The Muon Magnetic Moment in the TeV Scale Seesaw Models

Wei Chao

Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

Abstract

The reported discrepancy of the muon abnormal magnetic moment $a_\mu$ has impacts on the low energy phenomenology. In this paper we calculate the corrections to $a_\mu$ in the standard model extended by the TeV scale seesaw models. We show that the correction induced by the type-I seesaw model is negative and of the order $O(10^{-11})$, which can be neglected compared with $a_\mu^{SM}$. The correction induced by the type-II seesaw model, which depends on the mass of the Higgs triplet $m_\Delta$ and the Yukawa coupling $Y_\Delta$, can be of the order $O(10^{-10})$ and compensate for the discrepancy between $a_\mu^{SM}$ and $a_\mu^{exp}$. The correction induced by the type-III seesaw model is also negative and can be of the order $O(10^{-10})$. 

*Electronic address: chaowei@ihep.ac.cn
I. INTRODUCTION

For a spin 1/2 particle, the relation between its magnetic moment and its spin reads $\mu = g(e/2m)\vec{s}$. The Dirac equation predicts for the gyromagnetic factor $g = 2$, but radiative corrections to the lepton-photon-lepton vertex in quantum field theory may switch the value slightly. The abnormal magnetic moment is then defined as $a = (g - 2)/2$.

There has been a long history in measuring and calculating the muon abnormal magnetic moment $a_{\mu}$. In particular the steadily improving precision of both the measurements and the predictions of $a_{\mu}$ and the disagreement observed between the two have made the study of $a_{\mu}$ one of the most active research fields in particle physics in recent years. The final result of the “Muon g-2 Experiment” (E821) for $a_{\mu}$ reads \[ a_{\mu}^{exp} = (11659208 \pm 6) \times 10^{-10}, \] which deviates from the standard model (SM) prediction:

\[ \Delta a_{\mu} = a_{\mu}^{exp} - a_{\mu}^{SM} = 22(10) \times 10^{-10}. \]

Many new physics scenarios have been proposed to interpret the non-vanishing and positive value of $\Delta a_{\mu}$\[^{[2]}\]. Meanwhile new physics proposed to solve some other problems may potentially contribute to $\Delta a_{\mu}$.

On the neutrino sector, the discovery of neutrino oscillations has confirmed the theoretical expectation that neutrinos are massive and lepton flavors are mixed, providing the first evidence for physics beyond the SM in particle physics. The most appealing and natural idea for generating small neutrino masses is the seesaw mechanisms \[^{[3,4,5]}\], which rely on the existence of heavy particles such as right-handed Majorana neutrinos, triplet scalar or triplet fermions. A salient feature of the seesaw mechanisms is that the thermal leptogenesis mechanism \[^{[6]}\] can work well to account for the cosmological baryon number asymmetry. A direct test of the seesaw mechanisms would involve the detection of those heavy particles at a collider and the measurement of their Yukawa couplings with the electroweak doublets. If such Yukawa couplings are similar to the other fermion Yukawa couplings, the masses of those heavy particles turn out to be too high to be experimentally accessible.

To submit to the experiment, some kinds of TeV scale seesaw models \[^{[7,8,9,10,11]}\] were proposed, in which the masses of the heavy particles are set at the electroweak scale.
The key point of such seesaw scenarios is to adjust the structures of heavy particles’ Yukawa couplings to guarantee that $M_\nu$ (i.e., the mass matrix of light Majorana neutrinos) equals to zero at the tree level. Then tiny but non-vanishing neutrino masses can be ascribed to slight perturbations or radiative corrections to $M_\nu$ in the next-to-leading order approximation. A prominent feature of such kinds of seesaw scenarios is that the interactions of heavy particles with the SM gauge bosons and Higgs are not necessary suppressed, leading to very interesting lepton-number-violating phenomenology mediated by heavy particles at high-energy colliders such as the Tevatron, the LHC and the ILC.

TeV scale seesaw scenarios may lead to large unitarity violation of the lepton mixing matrix (MNS). However, a global analysis of current neutrino oscillation data and precision electroweak data yields very stringent constraints on the non-unitarity of the MNS matrix. Therefore a systematic investigation of the low energy phenomenology induced by such seesaw scenarios is necessary and important.

In this paper, we will calculate the corrections to $a_\mu$ induced by the TeV scale heavy particles. We show that the correction to $a_\mu$ induced by heavy Majorana neutrinos is of the order $-\mathcal{O}(10^{-11})$ in the type-I seesaw model. Corrections induced by the doubly charged Higgs boson and singly charged Higgs boson can be of the order $\mathcal{O}(10^{-10})$ in the type-II seesaw model. Therefore $\Delta a_\mu$ can be completely saturated by $\Delta a_\mu^{II}$. Whereas, the correction induced by triplet fermions is of the order $-\mathcal{O}(10^{-10})$ in the type-III seesaw model.

The outline of the paper is as follows. In section II we describe some basics of the TeV scale seesaw scenarios. Section III is devoted to the calculation of corrections to $a_\mu$ induced by various seesaw models. Some conclusions are drawn in section IV.

II. SOME BASICS OF THE TEV SCALE SEESAW MODELS

We regularize our notations and conventions in this section by reviewing some basics of the TeV seesaw scenario. After gauge symmetry spontaneous breaking, the neutrino mass terms turn out to be

$$\mathcal{L}_{\text{mass}} = \frac{1}{2}(\nu_L N_R)^c \begin{pmatrix} M_L & M_D^T \\ M_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix} + \text{h.c.} ,$$

In the type-I seesaw mechanism, $N$ can interact with the SM gauge bosons and Higgs through its mixing with the light SM SU(2) $\nu_L$. 

\[\]
where $\nu_L^c \equiv C\nu_L^T$ with $C$ being the charge conjugation matrix, likewise for $N_R^c$. The overall $6 \times 6$ neutrino mass matrix in $\mathcal{L}_{\text{mass}}$, denoted as $\mathcal{M}$, can be diagonalized by the unitary transformation $U^\dagger \mathcal{M} U^* = \hat{\mathcal{M}}$; or explicitly,

$$
\begin{pmatrix}
V & R \\
S & U
\end{pmatrix}^\dagger \begin{pmatrix}
M_L & M_D \\
M_D^T & M_R
\end{pmatrix} \begin{pmatrix}
V & R \\
S & U
\end{pmatrix}^* = \begin{pmatrix}
\hat{M}_\nu & 0 \\
0 & \hat{M}_N
\end{pmatrix},
$$

(4)

where $\hat{M}_\nu = \text{Diag}\{m_1, m_2, m_3\}$ and $\hat{M}_N = \text{Diag}\{M_1, M_2, M_3\}$ with $m_i$ and $M_i$ (for $i = 1, 2, 3$) being the light and heavy Majorana neutrino masses, respectively. Note that the $3 \times 3$ rotation matrices $V$, $U$, $R$ and $S$ are non-unitary, but they are correlated with one another due to the unitarity of $U$.

In the basis where the flavor eigenstates of three charged leptons are identified with their mass eigenstates, the standard charged-current interactions between $\nu_\alpha$ and $l_L$ (for $l = e, \mu, \tau$) can be written as

$$
-L_{\text{cc}} = \frac{g}{\sqrt{2}} \left[ \overline{l_L} V \gamma^\mu \nu_i W^-_\mu + \overline{l_L} R \gamma^\mu N_i W^-_\mu \right] + \text{h.c.} .
$$

(5)

It becomes clear that $V$ describes the charged-current interactions of three light Majorana neutrinos, while $R$ is relevant to the charged-current interactions of three heavy Majorana neutrinos. One can similarly write out the interactions between the Majorana neutrinos and the neutral gauge boson (or Higgs) in the chosen flavor basis $^7$:

$$
\mathcal{L}_Z = -\frac{g}{2c_W} \overline{\nu_L} R \gamma^\mu P_L N_i Z_\mu + \text{h.c.} ,
$$

(6)

$$
\mathcal{L}_H = -\frac{g M_i}{2 M_W} \overline{\nu_L} R P_R N_i h^0 + \text{h.c.} .
$$

(7)

There are constraints on the non-unitarity of $VV^\dagger$ from electroweak decays. Ratios of $\mu$, $\tau$, $W$ and $\pi$ decays, used often in order to test universality, can be interpreted as tests of lepton mixing unitarity. They result in constraints for the diagonal elements of $VV^\dagger$. The lepton-flavor-violating processes, which occur at the one-loop level, constrain the off-diagonal elements of $VV^\dagger$. A global fit to the constraints listed above results in $^12$

$$
VV^\dagger \approx \begin{pmatrix}
0.994 \pm 0.005 & < 7.0 \cdot 10^{-3} & < 1.6 \cdot 10^{-2} \\
< 7.0 \cdot 10^{-5} & 0.995 \pm 0.005 & < 1.0 \cdot 10^{-2} \\
< 1.6 \cdot 10^{-2} & < 1.0 \cdot 10^{-2} & 0.995 \pm 0.005
\end{pmatrix},
$$

(8)

at the 90% confidence level. It is clear that the deviation of $VV^\dagger$ from the identity matrix can be as large as a few percents. Therefore the low energy phenomenology induced by heavy neutrinos is not negligible.
III. \( a_\mu \) IN VARIOUS SEESAW MODELS

The most general form for the photon-muon vertex function \( \Gamma_\mu \), which is consistent with Lorentz covariance, can be written as \[2, 13\]

\[
\bar{u}(p_2)\Gamma_\mu u(p_1) = \bar{u}(p_2) \left[ F_1(q^2)\gamma^\mu - \frac{i}{2m_\mu} F_2(q^2)\sigma^{\mu\nu}q_\nu + \frac{1}{m_\mu} F_3(q^2)q^\mu \right] \gamma_5 \left( G_1(q^2)\gamma^\mu - \frac{i}{2m_\mu} G_2(q^2)\sigma^{\mu\nu}q_\nu + \frac{1}{m_\mu} G_3(q^2)q^\mu \right) u(p_1),
\]

where \( q = p_2 - p_1 \) and \( m_\mu \) is the mass of muon. The anomalous magnetic moment of the muon is related to \( \Gamma_\mu \) as follows: \( a_\mu = F_2(0) \).

The SM prediction of \( a_\mu \) is generally divided into three parts: \( a^{\text{SM}}_\mu = a^{\text{QED}}_\mu + a^{\text{EW}}_\mu + a^{\text{Had}}_\mu \). The QED part includes all photonic and leptonic \((e, \mu, \tau)\) loops starting with the classic \( \alpha/2\pi \) Schwinger contribution. Loop contributions involving \( W^\pm, Z \) or Higgs particles are collectively labeled as \( a^{\text{EW}}_\mu \). The hadronic part includes the contributions from the quark and gluon loops. There are contributions induced by various heavy particle loops, which are contained in TeV scale seesaw models. We will calculate them in the following.

A. \( a_\mu \) in the type-I seesaw scenario

Assuming that light but non-zero neutrino masses are generated by the type-I seesaw mechanism, we need to extend the SM with three right-handed Majorana neutrinos. The relevant Lagrangian can be written as

\[
\mathcal{L}_1 = \mathcal{L}_{\text{SM}} - \bar{l}_L Y_\nu \tilde{H} N_R - \frac{1}{2} N_R^c M_R N_R + \text{h.c.},
\]

where \( M_R \) is masses of the right-handed neutrinos. Integrating out right-handed Majorana neutrinos results in a light neutrino Majorana mass matrix of the form: \( M_\nu = -v^2 Y_\nu M_R^{-1} Y_\nu^T \).

In this model the MNS matrix is non-unitary and the heavy Majorana neutrinos interact with charged leptons through their mixing with light neutrinos, which was already shown in Eq. (5). As a result, the muon abnormal magnetic moment receives contribution from the heavy Majorana neutrino and \( W \) boson loop. The relevant diagram is shown in Fig. 1 (a), which gives the following correction to \( a_\mu \):

\[
\Delta a^I_\mu = \frac{G_F m_\mu^2}{8\sqrt{2}\pi^2} (RR^\dagger)_{\mu\nu} \left[ I \left( M^2_W, M^2_1 \right) - \frac{10}{3} \right],
\]

5
where $I(M_W^2, M_i^2)$ can be written as

$$I(M_W^2, M_i^2) = M_W^2 \int \frac{dx}{m^2 \mu^2 + x(M_W^2 - M_i^2 - m^2) + M_i^2}.$$  \(12\)

Suppose that masses of the right-handed Majorana neutrinos are degenerate. We plot $\Delta a_{\mu}$ in Fig. 2 by assuming that $RR^\dagger \sim 1\%$ and the masses of heavy neutrinos lie in the range $200\text{GeV} \leq M_1 \leq 500\text{GeV}$, which are potentially accessible at the LHC. We can find from the figure that $\Delta a_{\mu}^1$ is negative and not sensitive to $M_1$. Besides, $\Delta a_{\mu}^1$ is too small to change the SM prediction significantly.

**B. $\Delta a_{\mu}$ in the type-II seesaw scenario**

We now proceed to the TeV scale type-II seesaw scenario \([11]\). In this scenario an extra scalar triplet ($Y = 2$) together with some heavy Majorana neutrinos is added to the SM. The most general Lagrangian for this model is

$$L_{\text{II}} = \text{Tr} \left[ (D^\mu \Delta)^\dagger D_\mu \Delta \right] - m^2_\Delta \text{Tr} \left( \Delta^\dagger \Delta \right) - \frac{1}{2} \overline{L} Y_\Delta i \sigma_2 l_\ell^c - \overline{L} Y_\nu \tilde{H} N_R - \frac{1}{2} N_R^c M_R N_R + h.c.$$  \(13\)

where $\Delta$ represents the Higgs triplet. After integrating out heavy fields and the spontaneous gauge symmetry breaking, one obtains the effective mass matrix for three light neutrinos:

$$M_\nu \approx v \Delta Y_\Delta - v^2 Y_\nu M_{-1}^{-1} Y_\nu^T,$$

with $v$ and $v_\Delta$ being the vacuum expectation values (vev’s) of the neutral components of $H$ and $\Delta$, respectively. The smallness of $M_\nu$ is ascribed to a significant but incomplete cancellation between $v \Delta Y_\Delta$ and $v^2 Y_\nu M_{-1}^{-1} Y_\nu^T$ terms. There are totally seven physical Higgs bosons in this model: doubly-charged $\Delta^{++}$ and $\Delta^{--}$, singly-charged $\delta^+$ and $\delta^-$, neutral $A^0$ (CP-odd), and neutral $h^0$ and $H^0$ (CP-even), where $h^0$ is the SM-like Higgs boson. Doubly charged Higgs boson and charged lepton loops together with singly charged Higgs boson and neutrino ($W$ boson) loops may contribute to $a_{\mu}$. The relevant diagrams are shown in Fig. 1 (b) – (f). Direct calculation results in

$$a_{\mu}^\Delta = \frac{1}{16\pi^2} \sum_{\alpha = e, \mu, \tau} |(Y_\Delta)_{\mu\alpha}|^2 \left[ I_1(m_\alpha^2, m_\Delta^2) + I_2(m_\alpha^2, m_\Delta^2) \right],$$  \(14\)

$$a_{\mu}^\delta = \frac{1}{16\pi^2} \sum_{\alpha} |(Y_\Delta)_{\mu\alpha}|^2 (VV^\dagger)^\alpha I_3(m_\alpha^2, m_\delta^2),$$  \(15\)

with

$$I_1(m_\alpha^2, m_\Delta^2) = m_\mu^2 \int dx \frac{x(1 - x^2)}{x m_\Delta^2 + (1 - x)m_\alpha^2 + (x^2 - x)m_\mu^2}.$$
where \( \alpha (\alpha = e, \mu, \tau) \) reads as the mass of the charged lepton. When writing down Eq. (15), we have ignored the contributions of diagrams (e) and (f) in Fig. 1, which are suppressed by the masses of light Majorana neutrinos. The total corrections to \( a_\mu \) motivated by the type-II seesaw scenario is defined by the sum of Eqs. (11), (14) and (15): \( \Delta a_\mu^{II} = a_\mu^\Delta + a_\mu^\delta + \Delta a_\mu^I \).

Notice that \( M_L \) can be reconstructed via \( M_L = V\hat{M}_\nu V^T + R\hat{M}_N R^T \approx R\hat{M}_N R^T \) [11], which must be a good approximation. The element of the Yukawa coupling matrix \( (Y_\Delta) \) turns out to be

\[
(Y_\Delta)_{\alpha\beta} = \frac{(M_L)_{\alpha\beta}}{v_\Delta} \approx \sum_{i=1}^3 \frac{R_{\alpha i} R_{\beta i} M_i}{v_\Delta},
\]

where the subscripts \( \alpha \) and \( \beta \) run over \( e, \mu \) and \( \tau \). This result implies that the muon magnetic moment depends on both \( R \) and \( M_i \). \( v_\Delta \) may affect the gauge boson masses in such a way that

\[
\rho \equiv \frac{M_W^2}{(M_Z^2 \cos^2 \theta_W)} = \frac{(v^2 + 2v_\Delta^2)}{(v^2 + 4v_\Delta^2)} \text{ holds.}
\]

By using experimental constraint on the \( \rho \)-parameter [14], one gets \( \kappa \equiv \sqrt{2} \frac{v_\Delta}{v} < 0.01 \) and \( v_\Delta < 2.5 \text{ GeV} \). We work in the minimal type-II seesaw scenario [15] and set \( v_\Delta = 1 \text{ GeV} \) in our numerical analysis. Let us parametrize the \( 3 \times 1 \) complex matrix \( R \) in terms of three rotation angles and three phase angles [16]: \( R = (\hat{s}_{14}, \hat{s}_{24}, \hat{s}_{34})^T \), where \( c_{ij} \equiv \cos \theta_{ij} \) and \( \hat{s}_{ij} \equiv e^{i\delta_{ij}} s_{ij} \) with \( s_{ij} \equiv \sin \theta_{ij} \) for \( ij = 14, 24, 34 \). Combining all electroweak precision constraints, we may choose a self-consistent parameter space of three mixing angles: \( s_{14} \approx 0, s_{24} \in [0, 0.1] \) and \( s_{34} \in [0, 0.1] \). In Fig. 3 we plot \( \Delta a_\mu^{II} \) as a function of \( m_\Delta \), setting \( R \) to its largest allowed values. The solid, dotted and dashed lines correspond to \( M_1 = 50, 200, 500 \text{ GeV} \), separately.

The short dotted line corresponds to \( \Delta a_\mu \). It is clear that \( \Delta a_\mu^{II} \) is proportional to \( M_1 \) and the deviation of \( a_\mu \) from the SM prediction may be fully saturated by \( \Delta a_\mu^{II} \). Suppose that \( m_\Delta \) lies in the range \( 200 \text{ GeV} \leq m_\Delta \leq 500 \text{ GeV} \). The experimental result of \( a_\mu \) constrains the mass of the heavy Majorana neutrino to lie below 310.5 GeV.

\[ \text{C. } a_\mu \text{ in the type-III seesaw scenario} \]

Let us calculate \( a_\mu \) in the type-III seesaw scenario [5], which extends the SM with \( SU(2)_L \) triplet of fermions with zero hypercharge. In this model at least two such triplets (or one
triplet plus one singlet) are necessary in order to generate non-vanishing light neutrino masses. The relevant Lagrangian can be written as

$$L_{III} = \text{Tr}[\overline{\Psi} i D \Psi] - \frac{1}{2} \text{Tr}[\overline{\Psi} m_\Psi \Psi^C] - \sqrt{2} t_L \phi Y_\Psi \Psi + \text{h.c.},$$

where $m_\Psi$ is the mass of triplet fermion and $\Psi$ can be written as

$$\Psi = \begin{pmatrix} \Psi^0/\sqrt{2} & \Psi^+ \\ \Psi^- & -\Psi^0/\sqrt{2} \end{pmatrix}.$$

Integrating out triplet fermions at the tree level results in a dimension five effective operator which leads to a light neutrino Majorana mass matrix of the form: $M_{\nu}^{III} = -\frac{\sqrt{2}}{2} Y_\Psi m_\Psi^{-1} Y_\Psi^T$.

The possibility of testing type-III seesaw at the LHC is discussed in many articles [17], in which lepton-number-violating and (or) lepton-flavor-violating signals induced by the triplet fermions are discussed. Singly charged heavy field $\Psi^-$ may contribute to $a_\mu$. The relevant diagram is shown in Fig. 1 (g). Direct calculation results in

$$\Delta a_\mu^{III} = \frac{1}{8\pi^2} (Y_\Psi Y_\Psi^T)_{\mu\mu} I_4(m_H^2, m_\Psi^2),$$

where

$$I_4(m_H^2, m_\Psi^2) = m_\mu^2 \int dx \frac{x(1-x)^2}{x - x^2 m_\mu^2 + (x - 1)m_\Psi^2 - x m_H^2}.$$

It is clear that $I_4 < 0$, which means $\Delta a_\mu^{III} < 0$. Suppose that there is structure cancelation in $M_{\nu}^{III}$, just like what happens in TeV scale type-I and type-II seesaw models. Then $Y_\Psi \sim 1$ and $\Delta a_\mu^{III} \sim -\mathcal{O}(10^{-10})$, which is not negligible but theoretically unfavorable.

In summary, we have evaluated the corrections to $a_\mu$ induced by heavy Majorana neutrinos, triplet scalar and triplet fermions. To definitely illustrate the effect of different seesaw scenarios, we summarize our results in table I.

| TeV scale seesaw models | extra heavy particles | $\Delta a_\mu$ |
|------------------------|-----------------------|---------------|
| Type-I seesaw          | right-handed neutrinos | $-\mathcal{O}(10^{-11})$ |
| Type-II seesaw         | right-handed neutrinos+Higgs triplet | $\mathcal{O}(10^{-10})$ |
| Type-III seesaw        | triplet fermions       | $-\mathcal{O}(10^{-10})$ |

TABLE I: The corrections to $a_\mu$ induced by TeV scale heavy particles, which are contained in various seesaw models.
IV. CONCLUSION

Motivated by the conjecture that new physics at the TeV scale is responsible for the electroweak symmetry breaking and origin of neutrino masses, a series of TeV seesaw models were proposed. These TeV scale seesaw scenarios, in which sufficient lepton number (flavor) violation signals are induced, are testable at the LHC and (or) ILC. Meanwhile extra heavy particles in these models may induce interesting low energy phenomena. In this article, we have evaluated the corrections to $a_\mu$ induced by heavy Majorana neutrinos, triplet scalar and triplet fermions, which are separately included in type-I, II and III seesaw models. Our results show that the correction induced by the heavy neutrinos is ignorable compared with $a_{\mu}^{\text{SM}}$. Corrections induced by the doubly charged Higgs boson and singly charged Higgs boson may be of the order $O(10^{-10})$ and $\Delta a_\mu$ can be completely saturated by $\Delta a_{\mu}^{\text{II}}$. Whereas the correction induced by triplet fermions can be of the order $-O(10^{-10})$, which is theoretically unfavorable. In conclusion, TeV scale type-II and type-III seesaw scenarios can significantly contribute to $a_\mu$. The running of the LHC may potentially verify which mechanism is responsible for $\Delta a_\mu$ and the origin of neutrino masses.

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FIG. 1: One-loop Feynman diagrams contributing to $a_\mu$. Diagram (a) comes from the heavy neutrino and $W$ boson loop. Diagrams (b) and (c) come from doubly charged Higgs and charged lepton loops. Diagrams (d), (e), (f) come from singly charged Higgs and neutrino ($W$ boson) loops. Diagram (g) comes from the triplet fermion and SM-like Higgs boson loop.
FIG. 2: $\Delta a_{\mu}^I$ as a function of $M_1$, with $RR^I \sim 0.01$ and $200\text{GeV} \leq M_1 \leq 500\text{GeV}$.

FIG. 3: $\Delta a_{\mu}^I$ as a function of $m_\Delta$. The solid, dotted and dashed lines correspond to $M_1 = 50$, $200$, $500\text{ GeV}$, separately. The short dotted line corresponds to $\Delta a_{\mu}$. 