Theoretical investigation of unsteady MHD flow within non-stationary porous plates

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A B S T R A C T

Porous-permeable structures with magnetohydrodynamics (MHD) flow have a lot of applications in various disciplines of engineering. Transport of ferro-fluid in magnetic field has attained more focus in recent years. In this work, an analysis is accomplished to investigate the MHD transport of in-compressible viscous ferro-fluid (electrically conductive) amid two movable porous-permeable plates (PPP). Here asymptotic method is then selected to solve the model. In this study, the effects of diverse considerable parameters and constraints like surface permeability, Reynold’s factor, and Hartmann’s factor are elaborated in details.

1. Introduction

Porous-permeable structured fins or plates (PPP) are the principal operational mechanism for enhancing the working and efficacy of transport properties and dissipation because of their many physical characteristics. From several years, the fluidity of the unsteady viscous fluid particles owing to its numerous methodological and systematic submissions has been a significant topic in the numerous widespread investigations of flowing particles. Examples are rotational machinery, heat exchangers, computers based storage manoeuvres, greases of porous-permeable machine parts and anywhere the surfaces and discs are disconnected for grease that is inserted within the discs (Khan et al., 2019; Joseph and Tao, 1966; O’Connor, 1968). MHD stream on rotating disc with the association of feeble magnetic influences has been deliberated (Rizvi, 1962). An unsteady MHD transport on spinning disks has been investigated as well (Mohyuddin, 2007). The incompressible viscous fluid stream amid two spinning and immobile naturally porous disks has also been studied (Purohit and Bansal, 1995). Then analytic outcomes of MHD stream, rotating amid two porous permeable plates were particularized (Ganji et al., 2014). As fluid rotation is depending upon electrical conduction in the occurrence of a magnet, it is continuously associated to the MHD. And numerous solicitations of MHD structures are realized like the design and construction of temperature exchangers, filters, accelerators, thermal management of reactors, etc. (Mohyuddin, 2007; Purohit and Bansal, 1995; Ganji et al., 2014).

Some Homoptic remarks on non-Newtonian fluids in porous-permeable structures have been studied earlier in (Hayat and Khan, 2005). Idea to utilise the magnetic fluid particle gave many important results for two permeable disks by employing HPM as deliberated in (Domaìrry and Aziz, 2009). Nazir et al. (Nazir and Mahmood, 2011) also observed the viscous flow through thermal changes amid two spinning PPP by employing the idea of appropriate resemblance transformation. Here, impression of suitable boundary condition was originally recommended by Uchida et al. (Uchida and Aoki, 1977) to grasp the dissimilar features of fluid performances comparable to the constricating or increasing containers. Ohki (Ohki, 1980) displays consideration to time-related velocity in a semi-infinite PPP, with the measurement of elastic wall differs with time. Barron et al. (Barron et al., 2000) were the innovators of experimenting on the unsteady flow in a conduit of quadrangular form within permeable shells and acknowledged the noticeable growth by refined process of CO2 to model the inoculation procedure at the exteriors.

Majdalani et al. (Majdalani et al., 2002; Majdalani and Zhou, 2003; Dauenhauer and Majdalani, 2003) investigated on dissimilar stream fields associated to the escalating or shrinking boards and presented numerical results. Additional, liquids stream by using the ADM in a gradually distorting passage with reduced porousness was deliberated by Asghar et al. (Asghar et al., 2010). It is expressed in preceding studies that liquid’s flow is limited by two rotating permeable walls and delivers the estimated results by the HAM (Dinarvand and Rashidi,
Fig. 1. Sketch for MHD flow within non-stationary porous plates.

2010; Si et al., 2011; Xu et al., 2010). Xu et al. (Xu et al., 2010) extended of the Majdalani’s work by accumulation the factor on unsteady flow. Ghaffar et al. (Ghaffar et al., 2015) offered the outcomes of time-dependent stream amid two movable porous plates with adjustable wall enlargement. Stimulated by the investigations stated overhead, the determination of the existing effort is to investigate the OHAM of the MHD based unsteady stream of a viscous liquid amid two PPP.

Here, we have neglected the influences of body forces. The stream is proportionally motivated by inoculation within non-Stationary PPP with an identified flow speed. The estimated results are acquired for speed contours by an influential technique named the OHAM (Xu et al., 2010; Marinca and Herişanu, 2010; Iqbal et al., 2010; Javed et al., 2010; Dauenhauer and Majdalani, 2003) that has previously been implemented on a great amount of non-linear models proficiently and efficiently. Additional flow contours are investigated for diverse factors. This research paper is organized into four sub-sections. In sub-section 2; the formulation of governing equation which is based upon the heat transfer equation is presented. In sub-section 3; on the basis of OHAM, the solution’s computational remarks are given. Also in sub-section 4, the results are discussed, with the illustration of some study-cases as examples. Finally, summarized the whole work.

2. Material and methods

Consider the model as shown in Fig. 1, which consists of incompressible, and electrically steered liquid retained within two circular porous plates moving orthogonally. These plates are along z-axis at positions \( z_0 = -H(s) \) and \( z_1 = H(s) \) in the downward and upward directions respectively with respect to the origin. The plates are moving at the rate \( H(s) \) with constant angular velocity \( \Omega \). \( S_0 \) is the strength for the magnetic field which is uniform in the plate. In the light of above assumptions, for the motion in cylindrical polar coordinates of axisymmetric flow of MHD and also the symmetric transformations using the governing equations of MHD give rise to following boundary value problem as in (Khan et al., 2019).

\[
\frac{d^3 f}{d\eta^3} + a \left( 3 \frac{d^2 f}{d\eta^2} + \frac{d^3 f}{d\eta^3} \right) - 2Re \frac{d^3 f}{d\eta^3} - M^2 \frac{d^2 f}{d\eta^2} = 0
\]

with conditions

\[
f(1) = 1, \quad f(-1) = -1, \quad \frac{df}{d\eta}(1) = 0, \quad \frac{df}{d\eta}(-1) = 0.
\]

where \( f(\eta) = \frac{G(\eta, s)}{R} \) is dimensionless stream function and \( \eta = \frac{z}{H(s)} \) is the similarity variable. \( G(\eta, s) \) is the function defined with the involvement of velocity components \( u \) and \( w \) in the \( r \)- and \( z \)-directions as in (Khan et al., 2019).

The expansion and contraction of flow depends upon the positive and negative values of \( \alpha = \frac{H H}{\nu} \), which is known as the wall expansion ratio, where \( \nu \) is the kinematic viscosity. The Hartmann parameter is \( M = \frac{\alpha s \nu H^2}{\rho} \), where density and electrical conductivity are denoted by \( \rho \) and \( \sigma \) respectively. \( R = \frac{A H H}{2\nu} \) is permeable Reynold number, where \( A = 2v_{w} \) is the rate of wall permeability and \( v_{w} \) is suction velocity. The conditions of injections and suction depend upon the positive and negative values of \( R \) (Xu et al., 2010; Dauenhauer and Majdalani, 2003; Javed et al., 2014). Now, we will analyze and find solution of the BVP (1) numerically by using OHAM. Fig. 2 explains the formulation of MHD Model.

3. OHAM formulation

In the light of optimal homotopy asymptotic method (Xu et al., 2010; Ghaffar et al., 2015; Marinca and Herişanu, 2010; Iqbal et al., 2010; Javed et al., 2010; Dauenhauer and Majdalani, 2003; Javed et al., 2014), the form of equation is as follows:

\[
D(v(s)) + F(v) = 0, \text{ for } s \in \Omega.
\]

Here \( \Omega \) is the domain. The operator \( D(v) \) in equation (3) is

\[
D(v) = L(v) + N(v).
\]

The optimal homotopy constructed by OHAM is as follows

\[
\phi (x; p) : \Omega \times (0, 1] \rightarrow R
\]

with

\[
(1 - p) \left[ L(\phi (x; p)) + F(x) \right] - H(p) [D(\phi (x; p)) + F(x)] = 0.
\]

Here the embedding parameter \( p \) has the property \( p \in [0, 1] \) and

\[
H(p) = pC_1 + p^2 C_2 + p^3 C_3 + \ldots
\]

is known as an auxiliary function with \( H(0) = 0 \) in the equation (4) and \( H(p) \neq 0 \) with \( p \neq 0 \). The unknowns values of constants \( C_1, C_2 \ldots \) are to be determined. The following approximate solution is by expansion of \( \phi (x; p, C_i) \) with the use of Taylor’s series about \( p \)

\[
\phi (x; p, C_i) = \phi_0 (x) + \sum_{k=1}^{\infty} \phi_k (x; C_i) p^k, \quad i = 1, 2, \ldots
\]

The convergence of series in equation (5) mainly depends upon the constants \( C_1, C_2 \ldots \). The following is the result if series is convergent at \( p = 1 \):

\[
v(x; C_i) = \phi_0 (x) + \sum_{k=1}^{\infty} \phi_k (x; C_i).
\]

The residual expression obtained by substituting equation (6) into (3) is

\[
R(x; C_i) = L(v(x; C_i)) + F(x) + N(v(x; C_i)).
\]

Note that \( R(x; C_i) \) results the exact solution if \( R(x; C_i) = 0 \), which in nonlinear problems not generally happens. Using the method as mentioned in (Xu et al., 2010; Ghaffar et al., 2015; Marinca and Herişanu, 2010; Iqbal et al., 2010; Javed et al., 2010; Dauenhauer and Majdalani, 2003; Javed et al., 2014). The values of the constants \( C_i \), \( i = 1, 2, \ldots, m \), can be determined.
4. Mathematical formulation of MHD flow analysis

Here, in this section by following the flow as depicted in Fig. 2, we will formulate and analyze OHAM based formulation to nonlinear ordinary differential equation (1) as follows:

\[
(1 - p) \frac{d^4f}{d\eta^4} - \left(C_1 p + C_2 p^2 + C_3 p^3\right) \left[\frac{d^3f}{d\eta^3} + \alpha(3 \frac{d^2f}{d\eta^2} + \eta \frac{df}{d\eta})\right] + 2Rf \frac{d^2f}{d\eta^2} - M^2 \frac{df}{d\eta} = 0
\]

(7)

We consider \( f(\eta) \) as follows:

\[
f(\eta) = f_0(\eta) + pf_1(\eta) + p^2f_2(\eta) + p^3f_3(\eta)
\]

(8)

Substituting \( f(\eta) \) from (8) into (7), and after simplification and arranging in the form such that based on powers of \( p \)-terms, it follows that

\[
p^0: \quad \frac{d^4f_0}{d\eta^4} = 0, \quad f_0(1) = 1, \quad f_0(-1) = 1, \quad \frac{df_0}{d\eta}(1) = 0, \quad \frac{df_0}{d\eta}(-1) = 0.
\]

(9)

\[
p^1: \quad C_1 M^2 \frac{d^2f_0}{d\eta^2} + 2C_1 Rf_0(\eta) \frac{d^3f_0}{d\eta^3} - C_1 \frac{d^4f_0}{d\eta^4} - aC_1 \frac{df_0}{d\eta} - 3aC_1 \frac{d^2f_0}{d\eta^2} = 0,
\]

\[
-\frac{d^3f_0}{d\eta^3} + \frac{d^2f_1}{d\eta^2} = 0,
\]

\[
f_1(1) = 0, \quad f_1(-1) = 0, \quad \frac{df_1}{d\eta}(1) = 0, \quad \frac{df_1}{d\eta}(-1) = 0.
\]

(10)

\[
p^2: \quad C_2 M^2 \frac{d^2f_0}{d\eta^2} + C_1 M^2 \frac{d^2f_1}{d\eta^2} + 2C_1 R \frac{d^3f_0}{d\eta^3} f_0(\eta) + 2C_2 Rf_0(\eta) \frac{d^3f_0}{d\eta^3} = 0,
\]

\[
+ 2C_1 Rf_0(\eta) \frac{d^3f_1}{d\eta^3} - C_1 \frac{df_1}{d\eta} - aC_1 \frac{d^2f_0}{d\eta^2} - 3aC_1 \frac{df_0}{d\eta} - C_1 \frac{df_1}{d\eta} - \frac{d^2f_0}{d\eta^2} = 0,
\]

\[
- aC_1 \frac{df_1}{d\eta} - 3aC_1 \frac{df_1}{d\eta} + \frac{d^3f_2}{d\eta^3} = 0,
\]

\[
f_2(1) = 0, \quad f_2(-1) = 0, \quad \frac{df_2}{d\eta}(1) = 0, \quad \frac{df_2}{d\eta}(-1) = 0.
\]

(11)

\[
p^3: \quad C_3 M^2 \frac{d^2f_0}{d\eta^2} + C_2 M^2 \frac{d^2f_1}{d\eta^2} + C_1 M^2 \frac{d^2f_2}{d\eta^2} + 2C_2 R \frac{d^3f_0}{d\eta^3} f_0(\eta) + \frac{d^4f_3}{d\eta^4} = 0,
\]

\[
+ 2C_2 R \frac{d^3f_0}{d\eta^3} f_0(\eta) + 2C_1 R \frac{d^3f_0}{d\eta^3} \frac{d^3f_0}{d\eta^3} + 2C_2 R \frac{d^3f_0}{d\eta^3} \frac{d^4f_3}{d\eta^4} = 0.
\]
Fig. 3. Response of \( f(\eta) \) at different choice of values of \( R \) at \( \alpha = -1. M = 2 \) depicting the interesting nature of \( f(\eta) \) with following varying range values of \( \eta \). (a) \( \eta \) ranges from -0.5 to 0.5, (b) \( \eta \) ranges from -1.0 to 1.0, (c) \( \eta \) ranges from -1.5 to +1.5, (d) \( \eta \) ranges from -2.0 to +2.0, (e) \( \eta \) ranges from -2.5 to +2.5, (f) \( \eta \) ranges from -8.0 to +8.0.

With the help of boundary conditions, the equations (9), (10), (11) and (12) yield the following

\[
f_0(\eta) = \frac{1}{2} (3\eta - \eta^3);
\]

\[
f_1(\eta) = \frac{1}{280} \left( 7C_1M^2\eta^5 - 14C_1M^2\eta^3 + 7C_1M^2\eta + C_1(-R\eta^2 + 21C_1R\eta^3) - 39C_1R\eta^3 + 19C_1R\eta - 28aC_1\eta^5 + 56aC_1\eta - 28aC_1\eta \right)
\]

\[
f_2(\eta) = -115C_1^2M^4\eta^7 + 4851C_1^2M^4\eta^5 - 6237C_1^2M^4\eta^3 + 2541C_1^2M^4\eta
\]

\[
- 15246C_1^2M^2R\eta^5 + 30492C_1^2M^2R\eta^3 - 19866C_1^2M^2R\eta^5
\]

\[
+ 3465C_1^2M^2R\eta + 11550aC_1^2M^2\eta^3 - 38808aC_1^2M^2\eta^5
\]

\[
+ 48510C_1^2M^2\eta^5 + 48510C_1M^2\eta^7 + 48510C_1M^2\eta^7
\]

\[
+ 42966aC_1^2M^2\eta^5 - 97020C_1^2M^2\eta^3 - 97020C_1^2M^2\eta^3
\]

\[
- 97020C_1^2M^2\eta^3 - 15708aC_1^2M^2\eta^7 + 48510C_1^2M^2\eta
\]

\[
+ 48510C_1M^2\eta + 48510C_1M^2\eta - 189C_1^2R^2\eta^{11} + 4620C_1^2R^2\eta^9
\]

\[
- 35046C_1^2R^2\eta^7 + 47124C_1^2R^2\eta^5 - 6645C_1^2R^2\eta^3 - 9864C_1^2R^2
\]

\[
- 5005aC_1^2R^2\eta^5 + 67914aC_1^2R^2\eta^3 - 6930C_1^2R^2\eta^7 - 6930C_1^2R^2\eta^7
\]

\[
- 6930C_1^2R^2\eta^7 - 121968aC_1^2R^2\eta^5 + 145530C_1^2R^2\eta^3 + 145530C_1^2R^2\eta^5
\]

\[
+ 145530C_2R^2\eta^5 + 60214aC_1^2R^2\eta^3 - 270270C_1^2R^2\eta^3 - 270270C_1^2R^2\eta^3
\]

\[
- 270270C_2R^2\eta^3 - 1155aC_1^2R^2 + 131670C_1^2R^2 + 131670C_1^2R^2
\]

\[
+ 131670C_2R^2 - 27720aC_1^2\eta^7 + 77616aC_1^2\eta^5 - 194040aC_1^2\eta^3
\]
Fig. 4. Response of $f'(\eta)$ at different choice of values of $R$ at $\alpha = -1, M = 4$ depicting the interesting nature of $f(\eta)$ with following varying range values of $\eta$. (a) $\eta$ ranges from -0.5 to 0.5, (b) $\eta$ ranges from -1.0 to 1.0, (c) $\eta$ ranges from -1.5 to +1.5, (d) $\eta$ ranges from -2.0 to +2.0, (e) $\eta$ ranges from -2.5 to +2.5, (f) $\eta$ ranges from -6.0 to +6.0.

Substituting values in (8), gives the approximate solution of first order of (1).

$$f(\eta) = -\frac{1}{1940400} \left[ 13860 C_1 (\eta^2 - 1)^2 \left( 28 \alpha - 7 M^2 + R (\eta^2 - 19) \right) + 6930 \left( C_2 (\eta^2 - 1)^2 \left( -28 \alpha + 7 M^2 - R (\eta^2 - 19) \right) - 140 (\eta^2 - 3) \right) + C_2 (\eta^2 - 1)^2 \left( 231 M^2 (5 \eta^2 - 11) - 231 M^2 (68 \alpha + R (5 \eta^4 - 56 \eta^2) + 15 + 50 \eta^2 + 210) \right) + 3 R^2 \left( 63 \eta^6 - 1414 \eta^4 + 8791 \eta^2 + 3288 \right) + 77 R \left( 15 (\alpha - 114) + 65 \alpha \eta^2 + (90 - 752 \alpha) \eta^2 \right) \right]$$

$$(15)$$

Now for finding values of the constants $C_1$ and $C_2$ shown in Eq. (16), by the method of least squares (Xu et al., 2010; Ghaffar et al., 2015; Marinca and Herişanu, 2010; Iqbal et al., 2010; Javed et al., 2010; Dauenhauer and Majdalani, 2003; Javed et al., 2014) implies that with

$$\frac{\partial J}{\partial C_i} = 0$$

(17)

gives the values of constants $C_1$ and $C_2$, where

$$J = \int_0^1 R^2 ds$$

(18)

and here $R$ for the equation (1) is

$$R_\alpha = \frac{d^4 f}{d \eta^4} + \alpha \left( \frac{d^2 f}{d \eta^2} + \frac{d^3 f}{d \eta^2} \right) - 2 R \frac{d^3 f}{d \eta^2} - M^2 \frac{d^2 f}{d \eta^2}$$

(19)
Fig. 5. Response of $f''(\eta)$ at different choice of values of $R$ at $\alpha = -1, M = 6$ depicting the interesting nature of $f(\eta)$ with following varying range values of $\eta$. (a) $\eta$ ranges from -0.5 to 0.5, (b) $\eta$ ranges from -1.0 to 1.0, (c) $\eta$ ranges from -1.5 to +1.5, (d) $\eta$ ranges from -2.0 to +2.0, (e) $\eta$ ranges from -2.5 to +2.5, (f) $\eta$ ranges from -5.0 to +5.0.

| Table 1. Values of $R$ and $C_i$, $C_j$, $a = -1$, $M = 2$ |
|---------------------------------------------------------------|
| $R$  | $C_i$  | $C_j$  |
| 2    | -0.983721 | 641852  |
| 4    | 0.473149  | -0.889906 |
| 6    | 0.299114  | -0.408033 |
| 8    | 0.231707  | -0.274439 |
| 10   | 0.192113  | -0.209587 |
| 14   | 0.143978  | -0.142184 |

With $\alpha = -1$, $M = 2$, and for varying values of $R$, the equations (17) and (18) give the following values of constants $C_i, i = 1, 2$ shown in Table 1. With these constants, the values of $f(\eta)$ are obtained as shown in the equations (20), (21), (22), (23), (24), (25) and corresponding results are shown in Fig. 3. Also the behaviour of $f'(\eta)$, $f''(\eta)$ and $f'''(\eta)$ with $\alpha = -1$ and varying values of $R$ are represented in Figs. 4, 5 and 6 for $M = 4$, $M = 6$ and $M = 8$ respectively.

$$f_{(R=2)}(\eta) = -0.000377029\eta^{11} + 0.0188165\eta^{9} - 0.242004\eta^{7} + 0.366889\eta^{5} - 0.581148\eta^{3} + 1.43782\eta,$$  

(20)

$$f_{(R=4)}(\eta) = -0.000348888\eta^{11} + 0.0129702\eta^{9} - 0.138972\eta^{7} + 0.380014\eta^{5} - 0.893248\eta^{3} + 1.63958\eta,$$  

(21)

$$f_{(R=6)}(\eta) = -0.000313722\eta^{11} + 0.0103315\eta^{9} - 0.104086\eta^{7} + 0.341802\eta^{5} - 0.911104\eta^{3} + 1.66337\eta,$$  

(22)

$$f_{(R=8)}(\eta) = -0.000334679\eta^{11} + 0.0103115\eta^{9} - 0.10008\eta^{7} + 0.340161\eta^{5} - 0.919655\eta^{3} + 1.6696\eta,$$  

(23)

$$f_{(R=10)}(\eta) = -0.000359489\eta^{11} + 0.0106182\eta^{9} - 0.100492\eta^{7} + 0.342972\eta^{5} - 0.925144\eta^{3} + 1.6724\eta,$$  

(24)

$$f_{(R=14)}(\eta) = -0.000395749\eta^{11} + 0.0111134\eta^{9} - 0.101974\eta^{7} + 0.346602\eta^{5} - 0.929757\eta^{3} + 1.67441\eta.$$  

(25)
Fig. 6. Response of $f''''(\eta)$ at different choice of values of $R$ at $\alpha = -1, M = 8$ depicting the interesting nature of $f(\eta)$ with following varying range values of $\eta$. (a) $\eta$ ranges from -0.6 to 0.6, (b) $\eta$ ranges from -1.0 to 1.0, (c) $\eta$ ranges from -1.5 to +1.5, (d) $\eta$ ranges from -2.0 to +2.0, (e) $\eta$ ranges from -2.5 to +2.5, (f) $\eta$ ranges from -6.0 to +6.0.

| $M$  | $C_1$   | $C_2$   |
|------|---------|---------|
| 0    | -0.766337 | 1.46101 |
| 2    | -0.728105 | 1.29109 |
| 4    | -0.574535 | 0.857774 |
| 6    | -0.574535 | 0.857774 |
| 8    | -0.254526 | 0.319162 |
| 10   | -0.165582 | 0.209974 |

Now with the selection $\alpha = 1, R = 4$, and with varying values of $M$, the equations (17) and (18) give the following values of constants $C_i, i = 1, 2$ shown in Table 2. Now with these constants values, the values of $f(\eta)$ are calculated as shown in the equations (26), (27), (28), (29), (30), (31) and corresponding results are shown in Fig. 7. Also the behaviour of $f'(\eta), f''(\eta)$ and $f''''(\eta)$ with $\alpha = 1, R = 4$ and varying values of $M$ are represented in Figs. 8, 9 and 10 respectively.
The essential factors convoluted in this paper are the Re, α and M. And the shear stresses are investigated by quantity of \( f(\eta) \) for planes of flat surfaces.

Some different studies are given: First study: In instance of rotating plates are going outwards. Here, we take \( \alpha = 1 \). Explanation of satisfactory region of curvatures is stated in Table 1 for dissimilar values of Re and M concurrently. Second study: When the PPP are impending closer to each other. Here \( \alpha = -1 \). The explanation of graphs within range is revealed in Table 2.

The influence of dissimilar factors distribution of velocity profiles has been presented for the above mentioned studies in Figs. 3, 4, 5, 6, 7, 8, 9 and 10. Profile s of axial, radial velocity and applied stresses are given as well. From above results, it is deduced that a greater value of the Hartmann number reduces axial velocity contour, although for the entire dominion, separately axial velocity contour upsurges extent. From Figs. 3, 4, 5 and 6, in case of \( \alpha = -1 \), it is clear that radial velocity contours exhibit mirror response and increasing M reduces the contour close the PPP so by affecting the velocity significantly. In fact, a greater the worth of M frequently decelerates the fluid particles owing to the conflicting power recognized as the Lorentz force. In 7, 8, 9 and 10, \( \alpha = 1 \) it is clear that special effects are fast owing to a great \( R \) and for contraction case elaboration is the same but contour abruptly flattens close the midpoint of the field. Additional, it is significant to communicate that the current profile be subject to mostly on \( R \). Thus, if there is surge in \( R \) values against magnetization, we observe results as given in Figs. 3, 4, 5, 6, 7, 8, 9 and 10. Figs. 3, 4, 5, 6, 7, 8, 9 and 10 express about the collection of different \( R \) in growing order of \( M \) where major part of changes happen in the speed profile, i.e. the contour upsurges sooner at surfaces. After the values of \( R = 8 \), the contours start to crash their parabolic performance at the mid-point of the dominion.

5. Results and discussion

In this investigation, study is carried out in order to analyse behaviour of the unsteady MHD ferroparticles flow for the incompressible and two-dimensional viscid flow for the geometry of two orthogonal movable porous-permeable plates. Values of axial velocity upsurges in the entire plane and dominion. Circular velocity contour also shows some upsurge near mid point and declines on the exteriors. From the
above results, it is deduced that greater values of the Hartmann number actually reduce axial speed. It is evident from radial velocity contours show symmetric responses. In fact, the greater values of Hartmann number frequently changing and decelerate the fluid particles owing to the conflicting power recognized as the Lorentz force.

**Data availability statement**

Data will be made available on request.

**Declaration of interests statement**

The authors declare no conflict of interest.

**Additional information**

No additional information is available for this paper.

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Fig. 9. Response of $f''(\eta)$ at different choice of values of $M$ at $a = 1, R = 4$ depicting the interesting nature of $f''(\eta)$ with following varying range values of $\eta$. (a) $\eta$ ranges from -0.5 to +0.5, (b) $\eta$ ranges from -1.0 to +1.0, (c) $\eta$ ranges from -1.5 to +1.5, (d) $\eta$ ranges from -5.0 to +5.0, (e) $\eta$ ranges from -7.0 to +7.0, (f) $\eta$ ranges from -11.0 to +11.0.

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Fig. 10. Response of $f'''(\eta)$ at different choice of values of $M$ at $\alpha = 1, R = 4$ depicting the interesting nature of $f(\eta)$ with following varying range values of $\eta$. (a) $\eta$ ranges from -0.5 to +0.5, (b) $\eta$ ranges from -1.0 to 1.0, (c) $\eta$ ranges from -2.0 to +2.0, (d) $\eta$ ranges from -4.0 to +4.0, (e) $\eta$ ranges from -6.0 to +6.0, (f) $\eta$ ranges from -20.0 to +20.0.

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