Comment on “Damping of neutrino oscillations, decoherence and the lengths of neutrino wave packets”

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Abstract: We point out three apparent inconsistencies in the treatment of oscillation coherence from reactor neutrino and source neutrino experiments in recent paper [1]. First, that the dependence of the oscillation probability upon the subsequent interactions of entangled recoil particles implies causality violations and in some situations superluminal signaling; second, that integrating over a non-orthogonal basis for the entangled recoil leads to unphysical effects; and third, that the question of what interactions serve to measure the position of the initial state particle remains ambiguous. These points taken together appear to undermine the claim made therein that the effects of wave packet separation must be strictly unobservable in reactor and radioactive source based neutrino experiments.

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1 Causality violations

Ref. [1] considers the wave packet length resultant coherence properties of oscillating neutrinos emerging from nuclear decays. The formalism used there distinguishes between two scenarios: the decay product(s) accompanying the neutrino escape undetected, or they interact with source material, representing a measurement. The frequency of interaction of the decay product(s), either electron or nuclear recoil in this case, with the material is considered as a timescale over which neutrino emission coherence is interrupted. It is thus proposed to be an influential timescale (and hence distance scale) upon the neutrino wave packet length, and in some cases the critical timescale that determines oscillation coherence.

While superficially plausible, this picture presents a serious problem, in that it implies causality violations that in principle allow for superluminal propagation of signals between recoil and neutrino. An event happening to nuclear or electronic recoil at time $t_R$ has a corresponding future light-cone (the region of spacetime in which it can causally influence any observable at the detector) which meets the detector location $L$ at time $t_D = t_R + \frac{L}{c}$. Given that the recoil moves slowly relative to the neutrino, the light cone emanating from its interaction point almost certainly reaches the detector baseline when the neutrino detection process is already complete. This is shown overlaid on the spacetime diagram of Ref. [1] in Fig. 1, using the nuclear recoil for illustration. If the neutrino oscillation probability is indeed influenced by detection-or-not or interaction-or-not of the recoiling particles, causality has been violated. An equivalent statement is that since the entangled partners emerge from a common vertex with the neutrino and travel at less than $c$, they are space-like separated from it and so their subsequent behavior cannot causally impact the oscillation phenomenology.

To make this argument less abstract, let us contrive of an unrealistic but physically consistent thought experiment where superluminal signaling would be manifest, were the approach of Ref. [1] correct. We consider a gaseous source that generates neutrinos via electron capture, and contains a sufficient number atoms that a high flux of neutrinos is continuously being emitted. The choice of electron capture for this example means we can ignore the effects of an emitted electron and focus only on the daughter nucleus. We collect
Figure 1. Light cone of the nuclear recoil interaction overlaid on the spacetime diagram from Ref. [1]. If interactions of the recoil at time $t_{N'}$ may influence oscillations of the neutrinos at $L$, causality is violated.

enough neutrinos with an idealized detector to monitor the oscillation probability at $L$ at any time. We can control the source density $\rho$ by compressing the gas with a piston. Our procedure will be as follows:

1. Start the experiment with the gas at a sufficiently low density that interactions of the final state nuclei do not cause any observable decoherence effects (Fig. 2, left).

2. At some moment $t_0$, we make a decision: either compress the piston leading to an increase in the gas density, or leave the gas in its present low-density state. If we compress the piston, the density $\rho(t > t_0)$ becomes high enough to induce significant wave-packet separation effects on the oscillation probability for neutrinos at distance $L$ (Fig. 2, right).

3. By choosing to compress or not compress the cylinder we may send a bit of information via the neutrino oscillation probability. We ask: at what time does this bit of information arrive?

The formalism of Ref. [1] provides an answer to this question. Neutrinos detected at time $t'$ were emitted at $t' - \frac{L}{c} + \mathcal{O}(\frac{m}{E})$. Since we choose to begin our experiment with gas at a sufficiently low density that the mean-free time $t_{N'} \gg \frac{L}{c}$, the recoils are still travelling even after the neutrinos have been detected.

Upon compressing the piston at time $t_0$, we transition to the incoherent mode. If the interactions of the entangled recoil truly impact the oscillation probability, the relevant neutrinos that are decohered were those emitted at $t_0 - t_{N'}$, and then detected at $t_D = t_0 - t_{N'} + \frac{L}{c} + \mathcal{O}(\frac{m}{E})$. Since $t_{N'} \gg \frac{L}{c}$, the detection time $t_D < t_0$. If we measure the
Figure 2. Thought experiment where the formalism of Ref. [1] implies backward propagation of information in time. The left panel shows the effect of not compressing the piston; the right of compressing it. According to the formalism of Ref. [1], this mechanism can be used to send information backwards in time via the neutrino oscillation probability. All omitted labels are equivalent to those in Fig. 1.

oscillation probability of these neutrinos and find it influenced by the interactions of the final state nuclear recoil, we have succeeded in sending a signal backwards in time.

2 Choice of basis for tracing out an entangled partner, and analogies to the EPR experiment

The system as described has striking similarities to the original EPR experiment [2]. Here we explore some similarities and differences, and this ultimately proves enlightening as to the origin of the apparent causality violation of Ref. [1]. Some of our treatment here resembles the discussion in Appendix A of Ref. [3].

In the original EPR experiment, two qubits $A$ and $B$ are prepared in an entangled state and moved to a large spatial separation:

$$|\psi\rangle = \frac{1}{\sqrt{2}} |\uparrow Z\rangle_A \otimes |\downarrow Z\rangle_B + |\downarrow Z\rangle_A \otimes |\uparrow Z\rangle_B,$$  \hspace{1cm} (2.1)

Measurement of system $A$ influences system $B$ via wave function collapse. For example, if $A$ measures spin in the $\hat{z}$ direction and obtains $\downarrow Z$, the wave function of $B$ becomes:

$$|\psi\rangle_B \rightarrow |\uparrow Z\rangle_B,$$ \hspace{1cm} (2.2)

This final state for $B$ is only possible if $A$ has measured along $\hat{z}$. Conversely, if $A$ measures the spin along $\hat{x}$ direction and obtains $\downarrow X$, the $B$ wave-function would become:

$$|\psi\rangle_B \rightarrow |\uparrow X\rangle_B = \frac{|\uparrow Z\rangle_B + |\downarrow Z\rangle_B}{\sqrt{2}},$$ \hspace{1cm} (2.3)
It seems therefore that $A$ can encode information in $B$’s spin at a large physical distance by choosing what quantity to measure. This “spooky action at a distance” invited arguments against the plausibility of quantum mechanics, which were the motivation for Einstein, Podolsky and Rosen’s seminal analysis. The original EPR discussion resolved how this apparent information transfer can be reconciled with requirements of causality. The resolution to this foundational question is that $B$ cannot access the newly encoded information without an additional, classical signal sent from $A$ to $B$, which must by definition travel slower than light. This is because no matter what $A$ has done or not done to her qubit, given an initial state as in Eq. 2.1, $B$ will find a 50% probability of spin up as long as he measures his qubit alone while ignoring the outcome of $A$’s measurements. Only in cases where correlation measurements are performed that compare outcomes of measurements at $A$ to those at $B$ can the non-trivial consequences of quantum entanglement be observed.

This is easy to prove. We posit that $A$ makes some measurement defined by Hermitian operator $M$, with possible outcomes $|\lambda_i\rangle$ for i=1,2, where $M|\lambda_i\rangle = \lambda_i|\lambda_i\rangle$. The probability for $B$ to find an answer $|\uparrow\rangle$ is given by a probability-weighted sum over outcomes of $M$ on $A$,

$$P(\uparrow_B) = \sum_i \langle \lambda_i| \otimes \langle \uparrow_B | \psi \rangle \langle \psi | \langle \lambda_i | \otimes | \uparrow_B \rangle).$$

(2.4)

The mathematics is least tedious in the density matrix picture, though an identical proof can be completed with wave functions. We introduce the bipartite density matrix for the $AB$ system $\rho = |\psi\rangle \langle \psi|$, and the reduced density matrix for system $B$ is obtained by “tracing out” the entangled $A$ subsystem:

$$\rho_B = \sum_i \langle \lambda_i | \psi \rangle \langle \psi | \lambda_i \rangle \equiv \text{Tr}_A [\rho].$$

(2.5)

The probability Eq. 2.4 for $B$ to obtain $\uparrow$ is then conveniently expressed as

$$P(\uparrow_B) = \langle \uparrow_B | \rho_B | \uparrow_B \rangle.$$  

(2.6)

The probability for any other measurement we can imagine $B$ making on his qubit alone can be found analogously, using $\rho_B$. The construction accounts implicitly for the sum over outcomes of measurement $M$ on $A$.

The important principle that ultimately leads to resolution of the EPR paradox is that the reduced density matrix $\rho_B$ is independent of which $M$ has been made on $A$. For a different $M$, we would have different measurement outcomes, related to the original ones by $|\lambda'_i\rangle = U_{ij} |\lambda_j\rangle$ for some unitary $U$. The new reduced density matrix is $\rho'_B$:

$$\rho'_B = \langle \lambda_i | U^\dagger | \psi \rangle \langle \psi | U | \lambda_i \rangle.$$  

(2.7)

Inserting two complete sets of states into this expression we find:

$$\rho'_B = \langle \lambda_i | U^\dagger | \lambda_a \rangle \langle \lambda_a | \psi \rangle \langle \psi | \lambda_b \rangle \langle \lambda_b | U | \lambda_i \rangle$$

(2.8)

$$= U_{bi} U_{ia} \langle \lambda_a | \psi \rangle \langle \psi | \lambda_b \rangle$$

(2.9)

$$= \delta_{ba} \langle \lambda_a | \psi \rangle \langle \psi | \lambda_b \rangle$$

(2.10)

$$= \rho_B.$$  

(2.11)
and so, no matter what A does or does not do to her qubit, measurements made by B on his entangled qubit without knowledge of the outcome of A’s manipulations will have the same probabilities. This prohibits superluminal signaling, and reconciles the instantaneous nature of wave function collapse with the requirements of causality. This conclusion derives from properly summing B’s predictions over the probabilistic outcomes of A’s measurement (whatever they may be), as realized in the partial trace operation.

How is the neutrino-recoil system similar or different to the EPR system? We can replace A with the neutrino, B with the recoil, and move from 2D Hilbert spaces to continuous ones. Any measurement of the neutrino alone (say, its flavor $\beta$ at baseline $L$) can be made by probabilistically summing over possible outcomes for the recoil, which we express in a basis $|\lambda\rangle$ for some continuous $\lambda$:

$$P(\beta; L) = \int d\lambda \langle (\beta| \otimes \langle \beta, L|) \psi \rangle \langle (\beta\rangle \otimes |\beta, L\rangle \rangle = \langle \beta, L| \rho_\nu |\beta, L\rangle, \quad (2.12)$$

where the neutrino reduced density matrix is

$$\rho_\nu = \text{Tr}_\lambda [\rho] = \int d\lambda \langle \lambda| \rho \rangle \langle \lambda\rangle. \quad (2.13)$$

By the same argument as above, the neutrino reduced density matrix $\rho_\nu$ is independent of what we choose to measure about the recoil, since $\rho'_\nu$, for any choice of recoil basis states $U|\lambda\rangle$ is independent of $U$. It is also independent of the subsequent time evolution of the recoil, because the time evolution operator for the recoil is itself just another unitary operation:

$$|\lambda(t)\rangle = U_R(t - t_0)|\lambda\rangle. \quad (2.14)$$

If we change the nature of the time evolution of the recoil (for example by changing the medium into which it is emitted), we then switch out the $U_R$ operator for a different one. According to the above proof, $\rho_\nu$ must be unaffected and the neutrino oscillation probability must remain unchanged. The conclusion is that neither the time evolution nor the measurement of the recoil can influence the overall oscillation probability of the neutrino as long as we do not make correlation measurements - just as in the EPR experiment.

How then do we reconcile this with the proofs of Ref. [1] that suggest that the total oscillation probability is indeed influenced by the subsequent interactions of the recoil? To be maximally explicit, the claim there is that beyond some distance $L_{coh}$, oscillations of the neutrino alone would be observable were the recoil emitted into empty space, but unobservable if it were emitted into sufficiently dense matter, thus introducing an explicit dependence of $\rho_\nu$ on the operator $U_R(t - t_0)$.

This problem seems to be introduced by choosing to project the recoil onto a Gaussian wave-packet. Notably such functions do not represent an orthonormal basis of final states for the recoil. As such, the process of determining which Gaussian wave packet the recoil lands in is not a measurement in the sense described by the postulates of quantum mechanics. There, measurements are supposed to be described by Hermitian operators, whose eigenstates are necessarily orthogonal. The problem here is that taking a sum over sets of non-orthogonal final states is not a valid way to probabilistically sum the possible
fates of the recoil, and this introduces unphysical effects into the oscillation probability prediction.

We note that it is possible to introduce this same malady in the original EPR scenario by using non-orthogonal measurement states. Imagine that we do not force $A$ to make a measurement in the strict quantum mechanical sense, but instead allow $A$ to ask, which of the two (non-orthogonal) states below is her qubit found in?

\begin{align}
|\mu_1\rangle &= |\uparrow\rangle, \\
|\mu_2\rangle &= \alpha |\uparrow\rangle + \sqrt{1-\alpha^2} |\downarrow\rangle, \quad 0 \leq \alpha \leq 1.
\end{align}

(2.15) (2.16)

If we now we sum $B$’s probabilities for spin up over $A$’s probabilities for the outcomes $\mu_1$ and $\mu_2$, we find

\begin{align}
P(\uparrow_B) &= \sum_i \langle \mu_i | \otimes \langle \uparrow_B | \psi \rangle \langle \psi | (|\mu_i\rangle \otimes \uparrow_B) \\
&= \langle \uparrow_B | \tilde{\rho}_B | \uparrow_B \rangle,
\end{align}

(2.17) (2.18)

The object $\tilde{\rho}_B$ is not a reduced density matrix, but some analogous object constructed with non-orthogonal basis states,

\begin{equation}
\tilde{\rho}_B = \langle \mu_i | \psi \rangle \langle \psi | \mu_i \rangle.
\end{equation}

(2.19)

The matrix $X$ that relates the new basis states to the original ones is now non-unitary unless $\alpha = 0$:

\begin{equation}
|\mu_i\rangle = X(\alpha)_{ij} |\lambda_j\rangle, \quad X(\alpha)X^\dagger(\alpha) = \begin{pmatrix}
1 + \alpha^2 & \alpha \sqrt{1 - \alpha^2} \\
\alpha \sqrt{1 - \alpha^2} & 1 - \alpha^2
\end{pmatrix}.
\end{equation}

(2.20)

As a result we find

\begin{equation}
\tilde{\rho}_B = X_{bi}(\alpha)X_{ia}^\dagger(\alpha) \langle \lambda_a | \psi \rangle \langle \psi | \lambda_b \rangle \\
\neq \rho_B,
\end{equation}

(2.21) (2.22)

which now depends on $\alpha$. This dependence on $\alpha$ is also manifest in the probabilities $P(\uparrow_B)$ of Eq. 2.18. As such, superluminal communication between $A$ and $B$ now appears to be allowed by suitable choice of $\alpha$ by $A$.

This apparent causality violation emerges from mistaken assumption that Eq. 2.17 may be considered as a sum over probabilistic outcomes at $A$. There is no quantum mechanical measurement that $A$ could make that would have possible outcomes $|\mu_1\rangle$ and $|\mu_2\rangle$, since they are not orthogonal states. As a consequence, summing over their probabilities is an improper construction to integrate over possible final states of the distant $A$ system. This means the probabilities calculated for the measurements at $B$ are invalid. The origin of the apparent causality violation in the neutrino-recoil system appears to be the same as in this toy example.
3 Localization through interaction with the environment

The above examples illustrate that the neutrino oscillation probability cannot possibly depend on the interactions of the recoiling nucleus after it leaves the decay, or on the interactions of any other entangled daughter particles with surrounding material.

On the other hand, that the entangled final-state particles do both exist and carry away information about the neutrino away from the decay does impact neutrino coherence properties. Such information escapes into the universe recording information about the neutrino whether they are observed or not, and suppresses oscillation coherence in certain circumstances. The key point is that, while it is always important that that recoil particle(s) were emitted in the decay, what happens to them after production cannot influence the neutrino-only oscillation phenomenology, without violating causality. Tracing over the recoil final states in any orthogonal basis will generate the same oscillation probabilities for the neutrino, and this appears to be the the only well defined way to probabilistically sum over final states of the recoil.

While environmental interactions of the daughter particles cannot influence coherence, those of the parent can and do. Entanglements generated between the parent and others through collisions serve (colloquially) to “measure” the position of the emitter prior to its decay with a precision dictated by the momentum transfer in the interaction [4]. Those interactions are in the past light cone of the neutrino for every observer, so no causality problems are implied by their influence. The Weisskopf-Van-Vleck formalism of Ref. [5] as cited in Ref. [1] in fact describes the effects of these initial-state interactions on determining line shapes, and not those of the final-state recoil after the decay.

The question of which interactions do serve to measure the parent position appears to be not as clear-cut as described in Ref. [1], and there remains ambiguity in the relevant distance scale. While atomic collisions are advanced there as the relevant localizations, we may also consider neutrino production as proceeding via the process $n \rightarrow p + e + \nu_e$ for neutrons bound inside a nucleus. Thus another reasonable localization scale appears to be that induced by the interaction among nucleons inside the nucleus itself. After the decay the remaining N-1 entangled nucleons are left behind, and their position encodes the location of the original emitter, localizing it to something of order the size of the nucleus itself. This would seem to imply a far smaller wave packet width and more dramatic decoherence effects than the modest localization scale from atomic collisions of either the parent or daughter particles. A detailed treatment of the implied oscillation phenomenology is outside the scope of the present comment, but is the subject of a forthcoming paper.

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