Performance Evaluation of LDPC Coding and Iterative Decoding System in BPM R/W Channel Affected by Head Field Gradient, Media SFD and Demagnetization Field

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Abstract

We evaluate the performance of the write-margin for the low-density parity-check (LDPC) coding and iterative decoding system in the bit-patterned media (BPM) R/W channel affected by the write-head field gradient, the media switching field distribution (SFD), the demagnetization field from adjacent islands and the island position deviation. It is clarified that the LDPC coding and iterative decoding system in R/W channel using BPM at 3 Tbit/inch$^2$ has a write-margin of about 20%.

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1. Introduction

Bit-patterned media (BPM) \cite{1} is a one of the most promising magnetic recording media to achieve over 5 Tbit/inch$^2$ \cite{2}. The magnetic recording system using the BPM has a serious problem of the write-error affected by the write-head field gradient, the media switching field distribution (SFD), the demagnetization field from adjacent islands and the island position \cite{3}. Low-density parity-check (LDPC) code \cite{4} attracts much attention as a powerful error-correction code when it is combined with an iterative decoding system.

In this paper, we evaluate the performance of the write-margin for the LDPC coding and iterative decoding system in the BPM R/W channel affected by the write-head field gradient, the media SFD, the demagnetization field from adjacent islands and the island position deviation at a recording density of 3 Tbit/inch$^2$.

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2. LDPC coding and iterative decoding system

Fig. 1 shows the block diagram of the LDPC coding and iterative decoding system in magnetic recording system using the BPM. The system consists of an LDPC encoder, a BPM R/W channel, an equalizer, an a posteriori probability (APP) decoder, a sum-product (SP) decoder, and a hard decision circuit. The regular-(26, 3) LDPC code under 4K byte sector format is employed. The user data sequence is encoded with respect to each sector unit. The recording sequence is NRZ-recoded on the double-layered BPM with a soft under-layer [5].

Fig. 2 shows the island arrangement of the BPM. The open and filled squares show the islands where “0” and “1” are recorded, respectively. We assume a recording density of about 3 Tbit/inch$^2$ with 2300 kBPI and 1300 kTPI. The square island is 7 nm on a side. The island pitch $x_{ip}$ and the track pitch are fixed to 11 and 19.5 nm as the typical values, respectively. We also assume that the magnetic field is not fluctuated by the magnetization on the adjacent tracks because the reading head has the side shield, and the read head equals the track pitch.

Fig. 3 shows the isolated waveform. The solid and dashed lines show the waveforms from an isolated island with the infinite length and an isolated island with a length of 7 nm. We assume that the amplitude of the waveform reproduced from an isolated is proportional to the island width normalized by island length. An isolated reproducing waveform from an island at the reading point is assumed to be a hyperbolic tangent-function-like waveform given by

$$h(t) = \frac{A}{2} \left( \tanh \left( \frac{\ln 3}{T_{50}} t \right) + 1 \right),$$  \hspace{1cm} (1)

where $A$ is the saturation level, $T_{50}$ is the rising time of $h(t)$ from $A/4$ to $3A/4$. Here, we define the normalized linear density as $K = T_{50}/T_c$, where $T_c$ is a channel bit interval. We assume that $K = 1.2$ corresponds to the linear density of 2300 kBPI shown in Fig. 3. We assume that the reading noise is composed of a system noise and a media noise. The system noise is an additive white Gaussian noise (AWGN) whose power in bandwidth equal to a channel bit rate $f_c = 1/T_c$ is expressed as $\sigma_S^2$. Then the signal-to-noise ratio (SNR) for the system noise is defined as

![Fig.1 Block diagram of LDPC coding and iterative decoding system.](image1)

![Fig.2 Island arrangement of BPM.](image2)
\[ \text{SNR}_S = 20 \log_{10} \frac{A}{\sigma_S} \text{ [dB]} \]  

(2)

Here, the media noise is not included in the SNR\(_S\). We also assume that the island position and size variations follow Gaussian distributions with standard deviations \( \sigma_{\text{position}} \) and \( \sigma_{\text{size}} \) normalized by \( x_{ip} \) [6], respectively. In the case of 3 Tbit/inch\(^2\), \( \sigma_{\text{position}} \) and \( \sigma_{\text{size}} \) are assumed to be 0.03 and 0.1, respectively. The read-back waveform is equalized to a PR1 [7] target by the equalizer. The equalizer is composed of a low-pass filter (LPF) with a normalized cut-off frequency \( x_h \) by \( f_c \) and a transversal filter with \( N_t \) taps. The generalized (G) PR1 [8] channel output is obtained by subtracting the output of \( M \)th-order noise predictor from the PR1 channel output. The GPR1 channel output is input to the APP decoder. The APP decoder calculates the log-likelihood ratio (LLR) sequence from both the output sequence of the GPR channel and the extrinsic information provided by the sum-product (SP) decoder [9]. However, the extrinsic information is not used at the first decoding. The iterative decoder consists of an APP decoder and an SP decoder. Here, \( i_{sp} \) and \( i_{in} \) stand for the number of iterations in the SP decoder and the iterative decoder, respectively. In this paper, we set both iteration limits to 5. After given iterations, the data sequence is determined by a hard decision.

3. Recording model for BPM

Fig. 4 shows the writing scheme for the BPM. The main pole of the writing head is located above the islands, and its trailing edge is at the center of island #3. We assume that each island has the inherent fluctuations such as the SFD \( \Delta H_C \) which is the deviation from the standard media coercivity \( H_C \), the demagnetizing field \( H_d \), and the island position deviation \( \Delta x \) from a center position which follows a Gaussian distribution with the standard deviation \( \sigma_{\text{position}} \). The impressed head field \( H_{max} \) for each island decreases with the distance from the trailing edge of head according to the head field gradient \( dH/dx \). We assume that the perpendicular component of

![Fig.3 Isolated waveform (K = 1.2).](image1)

![Fig.4 Writing scheme for BPM.](image2)
write head field influences the reversal of island, where the field distribution is constant in the cross-track direction and decays with the constant gradient $dH/dx$ in the down-track direction. Now, we focus attention on the island #2 when the writing target is the island #3. The magnetization of island #2 is switched to the same direction as the head field if it meets the following equation:

$$H_{\text{max}} - (\Delta x_2 + x_{ip}) \times \frac{dH}{dx} \geq H_C + \Delta H_{C2} + H_{d1} + H_{d3}$$

(3)

where the polar directions of $H_{d1}$ and $H_{d3}$ are decided by the last magnetization condition [3]. Thus, the writing of island #3 may cause the write error in the island #2. In this paper, $H_{\text{max}}$ and $H_C$ are set to 12kOe and 9kOe, respectively.

Fig. 5 shows the write error rate (Write ER) performance which is obtained by counting the write error at the writing process. The vertical axis shows the Write ER and the horizontal axis shows the normalized write-clock offset $\varphi$ which is the amount of offset between ideal write-clock timing and ideal island center position. The parameters are set to $\Delta H_C/H_C = 0.05$ and $H_d/H_C = 0.01$. The symbols $\bigcirc$, $\triangle$, and $\square$ show the performances for $dH/dx = 350$, 450 and 550 Oe/nm, respectively. As can be seen in the figure, the head field gradient grater than 350 Oe/nm is needed to achieve a Write ER of $10^{-5}$ [3]. Fig. 6 shows the possible write margin for the case of $\Delta H_C/H_C = 0.05$ and $H_d/H_C = 0.01$. The vertical axis shows the normalized write margin by $x_{ip}$ at Write ER = $10^{-5}$, and the horizontal axis shows the head field gradient $dH/dx$. For a normalized write margin of 0.2, the head field gradient is approximately 450 Oe/nm. Hereafter, we employ the head field gradient $dH/dx = 450$. 
4. Performance evaluation

Fig. 7 shows the bit error rate (BER) performance for $\text{SNR}_S$. The parameters are set to $dH/dx = 450$, $\Delta H_C/H_C = 0.05$, $H_d/H_C = 0.01$, $K = 1.2$, $x_h = 0.4$, $N_t = N_{\text{opt}}$, $M = 3$. Here, $N_{\text{opt}}$ is the optimum number of taps of the transversal filter which gives the minimum BER at each $\text{SNR}_S$. The symbols △ and ○ show the channel error rate (Channel ER) and BER. As can be seen in the figure, the error-free performance is achieved at $\text{SNR}_S = 16.5\text{dB}$. Thus, we set $\text{SNR}_S = 16.5\text{dB}$ in the following evaluation of the write-error performance.

Fig. 8 shows the relationship between the BER performance and $\varphi$, where $dH/dx = 450$, $\Delta H_C/H_C = 0.05$, $H_d/H_C = 0.01$, $K = 1.2$, $\text{SNR}_S = 16.5\text{dB}$, $x_h = 0.4$, $N_t = N_{\text{opt}}$ and $M = 3$. The symbols ○, △ and × show the BER, the Channel ER and the Write ER performances, respectively. In the figure, the LDPC coding and iterative decoding system can keep error-free over $\varphi = 0.0 \sim 0.20$ at a Write ER of about $2 \times 10^{-5}$. Thus, the system has a write margin of about 20%.

5. Conclusion

We have evaluated the performance of the LDPC coding and iterative decoding system in a BPM R/W channel affected by the write-head field gradient, the media SFD, the demagnetization field from adjacent islands and the island position deviation at 3Tbit/inch². The results show that the LDPC coding and iterative decoding system has a write margin of about 20% in the BPM R/W channel with a write error rate of $2 \times 10^{-5}$.
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