Application of FE-SEA hybrid method in vibro-acoustic environment prediction of complex structure

Z H Qin, Z P Zhang, F Ren, H B Li, Z H Liu, Z Zhang, K Yuan, Y C Wang
Science and Technology on Reliability and Environment Engineering Laboratory, Beijing Institute of Structure and Environment Engineering, Beijing 100076, China

E-mail: zhh-qin@163.com

Abstract. Spacecraft sustains the complex and severe vibro-acoustic environments during the flight. Acoustic test is different from random vibration test, but they can complement the energy in different frequency band for each other. Vibro-acoustic environment prediction of structure can provide supports for measuring point arrangement and test condition defining. In this paper, the application of FE-SEA hybrid method in vibro-acoustic environment prediction of complex structure is studied. Structural response is predicted through reasonable loading method and modelling method. Vibro-acoustic test results show the validity of the prediction method.

1. Introduction
Spacecraft sustains the complex and severe vibro-acoustic environments during the flight, such as acoustic environment in light-off and transonic phase and vibration environment in light-off phase. There are two main vibration sources. One source is fluctuating thrust transmitted from engine to rocket, which can be simulated by shaker in low frequency band, but in high frequency band the excitation of shaker can not satisfy requirement for the limit of shaker and clamp fixture. The other source is aerodynamic force applied on the surface of rocket, which can simulate by ground acoustic test but not shaker. Acoustic test and random vibration test have essential difference, but they can complement each other. Combined test of vibration and acoustic will supply sufficient energy in different frequency band and much more really simulate the vibration source. Meanwhile, vibro-acoustic test can improve effectiveness of ground test. Vibro-acoustic environment prediction can supply supports for transducer arrangement and condition determination of test.

Yan compared the vibro-acoustic environment test, the acoustic test and the random vibration test of a spacecraft antenna, and analyzed the coupling effect of vibro-acoustic test. It was shown that the structure response was excited insufficiently in the low frequency region in the acoustic test and was attenuated in the high frequency region in the vibration test. Yang studied a vibro-acoustic environment test for the solar panel of a spacecraft. Two typical vibro-acoustic analysis methods, the hybrid FE-SEA and the coupled FE/BEM, were adopted to analyze the solar panel structure response under a random vibration and/or acoustic excitation condition. The components were tested effectively by the vibro-acoustic environment test. Zou applied a new technique based on the hybrid FE-SEA
method to the prediction of the random vibration environment, and studied the random vibration as specified by the predictive results[8].

In this paper, the application of FE-SEA hybrid method in vibro-acoustic environment prediction of complex structure is studied. Structural response is predicted through reasonable loading method and modelling method. To validate the prediction method, vibro-acoustic test are carried out.

2. Excitation Simulation

2.1. Random excitation simulation

Random vibration on shaker is a common dynamic environment method for spacecraft and provides important basis to reliability qualification for complex structure. The specimens are connected on shaker by rigid clamp fixture. The control spectrum of shaker is usually velocity spectrum or acceleration spectrum. The velocity spectrum or acceleration spectrum can be transmitted into excitation force of structure by large mass method. The principle is shown in Figure 1.

In Figure 1, $M_0$ is much larger than $m$, $\ddot{u}$ is the acceleration of $M_0$. The connection between $M_0$ and $m$ is assumed to be rigid. If the imported excitation of $M_0$ and the response of $m$ are both Sine function:

\[
\text{Imported excitation function: } P = p(\omega)e^{j\omega t} \\
\text{response function: } X = x(\omega)e^{j\omega t}
\]

to ensure the acceleration response of structure is $\ddot{u}$, the excitation force imposed should be:

\[
P = (M_0 + m)\ddot{u}
\]

For the case of $m \ll M_0$, $P \approx M_0\ddot{u}$, then $(M_0 + m)$ can be substituted by $M_0$. Therefore the dynamic equation of the system can be written as:

\[
\{M\}\ddot{x}(t) + \{B\}x(t) + \{K\}x(t) = \{P(\omega)e^{j\omega t}\}
\]

where $\{M\}$ is mass matrix, $\{B\}$ is damping matrix, $\{K\}$ is stiffness matrix.

$x(t)$ can be discretized as:

\[
\{x\} = [\phi]\{\xi(\omega)\}e^{j\omega t}
\]

where $[\phi]$ is modal matrix of system. Using equation (5), the displacement can be transformed from physical coordinate into modal coordinate. Submitting equation (5) into equation (4) and divided by $e^{j\omega t}$ on both sides, the following equation can be obtained:

\[
-\omega^2[M]\{\phi\}\{\xi(\omega)\} + i\omega[B][\phi]\{\xi(\omega)\} + [K][\phi]\{\xi(\omega)\} = \{P(\omega)\}
\]

Multiplying by $[\phi]^T$, equation (6) changes into:

\[
-\omega^2[\phi]^T[M][\phi]\{\xi(\omega)\} + i\omega[\phi]^T[B][\phi]\{\xi(\omega)\} + [\phi]^T[K][\phi]\{\xi(\omega)\} = [\phi]^T\{P(\omega)\}
\]

If damping matrix is orthogonal, equation (7) can be written as:
\[ -\omega^2 m_j \xi_j(\omega) + i \omega b_j \xi_j(\omega) + k_j \xi_j(\omega) = p_j(\omega) \]  

where \( m_j \) is \( j \)-order modal mass, \( b_j \) is \( j \)-order modal damping, \( k_j \) is \( j \)-order modal stiffness, \( p_j \) is \( j \)-order modal force. In equation (8), the response for \( j \)-order modal is:

\[ \xi_j(\omega) = \frac{p_j(\omega)}{-\omega^2 m_j + i \omega b_j + k_j} \]  

Submitting equation (9) into equation (5), the response of system in physical coordinate can be obtained. It is noted that the large mass is usually \( 10^3 \sim 10^8 \) times of mass of structure.

2.2. Acoustic excitation simulation

Usually, reverberation field can be simulated by diffused acoustic field. For diffused acoustic field, spatial correlation effect is considered.

The input of diffused acoustic field is root-mean-square pressure spectrum in the frequency band, which represents the surface pressure of plate and shell structures.

For low frequency, Finite Element (FE) method can be used to structural response analysis. For middle frequency, Finite Element- Statistical Energy Analysis (FE-SEA) hybrid method can be used\cite{9}-.\cite{13}.

3. Modelling method of complex structure in vibro-acoustic environment

The complex structure consists of solar panel, radome, bearing cylinder, grid beam, equipments and satellite adapter. There are many structural forms, including plate, shell and solid. In Figure 2, the red region presents rigid connection between the bottom of satellite adapter and excitation point. The restrained boundary condition is imposed on this point. Then a large mass is put on this point to simulate shaker. The excitation force obtained from equation (3) is also imposed on this point to simulate shaker input. The reverberation field is simulated by diffused acoustic field, respectively applied on FE face and SEA subsystem. During calculation, the acoustic load is transformed into power\cite{9}. For middle frequency (31.5-2000Hz), the model is constructed by FE-SEA hybrid method. By modal density and vibration transmission analysis, the modal density of solar panel and radome is larger than 3, so they are modelled by SEA method, and others are modelled by FE method. There are 10 FE subsystems and 3 SEA subsystems, where FE and SEA structure model is 2D and acoustic model is 3D. The virtual sensors are put on measure points to obtain simulated vibration response of structure. The model of the complex structure is shown in Figure 2.

4. Vibro-acoustic test of complex structure

To validate the prediction method of complex structure in vibro-acoustic environment, the test is carried out and the vibration response of each measure point is obtained. The test system consists of reverberation room, shaker, control and measure equipments (as shown in Figure 3-Figure 5). There are 3 and 6 measurement points on the solar panel and radome respectively. For the solar panel and
radome are modelled by SEA method, the response is obtained by averaging the measurement results of all point on the same subsystem.

![Figure 3. Vibro-acoustic test system of complex structure.](image)

![Figure 4. Measure points of vibro-acoustic test.](image)

![Figure 5. Control and measure equipments of vibro-acoustic test.](image)

The vibration condition is shown in Figure 6, and the acoustic condition is shown in Figure 7.

![Figure 6. Vibration condition of vibro-acoustic test.](image)

![Figure 7. Acoustic condition of vibro-acoustic test.](image)
5. Vibro-acoustic response analysis of complex structure

The response of the structure in 1/3Oct frequency band is simulated for the model in Figure 2. The simulated results are compared with test ones, shown in Figure 8.

Figure 8. Comparison of simulated results and test ones.
Table 1. Error between simulated results and test ones.

| Measuring points | Simulation(RMS) | Test(RMS) | error |
|------------------|-----------------|-----------|-------|
| 1                | 15.93g          | 19.07g    | -1.2 dB |
| 2                | 21.32g          | 23.02g    | -0.7 dB |
| 3                | 26.17g          | 21.25g    | 1.8 dB  |
| 4                | 17.76g          | 22.01g    | -1.9 dB |
| 5                | 17.13g          | 12.50g    | 2.7 dB  |
| 6                | 7.14g           | 7.12g     | 0.02 dB |
| 7                | 7.13g           | 6.35g     | 1.0 dB  |
| 8                | 19.17g          | 20.08g    | -0.4 dB |

Figure 8 and Table 1 show that the simulated results are well agreed with the test ones. The peaks of the response are captured. The differences between simulated results and test ones are within 3 dB. The prediction precision of FE-SEA hybrid method in vibro-acoustic environment of complex structure is validated.

6. Conclusions

In this paper, application of FE-SEA hybrid method in vibro-acoustic environment prediction of complex structure is studied. The simulation methods of random excitation and acoustic excitation are given. By modal density and vibration transmission analysis, the model of the complex structure is constructed. Structural response is predicted after reasonable loading and modelling. The prediction precision of FE-SEA hybrid method in vibro-acoustic environment of complex structure is validated through test. The peaks of the response are captured. The differences between simulated results and test ones are within 3 dB.

References

[1] Li C L, Chen Q H, Pu Y F. Spacecraft vibration test optimization in system and component level, *Spacecraft Environment Engineering* 24(3) 187-189
[2] Scharton T D. Vibration and acoustic testing of spacecraft, *Journal of Sound and Vibration* 36 14-19
[3] Forgrave J C, Man K F, Newell J M. Acoustic and random vibration test for low-cost missions, *Institute of Environmental Sciences: The 44th Annual Technical Meeting*, Phoenix, Arizona
[4] Larko J M, Coton V. Vibro-acoustic response of the NASA ACTS spacecraft antenna to launch acoustic excitation, *NASA/TM-2008-215168*
[5] Wang J, Ning W, Zhang J H. Simulation and analysis of the cabin typical structure under vibro-acoustic combined environment, *2009 International Conference on Measuring Technology and Mechatronics Automation*. Changsha China 327-330
[6] Yan T F, Zhang J G, Fang G Q, et al. A comparison between vibro-acoustic environment test and single environment test for a spacecraft antenna, *Spacecraft Environment Engineering* 31(2) 154-157
[7] Yang J, Zhang J G, Fang G Q, et al. Vibro-acoustic environment test and its simulation for spacecraft components, *Spacecraft Environment Engineering* 31(4) 369-373
[8] Zou Y J, Zhang J, Han Z Y. Random vibration specification for spacecraft components based on the hybrid FE-SEA method, *Spacecraft Environment Engineering* 27(4) 456-461
[9] Yao D Y, Wang Q Z. Theory and application of Statistical Energy Analysis method. *Beijing Institute of Technology Press*, China
[10] Soize C. A model and numerical method in the medium frequency range for vibro-acoustic predictions using the theory of structural fuzzy, *Journal of the Acoustical Society of America* 94 849-865.
[11] Langley R S, Bremner P. A hybrid method for the vibration analysis of complex structural-acoustic systems, *Journal of the Acoustical Society of America*, 105(3) 1657-1671.

[12] Shorter P J, Langley R S. On the reciprocity relationship between direct field radiation and diffuse reverberant loading, *Journal of the Acoustical Society of America* 117(1) 85-95.

[13] Shorter P J, Langley R S. Vibro-Acoustic analysis of complex systems, *Journal of Sound and Vibration* 288(3) 669-699.