Tricking Adversarial Attacks To Fail

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Abstract

Recent adversarial defense approaches have failed. Untargeted gradient-based attacks cause classifiers to choose any wrong class. Our novel white-box defense tricks untargeted attacks into becoming attacks targeted at designated target classes. From these target classes, we can derive the real classes. Our Target Training defense tricks the minimization at the core of untargeted, gradient-based adversarial attacks: minimize the sum of (1) perturbation and (2) classifier adversarial loss. Target Training changes the classifier minimally, and trains it with additional duplicated points (at 0 distance) labeled with designated classes. These differently-labeled duplicated samples minimize both terms (1) and (2) of the minimization, steering attack convergence to samples of designated classes, from which correct classification is derived. Importantly, Target Training eliminates the need to know the attack and the overhead of generating adversarial samples of attacks that minimize perturbations. We obtain an 86.2% accuracy for CW-L_2(κ=0) in CIFAR10, exceeding even unsecured classifier accuracy on non-adversarial samples. Target Training presents a fundamental change in adversarial defense strategy.

1 Introduction

Neural network classifiers are vulnerable to malicious adversarial samples that appear indistinguishable from original samples [43], for example, an adversarial attack can make a traffic stop sign appear like a speed limit sign [17] to a classifier. An adversarial sample created using one classifier can also fool other classifiers [43, 5], even ones with different structure and parameters [43, 19, 37, 46]. This transferability of adversarial attacks [37] matters because it means that classifier access is not necessary for attacks. The increasing deployment of neural network classifiers in security and safety-critical domains such as traffic [17], autonomous driving [11], healthcare [18], and malware detection [15] makes countering adversarial attacks important.

Gradient-based attacks use the classifier gradient to generate adversarial samples from non-adversarial samples. Gradient-based attacks minimize at the same time classifier adversarial loss and perturbation [43], though attacks can relax this minimization to allow for bigger perturbations, for example Carlini&Wagner (CW) [11] for κ >0, Projected Gradient Descent (PGD) [28], FastGradientMethod (FGSM) [19]. Other adversarial attacks include DeepFool [31], Zeroth order optimization (ZOO) [13], Universal Adversarial Perturbation (UAP) [50].

Many recent proposed defenses have been broken [2, 8, 9, 10, 44]. They fall largely into these categories: (1) adversarial sample detection, (2) gradient masking and obfuscation, (3) ensemble, (4) customized loss. Detection defenses [29, 27, 26, 24] aim to detect, correct or reject adversarial samples. Many detection defenses have been broken [10, 9, 44]. Gradient obfuscation is aimed at preventing gradient-based attacks from access to the gradient and can be achieved by shattering gradients [20, 47, 41], randomness [16, 26] or vanishing or exploding gradients [36, 42, 40]. Many gradient obfuscation methods have also been successfully defeated [8, 2, 44]. Ensemble defenses [45, 47, 34, 41] have also been broken [8, 44], unable to outperform their best performing component.
Customized attack losses defeat defenses \[44\] with customized losses \[33, 47\] but also, for example ensembles \[41\]. Even though it has not been defeated, Adversarial Training \[43, 24, 28\] assumes that the attack is known in advance and takes time to generate adversarial samples at every iteration. The inability of recent defenses to counter adversarial attacks calls for new kinds of defensive approaches.

In this paper, we propose an adversarial defense that turns untargeted gradient-based attacks into attacks targeted at designated classes. Then our defense derives correct classification from the designated classes. Our Target Training defense is based on the minimization \[43\] at the core of untargeted gradient-based attacks. Target Training minimizes both terms simultaneously - (1) perturbation, and (2) classifier adversarial loss - by training the classifier with nearby points that misclassify to designated classes. Thus, Target Training guides attacks to converge to adversarial samples from designated classes. We adapt Target Training for attacks that exclude perturbation from their minimization. Both approaches can be combined to defend against both types of attacks.

We make the following contributions:

- We develop Target Training - a novel, white-box adversarial defense that converts untargeted gradient-based attacks into attacks targeted at designated, target classes, from which correct classes are derived. Target Training is based on the minimization at the core of untargeted gradient-based adversarial attacks.
- We eliminate the need to know the attack or to generate adversarial samples of a whole category of attacks. We observe that for attacks that minimize perturbation, original samples can be used instead of adversarial samples. Original samples have 0 perturbation from themselves, the perturbation cannot be minimized further. We divide attacks into two categories: attacks that minimize perturbation; and attacks that do not.
- Target Training surpasses default accuracy of 84.3% on non-adversarial samples in CIFAR10 for most attacks that minimize perturbation. We achieve: 86.2% for CW-$L_2$($\kappa=0$), 84.2% for CW-$L_\infty$($\kappa=0$), 86.6% for DeepFool, 89.0% for ZOO and 86.8% for UAP. For MNIST, we achieve 96.6% for CW-$L_2$($\kappa=0$), 96.3% for CW-$L_\infty$($\kappa=0$), 94.9% for DeepFool, 93.0% for ZOO and 98.6% for UAP.

2 Related work

Here, we present the state-of-the-art in adversarial attacks and defenses.

**Notation** A $k$-class, neural network classifier with $\theta$ parameters is denoted by function $f(x)$ of input $x \in \mathbb{R}^d$ that outputs $y \in \mathbb{R}^k$, where $d$ is the sample dimensionality and $k$ is number of classes. An adversarial sample is denoted by $x_{adv}$. Standard $k$-class softmax cross-entropy loss function is used to calculate the $y$ classifier output viewed as a probability distribution, each $y_i$ denoting the probability that the input belongs to class $i$, with $0 \leq y_i \leq 1$ and $y_1 + y_2 + ... + y_k = 1$. At inference, the highest probability class predicted $C(x) = \arg \max_i y_i$.

**Distance metrics** Adversarial attacks and defenses quantify similarity between images using norms as distance metrics, for example $L_0$ (not a real norm in the mathematical sense) - the number of pixels changed in an image, $L_2$ - the Euclidean distance, and $L_\infty$ - the maximum change to any pixel. Many attacks and defenses are not limited to any one distance metric \[11, 31, 30\].

2.1 Adversarial attacks

The problem of generating adversarial samples was formulated by Szegedy et al. as a constrained minimization of the perturbation under an $L_p$ norm, such that the classification of the perturbed sample changes \[43\]. Because this formulation can be hard to solve, Szegedy et al. \[43\] did a reformulation of the problem as a gradient-based, two-term minimization of the sum of the perturbation and the classifier loss:

$$\minimize c \cdot \|x_{adv} - x\|_2^2 + \text{loss}_f(x_{adv}, l)\text{ subject to } x_{adv} \in [0, 1]^n,$$

(Minimization 1)
Multi-step attacks
Attack perturbation can be calculated in more than one step. UAP [30] finds et al.
Athalye Gradient masking and obfuscation
Many defenses that mask or obfuscate the classifier gradient that
Such defenses aim to detect, then correct or reject adversarial samples. So far,
Detection defenses
The current strongest attack, CW [11], changes the basic Minimization 1 by passing the c parameter to the second term and using it to tune the relative importance of the terms. CW also introduces a confidence parameter into its minimization for the confidence of the adversarial samples. High values of confidence push CW to find adversarial samples with higher confidence that no longer minimize perturbation. With a further change of variable, CW obtains an unconstrained minimization problem that allows it to optimize directly through back-propagation.

Implicitly following Minimization 1 Moosavi-Dezfooli et al. define adversarial perturbation as the minimal perturbation sufficient to cause misclassification in DeepFool [31]. DeepFool’s algorithm uses the gradient to approximate linear classifier boundaries and to calculate the smallest perturbation as the smallest distance to the boundaries.

Black-box attacks
Black-box attacks encompass attacks that assume no access to classifier gradients. Such attacks with access to output class probabilities are called score-based attacks, for example the ZOO attack [13], a black-box variant of CW [11]. Attacks that assume access to only the final class label decision of the classifier are called decision-based attacks, for example the Boundary [6] and the HopSkipJumpAttack [12] attacks.

Multi-step attacks
Attack perturbation can be calculated in more than one step. UAP [30] finds only one universal perturbation by iterating several times over the training samples to find the minimal perturbation to move to the classifier boundary. UAP aggregates all perturbations into a universal perturbation. The BIM attack [24] extends FGSM [19] by applying it iteratively with a smaller step size α. The PGD attack [28] is an iterative method with an α parameter for step-size perturbation magnitude. PGD starts at a random point x₀, projects the perturbation on an L_p-ball B of a specified radius at each iteration, and clips the adversarial sample values: \( x(j + 1) = \text{Proj}_B(x(j) + \alpha \cdot \text{sign}(\nabla_x \text{loss}(\theta, x(j), y))) \).

2.2 Adversarial defenses
Szegedy et al. used Adversarial Training [43,24,28] defense to populate low probability blind spots with adversarial samples labelled correctly. Adversarial Training is one of the few non-broken defenses. Its drawback is that it needs to know the attack in advance and to train the classifier with adversarial samples of the attack.

Detection defenses
Such defenses aim to detect, then correct or reject adversarial samples. So far, adversarial samples have defied detection efforts as many detection defenses have been defeated, for example ten diverse detection methods (other network, PCA, statistical properties) by attack loss customization in [9]; attack customization against [22] by Hu et al. in [44]; attack transferability [10] against MagNet [29]: deep feature adversaries [39] against [38] by Roth et al.

Gradient masking and obfuscation
Many defenses that mask or obfuscate the classifier gradient that gradient-based attacks rely on have been defeated [8,2]. Athalye et al. [2] identify three types of gradient obfuscation: (1) Shattered gradients - incorrect gradients caused by non-differentiable components or numerical instability, for example [20] by Guo et al. with multiple input transformations. Athalye et al. compute the backward pass with a function approximation that is differentiable using Backward Pass Differentiable Approximation [2]. (2) Stochastic gradients in randomized defenses are overcome with Expectation Over Transformation [3] by Athalye et al. Examples of this defense are Stochastic Activation Pruning [16] which drops layer neurons based on a weighted distribution and [48] by Xie et al. which adds a randomized layer to the input of the classifier. (3) Vanishing or exploding gradients are used, for example, in Defensive Distillation (DD) [36] which reduces
the amplitude of gradients of the loss function, PixelDefend [42], Defense-GAN [40]. Vanishing or exploding gradients are broken with parameters that avoid vanishing or exploding gradients [8].

**Complex defenses** Defenses combining several defeated approaches, for example Li *et al.* [26] using detection, randomization, multiple models and losses, can be defeated by focusing on the main defense components [44], [47], [34], [41] are defeated ensemble defenses combined with numerical instability [47] or regularization [34] or mixed precision on weights and activations [41]. [4] uses a Fourier transform to compress inputs, [33] by Pang *et al.* proposes a new loss function but is defeated with a customized loss in the attack.

**Summary** Many defense approaches have been broken. They mainly focus on changing the classifier. Instead, our defense focuses on changing how attacks behave, with minimal changes to the classifier. Target Training is the first defense that is based on the Minimization 1 at the core of untargeted gradient-based adversarial attacks.

### 3 Target Training

Target Training eliminates the need to know the attack or to generate adversarial samples of attacks that minimize perturbation. Our defense turns untargeted attacks into attacks targeted at designated target classes, then derives correct classification. Target Training undermines Minimization 1 [2] of gradient-based adversarial attacks by training the classifier with exactly the points that attacks look for: nearby points (at 0 distance) that minimize adversarial loss. For attacks that relax the minimization by removing the perturbation from it, we adjust Target Training.

Target Training defends by training a classifier so that attacks converge to adversarial samples of designated classes. Untargeted gradient-based attacks are based on Minimization 1 of the sum of (1) perturbation and (2) classifier adversarial loss. Target Training trains the classifier with samples of designated classes that minimize both terms of the minimization at the same time, turning untargeted attacks into attacks targeted at designated target classes. From the designated classes, derivation of correct classification is straightforward.

Here, we give the intuition for categorizing adversarial attacks into: attacks that minimize perturbation; and attacks that do not minimize perturbation. For attacks that minimize perturbation, the minimization of term (1) perturbation allows Target Training to completely eliminate the need for knowledge about the attacks or using their adversarial samples in training. Term (1) minimization is reached at original samples because they have 0 perturbation from themselves. In attacks that do not minimize perturbation, Target Training needs adversarial samples during training to minimize term (1) perturbation of generated adversarial samples.

For Target Training, it is important to cause attacks to minimize both terms of Minimization 1 simultaneously at samples from designated classes. Following, we outline how Target Training minimizes term (1) perturbation for each category of attacks, and how it minimizes term (2) classifier adversarial loss. Further, we explain how Target Training approaches for both categories of attack can be combined together.

**Minimization term (1) - perturbation** Against attacks that minimize perturbation, such as CW-$L_2(\kappa = 0)$, CW-$L_\infty(\kappa = 0)$, and DeepFool, Target Training uses duplicates of original samples in each batch instead of adversarial samples, since no other points can have smaller distance from original samples than the original samples themselves. This removes completely the need and overhead of calculating adversarial samples against all attacks of this type. Having reduced term (1) perturbation to 0, the minimization reduces to term (2) only. Algorithm 1 shows classifier training against attacks that minimize perturbations.

Against attacks that do not minimize perturbation, such as CW-$L_2(\kappa > 0)$, PGD and FGSM, Target Training adjusts by training with additional adversarial samples from the attack. The adjusted Algorithm 2 is shown in Appendix A.

**Term (2) of the minimization - classifier adversarial loss** Let us imagine that Minimization 1 of gradient-based attacks only had term (2). If this were just classifier loss without the adversarial requirement, attacks would converge to samples from the class with the highest probability, the real class. The reason is that the real class minimizes loss in a classifier that has converged. Since term (2) minimizes classifier adversarial loss, attacks would converge to the class with the second highest
Algorithm 1: Target Training of classifier $N$ against attacks that minimize perturbation.

**Result:** Target-trained classifier $N$

1. Size of the training batch is $m$, number of classes in the dataset is $k$;
2. Initialize network $N$ with double number of output classes, $2k$, keep all else in $N$ the same;
3. **repeat**
   4. Read random batch $B = \{x^1, \ldots, x^m\}$ and its ground truth $G = \{y^1, \ldots, y^m\}$;
   5. Duplicate batch $B$. The new batch is $B' = \{x^1, \ldots, x^m, x^1, \ldots, x^m\}$;
   6. Duplicate the ground truth and increase ground truth values by $k$. The ground truth becomes $G' = \{y^1, \ldots, y^m, y^1+k, \ldots, y^m+k\}$;
   7. Do one training step of network $N$ using batch $B'$ and ground truth $G'$;
4. **until** training converged;

**Training without adversarial samples**

| Labels | Original batch |
|--------|----------------|
| 4      | 3              |
| 3      | 5              |
| 5      | 8              |
| 8      | 7              |

**Training with adversarial samples**

| Labels | Original batch |
|--------|----------------|
| 4      | 3              |
| 3      | 5              |
| 5      | 8              |
| 8      | 7              |

| Labels | Duplicated samples |
|--------|--------------------|
| 14     | 13                |
| 13     | 15                |
| 15     | 18                |
| 18     | 17                |

| Labels | Adversarial sample |
|--------|--------------------|
| 17     | 17                |

**Inference**

- Outputs vector $y$
- Probabilities: $y_0 \sim 0.5$, $y_7 \sim 0.5$
- Class 17 has the highest probability of all adversarial classes. Attacks converge to samples of class 17 to minimize adversarial loss.

Figure 1: Target Training with and without adversarial samples, and output probabilities at inference. Example images are from the MNIST dataset, smaller batch size shown for brevity. Inference output probability values for MNIST and CIFAR10 images are shown in Appendix C, Table 5 and Table 6.

Probability - any of the adversarial classes with the highest probability. In a normal multi-class classifier, only the first highest probability is distinguished from the rest - a value close to 1 for the true-label class. The rest of the classes have $\sim 0$ probability value without any distinction between them. If we could control which classes have the the top two highest probability classes, we could control the minimization of term (2).

Figure 1 shows that as a result of training with batches with additional samples that are assigned to designated target classes, the classifier has two high probability output classes at inference: original class and designated class. Since attacks minimize classifier adversarial loss, attacks converge to adversarial samples from the designated class in order to minimize term (2) of Minimization 1. The same samples also minimize term (1) since designated classes were assigned to duplicated original samples in training. As a result, attacks converge to adversarial samples from the designated classes.

**Model structure and inference**

The only change to classifier structure is doubling the number of output classes from $k$ to $2k$. The loss function remains standard softmax cross entropy. Target Training has no norm limitation because it minimizes the perturbation to 0, which translates to $L_p$ norms of 0, for any $p$. For example, Target Training defends against CW-$L_2$ as well as CW-$L_\infty$ attacks. Inference calculation is: $C(x) = \arg \max_i (y_i + y_{i+k}), i \in [0 \ldots (k - 1)]$. 

5
3.1 Simultaneous defense against both categories of attack

Target Training can be extended to counter at the same time attacks that minimize perturbations and attacks that do not. An example would be to defend against attacks that minimize perturbation, and the CW-$L_2(\kappa = 40)$ attack which does not minimize perturbation. To counter both at the same time, Target Training would triple, instead of duplicate, the batch. One set of extra samples would be original samples. The other set of extra samples would be CW-$L_2(\kappa = 40)$ adversarial samples. For the labels, there would be two sets of designated classes: one set for the convergence of attacks that minimize perturbation, and the other one for the convergence of the CW-$L_2(\kappa = 40)$ attack. At inference, the correct class would be: $C(x) = \arg \max_i (y_i + y_{i+k} + y_{i+2k}), i \in [0 \ldots (k-1)]$.

This could be extended even further to accommodate more attacks that do not minimize perturbation.

4 Experiments and results

Our Target Training defense leverages the fact that some attacks minimize perturbation. To counter these attacks, we replace adversarial samples with original samples because they have perturbation 0 from themselves. Target Training does not use adversarial samples against attacks that minimize perturbation, but uses them against attacks that do not minimize perturbation. As a result, we conduct a separate set of experiments for each type of attack.

Threat model We assume that the adversary goal is to generate adversarial samples that cause untargeted misclassification. We perform white-box evaluations, assuming the adversary has complete knowledge of the classifier and how the defense works. In terms of capabilities, we assume that the adversary is gradient-based, has access to the CIFAR10 and MNIST image domains and is able to manipulate pixels. For attacks that minimize perturbations, no adversarial samples are used in training and no further assumption is made about attacks. For attacks that do not minimize perturbations, we assume that the attack is of the same kind as the attack used to generate the adversarial samples used during training. Further, we assume that perturbations are $L_p$-constrained.

Attack parameters For CW, 1, 000 iterations by default but we run experiments with up to 100,000 iterations, confidence values are 0 or 40. For PGD, we use the same attack parameters as Madry et al. in [28]. For MNIST, there are 40 steps of size 0.01, and $\epsilon = 0.3$. For CIFAR10, there are 7 steps of size 2, and $\epsilon = 8$. For ZOO attack, we use parameters specified in the ZOO attack paper [13], 1000 and 3000 iterations for CIFAR10 and MNIST, initial constant value is 0.01, 200 adversarial samples selected randomly from the testing images of CIFAR10 and MNIST. For FGSM, $\epsilon = 0.3$, as in [28].

Datasets MNIST [25] and CIFAR10 [23] are 10-class datasets used throughout previous work. MNIST [25] has 60K, $28 \times 28 \times 1$ digit images. CIFAR10 [23] has 70K, $32 \times 32 \times 3$ images. All evaluations are with testing samples.

Classifier models We purposefully do not use high capacity models, such as ResNet [21], to show that Target Training does not necessitate high model capacity. The architectures of MNIST and CIFAR datasets are shown in Appendix C, Table 3. No data augmentation used. We achieve 99.1% for MNIST and 84.3% for CIFAR10.

Tools We generate adversarial samples with CleverHans 3.0.1 [35] for the CW [11], DeepFool [31], and FGSM [19] attacks and the IBM Adversarial Robustness 360 Toolbox (ART) toolbox 1.2 [32] for the other attacks. Target Training is written in Python 3.7.3, using Keras 2.2.4 [14].

4.1 Target Training without adversarial samples against attacks that minimize perturbation

Target Training counts adversarial attacks that minimize perturbation without using adversarial samples. The non-broken Adversarial Training defense cannot be used here because it cannot work without adversarial samples. We use an unsecured classifier as baseline because other defenses have been defeated [10, 9, 8, 2, 44] successfully.

Table 1 shows that Target Training exceeds by far accuracies by unsecured classifier on adversarial samples in both CIFAR10 and MNIST. Target Training defends against attacks that minimize perturbation without prior knowledge of such attacks and without using their adversarial samples. In CIFAR10, Target Training exceeds even the accuracy of the unsecured classifier on non-adversarial samples (84.3%) for most attacks. Against the ZOO black-box attack, Target Training defense
Table 1: Target Training defends against attacks that minimize perturbations without using adversarial samples. In addition, Target Training exceeds the baseline by far, and even the accuracy of unsecured classifier on non-adversarial samples in CIFAR10. Target Training defends against attacks of different norms, against black-box attacks, and does not decrease performance for attacks with more iterations.

| Attack                  | Target Training | Unsecured Classifier | Target Training | Unsecured Classifier |
|-------------------------|-----------------|----------------------|-----------------|----------------------|
| CW-$L_2$, $\kappa = 0$, iterations=1K | 85.6% | 8.8% | 96.3% | 0.8% |
| CW-$L_2$, $\kappa = 0$, iterations=10K | 86.1% | 8.7% | 96.6% | 0.8% |
| CW-$L_2$, $\kappa = 0$, iterations=100K | 86.2% | 8.9% | 96.6% | 0.8% |
| CW-$L_\infty$, $\kappa = 0$, iterations=1K | 84.2% | 42.0% | 96.3% | 82.1% |
| DeepFool                | 86.6% | 9.2% | 94.9% | 1.3% |
| ZOO                     | 89.0% | 81.5% | 93.0% | 96.0% |
| UAP                     | 86.8% | 17.24% | 98.6% | 42.1% |

CIFAR10 (84.3%) | MNIST (99.1%)

maintains its performance. Target Training defends against attacks of different norms, for example $L_2$ and $L_\infty$. Finally, Target Training improves accuracy when the attack runs more iterations. With CW-$L_2$ attack iterations from 1K-100K, accuracy increases for CIFAR10 from 85.6% to 86.2%, for MNIST from 96.3% to 96.6%.

4.2 Target Training against adversarial attacks that do not minimize perturbation

Against adversarial attacks that do not minimize perturbation, Target Training uses adversarial samples and performs slightly better than Adversarial Training. We choose Adversarial Training as a baseline because it is a non-broken adversarial defense, more details in Related work. Our implementation of Adversarial Training is based on [24] by Kurakin et al., shown in Algorithm 3 in Appendix B.

Table 7 in Appendix C shows that Target Training defends against attacks that do not minimize perturbation, exceeding by far accuracies of the unsecured classifier. Furthermore, Target Training performs slightly better than Adversarial Training against these attacks. Target Training achieves accuracies starting from 72.1% in CIFAR10, and 91.7% for MNIST. In addition, Target Training defends against multi-step attacks, in this case the PGD attack.

4.3 Summary of results

With our experiments in Section 4.1, we show that we substantially improve performance against attacks that minimize perturbation without using adversarial samples. In Section 4.2, we show that at the same time, Target Training maintains performance against attacks that do not minimize perturbation, compared to previous non-broken defense. Target Training can combine both approaches and defend simultaneously against both types of attack, as we describe in Section 3.1.

4.4 Transferability analysis

For a defense to be strong, we need to show that it breaks the transferability of attacks [7]. A good source of adversarial samples for transferability is the unsecured classifier. We experiment on the transferability of attacks from the unsecured classifier to a classifier secured with Target Training.

Importantly, Table 2 shows that Target Training breaks the transferability of adversarial samples generated by attacks that do not minimize perturbation: CW-$L_2(\kappa = 0)$, CW-$L_\infty(\kappa = 0)$ and DeepFool. Target Training maintains high accuracies in CIFAR10 and MNIST against adversarial samples generated with the unsecured classifier.

Against attacks that do not minimize perturbation, CW-$L_2(\kappa = 40)$ and PGD, Target Training breaks the transferability of attacks for MNIST but not for CIFAR10. This indicates that we might need to look for samples that minimize perturbation better against this category of attacks.
Table 2: Target Training breaks the transferability of attacks from the unsecured classifier by maintaining high accuracy against attacks generated using the unsecured classifier in attacks that minimize perturbation. For attacks that do not minimize perturbation, Target Training breaks the transferability in MNIST only.

| Attack                  | CIFAR10 (84.3%) | MNIST (99.1%) |
|-------------------------|-----------------|---------------|
| CW-\(L_2(\kappa = 0), \text{iterations}=1K\) | 69.9% 8.8% | 78.3% 0.8% |
| CW-\(L_\infty(\kappa = 0), \text{iterations}=1K\) | 76.6% 42.0% | 93.5% 82.1% |
| DeepFool                | 74.8% 9.2%      | 96.5% 1.3%    |
| CW-\(L_2(\kappa = 40), \text{iterations}=1K\) | 34.7% 8.5% | 95.1% 0.7% |
| PGD                     | 36.8% 32.7%     | 92.2% 79.7%   |

4.5 Adaptive evaluation

Many recent defenses have failed to anticipate attacks that have defeated them \([7, 9, 2]\). To avoid that, we perform an adaptive evaluation \([7, 44]\) of our Target Training defense.

**What attack could defeat the Target Training defense?** Attacks that are either targeted or not gradient-based, both outside the threat model. Most current attacks, including the strongest ones, CW and PGD, are gradient-based. Finding adversarial samples without the gradient is a hard problem \([43]\).

**Could Target Training be defeated by methods used to break other defenses?** Attack approaches \([10, 9, 8, 2, 44]\) used to defeat most current defenses cannot break Target Training defense because we use none of the previous defenses, such as: adversarial sample detection, preprocessing, obfuscation (shattered, vanishing or exploding gradients, or randomization), ensemble, customized loss, subcomponent, non-differentiable component, or special model layers. We also keep the loss function simple - standard softmax cross-entropy and no additional loss.

Iterative attacks decrease Target Training accuracy more than single-step attacks, which suggests that our defense is working correctly \([7]\). Target Training defends against black-box ZOO attack, which means that we are not doing gradient masking or obfuscation \([7]\). Non-transferability of attacks also points to non-masking. Increasing iterations for CW-\(L_2(\kappa = 0)\) 100-fold from 1K to 100K increases the defense accuracy. In CIFAR10 accuracy increases from 85.6% to 86.2%, in MNIST from 96.3% to 96.6%. This is explained by the fact that Target Training tricks attacks into designated classes. Target Training also maintains performance on original samples, as shown in Appendix C, Table 4. We will release the code and trained models upon acceptance.

5 Discussion and conclusions

Target Training presents a fundamental shift in adversarial defense in two ways. First, our defense is the only defense able to convert untargeted gradient-based attacks to attacks targeted at designated classes. From the designated classes, correct classification is derived. Second, Target Training eliminates the need to know the attack in advance, and the overhead of adversarial samples, for attacks that minimize perturbation. In contrast, the previous non-broken Adversarial Training defense needs to know the attack and to generate adversarial samples of the attack during training. This is a limitation because in real applications, the attack might not be known.

Target Training achieves high accuracy against adversarial samples and breaks the transferability of adversarial attacks. We achieve even better accuracy than 84.3% accuracy of unsecured classifier on non-adversarial samples in CIFAR10. For example, 86.2% for CW-\(L_2(\kappa = 0)\), 84.2% for CW-\(L_\infty(\kappa = 0)\), 86.6% for DeepFool, 89.0% for ZOO and 86.8% for UAP. We show that Target Training breaks the transferability of adversarial samples in attacks that minimize perturbation. Target Training also breaks the transferability of adversarial samples in attacks that do not minimize perturbation in MNIST. Target Training also maintains performance on original, non-adversarial samples.

In conclusion, we show that Target Training succeeds by switching the focus from changing the classifier to changing indirectly how attacks behave.
Broader impact

Machine learning solutions in general, and neural network classifiers in particular, are increasingly being deployed into safety-critical domains, for example self-driving cars. If attacks on such applications are possible, this impacts the safety of the systems that deploy them and the people that use them. Therefore, it is crucial to have neural network classifiers that are robust to adversarial attacks.

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Appendix A  Target Training algorithm against attacks that do not minimize perturbation

Algorithm 2: Target Training of classifier $N$ using adversarial samples.

Result: Target-Trained classifier $N$

1. Size of the training batch is $m$, number of classes in the dataset is $k$;
2. Initialize network $N$ with $2k$ output classes;
3. ATTACK is an adversarial attack;
4. repeat
5. Read random batch $B = \{x^1, ..., x^m\}$ with ground truth $G = \{y^1, ..., y^m\}$ from training set;
6. Generate adversarial samples $A = ATTACK(B)$ using current state of $N$;
7. The new batch is $B' = B \cup A = \{x^1, ..., x^m, x^1_{adv}, ..., x^m_{adv}\}$;
8. Duplicate the ground truth and increase the duplicate values by $k$. The ground truth becomes $G' = \{y^1, ..., y^m, y^1 + k, ..., y^m + k\}$;
9. Do one training step of network $N$ using batch $B'$ and ground truth $G'$;
10. until training converged;

Appendix B  Adversarial Training algorithm we use for comparison

Algorithm 3: Adversarial Training of classifier $N$ using adversarial samples.

Result: Adversarially-Trained network $N$

1. Size of the training batch is $m$, number of classes in the dataset is $k$;
2. Initialize network $N$ with $k$ output classes;
3. repeat
4. Read random batch $B = \{x^1, ..., x^m\}$ with ground truth $G = \{y^1, ..., y^m\}$ from training set;
5. Generate adversarial samples $\{x^1_{adv}, ..., x^m_{adv}\}$ from batch using current state of $N$;
6. Make new batch $B' = \{x^1, ..., x^m, x^1_{adv}, ..., x^m_{adv}\}$;
7. Make new ground truth $G' = \{y^1, ..., y^m, y^1, ..., y^m\}$;
8. Do one training step of network $N$ using batch $B'$ and ground truth $G'$;
9. until training converged;
Appendix C  Additional tables

Table 3: Architectures of Target Training classifiers for CIFAR10 and MNIST datasets. For the convolutional layers, we use $L_2$ kernel regularizer. Notice that the final Dense.Softmax layers in both models have 20 output classes, twice the number of dataset classes. The default, unsecured classifiers have the same architectures, except the final layers have 10 output classes: Dense.Softmax 10.

| CIFAR10          | MNIST          |
|------------------|----------------|
| Conv.ELU 3x3x32  | Conv.ReLU 3x3x32 |
| BatchNorm        | BatchNorm      |
| Conv.ELU 3x3x32  | Conv.ReLU 3x3x64 |
| BatchNorm        | BatchNorm      |
| MaxPool 2x2      | MaxPool 2x2    |
| Dropout 0.2      | Dropout 0.25   |
| Conv.ELU 3x3x64  | Dense 128      |
| BatchNorm        | Dropout 0.5    |
| Conv.ELU 3x3x64  | Dense.Softmax 20 |
| BatchNorm        | MaxPool 2x2    |
| Dropout 0.3      | Dropout 0.4    |
| Conv.ELU 3x3x128 |                |
| BatchNorm        | Dense.Softmax 20 |
| Conv.ELU 3x3x128 |                |
| BatchNorm        | MaxPool 2x2    |
| Dropout 0.4      |                |

Table 4: Comparing Target Training and Adversarial Training accuracy on original samples. Adversarial Training is not applicable (NA) in the first row because it needs adversarial samples.

| Adv. samples in training | CIFAR10 (84.3%) | MNIST (99.1%) |
|--------------------------|-----------------|---------------|
|                          | Target Training | Advers. Training | No Defense | Target Training | Advers. Training | No Defense |
| none (against attacks w/o perturb.) | 86.7% | NA | 84.3% | 98.6% | NA | 84.3% |
| CW-$L_2$ ($\kappa = 40$) | 77.7% | 77.4% | 84.3% | 98.0% | 98.0% | 99.1% |
| PGD                      | 76.3% | 76.9% | 84.3% | 98.3% | 98.4% | 99.1% |
| FGSM($\epsilon = 0.3$)   | 77.6% | 76.6% | 84.3% | 98.6% | 98.6% | 99.1% |
Table 5: Class output probabilities for Target Training on original, and adversarial samples from MNIST. Adversarial samples generated with CW-$L_2(\kappa = 0)$. Zero probability values and probability values rounded to zero have been omitted.

| Labels | Original images | Adversarial images |
|--------|----------------|--------------------|
| 0      | 0.508          | 0.500              |
| 1      | 0.435          | 0.503              |
| 2      | 0.616          | 0.497              |
| 3      | 0.683          | 0.500              |
| 4      | 0.776          | 0.492              |
| 5      | 0.754          | 0.492              |
| 6      | 0.622          | 0.500              |
| 7      | 0.652          | 0.500              |
| 8      | 0.614          | 0.492              |
| 9      | 0.524          | 0.492              |
| 10     | 0.492          | 0.500              |
| 11     | 0.565          | 0.500              |
| 12     | 0.384          | 0.500              |
| 13     | 0.224          | 0.492              |
| 14     | 0.246          | 0.492              |
| 15     | 0.378          | 0.492              |
| 16     | 0.348          | 0.492              |
| 17     | 0.386          | 0.492              |
| 18     |                | 0.476              |
| 19     |                | 0.570              |
Table 6: Class output probabilities for Target Training on original, and adversarial samples from CIFAR10. Adversarial samples generated with CW-$L_2$($\kappa = 0$). Zero probability values and probability values rounded to zero have been omitted. The two highest class probabilities for each image are made bold. The deer (fifth image) appears to be misclassified as a horse.

| Labels | Original images | Adversarial images |
|--------|----------------|-------------------|
|        | air-           | automobile        |
|        | plane          | bird              |
|        |                | cat               |
|        |                | deer              |
|        |                | dog               |
|        |                | frog              |
|        |                | horse             |
|        |                | ship              |
|        |                | truck             |
| 0      | 0.405          | 0.002             |
| 1      | 0.455          | 0.007             |
| 2      | 0.562          | 0.004             |
| 3      | 0.602          | 0.083             |
| 4      | 0.006          | 0.482             |
| 5      | 0.387          | 0.527             |
| 6      | 0.556          | 0.537             |
| 7      |                | 0.004             |
| 8      |                | 0.471             |
| 9      |                | 0.005             |
| 10     | 0.583          | 0.002             |
| 11     | 0.545          | 0.434             |
| 12     | 0.005          | 0.438             |
| 13     | 0.398          | 0.005             |
| 14     | 0.497          | 0.004             |
| 15     |                | 0.456             |
| 16     | 0.518          | 0.473             |
| 17     | 0.455          | 0.444             |
| 18     | 0.541          | 0.455             |
| 19     |                | 0.479             |
Table 7: Target Training performs slightly better than Adversarial Training against attacks that do not minimize perturbation, both utilizing adversarial samples in training. We also compare with unsecured classifier performance. Further results in Appendix C, Table 8 show that both Target Training and Adversarial Training provide defense against some attacks they have not been trained for, but not all.

| Adv. samples in training (TT and AT) | Adv. samples in testing | CIFAR10 (84.3%) | MNIST (99.1%) |
|-------------------------------------|------------------------|-----------------|---------------|
|                                     | Target Training         | Advers. Training | Unsecured Classif. |
| CW-$L_2 (κ = 40)$                   | 77.7%                  | 77.4%           | 8.5%           |
| PGD                                 | 76.3%                  | 76.2%           | 32.7%          |
| FGSM($\epsilon = 0.3$)              | 72.1%                  | 71.8%           | 11.8%          |
|                                     | 98.0%                  | 98.0%           | 0.7%           |
|                                     | 92.3%                  | 91.7%           | 79.7%          |
|                                     | 98.0%                  | 98.4%           | 10.0%          |

Table 8: Expanded comparison of Target Training and Adversarial Training against attacks that do not minimize perturbation. Here, we show also performance against attacks, the adversarial samples of which have not been used in training. Both Target Training and Adversarial Training defend against some attacks that they have not been trained for, but not all. We also compare with unsecured classifier performance.

| Adv. samples in training (TT and AT) | Adv. samples in testing | CIFAR10 (84.3%) | MNIST (99.1%) |
|-------------------------------------|------------------------|-----------------|---------------|
|                                     | Target Training         | Advers. Training | No Defense |
|                                     |                         |                 |              |
| CW-$L_2 (κ = 40)$                   | 77.7%                  | 77.4%           | 8.5%           |
| PGD                                 | 76.3%                  | 76.2%           | 32.7%          |
| DeepFool                            | 75.8%                  | 13.2%           | 9.2%           |
| FGSM($\epsilon = 0.3$)              | 10.0%                  | 9.9%            | 11.8%          |
| FGSM($\epsilon = 0.01$)             | 48.9%                  | 36.4%           | 40.4%          |
|                                     | 98.0%                  | 98.0%           | 0.7%           |
|                                     | 97.4%                  | 1.5%            | 8.8%           |
|                                     | 97.6%                  | 1.6%            | 1.3%           |
|                                     | 96.6%                  | 1.5%            | 79.7%          |
|                                     | 56.6%                  | 15.8%           | 10.0%          |
|                                     | 97.7%                  | 97.8%           | 98.6%          |

| PGD                                 | 76.3%                  | 76.2%           | 32.7%          |
| CW-$L_2 (κ = 40)$                   | 7.3%                   | 57.3%           | 8.5%           |
| CW-$L_2 (κ = 0)$                    | 12.8%                  | 12.7%           | 8.8%           |
| DeepFool                            | 15.0%                  | 13.0%           | 9.2%           |
| FGSM($\epsilon = 0.3$)              | 10.7%                  | 10.2%           | 11.8%          |
| FGSM($\epsilon = 0.01$)             | 39.8%                  | 41.5%           | 40.4%          |
|                                     | 92.3%                  | 91.7%           | 79.7%          |
|                                     | 83.2%                  | 98.4%           | 0.7%           |
|                                     | 94.3%                  | 22.7%           | 8.8%           |
|                                     | 86.5%                  | 4.7%            | 1.3%           |
|                                     | 79.9%                  | 95.4%           | 10.0%          |
|                                     | 98.2%                  | 98.4%           | 98.6%          |

| FGSM($\epsilon = 0.3$)              | 72.1%                  | 71.8%           | 11.8%          |
| FGSM($\epsilon = 0.01$)             | 40.8%                  | 42.1%           | 40.4%          |
| CW-$L_2 (κ = 40)$                   | 49.9%                  | 74.2%           | 8.5%           |
| CW-$L_2 (κ = 0)$                    | 12.5%                  | 12.7%           | 8.8%           |
| DeepFool                            | 12.7%                  | 12.8%           | 9.2%           |
| PGD                                 | 17.2%                  | 1.2%            | 32.7%          |
|                                     | 98.0%                  | 98.4%           | 10.0%          |
|                                     | 98.5%                  | 98.5%           | 98.6%          |
|                                     | 58.8%                  | 1.1%            | 0.7%           |
|                                     | 51.8%                  | 1.1%            | 8.8%           |
|                                     | 48.3%                  | 1.2%            | 1.3%           |
|                                     | 72.6%                  | 42.5%           | 79.7%          |