As a strangeness $S = -1$ and baryon number $B = 2$ system, the two-body bound state of $\Lambda^* = \Lambda(1405)$ and a nucleon is studied. To solve the $\Lambda^*N$ system, we construct the $\Lambda^*N$ potential by extending the Jülich model with couplings estimated in the chiral unitary approach. We have the $\Lambda^*N$ quasi-bound state with the mass, $M_{\Lambda^*N} \sim 2364\text{MeV}$ which is shallowly bound about 9.5 MeV from the $\bar{K}NN$ threshold. Decay width of the fall apart process, where the $\Lambda^*N$ resonance decays to $\pi\Sigma N$ with a nucleon being as a spectator, is estimated to be $\Gamma_{F.A} \sim 49\text{MeV}$.

One of the most interesting topics of strange nuclear physics is possible existence of the $\bar{K}$ bound state in nuclei. The interaction of $\bar{K}$ and a nucleon is known to be attractive to support a quasi-bound state, which is called $\Lambda^* = \Lambda(1405)$, slightly below the $\bar{K}N$ threshold. By the $\bar{K}N$ attraction, the $\bar{K}NN$ three-body system is also considered to make a bound state. From a theoretical viewpoint, it is difficult to solve the $\bar{K}NN$ system due to the existence of the $\Lambda^*$ resonance and the $\bar{K}$ absorption by two nucleons. Meanwhile, based on several theoretical arguments, we can take the viewpoint that $\Lambda^*$ is regarded as a fundamental particle in the $\bar{K}NN$ bound system. This picture has an advantage that the inclusion of the $YN(Y = \Sigma, \Lambda)$ channels, which represent the two-nucleon absorption processes, is relatively easy, compared with the $\bar{K}NN$ picture. To search for a possible bound state of the $\Lambda^*N$ system, we construct the $\Lambda^*N$ potential. It is the most fundamental interaction in the “$\Lambda^*$-hypernuclei”, which we call the nuclei with a $\Lambda^*$ [1]. The $\Lambda^*N$ system is labeled by two quantum numbers, the total spin $S$ and the orbital angular momentum $L$, and we consider $S = 0, 1$ and $L = 0$ cases as candidates of the ground state.

We construct the $\Lambda^*N$ potential by extending the Jülich model [2, 3], which is one of the hyperon-nucleon one-boson-exchange potential models. Since the isospin of $\Lambda^*$ is zero, isoscalar mesons $X(X = \sigma, \omega)$ are exchanged, while the pseudoscalar $\eta$ exchange is omitted as its coupling to the nucleon is small. In addition, considering the $\bar{K}$ exchange which contributes with an interchange of $\Lambda^*$ and nucleon, the $\Lambda^*N$ potential can be written by

$$ V = V_\sigma + V_\omega + V_{\bar{K}}, $$

where $V_\sigma$, $V_\omega$ and $V_{\bar{K}}$ represent $\sigma$, $\omega$ and $\bar{K}$ meson exchange contributions respectively.

For the lack of the information of $\Lambda^*$, vertex properties concerning $\Lambda^*$ are not clear. To estimate relevant parameters, coupling constants, we adopt the microscopic structure of $\Lambda^*$ obtained by the chiral unitary approach. In the chiral unitary model, $\Lambda^*$ is described as a resonance in a coupled-channel meson and baryon ($\pi\Sigma, \bar{K}N, \eta\Lambda, K\Xi$) multiple scattering, and we take the $\Lambda^*$ meson baryon coupling constants from Refs. [4, 5]. The $\Lambda^*\bar{K}N$ coupling constant is directly obtained in the chiral unitary model, while the $\Lambda^*\Lambda^*X(X = \sigma, \omega)$ couplings are to
be estimated. Since the exchanged meson couples to the meson and baryon in the multiple scattering, the $\Lambda^*\Lambda^*X$ coupling can be estimated by evaluating microscopic contributions which correspond to one-loop diagrams shown in Fig. 1. In the estimation, we take only dominant contributions, $\pi\Sigma$ and $\bar{K}N$ for making mechanism more visible.

Moreover, from the analysis of $\Lambda^*$ with the chiral unitary approach, $\Lambda^*$ is a superposition of two states. Therefore, the $\Lambda^*N$ system consists of two states, $\Lambda^*_1N$ and $\Lambda^*_2N$. Here, we call higher(lower) energy state $\Lambda^*_1(\Lambda^*_2)$. Then, we solve the two-channel coupled Schrödinger equation given by

$$H\psi = E\psi, \quad H = T + V,$$

(2)

with

$$T = \begin{pmatrix} T_1 & 0 \\ 0 & T_2 \end{pmatrix}, \quad V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}, \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix},$$

(3)

where $T_a$, $V_{aa}$ and $\psi_a$ stand for the kinetic energy term, potential term and wave function of $\Lambda^*_a$, while, off-diagonal components, $V_{12}$ and $V_{21}$, contribute to the transition of the $\Lambda^*_1N$ system and the $\Lambda^*_2N$ system.

In order to convert the $\bar{K}$ exchange amplitude into the potential, an exchange factor should be applied, which introduces the spin dependence. Since the $\Lambda^*\bar{K}N$ vertex is a scalar type and the scalar exchange in the $NN$ potential is an attractive force, the $\bar{K}$ exchange contribution is attractive(repulsive) for total spin $S = 0(S = 1)$. Nonzero energy transfer, due to the difference of the mass between $\Lambda^*$ and nucleon, is included by the use of an effective $\bar{K}$ mass given by

$$\tilde{m}_K = \sqrt{m_{\bar{K}}^2 - (M_{\Lambda^*} - M_N)^2},$$

(4)

where $m_{\bar{K}}$, $M_{\Lambda^*}$ and $M_N$ stand for $\bar{K}$, $\Lambda^*$ and nucleon masses respectively. Since the effective $\bar{K}$ mass become lighter as the mass of $\Lambda^*$ gets closer to the $\bar{K}N$ threshold, the $\bar{K}$ exchange is stronger in the $\Lambda^*_1N$ system than in the $\Lambda^*_2N$ system.

In the $\Lambda^*N$ potential, isoscalar meson exchange contributions, the attractive $\sigma$ exchange and the repulsive $\omega$ exchange, are largely cancelled out each other, except for the repulsive core at the very short range region. On the other hand, the $\bar{K}$ exchange contribution is a long range interaction by the light effective $\bar{K}$ mass. Therefore, total spin, which controls the sign of the $\bar{K}$ exchange contribution, determine whether the $\Lambda^*N$ potential has attraction or not. In Fig. 2, we show the diagonal components of the obtained $\Lambda^*N$ potential in $S = 0$. In both case, the
potential has an attractive pocket at the intermediate range region due to the attraction in the \( \bar{K} \) exchange, which is not the case in \( S = 1 \). It can be seen that the \( \Lambda^*N \) potential is more attractive because of the lighter effective \( \bar{K} \) mass.

We solve the full coupled channel Schrödinger equation with the obtained \( \Lambda^*N \) potential, and study the bound state of the \( \Lambda^*N \) system. As a first step, to study the property of each \( \Lambda^*aN \) system, we search for a possible bound state of each \( \Lambda^*aN \) channel separately by switching off the off-diagonal components of the \( \Lambda^*N \) potential, \( V_{12} = V_{21} = 0 \). In the total spin \( S = 1 \), the \( \Lambda^*N \) potential has no attraction to make any bound states of the \( \Lambda^*N \) system. While, in \( S = 0 \), we find the bound state of the \( \Lambda_1^*N \) system only, with the binding energy \( B = 9.5 \text{ MeV} \) measured from the \( KNN \) threshold. The wave function in coordinate space of the \( \Lambda_1^*N \) bound state is shown in Fig. 3. We can see that the wave function has a long tail due to small binding energy.

In the next place, performing the full coupled channel calculation, the \( \Lambda_1^*N \) bound state acquires a finite width through the coupling to the \( \Lambda_2^*N \) channel. We find the resonance state of the \( \Lambda^*N \) system by using the real scaling method, with the two-body mass of the resonance, 

\[
M_{\Lambda^*N} \sim 2364 \text{ MeV}.
\] (5)

Since the resonance wave function is not normalized, it is not straightforward to extract the properties of the \( \Lambda^*N \) quasi-bound state from the resonance solution. However, the coupling to the \( \Lambda_2^*N \) channel does not change the mass of the (quasi-)bound state very much, so we expect that the \( \Lambda_1^*N \) component dominates the \( \Lambda^*N \) quasi-bound state. We thus use the wave function

\[\text{Figure 2.} \ \Lambda^*N \ \text{potential for } S = 0. \text{ Left(Right) panel of the figure corresponds to } V_{11}(V_{22}).\]

\[\text{Figure 3.} \ \text{Wave function of the } \Lambda_1^*N \ \text{bound state.}\]
Table 1. Comparison with other chiral models, [4, 5], [6], [7] and [8, 9].

| Refs. | \(M_{\Lambda^*N}(\text{MeV})\) | \(\sqrt{\langle r^2 \rangle}(\text{fm})\) | \(\Gamma_{F.A} (\text{MeV})\) |
|-------|-----------------|-----------------|-----------------|
| [4, 5] | 2364            | 5.1             | 49              |
| [6]    | 2364            | 6.3             | 49              |
| [7]    | 2371            | 5.2             | 28              |
| [8, 9] | 2364            | 3.2             | 59              |

of the bound \(\Lambda^*_1N\) system to estimate the property of the quasi-bound state. The mean-distance of the \(\Lambda^*N\) system is estimated as, \(\sqrt{\langle r^2 \rangle_{\Lambda^*N}} \sim \left[ \int d^3r r^2 |\psi_1|^2 \right]^{1/2} \sim 5.1\text{fm}\), which indicates the loosely bound of the \(\Lambda^*N\) system. The decay width can also be obtained using the wave function, together with some appropriate matrix element for the decay process. The \(\Lambda^*N\) quasi-bound state can decay into the three-body \(\pi Y N\) states and two-body \(Y N\) states. Among them, here we estimate the decay width, \(\Gamma_{F.A}\), for the fall apart process where \(\Lambda^*\) in the quasi-bound state of the \(\Lambda^*N\) system decays to \(\pi \Sigma\) with a nucleon being as a spectator, with \(\Gamma_{F.A} \sim 49\text{MeV}\).

We construct the \(\Lambda^*N\) potential based on the particular chiral unitary model, [4, 5]. To study ambiguities due to the properties of \(\Lambda^*\), we apply the \(\Lambda^*_a\) parameters, pole positions of \(\Lambda^*_a\) and meson-baryon couplings, taken from different models in Refs. [6], [7] and [8, 9], to our potential model. Solving the Schrödinger equation, we have results which are not so different between the models qualitatively. Namely, the two-body system of \(\Lambda^*\) and a nucleon develops a shallow and loose quasi-bound state, and the decay width is narrow in the fall apart process for its large phase space. The latter property may be related to the small coupling of the \(\Lambda^*_1\) state to the \(\pi \Sigma\) channel. Numerical results are listed in Table 1. Note that the \(\Lambda^*_1N\) threshold is different between the chiral unitary models, due to the different \(\Lambda^*_a\) mass. Thus, the same two-body masses with the models [4, 5] and [8, 9] come from the different \(\Lambda^*_1N\) binding energy, which is related to each mean-distance. For the same reason, the model [7] gives the smallest decay width for its largest phase space, because of its smallest \(\Lambda^*_1\pi \Sigma\) coupling.

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