On the importance of testing gravity at distances less than 1cm

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ABSTRACT

If the mechanism responsible for the smallness of the vacuum energy is consistent with local quantum field theory, general arguments suggest the existence of at least one unobserved scalar particle with Compton wavelength bounded from below by one tenth of a millimeter. We show that this bound is saturated if vacuum energy is a substantial component of the energy density of the universe. Therefore, the success of cosmological models with a significant vacuum energy component suggests the existence of new macroscopic forces with range in the sub-millimeter region. There are virtually no experimental constraints on the existence of quanta with this range of interaction.

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There are no significant experimental constraints on new gravitational-strength forces at distances shorter than 1 cm. The present situation is illustrated in Figure 1: a plot of wavelength ($\lambda$) vs. the strength of a weak Yukawa force relative to gravity ($\alpha$) [1]. In order to identify the characteristic scale in question, note that the dashed vertical lines cutting through the figure correspond to the Compton wavelength associated with the critical energy density of the universe. The critical energy density of the universe is given by $\rho_C = \frac{3H_0^2}{8\pi G_N} = (3.0\sqrt{h} \times 10^{-3} \text{ eV})^4$, where Hubble’s constant, $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$, is consistent with observations if $h = 0.4 - 0.9$. *A priori* the overlap of this characteristic scale with the experimentally unexplored region in Figure 1 is not particularly interesting. However, in this essay we will argue that if a substantial fraction of the energy density of the universe is in the form of vacuum energy, then the overlap of the dashed lines with the experimentally unexplored region of Figure 1 becomes a coincidence of fundamental interest. Our argument is based on two assumptions:

(A) *Local quantum field theory always works.*

(B) *In the present epoch a substantial component of the energy density of the universe is vacuum energy.*

Several comments are in order. Assumption (A) is simply the statement that *all* experimentally accessible physical phenomena can be described using effective quantum field theories [2]. The motivation for (B) is well known; cosmological models with $\rho_{\text{VAC}} \sim \rho_C$ have many attractive features [3]. In particular, cold dark matter models with $\Omega_\Lambda = \rho_{\text{VAC}}/\rho_C \simeq 0.65$ ($\Omega_{\text{TOT}} = 1$) are consistent with a wide variety of observations; for example, vacuum energy of this magnitude settles the age paradox and allows for a flat universe without contradicting measures of matter density [3]. We will argue that if (A) is not violated, then the smallness of the cosmological constant implies the existence of quanta not yet seen experimentally [4]. This argument—which we call the Banks-Susskind theorem—is powerful since there is *no evidence whatsoever* that (A) is violated in nature. We will argue on the basis of this theorem that if (B) is true, then the range of these unobserved quanta should fall in the experimentally unexplored region of Figure 1.
Effective quantum field theories—the current paradigm in particle physics—imply the existence of many disparate contributions to the vacuum energy—from zero-point fluctuations of the electromagnetic field to non-perturbative phenomena like spontaneous breaking of chiral symmetry in quantum chromodynamics. Here we will be concerned with the vacuum energy which is relevant at late times and large distance scales and which therefore plays a part in the evolution of the universe in the present epoch. This vacuum energy is constrained experimentally and can be defined in the context of the relevant effective quantum field theory. Since the relevant scales are macroscopic, the effective theory we are interested in has all massive particles in nature integrated out (see Figure 2). The lightest massive particle is the electron, and so the cutoff of the effective theory can be taken as the electron mass. This effective theory encodes the interactions of photons, gravitons and neutrinos in a manner consistent with the assumed symmetries. Here for simplicity we will neglect the neutrino and photon interactions—except for their possible contributions to the vacuum energy.

The action of gravity is determined by invariance under general coordinate transformations:

\[ S[g] = \int d^4x \sqrt{|g|} \left[ -\Lambda - \frac{M_p^2}{16\pi} R + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} + O(\partial^6) \right], \]  

(1)

where \( g \equiv -\det g_{\mu\nu} \), \( R_{\mu\nu} \) is the Ricci tensor, and \( R \) is the curvature scalar. The last term denotes invariant contributions with six or more derivatives of the metric field. Experiment determines \( M_p \simeq 10^{19}\)GeV. There are experimental bounds on \( \alpha \) and \( \beta \): \( |\alpha|, |\beta| \leq 10^{74} \). These bounds are weak because these terms are suppressed by powers of \( q/M_p \) where \( q \) is a characteristic momentum in the low-energy effective theory. A rough observational bound on \( \Lambda \), the vacuum energy or cosmological constant, is

\[ |\Lambda| \leq \rho_c \simeq 10^{-47}\text{GeV}^4. \]  

(2)

The cosmological constant problem is the simple fact that \( \Lambda \) is much smaller than any characteristic mass scale in elementary particle physics, whereas basic field theory dimensional analysis does not rule out \( \Lambda \sim M_p^4 \simeq 10^{74}\text{GeV}^4 \). Perhaps more discouraging than
the size of the discrepancy is the number of disparate contributions which evidently sum to a small number.

What about the value of $\Lambda$ in the macroscopic effective theory? Consider the ladder of effective field theories illustrated in Figure 2. Each rung of the ladder corresponds to a threshold at which a massive particle is integrated out or some non-perturbative phenomenon takes place. In the macroscopic effective theory (region III) we can write the total cosmological constant schematically as

$$\Lambda = \Lambda^{\text{III}} + \Lambda^{\text{II}} + \Lambda^{\text{I}} + \ldots$$

(3)

Since there is no symmetry to forbid it, we would expect that zero-point fluctuations of quantum fields in each effective theory give a contribution to $\Lambda$ of order the momentum cutoff. On purely dimensional grounds —ignoring geometrical factors of 2 and $\pi$— we expect

$$\Lambda^{\text{III}} \sim m_e^4 \simeq 10^{-13}\text{GeV}^4,$$

(4)

which is 32 orders of magnitude larger than the upper bound, and

$$\Lambda^{\text{II}} \sim m_\mu^4 \simeq 10^{-4}\text{GeV}^4,$$

(5)

which is 43 orders of magnitude larger than the upper bound. Even if we assume that the effective cosmological constant in region I vanishes, we have a severe cosmological constant problem in the macroscopic effective theory. We can make $\Lambda^{\text{II}}$ and $\Lambda^{\text{III}}$ cancel by introducing an arbitrary coefficient tuned to one place in a billion. One might think that a symmetry is capable of explaining this sort of correlation. However, symmetry generators which act locally on the fields carry no energy and momentum and cannot relate the vacuum energy associated with distinct effective field theories [4]. Evidently, the only way around this impasse which is consistent with local quantum field theory is to acknowledge the existence of quanta which have not been seen experimentally, and which have therefore been inadvertently left out of the effective theory description. This is the essence of the Banks-Susskind theorem [4]. We denote this field or fields collectively as $\phi$. Presumably $\phi$ carries vacuum quantum numbers as it must by some means act as a
source of the energy-momentum tensor in the vacuum \cite{3}. It is important to stress that no satisfactory mechanism involving $\phi$ has been found \cite{3}. Here we have argued that if this unknown mechanism is consistent with local quantum field theory, then $\phi$ should be a fundamental ingredient.

In this picture the cosmological constant relevant to cosmology is an effect arising from the decoupling of $\phi$. That is, at distance scales shorter than $\phi$’s Compton wavelength, the dynamics of $\phi$, by assumption, ensure a vanishing cosmological constant. On the other hand, when probing distances greater than $\phi$’s Compton wavelength, $\phi$ gets integrated out and there is no longer a mechanism to prevent gravitational and electromagnetic fluctuations in the vacuum. Hence at these distance scales we expect

$$\Lambda \sim m_{\phi}^4,$$

which together with Eq. (2) implies that $m_{\phi} \leq 3.0\sqrt{h} \times 10^{-3}$ eV, with associated Compton wavelength $\lambda_{\phi} \geq 6.6(h)^{-\frac{1}{2}} \times 10^{-5}$ m. There is an interesting consequence of this scenario for $\Lambda$. The only strictly massless particles in nature are associated with gauge invariance or general coordinate invariance and therefore transform as vectors and tensors under the Lorentz group. There are no strictly massless scalars. The only natural light scalars are Goldstone bosons, which arise from the spontaneous breakdown of global symmetries. Such symmetries are not exact in nature, and so in this conservative picture $\Lambda$ necessarily takes a non-zero value.

On the basis of assumption (B) we give the conservative lower bound $|\Lambda| \geq (0.1)\rho_C$, which in turn bounds the mass of $\phi$:

$$1.7\sqrt{h} \times 10^{-3} \text{ eV} \leq m_{\phi} \leq 3.0\sqrt{h} \times 10^{-3} \text{ eV}. \quad (7)$$

This places $\phi$’s range around the dashed lines in the experimentally unexplored region of the parameter space illustrated in Figure 1. Of course the coupling strength of $\phi$ to matter is also a relevant parameter. In order that $\phi$ eliminate vacuum fluctuations up to the Planck scale, $\phi$ must originate at the Planck scale. Hence it is natural that $\phi$ couple weakly to matter; if we assume the simple Yukawa form:
where $N$ is the nucleon field, we obtain $g^2/4\pi = \alpha$, where $\alpha$ is the usual coupling parameter \cite{Ref. 7}. On the basis of naive dimensional analysis $g$ — and therefore $\alpha$ — is expected to be of order one. However, the coupling strength can vary substantially with detailed dynamical assumptions \cite{Ref. 7}.

What is the present status of experimental searches for new gravitational strength forces in the sub-cm region? Existing limits are illustrated in Figure 1. The curve labelled Sparnaay is deduced from a classic electromagnetic Casimir force measurement \cite{Ref. 8}. This experiment measured the attractive force between parallel plates at separations of roughly $10^{-1} \mu m$ to $10 \mu m$. Bounds on $\alpha$ were extracted from Sparnaay data by conjecturing a force due to a Yukawa interaction between parallel plates (see Ref. 1). The curve labelled Hoskins et. al. is deduced from the Cavendish-type experiment of Ref. 9. This experiment searched for deviations from the inverse-square law in the $2 - 5 \text{ cm}$ region. The bounds on $\alpha$ are an extrapolation of these results to shorter distances. It is clear that there are no significant bounds in the sub-cm region.

Cryogenic mechanical oscillator techniques have been proposed \cite{Ref. 1} which would improve existing limits on the strength of a Yukawa force with a range of $100 \mu m$ by up to $10^{10}$. This is precisely the range which our theoretical argument finds most interesting. The dotted curve in Figure 1 indicates the sensitivity of this experiment. The important background effects are due to: vibrations generated by the motion of the source mass, Newtonian background due to edge effects and geometry defects, and magnetic and electrostatic forces \cite{Ref. 1}. The analysis leading to the dotted curve is given in Ref. 1. There also exists independent theoretical motivation for probing this region; it has recently been argued \cite{Ref. 7} that masses and couplings of scalar fields which arise in certain classes of supersymmetric theories fall naturally into the region of parameter space accessible to the cryogenic oscillator.

In summary, if local field theory always works, the observational fact that the cosmological constant is small implies the existence of quanta that have not been observed. This
is probably the most conservative statement one can make about the cosmological constant problem. We have argued on the basis of this “theorem” that a small non-vanishing cosmological constant of order the critical energy density of the universe suggests the existence of new macroscopic forces in the 100 $\mu$m region. On the basis of general physical principles and established cosmological observations, we hope to have convinced the reader that experimental tests of the inverse-square law in the sub-cm range should be vigorously pursued.

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Figure 1: Wavelength ($\lambda$) vs. the strength of a weak Yukawa force relative to gravity ($\alpha$). The region above the solid line is excluded by Sparnaay (electromagnetic Casimir force measurements [8]) and by Hoskins et al (Cavendish-type experiment [9]). The region between the solid line and the dotted line is accessible to the cryogenic mechanical oscillator [1]. The region below the dotted line is inaccessible due to Newtonian and electrostatic backgrounds. The dashed lines represent the Compton wavelength associated with the critical energy density of the universe for $h = 0.4$ (right) and $h = 0.9$ (left).
Figure 2: The ladder of effective field theories. All massive particles in nature are represented by a rung on the ladder. The chiral symmetry breaking scale, $\Lambda_\chi$, is an example of a rung on the ladder arising from non-perturbative phenomena. Here we are interested in the lowest rungs. Region III is the effective theory with all massive particles integrated out and is therefore relevant to macroscopic phenomena.