Theoretical Study of Quasi-Three-Level Frequency Doubling Laser

Zhaoyu Ma1*, Zhi Li1, Lin Wang1, Xiaotong Mu2, Baoying Shang2

1 CRRC Changchun Railway vehicles CO, LTD, No.435 Qingyin Road, Changchun, China.
2 Foshan Shunde Hangce electromagnetic compatibility technology CO. LTD, No.12, West road, Shunde district, Foshan, China.
*Corresponding author’s e-mail: ledisun@163.com

Abstract. Considering the spatial distribution of pumping beam, inverted particle number density, fundamental frequency and photon number density in the cavity of the quasi-three-level frequency-doubled laser, the solution of the rate equation depends on four dimensionless synthetic parameters by normalized correlation parameters. By solving the rate equation numerically, a set of general solutions of the equation are obtained. With these curves and expressions, the design of a quasi-three-level Q-switched intracavity frequency-doubled laser and the prediction of the pulse characteristics could be achieved.

1. Introduction

As an advanced coherent light source, small all-solid-state blue laser has the advantages of small size, compact structure, long life, high efficiency and reliable operation [1-2]. It has important application value in the fields of laser biomedicine, high density optical storage, ultra-short pulse, digital video technology, laser spectroscopy, laser printing and so on. Among them, high power blue laser is used in laser large screen television. Laser colour display, laser entertainment performance, marine applications and underwater resource exploration have broader application prospects.

The quasi-three-level system pumped by laser diode and doubled by non-linear crystal to obtain high power and high beam quality all-solid-state blue laser output is the most effective way to obtain all-solid-state blue laser at present. Rate equations are efficient tools for analysing the performance of a Q-switched laser. The rate equations describing intracavity quasi-three-level lasers have been studied [3-6]. Q. P. Wang et al. deduced the theory of intracavity frequency doubling quasi-three-level CW laser considering spatial distribution [7]. In order to make more accurate theoretical analysis of intracavity frequency doubled lasers, especially to obtain the relationship between output power and the cross-section ratio of pumped beam to laser beam, it is necessary to consider the spatial distribution of pumped beam, inverted particle number density, fundamental frequency and frequency doubled photon number density in the rate equation of intracavity frequency doubled lasers. In this paper, a theoretical model of intracavity frequency doubled quasi-three-level Q-switched laser is established considering spatial distribution.
2. Rate equation

In a quasi-three-level laser, the distribution of the number of particles in the upper and lower levels of the laser is Boltzmann distribution. Distribution constants are \( f_a \) and \( f_b \). In the absence of pumping, the lower level particle number density \( n_{d0} \) is expressed as

\[
n_{d0} = f_a n_0.  \tag{1}
\]

Assuming that the laser operates in TEM\(_{00}\) mode, the photon number density \( \phi (r, t) \) of the fundamental frequency light in the laser crystal and the photon number density \( \phi_d (r, t) \) of the fundamental frequency light at the frequency doubling crystal can be expressed as

\[
\phi (r, t) = \phi (0, t) \exp \left(-\frac{2r^2}{\omega_0^2} \right), \tag{2}
\]

\[
\phi_d (r, t) = \phi_d (0, t) \exp \left(-\frac{2r^2}{\omega_d^2} \right), \tag{3}
\]

where, \( r \) is the radial radius, \( \omega_0 \) and \( \omega_d \) are the radius of fundamental frequency light at the laser crystal and frequency doubling crystal respectively. Because of the continuity of laser energy in the cavity, the following relations can be obtained

\[
\omega_0^2 \phi (0, t) = \omega_d^2 \phi_d (0, t) \tag{4}
\]

When the frequency doubling efficiency is low, the power of the fundamental frequency light passing through the frequency doubling crystal is [6]

\[
P_{\text{SHG}} = \frac{\rho \pi \eta \nu c h v_f \phi (0, t)^2}{16 \omega_d^4}, \tag{5}
\]

where, \( \rho \) depends on the structure of the resonator and the mode of output of doubled light. \( c \) is the speed of light in vacuum, and \( \nu \) is the frequency of fundamental light. \( \eta \) indicates the conversion ability of doubled crystal to fundamental light.

\[
\eta = \frac{\omega_0^2 d_{\text{eff}}^2 l^2}{c^2 \varepsilon_0 n_0^2 n_d^2 n_e^2 (l_0^2 \Delta k / 2^2)}, \tag{6}
\]

where, \( \omega_0 \) is the angular frequency of fundamental frequency light, \( d_{\text{eff}} \) is the effective non-linear coefficient of doubled frequency crystal, \( \varepsilon_0 \) is the dielectric constant in free space, \( n_0 \omega_0, n_d \omega_d, n_e \omega_e \) are the refractive index of doubled frequency and fundamental frequency light of doubled frequency crystal respectively, and \( \Delta k \) is the phase mismatch.

In laser medium, the rate of change of fundamental frequency light caused by frequency doubling \( \frac{d \phi (r, t)}{dt} \) satisfies the following relationship

\[
P_{\text{SHG}} = h \nu' \int_0^\infty \frac{d \phi (r, t)}{dt} L_{\text{SHG}} 2\pi r dr, \tag{7}
\]

where, \( L' \) is the optical length of resonator.

From equations (4) and (6), it can be obtained

\[
\int_0^\infty \frac{d \phi (r, t)}{dt} L_{\text{SHG}} 2\pi r dr = \frac{\rho \pi \eta \nu c h v_f (\phi (0, t))^2}{16 \omega_d^4} \omega_0^4 = \frac{c}{2l'} \frac{\rho \pi \eta \nu c h v_f \omega_0^2}{2 \omega_d^2} \int_0^\infty \phi_d^2 (r, t) 2\pi r dr. \tag{8}
\]

Therefore, in the case of quasi-three-level active Q-switching operation, the effect of spontaneous emission and pumping on the pulse is neglected. The time-dependent integration of fundamental frequency light in the cross section of the beam and the term describing the generation of frequency-doubled light are added. The rate equation describing the Q-switching operation of intracavity frequency-doubled quasi-three-level laser can be obtained as

\[
\int_0^\infty \frac{d \phi (r, t)}{dt} 2\pi r dr = \int_0^\infty \frac{\phi (r, t)}{t_r} \left[ 2\sigma \left[ n_b (r, t) - n_a (r, t) \right] t - \ln (\tau) \right] L - L_o \} 2\pi r dr
\]
\begin{equation}
-\frac{c}{2l'} \frac{\rho \sigma \hbar \nu}{2 \omega_p^2} \int_0^\infty \phi_b^2(r,t) 2\pi rdr
\end{equation}

\begin{equation}
\frac{dn_b(r,t)}{dt} = -f_p \sigma c \varphi_b(r,t) \left[ n_b(r,t) - n_a(r,t) \right] \tag{9}
\end{equation}

\begin{equation}
\frac{dn_a(r,t)}{dt} = f_p \sigma c \varphi_b(r,t) \left[ n_b(r,t) - n_a(r,t) \right] \tag{10}
\end{equation}

where, \( n(r, t) \) is the density of inverted particles, \( \sigma \) and \( l \) are the stimulated emission cross sections and lengths of laser crystals, \( t_{\text{cav}} = 2l'/c \) is the time of light traveling in the laser cavity, \( R_l \) is the reflection coefficient of output mirror to the fundamental frequency light, and \( L \) is the loss of fundamental frequency light propagating in the cavity for one round.

Assuming that the initial particle number density of upper and lower levels of laser is Gaussian distribution, the initial particle number density of upper and lower levels is expressed as

\begin{equation}
n_b(r,0) = n_b(0,0) \exp\left(-\frac{2r^2}{\omega_p^2}\right), \tag{12}
\end{equation}

\begin{equation}
n_a(r,0) = n_a(0,0). \tag{13}
\end{equation}

By substituting equations (2), (3), and (12) into equations (10) and (11), the inverted particle number function is

\begin{equation}
n_b(r,t) - n_a(r,t) = \left[ n_b(0,0) \exp\left(-\frac{2r^2}{\omega_p^2}\right) - n_a(0) \right] \exp\left[-\gamma \sigma c \int_0^t \varphi_b(0,t) dt \exp\left(-\frac{2r^2}{\omega_p^2}\right)\right], \tag{14}
\end{equation}

where, loss factor gamma \( \gamma = f_p + f_b \) denotes the number of inverted particles consumed to produce a photon, \( n_b(0,0) \) is determined by the pump power and repetition frequency of the pump source, ignoring the influence of the number of the remaining inverted particles in the previous pulse. Before the formation of the pulse, the relationship between the number of inverted particles is expressed as

\begin{equation}
\frac{dn_b(r,t)}{dt} = r(r,t) - \frac{n_a(r,t)}{\tau_b}. \tag{15}
\end{equation}

where, \( \tau_b \) is the lifetime of upper level in laser medium, \( r(r,t) = \eta_p f_b P_{abs}(0,t) / h\nu_p \), \( \eta_p \) is the pumping quantum efficiency, \( P_{abs}(0,t) \) is the pumping power density absorbed per unit volume on the optical axis, and \( \nu_p \) is the pumping optical frequency. The relationship between \( P_{abs}(0,t) \) and the total absorption pump power of laser medium \( P_{abs} \) is

\begin{equation}
P_{abs} = \int_0^\infty P_{abs}(0,t) \exp\left(-\frac{2r^2}{\omega_p^2}\right) 2\pi rdrdz = \frac{1}{2} \pi \omega_p^2 l P_{abs}(0,t), \tag{16}
\end{equation}

From equations (15) and (16),

\begin{equation}
n_b(0,0) = \frac{2\eta_p f_b P_{abs} \tau_b}{\pi \omega_p^2 h\nu_p} \left( 1 - \exp\left(-\frac{1}{f_p \tau_b}\right) \right), \tag{17}
\end{equation}

Substitute equations (2) and (14) into equation (9),

\begin{equation}
\frac{d\varphi_b(0,t)}{dt} = \frac{4\sigma l \varphi_b(0,t)}{\omega_p^2 t_r} \int_0^\infty \left[ n_b(0,0) \exp\left(-\frac{2r^2}{\omega_p^2}\right) - n_a(0) \right] \\
\cdot \exp\left[-\gamma \sigma c \int_0^t \varphi_b(0,t) dt \cdot \exp\left(-\frac{2r^2}{\omega_p^2}\right)\right] \exp\left(-\frac{2r^2}{\omega_p^2}\right) 2\pi rdr \\
- \frac{\varphi_b(0,t)}{t_r} \left[ \ln\left(\frac{1}{R_l}\right) + L_\gamma \right] - \frac{c}{2l'} \frac{\rho \sigma \hbar \nu \omega_p^2}{2 \omega_p^2} \varphi_b^2(r,t) \tag{18}
\end{equation}
The above equation is the differential equation used to describe of an intracavity frequency doubled quasi-three-level Q-switched laser, and the output fundamental optical power of the laser is

\[
P_{d_{\text{out}}}(t) = (1 - R_d) h \nu_1 \int_0^t \frac{d\phi(t,t')}{dt} dt' + \frac{2 \pi \eta_{\text{SHG}}}{16} \frac{\alpha_1}{\alpha_0} (h \nu_1 \phi_1^2(0,t)).
\]  

(19)

The single pulse energy of the output frequency doubled light is

\[
E_{d_{\text{out}}} = \int_0^t P_{d_{\text{out}}}(t) dt = (1 - R_d) \frac{2 \pi \eta_{\text{SHG}}}{16} \frac{\alpha_1}{\alpha_0} (h \nu_1 \phi_1^2(0,t)).
\]  

(20)

The average power of the output frequency doubled light is

\[
P_d = \int_0^t P_{d_{\text{out}}}(t) dt = f_p (1 - R_d) \frac{2 \pi \eta_{\text{SHG}}}{16} \frac{\alpha_1}{\alpha_0} (h \nu_1 \phi_1^2(0,t)).
\]  

(21)

The comprehensive parameters \(M, N, \beta, \) and \(\eta_{\text{SHG}},\) normalized time \(\tau,\) normalized photon number density \(\Phi(0, \tau)\) are introduced

\[
M = \frac{2 \sigma n_p \lambda}{\ln \left(\frac{1}{R_p}\right) + L_p}, N = \frac{2 \sigma n_p (0,0) l - 2 \sigma n_p l \left(1 + \frac{\omega_0^2}{\omega_p^2}\right)}{\ln \left(\frac{1}{R_p}\right) + L_p} \left(1 + \frac{\omega_0^2}{\omega_p^2}\right) , \eta_{\text{SHG}} = \frac{\rho \eta h \nu_1 \phi_1^2}{8 \gamma \sigma l'}, \beta = \frac{1}{1 + (\omega_0 / \omega_p)^2}, \tau = \frac{t}{t_i} \left[\ln \left(\frac{1}{R_p}\right) + L_p\right], \Phi(r, \tau) = \phi(r, \tau) \frac{2 \gamma \sigma l'}{\ln \left(\frac{1}{R_p}\right) + L_p}.
\]  

(22)

\(M\) is the ratio of the lower-level reabsorption loss to the resonator loss. \(N\) is the ratio of the initial inversion particle number density to the threshold inversion particle number density. \(\eta_{\text{SHG}}\) is the parameter of the frequency doubling energy.

By substituting the above parameters into equation (18), the normalized rate equation is obtained as

\[
\frac{d\Phi(0, \tau)}{d\tau} = \Phi(0, \tau)(M + N) \int_0^1 \exp \left(-A(\tau)y^{M}\right) dy
\]

\(-M \Phi(0, \tau) \frac{1 - \exp\left[-A(\tau)\right]}{A(\tau)} - \Phi(0, \tau) - \eta_{\text{SHG}} \Phi^2(0, \tau)\)

(24)

where,

\[
A(\tau) = \int_0^\tau \Phi(0, \rho) d\rho.
\]  

(25)

The peak power and single pulse energy of the output frequency doubled light are expressed by the normalized photon number density as

\[
P_{\text{shg}_{\text{out}}} = (1 - R_d) h \nu_1 \phi_1^2 \frac{\rho \eta h \nu_1 \phi_1^2}{8 \gamma \sigma l'} \left[\ln \left(\frac{1}{R_p}\right) + L_p\right] \eta_{\text{SHG}} \Phi^2 = (1 - R_d) \frac{h \nu_1 \phi_1^2}{4 \gamma \sigma l'} \left[\ln \left(\frac{1}{R_p}\right) + L_p\right] e_{\text{SHG}_{\text{out}}},
\]  

(26)

\[
E_{\text{shg}_{\text{out}}} = (1 - R_d) h \nu_1 \phi_1^2 \frac{\rho \eta h \nu_1 \phi_1^2}{4 \gamma \sigma} \left[\ln \left(\frac{1}{R_p}\right) + L_p\right] \int_0^\tau \eta_{\text{SHG}} \Phi^2(0, \tau) d\tau = (1 - R_d) \frac{h \nu_1 \phi_1^2}{4 \gamma \sigma} \left[\ln \left(\frac{1}{R_p}\right) + L_p\right] e_{\text{SHG}_{\text{out}}}
\]

(27)

where, \(e_{\text{SHG}_{\text{out}}} = \eta_{\text{SHG}} \Phi^2(0, \tau)\) which is the normalized frequency doubling optical output power [8].
3. Solutions of the rate equation

Figure 1 The relation between $\int_0^\infty e_{SH} \, \mathrm{d}\tau$ and $N$ for different $M$ in the case of $\omega_p/\omega_l=1$ and $\eta_{\text{SHG}}=0.1$. (a) $M=0$, (b) $M=0.5$, (c) $M=1.0$, (d) $M=1.5$, (e) $M=2.0$, (f) $M=2.5$.

Figure 2 The relation between $(e_{\text{SH}})_m$ and $N$ for different $M$ in the case of $\omega_p/\omega_l=1$ and $\eta_{\text{SHG}}=0.1$. (a) $M=0$, (b) $M=0.5$, (c) $M=1.0$, (d) $M=1.5$, (e) $M=2.0$, (f) $M=2.5$.

Figure 1 and 2 show the dependence of $(e_{\text{SH}})_m$ and $\int_0^\infty e_{\text{SH}} \, \mathrm{d}\tau$ on $N$ for different $M$ in the case of $\omega_p/\omega_l=1$ and $\eta_{\text{SHG}}=0.1$. It can be seen that $(e_{\text{SH}})_m$ and $\int_0^\infty e_{\text{SH}} \, \mathrm{d}\tau$ are increasing with the increase of $N$. The larger the $M$, the smaller the value of $(e_{\text{SH}})_m$ and $\int_0^\infty e_{\text{SH}} \, \mathrm{d}\tau$. 
Figure 3 The relation between $\int_0^\infty e_{\text{SH}} \, d\tau$ and $\eta_{\text{SHG}}$ for different $\omega_p/\omega_l$ and $N$ in case of $M=1$. (a) $\omega_p/\omega_l=\infty$, $N=10$, (b) $\omega_p/\omega_l=2$, $N=10$, (c) $\omega_p/\omega_l=1$, $N=10$, (d) $\omega_p/\omega_l=\infty$, $N=5$, (e) $\omega_p/\omega_l=2$, $N=5$, (f) $\omega_p/\omega_l=1$, $N=5$.

Figure 4 The relation between $(e_{\text{SH}})_m$ and $\eta_{\text{SHG}}$ for different $\omega_p/\omega_l$ and $N$ in case of $M=1$. (a) $\omega_p/\omega_l=\infty$, $N=10$, (b) $\omega_p/\omega_l=2$, $N=10$, (c) $\omega_p/\omega_l=1$, $N=10$, (d) $\omega_p/\omega_l=\infty$, $N=5$, (e) $\omega_p/\omega_l=2$, $N=5$, (f) $\omega_p/\omega_l=1$, $N=5$.

The dependences of $(e_{\text{SH}})_m$ and $\int_0^\infty e_{\text{SH}} \, d\tau$ on $\eta_{\text{SHG}}$ for different $\omega_p/\omega_l$ and $N$ in case of $M=1$ are shown in Figure 3 and 4. Not that the bigger $\eta_{\text{SHG}}$ is, the larger values of $(e_{\text{SH}})_m$ and $\int_0^\infty e_{\text{SH}} \, d\tau$ are. $\int_0^\infty e_{\text{SH}} \, d\tau$ first increases with the increase of $\eta_{\text{SHG}}$, then remains unchanged within a certain range, and then decreases with the increase of $\eta_{\text{SHG}}$. For $(e_{\text{SH}})_m$, $\eta_{\text{SHG}}$ has an optimum value for different cases.

4. Conclusion
In this paper, the normalized quasi-three-level frequency doubling rate equation is obtained. In the rate equation, the intracavity light intensity and the pump beam are assumed to be Gauss distribution. The normalized correlation parameters show that the solution of the rate equation depends on four dimensionless comprehensive parameters: the ratio of the lower-level reabsorption loss to the resonator loss, the ratio of the initial inversion particle number density to the threshold inversion particle number density, the parameter of the frequency doubling energy, and the cross section ratio of the pump beam to the laser beam. A set of universal curves about the relationship between the general solution of the equation and the four comprehensive parameters are given.
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