Spatial Relation Resolution and Spatial Relation Abstraction

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1 Introduction

Map generalization, a process of transformation from high resolution to low resolution, can be thought as a mathematical set mapping procedure between two databases. Since the spatial database (set) contains two categories of information: spatial entity and spatial relation, the mapping procedure certainly involves spatial entity abstraction and spatial relation abstraction. Traditionally, we focus on spatial entity generalization, achieving set of generalization operators for spatial entity, but neglect the relation generalization. To resolve the conflicts between spatial entities is a typical spatial relation abstraction.

As the entity abstraction, the spatial relation abstraction is also based on resolution change. Spatial relation resolution (SRR) can be defined as the minimum identifiable semantic description of the spatial relation. Spatial relation representation, including topological, direction and distance relation, has different similarity to each other. The similar relations can be grouped as a component of higher level semantic description. So the spatial relation description is a hierarchical tree structure, and SRR describes the hierarchical level, represented as the node depth in the tree structure. In this paper, we will give three methods for constructing the hierarchical relation tree for topological, directional and distance relation, respectively. The topological hierarchical relation tree is based on Egenhofer's nine intersection descriptions.
2 Representation of three kinds of spatial relation resolutions

According to the relational algebra definition, spatial relation is described as a Cartesian tuple of spatial entities, so the object in spatial relation mapping is a spatial entity pair rather than the spatial entity itself. It is different from the transformation of geometric space, such as affine transformation, in which the entity itself is oriented. The expression \( e_1 r e_2 \) denotes the spatial entity \( e_1 \) having relation \( r \) with the entity \( e_2 \). The mapping that the original relation \( r \) is mapped as \( r' \) between \( e_1 \) and \( e_2 \) can be represented as:

\[
e_{1r'}e_2 = g(e_1r e_2)
\]

The three basic spatial relations, topological, distance and orientation relations, have different semantic descriptions. However, the way to combine them to get an integrated description for spatial cognition has not been found by now. So we can not find the mathematical function of mapping \( g \) just like an affine transformation which can be represented as one matrix to integrate the three independent transformations, translation, rotation and scaling.

Unlike the general spatial relation mapping, the relation mapping contained in generalization is the transformation from detailed state to abstract state. In this sense, we call it spatial relation abstraction. Displacement operation is a typical spatial relation mapping in generalization, through entity position adjustment to resolve spatial relation conflicts. In this procedure the operated object is a spatial entity pair rather than an independent entity.

In the geographic model, we have spatial resolution \( 6 \), attribute resolution \( r \) as well as temporal resolution. Peng (1995) classified spatial resolution as spatial size resolution, spatial feature resolution and spatial distance resolution\(^{[17]} \). These resolutions aim at spatial entity abstraction. Spatial relation abstraction is also based on resolution change. So a new resolution concept, spatial relation resolution has to be built. Spatial relation resolution (SRR) is defined as the minimum identifiable semantic description of spatial relation. Spatial relation representations including topological, orientation and distance relation, have different similarity to each other. It means that some relations are close to each other while others are far away. The close relations can be further grouped as a higher level semantic description. So the spatial relation description is a hierarchical tree structure. The SRR describes the hierarchical level, which is represented as the node depth in the tree structure. The SRR description depends on the model of spatial relation representations. Next we will give three methods for constructing a relationship hierarchical tree for topological, distance and orientation relations, respectively.

2.1 Topological relation resolution

The nine intersection representations of topological relation\(^{[24]} \) can get \( 2^9 = 512 \) sorts of relations between two spatial objects. However, the valid number of the relations is less than that number after the meaningless relations have been removed. Among the remaining relations, according to the steps of changing one state to another state, Egenhofer and Mark (1995) built the conceptual neighborhood graph of topological relation\(^{[17]} \). The connection between neighbor representations results in a network to describe adjacency relation of topological relation. In this model, the less the steps from one relation to another are, the closer the similarity between them is. On the basis of this model, we can construct the hierarchical tree to represent the semantic level of a topological relation. According to certain cognition standards, some neighbor relations in the neighbor network are grouped into high level description. As shown in Fig.1, the original 19 relations between line and area can be grouped into four higher level rela-
tions: inside, outside, going across and on boundary. Within each group, the relation is no longer to be distinguished from each other under the lower resolution standard. For some purpose, four distinguished relation representations are enough. Under control of this resolution we can execute spatial relation mapping to get abstract representation.

Fig. 1 Nineteen topological relationships between line and area object can be mapped as four descriptions with resolution decreasing

2.2 Distance relation resolution

Compared with the topological relation research, the distance relation research is less active and has few achievements in qualitative description. In absolute quantitative representation of the distance between two objects the sentence of Euclidean distance can be used. However in the distance relation representation, what means that object A is near object B depends not only on their absolute positions (and the metric distance between them) but also on their relative sizes and shapes, the position of other objects, the frame of reference. The context environment plays an important role in distance relation representation. We present a method based on Voronoi diagram (VD) to represent distance representation, and on the basis of this representation the distance relation resolution will be discussed.

Each spatial entity in scene environment has a certain influence region to which this object is closer than any other object. Of course, when determining the influence region, one has to consider the existence of neighbor objects. Assuming that the space is isotropic, we can use the Voronoi diagram partitioning of the area, acting as the spatial entity influence region. The boundary of VD cell polygon equally partitions the space between two neighbor left/right entities. The VD partitioning can be thought as the result of each entity equally competing outward for growth range. If two VD cells share a common boundary, we can say that the two entities belonging to the cell polygons are adjacent, even if their metric distance is far. On the basis of this idea, we can use the relation between VD cells to represent the distance relation between spatial entities. Making use of the adjacency transmitting property, we define a variable adjacent degree to describe distance relation and through progressive extension algorithm to obtain the adjacency degree value of all objects with respect to object A.

On the basis of the adjacency degree, we select and connect parts of VD cell boundaries getting the contour line which separates objects with adjacency degree n from those with adjacent degree n + 1, getting the results shown in Fig. 2 and Fig. 3. The objects within the loop between two neighbor contour lines have the same adjacent degree with respect to object A. So we call this kind of contour the iso-distance-relationship contour, just like the altitude contour of terrain representation. The smaller the value of adjacency degree is, the closer distance relation the two objects have to each other. Obviously this contour is different from the iso-distance contour which is represented as progressive circle buffers with the same center and increasing radius. The iso-distance-relation model considers the context environment and the spatial distribution. An object far away in metric distance may possibly have very low adjacency degree and close distance relation with the reference object. The reference object can be an object set rather than a single object, such
as the objects adjacent to outside boundary shown in Fig. 3.

Fig. 2  The iso-distance-relation contour with respect to the central object A, left with interval adjacency degree value one unit and right with two units.

Fig. 3  The iso-distance-relation contour with respect to the boundary b, left with interval adjacency degree value one unit and right with two units.

Having the iso-distance-relation model, we can now discuss the distance relation resolution. In terrain contour representation, we use altitude intervals to express resolution. The smaller the interval altitude is, the higher the resolution is. In the same way, we use the adjacency degree interval to describe the distance relation resolution. Selecting one from every two neighbor contour lines we get the distance relation representation whose resolution decreases by half as shown in Fig. 2(b) and Fig. 3(b). Higher resolution corresponds to more grades in the distance relation representation. For example, in the representation of one unit adjacency degree interval we use the following semantic expression containing seven grades, very close, close, medium close, medium, medium far, far, very far. But for the representation of interval adjacency degree of two units, the relations will be grouped into an updated semantic description with the following three grades, close, medium, far.

On the basis of this model, if the object moves within the loop, it does not change its distance relation with the reference object. In case of considering the distance relation only, it is not necessary to execute the relation mapping to correct the distance relation. But if the object moves across the loop, the distance relation mapping is required to correct
the destroyed distance relation. The lower the resolution is, the wider the loop is and the less the chances of destroying the original relation are.

2.3 Orientation relation resolution

Frank presented two methods of cardinal orientation direction representation, one is based on triangular areas and the other is on projection. Here we give the description of the orientation relation resolution based on the cardinal direction model. The semantic description of cardinal direction has hierarchical properties. We have four distinguished direction relations, north, west, south, east with each covering 2π/4 sector range. By further separating, we can get eight direction relationships: north, northeast, east, southeast, south, southwest, west, northwest with each covering 2π/8 sector range. The separation can go on and get more detailed direction relation descriptions. The angle range covered by one direction can be defined as an orientation relation resolution.

3 Spatial relation abstraction behavior in map generalization

The constraints of generalization are usually statements related to spatial, attribute and temporal resolution. Weibel and Dutton introduced four types of distinguished constraints: graphical, topological, structural and Gestalt. From the point of view of mapping transformation, the constraints can be categorized as spatial entity associated constraints and spatial relation associated constraints. The latter relates not only to topological relation which appears in Weibel and Dutton’s classification but also to distance relation and orientation relation, its statement format usually reflects “remaining spatial relation unchanged” or “avoiding the appearance of undistinguishable relation”. The comparison of spatial relation equality has to be based on SRR just like the equality judgment between two float numbers, in which the considered precision should be predefined. Under a certain resolution, according to the category of destroyed constraints, corresponding spatial entity mapping and spatial relation mapping are required. From one state to another state during map generalization, the relation between spatial entities may have changed in a strict sense. But under a low resolution, the cognition neglects most of the small changes as if they remain in the original state. Only for those distinct relation changes, post-processing is required to adjust the spatial position. Post-processing is usually called displacement in traditional map generalization. On the basis of the spatial mapping model in this paper, the displacement is just one of the concrete forms of spatial relation mapping used to satisfy a constraint.

We define the map generalization algebra system \( < E, R, E', R', f, g > \), \( E, R, E', R' \) to denote the spatial entity set, the spatial relation set of the original map, the spatial entity set, the spatial relation set of the new map, respectively; \( f \) and \( g \) denote the spatial entity mapping and the spatial relation mapping, respectively. For two original entities \( e_i, e_j \in E \), there is the relationship \( r \in R, e_i r e_j \). After mapping \( e_i \mapsto f(e_i) \in E', e_j \mapsto f(e_j) \in E' \), the new mapped entities have the new relation \( r' \in R', f(e_i) r' f(e_j) \). The spatial relation mapping \( e_i e_j \mapsto g(e_i e_j) = f(e_i) r' f(e_j) \) realizes the relation conversion from \( r \) to \( r' \).

3.1 Spatial entity against constraint

Spatial entity representation destroys the constraints associated with size resolution, feature resolution or attribute class resolution, and the spatial entity mapping \( f \) is required to abstract and get simple representation. If the spatial relationship \( r' \) between abstracted entities equals to the original \( r \), \( f(e_i) r' f(e_j) = e_i e_j \), and the relation \( r' \) or \( r' \) is not against the relation constraint, then the relation mapping is not necessary (Fig. 4(a)). Otherwise, \( f(e_i) r' f(e_j) \neq e_i e_j \), it means that the constraint “remaining origi-
nal spatial relation” is destroyed, and the relation mapping $g$ is required to convert $r'$ to $r$, just like the example in Fig. 4(b), in which the relation mapping $g$ corrects “overlap” relation between two simplified buildings returning to the original relation “touch”.

\[ g(e_i r' e_j) \rightarrow e_i r e_j. \]

The street generalization of classifying streets into grades according to streets’ width is one of these cases. Changing each polygon of lake cluster from the disjoint to exact touch also belongs to the spatial relation mapping. What drives the map generalization is the relation constraint rather than the entity constraint. For this kind of mapping, some of them can be obtained by the operation displacement, but others such as street classification generalization and lake cluster generalization cannot be described as displacement. From the classification of 20 generalization operators presented by Mackness(1994)\(^{(k)}\) or 12 operators as classified by McMaster & Shea (1989)\(^{(c)}\), it is difficult to find a proper operator to explain this kind of generalization. The reason is that the operator classification only considers entity oriented operation, neglecting the relation side.

In the spatial relation mapping, SRR plays the main role. In Fig. 3, when the street is widened, the boundary $b$ moves and destroys the relation between the street edge and the neighbor buildings. On the basis of the iso-distance-relation model, the displacement problem between the street edge and the buildings can be resolved through the adjacency degree loop analysis and the concept field in physics can be borrowed. The iso-distance-relation contour is similar to the iso-dynamic of magnetic field.

3.3 Both spatial entity and spatial relation against constraints

This is the mixture of the two former cases. The relation mapping has to take into account constraints from two sources: 1) the destroyed original relation possibly resulting from the entity mapping, 2) the undistinguished relations in existing relation representation. Generally, spatial entity mapping $f$ is executed first. Subsequently the relation mapping $g(f(e_i) r' f(e_j) ) \rightarrow f(e_i) r f(e_j)$, performs the relation abstraction, on the other hand, the damaged relation is corrected. Sometimes, the relation abstraction has implicitly satisfied the constraint “remaining relationship unchanged” under low resolution recognition. This process contains two comparisons: the parallel state between neighbors and the historical state between after and before mapping.

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