Models of Walking Technicolor on the Lattice

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We study models for the Higgs sector in which the Higgs is composite.

In particular, we study Technicolor models – QCD-like theories with massless fermions, where the Goldstone pion-like excitations play the role of the Higgs field, giving mass to the $W^\pm$ and $Z$.

Of particular interest are walking-Technicolor models, where there is a range of mass scales over which the running coupling evolves very slowly. Such models can avoid the phenomenological problems with naive Technicolor.

QCD with 2 colour-sextet quarks is a candidate walking-Technicolor model.

Need to distinguish whether this theory walks or is conformal.

Attractive because it has just the right number of Goldstone bosons (3) to give mass to the $W^\pm$ and $Z$. 
Other groups are studying this model: Lattice Higgs Collaboration and DeGrand, Shamir & Svetitsky.

We study this theory at finite temperature to see if the coupling at the chiral transition evolves as predicted by asymptotic freedom for a finite-temperature transition.

QCD with 3 colour-sextet quarks, which is believed to be conformal, is studied for comparison.

We simulate these theories, latticized with unimproved staggered fermions, using the RHMC method.

Does QCD with 2 colour-sextet quarks have a light Higgs with standard-model properties? What other light particles are in its spectrum? Can any of its particles be dark-matter candidates? What about its $S$, $T$, and $U$ parameter(s)?

We have also considered $SU(2)$ Yang-Mills with 3 Majorana/Weyl fermions. However, it is unclear how to embed the weak gauge group into this theory to give masses to the $W$'s and $Z$. 
QCD with 2 colour-sextet quarks at finite temperature

We simulate QCD with 2 color-sextet quarks at finite temperature by simulating on an $N_s^3 \times N_t$ lattice with $N_s \gg N_t$. Since $T = 1/N_t a$, increasing $N_t$ with $T$ fixed decreases $a$. Assuming the chiral phase transition is a finite-temperature transition, yields a convenient $T, T_\chi$. Measuring $g$ or $\beta = 6/g^2$ at $T_\chi$ gives a running coupling at a sequence of $a$s which approach zero as $N_t \to \infty$.

$$N_t = 12$$

Much of the past year has been devoted to increasing the statistics for our simulations on $24^3 \times 12$ lattices at quark masses $m = 0.0025$ and $m = 0.005$, close to the chiral transition.

For our largest mass $m = 0.01$ we have extended our simulations at low $\beta$s to determine the position of the deconfinement transition.

Because the $\beta$ dependence of the chiral condensate is so smooth for the masses we use, we determine the position $\beta_\chi$ of this
transition from the peaks in the (disconnected) chiral susceptibility:

\[
\chi_{\bar{\psi}\psi} = V \left[ \langle (\bar{\psi}\psi)^2 \rangle - \langle \bar{\psi}\psi \rangle^2 \right]
\]

extrapolated to \( m = 0 \). \( V \) is the space-time volume.

For \( m = 0.01 \) in the range \( 5.7 \leq \beta \leq 5.9 \), near the deconfinement transition, we run for 50,000 trajectories for each \( \beta \) with \( \beta \)s spaced by 0.02. In the range \( 6.6 \leq \beta \leq 6.9 \), near the chiral transition we run for 25,000 trajectories per \( \beta \) with \( \beta \)s spaced by 0.02. Elsewhere in the range \( 5.7 \leq \beta \leq 7.2 \) we run for 10,000 trajectories for \( \beta \)s spaced by 0.1.

For \( m = 0.005 \) in the range \( 6.6 < \beta \leq 6.9 \), we run for 50,000 trajectories per \( \beta \) at \( \beta \)s spaced by 0.02. At \( \beta = 6.6 \) we run for 100,000 trajectories. Elsewhere in the range \( 6.4 \leq \beta \leq 7.2 \), we run 10,000 trajectories per \( \beta \) for \( \beta \)s spaced by 0.1.

For \( m = 0.0025 \) in the range \( 6.7 \leq \beta \leq 6.9 \), we will run for 100,000 trajectories per \( \beta \) with \( \beta \)s spaced by 0.02. In the range \( 6.6 \leq \beta < 6.7 \), we will run for 50,000 trajectories per
$\beta$. These runs are nearing completion. Elsewhere in the range $6.5 \leq \beta \leq 7.2$ we run 10,000 trajectories per $\beta$ at $\beta$s spaced by 0.1.

Figure 1 shows the chiral condensates measured in these simulations. Note that while these suggest that this condensate will vanish in the chiral limit for large enough $\beta$ values, they do not allow a precise determination of $\beta_\chi$.

Figure 2 shows the chiral susceptibilities from these runs. The peak of the $m = 0.0025$ susceptibility yields an estimate of $\beta_\chi$, namely $\beta_\chi = 6.77(1)$.

Combining this with our $N_t = 8$ results yields:

$$\beta_\chi(N_t = 12) - \beta_\chi(N_t = 8) = 0.08(2) ,$$

significantly smaller than the 2-loop perturbative prediction:

$$\beta_\chi(N_t = 12) - \beta_\chi(N_t = 8) \approx 0.12 .$$
Figure 1: Chiral condensates on a $24^3 \times 12$ lattice.
Figure 2: Chiral susceptibilities on a $24^3 \times 12$ lattice.
Figure 3 shows the Wilson Lines from these simulations.
Figure 4 shows histograms of the magnitudes of Wilson Lines for $m = 0.01$ near to the deconfinement transition. From this we deduce that $\beta_d = 5.81(1)$ for $m = 0.01$. This should be close to the value for $m = 0$.

| $N_t$ | $\beta_d$  | $\beta_\chi$ |
|-------|-------------|--------------|
| 4     | 5.40(1)     | 6.3(1)       |
| 6     | 5.54(1)     | 6.60(2)      |
| 8     | 5.65(1)     | 6.69(1)      |
| 12    | 5.81(1)     | 6.77(1)      |

Table 1: $N_f = 2$ deconfinement and chiral transitions for $N_t = 4, 6, 8, 12$. 
Figure 3: Wilson Lines (Polyakov Loops) on a $24^3 \times 12$ lattice: States with real Wilson Lines only.
Figure 4: Histograms of magnitudes of Wilson Lines for $\beta$s close to the deconfinement transition for $m = 0.01$. 

**24^3 \times 12$ lattice $m=0.01**
QCD with 2 colour-sextet quarks at zero temperature

Planned simulations and measurements

Start with simulations on a $36^3 \times 72$ lattice at $\beta = 6.1(?)$, at several $m_s$. [Deconfinement transition for $\beta_d(N_t = 36) \sim 6.25-6.4$]

Unfortunately, $\beta = 6.1$ is still too small to access the continuum limit. However, we hope that we will be able to get results which are qualitatively correct.

Repeat on $48^3 \times 96$ lattice.

Measure $f_\pi$ and spectrum of local mesons (connected).

Measure non-local pion spectrum. How large is taste breaking?

Measure glueball spectrum. Are glueballs light?

Measure meson spectrum including disconnected terms. Is there a light $\eta/\eta'$?

Is there a light $0^{++}$ state with sufficient fermion content to be the Higgs? Is its mass $\approx \frac{1}{2}f_\pi$?
Does this Higgs have the right couplings to $W^\pm$, $Z$ and $\gamma$?

Measure $S$ parameter contributions.

Determine scaling behaviour of the chiral condensate to extract $\gamma m$.

Determine mass dependence of meson (and glueball) masses, and scaling behaviour.

Measure the $Q\bar{Q}$ potential.

Examine the $\beta$ dependence of the masses.
QCD with 3 colour-sextet quarks at finite temperature

We simulate lattice QCD with 3 colour-sextet quarks at finite temperature for comparison with the 2-flavour case. This theory is believed to be conformal with an infrared fixed point.

The chiral transition should be a bulk transition fixed at a finite constant $\beta_\chi$ for $N_t$ sufficiently large.

We have simulated this theory at $N_t = 4$, $6$ and $8$, and are now starting $N_t = 12$ simulations.

For $N_t = 6$ we simulate on a $12^3 \times 6$ lattice at $m = 0.02$, $m = 0.01$ and $m = 0.005$.

Close to the chiral transition, $6.2 \leq \beta \leq 6.4$ at the lowest quark mass ($m = 0.005$) we simulate at $\beta$s separated by 0.02, with 100,000 trajectories per $\beta$.

We estimate the position of the chiral transition as the peak in the chiral susceptibility for $m = 0.005$.

For $N_t = 8$ we simulate on a $16^3 \times 8$ lattice at $m = 0.01$ and
$m = 0.005$.
Close to the chiral transition, $6.28 \leq \beta \leq 6.5$ at the lowest mass ($m = 0.005$) we simulate at $\beta$s separated by 0.02. with 100,000 trajectories per $\beta$.
We estimate the position of the chiral transition as the peak in the chiral susceptibility for $m = 0.005$.
The results for the positions of the chiral and deconfinement transitions for $N_t = 4, 6$ and 8 are given in table 2.

| $N_t$ | $\beta_d$   | $\beta_\chi$ |
|-------|-------------|--------------|
| 4     | 5.275(10)   | 6.0(1)       |
| 6     | 5.375(10)   | 6.278(2)     |
| 8     | 5.45(10)    | 6.37(1)      |

Table 2: $N_f = 3$ deconfinement and chiral transitions for $N_t = 4, 6, 8$. In each case we have attempted an extrapolation to the chiral limit.

Since

$$\beta_\chi(N_t = 8) - \beta_\chi(N_t = 6) = 0.09(1)$$

we have yet to see evidence of a bulk transition.
We are therefore starting $N_t = 12$ simulations on a $24^3 \times 12$ lattice.

Figure 5 shows the $m = 0.005$ chiral susceptibilities for $N_t = 6$, $N_t = 8$ and preliminary results for $N_t = 12$.

Figure 6 shows the chiral condensates, both unsubtracted and subtracted for our $16^3 \times 8$ simulations. The subtracted condensates use the definition of the Lattice Higgs Collaboration:

$$\langle \bar{\psi} \psi \rangle_{sub} = \langle \bar{\psi} \psi \rangle - \left( m_V \frac{\partial}{\partial m_V} \langle \bar{\psi} \psi \rangle \right)_{m_V=m}.$$

Note, although it is clearer that the subtracted condensate will vanish in the continuum limit for $\beta$ sufficiently large than is the case for the unsubtracted condensate, it still does not yield an accurate estimate of $\beta_\chi$. 
Figure 5: Chiral susceptibilities for $N_f = 3$, $m = 0.005$ on $12^3 \times 6$, $16^3 \times 8$ and $24^3 \times 12$ lattices.
Figure 6: Chiral condensates on a $16^3 \times 8$ lattice for $m = 0.005$ and $m = 0.01$. The red graphs are unsubtracted, lattice regulated condensates. The blue graphs have been subtracted using the method of the Lattice Higgs Collaboration.
\[ \mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \psi^\dagger i \sigma^\mu \overleftrightarrow{D}_\mu \psi + \frac{m}{2} \left[ \psi^T i \sigma_2 \psi - \psi^\dagger i \sigma_2 \psi^* \right] \]

where \( \psi \) is a 3-vector in colour space and in flavour space.

If \( m = 0 \), the chiral flavour symmetry is \( SU(3) \).

The Majorana mass term reduces this flavour symmetry to the real elements of \( SU(3) \), i.e. to \( SO(3) \).

Thus when \( m = 0 \) and the chiral symmetry breaks spontaneously, the chiral condensate is \( \langle \psi^T i \sigma_2 \psi - \psi^\dagger i \sigma_2 \psi^* \rangle \). and the spontaneous symmetry breaking pattern is

\[ SU(3) \rightarrow SO(3) \]

The unbroken generators of \( SU(3) \) are the 3 imaginary generators. These form a spin-1 representation under the unbroken \( SO(3) \).
The 5 broken generators are the 5 real generators. They, as well as the 5 corresponding Goldstone bosons, form a spin-2 representation of $SO(3)$.

The problem occurs when one tries to embed the weak $SU(2) \times U(1)$ group in such a way as to give masses to $W^{\pm}$ and $Z$.

This is easiest to see if we consider the case where the Weinberg angle is zero. Then we need to embed $SU(2)$ in such a way that all 3 components are broken spontaneously. Thus we would need to make a set of $SU(2)$ generators from the 5 real $SU(3)$ generators. However, the $SU(2)$ algebra requires that at least one of its generators is complex, so this is impossible.

The only Weinberg angle which would work is $\pi/2$ where the photon is pure $SU(2)$ and the $Z$ is pure $U(1)$.
Discussion and Conclusions

- We simulate lattice QCD with 2 colour-sextet quarks at finite temperature to distinguish whether it is QCD-like and walks, or if it is a conformal field theory.
- We run on lattices with $N_t = 4, 6, 8, 12$. $\beta_\chi$ increases by $0.08(2)$ between $N_t = 8$ and $N_t = 12$. While this increase favours the walking scenario, this increase is significantly smaller than the 2-loop prediction of $\approx 0.12$.
- Is this because 2-loop perturbation theory is inadequate for this lattice action and $\beta$? Are there sizable finite volume corrections? Will the theory finally prove to be conformal?
- If walking, this theory is a promising walking-technicolor theory. We have outlined a program for checking its zero-temperature properties. Does it have a light Higgs? Does it satisfy the precision electroweak constraints? Does it have a Dark Matter candidate? What about its particle spectrum?...........
- We simulate QCD with 3 colour-sextet quarks which should be
conformal. The increase in $\beta_\chi$ between $N_t = 6$ and $N_t = 8$ is still appreciable (0.09(1)), so we don’t yet have evidence for $\beta_\chi$ approaching a finite constant as $N_t \to \infty$.

- We are now simulating at $N_t = 12$. This shows some promise.
- QCD$_2$ with 3 Majorana/Weyl quarks does not appear to be a Technicolor candidate.

These simulations were performed on Hopper, Edison and Carver at NERSC, Kraken at NICS, Stampede at TACC, and Fusion and Blues at LCRC, Argonne.