A New Type of Sliding Mode Observer for Permanent Magnet Synchronous Motor Control

Sheng Cheng a, Yu-Fa Xu b*

School of Electrical Engineering, Shanghai Dianji University, Shanghai, China

aemail:cs1942218015@163.com
b*email: xuyf@sdju.edu.cn

Abstract: Aiming at the high-frequency oscillation and estimation accuracy problems of traditional sliding mode observers in the control process of permanent magnet synchronous motors, a sensorless control method for permanent magnet synchronous motors based on a new approaching law sliding mode observer is proposed. Based on the construction of a permanent magnet synchronous motor two-phase static coordinate system model, a sliding mode observer is used to estimate the back electromotive force, and then the rotor speed and position information are obtained. Finally, a simulation experiment is carried out. The results show that the new sliding mode observer based on the new approaching law effectively reduces the high frequency chattering of the system, improves the estimation accuracy of the system, and has better control performance.

1. Introduction
Permanent Magnet Synchronous Motor (PMSM) has attracted more and more attention because of its combination of power density and high torque density, and superior speed regulation performance. Especially in the current new energy field. Because the traditional mechanical sensor will not only increase the cost and volume, it is easy to be damaged in some special environments, resulting in failure to work normally. Therefore, the control method of sensorless control technology of permanent magnet synchronous motor has become the current research focus.

At present, PMSM sensorless control methods are roughly divided into two categories: one is the signal injection method is mainly suitable for zero-speed and extremely low-speed conditions, including rotating high-frequency voltage signal injection method and pulse high-frequency voltage signal injection method [1]. The other type is mainly suitable for medium and high speed conditions. Commonly used are model reference adaptive method [2], extended Kalman filter method [3] and sliding mode observer method [4-10]. Literature [11] uses a piecewise function to replace the traditional sign function to reduce the chattering of the system to a certain extent, and introduces a division link in the traditional phase-locked loop to improve the accuracy of the observer. Literature [12] adjusts the sliding mode gain through fuzzy control, and designs an adaptive low-pass filter, which reduces the chattering and phase lag problems of the system. Literature [13] selects a continuous switching function at the zero point, combined with an exponential approaching law, to improve the chattering of the system. Literature [14] suppressed the chattering of the system by designing a new exponential approaching law and adaptively changing sliding mode gain, and reduced the error of the rotor position estimation when the motor was running stably.
This paper proposes a sliding mode observer based on a new approaching law. The sliding mode observer uses a special power function $f_a(x, \mu, \delta)^{[15-16]}$ to design a new approaching law, and uses the Lyapunov function to prove its stability. Finally, through simulation comparison and verification, the results show that the method improves the observation accuracy, significantly reduces the high-frequency chattering of the system, and has good control performance.

2. Mathematical model of permanent magnet synchronous motor

Assuming that the stator windings of the permanent magnet synchronous motor are three-phase symmetrical, the magnetic circuit is linear, there is no damping winding on the rotor, the core loss and eddy current loss are ignored, and the core saturation is ignored, then the surface mount permanent magnet synchronous motor is in a two-phase coordinate system. The mathematical model is:

$$
\begin{align*}
\frac{d\hat{i}_a}{dt} &= \frac{1}{L_s}(-R\hat{i}_a + u_a - E_a) \\
\frac{d\hat{i}_\beta}{dt} &= \frac{1}{L_s}(-R\hat{i}_\beta + u_\beta - E_\beta) \\
E_a &= -\psi_f \sin \theta \\
E_\beta &= \psi_f w_e \cos \theta
\end{align*}
$$

Where, $i_a, i_\beta$ is the component of the stator current on the $\alpha, \beta$ axis, $R$ is the stator resistance, $L_s$ is the stator inductance, $u_a, u_\beta$ is the voltage component of the stator voltage on the $\alpha, \beta$ axis, $E_a, E_\beta$ is the induced electromotive force of the $\alpha, \beta$ axis, $\psi_f$ is the permanent magnet flux linkage, $w_e$ is the electrical angular velocity, $\theta$ is the rotor position angle.

It can be seen from equation (1) that the back EMF contains the speed information and position information of the rotor. Therefore, the estimated values of the induced electromotive force of the $\alpha - \beta$ axis can be obtained to obtain the speed and rotor position.

3. Traditional sliding mode observer

In order to obtain the value of induced electromotive force, the traditional sliding mode observer is designed as:

$$
\begin{align*}
\frac{d\tilde{i}_a}{dt} &= \frac{1}{L_s}(-R\tilde{i}_a + u_a - V_a) \\
\frac{d\tilde{i}_\beta}{dt} &= \frac{1}{L_s}(-R\tilde{i}_\beta + u_\beta - V_\beta) \\
V_a &= \lambda \text{sgn} (\hat{i}_d - i_d) \\
V_\beta &= \lambda \text{sgn} (\hat{i}_q - i_q)
\end{align*}
$$

Among them, $\tilde{i}_a, \tilde{i}_\beta$ are the current observation values of the stator $\alpha$ axis and $\beta$ axis respectively; $\lambda$ is the sliding mode gain.

From equation (1) and equation (3), the state equation of the current error system can be obtained as:

$$
\begin{align*}
\frac{d\tilde{i}_a}{dt} &= \frac{1}{L_s}(-R\tilde{i}_a - V_a + E_a) \\
\frac{d\tilde{i}_\beta}{dt} &= \frac{1}{L_s}(-R\tilde{i}_\beta - V_\beta + E_\beta)
\end{align*}
$$

Among them, $\tilde{i}_a = \hat{i}_a - i_a, \tilde{i}_\beta = \hat{i}_\beta - i_\beta$ are current observation errors. The sliding mode surface
function is defined as $\tilde{I} = [\tilde{I}_a \ \tilde{I}_b]^T$. When the observer enters the sliding mode, $\tilde{I} = \theta$. According to the equivalent principle of sliding mode control, get:

$$
\begin{align*}
E_a &= V_a = \lambda \text{sgn}(\tilde{I}_a - i_a) \\
E_b &= V_b = \lambda \text{sgn}(\tilde{I}_b - i_b)
\end{align*}
$$

(6)

In order to obtain a continuous back-EMF, it is necessary to perform low-pass filtering on equation (6), and the filtered induced electromotive force is:

$$
\begin{align*}
\hat{E}_a &= \frac{w_c}{s + w_c} V_a \\
\hat{E}_b &= \frac{w_c}{s + w_c} V_b
\end{align*}
$$

(7)

In the formula, $w_c$ is the cut-off frequency of the low-pass filter, and the estimated value of the back electromotive force can be obtained. From the formula (2), the observed values of the rotor position and speed can be obtained:

$$
\begin{align*}
\hat{\theta} &= -\arctan\left(\frac{\hat{E}_a}{\hat{E}_b}\right) \\
w_c &= \sqrt{\frac{\hat{E}_a^2 + \hat{E}_b^2}{\psi f}}
\end{align*}
$$

(8)

Since the use of a low-pass filter will cause phase lag and amplitude conversion, it is necessary to compensate the rotor position and angle to obtain more accurate rotor position information, namely:

$$
\hat{\theta} = -\arctan\left(\frac{\hat{E}_a}{\hat{E}_b}\right) + \arctan\left(\frac{\hat{w}_c}{w_c}\right)
$$

(9)

4. New sliding mode observer

Traditional sliding mode observers mainly use symbolic functions, and the discontinuity of symbolic functions at the origin leads to system chattering. This paper uses a special power function $favl(x, \mu, \delta)$ to design a new type of approaching law. The nonlinear function $favl(x, \mu, \delta)$ is:

$$
\begin{align*}
favl(x, \mu, \delta) &= \begin{cases} 
|x|^{\alpha} \text{sgn}(x), & |x| > \delta \\
\frac{x}{\delta^{1-\alpha}}, & |x| \leq \delta
\end{cases}
\end{align*}
$$

(10)

The image is shown in Figure 1:

![Figure 1 Image of function fal(x, mu, delta)](image)

Among them, $0 < \delta < 1, \alpha > 0, \delta$ is the switching factor. It can be seen from equation (10) that when the value of the independent variable $x$ is greater than $\delta$, the gain $\alpha$ of the independent variable $\delta^{1-\alpha}$ is larger; otherwise, the gain is smaller.
Take the law of approaching:
\[ \dot{s} = -\varepsilon \cdot \text{fal}(s, \mu, \delta) - k \cdot \text{arsh}(s) \]  \tag{11}

Among them, \( s = \hat{i} = i - i \) is the sliding mode function, \( \hat{i} \) is the observation error of the stator current, \( i \) is the observation value of the stator current, \( i \) is the actual value of the stator current, and \( \varepsilon > 0, k > 0, \text{arsh}(s) \) is the inverse hyperbolic sine function.

In formula (11), when \( |s| \leq \delta \), the \( \varepsilon \cdot \text{fal}(s, \mu, \delta) \) term accelerates the approach speed, ensuring that the system can quickly approach the sliding mode surface, and the \( \text{fal}(s, \mu, \delta) \) function is a continuous function near the origin, which can effectively reduce the high-frequency oscillation problem of the system. At the same time, the hyperbolic tangent is a smooth continuous function. When \( |s| > \delta \), the \( k \cdot \text{arsh}(s) \) term can not only provide a faster approach to the sliding surface, but also have a smoothing and limiting effect, so that the value of \( e \) will not be too large. The sliding mode controls the amplitude of the input signal. The design of the new approaching law sliding mode observer is as follows:

\[
\begin{align*}
\frac{d\hat{i}_\alpha}{dt} = & -\frac{R_e \hat{i}_\alpha}{L_a} + \frac{u_a}{L_a} - \varepsilon \cdot \text{fal}(\hat{i}_{\alpha} - i_{\alpha}, \mu, \delta) + k \cdot \text{arsh}(\hat{i}_{\alpha} - i_{\alpha}) \\
\frac{d\hat{i}_\beta}{dt} = & -\frac{R_e \hat{i}_\beta}{L_a} + \frac{u_\beta}{L_a} - \varepsilon \cdot \text{fal}(\hat{i}_{\beta} - i_{\beta}, \mu, \delta) + k \cdot \text{arsh}(\hat{i}_{\beta} - i_{\beta})
\end{align*}
\]  \tag{12}

The error equation of sliding mode observer is obtained by subtracting equation (10) and equation (1):

\[
\begin{align*}
\frac{d\hat{i}_\alpha}{dt} = & -\frac{R_e \hat{i}_\alpha}{L_a} + \frac{1}{L_a}(E_{\alpha} - (\varepsilon \cdot \text{fal}(\hat{i}_{\alpha}, \mu, \delta) + k \cdot \text{arsh}(\hat{i}_{\alpha}))) \\
\frac{d\hat{i}_\beta}{dt} = & -\frac{R_e \hat{i}_\beta}{L_a} + \frac{1}{L_a}(E_{\beta} - (\varepsilon \cdot \text{fal}(\hat{i}_{\beta}, \mu, \delta) + k \cdot \text{arsh}(\hat{i}_{\beta})))
\end{align*}
\]  \tag{13}

To verify its stability, define the Lyapunov function as:
\[ V = \frac{1}{2} s^2 \]  \tag{14}

According to the stability condition, it is necessary to satisfy \( \dot{V} < 0 \), derivate the formula (14), and then bring the formula (11) into the derivation formula to obtain:
\[ \dot{V} = \dot{s} \dot{s} = -\varepsilon \cdot \text{fal}(s, \mu, \delta) - k \cdot s \cdot \text{arsh}(s) \]  \tag{15}

From the characteristics of functions \( \text{fal}(s, \mu, \delta) \) and \( \text{arsh}(s) \), we can see that the independent variable has the same sign as the function value, and \( \varepsilon > 0, k > 0 \), so \( \dot{V} < 0 \) is satisfied, that is, the system is stable.

5. Simulation results and analysis
The sensorless control block diagram of the permanent magnet synchronous motor in the synchronous rotating coordinate system based on the new sliding mode observer is shown in Figure 2. Use Matlab/Simulink to build a simulation model, and select \( i_d = 0 \) control strategy. The corresponding motor parameters are shown in Table 1.
Figure 2 Block diagram of sensorless control

Table 1 Motor parameters

| Parameter | Value |
|-----------|-------|
| $R$ / $\Omega$ | 2.875 |
| $L_s$ / $mH$ | 0.0085 |
| $\psi_f$ / $wb$ | 0.175 |
| $J$ / (kg $\cdot$ m$^2$) | 0.001 |
| $B$ / (N $\cdot$ m) | 0 |
| $P_n$ | 4 |
| $N_{ref}$ / (r $\cdot$ min) | 1000 |

Figure 3 shows the actual speed, estimated speed and speed error waveforms using a traditional sliding mode observer. Figure 4 shows the actual speed, estimated speed and speed error waveform using the new sliding mode observer. It can be clearly seen from the figure that the chattering phenomenon of the traditional sliding mode observer system is obvious, and the speed error is about $\pm 10$ r $\cdot$ min after stabilization. Based on the new sliding mode observer, the chattering phenomenon of the system is effectively reduced, and the actual speed can be better followed, the response speed is faster, and the speed error after stability is about $\pm 2$ r $\cdot$ min, which improves the observation accuracy of the system.

(a) Actual and estimated speed
(b) Speed error

Figure 3 Rotational speed information waveform of traditional sliding mode observer
Figure 4 shows the estimated value, actual value and estimated error waveform of the rotor position using a traditional sliding mode observer, and Figure 6 shows the estimated value, actual value and estimated error waveform of the rotor position using a new sliding mode observer. It can be seen from the figure that the chattering phenomenon of the system when the new sliding mode observer is adopted is obviously lower than that of the traditional sliding mode observer, and the observation accuracy is higher.

Figure 5 shows the estimated value, actual value and estimated error waveform of the rotor position using a traditional sliding mode observer, and Figure 6 shows the estimated value, actual value and estimated error waveform of the rotor position using a new sliding mode observer. It can be seen from the figure that the chattering phenomenon of the system when the new sliding mode observer is adopted is obviously lower than that of the traditional sliding mode observer, and the observation accuracy is higher.

6. Conclusion
Aiming at the high-frequency chattering and observation error problems of traditional sliding mode observers, this paper uses a special power function \( f(x, \mu, \delta) \) as the approaching law to construct a new sliding mode observer. The advantage of this law of approach is that when \( |x| \leq \delta \) , \( \varepsilon \cdot f(x, \mu, \delta) \) can increase the approach speed, thereby ensuring that the system state can be arbitrarily limited Time to reach the sliding mode surface \( s = 0 \). And, near the origin, \( k \cdot arsh(s) \) is a smooth continuous function, which can effectively reduce the high-frequency tremor of the control input.
At the same time, the inverse hyperbolic sine function is smooth and continuous. When \( |s| > \delta \), the \( k \cdot \text{arsh}(s) \) term can ensure that the system state approaches the sliding mode at a greater speed. In addition, the inverse hyperbolic sine function \( \text{arsh}(s) \) plays a smoothing and limiting role. The value of the \( k \cdot \text{arsh}(s) \) term in the reaching law cannot be too large, so as to reduce the amplitude of the sliding mode control input signal. The simulation results show that this new sliding mode observer is effectively weakened. The system’s high-frequency chattering problem can estimate the speed and rotor position more accurately than traditional sliding mode observers, and has good control performance.

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