), the tape is subjected to an abrupt stretching in the peeling section (see
Fig. 1. For this reason, regardless of the peeling velocity $V_0$, the material response close to the peeling front is governed by the high-frequency viscoelastic response, which makes the tape locally behave as a perfectly elastic material with elastic modulus $E_\infty$.

Notably, in real conditions, the abrupt change of the tape stress during peeling would be smoothed, as it must occur on a finite length scale across the peeling section. Since the size of this "process zone" can be estimated of order unity of the tape thickness $d$ (see also Refs. 52, 66), the tape excitation frequency during peeling is $\omega \approx V_0/d$, so that at very low peeling velocities, i.e. when $V_0 \ll d/t_c$, the tape response would be governed by the low-frequency viscoelastic modulus $E_0$. However, since we usually expect that $V_0 \gg d/t_c$, this would not qualitatively affect the physical picture of the peeling behavior provided so far.

**Frictional sliding interface**

The discussion provided in the previous section is based on the assumption that the tape firmly sticks to the rigid substrate, and the tangential component of the peeling force $P$, remotely acting on the tape tip, is locally balanced by a point reaction force acting in the peeling section. However, it has been shown that a certain amount of relative sliding occurs in real interfaces 38, 58, 67, so that the tangential component of the peeling force $P$ is balanced by the frictional shear stresses arising at the interface between the tape and the rigid substrate. In this case, assuming a uniform interfacial shear stress $\tau$, a portion of the adhering tape of length $a = P \cos \theta / \tau b$ is gradually stretched and slides against the substrate. Such a physical scenario is shown in Fig. 2.

![Diagram of peeling process with frictional sliding](image)

**FIG. 2:** The scheme of the peeling process of a thin viscoelastic layer from a rigid substrate in the presence of relative sliding at the interface. Notably, $\tau$ is the frictional shear stress. In the lower part, qualitative diagrams of the tape stress $\sigma$ (red) and deformation $\varepsilon$ (blue) are shown.

The work per unit time done by interfacial frictional stresses is

$$W_T = - \int_{-a}^{0} V_s(x) \tau b dx = -\tau b V_0 \int_{-a}^{0} \varepsilon(x) dx$$

(12)
where $V_S(x) = V_0 \varepsilon(x)$ is the sliding velocity distribution at the interface. Of course, both the stress and deformation distributions along the tape are modified due to the presence of the tangential tractions $\tau$. Indeed, we have

$$
\sigma(x) = \frac{\tau}{d}(x + a); \quad -a \leq x < 0 \\
\sigma(x) = \sigma_0; \quad x > 0
$$

(13)

where $\sigma_0 \cos \theta = \tau a / d$. Similarly, from Eq. (13), recalling, Eq. (5), one obtains

$$
\varepsilon(x) = \frac{\tau}{E_0 d}(a + x) - \frac{\tau \lambda}{E_1 d} \left[ 1 - \exp \left( -\frac{x + a}{\lambda} \right) \right]; \quad -a \leq x < 0 \\
\varepsilon(x) = \frac{\sigma_0}{E_0} - \frac{\sigma_0}{E_1} \left[ 1 - \frac{\tau a}{\sigma_0 d} + \frac{\tau \lambda}{\sigma_0 d} \left[ 1 - \exp \left( -\frac{a}{\lambda} \right) \right] \right] \exp \left( -x/\lambda \right); \quad x > 0
$$

(14)

Recalling Eq. (12) and using Eqs. (13, 14) we have

$$
W_T = -bdV_0 \frac{\sigma_0^2}{2E_0} \cos^2 \theta \left\{ 1 - 2 \frac{\kappa - 1}{\kappa} \frac{\lambda}{a} \left[ 1 - \frac{\lambda}{a} + \frac{\lambda}{a} \exp \left( -\frac{a}{\lambda} \right) \right] \right\}
$$

(15)

In Eq. (15), we note that for $a/\lambda \to \infty$ we get $W_T \to -\frac{1}{2}bdV_0(\sigma_0^2/E_0) \cos^2 \theta$, which involves the low frequency modulus $E_0$; whereas, for $a/\lambda \to 0$ we get $W_T \to -\frac{1}{2}bdV_0(\sigma_0^2/E_\infty) \cos^2 \theta$, which involves the high frequency modulus $E_\infty$. Moreover, $W_T(a/\lambda \to \infty) = \kappa W_T(a/\lambda \to 0)$.

This time, the work per unit time done by tape internal stresses is

$$
W_I = -bdV_0 \int_{-\infty}^{+\infty} \sigma(x) \varepsilon'(x) \, dx = \\
- \frac{1}{2} \int_{-\infty}^{+\infty} \sigma_0^2 \frac{\cos^2 \theta}{E_0} \left\{ 1 - 2 \frac{\kappa - 1}{\kappa} \frac{\lambda}{a} \left[ 1 - \frac{\lambda}{a} + \frac{\lambda}{a} \exp \left( -\frac{a}{\lambda} \right) \right] \right\}
$$

(16)

Finally, recalling that, in this case, Eq. (1) modifies in

$$
W_E + W_I + W_S + W_T = 0
$$

(17)

and using Eqs. (13,15,16) into Eq. (17), the final peeling equilibrium equation for a viscoelastic tape in the presence of frictional sliding at the interface is given by

$$
\frac{\sigma_0^2}{2E_0} \left\{ (1 - \cos^2 \theta) - \frac{\kappa - 1}{\kappa} (1 - \cos \theta) \left[ 1 + 2 \cos \theta \left( \frac{\lambda}{a} \left( 1 - \exp \left( -\frac{a}{\lambda} \right) \right) - \frac{1}{2} \right) \right] \right\} + \sigma_0(1 - \cos \theta) - \frac{\Delta \gamma}{d} = 0
$$

(18)
III. NUMERICAL RESULTS

Let us introduce the following dimensionless parameters: \( \tilde{\sigma}_0 = \sigma_0/E_0 \), \( \tilde{\tau} = \tau/E_0 \), \( \Delta \tilde{\gamma} = \Delta \gamma/(E_0d) \) and \( \tilde{V}_0 = V_0t_c/d \). Note that \( a/\lambda = \tilde{\sigma}_0 \cos \theta / \left( \tilde{V}_0 \tilde{\tau} \right) \). Therefore, Eq. (18) becomes

\[
\frac{1}{2} \tilde{\sigma}_0^2 \left\{ (1 - \cos^2 \theta) - \frac{\kappa - 1}{\kappa} (1 - \cos \theta) \left[ 1 + 2 \cos \theta \left( \tilde{V}_0 \tilde{\tau} \left( 1 - \exp \left( - \frac{\tilde{\sigma}_0 \cos \theta}{\tilde{V}_0 \tilde{\tau}} \right) \right) - \frac{1}{2} \right) \right] \right\} + \tilde{\sigma}_0 (1 - \cos \theta) = \Delta \tilde{\gamma}
\]  

(19)

In the limiting cases of \( \tilde{V}_0 \gg 1 \) and \( \tilde{\tau} \gg 1 \), Eq. (18) gives

\[
\frac{1}{2} \tilde{\sigma}_0^2 \left( 1 - \cos^2 \theta \right) + \sigma_0 (1 - \cos \theta) = \frac{\Delta \gamma}{d}
\]  

(20)

which clearly differs from Eq. (9), showing that the energy dissipation due to frictional sliding at the interface is proportional to \( \frac{1}{2} \left( \sigma_0^2/E_\infty \right) \cos^2 \theta \), which leads to much tougher peeling behavior at small peeling angle, as the peeling stress \( \sigma_0 \) diverges as \( \sigma_K/\theta \). This result has been already observed in Refs. [41, 68] for purely elastic tapes \((E \text{ is replaced by } E_\infty)\), and it can be interpreted as the emergence of an infinitely tough peeling behavior. Incidentally, it is worth noticing that ultra-tough peeling has been also predicted to occur when the tape is elastic and the substrate viscoelastic [30, 51].

Similarly, in the limiting case of \( \tilde{V}_0 \tilde{\tau} \ll 1 \) with \( \tilde{V}_0 \gg 1 \) (i.e. \( V_0 \gg d/t_c \)), Eq. (18) becomes

\[
\frac{1}{2} \frac{\tilde{\sigma}_0^2}{E_0} \left( 1 - \cos^2 \theta \right) + \sigma_0 (1 - \cos \theta) = \frac{\Delta \gamma}{d}
\]

(21)

where the tape response in the adhered portion subjected to frictional shear stresses is governed by the low frequency viscoelastic modulus \( E_0 \). However, in Eq. (21), the additional term

\[
\frac{\sigma_0^2 (1 - \cos \theta)^2}{2E_1} = \frac{D_f}{V_0bd} = (1 - \cos \theta)^2 \frac{D_s}{V_0bd}
\]

(22)

takes into account for the viscoelastic energy dissipation per unit time \( D_f \), triggered by the stress step change \( \Delta \sigma = \sigma_0 - \sigma_0 \cos \theta \), which still occurs at the peeling front. Indeed, Eq. (8) is still valid provided that \( \sigma_0 \) is replaced by \( \Delta \sigma \). Notice that, as already discussed before, for \( \tilde{V}_0 \ll 1 \) (i.e. \( V_0 \ll d/t_c \)) the term \( D_f \) must also vanish, as even in the peeling section the tape behaves elastically with modulus \( E_0 \). Therefore, for \( \tilde{\tau} \ll 1 \) and \( \tilde{V}_0 \ll 1 \) (i.e. \( V_0 \ll d/t_c \)), Eq. (18) becomes

\[
\frac{1}{2} \frac{\sigma_0^2}{E_0} \left( 1 - \cos^2 \theta \right) + \sigma_0 (1 - \cos \theta) = \frac{\Delta \gamma}{d}
\]

(23)

which holds true for purely elastic tapes \((E = E_0)\) in the presence of interfacial frictional sliding (see Refs [41, 68]).

Figures [3] show the dimensionless peeling stress \( \tilde{\sigma}_0 \) as a function of the peeling angle \( \theta \), for both stuck and sliding interfaces and different values of the parameter \( \kappa = E_\infty/E_0 \) [Fig. 3a], and of the energy of adhesion \( \Delta \gamma \) [Fig. 3b].

As already discussed, in the stuck case (dashed lines in both figures) we recover the well-known elastic Kendall’s solution, where the elastic modulus is replaced by the high-frequency
FIG. 3: The dimensionless peeling stress $\tilde{\sigma}_0$ as a function of the peeling angle $\theta$, for different values of (a) the viscoelasticity parameter $\kappa = E_\infty/E_0$; and (b) the dimensionless energy of adhesion $\Delta \tilde{\gamma}$. The dashed curves refer to the case of stuck interface between the tape and the rigid substrate, whereas continuous curves refer to frictionally sliding interfaces.

viscoelastic modulus $E_\infty$ [see Eq. (2)]. Given the values of the low-frequency viscoelastic modulus $E_0$, the peeling angle $\theta$ and work of adhesion $\Delta \gamma$, the peeling force increases with the parameter $\kappa = E_\infty/E_0$.

On the other hand, in case of frictional sliding at the interface a very different scenario emerges. This time, the peeling process is governed by Eq. (18), which, regardless of the $\kappa$ value, leads to unbounded peeling forces for vanishing peeling angle $\theta$ (see continuous lines in Figures 3a and 3b). In this case, the dimensionless peeling stress obeys the equation $\tilde{\sigma}_0 = \sqrt{2\Delta \tilde{\gamma}}/\theta$ for $\theta \to 0$. Interestingly, such a result is in agreement with several experimental observations on the peeling behavior of insects pads in the presence of relative frictional sliding between the fibrils and the substrate [43, 55, 67]. Figure 3b presents the effect of the dimensionless energy of adhesion $\Delta \tilde{\gamma}$ on the peeling behavior. As expected, regardless of the specific interface behavior, increasing $\Delta \tilde{\gamma}$ leads to an overall tougher peeling behavior, as the necessary stress $\tilde{\sigma}_0$ to sustain the peeling process increases [27].

Figure 4 shows the dimensionless peeling stress $\tilde{\sigma}_0$ as a function of the peeling angle $\theta$. This time, different values of the dimensionless parameter $\tilde{V}_0\tilde{\tau}$ are considered. In the same figure, we also report the purely elastic solution in the presence of frictional sliding (with elastic modulus $E = E_0$). We observe that, for relatively small values of the parameter $\tilde{V}_0\tilde{\tau}$ (red curve) and moderately large peeling angles $\theta$, the value of $\tilde{\sigma}_0$, observed in presence of frictional sliding, is lower than the value predicted in the case of stuck interface (black dashed curve). This is related to the different mechanisms of energy dissipation occurring in each case. Indeed, for a stuck interface, the only source of energy dissipation arises from the viscoelastic creep occurring in the detached branch of tape (i.e. for $x > 0$), which is independent on $\theta$. On the contrary, when dealing with interfaces where frictional relative motion occurs between the tape and the substrate, two additional sources of energy dissipation can be identified: (i) the work done by the frictional shear stress at the interface; and (ii) the viscoelastic creep occurring in the portion of the tape adhered to the substrate.
FIG. 4: The dimensionless peeling stress $\tilde{\sigma}_0$ as a function of the peeling angle $\theta$, for different values of the dimensionless parameter $V_0 \tilde{\tau}$. The dashed curve refers to the case of stuck interface between the tape and the rigid substrate, whereas continuous curves refer to frictionally sliding interfaces. In the same figure, we also plot for comparison the peeling behavior of an elastic tape in frictional sliding. Results refer to $V_0 \gg 1$.

and stretched by the interfacial frictional shear stresses (i.e. for $-a \leq x < 0$). However, for $V_0 \tilde{\tau} < 1$ and $0 \ll \theta \lesssim \pi/2$, both these terms vanish as the no viscoelastic creep occurs in the adhered sliding portion of the tape (i.e. the tape response is governed by $E_0$, see Eq. (21)) and the term $\cos^2 \theta \to 0$ (i.e. the work done by frictional shear stress is negligible, see Eq. (15)). Therefore, under these conditions, even in the case of frictionally sliding interfaces, the only source of the energy dissipation is the viscoelastic creep occurring in the detached tape, which can be quantified as $D_f$ through Eq. (22). Since $D_f = (1 - \cos \theta)^2 D_s < D_s$, for $0 < \theta < \pi/2$, a lower peeling force is predicted in the frictional sliding case compared to stuck interfaces, as indeed shown in Fig. 4. Notably, the effect of the energy dissipation term $D_f$ on the overall peeling behavior can be appreciated by comparing the low speed viscoelastic case (red curve) against the elastic limit (black continuous curve). As already discussed in commenting Eq. (21), a physical explanation of this phenomenon can be found by observing that, in the case of frictional sliding interfaces, the step change occurring in the tape stress at the peeling front is lower than in the case of stuck interfaces, as in the former case the tape in the adhered portion close to the peeling front is pre-stressed by the frictional shear stress by a quantity $\sigma_0 \cos \theta$. Thus, since for $V_0 \tilde{\tau} < 1$ the tape pre-stress occurs at a very low excitation frequency (i.e. the tape response does not present any creep), the resulting energy dissipation due to the viscoelastic creep (only occurring in the detached strip) is smaller for frictional sliding interfaces compared to the stuck case, in turn leading to smaller peeling forces.

Figure 5 shows the dimensionless peeling stress $\tilde{\sigma}_0$ as a function of the dimensionless parameter $V_0 \tilde{\tau}$ for a given value of $\theta$. All the cases refer to $V_0 > 1$. As expected three different regimes can be observed depending on the value of $V_0 \tilde{\tau}$. For $V_0 \tilde{\tau} \ll 1$ an asymptotic plateau for $\tilde{\sigma}_0$ is observed as predicted by Eq. (21), which depends on the value of $\kappa$. For $V_0 \tilde{\tau} \gg 1$, the peeling behavior is governed by Eq. (20) and depends on the high frequency viscoelastic modulus $E_\infty$. Again, a plateau is observed for $\tilde{\sigma}_0$, whose value saturates as for $\kappa \to \infty$ as the Rivlin’s solution is approached in the case of infinitely stiff tapes. At intermediate values of $V_0 \tilde{\tau}$ the hysteretic viscoelastic behavior of the tape plays a key role so
FIG. 5: The dimensionless peeling stress $\tilde{\sigma}_0$ as a function of the dimensionless parameter $\tilde{V}_0\tilde{\tau}$, for different values of the parameter $\kappa = E_\infty / E_0$. In the figure, both the high and low speed plateau are highlighted. Results refer to $\tilde{V}_0 \gg 1$.

that, in this transition region, the peeling force increases with the peeling rate by following a power law $\tilde{\sigma}_0 \approx \left(\tilde{V}_0\tilde{\tau}\right)^n$ where the exponent $n$ depends on the parameter $\kappa$.

IV. CONCLUSIONS

In this study, we investigate the peeling behavior of a thin viscoelastic tape peeled away from a rigid substrate. Specifically, we consider two alternative scenarios: one, with the interface between the tape and the rigid substrate under stuck adhesion (i.e. no sliding occurs); the other, assuming relative sliding on a portion of the interface in the presence of frictional shear stresses.

We found that, in stuck interfaces, the overall viscoelastic peeling behavior is independent of the peeling velocity, provided that the peeling velocity $\tilde{V}_0 \gg d/t_c$ (where $d$ is the thickness of the tape and $t_c$ is the creep characteristic time of the viscoelastic material), and the peeling force takes the value predicted by Kendall’s peeling model with the elastic modulus given by the high-frequency viscoelastic modulus $E_\infty$ of the tape material. Under these conditions, the energy dissipation associated with the viscoelastic creep of the tape is entirely localized in the detached portion of the tape.

In the presence of frictional sliding at the interface additional sources of energy dissipation come into play, which are associated with both the work done by frictional shear stress and the viscoelastic hysteresis occurring in the portion of the adhering tape subjected to frictional shear stresses. In such conditions, the peeling force is predicted to continuously increase as the peeling angle is decreased, leading to unbounded value for a vanishing peeling angle. Also, the viscoelastic hysteretic behavior of the tape strongly affects the dependence of the peeling force on the peeling velocity. Indeed, for any given value of the peeling angle, three regions can be identified: (i) the low velocity region, where a low plateau is reported for the peeling force; (ii) the transition region, where the peeling force increases as a power law of the peeling velocity, and (iii) the high velocity region, where a high plateau of the peeling force occurs.
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