Abstract

Theoretical and interpretative study on the subject of photodetachment of $H^-$ near a partial reflecting surface is presented, and the absorption effect of the surface is investigated on the total and differential cross sections using a theoretical imaging method. To understand the absorption effect, a reflection parameter $K$ is introduced as a multiplicative factor to the outgoing detached-electron wave of $H^-$ propagating toward the wall. The reflection parameter measures how much electron wave would reflect from the surface; $K = 0$ corresponds to no reflection and $K = 1$ corresponds to the total reflection.

PACS number: 32.80.Gc

It has been observed both theoretically as well as experimentally that the photodetachment cross section of $H^-$ shows a smooth behavior in the free space [1, 2]; while in the presence of a reflecting surface it displays oscillations [3, 4]. These oscillations are similar as if they are in the presence of a static electric field [5, 6, 7, 8, 9, 10, 11].

Quite recently, Afaq and Du [12] have argued that this oscillatory effect in the photodetachment cross section of $H^-$ is because of two-path interference of the detached-electron wave from the negative ion. To the observation point, one path comes directly from the source $H^-$, while the second path appears to be coming from an image of the source behind the wall. In reference [12], the idea has been discussed without considering the absorption effect of a wall. What would happen on the photodetachment cross section of negative ion when the wall in use will be absorbing? This problem is still interesting and has to be discussed. I use a simple model for $H^-$ and provide quantitative answer to the problem.

Near an absorbing wall the physical picture of the photodetachment process may be described as: When the detached-electron wave is made incident on the wall, a part of it is absorbed by the wall and the other part is reflected with low intensity. This low intensity electron wave propagates away from the system and appears to be coming from an image behind the wall, and at a very large distance it interferes with the direct outgoing detached-electron wave. Consequently, we obtain an outgoing electron flux interference pattern on the screen. The photodetachment cross section is proportional to the integrated outgoing electron flux across a large enclosure in which the source $H^-$ sits.

An absorbing wall $0 \leq K \leq 1$ is used for the electron scattering and it is placed from $H^-$ at a distance more than 50 Bohr radii, so that the asymptotic approximations can be valid. The assumptions about the partial reflecting wall are the same as in reference [12]. For an observer at large distance from $H^-$, there are two components of detached-electron wave going from $H^-$ to the observer. The first component propagates directly from $H^-$ to the observer as if there is no wall; the second component first propagates toward the wall, after being partially reflected by the wall, it then propagates from the wall to the observer. In the theoretical imaging...
Figure 1: The schematic diagram of the photodetachment of $H^-$ near a wall. The negative ion $H^-$ is on the z axis, and it’s distance from the reflecting wall W is d. A z-polarized laser light is applied for the photodetachment. The detached electron waves at large distance include two components $\Psi_1$ and $\Psi_2$. The direct component $\Psi_1$ is the detached electron wave generated at the negative ion source S without the wall, the second component $\Psi_2$ is obtained from $\Psi_1$ via a reflection by the wall. The wave $\Psi_2$ appears to come from an image I of the source S behind the wall W. $\Psi_K$ represents absorbed component of detached-electron wave. The ± symbols indicate the sign of two lobes of P-orbital wave function.

method, the first component comes from $H^-$ directly, the second component appears coming from an image of the $H^-$ behind the wall. The detached-electron flux can be calculated from the above two component outgoing waves. By integrating the detached-electron flux for all angles, we are able to derive analytic formula for the total photodetachment cross section of $H^-$. Atomic units are used unless otherwise noted.

A schematic diagram for the photodetachment of $H^-$ near a partial reflecting wall is shown in Fig. 1. A hydrogen negative ion $H^-$ acting as a source (S) of detached electron wave is on z-axis, its distance from the wall (W) is d. The reflecting surface of the wall coincides with the x-y plane. A laser polarized in the z direction is applied for the photodetachment of $H^-$. Three components of detached electron wave $\Psi_1$, $\Psi_2$ and $\Psi_K$ are also shown. $\Psi_1$ is the direct component, $\Psi_2$ is the reflected component and $\Psi_K$ is the absorbed components by the wall. The reflected component appears as if it is from an image (I) behind the wall. The ± symbols indicate the sign of two lobes of P-orbital wave function.

The photodetachment process can be regarded as a two step process: in the first step, the negative ion absorbs one photon energy $E_{ph}$ and generates an outgoing electron wave; in the second step, this outgoing wave propagates to large distances. Let $\Psi_1$ be the direct outgoing electron wave after the photodetachment of $H^-$ in the absence of the wall and let $\theta_1$ be the angle between the detached electron and the z-axis. Half of this wave with $\theta_1$ smaller than $\pi/2$ propagates away from the negative ion to large distance. The other half of the wave with $\theta_1$ larger than $\pi/2$ first propagates to the wall, after being partially reflected by the wall, it then propagates to large distance. We call this reflected wave $\Psi_2$. A part of this incoming wave towards the wall is absorbed by the surface $\Psi_K$ which can be measured by introducing reflection parameter $K$ as a multiplicative factor to the wave moving toward the wall. The total outgoing electron wave $\Psi^+$ at large distance from the system is given by $\Psi^+ = \Psi_1 + \Psi_2$. 
The expression for the direct wave $\Psi_1$ has been derived before[17]. Using $(r_1, \theta_1, \phi_1)$ as the spherical coordinates of the electron with respect to the source (S) and $(r_2, \theta_2, \phi_2)$ as the spherical coordinates of the electron with respect to image (I), we have

$$\Psi_1(r_1, \theta_1, \phi_1) = U(k, \theta_1, \phi_1) \frac{\exp(ikr_1)}{kr_1},$$

$$\Psi_2(r_2, \pi - \theta_2, \phi_2) = U(k, \pi - \theta_2, \phi_2)K \frac{\exp(i(kr_2 - \mu\pi/2))}{kr_2}. \tag{1}$$

Where $k = \sqrt{2E}$, and E is the detached-electron energy, $k_b$ is related to the binding energy $E_b$ of H$^-$ by $E_b = \frac{k_b^2}{2}$, B is a normalization constant and is equal to 0.31552, and K is the reflection parameter that accounts how much electron wave reflects from the wall. $U(k, \theta, \phi)$ is an angular factor, and for laser polarization parallel to z-axis it can be written as $U(k, \theta_1, \phi_1) = \frac{4k^2B_i}{(k_b^2 + k^2)^2} \cos\theta_1$, $U(k, \theta_2, \phi_2) = U(k, \pi - \theta_2) = -\frac{4k^2B_i}{(k_b^2 + k^2)^2} \cos\theta_2$.

Eqs. (1) becomes

$$\Psi_1(r_1, \theta_1, \phi_1) = \frac{4k^2B_i}{(k_b^2 + k^2)^2} \cos(\theta_1) \frac{\exp(ikr_1)}{kr_1},$$

$$\Psi_2(r_2, \theta_2, \phi_2) = -\frac{4k^2B_i}{(k_b^2 + k^2)^2}K \cos(\theta_2) \frac{\exp(i(kr_2 - \mu\pi/2))}{kr_2}. \tag{2}$$

Since $r_1$ and $r_2$ are large compared to the distance between H$^-$ and the wall d, we can simplify further. Let $(r, \theta, \phi)$ be the spherical coordinates of the detached-electron relative to the origin. For phase terms, we approximate as $r_1 \approx r - d \cos \theta$, $r_2 \approx r + d \cos \theta$, and in all other places in Eqs. (2), we use $r_1 \approx r_2 \approx r$, and $\theta_1 \approx \theta_2 \approx \theta$. With these approximations, the outgoing detached electron wave $\Psi^+$ from the system is given by

$$\Psi^+(r, \theta, \phi) = \frac{4k^2B_i}{(k_b^2 + k^2)^2} \cos(\theta) \left[e^{-ikd\cos \theta} - Ke^{i(kd\cos \theta - \mu\pi/2)}\right] \frac{\exp(ikr)}{kr}. \tag{3}$$

Eq. (3) represents the outgoing electron wave produced in the detachment of H$^-$ near a partial reflecting wall. We now find electron flux distribution on a screen at large distance and then total photodetachment cross section. The electron flux is defined as [12]

$$\vec{j}(r, \theta, \phi) = \frac{i}{2}(\Psi^+ \nabla \Psi^+ - \Psi^+ \Psi^+ \nabla). \tag{4}$$

Using the expression for $\Psi^+(r, \theta, \phi)$ in Eq. (3) and flux formula in Eq. (4), we obtain the electron flux distribution along the radial direction

$$\vec{j}_r(r, \theta, \phi) = \frac{16k^3B^2}{(k_b^2 + k^2)^4} \cos^2(\theta) \left[1 + K^2 + 2K \cos(2kd\cos \theta + \pi - \mu\pi/2)\right]. \tag{5}$$

We now calculate the total photodetachment cross section of negative ion near a partial reflecting wall. Imagine a large surface $\Gamma$ such as the surface of a semi-sphere enclosing the source region, a generalized differential cross section $\frac{d\sigma(q)}{ds}$ may be defined on the surface from the electron flux crossing the surface [12], $\frac{d\sigma(q)}{ds} = \frac{2\pi E_b}{c} \vec{j} \cdot \hat{n}$, where c is the speed of light approximately equal to 137 a. u., q is the coordinate on the surface $\Gamma$, $\hat{n}$ is the exterior normal vector at q, $ds = r^2 \sin \theta d\theta d\phi$ is the differential area on the spherical surface. The total cross section may then be obtained by integrating the differential cross section over the surface, $\sigma(q) = \int_{\Gamma} \frac{d\sigma(q)}{ds} ds$. Therefore, the first part of total photodetachment cross section of negative hydrogen ion near a partial reflecting wall is

$$\sigma_1(E) = \frac{\sigma_0(E)}{2} \left[1 + K^2 - 6K A_1(2d\sqrt{2E})\right]. \tag{6}$$
with
\[
A_1(u) = \left[ \frac{\sin(u - \mu \pi/2)}{u} + 2 \frac{\cos(u - \mu \pi/2)}{u^2} - 2 \frac{\sin(u - \mu \pi/2)}{u^3} - 2 \frac{\sin(\mu \pi/2)}{u^3} \right].
\]

Where, \( \sigma_0(E) = \frac{16\sqrt{2} \pi^2 E^{3/2}}{3(3E_b + E)^3} \), is the photodetachment cross section of \( H^- \) in the absence of reflecting wall, the argument \( u = 2d\sqrt{2E} \) of \( A_1 \) in Eq. (6) is equal to the action of the detached-electron going from the negative ion to the partial reflecting wall and back to the negative ion.

When the electron wave incidents on the surface of a wall, a part of it is absorbed. Let we denote this part be \( \Psi_K \) such that the sum of the reflected part and the absorbed part would be equal to the incoming electron wave to the wall. We introduce an absorption parameter \( T \) that measures how much of the electron wave is absorbed by the surface of the wall such that \( T^2 + K^2 = 1 \). The absorbed part of the detached-electron wave is then given by \( \Psi_K(r, \theta, \phi) = T\Psi(r, \theta, \phi) \), where \( \Psi(r, \theta, \phi) \) is an electron wave from the source \( H^- \) toward the wall. To calculate total cross section for the absorbed part of the detached-electron wave, we performed similar steps as for \( \Psi^+(r, \theta, \phi) \) but integration limits for \( \theta \) would be from \( \pi/2 \) to \( \pi \). Hence, the second part of the total cross section comes out
\[
\sigma_2(E) = \sigma_0(E) \left[ \frac{K^2 - 1}{2} \right].
\]

After adding the Eq. (6) and the Eq. (7), the total photodetachment cross section for \( H^- \) near a partial reflecting surface becomes
\[
\sigma(E, K) = \sigma_0(E)A(2d\sqrt{2E})a_0^2.
\]

Where \( A(u) \) is the modulation function and is defined by
\[
A(u) = 1 - 3K \left[ \frac{\sin(u - \mu \pi/2)}{u} + 2 \frac{\cos(u - \mu \pi/2)}{u^2} - 2 \frac{\sin(u - \mu \pi/2)}{u^3} - 2 \frac{\sin(\mu \pi/2)}{u^3} \right].
\]

It is clear that Eq. (8) reduces to the case as there is no wall for \( K = 0 \). Hence for this particular condition, the wall acts like a transparent medium for electron waves. For \( K = 1 \) Eq. (8) reduces to the results by Afaq and Du [12].

Fig. 2 using Eq. (9) shows the behavior of the modulation function \( A(u) \) for different values of \( K = 1, 0.7, 0.4 \) and \( \mu = 1, 1.5, 2 \). In Fig. 1(a), the soft wall case [12] is represented by thick solid line and the hard wall case [12] is represented by solid line, doted lines represent for the intermediary case. In Fig. 2 (a)-(c), we observe that the amplitude of oscillation decreases and phase changes due to change in values of \( K \) and \( \mu \). Fig. 3 using Eq. (8) with the exact value of modulation function in Eq. (9) shows total photodetachment cross section for the same values of \( K \) and \( \mu \) as in Fig. 2. For \( K = 1 \), the oscillation amplitude is large Fig. 2(a) and for \( K = 0.4 \), it becomes very small Fig. 2(c). It shows that oscillations in the photodetachment cross section can effectively be controlled by reflection parameter \( K \).

The reason is, the electron wave that initially propagates toward wall, a part of it is absorbed. Due to this absorption, the reflected part will possess low intensity electron wave. This reflected part appears coming from the image behind the wall. Two path interference occurs on a screen placed at very large distance from the system. Consequently, we observe a decrease in the oscillation amplitude of photodetachment cross section.

Assuming a screen perpendicular to z-axis is placed at a distance \( L \) from the wall, \( L \) is much greater than \( d \) and usually equal to thousands atomic units in the experiments [18, 19]. The flux distributions on the screen is cylindrically symmetric, it depends only on the distance between
Figure 2: The modulation function $A(x)$ is represented by using Eq. (9) for different values of $K$ and $\mu$. One can observe the change in oscillation amplitude and in phase with the change of values of $K$ and of $\mu$ respectively.
Figure 3: The total photodetachment cross section using Eq. (8) with the modulation function in Eq. (9) for values of $K = 1, 0.7, 0.4$ and $\mu = 1, 1.5, 2$. In these calculations, we used $d = 100$ Bohr radii.
Figure 4: The electron flux distributions using Eq. (10) on the screen for values of $K = 1, 0.5, 0.1$ and $\mu = 1, 2$. We have fixed photon energy $E_{ph} = 1eV$ and the distance between the wall and the screen $L = 10000$ Bohr radii.

any point $(x, y)$ on the screen and the z-axis $\rho = \sqrt{x^2 + y^2}$. By projecting the radial flux in Eq. (5) along z-direction and then adding flux due to absorbed electron wave function, the flux crossing the screen is

$$j_z(\rho) = 32k^3B^2 \frac{L^3}{(k^2 + k^2)^4} \left(1 + K \cos(\mu \pi/2) + 1 / \sqrt{\rho^2 + L^2} + \pi - \mu \pi/2) \right). \quad (10)$$

Fig. (4) using Eq. (10) represents the differential cross section across z-axis for different values of $K = 1, 0.5, 0.1$ and $\mu = 1, 2$. We have fixed photon energy $E_{ph} = 1eV$ and the distance between the wall and the screen $L = 10000$ Bohr radii. Fig. (4) represents the wall-induced interference for the differential cross section, which may be observable in the photodetachment microscopy experiments [18, 19].

In summary, the photodetachment of $H^-$ near a partial reflecting surface was studied using the theoretical imaging method. This method made possible the derivation of analytical formulas of total and differential cross sections in a straightforward manner. It is concluded that there is a strong relation between the reflection parameter $K$ and the oscillation of the cross sections. The
reflection parameter measures, how much of the electron wave would reflect from the wall. For \( K = 0 \) we get no oscillation, and for \( K = 1 \) we get maximum oscillation in the cross sections. On analyzing the photodetachment spectra for different values of \( K \), it may be possible to characterize surfaces using an electron beam as a probe just as various imaging and diffraction techniques, which have been developed for surface study [20, 21]. The detached-electron flux distributions on a screen placed at large distance from the negative ion is also obtained. The distributions displayed strong interference patterns. These patterns may be observable just as in the photodetachment microscopy experiments for negative ions in the presence of a static electric field [18, 19, 22]. I hope that this theoretical study of photodetachment of \( \text{H}^- \) near a partial reflecting surface will stimulate experiments, and may be useful in studying the surfaces.

I would like to thank Prof. M. L. Du for his useful discussion and comments.

References

[1] Smith S J and Burch D S 1959 Phys. Rev. Lett. 2 165.
[2] Ohmura T and Ohmura H 1960 Phys. Rev. 118 154.
[3] Yang G, Zheng Y and Chi X 2006 J. Phys. B 39 1855.
[4] Yang G, Zheng Y and Chi X 2006 Phys. Rev. A. 73 043413.
[5] Fabrikant I I 1980 Sov. Phys.-JETP 52 1045.
[6] Bryant H C et al 1987 Phys. Rev. Lett. 58 2412.
[7] Stewart J E et al 1988 Phys. Rev. A 38 5628.
[8] Rau A R P and Wong H 1988 Phys. Rev. A 37 632.
[9] Greene C H and Rouze N 1988 Z. Phys. D 9 219.
[10] Du M L and Delos J B 1988 Phys. Rev. A 38 5609.
[11] Kondratovich V D and Ostrovsky V N 1990 J. Phys. B 23 21.
[12] Afaq A and Du M L 2007 J. Phys. B: At. Mol. Opt. Phys. 40 1309.
[13] Du M L and Delos J B 1987 Phys. Rev. Lett. 58 1731.
[14] Du M L and Delos J B 1988 Phys. Rev. A. 38 1896.
[15] Du M L and Delos J B 1988 Phys. Rev. A 38 1913.
[16] Bracher C et al 2005 Phys. Lett. A 347 62.
[17] Du M L 1989 Phys. Rev. A 40 4983.
[18] Blondel C, Delsart C and Dulieu F 1996 Phys. Rev. Lett. 77 3755.
[19] Blondel C et al 2005 Eur. Phys. J. D 33 335.
[20] Yagi K 1987 J. Appl. Cryst. 20 147.
[21] Minoda H and Yagi K 1999 Phys. Rev. B 60 2715.
[22] Du M L 2004 Phys. Rev. A 70 055402.