Threshold resummation of Drell-Yan rapidity distributions

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Abstract

We present a derivation of the threshold resummation formula for the Drell-Yan rapidity distribution. Our argument is valid for all values of rapidity and to all orders in perturbative QCD and can be applied to all Drell-Yan processes in a universal way, i.e. both for the production of a virtual photon $\gamma^*$ and the production of a vector boson $W^\pm$, $Z^0$. We show that for the fixed-target experiment E866/NuSea used in current parton fits, the NLL resummation corrections are comparable to NLO fixed-order corrections and are crucial to obtain agreement with the data.
In perturbative QCD, it is well known that, when one approaches to the boundary of the phase space, the cross section receives logarithmically-enhanced contributions at all orders. These large terms have been resummed a long time ago for the classes of inclusive hadronic processes of the type of deep-inelastic and Drell-Yan [1, 2] to next-to-leading-logarithmic order (NLL). More recently, the next-to-next-to-leading-logarithmic (NNLL) accuracy has been reached [3]. Threshold resummation of inclusive processes can affect significantly cross sections and the extraction of parton densities [4, 5]. For the case of small transverse momentum distributions in Drell-Yan processes, it has been shown that resummation is necessary to reproduce the correct behavior of the cross section [6].

The differential rapidity Drell-Yan cross section is used for the extraction of the ratio $\bar{d}/\bar{u}$ of parton densities. The accurate knowledge of these functions is needed to study Higgs boson production and the asymmetry $W^\pm$. The resummation of Drell-Yan rapidity distributions was first considered in 1992 [7]. At that time, it was suggested a resummation formula for the case of zero rapidity. Very recently, thanks to the analysis of the full NLO calculation of the Drell-Yan rapidity distribution, it has been shown [8], that the result given in [7] is valid at NLL for all rapidities.

In this Letter, we will give a simple proof of an all-order resummation formula valid for all values of rapidity. To do this, we will use the technique of the double Fourier-Mellin moments developed in [9]. In particular, we will show that the resummation can be reduced to that of the rapidity-integrated process, which is given in terms of a dimensionless universal function for both DY and $W^\pm$ and $Z^0$ production, and has been largely studied [1 2] even to all logarithmic orders [10]. Finally, we implement numerically the resummation formula and give predictions of the full rapidity-dependent NLL Drell-Yan cross section for the case of the fixed-target E866/NuSea experiment. We find that resummation at the NLL level is necessary and that its agreement with the experimental data is better than the NNLO calculation [11].

We consider the general Drell-Yan process in which the collisions of two hadrons ($H_1$ and $H_2$) produce a virtual photon $\gamma^*$ (or an on-shell vector boson $V$) and any collection of hadrons ($X$):

$$H_1(P_1) + H_2(P_2) \rightarrow \gamma^*(V)(Q) + X(K).$$

In particular, we are interested in the differential cross section $\frac{d\sigma}{dQ^2 dY}(x, Q^2, Y)$, where $Q^2$ is the invariant mass of the photon or of the vector boson, $x$ is defined as usual as the fraction of invariant mass that the hadrons transfer to the photon (or to the vector boson) and $Y$ is the rapidity of $\gamma^*(V)$ in the hadronic center-of-mass:

$$x \equiv \frac{Q^2}{S}, \quad S = (P_1 + P_2)^2, \quad Y \equiv \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right),$$

where $E$ and $p_z$ are the energy and the momentum along the collisional axis of $\gamma^*(V)$ respectively. At the partonic level, a parton 1(2) in the hadron $H_1$ ($H_2$) carries a longitudinal momentum $p_1 = x_1 P_1$ ($P_2 = x_2 P_2$). Thus, the rapidity in the partonic center-of-mass ($y$) is obtained performing a boost of $Y$ between the two frames:

$$y = Y - \frac{1}{2} \ln \left( \frac{x_1}{x_2} \right).$$
In order to understand the kinematic configurations in terms of rapidity, it is convenient to define a new variable $u$,

$$u \equiv \frac{Q \cdot p_1}{Q \cdot p_2} = e^{-2y} = \frac{x_1}{x_2} e^{-2Y}. \quad (4)$$

which can assume all the values in the closed interval,

$$z \leq u \leq \frac{1}{z}, \quad (5)$$

with

$$z = \frac{Q^2}{2p_1 \cdot p_2} = \frac{Q^2}{(p_1 + p_2)^2} = \frac{x}{x_1 x_2}. \quad (6)$$

The upper and lower bounds in eq.(5) are reached when the extra radiation is emitted collinear to the incoming parton 1 and 2 respectively. Eqs. (3,4) allow us to rewrite the relation in eq.(5) as a relation for the upper and lower bounds of the partonic center-of-mass rapidity:

$$\frac{1}{2} \ln z \leq y \leq \frac{1}{2} \ln \frac{1}{z}. \quad (7)$$

Substituting eqs. (4, 6) into the two conditions $u \geq z$ and $u \leq 1/z$, we obtain the lower kinematic bound for $x_1$ and $x_2$:

$$x_1 \geq \sqrt{xe}^Y \equiv x_1^0, \quad x_2 \geq \sqrt{xe}^{-Y} \equiv x_2^0 \quad (8)$$

and the obvious requirement $x_{1(2)}^0 \leq 1$ implies that the hadronic rapidity has a lower and an upper bound:

$$\frac{1}{2} \ln x \leq Y \leq \frac{1}{2} \ln \frac{1}{x}. \quad (9)$$

The variable $z$ in eq.(6) can be viewed as the fraction of invariant mass that the incoming partons transfer to $\gamma^*(V)$, and, hence, the threshold limit is reached when $z$ approaches to 1.

According to the standard factorization of collinear singularities of perturbative QCD, the expression for the hadronic differential cross section in rapidity has the form,

$$\frac{d\sigma}{dQ^2 dy} = \sum_{i,j} \int_{x_1^0}^1 dx_1 \int_{x_2^0}^1 dx_2 F_i^{H_1}(x_1, \mu^2) F_j^{H_2}(x_2, \mu^2) \frac{d\tilde{\sigma}_{ij}}{dQ^2 dy} \left( x_1, x_2, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), y \right), \quad (10)$$

where $y$ depends on $Y$, $x_1$ and $x_2$ according to eq.(3). The sum runs over all possible partonic subprocesses, $F_i^{(1)}$, $F_j^{(2)}$ are respectively the parton densities of the hadron $H_1$ and $H_2$, $\mu$ is the factorization scale (chosen equal to renormalization scale for simplicity) and $d\tilde{\sigma}_{ij}/(dQ^2 dy)$ is the partonic cross section. In the threshold limit the gluon-quark channels are suppressed by powers of $(1 - z)$ and, so, in order to study resummation, we will consider only the
quark-anti-quark contributions of the sum in eq.(10). These last terms are related to the same dimensionless coefficient function \( C(z, Q^2/\mu^2, \alpha_s(\mu^2), y) \) through the relations

\[
x_1 x_2 \frac{d\hat{\sigma}^{\gamma^*}}{dQ^2 dy}(x_1, x_2, Q^2/\mu^2, \alpha_s(\mu^2), y) = \frac{4\pi\alpha^2 c_{qq'}}{9Q^2S} C\left(\frac{z}{\mu^2}, \alpha_s(\mu^2), y\right),
\]

(11)

for the virtual photon vertex and

\[
x_1 x_2 \frac{d\hat{\sigma}^{V_{qq'}}}{dQ^2 dy}(x_1, x_2, Q^2/\mu^2, \alpha_s(\mu^2), y) = \frac{\pi G_F Q^2 \sqrt{2} c_{qq'}}{3S} \delta(Q^2 - M_V^2) C\left(\frac{z}{\mu^2}, \alpha_s(\mu^2), y\right),
\]

(12)

for the real vector boson vertex. Here \( G_F \) is the Fermi constant, \( M_V \) is the mass of the produced vector boson. The coefficients \( c_{qq'} \) are given by:

\[
c_{qq'} = \frac{Q^2}{\mu^2} \delta_{qq'} \quad \text{for } \gamma^*, \quad \text{for } W^\pm,
\]

(13)\quad (14)

\[
c_{qq'} = |V_{qq'}|^2 \quad \text{for } W^0,
\]

(15)

where \( Q^2_q \) is the square charge of the quark \( q \), \( V_{qq'} \) are the CKM mixing factors for the quark flavors \( q, q' \) and

\[
g_v^q = \frac{1}{2} - \frac{4}{3} \sin^2 \theta_W, \quad g_a^q = \frac{1}{2} \quad \text{for an up-type quark},
\]

(16)

\[
g_v^q = -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W, \quad g_a^q = -\frac{1}{2} \quad \text{for a down-type quark},
\]

(17)

with \( \theta_W \) the Weinberg weak mixing angle. Thus, we are left with a dimensionless cross section of the form:

\[
\sigma(x, Q^2, Y) \equiv \int_{x_1}^1 \frac{dx_1}{x_1} \int_{x_2}^1 \frac{dx_2}{x_2} F_1^{H_1}(x_1, \mu^2) F_2^{H_2}(x_2, \mu^2) C\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), y\right),
\]

(18)

where \( F_1 \) and \( F_2 \) are quark or anti-quark parton densities in the hadron \( H_1 \) and \( H_2 \) respectively. This shows the universality of resummation in Drell-Yan processes in the sense that only the quantity defined in eq.(18) has to be resummed.

We shall now show that the resummed expression of eq.(18) is obtained by simply replacing the coefficient function \( C(z, Q^2/\mu^2, \alpha_s(\mu^2), y) \) with its integral over \( y \), resummed to the desired logarithmic accuracy. To show this, we recall that resummation is usually performed in the space of the variable \( N \), which is the Mellin conjugate of \( x \), since Mellin transformation turns convolution products into ordinary products. In the case of the rapidity distribution, however, this is not sufficient. In fact, we see that the Mellin transform with respect to \( x \),

\[
\sigma(N, Q^2, Y) \equiv \int_0^1 dx x^{N-1} \sigma(x, Q^2, Y),
\]

(19)

does not diagonalize the double integral in eq.(18), because the partonic center-of-mass rapidity \( y \) depends on \( x_1 \) and \( x_2 \) through eq.(3). The ordinary product in Mellin space can be recovered
performing the Mellin transform with respect to $x$ of the Fourier transform with respect to $Y$. Using eqs. (9,7) and the fact that the coefficient function must be symmetric in $y$, we find

$$
\sigma(N, Q^2, M) \equiv \int_0^1 dx x^{N-1} \int_{\log \sqrt{x}}^{\log 1/\sqrt{x}} dY e^{iMY} \sigma(x, Q^2, Y) = F_1^{H_1}(N + iM/2, \mu^2) F_2^{H_2}(N - iM/2, \mu^2) C\left(N, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), M\right),
$$

(20)

where

$$
F_i^{H_i}(N \pm iM/2, \mu^2) = \int_0^1 dx x^{N-1 \pm iM/2} F_i^{H_i}(x, \mu^2),
$$

(22)

$$
C\left(N, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), M\right) = 2 \int_0^1 dz z^{N-1} \int_0^{\log 1/\sqrt{z}} dy \cos(My) C\left(z, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), y\right).
$$

(23)

The dependence on $M$, the Fourier conjugate of the rapidity $y$, originates from the parton densities, that depend on $N \pm iM/2$, and from the factor of $\cos(My)$ in the integrand of eq.(23). This last dependence, however, is irrelevant in the large-$N$ limit. Indeed, one can expand $\cos(My)$ in powers of $y$,

$$
\cos(My) = 1 - \frac{M^2 y^2}{2} + O(M^4 y^4).
$$

(24)

and observe that the first term of this expansion leads to a convergent integral (the rapidity-integrated cross section), while the following terms are suppressed by powers of $(1 - z)$, since the upper integration bound is

$$
\ln \frac{1}{\sqrt{z}} = \frac{1}{2}(1 - z) + O((1 - z)^2).
$$

(25)

Hence, up to terms suppressed by factors $1/N$, eq.(23) is equal to the Mellin transform of the rapidity-integrated Drell-Yan coefficient function that we call $C_I(N, Q^2/\mu^2, \alpha_s(\mu^2))$. This completes our proof. We get

$$
\sigma^{res}(N, Q^2, M) = F_1^{H_1}(N + iM/2, \mu^2) F_2^{H_2}(N - iM/2, \mu^2) C_I^{res}\left(N, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right).
$$

(26)

This is the main theoretical result of our Letter: it shows that, near threshold, the Mellin-Fourier transform of the coefficient function does not depend on the Fourier moments and that this is valid to all orders of QCD perturbation theory. Furthermore this result remains valid for all values of hadronic center-of-mass rapidity, because we have introduced a suitable integral transform over rapidity. The resummed rapidity-integrated Drell-Yan coefficient function to NLL is well known [1, 2] and, using the notation of [10], it is given in a compact form (in the \(\overline{\text{MS}}\) scheme) by

$$
C_I^{res}\left(N, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) = \exp\left\{- \int_1^{N^2} \frac{dn}{n} \left[ \int_{\mu^2}^{Q^2} \frac{dk^2}{k^2} \left( A_1 \alpha_s(\frac{k^2}{n}) + A_2 \alpha_s^2(\frac{k^2}{n}) \right) + B_1 \alpha_s(\frac{Q^2}{n}) \right] \right\},
$$

(27)
where
\[
A_1 = \frac{C_F}{\pi}, \quad A_2 = \frac{C_F}{2\pi^2} \left[ C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} N_f \right], \quad B_1 = -\frac{\gamma_E A_1}{2\pi}
\]
(28)
with \(C_F = 4/3, \ C_A = 3, \ N_f\) the number of flavors and with the Euler gamma \(\gamma_E = 0.5772\ldots\). The use of only the first coefficient \(A_1\) allows us to resum all the LL contributions \(\alpha_s^k \log^{k+1}(N)\) and the use of all the three coefficients in eq. (28) enable us to add also the NLL terms \(\alpha_s^k \log^k(N)\).

A NLL expression of the rapidity distribution is obtained by taking the inverse Mellin and Fourier transform of \(\sigma^{res}(N, Q^2, M)\). This procedure requires the use of some specific prescription \[12, 13, 14\] in order to overcome the problem of the Landau singularity in \(\alpha_s(Q^2/N^2)\). Here, we adopt the “Minimal Prescription” proposed in \[12\], which is simply obtained choosing the integration contour of the inverse Mellin transform in such a way that all the poles of the integrand are to the left, except the Landau pole. Furthermore, in order to improve numerical convergence and to avoid the singularities of the parton densities of eq. (26) which are computed out of the real axis, we perform the \(N\)-integral along a path \(\Gamma\) given by:
\[
\begin{align*}
\Gamma &= \Gamma_1 + \Gamma_2 + \Gamma_3 \\
\Gamma_1(t) &= C_{MP} - \frac{iM}{2} + t(1 + i), \quad t \in (-\infty, 0) \\
\Gamma_2(s) &= C_{MP} + is\frac{M}{2}, \quad s \in (-1, 1) \\
\Gamma_3(t) &= C_{MP} + i\frac{M}{2} - t(1 - i), \quad t \in (0, +\infty)
\end{align*}
\]
(29)
(30)
(31)
(32)
where \(C_{MP}\) is a positive number below the Landau pole of \(\alpha_s(Q^2/N^2)\). Performing the changes of variable \(M = -\ln m\) and \(t = -\ln s\), the double inverse transform over the curve \(\Gamma\) becomes:
\[
\sigma^{res}(x, Q^2, Y) = \frac{1}{\pi} \int_0^1 \frac{dm}{m} \cos(-Y \ln m)\sigma^{res}(x, Q^2, -\log m),
\]
(33)
where \(\sigma^{res}(x, Q^2, M)\) is given by
\[
\sigma^{res}(x, Q^2, M) = \frac{1}{\pi} \int_0^1 \frac{ds}{s} \text{Re} \left[ x^{-C_{MP} - \ln s + i(M/2+1)} \sigma^{res}(C_{MP} + \ln s - i(M/2 + 1), Q^2, M)(1 - i) + \frac{sM}{2} x^{-C_{MP} - isM/2} \sigma^{res}(C_{MP} + isM/2, Q^2, M) \right].
\]
(34)
Eqs. (33, 34) are the expressions that we use to evaluate numerically the resummed dimensionless cross section in the variables \(x\) and \(Y\). The explicit expression of eq. (27) is easily obtained performing the integrals and is given by
\[
C_I^{res}(N, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)) = \exp \{ \ln N g_1(\lambda) + g_2(\lambda) \},
\]
(35)
where
\[
g_1(\lambda) = \frac{A_1}{\beta_0 \lambda}[2\lambda + (1 - 2\lambda) \log(1 - 2\lambda)]
\]
\[
g_2(\lambda) = -\frac{2A_1}{\beta_0} \gamma_E \log(1 - 2\lambda) + \frac{A_1}{\beta_0^2} [2\lambda + \log(1 - 2\lambda) + \frac{1}{2} \log^2(1 - 2\lambda)]
- \frac{A_2}{\beta_0^2} [2\lambda + \log(1 - 2\lambda)] + \log \left( \frac{Q^2}{\mu^2} \right) \frac{A_1}{\beta_0} \log(1 - 2\lambda)
\]
and
\[
\lambda = \beta_0 \alpha_s(\mu^2) \ln N, \quad \beta_0 = \frac{1}{4\pi} \left( 11 - \frac{2}{3} N_f \right), \quad \beta_1 = \frac{1}{16\pi^2} \left( 102 - \frac{38}{3} N_f \right).
\]

Here, we choose the factorization scale equal to the renormalization scale for simplicity. To study the dependence on the renormalization scale one has simply to express \( \alpha_s(\mu^2) \) in terms of \( \alpha_s \). Furthermore, we need the analytic continuations to the whole complex plane of the Mellin-transformed parton densities that appear in eq.(26). In order to overcome this problem, we have to evolve up a partonic fit taken at a certain scale solving the DGLAP evolution equations in Mellin space \[15\]. The LO and NLO expressions of the splitting functions are reported in \[16\] and their analytic continuations are given in \[17\] and \[18\].

Finally, we want to obtain a NLO determination of the cross section improved with NLL resummation. In order to do this, we must keep the resummed dimensionless part of the cross section eq.(33), multiply it by the correct dimensional prefactors looking eqs.(11-17), add the full NLO cross section and subtract the double-counted logarithmic enhanced contributions. This matching has to be done in the \( x \) and \( Y \) spaces, because we are not able to calculate the Mellin-Fourier moments of the full NLO cross section analytically. Thus, we have
\[
\frac{d\sigma}{dQ^2 dY} = \frac{d\sigma^{FO}}{dQ^2 dY} + \frac{d\sigma^{res}}{dQ^2 dY} - \left[ \frac{d\sigma^{res}}{dQ^2 dY} \right]_{\alpha_s=0} - \alpha_s \left[ \frac{\partial}{\partial \alpha_s} \left( \frac{d\sigma^{res}}{dQ^2 dY} \right) \right]_{\alpha_s=0}
\]

The first term is the full NLO cross section reported in \[8, 19, 20, 21\], which includes even the quark-gluon channel. The third and the fourth terms in eq.(39) are obtained in the same way as the second one, but with the substitutions
\[
C_I^{res} \left( N, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right) \rightarrow 1,
\]
\[
C_I^{res} \left( N, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right) \rightarrow \alpha_s(\mu^2) 2A_1 \left\{ \ln^2 N + \ln N \left[ 2\gamma_E - \log \left( \frac{Q^2}{\mu^2} \right) \right] \right\},
\]
respectively. The terms that appear in eq.(41) are exactly the \( O(\alpha_s) \) logarithmic enhanced contributions in the \( \overline{MS} \) scheme.

We note that the final expression eq.(39) is relevant even when the variable \( x \) is not large. In fact, the cross section can get the dominant contributions from the integral in eq.(18) for values of \( z \) eq.(3) that are near the threshold even when \( x \) is not close to one, because of the strong suppression of parton densities \( F_i(x_i, \mu^2) \) when \( x_i \) are large.
Figure 1: Y-dependence of $d^2\sigma/(dQ^2 dY)$ in units of pb/GeV$^2$. The curves are, from top to bottom, the NLO result (red band), the LO+LL resummation (blue band) and the LO (black band). The bands are obtained varying the factorization scale between $\mu^2 = 2Q^2$ and $\mu^2 = 1/2Q^2$.

To show the importance of this resummation, we have calculated the Drell-Yan rapidity distribution for proton-proton collisions at the Fermilab fixed-target experiment E866/NuSea [22]. The center-of-mass energy has been fixed at $\sqrt{S} = 38.76$ GeV and the invariant mass of the virtual photon $\gamma^*$ has been chosen to be $Q^2 = 64$ GeV$^2$ in analogy with [11]. Clearly the contribution of the virtual $Z^0$ can be neglected, because its mass is much bigger than $Q^2$. In this case $x = 0.04260$ and the upper and lower bound of the hadronic rapidity $Y$ eq.(9) are given by $\pm 1.57795$. We have evolved up the MRST 2001 parton distributions (taken at $\mu^2 = 1$ GeV$^2$) as in [11], where the NNLO calculation is performed. However, results obtained using more modern parton sets should not be very different. The LO parton set is given in [23] with $\alpha_s^{LO}(m_Z) = 0.130$ and the NLO set is given in [24] with $\alpha_s^{NLO}(m_Z) = 0.119$. The evolution of parton densities at the scale $\mu^2$ has been performed in the variable flavor number scheme. The quarks has been considered massless and, at the scale of the transition of the flavor number ($N_f \rightarrow N_f + 1$), the new flavor is generated dynamically. The resummation formula eq.(26) together with eqs.(35,38) has been used with the number of flavors $N_f = 4$.

In figure 1 we plot the rapidity-dependence of the cross section at LO, NLO and LO improved with LL resummation. The effect of LL resummation is small compared to the effect of the full NLO correction. We see that, at leading order, the impact of the resummation is negligible in comparison to the NLO fixed-order correction. This means that the NLL resummation is necessary.
Figure 2: Y-dependence of $d^2\sigma/(dQ^2dY)$ in units of pb/GeV$^2$. The curves are, from top to bottom, the NLO result (red band), the NLO+NLL resummation (green band) and the LO (black band). The bands are obtained as in figure 1.

Figure 3: Y dependence of $d^2\sigma/(dQ^2dY)$ in units of pb/GeV$^2$. The curves are, from top to bottom, the NLO result (red band) and the NLO+NLL resummation (green band) together with the E866/NuSea data. The bands are obtained as in figure 1.
The LO, the NLO and its NLL improvement cross sections are shown in figure 2. The effect of the NLL resummation in the central rapidity region is almost as large as the NLO correction, but it reduces the cross section instead of enhancing it for not large values of rapidity. Going from the LO result to the NLO with NLL resummation, we note a reduction of the dependence on the factorization scale i.e. a reduction of the theoretical error. It is interesting to observe that logarithmically enhanced and constant terms account for more than 80% of the NLO contribution for all relevant rapidities. Therefore, they have the same sign. Nevertheless a suppression arises due to the shift in the complex plane of the dominant contribution of the resummed exponent. This suppression starts at order $O(\alpha_s^2)$.

In figure 3, we report the experimental data of [22] converted to the $Y$ variable, together with our NLO and NLL resummed predictions. The agreement with data is good and a great improvement for not large rapidity is obtained with respect to the NLO calculation. We note also that the NLL resummation gives better result than the NNLO calculation performed in [11]. The NNLO prediction has a worse agreement with data than the NLO one for not large values of rapidity. This result suggests that, for the case of rapidity distributions, NLL resummation is more important than high-fixed-order calculation and that it can be so even at higher center-of-mass energies.

To summarize, we have proved a resummation formula for the Drell-Yan rapidity distributions to all logarithmic accuracy and valid for all values of rapidity. Isolating a universal dimensionless coefficient function, which is exactly that ones of the Drell-Yan rapidity-integrated, we have shown a general procedure to obtain resummed results to NLL for the rapidity distributions of a virtual photon $\gamma^*$ or of a real vector boson $W^\pm, Z^0$. Furthermore, we have outlined a general method to calculate numerical predictions and analyzed the impact of resummation for the fixed-target experiment E866/NuSea. This shows that NLL resummation has an important effects on predictions of differential rapidity cross sections giving an agreement with data that is better than NNLO full calculations.

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