Testing Quantum Mechanics in Neutrino Oscillation

Fengcai Ma 1 and Haiming Hu 2,3
1 Department of Physics, Liaoning University, Shenyang 110036, P.R.China
2 CCAST (World Laboratory), Beijing 100080, P.R.China
3 Institute of High Energy Physics, Academia Sinica,
P.O.Box 918, Beijing 100039, P.R.China

Abstract

A scenario of testing quantum mechanics in neutrino oscillation is presented. The quantum mechanics violation (QMV) that is motivated by arguments based on quantum gravity is investigated in neutrino system. It is found that the evolution equation of density matrix including QMV effect is analytically resolvable for neutrino propagate in vacuum or in matter under adiabatic approximation. The analytical formulas have been derived. Some bounds on the related parameters have been obtained from neutrino experiments.

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1 Introduction

It is most essential and important issue that whether neutrino has nonzero mass and whether there is neutrino oscillation because which is related to the theoretical structure of standard model (SM) of particle physics and is also related to astrophysics and cosmology. Recent experiments[1]-[5] have shown that there are signals of neutrino oscillation. If it is confirmed, neutrino system may not only be a window beyond SM but also be a fruitful system for testing quantum mechanics and probing discrete symmetries of Nature as in $K^0\bar{K}^0$ system.

The suggestion that quantum coherence might be lost at the microscopic level was made by Hawking[6], which suggested that asymptotic scattering should be described in term of a superscattering operator $S$, relating initial and final density matrices, that does not factorize as a product of $S-$ and $S^+$- matrix elements

$$\rho_{\text{out}} = S\rho_{\text{in}}, \quad S \neq SS^+. \quad (1)$$

The loss of quantum coherence was thought to be a consequence of microscopic quantum-gravitational fluctuations in the space- time background. Ellis, Hagelin, Nanopoulos and Srednicki (EHNS)[7] then pointed out that if Eq.(1) is correct for asymptotic scattering, there should be a corresponding effect in the quantum mechanics Liouville equation that describes the time evolution of the density matrix $\rho(t)$

$$\partial_t \rho(t) = i[\rho, H] + i\delta H \rho \quad (2)$$

The extra term may evolve a pure state into mixed state and result in quantum mechanics violation (QMV). The $\delta H$ was parameterized for simple two level system by EHNS[7]
and was used in neutron and $K^0\bar{K}^0$ systems. Then the QMV was through studied by the authors of references[8],[9],[10] for $K^0\bar{K}^0$ system, and by the authors of Ref.[11] for $B\bar{B}$ system. Some restrictions on QMV parameters have been given.

A beam of two flavor neutrinos is a simple two-state system. If mass eigenstate is not degeneracy with weak interaction eigenstate, then mixing between two flavor will exist and neutrino oscillation will occur. As in $K^0\bar{K}^0$ system, the quantum mechanics violation originate from quantum gravitational fluctuation may also exist in neutrino system. We investigate QMV in neutrino system in term of the formalism proposed by EHNS[7] and find that neutrino oscillation probability can be modified by QMV effect. Thus the neutrino oscillation experiments may present a precise testing for quantum mechanics.

The main goal of this paper is that establish a framework of QMV in neutrino system, derive the detailed formulas of neutrino oscillation probability including QMV effect, and present some bounds on QMV parameters. For completeness we will first derive the formulas of neutrino oscillation probabilities described by density matrix formalism within conventional quantum mechanics in section II. The QMV for neutrino oscillation in vacuum and in matter will be investigated in sections III and IV respectively. We will apply the formulas derived in previous section to current neutrino experiments and draw out certain restrictions on QMV parameters in section V. Our conclusions are presented in section VI.

2 Neutrino oscillation described in term of density-matrix formalism

We now study a system consisted of two-flavor neutrinos in vacuum. The Hamiltonian of this system, in mass eigenstate, is diagonal, $H = diag(E_1, E_2)$, here $E_1$ and $E_2$ are energy eigenvalues of neutrinos. In some practical problems, for instance solar neutrino detection or atmospheric neutrino observation, $m_j \ll E_j$ are always valid, then $E_i \approx E + \frac{m^2_i}{2E}$, $i = 1, 2$, are good approximations, here $E \equiv |p|$ is the magnitude of three momentum of neutrinos. We describe the time evolution of the neutrino system by Liouville equation in conventional quantum mechanics

$$\frac{d}{dt}\rho(t) = i[\rho, H],$$

where $\rho$ is the density matrix of the system. Following EHNS[7], we expand the Hamiltonian and density matrix in Pauli matrix basis

$$H = \frac{1}{2} h_\alpha \sigma_\alpha, \quad \rho = \frac{1}{2} \rho_\beta \sigma_\beta, \quad \alpha, \beta = 0, 1, 2, 3,$$

where $\sigma_0$ is unit matrix and $\sigma_j$ ($j = 1, 2, 3$) are Pauli matrices. The evolution of the components of density matrix obey

$$\partial_t \rho_\alpha = h_{\alpha\beta} \rho_\beta.$$
We assume that there are only electron neutrinos in the system at the moment $t = 0$. Considering this initial condition and solving the differential equation of Eq.(5), we get the density matrix

$$\rho(t) = \begin{pmatrix} \cos^2 \theta & \frac{1}{2} \sin 2 \theta e^{-i \frac{\pi}{2} t} \\ \frac{1}{2} \sin 2 \theta e^{i \frac{\pi}{2} t} & \sin^2 \theta \end{pmatrix},$$  

(6)

where the $\theta$ is the vacuum mixing angle of neutrinos. The probability that electron neutrino oscillate to muon neutrino is given by computing the expectation value of $\text{Tr}(O\rho)$ of an observable

$$O(\nu_\mu) = \begin{pmatrix} \sin^2 \theta & -\frac{1}{2} \sin 2 \theta \\ -\frac{1}{2} \sin 2 \theta & \cos^2 \theta \end{pmatrix}. $$  

(7)

It is easy to check that the oscillation probability

$$P(\nu_e \rightarrow \nu_\mu) = \text{Tr}[O(\nu_\mu)\rho(t)] $$  

(8)

is consistent with that obtained from solving propagation equation of neutrino\cite{12}.

If neutrino propagate in matter, the Hamiltonian is corrected as\cite{13}

$$\bar{H} = E + \frac{m_1^2 - m_2^2}{4E} - \frac{1}{\sqrt{2}} G_F N_n + \frac{1}{2} \bar{M}^2, $$  

(9)

where

$$\bar{M}^2 \equiv \frac{1}{2} \begin{pmatrix} -\Delta \cos 2 \theta + 2A & \Delta \sin 2 \theta \\ \Delta \sin 2 \theta & \Delta \cos 2 \theta \end{pmatrix}, \quad A = 2\sqrt{2} G_F N_e E, $$  

(10)

the $N_e$ and $N_n$ are the number densities of electrons and neutrons in matter. The Hamiltonian can be diagonalized by an unitary matrix $\bar{U}(\bar{\theta})$, here the $\bar{\theta}$ is effective mixing angle of neutrinos in matter, which is determined by

$$\tan 2\bar{\theta} = \frac{\Delta \sin 2 \theta}{\Delta \cos 2 \theta - A}. $$  

(11)

The diagonalized Hamiltonian can be expanded in Pauli matrix basis as

$$\bar{H}_d = \frac{\Sigma}{2} \sigma_0 - \frac{1}{2} \lambda \sigma_3, $$  

(12)

here

$$\Sigma \equiv 2[E - \frac{1}{\sqrt{2}} G_F N_n + \frac{1}{4E} (m_1^2 + m_2^2 + A)], $$  

(13)

$$\lambda \equiv \frac{1}{2E} \sqrt{\Delta \cos 2 \theta - A}^2 + \Delta \sin^2 2 \theta. $$  

(14)

In this situation, the $h_{\alpha\beta}$ in Eq.(5) becomes

$$h_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & -\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. $$  

(15)
In general, it is difficult to solve analytically the evolution equation of the components of density matrix, since the $\lambda$ is a complicated function of time. However, it is shown (see Appendix A) that if adiabatic condition is valid, then the approximation

$$\left| \frac{1}{\lambda} \frac{d\lambda}{Edt} \right| \ll 1$$

(16)

is available, the term being proportional to $(d\lambda/dt)$ can be neglected during solve the differential equation of density matrix. The evolution equation of density matrix can be analytically solvable. We still assume that there is only electron neutrino in system and the effective mixing angle is $\bar{\theta}$ at the moment $t = 0$. Under this initial condition the density matrix can be evaluated out

$$\rho(t) = \left( \begin{array}{cc} \cos^2 \bar{\theta} & -\frac{1}{2} \sin 2\bar{\theta} \exp(i \int_0^t \lambda dt') \\ \frac{1}{2} \sin 2\bar{\theta} \exp(-i \int_0^t \lambda dt') & \sin^2 \bar{\theta} \end{array} \right).$$

(17)

If neutrinos are detected in vacuum, the oscillation probability is calculated as

$$P(\nu_e \rightarrow \nu_\mu) = \frac{1}{2} \left( 1 - \cos 2\theta \cos 2\bar{\theta} \right) - \frac{1}{2} \sin 2\theta \sin 2\bar{\theta} \cos \left( \int_0^t \lambda dt' \right).$$

(18)

Which is just the well known MSW solution of solar neutrino problem.

3 QMV for neutrino oscillation in vacuum

We now introduce QMV in neutrino oscillation. Following EHNS the extra term in Eq.(2) may be parameterized as a symmetrical $4 \times 4$ matrix $h'_{\alpha\beta}$, then the evolution equation of the components of density matrix is rewritten as

$$\partial_t \rho_\alpha = (h_{\alpha\beta} + h'_{\alpha\beta}) \rho_\beta.$$  

(19)

The matrix $h'_{\alpha\beta}$ must obey some restrictions. The probability is conserved, entropy must be real and never decrease, which imply that $h'_{\alpha_0} = h'_{0\beta} = 0$ and $h'_{\alpha\beta}$ is negative semidefinite. The energy conservation in neutrino oscillation demands that $h'_{30} = h'_{03} = 0$ have to be imposed. The $h'_{\alpha\beta}$ is thus parameterized as

$$h'_{\alpha\beta} = \left( \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & -\alpha & -\beta & 0 \\ 0 & -\beta & -\gamma & 0 \\ 0 & 0 & 0 & 0 \end{array} \right),$$

(20)

$$\alpha > 0, \quad \gamma > 0, \quad \alpha \gamma > \beta^2.$$  

By solving the Eq.(19) we get the density matrix

$$\rho(t) = \left( \begin{array}{cc} \cos^2 \bar{\theta} & \frac{1}{2} \sin 2\bar{\theta} \left( \rho_1 - i \rho_2 \right) \\ \frac{1}{2} \sin 2\bar{\theta} (\rho_1 + i \rho_2) & \sin^2 \bar{\theta} \end{array} \right),$$

(21)
where the $\rho_1$ and $\rho_2$ have different forms due to the magnitude relation between the difference of neutrino mass square and the QMV parameters, which will be discussed as follows.

(i) If the condition

$$(\alpha - \gamma)^2 + 4\beta^2 < \frac{\Delta}{E}$$

(22)

is satisfied, then $\rho$ has a oscillation-like solution

$$\rho_1 = \cos\delta_1 t \ e^{-\frac{\alpha + \gamma}{2} t},$$

$$\rho_2 = \frac{1}{\frac{\Delta}{2E} - \beta}(\frac{\alpha - \gamma}{2}\cos\delta_1 t - \delta_1 \sin\delta_1 t) e^{-\frac{\alpha + \gamma}{2} t},$$

(23)

where

$$\delta_1 \equiv \frac{\Delta}{2E} \sqrt{1 - \frac{(\alpha - \gamma)^2 + 4\beta^2}{(\Delta/E)^2}}.$$ 

(24)

The oscillation probability is

$$P(\nu_e \rightarrow \nu_\mu) = \frac{1}{2} \sin^2 2\theta(1 - \cos\delta_1 t \cdot e^{-\frac{\alpha + \gamma}{2} t}).$$

(25)

(ii). If the relation

$$(\alpha - \gamma)^2 + 4\beta^2 \geq \frac{\Delta}{E}$$

(26)

is valid, the $\rho$ has an exponential-like solution

$$\rho_1 = [\exp(\frac{\delta_2}{2} t) + \exp(-\frac{\delta_2}{2} t)] e^{-\frac{\alpha + \gamma}{2} t},$$

$$\rho_2 = \frac{1}{\frac{\Delta}{2E} - 2\beta}[(\alpha - \gamma + \delta_2)\exp(\frac{\delta_2}{2} t) + (\alpha - \gamma - \delta_2)\exp(-\frac{\delta_2}{2} t)] e^{-\frac{\alpha + \gamma}{2} t},$$

(27)

here

$$\delta_2 \equiv \sqrt{(\alpha - \gamma)^2 + 4\beta^2 - \frac{\Delta}{E}^2}.$$ 

(28)

The probability of neutrino oscillation becomes

$$P(\nu_e \rightarrow \nu_\mu) = \frac{1}{2} \sin^2 2\theta[1 - \frac{1}{2}(\exp(-\frac{\alpha + \gamma + \delta_2}{2} t) + \exp(-\frac{\alpha + \gamma - \delta_2}{2} t))].$$

(29)

It is easy to show that $Tr\rho^2 \neq 1$, the pure state may evolve into mixed state. With the $\alpha, \beta, \gamma \rightarrow 0$, the $\rho(t)$ return to Eq.(6) in these two situations, quantum mechanics recovers.
4 QMV for neutrino oscillation in matter

In the situation of neutrino oscillation in matter, we add the parameterized QMV term of Eq.(20) into the Hamiltonian of Eq.(15) and get the evolution equation of the components of density matrix

$$
\frac{\partial}{\partial t} \begin{pmatrix}
\rho_0 \\
\rho_1 \\
\rho_2 \\
\rho_3
\end{pmatrix} = 
\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & -\alpha & \lambda - \beta & 0 \\
0 & -\lambda - \beta & -\gamma & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\rho_0 \\
\rho_1 \\
\rho_2 \\
\rho_3
\end{pmatrix}
$$

Taking the adiabatic approximation and the initial condition that only electron neutrinos exist in the system into account, solving differential equations Eq.(30), we obtain the density matrix

$$
\rho(t) = \begin{pmatrix}
\cos^2\bar{\theta} \\
\frac{1}{2}\sin 2\bar{\theta}(\rho_1 + i\rho_2) \\
\frac{1}{2}\sin 2\bar{\theta}(\rho_1 - i\rho_2)
\end{pmatrix},
$$

where $\bar{\theta}$ is the effective mixing angle of neutrinos in matter at the initial moment. The $\rho_1$ and $\rho_2$ are given as below.

(i) In the situation of

$$(\alpha - \gamma)^2 + 4\beta^2 < 4\lambda^2,$$

then

$$
\rho_1 = \cos(\int_0^t \delta_3 dt') \cdot e^{-\frac{\alpha + \gamma}{2} t},
$$

$$
\rho_2 = \frac{1}{\lambda - \beta} \left[ \frac{\alpha - \gamma}{2} \cos(\int_0^t \delta_3 dt') - \delta_3 \sin(\int_0^t \delta_3 dt') \right] e^{-\frac{\alpha + \gamma}{2} t},
$$

where

$$
\delta_3 \equiv \frac{1}{2} \sqrt{4\lambda^2 - (\alpha - \gamma)^2 - 4\beta^2}.
$$

The neutrino oscillation probability is calculated as

$$
P(\nu_e \rightarrow \nu_\mu) = \frac{1}{2} (1 - \cos 2\theta \cos 2\bar{\theta}) - \frac{1}{2} \sin 2\theta \sin 2\bar{\theta} \cdot \cos(\int_0^t \delta_3 dt') e^{-\frac{\alpha + \gamma}{2} t}.
$$

(ii) In another situation

$$(\alpha - \gamma)^2 + 4\beta^2 \geq 4\lambda^2,$$

then

$$
\rho_1 = \frac{1}{2} \cosh(\int_0^t \delta_4 dt') e^{-\frac{\alpha + \gamma}{2} t},
$$

$$
\rho_2 = \frac{1}{2(\lambda - \beta)} [(\alpha - \gamma + \delta_4) \exp(\int_0^t \delta_4 dt') + (\alpha - \gamma - \delta_4) \exp(-\int_0^t \delta_4 dt')] e^{-\frac{\alpha + \gamma}{2} t},
$$

where

$$
\delta_4 \equiv \frac{1}{2} \sqrt{(\alpha - \gamma)^2 + 4\beta^2 - 4\lambda^2}.
$$
The neutrino oscillation probability becomes

$$P(\nu_e \rightarrow \nu_\mu) = \frac{1}{2}(1 - \cos 2\theta \cos 2\bar{\theta}) - \frac{1}{4} \sin 2\theta \sin 2\bar{\theta} \cosh(\int_0^t \delta_4 dt') e^{-\frac{2+6}{2}t}. \quad (39)$$

It is not difficult to see that with the QVM parameter $\alpha, \beta, \gamma \rightarrow 0$, the probability $P(\nu_e \rightarrow \nu_\mu)$ back to Eq.(18) in the two situations, which is adiabatic MSW solution of solar neutrino problem. It is easy to calculate that $Tr|\rho|^2 \leq 1$. In this case, pure state may evolve into mixed state, and thus quantum mechanics is violated.

5 An estimation for the bounds on QMV parameters

We have derived the formulas of QMV effect in neutrino oscillation, which is to be tested in experiments. A complete analysis requires a detailed understanding of all neutrino experiments which goes beyond the scope of this paper. We present here only a rough estimate of maximum order of magnitude of the QMV parameters as an illuminating example of testing quantum mechanics in neutrino oscillation. It is estimated theoretically that the maximum possible order of magnitude for QMV parameters $\alpha$, $|\beta|$, or $\gamma$ is $O(E^2/m_{Pl})$, which in $K^0\bar{K}^0$ system is $\sim 10^{-19} GeV$, where $E$ is a typical energy scale in the system under discussion, and $M_{Pl}$ is the Plank energy scale. The bounds on these parameters from $K^0\bar{K}^0$ experiments have been obtained in Refs.$[8],[9]$ and $[10]$, the last one gives

$$\alpha_{K^0\bar{K}^0} \leq 4 \times 10^{-17} \text{ GeV}, \quad |\beta_{K^0\bar{K}^0}| \leq 3 \times 10^{-19} \text{ GeV}, \quad \gamma_{K^0\bar{K}^0} \leq 7 \times 10^{-21} \text{ GeV}. \quad (40)$$

Similary, in neutrino system, it is expected theoretically that the order of magnitude of the maximum parameter is $O(E_\nu^2/m_{Pl})$, where $E_\nu$ is a typical energy scale of neutrino system. This is the order of $\sim 10^{-22} GeV$ for solar neutrinos or reactor neutrinos. In general, we may reasonably assume that the order of the maximum parameters in neutrino system is $O(E_\nu^2/m_{Pl}) \sim (E_{K^0\bar{K}^0})^2 \alpha_{K^0\bar{K}^0}$, and that the magnitude relation among these parameters is retained in neutrino system, i.e., $\alpha \gg |\beta| \gg \gamma$.

In the formulas of neutrino oscillation probability, the QMV parameters appear always in company with time $t$, for example $\alpha t$. There is no enough large time to have the $\alpha t \sim 1$ in current territorial experiments. Therefore, we discuss here only solar neutrino experiments because of large neutrino propagation time. For the situation of neutrino oscillation in vacuum which is a possible solution of solar neutrino problem$[16]$. From Eq.(25), we get

$$\alpha + \gamma \leq \frac{2}{l} \ln |\frac{\sin^2 2\theta - 2P}{\sin^2 2\bar{\theta}}|, \quad (41)$$

where (and hereafter) the $P(\nu_e \rightarrow \nu_\mu)$ is simply written as $P$. If the condition Eq.(26) is satisfied, we get

$$\alpha + \gamma > \frac{\Delta}{E}. \quad (42)$$
From above arguments of the order of magnitude of the maximum parameters and the order of maximum parameter of $K^0\bar{K}^0$ system in Eq.(40), we get an upper limit of another allowed region

$$ (\alpha + \gamma)_{max} \leq 1.6 \times 10^{-20} \text{GeV} \quad (43) $$

Two allowed regions of the $\alpha + \gamma$ are decided by Eq.(41)-(43).

We studied the upper limit of $\alpha + \gamma$ given by Eq.(41) vary with the $\sin^2 2\theta$, the numerical results is shown in Fig.1. The lower and upper limits of the second allowed region are also shown in same figure. The neutrino oscillation probability $P$ is taken from the Kamiokande experimental data and the prediction of standard solar model (SSM) [17]. The region of $\sin^2 2\theta$ is given by recent fit [18] of world solar neutrino experiments. The $\Delta$ is taken as the center value of $\Delta \sim 6 \times 10^{-11} \text{GeV}^2$, and $E \simeq 10\text{MeV}$ is used.

In other situation, we discuss neutrino oscillation in matter, which is another possible solution (MSW effect) [15] of solar neutrino problem. From Eq.(35) we get

$$ \alpha + \gamma \leq -\frac{2}{t} \ln \left| \frac{1 - \cos 2\theta \cos 2\bar{\theta} - 2P}{\sin 2\theta \sin 2\bar{\theta}} \right|. \quad (44) $$

From Eq.(37) and Eq.(21), we get

$$ \alpha + \gamma > 2\lambda. \quad (45) $$

Two allowed regions of $\alpha + \gamma$ are given by Eqs.(43)-(45) for the case of including matter effect. From the Kamiokande experimental data, SSM predictions, as well as the fit [18] of experimental data to the adiabatic MSW solution of solar neutrino problem, we obtain the numerical result of variation of the parameter $\alpha + \gamma$ with the $\sin^2 2\theta$, which is shown in Fig.2. In numerical calculation, we used the center value of the $\Delta$ given in Ref. [15], $\Delta \sim 1.6 \times 10^{-5} \text{eV}^2$, and $E \simeq 10\text{MeV}$. We also apply the average value of the $\sin^2 2\theta$ over the radius of the Sun. The electron distribution in the Sun is taken from Ref.[20].

It should be emphasized that our numerical result is only an example far from an exact analysis. The aim is to show how the information of bounds on QMV parameters is extracted from neutrino oscillation experiments. The value of the bounds are different if the different experimental data are used. A complete numerical analysis based on our analytical formulas is necessary. It is noted that the QMV for neutrino system in vacuum was discussed in Ref.[19]. However, there is no analytical result and no any information about bounds on parameters in their discussions. Moreover, what discussion in Ref.[19] is the case of neutrino oscillation in vacuum, but the neutrino mixing parameters used in their calculation are taken form the values of MSW effect given in Ref.[21], there is no any room of small mixing angle to be allowed by experiments for vacuum oscillation solution of solar neutrino problem.

6 Conclusions

Neutrinos may be a useful system to precisely test quantum mechanics. the Liouville equation that describes the time evolution of density matrix of neutrino system can be modified by adding an extra term, which may be parameterized as a $4 \times 4$ matrix. The
modified evolution equation of density matrix can be solved analytically if neutrinos propagate in vacuum or if neutrinos propagate in non-uniform matter but adiabatic condition is available. The analytical expressions of neutrino oscillation probability with or without QMV effect have been derived. Based on theoretical analysis and neutrino experiment results we have extracted the bounds on the QMV parameters. Two allowed region of $\alpha + \gamma$ have been obtained. It is expected that more precise restrictions on QMV parameters may be obtained from future neutrino experiments and complete numerical analysis.

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Appendix A: Generalized adiabatic condition
In this appendix we will demonstrate that if the adiabatic condition is valid the term being proportional to $(d\lambda/dt)$ can be neglected when we solve the evolution equation of the components of density matrix. From the Eq(14) we get

$$\frac{1}{\lambda} \frac{d\lambda}{dt} = \frac{1}{\sqrt{(\cos 2\theta - A/\Delta)^2 + \sin^2 2\theta}} \frac{d\lambda}{E \lambda dt}.$$  \hspace{1cm} (A1)

We define two dimensionless parameters $C$ and $G$ as

$$C \equiv \frac{1}{E \lambda dt} \frac{d\lambda}{dt},$$  \hspace{1cm} (A2)

where $E$ is a typical energy scale in the system under discussion, which is taken as neutrino energy in the system considered here.

$$G \equiv \frac{|\sin 2\theta \cos 2\bar{\theta}|}{|\sin 2\bar{\theta}|},$$  \hspace{1cm} (A3)

where $\bar{m}_{2,1}^2$ are the eigenvalues of the matrix $\bar{M}^2$ defined in Eq.(10). The adiabatic condition is expressed \[ as $G \ll 1$. From Eqs.(11) and (A1)-(A3), we get

$$\frac{C}{G} = \frac{\Delta}{E^2} \frac{|\sin 2\theta \cos 2\bar{\theta}|}{\sin^2 2\bar{\theta}}.$$  \hspace{1cm} (A4)

For the problems related to current neutrino experiments, we estimate the maximum value of the righthanded of Eq.(A4). $\Delta_{max} \leq 10eV^2, E_{min} \sim 1MeV$, and $(|\sin 2\theta \cos 2\bar{\theta}|/\sin^2 2\bar{\theta})_{max} \leq 10^3$, We thus get

$$\frac{C}{G} \leq 10^{-8}.$$  \hspace{1cm} (A5)

If adiabatic condition is valid, $G \ll 1$, we have $C \ll 1$. Therefore, we get the generalized adiabatic conditions in density matrix description of neutrino system. we can safely neglected the term containing $(d\lambda/dt)$ during we solve the differential equation of density matrix as long as the adiabatic condition is satisfied. Under adiabatic approximation the analytical solution of the evolution equation can be obtained.


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Figure 1:
Fig. 2 The limits of the parameters $\alpha + \gamma$ vary with $\sin^22\theta$ for neutrino oscillation in vacuum. The curve III is an upper limit for the case of $(\alpha - \gamma)^2 + 4\beta^2 < \frac{\Delta m^2}{2E}$. The line II is a lower limit for the case that above condition is not valid. The line I is an upper limit from the theoretical arguments and the experimental upper limit of the maximum parameter in $K^0\bar{K}^0$ system. The arrows indicate the two allowed regions.

Fig. 2 Same as Fig. 1 except for the neutrino oscillation in matter.