THE THIRD TYPE OF FERMION MIXING IN THE LEPTON AND QUARK INTERACTIONS WITH LEPTOQUARKS

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The low-energy manifestations of a minimal extension of the electroweak standard model based on the quark-lepton symmetry \( SU(4)_V \otimes SU(2)_L \otimes G_R \) of the Pati–Salam type are analyzed. Given this symmetry the third type of mixing in the interactions of the \( SU(4)_V \) leptoquarks with quarks and leptons is shown to be required. An additional arbitrariness of the mixing parameters could allow, in principle, to decrease noticeably the lower bound on the vector leptoquark mass originated from the low-energy rare processes, strongly suppressed in the standard model.

Keywords: quark-lepton symmetry; vector leptoquark; fermion mixing

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1. Introduction

While the LHC methodically examines the energy scale of the electroweak theory and above, it is time to recall the two criteria for evaluating a physical theory, mentioned by A. Einstein.\(^1\) The first point of view is obvious: a theory must not contradict empirical facts, and it is called the “external confirmation”. The test of this criterion both for the standard model and its various extensions is now engaged in the LHC. The second point of view called the “inner perfection” of the theory, may be very important to refine the search area for new physics.

All existing experimental data in particle physics are in good agreement with the standard model predictions. However, the problems exist which could not be resolved within the standard model and it is obviously not a complete or final theory. It is unquestionable that the standard model should be the low-energy limit of some higher symmetry. The question is what could be this symmetry. And the main question is, what is the mass scale of this symmetry restoration. A gloomy prospect is the restoration of this higher symmetry at once on a very high mass scale, the so-called gauge desert. A concept of a consecutive symmetry restoration is much more attractive. It looks natural in this case to suppose a correspondence of
the hierarchy of symmetries and the hierarchy of the mass scales of their restoration. Now we are on the first step of some stairway of symmetries and we try to guess what could be the next one. If we consider some well-known higher symmetries from this point of view, two questions are pertinent. First, isn’t the supersymmetry as the symmetry of bosons and fermions, higher than the symmetry within the fermion sector, namely, the quark-lepton symmetry or the symmetry within the boson sector, namely, the left-right symmetry. Second, wouldn’t the supersymmetry restoration be connected with a higher mass scale than the others? The recent searches for supersymmetry carried out at the Tevatron and the LHC colliders shown that no significant deviations from the standard model predictions have been found, the vast parameter space available for supersymmetry has been substantially reduced and the most probable scenarios predicted by electroweak precision tests are now excluded or under some constraints after the new stringent limits.

We should like to analyse a possibility when the quark-lepton symmetry is the next step beyond the standard model. Along with the “inner perfection” argument for this theory, there exists a direct evidence in favor of it. The puzzle of fermion generations is recognized as one of the most outstanding problems of present particle physics, and may be the main justification for the need to go beyond the standard model. Namely, the cancellation of triangle axial anomalies which is necessary for the standard model to be renormalized, requires that fermions be grouped into generations. This association provides an equation \( \sum_f T_{3f} Q_f^2 = 0 \), where the summation is taken over all fermions of a generation, both quarks of three colors and leptons, \( T_{3f} \) is the 3d component of the weak isospin, and \( Q_f \) is the electric charge of a fermion. Due to this equation, the divergent axial-vector part of the triangle \( Z \gamma \gamma \) diagram with a fermion loop vanishes.

The model where a combination of quarks and leptons into generations looked the most natural, proposed by J.C. Pati and A. Salam was based on the quark-lepton symmetry. The lepton number was treated in the model as the fourth color. As the minimal gauge group realizing this symmetry, one can consider the semisimple group \( SU(4)_V \otimes SU(2)_L \otimes G_R \). To begin with, one can take the group \( U(1)_R \) as \( G_R \). The fermions were combined into the fundamental representations of the \( SU(4)_V \) subgroup, the neutrinos with the up quarks and the charged leptons with the down quarks:

\[
\begin{pmatrix}
  u^1_i \\
  u^2_i \\
  u^3_i \\
  \nu_i
\end{pmatrix}, \quad \begin{pmatrix}
  d^1_i \\
  d^2_i \\
  d^3_i \\
  \ell_i
\end{pmatrix}, \quad i = 1, 2, 3 \ldots (?)
\]

(1)

where the superscripts 1,2,3 number colors and the subscript \( i \) numbers fermion generations, i.e. \( u_i \) denotes \( u, c, t, \ldots \) and \( d_i \) denotes \( d, s, b, \ldots \).

The left-handed fermions form fundamental representations of the \( SU(2)_L \) sub-
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group:
\[
\begin{pmatrix}
  u^c \\
  d^c \\
  c \\
  \nu \\
  \ell
\end{pmatrix}_L,
\begin{pmatrix}
  \nu \\
  \ell
\end{pmatrix}_L.
\] (2)

One should keep in mind that when considering the mass eigenstates, it is necessary to take into account the mixing of fermion states (1), (2), to be analysed below.

Let us remind that such an extension of the standard model has a number of attractive features.

(1) As it was mentioned above, definite quark-lepton symmetry is necessary in order that the standard model be renormalized: cancellation of triangle anomalies requires that fermions be grouped into generations.

(2) There is no proton decay because the lepton charge treated as the fourth color is strictly conserved.

(3) Rigid assignment of quarks and leptons to representations (1) leads to a natural explanation for a fractional quark hypercharge. Indeed, the traceless 15-th generator \( T_{15}^V \) of the \( SU(4)_V \) subgroup can be represented in the form
\[
T_{15}^V = \sqrt{\frac{3}{8}} \text{diag} \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1 \right) = \sqrt{\frac{3}{8}} Y_V .
\] (3)

It is remarkable that the values of the standard model hypercharge of the left-handed quarks and leptons combined into the \( SU(2)_L \) doublets turn out to be placed on the diagonal. Let us call it the vector hypercharge, \( Y_V \), and assume that it belongs to both the left- and right-handed fermions.

(4) Let us suppose that \( G_R = U(1)_R \). The well-known values of the standard model hypercharge of the left and right, and up and down quarks and leptons are:
\[
Y_{SM} = \begin{cases}
\left( \frac{1}{3} \right) & \text{for } q_L; \\
\left( \frac{-1}{3} \right) & \text{for } \ell_L \\
\left( \frac{4}{3} \right) & \text{for } q_R; \\
\left( \frac{-2}{3} \right) & \text{for } \ell_R
\end{cases}
\] (4)

Then, from the equation \( Y_{SM} = Y_V + Y_R \), taking Eq. (3) into account, one obtains that the values of the right hypercharge \( Y_R \) occur to be equal \( \pm 1 \) for the up and down fermions correspondingly, both quarks and leptons. It is tempting to interpret this circumstance as the indication that the right-hand hypercharge is the doubled third component of the right-hand isospin. Thus, the subgroup \( G_R \) may be \( SU(2)_R \).

“Under these circumstances one would be surprised if Nature had made no use of it”, as P. Dirac wrote on another occasion.

The most exotic object of the Pati–Salam type symmetry is the charged and colored gauge X boson named leptoquark. Its mass \( M_X \) should be the scale of
breaking of $SU(4)_V$ to $SU(3)_c$. Bounds on the vector leptoquark mass are obtained both directly and indirectly, see Ref. [10] pp. 490-494, and Ref. [11]. The recent direct search [12] for vector leptoquarks using $\tau^+ \tau^- b\bar{b}$ events in $p\bar{p}$ collisions at $E_{cm} = 1.96$ TeV have provided the lower mass limit at a level of 250–300 GeV, depending on the coupling assumed. Much more stringent indirect limits are calculated from the bounds on the leptoquark-induced four-fermion interactions, which are obtained from low-energy experiments. There is an extensive series of papers where such indirect limits on the vector leptoquark mass were estimated, see e.g. Refs. [13–23]. The most stringent bound [10] were obtained from the data on the $\pi \to e\nu$ decay and from the upper limits on the $K^0_L \to e\mu$ and $B^0 \to e\tau$ decays. However, those estimations were not comprehensive because the phenomenon of a mixing in the lepton-quark currents was not considered there. It will be shown that such a mixing inevitably occurs in the theory.

An important part of the model under consideration is its scalar sector, which also contains exotic objects such as scalar leptoquarks. We do not concern here the scalar sector, which could be much more ambiguous than the gauge one. Such an analysis can be found e.g. in Refs. [22, 23, 24].

The paper is organized as follows. In Sec. 2, it is argued that three types of fermion mixing inevitably arise at the loop level if initially fermions are taken without mixing. The effective four-fermion Lagrangian caused by the leptoquark interactions with quarks and leptons is presented in Sec. 3. In Sec. 4, the constraints on the parameters of the scheme are obtained based on the data from different low-energy processes which are strongly suppressed or forbidden in the standard model. Combined constraint on the vector leptoquark mass from the $\pi, K, \tau$ and $B$ decays is obtained in Sec. 5. In Sec. 6, combined constraint on the vector leptoquark mass from the same processes is obtained in the case of different mixings for left-handed and right-handed fermions. In Sec. 7, one more mixing independent bound on the vector leptoquark mass is presented, coming from the decay $\pi^0 \to \nu\bar{\nu}$.

2. The third type of fermion mixing

As the result of the Higgs mechanism in the Pati–Salam model, fractionally charged colored gauge $X$-bosons, vector leptoquarks appear. Leptoquarks are responsible for transitions between quarks and leptons. The scale of the breakdown of $SU(4)_V$ symmetry to $SU(3)_c$ is the leptoquark mass $M_X$. The three fermion generations are grouped into the following $\{4, 2\}$ representations of the $SU(4)_V \otimes SU(2)_L$ group:

$$\begin{pmatrix}
u_i \\ \ell_i
\end{pmatrix}$$

where $c$ is the color index to be further omitted. It is known that there exists the mixing of quarks in weak charged currents, which is described by the Cabibbo-Kobayashi-Maskawa matrix. Therefore, at least one of the states in (5), $u$ or $d$, is not diagonal in mass. It can easily be seen that, because of mixing that arises at
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Fig. 1. Feynman diagram illustrating the appearance of fermion mixings.

the loop level, none of the components is generally a mass eigenstate. As usual, we assume that all the states in (5), with the exception of $d$, are initially diagonal in mass. This leads to nondiagonal transitions $\ell \to X + d(s, b)$ through a quark-leptoquark loop, see Fig. 1. As this diagram is divergent, the corresponding counterterm should exist at the tree level. This means that the lepton states $\ell$ in (5) are not the mass eigenstates, and there is mixing in the lepton sector. Other nondiagonal transitions arise in a similar way. Hence, in order that the theory be renormalizable, it is necessary to introduce all kinds of mixing even at the tree level. As all the fermion representations are identical, they can always be regrouped in such a way that one state is diagonal in mass. The most natural way is to diagonalize charged leptons. In this case, fermion representations can be written in the form

$$
\begin{pmatrix}
u \\
u_e \\
u_{\mu} \\
u_{\tau} \\
 u_e \\
u_{\mu} \\
u_{\tau} \\
 d_e \\
u_{\mu} \\
u_{\tau} \\
d_e \\
 d_e \\
u_{\mu} \\
u_{\tau} \\
d_e
\end{pmatrix}
= 
K_{\ell} \begin{pmatrix} \nu_1 \\
 \nu_2 \\
 \nu_3 \\
 u_1 \\
 u_2 \\
 u_3 \\
 d_1 \\
 d_2 \\
 d_3
\end{pmatrix},
$$

(6)

Here, the quarks and neutrinos subscripts $\ell = e, \mu, \tau$ label the states which are not mass eigenstates and which enter into the same representation as the charged lepton $\ell$:

$$
\nu_\ell = \sum_i K_{\ell i} \nu_i, 
\ u_\ell = \sum_p U_{\ell p} u_p, 
\ d_\ell = \sum_n D_{\ell n} d_n.
$$

(7)

Here, $K_{\ell i}$ is the unitary leptonic mixing matrix by Pontecorvo–Maki–Nakagawa–Sakata.25–28 The matrices $U_{\ell p}$ and $D_{\ell n}$ are the unitary mixing matrices in the interactions of leptoquarks with the $u$ and $d$ fermions correspondingly, both quarks and leptons. The states $\nu_i, u_p$ and $d_n$ are the mass eigenstates:

$$
\nu_i = (\nu_1, \nu_2, \nu_3),
\ u_p = (u_1, u_2, u_3) = (u, c, t),
\ d_n = (d_1, d_2, d_3) = (d, s, b).
$$

(8)

Thus, there are generally three types of mixing in this scheme.

In our notation, the well-known Lagrangian describing the interaction of charge weak currents with $W$-bosons takes the form

$$
\mathcal{L}_W = \frac{g}{2\sqrt{2}} \left[ (\bar{\nu}_{\ell} O_{\alpha} \ell) + (\bar{u}_{\ell} O_{\alpha} d_{\ell}) \right] W_{\alpha}^+ + \text{h.c.}
$$

$$
= \frac{g}{2\sqrt{2}} \left[ K_{\ell i}^* (\bar{\nu}_{i} O_{\alpha} \ell) + U_{\ell p}^* D_{\ell n} (\bar{u}_{p} O_{\alpha} d_{n}) \right] W_{\alpha}^+ + \text{h.c.},
$$

(9)
where $g$ is the constant of the $SU(2)_L$ group and $O_\alpha = \gamma_\alpha (1 - \gamma_5)$. It follows that the standard Cabibbo–Kobayashi–Maskawa matrix is $V = U^D$. This is the only available information about the matrices $U$ and $D$ of mixing in the leptoquark sector. The matrix $\mathcal{K}$ describing a mixing in the lepton sector is the subject of intensive experimental studies.

Following the spontaneous breakdown of the $SU(4)_V$ symmetry to $SU(3)_c$ on the scale of $M_X$, six massive vector bosons forming three charged colored leptoquarks, decouple from the 15-plet of gauge fields. The interaction of these leptoquarks with fermions has the form

$$L_X = \frac{g_S (M_X)}{\sqrt{2}} \left[ D_{\ell n} (\bar{F}_\alpha d_n^c) + (K^\dagger U)_{i p} (\bar{\nu}_i \gamma_\alpha u_p^c) \right] X_i^c + \text{h.c.},$$

(10)

where the color superscript $c$ is written explicitly once again. The coupling constant $g_S (M_X)$ is expressed in terms of the strong-interaction constant $\alpha_S$ on the scale of the leptoquark mass $M_X$ as $g_S^2 (M_X) / 4\pi = \alpha_S (M_X)$.

### 3. Effective Lagrangian with allowance for QCD corrections

If the momentum transfer satisfies the condition $q^2 \ll M_X^2$, the Lagrangian (10) leads to the effective four-fermion vector-vector interaction between quarks and leptons. By applying the Fierz transformation, we can isolate the lepton and quark currents (scalar, pseudoscalar, vector and axial-vector currents) in the effective Lagrangian. In constructing the effective Lagrangian of leptoquark interactions, it is necessary to take into account the QCD corrections, which can easily be estimated, see e.g. Refs. 29, 30. In the case under study, we can use the approximation of leading logarithms because $\ln (M_X / \mu) \gg 1$, where $\mu \sim 1$ GeV is the typical hadronic scale. As the result of taking the QCD corrections into account, the scalar and pseudoscalar coupling constants acquire the enhancement factor

$$Q (\mu) = \left( \frac{\alpha_S (\mu)}{\alpha_S (M_X)} \right)^{\frac{4\tilde{b}}{3}},$$

(11)

where $\alpha_S (\mu)$ is the strong-interaction constant on the scale $\mu$, $\tilde{b} = 11 - 2/3 (\bar{n}_f)$, and $\bar{n}_f$ is the mean number of quark flavors on the scales $\mu^2 \leq q^2 \leq M_X^2$; for $M_X^2 \gg m_t^2$, we have $\tilde{b} \simeq 7$.

Let us investigate the contribution to low-energy processes from the interaction Lagrangian (10) involving leptoquarks and find constraints on the parameters of the scheme from available experimental data. It will be shown below that the most stringent constraints on the vector-leptoquark mass $M_X$ and on the elements of the mixing matrix $D$ follow from the data on rare $\pi$ and $K$ decays and on $\mu e$ conversion on nuclei.

Possible constraints on the masses and coupling constants of vector leptoquarks from experimental data on rare $\pi$ and $K$ meson decays were analyzed in Refs. [13] [23]. One approach [13, 15, 16] was based on using the phenomenological model-independent Lagrangians describing the interactions of leptoquarks with quarks and leptons.
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Fig. 2. Feynman diagrams for the $\pi^+(ud) \rightarrow \ell^+\nu_\ell$ decay via the $W$-boson and leptoquark $X$ exchange. Substituting the $c$ quark instead of $u$ and other down antiquarks $\bar{s}, \bar{b}$ instead of $\bar{d}$, one obtains the diagrams for lepton decays of various charged mesons, where the $W$ and $X$ boson contributions interfere.

Pati–Salam quark-lepton symmetry was considered in Refs. [14, 17–23]. QCD corrections were included into an analysis in Refs. [17–19]. The authors of Ref. [17] considered the possibility of mixing in quark-lepton currents, but they analyzed only specific cases in which each charged lepton is associated with one quark generation. In our notation, this corresponds to the matrices $\mathcal{D}$ that are obtained from the unit matrix by making all possible permutation of columns.

In the description of the $\pi$- and $K$-meson interactions, it is sufficient to retain only the scalar and pseudoscalar coupling constants in the effective Lagrangian. Really, these couplings are more significant in the amplitudes, because they are enhanced, first, by QCD corrections, and second, by the smallness of the current-quark masses arising in the amplitude denominators. The corresponding part of the effective Lagrangian can be represented as

$$\Delta \mathcal{L}_{\pi,K} = -\frac{2\pi\alpha_s (M_X)}{M_X^2} Q(\mu) \left[ \mathcal{D}_{\ell n} (\bar{u} \gamma_{\ell 5} \nu) (\bar{u} p \gamma_{5} d_n) + \text{h.c.} - (\gamma_5 \rightarrow 1) \right]$$

$$-\frac{2\pi\alpha_s (M_X)}{M_X^2} Q(\mu) \left[ \mathcal{D}_{\ell n} \mathcal{D}^*_{\ell' n'} (\bar{\ell} \gamma_{\ell' 5} \ell') (\bar{d} n' \gamma_5 d_n) + (\mathcal{K}^\dagger \mathcal{U})_{i p} (\bar{u} \gamma_{\ell 5} \nu) (\bar{u} p' \gamma_5 u_p) - (\gamma_5 \rightarrow 1) \right].$$

This Lagrangian contributes to the rare $\pi, K, \tau$ and $B$ decays, which are strongly suppressed or forbidden in the standard model.

4. Constraints on the parameters of the scheme from low-energy processes

4.1. $\mu e$ Universality in $\pi\ell_2$ and $K\ell_2$ Decays

Analysis reveals that, in contrast to $W$-boson contribution, the leptoquark contribution to the decay $\pi \rightarrow e\nu$, see Fig. 2, does not involve suppression that is due to the electron mass. The corresponding part of the amplitude can be represented in
the form

$$\Delta M^X_{\pi e\nu} = -\frac{2\pi\alpha_s(M_X)}{M_X^2}D_{cd}(U^\dagger K)_{ui} \frac{f_\pi m^2_Q(\mu)}{m_u(\mu) + m_d(\mu)} (\bar{e}\gamma_5\nu_i),$$  \hspace{1cm} (13)$$

where $f_\pi \simeq 132$ MeV is the constant of the $\pi l\nu$ decay, and $m_{u,d}(\mu)$ are the running quark masses on the scale $\mu$. The ratio $Q(\mu)/m(\mu)$ is a renormalization-group invariant because the function $Q(\mu)$ determines the law of variation of the running mass. The known values of the current-quark masses, $m_u = (1.7 - 3.3)$ MeV, $m_d = (4.1 - 5.8)$ MeV, and $m_s = 101^{+29}_{-21}$ MeV, correspond to the scale $\mu \simeq 2$ GeV. For evaluations, we take the central values of them. The contribution to the amplitude from the $W$-boson exchange has the form

$$\Delta M^W_{\pi e\nu} = -\frac{f_\pi G_F}{\sqrt{2}} m_e K_{eu} V_{ud} [\bar{e}(1 - \gamma_5)\nu_i].$$  \hspace{1cm} (14)$$

Taking into account the interference of the amplitudes (13) and (14), we find that the ratio $\Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu) = R_\pi$ of the decay widths is given by

$$R_\pi = R^W_{\pi} \left[ 1 - \frac{2\sqrt{2}\pi\alpha_s(M_X) m^2_Q}{G_F M_X^2 m_e(m_u + m_d)} \Re \left( \frac{D_{cd}U^*_{eu}}{V_{ud}} \right) \right],$$  \hspace{1cm} (15)$$

where $R^W_{\pi} = (1.2345 \pm 0.0010) \times 10^{-4}$ is the value of this ratio in the standard model.\footnote{BRINGING TOGETHER THE RESULTS OF $R_\pi$ MEASUREMENTS AT TRIUMF\ref{32,33} AND IN THE PAUL SCHERRER INSTITUTE\ref{34} WE ARRIVE AT THE CONCLUSION THAT THE LEPTOQUARK MASSobeys the constraint

$$M_X > (210 \text{ TeV}) |\Re (D_{cd}U^*_{eu})|^{1/2}. \hspace{1cm} (16)$$

The vector-leptoquark contribution can also disturb the ratio $R_K$ of $\mu e$ universality for the decays $K \to e\nu$ and $K \to \mu\nu$. Feynman diagrams for these processes can be obtained from the diagrams in Fig. 2, replacing $d$ by $s$. In an analogy with the analysis of the ratio of $\pi \ell^2$-decay widths, we find that the experimental data on $R_K$ yield the constraint

$$M_X > (150 \text{ TeV}) |\Re (D_{cs}U^*_{eu})|^{1/2}. \hspace{1cm} (17)$$

In this analysis, it was essential to take into account the interference of the $W$-boson and leptoquark contributions to the amplitudes, which was not considered in Ref.\cite{17} Really, because of mixing, the phenomenological neutrino $\nu_u$ produced in the leptoquark interaction is a superposition of the phenomenological neutrinos produced in the weak interaction:

$$\nu_u = U^*_{eu}\nu_e + U^*_{mu}\nu_\mu + U^*_{m\tau}\nu_\tau.$$  \hspace{1cm} (18)$$

Because the experimental interval for $R_K$ (see Table\ref{11}) falls below the theoretical value of this ratio in the standard model ($R_{KW} = 2.57 \times 10^{-5}$) and because the interference of the above amplitudes is destructive, the inclusion of this interference substantially changes the estimate of the leptoquark mass.
Among rare $K^0\pi\mu e$ decays, that can occur at the tree level in the model under study, $K^+ \rightarrow \pi^+\mu^+e^-$ and $K^+ \rightarrow \pi^+\mu^-e^-$ yield the most stringent constraints. The amplitude of the decay $K^+ \rightarrow \pi^+\mu^+e^-$ can be represented in the form

$$
\mathcal{M}_{K^+\mu\pi} = -\frac{2\pi\alpha_S(M_X)}{M_X^2} \frac{f_+(q^2)}{m^2_{K^+} - m^2_{\mu}} + \frac{f_-(q^2)}{m^2_{K^+} - m^2_{\mu}} q^2 Q D_{\mu\pi} \bar{\epsilon}\mu, \quad (23)
$$

4.3. Rare $K^+$ Decays

The amplitude of one more rare $K^0_L$ decay, into an electron and a positron through an intermediate leptoquark, can also be obtained from (19) by making the substitution $e \rightarrow \mu$. We finally obtain

$$
M_X > (1100 \, \text{TeV}) \left| \text{Re} \left( D_{\mu\pi} D_{\mu\pi}^* \right) \right|^{1/2}. \quad (21)
$$

Experimental values of $\text{Br} \left( K^0_L \rightarrow \mu^+\mu^- \right)$ closely approach the unitary limit $\text{Br}_{\text{abs}} = 6.8 \times 10^{-9}$. Therefore, the effective leptoquark contribution to $\text{Br} \left( K^0_L \rightarrow \mu^+\mu^- \right)$ is unlikely to exceed $1 \times 10^{-10}$. The amplitude of the process is obtained from (19) by making the substitution $e \rightarrow \mu$. We finally obtain

$$
M_X > (2400 \, \text{TeV}) \left| \text{Re} \left( D_{\mu\pi} D_{\mu\pi}^* \right) \right|^{1/2}. \quad (22)
$$
Table 1. Constraints on the leptoquark masses and on the elements of the mixing matrices from experimental data on rare $\pi$ and $K$ decays and on $\mu e$ conversion on nuclei.

| Experimental limit | Ref. | Bound |
|-------------------|------|-------|
| $\Gamma (\pi^+ \rightarrow e^+ \nu_e) / \Gamma (\pi^- \rightarrow \mu^+ \nu_\mu) = (1.2310 \pm 0.0037) \times 10^{-4}$ | 32 33 | $M_X > (210 \text{ TeV}) |D_{cd}D_{\mu s}|^{1/2}$ |
| $\Gamma (K^+ \rightarrow e^+ \nu_e) / \Gamma (K^- \rightarrow \mu^+ \nu_\mu) = (2.493 \pm 0.301) \times 10^{-5}$ | 34 | $M_X > (150 \text{ TeV}) |D_{cd}D_{\mu s}|^{1/2}$ |
| $Br(K^+ \rightarrow \pi^+ \mu^+ e^-) < 1.3 \times 10^{-11}$ | 35 | $M_X > (240 \text{ TeV}) |D_{cd}D_{\mu s}|^{1/2}$ |
| $Br(K^+ \rightarrow \pi^+ \mu^- e^+) < 5.2 \times 10^{-10}$ | 36 | $M_X > (100 \text{ TeV}) |D_{cd}D_{\mu s}|^{1/2}$ |
| $Br(K_L^0 \rightarrow \mu^+ \mu^-) = (6.84 \pm 0.11) \times 10^{-9}$ | 37 38 39 | $M_X > (1100 \text{ TeV}) |D_{cd}D_{\mu s}|^{1/2}$ |
| $Br(K_L^0 \rightarrow e^+ \mu^-) < 4.7 \times 10^{-12}$ | 40 | $M_X > (2100 \text{ TeV}) |D_{cd}D_{\mu s}|^{1/2}$ |
| $Br(K_L^0 \rightarrow e^+ e^-) = (9.6 \pm 1) \times 10^{-12}$ | 41 | $M_X > (2400 \text{ TeV}) |D_{cd}D_{\mu s}|^{1/2}$ |
| $\sigma (\mu^- Au \rightarrow e^- Au) / \sigma (\mu^- Au \rightarrow \text{capture}) < 0.7 \times 10^{-12}$ | 42 | $M_X > (1000 \text{ TeV}) |D_{cd}D_{\mu s}|^{1/2}$ |

where $q$ is the 4-momentum of the lepton pair, and $f^{+0}_{+, -}$ are the known form factors of the $K_L^0$ decay. The amplitude of the decay $K^+ \rightarrow \pi^+ \mu^- e^+$ is obtained from [23] by means of the substitution $e \leftrightarrow \mu$. The resulting constraints on the leptoquark mass involve the same elements as those present in [20], but they appear separately:

$$M_X > (240 \text{ TeV}) |D_{cd}D_{\mu s}|^{1/2}, \quad M_X > (100 \text{ TeV}) |D_{cd}D_{\mu s}|^{1/2}. \quad (24)$$

### 4.4. $\mu e$ Conversion on a Nucleus

This is one more low-energy process, which can proceed through leptoquarks. Coherent $\mu e$ conversion, which leaves the nucleus in the ground state and which leads to the production of monoenergetic electrons with highest possible energy $\simeq m_\mu$, is most convenient for observation. The effective Lagrangian of the coherent $\mu e$ conversion involves only scalar and vector quark currents. In the model under study, it has the form

$$\Delta L_{\mu e} = -\frac{2\pi\alpha_S (M_X)}{M_X^2} D_{cd}D_{\mu s} \left[ \frac{1}{2} (\bar{e}_\gamma \gamma_\mu) (\bar{d} \gamma_\alpha d) - (\bar{e}_\mu) (\bar{d}d) Q (\mu) \right]. \quad (25)$$
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| Experimental limit | Ref. | Bound |
|--------------------|------|-------|
| $\text{Br}(\tau^- \to e^- K^0_S) < 2.6 \times 10^{-8}$ | 44 | $\frac{M_X}{|D_{ee}D_{\tau d}^* - D_{ed}D_{\tau s}^*|^{1/2}} > 10 \text{ TeV}$ |
| $\text{Br}(\tau^- \to \mu^- K^0_S) < 2.3 \times 10^{-8}$ | 44 | $\frac{M_X}{|D_{\mu s}D_{\tau d}^* - D_{\mu d}D_{\tau s}^*|^{1/2}} > 11 \text{ TeV}$ |
| $\text{Br}(\tau^- \to e^- \pi^0) < 8.0 \times 10^{-8}$ | 45 | $\frac{M_X}{|D_{ed}D_{\tau d}^*|^{1/2}} > 7 \text{ TeV}$ |
| $\text{Br}(\tau^- \to \mu^- \pi^0) < 1.1 \times 10^{-7}$ | 45 | $\frac{M_X}{|D_{\mu d}D_{\tau d}^*|^{1/2}} > 6 \text{ TeV}$ |

Using the computational technique developed in Ref. 43 for the effective quark-lepton interaction of the form (25), we estimated the branching ratio of $\mu e$ conversion on gold. By applying the result to the experimental data reported in Ref. 42, we arrived at the constraint

$$M_X > (1000 \text{ TeV}) |D_{ed}D_{\mu d}|^{1/2}.$$ (26)

The constraints that we obtained for the parameters of our model from experimental data on rare $\pi$ and $K$ decays and on $\mu e$ conversion on nuclei are summarized in Table 1. It can be seen that all the constraints involve the elements of the unknown unitary mixing matrices $\mathcal{D}$ and $\mathcal{U}$, which are related by the single condition $\mathcal{U}^\dagger \mathcal{D} = V$. Therefore, the possibility cannot be ruled out for the constraints on the vector leptoquark mass $M_X$ to be much weaker than the numbers in Table 1. For example, this is the case if some of the elements $D_{ed}$, $D_{\mu d}$, $D_{es}$ and $D_{\mu s}$ are small enough. Given the unitarity of the matrix $\mathcal{D}$, this would mean that its elements presented in the interactions of the $\tau$-lepton and $b$-quark should be close to unity. In this case, leptoquarks might make more significant contributions to $\tau$ and $B$ decays.

### 4.5. $\tau$ Decays

The current accuracy of experimental data on the $\tau$ and $B$ decays is poorer than in the processes considered above. Nevertheless it is possible to obtain some constraints on the elements of the matrix $\mathcal{D}$ and on the leptoquark mass from the data on decays that are strongly suppressed or forbidden in the standard model.

The amplitude of the decay $\tau^- \to \mu^- K^0_S$ proceeding through a leptoquark is given by

$$M_{\tau \mu K} = \frac{\pi \alpha_s (M_X) f_K}{\sqrt{2} M_X^2} (D_{\mu s}D_{\tau d}^* - D_{\mu d}D_{\tau s}^*) \left( m_{\tau} + m_{\mu} - \frac{2m_K^2Q}{m_s + m_d} \right) (\bar{\mu}\gamma_5\tau).$$ (27)
Table 3. Constraints on the model parameters from the data on rare $B^+$ decays.

| Experimental limit                              | Ref. | Bound                                      |
|------------------------------------------------|------|--------------------------------------------|
| $\text{Br}(B^+ \to K^+e^-\mu^+) < 1.3 \times 10^{-7}$ | 47   | $\frac{M_X}{|D_{es}D_{eb}|^{1/2}} > 38 \text{ TeV}$ |
| $\text{Br}(B^+ \to K^+e^+\mu^-) < 0.91 \times 10^{-7}$ | 47   | $\frac{M_X}{|D_{es}D_{eb}|^{1/2}} > 42 \text{ TeV}$ |
| $\text{Br}(B^+ \to \pi^+e^-\mu^-) < 0.92 \times 10^{-7}$ | 47   | $\frac{M_X}{|D_{es}D_{eb}|^{1/2}} > 42 \text{ TeV}$ |
| $\text{Br}(B^+ \to K^+\mu^+\tau^-) < 7.7 \times 10^{-5}$ | 49   | $\frac{M_X}{(|D_{es}D_{eb}|^2 + |D_{es}D_{eb}|^2)^{1/4}} > 8 \text{ TeV}$ |

The amplitude of the decay $\tau^- \to e^- K^0_S$ is obtained from (27) as the result of the substitution $\mu \to e$. From the experimental upper limits on the widths of these decays, we find the constraints which are presented in the Table 2.

Making corresponding substitutions, one can obtain in a similar way the constraints from the processes $\tau^- \to e^- \pi^0$ and $\tau^- \to \mu^- \pi^0$, see Table 2.

4.6. Rare $B$-Meson Decays

Let us consider the $B$ decays, for which there are experimental constraints, and which may proceed through an intermediate leptoquark. The decay $B^+ \to K^+e^-\mu^+$ is an example of the process being forbidden in the standard model. The amplitude of this decay via the vector leptoquark can be represented in the form

$$
M_X^{B K e\mu} = -\frac{2\sqrt{2}\pi\alpha_S(M_X)}{M_X^2} D_{es}D_{eb} \left[ \frac{1}{2}\frac{f^0_+}{f^0_+ - f^0_-} (q^2) (p_B + p_K) \gamma (\bar{e}\gamma\mu) Q(\mu_B) (\bar{e}\mu) \right],
$$

(28)

where $p_B$ and $p_K$ are the 4-momenta of the $B$- and $K$-mesons; the remaining notation is identical to that in equation (28). We assume that the form factors $f^0_{+, -}$ are of the same order of magnitude as in the $K_{e3}^0$ decay. This assumption is in agreement with the results obtained from the analysis of the decay $B^+ \to D^0 \ell^+\nu$.

The amplitude of the decay $B^+ \to K^+e^+\mu^-$ is obtained from Eq. (28) by means of the interchange $e \leftrightarrow \mu$. Using the experimental data reported in Ref. 17, we arrive at the constraints

$$
M_X > (38 \text{ TeV}) |D_{es}D_{eb}|^{1/2},
M_X > (42 \text{ TeV}) |D_{es}D_{eb}|^{1/2}.
$$

(29)

Making corresponding substitutions, one can also obtain in a similar way as for the decays of the $K^+$ meson, the respective constraints from other processes of $B^+$ decay (see Table 3).
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Table 4. Constraints on the model parameters from experimental data on rare $B^0$ and $B^0_s$ decays.

| Experimental limit | Ref. | Bound |
|--------------------|------|-------|
| $\text{Br}(B^0 \rightarrow e^+e^-) < 8.3 \times 10^{-8}$ | 52 | $\frac{M_X}{|D_{ee}D_{eb}|^{1/2}} > 51 \text{ TeV}$ |
| $\text{Br}(B^0 \rightarrow \mu^+\mu^-) < 1.5 \times 10^{-8}$ | 53 | $\frac{M_X}{|D_{\mu d}D_{\mu b}|^{1/2}} > 79 \text{ TeV}$ |
| $\text{Br}(B^0 \rightarrow \tau^+\tau^-) < 4.1 \times 10^{-3}$ | 54 | $\frac{M_X}{|D_{\tau d}D_{\tau b}|^{1/2}} > 3 \text{ TeV}$ |
| $\text{Br}(B^0 \rightarrow e^+\mu^-) < 6.4 \times 10^{-8}$ | 52 | $\frac{M_X}{|D_{\mu d}D_{eb}|^{1/2}} > 55 \text{ TeV}$ |
| $\text{Br}(B^0 \rightarrow e^+\tau^-) < 2.8 \times 10^{-5}$ | 55 | $\frac{M_X}{|D_{\tau d}D_{eb}|^{1/2}} > 11 \text{ TeV}$ |
| $\text{Br}(B^0 \rightarrow \mu^+\tau^-) < 2.2 \times 10^{-5}$ | 55 | $\frac{M_X}{|D_{\tau d}D_{eb}|^{1/2}} > 12 \text{ TeV}$ |
| $\text{Br}(B^0_s \rightarrow e^+e^-) < 2.8 \times 10^{-7}$ | 52 | $\frac{M_X}{|D_{es}D_{eb}|^{1/2}} > 38 \text{ TeV}$ |
| $\text{Br}(B^0_s \rightarrow \mu^+\mu^-) < 4.2 \times 10^{-8}$ | 50 | $\frac{M_X}{|D_{\mu d}D_{\mu b}|^{1/2}} > 61 \text{ TeV}$ |
| $\text{Br}(B^0_s \rightarrow e^+\mu^-) < 2.0 \times 10^{-7}$ | 52 | $\frac{M_X}{|D_{\mu d}D_{eb}|^{1/2}} > 41 \text{ TeV}$ |

The amplitudes of the $B^0$ and $B^0_s$ decay processes into charged lepton pairs can be calculated in a similar way as the amplitude (19). For the $B$ meson decay constant we take $f_B = 220 \text{ MeV}$, see e.g. Refs. 50, 51. From these processes the constraints on the leptoquark mass and the $D$ matrix elements are obtained. They are collected in Table 4.

5. Combined constraint from the $\pi$, $K$, $\tau$, $B$ decays

All the constraints on the vector leptoquark mass collected in Tables 1–4 contain the elements of the unitary $D$ matrix:

$$D_{\ell n} = \begin{pmatrix} D_{ed} & D_{es} & D_{eb} \\ D_{\mu d} & D_{\mu s} & D_{\mu b} \\ D_{\tau d} & D_{\tau s} & D_{\tau b} \end{pmatrix}. \quad (30)$$

There are also two constraints in Table 1 containing the element of the $U$ matrix:

$$U_{eu} = D_{ed}V_{ud}^* + D_{es}V_{us}^* + D_{eb}V_{ub}^* \approx 0.974 D_{ed} + 0.225 D_{es} + 0.004 D_{eb}, \quad (31)$$
where $V$ is the Cabibbo–Kobayashi–Maskawa quark-mixing matrix, see Ref. [10] p. 150.

Let us try to establish, varying the unknown elements of the $D$ matrix, the lowest limit on the vector leptoquark mass, being in agreement with all the constraints presented in Tables [11]–[14]. The similar approach was developed in Ref. [17].

It is possible for the strongest constraint on $M_X$ arising from the limit on $\text{Br}(K_L^0 \rightarrow e^\pm \mu^\mp)$, see Table [11] to be much lower than 2100 TeV if the matrix elements $D_{ed}$ and $D_{es}$ are small enough. For evaluation, let us take them to be zero. In this case, all the estimates which are presented in the right column of Table [11] disappear, except the one arising from the limit on $\text{Br}(K_L^0 \rightarrow \mu^+ \mu^-)$. Because of unitarity of the $D$ matrix, the elements $D_{\mu b}$ and $D_{\tau b}$ are also equal to zero. The remaining $(2 \times 2)$-matrix can be parameterized by one angle. The insertion of the phase factor allows to eliminate the restriction arising from the limit on $\text{Br}(K_L^0 \rightarrow \mu^+ \mu^-)$ which contains the real part of the $D$ matrix elements product. For example, it is possible to take the $D$ matrix in the form:

$$
D_{\ell n} \simeq \begin{pmatrix}
0 & 0 & 1 \\
\cos \varphi & i \sin \varphi & 0 \\
i \sin \varphi & \cos \varphi & 0
\end{pmatrix}.
$$

(32)

As the analysis shows, in this case there appear the following constraints from the remaining limits of Tables [12]–[14] on the branching ratios of the processes:

i) $\tau^- \rightarrow \mu^- K_S^0$

$$
M_X > 11 \text{ TeV} \left| \cos 2\varphi \right|^{1/2},
$$

(33)

ii) $\tau^- \rightarrow \mu^- \pi^0$

$$
M_X > 6 \text{ TeV} \left| \sin 2\varphi \right|^{1/2},
$$

(34)

iii) $B^0 \rightarrow e^+ \mu^-$

$$
M_X > 55 \text{ TeV} \left| \cos \varphi \right|^{1/2},
$$

(35)

iv) $B_s^0 \rightarrow e^+ \mu^-$

$$
M_X > 41 \text{ TeV} \left| \sin \varphi \right|^{1/2}.
$$

(36)

Here, the weaker constraints of the same type are omitted. Combining these constraints one obtains the final limit on the vector leptoquark mass from low-energy processes:

$$
M_X > 38 \text{ TeV}.
$$

(37)

6. Different mixings for left-handed and right-handed fermions

We consider a possibility when the quark-lepton symmetry is the next step beyond the standard model. Then the left-right symmetry which is believed to exist in
right-handed fermions. This possibility and some its consequences were also considered in Refs. 20–23. The interaction Lagrangian of leptoquarks with fermions takes the form instead of Eq. (10):

\[
\mathcal{L}_{K} = \frac{g_s (M_X)}{2\sqrt{2}} \left[ D_{\ell n}^{(L)} (\bar{\nu}_n O_{\alpha} u_p) + D_{\ell n}^{(R)} (\bar{\nu}_n O'_{\alpha} u_p) \right] X_{\alpha} + \text{h.c.,} \quad (38)
\]

where \(O_{\alpha} = \gamma_\alpha (1 - \gamma_5)\), \(O'_{\alpha} = \gamma_\alpha (1 + \gamma_5)\).

The constraints on the model parameters from experimental data on rare \(\pi\) and \(K\) decays collected in Table 1 in the case of different mixings take the forms presented in Table 5. If one would wish to reduce the limits of Table 5 on \(M_X\) from thousands and hundreds to tens of TeV by varying the elements of the \(D^{(L)}\) and \(D^{(R)}\) matrices, it seems that the elements \(D_{ed}^{(L)}\) and \(D_{ed}^{(R)}\) should be taken small.

| Experimental data | Bound |
|-------------------|-------|
| \(\frac{\Gamma (\pi^+ \rightarrow e^+ \nu_e)}{\Gamma (\pi^+ \rightarrow \mu^+ \nu_\mu)}\) | \(\frac{M_X}{|\text{Re}(D_{ed}^{(R)} D_{ed}^{(L)*})|^{1/2}} > 210\) TeV |
| \(\frac{\Gamma (K^+ \rightarrow e^+ \nu_e)}{\Gamma (K^+ \rightarrow \mu^+ \nu_\mu)}\) | \(\frac{M_X}{|\text{Re}(D_{es}^{(R)} D_{es}^{(L)*})|^{1/2}} > 150\) TeV |
| \(\text{Br}(K^+ \rightarrow \pi^+ \mu^- e^-)\) | \(\frac{M_X}{\left| D_{ed}^{(L)} D_{\mu e}^{(R)} \right|^2 + \left| D_{es}^{(L)} D_{\mu e}^{(R)} \right|^2}^{1/2} > 200\) TeV |
| \(\text{Br}(K^+ \rightarrow \pi^+ \mu^- e^-)\) | \(\frac{M_X}{\left| D_{es}^{(L)} D_{\mu e}^{(R)} + D_{es}^{(R)} D_{\mu e}^{(L)} \right|^2}^{1/2} > 84\) TeV |
| \(\text{Br}(K^+ \rightarrow \mu^- \mu^-)\) | \(\frac{M_X}{\left| D_{ed}^{(L)} D_{\mu e}^{(R)} \right|^2 + D_{es}^{(L)} D_{\mu e}^{(R)} \right|^2} + \{L \leftrightarrow R\}^{1/2} > 780\) TeV |
| \(\text{Br}(K^+ \rightarrow e^+ \mu^-)\) | \(\frac{M_X}{\left| D_{ed}^{(L)} D_{ed}^{(R)} + D_{ed}^{(R)} D_{ed}^{(L)} \right|^2} > 1770\) TeV |
| \(\text{Br}(K^+ \rightarrow e^- e^-)\) | \(\frac{M_X}{\left| D_{ed}^{(L)} D_{ed}^{(R)} + D_{ed}^{(R)} D_{ed}^{(L)} \right|^2} > 1700\) TeV |

Nature, should restore at higher mass scale. But this means that the left-right symmetry should be broken at the scale \(M_X\). It is worthwhile to consider the matrices \(D^{(L)}, U^{(L)}\) and \(D^{(R)}, U^{(R)}\) which are in a general case different for left-handed and right-handed fermions. This possibility and some its consequences were also considered in Refs. 20–23. The interaction Lagrangian of leptoquarks with fermions takes the form instead of Eq. (10):
in any case. For evaluation, let them be zero. Then, the most strong restriction of Table 5 from the limit on \( \text{Br}(K^0_L \to e^\pm \mu^\mp) \) takes the form:

\[
M_X > \left( \frac{\left| D_{es}^{(L)} D_{\mu d}^{(R)} \right|^2 + \left| D_{es}^{(R)} D_{\mu d}^{(L)} \right|^2}{2} \right)^{1/4} > 1770 \text{ TeV}.
\] (39)

There are two possibilities to eliminate this bound together with other bounds of Table 5 which we call the symmetric and the asymmetric cases.

The symmetric case is realized when both of the matrices \( D(L) \) and \( D(R) \) are taken in the form of Eq. (32) with the angles \( \varphi_L \) and \( \varphi_R \). In this case the restriction from the limit on \( \text{Br}(K^0_L \to \mu^+ \mu^-) \) takes the form:

\[
M_X > 780 \text{ TeV} |\sin (\varphi_L - \varphi_R)|^{1/2},
\] (40)

and the angles should be close to each other or differ by \( \pi \), in any case we come back to the results of Sec. 5.

The asymmetric case is realized when the matrices are taken in the form:

\[
D^{(L)}_{\ell n} \simeq \begin{pmatrix} 0 & \cos \chi_L & \sin \chi_L \\ 0 - \sin \chi_L & \cos \chi_L \\ 1 & 0 & 0 \end{pmatrix}, \quad D^{(R)}_{\ell n} \simeq \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.
\] (41)

In the Table 4 which has provided one more group of essential restrictions in Sec. 5, the following substitutions should be made in the bounds:

\[
|D_{\ell q} D_{\ell' b}| \Rightarrow \frac{1}{\sqrt{2}} \left( \left| D_{\ell q}^{(L)} D_{\ell' b}^{(R)} \right|^2 + \left| D_{\ell q}^{(R)} D_{\ell' b}^{(L)} \right|^2 \right)^{1/2},
\] (42)

where \( \ell, \ell' = e, \mu, \tau \) and \( q = d, s \). As the analysis shows, the most stringent constraints arise from the following limits on the branching ratios of the processes:

i) \( B_s^0 \to \mu^+ \mu^- \)

\[
M_X > 51 \text{ TeV} |\cos \chi_L|^{1/2},
\] (43)

ii) \( B_s^0 \to e^+ \mu^- \)

\[
M_X > 41 \text{ TeV} |\sin \chi_L|^{1/2}.
\] (44)

Combining these constraints, one obtains the final limit on the vector leptoquark mass from low-energy processes in the case of different mixing matrices for left-handed and right-handed fermions, which coincides, with a good accuracy, with the limit (37) obtained in the left-right-symmetric case:

\[
M_X > 38 \text{ TeV}.
\] (45)
7. Constraint from the decay $\pi^0 \rightarrow \nu\bar{\nu}$

Only one process was found in which the lower limit on the leptoquark mass was independent of mixing parameters, the decay $\pi^0 \rightarrow \nu\bar{\nu}$.

In the standard model, the width of this process is proportional to $m_\nu^2$, but it can also proceed through the leptoquark exchange; in the latter case, there is no suppression associated with the smallness of the neutrino mass. The corresponding amplitude of the process has the form

$$M_{\pi\nu\nu}^X = \frac{\pi \alpha_S (M_X) f_\pi m_\nu^2 Q}{\sqrt{2} M_X^2 m_u} \left( K^\dagger U \right)_{ij} \left( U^\dagger K \right)_{uj} (\bar{\nu}_i \gamma_5 \nu_j),$$

and the decay probability summed over all neutrino flavors $i$ and $j$ does not depend on mixing.

From the accelerator data for the decay $\pi^0 \rightarrow \nu\bar{\nu}$:

$$\text{Br}(\pi^0 \rightarrow \nu\bar{\nu}) < 2.7 \times 10^{-7},$$

we obtain a constraint:

$$M_X > 600 \text{ GeV}. \quad (48)$$

In Refs. 59, 60 the almost coinciding astrophysical and cosmological estimations of the width of this decay were found, being much stronger than the accelerator limit (47),

$$\text{Br}(\pi^0 \rightarrow \nu\bar{\nu}) < 3 \times 10^{-13}. \quad (49)$$

The resulting constraint on the leptoquark mass was estimated to be:

$$M_X > 18 \text{ TeV}. \quad (50)$$

The astrophysical estimation was based on evaluating the excess energy-loss rate from SN 1987A if the process $\gamma\gamma \rightarrow \pi^0 \rightarrow \nu\bar{\nu}$ via the pion-pole mechanism occurred, permitted if the neutrinos had a right-handed component. In turn, the cosmological limit on the width of the decay $\pi^0 \rightarrow \nu\bar{\nu}$ was established in Ref. 60, where the production was considered of right-handed neutrinos produced from the cosmic thermal background at the temperature of about the pion mass through the reaction $\gamma\gamma \rightarrow \pi^0 \rightarrow \nu\bar{\nu}$.

However, in Ref. 61 it was mentioned that the astrophysical limit must be relaxed in several orders of magnitude, because the effect of nuclear absorption of a pion in a supernova core was not considered in Ref. 59. In turn, a criticism has been expressed in Ref. 62 on the cosmological limit also. At the temperature where the pion mechanism is at resonance, the strong interaction among pions occurs much faster than the pion decay: the rate of $\pi - \pi$ scattering, $\Gamma_{\pi - \pi} \sim 0.2 \text{ MeV}$, dominates the pion lifetime in the dense medium, resulting in a suppression of several orders of magnitude in the rate of neutrino production.

Therefore, only the laboratory limit for the decay $\pi^0 \rightarrow \nu\bar{\nu}$ should be considered as reliable, to establish the bound $M_X > 0.6 \text{ TeV}$. 


8. Conclusion

Thus, the detailed analysis of the available experimental data on rare $\pi$, $K$, $\tau$ and $B$ decays and on the $\mu e$ conversion yields constraints on the vector leptoquark mass that always involve the elements of the unknown mixing matrix $D$. Combining the constraints from the experimental data on the low-energy processes presented in Tables 1–4, we have obtained in the case of identical mixings for left-handed and right-handed fermions the following lowest limit on the vector leptoquark mass: $M_X > 38$ TeV. The lowest limit obtained in the case of different mixing matrices for left-handed and right-handed fermions appears to be the same.

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