Melting of Wigner crystal in high-mobility n-GaAs/AlGaAs heterostructures at filling factors $0.18 > \nu > 0.125$: Acoustic studies.

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Using acoustic methods the complex high-frequency conductance of high-mobility n-GaAs/AlGaAs heterostructures was determined in magnetic fields $12 \div 18$ T. Based on the observed frequency and temperature dependences we conclude that in the investigated magnetic field range and at sufficiently low temperatures, $T \lesssim 200$ mK, the electron system forms a Wigner crystal deformed due to pinning by disorder. At some temperature, which depends on the electron filling factor, the temperature dependences of both components of the complex conductance get substantially changed. We have ascribed this rapid change of the conduction mechanism to melting of the Wigner crystal and study the dependence of the so-defined melting temperature on the electron filling factor.

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I. INTRODUCTION

Transport properties of a two-dimensional electron system (2DES) in high magnetic fields ($B$) are governed by an interplay between electron-electron interaction and their interaction with impurities. Both interactions depend on the typical size of the electron wave function, which is parameterized by the magnetic length, $l_B = \sqrt{\hbar/eB}$. At high $B$, $l_B \rightarrow 0$ and electrons act as classical point particles. Without disorder, such particles tend to form a triangular lattice - a Wigner crystal (WC) - stabilized by electron repulsion. The wave function overlap decreases with increase of $B$. Its role is quantitatively characterized by the ratio between $l_B$ and the lattice constant, $a$, of the WC. The ratio $l_B/a$ is related to the Landau filling factor, $\nu$, as $\nu = nh/eB = (4\pi/\sqrt{3})(l_B/a)^2$. At sufficiently high $\nu$ the WC ground state is predicted to undergo a transition to the fractional quantum Hall effect (FQHE) state, see, e.g., Ref. 2 for a review.

Since 2DESs at high magnetic fields are insulators it is concluded that WC is pinned by disorder. The disorder leads to texturing of the electron system into domains, typical size $L$ of which (the so-called Larkin-Ovchinnikov length) can be estimated comparing the cost in shear elastic energy and the gain due to disorder. This conclusion is supported by observation of well-defined resonances in the microwave absorption spectrum. In the pinning mode, parts of WC oscillate within the disorder-induced potential, which defines the so-called pinning frequency, $\omega_p$.

These oscillations get mixed with the cyclotron motion in the magnetic field resulting in absorption peaks at some frequencies, $f_{pk}$. In the classical, high-$B$ limit, where $l_B$ is much smaller than any feature of the disorder, and also small enough that the wave function overlap of neighboring electrons can be neglected, $f_{pk} \propto B^{-1}$.

The perfection of the WC order in 2DES has been addressed previously using time-resolved photoluminescence provided evidence for triangular crystalline ordering in the high-$B$ regime. In double quantum wells, evidence for ordering came from commensurability effects. In the context of the model described in Ref. 8 the domain size has been estimated previously from early microwave surface acoustic wave and nonlinear I-V data.

Previously we have studied dependences of complex conductance, $\sigma^{AC}(\omega) \equiv \sigma_1(\omega) - i\sigma_2(\omega)$, on frequency, temperature, and magnetic field in the vicinity of the filling factor $1/5$, namely for $0.19 < \nu < 0.21$. The complex conductance was extracted from simultaneous measurements of magnetic field dependences of attenuation and variation of velocity of surface acoustic waves (SAW) propagating in the vicinity of the sample surface. The results were interpreted as evidence of formation of a pinned Wigner crystal (WC). This conclusion was based on an observed maximum in the frequency dependence of $\sigma_1$ at $f \equiv \omega/2\pi \sim 100$ MHz coinciding with a change of the sign of $\sigma_2(\omega)$. This results allowed us to estimate the domain size in the pinned WC.

In this paper we study the dependences of complex conductance on the frequency, the temperature and the SAW electric field intensity in the same structure, but in higher magnetic field $12 < B < 18$ T corresponding to $0.18 > \nu > 0.125$, respectively. The measurements are made for temperatures $T = (40 \div 340)$ mK and SAW frequencies $f = (30 \div 300)$ MHz.

The paper is organized as follows. In Sec. II A we describe the experimental setup and the samples. The experimental results are reported in Secs. II B and II C. They are discussed in Sec. III.
SAW attenuation and deviation of its velocity. Interacts with the charge carriers. This interaction causes the piezoelectric field penetrates into the sample, the in-plane longitudinal component of the field and the vacuum and of the sample, respectively. The finite vacuum clearance a = 5 × 10^{-5} cm between the sample surface and the LiNbO₃ surface was determined from the saturation value of the SAW velocity in strong magnetic fields at T = 380 K; d = 845 nm is the finite distance between the sample surface and the 2DES layer. The SAW velocity is v₀ = 3 × 10^5 cm/s.

The frequency dependences of the components σᵢ shown in Fig. 2 is a characteristic of the Wigner crystal pinned by disorder with pinning frequency ~ 86 MHz in this case.

Shown in Fig. 3 are the frequency dependences of σ₁ for different filling factors. The curves have maxima at f ≈ 86 MHz, their amplitudes are decreasing when the magnetic field increases, see inset.

The same σ₁ data are presented in Fig. 4 as the temperature dependences at various filling factors. The each curve has a maximum, which decreases and shifts towards higher temperatures with decrease of the filling factor. Such a behavior is also observed at other frequencies.

On the left of the maxima, the temperature dependences of σ₁ are clearly dielectric; in these regions |σ₂| > σ₁. This fact, as well as the frequency dependences of σ₁ in the magnetic field interval between 12 and 18 T can be attributed to a pinned mode of WC. On the right of the maxima, |σ₂| rapidly decreases with temperature increase. σ₁ also decreases with temperature, but much slower then |σ₂|, and at high temperatures the condition |σ₂| < σ₁ is valid. Thus, it is natural to ascribe the maximum - the temperature at which the conduction mechanism rapidly changes - to the WC melting point, T_m, for a given filling factor.

So-obtained dependences T_m(ν) for different frequencies are shown in Fig. 5 as dataset 1. The dataset 2 pre-
FIG. 3. (Color online) Frequency dependences of $\sigma_1$ for different filling factors (shown near the curves). Inset: Magnetic field dependence of $\sigma_1(86\ MHz)$. $T=40\ mK$.

FIG. 4. (Color online) Temperature dependences of $\sigma_1$ for different filling factors, $\nu$: 1 - 0.18, 2 - 0.17, 3 - 0.16, 4 - 0.145, 5 - 0.125. $f = 28.5\ MHz$. Inset: Temperature dependences of $\sigma_1$ and $|\sigma_2|$ for $\nu=0.13$, $f = 28.5\ MHz$. Lines are guides for an eye.

FIG. 5. (Color online) 1 - Dependence of the “melting temperature”, $T_m$, on the filling factor $\nu$ for different frequencies, $f$: ■ - 28.5 MHz, ▼ - 86 MHz, ● - 142 MHz. 2 - Melting temperature, $T_m$, from Ref. 16 determined as the value where resonances disappear.

FIG. 6. (Color online) Dependences of $\sigma_1$ on the SAW electric field amplitude, $E$, for different filling factors $\nu$. $f = 28.5\ MHz, T = 40\ mK$.

C. Results: Nonlinear response

Shown in Fig. 6 are dependences of $\sigma_1$ on the amplitude of the electric field, $E$, produced by the SAW for several filling factors $0.125 \leq \nu \leq 0.18$. The electric field was determined according to Eq. (2) from Ref. 17, see also Ref. 18. The electric field dependences of $\sigma_1$ are similar to the temperature dependences shown in Fig. 4. Therefore, increase in the SAW amplitude acts as an increase of the temperature.

The electric field dependences of $|\sigma_2|$ for different $\nu$ are shown in Fig. 7. Notice that the dependences $\sigma_2(E)$ for different filling factors collapse on the same curve. For convenience, both components $\sigma_1$ and $|\sigma_2|$ at frequency 28.5 MHz and $\nu = 0.125$ are presented in the same graph, see the inset. On the left of the maximum
of \(\sigma_1(E)\), \(|\sigma_2| > \sigma_1\). This behavior is compatible with the prediction for a Wigner crystal. On the right of the maximum, \(|\sigma_2|\) rapidly drops and becomes much less than \(\sigma_1\). It indicates a change of the AC conduction mechanism. Assuming that an intense SAW increases the temperature of the electron system we ascribe this behavior to melting of the Wigner crystal. The behaviors of \(\sigma_{1,2}\) are similar for different frequencies with the exception of the frequency \(f = 142\) MHz at which \(\sigma_2 > 0\) at all used intensities.

### III. DISCUSSION

The behavior of \(\sigma(\omega)\) shown in Fig. 2 is typical for a pinned mode of a Wigner crystal \(19,20,22\) see also Refs. 2 and 6 for a review. The crystal manifests itself in observed resonances in \(\sigma_1(\omega)\) which has been interpreted as a signature of a solid and explained as due to the pinning mode (the disorder gapped lower branch of the magnetophonon) \(\sigma_{\perp}\) of WC crystalline domains oscillating collectively within the disorder potential. The WC states compete with the fractional quantum Hall effect (FQHE) states. Based on several experiments and calculations it is concluded that at \(\nu = 1/5\) the FQHE dominates while at \(\nu\) slightly less or slightly higher than 1/5 the WC state wins, see, e.g., Fig. 9 from Ref. 2.

The dynamic response of a weakly pinned Wigner crystal at not too small frequencies is dominated by the collective excitations \(19,20,22\) where an inhomogeneously broadened absorption line (the so-called pinning mode) appears \(6,25\). It corresponds to collective vibrations of correlated segments of the Wigner crystal around their equilibrium positions formed by the random pinning potential. The mode is centered at some disorder- and magnetic-field-dependent frequency, \(\omega_p\) (so-called pinning frequency), with a width being determined by a complicated interplay between different collective excitations in the Wigner crystal. There are modes of two types: transverse (magnetophonons) and longitudinal (magnetoplaxmons). The latter include fluctuations in electron density. An important point is that pinning modifies both modes, and the final result depends on the strength and the correlation length, \(\xi\), of the random potential. Depending in the strength and the correlation length of the random potential, the frequency, \(\omega_p\) may either increase, or decrease when the magnetic field rises.

The ratio \(\omega_p/\omega_c\), where \(\omega_c\) is the cyclotron frequency, can be arbitrary. Depending on the interplay between the ratio \(\omega_p/\omega_c\) and the ratio \(\eta \equiv \sqrt{\lambda/\beta}\) between the shear (\(\beta\)) and bulk (\(\lambda\)) elastic moduli of the Wigner crystal, one can specify two regimes where the behaviors of \(\sigma_{AC}\) are different:

\[
(a) \ 1 \ll \omega_c/\omega_p \ll \eta, \quad (b) \ 1 \ll \eta \ll \omega_c/\omega_p .
\]

Here \(\omega_{p0}\) is the pinning frequency at \(B = 0\). As a result, the variety of different behaviors is very rich. Assuming \(\xi \gg l_B = (\hbar/eB)^{1/2}\) one can cast the expression for \(\sigma_{\perp}(\omega)\) from Ref. 19 into the form

\[
\sigma(\omega) = -i \frac{\epsilon^2 n_0}{m^{*}\omega_p^2} \frac{1 - iu(\omega)}{[1 - iu(\omega)]^2 - (\omega \omega_p/\omega_{p0})^2} ,
\]

where the function \(u(\omega)\) is different for regimes (a) and (b).

Let us consider the regime (b) since only this regime seems to be compatible with our experimental results. Then

\[
\sigma(\omega) \sim \int (\omega/\Omega)^{2s}, \quad \omega \ll \Omega, \quad \Omega \ll \omega \ll \omega_c .
\]

Here \(\Omega \sim \omega_c^2 \eta/\omega_{p0}\) while \(s\) is some critical exponent. According to Ref. 19, \(s = 3/2\).

Assuming the regime (b1) we can cast Eq. 2 in the form \(\sigma(\omega) \equiv \sigma_0 s(\omega/\Omega)\) where

\[
\sigma_0 = \frac{\epsilon^2 n_0^2}{2m^{*}\omega_c} , \quad s(\tilde{\omega}) = -2 \left(\frac{\omega}{\tilde{\omega}} - 1 - \tilde{\omega}^2\right) ,
\]

with \(\tilde{\omega} = \omega/\Omega\). This function is normalized in order to have its maximum \(\eta\)-independent. Graphs of real and imaginary parts of \(s(\omega/\Omega)\) for \(\eta = 4, 5\) and 6 are shown in Fig. 5.

Equation 4 predicts decrease of the maximum magnitude of \(\sigma_1(\omega)\) with increase of magnetic field. This prediction is compatible with our experiment, see the inset in Fig. 5. However, the predicted behavior of maximum frequency as \(\omega_p \propto \omega^{-1}\) is not observed – the resonant frequency is almost independent of magnetic field, as seen in Fig. 4. One needs to note, however, that the specificity of our experimental technique does not allow to trace the impact of small change in frequency on the dependence \(\sigma\) on \(\omega\).

![Image](image-url)
Unfortunately, the experimental data shown in Fig. 2 do not provide an accurate structure of the maximum, and therefore do not allow fitting the model with high accuracy. Assuming \( \eta = 5 \) that gives approximately correct shape of the curves in Fig. 2 and taking into account that the maximum of \( \sigma_1(\omega) \) occurring at \( \omega = 44 \) MHz corresponds to \( \omega/\Omega = 0.44 \), we conclude that \( \Omega = \omega/\omega_{\max}/0.44 \approx 1.2 \times 10^9 \text{ s}^{-1} \). The quantity \( \omega_p \equiv \omega_{\max}/\omega_{\max} = 0.44 \omega_{\max}/\eta/\omega_0 \) plays the role of the pinning frequency in the magnetic field.

The frequency \( \omega_{\rho0} \) can then be determined as

\[
\omega_{\rho0} = \sqrt{\omega_0/\eta}.
\]

Substituting \( \eta = 5 \), \( \rho = 1.2 \times 10^9 \text{ s}^{-1} \), \( \omega_0 = 3.2 \times 10^{13} \text{ s}^{-1} \) (\( B = 12.2 \mathrm{T} \), \( \nu = 0.18 \)) we obtain

\[
\omega_{\rho0} = 8.7 \times 10^{10} \text{ s}^{-1}.
\]

Therefore, the regime (b) of Eq. (1) is the case, as we expected.

Estimating the Larkin length, i.e., the WC domain correlation length, as

\[
L = 2\pi c_1/\omega_{\rho0},
\]

where \( c_1 = (\beta/(\eta \mu n^2))^{1/2} \approx 4 \times 10^6 \text{ cm/s} \) is the velocity of the WC transverse mode for our electron density \( n \) we obtain \( L \approx 3 \times 10^{-4} \text{ cm} \) that is much larger than both the distance between the electrons \( \sigma = 4.8 \times 10^{-6} \text{ cm} \) and the magnetic length \( l_B = 7.3 \times 10^{-7} \text{ cm} \),

\[
L \gg a \gg l_B.
\]

These inequalities justify using the theory of the Wigner glass for our estimates.

In conclusion, we have measured the absorption and the velocity of SAWs in high-mobility samples n-GaAs/AlGaAs in magnetic fields 12 \( \div \) 18 T (i.e., at filling factors \( \nu \approx 0.18 \div 0.125 \)). From the measurement results the complex AC conductance, \( \sigma^\text{AC}(\omega) \equiv \sigma_1(\omega) - i\sigma_2(\omega) \) was found, and its dependences on frequency, temperature and the amplitude of the SAW-induced electric field were discussed. We conclude that in the studied interval of the magnetic field and \( T < 200 \text{ mK} \) the electronic system forms a pinned Wigner crystal, the so-called Wigner glass. The estimate of the correlation (Larkin) length of the Wigner glass is \( \simeq 3 \mu \text{m} \) at \( B = 12.2 \text{ T} \).

We have also defined an effective melting temperature, \( T_m \), as the temperature corresponding to the maximum in the temperature dependence of \( \sigma_1 \), or rapid decrease of \( |\sigma_2| \). These behaviors indicate a rapid change in the conductance mechanism – from the dielectric behavior at \( T < T_m \) to the metallic one at \( T > T_m \).

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