Algorithm of linear discrete filtering with fuzzy modification of structure

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Abstract. The paper considers a task of processing measurements to determine the state vector of a dynamic object, for example, an aircraft. A linear discrete filtering algorithm based on the Kalman filter is proposed. It is capable of modifying its structure using fuzzy expert system depending on the intensity of the object maneuver. The performance and efficiency of the algorithm are confirmed by the results of numerical simulation.

1. Introduction
When solving problems of controlling dynamic objects, for example, aircraft of various classes, it is necessary to determine their state vector based on the measurement vector available to the observer. The vector can contain errors of various types. To reduce the influence of measurement errors on the resulting estimate, in practice the processing algorithms such as classical and advanced Kalman filters, nonlinear filter, \( \alpha - \beta \) -filter, etc. are widely used [1-25]. The quality of estimates generated by these filters is largely determined by the adequacy of the state vector models used. The adequacy of models, as a rule, is degraded by a random (from the point of view of the observer) maneuver of the aircraft. Using an inadequate model in the filter leads to its divergence. To prevent this, the following methods of filter adaptation to the maneuver [1-3, 5, 6] are used in the theory and practice of dynamic filtering: periodically adjusting the filter gain to the initial value, increasing the dispersion of the shaping noise, increasing the covariance matrix of the state vector estimation errors, expanding the state vector, alternate using the multiple filters, using the multi-model filters. The use of the considered methods, with the exception of the latter, involves solving the problem to detect a maneuver. For this reason, the adaptive filtering algorithms contain three main modules: a maneuver detection module; filter settings module; filter module [1-3]. The existing variety of dynamic filtering algorithms is due to the features of the implementation of these modules. For example, ref. [6] proposes an algorithm to adjust the Kalman filter by changing the values of the elements of the matrix of the intensities of the forming noises. A feature of this algorithm is the iteration adjustment, causing the dynamic filtering errors to arise. Thus, the problem to develop an algorithm adaptive to maneuver intensity is still relevant.

The purpose of the work is to reduce the dynamic filtering error of trajectory measurements under the conditions of the aircraft maneuver.

The problem to be solved is to develop a linear discrete filtering algorithm capable to modify its structure depending on the intensity of maneuver using a fuzzy expert system.

2. Mathematical formulation of the problem
Let the motion of a dynamic object on the time interval \([t_0, T]\) is described by the difference equation [1]
and the observation equation has the form
\[ Z_j = H_j X_j + N_{z_j}, \quad j = 1, 2, \ldots \]
where \( X_j = X(t_j) = [x_{s_j}, s = 1, q]^T \) is the vector of the object motion parameters, \( Z_j = [z_{s_j}, s = 1, p]^T \) is the vector of measurements, \( \Phi_j = \psi_s l_j, l = s = 1, q, \Gamma_j = \psi_k l_j, s = 1, q, k = 1, m, H_j = b_k s_j, k = 1, p, s = 1, q \) are the known functional matrices, \( A_{x_j} = [a_{ss_j}, s = 1, m]^T \) is the maneuver intensity vector, which elements belong to a priori unknown ranges \( a_{ss} \in [a_{x_{min}}, a_{x_{max}}] \), \( s = 1, m \) and represent the object accelerations with respect to the corresponding coordinate. \( N_{s_j} = [n_{ss_j}, s = 1, m]^T \), \( N_{z_j} = [n_{zz_j}, s = 1, p]^T \) are random noises of the object (1) and the observation channel (2), respectively, which have zero mathematical expectations and correlation matrices \( Q_j = \text{diag} \left[ q_{ss_j}, s = 1, p \right] \).

R_j = \text{diag} \left[ r_{ss_j}, s = 1, p \right].

It is required: to obtain based on the results of current observations \( Z_j \) the optimal (in terms of the mean-square) estimate \( \hat{X}_j \) of the filtration of the state vector (1) under conditions of a priori uncertainty with respect to the values of the elements of the acceleration vector \( A_{x_j} \).

2 Solution to the problem

Suppose that to form an estimate \( \hat{X}_j \) of the vector of motion parameters of the dynamic object a finite set of operators \( \{ R_k, k = 1, M \} \) is used, and each operator is the Kalman filter \([1, 6]\) adjusted to a specific value of the acceleration vector \( A_{x_j} \in \{ A_1^x, A_2^x, \ldots, A_M^x \} \):
\[ \hat{X}_j^k = \tilde{X}_j^k + K_j [Z_j - H_j \tilde{X}_j^k], \]
\[ \tilde{X}_j^k = \Phi_j X_{j-1}^k + \Gamma_j A_{x_j}^k, \]
\[ P_{j|j-1} = \Phi_j P_{j-1} \Phi_j^T + \Gamma_j Q_j \Gamma_j^T, \]
\[ K_j = P_{j|j-1} H_j^T [H_j P_{j|j-1} H_j^T + R_j]^{-1}, \]
\[ P_j = [I - K_j H_j] P_{j|j-1}, \]
where \( \tilde{X}_j^k \) is the estimate of the prediction of the state vector at the time \( j \), \( P_{j|j-1} \) is the symmetric prediction error matrix, \( P_j \) is the covariance matrix of filtering errors \( X_j - \tilde{X}_j \), \( K_j \) is the filter gain, \( I \) is the identity matrix.

At any given time only one filter is used, this means that the filters are switched depending on the information about the value of \( A_{x_j}^k \).

It can be seen that for the case when in model (1) \( A_{x_j} \notin \{ A_1^x, A_2^x, \ldots, A_M^x \} \) the estimate at the output of the filter \( \hat{X}_j^k \) contains a significant dynamic error due to the inadequacy of the model (4) to the real process.

To ensure the sustainability of the estimation \( \hat{X}_j \) it is necessary to develop an adaptive filtering algorithm that takes into account the uncertainty of the value \( A_{x_j} \).

This algorithm should detect the divergence of the estimate \( \hat{X}_j \) and modify the filter structure.

Let us successively consider the possible solutions to these problems.

The problem to detect the divergence. To solve this problem, we need to introduce the divergence index. To increase the reliability of the solution to the problem of the divergence detection, we will simultaneously use two particular indices.

The first partial indicator. Let the updating process (residual) \( e_j = Z_j - H_j \tilde{X}_{j|j-1} \) is used as the decisive statistics, and for the threshold \( S_0 \) the value
is used [1], where $\Psi_{lij}, \ l = 1, p$ is the diagonal elements of the matrix $\Psi_f = H_f R_i H_f^T + R_i$; $c$ is a positive constant which value is determined by the probability $P_{\alpha}$ of the value $\varepsilon_{ij}$ to fall into the interval $[-S_{0ij}, S_{0ij}]$. For example, when $P_{\alpha} = 0.9973$ we have $c = 3$.

In view of (8), the divergence can be detected by the condition [1]

$$|\varepsilon_{ij}| > S_{0ij}. \tag{9}$$

From (9) it follows that it is reasonable to use the relative value of the residual of the form

$$|\delta \varepsilon_{ij}| = \frac{|\varepsilon_{ij}|}{S_{0ij}}. \tag{10}$$

The second partial indicator. Indicator (10) takes into account both the abnormal random error and the dynamic error due to the inadequacy of the model (4) to the real process. To better take into account the dynamic error we introduce, by analogy with ref. [6], the modulus of the arithmetic mean of the residual $|\bar{\varepsilon}_j|$ (for simplicity, without loss of generality, we assume that the residual is scalar):

$$|\bar{\varepsilon}_j| = \frac{\sum_{i=1}^{n} |\varepsilon_{ij}|}{n}, \tag{11}$$

where $n$ is the number of consecutive measurements used to detect the divergence of the filter.

The calculation of indices (11) is possible when $j \geq n$, i.e. when no less than $n$ observations of the form (2) is performed.

By analogy with (10) we transform (11) to the form:

$$|\delta \bar{\varepsilon}_j| = \frac{\sum_{i=1}^{n} |\varepsilon_{ij}|}{n\sigma_{prg}}, \tag{12}$$

where $\sigma_{prg}$ is some admissible value of the standard deviation (MSE) of the error $\varepsilon_{ij}, j \geq n$.

The problem to modify the filter structure. For example, let the evaluation system contain two filters $\{R_k, \ k = 1, 2\}$, the first filter $(k = 1)$ is set to the lowest acceleration value $A_{\min}^1 = A_{\min}$ and the second filter $(k = 2)$ is set to the highest acceleration value $A_{\max}^2 = A_{\max}$. In this case, the actual value of acceleration in model (1) can take any value from the known interval $A_{\min}^1 \in [A_{\min}^2]$. To modify the filter structure, we use the operatively advising expert system (OAES), which is based on the use of fuzzy logical inference [26]. The use of OAES is reasonable due to the uncertainty of setting the values of the acceleration vector $A_{\phi}$. We represent the process of the filter operation in the form of a tuple of some problem situations (PrS) [26-28]. Any PrS can be described by a situational vector $SV = [sv_\xi, \ \xi = 1, 2]^T$, and each coordinate $sv_\xi$ of the vector is a linguistic variable with a given set of terms $SV_\xi = \{SV_1^\xi, \ l = 1, m_\xi\}$. We believe that for some particular implementations of the situational vector $sv$ there are precedents of successful solving the current PrS, characterized by a certain precedent vector $pv = \{pv_m, \ m = 1, m_{pv}\}$, and each coordinate $pv_m$ of the vector is also a linguistic variable with a given set of terms $PV_m, \ p = 1, n_m$. Let us introduce for the considered filtering system consisting of the finite set of operators $\{R_k, \ k = 1, 2\}$ of the form (3)–(7) a situational vector $SV = [sv_\xi, \ \xi = 1, 4]^T$ with elements: $sv_1$ – “relative value of the residual of the first filter $|\delta \varepsilon_{ij}^1|$”, $sv_2$ – is “value of the relative error modulus of the first filter $|\delta \varepsilon_{ij}^2|$”, $sv_3$ – is “relative value of the residual of the second filter $|\delta \varepsilon_{ij}^2|$”, $sv_4$ – is “value of the relative error modulus of the second filter $|\delta \varepsilon_{ij}^2|$”.

Let the variables of the situational vector are described by the following term-sets:
We assume that linguistic variables \( SV_\xi \), \( \xi = 1, 4 \) are given on the universe \( E = [0, \infty] \), and the terms are described by the membership functions \( \mu_{SV_\xi} \in \{ \mu_{SV_l^\xi}, l = 1, \infty, \xi = 1, 4 \} \), presented in Figures 1 and 2.

\[
SV_1 = SV_3 = \left\{ \begin{array}{c}
"Very low" \left( OH(\{\delta_{e_1}\}) \right) \\
"Low" \left( H(\{\delta_{e_1}\}) \right) \\
"Average" \left( CP(\{\delta_{e_1}\}) \right) \\
"High" \left( B(\{\delta_{e_1}\}) \right)
\end{array} \right.
\]

\[
SV_2 = SV_4 = \left\{ \begin{array}{c}
"Very low" \left( OH(\{\delta_{e_2}\}) \right) \\
"Low" \left( H(\{\delta_{e_2}\}) \right) \\
"Average" \left( CP(\{\delta_{e_2}\}) \right) \\
"High" \left( B(\{\delta_{e_2}\}) \right)
\end{array} \right.
\]

Let the following precedents be known for this class of PrS: the first model of the form (4) \((k = 1)\) is “very close” to the actual process, it is reasonable to use the estimation formed by the first filter, that is: \( \mathbf{\hat{x}}_j = \mathbf{\hat{x}}_j^1; PV_2 \) – the first model of the form (4) \((k = 1)\) is “closer” to the actual process.
compared to the second model ($k = 2$), it is reasonable to use a weighted estimate $\tilde{X}_j = 0.7\tilde{X}_j^1 + 0.3\tilde{X}_j^2$; $pv_3$ – the second model of the form (4) ($k = 2$) is “closer” to the actual process, compared to the first model ($k = 1$), it is reasonable to use a weighted estimate $\tilde{X}_j = 0.3\tilde{X}_j^1 + 0.7\tilde{X}_j^2$; $pv_4$ – the second model of the form (4) ($k = 2$) is “very close” to the actual process; it is reasonable to use the evaluation formed by the second filter, that is: $\tilde{X}_j = \tilde{X}_j^2$.

Taking into account the previously introduced situational vector and the known precedents, the system of rules $R_{m_{prm}}^m$, $m = \overline{1, m_{pv}}$, $r_m = \overline{1, N_m}$, describing the mechanism to solve the current PrS, takes the form:

\[
R_{m_{prm}}^m = \begin{cases} 
1, \text{if}
\begin{pmatrix}
sv_1 = OH(\delta \xi^1_{j1}) \\
sv_2 = OH(\delta \xi^1_{j2}) \\
sv_3 = BV(\delta \xi^1_{j3}) \\
sv_4 = BV(\delta \xi^1_{j4})
\end{pmatrix}, \text{then} (pv_1),
\end{cases}
\]

\[
R_{m_{prm}}^m = \begin{cases} 
2, \text{if}
\begin{pmatrix}
sv_1 = H(\delta \xi^1_{j1}) \\
sv_2 = H(\delta \xi^1_{j2}) \\
sv_3 = CP(\delta \xi^2_{j3}) \\
sv_4 = CP(\delta \xi^2_{j4})
\end{pmatrix}, \text{then} (pv_2),
\end{cases}
\]

\[
R_{m_{prm}}^m = \begin{cases} 
3, \text{if}
\begin{pmatrix}
sv_1 = CP(\delta \xi^1_{j1}) \\
sv_2 = CP(\delta \xi^1_{j2}) \\
sv_3 = H(\delta \xi^2_{j3}) \\
sv_4 = H(\delta \xi^2_{j4})
\end{pmatrix}, \text{then} (pv_3),
\end{cases}
\]

\[
R_{m_{prm}}^m = \begin{cases} 
4, \text{if}
\begin{pmatrix}
sv_1 = B(\delta \xi^1_{j1}) \\
sv_2 = B(\delta \xi^2_{j2}) \\
sv_3 = OH(\delta \xi^3_{j3}) \\
sv_4 = OH(\delta \xi^3_{j4})
\end{pmatrix}, \text{then} (pv_4).
\end{cases}
\]

To calculate the membership function of a precedent $pv_m$ we use the Mamdani-Zade rule [26-28]:

\[
\mu_{pv_m}(sv_1, sv_2, ..., sv_k) = \max_{r_m} \min_{l} \mu_{sv_{l}(m_{prm})}(sv_{l}),
\]

(13)

where $\mu_{sv_{l}(m_{prm})} \in \{\mu_{sv_{l}^1}, l = \overline{1, m_{prm}}\}$ is the membership function of the linguistic variable $sv_{l}$, which is part of the production rule $R_{m_{prm}}^m$.

Taking into account the equation (13), the most preferable precedent to solve observed PrS can be defined as follows:

\[
pv_m^* = \arg \max_m \mu_{pv_m}(sv_1, sv_2, ..., sv_k).
\]

(14)

The scheme OAES to modify the structure of the filter, is presented in Figure 3.
The system functions as follows: the specific values of the elements of the situational vector $\mathbf{s} \mathbf{v}^*$, formed in the module for calculating indices and modifying the filter structure (MCIMFS), are fed to the phasification block, where they are converted into fuzzy sets. The obtained data are input to the fuzzy inference block that implements the algorithm of choice of the most preferable precedent $pW^* \in \{pW_1, pW_2, pW_3, pW_4\}$ based on expressions (13), (14), using information from the knowledge base containing fuzzy production rules, as well as the form and parameters of the membership functions (see Figure 1 and Figure 2). The number of the most preferred precedent chosen to solve the observed PrS is used in the MCIMFS to switch one of the four calculated arrays of estimates $\{X_j^1\}$, $\{0.7X_j^1 + 0.3X_j^2\}$, $\{0.3X_j^1 + 0.7X_j^2\}$, $\{X_j^2\}$ to the output of the filtering device.

Thus, the OAES provides a modification of the filter structure depending on the change in the model of the estimated information process of the form (1).

To confirm the performance of the proposed algorithm, mathematical modeling was performed. It is assumed that the model of movement of the maneuvering aircraft has the form:

$$X_j = \begin{cases} \phi X_{j-1} + \Gamma n_{x_{j-1}}, & 0 \leq j < 200, \\ \phi X_{j-1} + \Gamma (2g + n_{x_{j-1}}), & 200 \leq j < 400, \\ \phi X_{j-1} + \Gamma (4g + n_{x_{j-1}}), & 400 \leq j < 600, \end{cases}$$

where

$$\phi = \begin{bmatrix} 1 \\ \frac{\tau}{T} \end{bmatrix}, \Gamma = \begin{bmatrix} \frac{\tau^2}{2} \\ \tau \end{bmatrix}, \tau = t_{j+1} - t_j = 1c, j = 0,600.$$  

To estimate the state vector $\hat{X}_j$ we use the system containing two filters $\{R_k, k = T, 2\}$. The first filter ($k = 1$) is set to the lowest acceleration $A_{xj}^1 = 0$, and the second filter ($k = 2$) is set to the highest acceleration $A_{xj}^2 = 4g, g = 9.8 \text{ m/s}^2$. The actual value of acceleration in model (1), as can be seen from (15), take three different values $A_{xj} \in \{0, 2g, 4g\}$. 

Figure 3. Structure of the intelligent measurement processing system.
As a result of the simulation, we find that the application of the developed algorithm allows to reduce the dynamic estimation error by 27% compared with the multi-structured filtering algorithm based on the choice of only one particular filter at a time [1].

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