Relating inclusive $e^+e^-$ annihilation to electroproduction sum rules in Quantum Chromodynamics

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The Broadhurst-Kataev conjecture, that the “discrepancy” in the connection with the $\pi^0 \to \gamma\gamma$ anomaly equals the beta function $\beta(\pi)$ times a power series in the effective coupling $\alpha$, is proven to all orders of perturbative quantum chromodynamics. The use of nested short-distance expansions is justified via Weinberg’s power-counting theorem.

There has been a revival of interest in the relation

$$3S = KR'$$

between the anomalous constant $S$ governing $\pi^0 \to \gamma\gamma$ decay, the isovector part $R'$ of the cross-section ratio

$$R = \{e^+e^- \to \text{hadrons}\}/\{e^+e^- \to \mu^+\mu^-\}$$

at large centre-of-mass energy $\sqrt{s}/2$, and the lowest isovector moment of the first spin-dependent structure function $g_1(x,Q^2)$ for inclusive electroproduction at large momentum transfer $Q$:

$$\int_0^1 dx \, g_1^{ep-en}(x,Q^2) = \frac{1}{6} \left[ \frac{g_A}{g_V} \right] K(Q^2) + O(Q^{-2})$$

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The result (1) was derived in a non-perturbative fashion before the advent of quantum chromodynamics (QCD), with the hadronic energy-momentum tensor $\theta_{\mu\nu}$ assumed to have a soft trace:

$$\dim \theta_{\mu}\theta_{\mu} < 4, \text{ pre-QCD}$$

(4)

Of course, there is a trace anomaly [5, 12] in QCD [13] which violates (4). Nevertheless, the relation (1) remains valid for QCD in leading logarithm approximation: $K \rightarrow 1$ and $R' \rightarrow N_c/2$, where $N_c$ is the number of colours. The choice $N_c = 3$ fits the observed value $S_{\text{expt}} \approx 0.5$.

Broadhurst and Kataev [1] tried extending (1) to the leading QCD power, i.e. to corrections like $K(Q^2)$ in (3) and $D(Q^2)$ in the Adler function [14]

$$Q^2 \int_{4m_r^2}^{\infty} ds R(s)/(s + Q^2)^2 = D(Q^2)R(\infty) + O(Q^{-2})$$

(5)

The axial-vector current was required to be flavour non-singlet as in [5], but the vector currents could be either flavour singlet ($F = S$) or non-singlet ($F = NS$). Products $K_F D_F$ were tested using existing multi-loop results [15, 16] for the $F = S, NS$ versions of $K$ and $D$. Here $D_{NS}$ and $D_S$ are the factors in (3) produced by the isovector and baryon currents, $K_{NS}$ is the factor $K(Q^2)$ in the Bjorken sum rule (3), and $K_S$ gives the leading power in the sum rule of Gross and Llewellyn Smith [17] (but not that of Ellis and Jaffe [18]).

Also considered [1] were the $Q$-independent corrections $K^*$ and $D^*$ due to subsets of Feynman diagrams in which the gauge coupling $g$ is not renormalized. Actually, the comparison was with an Abelian calculation [19], but one can imagine a conformal non-Abelian extension obtained by neglecting self energies in an axial gauge.

Broadhurst and Kataev [1] found, in agreement with early work of Adler, Callan, Gross and Jackiw [20], that $K^*$ and $D^*$ satisfy (1) to the highest order of calculation available, i.e.

$$K_F^*(\alpha_s)D_F^*(\alpha_s) = 1, \quad F = S, NS$$

(6)

up to terms $O(\alpha_s^4)$ in $\alpha_s = g^2/4\pi$. With all diagrams included, $K$ and $D$ become power series in the effective coupling constant $\overline{\alpha} = \overline{\alpha}(Q^2/\mu^2, \alpha_s)$ defined by

$$\log(Q^2/\mu^2) = \int_{\alpha_s}^{\overline{\alpha}} d\alpha'/\beta(\alpha')$$

(7)

where $\mu$ denotes a suitable renormalization scale. Then Broadhurst and Kataev found that the relation (1) is violated by a term proportional to $\beta(\overline{\alpha})$,

$$K_F(\overline{\alpha})D_F(\overline{\alpha}) = 1 + \beta(\overline{\alpha}) \times \{\text{power series in } \overline{\alpha}\}, \quad F = S, NS$$

(8)

again to the $O(\overline{\alpha}^4)$ accuracy of current calculations. They christened the extra term “the Crewther discrepancy”.

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3In effect, conformal invariance was supposed to be softly broken and so valid for leading powers at short distances. The immediate precursors of [5] were papers by Schreier [9] and Migdal [10]. Some conformal aspects of [5] were found independently by Ferrara et al. [11].
In this letter, equations (8) and (10) will be shown to hold to all orders of perturbation theory. (An independent derivation of (8) by D. Müller, which I have not seen, is expected in the near future [21].) The derivation runs along the lines of [14], except that the pre-QCD assumption (12) is replaced by an analysis based on renormalized conformal Ward identities [22] and the QCD trace anomaly [13]

$$\theta^\mu = \frac{\beta(\alpha_s)}{\alpha_s} F^2 + \text{quark-mass terms}$$  \hspace{1cm} (9)

Here $F^2$ denotes the renormalized square of the gluonic field strength tensor $F^{\mu\nu}_{\mu\nu}$.

Let $J_\mu(x)$ and $J_{\mu5}(x)$ be the electromagnetic current and the isovector axial-vector current in QCD. As an operator on hadron states, $J_{\mu5}$ is almost conserved: its divergence $\partial^\mu J_{\mu5} = \Delta(x)$ is proportional to the light-quark masses $m_u, m_d$. In any order of perturbation theory, $\Delta$ carries a dynamical dimension of 3 modified by QCD logarithms. The condition $\dim \Delta < 4$ is satisfied, so Wilson’s prescription [23] for the anomalous constant $S$ can be adopted without change:\footnote{Perturbatively, photon channels have no mass gap because of the three-gluon threshold (for example), but the effect is infra-red finite and hence not a problem. In the pseudoscalar channel, the light $q\bar{q}$ pair provides a factor $\sim (m_u + m_d)^{-1}$ [24] which cancels the explicit mass dependence of $\Delta$ and so produces a result for $S$ independent of $m_u$ and $m_d.$}

$$S = \frac{\pi^2}{12} \epsilon^{\mu\nu\alpha\beta} \int_{\mathcal{R}_\epsilon} d^4x d^4y x_\mu y_\nu T\langle \text{vac} | J_\alpha(x) J_\beta (0) \Delta(y) | \text{vac} \rangle + O(\epsilon) \hspace{1cm} (10)$$

Here $\mathcal{R}_\epsilon$ is the entire eight-dimensional volume, except that narrow regions of width $O(\epsilon)$ containing coincident points $x_\mu = 0, y_\mu = 0$, or $x_\mu = y_\mu$ are excluded (Fig. 1). By definition, $S$ remains independent of $\epsilon$.

Wilson noticed that (10) is the volume integral of an eight-divergence which can be converted into an integral of the VVA amplitude $T\langle J_\alpha J_\beta J_{5\gamma} \rangle$ over the
short-distance surface $S_\epsilon$ bounding $R_\epsilon$. Only the leading power $C_{\alpha\beta\gamma}$ multiplying the identity operator $I$ in the expansion

$$T\{J_\alpha(x)J_\beta(0)J_\gamma(y)\} \sim C_{\alpha\beta\gamma}(x,y)I + \ldots, \quad x,y \to 0$$

(11)
can contribute in the limit $\epsilon \to 0$:

$$S = \frac{\pi^2}{12} \epsilon^{\mu\nu\alpha\beta} \int S_\epsilon \left\{d^4x d^4y x_\mu y_\nu C_{\alpha\beta\gamma} - y_\beta C_{\alpha\gamma\nu} - y_\gamma C_{\alpha\nu\beta} + dS_y^\gamma d^4y y_\beta (x_\alpha C_{\gamma\mu\nu} - x_\mu C_{\alpha\gamma\nu}) \right\} + O(\epsilon)$$

(12)

In QCD perturbation theory, $C_{\alpha\beta\gamma}$ consists of:

(a) a lowest-order contribution $S\Delta_{\alpha\beta\gamma}(x,y)$ from the two bare triangle diagrams. Schreier [9] showed that the $x,y$ dependence of $\Delta_{\alpha\beta\gamma}$ is allowed uniquely by conformal invariance. The resulting integral in (12) was performed as part of the non-perturbative analysis of [5] and produced the expected answer.

(b) amplitudes which break conformal invariance as a result of internal coupling constant renormalization. The simplest example is shown in Fig. 2. The Adler-Bardeen theorem [7,25–27] requires that these amplitudes do not contribute to $S$, even though some contributions at four loops and beyond are logarithmically more singular at $x,y \sim 0$ than the bare triangle diagrams.

In [5], the asymptotic three-point amplitude $C_{\alpha\beta\gamma}$ was analysed by substituting an expansion of the form

$$T\{J_\alpha(x)J_\beta(0)\} = C_{\alpha\beta}^R(x)I + C_{\alpha\beta}^K(x)J_\mu(0) + \ldots, \quad x_\mu \to 0$$

(13)
in $T\{J_\alpha J_\beta J_\gamma\}$ and then expanding as follows:

$$T\{J_\mu(0)J_\gamma(y)\} = C_{\mu\gamma}^R(y)I + \ldots, \quad y_\mu \to 0$$

(14)

Matching the expansions (11), (13) and (14) gave the constraint

$$C_{\alpha\beta\gamma}(x,y) \to C_{\alpha\beta}^R(x)C_{\mu\gamma}^R(y), \quad x \ll y$$

(15)

The superscripts $K$ and $R'$ label amplitudes related to the Bjorken sum rule [3] and the isovector part of [2]. The limit $y \ll x$, where $T\{J_\beta(0)J_\gamma(y)\}$ is expanded first, produces the connection with the Gross-Llewellyn Smith sum rule [17].

The technique leading to (13) depends on an interchange of nested short-distance limits $\rho_1 \to 0$ and $\rho_2 \to 0$ for operator products

$$T\left\{\prod_i A_i(\rho_1 \rho_2 \hat{x}_i)B(0) \prod_j C_j(\rho_2 \rho_j)\right\}$$

(16)

with $\hat{x}_i$ and $\hat{y}_j$ held fixed. Unlike most limit interchanges at short-distances, this always works, in any order of renormalized perturbation theory. The required uniformity property is contained in Weinberg’s power-counting theorem [28].
Weinberg introduced a $4N$-dimensional vector $P$ to describe the asymptotic behaviour of Euclidean amplitudes depending on $N$ four-dimensional momenta:

$$P = L_1 \eta_1 \eta_2 \ldots \eta_m + L_2 \eta_2 \ldots \eta_m + \ldots + L_m \eta_m + C \quad (17)$$

Here $\eta_1 \ldots \eta_m$ are large positive parameters corresponding to an arbitrary set of $m$ independent fixed vectors $L_1, \ldots, L_m$ ($m \leq 4N$), and $C$ is a bounded vector. The theorem states that all amplitudes belong to asymptotic classes labelled by characteristic asymptotic powers (and powers of logarithms \[29\]) as $\eta_1 \ldots \eta_m$ tend independently to infinity. In other words, the $\eta_i \to \infty$ limits are uniform with respect to each other: they can be carried out in any order.

Evidently

$$\eta_1 = \rho_1^{-1}, \quad \eta_2 = \rho_2^{-1} \quad (18)$$

are special cases of Weinberg’s asymptotic $\eta$ parameters. For any Green’s function containing \[16\] as a sub-product, we can subtract off terms in

$$T \{ \prod_i A_i(\rho_1 \rho_2 x_i) B(0) \} \sim \sum_m C_m(\rho_1 \rho_2, \{x_i\}) O'_m(0), \quad \rho_1 \to 0 \quad (19)$$

and

$$T \{ O'_m(0) \prod_j C_j(\rho_2 y_j) \} \sim \sum_n C_{mn}(\rho_2, \{y_j\}) O_m(0), \quad \rho_2 \to 0 \quad (20)$$

to remove as many asymptotic powers in $\rho_1$ and $\rho_2$ as we wish, and be sure that, relative to \[16\] in Euclidean space, the remainder

$$R_{MN} = T \{ \prod_i A_i B \prod_j C_j \} - \sum_{m=1}^M \sum_{n=1}^N C_{mn} C_n O_n \quad (21)$$

will be $O(\rho_1^{-M} \rho_2^{-M-N} \times \{\text{logs of } \rho_1 \text{ and } \rho_2\})$ as $\rho_1$ and $\rho_2$ tend independently to 0. Therefore the coefficient functions $f_n$ in the expansion $\sum_n f_n O_n$ of \[16\] for $\rho_2 \to 0$ must obey the rule

$$f_n \sim \sum_n C_m C_{mn}, \quad \rho_1 \to 0 \quad (22)$$

The interchange of the limits $\rho_1 \to 0$ and $\rho_2 \to 0$ to obtain conditions such as \[13\] is thus justified. Note that conformal invariance is not assumed — the result is absolutely general.

For conformal subsets of graphs, the analysis of \[3\] is fully applicable, with $R'$ and $K$ replaced by $\frac{1}{2} N \alpha_s D_{F}(\alpha_s)$ and $K_{F}(\alpha_s)$. Hence equation \[3\] is valid to all orders of perturbation theory.

In general, internal coupling constant renormalization breaks conformal invariance in leading powers and causes logarithms of $\rho_1$ and $\rho_2$ to appear. This occurs first for three-loop diagrams (Fig. 3):

$$T \langle J_\alpha(\rho_1 \rho_2 \hat{x}) J_\beta(0) J_{\gamma 5}(\rho_2 \hat{y}) \rangle_{3 \text{-loop}} = O(\rho_2^{-9} \rho_1^{-3} \log \rho_1) \quad (23)$$

\[5\]This should have been discussed in 1972. At the time, I tried to find a natural extension of axiomatic field theory to incorporate Weinberg’s asymptotic classes, but the project proved to be too ambitious — hence the mysterious reference 10 in \[5\] which never appeared.
Effects of this type are controlled by conformal Ward identities [22] in which the current
\[ K_{\mu
u} = (2x^\lambda x_\nu - \delta_\nu^\lambda x^2)\theta_{\mu\lambda}(x) \] (24)
has a divergence containing the QCD trace anomaly [13]:
\[ \partial^\mu K_{\mu
u} = 2x_\nu \frac{\beta(\alpha_s)}{\alpha_s} F^2 + \text{quark-mass terms} \] (25)

Let \( D_\nu \) denote the matrix differential operator which induces an infinitesimal conformal transformation on a current:
\[ D_\nu J_\alpha(x) = 2x_\nu (3 + x \cdot \partial) J_\alpha - x^2 \partial_\nu J_\alpha + 2g_{\nu\alpha} x \cdot J - 2x_\alpha J_\nu \] (26)

In order to write down the conformal Ward identity for the VVA amplitude, it is necessary to have \( D_\nu \) act separately on \( J_\alpha(x) \), \( J_\beta(x') \) (with \( x' \) temporarily not set to zero) and \( J_\gamma_5(y) \), and then take the sum. The result is a first-order differential equation in \( x, x', y \) space:
\[ (\sum_{\text{currents}} D_\nu) T\langle J_\alpha(x) J_\beta(x') J_\gamma_5(y) \rangle = \frac{\beta(\alpha_s)}{\alpha_s} T\langle J_\alpha J_\beta J_\gamma_5 \int d^4z \, 2z_\nu F^2(z) \rangle + \text{non-leading powers} \] (27)

For the moment, regions in which \( x, x', y \) coincide are excluded.

The solution of (27) consists of a particular integral and a homogeneous part. Schreier’s work [9] requires the homogeneous part to be proportional to the bare triangle amplitude \( \Delta_{\alpha\beta\gamma} \). So, setting \( x' = 0 \) once more, we find that the leading power \( C_{\alpha\beta\gamma} \) in (11) is given by
\[ C_{\alpha\beta\gamma} = c \Delta_{\alpha\beta\gamma}(x, y) + \frac{\beta(\alpha_s)}{\alpha_s} \overline{\Delta}_{\alpha\beta\gamma}(x, y; \alpha_s) \] (28)
where \( \overline{\Delta}_{\alpha\beta\gamma} \) is a power series in \( \alpha_s \) and \( c \) is a constant of integration.

Equation (28) can be substituted into the surface integral (12), with the result
\[ S = c + \frac{\beta(\alpha_s)}{\alpha_s} \overline{S}(\alpha_s) \] (29)
where \( \overline{S} \) is the contribution due to \( \overline{\Delta}_{\alpha\beta\gamma} \). This allows us to eliminate \( c \) from (28):
\[ C_{\alpha\beta\gamma} = S \Delta_{\alpha\beta\gamma}(x, y) + \frac{\beta(\alpha_s)}{\alpha_s} \{ \overline{\Delta}_{\alpha\beta\gamma}(x, y; \alpha_s) - \overline{S}(\alpha_s) \} \Delta_{\alpha\beta\gamma}(x, y) \] (30)

The next step is to substitute the \( x \ll y \) constraint (14). This is less straightforward than in [1]: unlike [1], the desired result (8) contains momentum dependent amplitudes which include contributions from coincident points \( x = 0 = y \). At such points, operator product expansions need not be generally valid because of renormalization ambiguities proportional to \( \delta^4 \) functions and their derivatives [30]. In this case, the problem is solved by making a second use of electromagnetic gauge invariance (the first being Wilson’s prescription (10)), and projecting out the Adler function.
Since $C_{\alpha\beta\gamma}$, $\Delta_{\alpha\beta\gamma}$ and $\overline{\Delta}_{\alpha\beta\gamma}$ are superficially linearly divergent, they may have ambiguities linear in momenta, i.e. proportional to $(\partial_x$ or $\partial_y)\delta^4(x)\delta^4(y)$ in coordinate space. As is well known, these ambiguities can be removed by imposing electromagnetic gauge invariance as renormalization conditions of the form

$$\partial_x^\alpha C_{\alpha\beta\gamma} = 0 = (\partial_x + \partial_y)^\beta C_{\alpha\beta\gamma}$$

(31)

The resulting amplitudes are then defined uniquely for all $x, y$, with non-canonical (anomalous) results for $\partial_y C_{\alpha\beta\gamma}$ and $\partial_y \Delta_{\alpha\beta\gamma}$. (Note that Lorentz covariance is also imposed as a renormalization condition; canonical constructions such as $T^\nu$-products should be avoided.)

With $C_{\alpha\beta\gamma}$ thus well defined, its $x \ll y$ limit will also respect electromagnetic gauge invariance. There is no problem with $C_{\alpha\beta\gamma}$: it converges superficially, so it can be extended to $x = 0$ without ambiguity, with current conservation maintained for the indices $\alpha$ and $\beta$. However the amplitude $C_{\mu\nu\gamma}(y)$ has a superficial quadratic divergence. Electromagnetic gauge invariance reduces this to a superficial logarithmic divergence,

$$C_{\mu\nu\gamma}(y) = (g_{\mu\gamma}\partial^2 - \partial_\mu\partial_\gamma)\Pi_{\mu\nu}(y)$$

(32)

but does not specify the subtraction procedure which fixes the $\delta^4(y)$ term in $\Pi_{\mu\nu}(y)$. The actual subtraction procedure is determined implicitly via the $x \ll y$ limit of $C_{\alpha\beta\gamma}$; presumably it has a complicated $\alpha_s$ dependence, so this information is not very useful.

Fortunately, we do not need this information. One of the advantages of the Adler function is that it does not depend on the subtraction procedure used to renormalize the hadronic vacuum polarization $[14]$. In the leading power

$$D_{NS}(-q^2)K_F(\alpha_s)D_F(\alpha_s) = 1 + \beta(\alpha_s)K_F(P^2, \alpha_s)D_F(Q^2, \alpha_s)$$

(35)

where $K_F$ and $D_F$ are power series in $\alpha_s$.

The final step is to set $P = Q$ and substitute

$$\beta(\alpha_s) = \beta(\alpha)(\partial^2 / \partial \alpha_s)^{-1}$$

(36)
to obtain

\[ K_{\mathcal{F}}(\alpha) D_{\mathcal{F}}(\alpha) = 1 + \beta(\alpha) \mathcal{P}_{\mathcal{F}}(Q^2, \alpha_s) \]  \hspace{1cm} (37)  \\
\[ \mathcal{P}_{\mathcal{F}} = K_{\mathcal{F}}(Q^2, \alpha_s) D_{\mathcal{F}}(Q^2, \alpha_s)(\partial \alpha / \partial \alpha_s)^{-1} \]  \hspace{1cm} (38) 

for \( \mathcal{F} = S, NS \). Since \( \mathcal{P}_{\mathcal{F}} \) is a power series in \( \alpha_s \) and the rest of (37) depends only on \( \alpha \), we have

\[ \mathcal{P}_{\mathcal{F}} = \text{power series in } \alpha \]  \hspace{1cm} (39) 

Hence equation (3) is proven to all orders in perturbation theory.

It has been observed [3] that the first two terms of \( \mathcal{P}_{\mathcal{F}} \) can be removed by using different commensurate scales \( Q, Q^* \) for \( K \) and \( D \):

\[ K_{\mathcal{F}}(Q) D_{\mathcal{F}}(Q^*) = 1 + O(\alpha_s^4) \]  \hspace{1cm} (40) 

It remains an open question whether this result can be extended to higher orders or not.

Finally [4], can this work be extended to include the singlet axial-vector operator and hence the leading power in the Ellis-Jaffe sum rule [18]? It seems not. The problem is [31] that the analogue of (12) involves a leading power with \( J_{\mu_5} \) replaced in (11) by the gauge-dependent symmetry current of Adler [7] and Bardeen [26].

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