On Dedicated CDCL Strategies for PB Solvers

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Abstract

Current implementations of pseudo-Boolean (PB) solvers working on native PB constraints are based on the CDCL architecture which empowers highly efficient modern SAT solvers. In particular, such PB solvers not only implement a (cutting-planes-based) conflict analysis procedure, but also complementary strategies for components that are crucial for the efficiency of CDCL, namely branching heuristics, learned constraint deletion and restarts. However, these strategies are mostly reused by PB solvers without considering the particular form of the PB constraints they deal with. In this paper, we present and evaluate different ways of adapting CDCL strategies to take the specificities of PB constraints into account while preserving the behavior they have in the clausal setting. We implemented these strategies in two different solvers, namely Sat4j (for which we consider three configurations) and RoundingSat. Our experiments show that these dedicated strategies allow to improve, sometimes significantly, the performance of these solvers, both on decision and optimization problems.

1 Introduction

The success of so-called modern SAT solvers has motivated the generalization of the conflict-driven clause learning (CDCL) architecture \cite{MSS99,MMZ01,ES04} to solve pseudo-Boolean (PB) problems \cite{RM09}. The main motivation behind the development of PB solvers is that classical SAT solvers are based

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on the resolution proof system, which is a weak proof system: instances that are hard for resolution (for instance those requiring counting capabilities, such as pigeonhole principle formulae \cite{Hak85} are hard for SAT solvers. A stronger alternative is the cutting planes proof system \cite{Gom58,Hoo88,Nor15}, which allows, for instance, to solve pigeonhole principle formulae with a linear number of derivation steps. Generally speaking, this proof system p-simulates resolution: any resolution proof can be simulated by a polynomial size cutting planes proof \cite{CCT87}. In theory, PB solvers should thus be able to find shorter unsatisfiability proofs, and thus be more efficient than classical SAT solvers. In practice however, current PB solvers fail to keep the promises of the theory. In particular, most PB solvers \cite{DG02,CK05,SS06,LBP10} implement a subset of the cutting planes proof system known as generalized resolution \cite{Hoo88}. This subset is convenient as it allows to extend the CDCL algorithm to PB constraints. As soon as a constraint becomes conflicting, the generalized resolution rule is applied between this constraint and the reason for the propagation of one of its literals to derive a new conflicting constraint. This operation is repeated until an assertive constraint is eventually derived. However, solvers implementing this procedure do not exploit the full power of the cutting planes proof system \cite{VEG+18}, and are still behind resolution-based solvers in PB competitions \cite{Rou20}.

Despite the recent improvements brought by RoundingSat \cite{EN18} with the use of the division rule during conflict analysis, current implementations of cutting planes still have a critical drawback: they degenerate to resolution when given a CNF as input. Moreover, such implementations are more complex than just replacing resolution during conflict analysis by generalized resolution: finding which rules to apply and when is not that obvious \cite{GNY19,LMW20}. In particular, PB solvers need to take care about the specific properties of PB constraints and of the cutting planes proof system to fit in the CDCL architecture. Additionally, CDCL comes with many other features, without which the performance of the solver may become very bad (see, e.g., \cite{EGCG+18}). To the best of our knowledge, little work has been done on extending these components for PB solvers: they are mostly reused from their definition in classical SAT solvers, and adapted just enough to work in the solver, without considering their effective impact in the context of PB solving. In this paper, we focus on such features, namely branching heuristics, learned constraint deletion strategies and restart schemes. We implemented different new strategies for these features, designed to consider the characteristics of PB constraints. Our experiments show that they allow to improve, sometimes significantly, the performance of different PB solvers, both on decision and optimization instances.

## 2 Preliminaries

We consider a propositional setting defined on a finite set of propositional variables $\mathcal{V}$. A literal $\ell$ is a variable $v \in \mathcal{V}$ or its negation $\bar{v}$. Boolean values are represented by the integers 1 (true) and 0 (false), so that $\bar{v} = 1 - v$. 
A pseudo-Boolean (PB) constraint is an integral linear equation or inequality over Boolean variables of the form \( \sum_{i=1}^{n} \alpha_i \ell_i \triangleq \delta \), in which the coefficients \( \alpha_i \) and the degree \( \delta \) are integers, \( \ell_i \) are literals and \( \triangle \in \{<, \leq, =, \geq, >\} \). Such a constraint can be normalized in linear time into a conjunction of constraints of the form \( \sum_{i=1}^{n} \alpha_i \ell_i \geq \delta \) in which the coefficients and the degree are all positive integers. In the following, we thus assume that all PB constraints are normalized. A cardinality constraint is a PB constraint in which all coefficients are equal to 1 and a clause is a cardinality constraint of degree 1. This definition illustrates that PB constraints are a generalization of clauses, and that clausal reasoning is thus a special case of PB reasoning.

PB solvers have thus been designed to extend the CDCL algorithm of classical SAT solvers. In particular, when looking for a solution, PB solvers have to assign variables. In the following, we use the notation \( \ell(V@D) \) to represent that literal \( \ell \) has been assigned value \( V \) at decision level \( D \), and \( \ell(?@?) \) to represent that \( \ell \) is unassigned. Assigning variables is achieved either by making a decision or by propagating a truth value for a variable. In this context, the normalized form of PB constraints is particularly useful for detecting propagations: as for clauses, propagations are triggered after the falsification of some literals in the constraint. However, contrary to clauses, a PB constraint may propagate a literal even if some other literals in this constraint are unassigned or satisfied, as shown in the following example.

**Example 1** The PB constraint \( 5a(0@3)+5b(?@?)+c(?@?)+d(?@?)+e(0@1)+f(1@2) \geq 6 \) propagates the literal \( b \) under the current partial assignment. If \( b \) is assigned to 0, giving \( 5a(0@3)+5b(0@3)+c(?@?)+d(?@?)+e(0@1)+f(1@2) \geq 6 \), the constraint becomes conflicting. In both cases, observe that \( f \) is satisfied and \( c \) and \( d \) are unassigned.

After propagations are triggered, it may happen that a constraint becomes conflicting. When this is the case, PB solvers perform a conflict analysis similar to that of SAT solvers, and successively apply the cancellation rule between the conflicting constraint and the reason for the propagation of some of its literals, so as to eliminate these literals. However, doing so does not guarantee to preserve the conflict, and several approaches based on the (partial) weakening rule have been introduced \([\text{Dix04, EN18, LMW20}]\) to provide such a guarantee, by (locally) assuming that some literals are assigned to 1. Some solvers such as Sat4j-GeneralizedResolution \([\text{LBP10}]\) apply this rule iteratively until the conflict is guaranteed to be preserved, while others such as RoundingSat \([\text{EN18}]\), Sat4j-RoundingSat and Sat4j-PartialRoundingSat \([\text{LMW20}]\) apply it on all literals that are not falsified and not divisible by the coefficient of the literals to eliminate, before applying the division rule.

### 3 Branching Heuristics

An important component in a SAT solver is its branching heuristic: to find efficiently a solution or an unsatisfiability proof, the solver has to choose the
right variables on which to make decisions. Currently, most SAT solvers rely on VSIDS [MMZ+01] or one of its variants [BF15], or the more recent LRB [LGPC16]. We focus on the former, as it is the one adopted by the native PB solvers we considered.

The most popular variant of VSIDS is exponential VSIDS (EVSIDS), introduced in MiniSat [ES04]. In this heuristic, a value $g$ is chosen between $1.01$ and $1.2$ at the beginning of the execution of the solver. When a variable is encountered during the analysis of the $i$-th conflict, this variable is bumped, i.e., its score is updated by adding $g^i$ to its current score. When it comes to selecting a variable, the solver chooses the variable with the highest score. We remark that, as the original VSIDS, EVSIDS is designed to favor variables appearing in recent conflicts. Moreover, modern implementations of VSIDS not only update the score of variables appearing in the learned clauses, but also that of variables appearing in all clauses used to produce them. This approach aims to favor the selection of variables that are involved in recent conflicts.

### 3.1 VSIDS in PB Solvers

Current PB solvers rely thus on the VSIDS heuristic (or one of its variants) to decide which variable should be assigned next. In practice, this heuristic may be used as is by PB solvers, even though doing so does not allow to take into account all the information given by a PB constraint, as observed in [CK05] (which, however, does not explicitly provide a more suitable heuristic). This is why different variants of this heuristic have been proposed. In [Dix04, Section 4.5], it is proposed to add, for each variable appearing in a cardinality constraint of the original problem (i.e., not for learned constraints) the degree of this constraint to the initial score of the corresponding variables. This approach actually counts the occurrences of the variable in the clauses that are represented by the cardinality constraint.

**Example 2 (from [Dix04, Section 4.5])** If the cardinality constraint $a + b + c \geq 2$ is present in the original constraint database, the score of each of its variables is increased by 2. Indeed, this constraint is equivalent to the conjunction of the clauses $a + b \geq 1$, $a + c \geq 1$ and $b + c \geq 1$. If this constraint is learned, the corresponding scores are only increased by 1.

Despite providing a more specific heuristic than the original VSIDS heuristic when considering PB problems, this heuristic is not completely satisfactory, as it does not fit well in modern implementations of VSIDS, and especially of EVSIDS. First, as only the original constraints are considered, the heuristic does not bring any improvement over the classical implementation of the heuristic, which essentially relies on the bumping of variables involved in recent conflicts. Second, the particular form of general PB constraints is not taken into account by this heuristic. The main reason for only considering cardinality constraints in this case is that computing the number of clauses in which a literal of a PB constraint appears is hard in general. Another alternative,
implemented in Pueblo [SS06], is estimating the relative importance of a literal in a constraint, by computing the ratio of its coefficient by the degree of the constraint. This value is then added to the VSIDS score of the variable. On the contrary, Sat4j [LBP10] and RoundingSat [EN18] both implement a more classical EVSIDS heuristic, by bumping each variable encountered during conflict analysis. However, some implementation details are worth noting for these two solvers. In particular, Sat4j bumps these variables each time they appear in a reason, while RoundingSat bumps them only once (as in MiniSat [ES04]), except if the variable is eliminated during conflict analysis, in which case it is bumped twice.

3.2 Towards Better VSIDS for PB Solvers

As mentioned above, current implementations of the VSIDS heuristic in SAT solvers, and in particular the EVSIDS heuristic, are designed to favor the selection of variables that are involved in recent conflicts. When only considering clauses, identifying such literals is straightforward: the literals involved in a conflict are those appearing in the clauses encountered during conflict analysis. However, this is no longer the case when PB constraints are considered. Indeed, given a PB constraint, the literals it contains may not play the same role in the constraint, and thus may not have the same influence in the conflicts in which this constraint is involved. In order to take into account this asymmetry between the literals when computing VSIDS scores, we introduce different ways of bumping the variables appearing in the constraints encountered during conflict analysis. The main reason for the asymmetry of the literals in a PB constraint is the presence of coefficients in the constraint. To take these literals into account, we generalize the heuristics proposed in the PB solvers pbChaff [DG02, Dix04, Section 4.5] and Pueblo [SS06] by defining the following bumping strategies:

- The bump-degree strategy multiplies the increment by the degree of the constraint, as a naive generalization of pbChaff’s approach, which only considers the degree of the original cardinality constraints.
- The bump-coefficient strategy multiplies the increment by the coefficient of the literal being bumped, as a tentative measure of the importance of the corresponding variable.
- The bump-ratio-coefficient-degree strategy multiplies the increment by the ratio of the coefficient of the literal by the degree of the constraint, as proposed in Pueblo.
- The bump-ratio-degree-coefficient strategy multiplies the increment by the ratio of the degree of the constraint by the coefficient of the literal, as a generalization of pbChaff’s strategy taking into account the relative importance of the variable in the constraint.

Let us illustrate these different strategies by the following example.
Example 3 When bumping the variable $a$ from the constraint $5a + 5b + c + d + e + f \geq 6$, the increment is multiplied by:

- 6 in the case of bump-degree,
- 5 in the case of bump-coefficient,
- $5/6$ in the case of bump-ratio-coefficient-degree (as in Pueblo), and
- $6/5$ in the case of bump-ratio-degree-coefficient

before being added to the variable’s score.

Another key observation to take into account to detect literals that are actually involved in a conflict is to consider the impact of the current assignment. Indeed, in classical SAT solvers, all variables appearing in the clauses encountered during conflict analysis are always assigned, and all but one are actually falsified. However, in PB constraints, this is not always the case (see Example 1), and falsified literals may even be ineffective [LMW20, Section 3.1].

Definition 1 (Effective Literal) Given a conflicting (resp. assertive) PB constraint $\chi$, a literal $\ell$ of $\chi$ is said to be effective in $\chi$ if it is falsified and satisfying it would not preserve the conflict (resp. propagation). We say that $\ell$ is ineffective when it is not effective.

Remark 1 To identify ineffective literals in a constraint, we use a greedy algorithm that works as follows. The literals of the constraint are successively (and implicitly) weakened away, and only those for which the weakening does not preserve the conflict (resp. propagations) are kept. This operation, yields an (implicit) clause that is both implied by the constraint and conflicting (resp. assertive). Its literals are those considered as effective. Note that this approach is similar to that used by SATIRE [WKS01] or Sat4j-Resolution [LBP10] to derive clauses during conflict analysis.

Even though they may be encountered during conflict analysis, ineffective literals do not play any role in the conflict, and neither do the corresponding variables. We thus introduce three other bumping strategies taking into account the current assignment:

- The bump-assigned strategy bumps only assigned variables appearing in the constraints encountered during conflict analysis.
- The bump-falsified strategy bumps only variables whose literals appear as falsified in the constraints encountered during conflict analysis.
- The bump-effective strategy bumps only variables whose literals are effective in the constraints encountered during conflict analysis.
Example 4 When bumping the variables of the constraint $5a(0@3) + 5b(1@3) + c(?@?) + d(?@?) + e(0@1) + f(1@2) \geq 6$,

- the strategy bump-assigned bumps the variables $a$, $b$, $e$ and $f$,
- the strategy bump-falsified bumps the variables $a$ and $e$, and
- the strategy bump-effective bumps only the variable $a$.

4 Learned Constraint Deletion

PB solvers, similarly to SAT solvers, need to regularly delete learned constraints during their execution. Indeed, storing these constraints may not only increase the memory required by the solver, but may also slow down unit propagation. In this context, the key element is to detect which constraints to remove. In PB solvers, this feature is mostly inherited directly from SAT solvers. For instance, Pueblo \cite{SS06} uses MiniSat’s learned constraint deletion, based on the activity of learned constraints (the less active constraints are removed first), Sat4j \cite{LBP10} uses also an activity-based strategy but more aggressively as in Glucose \cite{AS09}, while RoundingSat \cite{EN18} considers a custom hybrid approach, based on both the LBD and the activity measures (the latter is used as a tie-break rule when the former gives identical measures). In other PB solvers, such as pbChaff \cite{DG02} and Galena \cite{CK05}, the learned constraint deletion in use (if any) is not documented. In \cite{CK05}, a perspective is however mentioned to weaken learned constraints instead of removing them. However, note that while measures such as those based on the activity may be reused as they are by PB solvers (they do not take into account the representation nor the semantics of the constraints they evaluate), for other evaluation schemes, paying attention to the particular form of PB constraints may be more relevant to properly evaluate the quality of the constraints. This section focuses on two main approaches towards this direction.

4.1 Size-Based Measures

In classical SAT solvers, size-based measures delete the largest clauses in the database, i.e., those containing many literals. The intuition behind this evaluation scheme is that large clauses are weak, especially from a propagation viewpoint: a propagation can only be triggered after many literals have become falsified. When considering PB constraints, this is not the case anymore. Indeed, recall that PB constraints may propagate literals while some other literals remain unassigned, and that the number of literals in a PB constraint does not necessarily reflect its strength.

Another reason that motivated the use of size-based measures in SAT solving is that large clauses are expensive to handle, which is also true for PB constraints. In particular, in such constraints, the size also takes into account the size of the coefficients, which is not negligible: as coefficients may become
very large during conflict analysis, arbitrary precision encoding is required to represent these coefficients. As we already discussed, this representation slows down arithmetic operations, and thus the conflict analysis performed by the solver. Different approaches have been studied to limit the growth of the coefficient, such as those based on the division \cite{EN18} or the weakening \cite{LMW20} rules. However, these approaches lead to the inference of weaker constraints. By using a quality measure that takes into account the size of the coefficients, we can favor the learning of constraints with “small” coefficients. Towards this direction, we introduce quality measures based on the degree of the learned constraints, as described below:

- The *degree* quality measure evaluates the quality of a learned constraint by the *value* of its degree.
- The *degree-bits* quality measure evaluates the quality of a learned constraint by the *minimum number of bits* required to represents its degree.

In both cases, the smaller the degree, the better the constraint. Indeed, it is well-known that the degree of a PB constraint can be used as an upper bound of the coefficients of the constraints (because of the saturation rule), so that considering only the degree is enough for the purpose of this measure.

**Example 5** The degree-based quality measures for the constraint $5a + 5b + c + d + e + f \geq 6$ are:

- 6 in the case of degree, and
- 3 in the case of degree-bits (as the binary representation of 6, i.e., 110, needs 3 bits).

### 4.2 LBD-Based Measures

Another alternative to measure the quality of learned clauses in SAT solvers is the so-called *LBD* \cite{AS09}.

**Definition 2** (*LBD*) Consider a clause $\gamma$ and the current assignment of its literals. Let $\pi$ be a partition of these literals, such that literals are partitioned w.r.t. their decision levels. The LBD of $\gamma$ is the number of classes in $\pi$.

The LBD of a clause is first computed when this clause is learned, and is then updated each time the clause is used as a reason. In this context, the notion of LBD relies on the fact that all literals in a conflicting clause are falsified, and when the clause is used as a reason, only one literal is not falsified (the propagated literal), but its decision level is also that of another (falsified) literal, which has triggered the propagation. When PB constraints are considered, this is not the case anymore. As such, LBD is not well-defined for such constraints. To consider it as a quality measure for learned PB constraints, we thus need to take into account the literals that are unassigned in these constraints. To
do so, we introduce five different definitions of this measure. First, we consider a sort of default definition of $LBD$ for PB constraints, which only takes into account assigned literals. This definition of $LBD$ was used for instance in the first version of RoundingSat.

**Definition 3 ($LBD_a$)** Consider a PB constraint $\chi$ and the current assignment of its assigned literals. Let $\pi$ be a partition of these literals, such that literals are partitioned w.r.t. their decision levels. The $LBD_a$ of $\chi$ is the number of classes in $\pi$ ("a" stands for "assigned").

Unassigned literals may be considered as if they were assigned to a “dummy” decision level. This decision level may be the same for all literals, or not.

**Definition 4 ($LBD_s$)** Consider a PB constraint $\chi$ and the current assignment of its assigned literals. Let $\pi$ be a partition of these literals, such that literals are partitioned w.r.t. their decision levels. Let $n$ be the number of classes in $\pi$. The $LBD_s$ of $\chi$ is $n$ if all literals in $\chi$ are assigned, and $n + 1$ otherwise ("s" stands for “same”).

**Definition 5 ($LBD_d$)** Consider a PB constraint $\chi$ and the current assignment of its assigned literals. Let $\pi$ be a partition of these literals, such that literals are partitioned w.r.t. their decision levels. Let $n$ be the number of classes in $\pi$. The $LBD_d$ of $\chi$ is $n + u$, where $u$ is the number of unassigned literals in $\chi$ ("d" stands for “different”).

Another possible extension of $LBD$ is to only consider falsified literals, as in the current version of RoundingSat:

**Definition 6 ($LBD_f$)** Consider a PB constraint $\chi$ and the current assignment of its falsified literals. Let $\pi$ be a partition of these literals, such that literals are partitioned w.r.t. their decision levels. The $LBD_f$ of $\chi$ is the number of classes in $\pi$ ("f" stands for “falsified”).

The definition above is based on the observation that, when a clause is learned, all literals in this clause are falsified. However, it may happen that falsified literals in a PB constraint are actually ineffective (while this is never the case in a clause). As these literals are not involved in the conflict, we should not consider them either. We thus define another extension of $LBD$ that only considers effective literals:

**Definition 7 ($LBD_e$)** Consider a PB constraint $\chi$ and the current assignment of its effective literals. Let $\pi$ be a partition of these literals, such that literals are partitioned w.r.t. their decision levels. The $LBD_e$ of $\chi$ is the number of classes in $\pi$ ("e" stands for “effective”).
Example 6 The LBD-based quality measures for the constraint $\chi$ given by $5a(0\oplus 3) + 5b(1\oplus 3) + c(?@?) + d(?@?) + e(0\oplus 1) + f(1\oplus 2) \geq 6$ are:

- $LBD_a(\chi) = |\{(a, b), \{e\}, \{f\}\}| = 3$
- $LBD_s(\chi) = |\{(a, b), \{c, d\}, \{e\}, \{f\}\}| = 4$
- $LBD_d(\chi) = |\{(a, b), \{c, d\}, \{e\}, \{f\}\}| = 5$
- $LBD_f(\chi) = |\{(a), \{e\}\}| = 2$
- $LBD_e(\chi) = |\{(a)\}| = 1$

We remark that the definitions of LBD introduced in this section are extensions of the original definition of LBD (as given by Definition 3), in the sense that they all coincide when learning clauses.

4.3 Deleting PB Constraints

Taking advantage of the measures described above, we define the following deletion strategies, which are applied each time the learned clause database is reduced:

- $delete$-$degree$, which deletes the constraints with the highest degree,
- $delete$-$degree$-$bits$, which deletes the constraints with the largest degree,
- $delete$-$lbd$-$a$, which deletes the constraints with the highest $LBD_a$,
- $delete$-$lbd$-$s$, which deletes the constraints with the highest $LBD_s$,
- $delete$-$lbd$-$d$, which deletes the constraints with the highest $LBD_d$,
- $delete$-$lbd$-$f$, which deletes the constraints with the highest $LBD_f$, and
- $delete$-$lbd$-$e$, which deletes the constraints with the highest $LBD_e$.

5 Restarts

Restarts are a very powerful feature of CDCL SAT solvers [GSK98]. Even though this feature is not completely understood, it seems required to exploit more power of the resolution proof system [PD11,EGNV18,AFT11]. Restarting is mainly forgetting all decisions made by the solver, and go back to the root decision level. The main advantage of doing so is that wrong decisions made at the very beginning of the search can be cancelled to avoid being stuck in a subpart of the search space. To this end, many restart schemes have been proposed [BF19], either static such as those based on the Luby series [LSZ93,Hua07] or dynamic, as in PicoSAT [Bie08a] or Glucose [AS12]. In this section, we focus on the latter, considering restart strategies based on the quality of learned constraints. Such restarts are not exploited in current PB solvers. In solvers such as pbChaff [DG02] or Galena [CK05], it is not clear whether restarts are implemented or not, as they do not mention this feature. As Pueblo [SS06] is heavily based on MiniSat [ES04], it is most likely to inherit its restart policy,
even though no mention of this feature is made in [SS06] either. Regarding more recent solver, Sat4j [LBP10] implements PicoSAT’s static and aggressive restart scheme [Bie08] and RoundingSat [EN18] uses a Luby-based restart policy [LSZ93, Hua07]. Note that a common point to these two strategies is that they do not take into account the constraints that are being considered, as they are both static policies. They may thus be reused without any modification since they are independent from the type of the constraints being considered.

In this section, we propose instead to follow Glucose’s restart policy [AS12]. In this solver, the decision of whether a restart should be performed depends on the quality of the constraints that are currently being learned: when this quality decreases, the solver is most likely exploring the wrong search space. As of Glucose, the quality of learned clauses is measured with their LBD (see Definition 2). To measure the decrease in the quality of learned clauses, the average LBD is computed over the most recent clauses (in practice, the last 100 clauses). Whenever this average is greater than 70% of the average LBD computed over all learned clauses, a restart should be performed. Glucose also implements a wide variety of tricks to improve its restart policy (such as restart blocking) that are beyond the scope of this paper.

We thus define 7 restart strategies, that exploit the quality measures defined in Section 4, namely restart-degree, restart-degree-bits, restart-lbd-a, restart-lbd-s, restart-lbd-d, restart-lbd-f and restart-lbd-e.

6 Experimental Results

This section presents an empirical evaluation of the different strategies presented in this paper implemented in two PB solvers, namely Sat4j [LBP10] and RoundingSat [EN18]. All experiments have been executed on a cluster of computers equipped with quadcore bi-processors Intel XEON X5550 (2.66 GHz, 8 MB cache). The time limit was set to 1200 seconds and the memory limit to 32 GB. For space reasons, this section does not report the results of all individual strategies presented in this paper, but focuses on the performance of those providing the best improvements to the considered solvers. The interested reader may still have a look to the publicly available detailed results of our experiments [LBW21].

6.1 Solver Configurations

Let us first describe our implementation of the different strategies in Sat4j [LBP10], which are available in its repository. For this solver, we considered three main configurations, namely Sat4j-GeneralizedResolution, Sat4j-RoundingSat and Sat4j-PartialRoundingSat [LMW20].

https://gitlab.ow2.org/sat4j/sat4j/tree/cdcl-strategies
For these three configurations, the default strategies are given below:

- the branching heuristic bumps all variables appearing in each constraint encountered during conflict analysis each time they are encountered,
- learned constraints are stored in a \textit{mono-tiered} database, and are regularly deleted using \textit{MiniSat}'s learned constraint deletion strategy \cite{ES04}, based on the \textit{activity} of learned constraints (i.e., the constraints to remove are those that are less involved in recent conflicts), and
- the restart policy is that of \textit{PicoSAT} \cite{Bie08b}.

Based on our experiments, the \textbf{best} combination of strategies for \textit{Sat4j-GeneralizedResolution} is \textit{bump-effective}, \textit{delete-lbd-s} and \textit{restart-degree}, while the \textbf{best} combination for both \textit{Sat4j-RoundingSat} and \textit{Sat4j-PartialRoundingSat} is \textit{bump-assigned}, \textit{delete-degree-bits} and the static restart policy of \textit{PicoSAT} \cite{Bie08b}.

For \textit{RoundingSat} \cite{EN18}, our implementation is available in a dedicated repository\footnote{https://gitlab.com/pb-cdcl-strategies/roundingsat/-/tree/cdcl-strategies}. We refactored this solver starting from commit a17b7d0e (denoted \texttt{master} in the following) to support the use of the different strategies presented in this paper. The \textbf{default} configuration of this solver corresponds to the refactored version of \textit{RoundingSat} set up with the default strategies originally used by this solver, i.e.:

- the branching heuristic bumps all variables appearing in each constraint encountered during conflict analysis once, and twice when eliminated,
- learned constraints are stored in a \textit{mono-tiered} database, and regularly deleted using the \textit{LBD$_f$} of the constraints and their activity as a tie-break, and
- the restart policy uses the Luby series (with factor 100) \cite{Hua07}.

The \textbf{best} combination of strategies for this solver, according to our experiments, is \textit{bump-assigned} (with a bumping on the variables each time they are encountered), \textit{delete-lbd-e} and \textit{restart-lbd-e}.

### 6.2 Decision Problems

We first consider the performance of the different solvers on decision problems. To this end, we ran the different solvers on the whole set of decision benchmarks containing “small” integers used in the PB competitions since the first edition \cite{MR06}, for a total of 5582 instances. Figure \ref{fig:decision} gives the results of the different solvers on these inputs, with their \textbf{default} and \textbf{best} configurations.

The cactus plot shows that the different configurations of \textit{Sat4j} are significantly improved by the use of our dedicated strategies. Quite interestingly, we can also observe that \textit{Sat4j-GeneralizedResolution} with the best combination of the strategies beats both implementations of \textit{RoundingSat} in \textit{Sat4j} with their
default strategies. In the case of RoundingSat, we can also note a small improvement over its default configuration, but this improvement is not as significant as in Sat4j. Let us remark that combining the best strategies is not enough to get the best of all the strategies we investigated. In particular, for each feature we considered, the Virtual Best Solver (VBS) of the different strategies, i.e., the one obtained by selecting the best performing strategies on each individual input, has far better performance than each individual strategy, and this applies to all configurations of Sat4j and RoundingSat. This suggests that no strategy is better than the other on all benchmarks, and that they are actually complementary.

6.3 Optimization Problems

Let us now consider the performance of the different solvers on optimization problems, by using as input the whole set of optimization benchmarks containing “small” integers used in the PB competitions since the first edition [MR06], for a total of 4374 instances. Considering the huge amount of computation time needed to perform our exhaustive experiments on decision problems (more than 8 years of CPU time), we focused for these experiments on the best configurations of the different solvers we identified on decision problems (which still took about 9 months of CPU computation time). Figure 2 shows the results we obtained for these configurations.

Similarly to decision problems, we can observe on the cactus plots that all solvers are improved by the dedicated strategies on optimization problems, with a particularly significant improvement to Sat4j-GeneralizedResolution.
Figure 2: Cactus plots of different configurations of Sat4j and RoundingSat on optimization problems. For more readability, the first (easy) 2000 instances are cut out.

6.4 Discussion

Let us now make a more detailed analysis of our experimental results.

Not so surprisingly, the strategy that has the most important impact, especially in Sat4j, is the bumping strategy, i.e., the branching heuristic. On the one hand, our experiments showed that the strategies bump-degree and bump-ratio-degree-coefficient have really poor performance in all considered solvers (including RoundingSat). As described in [Dix04, Section 4.5], these strategies are designed to estimate the number of clauses that are represented by the PB constraint whose literals are being bumped. However, when a conflict occurs, not all these clauses are actually involved in the conflict, and thus some variables get “more bumped” than they should be.

On the other hand, assignment-based bumping strategies are, among all individual strategies, those having the biggest impact on the performance of Sat4j. For instance, we observed that Sat4j-GeneralizedResolution solves the (optimization) instances of the factor family much faster thanks to the bump-effective strategy (changing the learned constraint deletion or restart strategies makes almost no difference on this family). We made further investigations to understand why there was such an improvement, and it appears that the production of irrelevant literals (i.e., literals that occur in a PB constraint, but never affect its truth value, whatever their assignment) penalize the solver on this particular family. It is known that such literals may impact the size of the proof built by PB solvers [LMMW20]. Our experiments here also show that they may pollute the solver’s heuristic, as bump-effective never bumps irrelevant literals (they are always ineffective). This also proposes another way to deal with such literals.

The big impact of the bumping strategies in Sat4j may also explain why
the gain in \textit{RoundingSat} is not so significant. Indeed, the aggressive weakening performed by \textit{RoundingSat} tends, in a sense, to already identify the literals that are already involved in the conflict. This is particularly visible if we look at the behavior of the different bumping strategies in \textit{RoundingSat}: there is almost no difference between them. This suggests that the gain in this solver comes mostly from the learned constraint deletion strategy or the restart policy, which improve the default strategies without being significantly better.

In particular, we observed that, in \textit{Sat4j}, performing no deletion at all is actually better than the (default) activity-based deletion strategy. This may be explained by the fact that PB solvers are often slower in practice than SAT solvers, especially because the operations they need to perform, such as detecting propagations and applying the cancellation rule, are more complex than their counterpart in SAT solvers. This means that the number of conflicts per second in a PB solver is lower than that in a SAT solver, and so is the number of learned constraints. As a consequence, PB solvers do not need to clean their learned constraint database as regularly as a SAT solver.

Regarding the restart policies, there is no big difference between the strategies, except for \texttt{degree-bits}, which does not have good performance compared to the others, and especially to \texttt{degree}. This may be explained by the fact that degrees with the same number of bits may take very different values. These are taken into account by the latter while the former does not distinguish them. Nevertheless, there is clearly room for improvement as the VBS performs much better than the individual strategies.

It is also important to note that the different strategies we considered in this paper are often tightly linked in the solver, and may thus interact with each other. This is particularly true for the learned constraint deletion and restart policies, since they use the same quality measures. While using them independently does not necessarily have a big impact on the solver (this is particularly true for the learned constraint deletion strategy), combining them often allows to get better performance. For instance, in \textit{RoundingSat}, while the best (individual) strategies are \textit{PicoSAT}'s restart policy and the deletion based on the \textit{LBD}, the best gain is actually obtained by using the \textit{LBD} quality measure both for learned constraint deletion and restarts.

Another consequence of the tight link between the different strategies and the solver itself is that implementation details may have unintended side effects on the performance of the solver. For instance, to implement the new strategies in \textit{RoundingSat}, we had to adapt the code and change some data structures in the branching heuristic (by replacing an \textit{ordered set} with an \textit{(unordered) hash map}), resulting in the same literals being bumped, but \textit{in a different order}. As the insertion/update order of the variables is used as a tie-break by EVSIDS, the order in which the literals are selected varies between the \texttt{master} and \texttt{default} configuration of the solver, which increases the difficulty to interpret the results of \textit{RoundingSat}, especially on optimization problems.

To conclude this analysis, let us summarize the main outcomes of our experiments. The biggest impact on the solver is obtained by carefully adapting the bumping strategy: while considering coefficients in this case worsens all
tested solvers, considering the current partial assignment may drastically improve them. Regarding constraint deletion, using the activity based measure (which is the default in Sat4j, and only a tie-break in RoundingSat) has really poor performance. The other strategies have a lesser impact on the solver, and seem more closely dependent on the proof system of the solver to bring improvement. However, if one needs to set up strategies that work well for all different proof systems, it would be bump-assigned for the bumping strategies, delete-lbd-s for the deletion strategy and either degree-based or PicoSAT’s restart policy (depending of whether big degrees are expected to be produced or not, respectively).

7 Conclusion

In this paper, we introduced different branching heuristics, learned constraint deletion and restart strategies dedicated to native PB solving. These strategies are generalizations of those classically implemented in SAT solvers, and are designed to take into account the properties of PB constraints to better fit in the CDCL architecture. Our experiments revealed that one of the key aspects of PB constraints to take into account is the current assignment of their literals. This is particularly true for the EVSIDS-based heuristics, but also for the learned constraint deletion strategies and the restart policies through the use of new LBD-based measures. When combined, these strategies allow to improve the PB solvers RoundingSat and Sat4j, with a particularly significant improvement for the latter, both on decision and optimization problems.

Nevertheless, none of these strategies performs better than the others on all benchmarks: their VBS clearly beats each individual strategy, even when considering their combination. Yet, the strategies introduced in this paper show that better adapting SAT strategies may improve the performance of PB solvers. A perspective for future research is to find better ways to adapt such strategies, and to define new strategies that are specifically designed for PB solving or PB optimization (rather than adapting existing strategies). Another avenue to explore is to find how to properly combine these strategies to get their best, while taking into account the interactions between these different strategies. In particular, it is not clear that combining all single best strategies provides the best combination. A possible approach to identify such a combination is to use dynamic algorithm configuration to select the most appropriate strategies according to the state of the solver [BBE+20].

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