Theoretical $\Sigma$-$D$ relations for shell-type galactic supernova remnants

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Abstract. Relations between radio surface brightness ($\Sigma$) and diameter ($D$) of supernova remnants (SNRs) are important in astronomy. In this paper, following the work Duric and Seaquist (ApJ 301:308, 1986) at adiabatic phase, we carefully investigate shell-type supernova remnants at radiative phase, and obtain theoretical $\Sigma$-$D$ relation at radiative phase of shell-type supernova remnants at 1 GHz. By using these theoretical $\Sigma$-$D$ relations at adiabatic phase and radiative phase, we also roughly determine phases of some supernova remnant from observation data.

Keywords. Shell-type supernova remnants—radiative phase—surface brightness—$\Sigma$-$D$ relation.

1. Introduction

Relations between radio surface brightness ($\Sigma$) and diameter ($D$) of supernova remnants (SNRs) are important in astronomy, and are usually used to determine distance of a SNR (Poveda and Woltjer 1968; Clark and Caswell 1976; Lozinskaya 1981; Huang and Thaddeus 1985; Duric and Seaquist 1986; Guseinov et al. 2003). There have been many works via statistical or analytical approaches to investigate $\Sigma$-$D$ relations (e.g., Poveda and Woltjer 1968; Clark and Caswell 1976; Mills et al. 1984; Huang and Thaddeus 1985; Arbutina et al. 2004; Xu et al. 2005; Pavlovic et al. 2014, etc.). Among statistical results of $\Sigma$-$D$ relations, one straight line is often obtained by authors (e.g., Poveda and Woltjer 1968; Huang and Thaddeus 1985; Arbutina et al. 2004; Pavlovic et al. 2013, 2014), while a broken fit line or a transition point is also usually seen. For example, Clark and Caswell (1976) and Allakhverdiyev et al. (1983, 1985) obtained a broken fit line in their statistical works. At 408 MHz, Clark and Caswell (1976) had a broken line with slopes of $\beta = -2.7/10$ ($\Sigma \propto D^{\beta}$) at $D \leq 32$ pc/$D \geq 32$ pc, while Allakhverdiyev et al. (1983) had 30 pc at 408 MHz and 32 pc at 1 GHz for 15 shell-type remnants. For analytical $\Sigma$-$D$ relations, Duric and Seaquist (1986) derived

$$\Sigma(D) = 4 \times 10^{-14} D^{-5}, \quad D < 1 \text{ pc}$$

$$\Sigma(D) = 4 \times 10^{-15} D^{-3.5}, \quad D \geq 1 \text{ pc}$$

On the other hand, galactic supernova remnants are usually classified into three types: Shell-type, Plerion-type and Composite-type. In our paper, for simplicity, we just focus on investigating shell-type galactic supernova remnants. Usually, shell-type galactic supernova remnants have four evolution stages: the free expansion phase, adiabatic or Sedov phase, radiative or snowplough phase and the dissipation phase. Nearly all of detected shell-type SNRs are at adiabatic phase or radiative phase, since almost none is observed in the 1st and 4th phases due to fact that shell-type SNRs at these two phases are usually practically undetectable. Therefore, $\Sigma$-$D$ relations of shell-type supernova remnants at adiabatic phase and radiative phase are interesting issues to investigate. Indeed, Duric and Seaquist (1986) have analytical derived $\Sigma$-$D$ relation of shell-type supernova remnants at adiabatic phase, i.e. the above equation (2). In our paper, we will mainly focus on analytical investigations on $\Sigma$-$D$ relations of shell-type supernova remnants. We obtain theoretical $\Sigma$-$D$ relation at radiative phase of shell-type supernova remnants at 1 GHz, which is simply followed the work Duric and Seaquist (1986). We also have collected 57 shell-type galactic supernova remnants data where some data have been
updated according to Green (2004, 2009 and 2014) and other new references. By using these theoretical $\Sigma$-$D$ relations at adiabatic phase and radiative phase, we have roughly determined the phases of some supernova remnant among these 57 shell-type galactic supernova remnants data.

This article is organized as follows. In Section 2, after a brief review of the work Duric and Seaquist (1986) at adiabatic phase, we simply follow their work and further analytically investigate $\Sigma$-$D$ relation at radiative phase of shell-type galactic supernova remnants at 1 GHz. In Section 3, we have collected 57 shell-type galactic supernova remnants data. By using these two theoretical $\Sigma$-$D$ relations at adiabatic phase and radiative phase, we roughly determine the phases of some supernova remnant among these data. Finally, a brief conclusion and discussion are given in Section 4.

2. Theoretical $\Sigma$-$D$ relations at adiabatic phase and radiative phase

In this section, we mainly focus on theoretical $\Sigma$-$D$ relations of shell-type supernova remnants. After making a brief review on the analytical work Duric and Seaquist (1986) at adiabatic phase, we theoretically derive $\Sigma$-$D$ relation at radiative phase of shell-type supernova remnants.

2.1 A brief review: work Duric and Seaquist (1986)

Taking the linear diameter ($D$) of remnant in pc, time ($t$) in s, SNR initial explosion energy ($E_0$) in the unit of $10^{51}$ ergs with $E_0 = \varepsilon S_1 \times 10^{51}$, and ISM electron density ($n_0$) in cm$^{-3}$, from the standard Sedov solution, one has the following equation (Bignami et al. 1988; Zaninetti 2000; Völk et al. 2002; Ptuskin and Zirakashvili 2003)

$$D(t) = A_0 t^{2/5}.$$  (3)

where the coefficient is

$$A_0 = 5.4 \times 10^{-4} (\frac{S_1}{n_0})^{1/5}. $$  (4)

The shock wave velocity should be

$$\nu(t) = \frac{1}{2} \frac{d}{dt} D(t) = \frac{1}{5} A_0 t^{-3/5}. $$  (5)

At the adiabatic phase, the thickness of remnant is proportional to $D$, and the shell volume which contains all the radio-emitting particles is

$$V(D) = C_0 D^3,$$  (6)

here $C_0 = \frac{\pi}{6} (1 - \frac{D_i}{D_o})^3 \simeq 0.37$ is the volume coefficient (Milne 1970). Notice that condition $D_i/D_o \sim 2/3$ has been assumed and $D_i$ and $D_o$ are the inner and outer diameter of the remnant shell respectively. Combining Equations (3) and (6), one obtains the volume of the shell with respect to $t$

$$V(t) = C_0 A_0^3 t^{6/5}.$$  (7)

As the shock waves of remnant travel, the ambient magnetic field $B$ at the adiabatic phase will decrease with $D$ according to (Duric and Seaquist 1986)

$$B(D) = B_0 \left( \frac{D_0}{D} \right)^2.$$  (8)

Substituting Equation (3) to it, we have

$$B(t) = B_0 D_0^2 A_0^{-2} t^{-4/5}.$$  (9)

Ginzburg and Syrovatskii (1965) and Bell (1978) show that the radio emissivity $\varepsilon(B, \nu)$ of a shocked gas which are affected by a magnetic field to produce the synchrotron emission is expressed as (Arbutina et al. 2012)

$$\varepsilon(\nu) = 2.94 \times 10^{-34} (1.435 \times 10^5)^{0.75-\alpha} \xi(2\alpha + 1) \times \left( \frac{n_0}{cm^{-3}} \right)^{\alpha} \left( \frac{\nu}{10^4 km s^{-1}} \right)^{4\alpha} \left( \frac{B}{10^{-4} G} \right)^{\alpha+1} \times \left( 1 + \left( \frac{\nu}{7000 km s^{-1}} \right)^{-2} \right)^{\alpha} \left( \frac{\nu}{GHz} \right)^{-\alpha},$$  (10)

where $\psi_e = 4$ and $\phi_e = 10^{-3}$ have been set as same as in Bell (1978), and unit of $\varepsilon(\nu)$ is $WHz^{-1}m^{-3}$. $\xi(\mu) = 11.7a(\mu)$, and $a(\mu)$ is the function tabulated by Ginzburg and Syrovatskii (1965). The velocities of shock waves in the second and third phase of SNRs are typically far less than 7000 $km s^{-1}$. Thus, Equation (10) can be further simplified

$$\varepsilon(\nu) = 2.94 \times 10^{-34} \times (1.435 \times 10^5)^{0.75-\alpha} \xi(2\alpha + 1) \times \left( \frac{\alpha}{0.75} \right)^{\alpha} \left( \frac{n_0}{cm^{-3}} \right)^{\alpha} \left( \frac{\nu}{7000 km/s} \right)^{2\alpha} \times \left( \frac{B}{10^{-4} G} \right)^{\alpha+1} \left( \frac{\nu}{GHz} \right)^{-\alpha} \times \left( \frac{B}{10^{-4} G} \right)^{\alpha+1} \left( \frac{\nu}{GHz} \right)^{-\alpha} \times \left( \frac{\nu}{8 \times 10^{-10} pc/s} \right)^{2\alpha} \times \left( \frac{W Hz^{-1} m^{-3}}{GHz} \right)^{-\alpha} \times \left( \frac{W Hz^{-1} m^{-3}}{GHz} \right)^{-\alpha} \times \left( \frac{\nu}{GHz} \right)^{-\alpha} \times \left( \frac{W Hz^{-1} m^{-3}}{GHz} \right)^{-\alpha} \times \left( \frac{W Hz^{-1} m^{-3}}{GHz} \right)^{-\alpha}, \quad (11)$$

Note that, in the second line of this equation, shock velocity $\nu$ in unit of $pc/s$ has been considered for the later convenience of discussion. Taking account of Equations (3) and (9) and the average value of the remnants spectral index $\alpha = 0.5$, we can get
\[ \epsilon(D) = 2.25 \times 10^{-34} \left( \frac{D_0}{D} \right)^3 \left( \frac{B_0}{10^{-4} G} \right)^{3/2} \times \left( \frac{1}{5} \frac{A_0^{5/2}}{D^{3-2}/(2.3 \times 10^{-10} \text{ pc/s})} \right). \] (12)

which is at 1 GHz and \( a(2) = 0.103 \) has been used in Ginzburg and Syrovatskii (1965). If the shell volume is considered to be encompassed by the radiating electrons, the surface brightness of remnant can be written as (Duric and Seaquist 1986)

\[ \Sigma(t) = \frac{\epsilon(t)V(t)}{\pi^2 D^2(t)}. \] (13)

Inserting Equations (3)(4)(7) and (12) into it, we obtain

\[ \Sigma(D) = 2.25 \times 10^{-34} \frac{C_0 D_0^3}{\pi^2 D^2} \left( \frac{B_0}{10^{-4} G} \right)^{3/2} \times \left( \frac{1}{5} \frac{A_0^{5/2}}{D^{-3-2}/(2.3 \times 10^{-10} \text{ pc/s})} \right). \] (14)

Finally, one can get

\[ \Sigma(D) = m_a D_{pc}^{-3.5} (W m^{-2} Hz^{-1} sr^{-1}), \] (15)

where

\[ m_a = 2.25 \times 10^{-34} \frac{C_0 D_0^3}{\pi^2} \left( \frac{B_0}{10^{-4} G} \right)^{3/2} \times \left( \frac{1}{5} \frac{A_0^{5/2}}{D^{-3-2}/(2.3 \times 10^{-10} \text{ pc/s})} \right) \times 3.08 \times 10^{16} = 3.88 \times 10^{-17}, \] (16)

and \( \Sigma(D) \) in Equation (14) in unit of \( (W m^{-3} Hz^{-1} pc sr^{-1}) \) and \( 1 pc = 3.08 \times 10^{16} m \) have been considered, and some typical values of physical parameters of SNRs are taken: ISM density \( n_0 = 0.1 \text{ cm}^{-3} \), SNR initial explosion energy \( E_0 = 10^{51} \text{ erg} \), the diameter and ISM magnetic field of remnant at the beginning of Sedov phase \( D_0 = 2 \text{ pc} \) and \( B_0 = 10^{-4} \text{ G} \), etc. Therefore, the analytically derived line of \( \Sigma-D \) relation at the second phase of shell-type SNR is

\[ \Sigma(D) = 3.88 \times 10^{-17} D_{pc}^{-3.5} (W m^{-2} Hz^{-1} sr^{-1}). \] (17)

Note that, different typical values have been chosen, and hence coefficient in Equation (17) is a little different from that in work Duric and Seaquist (1986), but the power-law has the same exponent.

2.2 Analytical \( \Sigma-D \) relation at the radiative phase

It should be pointed out that the above work Duric and Seaquist (1986) just analytically investigate the adiabatic phase of shell-type SNRs. In fact, we can also simply follow their work to analytically investigate \( \Sigma-D \) relation at radiative phase of shell-type SNRs. After setting the same choices of units as those in Section 2.1, the equation for shell-type SNRs at the radiative stage is (McKee and Ostriker 1977)

\[ D(t) = A_1 t^{2/7}, \] (18)

where \( A_1 \) is a constant

\[ A_1 = 0.03 \left( \frac{C_1}{n_0} \right)^{1/7}. \] (19)

From which, we obtain the velocity of shock wave at the radiative phase

\[ v(t) = \frac{1}{7} A_1 t^{-5/7}. \] (20)

Same as the adiabatic phase, the volume of shell can be

\[ V(D) = C_1 D^3. \] (21)

If we roughly take \( D_1/D_0 \sim 3/4 \), then the coefficient will be \( C_1 = \pi/6 (1 - (D_1/D_0)^3) \geq 0.3 \). Changing the variant \( D \) to \( t \), one can rewrite the volume of shell as

\[ V(t) = C_1 A_1 t^{6/7}. \] (22)

Note that the ambient magnetic field \( B \) of a remnant decreases with the diameter \( D \) at the adiabatic phase following Equation (8), while at the dissipation-phase it is \( B(D) = B_1(D_1/D)^0 \). Therefore, we can assume that the ambient magnetic field \( B \) at the radiative phase can be expressed as

\[ B(D) = \left( \frac{D_1}{D} \right)^\beta B_1, \] (23)

where the parameter \( \beta \) ranges from 0 to 2. After substituting (18) to it, one gets

\[ B(t) = \left( \frac{D_1}{A_1} \right)^\beta B_1 t^{-2\beta/7}. \] (24)

Therefore, following the same steps as the above section and still taking \( n_0 = 0.1 \text{ cm}^{-3} \), we can obtain \( \Sigma - D \) relation at radiative phase at 1 GHz

\[ \Sigma(D) = 2.25 \times 10^{-34} \frac{C_1 D_1^3}{\pi^2 D^2} \left( \frac{B_1 D_1^\beta D^{-\beta}}{10^{-4} G} \right)^{3/2} \times \left( \frac{1}{5} \frac{A_0^{7/2}}{D^{-5/2}/(2.3 \times 10^{-10} \text{ pc/s})} \right). \] (25)

and this form is simply rewritten as

\[ \Sigma(D) = m_r D^{3-\frac{3}{2}(1+\beta)} (W m^{-2} Hz^{-1} sr^{-1}), \] (26)

where

\[ m_r = 2.25 \times 10^{-34} \frac{C_1 D_1^3}{\pi^2} \left( \frac{B_1}{10^{-4} G} \right)^{3/2} \times \left( \frac{1}{5} \frac{A_0^{7/2}}{(2.3 \times 10^{-10} \text{ pc/s})} \right) \times 3.08 \times 10^{16}. \] (27)
3. Roughly determine phases of some supernova remnants by using theoretical $\Sigma-D$ relations

For these theoretical $\Sigma-D$ relations at adiabatic and radiative phases, it will be highly interesting to identify some reasonable supernova remnants in adiabatic or radiative stages from observation data. Therefore, at first, some typical values of supernova remnants at radiative phase are also set as $B_1 = 10^{-6} G$, $D_1 = 20$ pc, $\beta = 1$. Then, theoretical $\Sigma-D$ relation at radiative phase is

$$\Sigma(D) = 1.86 \times 10^{-16} D_{pc}^{-3} \left( W m^{-2} H z^{-1} s r^{-1} \right).$$

(28)

Second, 57 shell-type supernova remnants data in Galaxy at 1 GHz have been collected and listed in table 2. From these data, the 1 GHz surface brightness $\Sigma_{1GHz}$ is obtained by (Clark and Caswell 1976)

$$\Sigma_{1GHz} = 1.505 \frac{S_{1GHz}}{\theta^2} \times 10^{-19} \left( W m^{-2} H z^{-1} s r^{-1} \right),$$

(29)

where $S_{1GHz}$ is the 1 GHz flux density in jansky ($1 Jy \equiv 10^{-26} W m^{-2} H z^{-1}$), and $\theta$ is the angular diameter in minutes of arc.

Finally, comparing $\Sigma_{1GHz}$ with theoretical $\Sigma-D$ relations both at adiabatic phase and radiative phase, we roughly determine phases of supernova remnants. From these comparisons, we find out that some of these supernova remnants indeed can be identified in adiabatic phase or radiative phase, which has been listed in Table 1.

4. Conclusion and discussion

In this paper, we have analytically investigated $\Sigma-D$ relations of shell-type supernova remnants both at adiabatic phase and radiative phase. For convenience to compare with observation data, we have chosen some typical values of shell-type supernova remnants and also collected 57 shell-type supernova remnants data. By using these theoretical $\Sigma-D$ relations and observation data, we have roughly identified some shell-type supernova remnants in adiabatic phase or radiative phase.
Table 2. Some physical parameters of 57 shell-type Galactic SNRs.

| Source  | Age/year | Dist./pc | Dia./pc | Size/arcmin | $S_{1GHz}$/Jy | References                                                                 |
|---------|----------|----------|---------|-------------|---------------|---------------------------------------------------------------------------|
| G4.5+6.8 | 380      | 2900     | 3       | 3           | 19            | Hatsukade et al. (1990) and Green (2004)                                  |
| G7.7−3.7 |          | 4500     | 29      | 22          | 11            | Milne et al. (1986)                                                      |
| G8.7−0.1 | 15800    | 3900     | 51      | 45          | 80            | Gorham et al. (1996)                                                     |
| G18.8+0.3 | 16000    | 12000    | 57      | 17×11       | 33            | Green (2004) and Tian et al. (2007)                                      |
| G27.4+0.0 | 2700     | 6800     | 8       | 4           | 6             | Green (2004) and Caswell et al. (1982)                                   |
| G31.9+0.0 | 4500     | 7200     | 13      | 7×5         | 25            | Chen and Slane (2001) and Green (2014)                                   |
| G32.8−0.1 |          | 7100     | 35      | 17          | 11            | Koralesky et al. (1998)                                                  |
| G33.6+0.1 | 9000     | 7800     | 23      | 10          | 20            | Seward et al. (2003), Seward and Velusamy (1995) and Green (2004, 2014) |
| G39.2−0.3 | 1000     | 11000    | 22      | 8×6         | 18            | Green (2014) and Caswell et al. (1982)                                   |
| G41.1−0.3 | 1400     | 8000     | 8       | 4.5×2.5     | 25            | Chen et al. (1999), Caswell et al. (1982), Binette et al. (1982) and Green (2014) |
| G43.3−0.2 | 3000     | 10000    | 10      | 4×3         | 38            | Lacey et al. (2001) and Zhu et al. (2014)                                |
| G49.2−0.7 | 30000    | 6000     | 52      | 30          | 160           | Koo et al. (1995) and Green (2004)                                       |
| G53.6−2.2 | 15000    | 2800     | 24      | 33×28       | 8             | Saken et al. (1995) and Green (2004)                                     |
| G55.0+0.3 | 1100000  | 14000    | 71      | 20×15       | 0.5           | Matthews et al. (1998)                                                   |
| G65.3+5.7 | 14000    | 1000     | 78      | 310×240     | 42            | Green (2014) and Rosado (1981)                                           |
| G73.9+0.9 | 10000    | 1300     | 8       | 27          | 9             | Lorimer et al. (1998), Green (2014) and Leahy (1989)                    |
| G74.0−8.5 | 14000    | 400      | 23      | 230×160     | 210           | Levenson et al. (1999), Stil and Irwin (2001) and Green (2004)           |
| G78.2+2.1 | 50000    | 1500     | 26      | 60          | 320           | Lorimer et al. (1998), Koo and Heiles (1991) and Green (2014)           |
| G84.2−0.8 | 11000    | 4500     | 23      | 20×16       | 11            | Matthews and Shaver (1980), Matthews et al. (1977) and Green (2004)     |
| G89.0+4.7 | 19000    | 800      | 24      | 120×90      | 220           | Leahy and Aschenbach (1996)                                              |
| G93.3+6.9 | 5000     | 2200     | 15      | 27×20       | 9             | Landecker et al. (1999) and Green (2004)                                 |
| G93.7−0.2 |          | 1500     | 35      | 80          | 65            | Uyaniker et al. (2002)                                                   |
| G109.1−1.0 | 17000    | 4000     | 24      | 28          | 22            | Fesen and Horford (1995), Green (2004, 2014), Hughes et al. (1981) and   |
|          |          |          |         |             |               | Tian and Leahy (2012)                                                    |
| G111.7−2.1 | 320      | 3400     | 5       | 5           | 2720          | Thorstensen et al. (2001)                                                |
| G114.3+0.3 | 41000    | 700      | 15      | 90×55       | 5.5           | Mavromatakis et al. (2002) and Green (2004, 2014)                        |
| G116.5+1.1 | 280000   | 1600     | 32      | 80×60       | 10            | Green (2004, 2014) and Reich and Braunsfurth (1981)                      |
| G116.9+0.2 | 44000    | 1600     | 16      | 34          | 8             | Koo and Heiles (1991) and Green (2004, 2014)                             |
| G116.9+0.2 | 24500    | 1400     | 37      | 90          | 36            | Mavromatakis et al. (2000)                                               |
| G117.1+1.4 | 410      | 2300     | 5       | 8           | 56            | Hatsukade et al. (1990) and Green (2004)                                 |
| G127.1+0.5 | 85000    | 5250     | 69      | 45          | 12            | Green (2014) and Fürst et al. (1984)                                     |
| G132.7+1.3 | 21000    | 2200     | 51      | 80          | 45            | Green (2004) and Galas et al. (1980)                                     |
| Source     | Age/year | Dist./pc | Dia./pc | size/arcmin | $S_{1GHz}$/Jy | References                                                                 |
|------------|----------|----------|---------|-------------|---------------|-----------------------------------------------------------------------------|
| G156.2+5.7 | 26000    | 2000     | 64      | 110         | 5             | Reich *et al.* (1992)                                                       |
| G160.9+2.6 | 7700     | 1000     | 38      | 140×120     | 110           | Leahy and Aschenbach (1995)                                                |
| G166.0+4.3 | 81000    | 4500     | 57      | 55×35       | 7             | Koo and Heiles (1991), Green (2004) and Leahy (1989)                       |
| G166.2+2.5 | 150000   | 8000     | 186     | 90×70       | 11            | Green (2014) and Routledge *et al.* (1986)                                 |
| G182.4+4.3 | 3800     | 3000     | 44      | 50          | 0.4           | Kothes *et al.* (1998) and Green (2014)                                    |
| G205.5+0.5 | 50000    | 1600     | 102     | 220         | 140           | Case and Bhattacharya (1999) and Green (2014)                               |
| G206.9+3.2 | 60000    | 7000     | 102     | 60×40       | 6             | Green (2014) and Leahy (1986)                                              |
| G260.4−3.4 | 3400     | 2200     | 35      | 60×50       | 130           | Berthiaume *et al.* (1994) and Rosado and González (1981)                  |
| G266.2−1.2 | 680      | 1500     | 52      | 120         | 50            | Kargaltsev *et al.* (2002) and Aschenbach *et al.* (1999)                  |
| G272.2−3.2 | 6000     | 1800     | 8       | 15          | 0.4           | Duncan *et al.* (1997)                                                     |
| G284.3−1.8 | 10000    | 2900     | 20      | 24          | 11            | Green (2014) and Ruiz and May (1986)                                       |
| G296.5+10.0| 20000    | 2000     | 44      | 90×65       | 48            | Green (2014) and Matsui *et al.* (1988)                                    |
| G296.8−0.3 | 160000   | 9600     | 47      | 20×14       | 9             | Gaensler and Johnston (1995) and Green (2004)                               |
| G299.2−2.9 | 5000     | 500      | 2       | 18×11       | 0.5           | Slane *et al.* (1996)                                                      |
| G309.2−0.6 | 2500     | 4000     | 16      | 15×12       | 7             | Rakowski *et al.* (2001)                                                   |
| G315.4−2.3 | 2000     | 2300     | 28      | 42          | 49            | Dickel *et al.* (2001) and Green (2004)                                    |
| G321.9−0.3 | 200000   | 9000     | 70      | 31×23       | 13            | Green (2014), Shull *et al.* (1989) and Salter *et al.* (1989)             |
| G327.4+0.4 | −        | 4800     | 29      | 21          | 30            | Seward *et al.* (1996), Green (2004, 2014) and Weiler and Sramek (1988)  |
| G327.6+14.6| 980      | 2200     | 19      | 30          | 19            | Green (2004) and Srinivasan *et al.* (1984)                                 |
| G330.0+15.0| −        | 1200     | 63      | 180         | 350           | Knödlseder *et al.* (1996)                                                 |
| G332.4−0.4 | 2000     | 3100     | 9       | 10          | 28            | Carter *et al.* (1997), Green (2004) and Meaburn and Allan (1986)          |
| G337.2−0.7 | 3250     | 15000    | 26      | 6           | 1.5           | Rakowski *et al.* (2001) and Green (2014)                                  |
| G337.8−0.1 | −        | 12300    | 27      | 9×6         | 18            | Koralessky *et al.* (1998)                                                 |
| G346.6−0.2 | −        | 8200     | 19      | 8           | 8             | Koralesky *et al.* (1998) and Dubner *et al.* (1993)                       |
| G349.7+0.2 | 14000    | 11500    | 9       | 2.5×2       | 20            | Reynoso and Mangum (2001), Green (2004) and Tian and Leahy (2014)          |
| G352.7−0.1 | 2200     | 8500     | 17      | 8×6         | 4             | Kinugasa *et al.* (1998)                                                   |

*a*Many of the radio SNRs have more than one published value for distance and age. For these, we either chose the most recent estimates or used an average of the available estimates, or the most commonly adopted value.

*b*Diameters were calculated using from distances together with the angular sizes in Green (2004, 2009 and 2014) catalogue. In addition, some data have been updated according to the new results in Green (2014) and other new references.

*c*Some data regarding G349.7+0.2 (Tian and Leahy 2014), G43.3−0.2 (Zhu *et al.* 2014), G18.8+0.3 (Tian *et al.* 2007) and G109.1−1.0 (Tian and Leahy 2012) have been updated.
Some discussions related to our results are in the following. First, shock compression ratio may differ among supernova remnants, and hence spectral index $\alpha$ will be different from $\alpha = 0.5$. For simplicity and convenience, we also investigate the case with $\alpha = 0.75$, and theoretical $\Sigma$-$D$ relation at adiabatic phase

$$\Sigma(D) = 1.06 \times 10^{-17} D_{pc}^{-19/4} (W m^{-2} Hz^{-1} sr^{-1}),$$

(30)

while at radiative phase

$$\Sigma(D) = 5.52 \times 10^{-16} D_{pc}^{-9/2} (W m^{-2} Hz^{-1} sr^{-1}).$$

(31)

From these equations, it will be easily found that effects from spectral index $\alpha$ on $\Sigma$-$D$ relations are in fact huge, i.e. different power exponents. Second, not only spectral index $\alpha$, our results are also dependent on other parameters such as volume coefficient $C_0$, mean electron density $n_0$, SNR initial explosion energy $E_0$, magnetic field at the beginning of the evolving second-stage and third-stage $B_0$ and $B_1$, and parameters $D_0$ and $D_1$. If those parameters are changed, our results may be also different. Therefore, further effects of parameters on theoretical $\Sigma$-$D$ relations will be an interesting open issue, while maybe they are also the main reason that why just some of supernova remnants are roughly identified at adiabatic phase or radiative phase among 57 supernova remnants, and hence our results just shed some insights onto the possibility to identify the phase of supernova remnant through comparisons theoretical $\Sigma$-$D$ relations with observation data. Third, the true physical process of supernova remnant at radiative phase is complicated, and we have assumed that the synchrotron radiation equation (11) is still valid in radiative stage. But yet, whether this assumption is correct or not is still an issue (e.g., discussions in Asvarov 2006). Comparison with observational data seem to support this assumption. Fourth, in principle, there is a simple and direct method to identify the phase of supernova remnant, i.e. comparisons $D(t)$ relations with observation data. However, this method may be not better than our method, and the reason may be that there is a larger uncertainty to decide the age of a supernova remnant than diameter. Finally, our results also predict that there will be a transition point between these two theoretical $\Sigma$-$D$ relations. However, since $D(t)$ relations used in our paper are just statistical and not precise, the details of this transition point are still lacking.

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