Soft docking with damping of two spacecraft using a tether

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Abstract. The paper focuses on the dynamics of a rendezvous of a chaser and a large space debris connected by a viscoelastic tether. It is assumed that constant thrust acts on the chaser and a control is realized by changing the length of the tether. The goal is to find the control law that provides a soft docking of two bodies on an orbit. Also, we considered a three-body system with an additional damping device. The results are expected to be applied for controlling a rendezvous of two bodies using a tether.

1. Introduction
In the recent years, the problems of space debris came to the forefront of modern astronautics. According to the forecast made in [1, 2], space debris can put an end to further space exploration. There are more than 15,000 large objects on the orbits around the Earth. Only 7% of these are active spacecraft, 17% are defunct satellites and 13% are old upper stages. All these objects are tracked. An active spacecraft or a space station can avoid collision with such objects [3-6]. Collisions of large space debris with other debris can significantly increase numbers of the small debris on the Earth orbit. There is a large number of papers focused on this problem [3-23]. Many modern spacecraft are equipped with means of de-orbiting. Various approaches have been offered to remove defunct satellites and old upper stages of spacecraft [3-4, 7, 10, 14, 22-23]. These approaches include the use of tether systems [3-4, 16]. A tether can be used as a means of the soft docking of an active spacecraft (chaser) and defunct satellites or old upper stages (space debris) [22]. In this case the chaser and the space debris are pulled together using the tether.

This study focuses on investigating of the stages of the rendezvous and soft docking of space debris with the chaser as shown in Figure 1. That is tether retraction stage when the length of the tether is decreased assisting to the soft docking of the chaser and the space debris. Our goal is to study the dynamic of the maneuver of the rendezvous and to find the control law, which would allow one to realize soft dockings of space debris with the chaser.

The paper includes two main parts. In Section 2, a mathematical model of a two-body system is developed. In Section 3, a damping device is considered as an additional part of the mechanical system aimed to eliminate the longitudinal oscillations of the tether. A mathematical model of a three-body system is developed. This model includes the chaser, space debris, a weightless viscoelastic tether and a damping device. As follows from our numerical analysis, at the end of the maneuver of the rendezvous longitudinal oscillations of the tether occur.
2. Equation of motion

The two-body system includes two points, \( S_1 \) (chaser) and \( S_2 \) (the space debris), which are connected by the weightless viscoelastic tether having initial length \( l_0 \), Figure 2(a). The origin of the inertial frame \( S \) coincides with the initial position of point \( S_1 \). The unit vector \( \mathbf{i} \) is in the direction of the tangent to the orbit in point \( S \), Figure 3. The unit vector \( \mathbf{j} \) is in the direction from point \( S \) to the Earth's center. The unit vector \( \mathbf{k} \) completes the system of Cartesian coordinates. The positions of points \( S_1 \) and \( S_2 \) can be defined by radius vectors \( \mathbf{r}_1 = \{x_1, y_1, z_1\}^T \) and \( \mathbf{r}_2 = \{x_2, y_2, z_2\}^T \), respectively. We assume that the thrust chaser \( \mathbf{F} \) coincides with the unit vector \( \mathbf{i} \). The viscoelastic force of the tether provides the mechanical coupling between chaser \( S_1 \) and space debris \( S_2 \).

![Figure 1](image1.png)

**Figure 1.** The stages of an active debris removal mission.

The equation of motion of the system can be written as [24]:

\[
\frac{d^2 \mathbf{r}_n}{dt^2} = \frac{1}{m_n} \left( \mathbf{F}^g_n + \Phi^e_n + \Phi^c_n + \mathbf{F}_n \right), \quad n = 1, 2
\]

(1)

![Figure 2](image2.png)

**Figure 2.** A two-body system: (a) - basic scheme of the two-body system, (b) - the forces.

where indices \( n = 1, 2 \) refer to the chaser and the space debris; \( m_1 \) and \( m_2 \) are masses of the material points \( S_1 \) and \( S_2 \); \( \mathbf{f}_1 \) and \( \mathbf{f}_2 \) are the viscoelastic forces of the tether interaction acting on the chaser and the space debris respectively; \( \mathbf{r}_n = \mathbf{r} + \mathbf{r}_n \) vectors showing the distance from the center of the Earth to the chaser and the space debris; \( \mathbf{r} = \{0, -r, 0\}^T \) is the vector showing the distance between the center of the Earth and point \( S \). The forces of inertia \( \Phi^e_n \), \( \Phi^c_n \) and gravitational force \( \mathbf{F}^g_n \) are inferred from the following equations:
\[
F'_n = -\mu \frac{m_n \mathbf{r}_n}{|\mathbf{r}_n|} \quad \Phi'_n = -m_n \mathbf{a}_n^e \quad \Phi'^e_n = -m_n \mathbf{a}_n^e = -2m_n \left( \mathbf{\omega} \times \frac{d\mathbf{p}_n}{dt} \right)
\]

where \( \mathbf{a}_n^e = \mathbf{a}_n - \mathbf{h}_n \omega^2 \) is the bulk acceleration, \( \mathbf{\omega} = \{0, 0, \omega\}^T \) is the angular velocity of the coordinate system (\( \omega = \text{const} \)), \( \mathbf{h}_n = \{x_n, y_n, 0\}^T \) is the vector showing the axis of rotation of points \( S_n \) around point \( S \), \( \mathbf{a}_n = -\mathbf{r} \omega \) is the acceleration of point \( S \).

\[2\]

Figure 3. System coordinates.

Having substituted (2) into (1) the equations of motion can be written as:

\[
\frac{d^2 \mathbf{p}_n}{dt^2} = -\mu \frac{m_n \mathbf{r}_n}{|\mathbf{r}_n|} + \mathbf{r} \omega^2 + \mathbf{h}_n \omega^2 - 2 \left( \mathbf{\omega} \times \frac{d\mathbf{p}_n}{dt} \right) + \frac{\mathbf{f}_n}{m_n}, \quad n = 1, 2
\]

(3)

Remembering that the orbit of the moving system is circular, we can rewrite (3) as:

\[
\frac{d^2 \mathbf{p}_n}{dt^2} = -3\omega^2 \frac{y}{r} \mathbf{r} - \omega^2 (\mathbf{r} - \mathbf{h}_n) - 2 \left( \mathbf{\omega} \times \frac{d\mathbf{p}_n}{dt} \right) + \frac{\mathbf{f}_n}{m_n}, \quad n = 1, 2
\]

(4)

This is equivalent to the classical Clohessy-Wiltshire-Hill equations [24]. Using (4) and introducing the new variable

\[
\Delta = \left\{\Delta_x, \Delta_y, \Delta_z\right\} = \mathbf{p}_1 - \mathbf{p}_2 = \{x_1 - x_2, y_1 - y_2, z_1 - z_2\}^T
\]

(5)

we obtain the following equation:

\[
\frac{d^2 \Delta}{dt^2} = -3\omega^2 \frac{\Delta_y}{r} \mathbf{r} - \omega^2 \Delta_z - 2 \left( \mathbf{\omega} \times \frac{d\Delta}{dt} \right) + \left( \frac{\mathbf{f}_1}{m_1} - \frac{\mathbf{f}_2}{m_2} \right)
\]

(6)

2.1. The viscoelastic force of the tether

Forces \( \mathbf{f}_n \) are determined by [25]

\[
\mathbf{f}_1 = \mathbf{F} \cdot \mathbf{i} - T_i \frac{\Delta}{|\Delta|} \quad \mathbf{f}_2 = T_i \frac{\Delta}{|\Delta|}
\]

(7)

where \( T_i \) is the value viscoelastic force with which the tether acts on the chaser, \( \mathbf{F} \) is the thrust of the chaser. The influence of viscoelastic properties of the tether has a significant impact on the dynamics of the system. The addition of viscoelastic force \( T_i \) allows to estimate the possible longitudinal oscillation of the tether, which can interfere with the soft docking of the chaser and the space debris. The direction \( T_i \) coincides with the direction of vector \( \Delta \) as shown in Figure 2 (b). Forces \( T_i \) are defined as [25]

\[
T_i = \begin{cases} 
  k \mathbf{\varepsilon}_i + c_i \left( \frac{d\varepsilon_i}{dt} \right), & |\Delta| \geq l \\
  0, & |\Delta| < l 
\end{cases}
\]

(8)
where $k_t$ and $c_t$ are stiffness and damping ratio, respectively; $l$ is the length control law; $\varepsilon_t$ is the tether relative deformation, $(d\varepsilon_t/l\,dt)$ is the rate-of-strain. The latter two parameters are estimated from the following equations:

\[
\varepsilon_t = \frac{|A|}{l} - 1, \quad (d\varepsilon_t/l\,dt) = \frac{\Delta \cdot (d\Delta/l\,dt)}{|A|l} - \frac{\Delta (dl/l\,dt)}{l^2}
\]  

(9)

2.2. Length control strategy

For soft docking, in work [26] the length control law of the tether was proposed:

\[
l = \frac{l_0}{2} \left(1 + \cos \varphi t\right)
\]  

(10)

where $\varphi = \pi / t_k$, $t_k$ is the duration of the maneuver, $l_0$ is an initial length of the tether. Expression (10) satisfies the requirement that $dl(t_k)/dt = 0$.

If the tether is rigid then the velocity of the space debris relative to the chaser are equal to zero. At the end of the maneuver the final length of the tether cannot be equal to zero (when $l = 0$ Expression (9) contains uncertainty). Hence, we reformulate the abovementioned law as:

\[
l = l(t) = L + \frac{(l_0 - L)}{2} \left(1 + \cos \varphi t\right),
\]  

(11)

where $L = \text{const}$ is the final length of the tether. In the work [26] it was shown that the introduced control law (11) allows the safe rendezvous manoeuvre of the chaser with the space debris. To control the length of the tether can be used and other control laws, in this work we do not consider them.

3. Three-body system

To reduce the oscillations of the system we introduce a damping device as its additional element (Figure 4). In this case, we have a three-body system which includes chaser $S_1$, space debris $S_2$ and damping device $S_3$. $S_2$ and $S_3$ connected by the viscoelastic tether which has initial length $l_0$. $S_1$ and $S_3$ are connected by the damping device which has initial length $\delta_0$ as shown in Fig.4. We focus on the rigid body of the chaser including two material points $S_1$ and $S_3$, where mass of point $S_1$ is considerably larger than that of point $S_3$ ($m_1 > m_3$). Points $S_1$ and $S_3$ are connected by the weightless viscoelastic bar, which plays the role of a damping device.

Based on the analogy with the two body system (6), the equations of motion of the three-body system can be written as:

\[
\frac{d^2p_n}{dt^2} = -3\omega^2 \frac{y_n}{r} \mathbf{r} - \omega^2 z_n - 2 \left( \mathbf{\omega} \times \frac{dp_n}{dt} \right) + \frac{f_n}{m_n}, \quad n = 1, 2, 3
\]  

(12)

Having introduced new variables

\[
\Delta = \{\Delta_x, \Delta_y, \Delta_z\} = p_3 - p_2 = \{x_3 - x_2, y_3 - y_2, z_3 - z_2\}^T
\]

\[
\delta = \{\delta_x, \delta_y, \delta_z\} = p_1 - p_3 = \{x_1 - x_3, y_1 - y_3, z_1 - z_3\}^T
\]  

(13)

Equation (12) can be rewritten as:

\[
\frac{d^2\Delta}{dt^2} = -3\omega^2 \frac{\Delta_z}{r} \mathbf{r} - \omega^2 \Delta_x - 2 \left( \mathbf{\omega} \times \frac{d\Delta}{dt} \right) + \left( \frac{f_1}{m_1} - \frac{f_3}{m_3} \right)
\]

\[
\frac{d^2\delta}{dt^2} = -3\omega^2 \frac{\delta_z}{r} \mathbf{r} - \omega^2 \delta_x - 2 \left( \mathbf{\omega} \times \frac{d\delta}{dt} \right) + \left( \frac{f_1}{m_1} - \frac{f_3}{m_3} \right)
\]  

(14)
For the three-body system, the viscolectic force of the tether (acting on points $S_1$ and $S_2$) and the viscoelastic force of the damping device (acting on points $S_2$ and $S_3$) are inferred from the following equations

\[ f_i = F_i \cdot i - T_d \frac{\delta}{|\delta|}, \quad f_2 = T_i \frac{\Delta}{|\Delta|}, \quad f_3 = T_d \frac{\delta}{|\delta|} - T_i \frac{\Delta}{|\Delta|}, \]

where $T_d$ is the value viscoelastic force produced by the damping device:

\[ T_d = \begin{cases} k_d \varepsilon_d + c_d (d \varepsilon_d / dt), & |\delta| \geq \delta_0 \\ 0, & |\delta| < \delta_0 \end{cases} \]

$\varepsilon_d$ and $(d \varepsilon_d / dt)$ are the relative deformation and rate-of-strain of the damping device, respectively,

\[ \varepsilon_d = \frac{|\delta|}{\delta_0} - 1, \quad (d \varepsilon_d / dt) = \frac{\delta \cdot (d \delta / dt)}{\delta_0 |\delta|}. \]

Force $T_i$ is given by (8) when $\Delta$ is calculated from Expression (13).

**Figure 4.** Schematic presentation of forces and distances acting in the three-body system.

### 4. Numerical analysis

To illustrate the performance of the system, numerical simulations were performed. Two-body and the three-body systems with parameters shown in Table 1 were considered. Motion of the systems without damping (6) and with it (14) were compared. The simulations were performed for 3000 seconds for the initial altitude of 800 km and the relative initial velocities of the chaser and space debris moving along a circular orbit (orbital velocity for a circular orbit of altitude 800 km) = 7.47 km/s. The rendezvous manoeuvre was performed within the first 2500 seconds, during which the tether length reduced to $L$. After the rendezvous manoeuvre ended, the whole system continued its orbital motion. Simulations were based on the numerical integration of equations (6) and (14) using the Runge–Kutta algorithm of the fourth order with variable step.

Table 2 shows the initial conditions for the systems. Figures 5-7 show the result of numerical simulations for two cases: without damping device and with it.

Figure 5 shows the time evolution of angle $\alpha$ defined as

\[ \alpha = \arctan \left( \frac{\Delta_y}{\Delta_z} \right) \]

Figure 5 (a) shows the case without damping (the first case), while Figure 5 (b) shows case with damping (the second case). Figure 5 shows that the amplitude of oscillations of angle $\alpha$ in the second case is almost two times less than in the first case. The value of angle $\alpha$ begins to grow at the end of the manoeuvre. Then it oscillates around zero.
Table 1. System parameters.

| Parameters                        | Value |
|-----------------------------------|-------|
| Initial tether length $l_0$, m   | 1000  |
| Final tether length $L$, m        | 0.1   |
| Initial length of the damping device $\delta_0$, m | 0.3   |
| Chaser mass $m_1$, kg             | 800   |
| Space debris mass $m_2$, kg       | 2000  |
| Mass of the damping device $m_3$, kg | 8     |
| Duration of the maneuver $t_s$, s | 2500  |
| Stiffness of the tether $k_t$, N  | 6000  |
| Damping ratio of the tether $c_t$, Ns | 4000  |
| Stiffness of the damping device $k_d$, N | 10    |
| Damping ratio of the damping device $c_d$, Ns | 100   |

Table 2. Initial conditions.

| $\Delta = \{\Delta_x, \Delta_y, \Delta_z\}^T$, m            | $\{1000, 0, 0\}^T$ |
|-------------------------------------------------------------|---------------------|
| $\frac{d\Delta}{dt} = \left\{ \frac{d\Delta_x}{dt}, \frac{d\Delta_y}{dt}, \frac{d\Delta_z}{dt} \right\}^T$, m/s | $\{0, 0, 0\}^T$ |
| $\delta = \{\delta_x, \delta_y, \delta_z\}^T$, m          | $\{0.3, 0, 0\}^T$  |
| $\frac{d\delta}{dt} = \left\{ \frac{d\delta_x}{dt}, \frac{d\delta_y}{dt}, \frac{d\delta_z}{dt} \right\}^T$, m/s | $\{0, 0, 0\}^T$ |

Figures 6-7 show time evolutions of velocities $(d\Delta_x / dt), (d\Delta_y / dt)$ and coordinates $\Delta_x, \Delta_y$ referring to the first and the second cases. Note that the motion of the system takes place in plane $S_{xy}$ due to the choice of the initial conditions: $\Delta_z = \delta_z = d\Delta_z / dt = d\delta_z / dt = 0$.

Figure 7 shows that after the end of the manoeuvre, values of velocities $(d\Delta_x / dt), (d\Delta_y / dt)$ start oscillating around zero in both cases. Figure 8 shows the oscillation of $\Delta_x, \Delta_y$ at the end manoeuvre in the both case. Note that the amplitudes of oscillations of $d\Delta_x / dt, d\Delta_y / dt, d\Delta_z / dt, d\delta_x / dt, d\delta_y / dt, d\delta_z / dt$ after the end of the manoeuvre in the second case are less than in the first case. As follows from Figures 6 (b, d) and 7 (b, d) the oscillations of the tether decreased when damping is introduced.

Figures 6 and 7 show that law (13) ensures good performance of the rendezvous and can be recommended for performing the rendezvous of the chaser and the space debris. As can you seen from Figures 6 and 7 after ending of the rendezvous, oscillation of the tether significantly less in the case with the damping device. Obtained result shows that use of the mechanical damper allows you to reduce the vibrations of the tether, but does not completely solve the problem of oscillations.
5. Conclusion
The dynamics of an active debris removal system, which consists of the chaser and space debris connected by the viscoelastic tether, has been investigated. The control law for the tether has been suggested. High frequency oscillations of the viscoelastic tether are predicted at the final docking phase. Therefore, it is recommended that a mechanical damper is added to the system for damping these oscillations.
Mathematical models for a two-body system, without the damping device, and the three-body system, with damping device, to study the dynamics during the rendezvous maneuver, have been developed. It is shown that, the manoeuvre during the rendezvous leads to high amplitude oscillations of the tether in the case without damping device. Adding the damping device allows one to eliminate or significantly reduce these oscillations.

We believe that the above approach to reduce the oscillations of the tether can be used for many applications for rendezvous of spacecraft.

6. References

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