The inclusive $b \rightarrow s \gamma$ decay in the noncommutative standard model

E O Itan
Physics Department, Middle East Technical University, Ankara, Turkey
E-mail: eiltan@heraklit.physics.metu.edu.tr

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Abstract. We study the new structures appearing due to noncommutative effects in the inclusive decay $b \rightarrow s \gamma^*$, in the standard model. We present the corresponding coefficients which carry the space–space and space–time noncommutativity.

1. Introduction

It is believed that the nature of space–time changes at very short distances of the order of the Planck length. The noncommutativity approach in space–time is a possible candidate to describe the physics at the Planck scale. The noncommutative (NC) structure in space–time can be introduced by taking NC coordinates $\hat{x}_\mu$ which satisfy the equation [1]

\[ [\hat{x}_\mu, \hat{x}_\nu] = i \theta_{\mu\nu}, \]  

where $\theta_{\mu\nu}$ is a real and antisymmetric tensor with the dimensions of length squared. Here $\theta_{\mu\nu}$ can be treated as a background field relative to which directions in space–time are distinguished.

The NC field theory is equivalent to the ordinary one except that the usual product is replaced by the $*$ product

\[ (f*g)(x) = e^{i\theta_{\mu\nu}\partial_\mu^a\partial_\nu^a} f(y)g(z)|_{y=z=x}. \]  

The commutation of the Hermitian operators $\hat{x}_\mu$ (see equation (1)) holds with this new product, namely,

\[ [\hat{x}_\mu, \hat{x}_\nu] = i \theta_{\mu\nu}. \]  

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The quantum field theory over NC spaces [2] has attracted great interest in recent years with the re-motivation due to the string theory arguments [3, 4]. NC field theories (NCFTs) are difficult to handle since they have nonlocal structure and the Lorentz symmetry is explicitly violated [5, 6]. The violation of the Lorentz symmetry is due to the constants $\theta_{\mu\nu}$ in equation (1). Since $\theta_{\mu\nu}$ is antisymmetric, the vectors $\theta_i = \epsilon_{ijk}\theta_{jk}$ and $\theta_0i$ are constant three-vectors in preferred directions in a given Lorentz frame.

NCFTs have been studied extensively in the literature. There has been a lot of work done on the renormalizability of NCFTs [7]. The unitarity in NC theories and the unitarity properties of spontaneously broken NC gauge theories have been discussed in [8] and [9] respectively. Bounding NC QCD due to the Lorentz violation has been studied in [6] and it was concluded that the collider limits were not competitive with low energy tests of Lorentz violation for bounding the scale of space–time noncommutativity. Furthermore, noncommutativity among extra dimensions for QED has been examined in [10]. NC quantum electrodynamics (NCQED) have been studied in [11] and the explicit calculation of electric dipole effects and anomalous magnetic moments has been done in [12]. In the non-Abelian case, the field theory is formulated on NC spaces as theories on commutative spaces, by expressing the noncommutativity using the $\ast$ product as in equation (2) [13]. The method proposed in [13] has been applied to the full standard model (SM) in [14] and recently a unique model for strong and electroweak interactions with their unification has been constructed in [15]. In a recent work [16], the SM forbidden $Z \rightarrow \gamma\gamma$ and $Z \rightarrow gg$ decays in the NCSM has been studied.

In our work, we construct the possible structures appearing for the process $b \rightarrow s\gamma^*$ in the NCSM up to the first order in $\theta$, using the consistent formalism of NCSM [14]. Since the decay under consideration occurs at least at the loop level, it is rich phenomenologically. The integration over heavy degrees of freedom brings an operator structure and the corresponding Wilson coefficients. These Wilson coefficients carry information about the model used and play an important role in the determination of the measurable physical parameters such as branching ratios, CP asymmetry etc. Here, we choose the rare $b \rightarrow s\gamma^*$ decay and try to extract the Wilson coefficients sensitive to the noncommutative effects, space–space noncommutativity, time–space noncommutativity and NC directions. This would be informative in the determination of NC effects with the more precise measurement of $b \rightarrow s$ transitions.

In the NCSM the additional vertices of quarks with the scalar particles (in our case the scalar particle is the unphysical Higgs boson $\phi^\pm$) is proportional to the parameter $\theta_{\mu\nu}p^\nu q^\nu$ where $p$ ($q$) is the quark ($\phi^\pm$) four-momentum vector. However, for the vertices of quarks with the vector particles, here $W$ boson or photon with four-momentum $q$, there exist new factors $\theta_{\alpha\beta}p^\alpha q^\beta$ and $\theta_{\alpha\beta}q^\alpha p^\beta$ in addition to $\theta_{\alpha\beta}q^\alpha p^\beta$. Similar behaviour appears for $\phi\phi\gamma$ and $WW\gamma$ vertices. Furthermore there are quark–quark–$\phi\gamma$ and quark–quark–$W\gamma$ four-point interactions which do not exist in the commutative SM (CSM). Therefore new structures appear, in addition to the ones which are based on the assumption that NC effects enter the expressions as an exponential factor $e^{-\frac{i}{2}\theta_{\mu\nu}p^\nu q^\nu}$, which is consistent in approximate phenomenology (see [17] and references therein). Notice that we do not present the operators which exist with the addition of the QCD corrections in the commutative part, since our aim is to obtain the new structures up to the first order in $\theta$ and the interference terms of these new structures with the CSM ones.
2. The noncommutative effects on the $b \to s\gamma^*$ decay

The inclusive $b \to s\gamma^*$ process appears at least at the loop level (see figures 1 and 2). Now, we present the possible structures appearing for this process in the NCSM up to the first order in $\theta$:

$$
Q_1 = \bar{s}(k_{\mu}\bar{k} - k^2\gamma_{\mu})Lb,
Q_2 = \frac{e}{8\pi^2}m_b\bar{s}\sigma_{\mu\nu}k^\nu Rb,
Q_3 = m_b\bar{s}\bar{k}_{\mu}Rb,
Q_4 = \bar{s}(k_{\mu}\bar{k} - k^2\theta_{\mu\nu}\gamma_{\nu})Lb,
Q_5 = \bar{s}(\bar{p}_{\mu}\bar{k} - \bar{p}\cdot k\gamma_{\mu})Lb,
Q_6 = m_b\bar{s}(k_{\mu}\bar{p} - k^2\gamma_{\mu})Rb,
Q_7 = m_b\bar{s}(\theta_{\mu\nu}\gamma_{\mu}\bar{k} - k\gamma_{\mu})Rb,
Q_8 = m_b\bar{s}(k_{\mu}\bar{k}\bar{k})Rb,
Q_9 = m_b\bar{s}(k_{\mu}\bar{k}\bar{k} - k^2\gamma_{\mu}\bar{k})Rb,
Q_{10} = \epsilon_{\mu\alpha\beta\gamma}k_{\beta}\bar{k}_{\alpha}\bar{s}\gamma_{\gamma}Lb,
Q_{11} = \epsilon_{\mu\alpha\beta\gamma}k_{\beta}\bar{p}_{\alpha}\bar{s}\gamma_{\gamma}Lb,
Q_{12} = (k^2\epsilon_{\mu\alpha\beta\gamma} - k_{\mu}k_{\beta}\epsilon_{\theta\alpha\beta\gamma})\theta_{\beta\gamma}\bar{s}\gamma_{\alpha}Lb,
Q_{13} = m_b\bar{k}_{\theta}\epsilon_{\mu\alpha\beta\gamma}\theta_{\beta\gamma}\bar{s}\gamma_{\alpha}Lb
$$

where $L (R) = \frac{1-\gamma_5}{2} (1+\gamma_5)$, $p (k)$ is the four-momentum vector of the $b$ quark (photon $\gamma^*$) and $\bar{\gamma}_{\mu} = \theta_{\mu\nu}q^\nu$. Here the first two structures exist in the CSM and the others are due to the NC effects. Notice that in the structures the $s$ quark mass is neglected. In the case of the approximate phenomenology, based on the assumption that NC effects enter the expressions as exponential factors, the structures $Q_i, i = 1, \ldots, 10$ are induced.

For the real photon case, namely the $b \to s\gamma$ decay, the structures $Q_2, Q_3, Q_5, Q_7, Q_{10}, Q_{11}$ and $Q_{13}$ appear and the decay width for this process in the $b$-quark rest frame reads as

$$
\Gamma = \Gamma_{CSM} + \Gamma_{New}
$$

where

$$
\Gamma_{CSM} = \frac{G_F^2\alpha_em_b^5}{32\pi^4}\left|\sum_i A_2(x_i)\right|^2,
\Gamma_{new} = \frac{G_F^2\alpha_em_b^3}{16\pi^4}\left(\epsilon_{\mu\alpha\beta\gamma}k_{\beta}p_{\alpha}\theta_{\beta\gamma}\Re\left[\sum_i A_2^\ast(x_i)\sum_i A_7(x_i)\right] - \left(\Im\left[\sum_i A_2(x_i)\sum_i A_7^\ast(x_i)\right]\right)\right) + 2\Im\left[\sum_i A_2(x_i)\sum_i A_7^\ast(x_i)\right] + \Re\left[\sum_i A_2^\ast(x_i)\sum_i A_11(x_i)\right]p\cdot\bar{k},
$$

Figure 1. Self-energy diagrams contribute to $b \to s\gamma^*$ in the NCSM. Wavy lines represent the electromagnetic field and dashed lines the $W^\pm$ and $\phi^\pm$ fields.
with $i = u, c, t$ and $x_i = m_i^2/m_W^2$. Here $A_j(x_i) = V_{ib}V_{is}^* C_j^{NC}(x_i)$ and $C_j^{NC}(x_i)$ are the coefficients corresponding to the existing structures. The coefficient $C_7^{NC}(x_i)$ is the well known Wilson coefficient $C_7(x_i)$.

It is obvious that the main contribution to the decay width comes from the CSM since the new part due to the NCSM is proportional to the extremely small parameter $\theta$. This new part is responsible for time–space and space–space noncommutativity. With the definitions

$$(\theta_T)_i = \theta_{ib}$$

and

$$((\theta_S)_i) = \epsilon_{ijk} \theta_{jk}, \quad i, j, k = 1, 2, 3,$$

in the $b$-quark rest frame, $\Gamma_{\text{New}}$ can be written as

$$\Gamma_{\text{new}} = \frac{G_F^2 \alpha_{em} m_b^3}{16 \pi^4} \left[ \text{Re} \left( \sum_i A_2^*(x_i) \sum_i A_7(x_i) \right) \vec{k} \cdot \vec{\theta}_S - \left( \text{Im} \left( \sum_i A_2(x_i) \sum_i A_7^*(x_i) \right) \right) \right]$$

$$+ 2 \text{Im} \left[ \sum_i A_2(x_i) \sum_i A_7^*(x_i) \right] + \text{Re} \left[ \sum_i A_2^*(x_i) \sum_i A_{11}(x_i) \right] \vec{k} \cdot \vec{\theta}_T. \quad (7)$$

This expression shows that the space–space noncommutativity is carried by the coefficients $C_2^{NC}(x_i)$ and $C_7^{NC}(x_i)$. In the case of real coefficients $C_{5,7,11}^{NC}(x_i)$, $C_2^{NC}(x_i)$ and $C_7^{NC}(x_i)$ play the main role in the time–space noncommutativity, since the imaginary parts of $A_j(x_i)$ come from the CKM matrix elements, which are extremely small.

In conclusion, we present the possible structures appearing for the process $b \to s \gamma^*$ in the NCSM up to the first order in $\theta$. Here, we do not show the additional operators in the CSM part and do not construct possibly new ones in the NC part, when the QCD corrections are included. It is obvious that the NC part of the decay width of the process under consideration is not easy to detect using present and even future sensibly arranged experiments, since it is
proportional to a small number, at the order of the optimistic magnitude, $|\theta| \sim 10^{-6}$ (GeV$^{-2}$), for our process. However, it brings a new source for the CP violating effects in addition to the complex CKM matrix elements in the SM, $V_{ub}$ in our case, and the effect of space–space (time–space) noncommutativity (see equation (7)) on the possible CP asymmetry seems interesting. Hopefully, with the precise experimental results of CP violating asymmetry and the consistent calculations of the coefficients of this process, it would be possible to understand the NC effects and to predict the NC direction $\vec{\theta}_S$, if the matrix $\theta_{\mu\nu}$ has constant components across the distances that are large compared with the NC scale.

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