Investigation of two methods of volume measurements based on hydrostatic comparison method

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Abstract. Volume of weights is needed for air buoyancy correction, and measurement methods based on hydrostatic comparison are widely used to detect volume of weights. Several different hydrostatic comparison methods are given in OIML R111. In NIM, two different devices had already been set up to measure the volume of weights, and one is based on a mechanical balance, and another is based on an electrical balance. In this paper, two methods are discussed. Detailed measurement procedure and calculation are investigated, and uncertainty evaluation are also given. The consistence of two methods is also discussed based on an example.

1. Introduction
In mass measurement of high accuracy, volume of weights need to be measured to correct the air bouncy effect. By far, several volume measure methods based on different principle such as hydrostatic weighing, geometric measurement, acoustic principle, change of air density are used to measure the volume of weights. Among these method, hydrostatic weighing method is one of traditional volume measuring methods with highest accuracy.
In NIM, two volume measurement devices had been setup to measure the volume of weights from 1 g to 20 kg based on hydrostatic weighing, an automatic measurement device of VC1005 is used for weights range from 1 kg to 1 g. For weights larger than 1 kg, a device based on mechanical balance is used. In this paper, two devices are investigated.

2. Principles

2.1. Volume measurement on mechanical balance

2.1.1. Measuring device
The schematic diagram and the photo of the measurement devices is shown in figure 1. The main part of the device is a mechanical balance (which maximum capacity is 20 kg). The weighing pan on one side is specially designed (the weighing pan of the right side in figure 1, has two pans linked by a metal wire, one pan is in the air, and another one is merged in water). In order to load the test weight on the weighing pan easily, the position of the right weighing pan is fixed, the position of the water tank can be moved vertically controlled by a handler.

2.1.2. Measuring Procedure
Weighing pan
water
Weighing pan
Counter weight
Test weight
Fig 1 Measuring device

Step 1: put the test weigh on upper pan, and put counter weight on another weighing pan, record the reading $I_a$. Since the balance is a mechanical balance, after the balance is in normal swing, when the indicating needle reach the limit of one side, record as $I_1$, next when the needle reach the limit of another side, record as $I_2$, and last when the needle is back to the limit of initial side, record as $I_3$, then

$$I = \frac{I_1 + 2I_2 + I_3}{4}$$

Step 2: put the test weigh on lower pan, and merged into water, at this time, some standard weights $m_B$ need to be loaded on the upper pan to reach equilibrium., and record the reading $I_w$.

Step 3: add sensitive weight $m_s$ on the upper pan, record the reading $I_s$.

Step 4: record the environment parameters, and calculate the volume of the test weight.

During the measurement, pure water is used, and small bubbles need to be eliminated.

The volume is calculated as follows:

$$m = m_B - V_B \rho_a + m_s(I_s - I_a)/(I_2 - I_a)$$

$$\rho = (1 + K_t(t_a - 20))\rho_a - (1 + K_t(t_a - 20))\rho_w$$

$$V_t = m/\rho$$

Where, $V_B$ is the volume of the standard weights loaded on the weighing pan. $\rho_a$ is the density of air, which can be get from air density formula CIPM-2007. $\rho_w$ is the density of water, which can be get from the table of density of the pure water, although it has relationship with temperature, but here we see it as an independent factor. $K_t$ is the volume expansion parameter, $t_a$ and $t_w$ are temperatures of water and air separately.

For uncertainty,

$$u^2(V_t) = c^2(m_B)u^2(m_B) + c^2(V_B)u^2(V_B) + c^2(m_s)u^2(m_s) + c^2(\rho_a)u^2(\rho_a) + c^2(\rho_w)u^2(\rho_w) + c^2(I_s)u^2(I_s)$$

$$+ c^2(I_a)u^2(I_a) + c^2(I_s)u^2(I_s) + c^2(t_a)u^2(t_a) + c^2(t_w)u^2(t_w)$$

In which,

$$c(m_a) = |1/\rho|, c(V_a) = -\rho_a/\rho, c(m_s) = (I_s - I_a)/\rho(I_s - I_a), c(\rho_a) = \frac{-V_B\rho + m(1 + K_t(t_a - 20))\rho_a}{\rho^2}$$

$$c(\rho_a) = -\frac{1 + K_t(t_a - 20)}{\rho^2}, c(I_s) = -\frac{m_s(I_a - I_s)}{\rho(I_s - I_a)^2}, c(I_a) = -\frac{m_s(I_a - I_s)}{\rho(I_s - I_a)^2}, c(I_a) = -\frac{m_s}{\rho(I_s - I_a)^2}$$

$$c(I_a) = \frac{m_s(I_a - I_s)}{\rho(I_s - I_a)^2}$$

2.2. Volume measurement on VC1005

2.2.1. Measuring device
VC1005 is a mass comparator used to measure the volume of weights no larger than 1 kg. The maximum capacity of the balance inside the device is 1109 g, and the readability is 0.01 mg.

2.2.2. Measuring Procedure
Step 1: measure the mass value of the test value $m_t$.
Step 2: put the reference weight (its volume is already known) and the test weight on magazines of VC1005 in pure water. Put the counter weight on the upper pan of the balance to make sure that the balance can indicate values when the reference weight or the test weight is loaded.
Step 3: use ABA or ABBA cycle to measure the indication difference between the reference weight and the test weight, record as $\Delta I$. A thermometer is used to measure the temperature of the water, record as $t$.
Step 4: calculating $V_t$, and associate uncertainty.

The formula to get $V_t$ is

$$V_t = V_0 [1 + K_r (t - 20)] + (m_r - m_i) (1 - \rho_0 / 8000) / (\rho_u - \rho_0) - \Delta I / (\rho_u - \rho_0)$$

(6)

$K_i$ and $K_r$ are coefficient of cubic expansion, they are assumed to be constant for each weight. $\rho_0$ is density of air as a reference value equal to 1.2 kg m$^{-3}$.

$$c(V_i) = \frac{1 + K_i (t - 20)}{1 + K_i (t - 20)}, \quad c(m_i) = \frac{1 - \rho_0 / 8000}{1 + K_i (t - 20) (\rho_u - \rho_0)}, \quad c(m_r) = -\frac{1 - \rho_0 / 8000}{1 + K_i (t - 20) (\rho_u - \rho_0)},$$

$$c(\Delta I) = \frac{-(m_r - m_i) (1 - \rho_0 / 8000) + \Delta I}{1 + K_i (t - 20) (\rho_u - \rho_0)^2},$$

$$c(t) = \frac{V_i K_i [1 + K_i (t - 20)] - V_0 K_i [1 + K_i (t - 20) + K_i [(m_r - m_i) (1 - \rho_0 / 8000) / (\rho_u - \rho_0) - \Delta I / (\rho_u - \rho_0)]}{[1 + K_i (t - 20)]^2}$$

3. Example and result

One E2 class weight of 1 kg was tested on these two volume measuring devices, and the relationship between these two results is investigated.

3.1. results based on mechanical balance

$I_a$ and $I_w$ were measured 10 times, and $I_s$ was measured 3 times. In step 2, one 100 g weight, one 20 g weight, two 2 g weights were put on the upper pan. In step 3, one 50 mg weights was used as sensitive weights. All records were shown in table 1.

| No. | $I_a$  | $I_w$   | $I_s$   |
|-----|--------|---------|---------|
| 1   | -2.1   | 1.375   | 0.4     |
| 2   | -2.1   | 1.525   |         |
| 3   | -2     | 1.55    |         |
| 4   | -1.95  | 1.425   |         |
| 5   | -2.125 | 1.375   |         |
| 6   | -1.975 | 1.4     | 0.375   |
| 7   | -2     | 1.4     |         |
| 8   | -1.95  | 1.35    |         |
| 9   | -1.95  | 1.325   |         |
The temperature of air was 21.76 °C, air density is 1.1840 kg/m³, the temperature of water is 21.10 °C measured with a glass thermometer which read ability is 0.1 °C. Since the weight is made of stainless steel, $K_t$ was assumed as 0.000045 K⁻¹. With all these information, the volume of this weight is calculated, the result is 124.5382 cm³, and from table 2, the uncertainty is 0.034 cm³ ($k=2$).

### Table 2. Uncertainty budgets for $V_t$

| Source | Value standard uncertainty | Sensitive coefficient | Uncertainty contribution |
|--------|----------------------------|-----------------------|-------------------------|
| $m_B$  | 5.84209E-05                | 1.003172437           | 5.86062E-05             |
| $V_B$  | 0.0015476                  | -1.187756165          | 0.001838171             |
| $m_s$  | 0.000904                  | 3.266329454           | 1.30653E-05             |
| $\rho_r$ | 0.000000002              | -15.37716387          | 3.07543E-07             |
| $\rho_w$ | 3.4641E-05                | -1.006404752          | 3.48629E-05             |
| $I_s$  | 0.094648472                | 0.156783814           | 0.014839348             |
| $I_u$  | 0.071807033                | -0.048152277          | 0.003457672             |
| $I_w$  | 0.072839245                | 0.113157851           | 0.008242332             |
| $\Delta V$ | 0.09                    | 6.65645E-06           | 5.9908E-07              |
| $\Delta w$ | 0.1                     | 0.00561099            | 0.00056106              |

3.2. results based on VC1005

The mass value of the weight is 1 kg +0.59 mg. 10 times ABA cycle was carried out on VC1005. The mean value of the indication difference is 269.0738 mg, the standard deviation is 0.028 mg, the temperature is 19.4842 °C, the standard deviation is 0.0005 °C. The volume of the reference weight is 124.8106 cm³, and uncertainty is 0.0004 cm³. With all these information, the volume of the weight is 124.5408 cm³, and the corresponding uncertainty is 0.0014 cm³ ($k=2$).

### Table 3. Uncertainty budgets for $V_t$

| Source | Value standard uncertainty | Sensitive coefficient | Uncertainty contribution |
|--------|----------------------------|-----------------------|-------------------------|
| $V_r$  | 0.0004                     | 1                     | 0.0004                  |
| $m_t$  | 0.5                        | 0.00100277            | 0.000501385             |
| $m_r$  | 0.1                        | -0.00100277           | -0.000100277            |
| $\Delta I$ | 0.02802                | 0.00100292            | 2.81018E-05             |
| $\rho_w$ | 0.001160474               | 0.000270732           | 3.14177E-07             |
| $t$    | 0.00053                    | -8.12263E-09          | -4.305E-12              |

$u(V_r)$ 0.0007 $U(V_r)$ ($k=2$) 0.0014

It is obviously consistency of the two results from these two methods.

### 4. conclusion

Two difference volume measuring methods were investigated in this paper. One is based on mechanical balance, and one is based on electrical balance. Detailed process and calculation were given, and uncertainty evaluations for each method were discussed. The volume of an sample are measured with each device, results show good consistency.

### 5. References

[1] OIML R111 Weights of classes E₁, E₂, F₁, F₂, M₁, M₁–2, M₂, M₂–3 and M₃ Part 1: Metrological and technical requirements (2004)

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[3] M.Ueki, S. Mizushima, Y.Nezu. A Mass Comparator Installed in an Air-tight Chamber. In proc. XVI IMEKO World Congress, (Austria ,Vienna, 2000). Vol. III,pp. 281-286.