Aerodynamic Analysis and Optimization of Airfoils Using Vortex Lattice Method

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Abstract: - Air transport is one of the fastest means of transport in the modern world. Aircraft’s efficiency depends on the efficiency of its subparts and its subsystems individually. For example, the gas turbine used as main source of thrust in aircrafts has to be largely efficient, which adds to the overall efficiency of the aircraft. The same way, another important part of the aircraft is the wing. Wings are the main reason for an air vehicle to fly. Aerodynamic parameters like lift, drag and the ratio of co-efficient of lift to drag which is $C_L/C_D$ ratio determines the efficiency of the aircraft wing. This $C_L/C_D$ ratio depends on the cross section of the wing, i.e. the airfoil. This paper emphasizes aerodynamic analysis on airfoils NACA 0018, NACA 2412, NACA 4412, USA 45, TSAGI-S12 and B737A used on different planform of an aircraft wing like Rectangular, Elliptical, Delta and Swept back wing. The results are analyzed and the best airfoil corresponding wing shape is reported. This best airfoil is further refined effectively to get high $C_L/C_D$ ratio. The complete analysis and airfoil optimization were carried on XFLR5 software which is governed by Naiver- Stokes equations. The airfoils were analyzed for a velocity of 50 m/s, at an angle of attack of 7°, up to a maximum Re (Reynolds Number) of $3 \times 10^6$, the wing chord length at root, $C_R=1$ m and total wing span, $L=10$ m was kept constant throughout the analysis for uniformity. Post analysis, the best three airfoils were further interpolated and refined to obtain higher $C_L/C_D$ ratio.

Keywords: Aerodynamic Analysis; Airfoils; XFLR5; Vortex Lattice Method (VLM); $C_L/C_D$ Ratio, Optimization

1. Introduction

Airfoils refer to the aerodynamic shape of the cross-section of the aircraft wing. The Wing is a prominent structure which suspends the bulky weight of aircraft in air. As the wing glides through the air [1], aerodynamic forces are produced. Component of force perpendicular to the movement is called lift and the component parallel and opposite to the motion of the wing is termed as drag. The aircraft requires more energy to overcome drag and hence airfoils are optimized with an intent to reduce drag force. Drag force adds on due to the friction between air molecules and the wing surface, this is known as skin friction. A phenomenon called Reynold’s effect [2] induces drag as the boundary layer thickens on the surface when the plane flies at slower speeds. The wing’s co efficient of lift being a dimensionless number depends on various design parameters like Aspect Ratio (AR), airfoil type and Reynold’s number. The pressure difference between the upper surface (suction surface) and the lower surface (suction surface) is the primary reason for lift generation as stated in Bernoulli’s Principle. The most important parameter to result in a higher lift is the Angle of Attack (AOA). Angle of attack is the angle between the chord line (straight line connecting leading and trailing edge) and the vector representing the motion of wind. Critical angle of attack is the maximum angle at which maximum lift can be observed. An increase in AOA increases lift up to the critical AOA after which the aircraft stalls. The distance between the upper and lower surfaces of the airfoil measured perpendicular to the camber line (American convention of measuring thickness) is referred to as thickness, which when increased increases the lift.
Another parameter which affects lift is the camber line. Camber line is the reference line equidistant from upper and lower surfaces, joining the leading and the trailing edge as shown in Fig. 01. Maximizing camber at specific sections of the airfoil increases lift. In addition, airspeed and area of the airfoil also adds on to the lift whereas lift decreases with higher altitudes.

![Airfoil Nomenclature](image)

**Figure 1. Airfoil Nomenclature**

2. Literature review

1. Numerical analysis on NACA 2412 airfoil was performed in ANSYS Fluent with a chord length of \( C_R = 0.3 \text{m} \) and total wing span of \( L = 1.6 \text{m} \), the coefficient of lift \( C_L = 0.16 \) and coefficient of drag \( C_D = 0.06 \) was obtained \[3\], and the same results were replicated in XFLR5 software for validation.

2. Aerodynamic analysis was performed on MH – 60 airfoil with varying airfoil thickness from 10.1% to 26.9% and max camber 1.7% to 36.6%. The analysis was performed using vortex lattice method (VLM) and the total wing span \( L = 1.9 \text{m} \) and chord length at root was \( C_R = 0.980 \text{m} \) \[4\].

3. Aerodynamic analysis was performed on 9 different airfoils for sailplane and the analysis was performed using XFLR5 software. The analysis was performed at Reynolds number \( 2e+05 \) to \( 5e+05 \). The results suggested that Wortmann FX S02-196 and FX-Wortmann FX61-184 airfoils were efficient for root section of the sailplane wing and GOE 533 airfoil was efficient for tip section of the wing \[5\].

4. Aerodynamic analysis was performed on Clark -Y airfoil using XFLR5 software, the analysis was performed at the Reynolds number \( 2.5e+05 \) to \( 5e+05 \). The chord length at the root is \( C_R = 0.27 \text{m} \), total wing span \( L = 1.8 \text{m} \) and angle of attack \(-5^\circ\) to \(20^\circ\). The analysis was carried out using Vortex Lattice Method (VLM) and Maximum coefficient of lift (\( C_L \)) was achieved at \(12^\circ\) angle of attack \[6\].

3. Wing Planforms

A wing planform is the shape of an aircraft wing as viewed from the top. The wing planforms have evolved drastically through generations. Some wing planforms analyzed are:

- Elliptical wing, which was used in Spitfire, a famous plane used in World War II.
- Rectangular wing, non-tapered and straight used in small aircrafts like Piper PA 38.
- Delta wing is a triangular wing used in modern fighter jets like Dassault Rafale.
- Swept back wings are angled backwards or forward from the root of the plane, mostly used in modern commercial aircrafts like Boeing 737.

4. XFLR5 Explanation

XFLR5 is an analysis tool for airfoils, wings and planes operating at low Reynolds number. It analyses airfoils and wing design based on Lifting Line theory, Vortex lattice method and 3D panel method \[7\].

5. Vortex Lattice Method (VLM)

The aerodynamic analysis was performed using VLM on XFLR5 software, VLM is a numerical method used in CFD, where the lifting surfaces such as a wing is modelled as an infinitely thin sheet...
of discrete vortices to compute lift and induced drag. The VLM is built on the theory of potential flow (ideal flow). Potential flow is a simplified representation of real flow experience in nature and this method neglects all viscous effects. Assumptions in Vortex Lattice Method:

- The flow is incompressible, inviscid and irrotational
- The lifting surfaces are thin
- Angle of attack and angle of slide slip are small

6. Methodology

A group of six airfoils were chosen for the aerodynamic analysis. These airfoils were analyzed on four different wing profiles, the aerodynamic characteristics were compared and the best 3 airfoils with the corresponding best wing profile were selected. These selected airfoils were further refined by interpolating to obtain best CL/CD value.

7. Airfoils Chosen

Totally six types of airfoils were chosen [8][9] for the analysis based on their varying camber and thickness. Table 1 contains the data of the airfoils depicted in Figures 2 to 7.

| Airfoil Name  | Thickness (%) | Camber (%) |
|---------------|---------------|------------|
| NACA 0018     | 18            | 0          |
| NACA 2412     | 12            | 2          |
| NACA 4412     | 12            | 4          |
| TSAGI S-12    | 11.9          | 1.99       |
| USA 45        | 14.53         | 4.15       |
| B737A         | 15.37         | 1.92       |

The analysis was performed using vortex lattice method (VLM), the wing root chord and total wing span is kept constant at \( C_R = 1 \text{m} \) and \( L = 10 \text{m} \) respectively. The density of air \( \rho = 1.225 \text{ kg/m}^3 \), kinematic viscosity of the air \( \nu = 1.789 \times 10^{-5} \text{ m}^2/\text{s} \), inlet velocity of air \( V = 50 \text{ m/s} \) (average cruise speed of Cessna 152 aircraft is 180 kmph) and Angle of Attack at \( 7^\circ \) are kept constant. The calculations are performed between the Reynolds number (Re) \( 1 \times 10^5 \) to \( 3 \times 10^6 \).
7. Four types of wing are chosen for the analysis, namely:
- Rectangular wing
- Elliptical wing
- Swept back wing
- Delta wing

7.1 Rectangular Wing
The chord length remains constant throughout the wing span length as shown in Figure. 8 and this type of wing is commonly used in small planes which are used for training pilots and planes used for recreational purposes. Example: Piper PA-38 Tomahawk, Cessna 152, etc.

| Table 2. Initial condition of rectangular wing profile |
|-----------------------------------------------|
| Chord length (CR) | 1 m |
| Mean Aerodynamic Chord (MAC) | 1 m |
| Wing area (A) | 10 m² |
| Aspect ratio | 10 |
| Velocity (V) | 50 m/s |
| Re at root | 2.79e+06 |
| Re at tip | 2.79e+06 |

![Figure 8. Rectangular wing profile](image)

7.1 Elliptical Wing
The wing profile looks like an ellipse when looked from the top, hence the name elliptical wing. Figure. 9 shows an elliptical wing planform. This type of wing is rarely used in aircraft because of its low manufacturability as it is a complex shape. Example: Supermarine Spitfire.

The equation to plot the ellipse is[10]:

\[ y = C_0 \sqrt{1 - \frac{y^2}{b^2}} \]

\[ \text{Distance from the wing root} \]
\[ b = \text{Total wing span} \]
\[ C_0 = \text{Chord length at root} \]
\[ C = \text{Chord length at ‘y’} \]

The equation for offset is:

\[ \text{offset} = \frac{1}{2} \left( C_0 - C \right) \]

\[ \text{Co} = \text{Chord length at root} \]
\[ C = \text{Chord length at ‘y’} \]

Using the above equations, the elliptical wing was designed. Table. 3 contains the co-ordinates for elliptical wing.
### Table 3. Plot coordinates for elliptical wing

| y  | C   | offset | y  | C   | offset |
|----|-----|--------|----|-----|--------|
| 0.00 | 1.000 | 0.000  | 3.70 | 0.673 | 0.164  |
| 0.50 | 0.995 | 0.003  | 3.80 | 0.650 | 0.175  |
| 1.00 | 0.980 | 0.010  | 3.90 | 0.626 | 0.187  |
| 1.25 | 0.968 | 0.016  | 4.00 | 0.600 | 0.200  |
| 1.50 | 0.954 | 0.023  | 4.05 | 0.586 | 0.207  |
| 1.75 | 0.937 | 0.032  | 4.10 | 0.572 | 0.214  |
| 2.00 | 0.917 | 0.042  | 4.15 | 0.558 | 0.221  |
| 2.10 | 0.908 | 0.046  | 4.20 | 0.543 | 0.229  |
| 2.20 | 0.898 | 0.051  | 4.25 | 0.527 | 0.237  |
| 2.30 | 0.888 | 0.056  | 4.30 | 0.510 | 0.245  |
| 2.40 | 0.877 | 0.061  | 4.35 | 0.493 | 0.253  |
| 2.50 | 0.866 | 0.067  | 4.40 | 0.475 | 0.263  |
| 2.60 | 0.854 | 0.073  | 4.45 | 0.456 | 0.272  |
| 2.70 | 0.842 | 0.079  | 4.50 | 0.436 | 0.282  |
| 2.80 | 0.828 | 0.086  | 4.55 | 0.415 | 0.293  |
| 2.90 | 0.815 | 0.093  | 4.60 | 0.392 | 0.304  |
| 3.00 | 0.800 | 0.100  | 4.65 | 0.368 | 0.316  |
| 3.10 | 0.785 | 0.108  | 4.70 | 0.341 | 0.329  |
| 3.20 | 0.768 | 0.116  | 4.75 | 0.312 | 0.344  |
| 3.30 | 0.751 | 0.124  | 4.80 | 0.280 | 0.360  |
| 3.40 | 0.733 | 0.133  | 4.85 | 0.243 | 0.378  |
| 3.50 | 0.714 | 0.143  | 4.90 | 0.199 | 0.401  |
| 3.60 | 0.694 | 0.153  | 4.95 | 0.141 | 0.429  |
| 5.00 | 0.000 | 0.000  | 5.00 | 0.000 | 0.500  |

### Table 4. Initial conditions of elliptical wing profile

| Chord length (C_R) | 1 m |
|--------------------|-----|
| Mean Aerodynamic Chord (MAC) | 0.849 m |
| Wing area (A) | 7.842 m² |
| Aspect ratio | 12.498 |
| Velocity (V) | 50 m/s |
| Re at root | 2.79e+06 |
| Re at tip | 3.94e+05 |

### Figure 9. Elliptical wing profile

7.3 **Sweptback Wing**

As the name suggests, the wings are swept backwards for aerodynamic drag reduction at an angle. for this analysis sweep angle of 37.5° is considered[11]. This type is most commonly used in commercial airplanes and cargo jets. Figure 10 is a swept back wing angled at 37.5 degrees Example: Boeing 737
Table 5. Initial conditions of Sweptback wing

|                     |          |
|---------------------|----------|
| Chord length at root ($C_R$) | 1 m      |
| Chord length at tip ($C_T$)   | 0.4 m    |
| Mean Aerodynamic Chord (MAC)  | 0.743 m  |
| Wing area (A)            | 7 m²     |
| Aspect ratio             | 14.286   |
| Root to tip sweep angle ($\phi$) | 37.5°   |
| Velocity (V)            | 50 m/s   |
| Re at root              | 2.79e+06 |
| Re at tip               | 1.11e+06 |

7.4 Delta Wing

Delta wing is type of trapezoidal shape wing which is commonly used in fighter jets for maneuverability. Delta wing planform is shown in Figure 11. Example: MiG-21

Table 6. Initial conditions of Delta Wing

|                     |          |
|---------------------|----------|
| Chord length ($C_R$) | 1 m      |
| Mean Aerodynamic Chord (MAC)  | 0.743 m  |
| Wing area (A)         | 7 m²     |
| Aspect ratio          | 14.286   |
| Velocity (V)          | 50 m/s   |
| Re at root            | 2.79e+06 |
| Re at tip             | 1.11e+06 |

8. $C_L/C_D$ Table and Graphs

After analysis, the following results in Table 7 were obtained.

Table 7. $C_L/C_D$ ratio of the airfoils with respective to wing planforms

| Profile names | Rectangular wing | Elliptical wing | Swept back wing | Delta wing |
|---------------|------------------|-----------------|-----------------|------------|
| NACA 0018     | 31.316           | 35.833          | 39.000          | 38.529     |
| NACA 2412     | 30.440           | 34.250          | 41.000          | 36.304     |
| NACA 4412     | 27.235           | 32.226          | 38.550          | 35.000     |
| USA 45        | 28.033           | 32.464          | 38.273          | 35.500     |
| TSAGI S-12    | 33.526           | 36.316          | 41.500          | 38.722     |
| B737A         | 27.920           | 32.696          | 35.556          | 34.864     |
Numerical data tabulated in Table 07 is pictorially represented in Graph 01. From the graph, we can conclude that TSAGI S-12 with swept back wing comparatively has the highest $C_l/C_D$ ratio and NACA 4412 with rectangular wing has the least $C_l/C_D$ ratio.

From the Table 7 it is evident that swept back wing profile coupled with TSAGI S-12 has the highest $C_l/C_D$ ratio among the airfoils and wing profiles chosen. The NACA 2412 and NACA 4412 results were also promising, so these 3 airfoils are further refined by interpolation.

### 9 Refinement

Refinement of airfoils or wing designs is undertaken to enhance lift and reduce drag or to obtain a higher $C_L/C_D$ ratio. This can be done either by increasing the pressure difference across the top and bottom surfaces of airfoil/wing or by distributing the pressure over larger length of airfoil. But parameters like camber, thickness and angle of attack also induces higher lift[12]. Certain parameters like camber and thickness of airfoils was modified to derive higher performances.

The three following airfoils were interpolated to obtain 3 test foils as we discussed above:

- TSAGI S-12
- NACA 2412
- NACA 4412

All the following test foils were used on swept back wing profile for analysis.

#### 9.1 Test foil 1

This airfoil was obtained by interpolating NACA 2412 and NACA 4412 airfoils. The thickness (%) and camber (%) was 10.2 and 3.01 respectively.

![Figure 12. Test foil 1](image)

#### 9.2 Test foil 2

This airfoil was obtained by interpolating NACA 4412 and TSAGI S-12 airfoils. The thickness (%) and camber (%) was 12.3 and 2.89 respectively.

![Figure 13. Test foil 2](image)
9.3 Test foil 3
This airfoil was obtained by interpolating NACA 2412 and TSAGI S-12 airfoils. The thickness (%) and camber (%) was 11.95 and 1.93 respectively.

![Figure 14. Test foil 3](image1.png)

After aerodynamic analysis of the above test foils yielded the following results:

| Airfoil name | \( \frac{C_L}{C_D} \) ratio |
|--------------|-----------------------------|
| Test foil 1  | 38.316                     |
| Test foil 2  | 41.765                     |
| Test foil 3  | 42.600                     |

![Figure 15. Lift and Downwash of test foil 3](image2.png)  
![Figure 16. Streamlines of test foil 3](image3.png)

Graph 2. \( \frac{C_L}{C_D} \) ratio of test foils with sweptback wing planform

10 Results and Conclusion
After analysis, it was found that Test foil 3 has the maximum \( \frac{C_L}{C_D} \) ratio when paired with swept back wing. In straight wing design such as rectangular wing the air flow is parallel to the wing, this is called the chord wise flow. The air above the wing accelerates and generate lift, but this type of wing design is not suitable for high speed flights as the wing plan form and the straight wing design generate more drag in high speed flight. This type of airflow reduces the critical Mach number of the plane. A swept will introduce a new type of flow along with chord wise flow, called span wise flow. The airflow is diverted into span wise flow which does not accelerate over the wing like the chord wise flow which allows the aircraft to fly at high speeds as the critical Mach number is further increased. This effect is more prominent when the sweep angle is higher.
Figure 16 shows a distortion at the wing tips of Swept back wing. Due to the pressure imbalance between upper and lower surfaces, the air tends to spiral at the wingtips. This is referred as wingtip vortices. A vortex is a flow pattern in which the elements of fluid follow circular paths about a point [13]. As the angle of attack increases during higher lift, there is higher pressure difference, as a result the effect of fast spinning vortices increases. These violent vortices increase turbulence in the flow and induce drag. Optimizing the wing design by introducing winglets at the wingtip reduces vortices. Winglets are small upright tails which avoid formation of vortices by reducing vortex interference with laminar airflow near wingtips.

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