New $N=4$ superfields and $\sigma$-models

by

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Abstract: In this note, we construct new representations of $D=2$, $N=4$ supersymmetry which do not involve chiral or twisted chiral multiplets. These multiplets may make it possible to circumvent no-go theorems about $N=4$ superspace formulations of WZWN-models.
1 Introduction

In this note, we introduce some new representations of $N = 4$ supersymmetry in $D = 2$. These are characterized by having no $N = 2$ component superfields that are chiral or twisted chiral. Such multiplets are important, because in the $N = 4$ theory, due to arguments based on the dimension of the superspace action (see for example [1, 2]), a good deal of the dynamics of the theory depends simply on the choice of multiplets. Indeed, previously known multiplets cannot be used to describe generic $N = 2, 4$ WZW-models [3], [4].

2 New Representations of $N = 4$ Supersymmetry

As may be seen from a dimensional analysis of the superspace measure and the superfield component content, to construct superspace actions for higher $N$ in $D = 2$ one needs to find invariant subspaces and corresponding restricted measures. Such subspaces are analogous to $N = 1, D = 4$ chiral and antichiral superspaces. In a series of papers [2, 4, 5, 6], to this end we have constructed and utilized a projective superspace. In the present $N = 4$ context it is introduced as follows:[1]

The complex $SU(2)$ doublet spinor derivatives $D_{a\pm}, \bar{D}^b_\pm$ that describe $N = 4$ supersymmetry obey the commutation relations

$$\{ D_{a\pm}, \bar{D}^b_\pm \} = i\delta^b_a \partial_{\pm} \tag{1}$$

(all others vanish). We will work with $N = 2$ superfields and identify $D_\pm \equiv D_{1\pm}$ as the $N = 2$ spinor covariant derivative and $Q_\pm \equiv D_{2\pm}$ as the generator of the non-manifest supersymmetries. As described in [4] we use two complex variables $\zeta$ and $\xi$ to define a set of anticommuting left and right derivatives:

$$\nabla_+ = D_+ + \zeta Q_+, \quad \nabla_- = D_- + \xi Q_- ,$$

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[1] A more detailed discussion may be found in [5].
\[ \nabla_+ = D_+ - \zeta^{-1}Q_+ , \quad \nabla_- = D_- - \xi^{-1}Q_- . \] (2)

A real structure \( R \) acts on \( \zeta \) and \( \xi \) by hermitian conjugation composed with the antipodal map, i.e.;

\[ R\zeta = -\bar{\zeta}^{-1} , \quad R\xi = -\bar{\xi}^{-1} \] (3)

Clearly \( \nabla_{\pm} = R\bar{\nabla}_{\pm} \), so \( R \) preserves the subspaces defined by the derivatives (2). We then consider superfields \( \eta(\zeta, \xi) \), specify the \( \zeta \) and \( \xi \) dependence (typically as a series expansion) and require that \( \eta \) is annihilated by the derivatives in (2). In general, if we write

\[ \eta = \sum \zeta^n \xi^m \eta_{nm} \] (4)

the \( N = 4 \) constraints that \( \eta \) is annihilated by the derivatives in (2) lead to the component relations

\[ D_+ \eta_{nm} + Q_+ \eta_{n-1,m} = 0, \quad \bar{D}_+ \eta_{nm} - \bar{Q}_+ \eta_{n+1,m} = 0 \]
\[ D_- \eta_{nm} + Q_- \eta_{n,m-1} = 0, \quad \bar{D}_- \eta_{nm} - \bar{Q}_- \eta_{n,m+1} = 0. \] (5)

We may also specify \( \eta \) further by, e.g., a reality condition such as

\[ \eta = R\bar{\eta} \] (6)

To construct \( N = 4 \) actions for \( N = 4 \) superfields we use a second set of linearly independent covariant spinor derivatives:

\[ \Delta_+ = D_+ - \zeta Q_+ , \quad \Delta_- = D_- - \xi Q_- , \]
\[ \bar{\Delta}_+ = \bar{D}_+ + \zeta^{-1} \bar{Q}_+ , \quad \bar{\Delta}_- = \bar{D}_- + \xi^{-1} \bar{Q}_- . \] (7)

An action may then be written as

\[ S = \frac{1}{16} \int d^2\sigma \int_C d\zeta \int_{C'} d\xi \Delta_+ \Delta_- \Delta_+ \Delta_- \mathcal{L}(\eta(\zeta, \xi); \zeta, \xi) \] (8)

where \( C \) and \( C' \) are some appropriate contours.

Using

\[ \Delta_+ = 2D_+ - \nabla_+ , \quad \Delta_- = 2D_- - \nabla_- , \]
\[
\bar{\Delta}_+ = 2\bar{D}_+ - \bar{\nabla}_+ , \quad \bar{\Delta}_- = 2\bar{D}_- - \bar{\nabla}_- \, ,
\]
and (of course) \( \nabla_\pm \eta = \nabla_\pm \bar{\eta} = 0 \), the \( N = 2 \) superspace form of the action (8) is:

\[
S = \int d^2\sigma D^2 \bar{D}^2 \int_C d\zeta \int_{C'} d\xi L (\eta(\zeta, \xi); \zeta, \xi) \quad (10)
\]

In [6] this general setting was applied to study a particular \( \eta \) obeying

\[
R\bar{\eta} = (-\zeta)^{-N} (\xi)^{-M} \eta , \quad (11)
\]

where the sum in (4) was restricted to be from 0 to \( N, M \). Among the component superfields we found the usual chiral and twisted chiral superfields, as well as semi-chiral and semi-antichiral fields, i.e. fields \( \phi \) and \( \bar{\phi} \) that obey only

\[
D_{1+}\phi \equiv D_+ \phi = 0 , \quad D_{1-}\bar{\phi} \equiv D_- \bar{\phi} = 0 . \quad (12)
\]

Here we further extend the set of \( N = 4 \) multiplets to include new types that contain no chiral or twisted chiral \( N = 2 \) component superfields.

We begin with a general multiplet of the type (4). If we further impose the reality condition (6), then the definition (3) implies

\[
\eta_{nm} = (-)^{n+m} \bar{\eta}_{-n,-m} . \quad (13)
\]

So far the expansion in (4) is quite general: \( n \) and \( m \) range over the integer numbers \( \mathbb{Z} \) from \(-\infty\) to \(+\infty\), and we have not worried about convergence properties, etc. Note that the constraints (5) are translation invariant; this leads us naturally to restrict the expansion by

\[
\eta_{n+k,m} = \eta_{nm} \quad (14)
\]

for some fixed \( k \in \mathbb{Z} \) (for \( \eta \) real in the sense of (13), \( k \) must be even). We shall call (14) a left cylindrical constraint. Similarly

\[
\eta_{n,m+k} = \eta_{nm} \quad (15)
\]
is a right cylindrical constraint and

\[
\eta_{n+k,m+l} = \eta_{nm} , \quad k, l \in \mathbb{Z} \quad (16)
\]
is called a toroidal constraint.

The components of a real $\eta$ obeying a toroidal constraint will obey the constraints (5). We see that this only restricts the components’ transformation properties under the nonmanifest supersymmetries. A dimensional analysis of the measure in the action (10) shows that we need $D_{\pm}$ constraints on the components to generate dynamics. The real toroidal $\eta$’s thus correspond to purely auxiliary multiplets.

Cylindrical $\eta$’s have some components that are semi-(anti)chiral and may hence be used to construct dynamical actions. Let us illustrate this in a particular case. We choose two real $\eta$’s with components $\eta_{nm}$ and $\chi_{nm}$ respectively. On the first we impose a left cylindrical constraint (14) with $k = 2$ and on the second we impose a right cylindrical constraint (15) with the same $k$. We also restrict the $m$ and $n$ range so that we have the following set of conditions

\begin{align}
\eta_{n+2,m} &= \eta_{nm}, \quad \eta_{nm} = 0, \quad |m| > 1 \\
\chi_{n,m+2} &= \chi_{nm}, \quad \chi_{nm} = 0, \quad |n| > 1,
\end{align}

(17)
in addition to the constraints (3) and (13) (that hold for both $\eta_{nm}$ and $\chi_{nm}$). Explicitly this yields

\begin{align*}
\eta_{11} &= \bar{\eta}_{-1,-1} , \quad \eta_{00} = \bar{\eta}_{00} , \quad \eta_{10} = -\bar{\eta}_{10} , \quad \eta_{01} = -\bar{\eta}_{0,-1} ; \\
D_- \eta_{n,-1} &= 0 , \quad \bar{D}_- \eta_{n,1} = 0 ; \\
\chi_{11} &= \bar{\chi}_{-1,1} , \quad \chi_{00} = \bar{\chi}_{00} , \quad \chi_{10} = -\bar{\chi}_{0,1} , \quad \chi_{01} = -\bar{\chi}_{-1,0} ; \\
D_+ \chi_{-1,m} &= 0 , \quad \bar{D}_+ \chi_{1m} = 0 . \quad (18)
\end{align*}

Note that $\eta_{00}$ and $\chi_{00}$ are real while $\eta_{10}$ and $\chi_{01}$ are imaginary. We now discuss what is required from a Lagrangian $L(\eta, \chi)$ for the action (10) to be $N = 4$ supersymmetric. From [7] we know that an action constructed out of ordinary $N = 2$ (anti)chiral superfields is $N = 4$ supersymmetric if and only if the Lagrangian satisfies a generalized Laplace equation. Here we expect to find some similar requirement.

Because of the issues of convergence that we have ignored, we cannot simply use the action (8) or its $N = 2$ reduction (10). We therefore look for
an invariant action directly. A non-manifest supersymmetry transformation generated by \( Q_+ \) acting on \( L(\eta, \chi) \) has the following effect

\[
Q_+ L(\eta, \chi) = -(L_{\eta_{n+1}m} D_+ \eta_{n+1,m} + L_{\chi_{n+1}m} D_+ \chi_{n+1,m})
\]  

(19)

where we have used the conditions (5). For the action to be invariant we require \( Q_+ L \) to be a total (super)derivative, which implies \( D_+ Q_+ L(\eta, \chi) = 0 \). This leads to

\[
L_{\eta_{n+1}m} \eta_{ij} D_+ \eta_{ij} D_+ \eta_{nm} + (L_{\chi_{ij} \eta_{n+1}m} - L_{\chi_{i-1,j} \eta_{nm}}) D_+ \chi_{ij} D_+ \eta_{nm}
\]

\[
+ L_{\chi_{n-1}m \chi_{ij}} D_+ \chi_{ij} D_+ \chi_{nm} = 0 \ .
\]

(20)

Exploring this equation using the explicit relations (18), we find a set of equations for the derivatives of \( L \). To present them and their solutions it is convenient to introduce the following linear combinations of the fields:

\[
\eta_1 = \eta_{11} + \eta_{01}, \quad \eta_0 = \eta_{00} + \eta_{10}, \quad \eta_2 = \eta_{11} - \eta_{01}
\]

(21)

\[
\chi_1 = \chi_{11} + \chi_{10}, \quad \chi_0 = \chi_{00} + \chi_{01}, \quad \chi_2 = \chi_{11} - \chi_{10} \ .
\]

(22)

Vanishing of the mixed derivatives term in (20) implies the following set of equations:

\[
L_{\chi^0 \eta_0} = - L_{\chi_{1} \eta_0} = - L_{\chi_{2} \eta_0} \ , \quad L_{\bar{\chi}^0 \eta_0} = - L_{\bar{\chi}_{1} \eta_0} = - L_{\bar{\chi}_{2} \eta_0}
\]

\[
L_{\chi^0 \eta_0} = - L_{\chi_{1} \eta_0} = - L_{\chi_{2} \eta_0} \ , \quad L_{\bar{\chi}^0 \eta_0} = - L_{\bar{\chi}_{1} \eta_0} = - L_{\bar{\chi}_{2} \eta_0}
\]

\[
L_{\chi^0 \eta_1} = - L_{\chi_{1} \eta_1} = - L_{\chi_{2} \eta_1} \ , \quad L_{\bar{\chi}^0 \eta_1} = - L_{\bar{\chi}_{1} \eta_1} = - L_{\bar{\chi}_{2} \eta_1}
\]

\[
L_{\chi^0 \eta_2} = - L_{\chi_{1} \eta_2} = - L_{\chi_{2} \eta_2} \ , \quad L_{\bar{\chi}^0 \eta_2} = - L_{\bar{\chi}_{1} \eta_2} = - L_{\bar{\chi}_{2} \eta_2}
\]

(23)

The pure \( \eta \) derivatives yield

\[
L_{\eta^0 \eta_0} = L_{\eta_{0} \eta_{1}} = L_{\eta_{0} \eta_{2}} = 0 \ , \\
L_{\eta_{1} \eta_0} = L_{\eta_{1} \eta_{1}} = L_{\eta_{1} \eta_{2}} = 0 \ , \\
L_{\eta_{2} \eta_0} = L_{\eta_{2} \eta_{1}} = L_{\eta_{2} \eta_{2}} = 0 \ .
\]

(24)

and pure \( \chi \) derivatives

\[
L_{\chi^0 \chi_0} = L_{\chi_{1} \chi_2} \ , \quad L_{\bar{\chi}^0 \chi_0} = L_{\bar{\chi}_{2} \chi_1} \ , \\
L_{\chi^0 \chi_1} = - L_{\bar{\chi}^0 \chi_2} \ , \quad L_{\chi^0 \chi_2} = - L_{\bar{\chi}^0 \chi_0} \ .
\]
\[ L_{\chi_0 \bar{\chi}_0} = -L_{\chi_1 \bar{\chi}_1} = -L_{\chi_2 \bar{\chi}_2} . \]  

(25)

There are also the relations that result from \( D_+ Q_- L = 0 \). They can be obtained from the above via the substitution \( \eta \leftrightarrow \chi \) (not all equations from the full set are independent; in particular, the complex conjugate relations have already been included).

A solution for the supersymmetric Lagrangian \( L \) is a linear combination of the following functions \( L_1, L_2, L_3, L_4 \) (and the corresponding complex conjugate expressions):

\[ L_1(\eta) = \int d\xi \, L_1\left(\xi \eta_1 + \eta_0 + \frac{1}{\xi} \bar{\eta}_2; \xi\right), \]  

(26)

\[ L_2(\chi) = \int d\zeta \, L_2\left(\zeta \chi_1 + \chi_0 + \frac{1}{\zeta} \bar{\chi}_2; \zeta\right), \]  

(27)

\[ L_3(\eta, \chi) = L_3\left(\eta_1 + \eta_0 + \bar{\eta}_2, \chi_1 + \chi_0 + \bar{\chi}_2\right), \]  

(28)

\[ L_4(\eta, \chi) = L_4\left(\eta_1 - \eta_0 + \bar{\eta}_2, \bar{\chi}_1 + \bar{\chi}_0 - \chi_2\right). \]  

(29)

Since the fields \( \vec{y}_i = (\chi_0, \bar{\chi}_0, \eta_0, \bar{\eta}_0) \) are unconstrained, their equations of motion will imply:

\[ L_{\chi_0} = L_{\bar{\chi}_0} = L_{\eta_0} = L_{\bar{\eta}_0} = 0 \]  

(30)

As long as \( L_{y_i y_j} \) is nondegenerate we can solve these for \( y_i \) in terms of the remaining fields \( \vec{x} = (\vec{\chi}, \vec{\eta}) \)

\[ \vec{\chi}_i = (\chi_1, \bar{\chi}_1, \chi_2, \bar{\chi}_2), \quad \vec{\eta}_i = (\eta_1, \bar{\eta}_1, \eta_2, \bar{\eta}_2) \]  

(31)

and replace \( y_i \) in the Lagrangian.

To interpret these results, we start from the \( N = 2 \) superspace reduction and continue down to \( N = 1 \) superspace in the standard way. \( N = 1 \)
superspace derivatives $\nabla$ and extra supersymmetry generators $Q$ are defined as

$$\nabla \equiv D + \bar{D} , \quad Q \equiv i(D - \bar{D}) , \quad (32)$$

Starting from (10) we obtain

$$S = \int d^2\sigma \left( \nabla_+ \nabla_+ Q_+ Q_- L(x_i, y_j(x_i)) \right)$$

$$= \int d^2\sigma \left( \nabla_+ \left( M_{x_i x_j}(x_k) Q_+ x_i Q_- x_j + L_{x_i} Q_+ Q_- x_i \right) \right) . \quad (33)$$

where

$$M_{x_i x_j}(x_k) = L_{x_i x_j} - L_{x_i y_k} \left( L_{y_i y_k} \right)^{-1} L_{y_i x_j} \quad (34)$$

We now use the constraints (18) to eliminate the extra supersymmetry generators $Q$ in terms of $N = 1$ $\nabla$-derivatives wherever possible and integrate by parts to rewrite the action as

$$S = \int d^2\sigma \left( \nabla_+ \left( \nabla_+ \chi \right) A_{ij} \nabla_+ \bar{\eta}_j + \nabla_+ \bar{\chi}_i B_{ij} Q_- \bar{\chi}_j + \nabla_+ \bar{\eta}_i C_{ij} Q_- \bar{\chi}_j \right.$$.

$$+ \nabla_+ \bar{\eta}_i D_{ij} \nabla_+ \bar{\pi}_j + \nabla_+ \bar{\pi}_i E_{ij} \nabla_+ \bar{\pi}_j + \nabla_+ \bar{\pi}_i F_{ij} Q_- \bar{\chi}_j \right) \quad (35)$$

where $A, B, C, D, E, F$ are 4 $\times$ 4 matrices:

$$A_{ij} = (IMI)_{\chi_i \eta_j} , \quad B_{ij} = [I, M]_{\chi_i \chi_j} ,$$

$$C_{ij} = -(MI)_{\eta_i \chi_j} , \quad D_{ij} = -[I, M]_{\eta_i \eta_j} ,$$

$$E_{ij} = -(IM)_{\eta_i \chi_j} , \quad F_{ij} = M_{\eta_i \chi_j} , \quad (36)$$

and

$$I = \left( \begin{array}{cc} J & 0 \\ 0 & J \end{array} \right) , \quad J = \left( \begin{array}{cccc} i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \end{array} \right) . \quad (37)$$
The fields with explicit $Q$ generators remaining are auxiliary spinors, and may be eliminated by completing the square to obtain:

$$S = \int d^2 \sigma \nabla_+ \nabla_- (\nabla_+ \tilde{\chi}_i A_{ij} \nabla_- \tilde{\eta}_j$$

$$- (\nabla_+ \tilde{\chi}_i B_{ik} + \nabla_+ \tilde{\eta}_i C_{ik}) (F_{ik})^{-1} (D_{lj} \nabla_- \tilde{\eta}_j + E_{lj} \nabla_- \tilde{\chi}_j) ) \quad (38)$$

The matrix $F_{ij}$ is invertible as long as

$$L_{\eta_0 \eta_1} \neq L_{\eta_0 \bar{\eta}_2} ,$$

$$L_{\chi_0 \chi_1} \neq L_{\chi_0 \bar{\chi}_2} ,$$

$$L_{\eta_0 \chi_0} \neq 0 , \quad L_{\eta_0 \bar{\chi}_0} \neq 0 , \quad (39)$$

(and their complex conjugates) are satisfied \footnote{This relations are not obvious, and were found using the algebraic manipulation program Maple.}. The explicit expressions for the action (26),(27),(28) and (29) show that these conditions are generically satisfied by our Lagrangian. We can write the final $N = 1$ superspace action in the form

$$S = \int d^2 \sigma \nabla_+ \nabla_- (\nabla_+ x_i T_{ij} (x_k) \nabla_- x_j) \quad , \quad (40)$$

where the matrix $T_{ij}$ has $4 \times 4$ blocks given by:

$$T = \begin{pmatrix} -BF^{-1}E & A - BF^{-1}D \\ -CF^{-1}E & -CF^{-1}D \end{pmatrix} \quad (41)$$

The extra supersymmetry generators $Q$ are given by:

$$Q_+ \tilde{x} = \begin{pmatrix} J \\ -F^{-1}B^t \end{pmatrix} \nabla_+ \tilde{x} \quad (42)$$

$$Q_- \tilde{x} = \begin{pmatrix} -F^{-1}E \\ -F^{-1}D \end{pmatrix} \nabla_- \tilde{x} \quad (43)$$
Here we have used the vector notation from (31) once again, and $J$ is defined in (37). The expressions for the remaining supersymmetry generators $Q_\pm$ and $\bar{Q}_\pm$ may be obtained from (5),(32) and (42),(43):

\[ Q_+ \vec{\eta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} D_+ \vec{\eta}, \]

\[ Q_+ \vec{\chi} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} (L_{yy})^{-1} (L_{yx}) D_+ \vec{x}, \quad (44) \]

\[ Q_- \vec{\eta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} D_- \vec{\eta}, \]

\[ Q_- \vec{\chi} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} (L_{yy})^{-1} (L_{qx}) D_- \vec{x}, \quad (45) \]

where $(L_{yx})$ is the matrix with entries $L_{yx,i,j}$, etc., and $D = \frac{1}{2} (\nabla - i \mathcal{Q})$. In $N = 1$ superspace, extra supersymmetries correspond to complex structures; we may read off the left and right complex structures $J^A_\pm$ from $Q^A_\pm \vec{x} \equiv J^A_\pm \nabla_\pm \vec{x}$ (with $\mathcal{Q}^A = \{ Q, (Q + \bar{Q}), i(Q - \bar{Q}) \}$ [2].

Other $N = 4$ multiplets of this type can be constructed by modifying the $\zeta$ and $\xi$ dependence. In particular, one can consider a complex multiplet with periodicity $k = 1$ (cf. (14)); a quick analysis shows no interesting new features.

3 Conclusions

We have found a new class of $N = 4$ multiplets that have no chiral or twisted chiral $N = 2$ component superfields. It is known that general
WZWN-models cannot be described in extended superspace in terms of only chiral and twisted chiral superfields [3]; it was hoped that multiplets, introduced in [6], with semi-chiral and semi-antichiral superfields as well as chiral and twisted chiral superfield could be used. Recently, we have shown that in many cases the description cannot involve any chiral or twisted chiral superfields [4]; the new multiplets introduced here are thus the only known candidates for describing WZWN-models in $N = 4$ superspace.

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