Extremal dilatonic black holes in 4D Gauss-Bonnet gravity

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This is a report of our recent investigation on the extremal dilatonic black holes in four dimensional Gauss-Bonnet gravity. We found that a global solution can exist only when the dilaton coupling is less than a critical value which can be determined numerically. Moreover, the black hole horizon is stretched by the Gauss-Bonnet correction and the entropy is twice the value given by Bekenstein-Hawking formula.

§1. Introduction

It is well known that the spherical symmetric charged black holes of the Einstein-Maxwell gravity are described by the Reissner-Nordström (RN) solutions. This class of solutions carries two conserved quantities, namely mass $M$ and electric charge $Q$, and they are asymptotically flat. There are two essential geometric characteristics of those spherical charged black holes. First, there exist a singular point (singularity) at $r = 0$ where the curvature diverges. Second, there are, in general, two apparent singular 2-surfaces located at $r_{\pm} = M \pm \sqrt{M^2 - Q^2}$. Indeed, these two surfaces are coordinate singularities which correspond to the outer (event) and inner (Cauchy) horizons. The existence of an (event) horizon is physically crucial, inspiring the cosmic censorship conjecture: the singularity should be protected by (event) horizon.

The black hole mechanism behaves like a thermal system. The connection was first observed by Bekenstein via comparing the area increasing theorem of black holes and the 2nd law for entropy in thermodynamics. Soon the other corresponding thermodynamical laws also were derived from black hole mechanisms. The corresponding macroscopic thermal quantities for a black hole, such as the Hawking temperature $T$ and entropy $S$, are given by

$$T = \frac{\kappa}{2\pi}, \quad S = \frac{A}{4}, \quad (1.1)$$

where $\kappa$ is the surface gravity on the horizon and $A$ is the area of horizon. The non-vanishing temperature indicates that the black hole is unstable and should emit thermal radiation (due to the quantum effect).

Generically, charged black holes can have two horizons: inner and outer. For the extremal limit ($M^2 = Q^2, r_+ = r_- = M$), the two horizons degenerate and the Hawking temperature vanishes ($\kappa = (r_+ - r_-)/2r_+^2$). However, the entropy (area of horizon) is non-vanishing ($r_H = M$) which represents the quantum degrees of freedom inside the black hole.

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§2. Dilatonic charged black holes

There are three possible generalizations of black hole solutions by introducing new ingredients: (i) dilaton fields, for example Brans-Dicke theory, (ii) extra dimensions, such as Kaluza-Klein theory, (iii) higher-rank form fields, extending black holes to black branes. Those new ingredients are all essential in low energy effective string theory. Moreover, the extremal black holes correspond to some BPS configurations which generically preserve partial supersymmetry. Here we will focus on extremal black holes with a dilaton field extension.

The action of the four dimensional Einstein-Maxwell-dilaton gravity is

$$ S = \frac{1}{16\pi} \int d^4 x \sqrt{-g} \left( R - 2(\partial\phi)^2 - e^{2a\phi} F_{[2]}^2 \right). \quad (2.1) $$

The metric of the spherical symmetric dilatonic black holes for the particular value of dilaton coupling $a = -1$ is

$$ ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + R^2(r) d\Omega_2^2, \quad f(r) = 1 - \frac{r^+}{r}, \quad R^2(r) = r(r - r^-), \quad (2.2) $$

and the mass and charge are related to the radii of the inner and outer horizons by $M = r^+/2, Q^2 = r^+r^-/2$. Here we leave out the explicit expressions for the dilaton and gauge fields, which are not essential in our later discussion. The complete solution and also the general solutions with arbitrary $a$ can be found in the references.

From the function $R(r)$ one can easy realize that, in the extremal limit, i.e. $r^+ = r^-$, the degenerated event horizon shrinks to a point, $R(r^+ = r^-) = 0$, which implies that the entropy of black hole vanishes. The result of zero entropy raises a puzzle that the expected quantum degrees of freedom inside the back holes seem to disappear completely. This discrepancy comes from the fact that general relativity is not sufficient to catch the fundamental point in this case, and higher curvature corrections are necessary to stretch the horizon and reproduce the correct entropy corresponding to the microstate degrees of freedom. We investigate this issue by considering the correction coming from the Gauss-Bonnet combination of quadratic curvature.\(^1\)

The action of four dimensional Gauss-Bonnet gravity can be obtained from (2.1) simply by replacing $F_{[2]}^2$ with $F_{[2]}^2 - \alpha L_{GB}$ where the Gauss-Bonnet term is defined by $L_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$ and $\alpha$ is its coupling. The ansatz for the metric and Maxwell field potential ($F_{[2]} = dA_{[1]}$) are

$$ ds^2 = -w(r)dt^2 + \frac{dr^2}{w(r)} + \rho^2(r) d\Omega_2^2, \quad A_{[1]} = -f(r) dt - q_m \cos \theta d\varphi. \quad (2.3) $$

Here $q_m$ is the magnetic charge parameter, and the electric part can be directly solved and then the electric charge parameter $q_e$ is introduced as

$$ f'(r) = q_e \rho^{-2} e^{-2a\phi}. \quad (2.4) $$
In Gauss-Bonnet gravity, the field equations remain second order in the metric, linear in the second derivative. However, the equations of motion become rather complicated. Our approach is to consider the analytical expansions near the horizon and at infinity, and then numerically interpolate these two sets of boundary data. So, let's first consider the series expansions around a particular value $r = r_H$ (supposed to be a horizon) in powers of $x = r - r_H$ (here $P(r) := e^{2a \phi(r)}$) as

$$w(r) = \sum_{k=2}^{\infty} w_k x^k, \quad \rho(r) = \sum_{k=0}^{\infty} \rho_k x^k, \quad P(r) = \sum_{k=0}^{\infty} P_k x^k. \quad (2.5)$$

The function $w$ starts from the quadratic term (vanishing of $w_0$ means that $r = r_H$ is a horizon, vanishing of $w_1$ means that the horizon is degenerate). The electric charge is then given by

$$q_e = \sqrt{\frac{4 \alpha + q_m^2}{2 \alpha + q_m^2} \rho_0^2}. \quad (2.6)$$

Any solution with finite horizon radius $\rho_0$ must have non-zero electric charge, depending on the magnetic charge. With fixed $\rho_0$, $q_e$ decreases with increasing $q_m$ and approaches zero in the limit $q_m \to \infty$.

For simplicity, we only focus on purely electric charged black holes. The near horizon expansion for the electric solution is

$$w(r) \approx \frac{1}{\rho_0^2} \left[ x - \frac{2(5a^2 - 3)}{3} \left( \frac{\alpha P_1}{a^2 \rho_0^2} \right) x^3 \right] + O(x^4), \quad (2.7)$$

$$\rho(r) \approx \rho_0 \left[ 1 + (a^2 - 1) \left( \frac{\alpha P_1}{a^2 \rho_0^2} \right) x - \frac{2a^2(a^4 - 6)}{(5a^2 - 3) (\alpha^2 \rho_0^2)^2} x^2 \right] + O(x^3), \quad (2.8)$$

$$P(r) \approx \frac{\rho_0^2}{\alpha} \left[ \frac{3}{4} + a^2 \left( \frac{\alpha P_1}{a^2 \rho_0^2} \right) x + \frac{a^2(a^4 - 5a^2 - 3)}{(5a^2 - 3) (\alpha^2 \rho_0^2)^2} x^2 \right] + O(x^3). \quad (2.9)$$

There are two independent parameters, $\rho_0, P_1$, in the expansion and the near horizon geometry is $AdS_2 \times S^2$. The asymptotic expansion, for asymptotically flat, is

$$w(r) = 1 - \frac{2M}{r} + \frac{\alpha Q_e^2}{r^2} + O(r^{-3}), \quad (2.10)$$

$$\rho(r) = r - \frac{D^2}{2r} - \frac{D(2MD - \alpha a Q_e^2)}{3r^2} + O(r^{-3}), \quad (2.11)$$

$$\phi(r) = \phi_\infty + \frac{D}{r} + \frac{2DM - \alpha a Q_e^2}{2r^2} + O(r^{-3}), \quad (2.12)$$

where $M, D, \phi_\infty$ are the mass, dilaton charge, asymptotic value of dilaton, and the electric charge is given by $Q_e = q_e e^{-a \phi_\infty}$.

Our numerical investigation\(^1\) shows that the value of $P_1$ is fixed (depending on $a, \alpha$) in order to get an asymptotically flat solution. The physical quantities mass $M$, dilaton charge $D$, and asymptotic value of dilaton $\phi_\infty$ are all determined only by the value of the parameter $\rho_0$ (i.e. charge). Moreover, we also found that the global solution can exist only when the dilaton coupling $a$ is less than a critical value.
$a_{cr} \simeq 0.488219703$. If $a > a_{cr}$ a singularity appears outside horizon. The physics behind the existence of such a critical value is unclear.

The Bekenstein-Hawking entropy-area formula in (1.1) breaks down for the theories including higher curvature terms. Formally, the general expression of entropy can be obtained via Wald’s Noether charge approach. However, for the black holes with a near horizon geometry of $AdS_2 \times S^{D-2}$, the calculation can be significantly simplified and the entropy can be obtained by the near horizon data via the entropy function proposed by Sen. Following Sen’s approach, the entropy of an extremal dilatonic Gauss-Bonnet black hole is $S = 2\pi \rho_0^2 = A/2$. Unexpectedly, the higher curvature terms contribute an equal amount of entropy as the Hilbert-Einstein action (scalar curvature).

§3. Discussion

By considering the extremal dilatonic black hole, we found the higher curvature corrections are required. A particular interesting case is to include the Gauss-Bonnet term in four dimensional dilaton gravity. In this case, the black hole solutions form a one-parameter family and exist in a finite range of the dilaton coupling constant $a$. The corresponding extremal solution in the Einstein-Maxwell-dilaton theory (2.1) has two parameters ($q_e$ and $\phi_\infty$). Thus, the asymptotic value of the dilaton is no longer a free parameter when the Gauss-Bonnet term is included.

The entropy calculated by Sen’s entropy function approach is twice the value from the Bekenstein-Hawking formula. This new entropy-area relation has been checked in theories with more general higher curvature corrections. The existence of the threshold value of the dilaton coupling constant under which the global solutions cease to exist is an interesting new phenomenon which may be related to the string-black hole transition. We think that our model can be regarded as simple toy model describing the string-black hole transition.

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References

1) C.-M. Chen, D. V. Gal’tsov and D. G. Orlov, Phys. Rev. D 75, 084030 (2007) [arXiv:hep-th/0701004].
2) G. W. Gibbons and K. Maeda, Nucl. Phys. B 298, 741 (1988); D. Garfinkle, G. T. Horowitz and A. Strominger, Phys. Rev. D 43, 3140 (1991) [Erratum-ibid. D 45, 3888 (1992)].
3) R. M. Wald, Phys. Rev. D 48, 3427 (1993) [arXiv:gr-qc/9307038].
4) A. Sen, Mod. Phys. Lett. A 10, 2081 (1995) [arXiv:hep-th/9504147]; JHEP 0505, 059 (2005) [arXiv:hep-th/0411255].
5) R.-G. Cai, C.-M. Chen, K. Maeda, N. Ohta and D.-W. Pang, “Entropy function and universality of entropy-area relation for small black holes,” arXiv:0712.4212 [hep-th].