Limits of sub-Doppler cooling for atoms with various recoil parameter

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We perform detailed analysis of sub-Doppler cooling limits for various atoms by direct solving quantum kinetic equation for atom density matrix in standing-wave light field generated by counterpropagating waves. It was demonstrated that the polarization gradient cooling effects are sensitive to atom recoil parameter (the ratio of recoil energy to natural linewidth) that results to limitation of sub-Doppler cooling and allows to outline the limits of well-known sub-Doppler cooling theory. We also give a comparison the cooling limits for well-known $\sigma_+ - \sigma_-$ and $\text{lin} \perp \text{lin}$ configurations.

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I. INTRODUCTION

Since the mid-1980s the laser cooling of atoms was rapidly developing field of laser and atomic physics. Nowadays laser cooled atoms are widely used in ultrahigh-resolution spectroscopy, developing new generation of time and frequency standards,[8–15] achieving Bose-Einstein condensation of neutral atoms[16], simulating models of quantum effects in condensed matter and interatomic collision investigation[22,23].

To this day various approaches to describe laser cooling have been developed. At the initial stage of research the semiclassical approaches were widely used[15–19]. These approaches allow describing the kinetics of atoms in terms of diffusion and forces acting on atoms, resulted from recoil processes due to absorption or emission of light field photons.

Semiclassical approaches are principally limited by a small value of a momentum transmitted to atoms from a single light field photon in comparison with the width of atom momentum distribution, $\hbar k/\Delta p \ll 1$, as well as by a small value of recoil parameter $\varepsilon_R = \omega_R/\gamma \ll 1$, that is the ratio of recoil energy $\hbar \omega_R = h^2 k^2/2M$ to the natural width of optical transition $\gamma$ used for laser cooling. Smallness of these two parameters ($\varepsilon_R$ and $\hbar k/\Delta p$) allows separating a fast evolution of internal degrees of freedom from a slow evolution of translational degrees of atoms. In this case the complex quantum kinetic equation for atom density matrix can be reduced to the Fokker-Planck equation for distribution function in coordinate and momentum spaces with the force acting on atom and diffusion coefficients[20–22]. The developing of semiclassical theory was of great importance to understand the basic principles of the laser cooling including the Doppler cooling[23–25] and the sub-Doppler cooling[20,26–29] mechanisms in optical molasses, i.e. in light fields formed by the pairs of counterpropagating waves.

An alternative way is to use quantum approaches allowing taking into account all recoil effects in interaction of atoms with resonance light field photons. Theoretical describing of such processes is quite complicated because the interaction between atoms and photons brings changing of both internal and motional degrees of freedom. In particular, instead of solving quantum kinetic equation for atom density matrix statistical Monte Carlo wave-function method was developed[20,21]. The number of variables involved in wave-function simulation is determined by the relevant Hilbert space dimension $N$ much smaller than the one required for calculations with density matrices ($\sim N^2$). However, it spends much time for computer to simulate and obtain appropriate statistics for calculating mean over the atom trajectories.

Kinetic quantum equation for atom density matrix contains much more variables than wave-function approach and at the beginning of investigation various approximation was used to solve it. For example, to describe limits of sub-Doppler laser cooling in $\text{lin} \perp \text{lin}$ field[22–24] approximation of low intensity limit was used allowing applying simplified equation for ground state atom density matrix. Moreover, in the papers mentioned above, it was use the secular approximation, $\sqrt{U_0/\hbar \omega_R} \ll |\delta|/\gamma$, supposing a gap between energy bands of optical potential to be much wider than band-width. Light shift $U_0$ is determined by the depth of optical potential, $\delta = \omega - \omega_0$ is detuning parameter of light field frequency $\omega$ from atomic resonance frequency $\omega_0$. In case of absence of optical potential, i.e. in a light field formed by a pair of counterpropagating waves with orthogonal circular polarizations $\sigma_+ - \sigma_-$, approaches developed in the papers[22–24] can not be used. However, the methods of p-families could be used in such cases[24,27].

In the papers[28,29], we offered an universal quantum approach allowing describing steady-state of atoms in 1D configuration formed by counterpropagating waves with arbitrary elliptical polarization interacting with arbitrary optical transition $\mathbb{J}_g \rightarrow \mathbb{J}_e$. This technique appears to be more general and face no restriction mentioned above. Moreover, in case of insufficient smallness of recoil parameter $\varepsilon_R$, the recoil effects were noted to become more sufficient, and significant discrepancy in temperatures was observed in contrast with predictions made by semiclassical and quantum theories[26,31–34].

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In this paper, we are performing detailed analysis of the limits of polarization-gradient sub-Doppler cooling of atoms with different ratio of recoil energy to the natural linewidth varied in wide range from extremely small values when well-known semiclassical and quantum models work properly to values close to $\varepsilon_R \simeq 1$ when the recoil effects carry critical weight. Our analysis allows to underline the limits of well-known sub-Doppler cooling theories for different polarization-gradient field configurations $\sigma_+ - \sigma_-$ and $\text{lin} \perp \text{lin}$.

## II. MASTER EQUATION

Let us consider the laser cooling of atoms with closed optical transition $J_g \rightarrow J_e$ where $J_g$ and $J_e$ are angular momenta of the ground ($g$) and excited ($e$) states. The atoms are resonantly interacting with 1D field formed by counterpropagating light waves along $z$ axis:

$$E(z, t) = E_0 \left( e_1 e^{-ikz} + e_2 e^{-ikz} \right) e^{-i\omega t} + \text{c.c.} \quad (1)$$

Here $E_0$ is the complex amplitude of the light waves, $\omega$ is the field frequency and $k = \omega/c$ is the wavevector. The polarization vectors $e_1$ and $e_2$ could be expressed in complex circular basis $e_{\pm} = \mp (e_x \pm e_y)/\sqrt{2}$ and $e_0 = e_z$:

$$e_n = \sum_{\sigma=0, \pm 1} e^n_\sigma e_\sigma, \quad n = 1, 2. \quad (2)$$

Here we will consider the following two configurations:

- $\text{lin} \perp \text{lin}$ configuration is defined by $e_1$ and $e_2$ with orthogonal linear polarization.$^{17}$

- $\sigma_+ - \sigma_-$ configuration is defined by two orthogonal circular polarization of $e_1 = e_+\sigma$ and $e_2 = e_-,\sigma$.\n
These simple configurations represent the light field with only one parameter having spatial dependence: the local ellipticity for $\text{lin} \perp \text{lin}$ configuration and the local polarization orientation angle for $\sigma_+ - \sigma_-$ configuration. The other field parameters like field intensity and phase have no spatial dependence fore these two configurations. Note that the general 1D field configuration with several parameters varied along the propagation axes can be represented by $\varepsilon_1 - \theta - \varepsilon_2$ field configuration formed by counterpropagating plane waves with arbitrary elliptical polarizations$^{13,19,35,36}$.

The kinetic evolution of a low-density atomic ensemble (when interatomic interaction could be neglected) is described by the quantum kinetic equation for the atomic density matrix $\hat{\rho}$ in the single-particle approximation

$$\frac{\partial \hat{\rho}}{\partial t} = -i\hbar \left[ \hat{H}, \hat{\rho} \right] + \hat{\Gamma}\{\hat{\rho}\}, \quad (3)$$

where $\hat{H}$ is the Hamiltonian, and the term $\hat{\Gamma}\{\hat{\rho}\}$ describes the relaxation in the process of spontaneous decay. The Hamiltonian can be divided into the sum

$$\hat{H} = \frac{\hat{p}^2}{2M} + \hat{H}_0 + \hat{V}, \quad (4)$$

where the first term is the kinetic energy operator; $\hat{H}_0 = -\hbar \delta \hat{P}_z$ is the Hamiltonian of the free atom in rotating wave approximation (RWA); $\delta = \omega - \omega_0$ is light field detuning from the atomic transition frequency $\omega_0$; and

$$\hat{P}_e = \sum_\mu |J_e, \mu\rangle \langle J_e, \mu|$$

is the projection operator to the exited sublevels $|J_e, \mu\rangle$ where $\mu$ is the angular momentum projection on the quantization axis ($-J_e \leq \mu \leq J_e$). The last term in (4) $\hat{V}$ describes the atom-light interaction which in electric dipole approximation takes a form

$$\hat{V} = \hat{V}_1 \exp(ikz) + \hat{V}_2 \exp(-ikz)$$

$\hat{V}_n = \frac{\hbar \Omega}{2} (\hat{D} \cdot e_n) = \frac{\hbar \Omega}{2} \sum_{\sigma=0, \pm 1} \hat{D}_\sigma e_n^\sigma \quad n = 1, 2. \quad (6)$$

Here $\Omega$ is the Rabi frequency. The circular components of operator $\hat{D}$ are expressed via the Clebsch-Gordan coefficients according to Wigner-Eckart theorem:

$$\hat{D}_\sigma = \sum_{\mu, m} C_{J_g, \mu, 2\sigma, 1, \sigma}^{|J_e, \mu\rangle \langle J_g, m|}. \quad (7)$$

The last term of the kinetic equation (4) describing the relaxation due to spontaneous decay taking into account photon recoil has a well-known form (see for example$^{30}$):

$$\hat{\Gamma}\{\hat{\rho}\} = -\frac{\gamma}{2} \left( \hat{P}_e \hat{\rho} \hat{P}_e + \hat{\rho} \hat{P}_e \right)$$

$$+ \left( \sum_{\xi=1,2} \left( \hat{D} \cdot e_\xi(k) \right)^\dagger e^{-ikr} \hat{\rho} e^{ikr} \left( \hat{D} \cdot e_\xi(k) \right) \right) \Omega_k \quad (8)$$

where $\langle \ldots \rangle_{\Omega_k}$ denotes averaging over the directions of emission of spontaneous photons having a momentum $h\hat{k}$ with two orthogonal polarizations $e_\xi(k)$.

To solve the kinetics of laser cooling it is convenient to use the coordinate representation for density matrix in which the spontaneous relaxation operator accounting recoil effects in 1D geometry takes the simplest form:

$$\hat{\Gamma}\{\hat{\rho}(z_1, z_2)\} = -\frac{\gamma}{2} \left( \hat{P}_e \hat{\rho}(z_1, z_2) + \hat{\rho}(z_1, z_2) \hat{P}_e \right)$$

$$+ \gamma \sum_{\sigma=0, \pm 1} \kappa_\sigma(q) \hat{D}_\sigma^\dagger \hat{\rho}(z_1, z_2) \hat{D}_\sigma \quad (9)$$

where $q = z_1 - z_2$ and functions $\kappa_{0, \pm 1}$ are

$$\kappa_0(q) = 3 \left( \frac{\sin(kq)}{(kq)^2} - \frac{\cos(kq)}{(kq)^3} \right)$$

$$\kappa_{\pm 1}(q) = 3 \left( \frac{\cos(kq)}{(kq)^2} + \frac{\sin(kq)}{kq} - \frac{\sin(kq)}{(kq)^3} \right). \quad (10)$$

For solving equation (3) in steady-state and analysis of the limits of sub-Doppler cooling we are using a generalized continuous fraction method suggested by us and described in detail using Wigner in$^{28,29}$ and coordinate representations in$^{30}$ for atomic density matrix. The steady-state solution of (3) occurs to be periodic in coordinate $z$ and allows factorizing on spacial harmonics:

$$\hat{\rho}(z, q) = \sum_n \hat{\rho}^{(n)}(q) e^{in\kappa_0}. \quad (11)$$
Thus the task is reduced to calculating of the amplitudes \( \hat{\rho}^{(n)}(q) \). The equation for steady-sate Fourier harmonics \( \hat{\rho}^{(n)} \) can be written in recurrent form with three terms:

\[
-n\frac{i}{\hbar} \frac{\partial}{\partial q} \hat{\rho}^{(n)}(q) = \mathcal{L}_0(\hat{\rho}^{(n)}) + \mathcal{L}_+\{\hat{\rho}^{(n-1)}\} + \mathcal{L}_-\{\hat{\rho}^{(n+1)}\},
\]

where operators \( \mathcal{L} \) are

\[
\mathcal{L}_+\{\hat{\rho}\} = -\frac{i}{\hbar} \left( \hat{W}_1 \hat{\rho} e^{i q k / 2} - \hat{\rho} \hat{W}_1 e^{-i q k / 2} \right)
\]

\[
\mathcal{L}_-\{\hat{\rho}\} = -\frac{i}{\hbar} \left( \hat{W}_2 \hat{\rho} e^{-i q k / 2} - \hat{\rho} \hat{W}_2 e^{i q k / 2} \right)
\]

\[
\mathcal{L}_0\{\hat{\rho}\} = -\frac{i}{\hbar} \left( \hat{H}_0 \hat{\rho} - \hat{\rho} \hat{H}_0 \right) - \hat{\Gamma}\{\hat{\rho}\}
\]

with matrix coefficients

\[
\hat{W}_1 = \begin{pmatrix} 0 & \hat{V}_1 \\ \hat{V}_1 & 0 \end{pmatrix}, \quad \hat{W}_2 = \begin{pmatrix} 0 & \hat{V}_2 \\ \hat{V}_2 & 0 \end{pmatrix}.
\]

Note that harmonics of atomic density matrix \( \hat{\rho}^{(n)} \) depend on variable \( q \) and contain the information on quantum correlations of atomic states between two points separated in space \( z_1 = z + q/2 \) and \( z_2 = z - q/2 \). As far as the correlation should decay with growing \( |q| \), we can cut it with large enough value \( q_{\max} \). In a Wigner representation \( q_{\max} \) defines delocalization in momentum space \( \Delta p \approx \pi / q_{\max} \). In our simulations, we typically used \( q_{\max} \leq 10 / k \), however, for some parameters of laser cooling the quantum correlation length is found to be quite large and requiring increasing \( q_{\max} \) up to \( \sim 100 / k \) to account for these effects correctly.

### III. STEADY-STATE OF LASER COOLING

The steady-state solution of quantum kinetic equation \([22, 23]\) is determined by the light field and atomic parameters. The light field parameters are: the intensity of light waves \( I \), optical frequency \( \omega \), and the spatial polarization configuration which is defined, as was noted above, by the polarizations of opposite light waves. The atomic parameters are the type of optical transition (which is defined by the total angular momenta of the ground and the exited states \( J_g \rightarrow J_e \)), the transition frequency \( \omega_0 \), the dipole moment of optical transition \( d \), the natural linewidth \( \gamma \), and the atom mass \( M \).

So, within the above list of parameters a few dimensionless parameters determined the steady-state solutions of master equation \([22, 23]\) can be separated. These are:

- \( \delta / \gamma \) is dimensionless detuning
- \( \Omega / \gamma \) is dimensionless Rabi frequency containing information on the light waves intensity \( \Omega / \gamma = \sqrt{I / 2I_{\text{sat}}} \) \( (I_{\text{sat}} = 2\pi^2 \gamma \hbar c / \lambda^3 \) is determined by the dipole moment of optical transition, see for example \([22, 23]\)
- \( \varepsilon_R = \omega_R / \gamma \) is recoil parameter

As well, the field light polarization configuration and the type of optical transition \( J_g \rightarrow J_e \) stay as additional parameters into represented list of dimensionless parameters of laser cooling task.

The third parameter \( \varepsilon_R = \hbar k^2 / (2M\gamma) \) is determined by atomic mass and optical transition used for realization of laser cooling (see the table \([1]\)). It is small enough \( \varepsilon_R \ll 1 \) and varies form \( \sim 10^{-4} - 10^{-1} \) for majority of atomic optical transitions used for laser cooling. It tends to reach extremely low values \( \varepsilon_R < 10^{-3} \) for alkali-line elements (like Cs and Rb are cooling with the use of D2 line). As well, it gets the values above \( 10^{-1} \) for narrow-line optical transition, for example, intercombination transitions \( ^1S_0 \rightarrow ^3P_1 \) of Sr, Cs, and Mg atoms.

Below we are demonstrating the influence of recoil parameter on the results of laser cooling in the case of \( \varepsilon_R \) being not extremely small and outlining the limits of sub-Doppler cooling for intermediate values of the recoil parameter \( 10^{-3} \leq \varepsilon_R \leq 10^{-1} \).

Emphasis should be placed on fact that in a field of low intensity at secular approximation of papers \([22, 23]\) describing Sisyphus cooling in a field of \( \text{lin} \perp \text{lin} \) configuration, the only one parameter

\[
U_0 = \frac{\left| \delta \right|}{3\omega_R (\delta^2 + \gamma^2 / 4)}
\]

defines the steady state solution. Therefore, in our analysis, we are using this parameter instead of \( \Omega / \gamma \). This will allow us to compare our results with the earlier results in the papers \([22, 23]\) obtained with some approximations. Thus the list of dimensionless parameters is following: \( \delta / \gamma, U_0, \varepsilon_R \).
A. Extremely low recoil parameter \( \varepsilon_R \leq 10^{-3} \)

In this section we demonstrate the results of steady-state solution of master equation \( (3) \) for extremely small recoil parameter \( \varepsilon_R \leq 10^{-3} \). This limit is well studied by many authors with the use of semiclassical approaches\(^{15} \) as well as the quantum approaches\(^{22,23} \). We made our analysis for transition \( J_g = 1 \rightarrow J_e = 2 \) admitting the sub-Doppler laser cooling for both types of light field configurations \( \text{lin} \perp \text{lin} \) and \( \sigma_+ - \sigma_- \). It is the simplest example allowing us to compare the cooling limits for these configurations.

![Figure 1](image1.png)

**FIG. 1.** (a) Average kinetic energy of cold atoms as function of \( U_0 \) for different detuning \( \delta \) in \( \text{lin} \perp \text{lin} \) field (a) and in \( \sigma_+ - \sigma_- \) field (c). Momentum distribution in of cold atoms for different \( U_0 \) at \( \delta = -2\gamma \) in \( \text{lin} \perp \text{lin} \) field (a) and \( \sigma_+ - \sigma_- \) field (d). Recoil parameter \( \varepsilon_R = 4 \times 10^{-4} \). Red dashed horizontal lines correspond to average kinetic energy of atoms with Gaussian momentum distribution and Doppler temperature \( k_B T_D = h\gamma/2 \).

![Figure 2](image2.png)

**FIG. 2.** (a) Average kinetic energy of cold atoms as function of \( U_0 \) for different detuning \( \delta \) in \( \sigma_+ - \sigma_- \) field (a) and in \( \sigma_+ - \sigma_- \) field (c). Momentum distribution in of cold atoms for different \( U_0 \) at \( \delta = -2\gamma \) in \( \text{lin} \perp \text{lin} \) field (a) and \( \sigma_+ - \sigma_- \) field (d). Recoil parameter \( \varepsilon_R = 10^{-3} \). Red dashed horizontal lines correspond to average kinetic energy of atoms with Gaussian momentum distribution and Doppler temperature \( k_B T_D = h\gamma/2 \).

In the case of \( \text{lin} \perp \text{lin} \) configuration, the result of average kinetic energy of cooled atoms in steady-state are represented in Fig.1(a) for recoil parameters \( \varepsilon_R = 4 \times 10^{-4} \) corresponding to Cs atoms cooling with use of D2 line, and for \( \varepsilon_R = 10^{-3} \) in Fig.2(a) for comparison. Note that the average kinetic energy tends to have a dependance only on \( U_0 \) parameter for enough large detunings that corresponds to secular approximation, \( \sqrt{U_0/\hbar\omega_R} \ll |\delta|/\gamma \), used in\(^{22,23} \). The minimum kinetic energy reaches the value \( E_{\text{min}} \approx 22\hbar\omega_R \) for \( \varepsilon_R = 4 \times 10^{-4} \) and \( E_{\text{min}} \approx 20\hbar\omega_R \) for \( \varepsilon_R = 10^{-3} \) that is slightly less in comparison with results of\(^{22,23} \). This discrepancy arises from differences of optical transitions \( J_g \rightarrow J_e \) under consideration: \( J_g = 1 \rightarrow J_e = 2 \) in our case, and \( J_g = 1/2 \rightarrow J_e = 3/2 \) in\(^{22,23} \). For the optical transition \( J_g = 1/2 \rightarrow J_e = 3/2 \) our results are fully consistent with the results of\(^{22,23} \) for large enough detunings\(^{10,14} \).

The momentum distribution of cold atoms Fig.1(b) and Fig.2(b) occurs not to be Gaussian functions and cannot be analyzed in terms of temperature, but in terms of energy only. Indeed, the averaged kinetic energy, \( E_{\text{kin}}/\hbar\omega_R \simeq 1/(4\varepsilon) \), corresponding to equilibrium momentum distribution described by Gaussian function with Doppler temperature \( k_B T_D \simeq \hbar\gamma/2 \), is \( E_{\text{kin}}/\hbar\omega_R \simeq 625 \) for Fig.1(a) and \( E_{\text{kin}}/\hbar\omega_R \simeq 250 \) for Fig.2(a). One can see that it is much above the minimum energy values in Fig.1(a) and Fig.2(a), that corresponds to well known sub-Doppler cooling effects\(^{12} \), having semiclassical interpretation\(^{10,14} \).

We got the similar results for the steady-state in \( \sigma_+ - \sigma_- \) light field configuration Fig.1(c) and Fig.2(c). We have found that the average kinetic energy of cold atoms also tends to have an unique dependence on \( U_0 \) for large detunings (as for \( \text{lin} \perp \text{lin} \)). In these figures, the red dashed horizontal lines correspond to the average kinetic energy of atoms with Gaussian momentum distribution and Doppler temperature \( k_B T_D = h\gamma/2 \). As it seen, for atoms with recoil parameter \( \varepsilon_R \) smaller than \( 10^{-3} \), the universal dependence on \( U_0 \) is achieved with enough large red detunings, \( |\delta|/\gamma \gtrsim 5 \). Also, for these detunings the sub-Doppler cooling gets minimum values.

B. Not enough small value of recoil parameter

The more significant discrepancy from well-known picture of sub-Doppler cooling theory we have found for the recoil parameters at larger values (\( \varepsilon_R > 10^{-3} \)). For example, the results of average kinetic energy as function of \( U_0 \) for different detunings for \( \varepsilon_R = 10^{-2} \) is shown in Fig.3. The red dashed horizontal lines correspond to average kinetic energy of atoms with Gaussian momentum distribution and Doppler temperature \( k_B T_D = h\gamma/2 \). There is some area of \( U_0 \) and \( \delta \) when the sub-Doppler cooling stays effective with use of light field of \( \text{lin} \perp \text{lin} \) configuration. However, the magneto-optical trap does not operate in \( \text{lin} \perp \text{lin} \) configuration. For \( \sigma_+ - \sigma_- \) field, the sub-Doppler cooling mechanisms lose their efficiency, when the average kinetic energy of cooled atoms always stays above the Doppler cooling limit despite the fact that all requirements for sub-Doppler cooling according to well-known semiclassical-
cal theories are fulfilled, as well as the recoil parameter is small enough for these theories to be valid, \( \varepsilon_R \ll 1 \). As seen from Fig. 3(c), the \( \sigma_+ - \sigma_- \) configuration does not provide the sub-Doppler cooling for these atoms. We suppose that our analysis explains the experiments on Mg atoms cooling with use of \( 3s3p^3P_2 \rightarrow 3s3d^3D_3 \) optical transition and recent results of Sr atoms cooling with use of \( 5s5p^3P_2 \rightarrow 5s4d^3D_3 \) optical transition, where temperatures above the Doppler limit were achieved only.

As it seen in Fig. 4 for more larger values of recoil parameter \( \varepsilon_R \approx 10^{-1} \), the sub-Doppler cooling effects becomes not effective even in \( \text{lin} \perp \text{lin} \) field configuration. In contrast to the case of extremely small values of \( \varepsilon_R \), the minimal average kinetic energy is achieved for the smaller detunings \( \delta \approx -2\gamma \) in \( \text{lin} \perp \text{lin} \) field and for \( \delta \approx -\gamma \) in \( \sigma_+ - \sigma_- \) field for various values \( U_0 \).

IV. CONCLUSION

We discussed the limits of sub-Doppler cooling of atoms in a fields formed by counterpropagating waves with orthogonal linear or circular polarizations. In our analysis we solved the master equation for atom density matrix taking into account the quantum recoil effects as well as the effects of saturation in light field. Our method allows getting the steady-state solution without limitations and approximations was used by other authors.

In our analysis we payed main attention to the cooling limits for atoms with different ratio of recoil parameter \( \varepsilon_R = \omega_R/\gamma \). In particular, we show that the well-known picture of sub-Doppler cooling is valid only in the limit of extremely small values of \( \varepsilon_R \lesssim 10^{-3} \). In this case, the average kinetic energy of cooled atoms for large detunings tends to dependence on light shift parameter \( U \) only (for \( \text{lin} \perp \text{lin} \) as well as \( \sigma_+ - \sigma_- \) field configurations). For larger values of \( \varepsilon_R \approx 10^{-2} \) the sub-Doppler cooling mechanisms becomes less effective especially in \( \sigma_+ - \sigma_- \) configuration usually used for magneto-optical trap. As well, the minimum of kinetic energy of cooled atoms is achieved for smaller red detunings \( |\delta/\gamma| \approx 1 \) in comparison with \( \varepsilon_R \lesssim 10^{-3} \) case \( (|\delta/\gamma| \gtrsim 5) \), i.e. in the case of \( \varepsilon_R \gtrsim 10^{-2} \) the optimal detuning becomes close to the optimal value \( \delta/\gamma = -1/2 \) for Doppler cooling limit of two-level atom.

In addition, we emphasize that the steady-state of cooled atoms in general is essentially non-equilibrium and therefore unable to be described in term of unit temperature. In low intensity regime of laser cooling, the “hot” and “cold” fractions of atoms can be described by different temperatures of these fractions. In this case, we can see a small portion of atoms with sub-Doppler temperatures while the major atoms is in “hot” fraction with temperature near the Doppler limit (see Figs. 1(b) and 3). Thus, for atoms with recoil parameter above \( \varepsilon_R \gtrsim 10^{-2} \) the fraction of “cold” atoms becomes nonessential and the main role in laser cooling plays the well-known Doppler laser cooling mechanism, despite of the Zeeman atomic degeneracy and presence of polarization gradient of the cooling field.

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