Floquet-engineering and simulating exceptional rings with a quantum spin system

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Time-periodic driving in the form of coherent radiation provides powerful tool for the manipulation of topological materials or synthetic quantum matter. In this paper we propose a scheme to realize non-Hermitian semimetals exhibiting exceptional rings in the spectra through Floquet engineering. A transition from a concentric pair of the rings to a dipolar pair is observed. The concentric pair only carries a quantized Berry phase while the dipolar pair possesses opposite Chern numbers in addition, signaling a topological Lifshitz transition of the Fermi surface. The transport properties of the system are addressed, and we find that this transition process is accompanied by the emergency of a nontrivial Hall conductivity. Furthermore, we explore the quantum simulation of non-Hermitian semimetals with a quantum spin system and the characterization of the topology via the long-time dynamics.

Introduction.– Exceptional points (EPs) are defective degeneracies in the energy spectra where the eigenstates of the Hamiltonian do not span a complete Hilbert space [1–3]. They are fascinating instances of non-Hermitian (NH) systems emerging as effective description of non-conserved systems such as solids with finite quasi-particle lifetimes [4–7], disordered Dirac fermions [8, 9], and artificial lattice with gain/loss or nonreciprocity [10–15]. Recently, the existence of different exceptional solutions, such as isolated EPs [16], exceptional rings (ERs) [17–19], exceptional links [20–22] and exceptional surfaces [23–25] in NH semimetals and their topological nature have been recognized [26–30]. The intriguing features of the topology include that new symmetries [28] and topology invariants such as energy vorticity [27] and discriminant number [31], anomalous skin effect [32–37], and the breakdown of the usual bulk-boundary correspondence [32, 38–41]. Furthermore, certain discrete symmetries are required to stabilize the 1D line nodes of a 3D Hermitian semimetal as a result of Bott periodicity for even-odd dichotomy [42, 43], while the exceptional line in NH semimetal is stable even without the imposition of any protecting symmetry [20, 21].

For conventional topological materials or synthetic quantum matters [44], a change in the structure of Fermi surface may happen when an external field is applied. For instance, the Dirac points in graphene can move under the changes of control parameters [45–48]; periodical drives can give rise to a dimension deformation from nodal line to Weyl points attached with a Z-monopole charge [49–52], incorporating the coherent manipulation of topological semimetals into the various contexts of Floquet topological matters [53, 54]. Furthermore, the non-Hermiticity has also been included in the field of Floquet states in topological insulators and superconductors [55–57]. In this paper, we propose the realization of exceptional ring semimetals by generalizing the Floquet-engineering methods developed in Ref. [49–52, 61, 62] to NH cases. The NH semimetal exhibiting double ERs is driven by a circularly polarized light. And a transition of the ERs from a concentric pair of which each one only carries a nontrivial Berry phase to a dipolar pair with opposite Chern numbers is observed. This signals an exotic topological Lifshitz transition which has no Hermitian counterpart. Furthermore, we find that this transition process is accompanied by the emergency of a nontrivial Hall conductivity, which provides experimentally observable evidence for the change of the Fermi surface.

On the other hand, the experimental demonstration and quantum simulation of the NH quantum mechanics has been attracted considerable interest in recent years. Particularly promising approaches using different platform have been successfully performed. One line of works is engineering the Lindblad master equation using acoustics [63, 64], optical [14, 15, 65, 66] or atomic [11, 44, 67, 68] systems. Other novel protocols adopt alternative methods with no need of controlling an open system [69–71] and recently successfully carried out with a single nitrogen-vacancy (NV) center in diamond [72, 73]. In these protocols the NH Hamiltonians emerge as an effective description of subsystem for a larger dilated Hermitian system. We adopt this dilation formalism to simulate the NH nodal-line semimetal. In principle, the eigen-energies of NH Hamiltonian can be revealed from the dynamical phase, and the topological properties of the exceptional degeneracies can be reconstructed from the time-averaged spin textures.

Floquet-engineering of exceptional ring.– We start with considering the model Hamiltonian exhibiting double exceptional rings,

\[
\hat{H}_k = \sum_k \hat{\Psi}_k^\dagger \hat{H}(k) \hat{\Psi}_k + \sum_{k,\sigma} i\hat{e}_{k,\sigma} \xi^\dagger + i\hat{e}_{k,\sigma}^\dagger \xi,
\]

where \(\hat{\Psi}_k = (\hat{c}_{k,a}, \hat{c}_{k,b})^T\) and \(a, b\) refer to the two orbitals involved; \(\hat{H}(k) = (m - Bk^2)\tau_z + vk_x \tau_x + i\gamma \tau_z\) with \(k^2 = k^2_x + k^2_y + k^2_z\) and \(\tau_{x,y,z}\) are Pauli matrices; \(\xi\) and \(\xi^\dagger\) are the Langevin noise operators, and they satisfy \(\langle \xi(\ell)\xi^\dagger(\ell') \rangle_{\text{noise}} = 2\gamma \delta_{\ell \ell'} \delta(\ell - \ell')\), \(\langle \xi^\dagger(\ell)\xi^\dagger(\ell') \rangle_{\text{noise}} = 0\) [78, 79].
We consider periodic drive by application of light beam which generates a vector potential \( \mathbf{A}(t) = A[0, \cos(\omega t), \sin(\omega t)] \). The time-dependent Hamiltonian under the external field can be obtained by the minimal coupling prescription \( \hat{\mathcal{H}}(t) = \hat{\mathcal{H}}(t) + e\mathbf{A} \). In the high frequency regime, the Floquet Hamiltonian can be computed in a perturbative manner. We follow the standard procedures to expand the prescribed Hamiltonian in orders of \( 1/\omega \) as \( \hat{\mathcal{H}}(t, k) = \sum_{\mathcal{H}_n} e^{\mathbf{A}t} \mathcal{H}_n \).

\[
\mathcal{H}_0(k) = [m - Bc^2A_s^2 - BK^2] \Psi^\dagger_k \tau_z \Psi_k + (vkz + i\gamma) \Psi^\dagger_k \tau_x \Psi_k,
\]

\[
\mathcal{H}_{\pm 1}(k) = -eA_0[B(k_y + ic^2k_z)]\Psi^\dagger_k \tau_z \Psi_k \pm ie^{\pm i\phi} v \Psi^\dagger_k \tau_z \Psi_k / 2,
\]

\[
\mathcal{H}_{\pm 2}(k) = -Be^2A_s^2(1 - e^{\pm 2i\phi}) \Psi^\dagger_k \tau_x \Psi_k / 4,
\]

and \( \mathcal{H}_n = 0 \) for \( |n| > 2 \). For the Floquet-engineering of a non-Hermitian system, the effective time-independent Hamiltonian takes a slightly different form \([81]\),

\[
\mathcal{H}_{\text{eff}}(k) = \mathcal{H}_0 + \sum_{n \geq 1} \frac{\mathcal{H}_{n+} \mathcal{H}_{-n}}{n\omega} + \sum_{n, \sigma} s \frac{2\gamma}{\omega^2} \mathcal{H}_{n, \sigma} c_{\sigma} \mathcal{H}_{n, \sigma} ,
\]

where \( s = \tau_z(c_{\sigma} |0\rangle) = \pm 1 \), and the last term describes a micromotion. The first two terms lead to

\[
\mathcal{H}_P = [\tilde{m} - BK^2] \tau_z + (vkz + i\gamma) \tau_z + \lambda k_y \tau_y ,
\]

where \( \lambda = -2e^2BvA^2\cos(\phi)/\omega \) and \( \tilde{m} = m - Be^2A_s^2 \), while the micromotion term gives rise to nontrivial second-order contributions,

\[
\sum_{\sigma} \mathcal{H}_{1, \sigma} \mathcal{H}_{1, \sigma} + \sum_{\sigma} \mathcal{H}_{-1, \sigma} \mathcal{H}_{-1, \sigma} + \frac{c^2A^2B^2(k_y^2 + k_z^2 + 2k_yk_z\sin(\phi))}{4} \Psi^\dagger_k \tau_x \Psi_k + \frac{c^2A^2B^2(k_y + k_y\sin(\phi))}{4} \Psi^\dagger_k \tau_z \Psi_k.
\]

Then the full form of the effective Hamiltonian is expressed as \( \mathcal{H}_{\text{eff}}(k) = \sum_k \Psi^\dagger_kH_k\Psi_k \) with

\[
H_k = [\tilde{m} - BK^2 - i(\gamma_1k_y\sin(\phi) + \gamma_1k_z)] \tau_z + \lambda k_y \tau_y + [vkz + i(\gamma_2 - \gamma_3(k_y^2 + k_z^2 + 2k_yk_z\sin(\phi)))] \tau_z ,
\]

where \( \gamma_1 = 4e^2A^2Bv\gamma/\omega^2 \) and \( \gamma_2 = \gamma + e^2A^2v^2/\omega^2 \) and \( \gamma_3 = 4e^2A^2B^2\gamma/\omega^2 \). We can rewrite the Hamiltonian in a more compact form,

\[
H_k = d(k) \cdot \sigma, \quad d \in \mathbb{C}^3 ,
\]

where the vector \( d \) can be decomposed into real and imaginary parts according to \( d = d_R + id_I \). Then the eigenvalues and thus energies take a general form,

\[
E^2 \pm = d_R^2 - d_I^2 + 2id_R \cdot d_I .
\]

For the non-Hermitian system we consider, \( E_{\pm} = \sqrt{A(\kappa)e^{i\theta}/2} \), where \( A(\kappa) = (d_R^2 - d_I^2)^2 + 4(d_R \cdot d_I)^2 \) and \( \theta = \arctan((d_R^2 - d_I^2)/(2d_R \cdot d_I)) \). Thus the eigenvalues of Hamiltonian possess two branches then acquire a nontrivial energy vorticity.

We calculate the band structure from the Floquet Hamiltonian defined in Eq. (6). The results are shown in Fig. 1. The nodal points in the spectrum are described by solutions to the equations,

\[
d_R^2 - d_I^2 = 0, \quad d_R \cdot d_I = 0.
\]

In the high-frequency limit \( \omega \gg 1 \), the term dependent on \( \omega \) can be neglected and the energy spectrum is approximated by,

\[
E = \pm \sqrt{(\tilde{m} - BK^2)^2 + v^2k_z^2 - \gamma^2 + 2i\gamma k_z \gamma} .
\]

When \( \gamma < m \), two ERs characterized by \( BK^2 = \tilde{m} \pm \gamma \) lie in the \( k_z = 0 \) plane, as shown in Fig.1 (a). The inner ER shrinks as \( \gamma \) increases, and vanishes beyond a critical value of \( \gamma = \tilde{m} \) where it becomes an EP. As the driving frequency \( \omega \) decreases and the system goes into the low-frequency regime, the structure of the exceptional solutions dynamically changes. We consider \( \gamma = 0.5\tilde{m} \) as a typical example. The inner ER extends and cuts the outer ER to form an concentric dipolar pair, as illustrated in Fig. 1 (b). Fig.1 (c) shows the band-touching for \( \omega/A \approx 1 \) where higher-order corrections lead to more remarkable modifications. The condition \( d_R \cdot d_I = 0 \) gives two more solutions \( k_z = \pm \sqrt{m\gamma_1 - \gamma_2} \), except for \( k_z = 0 \). With \( k_z = 0 \), \( d_R^2 - d_I^2 = 0 \) reduces to,

\[
k_z^2 = \frac{m}{B} - k_y^2 \pm \frac{\sqrt{B^2(k_y^2 - 2\gamma_2\gamma_3k_z^2 + \gamma_3^2k_y^4 - \lambda^2k_y^2)}}{B^2}.
\]

In particular, for the critical non-Hermiticity \( \gamma = \tilde{m} \) where an ER accompanied with a single EP appears in the spectrum in high-frequency limit, the inner EP could be tuned into an ER under the periodic driving.
where \( \eta_{ij}(k) \equiv \arctan(\sigma_j/\sigma_i) \) and \( \sigma_j = \frac{1}{T} \int_0^T \langle \sigma_j(k, t) \rangle dt \). In the long-time limit, this dynamic winding number is equivalent to the winding number \( w = \lim_{T \to \infty} w_d \). Only the real part of the phase \( \eta_{ij}(k) \) has nontrivial contributions and it can be decomposed to the sum of two observables [80],

\[
\Re(\eta_{ij}(k)) = \frac{1}{2} (\phi_{ij}^{RR} + \phi_{ij}^{LL}) + n \frac{\pi}{2} ,
\]

where \( \phi_{ij}^{RR} = \arctan(\langle u|\sigma_i|\sigma_j u \rangle/\langle u|\sigma_j|u \rangle) \) and \( \phi_{ij}^{LL} = \arctan(\langle u|\sigma_i|\tilde{u} \rangle/\langle \tilde{u}|\sigma_j|\tilde{u} \rangle) \).

As shown in Fig. 2, the ERs are characterized by the winding number along an \( S^1 \) loop which interlinks with them. The fact that the winding number takes value out of \( \mathbb{Z}/2 \) can be attributed to the net vorticity of this NH system. Furthermore, numerical simulations show that the dynamical winding number has good agreement with the winding number, as shown in Fig. 2 (d)-(f).

**Topological characterization**—The exceptional ring and exceptional point are topological defects which can be characterized by a quantized Berry phase,

\[
\gamma_B = \oint_{2L} i \langle \tilde{u}(k) | \partial_k u(k) \rangle dk ,
\]

where \( \langle \tilde{u}(k) | \) and \( | u(k) \rangle \) are left and right eigenvectors of the Hamiltonian Eq. (6) respectively; the path \( 2L \) travels across the ring twice. The path \( 2C \) forms a closed loop on the Riemann surface defined by \( E(\theta) \) [17], and the system returns to its original state after wrapping the ER twice.

Without loss of generality, we consider a path in the \( k_y = 0 \) plane. For the two-band model we consider, the Berry phase can be associated to a winding number with the relation \( \gamma_B = 2\pi w \), where

\[
w = \frac{1}{2\pi} \oint_{2L} \partial_k \phi_{zz}(k) \, dk ,
\]

with \( \phi_{zz}(k) \equiv \arctan(h_z/h_z) \). The winding number can be experimentally detected with a dynamic approach from the long-time average of spin textures [76, 77]. The spin textures are defined as the expectation values of the Pauli matrices \( \langle \sigma_j(k, t) \rangle = \langle \tilde{u}_k|\sigma_j|u_k \rangle/\langle \tilde{u}_k|u_k \rangle \) in a biorthogonal formalism. The dynamic winding number is defined by the spin vector

\[
w_d = \frac{1}{2\pi} \oint_{2L} \partial_k \eta_{ij}(k) \, dk ,
\]

where \( \eta_{ij}(k) \equiv \arctan(\sigma_j/\sigma_i) \) and \( \sigma_j = \frac{1}{T} \int_0^T \langle \sigma_j(k, t) \rangle dt \). In the long-time limit, this dynamic winding number is equivalent to the winding number \( w = \lim_{T \to \infty} w_d \). Only the real part of the phase \( \eta_{ij}(k) \) has nontrivial contributions and it can be decomposed to the sum of two observables [80],

\[
\Re(\eta_{ij}(k)) = \frac{1}{2} (\phi_{ij}^{RR} + \phi_{ij}^{LL}) + n \frac{\pi}{2} ,
\]

where \( \phi_{ij}^{RR} = \arctan(\langle u|\sigma_i|\sigma_j u \rangle/\langle u|\sigma_j|u \rangle) \) and \( \phi_{ij}^{LL} = \arctan(\langle u|\sigma_i|\tilde{u} \rangle/\langle \tilde{u}|\sigma_j|\tilde{u} \rangle) \).

As shown in Fig. 2, the ERs are characterized by the winding number along an \( S^1 \) loop which interlinks with them. The fact that the winding number takes value out of \( \mathbb{Z}/2 \) can be attributed to the net vorticity of this NH system. Furthermore, numerical simulations show that the dynamical winding number has good agreement with the winding number, as shown in Fig. 2 (d)-(f).

When the system goes into the low-frequency regime, the double concentric ERs are deformed as a dipolar pair with two pole ERs which carry opposite topological charges \( w = \pm 1 \) defined on the loop encircling the whole ER [indicated in Fig. 3 (a)]. The net winding number implies that the ER is protected by a Chern number,

\[
C = \frac{1}{2\pi} \oint_{\partial S} \Omega_\theta(k) \cdot dS
\]

where \( \Omega_\theta(k) = i \langle \nabla_k \hat{u}_\theta(k) | \times | \nabla_k \hat{u}_0(k) \rangle \) is the Berry curvature and \( S \) encloses whole single ER, as illustrated in Fig. 3 (b). Calculation shows that \( C = \pm 1 \).

**Hall conductivity**—The light beam is incident along \( x \) direction, thus we are interested in the \( yz \)-component of the Hall response \( \sigma_{yz} \). As suggested by previous works [74, 75], the Hall conductivity of a generic two-band non-Hermitian Hamiltonian with form of Eq. (7) is given by

\[
\sigma_{yz} = -\frac{e^2}{h} \int \frac{d^3k}{(2\pi)^3} \text{Re} \left\{ \frac{\partial \hat{d} \cdot (\partial \hat{d} / \partial k_y) \times (\partial \hat{d} / \partial k_z) \pi}{2} \text{sgn} \left( \text{Re} \hat{d} \right) \right\} ,
\]

FIG. 2: (a), (b) and (c) respectively show the relative phase \( \phi_{zz}(k_x, k_z) \) with \( m = 1, B = 1, v = 1, \gamma = 0.5 \) and (a) \( \omega/A = 10 \), (b) \( \omega/A = 2 \) and (c) \( \omega/A = 1 \); (b), (c) and (d) respectively show the relative phase \( \eta_{zz}(k_x, k_z) \) of spin textures for an evolution time \( T = 20 \); (g) and (h) show the winding number extracted from the loops encircling the two exceptional points on the inner ER.

FIG. 3: (a) The winding number of the ER, which is 0.5 for integral loop which interlinks with the ER, and 1 for that encircling the ER, (b) The Chern number of the ER. The ER is analogous to the Weyl point in the Hermitian case, indicated as colored dot in (b).
where \( \hat{d} = d/|d| \). With components considered in Eq.(6), the Hall conductivity takes the following explicit form:

\[
\sigma_{yz} = -\frac{e^2}{\hbar} \int \frac{dk}{(2\pi)^3} \text{Re}[\gamma_1 \gamma_2 \lambda + \nu \lambda - B v k_x^2 \lambda - 
\gamma_1 \gamma_3 k_x^2 \lambda + B v k_y^2 \lambda + 2i B \gamma_2 k_x \lambda - 2i \gamma_3 m k_z \lambda + 
2i B \gamma_3 k_z^2 - \gamma_1 \gamma_3 k_x^2 \lambda + B v k_x^2 \lambda]/E_3^2 \frac{\pi}{2} \text{sgn}(\text{Re} \ d)].
\] (18)

We numerically calculate the integral, and the results are shown in Fig. 4. A transition of the Hall conductivity from 0 to finite values appears when the dipolar pair of the ERs forms, which distinguishes from the Hermitian case where \( \sigma_{yz} \) is always nonzero under the driven field. And \( \sigma_{yz} \) is proportional to the distance between the two ERs, which is reminiscent of the results in Ref. [49]. Furthermore, it is readily seen that the Hall conductivity decreases as the strength of non-Hermiticity increases.

![FIG. 4: The dependence of Hall conductivity on the frequency \( A/\omega \) of periodic driving. The parameters are \( \bar{m} = 1, B = 1, \nu = 1, \phi = 0 \). The Dashed lines show the value of \( \sigma_{yz} \) in Hermitian limit \( \sigma_0 = e^2/\pi \hbar \).](image)

**Quantum simulation with a quantum spin system.** To simulate the dynamics of the Hamiltonian \( \hat{H}_k \), an ancilla qubit is required to dilate \( \hat{H}_k \) into a Hermitian Hamiltonian \( \hat{H}_d(t) \) [72]. The evolution of the diluted system \( \hat{H}_d(t) \) is governed by the schrödinger equation,

\[
i \frac{d}{dt} |\Psi(t)\rangle = \hat{H}_d(t) |\Psi(t)\rangle,
\] (19)

where \( |\Psi(t)\rangle \) is the state of the combined system. The essential idea that allows the exclusive dilation is to restrict the measurement results to those with a specific output for the ancilla qubit. In such a post-selection scenario, it’s convenient to write the state \( |\Psi(t)\rangle \) in a form of

\[
|\Psi(t)\rangle = |\psi(t)\rangle |-\rangle_a + \eta(t) |\psi(t)\rangle |+\rangle_a,
\] (20)

where \( |-\rangle_a \) and \( |+\rangle_a \) form an orthonormal basis of the ancilla qubit and here are chosen to be the eigenstates of \( \tau_y \) with \( |-\rangle = |0\rangle - i|1\rangle)/\sqrt{2} \) and \( |+\rangle = |0\rangle + i|1\rangle)/\sqrt{2} \). And after the evolution, a \( -\pi/2 \) pulse is applied and only the measurement results with no jump outside the submanifold \( |\psi\rangle |1\rangle_a \) is post-selected.

![FIG. 5: (a) and (b) Parameters \( A_i(t) \) in the dilated Hamiltonian. In general \( B_i \neq 0 \), but for parameters considered here \( B_0 = 0 \). (c) and (d) The population on state \( |0\rangle \) at time \( t \). The solid lines indicate the analytic results solved for the Schrödinger equation \( i\hbar \partial_t \psi = \hat{H}_k \psi \); the circles indicate the numerical results solved for dilated Hamiltonian \( \hat{H}_d \). The parameters in \( \hat{H}_k \) are chosen as \( \bar{m} = v = B = 1 \) and \( a,c \) \( k_x^2 + k_y^2 = 0.25 \), \( k_z = 0 \), \( b,d \) \( k_x^2 + k_y^2 = 0.6 \), \( k_z = 0 \).

Note that the Hamiltonian \( \hat{H}_d(t) \) is not uniquely determined. One proper choice is

\[
\hat{H}_d(t) = \Lambda(t) \otimes \mathbb{I} + \Gamma(t) \otimes \sigma_z,
\] (21)

where \( \Lambda(t) = \{ i [\hat{H}_k(t) + i \frac{\sigma_z}{\hbar} \eta(t) + \eta(t) \hat{H}_k(t)] \eta(t) \} M^{-1}(t) \) and \( \Gamma(t) = i [\hat{H}_k(t) \eta(t) - \eta(t) \hat{H}_k(t) - i \frac{\sigma_z}{\hbar} \eta(t)] M^{-1}(t) \). The time-dependent operator \( M(t) \) takes the form \( M(t) = \eta(t) \eta + I \). And we can expand \( \Lambda(t) \) and \( \Gamma(t) \) in terms of the Pauli operators and rewrite \( \hat{H}_d(t) \) as

\[
\hat{H}_d = A_1(t) \sigma_x \otimes \mathbb{I} + A_2(t) \mathbb{I} \otimes \sigma_z + A_3(t) \sigma_y \otimes \sigma_z + A_4(t) \sigma_z \otimes \sigma_z + B_1(t) \mathbb{I} \otimes \mathbb{I} + B_2(t) \sigma_y \otimes \mathbb{I} + B_3(t) \sigma_z \otimes \mathbb{I} + B_4(t) \sigma_x \otimes \sigma_z,
\] (22)

Figures 5 (a) and (b) show the time-dependent parameters \( A_i \) \((i = 1-4)\). Without loss of generality, here we only take the Hamiltonian in high-frequency limit as an example. The four level system described by the Hamiltonian \( \hat{H}_d \) could be encoded in the ground state manifold of electron spin and nuclear spin in a NV center, or alternatively, other quantum platforms such as trapped ion, Rydberg atom, and superconducting circuit. As shown in Fig. 5 (c) and (d), we demonstrate that the dynamics of the NH nodal-line semimetal can be revealed in the post-selected state population, which allows us to reconstruct the topological information from the spin textures. In experiments, microwave pulses and two radio-frequency pulses could be applied to couple the ground states of a NV center. For the coupling strength tuned to ~ 600 kHz, the dynamical process in Fig. 5 (c,d) can happen within 10 \( \mu \)s, which is feasible with current technology.
Conclusions. In summary, we have proposed a scheme to realize tunable exceptional rings in terms of Floquet engineering. As the driven frequency changes, a dipolar pair of ERs protected by opposite Chern numbers can be created from the NH semimetal with double concentric ERs. This transition process is accompanied by the emergence of a non-zero Hall conductivity. Furthermore, we explore possible realization with synthetic quantum matter, which do not require controlling an open system. The proposed system would provide a promising platform for elaborating NH topology which might be elusive in nature.

Note added.—When we prepare our manuscript, we became aware of a related eprint by A. Banerjee and A. Narayan [82].

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Appendix A: Derivation details

In this section, we give the detailed derivation of the micromotion term of Eq. (3) in the main text. We recall the full form of the first-order component of the Floquet Hamiltonian,

\[ H_{+1} = -\Psi_k^\dagger [eAB(k_y - ie^{i\phi})\tau_x + ie^{i\phi}\frac{eV}{2}\tau_z] \Psi_k, \]  

\[ H_{-1} = -\Psi_k^\dagger [eAB(k_y + ie^{-i\phi})\tau_x - ie^{-i\phi}\frac{eV}{2}\tau_z] \Psi_k, \]  

expanded as

\[ H_{+1} = -[eAB(k_y - ie^{i\phi})(c_{k,a}^\dagger\hat{c}_{k,b} + c_{k,b}^\dagger\hat{c}_{k,a}) + ie^{i\phi}\frac{eV}{2}(c_{k,a}^\dagger\hat{c}_{k,a} - c_{k,b}^\dagger\hat{c}_{k,b})], \]

\[ H_{-1} = -[eAB(k_y + ie^{-i\phi})(c_{k,a}^\dagger\hat{c}_{k,b} + c_{k,b}^\dagger\hat{c}_{k,a}) - ie^{-i\phi}\frac{eV}{2}(c_{k,a}^\dagger\hat{c}_{k,a} - c_{k,b}^\dagger\hat{c}_{k,b})]. \]

And the commutators are given by

\[ [H_{+1}, \hat{c}_{k,a}] = eAB(k_y - i\frac{eV}{2}\tau_z)\hat{c}_{k,b} + ie^{i\phi}\frac{eV}{2}\hat{c}_{k,a}, \]  

\[ [H_{+1}, \hat{c}_{k,b}] = eAB(k_y - i\frac{eV}{2}\tau_z)\hat{c}_{k,a} + ie^{i\phi}\frac{eV}{2}\hat{c}_{k,b} , \]  

\[ [H_{+1}, \hat{c}_{k,a}^\dagger] = eAB(k_y + ie^{-i\phi})\hat{c}_{k,b} + ie^{i\phi}\frac{eV}{2}\hat{c}_{k,a}^\dagger , \]  

\[ [H_{+1}, \hat{c}_{k,b}^\dagger] = eAB(k_y + ie^{-i\phi})\hat{c}_{k,a} + ie^{i\phi}\frac{eV}{2}\hat{c}_{k,b}^\dagger . \]  

By inserting Eq. (A5) and (A6) into Eq. (3), we have

\[ \frac{2i\gamma}{\omega^2}[H_{+1}, \hat{c}_{k,a}] + [H_{+1}, \hat{c}_{k,a}] = \frac{2i\gamma}{\omega^2}[e^2A^2B^2(k_y^2 + k_z^2 - 2k_yk_z\sin\phi)c_{k,b}^\dagger\hat{c}_{k,b} + e^2A^2Bv^2\frac{eV}{4}(k_ye^{i\phi} - k_z)c_{k,a}^\dagger\hat{c}_{k,a} + e^2A^2Bv^2\frac{eV}{4}(k_ze^{-i\phi} + k_y)c_{k,a}^\dagger\hat{c}_{k,a}]. \]

By inserting Eq. (A7) and (A8) into Eq. (3), we have

\[ -\frac{2i\gamma}{\omega^2}[H_{+1}, \hat{c}_{k,b}] + [H_{+1}, \hat{c}_{k,b}] = \frac{2i\gamma}{\omega^2}[e^2A^2B^2(k_y^2 + k_z^2 - 2k_yk_z\sin\phi)c_{k,a}^\dagger\hat{c}_{k,a} - e^2A^2Bv^2\frac{eV}{4}(k_ye^{i\phi} - k_z)c_{k,a}^\dagger\hat{c}_{k,a} + e^2A^2Bv^2\frac{eV}{4}(k_ze^{-i\phi} + k_y)c_{k,a}^\dagger\hat{c}_{k,a}]. \]

Collecting all terms in Eq. (A9) and Eq. (A10) gives

\[ \sum_{\sigma} s\frac{2i\gamma}{\omega^2}[H_{+1}, \hat{c}_{k,\sigma}] + [H_{+1}, \hat{c}_{k,\sigma}] = \frac{2i\gamma}{\omega^2}[e^2A^2B^2(k_y^2 + k_z^2 - 2k_yk_z\sin\phi) - e^2A^2v^2\frac{eV}{4}][\Psi_k^\dagger\tau_x\Psi_k - 2\frac{2i\gamma}{\omega^2}[e^2A^2Bv^2\frac{eV}{4}(2k_y\sin\phi + k_z)\Psi_k^\dagger\tau_x\Psi_k]. \]

In the same way we can obtain that,

\[ [H_{-1}, \hat{c}_{k,a}] + [H_{-1}, \hat{c}_{k,a}] = [H_{+1}, \hat{c}_{k,a}] + [H_{+1}, \hat{c}_{k,a}] . \]
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