Scaling critical behavior of superconductors at zero magnetic field

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Abstract

We consider the scaling behavior in the critical domain of superconductors at zero external magnetic field. The first part of the paper is concerned with the Ginzburg-Landau model in the zero magnetic field Meissner phase. We discuss the scaling behavior of the superfluid density and we give an alternative proof of Josephson’s relation for a charged superfluid. This proof is obtained as a consequence of an exact renormalization group equation for the photon mass. We obtain Josephson’s relation directly in the form $\rho_s \sim t'^\nu$, that is, we do not need to assume that the hyperscaling relation holds. Next, we give an interpretation of a recent experiment performed in thin films of $YBa_2Cu_3O_{7-\delta}$. We argue that the measured mean field like behavior of the penetration depth exponent $\nu'$ is possibly associated with a non-trivial critical behavior and we predict the exponents $\nu = 1$ and $\alpha = -1$ for the correlation length and specific heat, respectively. In the second part of the paper we discuss the scaling behavior in the continuum dual Ginzburg-Landau model. After reviewing lattice duality in the Ginzburg-Landau model, we discuss the continuum dual version by considering a family of scalings characterized by a parameter $\zeta$ introduced such that $m_{h,0}^2 \sim t^\zeta$, where $m_{h,0}$ is the bare mass of the magnetic induction field. We discuss the difficulties in identifying the renormalized magnetic induction mass with the photon mass. We show that the only way to have a critical regime with $\nu' = \nu \approx 2/3$ is having $\zeta \approx 4/3$, that is, with $m_{h,0}$ having the scaling behavior of the renormalized photon mass.

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I. INTRODUCTION

The study of the critical behavior of the high-temperature superconductors (HTSC) has been initiated long ago [1]. The discovery of this remarkable class of materials has opened a new perspective for the general theory of critical phenomena. In fact, the situation for the HTSC is much more favorable than for the conventional metallic low-temperature superconductors. The reason is that the size of the critical region in the HTSC is large enough to make its critical properties experimentally accessible using today techniques. In the tested temperature region it has been obtained that $-0.03 < \alpha < 0$ for the specific heat exponent, $\nu \approx 0.67$ for the correlation length exponent and the amplitude ratio $A^+/A^- \approx 1.065$ [2,3]. These values are consistent with those found for $^4$He. This means that the critical region probed corresponds to an uncharged 3D XY universality class [4]. In this situation the superfluid density, $\rho_s$, scales as $\rho_s \sim \lambda^{-2}$, where $\lambda$ is the penetration depth. The experimental result $\nu \approx 0.67$ follows from a direct measurement of $\lambda$. The measured value of the penetration depth exponent is $\nu' \approx 0.33$ and from Josephson’s relation $\rho_s \sim \xi^{-1}$ the result $\nu \approx 0.67$ follows. Recent zero field experimental results obtained using very clean crystals of $YBa_2Cu_3O_7-\delta$ (YBCO) by Kamal et al. [5], confirm early measurements of the penetration depth near $T_c$, giving $\nu' = 0.33 \pm 0.01$. These measurements estimate the critical region as being nearly 5 K wide. While this seems to be true for three-dimensional crystals, it does not seem to be the case for thin films. Indeed, recent measurements of $\lambda$ in thin films of $YBa_2Cu_3O_7-\delta$ performed by Paget et al. [6] display a critical regime where the 3D XY behavior is absent and $\nu' = 1/2$, that is, a mean field like behavior. The critical region reported by them is however only 0.5 wide. The same result $\nu' = 1/2$ has been obtained earlier for thin films by Lin et al. [7]. However, the result of Lin et al. followed as a consequence of taking in account the finite size of the sample, otherwise the result $\nu' \approx 0.33$ of Kamal et al. [5] follows. We stress that none of these experiments were performed in the charged critical region. The true charged transition corresponds to a very small critical region and is presently inaccessible to the experimental probes. From duality arguments in the lattice model, it is obtained that the charged phase transition should correspond to an “inverted” XY behavior [8]. For this situation it has been argued by Herbut and Tesanović [9] that $\rho_s \sim \lambda^{-1}$ which implies that $\nu' = \nu$. Recent numerical study in the lattice model [10] confirms this prediction and gives $\nu' = \nu \approx 0.67 \approx 2/3$.

The result $\nu \approx 0.67 \approx 2/3$ has been obtained in recent years using perturbative renormalization group (RG) methods in fixed dimension $d = 3$ [11,14]. RG calculations performed in the seventies on the basis of the $\epsilon$-expansion leads to a flow where the charged infrared stable fixed point is absent if the number of components $n$ of the order parameter is less than 365.9 [13]. Since the physical case corresponds to $n = 2$, we have that no second order phase transition is predicted in that calculation. The authors of ref. [13] have predicted that a weak first order transition takes place. Further calculations using dimensional regularization confirmed this scenario even up to 2-loop order [14]. This picture seems to be appropriate for superconductors in the type I regime. However, for the type II regime this result is shown to be an artifact of the $\epsilon$-expansion [17]. The $\epsilon$-expansion can be improved by doing a Padé-Borel resummation [18]. In this case a charged infrared stable fixed point is found and it is obtained that $\nu \approx 0.771$ [18]. Non-perturbative calculations on the basis of the Wilson RG gives $\nu \approx 0.53$ by truncating the average action (the Legendre transform
of the Wilsonian effective action) in $|\phi|^4$ and $\nu \approx 0.58$ with a truncation in $|\phi|^8$ [14]. Thus, the perturbative RG in fixed dimension seems to give more interesting values of $\nu$ though it is less controlled. The non-perturbative calculations using Wilson RG of ref. [19] are also performed in fixed dimension but it seems to be very difficult to make further improvements with respect to the different truncations.

It is in general very difficult to find reliable approximations to study the critical domain of a charged transition. However, since the experimentally probed critical regime corresponds to a crossover near the neutral $XY$ universality class, theoretical studies of scaling behavior are often performed using a Ginzburg-Landau (GL) model coupled to an external magnetic field $H$ and neglecting gauge field fluctuations [20]. When the order parameter critical fluctuations are taken in account it is possible to study the different critical regimes in a phase diagram in the $H - T$ plane. The situation is particularly interesting for the HTSC where many new physical effects like vortex-lattice melting occurs [21,22].

In this paper we will consider the scaling behavior in the charged critical domain in superconductors at the zero magnetic field Meissner phase. In the first part of the paper we will discuss the scaling behavior of the Ginzburg-Landau model. We use an exact RG equation for the photon mass to rederive the Herbut and Tesanovic result $\nu' = \nu$. Since our derivation does not use the Josephson relation [23], we obtain as a consequence the Josephson relation for a charged superfluid directly in the form $\rho_s \sim t^{\nu'}$, while in the original Josephson’s paper this form follows only if it is assumed that hyperscaling holds since he has obtained actually that $\rho_s \sim t^{2/3-\eta \nu}$. Next, we give an alternative interpretation of a recent experimental result of Paget et al. obtained using YBCO thin films [6].

The second part of the paper is concerned with the continuum dual version of the GL model [38,11,39]. We review the lattice duality for the lattice GL model and the meaning of “inverted” $XY$ behavior [8]. Then we discuss the scaling behavior of the proposed continuum dual version of lattice duality. For this end, we consider a family of scalings $\Sigma_\zeta$ where the parameter $\zeta \geq 0$ is introduced through $m^2_{h,0} \sim t^\zeta$, with $m_{h,0}$ being the mass of the magnetic induction field. We obtain the scaling behavior for some relevant values of $\zeta$ by assuming that the renormalized counterpart of $m_{h,0}$ is the renormalized photon mass of the GL model in the Meissner phase. We show that the only way to obtain $\nu = \nu' \approx 2/3$ is using $\zeta \approx 4/3$. This means that $m_{h,0}$ has the same scaling behavior as the renormalized photon mass of the GL model. This analysis helps us to understand the difficulties in describing a correct “inverted” $XY$ behavior with the continuum dual model.

II. SCALING IN THE GL MODEL FOR $T < T_C$

In the following we will assume the existence of an infrared stable fixed point. This should be true at least in the type II regime, as lattice results have convincingly shown [8,10,40]. The existence of an infrared stable fixed point has also been established in recent years directly in the continuum model by using mainly RG techniques [3,4,11–14,19,17,18]. The bare action for the $d = 3$ GL model in the zero external magnetic field Meissner phase is given by

$$S(A_0, \phi_0; m^2_0, u_0, e_0) = \int d^3x \left[ \frac{1}{2} (\nabla \times A_0)^2 + \left| \left( \nabla - ie_0 A_0 \right) \phi_0 \right|^2 - m^2_0 |\phi_0|^2 + \frac{u_0}{2} |\phi_0|^4 \right],$$ (1)
where the subindex 0 denotes bare quantities and $m_0^2 > 0$. Here $m_0^2 \sim t$, $t$ being the reduced temperature. The renormalized action, $S_R$, is defined in a standard way [24], that is,

$$S_R(A, \phi; m^2, u, e) = S(Z_A^{1/2}A, Z_{\phi}^{1/2}\phi; Z_{\phi}^{(2)}Z_{\phi\phi}^{-1}m^2, Z_UZ_{\phi\phi}^{-2}u, Z_A^{1/2}e),$$

where the quantities without the zeroes are renormalized and we have defined the corresponding renormalization constants [25]. The renormalization constants above are the same as in the Coulomb phase, as dictated by the Ward identities [26]. We assume that the correlation functions are evaluated in a renormalized gauge, the so called $R_\alpha$ gauge [26], and that the Coulomb gauge limit has been taken in the end. We cannot fix the gauge $\alpha = 0$ from the very beginning because the unphysical fields constituting the $R_\alpha$ gauge would become massless, generating infrared divergences. The unitary gauge, on the other hand, does not have this problem since only physical fields are present. However, it is not renormalizable, even in $d = 3$.

In order to obtain the RG equations we must define a scaling, that is, the choice of the mass scale controlling the flow while specifying the variables which are kept fixed in this process. Here we note that we have two dimensionful couplings, namely, $e^2$ and $u$, both having dimension of mass. In the Meissner phase the photon acquires a mass, $m_A$, which together with the mass, $m$, define two mass scales in the problem. An useful dimensionless parameter is the ratio between these two masses, the Ginzburg parameter, $\kappa = m/m_A = (u/e^2)^{1/2}$. Since the critical point corresponds to $m = 0$, the renormalized mass $m$ is a good mass scale to control the flow. Thus, we will choose it as the fundamental mass scale. Let us define in this way the dimensionless couplings $\hat{u} \equiv um^{d-4} = Z_A^2Z_u^{-1}u_0m^{d-4}$ and $\hat{e}^2 \equiv e^2m^{d-4} = Z_A\hat{e}^2_0m^{d-4}$. Note that although the GL model has been written in fixed dimension $d = 3$, we have defined the dimensionless couplings in arbitrary dimension $2 < d \leq 4$. In order to complete the definition of our scaling we must specify the parameters that will be kept fixed. The standard choice corresponds to differentiate the renormalized dimensionless couplings with respect to $\ln(m)$ keeping the bare couplings and the ultraviolet cutoff fixed. Note that in this process neither $m_0$ nor $m_{A,0}$ are kept fixed. We define the RG functions

$$\eta_A = m\frac{\partial \ln Z_A}{\partial m},$$

$$(3)$$

$$\eta_\phi = m\frac{\partial \ln Z_\phi}{\partial m},$$

$$(4)$$

$$\eta_{\phi}^{(2)} = m\frac{\partial \ln Z_{\phi}^{(2)}}{\partial m}.$$  

$$(5)$$

At the infrared stable fixed point $\eta_A$ and $\eta_\phi$ gives respectively the anomalous dimensions of $A$ and $\phi$. The anomalous dimension of $|\phi|^2$ is given by the fixed point value of $\eta_{\phi}^{(2)} - \eta_\phi$. The fixed point values $\hat{e}^2_*$ and $\hat{u}_*$ are determined from the equations $\beta_{\hat{e}^2} \equiv m\hat{e}^2/\partial m = 0$ and $\beta_\alpha \equiv m\hat{u}/\partial m = 0$ and we take the solution corresponding to the infrared stable fixed point. The beta funcion $\beta_{\hat{e}^2}$ is given exactly by
$$\beta e^2 = (\eta_A + d - 4)e^2.$$  \hfill (6)

An immediate consequence of the above equation is that a charged fixed point corresponds to $\eta_A^* = 4 - d$ \[9,13,14,19\], $\eta_A^*$ being the fixed point value of $\eta_A$. From Eq. (6) and the definition of $\kappa$ we have the following exact equations:

$$m \frac{\partial \kappa^2}{\partial m} = \kappa^2 \left( \frac{\beta u}{u} + 4 - d - \eta_A \right),$$  \hfill (7)

$$m \frac{\partial m_A^2}{\partial m} = m_A^2 \left( d - 2 + \eta_A - \frac{\beta u}{u} \right).$$  \hfill (8)

From Eq. (8) we obtain easily that near the phase transition (that is, near the charged infrared stable fixed point) the photon mass scales as $m_A \sim m$. Since $m = \xi^{-1}$ and $m_A = \lambda^{-1}$, $\lambda$ being the penetration depth, we obtain $\nu' = \nu$. This is a rederivation of the result of Herbut and Tesanović \[9\]. Note that in our derivation no use has been made of the Josephson relation. Since $m_A^2 = e^2 \rho_s$ and from Eq. (6) $e^2 \sim m^{4-d} = \xi^{d-4}$, we obtain that $\rho_s \sim \xi^{2-d} \sim t^{\nu(d-2)}$. This constitutes a renormalization group proof of the Josephson’s relation for the charged superfluid. In the original Josephson’s paper this relation is proved for an uncharged superfluid and given in the form $\rho_s \sim t^{2\beta-\eta \nu}$. This follows easily by noting that $\rho_s = <|\phi|^2> = Z_{\phi}^{-1} <|\phi_0|^2>$. Since near the critical point $Z_{\phi} \sim m^n \sim t^{\nu n}$ and defining $\beta$ through $<|\phi_0|^2> \sim t^{2\beta}$, it follows that $\rho_s \sim t^{2\beta-\nu \eta}$. Thus, this last argument leading to Josephson’s relation it does not matter if the superfluid is charged or not. Our derivation made directly for the superconductor implies therefore

$$2\beta - \eta \nu = \nu(d-2).$$  \hfill (9)

As pointed out by Fisher et al. \[28\] in the context of uncharged superfluids, the relation with the exponent $\nu(d-2)$ instead of $2\beta - \eta \nu$ holds only if hyperscaling holds, that is, $d \nu = 2 - \alpha$. Our argument shows that hyperscaling holds for a superconductor. For an uncharged superfluid the hyperscaling relation can be proved by using scaling RG arguments of the same type we used here \[24\]. Of course, this type of proof is not rigorous in the sense that it assumes that the continuum limit of the lattice statistical mechanical model exists. Only the inequality $d \nu \geq 2 - \alpha$ can be rigorously proved \[27\]. A well known case where hyperscaling fails is mean field theory where $\nu = 1/2$ and $\alpha = 0$ independent of the dimension. In this case hyperscaling is fulfilled only at $d = 4$.

The Josephson’s relation is used experimentally to determine the value of $\nu$. Today it is possible to perform very accurate direct measurements of the penetration depth. The critical region probed is such that the gauge field fluctuations are unimportant and the critical fluctuations are those of the order parameter field and this means that $\eta_A^* = 0$. Then, from Eq. (8), we obtain that near the phase transition $m_A^2 \sim m^{d-2} \sim t^{\nu(d-2)}$, that is, $\rho_s \sim \lambda^{-2} \sim t^{\nu(d-2)}$. Thus, in such critical regime

$$\nu' = \frac{\nu(d-2)}{2}.$$  \hfill (10)
Experiments performed in YBCO crystals give that $\nu' = 0.33 \pm 0.01$ \cite{3,5}. Using Eq. (10) for $d = 3$ we obtain $\nu \approx 2/3$, consistent with the 3D XY behavior, that is, a $^4$He like behavior.

The situation seems to be however different for YBCO thin films. A recent measurement by Paget et al. \cite{6} performed in YBCO thin films gives $\nu' = 1/2$, that is, a mean field like behavior. This result has been obtained to within 0.2 to 0.5 K of $T_c$, indicating in this way a much smaller critical region as compared to the bulk YBCO \cite{3,3,5}. However, this mean-field like behavior could be interpreted as a non-mean field behavior in the following way. If we insist in using (10) to evaluate $\nu$ we obtain

$$\nu = \frac{1}{d-2},$$

and, assuming that hyperscaling holds,

$$\alpha = \frac{d - 4}{d - 2},$$

The above exponents are not classical and we have $\nu = 1$ and $\alpha = -1$ for $d = 3$. Note the similarity between the present critical regime with the $O(n)$ model for large $n$ and $2 < d \leq 4$.

The above exponents are just the exact exponents for the $O(n)$ model at large $n$ \cite{24}. The exponent $\nu$ as given in (11) is also obtained in a $O(n)$ non-linear $\sigma$-model in $d = 2 + \epsilon$ dimensions and $n > 2$ \cite{24}. Thus, it is possible to interpret the experiments of Paget et al. as corresponding to a non-classical situation characteristic of superconducting thin films. There are, however, some possible handicaps in this point of view. For instance, it is not expected a so expressive change in $\alpha$ for thin films relative to the bulk material. Thus, if $\alpha$ remains close to zero, the critical behavior probed in ref. \cite{6} corresponds in fact to mean-field. Another important point is the possibility of dimensional crossover behavior due to finite size effects arising from the thickness of the film. This situation has been extensively studied in superfluid $^4$He films \cite{23,31}. For superconducting YBCO thin films the situation is unclear because in the optimally doped case the coupling between the CuO planes is strong and the 2D fluctuations are probably dominated by the 3D fluctuations even for a small film thickness. However, in the underdoped cuprates the coupling between the CuO planes is much weaker and we can expect a strong influence of 2D fluctuations for sufficiently small film thickness.

To conclude this section, let us discuss the scaling behavior of the order parameter. This is a controversial matter both from the theoretical and experimental point of view. The theoretical controversy has its origin in the scaling behavior of the correlation function $W_0^{(2)}(x,y) = \langle \phi_0(x)\phi_0^*(y) \rangle$ at large distances, $|x - y| \to \infty$. For $|p| \to 0$ and $m = 0$ its Fourier transform $\tilde{W}_0^{(2)}$ behaves as $\tilde{W}_0^{(2)}(p) \sim |p|^\eta - 2$. Most calculations gives a value of $\eta$ in the range $-1 < \eta < 0$ \cite{9,13,15,16,18}. This does not contradicts the scaling relation $\beta = \nu(1 + \eta)/2$ (for $d = 3$) because $-1 < \eta < 0$ imply $\beta > 0$ as it should be (note that $\nu$ must be positive). However, a negative value of $\eta$ is pathological in many respects. For instance, it has been pointed out by Kiometzis and Schakel \cite{32} that $\eta < 0$ violates the positivity of the spectral weight in the Källen-Lehmann spectral representation of $W_0^{(2)}$ \cite{33}. In fact, this representation of $W_0^{(2)}$ implies that $0 < Z_{\phi} < 1$. Since near the critical point $Z_{\phi} \sim m^\eta$, we have necessarily that $\eta > 0$. Moreover, from the $|p| \to 0$ behavior of $\tilde{W}_0^{(2)}(p)$ we see that $\eta < 0$ makes the low momentum behavior worse than before renormalization \cite{34}.
This is in contradiction with the infrared stability of the fixed point. Another important point is that \( \eta \) is the fixed point value of \( \eta_\phi \), a quantity which is gauge dependent \[14\]. Thus, we can question the physical meaning of the \( \eta \) exponent and, as a consequence, of the order parameter itself. This situation is uncomfortable because in the Gorkov derivation of the GL model from the BCS theory \[33\], \( \phi \) is defined as being proportional to the gap function. In the microscopic theory the gap function has a precise physical meaning since it is responsible for the generation of a gap in the spectrum. The expectation value of \( \phi \) is thus proportional to the gap near the critical temperature. The gauge dependence of the superconducting order parameter has been discussed recently in \[14\] and it has been shown that \( \partial \eta_\phi / \partial \alpha \rightarrow 0 \) as the critical point is approached (\( \alpha \) is the gauge fixing parameter), the only gauge contribution left to \( \eta \) corresponding to the Coulomb gauge, \( \alpha = 0 \). This result agrees with an early analysis by Kennedy and King \[36\], although these authors used a different, but closely related, definition of order parameter. Thus, the gauge dependence is actually not an issue and the point to be solved in the approximations is the negativeness of \( \eta \).

### III. SCALING AND DUALITY

In this section we will study the scaling behavior of superconductors in a continuum dual Ginzburg-Landau (dGL) model. The dGL model has been proposed using plausible arguments on the dynamics of a vortex gas \[11,38,39\] and is assumed to be the continuum version of the dual GL model in the lattice \[37,3\]. Lattice duality studies in abelian gauge models \[8,37,42\] helped condensed matter theorists to obtain important conclusions concerning the superconducting phase transition. In particular, it has been used to establish that a second order phase transition should take place at least in the type II regime \[8\].

#### A. Duality in the lattice GL model

For the sake of clarity, we set up here the arguments given in several papers \[37,38,40,39,12\].

A lattice version of the GL model has a partition function given by

\[
Z(\beta, e) = \int_{-\pi}^{\pi} \left[ \prod_i d\theta_i / 2\pi \right] \int_{-\infty}^{\infty} \left[ \prod_{i,\mu} dA_{i\mu} \right] \exp \left[ \beta \sum_{i,\mu} \cos(\Delta_{i\mu} - eA_{i\mu}) - \frac{1}{2} \sum_i (\Delta \times A_i)^2 \right],
\]

(13)

where \( \Delta_{i\mu} = \theta_{i+\mu} - \theta_i \), that is, \( \Delta_{i\mu} \) is a lattice derivative. Note that in the above partition function the integral over \( A_{i\mu} \) is over the interval \((-\infty, \infty)\), meaning that the gauge group is \( \mathbb{R} \), a non-compact gauge group.

In the Villain approximation \[41\] we can rewrite (13) as

\[
Z(\beta, e) = \int_{-\pi}^{\pi} \left[ \prod_i d\theta_i / 2\pi \right] \int_{-\infty}^{\infty} \left[ \prod_{i,\mu} dA_{i\mu} \right] \sum_{m_{i\mu}} \exp \left[ -\frac{\beta}{2} \sum_{i,\mu} (\Delta_{i\mu} - eA_{i\mu} - 2\pi m_{i\mu})^2 - \frac{1}{2} \sum_i (\Delta \times A_i)^2 \right].
\]

(14)
Let us introduce an auxiliary field $b_{i\mu}$ such that

$$
\exp \left[ -\frac{\beta}{2} (\Delta_\mu \theta_i - eA_{i\mu} - 2\pi m_{i\mu})^2 \right] \propto \int_{-\infty}^{\infty} db_{i\mu} \exp \left[ -\frac{1}{2\beta} b_{i\mu}^2 + i(\Delta_\mu \theta_i - eA_{i\mu} - 2\pi m_{i\mu}) b_{i\mu} \right].
$$

(15)

In the following we will neglect factors of proportionality which are generally smooth functions of $\beta$. Thus, many equations will be assumed up to proportionality factors. By performing the $\theta$ integrals we have

$$
Z(\beta, e) = \int_{-\infty}^{\infty} \prod_{i,\mu} dA_{i\mu} \int_{-\infty}^{\infty} \prod_{i,\mu} db_{i\mu} \delta(\Delta \cdot b_i) \sum_{m_{i\mu}} \left[ \sum_{i\mu} \left( -\frac{1}{2\beta} b_{i\mu}^2 + ieA_{i\mu} b_{i\mu} - 2\pi im_{i\mu} b_{i\mu} \right) - \frac{1}{2} \sum_{i} (\Delta \times A_i)^2 \right].
$$

(16)

Applying the Poisson formula we obtain

$$
Z(\beta, e) = \int_{-\infty}^{\infty} \prod_{i,\mu} dA_{i\mu} \sum_{m_{i\mu}} \delta(\Delta \cdot m_{i\mu,0}) \exp \left[ \sum_{i\mu} \left( -\frac{1}{2\beta} n_{i\mu}^2 + ieA_{i\mu} n_{i\mu} \right) - \frac{1}{2} \sum_{i} (\Delta \times A_i)^2 \right].
$$

(17)

The constraint $\Delta \cdot n_i = 0$ in (17) implies that the links $n_{i\mu}$ form closed loops. These closed loops are interpreted as magnetic vortices [39,37]. It can be shown that the $XY$ model partition function in the Villain approximation (Eq. (14) with $e = 0$) can be cast in the form [37]

$$
Z_{XY}(\beta) = \int_{-\infty}^{\infty} \prod_{i\mu} da_{i\mu} \sum_{M_{i\mu}} \delta(\Delta \cdot M_{i\mu,0}) \exp \left[ \sum_{i\mu} \left( -\frac{1}{2\beta} (\Delta \times a_i)^2 + 2\pi i \sum_{i\mu} a_{i\mu} M_{i\mu} \right) \right],
$$

(18)

and we have $Z(\infty, e) = Z_{XY}(e^2/(2\pi)^2)$. $\beta < \infty$ corresponds to add a chemical potential for the loop variables in the dual $XY$ model. In this sense we can think of the lattice GL model in (14) as a generalized dual $XY$ model. Precisely, we have $Z_{XY}(\beta) = Z'_{XY}(\beta, 0)$ where $Z'_{XY}(\beta, K)$ is the partition function of a generalized dual $XY$ model given by

$$
Z'_{XY}(\beta, K) = \int_{-\infty}^{\infty} \prod_{i\mu} da_{i\mu} \sum_{M_{i\mu}} \delta(\Delta \cdot M_{i\mu,0}) \exp \left[ \sum_{i\mu} \left( -\frac{1}{2\beta} (\Delta \times a_i)^2 + 2\pi i \sum_{i\mu} a_{i\mu} M_{i\mu} - K \sum_{i\mu} M_{i\mu}^2 \right) \right].
$$

(19)

It is possible to study the phase transition of the above dual model by looking the phase diagram in the $\beta - K$ plane [4]. In this phase diagram the point $(\beta_c, 0)$ corresponds to the $XY$ critical point, $\beta_c$ being the inverse critical temperature. From Eq. (14) with $e = 0$ we obtain after integration of the $\theta$ variables

$$
Z_{XY}(\beta) = Z(\beta, 0) = \sum_{m_{i\mu}} \delta(\Delta \cdot M_{i\mu,0}) \exp \left( -\frac{1}{2\beta} \sum_{i\mu} m_{i\mu}^2 \right).
$$

(20)
On the other hand, integration of the $a_{ij\mu}$ in Eq. (19) yields

$$Z'_{XY}(\beta, K) = \sum_{M_{ij\mu}} \delta_{\Delta \cdot M_{ij\mu}} \exp \left[ -2\pi^2 \beta \sum_{i,j,\mu} M_{ij\mu} G(|x_i - x_j|) M_{j\mu} - K \sum_{ij\mu} M_{ij\mu}^2 \right],$$

(21)

where the lattice Green function $G$ behaves like $|x_i - x_j|^{-1}$ for $|x_i - x_j| \to \infty$. From Eqs. (20) and (24) we have that the point $(0, 1/2\beta_c)$ in the $\beta - K$ plane corresponds to the so-called “inverted” $XY$ transition. By doing the $A_{ij\mu}$ integration in (17) we obtain

$$Z(\beta, e) = Z'_{XY} \left( \frac{e^2}{4\pi^2}, \frac{1}{2\beta} \right),$$

(22)

which means that the lattice GL model will undergo an “inverted” $XY$ transition for $0 < e^2 < 4\pi^2 \beta_c$. Let us work out further Eq. (17). Since $\Delta \cdot n_i = 0$ we can put $n_i = \Delta \times l_i$. By using the Poisson formula to introduce a continuum field $h_{ij\mu}$ and integrating out the $A_{ij\mu}$’s, we obtain

$$Z(\beta, e) = \int_{-\infty}^{\infty} \left[ \prod_{i\mu} dh_{i\mu} \right] \sum_{m_{ij\mu}} \delta_{\Delta \cdot m_{ij\mu}} \exp \left\{ \sum_{i\mu} \left[ -\frac{1}{2\beta} (\Delta \times h_i)^2 - \frac{e^2}{2} h_i^2 \right] + 2\pi i \sum_{i\mu} m_{ij\mu} h_{i\mu} \right\}. $$

(23)

Note that putting $e = 0$ in (23) we obtain a partition function identical to (18), as it should be. Reinroducing the variable $\theta$ through

$$\delta_{\Delta \cdot m_{ij\mu}} = \int_{-\pi}^{\pi} \frac{d\theta_i}{2\pi} e^{i\theta_i (\Delta \cdot m_i)},$$

(24)

and using the Poisson formula to convert the integral over $h_{ij\mu}$ in a sum over $n_{ij\mu}$, we obtain

$$Z(\beta, e) = \int_{-\pi}^{\pi} \left[ \prod_i \frac{d\theta_i}{2\pi} \right] \sum_{n_{ij\mu}} \exp \left\{ -\frac{e^2}{8\pi^2} \sum_{ij\mu} (\Delta_{ij\mu} \theta_i - 2\pi n_{ij\mu})^2 - \frac{\beta}{2} \sum_i (\Delta \times n_i)^2 \right\}. $$

(25)

The partition function (23) is the dual representation of (14) used in the numerical simulations in ref. [8]. Note that the dual representation (23) has gauge group $Z$. We observe that the $Z$ given in (14) with $e = 0$ should be the same, up to proportionality constants, as the $Z$ given in (23) with $\beta = 0$ provided we put $\beta = e^2 / 4\pi^2$ in (14). It is clear that the temperature $\beta^{-1}$ plays the role of a dual charge satisfying the Dirac condition $e\beta^{-1} = 2\pi$.

By performing the $h_{ij\mu}$ integration in (23) we obtain an equation analogous to (21) with $K = 0$. The difference is that the Green function $G$ is replaced by a massive Green function $\tilde{G}(|x_i - x_j|)$ which behaves like $e^{-\beta e^2 |x_i - x_j|} / |x_i - x_j|$ for $|x_i - x_j|$ large. Thus, we can write

$$Z(\beta, e) = Z'_{XY} \left( \frac{e^2}{4\pi^2}, \frac{1}{2\beta} \right) = \sum_{m_{ij\mu}} \delta_{\Delta \cdot m_{ij\mu}} \exp \left[ -2\pi^2 \beta \sum_{ij,\mu} m_{ij\mu} \tilde{G}(|x_i - x_j|) m_{j\mu} \right]. $$

(26)

Therefore, adding a chemical potential to the loop variables in the dual $XY$ model is equivalent to replace the massless Green function $G$ by a massive one in a dual $XY$ model with
$K = 0$. In this duality map we have represented the partition function of the lattice GL model in such a way that the loop variables have zero chemical potential. This amounts in replacing the massless Green function by a massive one. This means that if we consider a scaling (continuum) limit of the dual model, it is necessary to consider a massive gauge field $h(x)$. This gauge field should satisfy the constraint $\nabla \cdot h = 0$ and should be coupled minimally to a field $\psi$ such that $|\psi|^2$ represents the density of magnetic vortices.

B. The continuum dual model

The continuum version of lattice duality, the dual Ginzburg-Landau model (dGL) has been proposed on the basis of plausible arguments concerning the dynamics of a vortex gas [11,38,39]. Attempts have been made to justify it as the continuum limit of the lattice dual model [39,12] but it does not exist to date a rigorous mathematical construction of the continuum limit of the lattice dual model. For instance, a possible way towards motivating the dGL model from the lattice dual model has been proposed by Herbut [12]. Instead of combining (23) and (24) to obtain (25), we can follow Peskin [37] and insert in (23)

$$1 = \lim_{t \to 0} \exp \left( -\frac{t}{2} \sum_{ij} m_{ij}^2 \right).$$

By using (24) and the identity

$$\sum_{m=-\infty}^{\infty} e^{-\frac{\pi}{t} m^2 + i \pi m} = \sqrt{\frac{2\pi}{t}} \sum_{M=-\infty}^{\infty} e^{-\frac{\pi}{2t} (x - 2\pi M)^2},$$

we obtain

$$Z(\beta, e) = \lim_{t \to 0} \left( \frac{2\pi}{t} \right)^{N/2} \int_{-\infty}^{\infty} \prod_{ij} dh_{ij} \int_{-\pi}^{\pi} \prod_{i} d\theta_{i} \sum_{m_{ij}} \exp \left[ -\sum_{ij} \frac{1}{2t} \left( \sum_{i} \Theta_{i} - 2\pi h_{ij} - 2\pi M_{ij} \right)^2 - \frac{1}{2\beta} (\Delta \times h_i)^2 - \frac{e^2}{2} h_i^2 \right].$$

The limit $t \to 0$ generates delta functions in the integrand which, when replaced by the integral representation and applying the Poisson formula allows us to recover (23). It has been proposed in [12] that by leaving $t$ finite Eq. (25) is analogous to (14), with the difference that in (25) the lattice gauge field is massive. Moreover, in virtue of (28) $t$ can be interpreted as a chemical potential for the loop variables. Thus, by keeping $t$ small but finite we can write the following continuum limit of (29) in terms of bare quantities:

$$S = \int d^3x \left[ \frac{1}{2} (\nabla \times h_0)^2 + \frac{m_{h,0}^2}{2} h_0^2 + |(\nabla - i m_{h,0} q_0 h_0) \psi_0|^2 + \mu_0^2 |\psi|^2 + \frac{\eta_0}{2} |\psi|^4 \right].$$

In (30), the bare dual charge, $q_0$, is related to the charge $e_0$ by the Dirac condition $q_0 e_0 = 2\pi$. The field $\psi$ is usually called a disorder parameter, as opposed to the order parameter in the GL model. We can of course criticize the motivation of (30) from (29) because it is based
on the hypothesis of a finite, though small $t$ in (29). In fact, (29) represents a lattice GL model only if $t \to 0$. A small nonzero $t$ constitutes an approximation and the continuum limit above should be regarded as an approximate continuum dual model. This should be contrasted with the exact duality map we have obtained in the lattice (up to a Villain approximation). The dGL model given by (30) has been also motivated by arguing directly in the continuum but using the London limit of the GL model, that is, by assuming that $\phi(x) = \tilde{\phi} e^{i\theta}$ with $\tilde{\phi}$ constant [39]. All of this is approximate and we cannot really say that the dual map in the continuum is exact. The construction of continuum limits is a highly non-trivial matter, even for a simple scalar model [33].

Although the dGL model gives a respectable value for the correlation length exponent [11][12], it does not give $\nu' = \nu$ when $m_{h,0}$ is identified with the bare photon mass $m_{A,0}$ of the GL model [11]. Depending on the choice of scaling (to be precised later), we can find a different value of $\nu'$, as will be shown in the next paragraphs. We will consider a family of scalings $\Sigma_\zeta$ where $\zeta \geq 0$ is a parameter. For each $\zeta$ we find a different phase transition regime that must correspond to a possible critical behavior in the GL model. By considering this family of scalings, we will show that it is not possible to describe an “inverted” $XY$ behavior with $m_{h,0} = m_{A,0}$ and, at the same time, $m_{h} = m_{A}$, $m_{h}$ being the renormalized counterpart of $m_{h,0}$.

We define the renormalized fields $h = Z_{h}^{-1/2} h_{0}$ and $\psi = Z_{\psi}^{-1/2} \psi_{0}$. From the Ward identities we obtain that the term $m_{h,0}^{2} h_{0}^{2}/2$ does not renormalize, implying in this way that $m_{h}^{2} = Z_{h} m_{h,0}^{2}$. The renormalization of the remaining parameters follows easily: $\mu^{2} = (Z_{\psi}^{(2)})^{-1} Z_{\psi} \mu_{0}^{2}$, $v = Z_{v}^{-1} Z_{\psi}^{2} \psi_{0}$ and $q = q_{0}$. We note that the dual charge remains bare. Let us define the quantity $g_{0}^{2} = m_{h,0}^{2} \psi_{0}^{2}$ which renormalizes as $g^{2} = Z_{h} g_{0}^{2}$. The dimensionless couplings relevant to the problem are $\hat{g}^{2} = g^{2}/\mu$ and $\hat{v} = v/\mu$. We will perform our RG analysis by introducing a scaling hypothesis for $m_{h,0}^{2}$. We assume that $m_{h,0}^{2} \propto \kappa^{\zeta}$, where $\zeta \geq 0$. This scaling hypothesis will define under a RG a family of scalings $\Sigma_\zeta$. It is useful to define the ratio $\kappa_{d} = \mu/m_{h}$, analogous to the Ginzburg parameter $\kappa$ in the GL model. For arbitrary $\zeta \geq 0$ we obtain the flow equations:

$$\mu \frac{\partial \kappa_{d}^{2}}{\partial \mu} = [2 - \eta_{h} - \zeta(2 + \eta_{\psi}^{(2)} - \eta_{\psi})] \kappa_{d}^{2},$$

$$\mu \frac{\partial m_{h}^{2}}{\partial \mu} = [\eta_{h} + \zeta(2 + \eta_{\psi}^{(2)} - \eta_{\psi})] m_{h}^{2},$$

$$\mu \frac{\partial \hat{g}^{2}}{\partial \mu} = [\eta_{h} - 1 + \zeta(2 + \eta_{\psi}^{(2)} - \eta_{\psi})] \hat{g}^{2},$$

where the RG functions $\eta_{\psi}^{(2)}$, $\eta_{\psi}$ and $\eta_{h}$ are defined by

$$\eta_{\psi}^{(2)} = \mu \frac{\partial \ln Z_{\psi}^{(2)}}{\partial \mu},$$

$$\eta_{\psi} = \mu \frac{\partial \ln Z_{\psi}}{\partial \mu},$$

and
\[ \eta_h = \mu \frac{\partial \ln Z_h}{\partial \mu}. \]  

Near the phase transition \( \mu = \xi^{-1} \sim t^\nu \). As \( \zeta \) varies, the above equations describe different regimes of phase transitions in the case where an infrared stable fixed point exists. Let us analyse some relevant cases.

1. \( \Sigma_0 \) scaling

For \( \zeta = 0 \) we have that the infrared stable fixed point for Eq. (33) is a non-vanishing \( \hat{g}_s^2 \) satisfying \( \eta_h \equiv \eta_h(\hat{g}_s^2, \hat{\nu}_s) = 1, \beta_\nu(\hat{g}_s^2, \hat{\nu}_s) = 0 \) with \( \hat{\nu}_s \) being the fixed point value of the coupling \( \hat{\nu} \). From Eq. (32) we obtain that near the phase transition \( m_h \sim \mu^{1/2} \). By defining the exponent of \( m_h \) through \( m_h \sim t^{\nu_h} \), we obtain that \( \nu_h = \nu / 2 \). This suggests that \( \nu \rightarrow 0 \) as approaching the critical point. This is the case indeed, since from Eq. (31) we obtain that near the infrared stable fixed point \( \kappa_d \sim \mu^{1/2} \). From this behavior of \( \kappa_d \) we obtain that the fixed point value \( \hat{\nu}_s \) corresponds to the value in a \( XY \) model and \( \nu = \nu_{XY} \approx 2 / 3 \). The reason for this behavior comes from the fact that every power of the dual charge in \( \beta_\nu \) is multiplied by a function of \( \kappa_d \) which tends to zero as the critical point is approached. The same type of behavior has been already encountered in the literature [12,14]. If we identify \( m_h \) with the renormalized photon mass \( m_A \) we obtain \( \nu' = \nu_{XY} / 2 \), the half of the value expected for the superconducting transition in the “inverted” \( XY \) universality class [10]. However, this result corresponds to the universality class of the crossover regime governed by the neutral \( XY \) fixed point [4], which is the only one we have experimental access [13].

2. \( \Sigma_1 \) scaling

We consider now the scaling \( \Sigma_1 \) corresponding to \( \zeta = 1 \). In this case we obtain that the infrared stable fixed point corresponds to \( \hat{g}_s^2 = 0 \). The present situation matches with the one encountered in [11] where \( m_{h,0} \) is identified with the bare photon mass \( m_{A,0} \). We have still that \( \nu = \nu_{XY} \). However, if \( m_h = m_A \) we obtain that \( \nu_h = \nu' = 1 / 2 \) and the penetration depth exponent is classical. As we have already discussed in section II, the value \( \nu' = 1 / 2 \) has been measured by Lin et al. [7] and in a recent paper by Paget et al. [6].

3. \( \Sigma_\zeta \) scaling with \( \zeta \approx 4/3 \)

Clearly the scalings \( \Sigma_0 \) and \( \Sigma_1 \) of the dGL model are not in the “inverted” \( XY \) universality class but belong to different crossover regimes in the GL model. In order to have an “inverted” \( XY \) behavior we need to obtain \( \nu_h = \nu' = \nu_{XY} \), after identifying \( m_h \) with the renormalized photon mass in the GL model. It is easy to check that this is in fact the case for \( \zeta \approx 4/3 \), but in this case \( m_{h,0} \) clearly does not corresponds to the bare photon mass. Rather, \( m_{h,0} \) behaves like the renormalized photon mass. For \( \zeta \approx 4/3 \), the infrared stable fixed point corresponds to a neutral dual charge just as in the scaling \( \Sigma_1 \) and from Eq. (32) we obtain \( m_h \sim \mu \) near the fixed point. Note that in contrast with the other two cases, now we have that \( \kappa_d \) approaches a non-vanishing fixed point value in the infrared.
IV. CONCLUDING REMARKS

We have discussed in this paper some relevant aspects of the scaling behavior of superconductors in a zero magnetic field Meissner phase. We have obtained in the first part of the paper the scaling relation $\nu' = \nu$ as a consequence of an exact flow equation for the photon mass. The interesting point of our derivation is that since no use has been done of the Josephson relation, this relation is obtained directly in the form $\rho_s \sim t^\nu$. We argued that this implies the validity of the hyperscaling relation for a charged superfluid since the Josephson relation can be derived independently in the form $\rho_s \sim t^{2\beta-\nu\eta}$. In this part some emphasis has been given to the relation between theory and experiment, particularly in what concerns the usefulness of the Josephson relation in determining the exponent $\nu$. While this is not new, it helped us to propose an alternative interpretation of a recent experiment performed by Paget et al. [6] in YBCO thin films. We proposed that the correlation length and specific heat exponents are possibly non-classical and given respectively by $\nu = 1$ and $\alpha = -1$. In order to confirm this it is necessary to measure $\alpha$ directly in thin films, which is a very difficult task. We are aware however of the little probability of changing so dramatically the value of $\alpha$ with respect to the bulk sample value. Anyway, such a possibility cannot be completely discarded and we hope that some experimental effort could be made in this sense.

In the second part of the paper we tried to elucidate the scaling behavior of the continuum dual Ginzburg-Landau model. This has been done by the introduction of a scaling hypothesis on the bare magnetic induction mass. Our arguments in this part of the paper show that the continuum dual model still deserves more reflexion. We think that the dual map in the continuum is not as complete as it is in the lattice. We emphasize that there is no rigorous argument in favor of the usual continuum model [3]. A future theoretical perspective is to study different scalings in another possible choice for a continuum dual model. It consists of a continuum $XY$ model where a perturbatively non-renormalizable interaction of the form $(\psi^\dagger \nabla \psi - \psi \nabla \psi^\dagger)^2$ [3,2] is added. It is possible that such an interaction can be non-perturbatively renormalizable, like the situation encountered in the Gross-Neveu model in $d = 3$ [3] where the non-perturbative renormalizability has been rigorously demonstrated [4].

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