Theory of quasi-ballistic FET: steady-state regime and low-frequency noise.

M. Yelisieiev\textsuperscript{1} and V. A. Kochelap\textsuperscript{2}

\textsuperscript{1)Taras Shevchenko National University of Kyiv, Kyiv, Ukraine\textsuperscript{2)} Institute of Semiconductor Physics, NAS of Ukraine, Kyiv, Ukraine

We present the theoretical analysis of steady state regimes and low-frequency noises in quasi-ballistic FETs. The noise analysis is based on the Langevin approach, which accounts for the microscopic sources of fluctuations originated from intrachannel electron scattering. The general formulas for local fluctuations of the carrier concentration, velocity and electrostatic potential as well as, their distributions along the channel are found as functions of applied voltage/current. Two circuit regimes with stabilized current and stabilized voltage are considered. The noise intensities for the devices with different ballisticity are compared.

We suggest that the presented analysis makes better comprehension of physics of electron transport and fluctuations in quasi-ballistic FETs, improves their theoretical description and can be useful for device simulation and design.

I. INTRODUCTION

In short channel field effect transistors (FETs), electrons experience only a few collisions with defects and phonons during the transient time, while for typical electron concentrations electron-electron collisions are dominating and cause hydrodynamic behavior of the electron gas. For such a physical situation, Dyakonov and Shur have proposed to model the electron transport as that of a charged fluid, which is confined in a narrow layer and governed by hydrodynamic equations\textsuperscript{1}. The electrons are characterized by the area density, \(n\), and by the drift velocity, \(v\), induced by source-drain electric bias, \(\phi\). In the frame of the gradual channel approximation\textsuperscript{2}, the local potential is supposed to be proportional to the electron density.

The complete system of equations for the Dyakonov-Shur model of quasi-ballistic FET reads:

\[
\frac{\partial n}{\partial t} + j = 0, \quad j = nv, \quad \text{(2)}
\]

\[
\phi(x,z) = -\frac{4\pi e}{\kappa} n(x)z + \phi_g \quad (0 \leq z \leq h). \quad \text{(3)}
\]

These equations are for the frame of reference presented in Fig. 1 where geometry parameters of the FET under consideration are indicated; the conductive layer and the gate are situated at \(z = h\) and \(z = 0\), respectively. The voltage applied to the gate is \(\phi_g\); \(m\) and \(-e\) are the electron effective mass and the electron charge, \(\kappa\) is the dielectric constant, \(\tau\) is an electron relaxation time, \(j\) is the electron flux density.

In Dyakonov-Shur paper\textsuperscript{1} and numerous subsequent publications\textsuperscript{2}, Eqs. (1) - (3) have been applied mainly for time-dependent problems focusing to ultra-high frequency instabilities arising at specific boundary conditions (i.e., at suitable microwave environment).

Meanwhile, for steady-state conditions, Eqs. (1)-(3) permit the finding of exact analytical solutions. Analytical solutions independent on numerical methods are always important to make general statements, including qualitative conclusions. In this paper we obtain solutions for all variables \(v(x), n(x), \phi(x)\), present analysis of the current-voltage characteristics, particularly, we clarify the origin of what is called "pinch-off" effect. On the base of the steady-state solutions we studied non-equilibrium electron fluctuations in the FET channel.

In general, analysis of current- and/or voltage noises in devices, including FETs, is a very complex problem because of a number of factors affecting electron fluctuations, among them highly nonuniform carrier distributions and drift velocities along the active channels, both are induced by applied voltages, etc. These and other effects characteristic for nonlinear electron transport in quasi-ballistic FETs are significant and improved theoretical analysis is relevant from viewpoints of the device physics and device applications.

The model under consideration and analytical solutions facilitate the noise analysis. Below we present the analytical study of low-frequency noise in quasi-ballistic FETs. The analysis is based on the Langevin approach, which accounts for the microscopic sources of fluctuations originated from intrachannel electron scattering. The general formulas for local fluctuations of the carrier concentration, velocity and the electrostatic potential, and their distributions along the channel are found as functions of applied voltage/current. This enabled to reveal the effect of electron correlations under the metal gate on the fluctuations. Two circuit regimes are considered: (A) stabilized current regime (suppressed ac current fluc-
tions) and (B) stabilized voltage regime (suppressed ac voltage fluctuations). Respectively, the spectral densities of voltage-noise and current-noise in the FETs are derived. Comparing results for the devices with different ballisticity degrees, we have concluded, that at a given current, FETs with larger ballisticity of the active channels demonstrates larger low-frequency voltage noises for the circuit A and smaller current noises for the circuit B relatively to more dissipative channels.

II. THE STEADY-STATE SOLUTIONS

For the steady-state, Eq. (2) gives for the electron flux density: \( j = n \nu = j_0 \) with \( j_0 \) being an integration constant. It is convenient to solve the system (1)-(3) at a given \( j_0 \) and then to find the voltage drop on the device, \( \phi(L_x) \). The boundary condition for Eq. (1) is \( v(0) = j_0/\nu_s \), where \( \nu_s \) is the electron area density near the source (\( \nu_s = n(0) \)). For what follows, we introduce dimensionless variables and parameters:

\[
V = \frac{v}{\nu_s}, \quad N := \frac{n}{\nu_s}, \quad \xi = \frac{x}{L_x}, \\
\Phi(\xi) = \frac{[\phi(\xi, -h) - \phi(0)]}{\nu_s \kappa}, \quad u_{sc} = \frac{4\pi e^2 \hbar n_s}{\kappa}, \\
J = \frac{j_0}{j_{sc}}, \quad j_{sc} = \sqrt{\frac{4\pi e^2 h^3 n_s}{m \kappa}}, \quad B = \frac{L_x}{\tau} \sqrt{\frac{m \kappa}{4\pi e^2 h n_s}}.
\]

Here the scaling parameters for the potential, \( u_{sc} \), and the flux, \( j_{sc} \), account for the effect of interaction of the electrons with the metal gate. Factor \( B \) is the only parameter dependent on kinetic characteristic, which is the electron relaxation time, \( \tau \).

In these notations we obtain the equation for \( N \):

\[
N^3 - J^2 \frac{dN}{d\xi} = -JB 
\]

with \( N(0) = 1 \). Solution of Eq. 7 gives implicit dependence \( N(\xi) \):

\[
J^2 \left( 1 - \frac{1}{N} \right) + \frac{1 - N^2}{2} = JB \xi.
\]

Setting \( \xi = 1 \) in the latter equation, we find the dimensionless concentration at the drain, \( N(1) \), and the voltage drop on the constrictive channel, \( U = \Phi(1) - \Phi(0) = N(0) - N(1) \approx 1 - N(1) \). Then, Eq. 7 leads to the following relationship between \( J \) and \( U \):

\[
L(J, U) = \frac{U(2 - U)}{2J} - J \frac{U}{1 - U} = B.
\]

At a given \( J \), the function \( L(J, U) \) has maximum at \( U = 1 - J^{2/3} \). Thus, the following equation determines maximal possible \( J = J_c(B) \) and corresponding voltage drop \( U_c = 1 - J_c^{2/3} \) allowable in the model under consideration. Eq. 9 has solutions (two solutions) only at \( J \leq J_c(B) \). The case \( B \ll 1 \) corresponds to almost ballistic electron transport. For them \( J_c \approx 1 - \sqrt{3B/2} \). The opposite case, \( B \gg 1 \), is relevant to dissipative transport with \( J_c \approx 1/2B \). Yet, for parameters allowing the solutions there exists additional restriction: \( JB \leq 1 \). Of two branches of the dependence \( J(U) \) we shall select that corresponding to a positive flux \( (J, V > 0) \). This gives the dimensionless current-voltage characteristic:

\[
J = -\frac{B(1 - U)}{2U} + \sqrt{\frac{B^2(1 - U)^2}{4U^2} + \frac{(1 - U)(2 - U)}{2}}.
\]

In Fig. 3 we present the characteristics of the FETs for two values of \( B \) (\( B = 0.5 \) and \( B = 2 \)). The current-

![FIG. 2. Dimensionless saturation current \( J_c \) as a function of parameter \( B \)](image)

![FIG. 3. Dimensionless characteristics of quasi-ballistic FETs for \( B = 0.5 \), \( J_c = 0.415 \), panels (a), (b), (c) and \( B = 2 \), \( J_c = 0.187 \), panels (d), (e), (f), (a), (d): Current-voltage characteristics; current saturation portions are shown conditionally. (b), (e): Dimensionless velocities, \( V(\xi) \). (c), (f): Electron concentrations, \( N(\xi) \). Curves 1, 2 and 3 correspond to currents 0.5\( J_c \), 0.9\( J_c \) and \( J_c \). Curves 3 on panels (c), (f) clearly demonstrate the absence of real pinch-off effect in quasi-ballistic FETs.](image)
voltage characteristics allowed in the model are shown in the panels (a) for $J \leq J_c(0.5) = 0.41$, $U \leq U_c(0.5) = 0.43$ and $J \leq J_c(2) = 0.187$, $U \leq U_c(2) = 0.67$. At small voltage bias, Eq. (11) gives the following current dependence:

$$J \approx \frac{U}{B} \frac{U^2}{2B} - \frac{U^3}{B^3} \ldots$$

(12)

Thus, parameter $B$ is the dimensionless FET resistance in the linear regime. At arbitrary $J$ and $U$ the dimensionless differential resistance can be found in the parametrical form:

$$R_D = \frac{N(1)[1 - N(1)] [2J^2 - N(1)(1 + N(1))]}{2J^2 - N^2(1)}$$

(13)

where $N(1) = 1 - U$ and $J(U)$ is given by Eq. (11). At large $U$, i.e., beyond the model applicability, the currents should saturate. In Fig. 3(a) the saturation portions are shown conditionally. Note, from Fig. 2 it follows that the critical current $J_c$ decreases with increasing of parameter $B$ (i.e., at larger $L$ and/or smaller $\tau$).

In the panel (b) and (c) of these figures, distributions of the electron velocities and densities along the channel are presented. These distributions are linear near the source:

$$N(\xi) \approx 1 - \frac{JB}{1 - J^2} \xi, \quad V(\xi) = 1 + \frac{JB}{1 - J^2} \xi.$$

(14)

At larger $\xi$ the distributions, generally, are nonlinear (concave and convex dependencies, respectively). As $J \rightarrow J_c$, one can find:

$$N(\xi) \approx J_c^{2/3} + \frac{2}{3J_c^{1/3}} B^{1/2} \sqrt{1 - \xi},$$

$$V(\xi) \approx J_c^{1/3} - \frac{2}{3J_c^{2/3}} B^{1/2} \sqrt{1 - \xi}.$$

Similarly, the potential distributions near the source and the drain are

$$\Phi(\xi) \approx \Phi(0) + \frac{JB}{1 - J^2} \xi, \quad (\xi \ll 1)$$

$$\Phi(\xi) \approx J_c^{2/3} + \frac{2}{3J_c^{1/3}} B^{1/2} \sqrt{1 - \xi}, \quad (\xi \rightarrow 1, \ J \rightarrow J_c).$$

Therefore, we find that at the critical current, $J_c$, the electron density is nonzero everywhere in the conductive channel, and the velocity remains a finite value. The former means that in quasi-ballistic FETs actually there is no pinching-off of the conductive channel. The pinch-off effect ($N(1) \rightarrow 0$) evidences at large $B$ (long channel, $L \rightarrow \infty$, and/or strongly dissipative transport, $\tau \rightarrow 0$). While the electric field in the channel diverges in this limit: $\frac{d\Phi}{d\xi} \rightarrow \infty$ at $J \rightarrow J_c$.

III. LOW-FREQUENCY NOISE

Among different sources generating current/voltage noises in FETs, we focus on the low-frequency noise of an intrinsic nature, namely, that caused by random parameter of electron scattering in the conductive channel of FET. In the frame of the Langevin approach\textsuperscript{2,11} this type of noise can be evaluated by the use of linearized Eq. (1) supplemented with a random force $f(x, t)$. The ensemble average of the latter function is $\bar{f}(x, t) = 0$ and its properties are determined by a correlator $\bar{f}(x, t)\bar{f}(x', t')$, for which it is assumed that\textsuperscript{15}

$$\bar{f}(x, t)\bar{f}(x', t') = g(x, t) \delta(x - x') \delta(t - t').$$

(15)

with $g(x, t)$ defined by random scattering and a local density, and velocity distribution of the carriers. Here we consider the fluctuations around steady-state of the FET, thus dependence on time is absent: $g(x, t) = g(x)$. Introducing $f_\omega(x)$ as Fourier transformation of the force $f(x, t)$, one can transform Eq. (15) to the form:

$$f_\omega(x) f_\omega(x') = \frac{1}{2\pi} g(x) \delta(x - x') \delta(\omega + \omega').$$

(16)

We present the variables as $v = v_1 + n_1 \phi + \phi_1$, with $v_1, n_1, \phi_1$ being the fluctuation values. The Langevin equation reads

$$\frac{\partial v_1}{\partial t} + v_1 \frac{\partial v}{\partial x} + v \frac{\partial v_1}{\partial x} + \frac{v_1}{\tau} = \frac{e}{m} \frac{\partial \phi_1(x, t)}{dx} + \frac{1}{m} f_\omega(x, t).$$

(17)

Two other equations for $n_1$ and $\phi_1$ can be obtained from Eqs. (2), (3) substituting $n_2 \rightarrow n_1, \phi_2$ and $j \rightarrow j_1 = n_1 \nu + n_1 v$. It is convenient to introduce the dimensionless fluctuation values $N_\omega(\xi, t), V_\omega(\xi, t), \Phi_\omega(\xi, t)$, as done in relationships (4), (5). For the noise characteristics we define the Fourier transformations for all variables: $N_\omega(\xi, t), V_\omega(\xi, \phi_1(\xi, t), \Phi_\omega(\xi, t)$, as done in relationships (4), (5). For the noise characteristics we define the Fourier transformations for all variables: $N_\omega(\xi, t), V_\omega(\xi, t), \Phi_\omega(\xi, t), J_\omega, U_\omega$ and $f_\omega(\xi)$. Generally, temporal current/voltage fluctuations are dependent on microwave environment of the device. However the master equation for the fluctuations can be found for arbitrary external electric circuits, which provides $J_\omega \neq 0$ and $U_\omega \neq 0$. Then, the fluctuation flux normalized to $j_{sc}$ is

$$J_\omega = \frac{j_1}{j_{sc}} = V_\omega N + N_\omega V.$$

(18)

For low-frequency fluctuations one can drop derivatives with respect to time, as well as terms proportional to $\omega$ in equations for the Fourier components. This is valid for $\omega \tau \ll 1$ and $\omega \ll v_L/L_2$. As the result, we obtain the following equation in terms of $V_\omega$:

$$\frac{dV_\omega}{d\xi} - \frac{JB}{J^2 - N^2} \frac{3N^4}{J^2 - N^2} V_\omega = \frac{\tau}{mv_L} \frac{JBN}{J^2 - N^2} f_\omega(\xi) - \frac{JBN^3}{J^2 - N^2} J_\omega.$$

(19)

This is nonhomogeneous differential equation of the first order with $\xi$-dependent coefficients expressed through the steady-state solution $N(\xi)$:
with

\[ P(\xi) = JB \frac{3N^4(\xi)}{J^2 - N^3(\xi)} \]
\[ Q(\xi) = \frac{\tau n_s JBN(\xi)}{m j_0 J^2 - N^3(\xi)}, \quad W(\xi) = -\frac{JBN^3(\xi)}{[J^2 - N^3(\xi)]^2}. \]

Restricting ourselves by the intrinsic sources of the fluctuations, we set the boundary condition to this equation in the form: \( V_\omega(0) = 0 \) (short-circuited gate-drain part of the circuit at finite frequencies), for Eq. (20) this tells \( V_\omega(0) = J_\omega/N(0) = J_\omega. \) Now, we easily find the solution of Eq. (20):

\[ V_\omega(\xi) = J_\omega e^{\int_0^\xi d\xi' P(\xi')} + \int_0^\xi d\zeta [Q(\xi)f_\omega(\zeta) + W(\xi)J_\omega] e^{-\int_0^\zeta d\xi' P(\xi')} \quad (21) \]

The fluctuation flux, \( J_\omega, \) can be determined, when the external electric circuit is specified.

### A. The circuit with suppressed current fluctuations (the stabilized current regime).

For this case we set \( J_\omega = 0 \) and Eq. (21) can be rewritten in the form:

\[ V_\omega^A(\xi) = \int_0^\xi d\xi' Q(\xi)f_\omega(\xi)e^{-\int_0^\xi d\xi' P(\xi')} = \int_0^\xi d\zeta K(\xi, \zeta)f_\omega(\zeta). \quad (22) \]

(The results for fluctuations obtained in this Subsection are labeled by the upper mark \( 'A' \).) Using dependence \( N(\xi) \) implicitly given by Eq. (3) we can calculate the integrals in Eq. (21) and the value \( K(\xi, \zeta) \) in terms of \( N(\xi), N(\zeta). \) Indeed, according to Eq. (17)

\[ d\xi = \frac{J^2 - N^3}{JBN^2} dN \]

we can change integration over \( \xi' \) by that over \( N: \)

\[ \int_0^\xi P(\xi') d\xi' = \frac{\int_0^N(\xi) 3N^2 dN}{J^2 - N^3} = \ln \left[ \frac{J^2 - N^3(\xi)}{J^2 - N^3(\zeta)} \right] \]

and find

\[ K(\xi, \zeta) = \frac{\tau n_s JB}{m j_0} \frac{N(\zeta)}{(J^2 - N^3(\xi))} = K_N[N(\xi), N(\zeta)]. \quad (23) \]

The correlator for the velocity fluctuations is

\[ \frac{dV_\omega}{d\xi} - P(\xi)V_\omega = Q(\xi)f_\omega + W(\xi)J_\omega, \quad (20) \]

\[ \int_0^\xi f_\omega(\xi)f_\omega'(\xi') = \int_0^\xi d\xi' d\xi'' K(\xi, \zeta)K(\xi', \zeta')f_\omega(\zeta)f_\omega(\zeta'). \]

Here \( f_\omega(\xi)f_\omega'(\xi') \) can be obtained from Eqs. (16) and (17) by substitution \( x, x' \rightarrow \zeta L_x, \zeta' L_x: \)

\[ f_\omega(\xi)f_\omega'(\xi') = \frac{1}{2\pi} g(\xi)\delta(\xi - \xi')\delta(\omega + \omega') \]
with
\[ g(\zeta) \equiv \frac{1}{L_x} g(\zeta L_x) = \frac{2 D m^2}{\tau^2 n(\zeta) L_x L_y}. \]

The above obtained correlator can be rewritten as
\[ \frac{V_s^2(\xi)}{V_0^2(\xi)} = \frac{1}{2\pi} \delta(\omega + \omega') \int_0^\xi d\zeta \frac{\overline{g}(\zeta) K(\xi, \zeta) K(\xi', \zeta)}{\overline{g}(\zeta)}. \] (24)

Using Eqs. (A6), (A7) we obtain the spectral density of the electron velocity fluctuations in a point in the form:
\[ S^A_{V_s}(\xi) = 2 \int_0^\xi d\zeta \overline{g}(\zeta) [K(\xi, \zeta)]^2 = \frac{2}{J B} \int_1^N dN \frac{J^2 - N^3}{N^2} \overline{g}(N) (K_N[N(\xi), N])^2. \] (25)

Here \( \overline{g}(\zeta) \) is expressed through \( N(\zeta) \) using implicit dependence of Eq. (25); \( \overline{g}(N) \equiv \overline{g}(\zeta(N)) \). Calculation of the integral in Eq. (25) gives
\[ S^A_{V_s}(\xi) = \frac{4 D n_s}{j_0 L_x L_y} \Psi^A(\xi, J), \]
\[ \Psi^A(\xi, J) \equiv \frac{1 - N^3(\xi) + 3 J^2 n N(\xi)}{3 J^2 - N^4(\xi)} = \frac{2 N(\xi)}{J B} \right. \]
\[ \left. \Psi^A(\xi, J) \equiv \int_1^N dN \frac{J^2 - N^3}{N^2} \overline{g}(N) (K_N[N(\xi), N])^2. \] (26)

The spatial distribution of the spectral density of the fluctuations of the dimensional velocity, \( v_{\xi} \), is given by
\[ s^A_{v_{\xi}}(\xi) = \frac{4 D}{n_s L_x L_y} \Psi^A(\xi, J). \] (27)

At small \( J \) (or \( U \)), we find this spatial distribution in the simple form
\[ s^A_{v_{\xi}}(\xi) \approx \frac{4 D (J B)^2}{n_s L_x L_y} \left( \frac{1}{2} + 15 J B \xi^2 + \ldots \right) = \frac{4 D U^2}{n_s L_x L_y} \left[ \xi + \left( \frac{1}{2} \xi + \xi^2 \right) U + \ldots \right]. \]

That is, the spectral density of the velocity fluctuations increases along the electron flux and the rate of this increase is quadratic on \( J \) (or \( U \)) at small currents. For finite currents, the spatial distributions of the velocity fluctuations are presented in Fig. 4(a). The fluctuations increase considerably at the drain side of the conductive channel. At \( J \to J_c \) and \( \xi \to 1 \), Eq. 27 predicts infinitely high fluctuation intensity:
\[ s^A_{v_{\xi}}(\xi) \propto \frac{1}{\left[ J_c - J - 2 \sqrt{B (1 - \xi)} \right]^2}. \]

Eq. (26) facilitates finding the spectral density fluctuations of the concentration and the potential as functions of \( \xi \): \( s^A_{N_s}(\xi) = s^A_{\phi_s}(\xi) = N^4(\xi) s^A_{V_s}(\xi) \). These dependencies are presented in Fig. 4(b). In the dimensional form for the spectral density fluctuations of the potential we obtain
\[ s^A_{\phi_s}(\xi) = \left( \frac{A \epsilon c h m_s}{e^2} \right)^2 s^A_{N_s}(\xi) = \left( \frac{4 D m^2 L_x}{e^2 n_s \tau^2 L_y} \right) \Psi^A(\xi, J), \]
\[ s^A_{N_s}(\xi) = \frac{4 D m^2 L_x}{e^2 n_s \tau^2 L_y}. \] (28)

Bellow we will show that the quantity \( s^A_{N_s}(\xi) \) coincides with the Nyquist density of voltage-noise for the active channel. At small \( J \) (or \( U \)) we find
\[ s^A_{\phi_s}(\xi) \approx s^A_{N_s}(\xi) \left( J B \xi^2 \right) = s^A_{N_s}(\xi) (J + U \xi^2). \]

For finite currents the spatial distribution of the potential fluctuations in the channel is similar to that of previously studied spatial distributions of the velocity fluctuations, including infinite growth of these fluctuations at \( J \to J_c \) and \( \xi \to 1 \). Fluctuations of the total voltage drop in the FET, \( u_\omega = \phi_s(1) \), are of a finite value at small currents:
\[ s^A_{u_\omega} = s^A_{N_s}(1) \left( 1 + \frac{3}{2} U + \ldots \right) \] (29)

they increase with the current/voltage and become infinitely high at \( J \to J_c \), as illustrated in Fig. 5(a).

B. The circuit with suppressed voltage fluctuations (the voltage stabilized regime).

For this case the fluctuations of the total voltage drop is zero, \( U_\omega = \phi_s(1) = -N_\omega(1) = 0 \). This leads to the condition \( V_s^2(\xi) \to J_\omega/N(1) \), which can be satisfied at
\[ J_\omega = \frac{N(1)}{\Delta} \int_0^1 d\xi Q(\xi) L_s(\xi) e^{-\int_0^\xi d\zeta P(\zeta)} = \frac{N(1)}{\Delta} V_s^2(1), \]
\[ \Delta = 1 - N(1) \left[ e^{\int_0^1 d\zeta P(\zeta)} + \int_0^1 d\zeta W(\xi) e^{-\int_0^\xi d\zeta P(\zeta)} \right], \]

where \( V_s^2(1) \) is given by Eq. 22 for \( \xi = 1 \). The above relationships allow easy to find the correlator \( J_\omega/J_\omega \), and the spectral density of the total current fluctuations in the FET for the circuit with the voltage stabilized regime:
\[ S^B_{I_s} = \frac{N^2(1)}{\Delta^2} S^A_{V_s}(1) \] (30)
with \( S^B_{s}(\xi) \) defined by Eq. (25). (The results obtained in this Subsection are labeled by the upper mark \( 'B' \).)

Calculations give us the spectral density of the current fluctuations in the analytical form
\[ S^B_{I_s} = \frac{4 D n_s}{j_0 L_x L_y} \Psi^B(J), \] (31)
\[ \Psi^B(J) = \frac{4}{3} J B^3 \frac{N^2(1) [1 - N^3(1) + 3 J^2 n N(1)]}{[1 - N(1)]^2 [2 J^2 + N(1)(1 + N(1))]^2}. \]
Together with relationship \( N(1) = 1 - U \) and Eq. 11, these give the voltage/current dependence of the fluctuation deterministic fluctuations for the considered case. At small \( U \) (or \( J \)) Eq. (30) gives

\[
S_{J}^{B}(\omega) = \frac{4Da}{J_0^2L_yL_y} \left( 1 - \frac{1}{2}U \right) (J, U \to 0). \quad (32)
\]

For finite value of the applied current/voltage, the fluctuation spectral densities are illustrated by Figs. 5 (b). The dimensional form of the spectral density of the current fluctuations in the active channel of the width \( L_y \) can be recovered as follows:

\[
S_{J}^{B}(\omega) = e^2J_0^2L_y^2S_{J}^{N}(\omega). \quad (33)
\]

In particular, from Eq. (32) it follows, that at small currents/voltages the spectral density of current fluctuations is equal

\[
S_{J}^{N}(\omega) = 4De^2n_s \frac{L_y}{L_x}, \quad (34)
\]

which is the Nyquist result for the current fluctuations (see discussion below). As seen from Fig. 5 (b), in the contrast to the voltage fluctuations at the stabilized current regime shown in Fig. 5 (a), the current fluctuations in the voltage stabilized regime are always finite, decrease with growing current/voltage drop reaching a minima at the pinch-off voltage.

**IV. DISCUSSION AND CONCLUSIONS.**

The considered model of quasi-ballistic FET admits analytical solutions for spatially dependent electron concentration, \( N(\xi) \), drift velocity, \( V(\xi) \), and potential, \( \Phi(\xi) \), in implicit forms. For example, the coordinate dependence \( N(\xi) \) is given by Eq. 3 (all are dimensionless quantities scaled according to Eqs. (1) - (6)). These dependencies are functions of two dimensionless parameters, the flux (current) \( J \) and the factor \( B \) of Eq. 3. The real voltage and current density can be calculated by using the scaling parameters \( u_{sc} \) of Eq. 54 and \( j_{sc} \) of Eq. 43. The current-voltage characteristic of FET, \( J(U) \), is given by Eq. 12 with the only parameter \( B \), which has the meaning of the dimensionless resistance in the linear operation regime (see Eq. 14). Using notations 14, 15 and writing the total current in the channel as \( -eJ_0L_y \), we found that

\[
B = \frac{e^2n_sL_y}{m} \sqrt{\frac{m\kappa}{4\pi e^2\hbar n_s}}
\]

with

\[
R = \frac{mL_x}{e^2\tau n_sL_y} \quad (35)
\]

being the resistance of the conductive channel with carriers characterized by the scattering time \( \tau \) and the mobility \( \mu = e\tau/m \). The applied model predicts steady-state solutions only for a finite interval of the current \( J \leq J_c(B) \) and voltage \( U \leq U_c \). The value \( J_c \) is determined by Eq. 11, while \( U_c = J_c^2/4 \). In Fig. 2 the filled area of the \( \{J, B\} \)-plane presents allowed current for this model. At \( J > J_c \), when the model is not applicable, a current saturation regime should occur. Detailed discussion of the quasi-ballistic FET for \( J \geq J_c \) is presented in paper 4. Here we only recall that at \( J \to J_c \) electron concentration and velocity remain finite at a finite relaxation time \( \tau \). Thus, no real pinch effect is realized. However, approaching \( J_c \) the lateral electric field increases infinitely.

In the model, besides the parameter \( B \) two scaling parameters \( u_{sc} \) and \( j_{sc} \) are important. The latter parameters are determined by the electron characteristics, charge, mass, concentration, and dielectric surrounding and distance between the channel and the gate, \( h \). Evidently, these parameters reflect the essentiality of electron-metal gate interaction. For numerical estimates, we consider the FET with the GaAs conductive channel at 77 K and 300 K. We set \( m = 0.063m_0, \kappa = 10.9, n_s = 10^{12} cm^{-2} \) and \( h = 5 \times 10^{-3} cm \). Then, the scaling parameters for voltage and current density are...
\[
u_{sc} = 83 \text{ mV} \quad \text{and} \quad \epsilon j_{sc} = 7.3 \text{ A/cm}.
\]
For 77 K at the mobility \(\mu = 3 \times 10^5 \text{ cm}^2/\text{Vs}\), we find \(\tau \approx 10^{-11} \text{ s}\) and \(B = 1 \approx 3.4 \mu\). Thus, numerical results presented for \(B = 0.5\) and \(B = 2\) correspond to the channel lengths 1.7 \(\mu\) and 6.8 \(\mu\) for the low temperature. For 300 K and \(\mu = 800 \text{ cm}^2/\text{Vs}\), we find \(\tau \approx 3 \times 10^{-13} \text{ s}\) and \(B = 1\) at \(L_x = 0.1 \mu\). Correspondingly, the numerical results for \(B = 0.5\) and 2 are for \(L_x = 0.05 \mu\) and 0.2 \(\mu\).

The found steady-state solutions facilitate analysis of low-frequency electron fluctuations, which originate from random electron scattering in the FET channel. This analysis was performed applying the Langevin approach based on the linearized dynamic equation supplemented by the random force related to electron scattering (see Eq. (17)). Using the microscopic Langevin forces for bulk electrons characterized by a scattering time, \(\tau\), we derived the random force correlator for hydrodynamic equation of the electron flux in narrow active layer of FET (see Eq. (A4)). At low frequency fluctuations (\(\omega \tau \ll 1\)), the corresponding Langevin equation (19) was solved. Its general solution for spatially dependent fluctuations in the channel can be calculated analytically using the steady-state electron distribution \(N(\xi)\). Particular results of the fluctuation analysis are dependent on properties of the low-frequency ac circuit with the FET. Two cases of such circuits are considered: a circuit with stabilized current (A) and a circuit with stabilized voltage on the FET (B).

For the circuit A, the analytical expression for the spectral density of low frequency electron velocity fluctuations is given by Eq. (25). From this expression it follows that the velocity fluctuations increase along the channel and reach maximal values at the drain side of the FET. They also increase with the current through the device (see Fig. (4) (a)). At the current approaching to the critical value, \(J \rightarrow J_c\), the velocity fluctuations grow infinitely. Similar behavior is observed for spatial distributions of fluctuations of the electron concentration (i.e., charge fluctuations in the channel) and the local voltage (see Eq. (28) and Fig. (4) (b)). As for the fluctuations of the voltage drop on the device, at low current/voltage the spectral density of Eq. (29) coincides with the Nyquist formula. Indeed, \(S_n^A = 4Dm^2L_x/e^2n_x\tau^2L_y = 4kTR\) with \(R\) being the low voltage resistance of the active channel (see Eq. (29)). As the current increases, the voltage fluctuations also increase and become infinitely large at approaching the critical current value, \(J \rightarrow J_c\) (see Fig. (4) (a)).

For the circuit B, the spectral density of current fluctuations is given by expression (31) with \(N(1)\) determined from Eq. (8) at \(\xi = 1\) and Eq. (11). At low voltage Eqs. (32) and (33) recover the Nyquist result for the current fluctuations: \(s^B = 4kTR/R\). As the voltage increases intensity of the current fluctuations decreases reaching minimal value at \(J \rightarrow J_c\) (see Figs. 5 (b)). Though for the circuit B the total voltage drop does not fluctuate, the local concentration (charge) and the potential do fluctuate. Spatial distributions of these fluctuations are not monotonous with maximal intensity in a middle of the channel, as shown in Fig. 4 (c). Note, spatial distributions of charges and electrostatic potentials and their low-frequency fluctuations on nanoscale can be studied using different scanning microscopic methods for specially designed FET with access to surface of the active channel. For example, nanometer scale imaging of the surface potential can be received with the Kelvin probe force microscopy. Such a study can provide additional information on transport processes within the device.

Comparing results for two values of the parameter \(B\) presented in Figs. (3) - (5) and taking into account the Nyquist quantities (28), (34) relevant to these values, we can conclude, that at a given current the device with larger ballisticity of the channel \((B = 0.5)\) demonstrates larger low-frequency voltage noises for the circuit A and smaller current noises for the circuit B in comparison to more dissipative channel of \(B = 2\).

It is pertinent to note that often the Nyquist formulas are used to estimate the current or voltage noises under nonlinear operation regimes with substitution of the low-voltage/current resistance, \(R\), by the differential resistance \(R_D = du/d(-ej)\). For the FET model under consideration, the dimensionless differential resistance is given by Eq. (13). In Figs 5 (a), (b) we presented the results of such estimates. It can be seen that the noise characteristics obtained in this paper differ significantly from mentioned above estimates. Moreover, the correct analysis predicts higher fluctuation intensities for both A and B circuits.

Presented study of noises generated by internal random scattering processes under nonlinear electron transport replenishes the list of reported analytical studies of electron fluctuations in device structures with nonuniform distributions of carriers and fields induced by currents; examples include: noises in \(p-n\)-junctions, \(n^+ - n\)-junctions, excess noise in nonuniform conductive channels, shot noise in injection diodes, hot-electron and intervalley size effects for fluctuations, etc.

The analysis makes better comprehension of physics of electron transport and fluctuations in quasi-ballistic FETs, particularly, revealing the effect of electron correlations under the metal gate on electrical fluctuations. The results improve theoretical description of quasi-ballistic FETs, which is essential from viewpoints of the device simulation and design.

**AUTHOR DECLARATIONS**

Conflict of Interest The authors have no conflicts to disclose.
AUTHOR CONTRIBUTIONS

Mykola Yelisieiev: Investigation (equal); Writing - review and editing (equal). Vyacheslav A. Kochelap: Conceptualization (lead); Supervision (lead); Investigation (equal); Writing - original draft;

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Appendix A: Correlator of random force $f(x,t)$

The Langevin approach and the above introduced random force, $f(x,t)$, are valid under the following conditions. First, the ensemble average of a variable ($v_i$ in our case) over a small time interval $\Delta t$ can be described by the same macroscopic dynamic equation (1). Second, this time interval has to be much greater than correlation time of true microscopic Langevin source (the force $f(r,t)$ in our case). The same is assumed for coordinate dependence of the fluctuation source. These conditions allow to set that the averaged over ensemble correlator of the random force in the dynamic equation is proportional to $\delta(r-r')\delta(t-t')$. The general relationship for a random force is

$$f_i(r,t)f_j(r',t') = g_{ij}(r,t)\delta(r-r')\delta(t-t'), \quad (A1)$$

where $f_i(r,t)$ with $i,j=x,y,z$, are projections of this vector force, functions $g_{ij}(r,t)$ are determined by the local parameters (distribution over velocities, carrier densities, etc).

The latter general equation can be applied to derive the correlator presented in Eq. (10). To find the function $g(r)$ used in the equation for the fluctuations over the steady-state, let us take advantage of the well known expression for the correlator of flux fluctuations, $\delta J_{3D,i}(r,t)$, for three-dimensional (3D) electrons provided in:  

$$\delta J_{3D,i}(r,t)\delta J_{3D,j}(r',t') = 2Dn_{3D}(r)\delta_{ij}\delta(r-r')\delta(t-t'). \quad (A2)$$

Here $D$ is a diffusion coefficients, $n_{3D}(r)$ is the electron density. For nondegenerate electrons one can set $D = \tau_p^2\nu^2/3$ with $\tau_p(\leq \tau)$ and $\nu_T$ being the collision time and the thermal velocity of the electrons, respectively. The density of the electron flux through the cross-section of the conductive channel, $j$, (see Eq. (2)) can be obtained by integration of the bulk flux density, $J_i = n_{3D}\nu_i$, over $z$ and averaging over $y$:  

$$j = j_x = \frac{1}{L_y} \int dz \int_0^{L_y} dy J_{3D,x},$$

(see the device geometry in Fig. [1]). Applying similar procedure for Eq. (A2) we easily obtain the correlator of the local fluctuations of $j_x$:  

$$\delta j_x(x,t)\delta j_x(x',t') = \frac{2}{L_y} \int dz \int_0^{L_y} dy (x')\delta(t-t'). \quad (A3)$$

with $n$ being the area density:

$$n(x) = \frac{1}{L_y} \int dz \int_0^{L_y} dy n_{3D}(r).$$

Here we assume, that the bulk electron density $n_{3D}$ does not depend on $y, z$. To link the correlator of Eq. (A2) and Eq. (A3) we again remind that the correlator of microscopic Langevin sources are determined by the local parameters, $n$ and $D$. This allows us to apply Eq. (A7) for the uniform case and to obtain $\delta j_{x}^{(un)}(x,t) = \frac{n}{m} f(x,t)$ in the limit $\omega \rightarrow 0$. Now using the correlator of Eq. (A3) we find:

$$f(x,t)f(x',t') = \frac{2Dm^2}{\tau^2 n(x)L_y} \delta(x-x')\delta(t-t'), \quad (A4)$$

and

$$g(x) = \frac{2Dm^2}{\tau^2 n(x)L_y}. \quad (A5)$$

Finally, we remind the definition of the spectral density of fluctuations of certain physical value $X(t)$: if correlator of the fluctuations $\delta X(\omega)$ is

$$2\pi \delta X(\omega) = 2\Xi(\omega), \quad (A6)$$

then according to the Wiener Khintchine theorem, the spectral density of the fluctuations equals

$$S_{X,\omega} = 2\Xi(\omega). \quad (A7)$$

1M. Dyakonov and M. S. Shur, Shallow Water Analogy for a Ballistic Field Effect Transistor. New mechanism of Plasma Wave Generation by DC Current, Phys. Rev. Lett. 71, 2465 (1993).
2B G Streetman, S K Banerjee, Solid State Electronic Devices. Prentice Hall Internationsl, Inc. 5-th Edition, 2000.
3V. V. Mitin, V. A. Kochelap, M. A Strescio, Quantum electronics. Microelectrónica and Optoelectronics. Cambridge University Press, 1999.
4M. Dyakonov and M. Shur, Choking of electron flow: A mechanism of current saturation in field-effect transistors, Phys. Rev. B 51, 14341 (1995).
5M. Dyakonov and M. Shur, Novel terahertz devices using two-dimensional electron fluid, IEEE Trans. Electron Devices 43, 1640 (1996).
6F. J. Crowne, Contact boundary conditions and the Dyakonov-Shur instability in high electron mobility transistors, J. Appl. Phys. 82, 1242 (1997).
7F. J. Crowne, Dyakonov-Shur plasma excitations in the channel of a real high-electron mobility transistor, J. Appl. Phys. 87, 8056 (2000).
8F. J. Crowne, Microwave response of a high electron mobility transistor in the presence of a Dyakonov-Shur instability, J. Appl. Phys. 91, 5377 (2002).
9 K. M. van Vliet, Markov approach to density fluctuations due to transport and scattering. I. Mathematical formalism, J. Mat. Phys. 12, 1981 (1971); Markov approach to density fluctuations due to transport and scattering. II. Applications. ibid, 12, 1998 (1971).
10 Sh. Kogan, Electronic Noise and Fluctuations in Solids. Cambridge University Press, 1996.
11 L. D. Landau, E. M. Lifshitz, Statistical Physics. v. 5 of Course of Theoretical Physics, Elsevier, 1980.
12 W Melitz, J. Shen, A. C. Kommel, S. Lee, Kelvin probe force microscopy and its application, Surf. Sci. Rep. 66, 1 (2011).
13 K. M. van Vliet and J. R. Fassett " Fluctuations due to Electronic Transitions and Transport in Solids", in Fluctuation Phenomena in Solids (R.E. Burgess, Ed.), Academic Press, NY, 1965, p. 151.
14 O. M. Bulashenko, G. Gomila, J. M. Rubi, and V. A. Kochelap, Spatial correlations across $n^+ - n$ semiconductor junctions Appl. Phys. Lett. 70, 3248 (1997).
15 O. M. Bulashenko, G. Gomila, J. M. Rubi, and V. A. Kochelap, Extension of the impedance field method to the noise analysis of a semiconductor junction: Analytical approach, J. Appl. Phys. 83, 2610 (1998).
16 Gomila G., Bulashenko O.M., Rubi J.M., Local noise analysis of a Schottky contact: combined thermionic emission and diffusion theory, J. Appl. Phys, 83, 2619 (1998).
17 O. M. Bulashenko, J. M. Rubi, V. A. Kochelap, Excess noise caused by transverse inhomogeneity of conductiv channels, Appl. Phys. Lett., 73, 217 (1998).
18 Gonzalez T., Gonzalez C., Mateos J., Pardo D., Reggiani L., Bulashenko O.M., Rubi J.M., Universality of the 1/3 shot-noise suppression factor in nondegenerate diffusive conductors, Phys. Rev. Lett., 80, 2901 (1998).
19 T. Gonzalez, C. Gonzalez, J. Mateos, D. Pardo, L. Reggiani, Microscopic analysis of shot-noise suppression in nondegenerate diffusive conductors, Phys. Rev. B, 60, 2670 (1999);
20 Bulashenko O.M., Rubi J.M., Kochelap V.A., Sub-Poisson current noise in ballistic space-charge-limited diodes with linear I-V characteristics, Appl. Phys. Lett., 75, 2614 (1999).
21 O. M. Bulashenko, J. M. Rubi, V. A. Kochelap, Suppression of non-Poissonian shot noise by Coulomb correlations in ballistic conductors, Phys. Rev. B 62, 8184 (2000).
22 V. A. Kochelap and V. N. Sokolov, N. A. Zakhleniuk, Limitation and suppression of hot-electron fluctuations in sub-micrometer semiconductor structures, Phys. Rev. B, 48, 2304 (1993).
23 V. A. Kochelap and V. N. Sokolov, Size effects in fluctuation spectra of many-valley semiconductors, Phys. Rev. B 57, 15465 (1998).