Spin polarization of strongly interacting 2D electrons: the role of disorder.

S. A. Vitkalov and M.P. Sarachik

Physics Department, City College of the City University of New York, New York, New York 10031

T. M. Klapwijk
Delft University of Technology, Department of Applied Physics, 2628 CJ Delft, The Netherlands
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In high-mobility silicon MOSFET’s, the $g' m^*$ inferred indirectly from magnetoconductance and magnetoresistance measurements with the assumption that $g' \mu_B H_s = 2E_F$ are in surprisingly good agreement with $g' m^*$ obtained by direct measurement of Shubnikov-de Haas oscillations. The enhanced susceptibility $\chi^* \propto (g' m^*)$ exhibits critical behavior of the form $\chi^* \propto (n - n_0)^{-\alpha}$. We examine the significance of the field scale $H_s$ derived from transport measurements, and show that this field signals the onset of full spin polarization only in the absence of disorder. Our results suggest that disorder becomes increasingly important as the electron density is reduced toward the transition.

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Two dimensional systems of electrons and holes have been the focus of a great deal of attention during the last few years. In contrast with expectations for noninteracting or weakly interacting electrons in two dimensions, these strongly interacting systems exhibit metallic behavior in the absence of a magnetic field: above some characteristic electron (hole) density, $n_c$, their resistivities decrease with decreasing temperature. Whether there is a genuine metallic phase and a true metal-insulator transition in these materials continues to be the subject of lively debate.

Experimental results have been obtained in the 2D system of electrons in silicon MOSFET’s that indicate that the response to a magnetic field applied in the plane of the electrons increases dramatically as the electron density is decreased toward $n_c$. Based on a study of the scaled magnetoconductance as a function of temperature and electron density, Vitkalov et al. [11] have identified an energy scale $\Delta$ that decreases with decreasing density and extrapolates to zero in the limit $T \to 0$ at a density $n_0$ in the vicinity of $n_c$: this was interpreted as evidence of a quantum phase transition at $n_0$. From studies at very low temperatures of the magnetoresistance as a function of electron density, Shashkin et al. [1] inferred that the two-dimensional system of electrons in silicon inversion layers approaches a ferromagnetic instability at the critical density $n_c$ for the zero-field metal-insulator transition. From a determination of the enhanced spin susceptibility derived from Shubnikov-de Haas measurements down to low densities, Pudalov et al. [12] have claimed there is no spontaneous spin polarization for electron densities above $n = 8.34 \times 10^{10}$ cm$^{-2} \approx n_c$, although they could not exclude this for lower densities. The possibility that a magnetically ordered phase exists in the limit $T \to 0$ in dilute two-dimensional silicon inversion layers is intriguing and bears further investigation.

In this paper we show that there is very good agreement between values reported for $g' m^*$ as a function of electron density in high-mobility silicon MOSFET’s obtained directly from measurements of the Shubnikov-de Haas oscillations and those inferred indirectly from magnetoconductance and magnetoresistance measurements by two different groups using different methods of analysis and the assumption that $g' \mu_B H_s = 2E_F$. Here $g'$ is the enhanced $g$-factor, $m^*$ is the enhanced electron mass, $\mu_B$ is Boltzmann’s factor, $E_F$ is the Fermi energy, and $H_s$ is a characteristic field scale determined by different methods from in-plane magnetoconductance and magnetoresistance experiments. The enhanced susceptibility $\chi^* \propto (g' m^*)$ exhibits critical behavior of the form $\chi^* \propto (n - n_0)^{-\alpha}$. Data from the three experimental groups yield exponents $\alpha$ of 0.23, 0.24 and 0.27, and critical densities between 0.88 and $1.04 \times 10^{11}$ cm$^{-2}$. We examine the significance of the field scale $H_s$, and show that this field signals the onset of full spin polarization only in the absence of disorder. Our results suggest that disorder becomes increasingly important as the electron density is reduced toward the transition.

Measurements of Shubnikov-de Haas oscillations in high-mobility silicon MOSFET’s with high electron densities have shown that the magnetic field required to achieve complete polarization of the electron spins is approximately the same as that required to saturate the magnetoresistance to a constant value $\rho_{sat}$. For the relatively high densities used in these experiments, the field $H_s$ corresponding to saturation of the magnetoresistance is approximately the same as the field $H_s$ above which there is apparent saturation of the magnetoconductance. As we show below, this equivalence breaks down at lower densities. A clear example is illustrated in Fig. 1, where the resistivity and conductivity are shown as a function of in-plane magnetic field for a silicon MOSFET with electron density near the critical density, $n_c$. 


for the metal-insulator transition. The saturation field $H_\rho$ derived from the resistivity is considerably larger than the field $H_\sigma$ above which the conductivity saturates. This can be understood with reference to the band diagrams shown as insets to Fig. 1. In the absence of disorder, all electron states are extended, band-tailing plays a negligible role, and full spin polarization is achieved when the Zeeman energy is sufficient to completely depopulate the minority spin band:

$$g^* \mu_B H_{\text{band}} = 2E_F;$$

(1)

where $g^*$ is the enhanced $g$-factor, $\mu_B$ is Boltzmann’s factor, $H_{\text{band}}$ is the magnetic field required to fully polarize the system in the absence of disorder, and $E_F$ is the Fermi energy. Disorder is weak at high electron densities and one expects $H_{\text{band}} \approx H_\rho \approx H_\sigma$.

As the density is decreased and disorder and the band-tails become more important, complete spin alignment requires the application of a larger magnetic field to fully polarize the tail states as well as the extended states:

$$g^* \mu_B H_{\text{tail+band}} = 2E_F + \delta$$

(2)

where we’ve assumed the band tail has an effective energy width $\delta$.

Except very near the transition, the number of states in the band tails in the case of samples of reasonably high mobility is much smaller than the number of extended states; at the same time, the energy width $\delta$ becomes appreciable as the density decreases and the disorder increases. The field required to align the electrons in the higher mobility band states can thus differ substantially from the magnetic field needed to polarize all the electrons. While the (small number) of tail states make a minor contribution to the conductivity, the resistivity is considerably more sensitive to the low-mobility states in the tail of the distribution, and consequently $H_\rho > H_\sigma$ as is evident in Fig. 1. We suggest that $H_\sigma \approx H_{\text{band}}$ and $H_\rho \approx H_{\text{tail+band}}$.

The fractional difference between $H_\rho$ and $H_\sigma$, $\Delta H/H = (H_\rho - H_\sigma)/H_\rho$, is shown as a function of electron density in Fig. 2. $\Delta H/H$ increases rapidly with decreasing electron density when disorder becomes more dominant. The quantity $2/\sigma$ is plotted for comparison through the following argument. For weak scattering, the parameter $\delta$ is on the order of the scatter-
ing rate: \( \delta \sim \hbar/\tau \). With Eqs. 1 and 3, this gives
\( \Delta H/H = \delta/2E_F = \hbar/2(E_F\tau) \). Using the expression for
the Drude conductivity \( \sigma = ne^2\tau/m^* \), and the Fermi
energy \( E_F/\hbar = (nh)/g_0g_em^* \) with a valley degeneracy
\( g_v = 2 \) and spin degeneracy \( g_s = 2 \), one obtains
\( \Delta H/H = (e^2/\hbar)(2/\sigma) \). The correlation between \( \Delta H/H \)
and \( 2/\sigma \) is evident in Fig. 4.

In an earlier paper [10], we showed that the magneto-
conductance of silicon MOSFET’s can be scaled onto a
single curve by plotting \([\sigma(H) - \sigma(0)]/\sigma(H = \infty) - \sigma(0)] \)
as a function of \( H/H_s \). The parameter \( H_s \) obtained by
this method is proportional to \( H_s \) discussed above. For
high densities where disorder plays a small role, the
magnetic field \( H_s \) needed to saturate the conductivity is very
nearly equal to the field required to obtain full spin polar-
ization. At lower densities, the saturation fields ded-
uced from the resistivity and the conductivity are not
the same, and we have argued that the difference is asso-
ciated with the effect of electrons in the states in the
band tails. We’ve suggested that \( H_s \) is the magnetic field
required to polarize the band states; the Zeeman energy
and \( g^*m^* \) are then given by Eq. 3 with \( H_{band} = H_s \).
The tail states remain unpolarized in \( H = H_s \). However,
except perhaps very near the transition (or in samples of
very low mobility), they represent a small fraction of the
electrons, so that the system is close to full spin polar-
ization.

Fig. 3 shows \( 2n_0/m^*g^* = \chi_0/\chi^* \) as a function of electron
density \( n_s \) obtained from our data [10], by Shashkin
et al. [11], and Pudalov et al. [12]. Here \( \chi^*/\chi_0 \)
is the enhanced susceptibility normalized to its free elec-
tron value, and \( \chi_0/\chi^* \) is its inverse. The closed circles
denote values obtained from scaling our data for the
in-plane magnetoconductance and the assumption that
\( g^*\mu_BH_{\sigma} = 2E_F \); the open circles were obtained
by Shashkin et al. [11] from magnetoresistance measure-
ments using a different data-fitting procedure and the
same assumption as above; the squares are from direct
Shubnikov-de Haas measurements of Pudalov et al. [12].
The data of Shashkin et al. decrease somewhat more
rapidly at low densities than the others. However, the
three sets obtained by different groups using different
measurements and different methods of analysis agree
surprisingly well. Again, this indicates that the small
number of states in the band tails in high-mobility MOS-
FET’s play a negligible role. A fit to the critical form
\( \chi_0/\chi^* \propto (n-n_0)^\alpha \),
\( \alpha = 0.23, n_0 = 0.96 \times 10^{11} \text{ cm}^{-2} \); for the
magnetoconductance data of Shashkin et al. [11] \( \alpha = 0.27 \),
\( n_0 = 1.04 \times 10^{11} \text{ cm}^{-2} \); and for our data [10] \( \alpha = 0.24 \),
\( n_0 = 0.88 \times 10^{11} \text{ cm}^{-2} \).

We have argued above that for high-mobility samples,
the difference \( (H_\rho - H_s) \) is associated with the effect of
a small fraction of the electrons in the band tails. The
characteristic field \( H_s \) obtained in our earlier work was
determined from scaling the magnetoconductance, which
is a measure of the field required to align the band states
while leaving a few electrons in the tail states unpolar-
ized. Shashkin et al. determined a field scale by match-
ing magnetoresistance data at low magnetic fields; close
examination shows that this procedure does not produce
a match at high fields (note that their data is shown on
a logarithmic scale, which deemphasizes differences be-
tween the curves at high values of magnetic field). Both
methods are sensitive to the contribution of the extended
state and minimize the effect of the states in the band
tails. These procedures yield reliable measures for the
behavior of the system at high electron densities where
disorder does not play an important role. This accounts
for the surprisingly good agreement between the \( g^*m^* \)
observed from transport experiments and those found
by direct measurement of the Shubnikov-de Haas oscilla-
tions. At densities very near the transition (and for very
low mobility MOSFET’s) one should expect this corre-
spondence to break down as disorder becomes more dom-
inant. We suggest that an understanding of any phase
transition that occurs in this regime must incorporate the
effect of disorder in a central way.

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