A Method for Integrating Delayed Recharge Flux Through Unsaturated Zones into Analytical and Numerical Groundwater Flow Modeling

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Abstract The temporal delay in the water table’s response to precipitation is a common phenomenon. Using a mathematical model, we developed a function that represents the recharge flux through unsaturated zones of varying thickness to model the delayed response of groundwater level due to precipitation. The recharge flux from the developed function was integrated into the analytical and numerical models to estimate the actual groundwater levels in the unsaturated zone. The analytical and numerical models accurately determined the actual groundwater levels. Additionally, calibrated hydraulic parameters from numerical estimates reflected the values measured in the actual aquifer, thereby highlighting the potential application of the developed function to wide-ranging groundwater problems. Based on the recharge function, a recharge module can be developed for existing flow simulators to enhance groundwater modeling.

Plain Language Summary The delay (ranging from minutes to months) in the change in the groundwater level due to precipitation is common. If this delay is neglected, then erroneous estimates of the groundwater flow and solute transport are inevitable. Most of the models on ground water estimation are incapable of assessing delayed groundwater response (practically evaluated with great difficulty). To overcome these limitations, in this study, a statistical function representing the delayed recharge flux through the unsaturated zone is derived. To validate the function, delayed recharge fluxes were obtained by calibrating the developed function using actual groundwater level data of a few locations. Then, the fluxes were applied to analytical and numerical models for groundwater level estimation. The results reasonably reflected the actual groundwater levels and aquifer hydraulic properties, indicating that the developed function is useful in resolving real-world problems.

1. Introduction

The temporal delay in the water table response to precipitation is a common phenomenon that is caused by the hydraulic intervention of the unsaturated zones (Besbes & de Marsily, 1984; Cook et al., 2003; Kim et al., 2014; Jeong & Park, 2017; Jeong et al., 2017, 2018; Moench & Barlow, 2000; Morel-Seytoux, 1984; Obergfell et al., 2019; von Asmuth et al., 2002). Many simultaneous monitoring results of groundwater from various climatic conditions indicate that the number of areas wherein the groundwater level response has been delayed for more than a few days exceeds the number of areas without delays (Jeong et al., 2017). In areas with thick (hundreds of meters) unsaturation zones, such as the Jeju Island in South Korea (Won et al., 2005), delayed groundwater level response is reported to exceed a month despite the high permeability of the zone, which is composed of volcanic sediments and lava tunnels. In these areas, the delayed groundwater level response is a major factor to consider when estimating the groundwater flow and contaminant transport originating from the surface. The major factor that causes this delay is the finite permeability of the media that constitutes the unsaturated zones (Jeong et al., 2017). However, particularly for unsaturated zones that are not properly characterized, it is virtually impossible to identify and characterize the processes (i.e., detoured advection and a complex capillary barrier effect due to irregularly distributed permeability structures) associated with the infiltrated water, delayed and dispersed in the unsaturated zones, and to
reflect them in groundwater level estimation. Therefore, an alternative concept that encompasses the processes regarding water infiltrated through the unsaturated zone is necessary.

Numerical groundwater flow modeling is an indispensable method for simulating the groundwater flow and level changes in aquifers over large areas. Groundwater flow models have been improved to incorporate various hydraulic phenomena within the model, thereby making modeling convenient. However, this effort has been made mainly in saturated zones, and relatively little attention has been paid to delayed recharge flux in unsaturated zones. Hence, currently, groundwater flow models that target only the saturated zones are of limited use in the simulation of groundwater flow and level fluctuations in unconfined aquifers involving thick unsaturated zones. Conversely, if the numerical model is based on nonlinear Richards equation (e.g., HYDRUS, Kool & van Genuchten, 1989), changes in the capillary pressure ($p$), moisture content ($θ$), and relative permeability ($k_r$) can be incorporated to simultaneously evaluate unsaturated and saturated flows. While the nonlinear Richards equation is a rigorous method for representing unsaturated flows, there are many difficulties in solving the equation numerically owing to the limited size of the time step and convergence problems. Many approaches have been proposed to effectively solve the Richards equation. Among the approaches, soil moisture velocity equation (Ogden et al., 2017) is a novel method for approximating the Richards equation with an increased numerical stability that can be considered for both homogeneous and stratified unsaturated zones. For layered unsaturated zones, the Kirchhoff integral transformation with a truncated Taylor series expansion (Suk & Park, 2019) can be applied to achieve computational efficiency and accuracy in unsaturated flow simulation. However, in most previous studies on the practical implementation of the Richards equation to field scale, homogeneity in the entire unsaturated zone or within a layer was assumed. Furthermore, the Richards-equation-based model requires constitutive relationships between $p$, $θ$, or $p_r$ and $k_r$ (e.g., Brooks & Corey, 1964; Russo, 1988; van Genuchten, 1980). However, the determination of the model parameters of the relationships requires detailed hydraulic characterization of the unsaturated zone; this is difficult to implement for large areas. Consequently, practical regional-scale groundwater flow simulations using Richards-equation-based models cannot be performed by modelers who are not sufficiently experienced.

To avoid difficulties associated with the handling of rigorous unsaturated flows over unsaturated volumes, statistical transfer function approaches (Besbes & de Marsily, 1984; Jeong & Park, 2017; Jeong et al., 2018; Morel-Seytoux, 1984; Obergfell et al., 2019; von Asmuth et al., 2002) can be employed. Jeong et al. (2018) explicitly conceptualized the time-delayed and dispersed fluctuations in groundwater levels that was attributed to groundwater recharge transferred through the unsaturated zone. In their study, temporal characteristics of spatially integrated recharge flux through large-scale vadose zones were assumed to follow the gamma distribution function, wherein each stream tube connecting a part of the surface with a part of the water table has an independent and identically distributed advective-dispersive recharge process (refer Jeong et al., 2018, Figure 1a). Note that Nash (1957) applied the gamma distribution function to a surface water discharge model to conceptualize the basin as a series of connected reservoirs. In his model, $k$ is the number of reservoirs and $α$ is a parameter related to the storage capacity of reservoirs (Besbes & de Marsily, 1984); his model shares the same mathematical formulation with Jeong et al. (2018) but is conceptually different. Based on their methods, Jeong et al. (2018) developed a semi-analytical solution for determining the groundwater level fluctuation by employing the gamma distribution function as the transfer function of the unconfined aquifer to statistically represent the arrival time distribution for recharge fluxes. Although the solution developed by Jeong et al. (2018) is important for point evaluations of monitoring wells, extending this solution to a wide range of groundwater level estimation methods is limited because of the lack of consideration of spatial dimensions.

As an alternative to the conventional constitutive model that converts infiltration into recharge, in this study, a statistical transfer function that represents the delayed recharge flux through unsaturated zones is developed. In addition, we demonstrate the ability of the function to estimate groundwater level fluctuations by integrating the function into analytical and numerical groundwater flow models. In this study, MODFLOW-2005 (Harbaugh, 2005; McDonald & Harbaugh, 1988), which is a groundwater flow model widely used for numerical simulation, was adopted. This study focuses on highlighting the practical aspects of the delayed recharge flux in hydrogeological applications without providing a recharge module for a conventional model.
2. Method

According to Jeong et al. (2018), the groundwater level fluctuation caused by a single precipitation event involving the delayed recharge flux within an unconfined aquifer can be expressed as follows:

\[
h(t; P_0, \rho \phi, \alpha, k, \kappa) = \frac{\rho P_0}{\phi} \mathcal{L}^{-1} \left( \alpha^k \frac{1 - \exp(-p \Delta t_p)}{p(p + \alpha)^k(p + \kappa)} \right),
\]

(1)

where \( h(t; P_0, \rho \phi, \alpha, k, \kappa) \) is the discharge potential (L) indicating the variable water level over a minimum groundwater level \((H_{\text{min}})\) due to \( P_0 \), and \( P_0 \) is the cumulative precipitation rate \((\text{LT}^{-1})\) during the time \( 0 \leq t < \Delta t_p \) (typically, \( \Delta t_p = 1 \text{ day} \)). \( \rho \) is the recharge-to-precipitation ratio \((-)\), \( \phi \) is the fillable porosity \((-)\), \( \kappa \) is the water table recession rate coefficient \((\text{T}^{-1})\), \( \alpha \) is the inverse scale parameter of the gamma distribution \((-)\), \( k \) is a shape parameter of the gamma distribution \((-)\), \( p \) is the Laplace transform parameter equivalent to \( t \) in the time domain, and \( \mathcal{L}^{-1} \) denotes the inverse Laplace transformation. In this study, \( \alpha \) and \( k \) are in the range of 0–1 and 0–5, respectively. The term \( 1/(p + \kappa) \) in the equation is associated with the groundwater level recession due to hydraulic head dissipation (or diffusion) through the aquifer.

To estimate the groundwater level from time 0 to \( T \) for multiple precipitation events, the following superposition can be incorporated:

\[
H = \sum_{t=0}^{T} h(t; P_t) + H_{\text{min}},
\]

(2)

where \( H \) is the groundwater level (L) and \( H_{\text{min}} \) is the minimum groundwater level (L) affected by the local hydraulic boundary condition.

This equation is obtained using the convolution of the delayed drainage process in the unsaturated zone and the recharge response of the underlying saturated zone. Assuming no delay \((k = 0)\), the transfer process in the unsaturated zones is nullified and Equation 1 is transformed as follows:

\[
h = \frac{\rho P_0}{\phi} \mathcal{L}^{-1} \left[ \frac{1 - \exp(-p \Delta t_p)}{p} \cdot \frac{1}{p + \kappa} \right],
\]

(3)

which is identical to the solution by Park and Parker (2008). Based on the comparison of Equations 2 and 3, the discrete recharge rate through the unsaturated zone with \( \Delta t_p \) intervals due to cumulative precipitation occurring at a specified interval \((P_0, (\text{LT}^{-1}))\) can be obtained as follows:
Due to the complexity of the full text, I will provide a simplified and summarized version:

In Figure 2, groundwater levels showing delayed response (black squares) to precipitation events (blue bars), with calibration results obtained using Equation 1 (red solid lines) and estimated results obtained using the calibrated Equation 1 (red dash lines) based on data from the (a) GR2, (b) HC4, (c) SC3, and (d) YS2-monitoring wells. In the figures, the observed groundwater level represented by black squares is plotted every 5 days.

The calibration is performed using a constrained nonlinear local optimization algorithm involving the interior point method (Potra & Wright, 2000). For more information on the geology, hydrogeology, and climate conditions, readers can refer to the study by Jeong et al. (2018). The locations of the monitoring wells are provided in Supplementary Material I (Figure S1). In Figure S1, the well names for the data presented in Figures 2a–2d are GR2, HC4, SC3, and YS2, respectively. The average thickness of the unsaturated zones for the areas hosting wells GR2, HC4, SC3, and YS2 is 278, 115, 96, and 29 m, respectively. For all monitoring wells, the data were collected from 2011 to 2015.

In the implementation of this study, Equation 1 is directly evaluated using the numerical inverse Laplace transformation so that  is, rather than an integer, is used in the computation, unlike in Jeong et al. (2018). The calibration is performed using a constrained nonlinear local optimization algorithm involving the interior point method (Potra & Wright, 2000). For more information on the geology, hydrogeology, and climate conditions, readers can refer to the study by Jeong et al. (2018). The locations of the monitoring wells are provided in Supplementary Material I (Figure S1). In Figure S1, the well names for the data presented in Figures 2a–2d are GR2, HC4, SC3, and YS2, respectively. The average thickness of the unsaturated zones for the areas hosting wells GR2, HC4, SC3, and YS2 is 278, 115, 96, and 29 m, respectively. For all monitoring wells, the data were collected from 2011 to 2015.

Therefore, the semi-analytical solution of the groundwater level fluctuations in response to the cumulative precipitation recharge flux during 1 and  can be obtained as follows:

\[ h_a = f_a \left( \sum_{t=1}^{T} R_t, K, S_f, S_B, B, G \right), \]

where  and  indicate hydraulic conductivity (LT⁻¹), specific yield (−), specific storage coefficient (L⁻¹), information on the boundary conditions, and information on domain and monitoring well geometry, respectively. In Equation 5,  can be replaced by the inverse Laplace transform of  multiplied by  (zero-dimensional semi-analytical solution).

Alternatively, the cumulative recharge flux can be applied to a numerical model  to obtain the groundwater hydrograph during  and  equivalent to  using the following equation:

\[ h_a = f_a \left( \sum_{t=1}^{T} R_t, K, S_f, S_B, B, G \right). \]

where  can be any groundwater flow simulator, such as MODFLOW-2005. In Equation 6,  and  indicate hydraulic conductivity, specific yield, and specific storage coefficient matrices that correspond to the number of domain discretization, respectively.

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\[ R_k = \left( \frac{p}{p + \alpha} \right)^k \frac{1 - \exp(-p\Delta t)}{p}. \]
This study demonstrates that the developed unsaturated transfer function can be used along with the independent analytical (semi-analytical) or numerical model. Therefore, instead of considering accurately reflecting the actual aquifer heterogeneity, domain geometry, and boundary conditions in the numerical simulation, we focused on whether the analytical and numerical model can be used jointly with the developed transfer function in estimating the monitored groundwater levels. In the MODFLOW-2005 simulation, one layer was considered while composing the model domain and the simulation was two-dimensional. The area of the model domain was discretized into 40 × 40 grids with horizontal dimension of 4,000 m × 4,000 m. Therefore, the grid for the simulation is uniform and equidimensional (100 mm × 100 mm × 100 m). Notably, the applied domain geometry is the assumed aquifer geometry and not the geometry of the actual aquifer, including the monitoring wells. The eastern and western margins of the model domain were assumed to be specified head (Hmin), and no-flow boundary condition was assigned to the northern and southern margins.

As no hydraulic properties were available for the hypothetical aquifer, the transmissivity (T) and storativity (S) values were determined using the Nelder–Mead simplex method (Lagarias et al., 1998) in MATLAB (fminsearch) by referring to the actual groundwater level data. The storativity value was assumed to be close to the fillable porosity, i.e., \( S = \phi \). The estimation fitness to the actual data was measured using Nash-Sutcliffe model efficiency coefficient (NSEC, Nash & Sutcliffe, 1970), which is defined as follows:

\[
NSEC = 1 - \frac{\sum_{t=1}^{T} (h_{o,t} - h_{e,t})^2}{\sum_{t=1}^{T} (\bar{h}_{o} - h_{e,t})^2}
\]  

where \( h_{o,t} \) is the estimated head at time \( t \), \( h_{e,t} \) is the observed head at time \( t \), and \( \bar{h}_{o} \) is the mean observed head. The range of NSEC values is from \(-\infty\) to 1, and \( NSEC = 1 \) when the estimations are identical to the observed heads.

### 3. Results and Discussion

The recharge fluxes calculated using Equation 4 under two scenarios are shown in Figure 1, with the first (Figure 1a) and the second (Figure 1b) each comprising four cases. In the first scenario, \( \alpha \) varies while \( k \) is constant; whereas, in the second scenario, \( k \) changes while \( \alpha \) is constant. In the evaluations, the term \( \rho P_0 / \phi \) is set to unity and \( \Delta t_g \) is 1 day for all cases as daily cumulative precipitation measurement data are readily available. Moreover, in the first scenario, \( k = 2 \) for all four cases, while \( \alpha \) decreases sequentially from 0.2 to 0.05 at 0.05 intervals. In contrast, in the second scenario, \( \alpha = 0.1 \) for all four cases and \( k \) increases sequentially from 0.5 to 4 by a factor of 2. In the example, values for baseline \( k \) and \( \alpha \) were taken from Table S1 (Supporting information SII), where \( k \) is determined close to the mean and \( \alpha \) is close to the minimum of the values.

In Figures 1a and 1b, the sums of bars were close to 1 for all cases. From Figure 1a, it can be seen that \( \alpha \) primarily contributes to disperse the recharge ratio with a minor impact on the recharge flux temporal delay. At high \( \alpha \), the early recharge ratio is concentrated, while at low \( \alpha \), the ratio is dispersed and requires a longer time to attain the peak. This observation indicates that \( \alpha \) is related to the conductivity of the unsaturated zone like \( \kappa \) in Equation 1 is to the conductivity of the saturated zone (Park et al., 2011). Additionally, increased dispersion of the ratio can be linked to increasing storage capacity of the unsaturated zone. According to Figure 1b, \( k \) principally affects the temporal delay in recharge flux through the unsaturated zone. As \( k \) increases, the recharge ratio peak later emerges and the distribution is more dispersed. Although \( \alpha \) and \( k \) seem to have an inverse correlation, one parameter cannot completely nullify the effects of the other. Hence, in addition to \( \alpha \), \( k \) is another parameter that jointly represents the storage, conductivity, thickness, and permeability structure of the unsaturated zone. In Figure 1, the location of the mode (peak recharge time) can be determined by \((k−1)/\alpha\) when \( k \geq 1 \) (otherwise, the mode is at 0), and the variance (temporal dispersion of recharge fluxes) is \( k/\alpha^2 \). Equation 4 can account for the delayed recharge flux caused by continuous permeability changes and discontinuous heterogeneous structures. However, as the transfer function is a convolution of the box and gamma distribution functions, the multimodal recharge flux cannot be represented for one precipitation event or negatively skewed data. In fact, the delayed recharge flux through
the unsaturated zone rarely exhibits a multimodal or negatively skewed temporal distribution when observed at the water table (Wu et al., 1996).

The results obtained using the calibrated Equation 1 based on the historical groundwater levels for 3 years (days 1–1,096 corresponding to years 2011–2013) from a few monitoring wells in Jeju Island and estimates for the next two years (days 1,097–1,826 corresponding to years 2014–2015) are displayed in Figure 2. In the estimations, the precipitation rate for the year 2014–2015 and the calibrated model parameters using the data for 2011–2013 (Table 1) were applied. The calibration and estimation results are generally satisfactory, showing errors within acceptable margins. Based on NSEC values in Equation 7, the estimation accuracies applied to the GR2, HC4, SC3, and YS2 data are 0.94, 0.74, 0.85, and 0.82, respectively. The correlation coefficients between the estimation parts of the actual and estimated groundwater levels are 0.97, 0.88, 0.94, and 0.91 for the GR2, HC4, SC3, and YS2 monitoring wells, respectively. These values indicate that the groundwater levels of the targeted monitoring wells are reasonably estimated based on the calibration of Equation 1. Comparable estimation accuracies can be obtained using a data-driven model, such as the nonlinear autoregressive exogenous or long short-term memory model (Jeong & Park, 2019). However, the purpose of a physical model differs from the data-driven model because the physically based model will yield parameters with hydrological significance. The model parameters based on the calibration of Equation 1 for the actual data are shown in Table 1, with \( \phi \) denoting fillable porosity (Park, 2012). Collectively considering the calibration results in Figure 2 and other data (Table S1 and Figure S2), \( k \) and the unsaturated thickness exhibit a strong positive correlation (0.74) whereas \( \alpha \) and the unsaturated thickness exhibit a weak negative correlation (−0.35). The values of \( \rho/\phi \) and \( k_2 \) of HC4 were calibrated to be 7.8 and 0.0075 day\(^{-1}\), respectively, by Park et al. (2011) based on the model developed by Park and Parker (2008), which are close to the parameters in the current study.

In these case studies, the response of the aquifer to precipitation varies depending on the precipitation pattern and groundwater level. The changes in the hydraulic process are most evident for days 1,100–1,700 in Figure 2b and days 1,300–1,700 in Figure 2c. These time periods correspond to year 2014 and 2015. In the area including HC4, the daily mean precipitation rates from 2011 to 2015 were 0.0068, 0.0074, 0.0065, 0.0048, and 0.0061 m/day, respectively. Similarly, in the area including SC3, the daily mean precipitation rates were 0.0043, 0.0047, 0.0036, 0.0021, and 0.0049 m/day for years 2011 through 2015. These observations reveal that 2014 was a particularly dry year. Therefore, the overall observations indicate that the groundwater level forecasts by the developed model may not be accurate when the precipitation pattern during the estimation period is different from the model calibration period. Variations in the precipitation patterns and amounts cause imbalance in infiltration and evapotranspiration; this can alter the flow and containment amounts of infiltrated water by altering the moisture distribution in the unsaturated zone. Additionally, in an aquifer with vertically varying hydraulic properties, the fillable porosity at the water table may change due to the groundwater level recession. In such a situation, the extent of groundwater level change due to a given recharge volume can be varied.

In Figure 3, the recharge rate distributions based on Equation 4, assuming \( \rho/\phi = 1 \), a unit precipitation rate, and the calibrated model parameters of \( \alpha \) and \( k \) shown in Table 1 are depicted. From the figure, the recharge flux infiltrating the unsaturated zone before reaching the water table can be deduced. For example, the recharge flux of the YS2 monitoring location when passing through the unsaturated zone shows low dispersion and most of it reaches the water table during the precipitation event. On the contrary, in the GR2-monitoring location, dispersive recharge flux dominates, and the peak recharge flux arrives at the water table after 14 days of the precipitation event. From the examples, the degree of dispersion of the recharge flux is inversely correlated with the thickness of the unsaturated zone, where the thickness of the unsaturated zone is 278 and 29 m for GR2 and YS2, respectively. This inverse correlation is intuitively consistent—the greater the unsaturated thickness, the greater the variance of the time to reach the water table of the recharge flux through individual stream tubes. In regions with highly heterogeneous unsaturated zones, such as Jeju Island, this variance can be even greater.

The increased travel time and dispersion through the unsaturated zone may also affect the natural reduction of contaminants contained in the infiltrated water. In fact, Jeju Island is currently suffering from

| Table 1 Calibrated Model Parameters of Equation 1 Based on the Observed Groundwater Level Data From Monitoring Wells GR2, HC4, SC3, and YS2 |
|-----------------------|---------|-------|--------|--------|
| \( \rho/\phi \) (−)  | \( k \) (day\(^{-1}\)) | \( \alpha \) (−) | \( k_2 \) (−) | \( H_{\text{min}} \) (m) |
| GR2                  | 13.6    | 0.0046 | 0.26   | 4.4    | 136.0  |
| HC4                  | 6.3     | 0.0084 | 0.85   | 4.0    | 0.5   |
| SC3                  | 17.0    | 0.0088 | 0.13   | 0.8    | 0.3   |
| YS2                  | 23.2    | 0.0101 | 0.8    | 0.7    | 6.8   |

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The increased travel time and dispersion through the unsaturated zone may also affect the natural reduction of contaminants contained in the infiltrated water. In fact, Jeju Island is currently suffering from
groundwater nitrate contamination caused by unauthorized discharge of livestock slurry. If the infiltrated water is contaminated and contamination attenuates in the unsaturated zone by a first-order decay process with a rate coefficient of $-0.07 \text{ day}^{-1}$ (Ling & El-Kadi, 1998), the mass ratio ($M$) between the infiltrated and recharged water for the GR2, HC4, SC3, and YS2 monitoring locations are 0.36, 0.75, 0.72, and 0.97, respectively, based on the calculation using the following equation:

$$M = \frac{\phi}{\rho} \sum_{t=1}^{\infty} R_t \exp\left(-0.07(t-1)\right).$$

(8)

This observation indicates that the hydraulic characteristics of an unsaturated zone represented by the values of $\alpha$ and $k$ have the practical application of assessing the groundwater vulnerability caused by surface contamination. The impact can be validated using the calibrated parameters as in Table 1 and Equation 4.

The groundwater levels estimated using the analytical (Equation 5) and the numerical (Equation 6) approaches are compared in Figure 4. In the analytical approach, the recharge fluxes from Equation 4, which was based on the calibrated parameters of each monitoring well (Table 1), were inputted into Equation 5 with a substitution of $f_0$ by the inverse Laplace transform of $1/(p+k)$. Theoretically, the basics of this approach are identical to Equation 1; therefore, results consistent to those in Figure 2 are expected. In the numerical approach, the recharge flux from Equation 4, which was used in the analytical approach, was
Figure 4. Comparison of three approaches for estimating the groundwater level applied to the (a) GR2, (b) HC4, (c) SC3, and (d) YS2-monitoring wells. The observed groundwater levels are represented by black squares. The blue lines represent data from Equation 5, with the recharge rates calculated from Equation 4 and the model parameters in Table 1. The red lines are results from MODFLOW-2005 simulation of the recharge rates calculated using Equation 4 and the corresponding parameters in Table 1. The yellow lines are from MODFLOW-2005 simulation wherein the recharge rates are calculated by multiplying the precipitation and \( \frac{\rho}{\phi} \), where \( \rho/\phi \) are from Table 1 and \( S \) are from Table 2. In the figures, the observed groundwater level represented by black squares is plotted every 5 days.
employed as the input data (i.e., recharge package) for the numerical simulation model using MODFLOW-2005. If the inverse Laplace transform of $1/(p+\kappa)$ and the numerical model operate in a similar manner, the numerical approach would produce results similar to that of the analytical approach. Based on this premise, the cumulated recharge flux was applied to Equation 5 and MODFLOW-2005 to estimate the groundwater levels over time. In a typical groundwater flow simulation lacking a temporal transfer process (e.g., Equation 4), the changes in the groundwater level caused by precipitation are immediately reflected and groundwater fluctuations with a time delay cannot be represented as in the monitoring locations studied. Although the unsaturated zone flow package (Niswonger et al., 2006) is available for MODFLOW representing delayed recharge, the package was not applied because the detailed model parameters for the constitutive relationships of $p_\kappa$ and $p_\phi$ were not available for the study areas. However, it should be noted that experienced practitioners can calibrate the model parameters for existing constitutive models, and groundwater flow modeling can be performed based on these parameters.

In the MODFLOW-2005 simulation, the unconfined aquifer spans vertically from $H_{\text{min}} = 100$ m to $H_{\text{max}}$ in Table 1, and the area of the model domain was discretized into 40 x 40 grids with horizontal dimensions of 4,000 m x 4,000 m. The assumed observation locations (1950, 1950, and 86.033 m) correspond to the center grid (20, 20, and 1) of the model domain to avoid boundary effects.

To calibrate the MODFLOW-2005 model parameters (i.e., $T$ and $S$), the Nelder-Mead simplex method was applied on the basis of the observed head and estimated recharge rate from Equation 4. Additionally, in Table 2, the NSEC values between the observed and estimated heads based on each method are summarized. Although the actual aquifer hydraulic parameters directly obtained from the monitoring wells were not available, the $T$ and $S$ values measured at the pumping wells located in the immediate vicinity of GR2 were reported to be 43.8 m$^2$/day and 0.02, respectively. This observation indicates that the values determined in this study (the calibrated $T$ and $S$ of 39.2 m$^2$/day and 0.02, respectively) closely reflect the actual values. This indicates that the developed model can be used to obtain aquifer hydraulic properties using groundwater level time series. However, more cases need to be considered to confirm the ability to estimate the hydraulic property through the combination of the developed model and analytical or numerical model. Because groundwater level fluctuation is strongly influenced by nearby boundary conditions and local hydraulic properties, more reliable values can be obtained from the calibration by applying the actual conditions of the aquifer. Additionally, it may be impossible to determine $T$ and $S$ through the calibration of the developed model if groundwater level data for an appropriate duration (more than a year) is absent. Once the storativity value is determined with a low uncertainty, the recharge-to-precipitation ratio ($\rho$) can also be obtained by $\rho = S_p/\phi$. Assuming that 0.02 is a reliable storativity for aquifers, including the GR2 monitoring well, the recharge-to-precipitation ratio is determined to be approximately 0.27 (Table 1).

The hydrograph, which is obtained by applying the precipitation data to MODFLOW-2005 model (yellow lines), tends to precede the actual peak level (Figure 4). This tendency is most evident in Figure 4a, and the predicted peak groundwater level was several weeks ahead of the actual peak level. In most estimations, NSEC values were closer to 1 with the application of Equation 4, but in the case of YS2, which has the least unsaturated zone thickness, the estimations showed the same quality regardless of whether Equation 4 was applied. As evident in Figure 4 and Table 2, the estimation results of the analytical and numerical approaches generally correspond well with each other and show good agreement with the observed groundwater levels. However, estimates from the numerical approach are slightly different from those of the analytical approach for some time intervals. These differences are attributed to the fact that the assumed simulation domains in the numerical approach do not consider the heterogeneity of the aquifer and the relative distance between the monitoring wells and nearby hydraulic boundary conditions, whereas this information was implied in $\kappa$ of the analytical model (Park et al., 2011) in the case of the analytical approach. Given the good agreement of the approaches and their ability to estimate the actual data, the delayed recharge flux
calculated using Equation 4 is considered suitable as an input for the practical estimation of problems using analytical and numerical methods. The application of Equation 4 is considered particularly useful when estimating the groundwater level with thick unsaturated zones.

The model developed in this study has some limitations: the seasonal variability has been ignored. That is, the calibrated model parameters (i.e., $\rho/\phi$, $\alpha$, and $k$) are averaged not only spatially but also temporally. Consequently, seasonal or annual changes in climate may cause deviations from the average predicted behavior. Additionally, only precipitation was considered in this study as the source of recharge. However, evapotranspiration may contribute negatively to recharge, and the use of monitoring data, including artificial interference, may lead to inaccuracies in the model calibration. Finally, as this study used a statistical transfer function to represent temporal changes in recharge flux, the meaning of the calibrated model parameters is implicit and does not reflect detailed flows in unsaturated zones.

4. Summary and Conclusion

In this study, a function representing the recharge flux through unsaturated zones of varying thickness suitable for modeling the delayed response of groundwater level was developed. For efficiency, the function in the Laplace domain was directly evaluated using a numerical method, enabling the determination of a real shape parameter ($k$) and the inverse of the scale parameter ($\alpha$) of the gamma distribution function used in the calculations. To demonstrate the ability to express various types of delayed recharge flux in the developed function, multiple hypothetical cases were generated and the calibrated results obtained using actual data were analyzed. The estimation results of the analytical and numerical approaches corresponded with each other and showed good agreement with the observed groundwater levels. In the numerical approach, it was confirmed that hydraulic parameters that reflect the actual characteristics of the aquifer can be derived. Therefore, the function developed in this study for representing the delayed recharge flux through unsaturated zones exhibits application potential for wide-ranging practical groundwater problems with a multidimensional analytical or numerical solution. For example, based on the developed function, a recharge module can be developed in conventional groundwater flow simulators to cover the groundwater flow and contaminant transport modeling in large areas.

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All software programs were written in MATLAB. All the executable software and data used in this study are available through a public data repository (Park, E. (2020). A MATLAB code and associated data for calculating delayed recharge flux through unsaturated zones, HydroShare, http://www.hydroshare.org/resource/84a88cfc64ab4f8fafebd914a5101121).

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