Study of net-baryon higher moments in PNJL model at RHIC energies for the signature of the QCD critical point

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The non-monotonic behavior of the conserved quantities like net-baryon, net-charge and net-strangeness are believed to be the signatures of the QCD Critical End Point (CEP) as a function of the collision energy. We study the effect of the QCD critical point on moments of net-baryon in the Polyakov loop enhanced Nambu-Jona-Lasinio (PNJL) model of QCD. The study is performed at energies similar to RHIC beam energy scan (BES). Experimentally measuring conserved quantities is difficult due to systematic limitations, therefore net-proton, net-pion, net-kaon are measure as the proxy of $\Delta B$, $\Delta Q$, $\Delta S$. Thus the need for different models becomes predominant to estimate the value of different observables. The PNJL model of QCD, is such an effective model, which possesses the benefit of having characteristics similar to the observables. Higher-order moments like skewness ($S$), kurtosis ($\kappa$) and their products ($s\sigma$, $\kappa\sigma^2$) which are calculated in the PNJL model, are sensitive to the correlation length of the hot and dense medium, making them more prone to search for the critical point. We also compare the value of higher-order moment products with STAR and HRG data with different values to understand the existence of critical point.

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I. INTRODUCTION

Quantum Chromodynamics (QCD) describes the strong interaction based on $SU(3)$ gauge theory with quark in the fundamental and gluon in the adjoint replacement of the gauge group weak connection. The main aim of the Relativistic Heavy-Ion Collider (RHIC) Beam Energy Scan (BES) is to explore the bulk properties of QCD such as phase transition, QCD critical point, and thermalization of the matter. The most important characteristic of relativistic nucleus-nucleus collision is the large amount of energy deposited in a very small region of space for a very short duration of time. Hadrons confine quarks and gluons at ordinary temperature but a de-confinement phase of quarks and gluons takes place at very high temperature and density, creating a state called Quark-Gluon Plasma (QGP) [1]. It is noticed that some matter undergoes a change from hadronic state to QGP state initiating a first-order chiral phase transition. Lattice QCD calculation at vanishing chemical potential $\mu_B$ indicates a rapid but smooth cross-over transition at a large temperature $T$, while various models representing matter at vanishing $T$ predict a strong first-order phase transition at a large $\mu_B$. If various models are correct, then a critical point must be located where the transition changes from a smooth cross-over to first-order [2, 3]. The critical endpoint (CEP) at the end of the first-order line is a critical point where the phase transition is second-order [4, 5]. The properties of low-energy hadrons as well as the nature of the chiral phase transition at finite temperature and density have been studied intensively through different effective models [6, 7, 8]. The Nambu-Jona-Lasinio (NJL) model [9] describes the interaction of constituent quark fields. It exhibits a global $SU(3)$ symmetry that is a replacement of a local gauge $SU(3)$ color transformation of the QCD Lagrangian. This color confinement is lost in the NJL dynamics [10]. The idea is to extend the chiral Lagrangian such as the NJL model by introducing a coupling of quarks to a uniform temporal background of gauge field (the Polyakov loop) was an important step forward in this investigation. The PNJL model [11] exhibits two essential features of QCD, which are spontaneous chiral symmetry breaking and confinement-like property. The event-by-event fluctuations of conserved quantities such as net-charge,
net-baryon number, and net-strangeness are predicted to depend on the non-equilibrium correlation length $\xi$ [12, 13] which in the idealized thermodynamic limit, diverges at the critical point. A theoretical calculation suggests that $\xi$ may rise from $\sim 0.5$ to 3 fm in heavy-ion collision, constrained by the size of the system. Experimentally protons and anti-protons are measured with high efficiency and have been shown to be reliable proxies for baryons and anti-baryons. In heavy-ion collision experiments, the QCD critical point can be found via the non-monotonic behavior of many fluctuation observables as a function of the collision energy. Locating the point requires a scan of the phase diagram by varying temperature and chemical potential which can be performed by varying the initial collision energy $\sqrt{s}$. The event-by-event particles multiplicity fluctuations can be characterized by the moments of the event-by-event multiplicity distributions. The magnitude of fluctuations in conserved quantities like net-baryon, net-charge and net-kaon at finite temperature are distinctly different in the hadronic and QGP phase. Higher-order moments like mean ($M$), variance ($\sigma^2$), skewness ($S$), kurtosis ($\kappa$) depend on the higher power of $\xi$ i.e., $S \sim \xi^{4.5}$ and $\kappa \sim \xi^7$. The signature of the phase transition or Critical point is detectable if they survive the evolution of the system. Finite-size and time effects in heavy-ion collisions put constraints on the significance of the desired signals [14]. The moment ratios of experimental observables cancel the volume dependency of the system and can be directly compared to the ratios of susceptibilities from theoretical calculation. Hence it is proposed to study the higher order moments like [skewness $S = \langle (\delta N)^3 \rangle / \sigma^3$ and kurtosis $\kappa = \langle (\delta N)^4 \rangle / \sigma^4 - 3$ with $\delta N = N - \langle N \rangle$ and their ratios of the conserved quantities due to stronger dependence on $\xi$ [15, 16]. So, the non-monotonic appearance and then disappearance of the critical fluctuation on which we have described would be strong evidence for the critical point.

In this paper, we study the fluctuations of higher moments (mean ($M$), variance ($\sigma^2$), skewness ($S$), kurtosis ($\kappa$)) and their volume independent moment products ($s\sigma$, $\kappa\sigma^2$) of net-baryon in the PNJL model. The higher order moments and their volume independent moment products of the conserved quantities are expected to show fluctuations as they are significantly more sensitive to the proximity of the QCD phase diagram.

II. WHAT’S THE NEED FOR CALCULATION OF HIGHER-ORDER MOMENTS?

Physics of heavy-ion collisions is one of the several experimental windows of QCD phenomenology. There are many experimental programs like the Beam Energy Scan (BES) program at the Relativistic Heavy-Ion Collider (RHIC) which aims to study the detailed QCD phase structure [17, 18, 19, 20]. As any system in thermal and chemical equilibrium is characterized by the given values of temperature $T$ and baryon chemical potential $\mu_B$. The modern physicists map the phase diagram of strongly interacting matter (describe through QCD) through temperature ($T$) versus baryonic chemical potential ($\mu_B$) curve. With many studies, the existence of QCD critical point and the first-order phase boundary between quark-gluon and hadronic phases. The critical point is of great importance to modern physicists. As known, a phase transition is actually a thermodynamic singularity of the system. But as we decrease $\mu_B$ after the critical point, the transitions are smooth i.e. there is no singularity [21]. Theoretically, the determination of CP at a coordinate ($\mu_B$, $T$) is a definite task which involves analytical skill in the mathematical method that is yet to be achieved at present. The only choice left is to approach it using numerical methods. This is achieved through monte Carlo simulations. But, they also have limitations at the non-zero value of $\mu_B$. The simulation models are made to satisfy the vacuum phenomenology i.e., $T = \mu_B = 0$ [22]. Therefore, the next plan of action towards determination of CP is from experimental means. Even though the exact coordinate of CP is yet to be calculated theoretically, but it is suggested from different studies that the location of CP is within a regime which can be probed through heavy-ion collision experiments. The underlying physics for the detection of signatures of CP in the experiment is interpreted as the divergence of susceptibilities at the critical points. Some observable which is related to the susceptibilities are needed which would show some fluctuation at CP. As the moments of the distributions of conserved quantities such as net-baryon, net-charge, and net-strangeness, suppose to show a non-monotonic behaviour near the CEP which suggests good signatures of phase transition and CP. That’s the whole reason, the calculation of higher moments are necessary for the study of QCD-phase. Theoretical and experimental challenges faced during determination of CP is described in Ref. [23].
A. Calculation of higher-order moments from distribution of conserved quantities

In terms of statistics, a probability distribution is mainly characterized by various higher-order moments (Mean (M), variance (σ²), Skewness (S), kurtosis (κ)). This section is dedicated to the definition of central moments and cumulants. The distribution of the conserved quantity can be simply given by major emitted particle of the quantity. For example, net-proton distribution is used as a proxy for net-baryon quantity as the abundance of the proton is most among other emitted baryons (about 70%). Similarly, net-kaon distribution is used as a proxy of net-strangeness quantity ~ 65%. Experimentally, Net-proton distribution is find out by calculating event-by-event ∆Np as,

$$\Delta N_P = N_P - \bar{N}_P$$  \hspace{1cm} (1)

where $N_P$ is the proton number and $\bar{N}_P$ is the anti-proton number produced in the collision. Generally, $\Delta N$ is used to denote Net-particle number (in case of proton $\Delta N_P$) in one event. Thus, $\langle N \rangle$ represents the average of $N$ over all event ensemble. Here, angular brackets are used to denote the average of the event-wise distribution. And the deviation of $N$ from its average net-particle multiplicity (i.e. $M=\langle N \rangle$) is given by,

$$\delta N = N - M$$ \hspace{1cm} (2)

While defining the central moment, the average of net-particle distribution is denoted as,

$$\hat{\mu} = \langle N \rangle = M$$ \hspace{1cm} (3)

The hat is used to define the average operation. The $r^{th}$ central moment is given by,

$$\hat{\mu}_r = \langle (\delta N)^r \rangle$$ \hspace{1cm} (4)

Also,

$$\hat{\mu}_1 = \langle N \rangle - \hat{\mu} = \hat{\mu} - \hat{\mu} = 0$$ \hspace{1cm} (5)

The cumulants can be calculated from the moments as,

$$\hat{C}_1 = \hat{\mu}$$
$$\hat{C}_2 = \hat{\mu}_2$$
$$\hat{C}_3 = \hat{\mu}_3$$ \hspace{1cm} (6)

Above $n=3$, $\hat{C}_n$ is generalized through a recursion relation as [24],

$$\hat{C}_n = \hat{\mu}_n - \sum_{m=2}^{n-2} \binom{n-1}{m-1} \hat{C}_m \hat{\mu}(n-m)$$ \hspace{1cm} (7)

Thus, the skewness and kurtosis of the multiplicity distribution is denoted as,

$$\hat{M} = \hat{C}_1 ; \sigma^2 = \hat{C}_2 ; \hat{S} = \hat{\hat{C}}_3^{3/2} \hat{C}_2^{1/2} ; \hat{\kappa} = \hat{\hat{C}}_4 \hat{C}_2$$ \hspace{1cm} (8)

The moments define the characteristics of the net-particle multiplicity distribution. The first-order moment describes an expectation operator of the multiplicity density. The second-order moment is called variance and it gives the susceptibility of the measurements. The third-order moment measures the lopsidedness of the distribution. The normalization of the third-order moment is known as skewness. Skewness tells us about the direction of variation of the data set. The fourth-order moment compares the peakness or shortness and squatness, that is, shape, of a certain measurement to its normal distribution. The normalization of the fourth-order moment is known as heteroskedacticity or kurtosis. Actually, the kurtosis is given by normalized fourth-order moment minus 3. The subtraction of 3, which arises from the Gaussian distribution. The fifth-order moment measures the asymmetry sensitivity of the fourth-order moment and the sixth-order moment is generally associated with compound options. The detailed calculation of moments from the statistical point of view from the definition of Partition function is calculated analytically [24].
The non-linear sigma model (NLSM) is a phenomenological approach allowing the study of critical opalescence in nuclear systems. In this model, the moments are also related\cite{25, 26} to the correlation length of the system produced in heavy-ion collision, given by $\xi$. From, theoretical calculation it is found that for heavy-ion collision, $\xi \approx 2 - 3\text{ fm.}$ The variation of correlation length to the moments are given as,

$$\hat{\sigma}^2 \sim \xi^2; \hat{S} \sim \xi^{4.5}; \hat{\kappa} \sim \xi^7.$$ (9)

The variation of the correlation length to the moments is given in Ref.\cite{16},

B. Need for calculation of the product of moments

The moments are related to the thermodynamic susceptibilities, $\chi^{(n)}_i$, where $n$ is the order of the susceptibility and $i$ stands for the type of conserved quantum number. For net-proton distribution, $i$ becomes baryon quantum number ($B$). These susceptibilities are written in terms of cumulants ($C_n$) as,

$$\chi^{(n)}_i = \frac{1}{VT^3} C_n$$ (10)

where $V$ is the volume and $T$ is the temperature.

From Eq. (10) susceptibilities are dependent on volume i.e. the size of the system. This can also be observed that most of the cumulant values show a linear variation with the average number of participants (viz. $\langle N_{\text{part}} \rangle$) which is calculated from the globular model, this means a linear increase with $\langle N_{\text{part}} \rangle$ as the system volume increases. Refer to\cite{19, 20} for net-proton, net-kaon and net-Λ particle multiplicity distribution respectively. For volume independent susceptibility ratios, moment products or ratios of net-particle multiplicity distributions are calculated. They are related as,

$$\frac{\sigma^2}{M} = \frac{\chi^{(2)}_i}{\chi^{(1)}_i} S; \frac{\kappa \sigma^2}{S} = \frac{\chi^{(4)}_i}{\chi^{(2)}_i}$$ (11)

The calculation of product of moment is easily done by using the Eq. (8) as follows,

$$\frac{\sigma^2}{M} = \frac{C_2}{C_1}; \frac{\kappa \sigma^2}{S} = \frac{C_4}{C_2}$$ (12)

III. THE POLYAKOV LOOP ENHANCED NAMBU-JONA-LASINIO MODEL

At low temperatures, lattice and phenomenology say the state of the matter is confined. Whereas at high temperatures, perturbation theory is reliable and shows de-confinement. Therefore, there is a need to build a phenomenological model which describes both low and high temperature QCD behaviour in a single picture. The PNJL model is such a model that can serve this purpose. In this model, the gluon dynamics is reduced to the chiral point couplings between quarks and a background field that represents Polyakov loop dynamics. In the pure gauge system, the first-order de-confinement phase transition\cite{27} becomes discontinuous in the Polyakov loop, which turns out to be a smooth crossover when quarks are introduced. The Polyakov line is represented as\cite{28}

$$L(\vec{x}) = p \exp[i \int_0^\beta d\tau A_x(\vec{x}, \tau)]$$ (13)

where $A_x = iA_0$, is the temporal component of Euclidean gauge field ($\vec{A}, A_x$), in which strong coupling constant $g_s$ has been absorbed, $p$ denotes path ordering and $\beta = 1/T$ with Boltzmann constant $K_B = 1$. The Polyakov line $L(\vec{x})$ transform as a field with charge one under global $Z(3)$ symmetry. The Polyakov loop field\cite{29} is given by

$$\Phi = (Tr_L L)/N_c$$

and its conjugate $\bar{\Phi} = (Tr_L L^\dagger)/N_c$ (14)
The NJL model describes the interaction of constituent quark fields except for the substitution of a covariant derivative containing the temporal background gauge field $\Phi$. Thus the 2+1 flavor version of the PNJL model is described by the Lagrangian \cite{30},

$$
\mathcal{L} = \sum_{f=u,d,s} \bar{\psi}_f \gamma_\mu i D^\mu \psi_f - \sum_f m_f \bar{\psi}_f \psi_f + \sum_f \bar{\psi}_f \gamma_\mu \delta_{ij} \psi_f + \frac{g_u}{2} \sum_{a=0\ldots s} \left[ (\bar{\psi}_f \lambda^a \psi_f)^2 + (\bar{\psi}_f \gamma_\mu \Phi \psi_f)^2 \right] - g_\phi \text{det} \bar{\psi}_f P_L \psi_f + \text{det} \bar{\psi}_f P_R \psi_f \right] - u'[\Phi, \bar{\Phi}, T]
$$

where $f$ denotes the flavors u or d or s, respectively and for two flavors $g_d = 0$. $m_f$ represents the diagonal elements of mass matrix $(m_u, m_d, m_s)$ which is the current quark mass matrix, and $\lambda^a$ are the flavor $SU(3)$ Gell-Mann matrices $(a = 0, 1, \ldots, 8)$, with $\lambda^0 = \sqrt{\frac{2}{3}} I$. The matrices $P_{L,R} = (1 \pm \gamma_5)/2$ are respectively the left-handed and right-handed chiral projectors \cite{31}. The covariant derivative is defined as $D^\mu = \partial^\mu - i A^\mu$, with $A^\mu = \delta^\mu_0 A_0$ (Polyakov gauge); in Euclidean notation $A_0 = -i A_t$. In the PNJL model the Polyakov loop, which is the normalized trace of the Wilson line $L$, should become greater than unity above $2T_c$. To solve this problem, one has to take a proper Jacobian of transformation from the matrix valued field $L$ to $\phi$, which will then constrain the value of $\phi$ within 1. So, for reproducing lattice results, the Vandermonde term is to be introduced in the Polyakov loop potential. Thus the potential $u'$ with the Vandermonde term can be expressed as

$$
u'(\Phi, \bar{\Phi}, T) = \frac{u(\Phi, \bar{\Phi}, T)}{T^4} - \kappa d n[\Phi, \bar{\Phi}]
$$

where $u(\Phi)$ is the Landau-Ginzburg type potential commensurate with the $Z(3)$ global symmetry. We choose \cite{32},

$$
u(\Phi, \bar{\Phi}, T) = -\frac{b_2(T)}{2} \Phi \bar{\Phi} - \frac{b_3}{6} (\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4} (\Phi \bar{\Phi})^2
$$

with $b_2(T) = a_0 + a_1 \left( \frac{T_0}{T} \right)^{\frac{3}{2}} + a_2 \left( \frac{T_0}{T} \right)^{\frac{5}{2}} + a_3 \left( \frac{T_0}{T} \right)^3$ and $T_0$ is the critical temperature for de-confinement phase transitions according to pure gauge lattice theory, $b_i$ and $b_j$ are being constant. The term $\nu(\Phi, \bar{\Phi}, T)$ is the Vandermonde term which replicates the effect of $SU(3)$, where $j(\Phi, \bar{\Phi})$ is the Jacobian of transformation from the Wilson line $L$ to $(\Phi, \bar{\Phi})$ written as

$$
\nu(\Phi, \bar{\Phi}) = \left[ \frac{27}{24 \pi^2} (1 - 6\Phi \bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi \bar{\Phi})^2) \right]
$$

$j(\Phi, \bar{\Phi})$ is also known as the Vandermonde determinant and is not explicitly space-time dependent. The value of the dimensionless parameter $\kappa$ will be determined phenomenologically. The NJL part of the theory is analogous to the BCS theory of superconductor, where the pairing of two electrons leads to the condensation causing a gap in the energy spectrum. Similarly, in the chiral limit, the NJL model exhibits dynamical breaking of $SU(3)_L \times SU(3)_R$ symmetry to $SU(3)_V$. As a result, a composite operator picks up a nonzero vacuum expectation value leading to $\langle \bar{\psi}_f \psi_f \rangle$ condensation. The quark condensate is given by

$$
\langle \bar{\psi}_f \psi_f \rangle = -i N_c \mathcal{L}_{\gamma_{\mu}} \left( \text{tr} S_f (x - y) \right)
$$

where the trace is over color and spin states. The self-consistent gap equation for the constituent quark masses are

$$
M_f = m_f + g_u \sigma_f + g_d \sigma_{f+1} \sigma_{f+2}
$$

where $\sigma_f = \langle \bar{\psi}_f \psi_f \rangle$ denotes the chiral condensate of the quark with flavor $f$. Here if we consider $\sigma_f = \sigma_u$, then $\sigma_{f+1} = \sigma_d$ and $\sigma_{f+2} = \sigma_s$. Similarly if $\sigma_f = \sigma_d$, then $\sigma_{f+1} = \sigma_u$ and $\sigma_{f+2} = \sigma_s$; if $\sigma_f = \sigma_s$, then $\sigma_{f+1} = \sigma_u$ and $\sigma_{f+2} = \sigma_d$. The expression for $\sigma_f$ at zero temperature and zero chemical potential \cite{33} may be written as

$$
\sigma_f = -\frac{3M_f}{\pi^2} \int_0^\Lambda \frac{p^2}{\sqrt{p^2 + M_f^2}} dp
$$

where $\sigma_f = \langle \bar{\psi}_f \psi_f \rangle$.
where $\Lambda$ being three-momentum cutoff. Because of the dynamical breaking of chiral symmetry, $N_f^2 - 1$ Goldstone bosons appear [34]. To study the finite volume effects on the thermodynamics of strongly interacting matter the PNJL grand canonical potential in the mean-field approximation in the $SU(3)_f$ sector can be written as

$$\Omega'(\Phi, \bar{\Phi}, \sigma_f, T, \mu_f) = u'[\Phi, \bar{\Phi}, T] + 2g_s \sum_{f=u,d,s} (\sigma_f^2 - \frac{g_d}{2} \sigma_u \sigma_d \sigma_s) - T \sum_{n} \int_{\Lambda}^{\infty} \frac{d^3 p}{(2\pi)^3} T \ln S^{-1}(i\omega_n, \vec{p})$$

(22)

where $\omega_n = \pi T(2n + 1)$ are Matsubara frequencies for fermions. In momentum space the inverse quark propagator is given by

$$S^{-1} = \gamma_0(p^0 + \hat{\mu} - i A_4) - \vec{\gamma} \vec{p} - \hat{\mathcal{M}}.$$  

(23)

Using the identify $\text{Tr} \ln(X) = \ln \det(X)$, we get

$$\begin{align*}
\Omega' &= u'[\Phi, \bar{\Phi}, T] + 2g_s \sum_{f=u,d,s} (\sigma_f^2 - \frac{g_d}{2} \sigma_u \sigma_d \sigma_s) - 6 \sum_{f} \int_{\Lambda}^{\infty} \frac{d^3 p}{(2\pi)^3} E_{p_f} \Theta(\Lambda - |\vec{p}|) - 2 \\
&\sum_{f} T \int_{\Lambda}^{\infty} \frac{d^3 p}{(2\pi)^3} \ln[1 + 3(\Phi + \bar{\Phi} \exp(-E_{p_f} - \mu_f) + \exp(-3(E_{p_f} - \mu_f))] - 2 \\
&\sum_{f} T \int_{\Lambda}^{\infty} \frac{d^3 p}{(2\pi)^3} \ln[1 + 3(\Phi + \bar{\Phi} \exp(-E_{p_f} + \mu_f) + \exp(-3(E_{p_f} + \mu_f))] - 2
\end{align*}$$

(24)

$$\begin{align*}
\Omega' &= \Omega - \kappa T^4 \ln([\Phi, \bar{\Phi}])
\end{align*}$$

(25)

where $E_{p_f} = \sqrt{(p^2 + M_f^2)}$ is the single quasiparticle energy. In the above integrals, the vacuum integral has a cut-off $\Lambda$, whereas the medium dependent integrals have been extended to infinity.

**IV. RESULTS**

In this present paper, we have discussed the higher-order moments ($M, \sigma, S, \kappa$) of net-baryon and their volume independent products ($s\sigma, \kappa\sigma^2$) in the Polyakov loop enhanced Nambu-Jona-Lasinio (PNJL) model. In heavy ion collisions these moments and the susceptibility of the system are strongly depend on correlation length $\xi$ produce in the created medium. The signature of the critical point produce in medium of strongly interacting matter also has been discussed. Near the critical point, the correlation length diverge, therefore, the susceptibility and the moments. Hence, the non-monotonic behaviour of these moment can be treated as the signature of the critical point. As these moments are system volume dependent, moment products or cumulant ratios can be treated as volume independent of the system. Using the PNJL model, data sets for various cumulants ($C_1, C_2, C_3, C_4$) have been obtained at a fixed quark chemical potential for different energies similar to the RHIC Beam Energy Scan (BES) energies (7.7, 11.5, 14.5, 19.6, 27.0, 39.0, 62.4, 130.0 and 200 GeV). In the PNJL model, two sets of finite volume system have been taken to compare with the infinite volume system. The original model was built in infinite volume system. But as the fireball created in the heavy-ion collision has a finite volume, so finite volume system with the radius $R=2\text{fm}$ and $R=4\text{fm}$ can be considered for better comparison with the experimental results. The Lagrangian of the NJL model with three flavors has interaction terms with combination of four quarks and six quarks. Also later eight quarks interaction terms were introduced for vacuum stabilization. Therefore, there are four parameter sets with the combination of $2\text{fm6q}, 2\text{fm8q}, 4\text{fm6q}$ and $4\text{fm8q}$. Various cumulants ($C_1, C_2, C_3, C_4$) which are calculated in the PNJL model with respect to different energies (in GeV unit), are shown in Fig. 1. First-order cumulant ($C_1$) with respect to energies is shown in Fig. 1(a). In this Figure, $4\text{fm6q}$ (green) and $4\text{fm8q}$ (cyan) curves merge to a single line and decreases exponentially with increasing energy. $2\text{fm6q}$ (red) and $2\text{fm8q}$ (blue) curves also consolidate to a single line and increases firstly and
then decreases slowly at lower energies. Towards the higher energies, the gap between 2fm and 4fm curves diminishes slowly. Fig. 1(b) shows second-order cumulant \( (C_2) \) with respect to different energies. In this Figure, 4fm6q and 4fm8q curves merge to a single line and decreases at lower energy regime. Towards the higher energies, 2fm and 4fm both curves are independent of energy. Third-order cumulant \( (C_3) \) with respect to different energies is shown in Fig. 1(c). At lower energies, 4fm6q and 4fm8q curves are slightly separated from each other and then merge to a single line. This line decreases exponentially with energy. 2fm6q and 2fm8q curves also merge to a single line and decreases slowly at lower energy regime. Towards the higher energies, third-order cumulant has same nature like first-order cumulant. For these cumulants \( (C_1, C_2, C_3) \), any major fluctuations can not be observed with respect to energies. Fig. 1(d) shows fourth-order cumulant \( (C_4) \) with respect to different energies. In this Figure for 2fm6q (red) data, a dip occurs between 5 GeV to 25 GeV energies. After that this curve increases slowly with increasing energy. 2fm8q (blue) line shows small fluctuations at lower energies but beyond 60 GeV energy this curve is independent of energy. In case of 4fm8q (cyan) data, dip occur between 5 GeV to 25 GeV energies. Towards the higher energies, 4fm6q and 4fm8q curves are slowly uprising.

FIG. 1: (a) First-order cumulant \( (C_1) \) vs. Energy [GeV]. (b) Second-order cumulant \( (C_2) \) vs. Energy [GeV]. (c) Third-order cumulant \( (C_3) \) vs. Energy [GeV]. (d) Fourth-order cumulant \( (C_4) \) vs. Energy [GeV].

The ratio of first-order cumulant to second-order cumulant \( \frac{C_1}{C_2} \) with respect to different energies (left) and the ratio of third-order to second-order cumulant \( \frac{C_3}{C_2} \) or moment product \( (S\sigma) \) with respect to different energies (right) in PNJL model is shown in Fig. 2. If the energy increases, the ratio \( \frac{C_1}{C_2} \) decreases exponentially but the decreasing value for 4fm data is greater than 2fm data. At lower energies, the 2fm and 4fm curves with different quark numbers (6q and 8q) are slightly separated from each other. But at higher energies, the ratio for four data (2fm6q, 2fm8q, 4fm6q, 4fm8q) is almost the same i.e all lines in the graph are merge to a single line. If the energy decreases, there is some difference between 2fm and 4fm curves with different quark numbers (6q and 8q) are slightly separated from each other. But at higher energies, the ratio for four data (2fm6q, 2fm8q, 4fm6q, 4fm8q) is almost the same i.e all lines in the graph are merge to a single line. If the energy decreases, there is some difference between 2fm and 4fm curves with different quark numbers (6q and 8q) are widely separated from each other. For increasing beam energy, the value of \( S\sigma \) decreases quantitatively, i.e, the gap between 2fm and 4fm curves with different quark numbers (6q and
8q) slowly diminishes. HRG and STAR data for moment products $S\sigma$ and $\kappa\sigma^2$ have been collected from Ref. [19] and also added these data in the graphs with respect to different energies which are shown in Fig. 2. The comparison shows that in case of 2fm data, there is a sameness with hadron resonance gas model (HRG) structure and 4fm data has same tendency with STAR data with different values. Any major fluctuations can not be observed in this graph.

![Graph showing the ratio of first-order to second-order moments $C_1^2 (M/\sigma^2)$ as a function of center of mass energy [in GeV].](image1)

**FIG. 2:** Left: Ratio of first-order to second-order moments $C_1^2 (M/\sigma^2)$ as a function of center of mass energy [in GeV]. Right: Ratio of third-order to second-order moment $S\sigma (\frac{C_3}{C_2})$ as a function of center of mass energy [in GeV].

Fig. 3 shows the product of the moment $\kappa\sigma^2$ (i.e the ratio of the fourth-order cumulant to the second-order cumulant $\frac{C_4}{C_2}$) with respect to different energies. Below 60 GeV energy, this results show deviation from being constant. For 2fm6q and 2fm8q calculated lines some fluctuation around 15 GeV for 2fm6q and 2fm8q are seen. Towards the higher energy these curves shows almost independent behaviour with respect to the energy. For decreasing the system size from 4fm to 2fm, a dip kind of structure occurs between 5 GeV to 30 GeV energy, after that this curve increases slowly for 2fm6q and become constant for 2fm8q. For 4fm6q and 4fm8q curves, these are slowly increasing concerning lower energy and becomes independent of energy above 60 GeV. These results are compared with STAR experimental result of net-proton higher order moments calculation from RHIC BES-I data (black line) which can serve as the proxy of net baryon study of the same. Calculations are also compared with HRG model calculation (grey line). The fluctuation shown in this study has similar trend to the experimental measurement from STAR at RHIC. The STAR data is confirming this agreement as STAR data passes through similar fluctuations near the same region of concern. The 4fm data has lower values compared to the HRG model calculation and shows similar tendency.

![Graph showing the ratio of fourth-order to second-order moment $\kappa\sigma^2 (\frac{C_4}{C_2})$ as a function of center of mass energy [in GeV].](image2)

**FIG. 3:** Ratio of fourth-order to second-order moment $\kappa\sigma^2 (\frac{C_4}{C_2})$ as a function of center of mass energy [in GeV].
V. SUMMARY AND CONCLUSIONS

Through this work, we have endeavoured to locate the critical endpoint (CEP) in the QCD phase diagram using PNJL model. This work is of prime importance to high-energy physics all over the world. Our effort in finding the CEP of the QCD phase diagram will enable researchers to have a fair idea about the critical point of the early universe i.e., about the temperature and state of the matter just after Big Bang. Different approaches and progress have been undertaken by scientists world-wide to explore the CEP at different times. Technology has advanced radically and keeping track with it. We have not confined ourselves to theoretical approaches only. We have strived towards exploring CEP in the experimental domain. One such approach in the experimental world is the heavy-ion collider. Inside these colliders, we can collide ions, protons, and so on, to get the elementary constituents of particles. But the chief constraint lies in the fact that the elementary constituents produced by the instantaneous collision of particles are evanescent in nature and thus very difficult to observe directly. When the critical point is passed by thermodynamics condition of the matter created in heavy-ion collision, the expected signature is the non-monotonic variation of the observables with the colliding energy. Thus the different models are essential to estimate the value of different observables. In an extension of the Nambu-Jona-Lasinio model where the quarks interact with the temporal gluon field, represented by the Polyakov loop. In this model, a non-local interaction instead of point-like four-fermion coupling is employed. Due to the symmetry of the Lagrangian, the PNJL model belongs to the same universality class as expected for QCD. For this reason, this model can be considered as a testing ground for the critical phenomena related to the breaking of global $Z(3)$ and chiral symmetry. The interactions of the effective gluon field with quarks lead to a shift of the minimum to larger values of $\phi$. At high temperature the minimum corresponds to $\phi > 1$. This is a consequence of attractive interaction, in the presence of the Polyakov loop there is a strong ”binding” of the constituent quarks in the chirally broken phase. In our present study, we have elaborated the formalism of the PNJL model briefly in section III. In the following section, we have demonstrated our work to find the location of the critical point. We have plotted various moments and products of the moment including HRG and STAR data with respect to different energies. We have analyzed the nature of the curves and tried to find out the fluctuations of the curves. The fluctuation regions of the graphs give an estimation for the critical region. Non-monotonic variation of observable related to the moment of the distributions of conserved quantities such as net-baryon, net-charge, and net-strangeness number with energy $\sqrt{S_{NN}}$ is considered to be a good signature of critical point and phase transition. Higher-order moments are more susceptible to diverging correlation length and favourable quantities for the experimental search of an endpoint in the QCD phase diagram. The characteristic feature of critical point is the divergence of the correlation length which depends on higher-order moment, is also limited by the system size and finite time effects due to critical slowing down.

We have studied higher-order moments and moment products (or cumulant ratios) of net-baryon in the PNJL model at energies similar to RHIC beam energy scan (BES) energies. The order parameter ($\phi$) of the Polyakov loop in the PNJL model confirms the existence of the critical point and the critical region. From this study concluding that in case of 4fm data, there is a likeness with HRG structure with different values. In case of 2fm data, fluctuation is observed at lower energies which may be because of the influence of the critical region. STAR data is confirming this inference, as STAR data may passes through this critical region. This study confirm that the study of higher order moments of the conserved quantities is a good tool to locate the QCD critical point. In presence of critical point (CP) these higher-order moments would show non monotonic behavior as a function of energy. Increase in the value of fluctuation at lower energy rather than higher energy signifies that critical region may exists in lower energy regime, which is also have been found in the experimental results. This current study has similar dependency as a function of energy with STAR measurement of net-proton multiplicity and imply that the QCD critical region may exist at energies between 5 GeV to 25 GeV.

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