Relativistic three-partite non-locality

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Bell-like inequalities have been used in order to distinguish non-local quantum pure states by various authors. The behavior of such inequalities under Lorentz transformation has been a source of debate and controversies in the past. In this paper, we consider the two most commonly studied three-particle pure states, that of W and GHZ states which exhibit distinctly different type of entanglement. We discuss the various types of three-particle inequalities used in previous studies and point to their corresponding shortcomings and strengths. Our main result is that if one uses Czachor’s relativistic spin operator and Svetlichny’s inequality as the main measure of non-locality and uses the same angles in the rest frame (S) as well as the moving frame (S’), then maximally violated inequality in S will decrease in the moving frame, and will eventually lead to lack of non-locality (i.e. satisfaction of inequality) in the $v \to c$ limit. This is shown for both the GHZ and W states and in two different configurations which are commonly studied (Case I and Case II). Our results are in line with a more familiar case of two particle case. We also show that the satisfaction of Svetlichny’s inequality in the $v \to c$ limit is independent of initial particles’ velocity. Our study shows that whenever we use Czachor’s relativistic spin operator, results draws a clear picture of three-particle non-locality making its general properties consistent with previous studies on two-particle systems regardless of the W state or the GHZ state is involved. Throughout the paper, we also address the results of using Pauli’s operator in investigating the behavior of $|S_v|$ under LT for both of the GHZ and W states and two cases (Case I and Case II). Our investigation shows that the violation of $|S_v|$ in moving frame depends on the particle’s energy in the lab frame, which is in agreement with some previous works on two and three-particle systems. Our work may also help us to classify the results of using Czachor’s and Pauli’s operators to describe the spin entanglement and thus the system spin in relativistic information theory.

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I. INTRODUCTION

The non-local feature of the Quantum Mechanics was first pointed out in a paper by Einstein, Podolsky and Rosen, otherwise known as EPR \cite{EPR}. This non-locality may yield to the entanglement phenomenon which restrains decomposing quantum state of a system into a product state including the states of its basic constituents \cite{Bell}. Entangled states may violate Bell’s inequality, which provides a theoretical basis for investigating entanglement in a two-partite system \cite{Bell}. It was Aspect and coworkers who first verified such non-local behavior in their experiments \cite{Aspect}. Entanglement can be related to the concept of information via entropy which has attracted a large amount of attention in recent years due to its applications in quantum computation, teleportation and cryptography, among others \cite{Shor, Peres}.

Meanwhile, relativistic considerations attracted various authors early on. On one hand, one expects the amount of information to be independent of inertial observer \cite{Peres}, while on the other hand, probability distributions may depend on the frame of reference \cite{Peres}. Peres and co-workers have considered a single free spin-$\frac{1}{2}$ particle and shown that the reduced density matrix for its spin is not covariant under Lorentz transformation (LT). Briefly, they proved that the spin entropy depends on the frame due to Wigner rotation \cite{Peres}. Essentially, it was proved that the single particle system can possess non-locality \cite{Peres} and such non-locality changes under (LT) \cite{Peres}.

Perhaps more importantly, many authors have considered relativistic effects on bi-partite entanglement in a two-particle systems \cite{Peres, Peres2}. Typically one is concerned with how entanglement and/or non-locality is related in a lab frame (S) to a moving frame (S’) under LT. Authors in \cite{Collins, Peres3} have considered a situation where Bell’s inequality is maximally violated in S. Keeping the same setup of angles for Bell’s operator, they find that this function is a decreasing function of the boost velocity. Furthermore, as the speed of the boost approaches the speed of light, they

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find that Bell’s inequality is satisfied indicating lack of non-locality in the $v \rightarrow c$ limit. However, authors in Refs. 16–26 have shown that if one Lorentz transforms the quantum states as well as spin operators, then the maximal violation of Bell’s inequality in $S$ remains intact in the $S'$. However, more recently, some authors have shown that under certain LT, total entanglement (entropy) of the system depends on the frame 27, 28. Also, similar results have been shown in curved spacetime 29–33. It has also been shown that acceleration effects do not preserve the amount of entanglement 34–38. Therefore, the invariance of bi-partite entanglement seems somewhat suspect at the moment.

The case of a three-particle system offers interesting possibilities. On the one hand, it is a natural extension of one-particle and two-particle studies mentioned above. On the other hand, it opens the possibility of studying different types of (multi-partite) entanglement included in such systems, but not present in one or two-particle systems. It is well-known that the behavior of bi-partite entanglement is different from multi-partite entanglement 39–41. Therefore, the invariance of bi-partite entanglement seems somewhat suspect at the moment.

Extending Bell’s inequalities to a three-particle case was first done by Svetlichny (|$S_v$|) 42. Shortly after, Mermin (|$M$|) also offered another such generalization 43. Later on, Collins (|$M'$|) et al. also offered another generalization 44, 45. They appear as follows:

$$|S_v| = |E(ABC) + E(ABC') + E(AB'C) + E(A'BC)| - E(A'B'C') - E(A'B'C) - E(AB'C')| \leq 4,$$

$$|M| = |E(ABC') + E(AB'C) + E(A'BC) - E(A'B'C')| \leq 2,$$

$$|M'| = |E(ABC) - E(A'B'C') - E(AB'C') - E(AB'C')| \leq 2.$$

(3)

In the above equations, $A$ and $A'$ are possible measurements on particle 1. Same relations are valid for particles 2 and 3 with possible measurements $B$, $B'$ and $C$, $C'$, respectively. $E(ABC)$ represents the expectation value of the product measurement outcomes of the observable $A$, $B$ and $C$. For example,

$$E_{GHZ}(ABC) = \langle GHZ|\sigma(\vec{n}_1) \otimes \sigma(\vec{n}_2) \otimes \sigma(\vec{n}_3)|GHZ\rangle = \cos(\theta_1 + \theta_2 + \theta_3),$$

(4)

where $\sigma(\vec{n}_i)$ is the Pauli spin operator of the $i$th particle in the $xy$ plane and $\theta_i$ is the azimuthal angle of the vector $\vec{n}_i$.

It has been shown that maximal violation of $|S_v| = 4\sqrt{2}$ occurs for GHZ state for angles which satisfy $\Sigma \phi_i = (m + \frac{3}{2})\pi$ and $\Sigma \phi'_i = (m + \frac{3}{2})\pi$, with $(m = 0, \pm 1, \pm 2, \ldots)$ 46. For W state we restrict our measurement in the $xz$ plane where

$$E_W(ABC) = \langle W|\sigma(\vec{n}_1) \otimes \sigma(\vec{n}_2) \otimes \sigma(\vec{n}_3)|W\rangle = -\frac{3}{2} \cos(\theta_1 + \theta_2 + \theta_3) - \frac{1}{4} \cos \theta_1 \cos \theta_2 \cos \theta_3,$$

(5)

where $\theta_i$ specifies the polar angle measurement direction of the $i$th spin observable. For W state maximal violation of $S_v$ has been shown to be $|S_v| = 4.354$ which occur for $\theta_i = 35.264^\circ$ and $\theta'_i = \pi - \theta_i = 144.736^\circ$, $\forall i$ 46. Additionally, the maximal values of $|M|$ (as well as $|M'|$) have been shown to be equal to $|M| = |M'| = 4$ and $|M| = |M'| = 3.046$ for GHZ and W states, respectively 46.

However, there are some apparent inconsistencies between these three measures of non-locality. One can show that when $S_v$ is maximally violated, then $|M| = |M'| = |S_v|\frac{2}{3}$ which indicates that $|M|$ and $|M'|$ will not be maximally violated 46. In other words, no given set of angles will cause the three measures to obtain their maximum value. Furthermore, one can show that $|M|$ and $|M'|$ can show opposite behavior. For example, when $|M|$ is maximally violated, $|M'|$ is satisfied and vice versa 46. Roy has shown that the upper bound of the Mermin’s inequality should be corrected 47, and it was proven that the three-particle Bell’s like inequalities, such as $|M|$ and $|M'|$, which include four of the correlation functions ($E(ABC)$) can be violated by a hybrid local-nonlocal hidden variables model 43, 46. Since states which include genuine three-partite non-locality can only violate the $|S_v|$ inequality 43, 46, this inequality can be used to distinguish such states (such as the W and GHZ states) from other states which does not include genuine three-partite non-locality (such as three-particle systems with bi-partite non-locality). It is also believed that genuine three-partite entanglement is observed when either $|M|$ or $|M'|$ is violated by greater amount.
than $2\sqrt{2}$ \cite{48}, which is less than the value one gets for the W state ($|S_\alpha| = 4.354$) and the GHZ state in the three-particle non-local system when its value is a certain well-defined value of $|S_\alpha| = 4.354$ which is, in fact, the maximum value for W state \cite{39}. Indeed, only the GHZ state can violate $|S_\alpha|$ to its maximum possible value $(4\sqrt{2})$ while the W state cannot violate $|S_\alpha|$ to its maximum possible value $3\sqrt{2}$ \cite{48}. Therefore, because of above-mentioned shortcomings of $|M|$ and $|M'|$, it seems reasonable to use $|S_\alpha|$ in order to study the GHZ and W states \cite{39,46,50}.

We should note that the correlation functions $E(ABC)$, used in order to evaluate $|S_\alpha|$ (as well as $|M|$ and $|M'|$), are stronger than required to study general three-particle non-locality \cite{49,50}. This is a weakness for $|S_\alpha|$, and yields violation of no-signalling constraint which leads to grandfather-type paradoxes \cite{51}. This shortcoming of $|S_\alpha|$ can be eliminated by using bi-partite correlation functions which satisfy no-signalling constraint or time-ordered correlations \cite{50,52,53}.

Author in \cite{54} has used the $|M|$ inequality and studied the relativistic behavior of the entanglement of GHZ state, when moving observer $(S')$ uses measurements angles same as lab frame $(S)$. He has used Czachor’s relativistic spin operator \cite{55} and considered a special set of measurement apparatus that violates the $|M|$ inequality to its maximum possible value in the rest frame. He concludes that the violation of the $|M|$ inequality in the moving frame, when the speed of the boost increases $(\beta \rightarrow 1)$ and if the directions of the measurements are fixed, depends on the energy of the particles in the $S$ frame. This results disagrees with the two-particle non-local systems \cite{16,20}. Similar to the Bell state \cite{16,20}, Moradi et al. have shown that in the moving frame, by choosing special measurements, one can find maximum violation of the $|M|$ inequality \cite{50}. Finally, author compared his results with those of attempt in which two-particle pure entangled states (the Bell states) are studied \cite{22}, and concludes that the behaviors of non-locality stored in the Bell and GHZ states under LT differ from each other if the moving and lab frames use Czachor’s relativistic spin operator and the same special set of measurements violating the corresponding Bell-like inequality to its maximum violation amount in the lab frame. Similar results where authors have used Pauli operators as the relativistic spin operator instead of the Czachor’s relativistic spin operator, can be found in \cite{57}. Briefly, if the moving frame uses the same measurements as the lab frame and the Mermin’s inequality as a witness of the non-locality in the GHZ state, then the violation of the inequality in the $S'$ frame will depend on the energy of the particles in the $S$ frame as the velocity of the boost reaches that of light \cite{54}. Since the entanglement of the W state differs from that of the GHZ state, results obtained in \cite{54,57} cannot be generalized to the W state, and therefore, the behavior of entanglement stored in the W state under LT is completely unknown.

Moreover, if one compares the results of You et al. \cite{57} with those of two-particle studies, obtained in \cite{22,23,24}, he finds that the behavior of bi-partite non-locality under LT is similar to the behavior of non-locality stored in the GHZ under LT if the lab and moving frames use Pauli’s spin operator and the same set of measurements violating the corresponding Bell-like inequality to its maximum violation amount in the lab frame. In this situation, the corresponding Bell-like inequality is violated to its maximum violation amount in the $\beta \rightarrow 1$ limit for the low energy particles, and it is not violated for the high energy particles in the $\beta \rightarrow 1$ limit. Here, we should note that the results of studying the behavior of the Bell state under LT by considering Pauli’s spin operator and the special set of measurements violating the Bell’s inequality to its maximum violation amount in the lab frame \cite{22,23,24} differ from those of in which Czachor’s relativistic spin operator is considered instead of the Pauli operator \cite{22}. Indeed, in the moving frame and independently of particles energy, Bell’s inequality is preserved in the $\beta \rightarrow 1$ limit if one uses Czachor’s relativistic spin operator and the special set of measurements violating the Bell’s inequality to its maximum violation amount in the lab frame \cite{22}. It is also interesting to note again that although authors in refs. \cite{54,57} use different spin operators, their results are compatible with each other. The latter consistency comes from considering the $|M|$ inequality and the same set of measurements which violate the $|M|$ inequality to its maximum violation amount in the lab frame, by authors, to study the behavior of the GHZ state under LT \cite{55}. Therefore, there is an inconsistency between the generalization of two-particle studies to the three-particle studies whenever we compare the results of considering Czachor’s relativistic spin operator with those obtained by considering Pauli’s operator \cite{58}. In fact, it is shown that if one uses Pauli’s operator and the special set of measurements violating $|S_\alpha|$ to its maximum violation amount in the lab frame, then the violation amount of three-particle Bell-like inequalities depends on the particles energy \cite{58}. This latter point helps us to better understand the mentioned inconsistency. However, as we will show in the following, this inconsistency may be completely solved if one considers the GHZ and W states and uses Czachor’s operator to investigate the behavior of $|S_\alpha|$, $|M|$ and $|M'|$ under LT.

Our goal in this paper is to study the behavior of non-locality stored in the GHZ and W states under LT by using the $|S_\alpha|$ inequality, while the lab and moving frames use the same set of measurements that violate the $|S_\alpha|$ inequality to its maximum possible violation amount in the lab frame. We also resolve the above mentioned inconsistency. Since some authors \cite{54,57} have used $|M|$ in order to study the effects of LT on the GHZ state, we also consider the $|M|$ and $|M'|$ inequalities in order to make a comparison with their works. In addition, this analysis helps in clarifying previously discussed sensitivity of three-particle Bell-like inequalities regarding measurement directions \cite{16}. In order to achieve this, in section II, we use Czachor’s spin operator to calculate $|S_\alpha|$ (as well as $|M|$ and $|M'|$) for two typically
studied scenarios for both GHZ and W states in the $v \to c$ limit. The results of using Pauli’s operator in investigating the behavior of $|S_v|$ under LT are also addressed throughout the paper. We devote section III to a summary of our results and some concluding remarks.

II. THREE-PARTITE NON-LOCAL SYSTEM UNDER LT

In the $S$ frame, consider a spin-$\frac{1}{2}$ particle with a momentum vector $\vec{p}$ and a spin state $|\psi\rangle = \lambda|+\rangle + \kappa|-\rangle$. The total state of the particle can be written as:

$$|\xi\rangle = |\vec{p}\rangle|\psi\rangle. \quad (6)$$

In the moving frame ($S'$), the state of the system is:

$$|\xi'\rangle = |\Lambda\vec{p}\rangle D(W(\Lambda, p))|\psi\rangle. \quad (7)$$

In the above equation, $\Lambda\vec{p}$ denotes the momentum of the particle in the boosted frame and $D(W(\Lambda, p))$ is the Wigner representation of the Lorentz group for the spin-$\frac{1}{2}$ particle [59]:

$$D(W(\Lambda, p)) = \cos \Omega_p \frac{\Omega_p}{2} + i(\vec{\sigma} \cdot \vec{n}) \sin \Omega_p \frac{\Omega_p}{2}, \quad (8)$$

where $\sigma$ and $\Omega_p$ are the Pauli matrix and Wigner angle, respectively. Also, we have:

$$\vec{n} = \hat{e} \times \hat{p}. \quad (9)$$

where $\hat{e}$ denotes unit vector in the boost direction. $\hat{p}$ is the unit vector along the momentum direction of the particle in the $S$ frame. We consider the case in which the boost speed is along the $\hat{x}$ direction ($\vec{\beta} = \beta \hat{x}$) and the particle moves along the $z$ direction ($\vec{p} = p_0 \hat{z}$). In this case we have:

$$D(W(\Lambda, p)) = \cos \Omega_p \frac{\Omega_p}{2} - i\sigma_y \sin \Omega_p \frac{\Omega_p}{2}, \quad (10)$$

where

$$\tan \Omega_p = \frac{\sinh \alpha \sinh \delta}{\cosh \alpha + \cosh \delta}. \quad (11)$$

Here, $\cosh \delta = \frac{p_0}{m}$ and $\cosh \alpha = \sqrt{1 - \beta^2}$. Therefore, we have:

$$D(W(\Lambda, p))|+\rangle = \cos \Omega_p \frac{\Omega_p}{2} |+\rangle + \sin \Omega_p \frac{\Omega_p}{2} |-\rangle$$

$$D(W(\Lambda, p))|-\rangle = -\sin \Omega_p \frac{\Omega_p}{2} |+\rangle + \cos \Omega_p \frac{\Omega_p}{2} |-\rangle. \quad (12)$$

Also, the relativistic spin operator $\hat{A}$ is given by [55]:

$$\hat{A} = \frac{\sqrt{1 - \beta^2} \hat{A}_\perp + \hat{A}_\parallel \cdot \vec{\sigma}}{\sqrt{1 + \beta^2[(\hat{e} \cdot \hat{A})^2 - 1]}}. \quad (13)$$

The subscripts $\perp$ and $\parallel$ denote the perpendicular and the parallel components of the vector $\hat{A}$ to the boost direction. In the $S$ frame, consider a three-particle system as:

$$|\psi\rangle = \prod_i |p_i\rangle |GHZ\rangle, \quad (14)$$

where $p_i$ represents the 4-momentum of the $i^{th}$ particle in the laboratory frame. Inserting the special set of the angles $\phi_1 = \frac{\pi}{4}$ and $\phi_1' = \frac{\pi}{4}$ into the Eq. (14) and evaluating Eq. (13), we obtain the maximum violation $(4\sqrt{2})$ for $|S_v|$. Also for this set of measurements we have $|M'| = |M'| = 2\sqrt{2}$, which according to [48] indicates that the system only includes
bi-partite entanglement, which is clearly not the case for the GHZ state at hand here. This confirms our motif that $|S_v|$ is a more appropriate measure of three-particle non-locality than $|M|$ and $|M'|$.

We now set out to consider how $|S_v|$ behaves under LT when the measurement setup are chosen to maximize its value in the rest frame $(S)$ and keeping the condition that the same measurement setup is used in $S'$. We will see that under such conditions the various three-particle systems considered will show consistent and reasonable behavior, devoid of previous inconsistency when $|M|$ and/or $|M'|$ are used to evaluate non-locality under LT. Our results are also in line with well-known results of two-particle systems which have been studied using Bell’s inequality \[17\] \[27\].

Case I: We first consider the situation where all particles move along the $z$ axis with the same momenta, in the $S$ frame, i.e. $\vec{p_i} = p_0 \hat{z}$. In order to satisfy the no-signalling constraint, we consider a situation where the measurement apparatus of each particle is located with arbitrary non-zero distance from each other along the $z$ axis. In fact, their distances from each other should be spacelike during the measurements \[60\]. The Wigner rotation is the same as the Eq. (10). Using Eq. (13) we get:

$$E(ABC) = \langle\text{GHZ}^{\Lambda}\rangle = \sum_{i=A}^{C} \frac{1}{\sqrt{1-\beta^2}} \sin^2 \phi_i$$

\[15\]

where

$$|\text{GHZ}^{\Lambda}\rangle = D(W(\Lambda, p))|\text{GHZ}\rangle = \frac{1}{\sqrt{2}} (|++-\rangle - |---\rangle) + \frac{1}{\sqrt{3}} \left[ (\sin(\Omega_\beta) - \cos(\Omega_\beta))|---\rangle \right],$$

is the spin state in the $S'$ frame and $\gamma = \frac{1}{\sqrt{1-\beta^2}}$. Inserting $\phi_i = \frac{\pi}{4}$ and $\phi_i' = \frac{3\pi}{4}$ into Eq. (15), we find

$$E(ABC) = \frac{\cos\Omega_\beta}{\sqrt{2-\beta^2}}((\cos\Omega_\beta)^2 - 3(1 - \beta^2)) = -E(A'B'C')$$

$$E(A'B'C') = E(AB'C') = E(A'B'C) = E(AB'C) = -E(A'B'C').$$

(16)

We therefore obtain

$$|M| = |M'| = \frac{|S_v|}{2} = \frac{2\cos\Omega_\beta}{\sqrt{2-\beta^2}}((\cos\Omega_\beta)^2 + 1 - \beta^2)).$$

As a check, Eq. (18) reduces to the $S$ frame result \[4\sqrt{2}\] in the appropriate limit of $\beta \to 0$ and $\Omega_\beta \to 0$. In the ultra-relativistic limit ($\beta \to 1$) and independently of the particles’ initial energy, unlike \[54\] \[57\] \[58\], all of the above inequalities are satisfied, indicating the familiar result that Bell’s inequalities are satisfied in $S'$ as $\beta \to 1 \[22\]$.

Now, let us point to the behavior of $|S_v|$ when Pauli’s operator is considered instead of Czachor’s. It has been shown that \[58\]

$$|S_v| = 4\sqrt{2}(\cos(\frac{\Omega_\beta}{2}))^6 - (\sin(\frac{\Omega_\beta}{2}))^6),$$

which leads to

$$|S_v| \sim \sqrt{2}(\frac{1+3\Gamma^2}{\Gamma^3}),$$

in the $\beta \to 1$ limit. Here, $\Gamma = \frac{1}{\sqrt{1-\beta^2}}$ and $v_0$ are the energy factor and velocity of particles in the $S$ frame, respectively.

This result shows that the inequality is preserved by the high energy particles ($\Gamma \to \infty$) and it is violated to the same
value as the lab frame \((4\sqrt{2})\) by the low energy particles \((\Gamma \rightarrow 1)\). Such results are clearly in contrast to the case of Czachor’s operator.

By following the above recipe for the W state we get

\[
|W\rangle^A = \sqrt{3} \sin(\frac{\Omega p}{2}) \cos(\frac{\Omega p}{2}) [\cos(\frac{\Omega p}{2}) + \sin(\frac{\Omega p}{2})] \langle - + - + | + + + + - - - - | - - - - + + + + - - - - ]
\]

+ \[\frac{(\cos(\frac{\Omega p}{2}))^3 - 2 \cos(\frac{\Omega p}{2}) (\sin(\frac{\Omega p}{2}))^2}{\sqrt{3}} \langle W \rangle \]

+ \[\frac{[2 \sin(\frac{\Omega p}{2}) (\cos(\frac{\Omega p}{2}))^2 - (\sin(\frac{\Omega p}{2}))^3]}{\sqrt{3}} \langle W \rangle \]

(21)

for the spin state in the moving frame, and

\[
E_{W}(\theta_1 \theta_2 \theta_3) = \frac{1}{3} (A_{11}(\theta_1)A_{22}(\theta_2)A_{11}(\theta_3) + A_{11}(\theta_2)A_{11}(\theta_2)A_{22}(\theta_3)
\]

+ \[A_{22}(\theta_1)A_{11}(\theta_2)A_{11}(\theta_3) + A_{11}(\theta_1)A_{12}(\theta_2)A_{21}(\theta_1) + A_{11}(\theta_1)A_{21}(\theta_2)A_{12}(\theta_3)
\]

+ \[A_{11}(\theta_2)A_{12}(\theta_1)A_{21}(\theta_3) + A_{11}(\theta_2)A_{21}(\theta_1)A_{12}(\theta_3) + A_{11}(\theta_3)A_{12}(\theta_1)A_{21}(\theta_2)
\]

+ \[A_{11}(\theta_3)A_{21}(\theta_1)A_{12}(\theta_2))]\]

where we have

\[
A_{11}(\theta) = -A_{22}(\theta) = \frac{1}{\sqrt{1-\beta ^2 \sin ^2 \theta}} (\frac{\cos \theta \cos \Omega p}{\gamma} + \sin \theta \sin \Omega p),
\]

\[
A_{12}(\theta) = A_{21}(\theta) = \frac{1}{\sqrt{1-\beta ^2 \sin ^2 \theta}} (\frac{-\cos \theta \sin \Omega p}{\gamma} - \sin \theta \cos \Omega p).
\]

for the correlation function \(E_{W}(\theta_1 \theta_2 \theta_3)\). In the \(\gamma = 1, \Omega_p = 0\) limit (S frame) we have:

\[
A_{11}(\theta) = -A_{22}(\theta) = \cos \theta
\]

\[
A_{12}(\theta) = A_{21}(\theta) = \sin \theta,
\]

which leads to

\[
E_{W}(\theta_1 \theta_2 \theta_3) = -\cos \theta_1 \cos \theta_2 \cos \theta_3 + \frac{2}{3} (\cos \theta_1 \sin \theta_2 \sin \theta_3
\]

+ \[\cos \theta_2 \sin \theta_1 \sin \theta_3 + \cos \theta_3 \sin \theta_2 \sin \theta_1)\]

(25)

Which is the same as the S frame result previously obtained in Eq. 5. However when the speed of the boost reaches light velocity \((\beta \rightarrow 1)\), we have:

\[
E_{W}(\theta_1 \theta_2 \theta_3) \rightarrow \tan \theta_1 \tan \theta_2 \tan \theta_3 \sin \Omega_p (2 \cos ^2 \Omega_p - \sin ^2 \Omega_p),
\]

where \(\sin \Omega_p \rightarrow \frac{1}{\sqrt{1-\frac{1}{\Gamma^2}}\} and so,

\[
E_{W}(\theta_1 \theta_2 \theta_3) \rightarrow \tan \theta_1 \tan \theta_2 \tan \theta_3 \sqrt{1-\Gamma^2} (\frac{3}{\Gamma^2} - 1).
\]

(27)

In the above equation, \(\Gamma = \frac{1}{\sqrt{1-\frac{1}{\Gamma^2}}\} is again the factor of the energy of the particles in the S frame and \(v_0\) is the velocity of the particles in the S frame. We therefore get for \(\theta_1 = \theta\) and \(\theta_2 = \pi - \theta = \theta':

\[
|M| = |M'| = | - 2E_{W}(\theta_1 \theta_2 \theta_3)| = \frac{|S_c|}{2} \rightarrow 2 \tan ^3 \theta \frac{(3 - \Gamma ^2) \sqrt{\Gamma ^2 - 1}}{\Gamma ^3},
\]

which for \(\theta = 35.264°\) is always less than 2, showing non-violation of inequalities regardless of system’s energy \((\Gamma)\) in the rest frame.

We have therefore shown that when one uses a setup in which \(|S_c|\) is maximally violated in the S frame, LT reduces this amount gradually and that in the extreme relativistic case of \(\beta \rightarrow 1, |S_c|\) (as well as other) inequalities are satisfied indicating lack of non-locality. This is consistent with previous results in the two-particle systems studied.
We have shown this for both W and GHZ states, indicating that this result is general beyond two-particle systems and regardless of the type of the entanglement, whether the W or the GHZ state.

Once again, we point to the results of using Pauli’s operator instead of Czachor’s in order to investigate the behavior of $|S_v|$. It has been shown that

$$|S_v| = \frac{4}{3}[(\cos \Omega_p)^3 - \frac{7}{2}(\sin \Omega_p)^2 \cos \Omega_p][(\cos \theta)^3 + 3 \cos \theta - \cos 3\theta]].$$

Substituting $\theta = 35.264^\circ$ into this equation leads to

$$|S_v| \sim \frac{19.594}{\Gamma^3} - \frac{15.236}{\Gamma},$$

in the $\beta \rightarrow 1$ limit. Indeed, this equation tells us that $|S_v|$ is not violated by the high energy particles in the $\beta \rightarrow 1$ limit, while, the low energy particles violate this inequality to the same value as the lab frame whenever the boost speed reaches light velocity. As the GHZ case, we see that the results of considering Pauli’s operator differ from those obtained by considering Czachor’s operator. We next consider another commonly studied case along the same lines.

**Case II.** Here the particles are in the center-of-mass frame with the following momenta:

$$\vec{p}_1 = -p_0 \hat{z},$$
$$\vec{p}_2 = \frac{p_0}{2}(\hat{z} + \sqrt{3}\hat{y}),$$
$$\vec{p}_3 = \frac{p_0}{2}(\hat{z} - \sqrt{3}\hat{y}).$$

Following Eq. (8), we find

$$D(W(\Lambda, p_1)) = \cos \frac{\Omega_p}{2} + i\sigma_y \sin \frac{\Omega_p}{2},$$
$$D(W(\Lambda, p_2)) = D^*(W(\Lambda, p_3)) = \cos \frac{\Omega_p}{2} + i(\frac{\sqrt{3}}{2}\sigma_z - \frac{1}{2}\sigma_y) \sin \frac{\Omega_p}{2}.$$

In the moving frame, it is a matter of calculation to show that the GHZ and W states take the form

$$|GHZ^\Lambda\rangle = -\frac{1}{4\sqrt{2}}\sin^3(\frac{\Omega_p}{2})(|+++\rangle - |---\rangle)$$
$$+ \cos(\frac{\Omega_p}{2})(|+++\rangle + \frac{3}{4}\sin^2(\frac{\Omega_p}{2}))\langle GHZ\rangle$$
$$+ \frac{1}{\sqrt{2}}\sin^2(\frac{\Omega_p}{2})\cos(\frac{\Omega_p}{2})(|++-\rangle + |--+\rangle)$$
$$+ \sin(\frac{\Omega_p}{2})\cos(\frac{\Omega_p}{2})\langle \frac{3}{4}\sin^2(\frac{\Omega_p}{2})(|---\rangle + |++-\rangle)$$
$$+ \frac{1}{2}\sin(\frac{\Omega_p}{2})\cos(\frac{\Omega_p}{2})\langle \frac{3}{4}\sin(\frac{\Omega_p}{2})(|+++\rangle - |---\rangle)$$
$$+ \frac{1}{2}\sin^2(\frac{\Omega_p}{2})\langle \frac{\sqrt{3}}{2}\sin(\frac{\Omega_p}{2})(|---\rangle + |++-\rangle)$$
$$+ \frac{1}{2}\sin^2(\frac{\Omega_p}{2})\langle \frac{\sqrt{3}}{2}\sin(\frac{\Omega_p}{2})(|+++\rangle + |---\rangle)$$

In the $\beta \rightarrow 1$ limit, this equation tells us that $|S_v|$ is not violated by the high energy particles in the $\beta \rightarrow 1$ limit, while, the low energy particles violate this inequality to the same value as the lab frame whenever the boost speed reaches light velocity. As the GHZ case, we see that the results of considering Pauli’s operator differ from those obtained by considering Czachor’s operator. We next consider another commonly studied case along the same lines.

**Case II.** Here the particles are in the center-of-mass frame with the following momenta:

$$\vec{p}_1 = -p_0 \hat{z},$$
$$\vec{p}_2 = \frac{p_0}{2}(\hat{z} + \sqrt{3}\hat{y}),$$
$$\vec{p}_3 = \frac{p_0}{2}(\hat{z} - \sqrt{3}\hat{y}).$$

Following Eq. (8), we find

$$D(W(\Lambda, p_1)) = \cos \frac{\Omega_p}{2} + i\sigma_y \sin \frac{\Omega_p}{2},$$
$$D(W(\Lambda, p_2)) = D^*(W(\Lambda, p_3)) = \cos \frac{\Omega_p}{2} + i(\frac{\sqrt{3}}{2}\sigma_z - \frac{1}{2}\sigma_y) \sin \frac{\Omega_p}{2}.$$
and

\[ |W|^A = \frac{3}{4} \sin^3 \left( \frac{\Omega_p}{2} \right) |++ \rangle - \frac{1}{4} \sin^2 \left( \frac{\Omega_p}{2} \right) \cos \left( \frac{\Omega_p}{2} \right) |-- \rangle \]

\[ + \left[ \cos^2 \left( \frac{\Omega_p}{2} \right) + \frac{7}{4} \cos \left( \frac{\Omega_p}{2} \right) \sin^2 \left( \frac{\Omega_p}{2} \right) \right] |++ \rangle - ++ \]

\[ + \left[ - \sin \left( \frac{\Omega_p}{2} \right) \cos \left( \frac{\Omega_p}{2} \right) + i \frac{\sqrt{3}}{2} \sin \left( \frac{\Omega_p}{2} \right) \right]^2 + \frac{1}{4} \sin^3 \left( \frac{\Omega_p}{2} \right) \]

\[ + \frac{1}{2} \sin \left( \frac{\Omega_p}{2} \right) \cos^2 \left( \frac{\Omega_p}{2} \right) \cos \left( \frac{\Omega_p}{2} \right) \sin \left( \frac{\Omega_p}{2} \right) + \frac{1}{4} \sin^3 \left( \frac{\Omega_p}{2} \right) \]

\[ + \frac{1}{2} \sin \left( \frac{\Omega_p}{2} \right) \cos \left( \frac{\Omega_p}{2} \right) (\cos \left( \frac{\Omega_p}{2} \right) - i \frac{\sqrt{3}}{2} \sin \left( \frac{\Omega_p}{2} \right)) + \frac{1}{4} \sin^3 \left( \frac{\Omega_p}{2} \right) \]

\[ + \left[ - \frac{1}{4} \sin^2 \left( \frac{\Omega_p}{2} \right) \cos \left( \frac{\Omega_p}{2} \right) + \cos \left( \frac{\Omega_p}{2} \right) \cos \left( \frac{\Omega_p}{2} \right) - i \frac{\sqrt{3}}{2} \sin \left( \frac{\Omega_p}{2} \right) \right]^2 \]

\[ + \frac{1}{2} \sin^2 \left( \frac{\Omega_p}{2} \right) \cos \left( \frac{\Omega_p}{2} \right) - i \frac{\sqrt{3}}{2} \sin \left( \frac{\Omega_p}{2} \right) \]

\[ + \left[ \cos \left( \frac{\Omega_p}{2} \right) \cos \left( \frac{\Omega_p}{2} \right) + i \frac{\sqrt{3}}{2} \sin \left( \frac{\Omega_p}{2} \right) \right]^2 - \frac{1}{4} \sin^2 \left( \frac{\Omega_p}{2} \right) \cos \left( \frac{\Omega_p}{2} \right) \]

\[ + \frac{1}{2} \sin^2 \left( \frac{\Omega_p}{2} \right) \cos \left( \frac{\Omega_p}{2} \right) + i \frac{\sqrt{3}}{2} \sin \left( \frac{\Omega_p}{2} \right) \]

Calculations become tedious when one uses Czachor’s operator, but when \( \beta \to 1 \), by choosing measurements same as the Case I, we get

\[ |M| \rightarrow 2 \cos \Omega_p (\cos^2 \Omega_p + \frac{3}{4} \sin^2 \Omega_p) \]

\[ M \rightarrow -M', \quad |S_v| = |M + M'| \rightarrow 0, \]

for the \( |GHZ\) state. It is obvious that, independent of the energy of the particles in the \( S \) frame, \( |S_v| \) is satisfied. Using the above equation, one obtains:

\[ |M| = |M'| \rightarrow \frac{3\Gamma^2 + 1}{2\Gamma^3} \leq 2, \]

which unlike [54] shows that \( |M| \) and \( |M'| \) are satisfied in this limit.

For the \( W \) state, when \( \beta \to 1 \), we obtain

\[ |M| = |M'| = \frac{|S_v|}{2} \rightarrow \frac{3}{2} \tan^3 \theta (1 - \frac{1}{\Gamma^2})^2, \]

where \( \theta_i = \theta \) has been used as before. We conclude that, independent of the energy of the particles in the lab frame (\( \Gamma \)), the inequalities are satisfied when \( \beta \to 1 \) and \( \theta = 35.264^\circ \).

As in Case I, maximally violated \( |S_v| \) setup in the lab frame leads to the reduction of this violation in the \( S' \) under LT and leads to lack of non-locality in the extreme relativistic case of \( \beta \to 1 \), regardless of initial system’s energy or the type of entanglement involved.

Additionally, note that two-particle entangled system, in the center-of-mass frame, has been considered by Ahn et al. [22]. They get:

\[ B = \frac{2}{\sqrt{2 - \beta^2}} (\cos \Omega_p + \sqrt{1 - \beta^2}), \]

for Bell’s inequality in the moving frame. Clearly, in the lab frame (\( \beta \to 0, \Omega_p \to 0 \)), the maximum violation of the Bell’s inequality (\( 2\sqrt{2} \)) is obtained. Note that the similarity with our results is obvious which indicates that, if the
moving frame uses the same measurements as the lab frame, the degree of violation decreases under LT leading to lack of entanglement in the $\beta \to 1$ limit regardless of particles’ energy in the lab frame. These results show that the behavior of the three-particle non-local system under LT is the same as the two-particle entangled system, if we use $|S_v|$ and proper measurements which violate $|S_v|$ to its maximum possible value in the $S$ frame. This is important to note that our results indicate that the general behavior of entanglement in the Bell, GHZ and W states under LT is the same as each other, if one uses Czachor’s operator, the Bell inequality for the Bell states [22], and the special set of measurements violating $|S_v|$ to its maximum possible violation amount in the $S$ frame for the GHZ and W states.

Finally, we set out to obtain the behavior of $|S_v|$ for both the GHZ and W states, when Pauli’s operator is used instead of Czachor’s operator. In this situation, we get

$$E_{GHZ}(ABC) = a \cos(\phi_1 + \phi_2 + \phi_3) + b \cos(\phi_1 - \phi_2 - \phi_3) + i(\sin(\phi_1 + \phi_2 - \phi_3) + \sin(-\phi_1 + \phi_2 - \phi_3))f - 2 \cos(\phi_1 - \phi_2 + \phi_3)d + (- \cos(\phi_1 - \phi_2 + \phi_3) + \cos(\phi_1 + \phi_2 - \phi_3))e,$$

where $i = \sqrt{-1}$ and

$$a = -\frac{1}{16} \sin^6 \left(\frac{\Omega_p}{2}\right) + \cos^2 \left(\frac{\Omega_p}{2}\right) \left(\cos^2 \left(\frac{\Omega_p}{2}\right) + \frac{3}{4} \sin^2 \left(\frac{\Omega_p}{2}\right)^2\right),$$

$$b = \frac{1}{16} \sin^4 \left(\frac{\Omega_p}{2}\right) \cos^2 \left(\frac{\Omega_p}{2}\right) \left(\cos^2 \left(\frac{\Omega_p}{2}\right) + \frac{3}{4} \sin^2 \left(\frac{\Omega_p}{2}\right)^2\right),$$

$$f = \frac{1}{4} \sin^2 \left(\frac{\Omega_p}{2}\right) \cos \left(\frac{\Omega_p}{2}\right) \left(\cos^2 \left(\frac{\Omega_p}{2}\right) + \frac{3}{4} \sin^2 \left(\frac{\Omega_p}{2}\right)^2\right),$$

$$d = \frac{1}{4} \sin^3 \left(\frac{\Omega_p}{2}\right) \cos \left(\frac{\Omega_p}{2}\right) \left(\cos^2 \left(\frac{\Omega_p}{2}\right) + \frac{3}{4} \sin^2 \left(\frac{\Omega_p}{2}\right)\right),$$

$$e = \frac{1}{4} \sin^4 \left(\frac{\Omega_p}{2}\right) \cos^2 \left(\frac{\Omega_p}{2}\right) + \frac{3}{4} \sin^2 \left(\frac{\Omega_p}{2}\right).$$

Now, setting $\phi_i = \frac{\pi}{4}$ and $\phi_i' = \frac{3\pi}{4}$, after some calculations we get

$$E_{GHZ}(ABC) = -\frac{\sqrt{2}}{2} a + \frac{\sqrt{2}}{2} b - \sqrt{2} d = -E_{GHZ}(A'B'C'),$$

$$E_{GHZ}(A'BC) = -\frac{\sqrt{2}}{2} a + \frac{\sqrt{2}}{2} b + \sqrt{2} d,$$

$$E_{GHZ}(AB'C) = -\frac{\sqrt{2}}{2} a - \frac{\sqrt{2}}{2} b + i\sqrt{2} f - \sqrt{2} d,$$

$$E_{GHZ}(ABC') = -\frac{\sqrt{2}}{2} a - \frac{\sqrt{2}}{2} b + \sqrt{2} d,$$

$$E_{GHZ}(A'B'C) = +\frac{\sqrt{2}}{2} a + \frac{\sqrt{2}}{2} b - i\sqrt{2} f - \sqrt{2} d - \sqrt{2} e,$$

$$E_{GHZ}(AB'C') = +\frac{\sqrt{2}}{2} a - \frac{\sqrt{2}}{2} b - \sqrt{2} d,$$

$$E_{GHZ}(A'B'C') = +\frac{\sqrt{2}}{2} a + \frac{\sqrt{2}}{2} b + \sqrt{2} d + \sqrt{2} e,$$

which finally leads to:

$$|S_v| = \sqrt{32a^2 + 8f^2}.$$

which, as a check, covers the lab frame result ($4\sqrt{2}$) in the appropriate limit $\Omega_p \to 0$. Since in the $\beta \to 1$ limit, $f \to 0$ for the low energy particles, the $|S_v|$ inequality is violated to the same value as the lab frame ($4\sqrt{2}$), which is in agreement with previous study pointed out in the Case I [58]. It can also be checked that, in the $\beta \to 1$ limit, high energy particles does not violate $|S_v|$. In fact, in this situation we get $|S_v| \approx 2.121496$. This behavior is in line with some studies on two-particle systems [23, 24].

For the W state, calculations lead to

$$E_W(\theta_1 \theta_2 \theta_3) = \cos(\theta_1) \cos(\theta_2) \cos(\theta_3) A' + \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) B' + \sin(\theta_1) \cos(\theta_2) \sin(\theta_3) C' + \sin(\theta_1) \cos(\theta_2) \sin(\theta_3) D' + \sin(\theta_1) \cos(\theta_2) \cos(\theta_3) E' + \cos(\theta_1) \sin(\theta_2) \sin(\theta_3) F' + \cos(\theta_1) \sin(\theta_2) \cos(\theta_3) G' + \cos(\theta_1) \cos(\theta_2) \sin(\theta_3) H',$$
in which

\[ A' = a'^2 - b'^2 - c'^2 + d'^2 + e'^2 - g'^2 - h'^2 + f'^2 \]  \hspace{1cm} (45)

and

\[ a' = \frac{\sqrt{3} \sin^3(\frac{\Omega_p}{2})}{4} \]
\[ b' = -\frac{1}{4\sqrt{3}} \sin^3(\frac{\Omega_p}{2}) \cos(\frac{\Omega_p}{2}) \]
\[ c' = \frac{1}{\sqrt{3}} \sin^3(\frac{\Omega_p}{2}) \cos(\frac{\Omega_p}{2}) \sin^2(\frac{\Omega_p}{2}) \]
\[ d' = \frac{1}{\sqrt{3}} \frac{1}{2} \sin^3(\frac{\Omega_p}{2}) \frac{1}{2} \cos(\frac{\Omega_p}{2}) \frac{1}{2} \sin^2(\frac{\Omega_p}{2}) \]
\[ e' = \frac{1}{\sqrt{3}} \frac{1}{2} \sin^3(\frac{\Omega_p}{2}) \frac{1}{2} \cos(\frac{\Omega_p}{2}) \frac{1}{2} \sin^2(\frac{\Omega_p}{2}) \]
\[ f' = \frac{1}{\sqrt{3}} \frac{1}{2} \sin^3(\frac{\Omega_p}{2}) \frac{1}{2} \cos(\frac{\Omega_p}{2}) \frac{1}{2} \sin^2(\frac{\Omega_p}{2}) \]
\[ g' = \frac{1}{\sqrt{3}} \frac{1}{2} \sin^3(\frac{\Omega_p}{2}) \frac{1}{2} \cos(\frac{\Omega_p}{2}) \frac{1}{2} \sin^2(\frac{\Omega_p}{2}) \]
\[ h' = \frac{1}{\sqrt{3}} \frac{1}{2} \sin^3(\frac{\Omega_p}{2}) \frac{1}{2} \cos(\frac{\Omega_p}{2}) \frac{1}{2} \sin^2(\frac{\Omega_p}{2}) \]

Now, for the correlation functions we get

\[ E \ w(\theta_1 \theta_2 \theta_3) = 0.5443A' + 0.1924B' + 0.38(C' + G' + H') + 0.272(D' + E' + F') \]
\[ E \ w(\theta_1' \theta_2' \theta_3') = -0.5443A' + 0.1924B' + 0.38(C' + G' + H') - 0.272(D' + E' + F') \]
\[ E \ w(\theta_1' \theta_2' \theta_3') = -0.5443A' + 0.1924B' + 0.38(C' + G' + H') + 0.272(D' + E' + F') \]
\[ E \ w(\theta_1' \theta_2' \theta_3') = -0.5443A' + 0.1924B' + 0.38(C' + G' + H') - 0.272(D' + E' + F') \]
\[ E \ w(\theta_1' \theta_2' \theta_3') = -0.5443A' + 0.1924B' + 0.38(C' + G' + H') + 0.272(D' + E' + F') \]
\[ E \ w(\theta_1' \theta_2' \theta_3') = -0.5443A' + 0.1924B' + 0.38(C' + G' + H') - 0.272(D' + E' + F') \]
\[ E \ w(\theta_1' \theta_2' \theta_3') = -0.5443A' + 0.1924B' + 0.38(C' + G' + H') + 0.272(D' + E' + F') \]

which finally gives

\[ |S_v| = | -2.1772A' + 1.088(D' + E' + F') |. \]  \hspace{1cm} (48)
One may check to see that the result of lab frame (4.354) is recovered in the appropriate limit $\Omega_\omega = 0$. In addition, it is a matter of calculation to show that, in the $\beta \to 1$ limit, this inequality is preserved by the high energy particles ($\Gamma \to \infty$). Moreover, for the low energy particles ($\Gamma \to 1$) in the $\beta \to 1$ limit, the violation amount of $|S_v|$ is the same as the lab frame (4.354). Therefore, we can conclude that if one uses Pauli’s operator, Bell’s inequality for the Bell states $| S \rangle_{\Omega}$, and special set of measurements violating $|S_v|$ to its maximum violation amount in the lab frame, then the violation of the corresponding Bell inequality in the moving frame depends on the particles energy in the lab frame.

### III. SUMMARY AND CONCLUDING REMARKS

In this work we have considered various three-particle systems and have calculated the Bell-like inequalities for each systems in the moving frame ($S'$) under Lorentz transformation. Our main result is that Svetlichny’s inequality ($|S_v|$) when combined with Czachor’s relativistic spin operator gives consistent and reasonable results in line with various studies in two-particle systems $| S \rangle_{\Omega}$, devoid of inconsistencies when one uses other measures such as $|M|$ or $|M'|$. We also studied the results of using Pauli’s operator to investigate the behavior of $|S_v|$ under LT. The results of considering Pauli’s operator is in agreement with some previous studies on the two and three-particle systems $| S \rangle_{\Omega}$ and differ from those obtained by considering Czachor’s operator.

We are able to show that, whenever Czachor’s operator is considered, if one uses the same set of angles in $S$ as well as $S'$ and starts with such a setup that maximizes non-locality in the rest frame then the results are such that (i) non-locality decreases as a function of boost parameter $\beta$. (ii) in the extreme relativistic case of $\beta \to 1$ limit, the inequality is satisfied indicating lack of non-locality. (iii) in such a limit, though the value of the inequality itself depends on the initial particles’ energy, it is not violated no matter how that energy is chosen. (iv) all these results are true regardless of the type of entanglement present in the pure three-particle system: the GHZ or W state. (v) these results are true regardless of how one sets up the particles in the rest frame, net non-zero momentum (Case $I$) or center-of-mass frame (Case $II$). Furthermore, comparison of our results with that of Ref. [54], where a different set of measurements are used to evaluate $|M|$, indicates that the three-particle Bell-like inequalities are more sensitive to measurement set-ups than the bi-partite case of Bell’s inequality, a point has been emphasized in previous non-relativistic studies [46, 49].

The fact that our general results are consistent with previous studies in two-particle systems $| S \rangle_{\Omega}$, supports these results against some other results to their contrary $| M \rangle_{\Omega}$. However, it is clear that one can also find certain measurements that leave the correlation functions unchanged in the moving frame, thus leading to invariance of such inequalities $| S \rangle_{\Omega}$ and as finding particular cases which lead to maximization of their violation in certain moving frames $| M \rangle_{\Omega}$ and $| M' \rangle_{\Omega}$. Eventually, our results show that the inconsistency between the previous attempts mentioned in the introduction is the direct result of using the $|M|$ inequality and the set of measurements which violate the $|M|$ inequality to its maximum violation amount in the lab frame to study the behavior of the GHZ state under LT $| M \rangle_{\Omega}$. In fact, if one uses Czachor’s operator and the special set of measurements violating the $|S_v|$ inequality to its maximum possible violation amount in the lab frame the mentioned inconsistency will be eliminated independently of considering the $|S_v|$, $|M|$ or $|M'|$ inequality.

We also investigated the results of using Pauli’s operator instead of Czachor’s operator to study the behavior of $|S_v|$. Our study shows that in this situation the violation amount of $|S_v|$ in the moving frame depends on the particles energy in the lab frame. This result is in line with some previous studies $| S \rangle_{\Omega}$ and also helps in eliminating the mentioned inconsistency. Finally, we note that since our results for Pauli’s operator vs. Czachor’s operator lead to decidedly different type of behavior under LT, they provide a mechanism whereby a Stern-Gerlach experiment could be used to see which results are more consistent with experiments and therefore provide evidence for a more suitable spin operator.

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