Analysis of methods and means for estimating losses in magnetic components caused by proximity effect and skin effect

A V Bashkirov, V V Glotov, N Yu Veretennikov, V M Pitolin, A S Demikhova

Voronezh State Technical University, 14, Moskovsky Prospekt, Voronezh, 394026, Russian

E-mail: okipr.vgtu@rambler.ru, vadik-livny@mail.ru

Abstract. In this paper, the mechanism of the occurrence of high-frequency losses in multilayer windings of a magnetic component is considered. The analysis of known methods for calculating and optimizing high-frequency losses in transformer windings caused by the proximity effect and the skin effect is carried out. Most of the methods of modern analyses show an increase in the accuracy of modeling, but do not solve the optimization problem. The SFD (field squared derivative) method proposed by Sullivan, specially reduced costs for modeling the magnetic component when calculating losses, as well as the Dowell and Karsten methods, allowing one to analytically find the optimal value of the conductor thickness in the winding, are investigated. The analysis identified the strengths and weaknesses of the methods, as well as possible ways to improve them.

1. Introduction

With a decrease in the overall dimensions of switching power supplies, the problem of calculating and optimizing high-frequency losses in magnetic components during their design becomes more and more urgent. With an increase in the conversion frequency, losses caused by the skin effect and the proximity effect can significantly degrade the efficiency of the final device. In the literature [5, 6, 8] it is shown that with an increase in the number of layers of the windings of the magnetic component, the proximity effect tends to become dominant in the formation of losses in copper. Eddy currents are induced in the inner layers of a multilayer winding, caused by the magnetomotive force $F$ created by the currents of the previous layer. With each subsequent layer, the magnetomotive force

$$F = \oint H dl = NI$$

increases and, as a result, losses increase. So, for example, in a three-layer choke winding (fig. 1), the losses in the second and third layers exceed the losses in the outer layer several times. Due to the fact that this type of loss significantly depends on the topology and design features of the magnetic component, there is often a need to carry out evaluative calculations designed to select the optimal topology. Modern CAD systems using the finite element method (FEA, Finite Element Analysis) are capable of constructing with sufficient accuracy the distribution of fields and currents in the turns of the magnetic component of any topology. However, their use often requires a complete 3D modeling of the magnetic component, which is costly in terms of both time and computational resources. In
addition, with this approach, it becomes difficult to solve the problem of optimizing a component, which requires repeated modeling of it for different configurations. In this regard, in many articles, methods are proposed that are designed to simplify or completely eliminate the construction of models when calculating losses, which greatly simplifies the finding of the optimal topology of magnetic components during their design.

![Figure 1. Eddy currents in multilayer windings](image)

2. SFD method

There are methods that can significantly reduce the computational power required for modeling in finite elements and at the same time reduce the sufficient accuracy. These approaches include the SFD (squared-field-derivative) method proposed by Sullivan [1, 2] for calculating losses in cylindrical conductors. We consider a cylindrical conductor lying in a plane perpendicular to the magnetic field, as shown in Figure 2. We consider a loop formed by an eddy current of thickness $dx$ at a distance $x$ from the center of the conductor. The reverse current flows at a distance of $-x$, as shown in the figure.

![Figure 2. Calculation of losses in a conducting cylinder in a uniform magnetic field](image)

If the magnetic field is uniform and directed perpendicular to the axis of the conductor, then the EMF induced in this circuit is:

$$d\varepsilon = -\frac{d\Phi}{dt} = -2xl\frac{dB}{dt}$$

(2)

where $l$ is the length of the cylinder (going into the plane in figure 1).

Taking into account the smallness of $dx$, we write down the resistance of the circuit under consideration:

$$R = \frac{2l\rho}{2\sqrt{\frac{dx}{\pi} - x^2}dx}$$

(3)

where $\rho$ is the resistivity of the cylinder material. The power dissipated in this small area is:
\[ dP = \frac{(-d\phi_t)^2}{R} \]  

(4)

and can be integrated to find the instantaneous power dissipation in the conductor.

\[ P = \int_0^2 \left(-2\pi d \frac{dB}{dt}\right)^2 \frac{d^2 \pi - x^2}{\pi d^4} dx = \frac{\pi l d_0}{64} \left(\frac{dB}{dt}\right)^2. \]  

(5)

To find the average dissipated power in the entire winding, it is necessary to average the derivative of the magnetic induction in time and space [1]. Thus, the losses in the winding are equal to:

\[ P_j = \frac{\pi l_0 N_j d_0}{64} \left(\frac{dB}{dt}\right)^2. \]  

(6)

Where \( P_j \) - time \( j \), \( l_0 \) - average length of a turn in a winding \( j \), \( N_i \) - number of turns in a winding \( j \).

Thus, the average winding loss depends on the derivative of the magnetic induction squared. (Hence the name of the method - squared-field-derivative or SFD) [1]. To calculate the losses in the litz wire, \( N_j \) should be replaced by the product of the number of turns in the winding and the number of cores in one turn. \( d \) should be represented as the diameter of one core. It should be noted that this approach neglects the eddy currents circulating between the turns of the litz wire, but these currents are extremely small compared to the currents between the veins of the litz wire [1]. This approach improves the accuracy, compared to the Dowell method, by taking into account the field between the conductors within the same layer. However, the proposed method has a number of limitations. The thickness of the conductor should not exceed twice the thickness of the skin layer for a given frequency, and the calculation itself is relevant only for cylindrical conductors. In addition, it is only suitable for harmonic currents, while non-sinusoidal currents require a separate calculation for each harmonic.

3. Dowell's method

There is an analytical method that allows you to calculate and analyze the AC resistance for multilayer windings of a magnetic component, without resorting to 3D modeling [3, 4]. This method is based on the representation of the conductive layer of the winding as a flat rectangular conductor filling the core window along its entire height \( h \). If the conductive layer consists of several conductors of cylindrical cross-section, then it can be converted into a single rectangular conductor with an equivalent cross-section and equivalent conductivity [3, 7] (Fig. 3).

![Figure 3. Converting a flat conductor to a foil with an equivalent cross section](image)

D-diameter cylindrical wire is reduced to a square wire with a side:

\[ d = \frac{\pi}{\sqrt{4}} D. \]  

(7)

The fill factor of the window height becomes:
\[ \eta = \frac{N d}{n} \]  

(8)

where \( N \) is the number of turns in one layer of the winding. Thus, the equivalent conductivity of the layer takes the form:

\[ \sigma = \eta \sigma_0, \] 

(9)

where \( \sigma_0 \) — conductivity of the conductor material.

The theoretical approach to calculating high-frequency losses proposed by Dowell [3] is based on the condition that the magnetic field is parallel to the winding axis and has the same intensity along the \( Y \) and \( Z \) axes (fig. 3). If a closed-type core is used, then the field divergence at the edges of the windings decreases and the edge effects can be neglected, which will make it possible to consider the field uniform throughout the layers [4].

\[ \text{Figure 4. Conductive layer in the winding} \]

Maxwell’s equations for such a field take the following form [5]:

\[ \begin{align*}
\text{rot} \mathbf{E} &= -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \\
\text{rot} \mathbf{H} &= \sigma \mathbf{E}.
\end{align*} \] 

(10)  

(11)

Having written out the equations in terms of the coordinates of the field, changing according to the harmonic law, we obtain:

\[ \sigma E_z = \frac{\partial H_y}{\partial x} \] 

(12)

\[ j \omega \mu_0 H_y = \frac{\partial E_z}{\partial x}. \] 

(13)

Substituting the expression for \( E_z \) from (11) into formula (12), we obtain a differential equation for \( H_y \):

\[ \frac{\partial^2 H_y}{\partial x^2} = j \omega \mu_0 H_y. \] 

(14)

The general solution of the equation has the form:

\[ H_y(x) = Ae^{-mx} + Be^{mx} \] 

(15)

where \( m = \sqrt{j \omega \mu_0 \sigma} \).
Assuming that the entire field is between the windings and does not attenuate [5, 6], we can write the boundary conditions for the field at the boundaries of the n-th layer of a multilayer winding, as shown in Figure 4:

\[ H(x_n) = nH_0, \]  

(16)

where \( x_{no} \) and \( x_{ni} \) — coordinates of the outer and inner boundaries of the n layer, respectively, and \( H_0 = \frac{NI}{h} \) — field strength at the boundary of the first (outer) layer of the winding. Thus, solving a one-dimensional problem for a sinusoidal current, it is possible to obtain the Dowell formula, which reflects the ratio of AC and DC resistances:

\[ F_r = \frac{R_{ac}}{R_{dc}} = \Delta \left[ \frac{\text{sh}(2\Delta)+\sin(2\Delta)}{\text{ch}(2\Delta)-\cos(2\Delta)} + \frac{2(p^2-1)}{3} \frac{\text{sh}(\Delta)-\sin(\Delta)}{\text{ch}(\Delta)+\cos(\Delta)} \right]. \]  

(17)

Here \( p \) is the number of layers of the winding, and \( \Delta = \frac{d}{\delta} \) —ratio of the conductor thickness to the skin depth:

\[ \delta = \frac{1}{\sqrt{\pi \sigma_f}}. \]  

(18)

Later, this technique was supplemented by Karsten [7] for the analysis of currents of arbitrary shape. Calculating the thickness of the skin layer and the ratio of resistances (16) at each frequency, we can find the total losses in the winding:

\[ P = \sum_{i=0}^{n} \delta(NI_i)^2R_{dc}F_{ri}. \]  

(19)

Here \( F_{ri} \) is the ratio of AC and DC resistances, and \( I_i \) is the RMS current at the frequency of the i harmonic. In this case, the total number of harmonics \( n \) can vary depending on the steepness of the current fronts and the desired calculation accuracy. The disadvantages of this technique include lower accuracy for cylindrical conductors compared to the SFD method, as well as its inapplicability for calculating losses in transformers with an arbitrary winding configuration.

4. Conclusion

In this work, the main methods for calculating and assessing losses in the windings of magnetic components were analyzed. The Sullivan method (SFD method) described in the article makes it possible to determine with sufficient accuracy the losses in the windings of cylindrical conductors, significantly reducing the time for modeling the magnetic component. Solving a two-dimensional problem, it is possible to determine the losses in magnetic components with a fairly diverse geometry and different configurations of the windings. However, this approach, as well as similar ones [1, 2], still requires finding the distribution of the magnetic field in the winding section using CAD using the finite element method, which does not allow us to solve the optimization problem with sufficient convenience. On the other hand, Dowell's approach is quite simple. Reducing a three-dimensional problem to a one-dimensional one, it allows you to completely solve the problem by the analytical method, without resorting to numerical modeling. Despite the relatively low accuracy compared to the SFD method, the Dowell method allows you to solve the problem of optimizing the conductor thickness. In addition, it is easily scalable for currents of various shapes [7]. The disadvantages of the method include a decrease in accuracy for cylindrical conductors, as well as the impossibility of calculating magnetic components with a complex winding configuration or with alternating windings. In conclusion, it should be said that both methods have a significant drawback in the form of the impossibility of calculating losses in transformers with alternating current flow in the windings, such
as in a flyback converter, as well as their inapplicability for analyzing and optimizing losses in planar transformers implemented on a printed circuit board.

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