Mass spectrum of the vector hidden charmed and bottomed tetraquark states

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Abstract

In this article, we perform a systematic study of the mass spectrum of the vector hidden charmed and bottomed tetraquark states using the QCD sum rules.

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1 Introduction

The Babar, Belle, CLEO, D0, CDF and FOCUS collaborations have discovered (or confirmed) a large number of charmonium-like states $X_{3940}$, $X_{3872}$, $Y_{4260}$, $Y_{4008}$, $Y_{3940}$, $Y_{4325}$, $Y_{4360}$, $Y_{4660}$, etc, and revitalized the interest in the spectroscopy of the charmonium states [1, 2, 3, 4]. Many possible assignments for those states have been suggested, such as multiquark states (whether the molecular type or the diquark-antidiquark type), hybrid states, charmonium states modified by nearby thresholds, threshold cusps, etc [1, 2, 3, 4].

The $Z^+_{1}(4430)$ observed in the $\psi'\pi^+$ decay mode is the most interesting subject [5], it can’t be a pure $c\bar{c}$ state due to the positive charge. We can distinguish the multiquark states from the hybrids or charmonia with the criterion of non-zero charge. The two resonance-like structures (thereafter we will denote them as $Z_1(4050)$ and $Z_2(4250)$ respectively) in the $\pi^+\chi_{c1}$ invariant mass distribution near 4.1 GeV are also particularly interesting [6]. Their quark contents must be some special combinations of the $c\bar{c}ud$, just like the $Z^+_{1}(4430)$, they cannot be the conventional mesons.

In Refs.[7–8], we assume that the hidden charmed mesons $Z_1(4050)$ and $Z_2(4250)$ are vector (and scalar) tetraquark states, and study their masses using the QCD sum rules. The numerical results indicate that the masses of the vector hidden charmed tetraquark states are about $M_{Z_1} = (5.12 \pm 0.15) \text{ GeV}$ or $M_{Z_2} = (5.16 \pm 0.16) \text{ GeV}$, while the masses of the scalar hidden charmed tetraquark states are bout $M_{Z} = (4.19 \pm 0.17) \text{ GeV}$. The scalar hidden charmed tetraquark states may have smaller masses than the corresponding vector states. In Ref. [9], we study the mass spectrum of the scalar hidden charmed and bottomed tetraquark states using the QCD sum rules. In this article, we extend our previous work to study the mass spectrum of the vector hidden charmed and bottomed tetraquark states.

In the QCD sum rules, the operator product expansion is used to expand the time-ordered currents into a series of quark and gluon condensates which parameterize the long distance properties of the QCD vacuum. Based on the quark-hadron duality, we can obtain copious information about the hadronic parameters at the phenomenological side [10–11].

The mass is a fundamental parameter in describing a hadron, whether or not there exist those hidden tetraquark configurations is of great importance itself, because it provides a
new opportunity for a deeper understanding of low energy QCD. The scalar and vector hidden charmed \((c\bar{c})\) tetraquark states may be observed at the KEKB, Tevatron and LHC, while the scalar and vector hidden bottomed \((b\bar{b})\) tetraquark states may be observed at the LHCb, where the \(b\bar{b}\) pairs will be copiously produced with the cross section about 500 \(\mu b\).

The hidden charmed and bottomed tetraquark states \((Z)\) have the symbolic quark structures:

\[
\begin{align*}
Z^+ &= Q\bar{Q}u\bar{d}; \\
Z^0 &= \frac{1}{\sqrt{2}}Q\bar{Q}(u\bar{u} - d\bar{d}); \\
Z^- &= Q\bar{Q}d\bar{u}; \\
Z^+_s &= Q\bar{Q}u\bar{s}; \\
Z^-_s &= Q\bar{Q}s\bar{u}; \\
Z^0_s &= Q\bar{Q}d\bar{s}; \\
Z_\phi &= \frac{1}{\sqrt{2}}Q\bar{Q}(u\bar{u} + d\bar{d}); \\
Z_\phi &= Q\bar{Q}s\bar{s},
\end{align*}
\]

where the \(Q\) denote the heavy quarks \(c\) and \(b\).

We can construct the tetraquark states with the diquark and antidiquark pairs. The diquarks have five Dirac tensor structures, scalar \(C_{\gamma_5}\), pseudoscalar \(C\), vector \(C_{\gamma_\mu}\gamma_5\), axial vector \(C_{\gamma_\mu}\) and tensor \(C_{\sigma_\mu\nu}\). The structures \(C_{\gamma_\mu}\) and \(C_{\sigma_\mu\nu}\) are symmetric, the structures \(C_{\gamma_5}, C\) and \(C_{\gamma_\mu}\gamma_5\) are antisymmetric. The attractive interactions of one-gluon exchange favor formation of the diquarks in color antitriplet \(\bar{3}_c\), flavor antitriplet \(\bar{3}_f\) and spin singlet \(1_s\) \([12,13]\). In this article, we assume the vector hidden charmed and bottomed mesons \(Z\) consist of the \(C_{\gamma_5} - C_{\gamma_\mu}\gamma_5\) type and \(C - C_{\gamma_\mu}\) type diquark structures, and construct the interpolating currents \(J_\mu(x)\) and \(\eta_\mu(x)\):

\[
\begin{align*}
J_\mu^Z(x) &= \epsilon_{ijk}\epsilon_{imn} u_j^T(x)C_{\gamma_5}Q_k(x)\bar{Q}_m(x)\gamma_5\gamma_\mu C_d^T(x), \\
J_\mu^Z(x) &= \frac{\epsilon_{ijk}\epsilon_{imn}}{\sqrt{2}} [u_j^T(x)C_{\gamma_5}Q_k(x)\bar{Q}_m(x)\gamma_5\gamma_\mu C_{\bar{d}}^T(x) - (u \rightarrow d)], \\
J_\mu^Z(x) &= \epsilon_{ijk}\epsilon_{imn} u_j^T(x)C_{\gamma_5}Q_k(x)\bar{Q}_m(x)\gamma_\mu C_s^T(x), \\
J_\mu^Z(x) &= \epsilon_{ijk}\epsilon_{imn} d_j^T(x)C_{\gamma_5}Q_k(x)\bar{Q}_m(x)\gamma_5\gamma_\mu C_{\bar{s}}^T(x), \\
J_\mu^Z_\phi(x) &= \frac{\epsilon_{ijk}\epsilon_{imn}}{\sqrt{2}} [u_j^T(x)C_{\gamma_5}Q_k(x)\bar{Q}_m(x)\gamma_5\gamma_\mu C_{\bar{d}}^T(x) + (u \rightarrow d)], \\
J_\mu^Z_\phi(x) &= \epsilon_{ijk}\epsilon_{imn} s_j^T(x)C_{\gamma_5}Q_k(x)\bar{Q}_m(x)\gamma_\mu C_s^T(x), \\
\eta_\mu^Z(x) &= \epsilon_{ijk}\epsilon_{imn} u_j^T(x)CQ_k(x)\bar{Q}_m(x)\gamma_\mu C_d^T(x), \\
\eta_\mu^Z(x) &= \frac{\epsilon_{ijk}\epsilon_{imn}}{\sqrt{2}} [u_j^T(x)CQ_k(x)\bar{Q}_m(x)\gamma_\mu C_{\bar{d}}^T(x) - (u \rightarrow d)], \\
\eta_\mu^Z(x) &= \epsilon_{ijk}\epsilon_{imn} u_j^T(x)CQ_k(x)\bar{Q}_m(x)\gamma_\mu C_{\bar{s}}^T(x), \\
\eta_\mu^Z(x) &= \epsilon_{ijk}\epsilon_{imn} d_j^T(x)CQ_k(x)\bar{Q}_m(x)\gamma_5\gamma_\mu C_{\bar{s}}^T(x), \\
\eta_\mu^Z_\phi(x) &= \frac{\epsilon_{ijk}\epsilon_{imn}}{\sqrt{2}} [u_j^T(x)CQ_k(x)\bar{Q}_m(x)\gamma_5\gamma_\mu C_{\bar{d}}^T(x) + (u \rightarrow d)], \\
\eta_\mu^Z_\phi(x) &= \epsilon_{ijk}\epsilon_{imn} s_j^T(x)CQ_k(x)\bar{Q}_m(x)\gamma_\mu C_s^T(x),
\end{align*}
\]

where the \(i, j, k, \cdots\) are color indexes. In the isospin limit, the interpolating currents result in six distinct expressions, which are characterized by the Dirac structures of the interpolating currents and the number of the \(s\) quark they contain.
We can also interpolate the vector tetraquark states with the currents \( \hat{J}_\mu(x) \) and \( \hat{\eta}_\mu(x) \), which consist of \( C\gamma_\mu\gamma_5 - C\gamma_5 \) type and \( C\gamma_\mu - C \) type diquark structures, respectively:

\[
\begin{align*}
\hat{J}^{\mu}_{\eta}(x) &= \epsilon^{ijk}\epsilon^{mn}u^T_j(x)C\gamma^\mu\gamma_5Q_k(x)\bar{Q}_m(x)\gamma_5C\bar{t}^T_n(x), \\
\hat{J}^{\mu}_{Z^0}(x) &= \frac{\epsilon^{ijk}\epsilon^{mn}}{\sqrt{2}}\left[u^T_j(x)C\gamma^\mu\gamma_5Q_k(x)\bar{Q}_m(x)\gamma_5C\bar{u}^T_n(x) - (u \rightarrow d) \right], \\
\hat{J}^{\mu}_{Z^+}(x) &= \epsilon^{ijk}\epsilon^{mn}u^T_j(x)C\gamma^\mu\gamma_5Q_k(x)\bar{Q}_m(x)\gamma_5C\bar{s}^T_n(x), \\
\hat{J}^{\mu}_{Z^0}(x) &= \epsilon^{ijk}\epsilon^{mn}d^T_j(x)C\gamma^\mu\gamma_5Q_k(x)\bar{Q}_m(x)\gamma_5C\bar{d}^T_n(x), \\
\hat{J}^{\mu}_{Z^+}(x) &= \frac{\epsilon^{ijk}\epsilon^{mn}}{\sqrt{2}}\left[u^T_j(x)C\gamma^\mu\gamma_5Q_k(x)\bar{Q}_m(x)\gamma_5C\bar{u}^T_n(x) + (u \rightarrow d) \right], \\
\hat{J}^{\mu}_{Z^0}(x) &= \epsilon^{ijk}\epsilon^{mn}s^T_j(x)C\gamma^\mu\gamma_5Q_k(x)\bar{Q}_m(x)\gamma_5C\bar{s}^T_n(x), \\
\hat{J}^{\mu}_{Z^+}(x) &= \epsilon^{ijk}\epsilon^{mn}d^T_j(x)C\gamma^\mu\gamma_5Q_k(x)\bar{Q}_m(x)\gamma_5C\bar{d}^T_n(x), \\
\hat{J}^{\mu}_{Z^0}(x) &= \frac{\epsilon^{ijk}\epsilon^{mn}}{\sqrt{2}}\left[u^T_j(x)C\gamma^\mu\gamma_5Q_k(x)\bar{Q}_m(x)\gamma_5C\bar{u}^T_n(x) - (u \rightarrow d) \right], \\
\hat{J}^{\mu}_{Z^+}(x) &= \epsilon^{ijk}\epsilon^{mn}u^T_j(x)C\gamma^\mu\gamma_5Q_k(x)\bar{Q}_m(x)\gamma_5C\bar{s}^T_n(x), \\
\hat{J}^{\mu}_{Z^0}(x) &= \epsilon^{ijk}\epsilon^{mn}s^T_j(x)C\gamma^\mu\gamma_5Q_k(x)\bar{Q}_m(x)\gamma_5C\bar{d}^T_n(x). \\
\end{align*}
\]

Our analytical results indicate that the current \( J^\mu(x) (\eta^\mu(x)) \) and \( \hat{J}^\mu(x) (\hat{\eta}^\mu(x)) \) lead to the same expression. The special superpositions \( tJ^\mu(x) + (1-t)\hat{J}^\mu(x) \) and \( t\hat{\eta}^\mu(x) + (1-t)\hat{\eta}^\mu(x) \) cannot improve the predictions remarkably, where \( t = 0 - 1 \).

The article is arranged as follows: we derive the QCD sum rules for the \( Z \) in section 2; in section 3, numerical results and discussions; section 4 is reserved for conclusion.

## 2 QCD sum rules for the vector tetraquark states \( Z \)

In the following, we write down the two-point correlation functions \( \Pi_{\mu\nu}(p) \) in the QCD sum rules,

\[
\Pi_{\mu\nu}(p) = i \int d^4xe^{ip\cdot x}\langle 0|T\left\{ J/\eta_\mu(x)J/\eta_\nu^+(0) \right\}|0\rangle, \tag{4}
\]

where the \( J^\mu(x) (\eta^\mu(x)) \) denotes the interpolating currents \( J^\mu_{\eta}(x) (\eta^\mu(x)) \), \( J^\mu_{Z^0}(x) (\eta^\mu_{Z^0}(x)) \), \( J^\mu_{Z^+}(x) (\eta^\mu_{Z^+}(x)) \), etc.

We can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators \( J_\mu(x) \) and \( \eta_\mu(x) \) into the correlation functions \( \Pi_{\mu\nu}(p) \) to obtain the hadronic representation [10][11]. After isolating the ground state contribution from the pole term of the \( Z \), we get the following result,

\[
\Pi_{\mu\nu}(p) = \frac{\lambda^2_Z}{M^2_Z - p^2} \left[-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right] + \cdots, \tag{5}
\]

3
where the pole residue (or coupling) $\lambda_Z$ is defined by

$$
\lambda_Z e_\mu = \langle 0 | J/\eta_\mu(0) | Z(p) \rangle ,
$$

where the $e_\mu$ denotes the polarization vector.

After performing the standard procedure of the QCD sum rules, we obtain the following twelve sum rules:

$$
\lambda_\pm^2 e^{-M_\pm^2 s} = \int_{\Delta_{\pm s}} \rho_\pm^2(s) e^{-\frac{s}{M_\pm^2}} ,
$$

$$
\rho_{\bar{q}q}^\pm(s) = \frac{1}{3072\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\alpha d\beta \alpha \beta (1 - \alpha - \beta)^3 (s - \bar{m}_Q^2)^2 (35s^2 - 26s\bar{m}_Q^2 + 3m_\pi^4)
\pm \frac{m_Q \langle \bar{q}q \rangle}{32\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta (1 - \alpha - \beta) (s - \bar{m}_Q^2) [(4\beta - 3\alpha)s + (\alpha - 2\beta)\bar{m}_Q^2]
\pm \frac{m_Q \langle \bar{q}g_s \sigma Gq \rangle}{64\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta [(2\alpha - 3\beta)s - (\alpha - 2\beta)\bar{m}_Q^2]
\pm \frac{m_\pi^2 \langle \bar{q}q \rangle^2}{12\pi^2} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha ,
$$

$$
\rho_{qq}^\pm(s) = \frac{1}{3072\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\alpha d\beta \alpha \beta (1 - \alpha - \beta)^3 (s - \bar{m}_Q^2)^2 (35s^2 - 26s\bar{m}_Q^2 + 3m_\pi^4)
\pm \frac{m_Q m_\pi}{256\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta \alpha \beta (1 - \alpha - \beta)^2 (s - \bar{m}_Q^2)^2 (5s - 2\bar{m}_Q^2)
\pm \frac{m_\pi \langle ss \rangle}{64\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta \alpha \beta (s - \bar{m}_Q^2) (2s - \bar{m}_Q^2)
\pm \frac{m_Q \langle \bar{q}g_s \sigma Gq \rangle}{64\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\alpha (s - \bar{m}_Q^2)
\pm \frac{m_Q \langle \bar{q}g_s \sigma Gs \rangle}{64\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta \alpha \beta (s - \bar{m}_Q^2)
\pm \frac{m_\pi m_\pi \langle \bar{q}q \rangle}{192\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta \alpha \beta [8s - 3\bar{m}_Q^2 + s^2 \delta(s - \bar{m}_Q^2)]
\pm \frac{m_\pi m_\pi \langle \bar{q}q \rangle}{16\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta (s - \bar{m}_Q^2)
\pm \frac{m_\pi \langle \bar{q}q \rangle^2}{12\pi^2} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta \alpha \beta (2 + s \delta(s - \bar{m}_Q^2))
\pm \frac{m_\pi m_\pi \langle \bar{q}q \rangle}{24\pi^2} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta [2 + s \delta(s - \bar{m}_Q^2)] .
$$
\[
\rho(s) = \frac{1}{3072\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta \alpha \beta (1 - \alpha - \beta)^3 (s - \tilde{m}_Q^2)^2 (35s^2 - 26s\tilde{m}_Q^2 + 3\tilde{m}_Q^4) \\
\pm \frac{m_s m_Q}{256\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta (1 - \alpha - \beta)^2 (s - \tilde{m}_Q^2)^2 \left[ (4\alpha - 5\beta)s - (\alpha - 2\beta)\tilde{m}_Q^2 \right] \\
\pm \frac{m_s (s\bar{s})}{32\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta \alpha \beta (1 - \alpha - \beta) (15s^2 - 16s\tilde{m}_Q^2 + 3\tilde{m}_Q^4) \\
\pm \frac{m_Q (s\bar{s})}{16\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta (1 - \alpha - \beta) (s - \tilde{m}_Q^2) \left[ (4\beta - 3\alpha)s + (\alpha - 2\beta)\tilde{m}_Q^2 \right] \\
\pm \frac{m_Q (s\bar{s})}{64\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta \left[ (2\alpha - 3\beta)s + (\alpha - 2\beta)\tilde{m}_Q^2 \right] \\
\pm \frac{m_s (s\bar{s})}{96\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta \alpha \beta \left[ 8s - 3\tilde{m}_Q^2 + s^2\delta(s - \tilde{m}_Q^2) \right] \\
\pm \frac{m_s^2 Q (s\bar{s})}{8\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta (s - \tilde{m}_Q^2) \\
\pm \frac{m_Q^2 (s\bar{s})^2}{12\pi^2} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha - \frac{m_s m_Q^2 (s\bar{s})G_s}{32\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha, \\
\] (10)

where the \(i\) denote the \(c\bar{c}ss, c\bar{c}qs, c\bar{c}q\bar{q}, b\bar{b}ss, b\bar{b}q\bar{q}\) and \(bbq\bar{q}\) channels, respectively, the \(s_0^i\) are the corresponding continuum threshold parameters, the \(\pm\) denotes the current operators of the \(C\gamma_5 - C\gamma_5\gamma_5\) type, and \(C - C\gamma_5\) type respectively, and the \(M^2\) are the Borel parameters; \(\alpha_{\max} = 1 + \sqrt{\frac{1 - 4m_Q^2}{2}}, \quad \alpha_{\min} = 1 - \sqrt{\frac{1 - 4m_Q^2}{2}}, \quad \beta_{\min} = \frac{\alpha m_Q}{\alpha - \bar{m}_Q^2}, \quad \bar{m}_Q = \frac{(\alpha + \beta)\tilde{m}_Q^2}{\alpha \beta},\)

\(\bar{m}_Q = \frac{m_Q^2}{\alpha (1 - \alpha)}\). The thresholds \(\Delta_{\pm i}\) can be sorted into three sets, we introduce the \(q\bar{q}, q\bar{s}\) and \(ss\) to denote the light quark constituents in the vector tetraquark states to simplify the notation, \(\Delta_{q\bar{q}} = 4m_Q^2, \Delta_{q\bar{s}} = (2m_Q + m_s)^2, \Delta_{ss} = 4(m_Q + m_s)^2\).

We carry out operator product expansion to the vacuum condensates adding up to dimension-6. In calculation, we take assumption of vacuum saturation for high dimension vacuum condensates, they are always factorized to lower condensates with vacuum saturation in the QCD sum rules, factorization works well in large \(N_c\) limit. In this article, we take into account the contributions from the quark condensates, mixed condensates, and neglect the contributions from the gluon condensate. In calculation, we observe the contributions from the glue condensate are suppressed by large denominators and would not play any significant roles [14] [15] [16] [17] [18]. The term \(\langle q\bar{q}\rangle\langle s\bar{s}\rangle\langle qg\sigma Gq\rangle\langle s\bar{s}\rangle\sigma Gs\rangle\) is suppressed by a large numerical denominator and an additional factor \(1/M^2\) comparing with the term \(\langle q\bar{q}\rangle\langle s\bar{s}\rangle\langle q\rangle\langle s\bar{s}\rangle\), and can be neglected safely. Furthermore, we neglect the terms proportional to the \(m_q\) and \(m_t\), their contributions are of minor importance.

Differentiating the Eq.(7) with respect to \(1/M^2\), then eliminate the pole residues \(\lambda_{\pm i}\), we can obtain the sum rules for the masses of the \(Z\),

\[
M_{\pm i}^2 = \frac{\int_{\Delta_{\pm i}}^{s_0^i} ds \rho_{\pm i}^*(s)s e^{-s/M^2}}{\int_{\Delta_{\pm i}}^{s_0^i} ds \rho_{\pm i}^*(s) e^{-s/M^2}}. \\
\] (11)
3 Numerical results and discussions

The input parameters are taken to be the standard values $\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{GeV})^3$, $\langle \bar{s}s \rangle = (0.8 \pm 0.2) \langle \bar{q}q \rangle$, $\langle \bar{q}g_s \sigma Gq \rangle = m_Q^2 \langle \bar{q}q \rangle$, $\langle \bar{g}_s \sigma Gs \rangle = m_Q^2 \langle \bar{s}s \rangle$, $m_Q^2 = (0.8 \pm 0.2) \text{GeV}^2$, $m_s = (0.14 \pm 0.01) \text{GeV}$, $m_c = (1.35 \pm 0.10) \text{GeV}$ and $m_b = (4.8 \pm 0.1) \text{GeV}$ at the energy scale about $\mu = 1 \text{GeV}$ [10,11,21].

In the conventional QCD sum rules [10,11], there are two criteria (pole dominance and convergence of the operator product expansion) for choosing the Borel parameter $M^2$ and threshold parameter $s_0$. The light tetraquark states can not satisfy the two criteria, although it is not an indication non-existence of the light tetraquark states. We impose the two criteria on the heavy tetraquark states to choose the Borel parameter $M^2$ and threshold parameter $s_0$.

If the resonance-like structures $Z_1(4050)$ and $Z_2(4250)$ observed by the Belle collaboration in the $\pi^+\chi_{c1}$ invariant mass distribution near 4.1 GeV in the exclusive $B^0 \rightarrow K^-\pi^+\chi_{c1}$ decays are scalar tetraquark states [6], the central value of the threshold parameter can be taken as $s_{qq}^0 = (4.248 + 0.5)^2 \text{GeV}^2 \approx 23 \text{GeV}^2$ to take into account all possible contributions from the ground state, where we choose the energy gap between the ground states and the first radial excited states to be 0.5 GeV. The numerical result indicates that it is a good choice, the predicted mass is about $M_{qq} = (4.19 \pm 0.17) \text{GeV}$ [8], which is consistent with the experimental data.

However, the threshold parameter $s_0 = 23 \text{GeV}^2$ cannot result in a reasonable Borel window for the vector hidden charmed tetraquark state (taking the $Z_1(4050)$ and $Z_2(4250)$ as the vector tetraquark states), we have to postpone it tentatively to larger values. It is not an indication that non-existence of the vector hidden charmed tetraquark states below 4.7 GeV; in other words, the QCD sum rules alone cannot indicate (non-) existence of the multiquark states strictly.

For the tetraquark states consist of light flavors, if the perturbative terms have the main contribution (in the conventional QCD sum rules, the perturbative terms always have the main contribution), we can approximate the spectral density with the perturbative term (where the $A$ are some numerical coefficients) [22].

$$B_{sA} \Pi \sim A \int_0^\infty s^4 e^{-sM^2} ds = AM^{10} \int_0^\infty t^4 e^{-t} dt,$$

then take the pole dominance condition,

$$\frac{\int_0^{t_0} t^4 e^{-t} dt}{\int_0^\infty t^4 e^{-t} dt} \geq 50\%,$$

and obtain the approximated relation,

$$t_0 = \frac{s_0}{M^2} \geq 4.7.$$

The superpositions of different interpolating currents can only change the contributions from different terms in the operator product expansion, and improve convergence, they cannot change the leading behavior of the spectral density $\rho(s) \propto s^4$ of the perturbative term [22]. This relation is difficult to satisfy for the light flavor tetraquark states [14,15,10,17,18,19,20], if we take the Borel parameter has the typical value $M^2 = 1 \text{GeV}^2$, $s_0 \geq 4.7 \text{GeV}^2$, the threshold parameter is too large for the light tetraquark state candidates $f_0(980)$, $a_0(980)$, etc. The open (hidden) bottomed (charmed) tetraquark states may satisfy the relation, as they always have larger Borel parameter $M^2$ and threshold parameter $s_0$. Although the relation is derived for the light flavor quarks in the massless limit, the $c$ and $b$ are heavy quarks.
values of the masses and pole resides of the $Z$ we can take into account the perturbative $O(\alpha_s)$ corrections in order to improve the accuracy of the QCD sum rules. It is expected that those tetraquark states may be observed at the LHC in future.

The threshold parameters in the vector hidden charmed channels can be tentatively taken as $s_{cq}^0 = (32 \pm 1) \text{GeV}^2$, $s_{qs}^0 = (33 \pm 1) \text{GeV}^2$ and $s_{s}^0 = (34 \pm 1) \text{GeV}^2$. Taking into account the mass difference between the $c$ and $b$ quarks, threshold parameters in the vector hidden bottomed channels can be tentatively taken as $s_{c\bar{q}}^0 = (160 \pm 2) \text{GeV}^2$, $s_{q\bar{s}}^0 = (162 \pm 2) \text{GeV}^2$ and $s_{s}^0 = (164 \pm 2) \text{GeV}^2$. Here we take the same threshold parameter for the $C_{\gamma_5} - C_{\gamma_5\gamma_5}$ type and $C - C_{\gamma_5}$ type interpolating currents, the numerical results indicate that it is a good choice.

The contributions from different terms in the operator product expansion are shown in Figs.1-2. From the figures, we can see that the interpolating currents contain more $s$ quarks have better convergent behavior, convergence of the operator product expansion (i.e. the contributions from the four quark condensates are less than 20%) requires $M^2 \geq 3.5 \text{GeV}^2$ and $M^2 \geq 9.0 \text{GeV}^2$ for the $c\bar{c}q\bar{q}$ channel and $b\bar{b}q\bar{q}$ channel, respectively. In this article, we take the uniform Borel parameter $M_{\min}^2$, i.e. $M_{\min}^2 \geq 3.5 \text{GeV}^2$ and $M_{\min}^2 \geq 9.0 \text{GeV}^2$ for the $c\bar{c}$ channels and $b\bar{b}$ channels, respectively.

In Figs.3-4, we show the contributions of the pole terms with variation of the Borel parameters. The pole contributions are larger than 44% at the value $M^2 \leq 4.5 \text{GeV}^2$ in the $c\bar{c}$ channels and larger than 50% at the value $M^2 \leq 10.0 \text{GeV}^2$ in the $b\bar{b}$ channels. Again we take the uniform Borel parameter $M_{\max}^2$, i.e. $M_{\max}^2 \leq 4.5 \text{GeV}^2$ and $M_{\max}^2 \leq 10.0 \text{GeV}^2$ for the $c\bar{c}$ channels and $b\bar{b}$ channels, respectively. We can take smaller $M_{\max}^2$ to ensure the pole contributions are larger than 50% in all the $c\bar{c}$ channels, however, they can not improve the predictions remarkably, as the Borel platforms are rather flat, see Figs.5-6. In this article, the two criteria of the QCD sum rules are full filled \cite{10,11}.

Taking into account all uncertainties of the input parameters, finally we obtain the values of the masses and pole resides of the $Z$, which are shown in Figs.5-8 and Tables 1-2.

| tetraquark states | $C_{\gamma_5} - C_{\gamma_5\gamma_5}$ | $C - C_{\gamma_5}$ |
|------------------|--------------------------------------|---------------------|
| $c\bar{c}ss$     | $5.25 \pm 0.18$                     | $5.21 \pm 0.18$     |
| $c\bar{c}qs$     | $5.24 \pm 0.20$                     | $5.11 \pm 0.15$     |
| $c\bar{c}q\bar{q}$ | $5.12 \pm 0.15$                   | $5.10 \pm 0.16$     |
| $bb\bar{s}s$     | $12.24 \pm 0.22$                   | $12.21 \pm 0.20$    |
| $bbq\bar{s}$     | $12.31 \pm 0.27$                   | $12.10 \pm 0.18$    |
| $bbq\bar{q}$     | $12.15 \pm 0.15$                   | $12.14 \pm 0.16$    |

Table 1: The masses (in unit of GeV) for the vector tetraquark states.

4 Conclusion

In this article, we perform a systematic study of the mass spectrum of the vector hidden charmed and bottomed tetraquark states with the QCD sum rules, including the contributions of the vacuum condensates up to dimension six in the operator product expansion. We can take into account the perturbative $O(\alpha_s)$ corrections in order to improve the accuracy of the QCD sum rules. It is expected that those tetraquark states may be observed at the LHC in future.
Figure 1: The contributions from different terms with variation of the Borel parameter $M^2$ in the operator product expansion for the $C\gamma_5 - C\gamma_\mu \gamma_5$ type current operators. The $A$, $B$, $C$, $D$, $E$ and $F$ denote the $c\bar{c}s\bar{s}$, $c\bar{c}q\bar{s}$, $c\bar{c}q\bar{q}$, $b\bar{b}s\bar{s}$, $b\bar{b}q\bar{s}$ and $b\bar{b}q\bar{q}$ channels, respectively. The $\alpha$ and $\beta$ correspond to quark condensates + mixed condensates term and four quark condensates term, respectively; the contribution from the perturbative term is normalized to be 1.
Figure 2: The contributions from different terms with variation of the Borel parameter $M^2$ in the operator product expansion for the $C - C\gamma_\mu$ type current operators. The $A$, $B$, $C$, $D$, $E$ and $F$ denote the $c\bar{c}s\bar{s}$, $c\bar{c}q\bar{s}$, $c\bar{c}q\bar{q}$, $b\bar{b}s\bar{s}$, $b\bar{b}q\bar{s}$ and $b\bar{b}q\bar{q}$ channels, respectively. The $\alpha$ and $\beta$ correspond to quark condensates + mixed condensates term and four quark condensates term, respectively; the contribution from the perturbative term is normalized to be 1.
Table 2: The pole residues (in unit of $10^{-1}$ GeV$^5$) for the vector tetraquark states.
Figure 4: The contributions from the pole terms with variation of the Borel parameter $M^2$ for the $C - C_{\gamma\mu}$ type current operators. The $A$, $B$, $C$, $D$, $E$ and $F$ denote the $c\bar{c}s\bar{s}$, $c\bar{c}q\bar{s}$, $c\bar{c}q\bar{q}$, $b\bar{b}s\bar{s}$, $b\bar{b}q\bar{s}$ and $b\bar{b}q\bar{q}$ channels, respectively.
Figure 5: The masses of the vector tetraquark states with variation of the Borel parameter $M^2$ for the $C\gamma_5 - C\gamma_\mu\gamma_5$ type current operators. The A, B, C, D, E and F denote the $c\bar{c}s\bar{s}$, $c\bar{c}q\bar{s}$, $c\bar{c}q\bar{q}$, $b\bar{b}s\bar{s}$, $b\bar{b}q\bar{s}$ and $b\bar{b}q\bar{q}$ channels, respectively.
Figure 6: The masses of the vector tetraquark states with variation of the Borel parameter $M^2$ for the $C - C\gamma\mu$ type current operators. The A, B, C, D, E and F denote the $c\bar{c}s\bar{s}$, $c\bar{c}q\bar{s}$, $c\bar{c}q\bar{q}$, $b\bar{b}s\bar{s}$, $b\bar{b}q\bar{s}$ and $b\bar{b}q\bar{q}$ channels, respectively.
Figure 7: The pole residues of the vector tetraquark states with variation of the Borel parameter $M^2$ for the $C\gamma_5 - C\gamma_\mu\gamma_5$ type current operators. The A, B, C, D, E and F denote the $c\bar{c}s\bar{s}$, $c\bar{c}q\bar{s}$, $c\bar{c}q\bar{q}$, $b\bar{b}s\bar{s}$, $b\bar{b}q\bar{s}$ and $b\bar{b}q\bar{q}$ channels, respectively.
Figure 8: The pole residues of the vector tetraquark states with variation of the Borel parameter $M^2$ for the $C - C \gamma \mu$ type current opertors. The $A$, $B$, $C$, $D$, $E$ and $F$ denote the $c\bar{c}s\bar{s}$, $c\bar{c}q\bar{q}$, $c\bar{c}q\bar{q}$, $b\bar{b}s\bar{s}$, $b\bar{b}q\bar{s}$ and $b\bar{b}q\bar{q}$ channels, respectively.
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