Measurement of $\hat{q}$ in Relativistic Heavy Ion Collisions using di-hadron correlations.

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The propagation of partons from hard scattering through the Quark Gluon Plasma produced in A+A collisions at RHIC and the LHC is represented in theoretical analyses by the transport coefficient $\hat{q}$ and predicted to cause both energy loss of the outgoing partons, observed as suppression of particles or jets with large transverse momentum $p_T$, and broadening of the azimuthal correlations of the outgoing di-jets or di-hadrons from the outgoing parton-pair, which has not been observed. The widths of azimuthal correlations of di-hadrons with the same trigger particle $p_{Tt}$ and associated $p_{Ta}$ transverse momenta in p+p and Au+Au are so-far statistically indistinguishable as shown in recent as well as older di-hadron measurements and also with jet-hadron and hadron-jet measurements. The azimuthal width of the di-hadron correlations in p+p collisions, beyond the fragmentation transverse momentum, $p_T$, is dominated by $k_T$, the so-called intrinsic transverse momentum of a parton in a nucleon, which can be measured. The broadening should produce a larger $k_T$ in A+A than in p+p collisions. The present work introduces the observation that the $k_T$ measured in p+p for di-hadrons with $p_{Tt}$ and $p_{Ta}$ must be reduced to compensate for the energy loss of both the trigger and away parent partons when comparing to the $k_T$ measured with the same di-hadron $p_{Tt}$ and $p_{Ta}$ in Au+Au collisions. This idea is applied to a recent STAR di-hadron measurement, with result $\langle \hat{q} L \rangle = 2.1 \pm 0.6 \text{ GeV}^2$. This is more precise but in agreement with a theoretical calculation of $\langle \hat{q} L \rangle = 14_{-2}^{+3} \text{ GeV}^2$ using the same data. Assuming a length $L \approx 7 \text{ fm}$ for central Au+Au collisions the present result gives $\hat{q} \approx 0.30 \pm 0.09 \text{ GeV}^2/\text{fm}$, in fair agreement with the JET collaboration result from single hadron suppression of $\hat{q} \approx 1.2 \pm 0.3 \text{ GeV}^2/\text{fm}$ at an initial time $\tau_0 = 0.6 \text{ fm}/c$ in Au+Au collisions at $\sqrt{s_{NN}} = 200 \text{ GeV}$.

I. INTRODUCTION

In the original BDMPZ formalism [1, 2], the energy loss of an outgoing parton, $-dE/dx$, per unit length ($x$) of a medium with total length $L$, is proportional to the 4-momentum transfer-squared, $q^2$, and takes the form:

$$-dE/dx \simeq \alpha_s(q^2(L) = \alpha_s \mu^2 L/\lambda_{mfp} = \alpha_s \hat{q} L) \quad (1)$$

where $\mu$, is the mean momentum transfer per collision, and the transport coefficient $\hat{q} = \mu^2/\lambda_{mfp}$ is the 4-momentum-transfer-squared to the medium per mean free path, $\lambda_{mfp}$. Additionally, the accumulated momentum-squared, $\langle p_{T\perp W}^2 \rangle$ transverse to a parton traversing a length $L$ in the medium is well approximated by $\alpha_s \langle q^2(L) \rangle = \hat{q} L$. This results in a direct and simple relationship between the parton energy loss (Eq. 1) and the di-jet azimuthal broadening, $\langle p_{T\perp W}^2 \rangle/2$, because only one of the components of the accumulated momentum transverse to the outgoing parton is in the scattering plane, the other being along the beam axis for mid-rapidity di-jets.

It has long been established [3] that even in p+p collisions, or in the initial hard-scattered parton pair in A+A collisions, the mid-rapidity di-jets from hard-scattering are not back-to-back in azimuth but are acollinear with a net transverse momentum, $\langle p_{T\perp \text{pair}}^2 \rangle = 2 \langle k_T^2 \rangle$, where $\langle k_T \rangle$ is the average ‘intrinsic’ transverse momentum of a quark or gluon in a nucleon as defined by Feynman, Field and Fox [4]. Again, only the component of $\langle p_{T\perp \text{pair}}^2 \rangle$ perpendicular to the di-jet axis leads to acoplanarity. Thus in an A+A collision the relationship in Eq. 2 should hold:

$$\langle \hat{q} L \rangle/2 = \langle k_T^2 \rangle_{AA} - \langle k_T^2 \rangle_{pp} \quad (2)$$

for azimuthal correlations of a trigger particle with $p_{Tt}$ and away-side particles with $p_{Ta}$. It is important to note the ′ in $\langle k_T^2 \rangle_{pp}$, introduced here, which indicates that the $k_T$ measured in p+p collisions for di-hadrons with $p_{Tt}$ and $p_{Ta}$ must be reduced to compensate for the energy loss of both the trigger and away parent partons when comparing to the $k_T$ calculated with the same di-hadron $p_{Tt}$ and $p_{Ta}$ in Au+Au collisions.

Many experiments at RHIC, including recent experiments with di-hadron [5], jet-hadron [6] and di-jet [7] azimuthal correlations have searched for azimuthal broadening in Au+Au collisions compared to p+p collisions but have not found a significant difference in the azimuthal angular Gaussian width of the away-peak. Here we shall reexamine the STAR di-hadron measurement [4] in terms of the out of plane component, $p_{out}$ rather than the azimuthal angular width, taking account of the energy lost by the original parton-pair in Au+Au collisions when comparing to the p+p measurement.

II. HOW INFORMATION ABOUT THE INITIAL PARTONS CAN BE DERIVED FROM TWO-PARTICLE CORRELATIONS.

We shall calculate $\langle k_T^2 \rangle$ from p+p and Au+Au di-hadron measurements with the same trigger particle transverse momentum, $p_{Tt}$, away-side $p_{Ta}$ and $x_h =$
\( p_{T\alpha}/p_{T\beta} \). The di-hadrons are assumed to be fragments of jets with transverse momenta \( \vec{p}_{T\alpha} \) and \( \vec{p}_{T\beta} \), with ratio \( x_h = \vec{p}_{T\alpha}/\vec{p}_{T\beta} \), where \( z_t \approx p_{T\beta}/p_{T\alpha} \) is the fragmentation variable, the fraction of momentum of the trigger particle in the trigger jet, and \( j_T \) is the jet fragmentation transverse momentum. The standard equation at RHIC comes from PHENIX \[8\], which we write in a slightly different form in Eq. \[3\] \[
\sqrt{\langle k_T^2 \rangle} = \frac{x_h}{\langle z_t \rangle} \sqrt{\langle p_{out}^2 \rangle - (1 + x_h^2) \langle j_T^2 \rangle / 2}
\]

Here \( p_{out} \equiv p_{T\alpha} \sin \Delta \phi \) (see Fig. 1) and we have taken \( \langle j_{T,\alpha}^2 \rangle = \langle j_{T,\phi}^2 \rangle = \langle j_T^2 \rangle / 2 \). The variable \( x_h \) (which STAR calls \( z_T \)) is used as an approximation of the variable \( x_E = x_h \cos \phi \) of the original terminology from the CERN ISR where \( k_T \) was discovered and measured 40 years ago \[3\] \[4\] \[9\] \[10\].

**FIG. 1.** Azimuthal projection of di-jet with trigger particle \( p_{T\alpha} \) and associated away-side particle \( p_{T\beta} \), and the azimuthal components \( j_T \) of the fragmentation transverse momentum. The initial state \( k_T \) of a parton in each nucleon is shown schematically: one vertical which gives an azimuthal decorrelation of the jets and one horizontal which changes the transverse momentum of the trigger jet.

### A. Bjorken parent-child relation and ‘trigger-bias’ \[11\]

If the fragmentation function of the jet is a function only of the fragmentation variable \( z \) and not of the jet \( \vec{p} \), then the single particle cross section has the same power law shape, \( d^3\sigma / 2 \pi p_T dp_T dy \propto p_T^n \), as the parent jet cross section.

Furthermore, large values of \( \langle z_t \rangle = p_{T\alpha}/p_{T\beta} \) dominate the single-particle cross section (e.g. \( \pi^0 \)) used as the trigger for the di-hadron (e.g. \( \pi^0 + \text{h} \)) measurement. This is called trigger-bias but is valid also for the simple single-particle measurements. Calculations of \( \langle z_t \rangle \) vs. \( p_{T\alpha} \) for \( \pi^0 \) at \( \sqrt{s_N} = 200 \text{ GeV} \) are given in Ref. \[12\].

### B. The energy loss of the trigger jet from p+p to Au+Au can be measured.

At RHIC, in p+p and Au+Au collisions as a function of centrality the \( \pi^0 \) \( p_T \) spectra with \( 5 < p_T < 20 \text{ GeV/c} \) all follow the same power law with \( n \approx 8.10 \pm 0.05 \) \[14\]. From the Bj parent-child relation, the energy loss of the trigger jet is found by measuring \( \delta p_T/p_T^{PP} \), the shift in the \( \pi^0 \) spectra in Au+Au at a given \( p_T \) from the \( \langle T_{AA} \rangle \) corrected p+p cross section (Fig. 2) \[14\]. The small dropoff of \( \delta p_T/p_T^{PP} \) for \( p_T \geq 14 \text{ GeV/c} \) indicates a small increase of \( n \) with increasing \( p_T \).

It is important to note that the same value of \( n \) for the \( \pi^0 \) spectra in p+p and Au+Au collisions implies the same value of \( n \) for the original parton in p+p and the one that has lost energy in Au+Au. However \( \langle z_t \rangle \) for p+p and Au+Au measurements may differ slightly because the maximum possible parton energy \( \sqrt{s_{NN}}/2 \) is reduced by the energy loss. The effect on \( \langle z_t \rangle \) from p+p to Au+Au was estimated by increasing \( p_{T\alpha} \) in the calculation of \( \langle z_t \rangle \) in p+p collisions \[12\] by the largest \( \delta p_T/p_T^{PP} = 0.20 \) for centrality 0-10% (Fig 2) with result for \( p_{T\alpha} = 7.8 \text{ GeV/c} \), \( \langle z_t \rangle = 0.63 \pm 0.07 \), and for \( p_{T\alpha} = 7.8/0.80 = 8.78 \text{ GeV/c} \), \( \langle z_t \rangle = 0.66 \pm 0.06 \). Since the difference for the largest \( \delta p_T/p_T^{PP} = 0.20 \) is considerably less than the error in the calculation, we shall use the measured or calculated \( \langle z_t \rangle \) in p+p also for Au+Au with the same \( p_{T\alpha} \).

### C. The away particles from a hadron-trigger do not measure the fragmentation function \[8\]

It was generally assumed, as implied by Feynman, Field and Fox in 1977 \[1\], that the \( p_{T\alpha} \) (or \( x_E \), or \( x_h \)) distribution of away-side hadrons from a single hadron trigger with \( p_{T\alpha} \), corrected for \( \langle z_t \rangle \), would be the same as that from a jet-trigger and would measure the away-jet fragmentation function as it does for direct photon triggers \[15\]. However, attempts to try this at RHIC led to the discovery \[8\] that the \( x_E \) distribution does not measure the fragmentation function. The good news was that it measured the ratio of the away jet to the trigger.
FIG. 3. Fits to STAR $\pi^0$-h correlation functions for $12 < p_{Tt} < 20$ GeV/c [5] measured in central (0-12%) Au+Au collisions (left) and p+p collisions (right): (top) $1.2 < p_{Ta} < 3$ GeV/c; (bottom) $3 < p_{Ta} < 5$ GeV/c.

jet transverse momenta, $\hat{x}_h = \hat{p}_{Ta}/\hat{p}_{Tt}$, Eq. 4

$$\frac{dP_t}{dx_E} \bigg|_{\hat{p}_{Tt}} = N (n - 1) \frac{1}{\hat{x}_h} \frac{1}{(1 + \frac{\hat{x}_h}{\hat{x}_h})^n},$$

with the value of $n = 8.10 \pm 0.05$ fixed as determined in Ref. [13], where $n$ is the power-law of the inclusive $\pi^0$ spectrum and is observed to be the same in p+p and Au+Au collisions in the $p_{Tt}$ range of interest as noted in section II B above.

III. HOW TO APPLY THIS INFORMATION TO FIND $\hat{q}$ FROM P+P AND AU+AU DI-HADRON MEASUREMENTS

A recent STAR $\pi^0+h$ di-hadron measurement in p+p and Au+Au collision at $\sqrt{s_{NN}}=$200 GeV [5] is used to measure $\langle \hat{q}L \rangle$ by calculating $k_T$ in each case as in Eq. 2. For a di-jet produced in a hard scattering, the initial $\hat{p}_{Tt}$ and $\hat{p}_{Ta}$ will both be reduced by energy loss in the medium to become $\hat{p}'_{Tt}$ and $\hat{p}'_{Ta}$, that will be measured by the di-hadron correlations with $p_{Tt}$ and $p_{Ta}$ in Au+Au collisions. As both jets from the initial di-jet lose energy in the medium, the azimuthal angle between the di-jets from the $\langle k_T^2 \rangle$ in the original collision should not change unless the medium induces multiple scattering from $\hat{q}$. Thus, without $\hat{q}$ and assuming the same fragmentation transverse momentum $\langle j_T^2 \rangle$ in the original jets and those that have lost energy, the $p_{out}$ between the away hadron with $p_{Ta}$ and the trigger hadron with $p_{Tt}$ will not change (Fig. 1), but the $\langle k_T^2 \rangle$ will be reduced according to Eq. 3 because the ratio of the away to the trigger jets $\hat{x}'_h = \hat{p}'_{Ta}/\hat{p}'_{Tt}$ will be reduced. Thus the calculation of $k_T'$ from the di-hadron p+p measurement to compare with Au+Au measurement with the same di-hadron trigger $p_{Tt}$ and $p_{Ta}$ must use the values of $\hat{x}_h$, and $\langle z_T \rangle$ from the Au+Au measurement to compensate for the energy lost by the original dijet in p+p collisions.

IV. CALCULATION OF $\langle \hat{q}L \rangle$ FROM THE STAR MEASUREMENT [5].

A. Determine $\langle p_{out}^2 \rangle$ from the $\pi^0$-h correlation function

This is accomplished by fitting the $\pi^0$-h correlation functions for $12 < p_{Tt} < 20$ GeV/c [5] to a gaussian in sin $\Delta\phi$ for the away-side, $\pi/2 \leq \Delta\phi \leq 3\pi/2$ [8, 12, 16]; and a gaussian in $\Delta\phi$ for the trigger side $-\pi/2 \leq \Delta\phi \leq \pi/2$.
\[ \chi^2 = \sum_{i=1}^{n} \left( \frac{(y_i + \epsilon_i \sigma_i - y_i^{\text{fit}})^2}{\sigma_i^2} \right) + \epsilon_i^2 \quad , \quad (5) \]

B. Determine \( \hat{x}_h = \hat{p}_{RA} / \hat{p}_{Tt} \)

This is done by a fit of Eq. 4 to the STAR measurements of what they call the away-side \( z_T \) distribution \[ \chi \] (called the \( x_h \) or \( x_E \) distribution here) for \( 12 < p_{Tt} < 20 \) GeV/c in p+p and Au+Au 0-12% centrality collisions (Fig. 4). The fit \[ \chi \] takes account of the statistical and correlated systematic errors, \( \sigma_i \) and \( \sigma_{b_i} \), for each data point with \( dP / dx_E = y_i \) :

\[ \chi^2 = \sum_{i=1}^{n} \left( \frac{(y_i + \epsilon_i \sigma_i - y_i^{\text{fit}})^2}{\sigma_i^2} \right) + \epsilon_i^2 . \quad (5) \]

where \( \tilde{\sigma}_i \) is the statistical error, \( \sigma_i \), scaled by the shift in \( y_i \) such that the fractional error remains unchanged: \( \tilde{\sigma}_i = \sigma_i (1 + \epsilon_i \sigma_{b_i} / y_i) \), where \( \epsilon_i \) is to be fit.

The fit worked very well with a result for Au+Au of \( \hat{x}_h = 0.36 \pm 0.05 \) with \( \chi^2 / \text{dof} = 38.8 / 5 \) where the error has been corrected upward by \( \sqrt{\chi^2 / \text{dof}} \). This is consistent with the value \( \hat{x}_h = 0.48 \pm 0.10 \) for \( 9 < p_{Tt} < 12 \) GeV/c from a PHENIX measurement \[ \chi \] (see Fig. 4).

The value of \( \hat{x}_h \) for the p+p measurement, although not needed for determining \( \langle \hat{q}_L \rangle \) in the present method, was determined for the STAR p+p data with fitted result \( \hat{x}_{hp} = 0.84 \pm 0.04 \) which is in decent agreement with the result \( \hat{x}_{pp} = 0.73 \pm 0.04 \) for \( 9 < p_{Tt} < 12 \) GeV/c from the PHENIX measurement (Fig. 4).

C. Determine \( \langle z_i \rangle \)

This was the easiest part of the calculation because STAR \[ \chi \] had determined that \( \langle z_i \rangle = 0.80 \pm 0.05 \) in their p+p collisions for \( \pi^0 \) with \( 12 < p_{Tt} < 20 \) GeV/c.
D. Calculate $\langle k_T^2 \rangle_{AA}$, $\langle k_T^2 \rangle_{pp}$, $\langle \hat q L \rangle / 2$

The $\langle p_{\text{out}}^2 \rangle$ values from the fits to the correlation functions in p+p and Au+Au plus the results for $\hat x_A^A = 0.36 \pm 0.05$, $\langle z_i \rangle = 0.80 \pm 0.05$ above are used to calculate

$$\sqrt{\langle k_T^2 \rangle} \text{ using Eq. 3 with the value } \sqrt{\langle j_T^2 \rangle} = 0.62 \pm 0.04 \text{ GeV/c} \text{ for both p+p and Au+Au. Equation 6 is used for } \langle \hat q L \rangle / 2.$$ 

The results are given in Table I.

$$\langle \hat q L \rangle / 2 = \left[ \frac{\hat x_h}{\langle z_i \rangle} \right]^2 \left[ \frac{\langle p_{\text{out}}^2 \rangle_{AA} - \langle p_{\text{out}}^2 \rangle_{pp}}{x_h^2} \right]. \quad (6)$$

For completeness, the results for $\sqrt{\langle k_T^2 \rangle}$ with the p+p values $\hat x_{pp}^p = 0.84 \pm 0.04$, $\langle z_i \rangle = 0.80 \pm 0.05$ are given in Table II.

| TABLE I. Tabulations for $\hat q$-STAR $\pi^0$-h |
|-----------------|-----------------|-----------------|
| STAR PLB760 | $\sqrt{s_{NN}} = 200$ | $\langle p_{T1} \rangle$ | $\langle p_{T2} \rangle$ | $\langle p_{\text{out}}^2 \rangle$ |
| Reaction | GeV/c | GeV/c | (GeV/c)^2 |
| p+p | 14.71 | 1.72 | 0.263 ± 0.113 |
| p+p | 14.71 | 3.75 | 0.576 ± 0.167 |
| Au+Au 0-12% | 14.71 | 1.72 | 0.547 ± 0.163 |
| Au+Au 0-12% | 14.71 | 3.75 | 0.851 ± 0.203 |
| p+p comp 14.71 | 1.72 | 0.263 ± 0.113 |
| p+p comp | 14.71 | 3.75 | 0.576 ± 0.167 |
| Au+Au 0-12% | 2.28 ± 0.35 | 1.006 ± 0.18 | 8.41 ± 2.66 |
| Au+Au 0-12% | 1.42 ± 0.22 | 1.076 ± 0.18 | 1.71 ± 0.67 |

| TABLE II. Tabulations for $\hat q$-STAR $\pi^0$-h |
|-----------------|-----------------|-----------------|
| Reaction | GeV/c | GeV/c | (GeV/c)^2 |
| p+p | 14.71 | 1.72 | 2.34 ± 0.34 |
| p+p | 14.71 | 3.75 | 2.51 ± 0.31 |

V. DISCUSSION AND CONCLUSION

For the $12 < p_{T1} < 20$ ($\langle p_{T1} \rangle = 14.71$) GeV/c, $1.2 < p_{Ta} < 3$ ($\langle p_{Ta} \rangle = 1.72$) GeV/c bin, the result of $\langle \hat q L \rangle = 8.41 ± 2.66$ GeV^2 agrees with the Ref. 20 result, $\langle \hat q L \rangle = 14 ± 2$ GeV^2, but is not consistent with zero because of the much smaller error. The result for the $3 < p_{Ta} < 5$ ($\langle p_{Ta} \rangle = 3.75$) GeV/c bin, $\langle \hat q L \rangle = 1.71 ± 0.67$ GeV^2, is at the edge of agreement, $2.4 \sigma$ below the value in the lower $p_{Ta}$ bin, but also $2.6 \sigma$ from zero. If the different $p_{Ta}$ ranges do not change the original di-jet configuration, then the value of $\langle \hat q L \rangle$ should be equal in both ranges and can be weighted averaged with a result of $\langle \hat q L \rangle = 2.11 ± 0.64$ GeV^2. Taking a guess for $\langle L \rangle$ in an Au+Au central collision as 7 fm, half the diameter of an Au nucleus, the result would be $\hat q = 1.20 ± 0.38$ GeV^2/fm for the lowest $p_{Ta}$ bin, $\hat q = 0.24 ± 0.096$ GeV^2/fm for the higher $p_{Ta}$ bin, with weighted average $\hat q = 0.30 ± 0.09$ GeV^2/fm. These results are close to or lower than the result of the JET collaboration 21 $\hat q = 1.2 ± 0.3$ GeV^2/fm at $\tau_0 = 0.6$ fm/c.

The new method presented here gives results for $\langle \hat q L \rangle$ comparable with the theoretical calculations noted 20 21 but is more straightforward and transparent for experimentalists. This is possibly the first experimental evidence for the predicted di-jet azimuthal broadening 1 2. It is noteworthy that the value of $\hat x_A^A = \hat p_{Ta}/\hat p_{T1} ≈ 0.4$ combined with the 20% loss of $\hat p_{T1}$ for the trigger jet (Fig. 2), which is surface biased 22, implies that the away jet has lost $3 \times$ more energy than the trigger jet and thus traveled a longer distance so spent a longer time in the QGP. This may affect 23 the value of $\hat q$ used for comparison from the JET collaboration which used only single (trigger) hadrons for their calculation.

It is important to emphasize that the calculated values of $\langle \hat q L \rangle$ are proportional to the square of the value of $\hat x_h$ derived from the measured away-side $z_T$ (i.e. $x_E$) distribution using Eq. 4. Although in the literature for more than a decade in a well-cited paper 8 and referenced in an important QCD Resource Letter 24, Eq. 4 has neither been verified nor falsified by a measurement of di-jet correlations with a di-hadron trigger. Future measurements at RHIC 25 26 will be able to do this and thus greatly improve the understanding of di-jet and di-hadron azimuthal broadening.

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