Angular dependence of synchrotron radiation intensity

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Abstract

The detailed analysis of angular dependence of the synchrotron radiation (SR) is presented. In particular, we analyze the angular dependence of the integral SR-intensity and peculiarities of the angular dependence of the first harmonics SR. Studying spectral SR-intensities, we have discovered their unexpected angular behavior, completely different from that of the integral SR-intensity. Namely, for any given synchrotron frequency, maxima of the spectral SR-intensities recede from the orbit plane with increasing particle energy. Thus, in contrast with the integral SR-intensity, the spectral ones have the tendency to deconcentrate themselves on the orbit plane.

1 Introduction

At present the theory of synchrotron radiation (SR) is well developed and its predictions are in good agreement with experiment [1, 2, 3, 4]. We recall that the SR is created by charged particles, which are moving with velocities $\nu$ along circles of radius $R$ in an uniform magnetic field $H$,

$$R = \frac{\beta E}{eH} = \frac{m_0 c^2}{e H} \sqrt{\gamma^2 - 1} = \frac{\nu c}{e} , \quad \gamma = (1 - \beta^2)^{-1/2} = \frac{E}{m_0 c^2} \gg 1 .$$

(1)

Here $E$ is the particle energy, $e$ is the charge, and $m_0$ the rest mass. The radiation frequencies $\omega_\nu = \nu \omega_0$, $\nu = 1, 2, ...$, are multiples of the synchrotron frequency $\omega_0 = ceH/E$. The spectral SR-intensity (SR-intensity for a fixed radiation frequency) has maximum for harmonics with $\nu \sim \gamma^3$. Two limiting cases, the non-relativistic ($\beta \ll 1$, $E \simeq m_0 c^2$) and the relativistic limits ($\beta \sim$...
1, $E \gg m_0c^2$), are of particular interest. In the non-relativistic case, only the first harmonic $\omega_1 = \omega$ is effectively emitted. The SR-intensity has a maximum in the direction of the magnetic field. In the relativistic case, the integral SR-intensity (spectral SR-intensity summed over the spectrum) is concentrated in the orbit plane within a small angle $\Delta \theta \sim 1/\gamma \ll 1$. Thus, as the electron energy increases, the integral SR-intensity tends to be concentrated in the orbit plane. Any polarization component of the integral SR-intensity has the same behavior. These results were first derived in the framework of classical theory. Consideration in the framework of quantum theory does not change essentially results of the classical analysis, since quantum corrections are small \[1, 2, 3, 4\]. However, one ought to say that the analysis of angular dependence of the spectral and the integral SR-intensities was not done before in detail. Recently this work was done by us, and in the present article we present results of such an analysis. In Sect. II we analyze in detail angular dependence of the integral SR-intensity. In Sect. III. we study peculiarities of the angular dependence of the first harmonics SR. Studying spectral SR- intensities (see Sect.IV), we have discovered their unexpected angular behavior, completely different from that of the integral SR-intensity. Namely, one can see that for any given synchrotron frequency, maxima of the spectral SR-intensities recede from the orbit plane with increasing particle energy. There exist limiting angles (at $\beta \to 1$) for the maxima, which depend on the synchrotron frequency. Thus, in contrast with the integral SR-intensity, the spectral ones have the tendency to deconcentrate themselves on the orbit plane. The analysis is done in the framework of classical theory, but as was already mentioned above, quantum corrections cannot change the results essentially.

2 Angular dependence of integral SR- intensity

In the SR theory one introduces polarization components $W_i$, $i = 0, \pm 1, 2, 3$ of the integral SR-intensity \[1, 2, 3, 4\]. Here $W_{\pm 1}$ are the integral SR-intensities of the right (+1) and the left (−1) circular polarization components respectively, whereas $W_2$ and $W_3$ are the so called "$\sigma$" and "$\pi$" linear polarization components. The total integral SR-intensity $W_0$ is defined as $W_0 = W_1 + W_{-1} = W_\sigma + W_\pi$. In the framework of the classical theory of SR one can find:

$$W_i = V_0 \Phi_i(\beta), \quad V_0 = \frac{e \epsilon^2 \beta^4}{R^2} = \frac{e^4 H^2 \beta^2 (1 - \beta^2)}{m_0^2 c^3},$$

$$\Phi_i(\beta) = \int_0^\pi F_i(\beta, \theta) \sin \theta d\theta, \quad F_i(\beta, \theta) = \sum_{\nu=1}^\infty f_i(\nu, \beta; \theta),$$

$$f_0(\nu, \beta; \theta) = f_{-1}(\nu, \beta; \theta) + f_1(\nu, \beta; \theta) = f_2(\nu, \beta; \theta) + f_3(\nu, \beta; \theta). \quad (2)$$

Here $\theta$ is the angle between the $z$-axis and the radiation direction. The sum over $\nu$ is just the sum over the spectrum, such that the expressions inside the
sum represent spectral distributions. The functions \( f_i(\nu, \beta; \theta) \) have the form:

\[
\begin{align*}
  f_{+1}(\nu, \beta; \theta) &= \frac{\nu^2}{2} \left[ J'_\nu(z) + \frac{\cos \theta}{\beta \sin \theta} J_\nu(z) \right]^2, \quad z = \nu \beta \sin \theta, \\
  f_2(\nu, \beta; \theta) &= \nu^2 J'_\nu(z), \quad f_3(\nu, \beta; \theta) = \frac{\nu^2 \cos^2 \theta}{\beta^2 \sin^2 \theta} J'_\nu(z).
\end{align*}
\]  

(3)

Here \( J_\nu(x) \) are Bessel functions of integer indices. The following simple properties hold true:

\[
f_k(\nu, \beta; \theta) = f_k(\nu, \beta; \pi - \theta), \quad k = 0, 2, 3; \quad f_{-1}(\nu, \beta; \theta) = f_1(\nu, \beta; \pi - \theta).
\]  

(4)

Thus, it is enough to study the functions \( f_k(\nu, \beta; \theta) \), \( k = 0, 2, 3 \), at the interval \( 0 \leq \theta \leq \pi/2 \) only, and between the functions \( f_{\pm 1} \) it is enough to study \( f_1 \) only.

Exact analytic expressions for the functions \( F_k(\beta, \theta) \), \( k = 0, 2, 3 \) were already known [1, 3]:

\[
\begin{align*}
  F_2(\beta, \theta) &= \frac{7 - 3 \varepsilon}{16 \gamma^2}, \quad \varepsilon = 1 - \beta^2 \sin^2 \theta, \quad \frac{1}{\gamma^2} \leq \varepsilon < 1, \\
  F_3(\beta, \theta) &= \frac{(\gamma^2 - 1)(5 - \varepsilon)}{16(\gamma^2 - 1)\varepsilon^{7/2}}, \quad F_0(\beta, \theta) = \frac{(3 - 4 \gamma^2)\varepsilon^2 + 6(2 \gamma^2 - 1)\varepsilon - 5}{16(\gamma^2 - 1)\varepsilon^{7/2}}.
\end{align*}
\]  

(5)

Expressions for the functions \( F_{\pm 1} \) can be found in the form:

\[
F_{\pm 1}(\beta, \theta) = \frac{1}{2} F_0(\beta, \theta) \pm \Psi(\beta \sin \theta) \cos \theta, \quad \Psi(x) = \frac{1}{2x} \frac{d}{dx} \sum_{\nu=1}^{\infty} \nu J_\nu^2(\nu x).
\]  

(6)

One can find that for any fixed \( \beta \) all the functions \( F_i(\beta, \theta) \) have an extremum at \( \theta = 0 \). Moreover, the extremal values of these functions do not depend on \( \beta \),

\[
F_{-1}(\beta, 0) = 0, \quad 2F_0(\beta, 0) = 2F_1(\beta, 0) = 4F_2(\beta, 0) = 4F_3(\beta, 0) = 1.
\]  

(7)

The point \( \theta = \pi/2 \) provides an extremum for the functions \( F_k \), \( k = 0, 2, 3 \) only. Here we have:

\[
F_0(\beta, \pi/2) = F_2(\beta, \pi/2) = 2F_3(\beta, \pi/2) = \frac{1}{16} \gamma^3(7\gamma^2 - 3), \quad F_3(\beta, \pi/2) = 0.
\]  

(8)

Therefore, for \( F_2 \) the point \( \theta = \pi/2 \) is an absolute minimum. For any fixed \( \beta \) the function \( F_2(\beta, \theta) \) is a monotonically increasing function of \( \theta \) on the interval \( 0 \leq \theta \leq \pi/2 \). Thus, \( \theta = 0 \) is an absolute minimum and \( \theta = \pi/2 \) is an absolute maximum of this function. The maximum of the function \( F_2 \) increases as \( E^5 \) with increasing particle energy \( E \).

For \( \gamma \leq \gamma_0^{(1)} \), \( (\beta \leq \beta_0^{(1)}) \),

\[
\gamma_0^{(1)} = \sqrt{7/6} \approx 1.0801, \quad \beta_0^{(1)} = 1/\sqrt{7} \approx 0.378,
\]  

(9)

\( F_0 \) and \( F_1 \) are monotonically decreasing functions of \( \theta \) (\( F_0 \) on the interval \( 0 \leq \theta \leq \pi/2 \) and \( F_1 \) on the interval \( 0 \leq \theta \leq \pi \)). Thus, at \( \theta = 0 \) these functions have
an absolute maximum. The functions $F_0$ and $F_1$ have their absolute minima at $\theta = \pi/2$ and $\theta = \pi$ respectively. Besides, $F_1(\beta, \pi) = 0$. For $\gamma_0^{(1)} < \gamma < \gamma_0^{(2)}$, $(\beta_0^{(1)} < \beta < \beta_0^{(2)})$,

$$
\gamma_0^{(2)} = \frac{\sqrt{3} + 3\sqrt{2}}{5} \approx 1.1949, \quad \beta_0^{(2)} = \frac{2}{3}(\sqrt{6} - 2) \approx 0.5474, \quad (10)
$$

the points $\theta = 0$, $\pi/2$ are minima for $F_0$, and the point $\theta = \theta_0(\beta)$,

$$
\sin^2 \theta_0(\beta) = \frac{6\gamma^2(1 - 3\gamma^2) + 2\gamma^2\sqrt{15(15\gamma^4 - 22\gamma^2 + 9)}}{3(4\gamma^2 - 3)(\gamma^2 - 1)}, \quad 0 < \theta_0(\beta) < \pi/2,
$$

provides a maximum for $F_0$. For $\gamma_0^{(2)} < \gamma$, $(\beta_0^{(2)} < \beta < 1)$, the function $F_0$ has an absolute maximum at the point $\theta = \pi/2$.

Denoting via $\theta_0^{(m)}(\beta)$ all the maximum points of $F_0$, we may write:

$$
\theta_0^{(m)}(\beta) = \begin{cases} 
0, & \beta \leq \beta_0^{(1)} \\
\theta_0(\beta), & \beta_0^{(1)} < \beta < \beta_0^{(2)} \\
\pi/2, & \beta_0^{(2)} \leq \beta < 1
\end{cases}. \quad (12)
$$

The plot of the function $\theta_0^{(m)}(\beta)$ see below:

For any given $\beta \in (\beta_0^{(1)}, 1)$, the function $F_1$ has its maximum at the point $\theta = \theta_1(\beta)$, $0 < \theta_1(\beta) < \pi/2$. Denoting via $\theta_1^{(m)}(\beta)$ all the maximum points of $F_1$, we may write:

$$
\theta_1^{(m)}(\beta) = \begin{cases} 
0, & \beta \leq \beta_0^{(1)} \\
\theta_1(\beta), & \beta_0^{(1)} < \beta < 1
\end{cases}. \quad (13)
$$

At the moment, there is no analytical expression for $\theta_1(\beta)$ similar to (11) for $\theta_0(\beta)$. However, one can see that the function $\theta_1(\beta)$ is a monotonically increasing function of $\beta \in [\beta_0^{(1)}, 1]$. For $\beta \to 1$ there is an asymptotic form

$$
\theta_1(\beta) \approx \pi/2 - a_1/\gamma, \quad (14)
$$

Figure 1: The function $\theta_0^{(m)}(\beta)$. 

4
where $\alpha_1 \approx 0.2672$ is a root of the equation (see [1])
\[ 5\pi a_1 (5 + 12a_1^2) \sqrt{3} + 64(5a_1^2 - 1) \sqrt{1 + a_1^2} = 0. \] (15)

The plot of the function $\theta^{(m)}_1(\beta)$ see below:

For $\beta \leq \beta_3$ ($\gamma \leq \gamma_3$),
\[ \beta_3 = \frac{2}{\sqrt{15}} \approx 0.5164, \gamma_3 = \sqrt{\frac{15}{11}} \approx 1.1678, \] (16)

$F_3$ is a monotonically decreasing function on the interval $0 \leq \theta \leq \pi/2$. The point $\theta = 0$ provides the absolute maximum for this function. For $1 > \beta > \beta_3$, ($\gamma > \gamma_3$), the points $\theta = 0$ and $\theta = \theta_3(\beta)$ provide the minimum and the maximum respectively for $F_3$,
\[ \sin^2 \theta_3(\beta) = \frac{\sqrt{5(125\gamma^4 - 34\gamma^2 + 5) - 19\gamma^2 - 5}}{6(\gamma^2 - 1)}, \quad 0 < \theta_3(\beta) < \pi/2. \] (17)

Denoting via $\theta_3^{(m)}(\beta)$ all the maximum points of $F_3$, we may write:
\[ \theta_3^{(m)}(\beta) = \begin{cases} 0, & \beta \leq \beta_3 \\ \theta_3(\beta), & \beta_3 < \beta < 1 \end{cases}. \] (18)

For $\beta \rightarrow 1$ the following asymptotic expression holds true:
\[ \theta_3^{(m)} \approx \pi/2 - \frac{1}{\gamma} \sqrt{\frac{2}{5}}. \] (19)

The plot of the function $\theta_3^{(m)}(\beta)$ see below:

3 Angular dependence of spectral SR-intensity

3.1 First harmonic radiation

The angular distribution of SR from the first harmonic ($\nu = 1$) is distinctly different from that of the higher harmonics ($\nu \geq 2$). Previously it was known [1]
that: a) The first harmonic alone contributes essentially to the radiation in the directions \( \theta = 0, \pi \). b) In the nonrelativistic case (\( \beta \sim 0 \)), the radiation is maximal exactly in these directions.

Let us consider Eqs. (3) for the first harmonic,

\[
f_{\mp 1}(1, \beta; \theta) = \frac{1}{2} \left[ J'_1(z) \mp \frac{\cos \theta}{x^2} J_1(z) \right]^2, \quad z = \beta \sin \theta,
\]

\[
f_2(1, \beta; \theta) = J_1^2(z), \quad f_3(1, \beta; \theta) = \frac{\cos^2 \theta}{z^2} J_1^2(z).
\]

In the nonrelativistic case (\( \beta = 0 \)) we get:

\[
f_{\mp 1}(1, 0; \theta) = \frac{1}{8} (1 \mp \cos \theta)^2, \quad f_2(1, 0; \theta) = \frac{1}{4}, \quad f_3(1, 0; \theta) = \frac{\cos^2 \theta}{4}.
\]

Thus, in this case, the radiation components \( W_0, W_2, \) and \( W_3 \) peak at \( \theta = 0, \pi \), whereas \( W_\sigma \) does not depend on \( \theta \) at all.

Analyzing the expressions (20), one can see that the functions \( f_k(1, \beta; \theta) \), \( k = 0, 1, 2, 3 \), peak at \( \theta = 0 \) for any \( \beta \) (including \( \beta \to 1 \)). Thus, the corresponding radiation components \( W_k \) are maximal at \( \theta = 0 \) for any \( \beta \).

Besides, at any fixed \( \theta \neq 0, \pi \), the functions \( f_k(1, \beta; \theta) \), \( k = -1, 0, 1, 2, 3 \), decrease monotonically with increasing \( \beta \). Thus, the radiation from the first harmonic has the tendency to line up in the direction \( \theta = 0, \pi \) with increasing electron energy (\( \beta \to 1 \)). This behavior of the first harmonic radiation is completely opposite to that of the total SR-intensity in the ultrarelativistic case (as was already said, see the previous Section, the latter radiation tends to be concentrated in the orbit plane). All the functions (20) have finite limits as \( \beta \to 1 \). See below the plot of the function (20) at \( \beta = 0, 1 \).

### 3.2 Higher harmonic radiation

To study the angular dependence of higher harmonic (\( \nu > 1 \)) radiation, we have to analyze the angular dependence of the functions \( f_k(\nu, \beta; \theta) \) for \( \nu > 1 \).
First of all, one has to remark that all the functions $f_k(\nu, \beta; \theta)$, $\nu > 1$, vanish at $\theta = 0, \pi$. Thus, they have the mentioned absolute minimum at these points. By virtue of property (4) the functions $f_s(\nu, \beta; \theta)$, $s = 0, 2, 3$, have two symmetric maxima at the points

$$\theta^\nu_s(\beta) = \pi/2 \mp \delta_s(\nu, \beta), \quad s = 0, 2, 3. \quad (22)$$

The functions $f_{\pm 1}(\nu, \beta; \theta)$ have maxima at the points

$$\theta^\nu_{\pm 1}(\beta) = \pi/2 \mp \delta_1(\nu, \beta). \quad (23)$$

All $\delta_k(\nu, \beta)$, $k = 0, 1, 2, 3$ are non-decreasing functions of $\beta$ for any given $\nu$, and for any given $\beta$ they are non-increasing functions of $\nu$. At the same time,

$$0 \leq \delta_k(\nu, \beta) < \pi/2, \quad k = 0, 1, 2, 3, \quad (24)$$

and

$$\lim_{\beta \to 1} \delta_k(\nu, \beta) = \delta_k(\nu, 1) = \delta^\nu_k < \pi/2, \quad k = 0, 1, 2, 3. \quad (25)$$

The quantities $\delta^\nu_k$ are maxima for $\delta_k(\nu, \beta)$ at fixed $k, \nu$.

Thus, for each harmonic ($\nu > 1$) and for each polarization component, the angular distribution of the SR-intensity has its own maximum. All these maxima have the tendency to recede away from the orbit plane with increasing particle energy. Therefore, as in the case $\nu = 1$, the spectral SR-intensities for $\nu > 1$ have the tendency to deconcentration from the orbit plane with increasing particle energy.

Below we study the functions $\delta_k(\nu, \beta)$ in detail.

Let $k = 0, 2$. Then

$$\delta_k(\nu, \beta) = 0, \quad \beta < \beta^\nu_k, \quad k = 0, 2. \quad (26)$$
Here \( \beta_0^\nu \) and \( \beta_2^\nu \) are respectively roots of the transcendental equations

\[
2 J_\nu(\nu \beta) = \beta \left[ \sqrt{\nu^2 (1 - \beta^2)^2 - 4 + \nu (1 - \beta^2)} \right] J'_\nu(\nu \beta),
\]

and

\[
\nu (1 - \beta^2) J_\nu(\nu \beta) - \beta J'_\nu(\nu \beta) = 0.
\]

The condition \( \beta \leq 1 \) implies that \( \beta_0^\nu \) and \( \beta_2^\nu \) are unique. The following inequality holds true

\[
\beta_0^\nu < \beta_2^\nu.
\]

For \( \gamma_k^\nu = \left[ 1 - (\beta_k^\nu)^2 \right]^{-1/2} \), one can find the asymptotic (at \( \nu \gg 1 \)) expressions:

\[
\gamma_0^\nu \approx \left( \frac{\nu}{a_0} \right)^{1/3}, \quad \gamma_2^\nu \approx \left( \frac{\nu}{a_2} \right)^{2/3},
\]

\[
a_0 = 3z_0 \approx 0.7332, \quad a_2 = \left[ \frac{16\pi^3}{\Gamma^6(1/3)\sqrt{3}} \right]^{1/4} \approx 0.9382.
\]

Here \( z_0 \) is a root of the transcendental equation

\[
3z_0 K_{2/3}(z_0) - K_{1/3}(z_0) = 0, \quad z_0 \approx 0.2444,
\]

where \( K_\mu(x) \) are the Macdonald functions.

For \( \beta > \beta_0^\nu \), the function \( \delta_0(\nu, \beta) \) is defined as a solution of the transcendental equation

\[
2 J_\nu(\nu \beta \cos \delta_0) = \left\{ \nu [1 - \beta^2 + (1 + \beta^2) \sin^2 \delta_0] + \sqrt{\nu^2 [1 - \beta^2 + (1 + \beta^2) \sin^2 \delta_0]^2 - 4} \right\} J'_\nu(\nu \beta \cos \delta_0) \beta \cos \delta_0.
\]

(One can see that the equation (27) is a particular case of (31) at \( \delta_0 = 0 \)). Here \( \delta_0(\nu, \beta) \) is a monotonically increasing function of \( \beta \) for each given \( \nu \). The maximum value \( \delta_0^\nu \) of \( \delta_0(\nu, \beta) \) is a solution of the equation (31) for \( \beta = 1 \). There is the asymptotic (at \( \nu \gg 1 \)) expression

\[
\delta_0^\nu \approx \left( \frac{b_0}{\nu} \right)^{1/3}, \quad b_0 = 3p_0 \approx 0.3066,
\]

where \( p_0 \) is a root of the transcendental equation

\[
6p_0 K_{2/3}(p_0) - K_{1/3}(p_0) = 0, \quad p_0 \approx 0.1022.
\]

For \( \beta > \beta_2^\nu \), the function \( \delta_2(\nu, \beta) \) has the form:

\[
\delta_2(\nu, \beta) = \arccos(\beta_2^\nu / \beta), \quad \beta \in (\beta_2^\nu, 1).
\]

Therefore, the maximum value \( \delta_2^\nu \) of \( \delta_2(\nu, \beta) \) at the point \( \beta = 1 \) is:

\[
\delta_2^\nu = \delta_2(\nu, 1) = \arccos(\beta_2^\nu).
\]
According to (30), we have the asymptotic (at $\nu \gg 1$) expression
\[
\delta_2^\nu \approx 1/\gamma_2^\nu.
\] (36)

The functions $\delta_1(\nu, \beta)$ and $\delta_3(\nu, \beta)$ behave similarly to $\delta_0(\nu, \beta)$ and $\delta_2(\nu, \beta)$. Namely, $\delta_1(\nu, \beta)$ and $\delta_3(\nu, \beta)$ are defined as solutions of the transcendental equations
\[
(\nu \sin \delta_1 - 1 - \nu \beta^2 \sin \delta_1 \cos^2 \delta_1)J_\nu(\nu \beta \cos \delta_1)
= J'_\nu(\nu \beta \cos \delta_1) \beta (1 - \nu \sin \delta_1 \sin \delta_1 \cos \delta_1),
\] (37)
and
\[
J'_\nu(\nu \beta \cos \delta_3) \nu \beta \sin^2 \delta_3 \cos \delta_3 = J_\nu(\nu \beta \cos \delta_3),
\] (38)
respectively. For each given $\nu$, these functions are bounded and monotonically increasing functions of $\beta \in [0, 1]$,
\[
\arcsin(1/\nu) = \delta_1(\nu, 0) \leq \delta_1(\nu, \beta) \leq \delta_1(\nu, 1),
\]
\[
\arcsin(1/\sqrt{\nu}) = \delta_3(\nu, 0) \leq \delta_3(\nu, \beta) \leq \delta_3(\nu, 1).
\] (39)

For each given $\beta$, these functions decrease monotonically with increasing $\nu$.

At $\nu \gg 1$ we get the following asymptotic expressions:
\[
\delta_1^\nu \approx \left(\frac{b_1}{\nu}\right)^{1/3}, \quad b_1 = 3p_1 \approx 0.3933, \quad \delta_2^\nu \approx \left(\frac{a_0}{\nu}\right)^{1/3},
\] (40)
where $a_0$ is defined by (30), and $p_1 \approx 0.13114$ is a root of the transcendental equation
\[
(3p_1 - 1)K_{1/3}(p_1) + 3p_1K_{2/3}(p_1) = 0.
\] (41)

The following inequalities hold true:
\[
\delta_3(\nu, \beta) > \delta_1(\nu, \beta) > \delta_0(\nu, \beta) \geq \delta_2(\nu, \beta).
\] (42)

The threshold values $\gamma_0^\nu$, $\gamma_2^\nu$ and the extremal values $\delta_k^\nu$ (in Celsius) for some $\nu$, are given on the Table 1:
Table 1: Threshold values $\gamma^\nu_0$, $\gamma^\nu_2$ and limit values $\delta^\nu_k$ (in degrees)

The Fig. 5 presents the plots of $\delta_k(\nu, \beta)$ ($\delta_k$ are given in Celsius). The Fig. 6 presents the plots of $f_k(\nu, \beta; \theta)$ at $\beta = 1$ and at $\nu = 1 - 5, 10$. 

\[
\begin{array}{cccccccccccc}
\nu & 2 & 3 & 4 & 5 & 6 & 7 & 10 & 15 & 20 & 25 \\
\gamma^\nu_0 & 1.00 & 1.22 & 1.40 & 1.54 & 1.67 & 1.79 & 2.08 & 2.46 & 2.75 & 3.00 \\
\gamma^\nu_2 & 1.59 & 2.10 & 2.55 & 2.97 & 3.36 & 3.73 & 4.75 & 6.25 & 7.59 & 8.82 \\
\delta^\nu_0 & 45.50 & 36.22 & 31.29 & 28.11 & 25.84 & 24.10 & 20.66 & 17.48 & 15.59 & 14.30 \\
\delta^\nu_1 & 45.88 & 36.83 & 32.02 & 28.91 & 26.68 & 24.98 & 21.57 & 18.39 & 16.48 & 15.16 \\
\delta^\nu_2 & 38.84 & 28.44 & 23.06 & 19.67 & 17.30 & 15.54 & 12.14 & 9.20 & 7.57 & 6.51 \\
\delta^\nu_3 & 49.83 & 41.09 & 36.29 & 33.11 & 30.80 & 29.00 & 25.34 & 21.83 & 19.69 & 18.19 \\
\nu & 30 & 35 & 40 & 45 & 50 & 100 & 200 & 300 & 400 & 500 \\
\gamma^\nu_0 & 3.21 & 3.40 & 3.58 & 3.74 & 3.88 & 4.98 & 6.35 & 7.31 & 8.07 & 8.70 \\
\gamma^\nu_2 & 9.98 & 11.07 & 12.10 & 13.10 & 14.06 & 22.38 & 35.58 & 46.66 & 56.54 & 65.63 \\
\delta^\nu_0 & 12.34 & 12.59 & 11.98 & 11.47 & 11.03 & 8.60 & 6.74 & 5.86 & 5.31 & 4.92 \\
\delta^\nu_1 & 14.18 & 13.41 & 12.77 & 12.24 & 11.79 & 9.24 & 7.28 & 6.34 & 5.75 & 5.33 \\
\delta^\nu_2 & 5.75 & 5.18 & 4.74 & 4.38 & 4.08 & 2.56 & 1.61 & 1.23 & 1.01 & 0.87 \\
\delta^\nu_3 & 17.06 & 16.16 & 15.43 & 14.81 & 14.28 & 11.26 & 8.90 & 7.76 & 7.04 & 6.53
\end{array}
\]
Figure 5: The functions $\delta_k(\nu, \beta), (k = 0, 1, 2, 3)$ at $\nu = 1 - 10, 50$. 
Figure 6: The functions $f_k(\nu, 1; \theta), k = 0, 1, 2, 3$ at $\nu = 1, 2, 3, 4, 5, 10$.

Integrating a spectral SR-intensity of $k-$polarization component over all the directions, one can obtain the so called total spectral SR-intensity of $k-$polarization component. Let us denote via $\nu_{k}^{\max} = \varphi_k(\beta)$ the harmonic that has a maximal total spectral SR-intensity of $k-$polarization component. Then, for any given $\nu$ there exist functions $\beta_k(\nu)$ such that

$$\varphi_k(\beta) = \text{const} = \nu, \quad \beta \in [\beta_k(\nu), \beta_k(\nu + 1)].$$
In the article [5] the functions $\beta_k(\nu)$ were studied in detail. Results of such an analysis, being compared with the above consideration, allow us to conclude that the function $\delta_0(\nu, \beta)$ is not zero for $\nu = \nu_0^{\text{max}} = \varphi_0(\beta)$. The function $\delta_2(\nu, \beta)$ equals zero for $\nu = \nu_\sigma^{\text{max}} = \varphi_\sigma(\beta)$ that corresponds to $\sigma$–polarization component of the total spectral SR-intensity.

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