Electroweak corrections to the Drell-Yan process in the high dimuon mass range

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The complete electroweak radiative $\mathcal{O}(\alpha)$ corrections to the Drell-Yan process at large invariant dimuon mass have been studied. All formulas for the cross sections and kinematical restrictions are presented in explicit form, for the simplification of calculation and coding the $\theta$– and $\delta$–functions are actively used. The FORTRAN code READY for the numerical analysis in the high energy region corresponding to the future experiments at the CERN Large Hadron Collider has been constructed. To simulate the detector acceptance we used the standard CMS detector cuts. The radiative corrections are found to become large at high dimuon mass $M$, the complete corrections at "bare" setup change the dimuon mass distribution up to $\sim -5.6\%$ ($-23.2\%; -35.3\%$) at the LHC energy and $M = 1 (3; 5)\text{TeV}$.

I. INTRODUCTION

More than twenty years the Standard Model (SM) keeps for oneself the status of consistent and experimentally confirmed theory since the experimental data of past and present accelerators (LEP, SLC and Tevatron) have shown no significant deviation from the SM predictions up to energy scales of several hundred GeV. However, New Physics (NP) models: various left-right symmetric models, extended gauge theories including grand unification theories, models of composite gauge bosons [1], some extra dimension scenarios [2], extra neutral gauge bosons [3] and fermion compositeness models [4] predict various deviations out of the SM predictions and, therefore, their testing in the new energy scale (thousands GeV) is one of the main tasks of modern physics. The forthcoming experiments at the collider LHC provides such possibility and probably will shed the light on this important problem in the immediate future.

The experimental investigation of the continuum for the Drell-Yan production of a dilepton pair, i.e. data on the cross section and the forward-backward asymmetry of the reaction

$$pp \rightarrow \gamma, Z \rightarrow \mu^+\mu^- X \quad (1)$$

at large invariant mass of a dimuon pair (see [3] and references therein) is considered to be one of the powerful tool in the experiments at the LHC from the NP exploration standpoint.

The studies of the NP effects are impossible without the exact knowledge of the SM predictions including higher order QCD and ElectroWeak radiative Corrections (EWC). The EWC to the reaction (1) are studied well (see papers on pure QED corrections [6], and the QED and electroweak corrections in the Z-peak region and above in [7] and numerous papers cited there). The EWC result contains so-called Double Sudakov Logarithms (DSL) [8], i.e. the expressions which are growing with the scale of energy, and thus giving one of the main effect in the region of large invariant dimuon mass. By now extensive studies have been done in this area. For instance, the weak Sudakov expansion for general four-fermion processes has been studied in detail (see, for example, [9] and the recent paper [10] along with the extensive list of references therein). Obviously, the collinear logarithms of QED radiative corrections can compete with the DSL in the investigated region. This important issue has been studied at the one-loop level in [7], where both the QED and weak corrections have been calculated for $M \leq 2 \text{TeV}$, but it has yet remained unsolved in the region of $M > 2 \text{TeV}$ (with the exception of short numerical estimation of EWC to $pp \rightarrow e^+e^- X$ by program ZGRAD [11] in [12], see Fig.7 there). Other important contributions in the investigated reaction at high invariant masses are the higher-order corrections (two-loop electroweak logarithms, at least), which also have been studied in the works [10, 13] (see also the numerous papers cited therein). Weak boson emission contribution has been recently calculated in [12] and the contribution of higher-order corrections due to multiple photon emission has been computed in [14], these contributions are beyond the presented calculations.

Thus, for the future experiments at LHC aimed at the searches of NP in the reaction (1) it is urgent to know exactly the SM predictions, including the radiative background, i.e. the processes, which are experimentally indistinguishable from (1). The important task is the insertion of this background into the LHC Monte Carlo generators and they

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should be both accurate and fast. For them to be fast it is necessary to have a set of compact formulas for the EWC. They are obtained in our previous paper [15] using the Asymptotic Approach (AA) and improved in subsequent paper [16]. To speed up the bremsstrahlung calculation and increase its accuracy it is very important to have all formulas for the cross sections and kinematical restrictions in explicit form appropriate for adaptive multidimensional integration. We obtained such form here and reached the high speed and good accuracy of calculation using the $\theta$– and $\delta$–functions apparatus.

II. NOTATIONS AND THE CROSS SECTION WITH THE BORN KINEMATICS

Our notations are the following (see Fig.1,a): $p_1(p_2)$ is the 4-momentum of the first (second) quark or antiquark with the flavor $q$ and mass $m_q$; $k_1(k_2)$ is the 4-momentum of the final muon $\mu^+$ ($\mu^-$) with the mass $m$; $q = k_1 + k_2$ is the 4-momentum of the $i$-boson with the mass $m_i$ ($i = \gamma, Z$); $P_A$ ($P_B$) is the 4-momentum of initial nucleon $A$ ($B$).

We use the standard set of Mandelstam invariants for the partonic elastic scattering $s, t, u$:

$$ s = (p_1 + p_2)^2, \quad t = (p_1 - k_1)^2, \quad u = (k_1 - p_2)^2, $$

and the invariant $S = (P_A + P_B)^2$ for hadron scattering. The invariant mass of dimuon is $M = \sqrt{q^2}$.

For a start let us present the convolution formula for the total hadronic ($H$) cross section, where we used such abbreviations and indices: "Born" (index 0), V-contribution: (indices: BSE for boson self energies, HV for "heavy" vertices, "$\gamma\gamma$" for the IR-finite part of $\gamma\gamma$-boxes, "$\gamma Z$" for the IR-finite part of $\gamma Z$-boxes, "$ZZ$" for the ZZ-boxes, "$WW$" for the WW-boxes, "$fin$" for the sum of "Light" Vertices (LV), the IR-part of $\gamma\gamma$ and $\gamma Z$-boxes and emission of "soft" photon, with the energy less than $\omega$). The "$fin$"-part is IR-finite and described by Born kinematics. Also we used common index for V-contribution $V = 0$, BSE, HV, $b, fin$ and special index for boxes $b = \gamma\gamma, \gamma Z, ZZ, WW$. Thus, the hadronic cross section looks like

$$ \sigma_H^{\text{V}} = \frac{1}{3} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^0 dt \sum_{q=u,d,...} [f_q^A(x_1, Q^2)f_q^B(x_2, Q^2)\tilde{\sigma}_V^{\text{q}}(t) + f_{\bar{q}}^A(x_1, Q^2)f_{\bar{q}}^B(x_2, Q^2)\tilde{\sigma}_{\bar{q}}^{\text{q}}(t)]\theta(t + \hat{s})\theta_{M}\theta_D, $$

here the $f_q^H(x, Q^2)$ is the probability at energy scale $Q^2$ of constituent $q$ with the fraction $x$ of the hadron’s momentum in hadron $H$ finding, $\hat{s} = x_1x_2S$, the $\theta$-function under integral sign determined by the kinematics of parton reaction,
the factor

$$\theta_M = \theta(\hat{s} - M_1^2)\theta(M_2^2 - \hat{s})$$

(4)

provides the integration in interval of invariant mass $M_1 \leq M \leq M_2$ and the factor

$$\theta_D = \theta(\zeta^* - \cos \theta)\theta(\zeta^* + \cos \theta)\theta(\zeta^* - \cos \alpha)\theta(\zeta^* + \cos \alpha)\theta(p_T(\mu^+) - p_T^{\min})\theta(p_T(\mu^-) - p_T^{\min})$$

(5)

cuts the region of integration according detector geometry, the parameters $\zeta^*$ and $p_T^{\min}$ will be discussed below. The expressions for the angles ($\theta$ ($\alpha$) is the scattering angle of the muon with the 4-momenta $k_1$ ($k_2$) in the center mass system of hadrons) and energies (also in the centre of hadron mass system) can be obtained as special situation of radiation case (“radiative” invariants $v$, $z$, $u$, $\chi = 0$) from the formulas described below. For transverse components the expressions take place: $p_T(\mu^+) = k_{10} \sin \theta$, $p_T(\mu^-) = k_{20} \sin \alpha$.

Let us enumerate all quark cross sections in (3) using agreement $\sigma(t) \equiv d\sigma/dt$.

The Born cross section looks like

$$\sigma^{0\bar{q}}_B(t) = \frac{2\pi\alpha^2}{s^2} \sum_{i,j=\gamma,Z} D^i D^i (b^{ij}_+ t^2 + b^{ij}_- u^2),$$

(6)

where the non-radiative boson propagators look like

$$D^i = \frac{1}{s - m_j^2 + i m_j \Gamma_j},$$

(7)

$\Gamma_j$ is the $j$-boson width,

$$b^{n,k}_\pm = \lambda_{q+}^{n,k} \lambda_{l+}^{n,k} \pm \lambda_{q-}^{n,k} \lambda_{l-}^{n,k}$$

(8)

and the combinations of coupling constants for $f$-fermion with $i$- (or $j$-) boson have the form

$$\lambda_f^{i,j} = v_f v_f + a_f a_f, \quad \lambda_f^{i,j} = v_f a_f + a_f v_f,$$

(9)

where

$$v_f = -Q_f, \quad a_f = 0, \quad v_f^2 = \frac{I^3_f - 2 s_W Q_f}{2 s_W c_W}, \quad a_f^2 = \frac{I^3_f}{2 s_W c_W},$$

(10)

$Q_f$ is the electric charge of fermion $f$, $I^3_f$ is the third component of the weak isospin of fermion $f$, and $s_W$ ($c_W$) is the sine(cosine) of the weak mixing angle: $s_W = \sqrt{1 - c_W^2}$, $c_W = m_W/m_Z$.

The BSE-part is

$$\sigma^{0\bar{q}}_{\text{BSE}}(t) = -\frac{4\alpha^2 \pi}{s^2} \left[ \sum_{i,j=\gamma,Z} \Pi^i S D^i D^j \sum_{\chi=+,-} \lambda^{i,j}_\chi \lambda^{i,j}_\chi B_{\chi} + \Pi^{\gamma Z} S D^i \sum_{\chi=+,-} \left( \lambda^{\gamma,Z}_\chi \lambda^{Z,j}_\chi + \lambda^{Z,j}_\chi \lambda^{\gamma,j}_\chi \right) B_{\chi} \right].$$

(11)

Here $\Pi^{\gamma,Z,\gamma Z}_S$ are connected with the renormalized photon–, $Z$– and $\gamma Z$–self energies as

$$\Pi^{\gamma}_S = \frac{\tilde{\Sigma}^\gamma}{s}, \quad \Pi^{Z}_S = \frac{\tilde{\Sigma}^Z}{s - m_Z^2}, \quad \Pi^{\gamma Z}_S = \frac{\tilde{\Sigma}^\gamma Z}{s}.$$

The HV-part looks like

$$\sigma^{0\bar{q}}_{\text{HV}}(t) = \frac{4\pi\alpha^2}{s^2} \Re \sum_{i,j=\gamma,Z} D^i D^i \sum_{\chi=+,-} \left( \lambda^{F,i,j}_\chi \lambda^{i,j}_\chi + \lambda^{i,j}_\chi \lambda^{F,i,j}_\chi \right) B_{\chi},$$

(12)

where the form factors $\lambda^{F,i,j}_\pm$ are explained in [13].
The boxes can be presented as

\[
\sigma_b^{q\bar{q}}(t) = \frac{2\alpha^3}{s^2} \sum_{k=\gamma, Z} D_k^+(\delta^{b,k}(t, u, b_+, b_-) - \delta^{b,k}(u, t, b_-, b_+)),
\]

where functions \(\delta^{b,k}(t, u, b_+, b_-)\) and all prescriptions for them can be found in [13, 16].

The "fin"-part (the result of infrared singularity cancellation of \(\gamma \gamma, \gamma Z,\) and "soft" bremsstrahlung) is

\[
\sigma_{fin}^{q\bar{q}}(t) = \frac{\alpha}{\pi} \delta_{fin}^{q\bar{q}}(t),
\]

\[
\delta_{fin}^{q\bar{q}}(t) = J_0 \log \frac{2\omega}{\sqrt{s}} + Q_1^2 \left( \frac{3}{2} \log \frac{s}{m^2} - 2 + \frac{\pi^2}{3} \right) + Q_2^2 \left( \frac{3}{2} \log \frac{s}{m^2} - 2 + \frac{\pi^2}{3} \right)
\]

\[
- Q_1 Q_l (\log \frac{s}{m} + \frac{\pi^2}{3} + \log \frac{1}{u} + 4L_i \frac{u}{t} - t),
\]

\[
J_0 = 2(Q_1^2 (\log \frac{s}{m^2} - 1) - 2Q_2 Q_l \log \frac{1}{u} + Q_2^2 (\log \frac{s}{m^2} - 1)),
\]

where \(L_i\) denotes the Spence dilogarithm. Let us note that correction \(\delta_{fin, FSR}^{q\bar{q}}\) is well known and presented, for example, in paper [19], and the correction \(\delta_{fin, ISR}^{q\bar{q}}\) can be found in the following way

\[
\delta_{fin, ISR}^{q\bar{q}} = \delta_{fin, FSR}^{q\bar{q}}(m \rightarrow m_q, Q_l \rightarrow Q_q).
\]

To find the cross section for \(q\bar{q}\)-case, it is necessary to change \(t \leftrightarrow u\) in the Born part and \(Q_q Q_l \rightarrow -Q_q Q_l\) in "fin"-part. The "hat" in formula (3) means only \(s \rightarrow \hat{s} \).

III. "HARD" PHOTONS

Let us present the hadronic cross section induced by bremsstrahlung (Fig.1,i-Fig.1,l). Introducing the total phase space of reaction as

\[
I^0_{th}[A] = \int_0^1 dx_1 \int_0^1 dx_2 \int dtdvzdu \frac{1}{\pi \sqrt{R_{u_1}}} \theta(R_{u_1}) \theta^R_{M} \theta^R_{D} A,
\]

where \(z = 2k_1 p, v = 2k_2 p, z_1 = 2p_1 p, u_1 = 2p_2 p\) and \(p\) is the 4-momentum of a real bremsstrahlung photon.

The factors \(\theta^R_M\) and \(\theta^R_D\) look in "radiative" case slightly different in comparison with "non-radiative" ones:

\[
\theta^R_M = \theta(\hat{s} - z - v - M_1^2 \theta(M_2^2 - \hat{s} + z + v),
\]

and for \(\theta^R_D\) we use "non-radiative" expression \(\theta_D\) [10], we should change only the angles and energies:

\[
\cos \theta = 1 + \frac{t}{k_{10} x_1 \sqrt{S}}, \cos \alpha = 1 + \frac{u + z}{k_{20} x_1 \sqrt{S}},
\]

\[
k_{10} = \frac{1}{2 \sqrt{S}} \frac{t}{x_1 + u}, k_{20} = \frac{1}{2 \sqrt{S}} \frac{t + z_1}{x_1 + u},
\]

where \(u = v - \hat{s} - t\) and \(z_1 = z - u_1 + v\).

The physical region \(\Omega\) is determined by \(\theta(R_{u_1})\), where \(R_{u_1}\) (\(R_{u_1}\) is the Gram determinant multiplied by constant factor) is described by:

\[
R_{u_1} = -A_{u_1} u_1^2 - 2B_{u_1} u_1 - C_{u_1},
\]

\[
A_{u_1} = -4m^2 s + (s - v)^2,
\]
\[ B_{u_1} = v[m^2(3s - v) + (s - v)(m_q^2 - s - t + v)] + z[m^2(s - v) - m_q^2(s + v) + st + v(s + t - v)], \]
\[ C_{u_1} = z^2[(m^4 + m_q^4 - 2m^2(m_q^2 + t - v) - m_q^2(t + v)) + + 2zv[m^4 + m_q^4 + m^2(s - 2t) - m^2(2m_q^2 + s + 2t - 2v) + (t - v)(s + t - v)] + + v^2[m^4 - 2m^2(m_q^2 + s + t - v) + (m_q^2 - s - t + v)^2] \] (18)

Then the total bremsstrahlung cross section have form:
\[
\sigma_R^H = \alpha^3 T_0 \frac{\hat{s}^{-2}}{\Omega} \sum_{\chi=\gamma,\gamma} \sum_{q=u,d,...} \sum_{i,j=\chi,Z} \lambda_{q\chi}^{i,j} \lambda_{l\chi}^{i,j} \times \\
([f_q^A(x_1, Q^2)f_q^B(x_2, Q^2) + \chi f_q^A(x_1, Q^2) f_q^B(x_2, Q^2)][Q^2 R_l^q D^j D^{j*} + Q^2 R_q^{u\chi} \Pi_l^{j*} + \Pi_l^l] + \Pi_l^{j*} + D^j D^{j*}) \bigg|_{s \rightarrow \hat{s}}. \] (19)

Indices \(l, qk \) and \( int \) mean the origin of emitted photon: lepton, quark and lepton-quark interference, i.e. Final State Radiation (FSR), Initial State Radiation (ISR) and their INTERference (INT), correspondingly. The "radiative" boson propagators look like
\[
\Pi^j = \frac{1}{s - z - v - m_j^2 + im_j \Gamma_j} \] (20)
and the expressions \( R \) can be found in Appendix A.

Dissecting the region of integration with the help of function
\[
\theta_{\omega} = \theta\left(\frac{v + z}{2\sqrt{s}} - \omega\right),
\]
we divide the cross section \[19\] in two parts: first one is corresponding to "soft" photons with the energy less than \( \omega \) (it goes to IR singularity cancellation in formula \[14\] of Sect.I) and the second one is corresponding to "hard" photons with the energy more than \( \omega \). We realize the numerical integration of \[15\] (and, certainly, of \[3\]) by Monte Carlo routine based on the VEGAS algorithm \[20\].

IV. DISCUSSION OF NUMERICAL RESULTS

First, we want to demonstrate the independence of results on unphysical parameters: "soft"-"hard" photon separator \( \omega \) (Fig\[2\]) and the quark mass (Fig\[3\]). In these Figs. we can see the relative corrections
\[
\delta^C = \sigma_{C}^{H}/\sigma_{0}^{H} \] (21)
to cross sections integrated over interval of invariant dimuon mass \( 1 \text{ TeV} \leq M \leq 14 \text{ TeV} \) and assuming \( \zeta^* = 1 \) and \( p_T^{min} = 0 \). Fig[4] shows the \( \omega \)-dependence for FSR (left picture), ISR (middle picture) and INT-part (right picture) separately in wide range of \( \omega: 10^{-2} \text{ GeV} \leq \omega \leq 10 \text{ GeV} \). We can see also the obvious property for sums the SOFT and the HARD parts: \( |\text{FSR}| > |\text{ISR}| > |\text{INT}| \); all of them are negative.

For the decision of the quark mass singularity problem we used the MS scheme \[21\] and the procedure of linearization which is well-grounded in \[22\]. After all manipulations the part of cross section which must be subtracted to be free from the quark mass dependence has the form (here we used abbreviations Q.S.=QUARK SING. and \( q(x, Q^2) \equiv f_q(x, Q^2) \))
\[
\sigma_{Q.S.}^{H} = \frac{1}{3} \int_0^1 dx_1 \frac{1}{0} dx_2 \int_0^1 dz \int_0^1 dt \sum_{u,d,...} \left[ (q(x_1)\Delta\bar{q}(x_2)\theta(z - x_2) + \Delta q(x_1, z)\bar{q}(x_2)\theta(z - x_1)) \sigma_0^{q\bar{q}}(t) + + (q \leftrightarrow \bar{q}) \right] \theta(t + \hat{s}) \theta_M \theta_D, \] (22)
where

\[
\Delta q(x, z) = \frac{\alpha}{2\pi} Q_q^2 \left[ \frac{1}{z} q(x, M_{sc}^2) - q(x, M_s^2) \right] \left[ 1 + \frac{z^2}{1 - z} \left( \log \frac{M_{sc}^2}{m_q^2} - 2 \log(1 - z) - 1 \right) \right]
\]  

(23)

and \( M_{sc} \) is the factorization scale \[21\]. Fig.3 shows the \( m_q \)-independence for ISR part of cross section (the asterisks on plot are the points where the calculation was made, they are connected by straight lines). We can see that in the range of rather big values (10 – 100) of ratio \( m_q/m_u \) the difference (SOFT+HARD)-(QUARK SING.) is constant (i.e. independent on \( m_q \)). In the region of small \( m_q \) this property is slightly broken. The reason is simple: at small parameter of mass the calculation of mass singularity cross section demands of more time (it is better to say – more iterations in integration). In Fig.3 all of points are obtained with the same number of iterations, so in the region of
small \( m_q \) the result for HARD ISR part has not so good accuracy, in actual fact this part is slightly more. Surely increasing the accuracy (and simultaneously the running time of code) we provide the exact cancellation of \( m_q \), this obvious graph of less importance than Fig.13 and we do not present it here.

In the following using FORTRAN program READY [24] (READY is "Radiative corrEctions to lArge invariant mass Drell-Yan process") the scale of electroweak radiative corrections and their effect on the observables of the Drell-Yan processes for future CMS experiments will be discussed. In Fig.4 and Fig.5 it is shown the differential Born cross section and the relative corrections to it

\[
\delta^C_M = \frac{d\sigma^H_M}{dM} / \frac{d\sigma^H_0}{dM}
\]  

(24)

as a functions of \( M \). The pure weak (total electroweak) corrections are presented in left (right) picture of Fig.5. The translation from total to the differential cross sections realized with the help of trick presented in Appendix B.

We used the following set of prescriptions:

- investigated reaction is (1) with the energy of LHC \( \sqrt{s} = 14 \) TeV,
- the set of SM input electroweak parameters: \( \alpha = 1/137.03599911, m_Z = 91.1876 \) GeV, \( m_W = 80.37399 \) GeV, \( \Gamma_Z = 2.4924 \) GeV, \( \Gamma_W = 2.0836 \) GeV, \( m_H = 115 \) GeV,
- muon mass \( m_\mu = 0.105658369 \) GeV, masses of fermions for loop contributions to the BSE: \( m_e = 0.51099892 \) keV, \( m_\tau = 1.77699 \) GeV, \( m_u = 0.06983 \) GeV, \( m_d = 1.2 \) GeV, \( m_t = 174 \) GeV, \( m_s = 0.06984 \) GeV, \( m_c = 0.15 \) GeV, \( m_b = 4.6 \) GeV (the light quark masses provide \( \Delta C_{had} = m_\tau^2 = 0.0276 \)),
- 5 active flavors of quarks in proton, their masses as regulators of the collinear singularity \( m_q = 10 \times m_u \),
- non-diagonal elements of CKM matrix = 0, diagonal ones = 1,
- "soft"-"hard" photon separator \( \omega = 10 \) GeV,
- the MRST2004QED set of unpolarized parton distribution functions [23] (with the choice \( Q = M_{sc} = m_Z \)),
- we impose the experimental restriction conditions on the detected lepton angle \( -\zeta^* \leq \zeta \leq \zeta^* \) and on the rapidity \( |y(l)| \leq y(l)^* \), see [24]; for CMS detector the cut values of \( \zeta^* \) and \( y(l)^* \) are determined as

\[
y(l)^* = -\log \tan \frac{\theta^*}{2} = 2.5, \quad \zeta^* = \cos \theta^* \approx 0.986614,
\]

(25)

also we used the second standard CMS restriction \( p_T(l) \geq 20 \) GeV,
- here we used so-called "bare" setup for muons identification requirements (no smearing, no recombination of muon and photon).

Let us discuss briefly the effects of EWC induced by different contributions (in region \( M = 1 \) TeV). The BSE-contribution is positive and \( \sim 0.12 \), it is usual effect of BSE. The HV-part gives positive contribution \( \sim 0.07 \) in spite of the negative sigh of DSL \( -\log^2 m_\tau^2 / s = -l^2 Z(W)_+ \) in diagrams Fig.1,b and Fig.1,c with the Z and W as additional virtual particle. Analysis shows that the Single Sudakov Logs (SSL=\( l Z(W)_+ \)) of diagrams Fig.1,d and Fig.1,e play the very important role in the region of TeV’s M. To determine that we can compare, for example, the coefficients at the functions \( \Lambda_2(k^2, M_W) \) (it contains DSL and SSL) and \( \Lambda_3(k^2, M_W) \) (it contains only SSL) in formulas (6.8)-(6.12) from [17], the second one is sometimes much more than first one (up to 9 times), whereas \( |DSL/SSL| = |SSL| \approx 4.79 \) at \( M = 1 \) TeV. The combined effect of all HV becomes positive. Then, the WW boxes are uniquely negative and they are the dominant contributions. Let us explain it by the example of WW-diagram and \( \gamma \)-exchange Born diagram interference: extracting this part of cross section (denote it \( \sigma_{WW,\gamma}^{u_\bar{u}+\bar{u}u} \)) and retaining only \( u \)-type of quark contributions and leading power of Sudakov logs (there is no SSL in WW-boxes). Then (see formula (40) of [14])

\[
\sigma_{WW,\gamma}^{u_\bar{u}+\bar{u}u} \sim u\bar{u} \cdot \delta^{WW,\gamma}(u, b_+, b_-) + \bar{u}u \cdot \delta^{WW,\gamma}(u, b_+, b_-)
\]

(26)

here \( u(\bar{u}) \equiv f_{W,u}^{p(x_1/x_2)}(x_1/x_2) \). In that way we can see the fact: the terms \( u\bar{u} \) and \( \bar{u}u \) contain the same \( b \) and different invariants \( t \) and \( u \) as arguments of \( \delta^{WW,\gamma} \). Further, using \( b_+^{WW,\gamma} = -2(uW)Q_q \) and \( b_-^{WW,\gamma} = 0 \) (see [8], [19], and \( u^j = v^j + a^j \)) we can make sure that

\[
\sigma_{WW,\gamma}^{u_\bar{u}+\bar{u}u} \sim -2(uW)^2Q_u[\bar{u}u \cdot 2^j + \bar{u}u \cdot u^2] < 0.
\]

(27)
Corresponding contribution of d-type of quarks also less than zero and looks like

\[ \sigma_{W^+W^-\gamma}^{dd+\bar{d}d} \sim 2(v_W^{W^-})^2 Q_d [d\bar{d} \cdot t^2 l_{W,t}^2 + \bar{d}d \cdot u^2 l_{W,u}^2] < 0. \]  

(28)

The same situation takes place also for WW \times Z-case. At last, ZZ-, ISR-, INT- parts are small enough to give determinant effect (ISR gives \( \sim -0.019 \), INT gives \( \sim -0.008 \), ZZ is \( \sim 0.0003 \)) and FSR-part is negative and \( \sim -0.071 \), so the total effect of EWC is found to be negative \( \sim -0.056 \).
V. CONCLUSIONS

The complete electroweak radiative $O(\alpha)$ corrections to the Drell-Yan process at large invariant dimuon mass have been studied. For the shortening of code running time (keeping an enough accuracy) we simplify the calculation as much as possible (using AA and generalized functions). Using FORTRAN code READY the numerical analysis is performed in the high energy region corresponding to the future experiments at the CERN Large Hadron Collider. To simulate the detector acceptance we used the standard CMS detector cuts. The radiative corrections are found to become large at high dimuon mass $M$, the complete corrections at "bare" setup change the dimuon mass distribution up to $\sim -5.6\%$ ($-23.2\%$; $-35.3\%$) at the LHC energy and $M = 1$ (3; 5) TeV.

Some issues have become beyond the scope of the presented paper (the detailed numerical analysis of process $pp \rightarrow e^+e^-X$ and other interesting observables: total inclusive cross section, forward-backward asymmetries; al last, "calo" results – taking into consideration also smearing and recombination). All that will be the subjects of future investigation but, first of all, due the importance and complexity of investigated problem, we should cross-check with the results of other groups (programs SANC [22], ZGRAD [7,11], ...) to make sure that our result is correct. Author will be grateful to all interested groups for giving a chance to compare the results in that stage.

VI. ACKNOWLEDGMENTS

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[24] FORTRAN code READY is available by contacting to author via e-mail
The expressions for the $R$ have such form: for lepton emission (see Fig.1,i and Fig.1,j)

$$R_{l+}^{q\bar{q}} = 2s - 2\frac{m_q^2}{z} (2u^2 + 2u_1 s + 4u_1 t - 4u_1 v + s^2 + 2st - 2sv + 2t^2 - 4tv + 2v^2)$$

$$-2 \frac{8}{z} (-u_1^2 - u_1 s - 2u_1 t - s^2 - 2st - 2t^2) - \frac{8}{z} (4u_1 + 4s + 6t - 3v)$$

$$-2 \frac{m_q^2}{v^2} (s^2 + 2st + 2t^2) - \frac{1}{v} s (-z + 2u_1 + 2s + 2t),$$

$$R_{l-}^{q\bar{q}} = 2 \frac{m_q^2}{z} s (2u_1 + s + 2t - 2v) + 2 \frac{s^2}{z v} (-u_1 - s - 2t) + \frac{s}{z} (4s + 2t - v)$$

$$+ 2 \frac{m_q^2}{v^2} s (s + 2t) + \frac{s}{v} (-z + 2u_1 + 2s + 2t) - 2s,$$

(A1)

for quark emission (see Fig.1,k and Fig.1,l)

$$R_{q\bar{q}}^{q\bar{q}} = 2(z - s + v) - 2 \frac{m_q^2}{z_1} (z^2 + 2zt + s^2 + 2st - 2sv + 2t^2 - 2tv + v^2)$$

$$-2 \frac{8}{z_1 u_1} (-s^2 - 2st + sv - 2t^2 + 2tv - v^2)$$

$$-\frac{1}{z_1} (z^2 - zs + 2zt + 2s^2 + 2st - sv + 4t^2 - 2tv + v^2)$$

$$-2 \frac{m_q^2}{u_1} (z^2 - 2zs - 2zt + 2zv + s^2 + 2st - 2sv + 2t^2 - 2tv + v^2)$$

$$-\frac{1}{u_1} (z^2 - 3zs - 2zt + 2zv + 4s^2 + 6st - 5sv + 4t^2 - 6tv + 3v^2),$$

$$R_{q\bar{q}}^{q\bar{q}} = 2 \frac{m_q^2}{z_1} (-z^2 - 2zt + s^2 + 2st - 2sv - 2tv + v^2) + 2 \frac{s^2}{z_1 u_1} (-s - 2t + v)$$

$$+ \frac{1}{z_1} (-z^2 + zs - 2zt + 2s^2 + 6st - 3sv - 2tv + v^2)$$

$$+ 2 \frac{m_q^2}{u_1} (z^2 - 2zs - 2zt + 2zv + s^2 + 2st - 2sv - 2tv + v^2)$$

$$+ \frac{1}{u_1} (z^2 - 3zs - 2zt + 2zv + 4s^2 + 6st - 5sv - 2tv + v^2),$$

(A2)

for lepton-quark interference

$$R_{int+}^{q\bar{q}} = 2(z - u_1 - s - 4t + 3v) + \frac{t}{z_1} (2s^2 + 4st - 2sv + 4t^2 - 2tv + v^2)$$

$$+ \frac{1}{u_1} (2s^3 + 6s^2 t - 6s^2 v + 8st^2 - 14stv + 7sv^2 + 4t^3 - 10t^2 v + 9t^2 v - 3v^3)$$

$$+ \frac{2}{z_1} (u_1 s - u_1 v + s^2 + 2st - 3sv - 3tv + 2v^2)$$

$$+ \frac{1}{z_1 v} (z^2 s + z^2 t + 2zst + 2zt^2 + 2s^3 + 6s^2 t + 8st^2 + 4t^3) + \frac{1}{z_1} (zt - 2s^2 - 4st + sv + tv)$$
\[
R_{\text{int}}(q, q') = 2(z - 2s - v) - \frac{t}{z_v}(2s^2 + 4st - 2sv - 2tv + v^2)
\]
\[
- \frac{1}{zu_1}(2s^3 + 6s^2t - 6s^2v + 4st^2 - 10stv + 5sv^2 - 2t^2v + 3tv^2 - v^3)
\]
\[
- \frac{2s}{z}(s - v) - \frac{1}{zu_1}(-z^2s - z^2t - 2zst - 2st^2 + 2s^3 + 6s^2t + 4st^2)
\]
\[
- \frac{1}{zu_1}(-z^2 - 2zs - 2zt + 2s^2 + 4st)
\]
\[
- \frac{2s}{v}(z - s) - \frac{1}{u_1}(-z^2 + 3zs + 5zt - 3zv - 4s^2 - 12st + 8sv - 4t^2 + 7tv - 3v^2).
\] (A3)

**APPENDIX B: TRANSLATION FROM TOTAL TO THE DIFFERENTIAL CROSS SECTION**

To translate the total non-radiative (3) (or radiative (19)) cross section to the differential one we should just differentiate it on variable \(M\) using obvious rule

\[
\frac{d\sigma}{dM} = -\left.\frac{d\sigma}{dM_1}\right|_{M_1=M}.
\] (B1)

After that and taking into consideration the formula

\[
\frac{d\theta}{dx} = \delta(x)
\]

we can significantly simplify the form of cross section, as we are in a position to integrate analytically over one of variable (here we choose \(x_2\) in such a way

\[
\int_a^b f(x)\delta(x - z)dx = f(z)\theta(z - a)\theta(b - z).
\]

Finally, we get very simple recipe for translation from total to the differential cross section: so, for radiative case to pass

\[
\sigma^H_{\text{hard}} \rightarrow \frac{d\sigma^H_{\text{hard}}}{dM}
\]

we should in formula (19)

1. do not integrate over \(x_2\) and omit \(\theta(\hat{s} - z - v - M_1^2)\),

2. substitute \(x_2 \rightarrow (M^2 + z + v)(Sx_1)^{-1}\) (or \(\hat{s} \rightarrow M^2 + z + v)\),

3. multiply by factor \(2M(Sx_1)^{-1}\theta(Sx_1 - M^2 - z - v).\)

For nonradiative case the translation steps are the same but "radiative" invariants should equal to zeros: \(v = z = 0\) since in that case \(p \rightarrow 0\).