Securing emergent behaviour in swarm robotics

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Abstract

Swarm robotics is the study of how a large number of relatively simple robots can be designed so that a desired collective behaviour emerges from the local interactions among robots and between the robots and their environment. While many aspects of a swarm may be modelled as various types of ad hoc networks, and accordingly many aspects of security of the swarm may be achieved by conventional means, here we will focus on swarm emergent behaviour as something that most distinguishes swarm robotics from ad hoc networks. We discuss the challenges emergent behaviour poses on communications security, and by classifying a swarm by types of robots, types of communication channels, and types of adversaries, we examine what classes may be secured by traditional methods and focus on aspects that are most relevant to allowing emergent behaviour. We will examine how this can be secured by ensuring that communication is secure. We propose a simple solution using hash chains, and by modelling swarm communications using a series of random graphs, we show that this allows us to identify rogue robots with a high probability.

Keywords: swarm robotics, security protocols, distributed systems security, public key cryptography, digital signatures, hash chains, random graphs

1 Introduction

There are many variations on what the term “swarm robotics” means exactly. From [13, 12, 7, 10] we see that it is generally agreed that a swarm is a collection of a large number of autonomous mobile robots. The robots are generally resource-constrained and have low capability individually. They are
self-organised, and no synchronicity is assumed. The swarm exhibits collective emergent behaviour: a desired collective behaviour emerges from the local interactions among the robots and between the robots and the environment.

What differs in diverse applications and scenarios are the communication capabilities, the hierarchical organisation, and the presence or absence of a central control. Some swarms consist of homogeneous robots but some may allow a few groups of homogeneous robots or allow some robots to take on special roles in communications or control. There is generally no centralised control, although in some applications a global channel is needed for a central control to download information onto the robots, or for robots to report back findings from the field. Robots are generally assumed to have local sensing and communication capabilities but may communicate with their neighbours in different ways, for example, they may communicate directly in a peer-to-peer fashion, or they may observe physical traits of their neighbours, or they may communicate by leaving messages in the environment. These variations have impact on the provision of security to the swarm, and we will discuss them in more detail in Section 2.1.

In [12] basic behaviours of swarm robotics are enumerated, and in [10] it is argued that typical behaviours of swarm robots are either aggregation or dispersion, and in these behaviours it is the ability to distinguish robots of the same swarm that is key to the success of the swarm. This can only be done if there are some secret common to the swarm that is not available to adversaries or observers. In [10] the use of public key cryptography and key predistribution is briefly explored. We will consider this with more depth here.

Swarm robotics have applications in many areas, for example, military (such as mine clearance, access, surveillance), environmental monitoring, disaster relief and health care (such as medication provision and monitoring). We refer the reader to [13] for a more detailed picture. In many of these applications it is paramount that the swarm is protected so that the goals can be achieved. What “security” means differ from scenario to scenario, and is discussed in some detail in [7, 10]. In any case the desirable properties of a swarm in many applications are common: data is collected by many robots so the loss of individual robots has little impact on the success of the task. This redundancy makes the swarm reliable. The distributed nature of the swarm also ensures that there are no single points of failure. Any measures taken to increase security must not adversely affect these properties.

In [7, 10] a comparison is made between swarm robotics and other distributed networks such as mobile wireless sensor networks (WSN), mobile and vehicular ad hoc networks (MANETs and VANETs), multi-robot systems and software agents. We refer the readers to those references for more details. Here we concentrate on physical robots (hence the use of “robots” rather than “agents”) and emergent behaviour, a feature that is not emphasised in most other distributed networks. We will examine the security of the communications between robots, with a focus on preventing the disruption of emergent behaviour.
1.1 Stigmery and local sensing

We will rely heavily on peer-to-peer communications to secure emergent behaviour, but we will firstly discuss these two communications methods peculiar to swarm robotics.

Stigmery refers to the communication of robots via the environment. Robots modify the environment which in turn affects the behaviour of other robots. These messages left in the environment may “fade” over time and eventually disappear. This is a feature seldom discussed in other types of network. However, there are parallels in some applications in other networks. For example, one could consider the “environment” as a shared memory space such as a bulletin board. This is used commonly in voting schemes (for example, [1]). “Fading” messages can be modelled by using time-stamps or a time-discount function. For instance, in [11], a time-discount function is used to “fade” a reputation. All these have to be achieved while guaranteeing the authenticity or integrity of the messages. Much of this have been discussed in [10].

In some cases robot behaviour is determined not by direct communication with other robots or central control, but by what the robot can observe in its immediate surrounding, such as the configuration or behaviour (such as proximity or velocity of movement) of its neighbours. Some networks have this feature. For example, in VANET, a vehicle may use the speed or location information of other vehicles to determine its own behaviour. Conventionally, when we discuss security, we consider the confidentiality, integrity and authenticity of the information that is exchanged. Here the question of confidentiality does not arise, if the robots are observable by all. However, it is clear at least that the integrity and authenticity of the information obtained by observation is tied in with the legitimacy of the observed robots. It was argued in [10] that the authenticity of a robot cannot be guaranteed unless there is some shared secret within the swarm. We will discuss later how we can obtain some assurance of whether a robot is still legitimate or whether it is malfunctioning or has been subverted.

1.2 Emergent behaviour

The “emergent behaviour” aspect of swarm robotics presents one of the greatest challenge to security. Emergent behaviour arises from local interactions amongst robots and between robots and the environment. (We do not concern ourselves here in how to design individual behaviour in order to achieve the task at hand. We are concerned about the implication on communications security.) While it seems possible to guard against external adversaries using conventional peer-to-peer protocols of key distribution and authentication, guarding against internal adversaries would appear to be a lot trickier. Robots are mobile and compromised robots may spread their influence throughout the swarm, affecting local behaviours which in turn may disrupt emergent behaviour. How would one distinguish between emergent and malicious or malfunctioning erratic behaviour? This is discussed in some depth in [10], arguing that traditional anomaly-based and misuse-based intrusion detection methods do not work. We will consider how secure communications may be used to ameliorate the situation. It would
appear that the policing of malicious/erratic behaviour should not be too tight, so that some erratic behaviour is still allowed in case it is in fact emergent. This means that any solution would have to be flexible so that it can be tuned by the designer of the swarm.

2 Types of swarm and threats

2.1 Classifying the swarm

Before we describe our approach to securing swarm communications to preserve emergent behaviour, we give a rough classification of robotic swarms in terms of the homogeneity of its robots, the interaction between robots, and the interaction with a central control. We will discuss the adversaries in the next section.

1. **Homogeneity of robots.**

   (a) Homogeneous robots.
   It is most generally accepted that the swarm is composed of a large group or a few large groups of homogeneous robots.

   (b) Hierarchical structure.
   It is possible that some swarms may have “special” robots which are given a bigger role. This role may be that of control (the “master” robot may direct other robots to certain location or it may trigger an update of some kind in the neighbouring robots), or it may be that of greater capability (the “master” robot may have more keys so that it could communicate with more robots), or it may be that of a sink - it gathers all the data from its neighbours and reports back to a central control. (This is not an uncommon scenario in wireless sensor networks [14].)

2. **Interaction between robots.**

   We assume that there is no secret or hidden channels of communications, and that an adversary may eavesdrop on or interfere with all communications.

   (a) Direct communication.
   This may be a broadcast, or it may be a peer-to-peer communication. The channel may be a WLAN channel, a Bluetooth channel, an RFID channel, or it may use infrared or audio.

   (b) Stigmergy.
   Robots communicate using stigmergy, that is, via the environment. They leave messages for other robots by modifying the environment. This message will “fade” over time and eventually disappears.
(c) Local sensing.
   A robot may make decisions on how to behave by observing the physical traits of its
   neighbours, such as their proximity, velocity, or configuration.

3. Interaction with a central control.

Again we make the same assumption as above that there is no secret or hidden channels of
communications, and that an adversary may eavesdrop on or interfere with all communications.

(a) No interaction between robots and central control after deployment.
   This is commonly assumed, though it is not clear that it is entirely necessary, desirable, or
   practical.

(b) Central control to robots.
   It is sometimes accepted that there may be a central control entity which can broadcast
   messages to the swarm. These messages may include key management messages or software
   updates.
   This is generally a broadcast channel, though it is possible that in a more hierarchical
   swarm, the messages are relayed via the “higher” robots.

(c) Robots to central control.
   It is also possible that robots of the swarm send messages back to central control. This is
   required in some applications, such as that of search and rescue. This can be done either
   by beaconing, or by relaying messages along to the sink (some designated reporting robot)
   which is responsible for reporting back.

(d) Two-way communications between robots and central control.
   Certain applications may require that robots and central control remain in contact.

2.2 Communications and computation capabilities

Computational abilities of robots vary. We will assume that the robots in this paper have enough
capability to perform basic cryptographic computations. This is not unrealistic: advances in hardware
design and manufacturing has resulted in small embedded devices that are capable of executing public
key cryptography and other complex computational tasks [1]. For example, even the first generation
Raspberry Pi [2] provided a real-world performance roughly equivalent to a 300 MHz Pentium II of
1997–99 [2]. The robots used in [2, 9] are able to communicate at a range of up to 300m. Some swarm
robots are large machines used for precision agriculture and for these onboard computation is not a
great restriction. At the other end of the size spectrum, we have nanorobots. For example, the design

\footnote{https://www.raspberrypi.org/}
\footnote{https://en.wikipedia.org/wiki/Raspberry-Pi}
of an artificial mechanical red blood cell or “respirocyte” envisions that a 104 bit/sec nanocomputer meeting all its computational requirements, which is roughly about 1/50th the capacity of a 1976-vintage Apple II microprocessor-based PC [5].

2.3 Threats to the swarm

We assume that the aim of an adversary is to disrupt the swarm. It may do so by

- discovering secrets and confidential information, or
- impersonating or corrupting or introducing robots to masquerade as robots of the swarm to gather information, plant false information or to change the swarm’s behaviour by its robots’ own behaviour, or
- removing robots or information from the system.

We will not discuss the first threat in detail, since it is discussed in many other work (such as [7, 10]), and also we are more interested in what security can be vouchsafed in publicly observable behaviours. Instead we will focus on the second and third threats, specifically to emergent behaviour. We will consider different classes of adversaries as follows.

1. Insider/outsider. (What they know.)

An adversary who is an outsider has no access to the cryptographic keys and credentials of the robots. This type of adversary and the threats it poses and possible mitigation are discussed in depth in [10]. Most threats can be dealt with by having some sort of secrets known only to the swarm, and both public key cryptography and key predistribution for symmetric key cryptography can be used to prevent any attacks. It is not clear what an outsider in the context of local sensing is - we will assume either that it would simply look different and will be disregarded by robots of the swarm, or it would fail some sort of authentication process.

Insiders, in contrast, have access to keys and credentials. They may be corrupted robots, or they may have keys and credentials manufactured by an adversary. The use of threshold schemes and intrusion detection systems is discussed in [10], and it is found that neither of these solutions are ideal in the context of local sensing and emergent behaviour: apart from the issue of removing such an adversary, it is not easy to distinguish bad behaviour from emergent behaviour.

2. Active/passive. (What they can do.)

A passive adversary eavesdrops and tries to deduce secrets and information. Using encryption would prevent an outsider from doing this but this is ineffective against an insider. In the context
of local sensing, we assume that an insider passive adversary would simply observe and possibly
record the behaviour of the swarm robots.

An active adversary, in addition to eavesdropping, may modify or inject messages, or participate
in the swarm while behaving incorrectly. Again, having some form of public key or symmetric
key cryptography and using this to authenticate messages and robots would thwart an active
outsider. What is more difficult to address is an active insider behaving in a way to subvert the
purpose of the swarm. This is the issue we will deal with in this paper.

3. Local/global. (How many robots do they affect.)

A local adversary can only affect the robots local to it. It has no view of robots not local to it.
It does not know whether they are communicating or what they are communicating, nor does it
know their behaviour or action. A global adversary, on the other hand, sees the behaviour of the
entire swarm: the behaviour of the robots, and the presence of communication between robots.

This class of adversaries appear to be different from the usual kinds of adversaries that are
discussed in the literature [1] and [2] and seems quite pertinent to swarm security. Given that
emergent behaviour arises out of local interactions, it may be that a global adversary could
coordinate disruptions at different localities to effect a disruption of emergent behaviour more
efficiently. Hence, as a defence, we would like to ensure that local behaviours are not disrupted.

Since outsiders can mostly be dealt with using appropriate cryptographic mechanisms we will focus
here on insider adversaries. Passive insiders would appear to be hard to identify, and we will consider
what can be done when such adversaries are active. In dealing with local disruptions we hope to
prevent them propagating to disrupt global emergent behaviour.

2.4 Some examples

We will consider some examples to illustrate the usefulness of this classification of types of swarms
and threats to the swarm.

Suppose we only expect adversaries who are passive outsiders, and suppose our swarm consists of
homogeneous robots (1(a) of Section 2.1) with only direct communication between the robots (2(a)).
Suppose there is no interaction with central control (3(a)). One simple solution could be to equip each
robot with a single encryption key, to prevent eavesdropping.

Suppose however that the adversaries are active outsiders who might attempt to manipulate messages,
we could give each robot, in addition, a single signing key and its certificate, issued by central control
prior to deployment. These keys may not expire during the event or the action time.

Another possibility is to deploy our robots with random key predistribution [3]. There is a fixed
probability, decided upon before deployment, that two robots can authenticate and communicate with
each other. Such a solution is effective against active outsider adversaries. However, this does not allow adaptation if the environment should change. One may ask whether changing one of the conditions might allow more flexibility. Indeed, if a broadcast channel from central control to the robots should be made available (3(b)) then one could deploy the broadcast enhanced key predistribution scheme proposed in [8]:

1. Key pre-distribution: each robot is given a set of underlying keys prior to deployment. These keys are used only for the encryption and decryption of temporal keys.

2. Periodic broadcast from control to robots after deployment: send temporal keys for use in communication. Temporal keys are encrypted using underlying keys so that a robot learns a temporal key only if the temporal key is encrypted by an underlying key known to the robot. The distribution of temporal keys can be adjusted according to the desired connectivity and resilience at particular times.

3. Robots discover common keys by broadcasting identifiers of temporal keys.

In addition, if we choose the underlying key pre-distribution scheme carefully, we would be able to revoke a robot if it is known to be malfunctioning or captured. This gives the swarm some resilience against an active insider adversary. This example was discussed in the “Further Research” section of [10].

It would be interesting to study solutions to specific types of swarm and what might be adapted if certain conditions are tightened or relaxed.

# 3 A scheme to protect emergent behaviour

## 3.1 Overview

Suppose that our robots are capable of public key cryptography, and are given individual public and private keys. If the public keys are signed by the central control, this guards against external adversaries even though it is not secure against internal adversaries. Given that we can now authenticate a robot by its public key, how do we know if a robot is not malfunctioning or corrupt? If we know a robot is malfunctioning or corrupt it can be revoked. There are many solutions to the revocation problem, including revocation lists. However, how to decide whether a signer or a signing key should be put into the revocation list is often the trickier problem. In this work, we try to give one solution for how to identify and revoke a bad robot.

However, this on its own may not be sufficient. If a malicious or malfunctioning robot is not filtered out by the step above it could still behave badly to affect its neighbours’ behaviour. What we can
do about this is to take some sort of consensus from neighbours. For example, if a robot observes a few neighbours, and one of them behaves in a different way from the others, the robot could make a decision to follow the majority with some probability and follow the minority with some other probability. What these probabilities are would be up to the engineer of the swarm. Alternatively, a robot could consider another robot more trustworthy if it has encountered that robot regularly in the past, and consider a robot less trustworthy if it has not encountered it before, or only has a report of it from some other robot. Our goal is how to ensure that the information gathered by a robot is trustworthy.

By identifying and revoking bad robots with high probability, and preventing bad robots from having too much influence on local behaviour, we can maximise the chances of emergent behaviour.

We assume that robots are capable of public key cryptography and that they are equipped with a hardware clock synchronised before deployment. Clocks may drift and we allow some margin of error. We assume there is a central control which installs all the necessarily credentials and algorithms before deployment and has no more contact with the robots after deployment. We also assume the presence of an adversary who is an active insider with a global view, but we assume that the fraction of corrupt and malfunctioning robots is small.

To start with we assume that robots are deployed in one single area such that within some time interval $\Delta$ all robots would have exchanged information with some other robot. We consider a robot suspicious - they might be malfunctioning or they might have been tampered with - if they make false reports or if they are taken out of the systems for a certain number of time intervals. We aim to identify this behaviour: when two robots meet they make a record of the encounter, and exchange their history - here this means a record of what robots they have met in a specified number of past time intervals. We will see that this simple mechanism allows us to achieve our goal with high probability.

### 3.2 The scheme

We assume that central control has a signature scheme $SS(\text{sig}_{CC}, \text{ver}_{CC})$. Central control is assumed to be trusted. To set up a task for the swarm, it installs all the necessarily credentials and algorithms in the robots before deployment and has no more contact with them afterwards until the task is completed.

There are $N$ robots $R_1, \ldots, R_N$. Each robot $R_i$ has:

- a signature scheme $SS(\text{sig}_i, \text{ver}_i)$, and a certificate $\text{cert}_i$ from central control (so each robot has a unique verifiable identity associated with $(\text{ver}_i, \text{cert}_i)$);
- the signature verification algorithm of the central control $\text{ver}_{CC}$;
- a clock that is capable of measuring time intervals;
• a hash function $h$.

We assume that time is divided into intervals, and the first time interval is $t = 1$. Each robot $R_i$ maintains a signed list for time interval $t$, $\text{Hist}^t_i$, and we set $\text{Hist}^0_i = \emptyset$. Within a time interval $t$ a robot $R_i$ may meet other robots. When it does it records the encounter in $\text{Hist}^t_i$: if $R_i$ meets $R_j$ they exchange their history from the previous time interval, so $R_i$ gives $R_j$ the signed list $\text{Hist}^{t-1}_i$, and $R_j$ gives $R_i$ the signed list $\text{Hist}^{t-1}_j$. They also exchange their authenticated verification algorithms $(\text{ver}_i, \text{cert}_i)$ and $(\text{ver}_j, \text{cert}_j)$. Robot $R_i$ then checks the validity of the signed list $\text{Hist}^{t-1}_j$ it receives by verifying the signature on it. Similarly $R_j$ checks $\text{Hist}^{t-1}_i$. We assume that a time interval is short enough that a robot can only exchange history with its immediate neighbours once.

At the end of time interval $t$, $R_i$ constructs an event list $E^t_i$:

$$E^t_i = \left\{(R_{i1}, \text{Hist}^{t-1}_{i1}), (R_{i2}, \text{Hist}^{t-1}_{i2}), \ldots, (R_{ik}, \text{Hist}^{t-1}_{ik})\right\},$$

if $R_i$ encountered robots $R_{i1}, R_{i2}, \ldots, R_{ik}$ in time interval $t$, or $E^t_i = \emptyset$ if it did not encounter any other robots. If any $\text{Hist}^{t-1}_{ij}$ is missing or does not verify, then the entry $(R_{ij}, \text{Hist}^{t-1}_{ij})$ is omitted. A new history list is also constructed:

$$\text{Hist}^t_i = \left\{E^t_i, t, \text{Hist}^{t-1}_i, \text{sig}_i(h(E^t_i, t, \text{Hist}^{t-1}_i))\right\}.$$

So each robot constructs a chain of events. At the end of each time interval $t \geq 1$ a robot constructs a link of the chain $(E^t_i, \text{Hist}^t_i)$, where $E^t_i$ contains information about events that happened in the time interval $t$, and $\text{Hist}^t_i$ links $E^t_i$ to events that happened in previous time intervals described in $\text{Hist}^{t-1}_i$. In this way $R_i$ has a record of all the robots it has met as well as the robots that these robots claimed to have met. An encounter between two robots $R_i, R_j$ in time interval $t$ is accepted if $R_i$ has $\text{Hist}^{t-1}_j$ and $R_j$ has $\text{Hist}^{t-1}_i$.

Based on this record, $R_i$ can analyses the behaviour of each robot to discover a “bad” robot, such as a robot that makes false reports or that disappears for too long. The details of the analysis are given in the next subsection. After analysis, $R_i$ can put bad robots into its local revocation list. The policy on what kind of behaviour a robot has that leads to revocation would be decided by the engineer of the swarm and this is out scope of this paper.

After that task of the swarm finishes, the history lists of each robot will be collected by the central control and further analysis will be made. The whole swarm system can benefit from the above information collection during the job. As this procedure is straightforward, we do not discuss it further in the paper.

(We note that robots in collusion may either simply share each other’s private keys, or create lists in advance and distribute them on each other’s behalf. Hence a challenge/response protocol may not
add more security. However, while they can always vouch for each other, they can’t pretend to have met honest robots, because these lists have to be signed too.

4 Analysis of the above scheme

We model the activities of the $N$ robots in any time interval $t$ as vertices in a binomial random graph $G_t = G_t(N, p)$, with an edge between vertices if the corresponding robots have exchanged information within that interval, and this happens with probability $p$. We write $E(G_t)$ for the set of edges of $G_t$, and we write $N_{G_t}(R)$ for the set of vertices that are neighbours of $R$ in $G_t$, that is, the set of vertices that share an edge with $R$ in $G_t$. This gives all the robots that $R$ meets in time interval $t$. The expected mean degree of a vertex is $Np$, and this gives the expected number of robots a robot will meet in one interval (see [6]).

Example 4.1 (Numerical example) In [2] an experiment involving exploration using robots used $N = 25$ robots in a $1\text{km}^2$ area, which gives robot density of about 1 in $40\ 000 \text{m}^2$. If we consider the area as a $1\text{km} \times 1\text{km}$ square, subdivided into $200\text{m} \times 200\text{m}$ grids, with a robot in the centre of each grid, then since the robots were able to communicate at over 300m, this allows a robot to see the 8 robots in the neighbouring grids, out of 24 other robots, giving $p = 0.33$. At the upper end of the experiment, $N = 48$ robots were deployed in a larger area, giving $p = 0.17$ using the same assumptions. We will run a rudimentary MATHEMATICA program with random graphs of these parameters to confirm our theoretical analysis.

4.1 Correctness

We first discuss correctness of a swarm system and claim that a swarm system described in Subsection 3.2 holds two correctness properties, namely System Correctness and Local Correctness.

4.1.1 System Correctness

Theorem 4.2 (System Correctness) Under the assumption that every robot of the swarm is honest, a full collection of all robots’ individual history lists will be a true record of the communications among them.

Proof: This property can be argued straightforwardly. Let $N$ be the total number of robots in the swarm and $T$ be the total number of intervals during a task. Following the scheme description of Subsection 3.2 a history list made by robot $R_i$, $i \in \{1, 2, ..., N\}$ is:

$$
\text{Hist}_i^T = \left\{ E_i^T, T, \text{Hist}_i^{T-1}, \text{sig}_i(h(E_i^T, T, \text{Hist}_i^{T-1})) \right\},
$$
and a full collection of all robots’ individual history lists are

\[ \text{Hist}^T = \text{Hist}^T_1 \land \text{Hist}^T_2 \land \ldots \land \text{Hist}^T_N. \]

Because all robots are honest, every event where two robots, say \( R_i \) and \( R_j \), met each other must be recorded in \( \text{Hist}^T_i \) and \( \text{Hist}^T_j \), and each robot will correctly report every meeting that it was involved in. More specifically, if there is a record of \( R_i \) meeting \( R_j \) in interval \( t \) then \( R_i \) did indeed meet \( R_j \) in interval \( t \) and there is a record of \( R_j \) meeting \( R_i \), and vice versa: if \( R_i \) met \( R_j \) in interval \( t \) then there is a record in \( R_i \) (and \( R_j \))’s history. Records must be paired. There is neither any forged reports nor any missing reports. Therefore, \( \text{Hist}^T \) must be a true record of the communications among all the robots of the swarm. So the theorem follows.

4.1.2 Local Correctness

**Theorem 4.3 (Local Correctness)** Under the assumption that every robot of the swarm is honest and that the swarm communication patterns follow a binomial random graph as described at the start of Section 4, after a certain number of time intervals, the probability that each robot’s local history list will cover more or less the same information of any other individual robot’s record, which is a true record of the communications of the whole swarm during these intervals, is significantly high.

**Proof:** Because all the \( N \) robots are honest and their communication patterns follow a random graph \( G(N,p) \) in every interval, the probability that a robot does not meet another robot in \( \Delta \) intervals is \((1 - p)^{(1 + \frac{(\Delta - 1)N^2}{2})\Delta}\), as calculated in Section 4.4. Since \( 1 - p < 1 \), this probability tends to 0 quadratically with increasing \( \Delta \). Hence the probability that a robot has met all other robots after some \( \Delta \) intervals is reasonably high. We can therefore assume that in every \( \Delta \) continuous intervals, a robot will meet every other robot and exchanged its history list with them at least once. The robot then obtains a full collection of all robots’ individual history lists up to the point of time when it is the beginning of these \( \Delta \) intervals. Based on the discussion of system correction above, the robot should have a true record of the communications among all the robots of the swarm before that time. So the theorem follows.

To illustrate this property, let us recall the numerical example in Example 4.1 with \( N = 25 \), \( p = 0.33 \). In every \( \Delta = 3 \) continuous intervals, the probability that a robot does not meet another robot is \((2/3)^{28} \approx 1.49 \times 10^{-5}\). Hence the probability that a robot has met all other robots in 3 intervals in either case is very high \((\approx 0.99998)\) and the probability of having a true record of the communications of the whole swarm during these 3 intervals is high.

\[\square\]
4.2 Threat Model

A robot is bad if it makes false reports, or if it disappears for too long. We will analyse these two bad behaviours respectively in the following two subsections. We assume that a bad robot has the same capability as a good robot.

Our goals are to prevent bad robots from having too much influence on local behaviour during the execution of the task, and to identify bad robots. The first goal, as discussed in Section 3.1, can be achieve by identifying as many bad robots as we can during and after the execution of the task. To do this, we identify two suspicious behaviours:

1. If a robot disappears for too many intervals we suspect it of either malfunctioning or having been captured and subverted. We aim to identify such a robot.
2. A robot may make false reports in order to make a robot distrust the good robots around it or to disguise its own or its fellow bad robots’ status. We aim to prevent framing of good robots and to detect collusion.

We assume that a small fraction $\alpha$ of robots are corrupt or are malfunctioning (“bad”), $0 < \alpha \ll 1$.

4.3 Making false reports

Suppose $R_i$ is a bad robot and wants to make a false report about an honest robot $R_j$. There are two types of false reports:

1. $R_i$ claims to have met $R_j$ in time interval $t$ even though it has not.
   To claim this $R_i$ must prove that it has $\text{Hist}_i^{t-1}$ which contains $R_j$’s signature. This cannot be done if the signature scheme is secure.
   Also, if $R_j$ does not have $\text{Hist}_i^{t-1}$ in its own $\text{Hist}_j^t$, a record of the fake meeting between $R_i$ and $R_j$ at time interval $t$ will not be accepted by other honest robots later. To make the exchange record be paired and be accepted by others, $R_i$ also needs to let $R_j$ record $\text{Hist}_j^{t-1}$. This cannot be done by $R_i$ itself.
   Another possibility is that at time interval $t' > t$, some other, possibly corrupt, robot $R_k$ passes a legitimate $\text{Hist}_j^{t-1}$ to $R_i$ and a legitimate $\text{Hist}_i^{t-1}$ to $R_j$, but $R_i$ cannot incorporate $\text{Hist}_j^{t-1}$ into its history without modifying the hash chains, and this can be detected by other robots. Any attempt of $R_k$ to pass $\text{Hist}_i^{t-1}$ to $R_j$ at time $t$ will also be detected by $R_j$ when the signature is verified.
2. $R_i$ claims not to have met $R_j$ in time interval $t$ even though it has, in order to give the impression that $R_j$ is a suspicious robot and has disappeared for too long.

In this case $R_i$ simply does not record $R_j$’s chain $\text{Hist}_j^{t-1}$. If $R_i$ is the only robot to meet $R_j$ then it can try to convince other robots that $R_j$ has disappeared. However, if the expected mean degree $Np$ of the random graph $G(N, p)$ is greater than 1 then it is likely that $R_j$ would have met $Np-1$ other robots, and if the number of rogue robots is small then it is likely that this sort of attack would not have any significant effect on $R_j$.

It is possible also that $R_i$ refuses to give $\text{Hist}_i^{t-1}$ to $R_j$, but this would make it seem more likely that $R_i$ is suspicious, and would be the correct outcome.

Indeed, if we assume that the proportion of bad robots $\alpha$ is small, and we consider a record of an encounter trustworthy if at least $(1-\alpha)Np$ of them are paired, then this sort of attack would not succeed.

**Numerical example:** We consider the case when a proportion $\alpha$ of the robots do not record encounters with other robots. Our Mathematica program showed that, for $N = 25$, $p = 0.33$, $\Delta = 3$, even with up to $\alpha = 1/3$ of the robots corrupt, there is no effect on other robots being seen or reported seen. For $N = 48$, $p = 0.17$, all robots were reported seen in $\Delta = 4$ intervals with again $\alpha = 1/3$ of the robots corrupt ($\Delta = 3$ if there are no corrupt robots). This confirms the theory that as long as the mean degree of the random graphs is high enough, bad robots are not able to “frame” other robots.

In the situation where both $R_i$ and $R_j$ are corrupt, we may assume that they are in collusion, and share each other’s private keys. In this case $R_i$ may construct a $\text{Hist}_j^{t-1}$ and claim that $R_j$ has been seen. Indeed, robots in collusion may vouch for each other, even if they cannot fake uncorrupted robots’ lists.

However, we can then calculate the probability of a robot meeting another robot - $R_i$ meets $R_j$ with probability $p$, so $R_i$ meets $R_j$ in all $\Delta$ time intervals with probability $p^\Delta$, and if we see that $R_i$ meets $R_j$ too often then we may assume they are in collusion.

**Numerical example:** With $N = 25$, $p = 0.33$, and $\Delta = 3$ intervals, this gives the probability of $p^\Delta \approx 0.036$ for a pair of robots to meet in 3 time intervals. With $N = 48$, $p = 0.17$, the probability is approximately 0.005. Hence the probability of a pair of robots meeting in many intervals is low, and if a pair of robots meet too often we may treat them as suspicious.

### 4.4 Disappearance

A robot also becomes suspicious if it disappears for too many intervals. For example, let us consider the probability that a robot $R$ gets a report of another robot $R'$ within, say, $\Delta = 3$ time intervals. This does not happen if
• R does not meet R' at t = 3, 2 and 1, that is, (R, R') \notin E(G_3) and (R, R') \notin E(G_2) and (R, R') \notin E(G_1). This has probability \((1 - p)^3\).

• None of the robots R meets at t = 3 has met R' at t = 2 or t = 1, that is, (R'', R') \notin E(G_2) \cup E(G_1) for all R'' \in N_{G_3}(R). This has probability \((1 - p)^2|N_{G_3}(R)|\).

• None of the robots R meets at t = 2 has met R' at t = 1, that is, (R'', R') \notin E(G_1) for all R'' \in N_{G_2}(R). This has probability \((1 - p)|N_{G_2}(R)|\).

Hence the probability that R has a report of R' within \(\Delta = 3\) intervals is \(1 - (1 - p)^3 + 2|N_{G_2}(R)| + |N_{G_1}(R)| = 1 - (1 - p)^3 + 3Np\) since R is expected to have degree \(Np\). If this is sufficiently high that means that a legitimate robot would have been seen by some other robot in two time intervals with high probability, and therefore a robot not seen in three time intervals may be regarded as suspicious and blacklisted.

In general the probability that R has a report of R' within \(\Delta\) intervals is \(1 - (1 - p)^\Delta + ((\Delta - 1) + (\Delta - 2) + \cdots + 1)Np = 1 - (1 - p)^{1 + (\frac{\Delta - 1}{2})Np}\Delta\), which is increasing with \(\Delta\).

**Numerical example:** Using the same experiment as described above in Section 4.3, with \(N = 25\) and \(p = 0.33\) this gives the probability of 0.99998 for a robot having a report of another robot within 3 time intervals. With \(N = 48\) and \(p = 0.17\) the probability is 0.99403 within 3 intervals. Our Mathematica program confirms this.

### 5 Conclusion and further work

By modelling a swarm using random graphs, and ensuring that events are recorded securely in a hash chain, we can allow robots to identify “bad” robots with a high probability, while ensuring that “good” robots were not adversely affected. This goes some way towards protecting emergent behaviour, by limiting the influence of bad robots locally.

Note that here we have restricted ourselves to modelling the swarm as a binomial random graphs. In the situation where we desire the swarm to aggregate or disperse we may consider sequences of graphs where the number of edges increases or decreases through time, or where the likelihood of having an edge between two vertices increases or decreases depending on whether there is an edge previously. It will also be interesting to examine the properties of the swarm where bad robots are modified to have additional capability, such as broadcasting ability, or a higher communication range which may be modelled as a subset of vertices with higher connectivity.

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