Precision measurement of cosmic magnification from 21 cm emitting galaxies

Pengjie Zhang\textsuperscript{1*}, Ue-Li Pen\textsuperscript{2†}

\textsuperscript{1}NASA/Fermilab Astrophysics Group, Fermi National Accelerator Laboratory, Box 500, Batavia, IL 60510-050
\textsuperscript{2}Canadian Institute for Theoretical Astrophysics, University of Toronto, Toronto, Canada, M5S 3H8

\textsuperscript{*} E-mail: zhangpj@fnal.gov
\textsuperscript{†} E-mail: pen@cita.utoronto.ca

\begin{abstract}
We show how precision lensing measurements can be obtained through the lensing magnification effect in high redshift 21cm emission from galaxies. Normally, cosmic magnification measurements have been seriously complicated by galaxy clustering. With precise redshifts obtained from 21cm emission line wavelength, one can correlate galaxies at different source planes, or exclude close pairs to eliminate such contaminations.

We provide forecasts for future surveys, specifically the SKA and CLAR. SKA can achieve percent precision on the dark matter power spectrum and the galaxy dark matter cross correlation power spectrum, while CLAR can measure an accurate cross correlation power spectrum. The neutral hydrogen fraction was most likely significantly higher at high redshifts, which improves the number of observed galaxies significantly, such that also CLAR can measure the dark matter lensing power spectrum. SKA can also allow precise measurement of lensing bispectrum.

\textbf{Key words:} Cosmology: large-scale structure of Universe--radio lines: galaxies--galaxies: abundance
\end{abstract}

1 INTRODUCTION

Gravitational lensing measures the distortion of light by gravity originating from inhomogeneous distribution of matter. The physics of weak gravitational lensing is clean, perhaps comparable to the primary CMB. To the first order approximation, it involves only general relativity and collision-less dark matter dynamics. Gas physics only enters at small scales (Zhan & Knox 2004; White 2004) and can be nullled by throwing away small scale lensing information (Huterer & White 2005). Though the prediction of weak lensing statistics is complicated by nonlinear evolution of matter density fluctuation, high resolution, large box size N-body simulations are able to measure these statistics to high accuracy (e.g. Vale & White 2003; Merz et al. 2005). Thus, weak gravitational lensing is one of the most powerful and robust tools to constrain cosmology and study the large scale structure of the universe.

Weak gravitational lensing has been detected through cosmic shear (see Refregier 2003 for a recent review). Ongoing and upcoming large scale galaxy surveys such as CFHTLS\textsuperscript{1}, DES\textsuperscript{2}, LSST\textsuperscript{3}, Pan-STARRS\textsuperscript{4} and SNAP\textsuperscript{5} will reduce the statistical errors of the cosmic shear measurement to $\lesssim 1\%$ level. At that stage, systematic errors such as point spread function, galaxy intrinsic alignment, seeing and extinction (Hoekstra 2004; Jarvis et al. 2004; Vale et al. 2004; van Waerbeke et al. 2004) and errors in galaxy redshift measurement will be ultimate limiting factors. Given these possible systematics, it is worthwhile to search for independent method to measure the gravitational lensing, for consistency check and, for likely better statistical and systematic errors.

In this paper, we will show that the cosmic magnification of 21cm emitting galaxies is a competitive candidate.

Cosmic magnification is the lensing induced changes in galaxy number density. It introduces extra correlations in galaxy clustering (Kaiser 1992; Villumsen 1993; Moessner et al. 1993; Jain et al. 2003) and correlates galaxies (quasars) at widely separate redshifts (Moessner & Jain 1998). Cosmic magnification contains as much information of cosmology and matter clustering as cosmic shear.

\begin{itemize}
\item http://www.cfht.hawaii.edu/Science/CFHLS/
\item http://www.darkenergysurvey.org/
\item http://www.lsst.org/
\item http://pan-starrs.ifa.hawaii.edu/public/science/
\item http://snap.lbl.gov/
\end{itemize}
It has been robustly detected in quasar-galaxy lensing (Scranton et al. 2005) and references therein. The measurement of cosmic magnification does not require the accurate determination of galaxy shapes and is thus free of many systematics, such as point spread function and galaxy intrinsic alignment, entangled in the cosmic shear measurement. But it suffers from several obstacles. (1) It suffers stronger shot noise than cosmic shear measurement. For cosmic shear, shot noise comes from the intrinsic ellipticity fluctuations and is generally much smaller than intrinsic clustering of galaxies. But the results shown in this paper should not be changed significantly by more realistic choice of initial power index $n = 1$, as is consistent with WMAP result (Spergel et al. 2003). We take Canadian Large Adaptive Reflector (CLAR)\(^7\) and SKA as our targets to forecast the ability of future radio 21cm surveys to measure cosmic magnification. The instrumental parameters and survey patterns of these two surveys have not been completely fixed yet. For CLAR, we adopt a system temperature $T_{\text{sys}} = 30$ K, effective collecting area $A_{\text{eff}} = 5 \times 10^4$ m\(^2\) and field of view 1 deg\(^2\). For SKA, we adopt the same value of $T_{\text{sys}}$ and field of view and adopt $A_{\text{eff}} = 6 \times 10^5$ m\(^2\).

### 2 DETECTING GALAXIES IN RADIO SURVEYS

Though the universe is highly ionized, there are still large amounts of neutral gas present in galaxies. The typical HI mass is around $\sim 10^7 M_\odot$ (Zwaan et al. 1997). HI rich galaxies appear in the radio band by emitting the 21 cm hyperfine line resulting from the transition of atomic hydrogens from spin 1 ground state to spin zero ground state. The spontaneous transition rate is $A_{21} = 2.85 \times 10^{-15} s^{-1}$. Each emission line has negligible width. But since neutral gas has rotational and thermal motions, the integrated intrinsic line width is not negligible. It turns out that the line width of 21 cm emission is mainly determined by the rotation of HI gas, which has a typical velocity $\sim 100$ km/s. The Doppler effect by thermal motions cause a velocity width $w \sim c \sqrt{k_B T / m_H} \lesssim 10 \sqrt{T / 10^4 K}$ km/s. The actual emission line width is determined by many parameters, such as the total mass of galaxies, redshift and inclination angle of the HI disk. For simplicity, we assume that the combined intrinsic line width is $w = 100$ km/s. The choice of $w$ affects the prediction of detection efficiency of HI emitting galaxies. But the results shown in this paper should not be changed significantly by more realistic choice of $w$.

Since the velocity (frequency) resolution of radio surveys can be much higher than the intrinsic line width, the observed flux could have a nontrivial dependence on the bandwidth, whose modeling requires detailed description of HI distribution in galaxies. To avoid this complexity, we adopt

---

\(^6\) http://www.skatelescope.org/
\(^7\) http://www.clar.ca/
bandwidth to be larger or equal to redshifted 21 cm line width, which changes from \( w \) at \( z = 0 \) to \( w/(1+z) \) at \( z = 0 \), or in frequency space, to \( \Delta \nu = w_{21}/c(1+z) \), where \( w_{21} = 1.4 \) G\( \text{Hz} \) is the 21 cm frequency. The total 21 cm flux of HI rich galaxies is

\[
S_{21} = \frac{g_2 A_{21} N_{\text{HI}} E_{21}(z)}{g_1 + g_2 4\pi D_L^2(z) c(1+z)} \nu_{21}^{-1} = 0.023 \text{mJy} \frac{M_{\text{HI}}}{10^{10} M_\odot} \left( \frac{c/H_0}{\chi(z)} \right)^2 100 \text{km/s} \frac{1}{w(1+z)}.
\]

Here, \( g_1 = 1 \) and \( g_2 = 3 \) are the degeneracies of atomic hydrogen spin 0 and 1 ground state. \( E_{21} = h\nu_{21} \) is the energy of each 21 cm photon. \( N_{\text{HI}} = M_{\text{HI}}/m_{\text{HI}} \) is the total number of neutral hydrogen of a galaxy with total hydrogen mass \( M_{\text{HI}} \). \( D_L(z) = \chi(z)(1+z) \) is the luminosity distance and \( \chi \) is the comoving angular distance. \( H_0 = 100 \text{h} \) km/s/Mpc is the Hubble constant at present time.

Instrumental noise scales as \( \Delta \nu^{-1/2} \), where \( \Delta \nu \) is the bandwidth. So a larger bandwidth helps to beat down instrumental noise. But if the bandwidth is larger than the 21 cm line width, the signal is diluted and scales as \( 1/\Delta \nu \). Thus, the highest signal to noise ratio is gained when the bandwidth \( \Delta \nu \) is equal to the redshifted bandwidth of integrated 21 cm emission line, \( w_{21}/c(1+z) \). The system noise per beam is

\[
S_{\text{sys}} = \frac{\sqrt{2T_{\text{sys}}}}{\eta_c A_{\text{eff}}} \frac{K_B}{\sqrt{\Delta \nu}} \approx 0.032 \text{mJy} \frac{5 \times 10^4 \text{m}^2}{A_{\text{eff}}} \left( \frac{100 \text{km/s} \text{ hour}}{w/(1+z)} \right)^{1/2}.
\]

Here, \( \eta_c \) is the correlator efficiency, which is adopted as \( \eta_c = 0.9 \). The instrumental beam area is \( A_{\text{b}} \sim \lambda^2/A_{\text{eff}} \).

For CLAR, \( A_{b} \approx 10^2 (\nu_{21}/\nu)^2 \). For SKA, \( A_{b} \sim 10^2 (\nu_{21}/\nu)^2 \), 21 cm emitting regions have typical size \( \sim 30 \text{ kpc/h} \) or \( 1'' \) at \( z \approx 1 \). Thus, the size of 21 cm emitting regions is much smaller than the beam size. The observed flux is thus the total flux of each galaxies. In this case, the calculation of the detection threshold is straightforward.

If we choose those peaks with flux above \( n S_{\text{sys}} \) \( (n\sigma \) selection threshold), the minimum HI mass selected is

\[
M_{\text{HI,min}} = n \times 1.39 \times 10^{10} M_\odot \left( \frac{\chi(z)}{c/H_0} \right)^2 (1+z)^{3/2} \sqrt{\frac{w}{100 \text{km/s}}} \frac{\text{hour}}{T_{\text{sys}} 5 \times 10^4 \text{m}^2} \frac{1}{A_{\text{eff}}}. \]

We assume that the HI mass function follows the Schechter function found at \( z = 0 \) (Z\( \text{waan et al.} \) 1997)

\[
n(M,z) dM = n_0(z) \left( M/M_*(z) \right)^{-\gamma} \exp\left( -M/M_*(z) \right) dM. \]

We fix \( \gamma = 1.2 \). Z\( \text{waan et al.} \) (1997) found that \( M_*(z = 0) = 10^{8.55} h^{-3} M_\odot \) and \( n_0(z = 0) = 0.04 h^4 \text{Mpc}^{-3} \). There is little solid measurement of \( n_0(z) \) and \( M_*(z) \) other than at local volume. But for this form of the mass function, there exists a tight relation between \( \Omega_{\text{HI}} \), the cosmological neutral hydrogen density with respect to the present day critical density, and \( n_0 M_\star \):

\[
\Omega_{\text{HI}} h = 2.1 \times 10^{-4} \frac{n_0(z)}{n_0(z = 0)} M_*(z). \]

Figure 1. The predicted abundance of 21 cm emitting galaxies above \( 4\sigma \) detection threshold of CLAR 5 year survey. The solid lines are the cumulative number distribution \( N(< z) \) and the dashed lines are the differential distribution \( dN/dz \). The total number of observed galaxies is of the order \( 10^4 \) and the number of galaxies at \( z > 1 \) is of the order \( 10^6 \). We assume no evolution in the mass function. If realistic evolution model is considered, the total number of galaxies at \( z > 1 \) can increase by a factor of 5 or more.

Observations of damped Lyman-\( \alpha \) systems and Lyman-\( \alpha \) limit systems measure \( \Omega_{\text{HI}} \) from \( z = 0 \) to \( z = 4 \). Combining Eq. 4 and these observations, one can put constraints on \( n_0 \) and \( M_\star \). These observations found that \( \Omega_{\text{HI}} \) increases by a factor of 5 toward \( z \approx 3 \) and then decreases toward higher redshift (Z\( \text{waan et al.} \), 2003; Rao & Turnshek, 2000; Storrie-Lombardi & Wolfe, 2000; P\( \text{éroux et al.} \) 2003 and data compilation in P\( \text{éroux et al.} \) 2003; N\( \text{agamine et al.} \) 2004). Either the increase of \( n_0 \) or \( M_\star \) increases the detectability of 21 cm emitting galaxies. Thus, estimations based on the assumption of no evolution should be regarded as conservative results. We will show that even in this conservative case, future radio surveys such as CLAR and SKA still allow precise measurement of galaxy-galaxy lensing (\( \Omega_0 \)) and in the case of SKA, the precise measurement of lensing power spectrum (\( \theta \)) and lensing bispectrum (\( \bar{\theta} \)). If we adopt evolution models implied by and consistent with observations, even CLAR is able to measure the lensing power spectrum (\( \bar{\theta} \)).

The number of galaxies detected depends on the selection threshold \( n \). If we choose \( 4\sigma \) detection threshold \( (n = 4) \), CLAR could detect \( \sim 10^5-10^6 \) galaxies in a 5-year survey. A deeper survey (smaller sky coverage) detects a larger fraction of high redshift galaxies. But since the survey volume is smaller, the total number of high redshift galaxies is not necessarily higher. A survey area around 100 \( \text{deg}^2 \) is optimal to detect high \( z \) galaxies. For a 160 \( \text{deg}^2 \) survey area, \( \sim 10^6 \) galaxies at \( z > 1 \) can be detected. SKA is about 10
times more sensitive than CLAR and can detect two orders of magnitude more galaxies. For a 1600 deg$^2$ survey area, $\sim 10^6$ galaxies at $z > 1$ and $\sim 10^7$ galaxies at $z > 2$ can be detected. As a reminder, these estimations are extremely conservative. The number of galaxies detected at $z > 1$ can be enhanced by a factor of 5 or more.

Some peaks above the selection threshold are caused by noise. $N_{\text{noise}}$, the number of false peaks, has strong dependence on the detection threshold. If we choose $n = 1$ (1σ detection), the number of false peaks of noise is

$$N_{\text{noise}} \approx \frac{4\pi f_{\text{sky}}}{A_{\text{pixel}}} \frac{\nu}{v} \frac{\text{Erfc}(1.0/\sqrt{2})}{2} \lesssim 10^{13} \frac{f_{\text{sky}}}{A_{0}} 1/2.$$  

For CLAR deep surveys which cover $f_{\text{sky}} \sim 1\%$ of the sky, $N_{\text{noise}} \lesssim 10^5$, which is still less than the number of detected galaxies above 1σ threshold. For SKA with several thousand square degree sky coverage, $N_{\text{noise}}$ above 1σ is $\sim 10^6$, which is slightly less than the total number of galaxies above 1σ threshold. In CMB measurements, signal-to-noise per pixel is often chosen to be 1, which maximizes return on the power spectrum measurement. This would correspond to a detection threshold for which the false-positive rate is 50%. In this sense, we can choose $n = 1$ as our selection threshold. The selection threshold problem can be dealt with in a more sophisticated way. The survey measures a three-dimensional map of the sky. Each pixel in that map will have some significance of detection. Clearly the large number of low significance pixels do collectively contain information, if they can be averaged in a meaningful way. [Zhang & Pen 2005] describe one such algorithm to extract the luminosity function deep into the noise.

With these large samples of galaxies, the galaxy clustering can be precisely measured. This allows constraining cosmology through the baryon oscillation [Abdalla & Rawlings 2005]. The redshift distortion of galaxies allows the measurement of galaxy velocity power spectrum to high accuracy. In this paper we do not attempt to utilize all information of these galaxies. Our primary goal is to demonstrate the feasibility of using future 21cm radio survey to do precision lensing measurement. We notice that the properties of lensing magnification depends strongly on the mass threshold or equivalently $n$. We will explore different $n \geq 1$ to find suitable choice for different quantities. For $n \geq 3$, since the number of false peaks is much smaller than the number of detected galaxies, one can safely neglect all errors caused by false detections. But for $n \lesssim 3$, one has to take them into account.

3 CROSS CORRELATIONS OF DIFFERENT REDSHIFT BINS

3.1 Cosmic magnification preliminary

Cosmic magnification changes the galaxy number overdensity $\delta_g$ to

$$\delta_g^L = \delta_g + 2(\alpha - 1)\kappa + O(\kappa^2).$$

Here, $\alpha \equiv -\int (> F_c) F_c/f(F_c)$, $f(> F)$ is the number of galaxies brighter than $F$, $F_c$ is the flux limit adopted and $\kappa$ is the lensing convergence. Since $\langle \kappa^2 \rangle \sim 10^{-4}$, lensing does not change the averaged $f(> F)$. Thus $\alpha$ is effectively an observable.

The usual method to eliminate the intrinsic correlation of close galaxy pairs is to cross correlate galaxies in two separate redshift bins. Galaxy peculiar velocity shifts the position of galaxies in the redshift space by $c\Delta z \lesssim 10^3$ km/s. Galaxy correlation becomes negligible at scales $r \gtrsim 100 h^{-1}$ Mpc, which corresponds to $\Delta z \sim 0.03 [H(z)/H_0] [r/100h^{-1}$ Mpc]. Choosing $\Delta z \gtrsim 0.05$, residual correlations caused by intrinsic galaxy correlation can be safely neglected. With accurate measurement of 21cm emitting galaxy redshift, this can be done straightforwardly.

The distribution of foreground galaxies traces lenses of background galaxies which cause the magnification effect. Thus there exists a correlation between the background magnification and foreground galaxies. This galaxy-galaxy correlation is the correlation generally considered in the literature. There exists another correlation. Both foreground galaxies and background galaxies are lensed by intervening matter. Thus there exists the background magnification-foreground magnification correlation. The combined correlation is

$$\langle \delta_g^L(\theta_f, z_f)\delta_g(\theta_b, z_b) \rangle = 2(\alpha - 1)(\kappa_0 \delta(\theta_f, z_f)$$

$$+ 4(\alpha - 1)(\alpha - 1)(\kappa \kappa_0).$$

8 Zhang & Pen, 2005, in preparation

9 In this expression, we neglect all high order terms throughout this paper. These terms increase the lensing signal [Ménard et al. 2003b] and thus improve the correlation measurement.
Here the subscript $f$ and $b$ denote foreground and background respectively. The first term in the right side of the equation is the magnification-galaxy correlation and the second term is the magnification-magnification correlation.

The observed galaxy surface density is

$$\Sigma_g = \int_{z_{\text{min}}}^{z_{\text{max}}} n_g(z)(1 + \delta_g^b)dz.$$  \hspace{1cm} (9)

Its correlation function is Eq. \ref{eq:corr} weighted by the differential galaxy number distribution $n_g$ and clustering signal. By Limber’s approximation, the corresponding 2D angular power spectrum of the magnification-galaxy cross correlation is given by

$$\frac{\ell^2 C_{\mu g}^{\ell}}{2\pi} = \frac{3\Omega_m H_0^2}{2c^2} N_f \frac{\pi}{l} \times \int_{z_{f,\text{min}}}^{z_{f,\text{max}}} \Delta_m^2 \left( \frac{1}{\chi_f}, z_f \right) G_b(z_f)n_g(z_f)\chi_f dz_f.$$

Here, $\Omega_m$ is the present day matter density with respect to the cosmological critical density. $N_f = \int n_g(z_f)dz_f$ is the total number of foreground galaxies. $\Delta_m^2 = b_f^2\Delta_m^2$ is the galaxy-matter cross correlation power spectrum. We assume $b_f^2 = 1$ ($b_f$ is the galaxy bias and $r_f$ is the cross correlation coefficient between galaxies and dark matter).

The matter power spectrum $\Delta_m^2$ is calculated by the BBKS transfer function \cite{BBKS} and its nonlinear evolution is calculated by the Peacock-Dodds fitting formula \cite{PD}. $G_b$, the kernel of the background magnification effect, is given by

$$G_b(z) = \frac{1+z}{N_b} \int_{z_{b,\text{min}}}^{z_{b,\text{max}}} w(\chi, \chi_b)n_g(z_b)(\alpha(z_b) - 1)dz_b.$$  \hspace{1cm} (11)

Here, $w(\chi, \chi_b)$ is the lensing geometry function. For the flat universe we adopt, the geometry function is simplified to $w(\chi, \chi_b) = \chi(1 - \chi/\chi_b)$. The strength of the magnification effect relies on both the strength of lensing, which prefers background galaxies with higher $z$, and $\alpha - 1$, which prefers deeper slope of the mass function at the mass threshold.

The power spectrum of the corresponding magnification-magnification cross correlation is:

$$\frac{\ell^2 C_{\mu g}^{\ell}}{2\pi} = \left( \frac{3\Omega_m H_0^2}{2c^2} \right)^2 \frac{\pi}{l} \times \int_0^{z_{f,\text{max}}} \Delta_m^2 \left( \frac{1}{\chi_f}, z_f \right) G_f(z_f)G_f(z_f)\chi_f dz_f.$$  \hspace{1cm} (12)

Here, $G_f$, the kernel of foreground magnification effect, is given by

$$G_f(z) = \frac{1+z}{N_f} \int_{z_{f,\text{min}}}^{z_{f,\text{max}}} w(\chi, \chi_f)n_g(z_f)(\alpha(z_f) - 1)dz_f.$$  \hspace{1cm} (13)

where $N_f$ is the total number of foreground galaxies. The amplitude of $C_{\mu g}^{\ell}$ relies on both the lensing signal of foreground and background galaxies. The higher the $z_f$ and $z_b$, the stronger the correlation signal. It also depends strongly on $\alpha_f - 1$ and $\alpha_b - 1$. If the mass threshold is larger, $\alpha - 1$ is generally larger.

Depending on the choice of foreground bins and foreground galaxy selection criteria (which determines $\alpha$ and thus the strength of magnification effect), either $C_{\mu g}^{\ell}$ or $C_{\mu g}^{\ell}$ can dominate. In \ref{sec:mg} we discuss cases where $C_{\mu g}^{\ell}$ dominates and in \ref{sec:mm} we discuss cases where $C_{\mu g}^{\ell}$ dominates.

**Figure 3.** The predicted accuracy of cosmic magnification-galaxy cross correlation power spectrum $C_{\mu g}$ measured by CLAR 5 year survey. Foreground galaxies (0.5 < $z_f$ < 1.0) are selected with a $2\sigma$ selection threshold. The optimal survey coverage is around several hundred square degrees. For such configuration, $C_{\mu g}$ can be measured to $\sim 10%$ accuracy at $l$ around several thousand (lower panel, bin size $\Delta l = 0.2l$). $C_{\mu g}$ is several percent of $C_{\mu g}$ and does not show up in the plot. In top panel, we further split background galaxies into several redshift bins. This lensing tomography allows the measure of the evolution of matter distribution. It also allows the measure of relative change of the comoving distance as a function of $z$ to better than 10% in three redshift bins at $z_b > 1$.

### 3.2 Magnification-galaxy power spectrum

For a sufficiently wide foreground galaxy redshift distribution with low median redshift, the lensing effect and the magnification ($\alpha - 1$) effect are both small. Generally, in this case, $C_{\mu g}^{\ell}$ is much smaller than $C_{\mu g}^{\ell}$. In this section, we fix the foreground galaxy distribution ($0 < z_f < 1.0$) and vary the background galaxy redshift distribution. For this choice of foreground galaxy distribution, $C_{\mu g}$ is $\sim 1\%$ of $C_{\mu g}$ (Fig. \ref{fig:magnification}). The correlation signal peaks at $l \approx 10^5$, where the fluctuation is $\sim 10\%$.

$C_{\mu g}$ is the projection of $\Delta_m^2$ along the line of sight (Eq. \ref{eq:corr}). Given a cosmology, $\Delta_m^2(k, z)$ can be extracted using the inversion methods applied to galaxy surveys and lensing surveys \cite{Dodelson, Pen}.

Cosmological information is also carried in the geometry term of lensing, in our case, $\chi$ in $C_{\mu g}^{\ell}$ and $C_{\mu g}^{\ell}$. By fixing the foreground galaxy distribution and varying the background galaxy distribution, one can isolate $\chi_b$ from $\Delta_m^2$ and measure the dependence of $\chi_b$ on $z_b$. This method allows an independent and robust constraint on cosmology \cite{Jain, Zhang}.
decreases system noise per beam and thus increases the selection mass threshold of high z galaxies. Since the mass function is steeper at higher mass, \( \alpha(z_f) \) increases and \( C^{\nu} \) increases. (2) \( f_{\text{sky}} \) affects the relative distribution of galaxies. Larger \( f_{\text{sky}} \) survey detects relatively more low z galaxies. Since \( C^{\nu} \) is proportional to \( \Delta^2 \), weighted by the distribution of foreground galaxies and matter clustering is stronger at lower \( z \), larger sky coverage tends to increase \( C^{\nu} \). But on the other hand, the lensing effect is smaller for lower z galaxies. This has the effect to decrease \( w(\chi, z_f) \) and thus decrease \( C^{\nu} \). Furthermore, the noise terms \( C^{\nu}_b \) and \( C^{\nu}_f \) increase. (3) \( f_{\text{sky}} \) affects the cosmic variance. (4) \( f_{\text{sky}} \) affects the total number of foreground and background galaxies and thus changes the shot noise. The lower panels of fig. 3 & 4 show the dependence of \( \Delta C^{\nu} / \Delta C^{\nu} \) on sky coverage. If the sky coverage is too small, the cosmic variance is large. If the sky coverage is too large, too few background galaxies can be detected. Shot noise begins to dominate at relatively large scales. The choice of selection threshold \( n (\text{m} \text{s}) \) also affects the statistical errors. Larger \( n \) increases \( \alpha \) and thus increases \( C^{\nu} \). But it also decreases the number of detected galaxies and thus increases shot noise.

Both CLAR and SKA can measure \( C^{\nu} \) precisely (fig. 3 & 4). For CLAR, the optimal sky coverage is around several hundred square degree. \( C^{\nu} \) can be measured to \( \sim 20\% \) accuracy for bin size \( \Delta l = 0.2l \). It can also measure \( C^{\nu} \) at several redshift bins and allows isolating geometry. For SKA, the optimal sky coverage is around ten thousand square degree. \( C^{\nu} \) can be measured to \( \sim \% \) accuracy. The size of background redshift bins can be as narrow as \( \Delta z \lesssim 0.1 \). The change of \( \chi_b(z) \) can be precisely measured at \( z \gtrsim 1 \).

The best result from optical galaxy surveys by far is an \( 8\sigma \) detection by SDSS, using an optimal quasar weighting function \( \text{Scranton et al. 2005} \). CLAR will detect much less galaxies and cover much smaller sky area, but even CLAR can reach \( \sim 5\sigma \) at more than 10 independent bins of width \( \Delta l = 0.2l \). If adopted similar background galaxy weighting function, the result can be further improved. So CLAR and SKA can do much better than optical galaxy surveys. This is well explained by Eq. 14 (1) The number of \( z > 1 \) CLAR galaxies is of the order \( 10^6 \) and is much higher than the number of SDSS quasars (\( \sim 2 \times 10^5 \)). This significantly reduces the shot noise term \( C^{\nu}_{\text{shot}} \) in Eq. 14 (2) CLAR foreground galaxies have broad redshift distribution, which roughly matches the lensing kernel of background galaxies and thus amplifies the cross correlation signal. For SDSS, quasars lie at \( z \gtrsim 1 \) while most galaxies locate at \( z \ll 0.5 \). Since these quasars are mainly lensed by matter at \( z \gtrsim 0.5 \), the cross correlation signal is weak and \( C^{\nu} \ll C^{\nu} g f_{\text{sky}} \). This is likely the dominant reason why the measured cross correlations by 2dF and SDSS \( \text{Myers et al. 2003} \), \( \text{Scranton et al. 2005} \) do not have as good S/N as one would naively expect from their large galaxy and quasar samples.

It is difficult for optical surveys to detect large number of high z galaxies. But 21cm radio surveys can. This is an inherent advantage of 21cm radio surveys to measure cosmic magnification. Furthermore, since 21cm galaxies have precise redshift measurement, one can weight foreground galaxies deliberatively to optimize the cross correlation mea-

---

**Figure 4.** Similar as Fig. 3 but for SKA 5 year survey. Foreground galaxies are chosen to be at \( 0.7 < z_f < 1.0 \). To measure \( C^{\nu} \), optimal sky coverage should be around ten thousand degrees. Under such configuration, \( C^{\nu} \) can be measured to several percent accuracy at \( 1000 \lesssim l \lesssim 10^4 \). The dashed line in the lower panel is \( C^{\nu}/C^{\nu} g \) for 6400 deg$^2$ sky coverage. The dependence of comoving distance \( \chi(z) \) on redshift can be measured to be better than 1% in 7 redshift bins at \( z > 1 \).

The statistical error in the \( C^{\nu} \) measurement$^{10}$ is

$$\Delta C^{\nu} = \sqrt{\frac{C^{\nu}_g + (C^{\nu}_f + C^{\nu}_{\text{shot}})(C^{\nu}_f + C^{\nu}_{\text{shot}})}{(2l + 1)\Delta f_{\text{sky}}}}.$$  

Here, \( C^{\nu}_g \) is the auto correlation power spectrum of the background galaxies, which includes contributions from \( \Delta^2 \delta^2 \), \( \langle \delta \delta \rangle \), and \( \sigma^2 \). For high redshift background galaxies, \( \langle \delta \delta \rangle \) is negligible comparing to the intrinsic galaxy correlation \( \langle \delta \delta \rangle \) and thus has to be taken into account. \( C^{\nu}_{\text{shot}} = 4\pi f_{\text{sky}}[1 + N_{\text{noise}}/N_b] \) is the background shot noise power spectrum. The extra factor 1 + \( N_{\text{noise}}/N_b \) accounts for the contaminations of false peaks in the selected background sample. These false peaks do not correlate with each other, so their only effect is to increase the shot noise. For selection thresholds above 2-\( \sigma \), false positive rate is small and this extra factor can be neglected. \( C^{\nu}_f \) is the auto correlation power spectrum of the foreground galaxies.

Statistical errors strongly depend on \( f_{\text{sky}} \) (when fixing total observation time). This dependence is complicated. (1) \( f_{\text{sky}} \) affects \( \alpha(z_f) \). Shorter integration time per unit area is required in order to survey for a larger sky area. This in-

---

$^{10}$ At frequencies below 1.4 GHz, radio sources have smooth continuum spectra, so they can be subtracted away in the frequency space \( \text{Wang et al. 2005} \). Their only effect is to contribute to \( T_{\text{sys}} \) and cause a fluctuation in \( T_{\text{sys}} \) across the sky. This can introduce a false correlation, but it is routine in observation to eliminate this effect and thus we do not consider it in this paper.
We treat prediction of these quantities only relies on the matter clustering whose theoretical understanding is robust. The measurement of $C_{l}^{\mu \mu}$ allows constraining cosmology and studying matter clustering without the complexity of galaxy bias. We choose foreground galaxies at $1.6 \leq z_f \leq 2.0$ and background galaxies at $2.0 \leq z_b \leq 4.0$. We treat $C_{l}^{\mu g}$ as contaminations. Since $C_{l}^{\mu \mu}$ depends on $\alpha - 1$ of foreground galaxies while $C_{l}^{\mu g}$ does not, we vary the selection threshold of foreground galaxies to change $C_{l}^{\mu \mu}/C_{l}^{\mu g}$. We fix the selection threshold of background galaxies as 4$\sigma$. For 6$\sigma$ and 10$\sigma$ selection threshold of foreground, galaxies, systematic errors ($C_{l}^{\mu g}$) is comparable to statistical errors (error-bars of data points).

3.3 Magnification-Magnification power spectrum

The main strength of the cosmic shear power spectrum and bispectrum to constrain cosmology lies in the fact that the prediction of these quantities only relies on the matter clustering whose theoretical understanding is robust. The prediction of $C_{l}^{\mu \mu}$ is as straightforward as the prediction of the cosmic shear power spectrum, which does not require the knowledge of complicated galaxy bias, as $C_{l}^{\mu g}$ does. So, the measurement of $C_{l}^{\mu \mu}$ would allow robust constraints on cosmological parameters and dark matter clustering, as cosmic shear power spectrum does. In this section, we will show that SKA is straightforward to measure the magnification (foreground)-magnification (background) cross correlation power spectrum $C_{l}^{\mu \mu}$.

For the purpose of constraining cosmology using $C_{l}^{\mu \mu}$, $C_{l}^{\mu \mu}$ should be treated as contamination and marginalized over. $C_{l}^{\mu \mu}$ depends on the magnification strength of foreground galaxies, while $C_{l}^{\mu g}$ does not. By increasing the redshifts of foreground galaxies, the lensing signal increases and $\alpha - 1$ also increases, due to higher mass selection threshold at higher redshift, thus $C_{l}^{\mu \mu}$ increases with respect to $C_{l}^{\mu g}$. Since $C_{l}^{\mu g}$ is proportional to the strength of matter clustering, it decreases when increasing redshifts of foreground galaxies. But these requirements to increase $C_{l}^{\mu \mu}$ with respect to $C_{l}^{\mu g}$ can be at odds with the requirement to reduce statistical errors. For example, increasing the mass selection threshold or redshifts of foreground galaxies reduces $N_f$ and thus increases shot noise.

We try different foreground redshift bins, selection threshold and sky coverage to optimize the measurement of $C_{l}^{\mu \mu}$ such that $C_{l}^{\mu g}/C_{l}^{\mu \mu}$ is smaller or comparable to the statistical error. For SKA, it is indeed possible to measure $C_{l}^{\mu \mu}$ and control both systematic errors ($C_{l}^{\mu g}$) and statistical errors to 10% level (Fig. 4). Since such measurement requires large number of high-z galaxies, it is extremely difficult for optical surveys to realize it.

4 AUTO CORRELATION

With the precision measurement of galaxy redshift and large amount of high-z galaxies, one can extract the cosmic magnification from the galaxy auto correlation measurement. In the correlation estimator, we throw away pairs with redshift separation $|z_1 - z_2| \leq \Delta z_c$. We choose $\Delta z_c = 0.1$, which corresponds to comoving separation $r_c \approx 180 h^{-1}$ Mpc at $z = 1$. We disregard close pairs within redshift separation $\Delta z < 0.1$ and thus eliminate intrinsic galaxy clustering. We try different galaxy distribution. Lines with error bars, from bottom to top, correspond to $z > 1.5$, $z > 2.0$, and $z > 2.5$, respectively. For our choice of 4$\sigma$ selection threshold and $z > 1.5$, $C_{l}^{\mu \mu}$ dominates over $C_{l}^{\mu g}$. For higher redshift, the luminosity function is steeper at the limiting flux. Larger $\alpha - 1$ then increases $C_{l}^{\mu \mu}$ with respect to $C_{l}^{\mu g}$. On the other hand, high redshift galaxies are mainly lensed by low-z matter distribution. The higher the redshift, the less likely that galaxies can be lensed by matter distribution in the same redshift bins. This also increase $C_{l}^{\mu \mu}$ with respect to $C_{l}^{\mu g}$. We assume no evolution in the HI mass function. Realistic evolution scenarios would result in more galaxies and thus allow better measurement.
and $r_c \simeq 100h^{-1}$Mpc at $z = 2$. In Fourier space, this corresponds to cut off the power at $k_c \lesssim 1/r_c \lesssim 0.01h$/Mpc. Applying the Limber’s equation in Fourier space, the angular fluctuation at multipole $l$ is contributed by the spatial fluctuation at $k = l/\chi$. Then an effective cut off $k_e$ in Fourier space corresponds to an effective cutoff at $l_e = k_e \chi \lesssim 20$. So, under the Limber’s approximation, one can neglect the residual intrinsic correlation of galaxies at $l \gtrsim 20$.

One can further quantify the residual intrinsic clustering. The angular correlation it produces is

$$w_{IC}(\theta) = 2 \int_{r_{\text{min}}}^{r_{\text{max}}} n_s^2(z) \frac{dz}{dz} \times \int_{r_e}^{r_c} \xi_\delta(\sqrt{\chi^2 \theta^2 + (\Delta \chi)^2}, z) d\Delta \chi .$$

Here, $\xi_\delta$ is the galaxy correlation function. Numerical calculation shows that $|w_{IC}(\theta)|$ is smaller than $10^{-5}$ at all scales. For example, $|w_{IC}(\sim 1)|$ is less than $\sim 10^{-5}$ and $|w_{IC}(\sim 1^\circ)|$ is less than several $\times 10^{-7}$. One can further convert $w_{IC}(\theta)$ to the corresponding $C_l$. We find that $|l^2C_l/(2\pi)| \lesssim 10^{-5}$ at all scales. Specifically, at $l \sim 100$, $|l^2C_l/(2\pi)| \lesssim$ several $\times 10^{-6}$ and at $l \sim 1000$, $|l^2C_l/(2\pi)| \lesssim$ several $\times 10^{-6}$. The angular fluctuation caused by residual galaxy intrinsic clustering is roughly $1\%$ of $C^{\mu\nu}$ at $l \sim 100$ and much less than $1\%$ at smaller scales. So, the close pair removal procedure effectively eliminates all intrinsic galaxy clustering.

The auto correlation function is composed of two parts, the one arising from the auto correlation of cosmic magnification and the one from the cross correlation between cosmic magnification and $\delta_g$. The magnification auto correlation power spectrum is

$$\frac{l^2C^{\mu\mu}_l}{2\pi} = \left( \frac{3 \Omega_m H_0^2}{2\pi} \right)^2 \frac{\pi}{l} \int_{0}^{\Delta_m l/\chi(z)} \Delta_m^2 \frac{l}{\chi(z)} G(z) f_l^2(z) \chi d\chi .$$

where $N$ is the total number of galaxies in the corresponding redshift bin. This expression differs from Eq. 12 only by a factor $f_l^2(z)$, which arises from the close pair removal. $f_l(z)$ is given by

$$f_l(z) = \frac{(1 + z)}{NG(z)^2} \int w(\chi, \chi_1)(\alpha_1 - 1) w(\chi, \chi_2)(\alpha_2 - 1) \times n_g(z) n_g(z_\delta) \Theta(|z_\delta - z| - \Delta z_c) d z_\delta .$$

The function $\Theta(x) = 0$ if $x < 0$ and $\Theta(x) = 1$ if $x > 0$. $G(z)$ is defined analogous to $G_\delta$ and $G_f$.

The power spectrum of the magnification-galaxy cross correlation function is

$$\frac{l^2C^{\mu g}_l}{2\pi} = \frac{3 \Omega_m H_0^2}{2\pi} N^{-1} \frac{\pi}{l} \int_{r_{\text{min}}}^{r_{\text{max}}} \Delta_m^2 \frac{l}{\chi(z)} G(z) f_l(z) n_g(z) \chi d\chi .$$

The effect of close pair removal is carried by $f_l(z)$

$$f_l(z) = \frac{1 + z}{NG(z)} \int_{r_{\text{min}}}^{r_{\text{max}}} w(\chi, \chi_\delta) \times n_g(z_\delta)(\alpha(z_\delta) - 1) \Theta(|z_\delta - z| - \Delta z_c) d z_\delta .$$

Again, as explained in [11], for the purpose of constraining cosmology, we treat $C^{\mu g}$ as contaminations of $C^{\mu\mu}$. For the SKA, both the statistical errors of $C^{\mu\mu}$ and systematics errors ($C^{\mu g}$) can be controlled to better than $10\%$ level, if we only use $z \gtrsim 1$ galaxies (Fig. 5). For CLAR, the detectability of $C^{\mu\mu}$ is sensitive to the evolution of HI mass function. Assuming the conservative no evolution model, $C^{\mu\mu}$ can be detected at the several $\sigma$ level (fig. 5). Though this close pair removal method is successful for 21cm emitting galaxies, it is essentially infeasible for optical galaxies because of (1) insufficient number of high $z$ optical galaxies and (2) inaccurate photo-z redshifts and too expensive spectroscopic redshifts.

## 5 Cosmic Magnification Bispectrum

Lensing bispectrum contains valuable and often complimentary information on cosmology and the large scale structure, comparing to the 2-point correlation power spectrum [Barnes et al. 1997; Hu 1999; Barnes et al. 2002; Ménard et al. 2003; Pen et al. 2003a; Takada & Jain 2004]. But current data only allows low significance detection of the skewness at several angular scales [Barnes et al. 2002; Pen et al. 2003a; Jarvis et al. 2004]. 21cm radio surveys can do much better. In this section, we will show its feasibility by focusing on the bispectrum of galaxies in the same redshift bins. We throw away close pairs with $|z_1 - z_2| < 0.1$ where $z_i (i = 1, 2, 3)$ is the redshift of each galaxies.

The bispectrum comes from four parts, $(\mu\mu\mu)$, $(\mu g g)$, $(\mu g \mu)$ and $(g g g)$. $(\mu g \mu)$ and $(g g g)$ terms are negligible following similar argument in [11]. The $(\mu\mu\mu)$ term contributes a bispectrum

$$B_{\mu\mu\mu}(l_1, l_2, l_3) = \left( \frac{3 \Omega_m H_0^2}{2\pi} \right)^3 \int_{0}^{\Delta_m l/\chi} G^2(z) f_l^3(z) \chi^{-4} \times B_\delta(\frac{l_1}{\chi}, \frac{l_2}{\chi}, \frac{l_3}{\chi}) d\chi .$$

Here, $f_l(z)$ takes the effect of close pair removal into account. $B_\delta(k, k_1, k_2, k_3)$ is the matter density bispectrum, which is calculated adopting the fitting formula of [Scoccimarro & Couchman 2001].

The $(\mu g g)$ term contributes another bispectrum

$$B_{\mu g g}(l_1, l_2, l_3) = 3 \left( \frac{3 \Omega_m H_0^2}{2\pi} \right)^2 \int_{r_{\text{min}}}^{r_{\text{max}}} G^2(z) f_l^2(z) \chi^{-4} \times b_g B_\delta(\frac{l_1}{\chi}, \frac{l_2}{\chi}, \frac{l_3}{\chi}) n_g(z) d z .$$

Here, $f_l(z)$ takes the effect of close pair removal into account. $b_g$ is the galaxy bias in the above equation, though we adopt $b_g = 1$ in the estimation. The factor 3 comes from the permutation of $(\mu, \mu, \mu_0)$.

The shot noise of the bispectrum is

$$B_{\text{shot}}(l_1, l_2, l_3) = C_{\text{N}}^l + C_N(C_1 + C_2 + C_3) .$$

Here, $C_N = 4\pi f_{\text{sky}}/N$ is the shot noise power spectrum. Full evaluation of the sample variance of bispectrum involves integrating $6$-point nonlinear density correlation function. Since we do not have robust theoretical predictions or simulation results of $6$-pt nonlinear correlation function, we only consider the Gaussian sample variance. The statistical error of corresponding bispectrum $B_{\text{123}}$ is

$$\Delta B_{\text{123}} = \sqrt{\frac{2}{N_{\text{123}}}} (C_1 C_2 C_3 + B_{\text{shot}}^2) .$$
Figure 7. Forecasted measurement of bispectrum by SKA. We assume no evolution in the HI mass function. The solid lines are $B_{\mu\mu\mu}$ of equilateral configuration ($l_1 = l_2 = l_3$), the dot lines are $B_{\mu\mu\bar{g}}$ and the dash lines are the statistical error. For statistical error, we adopt the bin size $\Delta l = 0.2l$ and the angular bin size $10^\circ$. We try different galaxy redshift ranges and selection thresholds. The top left, top right, bottom left and bottom right panels are the results of (3σ, $z > 5$), (5σ, $z > 1.5$), (3σ, $z > 1.0$) and (5σ, $z > 1.6$), respectively.

Here, $C_\ell = C^{\mu\mu}(l_\ell) + C^{\mu g}(l_\ell)$. $N_{123}$ is the number of independent combination of $l_1, l_2, l_3$ used to obtain the averaged $B_{123}$. For a rectangle survey area with $x$ axis size $\theta_x$ and $y$ axis size $\theta_y$, independent modes are $l = (2\pi m/\theta_x, 2\pi n/\theta_y)$, where $m, n = 0, \pm 1, \pm 2, \ldots$. There is a constraint that $l_3 = -l_1 - l_2$, then the total number of independent combination is

$$N_{123} = \frac{d m_1 d n_1 d m_2 d n_2}{\Delta l_1 \Delta l_2 \Delta \theta_1 \Delta \theta_2}$$

where $\theta_1$ is the angle between $l_1$ and $l_2$. We only show the result of $l_1 \simeq l_2 \simeq l_3$, for which, we choose $\Delta l = 0.2l$ and $\Delta \theta = \pi/18 (10^\circ)$.

At large scales ($l \lesssim 1000$), cosmic variance prohibits the measurement of $B_{\mu\mu\mu}$ and $B_{\mu\mu\bar{g}}$. But at scales $l \sim 10^5$, either $B_{\mu\mu\mu}$ or $B_{\mu\mu\bar{g}}$ or both can be measured to be better than 10% accuracy by SKA (Fig. 7). As expected, higher $z$ and/or higher selection threshold result in higher lensing signal and amplify $B_{\mu\mu\mu}$ with respect to $B_{\mu\mu\bar{g}}$. The computation of three point function may appear computationally challenging, requiring the enumeration of $N^3$ triangles with $N \sim 10^8$. Recently, linear algorithms have been devised which resolve these problems (Zhang & Pei 2004). Again the requirements of precise redshift measurement and high $z$ galaxies prohibit the measurement of lensing bispectrum through cosmic magnification of optical galaxies.

6 EVOLUTION EFFECT

We have demonstrated the feasibility of measuring lensing power spectra and bispectrum in cross correlation of galaxies in two redshift bins and in auto correlation of galaxies in the same redshift bin. We caution on that these results (Fig. 7) should be regarded as conservative estimation of the power of radio surveys to measure cosmic magnification. There are several reasons. One is that in this paper we only use galaxies in certain redshift ranges and above certain selection threshold. Better measurement of $C^{\mu\mu}$ and $C^{\mu g}$ can be obtained by cross correlating galaxies above $1\sigma$ detection threshold at all redshifts. To estimate how much one gain requires the design of careful weighting on different redshifts and luminosity (HI mass). This work is beyond the scope of this paper. Another reason is that we have assumed no evolution in the HI mass function. Evolution effect is very likely to improve the accuracy of lensing measurement by providing many more detected galaxies. We will investigate the evolution effect in this section.

As discussed in (2) the observed $\Omega_{\text{HI}}h$ shows a factor of 5 increase from $z = 0$ to $z = 3$. Its evolution can be approximated as $g(z) = (1 + z)^{2.9} \exp(-z/1.3)$. Thus we have a constraint of $n_{\text{HI}}(z) = n_{\text{HI}}(z = 0)g(z)$. There is little solid constraint on the evolution of $n_{\text{HI}}$ or $M_\star$ separately. But the observation of damped Lyman-$\alpha$ systems and Lyman-Limit systems provides some indirect constraints. Damped Lyman-$\alpha$ systems have HI column density $N_{\text{HI}} \geq 2 \times 10^{20}$ cm$^{-2}$. If the size of the corresponding HI regions is $\sim 30$ kpc/h, then the total HI mass is $\sim 10^{10} M_\odot$. 

Figure 8. $C^{\mu\mu}$ and $C^{\mu g}$ that would be measured in the same redshift bin by CLAR. We try different evolution models, as explained in (2). Top left panel assumes no evolution and we choose galaxies selected above $2\sigma$ at $z > 1.5$. Other panels assume evolution models explained in the text. Top right panel uses galaxies above $3\sigma$ at $z > 1.5$. Bottom right panel uses galaxies above $2\sigma$ at $z > 2.0$. Bottom left panel uses galaxies above $1\sigma$ at $z > 2.5$. 


Thus these damped Lyman-α systems are likely part of corresponding massive HI (proto-)galaxies. On the other hand, Lyman-Limit systems have much smaller HI column density and are likely part of less massive HI galaxies. The ratio of damped Lyman-α systems abundance with respect to Lyman-limit systems decreases after $z = 3$. This implies that there may be fewer massive HI galaxies after $z \sim 3$ and thus an evolution of $M_\star(z)$.

Since the constraint to either $n_0(z)$ or $M_\star$ is weak and it is likely that both $n_0(z)$ and $M_\star(z)$ evolve, we explore three evolution scenarios. (A) No evolution in $M_\star(z)$, $n_0(z) = n_0(z = 0)g(z)$. (B) No evolution in $n_0(z)$, $M_\star(z) = M_\star(z = 0)g(z)$. (C), $n_0(z)/n_0(z = 0) = M_\star(z)/M_\star(z = 0) = g(z)^{1/2}$.

The number of $> 1$ galaxies increases by at least a factor of 5 for these evolution scenarios. Taken the evolution effect into account, even CLAR can measure $C^{\mu\nu}$ to $\sim 10\%$ accuracy (systematic and statistical, fig. 5).

7 DISCUSSION

We further address that the results shown in this paper only utilize a small fraction of cosmic magnification information contained in 21cm emitting galaxy distribution. (1) We only tried several bins of galaxy redshift distribution and several galaxy selection threshold to demonstrate that cosmic magnification can be measured to high accuracy. To utilize the full lensing information, one needs to divide galaxies into many redshift bins and selection threshold bins. One then measures (N-point) auto correlation functions of each bins and cross correlation functions between different bins. In principle, one can develop optimal weighting scheme to combine all measurements to get the best measurement of lensing statistics and lensing-galaxy statistics. (2) We did not attempt to separate the cosmic magnification auto correlations and the cosmic magnification-galaxy correlations. These two classes of correlation have different dependence on the selection threshold of galaxies. These dependences are straightforward to predict and can be applied to separate two components. Such component separation improves the robustness of constraining cosmology and large scale structure significantly. The cosmic magnification auto correlations and the geometry of cosmic magnification-galaxy correlations (Jain et al 2003; Zhang et al 2003) are ideal to constrain cosmology and matter clustering. The amplitude and angular scale dependence cosmic magnification-galaxy correlations are ideal to constrain halo occupation distribution. Advanced analysis methods are required to address the above two issues and to utilize full information of cosmic magnification information in 21cm emitting galaxy distribution. (3) We note that individual galaxies can be resolved with SKA, this allows the measurement of cosmic shear. It may also allow an independent measurement of cosmic magnification. SKA resolution allows the measurement of an inclination angle. If galaxies at high redshift also follow a Tully-Fisher relation, the lensing effect can also be large compared to shot noise and can be extracted. Since we do not know the evolution of the scatter in the Tully-Fisher at high redshift, we do not use this information in this paper. It is likely that real surveys can do significantly better than our estimates.

Utilizing all information of cosmic magnification information in 21cm emitting galaxy distribution, the relative error of cosmic magnification will be much smaller than what shown in this paper. Upon this precision era, one needs to improve the theoretical prediction to better than 1% accuracy and at the same time, understand possible 1% systematics.

Our prediction of cosmic magnification is simplified in two ways. (1) We only considered the leading order term of cosmic magnification (Eq. 1). Higher order terms are known to be capable of generating 10% effect (e.g. Ménard et al (2003a)) and have to be included to interpret data at the forecast accuracy. (2) For the lensing convergence, we neglected high order corrections caused by lens-lens coupling and deviation from Born’s approximation. These high order corrections are known to have several percent effect (Schneider et al 1998; Yale & White 2003; Dodelson & Zhang 2002) and should be taken into account.

Source-lens coupling (Bernardeau 1998; Hamann 2001) can have non-negligible effect to cosmic shear. It arises from the fact that measured cosmic shear is always weighted by the number of observed galaxies, which also trace the matter distribution. But this effect does not exist in cosmic magnification where we directly correlate the numbers of observed galaxies at two different redshifts and directions.

Several other approximations in our cosmic magnification measurement only introduce negligible corrections. (1) We have assumed that the luminosity function $f(> F)$ is the same everywhere and thus $\alpha$ is the same everywhere. This picture is over simplified. $f(> F)$ can have environmental dependence. Thus in principle $\langle \alpha \alpha \rangle \neq (\alpha)^2$. This affects the prediction of correlations where two or more cosmic magnification terms present (e.g. in $C^{\mu\nu}$, $B_{\mu\nu\nu}$ and $B_{\mu\nu\nu\nu}$). But this effect is very small, since $\alpha$ only depends on local environment, for two redshift bins with modest separation $\Delta z \gtrsim 0.05$, $\langle \alpha_1 \alpha_2 \rangle \simeq (\alpha_1/\alpha_2)$. The close pair removal procedure further guarantees that even for the same redshift bin, $\langle \alpha_1 \alpha_2 \rangle \simeq \langle \alpha_1 \rangle/\langle \alpha_2 \rangle$. Thus, one can safely neglect this effect. (2) Residual intrinsic clustering causes $\sim 1\%$ correction at $l \sim 100$ (11). Since cosmic variance at $l \sim 100$ is $\gtrsim 0.01h_{-1}^{-1/2}$. One needs to worry about this effect only for full sky surveys. Furthermore, it can be reduced by extrapolating galaxy correlation function $\xi_\alpha$ at smaller scales measured from the same survey to relevant scales ($\gtrsim 100h_{-1}^{-1}$Mpc).

8 CONCLUSIONS

We have made simple forecasts for future radio surveys to measure gravitational lensing. We found that radio surveys can be precise sources for lensing measurements, and that lensing magnification is measurable because redshifts are known and many galaxies can be detected. CLAR and SKA are expected to measure the dark matter power spectrum and galaxy-matter cross correlation to high accuracy. The estimates are conservative, since evolution in HI mass function, as suggested by observations, will improve the above results significantly. Also, many effects will increase the sensitivity. Unfortunately, these effects, which include Tully-

\footnote{But it causes $\lesssim 1\%$ correction at smaller scales.}
Fisher relations, are difficult to quantify at high redshifts, for which we neglect in this paper. More complete statistical information is available at lower signal to noise levels and using non-Gaussian statistics. We have made estimates of the three point statistics, which appear promising, and are expected to improve the information that can be gained by lensing.

Acknowledgments We thank Ron Ekers and Simon Johnston for explanation of radio experiments. We thank Scott Dodelson for helpful discussion. P.J. Zhang was supported by the DOE and the NASA grant NAG 5-10842 at Fermilab.

REFERENCES

Abdalla, F. B., & Rawlings, S. 2005, MNRAS, 360, 27
Bardeen, J. M., Bond, J. R., Kaiser, N., & Szalay, A. S. 1986, ApJ, 304, 15
Bernardeau, F., van Waerbeke, L., & Mellier, Y. 1997, Astron. & Astrophys., 322, 1
Bernardeau, F. 1998, Astron. & Astrophys., 388, 28
Bernardeau, F., Mellier, Y., & van Waerbeke, L. 2002, Astron. & Astrophys., 389, L28
Dodelson, S., et al. 2002, ApJ, 572, 140
Dodelson, S., & Zhang, P. 2005, astro-ph/0501063
Hamana, T. 2001, MNRAS, 326, 326
Hoekstra, H. 2004, MNRAS, 347, 1337
Hui, L. 1999, ApJL, 519, L9
Huterer, D., & White, M. 2005, astro-ph/0501451 submitted to PRD
Jain, B., Scranton, R., & Sheth, R. K. 2003, MNRAS, 345, 62
Jain, B., & Taylor, A. 2003, Physical Review Letters, 91, 141302
Jarvis, M., Bernstein, G., & Jain, B. 2004, MNRAS, 352, 338
Jarvis, M., Jain, B., 2004, astro-ph/0412234
Kaiser, N. 1992, ApJ, 388, 272
Maoli, R., Van Waerbeke, L., Mellier, Y., Schneider, P., Jain, B., Bernardeau, F., Erben, T., & Fort, B. 2001, Astron. & Astrophys., 368, 766
Ménard, B., Hamana, T., Bartelmann, M., & Yoshida, N. 2003, Astron. & Astrophys., 403, 817
Ménard, B., Bartelmann, M., & Mellier, Y. 2003, Astron. & Astrophys., 409, 411
Merz, H., Pen, U., & Trac, H. 2005, New Astronomy.
astro-ph/0402443
Moessner, R., Jain, B., & Villumsen, J. V. 1998, MNRAS, 294, 291
Moessner, R., & Jain, B. 1998, MNRAS, 294, L18
Myers, A. D., Outram, P. J., Shanks, T., Boyle, B. J., Croom, S. M., Loaring, N. S., Miller, L., & Smith, R. J. 2003, MNRAS, 342, 467
Nagamine, K., Springel, V., & Hernquist, L. 2004, MNRAS, 348, 421
Peacock, J. A., & Dodds, S. J. 1996, MNRAS, 280, L19
Pen, U., Zhang, T., van Waerbeke, L., Mellier, Y., Zhang, P., & Dubinski, J. 2003a, ApJ, 592, 664
Pen, U., Lu, T., van Waerbeke, L., & Mellier, Y. 2003b, MNRAS, 346, 994
Péroux, C., McMahon, R. G., Storrie-Lombardi, L. J., & Irwin, M. J. 2003, MNRAS, 346, 1103
Rao, S. M., & Turnshek, D. A. 2000, ApJS, 130, 1
Refregier, A. 2003, Annu. Rev. Astron. Astrophys., 41, 645
Schneider, P., van Waerbeke, L., Jain, B., & Kruse, G. 1998, MNRAS, 296, 873
Scoccimarro, R., & Couchman, H. M. P. 2001, MNRAS, 325, 1312
Scranton, R., et al. 2005, astro-ph/0504510 ApJ, accepted
Spergel, D. N., et al. 2003, ApJS, 148, 175
Storrie-Lombardi, L. J., & Wolfe, A. M. 2000, ApJ, 543, 552
Takada, M., & Jain, B. 2004, MNRAS, 348, 897
Vale, C., & White, M. 2003, ApJ, 592, 699
Vale, C., Hoekstra, H., van Waerbeke, L., & White, M. 2004, ApJL, 613, L1
Van Waerbeke, L., et al. 2001, Astron. & Astrophys., 374, 757
Van Waerbeke, L., Mellier, Y., Pelló, R., Pen, U.-L., McCracken, H. J., & Jain, B. 2002, Astron. & Astrophys., 393, 369
van Waerbeke, L., Mellier, Y. & Hoekstra, H. 2004, astro-ph/0406468
Villumsen, J. V. 1996, MNRAS, 281, 369
Wang, X. et al. 2005, astro-ph/0501081
White, M. 2004, Astroparticle Physics, 22, 211
Zhan, H., & Knox, L. 2004, ApJL, 616, L75
Zhang, J., Hui, L., & Stebbins, A. 2003, astro-ph/0312348 submitted to ApJ
Zhang, L. L., & Pen, U.-L. 2005, New Astronomy, 10, 569
Zhang, T. & Pen, U. 2005, astro-ph/0503064
Zhang, P. & Pen, U., 2005, astro-ph/0506740
Zwaan, M. A., Briggs, F. H., Sprayberry, D., & Sorar, E. 1997, ApJ, 490, 173