Gravitational-wave Detection With Matter-wave Interferometers Based On Standing Light Waves

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Outline

- Introduction to GW detection
- Matter-wave Interferometer Detectors
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Introduction

Ground-based Gravitational-wave Detectors

The existence of gravitational waves is an important prediction of general relativity. Its detection has been the research focus for a long time. Currently, there exist two types of detectors.
The earliest GW detectors
Working at higher frequencies
Low sensitivity and narrow bandwidth
Low expense

- Resonant mass detectors
| Detector         | Place                      | Year | Mass     | Resonance Frequencies |
|------------------|----------------------------|------|----------|-----------------------|
| ALLEGRO          | Louisiana State U. (USA)   | 1991 | 2296kg Al | 895 & 920Hz           |
| AURIGA           | Legnaro (Italy)            | 1997 | 2230kg Al | 912 & 930Hz           |
| EXPLORER         | CERN                       | 1984 | 2270kg Al | 905 & 921Hz           |
| NAUTILUS         | Frascati (Italy)           | 1995 | 2260kg Al | 908 & 924Hz           |
| miniGRAIL        | Leiden U. (Netherlands)    | being built | 1300kg CuAl(6%) | 2940 & 3030Hz |
| Mario Schenberg  | U. of Sao Paulo (Brazil)   | being built | 1150kg CuAl(6%) | 3200Hz |

Resonant mass detectors in the world
• Laser interferometers

This type of detectors has high sensitivity and broad bandwidth. But the expense is high.

| Detector  | Place                    | Year | Size   |
|-----------|--------------------------|------|--------|
| GEO600    | Ruthe (Germany)          | 2001 | 600m   |
| LIGO      | Hanford & Livingston (USA) | 2001 | 4000m  |
| TAMA300   | Tokyo (Japan)            | 1999 | 300m   |
| VIRGO     | Cascina (Italy)          | 2005 | 3000m  |

Laser interferometer detectors in the world
Atom Interferometers

Atom interferometers become more and more important in precision experiments in general relativity because of their high sensitivity.

The use of atom interferometers to detect gravitational-waves was put forward several decades ago. There are two classes of atom interferometer detectors.

- Those using the superposition of two different momentum states of atoms.

  Stanford research group: AGIS
  European research group.

- Those using the matter-wave duality property of atoms.
The detection of GW with matter-wave interferometers was first considered by Chiao and Speliotopoulos in 2004. They called it, MIGO (Matter-wave Interferometric Gravitational-wave Observatory).

Like light beams in laser interferometers, atoms passing through the two arms of the interferometer will pick up a phase shift due to an incoming GW. However, dealing with matter-waves implies some important differences.
Fig. 1. Schematic diagram of the MIGO configuration (From Chiao and Speliotopoulos 2004)
Their result of the phase shift:

\[ \Delta \phi_{MIGO}^{hor}(f) = -\frac{m}{\hbar} \pi L_\perp L_\parallel i f h_\times(f) e^{-i \pi f T} F_h(fT), \]

\[ F_h(fT) = 1 - 2e^{i\pi f T/2} \text{sinc} \left( \frac{\pi f T}{2} \right) + \left[ \text{sinc} \left( \frac{\pi f T}{2} \right) \right]^2 - \frac{1}{2} \frac{f^2}{f^2 - f_0^2 + i f_0 f/Q}, \]

Different from laser interferometers, the phase shift here depends on the derivative of the GW, and is proportional to the area of the interferometer.
Our Scheme of Modified MIGO

The limitation of Chiao and Speliopoulos’ scheme: the crystal mirrors can only be prepared for Helium atoms. On the other hand, the phase shift prefers to use heavy atoms.

Our proposal: using standing light waves to split, deflect and recombine the atom beams, instead of material gratings and crystal mirrors.
Fig. 2. Schematic diagram of our proposed configuration
• Calculation of the phase shift

The metric for a linearized GW propagating along the z-direction in the rigid coordinate frame:

\[ ds^2 = -dt^2 - \dot{h}_{ij}(t-z)x^i dt dx^j + \delta_{ij} dx^i dx^j + \dot{h}_{ij}(t-z)x^i dz dx^j + O(h^2_{ij}), \]

The geodesic equations:

\[ \frac{d^2 x^i}{dt^2} = \frac{1}{2} \ddot{h}_{ij} x^j + O(h^2_{ij}, v^2 h_{ij}). \]

The Lagrangian for a non-relativistic atom is

\[ L(x^i, \dot{x}^i) = \frac{m}{2} \left( \dot{x}^i \dot{x}_i - \dot{h}_{ij}(t)x^j \dot{x}^i - 2 \right). \]
For a given path, the phase at the final point is calculated to be

\[ \Phi(f) = \Phi(i) + \frac{1}{\hbar} S(i \rightarrow f), \]

where \( S(i \rightarrow f) \) is the action along the path.

For the \( h_\times \) polarization GW, the non-zero metric components are

\[ h_{xy} = h_{yx} = h \sin(\Omega t + \phi_0) \]
Decompose the coordinates and velocities into

\[ x^i = x_0^i + \delta x^i \]
\[ v^i = v_0^i + w^i \]

Choose proper conjunction conditions at the standing light waves

\[ \delta x^i \bigg|_{t \to T^-/2} = \delta x^i \bigg|_{t \to T^+/2} \]
\[ w^i \bigg|_{t \to T^-/2} + w^i \bigg|_{t \to T^+/2} = 0 \]
Calculate the phase for each arm, and take the difference, we get

\[
\frac{\hbar}{m} \Delta \Phi = \frac{1}{2} \Omega \hbar v_0^2 T^2 \sin \alpha \tan \alpha \cos \left( \frac{\Omega T}{2} + \phi_0 \right) \\
- \hbar v_0^2 T \sin \alpha \tan \alpha (1 - \sin \alpha) \sin \left( \frac{\Omega T}{2} \right) \cos \left( \frac{\Omega T}{2} + \phi_0 \right) \\
+ \hbar v_0^2 T \sin \alpha \tan \alpha (1 - \sin \alpha) (1 - \cos \left( \frac{\Omega T}{2} \right)) \sin \left( \frac{\Omega T}{2} + \phi_0 \right)
\]
Define

\[ \Delta \Phi_1 = mA \Omega \hbar \tan \alpha \cos \left( \frac{\Omega T}{2} + \phi_0 \right) / (\hbar \cos \alpha) \]

\[ \Delta \Phi_2 = \frac{m}{\hbar} \hbar v_0^2 T \sin \alpha \tan \alpha \sin \left( \frac{\Omega T}{2} + \phi_0 \right) = 2hdk_{dB} \tan \alpha \sin \left( \frac{\Omega T}{2} + \phi_0 \right) \]

If \( \Omega T \ll 1 \),

\[ \Delta \Phi \sim \Delta \Phi_1 \sin \alpha \]

If \( \Omega T \sim 1 \),

\[ \Delta \Phi \sim \Delta \Phi_1 + \Delta \Phi_2 \]

If \( \Omega T \gg 1 \),

\[ \Delta \Phi \sim \Delta \Phi_1 \]

where \( A = \frac{1}{2} v_0^2 T^2 \sin \alpha \cos \alpha \) is the area enclosed by the two arms, and \( k_{dB} = mv_0 / \hbar \) is the de Broglie wavenumber.
• Measurement of the phase shift

This phase shift can be measured by detecting oscillations in the number of atoms going into detectors A and B:

\[ N_A = \frac{N_0}{2} (1 + \cos((\mathbf{r}_{21} - \mathbf{r}_{32}) \cdot \mathbf{k}_g + \Delta \Phi)), \]
\[ N_B = \frac{N_0}{2} (1 - \cos((\mathbf{r}_{21} - \mathbf{r}_{32}) \cdot \mathbf{k}_g + \Delta \Phi)), \]

where \( \mathbf{r}_{21} = \mathbf{r}_2 - \mathbf{r}_1, \mathbf{r}_{32} = \mathbf{r}_3 - \mathbf{r}_2 \), \( \mathbf{r}_i \)'s denote the positions of light gratings. \( \mathbf{k}_g \) is the reciprocal vector associated with each light grating, which is assumed to be the same for all light gratings. Its norm is \( ||\mathbf{k}_g|| = 2\pi/a \), where \( a \) is the period of light gratings. \( N_0 \) is the total number of atoms from the supersonic source. In order to maximize the detection sensitivity, we set \( (\mathbf{r}_{21} - \mathbf{r}_{32}) \cdot \mathbf{k}_g = \pi/2 \). Then it follows that

\[ N_A = \frac{N_0}{2} (1 - \sin(\Delta \Phi)) \approx \frac{N_0}{2} (1 - \Delta \Phi), \]
\[ N_B = \frac{N_0}{2} (1 + \sin(\Delta \Phi)) \approx \frac{N_0}{2} (1 + \Delta \Phi). \]
Noise Analysis of Our MIGO

Shot Noise

Shot noise is the fundamental limitation on the sensitivity of any atom interferometer.

\[
\tilde{h}_{\text{shot}}(f) = \frac{2\hbar}{m|C(f)|\sqrt{\mathcal{R}}}
\]

where

\[
C(f) = [ -\pi v_0^2 T^2 \sin \alpha \tan \alpha \sin(\pi fT)f + v_0^2 T \sin \alpha \tan \alpha (1 - \sin \alpha) \sin^2(\pi fT) \\
+ v_0^2 T \sin \alpha \tan \alpha (1 - \sin \alpha)(1 - \cos(\pi fT)) \cos(\pi fT) ] \\
+ i[\pi v_0^2 T^2 \sin \alpha \tan \alpha \cos(\pi fT)f - v_0^2 T \sin \alpha \tan \alpha (1 - \sin \alpha) \sin(\pi fT) \cos(\pi fT) \\
+ v_0^2 T \sin \alpha \tan \alpha (1 - \sin \alpha)(1 - \cos(\pi fT)) \sin(\pi fT)].
\]
Seismic Noise

One effect of seismic noise is that it will change the phase shift. It is easy to see that this effect is small. On the other hand, seismic noise is one of the important factors that limits the sensitivity of laser interferometers.

\[
\tilde{x}_{\text{seismic}}(f) = \begin{cases} 
10^{-9} \text{m}/\sqrt{\text{Hz}} & \text{for } 1 \text{Hz} \leq f \leq 10 \text{Hz} \\
(10 \text{Hz}/f)^2 \cdot 10^{-9} \text{m}/\sqrt{\text{Hz}} & \text{for } f > 10 \text{Hz}
\end{cases}
\]

\[
\tilde{h}_{\text{seismic}}(f) = \left| \frac{2\pi}{a} \cdot \frac{\tilde{h}}{mC(f)} \right| \sqrt{4S_{\text{seismic}}(f)}
\]

\[
= \left| \frac{2\pi}{a} \cdot \frac{\tilde{h}}{mC(f)} \right| \cdot 2 \tilde{x}_{\text{seismic}}(f)
\]
Argon atom beam for example

Take

\[ L_{//} = 200 \text{m}, \quad v_0 = 1000 \text{m/s}, \quad a = 405 \text{nm} \]

Preliminary design:

\[ \alpha = 5 \times 10^{-3} \text{rad}, \quad \mathcal{R} \sim 10^{19} \text{atoms/s} \]

Optimized design:

\[ \alpha = 5 \times 10^{-2} \text{rad}, \quad \mathcal{R} \sim 10^{21} \text{atoms/s} \]
Fig. 3. Spectral density curve for shot noise

Fig. 4. Spectral density curve for seismic noise
Fig. 5. Curves for several astrophysical GW sources
Conclusion

Through this preliminary study, we find that the phase shift is dominated by terms proportional to the time derivative of the GW.

Taking into account future improvements on current technologies, we feel that our scheme can be a good candidate scheme for building future GW detectors.

My collaborators: Peng Ju, Baocheng Zhang, Mingsheng Zhan