Secure sharing of random bits over the Internet

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Although one-time pad encrypted files can be sent through Internet channels, the need for renewing shared secret keys have made this method unpractical. This work presents a scheme to turn practical the fast sharing of random keys over arbitrary Internet channels. Starting with a shared secret key sequence of length $K_0$ the users end up with a secure new sequence $K \gg K_0$. Using these sequences for posteriori message encryption the legitimate users have absolute security control without the need for third parties. Additionally, the security level does not depend on the unproven difficulty of factoring numbers in primes. In the proposed scheme a fast optical random source generates random bits and noise for key renewals. The transmitted signals are recorded signals that carries both the random binary signals to be exchanged and physical noise that cannot be eliminated by the attacker. These signals allow amplification over the Internet network without degrading security. The proposed system is also secure against a-posteriori known-plaintext attack on the key. Information-theoretic analysis is presented and bounds for secure operation are quantitatively determined.

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INTRODUCTION

Unconditionally secure one-time pad encryption has not find wide applicability in modern communications. The difficult for users to share long streams of secret keys beforehand has been an unsurmountable barrier preventing widespread use of one-time pad systems. Even beginning with a start sequence of shared secret keys, no practical amplification method to obtain new key sequences or key “refreshing” is available. This work proposes a solution for this problem where transmitted random bit sequences are protected physical noise. Secure operational bounds are imposed by the noise level used and a controlled number of bits transmitted. Posterior data encryption is done by X-or bit-by-bit the shared random bits with the message bits as in one-time pad encryption.

The method involves binary random sequences physically created (key sequence) $X = x_1, x_2, ...$ to be transmitted between the legitimate users A to B, and physical noise sequences $n = n_1, n_2, ...$ that are not controlled or reducible by any means and will be physically superposed on $X$. A and B share a starting secret key $K_0$. Also essential for the method is the use of physical non-orthogonal bases specified by shared keys $K$ to encode the message bits $X$, before the deterministic “recording” of the noisy signals described by $Y$. The transmitted signals are open to the attacker but improving signal resolution is impossible over the deterministic patterns. This simplifies the security considerations, distinctly from cases where the attacker have access to physical signals in the channel and may constantly improve its technology for signal resolution by performing homodyne, heterodyne and any other measurement.

It will be assumed that (statistical) physical noise $n$ can be added to a message bit sequence $X$ according to some rule $f_j(x_j, n_j)$ giving $Y = f_1(x_1, n_1), f_2(x_2, n_2), ...$ (Whenever only binary physical signals are implied, use of $f_j(x_j, n_j)$ will represent $f_j \equiv \oplus (= addition \mod 2)$). When analog physical signals are made discrete by analog-to-digital converters, a sum of a binary signal onto a discrete set will be assumed or other convenient rounding of the signal. $f_j$ describe operations on non-orthogonal bases specified by shared keys $K$ as will be explained. The addition process is performed at the emitter station and $Y$ becomes a binary file carrying random bits and the recorded noise. $Y$ is sent from user A to user B (or from B to A) through an insecure channel and can be tapped with perfection (any copy will be identical to each other) by the attacker. The amount of noise is assumed high and such that without any knowledge beyond $Y$, neither B (or A) or an attacker E could extract the sequence $X$ with a probability $P$ better than $P \simeq (1/2)^N$, where $N$ is the number of bits transmitted. The conditional entropy for these randomized signals satisfy

$$H(Y|K_0, X) \neq 0,$$ (1)

that guarantees that even knowing the message $X$ and the key $K_0$, the transmitted sequence $Y$ is not unique. This emphasizes the uncontrollable character of the physical noise present at each signal generation.

Assuming that A and B share some knowledge beforehand (the key $K_0$ or $K$), the amount of information between A (or B) and E differs. This information asymmetry can be expressed by

$$H_{AB}(X|Y, K)(\approx 0) \ll H_{E}(X|Y)(\approx H(X)).$$ (2)

The mutual information reflects that E has much less information on $H(X)$ than A and B:

$$I_{AB}(X; Y, K) = H(X) - H_{AB}(X|Y, K) \approx H(X),$$ (3)
\[ I_E(X, Y) = H(X) - H_E(X|Y) \approx 0. \] 

This asymmetry will be used by A and B to share secure information over the Internet. It will be shown that if A and B start sharing a secret key sequence \( K_0 \) they end up with a secure new key sequence \( K \gg K_0 \). Within bounds to be demonstrated, this makes bit-by-bit encryption (as a one-time pad) practical for fast Internet communications (data, image or sound). It should be emphasized that being secure does not imply that communications (data, image or sound) have to be protected, as in one-time pad communication. The system gives users A and B direct control to guarantee secure communication without use of third parties or certificates. Some may think of the method as an extra protective layer to current Internet encryption protocols and it may be used as such. In fact, the system operates on top of all IP layers and does not disturb current protocols in use by Internet providers, including security ones. Anyway, it should be emphasized that the proposed method relies on security created by physical noise. This way, its security level does not depend on advances in algorithms or computation. Also, the proposed protocol does not need to be modified to follow improvements in communication’s technology. Moreover, it works as a stand-alone system.

Random events of physical origin cannot be deterministically predicted and sometimes are classified in classical or quantum events (See examples of differences between quantum and classical random walks in \([2]\)). The point of view adopted here is that a recorded classical random event is just the record of a single realization among all the possible quantum trajectories possible \([3]\). All of these distinct classifications are not relevant to the practical aspects to be discussed here. However, what should be emphasized is that physical noise is completely different from pseudo noise generated in a deterministic process (e.g. hardware stream ciphers) where despite any complexity introduced, the deterministic generation mechanism can be searched, eventually discovered and used by the attacker.

**SIGNAL ENCODING**

Before introducing the communication protocol to be used, one should discuss the superposition of physical signals to deterministic binary signals. Any signal transmitted over Internet is physically prepared to be compatible with the channel being used. This way, e.g., voltage levels \( V_0 \) and \( V_1 \) in a computer may represent bits. These values may be understood as the simple encoding

\[ V^{(0)} \Rightarrow \begin{cases} V_0 & \text{bit 0} \\ V_1 & \text{bit 1} \end{cases} \]

Technical noise, e.g. electrical noise, in bit levels \( V_0 \) and \( V_1 \) are assumed low. Also, channel noise are assumed with a modest level. Errors caused by these noises are assumed to be possibly corrected by classical error-correction codes. Anyway, the end user is supposed to receive the bit sequence \( X \) (prepared by a sequence of \( V_0 \) and \( V_1 \)) as determined by the sender.

If one of these deterministic binary signals \( x_j \) is repeated over the channel, e.g. \( x_1 = x \) and \( x_2 = x \), one has the known property \( x_1 \oplus x_2 = 0 \). This property has to be compared to cases where a non-negligible amount of physical noise \( n_j \) (in analog or a discrete form) has been added to each emission. Writing \( y_1 = f_1(x_1, n_1) = f_1(x, n_1) \) and \( y_2 = f_2(x_2, n_2) = f_2(x, n_2) \) one has \( f(y_1, y_2) = \) neither 0 or 1 in general. This fundamental difference from the former case where \( x_1 \oplus x_2 = 0 \) emphasizes the uncontrollable effect of the noise.

The \( V^{(0)} \) encoding shown above allows binary values \( V_0 \) and \( V_1 \) to represent bits 0 and 1, respectively. These values are assumed to be determined without ambiguity. Instead of this unique encoding consider that two distinct encodings can be used to represent bits 0 and 1: Either \( V^{(0)} \) over which \( x_0^{(0)} \) and \( x_1^{(0)} \) represent the two bits 0 and 1 respectively, or \( V^{(1)} \), over which \( x_1^{(1)} = x_0^{(0)} + \epsilon \) and \( x_0^{(1)} = x_1^{(0)} + \epsilon \) (\( \epsilon \ll 1 \)) represent the two bits 1 or 0 (in a different order from the former assignment). These encodings represent physical signals as, for example, phase signals.

Assume noiseless transmission signals but where noise \( n_j \) has been introduced or added to each \( j \)th bit sent (This is equivalent to noiseless signals in a noisy channel). Consider that the user does not know which encoding \( V^{(0)} \) or \( V^{(1)} \) was used. With a noise level \( n_j \) superposed to signals in \( V^{(0)} \) or \( V^{(1)} \) and if \( |x_0^{(0)} - x_0^{(1)}| \gg n_j \gg \epsilon \), one cannot distinguish between signals 0 and 1 in \( V^{(0)} \) and \( V^{(1)} = V^{(0)} + \epsilon \) but one knows easily that a signal belongs either to the set “0 in \( V^{(0)} \) or 1 in \( V^{(1)} \)” or to the set “1 in \( V^{(0)} \) or 0 in \( V^{(1)} \)”. Also note that once the encoding used is known, there is no difficulty to identify between \( x_2 \) and \( x_2 + \epsilon \). In this case, it is straightforward to determine a bit 0 or 1 because values in a single encoding are widely separated and, therefore, distinguishable. One may say that without information on the encoding used, the bit values cannot be determined.

Physical noise processes will be detailed ahead but this indistinguishability of the signals without basis information is the clue for A and B to share random bits over the Internet in a secure way.

Encryption methods with randomized ciphers have been proven to be secure, e.g., when the attacker’s memory is limited \([4]\). More recently, physical noise has been used both in free propagation and fiber-optics based systems using M-ry levels \([5]\) for data encryption (\(\eta\) systems) and have been analyzed ever since. See a recent discussion in \([6]\). The system proposed here is distinct...
from those αη systems but the idea of covering information with physical noise underlies both logical structures. The proposed system is closer related to to the key distribution system presented in [6] but differs from it by the use of a deterministic channel to carry noisy-recorded information (Some other aspects have been shown in [6] and [7]) and the use of just two encryption bases [10]. Use of the (ideally) deterministic channel—or high intensity channels—makes this system slower than the αη systems [6] or [7] due to the recording stage but avoids amplification problems related to those systems.

**DISTRIBUTION PROTOCOL**

A brief description of protocol steps will be made, before a theoretic-security analysis that includes the system’s limitations is presented. Assume that shared sequence \(K_0\) gives encoding information, that is to say, which encoding \((V^{(0)}\) or \(V^{(1)}\) is being used at the \(j^{\text{th}}\) emission. Assume that \(K_0 = k_1^{(0)}, k_2^{(0)}, \ldots\) has length \(L_0\) and that the user A has a physical random generator PhRG able to generate random bits and noise in continuous levels. A generates a binary random sequence \(K_1 = k_1^{(1)}, k_2^{(1)}, \ldots k_{L_0}^{(1)}\) (say, binary voltage levels) and a sequence of \(K_0\) noisy-signals \(n\) (e.g., voltage levels in a continuum). The deterministic sequence (carrying recorded noise) \(Y_1 = k_1^{(0)} \oplus f_1(k_1^{(1)}, n_1^{(1)}), k_2^{(0)} \oplus f_2(k_2^{(1)}, n_2^{(1)}), \ldots f_N(n_1^{(1)}, n_1^{(1)})\) is then sent to B. First, one has to see if B is able to extract the fresh sequence \(K_1\) from \(Y_1\): B applies \(f(Y_1, K_0) \equiv f_1(k_1^{(1)}, n_1^{(1)}), f_2(k_2^{(1)}, n_2^{(1)}), \ldots f_N(n_1^{(1)}, n_1^{(1)})\). As B knows the encoding used and the signals representing bits 0 or 1 in a given encoding are easily identifiable. B obtains \(f_1(k_1^{(1)}, n_1^{(1)}) \rightarrow k_1^{(1)}, f_2(k_2^{(1)}, n_2^{(1)}) \rightarrow k_2^{(1)}, \ldots f_N(n_1^{(1)}, n_1^{(1)}) \rightarrow k_{N_1}^{(1)}\). B then obtains the new random sequence \(K_1\) generated by A.

Is the attacker also able to extract the same sequence \(K_1\)? Actually, this was a one-time pad with \(K_0\) with added noise and, therefore, it is known that the attacker cannot obtain \(K_1\). The security problem arises for further exchanges of random bits, e.g. if B wants to share further secret bits with A.

Assume that B also has a physical random generator PhRG able to generate random bits and noise in continuous levels (One could proceed with A being the sole one generating random signals but the problem is identical). B wants to send in a secure way a freshly generated key sequence \(K_2 = k_1^{(2)}, k_2^{(2)}, \ldots k_{L_0}^{(2)}\) from his PhRG to A. B record the signals \(Y_2 = k_1^{(2)} \oplus f_1(k_1^{(2)}, n_1^{(2)}), k_2^{(2)} \oplus f_2(k_2^{(2)}, n_2^{(2)}), \ldots f_N(n_1^{(2)}, n_1^{(2)})\) and sends it to A. As A knows \(K_1\) he(or she) applies \(Y_2 \oplus K_1\) and extracts \(K_2\). A and B now share the two new sequences \(K_1\) and \(K_2\). For speeding communication, even a simple rounding process to the nearest encoding position would produce a simple binary output for the operation \(f_j(k_j, n_j)\). The security of this process will be shown after a presentation of the complete distribution protocol.

The simple description presented show a key distribution from A to B and from B to A, with the net result that A and B share the fresh sequences \(K_1\) and \(K_2\). These steps can be seen as a first distribution cycle. A could again send another fresh sequence \(K_3\) to B and so on. This repeated procedure provides A and B with sequences \(K_1, K_2, K_3, K_4, \ldots\) This is the basic and simple key distribution protocol for the system.

A caveat should be made. Although the key sharing seems adequate to go without bounds, physical properties impose some constraints and length limitations as discussed ahead.

**PHYSICAL ENCODING**

A and B use PhRGs to generate physical signals creating the random bits that define the key sequences \(K\) and the continuous noise \(n\) necessary for the protocol. Being physical signals, precise variables have to be discussed and the noise source well characterized. Analog-to-digital interfaces will transform the physical signals onto binary sequences adequate for Internet transmission protocols. Optical sources for the noise signals can be chosen for fast speeds. PhRGs have been discussed in the literature and even commercial ones are now starting to be available. Increasing operational speeds are expected. Without going into details one could divide the PhRG in two parts, one generating random binary signals and another providing noise in a continuous physical variable (e.g., phase of a light field). These two signals are detected, adequately formatted and can be added.

Taking the phase of a light field as the physical variable of interest, one could assume laser light in a coherent state with average number of photons \(\langle n \rangle\) within one coherence time \((\langle n \rangle = |\alpha|^2 \gg 1)\) and phase \(\phi\). Phases \(\phi = 0\) could define the bit 0 while \(\phi = \pi\) could define the bit 1.

A concrete image of a possible phase encoding with non-orthogonal states is shown in Fig. 11. \(k = 0\) defines encoding of 0 and 1 as phase values 0 and \(\pi\), respectively. These values are widely separated and easily distinguishable. This case distinguishability will be quantified ahead. Distinctly, \(k = 1\) defines encoding of 0 and 1 with phase values \(\pi + \Delta \phi\) and \(0 + \Delta \phi\), respectively. These bits are also easily distinguishable in \(k = 1\). However, the poor distinguishability between 0s and 1s in distinct bases \(k = 0\) and \(k = 1\) is crucial for the proposed scheme; this will be quantitatively explained. A bit 0 on this sector is encoded by one of the phases

\[\phi_{0k} = k\Delta \phi + \pi \frac{1 - (-1)^k}{2}\] , \((k = 0, 1)\) , \((6)\)
The separation between bits in the same encoding is easily carried under condition \( \pi/2 \gg \sqrt{2/(\langle n \rangle)} \). The condition \( \sqrt{2/(\langle n \rangle)} \gg \Delta \phi \) implies that bits 0 (in encoding \( k = 0 \)) and 1 (in encoding \( k = 1 \)) (upper position in Fig. 1) cannot be easily identifiable and the same happens with sets of bit 1 (in encoding \( k = 0 \)) and bit 0 (in encoding \( k = 1 \)) (lower position in Fig. 1). However, for A, B and E, there are no difficulty to identify that a sent signal is encoded by \( k = 0 \) or \( k = 1 \). One may therefore assume that physical signals within the same encoding \( k \) have negligible overlap. The signal distinguishability could also be studied assuming non-negligible overlap between “upper” and “lower” states but results [9] are similar under the desired conditions.

**Signal distinguishability**

The attacker does not know the encoding provided to A or B by their shared knowledge on the basis used. An answer to the question “What is the attacker’s probability of error in bit identification without repeating a sent signal?” depends on the properties of the physical signals being used. Under the assumption that “upper” positions and “lower” positions in Fig. 1 can be identified with high precision both by the legitimate users as well as by the attacker, this question basically deals with distinguishability of the two close physical states in “upper” or “down” positions.

Binary identification of two states has a general answer using information theory: The average probability of error in identifying two states \( |\psi_0\rangle \) and \( |\psi_1\rangle \) is given by the Helstrom bound [12]

\[
P_e = \frac{1}{2} \left[ 1 - \sqrt{1 - |\langle \psi_0 | \psi_1 \rangle|^2} \right].
\] (10)

Here \( |\psi_0\rangle \) and \( |\psi_1\rangle \) are coherent states of light \( |\psi\rangle \) with same amplitude but distinct phases

\[
|\psi\rangle = |\alpha\rangle = |\alpha| e^{-i\phi} = e^{-\frac{1}{2} |\alpha|^2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle,
\] (11)

defined at the PhRG. \( |\psi_0\rangle \) define states in encoding \( k = 0 \), where bits 0 and 1 are given by

\[
|\psi_0\rangle = \begin{cases} 
|\alpha\rangle, & \text{for bit 0, and} \\
|\alpha\rangle, & \text{for bit 1,}
\end{cases}
\] (12)

\( |\psi_1\rangle \) define states in encoding \( k = 1 \), where bits 1 and 0 are given by

\[
|\psi_1\rangle = \begin{cases} 
|\alpha| e^{-i\frac{\Delta \phi}{2}}, & \text{for bit 1, and} \\
|\alpha| e^{-i\left(\frac{\Delta \phi}{2} + \pi \right)}, & \text{for bit 0,}
\end{cases}
\] (13)

where \( |\phi_{k=0} - \phi_{k=1}| = \Delta \phi \). \( |\langle \psi_0 | \psi_1 \rangle|^2 \) is calculated in a straightforward way and gives

\[
|\langle \psi_0 | \psi_1 \rangle|^2 = e^{-2\langle \langle n \rangle \rangle} [1 - \cos \frac{\Delta \phi}{2}].
\] (14)
For $\langle n \rangle \gg 1$ and $\Delta \phi \ll 1$,
\[
|\langle \psi_0 | \psi_1 \rangle|^2 \simeq e^{-\frac{\Delta \phi}{2} \langle \phi \rangle^2} \equiv e^{-\Delta \phi^2 / (2\sigma_f^2)} .
\]  
(15)

One should remind that in the proposed system the measuring procedure is defined by the users A and B and no physical attack can be launched by E can improve the deterministic signals that were already available to him(her). Thus, her knowledge on the signals cannot be increased by measurement techniques.

**INFORMATION LEAK AND LENGTH LIMITATION**

One should observe that each random bit defining the key sequence is once sent as a message by A (or B) and then resent as a key (encoding information) from B (or A) to A (or B). In a deterministic encryption this will lead straightforwardly to a breaking of the security. The noisy signals modify this situation dramatically: In both emissions, noise is superposed to the signals. In general, repetitions of coherent signal imply that a better resolution may be achieved that is proportional to the number of repetitions $r$. This improvement in resolution is equivalent to a single measurement with a signal $r \times$ more intense. To take into account the single repetition demanded by the protocol $\langle n \rangle$ is replaced by $2\langle n \rangle$ in $|\langle \psi_0 | \psi_1 \rangle|^2$. In other words, the protection level will then be considered for signal levels twice stronger than the one currently used. The final probability of error results
\[
P_e = \frac{1}{2} \left[ 1 - \sqrt{1 - e^{-\frac{\langle \phi \rangle^2}{2}\Delta \phi^2}} \right].
\]  
(16)

This error probability can be used to derive some of the proposed system’s limitations. The attacker’s probability of success $P_s (= 1 - P_e)$ to obtain the basis used in a single emission may be used to compare with the a-priori starting entropy $H_k$ of the encoding that carries one bit of the message to be sent (a random bit). If the attacker knows the encoding, the bit will also be known, with the same probability $\rightarrow 1$ as the legitimate user.
\[
H_{k,\text{bit}} = -p_0 \log p_0 - p_1 \log p_1 = 1 ,
\]  
(17)

where $p_0$ and $p_1$ are the a-priori probabilities for each encoding $k$ ($p_0 = p_1 = 1/2$) given by the PhRG. The entropy defined by success events is $H_s = -P_e \log P_e$. The entropy variation $\Delta H = H_{k,\text{bit}} - H_s$ statistically obtained—or leaked from bit measurements—show the statistical information acquired by the attacker with respect to the a-priori starting entropy:
\[
\Delta H_k = \left( H_{k,\text{bit}} - H_s \right) .
\]  
(18)

**Length limitation**

In order to be possible to obtain statistically a good amount of information on a single encoding used, $L$ bits have to be transmitted. $L$ thus establishes the length limitation for bits exchanged starting from $K_0$. It will be defined by
\[
L \times \left( \Delta H_k - \frac{1}{2} \right) = 1 .
\]  
(19)

Fig. 2 shows $\Delta H_k$ for some values of $\langle n \rangle$ and $\Delta \phi$. Value $\Delta H_k = 1/2$ is the limiting case where the two bases cannot be distinguished. $\Delta H_k$ deviations from this limiting value of 1/2 indicates that some amount of information on the basis used may potentially be leaking to the attacker. However, it is clear that the attacker cannot obtain bit-by-bit the encoding used.

![Fig. 2: $\Delta H_k$ as a function of $\langle n \rangle$ and $\Delta \phi$.](image)

Fig. 3 shows estimates for $L$ for a range of values $\langle n \rangle$ and $\Delta \phi$ satisfying $L \times \left( \Delta H_k - \frac{1}{2} \right) = 1$ ($\Delta \phi$ is given in powers of 2, indicating bit resolution for analog-to-digital converters). It should be emphasized that $\langle n \rangle$ is the mesoscopic average photon number in the PhRG while an optical signal in the transmission channel can be carried by very intense light—the deterministic signal.

It is assumed that error correction codes can correct for technical errors in the transmission/reception steps for the legitimate users. The leak estimate given by Eq. (19) do not imply that the information actually has leaked to the attacker. However, for security reasons, one takes for granted that this deviation indicate a statistical fraction of bits acquired by the attacker. A probability measure corresponds to condition (19): $p(1) = 1/L = (\Delta H_k - \frac{1}{2})$, expressing that one bit among $L$ bits may have been statistically compromised.

Privacy amplification procedures can be applied to the shared bits in order to reduce this hypothetical information gained by the attacker to negligible levels [11].
FIG. 3: Estimates for the minimum length of bits L exchanged between A and B that could give one bit of information about the bases used to the attacker.

These procedures are beyond the purposes of the present discussion but one can easily accept that A and B may discard a similar fraction of bits to statistically reduce the amount of information potentially leaked. Reducing this fraction of bits after a succession of bits are exchanged between A and B implies, e.g., that the number of bits to be exchanged will decrease at every emission. Eventually, a new shared key K_0 has to start the process again to make the system secure. Nevertheless, the starting key length K_0 was boosted in a secure way. Without further procedures, the physical noise allowed K ≥ 10^6 K_0, a substantial improvement over the classical one-time pad factor of 1. One may still argue that the ultimate security relies on K_0’s because if K_0 is known no secret will exist for the attacker. This is also true but does not invalidate the practical aspect of the system. K_0 length can even be made sufficiently long to frustrate any brute-force attack at any stage of technology. Therefore, the combination of physical noise and complexity makes this noisy-one-time pad practical for Internet uses.

**FRUSTRATING A-PÓSTERIORI KNOWN-PLAINTEXT ATTACKS**

**Known-plaintext attack**

Although the security of the process has been demonstrated, one should also point to a fragility of the system that has to be avoided when A and B are encrypting messages X between them. As it was shown, knowledge of one sequence of random bits lead to the knowledge of the following sequence for A and B. This makes the system vulnerable to known-plaintext attacks in the following way: E has a perfect record of sequences Y_1 and Y_2 and tries to recover any key sequence from them, K_2, K_1 or K_0. E will wait until A and B uses these sequences for encryption before trying to brake the system. A and B will encrypt a message using a new shared sequence, K_1 or K_2. This message could be a plaintext, say X = x_1, x_2, ... x_K_0 known to the attacker. Encrypting this message with say K_1 in a noiseless way, gives Y = x_1 ⊕ k_1(1), x_2 ⊕ k_2(1), ... x_K_0 ⊕ k_K_0(1). Performing the operation Y ⊕ X, E obtains K_1. The chain dependence of K_j on K_{j-1} allows E to find successive keys. Even addition of noise to the encrypted file does not eliminate this fragility, because the attacker can use his/her knowledge of X – as the key – to obtain K as a message. The situation is symmetric between B or the attacker: one that knows the key (X for E, and K for B) obtains the desired message (K for E, and X for B).

This kind of attack can be frustrated to the attacker with a simple strategy as explained ahead. In general, random generation processes are attractive to attackers. Even physical components (e.g. PhiRG) are targets for attackers that may try to substitute a true random sequence by pseudo-random bits generated by a seed key under his/her control. Electronic components can also be inserted to perform this task replacing the original generator; electric or electromagnetic signal may induce sequences for the attacker and so on. While these can be controlled by simple equipment surveillance, the known-plaintext attack is more subtle. A may not know, e.g., that some information to be transmitted to B is known to the attacker.

**Frustrating the known-plaintext attack**

This attack can be avoided by shuffling (permutation operations) the random bit sequence being transmitted from A to B (or B to A) followed by a re-shuffling by B (or A). A particular shuffling function could be chosen among members of a family of one-way functions by use of a short sequence of shared secret random bits. An even simpler way (or less costly) is to use the short sequence of bits to choose one among a list of pre-recorded permutations, what speeds the processing time. The shuffling function used changes from block to block of sequences exchanged. This creates a non-invertible structure for the attacker.

Although the number of bits n_x necessary to select one among K_0! permutations in K_0 bits has a too high cost, a reduced number of permutations K_0!/d within K_0! can be chosen to provide the pre-recorded list and still provide a negligible chance for the attacker to obtain a particular choice among K_0!/d. With this reduced list, the number of bits n_b necessary to assign a particular permutation is given by K_0!/d = 2^{n_b}. At the same time the probability p_d for the attacker to find this particular permutation choice is p_d = 1/(K_0!/d).

The a-posteriori known-plaintext attack can then be frustrated by the shuffled sequence. The random bit se-
sequence used in this process is also protected by the encryption method with the added noise.

MESSAGE AUTHENTICATION

Encryption performed by one user with the shared random bits can only be decrypted by the other legitimate user and the message obtained can therefore be understood as “authentic”. However, it may happen that particular messages need explicit message authentication without the need to decrypt. Fortunately, this can be done with a modest use of shared secret random bits. For example, Ref. [14] describes a message authentication code (MAC) where one key with \( k \) shared secret bits encode message blocks and generate a tag \( T \). No decryption is needed, the receiver applies the shared key to the received data stream to generate the tag \( T' \). Authenticity is given by \( T' = T \).

CONCLUSIONS

As a conclusion, it has been shown that Internet users will succeed in generating and sharing, in a fast way, a large number of secret keys to be used in bit-by-bit encryption (one-time pad). They have to start from a shared secret sequence of random bits obtained from a physical random generator. The physical noise in the signals openly transmitted is set to hide the random bits sent. No intrusion detection method is necessary. Privacy amplification protocols eliminate any fraction of information that may have eventually obtained by the attacker. As the security is not only based on mathematical complexities but depend on physical noise, scientific or technological advances will not harm this system. This is then very different from systems that would rely entirely, say, on the current lack of efficient algorithms to factor large numbers into their primes. The system is also secure against a posteriori known-plaintext attacks on the key. It was then shown that by sharing secure secret key sequences, a practical bit-by-bit encryption over the Internet can be implemented.

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