Effect of Shape of Cut-out on Natural Frequency of Square Plate

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Abstract. In this paper, the vibration analysis of a square plate with different cut-out shapes with the same area is investigated. Seven square plates of 150 mm x 150 mm size having central cut-out with all outer edges clamped (C-C-C-C) were considered for the study. Different shapes, namely triangle, square, pentagon, hexagon, heptagon, octagon and circle of the same cross-sectional area of 706.86 mm² were cut out centrally on each square plate. The modal analyses were carried out using finite element package ANSYS Workbench 15.0. The effect of the shape of cut-out on the natural frequency of the plate was studied. It is observed that the values of natural frequencies are increased by increasing the number of edges of the cut-out.

1. Introduction
Plates are one of the most common structural elements in many industries. The Plates are frequently subjected to vibration and controlling the frequency at which a plate vibrates is very important in the field of Engineering. So, it is very important to study the dynamic behaviour of such plate structures subjected to various excitations. The cut-outs are introduced in the plate to provide access, reduce weight, and alter the dynamic response of structures and are often provided to meet some functional or aesthetic requirements. These cut-outs alter the free vibration characteristics of the plates. For plates having multiple cut-outs, the change in frequency may have the variation to a great extent. Hence the study of plates with cut-out is also very important in the field of engineering.

It is important to know the natural frequencies of the component in the analysis of structure subjected to various excitation. Many researchers adopt various Analytical, Numerical and Experimental method for vibration analysis. The free vibration of rectangular plates with an arbitrarily-located hole of different shapes for a rectangular plate with variable thickness is by applying the discrete solution [1]. The ICCM method used for the analysis of a rectangular plate with the change in position of cut-outs of different sizes along its diagonal and major axis line under different aspect ratios of plates [2]. The squared plate of different material with varying boundary condition and the analysis is carried out in both experimental and numerical method to find the effects of different parameters like density, boundary conditions on free vibration characteristics of the plates [3]. The Rayleigh-Ritz [4] and its modified method [5] is adopted for the analysis of plate having different plate and hole domain under varying boundary conditions. The ICCM method for the vibration analysis of a rectangular plate having central cut-out [6]. The Rayleigh-Ritz method is used for analysis of rectangular plates with cut-outs and non-homogeneity [7]. Then the results are compared with both isotropic and orthotropic plates. The method of determining the effects of
openings on the fundamental frequencies in plates with different types of boundary conditions by extending the grid framework model [8] and the results are compared with other methods. The fundamental natural frequency of a thin elastic rectangular, isotropic and orthotropic, plate with clamped corners [9] using Rayleigh’s Analytical method. The Green function for the free vibration problem of the plate and it is translated into the eigenvalue problem of the matrix [10]. This method is used for the analysis of free vibration of thin and moderately thick rectangular plates with arbitrary variable thickness.

Literature survey shows that many researchers were adopted different methods for the vibration analysis of rectangular, square and circular plates for the different cut-out shapes of square, rectangle and circle under varying boundary condition. In most of the literature, the study is based on the different shape of the cut-out in the plates, not on the cut-out area of the plates. So, the main objective of the work is to carry out the modal analysis to find the effect of the shape of cut-out on the natural frequency of the square plates having the same cut-out area.

2. Numerical Analysis

The seven homogeneous, isotropic Aluminium plates with size 150mm×150mm×3mm having central cut-out with all outer edges clamped (C-C-C-C) are considered for the study. The properties of the Aluminium specimen are as shown in Table 1.

**Table 1. Mechanical properties of Aluminium.**

| Particulars          | Values          |
|----------------------|-----------------|
| Density              | 2700 Kg/m³      |
| Young’s Modulus      | 70 GPa          |
| Poisson’s Ratio      | 0.3             |

The cut-out is made centrally in the plate with different shapes with the same area 706.86mm². Since the different geometrical shapes must be maintained with the same area, the area is taken to the accuracy of two decimal points. Hence the area will be maintained with 706.86±0.39mm² by considering circular cut-out area as its reference. The cut-out specification is as in Table 2.

**Table 2. Cut-out Specifications.**

| Sl. No. | Cut-out Shape | Dimension       | Area                          |
|---------|---------------|-----------------|-------------------------------|
| 1       | Triangle      | Side (a) = 40.40 mm | $A = \frac{\sqrt{3}}{4}a^2 = 706.75 \text{ mm}^2$ |
| 2       | Square        | Side (a) = 26.59 mm | $A = a^2 = 707.03 \text{ mm}^2$ |
| 3       | Pentagon      | Side (a) = 20.27 mm | $A = \frac{1}{4}\sqrt{5(5 + 2\sqrt{5})} \times a^2 = 706.90 \text{ mm}^2$ |
| 4       | Hexagon       | Side (a) = 16.49 mm | $A = \frac{3\sqrt{3}}{2} \times a^2 = 706.47 \text{ mm}^2$ |
| 5       | Heptagon      | Side (a) = 13.95 mm | $A = \frac{7}{4} \times \frac{a^2}{\tan\left(\frac{\pi}{7}\right)} = 707.17 \text{ mm}^2$ |
| 6       | Octagon       | Side (a) = 12.10 mm | $A = 2(1 + \sqrt{2})a^2 = 706.93 \text{ mm}^2$ |
| 7       | Circular      | Radius (r) = 15 mm  | $A = \pi r^2 = 706.86 \text{ mm}^2$ |
The Modal analysis was conducted in the Finite analysis package ANSYS Workbench 15.0. In this the Plate is modelled with the size of 150×150×3mm and a cut-out has been made centrally for required dimensions. The properties of the material are entered in the Engineering data section. For meshing hexahedral element with size 10 is considered. Then the plate is fixed at all the outer edges. Finally the analysis is carried for the required amount of modes. The results obtained will be the Natural frequency and its corresponding Mode shape.

3. Results and Discussion
The Modal analysis is used to find the Natural frequencies and its corresponding Mode shapes. This helps us to find the dynamic behaviour of components or structures. For the above mentioned seven plates, it is observed that the values of natural frequencies are very less in plate having triangular cut-out, it is increased little more for the square and then it goes on increasing in the order of plate having cut-out shape of pentagon, hexagon, heptagon and octagon. Finally, it has the maximum natural frequency for the plate with circular cut-out and it is considered to have an infinite number of sides. The Frequency results are shown in Table 3.

Table 3. Frequencies (Hz) of plate with different cut-out shape.

| Frequency | Triangle | Square | Pentagon | Hexagon | Heptagon | Octagon | Circular |
|-----------|----------|--------|----------|---------|----------|---------|----------|
| 1st       | 1186.4   | 1197.7 | 1202.9   | 1202.9  | 1203.4   | 1203.9  | 1205.8   |
| 2nd       | 2289.0   | 2323.4 | 2336.3   | 2338.0  | 2338.6   | 2339.2  | 2346.9   |
| 3rd       | 2297.0   | 2323.7 | 2337.2   | 2338.3  | 2339.5   | 2339.8  | 2348.7   |
| 4th       | 3449.4   | 3442.9 | 3459.5   | 3460.1  | 3460.2   | 3459.5  | 3464.1   |
| 5th       | 4163.8   | 4188.8 | 4178.3   | 4175.1  | 4175.7   | 4177.8  | 4185.1   |
| 6th       | 4469.1   | 4525.5 | 4538.8   | 4542.5  | 4544.9   | 4545.4  | 4552.7   |

The results show that the natural frequency of the plate with cut-outs is increased by increasing the number of edges of the cut-out and has the maximum frequency for circular cut-out plate which is considered to have an infinite number of edges.

The mode shape results for the corresponding frequency results show similar mode shapes in each mode for different cut-out plates. Even though it has a similar mode shape, the orientation of its pattern may change according to the shape of the cut-out. This may be due to a very small variation in the frequency. Table 4 shows the mode shapes results of the first three modes. Similarly, for the rest of the modes, the mode shape follows the same trend.

Table 4. Mode shapes of plates having different shape of cut-outs.

| Shape of cut-out | 1st Mode Shape | 2nd Mode shape | 3rd Mode shape |
|------------------|----------------|----------------|----------------|
| Triangle         | ![Image](image1.png) | ![Image](image2.png) | ![Image](image3.png) |
Figures 1, 2 & 3 shows the graph of frequency versus different cut-out shape of the plate in first, second and third modes respectively. Where it can be seen that the frequencies of plates in each mode are increased as the number of edges of the cut-out is increased. Similarly, for the rest of modes, the graph of frequency versus the shape cut-out follows the same trend.
Figure 1. Frequency v/s the shape of cut-out in First mode.

Figure 2. Frequency v/s the shape of cut-out in Second mode.

Figure 3. Frequency v/s the shape of cut-out in Third mode.

Note: In Figures 1 to 3 the number 1 to 7 represents the cut-out shape of the plate in order Triangle, Square, Pentagon, Hexagon, Heptagon, Octagon and Circle respectively.
4. Conclusion
In this work, the effect of the shape of cut-out on the natural frequency of the plate is analysed. The seven homogeneous, isotropic aluminium square plates with 150×150×3 mm size having central cut-out of different shape with the same area of 706.86±0.39 mm² with all four edges clamped are considered. The modal analysis was carried out using a finite element analysis package ANSYS Workbench 15.0. The result shows that the plate having a triangular cut-out has very less frequency, further the frequency will go on increasing for the plates having a square, pentagon, hexagon, heptagon, octagon cut-out shapes and finally the frequency will be maximum for circular cut-out plate. So, it is observed that the natural frequency of the plate is increased by increasing the number of edges of the cut-outs. The circular cut-out plate shows the maximum frequency because it is considered to be having an infinite number of edges.

The plate with multiple cut-outs may have a very large variation in the frequency. If all the cut-outs are necessary, then the proper designing of the plate structure is done with the help of changing its cut-out shape according to its functionality. The present work may help in choosing the correct shape of the cut-out for the plate structures.

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