Theoretical Study of Signature-splitting and Signature-inversion in Doubly-odd Nuclei in $A\sim80$ Mass Region

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Abstract. Projected Shell model calculations have been performed using the angular-momentum projected two-quasiparticle states with the employment of a simple quadrupole-quadrupole + monopole–Pairing + quadrupole-pairing Hamiltonian to study the nuclear structure properties of doubly-odd $^{80}$Br and $^{82}$Rb isotones. The present calculations reproduce reasonably well the available experimental data on the yrast bands and also predict the new high spin states in these nuclei, where current data are still sparse. The phenomena of Signature-splitting and Signature–inversion have also been studied in detail within the context of Projected Shell Model.

1. Introduction

A unified theoretical description of the structure of doubly-even, odd-mass and doubly-odd nuclei is one of the main goals of the nuclear structure research. While a good amount of research work is reported on the study of nuclear structure of the doubly-even and odd-mass nuclei in the recent past, the structure of their doubly-odd counterparts is still not well understood. It is may be due to the reason that these doubly-odd nuclei have coexisting low-lying two-quasiparticle states which can have very small energy differences between them e.g., less than 100 keV. It is not easy for the theoretical models to reproduce such small differences, and hence understanding their intricate structure becomes difficult. However, with the advent of recent experimental techniques and computational advances, it has now become feasible to explore the nuclear structure of the odd-odd nuclei present in the Segre chart. If we talk, in particular, about the $A\sim70-80$ mass regions around the $pf$-shell, invigorating new data have been made available by the experiments, in recent years, on the odd-odd nuclei. This data suggests that the nuclei in this mass region exhibit various interesting phenomena such as rotational alignments, shape coexistence, signature-splitting and signature-inversion, etc. Out of these, the phenomena of signature-splitting and signature-inversion have grabbed a considerable amount of research attention in the recent past because of the reason that gaining knowledge about these phenomena could help one to understand the dynamics of nuclear structure in a particular mass region. The nuclei in mass regions 80, 100, 130 and 160 have been found to exhibit the phenomenon of signature splitting and signature inversion.

Signature is a quantum number which is associated with symmetry under the rotation of a deformed nucleus around the principal axis by 180° such that the rotational band splits into two sequences according to the signature. The shifting of the energy levels between both bands (sequences) at a given rotational frequency is called the signature splitting (SP), and it is characterized by a staggering in the energy. In the present work, we attempted to study this interesting phenomena in the mass region $A\sim80$ along
with the phenomenon of signature–inversion (where an expected favoured branch (lower in energy) becomes unfavoured at higher spins). It is evident from the literature that the signature inversion in the mass 70-80 region is related to the filling of the high-$j$ $g_{9/2}$ proton and $g_{9/2}$ neutron subshells [1] and is a sign of the transition from mainly single-particle excitations at low spins to more rotational (collective) motion at higher spins. We aimed at studying these characteristic features of mass 80 region to understand the role of the $g_{9/2}$ orbital along with the other nuclear structure properties. The nuclei chosen for present study are doubly-odd $N=45$ isotones, $^{80}\text{Br}$ and $^{82}\text{Rb}$.

2. Brief theory of Projected Shell Model

A brief explanation of the Projected Shell Model (PSM) [2] along with the important input parameters is given hereunder.

The PSM calculation generally begins with the deformed Nilsson single-particle states having deformation $\varepsilon_2$, and pairing correlations included by BCS calculations. As the result of the Nilsson-BCS calculations we get a set of quasiparticle (qp) states. The shell model bases is then constructed by building multi-qp states. In this process, the rotational symmetry of the states is broken which is then recovered by angular momentum projection technique so as to form a shell model basis in the lab frame. Lastly, a two-body shell model Hamiltonian is diagonalized in this projected space.

The qp subspace chosen for the present work is spanned by the basis set

$$|\phi_\kappa\rangle = \{a_\alpha^+ a_\beta^+ |0\rangle\}$$

(1)

where $a^+$’s are the quasiparticle (qp) creation operators, $\nu$’s ($\pi$’s) denote the neutron (proton) Nilsson quantum numbers which run over low-lying orbitals and $|0\rangle$ is the Nilsson + BCS vacuum (0-qp state). Each configuration in equation (1) consists of one quasineutron and one quasiproton. The indices $\nu$ and $\pi$ in eq. (1) are general; for example, a two-qp state can be of positive-parity if both quasiparticles $i$ and $j$ are from the same major $N$ shell, or of negative parity if $i$ and $j$ are from $N$ shells differing by $\Delta N=1$. For the current odd-odd nuclei, low-lying two-qp states with positive-parity are those in which both the neutron and the proton occupy the $N=4$ fp$g$ shell. The configuration space is obviously large in this case compared to the nearby odd-mass nuclei and usually several configurations contribute to the shell model wave function of a state with nearly equal weightage. This makes the numerical results very sensitive to the shell filling and the theoretical predictions for doubly-odd nuclei become far more challenging.

The PSM Hamiltonian used in the present calculations consists of the harmonic oscillator single-particle Hamiltonian and a sum of schematic (quadrupole-quadrupole ($Q.Q$) + Monopole Pairing + Quadrupole Pairing) forces and is of the form

$$\hat{H} = \hat{H}_0 - \frac{1}{2} \sum_{\nu} \hat{Q}_\nu \hat{Q}_\nu - G_{ij} \hat{P}_i \hat{P}_j - G_{ij} \sum_{\nu=1}^{\nu_{max}} \hat{p}_\nu \hat{p}_\nu$$

(2)

where $\hat{H}_0$ is the spherical single-particle Hamiltonian which in particular contains a proper spin-orbit force, whose strengths (i.e. the Nilsson parameters $\kappa$ and $\mu$) are taken from [3]. The second term in the Hamiltonian (2) is the $Q.Q$ interaction and the last two terms are the monopole and quadrupole pairing interactions, respectively.

It should also be noted that in the present calculations, the configuration space contains three major shells, $N=2, 3, 4$ for protons and $N=3, 4, 5$ for neutrons. Moreover, $Z=8$ and $N=20$ are taken as inert cores for protons and neutrons respectively. The Shell model space in the present calculations is truncated at a deformation, $\varepsilon_2 = 0.195$, for both these nuclei which is very close to the values given in Refs. [8, 52-54].

3. Projected Shell Model Results

3.1. Yrast Spectra
Yrast bands for $^{80}$Br and $^{82}$Rb up to the maximum spin of $18\hbar$ for are calculated and plotted in figures 1(a) and 1(b) respectively. These results are compared with the available experimental data in the same figures where the experimental data are taken from NNDC database [4,5,6]. On analysing this available data we found that the ground state band is positive-parity band in both these nuclei, with band-head at $1^+$ where the experimental data is available upto maximum spin $14^+$ for $^{80}$Br and $17^+$ for $^{82}$Rb. Our PSM calculations also predicts the ground state band-heads at $K^\pi = 1^+$, for both these nuclei. Further, from figures 1(a-b), it is clear that the calculated PSM data reproduced the available experimental data with a satisfactory degree of agreement where the maximum gap between the experimental and the calculated energy levels is ~0.2 MeV in case of $^{82}$Rb for $9^+$ state.

It is noted by Doring et al [1] that with the decrease in number of protons, i.e., while going from $^{82}$Rb to $^{78}$As, the $6^+$ state shifts from 191 keV in $^{82}$Rb to 357 keV in $^{80}$Br to 621.9 keV in $^{78}$As, indicating the fact that the $g_{9/2}$ proton orbital filling becomes energetically more expensive. In the present PSM calculations, we found that $6^+$ state in $^{82}$Rb occurs at 157 keV whereas in $^{80}$Br it is found to have the energy 370.7 keV, So one can say that our PSM results support the findings of Doring et al and also points out that the $g_{9/2}$ proton orbital does not play any major role for obtaining the low-lying states in these isotones, thereby, resulting in a less deformed nucleus.

3.2. Signature-splitting and signature-inversion

The signature-splitting and signature-inversion are best understood by plotting the quantity $[E(I) - E(I - 1)]/2I$ versus spin $I$ of the initial state [7]. The plot of $[E(I) - E(I - 1)]/2I$ vs $I$ for the N=45 $^{80}$Br and $^{82}$Rb isotones are shown in figures 2(a) and 2(b) respectively. It is clear from the figures that the signature splitting is very much present at the middle and higher spin range in these nuclei. Here, it is noticed that the even-spin states are lower in energy while the odd spin states are higher in energy for both of these nuclei in the low-spin region while the energy difference of odd-spin states becomes lower after the reversal in the phase of the staggering (signature inversion) takes place at the spin $I = 12\hbar$. Experimentally also a signature inversion is observed in both these nuclei around the intermediate spin of $11\hbar$ where the yrast band is found to be composed of two bands, Bands 1a and 1b which are identified as the $\alpha = 0$ and $\alpha =1$ signature partners, respectively.

It may be pointed out here that inversion of the signature in the vicinity of $11\hbar$ is a generally observed feature in the positive-parity yrast bands of doubly-odd nuclei in this mass 70-80 region. It is understood to occur due to the underlying $\pi g_{9/2} \otimes \nu g_{9/2}$ quasi-particle configuration [8] and results of our calculations also corroborate this. Moreover, for $^{82}$Rb, Shen et al [8] also performed the PSM calculations but in their work they were not able to reproduce the signature inversion at the

![Figure 1](image_url)
experimentally observed spin by keeping the deformation fixed and concluded that the signature splitting and especially the signature inversion in $^{82}$Rb are reproduced only by a varying $\gamma$ deformation with increasing spin. However, in our work we are able to reproduce this feature even by making use of fixed deformation and the assuming the axial symmetry.

![Figure 2](image-url)  
**Figure 2.** $(E(I) - E(I-1))$ as a function of spin for yrast band in a) $^{80}$Br and b) $^{82}$Rb.

4. Summary
In order to gain better knowledge of the structure of the doubly-odd nuclei, particularly those lying in $A=70-80$ mass region, PSM calculations have been performed on $N=45$ isotones, $^{80}$Br and $^{82}$Rb. The calculated data reproduces reasonably well the reported experimental data on the yrast bands and also predicted the high spin states in these nuclei, where current data are still sparse. The phenomena of signature splitting and signature-inversion are also studied and the role of the $\pi g_{9/2} \otimes \nu g_{9/2}$ configuration in producing the inversion of signature around the spin $11\hbar$ is also established through present calculations.

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References
[1] Doring J, Johns G D, Hartley D J, Kaye R A, Kemper K W, Sylvan G N and Tabor S L 1996 *Z. Phys. A* **354** 345
[2] Hara K and Sun Y 1995 *Int. J. Mod. Phys. E* **4** 637
[3] Zhang J Y, Larabee A J and Reidinger L L 1987 *J. Phys. G* **13** L75
[4] Singh B 2005 *Nucl. Data Sheets* **105** 223
[5] Ray I, Banerjee P, Bhattacharya S, Saha-Sarkar M, Muralithar S, Singh R P and Bhowmik R K 2000 *Nucl. Phys. A* **678** 258
[6] Tuli J K, Browne E 2019 *Nucl. Data Sheets* **157** 260
[7] Shen S, Han G, Wen S, Pan F, Zhu J, Gu J, Draayer J P, Wu X, Zhu L, He C, Li G, Yu B, Wen T and Yan Y 2010 *Phys. Rev. C* **82** 014306
[8] Shen SF, Wang FG, Peng N, Chen HG, Lu H and Wu YP 2006 *High Energy Phys. and Nucl. Phys.* **30** 1234