A Study on Stacking Sequence Design of Composite Fan Blades Using Multi-level Optimization

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Abstract. In this paper, the multi-level optimization method is proposed to design the optimal stacking sequence of composite fan blade more efficiently. The fan blade model used is a laminate with inconsistent stiffness because the thickness is not uniform throughout the model. This feature complicates the optimization problem and makes it difficult to apply optimal design algorithms that use only continuous or discrete variables. For this reason, we conducted the literature survey about combining the optimal design algorithm with continuous and discontinuous variables and developed optimization algorithm for the composite fan blade. Moreover, the design space of discrete variables is reduced through the optimal design of continuous variables. In the first-level, response surface method is used, with the design variables being the volume fraction of plies of the predefined angles of [0°/±45°/90°]. The optimization problem in the first-level aims at minimizing the tip displacement of the fan blade and constraints on other structural responses are imposed. At the second-level, the permutation genetic algorithm is used with the fiber orientation of each ply as design variables. In this level, we perform the optimal design of the stacking sequence using the results determined by prior level, while the composite design rule and compliance constraints are applied.

1. Introduction

Composite materials are widely used in the aerospace industry because of their advantages having high specific stiffness and strength. In addition, they can increase the structural performances by adjusting the fiber orientation angle and stacking sequence. Therefore, to reduce the weight and increase the rigidity, the conventional metal structure has been replaced by the composite structure. And research on an optimal design technique considering the characteristics of the composite material has been conducted. Especially, structures such as fan blade have a characteristic that geometrical shape significantly affects the aerodynamic performance. For this reason, it is more advantageous to apply the composite material to fan blade for improved structural performance without modifying the shape. Accordingly, Coroneos researched the optimal design of stacking sequence to minimize the weight of fan blade [1]. The composite fan blade is tapered laminate because of its twisted shape and uneven thickness, so the stiffness of laminate can be variable at each point of a plane. This type of structures called ‘variable stiffness composite structures,’ and it requires higher design cost to optimize due to a large number of design variables. Therefore, in recent studies, design problems of variable stiffness and related optimization methods were proposed [2]. For optimal design of stacking sequence, the genetic algorithm is most widely used because it is combinatorial problem of choosing the fiber
orientation from the permissible set for each ply. However, stacking sequence design is a complicated design problem that is often strongly coupled to the overall design of a structure [3]. In the case of variable stiffness composite design, the degree of freedom on the design becomes much higher, which is more difficult to find the optimal solution. In this reason, we adopted a multi-level optimization method to solve this problem. It combines continuous variable problem and discrete variable problem to explore the wide design space effectively that comes from variable stiffness design. Liu et al. minimized the weight of blended composite wing structure using multi-level optimization with constraints on buckling and strain [4]. Similarly, Herencia et al. researched the multi-level optimization using lamination parameters [5]. At the upper level, continuous optimization problems are defined with design variables as the number of plies of each orientation, lamination parameters, or stiffness terms. In this level, response surface method applied to create approximation model. Next, at the lower level, the genetic algorithm employed to solve discrete problems such as stacking sequence optimization using the results obtained from the upper level [6-9]. In this study, we developed an algorithm of multi-level optimization to design the optimal stacking sequence of composite fan blade more efficiently. In the first-level, the optimization problem considers the global structural responses and determine the volume fractions of plies of predefined angles of $[0^\circ/\pm 45^\circ/90^\circ]$. Then, response surface method and the gradient-based method was adopted to construct the approximation model for structural features and solve the optimization problem. Before the second-level, we conducted additional constraint for the next process using the concept of lamination parameters. Finally, at the second-level, the permutation genetic algorithm (permutation GA) employed for the design of local stacking sequences of the entire model. Using the volume fractions that specified from first-level, detailed stacking sequences design was performed.

2. Finite element model of the composite fan blade
The composite fan blade model has total of 112-plies symmetric laminate consisting of 56 plies with inconsistent shape. For this reason, this model has variable stiffness corresponding to the variable thickness. This feature makes the optimal design process complicated because it makes many design variables to consider. Therefore, we performed the task of dividing the ply-based model by sets of elements having the same thickness and stacking sequence, and defined them as ‘regions’. Figure 1 shows the two contours of the composite fan blade model, distribution of thickness and regions, respectively. In figure 1, different colors mean having different thickness and stacking sequence. The bottom of the blade was constrained at translational DOF, and the aerodynamic forces having magnitudes of 470 N and 650 N imposed on the airfoil, in the y and z directions of the global system. Additionally, a rotational velocity of 7000 rpm was applied to the rotation center of the blade.
3. Optimization strategies

In this paper, two-step decomposition of the problem is the basis of optimization strategy. The two-step decomposition consists of first searching for the optimal number of global plies of structure, before searching for the corresponding optimal stacking sequence in the second step. In the first-level, finite element analysis is conducted to analyze the relationship between each volume fraction of plies having angles $[0^\circ/\pm 45^\circ/90^\circ]$ and structural responses. To define the relationship between them, the response surface method employed to create the approximation model. At the second-level, we conducted optimization on the local stacking sequence using Genetic algorithm (GA). The results obtained from the first-level became a constraint that allows GA to find a solution in the desired range.

3.1. First-level optimization

At this step, a nonlinear constrained optimization is performed. The basic mathematical optimization problem can be expressed as follows:

Minimize : $f(x)$
Subject to : $G_i(x) \leq 0$ $i = 1,...,n_i$
$x_i^l \leq x_j \leq x_i^u$ $j = 1,...,n_i$
with $x = \{x_1, x_2, ...., x_n\}$

(1)

In this case, the objective function is the maximum value of tip displacement. The inequality constraints are the maximum failure index and the first natural frequency. In addition, the design variables are volume fractions of plies of predefined angles of $[0^\circ/\pm 45^\circ/90^\circ]$, with side constraints on the bounds of design variables. Additionally, the equality constraint that the sum of the volume fractions should be 1 is imposed. The volume fraction of plies used as design variable is calculated as follows:

$$v_i = \frac{h_i}{h_{total}} = \frac{n_i}{n_{total}}$$ $i = 0^\circ, \pm 45^\circ, 90^\circ$

(2)

When the thickness of plies is constant, the volume fraction is the ratio of the total number of plies to the number of plies with the corresponding fiber orientation angle. The volume fraction is the continuous variable and range is from 0 to 1. Furthermore, the lower bound is limited to 0.1 by the rule for the design of the composite structure in the aerospace industry. Thus, the first-level optimization is a continuous variable optimal design problem. In this study, approximation models created by the response surface method are used as an objective function and constraints for the structural response. That is, structural responses such as maximum displacement are formulated in terms of volume fraction $v_i$. Response surface method employed to obtain an approximation to a response function in terms of predictor variables. The response surface model is generally written as follows:

$$y = F(x_1, x_2, ...., x_n) + \varepsilon$$

(3)

Where $y$ is the response, $x_i$ $(i = 1,...,n)$ are predictor variables, and $\varepsilon$ is an error term. The function $F$ is normally selected to be a polynomial. For a quadratic polynomial, $F$ is written as follows:

$$y = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1, j \neq i}^{n} \beta_{ij} x_i x_j$$

(4)
Where $\beta$ represents unknown coefficients. When considering a case employing two variables and a quadratic polynomial, the response surface is expressed as:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2$$

(5)

For ease of notation, let $x_3 = x_2^2$, $x_4 = x_2^2$ and $x_5 = x_1 x_2$. Thereby, the equation becomes:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5$$

(6)

The unknown coefficients $\beta_i (i = 0, ..., 5)$ in the equation (6) are estimated by a linear multiple regression set of data points where the response surface. The set of data points are selected to achieve higher accuracy of the approximation, by design of experiments (DOE). To evaluate the performance of the approximation of the response surface, the adjusted coefficient of multiple determination $R^2_{adj}$ (R-square-adjusted) is used, and calculated as follow:

$$R^2_{adj} = 1 - \frac{SS_E}{S_{yy}} / (n-k-1)$$

(7)

Where $SS_E$ is the squared sum of errors, and $S_{yy}$ is the sum of squares. To obtain a higher value of $R^2_{adj}$, insignificant factors are removed from the approximation model [6]. Through the above procedure, the maximum displacement, failure index, and first natural frequency were fitted as a quadratic polynomial in terms of the volume fraction of plies. Moreover, the $R^2_{adj}$ of each approximation model were higher than 95%, so the approximation and prediction of the responses are reliable. The formulation of the optimal design problem for this stage is as follow:

Minimize : $f(v) : \text{max. displacement}$

Subject to : $G_1(v) \leq 0.7 : \text{max. Tsai-wu failure index}$

$G_2(v) \geq 210 : 1\text{st natural frequency}$

$0.1 \leq v_i \leq 1.0, \sum v_i = 1.0 \quad i = 0, 45, 90$  

find : $v = \{v_0, v_{45}, v_{90}\}$

(8)

Figure 2. Optimal solution searching process
Since this is a constrained nonlinear minimization problem, we used the gradient-based method. Figure 2 shows the process of searching an optimal solution and convergence to a feasible region.

3.2. Constraints associated with lamination parameters
The additional task performed to formulate constraints associated with lamination parameters. This constraint acts on the GA performed in the second-level optimization and can determine whether the designed stacking sequence meets the expected structural performance. For this task, the lamination parameters of each laminate within the model were calculated. For symmetric laminates, the stiffness terms of the laminates A, D matrices can be represented with lamination parameters. The in-plane, out-of-plane lamination parameters are defined as follows:

\[ V^A_{1,2,3,4} = \frac{1}{h} \int_0^h \begin{bmatrix} \cos 2\theta(z) \\ \sin 2\theta(z) \\ \cos 4\theta(z) \\ \sin 4\theta(z) \end{bmatrix} dz, \quad V^D_{1,2,3,4} = \frac{1}{h} \int_0^h \begin{bmatrix} \cos 2\theta(z) \\ \sin 2\theta(z) \\ \cos 4\theta(z) \\ \sin 4\theta(z) \end{bmatrix} dz \]  

(9)

Where \( h \) is the thickness of the laminate, \( z \) is coordinate of thickness direction, the origin locates at the end of the plate, \( \theta(z) \) is the fiber angle of location \( z \). The terms of laminate stiffness matrices A, D can be represented with these lamination parameters and material invariants. When we consider the case that the fiber angle is limited to the small set, the value of \( \sin 4\theta_k \) is always equal to zero, so all values of \( V^A_{4}, V^D_{4} \) are equal to zero. This makes the reduced lamination parameter vectors of three-dimension [9-10]. Furthermore, if the laminate is symmetric and balanced, \( V^A_{2}, V^D_{2} \) and \( V^A_{4}, V^D_{4} \) become zero. So two lamination parameters \( V^A_{1}, V^A_{3} \) are required to fully model the in-plane and out-of-plane stiffness. With composite fan blade model and given load cases, we consider the only in-plane case, so \( V^A_{1} \) and \( V^A_{3} \) represent the in-plane laminate parameters. If ply thickness is constant, in-plane lamination parameters are as follow:

\[ V^A_{1} = \sum_{k=1}^l v_k \cos 2\theta_k, \quad V^A_{3} = \sum_{k=1}^l v_k \cos 4\theta_k \]  

(10)

where \( v_k \) is the volume fraction of the layers with \( \pm\theta \) orientation angles [11]. Equation (10) denotes that if \( v_i \) was determined for each angles of \([0^\circ/\pm45^\circ/90^\circ]\), the boundaries of \( V^A_{1} \) and \( V^A_{3} \) can be determined. In this study, compliance was used as a criterion to confirm the suitability of the stacking sequence. Using such a standard finite element model, compliance is computed as:

\[ C = \frac{1}{2} F^T \cdot U \]  

(11)

where \( F \) and \( U \) are vectors of external forces and displacements, respectively. For in-plane problems, the minimum compliance design problem is formulated in its discrete form as:

\[ \min_{v_i} \frac{1}{2} N^T_i \cdot A^{-1}(V_i) \cdot N_i \quad (i = 1,...,n) \quad s.t. \quad \left( C_i \right) \]  

(12)
where $N_i$ is the vector of the stress resultants at node $i$, $V_i = \{V_{i_1}, V_{i_2}, V_{i_3}, V_{i_4}\}$ is the vector of the node lamination parameters, $A(V_i)$ is the in-plane stiffness of the laminate, and constraints $(C_i)$ define the feasible lamination parameters space [12-13]. With equation (12), it can be assumed that compliance can be expressed in terms of lamination parameters of each node. Through the above description, we performed a regression analysis of the compliance in terms of $V_1$ and $V_3$ obtained by stacking sequences with the volume fraction determined in the first step. $V_1$ and $V_3$ was calculated at the region stacked all plies in the fan blade model, and compliance was obtained by finite element analysis. The approximation model for compliance is fitted to a quadratic polynomial model, and the mean value of compliance that could be calculated from the determined volume fraction was the criterion. This constraint expressed as:

$$C(V_{k_1}, V_{k_2}) \leq \text{mean value of } C \text{ at } v = \{v_{0^\circ}, v_{\pm45^\circ}, v_{90^\circ}\} \quad (13)$$

### 3.3 Second-level optimization

At this level, Genetic algorithm (GA) employed for the local level stacking sequence optimization. Especially for this optimization problem, ply thickness and total global ply number are fixed and ply orientation angles are limited to $[0^\circ/\pm45^\circ/90^\circ]$. So design on the stacking sequence is then limited to permutation of given plies, but not to changes in the number of plies or each orientation. For this reason, a permutation genetic algorithm is an ideal tool than standard GA. Using perm GA, we do not need additional constraints on volume fractions and number of plies. As shown in below, permutation GA performs a slightly different mutation and crossover operation with standard GA:

1) **Mutation:** Two numbers are selected and swapped, e.g., second and fifth :

$$[1 \ 2 \ 3 \ 4 \ 5] \Rightarrow [1 \ 5 \ 3 \ 4 \ 2]$$

2) **Crossover:** Two numbers are selected and the numbers in between flipped, e.g., third and fifth :

$$[1 \ 2 \ 3 \ 4 \ 5] \Rightarrow [1 \ 2 \ 5 \ 4 \ 3]$$

The total plies with optimized volume fraction became an individual in the population. In the second-level, the number of plies with a fiber orientation of $[0^\circ/\pm45^\circ/90^\circ]$, respectively, rounded off according to the volume fraction determined in the first-level. The main target of second-level optimization is to determine the optimal stacking sequence for each region divided by the number of plies. The fitness function is a penalty function for violation of composite design rules. The adopted composite design rules are following as [14]:

1) Stacking sequences have to be symmetric about mid-plane.
2) Stacking sequences have to be balanced, with the same number of $+\theta$ and $-\theta$ plies (with $\theta$ different from 0 to 90)
3) A minimum of 10% plies in each of $0^\circ$, $\pm45^\circ$, and $90^\circ$ is required.
4) For damage-tolerance requirements, the outer plies should have patterns of $[\pm45^\circ]$, $[45^\circ/\theta^\circ/-45^\circ]$, or $[-45^\circ/\theta^\circ/45^\circ]$. The fiber orientation angle of outermost ply should not be $0^\circ$.
5) No more than three plies with the same orientation angle should be stacked together.

The penalty function did not include symmetric condition and minimum ply percentage condition. Because this model was symmetrically stacked and the lower limit of the volume fraction for each orientation angle was determined to be 10% at the first-level. In addition, because there are regions
with the different number of stacked plies, we designed the stacking sequence of each region to be balanced. Finally, using lamination parameters of stacking sequence, the approximated value of compliance is calculated. The stacking sequence that satisfies the compliance constraint, and has a smaller value of compliance is selected as the optimal stacking sequence.

4. Numerical results
In order to verify the proposals, the multi-level optimization method employed for the composite fan blade model described in Section 2. Each ‘region’ represents a set stacked with the same number of plies in the full model. The stacking sequence of each region is dependent on each other, and ply stacked in one region creates stacking sequence by stacking additional ply on the region with a smaller number of plies. And the material properties used in this study shown in Table 1.

Table 1. Material properties for analysis

| Property                        | Value  | Property                        | Value  |
|---------------------------------|--------|---------------------------------|--------|
| Elastic Modulus (E1)            | 159 GPa| Tensile Strength (Xt)           | 1.827 GPa|
| Elastic Modulus (E2)            | 8.96 GPa| Compressive Strength (Xc)       | 1.236 GPa|
| Shear Modulus (G12)             | 4.69 GPa| Tensile Strength (Yt)           | 0.044 GPa|
| Poisson’s ratio (v12)           | 0.316  | Compressive Strength (Yc)       | 0.199 GPa|
| Material density                | 1580 kg/m³| Shear Strength (S)              | 0.085 GPa|

At the first-level, we created approximation models of objective function and constraints in terms of volume fractions, using the response surface method. From these models and gradient-based method, the optimal solution was \( v = \{0.47, 0.43, 0.10\} \) which indicates that the total laminate consists of 47 %, 43 %, 10 % of plies with a fiber orientation of \([0°/±45°/90°]\), respectively. Since the total number of plies are 112, the full model contains 52 plies of 0°, 48 plies of ±45° and 12 plies of 90°. The approximation model for displacement predicted the maximum tip displacement of 2.441 mm when the plies are present with the optimized volume fraction. Next, at the second-level, the optimal stacking sequence was designed by the result from the first-level and permutation GA. Figure 3 shows the FEM results of optimized model. And Table 2 shows the comparison of the results for three models: The FEM results of the initial model, the results from first and second-level optimization.

Table 2. Comparison of results

|                  | Initial (FEM) | First-level | Second-level |
|------------------|---------------|-------------|--------------|
| Contour Plot     |               |             |              |
| Displacement     |               |             |              |
| Max: -2.42E+00   |               |             |              |
| Grids 1473       |               |             |              |
| Min: -6.00E+00   |               |             |              |
| Grids 5          |               |             |              |

Figure 3. FEM results of optimized model
Comparing the results of the finite element analysis between the initial model and the optimized model, the maximum displacement decreased by 11.8 % while the maximum failure index and first natural frequency did not violate the constraints applied in the first-level. Moreover, the approximation model created by response surface method predicts the structural response of the volume fraction reasonably. And Table 3 shows some of the optimal stacking sequences from permutation GA. In Table 3, we numbered each region in order from the minimum number of plies to the maximum number of plies. That is, plies are gradually build up from the thinnest laminate to the thickest laminate. As shown in Table 3, the stacking sequence designed in the second-level optimization satisfy the composite design rules. And they also satisfy the compliance constraint. Therefore, we can say that we have designed the optimal stacking sequence of composite fan blade using the proposed multi-level optimization method.

### 5. Conclusion

In this paper, the multi-level optimization method for designing the stacking sequence of a composite fan blade with variable stiffness was developed. At the first-level, finite element analysis was performed on the design points generated by the DOE to apply the response surface method. At the response surface approximation model, the responses were structural responses such as tip displacement, Tsai-wu failure index, and natural frequency. The variables were volume fractions of plies having predefined angles of $[0°/±45°/90°]$, respectively. These approximation models employed as the objective function or constraint functions for the first-level optimization problem. The obtained result through the first-level optimization was volume fractions of plies, which minimize the tip displacement while do not violate the constraints. In addition, we assumed that the structural compliance can be expressed as a function of lamination parameters $V_1$ and $V_3$ within a determined volume fraction $\nu = \{V_0, V_{±45}, V_{90}\}$. With this assumption, we formulated additional constraint for second-level optimization. This constraint determines whether the result of optimization shows the expected higher structural performance in the specified volume fraction. At the second-level, we adopted permutation GA instead of standard GA to find the optimal stacking sequence. The number of plies having angles of $[0°/±45°/90°]$ is constrained by the determined volume fraction. Since the penalty function evaluates for violation of composite design rules such as balanced condition and ply contiguity, so the object of the second level is to minimize the penalty function. The results from permutation GA evaluated again by compliance constraint, and finally, optimal stacking sequence is selected. From numerical results of applying this strategy to the composite fan blade model, the optimal volume fraction of plies was $\nu = \{0.43, 0.47, 0.1\}$, respectively. The tip displacement...
decreases about 11.8% after optimization, while all of the constraints were satisfied. Furthermore, we can check the effectiveness of the approximation model created by response surface method because it reduced the computational cost considerably. And the design space of second-level optimization was effectively reduced by the first-level optimization. Therefore, it is expected that the optimization of variable stiffness laminates such as fan blade could be performed by multi-level optimization method using the response surface method and permutation GA.

Acknowledgements

This research was supported under the framework of the Aerospace Technology Development Program (No.10074270) funded by the Ministry of Trade, Industry & Energy (MOTIE, Korea).

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