ON THE RELATIONSHIP BETWEEN THE PERIODIC AND APERIODIC VARIABILITY OF ACCRETING X-RAY PULSARS

DAVIDE LAZZATI
Osservatorio Astronomico di Brera, via E. Bianchi 46, I-22055 Merate (Lecco), Italy, and Università degli Studi di Milano—sede di Como, via Lucini 3, I-22100 Como, Italy; lazzati@merate.mi.astro.it

AND

LUIGI STELLA
Osservatorio Astronomico di Roma, via dell’Osservatorio 2, I-00040 Monteporzio Catone (Roma), Italy; stella@coma.mp.astro.it

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ABSTRACT

Besides the narrow peaks originating from the periodic signal, the power spectra of accreting X-ray pulsars display continuum components usually increasing toward low frequencies; these arise from the source aperiodic variability. Most studies up to the present have adopted the view that the periodic and aperiodic variations are independent. However, any aperiodic variability in the emission from the accretion column(s) toward the magnetic neutron star should be modulated at the X-ray pulsar period, by virtue of the same rotation-induced geometric effects that give rise to the periodic signal. We develop here a simple shot noise model to test the presence of a coupling between the periodic and aperiodic variability of X-ray pulsars. The model power spectrum is fitted to the power spectra of three X-ray pulsars, Vela X-1, 4U 1145–62, and Cen X-3, observed with EXOSAT. In the first two cases, we find that a highly significant coupling is present, as evidenced by a substantial broadening in the wings of the power spectrum peaks which are the result of the periodic modulation. We find also that these wings can mimic the presence of a knee in the continuum power spectrum components around the fundamental of the periodic modulation, therefore bringing into question the correlation reported by Takeshima between the X-ray pulsar frequency and the knee frequency, beyond which the continuum power spectral component steepens.

Subject headings: accretion, accretion disks — pulsars: general — X-rays: stars

1. INTRODUCTION

The presence of fast, aperiodic X-ray flux variations in accreting magnetic neutron stars became apparent with the discovery of 4.4 s pulsations from V0332+53, a transient X-ray binary that had tentatively been classified as a black hole candidate because of the similarity of its short-term variability with that of Cyg X-1 (Tanaka 1983; Stella et al. 1985; Makishima et al. 1990). A number of studies of the aperiodic variations of X-ray pulsars have been carried out since the discovery of these pulsations. In most cases these studies have relied upon power spectrum techniques in a way that parallels the application to nonpulsating X-ray binaries. Continuum power spectrum components arising from the source aperiodic variability are often identified, fitted to analytic models, and used to characterize the short-term variations in relation to other source properties (such as spectral or luminosity state). These continuum power spectrum components also provide an important element for classifications of different types of accreting X-ray sources (see, e.g., van der Klis 1995, and references therein).

In addition to the narrow peaks originating from the periodic modulation, the power spectra of many X-ray pulsars show distinct noise component(s) increasing toward low frequencies. In several of the slower wind-fed X-ray pulsars, the power increases in a power-law–like fashion down to the lowest frequencies sampled (usually $10^{-4}$ to $10^{-3}$ Hz). Some of the faster disk-fed X-ray pulsars show instead a (nearly) flat-topped noise component at low frequencies that steepens above a knee at a frequency of $v_{\text{knee}}$ (Nagase 1989; Soong & Swank 1989). To date, the most detailed and comprehensive study of the power spectra of X-ray pulsars has been carried out by Takeshima (1992), based on Ginga observations. This author reported that a flat-topped or a moderately steep noise component is present in virtually all X-ray pulsars shortward of the peak corresponding to the fundamental of the periodic modulation. In many cases the knee above which the power spectrum steepens lies close to the base of this peak and, consequently, a strong correlation is found between the pulse frequency $v_p$ and the knee frequency $v_{\text{knee}}$ centered around the relation $v_p \approx v_{\text{knee}}$ (Takeshima 1992).

Several other continuum power spectrum features extending over a more limited range of frequencies (usually one decade or less) are observed in X-ray pulsars: these include broad bumps, wiggles, and steep very low frequency excesses. Broad power spectrum peaks testifying to the presence of quasi-periodic oscillations have been clearly revealed in some cases, notably Cen X-3 (Nagase 1989; Soong & Swank 1989), EXO 2030+375 (Angelini, Stella, & Parmar 1989), 4U 1626–67 (Shinoda et al. 1990), and A0535+262 (Finger, Wilson, & Harmon 1996). The continuum power spectrum components of individual X-ray pulsars have been observed to change in relation to a variety of phenomena such as source flux, orbital phase, and spin-up versus spin-down (Angelini 1989; Angelini et al. 1989; Belloni & Hasinger 1990b; Parmar et al. 1989).

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Most studies so far have implicitly adopted the view that the periodic and aperiodic variability of X-ray pulsars are independent, and, consequently, the light curves are given by the sum of the two corresponding signals; similarly, the power spectra are given by the sum of the individual power spectra of the two signals (see, e.g., Angelini et al. 1989). On the other hand, it is well known that X-ray pulsars generate their periodic signal by virtue of geometric effects arising from the rotation-induced motion of the accretion column(s) through which the inflowing material is funneled onto the magnetic poles of the neutron star. Any aperiodic variation in the emissivity of the accretion column(s) should therefore be modulated by the periodic signal as well. Accordingly, Makishima (1988) suggested an approach in which the aperiodic variability of an X-ray pulsar is multiplied by (as opposed to summed with) the periodic modulation. The coupling between the aperiodic and periodic variations, if present, is expected to alter the shape of the power spectrum, as a result of the convolution of the corresponding Fourier transforms (see also Burderi, Robba, & Cusumano 1993).

In this work, we develop a simple model for testing the presence of a coupling between the aperiodic variability of X-ray pulsars and their periodic signal. We adopt a suitable shot noise model to work out analytically the expected power spectrum for an arbitrary coupling factor. The model power spectrum obtained in this way is then fitted to the power spectra of several X-ray pulsars. In two out of three cases, we find that a significant coupling is present between the shots and the periodic signal, as evidenced by a substantial broadening in the wings of the power spectrum peaks arising from the periodic modulation. In these two cases the continuum power still increases shortward of the periodic modulation peak(s), and the broad wings mimic the presence of a knee in the power continuum close to . By analogy this suggests that in other X-ray pulsars with extended red-noise components the power spectrum knee close to might also be an artifact produced by the broad wings of the coherent modulation peak(s); therefore, the reported correlation between and appears to be questionable.

Our paper is organized as follows: § 2 describes the analytic shot noise model that we have developed; the application of this model to the power spectra from the EXOSAT light curves of selected X-ray pulsars is described in § 3; and our results are discussed in § 4.

2. THE SHOT NOISE MODEL

Shot noise models provide a useful mathematical description of the accretion flow inhomogeneities that are believed to generate the variability of accreting compact stars (Terrell 1972; Weisskopf, Kahn, & Sutherland 1975; Sutherland, Weisskopf, & Kahn 1978; Shibazaki & Lamb 1987; Elsner, Shibazaki, & Weisskopf 1987; Abramowicz et al. 1991). Individual shots are often supposed to be representative of the emission from self-luminous blobs or clumps in the accretion process. In the context of accreting X-ray pulsars, models involving density fluctuations in the accreting plasma have been discussed by Burderi et al. (1993) and Hoshino & Takeshima (1993).

We assume that the X-ray pulsar signal consists of two different components: (1) a deterministic periodic component,

\[
f_1(t) = A + \sum_{n=1}^{N} B_n \sin(n\omega_0 t + \phi_n)
\]

where \(\omega_0 = 2\pi/P = 2\pi v_p\) is the fundamental of the pulsed signal of period \(P\), \(N\) is the number of its harmonics, and \(\phi_n\) are the corresponding phases; (2) an aperiodic component consisting of the superposition of a number of similar shots, which gives rise to the red-noise power spectra observed in most X-ray pulsars. In order to reproduce the complex characteristics of the red spectra that are frequently observed (such as, e.g., different power-law slopes in different frequency ranges), most authors adopt a simple (usually exponential) shot shape and consider suitable distributions of shot amplitudes and decay times (see, e.g., Belloni & Hasinger 1990a, and references therein). We adopt a somewhat complementary model in which all shots have the same shape and amplitude [described by the envelope function \(g(t)\)], and consequently their superposition is given by

\[
f_2(t) = \sum_j g(t - t_j),
\]

where \(t_j\) is the (random) start time of the \(j\)th shot. This model is more easily handled analytically and yet yields model power spectra of arbitrary complexity.

If the aperiodic component is coupled to the periodic one, then \(f_2\) is, at least in part, modulated by \(f_1\). Therefore we write

\[
f_2(t) = \left[ 1 + \frac{C}{A} \sum_{n=1}^{N} B_n \sin(n\omega_0 t + \phi_n) \right] f_2(t),
\]

with \(C\) a constant that controls the extent to which the periodic signal modulates the shots (the lower \(C\), the lower the modulation). In order to leave the fluence of each shot unaffected (at least on average), we have divided the amplitude of the deterministic function by its mean value \(A\). Note that \(A\) cannot be derived directly from the mean value of the signal since this comprises an additional term given by the mean of the shots. Therefore, we define a new constant, \(C = C/A\), which controls the degree of modulation but is inversely proportional to the mean signal. The products \(C B_t\) provide a measurement of the depth of the shot modulation in each harmonic.

The Fourier transform of the X-ray pulsar signal \(f_1(t) + f_2(t)\) is therefore

\[
F_\omega(\omega) = A \delta(\omega) + \sum_{n=1}^{N} \frac{B_n}{2i} e^{-i\omega t\phi_n} [\delta(\omega - n\omega_0) - \delta(\omega + n\omega_0)]
\]

\[
+ \left\{ \delta(\omega) + \frac{C}{2i} \sum_{n=1}^{N} B_n e^{-i\omega t\phi_n} [\delta(\omega - n\omega_0) - \delta(\omega + n\omega_0)] \right\} * \left[ G(\omega) \sum_j e^{-i\omega t_j} \right],
\]
where $G(\omega)$ is the Fourier transform of the shot envelope function $g(t)$.

For practical applications, the power spectrum must be calculated by taking into account the effects which are due to the binning, sampling, and finite duration of the light curves. Following van der Klis (1989), we describe a finite, equispaced, and binned light curve as (\(*)\) indicates a convolution

$$f(t) = [\{f_p(t)w(t)\} * b(t)]s(t) ,$$

where $b(t)$ is the binning function, $s(t)$ is the sampling function (a monodimensional lattice), and $w(t)$ is the window function. By using the convolution theorem, the Fourier transform $F(\omega)$ of $f(t)$ becomes

$$F(\omega) = 2\pi [\{F_p(\omega) * W(\omega)\}B(\omega)] * S(\omega) ,$$

where the Fourier transforms of $w(t)$ and $b(t)$ are, respectively,

$$W(\omega) = \frac{T}{2\pi} \text{sinc} \left( \frac{T}{2} \omega \right) \sim \delta(\omega) ,$$

$$B(\omega) = \frac{\Delta t}{2\pi} \text{sinc} \left( \frac{\Delta t}{2} \omega \right) ,$$

where $\text{sinc}(x) = x^{-1} \sin x$. $S(\omega)$ is the reciprocal lattice, and $T$ and $\Delta t$ indicate, respectively, the light curve duration and its binning time. Note that in equation (2) we have used the normalization

$$F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t)e^{-i\omega t} dt .$$

The convolution with $S(\omega)$ in equation (2) can be neglected since the aliasing introduced by the sampling is depressed by the sinc term which is due to the binning. Moreover, the observed power spectra (see § 3) are often dominated by the counting statistics noise well before the Nyquist frequency, and consequently the low signal to noise further reduces the possibility of detecting any alias. The expression for $F_p(\omega)$ in equation (2) can be simplified by neglecting the $\delta$-functions centered on negative frequencies. Therefore, for $F(\omega)$, we obtain

$$F(\omega) = \frac{\Delta t}{2\pi} \text{sinc} \left( \frac{\Delta t}{2} \omega \right) \left[ AT \text{sinc} \left( \frac{T}{2} \omega \right) + \frac{T}{2\pi} \sum_{n=1}^{N} B_n e^{-in\omega_0} \text{sinc} \left( \frac{T}{2} (\omega - n\omega_0) \right) \right] + 2\pi \left\{ \left[ G(\omega) \sum_{j} e^{-ij\omega} \right] + \frac{C}{2\pi} \sum_{n=1}^{N} B_n e^{-in\omega_0} G(\omega - n\omega_0) \sum_{j} e^{-i(\omega - n\omega_0)j} \right\} ,$$

where $W(\omega)$ was approximated with a $\delta$-function in the convolution with $G$ since the shot decay time is supposed to be much shorter than the observation duration $T$ [consequently $G(\omega)$ is much wider than $W(\omega)$].

The final step is to calculate the power spectrum $P(\omega)$. As usual, the simple definition $P(\omega) = |F(\omega)|^2$ cannot be adopted since it would include a strong dependence on the shot start times. Instead, the definition

$$P(\omega) = \langle |F(\omega)|^2 \rangle$$

must be used, where the angle brackets indicate the average over the ensemble of realizations of signals with the same deterministic periodic component and shot parameters, but with different $\{t_j\}$. The derivation of $P(\omega)$ is quite complex and is described in the Appendix. The result is

$$P(\omega) \approx \frac{\Delta t^2}{4\pi^2} \text{sinc}^2 \left( \frac{T}{2} \omega \right) \left\{ T^2 (A + \bar{E})^2 \text{sinc}^2 \left( \frac{T}{2} \omega \right) \right\} + \frac{T^2}{4} \sum_{n=1}^{N} B_n^2 \text{sinc}^2 \left( \frac{T}{2} (\omega - n\omega_0) \right)$$

$$+ 4\pi^2 T \left[ |G(\omega)|^2 + \frac{C^2}{4} \sum_{n=1}^{N} B_n^2 |G(\omega - n\omega_0)|^2 \right] + C^2 \pi^2 (T^2 + Tv)^2 \sum_{n=1}^{N} B_n^2 |G(\omega - n\omega_0)|^2 \text{sinc}^2 \left( \frac{T}{2} (\omega - n\omega_0)T \right)$$

$$+ 2\pi^2 T^2 v C \sum_{n=1}^{N} B_n^2 |G(\omega - n\omega_0)|^2 \text{sinc}^2 \left( \frac{T}{2} (\omega - n\omega_0)T \right),$$

where $v$ is the shot rate and $\bar{E}$ is the mean power in the shot component. It is worth emphasizing that the formula above is derived under the assumption that the observation duration $T$ is long enough that the cross-terms resulting from the relative
phases in the harmonics of the periodic signal can be neglected (see Appendix). The validity of this assumption is verified a posteriori in the application to the observed power spectra of selected X-ray pulsars presented in § 3.

Besides the multiplicative term which is due to the binning, the right-hand side of equation (5) comprises five additive terms; these are, respectively:

**Term (5a):** The term arising from the average level of the total signal, i.e., the deterministic periodic signal plus the shot noise. This term is sharply peaked around zero frequency.

**Term (5b):** The term containing the narrow peaks (width of $\sim 1/T$) that are due to the $N$ harmonics of the deterministic periodic signal.

**Term (5c):** A term consisting of the sum of the red-noise spectrum that is due to the envelope of the shots $[|G(\omega)|^2]$ plus broad wings (width of $\sim 1/\tau$, where $\tau$ is the characteristic decay time of the shots) centered around each narrow peak. These broad wings are characterized by the same functional form of the red-noise spectrum on both sides of each peak. They arise from the convolution of the deterministic signal peaks with the red-noise component due to the shots.

**Term (5d):** An additional term contributing to the narrow peaks (width of $\sim 1/T$) centered around the harmonics. This term is due to the fact that the modulation of all shots is in phase with respect to the deterministic periodic signal. In principle, this contribution to the narrow peaks could be distinguished from that which is due to the deterministic narrow peaks (term [5b]) by virtue of the dependence on the shot rate $\nu$.

**Term (5e):** A term, contributing to the amplitude of the narrow peaks that arises from the cross-product in the calculation of the squared modulus of the two terms in equation (3). Its amplitude is a factor of $\sim C\nu$ lower than the term in equation (5d).

Within our model, the presence of broad wings around the peaks (see term [5c]) unambiguously indicates that the shots are modulated by the periodic signal, i.e., that a coupling is present. Note that the functional form of the broad wings is dictated by the red-noise component, and consequently the modeling of the wings (which usually have a poor signal-to-noise ratio) is coupled to the modeling of the red-noise component. It should be emphasized that no assumption has yet been made about envelope function of the shots.

The possibility of revealing a modulation of the shots through the broad wings of the narrow peaks depends crucially on the ratio of the pulsar period to the characteristic decay time $\tau$ of the shots. To illustrate this point, we show in Figure 1 the model power spectra obtained for shots with an exponential envelope function and decay time $\tau = 300$ s. The different curves correspond to 50% modulated shots with a sinusoidal signal of period $P = 236, 471, 942, 1885, 3770$ s, respectively. It is apparent that when $2\pi P \gtrsim \tau$, the wings are so broad that they become virtually indistinguishable from the red noise. This is due to the fact that the shots are too short-lived to display a conspicuous periodic modulation.

![Figure 1](image_url)

**Fig. 1.—** Model power spectra (see terms [5a]–[5e]) for shots with an exponential envelope function of decay time $\tau = 300$ s. The shots are 50% modulated with a sinusoidal signal. Different curves correspond to different values of the period $P$; from top to bottom, this is $236, 471, 942, 1885, 3770$ s. The adopted light curve duration and binning time were $512,000$ s and 5 s, respectively. It is apparent that the width of broad wings below the sharp peak increases as $P$ decreases.
3. Application to the Power Spectra of Selected X-Ray Pulsars

3.1. Fitting Procedure

In order to test the coupling between the periodic and aperiodic variability, we used the model power spectrum in equation (5) to fit the power spectra obtained from the X-ray light curves of a few selected X-ray pulsars. The relevant formula was inserted in the QDP/PLT fitting program (Tennant 1981), which provides a nonlinear least-squares fitting based on the Marquardt method. A constant term was added to account for the presence of counting statistics (white) noise in the power spectra.

We used the 4–9 keV light curves of X-ray pulsars obtained with the EXOSAT Medium Energy detector array, because of their high statistical quality and (near) absence of interruptions (White & Peacock 1988). The data were extracted from the High Energy Astrophysics Database System at the Brera Astronomical Observatory (Tagliaferri & Stella 1993, 1994; see also White et al. 1995a). For each source a power spectrum was calculated over a number of separate observation intervals of the same length. For each interval, we checked that (1) the source count rate was approximately at the same level; (2) the light curves did not contain eclipses, absorption dips, or other nonstationary events; (3) the position of the narrow peaks that are due to the periodic pulsar signal was the same, to within the Fourier resolution $1/T$ of the power spectra (consequently the smearing due to the orbital Doppler effect and secular period derivative could be neglected).

As customary, we calculated the power spectra of each interval after subtracting the average count rate so that the contribution of term given in term (5a) can be neglected and the term itself excluded from the fit. These power spectra were then used to produce an average power spectrum for each X-ray pulsar. The statistical error of the power estimates was evaluated using the standard deviation of the average power for each Fourier frequency. Note that the interval duration was determined by compromising between two conflicting requirements: (1) that the intervals are sufficiently long to investigate the low-frequency end of the red noise and (2) that the number of intervals is sufficiently high (we averaged a number of $7$–$13$ individual spectra) to make the distribution of the power estimates in the average power spectrum close to a Gaussian distribution (as required to apply standard least-squares fitting techniques; see, e.g., Israel & Stella 1996, and references therein).

In the EXOSAT database, we identified three X-ray pulsars with observations meeting the above requirements. These are Vela X-1 ($P = 282.6$ s), 4U 1145 $- 62$ ($P = 292.1$ s), and Cen X-3 ($P = 4.8$ s) (see, e.g., Nagase 1989; White, Nagase, & Parmar 1995b, and references therein). Details of the relevant observations are given in Table 1, together with the interval length and binning time adopted in our analysis.

The fitting procedure for the average power spectrum of each X-ray pulsar consists of three steps.

1. First, we fitted our model, assuming that no coupling is present between the deterministic periodic signal and the shots. This can be done by setting $C = 0$. Consequently, terms (5d) and (5e) and the second part of term (5c) are excluded from the fit. Therefore this case includes only the red-noise component and the “narrow peaks” (on the assumption that the effects of the relative phases between the harmonics can be neglected). For $|G(\omega)|^2$ we find that a King-like model of the form

$$|G(\omega)|^2 = D[1 + (\omega/\omega_0)^2]^{-\alpha}$$

reproduced quite accurately the power law–like behavior and the low-frequency flattening that characterizes the red-noise spectra we analyzed.

2. The model used in step 1 is generalized to include the effects which are due to the relative phases between the harmonics. The relevant formula is given by equation (11) in Angelini et al. (1989). This step is designed to provide an a posteriori check of the assumptions under which equation (5) has been derived.

3. In the third step, the model that includes the coupling between the periodic signal and the red noise is used. In this case the coupling constant $C$ of equation (5) is used as a free parameter in the fit. The only contribution to the broad peak wings derives from the second part of term (5c). Note that the shape of the broad wings depends on $|G(\omega)|^2$, which in turn is mainly determined by the shape of the red noise. In practical applications of equation (5), it is found that the signal to noise of the observed power spectra is insufficient to isolate the contribution to the narrow peaks deriving also from terms (5d) and (5e). Therefore, we carry out the fit by neglecting these two terms. In this approach the shot rate cannot be obtained directly from the fit to the power spectrum. Moreover, the value of $C$, which is related to the ratio between the height of the narrow peak

| Name       | Time (yy.ddd) | Sequence | Exposure (s) | Count Rate (counts s$^{-1}$) | $N_{\text{int}}^a$ | $T_{\text{int}}^b$ (s) | $\Delta t$ (s) |
|------------|---------------|----------|--------------|-----------------------------|-----------------|---------------------|-----------------|
| Vela X-1   | 85.004        | 1402     | 37341        | 72.2                        | 7               | 5120                | 5               |
| 4U 1145 $- 62$ | 85.001    | 1324     | 34000        | 42.0                        | 13              | 12400               | 10              |
|            | 85.002        | 1326     | 21220        | 35.1                        | ...             | ...                 | ...             |
|            | 85.004        | 1329     | 24070        | 35.0                        | ...             | ...                 | ...             |
|            | 85.005        | 1332     | 23570        | 34.8                        | ...             | ...                 | ...             |
|            | 85.008        | 1339     | 25860        | 21.1                        | ...             | ...                 | ...             |
| Cen X-3    | 85.193        | 1697     | 40685        | 315.1                       | 11              | 2048                | 0.5             |

*a Number of intervals used to calculate the average power spectrum.

*b Duration of each interval.
and broad wing component, is underestimated. This is because, by neglecting terms (5d) and (5e), the height of the narrow peaks is attributed only to the deterministic component. A value of $C$ significantly different from zero would nevertheless reveal the presence of a coupling.

3.2. Vela X-1

The average power spectrum of Vela X-1 obtained from the 4–9 keV EXOSAT light curves is shown in Figure 2. The fit obtained on the hypothesis of no coupling (step 1) is shown in Figure 2a; the corresponding residuals are shown in the lower panel. The fit to the red noise provides a King model index of $a \approx 0.58$, whereas the break frequency $v_C = \omega_C/2\pi$ was too low to be measured. The fit included the first 11 harmonics of the pulse frequency $v_p = (3.5379 \pm 0.0007) \times 10^{-3}$ Hz. The corresponding $\chi^2$ is 325.9 for 189 degrees of freedom (hereafter dof). It is apparent that the peak wings are not well modeled. Including the effects of the relative phases between the harmonics (step 2) left the best fit and corresponding $\chi^2$ ($= 323.3$) virtually unaffected, while the number of dof decreased to 180. An $F$-test for the addition of nine free parameters confirmed that the improvement of the fit is not significant.

Having checked that the relative phases between the harmonics can be neglected, we proceeded to step 3 by allowing for a periodic modulation of the shots (the coupling constant $C$ is now treated as a free parameter). A much better fit is obtained in this case, with $\chi^2$/dof = 229.3/188. This corresponds to an $F$-test chance probability of less than $10^{-15}$ for the addition of one free parameter with respect to the fit of step 1. Figure 2b shows that the new fit reproduces far more accurately the broad wings of the peaks. For the red-noise component, the new fit provides a break frequency of $v_C = (2.0 \pm 1.3) \times 10^{-4}$ Hz and a King model index of $a = 0.72 \pm 0.05$ (errors are 90% confidence). The products $CB_n$ of the coupling constant and the amplitude of the harmonics obtained from the fit are listed in Table 2.

It is useful to define a parameter $R = C \sum_{n=1}^{N} B_n$, which is related to the degree of modulation of the pulse. In the case of a sinusoidal signal, $R$ represents the relative depth of the periodic modulation of the shots. For a periodic signal containing more than one harmonic, the shape of the shot modulation cannot be reconstructed because of the absence of information about the relative phases of the harmonics. Consequently, in this case $R$ does not measure the relative modulation of the shots, and values of $R > 1$ can be obtained. Nevertheless, the value of $R$ provides an indication of whether the shots are weakly ($R \approx 1$) or strongly ($R \approx 0$) modulated. For Vela X-1, we find a large value of $R = 3.2 \pm 0.2$, pointing to a large amplitude of the shot modulation.

3.3. 4U 1145–62

The average power spectrum of 4U 1145–62 was obtained from the X-ray light curves of separate EXOSAT observations since no single long observation was available (see Table 1). The fit obtained on the hypothesis of no coupling between the periodic and the red noise (step 1) is shown in Figure 3a (top). The first five harmonics of the pulsar modulation at $v_p = (3.4229 \pm 0.0006) \times 10^{-3}$ Hz were included in the fit. Note that in this case the red-noise component extends in a

![Figure 2](image)

**Fig. 2.**—Average power spectrum obtained from the EXOSAT light curves of Vela X-1, together with the best-fit models obtained in the case of nonmodulated shots (step 1, [a]) and modulated shots (step 3, [b]). The lower part of each panel shows the corresponding residuals.
TABLE 2

RESULTS FROM STEP 3 FITS

| Parameter | Vela X-1 | 4U 1145−62 | Cen X-3 |
|-----------|---------|------------|--------|
| R         | 3.2 ± 0.2 | 1.6 ± 0.2  | <0.66  |
| CB_1      | 0.61±0.11 | 0.79±0.17  | <0.37  |
| CB_2      | 0.63±0.10 | 0.29±0.09  | <0.23  |
| CB_3      | 0.10±0.03 | 0.07±0.03  | <0.06  |
| CB_4      | 0.41±0.07 | 0.25±0.06  | ...    |
| CB_5      | 0.44±0.07 | 0.17±0.05  | ...    |
| CB_6      | 0.23±0.04 | ...        | ...    |
| CB_7      | 0.26±0.04 | ...        | ...    |
| CB_8      | 0.14±0.03 | ...        | ...    |
| CB_9      | 0.10±0.03 | ...        | ...    |
| CB_{10}   | 0.11±0.03 | ...        | ...    |
| CB_{11}   | 0.14±0.03 | ...        | ...    |

The power spectrum of the EXOSAT light curves of Cen X-3 is different from those in the two cases presented above in that the red-noise component shows a clear flattening for frequencies shorter than the fundamental of the periodic modulation (see Fig. 4), implying that the shot duration is comparable to the X-ray pulsar period. Under these circumstances, any modulation of the shots with the periodic signal would give rise to wings around the power spectrum peaks sufficiently broad that they would be hardly distinguishable from the red noise itself (see § 2). We confined our analysis of the power spectrum of Cen X-3 to the range of frequencies between 0.02 and 0.8 Hz so that the King-like model of equation (6) provided a reasonably accurate fit.
fit of the red-noise component. The first three harmonics of the periodic modulation at $v_p = (2.0692 \pm 0.0008) \times 10^{-1}$ Hz were included in the fit.

On the hypothesis of no coupling, the fit (step 1) yields a $\chi^2$/dof of 2108/1589, for $x = 0.43 \pm 0.03$ and $v_c = 1.2 \pm 0.1$ Hz. The step 2 fit produced the same $\chi^2$, for 1580 dof; the effects of the relative phases between harmonics are therefore negligible also in the case of Cen X-3. Allowing for a periodic modulation of the shots (step 3) yielded a fit with a $\chi^2$ of 2108, for 1588 dof. This shows that no broad wings are detected around the peaks in the case of Cen X-3 (see Table 2 for upper limits on the $CB_n$).

Note, however, that since the expected shape of the wings is dictated by the shape of the red noise (see above), only a poor upper limit is found for $R$ (at the 90% confidence level). This indicates that a fairly strong coupling between the periodic and aperiodic variability might be present and remain undetected in Cen X-3.

4. DISCUSSION

Using a simple model consisting of the sum of a periodic signal plus random shots characterized by an arbitrary degree of modulation with the periodic signal, we have shown that in two out of three X-ray pulsars that we have analyzed (Vela X-1 and 4U 1145-62) there exists a strong coupling between the periodic and red-noise variability. This coupling is revealed through the broad wings that are found around the power spectrum peaks arising from the periodic X-ray pulsar signal. As the shape of the red-noise component dictates the shape of the broad wings, these are more easily detected when the red-noise power increases shortward of the pulsar frequency $v_p$. This is indeed the case for Vela X-1 and 4U 1145-62. On the contrary, the red-noise component of Cen X-3 flattens around so that any wings would be so broad that they would be very difficult to separate from the red-noise component. We find no evidence for such very broad wings around the power spectrum peaks of Cen X-3; the corresponding upper limit to the modulation of the shots is poor and still allows for a relatively strong coupling. This is likely the case also in other disk-fed X-ray pulsars that show unambiguously a flat-topped red-noise component steepening above frequencies of $\lesssim v_p$.

Our results therefore suggest that a coupling between the periodic and red-noise variability is frequently present in X-ray pulsars. This is not surprising since one would expect that any accretion flow inhomogeneity (or "shot") responsible for the red noise produces most of its X-ray luminosity in the accretion column close to the neutron star surface. The combination of rotation and radiative transfer effects should therefore produce a periodic modulation of the shots similar to that of any continuum X-ray accretion onto the polar caps. If, as we assumed in our model, the modulation across different shots is phase coherent, then narrower power spectrum peaks are also generated by the shot component (see § 2). One might conjecture that the whole X-ray flux is produced by the superposition of random modulated shots (i.e., no deterministic periodic signal is present). To ascertain whether this is in fact true is beyond the scope of the present paper.

An interesting consequence of our work concerns the correlation between the pulsar frequency and the knee frequency in the power spectrum continuum that was reported by Takeshima (1992). It is apparent from the power spectrum of Vela X-1 in Figure 2 that the broad wings around the peaks might mimic the presence of a break in the red-noise component around $v_p$. To address this point in a quantitative fashion, we adopted the power spectrum model used by Takeshima (1992). This model comprises the (1) the narrow peaks from the periodic component, (2) a power law, and (3) a flat component followed by a power law above a knee frequency $v_{\text{knee}}$. The fit to the power spectrum of Vela X-1 obtained in this way is given in Figure 5 (top), which also shows the separate contributions from the three model components. A value of $\chi^2$/dof of 297/186 was
obtained; this is considerably worse than the fit based on our modulated shot noise model ($\chi^2$/dof = 229/188). Figure 5 (bottom) also shows the latter fit, with the separate contributions from the red noise, the narrow peaks, and the broad wings. It is apparent that the broad wings provide a much more accurate fit of the power spectrum in between the narrow peaks. We conclude that the red-noise power spectrum of Vela X-1 shows no evidence of a knee around $v_p$. A similar conclusion, although with a lower statistical significance, is reached also for the power spectrum of 4U 1145−62. In this case we obtained a $\chi^2$/dof of 299/189, 284/186, and 272/188 for the step 1 fit, the Takeshima model, and the step 3 fit, respectively.

By analogy, this suggests that for other X-ray pulsars with red-noise components increasing shortward of $v_p$, the broad wings resulting from the modulated shots might also mimic the additional component with a knee at $v_{\text{knee}} \sim v_p$ that was introduced by Takeshima (1992). Therefore, this component is probably not required, and a red-noise knee, if present, would be at $v_{\text{knee}} \ll v_p$. On the other hand, Cen X-3 is probably typical of those X-ray pulsars for which the red noise indeed flattens below frequencies of $v_{\text{knee}} \gg v_p$; in that case, any broad wings would be (nearly) undetectable and their contaminating effect on the power spectrum continuum would be negligible. The discussion above indicates that, if the power spectrum model does not include the broad wings around the periodic modulation peaks, a bias toward the $v_{\text{knee}} \sim v_p$ relation is likely introduced. This is but an example of the caution that should be exercised in isolating the continuum power spectrum components that arise only from the aperiodic variability of X-ray pulsars. Indeed, the broad wings of the peaks originating from the coupling of the periodic and aperiodic variability, if not adequately modeled, can lead to inaccurate conclusions concerning the shape, amplitude, and frequency range of the continuum power spectrum components. A systematic reanalysis of the Ginga power spectra of X-ray pulsars should be carried out in the light of our present work. Valuable new information should be contained in the X-ray pulsar power spectra that are now being obtained by the Rossi X-ray Timing Explorer.

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APPENDIX

We begin the derivation of equation (5) by applying the definition given in equation (4) to equation (3). By averaging over the ensemble of different realizations of the start times of the shots, $\{t_j\}$, we obtain

$$P(\omega) = \frac{\Delta t^2}{4\pi^2} \left( \frac{\Delta t}{\omega} \right) \frac{\sin^2 \left( \frac{\Delta t}{2} \omega \right)}{\sin^2 \left( \frac{\Delta t}{2} \omega \right)} \left[ AT \, \sin \left( \frac{T}{2} \omega \right) + \frac{T}{2i} \sum_{n=1}^{N} B_n e^{-i\omega \phi_n} \sin \left[ \frac{T}{2} (\omega - n \omega_0) \right] \right]$$

$$+ 2\pi \left\{ G(\omega) \sum_j e^{-i\omega t_j} + C \sum_{n=1}^{N} B_n e^{-i\omega \phi_n} G(\omega - n \omega_0) \sum_j e^{-i(\omega - n \omega_0) t_j} \right\}^2.$$

![Fig. 5.—Comparison between the best fit to the average spectrum of Vela X-1 obtained with model of Takeshima (a) and the modulated shot model (step 3, b)]. The contribution from the three components of each model is also plotted. The lower part of each panel shows the corresponding residuals.
Since $T \omega_0 \gg 1$, the interference terms between different harmonics can be neglected. In this limit the sinc function is approximated by a $\delta$-function. A consequence of this is that also the relative phases between the harmonics are neglected. As described in § 3, this assumption is verified a posteriori in the fitting procedure. We therefore obtain

$$
P(\omega) = \frac{\Delta \omega^2}{4\pi^2} \sin^2 \left( \frac{\Delta \omega}{2} \right) A^2 T^2 \sin^2 \left( \frac{T}{2} \omega_0 \right) + T^2 \sum_{n=1}^{N} B_n^2 \sin^2 \left( \frac{T}{2} \omega - n \omega_0 \right) + 4\pi^2 \left( \frac{G(\omega)}{2} \sum_{j} e^{-i\omega t_j} + \frac{G(\omega)}{2} \sum_{j} B_n e^{-i\omega (\omega_0 - n \omega_0) t_j} \right)^2 + 4\pi n T \sin \left( \frac{T}{2} \omega_0 \right) \Re \left[ G(\omega) \sum_{j} e^{-i\omega t_j} \right]
$$

(A1a)

(A1b)

(A1c)

(A1d)

(A1e)

Note that the cross products in term (A1b) cannot be neglected because of the considerable width of $G(\omega)$. However, it can be shown that these cross products cancel each other exactly over the ensemble average. Term (A1b) becomes

$$
4\pi^2 \left\{ \left| G(\omega) \right|^2 \left( \left| \sum_{j} e^{-i\omega t_j} \right|^2 \right) + \frac{C^2}{4} \sum_{n=1}^{N} B_n^2 \left| G(\omega - n \omega_0) \right|^2 \left( \left| \sum_{j} e^{-i(\omega_0 - n \omega_0) t_j} \right|^2 \right) \right\}.
$$

In order to calculate $\left\langle \left| \sum_{j} e^{-i\omega t_j} \right|^2 \right\rangle$, we separate real and imaginary parts of the sum and work out the ensemble average separately. We obtain

$$
\left\langle \left| \sum_{j} e^{-i\omega t_j} \right|^2 \right\rangle = N + (N^2 - N) \left( \left\langle \cos \omega \theta \right| \right)^2 + \left\langle \sin \omega \theta \right| \right)^2.
$$

Note that, unlike the case of a random walk, the second terms become very large close to the resonance condition $\omega = \omega_0$ in equation (A1). If the mean over the ensemble is equal to the mean over time (i.e., the system is ergodic), we have

$$
\left\langle f \right| = f = \frac{1}{T} \int_{-T/2}^{T/2} f(t) dt,
$$

and

$$
\left\langle \left| \sum_{j} e^{-i\omega t_j} \right|^2 \right\rangle = N + (N^2 - N) \sin^2 \left( \frac{\pi T}{2} \right).
$$

Term (A1b) therefore becomes

$$
4\pi^2 T \left[ \left| G(\omega) \right|^2 + \frac{C^2}{4} \sum_{n=1}^{N} B_n^2 \left| G(\omega - n \omega_0) \right|^2 \right] + \pi^2 C^2 (T^2 \nu^2 - T^2) \sum_{n=1}^{N} B_n^2 \sin^2 \left( \frac{\omega - n \omega_0}{2} \right) |G(\omega - n \omega_0)|^2.
$$

Term (A1c) represents the contribution of the shot average to the zero-frequency power, while term (A1d) and (A1e) are simplified in a manner similar to term (A1b). We finally obtain equation (5).

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