Top-quark flavor-changing \( tqZ \) couplings and rare \( \Delta F = 1 \) processes

Chuan-Hung Chen\(^1\) and Takaaki Nomura\(^2\)

\(^1\)Department of Physics, National Cheng-Kung University, Tainan 70101, Taiwan
\(^2\)School of Physics, KIAS, Seoul 02455, Korea

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Abstract

We model-independently study the impacts of anomalous \( tqZ \) couplings (\( q = u, c \)), which lead to the \( t \to qZ \) decays, on low energy flavor physics. It is found that the \( tuZ \)-coupling effect can significantly affect the rare \( K \) and \( B \) decays, whereas the \( tcZ \)-coupling effect is small. Using the ATLAS’s branching ratio (BR) upper bound of \( \text{BR}(t \to uZ) < 1.7 \times 10^{-4} \), the influence of the anomalous \( tuZ \)-coupling on the rare decays can be found as follows: (a) The contribution to the Kaon direct CP violation can be up to \( \text{Re}(\epsilon'/\epsilon) \lesssim 0.8 \times 10^{-3} \); (b) \( \text{BR}(K^+ \to \pi^+\nu\bar{\nu}) \lesssim 12 \times 10^{-11} \) and \( \text{BR}(K_L \to \pi^0\nu\bar{\nu}) \lesssim 7.9 \times 10^{-11} \); (c) the BR for \( K_S \to \mu^+\mu^- \) including the long-distance effect can be enhanced by 11\% with respect to the standard model result, and (d) \( \text{BR}(B_d \to \mu^+\mu^-) \lesssim 1.97 \times 10^{-10} \). In addition, although \( \text{Re}(\epsilon'/\epsilon) \) cannot be synchronously enhanced with \( \text{BR}(K_L \to \pi^0\nu\bar{\nu}) \) and \( \text{BR}(K_S \to \mu^+\mu^-) \) in the same region of the CP-violating phase, the values of \( \text{Re}(\epsilon'/\epsilon) \), \( \text{BR}(K^+ \to \pi^+\nu\bar{\nu}) \), and \( \text{BR}(B_d \to \mu^+\mu^-) \) can be simultaneously increased.

*Electronic address: [physchen@mail.ncku.edu.tw](mailto:physchen@mail.ncku.edu.tw)
†Electronic address: [nomura@kias.re.kr](mailto:nomura@kias.re.kr)
I. INTRODUCTION

Top-quark flavor changing neutral currents (FCNCs) are extremely suppressed in the standard model (SM) due to the Glashow-Iliopoulos-Maiani (GIM) mechanism \cite{1}. The branching ratios (BRs) for the $t \rightarrow q(g, \gamma, Z, h)$ decays with $q = u, c$ in the SM are of order $10^{-12} - 10^{-17}$ \cite{2, 3}, and these results are far below the detection limits of LHC, where the expected sensitivity in the high luminosity (HL) LHC for an integrated luminosity of 3000 fb$^{-1}$ at $\sqrt{s} = 14$ TeV is in the range $10^{-5} - 10^{-4}$ \cite{4, 5}. Thus, the top-quark flavor-changing processes can serve as good candidates for investigating the new physics effects. Extensions of the SM, which can reach the HL-LHC sensitivity, can be found in \cite{6 – 10}.

Using the data collected with an integrated luminosity of 36.1 fb$^{-1}$ at $\sqrt{s} = 13$ TeV, ATLAS reported the current strictest upper limits on the BRs for $t \rightarrow qZ$ as \cite{11}:

$$BR(t \rightarrow uZ) < 1.7 \times 10^{-4},$$
$$BR(t \rightarrow cZ) < 2.4 \times 10^{-4}.$$  (1)

Based on the current upper bounds, we model-independently study the implications of anomalous $tqZ$ couplings in the low energy flavor physics. It is found that the $tqZ$ couplings through the $Z$-penguin diagram can significantly affect the rare decays in $K$ and $B$ systems, such as $\epsilon'/\epsilon$, $K \rightarrow \pi \nu \bar{\nu}$, $K_S \rightarrow \mu^+ \mu^-$, and $B_d \rightarrow \mu^+ \mu^-$. Since the gluon and photon in the top-FCNC decays are on-shell, the contributions from the dipole-operator transition currents are small. In this study we thus focus on the $t \rightarrow qZ$ decays, especially the $t \rightarrow uZ$ decay.

From a phenomenological perspective, the importance of investigating the influence of these rare decays are stated as follows: The inconsistency in $\epsilon'/\epsilon$ between theoretical calculations and experimental data was recently found based on two analyses: (i) The RBC-UKQCD collaboration obtained the lattice QCD result with \cite{12, 13}:

$$Re(\epsilon'/\epsilon) = 1.38(5.15)(4.59) \times 10^{-4},$$  (2)

where the numbers in brackets denote the errors. (ii) Using a large $N_c$ dual QCD \cite{14 – 18}, the authors in \cite{19, 20} obtained:

$$Re(\epsilon'/\epsilon)_{SM} = (1.9 \pm 4.5) \times 10^{-4}.$$  (3)
Both results show that the theoretical calculations exhibit an over 2$\sigma$ deviation from the experimental data of $Re(e'/\epsilon)_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4}$, measured by NA48 [21] and KTeV [22, 23]. Various extensions of the SM proposed to resolve the anomaly can be found in [24–48]. We find that the direct Kaon CP violation arisen from the $tuZ$-coupling can be $e'/\epsilon \lesssim 0.8 \times 10^{-8}$ when the bound of $BR(t \to uZ) < 1.7 \times 10^{-4}$ is satisfied.

Unlike $e'/\epsilon$, which strongly depends on the hadronic matrix elements, the calculations of $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$ are theoretically clean and the SM results can be found as [32]:

$$\begin{align*}
BR(K^+ \to \pi^+ \nu \bar{\nu}) &= (8.5^{+1.0}_{-1.2}) \times 10^{-11}, \\
BR(K_L \to \pi^0 \nu \bar{\nu}) &= (3.2^{+1.1}_{-0.7}) \times 10^{-11},
\end{align*}$$

where the QCD corrections at the next-to-leading-order (NLO) [54–56] and NNLO [57–59] and the electroweak corrections at the NLO [60–62] have been calculated. In addition to their sensitivity to new physics, $K_L \to \pi^0 \nu \bar{\nu}$ is a CP-violating process and its BR indicates the CP-violation effect. The current experimental situations are $BR(K^+ \to \pi^+ \nu \bar{\nu})_{\text{exp}} = (17.3^{+11.5}_{-10.5}) \times 10^{-11}$ [63] and $BR(K_L \to \pi^0 \nu \bar{\nu})_{\text{exp}} < 2.6 \times 10^{-8}$ [64]. The NA62 experiment at CERN is intended to measure the BR for $K^+ \to \pi^+ \nu \bar{\nu}$ with a 10% precision [49, 50], and the KOTO experiment at J-PARC will observe the $K_L \to \pi^0 \nu \bar{\nu}$ decay [51, 52]. In addition, the KLEVER experiment at CERN starting in Run-4 could observe the BR of $K_L \to \pi^0 \nu \bar{\nu}$ to 20% precision [53]. Recently, NA62 reported its first result using the 2016 taken data and found that one candidate event of $K^+ \to \pi^+ \nu \bar{\nu}$ could be observed, where the corresponding BR upper bound is given by $BR(K^+ \to \pi^+ \nu \bar{\nu}) < 14 \times 10^{-10}$ at a 95% confidence level (CL) [65]. We will show that the anomalous $tuZ$-coupling can lead to $BR(K^+ \to \pi^+ \nu \bar{\nu}) \lesssim 12 \times 10^{-11}$ and $BR(K_L \to \pi^0 \nu \bar{\nu}) \lesssim 7.9 \times 10^{-11}$. It can be seen that NA62, KOTO, and KLEVER experiments can further constrain the $tuZ$-coupling using the designed sensitivities.

Another important CP violating process is $K_S \to \mu^+ \mu^-$, where the SM prediction including the long-distance (LD) and short-distance (SD) effects is given as $BR(K_S \to \mu^+ \mu^-) = (5.2 \pm 1.5) \times 10^{-12}$ [66–68]. The current upper limit from LHCb is $BR(K_S \to \mu^+ \mu^-) < 0.8(1.0) \times 10^{-9}$ at a 90%(95%) CL. It is expected that using the LHC Run-2 data, the LHCb sensitivity can be improved to $[4, 200] \times 10^{-12}$ with 23 fb$^{-1}$ and to $[1, 100] \times 10^{-12}$ with 100 fb$^{-1}$ [69]. Although the $tuZ$-coupling can significantly enhance the SD contribution of
$K_S \rightarrow \mu^+\mu^-$, due to LD dominance, the increase of $BR(K_S \rightarrow \mu^+\mu^-)_{LD+SD}$ can be up to 11%.

It has been found that the $tuZ$-coupling-induced $Z$-penguin can significantly enhance the $B_d \rightarrow \mu^+\mu^-$ decay, where the SM prediction is given by $BR(B_d \rightarrow \mu^+\mu^-) = (1.06 \pm 0.09) \times 10^{-10}$. From the data, which combine the full Run I data with the results of 26.3 fb$^{-1}$ at $\sqrt{s} = 13$ TeV, ATLAS reported the upper limit as $BR(B_d \rightarrow \mu^+\mu^-) < 2.1 \times 10^{-10}$ [71]. In addition, the result combined CMS and LHCb was reported as $BR(B_d \rightarrow \mu^+\mu^-) = (3.9^{+1.6}_{-1.4}) \times 10^{-10}$ [72]. It can be seen that the measured sensitivity is close to the SM result.

We find that using the current upper limit of $BR(t \rightarrow uZ)$, the $BR(B_d \rightarrow \mu^+\mu^-)$ can be enhanced up to $1.97 \times 10^{-10}$, which is close to the ATLAS upper bound.

The paper is organized as follows: In Sec. II, we introduce the effective interactions for $t \rightarrow qZ$ and derive the relationship between the $tqZ$-coupling and $BR(t \rightarrow qZ)$. The $Z$-penguin FCNC processes induced via the anomalous $tqZ$ couplings are given in Sec. III. The influence on $\epsilon'/\epsilon$ is shown in the same section. The $tqZ$-coupling contribution to the other rare $K$ and $B$ decays is shown in Sec. IV. A summary is given in Sec. V.

II. ANOMALOUS $tqZ$ COUPLINGS AND THEIR CONSTRAINTS

We write the anomalous $tqZ$ interactions as [2]:

$$-\mathcal{L}_{tqZ} = \frac{g}{2c_W} \bar{q} \left( \xi^L_q P_L + \xi^R_q P_R \right) t Z^\mu + \frac{g}{2c_W} \bar{q} \left( \xi^u_q + \xi^a_q \gamma_5 \right) \frac{i\sigma_{\mu\nu} k^\nu}{m_t} t Z^\mu + H.c.,$$

where $g$ is the $SU(2)_L$ gauge coupling; $c_W = \cos \theta_W$ and $\theta_W$ is the Weinberg angle; $P_{L(R)} = (1 \mp \gamma_5)/2$, and $\xi^L_q$ and $\xi^a_q$ denote the dimensionless effective couplings and represent the new physics effects. In this study, we mainly concentrate the impacts of the $tqZ$ couplings on the low energy flavor physics, in which the rare $K$ and $B$ decays are induced through the penguin diagram. Thus, because of the $m_{K(B)}/m_t$ suppression factor, which arises from $k^\nu \sim O(m_{K(B)})$, the contributions of the dipole operators in Eq. (6) are both small and negligible. Hence, in the following analysis, we ignore the $\xi^a_q$ effects and only investigate the $\xi^L_q$ effects. In order to study the influence on the Kaon CP violation, we take $\xi^L_q$ as complex parameters, and the new CP violating phases are defined as $\zeta_q^\chi = |\zeta_q^\chi|e^{-i\phi_q^\chi}$ with $\chi = L, R$.

Using the interactions in Eq. (6), we can calculate the BR for $t \rightarrow qZ$ decay. Since our
purpose is to examine whether the anomalous \( tqZ \)-coupling can give sizable contributions to the rare \( K \) and \( B \) decays when the current upper bound of \( BR(t \to qZ) \) is satisfied, we express the parameters \( \zeta^L_R \) as a function of \( BR(t \to qZ) \) to be:

\[
\sqrt{\left| \zeta^L_q \right|^2 + \left| \zeta^R_q \right|^2} = \left( \frac{BR(t \to qZ)}{C_{tqZ}} \right)^{1/2},
\]

\[
C_{tqZ} = \frac{G_F m_t^3}{16\sqrt{2}\pi\Gamma_t} \left( 1 - \frac{m_Z^2}{m_t^2} \right)^2 \left( 1 + 2\frac{m_Z^2}{m_t^2} \right).
\]

(7)

For the numerical analysis, the relevant input values are shown in Table I. Using the numerical inputs, we obtain \( C_{tqZ} \approx 0.40 \). When \( BR(t \to u(c)Z) < 1.7(2.3) \times 10^{-4} \) measured by ATLAS are applied, the upper limits on \( \sqrt{\left| \zeta^L_u \right|^2 + \left| \zeta^R_u \right|^2} \) can be respectively obtained as:

\[
\sqrt{\left| \zeta^L_u \right|^2 + \left| \zeta^R_u \right|^2} < 0.019,
\]

\[
\sqrt{\left| \zeta^L_c \right|^2 + \left| \zeta^R_c \right|^2} < 0.022.
\]

(8)

Since the current measured results of the \( t \to (u, c)Z \) decays are close each other, the bounds on \( \zeta^u \) and \( \zeta^c \) are very similar. We note that BR cannot determine the CP phase; therefore, \( \theta^u \) and \( \theta^c \) are free parameters.

### TABLE I: Inputs for the numerical estimates.

| \( m_s \) = 1.09 GeV | \( m_d \) = 5.10 MeV | \( m_c \) = 1.3 GeV | \( m_t(m_t) = 165 \text{ GeV} \) |
| \( m_t^{\text{pole}} \) = 172 GeV | \( m_W \) = 80.38 GeV | \( \Gamma_t \) = 1.43 GeV | \( m_K \) = 0.498 GeV |
| \( m_{B_d} \) = 5.28 GeV | \( V_{ud, tb, cs} \approx 1 \) | \( V_{td} = 0.0088e^{-123^\circ} \) | \( V_{ts} = -0.041 \) |
| \( V_{us} = 0.225 \) | \( V_{cd} = -0.225 \) | \( \sin^2 \theta_W = 0.23 \) | \( f_\pi = 0.13 \text{ GeV} \) |
| \( f_K = 0.16 \text{ GeV} \) | \( f_B = 0.191 \text{ GeV} \) | \( |\epsilon_K| = 2.228 \times 10^{-3} \) | \( \tau_{K_S(B)} = 89.5(1.52) \times 10^{-12} \text{ s} \) |

III. ANOMALOUS \( tqZ \) EFFECTS ON \( \epsilon'/\epsilon \)

In this section, we discuss the \( tqZ \)-coupling contribution to the Kaon direct CP violation. The associated Feynman diagram is shown in Fig. I where \( q = u, c; q' \) and \( q'' \) are down type quarks, and \( f \) denotes any possible fermions. That is, the involved rare \( K \) and \( B \) processes in this study are the decays, such as \( K \to \pi\pi, K \to \pi\nu\bar{\nu}, \) and \( K_S(B_d) \to \ell^+\ell^- \). It is found that the contributions to \( K_L \to \pi\ell^+\ell^- \) and \( B \to \pi\ell^+\ell^- \) are not significant; therefore, we do not discuss the decays in this work.
FIG. 1: Sketched Feynman diagram for $q' \rightarrow q'' f \bar{f}$ induced by the tqZ coupling, where $q'$ and $q''$ denote the down-type quarks; $q = u, c,$ and $f$ can be any possible fermion.

Based on the tqZ couplings shown in Eq. (6), the effective Hamiltonian induced by the Z-penguin diagram for the $K \rightarrow \pi\pi$ decays at $\mu = m_W$ can be derived as:

$$
\mathcal{H}_{tqZ} = -\frac{G_F \lambda_t}{\sqrt{2}} \left( y_Z^3 Q_3 + y_Z^7 Q_7 + y_Z^9 Q_9 \right),
$$

where $\lambda_t = V_{ts}^* V_{td};$ the operators $Q_{3,7,9}$ are the same as the SM operators and are defined as:

$$
Q_3 = (\bar{s}d)_{V-A} \sum_{q'} (\bar{q}' q')_{V-A},
$$

$$
Q_7 = \frac{3}{2} (\bar{s}d)_{V-A} \sum_{q'} e_{q'} (\bar{q}' q')_{V+A},
$$

$$
Q_9 = \frac{3}{2} (\bar{s}d)_{V-A} \sum_{q'} e_{q'} (\bar{q}' q')_{V-A},
$$

with $e_{q'}$ being the electric charge of $q'$-quark, and the effective Wilson coefficients are expressed as:

$$
y_Z^3 = -\frac{\alpha}{24\pi s_W^2} I_Z(x_t) \eta_Z, \quad y_Z^7 = -\frac{\alpha}{6\pi} \eta_Z, \quad y_Z^9 = \left( 1 - \frac{1}{s_W^2} \right) y_Z^7, \quad \eta_Z = \sum_{q=u,c} \left( \frac{V_{qd}^* V_{qL}^L}{V_{td}} + \frac{V_{qs}^* V_{sL}^L}{V_{ts}} \right),
$$

with $\alpha = e^2/4\pi,$ $x_t = m_t^2/m_W^2,$ and $s_W = \sin \theta_W.$ The penguin-loop integral function is given as:

$$
I_Z(x_t) = -\frac{1}{4} + \frac{x_t \ln x_t}{2(x_t - 1)} \approx 0.693.
$$

Since $W$-boson can only couple to the left-handed quarks, the right-handed couplings $\zeta_{u,c}^R$ in the diagram have to appear with $m_{u(c)}$ and $m_t,$ in which the mass factors are from the mass insertion in the quark propagators inside the loop. When we drop the small factors
$m_{c,u}/m_W$, the effective Hamiltonian for $K \to \pi\pi$ only depends on $\zeta^{L}_{u,c}$. Since $|V_{ud}/V_{td}|$ is larger than $|V_{cs}/V_{ts}|$ by a factor of 4.67, the dominant contribution to the $\Delta S = 1$ processes is from the first term of $\eta_Z$ defined in Eq. (11). In addition, $V_{ud}$ is larger than $|V_{cd}|$ by a factor of $1/\lambda \sim 4.44$; therefore, the main contribution in the first term of $\eta_Z$ comes from the $V_{ud}\zeta^{L_u}/V_{td}$ effect. That is, the anomalous $tuZ$-coupling is the main effect in our study.

Using the isospin amplitudes, the Kaon direct CP violating parameter from new physics can be estimated using [20]:

$$Re \left( \frac{\epsilon'}{\epsilon} \right) \approx -\frac{\omega}{\sqrt{2}|\epsilon_K|} \left[ \frac{Im A_1}{Re A_0} - \frac{Im A_2}{Re A_2} \right],$$

where $\omega = Re A_2/Re A_0 \approx 1/22.35$ denotes the $\Delta I = 1/2$ rule, and $|\epsilon_K| \approx 2.228 \times 10^{-3}$ is the Kaon indirect CP violating parameter. It can be seen that in addition to the hadronic matrix element ratios, $\epsilon'/\epsilon$ also strongly depends on the Wilson coefficients at the $\mu = m_c$ scale. It is known that the main new physics contributions to $\epsilon'/\epsilon$ are from the $Q_6^{(0)}$ and $Q_8^{(0)}$ operators [25, 73]. Although these operators are not generated through the $tqZ$ couplings at $\mu = m_W$ in our case, they can be induced via the QCD radiative corrections. The Wilson coefficients at the $\mu = m_c$ scale can be obtained using the renormalization group (RG) evolution [74]. Thus, the induced effective Wilson coefficients for $Q_{6,8}$ operators at $\mu = m_c$ can be obtained as:

$$y_6^Z(m_c) \approx -0.08y_3^Z - 0.01y_7^Z + 0.07y_9^Z,$$

$$y_8^Z(m_c) \approx 0.63y_7^Z.$$  \hspace{1cm} (14)

It can be seen that $y_6^Z(m_c)$ is much smaller than $y_8^Z(m_c)$; that is, we can simply consider the $Q_8$ operator contribution.

According to the $K \to \pi\pi$ matrix elements and the formulation of $Re(\epsilon'/\epsilon)$ provided in [20], the $O_8$ contribution can be written as:

$$Re \left( \frac{\epsilon'}{\epsilon} \right)_p^Z \approx -a_8^{(3/2)} B_8^{(3/2)},$$

$$a_8^{(3/2)} = Im \left( \lambda_t y_8^Z(m_c) \right) \frac{r_2\langle Q_8\rangle_2}{B_8^{(3/2)} Re A_2}, \hspace{1cm} (15)$$

where $r_2 = \omega G_F/(2|\epsilon_K|) \approx 1.17 \times 10^{-4}$ GeV$^{-2}$, $B_8^{(3/2)} \approx 0.76$; $Re A_{2(0)}^{\exp} \approx 1.21(27.04) \times 10^{-8}$ GeV [75], and the matrix element of $\langle Q_8\rangle_2$ is defined as:

$$\langle Q_8\rangle_2 = \sqrt{2} \left( \frac{m_K^2}{m_s(\mu) + m_d(\mu)} \right)^2 f_{\pi} B_8^{3/2}. \hspace{1cm} (16)$$
Although the $Q_8$ operator can contribute to the isospin $I = 0$ state of $\pi\pi$, because its effect is a factor of 15 smaller than the isospin $I = 2$ state, we thus neglect its contribution.

Since the $t \to (u, c)Z$ decays have not yet been observed, in order to simplify their correlation to $\epsilon'/\epsilon$, we use $BR(t \to qZ) \equiv \text{Min}(BR(t \to cZ), BR(t \to uZ))$ instead of $BR(t \to u(c)Z)$ as the upper limit. The contours for $Re(\epsilon'/\epsilon)^Z_P$ (in units of $10^{-3}$) as a function of $BR(t \to qZ)$ and $\theta_L^u$ are shown in Fig. 2, where the solid and dashed lines denote the results with $\theta_L^c = -\theta_L^u$ and $\zeta_L^c = 0$, respectively, and the horizontal dashed line is the current upper limit of $BR(t \to qZ)$. It can be seen that the Kaon direct CP violation arisen from the anomalous $tuZ$-coupling can reach $0.8 \times 10^{-3}$, and the contribution from $tcZ$-coupling is only a minor effect. When the limit of $t \to qZ$ approaches $BR(t \to qZ) \sim 0.5 \times 10^{-4}$, the induced $\epsilon'/\epsilon$ can be as large as $Re(\epsilon'/\epsilon)^Z_P \sim 0.4 \times 10^{-3}$.

![FIG. 2: Contours for $Re(\epsilon'/\epsilon)^Z_P$ (in units of $10^{-3}$) as a function of $BR(t \to qZ)$ and $\theta_L^u$, where the solid and dashed lines denote the $\theta_L^c = -\theta_L^u$ and $\zeta_L^c = 0$ results, respectively. The $BR(t \to qZ)$ is defined as the minimal one between $BR(t \to uZ)$ and $BR(t \to cZ)$. The horizontal dashed line (red) is the current upper limit of $BR(t \to qZ)$.](image-url)
IV. Z-PENGUIN INDUCED (SEMI)-LEPTONIC K AND B DECAYS AND NUMERICAL ANALYSIS

The same Feynman diagram as that in Fig. 1 can be also applied to the rare leptonic and semi-leptonic \(K(B)\) decays when \(f\) is a neutrino or a charged lepton. Because \(|V_{us}/V_{ts}| \ll |V_{cs}/V_{ts}| \sim |V_{us}/V_{td}| \ll |V_{cd}/V_{td}|\), it can be found that the anomalous \(tu(c)Z\)-coupling contributions to the \(b \to s\ell\ell\) (\(\ell = \nu, \ell^-\)) processes can deviate from the SM result being less than 7\% in terms of amplitude. However, the influence of the \(tuZ\) coupling on \(d \to s\ell\ell\) and \(b \to d\ell\ell\) can be over 20\% at the amplitude level. Accordingly, in the following analysis, we concentrate the study on the rare decays, such as \(K \to \pi\nu\bar{\nu}\), \(K_S \to \mu^+\mu^-\), and \(B_d \to \mu^+\mu^-\), in which the channels are sensitive to the new physics effects and are theoretically clean.

According to the formulations in \[34\], we write the effective Hamiltonian for \(d_i \to d_j\ell\ell\) induced by the \(tuZ\) coupling as:

\[
\mathcal{H}_{d_i \to d_j\ell\ell} = -\frac{G_F V_{tdi}^* V_{tdj}}{\sqrt{2}} \alpha \pi C_L^Z \left[ \bar{d}_j \gamma_\mu P_L d_i \right] \left[ \bar{\nu}\gamma^\mu (1 - \gamma_5) \nu \right] - \frac{G_F V_{tdi}^* V_{tdj}}{\sqrt{2}} \alpha \pi \bar{d}_j \gamma_\mu P_L d_i \left[ C_9^Z \bar{\nu}\gamma^\mu + C_{10}^Z \bar{\nu}\gamma^\mu \gamma_5 \ell \right],
\]

(17)

where we have ignored the small contributions from the \(tcZ\)-coupling; \(d_i \to d_j\) could be the \(s \to d\) or \(b \to d\) transition, and the effective Wilson coefficients are given as:

\[
C_L^Z = C_{10}^Z \approx \frac{I_Z(x_t) c_{W}^Z V_{tdi}^* V_{tdj}}{4 s_W^2}, \quad C_9^Z \approx C_L^Z \left( -1 + 4 s_W^2 \right).
\]

(18)

Because \(-1 + 4 s_W^2 \approx -0.08\), the \(C_9^Z\) effect can indeed be neglected.

Based on the interactions in Eq. (17), the BRs for the \(K_L \to \pi^0\nu\bar{\nu}\) and \(K^+ \to \pi^+\nu\bar{\nu}\) decays can be formulated as \[25\]:

\[
BR(K_L \to \pi^0\nu\bar{\nu}) = \kappa_L \left| \frac{Im X_{\text{eff}}}{\lambda_5} \right|^2,
\]

\[
BR(K^+ \to \pi^+\nu\bar{\nu}) = \kappa_+ (1 + \Delta_{\text{EM}}) \left[ \left| \frac{Im X_{\text{eff}}}{\lambda_5} \right|^2 + \left| \frac{Re \lambda_{c}}{\lambda} P_c(X) + \frac{Re X_{\text{eff}}}{\lambda_5} \right|^2 \right],
\]

(19)

where \(\lambda_{c} = V_{cs}^* V_{cd}, \Delta_{\text{EM}} = -0.003; P_c(X) = 0.404 \pm 0.024\) denotes the charm-quark contribution \[76, 77\]; the values of \(\kappa_{L,+}\) are respectively given as \(\kappa_L = (2.231 \pm 0.013) \times 10^{-10}\) and \(\kappa_+ = (5.173 \pm 0.025) \times 10^{-11}\), and \(X_{\text{eff}}\) is defined as:

\[
X_{\text{eff}} = \lambda_t \left( X_{L}^{SM} - s_W^2 C_L^Z \right),
\]

(20)
with $X^\text{SM}_L = 1.481 \pm 0.009 \ [25]$. Since $K_L \to \pi^0 \nu \bar{\nu}$ is a CP violating process, its BR only depends on the imaginary part of $X_{\text{eff}}$. Another important CP violating process in $K$ decay is $K_S \to \mu^+ \mu^-$, where its BR from the SD contribution can be expressed as $[37]$:

$$BR(K_S \to \mu^+ \mu^-)_{\text{SD}} = \tau_{K_S} \frac{G_F^2 \alpha^2}{8\pi^3} m_K f_K^2 m_{\mu}^2 \left(1 - \frac{4m_{\mu}^2}{m_K^2} \right) \left| Im[\lambda_t (C^\text{SM}_{10} + C^Z_{10})] \right|^2,$$  \(21\)

with $C^\text{SM}_{10} \approx -4.21$. Including the LD effect $[66, 67]$, the BR for $K_S \to \mu^+ \mu^-$ can be estimated using $BR(K_S \to \mu^+ \mu^-)_{\text{LD+SD}} \approx 4.99_{\text{LD}} \times 10^{-12} + BR(K_S \to \mu^+ \mu^-)_{\text{SD}} [68]$. Moreover, it is found that the effective interactions in Eq. (17) can significantly affect the $B_d \to \mu^+ \mu^-$ decay, where its BR can be derived as:

$$BR(B_d \to \mu^+ \mu^-) = \tau_B \frac{G_F^2 \alpha^2}{16\pi^3} m_B f_B m_{\mu}^2 \left(1 - \frac{2m^2_{\mu}}{m_B^2} \right) \sqrt{1 - \frac{4m^2_{\mu}}{m_B^2}} \times \left| V_{td}^* V_{tb} (C^\text{SM}_{10} + C^Z_{10}) \right|^2.$$  \(22\)

Because $B_d \to \mu^+ \mu^-$ is not a pure CP violating process, the BR involves both the real and imaginary part of $V_{td}^* V_{tb} (C^\text{SM}_{10} + C^Z_{10})$. Note that the associated Wilson coefficient in $B_d \to \mu^+ \mu^-$ is $C^Z_{10}$, whereas it is $C^Z_{10}$ in the $K$ decays.

After formulating the BRs for the investigated processes, we now numerically analyze the $tuZ$-coupling effect on these decays. Since the involved parameter is the complex $\zeta^L_u = |\zeta^L_u| e^{-i\theta^L_u}$, we take $BR(t \to uZ)$ instead of $|\zeta^L_u|$. Thus, we show $BR(K_L \to \pi^0 \nu \bar{\nu})$ (in units of $10^{-11}$) as a function of $BR(t \to uZ)$ and $\theta^L_u$ in Fig. 3(a), where the CP phase is taken in the range of $\theta^L_u = [-\pi, \pi]$; the SM result is shown in the plot, and the horizontal line denotes the current upper limit of $BR(t \to uZ)$. It can be clearly seen that $BR(K_L \to \pi^0 \nu \bar{\nu})$ can be enhanced to $7 \times 10^{-11}$ in $\theta^L_u > 0$ when $BR(t \to uZ) < 1.7 \times 10^{-4}$ is satisfied. Moreover, the result of $BR(K_L \to \pi^0 \nu \bar{\nu}) \approx 5.3 \times 10^{-11}$ can be achieved when $BR(t \to uZ) = 0.5 \times 10^{-4}$ and $\theta^L_u = 2.1$ are used. Similarly, the influence of $\zeta^L_u$ on $BR(K^+ \to \pi^+ \nu \bar{\nu})$ is shown in Fig. 3(b). Since $BR(K^+ \to \pi^+ \nu \bar{\nu})$ involves the real and imaginary parts of $X_{\text{eff}}$, unlike the $K_L \to \pi^0 \nu \bar{\nu}$ decay, its BR cannot be enhanced manyfold due to the dominance of the real part. Nevertheless, the BR of $K^+ \to \pi^+ \nu \bar{\nu}$ can be maximally enhanced by 38%; even, with $BR(t \to uZ) = 0.5 \times 10^{-4}$ and $\theta^u = 2.1$, the $BR(K^+ \to \pi^+ \nu \bar{\nu})$ can still exhibit an increase of 15%. It can be also found that in addition to $|\zeta^L_u|$, the BRs of $K \to \pi \nu \bar{\nu}$ are also sensitive to the $\theta^L_u$ CP-phase. Although the observed $BR(K \to \pi \nu \bar{\nu})$ cannot constrain $BR(t \to uZ)$, the allowed range of $\theta^L_u$ can be further limited.
For the $K_s \to \mu^+\mu^-$ decay, in addition to the SD effect, the LD effect, which arises from the absorptive part of $K_s \to \gamma\gamma \to \mu^+\mu^-$, predominantly contributes to the $BR(K_s \to \mu^+\mu^-)$. Thus, if the new physics contribution is much smaller than the LD effect, the influence on $BR(K_s \to \mu^+\mu^-)_{LD+SD} = BR(K_s \to \mu^+\mu^-)_{LD} + BR(K_s \to \mu^+\mu^-)_{SD}$ from new physics may not be so significant. In order to show the $tuZ$-coupling effect, we plot the contours for $BR(K_s \to \mu^+\mu^-)_{LD+SD}$ (in units of $10^{-12}$) in Fig. 3(c). From the result, it can be clearly seen that $BR(K_s \to \mu^+\mu^-)_{LD+SD}$ can be at most enhanced by 11% with respect to the SM result, whereas the BR can be enhanced only $\sim 4.3\%$ when $BR(t \to uZ) = 0.5 \times 10^{-4}$ and $\theta_u^L = 2.1$.

As discussed earlier that the $tcZ$-coupling contribution to the $B_s \to \mu^+\mu^-$ process is small; however, similar to the case in $K^+ \to \pi^+\nu\bar{\nu}$ decay, the BR of $B_d \to \mu^+\mu^-$ can be significantly enhanced through the anomalous $tuZ$-coupling. We show the contours of $BR(B_d \to \mu^+\mu^-)$ (in units of $10^{-10}$) as a function of $BR(t \to uZ)$ and $\theta_u^L$ in Fig. 3(d). It can be seen that the maximum of the allowed $BR(B_d \to \mu^+\mu^-)$ can reach $1.97 \times 10^{-10}$, which is a factor of 1.8 larger than the SM result of $BR(B_d \to \mu^+\mu^-)_{SM} \approx 1.06 \times 10^{-10}$. Using $BR(t \to uZ) = 0.5 \times 10^{-4}$ and $\theta_u^L = 2.1$, the enhancement factor to $BR(B_d \to \mu^+\mu^-)_{SM}$ becomes 1.38. Since the maximum of $BR(B_d \to \mu^+\mu^-)$ has been close to the ATLAS upper bound of $2.1 \times 10^{-10}$, the constraint from the rare $B$ decay measured in the LHC could further constrain the allowed range of $\theta_u^L$.

V. SUMMARY

We studied the impacts of the anomalous $tqZ$ couplings in the low energy physics, especially the $tuZ$ coupling. It was found that the anomalous coupling can have significant contributions to $\epsilon'/\epsilon$, $BR(K \to \pi\nu\bar{\nu})$, $K_S \to \mu^+\mu^-$, and $B_d \to \mu^+\mu^-$. Although these decays have not yet been observed in experiments, with the exception of $\epsilon'/\epsilon$, their designed experiment sensitivities are good enough to test the SM. It was found that using the sensitivity of $BR(t \to uZ) \sim 5 \times 10^{-5}$ designed in HL-LHC, the resulted $BR(K \to \pi\nu\bar{\nu})$ and $BR(B_d \to \mu^+\mu^-)$ can be examined by the NA62, KOTO, KELVER, and LHC experiments.

According to our study, it was found that we cannot simultaneously enhance $Re(\epsilon'/\epsilon)$, $BR(K_L \to \pi^0\nu\bar{\nu})$, and $BR(K_S \to \mu^+\mu^-)$ in the same region of the CP violating phase, where the positive $Re(\epsilon'/\epsilon)$ requires $\theta_u^L < 0$, but the large $BR(K_L \to \pi^0\nu\bar{\nu})$ and $BR(K_S \to \mu^+\mu^-)$
FIG. 3: Contours of the branching ratio as a function of $BR(t \to uZ)$ and $\theta_u^L$ for (a) $K_L \to \pi^0 \nu \bar{\nu}$, (b) $K^+ \to \pi^+ \nu \bar{\nu}$, (c) $K_S \to \mu^+ \mu^-$, and (d) $B_d \to \mu^+ \mu^-$, where the corresponding SM result is also shown in each plot. The long-distance effect has been included in the $K_S \to \mu^+ \mu^-$ decay.

have to rely on $\theta_u^L > 0$. Since $BR(K^+ \to \pi^+ \nu \bar{\nu})$ and $BR(B_d \to \mu^+ \mu^-)$ involve both real and imaginary parts of Wilson coefficients, their BRs are not sensitive to the sign of $\theta_u^L$. Hence, $Re(\epsilon'/\epsilon)$, $BR(K^+ \to \pi^+ \nu \bar{\nu})$ and $BR(B_d \to \mu^+ \mu^-)$ can be enhanced at the same time.

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