SOLUTION OF INVERSE NON-STATIONARY BOUNDARY VALUE PROBLEMS OF DIFFRACTION OF PLANE PRESSURE WAVE ON CONVEX SURFACES BASED ON ANALYTICAL SOLUTION

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SOLUTION OF INVERSE NON-STATIONARY BOUNDARY
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ANALYTICAL SOLUTION

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Research in the field of unsteady interaction of shock waves propagating in continuous media with various deformable barriers are of considerable scientific interest, since so far there are only a few scientific works dealing with solving problems of this class only for the simplest special cases. In this work, on the basis of analytical solution, we study the inverse non-stationary boundary-value problem of diffraction of plane pressure wave on convex surface in form of parabolic cylinder immersed in liquid and exposed to plane acoustic pressure wave. The purpose of the work is to construct approximate models for the interaction of an acoustic wave in an ideal fluid with an undeformable obstacle, which may allow obtaining fundamental solutions in a closed form, formulating initial-boundary value problems of the motion of elastic shells taking into account the influence of external environment in form of integral relationships based on the constructed fundamental solutions, and developing methods for their solutions. The inverse boundary problem for determining the pressure jump (amplitude pressure) was also solved. In the inverse problem, the amplitude pressure is determined from the measured pressure in reflected and incident waves on the surface of the body using the least squares method. The experimental technique described in this work can be used to study diffraction by complex obstacles. Such measurements can be beneficial, for example, for monitoring the results of numerical simulations.

Key words: unsteady dynamics, fluid deformation, transitional surface functions, the inverse boundary value problems

INTRODUCTION

One of the most pressing problems of modern mechanics is the study of unsteady interaction of shock waves propagating in continuous media with various deformable barriers. Research in this area is of considerable interest both from the point of view of developing mathematical methods for solving initial boundary-value problems of mechanics, and for a number of technical applications, in particular, the calculation of thin-walled structural elements loaded by shock waves in a liquid.

At this point we study the inverse non-stationary boundary-value problems of diffraction of plane pressure wave on convex surfaces immersed in liquid and exposed to acoustic shock waves. As an example, we study the diffraction of direct pressure of plane wave on a convex surface in the form of parabolic cylinder. To determine the hydrodynamic pressure acting on an obstacle, we used a transition function, built on the basis and hypothesis of a thin layer [1-3]. In this case, approximate models of interaction of a wave in fluid with a rigid obstacle, which allows obtaining fundamental solutions in closed form, were built. The diffraction of weak shock waves in liquid was studied on the basis of approximate models [4].

During the study of various problems of continuum mechanics, two main approaches to the statement of problems naturally arise – direct and inverse [5-7]. A lot of works have been devoted to various problems of continuum mechanics, both direct and inverse [8-10]. In this work, we consider a method for solving the boundary inverse problem of determining the amplitude pressure. Numerous computational experiments have been carried out in which the experimental pressure values were determined from the solution of the direct problem with the addition of error.

MATERIALS AND METHODS

When stating and solving diffraction problems for an unsteady direct pressure of plane wave on a hard obstacle, the parameters of the incident wave are often not known and it is difficult to measure them in field and bench experiments [11, 12]. At the same time, the technique of measuring pressure on the surface of an obstacle is significantly developed [13]. A problem arises: by measuring the pressure on the surface of the body, to determine the parameters of the incident wave. The leading method in this research is the method of solving the boundary inverse problem of determining the amplitude pressure. By measuring the pressure on the surface of the body at spatio-temporal points using the analytical solution (least squares method), the amplitude pressure value is determined. Numerous computational experiments have been
carried out in which the experimental pressure values were determined from the solution of the direct problem with the addition of error. In this case, the accuracy in the obtained values does not exceed the accuracy in the experimental data.

The mathematical apparatus developed in the work are the transition functions – fundamental solutions to the unsteady initial-boundary-value problem of diffraction of an acoustic medium on a smooth convex surface. In particular, a transition function is used, built on the basis of the hypothesis of thin layer [14-16]. The use of transition functions provides a transition from solving the associated non-stationary problem of joint movement of the acoustic medium and the deformable obstacle to solving the problem only for the obstacle, the mathematical model of which takes into account interaction with the environment in form of integral relations [17]. The cores of integral terms of the equations of motion of the obstacle were formed on the basis of transition functions of the diffraction problem.

Therefore, the dimension of the problem was reduced, which makes it possible to significantly simplify the numerical solution on the basis of the finite element or finite difference approach, and in some important particular cases, construct analytical solutions and estimate the accuracy introduced by the accepted hypotheses. The mathematical formulation of direct problem has the following form [18] (Eqs. 1-3):

\[ \frac{\partial^2 \phi}{\partial \tau^2} = \Delta \phi \]  
\[ p = -\frac{\partial \phi}{\partial \tau} + \nabla \phi = \text{grad} \phi \]  
\[ \phi|_{\tau=0} = 0 \]

where \( \phi \) is the velocity potential in acoustic medium, \( p \) is the pressure in the reflected and incident waves, \( \nabla \) is the velocity vector of the acoustic medium, \( \Delta \) is the Laplace operator.

Then, the problem is solved by determining the pressure at the boundary of the body in a dimensionless form [19-21]. Furthermore, all linear dimensions are assigned to the focal distance \( \alpha \), velocities to the speed of sound in an acoustic medium \( c_0 \), quantities having the dimension of pressure to the complex \( \rho_0 c_0^2 \), time \( \tau \) to \( c_0/\alpha \) [22-24]. The pressures \( p \), in the reflected wave can be found using the transition function \( G(x, \tau) \) constructed in the framework of thin layer hypothesis (an asterisk denotes the convolution operation in time \( \tau \)) (Eqs. 4-5):

\[ p_1(\xi, \tau) = \frac{\partial \varphi_1(\xi, 0, \tau)}{\partial n} * G_p(\xi, \tau) \]  
\[ G_p(\xi, \tau) = \frac{\partial G(x, \tau)}{\partial \tau} \]

At this, the influence function \( G(x, \tau) \) satisfies the following initial-boundary-value problem (Eqs. 6-8):

\[ \frac{\partial^2 G}{\partial \tau^2} = \Delta G \]
The pressure behind the wave front in the coordinate system Ox=(i=1,2) is set by the relation [6] (Eq. 15) or (Eq. 16):

\[ p_i(x^i, τ) = p_0 H(τ - x^i) \quad (15) \]

\[ p_i(ξ, τ) = p_0 H(τ - \frac{ξ^2}{2}) \quad (16) \]

where \( p_i \) is the amplitude pressure.

The main curvature is determined by the formula (16), where the average curvature takes the form \( k(ξ)/2 \), and the components of normal vector are given by expressions (17) for the case of plane problem (Eqs. 17-19) [26-28]:

\[ k(ξ) = \frac{1}{(1 + ξ^2)^{3/2}} \quad (17) \]

\[ n_1^0 = \frac{-ξ}{\sqrt{1 + ξ^2}} \quad (18) \]

\[ n_2^0 = \frac{1}{\sqrt{1 + ξ^2}} \quad (19) \]

The pressure of the reflected wave is determined by the equality [6] (Eq. 20):

\[ p_i(ξ, τ) = -p_0 n_1^0 G_0(ξ, τ - \frac{ξ^2}{2}) \quad (20) \]

Figure 2 shows sections of the spatio-temporal total pressure (Eq. 21), upon action of a unit pressure jump \( p_0=1 \) by planes \( τ=\text{const} \):

\[ p(ξ, τ) = p_i(ξ, τ) + p_1(ξ, τ) \quad (21) \]

To determine \( p_0 \) using the least squares method, we compile the functional (Eq. 23):

\[ S(p_0) = \sum_{i=1}^{I} \sum_{k=1}^{K} \left( \tilde{p}_i^k - p(ξ_i, τ^k) \right)^2 \rightarrow 0 \quad (23) \]

where \( \tilde{p}_i^k \) are the experimental values of the total pressure on the surface of parabolic cylinder.

Calculating the gradient from the functional (23) by the parameter \( p_0 \), and equating it to zero, we get, taking into account (22):

\[ \frac{dS(p_0)}{dp_0} = \sum_{i=1}^{I} \sum_{k=1}^{K} \left( \tilde{p}_i^k - p_0 f(ξ_i, τ^k) \right) f(ξ_i, τ^k) = 0 \quad (24) \]

Then we express parameter \( p_0 \) from (24):

\[ p_0 = \frac{\sum_{i=1}^{I} \sum_{k=1}^{K} \tilde{p}_i^k F(ξ_i, τ^k)}{\sum_{i=1}^{I} \sum_{k=1}^{K} F(ξ_i, τ^k) F(ξ_i, τ^k)} \quad (25) \]

Formula (25) lets us calculate the value of the amplitude pressure \( p_0 \) with controlled accuracy, while the more experimental values we have, the higher is the accuracy of determining the parameter \( p_0 \).

**Simulation using the computational experiment**

To simulate the experimental values, we calculate the values of total pressure \( p(ξ, τ) \) according to formula (22) and add a random relative error in the range of 10% and 20%:

\[ p_0 = 12.3 \text{ (Eqs. 26-28):} \]

\[ \tilde{p}_i^k = p(ξ_i, τ^k) + δ \]

\[ ξ_i = \{ξ_1 = -0.5; ξ_2 = 0; ξ_3 = 0.5\}, i = \{1, 2, 3\} \]

\[ τ^k = \{τ^k = 0.3; τ^k = 1.0\}, k = \{1, 2\} \]

Values \( p_i^k \) are shown in Table 1.

| Table 1: Experimental values | \( p(ξ, τ) \) | \( p_i^k \) | \( p_i^k + δ_{10\%} \) | \( p_i^k + δ_{20\%} \) |
|-----------------------------|--------------|--------------|---------------------|---------------------|
| \( ξ_i = -0.5; τ^i = 0.3 \) | 22.632       | 20.66594     | 21.29192            |
| \( ξ_i = 0; τ^i = 0.3 \)   | 22.878       | 21.26062     | 21.99243            |
| \( ξ_i = 0.5; τ^i = 1.0 \)  | 20.664       | 24.26192     | 25.98572            |
| \( ξ_i = -0.5; τ^i = 1.0 \) | 20.172       | 18.76031     | 23.84168            |
| \( ξ_i = 0; τ^i = 1.0 \)   | 20.664       | 21.15547     | 23.29237            |

Having calculated the value \( p_0 \) using formula (26), we obtain:

- \( p_0 = 11.91049 \), with the accuracy of input data of 10% \( δ = 3.16\% \);
- \( p_0 = 13.03037 \), with the accuracy of input data of 20% \( δ = 5.94\% \).

Therefore, the accuracy of the obtained parameter does not exceed the accuracy in the experimental data.
CONCLUSIONS

Consequently, the problem of diffraction of direct pressure of plane wave on a convex surface in the form of a parabolic cylinder was studied. A fundamental solution of the problem of the acoustic wave diffraction pressure on a smooth convex obstacle in the form of a parabolic cylinder was constructed. An algorithm for solving the inverse problem of the boundary to determine the amplitude-stand pressures was offered. Based on the analytical solution, a calculation was made to determine the amplitude pressure. Computational experiments were performed in which the experimental values of pressure were determined from the direct problem solution with the addition of error.

For the inverse problem, the amplitude pressure was determined from experimental data (measured pressure in the reflected and incident waves on the surface of the body) using the least squares method. Computational experiments demonstrated that the amplitude pressure can be determined with controlled accuracy, despite the high (up to 20%) relative error in the experimental data.

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