Quantum gravity and cosmological observations

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Abstract. Quantum gravity places entirely new challenges on the formulation of a consistent theory as well as on an extraction of potentially observable effects. Quantum corrections due to the gravitational field are commonly expected to be tiny because of the smallness of the Planck length. However, a consistent formulation now shows that key features of quantum gravity imply magnification effects on correction terms which are especially important in cosmology with its long stretches of evolution. After a review of the salient features of recent canonical quantizations of gravity and their implications for the quantum structure of space-time a new example for potentially observable effects is given.

Keywords: Quantum gravity, early universe, curvature perturbation
PACS: 98.80.Cq, 98.80.Qc, 04.60.Pp

GRAVITY

The gravitational field is the only known fundamental force not yet quantized despite of more than six decades of research. Difficulties arise due to two key properties: Although gravity is the dominant player on cosmic scales, it is weak in usual regimes of particle physics. Strong quantum gravity effects, possibly accessible to observations, thus require large gravitational fields which are realized only in exotic situations such as the very early universe or black holes. But then, the classical field grows without bound, implying space-time singularities. Secondly, gravity is conceptually very different from other interactions due to the equivalence principle: gravity is a manifestation of space-time geometry. The full space-time metric \( g_{\mu\nu} \) is the physical object to be quantized, not perturbations \( h_{\mu\nu} \) on a background space-time such as Minkowski space \( \eta_{\mu\nu} \).

Thus, any quantization of gravity able to describe these phenomena faithfully must be non-perturbative and background independent for most applications. A new framework is required which does not refer to causality or vacuum states and other concepts which are available only once a metric has already been specified. One has to make use of the quantum structure of space and time itself.

Although mathematically involved, this is now available in broad form due to research in the past 15 years. It is not yet uniquely formulated, but several characteristic properties have been revealed. All this is essential for non-singular quantum space-times, making sense even at the big bang, but also for quantum corrections in strong field regimes as they might be observable as remnants of the very early universe. The stage is thus provided for the first potentially observable effects of quantum gravity.
POSSIBLE EFFECTS?

There is a dimensional argument which usually is taken as proof that quantum gravity effects are tiny, too tiny to be observable anytime soon. Given Planck’s constant $\hbar$ and Newton’s constant $G$, one can (in units where the speed of light is $c = 1$) define the Planck length $\ell_P = \frac{\sqrt{G \hbar}}{c} \approx 10^{-33}$ cm. Its value is tiny compared to any scales we can probe directly, or equivalently the Planck mass $M_P = \frac{\hbar G}{c^2} \approx 10^{19}$ GeV is huge compared to the mass of any elementary object. Only negligible correction terms are then expected from dimensionless combinations of available length scales, e.g. of the order $\ell_P H \approx 10^{60}$ in cosmology with the current Hubble length $H^{-1} = a^{-1}$. Indeed, correction terms in low energy effective actions, obtained by perturbative approaches on a background space-time, give only such negligible terms in equations of motion [1].

However, for quantum gravity the low energy effective action is too special unless one is interested only in scattering of gravitons. A low energy effective action is obtained by an expansion around the vacuum state of quantum field theory on a background. The concept of a vacuum itself changes in background independent approaches since a vacuum state, defined e.g. as the unique Poincaré invariant state of a quantum representation, refers to symmetries of a background space-time. (Or, as the ground state of the Hamiltonian a vacuum is uniquely defined only for time-independence, which requires a symmetry.) Moreover, the gravitational Hamiltonian as it arises from the action is always unbounded from below and thus lacks a ground state in the usual sense.

There is an additional expectation from quantum gravity, namely that space has a discrete structure on very small scales. One can think of this structure as an irregular lattice whose typical plaquette size $p$ is close to $\ell_P$. But unlike the Planck length, this is a geometrical parameter or field specifying the quantum gravity state and can thus be dynamical. This parameter brings in crucial information from quantum gravity, unlike $\ell_P$ which is determined simply by parameters of quantum mechanics and classical gravity.

In such a situation, there are three length parameters: the macroscopic scale $H^{-1}$, the fundamental scale $p$ and the dimensional Planck length $\ell_P$. Any dimensional argument must then fail since dimensionless combinations of three length parameters can have any value depending on which geometric mean $l_1^{x_1} l_2^{x_2} l_3^{x_3}$ with $x_1 + x_2 + x_3 = 0$ is relevant in a given situation. In other words, a large dimensionless parameter exists such as the number $N$ of lattice sites which may enter and magnify correction terms. The precise form of corrections can then only be determined by detailed calculations taking into account the discrete structure of space. (See also [2] for a critique of dimensional arguments in quantum gravity.)

SPACE-TIME STRUCTURE

A quantum theory for space-time structure is required, which is not unlike atomic pictures in condensed matter physics. From the two basic properties of general relativity we obtain two important lessons for the formulation of quantum gravity:

- Do not use a split $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ of the space-time metric as a background $\eta_{\mu\nu}$ with a perturbation $h_{\mu\nu}$, as this does not allow one to describe a lattice
structure. One rather has, at a fundamental level, metrics which are distributional and supported only on lattice links and vertices.

- Do not use low energy effective theory but suitable generalizations. Low energy theory does not apply due to the unboundedness of the Hamiltonian, and it would even be unclear what “low energy” refers to given that there are no observer independent concepts of energy in general relativity.

All these techniques are now available in the framework of loop quantum gravity [3, 4, 5]. In particular, cosmological effects are calculable with recent progress. The six main ingredients in this general scheme are described in what follows.

1. New variables

A canonical formulation of general relativity was originally developed in ADM variables [6] given by the spatial metric $q_{ab}$ and momenta related to extrinsic curvature $K_{ab}$ (or $\dot{q}_{ab}$). This refers to a chosen time coordinate, but the theory is independent of that choice if certain constraints are satisfied. These constraints are equivalent to Einstein’s equation and implement general covariance. However, when trying to quantize a theory whose dynamical variable is the metric, it is difficult to turn tensor components such as $q_{ab}$ itself into operators. In generally covariant systems one has to take into account arbitrary, non-linear changes of coordinates $x^0(x)$. A tensor such as $q_{ab}$ then transforms as $q_{a0}^0 = \left(\frac{\partial x^a}{\partial x^0}\right) q_{ab} \left(\frac{\partial x^b}{\partial x^0}\right)$ which leads to coordinate dependent factors. A quantization of gravity needs to represent the field $q_{ab}$ and its momenta as operators on a Hilbert space, but the definition must be independent of spatial coordinates which are not defined on the Hilbert space. This is the key difference to quantum field theories defined on, say, Minkowski space. This background space-time allows only Poincaré transformations $x^0(x)$ as coordinate changes, which are linear. The coordinate change of tensors is then a simple linear transformation by spatially constant matrices, which can easily be extended to operators.

This problem, as it turned out, can be circumvented by using a new set of variables [7, 8]: $q_{ab}$ is replaced by a co-triad $e^i_a$ (three co-vector fields such that $e^i_a e^j_b = q_{ab}$), which then defines the densitized triad $E^i_a = \det e^i_a e^j_b$. Similarly, one defines $K^i_a$ with $\Gamma^i_a = \epsilon^{ijk} e^j_a (\partial_k e^l_b e^i_l)$.

2. Basic objects

These are variables as in non-Abelian gauge theories, the Ashtekar–Barbero connection $A^i_a$ canonically conjugate to “electric fields” $E^i_a$: $\{A^i_a(x) ; E^b_j(y)\} \equiv \pi \delta^i_a \delta^b_j \delta(x-y)$.

The gauge group here is the group of triad rotations $E^i_a \Rightarrow R^i_a E^i_a$, $R \in SO(3)$, which leave the metric unchanged. Moreover, for any curve $e$ and surface $S$ in space we define holonomies and fluxes [9]

$$h_e(A) = \mathcal{P} \exp \int_e A^i_a \tau_i e^a dt ; \quad F_S(E) = \int_S d^2 y n_a E^a_i \tau_i$$

(1)
with the tangent vector $\dot{e}^a$ of $e$, the co-normal $n_a$ of $S$ and Pauli matrices $\tau_i$. These quantities have many advantages. (i) They do not have any indices and are thus scalar quantities. No complicated tensor transformations under non-linear coordinate transformations arise. (ii) A smearing is automatically included, resulting in well-defined Poisson brackets to become commutators, free of any delta functions. (iii) No reference is made to any metric other than that determined by $E^a_i$, and background independence of the classical theory is thereby respected.

3. Representation

One can thus construct a quantum representation of these smeared basic fields and then impose the necessary constraints as operators. A convenient way to do this is the connection representation, where holonomies are used as multiplication operators “creating” spin networks $T_{gij} k(A) = \prod_{v \in g} C_v \prod_{e \in g} \rho_j(e(A))$ as an orthonormal basis of $h_e$-dependent functions, where $g$ is an oriented graph with labels $j$ as SU(2) representations $\rho_j$ on edges, and $C$ as gauge invariant contraction matrices in vertices.

4. Discrete geometry

Fluxes are conjugate to holonomies and thus become derivative operators

$$\hat{F}_{S}^{fg} = 8\pi i g \sum_{S} d^2y_n a \frac{\delta}{\delta A^A_i(y)} f_g(A) = 8\pi i \sum_{e \in g} d^2y_n a \frac{\delta}{\delta A^A_i(y)} \partial f_g(A)$$

when acting on a state $f_g$ as a linear combination of spin networks. With $R_S d^2y_n a \delta h_e = \delta A^A_i(y)$, we have non-zero contributions only if $S$ intersects edges of $g$, and contributions are determined by $su(2)$ derivatives $J_i = \text{tr} (\tau_i h \partial = \partial h)$ acting on holonomies. As invariant derivatives on a compact group (identical to angular momentum operators), a discrete spectrum of fluxes and thus discrete spatial geometry results. This also extends to other geometrical operators such as the area operator, obtained as a quantization of $A(S) = R_S d^2y E^a_n e E^b_n p$, as

$$\hat{A}(S) f_{gij} = 4\pi \sum_{j \geq S \not\in g} q_j (j_p + 1) f_{gij}$$

(assuming no intersections in vertices of $g$). Similarly the volume operator has a discrete spectrum receiving contributions only from vertices in a region $[11, 12]$. The representation used so far is not only an explicitly known one, but is in fact, to a large degree, unique using the covariance under spatial diffeomorphisms $[14, 15]$. As it happens sometimes also in quantum field theories on a background, symmetry reduces the freedom in choosing a representation. The large symmetry group of diffeomorphisms present due to background independence selects a unique representation. This differs from usual Fock spaces for particle excitations, but Fock states can be reproduced as
distributional states. There is thus a tight kinematical setting, although quantization
ambiguities such as factor ordering will as usually arise for composite operators, in
particular the constraints, specifying the dynamics.

Example: Cubic lattice

Labels $j$ appearing on edges of graphs determine the geometry through flux values of
the densitized triad. They are half-integer and thus imply a minimum non-zero flux given
by $4\pi \frac{\gamma}{P}$. The states are all that is present in the quantum theory to determine geometry.
The lattice thus is space, and no continuum limit is to be taken as one would do it in
lattice gauge theories. From the labels we obtain lattice fluxes $p_{vI} = F_{SvI} = 8\pi \frac{\gamma}{P} j_{vI}$
depending on the direction $I$ of an edge and its starting vertex $v$. This is the state
dependent scale of geometry introduced before as the additional length scale provided
by discrete quantum gravity. This scale can be inhomogeneous if labels differ much for
different edges, and it is dynamical (as well as, possibly, the lattice structure such as the
number of vertices). States, in this way, determine which physical scales are relevant.
Recent developments from different directions within loop quantum gravity have now
converged to such structures [16, 17, 18].

5. Gravitational dynamics

Dynamics of space-time is determined by the Hamiltonian constraint which, when
solved, is supposed to show which special superpositions of lattices are allowed for
generally covariant states. This constraint is implemented through lattice Hamiltonians
which change the labels and possibly the graph when acting on a state. Thanks to
spatial discreteness, those operators are well-defined even with matter contributions:
there are no UV divergences [19, 20]. As even classical expressions are complicated non-
polynomial functions of the basic fields, the operators are only barely tractable. Although
they have been defined rigorously, they have quantization ambiguities (such as factor
ordering and several other choices) and do not easily reveal interesting solutions. But
they do display characteristic properties which follow from spatial discreteness and are
common to all available constructions. They can be tested with suitable approximation
schemes, which currently include symmetries [21, 22] or perturbations [18, 23].

More in detail, the classical expression of the constraint functional (to be imposed for
all spatial functions $N(x)$) is

$$H[N] = \frac{1}{16\pi G} \sum_0^Z d^3x N \Theta \varepsilon_{ijk} F^i_{ab} \frac{E^a E^b}{\det E} + \frac{4 \varepsilon^i \Gamma_i}{A^i} \left( A^i_{b} \frac{E^q E^b}{\det E} \right) \frac{1}{A} \quad (3)$$

which requires an inverse determinant of $E^a_i$. This is not available immediately due to
the fact that fluxes, and also the volume operator, have discrete spectra containing zero
and thus no inverse operators. But one can use the identity [19]

$$A^i_{a} \frac{E^q E^b}{\det E} \frac{1}{A} = 2\pi G \epsilon^{ijk} \varepsilon_{abc} \frac{E^q E^b}{\det E} \quad (4)$$
to obtain a well-defined quantization through a Poisson bracket, which then becomes a commutator of holonomies and volume. For the curvature components $F^i_{ab}$ we use $s_1^a s_2^b F^i_{ab} \tau_i = \Delta^{-1} \mathcal{H}_\alpha + O(\Delta)$ and write it in terms of a holonomy $\mathcal{H}_\alpha$ around a square loop of coordinate size $\Delta$ and with tangent vectors $s_1^a$ and $s_2^b$ at $v$ [24]. Finally, an extrinsic curvature operator for $A^i_a \Gamma^a_i$ in (3) can be derived as a double commutator.

6. Effective theory

In any quantum field theory, especially one with a highly complicated Hamiltonian, progress can be made only with suitable approximations to compute physical effects. One of the main tools in particle physics is the low energy effective action which allows powerful applications for instance in perturbation theory. The lattice Hamiltonians we have here are different from quantum field theory Hamiltonians on a background, and conceptually also from lattice gauge theory. For instance, they are unbounded from below already classically. No ground state is available to expand around as done in low energy effective actions. It is thus necessary to generalize effective theory which has been accomplished [25, 26]. Effective equations, in general terms, are obtained from an analysis of the coupled dynamics of $n$-point functions. (In quantum mechanics, these are spread and deformations of a wave packet back-reacting on the peak position.) Effective dynamics is given by the expectation value of the Hamiltonian in suitable semiclassical states, with a precise specification depending on the regime to be analyzed.

Applied to lattice Hamiltonians, we can already draw one important conclusion: local coefficients of the effective Hamiltonian appear as functions of $p H^2$ (after, e.g., using (4) with local lattice building blocks). Quantum effects are thus much larger than they would be in low energy effective theory due to the discrete structure [27].

Summary of Quantum effects

Typical properties of effective Hamiltonians as they are also known from effective actions are [26]: (i) Factors such as the quantized inverse metric determinant give rise to modified small-scale behavior of coefficients (possibly related to boundedness of classically diverging curvature expressions near singularities [28, 29]). (ii) Replacing local curvature and connection components by holonomies along extended loops implies non-locality or, when Taylor expanded in an effective Hamiltonian, higher order spatial derivatives. (iii) The coupling of $n$-point functions in general effective equations implies, as usually, new quantum degrees of freedom (related to higher time derivatives).

Properties (i) and (ii) are typical holonomy effects of the loop quantization which was forced upon us by background independence while (iii) is a genuine quantum effect. Both (ii) and (iii) correspond to higher curvature terms, while (i) corrects geometrical factors purely from quantum geometry.
APPLICATION: HOW BIG IS THE TYPICAL QUANTUM SCALE?

Lattice states as solutions to the Hamiltonian constraint are difficult to find even numerically. But orders of magnitude of corrections can be estimated based on two different roles played by the fundamental scale $p$. First, $p$ determines the number of lattice sites within a typical macroscopic scale which we take to be $H$ \(^{-1}\): for larger $p$, the lattice is coarser and discreteness corrections arise. Continuum physics requires $p \gg H$. Secondly, the size of $p$ signals quantum effects since it is proportional to the quantum number of a state. For smaller $p$, a lower “excitation level” is realized and thus one has larger quantum effects. Semiclassical physics requires $p \ll H^{2}$.

These are two opposite requirements, leaving an allowed range $\frac{2}{p} \gg p \gg H^{2} = \frac{3}{8 \pi G \rho}$ where we took the Hubble length as the macroscopic scale relevant for cosmology, and computed it in terms of energy density $\rho$ using the Friedmann equation. This range is wide in the late universe, where quantum corrections can be almost arbitrarily small. But it is more narrow in the early universe and during inflation where the typical energy scale implies $1 \ll \frac{2}{p} = 10^{6}$.

Direct observations of effects from $\frac{2}{p} = 10^{6}$ would not be observable soon, but magnifications can occur if they add up during long cosmic evolution. This is in fact realized \([27]\) as it follows from cosmological perturbation equations derived from the effective Hamiltonian \([30]\): One of the metric modes, $\Phi$, satisfies $\ddot{\Phi} + (1 + \nu) \dot{\Phi} = \eta + \epsilon \Phi = \eta^{2} = 0$ on large scales as a differential equation in conformal time $\eta$ and with $\nu$ being related to the matter equation of state $w$. The quantum correction $\epsilon$ changes the behavior crucially compared to the classical situation where $\epsilon = 0$. Solutions for constant $\nu$ are $\Phi(\eta) = \eta^{\lambda}$ with $\lambda = \frac{\nu}{2} + \frac{1}{2} \sqrt{\nu^{2} - 4 \epsilon}$. For $\epsilon = 0$, one mode is constant, one decays, which is known as conservation of curvature perturbations. Loop quantum gravity implies $\epsilon < 0$, such that the constant mode becomes slightly growing and curvature perturbations are not exactly preserved.

Since $\epsilon$ has a definite sign, which is a robust property in loop quantum gravity independently of quantization ambiguities \([31, 32]\), small corrections indeed add up during evolution. During inflation, conformal time for the largest visible modes changes by a factor $e^{60}$. Thus, a factor $e^{60} 1 + 10^{3} \epsilon$ results, magnifying the correction to at least $10^{4}$ for $\epsilon$, as estimated above, of the size $10^{6}$.

SUMMARY

Effects from the basic scale of quantum gravity in phenomenological equations can thus be computed with recent advances, and they reveal sometimes surprising implications. Classical modes (or also gravitons) are not fundamental but collective excitations out of the microscopic discrete state. Basic excitations are rather the scale parameters $p_{\nu, \lambda}$ which, when excited inhomogeneously, can give rise to the classical fields or, in some approximation, gravitons on large scales.

Systematic calculations are now possible, mainly due to advances in the understanding of effective theory, and can be applied to different regimes. While characteristic effects are robust under quantization ambiguities, ambiguity parameters might become
relevant for possible observations with more precise data. Cosmological observations thus have a good chance of revealing quantum gravity effects in the near future and to guide further constructions to complete the theory.

ACKNOWLEDGMENTS

The author thanks Hugo Morales-Técotl and Luis Urrutia for an invitation to give a plenary talk at the VIth Latin American Symposium on High Energy Physics (Puerto Vallarta, Mexico, Nov. 2006), on which this contribution is based. Results reported here were obtained through work funded by NSF grant PHY0554771.

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