Mathematical reflective thinking strategy in problem-solving viewed by cognitive style

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Abstract. Mathematical reflective thinking is a mathematical thinking activity involving the persistent consideration process of the use of knowledge and decision making in solving a problem. This study aims to describe mathematical reflective thinking strategy of students based on impulsive and reflective cognitive styles in solving the mathematical problems. To do so, first, we developed three tasks about the linear programming problems. Next, 35 students of senior high school in the 11th grade (16-17 years-old), were given tasks and continued with the giving one set of Familiar Figures Figure Test (MFFT). Then four students were selected as representatives of the other students. In-depth interviews were conducted with students to confirm the findings of the task’s answers. Finally, the data were analyzed to explain the mathematical reflective thinking strategy of students based on cognitive styles. We found that students with impulsive cognitive styles have a tendency to think reflectively on the stage of understanding and reflection, using inefficient strategies, not completing answers, and give up easily in the face of a difficult task. Whereas the students with reflective cognitive style have a tendency to think reflectively on the stage of reflection and critical reflection, using expert strategies, concluding answers, and solving difficult problems in accordance with logic; and trial and error.

1. Introduction

In supporting 21st Century Education, the Indonesian government established four main skills that must be mastered by students in learning. The four skills are known as 4C skills, namely critical thinking skills and problem-solving, communication skills, creativity and innovation, and collaboration [1]. Critical thinking skills and problem solving is one of the basic skills that must be mastered by students in facing the development of the current industrial era 4.0. This skill is also one of the skills students need to develop in learning mathematics [2,3]. Critical thinking is reflective thinking with reasons that focuses on decisions about what is believed or done [4]. While problem-solving is an attempt to find a solution of a problem that cannot immediately be resolved through prior knowledge formulations. Based on the approach, problem solving is divided into routine problems and non-routine problems [5-7].

Critical thinking skills and problem-solving also were supported by the reflective thinking skill [4]. Reflective thinking is thinking independently according to needs by considering the knowledge that owned and various assumptions and consequences that will occur in making a decision and conclusion [8-10]. The skill of reflective thinking in problem-solving is useful in minimizing the level of repetitive errors, encouraging someone to choose the best strategy and correcting errors from previously held
beliefs [9,11]. In general, the reflective thinking skill can make someone as a generation of learners, so that he can improve himself gradually [12].

Mathematical reflective thinking in this research is a mathematical thinking process that utilizes the knowledge possessed to consider, choose and decide on the best and right way to solve mathematical problems. There are four categories of mathematical reflective thinking used, namely (1) habitual action (non-reflective), is the use of strategies in problem-solving based on habits so that only requires a little thought; (2) understanding, is the use of strategies based on a simple understanding of related concepts without connecting it to other situations; (3) reflection, is the use of strategies that involve active, persistent and careful consideration of the concepts it believes in without considering other alternatives to test the correctness of the solution, and (4) critical reflection, is the use of strategies based on full awareness and comprehensive consideration of all concepts believed by considering other alternatives to test the correctness of the solution [13-15].

Another factor that influences success in problem-solving is cognitive style. Cognitive style is a variation of someone in processing and understanding information which includes how to analyze, feel, remember and think, reason, store, and utilize information to solve a problem. Cognitive style has characteristics tending to be consistent in thinking, remembering and solving problems [16,17]. Cognitive style divides into two categories, namely impulsive and reflective cognitive. Impulsive-reflective cognitive style is one of the cognitive style categories that combines decision-making time and the accuracy of solutions [18]. The characteristic of someone who has an impulsive cognitive style in decision making is acting quickly even though the accuracy level is low. Meanwhile, the characteristics of reflective cognitive style focuses on the accuracy of the solution and requires a longer time in decision making [19,20].

In Indonesia, reflective thinking has not developed optimally for students [15]. This is the reason for researchers to investigate the strategies of reflective thinking of some students who are already good so that later it can be applied and duplicated by other students. The thinking strategy that we mean is how to consider and analyze the selection of methods, and the achievement of solutions. The process of selecting and using solutions related to reflective thinking is divided into four stages, namely are habitual action, understanding, reflection, and critical reflection. Meanwhile the problem-solving process, we categorize it into two strategies, namely the procedural strategy with long and complicated stages, and the expert strategy with more efficient stages [21]. We also include the cognitive style variables of students' in the data analysis because it is closely related to the process of thinking, reasoning and analyzing.

2. Experimental method
This descriptive study aims to describe the reflective thinking strategies of senior high school students in solving linear programming problems viewed by cognitive style. This study was conducted in one of the high schools in Indonesia for the 11th graders (15-16-years-old) after studying the topic of linear programming. To investigate this problem, we developed three tasks related to the topic of linear programming (see table 1). Then, 35 students were given a test for 45 minutes and continued with the giving one set of Matching Familiar Figures Test (MFFT) developed by Warli [20] to determine the cognitive style of students. Then four students were selected to represent the other students based on cognitive style considerations, variations in test answers and information from the teacher. In-depth interviews were conducted with students to confirm the findings of the test answers. The last data was analyzed to reveal the reflective thinking strategies that students made in solving linear programming problems.

3. Results and discussion
Taking into variations of the students' answers in solving tasks the linear programming, we discuss some of the students' answers different in each task item. Taking into variations of the students' answers in solving tasks the linear programming, we discuss some of the students' answers different in each task item. Each student's answers were reviewed using their respective cognitive style categories. We found
that there is the diversity of mathematical reflective thinking strategies that students make in solving linear programming problems. On the first task, it found that the impulsive cognitive style group used two different strategies, while the reflective cognitive style group only used the same strategy. WDI and AGY’s as a group representative of the impulsive cognitive styles are shown in figure 1.

Table 1. The tasks of solving linear programming problems.

| No. | Tasks |
|-----|-------|
| 1. | A rice trader sells mixed rice with the following conditions. The first mixture consists of 1 kg of type A, 2 kg of type B and 1 kg of type C, sold at IDR 17,500. The second mixture consists of 2 kg of type A, 3 kg of type B and 1 kg of type C, sold at IDR 26,000. While the third mixture consists of 1 kg of type A and 2 kg of type C, sold at IDR 11,500. In all the three rice mixes, determine which type of rice is the most expensive. |
| 2. | Determine the right symbol to fill the blank box in the picture. |
| 3 | A trader buys three types of fishes namely grouper, milkfish, and mackerel fish from a fisherman with the price is IDR 26,000.00, IDR 18,000.00 and IDR 70,000.00 per kg respectively. The number of milkfish must be purchased three times from mackerel fish. If the trader needs as much as 36 kg of fish and the money provided is only IDR 1,056,000.00, determine how many kg of fish that the trader must buy for each type. |

On figure 1, it can be seen that WDI uses a complicated strategy that was dominated by the elimination method. Based on the interview, we found that she solved this problem according to her understanding. First, she formulated it into a mathematical model. Next, she solved it using her knowledge of linear equation system of three variables, without considering the efficiency of the stages of completion. Another finding from us was that WDI had not answered the questions on the first task. Even though in the interview, she said that her answer was complete. Based on Kember et al., WDI is in the understanding stage which is the weakest stage in the reflective thinking process and her answer was categorized as the procedural strategy [14].

![Figure 1. The representation of students’ answers of impulsive group on the first task.](image-url)
AGY, one of the students who has an impulsive cognitive style used a different strategy with WDI. AGY is more dominant using the substitution method in completing the first task. According to him, this method is easier to understand than the other methods. He also claimed to have no difficulty in understanding, formulating, considering and choosing a completion strategy in solving this problem. Regarding the achievement of the solution, we found that his answer was complete, even though he was not sure of the answer. In this case, the reflective thinking process carried out by AGY is at the stage of reflection and his answer was categorized as the procedural strategy.

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In this task, DPA's answers are the same as WDI. Besides that, the arguments she uses also have similarities. In this case, DPA has been able to think at the critical reflection stage and the answer was categorized in the expert strategy.

On the second task, all students have been able to think reflectively. Some things are different only in the use of methods, the efficiency of completion stages, selection of variables in the formulation of mathematical models and write the conclusions. The example are the students' answers on the impulsive and reflective cognitive style shown in figure 3.

**Figure 2.** DPA’s answer in the reflective group.

In the reflective group, we represent with the DPA's answer, as shown in figure 2. On the DPA’s answer, we can see that the method is the same as WDI. But on the step, DPA has done a little trick, so it is more concise and efficient. Based on the interview, we found that she completed the task with full consideration. The results of our interview are shown in the following transcript.

| R     | How did you complete the first task? |
|-------|-------------------------------------|
| DPA   | First, I tried to understand the problem. Then I formulated it into a mathematical model and tried to consider how to solve it, and finally, I got this method. |
| R     | At the elimination stage, why did not you eliminate the variable of C? Was not the coefficient the same? |
| DPA   | Yes, at first, I thought like that. But after I considered it, it would be faster if I eliminated the variable of B. Because in the third equation, of the model that I made, there were only two variables, A and C. |
| R     | Are you sure of your final answer? |
| DPA   | I'm very sure sir, because I checked it |

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AGY, the representative of the impulsive group, still uses the same method used in the first task. According to him, this method is the best although the stages are inefficient. In addition, we also found that he had not answered the second task. He only completed the algorithm for mathematical calculations without concluding it. Meanwhile, WDI also uses a strategy similar to that used in the first task. The argument given by WDI is still the same as the previous one. Between AGY and WDI when confirmed as to whether they were sure that the answer is correct or not, they got hesitated. In this case, AGY has been able to think at the reflection stage, and the WDI is still at the understanding stage. ANA, as a reflective group representative, shows a simple thinking process in solving the second task. Based on interviews, we found that she implements the analysis process with deep consideration in every step of her completion. So, she feels confident in her answer. She also wrote conclusions on the answer. In this case, she has been able to think reflective at the stage of critical reflection. Different with ANA, DPA performs the same procedure as WDI. The process of her completion in this task is not efficient. Based on interviews, we found that this caused by the use of the variable she chooses, which is using the same model on the problem. The example of DPA's answer can be seen in Figure 4.

In the third task, we found that the impulsive group had difficulty in solving it. AGY, one of its representatives, did not complete this task. Based on the interview, we found that he had difficulties in understanding and formulating mathematical models. For WDI, we found that she had used an efficient method in completing the third task. It is just that she forgot to conclude the mathematical solution, so the problem has not been answered completely. WDI’s answer can be seen in Figure 5. Based on the interviews, we found that the analysis process carried out by the WDI in the third task was at the stage of reflection, and the strategy used was categorized as the expert. WDI does not conclude the answer because it is in a hurry to finish it.
Figure 5. WDI’s answer in the impulsive group.

In the reflective group, we found that they were completed the third task with different strategies. The examples of reflective group answers can be seen in figure 6.

On figure 6, it can be seen that ANA did the same strategy with the WDI from the impulsive group. ANA also does not conclude the results of mathematical calculations. Based on the interview, we found that ANA also forgot to conclude because it was in a hurry to complete this task. Whereas DPA, carried out a different strategy from ANA, DPA completed the third task using logic with a trial and error strategy. From the results of the interviews, we found that DPA tried several numbers to fulfill the formulas of the mathematical models. Based on this process, DPA has been able to think reflectively at the stage of reflection but her strategy is not effective and efficient to apply to other students. It is because not all are students able to do a trial and error strategy effectively.

Based on the analysis results of all students’ answers and thinking processes, it was found that impulsive group students tend not to conclude the answer. Individuals who think impulsively have a tendency to be quick in making decisions even though their accuracy is low [19]. We also found that impulsive groups tend to think reflectively in terms of understanding and reflection, with procedural strategies. Compare with the fact in reflective students, they have a tendency to think reflectively on the stage of reflection and critical reflection with the expert strategy.

Generally, the summary of the four student answers’ analysis results based on mathematical problem-solving tests and cognitive style tests shown in table 2. On table 2, we can find that there is a diversity of students’ mathematical reflective thinking strategies in solving mathematical problems on the topic of linear programming that are reviewed based on their cognitive style.
Table 2. The results of student answers analysis.

| Tasks | Impulsive | Reflective |
|-------|-----------|------------|
|       | WDI       | AGY        | DPA       | ANA       |
| 1     | have been still weak to think reflectively, and the thinking strategy in solving the task is still procedurally without interpretation or conclusion | have been able to think reflectively, using the correct solution expertly, and there is an interpretation or conclusion | have been able to think reflectively, using the correct solution expertly, and there is an interpretation or conclusion |
| 2     | have been still weak to think reflectively, and the thinking strategy in solving the task is still procedurally without interpretation or conclusion | have been able to think reflectively, using the correct solution expertly, and there is an interpretation or conclusion | have been able to think reflectively, using the correct solution expertly, and there is an interpretation or conclusion |
| 3     | have been still weak to think reflectively, and the thinking strategy in solving the task is still procedurally without interpretation or conclusion | have been able to think reflectively, using the correct solution expertly, and there is an interpretation or conclusion | have been able to think reflectively, using the correct solution expertly, and there is an interpretation or conclusion |

4. Conclusion

Based on the results and discussion, we make two following conclusions. The first one is that students in impulsive groups have characteristics: reflective thinking with understanding and reflection, using inefficient strategies, not completing answers, and give up easily in the face of a difficult task. The last one is that students in the reflective group have characteristics: reflective thinking in reflection and critical reflection, using expert strategies, concluding answers, and solving difficult problems in accordance with logic; and trial and error.

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