Supernova neutrinos, from back of the envelope to supercomputer

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While an understanding of supernova explosions will require sophisticated large-scale simulations, it is nevertheless possible to outline the most basic features of the neutrino emission resulting from stellar core collapse with a pedestrian account that, through reliance upon broadly accessible physical ideas, remains simple and largely self-contained.

1. BACK OF THE ENVELOPE

Because the central features of a system can often be elucidated with simple models, and because neutrino emission is—from nature’s point of view of raw energetics—the central feature of core-collapse supernovae, one may hope to understand the basic features of supernova neutrino emission quickly and easily.

Shortly after the discovery of the neutron in the early 1930s Baade and Zwicky declared: “With all reserve we advance the view that supernovae represent the transitions from ordinary stars to neutron stars, which in their final stages consist of extremely closely packed neutrons” [9]. For the ‘core-collapse’ varieties of supernovae (Types Ib/Ic/II) this basic picture prevails today with strong theoretical and observational support.

The story of how an ‘ordinary star transitions to a neutron star,’ to paraphrase Baade and Zwicky, involves the emission of another electrically neutral particle ‘discovered’ (theoretically, if not experimentally) in the 1930s: the neutrino. Once conditions for their production are reached, the weakness of neutrino interactions implies that their emission is the most efficient means of cooling. Neutrino emission is responsible for the degeneracy of the precollapse stellar core of mass $M \approx 1.5 M_\odot$ (the Chandrasekhar mass); the core’s eventual instability, at least in part; and the energy loss of about $1.7 \times 10^{53}$ erg that ultimately allows collapse to proceed to radius $R_{\text{cold}} \approx 11$ km as a cold neutron star.

\[ 2E_N + E_R = -E_G, \]

where $E_N$ and $E_R$ are the contributions to a star’s internal energy in nonrelativistic and ultrarelativistic particles respectively, and

\[ E_G = -\frac{3GM^2}{5R} \]

is the total gravitational energy of a star of uniform density (assumed here throughout) with mass $M$ and radius $R$. (In this presentation $G$ is Newton’s constant; moreover, $\hbar = c = k = 1$.)

The assumption that the stellar core burns all the way to $^{56}$Fe—the top of the binding energy curve, from which no further energy can be extracted by fusion—allows an iron core temperature $T_{\text{Fe}} \approx 0.9$ MeV to be estimated as follows. Fusion requires tunneling through a Coulomb barrier. Take the uncertainty principle $\Delta x \Delta p \approx 1$...
as a measure of the potency of tunneling, with $\Delta p \approx \sqrt{2\mu Z_1 Z_2 \alpha / \Delta x}$ as a momentum scale corresponding to the energy height of the Coulomb barrier (here $\mu$ is the reduced mass of colliding nuclei of proton numbers $Z_1$ and $Z_2$, and $\alpha \approx 1/137$ is the fine structure constant). Take also $T \approx Z_1 Z_2 \alpha / \Delta x$ as the characteristic thermal energy per particle needed to bring the colliding nuclei within $\Delta x$ of each other. Elimination of $\Delta x$ yields $T = 2 Z_1^2 Z_2^2 \alpha^3 \mu$. Burning tends to proceed by successive capture of $\alpha$ particles (which have modest $Z$, and are tightly bound and therefore abundant), so taking $Z_1 = 24$, $Z_2 = 2$, and $\mu = m_\alpha (4/52)/(4 + 52)$ appropriate to a putative final $\alpha$ capture to $^{56}_{26}$Fe gives $T_{\alpha e} \approx 0.9$ MeV ($m_\alpha \approx 939$ MeV is a generic nucleon mass).

If the electrons are not already degenerate at this point, they must soon become so, thanks to neutrino emission. A temperature $T_{\nu e} \approx 0.9$ MeV is almost twice the electron mass $m_e$, so in the absence of an electron chemical potential of similar magnitude positrons will be thermally produced. But these $e^+$ will annihilate with $e^-$ into $\nu \bar{\nu}$ pairs that escape freely; hence we expect cooling by neutrinos. Continued increase in density with temperature regulated to $\sim 0.5$ – 1 MeV eventually results in electron degeneracy, which becomes the dominant source of pressure support. (Iron core central temperatures of $\sim 0.5$ – 1 MeV are in fact seen in stellar evolution codes.)

The consequences of degeneracy are dire. As the mass and density of the iron core increase during the final burning stage, the velocity of the degenerate electrons approaches the speed of light, at which point they can provide no further pressure support. To estimate the core’s maximum mass, take $E_N = 0$ and $E_R = (E_F)_e$ in Eq. (4), where

$$(E_F)_e = \frac{3\pi^{2/3}}{4} \left(\frac{3}{2}\right)^{2/3} \left(\frac{M Y_e}{m_N}\right)^{4/3} \frac{1}{R}$$

is the internal energy of an uniform core supported by a degenerate Fermi gas of relativistic electrons; the electron fraction $Y_e$ is the ratio of the net number density of electrons $n_e^-$ – $n_e^+$ (or, by charge neutrality, the number density of protons $n_p$) to the total baryon number density $n$. Note the cancellation of radius that then results in Eq. (4), a clear indication that something goes haywire when a star tries to support itself by relativistic degeneracy pressure alone. The resulting expression nevertheless yields an estimate of the mass of this extreme configuration, the so-called Chandrasekhar mass:

$$M \approx 1.5 M_\odot \left(\frac{Y_{e,0}}{0.46}\right)^2,$$

where $Y_{e,0} \approx 0.46$ is the ratio $Z/A$ of the proton and mass numbers of $^{56}_{26}$Fe. This may be taken as an estimate of the mass of the core upon collapse, and of the compact remnant that results therefrom. (Taking inhomogeneous stellar structure into account yields $M \approx 1.22 M_\odot (Y_{e,0}/0.46)^2$.)

Next consider the role of neutrinos in the ultimate instability of the core. As it approaches the Chandrasekhar mass, one proximate cause of the core’s instability—which in fact will be used here to define the onset of collapse—is electron capture, which saps pressure support from the core. Electron capture becomes possible when the electron Fermi energy

$$(\epsilon_F)_e = \frac{\pi^{1/3}}{2} \left(\frac{3}{2}\right)^{2/3} \left(\frac{M Y_e}{m_N}\right)^{1/3} \frac{1}{R}$$

rises above the energy threshold required to convert a proton to a neutron. Crudely considering a nucleus of proton number $Z$ and mass number $A$ as comprising degenerate Fermi gases of nonrelativistic protons and neutrons contained in a volume of radius $r_A = r_N A^{1/3}$ (where $r_N \approx 1.2$ fm), the typical emitted neutrino energy $\epsilon_{\nu e}$ is not the typical energy of an electron captured from its (ambient) degenerate Fermi sea, because the energy required to lift a bound proton from its Fermi sea (within the nucleus) to the top of the neutron Fermi sea (within the nucleus) must first be paid. Hence $\epsilon_{\nu e} \approx (\epsilon_F)_e - \Delta_0 - \Delta_{\nu e} (\epsilon_F)_{n,A,Z}$ (where $(\epsilon_F)_{n,A,Z}$ and $(\epsilon_F)_{n,A,Z}$ are the Fermi energies of the neutrons and protons bound in the nucleus), or

$$\epsilon_{\nu e} \approx (\epsilon_F)_e - \Delta \approx \frac{3}{4} \left(\frac{3\pi^{2/3}}{2}\right)^{2/3} \left[\frac{M Y_e}{m_N}\right]^{1/3} \frac{1}{R} \left(\frac{A - Z)^2}{m_N \rho_N R} \right)^{2/3}$$

$$\approx \frac{3}{4} \left(\frac{3\pi^{2/3}}{2}\right)^{2/3} \left[\frac{M Y_e}{m_N}\right]^{1/3} \frac{1}{R} \left(\frac{1 - Y_e)^2}{m_N \rho_N R} \right)^{2/3},$$

where $\Delta_0 = m_e - m_p - m_\nu \approx 0.8$ MeV and it is assumed that all baryons are locked up in nuclei characterized by the same $Z$ and $A$. Electron capture can proceed when the right-hand side is positive; using $(\epsilon_F)_e$ as given in Eq. (4) yields $R_0 \approx 810$ km at the onset of electron capture for $M = 1.5 M_\odot$ and $Y_{e,0} = 0.46$, corresponding to a density of $\rho_0 \approx 1.3 \times 10^{10}$ g/cm$^3$. Incidentally, we are now in a position to compute the entropy per bayon of the presupercollapse core: $s_0 \approx 0.85 – 1.5$ (corresponding to $T_0 \approx 0.5 – 1$ MeV) receives contributions from the nuclei of mass number $A = 56$, the relativistic degenerate electrons, and photons:

$$s_A = \frac{1}{A} \left(\frac{5}{2}\right)^{1/2} \ln \left[\frac{m_A T_0}{\rho_0} \left(\frac{m_A T_0}{2\pi}\right)^{3/2}\right]\}$$

$$\approx 0.31 – 0.33,$$

$$s_{e^-} = \pi^2 Y_{e,0} T_0 (\epsilon_F)_e \approx 0.52 – 1.04,$$

$$s_{\nu e} = \frac{4\pi^2 m_N T_0}{45} \rho_0 \approx 0.02 – 0.14,$$

where $m_A \approx m_N A$ is the mass of a nucleus of mass number $A$ and $(\epsilon_F)_e = (3\pi^{2/3} \rho_0 Y_{e,0}/m_N)^{1/3} \approx 4.4$ MeV is the
neutron star is well deserved! The single equation simultaneous solution of Eqs. (1) and (13) as specified above ǫ
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neutron star is (nucleus of astronomical proportions!) packed nucleons. A neutron star is not unlike an atomic core is (This is another consequence of the virial theorem: from 0

\[ \frac{R_{\text{cold}} \approx 11 \text{ km}}{M} \] (16)

with \( Y_e \) neglected. (The resulting baryon number density \( n \approx 0.25 \text{ fm}^{-3} \) is within a factor of two of the value 0.14 \text{ fm}^{-3} \) derived from from experimental measurements of the size of atomic nuclei, which also feature densely packed nucleons. A neutron star is not unlike an atomic nucleus of astronomical proportions!)

Note that neutrino emission is necessary not only to get collapse started, but also for it to proceed to completion. This is another consequence of the virial theorem: from Eq. (1) we see that the total energy of the precollapse core is \( (E_F)_e + E_G \approx 0 \), while the total energy of the cold neutron star is \( (E_F)_n + E_G \approx E_G/2 \). Hence an amount of energy equal in magnitude to half the final neutron star gravitational energy,

\[ E_\nu \approx 1.7 \times 10^{53} \text{ erg} \left( \frac{M}{1.5 M_\odot} \right)^2 \left( \frac{11 \text{ km}}{R} \right), \] (17)

must be lost to neutrinos (and a bit of it to the explosion) before the neutron star can reach its cold final state. In a sense these two themes are complementary, for degeneracy entails low entropy in general, and in particular a net electron number prevents the rise of a positron population that would allow \( \nu \bar{\nu} \) pair emission of all flavors by \( e^+e^- \) annihilation. That neutrino trapping will occur is shown by a comparison of the collapse and neutrino diffusion time scales,

![Figure 1](image_url)

Figure 1. In the crude model of gravitational collapse presented here, the neutrino diffusion time scale \( \tau_\nu \) exceeds the free-fall time scale \( \tau_f \) when the core radius has shrunk to about 180 km, which corresponds to a trapped lepton fraction of \( Y_{e,\text{trap}} \approx 0.33 \).
A current scattering off a heavy nucleus of mass number $A$ factors 8 (e.g., Appendix B of Ref. [13]). In this presentation the ‘neutral current’ constructed of quark and lepton fields (which changes electric charge by one unit) and the ‘charged current’ (thick) and characteristic energies (thin). Note the transition time scale when the core is at density $\rho \approx 10^{33}$, where the Fermi constant $G_F = 1.17 \times 10^{-11}$ MeV$^{-2}$ sets the overall scale of weak interaction amplitudes ($\beta$ decay being the prototype), and $(J_{cc})^\mu$ and $(J_{cc})^\mu$ are a ‘charged current’ (which changes electric charge by one unit) and ‘neutral current’ constructed of quark and lepton fields (e.g., Appendix B of Ref. [13]). In this presentation the factors $8G_F^2$ and $G_F^2/2$ are taken as crude estimates of leading factors in charged and neutral current processes respectively, with neutrino energy factors supplied to give the correct dimensionality for rate or cross sections.

Now to the collapse and neutrino diffusion time scales. Assuming that the collapse attending the core’s approach to the Chandrasekhar mass is catastrophic, such that the core is essentially in free-fall, the instantaneous contraction time scale when the core is at density $\rho$ is

$$\tau_C \approx (G\rho)^{-1/2} = \left(\frac{4\pi R^3}{3GM}\right)^{1/2}. \quad (18)$$

On the other hand the trapped electron neutrino diffusion time scale is

$$\tau_{\nu_e} \approx \frac{3R^2}{\lambda_{\nu_e}} \approx \frac{3R^2 \rho \sigma_{\nu_e A \rightarrow A \nu}}{m_N A} \approx \frac{9 A G_F^2 \epsilon_{\nu_e}^2 M}{8\pi m_N R}, \quad (19)$$

where $\lambda_{\nu_e}$ is the neutrino mean free path, $\sigma_{\nu_e A \rightarrow A \nu}$ is the charge current scattering off a heavy nucleus of mass number $A$ (the dominant source of opacity; the argument that the estimated $5.0 \times 10^{50}$ erg are released in $\nu_e$ before trapping; the gradual rise in total luminosity and cutoff at trapping, as well as the typical neutrino energy during infall, are depicted in the left panel of Fig. 2 at negative (i.e. pre-bounce) times). baryons remain locked in heavy nuclei through trapping is given in the next footnote). In accordance with the above remarks, a neutral current scattering cross section per nucleon $(G_F^2/2) \epsilon_{\nu_e}^2$ is suggested by dimensional analysis. Applied to an entire nucleus yields not just a factor $A$ corresponding to an incoherent sum over scatterings off individual nucleons, but a factor $A^2$ corresponding to collective scattering off the entire collection of nucleons—a coherent sum in the scattering amplitude. This is because the neutrino wavelength is larger than the size of the nucleus (as can easily be verified from Eq. (17) and $r_A = r_N A^{1/3}$ with $r_N \approx 1.2$ fm). With the nucleus then treated as a point particle, the cross section is partially analogous to Rutherford scattering of a charged particle from a point nucleus of proton number $Z$, with our factor of $A^2$ associated with ‘weak charge number’ $A$ corresponding to the factor $Z^2$ in the Rutherford scattering formula.

As collapse begins,

$$\tau_{\nu_e} \approx 0.11 \left(\frac{R}{810 \text{ km}}\right)^{3/2} \left(\frac{1.5 M_\odot}{M}\right)^{1/2}, \quad (20)$$

and $\tau_{\nu_e} \approx 0$ since $\epsilon_{\nu_e} \approx 0$ as given by Eq. (4) has here been taken to define the onset of collapse. While neutrinos initially escape easily, the decrease in $\tau_G$ and increase in $\nu_e$ as $R$ decreases during collapse imply that at some point neutrinos will be trapped (in addition to the explicit dependence of $\tau_{\nu_e}$ on $R$ in Eq. (19), see Eq. (5) for the increase in $(eF)_v$ and hence $\epsilon_{\nu_e}$ as $R$ decreases).

In order to estimate the trapped lepton fraction, note that as long as the $\nu_e$ from electron capture escape freely, the electron fraction changes according to

$$dY_e \approx -Y_e \Gamma_{\nu_e A \rightarrow Av} \, dt \approx 8Y_e G_F^2 \epsilon_{\nu_e}^2 \left(\frac{3\pi R}{G M}\right)^{1/2} dR. \quad (22)$$

The factor of $Y_e$ gives the fraction of baryons that are protons; the charged current capture rate per proton on nuclei of mass number $A$, $\Gamma_{\nu_e A \rightarrow Av} \approx 8G_F^2 \epsilon_{\nu_e}$, has been estimated in accordance with previous remarks (in contrast to neutrino scattering, there is no coherence on the nucleus as a whole); and $dt \approx -d\tau_G$ has been taken. Equation (22), with Eqs. (4) and (14), is an ordinary differential equation for $Y_e$ whose solution is shown in Fig. 4. Using this solution in connection with Eqs. (15) and (16), the trapped lepton fraction at the value of $Y_e$ where $\tau_{\nu_e} = \tau_r$. This is illustrated graphically in Fig. 4 for $M = 1.5 M_\odot$, $R_0 = 810$ km, and $A = 56$. The result is $Y_e \approx 0.33$. $\tau_{\nu_e}$ The total energy released in $\nu_e$ before trapping is obtained by integrating $dE_{\nu_e} = \epsilon_{\nu_e} (M/m_N)(dY_e/dR) dR$ from the initial radius to the trapping radius. The luminosity during infall is $L_{\nu_e} = dE_{\nu_e}/d\tau_G$, and is plotted...
The other point to make about collapse is that the entropy per baryon changes only modestly, which conditions what happens on dynamical time scales at the end of implosion. Instead in Fig. 2 as a function of time $t = -\tau_G$. The neutrino energy is given by Eq. (4). In all of this the solution $Y_c(R)$ as obtained in the previous footnote is used.

To see that the entropy per baryon changes only modestly during collapse, consider the first law of thermodynamics in the form

$$Tds = dq - \sum_i \mu_i dY_i.$$  

(23)

Here $dq = d(\epsilon/n) + p d(1/n)$ (where $\epsilon$ is the internal energy density, $n$ is the baryon number density, and $p$ is the pressure); but it is equal, as the label $dq$ suggests, to the heat per baryon exchanged with the environment. The second term represents the entropy per baryon generated by out-of-equilibrium ‘chemical’ changes to the core ($\mu_i$ are the chemical potentials including rest masses of the species $i$, and $dY_i$ are the changes in the numbers per baryon). There are arguments for the smallness of $ds$ both before and after trapping.

Before neutrino trapping both energy and lepton number are lost via electron capture, and the modesty of entropy increase results from a substantial cancellation between these contributions. The energy per baryon lost to electron capture neutrinos that freely escape is $dq \approx \epsilon_\nu_e dY_\nu_e$. Meanwhile the chemical change of electron capture involves $dY_e = dY_\nu_e = -dY_\nu_\nu$ (the first equality arising from charge conservation and the second from baryon number conservation), so that $\sum_i \mu_i dY_i = (\mu_\nu - \mu_e) dY_\nu_e$. Hence

$$Tds \approx (\epsilon_\nu_e + \mu_\nu - \mu_e) dY_\nu_e.$$  

(24)

It turns out that the leading factor cancels exactly with the prescription for electron capture neutrino energy of Eq. (7). In a more careful calculation the cancellation would not be exact, but it would still be substantial. An important consequence of the small change in entropy before trapping is that the baryons remain in nuclei (which feature the coherent scattering cross section $\propto A^2$): using $A = 1, R = 180$ km, and $T = 1 - 5$ MeV in Eq. (6) shows that $s \approx 4 - 6$ would be required for the nuclei to disassemble into free nucleons before trapping.

It can also be argued that the change in $s$ between trapping and core bounce will be modest, this time because the terms of Eq. (23) tend to vanish individually. First, $dq \approx 0$ since neutrinos can no longer carry away energy. Second, with neutrino trapping,

$$\sum_i \mu_i dY_i = (\mu_\nu + \mu_e - \mu_\nu_e - \mu_\nu_e) dY_\nu_e + \mu_\nu_e dY_\nu_e,$$  

(25)

where $Y_e = Y_\nu_e + Y_\nu_e$ is the trapped lepton fraction (in analogy with the definition of $Y_e$ given earlier, $Y_e \equiv (n_e - n_\nu_e)/n$). Between trapping and bounce, while

of a soft landing cushioned by abundant degrees of freedom capable of gently absorbing the kinetic energy of infall, a hard ‘bounce’ can be expected when nucleon degeneracy pressure kicks in at supranuclear densities. This is accompanied by a burst of $\nu_e$ emission (see the left panel of Fig. 2), involving an energy loss of about $5.9 \times 10^{51}$ erg and the sudden release of trapped electron lepton number. Moreover, an additional $6.8 \times 10^{51}$ erg

$Y_L = Y_{\nu_e,\text{trap}}$, the second term vanishes because $dY_\nu \approx 0$, and the first term also becomes small as $e^-, e^+, p, n, \nu_e$ and $\bar{\nu}_e$ come into chemical (‘$\beta$’ or ‘weak’) equilibrium.

The estimates of post-bounce neutrino emission presented below involve reasoning from the approximate global states of a neutron star in chemical equilibrium at finite temperature. In addition to thermal photons and $\nu_{\mu}\bar{\nu}_{\mu}$ and $\nu_{\tau}\bar{\nu}_{\tau}$ pairs, there must be allowance for arbitrary degeneracy in $\nu_e\bar{\nu}_e$ pairs, relativistic $e^+e^-$ pairs, and nonrelativistic neutrons and protons. For relativistic pairs of arbitrary degeneracy there are simple expressions for the differences of number densities and the sums of energy densities of particles and antiparticles, as well as the entropy. For the nonrelativistic nucleons the needed quantities can be derived from the Landau potential $\Omega(T, V, \mu) = -pV = E - TS - \mu N$. This can be written in terms of a polylogarithm function as $\Omega(T, V, \mu) = V(m^2T^3/2\pi^2)^{1/2} / [L_2(\mu/T)]$, whose symbolic derivatives and numerical evaluation can be handled by any self-respecting symbolic mathematics package (though the Sommerfeld expansion seems more reliable for $\mu/T > 10 - 20$). The global neutron star states are characterized by two parameters (the entropy per baryon $s$ and the lepton fraction $Y_L = Y_\nu_e + Y_\nu_e$) and seven unknowns ($R, T, Y_e, \mu_e, \nu_e, \bar{\nu}_e, \mu_\nu$, and $\mu_\nu_e$). The seven equations to be solved are the virial theorem of Eq. (4), chemical equilibrium, Eq. (11) with $\mu_\nu_e$ added to the left-hand side; the total entropy per baryon; and four ‘particle number equations’ defining $Y_e$, $Y_{\nu_e} = Y_L - Y_e$, $Y_{\bar{\nu}_e} = Y_e$, and $Y_\nu = 1 - Y_e$ in terms of the chemical potentials, temperature, etc.

A neutron star state (as described in the previous footnote) with $s = 1.5$ and $Y_L = Y_{\nu_e,\text{trap}}$ has a total energy $E_{\text{tot}}$ much lower than $-E_{\nu_e,\text{infall}}$ (the energy lost to $\nu_e$ escape during infall), so it cannot be the post-bounce state. (As noted earlier, the precollapse initial condition supported by purely relativistic energy is characterized by $E_{\text{tot}} = 0$, which follows from Eq. (11)). Examining a sequence of states with higher $s$ in a bid to reach $E_{\text{tot}} = -E_{\nu_e,\text{infall}}$ eventually yields a ‘burst state’ with $s \approx 5.9$ whereby $R \approx 303$ km is equal to its electron neutrino trapping radius. (As before, the neutrino trapping radii are given by equating a diffusion time scale $\tau_{\nu_e} = 3R^2/\Lambda$ with a dynamical time scale $\tau_{\text{dyn}} \approx (G\rho)^{-1/2}$). Here $\lambda_{\nu_e} \approx (n_\nu_s \sigma_{\nu_e} + n_\nu_s \sigma_{\nu_e})^{-1}$, where $\sigma_{\nu_e} \approx G\rho_{\nu_e}^2/2$ is the neutral current scattering on nucleons and $\sigma_{\nu_e} \approx G\rho_{\nu_e}^2/2$ is for charged current absorption on neutrons. For later use, note that $\lambda_{\nu_e} \approx (n_\nu_s \sigma_{\nu_e} + n_\nu_s \sigma_{\nu_e})^{-1}$, where $\rho_{\nu_e} \approx \sigma_{\nu_e}$, and $\lambda_{\nu_e,\nu_e,\nu_e,\nu_e} \approx (n_\nu_s \sigma_{\nu_e})^{-1}$. Hence as the core re-
or so is lost from the core before the dust settles after a dynamical time scale of about 24 ms post-bounce, in the left panel of Fig. 2. This full amount is attributed to neutrino emission equally divided among all flavors.

The neutrino emission encountered thus far accounts for only about 10% of the total energy to ultimately be lost, with the rest—about 1.5 × 10^{53} erg—temporarily stored as heat, thanks to the entropy generated by bounce and its aftermath. As the hot nascent neutron star shrinks (see Fig. 3), this energy is lost to neutrino emission of all flavors (albeit with a modest hierarchy of luminosities and average energies) over a period of many seconds, as depicted in Fig. 2—a recognizable if very rough caricature of light curves produced by detailed models.\(^{13}\)

bounds to this radius, \(\nu_e\) with \(\bar{\nu}_e \approx 3 \bar{v}_e / 4 \approx 7.1\) MeV escape freely in a burst in which a lepton fraction \(dY_L\) and energy \(E_{\nu_e,\text{burst}} \approx \bar{v}_e (M/m_N) dY_L\) are lost during a time \(t_{\text{dyn}} \approx 24\) ms that characterizes this 'burst state.' In order to find \(dY_L\), a sequence of states is searched by decreasing \(Y_L\) while adjusting \(s\) to stay just below the (varying) \(\nu_e\) trapping radius. All such states are found to be characterized by \(E_{\text{tot}} < -E_{\nu_e,\text{infall}} - E_{\nu_e,\text{burst}}\); all the way down to \(Y_c \approx 0\). The conclusion is that in this crude model the deleptonization accompanying the \(\nu_e\) burst is essentially complete, with \(E_{\nu_e,\text{burst}} \approx \bar{v}_e (M/m_N) \nu_e,\text{trap}\) evaluated at the initial 'burst state.'\(^{12}\)

At the end of the sequence of states with declining lepton number searched as described in the preceding footnote, we still do not have a state with \(E_{\text{tot}} = -E_{\nu_e,\text{infall}} - E_{\nu_e,\text{burst}}\); further energy loss is needed. But it is found at the end of this searched sequence that the population of thermal neutrinos of all flavors has begun to make a nontrivial contribution to \(E_{\text{tot}}\), so their emission is now a viable means of prompt energy loss so long as the star is outside the trapping radius of these flavors. Hence our 'post-bounce state' after the dust settles is that characterized by \(Y_L = Y_c\) and the highest entropy per baryon \(s\) decrease monotonically even as the temperature \(T\) increases before making its final plunge. Starting at 0.1 s post-bounce, diamonds mark time intervals of 0.2 s; starting at 1 s, squares mark time intervals of 2 s.

2. SUPERCOMPUTER

That some basic features of supernova neutrino emission can be motivated by a stick-figure-quality model provides a baseline degree of instant gratification. This is in contrast to the explosion manifest in an expanding supernova remnant’s kinetic energy, which, as a 1% subsidiary detail from nature’s perspective, has been requiring decades of detailed study by numerous people to understand.\(^{14}\)

5.3 to 0. The luminosities \(L_{\nu_i}\) and average energies \(\bar{v}_{\nu_i}\) for the various species are found by equating two different expressions for the neutrino luminosities necessary to allow a transition between two successive neutron star states. Defining \(dE_{\text{tot}}\) to be the difference in \(E_{\text{tot}}\) between two successive states in the sequence, and assuming that the star contrives to release an equal amount of energy in each flavor, one expression is the ratio of \(-dE_{\text{tot}}/6\) to the neutrino diffusion time scale \(\tau_\nu\) (given for the various species in a previous footnote) at that point in the sequence. A second expression for luminosity is \(L_{\nu_i} = 4\pi R^2 (7\pi^2/960)(\bar{v}_{\nu_i}/3)^4\), a neutrino version of the Stefan-Boltzmann law with \(T_{\text{eff.i}} \approx \bar{v}_{\nu_i}/3\). Once the \(\bar{v}_{\nu_i}\) and \(L_{\nu_i}\) are determined, the sequence is related to a time coordinate by defining \(dt = -dE_{\text{tot}}/(\sum_i L_{\nu_i})\) for each step of the sequence.

\(^{13}\)Such is the (probably unavoidable) result of our anthropocentric chauvinism. For decades, and not yet sure even
But stick figures do not ultimately satisfy, of course; we long for the voluptuous contours of a fully-fleshed-out model. In fact, under thorough dissection the simple model of the previous section might stand revealed as more retrospectively illustrative than predictively robust. This is one indication of the ultimate necessity of detailed simulations. Another is that the details of early neutrino emission and the explosion are more interrelated than some of my previous comments might suggest. Whether it be by playing the spoiler, acting as agent of explosion, or setting the conditions necessary to the operation of some other mechanism, neutrinos have played direct or indirect roles in the many explosion scenarios discussed over the years—and detailed simulation of collapse and the second or so after bounce will allow a neutrino signal detected from a Galactic supernova to serve as an unrivaled diagnostic tool. In its full glory neutrino transport is a time-dependent six-dimensional problem, as it requires the tracking of neutrino energy and angle distributions at every point in space. The computational resources necessary to solve the daunting fulness of this problem are still just over the horizon, but efforts to meet this problem in its full measure are already underway [19,20].

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