iLQGames.jl: Rapidly Designing and Solving Differential Games in Julia

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Abstract. In many problems that involve multiple decision making agents, optimal choices for each agent depend on the choices of others. Differential game theory provides a principled formalism for expressing these coupled interactions and recent work offers efficient approximations to solve these problems to non-cooperative equilibria. iLQGames.jl is a framework for designing and solving differential games, built around the iterative linear-quadratic method presented in \cite{3}. It is written in the Julia programming language to allow flexible prototyping and integration with other research software, while leveraging the high-performance nature of the language to allow real-time execution. The open-source software package can be found at \url{https://github.com/lassepe/iLQGames.jl}.

Keywords: differential games · multi-agent systems · Julia · open-source.

1 Introduction and Related Work

In order for a robot to be truly autonomous it needs to be capable of interacting with environments that are not isolated, but rather shared with other agents. Naturally, these shared environments are manipulated by multiple decision making agents at a time and different agents may pursue different objectives. For any non-trivial interaction scenario, the success of a given agent depends not only on its own actions but also on the decisions of others. Therefore, agents must consider the effects of their actions on the behavior of others.

Differential game theory offers an expressive theoretical framework for formulating these types of multi-agent problems in continuous domains. By solving such game to an appropriate equilibrium concept — e.g. a Nash equilibrium in the case of non-cooperative scenarios — one can recover complex interactive behaviors, even from simple and easy-to-understand objective functions \cite{3,6}. Due to their expressiveness and the interpretability of their solutions, differential games have recently received a lot of attention \cite{2,3,5,8,10}.

Despite the expressive power of game theoretic models, the complexity involved with describing and solving these problems impedes application to scenarios characterized by high-dimensional states, real-time constraints and fast planning rates, such as robotics. Here, the term \textit{complexity} is explicitly used with
threefold meaning: First, the algorithmic time and space complexity of solution methods; Second, the challenges involved with implementing these algorithms efficiently; And third, the conceptual complexity of the interface used to describe and set up the problem. While the first two aspects are crucial for quick solution of the problem, we emphasize that the latter aspect is particularly important to admit quick iteration of different designs, e.g. to experiment with different cost structures that encode the behavior of each player.

In terms of computational complexity recent work offers efficient approximations to non-cooperative games \cite{3, 8} and several works have demonstrated real-time performance of these algorithms in C++ implementations \cite{3, 8, 9, 10}. However, to the best of our knowledge, only \cite{3} provide a publicly available implementation of their solver\footnote{https://github.com/HJReachability/ilqgames} and little work has focused on providing flexible interfaces and tools for the design phase of differential games.

This work presents iLQGames.jl, a framework for designing and solving differential games using the iterative linear-quadratic (iLQ) method proposed in \cite{3}. iLQGames.jl is written in the Julia programming language \cite{1} and makes use of the language’s genericity to provide a flexible interface that admits quick iteration of different problem designs and keep up with execution times of a comparable C++ implementation. This paper describes the key aspects of the framework that enable its flexibility and performance, and make it an effective tool for differential game research. Besides introducing the framework itself, this discussion aims to provide helpful insight for the implementation of other solvers, tools and problem interfaces.

2 Architecture for Rapid Design and Solution

2.1 Rapid Design

When modelling a practical scenario of multi-agent interaction as a differential games, it may not be immediately clear what are suitable dynamics and costs to describe the problem. Therefore, iLQGames.jl provides a thin interface for describing differential games that allows users to set up a model in few lines of code.\footnote{See https://github.com/lassepe/iLQGames.jl for a code example.} Specifically, the user can define a differential game by providing the differential equation governing the system dynamics, a cost function for each player, the indices of the inputs that each player controls, and the horizon of the game. With this light-weight problem description, the user can invoke the iLQ solver with given initial conditions and initial strategies (see \cite{3} for the details of the solver). Most notably, even though the iLQ solver is based on successive linear-quadratic approximations of the game, the user does not need to hand-specify linearization of the dynamics or quadratization of the costs. Instead, iLQGames.jl by default uses automatic differentiation (AD) to compute these LQ approximations efficiently \cite{7}.
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(a) 3-Player Hallway Navigation

(b) 5-Player Free-Space Navigation Game

Fig. 1. Two differential games solved with iLQGames.jl: a 3-player collision avoidance problem in a hallway and a 5-player collision avoidance problem in free-space.

Furthermore, being written in pure Julia — a language with strong focus on scientific programming — iLQGames.jl can directly be used with various other packages from the ecosystem. For example, iLQGames.jl natively supports visualization of the state and input trajectories of a game solution (c.f. Fig. 1) or projections of the cost landscape for a given player to support the design process. Another example that demonstrates the advantages of interfacing with the Julia ecosystem is presented in [6], where iLQGames.jl is combined with ParticleFilters.jl to reason about behavioural uncertainty in differential games.

2.2 Rapid Solution

Despite being a high-level language that offers wide-ranging abstraction, the Julia compiler generates highly optimized code. Most notably for our use case, this allows iLQGames.jl to solve each LQ iterate very efficiently via a fully stack-allocated dynamic program implemented in a readable high-level style in less than 70 lines of code. In fact, for moderately sized games, this optimization allows our implementation to outperform the C++ implementation presented in [3]. While it must be noted that [3] does not make this optimization and in theory a C++ implementation can achieve similar performance, we argue that making comparable optimizations in C++ may not be possible without sacrificing readability.

Furthermore, the generic function dispatch mechanism used in Julia allows users to overload default implementations at various levels of the solver to make problem specific optimizations. For example, users can specify a custom method to perform LQ approximations once they have chosen a design for the problem.

Beyond that, iLQGames.jl supports exploitation of special structure of the dynamics to speed up computation. This aspect is realized via the LinearizationStyle trait concept. By default, a dynamical system is attributed the JacobianLinearization trait and AD or a user-defined linearization is used to obtain LQ approximations of the game. However, if the dynamics are feedback linearizable, the user can optionally specify the FeedbackLinearization trait for a model to invoke a specialized version of the solver presented in [4]. This trait concept can be easily extended to other special types of systems and thus
allows users to seamlessly customize the solver with small local changes without the need to overload other parts of the routine.

3 Performance

The performance of iLQGames.jl is evaluated by benchmarking it on three problems against the C++ implementation presented in [3]. For additional reference, a Python implementation of an LQ solver is benchmarked as well. The benchmark problems are a minimal LQ game, a nonlinear nonquadratic collision avoidance problem, and a feedback linearized version of this collision avoidance game.

Table 1 summarizes the benchmark results. The Python implementation for the LQ case is multiple orders of magnitude slower than the C++ version and iLQGames.jl and would not scale well to nonlinear nonquadratic problems. iLQGames.jl with AD achieves moderate runtime and is sufficiently fast to evaluate different problem designs. When utilizing manually specified LQ approximation, as is done in the C++ version, iLQGames.jl outperforms the baseline.

Table 1. Benchmark results. The tuple behind each problem indicates the number of players (P) and the dimensionality of the state (D). LQ-Python denotes a Python implementation of the dynamic program used at the inner loop of the iLQ game algorithm. iLQGames-C++ refers to the implementation used in [3]. iLQGames.jl-MD and iLQGames.jl-AD refer to our implementation, using manual differentiation and automatic differentiation, respectively. Each game is solved over a horizon of 100 time steps on a standard laptop.

|                      | LQ (2P, 2D) | Nonlinear (3P, 12D) | FBLinearized (3P, 12D) |
|----------------------|-------------|---------------------|------------------------|
| LQ-Python            | 20.800 ms   | n/a                 | n/a                    |
| iLQGames-C++         | 0.3490 ms   | 16.27 ms            | 13.25 ms               |
| iLQGames.jl-MD       | 0.0044 ms   | 7.19 ms             | 3.98 ms                |
| iLQGames.jl-AD       | n/a         | 63.57 ms            | 52.50 ms               |

4 Conclusion

iLQGames.jl is a framework for designing and solving differential games, built around the iterative linear-quadratic method presented in [3]. This manuscript provides an overview of the framework and discusses key design aspects that enable its flexibility and performance. iLQGames.jl provides a first step towards making differential games an easily accessible tool for multi-agent interaction research.

Future directions include abstraction of a high-level problem interface that can be shared between multiple solvers to simplify the process of benchmarking algorithms against one another.
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