Collectively-enhanced thermalization via multi-qubit collisions

Angsar Manatuly,1 Wolfgang Niedenzu,2 Ricardo Román-Ancheyta,1 Barış Çakmak,1,3 Özgür E. Müstecaplıoğlu,1,∗ and Gershon Kurizki4

1Department of Physics, Koç University, 34450 Sarıyer, İstanbul, Turkey
2Institut für Theoretische Physik, Universität Innsbruck, Technikerstraße 21a, A-6020 Innsbruck, Austria
3College of Engineering and Natural Sciences, Bahçeşehir University, Beşiktaş, İstanbul 34353, Turkey
4Department of Chemical Physics, Weizmann Institute of Science, Rehovot 7610001, Israel

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We investigate the evolution of a target qubit caused by its multiple random collisions with N-qubit clusters. Depending on the cluster state, the evolution of the target qubit may correspond to its effective interaction with a thermal bath, a coherent (laser) drive, or a squeezed bath. In cases where the target qubit relaxes to a thermal state its dynamics can exhibit a quantum advantage, whereby the target-qubit temperature can be scaled up proportionally to $N^2$ and the thermalization time can be shortened by a similar factor, provided the appropriate coherence in the cluster is prepared by non-thermal means. We dub these effects quantum super-thermalization due to its analogies to super-radiance. Experimental realizations of these effects are suggested.

I. INTRODUCTION

Heat flow or exchange between systems is an abundant resource, yet its control is subtle, particularly in the quantum domain [1–4]. A major motivation for the study of such control is the quest for quantum advantage in the exploitation of this resource for useful purposes, e.g., in heat engines [1–14] or heat diodes [15–17]. A prerequisite for such studies is our ability to control thermalization. According to the open-system approach, thermalization occurs whenever one can decompose the Hilbert space into a small subspace (“system”) and a much larger subspace (“bath”), assuming their interaction is weak enough to be treated perturbatively [18]. It is much less clear what are the criteria for thermalization in system-bath complexes subject to nonclassical state preparation, driving or measurements, even if they are describable by a master equation [18, 19]. Such is the case of the micromaser setup [20] wherein the cavity mode (the “system”) sequentially and randomly interacts with a “bath” composed of quantum objects (atoms [20] or atomic clusters [21, 22]) that are traced out after each interaction [19].

Here we examine the issue of thermalization and heat exchange for a composite system consisting of a target qubit (the system) that randomly, repeatedly, interacts with a cluster of N identical spin-1/2 (qubit) particles (the bath). In such a composite system asymptotic thermalization or its absence turns out to be strongly dependent on the initial state of the N-qubit cluster: We allow for multi-qubit coherences in the cluster bath and show that depending on the type of coherences the bath may drive the target qubit into either a thermal or a coherently-displaced state. Even in cases where the target qubit thermalizes, it can exhibit a quantum advantage, whereby the target-qubit temperature can be scaled up with $N^2$, and the thermalization time can be shortened by a similar factor, provided the appropriate coherence in the cluster is prepared by non-thermal means. We dub these effects quantum super-thermalization because of their similarity (but also differences) with superradiance. Experimental realizations of these effects are suggested.

In addition to the insights provided by this study into thermalization and non-thermalization conditions for composite quantum systems, the controllability of their thermalization rates by subspace-state preparation has potential technological significance that has only sporadically been explored so far: heat engines [1–14] and heat diodes [15–17] based on quantum systems require the ability to turn thermalization on and off as fast as possible, in order to maximize the heat flow (power) through the device.

The basic questions in this context that may be partly elucidated by our study are: What are the criteria for the division of energy exchange between a quantum system and a bath into heat and work [23, 24]? What are the criteria for quantum thermalization enhancement? A growing trend is aimed at identifying quantum coherence/entanglement as a resource that can provide quantum advantages (supremacy) in a thermodynamic process [25], so that the present study contributes to this trend. It extends previous works with featured baths consisting of entangled-qubit pairs [22] or triples [21] as resources that can yield quantum advantage in either heat-transfer/thermalization or work.

In Sec. II we develop a master equation for repeated random interactions of the target qubit with multi-qubit clusters. Classification of multi-qubit persistent quantum coherences in the cluster according to their effect on the target qubit is presented in Sec. III. In Sec. IV we focus on the role of heat-exchange coherences (HEC) in the cluster and their preparation as regards the scaling of the target-qubit temperature and thermalization time. We discuss the results and their possible realizations in Sec. V.
II. MASTER EQUATION

We adopt the commonly encountered central spin model [26–29] wherein a target qubit interacts at random times with a cluster of $N$ qubits, as shown in Fig. 1. The total Hamiltonian reads $H = H_{\text{free}} + H_{\text{int}}$, where

$$H_{\text{free}} = \frac{\hbar \omega_0}{2} \sigma_0^z + \frac{\hbar \omega_b}{2} \sum_{i=1}^{N} \sigma_i^z$$

is the free Hamiltonian of the target qubit and the $N$ bath qubits; $\sigma_i^z = |e_i\rangle \langle e_i| - |g_i\rangle \langle g_i|$ is the Pauli $z$ matrix for the $i$th qubit. For simplicity, we assume system-bath resonance, $\omega_0 = \omega_b$, and identical couplings such that

$$H_{\text{int}} = \hbar g \sum_{i=1}^{N} (\sigma_i^+ \sigma_0^- + \sigma_i^- \sigma_0^+) = \hbar g (J_z \sigma_0^- + J_- \sigma_0^+),$$

where $\sigma_i^- = |g_i\rangle \langle e_i|$ and $\sigma_i^+ = |e_i\rangle \langle g_i|$ are the individual lowering and raising operators and $J_k = \sum_{i=1}^{N} \sigma_i^k$ are the collective raising and lowering spin operators of the bath [18], respectively.

We consider the evolution of the system in the interaction picture with respect to the free Hamiltonian (1). During the interaction time $\tau$, the system evolves unitarily according to the propagator $U(\tau) = \exp(-iH_{\text{int}} \tau / \hbar)$ which, to second order in $g \tau$, reads

$$U(\tau) \approx 1 - i g \tau (J_+ \sigma_0^- + J_- \sigma_0^+) - \frac{(g \tau)^2}{2} (J_+ J_- \sigma_0^+ \sigma_0^- + J_- J_+ \sigma_0^+ \sigma_0^-).$$

Before each interaction, the system-bath density operator is supposed to be factorized, $\rho(t) = \rho_{q}(t) \otimes \rho_{b}$. This means that after each interaction the state of the bath is either reset or another replica of the bath is injected into the setup (see Sec. V). Since the target qubit interacts randomly with the bath qubits at a rate $p$, in a given time interval $\delta t$, the system either evolves according to Eq. (3) or remains invariant [19, 21, 30],

$$\rho(t + \delta t) = p \delta t U(\tau) \rho(t) U^\dagger(\tau) + (1 - p \delta t) \rho(t).$$

In the limit $\delta t \to 0$, the master equation describing the dynamics of the target qubit is obtained by tracing out the degrees of freedom of the bath qubits, yielding [30]

$$\dot{\rho}_q(t) = \text{Tr}_b [p(U(\tau) \rho(t) U^\dagger(\tau) - \rho(t))]$$

where the effective Hamiltonian and the Lindblad operators are

$$H_{\text{eff}} = \hbar g q \tau (\lambda \sigma^+ + \lambda^* \sigma^-)$$

$$\mathcal{L}_s \rho_q = \mu(\sigma^+ \rho_q \sigma^- + \sigma^- \rho_q \sigma^+)$$

$$\mathcal{L}_h \rho_q = \frac{\mu_{rA}}{2} (2 \sigma^- \rho_q \sigma^+ - \sigma^+ \sigma^- \rho_q - \rho_q \sigma^+ \sigma^-)$$

$$+ \frac{\mu_{rE}}{2} (2 \sigma^+ \rho_q \sigma^- - \sigma^- \sigma^+ \rho_q - \rho_q \sigma^- \sigma^+),$$

where $\mu = p (g \tau)^2$. The other coefficients are defined in Table I.

The structure of the master equation (5) is in full analogy to the one obtained for the interaction of a single cavity field mode (harmonic oscillator) with a beam of $N = 2$ or $N = 3$ two-level atoms in a micromaser setup [21, 22].

III. EFFECTS OF BATH COHERENCES ON THE TARGET-QUBIT EVOLUTION

Table I is a central result of this work. It relates the coefficients of the master equation (5) to expectation values of the collective bath spin. Their effect on the target qubit corresponds to an effective coherently-displaced, squeezed, or thermal bath.

| $\lambda$ | $\langle J_- \rangle$ | displaced bath |
| $\varepsilon$ | $\langle J_z^2 \rangle$ | squeezed bath |
| $r_c$ | $\langle J_+ J_- \rangle$ | thermal bath (HEC) |
| $r_d$ | $\langle J_+ J_+ \rangle$ | thermal bath (HEC) |

Table I. The coefficients in the master equation (5) are related to expectation values of the collective bath spin. Their effect on the target qubit corresponds to an effective coherently-displaced, squeezed, or thermal bath.
• **Squeezing coherences**: These coherences are associated with the mean values of the two-excitation transitions in the bath, \( \langle J_x^2 \rangle \). They occur in the Liouvillian (6b) which describes the interaction of the target qubit with an effective squeezed bath [18, 21, 30]. A qubit in a squeezed bath relaxes to an effective thermal state [18].

• **Heat-exchange coherences (HEC)**: These are coherences associated with \( \langle J_x \pm J_y \rangle \) that couple bath states with the same amount of excitations. They occur in the Liouvillian (6c) which describes the de-excitation or the excitation of the target qubit with rates \( \mu r_d \) and \( \mu r_e \), respectively. This Liouvillian thus describes the interaction of the target qubit with an effective thermal bath.

Thus, depending on the structure of the bath state, the target qubit may effectively be exposed to a coherently-displaced, squeezed or thermal environment (or any combination thereof). In contrast to earlier works [21, 22], we here found the rapport between the structure and coefficients of the master equation (5) and many-body expectation values of the collective bath spin (Table I). The insights from Table I expose the intimate connection between squeezing coherences and HECs, in analogy to [21]. Superradiance manifests itself by an \( N \) times faster relaxation rate than a single qubit such that the intensity scales as \( N^2 \) [18]. Analogous behavior will be shown here for collectively-enhanced thermalization.

We note that contrary to the micromaser case, the squeezing Liouvillian (6b) for a qubit also contributes to heating [18]. However, we maintain the distinction between squeezing coherences and HECs, in analogy to [21]. The insights from Table I allow us to answer for any \( N \) (rather than only for \( N \leq 3 \) as in Ref. [21]): How are the different coherences (displacement, squeezing and heat-exchange) distributed in the density matrix \( \rho_b \) of the bath?

In the product-state basis the HECs are located in the off-diagonals of blocks of size

\[
p_k = \binom{N}{k}
\]

in the main diagonal of \( \rho_b \). Here \( k = 0, 1, \ldots, N \) is the number of excitations of each state in block \( k \). These blocks are shown as solid red squares in Fig. 2 and their size (7) can in a straightforward manner be determined from Pascal’s triangle depicted in Fig. 3. The other coherences appearing in the master equation (5), i.e., the displacement (squeezing) coherences, pertain to states differing by one (two) excitations and are shown in blue (grey) in Fig. 2.

The remaining coherences are ineffective as they do not contribute to the second-order Lindblad master equation (5). Some of these ineffective coherences are scattered within the driving, squeezing and heat-exchange blocks (Fig. 4). The only ineffective coherences that appear in a HEC block correspond to the anti-diagonal of \( \rho_b \). These ineffective coherences also do not contribute to the generation of multi-partite correlations in a scenario where multi-qubit clusters repeatedly interact with independent qubits [33].

IV. QUBIT TEMPERATURE AND THERMALIZATION TIME

In this work we wish to focus on the effects of HECs in the bath state. To this end we consider the scenario where only the thermal component (6c) exists in the master equation (5) by setting \( \lambda = \varepsilon = 0 \). We then ask: How do different bath states influence the coefficients \( r_e \) and \( r_d \) and hence the final temperature and the thermalization
time of the target qubit?

In this scenario, the elements of the target qubit density matrix evolve as [18]

\[ \rho_{eg}(t) = \rho_{eg}(0)e^{-t/(2r_e)} \] (8a)
\[ \rho_{ee}(t) = \rho_{ee}(t) \] (8b)
\[ \rho_{gg}(t) = 1 - \rho_{ee}(t) \] (8c)
\[ \rho_{ee}(t) = \frac{r_e + c_0 e^{-t/r_d}}{r_e + r_d} \] (8d)

where \( c_0 = r_d \rho_{ee}(0) - r_e \rho_{gg}(0) \). From Table I, the characteristic qubit thermalization time \( t_{q}^{-1} := \mu(r_e + r_d) \) then evaluates to

\[ t_{q}^{-1} = \mu (\langle J_+ J_- \rangle + \langle J_- J_+ \rangle) \). \] (9)

The steady-state of Eqs. (8) is

\[ \rho_{ss} = \frac{1}{r_e + r_d} \begin{pmatrix} r_e & 0 \\ 0 & r_d \end{pmatrix}, \] (10)

which for \( r_d > r_e \) corresponds to a thermal state with the well-defined temperature

\[ T_q = -\frac{\hbar \omega_0}{k_B \ln \left( \frac{r_e}{r_d} \right)} = -\frac{\hbar \omega_0}{k_B \ln \left( \frac{\langle J_+ J_- \rangle}{\langle J_- J_+ \rangle} \right)}. \] (11)

It can be seen that both the final temperature (11) of the qubit and its thermalization time (9) explicitly depend on the many-body quantum state of the bath, as will be illustrated in several generic cases.

A. Incoherent bath qubits

First, we consider the simplest case in which every bath qubit is prepared in an incoherent mixture

\[ \rho_{i}^{\text{mix}} = p_g |g_i\rangle\langle g_i| + p_e |e_i\rangle\langle e_i| \] (12)

of its excited and ground state with probabilities \( p_e \) and \( p_g > p_e \), respectively, such that

\[ \rho_b = \bigotimes_{i=1}^{N} \rho_{i}^{\text{mix}}. \] (13)

The states (12) may have been individually prepared by a thermal environment at temperature \( T \), in this case \( p_e = p_g \exp[-\hbar \omega_0/(k_B T)] \).

For the state (13) we find \( r_e = Np_e \) and \( r_d = Np_g \) such that the target qubit temperature (11) evaluates to

\[ T_q = -\frac{\hbar \omega_0}{k_B \ln \left( \frac{r_e}{r_d} \right)} = \frac{1}{\mu N}. \] (14)

Hence, the target qubit thermalizes to the temperature of each bath qubit, but \( N \)-times faster. The decrease of the thermalization time with \( N^{-1} \) shows that no quantum advantage applies here.

A similar result was obtained in [34] for a bosonic single-mode field (oscillator) instead of the target qubit.

B. Thermally-prepared HECs

Next we consider a correlated state of the bath qubits with non-vanishing HECs. Such a state may either be prepared by thermal or non-thermal means. We first dwell on the former case which may be realized by collectively (rather than individually as above) coupling the \( N \) bath qubits to a thermal photon environment, thus giving rise to the master equation [18, 30, 35]

\[ \frac{\dot{n}}{2} (\hat{n} + 1) (2J_- \rho_{b} J_+ - J_+ J_- \rho_{b} - \rho_{b} J_+ J_-) \]
\[ + \frac{\gamma_0}{2} \hat{n} (2J_+ \rho_{b} J_- - J_- J_+ \rho_{b} - \rho_{b} J_- J_+), \] (16)

where \( \gamma_0 \) is the single-atom spontaneous emission rate and \( \hat{n} \) the mean number of photons in mode \( \omega_b \) of the environment at temperature \( T \). Assuming that all the bath qubits were initially prepared in their ground state, the dynamics can then only populate the fully-symmetric
Dicke states \[^{18}\] , which correspond to the HECs depicted in Fig. 2 (see Appendix A). The steady state of the bath qubits then reads
\[
\rho_b = \begin{pmatrix}
D_N & 0 & \ldots & 0 & 0 \\
0 & D_{N-1} & \ldots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \ldots & D_1 & 0 \\
0 & 0 & \ldots & 0 & D_0
\end{pmatrix},
\]
where \(\rho_b\) is the reduced density matrix of the bath qubits, \(D_k\) are matrices describing the states of the bath qubits, and \(d_k\) are coefficients of the matrix elements of \(D_k\).

With the \((p_k \times p_k)\)-dimensional matrices \(D_k = d_k U_k\) [cf. Eq. (7)]. Here \(U_k\) denotes the matrix whose elements are all 1 (which means that only the fully-symmetric Dicke states are populated) and the coefficients \(d_k\) evaluate to (see Appendix A)
\[
d_k = \frac{(1 - r)^p}{(1 - r^{N+1})^p}.
\]

where \(r := \bar{n}/(\bar{n} + 1)\).

We now consider the thermally-prepared block-diagonal state (17) as the bath for the target qubit. We then find \(r_e/r_d = r\), which only depends on \(\bar{n}\) (the explicit expressions are given in Appendix B). Consequently, the target qubit attains the same temperature \(T_\text{q}^\text{HEC} = T\) [Eq. (11)] as the environment that was used to prepare the coherent bath qubits in the state (17). Hence, such thermally-prepared bath clusters do not show any quantum superiority in the final temperature of the target qubit.

The thermalization time (9) of the target qubit evaluates to
\[
\tau_\text{q}^\text{HEC} = \frac{1}{\gamma_{\text{eff}}(2\bar{n} + 1)},
\]
where \(\gamma_{\text{eff}} := \mu r_e/\bar{n} = \mu r_d/\bar{n}(\bar{n} + 1)\) is the effective spontaneous emission rate that depends on the number of qubits (see Appendix B) and, basically, increases linearly with \(N\).

Hence, regardless of whether the bath consists of \(N\) independent thermal spins [Eq. (15)] or a thermal collective spin [Eq. (19)], the thermalization rate is sped up linearly in \(N\) in either case, which is also expected in a classical setting. Furthermore, the temperature attained by the target qubit always coincides with the environment temperature used to prepare the bath. This leads to the conclusion that a non-thermal generation of the HECs is required in order to exhibit possible quantum advantages of the bath.

C. Non-thermally prepared HECs: Dicke bath

We now consider a different type of \(N\)-qubit bath, namely, a fully-symmetric Dicke state with \(k\) excitations, such that only one of the blocks in the state (17) is populated,
\[
\rho_b = \begin{pmatrix}
0 & \ldots & \ldots & 0 \\
\vdots & \ddots & \ldots & 0 \\
\vdots & \ldots & D_k & \ddots \\
0 & 0 & \ldots & 0
\end{pmatrix},
\]
where normalization now implies \(D_k = U_k/p_k\). A possible preparation method of such states has been recently proposed in Ref. [36]. For the state (20) we find
\[
r_e = \langle J_\pm J_\pm \rangle = k(N - k + 1)
\]
\[
r_d = \langle J_- J_+ \rangle = (k + 1)(N - k).
\]

Thus, the steady-state temperature of the target qubit is
\[
T_\text{q}^\text{D} = -\frac{\hbar \omega_0}{k_B \ln \left(\frac{k(N - k + 1)}{\langle J_- J_+ \rangle (N - k)}\right)},
\]
which is positive as long as \(r_d > r_e\), yielding the non-inversion condition
\[
k \leq \lfloor N/2 \rfloor - 1.
\]

Namely, there must always be more bath qubits in the ground state than in the excited state.

The temperature (22) depends on the number of excitations \(k\) and the number of bath qubits \(N\). For large \(N\) and large \(k\) (such that \(r_d/r_e \gtrsim 1\)) the Laurent expansion yields
\[
\frac{k_B T_\text{q}^\text{D}}{\hbar \omega_0} \approx \frac{(k + 1)(N - k)}{(k + 1)(N - k) - k(N - k + 1)} - \frac{1}{2}.
\]

If we consider a block containing a relatively small number of excitations, i.e., away from the central block, for example \(k = N/4\) (assuming \(N\) is an integer multiple of 4), we find
\[
\frac{k_B T_\text{q}^\text{D}}{\hbar \omega_0} \approx 1 + \frac{3N}{8}.
\]

Hence, in this case, the scaling of the temperature is linear with the number of qubits in the bath cluster. On the other hand, increasing the number of excitations to \(k = N/2 - 1\), i.e., choosing the closest allowed block to the central one, yields
\[
\frac{k_B T_\text{q}^\text{D}}{\hbar \omega_0} \approx \frac{N^2 + 2N}{8} - \frac{1}{2}.
\]

Here we observe a clear quadratic scaling with \(N\). Hence, by varying the number of excitation in the bath, the temperature scaling may exhibit a strong collective quantum advantage by growing quadratically with the bath-ensemble size \(N\). This constitutes the second main result of this work.
(linear and quadratic means) of the collective spin of the bath (Table I), and showed that according to the class of state of the bath. We have established the dependence of the target qubit strongly depends on the quantum state of the bath. We have investigated a collisional model wherein a single target qubit repeatedly interacts with clusters of \( N \) bath qubits. We have observed that the evolution of the target qubit faster (optimally with \( t_{\text{opt}} \)) than their thermal (individual or collective) counterparts prepared bath states, the thermalization time may be sped-up quadratically in \( N \). For the examples above, \( k = N/4 \) and \( k = N/2 - 1 \), we find

\[
t_{q}^{D}(N/4) = \left[ \mu(N + 3N^2/8) \right]^{-1} \tag{28a}
\]

and

\[
t_{q}^{D}(N/2 - 1) = \left[ \mu(N + N^2/2 - 2) \right]^{-1}, \tag{28b}
\]

respectively. Therefore, coherences in the \( N \)-qubit bath prepared by non-thermal means allow us to thermalize the target qubit faster (optimally with \( N^{-2} \) dependence) than their thermal (individual or collective) counterparts (optimally with \( N^{-1} \) dependence), as shown in Fig. 5.

This favorable scaling is also manifest in the corresponding thermalization time (9), which for the Dicke state (20) is found to be

\[
t_{q}^{D}(k) = \frac{1}{\mu(N + 2kN - 2k^2)}. \tag{27}
\]

Hence, contrary to the previous cases of thermally-prepared bath states, the thermalization time may be sped-up quadratically in \( N \). For the examples above, \( k = N/4 \) and \( k = N/2 - 1 \), we find

\[
t_{q}^{D}(N/4) = \left[ \mu(N + 3N^2/8) \right]^{-1} \tag{28a}
\]

and

\[
t_{q}^{D}(N/2 - 1) = \left[ \mu(N + N^2/2 - 2) \right]^{-1}, \tag{28b}
\]

respectively. Therefore, coherences in the \( N \)-qubit bath prepared by non-thermal means allow us to thermalize the target qubit faster (optimally with \( N^{-2} \) dependence) than their thermal (individual or collective) counterparts (optimally with \( N^{-1} \) dependence), as shown in Fig. 5.

**V. CONCLUSIONS**

We have investigated a collisional model wherein a single target qubit repeatedly interacts with clusters of \( N \) bath qubits. We have observed that the evolution of the target qubit strongly depends on the quantum state of the bath. We have established the dependence of the qubit evolution on the first and second moments (linear and quadratic means) of the collective spin of the bath (Table I), and showed that according to the class of multi-qubit coherences in the bath clusters, the bath can thermalize the qubit, cause its coherent drive or simulate squeezed-bath effects on the target qubit.

Focusing on the case of a thermalized target qubit, we have considered different types of coherent and incoherent bath clusters; either thermally or non-thermally prepared. We have shown that thermally-prepared bath states cause thermalization of the target qubit to the temperature of the environment that was used to create them. Whether the bath consists of \( N \) independent thermal qubits [Eq. (13)] or thermally entangled qubits [Eq. (17)], in both cases a decrease of the target qubit thermalization time proportional to \( N^{-1} \) is found.

On the other hand, non-thermally-prepared bath states with only one non-zero Dicke block yield either linear [Eq. (25)] or quadratic [Eq. (26)] scaling of the target qubit temperature (24) with the number of qubits in the bath ensemble, depending on the number of excitations in the chosen block. The largest quantum advantage corresponds to quadratic scaling \( T \propto N^2 \). Likewise, the relaxation time may decrease quadratically, \( t_{\text{Q}} \propto N^{-2} \).

This \( N^2 \) quantum advantage obtained here is reminiscent of superradiance. Yet, whereas superradiance occurs for the collective interaction of an ensemble of qubits with a common bath [18], here it is the bath that exhibits collective behavior.

The Hamiltonian (2) effectively describes various systems, e.g., nuclear spin baths in quantum dots [37, 38], nitrogen-vacancy (NV) centers [39], nuclear magnetic resonance systems [40], microcavities hosting multi-atom systems [41, 42] or multiple superconducting qubits [36, 43], and molecular nanomagnets [44, 45].

However, it is not straightforward in the above-mentioned scenarios to mimic the effect of repeated impulsive interactions of a target qubit and non-thermally prepared \( N \)-qubit bath states. The required protocol would then consist of multiple steps, each step involving: controlled preparation of the \( N \)-qubit (entangled) bath state, abrupt on- and off-switching of its interaction with the target qubit followed by resetting of the \( N \)-qubit bath to its initial state.

There is, however, a simpler and more natural scenario where the described process can be implemented: It concerns a cold-atom cloud with, upon absorbing a few quanta, attains a superradiant (symmetric) Dicke state [46–48]. Following such state preparation, an impurity atom passing near or within this cloud will, to a good approximation, realize the described model.

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Upon transforming to the product-state basis by a unitary transformation $U$, the population $\rho_m$ of every fully-symmetric state $|N/2, m\rangle$ is transferred to the corresponding HEC block (Fig. 7). Each such block has dimensionality $(p_k \times p_k)$ [Eq. (7)] and may be written $D_k = d_k U_k$, where $U_k$ denotes the matrix whose elements are all 1. Hence, the population ratio $\rho_{m+1}/\rho_m$ is directly reflected in the ratio of the traces of two consecutive blocks,

$$\frac{\text{Tr}[D_{m+1}]}{\text{Tr}[D_m]} = \frac{\rho_{m+1}}{\rho_m} = \exp \left(-\frac{\hbar \omega_0}{k_B T}\right) = \frac{n}{n+1} =: r,$$

(A4)

From the normalization condition

$$1 = \text{Tr}(\rho_b) = \sum_{k=0}^{N} \text{Tr}[D_k] = \sum_{k=0}^{N} r^k \text{Tr}[D_0]$$

(A5)

follows

$$\text{Tr}[D_0] = \frac{1-r}{1-r^{N+1}}.$$  

(A6)

Since $D_k = d_k U_k$, we have $d_k = \text{Tr}[D_k]/\text{Tr}[U_k]$. Using $\text{Tr}[D_k] = r^k \text{Tr}[D_0]$ and $\text{Tr}[U_k] = p_k$ then yields

$$d_k = \frac{(1-r)r^k}{(1-r^{N+1})p_k}.$$  

(A7)

Appendix A: Thermal preparation of HECs

We may express the density matrix $\rho_b$ of the bath spins in either the product-state basis or the total-spin (Dicke) basis $|j, m\rangle$ where $j = 0, \ldots, N/2 \ (j = 1/2, \ldots, N/2)$ for even (odd) $N$ and $m = -j, \ldots, j$ [18]. Each $m$ corresponds to a distinct HEC block in the product-state basis. If, as in the main text, the atoms are initially prepared in their ground state $|g_1, g_2, \ldots, g_N\rangle \equiv |j = N/2, m = -N/2\rangle$, the dynamics governed by the master equation (16) do not mix the different $j$, i.e.,

$$\rho_b(0) = \frac{N}{2} \begin{pmatrix} N \ 2 \ 
N \ 2 \ 
- N \ 2 \end{pmatrix}$$

$$\rho_b(t) = \sum_{m=-N/2}^{N/2} \rho_m(t) \begin{pmatrix} N \ 2 \ 
N \ 2 \ 
m \end{pmatrix}.$$  

(A1)

The time evolution according to Eq. (16) then thermally populates every $|N/2, m\rangle$ such that in steady state (see Fig. 6)

$$\rho_m = \frac{1}{Z} \exp \left(-\frac{m\hbar \omega_0}{k_B T}\right)$$

(A2)

with

$$Z = \sum_{m=-N/2}^{N/2} \exp \left(-\frac{m\hbar \omega_0}{k_B T}\right).$$  

(A3)

Appendix B: Explicit form of the rates

For the state (17) the coefficients in Table I evaluate to [30]

$$r_c = \text{Tr}[J_- \rho_b J_+] = \sum_{k=0}^{N} d_k \text{Tr}[J_- U_k J_+]$$

$$= \sum_{k=1}^{N} d_k (N-k+1)^2 p_{k-1}$$

$$= \sum_{k=1}^{N} (1-r)r^k (N-k+1)(1-r^{N+1})$$

(B1)

and

$$r_d = \text{Tr}[J_+ \rho_b J_-] = \sum_{k=0}^{N} d_k \text{Tr}[J_+ U_k J_-]$$

$$= \sum_{k=0}^{N-1} d_k (k+1)^2 p_{k+1}$$

$$= \sum_{k=1}^{N} (1-r)r^{k-1} k(N-k+1)/(1-r^{N+1}).$$  

(B2)
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