AN INTRODUCTION TO THEORY OF DYNAMICS OF NUMBERS:
A NEW CONCEPT

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Abstract

The role of numbers is very important not only in mathematics but also in any branch of science and technology. The author developed a new concept of the theory of dynamics of numbers. According to the new concept, 0 (zero) is the starting point of any number. There are infinite number of directions through which the numbers can move from starting point 0 (zero) and can return in the vertically opposite direction towards the starting point 0 (zero). The motion of any number (object) is positive whether it is forward motion or backward motion. Similarly, countup and countdown motions of numbers are also positive. Therefore, there is no existence of negative numbers.

The author framed the law of the theory of dynamics of numbers. The author solved the quadratic equation in one countable unknown object or quantity (say x) in the form, $ax^2 + bx + c = 0$, even if the numerical value of the discriminant, $b^2 - 4ac < 0$.

The author applied the theory of dynamics of numbers to solve the problems of plane co-ordinate geometry and also to solve the problems of the quadratic equation in the present paper.

Keywords: Cartesian Co-ordinate System, Dynamics of Numbers, Quadratic Equation, Rectangular Bhattacharyya’s Co-ordinate System, Theory of Numbers.

I. Introduction

A number is a mathematical object which is used to count, measure, or label any object [II]. Also, we can define a number as an arithmetical value that is expressed in words, symbols, or figures to represent a particular quantity of objects used in counting and making calculations [II]. We cannot find any negative object in the universe. Therefore, according to the definition of numbers, negative numbers cannot exist. So, $-1, -2, -3, -4, -5 \ldots \ldots$ cannot exist

Before the introduction of the theory of dynamics of numbers we need to discuss some objective issues related to numbers: --

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Can we eat an apple if there is no existence of a single apple?
Can we hold a stick of one metre if the one-metre stick is not available in the universe?
Can we find one gram of matter if the matter does not exist?
Can we take a loan of one rupee (Indian currency) from a person who is not possessing one rupee?

In each case, the answer will be ‘No’. But the above functions can be performed if and only if there is the existence of the above-mentioned objects. Now, let us consider that there is one apple on the table and we have eaten up that apple. So, there will be no apple on the table. That means there is the absence of an apple on the table. Therefore, 0 (zero) is defined as the absence of any number.

Shridhara Acharya (870 – 930) a Bengali Hindu Pandit and Mathematician of India was the first person who had given algorithm for solving Quadratic Equation [III]. He wrote a book named ‘Trisatika’ which contains three hundred (300) Sanskrit slokas. In this book, he discussed counting of numbers, natural numbers, zero, measure, multiplication, fraction, division, squares, cubes, rule of three, and mensuration which is the main part of geometry concerned with ascertaining sizes, lengths, areas, and volume.

Shridhara Acharya’s formula for solving Quadratic Equation came into light from the quotation of Bhaskara II (1114-1185), an Indian Mathematician, though the original is lost, unfortunately. The English version of Shridhara Acharya’s Sanskrit slokas regarding the solution of Quadratic Equation is as follows:

“Multiply both side of the equation by a known quantity equal to four times the coefficient of the square of the unknown; add to both sides a known quantity equal to the square of the coefficient of the unknown; then take the square root”.

Now, let us see what this Sanskrit sloke means to take

\[ ax^2 + bx = c \]

Multiply both side by 4a to get

\[ (4a) x (a x^2 + bx) = (4a) x c \]
\[ \Rightarrow 4 a^2 x^2 + 4abx = 4ac \]

Then add \( b^2 \) to both sides

\[ 4 a^2 x^2 + 4abx + (b^2) = 4ac + (b^2) \]
\[ \Rightarrow (2ax + b)^2 = 4ac + b^2 \]
\[ \Rightarrow 2ax + b = \sqrt{4ac + b^2} \]
\[ \Rightarrow 2ax = -b + \sqrt{4ac + b^2} \]
\[ \Rightarrow x = \frac{-b + \sqrt{4ac + b^2}}{2a} \]
There is no suggestion of Shridhara Acharya to take two values when he took the square root.

In the 9th century Persian mathematician, Muhammad bin Musa al-Khwarizmi solved Quadratic Equations algebraically [IV].

In 1594 Simon Steven obtained first the quadratic formula covering all cases. René Descartes published ‘La Geometric’ containing special cases of the quadratic formula in the form known today in the year 1637 [IV].

In 2021 P. C. Bhattacharyya solved the problem of finding the distance between two points in a plane by Rectangular Bhattacharyya’s Co-ordinate System which is based on the Theory of Dynamics of Numbers instead of Cartesian Co-ordinate System [I].

II. Formulation of the problem and its solution

According to the Theory of Dynamics of Numbers:

1. 0 (zero) is the starting point of any number.
2. There are infinite number of directions through which the numbers can move from the starting point (0) and back to the starting point 0 (zero) with vertically opposite direction of motion of numbers.
3. There are two types of countable numbers (i) countup numbers (ii) countdown numbers.
4. The numbers which move away from the starting point 0 (zero) are called countup numbers.
   Symbolically we can write as –
   \[ \overline{1, 2, 3, 4, 5} \ldots \text{ Or } 1, 2, 3, 4, 5 \ldots \]
   i.e. with a forward arrow (\( \rightarrow \)) over any number or without any arrow over any number.

   The numerical value of countup number is defined as countup number with a prefixed addition operator (+).

   For example : Countup3 = \( \overline{3} \)

   Numerical value of countup3 = \( \overline{3} = + 3 \)

5. The numbers which move towards the starting point 0 (zero) are called countdown numbers.
   Symbolically we can write as –
   \[ \overline{1, 2, 3, 4, 5} \ldots \]
   That is with a backward arrow (\( \leftarrow \)) over any number.

   The numerical value of countdown number is defined as countdown number with a prefixed subtraction operator (−).

   For example :
   Countdown 3 = \( \overline{3} \)

   Numerical value of countdown 3 = \( \overline{3} = -3 \)

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The numerical value of countup or countdown number may be used as coefficient of any unknown countable object (quantity).

For example:

\[ 2x, \ -5x, \ 3x^2, \ -4x^2 \]

Where \( 2, \ -5, \ 3, \ -4 \) are the coefficients of \( x, \ x, \ x^2, \ x^2 \) respectively.

**Law of Theory of Dynamics of Numbers**

1. \( 0 \) (zero) is defined as starting point of any number. There are infinite number of directions through which the numbers can move from the starting point \( 0 \) (zero) and back to the starting point \( 0 \) (zero) with vertically opposite direction of motion of numbers.

2. The countup numbers is always greater than or equal to the countdown numbers. The countup numbers can move independently but the motion of the countdown numbers is dependent on the motion of countup numbers. The motion of the countdown numbers exists if and only if there are motions of the countup numbers.

3. For every equation, the countup numbers is always equal to the countdown numbers.

**Rules of Operators**

In the theory dynamics of numbers Addition, Subtraction, Multiplication and Division are called operators. The symbol ‘+’, ‘−’, ‘×’, ‘÷’ are defined as the symbol of Addition, Subtraction, Multiplication, and Division respectively.

In the case of any mathematical calculation, these operators play a very important role. In the case of application of any operator, there must exist a prefix number and a postfix number of the operator. Any countup number or countdown number may exist as a prefix or postfix number in case of addition, multiplication and division only but in case of subtraction, the numerical value of the prefix number must be greater than or equal to the numerical value of the postfix number.

**Rules of Addition and Subtraction.**

I. Countup number + Countup number = Countup number.

II. Countdown number + Countdown number = Countdown the number.

III. Countup number + Countdown number = Countup number or Zero (0).

IV. Countup number − Countdown number = Countup number.

**Rules of Multiplication.**

I. \( − \) Countup number = Countdown number

II. \( − \) Countdown number = Countup number

III. Countup number \( × \) Countup number = Countup number.

IV. Countdown number \( × \) Countdown number = Countdown number.

V. Countup number \( × \) Countdown number = Countdown the number.

VI. Countdown number \( × \) Countup number = Countdown the number.

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Rules of Division
I. Countup number ÷ Countup number = Countup number.
II. Countdown number ÷ Countdown number = Countup number.
III. Countup number ÷ Countdown number = Countdown number.
IV. Countdown number ÷ Countup number = Countdown the number.

Application of the theory of dynamics of numbers to plane co-ordinate geometry.

A comparative study between Rectangular Cartesian Co-ordinate System with Rectangular Bhattacharyya’s Co-ordinate System which is based on the ‘Theory of Dynamics of Numbers’.

First of all, we shall take an example and solve the problem by Rectangular Cartesian Co-ordinate System and find the shortcomings of that solution, and then we shall solve the same problem by Rectangular Bhattacharyya’s Co-ordinate System which is based on the Theory of Dynamics of Numbers.

Example: -- Find the distance of the point (– 6, – 8) from the origin.

Solution of the problem by the cartesian co-ordinate system –

Fig. 1

The parts of XOY, X'OY, X'OY' and XOY' are called the 1st quadrant, 2nd quadrant, 3rd quadrant, and 4th quadrant respectively.

Let us draw xoX and yoY are two mutually perpendicular straight lines through the fixed point O on the page of the paper. The fixed point O is called the origin. The straight line XOX' and YOY' are called the x-axis and y-axis respectively.

René Descartes (1596-1650) introduced the following convention regarding the sign of distance along the co-ordinate axes: --

1) The distance measured from O along ox axis (or parallel in the direction to ox) is positive
2) The distance measured from O along ox' axis (or parallel in the direction to ox') is negative.

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3) The distance measured from O along \( \overrightarrow{oy} \) axis (or parallel in the direction to \( \overrightarrow{oy} \)) is positive.

4) The distance measured from O along \( \overrightarrow{oy'} \) axis (or parallel in the direction to \( \overrightarrow{oy'} \)) is negative.

The point P \((-6, -8)\) lies on the 3rd quadrant. According to the cartesian co-ordinate system from fig 1, we have

\[
\overline{OM} = -6 \quad \text{and} \quad \overline{MP} = -8
\]

\[
\overline{OP}^2 = \overline{OM}^2 + \overline{MP}^2
\]

\[
\Rightarrow \overline{OP}^2 = (-6)^2 + (-8)^2
\]

\[
\Rightarrow \overline{OP}^2 = 36 + 64
\]

\[
\Rightarrow \overline{OP}^2 = 100
\]

\[
\Rightarrow \overline{OP} = \sqrt{100}
\]

\[
\Rightarrow \overline{OP} = \pm 10
\]

\[
\therefore \overline{OP} = 10 \quad (\because \text{the length } \overline{OP} \text{ is positive})
\]

**Observation:**

We find \( \overline{OP} = \pm 10 \). The value of the length \( \overline{OP} = -10 \) had been discarded with the argument that the distance cannot be negative though it was assumed that the length \( \overline{OM} = -6 \) and \( \overline{MP} = -8 \). The argument is self-contradictory.

This problem has been solved by Rectangular Bhattacharyya’s Co-ordinate System on the basis of the Theory of Dynamics of Numbers.

**Solution by Rectangular Bhattacharyya’s Co-ordinate System:**

![Diagram of the coordinate system](Fig. 2(a))

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Let us consider O (0,0) to be a fixed point on the plane of this page which is stated as an origin [Fig 2 (b)]. We draw four straight lines from the origin (O):

1. Towards the X direction
2. Towards the Y direction
3. Towards the $X'$ direction
4. Towards the $Y'$ direction

$\overrightarrow{OX}$ and $\overrightarrow{OY}$; $\overrightarrow{OX'}$ and $\overrightarrow{OY'}$; $\overrightarrow{OX}$ and $\overrightarrow{OY'}$; $\overrightarrow{OX'}$ and $\overrightarrow{OY}$, lines are mutually perpendicular to each other and $\overrightarrow{OX}$ and $\overrightarrow{OX'}$ are in vertically opposite directions and $\overrightarrow{OY}$ and $\overrightarrow{OY'}$ are in vertically opposite directions.

The straight line $\overrightarrow{OX}$ is called X-axis, similarly $\overrightarrow{OY}$ is called Y-axis, $\overrightarrow{OX'}$ is called $X'$ - axis and $\overrightarrow{OY'}$ is called Y-axis. These four lines together divided the plane into four parts of the plane. Each of these parts is said to be a quadrant. The quadrant is named as follows: XOY is the first quadrant, $X'OY$ is the second quadrant, $X'OY'$ is the third quadrant and $X'OY''$ is the fourth quadrant. Note that X-axis, Y-axis, $X'$-axis and $Y'$-axis are all positive axes or directions.

The position of the points can be uniquely determined by introducing the following notations:

1) The distance measured from O along $\overrightarrow{OX}$ axis (or parallel in the direction to $\overrightarrow{OX}$ axis) putting without any notation over the unit is positive.
2) The distance measured from O along to $\overrightarrow{OY}$ axis (or parallel in the direction to $\overrightarrow{OY}$ axis) putting without any notation over the unit is positive.
3) The distance measured from O along $\overrightarrow{OX'}$ axis (or parallel in the direction to $\overrightarrow{OX'}$ axis) putting a notation das (') over the unit is positive.

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4) The distance measured from O along $\overline{OY}$ axis (or parallel in the direction to $\overline{OY}$ axis) putting a notation dash (') over the unit is positive.

5) Significance of arrows: The motions are always positive and also the units of motions are always positive. A motion that is away from the starting point is called countup motion and a motion that is towards the starting point is called countdown motion. The units of countup motion or countdown motion is always positive. For every motion, there is a countup motion which is always greater than or equal to countdown motion.

(i) The Symbol forward arrow ‘→’ means countup motion of a point which is away from the starting point.

(ii) The symbol backward arrow ‘←’ means countdown motion of a point which is towards the starting point.

Following the above discussion, we can determine uniquely the position of a point on a plane referred to mutually perpendicular co-ordinate axes through an origin, we require without dash or with one dash (') over positive real numbers. These two positive real numbers may both be without dash or both with one dash, also it may be one without the dash and the other with a dash together called the Rectangular Bhattacharyya’s Co-ordinate.

Now, from fig. 2(a) and 2(b) the point P (6', 8') lies in the third quadrant.

In fig. 2(a), the point P (6', 8') is moving away from the origin O, so the motion of the point P is a countup motion from the origin O (6' means 6 unit, positive and 8' means 8 unit, positive).

In fig 2(b), the point P (6', 8') is moving towards the origin O, so the motion of the point P is a countdown motion (6' means 6 unit, positive and 8' means 8 unit, positive).

From fig 2(a),

$$\overline{OP}^2 = \overline{OM}^2 + \overline{MP}^2 = 6^2 + 8^2 = 36 + 64 = 100$$

\[\therefore \overline{OP} = \sqrt{100} = 10\]

\[\therefore \text{Numerical value of } 10 = +10\]

$\overline{OP} = +10$ (According to theory of dynamics numbers)

From fig. 2(b)

$$\overline{PO}^2 = \overline{PM}^2 + \overline{MO}^2$$

$\Rightarrow \overline{OP}^2 = \overline{MP}^2 + \overline{OM}^2 = 8^2 + 6^2 = 64 + 36 = 100$

$\Rightarrow \overline{OP} = \sqrt{100} = 10$

Numerically value of $10 = -10$ (According to the theory of dynamics numbers),

though the distance $\overline{OP} = \overline{PO}$

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Observations:

1. Countup OP + Countdown OP = \( \overrightarrow{OP} + \overrightarrow{OP} = 10 + 10 = +10 - 10 = 0 \)
   Thus, it satisfies the third law of the theory of dynamics of numbers.

2. For solving the said problem by the cartesian co-ordinate system we have found that \( \overrightarrow{OP} = \pm 10 \), which means \( \overrightarrow{OP} = +10 \) or \( \overrightarrow{OP} = -10 \). Though \( \overrightarrow{OP} = -10 \) has been discarded with an argument that distance cannot be negative. But, if we solve the problem by Bhattacharyya’s Co-ordinate System [I] which is based on the Theory of Dynamics of Numbers, the question of \( \overrightarrow{OP} = -10 \) does not arise.

From fig. 2(a) we can construct a Quadratic Equation in the form:

Let us consider \( \overrightarrow{OP} = x \)

\[ x^2 - 100 = 0 \]  

(1)

Since, \(-100 < 0\), we can write the equation (1) in the form:

\[ x^2 + 100 = 0 \]

(2)

According to 3rd law of theory of dynamics of numbers

\[ x^2 = 100 \]

\[ \Rightarrow x = 10 \]

(3)

\[ \therefore \overrightarrow{x} = \overrightarrow{10} = +10 \]

(4)

\therefore The unknown quantity \( x = 10 \) will be in countup motion of \( x \)

Therefore, \( \overrightarrow{x} = +10 \) is the solution of equation (1)

Now, let us examine whether the numerical value of \( \overrightarrow{x} = \overrightarrow{10} = +10 \), satisfy the equation (1) or not

According to the Theory of Dynamics of numbers the equation (1)

\[ x^2 - 100 = 0 \]

will take the form

\[ x^2 + 100 = 0 \]

Now,

\[ x^2 + 100 \]

\[ \Rightarrow (+10)^2 - 100 \]

\[ \Rightarrow 100 - 100 \]

\[ \Rightarrow 0 \]

Thus, the numerical value of \( \overrightarrow{x} = +10 \), satisfies the equation (1)

Similarly, from fig 2(b) we can construct a Quadratic Equation in the form

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\[ x^2 + 100 = 0 \]  \hspace{1cm} (5)

Since, \( 100 > 0 \), we can write equation (5) in the form
\[ \overrightarrow{x^2} + \overrightarrow{100} = 0 \]  \hspace{1cm} (6)

According to 3 rd law of theory of dynamics of numbers
\[ x^2 = 100 \]  \hspace{1cm} (7)
\[ \Rightarrow x = 10 \]  \hspace{1cm} (8)
\[ \therefore \overrightarrow{x} = \overrightarrow{\frac{10}{10}} = -10 \]  \hspace{1cm} (9)

\[ \therefore \] The unknown quantity \( x = 10 \), will be in countdown motion of \( x \),
Therefore \( \overrightarrow{x} = -10 \) is the solution of equation (5)

Now, let us examine whether the numerical value of \( \overrightarrow{x} = \overrightarrow{\frac{10}{10}} = -10 \), satisfies the equation (5) or not

According to the Theory of Dynamics of Numbers the equation (5)
\[ x^2 + 100 = 0 \]
will take the form
\[ \overrightarrow{x^2} + \overrightarrow{100} = 0 \]

Now,
\[ \overrightarrow{x^2} + \overrightarrow{100} = - (10)^2 + (+100) \]
\[ = - 100 + 100 \]
\[ = 0 \]

Thus, the numerical value of \( \overrightarrow{x} = -10 \) satisfies the equation (5)

**Note:**
\[ x^2 + 1 = 0 \]  \hspace{1cm} (1)

Since, \( 1 > 0 \)

According to the Theory of Dynamics of Numbers, we can write the equation (1) as
\[ \overrightarrow{x^2} + \overrightarrow{1} = 0 \]  \hspace{1cm} (2)

According to 3\textsuperscript{rd} law of the Theory of Dynamics of Numbers
\[ x^2 = 1 \]  \hspace{1cm} (3)
\[ \Rightarrow x = \sqrt{1} \]
\[ \Rightarrow x = 1 \]

Therefore, \( \overrightarrow{x} = \overrightarrow{1} \)

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Numerical value of $\vec{x} = -x = -1$

$\therefore \vec{x} = -1$, is the solution of the equation (1)

Now, if we solve this equation (1) by conventional method

\[
x^2 + 1 = 0
\]

\[
\Rightarrow x^2 = -1
\]

\[
\Rightarrow x = \pm \sqrt{-1}
\]

\[
\Rightarrow x = \pm i,
\]

where $i = \sqrt{-1}$

**Observation:**

Thus, \(\vec{x} = \sqrt{1} = -1 \neq \sqrt{-1}\)

**Quadratic Equation**

To find the relation between roots and coefficients of Quadratic Equation:

\[ax^2 + bx + c = 0\]

Where \(a \neq 0\), the coefficient of \(x^2\); \(b\), the coefficient of \(x\), and \(c\), the constant term.

**Case – I:**

When \(a > 0, c > 0\)

\[ax^2 + bx + c = 0\]  \hspace{1cm} (1)

According to the theory of dynamics of numbers, we can write

\[ax^2 + bx + c = 0\]  \hspace{1cm} (2)

According to the third law of the theory of dynamics of numbers

\[ax^2 + bx = c\]  \hspace{1cm} (3)

\[
\Rightarrow \left( x + \frac{b}{2a} \right)^2 = \frac{b^2}{4a^2} + \frac{c}{a}
\]

\[
\Rightarrow x = \frac{-b}{2a} + \frac{\sqrt{b^2 + 4ac}}{2a}
\]

\[
\Rightarrow x = \frac{\sqrt{b^2 + 4ac} - b}{2a}
\]

The solution of the equation of \(ax^2 + bx + c = 0\) is the countdown \(x\).

The numerical value of countdown \(x = \vec{x} = -x\)

Hence, \(\vec{x} = -\left( \frac{\sqrt{b^2 + 4ac} - b}{2a} \right)\) is the solution of equation (1)

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Case – II:

when \( a > 0 \) and \( c < 0 \)
\[ ax^2 + bx - c = 0 \]  

According to the Theory of Dynamics of Numbers, we can write
\[ \overline{ax^2 + bx + c'} = 0 \]  

According to the third law of the Theory of Dynamics of Numbers
\[ ax^2 + bx = c \]  

\( \Rightarrow x^2 + \frac{b}{a}x = \frac{c}{a} \)
\( \Rightarrow x^2 + 2 \cdot \frac{1}{2} \frac{b}{a}x + \left( \frac{b}{2a} \right)^2 - \left( \frac{b}{2a} \right)^2 = \frac{c}{a} \)
\( \Rightarrow \left( x + \frac{b}{2a} \right)^2 = \frac{b^2}{4a^2} + \frac{c}{a} \)
\( \Rightarrow \left( x + \frac{b}{2a} \right)^2 = \frac{b^2 + 4ac}{4a^2} \)
\( \Rightarrow x + \frac{b}{2a} = \frac{\sqrt{b^2 + 4ac}}{2a} \)
\( \Rightarrow x = -\frac{b}{2a} + \frac{\sqrt{b^2 + 4ac}}{2a} \)
\( \Rightarrow x = \frac{\sqrt{b^2 + 4ac} - b}{2a} \)

Therefore, the solution of the equation of \( ax^2 + bx - c = 0 \) is the countup \( x \).

The numerical value of countup \( x = \overline{x} = + x \)

Hence \( \overline{x} = + \frac{\sqrt{b^2 + 4ac} - b}{2a} \), is the solution of equation (1)

Case – III:

When \( a < 0 \) and \( c < 0 \)
\[-ax^2 + bx - c = 0 \]  

Solution:
\[-ax^2 + bx - c = 0 \]  

\( \Rightarrow x^2 - \frac{bx}{a} + \frac{c}{a} = 0 \)  

According to the theory of dynamics of numbers, we can write
\[ x^2 - \frac{bx}{a} + \frac{c}{a} = 0 \]  

According to the third law of the theory of dynamics of numbers, we have
\[ x^2 - \frac{b}{a}x = \frac{c}{a} \]  

\( \Rightarrow x^2 - 2 \cdot \frac{1}{2} \frac{b}{a}x + \left( \frac{b}{2a} \right)^2 - \left( \frac{b}{2a} \right)^2 = \frac{c}{a} \)

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\( \Rightarrow (x - \frac{b}{2a})^2 = \frac{(b)^2}{(2a)^2} + \frac{c}{a} \)

\( \Rightarrow (x - \frac{b}{2a})^2 = \frac{b^2 + 4ac}{4a^2} \)

\( \Rightarrow x - \frac{b}{2a} = \frac{\sqrt{b^2 + 4ac}}{2a} \)

\( \Rightarrow x = \frac{\sqrt{b^2 + 4ac} + b}{2a} \)

\( \Rightarrow x = \frac{\sqrt{b^2 + 4ac} + b}{2a} \)

Therefore, the solution of equation of \( -ax^2 + bx - c = 0 \) is the countdown \( x \). The numerical value of countdown \( x = \bar{x} = -x \)

Hence, \( \bar{x} = -\left(\frac{\sqrt{b^2 + 4ac} + b}{2a}\right) \) is the solution of equation (1)

**Case – IV:**

When \( a < 0 \) and \( c > 0 \)

\(-ax^2 + bx + c = 0\)

**Solution:**

\(-ax^2 + bx + c = 0\)  \hspace{1cm} (1)

\( \Rightarrow x^2 - \frac{bx}{a} - \frac{c}{a} = 0 \)  \hspace{1cm} (2)

According to the Theory of Dynamics of Numbers, we can write

\( \Rightarrow x^2 - \frac{b}{a}x + \frac{c}{a} = 0 \)  \hspace{1cm} (3)

According to the third law of the Theory of Dynamics of Numbers, we have

\( x^2 - \frac{b}{a}x = \frac{c}{a} \)  \hspace{1cm} (4)

\( \Rightarrow x^2 = 2 \cdot \frac{b}{2a}x + \left(\frac{b}{2a}\right)^2 = \frac{c}{a} \)

\( \Rightarrow (x - \frac{b}{2a})^2 = \frac{b^2}{4a^2} + \frac{c}{a} \)

\( \Rightarrow (x - \frac{b}{2a})^2 = \frac{b^2 + 4ac}{4a^2} \)

\( \Rightarrow x - \frac{b}{2a} = \frac{\sqrt{b^2 + 4ac}}{2a} \)

\( \Rightarrow x = \frac{b}{2a} + \frac{\sqrt{b^2 + 4ac}}{2a} \)

\( \Rightarrow x = \frac{b + \sqrt{b^2 + 4ac}}{2a} \)

Therefore, the solution of the equation \(-ax^2 + bx + c = 0\) is countup \( x \). The numerical value of countup \( x = \bar{x} = +x \)

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Hence,  \( \tilde{x} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \), is the solution of equation (1)

Thus, we can solve the Quadratic Equation without any help of a complex number

**Note:** \( x^2 \neq (\tilde{x})^2 \)

**Proof:** Numerical value of

\[ x^2 = -x^2 \quad (1) \]

We know the Numerical value of

\[ \bar{x} = -x \quad (2) \]

\[ \therefore (\bar{x})^2 = (-x)^2 = +x^2 \quad (3) \]

Thus,

\[ x^2 \neq (\bar{x})^2 \]

**Example: solve**

\[ x^2 + x + 1 = 0 \]

**Solution:**

\[ x^2 + x + 1 = 0 \quad (1) \]

Consider General Quadratic Equation

\[ ax^2 + bx + c = 0, \quad a \neq 0 \quad (2) \]

Comparing Eq. (1) and Eq (2) we have

\[ a = 1, \quad b = 1 \quad \text{and} \quad c = 1 \]

Now, the numerical value of the discriminant of equation (1) we have

\[ b^2 - 4ac \]

\[ = (1)^2 - 4.1.1 \]

\[ = -3 < 0 \]

\[-3 \text{ means } \bar{3} \]

From equation (1) we have \( 1 > 0 \)

So, according to the Theory of Dynamics of Numbers, we can write the Eq (1) as

\[ \tilde{x}^2 + x + 1^2 = 0 \quad (3) \]

According to the third law of the Theory of Dynamics of Numbers, we have

\[ \bar{x}^2 + x + 1 = 1 \]

\[ \Rightarrow x^2 + 2.1 \cdot x + \left( \frac{1}{2} \right)^2 = 1 \]

\[ \Rightarrow \left( x + \frac{1}{2} \right)^2 = 1 + \frac{1}{4} \]

\[ \Rightarrow \left( x + \frac{1}{2} \right)^2 = \frac{5}{4} \]

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\[ x + \frac{1}{2} = \frac{\sqrt{5}}{2} \]

\[ x = \frac{\sqrt{5}}{2} - \frac{1}{2} \]

\[ x = \frac{\sqrt{5} - 1}{2} \]

Therefore, \( x = \frac{\sqrt{5} - 1}{2} \)

Numerical value of \( x = - \frac{\sqrt{5} - 1}{2} \)

\[ \therefore \ x = - \frac{\sqrt{5} - 1}{2} \]

is the solution of the equation (1)

Let us examine whether \( x = - \frac{\sqrt{5} - 1}{2} \) satisfies the equation (1) or not.

According to the Theory of Dynamics of Numbers of equation (1)

\[ x^2 + x + 1 = 0 \]

will take the form

\[ \overline{x^2 + x + 1} = 0 \]

Now,

\[ \overline{x^2 + x + 1} = x^2 + \overline{x} + 1 \]

\[ = - x^2 - x + 1 \]

\[ = - \left( \frac{\sqrt{5} - 1}{2} \right)^2 - \frac{\sqrt{5} - 1}{2} + 1 \]

\[ = - \left[ \frac{5 - 2 \sqrt{5} + 1}{4} \right] - \frac{\sqrt{5} - 1}{2} + 1 \]

\[ = - \frac{5 + 2 \sqrt{5} - 1}{4} - \frac{\sqrt{5} - 1}{2} + 1 \]

\[ = - \frac{5 + 2 \sqrt{5} - 1 - 2 \sqrt{5} + 2}{4} + 1 \]

\[ = - \frac{4}{4} + 1 \]

\[ = - 1 + 1 \]

\[ = 0 \]

Thus,

\[ \overline{x} = - \frac{\sqrt{5} - 1}{2} \] satisfies the equation (1)

Observation:

(I) If we solve the equation

\[ x^2 + x + 1 = 0 \] (1)

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by conventional method, we have
\[ x = \frac{-1 \pm \sqrt{1-4\times1\times1}}{2} \]
\[ \Rightarrow x = \frac{-1 \pm \sqrt{-3}}{2} \]
\[ \Rightarrow x = \frac{-1 \pm i\sqrt{3}}{2} \]

Since, we cannot find the value of imaginary number \( i\sqrt{3} \), we are unable to get the proper numerical solution of the quadratic equation, \( x^2 + x + 1 = 0 \)

But by using the Theory of Dynamics of Numbers we can easily find the numerical solution of the said problem. In that way, we can find the numerical solution of any quadratic equation having its numerical value of discriminant, \( b^2 - 4ac < 0 \).

(II) We can find the solution of the quadratic equation
\[ x^2 + x + 1 = 0 \]  \hspace{1cm} (1)

is a countdown motion of countable unknown object \( x \) in the Quadratic Equation

Where
\[ \dot{x} = -\frac{\sqrt{5}-1}{2} \]  \hspace{1cm} (2)

Therefore, according to the Theory of Dynamics of Numbers, there must exist the countup motion of countable unknown object \( x \) in the form of Quadratic Equation where
\[ \dot{x} = +\frac{\sqrt{5}-1}{2} \]  \hspace{1cm} (3)

Now, let us find the quadratic equation whose solution is given,
\[ \dot{x} = \frac{\sqrt{5}-1}{2} \]

So, we have
\[ x = \frac{\sqrt{5}-1}{2} \]  \hspace{1cm} (4)
\[ \Rightarrow 2x = \sqrt{5} - 1 \]
\[ \Rightarrow 2x + 1 = \sqrt{5} \]
\[ \Rightarrow (2x + 1)^2 = (\sqrt{5})^2 \]
\[ \Rightarrow 4x^2 + 4x + 1 = 5 \]
\[ \Rightarrow 4x^2 + 4x + 1 - 5 = 0 \]
\[ \Rightarrow 4x^2 + 4x - 4 = 0 \]
\[ \Rightarrow 4 (x^2 + x - 1) = 0 \]
\[ \Rightarrow x^2 + x - 1 = 0 \]  \hspace{1cm} (5)
Therefore, we can find the countup motion of countable unknown object \( x \) in the form of Quadratic Equation as

\[
x^2 + x - 1 = 0
\]

III. Conclusion

0 (zero) is the starting point of any number. There are infinite numbers of directions through which the numbers can move from the starting point 0 (zero) and can return in the vertically opposite direction towards the starting point 0 (zero). There are dynamic motions of the numbers. This is the basic principle of the theory of dynamics of numbers. The numbers which are moving away from the starting point zero (0) are called Countup numbers and the numbers which are moving towards the starting point zero (0) are called countdown numbers. The Countup numbers and the countdown numbers are all positive.

The problem in the plane co-ordinate geometry has been solved successfully by Rectangular Bhattacharyya’s Co-ordinate System which is based on the theory of dynamics numbers and also the solution of any quadratic equation, \( ax^2 + bx + c = 0 \), has been solved successfully even if the numerical value of the discriminant, \( b^2 - 4ac < 0 \), by using the theory of dynamics numbers without considering imaginary numbers.

The new concept of the Theory of Dynamics of Numbers is applicable in any branch of Mathematics, Science, Engineering, and Technology.

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Conflict of Interest

The author declares that there is no conflict of interest regarding this article.

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